Analysis Line Approach for Assembly Tolerance Analysis

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Abstract. This work presents the analysis line approach for assembly tolerance analysis. Firstly, the product functional requirements and geometric variations of part positioning features can be converted into small displacements of a series of analysis points along the specified analysis directions. The small displacement torsor (SDT) theory is employed to establish the mathematical relations between the displacements of these analysis points. And then, the variation transfer relations and tolerance accumulation calculations of various orthogonal joints between different parts are studied to determine the maximum displacement of the analysis points on the functional surface. The final displacement accumulation results of the functional surface are compared with the geometric functional requirements of the product, so as to judge whether the product functional requirements are met. Finally, an example is given to demonstrate the effectiveness of the proposed analysis line approach.

1. Introduction

Tolerance analysis involves identifying the related tolerances in an assembly or a mechanism and calculating the total stack-up of these related tolerances. It is a powerful tool to predict the effects of dimensional and geometric tolerances that have been allocated to the components of an assembly on the assembly functional requirement (FR) itself 1. Many researches focus on three-dimensional tolerance design technology, especially three-dimensional tolerance analysis technology, and various tolerance analysis model have been developed, such as matrix model[3], vector loop model[3], unified Jacobian-torsor model[4], T-map model[5], etc. Anselmetti[6-7] proposed the analysis line method to perform tolerance analysis by using deviation transfer relation equations to establish the accumulation relations between each tolerance and the geometric FR. Later, Anselmetti extended this method to braze welded assemblies. 8. The purpose of this paper is to further explore the application of analysis line approach in assembly tolerance analysis, especially when the geometric tolerance principles such as: maximum material requirement (MMR), least material requirement (LMR), etc. are taken into account.

2. Analysis line approach

2.1 Definition of analysis line

Functional requirement usually specified by the designer is the variation of the dimension or angle between two geometric features, or the position or orientation tolerance of a geometric feature relative to the datum features. The surface feature of the FR specification is also called functional surface. For
tolerance analysis based on the analysis line approach, it is usually necessary to discretize the functional surfaces. That is, the geometrical variation of the functional surface is represented by the displacements of finite discrete points on the surface towards the discrete directions. These discrete points are called analysis points, the discrete directions are called analysis directions, and the analysis lines are defined by these analysis points and the analysis directions. Generally speaking, as long as the displacement of the analysis point on the functional surface at which the position changes most towards the analysis direction meets the tolerance specified by FR, the product design complies with the functional requirements.

2.2 Discretization of the functional surface

Different feature types and geometrical tolerance items on a functional surface have different discretization modes. Figure 1 displays the discretization modes of several typical functional surfaces. For the plane feature with a positional tolerance requirement, the analysis point P is distributed on the boundary of the plane, and the analysis directions p and -p are perpendicular to the plane. In particular, for the non-circular plane, the analysis point should be located at the vertex of the plane boundary, as shown in figure 1 (a). For the plane feature with an orientation requirement (such as parallelism tolerance, perpendicularity tolerance, etc.) as shown in figure 1 (b), the analysis directions are discretized to 8 p_i directions. For the cylinder feature with a positional tolerance requirement, the analysis points P_1 and P_2 are the two endpoints of the cylinder. If the tolerance zone is an area between two parallel planes, the analysis direction is the vector direction p and -p perpendicular to the cylindrical axis as shown in figure 1 (c); if the tolerance zone is a cylinder area, the analysis directions are the directions indicated by 8 vectors p_i perpendicular to the cylindrical axis shown in figure 1 (d).

![Figure 1. Discretization of ending surface.](image)

Here, the displacement of point P along p direction relative to its nominal position in R datum reference frame is denoted as e^R(P, p); the rotation angle of the analyzed geometric feature E (plane, axis, etc.) around p direction relative to its nominal position in R datum reference frame is denoted as ϕ^R(E, p).

3. Three-dimensional variations transfer of orthogonal joints

In order to make a part have a definite position during assembling process, we adopt the six-point positioning rule to fix the part and these six positioning points are respectively located on three mutually perpendicular planes, as shown in figure 2. The main positioning plane is defined by three points M_1, M_2, and M_3; the 2nd positioning plane is defined by points S_1 and S_2; the 3rd positioning plane is defined by one point T. The normal unit vectors of these three positioning surfaces are denoted
as \( \mathbf{m} \), \( \mathbf{s} \) and \( \mathbf{t} \), respectively. In this case, the displacement of the analysis point \( \mathbf{P} \) on the functional surface in the analysis direction \( \mathbf{p} \) is written as [7]:

\[
\varepsilon(\mathbf{P}, \mathbf{p}) = \varepsilon(\mathbf{M}, \mathbf{m}) + k_m \cdot \varepsilon(\mathbf{S}, \mathbf{s}) + k_t \cdot \varepsilon(\mathbf{T}, \mathbf{t}) + \varphi \cdot [\mathbf{0P} \times k_m \cdot \mathbf{(OM \times m)} - k_s \cdot \mathbf{(OS \times s)} - k_t \cdot \mathbf{(OT \times t)}]
\]

(1)

where the influence coefficients \( k_m = p_z / m_z \), \( k_s = p_x / s_x \), \( k_t = p_y / t_y \).

Equation (1) establishes the transfer relation between the displacement of the point on the functional surface of the part and the displacements of the feature points on these three positioning surfaces of this part. The influence coefficient shows the contribution rate of the each positioning surface variation to the displacement of the functional surface. Obviously, when the variation of the positioning surface increases, the displacement of the functional surface will also increase, and vice versa, so these three influence coefficients must be all positive. Accordingly, we can determine the directions of vectors \( \mathbf{m} \), \( \mathbf{s} \) and \( \mathbf{t} \). According to equation (1), in order to find the points \( \mathbf{M} \) and \( \mathbf{S} \) whatever \( \varphi \) rotation, we let:

\[
\mathbf{0P} \times k_m \cdot \mathbf{(OM \times m)} - k_s \cdot \mathbf{(OS \times s)} - k_t \cdot \mathbf{(OT \times t)} = 0
\]

(2)

According to equation (2), we can get the coordinates of points \( \mathbf{M} \) and \( \mathbf{S} \).

\[
\begin{align*}
\begin{cases}
x_m &= x_p - p_z(x_m - z_m)/p_z \\
y_m &= y_p - p_x(x_p - z_m)/p_x \\
z_m &= 0
\end{cases}
\end{align*}
\]

(3)

\[
\begin{align*}
\begin{cases}
x_s &= x_p - p_y(y_p - y_s)/p_y \\
y_s &= y_1 + (y_2 - y_1)(x_s - x_1)/(x_2 - x_1) \\
z_s &= z_1 + (z_2 - z_1)(x_s - x_1)/(x_2 - x_1)
\end{cases}
\end{align*}
\]

(4)

where point \( \mathbf{S} \) is on the straight line defined by point \( \mathbf{S}_1(x_1, y_1, z_1) \) and point \( \mathbf{S}_2(x_2, y_2, z_2) \).

Thus, the positions of these three points \( \mathbf{M} \), \( \mathbf{S} \), and \( \mathbf{T} \) can be determined. Theses points can also be called analysis points. The analysis point \( \mathbf{M} \) and vector \( \mathbf{m} \) define an analysis line which is called the main analysis line. Similarly, the analysis point \( \mathbf{S} \) and vector \( \mathbf{s} \) define the second analysis line; the analysis point \( \mathbf{T} \) and vector \( \mathbf{t} \) define the third analysis line. Then, as long as the maximum displacements at these three points are obtained, the maximum displacement of the analysis point \( \mathbf{P} \) on the functional surface can be calculated.

4. Variations transfer of the cylinder link

4.1 Displacement of analysis point on cylinder with MMR

For the cylinder shaft-hole fit with MMR as shown in figure 3, the maximum material size of the cylinder shaft is: \( d_M = d + t_d \); The maximum material virtual size of the cylinder hole defined by the
positional tolerance $t_{po}$ is: $D_M = D - t - t_{po}$; The maximum material virtual size of the cylinder hole defined by the parallelism tolerance $t_{pa}$ is: $D_O = D - t_D - t_{pa}$. The maximum displacements of points $M_1$ and $M_2$ are respectively:

$$\varepsilon (M_1, \mathbf{m}) = \frac{(D_M - d_M)}{2}$$

(5)

$$\varepsilon (M_2, \mathbf{m}) = \frac{(D_M - d_M)}{2} + \frac{(D_O - d_M) \cdot l_2}{l_1}$$

(6)

4.2 Displacement of analysis point on cylinder with LMR

For the cylinder shaft-hole fit with LMR as shown in figure 4, the displacement of the point on the cylinder shaft can be regarded as caused by the clearance, and this displacement has a maximum value when the clearance reaches its maximum. The least material size of the cylinder shaft is: $d_L = d - t_d$. The least material virtual size of the cylinder hole defined by the positional tolerance $t_{po}$ is: $D_L = D + t_D + t_{po}$. The least material virtual size of the cylinder hole defined by the parallelism tolerance $t_{pa}$ is: $D_O = D + t_D + t_{pa}$. The maximum displacements of points $M_1$ and $M_2$ are respectively:

$$\varepsilon (M_1, \mathbf{m}) = \frac{(D_L - d_L)}{2}$$

(7)

$$\varepsilon (M_2, \mathbf{m}) = \frac{(D_L - d_L)}{2} + \frac{(D_O - d_L) \cdot l_2}{l_1}$$

(8)

4.3 Variations transfer of the cylinder-plane joint

The joint types between parts in 3-D assemblies mainly include plane-plane, plane-cylinder, cylinder-cylinder, etc., and most of which are orthogonal joins. In this case, the established datum system is usually an orthogonal coordinate system. Next we further discuss the variation transfer of cylinder-plane joint.

As shown in figure 5, the centring pin is positioned by a cylindrical surface and a plane perpendicular to the axis of the cylindrical surface, the cylindrical surface and the plane are the main positioning surface and the second positioning surface, respectively. The positioning datum system is established by taking the cylinder axis as y-axis and the line perpendicular to the cylinder axis as z-axis. The main analysis point M (0, $y_m$, 0) is on the cylinder axis with the vector direction $\mathbf{m}$ (0, 0, $m_z$); the second analysis point S (0, $y_s$, $z_s$) is on the second positioning plane with the vector direction $\mathbf{s}$ (0, $s_y$, 0).

The displacement of the analysis point $P$ (0, $y_p$, $z_p$) on the functional surface along the vector direction $\mathbf{p}$ (0, $p_y$, $p_z$) will be:
\[ \varepsilon(P,p) = v \cdot p_y + w \cdot p_z + \phi \cdot (OP \times p) \]  

(9)

The displacement of the analysis M (0, y_m, 0) along the \( m \) (0, 0, m_z) will be:

\[ \varepsilon(M,m) = w \cdot m_z + \phi \cdot (OM \times m) \]

(10)

The displacement of the analysis S (0, y_s, z_s) along the \( s \) (0, s_y, 0) will be:

\[ \varepsilon(S,s) = v \cdot s_y + \phi \cdot (OS \times s) \]

(11)

Eliminating \( v \), \( w \) in the above equations (9)-(11), one will get:

\[ \varepsilon(P,p) = k_m \cdot \varepsilon(M,m) + k_s \cdot \varepsilon(S,s) + \phi \cdot [\mathbf{OP} \times p - k_m \cdot (\mathbf{OM} \times m) - k_s \cdot (\mathbf{OS} \times s)] \]

(12)

where \( k_m = p_y / m_z \cdot k_s = p_z / s_z \). We can orientate the vectors \( m \) and \( s \) by setting \( k_m \) and \( k_s \) positive.

Regardless of the value of \( \phi \), in order to find points \( M \) and \( S \), the rotation component of equation (12) needs to be eliminated. So one get:

\[ \mathbf{OP} \times p - k_m \cdot (\mathbf{OM} \times m) - k_s \cdot (\mathbf{OS} \times s) = 0 \]

(13)

When \( p_z \neq 0 \), \( z_p - z_i \neq 0 \), we will have:

\[ p_y / p_z = (y_p - y_m) / (z_p - z_i) \]

(14)

In this case, the three analysis lines intersect at one point as shown in figure 5. So one get:

\[ \varepsilon(P,p) = k_m \cdot \varepsilon(M,m) + k_s \cdot \varepsilon(S,s) = \sin \theta \cdot \varepsilon(M,m) + \cos \theta \cdot \varepsilon(S,s) \]

(15)

When \( p_z = 0 \), we have \( \theta = 0 \), the rotation component \( \phi \) is not equal to zero. The displacement of the analysis point \( P \) (0, \( y_p \), \( z_p \)) along the vector direction \( p \) (0, \( p_y \), \( p_z \)) will be:

\[ \varepsilon(P,p) = k_s \cdot \varepsilon(S,s) + \phi \cdot [\mathbf{SP} \times p = \varepsilon(S,s) + \phi \cdot (\mathbf{SP} \times p) = \varepsilon(S,s) + \phi \cdot a(\Delta, \mathbf{a}) \]

(16)

where \( l \) is the distance between \( P \) and \( S \) in the direction of the main analysis line; \( \Delta \) is the axis of the main cylinder surface; \( a(\Delta, \mathbf{a}) \) is the maximum rotation angle of axis \( \Delta \) around vector \( \mathbf{a} \). Similarly, for the plane-plane joint between parts, we can get the identical results as equations (15) and (16).

5. Illustrative example

5.1 Analysis of the functional requirement

As shown in figure 6, an example of the centring pin assembly consisted of three parts \(^9\) is employed to verify the effectiveness of the proposed analysis line approach. In this example, the ending features of the assembly FR are point \( P \) and datum plane A. We wish to control the variation of the vertical displacement of the centring pin endpoint \( P \) relative to datum plane A to guarantee the performance of this assembly. According to figure 6, the assembly FR is discretized into point displacement. We need to determine whether the maximum displacement of center pin endpoint \( P \) relative to datum A in the vertical direction meets the assembly FR.
5.2 Transfer and accumulation of variations

The displacement of point P in the p direction depends on its coaxiality deviation relative to the axis of part 1, denoted $\delta(P^1, z)$ and the variation of point $M^1$ on the axis of part 1 due to the default of positioning surface (see figure 7), denoted $\varepsilon(M^1, -z)$, one get:

$$\varepsilon(P, p) = \delta(P^1, z) + \varepsilon(M^1, -z) \quad (17)$$

where $\delta(P^1, z)$ is determined by the concentricity tolerance, so one gets: $\delta(P^1, z) = 0.015/2 = 0.0075$. The analysis line of point $M^1$ on the part 1 is along $-z$, the displacement of point $M^1$ depends on the deviation $\delta(M^1, -z)$ caused by the clearance and position default between part 1 and part 2 and the deviation of point $M^2$ on part 2, one gets:

$$\varepsilon(M^1, -z) = \delta(M^1, -z) + \varepsilon(M^2, -z) \quad (18)$$

Figure 7. The analysis points of the mechanism

The joint between part 1 and part 2 is a cylinder joint with MMR, the clearance and position default cause the deviation denoted as $\delta(M^1, -z)$. The maximum material size of the shaft is $d_M = 19.993$ mm; the maximum material virtual size of the hole defined by the position tolerance is $D_{po} = 19.984$ mm; the maximum material virtual size of the hole defined by the parallelism tolerance is $D_{pa} = 19.988$ mm. According to equation (15), one get:

$$\delta(M^1, -z) = (D_{po} - d_M)/2 - (D_{pa} - d_M) - 60/50 = 0.0015 \quad (19)$$

The displacement of point $M^2$ along $-z$ can be expressed as the variation of the hole of part 2 relative to the datum B and the deviation of the analysis point $G^2$ on part 2.
\[ \varepsilon(M^2, -z) = \delta(M^2, -z) + \varepsilon(G^2, -z) \]  
where \( \delta(M^2, -z) = 0.02 \). According to the analysis in section 4.3, part 2 is positioned by two planes of part 3, we have:

\[ \varepsilon(G^2, -z) = \delta(G^2, -z) + \delta(G^3, -z) + \varepsilon(H^3, -z) = 0.015/2 + 0.01 + 0.014/2 + 0.008 \times 100/10 = 0.1045 \]  

Substituting equations (18)-(21) into equation (17), we obtain the maximum displacement of P is: \( \varepsilon(P, p) = 0.1335 \).

6. Conclusions
Under the assumption of small displacement, the analysis line approach treats the functional tolerance for feature as small displacement variation, and thus converts geometric functional requirement and tolerance variation into small displacements of finite points along the direction of the analysis line. In the worst case, the three-dimensional geometric deviation transfers and accumulation relations of different assembly joints (cylinder-plane joint, plane-plane joint, etc.) are studied based on the SDT theory. In order to establish the variation transfer relations of the orthogonal assembly joints, it is necessary to consider the influence of geometric deviations of parts, assembly clearances and the priority order of positioning features between parts. After having analyzed the displacement variations of the typical positioning features, the maximum displacement expression of each point on the functional surface is obtained by using the transfer relation equation, and the three-dimensional tolerance analysis in worst case is realized. This work mainly studies the variation transfer relations and tolerance accumulation calculations of orthogonal assembly joints. The future research will further extend the analysis line approach to complex assembly joints to perform statistical tolerance analysis.

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