Reservoir fluid production optimization to sustain net-present value (NPV) using gradient-based Quasi-Newton method

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Abstract. After several years of production, the strong water drive reservoir can have high water cut. Unlike hydrocarbon, water has no commercial value, and tends to be disposed or separated. This leads to the decline in net present value (NPV) of reservoir. To sustain reservoir’s NPV for as long as possible, a gradient-based Quasi-Newton optimization in reservoir simulation is proposed. This mathematical method uses BFGS algorithm to find the minimum of the objective function of reservoir production. This method also assumes constant production and injection rate. Initially, the algorithm produces the base NPV (before optimized using Quasi-Newton method). After that, BFGS generates optimized NPV by iterating line search in the objective function until it meets the stopping criterion. This algorithm also shows the optimized oil and water production curve. These comparisons show how oil and water should be produced to sustain the NPV. The results show that the water cut value is decreased and NPV decline happens later than the non-optimized NPV. As a conclusion, to answer the sustainable energy needs, Quasi-Newton method using BFGS algorithm and its improvements are suitable to sustain the reservoir production in the long run.

1. Introduction
Decline of fossil fuel production is inevitable in the energy industry. For example, in Norway, oil production was so developed that the price hit low on 1990s. However, in 2001, their production started to decline [1]. Another case states that the giant oil fields in the world are not immune to the rapidly declining rate of production rate. It is found that over time, average decline rate of each giant oil fields is increasing due to increasing demand of energy supply [1]. Therefore, it is necessary to develop a more efficient way of exploring and producing more energy sources.

One of the methods used for increasing the oil production is Enhanced Oil Recovery (EOR). This method helps to obtain 60% - 70% of the oil where it cannot be produced. Because of this, EOR sees some potential in the future. There are several techniques in EOR, including chemical and gas floods, combustion, steam, and electric heating [2].

The particular EOR technique that we want to focus here is water injection. However, it is known that water injection often leads to high water production. Since water has no commercial value when produced, we want to minimize water production and maximize hydrocarbon production. This leads to the optimization of reservoir production. In turn, the net present value (NPV) of the reservoir can be sustained. Several researches about production optimization has been done, including gradient-based method, or using geostatistics and well-logging as supporting data [3]. However, these researches only
take account into a condition where the oil price is high and the petrophysical parameters of the reservoir is quite ideal.

In this paper, an established optimization method named Quasi-Newton is used to optimize the NPV in order to slow down the production decline rate. To further support the study, an algorithm in this method named Broyden-Fletcher-Goldfarb-Shanno (BFGS) is employed. This will be used in a non-ideal condition where the oil price and petrophysical properties of the reservoir, in the hindsight, look unprofitable.

2. Methodology
In this study, an open-source toolbox named MATLAB Reservoir Simulation Toolbox (MRST) was used. This toolbox operates on MATLAB environment, where the computation can be done more easily and cheaper than most industrial software designed for reservoir simulation. MRST is not intended to be a true full-scaled simulator, but only as a rapid prototyping of a reservoir modelling and simulation. MRST’s functionality is wide, such as upscaling, visualization, black-oil model, automatic differentiation (AD) module, and optimization using several mathematical algorithms [4]. This optimization, in particular, used AD, black-oil, and optimization modules.

The model used in this research is a simple synthetic faulted geological model with the dimension of 500x500 meters (Figure 1). The reservoir was made with petrophysical properties such that it represented a “difficult” reservoir with fairly low porosity and fairly tight permeability. The average porosity was set to be 0.1. The anisotropy was introduced to the permeability, with the permeability in the range of 0.1 mD. Inside the reservoir, the fluid was set to be two-phase oil and water with slightly compressible property.

Next, the wells were defined. Two producing wells and two water-injecting wells were modelled here. To simplify the simulation, the producing and injecting rate were assumed to be constant. This was also supported by another assumption that the quantity of fluids leaving the well is equal to the quantity of fluids entering the well. The rates correspond to the reservoir pore volume during the scheduled time. It was scheduled that the simulation would run for 1000 days (2 years, 287 days, and 3.43 hours). P1 and P2 indicates production wells, while I1 and I2 are descriptions for injection wells. Production wells are used to produce the oil, and the injection wells are used to inject the water into the formation.

![Figure 1. The model used for reservoir simulation.](image)

Since this simulation also heavily depends on oil price, water production cost, water injection cost, and discount factor, the parameters mentioned were initiated. The values can be seen in Table 1. Then, the simulation for the base case, NPV as a function of rock properties, schedules, was run. The base case corresponds to the condition where the optimization has not been applied yet. Figure 2 shows the
result of the initial running, with the x-axis and y-axis represent amount of days and amount of NPV in USD.

![Graph of base run evolution NPV](image)

**Figure 2.** Plot of base run evolution NPV

| Parameter                  | Value     |
|----------------------------|-----------|
| Oil price ($/STB)          | 40        |
| Water production cost ($/STB) | 6         |
| Water injection cost ($/STB) | 6         |
| Discount rate (/year)      | 0.05      |

*Table 1. Display of economical parameter used in this simulation*

Before the optimization can be done, constrains are needed to converge the problem to the desired solution and help the scaling process. In this study, box limit was defined for injector and producer to be 10 to 300 bar per day. All four wells were also set to that when the pressures all added up, the sum became zero. The total water injection constraint was also limited to be less than or equal to 500 bar per day. The objective function that we wanted to optimize here was the NPV, so we defined the function handle both for the objective function and the objective evaluation. BFGS algorithm was applied to this function handle and repeated until the gradient norm and objective change converged.

Based on this reservoir NPV case, the method used for optimizing the net-gradient based optimization can be structured as algorithm that consist of the local method for updating the step based on gradient information and strategy of globalization.

As alternative, we can update the Hessian rather than its inverse at every iteration. Several such updating methods are available; however, a popular method that has proven to be most effective in applications is Broyden-Fletcher-Goldfarb-Shanno (BFGS). The Hessian approximation mostly uses BFGS algorithm. This method mostly used in line-search methods. The BFGS method updates the Hessian on each iteration.

From here, optimized NPV can be compared to the base NPV and be analysed. For additional data, water cut, oil production, water production, and net cash flow can also be plotted to obtain information on how far the optimization goes. We chose to plot those graphs to complete our analysis.

### 3. Results and Discussion

Based on the methodology above, our computational work produced the comparison between base NPV and optimized NPV. In the following discussion, we will discuss about optimizing NPV, which is an discounted accumulated net cash flow [5]. In Figure 3, it can be easily seen that in general, the
NPV increases as the time passes, but eventually will decline at some point. However, there are differences between the base NPV and optimized NPV, mainly in its starting rate and how long it stays increasing until it starts declining. During the first 200 days, there are no differences between the base case and optimal case. In 200th day until 400th day, the base case seems to generate more NPV than the optimal case. However, we know that the Quasi-Newton method approximates a function as quadratic function near the extrema and updates the extrema value (maximum value in this case) to be more optimized [6]. This can be seen in the 600th day where the decline in the base case starts to happen. This is not the case when we use BFGS optimization. Not only this slows down the decline, but also enhances the maximum potential NPV. Thus, from our analysis, optimized NPV is more profitable since it is more sustainable in the long run. This is also comparable to [5]’s research where the aforementioned’s research also produced the same trend pattern. After the optimization, the NPV also increased in the end at that research.

Figure 3. Comparison between base NPV and optimal NPV

The next thing to do is to validate whether the optimization is valid. Here, what we want to minimize is the water production. So, during the running process that was done earlier, the water cut was also minimized and the comparison can be seen in Figure 4a and 4b. This also affects the oil production and water production as shown in Figure 5a and 5b.

Figure 4. (a) shows the water cut before optimized and (b) shows the water cut evolution after optimized using BFGS algorithm.
These figures explain that with BFGS optimization, the water cut can be truncated to below threshold (0.75). This is helpful especially when water is not very profitable to be produced. In the Figure 4(a), it is shown that the water cut keeps increasing as day goes. However, with the mathematical optimization, we can suppress the water cut so the value is not significantly higher than the desired threshold.

As a result, in the Figure 5(a), it can be seen that the oil production rate declines after certain days. However, with BFGS method, the production rates are adjusted so that during the 200-400th days, the rate is increasing to compensate the later declining rate. Also, examining Figure 5(b), the water production rate’s threshold is also lower compared to the unoptimized ones. All these results conclude that the BFGS method successfully modifies and optimizes how the water and oil production are run in the oil production process.

Lowering the water cut in this process is very important. According to [7], in one of their wells during 2014, the well intervention caused the increase in water cut phenomenon. The average water cut went up from 84% to 91.5%. Therefore, the oil production of the well decreased about 125 bopd. This case example shows that keeping the water cut low is a priority in the oil and gas industry. By this analysis, the company can decide next how to make sure the water cut is optimized.

In the end, all the changes in fluid production also affects the net cash flow from the corresponding reservoir. Figure 6 shows the net cash flow before and after optimized using the algorithm. It can be inferred that the loss in cash flow happens later in the optimized case than the non-optimized one. This proves the previous research done with the similar topic and objective.
4. Conclusion
In the sections above, we have demonstrated how the Quasi-Newton method using BFGS algorithm for optimization affects the production sector in the oil and gas industry. By using the standard method, the NPV evolution during the reservoir lifetime can be predicted. However, with the optimization, we can also pinpoint the time NPV starts declining. It can also be inferred that the loss happens slower in the optimized case. This is very important, especially in the oil and gas industry where the efficiency matters. With the most efficient budget, by using this optimization first, the mistakes in production process can be minimized and in turn saves a lot of money.

Compared to other commercial reservoir simulation software, this method offers a cheap and simple simulation to present how mathematical optimization should be done in a vast reservoir such as the case described above. The most challenging part of this research is to choose the proper value for the economic parameters, since the incorrect choice can lead to negative NPV, thus disabling the further use of this algorithm. In the future, this case can be solved with more efficient optimization methods that has not been tried a lot in the industry, such as Multiple Shooting method.

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