Modeling Merging Galaxies using MINGA – Improving Restricted N-body by Dynamical Friction

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1 Introduction

Finding the initial parameters of an interacting galactic system is still like looking for a needle in a haystack. One challenge is the large number of parameters describing the orbital and the galactic properties. Some of these parameters might be derived from detailed observations, e.g. from HI data cubes. However, to perform an effective search in a high dimensional parameter space, it is necessary to use fast simulations and sophisticated finding strategies in parameter space. We use the code MINGA (Theis 1999), where an improved restricted N-body code is coupled to a genetic algorithm (GA). Such a strategy has also been proposed by Wahde (1998). For the galaxy NGC 4449 Theis & Kohle (2001) showed, that the HI structure of a weakly interacting system can be reproduced.

1.1 Genetic Algorithm

MINGA uses a genetic algorithm based on pikaita (Charbonneau 1995). This kind of algorithms try to imitate nature regarding the evolution of species. Heredity and mutation of characteristics are used to adopt the simulations to the observations. Each model parameter is coded (normalised) to a gene (here we use 4 digits for a gene). All genes together form a single string, the chromosome, which is fully describing a complete interaction model. The realisation of heredity and the determination of the fitness¹ can be done using quite different techniques. Here we use a crossover operator that cuts two chromosomes at a random position and swaps the remaining ends. Better fitting models are more likely parents of the next generation of models. This process is used to evolve the models from generation to generation. The fitness of the models is usually raising, especially if elitism² is used. However, this evolution process could suffer from inbreeding³ and therefore mutation⁴ is applied. MINGA uses either constant or changing mutation rates (depending on symbols for inbreeding). A more detailed description of the GA is provided in Theis (1999) and Theis & Kohle (2001).

1.2 Restricted N-body

The first who applied the restricted N-body method to interacting galaxies were Pfleiderer & Siedentopf (1961) and Toomre & Toomre (1972) – hereafter TT72. This approach treats the galactic centres self-consistently, while the disk consists of (mass-free) test particles. The main advantage of the restricted N-body method is the reduction of the $O(N^2)$ problem of the original Newtonian equation of motion to about $O(NN_G)$, if $N_G$ denotes the number of galaxies. For point mass galaxies the set of equations is reduced to

$$\dot{r}_i = \frac{F_i}{m_i} = -G \sum_{k=1}^{N_G} \frac{M_k}{|r_i - R_k(t)|^3} \cdot (r_i - R_k(t)).$$

² The best model of a generation is forwarded, if no superior model was found in the next generation.
³ The optimisation process got stuck caused by a too homogeneous set of individual models.
⁴ With a low probability each chromosome entry might be changed.

¹ Fitness is a quantitative measure of the quality of a model.

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r, is the position of the i-th particle and m, its mass. \( R_k(t) \) describes the position of galaxy k at time t and \( M_k \) is its dynamical mass including dark matter. G represents the constant of gravitation.

Different to TT72, MINGA allows for a self-consistent description of (rigid) extended halos (Gerds 2001; Theis 2004). Though this treatment substantially influences the galactic orbits (and also increases the CPU time), the restricted N-body method, i.e. Eq. (1), can still be applied.

Another important process is dynamical friction. It describes the deceleration (due to scattering) of a perturber moving in a background of particles. Self-consistent modeling already accounts for dynamical friction, but it is missing in classical restricted N-body codes and, therefore, these codes were not able to remodel tightly interacting or merging systems.

1.3 Dynamical Friction

A simple formula for dynamical friction was derived by Chandrasekhar (1942) by using following assumptions: A point mass perturber is moving in an homogeneous, infinite background of particles and the mass of a background particle is negligible compared to the perturbers mass.

\[
\frac{dv_M}{dt} = -F(v_M, \sigma)\frac{\rho M}{v_M^3} \ln \Lambda \frac{v_M}{\beta} \quad (2)
\]

The acceleration \( dv_M/dt \) of a massive particle M depends on the background density \( \rho \), the mass of the perturber M and its velocity \( v_M \). The acceleration is pointing opposite to the direction of the velocity, hence causing an effective deceleration of the particle. For more details on the function \( F(v_M, \sigma) \), refer to Binney & Tremaine (1987). The Coulomb logarithm \( \ln \Lambda \) is the relation between the maximum impact parameter \( b_{max} \) and the impact parameter \( b_0 \) that leads to a 90° degree deflection:

\[
\Lambda = \frac{b_{max}^2}{G(M + m)} = \frac{b_{max}}{b_0}. \quad (3)
\]

\( V_0 \) denotes the velocity of the reduced particle (perturber and one background particle with mass \( m \)).

Recently, efforts have been made to improve shortcomings of the approach, Eq. (2). E.g. Hashimoto et al. (2003) and Spinnato et al. (2003) accounted for finite halo systems. Just & Peña-Rubia (2005) focused on the influence of a density gradient. Furthermore Jiang et al. (2008) claim, that a mass dependency should be applied to the Coulomb logarithm for merging time scales of dark matter galaxies in a cluster.

2 Method

We are using a set of self-consistent reference models to determine the appropriate formalism of the dynamical friction. We have chosen isothermal spheres to serve as host halo galaxies. Different satellites – point masses, isothermal spheres and disk galaxies – are merged with them. The reference models have been evolved using the gyrofalcON tree-code (Dehnen 2000). For an independent simulation we also used a direct code on a Grape6B - board (Sugimoto et al. 2000). 65 000 particles per halo have been used for the self-consistent models. The halo was truncated at 150 kpc resulting in a mass of \( 5.4 \cdot 10^{11} M_\odot \) and a velocity dispersion of \( \sigma = 62 \text{ km/s} \). The deviation between radial decay in our models and the reference models is used as a diagnostics. Detailed results of our studies will be published later. We varied the Coulomb logarithm, the strength and the direction of the dynamical friction force. The force itself is applied in a symmetric way to the equations of motion of the galaxy centres. The varied parameters (\( C_f, \beta \) and \( \ln \Lambda \)) are shown in Eq. (4).

\[
\frac{dv_M}{dt} = -Fc(v_M, \sigma)\frac{\rho M}{v_M^3} (\hat{v}_M \cos (\beta) + \hat{e}_z \sin (\beta)) \ln \Lambda. \quad (4)
\]

\( C_f \) is a simple scaling factor that allows for fitting. As galaxies have density gradients, the force might point not exactly opposite the velocity, therefore we introduce an orthogonal component which is adjustable via \( \beta \). Finally, \( \ln \Lambda \) is derived by different approaches, these also denote our models:

- **Model A** uses a constant Coulomb logarithm.
- **Model B** uses a distance-dependent Coulomb logarithm as described by Hashimoto et al. (2003):

\[
\ln \Lambda = \ln \left( \frac{r_M}{1.4b_0} \right) \quad (5)
\]

\( r_M \) is the distance satellite – halo centre.

- **Model C** uses an interpolation between two constant Coulomb logarithms (not presented here).
- **Model D** uses a mass- and distance-dependent Coulomb logarithm, similar to a description by Jiang et al. (2008):

\[
\ln \Lambda = \ln \left( 1 + \frac{M_{\text{halo}}(r_M)}{M} \right) \quad (6)
\]

\( M \) is the mass of the satellite and \( M_{\text{halo}}(r_M) \) is the mass of the host halo enclosed within the actual satellite’s radius \( r_M \).

3 Results

3.1 Isothermal satellite

We mainly tested the merging of isothermal satellites into isothermal halos. In total more than 250 000 restricted N-body models were compared to 20 self-consistent reference models (with different mass ratios). Here we present two examples for initially circular orbits, but different mass ratios, i.e. \( q = 1/30 \) and \( q = 1/3 \). The first example describes a low mass satellite. In that case all model approaches described in Sec. 2 are able to reproduce the radial decay of the satellite. In Fig. 1 we present the best fits for models with constant Coulomb logarithm (model A) and for a distance-dependent
was set up with *mkkd95* (Kuijken & Dubinski 1995) and integrated with *gyrfalcON*. The details of the model parameters can be found in Table 1. The determination of the orbital decay was done in the same manner than for the isothermal satellites. As we could already use our results from the isothermal satellites, we only needed to carry out a few tenth of simulations. We also compared the location of the disk particles, i.e. the observables. Fig. 3 shows the good match of the radial decay and the comparable formation of the trailing tidal arm. Minor mismatches like the distribution of particles in the trailing arm might be explained by the different initial setup of the disk. However, the leading arm of the disk galaxy could not be reproduced well. This shows, that we need to be careful about predictions for the innermost regions of the merger, derived from our models.

### 3.2 Disk satellite

We also carried out simulations using a disk-like satellite merging with an isothermal halo. A self-consistent model one (model B). The latter is superior because it is able to reproduce the complete merging process with a deviation\(^5\) of \(\delta_d(230) = 1.8 \cdot 10^{-2}\). This example already shows the limitations of a constant Coulomb logarithm, the innermost part of the merging sequence occurs to quickly.

The second example was done with a larger satellite mass (one third of the halo mass). In that case a constant Coulomb logarithm is not able to reproduce the merging process (Fig. 2). It either leads to a large underestimation of the merging time or to a different behaviour of the radial decay (as shown there), resulting in a deviation of \(\delta_d(185) = 6.8 \cdot 10^{-2}\). Nevertheless, we were able to improve the models by using a mass-dependent Coulomb logarithm (model D). With this approach we could remodel the radial decay for one revolution with a deviation of \(\delta_d(t_{\text{revolution}}) \leq 1.5 \cdot 10^{-2}\) and \(\delta_d(t_{\text{merge}}) = 2.8 \cdot 10^{-2}\) for the complete merging.

![Fig. 1](image1.png) Radial decay for an isothermal satellite within an isothermal halo for a mass ratio of \(q = 1/3\): Self-consistent reference model generated with *gyrfalcON* (green solid line); best model A (blue dashed line) and best model B (red dash-dotted line).

![Fig. 2](image2.png) Same as Fig. 1, but for a mass ratio of \(q = 1/3\): Self-consistent model (green solid line); best model A (blue dashed line) and best model D (red dash-dotted line).

### 3.3 Genetic Algorithm run

The last result we want to present is a complete GA run with the implementation of dynamical friction to improve the restricted N-body code of MINGA. The GA was provided with a reference model (representing a real observation), a merger of two disc galaxies with a mass ratio \(q = 1/3\). Eight free parameters were selected – see Table 2. We have used our mass-dependent Coulomb logarithm of Eq. (6), where the strength of the dynamical friction \(C_f\) was one of the free parameters. The results are shown in Fig. 4. Most of the parameters were recovered with errors of less than 10% cf. also Table 2.

### 4 Conclusions

We have improved the restricted N-body code by introducing dynamical friction. We varied the determination of the Coulomb logarithm as well as the strength and direction of the friction force. We compared our models to self-consistent simulations in order to find the best parameterisation. We have shown, that radial decays of mergers up to a mass ratio of \(q = 1/30\) can be reliably reproduced by using a constant or distance-dependent Coulomb logarithm. With the introduction of more sophisticated descriptions like a mass-

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5 \(\delta_d(t)\) is derived by integrating the quadratic difference between the compared radial decay curves over time \(t\).
Fig. 3  Model of a disk galaxy merging into an isothermal halo. Comparison between a self-consistent reference model (left) and an improved restricted N-body model (right). For the self-consistent model only disk particles are shown (16 000). The restricted model used 16 129 test particles. The initial distribution was set to meet optically the initial reference model. Merging was completed at time $t = 60.0$ TU or 1.7 Gyrs. CPU time was 36 min for the self-consistent and 5.6 sec for the restricted model.

Fig. 4  Result of a Genetic Algorithm run with MINGA: Particle distribution of the reference model shown in the upper left panel. The upper right panel shows the best model after generation 1. Lower left: best model found by the GA after 400 generations with 100 individuals each. Lower right: evolution of the fitness (defines how the original particle distribution is met) over 400 generations.

Table 2  Interaction parameters that should be recovered by the GA. Parameter name, value for the reference model and provided limits are listed in columns 1 to 4. The recovered parameters can be found in column 5 and their relative errors in column 6.

| name       | input | limits | recovered | rel. error |
|------------|-------|--------|-----------|------------|
| $M_{\text{halo}}$ | 1.80  | 0.54   | 2.70      | 1.70       | 5.7 $10^{-3}$ |
| $r_{\text{disk},1}$ | 20.00 | 10.00  | 30.00     | 19.04      | 4.8 $10^{-2}$ |
| $r_{\text{disk},2}$ | 3.00  | 1.00   | 10.00     | 3.23       | 7.5 $10^{-2}$ |
| $r_{\text{halo},2}$ | 3.00  | 1.00   | 10.00     | 3.02       | 7.7 $10^{-3}$ |
| $\Delta z$ | 0.00  | -2.00  | 2.00      | -0.83      | $-$ |
| $\Delta v_x$ | -0.1845 | -1.00 | 0.00      | -0.1500    | 1.9 $10^{-1}$ |
| $\Delta v_y$ | 0.0537 | 0.00   | 1.00      | 0.0650     | 2.1 $10^{-1}$ |
| $C_f$   | 0.50  | 0.10   | 1.00      | 0.46       | 8.6 $10^{-2}$ |

and distance-dependent Coulomb logarithm, we were able to remodel radial decays for mergers up to a mass ratio of $q = 1/3$. For these models it was also essential to use an orientation correction of the friction force. These improvements now account for a finite system with a density gradient. However, for equal mass mergers, we were not able to reproduce the orbital decay. Other neglected effects like mass loss might be the reason for failing remodeling. A few recent tests including mass loss already show promising results, so we might be able to improve the restricted N-body code, again.

Acknowledgements. It is a pleasure to thank Peter Teuben for supplying the NEMO N-body package and Walter Dehnen for providing the gyrfalcON tree-code. This work was supported by the German Science Foundation (DFG) under the grant TH 511/9-1, which is part of the DFG priority program 1177.

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