Mathematical model of two-phase flow processes in heat exchange tubes of the falling film evaporator

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Abstract. In this work, a mathematical model and a method for calculating the film evaporation apparatus were proposed. The mathematical model provides data on the distribution of hydrodynamic parameters of a two-phase flow along the length of the heat exchange tubes of the evaporator. The obtained data are necessary for the design of an efficient falling film evaporator.

1. Introduction
Vertical falling film evaporators are widely used in various industries for concentrating solutions, including thermolabile ones. The studies of the film hydrodynamics of the irrigation liquid are presented in a significant number of works [1-4]. The theoretical description of the heat transfer to the film of the irrigation liquid, when it is heated or cooled in a turbulent flow region, is difficult due to the complexity of this flow, which is accompanied by three-dimensional characteristic waves on the film surface [2, 3]. Therefore, most of the works are based on experimental research [5-7].

The aim of this work is designing a mathematical model of the processes occurring in a two-phase flow when it moves in the heat exchange tubes of falling film evaporators.

Proposed mathematical model will provide data on the distribution of hydrodynamic parameters of the two-phase flow along the length of the heat exchange tubes of the falling film evaporator. These data are necessary for the design of an efficient falling film evaporator, which has a lower metal content and high efficiency heat transfer. Based on the proposed mathematical model of a two-phase flow, a method for calculating the falling film evaporator is developed.

2. Modelling
The proposed mathematical model is a separate flow model. It is assumed that the phases move separately and the interaction between them occurs at the phase interface.

The mathematical model is semi-empirical. The difference scheme of the model assumes the solution of simple algebraic equations without using the method of finite approximations, i.e., the proposed model is not resource-intensive and can be implemented quite simply if automated calculations are necessary.

The movement of two-phase flows in the heat exchange tube of the falling film evaporator is shown in Figure 1. On the i-th section of the pipe, the length is Δl_i = l_i m; the heat transfer surface is F_i, m²; and the absolute pressure at the beginning of the i-th section is p_i, Pa. The physical properties of water and secondary steam at the beginning of the section have values corresponding to the values on the saturation line at absolute pressure p_i. The properties of water in the i-th section, which are necessary for further calculation, include: mass fraction of dry substances x_{c_i}; boiling point of the solution t_{i1}, °C; the density of the solution ρ_{i1}, kg/m³; the kinematic viscosity of the fluid ν_{i1}, m²/s.
The volume flow rates of the solution and secondary steam at the end of the i-th section under consideration are calculated according to the formulas:

\[ V_{1i} = \frac{G_{1i}}{\rho_{1i}}, \quad (1) \]
\[ V_{2i} = \frac{G_{2i}}{\rho_{2i}}, \quad (2) \]

where \( G_{1i}, G_{2i} \) – flow rates of liquid and secondary steam, respectively, kg/s.

Irrigation density at the i-th step:

\[ \Gamma_i = \frac{V_{1i}}{\Pi}, \quad (3) \]

where \( \Pi \) – wetted perimeter of the heat exchange tube of evaporator, m; \( \Pi = \pi d \), where \( d \) – the inner diameter of the tube, m.

The film thickness of the solution can be determined by empirical relationships [8]:

\[ \delta_i = \begin{cases} 3^{\frac{3v_{1i}^2}{4g}} Re_{1i}^{1/3}, & Re_{1i} < 1600 \\ 0.25 \sqrt[3]{\frac{1.2v_{1i}^2}{g}} Re_{1i}^{0.5}, & Re_{1i} \geq 1600 \end{cases} \quad (4) \]

\[ Re_{1i} = \frac{4\Gamma_i}{\nu_{1i}}, \quad (5) \]

where \( Re_{1i} \) – the Reynolds number for solution film.

Knowing the volume flow rate of the solution and the thickness of the solution film at the i-th step of the solution, it is possible to determine the film flow rate:

\[ W_{1i} = \frac{4V_{1i}}{\pi d^2 - \pi (d - 2\delta)^2} \quad (6) \]

The steam velocity is calculated similarly:

\[ W_{2i} = \frac{4V_{2i}}{\pi d^2} \quad (7) \]

The coefficients of friction of the solution film on the tube wall and the secondary vapour on the solution film are calculated according to the following dependences:

\[ \lambda_{1i} = \begin{cases} \frac{24}{Re_{1i}}, & Re_{1i} < 1600 \\ \frac{0.6}{\sqrt{Re_{1i}}}, & Re_{1i} \geq 1600 \end{cases} \quad (8) \]
\[ \lambda_{2i} = \begin{cases} \frac{64}{Re_{(1-2)_{i}}}, & Re_{(1-2)_{i}} < 2300 \\ 0.316 & Re_{(1-2)_{i}} \geq 2300 \end{cases} \] (9)

\[ Re_{(1-2)_{i}} = \frac{w_{(1-2)_{i}}^2 \rho_{2i}}{\mu_{2i}} \] (10)

where \( Re_{(1-2)_{i}} \) – the Reynolds number for the relative velocity of steam.

The masses of the film of solution and vapour simultaneously located on the \( i \)-th section can be calculated using the formulas:

\[ m_{1i} = \rho_{1i} l_{i} \frac{\pi d^2 - \pi (d - 2\delta)^2}{4}, \] (11)

\[ m_{2i} = \rho_{2i} l_{i} \frac{\pi d^2}{4} \] (12)

The absolute pressure at the end of the \( i \)-th section and at the beginning of the \( i+1 \)-th section, considering hydrodynamic losses, is:

\[ p_{i+1} = p_{i} - \Delta p_{i} \] (13)

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**Figure 2.** Algorithm of the main cycle of the mathematical model.
Pressure reduction in the section $\Delta p_i$, Pa, occurs due to pressure losses as a result of changes in the flow rate of the film and steam, overcoming local resistances and friction of the film against the tube walls and steam against the film surface.

There are no local resistances over most of the length of the heat exchange tube of evaporator. The main local resistances along the course of the film and solution movement are: resistances at the entrance to the tube from the solution chamber and resistances at the exit from the tube.

Special attention should be paid to the initial section at the entrance to the evaporator tube from the solution chamber. In the solution chamber of the evaporation apparatus, the solution is fed superheated, with a temperature exceeding the saturation temperature in the solution chamber by 5-10°C.

Getting into the solution chamber, the solution equalizes its pressure with the pressure of the surrounding space. The thermal effect of the enthalpy change is spent on self-evaporation of the solution. In this regard, the mass fraction of secondary steam is not equal to zero in the initial section of the tube.

3. Results
Based on the proposed mathematical model, a method for calculating falling film evaporator was derived, which allowed designing this evaporator with a low metal content and the high heat transfer coefficient. The main cycle of the proposed mathematical model is shown in Figure 2.

4. Conclusion
In this paper, we developed the mathematical model and the algorithm for calculating the processes occurring in the heat exchange tubes of falling film evaporator. The method for calculating the evaporation apparatus is proposed.

The proposed mathematical model is a system of equations that reflect the laws of conservation of the amount of motion, heat energy, and continuity of the two-phase flow in the heat exchange tube. The structure of the proposed model allows using it for computational design of falling film evaporator.

This model is necessary for the construction of the efficient falling film evaporator with low metal consumption and high heat transfer.

References
[1] Aursanda E 2019 *Int. J. Multiph. Flow* **116** 67
[2] Wang X 2019 *Chem. Eng. Res. Des.* **154** 303
[3] Semenov P A 1944 *Tech. Phys.* **14** 427 (in Russian)
[4] Kapica P L 1948 *J. Exper. Tech. Phys.* **18** 3 (in Russian)
[5] Leverash V I 1969 *Therm. Eng.* **3** 86 (in Russian)
[6] Marcenyuk A S 1981 *Film Heat and Mass Transfer Devices in the Food Industry* (Moscow: Light and Food Industry) (in Russian)
[7] Lazarev V D 1968 *Dairy Industry* **6** 20 (in Russian)
[8] Voroncov V G 1972 *Heat Transfer in Liquid Films* (Kiev: Technique) (in Russian)