Eutectic fracture model and damage localization band of composite ceramics with reinforced lamellae

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Abstract. The two-scale cell model is constructed based on the microstructure of composite ceramics with reinforced lamellae eutectic. The cohesive bond fracture stress between the matrix and reinforced phase is obtained considering the conditions of the rupture of the cohesive bond. The microcosmic damage fracture stress is obtained through introducing the damage variables and the cohesive bond fracture stress. The analytic results agree well with the experimental ones. According the Dugdale-Barenblatt model and regarding the microcosmic damage fracture stress of eutectic as residual intensity, the damage localization band model at crack tip is built. Quantitative analysis show that when the damage variable is high, the length of the damage localization band increases with the volume fraction of the reinforced lamellae increasing. The increases volume fraction of the lamellae leads to the declining residual intensity and the break of the strong binding force of the interphase. Reducing the material damage and selecting suitable volume fraction of reinforced lamellae phase can increase the fracture stress of eutectic and reduce the length of damage zone. So that the threshold of crack propagation and enhance the material strength would become high.

1. Introduction
Composite ceramic materials have excellent mechanical properties such as high temperature resistance, high strength, high hardness and corrosion resistance [1-3], which are widely used in ceramic armor and aerospace fields. Studies have found that the ZrO₂-enhanced rod-shaped eutectic complex ceramics prepared by Pastor and others using LHFZ method have very superior mechanical properties. The room temperature strength has reached 1.6GPa, and its mechanical properties are very weak when the temperature is higher than 1500°C. [4-6] It was found that the comprehensive mechanical properties of the composite ceramic materials were related to the microstructure of eutectic, the main component of the materials. In order to further analyze the influence of the microstructure of the laminated reinforced composite ceramics on the damage and fracture behavior of the materials, it is necessary to start with the microstructure of the ceramics.

In recent years, the constitutive relationship between the microstructure of different materials and the macroscopic mechanical properties of composite ceramic materials has been established. Based on the microscopic characteristics of interfacial composite materials, Ni [7] improved the effective self-consistent method, proposed a four-phase model method, and predicted the thermal expansion coefficient of eutectic matrix composite ceramics [8]. Literature [9] proposed a stochastic three-phase cellular model for the presence of local desorbed phase between particles and matrix in granular composites, predicted the stiffness of metal matrix composites. Literature [10], on the premise of...
considering the interfacial phase, established a set of performance prediction methods based on mesomechanics from the angles of stiffness performance, thermoelastic performance and strength performance. Zhao [11] used the four-phase model method to establish a four-phase model composed of lamellae inclusion, interfacial phase, matrix and effective medium, and deduced the effective elastic constant. Based on the flexibility increment prediction formula of lossy composite eutectic, Sun [12] established the mesoscopic strain field model of damaged eutectic. Tang [13], such as the multiphase particle composites on behalf of the unit in any of the particle as inclusions, using the method of equivalent inclusion, gives a multiphase particle composites macro elastic-plastic stress. Zou [14], based on the generalized self-consistent principle, derived the elastic field of hollow nanometer microsphere-filled composites under unidirectional loading.

In terms of micromechanics, the cohesive bond model of brittle materials proposed by Orowan [15] and Gilman [16] provides more profound physical understanding. Thomson [17], Camacho and ortize [18], Zhou [19] further improve the cohesive fracture model. Based on the microstructure of eutectic composite ceramics containing triangular symmetric fibers, the ultimate stress of the material was calculated considering the cohesive bond fracture condition and the effect of micro damage [20]. These studies provide a reference for the analysis of damage and fracture of composite ceramics with lamellar reinforced phase.

According to the meso damage theory of crack tip [21], microcracks grow steadily in the continuous damage zone at the crack tip, and then the microcracks will enter the damage localization zone where stress drop occurs and continue to expand. Xu [22] studied the damage localization behavior of rock under uniaxial compression by numerical simulation method. Zhao [23] established the damage localization bifurcation model of rock concrete materials. Zhou and Zhang [24,25] studied the damage localization in the process of rock tensile compression failure, and analyzed the conditions and causes of damage localization. Most of the above researches focus on the damage localization of rock like quasi brittle materials, but few on composite ceramics.

Based on the above theory, a two-scale cellular model is established to analyze the cohesive bond fracture conditions based on the microstructure of the composite ceramics. And the micro damage fracture stress model of the composite ceramics is built and the influence of micro damage on the effective properties of the material is analysed. The length of the damage localization band at the crack tip under different damage degrees is predicted, so as to analyze the composite ceramics with lamellar reinforcement, the theoretical basis is provided for the damage and fracture behavior.

2. Two-scale cell model
The eutectic composite ceramics with lamellar reinforced phase prepared by directional solidification technology have excellent mechanical properties at room temperature and high temperature [26], which is closely related to the microstructure of the materials. The microstructure of the composites is summarized as follows: (1) there is an interface between the matrix and the reinforced phase, and the interfacial phase has strong binding properties. (2) In the process of high temperature sintering, there are inevitable defects, such as microcracks, micropores and so on. In this paper, Al$_2$O$_3$–ZrO$_2$ eutectic composite ceramics are taken as the main research object, as shown in Figure 1.

According to the microstructure characteristics of eutectic matrix multiphase ceramics, a two-scale cellular model is established. One is that the rod-shaped eutectic with lamellar strengthening phase is arranged in the same direction, forming an ellipsoid like structure in the local region. This ellipsoid is placed in the matrix of particle structure and randomly distributed in space to form a two-scale cellular model of the material, as shown in Figure 2. The other is that the ZrO$_2$ reinforced phase is uniformly distributed in the Al$_2$O$_3$ matrix in a single eutectic, forming rod-shaped eutectic. The eutectic consists of micro cell and effective medium, as shown in Figure 3. In Figure 3, the elliptical reinforced phase can be regarded as the plane of the sheet reinforced phase, and the effective medium has the physical properties of eutectic.
3. Fracture stress model of eutectic micro damage

If cohesive bond fracture occurs in eutectic, the fracture stress of two-phase cohesive bond is the same at the phase boundary between lamellar reinforced phase and interfacial phase, and the functional relationship between bond bonding stress and separation displacement \( \delta \) can be expressed by half sine function [28], the cohesive bond fracture stress of eutectic is obtained as follows

\[
\sigma_{th} = \frac{(\gamma_0 + \gamma_b)E_bE_0}{(E_b + E_0)b_0}
\]

Where \( i=0 \) represents the matrix phase, \( i=b \) represents the reinforced phase.

It can be seen from reference [11] that the maximum principal stress in the eutectic matrix is the tensile stress along the 2-direction, and the minimum principal stress is the compressive stress along the 3-direction. According to the three-dimensional stress state, the external load stress field is

\[
\tau_{\text{max}} = \frac{\sigma_{22}^0 - \sigma_{33}^0}{2}
\]

Where \( \sigma_{22}^0 \) and \( \sigma_{33}^0 \) are the external stress of eutectic matrix with sheet reinforced phase, which can be obtained from reference [11].

When \( \tau_{\text{max}} \) reaching a certain value, the dislocation arrays pile up between obstacles with distance of \( d \). The larger the dislocation stacking concentration is near the interface with strong confinement, the dislocation source will be released continuously under the continuous action of shear stress, and a stress field will be generated around it. The maximum tensile stress produced by the stress field is
\[ \sigma_{\text{max}} = \frac{2}{\sqrt{3}} \left( \frac{d}{r} \right)^{\frac{1}{2}} (\tau_{\text{max}} - \tau_i) \]  

(3)

If the maximum tensile stress in the eutectic is equal to the cohesive bond fracture stress of the eutectic, the reinforced phase and the interface phase will not be destroyed first, and the matrix near the interface phase will appear bond fracture, that is to say, it will be destroyed at \( r = 2b_0 \). Let \( \sigma_{\text{max}} = \sigma_{bh} \), ignoring the lattice friction \( \tau_i \), combining equations (2) and (3), the lossless fracture stress of eutectic is obtained as

\[ \sigma_e = \frac{1}{\delta_{22}^0 - \delta_{33}^0} \sqrt{\frac{6E_bE_0(y_0 + y_b)}{h(E_b + E_0)}} \]  

(4)

Where, \( \delta_{22}^0 \) and \( \delta_{33}^0 \) are the dimensionless stresses in the matrix along and perpendicular to the loading direction, which can be obtained from reference [12].

Under the external load, the damage begins to appear in the eutectic with lamellar reinforced phase. Three damage variables are defined: \( d_i \) represents the effect of damage on the perpendicular direction of the lamellar reinforcement phase, \( d_2 \) and \( d_3 \) represents the effect of in-plane damage of the lamellar reinforcement on the transverse direction of the eutectic. The effect of shear coupling is not considered here.

For the eutectic with lamellar reinforced phase, when there is a crack in the matrix around the reinforcement phase, the damage of the eutectic along the vertical direction of the plate-shaped reinforced phase plane is very small [11], so the influence of the vertical direction of the plate-shaped reinforcement phase is ignored, i.e

\[ d_i = 0 \]  

(5)

Since \( d_2 \) and \( d_3 \) are on the transversely isotropic plane, therefore

\[ d_2 = d_3 \]  

(6)

It is assumed that the distribution of lamellar reinforcement in eutectic is as shown in Figure 4. The short axis and long axis of elliptical sheet reinforcement are \( a_1 \) and \( b_1 \) respectively.

It can be seen from Figure 4 that the volume content \( f_b \) of lamellar reinforced phase is:

\[ f_b = \frac{\pi a_1 b_1 h}{abh} \]  

(7)

Where \( h \) is the thickness of the lamellar enhanced phase, \( a_1 \) and \( b_1 \) are the lengths of the semi-major axis and the semi-minor axis of the elliptic section of the lamellar enhanced phase respectively. When the internal patchy enhanced phase of the eutectic cracks, the maximum cracking area is the patchy enhanced phase plane. At this point, the minimum cross section in a single cell in figure 4 is \( A_{\text{min}} = ab - \pi a_1 b_1 \), then the damage variable satisfies \( d_{2\text{max}} = 1 - A_{\text{min}} / A = f_b \). Then, the size range of damage variables \( d_2 \) is: \( 0 \leq d_2 \leq d_{2\text{max}} \). Under the applied load, the microscopic stress field [12] analysis of the eutectic shows that the strain \( \varepsilon_{22} \) along the reinforced lamellae plane is the main factor causing the damage of the eutectic, \( \varepsilon_{22} = \varepsilon_{\text{max}} \). Loading function \( \psi \) is defined according to the maximum linear strain theory, \( \psi = E_{22}\varepsilon_{22}/(1 - D_{22}) \). Where \( E_{22} \) is the elastic modulus along the direction of the eutectic crystal axis when lossless; \( D_{22} \) is the microscopic damage variable along the direction of the eutectic crystal axis. When the tensile stress \( \sigma \) is applied along the eutectic axis, the initial loading function is obtained under nondestructive conditions \( D_{22} = 0 \). \( \psi_0 = E_{22}\varepsilon_{22} = \sigma \). Regarding microscopic damage \( \varepsilon_{22}' = \sigma / E_{22} \). Where \( E_{22}' = (1 - d_2)E_{22} \) as determined by the formula in reference [26], \( E_{11} \), \( E_{22} \) and \( \mu_{12} \) are respectively the longitudinal, axial and shear elastic.
moduli along the eutectic. The mesoscopic damage variables were determined by the elastic modulus along the eutectic axis before and after the damage $D_{22} = 1 - E''_{22} / E_{22}$. In the process of eutectic fracture, neutral loading, namely $\psi = \sigma_e$, can be considered. When $\psi = \sigma_e$ there is no microscopic damage in the eutectic, the fracture condition is; when there is microscopic damage, the fracture condition is: $\psi = \sigma_{de} / (1 - D_{22})$.

Therefore, the damage fracture stress of the enhanced phase eutectic containing lamellae can be obtained as follows

$$\sigma_{de} = \sigma_e \left(1 - D_{22}\right) = \frac{E''_{22}}{E_{22} \left(\delta_{22}^0 - \delta_{33}^0\right)} \sqrt{\frac{6E_bE_0 (\gamma_0 + \gamma_b)}{h (E_b + E_0)}} \quad (8)$$

The parameters of the lamellar enhanced eutectic are: $E_0 = 402$ GPa, $\nu_0 = 0.233$; $E_p = 233$ GPa, $\nu_p = 0.31$; $E_a = 10E_0$, $\nu_a = \nu_0$, the interfacial phase thickness $\Delta = 1 \text{nm}$, $\gamma_0 = 1.06 \text{J/m}^2$, $\gamma_b = 1.829 \text{J/m}^2$, the thickness $f_a = 2\Delta f_b / h$, $h = 200 \text{nm}$, according to the relationship between the reinforced lamellae and the interfacial phase size, the interfacial phase volume fraction can be obtained.

According to Equation (8), the variation trend of the mesoscopic damage fracture strength of eutectic containing reinforced lamellae with the thickness of reinforced lamellae is shown in Figure 5.

As can be seen from Figure 5, when the damage variable in the eutectic containing reinforced lamellae is small, the larger the volume content of reinforced lamellae is, the greater the fracture stress of the material is due to the stronger constraint of the eutectic interface. When the damage variable is large, the interface's strong constraint will be damaged, and the fracture stress will be reduced.

![Figure 5](image_url)

**Figure 5.** Schematic representation of the variation of damage fracture stress with reinforced lamellae volume fraction.

It was experimentally measured [26] that when the volume fraction $f_l = 0.34$ and thickness of the lamellae inclusion $h = 0.7 \mu\text{m}$, the macroscopic fracture strength of the composite ceramics was 1.6 GPa. When $d_2 = d_{2\max}$, the theoretical calculation result is 1.466 GPa. When $d_2 = 2/3d_{2\max}$, the theoretical calculation result is 1.67 GPa. When $d_2 = 1/3d_{2\max}$, the theoretical calculation result is 1.94 GPa. The experimental results are within the range of theoretical calculation. At the time of taking $d_2 = d_{2\max}$, the sheet reinforced phase plane has cracked with the matrix, and the material has not completely failed to fracture after the cracking, so the theoretical calculation value is slightly lower than the experimental results. When $d_2 = 2/3d_{2\max}$ and $d_2 = 1/3d_{2\max}$, the theoretical calculation
result is slightly larger than the test result, because the actual material has other defects, particles and cracks outside the crystal throwing, so the theoretical calculation result is higher. It can be seen that the model of eutectic damage with sheet inclusion is reasonable.

4. Damage localization bond

The model of the damage localization was established (Figure 6) [25] based on the underlying theory of the Dugdale–Barenblatt model [29,30]. The coordinate system oxy is built on the tip of the damage localization band, supposing the length of this band is \( l \), and \( a \) is the radius of the microcrack... The length is related to the far-field stress intensity factor \( K_{\infty} \) and \( l \ll a \).

![Figure 6. Model of damage localization band at crack tip.](image)

Under the tensile stress \( \sigma_{\infty} \), the length of the damage localization zone is obtained

\[
l = \frac{3\pi h (E_b + E_0)}{64E_bE_0 (\gamma_0 + \gamma_b)} \left( \frac{K_{\infty} E_{22} (\sigma_{22}^0 - \sigma_{33}^0)}{E_{22}'} \right)^2
\]

(9)

Under the far-field stress intensity factor \( K_{\infty} \), if steady propagation of the crack occurs, the damage localization zone moves along the steady direction of the crack. The length \( l \) is influenced by the residual intensity mostly by equation (9) and the fracture stress of the damage eutectic. Assuming the far-field stress intensity factor \( K_{\infty} = 5 \text{ MPa-m}^{1/2} \) of the composite ceramics with lamellae enhanced eutectic, the thickness of the lamella \( h = 0.7 \mu m \), and when the damage variables are the maximum of 1/3, 2/3 and 1 times respectively, the change curve of the length of the damage localization zone and the volume fraction of the reinforced lamellae can be obtained from Equation (9), as shown in Figure 7.

![Figure 7. Relationship between the length of damage localization band and reinforced lamellae volume fraction.](image)

As can be seen from Figure 7, with the increase of the volume fraction of the lamellar enhanced phase, the length of the localized damage zone increases, and the greater the damage degree, the
longer the length. When \( d_2 = 1/3d_{2\text{max}} \), the length of the damage localization zone has a small change range. As \( d_2 = d_{2\text{max}} \), the length scale effect of damage localization zone was obvious. According to the comprehensive analysis, with the increase of the volume fraction of the enhanced lamellar phase, the residual strength decreases and the length of the localized damage zone increases. The greater the degree of damage, the smaller the residual strength, and the longer the length of the damage localized band, the maximum length of the damage localized band is \( l_{\text{max}} = 0.13 \mu \text{m} \).

5. Conclusion

(1) According to the microstructure of the eutectic containing lamellae and considering the fracture condition of the cohesive bond, damage variables were defined to obtain the microscopic damage and fracture stress of the eutectic.

(2) Based on the D-B model, the damage fracture stress of eutectic was introduced as the residual strength to establish the damage localization zone model of multiphase ceramics with lamellae reinforcement and conduct quantitative analysis. The results show that with the increase of the volume fraction of the enhanced lamellar phase, the residual strength decreases and the length of the localized damage zone increases. The greater the degree of damage, the smaller the residual strength and the longer the length of the localized damage zone.

(3) Reducing the degree of material damage and selecting the appropriate volume content of the patchy enhanced phase can increase the eutectic fracture stress, further reduce the length of the damage localization zone, improve the threshold value of crack instability growth, and help to enhance the strength of the material.

Acknowledgements

We acknowledge the support from the National Natural Science Foundation of China (Grant No. 11272355). The authors would like to thank the anonymous editor and reviewers for the instructive comments.

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