A COMPARISON BETWEEN AN ULTRA-RELATIVISTIC
\textit{Au}+\textit{Au} COLLISION AND THE PRIMORDIAL UNIVERSE

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Ultra-relativistic nucleus-nucleus collisions create a state of matter of high temperature
and small baryo-chemical potential, which is similar to the thermodynamical conditions
in the primordial universe. Recent analyses of \textit{Au}+\textit{Au} collisions at RHIC
declared the temperature, size and density of the system. Thus, a comparison
to the primordial universe can be attempted. In particular, two observables shall be
investigated, namely (1) the temperature at baryon freeze-out
($t\approx 10$ fm/c in the \textit{Au}+\textit{Au} collision) and (2) the matter density at the partonic
stage ($t\lesssim 1$ fm/c).

At the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory,
New York, USA, gold nuclei collide at a maximum center-of-mass energy $\sqrt{s}=200$ GeV.
The STAR experiment is one of four experiments \cite{1}, which investigate novel QCD
phenomena at high density and high temperature in these collisions. The main STAR
subdetector is a midrapidity ($|\eta|\lesssim 1.6$) Time Projection Chamber \cite{2} (TPC, $R=2$ m,
$L=4$ m) with $\approx 48,000,000$ pixels. In 3 years of data taking, high statistics
($10^6 \leq N \leq 10^7$ events on tape) for \textit{Au}+\textit{Au}, \textit{p}+\textit{p} and \textit{d}+\textit{Au} at
$\sqrt{s}=200$ GeV and \textit{Au}+\textit{Au} at $\sqrt{s}=130$ GeV were recorded\textsuperscript{a}.

1. Comparison of the Temperature

Generally, in a thermalized system, inclusive particle transverse mass spectra can
be described by a Boltzmann distribution
\[
dN/dm_T \sim m_T \exp(-m_T/m) \frac{1}{T}\]
where $N$ denotes the particle yield. The transverse mass is defined as
$m_T = \sqrt{p_T^2 + m^2}$. By using $dN/dm_T$ for $\pi^\pm$, $K^\pm$, protons and anti-protons,
the freeze-out temperature in an \textit{Au}+\textit{Au} collision was determined to be
$T=89\pm10$ MeV \cite{4}, which corresponds to $T=(1.03\pm0.12) \times 10^{12}$ K.
This temperature can be compared to other systems (Tab. 1). An interesting difference
is, that an \textit{Au}+\textit{Au} collision is matter dominated all the time. In the universe,
there has been at first a radiation dominated time period ($T \geq 1$ eV),
followed by a matter dominated period ($T \leq 1$ eV). Both periods differ in

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\textsuperscript{a}The \textit{p}+\textit{p} and \textit{d}+\textit{Au} data set serve as reference.
the time dependence of the temperature, i.e. $T \sim 1/t^{1/2}$ vs. $T \sim 1/t^{2/3}$, respectively. For a Au+Au collision, the time dependence can be derived by a simple thermodynamical ansatz $E/N \sim T$ and $N/V \sim T^{3} \sim 1/t$, which leads to a flatter time dependence $T \sim 1/t^{1/3}$ (using total energy $E$, particle number $N$ and volume $V$). The above mentioned temperature $T_{\text{kin}}=89\pm10$ MeV refers to kinetic freeze-out, i.e. the termination of elastic rescattering. The temperature of the earlier chemical freeze-out (i.e. the termination of inelastic rescattering) is higher $T_{\text{chem}}=156\pm6$ MeV [4]. As it does not depend upon the particle density, the chemical freeze-out temperature is a universal quantity, i.e. identical for the Au+Au collisions and the primordial universe.

2. Comparison of the Size
The size of the fireball in a Au+Au collision was measured precisely by $\pi^{\pm}\pi^{\pm}$ interferometry [7], i.e. in the beam direction $R_{\text{long}}=5.99 \pm 0.19$ (stat.) $\pm 0.36$ (syst.), and perpendicular to the beam direction $R_{\text{out}}=5.39 \pm 0.18$ (stat.) $\pm 0.28$ (syst.). This size corresponds to the time of pion freeze-out ($t \approx 10$ fm/c). As the fireball is expanding (Sec. 3), the size is a function of time. What was the size of the universe, when it had a temperature of $T=100$ MeV (corresponding to hadronic freeze-out)? As shown in Fig. 2, hadronic freeze-out is believed to have occurred in the primordial universe at $t \sim 10^{-3}$ s. As inflation is believed to have occurred at $10^{-36} \leq t \leq 10^{-33}$ s, it can be concluded, that during baryonic freeze-out the universe had already macroscopic dimensions, i.e. a horizon distance of $R \sim 10$ km.

3. Comparison of the Expansion Velocity
As mentioned in Sec. 1, inclusive particle $m_T$ spectra for a Au+Au collision can be fitted by a Boltzmann distribution. This ansatz is only approximately true, as it neglects kinetic terms, i.e. the inverse logarithmic slope is not only proportional to $1/T$, but $1/T+1/2mv^2$. Only for light particles the approximation $m \approx 0$ holds. If treated more quantitatively [4], an expansion velocity can be extracted from a simultaneous fit to $\pi$, $K$ and $p$ spectra. The result is $\beta = 0.59 \pm 0.05$ with $v = \beta c$. For the universe, at the time of baryon freeze-out with a horizon distance of $R \approx 10$ km (corresponding to $T \approx 100$ MeV), the expansion rate had been slower than the speed of light by a very large factor $1/M_{\text{Planck}}^2 \approx 10^{19}$ [8]. In addition, the topology of the primordial universe might have been very different from the topology of a RHIC collision. The general relativity line element $ds^2 \approx dr^2/(1-k)$ contains a curvature $k = 2GM/Rc^2$, using the gravitational constant $G$. $M$ and $R$

\[^b\text{Note that a priori } R_{\text{long}} \text{ and } R_{\text{out}} \text{ are measurements of homogeneity lengths, which then are interpreted as system size.}\]
denote the mass and the size of the system, respectively. Even though the universe appears to be flat \((k=0)\) in the present, the inflation model still allows a primordial non-flat geometry \([9]\). For a singularity at \(t=0\), one would have to assume \(k=+1\) for the period before inflation. In that case, the universe would have been a 3-dim sphere curved into the 4\(^{th}\) dimension, and thus infinite for an observer inside the universe. A particle travelling into one direction, finally - due to the curvature - comes back to the point, where it started. The system is closed. The RHIC collision topology is not a sphere, because the two initial state Au nuclei are moving along the beam axis with the speed of light. Thus, the system has a cylindrical component. Additionally, in case of the RHIC collision, space time is not curved. According to \([10]\), one can estimate \(k=10^{-22}\). Hence, the system is geometrically flat and thermodynamically open, and any particle can escape.

![Figure 2. Timeline of an Au+Au collision (left) and the primordial universe [5] [6] (right).](image)

### 4. Comparison of the Matter Density

For a RHIC Au+Au collision, the spatial matter density, i.e. the number of partons per volume \(\rho=N_{\text{partons}}/V\) can be estimated by a Bjorken ansatz \([11]\). If assuming that initial and final entropy are equal, the number of partons at \(t\leq 1\ \text{fm/c}\) is equal to the measured number of final state hadrons \(N_{\text{hadron}}\). The volume can be calculated by inserting the fireball
radius (Sec. 2) into a cylindrical volume (due to Lorentz boost in the beam direction), i.e. \( \rho \approx dN_{\text{parton}}/dy \cdot 1/(\pi R^2 t) \) using the Lorentz invariant rapidity \( y \). For this ansatz, highest densities are expected early in the collision (singularity for \( t \to 0 \)). For \( t = 0.2 \, \text{fm/c} \) the matter density is \( \rho \approx 20/\text{fm}^3 \), which corresponds to \( \approx 15 \times \rho_o \), the density of cold gold nuclei. At this high density, hadrons are definitely non-existent. In Tab. 2, the density is compared to other systems. What was the size of the universe at \( \rho = 15 \rho_o \)? For the scenario of inflation, an initial universe mass \( M \approx 25 \, \text{g} \) is generally assumed. For inflation from an initial radius of \( R_i = 10^{-40} \, \text{m} \) to a final radius of \( R_f = 1 \, \text{m} \), the density would change from \( \rho_i \approx 10^{118} \, \text{kg/cm}^3 \) (at \( t \approx 10^{-36} \, \text{s} \)) to \( \rho_f \approx 10^{-19} \, \text{kg/cm}^3 \) (at \( t \approx 10^{-33} \, \text{s} \)). Thus, RHIC density was achieved during inflation. In non-inflation models, the size of the universe in the Planck epoch (\( t \approx 10^{-43} \, \text{s} \)) is \( R \approx 10^{-5} \, \text{m} \). To achieve RHIC density, one would hypothetically\(^c\) have to fill it with \( m \approx 3600 \, \text{kg} \) gold. If the initial mass were less, RHIC density would never have been achieved.

| Table 1. Comparison of the temperatures in various systems. |
|--------------------|------------------|
| 1.4 \times 10^{34} \, \text{K} | Planck temperature |
| 1.0 \times 10^{12} \, \text{K} | Au+Au collision |
| 10^9 \, \text{K} | sun (core) |
| 15 \times 10^6 \, \text{K} | supernova |
| 55 \times 10^6 \, \text{K} | plasma fusion |
| 4 \times 10^6 \, \text{K} | laser fusion |

| Table 2. Comparison of the density in various systems. |
|------------------|------------------|
| 2 \times 10^{17} \, \text{kg/cm}^3 | Au nuclear density |
| 30 \times 10^{17} \, \text{kg/cm}^3 | Au+Au collision |
| \approx 20,000 \, \text{kg/cm}^3 | Au atomic density (solid) |
| \approx 1000 \, \text{kg/cm}^3 | metallic hydrogen |
| 1.1 \times 10^{-26} \, \text{kg/cm}^3 | universe critical density |

References
1. For a RHIC overview see [http://www.bnl.gov/rhic/](http://www.bnl.gov/rhic/)
2. M. Anderson at al., Nucl. Inst. Meth. A499(2003)659
3. J. S. Lange, Nucl. Phys. A718(2003)367
4. The STAR Collaboration, nucl-ex/0310004, subm. Phys. Rev. Lett.
5. [http://en.wikipedia.org/wiki/Timeline_of_the_Big_Bang](http://en.wikipedia.org/wiki/Timeline_of_the_Big_Bang)
6. D. H. Lyth, Introduction to Cosmology, 1993, astro-ph/9312022
7. The STAR Collaboration, Phys. Rev. Lett. 87(2001)082301, nucl-ex/0107008
8. K. Kajantie, European School of High-Energy Physics, CERN-97-03, p. 261
9. A. H. Guth, Phys. Rev. D23(1981)347
10. W. Busza, R. Jaffe, J. Sandweiss, F. Wilczek, Rev. Mod. Phys. 72(2000)1125
11. J.D. Bjorken, Phys. Rev. D27(1983)140
12. S. Hawking, Mon. Not. R. Astro. Soc. 152(1971)75

\(^c\)Even before the existence of mass in the context of particle mass, mass can be defined in the context of a gravitational horizon mass \( m = 6c^3t/G \) [12].