The Method of Collaborative Allocation of Slots and Trajectories Based on Route Resource

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Abstract. In order to solve the problem of cooperative allocation of route resources in the case of congestion or bad weather, this paper proposes a two-stage time slot coordinated allocation model. The two phases will be considered separately from the air traffic control department and the airlines, with the lowest total delay cost of the affected flights and the average flight delay cost between the airlines tend to be the same. In order to verify the model, this paper intends to use the greedy algorithm to solve. The validity of the model is tested by the simulation data of the route, so as to achieve the goal of balancing multiple flow constrained areas and improve the operation efficiency of the route.

1. Introduction
The Federal Aviation Administration (FAA) proposed the concept of the Collaborative Trajectory Options Program (CTOP) in September 2012 and successfully implemented the experiment in the United States in March 2014[1][2]. CTOP is a key tool in the NextGen project and is a new Traffic Management Plan (TMI) that balances the demand and capacity of airlines and terminal airspace[3]. CTOP combines many features of TMI such as Ground Delay Program (GDP), diversion, Airspace Flow Program (AFP), and airspace and traffic coordination management[4][8]. It also combines the time slot allocation mechanism with the diversion measures, fully considers the flight preference of the flight, increases the airline's autonomy[9], and realizes the goal of converting part of the ground waiting into airborne diversion, improving airspace operational efficiency and reducing overall delays[10]. Therefore, the research on CTOP has been widely concerned by scholars from various countries.

This paper aims to comprehensively consider the effectiveness and fairness, achieve the goal of minimizing the running cost of airlines and maximizing the route throughput of flight restricted areas under the conditions of CTOP, and establish a two-stage model of coordinated allocation of track and time slots. Delays and capacity-limited air routes are relatively balanced, which helps to ease route congestion and achieve efficient airspace operations.

2. Model description
2.1 Problem description and basic assumptions
When bad weather (such as thunderstorms) occurs or airspace is congested, it is easy to cause the airspace capacity to decrease, resulting in a Flow Constrained Area (FCA) on the route. Then the air traffic management department will initiate CTOP (Collaborative Trajectory Options Program). The program will assign corresponding time slots and trajectory to the flight preferences submitted by each
flight to ensure that delay costs and airline profits are maximized while meeting FCA capacity constraints. As shown in Figure 1, A is the departure airport, B is the destination airport, and the direction of the arrow indicates the direction of the trajectory. This article is modeled based on two trajectory options, one for the planned trajectory option and the other for the route change option. Each trajectory has one FCA, and each restricted flight has three options: flight option 1 for FCA001, flight option 2 for FCA002, and flight option 3 for no FCA.

Fig.1. Example of trajectory options

Modeling ideas: According to the FCA capacity and the published available time slots, minimizing the total delay of all flights will be the validity principle. The equalization of the time slots allocated to each flight and the equal amount of delays received will be the fairness principle. Maximizing airline profitability will be the effectiveness principle. Each flight is assigned a time slot based on the constraints of the principle of validity and fairness. The following are some of the basic assumptions for the establishment of the model:

1. We assume that exempted flights are not included in the affected flights, and the trajectory preference options and flight schedule information for the affected flights are known;
2. We assume that the FCA’s scope and its capacity are known;
3. We assume that the set of time slots available for different FCAs is known;
4. We assume that the trajectory option 3 is not considered in the first stage model. By default, all flights are submitted at least one of the trajectory options 1, 2, and at most two trajectory options are submitted.

2.2 Model parameter setting
In order to solve the above problem, this paper introduce the following decision variables:

- \( t_{ij} \): Time slot the i-th affected flight assigned to the j-th trajectory;
- \( \delta_{ij} \): Binary indicator whether flight i will take route j;
- \( t_{ij}^s \): Binary indicator of time slot whether flight i will take route j;
- I: Collection of affected flight, \( i \in I \);
- \( I_A \): Collection of A airline’s affected flight;
- S: Set of time slot, \( s \in S \);
- J: Set of trajectory, \( j \in J = \{1,2\} \);
- A: Cost factor of ground delay;
- B: Cost factor of airbone dealy (Generally take \( 2\alpha = \beta \), that means the cost of air delay is twice the cost of ground delay);
- \( e_{ij} \): The time of entry into the j-th trajectory FCA submitted by the i-th affected flight, ETA;
- \( \tau_i \): The earliest time of entry into the FCA of all the trajectories submitted by the i-th affected flight, IAT;
- \( c_{ij} \): Additional voyage cost of the j-th trajectory of the i-th affected flight;
- \( F_j \): Capacity requirements for the j-th trajectory FCA;
- \( \mu_{ij} \): Uncertainty cost of the i-th affected flight on the j-th trajectory;

2.3 Model Formulation

2.3.1 System effectiveness
The establishment of the objective function is mainly divided into two steps. The first step is that the
The objective function is the most efficient, that is, the total delay cost of the system is the lowest:

$$\min C_1 = \sum_{i \in I} [\alpha \sum_{j \in J} (t_{ij} - \tau_i) \cdot t_{ij}^s + \beta (\sum_{j \in J} \delta_{ij} \cdot c_{ij}) + \sum_{j \in J} \delta_{ij} \cdot \mu_{ij}] \quad (1)$$

Restrictions:

$$t_{ij} \geq e_{ij} \cdot \delta_{ij} \quad \forall i, i \in I; \forall j, j \in J$$ \quad (2)

$$\sum_{j \in J} \sum_{s \in S} t_{ij}^s = 1 \quad \forall i, i \in I$$ \quad (3)

$$\sum_{j \in J} \sum_{i \in I} t_{ij}^s \leq 1 \quad \forall s, s \in S$$ \quad (4)

$$\sum_{s \in S} t_{ij}^s = \delta_{ij} \quad \forall i, i \in I; \forall j, j \in J$$ \quad (5)

$$\sum_{i \in I} \delta_{ij} \leq F_j \quad \forall j, j \in J$$ \quad (6)

$$\tau_i = \min_{j \in J} \{e_{ij}\} \quad \tau_{i+1} \geq \tau_i \quad \forall i, i \in I$$ \quad (7)

In the above model, equation (1) is the objective function and is divided into three parts. They are the ground delay of flight, the air delay of flight and the flight cost delay of uncertainty. Equation (2) specifies that the allocated slot of the flight must be after the ETA time. Equation (3) stipulates that each flight is only assigned one slot. Equation (4) stipulates that only one flight can be arranged per slot. Equation (5) stipulates that when the flight has the j-th trajectory option, it can be assigned to the FCA slot of the trajectory. Equation (6) stipulates that the total number of flights allocated for each flight does not exceed the capacity requirement. Equation (7) stipulates that the flight schedules are arranged in the order of the earliest time to enter the FCA.

2.3.2 System fairness

The second step of the objective function is to resolve the issue of fairness between airlines. In traffic management, fairness is expressed as a fair distribution of delays or equitable allocation of resources to airspace users. Also define the following decision variables:

- \(h_{is}\): Cost factor, the "cost" of the i-th flight to obtain the time slot s;
- \(\omega_i\): Weight, related to flight i;

Objective function:

$$\min C_2 = \sum_{i \in I, j \in J} h_{is} t_{ij}^s \quad (8)$$

Restrictions:

$$h_{is} = \omega_i (t_{ij} - e_{ij})^{1+\varepsilon} \quad (9)$$

Where equation (8) is an objective function, indicating that the total cost of allocation of each flight slot is the smallest. Equation (9) is an expression of the cost coefficient, where \(\omega_i\) represents the weight associated with flight i; \(0 < \varepsilon < 1\). The parameter \(\varepsilon\) causes the flight delay cost to increase linearly, so the model tends to assign the appropriate delay to two flights. The weight \(\omega_i\) in the cost factor represents the cost associated with flight ground delays and air delays.

3. Algorithm design

This paper uses greedy algorithm for solving a two-stage model. The main idea is divided into two parts: The first part is to arrange all the flights affected by CTOP in ascending order according to the slots of the earliest available FCA. Then use the RBS (Ration by schedule) principle to assign slots to the lowest total delay cost of all affected flights. The second part is to allow slot exchange between the same airline or different airlines under the condition of the affected cancelled flight, so as to realize the fairness of slot allocation between airlines. The algorithm flow is shown in Figure 2.
4. Case analysis
This paper creates an example based on civil route simulation data to verify the performance of the model. The route of trajectory option 1 and 2 is planned to have 23 flights in the period from 1900Z to 2000Z on a certain day. Under the adverse effects of the weather, the two affected trajectories are respectively generating an FCA. Based on the currently available airspace capacity conditions and air traffic control experience, set three trajectories options: ① planned trajectory option, ② modified trajectory option (allocated time slot), ③ flight path options (only in the second stage and do not involve slot allocation). The relevant information of the route is shown in Table 1. The available time slot information of the track issued by the air traffic control department is shown in Table 2. The affected flight information is shown in Table 3.

![Algorithm flowchart]

Fig.2. Algorithm flowchart

| Trajectory property   | Capacity (rack/hour) | Flight delay cost (yuan/minute) |
|-----------------------|----------------------|---------------------------------|
| ① Planned trajectory option | 9                    | 2800                            |
| ② Diversion trajectory option | 12                   | 3500                            |
| ③ Flying trajectory   | 6                    | 3200                            |

| FCA1    | FCA2    |
|---------|---------|
| 19:10   | 19:12   |
| 19:16   | 19:15   |
| 19:22   | 19:17   |
| 19:28   | 19:20   |
| 19:34   | 19:24   |
| 19:40   | 19:29   |
19:46  19:34
19:52  19:39
19:58  19:44
20:04  19:49
20:10  19:54
20:16  19:59
20:22  20:04
20:28  20:09

Table 3. Flight information form

| Flight number | Affiliated airline | Model | Number of passengers | Earliest time to enter FCA1 | Earliest time to enter FCA2 | IAT |
|---------------|--------------------|-------|----------------------|-----------------------------|-----------------------------|-----|
| f1            | A                  | M     | 130                  | 19:10:00                    | 19:13:28                    | 19:10:00 |
| f2            | A                  | M     | 120                  | 19:10:30                    | 19:10:30                    | 19:10:30 |
| f3            | B                  | M     | 150                  | 19:15:47                    | 19:15:47                    | 19:15:47 |
| f4            | B                  | M     | 120                  | 19:17:22                    | 19:18:44                    | 19:17:22 |
| f5            | C                  | M     | 150                  | 19:18:46                    | 19:19:17                    | 19:18:46 |
| f6            | A                  | H     | 270                  | 19:18:52                    | 19:22:29                    | 19:18:52 |
| f7            | B                  | H     | 290                  | 19:21:00                    | 19:21:00                    | 19:21:00 |
| f8            | C                  | M     | 130                  | 19:22:46                    | 19:21:23                    | 19:21:23 |
| f9            | C                  | M     | 150                  | 19:25:49                    | 19:25:10                    | 19:25:10 |
| f10           | B                  | H     | 380                  | 19:26:05                    | 19:25:38                    | 19:25:38 |
| f11           | A                  | M     | 130                  | 19:33:31                    | 19:30:48                    | 19:30:48 |
| f12           | C                  | M     | 130                  | 19:35:39                    | 19:34:40                    | 19:34:40 |
| f13           | A                  | M     | 130                  | 19:35:00                    | 19:39:53                    | 19:35:00 |
| f14           | C                  | M     | 150                  | 19:40:36                    | 19:35:56                    | 19:35:56 |
| f15           | A                  | M     | 120                  | 19:37:30                    | 19:41:28                    | 19:37:30 |
| f16           | C                  | M     | 170                  | 19:40:00                    | 19:43:49                    | 19:40:00 |
| f17           | C                  | H     | 300                  | 19:46:17                    | 19:49:18                    | 19:46:17 |
| f18           | B                  | M     | 120                  | 19:48:45                    | 19:56:50                    | 19:48:45 |
| f19           | C                  | M     | 150                  | 19:49:34                    | 19:53:44                    | 19:49:34 |
| f20           | C                  | H     | 300                  | 19:55:38                    | 19:52:04                    | 19:52:04 |
| f21           | A                  | M     | 120                  | 19:54:47                    | 19:54:47                    | 19:54:47 |
| f22           | A                  | M     | 170                  | 19:55:00                    | 19:55:18                    | 19:55:00 |
| f23           | B                  | H     | 380                  | 19:56:38                    | 19:58:38                    | 19:56:38 |

Through the model and algorithm established in this paper, using Python to program and solve. Figure 3 is a trajectory assignment scheme for all affected flights with only the validity of the first phase considered. Figure 4 shows the optimization scheme of flight slot assignment under the condition of considering two phases. Regardless of fairness, the total flight delay of the slot-optimal assignment scheme based solely on the RBS principle is 295.48 min. After considering the second stage, the total flight delay of the optimization plan is 177.12 min. The optimization method can reduce the total delay time by 118.36 min and the total flight delay by 40.05%.
Fig. 3. Optimal assignment scheme for flight time slot trajectory before optimization

Fig. 4. Optimal assignment scheme for flight time slot trajectory after optimization

Using the models and algorithms built in this paper, the average flight delays of airlines are shown in Table 4.

| Unit(min) | Airline A | Airline B | Airline C |
|-----------|-----------|-----------|-----------|
| Before optimization | 12.07 | 14.14 | 12.68 |
| After optimization | 7.94 | 7.41 | 7.68 |

As shown in Figure 5, a bar chart of the average delay of airline flights is plotted. In the case of considering only the first phase, the average flight delay of each airline is more than the average delay of the second phase, and the average delay of each airline after optimization is also relatively close, which is a good reflection for fairness between the airlines.
5. Conclusion
This paper not only considers the validity, but also considers the principle of fairness. Based on the route resources, a cooperative allocation method for two-stage slot is established. Using the greedy algorithm, the global target with the smallest total flight delay and the same local optimal target for the average flight delay are realized. The results show that the model can better play the role of airlines in collaborative decision-making and promote the efficient use of route resources. Due to the limited author's ability, the model still has some shortcomings in the calculation speed of large-scale restricted airspace, which needs further improvement.

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