An Alternative Approach to Semi-inclusive DIS
and A Model Independent Determination of $D_{u,d,s}^{\pi^+}$

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Abstract

We discuss how to extract the polarized parton densities and fragmentation functions in a model independent way from SIDIS, and present results for $D_{u,d,s}^{\pi^+}$.

There are two types of processes that give information on polarized parton densities in nucleon. The polarized asymmetry in DIS of longitudinally polarized leptons on longitudinally polarized nucleons:

$$
\Delta A^{DIS}_N = \frac{2y(2-y)}{1 + (1-y^2)} \frac{\Delta \sigma^{DIS}_N}{\sigma^{DIS}_N} = \frac{\sum_{q,q'} e_q^2 \Delta q_i(x, Q^2)}{\sum_{q,q'} e_q^2 q_i(x, Q^2)}
$$

measures (both in LO and NLO in QCD) the combinations:

$$
\Delta u + \Delta \bar{u}, \quad \Delta d + \Delta \bar{d}, \quad \Delta s + \Delta \bar{s}, \quad \Delta G.
$$

Thus in principle, we cannot determine separately the valence and sea quark distributions, independently of the precision and the amount of data available.

The polarized asymmetry of semi-inclusive DIS (SIDIS) of longitudinally polarized leptons on longitudinally polarized nucleons

$$
\vec{e} + \vec{N} \rightarrow e + h + X,
$$

when a final hadron $h$ is detected

$$
\Delta A^h_N = \frac{2y(2-y)}{1 + (1-y^2)} \frac{\Delta \sigma^h_N}{\sigma^h_N} = \frac{\sum_{q,q'} e_q^2 \Delta q_i(x, Q^2) D^h_i(z, Q^2)}{\sum_{q,q'} e_q^2 q_i(x, Q^2) D^h_i(z, Q^2)}
$$

has the advantage that it measures the $\Delta q$ and $\Delta \bar{q}$ separately. However, a knowledge of the fragmentation functions (FFs) is required. The $D^h_q$ are supposed to be known from the inclusive process $e^+e^- \rightarrow h + X$, $h = \pi, K, p, ...$

$$
\sigma^h \propto \sum e_i^2 (D^h_q + D^h_{\bar{q}}).
$$

In principle, similar to DIS (both in LO and NLO), only the combination $D^h_q + D^h_{\bar{q}}$ can be determined. However, for the interpretation of SIDIS data, $D^h_q$ and $D^h_{\bar{q}}$ are needed

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separately. Usually additional theoretical assumptions about favoured and unfavoured transitions are made. This leads to rather different parametrizations of the FFs obtained from analyzing the $e^+e^-$ data \[2\]. In addition, in order to reduce the parameters, different assumptions about the polarized sea densities are made.

In this talk we consider what we can learn from SIDIS - both with polarized and unpolarized initial particles - without assuming any knowledge about FFs and without any assumptions about the polarized sea $\Delta \bar{q}$. We shall use information that comes directly from measurable quantities in the same experiment. A somewhat similar approach has been considered by S. Manayenkov \[1\].

2. As known quantities in our approach we consider the unpolarized parton densities and the polarized non-singlet combination $\Delta q_3 = (\Delta u + \Delta \bar{u}) - (\Delta d + \Delta \bar{d})$ that is determined directly from DIS:

$$g^p_1(x, Q^2) - g^n_1(x, Q^2) = \frac{1}{6} \Delta q_3 \otimes \left(1 + \frac{\alpha_s(Q^2)}{2\pi} \delta C_q\right)$$

either in LO or NLO. For $\Delta q_3$ we do not use its value obtained from the parametrizations of $(\Delta u + \Delta \bar{u})$ and $(\Delta d + \Delta \bar{d})$ since the latter are influenced by the less known quantities $\Delta s$ and $\Delta G$.

We work with three measurable quantities: the polarized asymmetries of DIS and SIDIS, $\Delta A_{N}^{DIS}$ and $\Delta A_{N}^{SIDIS}$, and the ratio

$$R_{N}^h = \frac{\sigma_{N}^h}{\sigma_{N}^{DIS}}$$

of the unpolarized SIDIS normalized to the inclusive DIS cross section.

3. In LO the polarized valence quark densities can be obtained, without any knowledge of the FFs assuming only isospin invariance, from simple algebraic equations. For $\pi^\pm$ production we have:

$$\Delta u_V(x, Q^2) = \frac{1}{15} \left\{4(4u_V - d_V)\Delta A_p^{\pi^+-\pi^-} + (4d_V - u_V)\Delta A_n^{\pi^+-\pi^-}\right\}$$

$$\Delta d_V(x, Q^2) = \frac{1}{15} \left\{4(4d_V - u_V)\Delta A_n^{\pi^+-\pi^-} + (4u_V - d_V)\Delta A_p^{\pi^+-\pi^-}\right\}$$

Analogously, $\Delta u_V$ and $\Delta d_V$ can be obtained also from $K^\pm$ and $\Lambda, \bar{\Lambda}$ production \[3\].

Once we know the valence quark densities, we can determine the SU(2) breaking of the polarized sea, without requiring any knowledge of the unknown $\Delta \bar{q}$ and $\Delta G$. We have:

$$[\Delta \bar{u}(x, Q^2) - \Delta \bar{d}(x, Q^2)]_{LO} = \frac{1}{2} \left[\Delta q_3(x, Q^2) + \Delta d_V(x, Q^2) - \Delta u_V(x, Q^2)\right]_{LO}.$$  

Given that $u_V(x, Q^2)$ and $d_V(x, Q^2)$ are well determined from inclusive DIS one can proceed further to obtain the non-singlet combinations of FF’s $D^h_q - D^h_{\bar{q}} \equiv D^{h-h}_{q}$. For $\pi^\pm$ we have

$$D^\pi^+-\pi^-(z, Q^2) = \frac{18 (F^p_1)_{LO} R^\pi^+-\pi^-_p}{4u_V - d_V}, \quad \text{or} \quad D^\pi^+-\pi^-(z, Q^2) = \frac{18 (F^n_1)_{LO} R^\pi^+-\pi^-_n}{4d_V - u_V}.$$
SU(2) and C invariance determines the other non-singlet combinations:
\[ D_d^{\pi^+\pi^-} = -D_u^{\pi^+\pi^-}, \quad D_s^{\pi^+\pi^-} = D_s^{\pi^+\pi^-} = 0. \] (11)

The analogous relations for \( D_d^{K^+K^-} \) and \( D_q^{\Lambda\bar{\Lambda}} \) are given in [3].

Via \((K^+, K^-)\) or \((\Lambda, \bar{\Lambda})\) production we can test the conventionally made assumptions \( s = \bar{s} \) and \( \Delta s = \Delta \bar{s} \). For the unpolarized case, assuming \( D_d^{K^+K^-} = 0 \) we have:
\[
(s - \bar{s})D_s^{K^+K^-} = 18 (F_1^p)_{LO} R_p^{K^+K^-} - 4u_V D_u^{K^+K^-} = 18 (F_1^n)_{LO} R_n^{K^+K^-} - 4d_V D_u^{K^+K^-}. \] (12)

As for the \( K \)-meson system \( s \) is a valence quark, \( D_s^{K^+K^-} \) should not be a small quantity. Then a zero value of \((12)\) would imply \( s - \bar{s} = 0 \).

Having thus determined \((s - \bar{s})D_s^{K^+K^-}\) we can proceed to determine \((\Delta s - \Delta \bar{s})D_s^{K^+K^-}:\)
\[
\Delta A_p^{K^+K^-} = \frac{4\Delta u_V D_u^{K^+K^-} + (\Delta s - \Delta \bar{s})D_s^{K^+K^-}}{4u_V D_u^{K^+K^-} + (s - \bar{s})D_s^{K^+K^-}}
\]
\[
\Delta A_n^{K^+K^-} = \frac{4\Delta d_V D_u^{K^+K^-} + (\Delta s - \Delta \bar{s})D_s^{K^+K^-}}{4d_V D_u^{K^+K^-} + (s - \bar{s})D_s^{K^+K^-}}, \] (13)

\( D_u^{K^+K^-} \) is assumed to be known through the analogue of \((11)\). The analogous relations for \( \Lambda, \bar{\Lambda} \) production can be found in [3].

The relations in this paragraph are true in LO approximation only. A characteristic feature of these expressions, \((8)\) and \((10)\), is that the RH sides, which, in principle, depend on \( (x, z, Q^2) \), should depend only on two of these, either \( (x, Q^2) \) – eq. \((8)\), or \( (z, Q^2) \) – eq. \((10)\), so that there is an independence of the third variable, which we call \( \text{passive} \) variable. Each expression should be tested for a dependence on the passive variable.

4. The analysis in NLO is more complicated as the simple products in the LO expressions are replaced by convolutions. This means that instead of solving algebraic equations, in NLO we have to deal with integral equations, i.e. one needs a fit to the data.

Using charge conjugation invariance one obtains, for semi-inclusive pion production
\[
R_p^{\pi^+\pi^-} = \frac{[4u_V - d_V][1 + \otimes(\alpha_s/2\pi)C_{qq} \otimes]D_u^{\pi^+\pi^-}}{18 F_1^p [1 + 2\gamma(y) R_p]}
\]
\[
R_n^{\pi^+\pi^-} = \frac{[4d_V - u_V][1 + \otimes(\alpha_s/2\pi)C_{qq} \otimes]D_u^{\pi^+\pi^-}}{18 F_1^n [1 + 2\gamma(y) R_n]}. \] (14)

The only unknown function in these expressions is \( D_u^{\pi^+\pi^-} (z, Q^2) \), which evolves as a non-singlet and does not mix with other FF’s. A \( \chi^2 \) analysis of either or both of the equations \((14)\) should thus determine \( D_u^{\pi^+\pi^-} \) in NLO without serious ambiguity.

Assuming \( D_u^{\pi^+\pi^-} \) is now known, one can then determine \( \Delta u_V \) and \( \Delta d_V \) in NLO via the equations
\[
\Delta A_p^{\pi^+\pi^-} = \frac{(4\Delta u_V - \Delta d_V)[1 + \otimes(\alpha_s/2\pi)C_{qq} \otimes]D_u^{\pi^+\pi^-}}{(4u_V - d_V)[1 + \otimes(\alpha_s/2\pi)C_{qq} \otimes]D_u^{\pi^+\pi^-}}. \] (15)
\[ \Delta A_{\pi^+\pi^-}^n = \frac{(4\Delta d_V - \Delta u_V)[1 + \otimes(\alpha_s/2\pi)\Delta C_{qq}\otimes]D_{u}^{\pi^+\pi^-}}{(4d_V - u_V)[1 + \otimes(\alpha_s/2\pi)C_{qq}\otimes]D_{u}^{\pi^+\pi^-}} \] (16)

where, of course, \( \Delta u_V \) and \( \Delta d_V \) evolve as non-singlets and do not mix with other densities. Eqs. (15) and (16) determine the densities \( \Delta u_V \) and \( \Delta d_V \) in NLO without any assumptions about the less known polarized gluon and sea densities.

Once \( \Delta u_V \) and \( \Delta d_V \) are known in NLO we can calculate \( [\Delta \bar{u}(x, Q^2) - \Delta \bar{d}(x, Q^2)]_{NLO} \) from (9), using the NLO expression (6) for \( \Delta q_3 \).

The expressions for \( s - \bar{s} \) and \( \Delta s - \Delta \bar{s} \) in NLO can be found in [3].

5. Recently the HERMES group has published very precise data for unpolarized SIDIS for \( \pi^\pm \) production [4]. There are 3 independent FFs \( D_{u}^{\pi^+}, D_{d}^{\pi^+} \) and \( D_{s}^{\pi^+} \). The SIDIS data provide two equations for them, and a third piece of information is thus required. We have shown [5] that, though the individual flavoured \( D_{q}^{\pi^+} \) in \( e^+e^- \to \pi^\pm + X \) are poorly known, the singlet combination \( D_{\Sigma}^{\pi^+} = 2 \left( D_{u}^{\pi^+} + D_{d}^{\pi^+} + D_{s}^{\pi^+} \right) \) is very well determined at the \( Z^0 \) peak. Then, evolving this down to \( Q^2 \) of the SIDIS experiment, it allows us to extract \( D_{u}^{\pi^+}, D_{d}^{\pi^+} \) and \( D_{s}^{\pi^+} \) without any assumptions about favoured and unfavoured transitions. The results are given in the Figure. The uncertainties, mainly affecting \( D_{\Sigma}^{\pi^+} \), come from the evolution of \( D_{\Sigma}^{\pi^+} \) at \( Q^2 \approx m_Z^2 \) down to \( D_{\Sigma}^{\pi^+} \) at \( Q^2 \approx \text{few}(GeV)^2 \), as it involves mixing with the poorly known \( D_{G}^{\pi^+} \). Both \( D_{u}^{\pi^+} \) and \( D_{d}^{\pi^+} \) are quite accurately determined. Quite surprising is the magnitude of the “unfavoured” fragmentation \( d \to \pi^+ \).

References

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Figure 1: $D_{u}^{\pi^{+}}$, $D_{d}^{\pi^{+}}$ and $D_{s}^{\pi^{+}}$ extracted as explained in the text.