\( \alpha \) clustering and flow in ultra-relativistic heavy-ion collisions

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Abstract. We show how ultra-relativistic collisions of light nuclei with heavy targets may be used to record snap-shots of the ground-state configurations and reveal information on cluster correlations. The development of collective flow in the formed fireball, which reflects the geometric correlations in the initial state, is essential for the method. As an illustration we analyze the \(^{12}\)C-\(^{208}\)Pb collisions.

This talk is based on [1], where the relevant details can be found. We propose a novel way of investigating the ground-state correlations in light nuclei, which provides a surprising bridge between the lowest-energy nuclear structure and highest-energy nuclear reactions, where collective flow of the fireball develops. This flow transmutes the correlations in the initial state (such as those due to the \( \alpha \) clusters) into specific measurable transverse-momentum asymmetries in the spectra of the produced hadrons.

While the concept of the \( \alpha \) clustering is more than 80 years old [2] (for reviews see, e.g., [3, 4, 5, 6, 7, 8]), numerous issues still remain open. Even the ground state structure of light nuclei, such as \(^{12}\)C, is a topic of active research (see various contributions to these proceedings). With this in mind, our method, inherently investigating multiparticle correlations leading to collective effects (flow) may provide a novel insight. We stress that since the clusterization phenomenon concerns multi-particle correlations (see, e.g., [9]), it is accessible directly only through observables which are many-body. Thus the typically studied one-body quantities, such as excitation spectra of the EM form factors, by definition cannot “prove” clusterization in a direct manner.

To model the collision process, we specifically need the distribution of centers of nucleons in \(^{12}\)C, which is nothing but the ground state nuclear wave function squared, \(|\Psi_A(\vec{x}_1, \ldots, \vec{x}_A)|^2\). While it would be best to incorporate realistic calculations (see, e.g., [13, 14]), in Ref. [1] we have applied a simple and practical procedure with \( \alpha \)-clustered (or unclustered for comparison) random distributions. In the \( \alpha \)-clustered case we randomly generate positions of the 12 nucleons, 4 in each cluster of a Gaussian shape and size \( r_\alpha \). The centers of the clusters are placed in an equilateral triangle of side length \( l \). The short-distance NN repulsion is incorporated by precluding the centers of each pair of nucleons to be closer than the expulsion distance of 0.9 fm [15]. The parameters \( l \) and \( r_\alpha \) are optimized such that the one particle density \( \rho(r) \) of BEC [11] or FMD [12] calculations are accurately reproduced (standard unfolding the proton density...
charge density from the charge distribution $\rho^{em}(r)$ is necessary), see Fig. 1. Note a large central depletion in the distributions, originating from the separation of the $\alpha$ clusters arranged in the triangular configuration. Besides, a fair reproduction of two particle densities $\rho_2(r)$ from multiclustered Jastrow correlated calculations [13] is observed. The radial distribution $4\pi r^2 \rho_2(r)$ peaks at the size of the triangle, $l \approx 3$ fm. Thus, we deal with realistic nuclear distributions.

The essence of our approach is as follows: $\alpha$ clusters lead to substantial intrinsic deformation of nucleon distributions in light nuclei. Since the time of the reaction in ultrarelativistic collisions is much shorter from any characteristic time of the nuclear structure (the Lorentz contraction factor for the colliding nuclei reaches 100 at RHIC!), an instant snapshot of the frozen light nucleus configuration is made, revealing the lumpy structure when present. Consequently, when a geometrically deformed nucleus hits a large target at almost the speed of light, the created fireball in the transverse plane inherits the shape of the light nucleus (cf. Fig. 2-4). With a large target, the created fireball is abundant enough to evolve collectively, in full analogy to ultrarelativistic collisions of two heavy nuclei as studied recently in colliders (RHIC, LHC) or fixed-target experiments (SPS). Due to the initial fireball deformation, a deformed flow pattern develops, leading to azimuthal asymmetry in the transverse momentum distributions.

Figure 1. Upper left: Normalized one-particle distributions in $^{12}$C. The electric charge density $\rho^{em}(r)/Z$ (thin lines) and the corresponding distribution of the centers of nucleons $\rho_1(r)$ (thick lines) in $^{12}$C for the data [10] and BEC [11] calculations (dashed lines), FMD [12] calculations (solid lines), and Jastrow correlated wave function [13] (dotted line). Upper right: Charge Form factor $F^{em}(q)$ in $^{12}$C as a function of the momentum transfer $q$ (in fm$^{-1}$) for BEC (dashed line) and FMD calculations (solid line). Lower left: Normalized two particle distribution $\rho_2(r)$ in $^{12}$C. We show our results for the fitted $\rho(r)$ in the FMD (thick line) and BEC cases (dashed line), and compare to the Jastrow correlated wave function [13] (dotted line). Lower right: The same as in lower left panel for the radial distribution $4\pi r^2 \rho_2(r)$. 
of the hadrons (mostly pions) produced in the collision. This asymmetry can be analyzed and measured event-by-event through well-established methods [16, 17, 18]. Here we focus on the promising $^{12}\text{C}+^{208}\text{Pb}$ system, as the $^{12}\text{C}$ has a large intrinsic triangularity [19, 12], resulting in large triangular flow, increasing strongly with the multiplicity of the produced hadrons [1].

In our study we apply GLISSANDO [20, 21] to model the early phase of the collision within the Glauber Monte Carlo approach. The eccentricity parameters $\epsilon_n$ are convenient measures of the harmonic components of the intrinsic deformation. They are defined for each collision (event) as the coefficients of the Fourier decomposition of the distribution in the transverse plane,

$$
\epsilon_n e^{in\Phi_n} = \frac{\sum_j w_j \rho_j^n e^{in\phi_j}}{\sum_j w_j \rho_j^n},
$$

where $j$ labels the sources in the event, $\rho_j$ is the transverse position of the source, $w_j$ its weight, $n$ indicates the rank, and, finally, $\Phi_n$ is the principal axis angle in the event. The $n = 2$ deformation is referred to as ellipticity, and $n = 3$ as triangularity. The notion of the source is used in the Glauber-model sense, and indicates the wounded nucleons [22] or binary collisions.
The collective dynamics of the intermediate evolution of an ultra-relativistic nuclear reactions is expected to be properly modeled with hydrodynamics (for recent reviews see, e.g., [23, 24] and references therein) or transport models [25]. These methods lead to an event-by-event transmutation of the initial-state anisotropies, quantified with the $\epsilon_n$ coefficients, into the harmonic flow coefficients of the transverse momentum distributions of the produced particles, known as $v_n$ [16, 17, 18]. Since it is well established that the proportionality $v_n \sim \epsilon_n$ holds (see, e.g., a recent analysis [26]), one may gain the relevant information on the physically relevant $v_n$ coefficients by studying the eccentricities $\epsilon_n$, which is way simpler, as no costly hydrodynamic or transport studies are needed (such full-fledged studies are underway).

Before showing our key results (here shown for the distributions reproducing the BEC case), let us present a very specific and crucial for understanding correlation between the “geometry” and the multiplicity of particles produced in the collision (which is proportional to the number of the wounded nucleons, $N_w$). The orientation of the “little triangle” describing the $^{12}$C nucleus with respect to the transverse plane is random in each event; sometimes the collision is flat-on, sometimes side-wise, or assumes any intermediate angle (cf. Fig. 5). In the flat-on case the damage created by the projectile in the target is largest, as the geometric cross section is highest (at ultra-relativistic energy the projectile nucleon wounds everything in its straight path!). At the same time, triangularity is highest and the ellipticity is lowest. For the side-wise orientation, the effects are opposite. Thus we find positive correlation between multiplicity and triangularity, and negative correlations between multiplicity and ellipticity.

This simple quantitative effect is clearly seen in actual simulations with GLISSANDO, presented in the left panel of Fig. 6. The displayed growth of the event-averaged $\epsilon_3$ and the falloff of the event-averaged $\epsilon_2$ are the advocated signatures of the $\alpha$ clusterization in $^{12}$C. The case of the unclustered $^{12}$C (i.e., with the nucleons distributed uniformly but with exactly the same one-body density as in the clustered case) is shown in the right panel of Fig. 6. We note a very similar behavior of $\left< \epsilon_2 \right>$ and $\left< \epsilon_3 \right>$ as functions of $N_w$. The fact that these are non-zero is due entirely to fluctuations [26] of the uniform system with a finite number of sources. In addition, we show the scaled standard deviations of the event-by-event values of $\epsilon_3$ and $\epsilon_2$. These measures are also sensitive to clusterization.

We note that the presented analysis is similar in spirit to the studies of the d-A collisions [27] and $^3$H/$^3$He-A collisions [28] (to be investigated experimentally at RHIC), however, the collective effects in a larger system, such as in $^{12}$C-$^{208}$Pb presented here, are expected to be much stronger, hence the shape-flow transmutation should be more visible. In particular, the separation of the geometric component of the flow from the non-flow effects due to fluctuations should be easier.

In summary, our method, which links the cluster features of lowest-energy nuclear structure physics with flow phenomena known from the highest-energy nuclear collisions, offers a novel possibility to study geometric correlations in the ground state. Viewed from the opposite direction, a detailed knowledge of the clustered nuclear wave functions (obtained, e.g., from the ab initio numerical analyses) will help to place important constraints on the models of the fireball evolution and thus gain information on the properties of the quark-gluon plasma.
Figure 6. Event-by-event average ellipticity and triangularity of the fireball, as well as their scaled standard deviations, plotted as functions of the number of wounded nucleons $N_w$. Left: the case where the $^{12}$C nucleus is clustered according to the BEC model. Right: uniform distribution (no clusters). SPS energies ($\sigma_{NN}^{\text{inel}} = 32$ mb), mixed Glauber model [1].

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