Hyperon Beta-Decay Analysis and the Recent KTeV Data

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Abstract. The analysis of hyperon semi-leptonic decay data is addressed with reference to SU(3) breaking and isospin mixing between $\Lambda^0$ and $\Sigma^0$. Various approaches to SU(3) breaking are discussed and compared. The phenomenological implications of $\Lambda^0-\Sigma^0$ mixing are not to be underestimated: it can induce vector couplings in decays otherwise purely axial and may also modify rates. In regard of the KTeV data on $\Xi^0 \rightarrow \Sigma^+ e^\nu$, predictions are presented and the impact of present a future data on the extraction of $F$ and $D$ is also examined. In addition, the implications of the new data for the use of octet baryon beta decays in determining $V_{us}$ are considered.
I INTRODUCTION

Hyperon semi-leptonic decay (HSD) data are the sole present source of information on the $F$ and $D$ parameters, vital for the analysis of polarised deep-inelastic scattering experiments. In addition, they may provide access to the Cabibbo-Kobayashi-Maskawa (CKM) matrix element, $V_{us}$. Recent data for $\Xi^0 \rightarrow \Sigma^+ e\bar{\nu}$ from the KTeV collaboration [1] at Fermilab have added new interest to this area.

Beside the accurately measured neutron $\beta$-decay rate and angular asymmetries, there is a body of data on the rest of the baryon octet [2]. In SU(3) such decays are described via two parameters, $F$ and $D$, relating to strong-interaction effects and two further parameters, $V_{ud}$ and $V_{us}$, the CKM matrix elements (heavy-flavour contributions may be neglected). The $F$ and $D$ parameters are important in connection with in the Ellis-Jaffe sum rule [3]; a 15% reduction in the ratio $F/D$ from its accepted value ($\sim 0.6$) would remove the discrepancy with polarised DIS data and alleviate the “proton-spin puzzle” [4].

As SU(3) is violated at about the 10% level, a reliable description of the breaking is important. A test of any scheme lies in the predictions made for new decays: such as, the process $\Xi^0 \rightarrow \Sigma^+ e\bar{\nu}$, which has now been measured by the KTeV collaboration [1,5]. In this talk, after outlining the data and an approach to SU(3) breaking, I shall present a prediction for this decay [6] and discuss future developments.

II THE HSD DATA

A number of hyperon semi-leptonic decays have been measured with varying degrees of accuracy and depth of information; fig. 1 depicts the measured baryon octet $\beta$-decays, indicating the type of data available, while the present world HSD data are collected in table 1. Note that several of the rates and asymmetries...
have now been measured to better than 5%. Moreover, I should add that a past discrepancy in the neutron $\beta$-decay data has now been resolved.

### TABLE 1. Present world HSD rate and angular-correlation data [2]. Numerical values marked $g_1/f_1$ are extracted from angular and spin correlations.

| Decay | Rate ($10^6$ s$^{-1}$) | $g_1/f_1$ | $g_1/f_1$ |
|-------|--------------------------|-----------|-----------|
| $n \rightarrow p$ | 1.1278 ± 0.0024 $^a$ | 1.2670 ± 0.0035 $^b$ | $F + D$ |
| $\Lambda^0 \rightarrow p$ | 3.161 ± 0.058 | 0.60 ± 0.13 | 0.718 ± 0.015 | $F + \frac{1}{2} D$ |
| $\Sigma^- \rightarrow n$ | 6.88 ± 0.23 | 3.04 ± 0.27 | -0.340 ± 0.017 | $F - D$ |
| $\Sigma^0 \rightarrow \Lambda^0$ | 0.387 ± 0.018 | -$\sqrt{3}D$ $^c$ |
| $\Sigma^+ \rightarrow \Lambda^0$ | 0.250 ± 0.063 | -$\sqrt{3}D$ $^c$ |
| $\Xi^- \rightarrow \Lambda^0$ | 3.35 ± 0.37 $^d$ | 2.1 ± 2.1 $^e$ | 0.25 ± 0.05 | $F - \frac{1}{2} D$ |
| $\Xi^- \rightarrow \Sigma^0$ | 0.53 ± 0.10 | $F + D$ |

$^a$ Rate given in units of $10^{-3}$ s$^{-1}$.

$^b$ Scale factor 1.9 included in error (PDG practice for discrepant data).

$^c$ Absolute expression for $g_1$ given (as $f_1 = 0$).

$^d$ Scale factor 2 included in error (as above).

$^e$ Data not used in these fits.

### III SU(3) BREAKING AND FIT RESULTS

SU(3) breaking is well described using centre-of-mass (CoM), or recoil, corrections [7–9]. One approach ($A$ here) is to account for the extended nature of the baryon by applying momentum smearing to its wave function. For the decay $A \rightarrow Bl\nu$, CoM corrections to $g_1$ lead to

$$g_1 = g_{1^SU(3)} \left[1 - \frac{\langle p^2 \rangle}{3m_A m_B} \left(\frac{1}{4} + \frac{3m_B}{8m_A} + \frac{3m_A}{8m_B}\right)\right].$$

A similar approach ($B$) relates the breaking to mass-splitting effects in the interaction Hamiltonian via first-order perturbation theory [10]. The correction here takes on the following form:

$$g_1 = g_{1^SU(3)} \left[1 - \epsilon (m_A + m_B)\right].$$

Both approaches normalise the corrections to the reference-point correction for $g_{1^{n\rightarrow p}}$ and depend on a single new parameter ($\langle p^2 \rangle$ or $\epsilon$). Corrections to $f_1$ are negligible in $A$ and assumed so in $B$, in accordance with the Ademollo-Gatto theorem. Any further global normalisation correction to the $|\Delta S=1|$ rates is disfavoured; in [7] a calculated value of \(\sim 8\%\), was used, this is excluded by present-day fits.

Table 2 displays the results of three fits: S (symmetric), A and B. Note that the value of $V_{ud}$ (and hence $V_{us}$, fixed here via CKM unitarity) is determined mainly by the super-allowed nuclear $ft$ values. However, when $V_{ud}$ and $V_{us}$ (with or without imposition of unitarity) are extracted from HSD data alone, all parameter values remain essentially unchanged. Thus, unitarity appears to be well respected.
TABLE 2. SU(3) symmetric and breaking fits, including \( V_{ud} \) from nuclear ft.

| Fit | \( V_{ud} \) | \( F \) | \( D \) | \( \chi^2/\text{DoF} \) |
|-----|-------------|---------|---------|-----------------|
| \( S \) | 0.9748 (4) | 0.466 (6) | 0.800 (6) | 2.3 |
| \( A \) | 0.9740 (4) | 0.460 (6) | 0.808 (6) | 0.8 |
| \( B \) | 0.9740 (4) | 0.459 (6) | 0.809 (6) | 0.8 |

IV \( \Lambda^0 \) AND \( \Sigma^0 \) MIXING

While at the level of isospin violation itself the effects are obviously small, their influence in HSD may, in fact, be significant. It has been pointed out [11] that isospin violation can induce mixing between \( \Lambda^0 \) and \( \Sigma^0 \), described via

\[
\Lambda^0 = \cos \phi \Lambda_8 + \sin \phi \Sigma_8, \\
\Sigma^0 = -\sin \phi \Lambda_8 + \cos \phi \Sigma_8.
\]

The suggested phenomenological mixing angle is \( \phi = -0.86^\circ \) [12]. Now, consider, e.g., the \( \Sigma^\pm \to \Lambda^0 \) decay: in exact SU(2) \( f_1 \) is zero, thus angular or spin correlations vanish here. If, however, the \( \Lambda^0 \) contains a small admixture of \( \Sigma^0 \), this is no longer true. While there is no strong signal in the fits for such mixing, intriguingly, the values returned are around \(-0.8^\circ \pm 0.8^\circ \) in both SU(3) symmetric and broken fits.

V A PREDICTION

Rates and parameters may now be predicted for any other decay in the octet; table 3 compares the predictions obtained for \( \Xi^0 \to \Sigma^+ e\bar{\nu} \) from the above three fits. Recall that \( g_1/f_1 = F + D \) for this decay, thus allowing for important cross checks. The variation between the two SU(3) breaking fits is well within statistical errors, I therefore combine the two, obtaining the following mean values:

\[ g_1/f_1 = 1.14 \pm 0.03 \pm 0.01 \quad \text{and} \quad \Gamma = (0.79 \pm 0.05 \pm 0.01) \cdot 10^6 \text{s}^{-1}, \]

where the second error estimates the systematic uncertainty due to the difference between fits, the corresponding branching ratio is \( BR = (2.29 \pm 0.12) \cdot 10^{-4} \).
Let me now briefly compare with a $1/N_{c}$ approach [13]; the quoted fit there results in a very low $F/D = 0.46$ and for $\Xi^{0} \rightarrow \Sigma^{+} e \bar{\nu}$ predicts

$$g_{1}/f_{1} = 0.91 \quad \text{and} \quad \Gamma = 0.68 \cdot 10^{6} \text{ s}^{-1} \quad (\text{fit } B \text{ of ref. [13]),}$$

Both values are smaller than those presented here, which are in turn smaller than naïve SU(3). To comprehend the difference between the predictions, note that the analysis of ref. [13] includes baryon-decuplet non-leptonic decay data, which dominate; and the overall fit (i.e., $\chi^{2}$) is poor. However, applied to the HSD data alone, the results are similar to those reported here [14]. These differences should be distinguishable by an experiment with good statistics, such as KTeV [1].

The preliminary KTeV results are as follows [1, 5]:

$$g_{1}/f_{1} = 1.24^{+0.20}_{-0.17} \pm 0.07 \quad \text{and} \quad BR = (2.54 \pm 0.11 \pm 0.16) \cdot 10^{-4},$$

the errors are expected to be at least halved when the full data set is analysed. Although not crucial to the $F$ and $D$ determination (owing to low sensitivity to the breaking scheme), depending upon the final central values, the KTeV data should have strong impact on the determination of $V_{us}$.

VI CONCLUSIONS

Before concluding, let me call attention to an all too often overlooked point: although easier to analyse (no extra corrections are necessary), the present data for angular correlations alone show no evidence of SU(3) breaking. Furthermore, compared to the full data set, they lack statistical power. Only full analyses can display the true picture [9]. Here I would comment that the observed consistency with unitarity (i.e., between $V_{ud}$ and $V_{us}$) and the fact that there is no disagreement between complete analyses (with the possible exception of [13]) suggest that the PDG [2] might reconsider this sector as a contending source for estimating $V_{us}$. Indeed, unitarity appears better satisfied here than in the $K_{3\ell}$ data.

A complete comprehension of SU(3) breaking is still wanting; witness the octet-decuplet discrepancy. Moreover, the system is as yet not truly over-constrained; in this context, I might also mention another decay (already measured but not accurately) for which large corrections are expected: namely, $\Xi^{-} \rightarrow \Sigma^{0} e \bar{\nu}$. There too, $g_{1}/f_{1} = F + D$, permitting additional sensitive cross checks, especially in combination with the KTeV results.

Concluding then, I would stress that, while the data do manifest significant departures from SU(3), and there is even modest evidence for SU(2) mixing, the mass-splitting driven schemes discussed here provide an adequate description. That said, there remains much to be understood, e.g., the long-standing question of second-class currents. Thus, any new precise data are more than welcome and the contribution of the KTeV collaboration will be invaluable.
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