SPECTRUM OF KINETIC-ALFVÉN TURBULENCE

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ABSTRACT

A numerical study of strong kinetic-Alfven turbulence at scales smaller than the ion gyroscale is presented, and a phenomenological model is proposed that argues that magnetic and density fluctuations are concentrated mostly in two-dimensional structures, which leads to their Fourier energy spectra \( E(k_\perp) \propto k_\perp^{-8/3} \), where \( k_\perp \) is the wavevector component normal to the strong background magnetic field. The results may provide an explanation for recent observations of magnetic and density fluctuations in the solar wind at sub-proton scales.

Key words: magnetic fields – magnetohydrodynamics (MHD) – solar wind – turbulence

Online-only material: color figure

1. INTRODUCTION

Possibly the simplest description of magnetic plasma turbulence at scales much larger than typical micro-scales (particle gyroradii, skin depth, etc.) is provided by one-fluid magnetohydrodynamics (MHD; e.g., Biskamp 2003; Kulsrud 2005). A characteristic feature of MHD turbulence is its anisotropic spectral energy transfer with respect to the background magnetic field. As a result, small-scale plasma fluctuations predominantly populate field-perpendicular wavevectors and turbulence is dominated by the shear-Alfven modes (Goldreich & Sridhar 1995). In the linear case these modes have the dispersion relation \( \omega = \kappa_z v_A \), where \( \kappa_z \) is the field-parallel wavenumber with respect to the background magnetic field \( B_0 \); \( v_A = B_0 / \sqrt{4 \pi \rho} \) is the Alfven velocity, and \( \rho \) is the fluid density.

At scales smaller than the ion gyroradius (or ion-acoustic radius if the electron temperature exceeds the ion temperature), the assumptions of one-fluid MHD break down, and the nature of turbulence changes. At such sub-proton scales, the shear-Alfven cascade is expected to transform into the cascade of strongly anisotropic kinetic-Alfven modes with a different linearized dispersion relation \( \omega \propto k_z k_\perp \). Sub-range, micro-scale plasma turbulence attracts considerable interest due to its importance in solar wind heating, magnetic reconnection in a variety of astrophysical systems, and laboratory experiments with strongly magnetized plasmas (e.g., Biskamp et al. 1999; Cho & Lazarian 2004; Kiyani et al. 2009; Gurcan et al. 2009; Chandran et al. 2010; Kletzing et al. 2010; Howes 2010; Salem et al. 2012). Subtle turbulence has been understated to the much lesser extent compared to its Alfvenic or MHD counterpart.

Our consideration is particularly motivated by recent measurements of sub-proton fluctuations in the solar wind. Although significant scatter exists among the earlier reported data Smith et al. (2006), recent observations suggest that magnetic and density fluctuations have the Fourier energy spectrum close to or possibly steeper than \( k^{-2.8} \) (e.g., Alexandrova et al. 2009, 2011; Sahraoui et al. 2009; Chen et al. 2010, 2012; Salem et al. 2012). The nature of such fluctuations is not fully understood as they are not described by existing models of either kinetic-Alfven or electron MHD turbulence, which predict the scaling \( k^{-7/3} \). Recently proposed explanations invoke the presence of kinetic-Alfven turbulence with the spectrum significantly steepened by Landau damping, weakening of turbulence, and wave-particle scattering, among others (e.g., Rudakov et al. 2011; Howes et al. 2011).

In this work we analyze the spectrum and structure of strong kinetic-Alfven turbulence using a two-fluid plasma description, which by its nature does not take into account Landau damping and other wave–particle interactions. We found that the steeper than \( k^{-7/3} \) energy spectrum persists in this case, in a form closely resembling the solar wind observations and the results of existing kinetic simulations. Our results suggest that the power-law energy spectrum of strong kinetic-Alfven turbulence is not an artifact of significant dissipation or non-universality, but rather an inherent property of nonlinear plasma dynamics. To describe this spectrum we propose a new model that assumes that the magnetic and density fluctuations tend to spontaneously organize into two-dimensional structures, thus leading to strong spatial and temporal intermittency of turbulence. Our model predicts that the energy spectrum of strong kinetic-Alfven turbulence has the scaling \( E(k_\perp) \propto k_\perp^{-8/3} \), which is demonstrated to be in good agreement with our numerical simulations and may explain the solar wind data.

2. KINETIC-ALFVÉN MODEL

In this section, we formulate a system of model equations governing kinetic-Alfven turbulence. These equations have been derived and studied in many works (e.g., Hazeltine 1983; Scott et al. 1985; Camargo et al. 1996; Terry et al. 2001; Schekochihin et al. 2009; Smith & Terry 2011). The basic assumptions are that a uniform background magnetic field is strong compared to magnetic fluctuations, \( B_0 \gg b \), and the turbulence is strongly anisotropic, \( k_\perp \ll k_z \), where \( k_z \) and \( k_\perp \) are typical wavenumbers of turbulent fluctuations in the field-parallel and field-perpendicular directions. To illustrate the essential physics and to set the notation, we start with the simplest case of small plasma beta (the ratio of thermal plasma energy to the magnetic energy); later we argue that the derived system of equations is also valid for \( \beta \sim 1 \), the case relevant for the solar wind studies mentioned above.

Since the electrons are strongly magnetized and their thermal speed exceeds the Alfven speed, an isothermal fluid description is possible for the electrons. The electrons are advected across the magnetic field by the “E cross B” drift, \( \mathbf{v}_e = \mathbf{E} \times \mathbf{B}_0 / B_0^2 \), while their field-parallel motion is related to the current
\[ J_\parallel = -e_n v_{\parallel} n_e, \] and the ion parallel motion can be neglected. For small \( \beta \) the field-parallel fluctuations of the magnetic field can be neglected, while its field-perpendicular component is expressed through the flux function \( b_\perp = \hat{z} \times \nabla \psi \), so that \( J_\parallel \approx J_z = (c/4\pi) \nabla_\perp \times b_\perp = (c/4\pi) \nabla^2 \psi \). The flux function is the (minus) field-parallel component of the vector potential, \( \psi = -A_\parallel. \)

The field-parallel force balance in the electron momentum equation gives \(-T_e \nabla_\parallel n_e - n_0 e b_\parallel = 0\), where the electric field is \( E = -\nabla \phi - (1/c) \partial_t A \). Supplemetnign this equation with the electron continuity equation, one obtains the system for the fluctuating parts of magnetic and density fields:

\[
\frac{1}{c} \frac{\partial}{\partial t} \psi - \nabla_\parallel \phi + \frac{T_e}{n_0 e} \nabla_\parallel n_e = 0, \tag{1}
\]
\[
\frac{\partial}{\partial t} n_e - \frac{c}{B_0} \nabla \phi \times \hat{z} \cdot \nabla n_e - \frac{1}{e} \nabla_\parallel J_\parallel = 0. \tag{2}
\]

We should note that the field-parallel gradient in these equations is the gradient along the total magnetic field, that is,

\[ \nabla_\parallel = \nabla_z + \frac{1}{B_0} \hat{z} \times \nabla \psi \cdot \nabla. \tag{3} \]

We will assume that the fluctuations are anisotropic with respect to the magnetic field in such a way that the so-called critical balance between the linear and nonlinear terms is satisfied, \( \nabla_\parallel \sim (1/B_0) \hat{z} \times \nabla \psi \cdot \nabla \); this condition is analogous to \( k_z B_0 \sim k_i b \) (e.g., Goldreich & Sridhar 1995; Cho & Lazarian 2004; Howes et al. 2011; TenBarge & Howes 2012). This is the condition of strong turbulence that we consider in this paper. Equations (1) and (2) are therefore essentially nonlinear and three-dimensional.

To close the system (1, 2) we still need to specify the electric potential \( \phi \), which is done as follows. We are interested in the sub-proton, dispersive kinetic-Alfvén waves, that is, we consider scales smaller than the ion gyroscale \( k_z b_\perp/\Omega_i \gg 1 \), where \( \Omega_i \sim (T_i/m_i)^{1/2} \) is the ion thermal speed and \( \Omega_i \) is the ion gyrofrequency. It is also convenient to introduce the ion-acoustic scale, \( \rho_i = v_i/\Omega_i \), with \( v_i \sim (T_i/m_i)^{1/2} \) the ion-acoustic speed. At such scales, the ions are not magnetized. Moreover, we will be interested in frequencies smaller than \( k v_{Ti} \), which implies the "Boltzmannian" response for the ion density fluctuations, \( n_i = -e\phi n_0/T_i \). The quasi-neutrality condition \( n_i = n_e \) then ensures that the second (advection) term in Equation (2) vanishes, while in Equation (1) the electric potential modifies the density term, that is, \( \nabla_\parallel \phi = -(T_i/n_0) \nabla n_e \).

Let us introduce the normalized electron density \( \tilde{n} = (1 + T_i/T_e)^{1/2} (v_e/v_n) n_e/n_0 \), the magnetic flux function \( \tilde{\psi} = (v_e/c) \psi/T_e \), and normalize the spatial scales to the ion-acoustic scale \( \rho_i \), and the timescale to \( (\rho_i/v_n)(1 + T_i/T_e)^{-1/2} \). We will use only the normalized variables (unless stated otherwise) and omit the over-tild sign. The magnetic and density fluctuations are then described by the system:

\[
\partial_t \psi + \nabla \tilde{n} = 0, \tag{4}
\]
\[
\partial_t n - \nabla \tilde{\psi} = 0. \tag{5}
\]

where \( \nabla \parallel = \nabla_z + \hat{z} \times \nabla \psi \cdot \nabla \). The presented ideal system conserves the total energy \( E \) and the cross-correlation \( H \):

\[
E = \int (|\nabla \psi|^2 + n^2) d^3 x, \tag{6}
\]
\[
H = \int \psi n d^3 x. \tag{7}
\]

The system (4, 5) possesses linear waves, \( n_k \propto \psi_k \propto \exp(-i \omega t + ik x) \). The linearization is done by neglecting the second term in the right-hand side of Equation (3), which gives the dispersion relation for the kinetic-Alfvén waves:

\[ \omega = k_z k_\perp. \tag{8} \]

The linear modes are characterized by the equipartition of density and magnetic fluctuations, \( n_k = \pm k_\perp \psi_k \).

To conclude this section we make two important comments. First, a similar consideration can be conducted without the assumption of small plasma beta. In this case the field-parallel fluctuations of the magnetic field should be taken into account in the derivation of (1, 2) (e.g., Howes et al. 2006; Schekochihin et al. 2009). The resulting system however has the structure identical to our system (4, 5) and it can be reduced to system (4, 5) by appropriate normalization of the variables. The low-beta assumption is therefore not essential for our discussion of scaling properties of kinetic-Alfvén turbulence, and our results are applicable, for instance, to the solar wind turbulence with \( \beta \sim 1 \).

Second, system (4, 5) is derived under the assumptions (in dimensional units) \( b \ll B_0, k_z \rho_i \gg 1, k_\perp \rho_e \ll 1, \) and \( \omega \ll k v_{Ti}, \) which do not imply strong inequalities between \( k_z \) and \( k_\perp \), and \( \omega \) and \( \Omega_i \). In particular, it can describe both weak and strong kinetic-Alfvén turbulence. In the case of strong turbulence, however, the critical balance condition requires that \( k_z \ll k_\perp \). We, however, note that Equations (4) and (5) admit a rescaling \( \partial_t \rightarrow -\epsilon \partial_t, \nabla \rightarrow \epsilon \nabla, n \rightarrow \epsilon n, \) and \( \psi \rightarrow \epsilon \psi \) with arbitrary \( \epsilon \), which preserves the critical balance. This reflects the fact that being derived in the limit of infinitely large electron gyrofrequency, Equations (4) and (5) lack any frequency scale. We may therefore always rescale the fields in these equations to satisfy \( k_z \sim n \sim \psi \sim 1 \). Such rescaling will be assumed in our numerical simulations below.

3. KINETIC-ALFVÉN TURBULENCE

PHENOMENOLOGY

The scaling of strong kinetic-Alfvén turbulence was addressed in a number of works (e.g., Howes et al. 2008; Schekochihin et al. 2009); see also Biskamp et al. (1999), Ng et al. (2003), and Cho & Lazarian (2004, 2009). It was argued that in strong turbulence, the critical balance condition, which ensures that both linear and nonlinear terms in (3) are of the same order, should be satisfied at all scales. We denote by \( n_3 \) and \( \psi_3 \) the typical (rms) fluctuations at the field-perpendicular scale \( \lambda \), and \( l \) the corresponding field-parallel scale of those fluctuations. Balancing linear and nonlinear terms in Equation (3) then gives \( l \sim \lambda^2/\psi_3, \) in which case the time of nonlinear interaction is comparable to the linear time (Equation (8)), \( \tau \sim 1/\omega \sim l \lambda \sim \lambda^2/\psi_3. \) Further, we estimate from Equations (4) and (5) that \( n_3 \sim \psi_3/\lambda. \) The energy associated with the scale \( \lambda \) can therefore be estimated as \( E_3 \sim n_3^2, \) and the condition of constant energy flux in the turbulent cascade leads to
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\( n^2_\parallel /\tau = \text{constant} \), which translates into the scaling for the turbulent fields \( n_\parallel \sim \psi_\parallel /\lambda \sim \lambda^{2/3} \). The Fourier energy spectrum of strong kinetic-Alfvén turbulence is then

\[
E_{\text{KA}}(k_\perp) \propto k^{-7/3}_\perp \quad \text{d}k_\perp. \tag{9}
\]

As we discussed in the Introduction, there is a puzzling disagreement of this scaling with the solar wind observations, where a spectrum closer to \( -2.8 \) is observed.

To address the spectrum of kinetic-Alfvén turbulence, we have conducted numerical simulations of systems (4, 5). Our results produce a spectrum that is different from Equation (9), and, interestingly, quite close to the observational data. Since our system does not include Landau damping and it is driven in the regime of strong turbulence, we propose that the observed scaling is not an artifact of non-universal or dissipative effects, rather, it is an inherent property of nonlinear turbulent dynamics. We then propose a model of kinetic-Alfvén turbulence, which predicts that the energy spectrum scales as \( E(k_\perp) \propto k^{-8/3}_\perp \), in good agreement with our numerical results.

4. NUMERICAL RESULTS

We supplement the system (4, 5) by a driving force and by small dissipation terms as follows:

\[
\partial_t \psi + \nabla_z n = \eta \nabla^2_\perp \psi + f, \tag{10}
\]

\[
\partial_t n - \nabla_\perp \nabla^2_\parallel \psi = \nu \nabla^2_\perp n. \tag{11}
\]

The force mimics energy supply from large-scale motion, while the dissipation terms (normalized plasma resistivity \( \eta \) and electron diffusivity \( \nu \)) remove the energy at small scales; the dissipation terms are mainly needed for numerical stability of the code.\(^4\) We solve these equations on a triply periodic cubic domain \((L^3, L = 1)\) using standard pseudo-spectral methods. The random force \( f \) is applied in Fourier space at wavenumbers \( 2\pi/L \leq k_\perp \leq 2(2\pi/L) \), \( k_z = 2\pi/L \). The Fourier coefficients outside the above range are zero and inside that range are Gaussian random numbers with amplitudes chosen so that \( |\nabla \psi|_{\text{rms}} \sim 1 \). The individual random values are refreshed independently on average every \( \tau = 0.1L/(2\pi |\nabla \psi|_{\text{rms}}) \). We choose \( \nu = \eta = 0.01 \). The strength of the nonlinear term relative to the dissipation term is then measured by the parameter \( R = \psi_{\text{rms}}/(L\nu) \sim n_{\text{rms}}/\nu \), which plays the role of the Reynolds number in this system.

We use a numerical resolution of \( 512^3 \) collocation points. The initial conditions are imported from a steady state snapshot obtained on \( 256^3 \) points. The system is then evolved until a new steady state is reached. The \( 512^3 \) simulations are run for about 35 large-scale dynamical times. The presented results correspond to statistical averages over approximately 60 last snapshots corresponding to about 15 dynamical times. Note that compared to the MHD equations where \( \omega \sim k_z \), the kinetic-Alfvén equations require significantly shorter time steps to accommodate high frequencies (Equation (8)), leading to tremendous increase in computational effort. In this respect the fluid model (10, 11) allows one to access the inertial intervals and averaging times currently unachievable in kinetic or gyrokinetic simulations. Figure 1 shows the energy spectrum of kinetic-Alfvén turbulence. The spectrum is steeper than \(-7/3\) and close to \(-8/3\).

5. A MODEL FOR KINETIC-ALFVÉN TURBULENCE

To understand the observed energy spectrum, let us discuss some characteristic properties of the dynamics described by the system (4, 5). First, consider the effect of the nonlinear terms (for that we can assume \( k_z = 0 \)). The nonlinear term in Equation (10) can be rewritten as \( \nabla n \times \hat{z} \cdot \nabla \psi \), implying that \( \psi \) is advected in the field-perpendicular direction with velocity \( \nabla n \times \hat{z} \). The field \( \psi \) thus gets striated, developing gradients aligned with the gradients of \( n \). This suggests that the magnetic field \( \nabla \psi \) tends to concentrate in two-dimensional structures. Let us now see what happens to those structures if the linear terms come into play (for that we can assume a non-zero \( k_z \)). The linear terms in (4, 5) tend to smear or break the initial perturbation into wave packets propagating in opposite directions along the local magnetic field, such that \( n \sim |\nabla \psi| \) inside those packets. Thus density tends to be in equipartition with the magnetic field and to concentrate in two-dimensional structures as well. We thus expect that as a result of nonlinear striation and linear propagation, both the density and the magnetic fluctuations become organized in highly intermittent, two-dimensional structures or sheets, elongated in the \( \hat{z} \)-direction. This is indeed consistent with our numerical observations presented in Figure 2.

We therefore assume that essential nonlinear interaction and energy cascade take place at such two-dimensional structures. Following standard procedure (e.g., Frisch 1995), we consider turbulent fluctuations of field-perpendicular size \( \lambda \). Since such fluctuations cover two-dimensional sheets, they occupy the volume fraction \( p_\lambda \propto \lambda \). The energy density of such fluctuations

\(^4\) We do not force the density fluctuations, allowing them to be nonlinearly generated. The choice of the large-scale forcing should not however affect the inertial interval, e.g., Mason et al. (2008).
therefore scales as $E_λ \propto n^2 p_λ$. The energy cascade time is estimated as before (cf. discussion preceding (9)), $\tau \sim 1/ω \sim l_κ/\psi_λ \sim \lambda^2 / n_λ$, and the condition of constant energy flux reads $E_∥ / \tau = \text{constant}$, which gives $n_λ \propto \lambda^{1/3}$. The scaling of the energy is then $E_∥ \propto \lambda^{5/3}$, and the Fourier energy spectrum scales as

$$E(k_⊥) \, dk_⊥ \propto k_⊥^{-8/3} \, dk_⊥.$$  \hfill (12)

This spectrum of kinetic-Alfvén turbulence is in excellent agreement with the numerical observation in Figure 1, and it is the main result of our work. It may provide a plausible explanation for the solar wind measurements of both magnetic and density fluctuations (e.g., Chen et al. 2010, 2012; Alexandrova et al. 2011). Balancing the linear wave frequency $ω \sim 1/(l_κ)$ with the inverse nonlinear interaction time $1/τ \sim n_λ / l_κ^2 \sim \lambda^{-5/3}$, we further derive the anisotropy of the turbulent fluctuations with respect to the (local) large-scale magnetic field: $l \sim \lambda^{2/3}$. If we formally introduce the local field-parallel wave number as $k_∥ \sim 1/l$, then the “field-parallel energy spectrum” corresponding to Equation (12) is $E(k_∥) \, dk_∥ \propto k_∥^{-7/2} \, dk_∥$. Such scaling can possibly be inferred from analyzing the $ω$-$k_∥$ correlation of the fluctuations (see, e.g., TenBarge & Howes 2012); we leave this question for further studies. As for the spectrum of the electric field, it is a factor of $k_∥^2$ flatter than Equation (12), $E_E \propto k_∥^{-2/3}$.

6. DISCUSSION

We have proposed a model for kinetic-Alfvén turbulence below the dispersion scale (ion-acoustic scale). Based on numerical simulations of the fluid equations (10) and (11) and on analytic modeling, we propose that the energy spectrum of such turbulence scales as $k_⊥^{-8/3}$, meaning that both magnetic and density fluctuations should have the same Fourier spectrum. This result is also consistent with in situ observations of the sub-proton solar wind fluctuations, and with the results of the gyrokinetic simulations where the spectra close to $k_⊥^{-7/3}$ are observed (Chen et al. 2010; Alexandrova et al. 2011; Howes et al. 2011).

Our model is complementary to the previously proposed explanations invoking Landau damping, the presence of weak kinetic-Alfvén turbulence, and effects of wave-particle scattering, among others (e.g., Rudakov et al. 2011; Howes et al. 2011). These explanations are interesting and the effects they point out may indeed affect a turbulent cascade. The difference of our approach (as compared to the gyrokinetic numerical studies, for example) is that it does not take into account Landau damping and other wave–particle effects. With those “spoilers” removed, the observed turbulent spectrum $k_⊥^{-8/3}$ is expected to arise from the nonlinear interaction, similar to the Kolmogorov spectrum of hydrodynamic turbulence. This also implies that kinetic dissipative effects may be less important at the sub-proton scales than was previously thought.

Our explanation points to an interesting property of the kinetic-Alfvén nonlinear dynamics, which tend to concentrate magnetic and density fluctuations in two-dimensional structures. This leads to strong spatio-temporal intermittency in the field distributions. In this work we have studied only the second-order statistics of the fluctuating fields (expressed through energy spectra); we plan to present more detailed discussion of sub-proton turbulence elsewhere.

Finally, we note that a system formally similar to our system of Equations (10) and (11) also appears in the limit of strong guide magnetic field in the so-called electron MHD, where the ions are assumed to be immobile. The wave modes in this case correspond to the so-called whistler waves and belong to a different branch of plasma dispersion relations (this branch continues into an MHD mode or compressional Alfvén mode in the limit of strong guide field).

Previous simulations of electron MHD were conducted mostly for two-dimensional cases and/or for relatively weak guide fields and/or for decaying cases, and the spectrum close to $k_⊥^{-7/3}$ was observed (e.g., Biskamp et al. 1999; Cho & Lazarian 2004, 2009; Ng et al. 2003; Dastgeer & Zank 2003). Due to the limited extent of the inertial interval, however, simulations with weak guide field may not reach the universal scaling regime that we address in this work. Decaying turbulence, on
the other hand, becomes progressively weaker since the guide field does not decay, which may also lead to flattening of the spectrum (compared to $-8/3$); see, e.g., Boldyrev (1995) and Galtier & Bhattacharjee (2003) for discussions of weak whistler turbulence. Our results suggest that in the limit of strong guide field, the spectrum of three-dimensional steadily driven strong electron-MHD turbulence should be modified as well.

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