Thermal rectification and spin-spin coupling of non-reciprocal localized and surface modes

Annika Ott and Svend-Age Biehs

Institut für Physik, Carl von Ossietzky Universität, D-26111 Oldenburg, Germany

(Dated: February 21, 2020)

We study the rectification of near-field radiative heat transfer between two InSb nano-particles due to the presence of non-reciprocal surface modes in a nearby InSb sample when an external magnetic field is applied and its dependence on the magnetic field strength. We reveal the spin-spin coupling mechanism of the localized particle resonances and the surface mode resonances which is substantiated by the directional heat flux in the given setup. We discuss further the interplay of the frequency shift, the propagation length, and local density of states on the strength and directionality of the rectification as well as the non-reciprocal heating effect of the nanoparticles.

I. INTRODUCTION

In the last few years, it could be shown that the non-reciprocal behaviour of magneto-optical materials like InSb has very interesting consequences for nanoscale thermal radiation. For example, fundamental effects like a persistent heat-current \[1, 2\], giant magneto-resistance \[3, 4\], thermal Hall effect \[5\] as well as a circular heat flux, angular momentum, and spin which do also persist in global equilibrium \[6\] were highlighted. As reviewed and discussed in detail in Ref. \[7\] magneto-optical materials and in particular these fundamental effects might have applications in the control of magnitude \[8–10\] and direction of radiative heat fluxes in nanoscale systems. Furthermore, it could be shown that non-reciprocal materials can also be utilized to introduce a near-field heat flux rectification. So far, most of the concepts for thermal rectification are based on the temperature dependence of the material properties \[11–16\] which can be very strong for phase-change materials like VO\(_2\) \[17\] which show up to date the strongest diode effect for thermal radiation \[18–21\]. Recently, it has been demonstrated that a thermal emitter and receiver can show an enhanced heat exchange by transporting the heat via the surface modes of a third body in their close vicinity \[22–30\] or by coupling to large wave-vector propagating modes in hyperbolic materials which is very similar to the Förster resonance energy transfer enhancement observed in plasmonic and hyperolic environments \[31–34\]. This coupling effect opens up a new possibility to rectify the radiative heat flux between a thermal emitter and receiver by introducing surfaces supporting non-reciprocal surface modes \[35–37\]. As has been shown by us, non-reciprocal surface modes allow to rectify the radiative heat flux very efficiently \[38\].

In this work, we will discuss our diode concept \[38\] as depicted in Fig. 1 in much greater detail. In this configuration two InSb nanoparticles are held in close vicinity to an InSb substrate. The nano-particles can exchange heat via direct coupling or coupling to the surface modes. To quantify this heat exchange, we first derive the many-body expressions for the power exchanged between \(N\) nano-particles in a given in general non-reciprocal environment as well as the many-body expression of the mean Poynting vector and discuss these quantities for the special case of \(N = 2\). We will show that the coupling to the surface modes is dictated by a spin-spin coupling mechanism which is behind the diode effect. We substantiate our interpretation by discussing the impact of the surface mode splitting, the propagation length, local density of states and the heat flux in the three-body structure as well as the thermal relaxation into the non-equilibrium steady state (NESS). We find, that the non-reciprocal heating of the nanoparticles can be on the order of 15% of the applied temperature difference.
II. HEAT FLUX

In order to investigate heat flux rectification we consider the system shown in Fig. 1. Two identical spherical nanoparticles 1 and 2 with radius $R$ and interparticle distance $d$ are placed in plane parallel to a substrate at a distance $z$. We will assume that the particles are so small that we can describe them as dipoles with a polarizability $\alpha_i$. This assumption is valid for particles much smaller than the dominant thermal wavelength which is about 10 $\mu$m in our case and if the distances $d$ between the particles and $z$ between the particles and the substrate are at least $4R$ [15, 16]. The particles and the substrate are made of a magneto-optical material supporting localized and surface resonances in the spectral window important for heat exchange at temperatures around 300 K. In this work we chose InSb for the particles and for the substrate. Moreover, the background is vacuum. We assume throughout the work that the particles and their environment or background (substrate and vacuum background) can be assumed to be in local thermal equilibrium at temperatures $T_1$, $T_2$, and $T_b$. This assumption is valid as long as the radiative heat flux is less important than the heat conduction inside the materials, which is definitely fulfilled in our configuration. Furthermore, the assumption is only valid on time scales which are much larger than the time scale of thermal relaxation of the materials. This is again true since the heat conduction inside the materials is much larger than the heat conduction by thermal radiation between the particles and between the particles and the substrate.

We can now determine the heat flux transferred between the two particles by calculating the net mean power received by particle $i$ defined as [17]

$$\langle P_i \rangle = \int_0^\infty \frac{d\omega}{2\pi} \Im \langle \tilde{p}_i(\omega) \cdot \tilde{E}^*_i(\omega) \rangle$$

with the Green’s functions $G_{ij} = G^{EE}(\vec{r}_i, \vec{r}_j)$ for the electric field due to electric sources as defined in Ref. [39] and explicitly given in Appendix A.

With these expressions the net mean power received by particle $i$ is

$$\langle P_i \rangle = \int_0^\infty \frac{d\omega}{2\pi} \Im \langle \tilde{p}_i(\omega) \cdot \tilde{E}^*_i(\omega) \rangle$$

and for the fluctuating dipole moments [17]

$$\langle \tilde{p}_i^{\text{fl}} \cdot \tilde{p}_j^{\text{fl}*} \rangle = 2\epsilon_0 \delta_{ij} \left( n_i + \frac{1}{2} \right) \chi$$

Here we have introduced the mean occupation number $n_i/b = n(T_{i/b}) = 1/(\exp(\hbar\omega/k_B T_{i/b}) - 1)$ with the reduced Planck constant $\hbar$ and Boltzmann constant $k_B$. With these definitions and relations we finally obtain

$$\langle P_i \rangle = 4\Im \int_0^\infty \frac{d\omega}{2\pi} \hbar \omega \sum_{j=1}^N (n_j - n_b) \chi_{ij}^{\text{fl}} \left( DT^{-1} \right)_{ij}$$

with the generalized susceptibility

$$\chi_{ij}^{\text{fl}} = \frac{\alpha_i - \alpha_i^{\dagger}}{2\hbar} - k_B^2 \Delta_{jj} G_{ii} - G_{ij} \Delta_{jj}$$

of the $i$-th particle.

In our special case of two particles which are in a plane parallel to the substrate we have $\chi_{ij} = \chi_{ij}^{\text{fl}}$ because $G_{11} = G_{22}$ due to translational symmetry. Furthermore, for $N = 2$ the mean power received by particle 1 is

$$\langle P_1 \rangle = 3 \int_0^\infty \frac{d\omega}{2\pi} \hbar \omega \left[ (n_1 - n_b) T_1^{\text{fl}} + (n_2 - n_b) T_1^{\text{fl}} \right]$$
introducing the transmission coefficients

\[ T^a_1 = \frac{4k_0^2}{3} \text{Im} \text{Tr} \left[ D^{-1}_{121} \chi \right. \]
\[ \times \left( G_{11} D^{-1}_{121} + G_{12} D^{-1}_{212} k_0^2 \right) \]  
\[ T^b_1 = \frac{4k_0^2}{3} \text{Im} \text{Tr} \left[ D^{-1}_{121} k_0^2 \chi \right. \]
\[ \times \left( G_{11} D^{-1}_{121} + G_{12} D^{-1}_{212} \right) \]  

(13)

(14)

with

\[ D_{ij} := I - k_0^2 \chi_{ij} \chi_{ji} \]  

(15)

The expression for \( \langle P_2 \rangle \) can be obtained by interchanging 1 \( \leftrightarrow \) 2. In the backward case with \( T_1 = T_b \) and \( T_2 > T_1 \), i.e. when particle 2 is heated with respect to its environment (see Fig. 1), then the power \( \langle P_1 \rangle \) received by particle 1 describes the heat flux from particle 2 to particle 1 which is obviously described by the transmission coefficient \( T^b_1 \). Similarly \( T^b_2 \), would describe the heat flux from particle 1 to particle 2 in the forward case that \( T_2 = T_b \) and \( T_1 > T_2 \) (see Fig. 1). As shown explicitly in Ref. [40], for reciprocal particles and substrate, i.e. if \( \chi^t = \chi^s \) and \( G_{ij} = G_{ji} \), we find \( T^b_1 = T^b_2 \). The heat flux in forward and backward direction is the same. Now, if the particles or the environment are non-reciprocal then \( T^b_1 \neq T^b_2 \) in general [40]. Hence, for non-reciprocal materials the heat flux in forward and backward direction are not the same. In the following we will show that when applying a magnetic field the non-reciprocal surface modes in the InSb sample will result in a large heat flux rectification.

### III. MATERIAL PROPERTIES

For an applied magnetic field in positive \( y \) direction the permittivity of InSb is given by

\[ \epsilon = \begin{pmatrix} \epsilon_1 & 0 & i\epsilon_2 \\ 0 & \epsilon_3 & 0 \\ -i\epsilon_2 & 0 & \epsilon_1 \end{pmatrix} \]  

(16)

with [42]

\[ \frac{\epsilon_1}{\epsilon_\infty} = \left( 1 + \frac{\omega^2 - \omega^2_T}{\omega^2 - \omega^2_L - i\Gamma} + \frac{\omega^2_p (\omega + i\gamma)}{\omega[\omega^2_c - (\omega + i\gamma)^2]} \right) \]  

(17)

\[ \frac{\epsilon_3}{\epsilon_\infty} = \left( 1 + \frac{\omega^2 - \omega^2_L - i\Gamma}{\omega^2 - \omega^2_T - i\Gamma} - \frac{\omega^2_p}{\omega(\omega + i\gamma)} \right) \]  

(18)

and

\[ \frac{\epsilon_2}{\epsilon_\infty} = \frac{\omega^2_p \omega_c}{\omega(\omega + i\gamma)^2 - \omega^2_c} \]  

(19)

with the cyclotron frequency \( \omega_c = eB/m^* \), the effective mass \( m^* = 7.29 \times 10^{-32} \text{ kg} \), the density of the free charge carriers \( n = 1.36 \times 10^{19} \text{ cm}^{-3} \) [41], the dielectric constant for infinite frequencies \( \epsilon_\infty = 15.68 \), the longitudinal and transversal optical phonon frequency \( \omega_0 = 3.62 \times 10^{13} \text{ rad/s} \) and \( \omega_T = 3.39 \times 10^{13} \text{ rad/s} \) [42]. With these parameters, the plasma frequency of the free carriers is \( \omega_p = \sqrt{\frac{ne^2}{m^* \epsilon_\infty}} = 1.86 \times 10^{14} \text{ rad/s} \). Furthermore, we use the phonon damping constant \( \Gamma = 5.65 \times 10^{11} \text{ rad/s} \), the free charge carrier damping constant \( \gamma = 10^{12} \text{ rad/s} \) [41].

From the above expressions for the permittivity it can be seen that the permittivity tensor is diagonal, if no magnetic field is applied (\( B = 0 \)) and therefore \( \epsilon = \epsilon^t \). On the other hand, if \( B \neq 0 \) the permittivity is non-reciprocal, i.e. \( \epsilon \neq \epsilon^t \), due to the Lorentz force acting on the electrons inside InSb. This will have an impact not only on the surface modes in the InSb sample but also on the localized resonances of the InSb nanoparticles.

### IV. LOCALIZED MAGNETO-OPTICAL PLASMONS OF THE NANOPARTICLES

In dipole approximation the polarizability of the particles is [13]

\[ \alpha = 4\pi R^3 (\xi - I)(\xi + 2I)^{-1}. \]  

(20)

Thus, due to the non-reciprocity of the permittivity the polarizability becomes non-reciprocal as well, if a magnetic field is applied we have \( \alpha \neq \alpha^t \). Furthermore, as discussed in detail in Ref. [3], the three-fold degenerate localized dipolar resonances at \( \omega_m=0, \pm 1 \) determined by the poles of \( \alpha \) with magnetic quantum numbers \( m = 0, \pm 1 \) split into three non-degenerated resonances where the splitting of the resonances with \( m = \pm 1 \) is mainly given by the cyclotron frequency \( \omega_c \). As shown in Ref. [6] these resonances are connected with a clockwise (counter-clockwise) radiative heat flux for \( m = -1 \) (\( m = +1 \)) as well as an angular momentum and spin which also persist in global thermal equilibrium and which are at the heart of the persistent heat current and thermal Hall effect in many-particle assemblies [11] [57]. As shown in Ref. [6] the mean spin for \( m = -1 \) (\( m = +1 \)) is parallel (anti-parallel) to the magnetic field resulting in a blue (red) shift so that the splitting can be understood as an analogue of the Zeeman splitting.

### V. MAGNETO-OPTICAL SURFACE MODES OF THE SUBSTRATE

The non-reciprocity introduced by the magnetic field also affects the surface modes of the InSb sample [35] [37]. To see this effect and to determine the heat transfer, we have determined the reflection matrix which has in the
Booker equation analytically. For analogously to the approach in Ref. [44] by solving the surface modes is given by the poles of vacuum light velocity. Now, the dispersion relation of the 

\[ r_{pp} = \left( \frac{k_0^2 - k_z k_x^2}{k_0^2 - k_z^2} \frac{k_0^2 k_x^2}{k_0^2 k_x^2} + k_z k_x (k_0^2 k_x^2 + k_z) \right) \]

introducing the wave vector components

\[ k_z = \sqrt{k_0^2 - k_x^2} \]
\[ k_z^- = \sqrt{k_0^2 k_x^2 - k_z^2} \]

with the Voigt permittivity

\[ \epsilon_v = \frac{1 - \epsilon_2}{\epsilon_1} \]

and the wavenumber in vacuum \( k_0 = \omega/c \) where \( c \) is the vacuum light velocity. Now, the dispersion relation of the surface modes is given by the poles of \( r_{pp} \). We obtain

\[ k_z^2 - k_0^2 \epsilon_1 - k_z k_x^2 \epsilon_1 - k_z k_x^2 \epsilon_2 = 0 \]

which is the same expression as in Ref. [37]. It can already be seen that this dispersion relation depends on the sign of \( k_z \). Hence, surface modes propagating to positive or negative x-direction have in general different dispersion relations if \( \epsilon_2 \neq 0 \), i.e. if a magnetic field is applied. Furthermore, in the quasi-static regime \( k_0^2 \approx k_0^2, k_0^2|\epsilon_v| \) we retrieve the result [37] \( \epsilon_1 + \epsilon_2 = -1 \) for \( k_x > 0 \) and \( \epsilon_1 - \epsilon_2 = -1 \) for \( k_x < 0 \) reflecting again the fact that the surface modes propagating in positive or negative x-direction are differently affected by the magnetic field which introduces this non-reciprocity. This non-reciprocity is more generally expressed by that fact that \( r_{pp}(k_x) \neq r_{pp}(-k_x) \) when a magnetic field is applied in y-direction. As a consequence also the Green’s tensor becomes non-reciprocal in this case and of course it is clear that the heat flux from particle 1 to particle 2 due to the coupling to the surface waves will be different from the heat flux from particle 2 to particle 1.

The non-reciprocal behaviour of the surface modes can be seen in Fig. 2 where we have plotted \( 1 - |r_{pp}|^2 \) for the propagating waves with \( k_x^2 \leq k_0^2 \) and \( \text{Im}(r_{pp}) \) for the evanescent waves with \( k_x^2 > k_0^2 \). These quantities reflect the absorption energy by reflection of incident propagating and evanescent waves. We have also plotted the dispersion relation of the surface modes in Eq. (26). It can be easily seen that the symmetry of \( r_{pp} \) with respect to \( k_x \) is broken when a magnetic field \( B \neq 0 \) is applied in y-direction. A splitting of the two resonances can be seen, where the surface modes travelling in positive x-direction are red-shifted whereas the surface modes travelling in negative x-direction are blue shifted [35]. As discussed in Ref. [49, 50] there is a spin-momentum locking of the surface waves. The surface waves for \( k_x < 0 \) have a spin in positive y-direction, i.e. in the direction of the magnetic field, whereas the surface waves for \( k_x > 0 \) have a spin in
negative y-direction, i.e., opposite to the magnetic field. From this one can intuitively understand the red-shift of the resonance frequency for surface waves with \( k_x > 0 \) and the blue shift for surface waves with \( k_x < 0 \) which is again simply analogous to the Zeeman effect.

VI. LOCAL DENSITY OF STATES

The fact, that the presence of the magnetic field introduces a non-reciprocity or asymmetry for the waves propagating in positive or negative x-direction motivates to devise the expression for the local density of states (LDOS) \( D(\omega, z) \) into two parts \( D^\pm(\omega, z) \) belonging to exactly such waves with \( k_x > 0 \) and \( k_x < 0 \). Thus, we define \( D^\pm(\omega, z) \) by starting from the well-known expression of the LDOS at a distance \( z \) above a semi-infinite medium \[ D(\omega, z) = \int_{-\infty}^{\infty} \frac{dk_x}{2\pi} \int_{-\infty}^{\infty} \frac{dk_y}{2\pi} f(\kappa, \omega, z) \] (27)

with \( \kappa^2 = k_x^2 + k_y^2 \) and
\[
\tilde{f}(\kappa, \omega, z) = \frac{\omega}{\pi c^2} \frac{1}{2\sqrt{k_0^2 - \kappa^2}} \left[ 4 + 2\kappa^2 \left( r_{ss} - r_{pp} e^{2i\sqrt{k_0^2 - \kappa^2}} \right) \right].
\] (28)

Strictly speaking this expression is only valid for media with \( r_{sp} = r_{ps} = 0 \). Since, we find for InSb with our choice of parameters that these depolarization components are negligible small compared to \( r_{ss} \) and \( r_{pp} \), we can also use the expression of the LDOS to characterize our InSb sample. Now, we define \( D^\pm(\omega, z) \) by considering only the contributions for the waves travelling in positive and negative x-direction
\[
D^+(\omega, z) = \int_{0}^{\infty} \frac{dk_x}{2\pi} \int_{-\infty}^{\infty} \frac{dk_y}{2\pi} f(\kappa, \omega, z),
\] (29)
\[
D^-(\omega, z) = \int_{-\infty}^{0} \frac{dk_x}{2\pi} \int_{-\infty}^{\infty} \frac{dk_y}{2\pi} f(\kappa, \omega, z).
\] (30)

In Fig. [6] we will use this quantity to discuss the heat flux rectification.

VII. HEAT TRANSFER MECHANISM

Due to the non-reciprocal behaviour of the surface modes the heat fluxes between the two particles can become asymmetric if the heat flux is dominated by the contribution of the surface modes. To study this effect, we consider now the configuration shown in Fig. [1] for the backward scenario with \( T_1 = T_b = 300 \text{K} \) and \( T_2 = 350 \text{K} \) and the forward scenario with \( T_2 = T_b = 300 \text{K} \) and \( T_1 = 350 \text{K} \) choosing \( z = 5R = 500 \text{nm} \). In Fig. [6] the net power \( P_1 = \langle P_1 \rangle \) received by particle 1 in the backward case and the net power \( P_2 = \langle P_2 \rangle \) received by particle 2 in the forward case are shown for different interparticle distances \( d \). Note, that these powers are normalized to the value \( P_0 \) where the substrate is replaced by vacuum. It can be easily seen that \( P_1 \neq P_2 \) if \( B \neq 0 \) and that the maximum of \( P_1/P_0 \) at position \( d_m \) moves to larger distances \( d \) out of the plotted region when increasing the magnetic field amplitude, whereas the maximum of \( P_2/P_0 \) moves to smaller distances.

The spectra \( P_1(\omega) \) and \( P_2(\omega) \) for the backward and forward case are shown in Fig. [3] for \( d = 2 \mu m \) and \( d = 15 \mu m \). It is apparent that for the forward direction \( P_{2,\omega} \) is dominated by the high-frequency resonance with magnetic quantum number \( m = -1 \) of the nanoparticles and for the backward direction \( P_{1,\omega} \) is dominated by the low-frequency resonance with magnetic quantum number \( m = +1 \). Furthermore, it can be observed that for \( d = 2 \mu m \) we have \( P_2 > P_1 \) and for \( d = 15 \mu m \) \( P_1 > P_2 \). Hence, there is a clear rectification of the heat flux which changes its direction when changing from near-field to far-field interparticle distances \( d \). Furthermore, since \( P_2 \) can only be due to the coupling to the surface waves travelling in positive (negative) x-direction, this suggest that the localized particle resonance with \( m = +1 \) having negative spin couples preferably to the surface wave with negative spin and the particle resonance with \( m = -1 \) having positive spin to the surface wave with positive spin. Hence, our results suggest that there is a selection rule allowing preferred coupling between particle and surface resonances with the same spin.

From this coupling mechanism, we can also understand the position of the maximum in \( P_1/P_0 \) and \( P_2/P_0 \). To this end, we consider now the propagation length of the surface modes, which is defined as
\[
\Lambda^\pm = \frac{1}{\pm 2 \text{Im}(k_x^\pm)}
\] (31)

where \( k_x^\pm \) is the complex solution \( k_x \) with \( \text{Re}(k_x) > 0 \) or \( \text{Re}(k_x) < 0 \) of the dispersion relation in Eq. [26] for a given real frequency \( \omega \). Note that this determines only the propagation length of surface waves with \( k_y = 0 \). In general, also surface waves with \( k_y \neq 0 \) which are included in our calculation have an impact on the heat transfer between the particles. Therefore, \( \Lambda^\pm \) is only a rough estimate of the length scale of the propagation length of the surface waves contributing to the full heat transfer.

In Fig. [5] we show a plot of this propagation length \( \Lambda^\pm \) for the surface waves travelling in positive and negative x-direction for \( B = 0 \text{T} \) and \( 2 \text{T} \) together with the spectral position of the three particle resonances with \( m = 0, \pm 1 \) for \( B = 2 \text{T} \). Note that for \( B = 0 \text{T} \) all resonances are at the same frequency as the \( m = 0 \) resonance. In Fig. [5] it can be observed that the propagation length of the surface wave travelling in positive (negative) x-direction which couples to the \( m = -1 \) (\( m = +1 \)) resonance has a much smaller (larger) propagation length of about \( 1 \mu m \) (100\( \mu m \)) for \( B = 2 \text{T} \) than the 20\( \mu m \) propagation length.
for $B = 0$ T. Furthermore, these values of the propagation length are in good agreement with the position $d_m$ of the maxima observed in Fig. 3 explaining why for small distance $P_2 > P_1$ and for large distances $P_1 > P_2$. Hence, from the coupling mechanism, the spectral shift of the particle resonances and the surface mode resonances we can understand the position of the maxima observed in $P_1 / P_0$ and $P_2 / P_0$ as function of the distance. A similar distance dependence has been observed for the positions of the maxima in Förster resonance energy transfer above plasmonic surfaces. Furthermore, it is clear from Fig. 3 that due to blue-shift of the $m = -1$ resonance and the red-shift of the surface modes travelling in positive x-direction, there can be no coupling anymore for large enough magnetic fields so that for large $B$ one clearly has $P_1 > P_2$ in the surface mode dominated heat transport regime.

To get a more complete picture, in Fig. 4 we show the spectral power $P_{1,\omega}$ and $P_{2,\omega}$ as function of frequency and interparticle distance $d$ together with the position of the three particle resonances, the propagation length $\Lambda^\pm$ and the LDOS $D^\pm$ from Eqs. (29) and (30). For $d \ll z$ it can be seen that all three resonances contribute to the heat transfer due to the fact that the heat flux is mainly directly transferred between both particles. For larger $d$ the heat transfer between both particles is more and more dominated by the coupling to the surface modes where the $m = +1$ resonance couples to the long range surface mode travelling to negative x-direction and the $m = -1$...
resonance couples to the short range surface mode traveling to the positive x-direction. For distances much larger than $\Lambda^\pm$ the surface mode contribution vanishes. That $P_2 > P_1$ for small distances $d$ can now also be understood by the fact that the LDOS $D^\pm$ is larger at $\omega_{m=1}$ than $\omega_{m=-1}$. For large distances $d > \Lambda^+$ only the long range surface modes can contribute and therefore $P_1 > P_2$.

VIII. HEAT FLUX RECTIFICATION

To quantify the heat flux rectification we define the rectification coefficient as

$$\eta = \frac{P_1 - P_2}{P_1}. \quad (32)$$

In Fig. 7 we plot the rectification coefficient as function of $d$ for different magnetic field strengths. It can be observed that for field amplitudes smaller than 3T the rectification coefficient is negative, because $P_2 > P_1$. For large field amplitudes like $B = 3T$ the rectification coefficient is purely positive, because $P_1 > P_2$. As discussed before, this is due to the fact that with increasing field strength the resonance $\omega_{m=-1}$ is blue shifted and the surface mode resonance is red-shifted leading to a decreasing propagation length $\Lambda^+$. This behaviour can here be observed in the shift of the position where $\eta = 0$ to smaller distances $d$ when the field amplitude is increased. For $B = 3T$ the particle resonance $\omega_{m=1}$ can simply not couple to a surface mode propagating to positive x-direction anymore, because the red-shift of the particle resonance and the blue shift of the surface mode resonance are too large. Hence, for 3T we find $P_1 > P_2$ for all distances. The curves for 2T and 3T converge for $d \gg \Lambda^+$ to a rectification coefficient which is close to 1 ($\eta = 0.996$) which means that $P_1 \gg P_2$ which is a clear diode effect. On the other hand, for relatively weak fields and small distances $d$ we have a “minimal” $\eta$ of about −6 which simply means that $P_2 \approx 7P_1$. If we would in this case define the rectification coefficient as $\tilde{\eta} = (P_2 - P_1)/P_2$ we would obtain $\tilde{\eta} \approx 0.86$. Hence, also in this case we have a large rectification, but in the other direction.

IX. MEAN POYNTING VECTOR

To have a deeper understanding of the heat flow and the spin-spin coupling we determine now the mean poynting vector $\langle \vec{S} \rangle$ due to the thermal radiation of the two particles in a given environment. As before we provide the general expression for an arbitrary number $N$ of nanoparticles, first, and then invoke the special case $N = 2$. With the electric field

$$\vec{E}(\mathbf{r}) = \omega^2 \mu_0 \sum_{i=0}^{N} G^E(\mathbf{r}, \mathbf{r}_i) \vec{p}_i + \vec{E}^b(\mathbf{r}) \quad (33)$$

produced by the thermal background radiation and the thermal dipole moments of the nanoparticles in Eq. 2.
we can determine directly the magnetic field by Faraday’s law \( \vec{H} = \nabla \times \vec{E}/(i\omega \mu_0) \). Then the mean Poynting vector \( \langle \vec{S}_{\omega} \rangle = 2\Re(\vec{E}_{\omega} \times \vec{H}_{\omega}) \) can be straightforwardly determined by using the fluctuation-dissipation theorem of the fluctuational dipole moments in Eqs. (9) and of the fields in Eq. (3). To obtain a day’s law we can determine directly the magnetic field by Faraday’s law. Therefore the heat transfer between the particles is fully determined by the first term of the Poynting vector. Since this term fulfills a positive or negative x-direction described by this third term. Here, in particular the non-reciprocal surface modes in the InSb substrate will produce a persistent heat flux in positive or negative x-direction described by this third term. Since this term fulfills \( \nabla \cdot \langle \vec{S} \rangle = 0 \) it does not contribute to heat transfer between the particles. Finally, the second term is an interference term between the background field and the particles. It describes the change of heat flow of the background field due to the presence of the particles. Since the second and third term persists in global equilibrium they describe the persistent heat flux of the interface and the nanoparticles, but they do not describe any heat transfer between the nanoparticles. Therefore the heat transfer between the particles is fully determined by the first term of the Poynting vector.

\[
\langle \vec{S}_{\omega}^{tr} \rangle = 4\hbar\omega^2 \mu_0 k_0^2 \sum_{\beta,\gamma=x,y,z} \epsilon_{\alpha\beta\gamma} \text{Re} \left[ \sum_{ij,k=1}^{N} (n_j - n_k) G_{0i}^{EE} T_{ij}^{-1} \chi_j (G_{0k}^{HE} T_{kl}^{-1}) \right]_{\beta,\gamma}. \tag{36}
\]

We have checked that for \( N = 2 \) the integration of the normal component of this Poynting vector on the surface of the nanoparticles 1 and 2 in the backward and forward scenario gives either \( P_1 \) or \( P_2 \). In Fig. 3 \( \langle \vec{S}_{\omega}^{tr} \rangle \) is shown for the different resonance frequencies \( \omega_{m=\pm 1} \) of the nanoparticles. Again as discussed for a single nanoparticle in Ref. [1] the mean Poynting vector is circulating around the nanoparticles clockwise (counter-clockwise) for \( m = -1 \) (\( m = +1 \)). Moreover, the influence of the substrate can be clearly identified. It can be easily seen that in the forward case the net heat transfer for the \( m = +1 \)-mode is much better then for the \( m = -1 \)-mode and for the backward case it is the other way round. From the clockwise circularity of the \( m = -1 \) particle resonance it is clear that it couples preferably to surface waves with \( k_x < 0 \), whereas the counter-clockwise circularity of the \( m = +1 \) particle resonance clearly suggests a preferred coupling to surface waves with \( k_x > 0 \). Hence, the mean Poynting vector visualises nicely the spin-spin coupling mechanism of the circular mode in the nanoparticle and the surface modes. Furthermore there seems to be a slight heat flow from the surface towards the nanoparticles for the \( m = -1 \) resonance in the backward case similar to the heat pumping found in Ref. [5].

\[
\rho n C V \frac{dT_k}{dt} = P_k(t, T_1, T_2, T_b), \tag{37}
\]

for \( k = 1, 2 \) with the heat capacity \( C = 200 \text{ J/kg K} \), a mass density \( \rho = 5775 \text{ kg/m}^3 \) and volume \( V \) of InSb nanoparticles [5].
Figure 8: Spectral Poynting vector $|\langle \vec{S}_{\omega} \rangle |$ in (W/m²) (colour scale) and its normalized direction (arrows) for the two InSb nano-particles (black) with a distance of $z = 5R = 500$ nm to the InSb substrate and interparticle distance $d = 1 \mu m$ for an applied magnetic field of $2T$. On the left side we show the backward case with $T_1 = T_b = 300$ K and $T_2 = 350$ K and on the right side the forward case with $T_2 = T_b = 300$ K and $T_1 = 350$ K.

Figure 9: NESS temperatures of the colder particle in the forward and backward direction without substrate $T_0 = T_1 = T_2$ and with substrate $T_1 \neq T_2$. We choose $d = z = 5R = 500$ nm.

In Fig. 9 we show the temperature of the colder particles in the NESS for the forward and backward case with and without substrate. First, it can be observed that $T_1 = T_2 \equiv T_0$ in forward and backward direction without substrate. Here, the particle temperature of the colder particle is slightly dropping from 309K to 308K when the magnetic field amplitude is increased. This temperature drop of 1K (10% of the applied temperature difference $\Delta T = 10K$) is the giant magnetic resistance effect [3, 4]. When the particles are brought in the close vicinity of the substrate at $z = 500$nm, then the temperature of the colder particle for $B = 0T$ drops from 309K to about 305K. Hence, a substantial part of the heat emitted by the warmer particle is dissipated in the substrate. Now, when turning on the magnetic field and increasing its amplitude, the temperatures first drop much faster than without surface to about 302K until $B = 2T$ and then for larger amplitudes they rise again up to 303K/304K. Hence the surface enhances the giant magnetic resistance effect which is for $B = 2T$ about 30% of the applied temperature difference, i.e. a temperature drop of 3K.
Figure 10: The temperature difference $(T_1 - T_2)/\Delta T$ for the NESS temperature of the colder particle in backward and forward direction normalized to the initially applied temperature difference of $\Delta T = 10K$. The inset shows the temperature evolution of $T_2$ (blue line) and $T_1$ (red line) in forward and backward case, resp., from the initial state of 300K to the NESS for the maximum rectification at 3T. The distance between the particles and between the particles and the substrate are $d = z = 5R = 500 \text{ nm}$.

It can also be seen, that with substrate $T_1 \neq T_2$ in general. In Fig. 10 we show the temperature difference $T_1 - T_2$ of the backward and forward NESS temperatures normalized to $\Delta T = 10K$. A maximum rectification of about $15\%$ can be observed for relatively large magnetic field amplitudes of $B = 3T$. For weak fields the effect is only -1.3\%. This is in agreement with the Hall effect, which is also relatively weak for InSb [7]. Hence, a clearly measurable non-reciprocity in the heating can be observed, but due to the fact that most of the heat is going into the substrate, this effect is rather small, but we have not made any optimization procedure. We find, that the non-reciprocal heating effect cannot be simply enhanced by for example increasing $d$ or decreasing $z$. By decreasing $z$ the surface mode coupling will be stronger, but also the amount of heat going into the substrate. Also decreasing/increasing $d$ is not a priori a good option. When decreasing $d$ then the non-reciprocality vanishes due to the fact that for $d \ll z$ the coupling via the surface vanishes. Increasing $d$ on the other hand, will result in large $P_1/P_0$ and $P_2/P_0$ as shown in Fig. 3 for $d = 1 \mu m$, for instance, but the absolute value of the heat flux drops enormously with $d$ so that the heating of the colder particles becomes inefficient. Detailed parameter studies which are out of the scope of our work of the impact of the distance $d$ and $z$ particles sizes $R$ and material properties are needed to find optimal materials and configurations to have a strong rectification effect.

XI. CONCLUSION

In summary, we have made a detailed discussion and investigation of the diode working principle of the non-reciprocal near-field diode in Ref. [38]. We showed that the rectification effect occurs due to the spin-sensitive coupling of the particle resonances and the surface modes. The transferred heat flux is maximal when the distance between the particle is on the order of the propagation length of the surface modes which is different for the surface modes travelling in positive or negative $x$ direction. Moreover, the strength and direction of the effect is highly dependent on the magnetic field, the distance between the particles and the substrate as well as the local density of states. Our investigation of the mean Poynting vector showed that the spin-sensitive coupling can be understood by the circularity of the particle resonances and the directionality of the surface mode resonances. When the spin of both resonances is the same, these directionalties match explaining the spin-spin coupling. In addition, we find large rectifications of the heat flux with our choice of parameters. Nonetheless, the effective assymmetry in the heating of the nanoparticles in non-equilibrium steady state is for relatively large fields maximally $15\%$ of the initially applied temperature difference between the warm and could nano-particle. We believe that the search for optimal parameters and proper materials can lead to a highly increased rectification effect which would make the here discussed concept interesting for future applications.

Appendix A: Dyadic Green’s functions

The Greens function of the electric field generated by the electric source currents is a sum of the vacuum $G_{ij}^0$ and the scattered contribution $G_{ij}^{sc}$ [25]:

$$G_{ij} = G_{ij}^0 + G_{ij}^{sc}. \quad (A1)$$

Here, we use the indices $i$ and $j$ to calculate the Greens function at position $\vec{r}_i = (x_i, y_i, z_i)T$ generated by a dipole at position $\vec{r}_j = (x_j, y_j, z_j)^T$.

For the Greens function in vacuum we use [9]

$$G_{ij}^0 = \frac{e^{ik_0 d}}{4\pi d} [a \mathbf{1} + b\mathbf{e}_d \otimes \mathbf{e}_d] \quad (A2)$$

with $d = |\vec{r}_i - \vec{r}_j|$ and

$$a = 1 + \frac{ik_0 d - 1}{k_0^2 d^2} \quad (A3)$$
$$b = \frac{3 - 3ik_0 d - k_0^2 d^2}{k_0^2 d^2}. \quad (A4)$$

The scattered contribution due to the presence of the flat surface is given by

$$G_{ij}^{sc} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dkx dk_y}{(2\pi)^2} e^{i(\xi x - \zeta y) - i\xi^k} \tilde{G}_{ij}^{sc}(k_x, k_y) \quad (A5)$$

with

$$\tilde{G}_{ij}^{sc}(k_x, k_y) = \frac{2ie^{ik_1 z(z_i + z_j)}}{2k_{i,z}} \sum_{k_i = \pm 1} r_{ik} \mathbf{a}_{k_i} \otimes \mathbf{a}_{k_i} \quad (A6)$$
using the polarization vectors for $s$ and $p$ polarization

$$\vec{a}_s^\pm = \frac{1}{\kappa} \begin{pmatrix} k_x \\
(k_y \\
0) \end{pmatrix}$$ (A7)

and

$$\vec{a}_p^\pm = \frac{1}{\kappa k_0} \begin{pmatrix} \mp k_{i,z} k_x \\
\mp k_{i,z} k_y \end{pmatrix}$$ (A8)

with $\vec{x}_i = (x_i, y_i)^T$, $\vec{r} = (k_x, k_y)^T$, and $k_{i,z} = \sqrt{k_0^2 - \kappa^2}$. The other Green’s tensors can be easily calculated from this expression [39].

Appendix B: Impact of phonon contribution

The effects discussed in this work highly depend on the material properties. We have chosen throughout the manuscript a parameter set for InSb with a clear dominating electric permittivity. However, for other parameter sets the phononic part may play an important role. Actually, the single surface mode band as seen in the reflection coefficients in Fig. 2 can split into several bands as found in our previous work on the diode effect in Ref. [38], for instance. The discussion given in this work can still be applied to this case, but of course the whole picture becomes more complex.

In order to contrast the impact of the phonons on the rectification coefficient we take now another set of parameters from Ref. [12] with effective mass $m^* = 1.99 \times 10^{-32}$ kg, density of the free charge carriers $n = 1.07 \times 10^{17}$ cm$^{-3}$, high frequency dielectric constant $\epsilon_\infty = 15.7$, longitudinal and transversal optical phonon frequency $\omega_L = 3.62 \times 10^{13}$ rad/s and $\omega_T = 3.39 \times 10^{13}$ rad/s. Furthermore, the plasma frequency of the free carriers is $\omega_p = \sqrt{\frac{\epsilon_\infty n e^2}{m^*}} = 3.15 \times 10^{13}$ rad/s. We use further the phonon damping constant $\Gamma = 5.65 \times 10^{11}$ rad/s and the free charge carrier damping constant $\gamma = 3.39 \times 10^{12}$ rad/s. For this set of parameters which we have used in Ref. [38] the rectification coefficient is shown in Fig. 11. In comparison to Fig. 7 it can be seen that in this case the field dependence is much different. In particular, in most cases $P_1 > P_2$. As could be a priori expected, the rectification effect strongly depends on the doping level of InSb and in particular the directionality. Hence, the rectification effect can be efficiently tailored by changing the doping level. Nonetheless, it should be kept in mind that for nano-particles also size effects might play a role. For example, for nano-particles of radius $R = 100$nm the number of electrons for $n = 1.07 \times 10^{17}$ cm$^{-3}$ is only 450, whereas for $n = 1.36 \times 10^{19}$ cm$^{-3}$ it is 56984, i.e. relatively high. Hence, the optical response of the nanoparticles with a comparably low free charge carrier density like $n = 1.07 \times 10^{17}$ cm$^{-3}$ might be quite different from the bulk response.

Acknowledgments

A. O. and S.-A. B. thank P. Ben-Abdallah, R. Messina, and A. Kittel for helpful discussion and comments. S.-A. B. acknowledges support from Heisenberg Programme of the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under the project No. 404073166.

Figure 11: Rectification coefficient $\eta$ for a parameter set of InSb as used in Ref. [38] taken from [12] as function of inter-particle distance $d$ for different strengths of the magnetic field and $z = 5R = 500$nm.
[8] E. Moncada-Villa, V. Fernández-Hurtado, F. J. García-Vidal, A. García-Martín and J.C. Cuevas, Phys. Rev. B 92, 125418 (2015).

[9] J. Song and Q. Cheng, Phys. Rev. B 94, 125419 (2016).

[10] H. Wu, Y. Huang, L. Cui, K. Zhu, Phys. Rev. Appl. 11, 054020 (2019).

[11] C. R. Otey, W. T. Lau, and S. Fan, Phys. Rev. Lett. 104, 154301 (2010).

[12] H. Iizuka and S. Fan, J. Appl. Phys. 112, 024304 (2012).

[13] S. Basu and M. Francoeur, Appl. Phys. Lett. 95, 231913 (2011).

[14] L. P. Wang and Z. M. Zhang, Nanoscale and Microscale Thermophysical Engineering 17, 337(2013).

[15] E. Nefzaoui, K. Joulain, J. Drevillon, and Y. Ezzahri, Appl. Phys. Lett. 104, 103905 (2014).

[16] J. Ordonez-Miranda, K. Joulain, D. De Sousa Meneses, Y. Ezzahri, and J. D revillon, J. Appl. Phys. 112, 093105 (2017).

[17] M. M. Qazilbash, M. Brehm, B. G. Chae, P.-C. Ho, G. O. Andreev, B.J. Kim, S.J. Yun, A.V. Balatsky, M.B. Maple, F. Keilmann, H.T. Kim, and D.N. Basov, Science 318, 1750 (2007).

[18] P. Ben-Abdallah and S.-A. Biehs, Appl. Phys. Lett. 103, 191907 (2013).

[19] Y. Yang, S. Basu, L. Wang, Appl. Phys. Lett. 103, 163101 (2013).

[20] K. Ito, K. Nishikawa, H. Iizuka, and H. Toshiyoshi, Appl. Phys. Lett. 105, 25350 (2014).

[21] A. Fiorino, D. Thompson, L. Zhu, R. Mittapally, S-A. Biehs, O. Bezencenet, N. El-Bondry, S. Bansropun, P. Ben-Abdallah, E. Meyhofer, P. Reddy, ACS Nano 12, 5774 (2018).

[22] K. Säskilathi, J. Oksanen J. Tulkki, Phys. Rev. B 89, 134301 (2014).

[23] K. Asheichyk, B. Müller M. Krüger, Phys. Rev. B 96, 155402 (2017).

[24] J. Dong, J. Zhan, and L. Liu, Phys. Rev. B 97, 075422 (2018).

[25] R. Messina, S.-A. Biehs, and P. Ben-Abdallah, Phys. Rev. B 97, 165437 (2018).

[26] Y. Zhang, M. Antezza, H.-L. Yi , H.-P. Tan, Phys. Rev. B 100, 085426 (2019).

[27] M.-J. He, H. Qi, Y.-T. Ren, Y.-J. Zhao, M. Antezza, Appl. Phys. Lett. 115, 263101 (2019).

[28] R. Messina, M. Antezza, P. Ben-Abdallah, Phys. Rev. Lett. 109, 244302 (2012).

[29] R. Messina, P. Ben-Abdallah, B. Guizal, M. Antezza, S.-A. Biehs, Phys. Rev. B 94, 104301 (2016).

[30] Y. Zhang, H.-L. Yi , H.-P. Tan, M. Antezza, Phys. Rev. B 100, 134305 (2019).

[31] S.-A. Biehs, G.S. Agarwal, Appl. Phys. Lett. 103, 243112 (2013).

[32] S.-A. Biehs, V. M. Menon, G. S. Agarwal, Phys. Rev. B 93, 245439 (2016).

[33] R. Deshmukh, S.-A. Biehs, E. Khwaja, T. Galfsky, G. S. Agarwal, V. M. Menon ACS Photonics 5, 2737 (2018).

[34] W. D. Newman, C. L. Cortes, A. Afshar, K. Cadieu, A. Meldrum, Z. Jacob, Science Adv. 4, eaar5278 (2018).

[35] J.J. Brion, R. F. Wallis, A. Hartstein, E. Burstein, Physical Review Letters 28, 1455 (1972).

[36] R. Wallis, J. Brion, E. Burstein and A. Hartstein, Phys. Rev. B. 9,8 (1974).

[37] K. Chiu and J. Quinn. Nuovo Cimento B 10,1 (1972).