Analysis of the stationary availability factor of fault-tolerant three-layer local area networks

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Abstract. This scientific paper is devoted to the fault-tolerant local area networks with three functional layers: core, distribution and access. The reliability analysis of three-layer networks using the reliability model of the group of identical and independent restorable elements and offered by the author formulas for calculation of the stationary availability factor of three-layer local area networks with single and multiple distribution subgroups are discussed. Finally, an example of calculation of the stationary availability factor of the fault-tolerant three-layer local area network with two distribution subgroups is presented.

1. Introduction

In the modern world, the local area networks [1-2] have become an essential part of human life and enterprise business processes. In present days, most enterprises use the local area networks, based on the three-layer architecture [3-4], which includes the core, distribution and access layers. Each of layers implements appropriate network functions and services and all together they provide high performance and security of the local area network.

Besides the technical characteristics of local area networks such as performance, security and scalability, the reliability parameters are also fundamental [5-6]. The reliability parameters directly affect the stability of the network and the availability of the network resources. One of the well-known reliability parameters is the availability factor, which is widely used for the local area networks. Therefore, analysis of the availability factor of local area networks is a quite urgent scientific task.

Within the scope of scientific research in the field of reliability of the computer networks [7, 8], the author carried out the reliability analysis of the three-layer local area networks using the reliability model of the group of identical and independent restorable elements [9-10] and obtained the formulas for calculation of the stationary availability factor of three-layer local area networks.

2. Structure of the three-layer local area networks

In accordance with the recommendations of leading manufacturers of the network hardware and software and developers of the network technologies and protocols, such as Cisco Systems Inc., three hierarchical layers are highlighted in the modern local area networks:

- Core layer.
- Distribution layer.
- Access layer.
The core layer is responsible for fast and reliable transmission of a large amount of data. The primary function of the core layer is fast network switching, because the traffic of the whole network is processed on the core layer. If an error occurs on the core layer, it affects all users of the network.

The distribution layer is also known as the layer of the user workgroups. The main functions of the distribution layer are network traffic routing and filtration for the workgroups. The workgroups, as a rule, belong to different virtual local area networks, and the distribution layer provides routing between the virtual local area networks. The rules of access of the workgroups network traffic to the core layer are also usually defined on the distribution layer.

Finally, the access layer implements the network access control for users and workgroups.

For providing the fault-tolerance on the core and distribution layers, there are used the groups of identical network switches connected to each other. The access layer switches are connected to each of the distribution layer switches. Moreover, it is possible to use multiple physical links between the switches to provide high reliability of the network connections.

Figure 1 shows an example of the three-layer fault-tolerant local area network.

For simplify the reliability analysis of the local area network, we will consider that the switches of the core, distribution and access layers are independent of the viewpoint of failure and repair events. Moreover, for simplification of the reliability analysis, we will not take into consideration the possibility of failure of the physical links between the network switches. Finally, within each layer, we will consider that the switches have the same reliability parameters.

Accordingly, at first, we will overview the well-known reliability model of the group of identical and independent elements based on the Markov birth-death chain. Next, based on this model, we will
derive formulas for calculation of the stationary availability factor of the three-layer networks with single and multiple distribution subgroups.

3. The reliability model of the group of identical and independent restorable elements
Let a group of the $n$ identical restorable elements with the failure rate $\lambda$ and repair rate $\mu$ is given. The elements may fail and can be repaired independently without any restrictions.

Let us discuss the reliability model based on the Markov birth-death chain. Figure 2 shows the Markov birth-death chain for the group of the $n$ identical and independent restorable elements.

The Markov chain contains states $i = 0 \ldots n$ and each of the states corresponds to the number of the inoperable elements. Accordingly, state 0 corresponds to the case, when all of the elements are operable, and state $n$ corresponds to the case when all of the elements are inoperable.

\[ P_i = \frac{C^n_i}{(1 + p)^n}; \quad i = 0 \ldots n; \quad p = \frac{\lambda}{\mu}. \]

4. The availability factor of the three-layer local area network with a single distribution group
Let us discuss the three-layer network with single distribution group, which contains $r$ core layer switches, $s$ distribution layer switches and $k$ access layer switches.

Switches of the core and distribution layers are connected. The access layer switches are not connected, but they are connected to each of the distribution layer switches. Figure 3 shows an example of the three-layer network with a single distribution group.

Failure of any of the access layer switches, as well as the impossibility of data transmission from anyone access layer switch to any other access layer switch, is considered as a failure of the whole network.

The failure rate of the core layer switches is $\lambda_C$ and repair rate is $\mu_C$.

The failure rate of the distribution layer switches is $\lambda_D$ and repair rate is $\mu_D$.

The failure rate of the access layer switches is $\lambda_A$ and repair rate is $\mu_A$. 

\[ \sum_{i=0}^{n} P_i = 1; \]
\[ 0 = -n\lambda P_0 + \mu P_1; \]
\[ 0 = n\lambda P_0 - (\mu + (n-1)\lambda)P_1 + 2\mu P_2; \]
\[ \vdots \]
\[ 0 = 2\lambda P_{n-2} - ((n-1)\mu + \lambda)P_{n-1} + n\mu P_n; \]
\[ 0 = \lambda P_{n-1} - n\mu P_n. \]
In the three-layer local area network with a single distribution group, we can highlight the following three groups of elements, independent from the viewpoint of failure and repair events:

- The group of the core layer switches.
- The group of the distribution layer switches.
- The group of access layer switches.

![Figure 3. Fault-tolerant local area network with a single distribution group.](image)

In the group of the core layer switches as far as each of the core layer switches is connected to each of the distribution layer switches, the first condition for the whole network operability is that at least one of the switches in the group of the core layer switches is operable. The stationary probability of that at least one of the core layer switches is operable is equal to the sum of the stationary probabilities of the states from 0 to \( r - 1 \) in the reliability model of the group of \( r \) independent elements with the failure rate \( \lambda_C \) and repair rate \( \mu_C \), and it can be calculated by the following formula:

\[
P_C = \sum_{i=0}^{r-1} P_i^{(C)} = 1 - P_r^{(C)} = 1 - \frac{\rho_C^r}{(1 + \rho_C)^r};
\]

\[
\rho_C = \frac{\lambda_C}{\mu_C}.
\] (2.1)

Next, in the group of distribution layer switches, as far as each of the distribution layer switches is connected to each of the core layer switches and each of the access layer switches, the second condition for the whole network operability is that at least one of the switches in the group of the distribution layer switches is operable. The stationary probability of that at least one of the distribution layer switches is operable is equal to sum of the stationary probabilities of the states from 0 to \( s - 1 \) in the reliability model of the group of \( s \) independent elements with the failure rate \( \lambda_D \) and repair rate \( \mu_D \), and it can be calculated by the following formula:

\[
P_D = \sum_{i=0}^{s-1} P_i^{(D)} = 1 - P_s^{(D)} = 1 - \frac{\rho_D^s}{(1 + \rho_D)^s};
\]

\[
\rho_D = \frac{\lambda_D}{\mu_D}.
\] (2.2)

Finally, in the group of access layer switches as far as the failure of any of the access layer switches is considered as the failure of the whole network, the third condition for the whole network operability is that all of the access layer switches are operable. The stationary probability of that all of the access layer switches are operable is equal to the stationary probability of the state 0 in the reliability model of the group of \( k \) independent elements with the failure rate \( \lambda_A \) and repair rate \( \mu_A \). The following formula can calculate it:
\[ P_\lambda = P_0^{(\lambda)} \frac{1}{(1 + \rho_\lambda)^k}; \]

\[ \rho_\lambda = \frac{\lambda_\lambda}{\mu_\lambda}. \]  

(2.3)

By taking into account all three conditions for the whole network operability, we can calculate the stationary availability factor of the network as the product of the probabilities \( P_C, P_D \) and \( P_\lambda \):

\[ K_{\text{NET}} = P_C P_D P_\lambda = \frac{((1 + \rho_C)^s - \rho_C^s)((1 + \rho_D)^s - \rho_D^s)}{(1 + \rho_C)^s(1 + \rho_D)^s}. \]  

(2.4)

5. The availability factor of the three-layer network with multiple distribution subgroups

Let us discuss the three-layer network with multiple distribution subgroups, which contains \( r \) core layer switches, \( m \) subgroups with \( s_j \) distribution layer switches and \( k_j \) access layer switches in each of the subgroups, where, \( j = 1 \ldots m \) is the index of the subgroup.

The core layer switches are connected to each other. The distribution layer switches are divided to \( m \) subgroups. Between the different subgroups, the distribution layer switches are not connected. Within each of the subgroups, the distribution layer switches is connected to each other, and each of them are also connected to each of the core layer switches. The access layer switches are not connected, but they are also divided into \( m \) subgroups and connected to each of the distribution layer switches within the appropriate subgroup. Figure 4 shows an example of the three-layer network with two distribution subgroups.

Failure of any of the access layer switches, as well as the impossibility of data transmission from anyone access layer switch to any other access layer switch, is considered as the failure of the whole network.

![Figure 4](image_url)

**Figure 4.** Fault-tolerant local area network with two distribution subgroups.

In the three-layer local area network with multiple distribution subgroups, we can highlight the following three groups of the elements, independent from the viewpoint of failure and repair events:

- The group of the core layer switches.
- The group of the \( m \) subgroups of the distribution layer switches.
- The group of the \( m \) subgroups of the access layer switches.

The subgroups are also independent of the viewpoint of failure and repair events.

The three-layer network with multiple subgroups is operable at conditions that at least one of the core layer switches is operable, in each of the subgroups at least one of the distribution layer switches is operable, and all of the access layer switches are operable.

Accordingly, the stationary availability factor of the network can be calculated as the product of the following probabilities and product of probabilities:
- Probability of that at least one of the \( r \) core layer switches is operable.
- Product for \( j = 1 \ldots m \) of the probabilities of that at least one of the \( s_j \) distribution layer switches is operable in the \( j \)-th subgroup.
- Product for \( j = 1 \ldots m \) of the probabilities of that all of the \( k_j \) access layer switches is operable in the \( j \)-th subgroup.

By taking into account all of the aforesaid, we obtain the following formula for calculation of the stationary availability factor of the three-layer network with multiple distribution subgroups:

\[
K_{\text{NET}} = \frac{(1 + \rho_C)^r - \rho_C^r}{(1 + \rho_C)^r} \prod_{j=1}^{m} \left[ \frac{(1 + \rho_D)^{s_j} - \rho_D^{s_j}}{(1 + \rho_D)^{s_j} (1 + \rho_A)^{k_j}} \right].
\]  

(3)

6. Example of calculation of the stationary availability factor

A three-layer network, containing \( r = 2 \) core layer switches and \( m = 2 \) distribution subgroups, is given. The first subgroup contains \( s_1 = 2 \) distribution layer switches and \( k_1 = 3 \) access layer switches. The second subgroup contains \( s_2 = 2 \) distribution layer switches and \( k_2 = 4 \) access layer switches. Figure 4 shows the topology of the given network.

The failure rate of the core layer switches is \( \lambda_C = 1/52560 \) hour\(^{-1} \) (on average one failure in 6 years), and repair rate is \( \mu_C = 1/24 \) hour\(^{-1} \) (on average one repair in 24 hours).

The failure rate of the distribution layer switches is \( \lambda_D = 1/26280 \) hour\(^{-1} \) (on average one failure in 3 years), and repair rate is \( \mu_D = 1/6 \) hour\(^{-1} \) (on average one repair in 6 hours).

The failure rate of the access layer switches is \( \lambda_A = 1/17520 \) hour\(^{-1} \) (on average one failure in 2 years), and the repair rate is \( \mu_A = 1 \) hour\(^{-1} \) (on average one repair in 1 hour).

Considering that \( \rho_C = \lambda_C / \mu_C = 1/2190, \rho_D = \lambda_D / \mu_D = 1/4380, \rho_A = \lambda_A / \mu_A = 1/17520 \) and using formula (3), we obtain the following value for the stationary availability factor of the given network:

\[
K_{\text{NET}} \approx 0.9996002.
\]

7. Conclusion

Thus, within the scope of this scientific research, the author carried out the reliability analysis of the three-layer local area networks using the reliability model of the group of identical and independent restorable elements and obtained the formulas for calculation of the stationary availability factor of three-layer networks with single and multiple distribution subgroups.

An example of calculation of the stationary availability factor of the fault-tolerant three-layer local area network with two distribution subgroups is also presented.

Scientific results, obtained by the author, can be used for the reliability analysis of fault-tolerant three-layer local area networks.

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