Harmonic Oscillator Prepotentials in SU(2) Lattice Gauge Theory

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Abstract

We write the SU(2) lattice gauge theory Hamiltonian in (d+1) dimensions in terms of prepotentials which are the SU(2) fundamental doublets of harmonic oscillators. The Hamiltonian in terms of prepotentials has $SU(2) \otimes U(1)$ local gauge invariance. In the strong coupling limit, the color confinement in this formulation is due to the U(1) gauge group. We further solve the $SU(2) \otimes U(1)$ Gauss law to characterize the physical Hilbert space in terms of a set of gauge invariant integers. We also obtain certain novel gauge invariant operators in terms of the above oscillators. The corresponding prepotential formulation of SU(N) lattice gauge theory is also simple and discussed.

1 Introduction

Gauge theories form the underlying framework for both strong and electroweak interactions. Therefore, it is desirable and important to understand them in terms of their most fundamental structures. In the simplest case of SU(2) gauge group, the fundamental operators are the SU(2) doublets. All higher representations are built out of them. Motivated by this simple fact, we reformulate the SU(2) lattice gauge theory in terms of harmonic oscillators (prepotentials) which, under gauge transformations, transform as SU(2) doublets. The above construction is based completely on the symmetry arguments. The SU(2) gauge invariance demands that the physical states satisfy the Gauss law constraint. On lattice, the Gauss law involves only electric fields (not vector potentials) and is linear. The non-linearity in the Gauss law in the corresponding continuum theory reflects itself in the SU(2) commutation relations of the lattice electric fields. On the other hand, the SU(2) Lie algebras and their representations are best analyzed using harmonic oscillators through the Jordan-Schwinger map [1]. Infact, this harmonic oscillator correspondence has been used in quantum physics and optics (in context of SU(2) coherent states) [2], nuclear physics (to describe high spin nucleus states) [3] and condensed matter physics (in context of spin chains with global SU(2) symmetry) [4]. However, the Jordan-Schwinger mapping has not been exploited in the context of theories with local SU(2) gauge invariance. In this work, we use this idea to study SU(2) lattice gauge theory by writing the SU(2) electric fields in terms Jordan-Schwinger bosons (in the present context, we call them prepotentials, see section 3). We find that the resulting formulation has the following novel features:

1. The Hamiltonian, in terms of prepotentials, has $SU(2) \otimes U(1)$ local gauge invariance.

Further, the prepotential doublets also form representations of the the new U(1) gauge group.

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2. As expected, the prepotential SU(2) doublets enable us to study the gauge invariance of the theory more critically. In particular, (a) we write certain novel gauge invariant operators in terms of prepotentials, (b) in this formulation, the physical Hilbert space can be characterized by a set of gauge invariant integers.

3. In the strong coupling limit, it is the U(1) gauge group in (1) which is responsible for color confinement.

4. The matter and prepotentials have similar gauge transformation properties and thus are on the same footing.

This transformation to prepotentials is also a non-abelian duality transformation and corresponds to transition from configuration space of SU(2) rigid rotator to the SU(2) angular momentum/electric field basis with the dual Hamiltonian having $SU(2) \otimes U(1)$ local gauge invariance. It is known that such duality transformations are extremely useful in theories having compact global or local symmetries as they extract out the topological degrees of freedom in terms of certain integer valued fields and thus isolate the effect of compactness. Some simple examples are 2-d $\chi y$ model [5] with global U(1) invariance and pure compact U(1) gauge theories on lattice [6]. In the latter case, the duality transformations lead to the dual description with magnetic monopoles representing the effect of compactness. In context of non-abelian lattice gauge theories, duality transformations are of special significance as it is widely believed that the topological degrees of freedom (e.g magnetic monopoles), due to compact nature of the gauge group, are responsible for confinement of color [6, 7, 8]. Towards this end, a duality transformation in the context of pure SU(2) lattice gauge in d=2+1 was constructed in [10, 11] and was the starting motivation for the present work.

The plan of the paper is as follows: In section (2), we start with an introduction to SU(2) lattice gauge theory in Hamiltonian formulation. This section is for the sake of completeness and setting up the notations. In Section (3), we describe our prepotential operators and the associated abelian gauge invariance. We must emphasize that this abelian gauge invariance is not a subgroup of SU(2). In Section (4), we study the SU(2) gauge transformation properties of the prepotentials. The Section (5) is devoted to the study of physical Hilbert space in terms of prepotential operators. At the end, we briefly mention some of the corresponding results for SU(N) lattice gauge theories.

## 2 The Hamiltonian Formulation

We start with SU(2) lattice gauge theory in (d+1) dimensions. The Hamiltonian is:

$$H = \sum_{n,i} tr E(n, i)^2 + K \sum_{\text{plquettes}} tr \left( U_{\text{plquette}} + U_{\text{plquette}}^\dagger \right). \quad (1)$$

where,

$$U_{\text{plquette}} = U(n, i) U(n + i, j) U^\dagger(n + j, i) U^\dagger(n, j); \quad E(ni) = E^a(n, i) \frac{\sigma^a}{2}.$$ 

and K is the coupling constant. The index n labels the site of a d-dimensional spatial lattice and i,j (=1,2,...d) denote the direction of the links. Each link (n,i) is associated with a symmetric
top, whose configuration (i.e. the rotation matrix from space fixed to body fixed frame) is given by the operator valued SU(2) matrix U(n,i) and \( E^a(n,i)(a = 1, 2, 3) \) are the SU(2) electric field operators and can be thought of as the angular momentum operators of the symmetric top in the body fixed frame. From now onwards, we associate the angular momentum operator \( E_a(n,i) \) to the left of the link (ni). The traces in (1) are over the spin half indices of the SU(2) gauge group. The quantization rules are given by [9, 11]:

\[
[E^a(n,i), U(n,i)] = \frac{\sigma^a}{2} U(n,i) \Rightarrow [E^a(n,i), E^b(n,i)] = -i\epsilon^{abc} E^c(n,i) \tag{2}
\]

The Hamiltonian (1) and (2) are invariant under:

\[
E(ni) \rightarrow V(n)E(ni)V^\dagger(n); \quad U(ni) \rightarrow V(n)U(ni)V^\dagger(n+i).
\]

Thus the generator of left gauge transformation on U(ni) are the electric fields E(ni). The right gauge transformations are generated by

\[
e(n, i) \equiv U^\dagger(n, i)E(n, i)U(n, i) \tag{4}
\]

and \( e^a(ni)(\equiv 2\text{tr}(e(ni)\sigma^a/2)) \) satisfies:

\[
[e^a(n, i), U(n, i)] = U(ni)\frac{\sigma^a}{2}; \quad [e^a(n, i), e^b(n, i)] = i\epsilon^{abc} e^c(n, i). \tag{5}
\]

The fields \( e^a(ni) \) can be interpreted as the angular momentum operators in the space fixed frame of the symmetrical top. They satisfy the constraints:

\[
\sum_{a=1}^{3} e^a(ni)e^a(ni) = \sum_{a=1}^{3} E^a(ni)E^a(ni) \tag{6}
\]

at each link (ni). Under SU(2) transformations:

\[
e(ni) \rightarrow V(n+i)e(ni)V^\dagger(n+i) \tag{7}
\]

Therefore, \( e^a(ni) \) should be associated with the right end of the link (ni). The SU(2) Gauss law at every site (n) is:

\[
\sum_{i=1}^{d} (E^a(n,i) - e^a(n-i,i)) = 0 \tag{8}
\]

The Gauss law (8) states that the sum of all the 2d angular momenta meeting at a site (n) is zero.

### 3 The Prepotentials and Abelian Gauge Invariance

We define the SU(2) prepotentials to be two independent doublets of harmonic oscillators: \( (a_\alpha, a_\alpha^\dagger) \) and \( (b_\alpha, b_\alpha^\dagger) \) on each link (ni). They satisfy:

\[
[a_\alpha, a_\beta^\dagger] = \delta_{\alpha,\beta}; \quad [b_\alpha, b_\beta^\dagger] = \delta_{\alpha,\beta} \quad \alpha, \beta = 1, 2. \tag{9}
\]
Using the Jordan-Schwinger boson representation of SU(2) Lie algebra [1], we write:

\[ E^\alpha(n, i) \equiv a^\dagger_\alpha(n)(ni)\frac{\sigma^\alpha}{2}a(ni); \quad e^\alpha(ni) \equiv b^\dagger_\beta(ni)\frac{\sigma^\alpha}{2}b(ni) \]

(10)

Note that in (10), \( \tilde{\sigma}_{\alpha\beta} \equiv \sigma^\beta_{3\alpha} \) is used to get the negative sign on the r.h.s of the equation (2).

From now onwards, instead of the electric field and the SU(2) link operators defined in (1) and (2), we will treat the harmonic oscillator prepotentials in (9) as the basic dynamical variables. We generally define the Hilbert space \( \mathcal{H} \) over the configurations of the SU(2) rigid rotator. It is obtained by link operators in the various representations of SU(2) group, e.g; the physical Hilbert space is described in terms of gauge invariant Wilson loop operators. In the present formulation, the prepotential harmonic oscillators create the dual Hilbert space \( \mathcal{\tilde{H}} \) as they are related to the SU(2) electric fields. In this sense, what follows should also be interpreted as dual formulation of the SU(2) lattice gauge theory.

The defining equations (10) for the SU(2) prepotentials imply \( U(1) \otimes U(1) \) gauge invariance on every link:

\[ a^\dagger_\alpha(ni) \rightarrow expi\theta(ni) a^\dagger_\alpha(ni); \quad b^\dagger_\alpha(ni) \rightarrow expi\phi(ni) b^\dagger_\alpha(ni). \]

(11)

In (11), \( \theta(ni) \) and \( \phi(ni) \) are the arbitrary phase angles at each link (ni). Note that the this \( U(1) \otimes U(1) \) group is not a subgroup of the SU(2) gauge group. The constraint (6) implies that the occupation numbers of the harmonic oscillator prepotentials are equal on each link (ni); i.e:

\[ \sum_{\alpha=1}^{2} a^\dagger_\alpha(ni)a_\alpha(ni) = \sum_{\beta=1}^{2} b^\dagger_\beta(ni)b_\beta(ni) \equiv N(ni) \]

(12)

Therefore, \( \mathcal{\tilde{H}} \) of pure SU(2) lattice gauge theory is characterized by the following orthonormal state vectors at each link:

\[ |n\quad N-n\rangle = \frac{(a^1_n)^N(a^2_n)^{N-n}(b^1_n)^N(b^2_n)^{N-n}}{\sqrt{n!}\sqrt{(N-n)!}\sqrt{n!}\sqrt{(N-n)!}}|0\quad 0\rangle. \]

(13)

The invariance of \( \mathcal{\tilde{H}} \) under (11) implies that the gauge group \( U(1) \otimes U(1) \) reduces to U(1) with \( \theta(ni) = -\phi(ni) \). The constraint (12) now becomes the Gauss law for this resulting abelian gauge invariance.

### 4 The SU(2) Gauge Invariance

Under the SU(2) gauge transformations (3 and 7):

\[ a^\dagger_\alpha(ni) \rightarrow V(n)_{\alpha\beta}a^\dagger_{\beta}(ni), \quad b^\dagger_\alpha(ni) \rightarrow b^\dagger_{\beta}(ni)(V^\dagger(n+i))_{\beta\alpha} \]

(14)

Thus, the two sets of prepotentials transform like SU(2) doublets, one from the left and the other from the right. We further define: \( \tilde{a}_\alpha \equiv \epsilon_{\alpha\beta}a_\beta \) and \( \tilde{b}_\alpha \equiv \epsilon_{\alpha\beta}b_\beta \). Under SU(2) gauge transformations, \( \tilde{a}_\alpha \) and \( \tilde{b}_\alpha \) transform as \( a^\dagger_\alpha(ni) \) and \( b^\dagger_\alpha(ni) \) respectively. Exploiting the above symmetry properties, we now directly write down the operator valued SU(2) matrix \( U(ni) \) in the Hilbert space \( \mathcal{\tilde{H}} \) as:

\[ U(ni)_{\alpha\beta} = F(ni)(a^\dagger_{\alpha}(ni)b^\dagger_{\beta}(ni) + \tilde{a}(ni)\tilde{b}(ni)_{\beta})F(ni) \]

(15)
In (15), \( F(ni) \equiv \frac{1}{\sqrt{N(ni)+1}} \) with \( N(ni) \) defined in (12). It is the normalization factor and is required for the operator valued SU(2) matrix to be unitary. More explicitly on a particular link:

\[
U = \frac{1}{\sqrt{a^\dagger.a + 1}} \left( \begin{array}{cc}
a_1^\dagger b_1^\dagger + a_2 b_2 & a_1^\dagger b_2^\dagger - a_2 b_1 \\
a_2^\dagger b_1^\dagger - a_1 b_2 & a_1^\dagger b_2^\dagger + a_2 b_1
\end{array} \right) \frac{1}{\sqrt{a^\dagger.a + 1}}
\]

This is the standard structure of a SU(2) matrix; i.e it is of the form: \( U = \begin{pmatrix} z_1 & -z_2^\dagger \\ z_2 & z_1^\dagger \end{pmatrix} \) with \( z_1^\dagger z_1 + z_2^\dagger z_2 = z_1 z_1^\dagger + z_2 z_2^\dagger = 1 \) on \( \tilde{H} \). Further, it can be explicitly checked that the operator valued matrix elements of \( U \) in (15) commute amongst themselves and (15) is consistent with the defining equation (4) for the generator of the right gauge transformations \( e(ni) \). The dual form of the SU(2) link operators in (15) can also be derived by the use of the Wigner Eckart theorem on the \( \tilde{H} \). The first and second terms in (15) change the value of the angular momentum by \( +\frac{1}{2} \) and \( -\frac{1}{2} \) units respectively\(^2\). Note that (15) is invariant under under the new U(1) gauge transformation (11) with \( \theta(ni) = -\phi(ni) \). Thus we have broken the SU(2) link operators \( U(ni) \) into the left \( (a_{\alpha}(ni)) \) and the right \( (b_{\alpha}(ni)) \) transforming prepotentials. This separation will help us to construct the gauge invariant states of the theory in the next section.

The Hamiltonian in (1) can now be written in its dual form:

\[
H = \sum_{ni} \frac{N(ni)}{2} \left( \frac{N(ni)}{2} + 1 \right) + \sum_{\text{plaquettes}} \text{tr} \left( U_{\text{plaquette}} + U^\dagger_{\text{plaquette}} \right).
\]

The first term in (16) depends on the number operator on all the links of the lattice. The second term is made up of the 4 links of the plaquettes given by (15). The dual Hamiltonian in (16) is trivially invariant under the \( SU(2) \otimes U(1) \) gauge transformations. In this dual formulation, we can write new gauge invariant operators which one can not write in terms of the original link fields. To see this we define two operators:

\[
U^{+\frac{1}{2}}(ni) \equiv a_{\alpha}(ni) b^\dagger_{\beta}(ni)_{\beta}, \quad U^{-\frac{1}{2}}(ni) \equiv \bar{a}(ni) \bar{b}(ni)_{\beta}
\]

and notice that 1) \( U^{\pm\frac{1}{2}}(ni) \) have the same gauge transformation properties as \( U(ni) \), 2) they are both invariant under the U(1) gauge transformation. Infact, these are the two terms appearing in (15). Therefore, one can define the fundamental gauge invariant operators as consisting of products of \( U^{\pm\frac{1}{2}}(ni) \) over the links of a directed closed loop. The standard Wilson loops in this dual language can be written as the sum of these basic gauge invariant operators but the reverse is not possible. Further, the coupling of matter with the gauge fields is also simple in this formulation. Let \( (q_{\alpha}(n), q_{\beta}(n)) \) be the doublets of matter creation annihilation operators at site \( n \). Under the SU(2) gauge transformations they transform as: \( q_{\alpha}(n) \rightarrow V_{\alpha\beta}(n) q_{\beta}(n) \)

The \( SU(2) \otimes U(1) \) singlet interaction terms are: \( \langle q^\dagger a^\dagger \rangle (b^\dagger q) \) and \( \langle q^\dagger a \rangle (b q) \). Note that these matter-prepotential couplings are novel as the minimal coupling with the original link variable \( U_{\alpha\beta} \) is only the sum of the two. Therefore, it is inadequate to incorporate all the possible gauge invariant interactions.

\(^2\)In terms of the Young tableau, the first term corresponds to adding a new box in the horizontal row and the second term corresponds to deleting a box from the horizontal row.
5 The Physical Hilbert Space $\tilde{H}^p$

In this section, we exploit the simple gauge transformation properties of the prepotentials to characterize the $SU(2) \otimes U(1)$ invariant physical Hilbert space denoted by $\tilde{H}^p$. We first solve the $SU(2)$ Gauss law. For this purpose, it is convenient to collect the set of 2d prepotential creation operators associated with the site $(n)$ as:

$$c^\dagger(n,i) \equiv \tilde{a}^\dagger(n,i)$$

$$c^\dagger(n,d+i) \equiv b^\dagger(n-i,i); \quad i = 1, 2, ..., d.$$

With the above relabelling, one can easily check:

1. The new prepotentials also satisfy harmonic oscillator algebra: 

$$[c^\alpha(n,\bar{i}), c^\beta(n,\bar{j})] = \delta^\alpha_\beta \delta_{\bar{i},\bar{j}}.$$

2. The $SU(2)$ Gauss law (8) at site $n$ is:

$$J_{\text{total}}^a(n) \equiv \sum_{\bar{i}=1}^{2d} c^\dagger(n,\bar{i}) \left( \frac{\sigma^a}{2} \right)_{\alpha\beta} c^\beta(n,\bar{i}) = 0$$

and it simply states that the sum of all the angular momenta meeting at the site $(n)$, $\tilde{J}_{\text{total}}(n)$, is zero.

3. Under $SU(2)$ gauge transformations, all the 2d operators $c^\dagger(n,\bar{i})$ transform from the right as $SU(2)$ doublets.

Thus, the problem of constructing the most general $SU(2)$ gauge invariant states at site $(n)$ reduces to constructing $SU(2)$ singlets out of 2d spin half prepotentials $c^\dagger_\alpha(n,\bar{i})$. We denote the physical Hilbert space at site $n$, consisting of all such invariants by $\tilde{H}^p_n$. Therefore, $\tilde{H}^p_n$ is characterized as:

$$|\vec{l}(n)\rangle \equiv \left| \begin{array}{cccc}
  l_{12} & l_{13} & \ldots & l_{1,2d} \\
  l_{23} & \ldots & l_{2,2d} \\
  \vdots & \ddots & \ddots & \ddots \\
  l_{2d-1,2d} & \ldots & \ldots & l_{2d-1,2d} \\
\end{array} \right| = \prod_{i,j>i}^{l_{ij}(n)} \left( c^\dagger(n,\bar{i}), c^\dagger(n,\bar{j}) \right)^{l_{ij}(n)} |0\rangle.$$  \tag{19}

In (19), $l_{ij}(n)$ ($\equiv l_{ji}(n)$) are $N_d = d(2d-1)$ +ve integers which are invariant under the $SU(2)$ gauge transformations. The states (19) $\in \tilde{H}^p_n$ are the eigenvectors of $c^\dagger(n,\bar{i}), c(n,\bar{i})$, $\bar{i} = 1, 2, ..., 2d$, with eigenvalue $\left( \sum_{j\neq i} l_{ij}(n) \right)$. Therefore, the operator $\prod_{i,j>i}^{l_{ij}(n)} \left( c^\dagger(n,\bar{i}), c^\dagger(n,\bar{j}) \right)^{l_{ij}(n)}$ in (19) creates $\frac{1}{2} \left( \sum_{j\neq i} l_{ij}(n) \right)$ units of electric flux on the link $(n,\bar{i})$. Thus, in terms of the prepotentials, the problem of solving the non-abelian $SU(2)$ Gauss law reduces to the problem of solving $U(1)$ Gauss law (see also [12]). The complete $SU(2) \otimes U(1)$ invariant Hilbert space can be written as:

$$\tilde{H}^p = \prod_n' \otimes \tilde{H}^p_n$$  \tag{20}

In (20), the direct product is taken over all the lattice sites such that $U(1)$ Gauss law (12) is satisfied. Note that this construction is dual description of the construction of the gauge
invariant states through the Wilson loop operators acting on the vacuum. Further, like Wilson loops, (19) describes an over complete basis. Therefore, the set of $N_d$ integers is not the minimal set required to characterize $\tilde{H}_n^p$. Infact, the minimum set consists of a set of $N_d - N_{d-1} - d = 3(d-1)$ integers per lattice site. These gauge invariant integers are associated with the physical transverse degrees of freedom of the three SU(2) gluons.

To see the color confinement in this formulation, we imagine a quark $q(n)$ and anti-quark $\bar{q}(n)$ pair located at lattice sites $n$ and $m$ respectively. Following the previous sections, we can construct the SU(2) color invariant states locally at $(n)$ and $(m)$ as $(b^\dagger q)$ and $(q^\dagger a^\dagger)$ respectively. It is the U(1) gauge invariance which forces us to connect them through a string of harmonic oscillator prepotentials and get a $SU(2) \otimes U(1)$ gauge invariant physical state. In the strong coupling limit, the energy of this configuration is proportional to the length of the string which is forced by the U(1) gauge invariance. This is probably related to the idea of 't Hooft [8] that in SU(N) gauge theory the $U(1)^{N-1}$ group is relevant for the color confinement mechanism. Infact, the prepotential formalism in the case of SU(N) lattice gauge theories should have 2 $(N-1)$ sets of prepotentials, transforming as $(N-1)$ fundamental representations of SU(N) [14]. Therefore, this formalism is invariant under $SU(N) \otimes (U(1))^{N-1}$ gauge group. This work will be reported elsewhere [14].

6 Discussion and Summary

The complete set of commuting operators in pure SU(2) lattice gauge theory are $E^a(ni)E^a(ni)$, $E^3(ni)$ and $e^3(ni)$. The common eigenvectors are denoted by $|j, m, \bar{m}>$ where $j, m$ and $\bar{m}$ denote the eigenvalues of the above three operators respectively. We note that the states in (13) can also be constructed using the standard link operators $U_{\alpha\beta}$. Infact,

$$|j, m, \bar{m}> = \sum_{i_1, i_2, ..., i_{2j} \in S_{2j}} U_{m_1 \bar{m}_1} U_{m_2 \bar{m}_2} ... U_{m_{2j} \bar{m}_{2j}} |0>$$

In (21), $(m_1, m_2, ..., m_{2j})$ and $(\bar{m}_1, \bar{m}_2, ..., \bar{m}_{2j})$ are the two sets of $\pm \frac{1}{2}$ with constraints: $m_1 + m_2 + ..m_{2j} = m$, $\bar{m}_1 + \bar{m}_2 + ..\bar{m}_{2j} = \bar{m}$ and $S_{2j}$ is the permutation group of order $2j$. The correspondence with (13) is: $2j = N, m = 2n - \frac{N}{2}$ and $\bar{m} = 2\bar{n} - \frac{N}{2}$. The construction (21) becomes more and more complicated as $j$ increases. Thus, the the characterization of the Hilbert space of the lattice gauge theories through the prepotential formulation presented in this work is much simpler than the standard formulation.

In this work, we have presented a new platform in terms of harmonic oscillator prepotentials to analyze the non-abelian lattice gauge theories. The formulation is in terms of the dynamical variables which belong to the most fundamental representation(s) of the gauge group. The novel features are already summarized in section (1).

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