Effective gauge group of pure loop quantum gravity is $SO(3)$: New estimate of the Immirzi parameter

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We argue that the effective gauge group for pure four-dimensional loop quantum gravity (LQG) is $SO(3)$ (or $SO(3, C)$) instead of $SU(2)$ (or $SL(2, C)$). As a result, links with half-integer spins in spin network states are not realized for pure LQG, implying a modification of the spectra of area and volume operators. Our observations imply a new value of $\gamma \approx 0.170$ for the Immirzi parameter which is obtained from matching the Bekenstein-Hawking entropy to the number of states from LQG calculations. Moreover, even if the dominant contribution to the entropy is not assumed to come from configurations with the minimum spins, the results of both pure LQG and the supersymmetric extension of LQG can be made compatible when only integer spins are realized for the former, while the latter also contains half-integer spins, together with an Immirzi parameter for the supersymmetric case which is twice the value of the $SO(3)$ theory. We also verify that the $-\frac{1}{2}$ coefficient of logarithmic correction to the Bekenstein-Hawking entropy formula is robust, independent of whether only integer, or also half-integer spins, are realized.

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I. INTRODUCTION

The simplification of the constraints and introduction of gauge variables, and subsequently loop variables, and spin network states, have brought excitement to the non-perturbative and background-independent program of canonical quantization of four-dimensional gravity. This program is often referred to as “loop quantum gravity” (LQG). The construction of loop and spin network states have moreover proved fruitful, and have yielded discrete spectra for the well-defined area and volume operators. In the literature, it is common to encounter the alternative use of spinorial variables, and to identify the gauge group of the theory as $SU(2)$ (or its complexification $SL(2, C)$), although $SO(3)$ (respectively $SO(3, C)$) have the same $su(2)$ Lie algebra. This is also manifest in the construction of spin networks and the spectra of area and volume operators. Explicitly, links in spin networks are usually labeled with representations of $SU(2)$ with integer and half-integer spins; consequently the area operator has eigenvalues $A(j) = 8\pi\gamma l_p^2 \sqrt{j(j+1)}$, where $l_p$ is the Planck length and $\gamma$ is the Immirzi parameter which reflects a freedom of choice in the theory. However, when the Immirzi parameter is fixed by comparing, as proposed in Ref. [4], the LQG results for the Bekenstein-Hawking entropy and the area spectrum to the corresponding quasi-normal mode calculations of a Schwarzschild black hole with the assumption that states with the minimum spin are dominant, this matching yields $j_{min} = 1$, instead of the expected $j_{min} = 1/2$ configurations, indirectly hinting that the gauge group should be $SO(3)$ instead. In the supersymmetric extension of LQG, the same procedure gave $j_{min} = 1/2$ instead and the value of $\gamma$ which is twice its value for pure LQG. More recent analyses with proper counting of states in Refs. [1, 7, 8, 9] have revealed that the dominance of minimum spin configurations is both questionable and underestimates the value of the Immirzi parameter. We shall discuss in Section IV the related computations and modifications in black hole physics in the light of these recent analyses and our conclusions, and shall also confirm that the coefficient of the area-dependent logarithmic correction to the Bekenstein-Hawking entropy formula is $-\frac{1}{2}$ and is independent of the gauge group.

We shall show that the effective gauge group of pure LQG without fermions is indeed $SO(3)$ (or its complexification), and is correspondingly lifted to its covering group $SU(2)$ (or its complexification) when fermions are present. Our arguments are based upon 1) retracing the steps which lead to the basic variables, 2) the fact that a gauge connection transforms according to the adjoint representation, and 3) the criterion that the effective gauge group of a theory is determined by its full physical contents. These observations are rather elementary, but they impact upon basic calculations in LQG and quantum geometry with spin network states. The Hilbert space of quantum gravity without fermionic matter should allow spin network states with links of integer representations only, and links with half-integer representations do not occur in pure LQG. We would like to stress that our conclusions on the relevant gauge groups are neither dependent upon, nor confined to, black hole situations.

II. BASIC CONJUGATE VARIABLES

We first retrace the steps which lead to the basic variables. In four, and only four dimensions, the Lorentz
algebra can be decomposed into two $su(2)$ (or $so(3)$) Lie algebras, with self- and anti-self-dual generators. It follows that finite-dimensional irreducible fields can be labeled by $(j_+, j_-)$ and transform according to $(2j_+ + 1) \times (2j_- + 1)$ representation wherein $j_\pm$ take positive integer or half-integer values. Self- and anti-self-dual two-forms, $\Sigma^\pm$, constructed from vierbein one-forms $e_A = e_{A\mu} dx^\mu$ through $\Sigma^\pm = \pm e_a \wedge e^b \Sigma^\pm_{ab}$, and the curvature, $F^\pm = \pm e_a \wedge F_a^\pm \pm e^b \wedge F_b^\pm$, of the self- and anti-self-dual spin connections, $A^\pm = \pm i \omega_0 + \pm \frac{1}{2} e_0 \wedge \omega_0$, are $(1,0)$ (respectively $(0,1)$) fields.

The above discussions highlight that regardless of whether one can easily convert anti-self-dual $SO(3)$ indices to $SU(2)$ primed spinor indices by contracting with Pauli matrices $\tau^a A^\sigma_a$. The dimension of the representation spaces is dependent upon the global structure of the gauge group; and one needs to examine the full physical contents of the theory to determine the actual gauge group. This latter point of view has been eloquently advocated, and explicitly illustrated, in Ref. [19] (for Yang-Mills gauge fields, see, in particular Section 1.4 of [10]). To wit, we should be reminded that a Lie algebra valued connection 1-form always transforms according to the adjoint representation of the group as $A' = g A g^{-1} + i g d g^{-1}$; $g \in G$. The question is whether all $g \in G$ can be regarded as distinct elements of the group of transformations. Since the center of the group, $C$, commutes with all elements of the Lie algebra, it follows that, $g$ and $g$ multiplied by any element of the center, has the same effect. Thus as far as gauge potentials are concerned, the effective gauge group is not $G$ but $G/C$. If the theory contains only gauge potentials, the gauge group of the theory is therefore $G/C$. For instance in $SU(N)$ pure Yang-Mills theory, the gauge group is not $SU(N)$ but $SU(N)/Z_N$.

III. EFFECTIVE GAUGE GROUP OF LQG

The above discussions highlight that regardless of which one of the previous canonical formulations is employed, the gauge group is $SO(3)$, either the real group of spatial rotations, or its complexification, the full Lorentz group $SO(3,C)$. It is not necessary to invoke spinorial variables to arrive at the fundamental variables, although one can easily convert anti-self-dual $SO(3)$ indices to $SU(2)$ primed spinor indices by contracting with Pauli matrices $\tau^a A^\sigma_a$. The dimension of the representation spaces is dependent upon the global structure of the gauge group; and one needs to examine the full physical contents of the theory to determine the actual gauge group. This latter point of view has been eloquently advocated, and explicitly illustrated, in Ref. [19] (for Yang-Mills gauge fields, see, in particular Section 1.4 of [10]). To wit, we should be reminded that a Lie algebra valued connection 1-form always transforms according to the adjoint representation of the group as $A' = g A g^{-1} + i g d g^{-1}$; $g \in G$. The question is whether all $g \in G$ can be regarded as distinct elements of the group of transformations. Since the center of the group, $C$, commutes with all elements of the Lie algebra, it follows that, $g$ and $g$ multiplied by any element of the center, has the same effect. Thus as far as gauge potentials are concerned, the effective gauge group is not $G$ but $G/C$. If the theory contains only gauge potentials, the gauge group of the theory is therefore $G/C$. For instance in $SU(N)$ pure Yang-Mills theory, the gauge group is not $SU(N)$ but $SU(N)/Z_N$.

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under gauge transformations. They are instead chosen to carry the right combination of Wigner symbols to ensure gauge invariance under transformations of the holonomy link elements. Bearing in mind the transformation properties of the non-integrable phase factor, it follows that the gauge group of the quantum theory is still effectively $SO(3)$ or $SO(3,C)$ in the absence of fermions.

The configuration space is the space of $SO(3)$ gauge connections modulo the action of $SO(3)$ gauge group, and it is faithfully parametrized by holonomy elements of $SO(3)$ connections with integer spin representations, rather than $SU(2)$ holonomies which include half-integer spin representations. If half-integer representations are also allowed, the subsequent “doubling” of the spectrum of the composite area operator may be an artifact of the quantization procedure. There can furthermore be obstructions to the lifting of the $SO(3)$ gauge group to its covering space. This is perhaps not too serious from the 3-dimensional perspective since every orientable 3-dimensional manifold is a spin manifold. Unlike the $SO(3,1)$ Ashtekar-Sen connection, the Barbero-Immirzi connection cannot be regarded as the pullback of a connection in four dimensions$^{[20]}$. Thus when restricted to orientable 3-manifolds, $SU(2)$ Barbero-Immirzi connections with non-(anti)self-dual value of $\gamma \neq \pm i$ may appear to be consistent, but it is not entirely clear if other contradictions will arise from 4-dimensional considerations. In 4-dimensions there exists numerous manifolds which are not spin manifolds and explicit Einstein manifolds which do not permit spin structures are known. On the other hand, it can perhaps also be argued that usual concepts regarding manifolds need not apply to a theory of quantum geometry. In the absence of further consistency checks and empirical evidence, the possibility that half-integer spins are not realized in pure LQG should not be dismissed; and it is also more straightforward to adhere to the original configuration space and consider only $SO(3)$ connections and holonomies.

**IV. NEW ESTIMATE OF THE IMMIRZI PARAMETER**

Assuming the statistical dominance of configurations with the minimum spin, Dreyer argued that quasi-normal excitations should be related to the appearance of punctures with spin $j_{\text{min}}$. In order for the Bekenstein-Hawking entropy formula and the quasi-normal mode result to agree on the same answer for the Immirzi parameter, $\gamma = \ln 3/(2\pi\sqrt{2})$, he inferred that $j_{\text{min}} = 1$, thereby supporting the case for $SO(3)$$^{[2]}$. However, Dreyer’s arguments in favor of $SO(3)$ has lost their cogency in view of the more careful counting of states carried out in Refs.$^{[2,5,8,9]}$. We shall also demonstrate it is possible to reconcile $SO(3)$ with the Bekenstein-Hawking entropy matching, even if the conjecture of dominance of minimum spin configurations and its association with quasi-normal mode excitations are in error.

Domagala and Lewandowski$^{[8]}$ showed that the configurations should be governed by sequences labelled by

$$\sum_{i} \sqrt{|m_i|(|m_i|+1)} \leq a \equiv \frac{A}{8\pi\gamma_{\ell_p}^2}, \quad \sum_{i} m_i = 0; \quad (2)$$

with $m_i \in -j_1,...,j_1 \in \mathbb{N}/2$. The correspondence to quasi-normal modes is thus not straightforward. Meissner demonstrated that the number of states, $N(a)$, for a given area is therefore given by $N(a) \propto \frac{C_M}{\sqrt{2\pi\gamma M}} e^{2\pi\gamma M a}$ and the black hole entropy is consequently$^{[8]}$

$$S = \ln N(a) = \left(\frac{\gamma M}{\gamma}\right) A_{\ell_p} - \frac{1}{2} \ln(A_{\ell_p}^2) + \ln \frac{C_M}{\sqrt{2\pi\gamma M}}. \quad (3)$$

By matching this result to the Bekenstein-Hawking entropy formula for large black holes, $\gamma_M = \gamma$ is obtained. Moreover, the numerical value of $\gamma_M$ can be retrieved from the recursion formula for $N(a)$ which in the large black hole limit is $N(a) = \sum_{k=1}^{\infty} 2N(a - \sqrt{k(k+1)/4})$. For large black holes, the ansatz $N(a) \propto e^{2\pi\gamma M a}$ therefore yields $1 = \sum_{k=1}^{\infty} 2e^{-2\pi\gamma} \sqrt{k(k+1)/4}$, from which the numerical result $\gamma \approx 0.238$ is obtained for the $SU(2)$ theory$^{[8]}$.

Ghosh and Mitra$^{[8]}$ treat the punctures as distinct and count both $j$ and $m$ instead of only surface degrees of freedom. This yields the formula $S = \sum_{j \in \mathbb{N}/2} 2[(2j+1)/2] e^{-2\pi\gamma} \sqrt{j(j+1)} = 1$, together with an estimate for the number of states as $\ln N(A) \approx \ln((e^{-\pi\gamma})(A/\ell_p^2)^{-\frac{1}{2}})$. The symbol $[\ldots]$ in the sum denotes the integer part which arises from enforcing $m_i \neq 0$ in the degeneracy factor$^{[8]}$. Thus the Immirzi parameter is estimated from the preceding series as $\gamma \approx 0.262$ for the $SU(2)$ gauge group$^{[8]}$.

Our conclusions in the previous section is that the gauge group of pure LQG is $SO(3)$ instead of $SU(2)$. The analyses of Refs.$^{[8,5,8,9]}$ remain essentially valid, but the area spectrum should be confined to integer $j$ values, and $m_i$ should accordingly be restricted to $m_i \in \mathbb{N}$ instead. These adjustments imply that in the recursion relation for $\gamma_M$ and elsewhere in Ref.$^{[8]}$, the values of $k$ should be restricted to even integers. The expression for the number of states $N(a) = \frac{C_M}{\sqrt{4\pi\beta M}} e^{2\pi\gamma M a}$ remains the same, but the numerical value of $\gamma_M$ is now given by $1 = \sum_{k \in \mathbb{N}} 2e^{-2\pi\gamma} \sqrt{k(k+1)}$, giving $\gamma_{SO(3)} = \gamma_M \approx 0.138$ for the $SO(3)$ theory. Similarly, the improved value of $\gamma$ following Refs.$^{[8,9]}$ should now be obtained from $\sum_{j \in \mathbb{N}} 2(2j+1)/2 e^{-2\pi\gamma} \sqrt{j(j+1)} = 1$. These considerations yield the new value of $\gamma_{SO(3)} \approx 0.170$. The form of the expression for $N(a)$ and the dependence on the $\ln A$ term in $\ln N(a)$ are unaffected by whether only integer, or half-integer spins are also allowed, so the coefficient of logarithmic correction to the Bekenstein-Hawking formula is thus robust and remains $-1/2$.

What happens when fermions are incorporated into the theory? From the point of view of the classical
fundamental field variables, although the effective group of the gauge connection remains $SO(3)$, fermions are not invariant under the non-trivial element of the center (which corresponds to $2\pi$ rotations) of $SU(2)$, so now $SU(2)$ (or its complexification $SL(2,C)$) becomes the effective gauge group of the full physical contents of the theory. Analogously, the presence of quarks in Quantum Chromodynamics lifts the physical group from $SU(3)/Z_3$ for the theory of pure gluons to the full $SU(3)$ \[1\]. With fermions, half-integer-spin representations can be effectively realized in spin network states, as the wave functions are now $(A,\psi|\Psi)$. An example of a spin network with fermionic degrees of freedom and $SU(2)$ effective gauge group is one with vertices such that every link with $j = 1/2$ has a spin $1/2$ fermion at one of its ends and a vertex (now with $j = 1/2$ allowed) with the correct Wigner symbols at the other; and links with $j \in \mathbb{N}$ are joint only to other vertices at both ends.

The relationship between black hole entropy based upon loop quantization of $N = 1$ supergravity and quasi-normal mode excitations based upon the minimum spin contribution was studied in Ref.\[3\]. In the presence of supergauge fields with bosonic and fermionic degrees of freedom, the effective gauge group is lifted to the covering group. The area spectrum for a link of spin $j$ has been calculated to be $A_{SU GRA}(\tilde{j}) = 8\pi j^2 e^{\gamma_l j} \sum_{e^{\gamma_l j}} (j(\frac{1}{2} + j)) = 8\pi (\frac{\tilde{j}}{2})^2 e^{2\gamma_l \tilde{j}}$, wherein the non-trivial values are for $j \in \mathbb{N}/2$, or $j \in \mathbb{N}$; and the degeneracy of states of a puncture of spin $\tilde{j}$ is $(4\tilde{j} + 1)$. Thus following the analysis of Refs.\[3, 4\], the Immirzi parameter now obeys the modified equation

\[
1 = \sum_{\tilde{j} \in \mathbb{N}/2} 2(4\tilde{j} + 1)/2 e^{-2\pi \gamma \sqrt{\tilde{j}(\tilde{j} + 1)}} = \sum_{j \in \mathbb{N}} 2(2j + 1)/2 e^{-2\pi \gamma (2j+1)/2} \sqrt{j(j+1)}.
\]

This indicates that the result for the supersymmetric case will be the same as for the case of $SO(3)$, but with the Immirzi parameter $\gamma$ for $N = 1$ supergravity theory replaced by $\gamma = 2\gamma_{SO(3)} \approx 2(0,170)$, despite the fact that for the supersymmetric case half-integer spins are also allowed while for pure LQG only integer spins are realized. Note that this relation between the Immirzi parameters produces exactly the same area spectrum for both pure LQG without supersymmetry and its supersymmetric extension. It also acts as an intriguing consistency condition, since results for both cases were calculated and compared for the same black hole masses and thus same horizon areas of all sizes.

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