Establishment of taxi priority scheme model based on priority queuing system

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Abstract. With the continuous development of China's economy, the construction of civil aviation engineering has become a major topic in China. Once again, there is a great lack of research on how to give passengers priority to return to the airport. Based on the analysis data of three representative airport passengers taking taxis, this paper establishes the priority scheme model based on the priority queuing system. Finally, by establishing the "priority function" model and quantifying the priority index, a feasible "priority" arrangement scheme is obtained. It is found that the professionalism, taxi drivers who have carried long-distance passengers are more likely to gain more profits by staying in the urban area, and the taxi drivers who have carried passengers for a short distance back to the airport to continue carrying passengers may reduce the time cost consumption. Through the guidance and feasibility evaluation of how to give priority to taxi drivers based on the mathematical model, it can help the airport management department to formulate the optimization scheme scientifically and effectively, improve the working enthusiasm of taxi drivers and the travel efficiency of passengers, which has certain guiding significance for the scientific and efficient management of the airport.

1. Introduction
The continuous development of China's overall economy and the improvement of residents' consumption level have stimulated the development of air transport industry. Most residents choose to travel by air. Taxi is convenient and fast, which makes an important contribution to the dispersion of airport passenger flow.

Most airports in China separate the way of seeing off (departure) from receiving (arriving). Taxi drivers who see off passengers to the airport will face two choices: (a) go to the arrival area and wait in line for passengers to return to the urban area; and (b) directly return to the urban area to solicit passengers. The income of taxi carrying passengers in the airport is related to the mileage of passengers. The destination of passengers is far or near. Taxi drivers cannot choose passengers and refuse to carry passengers, but taxi drivers are allowed to go back and forth to carry passengers for many times. In this context, the management department needs to formulate "priority" rules to make up for the loss of income of short distance drivers, so as to keep the income of airport taxi drivers as balanced as possible.

At present, the research on priority schemes mainly focuses on the taxi waiting time and the specific benefits. Most of these schemes are judged by experience, and lack of systematic research on mathematical models and data. In this paper, on the basis of the existing airport passenger taxi travel...
data, further through the priority queuing system to establish the priority scheme model, from the theory of scientific and reasonable "priority" rules, making the optimization scheme more practical significance.

2. Literature review

2.1. Industry overview
When the driver chooses to go to the arrival area, line up and wait for passengers to return to the city. Taxis must queue up at the designated "storage pool" and queue up for passengers according to the "first come, then come" line. The waiting time depends on the number of taxis and passengers in the queue, and it needs to pay a certain time cost. When the driver chooses to empty directly and return to the city to attract passengers. Taxi drivers will pay no-load charges and may lose potential passenger revenue.

The number of flights arriving in a certain period of time and the number of vehicles in the "car pool" are certain information that drivers can observe. Usually, the driver's decision-making is related to his personal experience judgment, such as the number of arriving flights and the number of possible passengers in a certain season and a certain period of time.

2.2. Influencing factors of decision making
The fundamental purpose of taxi drivers who send passengers to the airport is to make the choice that can produce the maximum economic benefits. Then, the selection of strategic scheme can be transformed into the selection of scheme that can produce greater economic benefits. The relevant factors of taxi drivers' decision-making are the relevant factors of economic benefits generated by the two decisions.

For the decision-making of drivers choosing to queue up for passengers, drivers will get the benefits of carrying passengers, but increase the cost of time. This time cost is reflected in the loss of the benefits of returning to the urban area to attract passengers generated by queuing time. For the driver to choose empty car back to the urban area to attract passengers, it will also produce certain benefits and costs. The income generated is the time saved to earn the benefit of urban soliciting, and the cost is the fuel fee that should be borne by the empty car back to the urban area.

2.3. Model overview
On the premise of ensuring the safety of passengers and taxis, it is also necessary to consider the passenger carrying efficiency of the design method. On the one hand, the more "boarding points" are, the higher the passenger carrying efficiency is, but at the same time, the risk is also increased. On the other hand, too many parking spaces in each lane will affect the traffic efficiency of the system. Due to the uncertain boarding time of passengers, passengers with large luggage will greatly increase the boarding time. At this time, the cars behind cannot leave, resulting in the waste of parking spaces. Due to the frequent occurrence of passengers waiting for cars and other passengers, the arrival of passengers and taxis is random, and it is impossible to determine which is the "customer" and which is the "service desk", so the classical queuing theory cannot be used directly\(^1\).

Through the combination of 2017 Based on the data of three representative airport passengers taking taxis in the first quarter of, this paper first divides the short distance and long-distance according to the data, and then calculates the expected value of short-distance passenger carrying and long-distance passenger carrying income by using the actual taxi fare, so as to obtain the income difference between short-distance passenger carrying taxi and long-distance passenger carrying taxi, We give priority to the returning short distance taxi. Based on the priority queuing system, we establish the priority scheme model, and use the data to illustrate the feasibility of the model. Finally, this paper attempts to extend and innovate the model, through the establishment of "priority function" model, the priority index is quantified, and then a feasible "priority" arrangement scheme is obtained.

3. Research hypothesis
3.1. Regulations on taxi management
Implement the charging standard formulated by the price department, use the special bills supervised by the tax department, do not increase the charging items, change the charging standard or use other charging certificates without authorization, and do not refuse or overload passengers:

H1: The average number of taxis carrying passengers back to the urban area is one person, when one person is full, they leave the airport.
H2: Both passengers and taxis arrive alone and satisfy Poisson distribution.

3.2. Business scope
All taxis departing from the airport go to a designated point in the urban area. This designated point is collectively referred to as the urban area.

H3: The travel distance between the airport and the urban area remains unchanged

3.3. Operating cost
The fuel consumption of an empty taxi is equal to that of a manned taxi from the airport to the urban area.

H4: The time cost of taxis has nothing to do with the number of vehicles

4. Theoretical model

4.1. Priority queuing system
Priority queuing model is the extension and extension of single server queuing model, including the M/M/ n/m queuing model of preemptive priority and non preemptive priority, which is suitable for solving the queuing problem of different customer classification levels, and can effectively reflect the similarities and differences of each priority in the service system.

Model assumptions of non preemptive priority M/M/ n/m queuing system [2]:

(1) There are two types of taxis in the queuing system. The first type of taxis has non preemptive priority and high priority, while the second type has only low priority. When a high priority taxi arrives at the storage pool and there is a car at the boarding point to carry passengers, then he has no right to forcibly occupy the boarding point and can only take priority of the lower class taxi to enter the boarding point to carry passengers next time.

(2) The arrival intervals of the two priority classes obey the Poisson distribution with parameter \( \lambda_i \), and \( \lambda_i \) is the average arrival rate of the two types of taxis.

(3) The passenger time (the service time) of the two classes of taxis at the passenger boarding point obeys the exponential distribution of parameter \( \mu \). \( \mu \) is the average service rate of the two types of taxis, and the average service time can be deduced as \( \frac{1}{\mu} \).

(4) The service time required for each class of taxi is \( S_i (i = 1, 2) \), There are n boarding points (service desk).

\[
\bar{S}_i = E(S_i) = \frac{1}{n\mu} \quad (1)
\]

\[
\rho_i = \lambda_i \bar{S}_i = \frac{\lambda_i}{n\mu} \quad (i = 1, 2) \quad (2)
\]

(5) Suppose that the probability of taxi joining the queue after arriving at the queuing system is \( \alpha_1 (0 < \alpha_1 < 1) \), k is the number of cars in the "car pool"(k \to \infty, \alpha_1 \to 0). \( \Lambda_1^* \) is the number of first class taxis joining the queue in unit time, and \( \Lambda_2^* \) is the number of second class taxis joining the queue in unit time.
\[
\begin{align*}
\begin{cases}
\lambda_1^* &= \alpha_k \lambda_1 \\
\lambda_2^* &= \alpha_k \lambda_2
\end{cases}
\end{align*}
\] (3)

(6) The mean of the two classes of taxis arriving at the queuing system is independent of each other. Therefore, the probability of arriving at the queuing system of type \(i\) at any time is \(\frac{\lambda_i^*}{\lambda}(i=1, 2)\), and the probability of joining the queue is \(\frac{\lambda_i^*}{\lambda}\), and \(\lambda = \lambda_1 + \lambda_2\). Therefore, the average service time of any taxi in the system is:

\[
\bar{S} = E(S) = \frac{\lambda_i^*}{\lambda} E(S_1) + \frac{\lambda_i^*}{\lambda} E(S_2) = \frac{\alpha_k \lambda_1}{\lambda \eta \mu} + \frac{\alpha_k \lambda_2}{\lambda \eta \mu} = \frac{\alpha_k}{\lambda} \frac{1}{\eta \mu} (\lambda_1 + \lambda_2) = \frac{\alpha_k}{\eta \mu}
\] (4)

\[
\rho = \rho_1 + \rho_2 = \frac{\lambda_1 + \lambda_2}{\eta \mu}
\]

is the total service volume of the queuing system.

4.2. Establishment of model

It is assumed that the taxis with short distance passengers have non preemptive priority and high priority (the first category), and the taxis with long-distance passengers have low priority (the second category). Moreover, the arrival rule of the two types of taxis obeys Poisson distribution, while the time of taxis carrying passengers at the boarding point obeys the exponential distribution of \(\mu\). Therefore, the state of the whole queuing system can be reflected by the following mathematical formula:

Average waiting queue length of queuing system:

\[
L_q = \sum_{k=0}^{n-1} \frac{\rho^k}{k!} p_0 + \sum_{k=n}^{m} \frac{\rho^k}{k!} \prod_{i=1}^{k-n} \alpha_i p_0
\] (5)

Number of boarding points (service counters):

\[
L = \sum_{i=0}^{n-2} \frac{\rho_i^{l+1}}{l!} p_0 + n^{n+1} \sum_{k=n}^{m} \frac{\rho^k}{n^k n!} \prod_{i=1}^{k-n} \alpha_i p_0
\] (6)

The average queue length of the whole queuing system:

\[
L_s = L_q + L
\] (7)

Inside:

\[
\rho_s = \frac{\lambda_1 + \lambda_2}{\mu} \quad p_0 = \left[ \sum_{k=0}^{n-1} \frac{\rho^k}{k!} + \sum_{k=n}^{m} \frac{\rho^k}{n^k n!} \left( \prod_{i=1}^{k-n} \alpha_i \right) \right]^{-1}
\] (8)

Average waiting time for class I taxis:

\[
W_{q_1} = \rho \frac{(n \mu)^2 \sigma^2 + \alpha^2}{2n \mu \alpha \left(1 - \alpha \rho_1\right)}
\] (9)

Average waiting time for class II taxis:

\[
W_{q_2} = W_q \frac{1}{1 - (\rho_1 + \rho_2)} = \rho \frac{(n \mu)^2 \sigma^2 + \alpha^2}{2n \mu \alpha \left(1 - \alpha \rho_1\right)} \frac{1}{1 - (\rho_1 + \rho_2)}
\] (10)

5. Empirical analysis

5.1. Data collection

Based on the travel analysis data of three representative airport passengers taking taxis in the first quarter of 2017 [3], the types of short-distance and long-distance taxis are divided, and the average income difference between short-distance and long-distance taxis is obtained according to the actual taxi fare.
5.1.1. **Capital International Airport.** Based on the travel analysis data of passengers taking taxis at Capital International Airport in the first quarter of 2017, we draw a column chart of the data.

![Figure 1](image1.png)

**Figure 1.** Column chart of passenger travel analysis data of Capital International Airport. Combined with the column chart of airport travel analysis, the central distance of passengers to the airport is about 25km, and there are many destinations within 10km near the airport.

5.1.2. **Shanghai Hongqiao Airport.** Based on the travel analysis data of passengers taking taxis at Shanghai Hongqiao Airport in the first quarter of 2017, we draw a column chart of the data.

![Figure 2](image2.png)

**Figure 2.** Column chart of passenger travel analysis data of Shanghai Hongqiao Airport. Combined with the airport travel analysis column chart, the mileage distribution is relatively average, and the central distance from passengers to the airport is about 15km.

5.1.3. **Hangzhou Xiaoshan Airport.** Based on the travel analysis data of passengers taking taxis at Hangzhou Xiaoshan Airport in the first quarter of 2017, we draw a column chart of the data.

![Figure 3](image3.png)

**Figure 3.** Column chart of passenger travel analysis data of Hangzhou Xiaoshan airport. Combined with the airport travel analysis column chart, there is less passenger diversion around the airport. Compared with other airports, it mainly goes to the urban area about 30km away from Hangzhou Xiaoshan airport.

5.2. **Data processing**

According to the actual situation, we collect taxi pricing standards: within 3 km, the basic unit price is 2.3 yuan per kilometer; for the part over 15 km, 50% of the basic unit price is charged (3.95 yuan per
kilometer). Then we use the data of four airports to get the total expected value of taxi revenue $E_{all} = 86.543$, then calculate the expected value $E_{short} = 52.515$ of short distance taxi revenue. According to the calculation $E_{all} - E_{short} = 34.028$, we can see the gap between the revenue of short distance taxi and that of all taxis, so we give some "priority" to some short distance taxis who return again.

5.3. Analyze the feasibility of the model

Reference the collected data and reprocess it

$$\lambda_1 = 2.5 \text{ unit/min}, \quad \lambda_2 = 0.5 \text{ unit/min}, \quad \mu = 2 \text{ unit/min}, \quad m = 20, \quad n = 4, \quad \alpha_s = \frac{1}{k+1}$$

In this way can calculate and acquire:

$$\rho_s = \frac{\lambda_2 + \lambda_3}{\mu} = \frac{3}{2}, \quad \rho_1 = \frac{\lambda_1}{n_\mu} = \frac{5}{16}, \quad \rho_2 = \frac{\lambda_2}{n_\mu} = \frac{1}{16}, \quad \rho = \frac{\lambda_1 + \lambda_2}{n_\mu} = \frac{3}{8}$$

$$p_0 = \left[ \sum_{k=0}^{n-1} \frac{\rho^k}{k!} \right] + \sum_{k=n}^{m} \frac{\rho^k}{k!} \left( \prod_{i=1}^{k-n} \frac{1}{1 + \rho} \right) = 0.4297$$

① $S_i$ is the time required for class $i$ taxi

② Average time required for class I taxi

$$\overline{S}_i = E(S_i) = \frac{1}{n_\mu} = \frac{1}{16} \text{ min}$$

③ Average waiting time for class I taxis

Since the passenger time of each vehicle at the passenger boarding point obeys the exponential distribution, the variance of evaluation service time is $\sigma^2 = 0$. Queuing system capacity $m = 20$, when $k \to 20$,

$$\alpha_s \to \frac{1}{21}, \quad i.e., \quad \frac{1}{21} \leq \alpha_s \leq 1$$

$$W_{q1} = \rho \frac{(n\mu)^2 \sigma^2 + \alpha_s^2}{2n\mu \alpha_s (1 - \alpha_s \rho_1)} = \frac{\lambda_1}{n_\mu} \frac{\alpha_s^2}{2n\alpha_s (1 - \alpha_s \rho_1)} = \frac{\lambda_1}{n_\mu} \frac{\alpha_s}{2n\mu (1 - \alpha_s \rho_1)}$$

$$> \frac{1}{4 \times 4 \times 4 \times 4 \left( 1 - \frac{1}{21} \right)} = 0.00009356 \text{ min}$$

In the same way:

$$W_{q1} = \rho \frac{(n\mu)^2 \sigma^2 + \alpha_s^2}{2n\mu \alpha_s (1 - \alpha_s \rho_1)} = \frac{\lambda_1}{n_\mu} \frac{\alpha_s^2}{2n\alpha_s (1 - \alpha_s \rho_1)} = \frac{\lambda_1}{n_\mu} \frac{\alpha_s}{2n\mu (1 - \alpha_s \rho_1)}$$

$$< \frac{1}{4 \times 4 \times 4 \times 4 \left( 1 - \frac{1}{4} \right)} = 0.002083 \text{ min}$$

So $0.00009356 \text{ min} < W_{q1} < 0.002083 \text{ min}$

④ Average waiting time for class II taxis

$$W_{q2} = W_{q1} \frac{1}{1 - (\rho_1 + \rho_2)} = W_{q1} \frac{1}{1 - \frac{5}{16}} = \frac{8}{5} W_{q1}$$

So $0.0001497 \text{ min} < W_{q2} < 0.003333 \text{ min}$

According to an important news released by the Shanghai Municipal People's Government on August 8, 2017, "Pudong airport uses big data to solve the problem of taking a taxi at the airport for many years. The queue is reduced to 20 minutes". It is pointed out that when the intelligent management system of hub taxis replaces the manual detection of long and short distance taxis, passengers can almost realize "go with you". Therefore, the model based on priority queuing system is feasible and practical.

6. Discussion
6.1. Suggestions to airport taxi management

During the implementation of the "priority" arrangement scheme, sufficient space should be reserved between the storage pool and the boarding point of passengers, especially during the peak period of flight arrival. When the taxi with low priority in the queue is temporarily accepted, the waiting area should be arranged in time to avoid causing congestion in the storage pool and affecting the safety of vehicles and passengers. For example, voluntary railway station in the taxi station before the implementation of long-distance diversion, which can avoid the dispute and impact after the station.

In the implementation of "priority" arrangement, management and punishment should be strengthened to ensure fair competition between taxis and ensure the safety and stability of the whole airport. For example, Shanghai Pudong Airport cooperates with major taxi companies to make full use of the opportunity of installing GPS positioning system on taxis in the whole city, and develops a set of hub taxi intelligent management system, which completely solves the problem of difficult identification of manual short distance vehicles and facilitates taxi management of the whole airport [4].

For taxis that have not received passengers in the airport, compared with other short-distance and long-distance passenger taxis, the revenue of the last airport pick-up is 0 \( (Q = 0) \), and higher priority is given.

6.2. Suggestions to taxi drivers

For taxi drivers who have long-distance passengers: if they return to the airport to pick up passengers at this time, it is difficult to find the passengers whose destination is the airport. They may spend unnecessary empty car fuel consumption, and the priority ranking after arriving at the airport is lower. Whether it is the rush hour of flight arrival or not, it may consume too much time cost. Therefore, it is more likely to obtain more benefits by staying in the urban area to carry passengers.

For taxi drivers who have carried passengers for a short distance: at this time, they are not far away from the airport, and the priority ranking after returning is in the upper middle stage, so it is easy to carry passengers. Therefore, it is recommended to return to the airport to continue carrying passengers, and the consumption of time cost may be reduced when the flight reaches the peak.

7. Conclusions

According to the above analysis, there are travel differences and income gaps between short-distance and long-distance passenger taxis. If the income gap between short-distance and long-distance passenger taxis is to be narrowed gradually, we need to give the right "priority" to the short distance taxis that returns to the airport to carry passengers, rather than to let the short distance taxi leave immediately after returning to the airport.

Therefore, we can establish a qualitative to quantitative model to calculate the priority of each taxi entering the airport. We can evaluate and rank the priority of all vehicles in the queue, and tentatively pick up the taxi with low priority in the queue, so that the taxi with high priority behind the queue enter the passenger boarding point to pick up passengers first, which makes the priority rule more scientific and reasonable, and has practical significance.

References

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