Further Discussion on the Influence Radius of a Pumping Well: A Parameter with Little Scientific and Practical Significance That Can Easily Be Misleading

Yuanzheng Zhai, Xinyi Cao, Ya Jiang, Kangning Sun, Litang Hu, Yanguo Teng, Jinsheng Wang and Jie Li

Abstract: To facilitate understanding and calculation, hydrogeologists have introduced the influence radius. This parameter is now widely used, not only in the theoretical calculation and reasoning of well flow mechanics, but also in guiding production practice, and it has become an essential parameter in hydrogeology. However, the reasonableness of this parameter has always been disputed. This paper discusses the nature of the influence radius and the problems of its practical application based on mathematical reasoning and analogy starting from the Dupuit formula and Thiem formula. It is found that the influence radius is essentially the distance in the time–distance problem in physics; therefore, it is a function of time and velocity and is influenced by hydrogeological conditions and pumping conditions. Additionally, the influence radius is a variable and is essentially different from the hydrogeological parameters reflecting the natural properties of aquifers such as the porosity, specific yield, and hydraulic conductivity. Furthermore, the parameterized influence radius violates the continuity principle of fluids. In reality, there are no infinite horizontal aquifers, and most aquifers are replenished from external sources, which is very different from theory. The stable or seemingly stable groundwater level observed in practice is simply a coincidence that occurs under the influence of various practical factors, which cannot be considered to explain the rationality of applying this parameter in production calculations. Therefore, the influence radius cannot be used to evaluate the sustainable water supply capacity of aquifers, nor can it be used to guide the design of groundwater pollution remediation projects, the division of water source protection areas, and the scheme of riverbank filtration wells. Various ecological and environmental problems caused by groundwater exploitation are related to misleading information from the influence radius theory. Generally, the influence radius does not have scientific or practical significance, but it can easily be misleading, particularly for non-professionals. The influence radius should not be used in the sustainable development and protection of groundwater resources, let alone in theoretical models. From the perspective of regional overall planning, the calculation and evaluation of sustainable development and the utilization of groundwater resources should be investigated in a systematic manner.

Keywords: influence radius; Dupuit; Thiem; groundwater flow system; sustainable development
as the Dupuit formula [2]. In the Dupuit model, the aquifer is a finite volume cylinder placed in the sea, and the pumping well is located at the axis of the cylinder, which is an ideal model for high generalization. In reality, aquifers with a finite cylinder shape, constant head boundary at the side, and zero flux boundaries at the top and bottom are extremely rare, and actual aquifers are also difficult to generalize as assumed by the Dupuit model and calculated directly using the Dupuit formula. Consequently, the practical application of the Dupuit model has been largely limited since its inception. To solve this problem, Thiem [3] extended the Dupuit model to a horizontal infinite aquifer using an approximate hypothesis and thus established the Thiem model. This model has a parameter called the range of cone of depression. Thiem assumed that this parameter represents the horizontal distance from the pumping well to the point where the water level cannot actually be observed to drop; therefore, a large error will not occur in the replacement of the round island radius “R” with the parameter of the range of cone of depression [4]. Later, Todd [5] figuratively renamed the range of the cone of depression as the influence radius and argued that it was not necessarily observable, but rather an approximate empirical value [6], which is how the influence radius originated.

The influence radius may be confusing with regard to the Dupuit model and Thiem model, and it is believed that the round island radius (R) in the former is equivalent to the influence radius (R) in the latter. Additionally, it is believed that the influence radius is objective, immutable, and measurable [7], and has nothing to do with human impact, in a manner similar to some hydrogeological parameters such as the hydraulic conductivity, specific yield, and round island radius. This erroneous understanding has misled practical work and resulted to errors in the theories and methods of groundwater resource evaluation [8]. Thus, the development of groundwater resources and the prevention and control of groundwater pollution have been subject to misleading information. This is particularly the case for the division of groundwater source protection areas [6]. In recent years, the influence radius has occasionally been discussed in academia [9–11], but consensus has not been reached. Considering this situation in combination with the needs of theoretical research and practical application, this paper tries to use non-professional language to further discuss the issue of the influence radius through mathematical reasoning and analogy to clarify this issue.

2. Birth and Application History of Influence Radius

2.1. Birth: A Misunderstanding

Dupuit made the following assumptions when he established the stable well flow model (round island model for short, Figure 1(1)): the unconfined aquifer is a homogeneous and isotropic circular island aquifer with a horizontal lower confining bed, the lateral boundary of the well has a constant head and there is a completely penetrating well in the center of the aquifer, the aquifer does not have vertical infiltration recharge and evaporation, and the seepage is a steady flow that conforms to the linear law.

The Dupuit formula for the unconfined aquifer is derived under these hypothetical conditions:

\[ Q = 1.366k \frac{H_0^2 - h_w^2}{\ln \frac{R}{r_w}} \]  

where \( Q \) is the water yield of the pumping well, \([L^3T^{-1}]\); \( K \) is the hydraulic conductivity, \([LT^{-1}]\); \( H_0 \) is the thickness of the aquifer, \([L]\); \( h_w \) is the water level of the pumping well, \([L]\); \( R \) is the influence radius, \([L]\); and \( r_w \) is the radius of the pumping well, \([L]\).
Figure 1. Schematic diagram of Dupuit model and Thiem model. Q is the water yield of the pumping well, [L$^3$T$^{-1}$]; $s_w$ is the drawdown of the pumping well, [L]; h is the groundwater level at distance r from the pumping well, [L]; $H_0$ is the thickness of the aquifer, [L]; $h_w$ is the water level of the pumping well, [L]; $r_w$ is the radius of the pumping well, [L]; r is the distance between the pumping well and observation well, [L]; and R is the influence radius, [L].

The relationship between Q-$h_w$ and Q-R in Equation (1) is further analyzed in Figure 2. In the Q-$h_w$ relationship curve (Figure 2(1)), the lower confining bed of the aquifer is considered as the base level. When $h_w$ is zero, the drawdown in the well reaches the maximum, and the hydraulic gradient also reaches the maximum. Because the hydraulic conductivity of the aquifer remains unchanged, the flow velocity in the aquifer is also maximized according to Darcy’s law. If the water sectional area is not considered, the water yield of the pumping well will be maximized. When $h_w$ is equal to $H_0$, there is no drawdown in the well; therefore, the hydraulic gradient cannot be formed, the flow velocity is correspondingly zero, and the water yield of the pumping well is also zero. In the Q-R relationship curve (Figure 2(2)), when the radius of the round island (R) is the same as the radius of the pumping well ($r_w$), the pumping well is equivalent to pumping directly from the sea. In this case, the pumping well can extract an infinite amount of water without considering the limitation of pumping power. When the radius of the round island is infinite, the distance between the sea (source) and the pumping well is also infinite. In this case, even if the pumping time is sufficiently long, the seawater cannot reach the pumping well, and the amount of water from the sea in the pumping well will be zero.
Thiem proposed the stable well flow model of the influence radius based on the Dupuit model (Figure 1(2)). Thiem argued that, in this model, the R value in an aquifer that extends indefinitely in the horizontal direction can be approximated as the horizontal distance from the center of the pumping well to the point where the drawdown of the groundwater level is virtually unobservable. To satisfy the Dupuit hypothesis, Thiem proposed a formula for calculating the stable well flow with one observation well and two observation wells, respectively [12]. This is known as the Thiem formula for a pumping well in unconfined aquifer and is expressed as follows:

\[
\begin{align*}
\frac{h_2^2 - h_1^2}{h_{w}^2} &= \frac{Q}{\pi K} \ln \frac{r}{r_w} \\
\frac{h_2^2 - h_1^2}{h_{w}^2} &= \frac{Q}{\pi K} \ln \frac{r_2}{r_1}
\end{align*}
\]

where \( h \) is the groundwater level at distance \( r \) from the pumping well [L]; \( h_{w} \) is the water level of the pumping well, [L]; \( Q \) is the water yield of the pumping well, \([L^3T^{-1}]\); \( K \) is the hydraulic conductivity, \([LT^{-1}]\); \( r \) is the distance between the pumping well and observation well, [L]; \( r_w \) is the radius of the pumping well, [L]; \( h_1 \) and \( h_2 \) are the groundwater levels of the two observation wells, respectively, [L]; and \( r_1 \) and \( r_2 \) are the distances between the two observation wells and the pumping well, respectively, [L].

Although the Thiem model was established based on the Dupuit model, there are essential differences between them. First, the aquifer in the Dupuit model is cylindrical and has a well-defined boundary, which is surrounded by water. The Dupuit formula does not work without this boundary, while the aquifer in the Thiem model extends indefinitely in the horizontal direction and does not have a boundary. Thus, there are essential differences with regard to the assumed boundary conditions. Additionally, in the Dupuit model, \( R \) refers to the radius of the cylindrical aquifer and is a geometric parameter describing the volume of an object. Moreover, \( R \) has a fixed value, which means that the cylindrical aquifer has a fixed volume. In the Thiem model, \( R \) refers to the horizontal distance from the center of the pumping well to the point at which the drawdown of the groundwater level is virtually unobservable; therefore, it is not a geometric parameter. The values of \( R \) in different directions of the aquifer may also be different because of the heterogeneity of the aquifer, and the boundary of the cone of depression in an aquifer may not be a circle [4,13]. Additionally, in the Dupuit model, the exterior of the cylindrical aquifer is full of water, and the aquifer can receive a constant supply of water. After pumping in the aquifer for a certain period of time, the water yield of the aquifer is entirely supplied by recharge from the outside of the boundary and the water is inexhaustible. Thus, a steady flow can form in the aquifer. In the Thiem model, the range beyond which \( R \) is involved is still an aquifer. When water is pumped in a horizontal infinite aquifer without leakage and
external recharge, all of the water yield comes from the consumption of internal storage in the aquifer. Thus, it is impossible to form a stable flow in the aquifer [14]. Additionally, if water is pumped for a certain period of time, the water level in the pumping well will drop to the lower confining bed.

From the above analysis, Dupuit did not consider the influence radius, while Thiem only introduced the concept of the range of the cone of depression into the hypothesis. Later, Todd renamed this concept as the influence radius. Thiem also avoided the appearance of the influence radius in the formula and did not parameterize it. The present meaning of the influence radius is either the result of misunderstanding Dupuit and Thiem or laziness (saving time and effort).

2.2. Application: Crude Simplification

Based on the Dupuit model and Thiem model, studies have successively deduced various formulas for calculating the influence radius according to their own understanding (Table 1). Some of these formulas are semi-empirical [15] and do not only involve hydrogeological parameters, such as K, H0, and \( \mu \), but also time factors. This indicates that some studies have realized that the influence radius is not a fixed hydrogeological parameter, but instead changes with time. The others are empirical formulas [15], which not only contain hydrogeological parameters but also include pumping variables such as \( s_w \) and Q in the calculation of the influence radius. These formulas only consider one of the time variables and pumping condition variables, and some even consider the influence radius as a given hydrogeological parameter [15].

### Table 1. Equations for calculating the influence radius.

| Equation Name      | Equation | Application Condition                          | Author, Year       | Parameter                      |
|--------------------|----------|-----------------------------------------------|--------------------|--------------------------------|
| Weber equation     | \( R = 74 \sqrt[6]{\frac{6KH_0t}{\mu}} \) | Unconfined aquifer               | Schultze, 1924     | R: influence radius, [L]; K: hydraulic conductivity, [LT\(^{-1}\)]; H0: thickness of aquifer, [L]; t: time from beginning of pumping to formation of stable cone of depression of groundwater level, [T]; \( \mu \): specific yield; \( s_w \): drawdown of pumping well, [L]; Q: water yield of pumping well, [L\(^3\)T\(^{-1}\)]; I: hydraulic gradient of groundwater level |
| Kusakin equation   | \( R = 2s_w \sqrt{H_0K} \) | Unconfined or confined aquifer           | Chertousov, 1949   |                                |
|                    | \( R = 47 \sqrt[6]{\frac{6KH_0t}{\mu}} \) | Unconfined aquifer               | Aravin and Numerov, 1953 |
| Siechardt equation | \( R = 10s_w \sqrt{K} \) | Preliminary stage of pumping in unconfined or confined aquifer | Chertousov, 1962   |                                |
| Wilbur equation    | \( R = 3 \sqrt{\frac{KH_0}{\mu}} \) | Unconfined aquifer               | Chen, 1976         |                                |
| Kelgay equation    | \( R = \frac{Q}{2KH_0I} \) | Completely penetrating well in unconfined aquifer | Chen, 1976         |                                |

To facilitate calculation, some people have introduced the influence radius into some well flow calculation formulas for the forward calculation of variables such as the water level and flow rate, or for the inversion of hydrogeological parameters such as the hydraulic conductivity and water storage coefficient (Table 2).

In some practical applications, to further simplify the calculation, the influence radius is considered empirically; that is, a quantitative relationship is established between the hydraulic conductivity and the influence radius (Table 3). This means that once the hydraulic conductivity of the aquifer is known, the influence radius of the aquifer can be obtained from the empirical value table. In other words, the influence radius is a given hydrogeological parameter that is independent of the pumping conditions. In this empirical relationship, the influence radius becomes larger as the hydraulic conductivity increases.
Table 2. Analytic solution models and equations using the influence radius.

| Model/Equation Name          | Equations Group | Application Condition          | Author, Year | Parameter                      |
|------------------------------|-----------------|--------------------------------|--------------|--------------------------------|
| Plotnikov equation           |                 | Well group pumping             | Chen et al., 1976 | \( R \): influence radius, [L]; \( K \): hydraulic conductivity, [LT\(^{-1}\)]; \( H \): thickness of aquifer, [L]; \( t \): time from beginning of pumping to formation of stable cone of depression of groundwater level, [T]; \( \mu \): specific yield; \( s \): drawdown of pumping well, [L]; \( Q \): water yield of pumping well, [L\(^3\)T\(^{-1}\)]; \( I \): hydraulic gradient of groundwater level. |
| Dupuit–Forchheimer equation  |                 | Unconfined aquifer             | Poehls and Smith, 2009 | \( h_2 = H_2^2 - \frac{Q}{\pi K} \ln \frac{R}{r_w} \) |
| \( s_w \)-calculate equation |                 | Confined aquifer               | China Geological Survey, 2012 | \( Q = \frac{2\pi n s_w}{\ln R_{rw}} \) |
| Siechardt equation          |                 | Confined aquifer               | China Geological Survey, 2012 | \( T = \frac{Q}{\pi} \ln \frac{R}{r_w} \) |
| Wilbur equation             |                 | Unconfined aquifer             |                | \( K = \frac{Q}{\pi [H_2^2 - (H_0 - s)^2]} \ln \frac{R}{r_w} \) |

Table 3. Empirical relationship of \( K \) and \( R \).

| K (m/d) | R (m) |
|---------|-------|
| 0.5–1   | 25–50 |
| 1–5     | 50–100|
| 5–20    | 100–300|
| 20–50   | 300–400|
| 50–100  | 400–500|
| 75–150  | 500–600|
| 100–200 | 600–1500|
| 200–500 | 1500–3000|

This table is taken from [16]; \( K \) is the hydraulic conductivity, [LT\(^{-1}\)]; and \( R \) is the influence radius, [L].

The influence radius has been parameterized and is more widely applied in theoretical research and engineering practice [7,16], which makes relevant research and applications more convenient. However, as the research and applications become more extensive, various problems are emerging with regard to the influence radius as a hydrogeological parameter. Various studies have expressed different opinions regarding the influence radius (Table 4). Additionally, some studies have conceptually defined the influence radius as a hydrogeological parameter [15] and considered that it can be used as the basis for designing a reasonable distance between wells [12]. However, other studies have reported that this is not the case [7] and have argued that it is impossible to have a stable influence radius in an infinite aquifer [12,17] and that the time factor should be considered in the calculation of the influence radius [18]. If the \( R \) in the Dupuit and the \( R \) in the Thiem models are treated equally, this will lead to errors in the theory and methodology of groundwater resource evaluation [8].

The introduction and application of the influence radius has greatly simplified the calculations required by various engineering problems, and its empirical treatment has provided great convenience to non-professionals for carrying out relevant calculations and understanding groundwater problems. However, because coincidence in practice is not the same as correctness in theory, these crude simplifications not only harm the development of the discipline and specialty but also are misleading in practical approaches and cause irreparable losses.
3. Gap between Theory and Practice

3.1. Substance of Influence Radius: Distance

The influence radius R is essentially the distance in the time–distance problem in physics, namely \( R = S(v, t) \). Therefore, R is a function of time t and is also controlled by the velocity v and its distribution on the flow line. The influence radius is actually the influence range of the pumping well in the horizontal direction. Generally, the essence of the extension of the influence range is the velocity conduction in the process of mass transfer (water molecules) in porous media. When the distance to the pumping well is shorter, the velocity becomes higher. For a particle in an aquifer, as long as there is flow velocity to the well at that particle, the particle is within the influence range of the pumping well, unless the velocity of that particle is zero. From this viewpoint, it is reasonable to state that R is a function of velocity v. Moreover, it can also be seen from the Kusakin formulas (Table 1) that there is no stable influence radius, and the observed cone of depression will gradually expand with the extension of pumping time [19]. Thus, the influence radius is a function of time t [20,21]. Similar to water ripples (Figure 3(1)) and dominoes (Figure 3(2)), the range of influence will spread out with the advancement of time t.

![Figure 3](image_url)

**Figure 3.** Influence radius of ripples in water and dominoes (used as an analogy to describe the influence radius discussed in this paper). R is the influence radius, [L]; S is the distance in physics, [L]; and t is time, [T].

### Table 4. Some representative viewpoints on the influence radius (R).

| Viewpoint                                                                 | Reference |
|---------------------------------------------------------------------------|-----------|
| (1) Dupuit’s R is an abstract parameter that reflects the well supply conditions and is recommended as a reference recharge radius. | [4]       |
| (2) There is still a considerable amount of drawdown beyond the range that we used to think of as R. |           |
| R should be interpreted as a parameter indicating the distance beyond which the drawdown is negligible or unobservable. | [15]      |
| R does not exist in an infinite aquifer.                                  | [17]      |
| (1) In theory, R does not exist in a confined aquifer that extends indefinitely without overcurrent recharge. | [12]      |
| (2) In practice, R should be considered as the horizontal distance from the pumping well to the point where the water level cannot actually be observed to drop and can be used as the basis for designing reasonable distances between wells. |           |
| (1) Dupuit’s R is different from Thiem’s R.                              | [8]       |
| (2) Confusion between them has led to theoretical errors and incorrect methods of groundwater resource evaluation. |         |
| (1) The magnitude of R has assumed properties making it essentially the same as unsteady flow. | [18]      |
| (2) The Kusakin equation with a time factor should be applied to the calculation of R. |           |
| (1) Dupuit’s R is different from Thiem’s R.                              | [7]       |
| (2) Dupuit’s R is simply the radius of the round island, while Thiem’s R is a variable related to the cone of depression of the groundwater level. |         |
However, many studies have reported that the influence radius $R$ is a hydrogeological parameter reflecting the natural properties of aquifers, similar to parameters such as the porosity $n$, specific yield $\mu$, and hydraulic conductivity $K$. Therefore, the influence radius is a constant value that is not affected by the drawdown $s_w$ and water yield $Q$ [16]. Studies have given the empirical values of $R$ for aquifers with different particle structures (Table 3) and have considered that greater aquifer permeability—that is, larger aquifer particles—results in a larger $R$ value. When pumping water in an aquifer without external recharge, the cone of depression will expand with the increase of the water yield and the advancement of time. If the empirical influence radius value is used, such as in the case of the coarse gravel aquifers mentioned in some papers, the empirical value of the influence radius will be 1500–3000 m (Table 3). In other words, regardless of how large the amount of exploitation is and how long the exploitation period is, the cone of depression of the aquifer will not continue to expand outward after extending to 1500–3000 m. Accordingly, it is assumed that, in a certain pumping well group in an aquifer, the water yield of $n$ single pumping wells is $Q_{m1}$, $Q_{m2}$, ..., $Q_{mn}$, respectively, their influence radius is $R$ (Figure 4(1)), and a regional cone of depression will be formed. If the sum of the water yield of $n$ single wells in the well group is provided by a single well, then a cone of depression with an influence radius $R$ will be formed (Figure 4(2)). In this case, owing to the decrease in the number of wells, the area affected by pumping will be much smaller compared with when the well group is pumped. However, without external recharge, this phenomenon is completely impossible in an aquifer; otherwise, the aquifer will become an inexhaustible resource, which is contrary to common sense.

![Figure 4](image-url)

**Figure 4.** Influence radius in well group pumping and single well pumping in influence radius theory. $R$ is the influence radius, [L]; $Q_{mi}$ is the maximum water yield of pumping well $i$, [L$^3$T$^{-1}$]; and $n$ is the number of pumping wells.

Further, because $v = v (K, I)$ in $R = S = S (v, t)$, $R = S = S (v(K, I), t)$. For a specific aquifer, the hydraulic conductivity $K$ is constant, and the hydraulic gradient $I$ and $t$ are variables. Therefore, $R$ is affected by the two variables $I$ and $t$. Additionally, the hydraulic gradient $I$ is related to the drawdown $s_w$. For a particular aquifer with thickness $H_0$, each drawdown of a pumping well in the aquifer has a corresponding hydraulic gradient $I$. Notably, there are two extremes: one is that the drawdown of the well is zero. At this point, the hydraulic gradient is zero and the groundwater does not flow. In another case, the drawdown of the well reaches $H_0$; that is, the water level in the well drops to the lower confining bed. In this case, the hydraulic gradient reaches the maximum and the groundwater flow velocity also reaches the maximum. However, the drawdown $s_w$ is closely related to the water yield $Q$; that is, it increases with the water yield. Therefore, the influence radius $R$ is not only affected by the hydrogeological conditions reflecting the natural properties of the aquifer, such as the porosity, specific yield, and hydraulic conductivity, but is also controlled by the
pumping conditions. Therefore, it is inappropriate to consider the influence radius R in
the Thiem model as a fixed parameter of the aquifer, which means that this parameter is
constant for a specific aquifer and does not change with the changes of the water yield and
drawdown [7].

3.2. The Continuity Principle of Fluids Reverses the Rationality of the Influence Radius

In any system, fluids follow the continuity principle (conservation of mass), which
means that the amount of fluid entering a region of space in a unit of time is equal to
that leaving plus that stored within the region through density changes [22]. Because a
horizontal infinite aquifer does not exist in practice, in the early 1960s, Toth [23] considered
the continuity principle of fluids and proposed the concept of the multi-level groundwater
flow system of a basin under the assumption that the phreatic surface is the recharge
boundary and the fluctuation is similar to the ground. By the 1980s, the theoretical
framework had been essentially formed [24]. Toth [23,24] pointed out that there is a
difference in the elevation of the groundwater level in the basin, and a nested multi-level
groundwater flow system is self-organized under the influence of gravity.

The emergence of groundwater flow system theory, which is a new paradigm of
hydrogeology [25,26], has influenced the traditional groundwater movement theory, which
was developed based on the well flow, whereby the groundwater converges to artificial
potential sinks such as wells from potential sources [26]. Under the effect of the influence ra-
dius theory, it is commonly believed that groundwater always flows from potential sources
to adjacent potential sinks within the range of the influence radius. In fact, when there
are several potential sources and potential sinks with different strengths, the groundwater
conforms to the theory of the minimum rate of energy dissipation and moves from differ-
ent potential sources to different potential sinks [26]. Thus, it forms a multi-level nested
groundwater flow system; that is, a complex flow pattern consisting of local, intermediate,
and regional groundwater flow systems [23]. Of these, the scale of the regional ground-
water flow system is the largest, and its flow lines can extend at great distances. Thus,
the pumping well may be recharged by groundwater that is far away from it in practical
situations (Figure 5). This theory suggests that we cannot use the influence radius to define
the aquifer range, and water particles outside of this range do not move to the pumping
well. In other words, the practical application of the influence radius is unreasonable.

![Figure 5. Schematic flow lines of a practical site according to the distribution of apparent resistivity, with a hypothetical pumping well added (modified from [27]).](image)

3.3. Essential Difference between Theory and Practice

The seemingly stable phenomenon observed in practice known as the quasi-steady
state [15] is essentially different from the influence radius considered in theory. The term
“seemingly stable” means that, when pumping the water in an aquifer, the change of the
groundwater piezometric head close to the well gradually slows down as the pumping time increases. Thus, within a certain range, it is close to a stable state and has the same shape as the falling curve of stable flow [12]. For a long time, the seemingly stable phenomenon observed in practice was associated with the theoretical influence radius, and the influence radius has even been used to represent the influence range when the aquifer is in a seemingly stable state. This confirms the rationality of the influence radius parameter, which is a misunderstanding of the seemingly stable phenomenon and influence radius concept.

The stable or seemingly stable groundwater level observed in practice is simply a coincidence that occurs under the influence of various practical factors and a misapprehension caused by the low accuracy of the actual measurement of the groundwater level or by the recharge of the aquifer. Therefore, this phenomenon cannot be used to explain the rationality of the influence radius parameter in theoretical models. The steady state does not exist under pumping conditions without recharge in theory. According to the continuity principle of fluids, in the cone of depression generated by pumping water, the amounts of water passing through regions A, B, and C in a unit of time are equal. However, from region A to region C, the basal area gradually increases (Figure 6(1)); therefore, the height gradually decreases (Figure 6(2)). Similarly, the drawdown at infinity will be very small but not zero, because the water yield is a concept of volume and cannot be changed from three dimensions to two dimensions. The location of the pumping well is considered as the origin of the coordinates to establish the coordinate system, and the drawdown \( s_w \) is considered as a function of \( x \), namely \( z = s_w(x) \). When pumping water, the drawdown of the aquifer is larger when the distance to the pumping well is shorter. As \( x \) approaches \( \infty \), the drawdown tends towards zero (Figure 6). Assuming that the radius of the pumping well is \( r_w = 0 \), then the water yield of the pumping well is the volume of the rotating body obtained by rotating the curvilinear trapezoid bounded by a continuous curve \( z = s_w(x) \), \( z \)-axis, line \( x = +\infty \), and line \( z = 0 \) once around the \( z \)-axis. For convenience of description, the object of investigation was considered to be a unit width aquifer passing through the axis; then, the value of its volume is \( V = \int s_w(x) dx, (0, \infty) \). When water is pumped continuously, \( x \) tends towards \( \infty \); therefore, \( V \) also tends towards \( \infty \). This means that the cone of depression will expand infinitely; therefore, a stable state cannot be formed. What is commonly referred to as the seemingly stable state does not mean that the groundwater level is stable, but that the change of the water level cannot be observed, which thus gives the illusion of stability. The phenomenon whereby the change of the water level cannot be observed is caused by the means of observation and other factors (external recharge). This is similar to the detection limit in analytical chemistry, which is a relative concept. With the improvement of the observation means, the observed drawdown range of the water level will increase. Therefore, the influence radius \( R \) cannot be considered as an intrinsic parameter of the aquifer beyond which there is no drawdown of the water level.

Therefore, in theory, the influence range will continue to expand with the extension of pumping time in an infinite aquifer without external recharge. However, there are big differences between reality and theory: (1) in reality, there are no unbounded aquifers that extend indefinitely in the horizontal direction. Therefore, for an aquifer without external recharge, the boundary of the influence range is the boundary of the aquifer when the pumping time is sufficiently long. (2) In reality, there are few aquifers without external recharge. The phenomenon whereby the observed water level reaches a stable or quasi-stable state when pumping in reality is attributed to the measurement accuracy [28]. More importantly, this phenomenon is attributed to the fact that the aquifer may have obtained unknown external recharge, such as atmospheric precipitation, surface water, and agricultural irrigation water [29], as well as to the vertical leakage recharge caused by pumping [30]. This contradicts the assumption of a lack of external recharge in the theoretical model, which is often ignored and thus results in the illusion of water level stability. Coincidences that occur in practical situations have led to the false impression that the influence radius theory has been verified, which has resulted in the wide adoption
Thus, the improved Dupuit model, which introduces external recharge, is more in line with actual needs [31].

Figure 6. Formation of cone of depression of groundwater level during pumping. Q is the water yield of the pumping well, [L^3T^{-1}].

4. The Dilemma of Practice

4.1. Misleading the Management of Sustainable Groundwater Development

From both a theoretical and practical viewpoint, it is proven that the influence radius cannot be used to evaluate the sustainable water supply capacity of aquifers. The influence radius is introduced for the convenience of well flow mechanics calculation to easily determine the hydraulic conductivity of an aquifer or the yield capacity of the pumping well per unit of time [4]. The yield capacity of a pumping well is actually the reflection of the working efficiency of the groundwater collecting structure, while the water supply capacity of an aquifer is an attribute of the aquifer itself. Therefore, the two concepts should not be confused. In reality, there are no aquifers that extend indefinitely in the horizontal direction; that is, actual aquifers are bounded. For an aquifer surrounded by confining boundaries, the phenomenon whereby the pumping well in the aquifer has a large yield capacity per unit of time only indicates that the aquifer can provide sufficient water during that period. In the absence of external recharge, the volume of water in an aquifer is limited. As the pumping proceeds, the water in the aquifer decreases and may not be able to continuously provide sufficient water in the next period. This is similar to a glass of water with a capacity of 600 ML: if we use a straw to draw the water from the cup and do not add water to the cup during the process, the water in the cup will gradually decrease (Figure 7(1)), and we will only obtain 600 ML of water. Thus, we must keep adding water to the cup if we desire to obtain water continuously (Figure 7(2)). This situation accurately describes the situation of pumping water in an aquifer.
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Figure 7. Drawing water from cup (used as analogy for pumping in bounded aquifer). Q is the water yield of the pumping well, [L$^3$T$^{-1}$]; and t is the time, [T].

For aquifers with a recharge boundary and drainage boundary, the increase of recharge and the reduction of discharge will occur when the influence range of pumping water extends to these boundaries [9]; that is,

$$ V_{uw} = \Delta V_r - \Delta V_d - \Delta V_s $$

where $V_{uw}$ is the total water yield in a certain period, [L$^3$]; $\Delta V_r$ is the increment of recharge in the same period, [L$^3$]; $\Delta V_d$ is the increment of discharge in the same period, [L$^3$]; and $\Delta V_s$ is the increment of storage in the same period, [L$^3$]. A riverside well is a typical example. Under natural conditions, groundwater may recharge rivers. After the addition of pumping wells close to the river, the recharge relationship between them is the fact that the river recharges groundwater and the water being pumped comes from the river and the aquifer [32]. If the recharge and discharge of the aquifer remain unchanged before and after pumping water, then $V_{uw} = -\Delta V_s$. This indicates that all the water being pumped comes from the consumption of the aquifer storage, and it is impossible to produce an influence radius. To achieve a steady state, the sum of the increment of recharge and the
reduction of discharge (sustainable yield) should always be equal to the pumping yield in the pumping state. Notably, when evaluating the amount of groundwater resources, the safe yield is usually adopted, which means that the water yield does not exceed the natural recharge amount [33]. This ignores the exploitation potential of the aquifer, which includes the increment of recharge and the reduction of discharge caused by pumping.

Because the external recharge of an aquifer directly affects the water storage of the aquifer [34], the sustainable water supply capacity of an aquifer largely depends on its ability to receive the recharge [35] and has nothing to do with the yield capacity of the pumping well, unless the extraction of the pumping well can increase the external recharge of the aquifer. From the viewpoint of sustainable development, the external recharge of the aquifer in the area and the lateral recharge inside the aquifer cannot ensure a sustainable water supply from the aquifer. First, because the lateral recharge within an aquifer is ultimately derived from the aquifer itself, its water is limited; second, from the viewpoint of resource ownership, the external recharge of the aquifer in the distance belongs to the residents in the distance, and local residents (where pumping wells are located) do not have the right to occupy it [33]. Therefore, sustainable replenishments from the upper boundary of the aquifer, such as atmospheric precipitation infiltration, river leakage, and irrigation water infiltration, play a decisive role in the sustainable water supply capacity of the aquifer. Thus, the influence radius cannot be used to evaluate the sustainable water supply capacity of the aquifer but can only be used to assess the yield capacity of the pumping well.

In practice, the influence radius theory is used to guide the development of groundwater resources in many areas. The aquifer within the range of the influence radius is considered as a “treasure basin”, and it is thought that the groundwater resources are inexhaustible within this scope. However, this leads to a series of ecological and environmental problems such as the global decline of groundwater levels as a result of excessive exploitation [34,36–40], the occurrence of the cone of depression over a wide range in plain areas [41–45], the attenuation and even depletion of spring water at piedmont [46], and seawater intrusion caused by the excessive exploitation of groundwater in coastal plain areas [35,47–49]. In production practice, it is unreasonable to use the influence radius as a guideline. In engineering practice, however, various problems such as foundation pit dewatering and pumping by a well group are unavoidable and have a certain impact on the surrounding environment (such as land subsidence). Therefore, it is particularly important to develop a clear method for determining the scope of environmental impacts according to the sensitive targets and receptors around the site [6].

4.2. Misleading the Safeguarding of Groundwater Quality

The objective of water resource management is not only human water demand; rather, the quality of water is also an important factor to be considered in the process of the development of water resources [50–52]. The general deterioration of groundwater quality makes the remediation, treatment, and protection of groundwater particularly important [53–55]. Before conducting research on groundwater pollution remediation, the accurate identification of groundwater pollution sources and characteristics such as their intensity, distribution, and existence time is crucial for improving the efficiency of groundwater remediation research [56]. In most cases, pollution source analysis is carried out using the numerical simulation inversion method for groundwater. The main idea of these methods is to simulate the groundwater flow and pollutant transport process in reverse according to the monitoring data of the spatial and temporal distribution of pollutant concentration and determine the characteristics of pollution sources [55,57,58]. In the most widely used optimization simulation method [57], it is always assumed that the numbers and locations of pollution sources in the groundwater solute migration model are known, and the monitoring data of the spatial and temporal distribution of groundwater pollutant concentrations are obtained within a limited study area [57,59–61], which limits the influence range of the pollution sources within a certain space. However, in aquifer systems, pollutants
migrate with the groundwater [48]. Without considering the degradation and attenuation of pollutants in the aquifer, the pumping wells are recharged from the aquifers in the distance under long-term pumping conditions. At this time, the pollutants in the distance will still be affected by pumping and migrate with the groundwater until they reach the pumping well [62,63]. In other words, not all pollutants in the pumping wells come from sources close to the pumping wells, but some also come from the aquifers outside the range of the influence radius. Therefore, the analytical method of the pollution source, which is restricted by the existing influence radius theory, cannot ensure the accuracy of analytical results, and thus its reference value is reduced. Similar problems also exist in the remediation of groundwater pollution. In the ex-situ pump and treat technique [64,65], if the layout of the pumping well group is determined based on the influence radius during the pumping design and then parameters such as the water yield, drawdown of water level, and pumping time are calculated, the groundwater outside of the polluted range may be pumped out, which will affect the treatment efficiency and increase the treatment cost. Similarly, there is also a problem regarding the injection well’s influence radius when the groundwater is restored by in-situ remediation with the injection of remediation agents into the groundwater [66–68]. Without considering the chemical reactions of the remediation agents in the migration process with groundwater, these remediation agents may not be restricted to interact with pollutants within the designed influence range but may instead migrate to a further range along with the flow and thus change the primary groundwater environment and cause secondary pollution.

The influence radius is also widely used as the basis for the division of groundwater source protection areas to prevent the pollution of water sources [69,70]. However, the flow line follows the range of influence and should be covered by the influence radius. The delineation of a protection area near a pumping well artificially cuts off the flow line, which violates the flow continuity principle. Such protection measures cannot guarantee the sustainable supply from groundwater sources and lead to dangers that may compromise water quality. Additionally, the influence radius is often used as the basis for designing a reasonable distance between wells in the planning of riverbank filtration systems [12]. Although the external recharge of the river can stabilize the influence radius [32,71–73], leakage recharge may occur when pumping is carried out on the side of a incompletely penetrating river. Pollutants in the groundwater on the other side of the river also migrate to the pumping wells with the groundwater [74]. Therefore, pumping water in riverbank filtration systems, which are planned based on the influence radius, cannot ensure the water quality of pumped water and may affect the groundwater on the other side of the river.

Thus, it is reasonable to state that any methodology involving the influence radius must be reconsidered. Moreover, effective methods of groundwater remediation, treatment, and protection should be investigated to make the influence radius theory obsolete. To this end, new technology should be used, such as the MODPATH module in GMS, which is used to track the virtual particle beam of a pumping well to obtain the capture area of the well in a given time [75] and is considered to be an effective research tool.

5. Summary and Prospects

The introduction of the Dupuit model has placed a focus on the quantitative calculation of well flow. To solve the computing problem of actual aquifers, Thiem introduced the concept of the range of cone of depression, which Todd later renamed as the influence radius. In later production practice, the round island radius was confused with the influence radius, and the parameterized influence radius has been widely used to evaluate the sustainable water supply capacity of aquifers and in the planning of groundwater source areas, which has led to the emergence of a series of ecological and environmental problems.

The influence radius was originally used in the calculation of some hydrogeological parameters but, owing to various coincidences that occur in practical situations, it has been considered that the parameterized influence radius is reasonable and convenient for calculations pertaining to actual production problems, and this misconception has
perpetuated. However, by considering the continuity principle of flow, it can be proven that the parameterized influence radius does not exist. The influence radius is essentially the distance in the time–distance problem in physics and is influenced by the hydrogeological conditions and pumping conditions, which is different from the hydrogeological parameters reflecting the natural properties of aquifers, such as the porosity, specific yield, and hydraulic conductivity. In reality, infinite horizontal aquifers do not exist, and most aquifers are replenished from external sources, which is very different from the theoretical considerations. The stable or seemingly stable groundwater level observed in practice is a misapprehension caused by low accuracy in the actual measurement of the groundwater level or by aquifer recharge.

In the past, the well flow model was only used to solve local water supply problems, and attention was only given to local groundwater problems due to economic and technological limitations. The influence radius theory has provided incorrect guidance in the analysis of groundwater pollution sources, the division of groundwater source protection areas, and the planning of riverbank filtration. With the development of the social economy, in addition to scientific and technological developments, attention has gradually shifted to regional and even global groundwater flow systems. Moreover, focus is increasingly being placed on sustainable development and the protection of the ecological environment. Because groundwater is a local and regional resource and is increasingly considered as a global resource, the evaluation and rational development of groundwater resources should consider hydrogeologic units as a whole, as in the case of basin management for surface water. Instead of solving the local practical problems of production, the long-term, comprehensive, and systematic topics of sustainable development should be given attention, and problems in resource ecology and environmental disasters should be addressed holistically in a systematic and methodical manner. Additionally, some methods may be useful in groundwater resource evaluation. For example, the concept of the “scope of environmental impacts” is suitable for different industries. Additionally, the original theoretical model can be improved such that it can be applied to current aquifer calculations. Finally, the combination of numerical simulation with new technologies, such as isotopic methods, remote sensing, and big data, can improve the accuracy of models.

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**Abbreviations**

| Symbol | Description                              | Dimension |
|--------|------------------------------------------|-----------|
| R      | Influence radius                         | L         |
| K      | Hydraulic conductivity                   | LT⁻¹      |
| H₀     | Thickness of aquifer                      | L         |
| s_w    | Drawdown of pumping well                  | L         |
| Q      | Water yield of pumping well               | LT⁻¹      |
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| Symbol | Description | Units |
|--------|-------------|-------|
| \( r_w \) | Radius of pumping well | L |
| \( T \) | Transmissibility coefficient of aquifer | \( \text{L}^2 \text{T}^{-1} \) |
| \( t \) | Time from beginning of pumping to formation of stable cone of depression of groundwater level | T |
| \( \mu \) | Specific yield | / |
| \( I \) | Hydraulic gradient of groundwater level | / |
| \( r \) | Distance between pumping well and observation well | L |
| \( h \) | Groundwater level at distance \( r \) from pumping well | L |
| \( h_w \) | Water level of pumping well | L |
| \( e \) | Empirical coefficient | / |
| \( Q_0 \) | Water yield of single pumping well | \( \text{L}^3 \text{T}^{-1} \) |
| \( B \) | Width of aquifer | L |
| \( Q_m \) | Maximum water yield of pumping well | \( \text{L}^3 \text{T}^{-1} \) |
| \( S \) | Distance in physics | L |
| \( v \) | Seepage velocity | \( \text{LT}^{-1} \) |
| \( n \) | Porosity of porous media (aquifer) | / |

References

1. Darcy, H. Les Fontaines Publiques de la Ville de Dijon [The Public Fountains of the City of Dijon]; Victor Dalmont: Paris, France, 1856.
2. Dupuit, J. Études Théoriques et Pratiques sur le Mouvement des Eaux dans les Canaux Découverts et à Travers les Terrains Pernéables [Theoretical and Practical Studies on the Movement of Water in Open Channels and Permeable Ground]; Dunod: Paris, France, 1863.
3. Thiem, G.A. Hydrologische Methoden; Gebhardt: Leipzig, Germany, 1906.
4. Chen, Y.S.; Yao, D.S.; Hua, R.R. Try to discuss the influence radius. Geotech. Invest. Survey 1976, 6, 5–33. (In Chinese)
5. Todd, D.K. Ground Water Hydrology; Wiley: New York, NY, USA, 1959.
6. Wang, J.H.; Wang, F. Discussion on the range of groundwater depression cone, radius of influence and scope of environmental impacts during pumping. J. Hydraul. Eng. 2020, 51, 827–834. (In Chinese with English abstract)
7. Wang, X.M.; Wang, X.H.; Wen, W.; Li, G.Y. Model analysis of Dupuit’s steady well flow formula. Coal. Geol. Explor. 2014, 6, 73–75, (In Chinese with English abstract).
8. Chen, C.X.; Lin, M.; Cheng, J.M. Groundwater Dynamics, 5th ed.; Geological Publishing House: Beijing, China, 2011. (In Chinese)
9. Chen, C.X. Stable well flow model of influence radius and sustainable yield: Differences of a basic theoretical problem in Groundwater Dynamics—Discuss with academician Xue Yuqun. J. Hydraul. Eng. 2010, 41, 1003–1008. (In Chinese)
10. Xue, Y.Q. Discussion on the steady well flow model and Dupuit formula—Reply to professor Chen Chongxi’s article “Discussion”. J. Hydraul. Eng. 2011, 39, 1252–1256. (In Chinese)
11. Chen, C.X. Discuss on stable well flow model of influence radius and sustainable yield: Differences of a basic theoretical problem in Groundwater Dynamics—Reply to academician Xue Yuqun’s discussion. J. Hydraul. Eng. 2014, 45, 117–121. (In Chinese)
12. Xue, Y.Q.; Wu, J.C. Groundwater Dynamics, 3rd ed.; Geological Publishing House: Beijing, China, 2010. (In Chinese)
13. Wu, C.M.; Yeh, T.C.J.; Zhu, J.; Lee, T.H.; Hsu, N.S.; Chen, C.H.; Sancho, A.F. Traditional analysis of aquifer tests: Comparing apples to oranges? Water Resour. Res. 2005, 41. [CrossRef]
14. Theis, C.V. The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage. Eos Trans. AGU 1935, 16, 519–524. [CrossRef]
15. Bear, J. Hydraulics of Groundwater; McGraw-Hill, Inc.: New York, NY, USA, 1979.
16. China Geological Survey. Handbook of Hydrogeology, 2nd ed.; Geological Publishing House: Beijing, China, 2012. (In Chinese)
17. Zhang, H.R. The steady state of groundwater movement. Hydrogeol. Eng. Geol. 1986, 6, 18–21. (In Chinese)
18. Zhu, D.T. Discussion on Dupuit formula and Forchheimer formula. J. Hydraul. Eng. 2012, 43, 502–504. (In Chinese)
19. Zhang, H.R. A couple of interesting inference of Theis’s equation. Hydrogeol. Eng. Geol. 1985, 5, 30–32. (In Chinese)
20. Chang, P.Y.; Hsu, S.Y.; Chen, Y.W.; Liang, C.; Wen, F.; Lu, H.Y. Using the resistivity imaging method to monitor the dynamic effects on the vadose zone during pumping tests at the Pengtsuo site in Pingtung, Taiwan. Terr. Atmos. Ocean. Sci. 2016, 27, 059. [CrossRef]
21. Chang, P.Y.; Chang, L.C.; Hsu, S.Y.; Tsai, J.P.; Chen, W.F. Estimating the hydrogeological parameters of an unconfined aquifer with the time-lapse resistivity-imaging method during pumping tests: Case studies at the Pengtsuo and Dajou sites. Taiwan. J. Appl. Geophys. 2017, 144, 134–143. [CrossRef]
22. Craik, A.D.D. “Continuity and change”: Representing mass conservation in fluid mechanics. Arch. Hist. Exact Sci. 2013, 67, 43–80. [CrossRef]
23. Tóth, J. A theoretical analysis of groundwater flow in small drainage basin. J. Geophys. Res. 1963, 68, 4795–4812. [CrossRef]
24. Tóth, J. Cross-formation gravity flow of groundwater: A mechanism of the transport and accumulation of petroleum (The generalized hydraulic theory of petroleum migration). AAPG Stud. Geol. 1980, 10, 121–167.
25. Mädzl-Szönyi. From the Artesian Paradigm to Basin Hydraulics: The Contribution of József Tóth to Hungarian Hydrogeology; Publishing Company of Budapest University of Technology and Economics: Budapest, Hungary, 2008.
26. Zhang, R.Q.; Liang, X.; Jin, M.G.; Wan, L.; Yu, Q.C. *Fundamentals of Hydrogeology*, 7th ed.; Geological Publishing House: Beijing, China, 2018. (In Chinese)

27. Jiang, X.W.; Wan, L.; Wang, J.Z.; Yin, B.X.; Fu, W.X.; Lin, C.H. Field identification of groundwater flow systems and hydraulic traps in drainage basins using a geophysical method. *Geophys. Res. Lett.* 2014, 41, 2812–2819. [CrossRef]

28. Zha, Y.Y.; Shi, L.S.; Liang, Y.; Tso, C.H.M.; Zeng, W.Z.; Zhang, Y.G. Analytical sensitivity map of head observations on heterogeneous hydraulic parameters via the sensitivity equation method. *J. Hydrol.* 2020, 591, 125282. [CrossRef]

29. Liu, Y.P.; Yamanaka, T.; Zhou, X.; Tian, F.Q.; Ma, W.C. Combined use of tracer approach and numerical simulation to estimate groundwater recharge in an alluvial aquifer system: A case study of Nasunogahara area, central Japan. *J. Hydrol.* 2014, 519, 833–847. [CrossRef]

30. Yuan, R.Q.; Song, X.F.; Han, D.M.; Zhang, L.; Wang, S.Q. Upward recharge through groundwater depression cone in piedmont plain of North China Plain. *J. Hydrol.* 2013, 500, 1–11. [CrossRef]

31. Chen, C.X. Improvement of Dupuit model: With infiltration recharge. *Hydrogeol. Eng. Geol.* 2020, 47, 1–4. (In Chinese with English abstract).

32. Zhu, Y.G.; Zhai, Y.Z.; Du, Q.Q.; Teng, Y.G.; Wang, J.S.; Yang, G. The impact of well drawdowns on the mixing process of river water and groundwater and water quality in a riverside well field, Northeast China. *Hydrolog. Process.* 2019, 33, 945–961. [CrossRef]

33. Wood, W.W. Groundwater “durability” not “sustainability”? *Groundwater* 2020. [CrossRef]

34. Ahmed, K.; Shahid, S.; Demirel, M.C.; Nawaz, N.; Khan, N. The changing characteristics of groundwater sustainability in Pakistan from 2002 to 2016. *Hydrogeol. J.* 2019, 27, 2485–2496. [CrossRef]

35. Jia, X.Y.; Hou, D.Y.; Wang, L.W.; O’Connor, D.; Luo, J. The development of groundwater research in the past 40 years: A burgeoning trend in groundwater depletion and sustainable management. *J. Hydrol.* 2020, 587, 125006. [CrossRef]

36. White, E.K.; Costelloe, J.; Peterson, T.J.; Western, A.W.; Carrara, E. Do groundwater management plans work? Modelling the effectiveness of groundwater management scenarios. *Hydrogeol. J.* 2019, 27, 1–24. [CrossRef]

37. Famiglietti, J.S. The global groundwater crisis. *Nat. Clim. Chang.* 2014, 4, 945–948. [CrossRef]

38. Giordano, M. Global groundwater? Issues and solutions. *Ann. Rev. Environ. Resour.* 2009, 34, 153–178. [CrossRef]

39. Konikow, L.; Kendy, E. Groundwater depletion: A global problem. *Hydrogeol. J.* 2005, 13, 317–320. [CrossRef]

40. Wada, Y.; van Beek, L.P.; van Kempen, C.M.; Reckman, J.W.T.M.; Vasak, S.; Bierkens, M.F.P. Global depletion of groundwater resources. *Geophys. Res. Lett.* 2010, 37, L20402. [CrossRef]

41. Erban, L.E.; Gorelick, S.M.; Zebker, H.A.; Fendorf, S. Release of arsenic to deep groundwater in the Mekong Delta, Vietnam, linked to pumping-induced land subsidence. *Proc. Natl. Acad. Sci. USA* 2013, 110, 13751–13756. [CrossRef]

42. Galloway, D.L.; Burbey, T.J. Review: Regional land subsidence accompanying groundwater extraction. *Hydrogeol. J.* 2011, 19, 1459–1486. [CrossRef]

43. Gleeson, T.; Alley, W.M.; Allen, D.M.; Sophocleous, M.A.; Zhou, Y.X.; Taniguchi, M.; VanderSteen, J. Towards sustainable groundwater use: Setting long-term goals, backcasting, and managing adaptively. *Groundwater* 2012, 50, 19–26. [CrossRef]

44. Qian, H.; Chen, J.; Howard, K.W.F. Assessing groundwater pollution and potential remediation processes in a multi-layer aquifer system. *Environ. Pollut.* 2020, 263, 114669. [CrossRef] [PubMed]

45. Scanlon, B.R.; Reedy, R.C.; Gates, J.B.; Gowda, P.H. Impact of agroecosystems on groundwater resources in the Central High Plains, USA. *Agric. Ecosyst. Environ.* 2010, 139, 700–713. [CrossRef]

46. Feng, W.; Zhong, M.; Lemoine, J.M.; Biancale, R.; Hsu, H.T.; Xia, J. Evaluation of groundwater depletion in North China using the Gravity Recovery and Climate Experiment (GRACE) data and ground-based measurements. *Water Resour. Res.* 2013, 49, 2110–2118. [CrossRef]

47. Hakon, A. Application of multivariate statistical techniques in the assessment of groundwater quality in seawater intrusion area in Bafra Plain. *Turkey* 2013, 18, 2439.

48. Jia, Y.F.; Xi, B.D.; Jiang, Y.H.; Guo, H.M.; Yang, Y.; Lian, X.Y.; Han, S.B. Distribution, formation and human-induced evolution of geogenic contaminated groundwater in China: A review. *Sci. Total Environ.* 2018, 643, 967–993. [CrossRef] [PubMed]

49. Lei, S.; Jiao, J.J. Seawater intrusion and coastal aquifer management in China: A review. *Environ. Earth Sci.* 2014, 72, 2811–2819. [CrossRef]

50. Narasimhan, T.N. Hydrogeology in North America: Past and future. *Hydrogeol. J.* 2005, 13, 7–24. [CrossRef]

51. Chinese Academy of Science. *Development Strategy of Chinese Subjects: Groundwater Science*; Science Press: Beijing, China, 2019. (In Chinese)

52. Zhang, F.G.; Huang, G.X.; Hou, Q.X.; Liu, C.Y.; Zhang, Y.; Zhang, Q. Groundwater quality in the Pearl River Delta after the rapid expansion of industrialization and urbanization: Distributions, main impact indicators, and driving forces. *J. Hydrol.* 2019, 577, 124004. [CrossRef]

53. Kurwadkar, S. Emerging trends in groundwater pollution and quality. *Water Environ. Res.* 2014, 86, 1677–1691. [CrossRef]

54. Zhang, B.; Song, X.; Zhang, Y.; Han, D.; Tang, C.; Yu, Y.; Ma, Y. Hydrochemical characteristics and water quality assessment of surface water and groundwater in Songnen plain, Northeast China. *Water Res.* 2012, 46, 2737–2748. [CrossRef] [PubMed]

55. Zhou, Y.; Khu, S.T.; Xi, B.; Su, J.; Hao, F.; Wu, J.; Huo, S. Status and challenges of water pollution problems in China: Learning from the European experience. *Environ. Earth Sci.* 2014, 72, 1243–1254. [CrossRef]

56. Datta, B.; Prakash, O.; Cassou, P.; Valetaud, M. Optimal unknown pollution source characterization in a contaminated groundwater aquifer—evaluation of a developed dedicated software tool. *J. Geosci. Environ. Prot.* 2014, 2, 41–51. [CrossRef]
57. Ayvaz, M.T. A linked simulation—optimization model for solving the unknown groundwater pollution source identification problems. *J. Contam. Hydrol.* 2010, 117, 46–59. [CrossRef] [PubMed]
58. Sun, A.Y.; Painter, S.L.; Wittmeyer, G.W. A constrained robust least squares approach for contaminant release history identification. *Water Resour. Res.* 2006, 42, 263–269. [CrossRef]
59. Mirghani, B.Y.; Mahintheakumar, K.G.; Tryby, M.E.; Ranjithan, R.S.; Zechman, E.M. A parallel evolutionary strategy based simulation—optimization approach for solving groundwater source identification problems. *Adv. Water Resour.* 2009, 32, 1373–1385. [CrossRef]
60. Singh, R.M.; Datta, B.; Jain, A. Identification of unknown groundwater pollution sources using artificial neural networks. *J. Water Res. Plan. Manag.* 2004, 130, 506–514. [CrossRef]
61. Singh, R.M.; Datta, B. Identification of groundwater pollution sources using GA-based linked simulation optimization model. *J. Hydrol. Eng.* 2006, 11, 101–109. [CrossRef]
62. Hintze, S.; Gétan, G.; Hunkeler, D. Influence of surface water–groundwater interactions on the spatial distribution of pesticide metabolites in groundwater. *Sci. Total Environ.* 2020, 733, 139109. [CrossRef]
63. Zhang, H.; Jiang, X.W.; Wan, L.; Ke, S.; Liu, S.A.; Han, G.L.; Guo, H.M.; Dong, A.G. Fractionation of Mg isotopes by clay formation and calcite precipitation in groundwater with long residence times in a sandstone aquifer, Ordos Basin, China. *Geochim. Cosmochim. Acta* 2018, 237, 261–274. [CrossRef]
64. Kim, J.W. Optimal pumping time for a pump-and-treat determined from radial convergent tracer tests. *Geosci. J.* 2014, 18, 69–80. [CrossRef]
65. Bortone, I.; Erto, A.; Nardo, A.D.; Santonastaso, G.F.; Chianese, s.; Musmarra, D. Pump-and-treat configurations with vertical and horizontal wells to remediate an aquifer contaminated by hexavalent chromium. *J. Contam. Hydrol.* 2020, 235, 103725. [CrossRef]
66. Cecconet, D.; Sabha, F.; Devecseri, M.; Callegari, A.; Capodaglio, A.G. In situ groundwater remediation with bioelectrochemical systems: A critical review and future perspectives. *Environ. Int.* 2020, 137C, 105550. [CrossRef]
67. Margalef-Martí, R.; Carrey, R.; Vilades, M.; Carrey, R.; Viladés, M.; Jubany, I.; Vilanova, E.; Grau, A.; Soler, A.; Otero, N. Use of nitrogen and oxygen isotopes of dissolved nitrate to trace field-scale induced denitrification efficiency throughout an in-situ groundwater remediation strategy. *Sci. Total Environ.* 2019, 686, 709–718. [CrossRef] [PubMed]
68. Gierczak, R.F.D.; Devlin, J.F.; Rudolph, D.L. Field test of a cross-injection scheme for stimulating in situ denitrification near a municipal water supply well. *J. Contam. Hydrol.* 2007, 89, 48–70. [CrossRef] [PubMed]
69. Ministry of Environmental Protection, the People’s Republic of China. *Technical Guideline for Delineating Source Water Protection Areas (H) 338-2018*; China Environment Press: Beijing, China, 2018. (In Chinese)
70. Zhou, Y. Sources of water, travel times and protection areas for wells in semi-confined aquifers. *Hydrogeol. J.* 2011, 19, 1285–1291. [CrossRef]
71. Ameli, A.A.; Craig, J.R. Semi-analytical 3D solution for assessing radial collector well pumping impacts on groundwater–surface water interaction. *Hydrod. Res.* 2017, 49, 17–26. [CrossRef]
72. Valois, R.; Cousquer, Y.; Schmutz, M.; Pryet, A.; Delbart, C.; Dupuy, A. Characterizing stream-aquifer exchanges with self-potential measurements. *Groundwater 2018*, 56, 437–450. [CrossRef]
73. Zhu, Y.G.; Zhai, Y.Z.; Teng, Y.G.; Wang, G.Q.; Du, Q.Q.; Wang, J.S.; Yang, G. Water supply safety of riverbank filtration wells under the impact of surface water–groundwater interaction: Evidence from long-term field pumping tests. *Sci. Total Environ.* 2020, 711, 135141. [CrossRef] [PubMed]
74. Qian, H.; Zheng, X.L.; Fan, X.F. Numerical modeling of steady state 3-D groundwater flow beneath an incomplete river caused by riverside pumping. *J. Hydraul. Eng* 1999, 30, 32–37. (In Chinese with English abstract)
75. Liu, S.; Zhou, Y.; Tang, C.; McClain, M.; Wang, X.S. Assessment of alternative groundwater flow models for Beijing Plain, China. *J. Hydrol.* 2021, 596, 126065. [CrossRef]