Scaffolding Based Learning: Strategies For Developing Reflective Thinking Skills (A Case Study On Random Variable Material in Mathematics Statistics Courses)

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Abstract. Scaffolding is a learning model by providing assistance based on student difficulties. The purpose of this study is to describe the application of scaffolding-based learning to support students' reflective thinking skills in mathematics statistics courses. Research subjects were 43 5th semester students of mathematics education study programs. This type of research is a qualitative descriptive study with three research procedures carried out, namely: 1) a preliminary study, 2) planning, and 3) implementation. The instruments used were lecturer activity observation sheets and student activity observation sheets. The results showed that the components of scaffolding-based learning can be used to support students' reflective thinking skills in this case in learning mathematics statistics. Explaining and reviewing play a role in shaping pre-reflective situations, restructuring in shaping reflective situations, and developing conceptual thinking play a role in shaping post-reflective situations.

1. Introduction
Mathematics is the science that underlies the development of modern science and technology [1]. Mathematics has abstract studies [2]. This abstract nature demands the development of a mindset in learning mathematics. So we can say that mathematics plays an important role in developing human thought patterns [3]. This change in mindset has an impact on achieving the nature of mathematics learning, namely the realization of students who are skilled in solving a problem [4,5,6]. The use of an appropriate learning approach is one of the factors that can influence student success to become a good problem solver [7,8] because it impacts conducive classroom conditions.

The concept of scaffolding gained attention in the last decade [9,10]. Scaffolding is proven to support the teaching and learning process [11]. Scaffolding is a learning model by providing assistance based on difficulties experienced by students and slowly the assistance is reduced so that students can achieve independence in learning [12,13,14]. Zone of Proximal Development (ZPD) is a concept that started the concept of scaffolding [15]. The ZPD concept was introduced by Lev Semenovich Vygotsky in 1920 which defines ZPD as the area between the level of actual development and potential development. The actual development can be seen from the initial ability of students to solve the problems given. At this stage, students have entered ZPD and can solve problems. The resolution of this problem will be maximized if there is assistance (scaffolding) provided so students reach their potential development [16,17].
Scaffolding-based learning contains four components, namely explaining, reviewing, restructuring, and developing conceptual thinking \[12\]. Activities in explaining include telling and showing, namely the preparation for initial discussion material by involving a little student contribution. Reviewing is an activity refocusing on core issues. This activity is to anticipate when students are unable to identify important aspects contained in a concept or problem. The restructuring component is an activity of introducing modifications of ideas that arise so that not only results in students' understanding but also the ability to make meaning from a concept. Developing conceptual thinking occurs when a lecturer engages students in a wider conceptual discourse in student thinking.

The urgency of using scaffolding in learning activities is to develop students' higher-order thinking skills \[18,19,20\]. Reflective thinking is one of the higher-order thinking abilities \[21\]. Reflective thinking is the process of processing information internally based on existing knowledge that is not affected by external factors \[22,23\]. Based on the opinion of the experts, it can be concluded that reflective thinking is a mental process of responding to information from outside through the adjustment of knowledge that has been known so that students can explain, realize mistakes, and communicate verbally and in writing. Reflective thinking according to \[24\] has three dimensions, namely 1) the timing of reflection, 2) the object of reflection, 3) the levels of reflection where each of these dimensions supports in developing students' reflective thinking abilities. According to \[25\] reflective thinking has four stages namely habitual action which is an activity commonly done by students, understanding when students associate solutions with problems, reflection when students reconsider solutions by adjusting to the truth of the concept, and the last stage is critical thinking which is the highest stage. Whereas \[22\] divides three situations in reflective thinking, namely pre-reflective situation, post-reflective situation, and reflection situation.

Reflective thinking can be equated with other higher-order thinking abilities such as critical thinking skills, creative thinking, and problem-solving \[26\]. \[27\] states that reflective thinking is part of creative thinking. Reflective thinking is an ability that can describe the structure of a child's mathematical logic in a period of cognitive development \[28\]. According to \[29\] reflective thinking can be used to practice students' problem-solving abilities. Besides reflective thinking can support student learning outcomes \[30\]. Given the importance of reflective thinking skills in the learning process, it is necessary to have certain methods to support students' reflective thinking skills. In this case, the researcher tries to apply scaffolding-based learning to support students' reflective thinking skills. Based on this, the purpose of this study is to describe the application of scaffolding-based learning to support students' reflective thinking skills.

2. Method
This type of research used in this research is descriptive research with a qualitative approach. The research subjects were students in the 5th semester of the Mathematics Education Study Program at the University of Muhammadiyah Malang, totaling 43 students. The instrument used in the study was the lecturer activity observation sheet and the student activity observation sheet as an instrument for obtaining qualitative data. Observation sheet of lecturer's activities to obtain qualitative data in the form of scaffolding activities conducted by lecturers during learning activities. While the observation sheet of student activity is used to obtain data in the form of students' reflective thinking abilities. The supporting instrument is the Lesson Plan.

The research procedure consisted of three stages: 1) a preliminary study, 2) planning, and 3) implementation. Preliminary studies consist of activities examining scaffolding theory and reflective thinking. The planning stage is the stage of making lesson plans and research instruments adjusted to the scaffolding theory. The last stage is the implementation stage, namely the implementation of scaffolding-based learning by the prepared lesson plans. The data analysis technique used consists of three stages, namely 1) data reduction, which focuses on the data obtained, and selects important data with the sum of the results obtained. In this case, the focused data is data about the scaffolding stage given by the lecturer that supports the student's reflective thinking ability 2) presentation of data that is to present the results of the reduction data, 3) concluding.
3. Results and Discussion
The application of scaffolding-based learning in statistics courses is devoted to the material on continuous random variables. The following Table 1 is about the application of scaffolding-based learning to support reflective thinking skills.

| Material | Scaffolding Element | Reflective Thinking Situation |
|----------|---------------------|-------------------------------|
| The basic concept of a random variable | 1. Explaining 2. Reviewing in the form of parallel modeling | Pre-Reflective Situation |
| Application of the definition of random variables to continuous random variables | Restructuring in the form of negotiating meanings | Reflective Situation |
| Use of sample space to determine probability | Developing conceptual thinking | Post-Reflective Situation |

The lecturer starts learning by asking students to understand the definition of random variables. In scaffolding-based learning, it is important to always pay attention to student understanding simultaneously [31]. The following definition is a random variable.

“Let \( X \) be a function defined by sample space \( S \), in the set of real numbers \( R \), that is \( S \xrightarrow{X} R \) which links for each \( c \in S \) to \( x \in R \), written \( X(c) = x, \forall c \in S \). Then \( X \) is called a random variable while the range of \( X \) is written \( \mathcal{X} = \{ x : X(c) = x, c \in S \} \) is called a random variable space.”

At this stage, the lecturer explains by discussing classically. This is by research [32] which starts learning by providing information. At first, when the lecturer asked students about how far they understood the definition of random variables they had read, the students seemed confused. In this phase, students are in a pre-reflective situation. This can be seen from none of the students who can answer or explain. The action taken by the lecturer at that time was to explain the definition of random variables by connecting the concept of random variables with the concept of functions and the concept of random variables with the concept of sets. According to [33] students will easily understand a new concept if they can relate or analogize to a simpler concept.

The lecturer conducts a review by giving an example of a function equation that is \( f(2) = 5 \). Then the lecturer asks students which function equation is the domain and range. From the equation of the function the lecturer blends with the symbol \( X(c) = x, \forall c \in S \) which is in the definition of a random variable. The results of this reviewing stage students can understand the concept of random variables. This can be seen from one of the students being able to explain the concept of random variables correctly that random variables are a function where the domain is the sample space and range is a real number. To understand the concept of a random variable space the lecturer gives parallel modelling in the form of a simple example, for example an attempt to throw a coin twice and \( X \) is a random variable that states many appear face. According to [12] it can be used to assist students in understanding a concept. After that, the lecturer asks students to determine the random variable space. Students first determine the sample space of the experiment, then determine the random variable space by adjusting between the sample and the random variable \( X \) that has been defined. The results of student answers
are $A = \{1, 2\}$. To reinforce the concept in this phase, the lecturer asks one of the students to determine one subset of $A$. One of the students determines the set $B = \{1\}$. Then the lecturer asks the student to explain the meaning of number 1. The student answers number 1 shows the number of faces appearing that is one in number namely $mb$ and $bm$ written $C = \{mb, bm\}$ where the chance of the event is $\frac{2}{4}$. Lecturers provide an opportunity for students to deduce the opportunity conditions of $B$ and $C$. Students answering from the given trial cases can be concluded that $P(C) = P(B)$.

The next activity is the application of the definition of random variables that have been understood. The lecturer asks students to pay close attention to the application of the random variable to the continuous random variable.

$$S = \{c : 0 < c < 1\}$$

eg the probability of any occurrence $C$ from this sample space is defined by

$$P(C) = \int_c^1 dz$$

For example if $C = \{c : \frac{1}{3} < c < \frac{1}{2}\}$, then

$$P(C) = \int_{\frac{1}{3}}^{\frac{1}{2}} dz = \left[ z \right]_{\frac{1}{3}}^{\frac{1}{2}} = \frac{1}{6}$$

eg the random variable $X$ is defined as

$$x = X(c) = 3c + 2$$

Then random variable space $X$ is

$$\lambda = \{x : 2 < x < 5\}$$

![Figure 1. Continuous Random Variable Material](image)

Maya : Sorry mom, why did $C = \{c : \frac{1}{3} < c < \frac{1}{2}\}$ take the example?

Lecturer : Try to look again, what is meant by $C$?

Maya : $C$ is the event from the sample space $S$. Okay let me look again at the definition of the event.

Lecturer : Very nice. Furthermore, to make it easier to understand, also compare it with the example of a discrete random variable, namely in the experimental case of throwing a coin twice.

Students look at the definition of events and example in the case of throwing a coin twice.

Tere : Oh I know, $C$ has to be a subset of $S$.

Lecturer : What if I take $C_1 = \{c : \frac{1}{2} < c < \frac{3}{2}\}$?
Indra : Can’t ma’am, Because \(1 < c < \frac{3}{2}\) is not a member of \(S\).

Maya : Yes that means \(C_i\) is not a subset of \(S\).

Based on the dialogue above, in this activity students ask about the origin of \(C = \{c : \frac{1}{3} < c < \frac{1}{2}\}\). The lecturer gives restructuring in the form of negotiating meanings, namely asking the student to compare the \(C\) value found in the discrete random variable problem. After comparing students can understand the origin of \(C = \{c : \frac{1}{3} < c < \frac{1}{2}\}\). The students can conclude that \(C\) must be a subset of \(S\).

After that the lecturer gives another \(C\) value which is \(C_i = \{c : \frac{1}{2} < c < \frac{3}{2}\}\). Then students answer that they cannot choose \(C_i\) because it is not a subset of \(S\). This is in accordance with the research [34] namely that giving more concrete concepts will help students solve problems.

The next question is obtaining \(\lambda = \{x : 2 < x < 5\}\). The lecturer gives restructuring by giving inducement questions about the meaning of symbol \(A\) and its definitions. Then the lecturer asks students to connect the definition obtained with the problems faced by students. The result of this restructuring is that students are able to understand the origin of obtaining \(\lambda = \{x : 2 < x < 5\}\), namely by substituting \(C\) at \(X(c) = 3c + 2\). The impact of negotiating meanings is that students are in a reflective situation which means a situation where students have successfully formed a concept as a whole [35].

The scaffolding element given next is developing conceptual thinking by developing the problem from obtaining \(\lambda = \{x : 2 < x < 5\}\). What if taken \(\lambda = \{x : 2 < x < b\}\) where \(2 < b < 5\) and students are asked to determine the value of \(P(A)\).

Lecturer : You have succeeded in determining the random variable space from the sample space \(S\). If I have \(A \subseteq \lambda\) that is \(A = \{x : 2 < x < b\}\) where \(2 < b < 5\). Can you guys determine the probability of \(A\) ?

Bagas : If seen from the example in the case of discrete random variable, \(A\) is the set that describes the symbol for the sample space, while \(C\) the set that describe the actual space.

Maya : Bagas, do you mean that \(A\) is the same as \(C\) ?

Bagas : Yes I mean.

Indra : So if that means \(P(A) = P(C)\) ?

Bagas : It is true. Look again at the case with discrete random variables. We look at the steps to solve it.

Inaz : Yes really \(P(A) = P(C)\). The problem now is how to change the form of set \(A\) to set \(C\).

Maya : Guys, look at set \(A\), its contents are variable \(x\) and set \(C\) is variable \(c\). Just look at the random variable \(X\) which is defined as \(x = 3c + 2\).
Indra : Right. This means that \( C = \left\{ c : 0 < c < \frac{b-2}{3} \right\} \). Now we use formula
\[
P(C) = \int c \, dz \text{ to find the probability value of } C.
\]

Maya : Okay, so we get \( P(C) = \frac{b-2}{3} \).

In this problem the first thing students do is to change \( A \) to another form, \( C = \left\{ c : 0 < c < \frac{b-2}{3} \right\} \).

The reason for the change is because \( P(A) = P(C) \) and the information in the form of \( P(C) = \int c \, dz \).

This shows that students can make steps to solve problems based on information that is known by referring to the experience of previous learning. This situation indicates that the student is in a post-reflective situation [36].

4. Conclusion

Scaffolding-based learning is effective learning to be applied especially in learning that requires high analytical skills. The scaffolding component applied in the form of explaining, reviewing, restructuring and developing conceptual thinking can be used to anticipate when students are unable to identify important aspects contained in a concept or problem. The components of scaffolding-based learning can be used to support students' reflective thinking skills, in this case mathematics learning statistics. Explaining and reviewing play a role in shaping pre-reflective situations, restructuring in shaping reflective situations, and developing conceptual thinking play a role in shaping post-reflective situations.

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