Supersymmetric Sudakov corrections

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For superpartner masses not much heavier than the weak scale $M = M_W$, large logarithmic corrections of the Sudakov type arise at TeV energies. In this paper we discuss the general structure of electroweak supersymmetric (susy) Sudakov corrections in the framework of the infrared evolution equation method. We discuss Yukawa sector Ward-identities which lead to the exponentiation of the subleading (SL) logarithmic Yukawa enhanced Sudakov corrections in both the Standard Model (SM) as well as in softly broken supersymmetric extensions. The results are given to SL accuracy to all orders in perturbation theory for arbitrary external lines in the “light” susy-mass scenario. The susy-QCD limit for virtual corrections is presented. Phenomenological applications regarding the precise determination of the important parameter $\tan \beta$ through virtual corrections are discussed which are independent of the soft susy breaking mechanism to sub-subleading accuracy to all orders.

1. Introduction

In light of future precision experiments in the TeV-energy regime at such machines as the LHC, TESLA or CLIC, a lot of interest has been devoted recently to studying the high energy limit of spontaneously broken field theories [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. The main conclusion of these works, summarized in Ref. [14] including a variety of new phenomenological applications, is that one needs to include higher order electroweak radiative corrections through two loops at least to sub-subleading (SSL) logarithmic accuracy [5]. At present, only a full SL analysis in the SM to all orders is available [1, 14, 15, 16] in the context of the infrared evolution equation method [17]. This approach has been confirmed by explicit two loop calculations at the leading DL [1, 12, 13] and the SL angular dependent level [18]. A further SSL analysis for massless fermion production points to the necessity to include also these SSL contributions due to large cancellations between DL, SL and SSL terms [5].

In general, new physics responsible for electroweak symmetry breaking is expected in the TeV regime and the minimal supersymmetric SM (MSSM) remains an attractive candidate. If supersymmetry is relevant to the so called hierarchy problem, then the masses $m_s$ of the new superpartners cannot be much heavier than the weak scale $M = M_W \sim M_Z$. In a “light” susy mass scenario with $m_s \sim M$, similarly large radiative corrections can be expected as in the SM at TeV energies. At one loop this was confirmed by several works of the last few years [13, 20, 21].

While these corrections are of considerable phenomenological interest, it is, however, also important to understand theoretically the high energy limit of theories which are spontaneously broken such as the SM or the MSSM. In Ref. [22] an important step was taken towards the understanding of the higher order electroweak susy Sudakov corrections. Since the DL and angular dependent SL terms originate only from the exchange of gauge bosons, softly broken susy does not introduce novel terms at this level. The new particle content of the MSSM leads, however, to new universal (i.e. process independent) SL corrections of gauge and Yukawa origin. In Ref. [22] it was shown that the higher order resummation of these terms in exponential form is a consequence of Ward-identities since also the Yukawa sector is gauge invariant. The same reasoning lead to the exponentiation of SL Yukawa terms [1, 14].
The corrections must also be included via matching. It is clear that the gauge boson loop momentum \( l \) at one loop only yield a single logarithm of Yukawa terms in the SM and the MSSM to SL accuracy. For our purposes we need to investigate terms containing three large masses taking \( M \sim \lambda \) for clarity. The soft photon corrections must also be included via matching. The \( G_{r,l} \) denote the chiral Yukawa couplings and \( \omega_{r,l} = \frac{1}{2} (1 \pm \gamma_5) \). The gauge coupling is written in the symmetric basis. For our purposes we need to investigate terms containing three large logarithms in those diagrams. Since the fermion loops at one loop only yield a single logarithm it is clear that the gauge boson loop momentum \( l \) must be soft. Thus we need to show that the UV logarithm originating from the \( k \) integration is identical (up to the sign) in both diagrams. We can therefore neglect the loop momentum \( l \) inside the fermion loop. We find for the fermion loop vertex \( \Gamma^\mu(p_1^2, 0, p_2^2) \) belonging to Eq. 2:

\[
\frac{1}{k^2(k - p_1)^2(k - p_1)^2} \Tr[(G_r \omega_r + G_l \omega_l)]
\]

\[
(\not{k} - \not{p}_1)\gamma^\mu(\not{k} - \not{p}_1)(G_r \omega_r + G_l \omega_l)\not{k}
\]

\[
= \frac{4G_r G_l (2p_1^\mu (k^2 - p_1 k) + k^\mu (p_1^2 - k^2))}{k^2(k - p_1)^4} \tag{3}
\]

This we need to compare with the self energy loop \( \Sigma(p_1^2) \) from Eq. (3):

\[
\frac{\partial}{\partial p_{1\mu}} \Tr[(G_r \omega_r + G_l \omega_l)(\not{k} - \not{p}_1)(G_r \omega_r + G_l \omega_l)\not{k}]
\]

\[
= \frac{\partial}{\partial p_{1\mu}} 4G_r G_l (p_1 k - k^2)
\]

\[
= \frac{4G_r G_l 2p_1^\mu (k^2 - p_1 k) + k^\mu (p_1^2 - k^2)}{k^2(k - p_1)^4} \tag{4}
\]

In short we can write

\[
\frac{\partial}{\partial p_{1\mu}} \Sigma(p_1^2) = \Gamma^\mu(p_1^2, 0, p_2^2) \tag{5}
\]

where the full sum of all contributing self energy and vertex diagrams must be taken. Thus, we have established a Ward identity for arbitrary Yukawa couplings of scalars to fermions and thus, the identity of the UV singular contributions. The relative sign is such that the generated SL logarithms of the diagrams in Fig. 1 cancel each other. The existence of such an identity is not surprising since it expresses the fact that also the Yukawa sector is gauge invariant. We are thus left with gauge boson corrections to the original vertices in the on-shell renormalization scheme such as depicted in Fig. 2. At high energies we can therefore employ the non-Abelian version of Gribov’s bremsstrahlung theorem. The soft photon corrections are included via matching as discussed in Refs. [1, 4]. The arguments above can be applied analogously to other external legs and lead to the exponentiation of the Yukawa and SL gauge terms as mentioned above.

In the following we will give the complete virtual electroweak high energy corrections for arbitrary on-shell matrix elements to all orders.
In the following we give only the corrections for particles irreducible representations of the gauge group, of the symmetry group of the unbroken masses, don’t belong to irreducible representations (fields) of the unbroken theory. Note that, particles (fields) by

In the general case let us denote physical particles (fields) by \( f \) and particles (fields) of the unbroken theory by \( u \). Let the connection between them be denoted by \( f = \sum_u C^{fu} u \), where the sum is performed over appropriate particles (fields) of the unbroken theory. Note that, in general, physical particles, having definite masses, don’t belong to irreducible representations of the symmetry group of the unbroken theory (for example, the photon and Z bosons have no definite isospin). On the other hand, particles of the unbroken theory, belonging to irreducible representations of the gauge group, have no definite masses. Then for the amplitude \( \mathcal{M}_{f_1 \cdots f_n}(\{ p_k \}; \{ m_i \}; M, \lambda) \) with \( n \) physical particles \( f_i \) with momenta \( p_i \) and infrared cut-off \( \lambda \), the general case for virtual corrections is given by

\[
\mathcal{M}_{f_1 \cdots f_n}(\{ p_k \}; \{ m_i \}; M, \lambda) = \sum_{u_1 \cdots u_n} C^{f_1 u_1} C^{f_2 u_2} \cdots C^{f_n u_n} \mathcal{M}^{u_1 \cdots u_n}(\{ p_k \}; \{ m_i \}; M, \lambda)
\]

In the following we give only the corrections for a light susy mass scale \( m_s \sim M \) and for a heavy photon (\( \lambda = M \)) with all \( 2p_p p_k \gg M^2 \). In this case, we can easily work in the symmetric basis and give the results for these amplitudes. As discussed in Refs. [1, 14, 22, 24], the soft virtual and real QED corrections must be added by matching at the weak scale \( M \). It should be mentioned, however, that the Yukawa terms are independent of the matching terms.

Under these assumptions, we have for general on-shell matrix elements with \( n \)-arbitrary external lines the following resummed SL corrections:

\[
\mathcal{M}_{\text{SL}}^{u_1 \cdots u_n}(\{ p_k \}; m_s; M) = \exp \left\{ \sum_{k=1}^{n} \left( \frac{g^2(m_s^2)}{16\pi^2} I_k(I_k + 1) + \frac{g'^2(m_s^2) Y_k^2}{16\pi^2} \right) \log^2 \frac{s}{M^2} 
\]

\[
+ \left( \frac{g^2(m_s^2)}{64\pi^4} I_k(I_k + 1) \right) \beta_0 \log \frac{s}{m_s^2} 
\]

\[
+ \left( \frac{g'^2(m_s^2) Y_k^2}{64\pi^4} \right) \beta_0' \log \frac{s}{m_s^2} 
\]

\[
+ \left( \frac{g^2(m_s^2)}{16\pi^2} \delta_{i_k,B} + \frac{g'^2(m_s^2) Y_k^2}{16\pi^2} \right) \beta_0 \log \frac{s}{m_s^2} 
\]

\[
+ \left( \frac{g'^2(m_s^2)}{16\pi^2} \right) \log \frac{s}{m_s^2} 
\]

\[
- \frac{1}{8\pi^2} \sum_{l<k} \sum_{V_u = B, W^3} \left( \tilde{f}^{v_u}_{i_k,i_l} \tilde{f}^{v_u}_{i_l,i_k} \right) \log \frac{s}{M^2} \log \frac{2p_p p_k}{s} 
\]

where the index \( j \) can be any value of the set \( \{1, 2, 3\} \). The fields \( u \) have a well defined isospin, but for angular dependent terms involving CKM mixing effects, one has to include the extended isospin mixing appropriately in the corresponding couplings \( \tilde{f}^{v_u}_{i_k,i_l} \) of the symmetric basis. If some of the sparticles should be heavy, additional corrections of the form \( \log^2 \frac{m^2}{M^2} \) etc. would be important. Here we also assume that the asymptotic MSSM \( \beta \)-functions can be used with

\[
\beta_0 = \frac{3}{4} C_A - \frac{n_f}{2} - \frac{n_h}{8} 
\]

\[
\beta_0' = -\frac{5}{6} n_f - \frac{n_h}{8} 
\]
where $C_A = 2$, $n_g = 3$ and $n_h = 2$. In practice, one has to use the relevant numbers of active particles in the loops. These terms correspond to the RG-SL corrections just as in the case of the SM as discussed in Refs. [23] but now with the MSSM particle spectrum contributing. They originate only from RG terms within loops which without the RG contribution would give a DL correction.

It should be noted that the one-loop RG corrections do not exponentiate and are omitted in the above expressions. They are, however, completely determined by the renormalization group in softly broken supersymmetric theories such as the MSSM and sub-leading at higher than one loop order. They can be obtained by inserting the running one loop couplings

$$g^2(s) = \frac{g^2(m^2)}{1 + \frac{M^2}{m^2}} \ln \frac{s}{m^2}$$

$$g'^2(s) = \frac{g'^2(m^2)}{1 + \frac{M^2}{m^2}} \ln \frac{s}{m^2}$$

into the Born cross section. In addition, we also did not include terms discussed in Refs. [3, 23] that are related to the renormalization of the mixing coefficients $C_{ij}^{u,v}$, which in the MSSM could be matrices. The result in Eq. (10), is valid for arbitrary softly broken supersymmetric extensions of the SM with the appropriate changes in the $\beta$-functions. Taking the susy-QCD limit

$$\frac{g^2}{M^2} \rightarrow \alpha_s, g' \rightarrow 0, I_{g}(g + 1) = I_{g}(g + 1) \rightarrow C_A = 3, I_{g}(g + 1) = I_{g}(g + 1) \rightarrow C_{SM} = 4/3, n_h = 0, M = \lambda_g = m_g, C_{iyuk} = 0$$

of the various terms, Eq. (10) is also valid for the virtual susy-QCD results. It should be emphasized, however, that in this case the virtual corrections are not physical in the sense that the gluon mass is zero and thus we would need to add the virtual matching and real contributions before we could make predictions for collider experiments, while in the SM soft QED energy cuts can define an observable and the heavy gauge boson masses are physical. In any case, the form of the operator exponentiation in color space agrees with the dimensionally regularized terms in Ref. [24] for non-susy QCD.

The universal SL-corrections of Yukawa type depend on the external lines only and in the MSSM are given by

$$C_{iyuk}^{\nu} = -\frac{1}{4} \left( 1 + \delta_{\alpha, R} \right) \frac{\hat{m}_{\nu}^2 + \delta_{\alpha, L} \hat{m}_{\nu}^2}{M^2}$$

$$= C_{f_s}^{\nu}$$

$$C_{iyuk}^{\nu} = -\frac{3}{4} \left( \frac{m_i^2}{M^2} + \frac{m_{\tilde{b}}^2}{M^2} \right) = C_{\chi}^{\nu} = C_{H_{SM}}^{\nu}$$

$$C_{iyuk}^{\nu} = -\frac{3}{4} \left( \frac{m_{\tilde{b}}^2}{M^2} \cot^2 \beta + \frac{m_{\tilde{b}}^2}{M^2} \tan^2 \beta \right) = C_{\chi}^{\nu}$$

$$C_{iyuk}^{\nu} = -\frac{3}{4} \left( \frac{m_{\tilde{b}}^2}{M^2} \left[ 1 + \cot^2 \beta \right] \delta_{\alpha, L} \right.$$  

$$\left. + \frac{m_{\tilde{b}}^2}{M^2} \left[ 1 + \tan^2 \beta \right] \delta_{\alpha, R} \right)$$

The fermion chiralities ($\alpha$) are those of the fermions ($f$) whose superpartner is produced. In addition we denote $\hat{m}_{\tilde{f}} = m_f / \sin \beta$ if $\tilde{f} = \tilde{i}$ and $\hat{m}_{\tilde{f}} = m_f / \cos \beta$ if $\tilde{f} = \tilde{b}$. $\tilde{f}'$ denotes the corresponding isopartner of $\tilde{f}$. For particles other than those belonging to the third family of quarks/squarks, the Yukawa terms are negligible. Eq. (12) displays an exact supersymmetry in the sense that the same corrections are obtained for the fermionic and sfermionic sector in the regime above the electroweak scale $M$. The same holds also for the remaining relative SL corrections from the full result in Eq. (10) to all orders. Since we assume $m_{\tilde{f}} \sim M$, we omit mixing effects which could be important for larger mass gaps.

Note the additional factor of $3 = N_c$ in Eqs. (12), (13) and (14) compared to Eq. (11), leading to a significant dependence on the important parameter $\tan \beta = \frac{v_d}{v_u}$, the ratio of the two vacuum expectation values. More importantly, terms depending on soft breaking parameters like mass ratios enter only at the sub-leading (SSL) level. In Refs. [20, 22] this point was emphasized and used for a determination of $\tan \beta$, based on the above virtual electroweak corrections, through a one loop subtraction method, which at SSL level is independent of the soft susy breaking terms. At higher orders, there are terms of $O(\alpha^2 C_{soft} \log^2 s_{R}/M^2)$, however, if the condition $s \sim s' \gg M^2$ is fulfilled, the difference in
the cross section measurements at $s$ and $s'$ will be proportional to $O (a^n C_{\text{soft}} \log \frac{s}{s'} \log^{2n-3} \frac{s}{s'})$ which is of beyond the SSL approximation.

This model independence is a clear and important difference to other ways of measuring $\tan \beta$ like $\tilde{\tau}$-decays \cite{27, 28, 29} and should be utilized at future TeV linear colliders. The possible relative precision depends strongly on the accuracy of the measured cross sections. Assuming 10 one percent measurements between 0.8 and 3 TeV the precision ranges from 50 % for $\tan \beta \geq 10$, 25 % for $\tan \beta \geq 15$ and a few percent measurement for $\tan \beta \geq 25$.

In conclusion, we have presented fully general virtual SL results to all orders in the context of the MSSM for arbitrary on-shell matrix elements. The form of these corrections can be written in exponential operator form in the $n$-particle space in the symmetric basis in the light susy scenario. Subleading universal terms exponentiate due to Ward identities while angular dependent corrections are determined by Born-rotated matrix elements in complete analogy to the SM.

The size of these contributions depends crucially on the energy of future colliders and it is clear that two (at CLIC possibly three) loop electroweak corrections cannot be omitted at machines operating at TeV energies if the desired precision is at the percentile level. A precise determination of $\tan \beta$, independent of the soft breaking terms to SSL accuracy, should be utilized at future linear colliders. Work towards completing this program on the theoretical side is in progress \cite{30}.

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