CORRIGENDUM

Corrigendum: Perturbative calculations of quantum spin tunneling in effective spin systems with a transversal magnetic field and transversal anisotropy (2017 New J. Phys. 19 013032)

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1. Chapter 1: Introduction

Here we want to correct the wrong impression that no previous work on perturbation theory with two or more perturbations [1, 2] exists. Our intention was to express that the derivation of an energy splitting formula for a system with two perturbations ($B_x$ and $K$) is missing.

2. Chapter 5: Transversal magnetic field influence on integer spins

Wrong expression: $\Delta_{B_x K}/\Delta_K = 1$

Correct expression: $|\Delta_{B_x K}|/|\Delta_K| = 1$

Here we want to clarify that the ratio $|\Delta_{B_x K}|/|\Delta_K| = 1$ has the following property: $\pm \Delta_{B_x K} = \mp \Delta_K$. Equations (8) and (9) are not correct. We accidentally assumed that the ratio $|\Delta_{B_x K}|/|\Delta_K| = 1$ has the property: $\pm \Delta_{B_x K} = \pm \Delta_K$. But this is not the case.

Corrected version:

For certain values which satisfy equation (6) and for spin quantum numbers, for which $\pm \Delta_{B_x K} = \mp \Delta_K$, we can simply leave off the $\Delta_{B_x K}$ as well as the $\Delta_K$ contributions to $\Delta E_{\text{Split}}$ in equation (3). This leads to the following formula:

$$\Delta E_{\text{Split}} = \frac{2 \prod_{j=1}^{2} (S_x)_{j+1} \cdot (\Delta_{B_x})}{\prod_{j=1}^{2} |(E_{-j} - E_{+j})|}$$

$$\Delta_{B_x} = B_x^{2^{L-1}}$$

which is the perturbative solution for the energy splitting for a system described by the Hamiltonian

$$\hat{H} = -K_z \hat{S}_z - B_x \hat{S}_x$$

For such Hamiltonian we can use the well known proportion

$$\Delta E_{\text{Split}} \propto B_x \left( \frac{B_x}{K_z} \right)^{2^{L-1}}.$$  (9)

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Abstract
We present a perturbative approach for the resonant tunnel splittings of an arbitrary effective single spin system. The Hamiltonian of such a system contains a uniaxial anisotropy, a transversal magnetic field and a second-order transversal anisotropy. Further, we investigate the influence of the transversal magnetic field on the energy splittings for higher integer quantum spins and we introduce an exact formula, which defines values of the transversal magnetic field, the transversal anisotropy and the uniaxial anisotropy where the contribution of the transversal magnetic field to the energy splitting is at least equal to the contribution of the transversal anisotropy.

1. Introduction
Quantum tunneling in spin systems is the consequence of symmetry breaking of an unperturbed system by perturbations which leads to a loss of degeneracy of energy levels. At a mesoscopic scale the quantum tunneling has been extensively studied experimentally [1, 2, 13–17] by using single-molecular magnets and theoretically by path-integral methods [3–5, 12] and perturbative approaches [6–11]. Especially, the influence of a transversal magnetic field in combination with transversal anisotropies on the tunnel splitting energy has been studied experimentally [1, 2] and theoretically [3, 5]. The appearance of a transversal magnetic field for a system which already contains a transversal anisotropy can be important, because for half integer spin systems with a transversal anisotropy the appearance of a transversal magnetic field enables tunneling effects which otherwise would not be possible due to Kramers degeneracy theorem, which states that every energy level is at least doubly degenerate if it is a half-integer spin. This phenomenon is known as the spin parity effect [1, 2, 12], where a sole transversal anisotropy enables spin tunneling for integer spin systems but is not enough to enable tunneling in half-integer systems. It has also been shown that half-integer spin systems are much more sensitive to transversal magnetic fields than integer spin systems [2].

A further important effect is the quenched tunnel splitting, which describes the destructive interference of two quantum spin paths of opposite windings [3, 18] (Instantons). These destructive interferences lead to a vanishing of the energy splitting for certain values of the transversal magnetic field and are not related to the Kramers degeneracy theorem. So far, quenched tunnel splitting has been described through spin–coherent-state path integrals which leads to the two instanton path picture. Instantons, for example in a double well potential, can be interpreted as solutions of the path integral which occurs by minimizing the Euclidean action of the path integral. The potential energy described within the Euclidean action change sign under the Wick rotation and the minima of the double well potential transform into maxima which enables a classical approach by minimizing the action. Regarding this, the perturbative approach has the potential to reveal a more detailed quantum mechanical understanding of the quenched tunnel splitting, beyond the two instanton path picture. In order to understand the spin parity effect and the quenched tunnel splitting from a perturbative point of view a perturbative approach with two or more perturbations is necessary, which is still missing. In this paper, we close this description gap by deriving a tunnel splitting energy formula for a system which contains two perturbations, a transversal magnetic field and a transversal anisotropy. The goal is to give a detailed theoretical description of...
the quenched tunnel splitting and in general a description of the influence of the transversal magnetic field and the transversal anisotropy on the energy splitting from a perturbation theory point of view.

2. Perturbative approach

In order to describe the influence of a transversal magnetic field and a transversal anisotropy on the ground doublet energy splitting by a time dependent perturbative approach it is necessary to use two perturbations simultaneously [10]. The simplest effective Hamiltonian which is capable of describing quantum tunneling at a mesoscopic scale with two perturbations is

\[ \hat{H} = -K_x S_x^2 - \hat{S}_x \hat{B}_x + \hat{K} (S_x^2 - S_y^2), \]

where \( \hat{B}_x \) and \( \hat{K} \) represents the two perturbations. In order to perform a perturbative calculation with two perturbations it is necessary to define a shared coefficient for \( \hat{B}_x \) and \( \hat{K} \), otherwise it would not be possible to use the coefficient comparison

\[ \beta_x = \lambda B_x = \lambda \mu B \]

\[ \hat{K} = \lambda K. \]

The \( \lambda \) in equation (2) represent the shared coefficient. It is worth mentioning that the concept of the shared coefficient is not limited to two perturbations. From now on we performed the standard time dependent perturbative approach to obtain a formula for the energy splitting. The difficulty was to master the sheer complexity, which arises when two or more perturbations occur. A short introduction of the construction of a perturbative series with two perturbations is shown in appendix A. Due to the parity effect we obtained two formulas for the energy splitting and in general a description of the influence of the transversal magnetic field and the transversal anisotropy on the energy splitting from a perturbation theory point of view.

The formula for the half-integer spin case, which is defined by equation (4) differs mainly from the integer spin formula by the absence of the transverse anisotropy term \( \Delta_B \), which is forbidden due to the Kramers degeneracy theorem

\[ \Delta E_{\text{split}} = \frac{2 \prod_{j=1}^{2s} (S_x)_{j+1} \cdot (\Delta_B + \Delta_{B,K})}{\prod_{j=1}^{2s-1} |(E_{j+1} - E_{j})|} \]

\[ \Delta_B = B_x^{2s}, \]

\[ \Delta_{B,K} = \sum_{n=0}^{s-\frac{1}{2}} B_x^{2n+1} \cdot K^{s-n} \cdot (-1)^{2^s} \sum_{j=1}^{\beta+1} \cdot \sum_{j=1}^{\beta+2^s} \prod_{j=j-1+2^s=1}^{2^s} |(E_{j+1} - E_{j})|. \]

To confirm the accuracy of our derivations we have performed exact diagonalizations, where we estimated the energy splitting \( \Delta E_{\text{Diag}} \) of the ground doublet by the difference between the two highest eigenvalues and compared them with the results of the formulas of equations (3) and (4). The results, which are listed in table 1 show a very good agreement.
3. Tunneling paths

Before we begin with the analysis of equations (3) and (4), we want to define first what we mean by paths. In figures 1(a) and (b) we illustrate a matrix which only contains the off-diagonal elements of the Hamiltonian in equation (1) in a simplified form. In order to obtain an energy splitting of the ground doublet through a perturbative calculation it is necessary to expand the perturbative series to the order where we obtain a resonant case. The resonant case is a situation within the perturbative series where we obtain a perturbative term with the energy difference between the highest state $|s\rangle$ and its symmetric opposite state $|-s\rangle$. This series expansion has a specific iterative structure where we can choose a path to obtain a resonant case. A path which only contains energy contributions of the transversal anisotropy $K$ is shown in figure 1(a), where we expand the perturbative series only with transversal anisotropy energies to obtain a resonant situation. Such a path is called a pure path because it contains only energies from one term of the Hamiltonian in equation (1). In contrast to a pure path a so called mixed path is shown in figure 1(b) where we used two energy contributions from different sources to obtain a resonant case (one source is from the magnetic transversal field energy $B_x$ and the other is from the transversal anisotropy $K$). It should be mentioned at this point that a spin system will always take all possible paths, but as we will show it is useful to distinguish between those. With these definitions of the paths we are now in a position to explain the properties of equations (3) and (4).

4. Quenched tunnel splitting from a perturbative point of view

The perturbative approach enables us to make conclusions complementary to a path integral formalism [19]. Through equations (3) and (4) we are able to distinguish separately between the energy splitting contributions from the transversal magnetic field and the transversal anisotropy, represented by $\Delta_{B_x}$, $\Delta_K$, and $\Delta_{B_xK}$. With the ability to distinguish separately between the energy splitting contributions we can analyze here the quenched tunnel splitting from a perturbative theory point of view. With the ability to distinguish separately between the energy splitting contributions we want to analyze here the quenched tunnel splitting from a perturbative theory point of view. Figure 2(a) shows the well known effect of the quenched tunnel splitting [18], where under certain values of the transversal magnetic field

\begin{table}
\centering
\caption{Comparison between the energy splitting $\Delta E$ obtained by the formulas and by exact diagonalizations. Parameters for spin $s = 2$ and $s = 5/2$: $K = 0.001$ (meV), $B_x = 0.002$ (meV), $H_z = 0.035$ (T)). Parameters for spin $s = 5$: $K = 0.01$ (meV), $B_x = 0.02$ (meV), $H_z = 0.35$ (T)).}
\begin{tabular}{|c|c|c|}
\hline
Spin $s$ & $\Delta E_{\text{Formula}}$ & $\Delta E_{\text{Diag}}$ \\
\hline
2 & $2.9933 \times 10^{-6}$ & $2.99334 \times 10^{-6}$ \\
5/2 & $6.65625 \times 10^{-9}$ & $6.65917 \times 10^{-9}$ \\
5 & $1.50179 \times 10^{-10}$ & $1.50223 \times 10^{-10}$ \\
\hline
\end{tabular}
\end{table}

Figure 1. Illustration of tunneling paths from a perturbative point of view: (a) Here we demonstrate a pure path where we only used energy contributions from the transversal anisotropy to obtain a resonant case. (b) Here we show a mixed path where we used energy contributions from the transversal magnetic field energy $B_x$ and the transversal anisotropy $K$. 

Table 1.
the energy splitting of the ground doublet is vanishing. This is already explained through a two instanton paths model with destructive interference [3, 18]. The perturbative approach we used here enables us to analyze this effect from a more detailed perspective. Through equations (3) and (4) we can identify the mixed path $\Delta_{B_x K}$ as the trigger for the quenched tunnel splitting for half integer spins and even integer spins. This can be seen through the property that $\Delta_{B_x K}$ is an alternating series for an positive $K$. Since $\Delta_K$ in equation (3) is positive for even spin quantum numbers the only term which can cause negative contributions is consequently $\Delta_{B_x K}$. The scenario of odd integer spins leads to a combined cause of $\Delta_{B_x K}$ and $\Delta_K$ for the quenched tunnel splitting. Instead of describing the quenched tunnel splitting by only two instanton paths, we are now in a position to specify it more as the destructive interference of many paths extracted from a perturbative approach. The destructive interference nature arises primarily from the alternating mixed paths $\Delta_{B_x K}$ (half-integer and even integer spins) or a combination of the pure path $\Delta_K$ and the mixed $\Delta_{B_x K}$ paths (odd integer spins). So far we expanded the two instanton path model through a detailed many path model with the corresponding important contributions to the quenched tunnel splitting.

In order to demonstrate that equations (3) and (4) also describe the negative $K$ case correctly, we want to present this known behavior [18] in figure 2(b). Figure 2(b) shows the situation where the transversal anisotropy $K$ is negative. Here we see that no quenching of the tunnel splitting occurs, because the former alternating series $\Delta_{B_x K}$ in equations (3) and (4) is not alternating any more, due to the Hamiltonian in equation (1). The consequence is that only positive contributions to the energy splitting are present, which makes it impossible to obtain a zero energy splitting.

5. Transversal magnetic field influence on integer spins

The derivation of equations (3) and (4) enabled us to analyze the influence of the transversal magnetic field on the energy splitting of the ground doublet, for larger integer quantum spin systems, from another perspective. Our main concern related with equation (3) is to present conditions for obtaining a significant contribution of the transversal magnetic field to the quantum spin tunneling for integer spins. Figures 3(a) and (b) show an interesting property of the ratio $\Delta_{B_x K}/\Delta_K$ for several integer spin quantum numbers and for a positive $K$. In figure 3(a), where we used a transversal magnetic field energy of $B_x = 0.1$ (meV) ($H_x = 1.7$ (T)) and a transversal anisotropy energy of $K = 0.0002$ (meV) which is approximately three orders of magnitudes smaller than $B_x$, we see an increase of the ratio $\Delta_{B_x K}/\Delta_K$ up to a spin of $s = 3$. After $s = 3$ the ratio decreases until it oscillates asymptotically against one ($\lim_{s \to \infty} = 1$). This behavior is very fascinating because it indicates that for this set of parameters the contribution of the transversal magnetic field to the energy splitting is equal to the contribution of the transversal anisotropy for higher quantum spins. In appendix C we show a situation, which occurs for most parameters $B_x$, $K$, and $K_s$ where the ratio $\Delta_{B_x K}/\Delta_K$ is decreasing with increasing spin quantum number until the influence of the transversal magnetic field ($\Delta_{B_x K}$) on the energy splitting becomes negligible in contrast to that of transversal anisotropy ($\Delta_K$). Considering that for most parameters $B_x$, $K$, and $K_s$ within the perturbative regime the ratio $\Delta_{B_x K}/\Delta_K$ is decreasing with increasing spin quantum number the question arises...
The values which satisfy equation (3) until it decreases and oscillates asymptotically against one. Used parameters in figure 3(a): $B_k = 0.1$ (meV) $K = 0.0002$ (meV) and $K_z = 1$ (meV). (b) Here we show a plot of equation (6) (blue, pink, yellow and red curves), which defines the parameters $B_k$ (meV) and $K$ (meV) for $K_z = 1$ (meV) where we obtain a $\Delta_{B_k,K}/\Delta_k = 1$ ratio for higher integer spins. The curves separate areas, where we obtain $\Delta_{B_k,K}/\Delta_k = 1$ ratios (curves) and where we obtain $\Delta_{B_k,K}/\Delta_k = 1$ ratios (white areas between the curves).

if the parameter set used in figure 3(a) is a unique one or whether it is a general behavior for a group of parameters. In order to answer this question we permuted several parameters in equation (3) until we were able to create a value table of parameters with the property $\Delta_{B_k,K}/\Delta_k = 1$ (in the limiting case for large quantum spins)

$$\frac{\Delta_{B_k,K}}{\Delta_k} = \frac{\sum_{\alpha_{m}B_k=2\alpha_m}^{\alpha_{m}} K^{2\alpha_{m}} \sum_{i,j} a_{ij} \sum_{i,j=1}^{\alpha_{m}-1} \sum_{i,j=1}^{\alpha_{m}-1} \Pi_{j=1}^{\alpha_{m}-1} \Pi_{j=1}^{\alpha_{m}-1} |(E_{i-j} - E_{\dots})|}{(1)^{2\alpha_{k}K} \Pi_{j=1}^{\alpha_{m}} |(E_{i-j} - E_{\dots})|}.$$  \hspace{1cm} (5)

From this value table we were able to set up the following equation, which reproduce the exact values of $B_k$, $K$ and $K_z$ to obtain a $\Delta_{B_k,K}/\Delta_k = 1$ ratio in the limiting case of higher integer spins:

$$\frac{1}{\alpha_m K_z} \cdot B_k^2 = K,$$  \hspace{1cm} (6)

where

$$\alpha_m = 2 + \sum_{m=0}^{\infty} 16m, \quad m \in \mathbb{N}_0$$  \hspace{1cm} (7)

The values which satisfy equation (6) lead to a situation where the contribution to the energy splitting of the mixed $\Delta_{B_k,K}$ and the pure $\Delta_k$ paths are equal in the limiting case of large quantum spins ($\Delta_{B_k,K} = \Delta_k$), as demonstrated in figure 3(a). Since the pure $\Delta_k$ path is negligible for larger spins, only the $\Delta_{B_k,K}$ and $\Delta_k$ path contributions are important. For certain values which satisfy equation (6) and for spin quantum numbers, for which $\Delta_{B_k,K} = \Delta_k$ we can simply double the $\Delta_k$ path in equation (3) and leave off the $\Delta_{B_k,K}$ as well as $\Delta_k$ contributions to $\Delta E_{\text{split}}$. This leads to the following formula,

$$\Delta E_{\text{split}} = \begin{vmatrix} 2 \Pi_{j=1}^{\alpha_{m}} |S_j| & 2(\Delta_k) \\ \Pi_{j=1}^{\alpha_{m}} |(E_{i-j} - E_{\dots})| & \end{vmatrix}$$

$$\Delta_k = (-1)^{2\alpha_{k}K} \Pi_{j=1}^{\alpha_{m}} |(E_{i-j} - E_{\dots})|$$  \hspace{1cm} (8)

which is the perturbative solution for the energy splitting for a system described by the Hamiltonian $\hat{H} = -K_z \hat{S}_z^2 + 2 \cdot \hat{K} (\hat{S}_x^2 - \hat{S}_y^2)$. For such Hamiltonian we can use the well known proportion

$$\Delta E_{\text{split}} \propto 2K \left( \frac{2K}{K_z} \right)^{j-1}.$$  \hspace{1cm} (9)
It is very important to mention that the proportion in equation (9) is only valid for cases where $\Delta_{B,K} \approx \Delta_K$. An example is given by $s \geq 9$ in figure 3(a). If the values of $B_s$ and $K$ satisfy equation (6) they lie on curves shown in figure 3(b), otherwise they belong to the areas between the curves. An example of parameters $B_x$ and $K_s$ from the areas between the curves is shown in appendix C. In summary, we found parabolic expressions (equation (6)), which enables us to estimate values of the transversal magnetic field, the transversal anisotropy and the uniaxial anisotropy where the contribution of the transversal magnetic field to the energy splitting for an arbitrary integer spin is equal to the contribution of the transversal anisotropy in the limiting case of higher integer spins.

The situation changes drastically when the transversal anisotropy energy $K$ is negative (hard axis), because then the series $\Delta_{B,K}$ does not alternate and there are no negative contributions to the energy splitting. The consequence is that the ratio $\Delta_{B,K}/\Delta_K$ does not converge to one, what means that the contribution of the transversal magnetic field on the energy splitting can be orders of magnitude larger than all other contributions for higher integer spins. This is shown in figure B1 in appendix B.

6. Conclusion

In summary we derived an energy splitting formula of the ground doublet by using a perturbative approach with two perturbations for a Hamiltonian which contains a uniaxial anisotropy, a transversal magnetic field, and a transversal anisotropy. The formula we derived enables a detailed understanding of the quenched tunnel splitting and enables us to estimate exact parabolic equations in which the influence of the transversal magnetic field on the energy splitting is significant for integer spins and a positive $K$. The situation changes drastically for a negative $K$, where the contribution of the transversal magnetic field to the energy splitting can be orders of magnitude larger for higher integer spins than the contribution of the transversal anisotropy.

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Appendix A. Perturbative series

Here we would like to introduce in a short form the construction of a perturbative series with two perturbations

$$\hat{H} = -K_z \hat{S}_z^2 - \hat{S}_x B_x + \hat{K} (\hat{S}_+^2 - \hat{S}_-^2),$$

(A.1)

We start with a Hamiltonian which contains two perturbations $\hat{B}_x$ and $\hat{K}$

$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = \hat{H} |\Psi(t)\rangle,$$

(A.2)

$$|\Psi(t)\rangle = \sum_{n=0}^{2s} a(t)_{s-n} e^{-iE_{s-n}t/\hbar} |\psi_{s-n}\rangle.$$  (A.3)

We define the coefficient $a$ from a general definition of a wave-function as time-dependent $a(t)$

$$\hat{B}_x = \lambda B_x = \lambda \mu_B H_S,$$

$$\hat{K} = \lambda K.$$  (A.4)

By inserting the wave-function with the time-dependent coefficient (equation (A.3)) in the Schrödinger equation equation (A.2),

$$i\hbar \frac{\partial}{\partial t} \left( \sum_{n=0}^{2s} a(t)_{s-n} e^{-iE_{s-n}t/\hbar} |\psi_{s-n}\rangle \right) = \hat{H} \left( \sum_{n=0}^{2s} a(t)_{s-n} e^{-iE_{s-n}t/\hbar} |\psi_{s-n}\rangle \right)$$

(A.5)

we obtain equation (A.5),

$$i\hbar \cdot e^{-iE_{s-n}t/\hbar} \frac{\partial}{\partial t} a(t)_{s-n} = E_n a(t)_{s-n} e^{-iE_{s-n}t/\hbar} - \left( \hat{S}_+ a(t)_{s-n} - \hat{S}_- a(t)_{s-n} e^{-iE_{s-n}t/\hbar} \right)$$

$$i\hbar \cdot e^{-iE_{s-n}t/\hbar} \frac{\partial}{\partial t} a(t)_{s-1} = ...$$

(A.6)
By expanding equation (A.5) we obtain a system of ordinary differential equations, demonstrated through

\[ i\hbar \frac{\partial}{\partial t} a(t) = -\left( S_1 \right)_{1,2} \lambda B_{x} a(t) e^{i(E_{2} - E_{1}) t/\hbar} + 2\left( S_2 \right)_{1,3} \lambda K a(t) e^{i(E_{2} - E_{1}) t/\hbar} \]

\[ i\hbar \frac{\partial}{\partial t} a(t)_{-1} = \ldots \]

(A.7)

After rearranging equation (A.6) and using the definition of equation (A.4) we obtain a system of differential equations with an expansion of the time-dependent coefficient \( a(t) \), demonstrated through equation (A.7)

\[ a(t)_{-n} = a(t)_{-n}^{(0)} + \lambda a(t)_{-n}^{(1)} + \ldots = \sum_{i=0}^{\infty} \lambda a(t)_{-n}^{(i)} \]

(A.8)

Now we utilize the standard perturbation theory power series ansatz (equation (A.8)) by inserting it in equation (A.7) and rearranging the \( \lambda \)'s

\[ \frac{\partial}{\partial t} a(t)_{-n}^{(0)} = 0 \]

\[ \frac{\partial}{\partial t} a(t)_{-n}^{(1)} = \frac{i}{\hbar} \left( \left( S_1 \right)_{1,2} B_{x} a(t)_{-n}^{(0)} e^{i(E_{2} - E_{1}) t/\hbar} - 2\left( S_2 \right)_{1,3} K a(t)_{-n}^{(0)} e^{i(E_{2} - E_{1}) t/\hbar} \right) \]

\[ \vdots \]

\[ \frac{\partial}{\partial t} a(t)_{-n}^{(0)} = 0 \]

(A.9)

The next step is to use the coefficient comparison to obtain systems of differential equations for every order of the time-dependent coefficient \( a(t)_{-n}^{(i)} \) from the power series ansatz in equation (A.8). This is implied through equation (A.9).

From here onwards we expand the series in the same way as we would do for the Fermi’s golden rule, by repeating the integration and the inserting of the coefficients \( a(t)_{-n}^{(i)} \) in equation (A.9) until the resonant case. The only difference from the situation with only one perturbation is that we now have much more direct paths which generate a resonant case.

Appendix B. Negative \( K \)

Figure B1. Here we show the evolution of the \( \Delta_{B,K}/\Delta_{K} \) ratio for spin systems from \( s = 2 \) to \( s = 15 \). We see that for the same parameter set of \( B_{x} \), \( K \) and \( K_{z} \), as in figure 3, with the sole difference that the transversal anisotropy \( K \) is negative, the ratio is constantly increasing with increasing spin quantum number. The asymptotic oscillation against one does not appear. Used parameters: \( B_{x} = 0.1 \) (meV) \( K = -0.0002 \) (meV) and \( K_{z} = 1 \) (meV).
Appendix C. Transversal magnetic field influence on integer spins

In most cases a quantum spin system described by the Hamiltonian
\[
\hat{H} = -K z \hat{S}_z^2 - \hat{S}_x \hat{B}_x + \hat{K} (\hat{S}_z^2 - \hat{S}_x^2)
\]
shows the following behavior: for a given set of parameters \( B_x, K, \) and \( K_z \), the ratio \( \Delta_{B_x, K} / \Delta_K \) is decreasing with increasing quantum spin number \( s \). Parameters used in figure C1: \( B_x = 0.1 \) (meV) \( (H_x = 1.75 \) (T)), \( K = 0.00077 \) (meV) and \( K_z = 1 \) (meV).

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