Modelling Mortality and Discharge of Hospitalized Stroke Patients using a Phase-Type Recovery Model

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Motivation

- Strokes cause severe impediments for those afflicted
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- Quick treatment often decisive in degree of recovery
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- Quick treatment often decisive in degree of recovery
- Modelling patient recovery LOS is needed to limit cost while ensuring adequate provision of health care resources
Strokes are largely grouped into three distinct types:

- **Haemorrhagic strokes** occur when there is bleeding in the brain. These are the most severe, and mortality levels are high.

- **Cerebral Infarctions** occur when there is a clot in a vein. If clot-busting drugs are administered quickly, recovery prospects can be very good.

- **Transient Ischemic Attacks (TIAs)** are the least severe of all, and are often referred to as 'mini-strokes'.
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Relevant Literature on LOS Modelling

- Faddy & McClean (2000) address LOS of geriatric patients.
- Marshall & McClean (2003) introduced idea of conditional PH models for LOS modelling.
- Heterogeneity by such factors as age, type of stroke, etc considered by Marshall & McClean (2004), Faddy & McClean (2000), Harper et al (2012) to explain differences in patient flow characteristics.
### Summary Statistics for Our Dataset

**Table:** Summary by Type of Stroke and Mode of Discharge

| Mode of Discharge | Haemorrhagic | Infarction | TIA |
|-------------------|--------------|------------|-----|
| Death             | 65           | 125        | 13  |
| Nursing Home      | 5            | 59         | 8   |
| Usual Residence   | 69           | 432        | 389 |

| Mode of Discharge | Haemorrhagic | Infarction | TIA |
|-------------------|--------------|------------|-----|
| Death             | 18.3         | 34.6       | 37.5|
| Nursing Home      | 85.5         | 83.7       | 25.8|
| Usual Residence   | 51.3         | 31.9       | 8.2 |
Our Phase-type Model for Stroke Recovery

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- In contrast, Infarctions are rarely ‘severely ill’; for parsimony, we envisaged them as sharing the ‘moderately ill’, and ‘normal recovery’ stages with the Haemorrhagic patients.
Transient Ischemic Attacks (TIAs) are even less severe, and are occasionally never diagnosed. Plots of the data revealed that a hyper-exponential mixture seemed appropriate.

The (relatively) more severe TIAs shared the 'normal recovery' stage with the foregoing groups, while the really short TIAs had an even shorter mean duration.
The Resulting State Transition Diagram

- Haemorrhagic
  - Phase 1
  - Death

- Cerebral Infarction
  - Phase 2
  - Nursing Home

- TIA
  - Phase 3
  - Usual Residence
  - Phase 4

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Parameters Used in our Model

- Transition rates that are independent of age include the mortality rates $\mu_i$, as well as discharge rates $\nu_i$ to nursing home and $\rho_i$ to regular residence; $i = 1, 2, 3$.

- Parameters that depend upon patient age $x$ include the probability $p(x)$ that the TIA recovery starts in stage 4, and the transition rate $\lambda_i(x)$ denotes the rate of transition from state $i$ to $i + 1$ where $i = 1, 2$.

- The probability takes the form $p(x) = e^{-\exp(\theta_0 + \theta_1 x)}$. The transition rate takes the form $\lambda_i(x) = e^{\gamma_i + \beta_i x}; i = 1, 2$. 
A Phase-type Construct That Sheds Insight

Let $T = (t_{ij})$ be a $4 \times 4$ matrix of transition rates among transient states and $T_A = (t_{ij}); \ i = 1, 2, 3, 4; \ j = 5, 6, 7$ be a $4 \times 3$ matrix of absorption rates to the various discharge modes (death, nursing home, and usual residence, resp.). Given an initial distribution of recovery phases $\alpha$, one finds

$$f_X(x | \alpha, T, T_A) = \alpha' \exp(T x) T_A 1_3, \ x \geq 0.$$  \hfill (1)

The $4 \times 3$ matrix $P = (-T)^{-1}T_A$ can be interpreted as the probability of absorption into the various discharge modes (death, nursing home, or regular residence), for each of the recovery phases.
Theoretical Constructs for Parameter Estimation

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- We start by considering only the TIA patients to determine initial estimates of the final stages.
- We then add the Infarction patients to the mix, and re-estimate the final-stage parameters while gaining initial estimates for the 'moderately ill' stage.
- We finally add the Haemorrhagic patients to the mix, and re-estimate all the foregoing parameters as well as for the 'seriously ill' stage.
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### Parameter Estimates

| Parameter | Estimate  | Std Error | Z-Stat   | p-value |
|-----------|-----------|-----------|----------|---------|
| $\gamma_1$ | 6.63570   | 1.21893   | 5.44388  | 0.00000 |
| $\beta_1$  | -0.03652  | 0.01631   | -2.23902 | 0.02515 |
| $\gamma_2$ | -3.06931  | 1.22697   | -2.50153 | 0.01237 |
| $\beta_2$  | 0.07153   | 0.01667   | 4.29057  | 0.00002 |
| $\theta_0$ | -8.66118  | 1.48644   | -5.82680 | 0.00000 |
| $\theta_1$ | 0.08801   | 0.01828   | 4.81391  | 0.00000 |
| $\mu_1$   | 22.10156  | 4.95434   | 4.46105  | 0.00001 |
| $\mu_2$   | 2.48820   | 0.37993   | 6.54912  | 0.00000 |
| $\mu_3$   | 1.56162   | 0.20294   | 7.69509  | 0.00000 |
| $\nu_3$   | 1.27849   | 0.17391   | 7.35165  | 0.00000 |
| $\rho_2$  | 11.76860  | 0.99634   | 11.81180 | 0.00000 |
| $\rho_3$  | 3.41989   | 0.38393   | 8.90762  | 0.00000 |
| $\rho_4$  | 63.92514  | 4.11394   | 15.53865 | 0.00000 |
## Ultimate Destination Percentage by Age and Type of Stroke

|                | Age 65          | Age 85          |
|----------------|-----------------|-----------------|
|                | Death | Nursing Home | Usual Residence | Death | Nursing Home | Usual Residence |
| Haemorrhagic   | 38.5   | 4.0           | 57.5            | 52.5   | 7.3           | 40.1            |
| Cerebral Infarction | 19.4   | 5.2           | 75.5            | 21.9   | 12.0          | 66.1            |
| TIA complex    | 24.9   | 20.4          | 54.6            | 24.9   | 20.4          | 54.6            |
| TIA simple     | 0      | 0             | 100.0           | 0      | 0             | 100.0           |
Cumulative probability of discharge by type of stroke and destination

Haemorrhagic

Cerebral Infarction

TIA