Scalable quantum processor noise characterization

Kathleen E. Hamilton  Tyler Kharazi  Titus Morris
Alexander J. McCaskey  Ryan S. Bennink  and Raphael C. Pooser

Abstract—Measurement fidelity matrices (MFMs) (also called error kernels) are a natural way to characterize state preparation and measurement errors in near-term quantum hardware. They can be employed in post processing to mitigate errors and substantially increase the effective accuracy of quantum hardware. However, the feasibility of using MFMs is currently limited as the experimental cost of determining the MFM for a device grows exponentially with the number of qubits. In this work we present a scalable way to construct approximate MFMs for many-qubit devices based on cumulant expansions. Our method can also be used to characterize various types of correlation error.

Index Terms—quantum computing, NISQ computing, error mitigation, noise characterization

I. INTRODUCTION

The current era of quantum computing has been characterized as the “noisy intermediate scale quantum” (NISQ) era [1]. While fault tolerance and error corrected qubits are necessary for many large-scale quantum algorithms, recent studies have suggested that quantum processors with 50-60 qubits and sufficiently low error rates can out-perform classical computers at certain problems [2], [3], [4]. Devices with several 10’s of qubits are now publicly accessible and allow diverse end users to explore a variety of applications. As the size of available devices and the applications deployed continue to grow, there is a need for scalable benchmarks and metrics that can be executed by both hardware developers end users to quantify device errors and application performance [5], [6].

Quantum processors exhibit multiple error types, including state preparation and measurement (SPAM) error, gate errors, and cross-talk. These errors have motivated the development of error characterization and mitigation methods, each of which pose unique challenges in scalability. Extrapolating zero-noise behavior can be difficult for densely parameterized circuits [7], [8], [9]; pulse-level control methods may not be available to end users [10]; and tomographic-based approaches have large computational overhead [11]. Error mitigation routines are available in the IBM Qiskit Ignis library [12].

Matrix-based methods of noise characterization are commonly employed in quantum state tomography, where the process matrix and measurement fidelity matrix (MFM) are used to characterize gate fidelity [13], [14], and have also been used to mitigate multi-qubit errors [15]. Measurement errors have been commonly treated as independent [16], [17], with $n$-qubit matrices built from single qubit measurements [14]. In this work we present a scalable matrix-based characterization of SPAM error which uses MFMs to store the distribution of results obtained when each computational state is prepared and subsequently measured.

MFMs capture the net effect of multiple sources of error in qubit initialization, gates, and measurement. Recent studies have shown that MFMs can be used to characterize the individual sources of noise on hardware [18]. However a significant challenge to this approach is the exponential scaling of the number of experiments needed to determine a full $n$-qubit MFM. While determining a MFM is significantly less costly than quantum tomography [11] or Richardson extrapolation [7], to construct a $2^n \times 2^n$ MFM a minimum of $2^n$ different circuits must each be executed many times to accumulate significant statistics. This is cost-prohibitive for remote users who can access the device only intermittently and for limited amounts at a time. Previous studies have introduced the approach of constructing MFMs from single-qubit measurements [14]. While such an approach is scalable, it cannot capture error correlations that are widely observed.

Here we present scalable methods for constructing many-qubit MFMs and demonstrate these methods on quantum processors with superconducting qubits. These methods are based on the cumulant expansion which is commonly employed in the study of correlated systems across many physics disciplines [19]. This approach provides a systematic method to construct MFMs that incorporate qubit correlations. The MFMs constructed by our methods stand to help diagnose hardware errors, better identify the best qubits on which to execute a given program, and improve the effectiveness of error mitigation.

In Section II we describe the methods used to generate state preparation circuits and characterize correlated SPAM errors. In Sections III and IV we apply our methods to multiple IBM qubit devices and present results showing how error correlations can be quantified by comparing the full MFM of a set of qubits to an approximation of the MFM constructed from measurements on smaller qubit subsystems. In Section V we discuss these results, and present our conclusions in Section VI.
II. Methods

A. Circuits for Measuring MFM

A circuit $U$ is a sequence of one- and two-qubit gates that rotates an $n$-qubit register $|0\rangle^\otimes n$ to a final quantum state $|\psi\rangle$. This state is sampled (measured in the computational basis) yielding a string of $n$ bits. Preparing and sampling the state $n_s \gg 1$ times yields an estimate of the probability distribution $p$ over all $2^n$ possible bitstrings $x \in \{0,1\}^n$.

We use circuits $\{U(x_i)\}$ that are constructed to return a specific basis state $|x_i\rangle$, such that in the absence of any hardware errors, sampling would return the bit string $x_i$ with probability 1. When hardware errors occur, sampling the state $U(x_i)|0\rangle^\otimes n$ may sometimes yield counts in states other than the target state $x_i$, i.e., $p(x_i) \leq 1$ and $p(x_j) > 0$ for some $x_j \neq x_i$. The distribution of counts in non-target states is due to a combination of lower-order moments. The distribution of counts in non-target states is quantified by the $m$-th order moment that is not dependent on multiple sources of noise including initialization error, gate errors, and measurement errors.

A MFM is a matrix $K$ whose entries $K_{ij} = p(x_j|x_i)$ are the conditional probabilities that sampling state $U(x_i)|0\rangle^\otimes n$ yields $x_j$. The distribution of counts in non-target states is a sequence of one- and two-qubit gates that may sometimes yield counts in states other than the target state $x_i$, i.e., $p(x_i) \leq 1$ and $p(x_j) > 0$ for some $x_j \neq x_i$. The distribution of counts in non-target states is due to a combination of lower-order moments. The distribution of counts in non-target states is quantified by the $m$-th order moment that is not dependent on multiple sources of noise including initialization error, gate errors, and measurement errors.

A MFM is a matrix $K$ whose entries $K_{ij} = p(x_j|x_i)$ are the conditional probabilities that sampling state $U(x_i)|0\rangle^\otimes n$ yields $x_j$. The distribution of counts in non-target states is a sequence of one- and two-qubit gates that may sometimes yield counts in states other than the target state $x_i$, i.e., $p(x_i) \leq 1$ and $p(x_j) > 0$ for some $x_j \neq x_i$. The distribution of counts in non-target states is due to a combination of lower-order moments. The distribution of counts in non-target states is quantified by the $m$-th order moment that is not dependent on multiple sources of noise including initialization error, gate errors, and measurement errors.

A MFM is a matrix $K$ whose entries $K_{ij} = p(x_j|x_i)$ are the conditional probabilities that sampling state $U(x_i)|0\rangle^\otimes n$ yields $x_j$. The distribution of counts in non-target states is a sequence of one- and two-qubit gates that may sometimes yield counts in states other than the target state $x_i$, i.e., $p(x_i) \leq 1$ and $p(x_j) > 0$ for some $x_j \neq x_i$. The distribution of counts in non-target states is due to a combination of lower-order moments. The distribution of counts in non-target states is quantified by the $m$-th order moment that is not dependent on multiple sources of noise including initialization error, gate errors, and measurement errors.

Many methods for constructing MFM are being developed. The most direct method is to construct each row $i$ of the matrix by repeatedly initializing an $n$-qubit register in the state $|0\rangle^\otimes n$, applying $X$ gates to individual qubits to nominally prepare the state $|x_i\rangle$ and measuring all $n$ qubits in the computational basis. This approach is not scalable: if there are $2^n$ possible target states, $2^n$ circuits must be run to estimate the full MFM.

In the remainder of this section we describe several methods to approximate many-qubit MFM by combining MFM of much smaller subsystems. We use $K$ to denote the full MFM of an $n$ qubit system, $K_Q$ to denote the MFM of qubit or subsystem $Q$, and $K$ to denote an approximation to $K$ obtained by combining subsystem MFMs.

B. Cumulant expansion for MFM

A cumulant expansion [19] relates the moments of a function to a set of generating coefficients called cumulants. If the moments of $n$ random variables are encoded as the coefficients of a multivariate polynomial $M(t_1, \ldots, t_n)$, the cumulants are the coefficients of the generating function $\lambda(t_1, \ldots, t_n)$ defined by

$$M(t_1, \ldots, t_n) = e^{\lambda(t_1, \ldots, t_n)}.$$  \hspace{1cm} (1)

Each $m$-th order moment is a function of cumulants up to order $m$. The advantage of the cumulant expansion is that only a few cumulants are needed when correlations among all the variables are due primarily to the combined effect of few-variable correlations. Intuitively, an $m$-th order cumulant quantifies the component of an $m$-th order moment that is not due to a combination of lower-order moments.

The cumulant expansion can be applied to MFM construction. An $n$-qubit MFM with elements $K_{ij} = p(x_j|x_i)$ can be viewed as a function of $2n$ variables. The low order (few-qubit) moments of $K$ can be used to calculate low order cumulants, which then generate a full conditional probability matrix $\tilde{K}$. If $K$ is dominated by few-qubit correlations, the generated matrix $\tilde{K}$ will be close to $K$. In this case $K$ may be efficiently estimated by a polynomial number of few-qubit circuits and measurements. Furthermore, extrapolated MFM may be compared to explicitly measured MFM to characterize correlated errors in NISQ devices. While such characterization would be unscalable for arbitrarily high order correlations, modern devices tend to be limited in the degree of correlated noise between qubits by the connectivity, distance from one another, and the readout resonator configuration (in the case of superconducting devices). Therefore our method can be used to characterize correlated noise within small collections of qubits and within subregions of a chip.

For a circuit $U(x_i) = u_0 \otimes u_1 \otimes \ldots \otimes u_{n-1}$, sampling from the final prepared state returns $\{u_0u_1 \ldots u_{n-1}\}$. In the absence of any correlated noise or error in the hardware then we could decompose the final distribution as $\langle U_0U_1 \ldots U_{n-1} \rangle = \langle U_0 \rangle \langle U_1 \rangle \ldots \langle U_{n-1} \rangle$ and these matrices could be constructed from the $n$ single qubit conditional probabilities.

We use $\lambda_Q$ to denote a cumulant coefficient involving a subset $Q$ of qubits. The single-qubit cumulants and conditional probabilities for a qubit $a$ are the same:

$$\lambda_0(0|0) = p_a(0|0) \quad \lambda_0(1|0) = p_a(1|0)$$
$$\lambda_0(1|1) = p_a(1|1).$$  \hspace{1cm} (2)

Whereas the two-qubit cumulants are computed from the 1- and 2-qubit conditional probabilities:

$$\lambda_{ab}(x_j^{(a)}x_j^{(b)}|x_i^{(a)}x_i^{(b)}) = p_a(x_j^{(a)}x_j^{(b)})p_b(x_j^{(a)}x_j^{(b)}) - p_a(x_j^{(a)})p_b(x_j^{(b)}).$$  \hspace{1cm} (3)

C. Cumulant construction of MFM

We assume that 1- and 2-qubit terms are the dominant terms when constructing an $n$-qubit MFM $\tilde{K}_{i\ldots j\ldots k}$, thus higher order terms in the cumulant expansion are constructed using the terms in Eqs. 2 and 3. We show the third-order cumulant term as an example. Starting from the 3-qubit conditional probabilities:

$$\lambda_{abc}(x_j^{(a)}x_j^{(b)}x_j^{(c)}|x_i^{(a)}x_i^{(b)}x_i^{(c)}) = p_{abc}(x_j^{(a)}x_j^{(b)}x_j^{(c)})|x_i^{(a)}x_i^{(b)}x_i^{(c)}) = p_{abc}(x_j^{(a)}x_j^{(b)}x_j^{(c)})$$
$$- \left[p_a(x_j^{(a)})p_b(x_j^{(b)})p_c(x_j^{(c)})x_j^{(a)}x_j^{(b)}x_j^{(c)}\right]$$
$$+ p_a(x_j^{(a)})p_b(x_j^{(b)}x_j^{(c)})p_c(x_j^{(a)}x_j^{(b)}x_j^{(c)})$$
$$+ p_b(x_j^{(b)}x_j^{(a)})p_c(x_j^{(c)})x_j^{(a)}x_j^{(b)}x_j^{(c)}$$
$$+ 2p_a(x_j^{(a)})p_b(x_j^{(b)}x_j^{(c)})p_c(x_j^{(a)}x_j^{(b)}x_j^{(c)}),$$  \hspace{1cm} (4)

then setting the left-hand side of Eq. 4 to zero we can compute element-by-element the conditional probabilities $\tilde{p}$ of a composite MFM $\tilde{K}_{abc}$ in terms of the 1- and 2-qubit cumulants in Eqs 2 and 3.
D. Extracting MFMs

The 1- and 2-qubit MFMs measured on hardware are the basic elements used in Eq. 3. However this approach can fail to capture higher-order correlated noise (see Section V). Throughout this work we utilize two additional methods for generating noisy estimates of 2-qubit MFMs. The first method exploits classical statistics, the second method uses additional hardware qubits.

The MFM is a matrix of classical conditional probabilities \( p(x_j|x_i) \) that can be marginalized in order to estimate the conditional probabilities on a \( m \)-qubit sub-system. If the full MFM measured over \( n \)-qubits is known, we can extract an estimated MFM on \( m < n \) qubits by marginalizing over qubit input and output values, for example:

\[
p_{ac}(x_j|x_i) \propto \sum_{x_i^{(b)},x_j^{(b)}} p_{abc}(x_j^{(a)}x_j^{(b)}x_j^{(c)}|x_i^{(a)}x_i^{(b)}x_i^{(c)}).
\]

This method is used in Section V to demonstrate the cumulant method for constructing the full MFM using estimated sub-system MFMs.

A second approach to estimating subsystem MFMs is an adaptation of the spectator qubit method introduced in [20], [21]. The \((\binom{n}{2})\) 2-qubit terms for a \( n \)-qubit layout are measured with Hadamard gates executed on the remaining \((n-2)\) qubits. Then from the full measured distribution over \( 2^n \) states we extract 2-qubit MFMs. The addition of spectator qubits does not increase the number of circuits needed to evaluate each 2-qubit MFM; the cost for measuring all 2-qubit terms is \((4\binom{n}{2})\).

E. MFM cluster products

An alternative construction of a full \( n \)-qubit MFM uses \( m \)-qubit MFMs of clusters (non-overlapping qubit subsets of arbitrary size). This approximation may be motivated by physical principles, e.g. based on physical distance or connectedness of qubits.

Consider a full MFM that is decomposed into two clusters \( A \) and \( B \). The definition of the 2-body cumulant, Eq. 3, remains valid if \( a \) and \( b \) refer to clusters and \( K_A \) and \( K_B \) are the independently measured MFMs for clusters \( A \) and \( B \), respectively. The lowest order approximation to the compound MFM would be \( \tilde{K}_{AB} = K_A \otimes K_B \), which captures correlations of all orders within each cluster, but ignores correlations between clusters. In this case, the two-body cumulant \( \lambda_{AB} \) (i.e., the difference between \( K_{AB} \) and \( K_A \otimes K_B \)) captures correlations between clusters \( A \) and \( B \). Its norm (see eq. 7) is then a metric for the degree of correlation between \( A \) and \( B \).

In Section IV we investigate \( n \)-qubit MFMs constructed from clusters of size \( \approx n/2 \).

F. Metrics

We quantify the accuracy of a full MFM reconstruction using 3 measures which are computed between matrix elements \( \| (K - I) \|_F, \| (\tilde{K} - K) \|_F \) or between the diagonal matrix elements \( \Delta(f) \).

The MFM reduces to the identity matrix in the absence of errors. The general amount of hardware error in a MFM has been quantified previously by \( \| (K - I) \|_F \), where \( \| : \|_F \) denotes the Frobenius matrix norm [22]. To assist in comparison between MFMs of different size, we note that \( \| (K - I) \|_F \) is related to the RMSE of \( (K - I) \) by a factor of \( 1/\sqrt{2^n} \). We also define \( \| (\tilde{K} - K) \|_F \) to quantify the closeness of \( \tilde{K} \) and \( K \). For some purposes the type of error may not be of interest and only the fidelity \( f \) of each target state \( x_i \) is of interest. (Our fidelity is equivalent to the probability of successful trial (PST) introduced in [23].) For this we use \( \Delta(f) \), the RMSE of the fidelity differences,

\[
\Delta f = \frac{1}{2^n} \sum_{i=1}^{2^n} (f_i^\tilde{K} - f_i^K)^2.
\]

A similar metric has been used in [24].

The scalar correlation factor (SCF) is a separate metric which quantifies the degree of correlation between two qubits \( (a,b) \) or clusters \( (A,B) \) using the Frobenius norm of the two-body cumulant matrix introduced in Eq. 3:

\[
\Lambda_{AB} \equiv \| \lambda_{AB} \|_F = \sqrt{\sum_{i,j} |\lambda_{AB}(i,j)|^2}
\]

where the sum is over distinct input and output states of composite system \( AB \). \( \| \lambda_{AB} \|_F \) may be understood as (proportional to) the root-mean-square error of conditional probabilities when \( K_{AB} \) is approximated by \( K_A \otimes K_B \), Theorem 1 of [19] and its corollary imply that \( \Lambda_{00} = 0 \) if and only if \( A \) and \( B \) are statistically independent. We will use \( \Lambda_{AB} = 0 \) to quantify correlations between qubit clusters in Section IV and individual qubits in Section V. In Section V we also define an upper bound on the SCF value.

G. Uncertainty Analysis

The main source of uncertainty is the randomness of quantum measurement outcomes. Each row of a MFM is generated by preparing a specific quantum state - the true output distribution \( p \) is obtained by preparing and sampling the state many times and estimating \( p(x_j|x_i) \) as the fraction of samples that were \( x_j \) when the state \( x_i \) was prepared. If \( n_s \) samples are taken, the variance of \( p(x_j|x_i) \) is

\[
\sigma(p(x_j|x_i)) = \frac{1}{n_s}p(x_j|x_i)(1 - p(x_j|x_i)).
\]

Uncertainty in the elements of \( K \) translates into uncertainty in the elements of \( \lambda_{ab} \). To distinguish correlations in the conditional probabilities constructed via the two-body cumulant (3) from statistical noise we use standard propagation of uncertainty to define a lower bound on the terms in the SCF.

For instance, if the MFMs \( K_a, K_b, \) and \( K_{ab} \) for a pair of qubits \( a,b \) are estimated from independent measurements, it can be shown that the uncertainty in \( \lambda_{ab}(j|i) \) is (to lowest order in statistical fluctuations) upper bounded by

\[
\sigma(\lambda_{ab}(j|i)) \leq \left( \frac{p_{ab}(x_j|x_i) + p_{ab}(x_j|x_i) + p_b(x_j|x_i)}{n_s} \right)^{1/2}.
\]

432
We consider the correlation between a pair of qubits \((a,b)\) to be statistically significant if the entries of the SCF satisfy \(\Lambda_{ab} > \sigma(\Lambda_{ab})\).

If \(K_a\) and \(K_b\) are extracted from a larger MFM (using either method introduced in Section II-D) instead of being measured in separate experiments then the cumulant uncertainties in this case are approximately the same as in the previous case and satisfy the bounds above. However, in this case each cumulant \(\lambda_{ab}(j|i)\) must be scaled by \((1 - 1/n_s)^{-1}\) to account for statistical correlations between the joint and marginal distributions and obtain an unbiased estimate.

As the number of qubits grows, it is intuitive that a higher shot size will be needed to ensure that the final state is sufficiently sampled. In order to assess the relative error introduced in reconstructed and directly measured MFM for a given shot size, we sampled single-qubit MFM and \(n\)-qubit MFM from a simulated quasi-ideal chip consisting of independent, identical qubits with the same \(p(x_j|x_i)\). This allows for the comparison of \(K\) and \(K_t\) to the ideal MFM, \(K_t\) obtained from \(p(x_j|x_i)\). For collections of up to 20 qubits, with \(.9 < p(0|0) = p(1|1) < .98\), and fixed shot size \(n_s = 8192\), we performed this sampling until a stable distribution of \(\| K - K_t \|_F\) and \(\| K - K_t \|_F\) are obtained and then used to gauge the relative uncertainty.

Our analysis indicated that it takes fewer shots, as well as many fewer circuits, to accurately reconstruct a large MFM from several small MFM over directly measuring the large MFM. While this analysis was limited to reconstructions built from single-qubit kernels, we expect that a similar analysis of more complicated reconstructions would yield similar findings.

**H. Computational Cost**

We define computational cost as the number of circuits that need to be executed on the hardware, assuming a fixed number of samples \(n_s\) for each circuit. Each single qubit MFM requires 2 circuits to be executed, each 2-qubit MFM requires 4 circuits, and the direct measurement of an \(n\)-qubit MFM requires \(2^n\) circuits. Fig. 1 shows how the computational cost required to construct an \(n\)-qubit MFM scales for different construction methods. For each method the number of circuits required are: \(2^n\) for the measurement of the full MFM; \(4\binom{n}{2}\) for construction using cumulants measured up to order 2; and \(8\binom{n}{3}\) for construction using cumulants measured up to order 3. We also include the scaling of the method which constructs an MFM from two similar-sized clusters of size \(k\) and \(n - k\), with cost \(2^k + 2^{n-k}\).

Even though adding higher order terms into the cumulant expansion incurs additional cost, the number of qubits in the MFM exceeds 8 qubits both cumulant expansion methods and the two-cluster method have a lower computational overhead than the exact MFM construction (see Fig. 1). However, for MFM on \(n < 5\) qubits the exact construction is less expensive than the cumulant methods.

To measure a full 5-qubit MFM the circuit cost is \(2^5 = 32\). Measuring all single qubit MFM \((q_a)\) requires 10 circuits, while a construction based on measurement of all 2-qubit subsystems has a cost of 40 circuits. The cost of a MFM constructed from a 2-qubit MFM and 3-qubit MFM is 12, while the cost of a MFM constructed from a 1-qubit MFM and 4-qubit MFM is 18.

1. **Hardware layouts**

The data and results presented in this paper cover the construction of MFM on up to 8 qubits using shallow circuits. Results on 5-qubit MFM were constructed in Qiskit. Results on 6 and 8-qubit MFM were measured using an XACC implementation [25] (for details see Appendix A). MFM are defined with respect to the computational basis (Z basis) using circuits constructed with only the native X gate implemented via the fixed rotation gate \(\text{u3}(\pi, 0, \pi)\). In Section III we report data measured with spectator qubits (see Section II-D) in which Hadamard gates are executed on the unmeasured qubits (“spectator qubits”) to randomize any potential influence they might have in the extracted MFM.

The MFM results presented in Secs. III and IV were generated on multiple superconducting qubit devices (QPUs) available from IBM. Most devices were accessed via cloud-based priority-queues, except for ibmq_valencia which was accessed via a dedicated queue. In this paper each QPU is represented by a graph that schematically illustrates the physical layout of the qubits and controllable couplings between them. Using the 5-qubit devices ibmq_valencia, and ibmq_5_yorktown we measured 5-qubit MFM that cover the entire QPU (see Fig. 2). On the 28 qubit device ibmq_cambridge we measured MFM for selected 5-qubit subsets (see Fig. 3). On the 20-qubit devices ibmq_boeblingen and ibmq_johannesburg we measured 6- and 8-qubit MFM.
III. CUMULANT SERIES CONSTRUCTION OF MFM

In this section we report reconstructions of 5 and 6 qubit MFM s using measurements on 1 and 2 qubits. We report the accuracy of our reconstructions using $\| (K - I) \|_F$ and $\Delta(f)$ values.

Our results were obtained using circuits executed on the 5-qubit devices ibmq_valencia and ibmq_5_yorktown, the 24-qubit device ibmq_cambridge and the 20-qubit device ibmq_johannesburg. On ibmq_cambridge we used three separate qubit subsets, shown in Fig. 3. On ibmq_johannesburg we used an 8-qubit layout ($J_8$) shown in Fig. 7.

For each target $n$-qubit MFM we measured all 1-qubit MFMs and all $\binom{n}{2}$ 2-qubit MFMs. We also measured the full 5-qubit MFM of each device for comparison. An example of a full measured MFM, measured on ibmq_valencia is shown in Fig. 4.

As an initial test we constructed approximate MFMs using only vendor-provided single-qubit calibration data. For each qubit the conditional probabilities $p(1|0)$ (the probability of measuring a qubit in state $|1⟩$ when prepared in state $|0⟩$) and $p(0|1)$ (the probability of measuring a qubit in state $|0⟩$ when prepared in state $|1⟩$) can be retrieved using the properties methods associated with the backends 1. The 1-qubit MFM constructed from these two values is

$$K_{VC}^{VC} = \left( \begin{array}{cc} 1 - p(1|0) & p(1|0) \\ p(0|1) & 1 - p(0|1) \end{array} \right).$$  \hspace{1cm} (11)

(We use the superscript $VC$ to denote an MFM constructed from vendor calibration data, as opposed to one directly measured on hardware.) In our experiments, calibration information was collected and stored at the start of each set of measurements of the full 5-qubit MFM. The 5-qubit MFM was then approximated as $K_0^{VC} \otimes \cdots \otimes K_4^{VC}$. Albeit simplistic, constructing an MFM from calibration data is effectively free to the end user, since the necessary information can be obtained without any circuit executions.

\footnote{Access provided by IBM Quantum through the IBM Quantum Provider.}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Device & $\| (K - I) \|_F$ & $\| (K - K) \|_F$ & $\Delta(f)$ \\
\hline
\hline
ibmq_valencia & 1.13 & 0.0 & 0.0 \\
\hline
$K_{Q=0,1,2,3,4}^{VC} \otimes \cdots \otimes K_4^{VC}$ & 0.658 & 0.560 & 0.084 \\
$K_0 \otimes \cdots \otimes K_4$ & 0.980 & 0.285 & 0.036 \\
$K_{(Q=0,1,2,3,4)}^{Q=0,1,2,3,4}$ & 0.997 & 0.279 & 0.036 \\
\hline
ibmq_5_yorktown & 1.296 & 0.0 & 0.0 \\
$K_{Q=0,1,2,3,4}^{VC} \otimes \cdots \otimes K_4^{VC}$ & 0.634 & 0.897 & 0.122 \\
$K_0 \otimes \cdots \otimes K_4$ & 0.550 & 0.969 & 0.136 \\
$K_{(Q=0,1,2,3,4)}^{Q=0,1,2,3,4}$ & 0.615 & 0.926 & 0.126 \\
\hline
\end{tabular}
\caption{Accuracy of 5-qubit MFMs constructed from 1- and 2-qubit MFMs directly measured without spectator qubits.}
\end{table}

Next, we constructed MFMs using the second-order cumulant expansion (Section II-B) and all 1- and 2-qubit MFMs. The kernel constructed in this way is denoted $K_{Q=0,1,2,3,4}^{Q=0,1,2,3,4}$, where $Q = \{0, 1, 2, 3, 4\}$ denotes the full set of qubits. The $\| (K - I) \|_F$, $\| (K - K) \|_F$ and $\Delta(f)$ values for each type of approximate MFM generated for each device are presented in Tables I and II.

Finally, we combined the cumulant expansion method with the addition of spectator qubits in the circuits used to generate the 1 and 2-qubit subsystem MFMs (as described in Section II-D). These results are reported in Table III.
### TABLE II
ACCURACY OF 5 AND 8-QUBIT MFMS CONSTRUCTED FROM 1- AND 2-QUBIT MFMS

| Construction | \(\| (K-1) \|_F \) | \(\| (K-K) \|_F \) | \(\Delta(f) \) |
|--------------|----------------------|----------------------|-------------|
| \(K \equiv K_{ibmq_{_cambridge}, region T_1 = \{8,9,10,5,0\}}\) | \(3.616\) | \(0.0\) | \(0.0\) |
| \(\bigotimes_{q \in T_1} K_q^{VC}\) | \(2.650\) | \(1.367\) | \(0.201\) |
| \(K + K_{\{0,1\} + \{2\}}\) | \(2.817\) | \(1.219\) | \(0.138\) |

| Construction | \(\| (K-1) \|_F \) | \(\| (K-K) \|_F \) | \(\Delta(f) \) |
|--------------|----------------------|----------------------|-------------|
| \(K \equiv K_{ibmq_{_cambridge}, region T_2 = \{12,13,14,6,4\}}\) | \(1.781\) | \(0.0\) | \(0.0\) |
| \(\bigotimes_{q \in T_2} K_q^{VC}\) | \(1.371\) | \(0.564\) | \(0.072\) |
| \(K + K_{\{0,1\} + \{2\}}\) | \(1.514\) | \(0.409\) | \(0.051\) |

| Construction | \(\| (K-1) \|_F \) | \(\| (K-K) \|_F \) | \(\Delta(f) \) |
|--------------|----------------------|----------------------|-------------|
| \(K \equiv K_{ibmq_{_joannesburg}, qubits J_8 = \{6,5,10,11,8,9,13,14\}}\) | \(2.247\) | \(0.0\) | \(0.0\) |
| \(\bigotimes_{q \in J_8} K_q^{VC}\) | \(2.129\) | \(0.137\) | \(0.019\) |
| \(K + K_{\{0,1\} + \{2\}}\) | \(2.263\) | \(0.170\) | \(0.017\) |

### IV. CLUSTER PRODUCT CONSTRUCTION OF MFM

In this section we reconstruct 5, 6 and 8 qubit MFMs using the cluster product method described in Sec. II-E. We report the accuracy of our reconstructions using the \(\| (K-1) \|_F \) metric and the SCF between clusters (see Eq. 7).

Figure 5 shows examples of (3,2) qubit clusters for each of the 5 qubit layouts: tree, bowtie and a simple chain. We construct our cluster sets such that they respect the bit ordering of the 5-qubit MFM. The metrics for the MFMs constructed from these clusters are plotted in Fig. 6.

![Examples of (3,2) clusters: (a) bowtie (b) tree (c) simple chain.](image)

Fig. 5. Examples of (3,2) clusters: (a) bowtie (b) tree (c) simple chain.

On ibmq_boeblingen and ibmq_johannesburg we reconstructed 6 and 8 qubit MFMs using clusters of 3 and 4 qubits. When choosing qubit clusters on the hardware we

### TABLE III
ACCURACY OF 5-QUBIT AND 8-QUBIT MFMS CONSTRUCTED FROM 1- AND 2-QUBIT MFMS MEASURED WITH SPECTATOR QUBITS.

| Construction | \(\| (K-1) \|_F \) | \(\| (K-K) \|_F \) | \(\Delta(f) \) |
|--------------|----------------------|----------------------|-------------|
| \(K \equiv K_{ibmq_{_valencia}}\) | \(1.65\) | \(0.0\) | \(0.0\) |
| \(\bigotimes_{q \in C_0} K_q^{VC}\) | \(0.674\) | \(1.105\) | \(0.168\) |
| \(K + K_{\{0,1\} + \{2\}}\) | \(1.512\) | \(0.663\) | \(0.071\) |

| Construction | \(\| (K-1) \|_F \) | \(\| (K-K) \|_F \) | \(\Delta(f) \) |
|--------------|----------------------|----------------------|-------------|
| \(K \equiv K_{ibmq_{_5_yorktown}}\) | \(2.06\) | \(0.0\) | \(0.0\) |
| \(\bigotimes_{q \in C_0} K_q^{VC}\) | \(2.30\) | \(0.322\) | \(0.045\) |
| \(K + K_{\{0,1\} + \{2\}}\) | \(1.96\) | \(0.165\) | \(0.019\) |

| Construction | \(\| (K-1) \|_F \) | \(\| (K-K) \|_F \) | \(\Delta(f) \) |
|--------------|----------------------|----------------------|-------------|
| \(K \equiv K_{ibmq_{_cambridge (T_1)}}\) | \(2.879\) | \(0.0\) | \(0.0\) |
| \(\bigotimes_{q \in T_1} K_q^{VC}\) | \(2.426\) | \(0.958\) | \(0.118\) |
| \(K + K_{\{0,1\} + \{2\}}\) | \(3.553\) | \(0.874\) | \(0.116\) |

| Construction | \(\| (K-1) \|_F \) | \(\| (K-K) \|_F \) | \(\Delta(f) \) |
|--------------|----------------------|----------------------|-------------|
| \(K \equiv K_{ibmq_{_joannesburg, qubits J_8 = \{6,5,10,11,8,9,13,14\}}\) | \(3.889\) | \(0.0\) | \(0.0\) |
| \(\bigotimes_{q \in J_8} K_q^{VC}\) | \(4.639\) | \(1.382\) | \(0.056\) |
| \(K + K_{\{0,1\} + \{2\}}\) | \(4.697\) | \(1.0838\) | \(0.003\) |

| Construction | \(\| (K-1) \|_F \) | \(\| (K-K) \|_F \) | \(\Delta(f) \) |
|--------------|----------------------|----------------------|-------------|
| \(K + K_{\{0,1\} + \{2\}}\) | \(3.889\) | \(4.147\) | \(0.003\) |
fixed the layout to be the same (see Fig. 7). While all 3-qubit clusters are chains, the 4-qubit clusters can be trees or chains.

The data collected on ibmq_johannesburg is plotted in Figs. 8 and 9 and the metrics evaluated using data collected on ibmq_boeblingen is plotted in Figs. 10 and 11.

We re-introduce the SCF described in Section II to quantify correlations between regions on a QPU using a \( n \) qubit layout decomposed into 2 disjoint clusters \( A, B \) each consisting of approximately \( n/2 \) qubits. Consider a layout of \( n \) qubits that is decomposed into two disjoint clusters \( A, B \), each consisting of approximately \( n/2 \) qubits. Eq. 3 may be applied to the two clusters, yielding the cluster cumulant

\[
\lambda_{AB}(j|i) = p_{AB}(x_{j}^{(A)} x_{j}^{(B)} | x_{i}^{(A)} x_{i}^{(B)}) - p_{A}(x_{j}^{(A)} | x_{i}^{(A)}) p_{B}(x_{j}^{(B)} | x_{i}^{(B)})
\]

and the corresponding SCF \( \Lambda_{AB} = \|\lambda_{AB}\|_{F} \). In the figure captions we also report an upper bound to the SCF value, but further explanation is deferred to Section V.

In Figs. 8,9,10,11 we plot the SCF value \( \Lambda_{AB} \) on the second y-axis. The full MFM is measured with a specific ordering of hardware qubits, which must be respected in the ordering of the clusters. In order to preserve the proper qubit ordering the rows and columns of the full MFM may have to be transposed. For example, using the layout \( \{0,1,2,15,16,17\} \) from Fig 8, \( \Lambda_{AB} = \|\hat{K}_{\{0,1,2,15,16,17\}} - K_{\{0,1,2\}} \otimes K_{\{15,16,17\}}\|_{F} \). However to evaluate \( \Lambda_{AB} \) the rows and columns of \( \hat{K}_{\{0,1,2,15,16,17\}} \) must be transposed to yield \( K_{\{15,16,17,0,1,2\}} \).

V. DISCUSSION

In Sections III and IV we constructed \( n \)-qubit MFMs using tensor products of single-qubit MFMs, a cumulant-based method, and also with cluster MFMs. The accuracy of the constructed MFMs was quantified by comparing the hardware noise measured by \( \|\hat{K} - I\|_{F} \). There is an upper bound to \( \|\hat{K} - I\|_{F} \), introduced previously in [22]: if each row of \( \hat{K} \) is a uniform distribution over all \( 2^{n} \) states, then the hardware noise according to \( \|\hat{K} - I\|_{F} \) is: \( \|\hat{K} - I\|_{F} = \sqrt{(2^{n} - 1)} \). The maximum value of the SCF can be derived in a similar manner. The largest value of the SCF will be measured for a system which has ideal single qubit cumulants (see Eq. 3).
This can be seen for example, in Fig. 11 where the value of $|\langle \bar{K} - \mathbb{I} \rangle|_F$ resulted in a MFM with a much lower value of $|\langle \bar{K} - \mathbb{I} \rangle|_F$. Likewise adding spectator qubits in the 1 and 2-qubit MFM measurements did not lead to substantive improvements in the $|\langle \bar{K} - \mathbb{I} \rangle|_F$ value for reconstructions using $K^{(2)}_{(q)}$ for ibmq_valencia, ibmq_5_yorktown, and ibmq_cambridge ($C_0, T_2$). We note that the addition of spectator qubits can also lead to a reconstructed MFM with much higher values of $|\langle \bar{K} - \mathbb{I} \rangle|_F$, as in the case of ibmq_johannesburg ($J_q$). The effects of spectator qubits will be discussed further in Sec. V-B.

On the other hand, we found that incorporating MFMs of larger clusters could yield significantly more accurate MFM reconstructions. For example, on ibmq_valencia, ibmq_5_yorktown, and ibmq_cambridge ($C_0$), cluster-based MFMs achieved $|\Delta K_{\bar{K}}|_F < 0.1$, whereas the cumulant-based MFMs had $0.2 < |\Delta K_{\bar{K}}|_F < 1.0$ (cf. Tables I, II, and III). We note that the key to obtaining such improvement was placing highly correlated subsets of qubits (e.g. qubits 1,3,4 in ibmq_valencia) into the same cluster.

2) $(\lambda_0(0|0) = \lambda_1(1|1) = 1.0)$ but the 2-qubit conditional probabilities $(p_{ab}(x^a_j, x^b_j | x^a_i, x^b_i)$ are uniformly distributed over the 4 output states.

Both $|\langle \bar{K} - \mathbb{I} \rangle|_F$ and the SCF are bounded over the same range of values $[0, \sqrt{2^n - 1}]$, but the interpretation of the upper bound is different for each metric. While the $|\langle \bar{K} - \mathbb{I} \rangle|_F$ bound can be saturated regardless of correlations, saturating the SCF metric bound denotes maximal two qubit correlations. This can be seen for example, in Fig. 11 where the value of $|\langle \bar{K} - \mathbb{I} \rangle|_F$ indicates relatively high overall hardware noise but the SCF remains low, indicating relatively low correlations between the chosen qubit subsets. Thus one can use $|\langle \bar{K} - \mathbb{I} \rangle|_F$ to quantify noise in a collection of qubits, while the SCF can be used to uncover correlations on that collection or subsets.

A. Direct measurement of sub-system MFMs

Overall, the accuracy of the MFM construction method is not dependent on the noise in the full MFM. Rather the accuracy of the reconstruction method is dependent on the degree of correlations that are captured in the sub-system MFMs. If a sub-system MFM of $m$ qubits is measured without the addition of spectator qubits, then the highest degree of correlations contained in that MFM is order $m$. Reconstructed MFMs using single qubit measurements were the least accurate- $K^{(2)}_m$ or $K_m$ underestimated the general noise of the full MFM (see Tables I and II).

Incorporating 2-qubit MFMs in the construction of larger MFMs did not lead to conclusive improvement in accuracy- for ibmq_cambridge ($C_0, T_1$) and ibmq_johannesburg ($J_q$) the construction using $K^{(2)}_{(q)}$ resulted in a MFM with a much lower value of $|\langle \bar{K} - \mathbb{I} \rangle|_F$. Likewise adding spectator qubits in the 1 and 2-qubit MFM measurements did not lead to substantive improvements in the $|\langle \bar{K} - \mathbb{I} \rangle|_F$ value for reconstructions using $K^{(2)}_{(q)}$ for ibmq_valencia, ibmq_5_yorktown, and ibmq_cambridge ($C_0, T_2$). We note that the addition of spectator qubits can also lead to a reconstructed MFM with much higher values of $|\langle \bar{K} - \mathbb{I} \rangle|_F$, as in the case of ibmq_johannesburg ($J_q$). The effects of spectator qubits will be discussed further in Sec. V-B.

B. SCF-based identification of qubit correlations

In Section IV we reported results on $\Lambda_{ab}$ as a measure of correlation between qubit subsets. In this section we calculate the SCF between individual pairs of qubits in a $n$-qubit layout. In the absence of correlations all entries of $\Lambda_{ab}$ would be zero, but statistical noise can result in non-zero matrix values. Using the uncertainty propagation from Sec. II-G we state that two regions are correlated if the SCF satisfies: $\Lambda_{ab} > \sigma(\Lambda_{ab})$. However, as mentioned in the above section, 1- and 2-qubit MFMs directly measured can only capture 1- and 2-qubit correlations. Combining the $\Lambda_{ab}$ matrix with spectator and extracted kernel methods outlined in II-D allows us to identify correlated regions and qubits on a chip.

We use MFMs measured on ibmq_valencia as an example and compute $\Lambda_{ab}$ for all qubit pairs $a,b$. As in Section IV: we state that two qubits are correlated if the SCF satisfies: $\Lambda_{ab} > \sigma(\Lambda_{ab})$. In Fig. 12 we plot the statistically significant degree of correlation $|\Lambda_{ab} - \sigma(\Lambda_{ab})|$. Directly measuring the 1 and 2 qubit MFM terms used in Eq. 3 shows weak correlations between qubit 0 and 4, and qubits 2 and 4. However from the results reported in Table I the 5-qubit MFM constructed with only 1 and 2 qubit MFMs did not agree with the directly measured 5-qubit MFM. If the 1 and 2-qubit MFMs are extracted from measurements made with spectator qubits then we see that the $|\Lambda_2 - \sigma(\Lambda_2)|$ matrix shows a strong correlation between qubits $\{1,2,3,4\}$. In Fig. 6 the (1,4) cluster MFM is formed by measuring a
The inclusion of spectator qubits in the measurement of each 4-qubit cluster improves the final state fidelities, compared to measurements of each 4-qubit cluster without spectators. On the other hand, the inclusion of spectators in the measurements of 1 and 2-qubit MFMs used in the cumulant reconstruction showed no clear improvement compared to measurements without spectators.

VI. CONCLUSIONS

In this work we have introduced a matrix-based method for characterization of qubit correlations. This can be utilized as a high-level diagnostic to identify qubit correlations a quantum device: both the location (between which hardware qubits) and the strength (magnitude). We have demonstrated our method on several superconducting qubit platforms. While there are multiple sources of qubit correlations, at the level of our approach we can distinguish between: statistical noise, measurement noise, and gate-level noise. On the devices studied, MFMs based on independent error models can be improved by measuring and incorporating correlations in small clusters of three or more qubits.

The cumulant analysis introduced in this paper can be used to find highly correlated qubit subsets on a QPU to avoid using them in experiments to minimize correlated error and in practice, this data can be measured independent of constructing a specific MFM. In this work we provide the user a matrix-based method to distinguish between generic hardware noise and correlated noise. However, there are multiple sources of qubit correlations and noise and fully characterizing the exact source of qubit-qubit correlations remains work for a future study. Additionally, the demonstrations in this paper has focused on identifying qubit correlations using a finite set of available quantum gates (X, H), measured in the |0⟩,|1⟩ basis. Future work will focus on distinguishing X gate noise from readout noise and identifying correlations in other bases.

ACKNOWLEDGEMENTS

The authors would like to thank Vicente Leyton-Ortega for insightful discussions about noise characterization and error mitigation. The authors would like to thank IBM for providing information about the IBM Q system.

This work was supported as part of the ASCR Quantum Testbed Pathfinder Program at Oak Ridge National Laboratory under FWP #ERKJ332. This research used quantum computing system resources of the Oak Ridge Leadership Computing Facility, which is a DOE Office of Science User Facility supported under Contract DE-AC05-00OR22725. Oak Ridge National Laboratory manages access to the IBM Q System as part of the IBM Q Network.
CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

APPENDIX A

XACC IMPLEMENTATION

The eXtreme-scale ACCelerator programming framework (XACC) provides a system-level, quantum-classical software infrastructure that is extensible, modular, and hardware agnostic [25]. XACC enables the programmability of quantum kernels that may be executed in a uniform manner on backends provided by IBM, Rigetti, D-Wave, IonQ, and a number of numerical simulators. Moreover, there exists extensive support for typical hybrid variational workflows through a uniform Algorithm interface.

Our implementation remains general with regards to the circuit ansatz, the backend quantum computer targeted, the loss function, and gradient computation strategy. Moreover, XACC provides extensibility with regards to typical error mitigation strategies that apply general pre- and post-processing of quantum execution results. XACC provides this through standard object-oriented decoration of the provided backend [26]. For this work, we have extended this mechanism with support for automated error kernel generation and mitigation.

Fig. 15 demonstrates a simple four qubit example of how one might implement the MFM generation described in this work. Programmers begin by specifying the desired Accelerator backend, here we select ibmq_boeblingen, the IBM Boeblingen machine. We then allocate four qubits using the C-like quantum malloc, qalloc. This specifies we are running on a four qubit register and the qbits variable will later serve to hold the computation results as well as additional information about the computation.

After specifying the backend and the qubit buffer, we now decorate the backend to endow the execution with the specified error mitigation strategy. The command line arguments , 'genKernel' and 'layout' specify whether one would like to generate a new error kernel on this run or use one from a previous run, and layout determines the mapping from logical qubits to physical qubits. We then specify the circuit we wish to execute. The accelerator decorator will then invoke the accelerator and generate the error kernel on the specified backend from a subroutine defined in the accelerator decorator.

import xacc

# Get the QPU and allocate a single qubit
qpu = xacc.getAccelerator('ibm:ibmq_boeblingen')
qbits = xacc.qalloc(3)
layout = [2, 4, 6]

# Decorate the QPU with the assignment-error-kernel
# error mitigation strategy
qpu = xacc.getAcceleratorDecorator(
    {'assignment-error-kernel', qpu,
     {'genKernel':True,
      'layout':layout,
      'cumulant':True
      'order': 1}})

# Create a simple quantum program
xacc.qasm('''
.compiler xasm
.circuit identity
.parameters x
.qbit q
.measure(q[0]);
.measure(q[1]);
.measure(q[2]);
''')

# Get the circuit, specify physical qubits
f = xacc.getCompiled('identity')
f.defaultPlacement(qpu, {'qubit-map':layout})

#must reshape (row wise) for matrix repr
kernel = qbits.getInformation("error-kernel")
original_counts = qbits.getInformation("unmitigated-counts")
REFERENCES

[1] John Preskill. Quantum computing in the NISQ era and beyond. Quantum, 2:79, 2018.
[2] Frank Arute, Kunal Arya, Ryan Babbush, Dave Bacon, Joseph C Bardin, Rami Barends, Rupak Biswas, Sergio Boixo, Fernando Brandao, David A Buell, et al. Quantum supremacy through practical variational quantum simulation. Nature, 574(7779):505–510, 2019.
[3] Michael J. Brenner, Ashley Montanaro, and Dan J. Shepherd. Achieving quantum supremacy with sparse and noisy commuting quantum computations. Quantum, 1:8, April 2017. Publisher: Verein zur Förderung des Open Access Publizierens in den Quantenwissenschaften.
[4] Scott Aaronson and Lijie Chen. Complexity-Theoretic Foundations of Quantum Supervised Learning. arXiv:1612.05903 [quant-ph], December 2016. arXiv: 1612.05903.
[5] Kristel Michielsen, Madita Nocon, Dennis Willsch, Fengping Jin, Thomas Lippert, and Hans De Raedt. Benchmarking gate-based quantum computers. Computer Physics Communications, 220:44–55, 2017.
[6] K Wright, KM Beck, S Debnath, N Grzesiak, J-S Chen, NC Pisenti, M Chmielewski, C Collins, et al. Benchmarking an 11-qubit quantum computer. Nature Communications, 10(1):1–6, 2019.
[7] Kristan Temme, Sergey Bravyi, and Jay M Gambetta. Error mitigation for short-depth quantum circuits. Physical review letters, 119(18):180509, 2017.
[8] Ying Li and Simon C Benjamin. Efficient variational quantum simulator incorporating active error minimization. Physical Review X, 7(2):021050, 2017.
[9] Vojtech Havlíček, Antonio D Córdoles, Kristan Temme, Aram W Harrow, Abhinav Kandala, Jerry M Chow, and Jay M Gambetta. Supervised learning with quantum-enhanced feature spaces. Nature, 576(7747):209, 2019.
[10] Abhinav Kandala, Kristan Temme, Antonio D. Córdoles, Antonio Mezzacapo, Jerry M. Chow, and Jay M. Gambetta. Error mitigation extends the computational reach of a noisy quantum processor. Nature, 567:491–495, 2019.
[11] Suguru Endo, Simon C Benjamin, and Ying Li. Practical quantum error mitigation for near-future applications. Physical Review X, 8(3):031027, 2018.
[12] Gadi Aleksandrowicz, Thomas Alexander, Panagiotis Barkoutos, Luciano Bello, Yael Ben-Haim, David Bucher, Francisco Jose Cabrera-Hernández, Jorge Carballo-Franquis, Adrian Chen, Chun-Fu Chen, Jerry M. Chow, Antonio D. Córdoles-Gonzales, Abigail J. Cross, Andrew C. Cross, Juan Cruz-Benito, Chris Culver, Salvador De La Puente González, Enrique De La Torre, Delton Ding, Eugene Dumitrescu, Ivan Duran, Pieter Eendebak, Mark Everitt, Ismael Faro Sertage, Albert Frisch, Andreas Fuhrer, Jay Gambetta, Borja Gódog Gago, Juan Gomez-Mosquera, Donny Greenberg, Ikko Hamamura, Vojtech Havlicek, Joe Hellmers, Lukasz Herok, Hiroshi Horii, Shaoan Hu, Takashi Imamichi, Toshinari Itoko, Ali Javadi-Abhari, Naoki Kanazawa, Anton Karazeev, Kevin Kruslich, Peng Liu, Yang Luh, Yunho Maeng, Manoel Marques, Toshinari Itoko, Ali Javadi-Abhari, Naoki Kanazawa, Anton Karazeev, Kevin Kruslich, Peng Liu, Yang Luh, Yunho Maeng, Manoel Marques, Swamit S Tannu and Moinuddin K Qureshi. Mitigating measurement errors in multi-qubit quantum gates. Phys. Rev. X, 9:011046, 2019.
[13] Abhinav Kandala, Antonio Mezzacapo, Kristan Temme, Maika Takita, Markus Brink, Jerry M Chow, and Jay M Gambetta. Hardware-efficient variational quantum eigensolver for small molecules and quantum magnets. Nature, 549(7671):242, 2017.
[14] Kenneth Rudinger, Timothy Proctor, Dylan Langharst, Mohan Sarovar, Kevin Young, and Robin Blume-Kohout. Probing context-dependent errors in quantum processors. Phys. Rev. X, 9:021045, 2019.
[15] Ryogo Kubo. Generalized cumulant expansion method. Journal of the Physical Society of Japan, 17(7):1100, 1962.
[16] Mingyu Sun and Michael R. Geller. Efficient characterization of correlated ramp errors, 2018.
[17] Michael R. Geller and Mingyu Sun. Efficient correction of multiquit measurement errors, 2020.
[18] Kathleen E Hamilton and Raphael C Pooser. Error-mitigated data-driven circuit learning on noisy quantum hardware. arXiv preprint arXiv:1911.13289, 2019.
[19] Swanit S Tamu and Moinuddin K Qureshi. Mitigating measurement errors in quantum computers by exploiting state-dependent bias. In Proceedings of the 52nd Annual IEEE/ACM International Symposium on Microarchitecture, pages 279–290, 2019.
[20] Megan N. Lilly and Travis S. Humble. Modeling noisy quantum circuits using experimental characterization, 2020.
[21] Alexander J McCaskey, Dmitry I Lyakh, Eugene F Dumitrescu, Sarah S Powers, and Travis S Humble. XACC: a system-level software infrastructure for heterogeneous quantum-classical computing. Quantum Science and Technology, 5(2):024002, Feb 2020.
[22] Erich Gamma, Richard Helm, Ralph Johnson, and John Vlissides. Design Patterns: Elements of Reusable Object-Oriented Software. Addison-Wesley Longman Publishing Co., Inc., USA, 1995.