Numerical Simulation of a Falling Droplet of Liquid Metal into a Liquid Layer in the Presence of a Uniform Vertical Magnetic Field

Toshio TAGAWA

Department of Aerospace Engineering, Tokyo Metropolitan University, Asahigaoka 6-6, Hino 191-0065, Japan
E-mail: ttagawa@cc.tmit.ac.jp

(Received on February 25, 2005; accepted on May 6, 2005)

Dynamics of a falling droplet of liquid metal into a horizontal liquid layer in the presence of a uniform vertical magnetic field is numerically studied. Non-dimensional governing equations for an axisymmetric cylindrical coordinate system, which can simulate incompressible, immiscible two-phase flows like bubble, droplet and free-surface flows, have been derived and solved numerically with a finite difference method using the HSMAC algorithm. The numerical results reveal that the electromagnetic force simply opposes spreading of the disturbance after collision.

KEY WORDS: two-phase flow; numerical simulation; magnetic field.

1. Introduction

Two-phase flows have been studied for their importance in various fields of industry and most of studies are related to water–air two-phase flows. Concerning such numerical studies, the VOF method, the level set method, the front capturing method and the lattice Boltzmann method are known to be useful methods. Harlow and Shannon1) numerically investigated the splash of a liquid drop into a pool with the MAC (Marker-and Cell) technique. They solved the full Navier–Stokes equations for a viscous, incompressible fluid but surface tension was not taken into account in the computation. Sussman et al.2) carried out computations of incompressible two-phase flows for the motion of air bubble in water and falling water drops in air by using a level set approach. Matsumoto and Tanahashi3) numerically simulated flows induced by the surface tension. They demonstrated phenomena such as oscillation of a droplet and capillary convection with curved boundary using the level set method. Sussman et al.3) carried out computations of incompressible two-phase flows for a viscous, incompressible fluid but surface tension was not taken into account in the computation. Matsumoto and Tanahashi3) numerically simulated flows induced by the surface tension. They demonstrated phenomena such as oscillation of a droplet and capillary convection with curved boundary using the level set method. Inamuro et al.4) recently succeeded in a numerical computation of incompressible two-phase flows with large density differences, such as capillary wave, droplet collision and bubble flow with a lattice Boltzmann method.

In the field of metal processing, various kinds of magnetic field, such as static, rotating and alternative ones, are applied to control flows of molten iron, semiconductors, and oxides. However, those flows encountered in such a metal processing have usually free surface and it is difficult to visualize and detect the flow motion inside the molten metal. Therefore, numerical models to simulate such MHD two-phase flows are required. Ueno et al.5) performed a numerical simulation of a deformed bubble rising in a magnetic fluid in the presence of a vertical uniform magnetic field with a front capturing method. Molokov6) studied a dynamics of a free-surface flow in a strong vertical magnetic field with an asymptotic analysis for high Hartmann number. Gao et al.7) simulated a movement of liquid metal droplet in a magnetic field using a VOF-CSF method. Tagawa8) carried out a numerical computation of spin-up from rest in a uniform axial magnetic field with taking deformation of free surface into account. In the present study, as a representative problem of two-phase flows, dynamics of a falling droplet of a liquid metal into a liquid layer under a uniform vertical magnetic field is numerically simulated in order to show the validity of a new numerical modeling of the MHD two-phase flow.

2. Model and Equations

2.1. Schematic Model

A schematic model is shown in Fig. 1. A cylindrical enclosure whose radius $a=3d$ and height $h=3d$ is filled with an incompressible, Newtonian, electro-conducting fluid (liquid metal) from the bottom with thickness of 0.1875$d$ (here, $d$ is the diameter of a droplet). A spherical metal droplet is released from the height of 1.5$d$ and accelerated downwards due to the gravity. A uniform vertical magnetic field...
field is imposed to investigate the effect of the Lorentz force on the dynamics after collision.

It is supposed that the flow and electromagnetic fields are axisymmetric, and neither the induced magnetic field nor the Joule heating is generated. In this situation, boundary conditions for electric current have nothing to do with the two-phase flow whether the enclosure walls are electrically insulating or conducting since the direction of the induced electric current density is azimuthal and the electric field does not exist.

2.2. Governing Equations

In the domain of the cylindrical enclosure, both liquid and gas phases as well as interfacial phase with finite thickness are solved simultaneously like a simple algorithm of single-phase flows. The phase is distinguished by the value of the index function \( \phi \), whether it is gas, liquid or interfacial one. In this study, it is assumed that the index function takes its value 0.5 on the gas phase and -0.5 on the liquid phase. The index function obeys the following advective equation using the axisymmetric cylindrical coordinate, which gives

\[
\frac{\partial \phi}{\partial t} + u \frac{\partial \phi}{\partial r} + w \frac{\partial \phi}{\partial z} = 0 \quad \ldots \ldots \ldots (1)
\]

This equation can be interpreted as a kind of density equation. The immiscible continuity equation is written with

\[
\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + w \frac{\partial \rho}{\partial z} = -\rho \left( \frac{\partial u}{\partial r} + u \frac{\partial}{\partial r} + \frac{\partial w}{\partial z} \right) \quad \ldots \ldots \ldots (2)
\]

If both the liquid and gas phases are assumed to be incompressible, the continuity equation, which can be derived by considering volume balance, is written as

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} = 0 \quad \ldots \ldots \ldots (3)
\]

From Eqs. (2) and (3), the following density equation is derived as

\[
\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + w \frac{\partial \rho}{\partial z} = 0 \quad \ldots \ldots \ldots (4)
\]

Equation (1) can be derived from Eq. (4). To simulate the two-phase flow precisely, it is necessary to solve the momentum equation not only for the liquid phase but also for the gas phase simultaneously. The Navier-Stokes equation can be written with vector notation as

\[
\rho \left( \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} \right) = -\nabla p + \mu \left( \nabla^2 \vec{u} - \frac{u}{r^2} \vec{e}_r \right) + \rho \vec{g} \quad \ldots \ldots \ldots (5)
\]

Where, \( \nabla = \nabla_r \vec{e}_r + \partial / \partial z \vec{e}_z \) and \( \nabla^2 = \partial^2 / \partial r^2 + (1/r) \partial / \partial r + \partial^2 / \partial z^2 \). When the fluid density difference is small, as seen in a computation of single-phase natural convection, the Boussinesq approximation is useful to reflect buoyant effect in such computation of fluid flow. However, the density difference in two-phase flows is too large to utilize the Boussinesq approximation and therefore it is necessary to seek an alternative method for solving the Navier-Stokes equation in two-phase flow problems with large density difference. To explain an alternative method, let us consider the momentum equation. The Eq. (5) can be written for the liquid and gas phases as follows:

\[
\frac{\partial \vec{u}_l}{\partial t} + \vec{u}_l \cdot \nabla \vec{u}_l + \frac{\partial \rho}{\partial r} + \frac{\partial \rho}{\partial z} = -\nabla p + \mu \left( \nabla^2 \vec{u}_l - \frac{u}{r^2} \vec{e}_r \right) - \rho \vec{g} \quad \ldots \ldots \ldots (6)
\]

\[
\frac{\partial \vec{u}_g}{\partial t} + \vec{u}_g \cdot \nabla \vec{u}_g + \frac{\partial \rho}{\partial r} + \frac{\partial \rho}{\partial z} = -\frac{1}{\rho} \nabla p + \frac{\mu}{\rho} \left( \nabla^2 \vec{u}_g - \frac{u}{r^2} \vec{e}_r \right) - \rho \vec{g} \quad \ldots \ldots \ldots (7)
\]

Where, \( \vec{f} \) indicates an external force. The Eqs. (6) and (7) are different from each other in terms of fluid properties. Thus it seems to be rather difficult to solve these two equations separately depending on the phase. However, if the physical properties such as density and viscosity vary across the interface continuously, one of possible and convenient methods is to establish a relationship between the density and the index function. The fluid density is supposed to be a function of \( \phi \) as

\[
\rho = \frac{1}{2} (\rho_l + \rho_g) - (\rho_l - \rho_g) \phi, \quad \text{or} \quad \phi = -\frac{\rho - 0.5(\rho_l + \rho_g)}{\rho_l - \rho_g} \quad \ldots \ldots \ldots (8)
\]

In the same way, the viscosity is supposed to be a function of \( \phi \) as

\[
\mu = \frac{1}{2} (\mu_l + \mu_g) - (\mu_l - \mu_g) \phi, \quad \text{or} \quad \phi = -\frac{\mu - 0.5(\mu_l + \mu_g)}{\mu_l - \mu_g} \quad \ldots \ldots \ldots (9)
\]

By having those relationships, the Eqs. (6) and (7) can be replaced with a momentum equation for any phase such as liquid, gas or interfacial one as long as Eqs. (8) and (9) are taken into account. The momentum equation is approximated with

\[
\frac{\partial \vec{u}_l}{\partial t} + \vec{u}_l \cdot \nabla \vec{u}_l + \frac{\partial \rho}{\partial r} + \frac{\partial \rho}{\partial z} = -\frac{1}{\rho} \nabla p + \frac{\mu}{\rho} \left( \nabla^2 \vec{u}_l - \frac{u}{r^2} \vec{e}_r \right) - \rho \vec{g} \quad \ldots \ldots \ldots (10)
\]

It is noted that a force term derived from the special variation of viscosity encountered in the interfacial phase is neglected.

Now, let us make a brief summary by comparing the buoyant convection flow and the two-phase flow. In the buoyant flow, the viscosity is usually supposed to be constant and the density is assumed to be a function of temperature using the thermal expansion coefficient only for the gravity term (Boussinesq approximation). The temperature field is governed by an energy equation. On the other hand, in the two-phase flow, both the viscosity and density are the function of the index function (see Eqs. (8) and (9)). The equation of the index function has been mentioned in Eq. (1). Anyway there are similarities except that there is no
diffusion term in Eq. (1) since an immiscible two-phase flow is assumed.

2.3. External Forces

When electric conducting materials move in a magnetic field, electric current is induced in the materials and the Lorentz force acts. In this paper, the Lorentz force is considered as an external force. This force is written with the product of electric current density and the magnetic induction, which gives

$$\vec{f} = \nabla \times \vec{B} = \sigma (-\nabla \psi + \vec{u} \times \vec{B}) \times \vec{B} \quad \text{......(11)}$$

In the present model system, the direction of the applied magnetic field is vertical and the assumption of axisymmetric flow does not make any electric field. Therefore, the Lorentz force is simply written with

$$\vec{f} = -\sigma \mu \frac{\partial \vec{B}}{\partial r} \hat{r} + (0) \hat{z} \quad \text{......(12)}$$

Where, $\sigma$ is the electric conductivity of the fluid phase. If the interfacial tension like an external force. This force is written with the gas phase. Therefore, as shown in Eqs. (8) and (9), the fluid phase is liquid, this value is finite, but it is zero for any uniform phase. Therefore, the body force due to surface tension does not act on the liquid and gas phases but acts on the interfacial phase.

2.4. Non-dimensional Equations

The non-dimensional governing equations for the axisymmetric cylindrical coordinate system are summarized as follows:

(Continuity equation)

$$\frac{\partial U}{\partial R} + \frac{U}{R} \frac{\partial W}{\partial Z} = 0 \quad \text{......(17)}$$

(Momentum equation)

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial R} + W \frac{\partial U}{\partial Z} = -\frac{1}{\rho} \hat{V} P + \frac{\mu}{\rho} \left( \nabla^2 U - \frac{U}{R^2} \hat{e}_r \right) - G e_z - \frac{\sigma}{\rho} \hat{\mu} H a^2 U e_r, \quad \text{......(18)}$$

(Index function)

$$\frac{\partial \phi}{\partial t} + U \frac{\partial \phi}{\partial R} + W \frac{\partial \phi}{\partial Z} = 0 \quad \text{......(19)}$$

The non-dimensional density, viscosity and conductivity appeared in Eq. (18) are defined as follows:

$$\tilde{\rho} = \frac{\rho}{\tilde{\rho}}, \quad \tilde{\mu} = \frac{\mu}{\tilde{\mu}}, \quad \tilde{\sigma} = \frac{\sigma}{\tilde{\sigma}}, \quad \text{for } \phi < 0.5$$

Where, the non-dimensional variables and numbers are defined as follows:
After having got a distribution of $H$ within the computational domain, the inverse transformation of tangent function is made to get a distribution of the index function $\phi$ as

$$\phi = \frac{1}{c\pi} \tan^{-1} H, \quad c = 0.9 \quad \text{.................(25)}$$

By this simple technique, the thickness of the interface is kept thin quite easily. The distribution of $\phi$ at the interface is shaper than that of $H$.

The second technique to be employed in the computation is to simplify the surface tension term in Eq. (18). The surface tension term is estimated from the function of $H$ since the sharp interface sometimes fails to estimate the surface tension correctly. The momentum equation is modified as

$$\frac{\partial \tilde{U}}{\partial \tau} + U \frac{\partial \tilde{U}}{\partial R} + \tilde{W} \frac{\partial \tilde{U}}{\partial Z} =$$

$$= -\frac{1}{\bar{\rho}} \tilde{\nabla} P + \frac{\bar{\rho}}{\bar{\rho}} \left( \nabla^2 \tilde{U} - \frac{U}{R^2} \tilde{e}_r \right) - G \tilde{e}_z - \frac{\sigma}{\bar{\rho}} \mu H a^2 \tilde{e}_r$$

$$- \Gamma \left( \frac{\partial^2 H}{\partial R^2} + \frac{1}{R} \frac{\partial H}{\partial R} + \frac{\partial^2 H}{\partial Z^2} \right) \tilde{\nabla} H \quad \text{.................(26)}$$

This simplification is valid only when the gradient of index function is constant across the interfacial region. By introducing this simplification, the thickness of the interface is almost automatically kept constant throughout the computation.

The third technique is to modify the relationships in Eqs. (20), (21) and (22). The modified equations are as follows:

$$\bar{\rho} = \begin{cases} \tilde{\rho}, & \text{for } \phi < -0.5 \\ \frac{1}{2} (\tilde{\rho} + 1) - \frac{1}{2} (\tilde{\rho} - 1) \sin(\pi \phi), & -0.5 \leq \phi \leq 0.5 \\ 1, & \text{for } 0.5 < \phi \end{cases}$$

$$\text{..........................(27)}$$

$$\bar{\mu} = \begin{cases} \tilde{\mu}, & \text{for } \phi < -0.5 \\ \frac{1}{2} (\tilde{\mu} + 1) - \frac{1}{2} (\tilde{\mu} - 1) \sin(\pi \phi), & -0.5 \leq \phi \leq 0.5 \\ 1, & \text{for } 0.5 < \phi \end{cases}$$

$$\text{..........................(28)}$$

$$\bar{\sigma} = \begin{cases} 1, & \text{for } \phi < -0.5 \\ \frac{1}{2} (1 + \tilde{\sigma}) - \frac{1}{2} (1 - \tilde{\sigma}) \sin(\pi \phi), & -0.5 \leq \phi \leq 0.5 \\ \frac{\sigma}{\bar{\sigma}}, & \text{for } 0.5 < \phi \end{cases}$$

$$\text{..........................(29)}$$

During the computation, if there are portions greater than 0.5 or less than −0.5 for the index function, such portions are overwritten with the value of 0.5 or −0.5 respectively.

2.6. Numerical Conditions

The initial shape of the liquid droplet is a sphere whose diameter is 1.0 with the dimensionless unit and whose position is 1.5 away from the bottom of enclosure. The thickness of liquid layer is 3/16. The diameter of enclosure is 6.0...
and the height is 3.0. For \( \tau < 0 \), the pressure is uniform throughout the computational domain and then suddenly the gravitational force is imposed at \( \tau = 0 \). The initial thickness of the interface is zero. The initial conditions are as follows:

\[
U = W = 0, \quad \phi = 0.5 \quad \text{for} \quad \left( \frac{x}{0.5} \right)^2 + \left( \frac{z-1.5}{0.5} \right)^2 > 1
\]

and \( z > 0.1875 \)

\[
\phi = -0.5 \quad \text{for} \quad \left( \frac{x}{0.5} \right)^2 + \left( \frac{z-1.5}{0.5} \right)^2 \leq 1
\]

and \( z < 0.1875 \) for \( \tau = 0 \)

Concerning the boundary conditions, the non-slip condition is employed at the walls and the contact angle is supposed to be 90 degree. The boundary conditions are as follows:

\[
U = W = 0, \quad \frac{\partial \phi}{\partial Z} = 0 \quad \text{at} \quad Z = 0.3
\]

\[
U = W = 0, \quad \frac{\partial \phi}{\partial R} = 0 \quad \text{at} \quad R = 3
\]

\[
U = \frac{\partial W}{\partial R} = 0, \quad \frac{\partial \phi}{\partial Z} = 0 \quad \text{at} \quad R = 0
\]

The numerical conditions are summarized in Table 1. The computational domain of a vertical square cross-section of the cylinder is divided into 256 times 256 square meshes.

### Table 1. Computational parameters with dimensionless unit.

| Parameters   | Value                  |
|--------------|------------------------|
| \( H_a \), Hartmann number | 0, 50, 100, 200, 500 |
| \( G \), Galilei number       | \( 3 \times 10^6 \)   |
| \( \Gamma \), Tension number   | \( 10^7 \)         |
| \( \rho \), Density ratio      | 100                  |
| \( \mu \), Viscosity ratio     | \( 100/9 \)          |
| \( \sigma \), Conductivity ratio | 0                    |
| Initial diameter of drop      | 1                    |
| Initial height of drop        | 1.5                  |
| Thickness of liquid layer     | 0.1875               |
| Diameter of enclosure         | 6                    |
| Height of enclosure           | 3                    |

3. Numerical Results

In this paper, numerical results obtained from Eqs. (17) and (23) to (29) are shown. Figures 2, 3, 4, 5 and 6 show the index function for various time instances for \( H_a = 0, 50, 100, 200 \) and 500 respectively. The black part represents the liquid phase (\( \phi < 0 \)) and the white part is the gas phase (\( \phi \geq 0 \)). The spherical droplet was accelerated downwards due to the gravitational force and collided with the flat liquid surface. When no magnetic field was applied in this system \( (H_a = 0) \), see Fig. 2), the dynamics after collision was very violent and a sheet jet was formed as reported in Ref. 1). At \( \tau = 15 \times 10^{-4} \), the tip of the sheet jet was removed from it and a small amount of liquid droplet was formed. In the assumption of axisymmetric flow, this should be like the shape of a ring or a doughnut. At \( \tau = 26 \times 10^{-4} \), the small
droplet was involved in the bottom moving liquid flow and thereafter the dynamic motion lasted quite a long time. When $Ha=50$ (Fig. 3), such removal from the liquid jet was not observed and the movement of liquid phase was moderately damped by the Lorentz force. As increase in the Hartmann number (Figs. 4–6), the imposition of uniform vertical magnetic field strongly inhibited the disturbance of liquid flow after the collision. When $Ha=500$, the spreading rate of liquid metal was substantially reduced and a swelling of free surface, whose height decreases very slowly, was observed. However, the falling speed of the droplet was not influenced as recognized by comparing Figs. 2 and 6.

**Figure 7** shows an instantaneous contour lines of the index function and velocity vectors for $Ha=0$ at $t=16\times10^{-4}$. The arrows indicate velocity vectors and the contour lines shows the index function. The number of lines is 15.

**4. Conclusions**

Numerical computations were carried out for a falling droplet into a liquid layer under a uniform vertical magnetic field with a new numerical modeling of MHD two-phase flow. The numerical result exhibits significant difference in the splash phenomena depending on the strength of magnetic field. As increase in the Hartmann number, the spreading rate of liquid metal was substantially reduced and a swelling of free surface, whose height decreases very slowly, was observed. However, the falling speed of the droplet was not influenced as recognized by comparing Figs. 2 and 6.

**Figure 7** shows an instantaneous contour lines of the index function and velocity vectors for $Ha=0$ at $t=16\times10^{-4}$. The number of lines is 15.

**Figure 8** shows an instantaneous contour lines of the index function and velocity vectors on (a) and filled contour lines of the azimuthal electric current density on (b) for $Ha=100$ at $t=13\times10^{-4}$. The number of contour lines is 9 for both (a) and (b). For any value of the Hartmann number, sharp interfaces were maintained throughout the computation. From (b), it is recognized that the electric current density is passing only in the liquid phase. The black portions indicate a large magnitude of the electric current while white ones indicate a small magnitude. Therefore, the Lorentz force can act only in the liquid phase depending on the strength of electric current in azimuthal direction.

**Fig. 7.** An instantaneous profile of the splash for $Ha=0$ at $t=16\times10^{-4}$. The arrows indicate velocity vectors and the contour lines shows the index function. The number of lines is 15.
just changing the initial condition and aspect ratio of enclosure. Besides it may be easy to apply this modeling to three-dimensional problems, although it will require a certain amount of computational time.

**Nomenclature**

- $a$: Radius of cylinder (m)
- $B$: Magnetic flux density $= B_0 \hat{e}_z$ (T)
- $B_0$: Imposed magnetic field strength (T)
- $c$: Constant value used in tangent function
- $d$: Diameter of initial spherical droplet (m)
- $e_n$: Unit vector normal to interface
- $e_r$: Unit vector along radial direction
- $e_z$: Unit vector along vertical direction
- $e_q$: Unit vector along azimuthal direction
- $f$: External force $= f_r \hat{e}_r + f_{z} \hat{e}_z$ (N/m$^3$)
- $f_r$: Radial component of external force (N/m$^3$)
- $f_z$: Vertical component of external force (N/m$^3$)
- $f_{sn}$: Surface force normal to interface (N/m$^3$)
- $f_{vn}$: Volume force normal to interface (N/m$^3$)
- $g$: Gravitational acceleration vector $= -g \hat{e}_z$ (m/s$^2$)
- $G$: Galilei number $= g \rho_G d^3 \mu_G^2$
- $h$: Height of cylinder (m)
- $H$: Transformed function
- $Ha$: Hartmann number $= \sqrt{\sigma_L / \mu_L B_0 d}$
- $J$: Electric current density $= j_a \hat{e}_a = -uB_0 \hat{e}_a$ (A/m$^2$)
- $p$: Pressure (Pa)
- $P$: Dimensionless pressure
- $r$: Radial coordinate (m)
- $R$: Dimensionless radial coordinate
- $t$: Time (s)
- $\tilde{u}$: Velocity $= u \hat{e}_r + w \hat{e}_z$ (m/s)
- $u$: Radial component of velocity (m/s)
- $\tilde{U}$: Dimensionless velocity $= U \hat{e}_r + W \hat{e}_z$
- $U$: Dimensionless radial component of velocity
- $w$: Vertical component of velocity (m/s)
- $W$: Dimensionless vertical component of velocity
- $z$: Vertical coordinate (m)
- $\bar{Z}$: Dimensionless vertical coordinate

**Greek letters**

- $\gamma$: Surface tension (N/m)
- $\bar{\Gamma}$: Tension number $= \gamma \rho_e d / \mu_G^2$
- $\kappa$: Curvature of interface (m$^{-1}$)
- $\mu$: Viscosity (Pa s)
- $\bar{\mu}$: Dimensionless viscosity $= \mu / \mu_G$
- $\bar{\mu}_G$: Viscosity ratio $= \mu / \mu_G$
- $\bar{\rho}$: Density (kg/m$^3$)
- $\bar{\rho}_G$: Dimensionless density $= \rho / \rho_G$
- $\bar{\rho}_L$: Density ratio $= \rho_L / \rho_G$
- $\sigma$: Electric conductivity ($\Omega^{-1}$ m$^{-1}$)
- $\bar{\sigma}$: Dimensionless electric conductivity $= \sigma / \sigma_L$
- $\bar{\sigma}_L$: Electric conductivity ratio $= \sigma / \sigma_L$
- $\tau$: Dimensionless time

---

Fig. 8. An instantaneous profile of the splash for $Ha=100$ at $\tau=13 \times 10^{-4}$. (a) Velocity vectors and contour lines of the index function. (b) Filled contour lines of the azimuthal electric current density. The number of lines is 9 for both (a) and (b).
\( \phi \): Index function

\( \psi \): Electric potential (V)

**Notations**

\[ \nabla: \frac{\partial}{\partial r} \cdot e_r + \frac{\partial}{\partial z} \cdot e_z \] or \[ \frac{\partial}{\partial R} \cdot e_r + \frac{\partial}{\partial Z} \cdot e_z \]

\[ \nabla^2: \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \] or \[ \frac{\partial^2}{\partial R^2} + \frac{1}{R} \frac{\partial}{\partial R} + \frac{\partial^2}{\partial Z^2} \]

**Subscripts**

G: Gas phase
L: Liquid phase

**REFERENCES**

1) F. H. Harlow and J. P. Shannon: *J. Appl. Phys.*, 38 (1967), 3855.
2) M. Sussman, P. Smereka and S. Osher: *J. Comput. Phys.*, 114 (1994), 146.
3) M. Matsumoto and T. Tanahashi: *Trans. JSCES*, Paper No. 20010031.
4) T. Inamuro, T. Ogata, S. Tajima and N. Konishi: *J. Comput. Phys.*, 198 (2004), 628.
5) K. Ueno, T. Nishita and S. Kamiyama: *J. Magn. Magn. Mater.*, 201 (1999), 281.
6) S. Molokov: Proc. of 5th Pamir Conf., Vol. 2, (2002), 65.
7) D. Gao, N. B. Morley and V. Dhir: *Trans. ASME J. Fluids Eng.*, 126 (2004), 120.
8) J. U. Brackbill, D. B. Kothe and C. Zemach: *J. Comput. Phys.*, 100 (1992), 335.
9) T. Tagawa: Proc. of Japan-France Cooperative Science Program Seminar on Evolving New Fields in Electromagnetic Processing of Materials, Keihanna, Japan, (2004), 101.
10) C. W. Hirt, B. D. Nichols and N. C. Romero: Los Alamos Scientific Laboratory Report, LA-5852, (1975).
11) T. Tagawa and H. Ozoe: *Numer. Heat Transfer, A*, 30 (1996) 271.
12) T. Yabe, T. Utsumi and Y. Ogata: CIP Method (Japanese), Morikita Shuppan Co., Ltd., Tokyo, (2003), 35.