Reshaping of Dirac cones by Floquet engineering

A. Díaz-Fernández¹, E. Díaz¹, Á. Gómez-León², G. Platero² and F. Domínguez-Adame¹

¹ GISC, Departamento de Física de Materiales, Universidad Complutense, E–28040 Madrid, Spain
² Instituto de Ciencia de Materiales de Madrid (ICMM-CSIC), Cantoblanco, E–28049 Madrid, Spain

E-mail: alvaro.diaz@ucm.es

Abstract.

Dirac cones seem to be ubiquitous to nontrivial materials, such as topological insulators and graphene. Manipulating the aforementioned Dirac cones is expected to have consequences on the quantum transport properties of these materials. In this work, we propose to Floquet-engineer Dirac cones at the surface of a three-dimensional topological insulator, as well as in graphene. We show that a large tunability of the Fermi velocity can be achieved, as a function of the polarization, direction and amplitude of the driving field. Using this external control, the Dirac cones in the quasienergy spectrum may become elliptic or massive, in accordance to experiments done in these materials. These results help us to understand the interplay of surface states and external ac driving fields in topological insulators.

PACS numbers: 73.20.At, 73.22.Dj, 81.05.Hd

Keywords: Floquet engineering, topological insulators, graphene.

Submitted to: New J. Phys.
1. Introduction

During the last two decades, new Dirac materials such as topological insulators, graphene and other carbon-based materials have emerged. These are foreseen to surpass the reach of semiconductors. Apart from their robustness to defects, stemming either from topological protection or symmetry, their linear dispersion is very much like that of photons, except for their quantum statistics and their much lower velocities. Different mechanisms have been put forward to modify the properties of these cones. For instance, breaking time-reversal symmetry in graphene leads to the quantum anomalous Hall effect, a system introduced by Haldane [1] in the 1980’s and experimentally realized very recently using ultracold atoms [2]. In this case, bandgaps open up in the otherwise gapless spectrum and the system becomes a topological insulator that can host chiral edge states. Other alternatives put their emphasis towards modifying the Fermi velocity [3–8], a crucial parameter in quantum transport [9]. As an example, applying static, uniform electric and magnetic fields to three-dimensional topological insulators such as Bi$_2$Se$_3$ widen the cone elliptically, so that the Fermi velocity is reduced in an anisotropic fashion [10][13].

Remarkably, however, the use of periodic drivings is dramatically expanding the possibilities in these Dirac materials. Indeed, examples are now not only found in solid state systems [14], but also in photonics [15] or even acoustics [16]. All these make use of what is known as Floquet’s theorem. Although the words are now mainstream in the scientific community, Floquet’s theorem is most well-known in its real space version, that is, Bloch’s theorem. Indeed, the discrete periodicity of a lattice in real space leads to the concepts of energy bands and Brillouin zones. The same knowledge can be directly transferred to the domain of discretely time-periodic systems. In this case, there are quasienergies, in analogy to the quasimomentum of Bloch’s theory, Floquet-Brillouin zones, and so forth [17][20]. In regard to the study of Dirac cones, it has been experimentally observed [21] and theoretically discussed [22][24] that these can be notably altered by applying time-periodic fields. In fact, depending on the polarization, it is possible to open up a gap in the quasienergy spectrum.

In this paper, we will use a model that was introduced in a series of seminal papers started off by Volkov and Pankratov [25–29] in the 1980’s and that is regaining very much interest lately in the context of surface states in three-dimensional topological insulators [30][31]. Different orientations of the applied field with respect to the surface, as well as different polarizations, lead to a variety of situations that should be observable in experiments, some of which have already been shown to exist [21]. In particular, an in-plane, circularly polarized field leads to gap openings, in accordance to the results of Ref. [21]. Other configurations, however, preserve the Dirac point, isotropically or anisotropically widening the Dirac cone [22]. We will show that the slope of the Dirac cone decreases quadratically with the field, very much like in the static case [10]. Finally, we will discuss briefly the case of graphene. Just like in the topological insulator case, an in-plane circularly polarized field leads to gap openings in the spectrum, in agreement
Reshaping of Dirac cones by Floquet engineering

3
to Ref. 24, whereas a linearly polarized field leads to a reduction of the slope. This feature has also been predicted for the static case in graphene armchair nanoribbons and metallic carbon nanotubes 10.

2. Topological Boundary

Topological materials can be characterized by an integer that is related to discrete symmetries of the bulk. For instance, Chern insulators are characterized by nonzero Chern numbers that arise when breaking time-reversal symmetry 32. The words topological insulators are usually reserved to systems that do preserve time-reversal symmetry and are generally classified according to $Z_2$ indices 32. Since an integer cannot change continuously, if two insulators of different topological index are placed together, at their interface there must be gapless modes. Otherwise, both systems would be connectable in a continuous way, implying that their invariants must be the same. As a result, the edge in two-dimensions or surface in three-dimensions formed in the contact region between these materials is known as a topological boundary 33. In this section, we will consider Bi$_2$Se$_3$, an outstanding candidate for the foreseen applications of these materials. This is in part because of its wide bandgap, which allows it to perform even at room temperature 32, but also because it is a well-known thermoelectric material and its experimental growth is now almost routine. The model is based on $k \cdot p$ theory and it was put forward by Volkov and Pankratov in the 1980s and it is currently recapturing very much attention 25,30,31.

In the orbital-spin basis, $\{\tau, \sigma\}$, the bulk Hamiltonian of Bi$_2$Se$_3$ is a Dirac-like Hamiltonian of the form 30,33

$$H = \alpha \cdot (k + A) + \beta,$$

where $\alpha = (\alpha_x, \alpha_y, \alpha_z)$, being $\alpha_j = \tau_x \otimes \sigma_j$ with $j = x, y, z$, $\beta = \tau_z \otimes 1_d$ the Dirac matrices, $\tau_j$ and $\sigma_j$ the Pauli matrices and $1_d$ the $d$-dimensional identity matrix. Hereafter we will set $\hbar = 1$. Energies will be expressed in units of half the bulk gap, $\Delta = E_G/2$, and there is a natural length scale, $d = v_F/\Delta$, where $v_F$ is the Fermi velocity. Momentum $k$ is therefore expressed in units of $1/d$ and the vector potential $A$ in units of $1/ed$, where $e$ is the elementary charge. In Bi$_2$Se$_3$, $E_G \simeq 350$ meV and $v_F \simeq 25$ eV nm, leading to $d \simeq 2$ nm. The spectrum of this Hamiltonian in the absence of driving fields, that is, if $A = 0$, corresponds to that of a massive Dirac fermion, $E(k) = \pm \sqrt{1 + k^2}$ with $k = |k|$, the two bands being doubly degenerate. In addition, the eigenstates of equation (1) are characterized by a non-vanishing $Z_2$ topological invariant given by $\nu = \text{sgn}(\Delta)$ 33.

In this case, a topological boundary is formed by introducing a position-dependent gap. This allows the system to have opposite bandgaps on each side of the boundary, changing the value of the $Z_2$ topological invariant. The actual meaning of this is that the gap, defined as a difference between band edges of a certain orbital character or parity, changes sign because of a band inversion. Therefore, if we form a boundary
between two systems with opposite bandgaps described by this Hamiltonian, there will be a change in the topological index and, as a result, there will be gapless modes at the boundary. Indeed, in the simplest case of a symmetric junction, the Hamiltonian above is modified to

\[ H = \alpha \cdot (k + A) + \beta \text{sgn}(z), \]  

where \( z \) is the coordinate along the growth direction. It is not particularly difficult to show that in this case there is a midgap state, localized at the boundary with a localization length of \( d \) and extended along the boundary plane. The dispersion in that plane is that of a single Dirac cone, \( E(k_\perp) = \pm k_\perp \). Here the subscript \( \perp \) indicates that the \( z \) component of a vector is zero. These cones can coexist with doubly degenerate massive Volkov-Pankratov states if the interface is sufficiently smooth [30], in contrast to the sharp interface considered in this paper. Interestingly enough, applying static external electric and magnetic fields, it is possible to anisotropically widen the cone, therefore leading to an effective reduction of the Fermi velocity [10–13]. In fact, it can be explicitly shown that the Fermi velocity decreases with the applied field in a quadratic manner [10]. As we will show below, specific configurations of the irradiated samples share this exact same characteristic.

3. Floquet engineering

If we apply a time-periodic driving to the system instead of static fields, a wider range of situations occur. It is known from the use of surface effective Hamiltonians that a circularly polarized field will lead to gap openings [24]. However, only in the case of graphene it has been shown that the Dirac cones become strongly anisotropic in the case of linearly polarized fields [22]. In the following, we shall show that these two features arise when the topological boundary Hamiltonian above is considered. Moreover, it allows us to consider out-of-plane configurations, which are not accessible to the aforementioned surface effective Hamiltonians.

In the spirit of Ref. [23] we consider the system size to be small enough so as to ignore any spatial dependence of the field. In that case, we can choose the vector potential components to be

\[ A_j(t) = a_j e^{i\omega t} + a_j^* e^{-i\omega t}, \]  

where \( a_j = (f_j/2\omega) \exp(i\theta_j) \). Here, \( f_j \) are the components of the electric field, \( F(t) = -\partial_t A(t) \), measured in units of \( \Delta/ed \), \( \omega \) is the driving frequency measured in units of \( \Delta \), and \( \theta_j \) are phases which can be tuned to obtain different polarizations. The symmetries of this problem allow us to introduce three good quantum numbers. On the one hand, as a consequence of continuous translational symmetry in the \( XY \) plane, the in-plane momenta \( k = (k_x, k_y, 0) \) are good quantum numbers. On the other hand, discrete translational symmetry in time leads to the quasienergies, a central concept in Floquet theory. The discreteness of this symmetry restricts the quasienergies to the first
Floquet-Brillouin zone, \( \varepsilon \in [-\omega/2, \omega/2] \), very much like the quasimomentum in a lattice. All in all, it is possible to express the envelope function upon which the Hamiltonian acts as follows

\[
\Psi(r, t) = e^{-i\varepsilon t} e^{ik \cdot r} \Phi(z, t),
\]

(4)

where \( \Phi(z, t) = \Phi(z, t + T) \) and \( T = 2\pi/\omega \). Notice that the problem is now very much simplified. Indeed, there is now only a \( z \)-dependence and the problem is reduced to a unit cell of size \( T \) along the time axis. Hence, the equation to be solved for \( \Phi(z, t) \) is given by

\[
\varepsilon \Phi(r, t) = (H - i\partial_t) \Phi(r, t).
\]

(5)

Taking advantage of the periodicity of \( \Phi(z, t) \), we can Fourier expand

\[
\Phi(z, t) = \sum_{l=-\infty}^{\infty} \varphi_l(z) e^{-il\omega t}.
\]

(6)

Indeed, it is possible to find straightforwardly an equation for the Fourier components

\[
\varepsilon \varphi_l(z) = [\alpha \cdot k + \beta \text{sgn}(z) - l\omega \mathbb{1}_4] \varphi_l(z) + J \varphi_{l+1}(z) + J^\dagger \varphi_{l-1}(z),
\]

(7)

where \( J = \alpha \cdot a \), with \( a \) a vector whose components are the previously defined \( a_j \)'s.

Several comments are in order before continuing. The first is that, if we remove the field by setting \( J = 0 \), the result is similar to that of free electrons when we imagine folding the energies by artificially introducing Brillouin zones. That is, the spectrum in the first Floquet-Brillouin zone can be obtained by repeatedly folding the spectrum for the driving-free case. For instance, for the topological boundary, the first Floquet-Brillouin zone displays evenly spaced cones, where the separation between consecutive Dirac points is \( \omega \). Similar to the free electrons’ case where the presence of a potential may open up energy gaps at the edges of the Brillouin zone, the presence of a non-zero \( J \) leads to avoided crossings at the edges of the Floquet-Brillouin zone [21]. The second point to notice is that, in the absence of boundary, that is, if there is no \( z \)-dependence, the equation is similar to that of a nearest-neighbors tight-binding problem with four orbitals per lattice site and a site dependent on-site energy due to the factor \( l\omega \). This case is readily solved by diagonalization of a block-tridiagonal matrix. Third, time-reversal symmetry is broken only if a circularly polarized laser field is applied. Indeed, if the laser is linearly polarized, we can always choose the phases to be zero and, as a result, \( J \) would be real. Alternatively, we can write \( J = \exp(i\theta) \tilde{J} \), where \( \tilde{J} \) is Hermitian and the phase factor can be eliminated via a gauge transformation of the form \( \varphi_l \rightarrow \exp[-i(l-1)\theta] \varphi_l \). Therefore, it is expected that a circularly polarized field will lead to gap openings, whereas a linearly polarized field will not. We shall see in the following that this is indeed the case when the field is properly oriented.

In order to make further progress in the topological boundary case, it becomes necessary to discretize the Hamiltonian in the \( z \)-direction. Following Ref. [34], it is convenient to perform an alternate sampling of the components of \( \varphi_l \). That is, we will consider the discrete lattice in the \( z \)-direction to be composed of two sublattices, one
for the even sites and one for the odd sites. The first and fourth components of $\varphi_l$ will be sampled in the even sites, whereas the second and third components in the odd ones. This is explained in further detail in the Supplementary Information.

4. Results and Discussion

In this section, we shall discuss four different cases of orientation and polarization of the incident field: in- and out-of-plane, linearly and circularly polarized fields. In all cases we shall consider frequencies that are larger than the bulk bandgap in order to clearly observe the effects of driving for a large range of momenta. Also, we have placed the system in a box that is considerably larger than $d$. In order to obtain meaningful results, it is necessary to notice that the infinite continuum of states above and below the bulk gap will lead to a quasienergy spectrum in the first Floquet-Brillouin zone that will be completely full due to the band folding effects. However, these states do not interact with the topological surface states since they are well separated in space for sufficiently small fields. This can be understood easily in the static case: the band edges bend due to the presence of the field, allowing the continuum states to have energies within the bulk energy gap. However, these states are far from the boundary if the field is small enough and, as a result, they do not interact with the very localized surface states. This discussion implies that the discretization step has to be chosen in such a way as to avoid states from the continuum entering the gap in order to fully assess the effect of the driving on the Dirac cone. Hereafter, non-zero field amplitudes will be the same in all directions and we shall denote them collectively by $f$.

Before considering each of the four cases in detail, it is worth to list the results very briefly. First, only for the circularly polarized in-plane field there is a gap opening. This make sense because we are only breaking time reversal symmetry in such a situation, since the cone lives within the plane. For the circularly polarized out-of-plane field, there is no gap opening because the projection of the field onto the $XY$–plane is that of a linearly polarized field. Second, in all four cases, the resulting cone, or the double-sheeted hyperboloid in the case of a circularly polarized in-plane field, widen isotropically or anisotropically, depending on the orientation and polarization, upon increasing the field. In the static case we would refer to the slope as the Fermi velocity and we shall do the same in the graphs below for simplicity of notation. That is, we will denote the reduction of the slope as a change in the Fermi velocity. We shall see below that it is possible to perfectly fit the change in Fermi velocity as a function of $f/\omega$ to a quadratic function of the form $v_F(f)/v_F(0) = 1 - \gamma(f/\omega)^2$, where $\gamma$ depends on the orientation, the polarization and the frequency of the driving field. Remarkably, this is the exact same behaviour as in the static case of a uniform electric field perpendicular to the boundary. This is related with the fact that, for large enough frequencies of the driving field, the effective Hamiltonian can be approximated by a stroboscopic one. Hence we expect deviations from this behavior to appear, as the driving frequency is decreased and inter-sidebands transitions become more important.
Reshaping of Dirac cones by Floquet engineering

Having said that, let us consider those orientations that have already been reported in the literature for graphene and for effective surface Hamiltonians of topological insulators \[22, 24\]. That is, we consider in-plane fields with linear and circular polarizations. In the first case, the Dirac cones become anisotropic. Just like in graphene, the cone only widens in the perpendicular direction to the field, therefore leading to an effective reduction of the velocity in that direction. This is qualitatively shown in figure 1(a) and quantitatively in figure 1(b). Moreover, in the latter we can see the perfect adjustment to a quadratic fit, as mentioned previously. A similar result has been obtained in the context of static, crossed electric and magnetic fields \[13\]. In this case, if the electric field is zero and we apply an in-plane magnetic field the cone widens only in the perpendicular direction to that of the applied field.

![Figure 1](image)

**Figure 1.** (a) Dispersion relations in a topological boundary with no field and with an in-plane linearly polarized field. The Dirac cone widens anisotropically and the slope decreases quadratically with the field amplitude. Widening only occurs in the direction perpendicular to the applied field. (b) Velocity as a function of $f/\omega$ for different values of $\omega$. Solid lines correspond to a quadratic fit of the form $1 - \gamma(f/\omega)^2$, $\gamma$ being a fitting parameter.

If the in-plane field is circularly polarized, a gap $2\delta$ opens up, as shown qualitatively in figure 2(a) and quantitatively as a function of $f/\omega$ in figure 2(b). This has been already known to occur by means of perturbation theory \[24\] and has been demonstrated in experiments \[21\] as well. However, to our knowledge, it has not been discussed that the resulting double-sheeted hyperboloid widens isotropically as well. In figure 2(c) we show its dependence on $f/\omega$. Moreover, both the velocity and the gap can be fitted to quadratic power laws of the form $1 - \gamma(f/\omega)^2$ and $\lambda(f/\omega)^2$, respectively, with $\gamma$ and $\lambda$ two fitting coefficients. The gap that opens up can be tuned by modifying the relative phase between the $x$ and $y$ components, $\delta\varphi_{xy} = \varphi_x - \varphi_y$ and by increasing the field. This is shown in figure 3. Indeed, if the phase difference is set to zero or $\pi$ the gap must close, but even the smallest non-zero phase difference breaks time-reversal symmetry and a gap opens up. Therefore, it is expected that the maximum gap will be at $\delta\varphi_{xy} = \pi/2$ and will increase with the field.

Finally, we turn our discussion to those cases where there is at least one out-of-plane component of the field. Let us begin with the study of a linearly polarized field.
Figure 2. (a) Dispersion relations in a topological boundary with no field and with an in-plane circularly polarized field. A gap opens up and the massive Dirac spectrum widens isotropically, the slope decreasing quadratically with the field amplitude. (c) Gap as a function of $f/\omega$ for different values of $\omega$. Solid lines correspond to a quadratic fit of the form $\lambda(f/\omega)^2$. (c) Velocity as a function of $f/\omega$ for different values of $\omega$. Solid lines correspond to a quadratic fit of the form $1 - \gamma(f/\omega)^2$. $\lambda$ and $\gamma$ are two fitting parameters.

Figure 3. Gap, $2\delta$, as a function of $f/\omega$ and the dephasing between the $x$ and $y$ components for an in-plane configuration in a topological boundary for $\omega = 5$.

In this situation, we observe the same physics as that of the static case where the field is perpendicular to the boundary [10, 11]. Indeed, the cone widens isotropically, therefore leading to an isotropic reduction of the velocity. Moreover, just like in the static case, the velocity decreases with the field following a quadratic power law of the form $1 - \gamma(f/\omega)^2$, as mentioned earlier in the text. This is displayed schematically in figure 4(a) and quantitatively in figure 4(b). For the circularly polarized out-of-plane field, however, we observe that the cone widens anisotropically, as shown qualitatively in figure 5(a) and quantitatively in figures 5(b) and 5(c). This can be explained with the results from the linearly polarized in- and out-of-plane fields. Indeed, the in-plane component leads to a reduction in the direction perpendicular to that of the field, leaving the parallel direction untouched. However, the out-of-plane component widens the cone isotropically, therefore leading to a reduction in the direction that had not been widened.
Reshaping of Dirac cones by Floquet engineering

Figure 4. (a) Dispersion relations in a topological boundary with no field and with an out-of-plane linearly polarized field. The Dirac cone widens isotropically and the slope decreases quadratically with the field amplitude. (b) Velocity as a function of $f/\omega$ for different values of $\omega$. Solid lines correspond to a quadratic fit of the form $1 - \gamma(f/\omega)^2$, $\gamma$ being a fitting parameter.

before and increasing the reduction in the direction that had already been affected.

Figure 5. (a) Dispersion relations in a topological boundary with no field and with an out-of-plane circularly polarized field. The Dirac cone widens anisotropically and the slope decreases quadratically with the field amplitude. (b) Perpendicular and (c) parallel velocities to the in-plane field projection as a function of $f/\omega$ for different values of $\omega$. Solid lines correspond to a quadratic fit of the form $1 - \gamma(f/\omega)^2$, $\gamma$ being a fitting parameter.

5. Graphene

Floquet engineering of graphene has already been reported on several occasions, using both tight-binding [23, 35] and continuum descriptions [22, 24]. For completeness, we believe it to be useful to discuss very briefly the properties of graphene under circularly and linearly polarized fields. Indeed, we will show that this Dirac material behaves in exactly the same manner as the topological boundary discussed in previous sections, leading us to postulate that the results obtained in this article should be applicable to
other Dirac materials as well. For graphene, a low energy Hamiltonian can be written in the valley-sublattice basis, \( \{\tau, \sigma\} \). In that case, the envelope functions satisfy a Weyl equation (\( \hbar = 1 \))

\[
i \frac{\partial}{\partial t} \Phi(t) = [v_F \tau_z (k_x + eA_x)\sigma_x + (k_y + eA_y)\sigma_y] \Phi(t) ,
\]

where \( v_F = 65.8 \text{ eV nm} \) is the Fermi velocity and \( e > 0 \) is the electron charge. In bulk graphene, we can focus on a single valley, meaning that we can set \( \tau_z = 1 \) in the previous equation and consider \( \Phi \) as having two components instead of a four. It turns out that it is possible to obtain analytic solutions at the Dirac point for the two cases of interest, that is, linearly polarized and circularly polarized fields. Indeed, the two components of the envelope function satisfy

\[
i \frac{\partial}{\partial t} \begin{pmatrix} \phi_A \\ \phi_B \end{pmatrix} = \begin{pmatrix} 0 & A_- \\ A_+ & 0 \end{pmatrix} \begin{pmatrix} \phi_A \\ \phi_B \end{pmatrix},
\]

where \( A_{\pm} = eA_F \cos(\omega t)(1 \pm i) \) for the linearly polarized field and \( A_{\pm} = eA_F \exp(\pm i\omega t) \) for the circularly polarized one.

In the linearly polarized case, it is not difficult to show that the solution to equation (9) satisfies \( \Phi(T) = \Phi(0) \). If we take into account Floquet’s theorem \( \Phi(T) = \exp(-i\varepsilon T)\Phi(0) \), we can see that the quasienergies at the Dirac point are given by \( \varepsilon = l\omega \), with \( l \) an integer. Therefore, we can see that in the linearly polarized case there is not a gap opening, regardless of the intensity of the laser field. However, for the circularly polarized case, it is possible to show that \( \Phi(t) = M(t)\varphi \), where \( \varphi \) is a constant two-vector containing the integration constants and \( M(t) \) is a matrix given by

\[
M(t) = \begin{pmatrix} e^{-i\Omega_- t} & e^{-i\Omega_+ t} \\ (\Omega_- / eA_F) e^{i\Omega_+ t} & (\Omega_+ / eA_F) e^{i\Omega_- t} \end{pmatrix},
\]

where we have introduced the following quantities

\[
\Omega_{\pm} = \frac{\omega}{2} \left( 1 \pm \sqrt{1 + (2eA_F/\omega)^2} \right).
\]

Floquet’s theorem in this case yields \( \det[M(T) - \exp(-i\varepsilon T)M(0)] = 0 \). The resulting equation for the quasienergies at the Dirac point is given by

\[
\cos(\varepsilon T) = \frac{\Omega_+ \cos(\Omega_- T) - \Omega_- \cos(\Omega_+ T)}{\Omega_+ - \Omega_-}.
\]

Notice that \( \varepsilon \) is periodic with period \( \omega \), as it should be, and the spectrum is particle-hole symmetric, that is, \( \varepsilon \) and \(-\varepsilon\) both satisfy the equation above. Also, if there is no driving, \( \Omega_+ = \omega \) and \( \Omega_- = 0 \), so that \( \varepsilon = l\omega \) with \( l \) an integer, the result that is expected from artificially folding the bulk band structure into Floquet-Brillouin zones. As long as \( x \equiv 2eA_F/\omega \) becomes slightly larger than zero, there is a gap opening. This gap increases in size as \( x \) increases, until \( \varepsilon \) hits the Floquet-Brillouin zone edge at \( \omega/2 \). When that happens, the gap begins to close until it reaches a critical \( x \) where it is
closed. This process repeats itself and it is actually possible to find a closed expression for the values of \( x \) where the gap closes, which are given by

\[
x_m^2 = 4m(m - 1) ,
\]

and those where \( \varepsilon \) hits the zone boundary,

\[
x_m^2 = 4m^2 - 1 ,
\]

with \( m \) a positive integer. Notably, an expansion around \( x = 0 \) in equation (12) shows that \( \varepsilon/\omega \) increases quadratically with \( x \) as \( \varepsilon/\omega \simeq x^2/4 \), meaning that the gap increases with \( x^2 \) for low fields, just like in the topological boundary (see figure 2(b)).

In order to obtain the dispersion relations, we can proceed as in the previous sections. In that case, taking into account equation (3), we can write equation (8) as follows,

\[
i \frac{\partial}{\partial t} \Phi(t) = \left[ v_F \sigma \cdot k + e^{i\omega t} \omega V + e^{-i\omega t} \omega V^\dagger \right] \Phi(t) ,
\]

where \( \sigma = (\sigma_x, \sigma_y) \), \( V = a_x \sigma_x + a_y \sigma_y \) and \( a_j = (eF_j v_F / 2\omega^2) \exp(i\theta_j) \), \( F_j \) being the electric field amplitudes and \( \theta_j \) phases that can be tuned to obtain different polarizations. We propose a solution that satisfies Floquet’s theorem

\[
\Phi(t) = e^{-i\epsilon t} \sum_{l=-\infty}^{\infty} e^{-il\omega t} \varphi_l .
\]

The resulting equation is similar to that of a two-orbital, nearest-neighbour tight-binding model with a site-dependent on-site energy,

\[
\frac{\varepsilon}{\omega} \varphi_l = \left[ \frac{v_F}{\omega} \sigma \cdot k - l\mathbb{I}_2 \right] \varphi_l + V \varphi_{l+1} + V^\dagger \varphi_{l-1} .
\]

Just like in the topological boundary, time-reversal symmetry is broken only in the circularly polarized case, since \( V \) can be chosen to be real in the linearly polarized case, but not in the circularly polarized one. This explains the quasienergy gap that is opened at the Dirac point in the circularly polarized case. This equation can be straightforwardly diagonalized and for \( k = 0 \) we obtain the same results as expected from the analytic calculations. For \( k \neq 0 \) we obtain similar results as to the ones presented earlier, as we anticipated already. Indeed, the cone widens anisotropically for the linearly polarized case, it only widens in the direction perpendicular to the field, and it becomes a double-sheeted hyperboloid with a slope that decreases as the field increases. Remarkably, both the velocity in the linear case and the gap in the circularly polarized case can be fitted to a quadratic function, as in our previous study, but this is not the case for the velocity in the circularly polarized field, in contrast to the results of the previous section. This is shown as a function of \( x \) in figure 6, where frequencies are chosen to be in the visible light range.
6. Conclusions

In this work we have shown that the Dirac cones arising at the surface of topological materials or those from graphene can be altered by using a periodic driving. In fact, it was already known that circularly polarized light breaks time-reversal symmetry and therefore opens up a gap in the otherwise gapless Dirac cones [21, 24]. Both the topology [35] and the edge states that appear in graphene have been studied as well [23]. Here, we have discussed the case of a topological boundary in such a way that we can consider other configurations for the fields. Indeed, we can apply out-of-plane fields and show that the cone can widen isotropically or anisotropically, depending on the polarization. Moreover, we have observed that the reduction in the velocity squares with the applied field, a feature that is found also in the case of static fields [10, 11]. In addition, a time-periodic driving simplifies the experimental setup by eliminating the need for a magnetic field in order to obtain an anisotropic renormalization of the velocity [13]. All of our findings should be easily probed by means of time- and angle-resolved photoemission spectroscopy, as discussed in Ref. [21]. We believe that our results could have an effect also in transport measurements, since a change in the velocity can lead to important reductions of the transmission. This is known to occur for graphene on top of a patterned substrate that effectively changes the Fermi velocity [9]. Using external fields, this could be achieved and lead to further control than the aforementioned setup, since the fields can be changed dynamically whereas the patterned substrate is fixed.
Reshaping of Dirac cones by Floquet engineering

Acknowledgments

The authors thank P. Rodríguez for very enlightening discussions. This research has been supported by MINECO (Grants MAT2016-75955 and MAT2017-86717-P). A. D.-F. acknowledges support from the UCM-Santander Program (Grant CT27/16-CT28/16) and A. G.-L. acknowledges the Juan de la Cierva program.

References

[1] Haldane F D M 1988 Phys. Rev. Lett. 61 2015
[2] Jotzu G, Messer M, Desbuquois R, Lebrat M, Uehlinger T, Greif D and Esslinger T 2014 Nature 515 237
[3] Li G, Luican A, Lopes dos Santos J M B, Castro Neto A H, Reina A, Kong J and Andrei E Y 2009 Nat. Phys. 6 109
[4] Trambly de Laissardière G, Mayou D and Magaud L 2010 Nano Lett. 10 804
[5] Hicks J, Sprinkle M, Shepperd K, Wang F, Tejeda A, Taleb-Ibrahimi A, Bertran F, Le Fèvre P, de Heer W A, Berger C and Conrad E H 2011 Phys. Rev. B 83 205403
[6] Hwang C, Siegel D, Ismach A, Zhang Y, Zettl A and Lanzara A 2012 Sci. Rep. 2 590
[7] Elias D C, Gorbachev R V, Mayorov A S, Morozov S V, Zhukov A A, Blake P, Ponomarenko L A, Grigorieva I V, Novoselov K S, Guinea F and Geim A K 2011 Nat. Phys. 7 701
[8] Miao L, Wang Z F, Ming W, Yao M Y, Wang M, Yang F, Song Y R, Zhu F, Fedorov A V, Sun Z, Gao C L, Liu X, Xue Q K, Liu C X, Liu F, Qian D and Jia J F 2013 Proc. Natl. Acad. Sci. 110 2758
[9] Lima J R F, Pereira L F C and Bezerra C G 2016 J. Appl. Phys. 119 244301
[10] Díaz-Fernández A, Chico L, González J W and Domínguez-Adame F 2017 Sci. Rep. 7 8058
[11] Díaz-Fernández A and Domínguez-Adame F 2017 Physica E 93 230
[12] Díaz-Fernández A, Chico L and Domínguez-Adame F 2017 J. Phys. Condens. Matter 29 475301
[13] Díaz-Fernández A, del Valle N and Domínguez-Adame F 2018 Beilstein J. Nanotechnol. 9 1405
[14] Lindner N H, Refael G and Galitski V 2011 Nat. Phys. 7 490
[15] Rechtsman M C, Zeuner J M, Plotnik Y, Lumy E, Podolsky D, Dreisow F, Nolte S, Segev M and Szameit A 2013 Nature 496 196
[16] Fleury R, Khanikaev A B and Ali A 2016 Nat. Commun. 7 11744
[17] Grifoni M and Hänggi P 1998 Phys. Rep. 304 229
[18] Platero G and Aguado R 2004 Phys. Rep. 395 1
[19] Gómez-León A and Platero G 2013 Phys. Rev. Lett. 110 200403
[20] Cayssol J, Dóra B, Simon F and Moessner R 2013 Phys. Status Solidi RRL 7 101
[21] Wang Y H, Steinberg H, Jarillo-Herrero P and Gedik N 2013 Science 342 453
[22] Syzranov S V, Rodionov Y I, Kugel K I and Nori F 2013 Phys. Rev. B 88 241112
[23] Usaj G, Perez-Piskunow P M, Foa Torres L E F and Balseiro C A 2014 Phys. Rev. B 90 115423
[24] Kitagawa T, Oka T, Brataas A, Fu L and Demler E 2011 Phys. Rev. B 84 235108
[25] Volkov B A and Pankratov O A 1985 Sov. Phys. JETP 62 178
[26] Korenman V and Drew H D 1987 Phys. Rev. B 35 6446
[27] Agassi D and Korenman V 1988 Phys. Rev. B 37 10095
[28] Pankratov O A 1990 Semicond. Sci. Technol. 5 S204
[29] Domínguez-Adame F 1994 Phys. Status Solidi B 186 K49
[30] Tchoumakov S, Jouffrey V, Inhofer A, Bocquillon E, Plaçais B, Carpentier D and Goerbig M O 2017 Phys. Rev. B 96 201302
[31] Inhofer A, Tchoumakov S, Assaf B A, Féve G, Berroir J M, Jouffrey V, Carpentier D, Goerbig
M O, Plaçais B, Bendias K, Mahler D M, Bocquillon E, Schlereth R, Brüne C, Buhmann H and Molenkamp L W 2017 Phys. Rev. B 96 195104

[32] Hasan M Z and Kane C L 2010 Rev. Mod. Phys. 82 3045
[33] Zhang F, Kane C L and Mele E J 2012 Phys. Rev. B 86 081303
[34] Díaz E, Miralles K, Domínguez-Adame F and Gaul C 2014 Appl. Phys. Lett. 105 103109
[35] Delplace P, Gómez-León A and Platero G 2013 Phys. Rev. B 88 245422