Supersymmetric D-term Inflation, Reheating and Affleck-Dine Baryogenesis

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Abstract

The phenomenology of supersymmetric models of inflation, where the inflationary vacuum energy is dominated by $D$-terms of a U(1), is investigated. Particular attention is paid to the questions of how to arrange for sufficient e-folds of inflation to occur, what kind of thermal history is expected after the end of inflation, and how to implement successful baryogenesis. Such models are argued to require a more restrictive symmetry structure than previously thought. In particular, it is non-trivial that the decays of the fields driving $D$-inflation can reheat the universe in such a way as to avoid the strong gravitino production constraints. We also show how the initial conditions for Affleck-Dine baryogenesis can arise in these models and that the simplest flat directions along which baryon number is generated can often be ruled out by the constraints coming from decoherence of the condensate in a hot environment. At the end, we find that successful reheating and baryogenesis can take place in a large subset of $D$-inflationary models.
1 Introduction

Successful slow-roll inflation requires that the inflaton potential be sufficiently flat [1]. Since theories with unbroken supersymmetry naturally possess moduli spaces of exactly flat directions (the flatness being preserved by quantum corrections), and because supersymmetric theories are the theoretically most motivated extension of the Standard Model (SM), supersymmetric theories of inflation have been much studied. In supersymmetric theories there are two possible sources of a non-zero potential energy: $F$-term contributions arising from terms in the superpotential, and $D$-terms that arise from the supersymmetrization of the gauge kinetic energy; either can in principle lead to the false vacuum density necessary for inflation.

A significant problem for $F$-term inflation arises from the simple fact that the non-zero energy density during the inflationary period spontaneously breaks supersymmetry; moreover this breaking feeds back into the inflaton potential, ruining the required flatness [2, 3, 4]. Specifically, suppose the field whose non-zero $F$-term breaks supersymmetry is $\psi$, with $\langle \psi \rangle = \theta^2 F_\psi$, while the inflaton field itself is $\phi$. No symmetry can prevent the appearance of the term

$$\Delta \mathcal{L} = \frac{1}{M_*^2} \int d^4 \theta \phi \phi \psi \psi \equiv \frac{|F_\psi|^2}{M_*^2} \phi \phi,$$

(1)

where, as suggested by supergravity, we have taken the reduced Planck mass $M_* = M_{\text{Planck}}/\sqrt{8\pi}$ as the typical scale of higher dimension operators. (We will also assume throughout that supergravity is the dominant messenger of supersymmetry-breaking to the visible world.) This effective mass squared term for $\phi$ is the same order of magnitude as the Hubble constant squared, $H^2$, during inflation, since $H$ is related to the total energy density, $\rho$, via $H^2 = \rho/3M_*^2$, and $\rho \simeq |F_\psi|^2$. Thus it is difficult to satisfy the slow-roll conditions for the inflaton field

$$\epsilon \equiv \frac{M_*^2}{2} \left( \frac{d \ln V}{d \phi} \right)^2 \ll 1,$$

(2)

$$|\eta| \equiv \frac{M_*^2}{V} \left| \frac{d^2 V}{d \phi^2} \right| \ll 1,$$

(3)

since $\eta$ picks up corrections of $\mathcal{O}(1)$. This problem is quite generic. For instance in supergravity theories the coupling in Eq. (1) naturally appears in the scalar potential

$$V = \exp \left( \frac{K}{M_*^2} \right) \left( W_i K_{ji} W^j - \frac{3|W|^2}{M_*^2} \right) + D\text{-terms},$$

(4)

as a cross term between the Kähler potential $K = \phi \phi \phi + \ldots$ for the inflaton field, and the expectation value of the generalized $F$-term for $\psi$, $W_\psi = \partial_\psi W + W \partial_\psi K/M_*^2$. 

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However if it is assumed that $D$-terms provide the dominant contribution to the vacuum energy \[5\], then this problem is greatly ameliorated, the basic reason being that a direct Kähler potential coupling of the supersymmetry breaking $D$-term to the inflaton direction, analogous to that for the $F$-term in Eq. \[1\], is forbidden by gauge invariance.

In this paper we will be concerned with some issues that are raised in constructing more fully realized models of $D$-term inflation. In Section 2 we outline the basic structure of $D$-term inflationary models, and we point out that the constraints that such a model must obey are more severe than usually assumed. In Section 3 we discuss reheating in $D$-term models and how one should expect it to proceed.

One of the most attractive aspects of $F$-term models of inflation is the possibility of using the Affleck-Dine (AD) mechanism \[6\] for generating the observed baryon asymmetry. In Section 4 we argue that a variation of the AD baryogenesis schemes due to Murayama and Yanagida \[7\] and Dine, Randall and Thomas \[4\] can be implemented in $D$-term inflationary models. Section 5 contains our conclusions, and the Appendix discusses in some detail the possible decoherence of the AD flat direction in a hot environment.

2 The basic model and constraints

Consider a toy model of $D$-term inflation \[8, 5\], whose field content includes, a neutral chiral superfield, $S$, and superfields $\psi_{\pm}$ with charges $\pm 1$ under a U(1) symmetry, $U(1)_{\text{FI}}$. The superpotential $W = \lambda S\psi_+\psi_-$, leads to the tree-level scalar potential

$$V = |\lambda|^2 \left(|\psi_+\psi_-|^2 + |S\psi_+|^2 + |S\psi_-|^2\right) + \frac{g^2}{2} \left(|\psi_+|^2 - |\psi_-|^2 + \xi^2\right)^2. \quad (5)$$

Here, a Fayet-Iliopoulos (FI) term, $\xi^2$, which we define to be positive, has also been included. The supersymmetry preserving global minimum of this potential is at $S = 0$, $\psi_+ = 0$, $\psi_- = \xi$. However if $S$ is initially displaced sufficiently far from its minimum, $S > S_{\text{crit}} \equiv g\xi/\lambda$, then the local minimum $\psi_+ = \psi_- = 0$ has non-zero energy density $\rho = g^2\xi^4/2$ that, to this order, is independent of the value of the field $S$. Most importantly, the flatness of the potential for $S$ is not disastrously disturbed by the inflationary epoch supersymmetry breaking, since the $U(1)_{\text{FI}}$ vector superfield couples only to the Kähler potential of charged chiral fields. Moreover, it has been argued \[10\] that possible dangerous non-minimal terms in the gauge kinetic function depending on $S$ can quite easily be forbidden by a combination of symmetries and holomorphy, although we will see in the following that the problem is more serious than previously thought.

The leading inflation-induced curvature of the effective potential for $S$ is usually assumed to be due to the superpotential couplings between $S$ and charged chiral fields that couple in turn to the $U(1)_{\text{FI}}$ $D$-term. In particular, at 1-loop, supersymmetry breaking leads to the splitting in mass of the bosonic components
of $\psi_\pm$ from their fermionic partners, and the potential for $S$ receives a correction
$\Delta V = C \alpha^2 \xi^4 \ln(|\lambda S|/\mu)$, where $\mu$ is the momentum scale, $\alpha = g^2/4\pi$, and $C \geq 1$
counts the multiplicity of $\psi_\pm$ pairs in more general toy models. As a result the in-
duced inflationary-epoch curvature of the inflaton potential is suppressed relative to
the Hubble constant by small gauge coupling dependent loop factors. Thus this model
is an implementation of the hybrid inflationary models of Refs. [11], where $S$ is the
field that slow rolls due to a nearly flat potential.

Actually, the situation is not quite so simple as just stated. For the logarithmically
dependent $S$ potential, $|\eta| \ll 1$ is the first of the slow-roll conditions to fail. Solving
for the final value $S_f$ by imposing $|\eta(S_f)| = 1$ gives $S_f = \sqrt{C \alpha M_*^2/2\pi}$; for typical
values of the parameters this is one or two orders of magnitude larger than $S_{\text{crit}}$.
Moreover, the slow-roll of $S$ must initiate at substantially larger values if there is
to be enough e-folds of expansion to solve the flatness and horizon problems of
the standard cosmology. Requiring $N = 55$ e-folds leads, under the assumption that the
logarithmic $S$ potential dominates, to an initial value

$$S_{55} = \left(\frac{55C\alpha M_*^2}{\pi} + S_f^2\right)^{1/2}.$$  \hfill (6)

For typical values of the gauge coupling, $S_{55}^2 \simeq 0.8C M_*^2$. It is now necessary to ask if,
given such large initial values, it is reasonable to suppose that the dominant curvature
of $V(S)$ is due to 1-loop effects for the entire evolution of the field. (Recall in this
regard that approximately 55 e-folds before the end of inflation the fluctuations on
the largest scales we currently observe were just leaving the horizon; the observational
bound on the spectral index $n$ of these fluctuations is $|n - 1| \lesssim 0.2$. Since the spectral
index of adiabatic density fluctuations is determined by the slow-roll parameters eval-
uated at the appropriate epoch, $n - 1 = 2\eta_{55} - 6\epsilon_{55}$, deviations from a flat potential
can be important. In this paper we will not consider the CMBR spectrum in detail
because, among other effects, these models have cosmic string solutions which can
significantly alter the power spectrum [1].)

Consider, for example, the addition of a superpotential term of the form

$$\Delta W = \kappa S^m/M_*^{(m-3)}.$$  \hfill (7)

The resulting shift in $\eta$ is

$$\Delta \eta(S) \simeq \frac{2m^2(2m-2)(2m-3)\kappa^2 S^4}{g^2 \xi^4} \left(\frac{S}{M_*}\right)^{2(m-4)}. \hfill (8)$$

Given the value $S_{55} \simeq M_*$, the term in parentheses is $O(1)$ and the expression is
dominated by the large enhancement $S_{55}^4/\xi^4 \sim 2 \times 10^{10}$ (we will see shortly how the
scale of $\xi$ is set to be about $6.6 \times 10^{15}$ GeV by CMBR measurements). Only for values
of $\kappa < 10^{-5}$ do we get $\Delta \eta < 1$. The smallest natural values we might expect for $\kappa$, if
it is not identically zero by some symmetry, are $\kappa \sim 1/m!$. Such values might arise
by matching this non-renormalizable superpotential coupling onto some underlying renormalizable theory in which heavy states have been integrated out. Then the bound $\kappa < 10^{-5}$ requires that we must forbid all operators of the form $S^m$ for at least $m \leq 9$ in order to keep $\Delta \eta < 1$. Note that increasing the multiplicity factor $C$ just makes the problem worse.

Further, as noted in Ref. [10], there are also dangerous non-minimal gauge-kinetic terms for $U(1)_{FI}$ of the form

$$S^k W^\alpha W^\alpha / M^k_t + \text{h.c.}$$

The contribution of these operators to $\eta_{55}$ is in turn

$$\Delta \eta_{55} \simeq \frac{1}{(k-2)!} \left( \frac{S_{55}}{M_*} \right)^{(k-2)},$$

also $O(1)$ unless all operators with $k \lesssim 6$ are forbidden.

The obvious way to forbid these two sets of operators while preserving the operator $S\psi_+ \psi_-$ is by a discrete symmetry of sufficiently high order or by an R-symmetry. The exact form of this symmetry will have a significant impact on the couplings of $S$ to light MSSM fields, and therefore on the reheat temperature resulting from decay of the coherent oscillations of the $S$ field. We will therefore address this issue more fully in Section 3.

Note, however, that one potential problem with a global symmetry such as R-symmetry is the worry that they may be violated by quantum gravitational effects [12]. The alternative of a discrete gauge symmetry [13] might therefore be preferable, since such symmetries are protected from violation by quantum gravity effects. Moreover, string theory very commonly possesses such symmetries, in contrast to the case of (exact) global symmetries.

Apart from the aesthetic and theoretical appeal of forbidding the dangerous $S^m$ and $S^k W^\alpha W^\alpha / M^k_t$ operators by a discrete gauge symmetry, there is another intriguing possibility that such a structure allows. Unlike the case of a continuous symmetry, at some order the dangerous operators will eventually arise and the potential will not be flat for $S$ greater than some value $S_W$. We must require that the plateau at $S < S_W$ is long enough to get at least 55 e-folds of inflation, which places a lower bound on $S_W$ ($S_W > S_{55}$) and thus on the order of the operator which lifts the $S$ potential. If, as traditionally assumed, the potential is flat well past $S = S_{55}$, then the universe inflates for many more than 55 e-folds and thus the spectral index $n$ is very close to 1 on all scales, up to and including superhorizon-size scales. This very long period of inflation simultaneously drives $\Omega$ so close to unity that we should currently observe a flat universe. However, if $S_W$ is just such that the universe inflates for only 55 e-folds ($i.e., \varepsilon(S)$ and $|\eta(S)| \geq 1$ for $S > S_{55}$), then $n$ is constant on small scales but deviates on scales comparable to our current horizon. In particular, the non-trivial structure of the potential at $S \approx S_W$ is imprinted on the spectral index via the increase of the functions $\eta(S)$ and $\varepsilon(S)$ at $S \approx S_W$. Since this spectrum was imprinted at the
very start of inflation, it is observable today on the largest scales. This then would be an example of what we might call “just-so inflation” in that we have a period of inflation just long enough to solve the horizon and flatness problems, but \textit{not so long} as to have the current value, $\Omega_0$, of $\Omega = \rho/\rho_{\text{crit}}$, approximate 1 \textit{exponentially} closely. In other words with two sources of the $S$ potential – the logarithmic dependence from loop corrections, and the power dependence from higher-dimension operators – it only requires a discrete choice of the order of the symmetry group to ensure that we currently measure $\Omega_0 < 1$, but not very far away from $\Omega_0 = 1$. This “just-so” mechanism, based on two different types of contribution to the potential of the inflaton, with one controlled by a discrete symmetry, appears to be one of nicest explanations of how an open universe could result from (and be consistent with) an inflationary early universe. (For implementations of open inflationary universes, see, e.g. \cite{14}.)

Notwithstanding these issues, crucial to the formulation of this model was the FI term $\xi$. The most attractive origin for this term is the so-called anomalous U(1)$_X$ symmetry of some string compactifications, whose apparent field-theoretic anomalies are cancelled by a 4-dimensional version of the Green-Schwarz mechanism involving shifts in the model-independent axion \cite{15}. In such a compactification a FI term is automatically generated, and its magnitude has been calculated to be \cite{15}

$$\xi^2 = \frac{\text{Tr}(Q_X)}{192\pi^2} g_{\text{str}}^2 M_*^2.$$  \hspace{1cm} (11)

Here the model-dependent factor Tr($Q_X$) is proportional to the total gravitational anomaly; in typical semi-realistic string models Tr($Q_X$) $\sim O(50)$. Now, the magnitude of the FI term sets the size of the Hubble constant during inflation, and in turn the magnitude of the fluctuations in the cosmic microwave background radiation (CMBR). Normalizing to the observed CMBR fluctuations requires \cite{16}, $\xi_{\text{CMBR}} = 6.6 \times 10^{15}$ GeV, independent of $g$. (This assumes that the logarithmically dependent $S$ potential receives no significant corrections at $S_{55}$. In the general case $(V_{55}/\epsilon_{55})^{1/4} = 6.7 \times 10^{16}$ GeV.) Unfortunately this is smaller than the prediction, Eq. (11), of string theory by a sizeable amount. Although some improvement of this problem is possible by increasing the 1-loop coefficient $C$ in $\Delta V(S)$, it is argued in Ref. \cite{16} that the two values cannot be brought into agreement unless the string prediction is lowered, or some other mechanism for generating $\xi$ (without simultaneously large $F$-terms) is found. At present this is an open problem.

One possibility in this regard is the suggestion that the expectation value of the dilaton, which sets the strengths of all couplings, is shifted during the inflationary epoch from its current value, thus allowing a significantly smaller $\xi$ \cite{17}. To agree with the CMBR normalization, an inflationary value of $g_{\text{str}}$ $\sim 1/50$ would be needed. It is interesting to note in this context that reducing the effective value of $g$ during inflation also helps alleviate the problem with the higher-dimensional superpotential and gauge-kinetic functions, as can be seen by counting powers of $g$ in the expression Eq. (11) for $S_{55}$. In the following we will just \textit{assume} that some variant leads to
the correct value of $\xi$ (which we take to be $6.6 \times 10^{15}$ GeV), and focus on the post-inflationary phenomenology of reheating and baryogenesis in $D$-term models.

3 Reheating

Even if the magnitude of $\xi$ can be made consistent with the CMBR data, successful reheating is still non-trivial. First there are the usual constraints on the reheat temperature arising from the overproduction of gravitinos, although these constraints are somewhat modified in $D$-term models, since there are typically at least two well separated stages of reheating. In addition, the particular mechanism by which one gets sufficient baryogenesis is greatly influenced by the details of reheating.

After inflation ends, the vacuum energy that drove inflation moves to potential and kinetic energy for condensates of fields oscillating around their various minima. In the model outlined above, the original vacuum energy, $V = \frac{1}{2}g^2\xi^4$, is now being carried by some combination of the $\psi_-$ and $S$ fields. The relative portion of the energy in each oscillating condensate depends on their effective masses after inflation, which are $\lambda\xi$ and $\sqrt{2}g\xi$ respectively. For $\lambda^2 = 2g^2$ they share the energy evenly; we will assume that $\lambda \sim g$ so that the contributions of the two condensates to the total energy density of the universe are of the same order. Moreover, because both condensates have quadratic potentials, their energy densities will both scale as matter $(i.e., R^{-3})$ as the universe expands, and so the ratio of their energy densities remains constant.

The universe remains cold until $H$ drops below the decay width of either one of the $S$ or $\psi_-$ fields, at which point that field will decay, dumping its stored energy and reheating the universe. At a later time the second condensate decays, dumping further entropy and diluting any products of the first stage of reheating. The amount of reheating is controlled by the widths of these fields, which in turn depend on their masses and couplings. In particular, at the reheat time, $t = t_R$, when $H \sim \Gamma_A$ (here, $A$ is a generic condensate field), the decay of the $A$ field dumps energy density

$$\rho_A(t_R) = 3\Gamma_A^2 M_*^2 f_A$$

into the vacuum, where $f_A$ is the fraction of $\rho_{\text{tot}}$ stored in the $A$ condensate. The corresponding reheat temperature is

$$T_R \simeq \left( \frac{30\rho_A(t_R)}{\pi^2 g_*(t_R)} \right)^{1/4} \simeq 0.4 f_A^{1/4} \sqrt{\Gamma_A M_*}.$$  \hspace{1cm} (13)

where $g_*(t_R)$ is the number of relativistic degrees of freedom at $t = t_R$; one expects that $g_*(t_R) \sim \text{few} \times 10^2$.

What are the bounds on $T_R$? There are a number of model-dependent limits; for example, in a GUT one requires $T_R \ll M_{\text{GUT}}$ in order to avoid creating stable
monopoles. However there are more stringent bounds coming from gravitino production in supergravity theories. For unstable gravitinos in the mass range 100 GeV to 1 TeV, one requires \( T_R \lesssim 10^{7-9} \text{ GeV} \) \[18\]. Also there is lower bound on \( T_R \) of approximately 6 MeV due to the fact that the universe must reheat sufficiently that conventional Big Bang Nucleosynthesis (BBN) is possible \[19\]. Note that the quoted gravitino bound on \( T_R \) assumes that there is no later entropy dump which dilutes the gravitino number density. In \( D \)-term models there are a number of stages of reheating, each with its own entropy release, so this dilution must be taken into account.

The amount by which the gravitino relic of the first stage of reheating is diluted by the second stage is easily calculated. Suppose a (generic) \( X_1 \) condensate decays at \( H_1 \simeq \Gamma_1 \) (at which time, for simplicity, we take \( \rho_1 \simeq \rho_2 \)) with reheat temperature \( T_1(t_1) \), while the \( X_2 \) condensate decays much later at time \( t_2 \) when \( H_2 \simeq \Gamma_2 \). Since the energy density in the \( X_2 \) condensate scales like non-relativistic matter \( \rho_{X_2} \sim R^{-3} \), it is easy to show that at the epoch of \( X_2 \) decay the radiation temperature has fallen from \( T_1(t_1) \) to \( T_1(t_2) = T_1(t_1) \left( \frac{\Gamma_2}{\Gamma_1} \right)^{2/3} \). Thus the ratio of entropy densities just before \( X_2 \) decay (when the entropy is almost entirely in the form of red-shifted radiation from \( X_1 \) decay) to that just after is given by

\[
\frac{s_{\text{before},2}}{s_{\text{after},2}} = \frac{g_{1*}(\Gamma_1^2 M_*^2/g_{1*})^{3/4}(\Gamma_2/\Gamma_1)^2}{g_{2*}(\Gamma_2^2 M_*^2/g_{2*})^{3/4}} \approx \sqrt{\frac{\Gamma_2}{\Gamma_1}} \simeq \frac{T_2(t_2)}{T_1(t_1)},
\]

where \( g_{1*} \equiv g_*(t_1) \), etc.. (In the last two equalities we have dropped the weak dependence on the change of the effective number of relativistic degrees of freedom, \( g_* \).) In the case of a single stage of reheating, the yield of gravitinos, \( Y_{3/2} \equiv n_{3/2}/s \), is well approximated by the simple linear form, \( Y_{3/2}(T \ll 1 \text{ MeV}) \approx 2.14 \times 10^{-11}(T_R/10^{10} \text{ GeV}) \) \[18\]. This, together with Eq. (14), shows that the diluted gravitino yield from the first epoch of reheating is in our case equal to the yield from the second epoch of reheating (up to \( O(1) \) coefficients); \( i.e., \) just after \( t_2 \),

\[
\frac{s_{\text{before},2}}{s_{\text{after},2}} Y_{3/2}^{(1)} \simeq Y_{3/2}^{(2)}.
\]

Thus in the models we are discussing, the gravitino constraint on the reheat temperature should be applied to the later of the two epochs of coherent field oscillation decay.

In any case, since the post-inflationary masses of both \( S \) and \( \psi_- \) are large (of order of \( \xi \simeq 6 \times 10^{15} \text{ GeV} \)), even small couplings can lead to untenable reheat temperatures. Thus it is necessary to investigate the couplings of \( \psi_- \) and \( S \) to light fields, given the constraints of \( D \)-term inflation.
3.1 Immediate decay of $\psi_-$ condensate

The most constrained decays involve the $\psi_-$ oscillations. Let us first consider the case in which the non-zero FI term is generated through cancellation of the anomaly of a pseudo-anomalous $U(1)_X$. Recall that in order for the four-dimensional version of the Green-Schwarz mechanism to operate, there must exist non-zero mixed $U(1)_X G^2_A$ anomalies with all gauge groups $G_A$. In particular there must exist fields that are simultaneously charged under both $U(1)_X$ and each of the subgroups of the Standard Model, and that are chiral with respect to this combination of groups. Therefore, the anomalous $D$-term must receive contributions from fields which are charged under $G_{SM}$, and which have $O(1)$ charges under $U(1)_X$. The $D$-term therefore has the form:

$$D = g \left( |\psi_+|^2 - |\psi_-|^2 + \sum_i q_i |Q_i|^2 + \xi^2 \right)$$  \hspace{1cm} (16)

where the $Q_i$ are fields charged under $G_{SM}$ which have charge $q_i$ under the anomalous $U(1)_X$. (The charges must satisfy $q_i > 0$ so that at the post-inflationary minimum none of the fields $Q_i$ gain expectation values of order $\xi$, since otherwise the Standard Model gauge group is broken at an unacceptably high scale.) If the usual MSSM fields, $\phi_i$, are included among the $Q_i$ then they couple to fluctuations of the $\psi_-$ field around its expectation value with strength

$$\Delta L = g^2 \xi \sum_{FI} q_i |\phi_i|^2 (\delta \psi_- + \delta \psi_+).$$  \hspace{1cm} (17)

The resulting decay width for $\psi_-$ is $\Gamma_{\psi_-} \sim g^2 \xi/16\pi$ and so $\psi_-$ immediately decays, giving $T_R \sim 10^{15} - 10^{16}$ GeV. From our earlier discussion of two-stage reheating, it is clear that this violates the gravitino bound unless the $S$ field decays with $T_R < 10^9$ GeV.

Moreover, such a large $\psi_-$ reheat temperature does lead to the evaporation of the $S$-condensate into an incoherent collection of $S$-particles. Recall that the MSSM fields $\phi_i$ and the $\psi_-$ interact via exchange of the $U(1)_X$ gauge field. After $U(1)_X$ breaks via $\langle \psi_- \rangle = \xi$, the gauge field can be integrated out, leaving an effective Kähler potential term of the form

$$\Delta K = \frac{\phi^\dagger \phi}{\xi^2}.$$  \hspace{1cm} (18)

But because of the form of the superpotential ($W = \lambda S \psi_+ \psi_-$), $\langle F_{\psi_+} \rangle = \lambda S \langle \psi_- \rangle = \lambda \xi S$. Eq. (18) then produces an interaction in the effective Lagrangian between $\phi_i$ and $S$ scalars:

$$\Delta L = \frac{|\phi_i|^2 |F_{\psi_+}|^2}{\xi^2} = \lambda^2 |\phi|^2 |S|^2.$$  \hspace{1cm} (19)
The corresponding scattering rate for an $S$-scalar in a bath of MSSM fields at temperature $T$ is then

$$\Gamma_{\text{scatt}} = n_\phi \langle v \sigma \rangle \sim \frac{\lambda^4}{16\pi T^2} T^3.$$  \hfill (20)

If $\psi_-$ decays immediately after the end of inflation, as is the case if the MSSM fields have $\mathcal{O}(1) U(1)_X$ charges, then $H \sim \sqrt{\rho}/M_* \sim T_2^2/M_* \ll T$ since $T \lesssim \xi$. Thus $\Gamma_{\text{scatt}} \gg H$ and the $S$-condensate is evaporated into an incoherent collection of $S$-particles which then later decay at $t \simeq \Gamma_S^{-1}$, where $\Gamma_S$ is the $S$ decay width. As long as the individual $S$-particles decay with reheat temperature $\lesssim 10^9 \text{GeV}$, the fact that the condensate has been destroyed does not violate the gravitino bounds. We will return to $S$-decays in Section 3.3. We will also see in Section 4 that this very high reheat temperature for $\psi_-$ can also disrupt certain implementations of AD baryogenesis.

### 3.2 Delayed decay of $\psi_-$ condensate through kinetic mixing

There is one way to satisfy the anomaly conditions for anomalous $U(1)_X$ models which does not lead to immediate $\psi_-$ decay: assume that the $Q_i$ are all vector-like with respect to $G_{SM}$, but chiral with respect to $U(1)_X$. Thus the fields may naturally acquire masses of $\mathcal{O}(\xi)$, and $\psi_-$ oscillations can no longer decay into them. (Treating the decay of the condensate as the decay of a collection of individual condensate particles. Solutions to the coupled $S$, $\psi_-$ equations of motion corresponding to parametric resonance do in principle allow for decay of a condensate into heavier fields, but recent work [20] indicates that this is in fact very difficult to achieve in an expanding universe.) It may also be that the FI term which drives inflation arises from a non-anomalous $U(1)_{FI}$, in which case there is no need for any fields charged under the $U(1)_{FI}$ to also carry MSSM charges.

This would appear to be an acceptable solution: the $U(1)_{FI}$ can drive inflation but decouples from all low-energy physics since no light field can be charged under it. This is not what one might have hoped (since there are, for example, many interesting models which use pseudo-anomalous $U(1)$’s to explain low-energy phenomena), but it at least seems consistent.

In fact it may not be, since we have ignored an important effect that seems unavoidable in models with FI terms. This effect is kinetic mixing between the $U(1)_{FI}$ and hypercharge, $U(1)_Y$, gauge symmetries [21, 22]. Recall that in a theory with two $U(1)$ factors, the most general renormalizable Lagrangian contains a gauge-invariant term which mixes the gauge field strengths of the two $U(1)$’s. In the basis in which the interaction terms have the canonical form, the pure gauge part of the supersymmetric

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*Implicit in this calculation is that the MSSM fields are relativistic; that is, they do not receive masses $\sim T$. Such masses do not arise from the $D$-term simply because it is cancelled off by $\langle \psi_- \rangle$. Similarly, such large masses for the first two MSSM generations cannot be generated by some large AD condensate vev because of their small Yukawa couplings.*
Lagrangian for an arbitrary $U(1)_a \times U(1)_b$ theory is

$$L_{\text{gauge}} = \frac{1}{32} \int d^2 \theta \left\{ W_a W_a + W_b W_b - 2 \chi W_a W_b \right\} \quad (21)$$

where $W_a$ and $W_b$ are the chiral gauge field strength superfields for the two gauge symmetries ($W = \overline{T}^bDV$ for a vector superfield $V$). (We assume in the following that the kinetic mixing parameter $\chi \ll 1$.) To bring the pure gauge portion of the Lagrangian to canonical form, a shift of one of the gauge fields is necessary:

$$V_b^\mu \rightarrow V_b'^\mu = V_b^\mu - \chi V_a^\mu \quad (22)$$

which implies $W_b \rightarrow W_b' = W_b - \chi W_a$. This particular basis is dictated by the assumption that $U(1)_a$ is broken by the expectation value of some field with non-zero $U(1)_a$ charge. Thus in the case of interest, we identify $U(1)_a$ as the $U(1)$ with the FI term, $U(1)_{FI}$, and $U(1)_b$ as $U(1)_Y$.

In this basis the gauge Lagrangian is diagonal, however, the interaction piece is modified. In particular suppose initially that there was a sector of fields $\Phi_i$ charged only under $U(1)_{FI}$, and another sector of fields $\phi_i$ charged only under $U(1)_Y$ (the MSSM fields in our case), then the basis change introduces a number of new interactions between the two sectors. Most importantly for the present purposes, upon solving for the $D$-terms, one finds \[22\]:

$$D_{FI} = -g \sum_i q_i |\Phi_i|^2 - g_y \chi \sum_i y_i |\phi_i|^2 + \xi^2$$

$$D'_y = -g_y \sum_i y_i |\phi_i|^2,$$  \quad (23)

namely, that the $U(1)_{FI}$ $D$-term picks up an admixture of the hypercharge $D$-term (here $y_i$ is the hypercharge of the various fields).

As discussed in detail in Refs. [21, 22], non-zero $\chi$ can arise from threshold effects, either field-theoretic or string theoretic, or renormalization group running of the couplings in the low-energy field theory. The general requirement that this occurs is that there exist particles charged under both $U(1)$’s such that, in an effective field theory language where we integrate out modes once we drop below their mass, $\text{Tr}(Q_a Q_b) \neq 0$ for some range of momentum scale.

There are two symmetries that naturally forbid the existence of kinetic mixing. The first is simply gauge invariance if one of the $U(1)$’s actually sits inside an unbroken non-Abelian group (and in this case $\text{Tr}(Q_a Q_b) = \text{Tr}(Q_a) \text{Tr}(Q_b) = 0$ for each, degenerate, mass level). The second symmetry that forbids kinetic mixing is an unbroken “charge conjugation” symmetry that acts on only one of the two $U(1)$’s, $A_\mu \rightarrow -A_\mu$, which again enforces the vanishing of the trace $\[22\]$.

However in the case of $U(1)_Y$ mixing with $U(1)_{FI}$, neither of the above symmetries can be exact. In particular, charge conjugation is certainly not an unbroken symmetry of the hypercharge interaction, and, most importantly, a charge conjugation symmetry
acting on \(U(1)_{FI}\) would also forbid the \(FI\) term itself. Moreover, in the case of the pseudo-anomalous \(U(1)_X\) symmetry, there must exist particles charged under both \(U(1)\)'s. Therefore non-zero kinetic mixing is typically generated in the case of interest.

Given that \(\chi\) is not exactly zero due to some unbroken symmetry, the natural expectation for \(\chi\) is that it is generated by a one-loop threshold effect and thus \(10^{-3} \lesssim \chi \lesssim 10^{-2}\). This estimate is applicable to string theoretic threshold corrections as well as field theory threshold effects \cite{22}. A reduction in the value of \(\chi\) generated by loop effects is possible if all states simultaneously charged under \(U(1)_Y\) and \(U(1)_{FI}\) are, as far as their quantum numbers are concerned, effectively in multiplets of some non-Abelian group that contains one of the \(U(1)\)'s. The fact that this non-Abelian gauge symmetry must be broken implies that the states in the multiplets are not naturally degenerate, and thus \(\chi\) will again be generated, but with a value now suppressed by the splitting in the masses of the states relative to their common mass. In this way \(\chi\) can be reduced to a 2 or 3-loop effect (if, respectively, one or both sets of \(U(1)\) quantum numbers embed in some effective non-Abelian multiplet): \(10^{-6} \lesssim \chi \lesssim 10^{-4}\).

The end result, therefore, in the case of \(D\)-term inflation, is that the full potential contains a contribution from the modified \(U(1)_{FI}\) \(D\)-term, now including an admixture of the usual hypercharge \(D\)-term, as well as the hypercharge \(D\)-term itself:

\[
V_D = \frac{g^2}{2} \left( |\psi_+|^2 - |\psi_-|^2 + \chi \sum_i y_i |\phi_i|^2 + \xi^2 \right)^2 + \frac{g_\chi^2}{2} \left( \sum_i y_i |\phi_i|^2 \right)^2. \tag{24}
\]

Here, \(\phi_i\) are the usual MSSM fields of hypercharge \(y_i\) (we have dropped contributions from any heavy fields which are vector-like under with respect to the MSSM since they play no role in the following). The presence of the extra terms in the \(U(1)_{FI}\) \(D\)-term does not destroy the possibility of inflation in this model. For \(\langle S \rangle\) sufficiently large, the local minimum is still at \(\langle \psi_+ \rangle = \langle \psi_- \rangle = 0\), as a consequence of the presence of the hypercharge \(D\)-term, while the expectation values of the MSSM fields \(\phi_i\) satisfy the constraint

\[
\sum_i y_i \langle |\phi_i|^2 \rangle = \frac{\chi g^2 \xi^2 / 4}{\chi^2 g^2 / 4 + g_\chi^2}. \tag{25}
\]

At this local minimum the combined \(D\)-terms are non-zero, and the energy density is

\[
\langle V \rangle = \frac{g^2 g_\chi^2 \xi^4}{\chi^2 g^2 / 2 + 2 g_\chi^2}. \tag{26}
\]

Moreover, at the true post-inflationary minimum we still have \(\langle \psi_- \rangle = \xi\), despite the fact that the \(U(1)_{FI}\) \(D\)-term now inevitably contains fields other than \(\psi_-\) with negative charge. Thus the alteration of the potential imposed by kinetic mixing still permits inflation as before.

However, kinetic mixing now allows \(\psi_-\) to decay to pairs of light MSSM scalars through a coupling of strength \(g_\chi^2 \chi\). Note that such mixing allows \(\psi_-\) to couple directly to light MSSM fields even if no such coupling existed in the bare Lagrangian.
In Section 4, we will see that AD baryogenesis occurs at $T \simeq 10^{10-11}$ GeV. We have already argued that the natural range for $\chi$ is $10^{-6} \lesssim \chi \lesssim 10^{-2}$. Over most of this range, $\psi_-$ decays before AD baryogenesis, i.e., $T_R^{(\psi)} > T_{AD}$, and the resulting physics is very similar to the immediate decay scenario outlined in the previous section.

However if $\chi \lesssim 10^{-6}$, then $\psi_-$ decays after baryogenesis, allowing further implementations of the AD scenario. How would $\chi \lesssim 10^{-6}$ arise in a realistic model? First, if hypercharge is embedded in a GUT at the unification scale (rather than string unifying at some possibly higher scale), there are no significant renormalization group enhanced contributions to $\chi$; only threshold corrections to the initial (zero) value of $\chi$ are relevant. Because of the tracelessness of the hypercharge for any GUT multiplet, the GUT symmetry can naturally reduce these contributions to $10^{-4}$, the only exceptions being if split multiplets, such as the GUT Higgs $(\overline{5} + \overline{5})$, which contain light doublets and heavy triplets, are charged under $U(1)_{FI}$. However from our earlier analysis we know that no light MSSM states, such as the doublets, can have “bare” $U(1)_{FI}$ charges, and so this exception does not arise. To further reduce $\chi$ we can impose an (approximate) “charge-conjugation” symmetry. For instance, if the spectrum of states charged under $U(1)_{FI}$ is of the form $(\overline{5}_q + 5_q)$, where the subscript is the charge, then so long as the states are approximately degenerate in mass the threshold induced $\chi$ is further suppressed. In this way, a value of $\chi \lesssim 10^{-6}$ is possible. However, it seems very difficult to imagine values for $\chi$ very far below this limit.

3.3 Decay of $S$

We now turn to the fate of the $S$ field, which depends upon its available decay modes. The $S$ and $\psi_+$ fields are nearly degenerate, while the mass of the $\psi_-$ field is suppressed/enhanced by $g/\lambda$. In either case, if $S$ is kinematically allowed to decay to $\psi_+$ or $\psi_-$, it will do so with $\Gamma_S \sim \xi$, producing $T_R \sim 10^{15-16}$ GeV. We already argued that $\psi_-$ is likely to decay early; since we cannot have both $\psi_-$ and $S$ decaying quickly, we conclude that $\Gamma_S \sim \xi$ violates the gravitino bound. (The later decay of the AD condensate cannot dump enough entropy to avoid this conclusion.) Ruling out these decays kinematically, one is left with decays of $S$ to light matter. However, the constraints discussed in Section 2 on superpotential terms of the form $S^k$, or non-minimal gauge-kinetic terms of the form $S^mW^\alpha W_\alpha$, indicate that the couplings of $S$ are strongly restricted by symmetries. As mentioned in Section 2, an attractive choice is to use R-symmetry to limit the couplings of $S$.

Thus consider $R(S) \neq 0$. Since the usual MSSM superpotential has R-charge $R(W_{MSSM}) = 2$, no superpotential couplings are allowed between $W_{MSSM}$ and $S^k$ (at least until $R$ is broken at an effective scale $O(m_{3/2})$ within the visible sector). However, $S$ can couple to $r$th-order invariants composed of MSSM fields for $r > 3$. For example, if we assign continuous $R$-charges $R(S) = -2$, $R(\psi_+) = 2$, and all non-Higgs MSSM fields $R = 1$, then the there exists the term $SSQQQL/M^2$ in the superpotential. Alternatively, if we take $R(Q, L, u^c, d^c, e^c) = 1/2$ and $R(H_u, H_d) = 1$, $
then the term \( S(H_u H_d)^2/M_s^2 \) is the lowest-dimension operator. For general \( r \) the leading superpotential terms involving \( S \) are schematically of the form

\[
W = \lambda S \psi_+ \psi_- + \kappa \frac{S \phi^r}{M_s^{(r-2)}},
\]

where the \( \kappa \) coupling leads to a \( r \)-body decay of the scalar component of \( S \) to \( (r-2) \) scalars and two fermions. The resulting visible sector reheat temperature is

\[
T_R^{(S)} \simeq \frac{\kappa \lambda^{(r-3/2)}}{(3 \times 10^2)^{(4-r)}} \left( \frac{P_4}{P_r} \right)^{1/2} \times (4 \times 10^8 \text{GeV}),
\]

where \( P_4/P_r \) is the ratio of the 4-body phase space factor to the \( r \)-body, and we have taken \( \xi = 6.6 \times 10^{15} \text{GeV} \). If \( \kappa, \lambda \) are \( \mathcal{O}(1) \), then for \( r = 4 \) the reheat temperature is at the upper end of the gravitino bound; for \( r > 4 \), \( T_R^{(S)} \) is well within the constraints imposed by gravitino production. Notice that for all \( r \geq 4 \) the decay of the \( S \)-fields occurs after the epoch of AD baryogenesis at \( H \simeq m_{3/2} \). (Because we assume supergravity mediation, \( m_{3/2} \simeq m_{\text{weak}} \simeq 1 \text{ TeV} \).)

Also note that such an R-symmetry acting on \( S \) and \( \psi_- \) has the further advantage of explaining both the absence of the \( S^m W^a W_a \) operators, and a direct mass term \( M \psi_+ \psi_- \). Such a mass term, if taken to be \( \mathcal{O}(M_s) \), would lead to the breaking of supersymmetry at an unacceptably high scale at the end of inflation. We have also checked that, for \( r = 4 \), Kähler potential couplings of \( S \) to MSSM fields never parametrically dominate over the superpotential interactions, and thus the reheat temperature is naturally maintained in the \( 10^8 \text{GeV} \) range.

In summary, for generic \( U(1)_{FI} \) charges and couplings, we expect the \( \psi_- \) condensate to decay immediately after the end of inflation, reheating the universe to a temperature \( T_R^{(\psi)} \simeq 10^{15} \text{GeV} \). The bath of light MSSM fields scatter off the \( S \) condensate, evaporating it into a plasma of incoherent \( S \) scalars. These scalars, however, must not decay until \( H \simeq \Gamma_s \lesssim 1 \text{ GeV} \) to give a final \( T_R \lesssim 10^9 \text{ GeV} \) consistent with the gravitino production bounds. We have shown that given the \( R \)-symmetries necessary for successful inflation, such small widths for \( S \) are actually quite natural. As we will see in the next section, baryogenesis occurs at \( H \simeq 1 \text{ TeV} \) and thus \( S \)-scalars decay after baryogenesis is complete. There is also the possibility that \( \psi_- \) has no bare couplings to light MSSM fields in which case its decays will be dominated by kinetic mixing effects. Here, if \( \chi \gtrsim 10^{-6} \), the thermal history is identical to that described previously. But if \( \chi \lesssim 10^{-6} \) then neither of the coherent \( S \) nor \( \psi_- \) condensates decays or evaporates until after baryogenesis has already occurred.

## 4 Affleck-Dine baryogenesis

We now turn to the question of whether AD baryogenesis can be successfully implemented in the \( D \)-term inflationary scenarios. We will argue that an adaption of
the versions of AD baryogenesis studied by Murayama and Yanagida [7] and Dine, Randall and Thomas [4] can lead to successful baryogenesis in the case of $D$-term inflation.

In general we will not make a distinction between direct AD baryogenesis and baryogenesis via leptogenesis. As long as the temperature of the SM degrees of freedom after the decay of the AD condensate is greater than the weak scale, non-perturbative sphaleron processes efficiently convert lepton number to baryon number, destroying $(B+L)$, but leaving asymmetries in both $L$ and $B$ as long as non-zero $(B-L)$ was also produced by the evolution of the condensate (as is typically the case).

Let us recall the basic idea of the implementations of AD baryogenesis as outlined in Refs. [4, 23]. First, an MSSM flat direction lifted by a (baryon or lepton number violating) higher-dimension operator takes on a large expectation value in the post-inflationary universe due to a negative effective mass-squared term, proportional to $H^2$ (coming from terms similar to that discussed in Eq. (1)). Due to inflation, the phase of this condensate is uniform over a region at least as large as our currently observable universe. When this $H$-dependent mass drops below the usual $T=0$ supersymmetry breaking soft mass of the flat direction (i.e., when $H^2 < m_{3/2}^2$), the condensate begins to oscillate. Furthermore, at this epoch, the magnitude of the baryon or lepton number violating $A$-terms in the potential (proportional to the $B$ or $L$ violating terms in the superpotential) are of the same magnitude as the $B$ or $L$ conserving terms. The relative phase between the $A$-terms and the condensate is a source of CP-violation that can be naturally large. In such a situation the oscillating condensate picks up a $B$ or $L$ number per mode of $O(1)$. Finally the AD condensate decays in a $B$ or $L$ conserving way, releasing these quantum numbers into light MSSM degrees of freedom. Note, that for this mechanism to apply, it is not necessary for the AD condensate to have a large expectation value either during, or immediately after, inflation. It is only important that at $H \simeq m_{3/2}$ it have a large vev and well-defined phase over volumes comparable to the currently observable universe.

As enumerated in Refs. [4, 23], there are many possible flat directions in the MSSM that could be involved in AD baryogenesis. We will label a generic MSSM flat direction by $\phi$. Examples of such flat directions are provided by $L H_u \sim \phi^2$ (in terms of doublet components: $L = (\phi, 0)$, $H_u = (0, \phi)$), or by $Q_1 L_1 d_2 \sim \phi^3$.

The magnitude of the expectation values of the MSSM flat directions after inflation are primarily fixed by higher dimension operators in the K"ahler potential that couple the flat direction to other fields. In particular consider the couplings

$$\Delta \mathcal{L} = \int d^4 \theta (c_1 S^\dagger S + c_2 \psi_+^\dagger \psi_+ + c_3 \psi_-^\dagger \psi_- + \cdots ) \frac{\varphi^+ \varphi}{M_*^2},$$  

(29)

where the $c_i$’s might be expected to be $O(1)$ constants of either sign. There is no symmetry of the low-energy effective theory that can forbid these couplings from appearing. In terms of components, Eq. (29) includes the terms, $[c_1 (|\partial S|^2 + |F_S|^2) +
\( c_2 (|\partial \psi|^2 + |F_\psi|^2 + ...) |\varphi|^2 / M_*^2 \), involving both the kinetic and potential energies of the various degrees of freedom. The only contribution to the energy that is not included in this direct coupling is that arising from \( D \)-terms, since such a coupling is disallowed by gauge invariance. In the case where all chiral superfields in the model couple with equal strength, \( c_i = c \), Eq. (29) couples the total energy density, minus the \( D \)-term contribution, to \( |\varphi|^2 \), leading to an effective mass-squared \(-c(\rho_{\text{tot}} - \rho_D)/M_*^2\) for the scalar component of \( \varphi \). Of course, in general, there is no symmetry that guarantees that all the \( c_i \)'s are of the same magnitude, or even sign, but in the case of interest to us, only the \( S \) field (and perhaps \( \psi_- \) if it is long-lived) contain any significant energy density in the post-inflationary universe. As long as \( c_S \) (and perhaps \( c_{\psi_-} \) also \(^1\)) is of the appropriate sign and \( O(1) \), then it is a reasonable approximation to take \( |\varphi|^2 \) to couple to \((\rho_{\text{tot}} - \rho_D)\).

If we take the leading non-renormalizable (and \( B - L \) violating) superpotential term that lifts the flat direction to be \( h |\varphi|^n / M_*^{n-3} \), the structure of the scalar potential for the flat directions is then

\[
V = \left( m_{3/2}^2 + \Delta m^2 \right) |\varphi|^2 + \frac{m_{3/2}}{M_*^{n-3}} (Ah\varphi^n + \text{h.c.}) + \frac{|h|^2 |\varphi|^{2(n-1)}}{M_*^{2(n-3)}}
\]  

(30)

This includes the explicit soft mass-squareds coming from supersymmetry breaking, the \( |\varphi|^{2(n-1)} \) term arising from the leading non-renormalizable superpotential term, the associated \( A \)-terms for supergravity mediation (with the appropriate power of the gravitino mass \( m_{3/2} \) factored out), and, importantly, the \( \Delta m^2 |\varphi|^2 \) term from expanding out the non-minimal Kähler potential Eq. (29). Notice that there are no \( A \)-terms proportional to \( H \) arising from the finite energy density of the universe after inflation. In an \( F \)-inflationary scenario, such terms do typically arise, but in \( D \)-inflation the form of the superpotential does not produce terms of this type because \( \langle \psi_+ \rangle = 0 \) and thus \( \langle W \rangle = 0 \) at all times.

Lacking any other information, it is natural to expect \( h \) in Eq. (30) to be \( O(1) \). In particular, there is one non-renormalizable term about which we possess some experimental information. As noted in Refs. [4, 3], the operator that lifts the \( LH_u \) flat direction, \( h(LH_u)^2 / M_* \), is the same MSSM operator that gives rise to Majorana masses for the left handed neutrinos. Thus, in this case, the coefficients \( h \) are related to the neutrino mass spectrum. To be precise, there are three \( L_i H_u \) flat directions \((i = e, \mu, \tau)\), and the superpotential operator is \( h_{ij} L_i^a L_j^b H_u^c H_u^d \epsilon_{ae} \epsilon_{bd} \), with \( a, b, c, d \) SU(2) indices. Since the flatter the AD potential the larger the resulting baryon asymmetry, it is the smallest of the couplings in the potential that dominate. If \( m_{ij} \) is the neutrino Majorana mass matrix then the term in the potential for the \( L_i H_u \equiv \phi_i^0 \) direction is \( \sum_j |h_{ij}|^2 |\phi_i|^6 / M_*^2 \), where \( h_{ij} = M_* m_{ij} / v^2 \), and \( v = 175 \text{ GeV} \). Given our expectations for neutrino masses and mixing angles from the MSW solution to the solar neutrino

\(^1\)Unequal coupling coefficients \( c_i \) cause oscillations around some mean value, and can potentially lead to quite complicated behavior at intermediate times, where energy is exchanged between the \( S, \psi_- \) and AD modes.
problem together with the see-saw mechanism, the $L_e H_u$ direction is the most important, with the off-diagonal coupling $h_{e\mu} \sim (10^{18} \text{ GeV})(10^{-4} \text{ eV})/(175 \text{ GeV})^2 \simeq 4$ being dominant\footnote{For the other $L_{\mu,\tau} H_u$ directions the couplings are substantially larger, and therefore produce a much lower final baryon to entropy ratio than the dominant $L_e H_u$ direction.}. Fortunately this is in line with our naive expectations and we will henceforth assume that the dominant $h$’s are $O(1)$.

In any case, after the inflationary phase has ended, the energy density formerly in the $D$-terms is converted to $F$-term and kinetic energy of the $S$ and $\psi_-$ fields, and thus the $\Delta m^2$ term can naturally have the value $\rho_{\text{tot}}/M^2_* \simeq H^2$. For successful AD baryogenesis, the overall sign of the $H$-dependent mass-squared term must be negative; we assume in the following that the sign of the non-minimal Kähler terms are such that this is the case.

For $H^2 \gg m_3^2/2$, the $-H^2|\varphi|^2$ term completely dominates the other mass terms in Eq. (30), and $\langle \varphi \rangle$ is determined by the balance between the negative $H$-dependent (and thus time-dependent) effective mass and the leading non-renormalizable term that lifts the flat direction

$$\langle |\varphi(t)|^2 \rangle \simeq \left( \frac{H(t)^2 M_2^{2(n-3)}}{h^2} \right)^{1/(n-2)}.$$  

(31)

The oscillations of $\varphi$ around this expectation value are critically damped for all values of $H > m_{3/2}$, so $\langle |\varphi|^2 \rangle$ accurately tracks Eq. (31). In addition, the existence of this minimum for $|\varphi|^2$ eliminates any initial value for $|\varphi|$ which may have been generated during inflation by, e.g., quantum fluctuations. Thus it is not enough to consider quantum fluctuations alone as setting non-zero values for $|\varphi|$, since any such values will be washed out by the post-inflationary potential in Eq. (31).

This statement is also true of any initial value for $|\varphi|$ set by the inflationary $D$-term. For example, one could imagine the following seemingly attractive scenario (which is a concrete realization of the suggestion in \cite{24}): If the fields of the AD condensate are originally uncharged under $U(1)_{FI}$ but pick up a small charge through kinetic mixing, then according to Eqs. (23)–(24) the AD condensate will be displaced from the origin at the end of inflation. However as soon as inflation ends and the AD condensate gains an effective mass term $m_\varphi^2 \sim \pm H^2$, its equation of motion will drive $\varphi$ to its new minimum, which is either at the origin (if $m_\varphi^2 > 0$) or given by Eq. (31) (if $m_\varphi^2 < 0$). Since the equation of motion is critically damped, the field will go to its new minimum in roughly a Hubble time. Therefore by the time of baryogenesis, $|\varphi|$ will have lost all information about its value during inflation.

The same, however, is not true for the phase of the AD field. Because in $D$-term inflationary models there are no $H$-dependent $A$-terms in the post-inflationary potential, the equation of motion for $\arg(\varphi)$ is over-damped and $\arg(\varphi)$ does not change until $H \simeq m_{3/2}$. (The only term in the potential for $\arg(\varphi)$ are the usual supergravity $A$-terms. These lead to terms in the equation of motion for $\arg(\varphi)$...
which are small compared to the friction term until \( H \simeq m_{3/2} \). Therefore the value of the phase of \( \varphi \) and its correlation length as set during the inflationary epoch are the values relevant for AD baryogenesis, quite unlike the case of the magnitude of \( \varphi \). In particular, if we are to get a net \( B \) or \( L \), the correlation length for \( \text{arg}(\varphi) \) must be greater than the current Hubble radius of the universe.

What sets the correlation length for the phase of \( \varphi \)? There are two cases to consider: one in which \( |\varphi| \) is massive and thus \( \langle \varphi \rangle = 0 \), and the other in which \( |\varphi| \) is massless and thus \( \langle \varphi \rangle \) can be large. In the first case, which for instance occurs when the U(1)_{FI} charges for \( \varphi \) are positive and \( O(1) \), the correlation length of quantum de Sitter fluctuations is \[ \ell = H^{-1}\text{infl}\exp\left(\frac{3H^2\text{infl}}{2m^2_{\varphi}}\right) \tag{32} \]

where \( m^2_{\varphi} \) is the mass-squared of \( \varphi \). Because \( \langle \varphi \rangle \) is zero classically the phase is ill-defined at the classical level. Therefore any quantum fluctuations of \( \varphi \) pick up random phases with correlation lengths given above. For \( H_{\text{infl}} = g^2\xi_4/6M^2_* \) and \( m^2_{\varphi} \simeq g^2\xi^2 \), the exponent in the correlation length is much smaller than the factor of 55 necessary to fit the entire observable universe into one correlation volume. AD baryogenesis cannot possibly work in this case! Thus we derive the result that the AD field must be a flat direction not only of the MSSM D-terms but also of the U(1)_{FI} D-terms even though the U(1)_{FI} interactions have been integrated out at energies far above those at which baryogenesis is occurring. Notice that a special example of this occurs for AD composites in which the individual fields have bare \( q = 0 \), but pick up non-zero \( q \) via kinetic mixing. Because the AD composite is by definition flat under hypercharge, it will also be flat under U(1)_{FI} even if the component fields are now charged.

For the second, massless case (i.e., \( m^2_{\varphi} \simeq m_{3/2} \ll H \)) the correlation length for de Sitter fluctuations is enormous, much larger than the observable universe. Thus any phase generated for \( \varphi \) will be coherent and can lead to AD baryogenesis. This phase however is completely random and in particular has no reason to be at the minimum selected by the the supergravity A-terms. And since the equations of motion for \( \text{arg}(\varphi) \) are over-damped until \( H \simeq m_{3/2} \), the value set by de Sitter fluctuations is the appropriate initial value for AD baryogenesis. This is to be contrasted with the situation for the magnitude of \( \varphi \), for which the value resulting from de Sitter fluctuations is washed out by the \(-H^2|\varphi|^2\) terms that arise after inflation ends.

However, there are in principle other processes which could destroy the coherence of the AD condensate in the post-inflationary period, in particular scattering off a thermal bath of \( \psi_- \) or \( S \) decay products. If these processes are efficient, then coherence length for the AD field will only be \( \ell \simeq T^{-1} \), much smaller than the radius of the observable universe, and so no net baryon number will be created. In the Appendix we derive the result that such scattering processes are reasonably efficient for AD flat directions lifted at \( n = 4 \) (e.g., the LH_u direction), but completely inefficient for \( n > 4 \), assuming \( \psi_- \) decays before the epoch of baryogenesis. Thus we view the \( n = 4 \)
AD directions as disfavored unless one can arrange $\Gamma_{\psi^{-}} < m_{3/2}$. Such a suppression of the $\psi^{-}$ width is possible, but as argued in Section 3 this requires that none of the MSSM fields have bare U(1)$_{FI}$ charges and that the kinetic mixing be rather small ($\chi < 10^{-6}$).

Now we turn to the actual baryon asymmetry production mechanism. After the end of inflation, the system evolves, with the total energy density, Hubble constant, and value of $\langle |\varphi(t)|^2 \rangle$, gradually decreasing until the supergravity terms in the potential Eq. (30) for the AD direction become comparable to the $-H^2 |\varphi|^2$ term. At this time ($H \simeq m_{3/2}$) the AD field starts oscillating around zero. Most importantly, at this stage all the terms in the potential for $\varphi$ are of comparable magnitude, both baryon (or lepton) number conserving and violating. Also the phase of the $A$ terms together with the a priori random initial phase of the $\varphi$ direction provides significant CP-violating phases in the evolution of the $\varphi$ direction as it oscillates around zero. In these circumstances the AD condensate acquires a baryon or lepton number per condensate particle of magnitude $\varepsilon \sim \mathcal{O}(1)$. The resulting baryon to entropy ratio is given by

$$\frac{n_b}{s} \simeq \frac{n_b}{m_{\varphi} n_{\varphi}} \frac{T_R^{(2)} \rho_{\text{AD}}}{\rho_{\text{tot}}},$$

where the first factor gives the baryon (or lepton) number per particle in the AD condensate, and the second and third factors arise from converting energy densities to entropy and number densities via $\rho_{\text{AD}} = m_{\varphi} n_{\varphi}$ and $\rho_{\text{tot}} \simeq T_R s$. As described in Section 3, the dilution in the baryon to entropy ratio due to a second stage of reheating is (up to a factor of $\mathcal{O}(1)$) taken account of by using the second stage reheat temperature $T_R^{(2)}$ in the expression Eq. (33). Substituting $\rho_{\text{AD}}/\rho_{\text{tot}}$, and taking $m_{\varphi} \sim m_{3/2}$, gives

$$\frac{n_b}{s} \simeq \varepsilon \frac{T_R^{(2)}}{m_{3/2}} \left( \frac{m_{3/2}^2}{h^2 M_{\ast}^2 (n-2)} \right)^{1/(n-2)}.$$  

Requiring this to reproduce the observed baryon to entropy ratio, $n_b/s \simeq (3 - 10) \times 10^{-11}$, leads to a correlation between the order, $n$, at which the flat direction is lifted, and $T_R^{(2)}$ (equivalently the decay width of $S$).\footnote{In principle, there is another constraint on our baryogenesis models that should be considered. The superpotential operator $h(LH_u)^2/M_\ast$ is the MSSM operator that gives rise to Majorana masses for the left handed neutrinos. This operator, together with sphaleron processes, can destroy any asymmetries in both $B$ and $L$, if the lepton-number violating interactions it mediates are in equilibrium after AD baryogenesis and reheat \cite{20}. The resulting constraint on the post-baryogenesis reheat temperature is very weak $T_R \lesssim 10^{16}$ GeV for an MSW-inspired spectrum of neutrino masses and mixings.}

Given this, and further assuming that there is no later dilution of the baryon number (we will consider this possibility below), we now summarize the resulting baryon to entropy ratio as a function of the order at which the AD potential is lifted:

- For $n = 4$ we have already argued that reheating through $\psi^{-}$ decays tends
to decohere the AD condensate and therefore if $\psi_-$ decays before the epoch of baryogenesis (at $H \simeq m_{3/2}$) no net baryon number is created. However, if $\psi_-$ can be made sufficiently long-lived (i.e., $\Gamma_\psi < m_{3/2}$), then baryogenesis takes place in a cold universe (note that this requires the kinetic mixing $\chi < 10^{-6}$). In order to generate a final $n_b/s \simeq (3 - 10) \times 10^{-11}$ then requires $T_R^{(2)} \simeq 10^7 - 10^8$ GeV. This is near the upper limit possible given the gravitino constraint, and nicely fits in with the reheating temperature expected from $S$-decay, Eq. (28), if we take the leading allowed coupling of $S$ to MSSM fields to be of the form $W = S\phi^4/M^2$.

- For $n = 5$, $\psi_-$ decays can occur either before or after baryogenesis without adversely affecting the final baryon to entropy ratio. To derive the measured $n_b/s$ along an $n = 5$ direction requires $T_R^{(2)} \simeq 10^3$ GeV. Notice that this is quite small compared to some of our estimates of typical $S$ reheat temperatures. Thus baryogenesis along an $n = 5$ (or $n > 5$) flat direction runs the risk of overproducing baryons which must then be diluted by an additional dump of entropy at late times. In particular if $T_R^{(2)} \simeq 10^8$ GeV, then the baryons are overproduced by a factor of $O(10^5)$ without a later dilution. On the other hand, if the leading coupling of $S$ to MSSM fields occurs at $r = 6$, then this scenario leads to $n_b/s$ of the correct magnitude.

- For $n = 6$, the necessary reheat temperature is $T_R^{(2)} \simeq 1$ GeV, though larger temperatures again overproduce baryons. For our naive estimate $T_R^{(2)}$ this overproduces baryons by $O(10^8)$; although, again, $S$ decays might be greatly suppressed.

- For $n = 7$, $T_R^{(2)}$ approaches the BBN bound of 6 MeV, and so for $n > 7$ it is always necessary to have a late time entropy dump in order to avoid large $n_b/s$.

Notice that for the $n = 4$ and $n = 5$ directions, the final reheat temperature is above the weak scale and so sphaleron processes are active. Therefore in these two cases, baryon number can be created through leptogenesis followed by sphaleron-induced conversion to baryons as long as the AD condensate had net $B - L$.

Finally one should always bear in mind that supersymmetry models, and in particular supersymmetry-breaking models, generically bring with them other moduli from either the hidden sector or from the string sector which have naturally late decays [27], spoiling the success of Big-Bang nucleosynthesis. If the necessary dilution of these moduli occurs after baryogenesis, the large increase in the entropy will also dilute the baryon number by many orders of magnitude. Therefore it is quite possible that baryogenesis must be over-efficient in the early universe in order to produce the current $n_b/s$ ratio of roughly $10^{-10}$ as a “post-dilution” value. It is comforting to see that, except along the $n = 4$ direction, it is possible to greatly overproduce baryons in the AD scenarios we have investigated. This is an option that other forms of baryogenesis, e.g. electroweak baryogenesis, do not provide.
5 Conclusions

We have demonstrated in this paper several important results about $D$-term inflation. First, we have found that inflation ends in these models at field values of $S$ far above its critical value, and that in order to have sufficient e-folds of inflation, the initial value must be quite large and the potential unusually flat. In particular we found that the potential for $S$ must be flat out to dimension-10 terms in the superpotential, a requirement that seems to point to the existence of $R$-symmetries which protect the flatness of the $S$ potential.

Second, we have found that reheating in these models is generically immediate because of the $O(1)$ couplings of the $\psi_-$ field to light matter. The corresponding first-stage reheat temperature is about $10^{15}$ GeV. Even if we tried to further push down that reheat temperature by constraining the couplings of $\psi_-$ to light matter, we found that it would be an unnatural tuning of the model for its temperature to fall below about $10^{10}$ GeV. Nevertheless, the $D$-term models very nicely overcome the gravitino constraints since they possess a second condensate (the $S$-field) whose suppressed decay width naturally leads to a reheat temperature $T_R < 10^9$ GeV.

Finally, we have shown that Affleck-Dine baryogenesis can proceed in models of $D$-term inflation in much the same way in which one expected it in models of $F$-term inflation. That is, even though there are not $F$-terms present during inflation, they do appear at the end of inflation in the form of either oscillating $\psi_-$ and $S$ fields, or in the finite energy densities present in the plasma after their decays. This energy density can, given a needed sign in the Kähler potential, produce negative effective squared-masses for the AD condensate, pushing its modulus away from its true minimum at the origin until $H \simeq m_{\text{weak}}$. Meanwhile the phase of the AD condensate has been set by de Sitter fluctuations which are inflated to superhorizon-size correlation lengths. The mismatch between this random phase and the phase preferred by the low-temperature scalar potential provides the $CP$ violation necessary for baryogenesis.

We find that the high-temperature environment produced by the $\psi_-$ decays can decohere the AD condensate if its potential is only lifted at $n = 4$ in the superpotential; however, AD flat directions lifted at $n > 4$ stay coherent over super-horizon volumes and thus produce substantial baryon number.

Thus $D$-term inflationary models can accommodate successful phenomenology, including very flat slow-roll potentials mandated by symmetries, reheating consistent with gravitino and nucleosynthesis constraints, and an efficient and appealing baryogenesis mechanism.

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Appendix

In this Appendix we will show that the thermal bath of MSSM particles created by decay of the $\psi_-$ field can decohere the phase of the AD condensate when it is lifted at $n = 4$ order, but not if lifted at $n \geq 5$.

Consider first scattering of thermal MSSM fields off of the AD condensate via gauge interactions. Any gauge boson which couples to the AD field will get a mass $g \langle \varphi \rangle$ so that the thermally averaged scattering rate per AD mode is approximately

$$\Gamma_{\text{scatt}} = n_\phi \langle v \sigma \rangle \sim \frac{g^4 T^2}{16\pi (T^2 - g^2 \langle \varphi \rangle^2)^2} T^3.$$  \hfill (35)

where we have assumed that the particles in the thermal bath are relativistic. (This should always be the case since, e.g., the selectron is always light compared to $T$ or $\langle \varphi \rangle$ and so is copiously produced in the thermal bath.)

For $n = 4$ flat directions, $g \langle \varphi \rangle \sim T \sim \sqrt{H M_*}$ for $\psi_-$ decaying at $H \simeq \Gamma_{\psi_-}$ (we assume $h \sim O(1)$ throughout this appendix). If $\psi_-$ decays immediately after the end of inflation then $\Gamma_{\text{scatt}} \simeq T/16\pi \simeq \xi/16\pi > H \simeq \xi^2/M_*$ and thus there are many scattering per Hubble time per AD mode and the AD condensate is evaporated into an incoherent collection of individual particles. If the $\psi_-$ decays late, but prior to $H \simeq m_{3/2}$, the scattering rate is even larger compared to $H$ and so again the AD condensate is evaporated. Of course, if $\psi_-$ decays after $H \simeq m_{3/2}$, that is, after baryogenesis, then there is no difficulty getting the currently observed baryon asymmetry.

For $n = 5$ flat directions, $g \langle \varphi \rangle \sim (H M_*^2)^{1/3} \gg T \sim \sqrt{H M_*}$ so that $\Gamma_{\text{scatt}} \simeq T^5/16\pi \varphi^4$. It is easy to show that for $\psi_-$ decays at any time after the end of inflation, $\Gamma_{\text{scatt}} \ll H$ and so the AD condensate is not evaporated by gauge interactions. This result holds also for all flat directions with $n \geq 6$.

There is one other scattering process which could evaporate the AD condensate coming from superpotential couplings: $\mathcal{L} = \lambda^2 |\phi|^2 |\varphi|^2$ where $\lambda$ is a usual Yukawa coupling. It is useful to consider the two limits in which the MSSM particles in the thermal bath are relativistic or non-relativistic. These correspond respectively to small $\lambda \ll 1$ (typical of the first two generations of MSSM squarks and sleptons) and to large $\lambda \sim 1$ (typical of the third generation) since we expect that particles in the thermal bath to acquire masses through the vev of $\varphi$ which are of order $\lambda \langle \varphi \rangle$.

Consider first the case of a non-relativistic thermal bath. The non-relativistic scattering rate is calculated using $n_\phi = \rho_\phi / m_\phi$, relative velocity $v_\phi \simeq \sqrt{T/m_\phi}$ and $\sigma \simeq \lambda^4 / 16\pi m_\phi^2$. Specifying an $n = 5$ flat direction and using the expression for $\langle \varphi \rangle$ in Eq. (31), one finds

$$\Gamma_{\text{scatt}} \simeq \lambda^{1/2}H(H/M_*)^{1/12} / 16\pi$$  \hfill (36)

(again we are calculating at the point of reheat where $T \simeq \sqrt{H M_*}$). Thus $\Gamma_{\text{scatt}} < H$ and the AD condensate is not evaporated; this is even more true as the reheat period approaches (from above) $H \simeq m_{3/2}$. Thus the non-relativistic component of
the thermal bath does not scatter off the AD condensate with sufficient strength to decohere the phase of the condensate. This argument generalizes for all $n \geq 5$.

For a relativistic thermal bath, the “worst case” occurs when $\psi_-$ decays right before baryogenesis (so $H \simeq m_{3/2}$). Then

$$\Gamma_{\text{scatt}} \simeq \frac{\lambda^4}{16\pi T^2} T^3 \simeq \frac{\lambda^4}{16\pi} \sqrt{m_{3/2} M_*}$$

(37)

which is less than $H$ as long as $\lambda \lesssim 10^{-2}$. (Note that this does not depend on the order at which the AD flat direction is lifted.) But this is essentially the same condition for the $\phi$ fields to be relativistic. In particular, for immediate decay of $\psi_-$, one has $T > \lambda \langle \varphi \rangle$ precisely for $\lambda \lesssim 10^{-2}$. Therefore we conclude that the relativistic component of the thermal bath also does not efficiently decohere the phase of an AD condensate lifted at order $n \geq 5$.

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