Bound on $Z'$ Mass from CDMS II in the Dark Left-Right Gauge Model II

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Abstract

With the recent possible signal of dark matter from the CDMS II experiment, the $Z'$ mass of a new version of the dark left-right gauge model (DLRM II) is predicted to be at around a TeV. As such, it has an excellent discovery prognosis at the operating Large Hadron Collider.
Introduction: One year ago, we proposed [1] that dark-matter fermions (scotinos) are naturally present in an unconventional left-right gauge extension of the standard model (SM) of particle interactions, which we call the dark left-right model (DLRM). It is a nonsupersymmetric variation of the alternative left-right model (ALRM) discussed already 23 years ago [2, 3]. One important difference of both the DLRM and the ALRM with the conventional left-right model (LRM) [4] is the fact that tree-level flavor-changing neutral currents [5] are naturally absent so that the $SU(2)_{R}$ breaking scale may easily be at around a TeV, allowing both the charged $W_{R}^{\pm}$ and the extra neutral $Z'$ gauge bosons to be observable at the large hadron collider (LHC). Interesting phenomenology of $Z'$ decay into scalar bosons in the DLRM has just recently been discussed [6].

In this paper, we propose a new variant of this extension which we call DLRM II. [Other more exotic variants are also possible [7].] Instead of having Majorana scotinos as dark matter, we now have Dirac scotinos. Their interactions with nuclei through the $Z'$ are thus relevant for understanding the recent result of the dark-matter direct-search experiment CDMS II [8]. It will be shown that the $Z'$ mass may indeed be around a TeV, and its discovery prognosis at the LHC is excellent.

Model: Consider the gauge group $SU(3)_{C} \times SU(2)_{L} \times SU(2)_{R} \times U(1)$. The conventional leptonic assignments are $\psi_{L} = (\nu, e)_{L} \sim (1, 2, 1, -1/2)$ and $\psi_{R} = (\nu, e)_{R} \sim (1, 1, 2, -1/2)$. Hence $\nu$ and $e$ obtain Dirac masses through the Yukawa terms $\bar{\psi}_{L} \Phi \psi_{R}$ and $\bar{\psi}_{L} \tilde{\Phi} \psi_{R}$, where $\Phi = (\phi_{0}, \phi_{1}; \phi_{2}^{+}, \phi_{2}^{0}) \sim (1, 2, 2, 0)$ is a Higgs bidoublet and $\tilde{\Phi} = \sigma_{2} \Phi^{*} \sigma_{2} = (\phi_{2}^{0}, -\phi_{2}^{-}; -\phi_{1}^{+}, \phi_{1}^{0})$ transforms in the same way. Both $\langle \phi_{1}^{0} \rangle$ and $\langle \phi_{2}^{0} \rangle$ contribute to $m_{\nu}$ and $m_{e}$, and similarly $m_{u}$ and $m_{d}$ in the quark sector, resulting thus in the appearance of tree-level flavor-changing neutral currents.

Suppose the term $\bar{\psi}_{L} \tilde{\Phi} \psi_{R}$ is forbidden by a symmetry, then the same symmetry may be used to maintain $\langle \phi_{1}^{0} \rangle = 0$ and only $e$ gets a mass through $\langle \phi_{2}^{0} \rangle \neq 0$. At the same time, $\nu_{L}$...
and $\nu_R$ are not Dirac mass partners, so they could in fact be completely different particles with independent masses of their own. Whereas $\nu_L$ is clearly the neutrino we observe in the usual weak interactions, $\nu_R$ can now be something else entirely. Here we rename $\nu_R$ as $n_R$ and show that it may in fact be a scotino, i.e. a fermionic dark-matter candidate.

In our previous proposal [1], we imposed a new global $U(1)$ symmetry $S$ in such a way that the spontaneous breaking of $SU(2)_R \times S$ will leave the combination $L = S - T_{3R}$ unbroken. We then showed that $L$ is a generalized lepton number, with $L = 1$ for the known leptons, and $L = 0$ for all known particles which are not leptons. Here we consider instead the case $L = S + T_{3R}$. Our model is nonsupersymmetric, but it may be rendered supersymmetric by the usual procedure which takes the SM to the MSSM (minimal supersymmetric standard model). Under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1) \times S$, the fermions transform as shown in Table 1. Note the necessary appearance of the exotic quark $h$, which will turn out to carry lepton number as well.

| Fermion | $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)$ | $S$ |
|---------|---------------------------------|-----|
| $\psi_L = (\nu, e)_L$ | $(1, 2, 1, -1/2)$ | 1 |
| $\psi_R = (n, e)_R$ | $(1, 1, 2, -1/2)$ | 3/2 |
| $\nu_R$ | $(1, 1, 1, 0)$ | 1 |
| $n_L$ | $(1, 1, 1, 0)$ | 2 |
| $Q_L = (u, d)_L$ | $(3, 2, 1, 1/6)$ | 0 |
| $Q_R = (u, h)_R$ | $(3, 1, 2, 1/6)$ | $-1/2$ |
| $d_R$ | $(3, 1, 1, -1/3)$ | 0 |
| $h_L$ | $(3, 1, 1, -1/3)$ | $-1$ |

The scalar sector consists of one bidoublet and two doublets:

$$
\Phi = \begin{pmatrix}
\phi_1^0 & \phi_2^+ \\
\phi_1^- & \phi_2^0
\end{pmatrix}, \quad \Phi_L = \begin{pmatrix}
\phi_L^+ \\
\phi_L^0
\end{pmatrix}, \quad \Phi_R = \begin{pmatrix}
\phi_R^+ \\
\phi_R^0
\end{pmatrix}.
$$
Table 2: Scalar content of proposed model.

| Scalar          | $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)$ | $S$  |
|-----------------|--------------------------------------------------|------|
| $\Phi$          | $(1,2,2,0)$                                      | $-1/2$ |
| $\Phi = \sigma_2 \Phi^* \sigma_2$ | $(1,2,2,0)$                                      | $1/2$ |
| $\Phi_L$        | $(1,2,1,1/2)$                                   | $0$  |
| $\Phi_R$        | $(1,1,2,1/2)$                                   | $1/2$ |

Their assignments under $S$ are listed in Table 2.

The Yukawa terms allowed by $S$ are then $\bar{\psi}_L \Phi \psi_R$, $\bar{\psi}_L \tilde{\Phi} \nu_R$, $\bar{\psi}_R \Phi \nu_L$, $\bar{\psi}_L \tilde{\Phi} Q_R$, $\bar{Q}_L \Phi d_R$, and $\bar{Q}_R \Phi h_L$, whereas $\bar{\psi}_L \Phi \psi_R$, $\bar{\nu}_L \nu_R$, $\bar{Q}_L \Phi Q_R$, and $\bar{d}_L d_R$ are forbidden. Hence $m_e$, $m_\nu$ come from $v_2 = \langle \phi_2^0 \rangle$; $m_\nu$, $m_d$ come from $v_3 = \langle \phi_L^0 \rangle$; and $m_n$, $m_h$ come from $v_4 = \langle \phi_R^0 \rangle$. This structure shows clearly that flavor-changing neutral currents are guaranteed to be absent at tree level.

As it stands, both the neutrino $\nu$ and the scotino $n$ are Dirac fermions, and lepton number $L$ is conserved. If we now introduce a soft term $\nu_R \nu_R$ which breaks $L$ by two units, then $\nu_L$ gets a Majorana mass through the canonical seesaw mechanism, as is usually assumed. As for $n$, it remains a Dirac fermion, being protected by a residual global $U(1)$ symmetry, under which $n$, $W_R^+$ transform as 1, and $h$, $\phi_1^{0-}$ transform as $-1$.

**Gauge sector:** Since $e$ has $L = 1$ and $n$ has $L = 2$, the $W_R^+$ of this model must have $L = S + T_{3R} = 0 + 1 = 1$. This also means that unlike the conventional LRM, $W_R^+$ does not mix with the $W_L^+$ of the SM at all. This important property allows the $SU(2)_R$ breaking scale to be much lower than it would be otherwise, as explained already 23 years ago [2, 3]. Let $e/g_L = s_L = \sin \theta_W$ and $s_R = e/g_R$, with $c_{L,R} = \sqrt{1 - s_{L,R}^2}$, then $g_B = e/\sqrt{s_L^2 - s_R^2}$ and
the neutral gauge bosons of the DLRM (as well as the ALRM) are given by

\[
\begin{pmatrix}
A \\
Z \\
Z'
\end{pmatrix} =
\begin{pmatrix}
(s_L & s_R & \sqrt{c_L^2 - s_R^2} \\
c_L & -s_L s_R/c_L & -s_L \sqrt{c_L^2 - s_R^2}/c_L \\
0 & \sqrt{c_L^2 - s_R^2}/c_L & -s_R/c_L
\end{pmatrix}
\begin{pmatrix}
W_0^L \\
W_0^R \\
B
\end{pmatrix}.
\] (2)

Whereas \(Z\) couples to the current \(J_{3L} - s_L^2 J_{em}\) with coupling \(e/s_L c_L\) as in the SM, \(Z'\) couples to the current

\[
J_{Z'} = s_R^2 J_{3L} + c_L^2 J_{3R} - s_L^2 J_{em}
\] (3)

with coupling \(g_{Z'} = e/s_R c_L \sqrt{c_L^2 - s_R^2}\).

The masses of the gauge bosons are given by

\[
M_{W_L}^2 = \frac{e^2}{2s_L^2} (v_2^2 + v_3^2), \quad M_Z^2 = \frac{M_{W_L}^2}{c_L^2}, \quad M_{W_R}^2 = \frac{e^2}{2s_R^2} (v_4^2 + v_2^2),
\] (4)

\[
M_{Z'}^2 = \frac{e^2 c_L^2}{2s_R^2 (c_L^2 - s_R^2)} (v_4^2 + v_2^2) - \frac{s_L^2 s_R^2 M_{W_L}^2}{c_L^2 (c_L^2 - s_R^2)},
\] (5)

where zero \(Z - Z'\) mixing has been assumed, using the condition \[\frac{v_2^2}{v_2^2 + v_3^2} = s_R^2/c_L^2\].

**Direct search constraint from CDMS II**: The \(Z'\) couplings to \(u, d, n\) (in units of \(g_{Z'}\)) are given by

\[
u_L = -\frac{1}{6} s_R^2, \quad u_R = \frac{1}{2} c_L^2 - \frac{2}{3} s_R^2, \quad u_V = \frac{1}{4} c_L^2 - \frac{5}{12} s_R^2,
\] (6)

\[
d_L = -\frac{1}{6} s_R^2, \quad d_R = \frac{1}{3} s_R^2, \quad d_V = \frac{1}{12} s_R^2,
\] (7)

\[
n_L = 0, \quad n_R = \frac{1}{2} c_L^2, \quad n_V = \frac{1}{4} c_L^2.
\] (8)

The effective Lagrangian for elastic scattering of the scotino \(n\) off quarks is then given by

\[
\mathcal{L} = \frac{g_{Z'}^2 n_V}{M_{Z'}^2} (\bar{n} \gamma_\mu n)(u_V \bar{u} \gamma^\mu u + d_V \bar{d} \gamma^\mu d).
\] (9)

In the original DLRM \[\|\), \(n\) is a Majorana scotino, so it does not contribute to the s-wave elastic spin-independent scattering cross section in the nonrelativistic limit. Here \(n\) is a Dirac scotino, so it will contribute. Let

\[
f_P = g_{Z'}^2 n_V (2u_V + d_V)/M_{Z'}^2, \quad f_N = g_{Z'}^2 n_V (u_V + 2d_V)/M_{Z'}^2,
\] (10)
then its elastic cross section per nucleon is given by 

$$\sigma_0 = \frac{4m_r^2}{\pi} \left[ Z f_P + (A - Z) f_N \right]^2 \frac{1}{A^2},$$

(11)

where $Z$ and $A$ are the atomic and mass numbers of the target nucleus, and $m_r = m_n m_P/(m_n + m_P) \simeq m_P$. The CDMS II collaboration [8] observed two possible signal events with an expected background of 0.6 ± 0.1. Using $^{73}$Ge, i.e. $Z = 32$ and $A - Z = 41$, as a representative estimate of $\sigma_0$, this result could also be considered as an upper bound, i.e.

$$\sigma_0 = \frac{\pi \alpha^2 m_P^2 (105 c_L^2 - 137 s_R^2)^2}{(146)^2 s_R^4 (c_L^2 - s_R^2)^2 M_{Z'}^2} < 3.8 \times 10^{-8} \text{ pb},$$

(12)

which occurs at $m_n = 70$ GeV.

**Phenomenological analysis**: We consider the range $e^2 < s_R^2 < c_L^2 - e^2$, where the lower bound corresponds to $g_R = 1$ and the upper bound to $g_B = 1$. The values of $g_{Z'}$ and $\Gamma_{Z'}/M_{Z'}$ are plotted in Fig. 1(a) and (b), where $Z'$ is assumed to decay only into SM fermions.

![Figure 1](image)

Figure 1: (a) $g_{Z'}$ vs $s_R^2$. (b) $\Gamma_{Z'}/M_{Z'}$ vs $s_R^2$ for SM fermions decay products only in the cases $M_{Z'} = 500$ GeV (blue solid) and $M_{Z'} \to \infty$ (red dashed).

We compute the production and decay of $Z'$ to $e^+e^-$ at the Tevatron as a function of $M_{Z'}$ for various values of $s_R^2$ and compare it to data [10] at $E_{cm} = 1.96$ TeV and an integrated
luminosity of 2.5 fb$^{-1}$ in Fig. 2(a). We then plot the exclusion limits on $M_{Z'}$ from both the new CDMS II data and the Tevatron as a function of $s_R^2$ in Fig. 2(b). Note that the CDMS II bound is stronger than the Tevatron bound for $s_R^2 < 0.5$. Note also that due to the accidental cancellation in the numerator of $\sigma_0$ in Eq. (12), the observed events at CDMS II cannot be interpreted as signals of dark matter in this model if $s_R^2 > 0.5$, because they would be excluded by the Tevatron data.

Figure 2: (a) Lower bound on the $Z'$ mass in this model from Tevatron dielectron search. (b) $M_{Z'}$ vs $s_R^2$ from the CDMS II (blue dashed) and Tevatron (red solid) bounds. The dotted segments assume a simple extrapolation of the Tevatron data.

Given that $M_{Z'}$ is allowed to be in the TeV range, its discovery prognosis is excellent at the LHC. We show in Fig. 3 its discovery reach (assuming $E_{cm} = 14$ TeV) by 10 dilepton events (either dielectron or dimuon) which satisfy the following basic cuts on their transverse momenta, rapidities, and invariant mass: $p_T > 20$ GeV (each lepton), $|\eta| < 2.4$ (each lepton), $|M_{\ell\bar{\ell}} - M_{Z'}| < 3\Gamma_{Z'}$.

Using these cuts, the dominant SM background from $\gamma/Z$ (Drell-Yan) is negligible. With an integrated luminosity of 1 fb$^{-1}$, the $Z'$ of DLRM II with $M_{Z'} \sim 2$ TeV may then be discovered at the LHC.
Dark-matter relic abundance: In this model, the dark-matter relic abundance is presumably determined by the annihilation $n\bar{n} \rightarrow Z' \rightarrow \text{SM fermions}$. The thermally averaged cross section multiplied by relative velocity is approximately given by

$$\langle \sigma v_{\text{rel}} \rangle_{Z'} = \frac{g_{Z'}^4 c_L m_n^2 \sum f L + f R^2}{32 \pi (4m_n^2 - M_{Z'}^2)^2},$$

(13)

where the sum over fermions should include a factor of 3 for quarks and an overall factor of 3 for families. Fixing the above at 1 pb as a typical value to satisfy the requirement of dark-matter relic abundance, it can easily be shown that for $m_n = 70$ GeV, the required $M_{Z'}$ is very much below the CDMS II bound. [For example, for $s_R^2 = 0.4$, $M_{Z'} = 267$ GeV would be required.] In other words, the $n\bar{n} \rightarrow Z'$ annihilation cross section would be too small to account for the observed dark-matter relic abundance. To remedy this situation, the mechanism proposed in the original DLRM may be invoked, i.e. $n\bar{n} \rightarrow l^-l^+$ through $\Delta^0_R$ exchange. However, this requires adding the $SU(2)_R$ scalar triplet ($\Delta^{++}_R$, $\Delta^+_R$, $\Delta^0_R$), which is not necessary in our present version and thus not very much motivated. The alternative is to consider a larger value of $m_n$. 

Figure 3: Luminosity for $Z'$ discovery by 10 dielectron events at LHC. Small circles are Tevatron limits.
The CDMS II bound on $\sigma_0$ is very well approximated in the range $0.3 < m_n < 1.0$ TeV by the expression

$$\sigma_0 < 2.2 \times 10^{-7} \text{ pb } (m_n/1 \text{ TeV})^{0.86}.$$  \hspace{1cm} (14)

Using this on the right-hand side of Eq. (12), we plot in Fig. 4(a) and (b) the $M_{Z'}$ bounds for $m_n = 400$ and 600 GeV, as well as the solutions of $M_{Z'}$ (with $M_{Z'} > 2m_n$) to Eq. (13) for 1 pb. We see that there are indeed consistent solutions (where the solid curve is higher than the dash curve) for a range of $s^2_R$ in each case. If $m_n$ falls below 300 GeV, then there is no solution because $M_{Z'}$ would then be excluded by the Tevatron bound. We note also that only a modest resonance enhancement is needed from the denominator of Eq. (13).

The $n\bar{n}$ annihilation to $l^+l^-$ through $W_R$ exchange also contributes to the dark-matter relic abundance, but its value is an order of magnitude less, i.e.

$$\langle \sigma v_{\text{rel}} \rangle_{W_R} = \frac{3g_R^4m_n^2}{64\pi(m_n^2 + M_{W_R}^2)^2}.$$ \hspace{1cm} (15)

Figure 4: (a) For $m_n = 400$ GeV, the CDMS II bound on $M_{Z'}$ (blue dashed) and the value of $M_{Z'}$ (red) from $\langle \sigma v_{\text{rel}} \rangle_{Z'} = 1$ pb vs $s^2_R$; (b) same as in (a) for $m_n = 600$ GeV.
Lepton flavor violation: Unlike the original DLRM, where a scalar triplet \((\Delta_R^{++}, \Delta_R^+, \Delta_R^0)\) may mediate lepton flavor violating processes such as \(\mu \to eee\) at tree level, and must be forbidden by hand, the DLRM II is safe because it has no such interactions. Nevertheless, lepton (as well quark) flavor violation occurs in one loop in the \(SU(2)_R\) sector, in complete analogy to that of the SM in the \(SU(2)_L\) sector. The branching fraction of \(\mu \to e\gamma\) is then

\[
B(\mu \to e\gamma) = \frac{3\alpha|\delta_R|^2}{64\pi} \left( \frac{s_L^2 M_W^2}{s_R^2 M_W^2} \right)^2 < 1.2 \times 10^{-11},
\]

where the experimental upper bound has also been displayed, and \(\delta_R\) is the analog of the well-known suppression factor \(\delta_L = \sum_i U^*_{ei} U_{\mu i} (m^2_{\nu_i}/M^2_W)\) in the SM. For \(s^2_R = s^2_L\), we have \(M_{W_R} = 1.5\) TeV, then \(|\delta_R| < 0.116\). Since the flavor structure of scotino mixing and their mass-squared differences are unknown, this upper bound could be saturated, and the observation of \(\mu \to e\gamma\) may be imminent. The same holds for other lepton flavor violating processes such as \(\mu - e\) conversion in nuclei. Note that the contribution to the muon anomalous magnetic moment here is about \(10^{-10}\), well below the experimental sensitivity. A more comprehensive study, including contributions to \(D^0 - \bar{D}^0\) mixing [11], will be given elsewhere.

Connecting the \(Z'\) and dark-matter searches: As the LHC begins its operation, one of its first possible discoveries could be a \(Z'\) through the process \(q\bar{q} \to Z' \to l^+l^-\). There are many \(Z'\) models, and some of them could also be invoked [12] to explain the CDMS II results. However, the coupling of the dark matter to the \(Z'\) in these models is in general not related to the \(Z'\) leptonic couplings. Here they are intimately connected and predicted as a function of only \(s^2_R\). In fact, if we assume \(s^2_R = s^2_L\) (i.e. left-right symmetry), then there is no free parameter. Our numerical analysis in this paper is only a rough estimate for illustration, but it points to the important assertion that the \(Z'\) interactions in this model are fixed with respect to direct dark-matter search and the detection of \(Z'\) itself at an accelerator. In these exciting times of having both the functioning LHC and ongoing dark-matter search experiments, the dark-matter mystery in astroparticle physics may be near a solution.
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