MIXING OF SCALAR GLUEBALL AND SCALAR QUARKONIA

HAI-YANG CHENG*

Institute of Physics, Academia Sinica, Taipei, Taiwan 115, Republic of China
*E-mail: phcheng@phys.sinica.edu.tw

The isosinglet scalar mesons \( f_0(1710) \), \( f_0(1500) \), \( f_0(1370) \) and their mixing are studied. Two recent lattice results are employed as the starting point; one is the approximate SU(3) symmetry in the scalar sector above 1 GeV for the connected insertion part without \( q\bar{q} \) annihilation, and the other is the scalar glueball mass at 1710 MeV in the quenched approximation. In the SU(3) symmetry limit, \( f_0(1500) \) becomes a pure SU(3) octet and is degenerate with \( a_0(1450) \), while \( f_0(1370) \) is mainly an SU(3) singlet with a slight mixing with the scalar glueball which is the primary component of \( f_0(1710) \). These features remain essentially unchanged even when SU(3) breaking is taken into account. The observed enhancement of \( \omega f_0(1710) \) production over \( f_0(1710) \) in hadronic \( J/\psi \) decays and the copious \( f_0(1710) \) production in radiative \( J/\psi \) decays lend further support to the prominent glueball nature of \( f_0(1710) \).

1. INTRODUCTION

Among the isosinglet scalar mesons \( f_0(1710) \), \( f_0(1500) \) and \( f_0(1370) \), it has been quite controversial as to which of these is the dominant scalar glueball. It has been suggested that \( f_0(1500) \) is primarily a scalar glueball, due partly to the fact that \( f_0(1500) \), discovered in \( p\bar{p} \) annihilation at LEAR, has decays to \( \eta \eta \) and \( \eta \eta' \) which are relatively large compared to that of \( \pi \pi \), and that the earlier quenched lattice calculations predict the scalar glueball mass to be around 1550 MeV. Furthermore, because of the small production of \( \pi \pi \) in \( f_0(1710) \) decay compared to that of \( K\bar{K} \), it has been thought that \( f_0(1710) \) is primarily \( s\bar{s} \) dominated. In contrast, the smaller production rate of \( K\bar{K} \) relative to \( \pi \pi \) in \( f_0(1370) \) decay leads to the conjecture that \( f_0(1370) \) is governed by the non-strange light quark content.

Based on the above observations, a flavor-mixing scheme is proposed to consider the glueball and \( q\bar{q} \) mixing in the neutral scalar mesons \( f_0(1710) \), \( f_0(1500) \) and \( f_0(1370) \). Best \( \chi^2 \) fits to the measured scalar meson masses and their branching ratios of strong decays have been performed in several references by Amsler, Close and Kirk, Close and Zhao, and He et al. A typical mixing matrix in this scheme is

\[
\begin{pmatrix}
  f_0(1370) \\
  f_0(1500) \\
  f_0(1710)
\end{pmatrix} =
\begin{pmatrix}
  -0.91 & -0.07 & 0.40 \\
  -0.41 & 0.35 & -0.84 \\
  0.09 & 0.93 & 0.36
\end{pmatrix}
\begin{pmatrix}
  |N> \\
  |S> \\
  |G>
\end{pmatrix}.
\]

A common feature of these analyses is that, before mixing, the \( s\bar{s} \) mass \( M_S \) is larger than the glueball mass \( M_G \), which, in turn, is larger than the \( N(\equiv (u\bar{u} + d\bar{d})/\sqrt{2}) \) mass \( M_N \), with \( M_G \) close to 1500 MeV and \( M_S - M_N \) of the order of 200 \( \sim 300 \) MeV. However, there are at least four serious problems with this scenario: (i) The isovector scalar meson \( a_0(1450) \) is now confirmed to be the \( q\bar{q} \) meson in the lattice calculation. As such, the degeneracy of \( a_0(1450) \) and \( K_0^*(1430) \), which has a strange quark, cannot be explained if \( M_S \) is larger than \( M_N \) by \( \sim 250 \) MeV. (ii) The most recent quenched lattice calculation with improved action and lattice spacings extrapolated to the continuum favors a larger scalar glueball mass close to 1700 MeV. (iii) If \( f_0(1710) \) is dominated by the \( s\bar{s} \) content, the decay \( J/\psi \rightarrow \phi f_0(1710) \) is expected to have a rate larger than that of \( J/\psi \rightarrow \omega f_0(1710) \). Experimentally, it is other way around: the rate for \( \omega f_0(1710) \) production is about 6 times that of \( J/\psi \rightarrow \phi f_0(1710) \).
\( \phi f_0(1710) \). (iv) It is well known that the radiative decay \( J/\psi \to \gamma f_0 \) is an ideal place to test the glueball content of \( f_0 \). If \( f_0(1500) \) has the largest scalar glueball component, one expects the \( \Gamma(J/\psi \to \gamma f_0(1500)) \) decay rate to be substantially larger than that of \( \Gamma(J/\psi \to \gamma f_0(1710)) \). Again, experimentally, the opposite is true. Simply based on the above experimental observations, one will naively expect that \( \gamma \gg \alpha > \beta \) in the wave function of \( f_0(1710) = \alpha|N\rangle + \beta|S\rangle + \gamma|G\rangle \).

In our recent work [9], we have employed two recent lattice results as the input for the mass matrix which is essentially the starting point for the mixing model between scalar mesons and the glueball. First of all, an improved quenched lattice calculation of the glueball spectrum at the infinite volume and continuum limits based on much larger and finer lattices have been carried out [7]. The mass of the scalar glueball is calculated to be \( m(0^{++}) = 1710 \pm 50 \pm 80 \) MeV. This suggests that \( M_G \) should be close to 1700 MeV rather than 1500 MeV from the earlier lattice calculations [3]. Second, the recent quenched lattice calculation of the isovector scalar meson \( a_0 \) mass has been carried out for a range of low quark masses [6]. It is found that, when the quark mass is smaller than that of the strange, \( a_0 \) mass is almost independent of the quark mass, in contrast to those of \( a_1 \) and other hadrons that have been calculated on the lattice (see Fig. 1). The chiral extrapolated mass \( a_0 = 1.42 \pm 0.13 \) GeV suggests that \( a_0(1450) \) is a \( q\bar{q} \) state. Furthermore, \( K_0^*(1430)^+ \), an us meson, is calculated to be \( 1.41 \pm 0.12 \) GeV and the corresponding scalar \( s\bar{s} \) state from the connected insertion is \( 1.46 \pm 0.05 \) GeV. This explains the fact that \( K_0^*(1430) \) is basically degenerate with \( a_0(1450) \) despite having one strange quark. This unusual behavior is not understood as far as we know and it serves as a challenge to the existing hadronic models. In any case, these lattice results hint at an SU(3) symmetry in the scalar meson sector. Indeed, the near degeneracy of \( K_0^*(1430) \), \( a_0(1470) \), and \( f_0(1500) \) implies that, to first order approximation, flavor SU(3) is a good symmetry for the scalar mesons above 1 GeV.

2. Mixing Matrix

We shall use \(|U\rangle, |D\rangle, |S\rangle \) to denote the quarkonium states \(|u\bar{u}\rangle, |d\bar{d}\rangle \) and \(|s\bar{s}\rangle \), and \(|G\rangle \) to denote the pure scalar glueball state. In this basis, the mass matrix reads

\[
M = \begin{pmatrix}
M_U & 0 & 0 & 0 \\
0 & M_D & 0 & 0 \\
0 & 0 & M_S & 0 \\
x & x & x_s & y \\
x & x & x_s & y \\
x_s & x_s & x_s & y_s \\
y & y & y_s & y
\end{pmatrix} + \begin{pmatrix}
\alpha & \beta & \gamma \\
\alpha & \beta & \gamma \\
\alpha & \beta & \gamma \\
x & y & x & y \\
x & y & x & y \\
x_s & y_s & x_s & y_s \\
y & x & y & y
\end{pmatrix},
\]

where the parameter \( x \) denotes the mixing between different \( q\bar{q} \) states through quark-antiquark annihilation and \( y \) stands for the glueball-quarkonia mixing strength. Possible SU(3) breaking effects are characterized by the subscripts “s” and “ss”. As noticed in passing, lattice calculations [6] of the \( a_0(1450) \) and \( K_0^*(1430) \) masses indicate a good SU(3) symmetry for the scalar meson sector above 1 GeV. This means that \( M_S \) should be close to \( M_U \) and \( M_D \). Also the glueball mass \( M_G \) should be close to the scalar glueball mass 1710 \( \pm 50 \pm 80 \) MeV from the lattice QCD calculation in the pure gauge sector [7].

We shall begin by considering exact SU(3) symmetry as a first approximation, namely, \( M_S = M_U = M_D = M \) and \( x_s = x \) and \( y_s = y \). In this case, two of the mass eigenstates are identified with \( a_0(1450) \)
and \( f_0(1500) \) which are degenerate with the mass \( M \). Taking \( M \) to be the experimental mass of \( 1474 \pm 19 \text{ MeV} \), it is a good approximation for the mass of \( f_0(1500) \) at \( 1507 \pm 5 \text{ MeV} \). Thus, in the limit of exact SU(3) symmetry, \( f_0(1500) \) is the SU(3) isosinglet octet state \( |f_{\text{octet}} \rangle \) and is degenerate with \( a_0(1450) \). In the absence of glueball-quarkonium mixing, i.e. \( y = 0 \), \( f_0(1370) \) becomes a pure SU(3) singlet \( |f_{\text{singlet}} \rangle \) and \( f_0(1710) \) the pure glueball \( |G \rangle \). The \( f_0(1370) \) mass is given by \( m_{f_0(1370)} = M + 3x \). Taking the experimental \( f_0(1370) \) mass to be 1370 MeV, the quark-antiquark mixing matrix element \( x \) through annihilation is found to be \(-33 \text{ MeV} \). When the glueball-quarkonium mixing \( y \) is turned on, there will be some mixing between the glueball and the SU(3)-singlet \( q\bar{q} \). If \( y \) has the same magnitude as \( x \), i.e. \( 33 \text{ MeV} \), then \( 3y^2 \ll \Delta^2 \) where \( \Delta \) is half of the mass difference between \( M_G \) and \( M + 3x \), which is \( \sim 170 \text{ MeV} \). In this case, the mass shift of \( f_0(1370) \) and \( f_0(1710) \) due to mixing is only \( \sim 3y^2/2\Delta = 9.6 \text{ MeV} \). In the wavefunctions of the mixed states, the coefficient of the minor component is of order \( \sqrt{3}y/(2\Delta) = 0.17 \) which corresponds to \( \sim 3\% \) mixing.

As discussed before, SU(3) symmetry leads naturally to the near degeneracy of \( a_0(1450) \), \( K_0^*(1430) \) and \( f_0(1500) \). However, in order to accommodate the observed branching ratios of strong decays, SU(3) symmetry must be broken to certain degree in the mass matrix and/or in the decay amplitudes. One also needs \( M_S > M_U = M_D \) in order to lift the degeneracy of \( a_0(1450) \) and \( f_0(1500) \).

To explain the large disparity between \( \pi\pi \) and \( K\bar{K} \) production in scalar glueball decays, Chanowitz 11 advocated that a pure scalar glueball cannot decay into quark-antiquark in the chiral limit, i.e.

\[
A(G \to q\bar{q}) \propto m_q. \tag{1}
\]

Since the current strange quark mass is an order of magnitude larger than \( m_u \) and \( m_d \), decay to \( K\bar{K} \) is largely favored over \( \pi\pi \). Furthermore, it has been pointed out that chiral suppression will manifest itself at the hadron level 12. To this end, it is suggested 12 that \( m_q \) in Eq. (1) should be interpreted as the scale of chiral symmetry breaking since chiral symmetry is broken not only by finite quark masses but is also broken spontaneously. Consequently, chiral suppression for the ratio \( \Gamma(G \to \pi\pi)/\Gamma(G \to K\bar{K}) \) is not so strong as the current quark mass ratio \( m_u/m_s \).

Guided by the lattice calculations for chiral suppression in \( G \to PP \), the fitted masses and branching ratios are summarized in Table 1, while the predicted decay properties of scalar mesons are exhibited in Table 2. The mixing matrix obtained in our model has the form:

\[
\begin{pmatrix}
f_0(1370) \\
f_0(1500) \\
f_0(1710)
\end{pmatrix} = \begin{pmatrix}
0.78 & 0.51 & -0.36 \\
-0.54 & 0.84 & 0.03 \\
0.32 & 0.18 & 0.93
\end{pmatrix} \begin{pmatrix}
|N| \\
|S| \\
|G|
\end{pmatrix}.
\]

It is evident that \( f_0(1710) \) is composed primarily of the scalar glueball, \( f_0(1500) \) is close to an SU(3) octet, and \( f_0(1370) \) consists of an approximate SU(3) singlet with some glueball component (\( \sim 10\% \)). Unlike \( f_0(1370) \), the glueball content of \( f_0(1500) \) is very tiny because an SU(3) octet does not mix with the scalar glueball.

Several remarks are in order. (i) Although \( f_0(1500) \) has a large \( s\bar{s} \) content, the ratio of \( K\bar{K}/\pi\pi \) is small due to the destructive interference between the \( n\bar{n} \) and \( s\bar{s} \) components for \( K\bar{K} \) production. (ii) In the absence of chiral suppression in \( G \to PP \) decay, the \( f_0(1710) \) width is predicted to be less than 1 MeV and hence is ruled out by experiment. This is a strong indication in favor of chiral suppression of \( G \to \pi\pi \) relative to \( G \to K\bar{K} \). (iii) Because the \( n\bar{n} \) content is more copious than \( s\bar{s} \) in \( f_0(1710) \), it is natural that \( J/\psi \to \omega f_0(1710) \) has a rate larger than \( J/\psi \to \phi f_0(1710) \). (iv) If \( f_0(1710) \) is
Table 1. Fitted masses and branching ratios. The chiral suppression in $G \to PP$ decay is taken to be $r_s = 1.55$.

| Decay          | Experiment | fit  |
|----------------|------------|------|
| $M_{f_0(1710)}$ (MeV) | 1718 ± 6   | 1718 |
| $M_{f_0(1500)}$ (MeV) | 1507 ± 5   | 1504 |
| $M_{J/\psi}$ (MeV) | 1350 ± 150 | 1346 |
| $\Gamma(f_0(1500) \to pp)$ | 0.145 ± 0.027 | 0.081 |
| $\Gamma(f_0(1500) \to \pi\pi)$ | 0.246 ± 0.026 | 0.27 |
| $\Gamma(f_0(1710) \to \pi\pi)$ | 0.30 ± 0.20 | 0.34 |
| $\Gamma(f_0(1710) \to KK)$ | 0.48 ± 0.15 | 0.51 |
| $\Gamma(f_0(1710) \to \eta\eta)$ | 0.88 ± 0.23 | 1.12 |
| $\Gamma(f_2(1270) \to \pi\pi)$ | 0.34 ± 0.06 | 0.46 |
| $\Gamma(f_2(1270) \to KK)$ | 0.054 ± 0.005 | 0.057 |
| $\chi^2$/d.o.f. | 2.5        |      |

Table 2. Predicted decay properties of scalar mesons.

| Decay          | Experiment | fit  |
|----------------|------------|------|
| $\Gamma(f_0(1370) \to KK)$ | 0.79 |      |
| $\Gamma(f_0(1370) \to \pi\pi)$ | 0.35 ± 0.30 | 0.12 |
| $\Gamma(f_0(1370) \to \eta\eta)$ | 0.003 ± 0.001 | 0.005 |
| $\Gamma(f_2(1270) \to \pi\pi)$ | 6.6 ± 2.7 | 4.1 |
| $\Gamma(f_2(1270) \to KK)$ | 0.47 |      |
| $\Gamma(f_0(1370) \to \eta\eta)$ | 2.56 |      |
| $\Gamma(f_0(1370) \to PP)$ | <137 ± 8 | 133 |
| $\Gamma_{f_0(1710) \to PP}$ | 146 |      |

composed mainly of the scalar glueball, it should be the most prominent scalar produced in radiative $J/\psi$ decay. Hence, it is expected that $\Gamma(J/\psi \to f_0(1710)) \gg \Gamma(J/\psi \to f_0(1500))$, a relation borne out by experiment. (v) It has been argued that the non-observation of $f_0(1710)$ in $p\bar{p}$ annihilation at Crystal Barrel implies an $s\bar{s}$ structure for the $f_0(1710)$. However, both $f_0(1710)$ and $f_0(1500)$ are observed in a recent study of the reaction $p\bar{p} \to \eta\eta\rho^0$ at Fermilab. (vi) The $2\gamma$ coupling to scalar quarkonia has been studied in detail in. In our mixing model, the relative $2\gamma$ coupling strength is $f_0(1370) : f_0(1500) : f_0(1710) = 9.3 : 1.0 : 1.5$. Hence $f_0(1500)$ has the smallest $2\gamma$ coupling of the three states.

Acknowledgments

I’m grateful to Keh-Fei Liu and Chun-Khiang Chua for collaboration on this interesting topic.

References

1. C. Amsler and F.E. Close, Phys. Lett. B 353, 385 (1995); Phys. Rev. D 53, 295 (1996); F.E. Close and A. Kirk, Phys. Lett. B 483, 345 (2000).
2. C. Amsler et al., Phys. Lett. B 342, 433 (1995); Phys. Lett. B 340, 259 (1994).
3. G. Bali et al. (UKQCD), Phys. Lett. B 309, 378 (1993); C. Michael and M. Teper, Nucl. Phys. B 314, 347 (1989).
4. F.E. Close and Q. Zhao, Phys. Rev. D 71, 094022 (2005).
5. X.G. He, X.Q. Li, X. Liu, and X.Q. Zeng, Phys. Rev. D 73, 051502 (2006); ibid. D 73, 114026 (2006).
6. N. Mathur et al., hep-ph/0607110.
7. Y. Chen et al., Phys. Rev. D 73, 014516 (2006).
8. C. Morningstar and M. Peardon, Phys. Rev. D 56, 3043 (1997); Phys. Rev. D 60, 034509 (1999).
9. H.Y. Cheng, C.K. Chua, and K.F. Liu, hep-ph/0607206.
10. Particle Data Group, Y.M. Yao et al., J. Phys. G 33, 1 (2006).
11. M.S. Chanowitz, Phys. Rev. Lett. 95, 172001 (2005).
12. K.T. Chao, X.G. He, and J.P. Ma, hep-ph/0512327.
13. J. Sexton, A. Vaccarino, and D. Weingarten, Phys. Rev. Lett. 75, 4563 (1995).
14. C. Amsler et al. Eur. Phys. J. C 23, 29 (2002).
15. Uman et al., Phys. Rev. D 73, 052009 (2006).
16. F.E. Close, G.R. Farrar, and Z. Li, Phys. Rev. D 55, 5749 (1997).