Reggeon Non–Factorizability
and the $J = 0$ Fixed Pole in DVCS

Stanley J. Brodsky, Felipe J. Llanes-Estrada, J. Timothy Londergan and Adam P. Szczepaniak

1- SLAC National Laboratory, Stanford University
2575 Sand Hill Road, 94025 Menlo Park, CA, USA
2- Universidad Complutense de Madrid, Departamento de Física Teórica I
Avda. Complutense s/n, 28040 Madrid, Spain
3- Indiana University, Nuclear Theory Center and Physics Department
Bloomington, IN 47405, USA.

We argue that deeply virtual Compton scattering will display Regge behavior $\nu^\gamma R(t)$ at high energy at fixed-$t$, even at high photon virtuality, not necessarily conventional scaling. A way to see this is to track the Reggeon contributions to quark-nucleon scattering and notice that the resulting Generalized Parton Distributions would have divergent behavior at the break–points. In addition, we show that the direct two-photon to quark coupling will be accessible at large $t$ where it dominates the DVCS amplitude for large energies. This contribution, the $J = 0$ fixed–pole, should be part of the future DVCS experimental programs at Jlab or LHeC.

It is commonly believed that the DVCS (Deeply Virtual Compton Scattering) amplitude scales; i.e., at high energy, its energy $\nu = (s-u)/4$ and photon virtuality $Q^2$ dependence enters only through the scaling variable $\xi = Q^2/(2\nu)$. However, we have argued that instead, DVCS and possibly other hard exclusive processes will have the characteristic Regge behavior $T \propto \nu^\alpha(t)$ at large $s$ at fixed $t$ and photon virtuality $Q^2$. This distinction has been noted in the past (see, for example, the work of Bjorken and Kogut \cite{2} and the $t$–channel analysis in ref. \cite{3}). Here we address the consequences of Regge behavior for Generalized Parton Distributions (GPD’s) \cite{4}.

Conventional analyses of DVCS are based on the collinear factorization theorem \cite{6} which expresses the Compton amplitude in terms of the convolution of hard–scattering kernel and a soft hadronic amplitude, which for the helicity conserving case, is the $H$ GPD \cite{7},

$$T_+ (\xi, t) = - \int_{-1}^1 dx H(x, \xi, t) \left( \frac{1}{x+\xi - i\epsilon} + \frac{1}{x-\xi + i\epsilon} \right)$$

This expression is valid whenever the GPD $H$ is continuous at the "break–points" $x = \pm \xi$. This continuity is an assumption in the derivation of the factorization theorem \cite{6}. In the handbag diagram, the longitudinal momentum fraction of the extracted and returning quark are respectively $k^+/P^+ = x-\xi$ and $x+\xi$. Thus the break–points correspond to either quark having zero momentum fraction and infinite light cone energy. Thus the parton-nucleon amplitude become singular at the break–points if it has energy dependence suggested by Regge scattering. Indeed in the case of DIS, it is well–known that structure functions have

\textsuperscript{a}Talk presented by FJLE at DIS09, Madrid, April 29\textsuperscript{th} 2009, see slides \cite{1}. Manuscript number SLAC-PUB-13694; \texttt{arXiv:0906.5515}.

\textsuperscript{b}The origin of Regge behavior in forward Compton scattering and deep inelastic scattering based on the handbag diagram dates back to the covariant parton model of Landshoff, Polkinghorne and Short \cite{4}. The analysis was extended to virtual Compton scattering in ref. \cite{3}.

\texttt{DIS 2009}

Work supported in part by US Department of Energy contract DE-AC02-76SF00515
Section of the text: Figure 1: Left panel: low–t Generalized Parton Distributions may present Regge divergences at the break–points \( x = \pm \xi, H(x, \xi, t) \propto (x - \xi)^{-\alpha(t)} \). Right panel: high–t GPD’s are the sum of an analytic part plus a non–analytic part that vanishes at the break–points. Collinear factorization then holds, in a regime where \( Q^2 >> -t > M^2_N \).

In addition to intuitive arguments, we have formally derived the break–point divergent behavior \( H(x, \xi, t) \propto (x \pm \xi)^{-\alpha} \) in [10]. We now briefly recall the demonstration. The hadron part of the Compton amplitude is a spin–tensor, the matrix element of a product of currents

\[
T_{\mu \nu} = i \int d^4 z e^{i k \cdot z} \langle p' \lambda' | T J_\mu(z/2) J_\nu(-z/2) | p \lambda \rangle.
\]

For large \( Q^2 \) the \( z \)-integral peaks at \( z^2 \sim 1/Q^2 \) and using the leading order operator product expansion of QCD we replace the product of the two currents by a product of two quark field operators and a free propagator between the photon interaction points \((z/2, -z/2)\)

\[
T_{\mu \nu} = -i e_q^2 \int \frac{d^4 k}{(2\pi)^4} A_{\beta \alpha}(k, \Delta, p, \lambda, \lambda') \left\{ \left( \frac{\gamma^\mu \left( \frac{q + q'}{2} \right) \gamma^\nu}{\left( \frac{q + q'}{2} + k \right)^2 + i\epsilon} - \left( \frac{\gamma^\nu \left( -\frac{q + q'}{2} \right) \gamma^\mu}{\left( \frac{q + q'}{2} - k \right)^2 + i\epsilon} \right) \right\}
\]

Here \( A \) is the parton-nucleon scattering amplitude [11] with quark propagators included

\[
A_{\beta \alpha} = -i \int d^4 z e^{-ikz} \langle p' \lambda'| T \bar{\psi}_{\alpha}(z/2) \psi_{\beta}(-z/2) | p \lambda \rangle.
\]
Figure 2: DVCS differential cross section as a function of the energy for constant scaling variable $\xi$ fixed by the H1 kinematics [9]. $Q^2 = 8 \text{ GeV}^2$, $W = 82 \text{ GeV}$. We argue that the DVCS amplitude at large $s$, $Q^2$ is Regge–behaved. This implies scaling violations which become less prominent for larger $t$. (The quantity plotted, $2Q^2W^2d\sigma/dt$ should scale at LO.) When $t$ is large enough such that $\alpha(t) < \alpha_0$, all Reggeons have receded. The resulting amplitude at large $s$ is then a real, $Q^2$–independent constant, the $J = 0$ fixed pole, which reveals the two–photon coupling of quarks (right).

From the GPD point of view, this arises because $H$ is a projection of the parton-nucleon amplitude $H(x, \xi, t) \propto p^+ \int \frac{d^4k}{(2\pi)^4} \delta(xp^+ - k^+) A$, so the Regge behavior of $A$ (natural for a hadron-hadron scattering amplitude) is inherited by the GPD in Eq. (1). However, Eq. (1) and (5) come together at large $-t$. In that case $\alpha_R(-t) < \alpha_0$ for conventional Reggeons and their contribution to the amplitude decreases with $s$. But a contribution remains with $\alpha_R = 0$, independent of $t$ and photon virtuality; it dominates the amplitude at sizeable $-t$, turning it to a real constant: the $J = 0$ fixed pole (see fig. 2). This contribution is familiar in atomic physics, corresponding to high energy Compton scattering on the atomic electrons. This constant acts as a subtraction in the dispersion relation for the Compton amplitude [13]

$$T_+(\xi, t) = C_\infty(t) + \frac{\xi^2}{\pi} \int_0^1 2dx \frac{\text{Im} T_+(x,t)}{x - \xi^2 - i\epsilon}.$$  

(A different issue is the inadequacy of the handbag diagram to represent leading-twist diffractive DIS, even in light–cone gauge [14].)

DIS 2009
The value of $C_\infty$ is not fixed by the dispersive integral, and its necessity is revealed by the dynamical insight that the target nucleon’s components which carry charge are elementary, so that a seagull-like $\gamma \gamma qq$ coupling is active.\(^4\)

The $J = 0$ contribution arises from a term in the numerator of the Feynman propagator in the case of spin-$1/2$ quarks which cancels the energy denominator; it thus has no imaginary part and no $s$ dependence (hence its real, constant nature). The resulting seagull–like interaction extends over $t - z$, conjugate to $k^+$ and $k^+$. This gives it a characteristic $\gamma^+/x$ dependence:

$$\begin{align*}
\frac{k^+ + q^+ + m}{(k + q)^2 - m^2 + i\epsilon} &\rightarrow \frac{\gamma^+}{2p^+} \left( \frac{1}{x} + \frac{\xi}{x} + \frac{1}{x - \xi + i\epsilon} \right) = \frac{\gamma^+}{2p^+} \frac{1}{x - \xi + i\epsilon} \\
- \frac{k^- - q^- + m}{(k - q)^2 - m^2 + i\epsilon} &\rightarrow \frac{\gamma^+}{2p^-} \left( \frac{1}{x} - \frac{\xi}{x} + \frac{1}{x + \xi - i\epsilon} \right) = \frac{\gamma^+}{2p^-} \frac{1}{x + \xi - i\epsilon} \text{.}
\end{align*}$$

(7)

Comparing Eq.(1), (6), we find at high enough energy

$$C_\infty(t) = \lim_{\xi \to 0} T_+(\xi, t) = -2 \int_{-1}^{1} dx H(x, 0, t) = -2 F_{1/2}(t) \text{.}$$

(8)

The real part of the DVCS amplitude can be identified by interference with the Bethe-Heitler amplitude, thus accessing the $1/x$ form factor of the nucleon \[15\]. In our recent work \[13\], we have been able to establish how the same fixed pole amplitude also appears in real as well as doubly virtual Compton scattering, showing its universal character. This extension of the $1/x$ form factor to small $-t$ is achieved by analytic continuation in $t$, defining a valence part that is free of Regge behavior $H_v(x, 0, t) \equiv H(x, 0, t) - H_R(x, t)$,

$$H_R(x, t) \equiv \theta(x) \sum_{\alpha > 0} \frac{\gamma_{\alpha}(t)}{x^{\alpha(t)}} - \theta(-x) \sum_{\alpha > 0} \frac{\bar{\gamma}_{\alpha}(t)}{(-x)^{\alpha(t)}} \text{.}$$

(9)

$$F_{1/2}(t) \equiv \int_{-1}^{1} dx H_v(x, 0, t) - \sum_{\alpha > 0} \frac{\gamma_{\alpha}(t)}{x^{\alpha(t)}} - \sum_{\alpha > 0} \frac{\bar{\gamma}_{\alpha}(t)}{(-x)^{\alpha(t)}} \text{.}$$

(10)

In conclusion, we have argued that DVCS at fixed $-t$ is Regge–behaved and at large $-t$ one can extract from it a $J = 0$ fixed pole contribution which is a distinct part of the handbag diagram for spin $1/2$ constituents. In the case of spin zero quarks, the handbag diagram is not sufficient to obtain the DVCS amplitude; the seagull is necessary. This gives an energy independent $J = 0$ fixed pole contribution to the DVCS amplitude which is independent of the incident or outgoing photon virtualities at fixed $t$, which is not given directly by the GPD–based handbag diagram.

**Acknowledgments**

Presented by FJLE. Financial support from grants FPA 2008-00592/FPA, FIS2008-01323, PR27/05-13955-BSCH (Spain), and DE-FG0287ER40365, [nsf-phy0555232](http://www.nsf.gov) (USA).

\(^4\)A subtraction term is needed for Compton scattering since the low energy amplitude is fixed by the negative-signed Thomson term; the unsubtracted dispersion relation has the wrong sign. $C_\infty$ receives contributions from the Thomson term and the constant contribution from the dispersive integral, which curiously enough, Damashek and Gilman\[12\] found to be small.

\(^5\)Hints of the fixed pole behavior can be seen in the Cornell and Jefferson lab data \[16, 17\].
References

[1] Slides presented at the conference: http://indico.cern.ch/contributionDisplay.py?contribId=101&sessionId=0&confId=53294

[2] J. D. Bjorken and J. B. Kogut, Phys. Rev. D 8 (1973) 1341.

[3] A. P. Szczepaniak and J. T. Londergan, Phys. Lett. B 643 (2006) 17.

[4] P. V. Landshoff, J. C. Polkinghorne and R. D. Short, Nucl. Phys. B 28 (1971) 225.

[5] S. J. Brodsky, F. E. Close and J. F. Gunion, Phys. Rev. D 8 (1973) 3678.

[6] J. C. Collins, L. Frankfurt and M. Strikman, Phys. Rev. D 56 (1997) 2982; J. C. Collins and A. Freund, Phys. Rev. D 59 (1999) 074009.

[7] X. D. Ji, J. Phys. G 24 (1998) 1181; A. V. Radyushkin, Phys. Rev. D 56 (1997) 5524; M. Diehl, Phys. Rept. 388 (2003) 41.

[8] D. D. Coon, et al, Phys. Rev. D 18 (1978) 1451.

[9] A. Aktas et al. [H1 Collaboration], Eur. Phys. J. C 44 (2005) 1.

[10] A. P. Szczepaniak, J. T. Londergan and F. J. Llanes-Estrada, arXiv:0707.1239 [hep-ph]. To appear in APB.

[11] S. J. Brodsky and F. J. Llanes-Estrada, Eur. Phys. J. C 46 (2006) 751.

[12] M. Damashek and F. J. Gilman, Phys. Rev. D 1 (1970) 1319.

[13] S. J. Brodsky, F. J. Llanes-Estrada and A. P. Szczepaniak, arXiv:0812.0395 [hep-ph], to appear in Phys. Rev. D.

[14] S. J. Brodsky et al, Phys. Rev. D 65 (2002) 114025.

[15] S. J. Brodsky, F. J. Llanes-Estrada and A. P. Szczepaniak, In the Proceedings of 11th International Conference on Meson-Nucleon Physics and the Structure of the Nucleon (MENU 2007), Julich, Germany, 10-14 Sep 2007, pp 149 [arXiv:0710.0981 [nucl-th]].

[16] M. A. Shupe et al., Phys. Rev. D 19 (1979) (1921).

[17] A. Danagoulian et al. [Hall A Collaboration], Phys. Rev. Lett. 98 (2007) 152001.

[18] S. J. Brodsky, M. Diehl and D. S. Hwang, Nucl. Phys. B 596 (2001) 99.

DIS2009