1. INTRODUCTION

We describe the calculation of the hadronic matrix elements that are required for the extraction of the $V_{cb}$ and $V_{ub}$ CKM matrix elements from experimental data [1]. To reach the bottom quark mass our strategy is to interpolate between results from relativistic quarks with $m_q \leq m_c$ and results from lattice HQET [2]. Here we discuss only the HQET simulations, as our clover form factor simulations have only just started.

All of our simulations use $n_f = 2$ dynamical staggered configurations with a volume $16^3 \times 48$ and $\beta = 5.445$.

2. ISGUR-WISE FUNCTION

The Isgur-Wise function is the QCD matrix element required in the extraction of $V_{cb}$ from experimental data. Experimental measurements of the slope of the Isgur-Wise function vary from 0.31 to 1.17, and the variations in theoretical predictions are nearly as large [3]. Initial attempts to calculate the Isgur-Wise function in lattice HQET had problems either with the signal to noise ratio [4] or the renormalization factors [7]. The first complete calculation has been done recently by the Kentucky group [8].

We use the same method as the Kentucky group (see also [4,7]). We ran at all permutations of the following velocities: (0,0,0), (0.1,0,0), (0.25,0,0) and (0.5,0,0). Our sample size is 80 configurations, and our Wilson $\kappa$ values are 0.160 and 0.163. A relative smearing function of $e^{-0.67r}$ was used between the quarks in the $B$ meson. In Fig. 1 we plot the bare Isgur-Wise function for various time separations between the current and the $B$ source. If the ground state has been isolated, then the Isgur-Wise function should be independent of this separation. The data for $\Delta t = 2, 3, 4$ are consistent within present errors.

It is traditional to report the slope of the Isgur-Wise function as a function of the dot product of...
Figure 1. Unrenormalized Isgur-Wise function

\[ \Delta t \quad \kappa = 0.160 \]

\[ \Delta t = 1 \quad \kappa = 0.163 \]

\[ \Delta t = 2 \quad \text{good} \]

\[ \Delta t = 3 \quad \text{chirality} \]

\[ \Delta t = 4 \quad \text{if needed} \]

\[ \kappa = 0.163 \]

Table 1:

| \( \Delta t \) | \( \kappa = 0.160 \) | \( \kappa = 0.163 \) |
|----------------|----------------|----------------|
|                | \( \rho^2 \) | \( \chi^2/df \) | \( \rho^2 \) | \( \chi^2/df \) |
| 2              | 0.415(8)     | 200/44         | 0.412(9)     | 171/44         |
| 3              | 0.48(4)      | 31/44          | 0.47(4)      | 24/44          |
| 4              | 0.48(12)     | 18/44          | 0.48(13)     | 17/44          |

Table 2:

| Vel | NP | \( PT_{bare} \) | \( PT_{boost} \) | tadpole |
|-----|----|----------------|----------------|--------|
| 0.1 | 0.04(2) | 0.074 | 0.051 | 0.085 |
| 0.25 | 0.13(2) | 0.18 | 0.12 | 0.21 |
| 0.5 | 0.25(3) | 0.34 | 0.23 | 0.41 |

Various estimates of the HQET velocity renormalization

Assuming negligible quark mass dependence, our best estimate is therefore \( \rho^2 = 0.48(13) \) at the physical light quark mass. For comparison, at \( \beta = 6.0 \) the Kentucky group gets a bare \( \rho^2 = 0.56 \) and Hashimoto and Matsufuru \( \text{[5]} \) get \( \rho^2 \sim 0.54 \) (where we have approximately removed the effect of tadpole improvement from their result \( \text{[5]} \)).

All the simulations of lattice HQET \( \text{[6,7]} \) show a very weak dependence of the slope on the light quark mass. However, the UKQCD collaboration \( \text{[5]} \) found a statistically significant decrease in \( \rho^2 \) with light quark mass in their simulations that used clover quarks for the \( b \) quark. We also tried fitting our bare Isgur-Wise function data to a fit model that had quadratic corrections of \( \omega \) in Eq. \( \text{(1)} \) Acceptable fits were found with approximately the same slope as in Table \( \text{[5]} \) and positive curvature.

The velocity of the lattice HQET action is renormalized \( \text{[6]} \) because the action breaks Lorentz symmetry. As we described last year \( \text{[1]} \), we have tried to estimate the renormalized velocity from the dispersion relation of an HQET meson at finite residual momentum \( \text{[7]} \). The renormalized velocity can be implicitly defined from

\[
E(p, v^R) - E(0, v^R) = \frac{v^R p}{v_R^0} \tag{2}
\]

where \( E(p, v^R) \) is the energy of the HQET meson at finite residual momentum \( (p) \). In Table \( \text{2} \) we show the results of the non-perturbative velocity renormalization. We fit all the correlators in the time region 5 to 13 and obtained correlated \( \chi^2/df \) slightly less than one. For comparison, we also show the results for the perturbative renormalization calculated by Mandula and Ogilvie \( \text{[5,6]} \), using both a boosted \( (g^2/\mu^4) PT_{boost} \) and bare coupling \( PT_{bare} \), as well as using the tree-level tadpole improved estimate \( \text{[8]} \).

The results in Table \( \text{2} \) show that the velocity renormalization is large. These results would suggest that perturbation theory with a boosted coupling agrees best with the non-perturbative result. However, other analyses found better agreement between the tree-level tadpole scheme \( \text{[6]} \) and the non-perturbative calculations \( \text{[5,7]} \). This issue is under investigation.

\[
\xi(\omega) = 1 - \rho^2(\omega - 1) \tag{1}
\]
3. $B \to \pi$ FORM FACTOR

The observation of the decay $B \to \pi + l^+ \nu_l$ allows a determination of $V_{ub}$, if the relevant QCD form factors can be calculated. There have been a number of lattice QCD calculations of the required form factors (see [1,12] for reviews). However, previous approaches suffer from the drawback that calculations are done at large $q^2$, thus requiring a large extrapolation to $q^2 \approx 0$, where measurements are currently made. To reach a low $q^2$ requires large meson velocities—not easily achieved for heavy mesons in the NRQCD- or propagating-quark-approaches. As we have shown [11], a good signal can be obtained for the HQET $B$ meson with a large velocity ($v \approx 0.8$), so we propose the use of HQET—light simulations to explore lower $q^2$ (see [13] for similar ideas). It is not clear that HQET will be a good approximation to the dynamics of the $B$ meson, at these values of $q^2$, nor that a sufficiently good signal will be obtained. However, results should be very useful in helping to reduce the heavy quark extrapolation errors over simulations that only use clover quarks.

Because $B \to$ light meson form factors have never been studied before using lattice HQET (although the static limit was studied in [14]), we have computed the matrix element for $B \to \pi + l^+ \nu_l$ using HQET to check for a signal. We use the setup described in [14] with the heavy clover quark replaced by a HQET quark. In Fig. 2 we plot the ratio of three point functions to two point functions

$$\frac{\langle C_3(t; t_f) \rangle}{\langle C_2(t) \rangle \langle C_2(t_f - t_B) \rangle}$$

that is proportional to the $\langle B | J_\mu | \pi \rangle$ matrix element, as a function of the operator time $t$. The $B$ source is fixed at $t_f = 23$, and the light meson source is fixed at $t = 0$.

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