We show that by introducing appropriate local $Z_N$ ($N \geq 13$) symmetries in electroweak models it is possible to implement an automatic Peccei-Quinn symmetry keeping at the same time the axion protected against gravitational effects. Although we consider here only an extension of the standard model and a particular 3-3-1 model, the strategy can be used in any kind of electroweak model. An interesting feature of this 3-3-1 model is that if: i) we add right-handed neutrinos, ii) the conservation of the total lepton number, and iii) a $Z_2$ symmetry, the $Z_{13}$ and the chiral Peccei-Quinn $U(1)_{\text{PQ}}$ are both accidental symmetries in the sense that they are not imposed on the Lagrangian but they are just the consequence of the particle content of the model, its gauge invariance, renormalizability and Lorentz invariance. In addition, this model has no domain wall problem.

PACS numbers: 14.80.Mz; 12.60.Fr; 11.30.Er

I. INTRODUCTION

It is well known that an elegant way to solve the strong CP problem is by introducing a chiral $U(1)_{\text{PQ}}$ [1] symmetry which also implies the existence of a pseudo-Goldstone boson—the axion [2]. This particle becomes an interesting candidate for dark matter if its mass is of the order of $10^{-5}$ eV [3–5]. It was also recently argued [6] that an axion–photon oscillation can explain the observed dimming supernovas [7] if the axion has a rather small mass: $\sim 10^{-16}$ eV. Whatever of these possibilities (if any) is realized in Nature, the existence of a light invisible axion can be spoiled by gravity, since it induces renormalizable and non-renormalizable effective interactions which break explicitly any global symmetry, in particular the $U(1)_{\text{PQ}}$ symmetry, and the axion can gain a mass which is greater than the mass coming from instanton effects [8]. This can be avoided if the dimension of the effective operators is $d \sim 12$. Hence, unless $d$ is high enough, invisible axion models do not solve the strong CP problem in a natural way. Here we will show how in two electroweak models the axion is protected against gravity effects, however the strategy can be used in any electroweak model.

It was pointed out several years ago by Krauss and Wilczek that a local gauge symmetry, say $U(1)$, can masquerade as discrete symmetries, $Z_N \subset U(1)$, to an observer equipped with only low-energy probes [9]. It means that these symmetries evade the no-hair theorem [10] i.e., unlike continuous symmetries they must be respected even for gravitational interactions. The only implication of the original gauge symmetry for the low energy effective theory is the absence of interaction terms forbidden by the $Z_N$ symmetry. For instance if there were more charged scalar fields in the theory, the discrete symmetry would forbid many couplings that were otherwise possible.

Here we will not worry about the origin of this $Z_N$ (or $U(1)$) symmetry [11–15]. For instance, it might be that the $Z_N$ symmetries come from a fifth dimension as it was shown for the case of $U(1)_Y$ in Ref. [12]. We will simply assume that at very high energies we have a model of the form $U(1) \otimes G_{\text{EW}}$ where $U(1)$ is a local symmetry (maybe a subgroup of a larger symmetry) which is broken to $Z_N$ at a high energy scale and $G_{\text{EW}}$ is an electroweak model i.e., $U(1) \otimes G_{\text{EW}} \rightarrow Z_N \otimes G_{\text{EW}}$. We
will use the existence of these local $Z_N$ symmetries in order to protect the axion against gravitational effects. We get this by enlarging, if necessary, the representation content of the model, so that we can impose symmetries with $N \geq 13$. In addition the $U(1)_{PQ}$ symmetry (and under some conditions also the $Z_N$ symmetry) is an automatic symmetry, in the sense that it is not imposed on the Lagrangian but it is just the consequence of the particle content of the model, its gauge invariance, renormalizability and Lorentz invariance.

In this circumstances we show how a naturally light invisible axion (it is almost singlet $\phi$ under the gauge symmetry), which is also protected against effects of quantum gravity, can be obtained in the context of electroweak models as follows. Effective operators like $\phi^{N-1}/M_{Pl}^{(N-1)-4}$ are automatically suppressed by the (local) $Z_N$ symmetry. At the same time this symmetry makes the $U(1)_{PQ}$ symmetry an automatic symmetry of the classical Lagrangian or, as in the 3-3-1 model considered here, both $Z_N$ and $U(1)_{PQ}$ are already automatic symmetries of the model under the conditions discussed in Sec. III. For instance, a $Z_{13}$ symmetry implies that the first non-forbidden operator is of dimension thirteen and it implies a contribution to the axion mass square of $\langle v^2 \rangle^{11}/(M_{Pl}^9 \sim 10^{-21}eV^2$ or $10^{-17}m_\phi^2$, if $m_{\phi} \sim A^2_{QCD}/v \approx 10^{-5}$ eV is the instantons induced mass (we have used $M_{Pl} = 10^{19}$ GeV and $v = 10^{12}$ GeV). The naturalness of the PQ solution to the $\theta$-strong problem is not spoiled since in this case we have $\theta_{eff} \propto v_{\phi}/M_{Pl}^{N-1} \sim 10^{-11}$ for $N = 13$. Recently we applied this strategy in the context of an extension of the electroweak standard model [17], now we will apply this procedure to a model with 3-3-1 symmetry.

Hence, we see that it is necessary to search for models which have a representation content large enough to allow the implementation of a discrete symmetry $Z_N$ with $N \geq 13$. In the context of a $SU(2)_L \otimes U(1)_Y$ model we have enlarged the representation content by adding several Higgs doublets, right-handed neutrinos and the scalar singlet necessary to make the axion invisible. Hence, it is possible to accommodate a $Z_{13}$ symmetry while keeping a general mixing among fermions of the same charge [17]. Larger $Z_N$ symmetries are possible if we add more scalar doublets in such a way as to generate appropriate texture of the fermionic mass matrices. On the other hand we will show in this work that in a 3-3-1 model, the minimal representation content plus right-handed neutrinos, admit enough large discrete symmetries. In these models the addition of a singlet (or a decuplet [18]) is the reason for maintaining the axion invisible, however, in the 3-3-1 model the axion picture is a mixture of the Dine et al invisible axion [20] and the Kim’s heavy quark axion model [21]. Nevertheless, unlike the model of Ref. [21], the exotic quarks are already present in the minimal version of the model.

This paper is organized as follows. In Sec. II we review briefly the new invisible axion in the context of an extension of the standard model [17]. In Sec. III we consider one 3-3-1 model. Our conclusions and some phenomenological consequences are in the last section.

**II. AN EXTENSION OF THE STANDARD MODEL**

Let us consider the invisible axion in an extension of the $SU(2)_L \otimes U(1)_Y$ model. The representation content is the following: $Q_L = (u d)_L^T \sim (2, 1/3), L_L = (\nu_l)_L^T \sim (1, -1)$ denote any quark and lepton doublet; $u_R \sim (1, 4/3), d_R \sim (1, -2/3), l_R \sim (1, -2), \nu_R \sim (1, 0)$ are the right-handed components; and we will assume that each charge sector gain mass from a different scalar doublet [19] i.e., $\Phi_u, \Phi_d, \Phi_l$ and $\Phi_\nu$ generate Dirac masses for $u-$like, $d-$like quarks, charged leptons and neutrinos, respectively (all of them of the form $2, +1 = (\phi^+, \phi^0)^T$). We also add a neutral complex singlet $\phi \sim (1, 0)$ as in Refs. [20,21], a singly charged singlet $h^+ \sim (1, +2)$, as in the Zee’s model [22] and finally, a triplet $T \sim (3, +2)$ as in Ref. [23]. The introduction of right-handed neutrinos seems a natural option in any electroweak model if neutrinos are massive particles, as strongly suggested by solar [24], reactor [25] and atmospheric [26] neutrino data.

Next, we will impose the following (local in the sense discussed above) $Z_{13}$ symmetry among those fields:

\[
\begin{align*}
Q_L &\rightarrow \omega_5 Q_L, \quad u_R \rightarrow \omega_3 u_R, \quad d_R \rightarrow \omega_5^{-1} d_R, \\
L &\rightarrow \omega_6 L, \quad \nu_R \rightarrow \omega_6^{2} \nu_R, \quad l_R \rightarrow \omega_5 l_R, \\
\Phi_u &\rightarrow \omega_2^{-1} \Phi_u, \quad \Phi_d \rightarrow \omega_3^{-1} \Phi_d, \quad \Phi_l \rightarrow \omega_4 \Phi_l, \\
\Phi_\nu &\rightarrow \omega_6^{-1} \Phi_\nu, \quad \phi \rightarrow \omega_1^{-1} \phi, \quad T \rightarrow \omega_5^{-1} T, \\
h^+ &\rightarrow \omega_1 h^+ \quad (1)
\end{align*}
\]

with $\omega_k = e^{2\pi i k/13}$, $k = 0, 1, \ldots, 6$. With this representation content and the $Z_{13}$ symmetry defined in Eq. (1) the allowed Yukawa interactions and the scalar potential is automatically invariant under a $U(1)_{PQ}$ chiral symmetry. The PQ charges are quantized after imposing an extra $Z_3$ symmetry with parameters denoted by $\tilde{\omega}_0, \tilde{\omega}_1$ and $\tilde{\omega}_1^{-1}$. Under $Z_3$ the fields transform as follows:

\[
\begin{align*}
\Phi_u, \Phi_l, T, &\nu_R \rightarrow \tilde{\omega}_1 (\Phi_u, \Phi_l, T, \nu_R), \\
\Phi_\nu, \phi, &u_R, l_R \rightarrow \tilde{\omega}_1^{-1} (\Phi_\nu, \phi, u_R, l_R), \quad (2)
\end{align*}
\]

while all the other fields remain invariant i.e., transform with $\tilde{\omega}_0$. As we said before it is possible to implement larger $Z_N$ symmetries if more scalar doublets are added in such a way as to generate appropriate texture of the fermionic mass matrices.

The PQ assignment is the following:
The axion is invisible since it is almost singlet as in Ref. [20,21], the scalar triplet is only a small correction for it.

Some axion models [27] lead to the formation of domain walls in the evolution of the universe which could be inconsistent with the standard cosmology [28]. The domain wall number is defined as [29]

\[ N_{DW} = \left| \sum_{f=L} \text{Tr} X_f T^2 \delta_{f,R} - \sum_{f=R} \text{Tr} X_f T^2 \delta_{f,L} \right|, \]

where \( X_f \) denotes the PQ charge of the quark \( f \) and there is no summation on \( a \), we have \( \text{Tr} T_a T_b = (1/2) \delta_{ab} \) for \( 3 \) and \( 3^* \). Using the PQ assignment in Eq. (3) we obtain \( N_{DW} = |(9/5) X_d| \). If \( X_d = \pm 5 \) we have \( N_{DW} = 9 \). (We choose \( X_d = -5 \) in order to have \( \bar{\theta} \rightarrow \bar{\theta} + 2\pi k, k = 0, \ldots 8 \).) The \( N = 9 \) vacua can be characterized by

\[
\begin{align*}
\langle u^j_{1L} u^i_{2R} \rangle &= \mu_3 e^{-i2\pi(3k/9)}, \quad \langle d^1_{1L} d^1_{2R} \rangle = \mu_3 e^{i2\pi(5k/9)}, \\
\langle \phi^0_{1L} \rangle &= v_u e^{i(\beta_u - 2\pi(3k/9))}, \quad \langle \phi^0_{1R} \rangle = v_u e^{i(\beta_u - 2\pi(3k/9))}, \\
\langle \phi^0_{0L} \rangle &= v_e e^{i(\beta_e - 2\pi(4k/9))}, \quad \langle \phi^0_{0R} \rangle = v_e e^{i(\beta_e - 2\pi(4k/9))}, \quad \langle T^0 \rangle = v^T e^{i(\beta_e - 2\pi(4k/9))}, \quad \langle \phi \rangle = v_\phi e^{i(\beta_e - 2\pi(3k/9))},
\end{align*}
\]

where \( k = 0, \ldots 8 \).

In this extension of the standard electroweak model only the \( U(1)_{\text{PQ}} \) is and automatic symmetry and its charges are quantized only if we add an extra \( Z_3 \) symmetry. As we showed before there are genuine discrete symmetries that are not broken by the instanton effects so, this model suffers of the domain wall problem and it must be solved by any of the way proposed in literature [29,30]. Moreover, in this model the \( Z_3 \) symmetry is not automatic. For more details see Ref. [17].

III. THE AXION IN A 3-3-1 MODEL

The so called 3-3-1 models are interesting candidates for the physics at the TeV scale [31–34]. In fact, some years ago Pal [18] pointed out that the strong-CP question is solved elegantly in those models. The point was that Yukawa couplings of these models automatically contain a Peccei-Quinn symmetry [1] if a simple discrete symmetry was also imposed in order to avoid a trilinear term in the scalar potential. Here we will consider one particular 3-3-1 model in which only three scalar triplets are needed [32] but we introduce also an scalar singlet, \( \phi \sim (1,1,0) \). In this 3-3-1 model if we add, \( i_j \) right-handed neutrinos, \( ii \) the conservation of the total lepton number \( L \) and, \( iii \) a \( Z_2 \) symmetry defined below, we have that both \( Z_{13} \) and \( U(1)_{\text{PQ}} \) are accidental symmetries of the classical Lagrangian (in the sense discussed in Sec. I).

Before considering the implementation of a naturally light and invisible axion in the context of the 3-3-1 model of Ref. [32], let us briefly review the model.

In the quark sector we have:

\[ Q_{mL} = (d_m, u_m, m_L)^T \sim (3, 3^*, -1/3), m = 1, 2; \quad Q_{dL} = (u_3, d_3, J)^T \sim (3, 3, 2/3), \]

and the respective right-handed components in singlets \( u_{aR} \sim (3, 1, 2/3), d_{aR} \sim (3, 1, -1/3), a = 1, 2, 3; \quad J_{aR} \sim (3, 1, 5/3), j_{aR} \sim (3, 1, -4/3). \)

In the scalar sector, this model has only three triplets

\[ \chi = (\chi^-, \chi^-, \chi^0)^T, \quad \rho = (\rho^+, \rho^0, \rho^{++})^T, \quad \eta = (\eta^0, \eta^-, \eta_2^0)^T, \]

transforming as \( (1, 3, -1) \), \( (1, 3, 1) \) and \( (1, 3, 0) \), respectively. Finally, in this model leptons transform as triplets \( (3, 0) \) where \( a = e, \mu, \tau \):

\[ \Psi_{aL} = (\nu_a, l_a, E_{a})^T \]

and the respective right-handed singlets \( \nu_{aR} \sim (1, 1, 0), \quad l_{aR} \sim (1, 1, -1), \quad E_{aR} \sim (1, 1, +1). \)

and we have added right-handed neutrinos which are not present in the minimal version of the model. Hence, we see that the model has fifteen multiplets, including right-handed neutrinos and the singlet \( \phi \) (in fact, as usual we have to add a scalar singlet \( \phi \sim (1, 1, 0) \) in order to make the axion invisible [20,21]) and it will admit, under the three condition introduced above, automatic \( Z_{13} \) and \( U(1)_{\text{PQ}} \) symmetries as we will show in the following.

With the quark and scalar multiplets above we have the Yukawa interactions

\[ -\mathcal{L}_Y = \mathcal{G}_{13L}(F_{13a} u_{aR} \rho^* + F_{13a} d_{aR} \eta^*) + \mathcal{G}_{33L}(G_{13a} u_{aR} \eta + \bar{G}_{13a} d_{aR} \rho) + \lambda_1 \mathcal{G}_{13L} J_{1R} \chi + \lambda_{im} \mathcal{G}_{13L} j_{mR} \chi^* + H.c., \]

where repeated indices mean summation. In the lepton sector we have

\[ -\mathcal{L}_Y = G_{13ab}(\bar{\Psi}_{aL} \nu_b \eta + \bar{G}_{13ab}(\bar{\Psi}_{aL} l_b \rho + \bar{G}_{13ab}(\bar{\Psi}_{aL} E_b \chi + H.c. \]

In both Yukawa interactions above a general mixing in each charge sector is allowed. If we want to implement a given texture for the quarks and lepton mass matrices we have to introduce more scalar triplets and a larger \( Z_2 \) symmetry will be possible in the model. Interesting possibilities are the cases where \( N \) is a prime number (see below).

The most general \( L-, Z_2- \) and gauge invariant scalar potential is
\[ V_{331} = \sum_{x=\eta, \rho, \chi, \phi} \mu_x^2 T^j_x T_x + \lambda_1 (\eta^j \eta^j) + \lambda_2 (\rho^j \rho^j) + \lambda_3 (\chi^j \chi^j) + (\eta^j \eta^j) [\lambda_4 (\rho^j \rho^j) + \lambda_5 (\chi^j \chi^j)] \]

\[ + \lambda_6 (\rho^j \rho^j) (\chi^j \chi^j) + \lambda_7 (\rho^j \eta^j) (\rho^j \eta^j) + \lambda_8 (\chi^j \eta^j) (\eta^j \chi^j) + \lambda_9 (\rho^j \chi^j) (\chi^j \rho^j) \]

\[ + \lambda_\phi (\phi^j \phi^j)^2 + \phi^j \phi^j \sum_{k=\eta, \rho, \chi} \lambda_{\phi k} T^j_x T_k + (\lambda_{10} \phi \epsilon^{ijk} \eta \rho \chi + H. c.), \]

where \( \mu' \)s are parameters with dimension of mass, \( \chi' \)s are dimensionless parameters and we have denoted \( T_x = \eta, \rho, \chi, \phi \) and \( T_k = \eta, \rho, \chi \). Now we can explain the motivation of the three conditions assumed at the beginning of the section: \( i \) with the present experimental data [24–26] in any electroweak model right-handed neutrinos are no longer avoided under the assumption that neutrinos are massless; \( ii \) in Eq. (11) it is still possible a Majorana mass term among right-handed neutrinos, say \( M_R (\nu^c) R \eta^\prime \); on the other hand in the scalar potential it possible to have the quartic term \( \chi^j \eta^j \eta^j \eta \) which also violates the total lepton number. Both terms are avoided by imposing the conservation of the total lepton number \( L \); \( iii \) the trilinear term in the scalar potential \( \eta \rho \chi \) is avoided if we impose a \( Z_2 \) symmetry under which \( J_R, J_m R, \chi, \phi \) are odd and all the other fields are even. In this conditions, the Yukawa interactions in Eqs. (11), (12) and the scalar potential in Eq. (13) are automatically invariant under the (local) \( Z_{13} \) symmetry:

\[ Q_{3L} \rightarrow \omega_5^{-} Q_{3L}, \quad Q_{3L} \rightarrow \omega_5 Q_{3L}, \quad u_{1R} \rightarrow \omega_1 u_{1R}, \quad d_{1R} \rightarrow \omega_1^{-} d_{1R}, \quad J_R \rightarrow \omega_3 J_R, \quad j_{mR} \rightarrow \omega_3^{-} j_{mR}, \quad \Psi_L \rightarrow \omega_0 \Psi_L, \quad l_R \rightarrow \omega_0^{-} l_R, \quad \nu_R \rightarrow \omega_4^{-} \nu_R, \quad E_R \rightarrow \omega_2^{-} E_R, \quad \eta \rightarrow \omega_4 \eta, \quad \rho \rightarrow \omega_0 \rho, \quad \chi \rightarrow \omega_2 \chi, \quad \phi \rightarrow \omega_1 \phi, \]

where \( \omega_k = e^{2\pi i k/13}, k = 0, \ldots, 6 \). Notice that if \( N \) is a prime number the singlet \( \phi \) can transform under this symmetry with any assignment (but the trivial one), otherwise we have to be careful with the way we choose the singlet \( \phi \) transforms under the \( Z_N \) symmetry. This symmetry implies that the lowest order effective operator which contributes to the axion mass is \( \phi^{13}/M_P^2 \) which gives a mass of the order of \( (v_\phi)^{11}/M_P^2 \) and also keeps the \( \theta \) parameter small as discussed previously.

It happens that as the \( Z_{13} \) symmetry, the \( U(1)_{\tilde{P}} \) is also automatic i.e., a consequence of the gauge symmetry and renormalizability in the model, in the interactions in Eqs. (11), (12) and (13). Let us see the PQ charge assignment for the fermions in the model:

\[ u'_L = e^{-i\alpha X_u} u_L, \quad d'_L = e^{-i\alpha X_d} d_L, \quad l'_L = e^{-i\alpha X_l} l_L, \quad \nu'_L = e^{-i\alpha X_\nu} \nu_L, \quad j'_L = e^{-i\alpha X_j} j_L, \quad J'_L = e^{-i\alpha X_J} J_L, \quad E'_L = e^{-i\alpha X_E} E_L, \]

and in the scalar sector we have the following PQ charges:

\[ \eta^0 = e^{-2i\alpha X_u} \eta^0 = e^{2i\alpha X_u} \eta^0, \quad \eta^0 = \eta^0 \]

\[ \eta^0 = e^{-i\alpha (X_u + X_d)} \eta^0 = e^{+i\alpha (X_u + X_d)} \eta^0, \quad \eta^0 = \eta^0 \]

\[ \eta^0 = e^{-2i\alpha X_u} \eta^0 = e^{-2i\alpha X_d} \eta^0, \quad \rho^0 = \rho^0 \]

\[ \rho^0 = e^{-i\alpha (X_u + X_d)} \rho^0 = e^{+i\alpha (X_u + X_d)} \rho^0, \quad \rho^0 = \rho^0 \]

\[ \rho^{++} = e^{-i\alpha (X_u + X_d)} \rho^{++} = e^{+i\alpha (X_u + X_d)} \rho^{++}, \quad \rho^{++} = \rho^{++} \]

\[ \rho^{--} = e^{-i\alpha (X_u + X_d)} \rho^{--} = e^{+i\alpha (X_u + X_d)} \rho^{--}, \quad \rho^{--} = \rho^{--} \]

\[ \chi^0 = e^{-2i\alpha X} \chi^0 = e^{2i\alpha X} \chi^0, \quad \phi' = e^{-2i\alpha X} \phi. \]

From Eqs. (15) and (16) we obtain the following relations

\[ X_d = -X_u = X_l = -X_\nu, \quad X_j = X_J = -X_E. \]

In the present model, although we have two independent PQ charges (say, \( X_u \) and \( X_j \)), the known quark contributions to \( \theta \) which are proportional to \( X_d \) cancel out exactly. Only \( X_j \) is important for solving the CP problem:

\[ \theta \rightarrow \theta - 2\alpha \sum_{\text{all quarks}} X_j = \theta - 2\alpha X_j. \]

Hence we can assume that \( X_u = 0 \) and the only relevant PQ transformations are

\[ j_L' = e^{-i\alpha X_j} j_L, \quad J_L' = e^{i\alpha X_j} J_L, \quad E_L' = e^{-i\alpha X_E} E_L, \quad \eta^0 = e^{i\alpha X_u} \eta^0, \quad \eta^0 = e^{-i\alpha X_u} \eta^0, \quad \rho^{--} = e^{i\alpha X_d} \rho^{--}, \quad \rho^{--} = e^{-i\alpha X_d} \rho^{--}, \quad \chi^0 = e^{-2i\alpha X} \chi^0, \quad \phi' = e^{-2i\alpha X} \phi. \]

Notice that like in Ref. [21] we have an invisible axion (it is almost singlet, see below) and the PQ charge which solves the strong CP problem is the charge of the exotic quarks. However, unlike Ref. [21] \( \phi' \) the heavy quarks is already present in the minimal version of this 3-3-1 model. The condition \( X_u = -X_d \) is not allowed in the context of the standard \( SU(2)_L \otimes U(1)_Y \) model since in this case it is not possible to shift the \( \eta^0 \).
We have seen above that the known quarks contributions to $\theta$ cancel out exactly. This also happens in the domain wall number, $N_{DW}$, defined in Eq. (4). Using the PQ charges in Eq. (15) and (16) we obtain $N_{DW} = X_j$ and since we have always chosen $X_j = 1$ we see that as in the Kim’s axion model, there is no domain wall problem in this 3-3-1 axion model. We stress that the contribution of the known quarks cancel out exactly even if we assume that $X_d \neq 0$. Moreover, we will see in Sec. IV that also the coupling of the axion with photon does not depend at all on the PQ charge of the usual quarks and leptons since there is an exact cancellation among them. However if $X_d \neq 0$ there are still couplings with the usual fermions. Notwithstanding since $X_d$ does not play any role in the solution of the strong $CP$ problem we can assume at the very start that $X_d = 0$ i.e., that the nontrivial PQ transformations are those in Eq. (19). It means that at the tree level there is no coupling of the axion with the known quarks and charged leptons.

We can verified that in fact the axion is almost singlet. After redefining the neutral fields as usual $T_0^k = (v_k + Re T_k^0 + i Im T_k^0)/\sqrt{2}$, $\phi = (v_0 + Re \phi + i Im \phi)/\sqrt{2}$, with $k = \eta, \rho, \chi$, we obtain the constraint equations $t_\phi = 0$, (where $x = \eta, \rho, \chi, \phi$):

$$
t_\eta = \text{Re}[\mu_\eta^2 v_\eta + \lambda_1 |v_\eta|^2 v_\eta + \frac{1}{2} (\lambda_4 |v_\rho|^2 + \lambda_5 |v_\chi|^2) v_\eta + \frac{\lambda_{10}}{2} v_\rho v_\chi v_\phi + \lambda_\phi |v_\phi|^2 v_\eta],
$$

$$
t_\rho = \text{Re}[\mu_\rho^2 v_\rho + \lambda_2 |v_\rho|^2 v_\rho + \frac{1}{2} (\lambda_4 |v_\eta|^2 + \lambda_6 |v_\chi|^2) v_\rho + \frac{\lambda_{10}}{2} v_\eta v_\chi v_\phi + \lambda_\phi |v_\phi|^2 v_\rho],
$$

$$
t_\chi = \text{Re}[\mu_\chi^2 v_\chi + \lambda_3 |v_\chi|^2 v_\chi + \frac{1}{2} (\lambda_5 |v_\eta|^2 + \lambda_6 |v_\rho|^2) v_\chi + \frac{\lambda_{10}}{2} v_\eta v_\rho v_\phi + \lambda_\phi |v_\phi|^2 v_\chi],
$$

$$
t_\phi = \text{Re}[\mu_\phi^2 v_\phi + \lambda_\phi |v_\phi|^2 v_\phi + \frac{\lambda_{10}}{2} v_\eta v_\rho v_\chi + \frac{1}{2} (\lambda_\phi |v_\eta|^2 + \lambda_\phi |v_\rho|^2 + \lambda_\phi |v_\chi|^2) v_\phi],
$$

$$
\text{Im}(v_\phi v_\rho v_\chi v_\phi) = 0.
$$

(20)

Notice that the solution with $v_\eta, v_\rho, v_\chi, v_\phi \neq 0$ is allowed. For instance, just for an illustration, assuming for simplicity that all VEVs and parameters (but $\mu_\phi$ and $\mu_\chi$) are real, $\lambda_1 v_\eta^2, \lambda_2 v_\rho^2, |\lambda_\phi| v_\phi v_k v_k \ll |\lambda_{10} v_k v_k|$, we obtain

$$
v_\phi \approx -\frac{\mu_\phi^2}{\lambda_\phi}, \quad v_\chi \approx -\frac{\mu_\chi^2}{\lambda_3},
$$

$$
v_\eta \approx -\lambda_{10} \frac{v_\rho v_\chi v_\phi}{\mu_\eta^2}, \quad v_\rho \approx -\lambda_{10} \frac{v_\eta v_\chi v_\phi}{\mu_\rho^2},
$$

and the self-consistent condition $\lambda_{10}^2 v_\chi^2 v_\phi^2 \approx \mu_\eta^2 \mu_\rho^2$. With all VEVs real the pseudo scalar mass eigenstates, in the basis $\text{Im}(\eta^0, \rho^0, \chi^0, \phi)$, are given by:

$$
G^0_1 = \frac{1}{(v_\eta^2 + v_\rho^2)^{1/2}} (-v_\eta, v_\rho, 0, 0),
$$

$$
G^0_2 = \frac{1}{\sqrt{N}} \left( \frac{s_1}{s_2} v_\eta^2 v_\eta, \frac{s_1}{s_2} v_\eta^2 v_\rho, \frac{s_2}{s_1} v_\rho^2 v_\chi, \frac{s_2}{s_1} v_\rho^2 v_\phi \right),
$$

$$
a = \frac{1}{(v_\chi^2 + v_\phi^2)^{1/2}} (0, 0, -v_\chi, v_\phi),
$$

$$
A^0 = \frac{1}{\sqrt{N}} (v_\rho v_\chi v_\phi, v_\eta v_\chi v_\phi, v_\eta v_\rho v_\phi, v_\eta v_\rho v_\chi). \quad (22)
$$

where $s_1 = (v_\chi^2 + v_\phi^2)^{1/2}$, $s_2 = (v_\eta^2 + v_\rho^2)^{1/2}$ and $N = v_\eta^2 (v_\rho^2 (v_\chi^2 + v_\phi^2) + v_\rho^2 v_\chi^2) + v_\rho^2 v_\phi^2$. $G^0_{1, 2}$ are genuine Goldstone bosons (eaten when the gauge bosons $Z, Z'$ become massive), $a$ is the axion pseudo-Goldstone. If $|v_\phi| \gg |v_\eta|$ we see that $a \approx \text{Im} \phi$ and we have an invisible axion [20]. The usual restriction coming from red giant implies $|v_\phi| > 10^9$ GeV [35]; $A^0$ is a heavy pseudo-scalar with $m_A^2 = -(\lambda_{10})^2 (v_\rho^2 (v_\chi^2 + v_\phi^2) + v_\phi^2 v_\chi^2) + v_\rho^2 v_\chi^2 v_\phi^2 |v_\phi|^2$, with $\lambda_{10} < 0$ and we have defined $C = v_\phi v_\eta v_\rho v_\chi$.

IV. CONCLUSION

We have build invisible axion models in which the axion is naturally light (protected against quantum gravity effects) because of a $Z_{13}$ discrete symmetry. In the context of a $SU(2) \otimes U(1)$ model this symmetry and a $Z_3$ have to be imposed and new fields have to be added. On the other hand, in a 3-3-1 model with right-handed neutrinos added, the $Z_{13}$ is automatic if we impose the conservation of the total lepton number and a $Z_2$ symmetry. Moreover, in both models $U(1)_{PQ}$ is an accidental symmetry. It means that at low energy the gauge symmetries are $SU(3)_{L} \otimes SU(2)_{L} \otimes U(1)_{Y} \otimes Z_{13} \otimes Z_3$ or $SU(3)_C \otimes SU(3)_L \otimes U(1)_{Y} \otimes Z_{13} \otimes Z_2$. Notice however that in the context of the standard model even by imposing $L$ conservation and the $Z_3$ symmetry, the $Z_{13}$ symmetry is not automatic but in the 3-3-1 model $L$ conservation and $Z_2$ make $Z_{13}$ an automatic symmetry.

Hence, we have implemented an invisible and naturally light axion in a multi Higgs extension of the standard model and also in a 3-3-1 model. Unlike the axion of the first model, in the 3-3-1 model considered here the minimal representation content (plus right-handed neutrinos) is already enough to implement a local $Z_{13}$ symmetry. As we said before, the interesting discrete
symmetries are those in which \( N \) is a prime number. In this case \( Z_N \) has no subgroup except itself and the identity [36]. Hence, the next interesting symmetry should be \( Z_{17} \) which will allow even lighter axion, i.e., \( m_a^{(\text{gravity})} = (v_a)^{15}/M_{\text{Pl}}^{13} \sim 10^{-67} \text{eV}^2 \) or \( 10^{-57} m_a^2 \), similar we have in this case \( \theta^{(\text{gravity})} \sim 10^{-43} \).

We will consider now some phenomenological consequences of the axion in the 3-3-1 model. (The case of the model of Sec. II has been considered in Ref. [17,]). In general the axion-photon coupling is given by

\[
c_{a\gamma\gamma} = \tilde{c}_{a\gamma\gamma} - 1.95, \tag{23}
\]

where the first term is defined as

\[
\tilde{c}_{a\gamma\gamma} = \frac{1}{N_{DW}} \sum_{\text{all fermions}} X_f Q_f^2, \tag{24}
\]

with \( N_{DW} = 1 \) in this model since it has no domain wall problem as in the Kim's model [21]; \( X_f \) and \( Q_f \) are the PQ and electromagnetic charge, respectively, of the fermion \( f \). The term \( -1.95 \) comes from the light quark PQ anomalies and it does exist only if these quarks carry PQ charges. In Eq. (24) the contributions proportional to the PQ and electromagnetic charge, respectively, of the known fermions cancel out exactly even if we had assumed that \( X_d \neq 0 \). So we have in general that \( c_{a\gamma\gamma} = - (2/3) X_f - 1.95 \), or \( \tilde{c}_{a\gamma\gamma} = 2.62(-1.28) \) for \( X_f = +1(-1) \). In our case in particular, since \( X_d = 0 \), we have just the first contribution in Eq. (23) i.e., \( c_{a\gamma\gamma} = \tilde{c}_{a\gamma\gamma} = \pm (2/3) \). We stress that the contributions of the PQ charges of the known quarks cancel out exactly in \( \theta \) in Eq. (18), in the domain wall number given in Eq. (4), and also in Eq. (24). Thus, if we assign PQ charges to those quarks it only implies in a coupling with the axion at the tree level. On the other hand, if we assume that only the exotic fermions of the model carry PQ charge the coupling with the known quarks and leptons arises at higher order in perturbation theory.

We have imposed at the very start the conservation of the total lepton number and a \( Z_2 \) symmetry. Another possibility is to impose the discrete symmetry \( Z_{13} \). In this case only the \( U(1)_{\text{PQ}} \) symmetry is automatic and the quartic \( L \)-violating term \( \chi^\dagger \eta \phi \eta \) is allowed. This term implies a new relation among the PQ charges of the known particles and the exotic ones: \( X_f = 3X_d \). Taking also into account Eqs. (17), it can be shown that there is a surviving symmetry \( Z_2 \subset U(1)_{\text{PQ}} \) which implies a domain wall problem [37].

The 3-3-1 model in which the leptons transform as \( \Psi_{aL} = (\nu_a, \ell_a, \ell_a^c) \) needs also the introduction of a scalar sextet \( S \sim (6, 0) \) (or singlet charged leptons \( E \sim (1, 1) \)) [38,39]. In this case the Yukawa interaction in the quark sector is given by Eq. (11) and in the leptonic sector we have

\[
-\mathcal{L}_Y' = G_{ab}^{\nu} \overline{\Psi}_a \nu_b \phi R \eta + G_{ab}^{\nu} \overline{\Psi}_a \nu_b \eta_S + G_{ab}^{\ell} \xi_{ijk} \overline{\Psi}_i \ell_j \eta_k + H.c. \tag{25}
\]

where \( G_{ab}^{\nu} \) is an arbitrary \( 3 \times 3 \) matrix while \( G_{ab}(G') \) is a symmetric (antisymmetric) matrix and we have omitted some \( SU(3) \) indices. Notice that this model has only thirteen multiplets (including right-handed neutrinos and the singlet \( \phi \)) so we can not have a symmetry as large as \( Z_{13} \). However by adding more scalar multiplets like in Ref. [40] it may be possible to implement, automatically, a large enough \( Z_N \) symmetry. The supersymmetric version of the model can also be considered since in this model, without considering right-handed neutrinos, there are 23 chiral superfields (in the same case, the MSSM has 14 chiral superfields) [41].

In the models considered in this work the axion couples to neutrinos too. This coupling may have astrophysics and/or cosmological consequences; we can also implement hard [42], soft [43] or spontaneous [44] CP violation.

**ACKNOWLEDGMENTS**

This work was supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), Conselho Nacional de Ciência e Tecnologia (CNPq) and by Programa de Apoio a Núcleos de Excelência (PRONEX).

[1] R. D. Peccei and H. Quinn, Phys. Rev. Lett. 38, 1440 (1977); R. D. Peccei and H. Quinn, Phys. Rev. D 16, 1791 (1977).
[2] S. Weinberg, Phys. Rev. Lett. 40, 223 (1978); F. Wilczek, ibid. 40, 279 (1978).
[3] M. Srednicki, Eur. Phys. J. C15, 143 (2000).
[4] Y. E. Kim, Phys. Rep. 150, 1 (1987).
[5] H.-Y. Cheng, ibid. 158, 1 (1998); G. G. Raffelt, Ann. Rev. Nucl. Part. Sci. 49, 163 (1999); Phys. Rep. 198, 1 (1990); M. S. Turner, ibid. 197, 67 (1991).
[6] C. Csáki, N. Kaloper and J. Terning, Phys. Rev. Lett. 88, 161302 (2002), hep-ph/0112212. See also E. Mörtsell, L. Bergström and A. Goodbar, Phys. Rev. D 66, 047702 (2002), astro-ph/0202153; Y. Grossman, S. Roy and J. Zupan, Phys. Lett. B543, 23 (2002), hep-ph/0204216.
[7] A. G. Riess et al. (Supernova Search Team Collaboration), Astron. J. 116, 1009 (1998); S. Perlmutter et al. (Supernova Cosmology Project Collaboration), Astrophys. J. 517, 565 (1999).
[8] H. Georgi, L. J. Hall and M. B. Wise, Nucl. Phys. B192, 409 (1981); R. Holman et al., Phys. Lett. B282, 132 (1992); M. Kamionkowski and J. March-Russell, Phys. Lett. B282, 137 (1992); S. M. Barr and D. Seckel, Phys. Rev. D 46, 539 (1992); R. Kallosh, A. Linde, D. Linde and L. Susskind, Phys. Rev. D 52, 912 (1995); R. D. Peccei, hep-ph/0009030.
[9] L. M. Krauss and F. Wilczek, Phys. Rev. Lett. 62, 1221
(1989). See also S. Coleman, J. Preskill and F. Wilczek, Nucl. Phys. B378, 175 (1992).

[10] T. Banks, Nucl. Phys. B323, 90 (1988); M. G. Alford, J. March-Russell and F. Wilczek, Nucl. Phys. B337, 695 (1990); J. Preskill and L. M. Krauss, ibid B341, 50 (1990); M. G. Alford, S. Coleman and J. March-Russell, ibid. B351, 735 (1991).

[11] M. Dine, Proceedings of the Cincinnati Symposium in Quantum gravity and beyond, 157-169, 1992, hep-ph/9207045; and Phys. Rev. D 45, 1424 (1992).

[12] C. T. Hill and A. K. Leibovich, Phys. Rev. D 66, 075010 (2002), hep-ph/0205237; ibid 66, 016006 (2002), hep-ph/020557.

[13] G. Lazarides, C. Panagiotakopoulos and Q. Shafi, Phys. Rev. Lett. 56, 432 (1986).

[14] E. Ma, M. Raidal and U. Sarkar, Phys. Lett. B504, 296 (2001).

[15] S. Chang, S. Tazawa and M. Yamagushi, Phys. Rev. D 66, 084005 (2000), hep-ph/0205237.

[16] R. D. Peccei, Nucl. Phys. B (Proc. Suppl.) 72, 3 (1999).

[17] Alex G. Dias, V. Pleitez and M. D. Tonasse, hep-ph/0210172.

[18] P. B. Pal, Phys. Rev. D 52, 1659 (1995).

[19] S. L. Glashow and S. Weinberg, Phys. Rev. D 15, 1958 (1977).

[20] M. Dine, W. Fischler and M. Srednicki, Phys. Lett. 104B, 199 (1981).

[21] J. E. Kim, Phys. Rev. Lett. 43, 103 (1979).

[22] A. Zee, Phys. Lett. 93B, 389 (1980).

[23] T. P. Cheng and L-F. Li, Phys. Rev. D 22, 2860 (1980).

[24] B. T. Cleveland et al. (Homestake Collaboration), Nucl. Phys. B (Proc. Suppl.) 38, 47 (1995); K.S. Hirata et al. (Kamiokande Collaboration), Phys. Rev. 44, 2241 (1991); S. Fukuda et al. (Super-Kamiokande Collaboration), Phys. Rev. Lett. 86, 5651 (2001); W. Hampel et al. (GALLEX Collaboration), Phys. Lett. B477, 127 (1999)

[25] J. N. Abdurashitov et al. (SAGE Collaboration), Phys. Rev. Lett. 77, 4708 (1996); Q. R. Ahmed et al. (SNO Collaboration), Phys. Rev. Lett. 87, 071301 (2001).

[26] K. Eguchi et al. (KumLAND Collaboration), Phys. Rev. Lett. 90, 021802 (2003), hep-ex/0212021.

[27] Y. Fukuda et al. (Super-Kamiokande Collaboration), Phys. Rev. Lett. 81, 1562 (1998); ibid 81, 1158 (1998); Phys. Lett. B436, 33 (1998); K.S. Hirata et al., Phys. Lett. B280, 146 (1992); R. Becker-Szendy et al., Phys. Rev. D 46, 3720 (1992); W. W. M. Allison et al., Phys. Lett. B 391, 491 (1997); Y. Fukuda et al., Phys. Lett. B 335, 237 (1994).

[28] P. Sikivie, Phys. Rev. Lett. 48, 1156 (1982).

[29] Y. B. Zeldovich, I. Y. Kobzarev and L. B. Okun, Sov. Phys. JETP, 40, 1 (1975).

[30] H. Georgi and M. B. Wise, ibid, 116B, 123 (1982)

[31] G. Lazarides and Q. Shafi, Phys. Lett. 115B, 21 (1982); S. M. Barr, D. B. Reiss and A. Zee, Phys. Lett. 116B, 227 (1982).

[32] F. Pisano and V. Pleitez, Phys. Rev. D 46, 410 (1992); P. Frampton, Phys. Rev. Lett. 69, 2889 (1992); R. Foot, O. F. Hernandez, F. Pisano and V. Pleitez, Phys. Rev. D 47, 4158 (1993).

[33] V. Pleitez and M. D. Tonasse, Phys. Rev. D 48, 2353 (1993).

[34] J. C. Montero, F. Pisano and V. Pleitez, Phys. Rev. D 47, 2918, (1993); R. Foot, H. N. Long and T. A. Tran, Phys. Rev. D 50, R34, (1994).

[35] Electroweak models with SU(3) symmetry have some predictive power concerning the observed value of the weak mixing angle and can be embedded in theories of TeV-gravity, see S. Dimopoulos and D. E. Kaplan, Phys. Lett. B531, 127 (2002), hep-ph/0211148; S. Dimopoulos, D. E. Kaplan and N. Weiner Phys. Lett. B534, 124 (2002), hep-ph/0202136; I. Gogoladze, Y. Mimura and S. Nandi, hep-ph/0210320 and references therein.

[36] D. A. Dicus, E. W. Colb, V. L. Teplitz and R. Wagoner, Phys. Rev. D 18, 1829 (1978); Phys. Rev. D 22, 839 (1980).

[37] M. Hall Jr. The Theory of Groups, MacMillan Company, New York, 1959.

[38] G. Lazarides, C. Panagiotakopoulos and Q. Shafi, Phys. Rev. Lett. 56, 432 (1986).

[39] T. V. Duong and E. Ma, Phys. Lett. B306, 307 (1993).

[40] J. C. Montero, C. A. de S. Pires, V. Pleitez, Phys. Rev. D 65, 093017 (2002), hep-ph/0103096.

[41] J. C. Montero, C. A. de S. Pires and V. Pleitez, Phys. Rev. D, 65, 113003 (2002), hep-ph/0112203.

[42] J. C. Montero, V. Pleitez and M. C. Rodriguez, Phys. Rev. D 65, 035006 (2002). See also Ref. [38] and R. A. Diaz, R. Martinez, J. A. Rodriguez, Phys. Lett. B552, 287 (2003), hep-ph/0208176 for other supersymmetric 3-3-1 models.

[43] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973); S. Weinberg, Phys. Lett. 37, 657 (1976).

[44] J. C. Montero, V. Pleitez and O. Ravinez, Phys. Rev. D 60, 076003 (1999).

[45] T. D. Lee, Phys. Rev. D 15, 3394 (1977); L. Epele, H. Fanchiotti, C. García Canal, D. Gómez Dunn, Phys. Lett. B343, 291 (1995).