On frequency errors of nanomechanical-resonators-based-on quantum computing

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We study the consequence of the frequency errors of individual oscillators on the scalability of quantum computing based on nanomechanical resonators. We show the fidelity change of the quantum operation due to the frequency shifts numerically. We present a method to perfectly compensate for these negative effects. Our method is robust to whatever large frequency errors.

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I. INTRODUCTION

Scalability is one of the most important issues in the realization of quantum computing. For example, in order to factorize a 200-digit number using a quantum computer, one needs to manipulate thousands of qubits.6 However, in realistic physical systems, there are many imperfections, such as the quantum decoherence, the device errors, and so on. These imperfections seriously limit the power of a quantum computing device. In particular, in a large scale quantum computing, the small error of each individual device can accumulate and this may lead to the failure of the final result.2

In the recent years, methods for scalable quantum computing based on artificial quantum systems have been extensively studied1,2,3,6. A promising scalable quantum computing architecture based on spin system of nanomechanical resonators (NAMRs) was proposed by P. Rabl et al.6. The spins have a long decoherence time and the NAMRs can be fabricated on a large scale. The quantum motions of the NAMRs can strongly interact with the spins and induce strong couplings between the spins4,7,8.

However, the inevitable fact is that the frequencies of the NAMRs can’t be exactly the same in practice. There are always frequency errors due to imperfect fabrications. Experimental results show that there can be ±1.0% deviations from the averaged frequency.10 These quantum imperfections may cause exponential suppression of quantum computations11, thus it is an important issue to find out quantitatively the impacts of these frequency errors and the efficient method to compensate for the negative effects due to the device errors.

In this article, we consider a scalable quantum computing architecture consisting of N spin qubits whose interactions are mediated by an array of N NAMRs 8,9. The frequency errors of the NAMRs shift the frequencies of the collective modes, the NAMR-spin coupling strengths and result in fluctuations in the spin-spin couplings. For a given evolution time, the spins’ final state depends on the coupling strengths among them. Supposing a certain duration is needed as the right evolution time to produce a certain state in the ideal case of no frequency errors, the same evolution will produce a wrong state in the actual case with frequency errors, due to the fluctuations of the spin-spin couplings which would reduce the quality of the quantum computing in the spin-spin interactions mediated by the NAMRs. Here, we analyze the impacts of the frequency errors on the quantum operation fidelity and figure out a limitation of the scalability of the quantum computing architecture. Our numerical simulations show that the quantum operation fidelity decreases rapidly with the number of the NAMR-spin elements in the present of frequency errors and these frequency errors finally limit the scale of the system. Based on the analysis, we propose a method to compensate for the negative effects of whatever large frequency differences.

This article is arranged as follows: In Sec. II the model is given first and then the method of diagonalizing the quadratic boson Hamiltonian is briefly reviewed. Next, in Sec. III we show the consequence of frequency errors on quantum operation fidelity by numerical simulation. We study how the influence of the frequency errors change with the number of the NAMRs N. In Sec. IV we present our theoretical method to resist the frequency errors. Finally, we present discussions and conclusions in Sec. V.

II. MODEL AND METHOD

A. Model

As shown in Fig. 1 we consider a system consisting of an array of N NAMRs, which are charged and interact capacitively with nearby wires interconnecting them. A magnetic tip is attached on the free end of each NAMR. An NAMR with fundamental frequency ωi and effective mass m magnetically couples to an electronic spin qubit associated with a nitrogen-vacancy (N-V) center located
in the substrate below [2] [12]. Each spin is driven by a local microwave to form a pair of dressed states in order to match the NAMR frequency [2]. The interaction Hamiltonian between the NAMR and the corresponding spin qubit is \( H_{fr}^i = \frac{1}{2}(a_i^+ + a_i)s_z^i \). Here \( a_i \) (\( a_i^+ \)) is the natural creation (annihilation) operator of the \( i \)th NAMR. \( s_z^i \) is the Pauli-z operator of the \( i \)th spin. The coupling strength \( \lambda = g_{ph}\mu_B G_m a_0 / \hbar \) with \( g_{ph} = 2 \), the Bohr magneton \( \mu_B \), the magnetic field gradient \( G_m \) and the amplitude of zero-point fluctuations \( a_0 = \sqrt{\hbar/(2m\omega_i)} \).

The Hamiltonian of these \( N \) coupled NAMRs is (setting \( \hbar = 1 \)) [3]:

\[
H_{ph} = \sum_{i=1}^{N} \omega_i a_i^+ a_i + \frac{1}{2} \sum_{i,j} g_{ij}(a_i^+ a_j^+ a_j a_i),
\]

with \( g_{ij} = a_0^2(\sqrt{2} U_{el}/(\partial z_i/\partial z_j))\|_{z_i = 0, z_j = 0} \). Here \( U_{el} = -(U_{el}^2/2) C_\Sigma C_w/(C_\Sigma + C_w) \) is the electrostatic energy between two nearest-neighbour NAMRs connected by a wire of self-capacitance \( C_w \) with \( C_\Sigma = C_i(z_i) + C_j(z_j) \), \( C(z) \approx C(1 - z_i/h) \). \( U_{el} \) is the applied gate voltage on each NAMR, \( h \) is the electrode spacing, \( z_i \) is the tip position of the NAMR and \( C \) is a constant. The first item of \( H_{ph} \) is the free Hamiltonian of the NAMRs and the second item is the interactions between them. The interaction \( g_{ij} \) consists of two parts: the self-coupling \( g_{ii} \) and the coupling between different NAMRs \( g_{ij} \) (\( i \neq j \)).

If the Hamiltonian \( H_{ph} \) is diagonalized by defining collective modes, the total NAMR-spin coupling Hamiltonian (the sum of \( H_{fr}^i \) over \( i \)) can be rewritten as the coupling between the \( z \)-components of the electronic spins and the collective modes of the NAMRs. After a transformation, the effective spin-spin interactions mediated by the NAMRs can be obtained and have the form \( H_{eff} = \sum_{i,j} M_{ij} s_z^i s_z^j \) with \( M_{ij} \) the coupling strength between the spin \( i \) and the spin \( j \), which could be used for scalable quantum computation [6].

B. Diagonalizing the quadratic boson Hamiltonian

When the frequencies of the NAMRs are exactly the same (denoted by \( \omega_i \)) and the nearest-neighbour NAMR coupling is a constant \( g \), the frequencies of collective modes can be obtained by solving the eigenvalue equations analytically and are given by \( \omega_n = \sqrt{\omega_i^2 + 4g \omega_i \cos[(n + 1)\pi/N]} \). In practice, the frequencies of all the NAMRs can’t be exactly the same due to imperfect fabrications. These systematic frequency errors of each NAMR are independent and are not the same, the frequencies of the collective modes of \( N \) coupled NAMRs \( \tilde{\omega}_n \) can’t be simply calculated by the perturbation method. A useful method is described as follows.

The Hamiltonian [4] can be rewritten as

\[
H_{ph} = \sum_{k=1}^{N} \tilde{\omega}_k b_k^+ b_k,
\]

with \( b_k^+ \) (\( b_k \)) the creation (annihilation) operators for the phonons of the collective modes and \( \tilde{\omega} \equiv [\omega_1, \cdots, \omega_N, \omega_1^N, \cdots, \omega_N^N]^T \) the natural creation (annihilation) operators of all the NAMRs. Here \( D = \begin{bmatrix} A & B \\ B & A \end{bmatrix} \), \( A \) is an \( N \times N \) symmetric real matrix governed by the coupling model between NAMRs.

Introducing an auxiliary matrix \( D = A^2 - B^2 \), the para-values and para-vectors [13] (and also the diagonalizing para-unitary matrix [14]) of \( D \) can be constructed from the eigenvalues (denoted by \( d_k \) with \( k = 1 \ldots N \)) and eigenvectors (denoted by \( \xi_k \)) of \( D \) [13]. Then the \( 2N \times 2N \) transformation matrix is given by \( J^{-1} = \begin{bmatrix} \xi_1 & \cdots & \xi_N & \cdots & \xi_N & \cdots & \xi_1 \end{bmatrix} \) with \( \xi_1 \) and \( \xi_N+k \) [14] are \( 2N \)-column vectors [13]. The transformation matrix should satisfy \((J^{-1})^T \tilde{\omega} J^{-1} = \text{diag}(d_1^{1/2}, d_2^{1/2} , \cdots, d_N^{1/2}, d_1^{1/2}, d_2^{1/2}, \cdots, d_N^{1/2}) \) to preserve the commutation relations of bosonic operators. The collective frequencies of these NAMRs are

\[
\tilde{\omega}_k = 2d_k^{1/2}.
\]

By denoting \( \tilde{\mathbf{b}} = [b_1^+, \cdots, b_N^+, b_N^+ N, \cdots, b_1^+ N]^T \), we obtain the relation \( \tilde{\mathbf{a}} = J^{-1} \tilde{\mathbf{b}} \). The collective frequencies are shifted by the frequency errors and the numerically result for \( N = 11 \) is as shown in Fig. [2].

For the system we considered, we only need to consider the nearest-neighbour NAMR-NAMR interactions. The coupling strengths between nearest-neighbour NAMRs are assumed to be the same and denoted by \( g_{i,i+1} \).
The collective frequencies of the NAMRs are shifted by the random frequency errors for $N = 11$ with numerical calculation. The parameters used here are: $\omega_r/(2\pi) = 1$ MHz, the coupling strength: $g/(2\pi) = 500$ kHz. The left panel (a) is the result by running the program one time while the right panel (b) is the average by running the program 200 times. The offsets of the collective frequencies are optimistic estimations.

The matrix $A$ and $B$ are given by

$$A = B + \text{diag} \left( \frac{\omega_1}{2}, \frac{\omega_2}{2}, \ldots, \frac{\omega_N}{2} \right), \quad (4a)$$

$$B = g \begin{pmatrix} 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix}_{N \times N} \quad (4b)$$

III. FIDELITY ANALYSIS

Based on the model and method above, we analyze the effects of the frequency errors. We shall calculate the fidelity of the assumed state from the ideal model without any frequency error and the actual state from the more realistic model with independent frequency errors in each individual NAMR.

Denoting the frequency of each NAMR $\omega_i = \omega_r + \epsilon_i$ with $\omega_r$ the averaged frequencies of all the NAMRs and $\epsilon_i$ the frequency error, the coupling strength between the NAMR and the corresponding spin can be written as $\lambda_i = \lambda \sqrt{\omega_i/\omega_r}$, which means that the frequency errors generally fluctuate the NAMR-spin couplings. The NAMR-spin coupling can be rewritten as

$$\sum_i^N \frac{\lambda_i}{2} (a_i + a_i^\dagger) \sigma_z^i = \sum_{n,i=1}^N \lambda_{n,i} (b_n + b_n^\dagger) \sigma_z^i, \quad (5)$$

with

$$\lambda_{n,i} = \lambda_i (J_{i,n}^{-1} + J_{i,n+N}^{-1}), \quad (6)$$

the coupling strength between the $n^{th}$ collective mode and the $z$ component of the $i^{th}$ spin. Here $\{J_{i,j}\}$ are the matrix elements of $J^{-1}$. Therefore the effective spin-spin coupling is $M_{i,j} = \sum_n \lambda_{n,i} \lambda_{n,j}/(4\omega_n)$. For a given evolution time $t_{g}$, the spin-entangling operation can be described by $U_{g}(t_{g}) = \exp(i\sum_{i,j} M_{i,j} \sigma_z^i \sigma_z^j / t_{g})$.

For $N$ NAMRs with an initial state $|\psi_{n}=\rangle = [(|0\rangle + |1\rangle)/\sqrt{2}]^\otimes N$, the relative frequency errors are denoted by $\Delta_i = \epsilon_i/\omega_r$. After a given evolution time $t_{g}$, the spins’ state becomes $|\phi_o(N, \{\Delta_i\}) = U_{g}(t_{g}) |\psi_{n}=\rangle$. The fidelity between the actual state and the ideal state is

$$F(N, \{\Delta_i\}) = |\langle \phi_o(N, \{\Delta_i\}) |\phi_o(N, \{\Delta_i = 0\}) \rangle| \quad (7)$$

We consider an array of Si-NAMRs with an averaged frequency $\omega_r/(2\pi) = 1$ MHz, $a_0 \approx 1.86 \times 10^{-13}$ m and $g/(2\pi) = 500$ kHz [6]. For gradient $G_m = 9.6 \times 10^6$ T m$^{-1}$, the resulting coupling strength between an NAMR with frequency $\omega_r$ and a spin is about $\lambda/(2\pi) \approx 50$ kHz. In this case, the effective nearest-neighbour spin-spin coupling is about a few kilohertz. We choose the evolution time $t_g = 0.3$ ms to complete one quantum operation. (For example, given $N = 2$, if we want obtain the entangled state $\{(00) + |11\rangle + i(01) + i(10)\}/2$ from the initial state $\|(00) + |11\rangle\)/\sqrt{2}^\otimes 2$, the evolution time and the two spin coupling strength $M$ should satisfy $t_g = \pi/4 |M|$. The spin decoherence time $T_2 \approx 6$ ms [15] and the dephasing time induced by nuclear-spin fluctuations $T_2^* \approx 0.35$ ms for N-V centers [18] are observed experimentally.)
Our numerical simulation shows that the fidelity $F(N, \{\Delta_i\})$ decreases with the number of the NAMRs $N$, as sketched in Fig. 3. In Fig. 3 the red solid star gives $F(2, \Delta_m = 5.0\%) \approx 0.988$ while $F(11, \Delta_m = 5.0\%) \approx 0.971$. $F(2, \Delta_m = 1.0\%) \approx 0.9996$ while $F(2, \Delta_m = 5.0\%) \approx 0.9879$. Here $F(N, \Delta_m = 5.0\%)$ is the value for $F(N, \{\Delta_i\})$ where the values of each $\Delta_i$ are randomly chosen from the range $[-\Delta_m, +\Delta_m]$. This shows that, as the number of qubits increases, the errors in the target state rise, hence one may end up with a wrong result with large probability in a large scale quantum computation, even though the frequency errors of each individual NAMR are small.

Fig. 3 also shows that $F$ is approximately linear with $N$. For example, suppose $\Delta_m = 1.0\%$. For $N = 2 \cdots 11$, the linear fitting gives the relationship between $F$ and $N$: $F = -0.000080952 \times N + 0.99976$ with the correlation coefficient $r = 0.99$ as shown in Fig. 3. For a given $F_0$, if we want the fidelity $F > F_0$, the length of a spin chain should be less than about $[10^4 \times (0.99976 - F_0)/0.80952]$ spins. The symbol $|P|$ denotes the maximal integer not greater than $P$. For example, we obtain $N < 4.6 \times 10^3$ for $F_0 = 2/3$. It can be seen that the frequency errors do limit the maximum length of the spin chain if we want to produce a faithful resultant state and finally limit the extensibility of the fault-tolerant quantum computing.

IV. COMPENSATION METHOD

As analyzed above, the frequency errors can lead to fidelity decease of the quantum state evolution. The deviation cannot be compensated by simply adjusting the evolution time $t_g$ when $N > 2$, as shown in Fig. 5. In what follows, we propose a method to solve this problem. The main idea is that the errors caused by frequency differences can be perfectly compensated for by controlling the interaction time of each two adjacent spin qubits block. With this compensation method, one can produce arbitrarily large-scale states with whatever large frequency error of each NAMR.

For a one-dimensional $N$ NAMR-spin chain, an auxiliary switchable control voltage gate is added between every two nearest-neighbour NAMRs to obtain switchable coupling between them, as depicted in Fig. 6. When the control voltage is zero, the nearest-neighbour NAMRs couple with each other and induce effective spin-spin interactions; and when the control voltage is switched on and equals to the NAMR’s gate voltage, the coupling of the two NAMRs is switched off, hence the NAMRs induce no effective spin-spin interactions. In addition, we assume that we have obtained the fundamental frequencies of the NAMRs by average after multiple measurements (the systematic errors still exist due to the limitations of the fabrication techniques and the random measurement frequency errors are eliminated by average). The effective
coupling strength between every two nearest-neighbour spins in the case of frequency errors can be theoretically calculated by the formula $M_{i,i+1}$ by assuming that the other NAMRs and spins do not exist.

For simplicity, we assume $N$ is an even number (for an odd $N$, we only need to regard the last spin as an already entangled spin pair). One can achieve the perfect result through the following procedures as sketched in Fig. 7 (1) switch on the coupling between the $(2i - 1)^{\text{th}}$ and the $(2i)^{\text{th}}$ spins for $i = 1 \cdots N/2$ and switch off the coupling between the $(2i)^{\text{th}}$ and the $(2i + 1)^{\text{th}}$ spins for $i = 1 \cdots (N/2 - 1)$. (2) control the evolution times between each two-spin pair to obtain the ideal entanglement between them and then switch off the coupling, respectively. (3) switch on the coupling between the $(2i)^{\text{th}}$ and the $(2i + 1)^{\text{th}}$ spins for $i = 1 \cdots (N/2 - 1)$ when all the couplings are switched off in (2). (4) repeat (2). The total time needed for all these processes is approximately two times of that in the case of no frequency errors. In addition, this method also avoid the influence of effective sub-nearest-neighbour interactions between the spins.

Proof. The operators of different spins communicate with each other. In the case of no frequency errors, the NAMR-spin couplings reduce to a constant and denoted by $\lambda$. The effective nearest-neighbour spin-spin couplings induced by the NAMRs are equal to each other and denoted by $M$. According to the Ising model, the evolution operator of the spin chain for a given evolution time $t_\text{g}$ is:

$$U_\text{g}(t_\text{g}) = \exp \left\{ i \sum_{i=1}^{N-1} M \sigma_z^i \sigma_z^{i+1} t_\text{g} \right\} \prod_{i=1}^{N-1} \exp \left\{ iM \sigma_z^i \sigma_z^{i+1} t_\text{g} \right\}.$$

In the case of frequency errors, the NAMR-spin couplings and the collective modes of the NAMRs are shifted by the frequency errors. Denoting the coupling strength between the $i^{\text{th}}$ and $(i + 1)^{\text{th}}$ spins as $M_{i,i+1}$, the ideal entanglement between two spins can be obtained by adjusting the evolution time in the two spin case as shown in Fig. 5 and the time needed is denoted by $t_{i,i+1}$. When all the four steps are completed, the evolution operator of these spins can be described by:

$$U_\text{g}(t_1, \cdots, t_{N-1}, N) = \prod_{i=1}^{N-1} \exp \{iM_{i,i+1}\sigma_z^i \sigma_z^{i+1} t_{i,i+1}\}.$$

By precisely controlling the evolution time $t_{i,i+1} = (Mt^2_\text{g})/M_{i,i+1}$, the evolution operators in the two cases are equal to each other and the two final states must be the same if the system evolves from the same initial state.

We should point out that the entanglement of the spins prepared by the following steps is not perfect: (1) switch on all the nearest-neighbour NAMR-NAMR couplings. (2) switch off the coupling when the nearest-neighbour spins involve to the maximum entanglement states (the effective interaction between the nearest-neighbour spins can be numerically calculated), respectively. A spin interacts with all the other spins through the collective modes of the NAMRs when the control voltage is zero. These interactions inevitably include sub nearest-neighbour interactions and so on. These sub nearest-neighbour interactions decay as $M_{i,i+m} \sim (g/\omega)^{(m-1)}$, but their influences are generally comparable with those due to frequency errors. Unwanted entanglement between the spins which are not nearest-neighbours is generated and the Ising interaction produced by the above two steps is unsatisfactory.
V. DISCUSSIONS AND CONCLUSIONS

Although only the one-dimensional case is discussed in the context, the similar analysis on the two-dimensional NAMRs-spins quantum computing architecture \[6\] can also be handled by using the same method. In the two-dimensional configuration, NAMRs are ordered on a two-dimensional lattice and couple to their four neighbours electrostatically. In the two-dimensional case, the effects of the frequency errors can be analysed by adjusting the matrix \(B\) in equation \((11)\) to be a general matrix instead of a tridiagonal matrix when considering the nearest-neighbour couplings. In the compensation method, a switchable voltage gate should be added between each two nearest-neighbour NAMRs.

In summary, we study the impacts of the frequency differences on the scalability of a promising NAMRs-based-on quantum computing architecture. The influences on the quantum operation fidelity are analyzed in detail, and a method is given to compensate for the negative effects of these frequency differences.

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[13] Para-values and para-vectors. Let \(\mathcal{D} = \begin{bmatrix} X & Y \\ Z & W \end{bmatrix}\) with \(X, Y, Z, W\) four \(N \times N\) matrices. If \(\mathcal{D} [u_\rho, v_\rho]^T = \lambda_\rho [u_\rho, -v_\rho]^T\) is satisfied, where \(u_\rho, v_\rho\) are \(N\)-column vectors with \(\rho = 1, 2, \ldots, 2N\), we call \(\lambda_\rho\) the para-values and \([u_\rho, v_\rho]^T\) the para-vectors of the matrix \(\mathcal{D}\).
[14] Para-unitary matrix. Define the para-unit matrix \(I \equiv \text{diag}(1, 1, \ldots, 1, -1, -1, \ldots, -1)_{2N \times 2N}\). We call \(I\) a \(2N \times 2N\) para-unitary matrix if \(I^T I = I\).
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