Formation of Dark Matter Halos

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Abstract. This article concerns the formation and structure of dark matter halos, including (1) their radial density profiles, (2) their abundance, and (3) their merger rates. The last topic may be relevant to the nature of the small, bright, high-redshift galaxies discovered by the Lyman break technique. (1) Study of a statistical sample of galaxy-mass dark halos in high-resolution Adaptive Refinement Tree simulations shows that they have a central density profile \( \rho(r) \propto r^{-\gamma} \) with \( \gamma \approx 0.2 \), in agreement with data on dark-matter-dominated disk galaxies. We present recent, higher resolution results on this. (2) Another important new result is that the Press-Schechter approximation predicts about twice as many galaxy-mass halos at \( z = 0 \) as are present in large dissipationless N-body simulations; more generally, PS overpredicts the abundance of \( M \lesssim 10^{-1}M_* \) halos at all redshifts. (3) Finally, we discuss the assembly of these halos, in particular the merger rate of (sub-)halos at high redshift and the distribution of the starbursts that these mergers are likely to trigger. If most of the Lyman-break galaxies are such starbursts, this perhaps resolves the apparent paradox that these galaxies appear to cluster like massive halos (\( \sim 10^{12}M_\odot \)), while their relatively low linewidths and their spectral energy distributions suggest that they have relatively low mass (few \( \times 10^{10}M_\odot \)) and young ages (few \( \times 10^8 \) yr). It also predicts much more star formation at high redshift in CDM-type hierarchical models for structure formation than if only quiescent star formation is included.

1. Radial Profiles of Dark Matter Dominated Galaxies

Navarro, Frenk, & White (1995) proposed the following simple radial density profile

\[
\rho_{\text{NFW}}(r) = \frac{\rho_0}{(r/r_s)(1 + r/r_s)^2}
\]

for dark matter halos corresponding to X-ray clusters. In very influential papers (hereafter referred to as NFW96 and NFW97) they subsequently showed that \( \rho_{\text{NFW}}(r) \) is a good fit to profiles of dark matter halos in SCDM and \( \Lambda \)CDM models, and in CDM models with power law \( P(k) = Ak^n \) fluctuation spectra.
with \( n = -0.5 \) to -1.5. They characterized the halos by a concentration parameter 
\[
c \equiv \frac{r_{200}}{r_s},
\]
where \( r_{200} \) refers to the radius within which the average overdensity is 200 times critical density. For \( \Omega_{\text{matter}} = 1 \) models, \( r_{200} \) is approximately the same as the virial radius. The mass enclosed is called \( M_{200} \), and it is useful to express this in units of the nonlinear mass \( M_* \), which is defined for \( \Omega_{\text{matter}} = 1 \) models by 
\[
\Delta_0(M) = \delta_0(1 + z),
\]
where \( \Delta_0(M) \) is the rms fluctuation of the mass in a sphere of average mass \( M \) (calculated from the linear power spectrum) and \( \delta_0 = 1.69 \) for top-hat collapse. Finally, NFW96 argued that halos with lower \( M_{200}/M_* \) are less concentrated because they form earlier.

The \( \rho_{\text{NFW}}(r) \) profile seems consistent with data on clusters, and the \( \rho \propto 1/r \) behavior at small \( r \) was consistent with earlier high-resolution N-body simulations (e.g., Dubinski & Carlberg 1991). But, as pointed out by Flores & Primack (1994) and Moore (1994), and acknowledged by NFW96, \( \rho \propto 1/r \) for small \( r \) is inconsistent with data on dark matter dominated dwarf irregular galaxies. Moreover, Burkert (1995) showed that the four galaxies considered by Moore (1994) have essentially the same rotation curve shape, corresponding to

\[
\rho_B(r) = \frac{\rho_b}{(1 + r/r_b)(1 + (r/r_b)^2)^2},
\]

(2)

This makes it implausible that a complicated starburst process leading to non-adiabatic expulsion of much of the central baryonic content of the galaxy, such as proposed by Navarro, Eke, & Frenk (1996), could account for the apparent inconsistency between simulations and real galaxies. The implausibility that the resolution of the discrepancy lies in this direction was further increased when Kravtsov, Klypin, Bullock, & Primack (1998, hereafter KKBP98) showed that a larger set of ten dark matter dominated dwarf irregular galaxies, and also a set of seven dark matter dominated low surface brightness (LSB) galaxies, all have the same rotation curve shape, again corresponding to \( \rho_B(r) \). This is shown in Figure 1. In our sample we included only those galaxies in which the dark matter component was shown to constitute \( \gtrsim 85\% \) of the total mass inside the last measured point of the rotation curve (in most cases with the maximum disk assumption). These dark matter dominated galaxies offer a unique opportunity for probing directly the density structure of dark matter halos which can be then compared with predictions of theoretical models. It hardly seems possible that all of these galaxies could have had the same sort of complicated conspiracy between dark matter and star formation, nor that the LSBs could have lost a significant fraction of their central baryons. A more plausible interpretation of the self-similarity of the radial mass distribution in these dark matter dominated galaxies is that it reflects an underlying similarity in the dark matter distribution.

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1 The analytically implied rotation curve shape is

\[
V_B^2(r) = 2V_b^2(r_b) \frac{\ln[(1 + r/r_b)^2(1 + (r/r_b)^2)] - 2\tan^{-1}[r/r_b]}{r/r_b},
\]

where \( V(r) \propto r \) for small \( r \). The peak in velocity occurs at \( r_{\text{max}} \approx 3.3r_b \), and \( V_{\text{max}} = V_B(r_{\text{max}}) \approx 2.4V_B(r_b) \).
McGaugh & de Blok (1998) emphasized that the sharp $r^{1/2}$ rise in central circular velocity predicted by the NFW $\propto r^{-1}$ density profile is in striking disagreement with the roughly linear rise in rotation velocity observed in LSB galaxies: “Even treating both $c$ and $V_{200}$ as completely free parameters, no fit can be obtained.”

Figure 1. Rotation curves of (a) ten dwarf irregular and (b) seven low surface brightness galaxies (symbols) with measured rotation curves and published mass models for stellar, gas, and dark matter components, normalized to the best fit values of $r_0$ and rotational velocity $v_0$ at $r_0$ predicted by density profile $\rho_{DM}(r)$ with $(\alpha, \beta, \gamma) = (2, 3, 0.2)$, which is represented by the solid line; $\rho_B(r)$ is nearly identical. (From KKBP98.)

KKBP98 fit the observed rotation curves with a more general profile,

$$\rho_{DM}(r) = \frac{\rho_0}{(r/r_0)^\gamma[1 + (r/r_0)^\alpha]^{(\beta-\gamma)/\alpha}}, \quad V(r) = V_t \left[ \frac{(r/r_t)^g}{1 + (r/r_t)^a} \right]^{(g+b)/a}, \quad (3)$$

in which $\rho(r \ll r_0) \propto r^{-\gamma}$, $\rho(r \gg r_0) \propto r^{-\beta}$, and $\alpha$ characterizes the sharpness of the change in logarithmic slope. This is equivalent to $\rho_{NFW}(r)$ for $(\alpha, \beta, \gamma) = (1, 3, 1)$, and to the so-called isothermal profile with a core $r_0$ for
\((\alpha, \beta, \gamma) = (2, 2, 0)\). Parameters \(r_t\) and \(V_t\) of the corresponding rotation curve are the effective “turnover” radius and velocity, and \(a\) parameterizes the sharpness of the turnover. Because of the relatively small range of radii probed by measured rotation curves, such measurements cannot be used to constrain all five parameters of \(\rho_{DM}\), so we fixed the outer logarithmic slope to the value suggested by the theoretical models \(\beta = 3, b = 0.34\). The plausible value of the parameter \(\alpha = 2\) was determined using galaxy rotation curves that do show a turnover. We then found that \(\gamma \approx 0.2\) fits the observed rotation curves of our sample of 17 galaxies. The corresponding best-fit slopes of the velocity profile are \((a, b, g) = (1.50, 0.34, 0.9)\). Note that \(g = 1 - \gamma/2\). With parameters \(\alpha, \beta,\) and \(\gamma\) (or \(a, b, g\)) fixed, we fitted the data for the remaining two free parameters: \(\rho_0\) and \(r_0\) (or \(V_t\) and \(r_t\), or in terms of Burkert’s profile, \(\rho_b\) and \(r_b\)). The resulting structural relations show a decrease in the characteristic density with increasing characteristic radii, or an increase in maximum rotation velocity with increase in the radius at which the maximum occurs. Matching these observational relations is a challenge for any theory that aspires to explain the observed rotation curves.

2. Radial Profiles From Simulations

Does the disagreement between CDM-type simulations and the observed rotation curves of dark matter dominated galaxies at small radii mean that the dark matter in these galaxies is not mostly cold dark matter? That would be the implication if the discrepancy were real.

We have used the Adaptive Refinement Tree (ART) \(N\)-body code (Kravtsov et al., 1997), which adaptively refines spatial and temporal resolution in high density environments, to simulate the evolution of collisionless dark matter in the three cosmological structure formation models:\footnote{The simulations followed trajectories of 128\(^3\) CDM particles in a box of size of \(L_{box} = 7.5h^{-1}\) Mpc. The CHDM simulation had \(3 \times 128^3\) particles due to the addition of two equal-mass neutrino species.} (a) the standard cold dark matter (CDM) model \((\sigma_8 = 0.67, h = 0.5)\); (b) a low-density CDM model with cosmological constant (\(\Lambda\)CDM) with parameters favored by the high-redshift supernovae data \((\Omega_{\text{matter}} = 0.3, h = 0.7, \sigma_8 = 1)\); and (c) a cold+hot dark matter model with two types of neutrino (CHDM) with favorite parameters \(\Omega_{\nu} = 0.2, h = 0.5\) (Primack et al. 1995, Gawiser & Silk 1998, but cf. Primack & Gross 1998). For the dark matter halos used in KKBP98, the spatial resolution is equal to \(\approx 0.5 - 2h^{-1}\) kpc, and for each of the analyzed halos we have taken into account only those regions of the density and circular velocity profiles that correspond to scales at least twice as large as the formal resolution. The reliability of the simulated density and velocity profiles was tested by comparing results of the simulations with different resolutions and time steps (Kravtsov et al. 1997). A sample of \(\sim 50\) halos was extracted from each simulation, and rotation curves of halos were fitted with the same density distribution as the data (the same set of \(\alpha, \beta,\) and \(\gamma\)). The halo rotation curves were then renormalized to their best fit values of \(r_0\) and \(v_0 = v(r_0)\) and these renormalized rotation curves (i.e. \(v/v_0(r/r_0)\)) were averaged over the whole sample for each model.
Figure 2. The average normalized dark matter velocity profiles for halos formed in (a) CDM ($\sigma_8 = 0.67, h = 0.5$), (b) ΛCDM ($\Omega_{\text{matter}} = 0.3, h = 0.7, \sigma_8 = 1$) and (c) CHDM ($\Omega_{\nu} = 0.2$ in $N_{\nu} = 2$ neutrino species, $h = 0.5$) models with corresponding profiles of the dwarf galaxies from our sample. The dotted lines show the 2σ envelope representing scatter of individual halo profiles around the average. Although the velocity profiles of the hierarchically formed dark matter halos are on average consistent with the shape of observed rotation curves, the scatter in the inner regions of the halo velocity profiles is substantial. (From KKBP98.)

Figure 2 shows the average normalized dark matter velocity profiles for halos formed in each of the three cosmological structure formation models compared with the rotation curves of the dark matter dominated galaxies in our sample. We find that on average, the velocity profiles of the halos formed in the hierarchical structure formation models and observed dark matter halos are in reasonably good agreement. Why did we not see the significant discrepancy between numerical simulations and rotation curve measurements indicated by previous work? One possible explanation is that we have not used an (extrapolated) analytic model fit, but have compared the data with the average shape of the dark matter halos directly. We also find that dark matter dominated dwarf and LSB galaxies show structural correlations between their characteristic density, $\rho_0$, and radius, $r_0$, consistent with the correlations of our simulated dark matter halos: physically smaller halos are denser. We find a similar correlation between the maximum of the rotation curve, $v_{\text{max}}$, and the corresponding radius $r_{\text{max}}$ (see KKBP98 for details, and also Figure 3 below). This increases our confidence that the agreement between the simulations and the observed dark matter dominated galaxies is not a fluke.
In recent work, finished after KKBP98, we have run and analyzed a 128$^3$ particle CDM ART simulation ($\sigma_8 = 1.0$) in a box of 2.5 $h^{-1}$ Mpc, 1/3 the linear size of that used for the simulations shown in Figure 2. Figure 3 shows the well-resolved halos fit to the Burkert profile; as already mentioned, the $\rho_{DM}$ profile with $\gamma \approx 0.2$ is essentially identical. Again we have verified that the structural relations, such as the correlation between the maximum rotation velocity and the radius at which this maximum occurs, are in reasonable agreement with our sample of dwarf and LSB galaxies — see Figure 4.

![Figure 3](image)

Figure 3. The density profiles of halos in a CDM simulation with box size 2.5 $h^{-1}$ Mpc, fit to Burkert’s profile $\rho_B$ (solid curve). For each halo, the smallest radial density bin is larger than two formal resolution elements and contains at least 10 particles. The failure of the NFW profile (dashed curve) at small radii is apparent, but $\rho_{NFW}$ agrees well with the simulations at larger radii where the isothermal profile with a core (dot-dash curve) fails because it only falls as $r^{-2}$.

Moore et al. (1998) reports results from two very high resolution simulations of clusters, which had steep central density profiles. It is not clear that our results are in disagreement, since we consider galaxy-mass halos and find that a statistical sample of halos has a range of central profiles. But it is important to understand why different simulations give different results, and in particular to understand the effects of different resolution and different simulation techniques,
Figure 4. Observed versus simulated maximum rotation velocity as a function of the radius at which it occurs. Simulated dark matter halos are from our CDM simulation in a box of 2.5 $h^{-1}$ Mpc (x symbols) and from the SCDM simulation from KKBP98. These are compared to $v_{\text{max}}$ vs. $r_{\text{max}}$ determined from $\rho_{\text{DM}}$ fits to our sample of dwarf and LSB galaxies.

of two-body relaxation (which is suppressed in the ART approach), and of the selection of halos to be simulated and analyzed.

It would appear to be better to compare a sample of galaxies with a statistical sample of galaxy-mass halos, as we have done. This has many other advantages. In work now being written up for publication, Bullock et al. have analyzed large ART simulations at many redshifts, to study the radial distribution of mass and angular momentum in dark matter halos at a given epoch, and also the evolution of these properties. We find that the dispersion of the concentration is roughly log-normal and large, and that while it can be explained at $z = 0$ by the argument presented by NFW96, this does not work at higher redshifts. In addition, halos in higher density environments tend to be more concentrated than isolated halos, suggesting that halo selection criteria may indeed be important for interpreting the conflicting results mentioned above. Since the dispersion of the concentration at $z = 0$ in a given cosmology (Bullock et al. 1998) is larger than the difference between different cosmologies (which comes largely from the difference between $r_{200}$ and the true virial radius), it is incorrect
to draw conclusions about cosmology from a few galaxies (as done, for example, by Navarro 1998, who argues that a few low-concentration galaxies favor $\Omega_{\text{matter}}$ very low).

One problem that may be resolved in the light of new data is the apparently greater dispersion of the inner radial profiles of the simulations than of the dark matter dominated galaxies in our sample. The work of Swaters et al. (1997) discussed at this meeting, and also of Côté, Freeman, & Carignan (1997), appears to indicate that the dispersion of the properties of dark matter dominated galaxies may be greater than our sample suggested. However, as our sample in KKBP98 represented only the $\sim 50$ most massive halos in our simulations, our scatter may be artificially small. A major challenge to theorists remains, to explain why the dark matter profiles of galaxies and simulations have the radial dependences observed. Regarding the inner profiles, Syer & White (1998) argued that the closer the power spectrum approximates the asymptotic CDM slope of -3, where halos of all sizes collapse at roughly the same epoch and therefore have the same density, the shallower the resulting profile, while on cluster scales where smaller-mass halos collapse earlier at higher density and are subsequently incorporated into larger-mass halos, these will go to the center and give rise to a steeper radial density profile. This seems consistent with our results, as mentioned in KKBP98. We should then understand why the results of Huss, Jain, & Steinmetz (1998) seem inconsistent with this argument. Regarding why the outer radial dependence is roughly $r^{-3}$, recent work by Henriksen & Widrow (1998) may be relevant.

3. Press-Schechter

The Press-Schechter (1974, hereafter PS) formula for the number density of dark matter halos as a function of their mass is based on two simple assumptions – Gaussian statistics for density fluctuations, and spherical top-hat collapse of these fluctuations. (See, e.g., Peebles (1993), pp. 630-635, or White (1996) for modern treatments.) Since both of these assumptions are known to be wrong, or at best oversimplified, the wonder is not that the PS approximation is not perfect, but rather that it works at all. In fact, the PS formula predicts the number density of virialized cluster-mass halos in N-body simulations remarkably well. However, several groups recently noticed a discrepancy: $N_{\text{PS}}(> M) \approx 2N_{\text{simulations}}(> M)$ for $M \lesssim M_*/10$. Since this has been seen in many simulations using different methods of simulating and identifying halos, it should be taken seriously. For example, Gross et al. (1998), using high-resolution particle mesh (PM) simulations and both spherical and ellipsoidal overdensity halo finders, found that the number density of galaxy-mass halos at the current epoch is overestimated by PS by about a factor of 2 in many currently popular CDM-type cosmological models, regardless of the collapse parameter $\delta_c$ used in the PS formula. Gross (1997), Appendix G, showed that the discrepancy of the PS number density is about this big for higher redshifts also, as long as $M \lesssim M_*/10$. Kauffmann et al. (1998) found the same factor of $\sim 2$ discrepancy at the current epoch for both the $\tau$CDM and $\Lambda$CDM models, and Somerville, Lemson, Kolatt, & Dekel (1998) generalized this to the halo merging trees of the Extended PS theory. This work is based on AP$^{3}$M simulations.
and a friends-of-friends halo finder, as is related work on larger mass halos, the abundance of which is underpredicted by the PS approximation (Governato et al. 1998). Finally, Sigad et al. (1998) have found the same phenomenon in the ART simulations, using a version of the bound density maximum (BDM) halo finder (Klypin et al. 1997). A similar result was actually found independently in an analytic calculation based on approximations relevant to the highly nonlinear regime (Valageas & Schaeffer 1997).

The overprediction of the number density of galaxy-mass halos at low redshift in the PS approximation has several implications, including an amelioration of the overprediction by semi-analytic models of galaxy formation (based on the extended PS theory) of the luminosity function of galaxies and the star formation rate at low redshift (see, e.g., Somerville & Primack 1998ab). The underprediction by PS of the number density of massive halos, especially at high redshifts, means that strong conclusions about the density of the universe based on observations of clusters compared with PS predictions should be treated with caution (Governato et al. 1998).

4. Lyman Break Galaxies as Merger-Triggered Starbursts

Semi-analytic models of galaxy formation were pioneered by White & Frenk (1991), Kauffmann, White, & Guiderdoni (1993), and Cole et al. (1994); see Somerville & Primack (1998a) for a review. Such models follow the evolution of the dark and luminous contents of the universe using simple approximations to treat gas cooling, star formation, and feedback within dark halos, and Extended PS theory to predict the merger rate of halos in order to construct merger trees (e.g., Somerville & Kolatt 1998). When halos merge, their luminous contents are usually assumed to merge only as dynamical friction brings smaller galaxies to the large galaxy at the center of the new halo, and most of the star formation even at high redshift is quiescent (e.g., Baugh et al. 1998). However, both observational and theoretical arguments suggest that many of the small but very bright galaxies now being identified in very large numbers at redshifts \( z \gtrsim 3 \) by the Lyman break method (Steidel et al. 1996, Dickinson 1998) are low-mass starbursts rather than large galaxies that have been quiescently forming stars for a long time (Lowenthal et al. 1997; Sawicki & Yee 1998; and Somerville, Primack, & Faber 1998, hereafter SPF98). SPF98 assumed that random collisions of dark matter subhalos, as well as decay of orbits due to dynamical friction, would trigger mergers of (proto-)galaxies that could lead to starbursts, and they based their semi-analytic modeling of the number of such mergers on recent dissipationless simulations of the mergers of dark halos (Makino & Hut 1997), and of the star formation efficiency and timescale on hydrodynamic simulations of starbursts triggered by mergers (Mihos & Hernquist 1994, 1996). The result was that most of the star formation in CDM-type hierarchical models at redshifts \( z \gtrsim 2 \) occurs in merger-triggered starbursts, and that most of the Lyman break galaxies (LBGs) are expected to be such starbursts. This perhaps resolves the apparent paradox that the LBGs appear to cluster like massive halos (Steidel et al. 1998, Giavalisco et al. 1998, Adelberger et al. 1998; cf. Wechsler et al. 1998) while their relatively low linewidths (Pettini et al. 1998) and their spectral energy distributions (Sawicki & Yee 1998) suggest that they have relatively low
mass \((\text{few} \times 10^{10} M_{\odot})\) and young ages \((\text{few} \times 10^8 \text{ yr})\). Including merger-triggered starbursts also predicts much more star formation at high redshift in CDM-type hierarchical models for structure formation than if only quiescent star formation is included (Somerville & Primack 1998b).

Do the merger rates actually grow so large at high redshift that this merger-triggered starburst scenario is plausible? With many stored timesteps from large ART simulations, we have begun to study the merger rate of dark matter halos as a function of redshift. This sort of study requires careful definitions of halos and subhalos, and of the criteria for identifying mergers, which will be given in papers now in preparation (Kolatt et al. 1998ab, Bullock 1999). But to summarize the situation briefly, the answer is yes. The collision rate of halos and subhalos grows in physical coordinates roughly as \((1 + z)^3\) up to a redshift that depends on the mass of the halo. The comoving collision rate of \(\gtrsim 10^{10} h^{-1} M_{\odot}\) halos peaks in the \(\Lambda\)CDM simulations at \(z \sim 3\), at a rate high enough to account for the observed number density of LBGs (Kolatt et al. 1998a). Indeed, the comoving number density of LBGs with AB magnitude brighter than 25.5 is predicted to be almost as high at \(z = 4\) as at \(z = 3\). We also find that the mergers occur mainly in and near the most massive halos, so that the high bias of the bright LBGs arises naturally.

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