The non-anticommutative supersymmetric $U_1$ gauge theory

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Abstract

We discuss the non-anticommutative ($\mathcal{N} = \frac{1}{2}$) supersymmetric $U_1$ gauge theory in four dimensions, including a superpotential. We perform the one-loop renormalisation of the model, including the complete set of terms necessary for renormalisability, showing in detail how the eliminated and uneliminated forms of the theory lead to equivalent results.
1 Introduction

Deformed quantum field theories have been subject to renewed attention in recent years due to their natural appearance in string theory. Initial investigations focussed on theories on non-commutative spacetime in which the commutator of the spacetime coordinates becomes non-zero. More recently [1–9], non-anticommutative supersymmetric theories have been constructed by deforming the anticommutators of the Grassmann coordinates $\theta^\alpha$ (while leaving the anticommutators of the $\overline{\theta}^\dot{\alpha}$ unaltered). Consequently, the anticommutators of the supersymmetry generators $\overline{Q}_\dot{\alpha}$ are deformed while those of the $Q_\alpha$ are unchanged. It is straightforward to construct non-anticommutative versions of ordinary supersymmetric theories by taking the superspace action and replacing ordinary products by the Moyal $\ast$-product [10] which implements the non-anticommutativity. Non-anticommutative versions of the Wess-Zumino model and supersymmetric gauge theories have been formulated in four dimensions [10, 11] and their renormalisability discussed [12–16], with explicit computations up to two loops [17] for the Wess-Zumino model and one loop for gauge theories [18–22]. Even more recently, non-anticommutative theories in two dimensions have been constructed [23, 25–28], and their one-loop divergences computed [24, 29]. In Ref. [30] we returned to a closer examination of the non-anticommutative Wess-Zumino model (with a superpotential) in four dimensions, and showed that to correctly obtain results for the theory where the auxiliary fields have been eliminated, from the corresponding results for the uneliminated theory, it is necessary to include in the classical action separate couplings for all the terms which may be generated by the renormalisation process.

It seems natural to extend the above calculations to the gauged case, for which we seek the simplest possible gauged extension of the Wess-Zumino model with a (trilinear) superpotential. General gauged non-commutative theories were considered earlier [18–22], and in particular gauged interacting theories in Ref. [22]; however there we only considered a trilinear superpotential in the adjoint $SU_N$ case, and a mass term in the fundamental $U_N$ case. The simplest model with a trilinear superpotential is the three-field Wess-Zumino model with a $U_1$ gauge invariance, and it is this model we shall consider here. We shall consider the one-loop renormalisation of this model in its entirety; the divergent contributions in the absence of a superpotential can be extracted from Refs. [18], [19], while even some of the contributions with a superpotential may be extracted from Ref. [22] by judicious adaptation of the results there presented for the case of the fundamental $U_N$ case with mass terms; while a number of the divergent contributions will require a fresh diagrammatic computation. We start by considering the uneliminated theory and then proceed to compare with the results from the corresponding theory with the auxiliary fields eliminated.

2 Action

In this section we shall give the action for an $\mathcal{N} = \frac{1}{2}$ supersymmetric $U_1$ gauge theory coupled to chiral matter with a superpotential [10] [11] [22]. This is obtained by the re-
duction to components of the deformed, i.e. non-anticommutative, action in superspace. A $U_1$ gauge-invariant superpotential requires at least three chiral fields; we shall take exactly three, with scalar, fermion, auxiliary components denoted $\phi_i$, $\psi_i$, $F_i$, $i = 1, 2, 3$. The corresponding $U_1$ charges are denoted $q_i$, $i = 1, 2, 3$. For simplicity we shall consider a massless superpotential. For convenience we split the action into kinetic and potential terms, namely

$$S_0 = S_{\text{kin}} + S_{\text{pot}}$$  \hspace{1cm} (1)

where

$$S_{\text{kin}} = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} - i \overline{\lambda} \sigma^\mu (D_\mu \lambda) + \frac{1}{2} D^2 + igC^{\mu\nu} F_{\mu\nu} \overline{\lambda} \lambda + \overline{\psi}_i \sigma^\mu (D_\mu \psi)_i - (D_\nu \overline{\psi}_i) (D_\mu \phi)_i \right.$$

$$+ \sqrt{2} g C^{\mu\nu} (D_\mu \overline{\phi}_i) \overline{\lambda} \sigma_\nu \psi_i + igC^{\mu\nu} \overline{\phi}_i F_{\mu\nu} F_i + \frac{1}{4} |C|^2 g^2 F_i \overline{\phi}_i \overline{\lambda} \lambda + \sum_i \left\{ g q_i \overline{\phi}_i D \phi_i + i \sqrt{2} g q_i (\overline{\phi}_i \lambda \psi_i - \overline{\psi}_i \lambda \phi_i) \right.$$

$$- \gamma_i C^{\mu\nu} g \left[ \sqrt{2} (D_\mu \overline{\phi}_i) \overline{\lambda} \sigma_\nu \psi_i + \sqrt{2} \overline{\phi}_i \overline{\lambda} \sigma_\nu (D_\mu \psi)_i + i \overline{\phi}_i F_{\mu\nu} F_i \right] \right) \right],$$  \hspace{1cm} (2)

and

$$S_{\text{pot}} = - \int d^4x \left\{ (F_i G_i - y \phi_1 \psi_2 \psi_3 - y \phi_2 \psi_3 \psi_1 - y \phi_3 \psi_1 \psi_2) + \text{h.c.} \right.$$

$$+ 2ig \overline{\psi}_1 \sigma^{\mu\nu} F_{\mu\nu} \overline{\phi}_1 \overline{\phi}_2 \overline{\phi}_3 - \frac{1}{4} y |C|^2 F_1 F_2 F_3 \right\},$$  \hspace{1cm} (3)

where

$$G_1 = y \phi_2 \phi_3,$$  \hspace{1cm} (4)

and similarly for $G_2$, $G_3$ (corresponding to a superpotential $W(\Phi) = y \Phi_1 \Phi_2 \Phi_3$). The covariant derivative is defined by

$$(D_\mu \phi)_i = (\partial_\mu + ig q_i A_\mu) \phi_i.$$  \hspace{1cm} (5)

In Eq. (2), $C^{\mu\nu}$ is related to the non-anticommutativity parameter $C^{\alpha\beta}$ by

$$C^{\mu\nu} = C^{\alpha\beta} \epsilon_{\beta\gamma} \sigma^{\mu\nu}_{\alpha},$$  \hspace{1cm} (6)

where

$$\sigma^{\mu\nu} = \frac{1}{4} (\sigma^{\mu\nu} \sigma^\mu - \sigma^\nu \sigma^\mu),$$

$$\overline{\sigma}^{\mu\nu} = \frac{1}{4} (\overline{\sigma}^{\mu\nu} \sigma^\mu - \overline{\sigma}^\nu \sigma^\mu),$$  \hspace{1cm} (7)

and

$$|C|^2 = C^{\mu\nu} C_{\mu\nu}.$$  \hspace{1cm} (8)

Our conventions are in accord with Ref. [10]; in particular,

$$\sigma^{\mu\nu} \overline{\sigma}^{\nu\mu} = -\eta^{\mu\nu} + 2 \sigma^{\mu\nu}.$$  \hspace{1cm} (9)
The definition of $|C|^2$ is similarly well-established although $C^2$ might be a preferable notation for this quantity.

For gauge invariance of $S_{\text{pot}}$ we require

$$q_1 + q_2 + q_3 = 0,$$  \hspace{1cm} (10)

while anomaly cancellation leads to

$$q_1 q_2 q_3 = 0$$  \hspace{1cm} (11)

so that the allowed set of charges is in fact $(q, -q, 0)$. This means that in fact the most general trilinear superpotential is in fact $W = y\Phi_1\Phi_2\Phi_3 + y'\Phi_3^2$ (assuming $\Phi_3$ to be the neutral field). We choose, however, to retain $W = y\Phi_1\Phi_2\Phi_3$ and to present formulae in a manner explicitly symmetric under $q_i$ permutations; for example for later convenience we denote

$$Q = q_1^2 + q_2^2 + q_3^2.$$  \hspace{1cm} (12)

Note also that it follows from Eqs. (10), (11) that superpotential mass terms are allowed in general; however as remarked earlier we will restrict ourselves to the massless case.

It is interesting to note that the constraints Eqs. (10), (11) mean that if we set $q_1 = -q_2 = q$ and $y = \sqrt{2}ggq$ then the undeformed theory has $\mathcal{N} = 2$ supersymmetry.

It is easy to show that $S_0$ is invariant under

\begin{align*}
\delta A_\mu &= -i\bar{\lambda}\sigma_\mu\epsilon, \\
\delta \lambda_\alpha &= i\epsilon_\alpha D + (\sigma^{\mu\nu}\epsilon)_\alpha [F_{\mu\nu} + \frac{1}{2}iC_{\mu\nu}\bar{\lambda}\lambda], \\
\delta D &= -\epsilon\sigma^\mu D_\mu \lambda, \\
\delta \phi_i &= \sqrt{2}\epsilon\psi_i, \quad \delta \bar{\phi}_i = 0, \\
\delta \psi_i^\alpha &= \sqrt{2}\epsilon^\alpha F_i, \quad \delta \bar{\psi}_{i\dot{\alpha}} = -i\sqrt{2}(D_\mu \bar{\phi}_i)(\epsilon\sigma^\mu)_{\dot{\alpha}}, \\
\delta F_i &= 0, \quad \delta \bar{F}_i = -i\sqrt{2}D_\mu \bar{\psi}_{i\dot{\alpha}}\epsilon - 2i g\bar{q}_i\psi_i\epsilon\lambda + 2C^{\mu\nu}gD_\mu(\bar{\psi}_{i\dot{\alpha}}\epsilon\sigma_{\nu}\bar{\lambda}). \hspace{1cm} (13)
\end{align*}

The set of terms multiplied by $\gamma_i$ are separately $\mathcal{N} = \frac{1}{2}$ invariant under the transformations of Eq. (13); they are not in fact produced by the reduction to components of the superspace action, but we have anticipated the need for them later when we renormalise the theory. It will be sufficient to take $\gamma_i$ to consist purely of divergent contributions. The $|C|^2F_1F_2F_3$ and $|C|^2F_i\bar{\phi}_i\bar{\lambda}\lambda$ terms in Eqs. (2), (3) are also each separately $\mathcal{N} = \frac{1}{2}$ invariant, and therefore could be omitted from our action without spoiling the $\mathcal{N} = \frac{1}{2}$ invariance. However, once we do include the $|C|^2F_1F_2F_3$ and $|C|^2F_i\bar{\phi}_i\bar{\lambda}\lambda$ terms, it is necessary for the renormalisation of the model to include all possible terms which may be generated, as was explained in the ungauged case in Ref. [30]. It is easy to list these terms [16] [22]. The action has a “pseudo R-symmetry” under

$$\phi_i \rightarrow e^{-i\alpha}\phi_i, \quad F_i \rightarrow e^{i\alpha}F_i, \quad \lambda \rightarrow e^{-i\alpha}\lambda, \quad C^{\alpha\beta} \rightarrow e^{-2i\alpha}C^{\alpha\beta}, \quad y \rightarrow e^{i\alpha}y,$$  \hspace{1cm} (14)

$F_i$, $\bar{\phi}_i$, $\bar{\lambda}$ and $\bar{\eta}$ transforming with opposite charges to $F_i$, $\phi_i$, $\lambda$ and $y$ respectively, and all other fields being neutral; and also a “pseudo chiral symmetry” under

$$\phi_i \rightarrow e^{i\gamma}\phi_i, \quad y \rightarrow e^{-3i\gamma}y,$$  \hspace{1cm} (15)

3
\( F_i \) and \( \psi_i \) transforming in a similar fashion to \( \phi_i \) and barred quantities transforming with opposite charges; the gauge fields being unaffected. The divergent terms which can arise subject to these invariances, for the massless \( U_1 \) case and suppressing the \( 1,2,3 \) subscripts, consist of (in addition to those already present in the action)

\[
|C|^2 F^2 \phi^2, \quad \overline{y}|C|^2 F \overline{\phi}^4, \quad y^2 |C|^2 \phi^6, \quad \overline{y}|C|^2 \lambda \overline{\lambda} \phi^3.
\] (16)

The combination

\[
\overline{y}^{-1}[F_1 \psi_2 (C \psi_3) + F_2 \psi_3 (C \psi_1) + F_3 \psi_1 (C \psi_2)]
\] (17)

(where \((C \psi)_\alpha = C_{\alpha \beta} \psi^\beta\)) is allowed by the above symmetries and \( \mathcal{N} = \frac{1}{2} \) invariant, but we shall see later that it is not in fact generated as a divergence in the \( U_1 \) theory (at least at one loop) if it is not already present in the classical Lagrangian, and so we choose to omit it. Terms of the generic form \( \overline{\phi}^2 \psi (C \psi) \) are allowed by the above symmetries but it is impossible to construct an \( \mathcal{N} = \frac{1}{2} \) invariant combination which includes them. We have included in (16) the appropriate factors of \( y \) for invariance under the pseudo-chiral symmetry. These factors are not uniquely determined since \( y \overline{y} \) is invariant under this symmetry; the choice we have made is both concise and motivated by later considerations.

We must include all the terms in (16) with their own coefficient in the action and therefore we are led to our complete action

\[
S = S_0 + S_{\text{gen}}
\] (18)

where \( S_0 \) is given in Eq. (1) and

\[
S_{\text{gen}} = \int d^4x \left[ \overline{y}^{-1}|C|^2 \left\{ (k_1 - \frac{\overline{y} y}{4}) F_1 F_2 F_3 + k_2 (F_1 F_2 \overline{G}_3 + F_2 F_3 \overline{G}_1 + F_3 F_1 \overline{G}_2) \\
+ k_3 (F_1 \overline{G}_2 \overline{G}_3 + F_2 \overline{G}_3 \overline{G}_1 + F_3 \overline{G}_1 \overline{G}_2) + k_4 \overline{G}_1 \overline{G}_2 \overline{G}_3 \right\} + |C|^2 \left\{ (K_1 - \frac{1}{4} g^2) \overline{F}_i \overline{\psi}_i + K_2 \overline{y} \overline{\phi}_1 \overline{\phi}_2 \overline{\phi}_3 \right\} \overline{\lambda} \overline{\lambda} \right].
\] (19)

(It is natural to impose the same cyclic symmetry on \( S_{\text{gen}} \) as already present in the superpotential). The \( F_1 F_2 F_3 \) and \( F_i \overline{\phi} \overline{\lambda} \overline{\lambda} \) terms are now effectively assigned an arbitrary coefficient since the fact that they are separately \( \mathcal{N} = \frac{1}{2} \) invariant (as are all the terms in \( S_{\text{gen}} \)) means there is no reason for their renormalisation to be accounted for purely by replacing quantities in \( S_0 \) by the corresponding bare ones; \( \mathcal{N} = \frac{1}{2} \) invariance will not preserve the values of their coefficients derived from the deformed superfield action.

We use the standard gauge-fixing term

\[
S_{\text{gf}} = \frac{1}{2\alpha} \int d^4x (\partial A)^2
\] (20)

with its associated ghost terms. The gauge propagator is given by

\[
\Delta_{\mu \nu} = -\frac{1}{p^2} \left( \eta_{\mu \nu} + (\alpha - 1) \frac{p_{\mu} p_{\nu}}{p^2} \right)
\] (21)

and the fermion propagator is

\[
\Delta_{\alpha \dot{\alpha}} = \frac{p_\mu \sigma_{\mu \alpha \dot{\alpha}}}{p^2},
\] (22)

where the momentum enters at the end of the propagator with the undotted index.
In this section we discuss the renormalisation of the gauged non-anticommutative Wess-Zumino model at one loop.

The divergent contributions from one-loop diagrams to terms in $S_{\text{kin}}$ can mostly be extracted from the results for the $SU_N \times U_1$ case presented in Refs. [18], [19], and so we shall just give the results (suppressing the well-known $C$-independent contributions) without tabulating the contributions from individual diagrams; an exception is the $y\overline{y}$-dependent divergences, since in Ref. [22], where we incorporated a superpotential, we did not consider the resulting new divergent contributions to terms in $S_{\text{kin}}$. The corresponding diagrams are depicted in Figs. 1, 2. The contribution from Fig. 1 is simply

$$-2\sqrt{2}yg\overline{y}C^{\mu\nu}\overline{\psi}_i\lambda\sigma_\nu\partial_\mu\psi_i,$$

where

$$L = \frac{1}{16\pi^2\epsilon}.$$  \hspace{1cm} (24)

The contributions from Fig. 2 are tabulated in Table 1, where

$$W_1 = i\sqrt{2}g\overline{y}gC^{\mu\nu}_A\mu\sum_i q_i\overline{\phi}_i\lambda\sigma_\nu\psi_i.$$  \hspace{1cm} (25)

(In this and all the following tables the factors of $L$ are suppressed.) Taking into account the contributions from Table 1 Eq. (23) and those which can be extracted from Ref. [19], we obtain

$$\Gamma_{\text{pole}}^{\text{kin}} = L \int d^4x \left[ -2ig^3QC^{\mu\nu}F_{\mu\nu}\overline{\phi}_i\lambda\lambda - 2\sqrt{2}ggy\overline{\psi}C^{\mu\nu}\overline{\phi}_i\lambda\sigma_\nu D_\mu\psi_i 
+ \sum_i \left( 2\sqrt{2}g^3q_i^2C^{\mu\nu}D_\mu\overline{\phi}_i\lambda\sigma_\nu\psi_i - 2ig^3C^{\mu\nu}q_i^2\overline{\phi}_iF_{\mu\nu}F_i \right) \right].$$  \hspace{1cm} (26)

The contributions to $S_{\text{pot}}$, however, need to be reassessed due to the different form for the potential, and we therefore show the relevant diagrams in Fig. 3 and list the corresponding contributions in Table 2. In Table 2 $W_2$ and $W_3$ are defined by

$$W_2 = ig^3C^{\mu\nu}F_{\mu\nu}\overline{\phi}_1\overline{\phi}_2\overline{\phi}_3,$$

$$W_3 = ig^3C^{\mu\nu}[q_i^2\partial_\mu\overline{\phi}_1\overline{\phi}_2\overline{\phi}_3 + q_2^2\partial_\mu\overline{\phi}_2\overline{\phi}_3\overline{\phi}_1 + q_3^2\partial_\mu\overline{\phi}_3\overline{\phi}_1\overline{\phi}_2]A_\nu.$$  \hspace{1cm} (27)
The contributions from Table 2 add to

\[ 10iQg^3L \int d^4x \overline{C} \epsilon^{\mu \nu} F_{\mu \nu} \overline{\phi}_1 \overline{\phi}_2 \overline{\phi}_3. \]  

(28)

Note that the contributions from Figs. 3(e)-(h) cancel those from 3(i)-(l); we shall subsequently omit several other pairs of diagrams where a similar cancellation occurs (in fact we have done so already, since a potential divergent \( y \overline{C} \epsilon^{\mu \nu} F_{\mu \nu} \overline{\phi} \) contribution cancels for this reason).

The divergent contributions to the \( F_1 F_2 F_3 \) and \( F_i \overline{\phi}_i \overline{\lambda} \overline{\lambda} \) terms will be given in detail shortly since these terms have now been assigned separate couplings in \( S_{\text{gen}} \) and so the divergences cannot be extracted from earlier work. The remaining divergent contributions are denoted by

\[
\Gamma_{\text{pole}}^{\text{rem}} = - \int d^4x \left[ |C|^2 \left\{ \overline{y}^{-1} X_1 F_1 F_2 F_3 + X_{2n} F_1 F_2 \overline{G}_3 + X_{3b} F_2 F_3 \overline{G}_1 + X_{2c} F_3 F_1 \overline{G}_2 
\begin{align*}
+ X_{3a} F_1 \overline{G}_2 \overline{G}_3 + X_{3b} F_2 \overline{G}_3 \overline{G}_1 + X_{3c} F_3 \overline{G}_1 \overline{G}_2 + X_{4d} \overline{G}_1 \overline{G}_2 \overline{G}_3 \\
+ X'_2(F_1^2 \overline{\phi}_1^2 + F_2^2 \overline{\phi}_2^2 + F_3^2 \overline{\phi}_3^2) + X''_2(q_1 \overline{\phi}_1 F_1 + q_2 \overline{\phi}_2 F_2 + q_3 \overline{\phi}_3 F_3)^2 \\
+ X_5 F_i \overline{\phi}_i + X'_5 \sum q_i^2 F_i \overline{\phi}_i + X_6 \overline{\phi}_1 \overline{\phi}_2 \overline{\phi}_3 \overline{\lambda} \overline{\lambda} \\
+ X_7(q_1^2 \overline{\phi}_1 \psi_1 + q_2^2 \overline{\phi}_2 \psi_2 + q_3^2 \overline{\phi}_3 \psi_3)(q_1 \overline{\phi}_1 C \psi_1 + q_2 \overline{\phi}_2 C \psi_2 + q_3 \overline{\phi}_3 C \psi_3) \right\} \right].
\]  

(29)

(Note the overall minus sign, introduced to avoid a proliferation of negative signs later on.) In Figs. 4-9 are depicted the divergent one-loop diagrams contributing to \( X_1 \), etc. Their divergent contributions are shown diagram by diagram in Tables 3-9 and given in Table 2: Divergent contributions from Fig. 3

|   | Contribution |
|---|-------------|
| a | \( 4W_2 + 8W_3 \) |
| b | \( 4W_3 \) |
| c | \( -2W_2 - 12W_3 \) |
| d | \( 8W_2 \) |
| e | \( 2\alpha W_2 \) |
| f | \( 2W_2 \) |
| g | \( -4W_2 - 8W_3 \) |
| h | \( 8W_3 \) |
| i | \( -2\alpha W_2 \) |
| j | \( -2W_2 \) |
| k | \( 4W_2 + 8W_3 \) |
| l | \( -8W_3 \) |

Table 2: Divergent contributions from Fig. 3
total by

\begin{align*}
X_1^{(1)} &= (6k_2 - 6g^2)y\overline{\psi}L, \\
X_2a &= \{4(k_1 + 2k_2 + 2k_3)y\overline{\psi} + 2(1 + \alpha)k_2q_1q_2g^2\}L, \\
X_2b &= \{4(k_1 + 2k_2 + 2k_3)y\overline{\psi} + 2(1 + \alpha)k_2q_2q_3g^2\}L, \\
X_2c &= \{4(k_1 + 2k_2 + 2k_3)y\overline{\psi} + 2(1 + \alpha)k_2q_3q_4g^2\}L, \\
X_3a &= \{2(3k_2 + 6k_3 + 4k_4)y\overline{\psi} + (1 + \alpha)[2(k_1 + 2k_2)q_2q_3 - Qk_3]g^2\}L, \\
X_3b &= \{2(3k_2 + 6k_3 + 4k_4)y\overline{\psi} + (1 + \alpha)[2(k_1 + 2k_2)q_3q_1 - Qk_3]g^2\}L, \\
X_3c &= \{2(3k_2 + 6k_3 + 4k_4)y\overline{\psi} + (1 + \alpha)[2(k_1 + 2k_2)q_1q_2 - Qk_3]g^2\}L, \\
X_4 &= -(1 + \alpha)(k_2 + 2k_3 + 2k_4)Qg^2L, \\
X_2^{(1)} &= 2(k_1 + 2k_2 + k_3)y\overline{\psi}L, \\
X_2^{(1)*} &= -\frac{1}{4}(1 + \alpha)g^4y\overline{\psi}, \\
X_5 &= \{4K_1 + 2K_2\}y\overline{\psi} - g^2y\overline{\psi}\}L, \\
X_5^{(1)*} &= g^2(8K_1 - 10g^2)L, \\
X_6 &= [2(7 - \alpha)K_1 + (7 - \alpha)K_2 + 14g^2]Qg^2L, \\
X_7^{(1)} &= 16g^4L. 
\end{align*}

The terms involving \(X_2', X_2^{(1)}\) and \(X_3'\) are not contained in the original action; while the term involving \(X_7\) is not \(\tilde{\mathcal{N}} = \frac{1}{2}\) invariant. However, we shall see later that all these terms may be removed (at least at one loop) by field redefinitions. Other diagrams which potentially contribute divergences turn out to be zero or to cancel. Fig. (11) is in fact zero by symmetry. The divergences from the diagrams of Fig. (11) are of the form

\[ \overline{\psi}^{-1}[(q_2 - q_3)F_1\psi_2(C\psi_3) + (q_3 - q_1)F_2\psi_3(C\psi_1) + (q_1 - q_2)F_3\psi_1(C\psi_2)] \]

which (in contrast to the similar combination in (17)) is also not \(\mathcal{N} = \frac{1}{2}\) invariant; moreover there is no field redefinition which could remove these terms and so they must and indeed do cancel.
Table 4: Divergent contributions from Fig. 5

|   | $X_{2a}$ | $X_{2b}$ | $X_{2c}$ | $X_{3a}$ | $X_{3b}$ | $X_{3c}$ | $X_1$ |
|---|---------|---------|---------|---------|---------|---------|------|
| a | $2\alpha k_2 q_2 g^2$ | $2\alpha k_2 q_2 g^2$ | $2\alpha k_2 q_3 g^2$ | $2\alpha k_2 q_1 g^2$ |
| b | $2k_2 q_2 g^2$ | $2k_2 q_2 g^2$ | $2k_2 q_3 g^2$ | $2k_2 q_1 g^2$ |
| c | $-\alpha k_3 Q g^2$ | $-\alpha k_3 Q g^2$ | $-\alpha k_3 Q g^2$ |
| d | $-k_3 Q g^2$ | $-k_3 Q g^2$ | $-k_3 Q g^2$ |
| e | $2k_1 q_2 q_3 g^2$ | $2k_1 q_3 q_1 g^2$ | $2k_1 q_1 q_2 g^2$ |
| f | $2k_1 q_2 q_3 g^2$ | $2k_1 q_3 q_1 g^2$ | $2k_1 q_1 q_2 g^2$ |
| g | $4\alpha k_2 q_2 q_3 g^2$ | $4\alpha k_2 q_3 q_1 g^2$ | $4\alpha k_2 q_1 q_2 g^2$ |
| h | $4k_2 q_2 g^2$ | $4k_2 q_3 g^2$ | $4k_2 q_1 g^2$ |
| i | $-2\alpha k_3 Q g^2$ |
| j | $-2k_3 Q g^2$ |
| k | $-\alpha k_2 g^2 Q$ |
| l | $-k_2 Q g^2$ |
| m | $-2\alpha k_3 Q g^2$ |
| n | $-2k_3 Q g^2$ |

Table 5: Divergent contributions from Fig. 6

|   | $X_1$ | $X''_2$ |
|---|------|-------|
| a | $-6g\bar{g}g^2$ |
| b | $-\frac{1}{4}\alpha g^4$ |
| c | $-\frac{1}{2}g^4$ |
| d | 0 |

Table 6: Divergent contributions from Fig. 7

|   | $X_5$ | $X_6$ | $X'_5$ |
|---|------|------|-------|
| a | $8g^2 K_1$ |
| b | $4K_1 g\bar{g}$ |
| c | $2K_2 g\bar{g}$ |
| d | $-2\alpha Q g^2 K_1$ |
| e | $-2g^2 Q K_1$ |
| f | $16Q g^2 K_1$ |
| g | $8Q g^2 K_2$ |
| h | $-\alpha Q g^2 K_2$ |
| i | $-Q g^2 K_2$ |
\begin{table}
\centering
\begin{tabular}{|c|c|c|}
\hline
& $X_5$ & $X_6$ & $X'_5$ \\
\hline
a & $-g^2y\gamma$ & & \\
b & & $-8g^4$ & \\
c & & $-2g^4$ & \\
d & & 0 & \\
e & & $8g^4Q$ & \\
f & & $\frac{1}{2}\alpha Qg^4$ & \\
g & & $\frac{1}{2}Qg^4$ & \\
h & & $\frac{1}{2}(3 + \alpha)Qg^4$ & \\
i & & $4Qg^4$ & \\
j & & $-\alpha Qg^4$ & \\
k & & 0 & \\
\hline
\end{tabular}
\caption{Divergent contributions from Fig. 8}
\end{table}

\begin{table}
\centering
\begin{tabular}{|c|c|}
\hline
& $X_7$ \\
\hline
a & $-4\alpha g^4$ \\
b & $4(3 + \alpha)g^4$ \\
c & $-4\alpha g^4$ \\
d & $4\alpha g^4$ \\
e & $4g^4$ \\
\hline
\end{tabular}
\caption{Divergent contributions from Fig. 9}
\end{table}
The divergences in Eq. (30) should be cancelled as usual by replacing the parameters \( y, \bar{y}, g, k_{1-4}, K_{1,2} \) and the fields \( \phi_i, \bar{\phi}_i, F_i, \bar{F}_i, \psi_i, \bar{\psi}_i, \lambda, \bar{\lambda} \) by corresponding appropriately-chosen bare quantities \( y_B, \bar{y}_B, k_{1B-4B}, K_{1B,2B}, \phi_{iB}, \bar{\phi}_{iB}, F_{iB}, \bar{F}_{iB}, \psi_{iB}, \bar{\psi}_{iB}, \lambda_B, \bar{\lambda}_B \), with the bare fields given by \( \phi_{iB} = Z_{\phi_i}^{\frac{1}{2}} \phi_i \), etc. However, as emphasised in Ref. [31], renormalisation of a gauged supersymmetric theory in the uneliminated case (i.e. without eliminating the auxiliary fields \( F_i \) and \( D \)) requires in general a non-linear renormalisation of \( F_i \) and \( D \); and in the general \( \mathcal{N} = \frac{1}{2} \) case in Ref. [22] we also required a non-linear renormalisation of the gaugino field. In our present case we find it necessary to take at one loop

\[
F_{1B}^{(1)} = \frac{1}{2} F_1^{(1)} \quad (31)
\]

with similar expressions for \( F_{2B,3B}^{(1)} \), and also

\[
\lambda^{(1)}_B = Z_{\lambda}^{\frac{1}{2}}(1) \lambda + i \sqrt{2} g \sum_i \rho_i^{(1)} \bar{\phi}_i (C \psi_i).
\]

Here, \( Z_F \) and \( Z_\lambda \), together with the renormalisation constants for the other fields have a loop expansion

\[
Z_F = 1 + \sum_{n \geq 1} Z_F^{(n)}
\]
etc, and at one loop we have

\[ 
\begin{align*}
Z_{\lambda}^{(1)} &= -2g^2LQ, \\
Z_{A}^{(1)} &= -2g^2LQ, \\
Z_{g}^{(1)} &= g^2LQ, \\
Z_{F}^{(1)} &= -2Ly_y, \\
Z_{\phi_i}^{(1)} &= 2L \left[-y_y + (1 - \alpha)g^2q_i^2 \right], \quad i = 1, 2, 3, \\
Z_{\psi_i}^{(1)} &= 2L \left[-y_y - (1 + \alpha)g^2q_i^2 \right], \quad i = 1, 2, 3.
\end{align*}
\]

The presence of \( \rho_i \) in the bare action produces terms

\[ 
\sum_i \rho_i g \left[ \sqrt{2}C_{\mu\nu}(D_\mu \phi_i \lambda\sigma_\nu \psi_i + \phi_i \lambda\sigma_\nu D_\mu \psi_i) + 2\phi_i \psi_i (\sum q_j \phi_j \psi_j) \right].
\]

The \( \rho_i \) in Eq. (33) are, like the \( \gamma_i \) in Eq. (2), purely divergent quantities, and at one loop we find we need to take

\[ 
\begin{align*}
\gamma_i^{(1)} &= (8g^2q_i^2 - 2y_y)L, \\
\rho_i^{(1)} &= 8g^2q_i^2L.
\end{align*}
\]

With this value for \( \rho_i \), the \( \mathcal{N} = 1/2 \) non-invariant terms involving \( Z_\tau \) in Eq. (29) are cancelled at one loop. In Eq. (32), \( \bar{R}, \bar{S}, \bar{T} \) represent possible additional renormalisations of \( F_i \) which are not determined by the requirements of renormalisability.

With the above expression for \( F_i^{(1)} \), the renormalisation of the Yukawa couplings is as expected from applying the non-renormalisation theorem in the superfield context, namely

\[ 
\begin{align*}
y_B &= \mu \frac{d}{d\mu} \left[ \sqrt{2}Z_{\phi_1} Z_{\phi_2} Z_{\phi_3} y \right], \\
\bar{y_B} &= \mu \frac{d}{d\mu} \left[ \sqrt{2}Z_{\phi_1} Z_{\phi_2} Z_{\phi_3} \bar{y} \right],
\end{align*}
\]

where \( \mu \) is the usual dimensional regularisation mass parameter, and \( Z_{\phi_i}, i = 1, 2, 3 \) are the renormalisation constants for the chiral superfields as computed in a supersymmetric gauge, namely (at one loop)

\[ 
Z_{\phi_i}^{(1)} = 2L \left[-y_y + 2g^2q_i^2 \right], \quad i = 1, 2, 3.
\]

The \( \beta \)-function for \( y \) is defined by \( \beta_y = \mu \frac{d}{d\mu} y \) with a similar expression for \( \beta_{\bar{y}} \) and then by virtue of Eqs. (38), (39),

\[ 
\beta_y^{(1)} = \frac{1}{16\pi^2} (3y_y - 2g^2Q)y,
\]

with a similar expression for \( \beta_{\bar{y}}^{(1)} \).

Note that if we set \( q_1 = -q_2 = q \) and \( y = \bar{y} = \sqrt{2}gq \) then Eq. (41) reduces to

\[ 
\beta_g^{(1)} = \frac{2q^2 g^3}{16\pi^2},
\]
which is indeed the one-loop gauge $\beta$-function, consistent with our earlier remark that the undeformed theory has $\mathcal{N} = 2$ supersymmetry in this case.

We find from Eqs. (19), (30), (32), (35), (37), (38),

$$
\begin{align*}
&k_{1B}^{(1)} = 6(k_1 + k_2 - g^2)y\bar{y}L - 3R^{(1)}, \\
&k_{2B}^{(1)} = 4(k_1 + 3k_2 + 2k_3)y\bar{y}L + R^{(1)} - S^{(1)}, \\
&k_{3B}^{(1)} = 2(k_1 + 5k_2 + 8k_3 + 4k_4)y\bar{y}L + S^{(1)} - T^{(1)}, \\
&k_{4B}^{(1)} = 3T^{(1)}, \\
&K_{1B}^{(1)} = ([6K_1 + 2K_2]y\bar{y} + 2Qg^2K_1 - g^2y\bar{y})L, \\
&K_{2B}^{(1)} = 2(12K_1 + 5K_2 + 2g^2)Qg^2 L.
\end{align*}
$$

(42)

To a large extent the renormalisation of $\mathbf{F}_{1,2,3}$ as given in Eq. (32) is determined by the requirement that the couplings $k_{1-4}$, $K_{1,2}$ are multiplicatively renormalised as described above. However we still have the freedom to choose $R^{(1)}$, $S^{(1)}$, $T^{(1)}$, which are the same for each $\mathbf{F}_{1,2,3}$. Choosing $R^{(1)} = S^{(1)} = T^{(1)} = 0$ in Eq. (32) leaves almost the minimal renormalisation of $\mathbf{F}_i$ possible to ensure multiplicative renormalisation; however we have included the terms with a factor $Q$ in Eq. (32) in order to remove $g^2k_i$-dependent terms in $k_{1-4B}$ (something which is only possible thanks to the particular form of the divergences, as will become clearer later when we discuss the eliminated theory).

Writing $\beta_{k_i} = \mu \frac{d}{d\mu} k_i$ (and similarly for $K_{1,2}$) and as usual requiring that $k_{iB}$ and $K_{1,2B}$ be independent of $\mu$ we then find that

$$
\begin{align*}
\beta_{k_1}^{(1)} &= \frac{1}{16\pi^2} [6(k_1 + k_2 - g^2)y\bar{y} - 3r], \\
\beta_{k_2}^{(1)} &= \frac{1}{16\pi^2} [4(k_1 + 3k_2 + 2k_3)y\bar{y} + r - s], \\
\beta_{k_3}^{(1)} &= \frac{1}{16\pi^2} [2(k_1 + 5k_2 + 8k_3 + 4k_4)y\bar{y} + s - t], \\
\beta_{k_4}^{(1)} &= \frac{3t}{16\pi^2}, \\
\beta_{K_1}^{(1)} &= \frac{1}{16\pi^2} ([6K_1 + 2K_2]y\bar{y} + 2Qg^2K_1 - g^2y\bar{y}), \\
\beta_{K_2}^{(1)} &= \frac{1}{16\pi^2} 2(12K_1 + 5K_2 + 2g^2)Qg^2,
\end{align*}
$$

(43)

writing $R^{(1)} = rL$, etc. We note that these $\beta$-functions are different in form from those derived in the ungauged case in Ref. [30]; of course our three-field superpotential is also somewhat different from that used in the ungauged case, and we have also had to include non-linear terms in $\mathbf{F}_{1B}$ (the $F_1^2\phi_1^2$ terms), which removed the $X_2'$ terms which would have spoiled renormalisability, but also contributed to $k_{3B}$. It seems impossible to use the freedom to choose $R^{(1)}$, $S^{(1)}$, $T^{(1)}$, in Eq. (32) to make the two sets of $\beta$-functions agree.
We now turn to the calculation in the eliminated theory. If we eliminate $F_i$ and $F_i$ from the action we find

$$ F_i = G_i, $$

$$ F_1 = G_1 - \mathcal{F}^{-1}|C|^2 \left[ k_1 F_2 F_3 + k_2 (F_2 G_3 + F_3 G_2) + k_3 G_2 G_3 \right] $$

$$ -igC^{\mu\nu} F_{\mu\nu} \phi_1 - \frac{1}{2}g^2 |C|^2 K_1 \phi_1 \bar{\lambda} \lambda, $$

(44)

(with corresponding expressions for $F_2$, $F_3$) and the action becomes

$$ S = \int d^4x \left[ -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - igC^{\mu\nu} \bar{\lambda} \lambda(D_\mu \lambda) + \frac{1}{2} D^2 \right] $$

$$ -igC^{\mu\nu} F_{\mu\nu} \bar{\lambda} \lambda - igC^{\mu\nu} \bar{\lambda} \lambda(D_\mu \psi)_i - i(D^\mu \bar{\phi})_i(D_\mu \phi)_i $$

$$ + g \sum \left\{ q_i \bar{\phi}_i D \phi_i + i\sqrt{2}gq_i(\bar{\phi}_i \lambda \psi_i - \bar{\psi}_i \lambda \phi_i) \right\} $$

$$ -\gamma_i C^{\mu\nu} g \left( \sqrt{2} D_\mu \bar{\phi}_i \lambda \sigma_\nu \psi_i + \sqrt{2} \bar{\phi}_i \lambda \sigma_\nu D_\mu \psi_i \right) \right] + \sqrt{2}gC^{\mu\nu} D_\mu \bar{\phi}_i \lambda \sigma_\nu \psi_i $$

$$ -G_1 G_i + y(\phi_1 \psi_2 \psi_3 + \phi_2 \psi_3 \psi_1 + \phi_3 \psi_1 \psi_2) + \bar{g}(\bar{\phi}_1 \bar{\psi}_2 \bar{\psi}_3 + \bar{\phi}_2 \bar{\psi}_3 \bar{\psi}_1 + \bar{\phi}_3 \bar{\psi}_1 \bar{\psi}_2) $$

$$ + ig\bar{g}(1 - \gamma_1 - \gamma_2 - \gamma_3)C^{\mu\nu} F_{\mu\nu} \phi_1 \phi_2 \phi_3 + \lambda_1 \bar{g}^{-1}|C|^2 \bar{G}^3 + \lambda_2 \bar{g}|C|^2 \phi_1 \phi_2 \phi_3 \lambda \bar{\lambda}] $$

(45)

where

$$ \lambda_1 = k_1 + 3(k_2 + k_3) + k_4, $$

$$ \lambda_2 = 3K_1 + K_2. $$

(46)

The renormalisation of the last three terms in Eq. (45) now needs to be reconsidered. First let us consider the $C^{\mu\nu} F_{\mu\nu} \phi_1 \phi_2 \phi_3$ term. Its coefficient has changed, and in particular we see, comparing Eqs. (3), (45), that its finite part has changed by a factor of $-\frac{1}{2}$. Moreover the diagrams Figs. 3(e)-(h) which cancelled the contributions from Figs. 3(i)-(l) are no longer present, while these latter contributions are multiplied by $-\frac{1}{2}$. Moreover, since the eliminated theory in Eq. (45) also contains a $G_i G_i$ vertex which was not present in the uneliminated case, there is a new diagram depicted in Fig. 12, giving a divergent contribution

$$ -6ig\bar{g}^2 C^{\mu\nu} \int d^4x F_{\mu\nu} \phi_1 \phi_2 \phi_3. $$

(47)

However, taking all these effects into account, it is straightforward to check that the divergences are still cancelled.

The remaining two terms need to be examined in more detail. We write the divergent contributions to these terms as

$$ \Gamma_{C,\text{dim}}^{\text{pole}} = -|C|^2 \int d^4x |Y_1 \bar{g}^{-1} G_1 G_2 G_3 + Y_2 \bar{g} \phi_1 \phi_2 \phi_3 \lambda \bar{\lambda}|, $$

(48)

(introducing an overall minus sign as in Eq. (29)). Most of the relevant contributions to $Y_1$ can be read off from those to $X_4$ in Table 3 with a $k_4$ (here replaced by $\lambda_1$). Similarly, most of the relevant contributions to $Y_2$ can be read off from those to $X_6$ in Table 6 with
Table 9: Divergent contributions from Fig. 13

|   | \( Y_1 \)   | \( Y_2 \) |
|---|-------------|-------------|
| a | \( 24y\eta\lambda_1 \) |             |
| b | \( -6g^2y\eta \)      |             |
| c | 0            |             |
| d | 0            |             |
| e | 6y\eta\lambda_2       |             |
| f | \( -8Qg^4 \)          |             |
| g | \( -2Qg^4 \)          |             |
| h | 0            |             |
| i | \( -3g^2y\eta \)      |             |

a \( K_2 \) (here replaced by \( \lambda_2 \)), and those to \( X_6 \) in Table 8. However, in the eliminated case there are also diagrams with a \( g\eta C^{\mu\nu} F_{\mu\nu}\phi_1 \bar{\phi}_2 \bar{\phi}_3 \) vertex. Such diagrams were previously cancelled by diagrams with an internal \( F \) propagator in a similar fashion to Figs. 3(e)-(h) and 3(i)-(l); but of course such diagrams are no longer present in the eliminated case. Again, there are further diagrams incorporating the \( \bar{\phi}_i G_i \) vertex which was not present in the uneliminated case. The result is that we now need to incorporate contributions from the diagrams shown in Fig. 13. The contributions are listed in Table 9 (note that the contributions from Figs. 13(j), (k) cancel).

We find from the eliminated diagrams that

\[
Y_1^{(1)} = 2[12y\eta\lambda_1 - (1 + \alpha)g^2Q\lambda_1 - 3g^2y\eta]L,
\]
\[
Y_2^{(1)} = [6y\eta\lambda_2 + (7 - \alpha)Qg^2\lambda_2 + 4Qg^4 - 3g^2y\eta]L,
\]

and so

\[
\beta_{\lambda_1}^{(1)} = \frac{1}{16\pi^2}(24\lambda_1y\eta - 6g^2y\eta),
\]
\[
\beta_{\lambda_2}^{(1)} = \frac{1}{16\pi^2}(6y\eta\lambda_2 + 10Qg^2\lambda_2 + 4Qg^4 - 3g^2y\eta).
\]

An important consistency check is that

\[
\lambda_{1B} = k_{1B} + k_{4B} + 3(k_{2B} + k_{3B}),
\]
\[
\lambda_{2B} = 3K_{1B} + K_{2B},
\]

and it is easy to confirm that this is satisfied at one loop using Eqs. (42) and (49). The fact that we were able to remove \( g^2k_i \) terms from \( k_{iB}^{(1)} \) in the uneliminated case is now seen as a consequence of the fact that \( \lambda_{1B}^{(1)} \) contains no \( g^2\lambda_1 \) terms.

The original deformed Wess-Zumino action of Eq. (1) corresponded to the values \( k_1 = y, K_1 = \frac{1}{4}g^2, k_{2-4} = K_2 = 0 \). However, our more general Lagrangian in Eq. (19)
is invariant under $\mathcal{N} = \frac{1}{2}$ transformations whatever the values of $k_{1-4}, K_{1,2}$; and we see from Eq. (43) that the choice $k_1 = y$, $K_1 = \frac{1}{4}g^2$, $k_{2-4} = K_2 = 0$ is not maintained by renormalisation; if we set $k_1 = y$, $K_1 = \frac{1}{4}g^2$, $k_{2-4} = K_2 = 0$ at one scale then different values are inevitably generated at other scales. In Ref. [30] we asked (for the ungauged case) if there is any set of values of $k_{1-4}$ (or at least any form for the deformed action) which is preserved by renormalisation and which would be in some sense natural. Requiring that

$$k_i = a_i(y\overline{y})^\rho, \quad i = 1 \ldots 4,$$

where $a_i, i = 1 \ldots 4$ are numbers (i.e. not functions of $y$, $\overline{y}$, or $g$, and hence scale independent), entails

$$\frac{\beta_1^{(1)}}{k_1} = \frac{\beta_2^{(1)}}{k_2} = \frac{\beta_3^{(1)}}{k_3} = \frac{\beta_4^{(1)}}{k_4} = \rho \left( \frac{\beta_y^{(1)}}{y} + \frac{\beta_{\overline{y}}^{(1)}}{\overline{y}} \right). \quad (52)$$

If we ask the same question here we shall find that the values of $k_{1-4}$ and $\rho_i$ must satisfy the sole condition

$$[(24 - 6\rho)y\overline{y} + 4\rho Qg^2] \lambda_1 = 6g^2y\overline{y} \quad (53)$$

which is the same condition we would find in the eliminated case using Eq. (50). In the ungauged case we once again find that the particular solutions

$$k_1 = -k_2 = k_3 = -k_4, \quad \rho = 0, \quad (54)$$

and also

$$k_1 = -\frac{3}{2}k_2 = 3k_3, \quad k_4 = 0, \quad \rho = \frac{1}{3}, \quad (55)$$

require no non-linear renormalisation of $F_i$.

It is tempting to feel that there is something particularly significant about the choices in Eqs. (54), (55) since they provided solutions in Ref. [30] at one and two loops without the need for any further renormalisation of $F_i$; and in fact they also solve our current model with the $\beta$-functions in Eq. (43), with $r = s = t = 0$, i.e. derived using the minimal renormalisation of the $F_i$ consistent with renormalisability.

### 4 Conclusions

We have performed a complete one-loop analysis of the renormalisation of the simplest gauged $U_1$ non-anticommutative Wess-Zumino model with a superpotential. We started with the action derived from the non-anticommutative superspace theory, but then found it necessary (working with the uneliminated form of the action, without eliminating auxiliary fields) also to include all possible terms which can be generated by renormalisation with their own couplings. We showed that this leads to results compatible with those obtained in the eliminated theory. Our main results are those in Eq. (43) (in the uneliminated case) and Eq. (50) (in the eliminated case). This is the first complete one-loop calculation for a general non-anticommutative supersymmetric gauge theory with a superpotential; as mentioned earlier, in Ref. [22] we omitted $y\overline{y}$ contributions to the
renormalisation of terms in $S_{\text{kin}}$. The renormalisation of the theory is much simpler than in the $SU_N \times U_1$ cases considered in Refs. [18, 19, 22], though once again we required a non-linear renormalisation of the gaugino $\lambda$, as parametrised by $\rho_i$ in Eq. (33), accompanied by a renormalisation parametrised by $\gamma_i$ in Eq. (2) (with $\rho_i$, $\gamma_i$ as given in Eq. (37)). These renormalisations were determined by consideration of the theory with a superpotential; however, the renormalisations contain $y$-independent pieces which yet would not have been required in the theory without a superpotential. It is somewhat reassuring that the $y$-independent part of the renormalisations for the $\rho_i$ and $\gamma_i$ is exactly as would be obtained from the $U_1$ part of the $SU_N \times U_1$ theory of Ref. [22], despite the fact that here we have considered a trilinear, three-field superpotential and there we considered a mass term (with two fields).

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Figure 1: One-loop diagram with a $C$ vertex and one gaugino, one $\psi$ and one $\bar{\phi}$ external legs (a blob representing the $C$ vertex and dashed, full, full/wavy lines representing scalar, fermion and gaugino fields respectively)

Figure 2: One-loop diagrams with a $C$ vertex and one gauge, one gaugino, one $\psi$ and one $\bar{\phi}$ external legs (wavy lines representing gauge fields)
Figure 3: One-loop diagrams with a $C$ vertex and three $\phi$ and one gauge-field external legs (double, zigzag lines representing chiral and gauge auxiliary fields respectively)
Figure 4: One-loop diagrams with a $|C|^2$ vertex, $F$ or $\bar{\varphi}$ external legs and purely $F$ or $\bar{\varphi}$ internal propagators
Figure 5: One-loop diagrams with a $|C|^2$ vertex, $F$ or $\bar{\phi}$ external legs and an internal gauge or $D$ propagator
Figure 6: One-loop diagrams with two $C^{\mu\nu}$ vertices, $F$ or $\phi$ external legs and an internal gauge or $D$ propagator.
Figure 7: One-loop diagrams with a $|C|^2$ vertex, and two gaugino and $F$ or $\overline{\phi}$ external legs.
Figure 8: One-loop diagrams with two $C^{\mu\nu}$ vertices, and two gaugino and $F$ or $\bar{\phi}$ external legs.
Figure 9: One-loop diagrams with two $\bar{\phi}$ and two $\psi$ external legs (and no Yukawa vertices)

Figure 10: One-loop diagram with two $\bar{\phi}$ and two $\psi$ external legs (and two Yukawa vertices)

Figure 11: One-loop diagrams with one $F$ and two $\psi$ external legs

Figure 12: Additional one-loop diagram for the eliminated case
Figure 13: Further one-loop diagrams for the eliminated case