Tests of weak equivalence principle with the gravitational wave signals in the LIGO-Virgo catalog GWTC-1

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The weak equivalence principle (WEP) is the cornerstone of gravitational theories. At the local scale, WEP has been tested to high accuracy by various experiments\textsuperscript{12}. On the intergalactic distance scale, WEP could be tested by comparing the arrival time of different messengers emitted from the same source\textsuperscript{3–5}. The gravitational time delay caused by massive galaxies are proportional to $\gamma + 1$, where the parameter $\gamma$ is unity in general relativity. The values of $\gamma$ for different massless particles should be different if WEP is violated, i.e., $\Delta \gamma$ is used to indicate the deviation from WEP. So far, $|\Delta \gamma|$ has been constrained to $\sim 10^{-10}$ with gamma-ray bursts, fast radio bursts and gravitational waves\textsuperscript{6}. Here we report a new estimation of $|\Delta \gamma|$ by using the gravitational wave (GW) data of binary black hole (BBH) coalescences in the LIGO-Virgo catalog GWTC-1\textsuperscript{7}. Our results show that $|\Delta \gamma|$ is not larger than $7.9 \times 10^{-15+2.6}_{-3.0}$ at 90\% confidence level for uniform logarithmic prior. For an alternative prior, the 90\% confidence interval of $\Delta \gamma$ is $[-1.0 \times 10^{-15}, + 1.4 \times 10^{-17}]$. WEP may be obeyed on the intergalactic distance scale for GWs.
WEP assumes that all freely falling bodies at the same space-time points will undergo the same acceleration in a given gravitational field, independent of their properties and rest mass. WEP, which is rather fundamental in physics, has withstood numerous experimental tests at the local scale. A measurement on the fractional difference in acceleration between two bodies is the Eötvös ratio \( \eta = \frac{2|a_1 - a_2|}{|a_1 + a_2|} \), where \( a_1 \) and \( a_2 \) are the free-fall accelerations of the two bodies. To date, the bounds on \( \eta \) have reached levels of \( 10^{-15} \) by MICROSCOPE’s satellite experiments.

Gravitational field can lead to an extra time delay on the propagation of photons and GWs which is called as Sharpio time delay. This delay is proportional to \( \gamma + 1 \), where \( \gamma \) is the parameterized post-Newtonian (PPN) parameter (\( \gamma = 1 \) in GR). If WEP is violated, different massless particles will have different values of \( \gamma \) when they freely fall in the gravitational field. For instance, the values of \( \gamma \) for signal 1 and signal 2 are \( \gamma_1 \) and \( \gamma_2 \) respectively, then \( |\Delta \gamma| (|\gamma_1 - \gamma_2|) \) can be used to quantitatively represent the derivation of WEP. There shall be \( |\Delta \gamma| = 0 \) in the case of GR.

Usually, this gravitational time delay is too small to be used to test WEP. However, in astrophysics, due to the huge mass of galaxies and super long distance of sources, it offers a unique opportunity to test WEP on the intergalactic scale. According to WEP, freely falling massless particles emitted from the same astrophysical source shall follow an identical geodesic and experience the same Shapiro time delay that caused by the presence of a gravitational potential. Considering two different signals emitted from the same source, the observed time-delays (\( \Delta t_{\text{obs}} \)) could be
greater than the difference of the Shapiro time delay of the two particles $\Delta t_{\text{gra}}$, which is proportional to $|\Delta \gamma|$. Therefore people could give an upper limit on $|\Delta \gamma|$ by observing $\Delta t_{\text{obs}}$ of different signals emitted from the same source. To date, several constraints on $|\Delta \gamma|$ have obtained using different astrophysical events, including emissions from supernova event SN1987A and fast radio bursts (FRBs). Recently, GWs have also been used to test WEP, the constraints on $|\Delta \gamma|$ obtained from GW150914 and GW170817 are $10^{-9}$ and $10^{-10}$ respectively. By now, Wei et al. gave a constraint of $10^{-13}$ if the gamma-ray burst events and neutrino events are really correlative in their model.

The aforementioned methods took $\Delta t_{\text{obs}}$ between low frequency and high frequency GWs or EM waves (or time delay between different messengers) as the maximum possible value of time-delay due to the violation of WEP. Because they did not remove the intrinsic time delay when these signals emitted (i.e. $\Delta t_{\text{e}}$), then only obtained rough upper limits on $|\Delta \gamma|$.

Fortunately, we can theoretical calculate the inspiral and merger of binary black holes, then the intrinsic time delay between the low and high frequency GWs in one event could be modeled accurately. This supplies us an opportunity to remove the intrinsic time delay and estimate the $\Delta \gamma$ better from the GW data, if we have waveform templates with this parameter. The violation of WEP will contribute $\Delta t_{\text{gra}}$ for GWs with different frequencies during the signals propagating through the galaxies, which causes the dephasing from the waveforms predicted by GR. In the present work, we ignore the extra intrinsic time delay due to violation of WEP on the waveforms at emission moment, and all dephasing of waveforms is introduced during the propagation.
of gravitational waves passing through the Galaxy. To get the constraints on $\Delta \gamma$, we construct a new gravitational waveform template by adding a modification term (see Method) on the IM-RPhenomPv2 waveforms\textsuperscript{[13,15]}, which is also used in LIGO-Virgo’s parameter estimation\textsuperscript{[7]}. Then we employ this new template to estimate the parameter $\Delta \gamma$ by using GW events in GWTC-1 by a Bayesian inference software named Bilby\textsuperscript{[16]}. We finally get the posteriors of $\Delta \gamma$ as follows.

**Results** The posterior distribution of $\Delta \gamma$ for GW events of BBHs in GWTC-1 (90% confirmation level) are shown in Figure.\textsuperscript{[1]} In Figure.\textsuperscript{[2]} we display the distribution of network SNR of our results and compare them with the LIGO-Virgo ones.

In the top panel of Figure.\textsuperscript{[1]} we assume logarithmic prior on $\pm \Delta \gamma$, which belongs to $[1.15 \times 10^{-18}, 1.15 \times 10^{-5}]$ ($\Delta \gamma > 0$) or $[-1.15 \times 10^{-5}, -1.15 \times 10^{-18}]$ ($\Delta \gamma < 0$). In the top panel, posteriors of positive $\Delta \gamma$ are red, and posteriors of negative $\Delta \gamma$ are green. By considering the value of $|\Delta \gamma|$, We could get a best constraint from GW170823, of which $|\Delta \gamma| = 7.9 \times 10^{-15}$. The bottom panel of Figure.\textsuperscript{[1]} demonstrates the posterior distribution of $\Delta \gamma$ for GW events of BBH in GWTC-1 (90% confirmation level) with another prior distribution. This prior of $\Delta \gamma$ obeys a modified logarithmic prior (see Method), which could cover both the negative and non-negative $\Delta \gamma$ values continuously, i.e., $\Delta \gamma \in [-1.15 \times 10^{-5}, 1.15 \times 10^{-5}]$. The best estimation is also from GW170823, 90% confidence interval of $\Delta \gamma$ is $[-1.0 \times 10^{-15}, +1.4 \times 10^{-17}]$. In Figure.\textsuperscript{[2]} the SNRs of our results are plotted versus the LIGO-Virgo ones. The prior and posterior distribution of $\Delta \gamma$ for GW170823 (90% confirmation level) is shown in Figure.\textsuperscript{[3]} In the top panels of Figure.\textsuperscript{[3]} logarithmic prior is assumed on the prior distribution of $\Delta \gamma$. In the bottom panel of Figure.\textsuperscript{[3]} the
prior distribution of $\Delta \gamma$ adopts a modified logarithmic prior (see Method), which could cover both the negative and non-negative value.

Figure 1: The posterior distribution of $\Delta \gamma$ for GW events of BBH in GWTC-1 (90% confirmation level). In the top panel, logarithmic prior is assumed on the prior distribution of $\pm \Delta \gamma$, which belongs to $[1.15 \times 10^{-18}, 1.15 \times 10^{-5}] (\Delta \gamma > 0)$ or $[-1.15 \times 10^{-5}, -1.15 \times 10^{-18}] (\Delta \gamma < 0)$. For points in red, $\Delta \gamma > 0$. For points in green, $\Delta \gamma < 0$. In the bottom panel, a modified logarithmic prior (See Method), which could cover both the negative and non-negative value, is assumed on the prior distribution of $\Delta \gamma$. $\Delta \gamma$ belongs to $[-1.15 \times 10^{-5}, 1.15 \times 10^{-5}]$. 
Figure 2: The distribution of network SNR for GW events of BBH in GWTC-1 (90% confirmation level). For points in red and green, logarithmic prior is assumed on the prior distribution of $\Delta \gamma$. For points in black, a modified logarithmic prior (See Method), which could cover both the negative and non-negative value, is assumed on the prior distribution of $\Delta \gamma$. Cyan points are given by the LIGO-Virgo Collaboration (See TABLE I. in Ref. [7]).

**Method** If WEP is violated, GWs with different frequencies may experience different Shapiro time delays $^8$. To give a constraint on the violation on WEP, we assume all the uncertainty of arrival time of GW is caused by the violation of WEP, which could be described by the difference of the Shapiro time delay of the two particles $\Delta t_{\text{gra}}$. For the same source, considering GWs emitted at $t_e$ and $t'_e$ with different frequency, which will be received at corresponding arrival times $t_a$ and $t'_a$. If there the difference of emitting time ($\Delta t_e = t_e - t'_e$) is so little that the cosmological inflation effect could be ignored, then the delay of arrival times of the two GWs ($\Delta t_a = t_a - t'_a$) is
Figure 3: The prior and posterior distribution of $\Delta \gamma$ for GW170823 (90% confirmation level). In the top panel, logarithmic prior is assumed on the prior distribution of $\pm \Delta \gamma$, which belongs to $[1.15 \times 10^{-18}, 1.15 \times 10^{-5}]$ ($\Delta \gamma > 0$) or $[-1.15 \times 10^{-5}, -1.15 \times 10^{-18}]$ ($\Delta \gamma < 0$). In the bottom panel, a modified logarithmic prior (See Method), which could cover both the negative and non-negative value, is assumed on the prior distribution of $\Delta \gamma$, which belongs to $[-1.15 \times 10^{-5}, 1.15 \times 10^{-5}]$.

\[
\Delta t_a = (1 + z) \Delta t_e + \Delta t_{\text{gra}},
\]  

(1)
where $z$ is the cosmological redshift, and $\Delta t_{\text{gra}}$ would be:

$$\Delta t_{\text{gra}} = \frac{\Delta \gamma}{c^3} \int_{r_o}^{r_e} U(r) \, dr,$$

(2)

where $\Delta \gamma (= \gamma - \gamma')$ may be negative or non-negative, $r_o$ and $r_e$ are the locations of observation and the source of GWs, $U(r)$ denotes the gravitational potential. For simplicity, in the present work we only consider the gravitational potential caused by Milky Way, and $\Delta t_{\text{gra}}$ would be:

$$\Delta t_{\text{gra}} = \Delta \gamma \left[ \frac{G M_{\text{MW}}}{c^3} \ln \left( \frac{d + (d^2 - b^2)^{1/2}}{b^2} \right) \right],$$

(3)

where the Milky Way mass $M_{\text{MW}} \approx 6 \times 10^{11} M_\odot$, $d$ denotes the distance from the source to the Milky Way center, $b$ represents the impact parameter of the GW paths relative to the center of Milky Way, and the distance from the Sun to the center of Milky Way $r_G \approx 8 \text{ kpc}$, $s_n = +1$ denotes the source is located along the direction of the Milky Way center and $s_n = -1$ denotes the source is located along the direction that pointing away from the Milky Way center. The positions of sources usually use celestial coordinates (right ascension $\beta$ and declination $\delta$), therefore we use a transform formula to convert celestial coordinate to $b$.

The Shapiro time delay difference $\Delta t_{\text{gra}}$ for one GW event in different frequencies could cause the dephasing of the waveforms comparing with GR’s templates. Therefore, we need a modified waveform template to include this effect. Our modified waveform in the frequency domain is:

$$\tilde{h}(f) = \tilde{A}(f)e^{i[\Psi_{\text{GR}}(f) + \delta \Psi(f)]},$$

(4)

where $\tilde{A}(f)$ denotes the complex amplitude, $\Psi_{\text{GR}}(f)$ denotes the complex phase that predicted by GR, and $\delta \Psi(f)$ is the modification term produced by the deviation of WEP. In this work, inspired
by gravitational waveforms with a time-delay-based dephasing that caused by modified dispersion relations\textsuperscript{17,18} and Eqn.3, we finally work out the modification term
\[
\delta \Psi (f) = \frac{\pi \Delta \gamma}{\Delta f} \left[ \frac{G M_{MW}}{c^3} \ln \left( \frac{d + (d^2 - b^2)^{1/2}}{b} \left[ r_G + s_n (r_G^2 - b^2)^{1/2} \right] \right) \right] (1 + z)^2 f^2,
\]
where \( \Delta f = f - f' \), and \( f, f' \) is two different frequencies of GWs in one event. We have the assumption that \( \Delta \gamma \propto \Delta f \). In this work, we set the highest frequency \( f = 150 \) Hz and lowest frequency \( f' = 35 \) Hz, then we use the waveform model Eqn.4 to estimate parameters of GW events in GWTC-1.

In the present work, we use a Bayesian parameter estimation software named Bilby\textsuperscript{16} to estimate the parameters with the above gravitational waveform templates. In Bayesian parameter estimation, the prior distributions of different parameters of GW source models are needed to provided firstly. For parameters except \( \Delta \gamma \), we use Bilby’s default parameter priors for BBH\textsuperscript{16}. For \( \Delta \gamma \), two kinds of prior distribution are used. One is the logarithmic prior, by which we could estimate the magnitude of \( \Delta \gamma \). However, the logarithmic priors of \( \Delta \gamma \) could not take the value zero and are fixed to be positive or negative. Therefore, we introduce a modified logarithmic prior that could cover both the negative and non-negative value. The modified logarithmic prior is described by
\[
\Delta \gamma (\alpha) = \begin{cases} 
10^{-\frac{1}{\alpha}} & (\alpha > 0) \\
0 & (\alpha = 0) \\
-10^{\frac{1}{\alpha}} & (\alpha < 0)
\end{cases}
\]
where \( \alpha \) is an uniform distribution parameter, then prior \( \Delta \gamma \) covers from the negative to positive value continuously.
Discussion Since the first direct detection of GWs\textsuperscript{19}, LIGO and Virgo collaboration has checked the consistency of several gravitational wave signals\textsuperscript{20,21} with the predictions of general relativity. By now, they give some constrains on modifications to the propagation of gravitational waves due to a modified dispersion relation, and they have not found any obvious inconsistency of the data with the predictions of GR\textsuperscript{21}. However, the WEP was still not tested with GW data analysis.

On the cosmology scale, by comparing the arrival time of GWs at different frequencies\textsuperscript{6,10} or with Gamma-ray burst in GW170817, the derivation of WEP $\Delta \gamma$ has been constrained not larger than $\sim 10^{-9}$. In these tests the observed time delay $\Delta t_{\text{obs}}$ for two signals from the same source are directly treated as the one caused by WEP violation. The delay of the arrival times of two signals is completely attributed to the estimation of $\Delta \gamma$. These assumptions gave roughly constraints on $\Delta \gamma$. This method could be improved by removing the intrinsic time delay if the physical mechanisms of the astrophysical events are known, such as GWs of the compact binary mergers.

In the present work, we study the intergalactic free-fall of GWs from all the GW events in GWTC-1, which is LIGO-Virgo’s first GW transient catalog of compact binary mergers. Because the emission-time differences $\Delta t_e$ of different frequency of GWs of one GW event are known exactly, any violation of WEP will contribute to the arrival time delay, and cause the dephasing of waveforms. This effect can be extracted in GW data. By considering the Shapiro time delay of the Milky Way’s gravitational field, we construct a waveform template with $\Delta \gamma$. With this template, we analyse the GW data of GWTC-1, and constrain the $|\Delta \gamma| \lesssim 10^{-15}$ at 90% confidence. For a modified logarithmic prior, the 90% confidence interval of $\Delta \gamma$ is $[-1.0 \times 10^{-15}, +1.4 \times 10^{-17}]$. 

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This will strongly imply that WEP is valid on the intergalactic scale for GWs.

There are two advantages in our model. First, the emission time delay of signals with different frequencies is known exactly. Second, the interstellar medium has no influence on the propagation of GWs. In addition, if we can include the gravitational potential of the host galaxies, the values of $\Delta \gamma$ should be constrained better. In the future, more and more GW events will be detected in the third observing run of LIGO-Virgo and future GW detections, we could expect that the constraint on WEP will be more stringent.

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**Author contributions**  W.B.H conceived the original idea of this project. W.B.H and S.C.Y constructed the waveform template, wrote the first draft. S.C.Y carried out the data analysis. G.W proposed the method of data analysis. All authors contributed to the text.

**Data availability**  The data sets generated during the current study are available from the corresponding author on reasonable request.

**Competing Interests**  The authors declare that they have no competing financial interests.

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