Unique Probe of Neutrino Electromagnetic Moments with Radiative Pair Emission

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The neutrino magnetic and electric moments are zero at tree level but can arise in radiative corrections. Any deviation from the Standard Model prediction would provide another indication of neutrino-related new physics in addition to the neutrino oscillation and masses. Especially, Dirac and Majorana neutrinos have quite different structures in their electromagnetic moments. Nevertheless, the recoil measurements and astrophysical stellar cooling can only constrain combinations of neutrino magnetic and electric moments with the limitation of not seeing their detailed structures. We propose using the atomic radiative emission of neutrino pair to serve as a unique probe of the neutrino electromagnetic moments with the advantage of not just separating the magnetic and electric moments but also identifying their individual elements. Both searching strategy and projected sensitivities are illustrated in this letter.

Introduction – In the Standard Model (SM) of particle physics, there is no tree-level coupling between neutrino (ν) and photon (A) [1]. However, the neutrino electromagnetic interactions are expected to arise from radiative corrections [2],

\[ H_M = \bar{\nu} \left[ -f_M(q^2)i\sigma_{\mu\nu}q^\nu + f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 \right] \nu A^\mu(q). \]  

(1)

The two terms account for the magnetic (\(\mu_\nu \equiv f_M(0)\)) and electric (\(\epsilon_\nu \equiv f_E(0)\)) dipole moments at vanishing momentum transfer, \(q^2 = 0\), respectively.

With three neutrinos, both \(\mu_\nu\) and \(\epsilon_\nu\) are \(3 \times 3\) hermitian matrices. For Majorana neutrinos, their electromagnetic moments are antisymmetric under permutation, \((\mu_\nu)_{ij} = -(\mu_\nu)_{ji}\) and \((\epsilon_\nu)_{ij} = -(\epsilon_\nu)_{ji}\), implying that only the off-diagonal transition moments exist [3–8]. Observing a non-zero diagonal element \((\mu_\nu)_{ii}\) or \((\epsilon_\nu)_{ii}\) is then a direct evidence of Dirac neutrinos. In addition, a nonzero diagonal \((\epsilon_\nu)_{ii}\) also indicates CP violation.

Generally speaking, the explicit form of \(\mu_\nu\) is model-dependent and its size is many orders smaller than the Bohr magneton \(\mu_B = e/2m_e\) [4, 5, 9–12]. Interestingly, if the neutrino mass and magnetic moment arise from the same effective operator, the magnetic moment for Majorana neutrino is typically 5 orders larger than the Dirac one [13, 14]. Currently, the experimental sensitivity is already around the threshold for discovering the Majorana neutrino magnetic moments.

The neutrino electromagnetic moments can be tested in various ways. Typically, the neutrino scattering cross section with electron peaks in the low momentum transfer region to provide a sizable signal in the electron recoil. Both solar and reactor neutrinos can be used for such recoil measurement. The best sensitivity comes from the reactor experiment GEMMA, \(\mu_\nu^{\text{eff}} < 2.9 \times 10^{-11} \mu_B\) [15], and the solar experiment Borexino, \(\mu_\nu^{\text{eff}} < 2.8 \times 10^{-11} \mu_B\) [16], both at 90% C.L. However, the recoil measurement probes not just the magnetic moment \(\mu_\nu\) but also the electric one \(\epsilon_\nu\) as a combination [17, 18],

\[(\mu_\nu^{\text{eff}})^2 \equiv \sum_{\mu} \sum_{\epsilon} |U_{\alpha \mu}^* U_{\beta \epsilon}(\mu_\nu_{\mu \epsilon} - i(\epsilon_\nu_{\mu \epsilon}))|^2,\]

(2)

where \(U_{\alpha \mu}\) is the neutrino mixing matrix element [1]. The sensitivity of scattering experiments only applies to the combination \(\mu_\nu^{\text{eff}}\) but not the individual electromagnetic moments due to possible cancellation among them. In other words, there is no unique probe of the neutrino magnetic or electric moment.

Similarly, the stellar cooling due to plasmon decay \((\gamma^* \rightarrow \bar{\nu} \nu)\) is also sensitive to a combination [2],

\[(\mu_\nu^{\text{pl}})^2 \equiv \sum_{ij} |(\mu_\nu_{ii})|^2 + |(\epsilon_\nu_{ii})|^2,\]

(3)

rather than an individual magnetic moment. The current bounds are \(\mu_\nu^{\text{pl}} < 2.2 \times 10^{-12} \mu_B\) from red giants [19], \(\mu_\nu^{\text{pl}} < 7 \times 10^{-12} \mu_B\) from white dwarfs pulsation [20], and \(\mu_\nu^{\text{pl}} < 2.9 \times 10^{-12} \mu_B\) from white dwarfs cooling [21, 22] at 90% C.L. Even though astrophysical bounds are stronger than the scattering counterparts, the stellar modelling contains various systematic uncertainties [23].

It is of interest to mention that the dark matter (DM) direct detection experiments can also constrain \(\mu_\nu^{\text{eff}}\) via the solar neutrino scattering with electron. In fact, the effective neutrino magnetic moment in Eq. (2) explains the recent Xenon1T data excess if \(\mu_\nu^{\text{eff}} \approx 2 \times 10^{-11} \mu_B\) [24] which is also consistent with the PandaX-II data [25, 26]. The excess has prompted many studies on the neutrino magnetic properties including active-to-active [27–29] and active-to-stereo magnetic moments [30, 31]. Future DM direct detection experiments can further improve the sensitivity [32–35].

Although having multiple experimental ways, the current probe of neutrino electromagnetic moments has intrinsic limitations. In addition to the fact that the aforementioned measurements cannot distinguish the magnetic moment from the electric counterpart, the presence...
of the mixing matrix leads to blind spots in the allowed parameter space [36]. Moreover, existing measurements can not truly probe the magnetic moment at zero momentum transfer but instead have an \( O(\text{keV}) \) threshold. It is desirable to find new ways of exploring the neutrino electromagnetic properties.

In this letter, we present a novel way to probe the neutrino magnetic and electric moments by using the proposed radiative emission of neutrino pair (RENP) [37–39]. Although the RENP transition is yet to be observed, the coherent superradiance has been demonstrated with two-photon emission from hydrogen molecules [40]. Further discussions on experimental realization and background suppression can be found in [41–43].

The RENP process with \( O(\text{eV}) \) momentum transfer is a perfect place for probing light mediator beams [44]. With massless photon being the mediator, the neutrino electromagnetic interactions fall exactly into this category. It allows the possibility of scanning the detailed structure of neutrino magnetic and electric moments in the mass eigenstate basis as a unique probe.

**Electromagnetic Emission of Neutrino Pair** – The radiative emission of a neutrino pair is an atomic transition from an excited state \( |e\rangle \) to the ground state \( |g\rangle \). With the direct transition \( |e\rangle \to |g\rangle + \gamma \) being forbidden, the emission arises at the second order in perturbation theory. The atom first goes from an excited state \( |e\rangle \) to a virtual state \( |v\rangle \) and then falls to the ground state \( |g\rangle \),

\[
|e\rangle \to |v\rangle + \bar{\nu}\nu \to |g\rangle + \gamma + \bar{\nu}\nu. \tag{4}
\]

This spontaneous process is very slow, but can be greatly enhanced by superradiance using a trigger laser beam [37, 45].

The total Hamiltonian describing the reaction contains three parts,

\[
H = H_0 + D_\gamma + H_W. \tag{5}
\]

The zeroth-order Hamiltonian, \( H_0 \), accounts for the electron state, \( H_0(a) = E_a(a) \) where \( a = v, e, \text{ or } g \). With energies \( E_v > E_e > E_g \), the two-step process \( |e\rangle \to |v\rangle \to |g\rangle \) renders \( |e\rangle \) meta-stable. By proper selection of \( |e\rangle \) and \( |g\rangle \), the whole transition is of \( M_1 \times E_1 \) type with one electric (E1) and one magnetic (M1) dipole transitions.

The photon is emitted from the second step, \( |v\rangle \to |g\rangle + \gamma \), by the E1-type electric dipole term \( D_\gamma \) [46]. The corresponding amplitude is,

\[
\langle g|D_\gamma|v\rangle \equiv \mathcal{M}_D e^{-i(q_v + q_\gamma) \cdot x}. \quad \mathcal{M}_D \equiv -d_{gv} \cdot E_0, \tag{6}
\]

where \( \omega \) and \( \mathbf{k} \) are the photon energy and momentum, respectively. The matrix element \( \mathcal{M}_D \) is a product of the dipole operator \( d_{gv} \) for the atomic transition \( |v\rangle \to |g\rangle \) and the photon electric field \( E_0 \).

On the other hand, the neutrino pair emission \( |e\rangle \to |v\rangle + \bar{\nu}_j\nu_i \) during the first step is of M1 type dictated by the weak Hamiltonian \( H_W \). In the SM, the leading contribution comes from the electroweak (EW) charged and neutral currents,

\[
|v\rangle H_W |e\rangle = \mathcal{M}_W e^{-i(p_v + p_e) \cdot x} \tag{7a}
\]

\[
\mathcal{M}_W = -a_{ij} \sqrt{2} G_F \langle v|e\gamma_5|e\rangle (\bar{u}_i \gamma_\mu v_j \gamma_\nu), \tag{7b}
\]

where the prefactor \( a_{ij} \equiv U_{ei} U_{ej}^\ast - \delta_{ij}/2 \) is a function of the neutrino mixing matrix elements \( U_{ei} \). Although both vector and axial-vector currents are present, only the axial part of the electron current contributes since the transition is of the M1 type [46].

Non-zero neutrino magnetic and electric moments in Eq. (1) can also contribute to the M1 type transition as depicted in Fig. 1. For Dirac neutrinos, the amplitude for the magnetic one is \( \langle v|H_M|e\rangle = \mathcal{M}_M e^{-i(p_v + p_e) \cdot x} \) with

\[
\mathcal{M}_M = \mu_B (\mu_\nu)_{ij} q_v q_\gamma^2 \langle v|\sigma^{\mu\nu}|e\rangle \bar{u}_i \sigma_\mu v_j, \tag{8}
\]

while the electric one \( \langle v|H_E|e\rangle = \mathcal{M}_E e^{-i(p_v + p_e) \cdot x} \) has

\[
\mathcal{M}_E = \mu_B (\epsilon_\nu)_{ij} q_v q_\gamma^2 \langle v|\sigma^{\mu\nu}|e\rangle \bar{u}_i \sigma_\nu v_j. \tag{9}
\]

The momentum transfer is defined as \( q \equiv p_v + p_e = (E_{eg} - \omega, -\mathbf{k}) \) with \( E_{eg} \equiv E_e - E_g \). For Majorana neutrinos, there is an extra contribution,

\[
\mathcal{M}_M^{(M)} = \mu_B (\mu_\nu)_{ij} q_v q_\gamma^2 \langle v|\sigma^{\mu\nu}|e\rangle \times \frac{(\mu_\nu)_{ij} \bar{u}_i \sigma_\mu u_j - (\mu_\nu)_{ji} \bar{u}_i \sigma_\mu v_j}{2}, \tag{10}
\]

and similarly for the electric moment case with the vertex replacement \( (\mu_\nu)_{ij} \to (\epsilon_\nu)_{ij} \) and the factor of 1/2 from the Lagrangian interaction normalization of Majorana neutrinos. Using \( (\mu_\nu)_{ij} = (\mu_\nu)_{ji} \) and \( \bar{u}_i \sigma_\mu v_i = -\bar{u}_i \sigma_\mu v_i \), the Majorana case is the same as its Dirac counterpart, \( \mathcal{M}_M^{(M)} = \)}
\( M_M \) for \( i \neq j \). It is also true for the electric moment case, \( M_E^{(M)} = M_E \).

For non-relativistic atomic states, only the spatial components of the atomic currents \( \langle \bar{v} \gamma_\mu \gamma_5 e | e \rangle \) in Eq. (7) and \( \langle \bar{v} | \sigma \gamma_5 e | e \rangle \) in Eq. (9) contribute significantly. They are proportional to the atomic spin operator \( S \) [47], \( \langle \bar{v} \gamma_\mu \gamma_5 e | e \rangle = 2S_{ve} \) and \( \langle \bar{v} | \sigma \gamma_5 e | e \rangle = -2e_{ij}kS_{ve}^k \). The summation over the electron spins \( m_e \) and \( m_o \) follows the identity [48],

\[
\frac{1}{2(2J_v + 1)} \sum_{m_e,m_o} S_{ve}^i S_{ve}^j = (2J_v + 1) \frac{C_{ve}}{3} \delta_{ij}. \tag{11}
\]

The other factors \( J_o \) are the total spin of the excited (\( a = e \)) and virtual (\( a = v \)) states. For \( Y_b \) and \( X_e \), \( (2J_v + 1)C_{ve} = 2 \) [46].

In addition to the SM contribution \( |M_W|^2 \) [38, 44, 46], the neutrino magnetic/electric moment first contributes an spin averaged term,

\[
|M_E^g|^2 = \frac{8C_{ge}(2J_v + 1)}{3} \frac{B^2 \omega^2}{q^4} \times \left[ |(\mu_{\nu})_{ij}|^2, (\epsilon_{\nu})_{ij}|^2 \right] \times \left[ q^2(m_i \pm m_j)^2 - (\Delta m_{ij}^2)^2 + 2q^2 |p_{\nu}|^2 \sin^2 \theta \right], \tag{12}
\]

where \( \theta \) is the angle between the photon and the neutrino momentum. Between the magnetic and electric moments, the mass eigenvalue \( m_i \) flips a sign which comes from the \( \gamma_5 \) matrix in the second term of Eq. (1). It is also possible to have interference between the SM and electromagnetic moment contributions,

\[
|\mathcal{M}_W|\mathcal{M}_E^g = \frac{8C_{ge}(2J_v + 1)}{3} \sqrt{2} G_F a_{ij} \mu_B \times \left( \mu_{\nu}^* \text{ or } \epsilon_{\nu}^* \right) (m_i \pm m_j) (E_{\nu} - E), \tag{13}
\]

with \( E \equiv (E_{eg} - \omega) q^2 + \Delta m_{ij}^2 q^2 / 2q^2 \). However, the interference between magnetic and electric moment contributions is zero after integrating over \( E_{\nu} \).

The differential emission rate [46, 48, 49] is,

\[
d\Gamma_{ij} = \Gamma_0 |\mathcal{M}_W + \mathcal{M}_M| = \frac{\Gamma_0}{(E_{eg} - \omega) \omega} \frac{8G_F^2 C_{ge}(2J_v + 1)}{\pi} \frac{dE_{\nu} \cdot E_{\nu}}{dE_{\nu}}, \tag{14}
\]

with \( E_{eg} = E_{\nu} - E \). The reference decay width \( \Gamma_0 \),

\[
\Gamma_0 = (2J_v + 1) \frac{n^2 C_{ge} G_F^2 |dE_{\nu} \cdot E_{\nu}|}{\pi}, \tag{15}
\]

regulates the total number of decays. In practice only a fraction \( \eta \) of the total volume \( V \) can be enhanced by \( n^2 n \), where \( n_a \) and \( n_r \) are the atomic and photon number densities.

The momentum conservation fixes the value of the integration range to be \( E - \omega \Delta_{ij} / 2 \leq E_{\nu} \leq E + \omega \Delta_{ij} / 2 \) where the relative energy width is \( \Delta_{ij} \equiv \sqrt{q^2 - (m_i + m_j)^2} + \sqrt{q^2 - (m_i - m_j)^2} / q^2 \). Since Eq. (13) is anti-symmetric over \( E_{\nu} - E \), the interference term cannot survive the neutrino energy integration. So the total decay rate contains only three parts \( \Gamma = \Gamma_0 + \Gamma_{ij}(11) \) or \( \Gamma \equiv \Gamma_{ij} + \Gamma_{ij} \) for the Yb atom as a function of the trigger laser frequency \( \omega \). The solid black line corresponds to vanishing neutrino magnetic/electric moment, \( (\mu_{\nu})_{ij} = (\epsilon_{\nu})_{ij} = 0 \), while the colored lines with one non-zero \( (\mu_{\nu})_{ij} \) or \( (\epsilon_{\nu})_{ij} \) are \( 3 \times 10^{-11} \mu_B \) at a time. The black dots correspond to the kinematic thresholds \( \omega_{ij}^{\text{max}} \).

For illustration, we take the normal ordering with \( m_1 = 0.01 \text{ eV} \) hypothesis.

![FIG. 2: The total spectral function \( I \equiv I_W + |(\mu_{\nu})_{ij}|^2 I_M \) (left) or \( I \equiv I_W + |(\epsilon_{\nu})_{ij}|^2 I_E \) (right) for the Yb atom as a function of the trigger laser frequency \( \omega \). The solid black line corresponds to vanishing neutrino magnetic/electric moment, \( (\mu_{\nu})_{ij} = (\epsilon_{\nu})_{ij} = 0 \), while the colored lines with one non-zero \( (\mu_{\nu})_{ij} \) or \( (\epsilon_{\nu})_{ij} \) are \( 3 \times 10^{-11} \mu_B \) at a time. The black dots correspond to the kinematic thresholds \( \omega_{ij}^{\text{max}} \).](image-url)
\( \omega < \omega_{\max} \) receives contribution from all elements \((\mu_\nu)_{ij}\) while the region \(\omega_{33}^{\max} < \omega < \omega_{23}^{\max}\) cannot be affected by \((\mu_\nu)_{33}\). Two independent measurements below and above \(\omega_{33}^{\max}\) can identify a nonzero \((\mu_\nu)_{33}\). Similarly, two independent measurements in the regions of \((\omega_{33}^{\max}, \omega_{23}^{\max})\) and \((\omega_{23}^{\max}, \omega_{12}^{\max})\) can identify \((\mu_\nu)_{23}\). Carrying out this procedure recursively, all the six \((\mu_\nu)_{ij}\) elements can be identified. The process is equivalent for \((\epsilon_\nu)\). For \(m_1 = 0.01\) eV, we take six trigger laser frequencies \(\omega_i = 1.069, 1.07, 1.0708, 1.0712, 1.0716, 1.07164\) eV.

In addition, the sign flip \(m_1 \rightarrow -m_1\) in Eq. (16) allows separating the magnetic momentum contribution from the electric one. The difference \(\Delta \mathcal{I} = \mathcal{I}_M - \mathcal{I}_E \propto 12m_i m_j / q^2\) is relatively significant and can even reach 100% near the threshold. With two measurements, one near and another away from threshold, it is possible to distinguish the magnetic and electric moments. As a conservative estimation, we assign a universal extra frequency \(\omega_0 = 1.068\) eV below \(\omega_{33}^{\max}\) to resolve the ambiguity.

We try to estimate the sensitivity on the neutrino electromagnetic moments by taking the conservative setup with \(n_\gamma = n_\alpha = 10^{21} \text{cm}^{-3}\) [46, 52]. The number of photon events for the trigger laser frequency \(\omega_i\) is

\[
N(\omega) \approx 173 \left( \frac{T}{\text{day}} \right) \left( \frac{V}{100 \text{ cm}^3} \right) \left( \frac{n_\alpha}{10^{21} \text{cm}^{-3}} \right)^3 \mathcal{I}(\omega),
\]

For an exposure of \(T = 10\) days and a volume of \(V = 100\) cm\(^3\) that are equally assigned for all the seven frequencies \(\omega_i\), we expect the SM background events to be \(N_{i=0...6} \approx (120, 107, 87, 82, 79, 39, 2.5)\), respectively, with a total of 512 events.

Fig. 3 shows the 90% C.L. sensitivity curves evaluated in Poisson statistics [44] versus the expected RENP event number that can be translated for future experiments with different configurations as required design targets. The sensitivity can reach \(\langle |(\mu_\nu)_{ij}| \rangle < (1.5 \sim 3.5) \times 10^{-11} \mu_B\) \((\langle |(\epsilon_\nu)_{ij}| \rangle < (2 \sim 9) \times 10^{-11} \mu_B)\) for a conservative number of 500 events and further touch \((0.8 \sim 2) \times 10^{-12} \mu_B\) \((1.1 \sim 5.5) \times 10^{-12} \mu_B)\) for 5000 events. Among the magnetic moment elements, \((\mu_\nu)_{11}\) has the best sensitivity while the worst case is \((\mu_\nu)_{33}\) due to two reasons. First, the \((\mu_\nu)_{33}\) curve is probed at only a single frequency, \(\omega_1 = 1.0688\) eV while \((\mu_\nu)_{11}\) contributes at all the 6 frequencies. So the event statistic is the smallest for \((\mu_\nu)_{33}\) and the largest for \((\mu_\nu)_{11}\). Secondly, the SM background is also larger for lower frequency which makes it harder to probe \((\mu_\nu)_{33}\) in comparison with other parameters. All electric dipole moments have relatively worse sensitivity than their magnetic counterparts. This is because the electric dipole contribution is suppressed by the sign flip of \(m_3\). For larger mass, the suppression is larger. The extreme case happens for the heaviest neutrino with mass \(m_3\) where the sensitivity of \((\epsilon_\nu)_{33}\) is around 2.75 times worse than \((\mu_\nu)_{33}\).

For comparison, we also show the sensitivity of the Borexino experiment [16] (green). The result is translated into neutrino magnetic moments in the mass basis by the collaboration using the constraint \(\mu_\nu^{\text{eff}} < 2.8 \times 10^{-11} \mu_B\), taking \((\epsilon_\nu)_{ij} = 0\) and only one non-zero \((\mu_\nu)_{ij}\) at a time. Although \((\mu_\nu)_{33}\) still has the worst sensitivity, the best one occurs for \((\mu_\nu)_{12} < 2.7 \times 10^{-11} \mu_B\) instead of \((\mu_\nu)_{11}\). The RENP experiment can exceed this limit for all components of \(\mu_\nu\) with 1300 events. The gray band shows the neutrino magnetic moment explanation for the Xenon1T anomaly, \(\mu_\nu^{\text{eff}} \in (0.9 \sim 3.5) \times 10^{-11} \mu_B\) [24]. Our proposed RENP setup can probe this region with 500 events.

For comparison, the astrophysical constraints have even smaller numbers at 90% C.L. with \(\mu_\nu < 2.9 \times 10^{-12} \mu_B\) (yellow-dotted) for white dwarfs [22] and \(\mu_\nu < 2.2 \times 10^{-12} \mu_B\) (red-dotted) for red giants [19]. Although we show these two sensitivities in Fig. 3 for comparison, one needs to keep in mind that there are various uncertainties for astrophysical measurements.

**Conclusions** – With \(\mathcal{O}(\text{eV})\) momentum transfer, the RENP process is sensitive to light mediator including the massless photon. This feature provides a sensitive probe of the neutrino electromagnetic moments. The sensitivity can reach \((1.5 \sim 3.5) \times 10^{-11} \mu_B\) for the magnetic moment and \((2 \sim 9) \times 10^{-11} \mu_B\) for the electric one with 500 events. Further reduction by a factor of 2 is possible with 5000 events. The six components of \(\mu_\nu\) or \(\epsilon_\nu\) in the mass basis appear in different frequency regions which allows frequency scan to identify each component step-
wisely. Once measured, the different dependence on the trigger laser frequency allows separation of the magnetic and electric moments. All these features make the RENP a unique probe of the neutrino electromagnetic moments and the fundamental new physics behind them.

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