Effect of buoyancy force on turbulent modes of complex heat transfer in an air-filled square cavity

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Abstract. Turbulent natural convection with the interaction of surface radiation in square enclosure has been numerically studied. The governing equations are solved contemporaneously by finite difference method to obtain the velocity, temperature and heat flux distributions. Turbulence has been modeled using the standard $k-\varepsilon$ model. The change of convective and radiative Nusselt numbers with respect to time has been described. The analysis is carried out over a wide range of Rayleigh number from $10^8$ to $10^{10}$. The effect of this key parameter on temperature and velocity distributions, convective and radiative Nusselt numbers has been investigated. The results clearly demonstrate a significant effect of buoyancy ratio on unsteady turbulent heat transfer.

1. Introduction
Heat and mass transfer induced by natural convection has been well studied for the last two decades due to its importance in many areas such as building insulations, solar energy collectors, nuclear reactor design and so on. Radiative heat transfer plays a significant role in heat transfer in enclosures. In the literature, many researches have been done in the area of numerical analysis of natural convection with the interaction of surface radiation [1-3].

Lari et al. [2] studied the effect of radiative heat transfer on natural convection heat transfer in a square cavity under normal room conditions. The flow characteristics of the cavity were analyzed at a broad range of Rayleigh numbers from $10^7$ to $10^6$. They showed that the radiation plays a significant role on temperature distribution and flow pattern in the cavity. Shati et al. [3] investigated the effects of natural convection with and without the interaction of surface radiation in square and rectangular enclosures. It was shown that the ratio between Rayleigh number and Nusselt number follows a similar trend.

The goal of this paper is to investigate the effect of Rayleigh number on unsteady turbulent modes of complex heat transfer in an enclosure.

2. Physical and mathematical models
The geometry of the problem is shown in Figure 1. This is a two-dimensional fluid flow in a square enclosure. Two vertical walls ($x=0$, $x=L+2h$) are kept at constant temperatures $T_h$ and $T_c$ respectively.

The external surface of the bottom wall ($y=0$) and top wall ($y=H+2h$) were considered to be adiabatic. The internal surfaces of the walls are suggested to be gray diffusive emitters and reflectors of radiation. Turbulence is modeled using the standard $k-\varepsilon$ model [4].
The continuity, Navier–Stokes and energy equations are given below:

inside the air cavity (2 in Figure 1):

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + 2 \frac{\partial}{\partial x} \left[ (v + v_r) \frac{\partial u}{\partial x} \right] + \frac{\partial}{\partial y} \left[ (v + v_r) \frac{\partial u}{\partial y} \right]
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\partial}{\partial x} \left[ (v + v_r) \frac{\partial v}{\partial x} \right] + 2 \frac{\partial}{\partial y} \left[ (v + v_r) \frac{\partial v}{\partial y} \right] + \beta \Delta T
\]

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial x} \left[ \left( \alpha_2 + \frac{v_r}{\sigma_r} \right) \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \left( \alpha_2 + \frac{v_r}{\sigma_r} \right) \frac{\partial T}{\partial y} \right]
\]

in the solid walls (1 in Figure 1):

\[
\frac{\partial k}{\partial t} + u \frac{\partial k}{\partial x} + v \frac{\partial k}{\partial y} = \frac{\partial}{\partial x} \left[ \left( \nu + \frac{v_r}{\sigma_k} \right) \frac{\partial k}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \left( \nu + \frac{v_r}{\sigma_k} \right) \frac{\partial k}{\partial y} \right] + P_t + G_k - \epsilon
\]

\[
\frac{\partial \epsilon}{\partial t} + u \frac{\partial \epsilon}{\partial x} + v \frac{\partial \epsilon}{\partial y} = \frac{\partial}{\partial x} \left[ \left( \nu + \frac{v_r}{\sigma_k} \right) \frac{\partial \epsilon}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \left( \nu + \frac{v_r}{\sigma_k} \right) \frac{\partial \epsilon}{\partial y} \right] +
\]

\[+ \left( c_{\epsilon f} \left( P_t + c_{\epsilon G_k} \right) - c_{\epsilon f} \epsilon \right) \frac{\epsilon}{k} \]
\[
\frac{\partial T}{\partial t} = \alpha_{1,2} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]  

(7)

Where

\[
P_i = \nu \left( \frac{\partial^2 u}{\partial x^2} + 2 \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} \right), \quad G_k = -\frac{\nu}{\sigma_y} g\beta \frac{\partial T}{\partial y}, \quad \nu = c_f \mu k^2 \frac{\xi}{\epsilon}
\]

For the \(k-\epsilon\) turbulence model, the constants used are [4]:

\[
c_\mu = 0.09, \quad c_{1e} = 1.44, \quad c_{2e} = 1.92, \quad c_{3e} = 0.8, \quad \sigma_k = 1.0, \quad f_\mu = f_1 = f_2 = 1.0, \quad \sigma_f = 1.0, \quad \sigma_\epsilon = 1.3.
\]

The initial and boundary conditions for the formulated governing equations (1) – (7) have been described in [5, 6]. The mathematical model has been formulated in terms of the dimensionless stream function, vorticity and temperature [4-6].

The following algebraic transformation is used for thickening of the computational grid:

\[
\xi = \frac{1}{2} \left[ 1 + \tan \left( \frac{\pi}{2} (2x-1) \gamma \right) \right] / \tan \left( \frac{\pi}{2} \gamma \right), \quad \eta = \frac{1}{2} \left[ 1 + \tan \left( \frac{\pi}{2} (2y-1) \gamma \right) \right] / \tan \left( \frac{\pi}{2} \gamma \right).
\]

The dimensionless net radiative heat flux \(Q_{rad}\) is determined as follows [6]:

\[
Q_{rad,k} = R_k - \sum_{i=1}^{N} F_{k,i} R_i,
\]

(8)

\[
R_k = \left( 1 - \epsilon'_k \right) \sum_{i=1}^{N} F_{k,i} R_i + \tilde{\epsilon}_k (1 - \zeta)^4 \left( \Theta_k + 0.5 \frac{1 + \zeta}{1 - \zeta} \right)^4,
\]

(9)

Where \(\nu\) – kinematic viscosity, \(\nu_t\) – turbulent viscosity, \(\alpha_{1,2}\) – thermal diffusivity ratio, \(\alpha_2\) – air thermal diffusivity, \(\alpha_i\) – thermal diffusivity of the wall material, \(\beta\) - coefficient of volumetric thermal expansion, \(\zeta\) - temperature parameter, \(\Theta\) - dimensionless temperature, \(R_k\) – radiosity of the \(k\)th element of an enclosure, \(F_{k,i}\) – view factor from \(k\)th element to the \(i\)th element of an enclosure, \(Q_{rad,k}\) – dimensionless net radiative heat flux of the \(k\)th element of an enclosure, \(\gamma\) - compaction parameter.

The governing equations (1)–(7) are solved by the finite difference method. The solution procedure is described in [4]. The developed computational code has been verified for several benchmark problems [4-6].

3. Results and discussion

Numerical analysis has been conducted in a wide range of the Rayleigh number from \(10^8\) to \(10^{10}\). Figure 2 shows the graphic dependences of the average convective and radiative Nusselt numbers at the left solid-fluid interface on the dimensionless time and the Rayleigh number.

It was shown that at dimensionless time \(\tau = 900\) the convective Nusselt number increases up to 2.1 times at changing of Ra from \(10^8\) up to \(10^{10}\). It is worth noting that \(\overline{\text{Nu}}_{\text{con}}(\tau) < \overline{\text{Nu}}_{\text{rad}}(\tau)\) at surface
emissivity $\epsilon = 0.6$ for a wide range of Rayleigh number. An increase in this parameter leads to an intensification of fluid flow inside the cavity.

![Figure 2. Variation of the average convective (a) and radiative (b) Nusselt numbers at the left vertical wall versus the dimensionless time and the Rayleigh number at surface emissivity $\epsilon = 0.6$ and thermal conductivity ratio $k_{1,2} = 100$.](image)

4. Conclusion

The distributions of streamlines and isotherms describing the main features of the flow pattern and heat transfer have been obtained. The influence of the buoyancy force on both velocity and temperature fields and the convective and radiative Nusselt numbers at solid–fluid interface has been analyzed. It has been found that thermal boundary layers are formed close to the vertical solid-fluid interfaces for high values of the Rayleigh number. It is therefore hoped that this type of investigation would lead to an improved knowledge in the area of numerical analysis heat transfer characteristics.

Acknowledgments

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References

[1] Vivek V, Sharma A K and Balaji C 2012 Int. J. Therm. Sci. 60 70-84
[2] Lari L, Baneshi M, Gandjalikhan N S A, Komiy A and Maruyama S 2011 Int. J. Heat Mass Tran. 54 5087-5099
[3] Shati A K A, Blakey S G and Beck S B M 2012 J. Eng. Sci. Technol. 7 257-279
[4] Miroshnichenko I and Sheremet M 2015 EPJ Web of Conf. 82 01057
[5] Martyushev S G, Miroshnichenko I V and Sheremet M A 2014 J. Engineering Physics and Thermophysics 87 pp 124-134
[6] Martyushev S G and Sheremet M A 2014 Int. J. Therm. Sci. 76 pp 51-67