THE ENERGY DEPENDENCE OF THE CENTROID FREQUENCY AND PHASE LAG OF THE QUASIPERIODIC OSCILLATIONS IN GRS 1915+105

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Abstract

We present a study of the centroid frequencies and phase lags of quasi-periodic oscillations (QPOs) as functions of photon energy for GRS 1915+105. It is found that the centroid frequencies of the 0.5–10 Hz QPOs and their phase lags are both energy dependent, and there exists an anticorrelation between the QPO frequency and phase lag. These new results challenge the popular QPO models, because none of them can fully explain the observed properties. We suggest that the observed QPO phase lags are partially due to the variation of the QPO frequency with energy, especially for those with frequency higher than 3.5 Hz.

Key words: accretion, accretion disks – black hole physics – stars: individual (GRS 1915+105) – stars: oscillations

1. Introduction

GRS 1915+105 is a low-mass X-ray black hole binary showing a rich diversity of X-ray light curve morphology and complex timing phenomena (Morgan et al. 1997; Cui 1999; Belloni et al. 2000; Ji et al. 2003). The variability of the source can be reduced to transitions between three basic states: a hard state corresponding to the non-observability of the innermost part of the accretion disk (state C), and two softer states with a fully observable disk but different temperatures (states B and A; Belloni et al. 1997a, 1997b, 2000). According to the appearance of light curves and color–color diagrams, the behaviors of the source can be further classified into 12 classes (Belloni et al. 2000). In addition to the above flux and spectral variabilities, abundant quasi-periodic oscillations (QPOs) are also observed in this system. The fundamental frequency of its QPO ranges from mHz to several hundred Hz, and some QPOs are detected up to the third harmonic frequencies (Morgan et al. 1997; Cui 1999). According to their (fundamental) frequencies, the QPOs of GRS 1915+105 can be divided into three classes: the low-frequency (0.5–2 Hz) QPOs, the intermediate-frequency (2–10 Hz) QPOs, and the high-frequency (10–100 Hz) QPOs. These QPOs occur in different states of the accretion disk of the source (Chakrabarti & Manickam 2000). The low-frequency and the high-frequency QPOs are often observed during the soft state of GRS 1915+105 (state B). The low-frequency QPOs are considered to be connected with disk instability, as the rapid disappearance and refill of the inner accretion disk (Belloni et al. 2000). The high-frequency QPO has been proposed to arise in the close vicinity of the black hole and been thought to reflect the general relativity properties of the black hole (Cui et al. 1998). Conversely, the intermediate-frequency QPOs only appear in state C of the source, and never appear in state B. Although it is suggested that the 0.5–2 Hz QPOs may originate from the compact jet (Fender 2006), the phase-resolved spectra and variation of the phase lags with frequencies show that the 0.5–10 Hz QPOs are all from the inner disk (Miller & Homan 2005; Reig et al. 2000). They are linked to the properties of the accretion disk since their centroid frequencies and fractional rms are correlated with the thermal flux and the apparent temperature of the disk (Markwardt et al. 1999; Trudolyubov et al. 1999; Muno et al. 1999; Sobczak et al. 2000). It is possible that the QPOs trace the Keplerian motion at the inner radius of the observable disk (Belloni et al. 2000). We note that the other two micro-quasars XTE J1550−564 (Wijnands et al. 1999; Cui et al. 2000) and GRO J1655−40 (Remillard et al. 1999; Cui et al. 1999) have different types of QPOs too. Study of the QPO properties in GRS 1915+105 can provide important information of the accretion flow in this source as well as other microquasars.

Complex phase/time lags have also been observed for the QPOs of GRS 1915+105. Cui (1999) found that the hard lags of the low-frequency QPOs in GRS 1915+105 alternate from negative to positive values as the frequency increases from the fundamental to higher harmonic frequencies, and the high-frequency QPOs always show hard phase lag. The intermediate-frequency QPOs were studied by Lin et al. (2000a) and Reig et al. (2000). They found that the hard lags of these QPOs, different from the phase lags of the low-frequency QPOs, alternate from positive to negative values as the frequency increases from 0.5 to 10 Hz. The similar timing characteristics were also observed in XTE J1550−564 (Cui et al. 2000). According to the phase lags of GRS 1915+105, the intermediate-frequency QPOs can be further classified into three types: (1) the 0.5–2 Hz QPOs, whose hard lags at the fundamental and first harmonic frequencies are both positive; (2) the 2–4.5 Hz QPOs, whose hard lags are negative at the fundamental frequency but positive at the first harmonic frequency; and (3) the 4.5–10 Hz QPOs, which show negative phase lags at both the fundamental and harmonics (Lin et al. 2000a; Reig et al. 2000).

It is suggested that the 0.5–10 Hz QPOs observed during state C provide a link between the optically thick accretion disk and the Comptonization region (Muno et al. 2001). Since the accretion disk has temperature structures, and the Comptonization region could upscatter the lower energy photons into higher energy ones, the energy dependence of different types of QPOs and their phase lags can set strong constraints on the current accretion disk models and provide another avenue to explore the origin of phase lags of GRS 1915+105. According to the drifting blob model, the centroid frequency of a QPO is a function of the photon energy. The energy dependence of the fractional rms and phase lags of the QPOs have been widely studied (Morgan et al. 1997; Cui et al. 1999; Lin et al. 2000a; Reig et al. 2000; Rodriguez et al. 2004). For ~4 Hz QPO of XTE J1550−564,


Table 1

| ObsID | Date       | Exposure (s) | \(f_c\) (Fitting Range) |
|-------|------------|--------------|-------------------------|
| 21-02 | 1996 Jul 7 | 2000         | 7.988 ± 0.051 (4.5–12)  |
| 22-00 | 1996 Jul 11| 2520         | 3.479 ± 0.005 (1.8–6.5) |
| 22-01 | 1996 Jul 11| 3320         | 2.770 ± 0.005 (1.7–9)   |
| 22-02 | 1996 Jul 11| 3312         | 2.554 ± 0.006 (0.16–4.2)|
| 23-00ab| 1996 Jul 14| 6640         | 3.544 ± 0.005 (12–10)   |
| 24-00ab| 1996 Jul 16| 6544         | 2.266 ± 0.006 (1.6–6.5)|
| 24-00cd| 1996 Jul 16| 6544         | 2.543 ± 0.005 (1.6–6.5)|
| 25-00 | 1996 Jul 19| 3328         | 1.125 ± 0.002 (0.2–1.8) |
| 27-00 | 1996 Jul 26| 8960         | 0.631 ± 0.002 (0.4–2)   |
| 28-00 | 1996 Aug 3 | 9984         | 0.966 ± 0.002 (0.2–2.9) |
| 29-00a | 1996 Aug 10| 2952         | 1.677 ± 0.004 (8.0–4.6) |
| 29-00b | 1996 Aug 10| 3392         | 1.866 ± 0.004 (8.0–4.6) |
| 29-00c | 1996 Aug 10| 3392         | 1.967 ± 0.004 (8.0–4.6) |
| 30-00 | 1996 Aug 18| 9960         | 4.871 ± 0.011 (1.8–10)  |
| 31-00a | 1996 Aug 25| 2208         | 4.092 ± 0.007 (2.6–6)   |
| 31-00b | 1996 Aug 25| 2912         | 4.434 ± 0.011 (2.5–8.8) |
| 31-00c | 1996 Aug 25| 2912         | 3.505 ± 0.008 (2.2–7.4) |
| 32-00 | 1996 Aug 31| 7394         | 5.960 ± 0.019 (3–10)    |
| 49-00 | 1997 Aug 8 | 3408         | 2.923 ± 0.007 (1.4–15)  |

Notes. 21-02: 10408-01-21-02; ab: a+b; *49-00: 20402-01-49-00; \(f_c\) is the QPO frequency at PCA channel 2–13 keV.

Sriram et al. (2007) found a frequency difference at two energy bands (2–20 keV and 20–50 keV). Choudhury et al. (2005) found that the centroid frequency of the 3 ± 4 Hz QPOs at 2–7 keV is higher than those in 20–50 keV in GRS 1915+105. However, as a parameter of the theoretical model of the QPOs, the relations between the QPO centroid frequency and photon energy in GRS 1915+105 and other microquasars have not been well studied yet.

Since photons with different energies are usually from regions with different physical properties, the energy dependence of QPOs can provide additional information that may be critical for a better understanding of the QPO origins. Therefore, we present in this paper a study of the energy dependence of the centroid frequencies of the QPOs. In Section 2, we describe the data and how they were analyzed. The main results are given in Section 3, their physical implications are discussed in Section 4, and Section 5 is a short summary of this work.

2. DATA REDUCTION AND ANALYSES

To evaluate the energy dependence of the QPO centroid frequency, we select the RXTE observations published in Morgan et al. (1997), which show the 0.5–10 Hz QPOs and have enough exposure time for each observation to evaluate the QPOs. In these observations (Table 1), GRS 1915+105 was in class \(\chi\) of state C in the classification by Belloni et al. (2000) or in the plateau state (Fender 2001), because these observations did not show strong variability, HR\(_{f}(13–60 keV/2–5 keV) > 0.1\), and the original disk contribution is expected to be very soft here. The timing and spectral properties of GRS 1915+105 in those observations have been widely studied previously. For example, Lin et al. (2000a) and Reig et al. (2000) have investigated the phase-frequency dependence of the QPOs, while Muno et al. (2001) and Trudolyubov et al. (1999) have reported its energy spectral properties. The QPO centroid frequency is relatively stable over each RXTE epoch in these observations. Therefore, they are very suitable to study the energy dependence of the centroid frequencies of the QPOs.

The data are reduced by using the FTOOLS package as described by Qu et al. (2001). The timing analyses include calculations of the power density spectrum (PDS) and the cross-power spectrum (CPS), using the binned mode and event mode data, respectively. According to the data modes of the observations, we extract the light curves of GRS 1915+105 with a time resolution of ~4 ms (2–8 s) in seven PCA energy bands defined in Table 2. Among these energy bands, the hardest energy band (Channels 50–103) has the lowest statistics, and its mean count rate is still about 320 counts s\(^{-1}\) with the model predicted background count rate of 16 counts s\(^{-1}\). In every energy channel, the quality factor \((Q = f/\Delta f_{\text{FWHM}})\) of the 6 Hz QPO that presented in the observation is greater than 4, permitting a detailed study on the relations between the QPO frequency, photon energy, and phase lag.

The PDS is fitted with a model including a power law to represent the continuum plus one or two Lorentzians to represent the QPOs. However, it is difficult to obtain a statistically acceptable fit to the PDS exactly between 1/16 and 16 Hz. For an example, we fit the 1/16 to 16 Hz PDS of observation (10408-01-32-00) that shows a 6 Hz QPO with a power law plus three Lorentzians, the reduced \(\chi^2\) is 6.5. If we limit the frequency range as 4–8 Hz, the fit is improved apparently, with \(\chi^2 < 1.6\). Thus, in order to get the accurate centroid frequency and the full width at half-maximum (FWHM) of the QPO in 2–13 keV, the frequency range is selected to cover the QPO and to make the reduced \(\chi^2\) close to the minimum (see Table 1), similar to that used by Cui (1999). The PDSs in the other energy bands are fitted in the same frequency range by the model forenamed. The errors of the model parameters are derived by varying the parameters until \(\Delta \chi^2 = 1\).

The phase lags \(\phi\) of the QPOs are calculated by averaging the phase lags over the frequency range from \(f_{\text{QPO}} – \text{FWHM}/2\) to \(f_{\text{QPO}} + \text{FWHM}/2\). Their errors are estimated from the standard deviation of the real and imaginary parts of the CPSs (Cui et al. 1997). Figure 1 shows two example PDSs in energy bands of 2–5 keV and 18–38 keV as well as the CPSs between these two energy bands. Apparently, the centroid frequency of the QPO around ~6 Hz has a higher value in the harder energy band. The inset in this figure also shows that the fitting method above-mentioned gives reasonable fits to the PDSs.

3. RESULTS

We find that the centroid frequencies of QPOs are related to photon energy. This relation evolves from a negative correlation to a positive one when the QPO frequency increases. Figure 2(a) shows such relations for a few typical QPOs. The energy dependence of the centroid frequency of the QPO can be fitted by a power law, and the fitted results are listed in Table 3. For QPOs with the centroid frequencies lower than 3 Hz, the centroid frequency decreases monotonically with photon energy, but...
the correlation becomes weaker with the centroid frequency increases. For QPOs with the centroid frequencies higher than 3 Hz, the centroid frequency increases significantly with photon energy, and the correlation also becomes stronger as shown by the correlation coefficients. However, for QPOs around 3 Hz, their centroid frequencies do not have a monotonic evolutionary trend with photon energy, while the values of the correlation coefficients turn over their sign from negative to positive.

The relation between phase lag and photon energy is opposite to that between QPO centroid frequency and photon energy. The results of fitting and correlation coefficient are listed also in Table 3 and displayed in Figure 2(b). When the QPO frequency is around 1 Hz, the phase lag is positively correlated with photon energy, and the two quantities become negatively correlated when the QPO frequency reaches above 3.5 Hz. These results are similar to the ones of Lin et al. (2000a) and Reig et al. (2000).

In Figure 3, we plot the centroid frequency variation $\Delta f$ and phase lag $\phi$ versus the QPO frequency for all the QPOs we detected in the observations. Both $\Delta f$ and phase lag are calculated between two energy bands of 2–5 keV and 13–18 keV only. It is shown that $\Delta f$ increases with the QPO frequency, while the phase lag decreases, indicating a negative correlation between $\Delta f$ and $\phi$ for the QPOs.

The negative correlation between $\Delta f$ and $\phi$ holds not only among different QPOs but also within a QPO. The $\Delta f$ and $\phi$ calculated between 2–5 keV and the other five higher energy channels (Table 2) for a few typical QPOs are shown in Figure 4.

The results show that $\Delta f$ and $\phi$ have an anticorrelation for QPOs with centroid frequency lower than 2 Hz or higher than...
energy bands 2–5 keV and 13–18 keV. The inset show the
results are listed in Table 4.

The phase lag $\phi^*$ is obtained between 13–18 keV and 2–5 keV energy bands; reduced $\chi^2 = \chi^2/(6-2)$.

| ObsID | $f_c$ (Hz) | $E_{\text{QPO}}$ (keV) | $\chi^2$ | Cor | $\Gamma$ (x10^{-3}) | $E_{\text{QPO}}$ (keV) | $\chi^2$ | Cor |
|-------|-----------|-----------------|----------|-----|-----------------|-----------------|----------|-----|
| 21-02 | 7.988 ± 0.051 | 45.0 ± 6.7 | 6.1 | 0.98 | -0.655 ± 0.026 | 1.17 ± 0.33 | 1.75 | -0.97 |
| 32-00 | 5.960 ± 0.019 | 49.0 ± 3.1 | 76.5 | 0.97 | -0.511 ± 0.020 | 0.97 ± 0.13 | 5.98 | -0.96 |
| 31-00a | 4.092 ± 0.007 | 3.1 ± 1.3 | 3.2 | 0.78 | -0.312 ± 0.023 | 0.69 ± 0.39 | 1.01 | -0.92 |
| 23-00 | 3.544 ± 0.005 | 4.3 ± 0.9 | 2.1 | 0.81 | -0.278 ± 0.015 | 0.78 ± 0.42 | 0.75 | -0.95 |
| 22-00 | 3.479 ± 0.005 | 3.9 ± 1.0 | 2.7 | 0.83 | -0.244 ± 0.021 | 0.80 ± 0.50 | 0.35 | -0.75 |
| 49-00 | 2.923 ± 0.007 | -1.2 ± 1.8 | 3.8 | -0.29 | -0.103 ± 0.011 | 0.45 ± 0.35 | 0.06 | -0.87 |
| 22-01 | 2.770 ± 0.005 | -1.1 ± 1.4 | 6.4 | -0.71 | -0.121 ± 0.011 | 0.61 ± 1.6 | 0.08 | -0.91 |
| 22-02 | 2.554 ± 0.006 | -6.6 ± 1.4 | 3.4 | -0.69 | -0.096 ± 0.013 | 0.63 ± 5.0 | 0.02 | -0.92 |
| 29-00c | 1.967 ± 0.004 | -3.2 ± 1.4 | 1.1 | -0.99 | 0.034 ± 0.013 | 0.62 $^{+0.00}_{-0.01}$ | 0.01 | 0.57 |
| 25-00 | 1.125 ± 0.002 | -4.2 ± 1.3 | 1.94 | -0.997 | 0.253 ± 0.011 | 0.65 ± 0.26 | 0.453 | 0.93 |
| 28-00 | 0.966 ± 0.002 | -3.2 ± 0.2 | 0.48 | -0.94 | 0.340 ± 0.013 | 0.66 ± 0.15 | 1.35 | 0.94 |
| 27-00 | 0.631 ± 0.002 | 0.52 ± 1.5 | 2.58 | -0.80 | 0.489 ± 0.024 | 0.067 ± 0.027 | 2.0 | 0.94 |

**Notes.** Cor: correlation coefficient; $\Gamma$ is the power index of the relation between $\Delta f$ or $\phi$ and photon energy $E$. The phase lag $\phi^*$ is obtained between 13–18 keV and 2–5 keV energy bands; reduced $\chi^2 = \chi^2/(6-2)$.

**Figure 3.** QPO frequency differences and phase lag vs. QPO frequency for energy bands 2–5 keV and 13–18 keV. The inset show the $\Delta f$ of the QPOs with frequencies less than 3 Hz.

3.5 Hz, and no correlation for QPOs between 2 and 3.5 Hz. We calculate the correlation coefficients and fit the relation with a linear function to all the QPOs we detected in this work. The results are listed in Table 4.

### 4. DISCUSSIONS

For the first time we find the centroid frequency evolution with photon energy for the 0.5–10 Hz QPOs of GRS 1915+105. We also find that the QPO phase lag is correlated with both photon energy and QPO frequency. The QPO centroid frequencies are shown to have an anticorrelation with the phase lags, as shown in Figure 4. These results set strong constraints on the current models of the origin of QPOs and phase lags in black hole binaries and make a direct challenge for theorists to explain the new observable phenomena.

Various models have been proposed to explain the timing phenomena in black hole binaries. It is generally believed that
the x-ray radiation of a black hole binary is contributed by three components: the soft x-ray radiation from the accretion disk, the hard components from the Compton cloud and/or jet (McClintock & Remillard 2006). The QPOs are suggested to be related to the accretion disk of the compact object, while the phase lags to the electron cloud. Particularly, the 0.5–10 Hz QPOs of GRS 1915+105 could occur in the inner region of the disk and are associated with disk instabilities (see van der Klis 2006 and McClintock & Remillard 2006 for review). However, GRS 1915+105 displays the number of X-ray states, many models for microquasar behavior are based on a limited number of its X-ray states. Based on our observational results, we only discuss the following four models commonly used to describe the origin of QPOs in compact objects: (1) the global disk oscillation (GDO) model; (2) the radial and orbital oscillation model (ROOM); (3) the accretion flow instability model (AFIM); and (4) the drift blob model (DBM).

4.1. Constraints from the Energy-dependent QPO Frequency

For orbital and epicyclic frequency models, the particles moving around the compact objects could have different oscillating frequencies in the inner region of the accretion disk: orbital, radial, and vertical epicyclic frequencies. Damping, or the superposition of many local frequencies can turn the intrinsically periodic disk oscillations into QPOs or broad noise. In the GDO model, the disk oscillation is a vertical mode, and the GDO frequency is expected to be also independent of the photon energy and should be seen in all the energy bands that disk emits (Titarchuk 2000). Thus, the GDO model cannot explain the observational phenomena of the intermediate-frequency QPOs.

For ROOM, the oscillation frequency is a function of the disk radius/temperature (Nowak & Wagoner 1993; Nowak 1994). The QPO frequency will vary with energy because photons with different energies are from different radii. These models can therefore explain qualitatively the evolution of the QPO frequency with energy. However, the emission from the disk is thermal. There should be a break of the QPO power represented by the rms at higher energy if the oscillations occur in the inner region of the disk, since the high energy emission is from the Compton cloud (Rodríguez et al. 2004). However, such a break has not been detected up to ~30 keV, making the disk oscillation model of the QPOs unlikely. Meanwhile, ROOM cannot explain the various correlations between the centroid frequency and photon energy for different QPOs. This model needs to be greatly modified in order to fit the observational results.

In the accretion flow instability model (AFIM), locally at each radius the disk fluctuates on different instability timescales, and the oscillations propagate in the disk (Nowak 1994). In this scenario, emission from the inner region of the disk tends to have a higher QPO frequency and a harder spectrum. This model can naturally explain the observed positive correlation between the centroid frequency and photon energy for the QPOs with frequency higher than ~3.5 Hz. But the observed negative correlation for QPOs less than 2 Hz contradicts the model prediction.

The energy dependence of the QPO frequency is expected by the drift blob model (DBM; Böttcher & Liang 1998, 1999; Hua et al. 1997). In this model, the Keplerian motion of the blobs could manifest itself in a QPO observationally, and any radial drift of the blobs would cause the QPO frequency to increase with energy. Similar to AFIM, DBM can explain the observed properties of the QPOs higher than ~3 Hz, but not for the QPOs less than 2 Hz. The relation between model and observed phenomena is summarized in Table 5.

Of the models we considered above, none can fully explain the energy dependences of QPOs obtained in this paper. The energy dependences of the QPO centroid frequencies provide additional information for theoretical models. And the new observable properties of the QPOs of the GRS 1915+105 also make a challenge to the new models of the QPOs, i.e., the new theoretical model should not only reproduce or explain the observable phenomena in this paper, but also explain the other properties such as the energy dependence of the rms and the phase lags (Cui 1999, Rodríguez et al. 2004).

4.2. Phase Lag Results and Their Constraints on Models

To explain the observational phase (time) lags in the compact X-ray objects, a lot of models are proposed (see Cui 1999; Poutanen 2001 for review, and reference therein). Among them, the so-called standard model and the perturbation propagation model are the two major ones. In the standard model, the time lag is considered as the diffusion timescale of the photon passing through the Comptonization region (Cui 1999; Poutanen 2001).

Table 4
Relation Between Frequency Differences (Δf) and Phase Lags (φ) of the QPOs

| ObsID | fc | kobs | χ² | Cor | kf |
|-------|----|------|----|-----|----|
| 21-02 | 7.988 ± 0.051 | −1.08 ± 0.30 | 1.47 | −0.99 | −0.79 |
| 32-00 | 5.960 ± 0.019 | −0.694 ± 0.093 | 0.677 | −0.99 | −1.1 |
| 31-00a | 4.092 ± 0.007 | −1.1 ± 7.4 | 4.15 | −0.90 | −1.5 |
| 23-00 | 3.544 ± 0.005 | −12.9 ± 6.8 | 0.7 | −0.94 | −1.8 |
| 22-00 | 3.479 ± 0.005 | −8.0 ± 8.0 | 2.0 | −0.59 | −1.8 |
| 49-00 | 2.923 ± 0.007 | 0.5 ± 8.4 | 0.10 | −0.08 | 2.1 |
| 22-01 | 2.770 ± 0.005 | 2.3 ± 6.4 | 0.15 | 0.48 | 2.3 |
| 22-02 | 2.554 ± 0.006 | 3.5 ± 17 | 0.03 | 0.64 | 2.5 |
| 29-00c | 1.967 ± 0.004 | −3.3 ± 18 | 0.01 | −0.78 | −3.2 |
| 25-00 | 1.125 ± 0.002 | −20.3 ± 8.5 | 1.38 | −0.90 | −5.6 |
| 28-00 | 0.966 ± 0.002 | −59 ± 13 | 0.86 | −0.97 | −6.5 |
| 27-00 | 0.631 ± 0.002 | −65 ± 15 | 9.1 | −0.81 | −10 |

Notes. kf = 2π/fQPO, kobs is the fitted slope of the relation between φ and Δf. Reduced χ² = χ²/(5–2).

Table 5
Model and Observation

| Model | Prediction (E ~ Δf) | Observation | Reference |
|-------|-------------------|-------------|-----------|
| GDO   | Δf = 0            | No          | Titarchuk 2000 |
| ROOMs | Δf ~ f(E)         | Partial     | Nowak & Wagoner 1993; Nowak 1994 |
| AFIMs | Δf ~ f(E)         | Partial     | Nowak 1994 |
| DBM   | Δf ~ f(E)         | Partial     | Böttcher & Liang 1998, 1999; Hua et al. 1997 |

Notes. GDO: the global disk oscillation. ROOMs: the radial and orbital oscillation models. AFIMs: the accretion flow instability models. DBM: the drift blob model. Partial: the models only explain partial observed phenomena.
The hard lag is the result of the process in which the soft (seed) photons gain energy from the hotter electron corona. For the perturbation propagation model, the soft phase lags can be explained by assuming that perturbations propagate from the inner disk to the outer disk. There may also be perturbations propagating inward from the outer edge of the disk, which generate hard phase lags (Lin et al. 2000b).

The observed phase lag behaviors in this paper challenge the above two models. Since the standard model only predicts hard lags, the measured soft lags in GRS 1915+105 are incompatible with the model (see also Cui 1999), and furthermore, the observed large time lag values require a huge corona size (~3 × 10^{10} cm) that is physically unrealistic (Hua et al. 1999). Although the perturbation propagation model can explain the observed soft and hard lags qualitatively, it is not clear how the propagation direction of the perturbation transits when QPO frequency passes 2 Hz, at which the phase lag of the QPO changes sign as revealed in this work. Probably the phase lags are of multiple origins.

The anticorrelations between the phase lags \( \phi \) and frequency variation (\( \Delta f \)) of the QPOs (see also Figure 4), and between \( \Delta f \) and \( \log(E) \), imply that the phase lags could be caused by the change of the QPO frequency, which may give a new approach to explain the phase lags in the compact X-ray objects. To better illustrate this point, we derived the relation between the phase lag and \( \Delta f \) as follows.

According to the definition of the cross-correlation function, \( \text{ccf}(\tau) = \int_{-\infty}^{\infty} h(t)(t-\tau)dt \), the cross power spectrum is
\[
\text{CPS}(v) = \int_{-\infty}^{\infty} \text{ccf}(\tau) e^{2\pi i vt} d\tau = S^*(v) H(v),
\]
where \( S^*(v) = \left| S^* \right| e^{-i\phi_0(v)} \) and \( H(v) = |H|e^{i\phi_0(v)} \). If the light curves with oscillations in two different energy bands could be described by \( s(t) = s(f t) \) and \( h(t) = h((f + \Delta f) t) \), then
\[
\text{CPS}(v) = \frac{1}{f + \Delta f} S^* \left( \frac{v}{f + \Delta f} \right) H \left( \frac{v}{f + \Delta f} \right) = A e^{i\phi(v)},
\]
where \( A \) is a constant. So the phase lag is
\[
\phi(v) = \phi_h \left( \frac{v}{f + \Delta f} \right) - \phi_s \left( \frac{v}{f} \right) \\
\approx \phi_h \left( \frac{v}{f} \left( 1 - \frac{\Delta f}{f} \right) \right) - \phi_s \left( \frac{v}{f} \right) \\
\approx \phi_h \left( \frac{v}{f} \right) + \phi_h |_f \times \frac{\Delta f}{f^2} - \phi_s \left( \frac{v}{f} \right).
\]

If let \( v = f \), the phase lag of the QPO with frequency \( f \) is
\[
\phi(f) = \phi_h (f) - \phi_s (f) + \phi_h (f) \times \frac{\Delta f}{f} \\
= \phi_s (f) + \phi_f (\Delta f),
\]
where \( \phi_s (f) = \phi_h (f) - \phi_s (f) \) is phase lag due to physical processes such as Comptonization and \( \phi_f (\Delta f) = \phi_i (f) \times \frac{\Delta f}{f} \) is caused by the change of the oscillation frequency of the QPO.

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