Abstract. The Faint Young Sun Paradox comes from the fact that solar luminosity (2÷4)\times10^9 years ago was insufficient to support the Earth’s temperature necessary for the efficient development of geological and biological evolution (particularly, for the existence of considerable volumes of liquid water). It remains unclear by now if the so-called greenhouse effect on the Earth can resolve this problem. An interesting alternative explanation was put forward recently by M. Krizek (New Ast. 2012, 17, 1), who suggested that planetary orbits expand with time due to the local Hubble effect, caused by the uniformly-distributed Dark Energy. Then, under a reasonable value of the local Hubble constant, it is easy to explain why the Earth was receiving an approximately constant amount of solar irradiation for a long period in the past and will continue to do so for a quite long time in future.

Introduction

The Faint Young Sun Paradox was recognized in the late 1950s, when the sufficiently accurate models of stellar evolution were constructed. As a result, it was found that luminosity of the Sun should change considerably at the time scale of geological and biological evolution of the Earth. For example, the well-known monograph [1] stated that the luminosity increased by 1.6 times for the period of 5\times10^9 years. Although a number of subsequent models gave a somewhat less variation in the luminosity (30÷40%), the problem persists. Namely, the average Earth’s temperature 2\times10^9 years ago is predicted to be so low that almost all water would be frozen. On the other hand, a lot of geological and planetological evidences suggest that there were extensive volumes of liquid water on the Earth (3÷4)\times10^9 years ago. Besides, the liquid water would be necessary for the emergence of life in the same period of time.

The commonly-used approach to resolve the Faint Young Sun Paradox, widely exploited since 1970s [2, 3], is the greenhouse effect, i.e., keeping the infrared radiation emitted from the Earth surface by the atmosphere. The efficiency of such process strongly depends on

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the atmospheric chemical composition (particularly, the minor constituents CO₂, CH₄, NH₃, etc.). Since their concentration in the early atmosphere is unknown, there is much uncertainty in the theoretical models. In general, it remains unclear by now if the greenhouse effect can be sufficiently efficient to compensate the reduced solar luminosity in the past [4].

Expansion of Planetary Orbits

An interesting alternative idea for resolution of the Faint Young Sun Paradox was suggested recently by M. Krček and L. Somer [5, 6]. They believe that the increasing solar luminosity could be compensated by the expansion of the Earth’s orbit, which is presumably caused by the local Hubble effect associated with the uniformly-distributed Dark Energy (or Λ-term).

In fact, the question if planetary orbits experience the universal cosmological expansion was posed by G.C. McVittie in the early 1930’s [7], and a quite large number of researchers worked on this problem in the subsequent few decades (e.g., review [8]). Although the most of them concluded that Hubble expansion should be strongly suppressed or absent at all at the sufficiently small scales, there is no well-established criterion for such suppression. Besides, some of the proposed criteria strongly contradict each other. For example, observers usually assume that Hubble expansion should be absent in the gravitationally-bound systems, i.e. in the regions of enhanced density; while the well-known Einstein–Straus model [9] claims that Hubble expansion is absent in the empty local neighborhood of the central body and is restored as the mass density increases up to the mean cosmological value. Moreover, the most of theoretical treatments are not applicable to the case of cosmological models dominated by the Dark Energy, which is assumed to be perfectly uniform and present everywhere [10].

Therefore, the hypothesis by M. Krček and L. Somer seems to be quite reasonable and, from our point of view, deserves a more detailed consideration. Of particular importance is the question if the local Hubble parameter \( H_0^{(loc)} \) equals the global one and, if not, what is its value?

To get some estimates, let us consider the solar luminosity increasing linearly with time:

\[
L(t) = L_0 + \left( \frac{\Delta L}{\Delta T} \right) t,
\]

where \( L_0 \) is the present-day luminosity, \( \Delta T = 5 \cdot 10^9 \) yr, and \( \Delta L/L_0 = 0.3, 0.4, 0.5, 0.6 \), which covers the entire range of the solar models discussed in the literature.

Next, we assume that the mean radius of the Earth’s orbit gradually increases with time according to the Hubble law:

\[
dR/dt = H_0^{(loc)} R.
\]

Assuming for simplicity that \( H_0^{(loc)} = \text{const} \) (which should be a reasonable approximation when the Dark Energy dominates), equation (2) can be easily integrated:

\[
R(t) = R_0 \exp\left[ H_0^{(loc)} t \right].
\]

Then, solar irradiation (the energy flux density) at the Earth, \( F = L/(4\pi R^2) \), will change with time as

\[
F(t) = F_0 \left[ 1 + \left( \frac{\Delta L}{L_0} \right) (t/\Delta T) \right] \exp\left[ -2\left( H_0^{(loc)}/H_0 \right) (H_0 t) \right],
\]

where \( H_0 \approx 70 \) (km/s)/Mpc \( \approx 0.071 \cdot 10^{-9} \) yr⁻¹, and time \( t \) can be conveniently expressed in the units of \( 10^9 \) yr. A number of corresponding temporal dependences for different models
Figure 1: Relative variation of the solar irradiance at the Earth as function of time for a few solar models (four panels, from top to bottom) at various values of the local Hubble constant: $H_0^{(loc)}/H_0 = 0.5$ (green curves), 0.6 (cyan), 0.7 (blue), 0.8 (magenta), and 0.9 (red). Black inclined lines correspond to the case without cosmological correction, $H_0^{(loc)}/H_0 = 0.0$.

of solar evolution, characterized by $\Delta L/L_0$, and various values of the local Hubble parameter $H_0^{(loc)}$ are presented in Figure 1.

**Discussion**

To resolve the Faint Young Sun Paradox, we should seek for such regimes of evolution when the relative flux density $F(t)/F_0$ deviates as small as possible (e.g., within a few percent) from unity in the entire time interval from $-(3/4)\cdot10^9$ yr up to the present time, $t = 0$. So, the following principal conclusions can be derived from the analysis of Figure 1:

1. For the solar model with a relatively small variation in the luminosity, $\Delta L/L_0 = 0.3$ (top panel), the most favorable regime is achieved at $H_0^{(loc)} = 0.5 H_0$. This is approximately the case considered by M. Křížek and L. Somer [5, 6], which provides a very stable solar
irradiation in the past and a quite gradual deviation in the future.

Unfortunately, this value of the local Hubble constant is poorly consistent with the conjecture that just the Dark Energy is responsible for the expansion of planetary orbits. Really, let us assume that (i) the local Hubble expansion is produced only by the perfectly-uniform Dark Energy, while the clumped substance manifests itself as Newtonian forces; and (ii) the global Hubble expansion is formed both by the Dark Energy and average contribution from the ordinary matter. Then, it can be shown [11] that the local Hubble parameter is related to the global one as

\[
\frac{H_0^{\text{(loc)}}}{H_0} = \left[ 1 + \frac{\Omega_{D0}}{\Omega_{A0}} \right]^{-1/2} \approx 1 - \frac{1}{2} \frac{\Omega_{D0}}{\Omega_{A0}},
\]

where \( \Omega_{A0} \) and \( \Omega_{D0} \) are densities of the Dark Energy (\( \Lambda \)-term) and ordinary substance (for the most part, Dark Matter). So, taking \( \Omega_{A0} \approx 0.7 \) and \( \Omega_{D0} \approx 0.3 \), we get \( H_0^{\text{(loc)}}/H_0 \approx 0.8 \), which is substantially different from the above-mentioned value 0.5.

2. On the other hand, as is seen in the third panel of Figure 1, the rate of local expansion \( H_0^{\text{(loc)}} = 0.8 H_0 \) enables us to get a sufficiently stable irradiation for the solar model with an increased variability, \( \Delta L/L_0 = 0.5 \) (but the overall stability is not so good as in the first case, especially, in the future period). However, just the second case seems to be more consistent with the cosmological argumentation.

3. Anyway, it should be kept in mind that the greenhouse effect is of considerable importance in the thermal history of the Earth’s atmosphere. So, the temporal variation of the solar irradiation cannot be immediately confronted with geological and biological evidences.

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Резюме. Парадокс “тусклого молодого Солнца” связан с тем, что его светимость \((2 \div 4) \times 10^9\) лет назад была недостаточной для поддержания на Земле температуры, необходимой для эффективного протекания геологической и биологической эволюции (в частности, для существования значительного количества воды в жидком состоянии). До сих пор остается неясным, может ли “парниковый” эффект в достаточной степени решить эту проблему. Интересное альтернативное объяснение было недавно предложено М. Кризеком (M. Křízek, New Ast. 2012, 17, 1), который предположил, что орбиты планет увеличиваются с течением времени за счет локального эффекта Хаббла, связанного с однородно распределенной “темной энергией”. Тогда, при разумном значении локальной постоянной Хаббла легко объяснить, почему Земля получала приблизительно постоянную плотность потока солнечного излучения на протяжении длительного периода в прошлом и будет получать ее еще достаточно долгое время в будущем.