Regarding the Distribution of Glue in the Pion

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Understanding why the scale of emergent hadron mass is obvious in the proton but hidden in the pion may rest on mapping the distribution functions (DFs) of all partons within the pion and comparing them with those in the proton; and since glue provides binding in quantum chromodynamics, the glue DF could play a special role. Producing reliable predictions for the proton’s DFs is difficult because the proton is a three-valence-body bound-state problem. As sketched herein, the situation for the pion, a two-valence-body problem, is much better, with continuum and lattice predictions for the valence-quark and glue DFs in agreement. This beginning of theory alignment is timely because experimental facilities now either in operation or planning promise to realize the longstanding goal of providing pion targets, thereby enabling precision experimental tests of rigorous theory predictions concerning Nature’s most fundamental Nambu–Goldstone bosons.

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1. Introduction. The proton was discovered in 1917.1 It is stable: looking back through the approximately 14-billion year history of the Universe, no proton in isolation has ever been seen to decay. This is one of the most profound features of Nature; and for science it means, inter alia, that protons can readily be used as targets or accelerated as probes for the exploration of other materials.2 These properties have been very profitably exploited, enabling the discovery of quarks3–5 and fostering the development of quantum chromodynamics (QCD) as the theory of strong interactions.6 When discussing proton structure, parton distribution functions (DFs) are often the focus.7 Each DF, \( p(x; \xi) \), describes the number density of a given parton type, \( p = \) valence-quark (\( q \)), sea-quark (\( s \)), or glue (\( g \)), within the proton as a function of the light-front fraction, \( x \in (0, 1) \), of the proton’s momentum that it carries at a resolving scale, \( \xi \). As with many quantities in quantum field theory, like the coupling and masses of elementary fields, these densities depend on the energy scale with which they are probed. Crucially, under certain kinematic conditions7 such as those prevailing in deep inelastic scattering (DIS) or Drell–Yan (DY) reactions8,9 DFs can be extracted from the data acquired.

Since the first empirical DIS studies, a prodigious number of experiments has been completed and analyzed with this purpose.10–13 The current status may be described as follows. (i) The wealth of data has enabled development of numerous QCD-related proton DF fits, which are in fair agreement amongst themselves on the domains favored by dense data. On the complementary domains, the fits can disagree markedly, with significant physics impact, such as uncertainty in the value of the large-\( x \) exponent on the proton’s valence-quark distributions.14–17 (ii) With phenomenology thus positioned, there is great need for rigorous QCD-connected theory input; but even after roughly fifty years of QCD, this is lacking. Many models have been used to compute valence-quark DFs, but similarities and disagreements speak primarily about the practitioners’ assumptions; few predictions are available from realistic solutions of the continuum three-valence-body bound-state problem in QCD,18,19 results from lattice-regularized QCD (lQCD) are not yet of sufficient precision to materially contribute to the improvement of DF fitting;20 and there are no calculations of the proton’s glue distributions. Plainly, even in the optimal case supplied by the proton, one cannot claim understanding of its DFs.

Whereas the proton is defined by its valence \( u + d \) quark content, mesons are a different form of hadron matter, being constituted from a valence-quark and -antiquark. The pion has been a known hadron for more than seventy years.21,22 Like all mesons, pions are unstable; and while the charged states, \( \pi^+ = ud \) (\( u \)-quark +\( d \)-antiquark) and \( \pi^- = d\bar{u} \), decay only via weak interactions, they do not live long enough to serve readily as targets. Hence, regarding parton DFs within the pion, the empirical situation is dire.

Information on the pion’s valence-quark DF was obtained in a series of pion-beam experiments at CERN23–25 and Fermilab (E615)26 more than thirty years ago and also using the Sullivan process27 (scattering from the proton’s pion cloud) at DESY twenty years ago.28,29 Phenomenological fits of these data30–34 yield DFs with notable mutual dissimilarities35 and, excepting Ref. 30, possessing large-\( x \) behavior in conflict with predictions derived...
from QCD. Specifically [Ref. [36], Sec. 5A]; at any resolving scale, $\zeta$, for which data may be interpreted in terms of DFs, the pion’s valence-quark DF is predicted to exhibit the following behavior:

$$q^\pi(x; \zeta_H) \propto x^2 \alpha(x) (1 - x)^{3\pi(\zeta)}$$ (1)

$\alpha(x)$ is a constant, i.e., independent of $x$. Contradicting this, the $x \approx 1$ behavior of the phenomenological DFs in Refs. [31–34] corresponds to $\beta_\pi(\zeta) \approx 1$.

A key difference between the study of proton and pion DFs is that QCD theory has made real progress with the pion during the past decade. Preceding this period, numerous models had been used to compute the pion’s valence-quark DF [10] with the outcome depending on the model chosen. Today, a unified understanding has been provided for the pion’s glue distribution [42,43,57,59–65] even for the pion’s glue distribution.

Today, a unified understanding has been provided for the pion’s valence-quark DF: $\varphi_\pi(x)$ is a concave function, which is simultaneously dilated and reduced in maximum magnitude relative to its asymptotic profile $\varphi_\pi(x) = 6\log(1 - x)$. [66–68] Both features owe to the phenomenon of emergent hadron mass, [69] the likely explanation for > 98% of the visible mass in the Universe.

Important, and distinct from modeling and data fitting, the hadron (initial) scale is not introduced as a parameter in Ref. [43]. Instead, the value of $\zeta_H$ is a prediction derived from the behavior of QCD’s process-independent (PI) effective charge. [72,73] QCD’s PI charge is accurately interpolated by the following function:

$$\hat{q}^\pi(x; \zeta_H) = 375.32 x^2 (1 - x)^2 \times \left[1 - 2.5088 \sqrt{x(1-x) + 2.0250 x(1-x)}\right]^2.$$ (4)

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2 GeV; and one may obtain the DFs at \( \zeta_2 \) via evolution from another scale by integrating the DGLAP equations.\(^{[79–82]}\) When completing this exercise, a prescription must be specified because the standard DGLAP equations involve QCD’s running coupling. In modeling and fitting, it is usual to adopt a purely perturbative-QCD perspective, implementing evolution with a DGLAP kernel calculated at a given order in perturbation theory. If the scale at which evolution begins is large enough, then leading-order (LO) evolution kernels may be adequate, at least in practice. If they fail, then next-to-leading-order (NLO) can be implemented, and so on, in principle.

Evidently, gluon partons carry a large fraction of the pion’s momentum at \( \zeta_2 \) due to leading logarithmic effects in the hard-scattering kernel; and one may obtain the DFs at \( \zeta_2 \) by integrating the one-loop DGLAP equations.\(^{[84]}\) Namely, adapting ideas from Refs. \([74,85,86]\), evolution in perturbation theory is implemented by using \( \hat{\alpha}(k^2) \) in Eq. (5) to integrate the one-loop DGLAP equations. This \( \hat{\alpha} \) scheme eliminates ambiguity from the resulting predictions because it renders moot any questions regarding the order of the evolution kernels. Working with the hadron scale DFs described above, one obtains the parameter-free predictions for the \( \zeta_2 = 2 \) GeV valence, sea, and glue DFs drawn in Figs. 1–5. Using those predictions, one calculates the following light-front momentum fractions:

\[
\begin{align*}
\langle x q^\pi_+ \rangle &= 0.24(2) \\
\langle x^2 q^\pi_+ \rangle &= 0.094(13) \\
\langle x^4 q^\pi_+ \rangle &= 0.047(08),
\end{align*}
\]

Larger values for the \( n \geq 2 \) moments indicate a harder DF, i.e., a function with greater support on the valence-quark domain than the prediction in Ref. \([43]\); consequently, less support on the complementary domain.

It is worth continuing with a discussion of the pointwise behavior of the pion’s valence-quark distribution, Fig. 1. Within uncertainties, the parameter-free prediction from Ref. \([43]\) agrees with the IQCD result in Ref. \([83]\); and both agree well on the valence-quark domain with the NLO analysis of E615 data\(^{[26]}\) described in Ref. \([30]\), which is only work thus far to include next-to-leading-logarithmic (NLL) threshold resummation effects when calculating the hard-scattering kernel. (N.B. Ref. \([30]\) used sea and glue distributions from Ref. \([32]\); hence, the valence distribution in Ref. \([30]\) is likely less reliable on \( x \lesssim 0.4 \).) Interestingly, the result in Ref. \([84]\), which reignited debate over the pion’s valence-quark DF and was obtained using algebraic Ansätze for the elements in the pion wave function, also agrees with the modern CSM prediction. The continuum prediction\(^{[41]}\), IQCD result,\(^{[83]}\) NLO+NLL fit\(^{[30]}\) and QCD-based model\(^{[84]}\) are consistent with Eq. (1). (See also Fig. 5(a) in Ref. \([43]\))

\[
\begin{align*}
\langle x^2 q^\pi_+ \rangle &= 0.027(05) \\
\langle x^3 q^\pi_+ \rangle &= 0.017(04) \\
\langle x^6 g^\pi_+ \rangle &= 0.011(03).
\end{align*}
\]

\( \)
fits must possess less support at low \( x \). These expectations were confirmed in Fig. 4(b) of Ref. [43], and the results are highlighted in Figs. 3 and 4 herein.

**Fig. 3.** Sea distribution, \( xg^S(x; \zeta_2 = 2\text{ GeV}) \): solid purple curve, prediction from Ref. [43]. The associated band expresses a conservative estimate of uncertainty in the prediction, obtained by varying \( \zeta_2 \) by \( \pm 10\% \). Comparisons are selected fits to data: dashed blue curve,\(^{[32]}\) dotted red curve and associated band,\(^{[34]}\) dot-dashed brown curve and band.\(^{[33]}\)

**Fig. 4.** Glue distribution, \( xg^G(x; \zeta_2 = 2\text{ GeV}) \): solid purple curve, prediction from Ref. [43]. Panel (a) highlights low-\( x \) and panel (b), large-\( x \). The band surrounding this curve expresses a conservative estimate of uncertainty in the prediction, obtained by varying \( \zeta_2 \) by \( \pm 10\% \). Comparisons are selected fits to data: dashed blue curve,\(^{[32]}\) dotted red curve and associated band,\(^{[33]}\) dot-dashed brown curve and band.\(^{[34]}\)

Regarding Fig. 4, it is worth emphasizing that at \( \zeta = \zeta_2 \) the pion’s predicted glue distribution is larger in magnitude than its valence distribution on \( x \lesssim 0.22 \). Such an outcome was to be expected. Indeed, as stated in connection with Eq. (5), at \( \zeta = \zeta_H \) all properties of the pion are expressed in the dressed-quark and -antiquark degrees-of-freedom that are generated dynamically in solving the continuum bound-state problem. Under evolution to \( \zeta > \zeta_H \), these quasiparticles steadily shed their cladding, thereby producing populations of sea and glue partons. The size of these populations increases with increasing \( \zeta \). Hence, all symmetry-preserving treatments of pion structure and dynamics must produce nonzero valence, sea, and glue DFs at any scale for which an analysis of experiments may be interpreted in terms of DFs. Furthermore, there is no scale at which all three vanish identically.

Figures 1–4 show that the pointwise form of each DF obtained via a fit to data which omits NLL effects disagrees markedly with the comparable CSM prediction.\(^{[44]}\) This stresses the need for both improved data analyses (Complementing Ref. [30], the potential impacts of including NLL effects when extracting DFs from data are also illustrated in Ref. [89], e.g., all DFs are softer at large-\( x \) and a greater fraction of the pion’s light-front momentum is carried by glue, with the cost paid largely by the sea fraction.) and additional rigorous DF predictions from QCD theory.

In this connection, recalling Fig. 1, the CSM prediction for the pion’s valence-quark DF\(^{[43]}\) is confirmed by the IQCD result in Ref. [83].

No lQCD results are available for the pion’s sea distribution; so, nothing can yet be directly concluded on this score. Regarding phenomenology, Fig. 3 highlights that the pion’s sea distribution is very poorly constrained by existing data analyses.

**Fig. 5.** \( xg^G(x; \zeta_2 = 2\text{ GeV})/\langle xg^G \rangle \): solid purple curve, prediction from Ref. [43]. Panel (a) highlights low-\( x \) and panel (b), large-\( x \). Comparison curve: lQCD result,\(^{[44]}\) for which the associated band expresses aspects of statistical error and systematic uncertainty associated with using an assumed functional form to fit the results.

On the other hand, lQCD results for the pion’s glue DF have recently become available.\(^{[44]}\) They are compared in Fig. 5 with the CSM prediction.\(^{[43]}\) Within uncertainties, there is pointwise agreement between
the two results on the entire depicted domains. Combined with similar agreement between CSM and IQCD predictions for the pion’s valence distribution (Fig. 1), one has evidence to suggest that the CSM prediction for the pion’s sea distribution, Fig. 3, is also reliable.

Such quantitative agreement between continuum and lattice results for pion DFs supports the perspective on the origin of hadron masses developed in Ref. [90]. Namely, the mass of the pion is much smaller than that of the \( \rho \)-meson (and the proton) owing to a symmetry-ensured cancelation in this near Nambu–Goldstone boson between the positive one-body-dressing content of the QCD trace anomaly and the negative binding energy produced by interactions. (See also Ref. [36], Sec. 2D)

4. Summary and Perspective. Through comparisons with continuum and lattice predictions for pion distribution functions (DFs), evidence is accumulating which indicates that the pion DFs inferred from analyses of existing data are pointwise inaccurate, possessing too much support on the valence-quark domain \((x \gtrsim 0.2)\) and too little on its complement. Additional evidence suggests that the discrepancies may be ameliorated, or even eliminated, by inclusion of next-to-leading-logarithmic threshold resummation effects in the hard-scattering kernels whose accurate representation is an essential element in the determination of DFs through data fitting. If these remarks are correct, then their implications for the DFs of nucleons and nuclei should similarly be considered; especially because imperfect extractions of such DFs may obscure or provide misleading signals of physics beyond the Standard Model. In any event, one must look forward to the era in which sound QCD-connected predictions are exploited in providing material constraints on data analyses. This time is approaching for pions and kaons.

As Nature’s most basic Nambu–Goldstone bosons, pions and kaons are unique. Being mesons – quark-antiquark bound-states – they also typify a form of hadron matter about whose structure very little is empirically known. That is changing, with new-generation facilities, in operation or planning, having the capacity to provide unprecedented access to these systems as targets.

Critically, theory is also making rapid progress, beginning to deliver robust predictions for the measurable properties of pions and kaons. So, it is reasonable to expect that sometime after the centenary of the prediction of the pion’s existence\[^{[21]}\] and before the centenary of its discovery\[^{[22]}\], science will finally have the information necessary to draw charts which reveal the pion’s structure in exquisite detail. This may finally explain why, inter alia, basic features of emergent hadron mass are hidden in the pion whereas they are blatant in the proton.

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References

[1] Rutherford E 1919 The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 27 581
[2] Cockcroft J D and Walton E T S 1932 Proc. Roy. Soc. Lond. A 136 619
[3] Taylor R E 1991 Rev. Mod. Phys. 63 573
[4] Kendall H W 1991 Rev. Mod. Phys. 63 597
[5] Friedman J I 1991 Rev. Mod. Phys. 63 615
[6] Marciano W J and Pagels H 1979 Nature 279 479
[7] Brock R et al. 1995 Rev. Mod. Phys. 67 157
[8] Friedman J I, Kendall H W and Taylor R E 1991 Rev. Mod. Phys. 63 629
[9] Drell S and Yan T M 1970 Phys. Rev. Lett. 25 316 [Erratum: 1970 Phys. Rev. Lett. 25 902]
[10] Holt R J and Roberts C D 2010 Rev. Mod. Phys. 82 2991
[11] Gao J, Harland-Lang L and Rojo J 2018 Phys. Rep. 742 1
[12] Geesaman D F and Reimer P E 2019 Rep. Prog. Phys. 82 006301
[13] Ethier J J and Nocera E R 2020 Ann. Rev. Nucl. Part. Sci. 70 43
[14] Ball R D, Nocera E R and Rojo J 2016 Eur. Phys. J. C 76 383
[15] Segarra E, Schmidt A, Kutz T, Higinbotham D, Piasetzky S, M. Schmidt. This work benefited from discussions with running DGLAP evolution codes; and simulations results; to J. Lan and Z.-B. Xing for assistance with running DGLAP evolution codes; and for constructive comments from V. Andreix, D. Binosi, W.-C. Chang, Z.-F. Cui, O. Denisov, M. Ding, R. ENT, F. Gao, T. Horn, W.-D. Nowak, S. Platechkov, C. Quintans, K. Raya, J. Rodriguez-Quintero and S. M. Schmidt. This work benefited from discussions and presentations during the “Workshop on Pion and Kaon Structure at the EIC”, hosted by the Center for Frontiers in Nuclear Science, 2–5 June 2020, and the ongoing series of CERN-hosted workshops on “Perceiving the Emergence of Hadron Mass through AMBER@CERN” – 11 December 2019 (I), 30 March–02 April 2020 (II), 06–07 August 2020 (III), 30 November–4 December 2020 (IV), 27–30 April 2021 (V).

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[4] Kendall H W 1991 Rev. Mod. Phys. 63 597
[5] Friedman J I 1991 Rev. Mod. Phys. 63 615
[6] Marciano W J and Pagels H 1979 Nature 279 479
[7] Brock R et al. 1995 Rev. Mod. Phys. 67 157
[8] Friedman J I, Kendall H W and Taylor R E 1991 Rev. Mod. Phys. 63 629
[9] Drell S and Yan T M 1970 Phys. Rev. Lett. 25 316 [Erratum: 1970 Phys. Rev. Lett. 25 902]
[10] Holt R J and Roberts C D 2010 Rev. Mod. Phys. 82 2991
[11] Gao J, Harland-Lang L and Rojo J 2018 Phys. Rep. 742 1
[12] Geesaman D F and Reimer P E 2019 Rep. Prog. Phys. 82 006301
[13] Ethier J J and Nocera E R 2020 Ann. Rev. Nucl. Part. Sci. 70 43
[14] Ball R D, Nocera E R and Rojo J 2016 Eur. Phys. J. C 76 383
[15] Segarra E, Schmidt A, Kutz T, Higinbotham D, Piasetzky S, M. Schmidt. This work benefited from discussions with running DGLAP evolution codes; and simulations results; to J. Lan and Z.-B. Xing for assistance with running DGLAP evolution codes; and for constructive comments from V. Andreix, D. Binosi, W.-C. Chang, Z.-F. Cui, O. Denisov, M. Ding, R. ENT, F. Gao, T. Horn, W.-D. Nowak, S. Platechkov, C. Quintans, K. Raya, J. Rodriguez-Quintero and S. M. Schmidt. This work benefited from discussions and presentations during the “Workshop on Pion and Kaon Structure at the EIC”, hosted by the Center for Frontiers in Nuclear Science, 2–5 June 2020, and the ongoing series of CERN-hosted workshops on “Perceiving the Emergence of Hadron Mass through AMBER@CERN” – 11 December 2019 (I), 30 March–02 April 2020 (II), 06–07 August 2020 (III), 30 November–4 December 2020 (IV), 27–30 April 2021 (V).

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[1] Rutherford E 1919 The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 27 581
