Study of Simplified Calculating Model on Friction Correlated Prestress Loss for PC Bridges

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Abstract. The friction correlated loss for prestress takes up a comparative large proportion in the total prestress loss, which is hard to control and brings unsafe factor to the Prestressed Concrete (PC) bridges. For better quantification, a simplified calculating model for friction correlated prestress loss was put forward in a universal way based on comparatively sophisticated prestressing loss method in current code. The determinant simplified coefficients may also provide some references in calculating the friction relevant losses for prestress of the space curve tendon in other PC bridges.

1. Introduction
The application of prestressed concrete bridges is becoming more and more widespread. Applying three-way prestress to the bridge structure during construction will greatly improve its safety and durability. On the other side, it leads to much more complex calculations for prestress losses, in which the friction correlated losses occupy a high proportion. Therefore, when the post-tensioned PC bridge structure is damaged, the friction relevant prestress losses including bending tunnel friction and anchorage deformation with reverse friction are essential.

We launch a research on simplified calculating model of friction correlated prestress loss for post-prestressing tendons in PC bridges. The bridges may have different geometric patterns of prestressed reinforcement, different types of structures, and different structural formation systems.

2. Unified coefficients on friction relevant losses
We define two items of coefficient corresponding to friction for unifying friction losses correlation. All these are fulfilled on the foundation of statistical analysis about tunnel friction coefficient $\kappa$ and coefficient of friction $\mu$.

2.1. Definition of corresponding influence parameter
For the purpose that unified algorithm is suitable for any tendon, influence parameter $\zeta$ is quoted to render the contribution of two friction correlated losses, which are composed of bending tunnel friction $\sigma_1(x)$ and anchorage deformation with reverse friction $\sigma_2(x)$.
In Figure 1., second-degree curvilinear equation in local coordinate system $\text{xy}$ is presented as:

$$\overline{y(x)} = \frac{f}{L^2} \overline{x}^2 - \frac{f}{L} \overline{x} + c \quad (1)$$

Curvature at any point is:

$$tg \theta_x = \frac{2f}{L^2} \overline{x} - \frac{f}{L} \approx \theta_x \quad (2)$$

Therefore, cumulative angle can be expressed as linear function of longitudinal coordinate:

$$\theta_x = \frac{2f}{L^2 \cdot \cos \theta_0} \overline{x} - \frac{f}{L} + \theta_0 - \frac{2fx_0}{L^2 \cdot \cos \theta_0} \quad (3)$$

Substituting curve length for cast shadow length $x$ approximately shown in Figure 2., it must satisfy the equilibrium equation:

$$\frac{dN}{dx} = -(\mu d \theta + \kappa dx) \approx -\zeta dx \quad (4)$$

Integrated on both sides, effective force can be determined as:

$$N(x) = N_{con} e^{-\zeta x} \quad (5)$$

Thus the friction loss function is got:

$$\sigma_{fl}(x) = \frac{N_{con} - N(x)}{A_p} = \sigma_{con} (1 - e^{-\zeta x}) \quad (6)$$

2.2. Error analysis

By Taylor series expansion method, first two items can be adopted. The exponential function $y = e^{-\zeta x}$ may be substituted for $y = 1 - \zeta x$. In order to complete the accuracy checking, error analysis on second order remainder formula $|R_2(x)| = \left| \frac{e^{-\zeta x} (\zeta x)^2}{2!} \right|$ is promoted. The error curve is presented in Figure 3.
Conclusion can be drawn that relative error is smaller than 3.7 percent provided that \( |\zeta x| \leq 0.25 \).

Through statistics on all kinds of prestressing steel tendons in a large number of existing bridges, it can meet the condition of \( |\zeta x| \leq 0.25 \) for each single tendon in common range. That is to say, the former simplified method can meet the precision need for practical project.

3. Simplified calculating model of friction correlated prestress loss

3.1. Elementary components for prestressed tendon

As far as existing long-span and mid-span PC bridges are concerned, it is common to see tendons with composite-curve type and mainly for the following reasons. Load acted on continuous beam bridge is linear one in most cases. Vice versa, linear resisting force may be acquired equivalently on those structures with assembled-curve tendon. Meanwhile, in order to meet the need in construction and service stage, different patterns of tendon may be employed at different spans.

Therefore, considering the complexity for geometric type, classification on geometric patterns should be achieved in detail before analysis on friction correlated prestress losses. Altogether, there are two types of elementary components categorized by geometric topology of common prestressed tendons shown in Figure 4.

![Diagrammatic sketch of elementary components for prestressed tendons.](image)

From the diagrammatic sketch, we can see that one is a familiar tendon shape named “SL-CU-SL”, which is in form of bottom slab tendon for simple supported beam bridge, tip slab tendon for continuous rigid-frame bridge and local tendon for some continuous beam bridge. The other is addressed as “SL-CU-CU”, which is in form of full-length tendon for some continuous beam bridge, full-length tendon for cantilever bridge and lateral tendon for box girder. All the other geometric types for prestressed tendons may be existed as the combination of these two elementary components.

3.2. Iterative algorithm

As to the reverse friction correlated loss function for composite prestress tendon, it is unable to utilize current specifications directly. A practical programming algorithm is promoted by means of simplifying frictional loss curve as segmental polygonal lines with different gradient ratios.

Effective prestress at the end of the first segment is corresponding to the initial stretching stress for the second segment, that is \( \sigma_{pe(2)} = \sigma_{con} e^{-\zeta l_1} \). In same argument, effective prestress at the end of the second segment is \( \sigma_{pe(3)} = \sigma_{pe(2)} e^{-\zeta l_2} = \sigma_{con} e^{-\zeta(l_1+l_2)} \). The rest may be deduced by analogy, as \( x \) is located in. As to segment \( i \), prestress loss of bending tunnel friction \( \sigma_{n_i}(x) \) may be expressed as:
\[ \sigma_{11}(x) = \sigma_{con} - \sigma_{pre(i)} e^{\varepsilon_1 x} = \sigma_{con} (1 - e^{-\sum_{j=1}^{i-1} \varepsilon_j x}) \]  

(7)

Thereafter discriminated equation is defined as:

\[ \sigma_{(k)}^* = \sum_{i=1}^{k} (L_i^2 - L_{i-1}^2) \cdot S_i \]

(8)

Where:

\( S_i \) being defined as the segmental gradient ratio, is equal to \( e^{-\sum_{j=1}^{i} \varepsilon_j} \) and \( L_i \) is projecting length on condition that \( L_0 = 0 \).

Varying \( k \) from 1 to \( n \), segment \( L_k \) is allocated when \( \sigma_{(k)}^* \leq \Delta \cdot E_p \) and \( \sigma_{(k+1)}^* > \Delta \cdot E_p \) are met at same time. Thus, length of influence for negative frictional loss \( l_f \) can be calculated through deforming coordination:

\[ L_f = \sqrt{\frac{\Delta \cdot E_p - \sum_{i=1}^{k-1} (L_i^2 - L_{i-1}^2) \cdot S_i}{S_k} + L_{k-1}^2} \]

(9)

Therefore, as \( x \) is located in segment \( i \), prestress loss of anchorage deformation with reverse friction \( \sigma_{12}(x) \) may be expressed as:

\[ \begin{cases} 
\sigma_{12}(x) = 2S_i (L_i - x) + \sum_{j=i+1}^{k-1} 2(L_{j+1} - L_j)S_j + 2(L_f - L_i)S_k & (x \leq L_f) \\
\sigma_{12}(x) = 0 & (x > L_f)
\end{cases} \]

(10)

4. Calculation example

In comparison, a symmetrical tendon for three-span continuous beam bridge is composed of 16 \( \phi_{15.2} \) strand and its stretching stress is \( \sigma_{con} = 1395 \text{MPa} \). The profile of quoted tendon is demonstrated in Figure 5.

Figure 5. Profile of quoted tendon.

Comparison results for prestressed strand are represented graphically in Figure 6.
Figure 6. Comparison result.

It can be seen that a match tendency on decaying curve between method in this paper and current code is acquired. All errors for effective prestress along longitudinal direction are limited below 5 percent, which is the initiative error threshold. At the same time, in the process of iteration, features on accuracy, convenience and utility of the simplified calculating model are revealed completely.

5. Conclusions

- Satisfactory accuracy is met for the proposed method and analytical module on special software platform is available, which enhances the practicability greatly.
- Friction correlated losses model is drawn out, which categorize prestress tendons with composite geometric topology according to a large number of practical projects and adequate accuracy for friction parameters prediction is met for the need of practical engineering.
- Being an essential research in over-all inspection and assessment on prestress condition, the model is expected to favor structural evaluation study on existing long-span PC Bridges.

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