QINI-BASED UPLIFT REGRESSION

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ABSTRACT

Uplift models provide a solution to the problem of isolating the marketing effect of a campaign. For customer churn reduction, uplift models are used to identify the customers who are likely to respond positively to a retention activity only if targeted, and to avoid wasting resources on customers that are very likely to switch to another company. We introduce a Qini-based uplift regression model to analyze a large insurance company’s retention marketing campaign. Our approach is based on logistic regression models. We show that a Qini-optimized uplift model acts as a regularizing factor for uplift, much as a penalized likelihood model does for regression. This results in interpretable parsimonious models with few relevant explanatory variables. Our results show that performing Qini-based variable selection significantly improves the uplift models performance.

Keywords casual inference · Kendall’s correlation · lasso · logistic regression · marketing · regularization

1 Introduction

This article proposes methodology that identifies characteristics associated with a home insurance policy that can be used to infer the link between marketing intervention and policy renewal rate. Using the resulting statistical model, the goal is to predict which customers the company should focus on, in order to deploy future retention campaigns.

A subscription-based company loses its customers when they stop doing business with their service. Also known as customer attrition, customer churn can be a drag on the business growth. It is less expensive to retain existing customers than to acquire new customers, so businesses put effort into marketing strategies to reduce customer attrition. Customer loyalty, on the other hand, is usually more profitable because the company have already earned the trust and loyalty of existing customers. Businesses mostly have a defined strategy for fighting customer churn over a period of time. Organizations are able to determine their success rate in customer loyalty and identify improvement strategies using available data and learning about churn.

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A customer base is a historical list of clients to whom a business sold products and services. With the increasing amount of data available, a company tries to find the causal effects of customer churn. The term causal, as in causal study, refers to a study that tries to discover a cause-effect relationship. The statement $A$ causes $B$ means that changing the value of $A$ will change the distribution of $B$. When $A$ causes $B$, $A$ and $B$ will be associated but the reverse is not, in general, true, since association does not necessarily imply causation. We will consider one of the two frameworks for discussing causation. The statistical framework for causal inference formally introduced by Rubin [1974] uses the notation of counterfactual random variables. This framework is also associated with the potential outcome framework [Neyman, 1923], also known as the Rubin causal model [Holland, 1986]. Suppose a company decides to deploy a marketing campaign, and that customers are randomly divided into two groups. The first group is targeted with a marketing initiative (treatment group), and the second group serves as control (or baseline). A potential outcome is the theoretical response each customer would have manifested, had it been assigned to a particular group. Under randomization, association and causation coincide and these outcomes are independent of the assignment other customers receive. In practice, potential outcomes for an individual cannot be observed. Each customer is only assigned to either treatment or control, making direct observations in the other condition (called the counterfactual condition) and the observed individual treatment effects, impossible [Holland, 1986].

In marketing, the responses of customers in the treatment and control groups are observed. This makes it possible to calculate and compare the response rate of the two groups. A campaign is considered successful if it is successful in increasing the response rate of the treated group relative to the response rate of the control group. The difference in response rate is the increase due to the campaign. To further increase the returns of future direct marketing campaigns, a predictive response model can be developed. Response models [Smith and Swinyard, 1982, Hanssens et al., 2003, Coussement et al., 2015] of client behavior are used to predict the probability that a client responds to a marketing campaign (e.g. renews subscription). Marketing campaigns using response models concentrate on clients with high probability of positive response. However, this strategy does not necessarily cause the renewal. In other words, the customers could renew their subscription without marketing effort. Therefore, it is important to extract the cause of the renewal, and isolate the effect of marketing.

Data from one of the leader north-American insurers is at our disposal to evaluate the performance of the methodology introduced in this work. This company is interested in designing retention strategies to minimize its policyholders’ attrition rate. For that purpose, during three months, an experimental loyalty campaign was implemented, from which policies coming up for renewal were randomly allocated into one of the following two groups: a treatment group, and a control group. Policyholders under the treatment group received an outbound courtesy call made by one of the company’s licensed insurance advisors, with the objective to reinforce the customers confidence in the company, to review their coverage and address any questions they might have about their renewal. No retention efforts were applied to the control group. The goal of the study is develop models that will be used to identify which clients are likely to benefit from a call at renewal, that is, clients that are likely to renew their policy only if they are called by an advisor during their renewal period. Also, clients that are not targeted will most likely renew their policy on their own, cancel their policy whether they receive a call or not, or cancel their policy only if they receive a call.

Table 1 shows the marketing campaign retention results. The observed difference in retention rates between the treated group and the control group is small, but there is some evidence of a slightly negative impact of the outbound call. Even if the difference is slightly negative, it may be the case that the campaign had positive retention effects on some subgroup of customers, but they were offset by negative effects on other subgroups. This can be explained by the fact that some customers are already dissatisfied with their insurance policies and have already decided to change them before receiving the call. This call can also trigger a behavior that encourages customers to look for better rates.

Table 2 describes some of the $p = 97$ available predictors (features) in the dataset, in addition to the treatment (Called or Control) and outcome (Renewed or Cancelled the policy) variables.

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**Table 1: Renewal rate by group for $n = 20,997$ home insurance policies.**

|                  | Control | Called | Overall |
|------------------|---------|--------|---------|
| Renewed policies | 2,253   | 18,018 | 20,271  |
| Cancelled policies | 72     | 654    | 726     |
| Renewal rate     | 96.90%  | 96.50% | 96.54%  |

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Table 2: Descriptive statistics of some available variables for \( n = 20,997 \) home insurance policies. With randomization, the difference of means between treatment and control groups is significantly not different from 0 for all available predictors. For privacy concerns, we hide some values with *.

| Sample size | Control | Called | Diff Mean | Diff SD | Domain |
|-------------|---------|--------|-----------|---------|--------|
|             | 18,672  | 2,325  |           |         |        |
| Credit Score| 756.93  | 756.92 | -0.00     | 1.46    | \( \mathbb{R}^+ \) |
| Age (Years) | 44.97   | 45.26  | 0.30      | 0.25    | \( \mathbb{R}^+ \) |
| Genger      |         |        |           |         |        |
| Male        | 0.59    | 0.60   | 0.01      | 0.01    | \{0, 1\} |
| Marital Status|       |        |           |         |        |
| Divorced    | 0.02    | 0.02   | 0.00      | 0.00    | \{0, 1\} |
| Married     | 0.69    | 0.69   | 0.00      | 0.01    | \{0, 1\} |
| Single      | 0.23    | 0.23   | 0.00      | 0.01    | \{0, 1\} |
| Seniority (Years) | 9.57 | 9.73 | 0.16 | 0.16 | \( \mathbb{R}^+ \) |
| Policy Premimm ($) | * | * | -7.44 | 16.14 | \( \mathbb{R}^+ \) |
| Territory   |         |        |           |         |        |
| Rural       | 0.06    | 0.06   | 0.00      | 0.01    | \{0, 1\} |
| Products    |         |        |           |         |        |
| Auto and Home| 0.86   | 0.85   | -0.01     | 0.01    | \{0, 1\} |
| Auto Policies Count | 1.05 | 1.04 | -0.01 | 0.01 | \( \mathbb{N} \) |
| Mortgage Count | 0.66 | 0.67 | 0.01 | 0.01 | \( \mathbb{N} \) |
| Residences Count | 1.07 | 1.08 | 0.01 | 0.01 | \( \mathbb{N} \) |
| Endorsement Count | 1.98 | 2.00 | 0.02 | 0.03 | \( \mathbb{N} \) |
| Neighbourhood’s Retention | 0.87 | 0.87 | 0.00 | 0.00 | \{0, 1\} |
| Type of Dwelling|         |        |           |         |        |
| Family House| 0.69    | 0.70   | 0.00      | 0.01    | \{0, 1\} |
| Duplex      | 0.03    | 0.03   | -0.00     | 0.00    | \{0, 1\} |
| Apartment   | 0.19    | 0.19   | -0.00     | 0.01    | \{0, 1\} |
| Year of Construction | 1982.82 | 1983.52 | 0.71 | 0.58 | \( \mathbb{N} \) |
| Extra Options|         |        |           |         |        |
| Option 1    | 0.20    | 0.20   | 0.00      | 0.01    | \{0, 1\} |
| Option 2    | 0.73    | 0.73   | 0.00      | 0.01    | \{0, 1\} |
| Option 3    | 0.71    | 0.72   | 0.01      | 0.01    | \{0, 1\} |

In a randomized experiment, researchers often focus on the estimation of average treatment effects; the effect of the marketing initiative on a particular client is determined from this estimate. However, there might be a proportion of the customers that responds favorably to the marketing campaign, and another proportion that does not, depending on whether or not individual treatment effects vary widely in the customer base. A decision based on an average treatment effect at the individual level would require an adjustment because of the heterogeneity in responses that can be originated by many factors.

The so-called uplift model [Radcliffe and Surry, 1999, Hand and Yu, 2001, Lo, 2002] provides a solution to the problem of isolating the marketing effect. Instead of modeling the class probabilities, uplift attempts to model the difference between conditional class probabilities in the treatment (e.g., a marketing campaign) and control groups. Uplift modeling aims at identifying groups on which a predetermined action will have the most positive effect.

As in all regression-based modeling, an issue in uplift modeling is the ease of interpretation of the results. The model becomes harder to interpret when the number of potential explanatory variables, that is the dimension of the explanatory variables increases. When the variable dimension is small, knowledge-based approaches to select the optimal set of variables can be effectively applied, provided that the data size is sufficiently small. When the number of potentially important variables is too large, it soon becomes too time-consuming to apply a manual variable selection process. In this case one may consider using automatic subset selection tools. Variable selection is an important step. It reduces the dimension of the model, avoids overfitting,
and improves model stability and accuracy [Guyon and Elisseeff, 2003]. Well-known variable selection techniques such as forward, backward, stepwise [Montgomery et al., 2012], stagewise [Hastie et al., 2007], lasso [Tibshirani, 1996], and LARS [Efron et al., 2004b], among others, are not designed for uplift models. One might need to adapt them to perform variable selection in this context. Instead, we introduced a statistic, the adjusted Qini, whose optimization acts as a regularizing factor for variable selection in uplift models.

The rest of the paper is organized as follows. We first present the current uplift models in Section 2 and Section 3 introduces the notation and details of Qini-based uplift regression. Sections 4 and 5 present the computational results of the proposed methodology on synthetic and real datasets. Final remarks and conclusion are given in Section 6.

2 Uplift modeling

Let $Y \in \{0, 1\}$ be a binary response variable, $x = (x_1, \ldots, x_p)$ a vector of explanatory variables (predictors), and $T \in \{0, 1\}$ the treatment indicator variable. The binary variable $T$ indicates if unit $i$ is exposed to treatment ($T = 1$) or control ($T = 0$). Suppose that $n$ independent units are observed $(y_i, x_i, t_i)$, $i = 1, \ldots, n$. Denote the potential outcomes under control and treatment by $\{Y_i \mid T_i = 0\}$ and $\{Y_i \mid T_i = 1\}$ respectively. For $i = 1, \ldots, n$, an uplift model estimates

$$u(x_i) = \Pr(Y_i = 1 \mid x_i, T_i = 1) - \Pr(Y_i = 1 \mid x_i, T_i = 0).$$

(1)

Uplift modeling was formally introduced in Radcliffe and Surry [1999] under the appellation of differential response modeling where a thorough motivation and several practical cases promoted uplift modeling in comparison with common regression or basic tree-based methods that were used to predict the probability of success for the treatment group. They showed that conventional models, which were referred to as response models, did not target the people who were the most positively influenced by the treatment. In Radcliffe and Surry [1999], the methodology introduced is a tree-based algorithm similar to CART [Breiman et al., 1984], but using a modified split criterion that suited the uplift purpose.

The intuitive approach to uplift modeling is to build two separated classification models. Hansotia and Rukstales [2001] used the two-model approach which consists in direct subtraction of models for the treated and untreated groups. The asset of this technique is its simplicity. However, in many cases this approach performs poorly [Radcliffe and Surry, 2011]. Both models focus on predicting the class probabilities instead of making the best effort to predict the uplift, i.e., the difference between two probabilities. General discussions following differential response modeling and the two-model approach appeared in Hansotia and Rukstales [2002] where the technique known as incremental value modeling was introduced. This uses the difference in response rates in the two groups (treatment and control) as the split criterion of a regression tree. Also, Lo [2002] introduced the true lift modeling using a single standard logistic regression model which explicitly added interaction terms between each explanatory variable and the treatment indicator. The interaction terms measure the additional effect of each explanatory variable because of treatment. The model yields an indirect estimation of the causal effect by subtracting the corresponding prediction probabilities, which are obtained by respectively setting the treatment indicator variable to treated and control in the fitted model.

Most current approaches that directly model the uplift causal effect are adaptations of classification and regression trees [Breiman et al., 1984]. Rzepkowski and Jaroszewicz [2010] propose a tree-based method based on generalizing classical tree-building split criteria and pruning methods. The approach is based on the idea of comparing the distributions of outcomes in treatment and control groups, using a divergence statistic. Two divergences are suggested, a modified Kullback-Leibler divergence and a modified Euclidean distance. This approach was later extended to the case of multiple treatments applied to personalized medicine [Rzepkowski and Jaroszewicz, 2012]. Using the same idea, Guelman et al. [2012] presents an adaptation of random forests [Breiman, 2001] to the uplift case. The adaptation uses splitting criteria defined in Rzepkowski and Jaroszewicz [2010]. Later on, motivated by the unbiased recursive partitioning method proposed by Hothorn et al. [2006], the same authors developed the framework of causal conditional inference forest [Guelman et al., 2015] that improved the performance of uplift random forests. The algorithm separates variable selection from splitting. Namely, for each terminal node in the tree, a statistical test of the global null hypothesis of no interaction effect between the treatment and any of the current subset of explanatory variables is performed. When the global null hypothesis is not rejected, the splitting process is stopped at that node. Otherwise, the variable with the smallest p-value is selected. A comparison between several ensemble methods [Soltys et al., 2015] shows that bagging [Breiman, 1996] and random forests emerge as better tools for uplift. Another tree-based method, the pessimistic uplift model of Shaar et al. [2016] have been suggested to improve performance in the presence of noise. Some other interesting methods have been
applied to uplift. A non-parametric method discussed in [Alemi et al., 2009, Su et al., 2012] uses a modified version of the k-nearest-neighbours classifier [Cover and Hart, 1967]. The idea is to estimate the uplift from the nearest neighbors containing at least one treated and one control observation. This method quickly becomes computationally expensive when dealing with large datasets, because the entire dataset has to be stored in order to predict the uplift for new observations. Uplift has also been incorporated into machine learning algorithms such as support vector machines specially conceived to maximize uplift in the context of medical data [Zaniewicz and Jaroszewicz, 2013, Kuusisto et al., 2014, Zaniewicz and Jaroszewicz, 2017]. For further details, including a concise overview of the uplift modeling literature, the reader is referred to the works of Kane et al. [2014], Gutierrez and Gérardy [2017] and Devriendt et al. [2018].

From a complexity point of view, parametric models are simpler than non-parametric ones such as regression trees, because for parametric models the number of parameters is kept small and fixed. Although for many analysts prediction is the main target, from a business point of view, model interpretation is very important. Knowing which variables and how these variables discriminate between groups of clients is one of the main goal of uplift modeling for marketing. For these reasons, in this work we focus on parametric models.

Although the uplift literature is vast, to the best of our knowledge, our work introduces the first methodological procedure for variable selection adapted explicitly for uplift. However, there exists some previous work that discusses the problem of variable selection. These are based, either in heuristics [Rzepakowski and Jaroszewicz, 2010, 2012, Sołtys et al., 2015], or make use of the classic lasso [Guelman et al., 2015, Gross and Tibshirani, 2016]. In addition, Larsen [2011] introduces the net weight of evidence, a modification of the weight of evidence [Thomas, 2000], as a measure of the predictive power of an explanatory variable. This statistic is used as the basis for defining the net information value, which is then used for variable selection. A drawback of this method is that it is difficult to predict or guess the number of important variables based solely on this measure. In our present work, we introduce a new variable selection procedure based on regularization, and designed explicitly for uplift regression models.

3 Qini-based logistic regression for uplift

Logistic regression is a well-known parametric model for binary response variables. Given a p-dimensional covariate $x_i, i \in \{1, \ldots, n\}$, and logistic regression coefficients $\theta_0 \in \mathbb{R}$, and $\beta \in \mathbb{R}^p$, the model is

$$p_i = p_i(\theta_0, \beta) = \Pr(Y_i = 1 \mid x_i, \theta_0, \beta) = \left(1 + \exp\{-\theta_0 + x_i^\top \beta\}\right)^{-1}$$

or, equivalently, $\logit(p_i) = \theta_0 + x_i^\top \beta$, where $\logit(p_i) = \log\{p_i/(1 - p_i)\}$. Throughout the paper, the superscript $\top$ will stand for the transpose of a column vector or matrix. In the uplift context, one need to add explicit interaction terms between each explanatory variable and the treatment indicator so that

$$p_i(\theta_0, \theta) = \Pr(Y_i = 1 \mid x_i, t_i, \theta_0, \theta) = \left(1 + \exp\{-\theta_0 + x_i^\top \beta + \gamma t_i + t_i x_i^\top \delta\}\right)^{-1},$$

where $\theta = (\beta, \gamma, \delta)$. Here, $\gamma$ denotes the treatment effect, $\beta$ is the vector of main effects and $\delta$ is the vector of interactions effects. The likelihood function associated with the uplift model is

$$\mathcal{L}(\theta_0, \theta) = \prod_{i=1}^n p_i(\theta_0, \theta)^{y_i} \{1 - p_i(\theta_0, \theta)\}^{(1-y_i)}.$$  

The maximum likelihood estimates of $(\theta_0, \theta)$ will be denoted by $(\hat{\theta}_0, \hat{\theta})$, with $\hat{\theta} = (\hat{\beta}, \hat{\gamma}, \hat{\delta})$. The predicted uplift associated with the covariate vector $x_{n+1}$ of a future individual is estimated by

$$\hat{u}(x_{n+1}) = \left(1 + \exp\{-\hat{\theta}_0 + x_{n+1}^\top \hat{\beta} + \hat{\gamma} + x_{n+1}^\top \hat{\delta}\}\right)^{-1} - \left(1 + \exp\{-\hat{\theta}_0 + x_{n+1}^\top \hat{\beta}\}\right)^{-1}.$$  

3.1 Adjusted Qini

Evaluating uplift models requires the construction of the Qini curve and the computation of the Qini coefficient [Radcliffe, 2007]. The motivation to consider the Qini curve comes from the belief that a good model should be able to select individuals with positive uplift first. More explicitly, for a given model, let $\hat{u}_{(1)} \geq \hat{u}_{(2)} \geq \ldots \geq \hat{u}_{(n)}$ be the sorted predicted uplifts. Let $\phi \in [0, 1]$ be a given proportion and let $N_\phi = \{i : \hat{u}_i \geq \hat{u}_{(\phi n)}\} \subset \{1, \ldots, n\}$ be the subset of individuals with the $\phi n \times 100\%$ highest predicted uplifts $\hat{u}_i$ (here $\lfloor s \rfloor$ denotes the smallest integer larger or equal to $s \in \mathbb{R}$). Because $N_\phi$ is a function of the
predicted uplifts, $N_\phi$ is a function of the fitted model. For a parametric model such as (4), $N_\phi$ is a function of the model’s parameters estimates, and should be denoted $N_\phi(\hat{\theta})$. To simplify the notation, we prefer to omit this specification.

As a function of the fraction of population targeted $\phi$, the incremental uplift is defined as

$$h(\phi) = \sum_{i \in N_\phi} y_i t_i - \sum_{i \in N_\phi} y_i (1 - t_i) \left\{ \sum_{i \in N_\phi} t_i / \sum_{i \in N_\phi} (1 - t_i) \right\},$$

with $h(0) = 0$. The incremental uplift has been normalized by the number of subjects treated in $N_\phi$. The relative incremental uplift $g(\phi)$ is given by

$$g(\phi) = h(\phi) / \sum_{i=1}^{n} t_i.$$

Note that $g(0) = 0$ and $g(1)$ is the overall sample observed uplift

$$g(1) = \left( \sum_{i=1}^{n} y_i t_i / \sum_{i=1}^{n} t_i \right) - \left( \sum_{i=1}^{n} y_i (1 - t_i) / \sum_{i=1}^{n} (1 - t_i) \right).$$

The Qini curve is constructed by plotting $g(\phi)$ as a function of $\phi \in [0, 1]$. This is illustrated in Figure 1. Figure 1 can be interpreted as follows. The $x$-axis represents the fraction of targeted individuals and the $y$-axis shows the incremental number of positive responses relative to the total number of targeted individuals.

The straight line between the points $(0, 0)$ and $(1, g(1))$ in Figure 1 represents a benchmark to compare the performance of the model to a strategy that would randomly target subjects. In other words, when the strategy is to treat individuals randomly, if a proportion $\phi$ of the population is treated, we expect to observe an uplift equal to $\phi$ times the global uplift. The Qini coefficient $q$ is a single index of model performance. It is defined as the area between the Qini curve and the straight line

$$q = \int_{0}^{1} Q(\phi) \, d\phi = \int_{0}^{1} \{g(\phi) - \phi g(1)\} \, d\phi,$$

where $Q(\phi) = g(\phi) - \phi g(1)$. This area can be numerically approximated using a Riemann method such as the trapezoid rule formula: the domain of $\phi \in [0, 1]$ is partitioned into $J$ panels, or $J + 1$ grid points $0 = \phi_1 < \phi_2 < \ldots < \phi_{J+1} = 1$, to approximate the Qini coefficient $q$ (5) by

$$\hat{q} = \frac{1}{2} \sum_{j=1}^{J} (\phi_{j+1} - \phi_j) \{Q(\phi_{j+1}) + Q(\phi_j)\}.$$

In general, when comparing several models, the preferred model is the one with the maximum Qini coefficient [Radcliffe, 2007].
In order to have a hint on what adequate values for the adjusted Qini coefficient. In practice, the sample is divided into quantiles (the number of bins grouping the individuals in decreasing uplift bins.

The adjusted Qini coefficient represents a trade-off between maximizing the area under the Qini curve and the correlation between the predicted uplift and the observed uplift. The Kendall’s uplift rank correlation is defined as

\[ \rho = \frac{2}{J(J-1)} \sum_{i<j} \text{sign}(\bar{u}_i - \bar{u}_j) \text{sign}(\bar{u}_i - \bar{u}_j), \]  

(7)

where \( \bar{u}_k \) is the average predicted uplift in bin \( k, k \in 1, \ldots, J \), and \( u_k \) is the observed uplift in the same bin. Combining (6) and (7), we define the adjusted Qini coefficient as

\[ \hat{q}_{adj} = \rho \max\{0, \hat{q}\}. \]  

(8)

The adjusted Qini coefficient represents a trade-off between maximizing the area under the Qini curve and grouping the individuals in decreasing uplift bins.

The number of bins \( J \) may be seen as a hyper-parameter. Its choice will certainly affect the computation of the adjusted Qini coefficient. In practice, the sample is divided into quantiles \( J = 5 \) or deciles \( J = 10 \). In order to have a hint on what adequate values for \( J \) are, suppose that the relative incremental uplift function \( g(\phi) \) is twice-differentiable, with bounded second derivative. Consider the trapezoid rule approximation to the integral \( g \) based on \( J \) bins. Let us assume that the bin sizes are proportional to \( 1/J \). It is well-known that under these assumptions the error of the approximation is order \( O(1/J^2) \). Since \( g(\cdot) \) is unknown, one need to estimate it with data. Let \( \hat{g}_j \) be the estimate of \( g(\phi_j), j = 1, \ldots, J \). Suppose that the \( \hat{g}_j \) is obtained as a mean of \( n/J \) random variables observed in the \( j \)-th bin. We suppose that these random variables are independent and identically distributed with mean \( g(\phi_j) \) and a certain finite variance. The weak law of large numbers says that \( \hat{g}_j \) converges to \( g(\phi_j) \), and the error in this approximation is of order \( O(1/J) \). It turns out that we need \( J \) to minimize \( \kappa_1/J^2 + \kappa_2 J/\sqrt{n} \), where \( \kappa_1, \kappa_2 \) are constants. The solution is \( J = O(n^{1/6}) \). So, for example, if \( n = O(1000) \), then the optimal \( J = O(3) \). Hence, the usual values of \( J = 5 \) and \( J = 10 \) seem reasonable to estimate the Qini [Radcliffe, 2007].

### 3.2 Qini-based regression

We propose to select a regression model that maximizes the adjusted Qini instead of a regression model that maximizes the likelihood. Because model selection corresponds to variable selection, the task is haunting and intractable if done in a straightforward manner when the number of variables to consider is large, e.g. \( p \approx 100 \), like in the case of the insurance data. To realistically search for a good model, we conceived a searching method based on an efficient sampling of the regression coefficients space combined with a lasso regularization of the log-likelihood. There is no explicit analytical expression for the adjusted Qini surface,
so unveiling it is not easy. The idea is to gradually uncover the adjusted Qini surface in a manner inspired by surface response designs. The goal is to find the global maximum or a reasonable local maximum of the adjusted Qini by exploring the surface near optimal values of the coefficients. These coefficient values are given by maximizing the lasso regularized log-likelihood. The exploration is done using Latin hypercube sampling structures [McKay et al., 2000] centered in the corresponding coefficients. The procedure is explained in the following sections.

3.2.1 Regularization

In the context of linear regression, the effectiveness of regularization has been amply supported practically and theoretically in several studies. In order to decrease the mean squared error of least squares estimates, ridge regression [Hoerl and Kennard, 1970] has been proposed as a trade-off between bias and variance. This technique adds an $L_2$-norm regularization term to the least squares loss. The lasso (least absolute shrinkage and selection operator) penalization technique [Tibshirani, 1996] uses an $L_1$-norm regularization which sets some of the regression coefficients to zero (sparse selection) while shrinking the rest. The elastic net penalization technique [Zou and Hastie, 2005] linearly combines the $L_1$ and $L_2$-norms to provide better prediction in the presence of collinearity. Other regularization techniques such as scad [Fan and Li, 2001] and bridge regression [Frank and Friedman, 1993], offer interesting theoretical properties, including consistency.

Here, we focus on sparse estimation of the coefficients. That is, the selection of a small subset of features to predict the response. This is often achieved with a $L_1$-norm regularization. Performing $L_1$-norm regularization is computationally challenging for very large feature dimensions $p$. Fan and Lv [2008] recommend sure screening to pre-select a subset of features having large absolute correlations with the response, to then run $L_1$-norm regularization on this subset. Fan and Lv [2008] show that this subset selection procedure keeps with high probability important features in the model, and drastically reduces the computational cost.

Given $\lambda \in \mathbb{R}^+$, in the context of linear regression, the lasso regularization [Tibshirani, 1996] finds the estimate of the coefficients $\hat{\beta}(\lambda)$ that maximizes the penalized log-likelihood, say $\ell(\beta) + \lambda \sum_{j=1}^p |\beta_j|$. Setting the regularization constant $\lambda = 0$ returns the least squares estimates which performs no shrinking and no selection. For $\lambda > 0$, the regression coefficients $\hat{\beta}(\lambda)$ are shrunk towards zero, and some of them are set to zero (sparse selection). Friedman et al. [2007] proposed a fast pathwise coordinate descent method to find $\hat{\beta}(\lambda)$, using the current estimates as warm starts. In practice, the value of $\lambda$ is unknown. Cross-validation is often used to search for a good value of the regularization constant. The least angle regression (or LARS algorithm) efficiently computes a path of values of $\hat{\beta}(\lambda)$ over a sequence of values of $\lambda = \lambda_1 < \cdots < \lambda_j < \cdots < \lambda_{\min(n,p)}$, for which the parameter dimension changes [Efron et al., 2004a].

The entire sequence of steps in the LARS algorithm with $p < n$ variables requires $O(p^3 + np^2)$ computations, which is the cost of a single least squares fit on $p$ variables. Extensions to generalized linear models with nonlinear loss functions require some form of approximation. In particular, for the logistic regression case, which is our model of interest, Friedman et al. [2010] extend the pathwise coordinate descent algorithm [Friedman et al., 2007] by first, approximating the log-likelihood (quadratic Taylor expansion about current estimates), and then using coordinate descent to solve the penalized weighted least-squares problem. The algorithm computes the path of solutions for a decreasing sequence of values for $\lambda = \lambda_{\min(n,p)} > \cdots > \lambda_j > \cdots > \lambda_1$, starting at the smallest value for which the entire vector $\hat{\beta} = 0$. The algorithm works on large datasets, and is publicly available through the R package glmnet [Friedman et al., 2009], which we use in this work. In what follows, we will refer to the sequence of regularizing constant values given by glmnet as the logistic-lasso sequence.

Shao [1996] show that sparsity and parameter consistency do not necessarily go together for $L_1$-norm regularization methods such as the lasso. Two approaches are suggested to address this issue: (i) estimate the model dimension $p$ consistently, to then use the estimated dimension to re-estimate the regression parameter $\beta$ [Belloni et al., 2013]; or (ii) use a non-convex regularization technique such as scad [Fan and Li, 2001]. In this work, we use the first approach. Choosing the right model dimension $m \in \{1, \ldots, p\}$, and selecting one of the $\lambda_j$‘s are inter-related problems. One can choose a value $\lambda_j$, and evaluate the model with the associated effective dimension induced by that $\lambda_j$. This procedure can be repeated for each value $\lambda_j$, given rise to a subset of different models. The best model along with its corresponding dimension is selected from this subset. This approach is known as two-stage selection. It guarantees both the statistical consistency of the model dimension, and the statistical consistency of the estimated parameters [Zou, 2006, Meinshausen, 2007, Belloni et al., 2013, Liu et al., 2013].
Recall the uplift model likelihood given in (4). The vector of parameters \( \theta = (\beta, \gamma, \delta) \) is a \( p' = (2p + 1) \)-dimensional vector. Because of the considerations mentioned in the previous sections, in order to select an appropriate sparse model for uplift, we adapt the lasso algorithm to explore a relatively small set of reasonable models, so as to avoid an exhaustive model search. The penalized uplift model log-likelihood is given by

\[
\ell(\theta_0, \theta | \lambda) = \sum_{i=1}^{n} \left( y_i \log \{ p_i(\theta_0, \theta) \} + (1 - y_i) \log \{ 1 - p_i(\theta_0, \theta) \} \right) + \lambda \| \theta \|_1, \tag{9}
\]

where \( p_i(\theta_0, \theta) \) is as in (3), and \( \| \cdot \|_1 \) stands for the \( L_1 \)-norm. For any given \( \lambda \), the parameters that maximize the penalized log-likelihood (9) are denoted by

\[
(\hat{\theta}_0(\lambda), \hat{\theta}(\lambda)) = \underset{\theta_0, \theta}{\text{argmax}} \ \ell(\theta_0, \theta | \lambda). \tag{10}
\]

Applying the pathwise coordinate descent algorithm to the uplift model, we get a sequence of critical regularization values \( \lambda_1 < \cdots < \lambda_{\text{min}(n, p')} \) and corresponding model parameters \( \{ (\hat{\theta}_0(\lambda_j), \hat{\theta}(\lambda_j)) \}_{j=1}^{\text{min}(n, p')} \) associated with different model dimensions \( m \in \{1, \ldots, p' \} \). Because the adjusted Qini function is not straightforward to optimize with respect to the parameters, one needs to explore the parameters space in order to find the maximum of the adjusted Qini.

Latin hypercube sampling (LHS) is a statistical method for quasi-random sampling based on a multivariate probability law inspired by the Monte Carlo method [McKay et al., 2000]. The method performs the sampling by ensuring that each sample is positioned in a space \( \Omega \) of dimension \( p \) as the only sample in each hyperplane of dimension \( p - 1 \) aligned with the coordinates that define its position. Each sample is therefore positioned according to the position of previously positioned samples to ensure that they do not have any common coordinates in the \( \Omega \) space. When sampling a function of \( p \) variables, the range of each variable is divided into \( M \) equally probable intervals. \( M \) sample points are then placed to satisfy the Latin hypercube requirements; this forces the number of divisions, \( M \), to be equal for each variable. Also this sampling scheme does not require more samples for more dimensions (variables); this independence is one of the main advantages of this sampling scheme.

We use LHS to find the coefficient parameters that maximize the adjusted Qini. For each \( \lambda_j, j = 1, \ldots, \text{min}(n, p') \), we generate a LHS comprising \( L \) points \( \{ \hat{\theta}(\lambda_j)_l \}_{l=1}^{L} \) in the neighborhood of \( \hat{\theta}(\lambda_j) \), and evaluate the adjusted Qini on each of these points. The optimal coefficients are estimated as those coefficients among the \( \{ \text{min}(n, p') \times L \} \) LHS points that maximize the adjusted Qini. Figure 3 illustrates the procedure.

### 3.2.3 A simpler estimate of the Qini-based uplift regression parameters

We also consider a simpler two-stage procedure to find a good uplift model. This one is based only on the penalized log-likelihood and does not require the posterior LHS-based search for the optimal coefficients.
Let \( \hat{q}_{adj}(\lambda) \) be the adjusted Qini coefficient associated with the model with parameters \((\hat{\theta}_0(\lambda), \hat{\theta}(\lambda))\). The first stage of the procedure solves

\[
\hat{\lambda} = \text{argmax} \ (\hat{q}_{adj}(\lambda_j) \ : \ j = 1, \ldots, \min\{n, p'\}),
\]

where as before, the sequence \( \lambda_1 < \cdots < \lambda_{\min\{n, p'\}} \) is the logistic-lasso sequence. On the second-stage, a reduced model that only include those explanatory variables associated with non-zero entries of the estimated parameter \( \hat{\theta}(\lambda) \) is fitted without penalization, that is, with \( \lambda \) set to zero. This yields the selected model. In our simulations, this model performs very well. It also serves to show that the value of the regularization parameter \( \hat{\lambda} \) that maximizes the Qini or adjusted Qini, is not necessarily the same as the one that maximizes the penalized log-likelihood.

## 4 Simulations

We conduct a simulation study to examine the performance of Qini-based uplift regression. More specifically, we study the ability of the method to perform variable selection by varying both the complexity of the data, and the number of predictors in the model. In order to create realistic scenarios, we based our artificial data generation on the home insurance policy data described in the introduction. We take advantage of the opportunity to have real data in order to generate realistic scenarios. We proceed as follows. First, we fit a non-parametric model on a random sample \( \mathcal{D} \) of the home insurance policy data. Based on the resulting model, we can extract the probabilities

\[
p_1(x) = \Pr(Y = 1 \mid x, T = 1), \quad \text{and} \quad p_0(x) = \Pr(Y = 1 \mid x, T = 0),
\]

for any given value \( x \). Then, we use these probabilities to generate synthetic data. We start by creating a bootstrap sample \( \mathcal{S} \) of size \( n_\mathcal{S} \) from \( \mathcal{D} \). For each observation \( x_i \in \mathcal{S} \), we generate a random vector \( \tilde{y}_i = (\tilde{y}_{i0}, \tilde{y}_{i1}) \), where \( \tilde{y}_{i0} \) is the binary outcome of a Bernoulli trial with success probability \( p_0(x_i) \), and \( \tilde{y}_{i1} \) is the binary outcome of a Bernoulli trial with success probability \( p_1(x_i) \). We then fit synthetic dataset \( \{x_i, t_i, \tilde{y}_i\}_{i=1}^{n_\mathcal{S}} \), which we are going to denote again by \( \mathcal{S} \), is the data of interest in the simulation. For each simulated dataset, we implement the following models:

(a) a multivariate logistic regression without penalization as in (4). This is the baseline model, and we will refer to it as Baseline.

(b) our Qini-based uplift regression model that uses several LHS structures to search for the optimal parameters (see Section 3.2.2). We denote this model by \( Q+LHS \). We consider three different optimizations with this estimation method:

   (a) \( Q+LHS_q \), which searches for the coefficients that maximize the Qini coefficient \( \hat{q} \) (5);
   
   (b) \( Q+LHS_\rho \), which searches for the coefficients that maximize the Kendall’s uplift rank correlation \( \rho \) (7);
   
   (c) \( Q+LHS_{q_{adj}} \), which searches for the coefficients that maximize the adjusted Qini coefficient \( \hat{q}_{adj} \) (8).

(c) our Qini-based uplift regression model that uses the simpler estimate of the regression parameters as explained in Section 3.2.3. We denote this model by \( Q+lasso \).

### Data generation

As discussed in Section 2, several tree-based methods have been suggested in the uplift literature. Here, we use the uplift random forest [Guelman et al., 2012] as the data generating process. We chose this method due to its simplicity, and because it is readily available in \( \text{R} \) through the package uplift [Guelman, 2014]. Algorithm 1 describes the associated methodology.

In our simulations, we vary two parameters: the depth of the trees used to fit the uplift random forests, and the number of variables \( k \) considered when fitting the uplift logistic models. Algorithm 2 details the procedure.

We define 9 scenarios by varying two parameters: (i) the depth of the uplift random forest trees used to generate the synthetic data is either 1, 2 or 3, and (ii) the number of covariates \( k \) considered for modeling is either 10, 20 or 30. For scenarios 1-3, the depth is 1, and we vary \( k = 10, 20, 30 \); for scenarios 4-6, the depth is 2; and for scenarios 7-9, the depth is 3. Each scenario was replicated 100 times. The averaged metrics \( \hat{q} \) (5), \( \rho \) (7), and \( q_{adj} \) (8) and their corresponding standard errors are reported in Tables 3, 4, and 5, respectively. Since the conclusions are similar for the three tree depths, we report only the results associated with depth 3, that is, for the most complex model. For each comparison group, we also report the corresponding performance...
shown in Table 3, the performances of the Q+lasso and the Q+LHS models are very similar.

Although the true model, that is, the random forest, presents the highest Qini coefficient ($\hat{q}$ = 1.00), it is not the best performer in terms of the Kendall’s correlation ($\rho$ = 0.86). In fact, it performs just like the baseline model. The Q+LHS models perform very well, attaining perfect correlations. Here again, as with the results shown in Table 3, the performances of the Q+lasso and the Q+LHS models are very similar.
Figure 4: Comparison between Q+lasso and Q+LHS$_{q_{adj}}$. Boxplots of the difference $q_{adj, Q+lasso} - q_{adj, Q+LHS_{q_{adj}}}$ as a function of the number of predictors used in the models over 100 simulations, $n = 5000$ observations. Black line represents the differences median. The skewness of the boxplots shows that when Q+LHS$_{q_{adj}}$ outperforms Q+lasso, the difference in terms of performance is significantly different from 0.

Table 5: Adjusted Qini coefficient ($\hat{q}_{adj}$) averaged over 100 simulations. The true model’s performance is 1.40 ($\pm$0.048), RF with $p = 97$ and depth = 3, $n = 5000$ observations.

| $k$ | Baseline | Q+lasso | Q+LHS$_{q}$ | Q+LHS$_{\rho}$ | Q+LHS$_{q_{adj}}$ |
|-----|----------|---------|-------------|----------------|------------------|
| 10  | 0.50 ($\pm$0.023) | 0.63 ($\pm$0.024) | 0.62 ($\pm$0.024) | 0.64 ($\pm$0.025) | 0.67 ($\pm$0.023) |
| 20  | 0.80 ($\pm$0.029) | 0.98 ($\pm$0.025) | 0.97 ($\pm$0.026) | 0.99 ($\pm$0.025) | 1.01 ($\pm$0.024) |
| 30  | 1.02 ($\pm$0.036) | 1.21 ($\pm$0.026) | 1.18 ($\pm$0.028) | 1.23 ($\pm$0.027) | 1.24 ($\pm$0.026) |

In Table 5, we compare the main statistic of interest, that is, the adjusted Qini coefficient. These results corroborate the findings from the previous tables. Guiding variable selection by this statistic leads to significant improvements from the results of the baseline model. Although, the difference in performance between Q+lasso and Q+LHS$_{q_{adj}}$ does not appear to be significant, a boxplot of the difference in performance between these two models in each simulation shows that the medians are more or less equal to zero. Therefore, 50% of the time, Q+LHS$_{q_{adj}}$ “outperforms” Q+lasso and 50% of the time Q+lasso “outperforms” Q+LHS$_{q_{adj}}$. But, in the first case, the difference in absolute value tends to be larger than in the second case where the difference is almost zero (see Figure 4).

In practice, the true uplift is unknown and impossible to compute at the individual level. However, because we are in a simulation setting, and based on the data generating model, we can easily compute the true uplift at the individual level, that is, $p_1(x_i) - p_0(x_i)$. We consider the Root Mean Square Error (RMSE) between the true uplift and the predicted uplift as a goodness-of-fit measure. The RMSE is the standard deviation of the residuals, and it is defined as

$$\text{RMSE} = \sqrt{\frac{1}{nS} \sum_{i=1}^{nS} (u_i - \hat{u}_i)^2} / nS,$$

where $u_i$ denotes the true uplift and $\hat{u}_i$, the predicted uplift. To get a relative comparison between the models, we consider the relative RMSE, which is the ratio between the RMSE of a given model $m$ and that RMSE of the baseline model. More explicitly, $\text{RRMSE}_m = \text{RMSE}_m / \text{RMSE}_{\text{Baseline}}$. Tables 6 and 7 show the RMSE statistics for the five uplift models and the true model. Results from Table 6 suggest that Q+lasso and Q+LHS improve the prediction at the individual level when compared to the baseline model. We also see, now from Table 7, that, in general, Q+LHS$_{\rho}$ appears to perform better than the Q+lasso and Q+LHS$_{q}$ models. However, this improvement does not seem significantly better than Q+LHS$_{q_{adj}}$ when 30 predictors are considered in the models. These findings, as well as the results of the previous tables, corroborate our original idea of using Kendall’s correlation and the Qini coefficient as a goodness-of-fit measure for uplift models.
Finally, we also compare the Qini-based uplift regression to the classic lasso approach where the regularization constant is chosen by cross-validation on the log-likelihood. We look at the value of \( \hat{\lambda} \in \{\lambda_1, \ldots, \lambda_{\min\{n,p'\}}\} \) in the logistic-lasso sequence that is chosen by cross-validation of the log-likelihood,
Figure 6: Barplot of the distribution of the Q+lasso rankings associated with $\hat{\lambda}$.

and report its ranking based on the sorted Q+lasso sequence $\lambda_{\min(n,p')} \leq \cdots \leq \lambda_{\pi}$. We repeated the simulation 100 times, each time using $n = 5000$ observations randomly selected from the full data set. The barplot in Figure 6 shows that only 7% of the time $\hat{\lambda}$ also maximizes $\hat{q}_{\text{adj}}$. Observe that 4% of the time $\hat{\lambda}$ is positioned 10 in the ranking, and 12% of the time, it is positioned 39. These results clearly show that choosing the regularization constant by cross-validation of the log-likelihood does not solve the problem of maximizing the adjusted Qini coefficient, and therefore, is not necessarily appropriate for uplift models.

5 Application

Recall the insurance data introduced in Section 1. The insurance company is interested in designing retention strategies to minimize its policyholders’ attrition rate. An experimental loyalty campaign was implemented, from which policies coming up for renewal were randomly allocated into one of the following two groups: treatment group, and control group. The goal of this section is to analyze the marketing campaign results so as to identify both the set of persuadables clients, and the set of clients that should not be disturbed.

We fit the Qini-based uplift regression to the data using the methodology described in the previous section and referred to as Q+LHS$_{q_{\text{adj}}}$. We also consider the Q+LHS$_{p}$ model that maximizes the Kendall’s uplift rank correlation, because the simulations results suggest that this model could potentially give the lowest errors in terms of individual uplift estimates (see Table 7). In order to choose the optimal value from the logistic-lasso sequence of regularization constant values $\{\lambda_1, \ldots, \lambda_{\min(n,p')}\}$, we use a 5-fold cross-validation on the adjusted Qini statistics. We compare the resulting models with the one yielded by applying the classical lasso approach, that is, with the model associated with the value of the regularization constant that maximizes the cross-validated log-likelihood. The two-stage approach was used in all the cases. The first stage estimates the best $\lambda$ in the logistic-lasso sequence by cross-validation. The second stage fits the non penalized logistic regression model with the subset of selected variables. The results of the first stage are presented in Figure 7. If we use the simple search method described in Section 3.2.3, which was denoted by Q+lasso in the previous section, the optimal value of the regularization constant for the Qini-based model is $\lambda_{q,1} = e^{-10.07} \text{(Q+lasso)}$, while for the lasso logistic regression is $\lambda_{\text{lasso}} = e^{-6.93}$. The numbers of selected variables are, respectively, 156 and 53 out of a total of 195 main and interaction effect terms. We can also select the final model using the usual one-standard-error (OSE) rule. For each of the values in the logistic-lasso sequence, we calculate the standard error of the adjusted Qini statistics from its five-fold estimates. The chosen model corresponds to the smallest model for which the adjusted Qini statistics is within one standard error of the highest point in the curve. The rationale here is based on the principle of parsimony: if several models appear to be equally good, then we might as well choose the simplest model, that is, the model with the smallest number of predictors.
Figure 7: 5-fold cross-validation results for the adjusted Qini coefficient (top panel), and for the log-likelihood (bottom panel). The cross-validation is shown as a function of the logarithm of the regularization constant.
In this case, applying the OSE rule leads to selection of the model associated with $\lambda_{q,2} = e^{-9.31}$ (Q+lasso OSE). The corresponding model only includes 130 terms, that is, a reduction of 17% in the dimension of Q+lasso model.

For the Q+LHS models, we also make use of the 5-fold cross-validation technique. For each $\lambda_j$, we perform a LHS search to directly maximize the adjusted Qini coefficient (8) for the Q+LHS$_{qadj}$ model or to maximize the Kendall’s uplift rank correlation (7) for the Q+LHS$_{\rho}$ model. In these cases, applying the LHS search leads to the selection of the models associated with $\lambda_{qadj} = e^{-10.50}$, and $\lambda_{\rho} = e^{-11.05}$, respectively.

Once we have selected the different values of the optimal regularization constant, we perform the second stage of the estimation of the uplift models using all available data. Figures 8 and Figures 9, 10 and 11 show the performance of the models in terms of the Qini curve and the uplift barplot, respectively. As expected, the Qini-based uplift regression models outperform the classic lasso approach, both in terms of overall adjusted Qini coefficient (see Table 8) and in terms of sorting the individuals in decreasing order of uplift (see Figures 9, 10 and 11).

![Qini curve](image1)

**Figure 8**: Performance of the final models based on the Qini curves.

![Uplift barplot](image2)

**Figure 9**: Performance of the LHS models based on uplift Kendall’s correlations. The left barplot corresponds to the Q+LHS$_{qadj}$ model and the right barplot to the Q+LHS$_{\rho}$ model.
Figure 10: Performance of the Q+lasso models based on uplift Kendall’s correlations. The left barplot corresponds to the Q+lasso model and the right barplot to the Q+lasso OSE model. Overall, the second model has more difficulty sorting the different groups.

Figure 11: Performance of the usual lasso penalized log-likelihood model based on uplift Kendall’s correlations. Even though the model is able to identify a group of individuals that should not be targeted (Quantile 5), the rest of the uplifts are not well estimated.

Based on the final models, we can identify both (i) the group of clients at the top 20% of predicted uplifts, that is, the clients to pursue in the marketing, and (ii) the group of clients at the bottom 20% of predicted uplifts, that is, the clients not to disturb with any marketing. The group of clients at the top 20% of predicted uplifts provides very strong return on investment cases when applied to retention activities. For example, by only targeting the persuadable customers in an outbound marketing campaign, the contact costs and hence the return per unit spend can be dramatically improved [Radcliffe and Surry, 2011].

We observe from Table 8 that the Q+LHS\(q_{adj}\) model finds the highest top 20% uplift group which presents an uplift of approximately 6%, while the bottom 20% uplift group shows an uplift of approximately −6%. Note that the overall uplift is approximately −0.5%. Even though the Q+LHS\(\rho\) and Q+lasso models perform as well as the Q+LHS\(q_{adj}\) model in terms of \(\hat{q}, \rho\) and \(\hat{q}_{adj}\), their top 20% uplift is slightly lower than the one from the Q+LHS\(q_{adj}\) model.

Table 8: Uplift comparison of the top and bottom 20% uplift groups within the five models: Q+LHS\(q_{adj}\), Q+LHS\(\rho\), Q+lasso, Q+lasso OSE and classical lasso.

| Method            | \(\hat{q}\) | \(\rho\) | \(\hat{q}_{adj}\) | Top 20% Uplift | Bottom 20% Uplift |
|-------------------|-------------|----------|-------------------|----------------|-------------------|
| Q+LHS\(q_{adj}\) | 1.03        | 1.00     | 1.03              | 5.94%          | −6.02%            |
| Q+LHS\(\rho\)    | 1.03        | 1.00     | 1.03              | 5.58%          | −6.45%            |
| Q+lasso           | 1.02        | 1.00     | 1.02              | 5.76%          | −5.93%            |
| Q+lasso OSE       | 0.90        | 0.80     | 0.72              | 4.52%          | −6.16%            |
| lasso             | 0.48        | 0.80     | 0.39              | 1.41%          | −4.38%            |
Because our model is a logistic model, we can interpret the results through its coefficients. The usual approach is that of odds ratios. Since the company is not interested in all the variables included in the model, we will analyze a subset with relevant interpretation for the business. In addition, for confidentiality reasons, we do not show the analysis of variables related to the insurance premium. The following variables are chosen by our model: client’s credit score, age, gender and marital status (single or not); client’s products: whether it is a single line (home) or a multi-line (automobile and home) account; client’s number of automobile policies, mortgages and residences; and whether the client has extra options (additional endorsements) in his/her account. For a model with \(p\) variables, the odds ratio \(\text{OR}_{X_j}(t)\) for a specific variable \(X_j\) is given by

\[
\frac{\Pr(Y = 1 | X_j = x_j + 1, T = t) / \Pr(Y = 0 | X_j = x_j + 1, T = t)}{\Pr(Y = 1 | X_j = x_j, T = t) / \Pr(Y = 0 | X_j = x_j, T = t)} = \frac{\exp(\hat{\beta}_j (x_j + 1) + \hat{\gamma}_j t (x_j + 1))}{\exp(\hat{\beta}_j x_j + \hat{\gamma}_j t x_j)} = \exp(\hat{\beta}_j - \hat{\gamma}_j t),
\]

where \(T\) is the treatment indicator, and where for a binary variable, such as extra options, \(x_j\) is set to 0 in the above expression. When the company does not call a client \((T = 0)\), the odds ratio is \(\text{OR}_{X_j}(0) = \exp(\hat{\beta}_j)\) and when the company calls a client \((T = 1)\), the odds ratio is \(\text{OR}_{X_j}(1) = \exp(\hat{\beta}_j) \exp(\hat{\gamma}_j) = \text{OR}_{X_j}(0) \exp(\hat{\gamma}_j)\). Table 9 gives the estimated odds ratios \(\text{OR}_{X_j}(0)\) and \(\text{OR}_{X_j}(1)\) with 95% confidence intervals. We can see, for example, that when the company does not call a client which has extra options in his/her policy, his/her odds ratio of renewing the policy is 0.35 while when the company calls that same client, the odds ratio becomes 1.28.

Table 9: Odds ratios and 95% confidence intervals estimated by the Qini-based uplift regression model (Q+LHS_{qadj}) for some of the selected variables, * represents significant coefficients.

|                                | exp(\(\hat{\beta}_j\)) | CI (95%)         | exp(\(\hat{\beta}_j + \hat{\gamma}_j\)) | CI (95%)         |
|--------------------------------|------------------------|-----------------|------------------------------------------|-----------------|
| Credit Score                   | 0.998                  | (0.994; 1.003)  | 1.001                                    | (0.999; 1.001)  |
| Age (Years)                    | 0.995                  | (0.969; 1.023)  | 0.998                                    | (0.989; 1.006)  |
| Gender                         |                        |                 |                                          |                 |
| Male                           | 1.297                  | (0.778; 2.163)  | 0.962                                    | (0.814; 1.135)  |
| Marital Status                 |                        |                 |                                          |                 |
| Single                         | 2.759                  | (0.956; 7.963)  | 0.697                                    | (0.452; 1.077)  |
| Products                       |                        |                 |                                          |                 |
| Auto and Home                  | 1.619                  | (0.586; 4.472)  | *1.418                                   | (1.017; 1.977)  |
| Auto Policies Count            | 2.106                  | (0.653; 6.790)  | *1.996                                   | (1.368; 2.918)  |
| Mortgage Count                 | 1.381                  | (0.789; 2.418)  | *1.366                                   | (1.137; 1.642)  |
| Residences Count               | 0.505                  | (0.172; 1.489)  | *1.788                                   | (1.122; 2.848)  |
| Extra Options                  | *0.350                 | (0.141; 0.874)  | 1.276                                    | (0.949; 1.715)  |

Next, we use the Q+LHS_{qadj} model predictions to describe in more detail the two extreme groups found by the model (top 20% and bottom 20% predicted uplifts). This furnishes the insurance company with typical profiles of clients that are persuadables (top 20%), and clients that should not be targeted (bottom 20%). Table 10 shows descriptive statistics of some selected predictors for both groups. A MANOVA comprising only these two groups for the selected variables, followed by ANOVAs involving individual selected variables separately, show that all mean differences were statistically significant \((p\text{-}value < 0.0005)\). Looking at the average profiles of persuadable and do not disturb clients, we can say that a persuadable client has a higher credit score and is slightly older than a client that should not be targeted. A persuadable client is less likely to be single and more likely to hold both company insurance products (i.e., home and auto policies). Also, this type of client holds more auto policies in his/her account, more mortgages, more residences in his/her name and is more likely to have extra coverage options. Thus, it seems that a persuadable client is a customer with many products to insure.

The correlation matrices associated with these two groups are displayed in image format in Figure 12. There are some obvious patterns that distinguish the two groups. For example, credit score is slightly correlated with client age for persuadable clients, but not for do-not-disturb clients. Client age is negatively correlated with marital status for do-not-disturb clients, but only slightly correlated for persuadables. Indeed, there are several differences in the marital status correlations in both groups. Also, the number of mortgages and residences are more correlated for persuadables than do-not-disturb, and the number of mortgages and whether or not a client has extra options are more correlated for do-not-disturb than persuadables.
The differences between these two groups can also be observed through the odds ratios. For any specific variable \( X_j \) which takes average values \( x^{(p)}_j \) (for persuadable clients), and \( x^{(d)}_j \) (for do-not-disturb clients), consider the odds ratio \( \text{OR}_{X_j}^{(p-d)}(t) \) between persuadable and do-not-disturb clients

\[
\frac{\Pr(Y = 1 \mid X_j = x^{(p)}_j, T = t)}{\Pr(Y = 0 \mid X_j = x^{(p)}_j, T = t)} / \frac{\Pr(Y = 1 \mid X_j = x^{(d)}_j, T = t)}{\Pr(Y = 0 \mid X_j = x^{(d)}_j, T = t)} = (\exp(\hat{\beta}_j) \exp(\hat{\gamma}_j t))^{x^{(p)}_j - x^{(d)}_j} = (\text{OR}_{X_j}(t))^{x^{(p)}_j - x^{(d)}_j},
\]

where \( T \) is the treatment indicator. Table 11 shows these odds ratios for the two values of \( T \in \{0, 1\} \). For example, if we only consider extra options, when the insurance company calls a client (i.e., \( T = 1 \)), the odds ratio between a persuadable client (Extra Options = 87%) and a do-not-disturb client (Extra Options = 41%)
is about 1.12 with a 95% confidence interval of [0.98; 1.28]. On the other hand, when the company does not call a customer (i.e., \( T = 0 \)), the odds ratio becomes 0.62 with a 95% confidence interval of [0.41; 0.94]. These results are quite logical in the sense that the odds of renewing the insurance policy are higher for the persuadable clients if the company calls, while the same odds are higher for the do-not-disturb clients if the company does not call.

Table 11: Odds ratios of the persuadable compared to the do not disturb clients (Eq. 13) and 95% confidence intervals estimated by the Qini-based uplift regression model \( (Q+LHS_{\text{adj}}) \) for some of the selected variables. The \textit{Diff} column represents the difference of group means \( x_j^{(p)} - x_j^{(d)} \) from Table 10.

|               | Diff | Control | CI (95%)          | Called | CI (95%)          |
|---------------|------|---------|-------------------|--------|-------------------|
| Overall       | -    | 0.52    | (0.26; 1.03)      | 1.81   | (1.43; 2.29)      |
| Credit Score  | 35   | 0.95    | (0.82; 1.09)      | 1.02   | (0.98; 1.07)      |
| Age (Years)   | 4.7  | 0.98    | (0.86; 1.11)      | 0.99   | (0.95; 1.03)      |
| Gender        |      |         |                   |        |                   |
| Male          | -11% | 0.97    | (0.92; 1.03)      | 1.00   | (0.99; 1.02)      |
| Single        | -30% | 0.73    | (0.54; 1.01)      | 1.11   | (0.98; 1.27)      |
| Products      |      |         |                   |        |                   |
| Auto and Home | 17%  | 1.09    | (0.91; 1.29)      | 1.06   | (1.00; 1.12)      |
| Auto Policies Count | 0.28 | 1.23    | (0.89; 1.71)      | 1.21   | (1.09; 1.35)      |
| Mortgage Count | 0.12 | 1.04    | (0.97; 1.11)      | 1.04   | (1.02; 1.06)      |
| Residences Count | 0.13 | 0.92    | (0.80; 1.05)      | 1.08   | (1.02; 1.15)      |
| Extra Options | 46%  | 0.62    | (0.41; 0.94)      | 1.12   | (0.98; 1.28)      |

Overall, we observe that when calling a client, the odds of renewing the insurance policy of persuadable clients are almost twice (1.81) the odds of do-not-disturb clients with a 95% confidence interval of [1.43; 2.29]. Conversely, when the company does not call a client, the odds of renewing the insurance policy of persuadable clients are half (0.52) the odds of do-not-disturb clients with a 95% confidence interval of [0.26; 1.03]. Hence, based on our model, by calling identified persuadable clients and not calling identified do-not-disturb clients in future marketing campaigns should result in increased retention rates for the company.

6 Conclusion

Our goal was to analyze the data of a marketing campaign conducted by an insurance company to retain customers at the end of their contract. A random group of policyholders received an outbound courtesy call made by one of the company’s licensed insurance advisors, with the objective to reinforce the customers confidence in the company, to review their coverage and address any questions they might have about their renewal. In the database at our disposal, an independent group of clients was observed and serves as control. In order to evaluate the causal effect of the courtesy call on the renewal or cancellation of the insurance policy of its clients, an uplift model needed to be applied.

We have developed a methodology for selecting variables in the context of uplift models. This is based on a new statistic specially conceived to evaluate uplift models. The statistic, the adjusted Qini, is based on the Qini coefficient. It takes into account the correlation between the observed uplift and the predicted uplift by a model. Maximizing the adjusted Qini to choose an adequate model for uplift acts as a regularizing factor to select parsimonious models, much as lasso does for regression models.

Since the Qini is a difficult statistic to compute, maximizing the adjusted Qini directly is not an easy task. Instead, we proposed to use lasso-type likelihood regularization to search the space of appropriate uplift models, so as to only consider relevant variables for uplift. Since the usual lasso is not designed for uplift models, we adapted it, by selecting the value \( \lambda \) of the lasso regularization constant that maximizes the adjusted Qini.

At first, this ensures that the selected variables (i.e., those associated with non-zero regression coefficients) are important variables for estimating uplift. Then, in a second step, using only the selected variables, we estimate the parameters that maximize the adjusted Qini by searching a Latin hypercube sampling (LHS)
surface around the lasso estimates. A variant of this procedure consists of estimating the parameters as those that maximize the likelihood associated with the model selected by $\lambda$.

Experimental evaluation showed that for the first stage of the Qini-optimized uplift regression, choosing the regularization constant from the logistic-lasso sequence by maximizing the adjusted Qini dramatically improves the performance of uplift models. This is the Q+lasso model. In addition, using a LHS search on the second stage leads to a direct maximization of the adjusted Qini coefficient, and to a further boost in the performance of the model. The resulting model is the Q+LHS$_{q_{adj}}$ model. The simulations show that the Q+lasso model may be considered as an alternative model when the LHS search may be too computationally expensive to realize. In addition, our empirical studies clearly show that the performance of a Qini-based regression model such as the Q+LHS$_{q_{adj}}$ model, is much better than the performance of the usual lasso penalized logistic regression model.

Both models Q+LHS$_{q_{adj}}$ and Q+lasso yielded similar results on the insurance company data. We selected several final models and compared them to the usual lasso regression approach. The results show that our method clearly surpasses the usual approach in terms of performance. We argue that this is due to the Qini-based methods performing variable selection explicitly build for optimizing uplift. Although, even if overall, the marketing campaign of the insurance company did not appear to be successful, the uplift models with the selection of the right variables identify a group of customers for which the campaign worked very well, and another group of customers for whom the call had a negative impact. For future campaigns, the company can target only those customers for whom the courtesy call will be useful and remove and investigate more the clients for whom the marketing campaign had a negative effect.

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