Gravitational baryogenesis in Gauss-Bonnet braneworld cosmology

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The mechanism of gravitational baryogenesis, based on the $CPT$-violating gravitational interaction between the derivative of the Ricci scalar curvature and the baryon-number current, is investigated in the context of the Gauss-Bonnet braneworld cosmology. We study the constraints on the fundamental five-dimensional gravity scale, the effective scale of $B$-violation and the decoupling temperature, for the above mechanism to generate an acceptable baryon asymmetry during the radiation-dominated era. The scenario of gravitational leptogenesis, where the lepton-number violating interactions are associated with the neutrino mass seesaw operator, is also considered.

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I. INTRODUCTION

The origin of the baryon asymmetry is an outstanding problem in particle physics and cosmology. Sufficient conditions for baryogenesis are the violation of baryon number, the violation of $C$ and $CP$ symmetries and the existence of nonequilibrium processes. Alternatively, if $CPT$ and baryon number are violated, a baryon asymmetry could arise even in thermal equilibrium. In Ref. [1], the effects on baryogenesis of certain $CPT$-violating terms arising in a string-based framework were investigated and it was shown that a large baryon asymmetry could be produced at the grand-unified scale. Recently, a new baryogenesis mechanism, where the baryon asymmetry is generated via a dynamical breaking of $CPT$ while maintaining thermal equilibrium, was proposed in Ref. [2]. The crucial ingredient is an interaction between the derivative of the Ricci scalar curvature $\mathcal{R}$ and the baryon-number ($B$) current $J^\mu$ (or any current that leads to a net $B - L$ charge in equilibrium, where $L$ is the lepton number, so that the asymmetry will not be erased by the electroweak anomaly):

$$\frac{1}{M_s^2} \int d^4 x \sqrt{-g} \left( \partial^\mu \mathcal{R} \right) J^\mu,$$  \hspace{1cm} (1)

where $M_s$ characterizes the scale of the interaction in the effective theory. Such an operator is expected to arise in the low-energy effective field theory of quantum gravity or in supergravity theories from a higher-dimensional operator.

The interaction in Eq. (1) violates $CP$ and, in an expanding universe, it also dynamically breaks $CPT$. If one requires the existence of $B$-violating processes in thermal equilibrium, then a net baryon asymmetry can survive after their decoupling at a temperature $T_D$:

$$\frac{n_B}{s} \propto \left. \left| \frac{\mathcal{R}}{M_s^2 T} \right| \right|_{T_D}.$$ \hspace{1cm} (2)

For this mechanism to work, a nonvanishing time derivative $\dot{\mathcal{R}} \neq 0$ is necessary.\(^\dagger\) Although in an expanding universe $\mathcal{R} \neq H^2$ is nonzero in four-dimensional general relativity (GR), its time derivative $\dot{\mathcal{R}} = 0$ during the radiation-dominated (RD) epoch. It turns out, however, that $\dot{\mathcal{R}} \neq 0$ can be easily realized in the braneworld scenario, which suggests that higher-dimensional gravity effects can offer a novel way to generate a baryon asymmetry through the dynamics of spacetime.

In the past few years, stimulated by the development of string theory, the braneworld ideas, and particularly, the Randall-Sundrum (RS) model, have been actively investigated. The RS braneworld cosmology is based on the five-dimensional Einstein-Hilbert action; at high energies, it is expected that this action will acquire quantum corrections, in the form of higher-order curvature invariants in the bulk action. String theory and holography indicate that such terms arise in the action at the perturbative level. In five dimensions, the Gauss-Bonnet (GB) invariant has special properties: it represents the unique combination that leads to second-order gravitational field equations linear in the second derivatives and is ghost-free. Moreover, the graviton zero mode remains localized in the GB braneworld and deviations from Newton’s law at low energies are less pronounced than in the RS case.

In this paper we examine gravitational baryogenesis in the context of GB braneworld cosmology. The case when the GB contribution is absent and cosmology is of RS type is also considered. We show that the observed baryon-to-entropy ratio can be successfully explained in both frameworks. The possibility that $B - L$ violation is associated with the neutrino mass seesaw operator is

\(^\dagger\) For a more general form of the derivative coupling of the Ricci scalar to ordinary matter, $\mathcal{L} \propto \partial^\mu f(\mathcal{R}) J^\mu$, see Ref. [3].
also studied. In the latter case, the limits coming from low-energy neutrino physics when combined with the GB inflationary constraints allow us to put bounds on the fundamental scale of gravity, the effective scale of $B$-violation and the decoupling temperature, which are required to generate an acceptable baryon asymmetry in the gravitational leptogenesis scenario.

II. GAUSS-BONNET BRANEWORLD

The five-dimensional bulk action for the Gauss-Bonnet braneworld scenario is given by

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \left[ -2\Lambda_5 + R + \alpha \left( R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd} \right) \right] - \int_{\text{brane}} d^4x \sqrt{-g} \lambda + S_{\text{mat}},$$

(3)

where $\alpha > 0$ is the GB coupling, $\lambda > 0$ is the brane tension, $\Lambda_5 < 0$ is the bulk cosmological constant and $S_{\text{mat}}$ denotes the matter action. The fundamental energy scale of gravity is the 5D scale $M_5$ with $\kappa_5^2 = 8\pi/M_5^2$, and $M_4$ is the 4D Planck scale with $\kappa_4^2 = 8\pi/M_4^2$.

The GB term may be viewed as the lowest-order stringy correction to the 5D Einstein-Hilbert action with $\alpha \ll \ell^2$, where $\ell$ is the bulk curvature scale, $|R| \sim 1/\ell^2$. The Randall-Sundrum type models are recovered for $\alpha = 0$. Moreover, for an anti-de Sitter bulk, it follows that $\Lambda_5 = -3\mu^2(2 - \beta)$, where $\mu = 1/\ell$ is the extra-dimensional energy scale and

$$\beta \equiv 4\alpha \mu^2 \ll 1.$$  

(4)

Imposing a $Z_2$ symmetry across the brane in an anti-de Sitter bulk and assuming that a perfect fluid matter source is confined to the brane, one gets the modified Friedmann equation \[12, 13\]

$$\kappa_5^2(\rho + \lambda) = 2\mu \sqrt{1 + \frac{H^2}{\mu^2} \left[ 3 - \beta + 2\beta \frac{H^2}{\mu^2} \right]}.$$  

(5)

This can be rewritten in the useful form \[17\]

$$H^2 = \frac{\mu^2}{\beta} \left[ (1 - \beta) \cosh \left( \frac{2\chi}{3} \right) - 1 \right],$$  

(6)

where $\chi$ is a dimensionless measure of the energy density $\rho$ on the brane such that

$$\rho + \lambda = m_\alpha^4 \sinh \chi,$$  

(7)

with

$$m_\alpha = \left[ \frac{8\mu^2(1 - \beta)^{3/2}}{\beta \kappa_5^2} \right]^{1/8}$$  

(8)

the characteristic GB energy scale. The GB high-energy regime (\(\sinh \chi \gg 1\)) corresponds then to $\rho \gg m_\alpha^4$. Notice also that we must have $m_\alpha > m_\lambda = \lambda^{1/4}$, where $m_\lambda$ is the characteristic RS energy scale. This in turn implies $\beta \lesssim 0.15$ \[13\].

The requirement that one should recover general relativity at low energies leads to the relation \[12, 14\]

$$\kappa_5^2 = \frac{\mu}{1 + \beta} \kappa_4^2.$$  

(9)

Since $\beta \ll 1$, we have $\mu \approx M_5^3/M_4^2$. Furthermore, the brane tension is fine-tuned to achieve a zero cosmological constant on the brane \[10\]:

$$\kappa_5^2 \lambda = 2\mu(3 - \beta).$$  

(10)

Expanding Eq. (6) in $\chi$, we find three regimes for the dynamical history of the brane universe \[15\]

$$\rho \gg m_\alpha^4 \Rightarrow H^2 \approx \left[ \frac{\mu \kappa_5^2}{4\beta} \rho \right]^{2/3}$$  

(GB),

$$m_\alpha^4 \gg \rho \gg m_\lambda^4 \Rightarrow H^2 \approx \kappa_4^2 \rho^2$$  

(RS),

$$\rho \ll m_\lambda^4 \Rightarrow H^2 \approx \kappa_4^2 \frac{3}{4} \rho$$  

(GR).  

Eqs. (11)-(13) are considerably simpler than the full Friedmann equation and in many practical cases one of the three regimes can be assumed. In this case, it is useful to consider a single patch with the effective Friedmann equation \[20\]

$$H^2 = \beta_q^4 \rho^q,$$  

(14)

where $q = 1, 2, 2/3$ for GR, RS and GB regimes, respectively. For each regime, the coefficients $\beta_q > 0$ are determined in accordance with Eqs. (11)-(13).

III. GRAVITATIONAL BARYOGENESIS

Let us now consider the gravitational baryogenesis mechanism in the GB braneworld. In an expanding universe, the interaction term in Eq. (1) gives rise to an effective chemical potential $\mu_0 \sim \dot{R}/M_4^2$ for baryons. In thermal equilibrium, the net baryon-number density does not vanish as long as $\mu_b \neq 0$, and for $m_b, \mu_b \ll T$ one has \[21\]

$$n_B \approx \frac{q_b}{6} \mu_b T^2,$$  

(15)

where $q_b$ is the number of intrinsic degrees of freedom of the baryon. During the RD epoch, this leads to a baryon-to-entropy ratio given by

$$\frac{n_B}{s} = -c \frac{\dot{R}}{M_4^2 T} T_D,$$  

(16)

with

$$c = \frac{15}{4\pi^2} g_{*s}.$$  

(17)
We have used $s = 2\pi^2 g_\ast T^3/45$ for the entropy density, where $g_\ast$ is the total number of degrees of freedom which contribute to the entropy of the universe.

The Ricci scalar in the Friedman-Robertson-Walker brane is defined in terms of the expansion law as

$$\mathcal{R} = -6 \left( \dot{H} + 2H^2 \right).$$

Using the full GB Friedmann Eq. (10), we get for the time derivative of the Ricci scalar,

$$\dot{\mathcal{R}} = -\frac{4\mu^2 (1+w)(1-\beta)}{\beta m_\alpha^4} H\rho \left[ r_1(\chi) + r_2(\chi) \right],$$

$$r_1(\chi) = \left[ -6 + \frac{9}{2}(1+w) \right] \sinh \left( \frac{2\chi}{3} \right),$$

$$r_2(\chi) = 3(1+w) \frac{\rho}{m_\alpha^4 \cosh \chi}$$

$$\times \left[ \cosh \left( \frac{2\chi}{3} \right) \frac{3}{2} \sinh \left( \frac{2\chi}{3} \right) \tanh \chi \right],$$

where $w = p/\rho$ is the equation of state. During the RD era, $w = 1/3$, $r_1(\chi) = 0$ and Eq. (19) simplifies to

$$\dot{\mathcal{R}} = -\frac{32\mu^2 (1-\beta)}{\beta m_\alpha^4} H\rho \frac{\cosh \chi}{\cosh^4 \chi}$$

$$\times \left[ \frac{2}{3} \cosh \left( \frac{2\chi}{3} \right) - \sinh \left( \frac{2\chi}{3} \right) \tanh \chi \right].$$

Combining Eqs. (10) and (20), it is then possible to compute the decoupling temperature, $T_D$, required to produce an acceptable baryon-to-entropy ratio $n_B/s$. In Fig. 1 we plot this temperature as a function of the scale $M_5$ for different values of the GB coupling $\beta$ and the fundamental gravity scale $M_5$, assuming the observed value $n_B/s \simeq 9 \times 10^{-11}$ [22]. We have used the fact that in the RD era the energy density is $\rho = \pi^2 g_* T^4/30$, where $g_*$ is the total number of relativistic degrees of freedom. In the standard model $g_* \approx g_\ast \approx 100$ above the electroweak scale and, assuming $g_0 \sim O(1)$, one gets $c \sim O(10^{-2})$.

In order to establish the transition between the different regimes, we may consider the simplified expansion law given by Eq. (14). In this case,

$$\dot{\mathcal{R}} = -48 q (q-1) H^3$$

and the baryon asymmetry reads as

$$\frac{n_B}{s} = 48 c q (q-1) \left( \frac{\pi^2 g_*}{30} \right)^{3q/2} \frac{T_D^{6q-1}}{M_5^8}.$$  

(22)

If the decoupling occurs in the RS regime, where $q = 2$ and $\beta_q = (\kappa_5^2/6\lambda)^{1/2}$, we obtain

$$T_D \simeq 3.2 \times 10^{-2} \left( M_5^2 M_\alpha^4 \right)^{1/11},$$

(23)

for $\beta \ll 1$. For the decoupling to occur in this regime, it is required that $\rho(T_D) \geq m_\chi$, which implies in turn that $M_\ast \geq M_\ast^{RS}$ with

$$M_\ast^{RS} \simeq 1.9 \times 10^5 \left( \frac{M_5}{M_4} \right)^{11/4} M_5.$$  

(24)

On the other hand, in the GB regime, where $q = 2/3$ and $\beta_q = (\mu^2 \kappa_5^2/4\lambda)^{1/3}$, the decoupling temperature is given by

$$T_D \simeq 1.6 \times 10^{-4} \left( \frac{\beta M_5^2 M_4^4}{M_5^6} \right)^{1/3},$$

(25)

so that the transition from the GB to the RS regime, defined by the condition $\rho(T_D) = m_\chi$, occurs for

$$M_\ast^{GB} \simeq 5.9 \times 10^4 \left( \frac{M_5}{M_4} \right)^{11/4} M_5.$$  

(26)

The transition values $M_\ast^{RS}$ and $M_\ast^{GB}$ are represented in Fig. 1 by the two vertical dashed lines. We see that the decoupling of the baryon-number violating interactions can generally occur in either (GB, RS or GR) regime. We also notice that the RS transition region shrinks as the Gauss-Bonnet coupling $\beta$ increases.

An important constraint on the decoupling temperature comes from reheating and inflation. Obviously, we must require $T_D < T_{rh} < M_I$, where $T_{rh}$ is the reheating temperature (at which the universe becomes radiation dominated) and $M_I$ is the inflation scale. In the conventional scenario, reheating occurs as the inflaton field oscillates around its minimum and decays into matter. In this case, $T_{rh}$ crucially depends on the details of the inflaton coupling to matter. Here we restrict our discussion to the inflation scale $M_I$. Assuming that inflation occurs in the high-energy (RS or GB) regime of the theory and that it is driven by a quadratic potential, we find $M_I \approx m_\chi \sinh^{1/4} \chi_5$, where $\chi_5$ is evaluated at the end of inflation (see the Appendix for details). In Fig. 1 we have plotted $M_I$ for different values of $\beta$ and $M_5$. We note that for $M_\ast \leq M_5$ the constraint $T_D < M_I$ is always verified.

We have also performed a random scan of the parameter space ($M_\ast, M_5, \beta$) in order to find the allowed region for the gravitational baryogenesis mechanism considered here to generate an observationally acceptable $n_B/s$ with $T_D < M_I < M_5$. The results are presented in Fig. 2. A similar analysis was done for the case when the GB terms are absent, i.e. $\beta = 0$, and braneworld cosmology is of RS type (see Fig. 3). We notice that in the RS case the gravity scale $M_5$ can take considerably lower values (cf. Fig. (3a)), only constrained to be larger than $10^5$ TeV, if one requires the theory to reduce to Newtonian gravity on scales larger than 1 mm. The above bound yields $T_D \gtrsim 10^5$ GeV.

Up to now we have not taken into account possible effects which could dilute the baryon asymmetry generated by the mechanism described above. It is well known that electroweak sphaleron transitions, which are unsuppressed at temperatures above the electroweak phase transition, are a potential source of dilution [24]. Sphaleron-induced baryon-asymmetry dilution occurs when $B - L$ vanishes. In this case, the $B$ and $L$-number densities will be typically diluted by a factor
0.02 m_τ^2/T_{\text{sph}}^2 \, \text{(3)}, where m_τ is the τ lepton mass and T_{\text{sph}} is the sphaleron freeze-out temperature. Assuming $T_{\text{sph}}$ to be the electroweak scale, one finds that the baryon asymmetry is diluted by a factor of about $10^{-6}$. Hence, according to Eq. (2), the scale $M_\ast$ would have to be, in this case, smaller by a factor of $10^{-3}$ to reproduce the correct value of $n_B/s$ via the gravitational interaction of Eq. (11). On the other hand, if $B-L \neq 0$, essentially no sphaleron dilution occurs. In the latter case, the baryon asymmetry generated will remain after the decoupling of the $(B-L)$-violating interactions. An example of this possibility will be presented in the next section.

**IV. GRAVITATIONAL LEPTOGENESIS**

In the standard model of electroweak interactions, the $B-L$ symmetry is exactly conserved. This symmetry is however violated in many of its extensions. In general, it is possible that the $B$-violating interactions are generated by an operator $O_B$ of mass dimension $D = 4 + n$. The rate of such interactions is $\Gamma_B \sim T^{2n+1}/M_B^{2n}$, where $M_B$ is the mass scale associated with the operator $O_B$. In the standard electroweak model the lowest-dimensional operator that violates $B-L$ is the dimension five operator

$$L_\phi = \frac{1}{M} \ell \ell \phi \phi + \text{H.c.}, \quad (27)$$

where $\ell$ and $\phi$ are the left-handed lepton and Higgs doublets, respectively; $M$ is the scale of new physics which induces $B-L$ violation. This interaction represents a typical term that gives rise to the seesaw mechanism and is responsible for the light neutrino masses $m_i \sim v^2/M$, $v \approx 174$ GeV. In the early universe the $L$-violating rate induced by the interaction (27) is

$$\Gamma_\phi = \frac{T^3}{M_B^2}, \quad M_B \approx \frac{10 v^2}{(\sum m_i^2)^{1/2}}. \quad (28)$$

The decoupling of the $(B-L)$-violating processes occurs when $\Gamma_\phi$ falls below the Hubble rate, i.e. when

![Graph showing decoupling temperatures and scale $M_B$ for different values of $\beta$ and the fundamental scale of gravity $M_5$. The graphs illustrate the transitions between GB, RS, and GR regimes, with dot-dashed lines indicating decoupling temperatures.](image-url)
by Eq. (16), and it determines the required scale of that produces an acceptable baryon asymmetry is fixed by the seesaw operator. In Fig. 1 we have plotted $M_{\nu}$ as a function of $\beta$ for different values of $\beta$. In Fig. 1 we have plotted $M_{\nu}$ as a function of $\beta$ for different values of $\beta$. We notice that the requirement $T_{D} < M_{\nu} < M_{5}$ imposes an upper bound on $M_{\nu}$. We find $M_{\nu} \lesssim 10^{10}$ GeV.

If the scale $M_{\nu}$ is associated with the neutrino mass seesaw operator, as in Eq. (28), then the value of this scale will be fixed by the light neutrino mass spectrum. The current cosmological limit coming from the Wilkinson Microwave Anisotropy Probe (WMAP) implies $\sum m_{\nu} \lesssim 0.69$ eV [22]. If neutrinos are quasidegenerate (QD) in mass, the above limit requires $m_{1} \simeq m_{2} \simeq m_{3} \simeq 0.23$ eV. In this case,$$M_{\nu}^{QD} \approx 7.6 \times 10^{14} \text{ GeV}.$$ Instead, if neutrinos masses are hierarchical (HI) with $m_{1} \simeq 0 \ll m_{2} \ll m_{3}$, then $m_{2} \simeq (\Delta m_{\text{sol}}^{2})^{1/2}$ and $m_{3} \simeq (\Delta m_{\text{atm}}^{2})^{1/2}$, where the squared mass differences measured in solar and atmospheric neutrino oscillation experiments are $\Delta m_{\text{sol}}^{2} \simeq 8.1 \times 10^{-5}$ eV$^{2}$ and $\Delta m_{\text{atm}}^{2} \simeq 2.2 \times 10^{-3}$ eV$^{2}$ [24], respectively. In the latter case,$$M_{\nu}^{HI} \approx 6.3 \times 10^{15} \text{ GeV}.$$ Eqs. (29) and (30) yield a decoupling temperature in the range$$T_{\nu}^{QD} \leq T_{D} \leq T_{\nu}^{HI}.$$
Clearly, the specific values of $T_{QD}^v$ and $T_{H1}^v$ depend on whether the decoupling of $\bar{B} - L$ violation occurs in GB, RS or GR regime and, thus, on the values of the Gauss-Bonnet coupling $\beta$ and the fundamental scale $M_5$. In standard cosmology with $H(T_D) \approx 1.66 g^{1/2}T_D^2/M_4$, one finds $T_{QD}^v \approx 7.9 \times 10^{11}$ GeV and $T_{H1}^v \approx 5.5 \times 10^{13}$ GeV. Some other examples are presented in Fig. 1. While in Fig. (1a) and (1b), the decoupling corresponding to $T_{QD}^v$ and $T_{H1}^v$ (horizontal dot-dashed lines) occurs in standard cosmology, in Fig. (1c) and (1d) such decoupling takes place in the high-energy Gauss-Bonnet regime for the case of hierarchical neutrinos. We also remark that for gravitational leptogenesis to be successful we must require $T_D < T_{QD}^v$, which implies the lower bound $M_\nu \gtrsim 100$ GeV for $\beta \lesssim 0.1$, as can be seen from the figure.

Let us now consider the inflation bound. Since $T_D < M_B$, the requirement $M_B < M_I$ is more stringent in this case. We find that this bound strongly constrains the scale of $\bar{B} - L$ violation and, consequently, the mechanism of gravitational leptogenesis. For instance, it can be seen that for the case presented in Fig. (1d), the above constraint implies the bound $T_D < T_{QD}^v$ and, therefore, the leptogenesis mechanism cannot generate the required baryon asymmetry. The allowed region for gravitational leptogenesis is presented in Fig. 2 (black dots). The horizontal dot-dashed lines correspond to the GR decoupling temperatures $T_{QD}^v$ and $T_{H1}^v$ (Fig. (2c)) and the scales $M_{QD}^v$ and $M_{H1}^v$ (Fig. (2d)) associated with the neutrino mass seesaw operator. We conclude that

$$10^{15} \text{ GeV} \lesssim M_5 \lesssim 10^{17} \text{ GeV},$$

$$10^2 \text{ GeV} \lesssim M_\nu \lesssim 10^{10} \text{ GeV}. \quad (32)$$

One can compare the above results with the ones that are obtained in the case when braneworld cosmology is of RS type, i.e. when $\beta = 0$. The allowed range of values for the parameters is shown in Fig. 3. We notice that a successful gravitational leptogenesis in RS cosmology requires

$$10^{16} \text{ GeV} \lesssim M_5 \lesssim 10^{17} \text{ GeV},$$

$$10^2 \text{ GeV} \lesssim M_\nu \lesssim 10^6 \text{ GeV}. \quad (33)$$

FIG. 3: The parameter space that generates the observed baryon-to-entropy ratio $n_B/s$ during the radiation era of Randall-Sundrum braneworld cosmology ($\beta = 0$). In this case, $M_I \approx 5 \times 10^{-6} M_5$. For other details, see caption of Fig. 2.
V. CONCLUSION

In this work we have considered the possibility that the observed baryon asymmetry arises via the spacetime dynamics of Gauss-Bonnet braneworld cosmology. The framework presented here is based on the CPT-violating gravitational interaction between the derivative of the Ricci scalar curvature and the $B$ (or $B - L$) current. We have shown that it is possible to generate the correct magnitude of the baryon asymmetry in different cosmological scenarios, depending on whether the decoupling of the $B$- or $(B - L)$-violating interactions occurs in standard cosmology or in the high-energy Randall-Sundrum or Gauss-Bonnet braneworld regimes.

We have also studied the case when baryogenesis occurs via leptogenesis, and the $B - L$ current is associated with the neutrino mass seesaw operator. In this framework, the produced $n_{B - L}$ asymmetry will be converted to a baryon asymmetry, once sphaleron transitions enter thermal equilibrium. We have seen that for this scenario to be viable, a rather high fundamental scale of gravity $M_5$ is required (cf. Eq. (22)), as well as an effective interaction scale $M_*$ above the electroweak scale but below $10^{10}$ GeV. At this point it is worth noticing that, although in four-dimensional gravity it is natural to expect $M_*$ of the order of the Planck mass $M_4$, this may not be necessarily the case. For instance, $M_* \approx (M_R M_A)^{1/2}$ could be possible, if the right-handed neutrino Majorana mass $M_R$ softly violates baryon number $R$. Moreover, this scale can be much lower, if the effective four-dimensional theory comes from a higher-dimensional theory. Indeed, AdS/CFT correspondence [23] and braneworld holography [3, 24] indicate that interaction terms such as given by Eq. (1) are expected in the effective action on the brane with $M_5$. If this is the case, the bound $M_5 \approx 10^{15} - 10^{17}$ GeV would then imply $M_* \approx 10^2 - 10^3$ GeV, well within the range allowed by gravitational baryogenesis and leptogenesis in the high-energy GB regime.

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APPENDIX: INFLATION IN A GAUSS-BONNET BRANEWORLD

In this Appendix we briefly review inflation in GB brane cosmology [17, 18, 27]. For simplicity we assume that inflation is driven by the simple quadratic potential

$$V(\phi) = V_0 \phi^2.$$  \hspace{1cm} (A.1)

We are interested in slow-roll inflation which occurs in the GB or RS high-energy regime. In this case, $V \approx \rho \gg \lambda$ and Eq. (7) implies $V \approx m_\lambda^4 \sinh \chi$. Moreover, the bound $V > m_\lambda^4$ together with the quantum gravity upper limit $V < M_5^4$ imply

$$\left(\frac{m_\lambda}{m_n}\right)^4 < \sinh \chi < \left(\frac{M_5}{m_n}\right)^4.$$  \hspace{1cm} (A.2)

The slow-roll parameters $\epsilon$ and $\eta$ are given by

$$\epsilon = \frac{16 \lambda V_0}{27 \kappa_4^2 m_n^8} f(\chi), \quad \eta = \frac{8 \lambda V_0}{9 \kappa_4^2 m_n^8} \frac{1}{g(\chi)},$$  \hspace{1cm} (A.3)

where

$$f(\chi) = g^{-2}(\chi) \tanh \chi \sinh \left(\frac{2 \chi}{3}\right),$$  \hspace{1cm} (A.4)

$$g(\chi) = \cosh \left(\frac{2 \chi}{3}\right) - 1.$$  \hspace{1cm} (A.5)

The number of e-folds of inflation is given by

$$N_* = \frac{3 \mu^2}{4 \beta V_0} \int_{\chi_*}^{\chi} g(\chi) \coth \chi d\chi \equiv \frac{9 \mu^2}{8 \beta V_0} I(\chi)|_{\chi_*},$$  \hspace{1cm} (A.6)

where

$$I(\chi) = g(\chi) - \ln \left[1 + \frac{2}{3} g(\chi)\right].$$  \hspace{1cm} (A.7)

$\chi_*$ is evaluated when cosmological scales leave the horizon and $\chi_e$ is evaluated at the end of inflation, when $\max\{\epsilon, \eta\} = 1$.

The amplitude of scalar perturbations is

$$A_S^2 = \frac{3^{5/2} \kappa_4 \mu^5}{16 \pi^2 V_0 \lambda^{1/2} \beta^{3/2}} \sinh \chi_*.$$  \hspace{1cm} (A.8)

Using the COBE normalized value $A_S \simeq 2 \times 10^{-5}$ for the density perturbations and $55 \leq N_* \leq 65$, we can obtain the scale of inflation $M_I = V^{1/4}(\phi_e) \approx m_\lambda \sinh^{1/4} \chi_e$. This scale is plotted in Fig. II for given values of $\beta$ and $M_5$, taking $N_* = 60$. A more complete analysis is presented in Fig. (2a) and (2b). Notice that, for consistency, one should require $m_\lambda < M_I < M_5$. 

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