Vibrations in engineering systems

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Abstract. The value of vibration, defined as the oscillatory phenomenon of a solid subjected to the action of force, appeared relatively recently. In fact, mechanical vibration became real only after the industrial revolution. Mechanical oscillatory phenomena, for example, due to working equipment and train movement, began to emerge only by the second half of the eighteenth century. The intensive use of new machines and mechanisms suddenly filled the world, until then, quiet and silent, with new intense noises and vibrations. Different ways of calculating the estimated oscillations of engineering structures, methods of reducing vibrations to the level allowed or their complete isolation, determine the purpose and main problems of the applied theory of mechanical vibrations. In engineering practice, we are almost always interested in predicting the response of a structure or mechanical system to an external effect. For example, we may need to predict the response of a bridge or high-rise building to wind loads, earthquakes, or ground motion. In mechanics and construction, resonant catastrophe describes the destruction of a building or a technical mechanism by induced vibrations at the resonant frequency of a system, which causes its oscillation. Periodic excitation optimally transfers vibration energy to the system and stores it there. Because of this repeated and additional energy input, the system sways more and more until its load limit is exceeded. Frequent reason for these disasters were periodic oscillations of bridges. Periodic fluctuations can be described as a body movement that regularly goes through an equilibrium position. Any oscillatory movement of a mechanical system relative to its equilibrium position is called vibration.

1. Relevance

The article discusses the value of the theory of oscillations of mechanical systems for various industries, such as engineering, transport systems and construction. The main purpose of the article is to draw attention to the dramatically increased role of this section of theoretical mechanics in the design of various engineering structures and structures and the need to further develop the theory of oscillations, which the practice of operating machines and structures in industry and transport strongly advances. As engineers continue to expand the boundaries of scientific inventions and research, along with great achievements, unfortunately, terrible failures come.

Another goal of the article is to draw attention to the fact that the engineers of any specialty have to solve very different problems of structural vibrations. In the technique, oscillations have a double meaning: oscillations are harmful to structures, which, naturally, should be suppressed, or at least by the possibility of reducing their influence, and useful oscillations such as vibrating the concrete mix,
vibro-transporting, "vibrating bending", etc.; although in this case, the vibrations that are useful give rise to harmful vibrations, which are transmitted by supporting structures.

2. Literature review

The term vibration as a mechanical phenomenon (for example, a vibrating thread) can be first found in the studies of Galileo and Newton (Principia Mathematica, 1687).

A systematic study of vibrational phenomena begins to deepen in the second half of the nineteenth century. In 1917, Watson published "Recent developments in the field of building acoustics" [1], and Tuttle and Morton presented the question: "Does a concrete structure reduce vibration?" [2]. Buildings due to large structures (piles, drilling) are being considered with particular attention [3], however interest in mechanical vibration is primarily associated with industry and transport, as a consequence of the development of the automotive, aviation and large marine industries. The main interest is to identify systems that can reduce vibration. As a result, in the 1930s, a huge collection of patents relating to systems and materials to reduce vibration in buildings and vehicles was deposited around the world. In 1934, Jacob Peter Den Hartog, a professor of mechanical engineering, published the book Mechanical Vibrations.

From 1940 to 1950, we can distinguish the first systematic studies on the perception of vibration and shock. An example is "Vibration Problem Analysis" [4].

In the 1990s, the scientific literature on the perception of vibration, mainly focused on experimental data in situ. As an example, "Building vibrations caused by traffic in Montreal" [5], in which extensive measurements and detailed analysis of the building are carried out, reports on vibrations caused by traffic in Montreal. In the early years of the 21st century, Haoa published an experimental paper, "Construction of vibration for soil displacement caused by motion" [6], in which he discussed the effect of ground displacement caused by traffic on the safety of building structures adjacent to busy roads. Among the latest published studies can be identified "Measurement of the foundation of the building and ground vibrations from elevated trains and subways" [7].

3. Main part

The simplest example of a vibration system is a mass-spring system. The body, which is a mass, begins to vibrate when it moves from a position of stable equilibrium [8,9]. The body continues to move forward and backward in its equilibrium, caused by the restoring force, in this case the restoring force is the elastic force of the spring. This elementary vibration system is called a mechanical generator [10].

The mass-spring system performs harmonic periodic motion with a constant amplitude of displacement, if the system does not fade [11]. The continuous system retains mechanical energy - the harmonic movement will continue to vibrate forever. The exchange of kinetic energy for potential and vice versa will continue. But an undamped system simplifies reality, because in the real world it is impossible to isolate a system — friction and air resistance divert mechanical energy from the system to the environment in the form of thermal energy. Thus, the total mechanical energy remaining in the system gradually decreases - the displacement amplitude also decreases with time.

Another typical problem is the isolation of the sensitive system from vibrations [12]. For example, a car's suspension is designed to isolate a sensitive system from road bumps. Electron microscopes are another example of sensitive instruments that need to be isolated from vibrations. Electron microscopes are designed to determine the characteristics of a size of several nanometers. If the sample vibrates with an amplitude of only a few nanometers, it will be impossible to see. Much attention is paid to isolating this type of instrument from vibrations. This is one of the reasons why they are almost always in the basement of a building: the basement vibrates much less than on the floors above.

Almost all machines produce vibrations, for example, internal combustion engines vibrate due to the periodic movement of pistons, rotating devices vibrate caused by unbalanced parts; car vibration is caused by the roughness of the road surface, etc. Human speech is a product of the vocal cords.
vibration, sound recognition is caused by vibrations of the eardrum, and many musical instruments are based on vibration.

Alternating current is the result of periodic oscillations of electric charges. If the alternating current flows through the induction coil, the magnetic field in the coil cavity changes [13-15]. Alternating current also changes the electric field in an electric capacitor. These examples show that vibration is a phenomenon not only in mechanical engineering, but also in many other physical fields (electricity, magnetism, etc.).

Vibrations can be divided into three categories:

- Free vibration of a system is vibration that occurs in the absence of external force. Sources of free vibration are the initial displacement of the system out of balance or the initial velocity of the system.
- Forced vibration caused by an external force acting on the system. In this case, the driving force continuously supplies energy to the system.
- Self-excited vibration is a periodic and deterministic oscillation. Under certain conditions, the equilibrium state in such an oscillatory system becomes unstable, and any disturbance leads to an increase in disturbances until any effect limits further growth. Unlike forced vibrations, the exciting force does not depend on vibrations and can be maintained even if the system does not vibrate. The force acting on a vibrating object is usually external to the system and does not depend on movement. However, there are systems in which the driving force is a function of motion variables (displacement, velocity, or acceleration) and thus varies with the movement produced. As an example, the vibration caused by friction (in the clutches and brakes of the vehicle, the interaction between the vehicle and the bridge) and the vibration caused by the flow (circular saws, compact discs, DVD discs, machining, fluid supply lines), fluttering of an airplane wing, etc.

Mechanical systems usually consist of structural elements that have a distributed mass and elasticity. Examples of these structural components are rods, beams, plates and shells. These structural components are considered as continual systems that have an infinite number of degrees of freedom, and therefore the vibration of real systems is governed by partial differential equations, which include variables that depend on time as well as spatial coordinates [16]. To study the vibration, it is preferable to simplify the real system to a discrete system with a finite number of degrees of freedom. The discrete system is represented by a concentrated mass and discrete elastic elements (translational and torsion springs) and discrete damping elements (viscous dotted points). These systems are governed by a system of ordinary second-order differential equations.

A cantilever beam is an example of a continuous system. Under certain conditions, this beam can be modeled as a simple discrete spring-mass system. To simulate the vibration of the cantilever beam, the end of the beam is selected as the reference point at which the beam characteristics are measured. Then an equivalent system is created with the answer - y(t), identical to the answer of the real system.

The spring constant - k equivalent system is identical to the cantilever beam constant and can be easily calculated using the beam deflection formulas. Calculation of the equivalent mass is necessary because all points along the length of the beam do not have the same characteristics as the end of the beam. This means that the equivalent mass - m cannot be determined simply by adding the mass m of the beam and the end m, but must be found by equating the energies of the two systems when they vibrate (this type of analysis is called clumping).

We will use the spring mass system as a model for a real engineering system. The spring-mass perception system can be considered as representing the only vibration mode in a real system, whose natural frequency and damping coefficient coincide with that in our spring-mass mode.

Consider three types of effects on the system of spring-mass effects.

An external influence models the behavior of a system affected by a time-varying force. An example is a marine structure subjected to wave loading.

The base excitation models the behavior of the vibration isolation system. The base of the spring makes a given movement, causing the mass to vibrate. This system can be used to simulate a vehicle suspension system or a construction response to an earthquake.
The excitation rotor simulates the effect of a rotating machine mounted on a flexible floor. A low-mass crank rotates at a constant angular velocity, causing the mass \( m \) to vibrate.

Of course, vibrating systems can be excited in other ways, but the equations of motion will always be reduced to one of the three cases we have considered.

In each case, we limit the analysis to harmonic excitation. For example, the external force applied to the first system is defined as

\[
F(t) = F_0 \cdot \sin \omega t
\]

The force varies harmoniously with amplitude and frequency. Similarly, the basic motion for the second system

\[
y(t) = Y_0 \cdot \sin \omega t
\]

and the distance between the small mass and the large mass \( m \) for the third system have the same shape.

We assume that at time \( t = 0 \) the initial position and speed of each system

\[
x = x_0 \quad \frac{dx}{dt} = v_0
\]

In each case, we want to calculate the mass displacement \( x \) from its static equilibrium configuration as a function of time \( t \). Of particular interest is the determination of the influence of amplitude and frequency of influence on the mass movement.

**Solve the equation of motion**

**Equation of motion for external influence**

Newton's law gives:

\[
m \frac{d^2x}{dt^2} = F(t) - kx - \frac{\lambda}{xt} \frac{dx}{dt}
\]

Rearrange and replace for \( F(t) \)

\[
\frac{m \cdot d^2x}{k \cdot dt^2} + \frac{\lambda}{k} \frac{dx}{dt} + x = -\frac{F_0}{k} \cdot \sin \omega t
\]

We find that if we install

\[
\omega_n = \sqrt{\frac{k}{m}}, \quad \zeta = \frac{\lambda}{2 \sqrt{km}}, \quad K = \frac{1}{k}
\]

our equation can be brought to mind

\[
\frac{1}{\omega_n^2} \frac{d^2x}{dt^2} + 2\zeta \frac{dx}{\omega_n dt} + x = KF_0 \cdot \sin \omega t
\]

**Equation of motion for basic excitation**

Force in the spring \( k \cdot (x-y) \).

\[
L_0 + x = y
\]

Newton's Second Law

\[
m \frac{d^2x}{dt^2} = -k(x-y) - \frac{\lambda}{k} \left( \frac{dx}{dt} - \frac{dy}{dt} \right)
\Rightarrow \frac{m \cdot d^2x}{k \cdot dt^2} + \lambda \frac{dx}{dt} + x = -\frac{\lambda}{k} \frac{dy}{dt}
\]

Perform the following replacements

\[
\omega_n = \sqrt{\frac{k}{m}}, \quad \zeta = \frac{\lambda}{2 \sqrt{km}}, \quad K = 1
\]

and the equation comes down to standard form

\[
\frac{1}{\omega_n^2} \frac{d^2x}{dt^2} + 2\zeta \frac{dx}{\omega_n dt} + x = K \left( y + \frac{2\zeta}{\omega_n} \frac{dy}{dt} \right)
\]
Given the initial conditions
\[ x = x_0, \quad \frac{dx}{dt} = v_0 \]
and basic movement
\[ y(t) = Y_0 \sin \omega t \]

**Equation of motion for the excitation of the rotor**

It should be noted that the horizontal mass acceleration is \( m_0 \)
\[ a = \frac{d^2}{dt^2} (L_0 + x + y) = \frac{d^2x}{dt^2} + \frac{d^2y}{dt^2} \]
Consequently, applying the second law of Newton in the horizontal direction for both masses:
\[ m \frac{d^2x}{dt^2} = H - kx - \lambda \frac{dx}{dt} \]
\[ m_0 \left( \frac{d^2x}{dt^2} + \frac{d^2y}{dt^2} \right) = -H \]
\[ \frac{m + m_0}{k} \frac{d^2x}{dt^2} + \frac{\lambda}{k} \frac{dx}{dt} + x = - \frac{m_0}{k} \frac{d^2y}{dt^2} \]

Make the following replacements
\[ \omega_n = \sqrt{\frac{k}{(m + m_0)}}, \quad \zeta = \frac{\lambda}{2 \sqrt{k(m + m_0)}}, \quad K = \frac{m_0}{m + m_0} \]
after which the equation of motion is reduced to
\[ \frac{1}{\omega_n^2} \frac{d^2x}{dt^2} + 2\zeta \frac{\omega_n}{\omega_n^2} \frac{dx}{dt} + x = - \frac{K}{\omega_n^2} \frac{d^2y}{dt^2} \]
\[ y(t) = Y_0 \sin \omega t \]

where \( Y_0 \) is the crank length
Thus, the duration of achieving a steady state depends on the properties of the system (as well as on the initial conditions).

Analyzing the forced oscillations, we always ignore the transient characteristics of the system and only calculate the behavior in a stationary mode.

Each solution has the form:
\[ x(t) = x_h(t) + x_p(t) \]
The steady state is only part of the solution and acts only if the time is long enough that the transitional term can be neglected.

Now we will discuss the consequences of the results.
The response in steady state is always harmonic and has the same frequency as the boost.
Thus, the vibration frequency is determined by the force, and not the properties of the spring-to-mass system. This is not like a free vibration reaction.
\[ x(t) = X_0 \sin(\omega t + \phi) \]
The amplitude of vibration strongly depends on the frequency of excitation and on the properties of the system of masses of springs
If the frequency of exposure is close to the natural frequency of the system, and the system is slightly damped, huge vibration amplitudes may occur. This phenomenon is known as resonance.

As a rule, engineers try to avoid resonance. Large amplitude oscillations imply large forces; and large forces cause material failure. Of course, there are exceptions to this rule. Musical instruments, for example, must resonate to amplify sound. Musicians who play string, wind and copper instruments...
have been training their lips for years or leaning their hands to bring the right vibration modes in their instruments to make them sound perfect.

Prevent system vibrations. Suppose that we need to stop the vibration of a structure or component, for example, to stop the swinging of a tall building. The constructions are always to some extent deformable, which is qualitatively represented by a spring in the mass-spring system. They always have mass, it is represented by the mass of the block. Finally, the damper represents the dissipation of energy. The forces acting on a system usually fluctuate over time. They are probably not entirely harmonious, but usually have a fairly well-defined frequency (for example, they visualize waves on the ocean or gusts of wind). Many vibrations are created by man, and in this case their frequency is known, for example, vehicles moving along the road tend to cause vibrations at a frequency of about 2 Hz, which corresponds to a car’s rebound from its suspension.

So how do we stop the vibration of the system?

This is a broad problem, closely related to the issues of dynamic loads and dissipative characteristics of structures and bases discussed above. At present, it is sometimes suggested that the applied theory of linear oscillations has completely exhausted itself [17-20]. One can hardly agree with this opinion. With the establishment of the fundamental possibility of solving the problem, the role of the theoretical mathematician ends, but the activity of the applied mechanic begins. Between the theoretical possibility and the practical solution of some problems of the linear theory of oscillations, in spite of the highly developed, computer technology, at present there is a great distance. A number of questions still need additional research.

4. Conclusion

Most vibrations are undesirable in engineering structures. Vibrations in machines and structures cause increased stresses, energy losses, additional wear, increased load on the bearings, fatigue, discomfort for passengers in vehicles and absorb energy from the system. Rotating machine parts need careful balancing to prevent vibration damage. The worst effect is the resonance of mechanical systems. Resonance can occur with forced vibration and can cause serious damage even at low loads. Therefore, the understanding of vibrations is very important for engineers.

5. Discussion

The article does not pretend to the full coverage of issues arising in the practice of designing and operating structures exposed to dynamic effects in various fields of technology. It draws attention only to some issues of the applied theory of oscillation of structures. It should be emphasized that the issues discussed in the article are not fundamentally new. The purpose of this discussion is to draw attention to those aspects of these issues that are important for practice and require further theoretical and experimental research.

Overview of the methods used

In practice, the following rules can be used to shift the natural frequency and minimize the vibration response of the system;
1. Adding stiffness increases the natural frequency.
2. Adding mass reduces its own frequency.
3. Increasing damping reduces peak response, but expands response range.
4. Reducing damping increases peak response, but narrows response range.
5. Reducing the amplitude of the impact reduces the response at resonance.

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