Algorithm for detecting a change in the motion mode of an object moving along a complex trajectory

A V Golubkov¹, A V Tsyganov¹, I O Petrishchev¹

¹Ulyanovsk State University of Education, Lenin square 4/5, Ulyanovsk, Russia, 432071

Abstract. The present paper solves the problem of quick detection of a change in the motion mode of an object moving along a complex trajectory. The process of the object movement is described by a hybrid stochastic model. To solve the problem, a sequential probability ratio criterion is applied. A distinctive feature of the proposed algorithm is the ability to make decisions on a limited set of values of the likelihood ratio function. The results of numerical experiments confirm the efficiency of the developed algorithm.

1. Introduction
The problems of mathematical modeling of moving objects trajectories, moving objects tracking and recognition, as well as those of target tracking, are a subject of current interest of modern scientific research due to the importance of practical applications that make use of solutions to these problems [1].

In real practical problems, the trajectory of an object is so complex that in the general case it is difficult to represent it as any specific mathematical model, even if it is nonlinear. Most often, various nonlinear stochastic models in continuous or discrete time are used to model motion trajectories under a priori uncertainty (see, for example [2, 3]). Under conditions of a priori uncertainty of motion parameters, that is, when only incomplete noisy measurements are available, one of the main approaches is to use nonlinear filtering methods [4, 5].

One of the approaches to modeling and parameter estimation of object motion is the use of hybrid models. By a hybrid stochastic model, we mean a set of discrete linear stochastic models, each of which is responsible for a certain motion mode of the object, that is, a section of the motion that can be represented by a linear model. Thus, a complex (in general case, nonlinear) trajectory of an object motion is approximated by a piecewise linear trajectory. The solution to the problem of modeling the trajectory of a marine moving object using linear stochastic models of rectilinear uniform and circular motion was obtained in [6], and then developed in [7]. The advantage of the approach to an object motion modeling is that a generally nonlinear mathematical model is replaced by a set of linear dynamic models for which optimal discrete Kalman filtering algorithms [8] can be used instead of using nonlinear filters (with inevitable calculation errors due to linearization) to estimate motion parameters in each piece of the trajectory. However, this approach inevitably entails the need to solve another problem — the fast detection of the moment of change in the motion mode (maneuvering) of a moving object.

The problem of detecting changes in the properties of random processes was first solved by E.S. Page [9]. The optimal rules for stopping observations including the well-known problem of
change point detection were obtained by A. N. Shyryaev [10]. Theoretical issues of this problem are also considered in [11, 12, 13] and others.

In [14] the method of detection and identification of faults in the class of linear stochastic control systems in the filtering process, guaranteed by the probability of type I and type II errors, is obtained. With $M$ possible modes of functioning of the system, the decision is made on a limited set containing $M$ likelihood ratio function values.

In this paper, we propose the development of the idea of modeling and estimating the motion of an object along a complex trajectory using a hybrid stochastic model, which is a set of discrete linear stochastic models responsible for various pieces of the trajectory of an object motion [7]–[16]. To solve the problem, the method obtained in [14] is used.

2. A model of an object motion along a complex trajectory

Suppose that the trajectory of an object can be divided into separate sufficiently long pieces, on each of which its motion can be represented by a linear stochastic model that describes either a uniform linear motion or a uniform circular motion counterclockwise/clockwise (left/right turn) with a given radius.

Let us consider three models of this kind. Then the motion of the object along the entire trajectory can be described by a hybrid stochastic model:

$$x(k) = \Phi^p x(k-1) + B^p + Gw(k-1), \quad p = 0, 1, 2,$$

where $k$ is a discrete time instance, $p$ is the number of the motion mode, $x = [x_1, x_2, x_3, x_4]^T \in \mathbb{R}^4$ is the vector of motion parameters of the object, in which $x_1$ is the coordinate of the object along the axis $Ox$ (m), $x_2$ is the velocity $v_x$ along the axis $Ox$ (m/s), $x_3$ is the coordinate of the object along the axis $Oy$ (m), $x_4$ is the velocity $v_y$ along the axis $Oy$ (m/s).

We write down all the matrices of the model (1).

- Uniform linear motion (the number of the motion mode $p = 0$):

\[
\Phi^0 = \Phi^0(\tau) = \begin{bmatrix} \Phi_l & 0 \\ 0 & \Phi_l \end{bmatrix}, \quad \Phi_l = \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix},
\]

\[
B^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T,
\]

where $\tau = t_k - t_{k-1}$ is a discrete-time step.

- Uniform circular counterclockwise motion with a given radius $r_1$ (the number of the motion mode $p = 1$) or uniform circular clockwise motion with a given radius $r_2$ (the number of the motion mode $p = 2$):

\[
\Phi^1 = \Phi^1(x_s, r_1, \tau) = \begin{bmatrix} \Phi^1_c & 0 \\ 0 & \Phi^1_c \end{bmatrix}, \quad \Phi^2 = \Phi^2(x_s, r_2, \tau) = \begin{bmatrix} \Phi^2_c & 0 \\ 0 & \Phi^2_c \end{bmatrix},
\]

\[
\Phi^1_c = \begin{bmatrix} \cos \omega_1 \tau & \omega_1^{-1} \sin \omega_1 \tau \\ -\omega_1 \sin \omega_1 \tau & \cos \omega_1 \tau \end{bmatrix},
\]

\[
\Phi^2_c = \begin{bmatrix} \cos \omega_2 \tau & \omega_2^{-1} \sin \omega_2 \tau \\ -\omega_2 \sin \omega_2 \tau & \cos \omega_2 \tau \end{bmatrix},
\]

\[
B^1 = B^1(x_s, r_1, \tau) = \begin{bmatrix} (x_{1,s} - \omega_1^{-1} x_{4,s})(1 - \cos \omega_1 \tau) \\ (\omega_1 x_{1,s} - \omega_1^{-1} x_{4,s}) \sin \omega_1 \tau \\ (x_{3,s} + \omega_1^{-1} x_{2,s})(1 - \cos \omega_1 \tau) \\ (\omega_1 x_{3,s} + x_{2,s}) \sin \omega_1 \tau \end{bmatrix},
\]
\[ B^2 = B^2(x_s, r_2, \tau) = \begin{bmatrix} (x_{1,s} + \omega_2^{-1}x_{4,s})(1 - \cos \omega_2 \tau) \\ (\omega_2 x_{1,s} + x_{4,s}) \sin \omega_2 \tau \\ (x_{3,s} - \omega_2^{-1}x_{2,s})(1 - \cos \omega_2 \tau) \\ (\omega_2 x_{3,s} - x_{2,s}) \sin \omega_2 \tau \end{bmatrix}. \]

Here the matrices \( B_1 \) and \( B_2 \) determine a left or right rotation, \( \tau \) is the sampling period, \( \omega_1 = |v_s|/r_1 > 0 \), \( \omega_2 = |v_s|/r_2 > 0 \) is the angular velocity at the moment of the motion mode change, the module of the velocity vector \( |v_s| = \sqrt{x_{1,s}^2 + x_{2,s}^2} \), \( x_{i,s} \) is the \( i \)-th element of the model state vector (1) at time \( t_s \).

- For all motion modes, the transfer matrix of discrete white noise \( w(k) \sim \mathcal{N}(0, Q) \) is

\[ G = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T. \]

The hybrid model (1) allows us to model the motion of an object along a complex trajectory using the algorithm described in [7].

Provided that only the spatial coordinates of the object are measured, the corresponding measurement model can be written as follows:

\[ z(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(k) + v(k), \tag{2} \]

where \( v(k) \) is the measurement error vector, \( \nu(k) \sim \mathcal{N}(0, R) \).

### 3. Detection of a change in the motion mode

Let us suppose that the moment of a possible transition of a system from one given mode to another is a priori unknown.

Consider two motion modes \( (p = 0, 1) \). We assume that the initial state of the system corresponds to the nominal motion mode \( p = 0 \). It is necessary according to the measurement results

\[ Z(i) = [z(1), \ldots, z(i)]^T, \quad i = 1, \ldots, N, \]

to confirm or deny the fact that the system switches to motion mode \( p = 1 \).

A solution to this problem can be obtained using Wald’s sequential probability ratio test. The choice of two hypotheses is determined by the decision rule:

\[
\begin{cases} 
\text{If } \lambda_k \geq A, & \text{the test is completed with the choice of the hypothesis } \mathcal{H}_1. \\
\text{If } \lambda_k \leq B, & \text{the test is completed with the choice of the hypothesis } \mathcal{H}_0. \\
\text{If } A > \lambda_k > B, \text{ the test is continued for the next } k.
\end{cases} \tag{3}
\]

Here

\[ \lambda_k = \frac{f_{N(1,i)|\mathcal{H}_1}(x(1), \ldots, x(i))}{f_{N(1,i)|\mathcal{H}_0}(x(1), \ldots, x(i))} \]

is a likelihood ratio, \( A = (1 - \beta)/\alpha \) and \( B = \beta/(1 - \alpha) \) are the upper and lower decision making thresholds in which \( \alpha \) and \( \beta \) are error probabilities of type I and II, \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \) are hypotheses corresponding to the motion mode \( p = 0 \) and \( p = 1 \), respectively; \( N(1,i)|\mathcal{H}_p = [\nu_p(1), \nu_p(2), \ldots, \nu_p(i)]^T \) is a sequence of residuals generated by Kalman filters, constructed in accordance with the existing hypotheses (\( p \) is the hypothesis number):

**I. Extrapolation:**
\[ \begin{array}{c}
\hat{x}_{k-p}^- = \Phi^p \hat{x}_{k-1,p}^- + B^p, \\
P_{k,p}^- = \Phi^p P_{k-1,p}^+ (\Phi^p)^T + GQG^T.
\end{array} \]

**II. Filtering:**
\[ \begin{array}{c}
K_{k,p} = P_{k,p}^- H^T (HP_{k,p}^- H^T + R)^{-1}, \\
P_{k,p}^+ = P_{k,p}^- - K_{k,p} HP_{k,p}^-,
\end{array} \]
\[ \hat{x}_{k,p}^+ = \hat{x}_{k,p}^- + K_{k,p} (z_k - H \hat{x}_{k,p}^-). \]
The residuals and their covariance matrix are calculated as follows:

\[ \Sigma^p(k) = H^T P_{k,p}^{-} H + R, \quad \nu^p(k) = z_k - H \tilde{x}_{k,p}. \]  

However, since the possible moment of the motion mode change is a priori unknown, instead of one alternative hypothesis \( \mathcal{H}_1 \) we have to introduce a set of hypotheses \( \mathcal{H}_1, \mathcal{H}_2, \ldots, \mathcal{H}_i \), which suggests a possible change in the motion mode at any given time from the beginning of the observation.

To solve the problem on a limited set of values of the likelihood ratio function, we use the following result:

Theorem 1. [14] Let the moment of occurrence of a possible motion mode change in the system (1), (2) be a discrete random variable \( \theta \), uniformly distributed over the segment \([0, i]\). Then the ratio of the likelihood functions in the decision rule (3) is calculated by the expressions:

\[ \lambda_k = \frac{1}{k} \sum_{j=1}^{k} \psi^j_k(k) \]  

where \( \psi^j_k(k) = \prod_{i=j}^{k} f_{\nu_j(i) | \mathcal{H}_i}(x(i)) f_{\nu_1(i) | \mathcal{H}_0}(x(i)) \).

Considering the fact that in an optimal filter each random residual vector \( \nu_j(i) | \mathcal{H}_p \) is normally distributed with zero mean and covariance matrix \( \Sigma_j(i) | \mathcal{H}_p \) calculated by equations (5) (subscript \( j \) means the discrete instance in time at which the filter \( F_j \) starts working), we can rewrite the expression for \( \psi^j_k(k) \) as:

\[ \psi^j_k(k) = \begin{cases} 1, & k < j, \\ \psi^1_j(k-1) \sqrt{\frac{\Sigma^1_j(k)}{\Sigma^0_j(k)}} \exp \left[ \frac{(\nu^0_j(k))^T (\Sigma^0_j(k))^{-1} \nu^0_j(k) - (\nu^1_j(k))^T (\Sigma^1_j(k))^{-1} \nu^1_j(k)}{2} \right], & k \geq j. \end{cases} \]  

The algorithm for detecting a change in the motion mode is written as follows:

1) Set error probabilities \( \alpha \) and \( \beta \) of type I and II.
2) Calculate the decision rule threshold values \( A = (1 - \beta)/\alpha \) and \( B = \beta/(1 - \alpha) \).
3) At each moment of time, a new filter which corresponds to the hypothesis that at the current moment of time a change in the motion mode occurs, is created.
4) Calculate \( \lambda_k \) by (4)–(7).
5) Check the criterion (3).

4. Numerical experiments

Let us carry out a computer simulation to verify the operability and efficiency of the proposed algorithm. First, it is necessary to obtain model measurements data of the object coordinates when it moves along a certain trajectory. Let us simulate the data of trajectory measurements with the following motion scheme: the object moves rectilinearly and uniformly for the first 20 cycles, then the object moves uniformly in a circular path for the next 20 cycles when turning right with a given turning radius \( r_2 = 5 \) m. The initial parameters of the object motion \( x = [0, 0, 0, 2]^T \), the covariances of the Gaussian noise in the state equation and in the measurement scheme are equal to \( Q = \text{diag}(0.001, 0.001) \) and \( R = \text{diag}(0.1, 0.1) \), respectively.

The computer simulation was carried out in Matlab. The results are presented in figure 1. It is seen that the likelihood ratio \( \lambda_k \) crosses the upper threshold \( A \), which means accepting the hypothesis of a change in the motion mode. The decision time took 12 discrete-time cycles.
5. Conclusion

The obtained method of detecting a change in the motion mode of an object in the process of measurement data filtering is computationally efficient and guaranteed by the probability of errors of type I and II. It was assumed that the moment of change in the motion mode is unknown. The solution is based on the representation of the trajectory of the object motion by a hybrid stochastic model, the application of the Wald’s sequential probability ratio test, and the Kalman filtering algorithm. The effectiveness of the method lies in the fact that the decision is made on a limited set of values of the likelihood ratio function. Further research will be aimed at solving the problem of identifying the motion mode of an object when the moment of changing the motion mode is unknown.

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Figure 1. Graphs a) object motion trajectories and measurements, b) likelihood ratio $\lambda_k$.
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