AN INTEGRATED INVENTORY MODEL WITH VARIABLE TRANSPORTATION COST, TWO-STAGE INSPECTION, AND DEFECTIVE ITEMS

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ABSTRACT. The paper deals with an integrated inventory model with make-to-order policy from buyer to vendor. A variable transportation cost is used as a power function of the delivery quantity for either tapering or considering proportional rate data to maintain single-setup multi-delivery (SSMD) policy with reduced transportation cost. A two-stage inspection process is introduced by the vendor to ensure the perfect quality of product even though the first inspection process indicates a constant defective rate of imperfect production is present during the long-run production system and all defective items are reworked with some fixed cost. The aim is to minimize the total cost of the integrated inventory model by using classical optimization technique. Two numerical examples, sensitivity analysis, and graphical representations are given to illustrate the model.

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Key words and phrases. Inventory, variable transportation cost, make-to-order policy, inspection, rework, disposal cost.

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1. Introduction. Supply chain management (SCM) can be used among suppliers, manufacturers, retailers, and customers like a network/chain for minimizing costs, for increasing quality, decreasing transportation, and others. In the literature, the supply chain model is used to mainly to minimize the total cost or to maximize the total profit throughout the network under the condition that demands of all players have to be met. The organizations that make up the supply chain are “linked” together through physical flows and information flows. Physical flows involve the transformation, movement, storage of goods and materials. These are the most visible parts of the supply chain. But another important factor as information flows which allow the various supply chain partners to coordinate their long-term plans, and to control the day-to-day flow of goods and material up and down the supply chain. Recently, Cárdenas-Barrón and Sana [4, 5] developed two excellent research models by using supply chain policy based on variable demand whereas the basic economic order quantity (EOQ) model with constant demand of Harris [13] completed one century and Cárdenas-Barrón et al. [3] wrote an article to honor of Ford Whitman Harris [13]. Taleizadeh et al. [42] discussed a joint optimization of price, replenishment frequency, replenishment cycle, and production rate in vendor managed system. EOQ models with partial backorder, special selling price and special sales for perishable products were developed by Taleizadeh et al. [43, 44] and Taleizadeh and Pentico [45]. The basic economic production quantity model (EPQ) for lot size with continuous delivery was formed by Hadley and Whitin [12], Silver et al. [33], Taleizadeh et al. [38, 37]. In those papers, both the production quantity and demand rates are finite and constant. Also there is no on-hand inventory when any replenishment cycle starts and the model focused on the make-to-order production system. An infinite replenishment rate in the inventory model was presented by Goyal [10]. The finite production rate with lot-for-lot production/delivery policy was proposed by Banerjee [1]. Goyal [11] extended the model of Banerjee [1] where the shipment of delivery quantity can perform multiple times. Sarker and Parijia [28] developed an inventory model to determine an optimal ordering policy for procurement of raw materials and the manufacturing batch size to minimize the total cost. To reduce setup cost and ultimately final cost of the system, a continuous investment is used by Sarker and Majumder [24], Sarkar et al. [22]. To avoid stock out, the periodic ordering (delivery) policy was introduced by Sarker and Parijia [29]. Recently, Mungan et al. [18] considered a dynamic delivery policy with a similar production/delivery policy. Taleizadeh et al. [41, 46, 35, 40, 34, 39] used different types of algorithms to optimize multi-product, multi-constraint problems with uncertainty.

The aim for any industry is to minimize transportation costs while also meeting demand for products. Transportation costs generally depend upon the distance between source and destination, ways of chosen transportation, volume, and quantity of the product to be shipped. Transportation cost is much very useful to consider for a real-life problem in SCM. When any product from a producer to a retailer or customer is transported then there must be the consideration of the transportation cost. Recently, Sarkar [19] developed a supply chain model based on variable transportation cost. He assumed three different types of stochastic deterioration and the buyer pays the variable transportation cost and handling cost. He obtained a closed form solution. To save the world environment, the effect of carbon emission cost was incorporated within the supply chain model by Sarkar et al. [32]. Sarkar et al.
considered a fixed cost and a variable carbon emission cost within three-echelon supply chain model.

Make-to-order is a production approach where products are not built until a confirmed order for products is received. i.e., it is a manufacturing process in which manufacturing starts only after a customer’s order is received. Similarly, in make-to-stock, products are manufactured based on demand forecasts. As the accuracy of the forecasts will prevent excess inventory and opportunity loss due to stock-out, the issues are very important in real life problems. But how do we obtain the forecast demands accurately? Thus, these two approaches are almost near to each other. But this model will study on the make-to-order approach only. The make-to-stock production system is applicable for managing most of standard products. The shipment lot size with known or fixed on-hand inventory was proposed by Lu [17]. Hill [14, 15] extended this model by introducing the periodic delivery quantity and variable production quantity. Both production and delivery schedule for a single-vender multi-buyer supply chain was considered by Chain and Kingsman [6]. But the model does not contain any idea of transportation cost. The model of Ertogral et al. [7] included the cost of transportation. The major contribution regarding the transportation cost can be found in Ben-Daya et al.’s [2] and Glock’s [8] model. Golhar and Sarker [9] considered periodic delivery frequency and decision variables as production quantity. Sarkar et al. [31], and Sarkar et al. [27] used the trade-credit policy and backorder price-discount in inventory model.

Inspection is the procedure to obtain the defective items. This is another important criteria in any production system to test products. When an order is received from the buyer or customer to the producer, then the producer starts the production. During the whole production process some defective items may be produced. Inspection is important to check that produced items regarding its perfectness. Generally, it is preferred that the inspection should properly done by the producer to remove complain from retailer or customers, before the delivery of an item. Recently, Wee and Chung [47] developed multi-stage inspections to confirm’s the product’s quality. Sarkar and Saren [30] incorporated an inspection policy with inspection errors and a warranty policy in their inventory model. For fixed lifetime products, three-stage inspections and quantity discount policy were introduced in a supply chain model by Sarkar [20].

After inspection, defective products are sent for rework. After reworking, the second stage inspection is considered and perfect items are sent for delivery to the market and defective items are disposed. Sarkar et al. [48] developed a SCM with manufacturing quality improvement. Several researchers, Sarkar et al. [21], Taleizadeh et al. [36], Sarker et al. [25], and Sarkar et al. [26] developed their models on reworking of defective items.

This paper extends the model of Lee and Fu [16]. They considered the production and delivery quantity as variable and periodic delivery quantity. But there is no idea of inspection. In any production system, there is no guarantee for all perfect production. The inspection policy is essential to check about the perfectness of products. Thus, this model develops two-stage inspections to make the research more realistic. This paper studies production, inspection and delivery of lot sizing policies in a producer buyer supply chain management. The production and demand rates are finite and constant. When an order is received from the buyer to the producer then the production immediately fills those orders. The whole lot inspections are conducted and defective items are sent to producer for reworking.
to make them perfect. Again second stage inspection are conducted to check those reworked items and it will be announced that the defective products, which are still present in the system, are disposed and perfect items are sent to buyer or customers. At the same time, when production and inspection are simultaneously working, then the perfect items can be delivered to the buyer or customer. In the total period, the model investigates the total cost to produce this lot size. Finally, the total cost is minimized to obtain the lot size, number of shipments, and shipment quantity.

Table 1. A summary of production-delivery lot sizing models.

| Author(s) | MTO/MTS strategy | Production quantity | Delivery quantity | Delivery cost | Inspection policy |
|-----------|------------------|---------------------|-------------------|--------------|------------------|
| Cárdenas–Barrón and Sana [1, 5] | MTO | Variable | – | – | – |
| Taleizadeh et al. [42] | MTO | Variable | – | – | – |
| Benerjee [1] and Goyal [11] | MTO | Variable | Costant | – | – |
| Goyal [10] | MTO | Costant | – | – | – |
| Sarkar and Parja [28, 29] | MTO | Variable | Costant | – | – |
| Sarkar and Majumder [24] | MTO | Costant | Variable | Considered | – |
| Sarkar et al. [32] | MTO | Costant | Variable | – | – |
| Gallhar and Sarkar [9] | MTO | Variable | Costant | – | – |
| Lu [17] | MTS | Variable | Costant | – | – |
| Hill [14, 15] | MTS | Variable | Variable | – | – |
| Chan and Kingsman [6] | MTS | Variable | Variable | – | – |
| Ben–Deya et al. [2] | MTS | Variable | Variable | – | – |
| Glock [8] | MTS | Variable | Variable | – | – |
| Ertogral et al. [7] | MTS | Variable | Variable | Considered | – |
| Wee and Chung [47] | MTS | Variable | Variable | Considered | Included |
| Lee and Fu [16] | MTS | Variable | Variable | Considered | – |
| This study | MTO | Variable | Variable | Considered | Included |

“−” indicates non-availability of the research contribution, “MTO” denotes make–to–order, and “MTS” denotes make–to–stock.

2. Problem definition, notation, and assumptions. This section contains problem definition, notation for the model, major assumptions.

2.1. Problem definition. The model is an integrated inventory model, where imperfect items are produced during long-run production. First-stage inspection is considered to detect defective items and the second-stage inspection is to verify reworked products of their quality like new products. The first inspection is considered just after production. The defective items are sent for reworking. After reworking the second time inspection is considered. The imperfect items (if any after second inspection) send for disposing and the perfect items will send to market. A make-to-order policy is introduced buyer to vendor. The aim is to reduce the total system cost for the integrated inventory model with delivery cost per shipment as the power function of the delivery quantity for either tapering or proportional rate data.

2.2. Notation. The following notation are used to develop the model.

Decision variables

- \( q \) delivery quantity to buyer.
- \( n \) number of shipment in the entire planning horizon (integer number).
Parameters $Q_0$ ordering lot size (units).
$Q$ perfect item (units) to sell in a cycle time $T$ i.e., $(Q = (1 - \alpha + \alpha\beta)Q_0 = uQ_0$,
where $u = (1 - \alpha + \alpha\beta)$. $p_0$ production rate (units/unit time).
$p$ production rate of perfect product i.e., $p = up_0$ (units/unit time).
$d$ demand rate (units/unit time).
$Q_p(t)$ total production units at time $t$.
$Q_d(t)$ total delivery units at time $t$.
$T$ replenishment cycle time (unit time).
$A_1$ setup cost of vendor ($$/setup).
$A_2$ handling cost of buyer ($$/unit time).
$h_1$ inventory carrying cost of vendor ($$/units/unit time).
$h_2$ inventory carrying charge of buyer ($$/units/unit time).
$k(q)$ delivery cost for vendor ($$/shipment$).
$k_0, k_1$ constants to adjust transportation cost.
$C_0$ inspection cost($$/units$).
$C_1$ rework cost ($$/units$).
$C_2$ disposal cost ($$/units$).
$\alpha$ rate of percentage of defective items in the start of production.
$\beta$ rate of percentage of perfect items in the rework items.

2.3. Assumptions.
1. This is an integrated inventory model for single type of items, where during production defective items are produced. The inspection is considered as: after 1st stage inspection, $\alpha$ percentage of production rate of defective items are detected. Thus, $(1 - \alpha)Q_0$ number of perfect items are within the system.
2. The defective items will send for rework and after reworking second stage inspection occurs. At this time, $\beta$ percentage perfect items of the defective items are detected and the rests are disposed. From this stage, $\beta\alpha Q_0$ quantities perfect items are obtained and $(1 - \beta)\alpha Q_0$ numbers of defective items are disposed.
3. The vendor delivers only perfect items to the market at a small quantity $q(q \leq Q)$. The perfect items have been transported to the to the market with a fixed period $\frac{d}{d}$, where $d(d \leq q)$ is the demand rate of buyer.
4. It follows a make-to-order policy. No reserved stock is assumed here. All products are produce based on the order.
5. Fixed setup cost is considered in the model.
6. No shortage is considered here.

3. Mathematical model. The model considers a two-echelon supply chain co-ordination between a buyer and a vendor. A production system (as a vendor or supplier) at the upper-echelon and a customer (as a retailer or buyer) at the lower-echelon are considered. When an order of $Q_0$ quantity is received from the buyer to the vendor, then the vendor starts the production at a rate $p_0$ of lot size quantity $Q_0$. To produce the quantity $Q_0$, the machine has to maintain long-run production process, in which the production of defective items may appear. We assume that $\alpha$ be percentage of production rate of defective items. Thus $\alpha Q_0$ quantities are defective items, which are sent to vendor for reworking and rest quantities $(1 - \alpha)Q_0$ are perfect items. At $\beta$ percentage of perfect quantities are found within on the rework items $\alpha Q_0$ and get the perfect quantities $\beta\alpha Q_0$. Rest $(1 - \beta)\alpha Q_0$ defective items are disposed. Thus, the total number of perfect items in the production
system are $Q = (1 - \alpha + \alpha \beta)Q_0 = uQ_0$, which are delivered to the buyer, where $u = (1 - \alpha + \alpha \beta)$. We assume that the buyer does not claim any type of demand for the shortage quantity ($Q_0 - Q$). In the similar manner, the production rate of perfect product is $p = uQ_0$.

On the production and inspection time, the vendor deliverers perfect products to buyer at a small quantity $q$ ($q \leq Q$) with a fixed period $\frac{Q}{d}$, where $d$ ($d \leq q$) is the demand rate of buyer. There are no reserved stock to meet the immediate demand, i.e., make-to-order production. Here the replenishment cycle period is $[0, \frac{Q}{d}]$. The replenishment cycle time $T = \frac{Q}{d}$ has two parts namely, one $t_1 = [0, \frac{Q}{p}]$ and another $t_2 = [\frac{Q}{p}, \frac{Q}{d}]$.

Now, this model derives the cost function as follows:

i) Here, a fixed setup cost $A_1$ is included for the production system.

ii) In time $t$, the inventory level at the production $I_1(t)$ is the surplus of the total production $Q_p(t)$ over the total delivery $Q_d(t)$, where $0 \leq t \leq \frac{Q}{d}$. At time $t$, during the replenishment cycle, the total production quantity $Q_p(t)$ can be expressed as follows:

$$Q_p(t) = \begin{cases} p_0 t, & 0 \leq t \leq \frac{Q}{p} \\ Q, & \frac{Q}{p} < t \leq \frac{Q}{d} \end{cases}$$

Hence, the area formed by $Q_p(t)$ on the replenishment cycle $T$ is

$$\int_0^{\frac{Q}{d}} Q_p(t) dt = \left( \frac{1}{d} - \frac{2u}{2pu} \right) Q^2$$
Here the first delivery quantity \( q \) is shipped at time \( \frac{Q}{d} \) and equally shifted \( n \) times. Thus the \( kth \) delivery is made at time \( \frac{kQ}{d} \), for \( k = 1, 2,...,n \), where \( n \) is the last shipment. At time \( t \), the total delivery quantity \( Q_d(t) \) can be expressed as \( Q_d(t) = kq \), where \( \frac{kQ}{d} \leq t \leq \frac{(k+1)Q}{d} \leq T, k = 1,........, (n-1) \).

The total delivery quantity on the replenishment cycle \( T \) is

\[
\int_0^{T=Q/d} Q_d(t)dt = \int_0^{nq/d} Q_d(t)dt = \frac{n(n-1)q^2}{2d}.
\]

Hence, the total on-hand inventory under the cycle \( T \) is the subtract of the area formed by \( Q_d(t) \) and the total delivery quantity i.e.,

\[
I_1 = \left( \frac{1}{d} - \frac{2u-1}{2p} \right) Q^2 - \frac{n(n-1)q^2}{2d}.
\]

The total inventory carrying cost is

\[
h_1 I_1 = h_1 \left[ \left( \frac{1}{d} - \frac{2u-1}{2p} \right) Q^2 - \frac{n(n-1)q^2}{2d} \right].
\]

iii) The delivery cost per shipment is the power function of the delivery quantity for either tapering or proportional rate data i.e., \( k(p) = k_0 + k_1q^a \), for \( k_0, k_1 \geq 0 \). The total delivery cost is \( nk(q) = n(k_0 + k_1q^a) \).

iv) For the buyer, there are \( n \) fixed handling cost to process the \( n \) received shipments and the average on-hand inventory level is \( I_2 = \frac{q}{2} \) in a single replenishment cycle \( \left( \frac{Q}{d} \right) \). Only the increased echelon value \( (h_2 - h_1) \) per unite per time is counted at the lower-echelon for the definition of echelon-value. Thus, the total cost calculated for the buyer is

\[
A_2n + \frac{Q(h_2 - h_1)q}{2d}.
\]

v) As the inspection cost \( C_0 \) per unit is considered on the total production quantity \( Q_0 \), the total inspection cost is \( C_0Q_0 \) units. In the second step, the rework cost \( C_1 \) per unit is used on the defective items \( \alpha Q_0 \). Thus, the total rework cost is \( C_1\alpha Q_0 \). The inspection is conducted on the reworked items \( \alpha Q_0 \) at the fixed inspection cost \( C_0 \) per unit and get the total inspection cost as \( C_0\alpha Q_0 \). At this step, we obtain \((1-\beta)\alpha Q_0 \) quantities are defective. At a fixed disposal cost \( C_2 \) per unit is effect on that defective items and get the total disposal cost is \( C_2(1-\beta)\alpha Q_0 \) units. Thus, the total costs for inspection, rework, and disposal are \([C_0Q_0 + C_1\alpha Q_0 + C_0\alpha Q_0 + C_2(1-\beta)\alpha Q_0]\).

Hence, the total cost per unite time is

\[
T = \frac{d}{Q} \left[ A_1 + h_1 \left\{ \left( \frac{1}{d} - \frac{2u-1}{2p} \right) Q^2 - \frac{n(n-1)q^2}{2d} \right\} + n(k_0 + k_1q^a) + A_2n \right. \\
+ \frac{Q(h_2 - h_1)q}{2d} + \left\{ C_0Q_0 + C_1\alpha Q_0 + C_0\alpha Q_0 + C_2(1-\beta)\alpha Q_0 \right\} \\
= A_1 + h_1 \left\{ \left( \frac{1-d(2u-1)}{2p} \right) Q - \frac{n(n-1)q^2}{2Q} \right\} + \frac{dn}{Q}(k_0 + k_1q^a) + \frac{A_2nd}{Q} \\
+ \frac{(h_2 - h_1)q}{2} + \frac{d}{u} \left\{ C_0(1+\alpha) + C_1\alpha + C_2(1-\beta)\alpha \right\}.
\]
We consider the total delivery quantity as perfect quantity \( Q \), which are equally delivered at a rate \( q \) and thus \( Q = nq \). Set \( Q = nq \) and after simplification, we obtain the total cost per unit time as

\[
TRC(q, n) = \frac{A_1 d}{nq} + \frac{h_1(d + pu - 2du)}{2pu}nq + \frac{d(A_2 + k_0)}{q} + \frac{h_2q}{2} + dk_1q^{(a-1)} + \frac{d}{u}\{C_0(1 + \alpha) + C_1\alpha + C_2(1 - \beta)\alpha\}.
\]

(1)

**Proposition 1.** When the number of delivery \( n \) in one replenishment cycle is an integer, the cost function (1) is convex with respect to the delivery quantity \( q \), for a given \( n \).

**Proof.** Without loss of generality suppose that \( q \) is a continuous variable and hence function (1) can be differentiate with respect to \( q \). Thus, the first and second order partial derivative of \( TRC(q, n) \) with respect to \( q \) is

\[
\frac{\partial TRC(q, n)}{\partial q} = -\frac{A_1 d}{nq^2} + \frac{h_1(d + pu - 2du)}{2pu}n - \frac{d(A_2 + k_0)}{q^2} + \frac{h_2}{2} + dk_1(a - 1)q^{(a-2)}
\]

and

\[
\frac{\partial^2 TRC(q, n)}{\partial q^2} = \frac{2A_1 d}{nq^3} + \frac{2d(A_2 + k_0)}{q^3} + dk_1(a - 1)(a - 2)q^{a-3}.
\]

The second order partial derivative is always greater than zero for all fixed positive numbers as the third term is very small quantity compare to sum of 1st two terms. Thus, the function in (1) is convex with respect to \( q \).

**Proposition 2.** When the delivery quantity \( q \) is fixed, then the cost function (1) is convex with respect to the number of delivery quantity \( n \).

**Proof.** Let \( n \) be continuous. Then, partially differentiate \( TRC(q, n) \) with respect to \( n \), we obtain

\[
\frac{\partial TRC(q, n)}{\partial n} = -\frac{A_1 d}{n^2q} + \frac{h_1(d + pu - 2du)}{2pu}n.
\]

Again partially differentiate \( \frac{\partial TRC(q, n)}{\partial n} \) with respect to \( n \) we find

\[
\frac{\partial^2 TRC(q, n)}{\partial n^2} = \frac{2A_1 d}{n^3q},
\]

which is greater than 0 for all positive numbers \( A_1, d, u \) and \( q \). Hence, the proposition proves.

Now, it is assumed that both the decision variables \( q \) and \( n \) are continuous in equation (1). Then the partial derivatives of \( TCR(q, n) \) with respect to \( q \) and \( n \), respectively are as follows:

\[
\frac{\partial TRC(q, n)}{\partial q} = -\frac{A_1 d}{nq^2} + \frac{h_1(d + pu - 2du)}{2pu}n - \frac{d(A_2 + k_0)}{q^2} + \frac{h_2}{2} + dk_1(a - 1)q^{(a-2)}
\]

(2)

\[
\frac{\partial TRC(q, n)}{\partial n} = -\frac{A_1 d}{n^2q} + \frac{h_1(d + pu - 2du)}{2pu}n
\]

(3)

By setting (3) equal to zero, we obtain

\[
q^2 = \frac{2A_1 dpu}{h_1(d + pu - 2du)n^2}
\]

(4)

Putting the value of \( q \) in (2) and considering \( a = 1 \), we have

\[
n^2 = \frac{A_1 h_2 pu}{h_1(A_2 + k_0)(d + pu - 2du)}
\]

(5)
From equations (4) and (5) we have the optimal value of \(q\) and \(n\) are

\[
q^* = \sqrt{\frac{2d(A_2 + k_0)}{h_2}}
\]
\[
n^* = \sqrt{\frac{A_1h_2pu}{h_1(A_2 + k_0)(d + pu - 2du)}}
\]

These values are same when \(a = 0\) i.e., at the maximum and minimum value of \(a\) there are same value of \(q^*\) and \(n^*\).

Again taking partially derivative of (2) and (3) with respect to \(q\) and \(n\) respectively, we have

\[
\frac{\partial^2 \text{TRC}(q,n)}{\partial q^2} = \frac{2A_1d}{nq^3} + \frac{2d(A_2 + k_0)}{q^3} + dk_1(a - 1)(a - 2)q^{(a-3)}
\]
\[
\frac{\partial^2 \text{TRC}(q,n)}{\partial n \partial q} = \frac{A_1d}{n^2q^2} + \frac{h_1(d + pu - 2du)}{2pu}
\]
\[
\frac{\partial^2 \text{TRC}(q,n)}{\partial n^2} = \frac{2A_1d}{n^3q}
\]

For sufficient condition of global minimization at \((q^*, n^*)\), we obtain

\[
\frac{\partial^2 \text{TRC}(q,n)}{\partial q^2} = \frac{2A_1d}{n^*q^3} + \frac{2d(A_2 + k_0)}{q^{(a-3)}} + dk_1(a - 1)(a - 2)q^{(a-3)} \geq 0
\]
\[
\frac{\partial^2 \text{TRC}(q,n)}{\partial n^2} = \frac{2A_1d}{n^{*3}q^*} \geq 0
\]

and

\[
\frac{\partial^2 \text{TRC}(q,n)}{\partial q^2} \frac{\partial^2 \text{TRC}(q,n)}{\partial n^2} - \frac{\partial^2 \text{TRC}(q,n)}{\partial n \partial q}
\]
\[
= \left[\frac{2d(A_2 + k_0)}{q^{(a-3)}} + dk_1(a - 1)(a - 2)q^{(a-3)}\right] \frac{2A_1d}{n^*q^*} \geq 0
\]

for all fixed constants as non-negative.

Thus, the minimum value of total cost function \(\text{TRC}(q,n)\) reaches when \(q = q^*\) and \(n = n^*\). The total minimum cost can be calculated from equation (1) by putting the value of \(q^*\) and \(n^*\).

4. Numerical examples. Example 1. We consider an example to illustrate the above model. Let us consider \(A_1 = $1700\)/setup, \(A_2 = $35/\text{month}, p_0 = 300\) units/month, \(d = 250\) units/month, \(h_1 = $300/\text{unit}/\text{month}, h_2 = $550/\text{unit}/\text{month}, C_0 = $2/\text{units}, C_1 = $2.5/\text{units}, C_2 = $2/\text{units}, \alpha = 15\%, \beta = 96\%, a = 0.8, k_0 = 75, k_1 = 15\). Then, the optimum \(q^* = 10.71\) units, \(n^* = 9\), and the minimum cost is $17706.6.

Sensitivity for the parameter \(a\) in the Example 1. We obtain Table 2 for the sensitiveness with the total cost of the parameter \(a\).
Figure 2. Graphical illustration of the unit time cost with respect to joint decision \((n, q)\) of Example 1.

Table 2. Cost sensitiveness based on the parameter \(a\) from Example 1

| \(a\) | \(TRC\) |
|------|--------|
| 0    | 15722.9 |
| 0.1  | 15816.6 |
| 0.2  | 15935.4 |
| 0.3  | 16085.9 |
| 0.4  | 16276.8 |
| 0.5  | 16518.7 |
| 0.6  | 16825.3 |
| 0.7  | 17214.0 |
| 0.8  | 17706.6 |
| 0.9  | 18331.1 |
| 1    | 19122.8 |

Example 2. For this example, we consider the following parametric values in appropriate units: \(A_1=\$1100\)/setup, \(A_2=\$40\)/month, \(p_0=225\) units/month, \(d=150\) units/month, \(h_1=\$400\)/unit/month, \(h_2=\$500\)/unit/month, \(C_0=\$2\)/units, \(C_1=\$3\)/units, \(C_2=\$2\)/units, \(\alpha = 13\%\), \(\beta = 98\%\), \(a = 0.7\), \(k_0=52\), \(k_1=15\). Then, the optimum \(q^*=7.56\) units, \(n^*=6\) and the minimum cost is \$13055.3.

Sensitivity analysis for the parameter \(a\) in the Example 2. Similarly as Example 1, the sensitiveness of the cost can be found based on the parameter \(a\) in Table 3.

From the above two examples, we obtain that the function \(TRC\) is increased for increasing of the parameter \(a\).
Figure 3. The curve of total cost function TRC when \( a \) varies as on Example 1.

Figure 4. Graphical illustration of the unit time cost with respect to joint decision \((n, q)\) of Example 2.
Table 3. Cost sensitiveness based on the parameter $a$ from Example 2

| $a$ | $TRC$  |
|-----|--------|
| 0   | 12126.6|
| 0.1 | 12193.3|
| 0.2 | 12275.0|
| 0.3 | 12375.0|
| 0.4 | 12497.4|
| 0.5 | 12647.3|
| 0.6 | 12830.7|
| 0.7 | 13055.3|
| 0.8 | 13330.3|
| 0.9 | 13666.9|
| 1   | 14078.9|

Figure 5. The curve of total cost function $TRC$ when $a$ varies as on Example 2.

5. Sensitivity analysis. The sensitivity analysis of the key parameters are given in Table 4.

Table 4: Sensitivity analysis of key parameters

| Parameters | Changes(in %) | $TRC$  | Parameters | Changes(in %) | $TRC$  |
|-----------|--------------|--------|-----------|--------------|--------|
| $A_1$     | -50%         | -12.45 | $h_1$     | -50%         | -12.70 |
|           | -25%         | -5.04  |           | -25%         | -6.35  |
|           | +25%         | 6.23   |           | +25%         | 6.35   |
|           | +50%         | 12.45  |           | +50%         | 12.70  |
| $A_2$     | -50%         | -2.31  | $h_2$     | -50%         | -8.32  |
|           | -25%         | -1.15  |           | -25%         | -4.16  |
|           | +25%         | 1.15   |           | +25%         | 4.16   |
|           | +50%         | 2.31   |           | +50%         | 8.32   |

Sensitivity analysis. With the changes of ($-50\%$, $-25\%$, $+25\%$, $+50\%$) the optimal values of parameters, total cost changes according to the Table 4.
1. When the value of vendor’s setup cost is increasing, the total cost is also increasing. The change is significant for setup cost. With the increasing value of handling cost of buyer, the total cost is increasing. In this model, setup cost is more sensitive than handling cost.

2. The total cost is changing in same rate with the changing of holding cost of vendor. When the holding cost increases, total cost increases and decreases with the decreasing value of holding cost of vendor. Also, when the holding cost of buyer is increasing, the total cost is increasing. Here, the holding cost of the vendor is more sensitive than the holding cost of buyer.

6. **Managerial insights.** The model consists of two times inspections. After 1st time inspection the defective items will send for rework and after reworking 2nd stage inspection will occur. After 2nd time inspection the imperfect items are disposed and the perfect items are delivered.

   In this situation, any production system can deliver only perfect items. The quality of the products are verified. Thus, quality as well as the brand image of the production system will increase. The profit of the vendor can be increased due to the up-to-the mark of the quality of the products. The demand of the products always increase due the quality of the product. Thus, the vendor always gains more profit at the optimum level.

7. **Conclusions.** This paper investigated a supply chain model with a vendor/supplier and a buyer. The vendor produced items which were shifted to buyer after two-stage inspection process by multi-shipment policy. The make-to-order policy was considered and two-stage inspection are conducted to make sure about the perfectness of all products which were sent to the buyer. The variable transportation cost was used in a form of power function in this model. By utilizing two-stage inspection policy, the vendor/supplier ensured quality of products to make the brand image in customer’s mind. The model was solved analytically. There is propositions to check the optimality of the cost function. The global minimum of the total cost function was found by analytical method. By the numerical study, we obtained better results from the existing literature. The sensitiveness of the power function is shown with the changes of total cost. The model can be extended by assuming multi-product as well as multi-echelon supply chain coordination. The another fruitful extension can be considered by assuming fuzziness within the costs. The model considered only MTO policy with the centralized integrated inventory model, but if there is an asymmetry of powers within players of the integrated model, then decentralized and centralized model have to consider with several game policies. This would be a more fruitful direction of the research model. A sales effort can be considered with asymmetric model from buyer’s side to increase more sales. This would be the ongoing future research. The defective rate also can be considered as random and then based on random rate, shortages may appear any time which would another new approach of this research.

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