Hybrid Finite Element Method for Reconstructing Electric Field Distribution of Dielectric Object Excited by Dipole Antenna

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Abstract: This paper presents an inverse method to reconstruct an antenna-excited electric field distribution of a dielectric object from measured field data, for the purpose of noninvasive SAR evaluation in the human phantom. The proposed method is a hybrid method combining the Boundary Integral Equation (BIE) and Finite Element Method (FEM). The BIE for the electromagnetic (EM) fields and equivalent currents of a radiating source are discretized using the method of moments (MoM) while EM fields in the scatterer and its surroundings are discretized using the FEM. The effectiveness of the method is tested numerically and experimentally using a dipole and a dielectric cuboid at 2.5 GHz. The numerical reconstruction achieves acceptably good agreement with that of reference as a whole. However, it is found from the experiment that the method is considerably sensitive to the measurement error. The factors that may have contributed to the errors are shortly discussed.

Keywords: Finite element method, method of moments, boundary integral equation, inverse problems, FE-BI, SAR

Classification: Electromagnetic compatibility (EMC)

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1 Introduction

The specific absorption rate (SAR) was established as a standard to ensure safe irradiation level of living bodies exposed to electromagnetic (EM) waves. The SAR is defined by \( \text{SAR} = \sigma \frac{|E|^2}{\rho} \), where \( \sigma \), \( E \), and \( \rho \) are conductivity, electric (E)-field, and mass density, respectively [1]. The \( \sigma \) and \( \rho \) are typically acquired by a separate prior measurement of the material, or simply from a data sheet. The widely used SAR measurement method is performed by insertion of electric field probe inside of a tank filled with liquid phantom that is illuminated by the antenna under test (AUT) [2]. This inherently invasive method, although straightforward, poses a set of disadvantages such as the instability of liquid phantom’s dielectric properties and considerably long time for data collection. Hence, a noninvasive SAR measurement is necessary to address the drawbacks of the conventional method. The purpose of this paper is to reconstruct the internal E-field of the phantom using two-dimensional probe data, with the measurement taking place externally on an arbitrary surface outside of the phantom. A mechanically and electrically stable solid phantom is surely suitable when we need to rotate or tilt the phantom in the experimental circumstance.

Previously, there was an attempt to develop such noninvasive SAR measurement system using boundary integral equation (BIE) by Omi et. al [3]. Here, they discretize all the surfaces’ elements in the manner of MoM and imposed appropriate boundary conditions on the equivalent surfaces. In this method, only a homogenous phantom was considered, because it is extremely difficult for BIE to take account of inhomogeneous dielectric medium such as the realistic bioequivalent phantom. This inherent disadvantage can be solved by merging the FEM with MoM/BIE, resulting in a hybridized FEM...
that is the Finite Element-Boundary Integral Method (FE-BI).

In FE-BI, the two-dimensional unknown function on the boundary surface and three-dimensional one in the closed region that encapsulates the scatterer and source are essentially discretized by the method of weighted residuals. The two types of numerical approach can be coupled or decoupled freely to form a complete system of equation. This paper presents the inverse problem application of FE-BI by reconstructing the E-field (internal and on the surface of dielectric) from receiver probe’s surface sampling data, $E_{\text{meas}}$. The $E_{\text{meas}}$ used for numerical calculation here comes from spherical near field of a commercial simulator, FEKO. We also attempted the FE-BI conversion using experimental data using the same procedure. The numerical calculation are compared with forward solution calculated by FEKO’s MoM.

2 Finite element-boundary integral formulations

This section describes the FE-BI formulations implemented in the reconstruction process. The theory of FEM and MoM are already well known and available in many textbooks [4, 5], hence the emphasis will be on discretization and how the final equation systems are obtained.

2.1 Overview

The FE-BI is a numerical method for an unbounded region where the FEM is used for analyzing the EM fields in the closed surface enclosing all target objects, meanwhile the fields outside of the boundary is calculated by MoM applied to BIE that satisfies the radiation condition. Hence, an absorbing boundary condition such as the Perfectly Matched Layer [4] is unnecessary.

The original scattering/radiation problem of FE-BI is shown in Fig.1(a) as described in [4, 6, 7]. A three-dimensional structure of arbitrary shape is placed in a closed fictitious surface, $S$ with its interior and exterior region is denoted as $V$ and $V_\infty$, respectively. The fields are generated by current sources from interior (source radiation case) and exterior (scattering case); these currents denoted by $(J_{\text{int}}, M_{\text{int}})$ and $(J_{\text{ext}}, M_{\text{ext}})$, respectively. FEM is employed to formulate fields inside of $S$ while BIE is enforced by coupling a tangential field continuity between the two regions. This leads to a solution of equivalent electric and magnetic currents $(J_{\text{eq}}, M_{\text{eq}})$ on $S$, the corresponding equivalent surface denoted as $S_{\text{eq}}$.

Let us consider an inverse problem as shown in Fig. 1(b), where both the source and scatterer are located inside of the closed sphere as two explicitly separate entities. Their surfaces are also considered independently, with $S_{\text{eq}}$ in this case is the surface that bounds the source. The radiated field are collected on a spherical measurement surface, $S_m$. The probe data is used as input to reconstruct the electric field inside of the shaded volume $V$ which includes the scatterer. The volumetric region is discretized by tetrahedrons of linear and quadratic element as shown in Fig. 1(c). The configuration of dimensions used in this study that directly corresponds to Fig. 1(b) is shown in Fig. 1(d).
2.2 Formulations for boundary integral

The EM fields are expressed by equivalent electric and magnetic currents $J_{eq}$ and $M_{eq}$ on $S_{eq}$. The two kinds of integral equations for E-field and H-field are separately satisfied due to the boundary conditions. This paper uses the combined field integral equation (CFIE) [4] for eliminating spurious solutions appearing in the closed region. The electric and magnetic field equations were combined with equal weight for simplicity, and then the CFIE equation are given as follows.

$$
\frac{1}{2} J_{eq}(\mathbf{r}) + \hat{n} \times \tilde{K}(J_{eq}) + \hat{n} \times L(M_{eq}) + \hat{n} \times \left[ \frac{1}{2} M_{eq}(\mathbf{r}) + \hat{n} \times \tilde{K}(M_{eq}) - \hat{n} \times L(M_{eq}) \right] = \hat{n} \times \bar{H}_{inc}(\mathbf{r}) - \hat{n} \times [\hat{n} \times E_{inc}(\mathbf{r})]
$$

where $L$ and $K$ are surface integral operators. $E_{inc}$ and $H_{inc}$ are incident EM fields radiated by source, and $\mathbf{H} = Z_{0} \mathbf{H}$. For discretization, $J_{eq}$ and $M_{eq}$ are expanded as weighted sum of elements consisting of small element patches as following

$$
\vec{J}_{S_{eq}}^{S_{eq}} = \hat{n} \times \bar{H}_{S_{eq}} \Delta_{S_{eq}}^{S_{eq}} = \sum_{i=1}^{n_{S_{eq}}} \bar{H}_{i}^{S_{eq}} A_{i}^{S_{eq}}
$$

$$
M_{S_{eq}}^{S_{eq}} = E_{S_{eq}}^{S_{eq}} \hat{n} = - \sum_{i=1}^{n_{S_{eq}}} E_{i}^{S_{eq}} A_{i}^{S_{eq}}
$$

where $\bar{H}_{i}^{S_{eq}}$ and $E_{i}^{S_{eq}}$ are the unknown expansion coefficients with respect to $\bar{H}_{inc}$ and $E_{inc}$, while $A_{i}^{S_{eq}}$ is the vector basis function. By substituting Eq. (2) and (3) into Eq. (1), and further reducing it to a matrix equation form,

$$
[P] \{ E \} + [Q] \{ \bar{H} \} = \{ b \} \tag{4}
$$
is obtained. It is not shown here because of their complexity, but all of the elements in the matrices \([P]\) and \([Q]\) and the vector \([b]\) are expressed by familiar integral forms as they commonly appear in the FEM textbook.

### 2.3 Formulations for finite element

The functional of the problem discussed here has been derived and is expressed as follows.

\[
F (\mathbf{E}) = \frac{1}{2} \iiint_V \left[ \frac{1}{\mu_r} (\nabla \times \mathbf{E}) \cdot (\nabla \times \mathbf{E}) - k_0^2 \varepsilon_r \mathbf{E} \cdot \mathbf{E} \right] dV + jk_0 \oint_S (\mathbf{E} \times \mathbf{H}) \cdot \hat{n} dS
\]

(5)

where \(V\) is the volume enclosed by \(S = S_{eq} + S_m\). The boundary condition on the perfect conductor is \(\hat{n} \times \mathbf{E} = 0\). Other boundary conditions are automatically satisfied using the functional expression of Eq. (5). To discretize the fields inside of \(S_m\), volume \(V\) is subdivided into small tetrahedral elements as that of Fig. 1(c). The surface patches of \(\mathbf{E}\) and \(\mathbf{H}\) are then expanded in a similar manner to Eq. (2) and (3) but the vector basis functions becomes \(N_{eq}^S\). For compatibility, \(\Lambda_{eq}^S = \hat{n} \times N_{eq}^S\) is satisfied in BIE formulations. After appropriate substitutions and putting the first variation of \(F\) to zero, that is, from \(\delta F = 0\), we obtain the matrix equation

\[
[K] \{E\} + [B] \{\bar{H}\} = \{0\}
\]

(6)

in which Eq. (4) and Eq. (6) forms the required simultaneous matrix equations for the FE-BI system. Solving these simultaneous equation of \(E\) and \(\bar{H}\) allows us to obtain the numerical solutions which will be presented in the next section.

### 3 Results and discussion

The geometry of the problem discussed here was previously shown in Fig. 1(d) which is almost the same as the model adopted in [3] for comparison. The operating frequency is 2.5 GHz which is the upper limit of the industrial, scientific and medical (ISM) radio band [8]. A half-wavelength horizontal dipole is the AUT that radiates inside of a bounded sphere with surface \(S_m\), the sphere’s radius given by \(R = 10\) cm. A cylinder with height \((h_s)\) and radius \((r_s)\) was selected as the equivalent surface of source \(S_{eq}\), which must be set slightly larger than that of antenna’s wire dimension. The scatterer is a lossless, single-layer dielectric cuboid of permittivity \(\varepsilon_r = 4\).

We have considered the practicality of the chosen setup. The AUT could be any kind of object radiating EM waves, for example a mobile phone which could be placed in proximity such as during a phone call, or from some distance to the user such as during the device standby mode. Here, we chose the second option by placing the AUT 6 cm away from the subject, but this distance is not a determinant factor for this inverse calculation. Meanwhile, \(E_{meas}\) in simulation is a full-sphere scanning with its spatial sampling points given as \(\Delta \theta = \Delta \phi = 5^\circ\).
3.1 Reconstruction using simulation data

The reconstructed electric field distributions for linear and quadratic elements are shown in Fig.2(a),(b) and Fig.2(c),(d) respectively. The forward scattering results calculated by the FEKO are also shown for reference. These results include some important points in SAR measurement using the proposed method. Since the theoretical relation between E and SAR is directly proportional as briefly mentioned in Section I, the spatial distribution of SAR can be recognized. Furthermore, if the E-field levels observed here exceeds a predetermined limit (also depending on the \( \sigma \) and \( \rho \)), it can be said that the device will consequently not pass the SAR safety requirements, and so on.

From the figures, the reconstruction results show acceptable agreement with that of reference as a whole, but there are large differences at some parts. In these parts, the condition number of the matrices in Eqs.(4) and (6) tends to increase. In Fig.2(a) and Fig.2(c), it can be seen the reconstruction results are null at the area that corresponds to \( S_{eq} \) in which the antenna is located. It is not shown here but we also calculated for \( R = 30 \) cm and the results indicate a similar tendency. It is also found that the reconstruction accuracy is inferior to the full-MoM version proposed in [3]. The reason is not clarified yet, but it may be related to the properties of the FE-BI matrices that is mainly caused from the equally weighted CFIE indicated in Eq. (1). Another choice of weighting factor for reducing the condition number of CFIE has been investigated for the case of conducting scatterers [9], so we intend to examine this in the near future.

3.2 Reconstruction using experimental data

In this subsection, we will show the experimental results. Due to limitation of the mechanical rotating arm’s hardware, we did experiment using radius of \( R = 30 \) cm for \( S_m \), and a hemisphere scan was deployed but with same angular sampling points of \( \Delta \theta = \Delta \phi = 5^\circ \). The transmitting and receiving dipole antenna is the Schwarzbeck UHA 9125D-144 (length, 6cm) and Schwarzbeck UHA 9125D-143 (length, 4.8cm), respectively. The scatterer is an FR-4 as its typical \( \varepsilon_r \) value is 4. The other parameters including the dimension of FR-4 is the same to that of Fig.1(d) and also subsection 3.1. The experimentally reconstructed electric field distributions are entirely inaccurate as shown in Fig. (3).

Since the measurement error significantly affects the reconstruction results, more specifically on the E-vector \( \{E\} \) at \( S_m \) as included in Eq. (6), we shall infer several error factors that may occur in practical measurements. The mechanical alignment of receiving probe is one of error-inducing factors. We have used a laser module to ensure the precise positioning of the probe. However, when the anechoic chamber is fully closed during the data collection, the rotating position cannot be monitored thus a slight misalignment could have happened. Probe correction is also an important factor, specifically for FE-BI conversion of experimental results. In the inverse conversion purely by MoM [3], the results of measured E-field is
Fig. 2. Results comparison when $E_{\text{meas}}$ is from simulator ($R = 10\text{cm}$).

compensated by a weighted plane wave expansion. However in the proposed FE-BI method, the conversion of direct E-field data affects the reconstruction quality as shown in experimental results. Hence, the specification of extremely precise antenna factor is needed for our experiment. In addition, although a full-sphere scan is necessary for this method, we could not comply to it due to the restrictions on the chamber’s space and other experimental setup such as the cable. We consider this as one of the factors that give rise to measurement errors, and intend to improve the scanning method [10].

Theoretically, the instability due to the ill-conditioned FE-BI matrices
also may have been increased more due to these factors.

Fig. 3. Results comparison when $E_{\text{meas}}$ is from experiment ($R = 30\text{cm}$)

### 4 Conclusion

In this paper, we have examined the applicability of FE-BI method in an inverse problem, namely for reconstructing the electric field distribution of a dielectric for a noninvasive SAR measurement system. The method was demonstrated numerically with acceptable agreement shown in the reconstruction results. However, for reconstruction using experimental results, it can be seen that the results does not agree well. The steps to reduce errors pertaining to measurement procedure as well as the modification of the FE-BI implementation will be studied in the future.