Violation of the Bell-CGLMP-like inequality for high-dimensional maximally non-separable angular-radial modes

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Following the development of two-dimensional spin-orbit hybrid entanglement, we propose a novel type of high-dimensional hybrid entanglement existed between the angular and radial degree of freedom of the improved Laguerre Gaussian (LG) mode. Experimentally, we observe strong violations of the Bell-CGLMP-like inequality for a maximally non-separable state from dimension 2 to 10, which provides new physical insights into the notion of classical high dimensional non-separable system. Additionally, our results show the Bell measurements can be a sufficient criterion for identifying the mode separability in a high dimensional space.

I. INTRODUCTION

Non-local entanglement, a remarkable non-classical correction between spatially separated systems, is not only discussed in the context of fundamental physics, i.e., Einstein, Podolsky and Rosen (EPR) paradox [1] and Bell’s inequality [2,4], but also widely applied to the fields of metrology [5], quantum computation [6] and communications as well [7,9]. Besides, there exists a type of local entanglement [10] between different degrees of freedom (DoFs) in single system [11,12], may also appear in classical systems, i.e., a classical nonseparable optical mode [13,15], and in single particle [16,19]. The ‘nonlocality’ is the biggest distinguish between non-local and local entanglements. The ‘nonlocality’ is not a necessary condition for implementing many quantum computing tasks [20], i.e., quantum walk [21,22] and some parallel searching algorithms [23,24], where the local entanglement can increase computation resources while keeping the physical number of particles constant.

One of the well-known local entanglement is the spin-orbit non-separable state [13,20,25–27], which has been widely studied in applications in optical metrology, sensing [12,23], communication protocols [25], and key devices for quantum computation, for example, CNOT gates [24,29]. Besides, due to the non-product mathematical form, i.e., $|H⟩L + |V⟩−L$, the spin-orbital non-separable state provides an effective tool to emulate the behaviors of the non-local entanglement [14,15]. For the non-local entanglement, violation of Bell inequality illustrates that the corresponding quantum state can’t be explained by the local hidden-variable theory (LHV). For the local non-separable mode, the violations of Bell-like inequality is an effective quantitative tool in studying classical optical coherence and mode (non-) separability [13,20,27]. Experimentally, a convenient way to realize non-separable mode is to employ spin angular momentum (SAM) and orbital angular momentum (OAM) DoFs of the light. However, the SAM has only two orthogonal eigenstates, which leads to poor scalability in dimensions for this of local non-separable state. For the Laguerre-Gaussian beam ($LG_n^m \rightarrow |L⟩|P⟩$), it has infinite dimensions in both angular mode L and radial mode P, therefore it has the potential to construct a high-dimensional hybrid angular-radial entanglement [30,31].

Over the past few decades, the angular mode has drawn much attention [32,35]. While the radial mode is not very attractive because of the difficulties in sort and detection, and there are some progress in both classical and quantum fields [30,36,40], i.e., exploring full-field quantum correction [30,31]. Very recently, some significant advances have been made to sort radial mode using accumulated Gouy phases [22,43], to measure full fields by a method of intensity-flattening [44], where a (de-) magnification telescope is employed to reduce crosstalk in cost of collection efficiency. These latest technologic advances indicate that it is possible to build a higher dimensional (HD) angular-radial non-separable state $|L_1⟩|P_1⟩ + |L_2⟩|P_2⟩ + ...$ to beyond the low dimensional limits of spin-orbital local entanglement mentioned before.

In this article, we construct a high-dimension angular-radial non-separable state (HD-ARNS) by using revised LG modes, where the HD-ARNS has the form of maximally entanglement state-like (MHD-ARNS): $\sum_{j=0}^{d-1} |j⟩_L|j⟩_P$. We observe the violation of the Bell-
CGLMP-like inequality \cite{43} from d=2 to 10. The results for violations of MHD-ARNS are in very good agreement with the quantum’s case shown in \cite{38}, which illustrates that our scheme can be a useful platform to classically simulate various high dimensional entanglement from two particles \cite{33,34,15,6}. For a two-dimensional spin-orbital laser beam, Bell’s measurements like Bell-like inequalities or Bell-type interference visibility is an effective criterion for classical nonseparability \cite{15,20}, which can be also used to identify a high dimensional non-separable mode. However, there are very few experimental researches due to the difficult in manipulation. Very recently, one tries to simulate the high-dimensional entanglement by position-position correction, while it is difficult to expand dimensions due to the complicated experimental setup and the low value of Bell-like inequality \cite{17}. In our works, for verifying the mode separability of the generated angular-radial modes, we demonstrate a dynamic process from the separation to nonseparation for a three dimension angular-radial non-separable mode, where the distributions of Bell-CGLMP-like inequality and interference visibility are discussed in detail.

\textbf{FIG. 1.} Spatial vector distribution for LG beam and setups for generation of a HD-ARNS. (a): The distribution of spatial vector, phase, and intensity for LG beam $|1\rangle_L|1\rangle_P$; (b): The optical layout for generation and detection HD-ARNS. The input laser source, a semiconductor laser at the 780 nm, is not plotted. HWP: half wave plate. PBS: polarization beam splitter. F1-4: convex lens with two inches. D: A power meter for detection.

For similarity, the LG beam $LG_{d=1}$ with both angular L and radial P modes can be written as $|1\rangle_L|1\rangle_P$. When both of them are zero, the beam degenerates a conventional Gaussian beam. Fig.1 (a) shown the spatial vector and intensity distribution of the state $|1\rangle_L|1\rangle_P$ from the view of 3-D perspective. The angular number L gives rise to the twist helical wavefront, which illustrates the light carrying a well-defined orbital angular momentum; the radial number P exhibits P+1 concentric rings on the wavefront and intensity, which describes as an intrinsic hyperbolic momentum charge \cite{43}. The manipulation of P mode is experimentally more difficult than L mode, where the overlap between different radial modes is strongly dependent on the beam waist of basis and propagation distance $z$ \cite{30,36,48}. The development of the amplitude-phase encoding technology \cite{49} can overcome these difficulties although the low reflection efficiency \cite{36,20}. Using the amplitude-phase encoding hologram, we successfully generate and detect the full field with both DoFs, which is shown in Fig.1 (b). The SLM-G is used to generate angular-radial non-separable state $|L\rangle|P\rangle$. The beam is exactly imaged on the surface of the another SLM-M for measurement by a 4f system. In order to void the higher radial mode exceeding the margin of SLMs, we employ the revised LG mode for constructing MHD-ARNS, where the beam for all the basic state has equal size, but different waists $w_0(L,P) = w_0(0,0)/\sqrt{L+P+1}$, and both of DoFs still keeps the well orthogonality (see supplementary). Recently, the improvable LG mode has shown many outstanding abilities in multi-mode superposition and quantum-key-distribution \cite{51,52}.

The results for manipulating a HD-ARNS is shown in Fig. 2. Fig. 2(a) depicts the modal decomposition density for six dimensional ARNS $|\psi\rangle_d=\frac{1}{\sqrt{6}}\sum_{j=0}^{5}|j\rangle_L|j\rangle_P$, where one needs to make 36 ($d^2$) projection measurements $\{\langle \varphi |\} = \{\langle m_L|j|\rangle_P\}$ with the help of the SLM-M (see Fig.1(b)). For charactering the crosstalk, one can define a power-visibility $V = \sum_i I_i/\sum_j I_j$, where $I_j$ represents the obtained average power $\langle |\varphi|\psi\rangle^2$. The power-visibility is 87.3% for six dimensional ARNS. Also, we measure the modal decomposition density for the single state with one DoF, which is plotted in Fig.2 (b) and (c). The power-visualibilities calculated by two matrix are 92.78% and 82.9% for angular and radial modes, respectively, where the visibility of the radial mode is a bit low due to the imperfect overlaps. Nevertheless, the high diagonal elements and small cross-talk illustrate the generation has well orthogonal and high quality.

The generation of MHD-ARNS in d dimensions can be written as $|\psi\rangle_d=\frac{1}{\sqrt{d}}\sum_{j=0}^{d-1}|j\rangle_L|j\rangle_P$, which has the same mathematical forms as the two particles \cite{34,33}. We find that the high dimensional inequality $S_d = \sum_{k=0}^{[d/2]-1}S_k(A_a,B_b)$ also holds for the demonstrated MHD-ARNS, where the observations of A $\theta_a^L = \frac{2\pi}{d} (v+a/2)$ and B $\theta_b^P = \frac{2\pi}{d} [-w + 1/4(-1)^{b}]$ are angular and radial DoFs, respectively, and two sets of label a and b have the discrete values 0 and 1. Therefore, the measurement
The key results for this paper are shown in Fig. 3, where Fig. 3(a) and (b) present the Bell-CGLMP-like interference curves in dimension of two and ten. For two-dimensional state \(|0\rangle_L |0\rangle_P + |1\rangle_L |1\rangle_P\), the measured intensity is similar to the results of two-dimensional biphoton entanglement state [33, 34]. In Fig. 3(a), we show two experimental interference curves by changing the value of radial phase \(\theta_R\) while fixing the value of angular phase \(\theta_L\) on 0 and \(\pi\). Fig. 3(b) shows the interference curve for the ten dimensional ARNS, where the radial phase is fixed on a constant value. For two subfigures, the theoretical fitted curves are plotted by solid lines, and the error bars were estimated by statistical simulations that assumed the data follows the Poisson’s distribution. Because the imperfect optical system results in some basic phase between low-order and high-order modes, the interference curve appears movement on the left, especially for higher dimensional state. The movement will give rise to a dislocation between sample and background (see Fig. 4(a) in [45]). For overcoming the dislocation, one needs to increase a little basic phase for both radial and angular parts. The Fig.3 (c) shows the values of Bell-CGLMP inequality \(S_d\) versus the dimensions \(d\) as measured by two-photon high dimensional entanglement (blue bars) with entanglement concentration in Ref. [33] and MHD-ARNs (red bars) in our setups. The gradual blue area \(S_d \leq 2\) on the bottom illustrates the state satisfies the LHV theory, and the green line on the top is the upper bound for violations of the maximally high dimensional entanglement states. In our systems, the dimension that violation of a Bell inequality for MHD-ARNs is up to 10, where the value obtains \(S_{d=10}=2.650 \pm 0.035\) that violates 18.6 standard deviations. An interesting feature is that the maximum limit for maximally high dimensional entangled states will be greater than two dimension’s \(2\sqrt{2}\). In our system, we find the such violations of \(S_{d=4}=2.883 \pm 0.017\) and \(S_{d=5}=2.912 \pm 0.018\) in four and five dimensions, where the standard deviations are 3.2 and 4.6 beyond \(2\sqrt{2}\), respectively. Because of the crosstalk between both of \(L\) and \(P\) modes (details in supplementary), the violations are weaker as the increase of dimension. Both of them can mathematically beyond a kind of limit, i.e., \(S_d=2\), although different physical meaning between them. The results for violations of MHD-ARNs are in very good agreement with the quantum’s case shown in [33], which illustrates that our scheme can be a useful platform to classically simulate high dimensional non-local entanglement.

For the high-dimensional non-local quantum entanglement, violation of the Bell-CGLMP inequality is an effective criterion for the existence of nonlocality, while it can be a valid criterion for the (non-)separability of the high-dimensional local entanglement, which have been demonstrated in two-dimensional classical entanglement systems [13, 20, 27]. For the separable mode, \(|S| \leq 2\), however, this conditions can be violated for some non-separable modes in two dimensions. Also, one can evaluate the separability by the visibilities measured from Bell-CGLMP type interference curves [13]. Now, we expand the results to high dimensional classical non-separable mode. Considering a three-dimensional ARNS, \(|\varphi_3\rangle = \sqrt{\epsilon_0} |0\rangle_L |0\rangle_P + \sqrt{\epsilon_1} |1\rangle_L |1\rangle_P + \sqrt{1 - \epsilon_0 - \epsilon_1} |2\rangle_L |2\rangle_P\), \(\epsilon_0\) and \(\epsilon_1\) are two real coefficients on the states of \(|0\rangle_L |0\rangle_P\) and \(|1\rangle_L |1\rangle_P\), a constraint should be acquired \(\epsilon_0 + \epsilon_1 \leq 1\). The theoretical simulation of \(S\) versus two coefficients are plotted on the right top of Fig. 4.
We first experimentally demonstrate the violation of a Bell-CGLMP-like inequality for high dimensional maximally non-separable state supports an effective criterion for mode (non-) separability by high-dimensional Bell’s measurement. The concepts can be easily expanded to the already prepared various HD quantum entanglement from two particles. Compared with the entanglement between multi photon in quantum level, the entanglement between DoFs in classical fields have well manipulation, higher detection rate, and strong robustness to noise, which enable us to implement some special computation protocols with highly efficient. For example, one can generate the group of high-dimensional Bell-like state with high fidelity, where the non-diagonal state and phase between entangled modes are easily to manipulate compared with the quantum’s [45].

For the nonzero situation, i.e., $\varepsilon_1=1/3$, the state will be non-separable regardless of the value of $\varepsilon_0$, where the high visibility should be maintained, which is shown in Fig. 4(d). An interesting fact is that the distributions of visibility are different from the value of Bell-CGLMP-like inequality. For example, in a three dimensional ARNS, the area for values of Bell-CGLMP-like inequality beyond one special boundary, i.e., $S_d \geq 2$ is more smaller than the visibility’s, i.e., $V \geq 71\%$ (see the red boundary in Fig. 4(a)), which depicts that the criterion for (non-) separability by the visibility values is more rigorous than the visibility.

**Conclusion** We first experimentally demonstrate the violation of a Bell-CGLMP-like inequality for high dimensional local non-separable modes with both angular and radial DoFs. On the one hand, the violations demonstrated in local single system are compared with the non-local two-photon high dimensional entangled state, which illustrate that our regime provides an effective platform to classically simulate non-local high dimensional entanglements. On the other hand, it can be an effective criterion for mode (non-) separability by high-dimensional Bell’s measurement. The concepts can be easily expanded the single photon level by changing the inputs. From the view of computing resource, this type of DoF entanglement $\sum_{j=0}^{d-1} |j\rangle_L |j\rangle_P$ from a single particle is equal to the already prepared various HD quantum entanglement from two particles. Compared with the entanglement between multi photon in quantum level, the entanglement between DoFs in classical fields have well manipulation, higher detection rate, and strong robustness to noise, which enable us to implement some special computation protocols with highly efficient. For example, one can generate the group of high-dimensional Bell-like state with high fidelity, where the non-diagonal state and phase between entangled modes are easily to manipulate compared with the quantum’s [45].

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