Formulation of steady-state void fraction through the principle of minimum entropy production

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Abstract
The complexity of the actual operation of thermal engineering systems comprises multiphase interfacial phenomena evolving out of equilibrium. Therefore, their generalised formulation can contribute towards better understanding and control of these phenomena, eventually pushing the existing related technologies beyond the state-of-the-art. In this respect, variational principles are significant for a more comprehensive physical representation and for closing the problem, while obtaining relatively simpler mathematical formulations. In this study, a general variational formulation of dissipative two-phase flows based on the minimum entropy production is developed. In particular, this study provides a general expression of the entropy generation rate, which introduces interfacial contributions due to surface tension between different phases, and is used to estimate two-phase flow fraction based on Prigogine’s theorem of minimum entropy generation. Subsequently, this formulation is investigated in terms of different assumptions and pressure drop models, and employed to clarify the implementation of Prigogine’s theorem to obtain the widely-accepted Zivi’s expression of void fraction and the effect of different assumptions on the deviation from his expression. A new expression is finally obtained to cover laminar flow conditions, which are implicitly excluded from the applicability of Zivi’s expression.

Keywords: Void fraction, Entropy generation, Prigogine’s theorem, Variational formulation, Zivi’s expression

Nomenclature
Symbols
A Cross-sectional area \( m^2 \)
D Channel diameter \( m \)
f Friction factor \(-\)
G Mass flux \( kg \cdot m^2 \cdot s^{-1} \)
h Specific entropy \( kJ \cdot kg^{-1} \)
\( \dot{m} \) Mass flow rate \( kg \cdot s^{-1} \)
p Pressure \( Pa \)
q Heat flux \( kW \cdot m^2 \)
Re Reynolds number \(-\)
s Specific enthalpy \( kJ \cdot kg^{-1} \cdot K^{-1} \)
\( \dot{S} \) Entropy generation rate \( kW \cdot K^{-1} \)
t Time \( s \)
T Temperature \( K \)
u Velocity \( m \cdot s^{-1} \)
V Volume \( m^3 \)
x Vapour quality \(-\)
z Stream-wise position \( m \)

Greek symbols
\( \alpha \) Heat transfer coefficient \( kW \cdot m^{-2} \cdot K^{-1} \)
\( \beta \) Interfacial area density \( m^{-1} \)
\( \epsilon \) Void fraction \(-\)
\( \phi \) Slip ratio \(-\)
\( \varphi \) Interface cross-section contour \( m \)
\( \mu \) Viscosity \( Pa \cdot s \)
\( \Omega \) Control volume \( m^3 \)
\( \rho \) Density \( kg \cdot m^{-3} \)
1. Introduction

This study originates from the acknowledgement of a gap between the technological progress of thermal engineering systems and their physical understanding, meaning there is not an established formulation of several among the complex physical phenomena involved. This is reflected by a significant difference between catalogue system performance at the rated condition and actual operative performance, which can be associated with inappropriate system operation based on trial and error procedures or oversimplified analysis at thermodynamic equilibrium. Because the complexity of actual operation involves multiphase interfacial phenomena dynamically evolving out of equilibrium, their general formulation within the theory of non-equilibrium thermodynamics could contribute to fill this gap, and eventually revolutionise the related technologies beyond the state-of-the-art.

Variational principles related to the second law of thermodynamics have been applied to engineering optimisation problems owing to their ability to provide additional conditions to the conservation principles (Giannetti et al., 2015), which can also be used to conduct optimisation procedures (Giannetti et al., 2016). Heat and mass transfer of diabatic devices using single-phase working fluids has been enhanced by spreading dissipative effects according to the entropy generation minimisation principle (Bejan, 1987; Grazzini and Gori, 1988). Owing to the higher variability encountered when dealing with two-phase flow problems, the simple application of momentum, energy, and mass balances, together with related boundary conditions, does not result in a sufficient number of equations for the large number of variables involved in these engineering problems. Therefore, variational principles and simplifying geometrical assumptions are significantly useful for closing the problem and obtaining simpler mathematical formulations in terms of generalised variables.

Although significant efforts have been applied to the theoretical investigation of non-equilibrium thermodynamics, the theory has not reached the level of completeness due to the lack of a generalised variational formulation (Gay-Balmaz and Yoshimura, 2019). The modern approach to thermodynamics initiated in the twentieth century by Duhem in his work titled "Energetique" (Kondepudi and Prigogine, 2014), is directed towards the quantification of the "uncompensated transformation", which is the so-called "entropy production" by irreversible processes. This procedure enables writing the formalism of entropy as an equality, rather than Clausius inequality, where besides thermodynamic states, irreversible processes appear explicitly. Therefore, the objective is the formulation of irreversible processes as an aftermath of the presence of thermodynamic forces associated with their potentials resulting in thermodynamic flows, which drive the system to a stable condition where the thermodynamic potentials are extremised.

This study applies to the category of systems where "local equilibrium" holds. A general expression of the entropy generation rate is explicitised as the basis for describing dissipative two-phase flows in conventional-sized to capillary channels through a variational principle indicating the system arrangement towards an extremum of the rate of entropy production. The implementation of Prigogine's theorem to express the steady-state void fraction is investigated to clarify the influence of various operating regimes, assumptions, and underlying physical relations. Consequently, considerations are provided with regard to the simplifying assumptions that must hold to reach the widely-accepted Zivi’s expression of the steady-state void fraction (Zivi, 1964) where entropy production is made identical to energy dissipation. Finally, a new expression is obtained to cover laminar flow conditions, which is shown to be implicitly
excluded from the applicability of Zivi’s expression. This exemplifies the necessity of a direct application of Prigogine’s theorem through the expression of the entropy generation rate, and opens up to the possibility of including different entropy sources for obtaining theoretical formulations with different degrees of resolution, which can be generally applied to different working fluids. In this manner, this study contributes to the theoretical understanding of recurrent multiphase phenomena within engineering thermal systems.

Additionally, such fundamental formulations are also regarded as the basis for demonstrating influential physical terms and developing semiempirical correlations via the calibration of empirical corrective coefficients (Giannetti et al., 2020) in order to attain the required degree of accuracy for engineering design and control of thermal systems.

2. Thermodynamic formulation

First, it is necessary to lay out the conservation laws and the second law of thermodynamics for their application to the systems with a phase interface and out of global equilibrium. This is carried out by relying on the assumption of local equilibrium or the principle of local validity of the second law, to establish a relationship between local entropy density and local macroscopic quantities. This implies that significant deviations from statistical equilibrium are restrained by a sufficient dominance of molecular collisional effects (Defay et al., 1976). The interface is assumed to divide the system into two continuous partial systems, and the possibility of surface entropy sources is considered (Meinhold-Heerlein, 1973).

2.1 Entropy generation rate of the capillary two-phase flow

Assume a saturated two-phase flow with a given total mass flow rate in a circular channel (Fig. 1), where the target control volume \( V(z_0) \) is associated with the phase interfacial areas \( \Sigma_0 \) (Eq. 2).

\[
V(z_0) = \int_0^{\frac{z_0}{\Omega}} \left( A_i + A_g \right) d\zeta
\]

The indexes \( i \) and \( j \) are representative of the individual phases (g "gas", l "liquid", s "solid") describing a certain interface.

\[
\Sigma(\varepsilon, z_0) = \int_0^{\frac{z_0}{\Omega}} \phi_{ij} d\zeta
\]

The fluid flow is assumed to be at saturation \( T_{sat} \) for the inlet pressure \( P_{sat} \), with inlet vapour quality \( x \), while experiencing a constant heat transfer rate (Eq. 3).

\[
\dot{\omega}_g = \frac{d}{dt} \omega_g
\]

As the target variable is the void fraction \( \varepsilon \) (Eq. 4) (or interfacial area density \( \beta_{ij} \), Eq. 5), which represents the local phase distribution within the channel.

\[
\varepsilon = \frac{A_g}{A} = \frac{A_g}{A_g + A_i}
\]

The interfacial area density is linearised around the axial position \( z \).

\[
\beta_{ij} = \frac{\Sigma_{ij}}{\Omega} \approx \frac{\phi_{ij}}{A}
\]
For given total and individual flow rates \((\dot{m}_t, \dot{m}_v, \dot{m}_l)\), assuming negligible kinetic and gravitational potential terms, and absence of internal heat sources or sinks as well as external shaft work (Revellin et al., 2009), the first law of thermodynamics, for the open system under consideration, is written as,

\[
\frac{\omega_q}{m_t} = dh_{tp}
\]

where \(dh_{tp}\) is the state variable defined as follows (Eq. 7) (Revellin et al., 2009):

\[
dh_{tp} = d \left( xh_g + (1-x)h_l \right)
\]

Under the assumption of a low curvature surface \((R = 1/\kappa > 1 \mu m)\), Gibbs equation yields the relations written in Eqs (8-9) (Revellin et al., 2009). It is worth noting that, as far as local equilibrium holds, Gibbs relation of the state quantities is representative of both reversible and irreversible processes, since it describes the change of the property entropy between two states independently of the type of process (Cengel, 2008; Kondepudi and Prigogine, 2014).

\[
dh_g = T_{sat} ds_g + v_g dp
\]

\[
dh_l = T_{sat} ds_l + v_l dp
\]

where the vapour quality \(x\) is defined by Eq. (10).

\[
x = \frac{\dot{m}_g}{\dot{m}_g + \dot{m}_l}
\]

The possibility of surface entropy sources is considered by relying on the concept of excess properties due to the different bonding of the molecules in proximity of a condensed interface (Mullins, 1963; Blakely, 1973; Meier, 2014). Accordingly, by considering the interfacial entropy term \(dS_i\), (Eq. 13) (Berg, 2009), the entropy balance applied to the control volume of length \(dz\) (Fig.1) yields Eq. (11).

\[
\dot{S}_{gen} = \dot{S}_{int} - \dot{S}_{ext} = d \left( \dot{m}_g s_g + \dot{m}_l s_l + \dot{S}_{gl} \right) - \frac{\omega_q}{T_w}
\]

where the external entropy generation rate \(\dot{S}_{ext}\) is exclusively provided by the heat transfer rate to the wall, as
defined in Eq. (12),

\[
\dot{S}_{\text{ext}} = \frac{\omega_q}{T_w}
\]  

(12)

It should be noted that, as previously observed in literature (i.e. Zhang et al., 2019; Revellin et al., 2012) the capillary work due to the surface tension is reversible and does not contribute to the entropy generation. The term reported in Eq. (13) is an approximate form of the dissipative part of the entropy variation at the phase interface resulting from the “heat of extension” (Berg, 2009).

\[
dS_{gl} = -\frac{\partial \sigma}{\partial T} \left( d\Sigma_{gl} \right) dt = -\frac{\partial \sigma}{\partial T} \left( \varphi_{gl} dz \right) dt = -\frac{\partial \sigma}{\partial T} u_{gl} \varphi_{gl} dt
\]  

(13)

For a more detailed representation, which exceeds the scope of this contribution, Napolitano (1978) demonstrates that this dissipative contribution of the surface entropy is related to the difference between the mean surface normal stress and the surface tension.

Introducing the first law (Eq. 7) and the Eq.s (8-9) in Eq. (11), yields the local entropy generation rate (Eq. 14).

\[
\dot{S}_{\text{gen}} = \frac{\omega_q}{T_{sat}} + \frac{m_i v_p}{T_{sat}} (-dp_p) - \frac{\partial \sigma}{\partial T} u_{gl} \varphi_{gl} - \frac{\omega_q}{T_w}
\]  

(14)

The temperature difference between the saturated fluid \(T_{sat}\) and the exchange surface \(T_w\) is represented as in Eq. (16).

\[
T_w = T_{sat} - \Delta T
\]  

(15)

Whereas the heat flux \(q\) is related to the heat transfer coefficient \(\alpha\) and the total heat transfer rate \(\omega_q\) over the exchange area \(\varphi dz\) (Eq. 16).

\[
q = \alpha(\varepsilon) \Delta T = \frac{\omega_q}{\varphi dz}
\]  

(16)

Accordingly, Eq. (14) can be rearranged as,

\[
\dot{S}_{\text{gen}} = \frac{\omega_q}{T_{sat}^2} \left( \frac{\Delta T}{1 + \frac{\Delta T}{T_{sat}}} \right) + \frac{m_i v_p}{T_{sat}} (-dp_p) - \frac{\partial \sigma}{\partial T} u_{gl} \varphi_{gl}
\]  

(17)

which can be simplified to Eq. (18) by considering common operative circumstances \((\Delta T \ll T_{sat})\).

\[
\dot{S}_{\text{gen}} = \frac{\omega_q \Delta T}{T_{sat}^2 \alpha} + \frac{m_i v_p}{T_{sat}} (-dp_p) - \frac{\partial \sigma}{\partial T} u_{gl} \varphi_{gl}
\]  

(18)

The entropy generation density is given by Eq. (19).
\[
\dot{\xi}_{\text{gen}} = \frac{q^2 \varphi}{T_{\text{sat}}^2 \alpha(\varepsilon) A} + \frac{m_i v_i p_i}{T_{\text{sat}} A} \left( -\frac{d\sigma_{gl}}{dz}(\varepsilon) \right) - \frac{\partial \sigma_{gl}}{\partial T} u_{gl} \frac{d\beta_{gl}}{dz}(\varepsilon) 
\]

where,
\[
d\beta_{q,i} = \frac{d\varphi_{q,i}}{dA} = \frac{d\varphi_{q,i} dz}{dV} = d\Sigma_{q,i} 
\]

Accordingly, the entropy generation per unit length of the channel \(dz\) is written as in Eq. (21).
\[
\dot{\xi}_{\text{gen}} A = \frac{q^2 \varphi}{T_{\text{sat}}^2 \alpha(\varepsilon)} + \frac{m_i v_i p_i}{T_{\text{sat}} A} \left( -\frac{d\sigma_{gl}}{dz}(\varepsilon) \right) - \frac{\partial \sigma_{gl}}{\partial T} u_{gl} \frac{d\varphi_{gl}}{dz}(\varepsilon) 
\]

### 2.2 Proof of \(\xi_{\text{gen}}\) positivity

As explained in the following section, the constitutive relationship for heat and momentum transfer enables us to ensure the positivity of the first two terms of the entropy generation density, as represented by Eq. (22).
\[
\dot{\xi}_{\text{gen}} = \dot{\xi}_q + \dot{\xi}_f + \dot{\xi}_i
\]

The terms related to heat transfer (Eq. (23)), friction (Eq. (24)), and capillarity (Eq. (25)) can be expressed as a quadratic form of the fluxes and thermodynamic forces, respectively:
\[
\dot{\xi}_q = \frac{\omega_q \Delta T}{T_{\text{sat}}^2} = \frac{1}{T_{\text{sat}}^2} (\alpha \Sigma \Delta T) \Delta T 
\]
\[
\dot{\xi}_f = \frac{1}{T_{\text{sat}}} \left( -f \cdot z \right) = \frac{1}{T_{\text{sat}}} \left( -(r \cdot z) \cdot z \right) 
\]

Specifically, the constitutive relation for the surface tension \(\sigma\) is often linearly expressed with respect to the absolute temperature \(T\) (Osipow, 1962): \(\sigma_{gl}(T) = f(T)\). The expression of entropy variation due to capillary systems with commonly negative values of the temperature derivative of surface tension shows that, from a thermodynamic perspective, a system at constant enthalpy and pressure will always tend to spontaneously contract its interface, and interface generation will be associated with positive entropy variations and heat release. In the framework of linear irreversible thermodynamics (de Groot and Mazur, 1962) constitutive relations for the dissipative part of surface diffusive fluxes are postulated as Eq. (30); here, \(\mu_{gl}\) is referred to the viscosity coefficient, which is a non-negative function of the sole temperature according to the postulate of positive entropy production (Napolitano, 1977).
\[
\dot{\xi}_i = -\frac{\partial \sigma_{gl}}{\partial T} d\beta_{gl} = -\frac{1}{T} \left( -\mu_{gl} \nabla \cdot \vec{z}_{gl} \right) \nabla \cdot \vec{z}_{gl} 
\]

### 2.3 Variational formulation through Prigogine's theorem

Beyond equilibrium, in the "linear region" of Onsager reciprocity laws, the Prigogine's theorem of minimum entropy production has been established (Prigogine, 1961), which implies that the entropy generation density represents a kind of universal thermodynamic potential. Provided the necessary relationships for expressing the heat transfer coefficient \(\alpha(\varepsilon)\), pressure gradient, interfacial velocity \(u_{gl}(\varepsilon)\), and interfacial geometry \(\varphi_{gl}(\varepsilon)\) or \(\beta(\varepsilon)\), it is possible to directly apply the principle of minimum entropy generation:
\[
\frac{\partial \hat{\xi}_{\text{gen}}}{\partial \epsilon} = 0
\]  

(26)

Therefore, once the necessary expressions of the heat transfer coefficient \(\alpha(\epsilon)\), pressure gradient, interfacial velocity \(u_{gl}(\epsilon)\), and interfacial geometry \(\varphi_{gl}(\epsilon)\) \((\text{or } \beta(\epsilon))\), it is conceptually possible to represent the phenomenon as the steady distribution of the local void fraction or the local interfacial density (Fig. 2).

![Fig. 2 Representation of two-phase flow interfaces within a channel with a circular cross-section (reproduced from Collier and Thome, 1994)](image)

3. Analytical void-fraction formulations

Considering the separated integration of the entropy density (Eq. 19) over the two separated phase volumes, (Fig. 3), in an adiabatic channel, the entropy generation rate per unit channel length can be written as Eq. (27).

![Fig. 3 Representation of separated phase volumes for separated integration](image)

\[
\int_{V} \dot{\xi}_{\text{int}} dV = \int_{V_g} \dot{\xi}_{\text{int},g} dV + \int_{V_l} \dot{\xi}_{\text{int},l} dV
\]  

(27)

3.1 Derivation of Zivi’s formula

The treatment of the problem presented by Zivi (Zivi, 1964) has been questioned in previous literature for the ambiguous relationship between minimum entropy production and minimum kinetic energy flux when applying Prigogine’s theorem of minimum entropy production. To clarify the implementation of Prigogine’s theorem to reach Zivi’s expression of void fraction, where minimal kinetic energy flux and minimal entropy generation are identical, the necessary hypotheses are discussed.

Considering the separated integration of the entropy density over the two separated phase volumes in an adiabatic channel with no capillary effects, the entropy generation rate per unit channel length can be written as Eq. (28).

\[
\dot{\xi}_{\text{gen}} A = \frac{m_l V_l}{T_{\text{sat}}} \left( \frac{dp}{dz}(\epsilon) \right) + \frac{m_g V_g}{T_{\text{sat}}} \left( \frac{dp}{dz}(\epsilon) \right)
\]  

(28)
Additionally, when the pressure gradient is expressed by the Darcy-Weisbach equation, and a given (constant) friction factor $f$, applied to a given friction interface $\varphi_i$, is considered, this condition is representative of a completely turbulent flow of each individual phase. This could be effectively restricted to considering completely turbulent, ideally separated, adiabatic flow, with no interfacial effects, constant friction factor, and given friction interface.

$$\frac{dp_i}{dz} = \tau_i \frac{\varphi_i}{A}$$  \hspace{1cm} (29)$$

$$\frac{dp_g}{dz} = \tau_g \frac{\varphi_g}{A}$$  \hspace{1cm} (30)$$

where the shear stress and average phase velocity are expressed as follows.

$$\tau_i = f \frac{\rho_i u_i^2}{2} \hspace{1cm} \tau_g = f \frac{\rho_g u_g^2}{2}$$ \hspace{1cm} (31)$$

$$u_g = \frac{Gx}{\rho_g \varepsilon} \hspace{1cm} u_i = \frac{G(1-x)}{\rho_i (1-\varepsilon)}$$ \hspace{1cm} (32)$$

Thus obtaining Eq. (33), which is exemplified in Fig. 4 for the case of saturated two-phase water at $T_{sat} = 475$ °C with a mass flux $G = 578$ kg·m$^{-2}$·s$^{-1}$ and friction factor $f = 0.05$.

Figure 4 presents the characteristics of the thermodynamic force related to the pressure gradient of the flow under the hypothesis considered to obtain Zivi’s result for different vapour quality values. The stationary point among the scenario of possible configurations in terms of the void fraction corresponds to the steady-state regime of the irreversible flow. The continuous path of the local minima is summarised in Fig. 5 as a function of different values of the density ratio between the liquid and vapour phases.

![Fig. 4 Entropy generation rate per unit length as a function of the local void fraction for the case of saturated two-phase water at $T_{sat} = 475$ °C with a mass flux $G = 578$ kg·m$^{-2}$·s$^{-1}$ and friction factor $f = 0.05$ ($\rho^* = 107$)](image)

$$\dot{\zeta}_{gen} A = \frac{2G^3}{T_{sat}} f \varphi_i \left[ \frac{x^3}{\rho_g \varepsilon^2} + \frac{(1-x)^3}{\rho_i (1-\varepsilon)^2} \right]$$ \hspace{1cm} (33)$$

In this particular case, Eq. (33) is proportional to the kinetic energy of the flow presented in Zivi’s work and yields Zivi’s result (Eq. 34) for minimum entropy production (Eq. 26). This formulation corresponds to the representation of the slip ratio given by Eq. (35).
\[ \varepsilon = \frac{1}{1 + \frac{1-x}{x} \left( \frac{\rho_s}{\rho_l} \right)^2} \]  
\[ \phi = \left( \frac{\rho^*}{\rho} \right)^{\frac{1}{3}} \]  

Fig. 5 Void fraction (from Eq. (40)) as a function of vapour quality for different values of the density ratio between the liquid and vapour phases \( \rho^* \), for the case of saturated two-phase water at \( T_{sat} = 475 \, ^\circ C \) with a mass flux \( G = 578 \, kg \cdot m^{-2} \cdot s^{-1} \) and friction factor \( f = 0.05 \) 

When plotted for different density ratios between the liquid and vapour phases \( \rho^* \) (as in Fig. 5), in general, higher values of \( \rho^* \) are associated with higher slip ratios (Eq. 35), yielding higher void fractions for a particular vapour quality.

### 3.2 Extension of Zivi's formula

In the case of a laminar, ideally separated, adiabatic flow with no interfacial effects, and given friction interface, the entropy generation rate per unit channel length can be written as Eq. (37) when the Darcy friction factor in the laminar range (Eq. 36) is used. This formulation provides evidence for the fact that a different treatment of the pressure gradient than that presented in Section 3.1 results in entropy generation rates that are not proportional to the kinetic energy of the flow, and the formulation presented by Zivi does not apply.

\[ f_l = \frac{64}{Re_l} \quad f_g = \frac{64}{Re_g} \]  
\[ \dot{\Omega}_{gen} A = \frac{128G^2}{T_{sat}} \frac{\phi_l \mu_l}{D \rho_s^2} \left[ \frac{x^2}{\varepsilon} + \frac{\mu^* (1-x)^2}{\rho^* \varepsilon^2 (1-\varepsilon)} \right] \]  

Equation (38) is obtained by minimising Eq. (37) with respect to the void fraction \( \varepsilon \). It can be noted that the laminar formulation of the entropy production rate gives a slip ratio that is related to the viscosity ratio between the liquid and vapour phases (Eq. 39).
\[ \varepsilon = \frac{1}{1 + \frac{1 - x}{x} \left( \frac{\mu^*}{\rho^2} \right)^2} \]  
\[ \phi = \left( \frac{\mu^*}{\rho^2} \right)^{\frac{1}{2}} \]  

(38)  

(39)

Figure 6 presents the characteristics of the thermodynamic force related to the pressure gradient of the flow under the hypothesis of a laminar separated flow.

Fig. 6 Entropy generation rate per unit channel length (Eq. 37) as a function of the local void fraction for the case of saturated two-phase water at \( T_{sat} = 475^\circ C \) with a mass flux \( G = 578 \, \text{kg} \cdot \text{m}^{-2} \cdot \text{s}^{-1} \) (\( \rho^* = 107 \))

Under this approach, the stationary point among the scenario of possible configurations in terms of void fraction corresponds to the steady state regime of the irreversible flow. The continuous path of the local minima is summarised in Fig. 7 as a function of different values of the density and viscosity ratios between the liquid and vapour phases.

In general, higher values of density ratios (Fig. 7 a) and lower viscosity ratios (Fig. 7 b) are associated with higher slip ratios (Eq. 45), yielding higher void fractions for a specified vapour quality.

Fig. 7 Void fraction (from Eq. 38) as a function of vapour quality for different values of (a) the density ratio between the liquid and vapour phases \( \rho^* \); (b) the viscosity ratio between the liquid and vapour phases \( \mu^* \), for the case of saturated two-phase water at \( T_{sat} = 475^\circ C \) with a mass flux \( G = 578 \, \text{kg} \cdot \text{m}^{-2} \cdot \text{s}^{-1} \)
The magnitude of the deviations between Zivi’s model (Eq. 34) and Eq. (38) is represented in Fig. 8, where higher void fractions are provided by the latter (in relation to higher resulting slip ratios obtained for $\mu^* = 1$). Higher deviations appear at density ratios in the order of $10^1$. However, as the value of $\mu^*$ is necessarily higher than unity for two-phase flows, the resulting prediction might get closer to that of Zivi, or even lower (as shown in Fig. 9, where some representative experimental results from previous literature are plotted along with the results of these two models).

Fig. 8 Comparison between the calculated void fraction from Zivi’s formula (Eq. 34) and Eq. (38) as a function of vapour quality for different values of the density ratio between the liquid and vapour phases $\rho^*$, for the case of saturated two-phase water at $T_{\text{sat}} = 475$ °C with a mass flux $G = 578$ kg·m$^{-2}$s$^{-1}$

![Fig. 8](image)

Fig. 9 Comparison between the calculated void fraction from Zivi’s formula (Eq. 34) and Eq. (38) with experimental data (a) of saturated R410A at $p = 1.9$ MPa, in a channel with diameter $D = 3$ mm, and mass flux $G = 200$ kg·m$^{-2}$s$^{-1}$ (Shedd, 2010); (b) of saturated R410A at $p = 3.1$ MPa, in a channel with diameter $D = 3$ mm, and mass flux $G = 600$ kg·m$^{-2}$s$^{-1}$ (Shedd, 2012)

![Fig. 9](image)

The comparison with experimental data illustrated in Fig. 9 is not meant to demonstrate the superiority of one formulation over the other. On the contrary, the purpose of this work is to show the possibility of obtaining different theoretical formulas by the direct application of Prigogine’s theorem of minimum entropy production in relation to the physical representation of the pressure gradient or other sources of entropy generation. Therefore, Eq. (38) results in a better prediction for the data in Fig. 9(a) obtained at lower mass flux condition, which is consistent with the laminar friction factor formulation in Eq. (36). Meanwhile, Zivi’s formulation (which requires a fully turbulent representation of the friction factor) gives better predictions at higher mass fluxes, such as the condition at which the data in Fig 9(b) were measured.

4. Conclusion

In this study, a general variational formulation of dissipative two-phase flows based on the minimum entropy
production has been developed by expressing the entropy generation rate in a general manner. This introduces interfacial contributions due to surface tension between different phases, and can be used to estimate the two-phase flow fraction on the basis of Prigogine's theorem of minimum entropy generation. Subsequently, this formulation has been investigated in terms of different assumptions to clarify the implementation of Prigogine's theorem to obtain the well-known Zivi's expression of void fraction. It is valid for a completely turbulent, ideally separated, adiabatic flow, with no interfacial effects, constant friction factor, and given friction interface.

Nonetheless, different modelling assumptions result in deviation from Zivi's expression, thus highlighting the necessity of a direct application of Prigogine's theorem through the expression of the entropy generation rate, while the equivalence to the kinetic energy dissipation presented by Zivi (1964) does not generally apply.

Moreover, a new expression obtained for laminar, ideally separated, adiabatic flow, with no interfacial effects, and given friction interface, has been obtained and discussed. It has shown a dependency of the slip ratio on the viscosity ratio rather than the density ratio between the liquid and vapour phases. Hence, the new expression expands the applicability of Zivi's method to capture the void fraction in laminar flow conditions.

This exemplifies the necessity of a direct application of Prigogine's theorem through the expression of the entropy generation rate, and opens up to the possibility of including different entropy sources for obtaining theoretical formulations with different degrees of resolution, which can be generally applied to different working fluids.

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