Dispersion of Surface Waves above Time-Varying Reactive Boundaries

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Abstract—In this presentation, we analytically derive the dispersion equation for surface waves traveling along reactive boundaries which are periodically modulated in time. In addition, we show numerical results for the dispersion curves and uniquely uncover that time-varying boundaries generate band gaps that can be controlled by engineering the modulation spectrum. Furthermore, we also point out an interesting effect of field amplification related to the existence of such band gaps for surface waves. The effect of amplification does not require the synchronization of signal and pumping waves. This unique property is very promising to be applied in surface-wave communications from microwave to optical frequencies.

Index Terms—Surface waves, temporal modulation, metasurfaces, dispersion relation.

I. INTRODUCTION

It is well known that volumetric photonic crystals have the efficacy of controlling propagating waves, e.g. [1], both in three and two dimensions, e.g. [2]. Recently, in parallel with thorough and general investigations of time-varying electromagnetic systems (see, e.g., Refs. [3]–[5]), three-dimensional (volumetric) temporal photonic crystals whose material properties are uniform in space but changing in time have attracted significant attention [6]. This is due to the intriguing effects that they have on propagating plane waves.

It appears that it is fundamentally important to explore properties of temporal metasurfaces which are 2D material sheets or boundaries with time-varying properties. In this case, the waves are surface waves, bound to the sheet or boundary. The eigenmode problem for waves along spatially uniform but time-modulated boundaries is one of the important canonical problems in electromagnetics of time-varying structures.

In this work, we solve this problem and discuss dispersion properties of surface waves over time-modulated reactive boundaries. As a simple canonical case, we consider a planar infinite boundary that is modeled by a surface capacitance which is spatially uniform over the surface and temporally modulated by an arbitrary periodical function. As a particular realization, one can consider, for example, a high-impedance surface of the type introduced by D. Sievenpiper [7], where the capacitance between patches is modulated using varactors.

In addition to derivation of the dispersion relation for surface waves over such time-varying boundary, we give numerical examples of dispersion plots. Also, we explain the conditions for appearing stop bands for propagation constants and reveal phenomena of exponential field growth, reflection amplification, and radiation of space waves from those time-modulated boundaries.

II. THEORY

Let us consider a spatially uniform and time-varying reactive boundary. Here, as an example, we assume a time-varying capacitive one which is represented by \( C(t) \). As is well known, such a boundary supports surface waves which have the transverse-electric (TE) polarization with respect to the propagation direction. In other words, in free space, the electric field is perpendicular to the propagation plane, as shown in Fig. 1. For a stationary boundary, the dispersion equation defines a relation between the frequency \( \omega \) and the propagation constant along the surface \( \beta \). Since the capacitive boundary periodically changes in time, the eigenmode \( \beta \) will contain components at frequencies \( \omega_n = \omega + n\omega_M \), \( n = 0, \pm1, \pm2, \ldots \). Here, \( \omega_M \) is the fundamental modulation frequency. The electric field is expressed as

\[
E = \sum_{n=-\infty}^{+\infty} E_n \exp(j\omega_n t) a_y(x),
\]

in which

\[
E_n = A_n \exp(j\beta z) \exp(-\alpha_n x). \tag{2}
\]

Here, \( A_n \) is the amplitude of the wave corresponding to each frequency harmonic, and \( \alpha_n \) denotes the attenuation constant along the normal direction, for each harmonic. The plane-wave dispersion equation for free space above the boundary sets the following relation for each harmonic:

\[
\beta^2 = \alpha_n^2 + \omega_n^2 \epsilon_0 \mu_0. \tag{3}
\]
It is worth mentioning that since we investigate surface waves, $\alpha_n$ must be a real value.

Similarly to the electric field, the tangential component of the magnetic field which is directed along the z-axis is given by

$$H_t = \sum_{n=-\infty}^{+\infty} H_n \exp(j\omega_n t) a_z,$$

where

$$H_n = B_n \exp(j\beta z) \exp(-\alpha_n x).$$

The Maxwell equations relate the amplitudes of the electric field and the tangential component of the magnetic field to each other. By applying Eq. (3) and doing some algebraic manipulations, we obtain a matrix relation $M \cdot A = B$. Here, $M$ is a matrix that has $2N + 1$ rows and columns, and it is a function of $\beta$ and $\omega_n$. The matrices $A$ and $B$ have only one column and $2N + 1$ rows, and they are representing the amplitudes.

In analogy with the circuit theory, where we explicitly express the relation between the electric current flowing through the time-varying capacitance $i(t)$ and the voltage over it $v(t)$ as

$$\int i(t) dt = C(t) v(t),$$

we simply write the relation between the tangential components of the electric and magnetic fields. In fact, this is the boundary condition in the dynamic scenario. By imposing the boundary condition, we obtain another matrix equation as $Y \cdot A = B$. The matrix $Y$ is a function of $\omega_n$ and the Fourier coefficients of the periodic function $C(t)$ which is expressed by the Fourier series in the exponential form. Similar modeling method has been used in our recent paper [8].

Now, we have two matrix equations $M \cdot A = B$ and $Y \cdot A = B$. Therefore, we conclude that $[Y - M] \cdot A = 0$. The determinant of the whole matrix in the square brackets must be zero in order to allow nonzero solutions for the electric field. Consequently, relation

$$\det [Y - M] = 0$$

determines the dispersion of the surface waves above time-varying capacitive boundaries. In the following, we give some particular examples and numerically investigate the dispersion curves.

III. NUMERICAL EXAMPLES AND DISCUSSION

First, we consider a capacitive boundary that is modulated in time harmonically, assuming, as an example, $C(t) = C_0 [1 + 0.3 \cos(\omega_M t) + 0.2 \cos(2\omega_M t)]$. To obtain the dispersion diagram, we specify the value of $\beta$, and solve the dispersion equation Eq. (7) for the corresponding eigenfrequencies. The corresponding dispersion curves are shown in Fig. 2. One can see that for a fixed value of the propagation constant $\beta$, there are many solutions for the eigenfrequencies $\omega$, meaning that an eigenmode contains components at many frequencies. This property means that we can excite the mode by external sources at many possible frequencies. The excitation frequency can even be above the light line (purple dot) which excites higher-order frequency harmonics below the light line (green dot). This indicates that, in time-varying structures, it is possible to launch surface waves with an incident plane wave, as is also reported in a recent work [9]. Most importantly, temporal modulation opens up a band gap in $k$-space, which is a dual phenomenon of spatial periodic structures where the band gap is at the frequency axis. Similar properties of bulk media have been reported in [9] [10]. Increasing the modulation depth, one can widen the band gap. Interestingly, the number of band gaps corresponds to the number of modulation tones. Figure 3 shows that by adding a second modulation tone, a second band gap opens up. Positions and widths of the gaps can be tuned by varying the modulation spectrum. These band gaps provide great possibilities to control the propagation of surface waves on a metasurface plane.

Next, we examine the wave behavior when the excited surface modes are inside a band gap. For a specified wavenumber $\beta$ in the band gap, the solved eigenfrequencies are complex numbers $\omega = \omega' \pm j\omega''$, meaning that the wave can be exponentially attenuated or amplified in time. Next, we use COMSOL to numerically simulate the time-varying structure in this regime. Figure 4(a) illustrates a constant-amplitude surface wave propagating along an unmodulated capacitive boundary. Then, the temporal modulation of the surface reactance is suddenly switched on. Evidently, as it can be seen
in Fig. 4(b), the surface mode is significantly amplified after modulating with a duration of $\Delta t = 5T_0$, where $T_0$ is the time period at the excitation frequency. In addition, there are higher-order harmonics generated in free space, forming a standing wave pattern along the surface, due to symmetry of the structure.

IV. CONCLUSIONS

Here, we have presented the dispersion equation and example dispersion plots for a temporally modulated electromagnetic boundary. The results show that time modulation induces band gaps in the two-dimensional $k$-space, which provides opportunities to control surface wave propagation. In the presentation, we will discuss in detail interesting properties of surface waves launched inside a band gap. Let us note that one can repeat the above path to achieve the dispersion equation associated with time-varying inductive boundaries. The difference is that the wave has a transverse-magnetic polarization (i.e., magnetic field is perpendicular to the propagation plane).

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REFERENCES

[1] J. D. Joannopoulos, R. D. Meade, and J. N. Winn, “Photonic Crystals: Molding the flow of light,” Princeton Univ. Press, Princeton, NJ, 1995.
[2] T. F. Krauss, R. M. De La Rue, and S. Brand, “Two-dimensional photonic-bandgap structures operating at near-infrared wavelengths,” Nature, vol. 383, no. 6602, pp. 699–702, 1996.
[3] X. Wang, A. Díaz-Rubio, H. Li, S. A. Tretyakov, and A. Alù, “Theory and design of multifunctional space-time metasurfaces,” Physical Review Applied, vol. 13, no. 4, p. 044040, 2020.
[4] M. S. Mirmoosa, T. Koutserimps, G. Ptitcyn, S. Tretyakov, and R. Fleury, “Dipole polarizability of time-varying particles,” arXiv preprint arXiv:2002.12297, 2020.
[5] M. S. Mirmoosa, G. Ptitcyn, V. S. Asadchy, and S. A. Tretyakov, “Time-varying reactive elements for extreme accumulation of electromagnetic energy,” Physical Review Applied, vol. 11, no. 1, p. 014024, 2019.
[6] J. R. Zurita-Sánchez, P. Halevi, and J. C. Cervantes-Gonzalez, “Reflection and transmission of a wave incident on a slab with a time-periodic dielectric function $\epsilon(t)$,” Physical Review A, vol. 79, no. 5, p. 053821, 2009.
[7] D. Sievenpiper, L. Zhang, R. F. Broas, N. G. Alexopolous, and E. Yablonovitch, “High-impedance electromagnetic surfaces with a forbidden frequency band,” IEEE Transactions on Microwave Theory and Techniques, vol. 47, no. 11, pp. 2059–2074, 1999.
[8] X. Wang, G. Ptitcyn, V. Asadchy, A. Díaz-Rubio, M. S. Mirmoosa, S. Fan, and S. A. Tretyakov, “Nonreciprocity in bianisotropic systems with uniform time modulation,” Physical Review Letters, vol. 125, no. 26, p. 266102, 2020.
[9] E. Galiffi, Y. T. Wang, Z. Lim, J. Pendry, A. Alù, and P. A. Huidobro, “Wood anomalies and surface-wave excitation with a time grating,” Physical Review Letters, vol. 125, no. 12, p. 127403, 2020.
[10] E. Lustig, Y. Sharabi, and M. Segev, “Topological aspects of photonic time crystals,” Optica, vol. 5, no. 11, pp. 1390–1395, 2018.