EXOTIC BARYONS AND MULTIBARYONS IN CHIRAL SOLITON MODELS.

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Recently observed baryonic resonance with positive strangeness is discussed. Mass and width of this resonance are in agreement with the chiral soliton model predictions. A number of other exotic states are predicted within this approach, some of them are probably observed in experiments. Existence of exotic multibaryons is expected as well, with positive strangeness or beauty, and negative charm. The possibility of binding of heavy anti-flavor is noted.

1 Introduction

In recent experiments [1, 2, 3] the baryonic resonance has been discovered with positive strangeness and rather small width, $\Gamma < 24$ Mev, and subsequent experiment [4] has confirmed this discovery. This resonance is observed independently in different reactions on different experimental setups in Japan, Russia, USA and FRG, therefore only few doubts remain now that it really exists.

This baryon, predicted theoretically in [6, 7, 8] originally called $Z^+ [8]$ and later $\Theta^+$, together with the well known resonances $\Lambda(1520)$ and $\Xi(1530)$ has one of the smallest widths among available baryon resonances. It has necessarily one quark-antiquark pair in its wave function since baryons made of 3 valence quarks only can have negative strangeness, $S < 0$.

Besides this, some hints have been obtained on detector CLAS in reaction of $\pi^+ \pi^-$ electroproduction on protons for existence of new resonance with zero strangeness, positive parity, strong coupling to the $\Delta \pi$ channel and weak to the $N\rho$ [9]. This resonance could belong to one of the multiplets of exotic baryons considered in [10]. Review of experimental situation, methods of detection of exotic and so called cryptoexotic states (states with hidden exotics) before discovery of $\Theta^+$ can be found, for example, in [11].

2 Multiplets of exotic baryons

Exotic, in specific meaning of this word, are baryonic states which cannot be made of $3B$ valence quarks ($B$ is the baryon number) and should contain one or more quark-antiquark pairs. Obviously, any state with positive strangeness is exotic one, as well as states with large enough negative strangeness, $S < -3B$. Besides, for any value of hypercharge $Y$ or strangeness $S < 0$ there are exotic states with large enough isospin, $I > (3B + S)/2$. It is due to the fact that nonzero isospin have only nonstrange quarks $u, d$, and the number of nonstrange valence quarks equals to $3B + S$. The newly found hyperon with positive strangeness and at least one quark-antiquark pair in the wave function is called also the pentaquark state. It is well known that baryons (hadrons, more generally) contain the so-called sea quarks and gluons which carry large fraction of their momenta. But in the $\Theta$-pentaquark the $q\bar{q}$-pair has definite quantum number, antistrangeness, therefore it is in fact valence quark-antiquark pair.

1 These data, as well as [5] where resonance was observed in analysis of neutrino/antineutrino interactions with nuclei, became available after the symposium.
From theoretical point of view the existence of such states was not unexpected. Such possibility was pointed out by a number of people within the quark models [12], as well as in the chiral soliton approach [13, 14]. Analysis of peculiarities of exotic baryons spectra, for arbitrary $B$-numbers, and estimates of energies for exotic $SU(3)$ multiplets was made in [15]. First numerical estimates of the masses of the antidecuplet components were made in [6, 14, 7]. Relatively small mass of the components of antidecuplet, in particular $\Theta^+$, was predicted in a number of papers [6, 14, 7, 8, 16], and strictly speaking, it was not enough grounds for this in [6, 14], because the mass splitting in the octet and decuplet of baryons was not described in these papers. In the paper [8] an assumption was important to provide the prediction $M_{\Theta^+} = 1530$ Mev, that the nucleon resonance $N^*(1.71)$ is the nonstrange component of the antidecuplet. The small width $\Gamma_{\Theta} \simeq 15$ Mev was obtained in [8] only.

Topological soliton models are very economical and effective in predicting the spectra of baryons and baryonic systems with various quantum numbers. The relativistic many-body problem to find the bound states in a system of three, five, etc. quarks and antiquarks is not solved in this way, of course. However, many unresolved questions of principle are circumvented so that calculations of spectra of baryonic states become possible without detalization of their internal structure. In such models baryons or baryonic systems (nuclei) appear as quantized classical (chiral) fields configurations obtained in the procedure of classical energy or mass minimization. Here important role plays the quantization condition [17]

$$Y_R = N_C B/3$$

where $Y_R$ is the "right" hypercharge, or hypercharge of the state in the body-fixed system, $N_C$ - the number of colors of underlying $QCD$, $B$ is baryon number coinciding with topological number characterizing the classical field configuration. For each $SU(3)$ multiplet $(p, q)$ the maximal hypercharge or triality $Y_{max} = (p + 2q)/3$, and relation should be fulfilled evidently $Y_{max} \geq Y_R$, or

$$\frac{p + 2q}{3} \geq \frac{N_C B}{3},$$

which means that

$$p + 2q = 3(B + m)$$

at $N_C = 3$, with $m$ being positive integer. This quantization condition has simple physical interpretation: we start from originally nonstrange configuration which remains nonstrange in the body fixed system. All other components of the $(p, q) SU(3)$ multiplet in the laboratory frame appear as a result of rotation of this configuration in $SU(3)$ configuration space, are described by Wigner final $SU(3)$-rotations functions, and each multiplet should contain original nonstrange state. It is natural to call the multiplets with $m = 0$ the minimal multiplets [15], for $B = 1$ the minimal multiplets are well known octet and decuplet, multiplets with smaller dimension are forbidden due to Guadagnini quantization condition [17] (recall that the number of components of the multiplet $N(p, q) = (p + 1)(q + 1)(p + q + 2)/2$).

The states with $m = 1$ contain at least one additional quark-antiquark pair. Indeed, the maximal hypercharge $Y_{max} = 2$ in this case, or strangeness $S = +1$ for the upper components of such multiplets, i.e. the pair $q\bar{q}$ should be present in the wave function, $q = u$ or $d$. Due to $SU(3)$ invariance of strong interactions all other components of such multiplet should contain additional quark-antiquark pair
[15]. One more restriction appears from the consideration of the isospin, really the components with maximal isospin. It is easily to check, that \{\overline{10}\}, \{27\} and \{35\}-multiplets are the pentaquark states, but the multiplet with maximal \(p\), \{28\}-plet with \((p,q) = (6,0)\) contains already 2 \(q\bar{q}\) pairs, i.e. it is septuquark. This follows from the fact that this multiplet contains the state with \(S = -5\) and the state with \(S = 1\), isospin \(I = 3\). All baryonic multiplets with \(B = m = 1\) are shown in Fig.1.

\[
\begin{align*}
\{\overline{10}\} & : J = 1/2 \\
\{27\} & : J = 3/2; 1/2 \\
\{35\} & : J = 5/2; 3/2 \\
\{28\} & : J = 5/2
\end{align*}
\]

Figure 1: The \(I_3-Y\) diagrams for the baryon multiplets with \(B = 1, m = 1\). Large full circles show the exotic states, smaller - the cryptoexotic states which can mix with nonexotic states from octet and decuplet.

The minimal value of hypercharge is \(Y_{\text{min}} = -(2p+q)/3\), the maximal isospin \(I_{\text{max}} = (p+q)/2\) at \(Y = (p-q)/3\). Such multiplets as \{27\}, \{35\} for \(m = 1\) and all multiplets for \(m = 2\), except the last one with \((p,q) = (9,0)\) in their internal points contain 2 or more states with different values of spin \(J\) (shown by double or triple circles in Fig.1).
3 The mass formula

In the collective coordinates quantization procedure one introduces the angular velocities of rotation of skyrmion in the $SU(3)$ configuration space, $\omega_k$, $k = 1,...8$: $A^i(t)A(t) = -i\omega_k\lambda_k/2$, $\lambda_k$ being Gell-Mann matrices, the collective coordinates matrix $A(t)$ is written usually in the form $A = A_{SU2}exp(\nu\lambda_3)A_{SU2}exp(\rho\lambda_8/\sqrt{3})$. The corresponding contribution to the lagrangian is quadratic form in these angular velocities, with momenta of inertia, isotopical (pionic) $\Theta_\pi$ and flavor, or kaonic $\Theta_K$ as coefficients [17]:

$$L_{rot} = \frac{1}{2}\Theta_\pi(\omega_1^2 + \omega_2^2 + \omega_3^2) + \frac{1}{2}\Theta_K(\omega_4^2 + ... + \omega_8^2) - \frac{N_c B}{2\sqrt{3}}\omega_8. \quad (4)$$

The expressions for these moments of inertia as functions of skyrmion profile are presented below. The quantization condition (1) discussed above follows from the presence of linear in angular velocity $\omega_8$ term in (4) originated from the Wess-Zumino-Witten term in the action of the model [18].

The hamiltonian of the model can be obtained from (4) by means of canonical quantization procedure [17]:

$$H = M_{cl} + \frac{1}{2\Theta_\pi}\vec{R}^2 + \frac{1}{2\Theta_K} \left[ C_2(SU_3) - \vec{R}^2 - \frac{N_c^2 B^2}{12} \right], \quad (5)$$

where the second order Casimir operator for the $SU(3)$ group, $C_2(SU_3) = \sum_{a=1}^8 R_a^2$, with eigenvalues for the $(p,q)$ multiplets $C_2(SU_3)_{p,q} = (p^2 + p q + q^2)/3 + p + q$, for the $SU(2)$ group, $C_2(SU2) = \vec{R}^2 = R_1^2 + R_2^2 + R_3^2 = J(J + 1) = I_R(I_R + 1)$.

The operators $R_\alpha = \partial L/\partial \omega_\alpha$ satisfy definite commutation relations which are generalization of the angular momentum commutation relations to the $SU(3)$ case [17]. Evidently, the linear in $\omega$ terms in lagrangian (4) are cancelled in hamiltonian (5). The equality of angular momentum (spin) $J$ and the so called right or body fixed isospin $I_R$ used in (5) takes place only for configurations of the "hedgehog" type when usual space and isospace rotations are equivalent. This equality is absent for configurations which provide the minimum of classical energy for greater baryon numbers, $B \geq 2$.

For minimal multiplets ($m = 0$) the right isospin $I_R = p/2$, and it is easy to check that coefficient of $1/2\Theta_K$ in (5) equals to

$$K = C_2(SU_3) - \vec{R}^2 - \frac{N_c^2 B^2}{12} = N_C B/2, \quad (6)$$

for arbitrary $N_C$. So, $K$ is the same for all multiplets with $m = 0$ [15], see Table 1- the property known long ago for the $B = 1$ case [17]. For nonminimal multiplets there are additional contributions to the energy proportional to $m/\Theta_K$ and $m^2/\Theta_K$, according to (5)[15]. It means that in the framework of chiral soliton approach the "weight" of quark-antiquark pair is defined by parameter $1/\Theta_K$, and this property of such models deserves better understanding.

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\[\text{Note:}\] It should be kept in mind that for $N_C$ different from 3 the minimal multiplets for baryons differ from octet and decuplet. They have $(p,q) = (1, (N_C - 1)/2), (3, (N_C - 3)/2), ... , (N_C, 0)$. 

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\(N(p, q)\), Casimir operator \(C_2(SU_3)\), spin \(J = I_R\), coefficient \(K\) for first two values of \(J\) for minimal \((m = 0)\) and nonminimal \((m = 1, 2)\) multiplets of baryons.

It follows from Table 1 that for each nonzero \(m\) the coefficient \(K(J_{\text{max}})\) decreases with increasing \(N(p, q)\), e.g. \(K_{3/2}(35) < K_{3/2}(27) < K_{1/2}(10)\). The following differences of the rotation energy can be obtained easily:

\[
M_{27,J=3/2} - M_{10} = \frac{3}{2\Theta_\pi},
\]

This relation is known since 1984 [17].

\[
M_{27,J=3/2} - M_{10} = \frac{3}{2\Theta_K},
\]

as it was stressed in [8],

\[
M_{27,J=3/2} - M_{10} = \frac{3}{2\Theta_\pi} - \frac{1}{2\Theta_K},
\]

\[
M_{35,J=5/2} - M_{27,J=3/2} = \frac{5}{2\Theta_\pi} - \frac{1}{2\Theta_K}.
\]

If the relation took place \(\Theta_K \ll \Theta_\pi\) then \(\{27\}\)-plet would be lighter than antidecuplet, and \(\{35\}\)-plet would be lighter than \(\{27\}\). In realistic case \(\Theta_K\) is approximately twice smaller than \(\Theta_\pi\) (see Table 2, next section), and therefore the components of antidecuplet are lighter than components of \(\{27\}\) with same values of strangeness. Beginning with some values of \(N(p, q)\) coefficient \(K\) increases strongly, as can be seen from Table 1, and this corresponds to the increase of the number of quark-antiquark pairs by another unity. The states with \(J < J_{\text{max}}\) have the energy considerably greater than that of \(J_{\text{max}}\) states, by this reason they could contain also greater amount of \(q\bar{q}\)-pairs.

The formula (5) is obtained in the rigid rotator approximation which is valid if the profile function of the skyrmion and therefore its dimensions and other properties are not changed when it is rotated in the configuration space. It is necessary for this, that the rotation time in the configuration space, \(\tau_{\text{rot}}\) is smaller than the time of its deformation \(\tau_{\text{deform}}\) under influence of the forces due to presence of the terms in lagrangian violating the flavor symmetry, i.e. \(m_k/m_{\pi} > 1, F_K/F_{\pi} > 1\), see also next Section. Rotation time can be estimated easily, \(\tau_{\text{rot}} \sim \pi/\omega\) with \(\omega \sim \sqrt{C_2(SU_3)/\Theta_K}\). It is more difficult to estimate \(\tau_{\text{deform}}\), one can state only that it is greater than

| \((p, q)\) | \(N(p, q)\) | \(m\) | \(C_2(SU_3)\) | \(J = I_R\) | \(K(J_{\text{max}})\) | \(K(J_{\text{max}} - 1)\) |
|-----------|-----------|------|----------------|-----------------|-----------------|-----------------|
| \((1, 1)\) | \{8\}     | 0    | 3              | 1/2             | 3/2             |                 |
| \((3, 0)\) | \{10\}    | 0    | 6              | 3/2             | 3/2             |                 |
| \((0, 3)\) | \{10\}    | 1    | 6              | 1/2             | 3/2+3           |                 |
| \((2, 2)\) | \{27\}    | 1    | 8              | 3/2; 1/2        | 3/2+2           | 3/2+5           |
| \((4, 1)\) | \{35\}    | 1    | 12             | 5/2; 3/2        | 3/2+1           | 3/2+6           |
| \((6, 0)\) | \{28\}    | 1    | 18             | 5/2             | 3/2+7           |                 |
| \((1, 4)\) | \{33\}    | 2    | 12             | 3/2; 1/2        | 3/2+6           | 3/2+9           |
| \((3, 3)\) | \{64\}    | 2    | 15             | 5/2; 3/2; 1/2   | 3/2+4           | 3/2+9           |
| \((5, 2)\) | \{81\}    | 2    | 20             | 7/2; 5/2; 3/2   | 3/2+2           | 3/2+9           |
| \((7, 1)\) | \{80\}    | 2    | 27             | 7/2; 5/2        | 3/2+9           | 3/2+16          |
| \((9, 0)\) | \{55\}    | 2    | 36             | 7/2             | 3/2+18          |                 |
the time needed for light to cross the skyrmion, \( \tau_{\text{travel}} \sim 2R_H \). So, the rigid rotator approximation is valid if \( \pi \Theta_K < 2R_H \sqrt{C_2(SU3)} \). Numerically \( \pi \Theta_K \approx 8 \text{GeV}^{-1} \) and \( 2R_H \sqrt{C_2(SU3)} \approx 12 \text{GeV}^{-1} \) for decuplet and antidecuplet of baryons.

The alternative is the “soft” or slow rotator approximation when it is assumed that for each value of the angle of rotation in “strange” direction \( \nu \) there is enough time for the soliton to be deformed under influence of the flavor symmetry breaking forces [19]. The realistic case is intermediate one, but for the baryons the rigid rotator approach is more justified, due to above estimate. With increasing \( B \)-number the slow rotator approach becomes more actual. The dependence of the moments of inertia on \( \nu \) is given by following expressions [19, 20]:

\[
\Theta_K(\nu) = \frac{1}{8} \int (1 - c_f) \left\{ F_K^2 \left( 1 - \frac{2 - c_f s_\nu^2}{2} \right) + F_\pi^2 \frac{2 - c_f}{2} s_\nu^2 + \frac{1}{c_f} \left[ f'^2 + \frac{2s_f^2}{r^2} \right] \right\} \, d^3r, \tag{12}
\]

\[
\Theta_\pi(\nu) = \frac{1}{6} \int s_\nu^2 \left[ F_\pi^2 + (F_K^2 - F_\pi^2)c_f s_\nu^2 + \frac{4}{c_f} \left( f'^2 + \frac{s_f^2}{r^2} \right) \right] \, d^3r. \tag{13}
\]

These formulas hold for configurations of hedgehog type described by one profile function \( f \), \( c_f = \cos f \), \( s_f = \sin f \); \( s_\nu = \sin \nu \). \( \Theta_K(\nu) \) decreases and \( \Theta_\pi(\nu) \) increases with increasing \( \nu \). Rigid rotator approximation corresponds to \( \nu = 0 \) since we start from nonstrange \( SU(2) \)-skyrmion. The decay constants \( F_\pi, F_K \) are taken from experiment: \( F_\pi \approx 186 \text{MeV} \); the model parameter (Skyrme constant) \( e \) is close to 4. The dependence on \( F_K \) in (12,13) appears due to nonadiabatic (time dependent) terms in the lagrangian which can have also other manifestations.

### 4 Spectrum of baryonic states

Expressions (5), (6) and numbers given in Table 1 are sufficient to calculate the spectrum of baryons without mass splitting inside of \( SU(3) \)-multiplets, as it was made e.g. in [14, 15]. The mass splitting due to the presence of flavor symmetry breaking terms plays a very substantial role [17, 7, 10]:

\[
H_{SB} = \frac{1 - D_{88}^{(8)}}{2} \Gamma_{SB} \tag{14}
\]

where the \( SU(3) \) rotation function \( D_{88}^{(8)}(\nu) = 1 - 3s_\nu^2/2 \),

\[
\Gamma_{SB} = \frac{2}{3} \left[ \left( \frac{F_K^2}{F_\pi^2} m_K^2 - m_\pi^2 \right) \Sigma + (F_K^2 - F_\pi^2) \tilde{\Sigma} \right] \tag{15}
\]

\[
\Sigma = \frac{F_\pi^2}{2} \int (1 - c_f) \, d^3r, \tag{16}
\]

\[
\tilde{\Sigma} = \frac{1}{4} \int c_f \left( f'^2 + \frac{2s_f^2}{r^2} \right) \, d^3r,
\]

kaon and pion masses \( m_K, m_\pi \) are taken from experiment. The quantity \( SC = < s_\nu^2 > /2 = < 1 - D_{88}^{(8)} > /3 \) averaged over the baryon \( SU(3) \) wave function defines its strangeness content. Without configuration mixing, i.e. when flavor symmetry breaking terms in the lagrangian are considered as small perturbation, \( < s_\nu^2 >_0 \) can be expressed simply in terms of the \( SU(3) \) Clebsh-Gordan coefficients. The values
of $<s_b^2>_{0}$ for the octet, decuplet, antidecuplet and some components of higher multiplets are presented in Table 2. In this approximation the components of $\{10\}$ and $\{1\}$ are placed equidistantly, and splittings of decuplet and antidecuplet are equal.

The spectrum of states with configuration mixing and diagonalization of the hamiltonian in the next order of perturbation theory in $H_{SB}$ is given in Table 2 (the code for calculation was presented by H.Walliser). The calculation results in the Skyrme model with only one adjustable parameter - Skyrme constant $e$ ($F_\pi = 186$ Mev - experimentally measured value) are shown as variants A and B. The values of $<s_b^2>$ become lower when configuration mixing takes place, and equidistant spacing of components inside of decuplet and especially antidecuplet is violated, see also Fig.2.

It should be stressed here that the chiral soliton approach in its present state can describe the differences of baryon or multibaryon masses [7, 8, 10, 19]. The absolute values of mass are controlled by loop corrections of the order of $\mathcal{N}_c^3 \sim 1$ which are estimated now for the case of $B = 1$ only [21]. Therefore, the value of nucleon mass in Table 2 and Fig.2 is taken to be equal to the observed value.

| Baryon | $<s_b^2>_0$ | A  | B  | C  | Data |
|--------|-------------|----|----|----|------|
| $\Lambda$ | 0.60         | 6.175 | 5.556 | 5.61 | -   |
| $\Sigma$ | 0.73         | 2.924 | 2.641 | 2.84 | -   |
| $\Xi$ | 0.80         | 1.369 | 1.244 | 1.45 | -   |

Table 2. Values of masses of the octet, decuplet, antidecuplet and some components of higher multiplets (with nucleon mass subtracted). A: $e = 3.96$; B: $e = 4.12$; C: fit with parameters $\Theta_K$, $\Theta_\pi$ and $\Gamma_{SB}$ [10], which are shown as well.

As it can be seen from Table 2, the agreement with data for pure Skyrme model with one parameter is not so good, but the observed mass of $\Theta^-$ is reproduced with some reservation. To get more reliable predictions for masses of other exotic states the more phenomenological approach was used in [10] where the observed value $M_{\Theta} = 1.54 Gev$ was included into the fit, and $\Theta_K$, $\Gamma_{SB}$ were the variated parameters (variant C in Table 2 and Fig.2). The position of some components of $\{27\}$, $\{35\}$ and $\{28\}$ plents is shown as well.

The variant D shown in Fig.2 takes into account the term in $H_{SB}$ which
The best description of the octet and decuplet masses was obtained at $\Delta = 0.4$. Such contribution was included also in [8] where the linear in hypercharge term $H_{SB}^Y = \beta Y$ with $\beta \approx -156 \, \text{MeV}$ plays an important role. Such term is absent in approach [7, 10].

It looks astonishing at first sight that the state $\Theta^+$ containing strange antiquark is lighter than nonstrange component of antidecuplet, $N^*(I = 1/2)$. But it is easy to understand if we recall that all antidecuplet components contain $q\bar{q}$ pair: $\Theta^+$ contains 4 light quarks and $\bar{s}$, $N^*$ contains 3 light quarks and $s\bar{s}$ pair with some weight, $\Sigma^*$ in [10] contains $u, d, s$ quarks and $s\bar{s}$, etc.

The mass splitting inside of decuplet is influenced essentially by its mixing with $\{27\}$-plet components [10], see Fig.1, which increases this splitting considerably - the effect ignored in [8]. The mixing of antidecuplet with the octet of baryons has some effect on the position of $N^*$ and $\Sigma^*$, the position of $\Theta^*$ and $\Xi^*_{3/2}$ is influenced by mixing with higher multiplets [10].

The component of $\{35\}$-plet with zero strangeness and $I = J = 5/2$ is of special interest because it has the smallest strangeness content (or $\delta^2$) - smaller than nucleon and $\Delta$. As a consequence of isospin conservation by strong interactions it can decay into $\Delta\pi$, but not to $N\pi$ or $N\rho$. According to the results presented in Table 2, the...
components of \{28\} -plet containing 2 \(q\bar{q}\) pairs, have the mass considerably greater than that of other multiplets on Fig.1.

All baryonic states considered here are obtained by means of quantization of soliton rotations in \(SU(3)\) configuration space, and have therefore positive parity. A qualitative discussion of the influence of other (nonzero) modes - vibration, breathing - as well as references to corresponding papers can be found in [10, 16]. The realistic situation can be more complicated than somewhat simplified picture presented here, since each rotation state can have vibrational excitations with characteristic energy of hundreds of \(MeV\).

If the matrix element of the decay \(\Theta^+ \to KN\) is written in a form

\[
M_{\Theta \to KN} = g_{\Theta KN} \bar{u}_N \gamma_5 u_{\Theta} \phi_{K}^1
\]

with \(u_N\) and \(u_{\Theta}\) - bispinors of final and initial baryons, then the decay width equals to

\[
\Gamma_{\Theta \to KN} = \frac{g_{\Theta KN}^2 \Delta_M^2 - m_K^2}{8\pi M^2} p_K^{em}
\]

where \(\Delta_M = M - m_N\), \(M\) is the mass of decaying baryon, \(p_K^{em}\) - the kaon momentum in the c.m. frame. For the decay constant we obtain then \(g_{\Theta KN} \approx 4.4\) if we take the value \(\Gamma_{\Theta \to KN} = 10 MeV\) as suggested by experimental data [1]-[5]. This should be compared with \(g_{\pi NN} \approx 13.5\). So, some suppression of the decay \(\Theta \to KN\) takes place, but not large and understandable, according to [8, 22].

5 Exotic multibaryons

There is no difference of principle, within the chiral soliton approach, between baryons and multibaryons, as it was demonstrated in previous sections. The latter are quantized configurations of chiral fields which correspond to the minima of classical energy for arbitrary baryon number. The equality between body-fixed isospin and spin of the quantized state, specific for hedgehog-type configuration, does not hold anymore.

It is easily to understand that minimal (nonexotic) multiplets for \(B = 2\) coincide with \(m = 1\) multiplets for \(B = 1\), i.e. they are antidecuplet, including the deuteron - isosinglet state, \{27\}-plet, including the isotriplet NN-state (so called singlet deuteron), \{35\} and \{28\}-plets. Similarly, the minimal multiplets for \(B = 3\) are those for \(B = 1\) and \(m = 2\), see Table 1.

Here we show several examples of lowest exotic multiplets with \(m = 1\): the \{35\}-plet for \(B = 2\), the \{25\}-plet for \(B = 3\) and \{80\}-plet for \(B = 4\), Fig.3. There is isodoublet of positive strangeness dibaryons, \(2D_S^+, \ 2He_S^{++}\) with minimal quark contents \((\bar{s}3u4d), (\bar{s}4u3d)\), which have the energy about 600\(MeV\) above 2\(N\)-threshold, according to calculation performed in [20] in the slow rotator approximation. The spectrum of all minimal dibaryons was calculated in [20] as well.

For \(B = 3\) there is positive strangeness tribaryon (isosinglet) \(3He_S^{++}\), its quark content is \((\bar{s}5u5d)\). The position of the components of this multiplet is not calculated yet. One can state, however, in the spirit of the version of the bound state model developed in [23, 24], that the difference of the masses of positive strangeness isosinglet and ground state of \(3He\)

\[
M_{3He_S} - M_{3He} = \bar{\omega}_{S,B=3} + O(1/N_c)
\]

\(^3\)The chemical symbol is ascribed according to the total charge of the baryonic state.
with $\bar{\omega}_S$ - the energy of antistrangeness excitation. For $B = 4$ there is positive strangeness isodoublet $^4He_{S}^{++} - ^4Li_{S}^{++}$ with minimal content ($\bar{s}u \bar{d}7u6d$) and $(\bar{s}7u6d)$. Similarly, we have

$$M_{^4He_S} - M_{^4He} = \bar{\omega}_{S,B=4} + O(1/N_c)$$  \hspace{1cm} (21)

The nonstrange components of such exotic multiplets (i.e. those with $Y = B$) have the difference of masses

$$M_{Y=B} - M_{B,ground st.} = \bar{\omega}_{S,B} + O(1/N_c),$$  \hspace{1cm} (22)

and further $\omega_{S,B}$ should be added for each unit of strangeness, but the whole method [23] works when strangeness is not large ($1 - 2$ units, not more). The energies of flavor and antiflavor excitation for multiskyrmion were calculated in [24] for baryon numbers up to 22, for $B > 8$ within rational map approximation [25], using the results obtained in comprehensive paper [26].

Their characteristic feature is that they depend slightly on $B$-number. It is known that the difference between antiflavor and flavor excitation energies [23]

$$\bar{\omega}_{F,B} - \omega_{F,B} = N_C B/(4\Theta_{F,B}),$$  \hspace{1cm} (23)

for any flavor (strangeness, charm or beauty) and baryon number. Since $\Theta_{F,B} \sim B$ roughly [24], this difference depends weakly on $B$-number and scales like $N_C^0 \sim 1$ [23]. Numerically $\bar{\omega}_S$ is close to 600MeV with small variations [24]. However, the $1/N_c$ corrections are not negligible, and this question deserves further study.

The qualitative treatment becomes very easy when the kaon mass is large enough. In this case one obtains [24]

$$\omega_{S,B} \simeq \frac{\bar{m}_K}{2} \sqrt{\frac{\Sigma_B}{\Theta_{K,B}}} - \frac{3B}{8\Theta_{K,B}}$$  \hspace{1cm} (24)

$$\bar{\omega}_{S,B} \simeq \frac{\bar{m}_K}{2} \sqrt{\frac{\Sigma_B}{\Theta_{K,B}}} + \frac{3B}{8\Theta_{K,B}}$$  \hspace{1cm} (25)

with $\bar{m}_K^2 = F_K^2 m_K^2/F_\pi^2 - m_\pi^2$, $\Sigma_B$ and $\Theta_{K,B}$ are given by expressions similar to (16) and (12). The ratio $r_{K,B} = \Sigma_B/\Theta_{K,B}$ decreases slightly with increasing $B$, it can be proved rigorously that $r_{K,B} < 4F_K^2/F_\pi^2$ [24], therefore we have always $\omega_K \leq m_K$, and strangeness is bound for any $B$-number, with slightly increasing binding.

For antistrangeness the treatment simplifies if $F_K = F_\pi$, and we take this equality for the moment. Then

$$\bar{\omega}_{K,B} \simeq \frac{m_K}{2} r_{K,B,1/2} + \frac{3B}{8\Theta_{K,B}}.$$  \hspace{1cm} (26)

Numerically $r_{K,B,1/2}$ decreases from 1.53 for $B = 1$ to 1.48 for $B = 4$ and $3B/(8\Theta_{K,B})$ is about 180 MeV [24] for $e = 4.12$, but really the first ratio depends on $e$ very weakly. So, we have

$$\bar{\omega}_K \simeq 0.76m_K + 180\text{ MeV}$$

for $B = 1$ and very close relations for other $B \leq 4$. Evidently, with increasing $m_K$ antistrangeness also becomes bound, similar to strangeness (more precise, for $m_K > \sim 750$ MeV). Corrections $\sim 1/N_c$ and $F_K/F_\pi = 1.22$ increase the critical value of $m_K$. These conclusions agree with those made recently in [27].
Anticharm and antibeauty have chances to be bound: we obtain the corresponding excitation energies \( \bar{\omega}_c \sim (1.75 - 1.8) \text{Gev} \) for \( B \) between 4 and 1, \( F_D/F_\pi \simeq 1.5 \), and for antibeauty \( \bar{\omega}_b \sim (4.9 - 5.0) \text{Gev} \) for the ratio \( F_b/F_\pi \sim 2 \). So, these energies are smaller than corresponding meson masses, but to make more definite conclusions the \( \sim 1/N_c \) corrections should be treated carefully.

The positive strangeness dibaryons should decay into \( K^0N \), tribaryons - into \( K^3N \) final states, etc. with a width of same order of magnitude as \( \Gamma_\Theta \). There are also exotic states with negative strangeness: dibaryons with \( S = -4 \), isospin \( I = 2 \), with electric charge in the interval from \( Q = -3 \) to \( Q = +1 \), and tribaryons with \( S = -5 \), \( I = 3 \) and charge from \( -4 \) up to \( +2 \), see Fig. 3. Tetrabaryons with \( S = -7 \) can have charge in the interval \( -5 \) to \( +2 \). As usually, it would be difficult to produce such states (one of possibilities are heavy ion collisions), but their detection could be easier: they decay mainly into \( \Xi \)-hyperons and pions. The large amount of exotic multibaryons looks embarrassing at first sight. One should keep in mind, however, that many of them are too broad (those which have energy by some hundreds of Mev above threshold) and can be hardly distinguishable from continuum.

To conclude this section, note that there are other predictions of states in chiral soliton models which are exotic in the common meaning of this word: for example, charmed or beautiful hypernuclei bound stronger than strange hypernuclei \[28\]. The supernarrow electromagnetically decaying dibaryon with width about \( \sim 1K \) ev below the \( NN\pi \) threshold \[29\] was observed in two experiments \[30, 31\], but not confirmed in \[32\] in the mass interval below 1914 Mev. Its searches certainly deserve further efforts.

## 6 Conclusions and prospects

The mass and width of recently detected baryon with positive strangeness, \( \Theta^+ \) are in agreement with predictions of the topological (chiral) soliton model \[6, 7, 8\] \footnote{As it was noted, from rigoristic point of view one could doubt in each of these predictions, therefore experimental confirmation was necessary.}. Possibly, another exotic baryon with zero strangeness has been observed \[9\]. To be sure that the observed \( \Theta^+ \) belongs to antidecuplet, the measurement of its spin and parity is necessary first of all, as well as establishing its partners in \( SU(3) \) multiplet (antidecuplet).

The searches for the state \( \Theta^+ \in \{27\} \) with isospin \( I = 1 \) are of interest. The double charged state \( \Theta^{++} \) could appear as a resonance in \( K^+p \) system. Since this state is by \( \sim (120 - 160) \text{ Mev} \) heavier than \( \Theta^+ \), its width should be at least \( 3 - 4 \) times greater than that of \( \Theta^+ \). The absence of such resonance could be a serious problem for the whole chiral soliton approach.

Let us note also that the mass splitting inside of antidecuplet obtained in \[7, 10\] is considerably smaller than in \[8\] where it is about 540 Mev. In addition, the deviation from equidistant law is large in \[7, 10, 6\] as a consequence of configuration mixing being taken into account. As a result, the value of mass of the hyperon with isospin \( I = 3/2 \), \( \Xi_{3/2} \) obtained in \[10\] is considerably smaller than in \[8\]. It is worth noting that its mass estimate made in \[22\] within antiquark-diquark-diquark model is close to our result \[10\]. The value of the mass of \( \Sigma^+ \in \{\bar{10}\} \) also is lower in \[10\] and is more close to \( \Sigma^+(1770) \) than to \( \Sigma^+(1880) \).

Many exotic resonances of interest have large values of isospin, therefore they cannot be observed in reactions of pion or kaon scattering on nucleons, but could
be seen in reactions of two and more pions (kaons) production, similar to reaction
studied in [9]. It could be attractive a possibility to identify the state of mass
$1.72\text{ Gev}$ observed in [9] with a component of $\{35\}$-plet with $S = 0, I = 5/2$. But the
isospin selection rule for reaction of electroproduction [9] with one-photon exchange
makes such identification difficult. Another possibility noted already in [9], is that
it is cryptoexotic component of $\{27\}$-plet with isospin $3/2$ and mass about $1.76\text{ Gev}$,
according to [10].

Of course, there is no contradiction between chiral soliton approach and the
quark (or quark-diquark, etc.) picture of baryons and baryon resonances, as it is
stated in some papers. Both approaches are dual, the first one describes baryons or
baryonic systems from large enough distances and allows to calculate such character-
istics where the details of internal structure of baryons are not essential, one of
such characteristics is just the mass of baryons.

The consequences of discovery of new baryon resonance are considered in
several recent papers [22],[23],[33]-[35] and others, many of them have been reviewed
and analysed in [35]. Hopefully, the results obtained in [1]-[5] and [9] open new
interesting chapter in physics of baryon resonances, and its new pages can be devoted
also to studies of baryonic systems with exotic properties, including (anti)charm and
beauty quantum numbers.

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Figure 3: The $I_3 - Y$ diagrams for the lowest exotic multibaryons: \{$35\}$-plet of dibaryons, \{$28\}$-plet of tribaryons and \{$80\}$-plet of tetrabaryons, with $m = 1$. Large full circles show the exotic states, smaller - the cryptoexotic states.