Fluctuations in produced charged particle multiplicities in relativistic nuclear collisions for simulated events

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Abstract. The event-by-event (E-by-E) fluctuations have been studied for relativistic heavy ion collisions and compared by theoretical prediction of available event generators. The multiplicity fluctuations are sensitive to QCD phase transition and to the presence of critical point in the QCD phase diagram. These fluctuations also provide baselines for other event-by-event (E-by-E) measurements. In the present article, a modest approach attempt has been made for a logical study of event-by-event (E-by-E) charge fluctuations in the relativistic nuclear collisions of proton-proton (p-p) and nucleus-nucleus (A-A) interactions. Finally, simulations by Monte Carlo Generators (MCGs) have been done and findings were found within good agreements with other works.

Introduction
The study of the early Universe in the standard big bang model necessarily requires also an intimate connection to particle physics. Already at a late time of the order of a minute, the nuclear and particle physics play an important role in determining the abundance of the light elements. It is commonly believed that ultrarelativistic nuclear collisions are possible test grounds of the formations of new phases of hadronic matter / quark gluon plasma (QGP) under extreme conditions [1-5]. Several prominent features have been already observed by various workers in the field of high energy physics, such as high multiplicity events large average transverse momentum, collective effects, the side-splash of the participants and the bounce-off of the spectator particles and the comparison with theoretical predictions and models. Early speculations of possible exotic states of matter focused on the astrophysical implications of abnormal states of dense nuclear matter. A schematic representation of the phase diagram of strongly interacting matter, showing the transition between hadronic matter and the quark-gluon plasma as a function of temperature and baryon chemical potential has been depicted in Figure 1 [6,7]. The phase diagram of strongly interacting matter gives the following informations:
(i) At low temperatures and baryon densities, the system can be described in terms of hadrons, nucleons, mesons and internally excited states of nucleons.
(ii) In the high-temperature (~170 MeV or 10^12 K), high-baryon density (~5-10 times density of nuclear matter) regions in which matter exists as a nuclear liquid, hadron gas, or quark-gluon plasma. (iii) The path followed by the early universe [1, 2, 8] as it cooled from the quark-gluon plasma phase to normal nuclear matter is shown by an arrow on the left. The dotted line leading to a black rectangular spot near the bottom indicates the path taken by a neutron star [8] as it forms. (iv) Heavy ion collisions follow a path between these two extremes, increasing both the temperature and baryon density. It may be verified in future experiments that the energy densities ~1-3 GeV/fm^3, equivalent to a temperature T_c ~ 150-200 MeV [1, 2, 8] or baryon density, \( \rho_c \approx 5-10 \) times nuclear matter density can indeed be reached in heavy ion collisions. A phase transition is said to be of first-order if there is at least one finite gap in the first derivatives of a suitable thermodynamic potential. A transition is said to be of second order if there is a power-like singularity in at least one of the second derivatives of the potential. The electric charge fluctuations are regarded as an important signal for a QGP phase transition. It has been predicted [9-12] that such fluctuations in the local phase space should be drastically reduced as the charges are more evenly distributed in the plasma. Furthermore, the charge fluctuations are influenced by the decay of hadronic resonances and therefore, in the absence of QGP, deviation of such fluctuations from the statistical behavior may be used to determine the abundance of particles (\( \rho \) and \( \omega \)) mesons. An attempt is, therefore made to carry out a systematic study of event-by-event (E-by-E) charge fluctuations in proton-proton (p-p) and nucleus-nucleus (A-A) collisions by simulating the events using some of the popular Monte Carlo Generators (MCGs), which are based on different physics aspects. Such studies may help establish the event-by-event (E-by-E) charge fluctuations as a robust variable and then to interpret the physical information contained in the measurement. Yet another advantage of MCGs data is that the analysis may be carried out in both full and limited phase spaces, which, in turn, would lead to test the efficiencies of the detectors of limited acceptance [13].

Figure 1. A schematic phase diagram of strongly interacting matter, showing the phase transition between hadronic matter and QGP as a function of temperature and baryonic chemical potential [Reference 6, 7].

Overview / simulation
In real life, machines produce events that are stored by the data acquisition system of a detector [14-16]. In the virtual reality, event generators like ultra-relativistic Quantum Molecular Dynamics model (UrQMD) [17, 18], FRITIOF [19] and HIJING [20] play the role of machines like the Tevatron and...
LHC, and detector simulation programs like GEANT 4 [21] the role of detectors like ATLAS or CMS. The real and virtual worlds can share the same event reconstruction framework and subsequent physics analysis. It is by understanding how an original physics input is distorted step-by-step in the better-controlled virtual world that an understanding can be gained of what may be going on in the real world. For approximate studies the detector simulation and reconstruction steps can be shortcut, so that generators can be used directly in the physics studies [14-16]. An example: how different programs can be combined in the event-generation chain has been shown in Figure 2.

![Event Generator Position Diagram](image)

**Figure 2.** An example: how different programs can be combined in the event-generation chain.

The key aspect of generators here is that they provide a detailed description of the final state so that, ideally, any experimental observable or combination of observables can be predicted and compared with data [14-16]. Thereby generators can be used at various stages of an experiment: when optimizing the detector and its trigger design to the intended physics program, when estimating the feasibility of a specific physics study, when devising analysis strategies, when evaluating acceptance corrections, and so on. However, it should always be kept in mind that generators are not perfect. They suffer from having to describe a broad range of physics, some of which is known from first principles, while other parts are modeled in different frameworks. Further, a generator actually acts as a vehicle of ideology, where ideas are disseminated in prepackaged form from theorists to experimentalists.

**Importance of Monte Carlo event generators**

In quantum mechanics [14-16], calculations provide the probability for different outcomes of a measurement. Event-by-event (E-by-E), it is impossible to know beforehand what will happen: anything that is at all allowed could be next. It is only when averaging over large event samples that the expected probability distributions emerge — provided we did the right calculation to high enough accuracy. In generators, (pseudo) random numbers are used to make choices intended to reproduce the quantum mechanical probabilities for different outcomes at various stages of the process.

**The mathematical details**

The charge fluctuations are generally studied in terms of two kind of measures [22, 23]. The first one is the D-measure of the net charge fluctuations, the direct measure of which is the variance such as: $V(Q) = \langle Q^2 \rangle - \langle Q \rangle^2$, where $Q = n^+ - n^-$; and the $n^+$, $n^-$ are the multiplicities of positively & negatively charged hadrons respectively, produced in a particular phase space of a particular event in nuclear collisions at ultra-relativistic energies. The other measure is the charge ratio, $R^+ = n^+/n^-$ or
vice versa \( R^- = n^-/n^+ \). Further, the D-measures of the charge ratio fluctuations are defined by following mathematical relation:

\[
D(R^\pm) = \langle n_{\text{ch}} \rangle V(R^\pm)^2 \quad \text{and the variance:} \quad V(R) = \langle R^2 \rangle - \langle R \rangle^2
\]

In the high multiplicity limit, the above measures are precisely equal to the following relation:

\[
D(R) = D(Q) = 4 V(Q) \langle n_{\text{ch}} \rangle
\]

If each produced particle is assigned randomly a charge +1 or -1 with equal probability, then \( V(Q) = \langle n_{\text{ch}} \rangle \) and \( D(Q) = 4 \); in order to account for a non-zero net charge due to baryon stopping and the charge conservation in the large pseudo-rapidity, \( \eta \) window. And, thereafter the following two corrections are applied to the D-measure by redefined parameter [22-24 and references therein] such as given below:

\[
D_{\text{corr}}(Q) = \frac{D(Q)}{B_1 B_2}
\]

where the parameters, \( B_1 = \frac{\langle n^+ \rangle_{\Delta \eta}}{\langle n^- \rangle_{\Delta \eta}} \) and \( B_2 = 1 - \frac{\langle n_{\text{ch}} \rangle_{\Delta \eta}}{\langle n_{\text{ch}} \rangle}
\)

It has been predicted that \( D(Q) = 1 \) for QGP, 2.9 for resonance gas 4 for uncorrelated pion gas [25-28].

**Results and discussions**

The various mathematical parameters; \( D(Q) \), \( D_{\text{corr}}(Q) \) and \( D(R^\pm) \) were calculated for the proton-proton (p-p) collisions in the energy range of \( \sqrt{s} = 200 \) GeV to 14.0 TeV by using the simulations of Monte Carlo Generators (MCGs) [17, 29], HIJING [20], HIJING/BB~ [30] and UrQMD [17, 18]. In the simulated data samples/sets total generated events were 250 events. The dependence behaviour of \( D(Q) \) on the width of pseudo-rapidity window, \( \Delta \eta \) for HIJING data sets at different relativistic energies has been plotted and shown in Figure 3.

In the same figure, similar plots for the events generated by assigning the random charge were also shown. Here to find much more interest, all the charged particles of a single event of data sets at energy 14.0 TeV, were randomly assigned a charge +1 or -1 with equal probabilities. From figure 3, it is clear that for this sample the \( D(Q) \) values were found \( \approx 4 \) irrespective of the fact that how small (or large) was the multiplicity in a chosen narrow (or wide) \( \eta \)-window. This result indicates that the event by event (E-by-E) analysis may be successfully applied to the narrow phase space bins having a very limited number of particles [22]. The values of parameter \( D(Q) \) in the pseudo-rapidity interval \( \Delta \eta \leq \)
0.5, for all the simulated data sets were found to be \( \approx 4.0 \), i.e., in the region of hadronic gas. These values, with the widening of \( \eta \)-window, decrease to \( \approx 1.0 \) and below this value, and one can expected for the quark gluon plasma (QGP) formation.

It is fascinating to note that the data produced from various collider experiments at different energies, SPS, RHIC and LHC, overlap and suggesting that there is no energy dependence at all. Almost similar pattern of variation of \( D(Q) \) with the size of the rapidity window (\( \eta \) or \( \Delta \eta \)) has been observed [24] in the case of PYTHIA simulations of \( \pi^+p \) collisions at \( \sqrt{s} = 22.0 \) GeV. In order to compare the results from the three MCGs considered, the \( D(Q) \) dependence on pseudo-rapidity width \( \Delta \eta \) have been shown for the proton-proton (p-p) interactions at energies \( \sqrt{s} = 200 \) GeV and 14.0 TeV and depicted in Figure 4.

![Figure 4](image)

**Figure 4.** The dependence of \( D(Q) \) on the width of pseudo-rapidity window, \( \Delta \eta \) for Monte Carlo Generators (MCGs) data sets.

It is clear from the Figure (4), that the UrQMD data exhibit a slight energy dependence of \( D(Q) \), while the simulated data sets of HIJING and HIJING/\( BB^- \) do not show such patterns. After applying the correction factor to the \( D(Q) \) values, as it was mentioned earlier in mathematical tools, the values of \( D \)-measure, \( D_{\text{corr}}(Q) \) were calculated and their dependence on the width of the pseudo-rapidity (\( \eta \)) window have been shown in Figure 5 for the HIJING data. And for the comparison purpose of same dependence on \( \eta \)-window, the findings from the different data sets of Monte Carlo Generators (MCGs) events were depicted in Figure 6. Further, from the Figures (2-6) one can conclude the following outcomes:

(i) It has been found from all above mentioned figures, that the values of parameters: \( D(Q) \approx D_{\text{corr}}(Q) \approx 4.0 \) in a small pseudo-rapidity, \( \eta \)-window for relativistic nuclear collisions. However, the \( D_{\text{corr}}(Q) \) values decrease to a little above 1.0 and thereafter tend to acquire a saturation in the larger \( \eta \)-windows. This specify that the influence of global charge conservation and leading particle stopping was well taken into account by the corrected measure, \( D_{\text{corr}}(Q) \).

(ii) In the present study the calculated values of \( D_{\text{corr}}(Q) \), for a particular \( \Delta \eta \) were found to decrease with energy and becomes more pronounced in the saturation region of \( \Delta \eta \). Such type of dependence might be due to the increasing number of charged particles at higher energies rather larger charge asymmetry.

(iii) For Monte Carlo Generators (MCGs) events, the calculated \( D(Q) \) values were nearly found same in all data sets.
Figure 5. The behavior of $D_{corr}(Q)$ on the width of pseudo-rapidity window, $\Delta \eta$ for HIJING data sets at different relativistic energies.

Figure 6. The behavior of $D_{corr}(Q)$ on the width of pseudo-rapidity window, $\Delta \eta$ for Monte Carlo Generators (MCGs) data sets.
The saturation in the values of $D_{\text{corr}}(Q)$ was found in the pseudo-rapidity window, $\Delta \eta \geq 4.0$. Whereas, as per another high energy physicists, Q.H. Zhang et. al., [23], the correction factor, $1 - (< n_{ch} >_\Delta \eta / < n_{ch} >) = 0$ for the entire kinematic phase space and cannot be used for the larger $\eta$-windows. The results corresponding to $\Delta \eta \geq 4.0$ and beyond have been depicted in Figures (5 & 6). The values of $D_{\text{corr}}(Q)$ for a smaller $\eta$-window are observed because of the fact that if the $\eta$-bin width is small enough, it will not pick up all the particles decaying from a resonance. Due to the presence of positively charged particles in the initial state, average number of positively charged particles in a particular $\eta$ window will be larger than that of negatively charged ones. This charge asymmetry becomes more noticeable with the widening of the $\eta$-windows, resulting in the fluctuations in the charge ratio, $R' = n'/n$ and vice versa $R = n/n'$ and that it is not a simple inverse relation. 

In order to test the predictions with the participant model at LHC energies and compare the findings with the SPS and RHIC energy data, the values of $D_{\text{corr}}(Q)$ have been draw as a function of $\Delta \eta$ for the HIJING data sets corresponding to p-p and A-A collisions at RHIC and LHC energies. These variations were shown in Figure 7. It may be noted in the figure that $D$-measure for p-p and Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV were nearly the same at least in the region of $\Delta \eta < 4.0$. This result incidentally, agrees fairly well with the one reported by PHENIX collaboration [31, 32]. However, in the present study, the values of $D$-measure for the p-p data are found to acquire relatively smaller values as compared to those obtained for the A-A data sets. The fluctuations in the $D_{\text{corr}}(Q)$ values corresponding to Pb-Pb data, as can be seen in the figure are due to the limited statistics. These findings, thus, reveal that the $D$-measures for both p-p and A-A exhibit significant energy dependence. These observations were found in good agreement with the idea [33-39] that at lower energies, a single nucleon-nucleon (N-N) collision is dominated by hadronic picture, whereas, at higher energies, such collision can experience the contents of nucleons, i.e., at higher energies gluons are expected to make larger contributions which would cause a decrease in the $D(Q)$ values with increasing incident energies. All above the findings in the present simulations / work were found in good agreement with the other high energy physicists Shakeel Ahmad et. al., (2012) [35].

![Figure 7. The behavior of $D_{\text{corr}}(Q)$ on the width of pseudo-rapidity window, $\Delta \eta$ for HIJING data along with p-p, Au-Au & Pb-Pb collisions at different relativistic energies.](image)

The impact parameter, (b) dependence of $D$-measure is examined by plotting the variations of $D_{\text{corr}}(Q)$ with b for Pb-Pb collisions at 5.5A TeV. These results are shown in Figure 8. Data points for each of the two central $\eta$-windows were from the five sets of HIJING events, each generated for a different impact parameter range, e.g. b = 0 – 1 fm, 1 - 2 fm, etc. It may be noticed in the figure that
the data points for both the η windows lie below the resonance gas and above that the one expected for a QGP. A slight centrality dependence may also be noticed such that the $D_{\text{corr}}(Q)$ increases with increasing centrality. It should be mentioned here that the $D$-measure observed in the present study, are relatively smaller as compared to those reported for Au-Au collisions at RHIC energies [13,14,27]. This difference in the $D$-measures might be due to the difference in gluon populations at RHIC and LHC energies.

**Conclusions and final remarks**

On the basis of present experimental / simulation study in relativistic heavy ion collisions, on can conclude the following worth meaning points:

The $D$-measures, $D(Q)$, after the correction, $D_{\text{corr}}(Q)$, were found to reveal significant energy dependence in the proton-proton (p-p) and nucleus-nucleus (A-A) collisions along with Monte Carlo Generators (MCGs). The results in terms of the net charge fluctuations may be regarded as a better measure in reading the change in the charge fluctuations as compared to the measure of $D(Q)$, of charge ratio fluctuations.

It was found in present study, that there is no essential difference in the values of $D$-measures, $D(Q)$, as already predicted by the different theoretical models.

The observed energy dependence of the $D$-measure in p-p collisions indicates that at higher energies gluons have higher contribution and reduce the values of the $D$-measure.

It may be pointed out that charge fluctuations are insightful to the parton number embedded in different theoretical models, if the re-scattering effects are not essential and hence the $D$-measure may be regarded as a signature of novel state (QGP) formation.

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