Brane world creation from flat or almost flat space in dynamical tension string theories

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Abstract There is great interest in the construction of brane worlds, where matter and gravity are forced to be effective only in a lower dimensional surface, the brane. How these could appear as a consequence of string theory is a crucial question and this has been widely discussed. Here we will examine a distinct scenario that appears in dynamical tension theories and where string tension is positive between two surfaces separated by a short distance and at the two surfaces themselves the string tensions become infinite, therefore producing an effective confinement of the strings and therefore of all matter and gravity to the space between these to surfaces, which is in fact a new type of stringy brane world scenario. The basic formulation for obtaining this scenario consist of assuming two types of strings characterized by a different constant of integration related to the spontaneous string tension generation. These string tension multiplied by the embedding metric define conformally related metrics that both satisfy Einsteins equation. The braneworlds appear very naturally when these two metrics are both flat spaces related by a special conformal transformation. The two types of string tensions are determined and they blow up at two close expanding surfaces. A puzzling aspect appears then: the construction is based on flat spaces, but then there are also strings with very large tension near the boundaries of the braneworld, so can the back reaction from the infinite tension strings destroy the flat space background? Fortunately that can be resolved using the mechanism Universe creation from almost flat (or empty) spaces, which incorporates a gas of very large string tensions in a membrane, studied before in 1+1 membranes in a 2+1 embedding space and now is generalized for a 1+(D-2) membrane moving in a 1+(D-1) space.

1 Introduction

In a previous publication [1] we have shown, in the context of theories where the string tension becomes a dynamical variable, using the modified measures formalism, which was previously used for a certain class of modified gravity theories under the names of Two Measures Theories or Non Riemannian Measures Theories, see for example [2–9] Leads to the modified measure approach to string theory [10,11], where rather than to put the string tension by hand it appears dynamically.

This approach has been studied in various previous works [12–18]. See also the treatment by Townsend and collaborators for dynamical string tension [19,20].

In [1,21] and references there we have also introduced the tension scalar, which is an additional background field that can be introduced into the theory for the bosonic case (and expected to be well defined for all types of superstrings as well) that changes the value of the tension of the extended object along its world sheet. Before studying issues that are very special of this paper we review some of the material contained in previous papers, first present the string theory with a modified measure and containing also gauge fields that like in the world sheet, the integration of the equation of motion of these gauge fields gives rise to a dynamically generated string tension, this string tension may differ from one string to the other.

Then we consider the coupling of gauge fields in the string world sheet to currents in this world sheet, as a consequence this coupling induces variations of the tension along the world sheet of the string. Then we consider a bulk scalar and how this scalar naturally can induce this world sheet current that couples to the internal gauge fields. The integration of the equation of motion of the internal gauge field lead to the remarkably simple equation that the local value of the tension along the string is given by $T = e\phi + T_i$, where $e$ is a coupling
constant that defines the coupling of the bulk scalar to the world sheet gauge fields and $T_i$ is an integration constant which can be different for each string in the universe.

Then each string is considered as an independent system that can be quantized. We take into account the string generation by introducing the tension as a function of the scalar field as a factor inside a Polyakov type action with such string tension, then the metric and the factor $g_{\mu\nu} + T_i$ enter together in this effective action, so if there was just one string the factor could be incorporated into the metric and the condition of world sheet conformal invariance will not say very much about the scalar $\phi$ , but if many strings are probing the same regions of space time, then considering a background metric $g_{\mu\nu}$, for each string the “string dependent metric” $(\phi + T_i)g_{\mu\nu}$ appears and in the absence of othe background fields, like dilaton and antisymmetric tensor fields, Einstein’s equations apply for each of the metrics $(\phi + T_i)g_{\mu\nu}$, considering two types of strings with $T_1 \neq T_2$. We call $g_{\mu\nu}$, the universal metric. In [1] the metrics $(\phi + T_i)g_{\mu\nu}$, for $i = 1, 2$ are taken to be Minkowski space and Minkowski space after a special conformal transformation. There are then solutions for the tensions of the two types of strings that imply a brane type, where the string tension becomes infinite at two expanding surfaces, so that all matter and gravity are constrained to be between those surfaces.

Here we want to discuss how, now from the point of view of of a gravitational theory, this phenomenon of arbitrarily large tensions can be consistent with the existence of flat spaces.

2 Are the flat space backgrounds consistent with the presence of very high tension strings?

The whole construction of the braneworld has been based on the conformal mapping between two flat spaces, this conformal mapping then defines the behavior of the string tensions and in principle it represents a vacuum solution where test strings acquire string tensions that diverge at two concentric and expanding surfaces, for details see [1].

Furthermore, as we start to populate the braneworld with actual strings, these strings will have infinite tension at the borders of the braneworld. A natural question one may ask at this point is the following: Are the flat space backgrounds of our construction consistent with the presence of very high Tension Strings or will the backreaction from the very large string tension destroy this basic feature of the model?

This question requires a non trivial answer because the presence of arbitrarily large string tensions would appear at first sight substantial back reaction from the space time and possibly large deviations from the construction based on the flat spaces in the previous sections, but is that so? As we will see, indeed, our picture it appears that the introduction of large tension strings is consistent with the matching of two flat or almost flat space times.

2.1 A general relativistic macroscopic string gas shells model with arbitrarily large tensions

A most important observation in this respect is that indeed two spaces that are almost flat can be matched with a surface with matter described by a string gas with arbitrarily large tensions [22]. There are obstacles to directly compare the braneworld solutions in dynamical tension strings and those found in [22], which are: (1) [22] describes a 2 + 1 dimensional brane moving in an embedding bulk space of 3 + 1 dimensions, while for string theories we must consider higher dimensions, (2) in [22] Einstein gravity is assumed with one metric to hold in the embedding bulk space, while the effective gravity theory for the dynamical tension strings two string metrics appear, (3) in [22] an infinitely thin brane is considered, while in dynamical tension strings the branes are thick, that is why the thin wall model will be referred as a macroscopic representation of the braneworld scenario.

A difference can be resolved in a simple way is generalizing the dimensions of the brane to $(D - 2) + 1$ and that of the embedding space to $(D - 1) + 1$. This we will do, while we hope the other aspects will not change the basic qualitative aspects of the comparison.

We consider then a surface or thin shell with $D - 2$ spatial dimensions, where in this shell a gas of strings with the equation of state that relates the surface pressure $p$ to the $\sigma$ being

$$p = -\frac{\sigma}{D - 2}$$

see for example a discussion of the string gas equation of state in 4D cosmology in [23]) and for an example involving string gas shells see [24], so for $D = 3$, we obtain that the surface becomes a line with $p = -\sigma$. This was a matching corresponding to a particular choice of the ones studied in [25], while the $D = 4$ corresponds to a membrane (2 + 1 dimensional brane) moving in 3 + 1 universe with a string gas matter in it [22]. In [23,25] the universe was meant to be the bulk space inside the bubble, while now, being interested in the braneworld picture, the bubble, that is the surface with the large string tensions itself is the Universe where we live. We must consider therefore higher dimensions to get a relevant braneworld scenario.

Applying a local conservation law of the energy momentum in the brane defined by Eq. (1) leads to the possibility of integrating $\sigma$,

$$\sigma = \sigma_0 \frac{r}{D - 3}$$

where $\sigma_0$ is a constant. As we can see, for $D = 3$, $\sigma = c$ constant, as considered as a particular case in [25] while for
$D = 3, \sigma = \frac{c_0}{r}$ as considered in [22]. These cases used the Israel matching conditions [26] for two space times separated by the string gas shell that we will now generalize to higher dimensions. Following [22] generalized now to higher dimensions, we consider two stationary metrics with rotational invariance of the form,

$$ds^2 = -A_+ dr^2 + \frac{dr^2}{A_+} + r^2 d\Omega^2_{D-2}$$

for the outside metric and

$$ds^2 = -A_- dr^2 + \frac{dr^2}{A_-} + r^2 d\Omega^2_{D-2}$$

for the inside metric. Here $d\Omega^2_{D-2}$ represents the contribution to the metric of the $D - 2$ angles relevant to the spherically symmetric solutions in $D$ space time dimensions. $A_+$ and $A_-$ are functions of $r$, different for the inside and the outside spaces, matched at a bubble defined by a trajectory

$$r = r(\tau)$$

Then the matching condition as a consequence of the Israel analysis [26] generalized to $D$ dimensions reads,

$$\sqrt{A_- + r^2} - \sqrt{A_+ + r^2} = \kappa \sigma r$$

where $\kappa$ is proportional to Newton constant in $D$ dimensions. The square roots are not necessarily positive, the sign can be negative for example for a wormhole matching as has been discussed in details in $D = 4$, which corresponds to a membrane (2 + 1 dimensional brane) moving in 3 + 1 universe with a string gas matter in it [22]. Another case where a difference a sum of the two terms is obtained, or what is equivalent, we can say that the second square root is negative is when considering a braneworld scenario where the radius grows as we go out from the brane on both sides, see for example [27]. The assignment of signs of the square roots when one of the spaces is a Schwarzschild space can be worked out rigorously by study the problem using Kruskal–Szekeres coordinates [28] where these expressions were used for the study of the dynamics of false vacuum bubbles and baby universe creation.

We will now study the case where inside we have flat space, that is

$$A_- = 1$$

and outside a $D$ dimensional Schwarzschild solution with maximal rotational invariance, which gives the Tangherlini solution [29]

$$A_+ = 1 - \frac{c_1}{r^{D-3}}$$

where $c_1$ is a constant. In the Tangherlini solution the radial fall off $\frac{1}{r}$ of the Newtonian potential is replaced by the $\frac{1}{r^{D-2}}$ behavior. These expressions have been used for $D = 3$ in [25], while the $D = 4$ was studied in [22], we now generalize for any dimension.

Solving from 6 for one of the square roots and then solving for the other square root and squaring again, we obtain the particle in a potential like equation,

$$r^2 + V_{\text{eff}}(r) = 0$$

where

$$V_{\text{eff}}(r) = -\left(\frac{r^{D-4}}{\kappa \sigma_0} - \frac{c_1}{2 \kappa \sigma_0 r} - \frac{\kappa \sigma_0}{2} r^{D-4} + \frac{r^{2D-8}}{(k \sigma_0)^2} \right)^2 \tag{7}$$

It is useful to look at the solutions where $V_{\text{eff}}(r) = 0$, from (8) and considering $c_1 = 0$, so we are matching two flat spaces, we get,

$$V_{\text{eff}}(r) = -\left(\frac{r^{D-4}}{\kappa \sigma_0} - \frac{\kappa \sigma_0}{2} r^{D-4} + \frac{r^{2D-8}}{(k \sigma_0)^2} \right)^2 = 0 \tag{9}$$

which is solved by

$$-\left(\frac{r^{D-4}}{\kappa \sigma_0} - \frac{\kappa \sigma_0}{2} r^{D-4} \right)^2 = -\frac{r^{2D-8}}{(k \sigma_0)^2} \tag{10}$$

multiplying by minus one and taking the minus square root in the right hand side, we get

$$\frac{r^{D-4}}{\kappa \sigma_0} - \frac{\kappa \sigma_0}{2} r^{D-4} = \frac{r^{D-4}}{(k \sigma_0)} \tag{11}$$

which leads to the solution for the maximum radius,

$$r_m = \left(\frac{(k \sigma_0)^2}{2} \right)^{\frac{1}{D-4}} = \left(\frac{\kappa \sigma_0}{4} \right)^{\frac{1}{D-4}} \tag{12}$$

for large $D$, that is $D$ bigger than four, the exponent is positive, so that the maximum radius goes to infinity as the string tensions goes to infinity.

The expression (8) can also be expressed as

$$V_{\text{eff}}(r) = 1 - \left(\frac{c_1}{2 \kappa \sigma_0 r} + \frac{\kappa \sigma_0}{2} r^{D-4} \right)^2 \tag{13}$$

This expression coincides with the expression obtained in [22] for the case where $D = 4$, as we see from the above expression, the potential, even for $c_1 \neq 0$ goes to a constant as $r \rightarrow \infty$, as depicted for a particular choice of parameters in Fig. 1.

This constant can be positive or negative depending on whether $\kappa \sigma_0$ is big or small, for $\kappa \sigma_0 > 2$ the asymptotic value of the potential is negative and the membrane approaches $\infty$ with constant velocity regardless of the value of $c_1$, so large string tensions produce indeed child universes even in the case the inside and outside spacetimes are flat (the inside space time is always flat, while the outside space is flat for $c_1 = 0$). The matching requires a wormhole as shown in [22].
Fig. 1 The potential for a particular choice of parameters in $D = 4$

Fig. 2 The potential for a particular choice of parameters in $D = 3$

Fig. 3 The potential for a particular choice of parameters in $D = 3$

Fig. 4 The potential for a particular choice of parameters in $D = 26$

For $D=3$ we have that for $r \to 0$ and $r \to \infty$ the potential is negative (see Fig. 2), or in the case of more interest to us, if $c_1 = 0$ in the limit $\sigma_0 \to \infty$ it will be negative everywhere except for an infinitesimal region around $r = 0$

The expression (13) allows also a particularly simple solution for the point where $V_{\text{eff}}(r) = 0$, the point of return of the bubble, even for $c_1 \neq 0$, which is particularly simple for $D = 5$, see Fig. 3 since in this case both terms inside the square are proportional to $\frac{1}{r}$, so we must choose

$$1 - \left( \frac{c_1}{2k\sigma_0 r} + \frac{\kappa \sigma_0}{2r} \right) = 1 - \frac{1}{r} \left( \frac{c_1}{2k\sigma_0} + \frac{\kappa \sigma_0}{2} \right) = 0$$

so that

$$r_m = \left( \frac{c_1}{2k\sigma_0} + \frac{\kappa \sigma_0}{2} \right)$$

So, we explicitly see that for $\kappa \sigma_0 \to \infty$, then $r_m \to \infty$ regardless of the mass (i.e. $c_1$), so infinite tension string gas shell can describe an expanding shell to infinity being connected by two flat spaces. This feature extends to all dimensions bigger than 4 as well. Finally, Fig. 4 shows the effective potential for $D = 26$.

2.2 Further possible improvements on the macroscopic general relativistic string gas model with arbitrarily large tensions

The toy model described in the previous section describes some of the rough features of what is taking place in the braneworld scenario, in particular the presence of arbitrarily large string tensions without giving up the flat space structure of the background metrics.

Although this basic feature is captured by the model, it is clear that this is only a first step, since (1) the braneworld does not start as a thin wall, this happens only at asymptotically large times, (2) in the real braneworld scenario, the internal wall is initially missing, the universe starts as a ball where the internal wall appears and then becomes very close to the outside wall, which suggests that the tensions will in fact grow in time, this should help the expansion of the shell to infinity in the gravity picture of the scenario, which require
indeed large string tensions to expand to infinity. These more refined models will be studied in a future paper.

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