A Lorentz invariance violating cosmology on the DGP brane

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Abstract. We study cosmological implications of a Lorentz invariance violating DGP-inspired braneworld scenario. A minimally coupled scalar field and a single, fixed-norm, Lorentz-violating timelike vector field within an interactive picture provide a wide parameter space which accounts for late-time acceleration and transition to phantom phase of the scalar field.

Keywords: cosmology with extra dimensions, cosmological applications of theories with extra dimensions
1 Introduction

Theories of extra spatial dimensions, in which the observed universe is realized as a brane embedded in a higher dimensional bulk, have attracted a lot of attention in the last few years. In this viewpoint, ordinary matters and gauge fields are trapped on the brane but gravitation and possibly non-standard matter can propagate through the entire spacetime [1–3]. In a cosmological perspective, braneworld scenarios have the capability to explain some of the new achievements of observational cosmology such as late-time positively accelerated expansion. Although some of these models predict deviations from the usual 4-dimensional gravity at short distances, the model proposed by Dvali, Gabadadze and Porrati (DGP) [1] is different in this regard since it predicts deviations from the standard 4-dimensional gravity over large distances. In fact, DGP scenario is infrared modification of general relativity.

On the other hand, impacts of Lorentz invariance violation (LIV) on cosmology have been studied by some authors [4, 5]. For instance, this issue has been studied in the context of scalar-vector-tensor theories [4]. It has been shown that Lorentz violating vector fields affect the dynamics of the inflationary models. One of the interesting feature of this scenario is the fact that exact Lorentz violating inflationary solutions are related to the absence of the inflaton potential. In this case, the inflation is completely associated with the Lorentz violation and depends on the value of the coupling parameters [5]. One important observation has been made recently in references [6] which accelerated expansion and crossing of the phantom divide line with one minimally coupled scalar field in the presence of a Lorentz invariance violating vector field has been shown as a result of interactive nature of the model. We know, an important consequence in quintessence model is the fact that a single minimally coupled scalar field is not suitable to explain crossing of the phantom divide line, $\omega = -1$ [7]. However, a single but non-minimally coupled scalar field is enough to cross the phantom divide line by its equation of state parameter $\omega$ [8]. It has been shown that within an interactive picture, a minimally coupled scalar field in the presence of a Lorentz violating vector field can evolve to take phantom phase [6]. This happens due to wider parameter space prepared by inclusion of Lorentz invariance violating vector field and its interaction with scalar field.

From another viewpoint, currently it is well-known (based on various observational data) that our universe has entered the stage of a positively accelerate expansion around the redshift $z < 1$; see [9] and references therein. The equation of state (EoS) parameter $\omega$
responsible for the acceleration of the Universe has been constrained to be close to $\omega = -1$. In this regard, the analysis of the properties of dark energy from recent observational data mildly favor models of dark energy with $\omega$ crossing $-1$ line in the near past. So, the phantom phase EoS with $\omega < -1$ is still mildly allowed by observations. Recently, there have been a number of attempts to realize the phantom phase EoS, the simplest model which realizes the phantom EoS is provided by a nonminimally coupled scalar field \cite{7}. Other examples are in the spirit of braneworld models of dark energy. In these models, crossing of the phantom divide line and late-time acceleration are studied extensively (see for instance \cite{10,11,12}).

With these preliminaries, construction of yet another theoretical framework which combines braneworld effects and Lorentz invariance violation to realize positively accelerate expansion and transition to phantom phase is an interesting challenge. With this motivation, in which follows, we construct a new dark energy model to realize crossing of the phantom divide line and explanation of other observational achievements such as late time acceleration. In this regard, we study cosmological dynamics of a Lorentz violating DGP-inspired braneworld scenario. By implementing local Lorentz violation in a gravitational setting due to the existence of a tensor field with a non-vanishing expectation value, and then coupling of this tensor field to gravitational sector and matter (a scalar field), we study late-time acceleration and transition to phantom phase of the scalar field. The simplest example of this approach is to consider a single timelike vector field with fixed norm. This vector field picks out a preferred frame at each point in space-time and any matter field coupled to it will experience a violation of local Lorentz invariance. A special case of this theory was firstly introduced as a mechanism for Lorentz-violation by Kostelecky and Samuel in ref. \cite{13}. In curved spacetime, however, there is no natural generalization of the notion of a constant vector field (since $\nabla_\mu u^\nu = 0$ generically has no solutions); we must therefore allow the vector field to have dynamics, and fix its norm by choosing an appropriate action for the field. We will show that as a result of Lorentz invariance violation, there is an interacting term in the dynamics of scalar field which affects cosmological dynamics of the model considerably and is responsible for transition to phantom phase of the scalar field.

Nevertheless, one point should be stressed here: we know that DGP braneworld scenario explains accelerated expansion of the universe via leakage of gravity to extra dimension \cite{14}. But in this scenario the EoS parameter of dark energy never crosses the $\omega(z) = -1$ line and universe eventually turns out to be de Sitter phase. On the other hand, in this setup by incorporating a single scalar field (ordinary or phantom) on the brane, one can show that EoS parameter of dark energy crosses the phantom divide line \cite{15}. The question then arises: why we need to further generalization of this braneworld setup? One important reason lies in the fact that DGP setup suffers from ghost instabilities and it is important to add new ingredients to original setup to overcome this shortcoming. We think Lorentz invariance violation may help us to construct a wider parameter space with potential to overcome this problem.

We begin by an overview of the basic equations of motion for the most general theory of a fixed-norm vector field $u^\mu$ with a generalized action $S = S_{\text{Bulk}} + S_{\text{Brane}}$ where $S_{\text{Brane}} = S_{\text{EH}} + S_\phi + S_m + S_u$. Then we generalize this setting to the model universe with DGP-inspired action and we investigate cosmological consequences of this setup.
2 A Lorentz violating DGP-inspired braneworld scenario

As it is well known, braneworlds are often studied within the framework of the 5D Einstein field equations projected onto the 4D brane \cite{16}. To study impact of Lorentz invariance violation on the cosmological dynamics of DGP setup, we consider a vector field $u^\mu$ along with the extra dimension. So, a local frame at a point in space-time is inevitably selected as the preferred frame. In other words, the existence of the brane defines a preferred direction in the bulk. We study the effects of local Lorentz violation on the dynamics of the brane by inclusion of this vector field in the action. This additional field modifies the 4D Einstein equations with cosmological implications which we investigate by studying the resulting Friedmann equation on the brane.

The action of the Lorentz violating DGP scenario in the presence of a minimally coupled scalar field and a vector field on the brane can be written as the sum of two distinct parts

$$ S = S_{\text{Bulk}} + S_{\text{Brane}}, \quad (2.1) $$

where $S_{\text{Bulk}}$ and $S_{\text{Brane}} \equiv S_{EH} + S_\phi + S_m + S_u$ are defined as follows

$$ S_{\text{Bulk}} = \int d^5x \frac{m_3^2}{2} \sqrt{-g} \mathcal{R}, \quad (2.2) $$

$$ S_{\text{Brane}} = \left[ \int d^4x \sqrt{-q} \left( \frac{m_3^2}{2} R[q] - \frac{1}{2} q^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) + m_3^2 \mathcal{K} + \mathcal{L}_m + \left[ - \beta_1 \nabla^\mu u^\nu \nabla_\mu u_\nu - \beta_2 \nabla^\mu u^\nu \nabla_\nu u_\mu - \beta_3 (\nabla_\mu u^\mu)^2 
- \beta_4 u^\mu \nabla_\mu u^\nu \nabla_\nu u_\alpha + \lambda (u^\mu u_\mu + 1) \right] \right] \right|_{y=0}, \quad (2.3) $$

where $y$ is coordinate of the fifth dimension and we assume brane is located at $y = 0$. $m_3^2$ and $m_4^2$ are fundamental scales in the bulk and brane respectively. $g_{AB}$ is five dimensional bulk metric with Ricci scalar $\mathcal{R}$, while $q_{\mu\nu}$ is induced metric on the brane with induced Ricci scalar $R$. $g_{AB}$ and $q_{\mu\nu}$ are related via $q_{\mu\nu} = \delta^{A}_\mu \delta^{B}_\nu g_{AB}$. $\mathcal{K}$ is trace of the mean extrinsic curvature of the brane defined as

$$ \mathcal{K}_{\mu\nu} = \frac{1}{2} \lim_{\epsilon \to 0} \left( \left[ K_{\mu\nu} \right]_{y=-\epsilon} + \left[ K_{\mu\nu} \right]_{y=+\epsilon} \right), \quad (2.4) $$

and corresponding term in the action is York-Gibbons-Hawking term \cite{17}. This action is allowed to contain any non-gravitational degrees of freedom in the framework of Lorentz violating scalar-vector-tensor theory of gravity. As usual, we assume $u^\mu u_\mu = -1$ and that the expectation value of vector field $u^\mu$ is $< 0 | u^\mu u_\mu | 0 >= -1 \cite{18}$. $\beta_i(\phi)$ ($i = 1, 2, 3, 4$) are arbitrary parameters with dimension of mass squared and $\lambda$ is a Lagrange multiplier. Note that $\sqrt{\beta_i}$ are mass scale of Lorentz symmetry breakdown \cite{4,18,19}. In which follows, we neglect quartic self-interaction term, $u^\mu u^\nu \nabla_\mu u^\alpha \nabla_\nu u_\alpha$. Some cosmological consequences of this term in the action are studied in refs. \cite{4,19} for ordinary 4D framework. In this setup, the preferred frame is selected through the constrained vector field $u^\mu$ and this leads to the violation of the Lorentz symmetry.

The ordinary matter part of the action is shown by Lagrangian $\mathcal{L}_m \equiv \mathcal{L}_m(q_{\mu\nu}, \psi)$ where $\psi$ is matter field and corresponding energy-momentum tensor is

$$ T_{\mu\nu} = -\frac{2}{\delta q_{\mu\nu}} \frac{\delta \mathcal{L}_m}{\delta q_{\mu\nu}} + q_{\mu\nu} \mathcal{L}_m. \quad (2.5) $$
The pure scalar field Lagrangian, \( L = -\frac{1}{2} q^{\mu \nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) \), yields the following energy-momentum tensor
\[
\tau_{\mu \nu} = \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} q_{\mu \nu} (\nabla \phi)^2 - q_{\mu \nu} V(\phi).
\] (2.6)

The energy-momentum tensor of the Lorentz violating vector field is defined as usual by
\[
T_{\mu \nu}^{(u)} = -2 \frac{\delta L^{(u)}}{\delta q_{\mu \nu}} + q_{\mu \nu} L^{(u)}.
\] (2.7)

The Bulk-brane Einstein’s equations calculated from action (2.1) are given by
\[
m_3^4 \left( R_{AB} - \frac{1}{2} g_{AB} R \right) + m_2^2 \delta A^\mu \delta B_\nu \left( R_{\mu \nu} - \frac{1}{2} q_{\mu \nu} R \right) \delta(y) = \delta A^\mu \delta B_\nu \Upsilon_{\mu \nu} \delta(y)
\] (2.8)

where \( \Upsilon_{\mu \nu} \equiv T_{\mu \nu} + \tau_{\mu \nu} + T_{\mu \nu}^{(u)} \). From equation (2.8) we find
\[
G_{AB} = R_{AB} - \frac{1}{2} g_{AB} R = 0
\] (2.9)
and
\[
G_{\mu \nu} = \left( R_{\mu \nu} - \frac{1}{2} q_{\mu \nu} R \right) = \frac{\Upsilon_{\mu \nu}}{m_3^2}
\] (2.10)
as Einstein’s equations in the bulk and brane respectively. The corresponding junction conditions relating the extrinsic curvature of the brane to its energy-momentum tensor, have the following form (see [20] and related references therein)
\[
\lim_{\epsilon \to 0} \left[ K_{\mu \nu} \right]_{y=\epsilon} = \frac{1}{m_3^2} \left[ \Upsilon_{\mu \nu} - \frac{1}{3} q_{\mu \nu} q^{\alpha \beta} \Upsilon_{\alpha \beta} \right]_{y=0} - \frac{m_2^2}{m_3^2} \left[ R_{\mu \nu} - \frac{1}{6} q_{\mu \nu} q^{\alpha \beta} R_{\alpha \beta} \right]_{y=0}.
\] (2.11)

We start with the following line element to derive cosmological implications of our model
\[
ds^2 = q_{\mu \nu} dx^\mu dx^\nu + b^2(y,t) dy^2 = -n^2(y,t) dt^2 + a^2(y,t) \gamma_{ij} dx^i dx^j + b^2(y,t) dy^2.
\] (2.12)

For such a metric, to solve Einstein equations in the presence of a fixed-norm vector field, the vector field must respect spatial isotropy, at least in the background (though perturbations will generically break the symmetry). Thus the only component that the vector can possess is the timelike component. Therefore, we take the constraint \( u^\mu = (\frac{1}{N}, 0, 0, 0) \) where \( N \) is a lapse function. After performing required algebra, we set \( n(0,t) = 1 \) and \( \mathcal{N} = 1 \) in which follows. The scale of the universe is determined by \( a(y,t) \) and \( \gamma_{ij} \) is a maximally symmetric 3-dimensional metric defined as
\[
\gamma_{ij} = \delta_{ij} + k \frac{x_i x_j}{1 - k r^2}
\] (2.13)
where \( k = -1, 0, 1 \) parameterizes the spatial curvature and \( r^2 = x_i x^i \). We assume that scalar field \( \phi \) depends only on the proper cosmic time of the brane and we adopt the gauge \( b^2(y,t) = 1 \) in Gaussian normal coordinates.

Now total energy density and pressure are given as follows
\[
\rho_{\text{tot}} = \rho_m + \rho_u + \rho_\phi
\] (2.14)
and

\[ p_{\text{tot}} = p_m + p_u + p_\phi. \]  

(2.15)

Here we assume that energy-momentum tensor of ordinary matter on the brane has a perfect fluid form with energy density \( \rho_m \) and pressure \( p_m \) so that \( T_{\mu\nu} = (\rho_m + p_m)N_\mu N_\nu + p_m g_{\mu\nu} \) where \( N_\mu \) is a unit timelike vector field representing the fluid four-velocity. We also assume a linear isothermal equation of state for the fluid \( p_m = (\gamma_m - 1)\rho_m \) that \( 1 \leq \gamma_m \leq 2 \). Energy density and pressure of minimally coupled scalar field are given as follows

\[ \rho_\phi = \left[ \frac{1}{2} \dot{\phi}^2 + n^2 V(\phi) \right]_{y=0}, \]  

(2.16)

and

\[ p_\phi = \left[ \frac{1}{2n^2} \dot{\phi}^2 - V(\phi) \right]_{y=0}, \]  

(2.17)

where a dot denotes the derivative with respect to cosmic time \( t \). The stress-energy for the vector field also takes the form of a perfect fluid, with energy density given by \( [4, 5] \)

\[ \rho_u = -3\beta H^2 \]  

(2.18)

and pressure

\[ p_u = \beta H^2 \left[ 3 + 2\frac{\dot{H}}{H^2} + 2\frac{\dot{\beta}}{H\beta} \right] \]  

(2.19)

where we have defined the parameter \( \beta \) as \( \beta \equiv (\beta_1 + 3\beta_2 + \beta_3) \) and \( H = \frac{\dot{a}(0,t)}{a(0,t)} \) is the Hubble parameter. In the absence of vector field, that is, when all \( \beta_i = 0 \), the above equations reduce to the conventional ones and in the case \( \beta = \text{const.} \), the above equations are lead to the equations given in \([19]\). For future references, the total equation of state parameter defined as \( \omega_{\text{tot}} = \frac{p_{\text{tot}}}{\rho_{\text{tot}}} \) or

\[ \omega_{\text{tot}} = \frac{p_m + p_u + p_\phi}{\rho_m + \rho_u + \rho_\phi} \]

takes the following form

\[ \omega_{\text{tot}} = \left[ \frac{p_m + \beta H^2 \left[ 3 + 2\frac{\dot{H}}{H^2} + 2\frac{\dot{\beta}}{H\beta} \right] + \frac{1}{2n^2} \dot{\phi}^2 - V(\phi)}{\rho_m - 3\beta H^2 + \frac{1}{2n^2} \dot{\phi}^2 + n^2 V(\phi)} \right]_{y=0}. \]  

(2.20)

Now, the effective Einstein equations on the brane are given as follows \([21]\)

\[ G_{\mu\nu} = \frac{\Pi_{\mu\nu}}{m_4^6} = \mathcal{E}_{\mu\nu}, \]  

(2.21)

where

\[ \Pi_{\mu\nu} = -\frac{1}{4} \eta_{\mu\rho} \eta_{\nu\sigma} + \frac{1}{12} \eta_{\mu\nu} + \frac{1}{8} g_{\mu\nu} \left( \eta_{\rho\sigma} \eta^{\rho\sigma} - \frac{1}{3} \eta^2 \right), \]  

(2.22)

and

\[ \mathcal{E}_{\mu\nu} = C_{IJKL} \Theta^I \Theta^K g_{\mu J} g_{\nu L} \]  

(2.23)

where \( C_{IJKL} \) is five dimensional Weyl tensor and \( \Theta_A \) is the spacelike unit vector normal to the brane. Using equation (2.21) we find

\[ G^0_0 = \frac{\Pi^0_0}{m_4^6} = \mathcal{E}^0_0 \]  

(2.24)
where for FRW universe we have
\[ G^0_0 = -3 \left( H^2 + \frac{k}{a^2} \right). \]  
(2.25)

Similarly, for space components we have
\[ G^i_j = \frac{\Pi^i_j}{m_4^2} - \mathcal{E}^i_j \]  
(2.26)

where
\[ G^i_j = -\left( 2\dot{H} + 3H^2 + \frac{k}{a^2} \right) \delta^i_j. \]  
(2.27)

Now, using equation (2.22) we find \( \Pi^0_0 = -\frac{1}{12} \left( \Upsilon^0_0 \right)^2 \) and \( \Pi^i_j = -\frac{1}{12} \Upsilon^0_0 \left( \Upsilon^0_0 - 2 \Upsilon^1_1 \right) \delta^i_j. \)

Also, the time and space components of the total energy-momentum tensor are given by
\[ \Upsilon^0_0 = -\rho_{\text{tot}} - \frac{m_2}{3} G^0_0 \]  
(2.28)
and
\[ \Upsilon^i_j = -p_{\text{tot}} \delta^i_j - \frac{m_2}{3} G^i_j, \]  
(2.29)
where \( \rho_{\text{tot}} \) and \( p_{\text{tot}} \) are given by (2.14) and (2.15). These equations lead us to the following effective Friedmann equation on the brane
\[ 3 \left( H^2 + \frac{k}{a^2} \right) = \mathcal{E}^0_0 + \frac{1}{12m_4^2} \left[ \rho_m - 3\beta H^2 + \frac{1}{2} \phi^2 + n^2 V(\phi) - 3m_2^2 \left( H^2 + \frac{k}{a^2} \right) \right]^2. \]  
(3.20)

3 Cosmological aspects of the model

Late-time positively accelerated expansion and transition to phantom phase of scalar field as a dark energy component are two interesting challenges of modern cosmology. Therefore, in which follows, we study late-time acceleration and dynamics of equation of state parameter, \( \omega_\phi(t) \) in this Lorentz violating DGP setup. The equation of state parameter of scalar field on the brane is given by
\[ \omega_\phi = \frac{p_\phi}{\rho_\phi} = \left[ \frac{1}{\pi a^2} \dot{\phi}^2 - V(\phi) \right]_{y=0} \]  
(3.1)

where from now on we set \( n(0, t) = 1 \). Dynamics of \( \omega_\phi(t) \) can be obtained in two different viewpoints. Firstly, Friedmann equation (2.30) in the absence of ordinary matter on the brane and with flat spatial geometry (\( k = 0 \)), can be rewritten as follows
\[ H^2 = \frac{\mathcal{E}_0}{3a^4} + \frac{1}{36m_4^6} \left[ -3\beta H^2 + \frac{1}{2} \dot{\phi}^2 + V(\phi) - 3m_2^2 H^2 \right]^2. \]  
(3.2)
We take $\dot{\mathcal{E}}_0 + 4H\mathcal{E}_0 = 0$ which an integration gives $\mathcal{E}_0 = \frac{\dot{\mathcal{E}}_0}{4}$ where $\mathcal{E}_0$ is integration constant [21]. Dynamics of scalar field can be deduced from this equation directly

$$\dot{\phi}^2 = 6H^2(\beta + m^2_\phi) - 2V(\phi) + 12m^3_\phi\epsilon\sqrt{H^2 + \frac{\mathcal{E}_0}{3a^4}}$$

(3.3)

where $\epsilon = \pm 1$, corresponding to two possible embedding of DGP braneworld. To determine exact dynamics of scalar field, we need to specify functional form of potential, $V(\phi)$. In this framework, we use two well-known potentials: $V(\phi) = \lambda\phi^2$ and an exponential potential as $V(\phi) = V_0 \exp\left(-\sqrt{\frac{16\pi}{pm^2}}\phi\right)$ [22].

Secondly, we see that equation (3.3) depends on the potential of scalar field. With this facility, we can obtain a potential-independent equation of dynamics for scalar field. The energy equation for vector field $\mathbf{u}$ is

$$\dot{\rho}_u + 3H(\rho_u + p_u) = +3H^2\dot{\beta}$$

(3.4)

and for the scalar field we find

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = -3H^2\dot{\beta}.$$ 

(3.5)

There is a non-conservation scheme in this setup due to energy-momentum transfer between scalar and vector fields. This is very similar to the case studied by Zimdahl et al. [23]. As they have shown, a coupling between a quintessence scalar field and a cold dark matter (CDM) fluid leads to a stable, constant ratio for the energy densities of both component compatible with a power law accelerated cosmic expansion. In fact this coupling is responsible for accelerated expansion and possible crossing of phantom divide line. In our Lorentz invariance violating scenario this coupling is present between scalar field and vector field leading to an interactive picture. Note that one can consider also an interaction between ordinary matter and vector field on the brane. Very recently, authors of ref. [24] have investigated the cosmological evolution of an interacting scalar field model in which the scalar field has an interaction with the background matter via Lorentz violation in 4-dimensional model.

Nevertheless, the total energy in the presence of both scalar and vector fields is conserved

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (\rho = \rho_u + \rho_\phi).$$

(3.6)

This energy conservation equation can also be obtained by equating the covariant divergence of the total energy-momentum tensor to zero, since the covariant divergence of the Einstein tensor is zero by its geometric construction. It follows from contraction of the geometric Bianchi identity.

We obtain dynamics of the scalar field by differentiating equation (2.16) with respect to $t$ and then using equation (3.5) to find

$$\ddot{\phi} + 3H\dot{\phi} + 3H^2\beta,\phi + V,\phi = 0.$$ 

(3.7)

By differentiating equation (3.2) with respect to $t$ and using equation (3.7) we have

$$\dot{\phi} = \pm 2m^3_\phi\left(\frac{2H_\phi}{\dot{\phi}H} + \frac{12\mathcal{E}_0}{\phi^2\dot{a}^2}\right)^{\frac{1}{2}} - 2H_\beta,\phi - 2\beta H_\dot{\phi} - m^2_\phi \frac{H_\phi}{H}$$

(3.8)
where we assume $H$ and $\beta$ are depended on $\phi$ in forthcoming arguments and for simplicity, we set $\mathcal{E}_0 = 0$. By substituting equation (3.8) into the Friedmann equation (3.2), the potential of the scalar field in our model takes one of the following forms

$$V(\phi) = \frac{1}{2} \frac{12H + 6m_3^2H^2m_4^3 - \dot{\phi}^2m_4^3 + 6H^2m_4^3\beta}{m_4^3}$$

(3.9)

and

$$V(\phi) = \frac{1}{2} \frac{-12H + 6m_3^2H^2m_4^3 - \dot{\phi}^2m_4^3 + 6H^2m_4^3\beta}{m_4^3}$$

(3.10)

where $\dot{\phi}$ can be obtained from equation (3.8). We emphasize that equation (3.8) has several solutions, here we just consider one real root of this equation to proceed further. On the other hand, equation governing on dynamics of $\phi$ itself has no analytical solution. So, one can try to find some intuition by numerical analysis of the parameter space of the model.

To obtain dynamics of equation of state parameter, we note that there are two governing equations on the dynamics of $H^2$ as follows

$$H^2 = \frac{1}{3} \frac{\rho_\phi m_3^2 + \beta \rho_\phi + 6m_4^6 + 2\sqrt{3}m_4^6\rho_\phi m_3^2 + 3m_4^6\beta \rho_\phi + 9m_4^{12}}{2\beta m_3^2 + \beta^2 + m_3^4}$$

(3.11)

and

$$H^2 = \frac{1}{3} \frac{\rho_\phi m_3^2 + \beta \rho_\phi + 6m_4^6 - 2\sqrt{3}m_4^6\rho_\phi m_3^2 + 3m_4^6\beta \rho_\phi + 9m_4^{12}}{2\beta m_3^2 + \beta^2 + m_3^4}$$

(3.12)

Using these two branches of the model and also equation (3.5), we find

$$E_1 + E_2(1 + \omega_\phi) = -\dot{\beta}E_3$$

(3.13)

where we have defined

$$E_1 = \frac{\dot{\rho}_\phi}{\rho_\phi} = \frac{\dot{H} + \beta \dot{H} m_3^3 + m_3^2 \dot{H} m_4^3}{2\epsilon_4 + \beta \dot{H} m_4^3 + m_3^2 \dot{H} m_4^3},$$

$$E_2 = \frac{3\epsilon_1 \left( m_4^3 + \epsilon_2 \sqrt{m_4^9 + H^2 (\beta + m_3^2) m_4^3 + 2H \epsilon_4 (\beta + m_3^2)} \right)}{(\beta + m_3^2)},$$

(3.14)

and

$$E_3 = \frac{2\epsilon_3 \sqrt{m_4^3 \left( m_4^9 + H^2 (\beta + m_3^2)^2 m_4^3 + 2H \epsilon_4 (\beta + m_3^2) \right)}}{H \left( m_4^3 (\beta + m_3^2)^2 + H + 2\epsilon_4 \right) (\beta + m_3^2)^2} + \frac{H^2 (\beta + m_3^2)^2 m_4^3 + 2H \epsilon_4 (\beta + m_3^2) + 2m_4^9}{H \left( m_4^3 (\beta + m_3^2)^2 + H + 2\epsilon_4 \right) (\beta + m_3^2)^2}.$$

(3.15)

In these relations we have defined $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_4 = \pm 1$ where have been appeared due to algebraic structure of the model. Although all of these quantities have the same value, we have to save them in forthcoming equations because of several permutations of signs in equations of cosmological dynamics. In fact, as we will show, suitable and simultaneous
choices of these quantities are important in analysis of parameters space. Now, equation of state parameter takes the following complicate form

\[
\omega_\phi = \frac{1}{6} \left[ -6 \epsilon_2 \left( \beta + m_3^2 \right) H \epsilon_1 \left( 1/2 m_4^3 \left( \beta + m_3^2 \right) H + \epsilon_4 \right) \times \sqrt{m_4^3 + H^2 \left( \beta + m_3^2 \right)^2 m_4^3 + 2 H \epsilon_4 \left( \beta + m_3^2 \right)} \right] \\
- 2 \beta_\phi \dot{\phi} \epsilon_3 \sqrt{m_4^3 \left( m_4^9 + H^2 \left( \beta + m_3^2 \right)^2 m_4^3 + 2 H \epsilon_4 \left( \beta + m_3^2 \right) \right)} m_4^3 \\
- 3 m_4^3 \left( \beta + m_3^2 \right)^2 \left( 2/3 \beta_\phi \dot{\phi} + \epsilon_1 m_4^3 \right) H^2 \\
- 6 \left( 1/3 H_\phi \dot{\phi} m_3^4 + 2/3 H_\phi \dot{\phi} \beta m_3^2 + \epsilon_1 \epsilon_4 + 1/3 \beta^2 H_\phi \dot{\phi} \right) m_4^3 + 1/3 \beta_\phi \dot{\phi} \epsilon_4 \right) \left( \beta + m_3^2 \right) H \\
- 2 \left( m_4^9 \beta_\phi + H_\phi \epsilon_4 \left( \beta + m_3^2 \right) \right) \dot{\phi} \left( m_4^3 + \epsilon_3 \sqrt{m_4^9 + H^2 \left( \beta + m_3^2 \right)^2 m_4^3 + 2 H \epsilon_4 \left( \beta + m_3^2 \right)} \right)^{-1} \\
\times \left( \beta + m_3^2 \right)^{-1} H^{-1} \epsilon_1^{-1} \left( 1/2 m_4^3 \left( \beta + m_3^2 \right) H + \epsilon_4 \right)^{-1}. \tag{3.17} \]

This equation will be used to perform numerical analysis of the model. As another important cosmological parameter, the deceleration parameter \( q \) which is defined as

\[
q = -\frac{\ddot{a}}{a^2} = -1 - \frac{\dot{H}}{H^2} \tag{3.18} \]

using equation (3.2), takes the following form

\[
q = \frac{1}{3} \left[ -2 \sqrt{6} \epsilon_5 \sqrt{\left( 6 m_4^6 H^4 + (2 H \beta + m_3^2) \left( -3 \beta H^2 + \dot{V}(\phi) + \ddot{\phi} \right) \right) m_4^6} \\
+ \left( -12 \beta^2 - 12 m_4^6 \right) H^4 - 12 \beta \left( m_3^2 - 1/2 \beta \right) H^3 + \left( -3 m_3^4 + 3 \beta m_3^2 \right) H^2 - 2 \beta \left( \dot{V}(\phi) + \ddot{\phi} \right) H \\
- m_3^2 \left( \dot{V}(\phi) + \ddot{\phi} \right) \right] \left[ (2 H \beta + m_3^2)^2 H^2 \right]^{-1}. \tag{3.19} \]

Recent observations of distant type Ia supernovae and other observational data [9] indicate that \( q \) is currently negative; that means the expansion of the universe is positively accelerated. This is an indication that the gravitational attraction of matter, on the cosmological scale, is more than contraction by negative pressure dark energy in the form of quintessence. In which follows, we study cosmological implications of our model focusing on late-time acceleration and scalar field dynamics as a candidate of dark energy and possible transition to phantom phase. We show that the Lorentz symmetry breaking scalar field can be treated as a good candidate for the role of the dark energy source.

Before proceeding further, we should address some important and related issues here. Firstly, one should be careful to choose the appropriate equation of state for components that are used to describe the universe energy-momentum content. As we have emphasized earlier, a suitable coupling between a quintessence scalar field and other matter content can leads to
a constant ratio of the energy densities of both components which are compatible with an accelerated expansion of the universe or crossing of the phantom divide line (for more details see [23] and reference therein). In this respect and for instance, the holographic dark energy models studied in ref. [7] have the phantom phase by adopting a native equation of state, whereas the authors in [25] have found accelerating phase only using the effective equation of state. Based on these arguments, we should explain what kind of equation of state is used for observing the nature of mixed fluids here. In our model, we have three sources of energy-momentum: 1- standard ordinary matter, 2- scalar field as a candidate of dark energy and 3- energy-momentum content depended on Lorentz violating vector field. Here we assume that standard matter has negligible contribution on the total energy-momentum content of the universe and we can consider a constant linear isothermal equation of state as \( p_m = (\gamma_m - 1)\rho_m \) that \( 1 \leq \gamma_m \leq 2 \) for it. For other two energy-momentum contents, it is possible to use the "trigger mechanism" to explain dynamical equation of state [26]. This means that we assume scalar- vector-tensor theory containing Lorentz invariance violation which acts like the hybrid inflation models. In this situation, scalar and vector field play the roles of inflaton and the "waterfall" field respectively [26]. In this regard, we can fine-tune parameter \( m \) and other parameters to obtain best fit model using the observational data. Of course, an attractor solutions and fine-tuning in Lorentz violation model for suitable inflation phase has been studied in ref. [5]. Therefore it is reasonable to expect that one of them will eventually dominate to explain inflation or accelerating phase and crossing of phantom divide line. We should emphasize that the model studied in this paper belongs to a wider class of Lorentz-violating theories exhibiting the phantom behavior (see for instance refs. [4, 5, 27] for a number of Lorentz-violating models). One can extend this framework to understand issues such as transient phantom stage, super-horizon ghosts and to deal with the question that how generic are the features obtained in this particular model of late-time de Sitter attractor. Some of these issues have been discussed in ref. [28]. One more direction in this framework is to modify our model in such a way that it find the capability to describe inflationary epoch rather than the late-time acceleration. As has been pointed in [28], this type of model may give rise to some distinct features in the CMB spectrum.

4 Numerical analysis of the parameters space

In this section we study late-time acceleration and possible transition to phantom phase of the scalar field. Incorporation of the Lorentz invariance violation in the model provides a wider parameter space (relative to the case with one Lorentz preserving scalar field) which leads to more suitable framework for explanation of these interesting cosmological aspects. To show the validity of this statement, we need to solve equations (3.17) and (3.19). In the first stage, we obtain dynamics of scalar field \( \phi \) using equation (3.3). This goal will be achieved only if the Hubble parameter \( H(\phi(t)) \) and the vector field coupling, \( \beta(\phi(t)) \) are known a priori. In which follows, our strategy is to choose some suitable and natural candidates for the Hubble parameter \( H(\phi(t)) \) and the vector field coupling \( \beta(\phi(t)) \). Then we focus on possible crossing of the phantom divide line and realization of the universe late-time acceleration. We obtain suitable domains of parameters space which admit late-time acceleration and crossing of the phantom divide line by equation of state parameter. Probably this Lorentz violating DGP-inspired model has some important consequences in the spirit of cosmology (such as possible realization of bouncing solutions) and particle physics, but here we focus only on late-time acceleration and transition to phantom phase of the scalar field.
We consider a general case where both the vector field coupling and the Hubble parameter are functions of scalar field $\phi$ defined as follows

$$H(\phi) = H_0 \phi^\xi, \quad \beta(\phi) = m \phi^\xi$$

(4.1)

where $H_0$ and $m$ are positive and constant parameters. Note that scalar field itself is dependent on the cosmic time, $t$. In this case, dynamics of scalar field with potential of the type $V(\phi) = \lambda' \phi^2$ is given by

$$\dot{\phi} = 6H_0^2 \phi^2(\lambda \phi^2 + m_\phi^2) - \lambda' \phi^2 + 12m_4^3 \epsilon H_0 \phi^\xi$$

(4.2)

and for potential of the type $V(\phi) = V_0 \exp \left( -\sqrt{\frac{16\pi}{p m^4_0}} \phi \right)$, we find

$$\dot{\phi} = 6H_0^2 \phi^2(\lambda \phi^2 + m_\phi^2) - V_0 \exp \left( -\sqrt{\frac{16\pi}{p m^4_0}} \phi \right) + 12m_4^3 \epsilon H_0 \phi^\xi.$$ 

(4.3)

The question then arises here: what choices of space parameters lead to analytical solutions of the equations (4.2) and (4.3)? The answer to this question is summarized in tables 1 and 2. We note that as table 2 shows, for exponential potential there is very limited possibility to obtain analytical solution for scalar field dynamics. Nevertheless, we can find analytical solution in some especial cases. For instance, form equation (4.2) with $\xi = -1$ and $\zeta = 1$ we find the following analytical solution for dynamics of scalar field

$$\phi(t) = \frac{1}{2} \left[ 36m_4^6 e^2 H_0^2 e^{2A_0} \sqrt{6H_0^2 m_\phi^2 - \lambda'} + 9H_0^4 m^2 e^{2A_0} \sqrt{6H_0^2 m_\phi^2 - \lambda'} \right]$$

$$-12 e^{(t+A_0)} \sqrt{6H_0^2 m_\phi^2 - \lambda'} + 9H_0^4 m^2 e^{2A_0} \sqrt{6H_0^2 m_\phi^2 - \lambda'}$$

$$-e^{2t} \sqrt{6H_0^2 m_\phi^2 - \lambda'} \lambda' - 6 e^{(t+A_0)} \sqrt{6H_0^2 m_\phi^2 - \lambda'} \sqrt{6H_0^2 m_\phi^2 - \lambda'}$$

$$+36H^3 m_4^3 e^{2A_0} \sqrt{6H_0^2 m_\phi^2 - \lambda'}$$

$$\times e^{-(t+A_0)} \sqrt{6H_0^2 m_\phi^2 - \lambda'} \frac{1}{\left( \left( 6H_0^2 m_\phi^2 - \lambda' \right)^{3/2} \right)^{1}}$$

(4.4)

where $A_0$ is an integration constant. We emphasize that this analytical solution of equation (4.2) is obtained under some especial choices of parameter space. Without these choices it is impossible to find closed analytical solution for $\phi(t)$. We use this solution for our forthcoming numerical analysis. Figure 1 shows dynamics of $\phi$ as described by equation (4.4) for two different cases.

Now by substituting equation (4.4) into equation (3.19), we can analyze dynamics of deceleration parameter $q$ with respect to cosmic time $t$. This has been shown in figures 2 and 3 and corresponding results are summarized in table 3.

In which follows, we consider equation of state as given by equation (47) with equation (4.4) for dynamics of scalar filed in order to study crossing of the phantom divide line in this setup. For this purpose, we set $\xi = -1$ and $\zeta = 1$ to be more specific. The results of numerical calculations are shown in figures 4, 5, 6 and 7. All of these figures show the
Figure 1: Dynamics of scalar field in two especial cases.

| values of $\xi$ and $\zeta$ | analytical solution for equation (4.2)? |
|-----------------------------|----------------------------------------|
| $\xi \neq 0$ and $\zeta < 0$ | no                                     |
| $\xi = 0$ and $\zeta = 0$  | yes                                    |
| $\xi = 1$ and $\zeta = 0$  | yes                                    |
| $\xi = 2$ and $\zeta = 0$  | yes                                    |
| $\xi \geq 1$ and $\zeta \geq 1$ | no                                    |
| $\xi = -1$ and $\zeta = 1$ | yes                                    |
| $\xi = -2$ and $\zeta = 1$ | yes                                    |
| other $\xi$ and $\zeta$    | should be examined                     |

Table 1: Acceptable range of $\xi$ and $\zeta$ to have analytical solution of the scalar field equation (4.2)

| values of $\xi$ and $\zeta$ | analytical solution for equation (4.3)? |
|-----------------------------|----------------------------------------|
| $\xi \neq 0$ and $\xi \neq 0$ | no                                     |
| $\xi = 0$ and $\zeta = 0$  | yes                                    |
| $\xi = 1$ and $\zeta = 0$  | no                                     |

Table 2: Acceptable range of $\xi$ and $\zeta$ to have analytical solution of the scalar field equation (4.3)

possibility of transition to the phantom phase of the scalar field. It is interesting to note that as these figures show, transition from quintessence to phantom phase and from phantom phase to quintessence is possible in this braneworld scenario with appropriate choices of model parameters. Since one minimally coupled scalar field in the presence of Lorentz invariance symmetry cannot realize phantom divide line crossing, observation of this crossing in our model is a result of interactive nature of the model due to Lorentz invariance viola-
Figure 2: Variation of deceleration parameter $q$ relative to cosmic time $t$ for $\epsilon = +1$ and $\epsilon_5 = \pm 1$ with scalar field potential of the type $V(\phi) = \lambda'\phi^2$.

Figure 3: Variation of deceleration parameter $q$ relative to cosmic time $t$ for $\epsilon = -1$ and $\epsilon_5 = \pm 1$ with scalar field potential of the type $V(\phi) = \lambda'\phi^2$.

tion. This is manifested in our model via existence of the term such as $-3\beta H^2$ in Friedmann equation (3.2) or $-3\dot{\beta} H^2$ in equation (3.5).

We should stress here that by neglecting ordinary matter content ($\rho_m = 0 = p_m$) in equation (2.20), we find

$$\omega_{\text{tot}} = \left[ \frac{\beta H^2 \left( 3 + 2\frac{\dot{\beta}}{H} + 2\frac{\dot{\beta}}{H\beta} \right) + \frac{1}{2n^2} \dot{\phi}^2 - V(\phi)}{-3\beta H^2 + \frac{1}{2} \dot{\phi}^2 + n^2 V(\phi)} \right]_{y=0}. \tag{4.5}$$

The same procedure as described above, leads us to the dynamics of $\omega_{\text{tot}}$ as shown in figure...
| $\epsilon_1$ | $\epsilon_2$ | deceleration parameter $q$ | late-time acceleration? |
|-----------|-----------|--------------------------|------------------------|
| +1        | +1        | negative                 | yes                    |
| +1        | -1        | negative                 | yes                    |
| -1        | +1        | negative                 | yes                    |
| -1        | -1        | negative but almost constant | yes                   |

Table 3: Summary of results from figures 2 and 3.

8. We see that for $\epsilon = +1$, $\omega_{\text{tot}}$ never crosses the phantom divide line.
for $\xi = -1$, $\zeta = 1$ and $\epsilon = 1$

for $\xi = -1$, $\zeta = 1$ and $\epsilon = -1$

Figure 8: Dynamics of $\omega_{\text{tot}}$. There is no crossing for $\epsilon = +1$ branch of the model.

5 Summary and discussion

In this paper we have studied a DGP-inspired braneworld scenario where the idea of Lorentz invariance violation has been incorporated by specifying a preferred frame through the introduction of a dynamical vector field normal to our brane. The Einstein field equations obtained on the brane are modified by additional terms emanating from the presence of the vector field. This model breaks the 4D Lorentz invariance in the gravitational sector. As a new mechanism for crossing of the phantom divide line by equation of state parameter, we have shown that by a suitable choice of parameters of the model, it is possible to have phantom divide line crossing and realizing late-time acceleration in this Lorentz invariance violating context.

More importantly, we need to show the stability of the solutions in this Lorentz violating DGP-inspired model. It has been shown that the self-accelerating branch of the DGP model contains a ghost at the linearized level [29]. The ghost carries negative energy density and it leads to the instability of the spacetime. The presence of the ghost can be related to the infinite volume of the extra dimension in the DGP setup. When there are ghost instabilities in the self-accelerating branch, it is natural to ask what are the results of solution decay. One possible answer to this question is as follows: since the normal branch solutions are ghost-free, one could think that the self-accelerating solutions may decay into the normal branch solutions. In fact for a given brane tension, the Hubble parameter in the self-accelerating universe is larger than that of the normal branch solutions. Then it is possible to have nucleation of bubbles of the normal branch in the environment of the self-accelerating branch solution. This is similar to the false vacuum decay in de Sitter spacetime. However, there are arguments against this kind of reasoning which suggest that the self-accelerating branch does not decay into the normal branch by forming normal branch bubbles [29]. It was also shown that the introduction of a Gauss-Bonnet term for the bulk does not help one to overcome this problem [30]. In fact, it is still unclear what the end state of the ghost instability is in the self-accelerated branch of DGP inspired setups (for more details see [29]). On the other hand, it seems that introduction of Lorentz violating vector field on the brane provides a
new degree of freedom which may provide a suitable basis to treat ghost instability. Field theories of vector fields are very restrictive and gauge invariant Lagrangians are the ones that guarantee that the zeroth component of the gauge field is not propagating and, therefore there are no ghosts. However, in the action of our model as given by equation (2.3), the brane Lagrangian for the vector field is not gauge invariant. Therefore, it is possible that this vector field action by itself can introduce a ghost, instead of curing the one of gravity. This is not necessarily the case in the presence of vector field condensates [31, 32]. On the other hand, for certain timelike vector theories with spontaneous Lorentz violation, suitable restrictions of the initial-value solutions are identified that yield ghost-free models with a positive Hamiltonian [33]. This restrictions can be imposed on parameters such as $\beta_i$ in our model (see also [19, 30]). Therefore, in our Lorentz invariance violating setup there is the capability to overcome instabilities both in gravitational and vector field sectors. Anyway, if the Lorentz violation annihilate the ghost, this will be a great progress in the cosmology. We think that our Lorentz invariance violating model on the DGP brane has the capability to solve ghosts instabilities problem due to its wider parameter space relative to other existing scenarios.

Within a similar viewpoint, very recently Zen et al. [24] have studied the cosmological evolution of an interacting scalar field model in which the scalar field has its interaction with dark matter, radiation, and baryon via Lorentz violation in 4D standard model. They proposed a model of interaction through the effective coupling parameter, $\bar{\beta}$, $Q_m = -\frac{\dot{\bar{\beta}}\rho_m}{\bar{\beta}}$. They also determined all critical points and studied their stability of Lorentz violation model. On the other hand, Mariz et al. [34] studied the fact that the Lorentz-breaking parameter can be treated as a natural explanation of the extremely small value of the cosmological constant. Thus, the Lorentz symmetry breaking introduces a mechanism for the arisal of a non-zero but very small cosmological constant, and therefore providing an acceptable solution for cosmological constant problem. We believe that Lorentz symmetry breaking fields can be treated as a good candidate for the role of the dark energy source and these types of models have the capability to address other cosmological issues with better adaptability than other models. Finally we should stress on some open issues in this field: one issue concerns that we have provided a scenario of a scalar field with Lorentz violation to realize the crossing of the phantom divide. One maybe curious to know whether this scenario can make some interesting predictions for observations. For example the interactive terms in this model is very important, but can this interaction affect the formation of large scale structure, or the occupation of dark matter? This is a very interesting issue, since as we know large scale structure and the measure of dark matter are usually strongly constrained by CMB observations. Moreover, we think this model is able to give a bouncing solution of the universe. The reason lies in the fact that as has been shown in ref. [35], any models with crossing of phantom divide line have the capability to realize a bouncing solution.

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