Theory of Charmonium Production

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I give an overview of the current status of the theory of charmonium production in hard-scattering processes and summarize the present state of comparisons between theory and experiment.

PRESENTED AT

Charm 2012
The 5th International Workshop on Charm Physics
Honolulu, Hawaii, May 14–17, 2012
1 Introduction

Since their discovery in the mid-1970s, heavy-quarkonium states have played an important role in helping us to understand QCD. Because heavy-quarkonium states are nonrelativistic, they allow the application of theoretical tools that simplify and constrain the analyses of nonperturbative effects. Hence, heavy-quarkonium states provide a unique laboratory in which to explore the interplay between perturbative and nonperturbative effects in QCD.

In this talk I begin by describing the theoretical framework for calculations of inclusive quarkonium production processes, with emphasis on the NRQCD factorization approach [1]. I then describe the current status of the comparison between theoretical predictions and experimental measurements.*

2 Theoretical Framework

A number of theoretical approaches have been proposed for the calculation of heavy-quarkonium production processes. These include the NRQCD factorization approach [1], the fragmentation approach [4, 5], the color-singlet model (CSM) [6, 7, 8, 9, 10], the color-evaporation model (CEM) [11, 12, 13, 14], and the $k_T$-factorization approach [15, 16, 17, 18]. Only NRQCD factorization and the fragmentation approach are believed to be methods that could be derived from QCD. NRQCD factorization is the default model for most current studies of quarkonium production.

2.1 NRQCD Factorization of the Inclusive Production Cross Section

Nonrelativistic QCD (NRQCD) is an effective field theory that describes the behavior of bound states of a heavy-quark ($Q$) and a heavy-antiquark ($\bar{Q}$) when the velocity $v$ of the $Q$ or $\bar{Q}$ in the $Q\bar{Q}$ rest frame is nonrelativistic ($v \ll 1$). Some years ago, Bodwin, Braaten and Lepage [1] conjectured that the inclusive cross section for producing a quarkonium at large momentum transfer ($p_T$ or $p^*$) can be written as a sum of products of “short-distance” coefficients times NRQCD matrix elements:

$$\sigma(H) = \sum_n F_n(\Lambda) \langle 0 | O^H_n(\Lambda) | 0 \rangle.$$

The “short-distance” coefficients $F_n(\Lambda)$ are essentially the process-dependent partonic cross sections to make a $Q\bar{Q}$ pair, convolved with the parton distributions of the incoming hadrons. The NRQCD long-distance matrix elements (LDMEs) $\langle 0 | O^H_n(\Lambda) | 0 \rangle$.

*See Refs. [2, 3] for some further details regarding these issues.
are the probability for a $Q\overline{Q}$ pair to evolve into a heavy quarkonium. $\Lambda$ is the factorization scale, which is the cutoff of the effective field theory. The NRQCD matrix elements are vacuum expectation values of four-fermion operators in NRQCD, but with a projection onto an intermediate state of the quarkonium $H$ plus anything:

$$O_n^H(\Lambda) = \langle 0 | \chi^\dagger \kappa_n \psi \left( \sum_X |H + X\rangle \langle H + X| \right) \psi^\dagger \kappa'_n \chi |0\rangle.$$  

(2)

Here, $\psi^\dagger$ and $\chi$ are two-component (Pauli) fields that create a heavy quark and a heavy antiquark, respectively, and $\kappa_n$ and $\kappa'_n$ are direct products of Pauli and color matrices.$^\dagger$

The short-distance coefficients have expansions in powers of $\alpha_s$. The LDMEs are nonperturbative, but they are conjectured to be universal, i.e., process independent. Only the color-singlet production LDMEs are simply related to the decay LDMEs. The LDMEs have a known scaling with $v$, where $v^2 \approx 0.23$ for the $J/\psi$ and $v^2 \approx 0.1$ for the $\Upsilon(1S)$. Hence, the NRQCD factorization formula is a double expansion in powers of $\alpha_s$ and $v$.

The current phenomenology of $J/\psi$, $\psi(2S)$, and $\Upsilon(nS)$ production uses LDMEs through relative order $v^4$:

$$\langle O^H(3S_1^{[1]}) \rangle \quad (O(v^0)),$$
$$\langle O^H(1S_0^{[8]}) \rangle \quad (O(v^3)),$$
$$\langle O^H(3S_1^{[8]}) \rangle \quad (O(v^4)),$$
$$\langle O^H(3P_1^{[8]}) \rangle \quad (O(v^4)).$$

(3)

Here, the superscripts $H$ stand for any spin-triplet, $S$-wave quarkonium state, including the $J/\psi$, the $\psi(2S)$, and the $\Upsilon(nS)$ states, all of which have different NRQCD matrix elements. The quantities in parentheses are the spin, orbital-angular-momentum, and color (singlet or octet) quantum numbers of the $Q\overline{Q}$ pair that evolves into the quarkonium state $H$.

A key feature of NRQCD factorization is that quarkonium production can occur through color-octet, as well as color-singlet, $Q\overline{Q}$ states. The color-singlet production LDMEs $\langle O^H(3S_1^{[1]}) \rangle$ can be determined from quarkonium electromagnetic decay rates (up to corrections of order $v^4$). However, the color-octet LDMEs must be determined through comparisons of theoretical predictions with measurements. Once they have been fixed through comparisons between theory and experiment in one or more processes, the LDMEs can be used, by virtue of the property of universality, to make predictions for other processes.

$^\dagger$It was pointed out by Nayak, Qiu, and Sterman that gauge invariance requires that the definitions of the NRQCD LDMEs include Wilson lines that run from the quark and antiquark fields to infinity. For simplicity, I have omitted these Wilson lines here.
One obtains the CSM by retaining, for a given process, only the contribution that
is associated with the color-singlet LDME of the lowest nontrivial order in $\nu$. The
CSM is theoretically inconsistent in that it leads to uncanceled infrared divergences
in calculations of production and decay processes for $P$-wave quarkonium states.

2.1.1 Status of a Proof of NRQCD Factorization

A proof of NRQCD factorization is complicated because gluons can dress the quarko-
nium basic production process in ways that apparently violate factorization in indi-
vidual Feynman diagrams. A proof of factorization would involve a demonstration
that diagrams in each order in $\alpha_s$ can be re-organized so that (1) all soft singularities
cancel or can be absorbed into NRQCD LDMEs and (2) all collinear singularities
and spectator interactions can be absorbed into parton distributions. Nayak Qiu
and Sterman have demonstrated factorization for the leading power in $1/p_T$ through
next-to-next-to-leading order (NNLO) in $\alpha_s$. However, it is not known if the proof
generalizes to all orders in $\alpha_s$. Note that an all-orders proof is essential because
potential violations of factorization involve soft gluons, for which $\alpha_s$ is not a good
expansion parameter. It seems likely that, if a proof of NRQCD factorization can be
established, it would hold only for values of $p_T$ that are greater than the heavy-quark
mass $m_Q$.

2.2 The Problem of Large Higher-Order Corrections

2.2.1 Higher-Order Corrections in $J/\psi$ and $\Upsilon(1S)$ Production

When one calculates quarkonium production cross sections differential in $p_T$ in the
NRQCD factorization approach, large corrections appear in higher orders in pertur-
bation theory. These large corrections were first noticed in calculations of quarkonium
production at hadron-hadron colliders through the color-singlet channel [19]. An ex-
ample of this phenomenon is shown in Fig. 1. The curve labeled “NNLO∗” is an
estimate that is based on only the real-emission contributions at NNLO in $\alpha_s$. Note
that the color-singlet contributions alone do not explain the data.

An example of large higher-order corrections in $\Upsilon(1S)$ production in the color-
singlet channel is shown in Fig. 2. The next-to-leading order (NLO) results that are
shown in this figure were confirmed by Gong and Wang [22]. In this case, the data
could be explained by NNLO∗ color-singlet production alone. However, given the
large error bars, there is still room for a substantial of color-octet contribution.

A very large correction in next-to-leading order in $\alpha_s$ also arises in $J/\psi$ production
through the color-octet $3P_J$ channel, where a negative correction has been found
[23, 24]. However, NLO corrections to the $S$-wave color-octet channels are small [25].
The corrections for production at the Tevatron are approximately factors of 1.235 for
the $1S_0$ color-octet channel and 1.139 for the $3S_1$ color-octet channel.
These results raise several questions: (1) Does the perturbation series converge? (2) How does one understand the different sizes of the higher-order corrections for different channels?
It has been suggested by Campbell, Maltoni, and Tramontano [26] that the large higher-order corrections to quarkonium production can be understood as follows: At high $p_T$, higher powers of $\alpha_s$ can be offset by a less rapid fall-off with $p_T$. For example, for production of the $J/\psi$ through the $S$-wave, color-singlet channel, the contribution at leading order (LO) in $\alpha_s$ goes as $\alpha_s^3 m_c^4 / p_T^8$. In contrast, the NLO contribution goes as $\alpha_s^4 m_c^2 / p_T^6$ and is enhanced at high $p_T$ relative to the LO contribution.\footnote{The NLO correction also contains a contribution, which arises from charm-quark fragmentation into a $J/\psi$, that goes as $\alpha_s^4 / p_T^4$. This charm-quark fragmentation contribution happens to be small numerically.} The NNLO contribution to $S$-wave, color-singlet production of the $J/\psi$ is further enhanced at high $p_T$, since it goes as $\alpha_s^5 / p_T^4$, owing to contributions in which a gluon fragments into the $J/\psi$. The leading power behavior of the cross section at high $p_T$ is $1/p_T^4$ [4, 5]. Hence, one expects no further large kinematic enhancements beyond NNLO.

The NNLO calculation of color-singlet, $S$-wave production of $S$-wave charmonium states is, so far, incomplete, as only the real-emission (NNLO$^*$) correction has been computed. It has been noticed by Ma, Wang, Chao [27] that the color-singlet NNLO$^*$ correction seems be dominated by contributions that are proportional to $\log^2(p_T^2/p_{\text{cut}}^2)$, where $p_{\text{cut}}^2 > m_Q^2$ is an artificial cutoff that is introduced to make the calculation finite. These contributions will cancel when virtual corrections are included in the complete NNLO calculation. Hence, the complete NNLO contribution is likely to be significantly smaller than the NNLO$^*$ contribution and cannot, by itself, explain the observed production cross sections of $S$-wave charmonium states.

Kinematic enhancements also account for the large corrections in the color-octet channels. The color-octet $^3S_1$ channel receives a small correction in NLO because it already has $1/p_T^4$ behavior in LO, owing to a gluon-fragmentation contribution. The color-octet $^3P_J$ channel receives a large correction in NLO because it first shows $1/p_T^4$ behavior in NLO, again owing to gluon fragmentation. The color-octet $^1S_0$ channel also shows first shows $1/p_T^4$ behavior in NLO, and that behavior arises from gluon fragmentation. However, the NLO correction to the color-octet $^1S_0$ channel is numerically small at moderate $p_T$ because the fragmentation process has little support near $z = 1$.\footnote{The absence of peaking near $z = 1$ may reflect the fact that the color-octet $^1S_0$ fragmentation process contains no soft divergences in full QCD [28]. Such soft divergences are ultimately absorbed into the NRQCD LDMEs, rendering the physical cross section finite, but the remnants of the soft divergences can produce a peaking near $z = 1$.}

Note that, at NLO, all of the color-octet channels that appear in $S$-wave quarkonium production already contain contributions that go as $1/p_T^4$. This suggests that no further large enhancements will occur in still higher orders in $\alpha_s$ in these channels. At leading-order (LO) in $\alpha_s$, only the $^3S_1$ color-octet channel has a $1/p_T^4$ behavior. This fact has been used to argue that the $^3S_1$ color-octet channel is dominant at large
However, at NLO, all of the color-octet channels have a $1/p_T^4$ behavior, and so the argument for the dominance of the $^3S_1$ color-octet channel at large $p_T$ is not valid.

### 2.3 The Fragmentation Approach

Kang, Qiu, and Sterman [4, 5] have suggested that one reorganize the perturbation expansions for inclusive quarkonium production cross sections according to the $p_T$ dependence of the various contributions. Specifically, they find that the leading behavior in $1/p_T$ in the production cross section $(1/p_T^4)$ comes from contributions in which single-parton production cross sections are convolved with the fragmentation functions for a single parton into a quarkonium:

$$d\hat{\sigma}_{A+B\to i+X} \otimes D_{i\to H}. \quad (4)$$

They also find that the first subleading behavior in $1/p_T$ in the production cross section ($m_Q^2/p_T^6$) comes from $Q\bar{Q}$ production cross sections convolved with fragmentation functions for a $Q\bar{Q}$ pair into a quarkonium:

$$d\hat{\sigma}_{A+B\to Q\bar{Q}+X} \otimes D_{Q\bar{Q}\to H}. \quad (5)$$

Nayak, Qiu, and Sterman have provided convincing arguments that these results hold to all orders in perturbation theory, up to corrections of order $m_Q^4/p_T^8$. The fragmentation approach of Kang, Qiu, and Sterman is well suited to the analysis of phenomenon of large higher-order corrections because it allows one to identify the specific sources of the kinematic enhancements at high $p_T$, which arise from the leading and first subleading powers in $1/p_T$ in the quarkonium production rates. Hence, the fragmentation approach should allow one to reduce the uncertainties in the theoretical predictions by focusing on those processes that produce the large corrections, calculating them to yet higher orders in $\alpha_s$, and resumming the associated logarithms of $p_T^2/m_Q^2$, which are large at high $p_T$.

It is important to check that the fragmentation formalism really does account for all of the large corrections. It has been confirmed that the fragmentation contribution reproduces most of the large correction in the color-singlet channel at NLO [5]. There is also an estimate of the fragmentation contribution in the $^3P_J$ color-octet channel that indicates that it gives a significant part of the NLO correction in that channel [28]. Several other tests of the fragmentation approach are in progress.

The values of the nonperturbative fragmentation functions that appear in this formalism are, so far, unknown. However, if NRQCD factorization holds, then the fragmentation functions of Kang, Qiu, and Sterman can be written as a sum of NRQCD LDMEs times perturbatively calculable short-distance coefficients. In that case, the fragmentation approach is equivalent to a reorganization of the NRQCD...
factorization formula (1) according to the behaviors $1/p_T^4$ and $m_Q^2/p_T^8$, with the stipulation that there are corrections of order $m_Q^4/p_T^8$ that are not accounted for in the fragmentation approach.

### 3 Comparisons of NRQCD Factorization with Experiment

#### 3.1 Overview

NLO corrections to charmonium production cross sections and polarizations have been computed for many production processes: $J/\psi$ and $\psi(2S)$ production cross sections and polarization at the Tevatron and the LHC; $J/\psi$ and $\psi(2S)$ production cross sections at RHIC; $J/\psi$ photoproduction cross sections and polarization at HERA; the $J/\psi + \eta_c$ production cross section, the $J/\psi + \phi$ production cross section, and the $J/\psi + X$ (non-$\phi$) production cross section in $e^+e^-$ annihilation at the $B$ factories. An NLO calculation of the $\chi_J$ production cross sections for the individual $J$ states also exists [29].

Generally, data and the NLO predictions of NRQCD factorization for quarkonium production agree, within errors.\(^\dagger\) There are three significant exceptions: (1) quarkonium polarization in hadron-hadron collisions, (2) the cross section for the production of $J/\psi + X$ (non-$\phi$) in $e^+e^-$ annihilation at Belle, and (3) the cross section for $J/\psi$ production in $\gamma\gamma$ scattering at LEP II. I will discuss each of these exceptions below, following a discussion of the extraction of the NRQCD LDMEs from hadroproduction and photoproduction data.

#### 3.2 Extraction of NRQCD LDMEs at NLO

I now describe an important issue that arises in extracting NRQCD matrix elements from fits of NLO NRQCD predictions to $J/\psi$ production data.\(^\dagger\)

Recently, two groups, Ma, Wang, and Chao [23, 27] and Butenschoen and Kniehl [24], have carried out the first complete calculations of $J/\psi$ hadroproduction at NLO. These calculations are complete in the sense that they include all of the color-singlet and color-octet channels that contribute through order $v^4$. [See Eq. (3).] The results of the two groups for the NRQCD short-distance coefficients agree. However, the fitted NRQCD matrix elements are very different.

Using the CDF Run II data, Ma, Wang, and Chao [23, 27] could fit only two linear

\(^\dagger\)See the global fit of NRQCD matrix elements of Butenschoen and Kniehl [30, 31] and Ref. [3] for some further details.
combinations of matrix elements unambiguously:

\[ M_{0, r_0} = \langle O^\psi (1 S_0^{[8]}) \rangle + \left( r_0 / m_c^2 \right) \langle O^\psi (3 P_0^{[8]}) \rangle = (7.4 \pm 1.9) \times 10^{-2} \text{ GeV}^3, \]
\[ M_{1, r_1} = \langle O^\psi (3 S_1^{[8]}) \rangle + \left( r_1 / m_c^2 \right) \langle O^\psi (3 P_0^{[8]}) \rangle = (0.05 \pm 0.03) \times 10^{-2} \text{ GeV}^3, \]  

(6)

where \( r_0 = 3.9 \) and \( r_1 = -0.56 \) were chosen on the basis of approximate relations between the short-distance coefficients.

Butenschön and Kniehl [30] used their NLO calculations for \( e\bar{p}, \gamma\gamma \), and \( e^+e^- \) production to fit all three color-octet LDMEs, using data from the Tevatron, LHC, RHIC, HERA, LEPII, and KEKB:

\[ \langle O^\psi (1 S_0^{[8]}) \rangle = (4.76 \pm 0.06) \times 10^{-2} \text{ GeV}^3, \]
\[ \langle O^\psi (3 S_1^{[8]}) \rangle = (0.265 \pm 0.014) \times 10^{-2} \text{ GeV}^3, \]
\[ \langle O^\psi (3 P_0^{[8]}) \rangle / m_c^2 = (-0.716 \pm 0.089) \times 10^{-2} \text{ GeV}^3, \]  

(7)

which implies that

\[ M_{0, r_0} = (2.17 \pm 0.56) \times 10^{-2} \text{ GeV}^3, \]
\[ M_{1, r_1} = (0.62 \pm 0.08) \times 10^{-2} \text{ GeV}^3. \]  

(8)

There are many small differences in the fitting procedures: (1) Ma, Wang, and Chao included feeddown from the \( \psi (2S) \) and the \( \chi_{cJ} \) states in the theoretical prediction, but Butenschön and Kniehl did not; (2) Ma, Wang, and Chao used a 2-parameter constrained fit, while Butenschön and Kniehl used a 3-parameter fit; (3) Ma, Wang, and Chao applied a cut \( p_T > 7 \text{ GeV} \) to the CDF data, while Butenschön and Kniehl applied a cut \( p_T > 3 \text{ GeV} \) to the CDF data. However, the most important difference between these fits is the use of the low-\( p_T \) H1 data by Butenschön and Kniehl. Indeed, the fit of Butenschön and Kniehl to the Tevatron and HERA data alone [24] gives a very similar result to that of the global fit:

\[ M_{0, r_0} = (2.5 \pm 0.08) \times 10^{-2} \text{ GeV}^3, \]
\[ M_{1, r_1} = (0.59 \pm 0.02) \times 10^{-2} \text{ GeV}^3. \]  

(9)

Note that, for the HERA data, \( p_T \) lies in the range \( 1 \text{ GeV} < p_T < 3 \text{ GeV} \). It is not clear that NRQCD factorization, if proven, would hold at such low values of \( p_T \).

Both fits describe the data within errors, but, as can be seen from Fig. 3, the shape of the Ma, Wang, and Chao fit agrees with the CDF data better than the shape of the Butenschön and Kniehl fit. The fit of Butenschön and Kniehl to the CDF data clearly overshoots the data at large \( p_T \), while the fit of Ma, Wang, and Chao does not. Resummation of large logarithms of \( p_T / m_c^2 \) at large \( p_T \) would lower the theoretical predictions because the fragmentation process softens the \( J/\psi \) momentum. This
resummation correction would improve the agreement of the fit of Butenschön and Kniehl with the CDF data and worsen the agreement of the fit of Ma, Wang, and Chao with the CDF data. Note that, in this figure (and in subsequent figures from the works of Butenschön and Kniehl), it is clear that the color-singlet contribution alone does not describe the data.

As can also be seen from Fig. 3, the prediction of Ma, Wang, and Chao deviates from the data at low $p_T$. Presumably, the prediction of Butenschön and Kniehl would show a similar deviation if it were extended to sufficiently low $p_T$. This deviation may be an indication that factorization is breaking down, and/or that large logarithms of $m_c^2/p_T^2$ must be resummed in the low-$p_T$ region. As can be seen in Fig. 4, there is also a slight discrepancy in shape in the fit of Butenschön and Kniehl to the H1 data, which might be attributable to a breakdown of factorization and/or a need to resum logarithms of $m_c^2/p_T^2$ in the low-$p_T$ region.

The fit of Ma, Wang, and Chao to the Tevatron data and the fit of Butenschön and Kniehl [24] to the Tevatron and HERA data have been used to make predictions for $J/\psi$ cross sections at the LHC. Comparisons to the LHCb data to the prediction of Ma, Wang, and Chao and to the prediction of Butenschön and Kniehl are shown in Fig. 5.

As can be seen, the prediction of Ma, Wang, and Chao yields a good fit to the data. The prediction of Butenschön and Kniehl also fits the data within errors, but
Figure 4: The global fit of Butenschön and Kniehl [30] compared to the H1 data [33, 34]. (Figure from Ref. [30].)

Figure 5: Comparisons of NLO predictions with the LHCb data [35]. Left: The prediction of Ma, Wang, and Chao. (Figure from Ref. [35].) Right: The prediction of Butenschön and Kniehl from a fit to the Tevatron and HERA data [24]. (Figure from Ref. [35].)

has the wrong shape: It overshoots the data at high $p_T$. This shape discrepancy at high $p_T$ is even more apparent in Fig. 6, which shows a prediction of Butenschön and Kniehl, which is based of their global fit, in comparison to Atlas data. Again, a resummation of large logarithms of $p_T^2/m_c^2$ at high $p_T$ may improve the shape of the Butenschön and Kniehl predictions, while worsening the shape of the Ma, Wang, and Chao predictions.
Comparisons of PHENIX data with the prediction of Ma, Wang, and Chao and the global fit of Butenschön and Kniehl are shown in Fig. 7. The prediction of Ma, Wang, and Chao agrees well with the data. This prediction takes into account the
effects of feeddown from the $\chi_{cJ}$ states and the $\psi(2S)$ in both the fit to the CDF data that is used to extract the LDMEs and in the prediction for the PHENIX data. The global fit of Butenschön and Kniehl undershoots the PHENIX data slightly, but still agrees within errors. Feeddown effects are not taken into account in the global fit of Butenschön and Kniehl, but they are expected to be small [24].

After this talk was presented, a new NLO calculation of $J/\psi$ hadroproduction by Gong, Wan, Wang, and Zhang [38] appeared. This calculation includes contributions at NLO from feeddown from the $\chi_{cJ}$ and $\psi(2S)$ states and extracts all three color-octet NRQCD LDMEs from a combined fit to the CDF Run II [20] and LHCb [35] measurements of the cross section, differential in $p_T$. The fit makes use only of data with $p_T \geq 7$ GeV. Gong et al. were able to determine three linear combinations of color-octet LDMEs with good accuracy. Their LDMEs yield

$$M_{0,r_0} = (6.0 \pm 1.0) \times 10^{-2} \text{ GeV}^3,$$

$$M_{1,r_1} = (0.07 \pm 0.01) \times 10^{-2} \text{ GeV}^3,$$

in good agreement with the fit of Ma, Wang, and Chao [Eq. (6)].

### 3.3 Polarization

Polarization of a $^3S_1$ quarkonium state is analyzed experimentally in terms of the angular distribution the lepton pairs into which it decays. The angular distribution is usually parametrized as

$$W(\theta, \phi) \propto 1 + \lambda_\theta \cos^2 \theta + \lambda_\phi \sin^2 \theta \cos(2\phi) + \lambda_{\theta\phi} \sin(2\theta) \cos \phi,$$

where $\theta$ and $\phi$ are the polar and azimuthal angles, respectively, of the positively charged lepton with respect to some coordinate system (often called the “frame”) and $\lambda_\theta$, $\lambda_\phi$, and $\lambda_{\theta\phi}$ are the measured polarization parameters. The parameter $\lambda_\phi$, which is also called $\alpha$ in the literature, is a measure of the amount of transverse or longitudinal polarization of the quarkonium: $\lambda_\phi = 1$ corresponds to completely transverse polarization, and $\lambda_\phi = -1$ corresponds to completely longitudinal polarization.

Quarkonium polarization at large $p_T$ has long been thought to provide a definitive test of the color-octet production mechanism. However, this is an idea that is based on arguments at LO in $\alpha_s$, and, as we will see, it may not be correct at NLO.

### 3.4 Polarization at LO in $\alpha_s$

At LO in $\alpha_s$, hadroproduction of a $^3S_1$ quarkonium at large $p_T$ is dominated by the process of gluon fragmentation into a color-octet $^3S_1$ $Q\overline{Q}$ pair. At large $p_T$ in the hadronic center-of-momentum (CM) frame, the gluon is nearly on mass shell, and, so, is transversely polarized. At LO in $\alpha_s$, that transverse polarization is transferred
completely to the $Q\bar{Q}$ pair in the perturbative fragmentation process. As the $Q\bar{Q}$ pair evolves nonperturbatively into a $^3S_1$ quarkonium through the emission of gluons that are soft in the $Q\bar{Q}$ rest frame, the transverse polarization is preserved at LO in $v$ because of the heavy-quark spin symmetry [39].\footnote{There are spin-flip interactions, which are suppressed as $v^3$. This suppression has been verified in a lattice calculation of decay matrix elements [40].} The transverse polarization is defined in the hadronic CM frame relative to the direction of the fragmenting gluon’s momentum, and is invariant under a boost in that direction. At LO in $\alpha_s$ and $v$, that boost vector coincides with the boost vector from the quarkonium rest frame to the hadronic CM frame, which defines the polar axis of the “helicity frame.” Hence, at LO in $\alpha_s$ and $v$, one expects $\lambda_0$ in the helicity frame to approach 1 as $p_T$ increases.

The LO prediction of transverse polarization at large $p_T$ has not been confirmed experimentally. As can be seen from the left-hand plot in Fig. 8, the LO prediction agrees, within errors, with the CDF Run I measurement, except for the two highest $p_T$ points. However, as can be seen from right-hand plot in Fig. 8, the LO prediction disagrees completely with the CDF Run II measurement. Feeddown contributions from the $\chi_c$ states (about 30%) and the $\psi(2S)$ state (about 10%) are included in both the data and the theoretical prediction. The CDF Run I and Run II measurements are also in disagreement with each other. The reason for this discrepancy has not been found.

The LO NRQCD factorization prediction for polarization of the $\Upsilon(1S)$ is compared with the CDF and D0 Run II data in Fig. 9. Both the CDF and D0 results are incompatible with the LO prediction, but in different regions of $p_T$. The CDF and

Figure 8: Comparisons of the LO NRQCD prediction for polarization of the $J/\psi$ [41] with CDF data. Left: The CDF Run I data [42]. (Figure provided by Jungil Lee.) Right: The CDF Run II data [20]. (Figure from Ref. [20].)
Figure 9: Comparison of the LO NRQCD prediction for polarization of the $\Upsilon$ [41] with the CDF Run II data [43] and the D0 Run II data [44]. (Figure provided by Hee Sok Chung, using Refs. [43, 44].)

D0 results are also incompatible with each other. No reason for the incompatibility of the experimental results has been found.

3.4.1 First Complete NLO Calculations of Polarization

The first NLO calculation of $J/\psi$ polarization in photoproduction was carried out by Butenschön and Kniehl [45]. This calculation is complete in the sense that it includes all of the production channels that appear in NRQCD through relative order $v^4$ [Eq. (3)]. It was preceded by a calculation at NLO for the $3S_1$ color-singlet channel for $J/\psi$ photoproduction [46].

The first complete NLO calculations of $J/\psi$ polarization in hadroproduction were carried out by Butenschön and Kniehl [47] and by Chao, Ma, Shao, Wang and Zhang [48]. These calculations also include all of the production channels that appear in NRQCD through relative order $v^4$. They were preceded by partial calculations at NLO for the $3S_1$ color-singlet channel for $J/\psi$ hadroproduction [22] and $\Upsilon(1S)$ hadroproduction [19] and by calculations at NLO for the $3S_1$ and $1S_0$ color-octet channels for $J/\psi$ hadroproduction [25] and $\Upsilon(1S')$ hadroproduction [49].

At NLO, the theoretical predictions for polarization change dramatically. Polarization in the color-singlet channel changes from transverse to longitudinal [19, 22]. However, this channel makes a very small contribution to the hadroproduction cross section. A much more important issue is that, at NLO at high $p_T$, there are large corrections to production in the $3P_J$ color-octet channel that are mostly transversely polarized. At NLO, the $3S_1$ color-octet channel is still transversely polarized at high $p_T$. However, owing to the large NLO corrections in the $3P_J$ color-octet channel, it is no longer the dominant channel for hadroproduction at high $p_T$, and it is not the only source of transverse polarization.
Fig. 10 shows the prediction of Butenschön and Kniehl for $J/\psi$ polarization in hadroproduction in comparison with the CDF Run I and Run II data and Alice data. The prediction is based on the LDMEs from the global fit of Butenschön and Kniehl [30]. It is in considerable disagreement with the data. On the other hand, as can be seen from Fig. 10, the NLO $J/\psi$ polarization prediction of Butenschön and Kniehl is in rough agreement with the Alice data. It should be noted, though, that the Alice data include $J/\psi$ production from $B$-meson decays and feeddown from the $\chi_{cJ}$ and $\psi(2S)$ states.

The LDMEs from the global fit of Butenschön and Kniehl can also be used to predict the polarization of the $J/\psi$ in photoproduction at HERA. That prediction is compared in Fig. 11 with H1 data. The data are roughly compatible with the theory at large $p_T$, but the error bars are large.

In contrast, Chao, Ma, Shao, Wang, and Zhang [48] determined the three color-octet NRQCD LDMEs by using data from the CDF Run II measurements of the $J/\psi$ cross section, differential in $p_T$, and the CDF Run I and Run II measurements of the $J/\psi$ polarization, differential in $p_T$. The resulting LDMEs are barely compatible with the LDMEs from the fit of Ma, Wang, and Chao [23, 27] to the production
cross section alone: The values of $M_{0,r_0}$ and $M_{1,r_1}$ that Chao et al. obtain differ from those of Ma, Wang, and Chao by about $2\sigma$. This tension between the two sets of LDMEs suggests that the LDMEs from the fit of Ma, Wang, and Chao might lead to a polarization prediction that deviates from the CDF Run II data, although perhaps with large uncertainties.

The LDMEs of Chao et al. lead to the prediction for $J/\psi$ polarization at CDF that is shown in Fig. 12. With this choice of LDMEs, the contributions of the color-octet $^3S_1$ and $^3P_J$ channels to the transverse polarization largely cancel, leaving a net polarization parameter $\lambda_\theta$ that is near zero, in agreement with the CDF Run II data. This choice of LDMEs still gives reasonable predictions for the $J/\psi$ production cross sections that have been measured by the Atlas and CMS Collaborations, as can be seen from Fig. 13. This reflects the fact that the $J/\psi$ LDMEs and, hence, the $J/\psi$ polarization in hadroproduction, are not well determined from fits to the production cross section alone. The Chao et al. LDMEs lead to a prediction of a slight longitudinal polarization at the LHC, as can be seen in Fig. 14. However, in Fig. 15, one can see that the Chao et al. LDMEs seem to be incompatible with the HERA photoproduction data, even at large $p_T$.

After this talk was presented, Gong et al. also completed a new NLO calculation. Figure 11: Comparison of the NLO NRQCD prediction of Butenschön and Kniehl [47] for the polarization of the $J/\psi$ with H1 data [34, 51]. (Figure from Ref. [47].)
Figure 12: Comparison of the NLO NRQCD prediction of Chao, Ma, Shao, Wang, and Zhang [48] for the polarization of the $J/\psi$ with the CDF Run I [42] and Run II [20] data. (Figure from Ref. [48].)

Figure 13: Comparisons of LHC data with the NLO NRQCD prediction for the $J/\psi$ cross section of Chao, Ma, Shao, Wang, and Zhang [48]. The prediction uses NRQCD LDMEs that are extracted from fits to the CDF measurements of the $J/\psi$ cross section [32] and the $J/\psi$ polarization [20, 42]. Left: The Atlas data [36]. Right: The CMS data [52]. (Figure from Ref. [48].)

of $J/\psi$ polarization in hadroproduction [38], including the effects at NLO of the feeddown contributions from the $\chi_{cJ}$ and $\psi(2S)$ states. Using the color-octet LDMEs from their fits to the CDF Run II and LHCb data, Gong et al. made predictions for the $J/\psi$ polarization. Their prediction for the polarization at CDF is intermediate between that of Butenschön and Kniehl [47] and that of Chao et al. [48], agreeing with the CDF Run I data [42], except for the highest-$p_T$ point, but disagreeing with the CDF Run II data [20]. Since the polarization predictions of Gong et al. rely only on data with $p_T \geq 7$ GeV, they are less subject to questions about the validity of factorization than the predictions of Butenschön and Kniehl [47].
Figure 14: NLO NRQCD prediction of Chao, Ma, Shao, Wang, and Zhang \cite{48} for the polarization of the $J/\psi$ at the LHC. (Figure from Ref. \cite{48}.)

Figure 15: Comparison of a prediction based on the LDMEs of Chao, Ma, Shao, Wang, and Zhang \cite{48} with the H1 data \cite{33, 34}. (Figure provided by Mathias Butenschön.)

### 3.5 $e^+e^- \rightarrow J/\psi + X$ (non-$c\bar{c}$)

The total cross section for the process $e^+e^- \rightarrow J/\psi + X$ (non-$c\bar{c}$) has been measured by the Belle Collaboration \cite{53}. Their result is

$$\sigma(e^+e^- \rightarrow J/\psi + X{\text{ (non-}}c\bar{c}) = 0.43 \pm 0.09 \pm 0.09 \text{ pb}. \quad (12)$$

NLO calculations using the NRQCD factorization approach have been carried out by Zhang, Ma, Wang, and Chao \cite{54} and by Butenschön and Kniehl \cite{30} and lead to the prediction

$$\sigma(e^+e^- \rightarrow J/\psi + X{\text{ (non-}}c\bar{c}) = 0.99^{+0.35}_{-0.17} \text{ pb} \quad (\mu = \sqrt{s}/2), \quad (13)$$
which is based on the NRQCD LDMEs from the global fit of Butenschön and Kniehl [30] and includes the effects of feeddown from the $\chi_{cJ}$ and $\psi(2S)$ states [54]. The comparison of the theoretical calculations with the measured cross section favors the value of $M_{0,ro}$ from the global fit of Butenschön and Kniehl, rather than the value from the fit of Ma, Wang, and Chao [23, 27].

There is tension between the Belle data and the theoretical prediction. However, it is worth noting that the most recent Belle measurements [53] imply that

$$\sigma(e^+e^- \to J/\psi + X) = \sigma(e^+e^- \to J/\psi + c\bar{c} + X) + \sigma(e^+e^- \to J/\psi + X(\text{non-}c\bar{c})) = 1.17 \pm 0.12^{+0.13}_{-0.12} \text{ pb},$$

while the BaBar Collaboration obtained [55]

$$\sigma(e^+e^- \to J/\psi + X) = 2.52 \pm 0.21 \pm 0.21 \text{ pb}. \quad (15)$$

This suggests the possibility that the Belle value of $\sigma(e^+e^- \to J/\psi + X(\text{non-}c\bar{c}))$ is too small. Furthermore, most of the Belle data are at $p_T < 3 \text{ GeV}$. Even if NRQCD factorization could be established at high $p_T$, it might not hold at such small values of $p_T$.

### 3.6 J/ψ Production in γγ Scattering at LEP II

Butenschön and Kniehl have also made an NLO calculation of $J/\psi$ production in $\gamma\gamma$ collisions at LEP II and incorporated it into their global fit [30]. This global fit is compared with DELPHI data in Fig. 16. The DELPHI data are slightly incompatible with the global fit. However, the error bars are large, especially at large $p_T$, and one should keep in mind that NRQCD factorization, even if it is established at high $p_T$, may not hold at such low values of $p_T$.

### 4 Conclusions

Studies of quarkonium production provide a unique laboratory in which to understand the interplay between the perturbative and nonperturbative aspects of QCD. They also provide an important testing ground for techniques for perturbative calculations of high-energy production cross sections.

At present, the primary method for theoretical calculations of quarkonium production is the NRQCD factorization approach. NRQCD factorization provides a systematic framework for computing quarkonium production. However, there is no proof of NRQCD factorization for quarkonium production beyond two loops. Furthermore, in spite of considerable theoretical and experimental progress over the last decade, a definitive test of NRQCD factorization has not yet been achieved.
Large corrections at NLO in $\alpha_s$ to inclusive quarkonium production are now believed to be understood in terms of kinematic enhancements. This understanding suggests that, for the color-octet production channels, which are believed to be much more important than the color-singlet production channels, there will be no large kinematic enhancements beyond NLO. If that is the case, then the existing NLO calculations of quarkonium production should at least be qualitatively correct. However, it seems likely that resummation of large logarithms will be needed in order to bring theory into good agreement with experiment at the large values of quarkonium $p_T$ that will be probed at the LHC.

The predictions of the color-singlet model fail to describe the data. In contrast, the predictions of NRQCD factorization are in agreement with most of the inclusive production data. There are several notable exceptions: (1) $J/\psi$ and $\Upsilon(2S)$ polarization at the Tevatron, (2) the $e^+e^- \rightarrow J/\psi + X (non-c\bar{c})$ total cross section, and (3) $J/\psi$ production in $\gamma\gamma$ scattering at LEP II. There are experimental discrepancies that cast some doubt on the polarization measurements and on the measurement of the $e^+e^- \rightarrow J/\psi + X (non-c\bar{c})$ total cross section. The measurement of $J/\psi$ production in $\gamma\gamma$ collisions has large experimental uncertainties. Furthermore, even if NRQCD factorization is established at high $p_T$, it might not hold at the low values of $p_T$ at which this measurement was made.

The $J/\psi$ cross sections measured at the Tevatron and HERA, the $J/\psi$ polarization measured by the CDF Collaboration in Run II, and NRQCD factorization seem to be mutually incompatible. Furthermore, the new polarization prediction of Gong et al. [38], which appeared after this talk was presented, suggests that the $J/\psi$ cross
sections measured by the CDF and LHCb collaborations, the \( J/\psi \) polarization measured by the CDF Collaboration in Run II, and NRQCD factorization are mutually incompatible. Some possible explanations for these incompatibilities are: (1) the CDF Run II \( J/\psi \) polarization measurement is wrong; (2) NRQCD factorization fails at the low values of \( p_T \) at which the HERA measurements were made; (3) additional corrections to the theoretical predictions are needed, such as those from higher orders in \( v \); (4) NRQCD factorization is incorrect.

Further advances in the theory of quarkonium production would help to sort out the various possibilities. These include a proof or a disproof of NRQCD factorization, calculations of corrections of higher order in \( \alpha_s \) and/or resummations of large logarithms of \( p_T^2/m_c^2 \) for the fragmentation processes that are dominant at large \( p_T \), and calculations of the rates of additional production processes that could help to pin down the NRQCD LDMEs and to provide further tests of the production mechanisms.

Additional experimental measurements are essential to the process of gaining an understanding of the quarkonium production mechanisms. Experimental efforts that would be of immediate value are (1) measurements \( J/\psi \) direct production cross sections at high \( p_T \); (2) measurements of all three \( J/\psi \) polarization parameters in different frames for direct production at high \( p_T \); (3) measurements of \( \chi_cJ \) cross sections and polarizations at high \( p_T \); (4) measurements of \( \Upsilon \) direct production cross sections at high \( p_T \); (5) measurements of additional high-\( p_T \) quarkonium production cross sections, such as the \( J/\psi + \text{jet} \) cross section at high \( p_T \).

Since NRQCD factorization, if it is correct, likely holds only for \( p_T \) substantially greater than \( m_Q \), it is essential that all measurements be made at the highest possible values of \( p_T \). It is also very important to make measurements of direct, rather than prompt, cross sections. The feeddown contributions in prompt cross sections greatly complicate the theoretical analyses, which may not be under good control even in the simpler case of direct production. The measurements of \( \chi_cJ \) cross sections and polarizations might be particularly interesting because the theory is more constrained: Only two NRQCD LDMEs enter at the leading non-trivial order in \( v \). \( \Upsilon \) cross sections and polarizations provide another important test of NRQCD factorization because the lower value of \( v^2 \) in the \( \Upsilon(1S) \), in comparison with the \( J/\psi \), means that the \( \Upsilon(1S) \) and \( J/\psi \) LDMEs have different relative sizes. It is clear that measurements of quarkonium production processes and polarizations that take advantage of the high \( p_T \) reach and high luminosity of the LHC will play a large role in advances in our understanding of quarkonium production mechanisms in the next few years.
ACKNOWLEDGEMENTS

I thank Mathias Butenschön, Kuang-Ta Chao, Bernd Kniehl, and Jian-Xiong Wang for many useful discussions. I also thank Jungil Lee, Hee Sok Chung, and Mathias Butenschön for providing some of the figures that were used in this talk. This work was supported by the U. S. Department of Energy, Division of High Energy Physics, under Contract No. DE-AC02-06CH11357.

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