Towards a realistic solution of the cosmological constant fine-tuning problem by Higgs inflation

冯朝君（Chao-Jun Feng）
上海师范大学
天体物理中心
2014.7.8 @郑州大学
Outline

• Higgs boson and Inflation

• Cosmological constant

• kill two birds with one stone

• Conclusion & discussion
Higgs confirmed to exist on 4 July 2012, by the ATLAS and CMS teams at LHC.
Inflation period driven by the so-called *inflaton*.

\[
S = \int d^4x \, \sqrt{-g} \, \mathcal{L} = \int d^4x \, \sqrt{-g} \left[ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) \right],
\]

M. Sasaki
1210.7880
Higgs Inflation

- Higgs boson cannot be directly the inflaton!

\[ m \approx 1.5 \times 10^{13} \left( \frac{N}{60} \right)^{-1} \left( \frac{10^9 A_s}{2.19} \right)^{1/2} \text{GeV} \]

\[ m_h \approx 125.9 \pm 0.4 \text{ GeV} \]
Available models

- non-minimal coupled inflation \( (\sim h^2 R) \)
  - F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B 659, 703 (2008)

- non-minimal kinetic coupled inflation \( (\sim G^{ab} \partial_a h \partial_b h) \)
  - C. Germani and A. Kehagias, Phys. Rev. Lett. 105, 011302 (2010)

- Higgs Galileon-inflation
  - K. Kamada, T. Kobayashi, et al. Phys. Rev. D 83, 083515 (2011)

- Running kinetic Higgs chaotic inflation
  - K. Nakayama and F. Takahashi, JCAP 1102, 010 (2011)

- ......
Running kinetic Higgs chaotic inflation

- Lagrange in unitary gauge

\[ \mathcal{L} = \frac{1}{2} \left( 1 + \xi \frac{h^2}{2} \right) (\partial h)^2 - \frac{\lambda_h}{4} (h^2 - v^2)^2. \]

- High energy limit

\[ h \geq \frac{1}{\sqrt{\xi}}. \]

\[ V(\phi) = \frac{1}{2} m^2 \phi^2 \quad \text{and} \quad m^2 \equiv \frac{4 \lambda_h}{\xi} M_{pl}^2. \]

\[ \phi \equiv \sqrt{\xi/8h^2}. \]
Cosmological constant

- Fine-tuning problem of C.C.

\[ \frac{\Lambda_0}{M_{pl}^2} = 3H_0^2 \Omega_{\Lambda_0}/M_{pl}^2 \approx \mathcal{O}(10^{-117}) \ll 1 \]

\[ H_0 \approx 2.1332h \times 10^{-42}\text{GeV with } h \approx 67 \]

\[ \Omega_{\Lambda_0} \approx 0.68 \]

- Dynamical dark energy (effective C.C.)
## Latest observations

**Planck**  \(95\%\) CL

\[
\begin{align*}
  n_s &= 0.9600 \pm 0.0071 \\
  r_{0.002} &< 0.11
\end{align*}
\]

\[
\begin{align*}
  n_s' &= -0.022^{+0.020}_{-0.021} \\
  n_s &= 0.95700 \pm 0.0075 \\
  r_{0.002} &< 0.26
\end{align*}
\]

**BICEP2**  \(68\%\) CL

\[
\begin{align*}
  r &= 0.20^{+0.07}_{-0.05}
\end{align*}
\]

Tension?
Large running index?

- For a single field inflation model

\[ n_s - 1 \sim O(10^{-2}) \quad n'_s \sim O(10^{-4}) \]

- If \( r \) is large, Planck gives:

\[ n'_s = -0.022^{+0.020}_{-0.021} \]

Qing Gao, Yungui Gong, Phys. Lett. B 734 (2014) 41-43
D. J. H. Chung, G. Shiu and M. Trodden, Phys. Rev. D 68, 063501 (2003)
R. Easther and H. Peiris, JCAP 0609, 010 (2006)
kill two birds with one stone

• Two birds
  - Cosmological constant fine-tuning problem
  - Large running of spectrum index

• One stone
  - Interaction $\Lambda(t)$
Variable C.C.

- Total energy-momentum tensor

\[ \tilde{T}_{\mu \nu} = T_{\mu \nu} - \Lambda g_{\mu \nu} \]

- If, it satisfies the conservation law

\[ \nabla^\mu \tilde{T}_{\mu \nu} = 0. \]

- No prior reason why C.C. should not vary

Review: J. M. Overduin, F. I. Cooperstock, Phys. Rev. D 58, 043506 (1998)
Dynamic equations I

C. J. Feng and X. Z. Li, arXiv:1405.3056 [astro-ph.CO].

- **Friedmann equation**

\[
3H^2 = \frac{1}{2}m^2 \phi^2 + \Lambda
\]

- **Conservation law**

\[
\nabla^\mu \tilde{T}_{\mu\nu} = 0
\]

\[
\dot{\rho}_\Lambda + \dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 0
\]
Dynamic equations II

- Interactions
  \[ \mathcal{L}_{\text{int}} \sim \phi \rho_\Lambda \]
  \[ \dot{\rho}_\Lambda + 3H(\rho_\phi + p_\phi) = Q, \]
  \[ \dot{\rho}_\Lambda = -Q, \]

- Coupling term
  \[ Q = -\alpha \phi \rho_\Lambda = -\alpha \phi \Lambda \]

\[ \alpha > 0 \] is a dimensionless constant

\[ 8\pi G = 1 \quad \rightarrow \quad \rho_\Lambda = \Lambda \]
Equation of motion I

• E.O.M
\[ \ddot{\phi} + 3H \dot{\phi} + m^2 \phi + \alpha \Lambda = 0 \]

• SL limit
\[ \dot{\phi} \approx -\frac{m^2 \phi + \alpha \Lambda}{3H} < 0 \quad \Rightarrow \quad Q > 0 \]
Equation of motion II

- E.O.M
  \[ \dot{\Lambda} = \alpha \dot{\phi} \Lambda \]

- Solution
  \[ \Lambda_f = \Lambda \exp \left[ -\alpha (\phi - \phi_f) \right] \approx \Lambda \exp \left( -\alpha \phi \right) \]
  subscript \( f \) denoted the values at the end of inflation

When \( \dot{Q} \approx 0 \) or \( \dot{\Lambda} \approx 0 \) the interaction term is almost vanished.

Therefore, C.C. becomes a constant after inflation,

\[ \Lambda_f \approx \Lambda_0 \]

and it will cause a second accelerating at later time that has been confirmed by the supernovae observations.
Slow-roll (SL) inflation

- SL parameters

\[ \epsilon \equiv - \frac{\dot{H}}{H^2} = \frac{2(m^2 \phi + \alpha \Lambda)^2}{(m^2 \phi^2 + 2\Lambda)^2}, \]

\[ \eta \equiv \frac{\dot{\epsilon}}{\epsilon H} = -4 \frac{m^2 + \alpha^2 \Lambda}{m^2 \phi^2 + 2\Lambda} + 4\epsilon. \]

- Inflation end when

\[ \epsilon \approx \frac{2}{\phi_f^2} \approx 1 \]
Power spectrum

- Scalar power spectrum

\[ \mathcal{P}_s = A_s \left( \frac{k}{k_*} \right)^{n_s - 1 + n'_s \ln(k/k_*)/2} \]

- With index and its running

\[ n_s - 1 \approx -2\epsilon - \eta, \]
\[ n'_s \approx -2\alpha^3 \Omega_\Lambda \sqrt{2\epsilon + 8\epsilon^2} - 8\epsilon \eta, \]

Interaction is also important to produce a relatively large running of the spectral index through the first term!
Rewrite formulas I

- Inflaton and its mass
  \[ \Omega_\Lambda = \Lambda / (3H^2) \]
  \[ m^2 = \frac{3\pi^2 r A_s}{4} \left( n_s - 1 + \frac{3r}{8} - 2\alpha^2 \Omega_\Lambda \right) \]

- Friedmann equation
  \[ \phi = 2 \left( \sqrt{\frac{r}{8} - \alpha \Omega_\Lambda} \right) \left( n_s - 1 + \frac{3r}{8} - 2\alpha^2 \Omega_\Lambda \right)^{-1} \]

- Solution of C.C.
  \[ \left( n_s - 1 + \frac{3r}{8} - 2\alpha^2 \Omega_\Lambda \right) \left[ \ln \left( \frac{2\Lambda_0}{3\pi^2 A_s} \right) - \ln (r \Omega_\Lambda) \right] = -2\alpha \left( \sqrt{\frac{r}{8} - \alpha \Omega_\Lambda} \right) \]
Rewrite formulas II

- Running of index

\[ n'_s = -2\alpha^3\Omega_\Lambda \sqrt{\frac{r}{8}} + \frac{r}{2} \left( n_s - 1 + \frac{3r}{16} \right) \]

- Number of e-folds

\[
N \equiv \int_{t_*}^{t_f} H dt = \int_{\phi_*}^{\phi_f} \frac{H}{\phi} d\phi = -\frac{1}{2} \int_{\phi_*}^{\phi_f} \frac{m^2\phi^2 + 2\Lambda}{m^2\phi + \alpha\Lambda} d\phi,
\]

where \( \phi_* \) is the value of the inflaton field at time \( t_* \) when there are \( N \) e-foldings to the end of inflation.
Case I: constant $\Lambda$

- Only enhance the tensor-scalar ratio

C. J. Feng and X. Z. Li, arXiv:1404.3817 [astro-ph.CO].
Case II: variable $\Lambda(t)$

C. J. Feng and X. Z. Li, arXiv:1405.3056 [astro-ph.CO].

- Solve the C.C. fine-tuning problem
• Solve the large running spectrum index problem
Conclusion

• Benefit of $\Lambda(t)$
  o Explain small C.C. at present
  o Predict large running of scalar spectral index
  o Make inflation temporary end
  o Could give a large tensor-to-scalar ratio

• So, why not to take it?
Thank you!