An Approach to Reasoning Topological Relations Between Areal Objects Under Randomness

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ABSTRACT This paper describes the geometric and statistical properties of areal object under randomness. In order to describe formally such an uncertain topological relation, a new formal model (i.e. 4ID model) is proposed. On the basis of this, the effects of positional uncertainty on topological relations between areal objects are investigated in detail. Some possibility functions for the determination of relations are constructed based on the assumption that randomness of point location complies with a normal distribution, and the concept of uncertain sets of topological relations under randomness is introduced.

KEYWORDS topological relations; randomness; positional error

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Introduction

One of the most fundamental properties of spatial objects in the real world is topological relationship, which has been widely investigated in GIS in recent years\(^1\)\(^2\). Existing work is based on crisp sets and involves general topology such as algebraic and point-set topology. However, there are inevitably errors or uncertainties in spatial data, which are used to represent reality world\(^3\)\(^4\). Those errors or uncertainties cause a suspicion for correctness of topological relations obtained by reasoning from observation data\(^5\)\(^6\). In this paper, we study the formal description of topological relations between areal objects represented by spatial data with random positional error. The questions which will be answered in this paper are: 1) what the classifications of topological relations are; 2) what the adaptive graphical structure for representing spatial objects with errors is; 3) how to calculate the randomness of the element in the 4-intersection model. Sequentially, our focus is on how to describe areal objects with random errors, and how to define the concept of random sets.

1 Representation of uncertain graphic data

1.1 Changes of graphic structure

Definition 1 For any random line, \(L_i\), all of its vertex should satisfy \(\text{Deg}(P_i) \geq 1\), in which \(\text{Deg}(P_i)\) denotes the connective degree related with vertex \(P_i\). If there exits an equation \(\text{Deg}(P_i) = 1\), the vertex \(P_i\) is the boundary point of the random line.

Definition 2 If there is a chain between two nodes, we call the two nodes connective. Furthermore, if all pairs of nodes in a planar graph are connective, the graph is connective.

Any graph in the plane, \(G\), is composed of nodes, edges and faces, and the numbers of these elements satisfy the following equation

\[ f + n - e = c + 1 \]  

where \(f\), \(n\), \(e\) are the numbers of the face, node and edge, respectively, and \(c\) is the number of...
connective branches of \( G \). If the planar graph is connective, i.e. \( c=1 \), Eq. (1) is simplified as

\[
f + n - e = 2
\]

Eq. (2) is the famous Euler theorem, which is often used for check of topological inconsistency. Below we apply it to analyze changes of graphic structure under the effect of uncertainty. In Fig. 1(a), it satisfies Eq. (2), but not for Fig. 1(b). This is mainly because uncertainty breaks the connectivity of adjacent graphs. From the viewpoint of spatial relations, a qualitative change occurs between Fig. 1(a) and 1(b). Their topological relations are described ‘overlap’ and ‘disjoint’, respectively. While in the real world topological relations between \( A_1 \) and \( A_2 \) is ‘meet’. Hence, it is very necessary to measure the uncertainty of topological relations.

\[
P = mP_i + (1 - m) P_{i+1}
\]

where \( m \) is a parameter of splitting ratio of distance, equaling to \( |P_i P_m|/|P_i P_{i+1}| \). For simplicity, we assume that positional uncertainty of a point complies with a normal distribution. In mathematical, a random point may be regarded as a random variable. According to Eq. (3), any interpolation point on line or area will also comply with a normal distribution. Furthermore, a whole line will be regarded as a normal random process, an area as a normal random field. And more, the projection of a normal random field in a 2-dimensional plane is a donut, which is also called as g-donut in this paper, as is seen in Fig. 2.

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1.2 Modeling randomness of graphic data

For uncertainty of positional data, there are considerable research documents. Here, we only focus on spatial entities with definite boundary locations in the real world. In general, its positional uncertainty is mainly from digitalization, scanning, field measurement and others. Therefore, we may assume that uncertainty of graphic data is random error. In spatial databases, a point, \( P_i \), is measured by \((x, y)\) in a 2-dimensional space, a line or arc \( L_i \), represented as a series of points, while an area, \( A \), consisted of a set of connective lines. And positional uncertainty of a point will affect location exactness of the line or the boundary of the area that the point is located on. Geometrically, any point on a line or an area may be linearly represented by its two adjacent endpoints, i.e.,
where $A_i$ and $\partial A_i$ ($i = 1, 2$) are interior and boundary of $A_i$, respectively. By using of the 4ID model, topological relations between two certain regions in a 2-D space can be divided into 8 families. These relations are named ‘disjoint’, ‘meet’, ‘overlap’, ‘contains’, ‘covers’, ‘inside’, ‘coveredby’, and ‘equal’, as listed in Fig. 3. It has been pointed out that topological relations may convert from one case to another with changes in geometry of one or two of the involved objects. That is to say that a qualitative change occurs if change in geometry of an object affects its topological relationship with respect to another object. Therefore, all of the eight relations in Fig. 3 can occur with a certain degree of changes in geometry, including: object location, orientation, shape, and size. In order to represent their occurrence rule, a concept of topological distance is presented here.

\[ d_{T}(\gamma_1, \gamma_2) = \sum_{i=1}^{4} |\delta(s_i^1) - \delta(s_i^2)| \]  

where $\gamma_1$ and $\gamma_2$ are corresponding elements in $\gamma_1$ and $\gamma_2$, respectively. Using Eq. (6), we can calculate the topological distance between any two relations shown in Fig. 3. For instance, the topological distance between ‘disjoint’ and ‘overlap’ is calculated as follows:

\[ d_{T}(\text{disjoint}, \text{overlap}) = |1 - 1| + |1 - 1| + |1 - 1| + |0 - 1| = 2 \]

Similarly, we may obtain all the distance for other relations, as listed in Table 1. It can be seen that the topological distance above defined satisfies the following properties:

- $d_T(\gamma_1, \gamma_2) = d_T(\gamma_2, \gamma_1)$
- $0 < d_T(\gamma_1, \gamma_2) = 4$
- $d_T(\gamma_1, \gamma_2) = 0$, if and only if $\gamma_1 = \gamma_2$
- $d_T(\gamma_1, \gamma_2) + d_T(\gamma_2, \gamma_3) = d_T(\gamma_1, \gamma_3)$

The same time, it is shown that the minimal distance between distinct topological relations are 1. Here we define such two relations with topological distance equal to 1 as neighborhood relations. That is to say that, if they satisfy the condition

\[ d_T(\gamma_1, \gamma_2) = 1 \]

$\gamma_1$ and $\gamma_2$ are then termed of neighborhood relations. Further, we link all pairs of neighborhood relations, thus the conceptual neighborhood graph is set up, as shown in Fig. 4. This is particularly useful to predict what is the most likely relations after a change in geometry of one or two objects occurs.

Under uncertainty circumstance, spatial data uncertainty, in essence, can be regarded as a small deformation of object boundary in location, size, shape, etc. It only possibly varies to topological relation from one kind to another, and

\[ d_T(\gamma_1, \gamma_2) = 1 \]
does not increase or decrease topological categories. As a result, for two imprecise regions, separable topological relations are still 8 families with the same names. In the following, 4ID model is taken as a basis of reasoning topological relations under uncertainty.

2.2 Reasoning topological relations from the observed data

It may be inconsistent between the topological relation by reasoning from observation data and its true value. As shown in Fig. 1, topological relations between $A_1$ and $A_2$ are described as ‘overlap’, or ‘disjoint’ under imposing a small perturbation on one or both of objects, while their true relation is ‘meet’. Sometimes, spatial data uncertainty does not have any effect on description of topological relation. One example is that two disjoint objects from each other, and in this case uncertainty may be ignored. Therefore, it is necessary to further investigate and set up a general model for topological relations, while considering location uncertainty.

The g-donut model is utilized to represent an imprecise region. And here we define the region which is demarcated by the exterior boundary of g-donut as exterior region, denoted by $g^+$, the region demarcated by the interior boundary of g-donut as interior region, denoted by $g^-$. On the basis of the 4ID model, four relations are defined as follows:

\[
\begin{align*}
\gamma(g^+ \cup g^-) &= <
\begin{array}{c}
\gamma(g_1^+, g_2^-), \\
\gamma(g_1^-, g_2^+)
\end{array}
\> \\
\gamma(g^+, g^-) &= <
\begin{array}{c}
\gamma(g_1^+, g_2^-), \\
\gamma(g_1^-, g_2^+)
\end{array}
\> \\
\gamma(g^+, g^+) &= <
\begin{array}{c}
\gamma(g_1^+, g_2^+), \\
\gamma(g_1^+, g_2^-)
\end{array}
\> \\
\gamma(g^-, g^-) &= <
\begin{array}{c}
\gamma(g_1^-, g_2^-), \\
\gamma(g_1^-, g_2^+)
\end{array}
\>
\end{align*}
\]

where $g_i^+$ and $g_i^-$ ($i = 1, 2$) are outer and inner region of g-donut of regions $A_i$. $(g_1^+, g_2^-)$ and $\partial(g_i^+)$ are the interior and the boundary of $g_i^+$. $(g_1^-, g_2^-)$ and $\partial(g_i^-)$ denote the interior and the boundary of $g_i^-$. Furthermore, all of the possible topological relations between $A_1$ and $A_2$ can be expressed as:

\[
\tilde{\gamma}(A_1, A_2) = <\gamma(g_1^+, g_2^-), \gamma(g_1^-, g_2^-) > U \\
<\gamma(g_1^+, g_2^-), \gamma(g_1^-, g_2^+) > U \\
<\gamma(g_1^+, g_2^+), \gamma(g_1^-, g_2-) > U \\
<\gamma(g_1^-, g_2^+), \gamma(g_1^-, g_2^-) > U \\
\gamma(A_1, A_2)
\]

In Eq. (8), $\gamma(A_1, A_2)$ is the observed topological relation, symbol $<$ denotes all possible relations. For instance, $<\gamma(g_1^+, g_2^-), \gamma(g_1^-, g_2^-)>$ is a set of the topological relations $\gamma(g_1^+, g_2^-), \gamma(g_1^-, g_2^-)$, and some relation $\gamma_i$ that must occurs from $\gamma(g_1^+, g_2^-)$ to $\gamma(g_1^+, g_2^-)$. The relation $\gamma_i$ can be determined according to the conceptual neighborhood graph presented in Section 2.1. A simple example is that, the relation $\gamma_i$ will be ‘meet’ if having $\gamma(g_1^+, g_2^-) = 'overlap' \gamma(g_1^+, g_2^-) = 'disjoint'$. Further, $<\gamma(g_1^+, g_2^-), \gamma(g_1^-, g_2^-)> = \{'overlap', \ 'meet', \ 'disjoint'\}$

Apparently, $\tilde{\gamma}(A_1, A_2)$ is likely to include more than one relation. In spatial analysis, we have to make a decision among all possible relations. It, therefore, is necessary to set up a criterion for such a decision. This issue will be further investigated.

3 Measuring the uncertainty of topological relations under randomness

3.1 Uncertainty of elements in 4ID model

Here, we take Fig. 1 as an example. By use of Eq. (8), all possible topological relations between $A_1$ and $A_2$ include ‘overlap’, ‘meet’ and ‘disjoint’, and their corresponding 4ID models are shown as follows, respectively.

\[
\begin{array}{c}
\begin{array}{c}
-\phi, -\phi \\
-\phi, -\phi \\
-\phi, -\phi
\end{array}
\end{array}
\]

overlap

\[
\begin{array}{c}
\begin{array}{c}
-\phi, -\phi \\
-\phi, -\phi \\
-\phi, -\phi
\end{array}
\end{array}
\]

meet

\[
\begin{array}{c}
\begin{array}{c}
-\phi, -\phi \\
-\phi, -\phi \\
-\phi, -\phi
\end{array}
\end{array}
\]

disjoint

On the basis of the conceptual neighborhood graph, the order of changes for the three topological relations by imposing a continuous deformation is, ‘overlap’ $\rightarrow$ ‘meet’ $\rightarrow$ ‘disjoint’. Correspondingly, only an element changes when each of qualitative changes occurs. By comparison of ‘overlap’ with ‘meet’, only the element...
in left-top corner is different. Thus, a determination of 'overlap' and 'meet' is to compare the possibility that the element in left-top corner takes $-\phi$ and $\phi$. Similarly, the determination of 'meet' and 'disjoint' is to compare the possibility that the element in right-down corner takes $-\phi$ and $\phi$. Below, we will construct a possibility function.

### 3.2 Basic possibility function

On the basis of the second section, we further made the assumption that random uncertainty complies with a circle normal distribution, i.e. $\sigma_x = \sigma_y = \sigma$, $\mu_x = 0$, and the probability density function is as follows,

$$f(x, y) = \frac{1}{2\pi\sigma^2} \exp\left[-\frac{(x-x_p)^2 + (y-y_p)^2}{2\sigma^2}\right]$$

(9)

where parameter $\sigma$ is the standard error of $P(x_p, y_p)$. In GIS databases, an areal object will be represented by an ordered set including a series of points. And positional uncertainty of any group of points will be propagated and affect accuracy of boundary of the area. By Eq. (3), the interpolated point is also of uncertainty, and complies with a normal distribution. At the same time, we can find from above-mentioned investigation that the uncertainty of topological relations between $A_1$ and $A_2$ is caused by uncertain relations between points belonging to $A_1$ and $A_2$, or uncertain relations between points belonging to $A_2$ and $A_1$. That is, such uncertain relations will be converted into uncertain relations between points belonging to $A_1$ and boundary points (including the vertex and the interpolated points) of $A_2$, vice versa.

By use of Eq. (9), the probability that any point falls in the equal density error circle is calculated as

$$P((x, y) \subseteq C_r) = \int_{C_r} \frac{1}{2\pi\sigma^2} \exp\left[-\frac{(x-x_p)^2 + (y-y_p)^2}{2\sigma^2}\right] dx dy = \int_{0}^{\rho} \int_{0}^{2\pi} \frac{1}{\sigma^2} \exp\left(-\frac{\rho^2}{2\sigma^2}\right) d\rho d\theta = 1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

(10)

where $C_r: (x-x_p)^2 + (y-y_p)^2 \leq r^2$, $r$ the radius of error circle. And more, the standard error of point $\sigma_p$ is represented as

$$\sigma_p = \sqrt{\sigma_x^2 + \sigma_y^2} = \sqrt{2}\sigma$$

Therefore, Eq. (10) is simplified into

$$P((x, y) \subseteq C_r) = 1 - \exp\left(-\frac{r^2}{\sigma_p^2}\right)$$

(11)

Further, we may define a quantitative function for determination of two points with a distance $d$ as follows.

$$\theta(\gamma = \text{equal}) = 1 - \lim_{r \to d} P((x, y) \subseteq C_r) = 1 - (1 - \exp\left(-\frac{d^2}{\sigma_p^2}\right)) = \exp\left(-\frac{d^2}{\sigma_p^2}\right)$$

(12)

In terms of Eq. (12), if the distance between two points is zero, then we may make the decision that the relation between two points is 'equal' with the possibility 1. If the distance is large enough (larger than $3\sigma_p$), then they are equal with the possibility 0, that is to say, they are unequal. Therefore, Eq. (12) can be transformed as;

$$\theta(\gamma \neq \text{equal}) = 1 - \exp\left(-\frac{d^2}{\sigma_p^2}\right)$$

(13)

Above definitions are also suitable for the determination of the relations between a point and a line. What is different is that $d$ is the shortest distance from the point to the line. For a point and an area, there are three basic relations, described by 'a point inside an area', 'a point on the boundary of an area', and 'a point outside an area', respectively. We here abbreviate these three relations as 'inside', 'on' and 'outside'. It is easily found that there is such orders as 'inside' $\rightarrow$ 'on' $\rightarrow$ 'outside', or 'outside' $\rightarrow$ 'on' $\rightarrow$ 'inside' when a continuous change occurs in the position of point or area. Therefore, there is a similar definition for the possibility function of $\gamma = \text{on}$ to Eq. (13), i.e.

$$\theta(\gamma = \text{on}) = \exp\left(-\frac{d^2}{\sigma_p^2}\right)$$

(14)

Thus,

$$\theta(\gamma \neq \text{on}) = 1 - \exp\left(-\frac{d^2}{\sigma_p^2}\right)$$

(15)

If the observed relation is 'outside', then to define
\[ \theta(\gamma = \text{outside}) = \theta(\gamma \neq \text{on}) \] (16)

\[ \theta(\gamma = \text{inside}) = \exp\left(-\left(\frac{d + 2a_p}{\sigma_p^2}\right)^2\right) \] (17)

Similarly, if the observed relation is ‘inside’, then to define

\[ \theta(\gamma = \text{inside}) = \theta(\gamma \neq \text{on}) \] (18)

\[ \theta(\gamma = \text{outside}) = \exp\left(-\left(\frac{d + 2a_p}{\sigma_p^2}\right)^2\right) \] (19)

### 3.3 Determining uncertain topological relations

Assuming that the observed topological relations between areal objects under randomness is \( \gamma_i \), which is obtained by reasoning spatial data stored in GIS databases. But it sometimes does not coincide with the real topological relation (\( \gamma \)). In light of basic principle of perturbation, we may think that \( \gamma_i \) is lying in the set that consists of the observation relation and its conceptual neighborhood. By use of Eq. (8), let \( \gamma(A_1, A_2) \) include \( n \) relations denoted by \( \gamma_1, \gamma_2, \cdots, \gamma_n \), let their relative possibility be \( \theta_1, \theta_2, \cdots, \theta_n \), respectively. Thus, we may represent the real topological relations between \( A_1 \) and \( A_2 \) as

\[ \gamma = \theta_1/\gamma_1 + \theta_2/\gamma_2 + \cdots + \theta_n/\gamma_n \] (20)

where \( \theta_j = \theta_j(\gamma = \gamma_i | \gamma_j) \). That is, it is the possibility of \( \gamma = \gamma_i \) in the case of \( \gamma_j \). Apparently, if

\[ \theta_j = \max_j(\theta_1, \theta_2, \cdots, \theta_n) \]

then, we determine the real topological relation between \( A_1 \) and \( A_2 \) as \( \gamma_i \). Below we will propose a new approach to making comparison of \( \theta_i (1 \leq i \leq n) \).

Let

\[ \gamma_i = \begin{bmatrix} a_{i1} & a_{i2} \\ a_{i1} & a_{i2} \end{bmatrix} \] \[ \gamma_j = \begin{bmatrix} a_{j1} & a_{j2} \\ a_{j1} & a_{j2} \end{bmatrix} \]

where, \( a_{km} \) (1 \( \leq k, m \leq 3 \)) are the elements in \( \gamma_i, \gamma_j \), taking values \( \varphi \) or \( \sigma \). And, \( \gamma_j \) differs from \( \gamma_i \) in value of one or more elements in 4ID model. Therefore, we define

\[ \theta_j(\gamma = \gamma_i | \gamma_j) = \frac{\prod_{k,m=1}^{3} \theta_j(a_{km})}{\prod_{k,m=1}^{3} \theta(a_{km})} \] (21)

Hence, \( \theta_j(\gamma = \gamma_i | \gamma_j) \) in Eq. (21) is also called relative possibility of \( \gamma_i \) versus \( \gamma_j \), here \( \gamma_j \) is regarded as the observation relation. \( \theta(a_{km}) \) is the possibility of taking values \( a_{km}, a_{km} \), for the corresponding elements in \( \gamma_i \) and \( \gamma_j \), respectively. They are calculated by use of basic possibility functions such as Eqs. (14)-(19). It is easily found that the relative possibility satisfies the following properties:

- \( 0 < \theta_j < +\infty \)
- \( \theta_j = (\theta_j)^{-1} \),
- For \( j = i \), \( \theta_j = 1 \).

### 4 Examples

In Fig. 5, there are two areal objects from different data layers, labeled by \( A_1 \) and \( A_2 \). For simplicity, we only list part of data in Table 1.

In terms of the algorithms in computation geometry, we can reason that the observed topological relation is ‘\( A_2 \) contains \( A_1 \)’. In Table 1, Part of original data

| Point | \( x/m \) | \( y/m \) | \( \sigma_x/m \) | \( \sigma_y/m \) | \( \rho_{xy} \) |
|-------|--------|--------|-------------|-------------|----------|
| 1 | 468.32 | 798.56 | 15.00 | 15.00 | 0 |
| 2 | 1338.82 | 798.56 | 15.00 | 15.00 | 0 |
| 3 | 834.52 | 810.14 | 15.00 | 15.00 | 0 |

At first, the approach is used for the generation of \( g^+_1, g^+_1, g^+_2 \) and \( g^+_3 \). In terms of 4ID model, the relations among them are respectively described as

\[ \gamma(g^+_1, g^+_2) = \text{‘contains’}, \]
\[ \gamma(g^+_1, g^+_3) = \text{‘contains’}, \]
\[ \gamma(g^+_1, g^+_2) = \text{‘overlap’}, \]
\[ \gamma(g^+_1, g^+_3) = \text{‘contains’}. \]

Therefore, having

\[ <\gamma(g^+_1, g^+_2), \gamma(g^+_1, g^+_3) >= \{\text{‘contains’}\}, \]
\[ <\gamma(g^+_1, g^+_2), \gamma(g^+_1, g^+_3) >= \{\text{‘contains’}, \text{‘covers’}, \text{‘overlap’}\}; \]
\[ <\gamma(g^+_1, g^+_2), \gamma(g^+_1, g^+_3) >= \{\text{‘contains’}, \text{‘covers’}, \text{‘overlap’}; \]
\[ <\gamma(g^+_1, g^+_2), \gamma(g^+_1, g^+_3) >= \{\text{‘contains’}. \]

By use of Eq. (8), all of possible topological relations between \( A_1 \) and \( A_2 \) are expressed as

\[ \gamma(A_1, A_2) = \{\text{‘contains’}, \text{‘covers’}, \text{‘overlap’} \}

Now we need to calculate their relative possibility, that is,

i) \( \theta(\gamma = \text{‘contains’} | \gamma = \text{‘contains’}) \);
ii) \( \theta(\gamma = \text{‘covers’} | \gamma = \text{‘contains’}) \);
iii) \( \theta(\gamma = \text{‘overlap’} | \gamma = \text{‘contains’}). \)
According to Eq. (21), it is obvious that there has
\[ \theta(\gamma = \text{'contains' | 'contains'}) = 1 \]

For ii), we can see from their 4ID model that only an element is different, that is, \( \partial A_1 \cap \partial A_2 \) takes value \( \phi \) for 'contains', while \( -\phi \) for 'covers'. Therefore, we have
\[ \theta(\gamma = \text{'covers' | 'contains'}) = \frac{\theta(\partial A_1 \cap \partial A_2 = -\phi)}{\theta(\partial A_1 \cap \partial A_2 = \phi)} \]

Further, it is equivalent to determine the possibility that points belonging to \( A_1 \) or \( A_2 \) locate on the boundary of \( A_2 \) or \( A_1 \). In Fig. 5, only the point 21 is possibly located on the boundary of \( A_1 \). And we can calculate the possibility by use of Eqs. (14)-(19), i.e.

\[ \theta(\partial A_1 \cap \partial A_2 = -\phi) = \exp(-\frac{d^2}{\sigma_p^2}) = \exp(-0.298) = 0.742 \]
\[ \theta(\partial A_1 \cap \partial A_2 = \phi) = 1 - \exp(-\frac{d^2}{\sigma_p^2}) = 0.258 \]

Thus, having
\[ \theta(\gamma = \text{'covers' | 'contains'}) = 2.876 \]

Here, \( d \) is the distance of point 21 to boundary of \( A_1 \), and equals to 11.58.

For iii), we may calculate the possibility by a similar approach to ii), having
\[ \theta(\gamma = \text{'overlap' | 'contains'}) = \frac{\theta(A_2 - A_1 = -\phi, \partial A_1 \cap \partial A_2 = -\phi)}{\theta(A_2 - A_1 = \phi \text{ or } -\phi, \partial A_1 \cap \partial A_2 = -\phi)} \]

Moreover, \( A_2 - A_1 = -\phi \) means that there exists at least point of \( A_2 \) outside \( A_1 \), while \( A_2 - A_1 = \phi \) means that all of points in \( A_2 \) fall inside \( A_1 \). For Fig. 5, we need only to determine whether point 21 is inside \( A_1 \) or outside \( A_1 \). Their possibility is, respectively,
\[ \theta(A_2 - A_1 = -\phi) = \exp\left(-\frac{(d + 2\sigma_p)^2}{\sigma_p^2}\right) \]
\[ \theta(A_2 - A_1 = \phi) = \exp(-6.482) = 0.0015 \]
\[ \theta(A_2 - A_1 = \phi) = 0.258 \]

Thus, having
\[ \theta(\gamma = \text{'overlap' | 'contains'}) = \frac{0.0015 \times 0.742}{0.258 \times 0.258} = 0.0167 \]

By comparisons of the three relative possibilities with each other, we may determine the topological relation between \( A_1 \) and \( A_2 \) to be 'covers'.

5 Conclusions

In this paper, positional uncertainty is analyzed and modeled statistically. With this model we make a detailed investigation on the effect of positional uncertainty on topological relations. A relative possibility-based approach for the determination of uncertain relations is proposed. A simple example is given for the illustration of the approach presented.

In this approach, the following new aspects are provided:

- Extended application of 4ID model presented in Reference [10], where it is used for formal description of topological relation under certainty, while is now adaptable under uncertainty.
- Basic possibility functions are derived, which is a bridge of positional uncertainty propagating to topological relation uncertainty.
- The approach presented can be simplified for use if epsilon band of probability distribution is taken for modeling location uncertainty of region boundary. And, the approach is also suitable for complex objects.

Further work should be to concentrate on how to process topological inconsistency between neighboring objects. It possibly needs to generate a new common boundary. This paper does not touch this aspect.

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