Multiscaling behavior in the volatility return intervals of Chinese indices

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Abstract – We investigate the probability distribution of the return intervals $\tau$ between successive 1-min volatilities of two Chinese indices exceeding a certain threshold $q$. The Kolmogorov-Smirnov (KS) tests show that the two indices exhibit multiscaling behavior in the distribution of $\tau$, which follows a stretched exponential form $f_q(\tau/\langle\tau\rangle) \sim e^{-a\tau/\langle\tau\rangle^\gamma}$ with different correlation exponent $\gamma$ for different threshold $q$, where $\langle\tau\rangle$ is the mean return interval corresponding to a certain value of $q$. An extended self-similarity analysis of the moments provides further evidence of multiscaling in the return intervals. Our results can be viewed as a support to the recent finding of Wang et al. (Phys. Rev. E, 77 (2008) 016109) that the volatility return intervals of stocks exhibit multiscaling behavior.

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Introduction. – The analysis of the waiting time between two successive events is helpful to understand the dynamics of stock markets, which has drawn much attention. A variety of waiting time variables have been raised by different definitions of event to characterize the stock markets from different view angles, such as the persistence probability [1–3], the exit time [4–9], and the intertrade duration [10–13]. Recently the return intervals between successive extreme events exceeding a certain threshold $q$ have been investigated for numerous complex systems, including rainfalls, floods, temperatures and earthquakes [14–18]. Similar analysis was subsequently carried out concerning the volatility return intervals, which are defined as the waiting times between successive volatilities exceeding a certain threshold in stock markets.

Yamasaki et al. and Wang et al. used the daily data and intraday data of US stocks to study the properties of the volatility return intervals [19–22]. They found that the distribution of return intervals $\tau$ between successive volatilities greater than a certain threshold $q$ showed scaling behavior. This scaling behavior is expected to be of great importance for the risk assessment of large price fluctuations. Similar scaling behavior was observed in the return intervals of daily and 1-min volatilities of thousands of Japanese stocks [23]. Qiu, Guo and Chen analyzed the high-frequency intraday data of four liquid stocks traded in the emerging Chinese market, and found that the return interval distributions of the Chinese stocks investigated also followed a scaling behavior [24].

In contrast, Lee et al. investigated the return intervals of 1-min volatility data of the Korean KOSPI index [25] and no scaling was observed. Wang et al. used the Trade & Quote Database of the 500 constituent stocks composing the S&P 500 Index and found a multiscaling behavior in the volatility return intervals [26]. A systematic deviation from scaling was observed in the cumulative distribution of return intervals, which implies that its probability distribution also deviates from scaling. Moreover, the $m$-th moment of the scaled return intervals showed a certain trend with the mean interval, which supports the finding that the return intervals exhibit multiscaling behavior. This finding was reinforced by further analysis of 1137 US common stocks [27]. Ren, Guo and Zhou used a high-frequency database [28] to study the interval returns of 30 most liquid stocks in Chinese stock market [29]. The Kolmogorov-Smirnov (KS) test was adopted to examine the possible collapse of the interval distributions for different threshold values. Only 12 individual stocks passed the KS test and showed a scaling behavior, while the remaining 18 stocks exhibited multiscaling behavior.
In this paper, we study the distribution of the volatility return intervals of two Chinese stock indices, i.e., Shanghai Stock Exchange Composite Index (SSEC) and Shenzhen Stock Exchange Composite Index (SZCI). We first use the KS test, and find that the return intervals of the two indices exhibit multiscaling behavior, consistent with the multiscaling behavior of some individual stocks which partially compose the indices. We also perform an extended self-similarity (ESS) analysis of the moments, and further confirm the multiscaling behavior of the return intervals. This result is qualitatively similar to that of the US stocks [26,27].

Preprocessing the data sets. – Our analysis is based on the high-frequency intraday data of two Chinese indices, the Shanghai Stock Exchange Composite Index (SSEC) and the Shenzhen Stock Exchange Composite Index (SZCI). Each composite index is constructed based on all the stocks listed on the corresponding exchange. The indices are recorded every six-to-eight seconds from January 2004 to June 2006. We define the volatility as the magnitude of logarithmic index return between two consecutive minutes, that is \( R(t) = \ln Y(t) - \ln Y(t-1) \), where the index \( Y \) is the closest tick to a minute mark. Thus the sampling time is one minute, and the volatility data size is about 140000.

The intraday volatilities of both indices exhibit a L-shaped intraday pattern [30], similar to the individual stocks [29,30]. When dealing with intraday data, this pattern should be removed [20–22,24,26]. Otherwise, the return intervals distribution will exhibit daily periodicity for large thresholds. The intraday pattern \( A(s) \) is defined as

\[
A(s) = \frac{1}{N} \sum_{i=1}^{N} R'(s),
\]

which is the volatility at a specific moment \( s \) of the trading day averaged over all \( N \) trading days and \( R'(s) \) is the volatility at time \( s \) of day \( i \). The intraday pattern is removed as follows:

\[
R'(t) = \frac{R(t)}{A(s)}.
\]

Then we normalize the volatility by dividing its standard deviation

\[
v(t) = \frac{R'(t)}{\sqrt{\langle [R'(t)]^2 \rangle - \langle R'(t) \rangle^2}}.
\]

Probability distribution of return intervals. –

Probability distribution of scaled return intervals. We study the return intervals \( \tau \) between successive volatilities exceeding a certain threshold \( q \). A series of return intervals are obtained for each particular threshold \( q \) and its number decreases with increasing threshold \( q \). For each value of \( q \), we can obtain empirically a probability distribution \( P_q(\tau) \) of the volatility return intervals, which is related to the probability distribution \( f_q(\tau/\langle \tau \rangle) \) of the scaled return intervals \( \tau/\langle \tau \rangle \) as follows:

\[
P_q(\tau) = \frac{1}{\langle \tau \rangle} f_q(\tau/\langle \tau \rangle),
\]

where \( \langle \tau \rangle \) is the mean return interval that depends on the threshold \( q \). If the function \( f_q(x) \) is independent of \( q \), there exists a universal function \( f(x) \) such that \( f_q(x) = f(x) \) for different values of \( q \). In other words, the probability distributions \( f_q(\tau/\langle \tau \rangle) \) of the scaled return intervals collapse onto a single curve \( f(\tau/\langle \tau \rangle) \) and the return intervals exhibit scaling behavior.

To investigate whether the return interval distributions of the two Chinese indices exhibit scaling behavior, we plot in fig. 1 the empirical probability distributions \( f_q(\tau/\langle \tau \rangle) \) for a wide range of thresholds \( q = 2, 3, 4, 5 \). For the Shanghai Composite Index, the curves for different thresholds \( q \) show systematic deviations from each other and do not collapse onto a single curve. For the Shenzhen Composite Index, the deviation is relatively weak, but one still can see some difference between the thresholds \( q = 2 \) and \( q = 5 \). This indicates that the distributions of return intervals for both indices could not be approximated by a scaling relation. With the increase of the threshold \( q \), there are more large scaled return intervals and the distribution becomes broader.

The observation that there is no scaling behavior in the volatility return interval distributions is consistent with the results of a previous study of individual Chinese stocks [29]. The Kolmogorov-Smirnov test shows that only
12 stocks out of 30 most liquid Chinese stocks exhibit scaling behaviors in the return interval distributions for different thresholds $q$, while the other 18 stocks do not show scaling behavior [29].

**Kolmogorov-Smirnov test of scaling in return interval distributions.** The eyeballing of the probability distributions offers a qualitative way of distinguishing scaling and nonscaling behaviors. Here we further adopt a quantitative approach based on the Kolmogorov-Smirnov test. The standard KS test is designed to test the hypothesis that the distribution of the empirical data is equal to a particular distribution by comparing their cumulative distribution functions (CDFs). Our hypothesis is that the two return interval distributions for any two different $q$ values do not differ at least in the common region of the scaled return intervals [23]. Suppose that $F_q$ is the CDF of return intervals for $q$, and $F_{q_j}$ is the CDF of return intervals for $q_j$, where $q_j \neq q$. We calculate the KS statistic by comparing the two CDFs in the overlapping region:

$$KS = \max \left( |F_{q_j} - F_q| \right), \quad q_j \neq q. \quad (5)$$

When the KS statistic is less than a critical value CV, the hypothesis is accepted and we can assume that the distribution for $q$ is coincident with the distribution for $q_j$. The critical value is $CV = c_0 \sqrt{n/m}[m+n]$, where $m$ and $n$ are the numbers of interval samples for $q$ and $q_j$ [31,32]. We can find the threshold $c_0 = 1.36$ at the significance level of $\alpha = 5\%$ from the tables [33,34].

In Table 1 is depicted the KS statistics and the corresponding critical values for the two indices. For the Shanghai Composite Index, $KS > CV$ for all $(q_i,q_j)$ pairs except $(q_4,q_1) = (4,5)$. It means that the distribution for $q=5$ coincides with the distribution for $q=4$, but significantly differs from the distributions for other $q$ values. For the Shenzhen Composite Index, though the distributions for $q=3,4,5$ are very close to each other, the distribution for $q=2$ differs from them as illustrated in fig. 1. Therefore, we can conclude that the distributions differs for different $q$ and do not collapse onto a single curve. The KS test confirms the result that the return interval distributions do not exhibit scaling behavior.

**Fitting the return interval distributions.** For those stock markets showing scaling behavior in the volatility return interval distributions, it is a consensus that the scaling form could be approximated by a stretched exponential function [20,21,23,24,26,27].

$$f_q(x) = f(x) = ce^{-(ax)^{\gamma}}, \quad (6)$$

where $c$ and $a$ are two parameters and $\gamma$ is the correlation exponent characterizing the long-term memory of volatilities. There are nevertheless exceptions. Based on the KS test and the weighted KS test, Ren, Guo and Zhou showed that the scaled return interval distributions of 6 stocks (out of the 12 stocks exhibiting scaling behavior) can be nicely fitted by a stretched exponential function with $\gamma = 0.31$ at the significance level of $5\%$ [29].

In this work, we have demonstrated that the return interval distributions of the two Chinese indices do not follow a scaling form. It is still interesting to check if the (scaled) return intervals follow a stretched exponential distribution expressed in eq. (6) but with different values of parameters $c$, $a$ and the correlation exponent $\gamma$ for different thresholds $q$. In this case, our hypothesis is that the empirical distribution is coincident with its best-fitted stretched exponential function. Similar to the KS test we have conducted for two empirical samples, we use the KS statistics to test whether the distribution for a certain threshold $q$ is identical to its best-fitted distribution in the overlapping region of the scaled return intervals. Let $F_q$ be the cumulative distribution for $q$ and $F_{SE}$ the cumulative distribution from integrating the fitted stretched exponential. The KS statistic defined in eq. (5) becomes

$$KS = \max \left( |F_q - F_{SE}| \right), \quad q \in \{2,3,4,5\}. \quad (7)$$

Then the bootstrapping approach is adopted [35,36]. To do this, we first generate 1000 synthetic samples from the best-fitted distribution and then reconstruct the cumulative distribution $F_{sim}$ of each simulated sample and its CDF $F_{sim,SE}$ from integrating the fitted stretched exponential. We calculate the values of KS between the fitted CDF and the simulated CDF using

$$KS_{sim} = \max \left( |F_{sim} - F_{sim,SE}| \right). \quad (8)$$

The $p$-value is determined by the frequency that $KS_{sim} > KS$. The tests are carried out for the two Chinese indices. The parameters of the fitted stretched exponential and resultant $p$-values for different $q$ are depicted in table 2.

The $p$ value could be regarded as the probability that the empirical distribution consists with its best fit.
Consider the significance level of 1%. If the \( p \)-value of an index for a certain threshold \( q \) is less than 1%, then the null hypothesis that the empirical PDF of this index can be well fitted by a stretched exponential is rejected. According to table 2, the null hypotheses for all the \( q \) values are accepted for both two Chinese indices. It is noteworthy to point out that the \( p \)-values for all the \( q \) values (except for \( q = 2 \) for the Shenzhen Composite Index) are very large, implying high goodness-of-fit of the stretched exponential to the empirical PDFs. At the significance level of 5%, the stretched exponential is rejected when \( q = 2 \) for the Shenzhen Composite Index. To show how good the stretched exponential fits the data, we illustrate in fig. 1 the fitted stretched exponential with the parameters listed in table 2. One observes that the empirical PDFs could be well fitted by a stretched exponential. In principle, the stretched exponential fits the empirical PDF better when the \( p \)-value is larger. For instance, the stretched exponential fits the empirical PDF for the Shanghai Composite Index better than the Shenzhen Composite Index when \( q = 2 \).

According to table 2, the parameters differ from one another, providing further evidence supporting our conclusion that the return interval distributions do not have a scaling form. On average, the exponent \( \gamma \) decreases with increasing threshold \( q \), which is in line with the US stocks [27].

**Moments of scaled return intervals.** – The distributions of return intervals exhibit multiscaling behavior and show a systematic tendency with the threshold \( q \). To further study this tendency of the interval distribution with \( q \), we follow the method in ref. [26] and compute the moments of the scaled return intervals \( x = \tau / \langle \tau \rangle \) defined as

\[
\mu_m = \langle (\tau / \langle \tau \rangle)^m \rangle^{1/m} = \left[ \int_0^\infty x^m f_q(x) dx \right]^{1/m}, \tag{9}
\]

where the mean interval \( \langle \tau \rangle \) is dependent on the threshold \( q \). When \( m = 1 \), we have \( \mu_1 = 1 \) by definition, independent of \( q \). If there is a scaling behavior such that \( f_q(x) = f(x) \), the \( m \)-th moment \( \mu_m \) is a univariate function of the order \( m \) and is independent of any other variables including the threshold \( q \) and the mean return interval \( \langle \tau \rangle \). On the contrary, the \( m \)-th moment \( \mu_m \) is not constant with respect to \( \langle \tau \rangle \) for \( m \neq 1 \), when there is no scaling in the return interval distributions.

**Dependence of moment on mean return interval.**

We first investigate the relation between \( \mu_m \) and \( \langle \tau \rangle \). To better quantify the dependence of \( \mu_m \) on \( \langle \tau \rangle \), we calculate the moments in a certain medium range of \( \langle \tau \rangle \) to avoid the finite-size effect and discreteness effect [26]. Figure 2 illustrates the moments \( \mu_m \) for \( m = 0.25, 0.5, 1, 5, 20 \) vs. \( \langle \tau \rangle \) for the two Chinese indices. We investigate \( \mu_m \) for a range of \( \langle \tau \rangle \) corresponding to \( 1 < q < 5 \). Each curve of the moments \( \mu_m \) significantly deviates from a horizontal line, displaying a shape qualitatively similar to that shown in ref. [26]: \( \mu_m \) decreases with the increase of \( \langle \tau \rangle \) when \( m < 1 \), and shows an increasing tendency with the increase of \( \langle \tau \rangle \) when \( m > 1 \). These two types of moment functions are delimited by the horizontal line \( \mu_1 = 1 \).

Take a careful look at \( \mu_m \) in fig. 2, for \( m < 1 \) (\( m > 1 \) \( \mu_m \) first decreases (increases) rapidly with the increase of \( \langle \tau \rangle \), and then starts to decrease (increase) relatively slowly at \( \langle \tau \rangle = 10 \). The discreteness of the records of \( \tau \) responds to the rapid increase (decrease) of \( \mu_m \) for small \( \langle \tau \rangle \) (\( \langle \tau \rangle > 10 \)). The moment for extremely large \( \langle \tau \rangle \), i.e., \( \langle \tau \rangle > 100 \), will increase (decrease) for \( m < 1 \) (\( m > 1 \)) due to the finite-size effect. We choose to study \( \mu_m \) in a medium region \( 10 \leq \langle \tau \rangle \leq 100 \), where the effects of finite size and discreteness are small and many of the curves show a power-law–like trend with \( \langle \tau \rangle \). We may use a power law to fit the moment in this medium region,

\[
\mu_m \sim \langle \tau \rangle^\alpha, \tag{10}
\]

If the PDF of return intervals follow a scaling form, \( \mu_m \) is independent of \( \langle \tau \rangle \) according to eq. (9) and the exponent \( \alpha \) should be some value very close to 0. If the exponent \( \alpha \) is significantly different from 0, it implies that the PDF of return intervals may show multiscaling behavior.

Figure 3 plots the exponent \( \alpha \) as a function of order \( m \) for the two Chinese indices. This figure shows that the exponent \( \alpha \) differs from 0 in a systematic fashion, and the magnitude of \( \alpha \) is larger than that of US stocks [26] for small and large \( m \) which indicates a relatively clearer multiscaling behavior. The exponents for the two indices...
are very close to each other when \( m \) is small. For \( m < 1 \), \( \alpha \) is negative. The exponent \( \alpha \) increases with \( m \) when \( m < 3 \) and decreases afterwards owing to the finite-size effect. For large order \( m \), the \( \alpha \) value for the Shenzhen Composite Index is greater than that for the Shanghai Composite Index. This implies that large \( \langle \tau \rangle \) tends to occur with greater probability for the Shenzhen Composite Index than the Shanghai Composite Index, since large \( \langle \tau \rangle \) contributes more for high order \( \mu_m \).

The above analysis according to eq. (10) can be related to the extended self-similarity (ESS) analysis [37], which reads

\[
\langle \tau^m \rangle \sim \langle \tau^n \rangle^{\xi(m,n)}. \tag{11}
\]

If the generalized variable of \( \mu_m \)

\[
\mu_{m,n} = \left\langle \left( \frac{\tau}{\langle \tau^n \rangle^{1/n}} \right)^m \right\rangle^{1/m} = \langle \tau^m \rangle^{1/m} \langle \tau^n \rangle^{1/n} \tag{12}
\]
scales as

\[
\mu_{m,n} \sim \langle \tau^n \rangle^{1/n}^\alpha \tag{13}
\]
together with eq. (11), we have

\[
(\alpha + 1)/n = \xi(m,n)/m. \tag{14}
\]

If the return interval distribution can be scaled as follows:

\[
P_q(\tau) = \frac{1}{\langle \tau^n \rangle^{1/n}} f \left( \frac{\tau}{\langle \tau^n \rangle^{1/n}} \right), \tag{15}
\]
we obtain that

\[
\xi(m,n) = m/n. \tag{16}
\]

In this case, we have

\[
\alpha = 0. \tag{17}
\]

In other words, \( \mu_{m,n} \) is independent of \( \langle \tau^n \rangle^{1/n} \). Our empirical test focuses on the case that \( n = 1 \). This ESS framework was also used to investigate scaling in the exit times in turbulence [38] and intertrade durations [39]. According to eq. (14)

\[
\alpha(m) = \xi(m,1)/m - 1. \tag{18}
\]

Since \( \xi(1,1) = 1 \), we have \( \alpha(1) = 0 \) when \( m = 1 \). This is well verified by fig. 3.

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**Dependence of moment on order \( m \).** The moments \( \mu_m \), not only display a significant dependence on \( \langle \tau \rangle \), but also show a systematic tendency with \( m \) as shown in fig. 2. It is interesting to investigate the relation between \( \mu_m \) and \( m \) directly. For a fixed \( \langle \tau \rangle \), one can study the moment of \( \tau \) of various orders \( m \). If the return interval distribution strictly obeys a scaling form, \( \mu_m \) should not depend on \( \langle \tau \rangle \), and the \( \mu_m \) curves for different \( \langle \tau \rangle \) should all collapse onto a single curve. We plot in fig. 4 the moment \( \mu_m \) for various thresholds \( q = 2, 3, 5, 8 \) for the Shenzhen Composite Index. One sees the curves for different \( \langle \tau \rangle \) exhibit substantial deviations from a single curve, which demonstrates the multiscaling behavior of return intervals. For small \( m \) (\( m < 1 \)), \( \mu_m \) shows a decreasing tendency with the increase of \( \langle \tau \rangle \), similar to that of the US stocks shown in ref. [26], for large \( m \) (\( m > 1 \)), \( \mu_m \) tends to increase with the increase of \( \langle \tau \rangle \) opposite to that of the US stocks which shows a decreasing tendency. This is not difficult to understand since small \( \tau \) dominates \( \mu_m \) for small order \( m \) and large \( \tau \) dominates \( \mu_m \) for large order \( m \). Our results are further validated by the forthcoming numerical simulations. The situation for the Shanghai Composite Index is very similar. We have demonstrated that the return interval distribution follows a stretched exponential form with different parameters for various thresholds \( q \) in the previous
section. For the data follow a stretched exponential distribution, we can calculate the analytical result of the moment $\mu_m$ by substituting eq. (6) to eq. (9) and considering the normalization condition of probability density. It follows immediately that \[ \mu = \frac{1}{a} \left[ \frac{\Gamma((m+1)/\gamma)}{\Gamma(1/\gamma)} \right]^{1/m}. \] (19)

In fig. 4(b), the analytical curves of $\mu_m$ vs. $m$ for three stretched exponential distributions fitted from empirical data of Shenzhen Composite Index for $q = 2, 3, 5$ are plotted. As one can see, the analytical results are similar to those of the empirical data, which supports the multiscaling of the empirical return intervals.

**Conclusion.** – We have studied the multiscaling properties of the distributions of volatility return intervals for two Chinese indices, together with their moments. The Kolmogorov-Smirnov test is adopted to examine the scaling behavior of the return interval distributions as well as the particular form of the distribution. We find that the return intervals of the two indices exhibit multiscaling behaviors, and their distributions for different thresholds $q$ can be well approximated by stretched exponential functions $f_q(x) \sim e^{-(ax)^q}$, but with different values of the correlation exponent $\gamma$. An ESS-like moment analysis confirms the existence of multiscaling rather than monoscaling. This result is consistent with a previous analysis on individual Chinese stocks [29] and helps us better understand the properties of volatility return intervals in the Chinese stock markets.

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