Production of exotic nuclei in quasifission-type reactions

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Abstract. The possibilities for producing neutron-rich isotopes \textsuperscript{82,84,86}Zn and \textsuperscript{86,88,90,92}Ge are demonstrated in the reactions \textsuperscript{48}Ca+\textsuperscript{238}U, \textsuperscript{244}Pu at incident energies near the Coulomb barrier. The production cross sections of new neutron-rich isotopes of nuclei with charge numbers \(Z=64–80\) are estimated as well. The dynamics of the binary reaction is considered as the diffusive multinucleon transfer between the interacting nuclei in the collisions when the excitation energy of the produced exotic isotope is lower than the threshold for the neutron emission. In the quasifission reactions \textsuperscript{48}Ca+\textsuperscript{244,246,248}Cm at beam energies close to the corresponding Coulomb barriers one can produce new isotopes of superheavies with \(Z = 103–108\), which mainly undergo fission.

1. Introduction

Besides the reactions at intermediate energies [1–7], the multinucleon transfer reactions at low energies are actively discussed to produce exotic nuclei [8–14]. These binary reactions have been known for producing exotic nuclei for many years [15–24]. In Refs. [12] the possibility have been shown to produce the neutron-rich nuclei close to drip-line in the transfer-type reactions \textsuperscript{48}Ca+\textsuperscript{232}Th, \textsuperscript{238}U, \textsuperscript{248}Cm at incident energies close to the Coulomb barrier. The \textsuperscript{238}U(5.5 MeV/nucleon)+\textsuperscript{48}Ca reaction has been used to produce the odd and even neutron-rich Ca isotopes and study their low-lying states [13]. In Refs. [9–11, 14] the neutron-rich nuclei with \(A=50–80\) have been studied through multinucleon transfer reactions by bombarding \textsuperscript{208}Pb and \textsuperscript{238}U targets with beams of \textsuperscript{48}Ca, \textsuperscript{58,64}Ni, \textsuperscript{70}Zn, and \textsuperscript{82}Se.

The multinucleon transfer reactions and quasifission-type reactions are expected to be efficient to synthesize exotic nuclei. We are going to demonstrate the possibilities for producing neutron-rich isotopes \textsuperscript{82,84,86}Zn and \textsuperscript{86,88,90,92}Ge in the reactions \textsuperscript{48}Ca+\textsuperscript{238}U, \textsuperscript{244}Pu at incident energies near the Coulomb barrier. In these reactions the contributions of fast nonequilibrium processes of the entrance channel are expected to be negligible. The dynamics of the binary reaction is considered as the diffusive multinucleon transfer between the interacting nuclei in the collisions when the excitation energy of the produced exotic isotope is lower than the threshold for the neutron emission.

As the first step, we deal with the production of neutron-rich Zn and Ge isotopes with neutron number \(N > 50\), which are the products of multinucleon transfer channel in the actinide-based reactions. Note that these reactions are intensively and successfully used to produce superheavy...
nuclei and products of multinucleon transfer process or quasifission which competes with the complete fusion process [25–28]. Within the same formalism, we will report for the future experiments the possibilities for producing new neutron-rich isotopes of nuclei with $Z=64–80$ as complementary to light fragments in the uranium-based multinucleon transfer reaction with a $^{48}$Ca beam at incident energy $E_{\text{c.m.}}=189$ MeV, the value of Coulomb barrier for the spherical nuclei. The possibilities for transfer-induced fission of new isotopes of heaviest nuclei with $Z=103–108$ will be discussed in the curium-based multinucleon transfer reactions with $^{48}$Ca beam at incident energies close to the corresponding Coulomb barriers.

As shown in Refs. [12], the diffusive multinucleon transfer-type reactions can be described as an evolution of the dinuclear system (DNS) which is formed in the entrance channel of the reaction after dissipation of the kinetic energy and angular momentum of the relative motion [15, 16, 29–37]. The dynamics of the process is considered as a diffusion of the DNS in the charge and mass asymmetry coordinates, which are defined here by the charge and neutron numbers $Z$ and $N$ of light nucleus of the DNS. During the evolution in charge and mass asymmetry coordinates, the excited DNS can decay into two fragments in relative distance $R$ between the centers of the DNS nuclei. So, within the DNS model the production of the exotic nucleus is treated as a three-step process. First, the initial DNS with light nucleus $(Z_i, N_i)$ is formed in the peripheral collision for a short time. Second, the DNS with light exotic nucleus $(Z, N)$ is produced by nucleon transfers. Then this DNS separates into two fragments.

2. Model

The cross section of the production of primary nucleus in the diffusive nucleon transfer reaction is written as a sum over all partial waves $J$

$$
\sigma_{Z,N}(E_{\text{c.m.}}) = \sum_J \sigma_{Z,N}(E_{\text{c.m.}}, J),
$$

$$
\sigma_{Z,N}(E_{\text{c.m.}}, J) = \int_0^{\pi/2} \int_0^{\pi/2} d\cos \Theta_1 d\cos \Theta_2 \sigma_c(E_{\text{c.m.}}, J, \Theta_i) Y_{Z,N}(E_{\text{c.m.}}, J, \Theta_i).
$$

Here, the average over the orientations of statically deformed interacting nuclei ($\Theta_i$ ($i=1,2$) are the orientation angles with respect to the collision axis) is taken into consideration [38].

For the correct description of $\sigma_{Z,N}$, the partial capture cross section $\sigma_c$ in the entrance channel and the formation-decay probability $Y_{Z,N}$ of the DNS configuration with charge and mass asymmetries given by $Z$ and $N$ should be properly calculated. The value of $\sigma_c(E_{\text{c.m.}}, J, \Theta_i) = \frac{\pi\hbar^2}{8mE_{\text{c.m.}}}(2J+1)T(E_{\text{c.m.}}, J, \Theta_i)$ defines the transition of the colliding nuclei over the Coulomb barrier with the probability $T$ and the formation of the DNS when the kinetic energy $E_{\text{c.m.}}$ and angular momentum $J$ of the relative motion are transformed into the excitation energy and angular momentum of the DNS. The capture (transition) probability $T(E_{\text{c.m.}}, J, \Theta_i) = (1 + \exp[2\pi(V_f(R_b, Z, N, \Theta_i) - E_{\text{c.m.}})/\hbar\omega_f(Z, N, \Theta_i)])^{-1}$ is calculated with the Hill-Wheeler formula. The effective nucleus-nucleus potential [29]

$$
V_f(R, Z, N, \Theta_i) = V_N(R, Z, N, \Theta_i) + V_C(R, Z, N, \Theta_i) + \hbar^2J(J+1)/(2J+1)
$$

is calculated as a sum of nuclear $V_N$, Coulomb $V_C$ and centrifugal interactions and approximated near the Coulomb barrier at $R = R_b$ by the inverted harmonic-oscillator potential with the barrier height $V_f(R_b, Z, N, \Theta_i)$ and frequency $\omega_f(Z, N, \Theta_i)$. In the entrance channel the moment of inertia is $\Sigma = \mu R^2$ but after the DNS formation it corresponds to the rigid body limit at sticking condition. The nuclear potential $V_N$ is calculated with the double-folding model using a nuclear radius parameter $r_0=1.15$ fm and a diffuseness $a=0.54$ fm for $^{48}$Ca and $a=0.56$ fm for the actinide targets [29]. The quadrupole deformation parameters of actinides are taken from Ref. [39].
The primary charge and mass yields of fragments can be expressed by the product of the formation probability \( P_{Z,N}(t) \) of the DNS configuration with charge and mass asymmetries given by \( Z \) and \( N \) (\( A = Z + N \) is the mass number of nucleus), and of the decay probability of this configuration in \( R \) represented by the one-dimensional Kramers rate \( \Lambda_{Z,N}^{f} \) [34]:

\[
Y_{Z,N} = \Lambda_{Z,N}^{f} \int_{0}^{t_{0}} P_{Z,N}(t)dt. \tag{3}
\]

Here, \( t_{0} \) is the time of reaction which is determined as in Ref. [34] from the normalization condition \( \sum_{Z,N} Y_{Z,N} = 0.98 \). The mass yield of transfer products is defined as follows

\[
Y_{A} = \sum_{Z} Y_{Z,A-Z}. \tag{4}
\]

Using the macroscopical method suggested in Ref. [34], one can find \( P_{Z,N} \) from the solution of the system of master equations

\[
\frac{d}{dt} P_{Z,N}(t) = \Delta_{Z+1,N}^{(-0)} P_{Z+1,N}(t) + \Delta_{Z-1,N}^{(+0)} P_{Z-1,N}(t) + \Delta_{Z,N+1}^{(0,-)} P_{Z,N+1}(t) + \Delta_{Z,N-1}^{(0,+)} P_{Z,N-1}(t) - (\Delta_{Z,N}^{(-0)} + \Delta_{Z,N}^{(+0)} + \Delta_{Z,N}^{(0,-)} + \Delta_{Z,N}^{(0,+)} + \Lambda_{Z,N}^{f} + \Lambda_{Z,N}^{is}) P_{Z,N}(t), \tag{5}
\]

with initial condition \( P_{Z,N}(0) = \delta_{Z,Z} \delta_{N,N} \) and the transport coefficients which characterize the proton and neutron transfer rates from a heavy to a light nucleus (\( \Delta_{Z,N}^{(+0)}, \Delta_{Z,N}^{(0,+)} \)) or in opposite direction (\( \Delta_{Z,N}^{(-0)}, \Delta_{Z,N}^{(0,-)} \)). In Eqs. (5) we take only the transitions \( Z \leftrightarrow Z \pm 1 \) and \( N \leftrightarrow N \pm 1 \) into account in the spirit of the independent-particle model. Here, \( \Lambda_{Z,N}^{f} \) is the fission rate of the DNS heavy nucleus [34]. The solution of the master equations (5) with the decay terms and the microscopically calculated transport coefficients ensures a realistic description of the DNS evolution in charge and mass asymmetry [34].

As in Ref. [40], the excitation energy of the initial DNS should be enough to form the DNS with the certain exotic nucleus, i.e. to overcome the energy threshold \( \Delta B_{Z,N,J} \) for this. The value of

\[
\Delta B_{Z,N,J} = U(R_{b}, Z, N, J) - U(R_{m}, Z_{i}, N_{i}, J) \tag{6}
\]

is defined using the DNS potential energy calculated as in Ref. [32]

\[
U(R, Z, N, J) = B_{L} + B_{H} + V_{j}(R, Z, N, \Theta_{i}), \tag{7}
\]

where \( B_{L} \) and \( B_{H} \) are the mass excesses of the light and heavy fragments, respectively. Here, the DNS potential energy is calculated at the touching distance \( R = R_{m} \approx R_{L}(1 + \beta_{L} Y_{20}(\Theta_{L})) + R_{H}(1 + \beta_{H} Y_{20}(\Theta_{H})) + 0.5 \text{ fm} \) (\( \beta_{L} \) and \( \beta_{H} \) are the quadrupole deformation parameters of the nuclei with radii \( R_{L} \) and \( R_{H} \) and the position of the Coulomb barrier \( R = R_{b} \approx R_{m} + 1.2 \text{ fm} \) for the systems \( ^{48}\text{Ca}^{+}{^{238}}\text{U} = ^{244}\text{Pu} \). Note that the values of \( R_{m} \) and \( R_{b} \) depend on charge and mass asymmetries. As follows from Eqs. (6) and (7) the \( Q_{gap} \) values influence the production cross sections because of the binary character of the reaction.

The excitation energy of the initial DNS is \( E^{*}(Z_{i}, N_{i}, J) = E_{c.m.} - V_{j}(R_{m}, Z_{i}, N_{i}, \Theta_{i}) \). With this value the excitation energy of the DNS with exotic nucleus (\( Z, N \)) is \( E^{*}(Z, N, J) = E^{*}(Z_{i}, N_{i}, J) - \Delta B_{Z,N,J} \). Assuming the situation of thermal equilibrium, the excitation energy
Figure 1. The calculated (solid lines) mass yield (in relative units) of the quasifission products as a function of the mass number of the light fragment for $^{48}\text{Ca}+^{238}\text{U}$ reaction at bombarding energy $E_{\text{c.m.}}=190.2$ MeV. The experimental data [41] are shown by solid points.

of the nucleus with mass $A = Z + N$ in this DNS is $E_L^* (Z, N, J) = E^* (Z, N, J) A / A_{\text{tot}}$, where $A_{\text{tot}}$ is the total mass number of the DNS. It is clear that the probability of formation of the DNS with exotic nucleus $(Z, N)$ increases with $E^* (Z_i, N_i, J)$. However, the increase of $E^* (Z_i, N_i, J)$ is possible up to the moment when $E_L^* (Z, N, J)$ reaches the neutron separation energy $S_n (Z, N)$. Further increase of $E^* (Z_i, N_i, J)$ would lead to the strong loss of neutron-rich nuclei because of the neutron emission. If the primary nucleus is excited, one should take into consideration its survival probability $W_{\text{sur}}$ in the deexcitation process to obtain the evaporation residue cross section as follows

$$
\sigma_{Z,N-x}^{ER}(E_{\text{c.m.}}) = \sum_{J} \sigma_{Z,N}(E_{\text{c.m.}}, J) W_{\text{sur}} (E_{\text{c.m.}}, J, x),
$$

(8)

where $x$ is the number of evaporated neutrons from the excited primary nucleus. $W_{\text{sur}}$ is treated as in Ref. [40] and takes into consideration the competition with other deexcitation channels.

In order to test our method of calculation of $\sigma_{Z,N-x}^{ER}(E_{\text{c.m.}})$ and $\sigma_{Z,N}(E_{\text{c.m.}})$, we treat the production of Ti in the multinucleon transfer reactions $^{58}\text{Ni}(E_{\text{c.m.}} = 256.8 \text{ MeV}) + ^{208}\text{Pb}$ [10] and $^{64}\text{Ni}(E_{\text{c.m.}} = 307.4 \text{ MeV}) + ^{238}\text{U}$ [9] at bombarding energies near the Coulomb barrier. In these reactions the available excitation energies supply 2 neutron evaporation from the primary Ti isotopes having the maximal yields. In the $^{58}\text{Ni} + ^{208}\text{Pb}$ reaction $^{50}\text{Ti}$ and $^{52}\text{Ti}$ are produced with the cross sections 1 and 0.2 mb [10], respectively, which are consistent with our calculated cross sections 0.6 and 0.35 mb, respectively. In the $^{64}\text{Ni}+^{238}\text{U}$ reaction the experimental [9] and theoretical production cross sections for $^{52}\text{Ti}$ are 0.5 and 1.6 mb, respectively. In the reaction $^{48}\text{Ca}(E_{\text{c.m.}} = 204 \text{ MeV}) + ^{248}\text{Cm} \rightarrow ^{400} + ^{254}\text{Fm} + 2n$, the calculated $\sigma_{Z,N}^{ER}(E_{\text{c.m.}})$ for $^{254}\text{Fm}$ is about 0.5 µb, which is close to the experimental result presented in Refs. [19]. In the $^{48}\text{Ca}(E_{\text{c.m.}} = 274.6 \text{ MeV}) + ^{238}\text{U}$ reaction the experimental [14] and calculated ratios of secondary yields $Y(^{62}\text{Fe})/Y(^{58}\text{Cr})$ for the neutron-rich $^{62}\text{Fe}$ and $^{58}\text{Cr}$ isotopes are about 0.2 and 0.3, respectively.
Figure 2. The same as in Fig. 1, but for $^{48}$Ca+$^{244}$Pu reaction at bombarding energy $E_{\text{c.m.}}=201$ MeV.

The multinucleon transfer products of the quasifission reactions $^{48}$Ca($E_{\text{c.m.}}=190.2$ MeV)+$^{238}$U and $^{48}$Ca($E_{\text{c.m.}}=201$ MeV)+$^{244}$Pu at incident energies close to the Coulomb barrier are correctly described within our model. Figures 1 and 2 show the mass yield $Y_A$ as functions of the mass number of the light fragment. The calculated data in Figs. 1 and 2 are related to the primary (before neutron emission) fragments. Therefore, the maxima and minima in the calculated functions $Y_A$ are more pronounced. The postneutron evaporation washes out some peculiarities of these functions. Taking into consideration the experimental uncertainties in the identification of quasifission and fusion-fission products, and the measurement of mass, the agreement between the calculated and experimental data [41] is quite good. In the experiment besides the quasifission and fusion-fission, the fission of the heavy nucleus in the DNS with a subsequent fusion of one of the fission fragments with the light nucleus of the DNS and ternary processes with the emission of a light particle are identified as two-body processes of complete momentum transfer. It should be noted that the small oscillations in experimental data are comparable with accuracy of the measurements [41]. The maximum yield of the quasifission fragments occurs around the nucleus $^{208}$Pb for the heavy fragment where the DNS potential energy has a deep minimum. The evolution of the DNS is hindered by this minimum to go to smaller mass asymmetry and, correspondingly, the decay probability from the configuration with $^{208}$Pb is increased. In the reaction $^{48}$Ca+$^{238}$U ($^{48}$Ca+$^{244}$Pu), the height of the peak around $A=80$ is 4.5 (3.5) times larger than the height of peaks in the symmetric mass region. The main contribution to the symmetric and near symmetric fragmentations comes from the multinucleon transfer process.

3. Production cross sections of new neutron-rich $^{84,86}$Zn and $^{90,92}$Ge isotopes

As shown above, the suggested method is suitable to predict the mass and charge yields and the production cross sections for the products of multinucleon transfer reactions. The calculated production cross sections of neutron-rich isotopes in the reactions $^{48}$Ca+$^{238}$U, $^{244}$Pu at incident energies near the Coulomb barrier are presented in Figs. 3-5. We treat only the reactions leading to excitation energies of light neutron-rich nuclei equal to or smaller than
their neutron separation energies \( (E_L^* (Z, N, J) \leq S_n (Z, N)) \). In this case \( W_{\text{sur}} = 1 \) and the primary and secondary yields coincide. In Figs. 3 and 4 the values of \( E_{c.m.} \) provide the condition \( E_L^* (Z, N, J) = S_n (Z, N) \). The predicted values of \( S_n (Z, N) \) for unknown nuclei are taken from the finite range liquid drop model [42]. If \( E_L^* (Z, N, J) > S_n (Z, N) \), the primary neutron-rich nuclei are transformed into the secondary nuclei with less number of neutrons because of the de-excitation by neutron emission. The DNS evolution in the reactions treated can be schematically presented in the following way: \( ^{48}\text{Ca} + ^{238}\text{U} \rightarrow ^{78,80}\text{Zn} + ^{208,206}\text{Pb} \rightarrow ^{82,84,86}\text{Zn} + ^{204,202,200}\text{Pb} \) and \( ^{48}\text{Ca} + ^{244}\text{Pu} \rightarrow ^{84,82}\text{Ge} + ^{208,210}\text{Pb} \rightarrow ^{86,88,90,92}\text{Ge} + ^{206,204,202,200}\text{Pb} \). The system initially moves to the deep minimum of the potential energy surface (energetically favorable) which is caused by the shell effects around the DNS with magic heavy \(^{208}\text{Pb} \) and light \(^{80}\text{Zn} \) or \(^{82}\text{Ge} \) nuclei then from this minimum it reaches the DNS with exotic light nucleus by fluctuations in mass asymmetry. For low excitation energy, the evolution of the dinuclear system towards symmetry is hindered by this minimum.

![Figure 3. The expected cross sections for the indicated neutron-rich isotopes of Zn produced in the \(^{48}\text{Ca} + ^{238}\text{U} \) reaction at values of \( E_{c.m.} \) providing the excitations of these isotopes to be equal to the corresponding thresholds for the neutron emission.](image)

The production cross section for \(^{82}\text{Zn} \) \((^{86}\text{Ge})\) is about 2 (3) orders of magnitude larger than the production cross section for \(^{86}\text{Zn} \) \((^{92}\text{Ge})\) (Figs. 3 and 4). Although \( P_{30,52} \gg P_{30,54} \) \((P_{32,54} \gg P_{32,56})\), the cross sections of production of \(^{82}\text{Zn} \) \((^{86}\text{Ge})\) and \(^{84}\text{Zn} \) \((^{88}\text{Ge})\) are comparable because in the case of \(^{82}\text{Zn} \) \((^{86}\text{Ge})\) the optimal bombarding energy is considerably below the Coulomb barrier for the spherical nuclei and, correspondingly, \( \sigma_c^{(82}\text{Zn} \) \((^{86}\text{Ge})\)\) \( \ll \sigma_c^{(84}\text{Zn} \) \((^{88}\text{Ge})\)\). At \( E_{c.m.} \) smaller than the value of this barrier, only the collisions at certain mutual orientations leads to the capture. Since the yields of \(^{82}\text{Zn} \) and \(^{86}\text{Ge} \) isotopes as primary products is suppressed by the capture process, the production of these isotopes as secondary products at higher bombarding energies seems be possible. However, the formation of the DNS with heavier isotopes of Zn or Ge occurs with smaller probability. Therefore, the productions of \(^{82}\text{Zn} \) and \(^{86}\text{Ge} \) as secondary products and as primary products seem to be with comparable cross sections.

The dependence of the production cross section of the neutron-rich \(^{84}\text{Zn} \) isotope versus \( E_{c.m.} \) is presented in Fig. 5. The overall trend of dependence of cross section on the neutron separation energy is good visible. The solid arrow indicates the value of \( E_{c.m.} \) at which value of \( E_L^* (Z, N, J) \) reaches \( S_n (Z, N) = 3.99 \) MeV. Since the predictions of \( S_n (Z, N) \) have some uncertainties, we indicate by dashed arrow the value of \( E_{c.m.} \) at which the value of \( E_L^* (Z, N, J) \) reaches \( 0.5 S_n (Z, N) \). One can see that the decrease of neutron binding energy by 2 MeV leads
Figure 4. The same as in Fig. 3, but for the indicated neutron-rich isotopes of Ge produced in the $^{48}\text{Ca}+^{244}\text{Pu}$ reaction.

Figure 5. The excitation function for producing $^{84}\text{Zn}$ in the multinucleon transfer reaction $^{48}\text{Ca}+^{238}\text{U}$. The solid (dashed) arrow indicates the expected cross section at the value of $E_{\text{c.m.}}$ providing the excitation of $^{84}\text{Zn}$ to be equal to the threshold (half of the threshold) for the neutron emission.

to the shift of $E_{\text{c.m.}}$ by about 7 MeV and decrease of the production cross section by about 2 orders of magnitude. This decrease of $\sigma_{Z,N}$ is mostly due to the strong decrease of the capture cross section with decreasing $E_{\text{c.m.}}$ below the spherical Coulomb barrier. The measurement of the excitation function would be useful to estimate $S_{n}(Z, N)$ in the neutron-rich nuclei since at $E_{\text{c.m.}}^{*}(Z, N, J) > S_{n}(Z, N)$ the excitation function strongly drops down.

In our calculations the theoretically predicted [42] mass excesses and neutron separation energies are used for neutron-rich nuclei. The uncertainties of these predictions mainly contribute to the uncertainty of our results. We assume that the excitation energy of DNS is shared between the DNS nuclei proportionally to their mass numbers. Since the isotopes of interest are formed from the projectile by the acceptance of nucleons, they can be slightly more excited than in the limit of thermal equilibrium. Taking into consideration these facts, we estimate the uncertainty within a factor of 3-5 in the calculated cross sections. If the neutron-rich isotope is close to the
region of known nuclei, then the predictions for it have less uncertainties and the cross section is estimated with higher accuracy.

4. Predicted yields of new neutron-rich isotopes of nuclei with $Z=64-80$

The calculated production cross sections of the primary isotopes in the reactions $^{48}$Ca+$^{238}$U at $E_{c.m.}=189$ MeV are presented in Figs. 6-13. The primary neutron-rich nuclei of interest are excited and transformed into the secondary nuclei with less number of neutrons without a loss of the cross section because of the neutron emission is dominant over other deexcitation channels. The neutron emission channels are indicated in Figs. 6-13 for primary neutron-rich isotopes. The predictions are done by assuming the excitation energy of the DNS is divided proportionally to the mass numbers of the fragments.

![Figure 6. Calculated production cross sections of the primary Hg and Au isotopes versus the mass number. Neutron evaporation channels for neutron-rich primary isotopes are indicated. The heaviest known isotopes are marked by arrows.](image)

Since the predicted production cross sections for new exotic isotopes $^{193}$W, $^{195,196}$Re, $^{198}$Os, and $^{200}$Ir are at the microbarn level, they can be easily identified. For these nuclei, the known heaviest isotopes are in the vicinities of maxima of the primary isotopic distributions (Figs. 7-9). Because the calculated production cross sections for new exotic isotopes $^{178}$Er, $^{180,181}$Tm, $^{182-184}$Yb, $^{185-187}$Lu, $^{190}$Hf, $^{190-193}$Ta, $^{194,196}$W, $^{197,199}$Re, $^{199,200}$Os, $^{201,202}$Ir, $^{203}$Pt (Figs. 7–12) are between microbarn and nanobarn levels, they can be also detected with the present experimental setups. The group of new isotopes $^{173}$Tb, $^{174,176}$Dy, $^{176,177,179}$Ho, $^{179-182}$Er, $^{182,183}$Tm, $^{185-187}$Yb, $^{188,189,191}$Lu, $^{191,192}$Hf, $^{195,196}$Ta, $^{197}$W, $^{200}$Re, $^{202}$Os, $^{204}$Ir, $^{204,206}$Pt, $^{206,208}$Au (Figs. 6-13) is produced with the cross sections in the interval between nanobarn and picobarn. The unknown neutron-rich isotopes of Hg can be produced in the reaction treated
with the cross sections less than 1 pb. In this case the known heaviest isotope $^{210}\text{Hg}$ is far to the right side from the maximum of the calculated isotopic distribution (see Fig. 6).

5. Transfer-induced fission of superheavy nuclei

In Fig. 14 the cross sections $\sigma_f$ of transfer-induced fission of superheavy nuclei are predicted in the reactions $^{48}\text{Ca} + ^{244,246,248}\text{Cm}$ at beam energies near the corresponding Coulomb barriers. The value of $\sigma_f$ drops down by approximately three orders of magnitude as $Z$ increases from 101 to 108. As the value of $E_{\text{c.m.}}$ increases by 20 MeV the values of $\sigma_f$ grow by about two orders of magnitude. The experimental data [19] as well as our analysis show the advantages of reactions with a smaller number of neutrons in the target for the synthesis of unknown isotopes of superheavy nuclei. The probability of a large number of nucleons transferring from the projectile to the target nucleus is correlated with the dependence of the DNS potential energy on $Z$. For $102 < Z < 110$, the DNS potential energy decreases as the total number of DNS neutrons decreases, therefore one can expect larger yields of superheavy nuclei in transfer type reactions with the $^{244,246}\text{Cm}$ nuclei than when the $^{248}\text{Cm}$ target is used. In the reaction $^{48}\text{Ca} + ^{248}\text{Cm}$, the calculated production cross section for the $^{254}\text{Fm}$ is close to the experimental results presented in Refs. [19] where the yields of transfermium nuclei were not measured.

Note that isotopes of nuclei with $Z=103–108$ listed in Fig. 14 were not observed yet in the experiments. The complementary to the fission fragments of these nuclei are the O, F, Ne, Na, Mg, Al, Si, and P nuclei. Figure 14 shows that fission of unknown isotopes with $Z=103–108$ can be measured with acceptable cross sections ($\sigma_f \approx 100 \text{ nb} - 100 \mu \text{b}$) in the reactions with $^{244,246,248}\text{Cm}$ targets and $^{48}\text{Ca}$ beam at energies which are about 28 MeV above the corresponding Coulomb barriers. If the excited superheavy nucleus undergoes fission in

Figure 7. The same as in Fig. 6, but for the primary Pt and Ir isotopes.

Figure 8. The same as in Fig. 6, but for the primary Os and Re isotopes.
the excitation energy and verification of these predictions would provide additional information about the absolute primary W and Ta isotopes.

The average excitation energies \( E_{\text{exc}} \) of the fissioning superheavy nuclei are presented in Fig. 15. It was assumed that the excitation energies of the DNS nuclei are proportional to their masses. Since the DNS potential energy increases towards \( Z \) larger than \( Z_t \), the excitation energy of the heavy fragment (the fissioning nucleus) decreases from (16–18) MeV to (8–11) MeV at the smaller bombarding energies treated in Fig. 14. Note that the peak/valley ratio of the mass distribution of the fission fragments strongly depends on the value of the excitation energy. In the transfer-induced fission one can study the multimodal structure of the mass distribution of fission fragments for certain isotopes of superheavy nuclei with \( Z = 101–108 \).

In the multinucleon transfer reactions \( ^{48}\text{Ca}+^{204,206,208}\text{Cm} \) at the beam energies \( E_{\text{c.m.}}=207, 205.5, 204 \) MeV, respectively, one can produce the new isotopes of superheavies (the evaporation residues) with \( Z=104–108 \) which are not reachable in the hot and cold complete fusion reactions with the stable projectiles and targets \([40]\). While \( \sigma_f \) drops down by approximately three orders of magnitude as \( Z \) increases from 102 to 108, the evaporation residue cross section decreases only about of 100 times due to the increase of survival probability in the \( 1n \) evaporation channel for nuclei approaching to the deformed subshell \( Z=108 \) and \( N=162 \) \([40]\). The experimental verification of these predictions would provide an additional information about the absolute values and the excitation energy- and \( Z \)-dependencies of shell correction in this range of

\[ \text{Figure 9. The same as in Fig. 6, but for the primary W and Ta isotopes.} \]

\[ \text{Figure 10. The same as in Fig. 6, but for the primary Hf and Lu isotopes.} \]
superheavy nuclei.

6. Production of heaviest nuclei in quasifission reactions

The master equations also describe configurations of dinuclear systems which are more asymmetric than the DNS in the entrance channel. The processes of formation and decay are ruled by the same mechanism of diffusion in the same relevant collective coordinates: mass and charge asymmetries and relative distance.

With asymmetric-exit-channel quasifission reactions leading to nuclei with charge numbers larger than the charge number of the target, one can produce isotopes that cannot be synthesized in complete fusion reactions. The direct production of transactinides in asymmetric-exit-channel quasifission reactions would give nuclei with $101 \leq Z \leq 108$ in the reactions $^{48}$Ca$^{+}$, $^{238}$U, $^{243}$Am, $^{244}$, $^{246}$Cm. The production of heavy actinides has been studied in the transfer-type reactions by bombarding of actinide targets with $^{16}$, $^{18}$O, $^{20}$, $^{22}$Ne, and $^{40}$, $^{44}$, $^{48}$Ca [19]. Nuclei with $Z > 102$ have not been observed because of the small cross sections or short lifetimes in the radiochemical identification of the nuclei.

The cross section $\sigma_{Z,N}$ of the production of a primary heavy nucleus with $Z = Z_H$ and $N = N_H$ ($H$ = heavy) in the asymmetric-exit-channel quasifission reaction is written in Eq. (1). The primary heavy nucleus is excited and evaporates neutrons in the deexcitation process. The evaporation residue cross section for the heavy nucleus with charge number $Z$ is obtained as in Eq. (8). The actinide targets proposed for such reactions are deformed. Therefore, the minimum value of the incident energy $E_{\text{c.m.}}^\text{min}$, at which the collisions of nuclei at all orientations become possible, is larger than the Coulomb barrier calculated for spherical nuclei. In the asymmetric-exit-channel quasifission reactions which occur slightly above the Coulomb barrier, only partial waves with $J \leq J_{\text{cap}} = 20$ contribute to the production of superheavy nuclei. For $J_{\text{cap}} = 20$, the

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**Figure 11.** The same as in Fig. 6, but for the primary Yb and Tm isotopes.
For $102 < Z < 110$, the potential energy decreases with the total number of neutrons of the DNS and a larger primary yield of superheavy nuclei is expected in the reactions with $^{244,246}$Cm rather than with $^{248}$Cm. This is demonstrated in Fig. 16, where the primary yields of the most probable isotopes of heavy nuclei are calculated with the master equations and with the statistical formula

$$Y_{Z,N} \approx 0.5 \exp \left( -\frac{B_R(Z,N) - B_{sym}^a(Z_i,N_i)}{T(Z_i,N_i)} \right),$$

where $B_R$ is given by $B_R(Z,N) = U(R_b,Z,N,J) - U(R_m,Z_i,N_i,J)$ and $T$ is temperature calculated by using the Fermi-gas expression $T = \sqrt{E^*/a}$ with the excitation energy $E^*$ of the initial DNS and the level-density parameter $a = A_{tot}/12$ MeV$^{-1}$. Similar results are obtained with both methods.

In Fig. 16 the excitation energies of primary heavy nuclei correspond to $E_{c.m.} = 204 - 207$ MeV. The excitation energy of the primary heavy nucleus is defined proportionally to its mass $A_H$: $E^*_H(Z,N) = (E^*(Z_i,N_i) - B_R(Z,N))A_H/(A_L + A_H)$. In this case $E^*_H(Z,N)$ is related to the maxima or to the right hand sides of excitation functions for one-neutron emission. For example, for $^{202}$No we find $E_H = 16$ MeV and $W_{sur}(1n) = 2.4 \times 10^{-4}$.

Although $Y_{Z,N}$ decreases by about 3 orders of magnitude with increasing $Z$ from 102 to 108, the evaporation residue cross section decreases only by about 30 times because of the increase of $W_{sur}$ with approaching the deformed subshell closure $Z=108$, $N=162$. The experimental data as well as our treatment indicate the preference of a smaller number of evaporated neutrons to produce superheavy nuclei. So, one can see that with the asymmetric-exit-channel quasifission

**Figure 12.** The same as in Fig. 6, but for the primary Er and Ho isotopes.

**Figure 13.** The same as in Fig. 6, but for the primary Dy and Tb isotopes.
Figure 14. The cross sections $\sigma_f$ of transfer-induced fission of indicated isotopes of superheavy nuclei (the mass number near the symbol) in the reactions $^{48}\text{Ca}(E_{\text{c.m.}}=207, 227 \text{ MeV})+^{244}\text{Cm}$ (triangles), $^{48}\text{Ca}(E_{\text{c.m.}}=205.5, 225.5 \text{ MeV})+^{246}\text{Cm}$ (circles), and $^{48}\text{Ca}(E_{\text{c.m.}}=204, 224 \text{ MeV})+^{248}\text{Cm}$ (squares). The results at larger bombarding energies are shown in the upper part.

Reactions on actinide targets unknown isotopes of superheavy nuclei can be produced with suitable cross sections. The quasifission reactions $^{48}\text{Ca}+^{244,246,248}\text{Cm}$ at beam energies close to the corresponding Coulomb barriers, one can produce the new isotopes of superheavies with $Z = 103 - 108$, which undergo fission (the fission width is much larger than the neutron emission width). The calculated results indicate that these quasifission reactions provide a very efficient tool for the study of new isotopes of superheavy nuclei that fill the gap between the isotopes produced in the cold and hot complete fusion reactions. The predicted cross sections of the fission, which follow multinucleon transfer are on the level (100 nb-100 $\mu$b). One can propose the experiments on the quasiternary fission in which the fission fragment mass and the angular distributions in coincidence with the complementary transfer products, which range from O to P ions can be measured. Since the fission barrier of the superheavy nuclei is mainly determined by the shell correction in the ground state, in these experiments, one can study the dependence of the value of shell correction on the average excitation energy, which is easily calculated, and $(Z, N)$ of the fissioning nucleus.
Figure 15. The average excitation energies of the fissioning superheavy nuclei produced in the reactions $^{48}$Ca($E_{\text{c.m.}}=207\ \text{MeV}$) + $^{244}$Cm (triangles), $^{48}$Ca($E_{\text{c.m.}}=205.5\ \text{MeV}$)+$^{246}$Cm (circles), and $^{48}$Ca($E_{\text{c.m.}}=204\ \text{MeV}$)+$^{248}$Cm (squares).

7. Summary
We demonstrated the possibilities for producing neutron-rich isotopes $^{82,84,86}$Zn and $^{86,88,90,92}$Ge in the reactions $^{48}$Ca+$^{238}$U,$^{244}$Pu at incident energies near the Coulomb barrier. Note that $^{84,86}$Zn and $^{90,92}$Ge isotopes were not observed yet in the experiments. One can also produce new heavy neutron-rich isotopes of nuclei with $Z=66$–$82$ as complementary fragments in the actinide-based multinucleon transfer reactions with $^{48}$Ca beam. The predicted cross sections are on the level (0.1–$160\ \text{pb}$). Therefore, the multinucleon transfer reaction at low energies provides an efficient tool for producing nuclei far from stability and may be the fruitful method to reach the neutron drip line. The production of isotopes treated can be a supplementary information in the experiments on production of superheavy nuclei which run a long time with the same reactions. The multinucleon transfer reactions can provide detailed information about the dynamics of dinuclear system in the mass and charge asymmetry degrees of freedom.

Due to the large neutron excess and smaller losses because of the quasifission near the entrance channel, the use of $^{48}$Ca projectile is more preferable than the use of heavier projectiles to reach the neutron-rich region of nuclide in the actinide-based reactions. It is apparent that the use of the heavier actinide target, for example, $^{244}$Pu or $^{248}$Cm, with $^{48}$Ca beam at energies near the Coulomb barrier one can reach more neutron-rich region of nuclide. Irradiating the heavier actinide targets by $^{48}$Ca beam for producing neutron-rich isotopes, we gain in the $Q$-value and the known heaviest isotopes are closer to the maxima of isotopic distributions.

In the multinucleon transfer reactions $^{48}$Ca+$^{244,246,248}$Cm at beam energies close to the corresponding Coulomb barriers one can produce new isotopes of superheavies with $Z=103$–$108$ which undergo fission (the fission width is much larger than the neutron emission width). The calculated results indicate that the multinucleon transfer reactions provide a very efficient tool for the study of new isotopes of superheavy nuclei which fill the gap between the isotopes produced in the cold and hot complete fusion reactions. The predicted cross sections of the fission following multinucleon transfer are on the level ($100\ \text{nb}$ – $100\ \mu\text{b}$). One can propose the experiments on the quasi-ternary-fission in which the fission fragment mass and angular distributions in coincidence with the complementary transfer products ranging from O to P ions can be measured. Since the fission barrier of the superheavy nuclei is mainly determined by the shell correction in the ground state, in these experiments one can study the dependence of the
Figure 16. The calculated primary yields $Y_{Z,N}$ (lower part) and evaporation residue cross sections $\sigma_{ER}$ (middle and upper parts) are shown by triangles, circles and squares for the reactions $^{48}\text{Ca} + ^{244,246,248}\text{Cm}$ ($E_{c.m.} = 207, 205.5$ and $204$ MeV, respectively). The heavy fragments after $1n$ evaporation are indicated in the upper part of the figure. The results obtained with (9) and (3) are indicated by closed and open symbols, respectively.

value of shell correction on the average excitation energy, which is easily calculated, and $(Z, N)$ of the fissioning nucleus.

In the complete fusion-fission reactions it is difficult to separate the true fission fragments from the quasifission fragments. This problem is absent in the transfer-induced fission reactions. Therefore, comparing mass distributions of fragments of the same fissioning nucleus produced in the complete fusion-fission and transfer-induced fission reactions, one can distinguish the quasifission products.

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