Network model evaluation of the effects of pore and particle size distributions on dynamic straining-dominant deep bed filtration in porous media

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Abstract. A straining-dominant filtration model based on a 2D square network is proposed. In this model, the colloid particle can be captured in a node only. The effects of network parameters (node size distribution parameters $\mu_n$ and $\sigma_n$ and pore throat size distribution parameters $\mu_b$ and $\sigma_b$), particle concentration of influent ($C_0$), and particle size distribution parameters ($\mu_r$ and $\sigma_r$) on straining-dominant deep bed filtration simulation are investigated. Four parameters ($C_0$, $\mu_n$, $\mu_b$, and $\mu_r$) significantly affect the simulated pressure drop and the normalized particle concentrations of effluent ($Ce/C_0$). The increase in parameters $\mu_n$ and $\mu_b$ decreases pressure drop and increases $Ce/C_0$, whereas the increase in parameters $\mu_r$ and $C_0$ increases pressure drop and decreases $Ce/C_0$. With the experimental condition and a 5% variation in parameters, parameters $\sigma_n$, $\sigma_b$, and $\sigma_r$ are lowly sensitive to $Ce/C_0$ and pressure drop. When the variation in $\sigma_r$ is sufficiently large, the model simulation is significantly affected. The simulated normalized effluent concentrations and pressure drop are consistent with the experimental data under appropriate simulated conditions.

1.Introduction

The transport and filtration of particles in porous media are common phenomena encountered in many chemical[1] and petroleum industrial[2] and natural processes[3]. In the petroleum industry, deep bed filtration and external filter cake formation may cause severe permeability decline and formation damage[4, 5].

The effect of several system parameters on permeability decay along the porous media has been investigated. Those parameters include fluid velocity, pore size distribution[6-9], packing material and colloid particle sizes[10-12], collector structure[13], and interaction between collector surfaces and particles[14-15].
The mechanism of colloid particles removed from porous media includes diffusion, bridging and electrical forces, gravity segregation, and straining[16-17]. Straining usually occurs when pore throats are significantly small to allow the passing of a given colloid particle. Recent findings have suggested that the rate of colloid straining within saturated porous media is sensitive to the size and shape of colloid particle, the ratio of particle diameter to sand-grain diameter, pore-scale hydrodynamics and pore water chemistry, and the shape and surface roughness of the solid matrix[18-20].

Advection–dispersion theory has frequently been used to describe the straining of spherical and non-spherical colloidal particles and non-uniform colloid mixtures in saturated porous media. The mathematical form of this model consists of the two following equations[20]:

$$\frac{\partial C}{\partial t} + \frac{\rho_b}{n} \frac{\partial S}{\partial t} = -\nu \frac{\partial C}{\partial z} + D \frac{\partial^2 C}{\partial z^2}, \quad (1)$$

$$\frac{\rho_b}{n} \frac{\partial S}{\partial t} = k_0 C e^{-S/\lambda}, \quad (2)$$

where C is the concentration of suspended colloids (mg/L), t is the time (h), $\rho_b$ is the bulk density of the solid matrix (g/L), n is the porosity (unitless), S is the concentration of strained colloids (mg/g), $\nu$ is the average linear pore water velocity (cm/h), z is the coordinate parallel to flow (cm), $D$ is the hydrodynamic dispersion coefficient (cm2/h), $k_0$ is the straining rate coefficient for clean-bed conditions (i.e., $S=0$ at $t=0$) (h −1), and $\lambda$ determines the degree of decrease in straining rate with time owing to particle accumulation inside pore spaces that can strain colloids. According to Shangping Xu[18], the latter parameter reflects but does not equal to colloid straining capacity (mg/g). The values of $k_0$ and $\lambda$ are then estimated in an inversion manner by minimizing the sum of squared differences between experimentally measured and model-predicted colloid break-through concentrations. This model is an empirical model, and its parameter estimation requires numerous experiments. The equation cannot describe the drop in pressure and the effects of pore and particle size distributions on the filtration.

The network model has been applied extensively to simulate the deposit formation in porous media and thus describe the effect of pore and particle size distributions on particle deposition behavior along the filter bed[21]. The network model utilizes the advantage of describing particle deposition and migration simultaneously and variation in permeability attributed to particle plugging directly. Chang et al.[22] applied the dynamics simulation method and the modified square network model to study the effect of different interaction energy curves of Derjaguin–Landau–Verwey–Overbeek (DLVO) theory on the permeability reduction in a filter bed. Żywczyk et al.[23] used a network model to evaluate the pressure drop and collection efficiency of particles in a filter composed of a few layers. The network model combined with percolation theory has been applied to process experimental data[24]. On the basis of the analysis on the feature in the infinite cluster, two power laws are proposed to describe the filtration coefficients close to the percolation threshold and far above the percolation threshold.

Our previous studies[25-27] investigated the effects of coordination numbers, capture scheme, and pore size distribution on the straining-dominant deep bed filtration (DBF) simulation in a 2D network under favorable conditions. Our previous results verified that two pore size distribution parameters significantly influence the simulation power law and experimental exponent calculation. The simulated results will be consistent with the experimental data if proper simulated conditions are applied. However, the structures of the 2D network and the actual 3D filter media differ. We assumed that the 3D grid exhibits better performance than the 2D grid in the simulation. After simulation, we found that the difference between 2D and 3D network modeling is insignificant. The effects of capture scheme and coordination number on the simulated normalized effluent concentration can be due to the variations in the total capture probability for different lattices with the same pore size distribution[28].

Previous studies[24-28] have focused on diluted suspensions in the straining-dominant filtration process under favorable conditions. Pore size distribution and flow field change because of the absence of pore blockage and given that the variation rule of pressure drop during filtration is
undetermined. These factors must be considered when studying the actual filtration process. The present work aims to study on non-diluted suspensions in the dynamic straining-dominant filtration process under favorable conditions. The network model of straining-dominant DBF process is established to evaluate the effects of pore and particle size distributions on straining-dominant deep bed filtration in porous media and thus investigate the role and mechanism of microscopic parameters in DBF. Finally, the normalized effluent concentrations and pressure drop are determined by comparing the numerical simulation results with the experimental data.

2. Experimental section

As shown in Fig. 1, the experimental apparatus primarily consists of three parts: (1) a packed column (packing space of 40 mm in diameter and 58 mm in length) provided with flanges and a distribution plate to avoid channeling. A stainless-steel mesh provided right between the distribution plate and the column supporting the glass beads that form the packing matrix; (2) an injection (dosing) pump with a flow rate range of 180 mL/h (linear velocity of $4.0 \times 10^{-5}$ m s$^{-1}$); (3) a U-tube manometer to measure the pressure drop across the packed column.

![Fig. 1 Schematic of the experimental apparatus and flow direction](image1.png)

![Fig. 2. Glass bead size distribution of medium determined using Malvern Mastersizer](image2.png)

The spherical glass beads for packing are sieved to reduce the particle size range to 58–550 μm. The size distribution of the sieved glass beads is examined with a laser particle size analyzer (Malvern Mastersizer 2000), and the results are shown in Fig. 2. The volumetric water content of the porous media is 18.3 mL. The monodisperse suspensions of carboxyl polystyrene used in the experiments are used as colloidal particles with a size range of 8.0–8.9 μm. Colloidal suspension (25 ppm), which is composed of 0.1 M NaOH and degassed ultrapure MilliQ water, is used to create a solution with a pH of 10. These carboxyl groups provide the net negative charge in an alkaline solution (pH 10), thereby resulting in mutual particle repulsion based on the calculation of the interaction energy using DLVO[29] theory.

Water is injected into the column at a chosen flow rate, and the pressure drop is recorded after the flow is stabilized (steady-state pressure drop). Then, the colloidal suspension is sent through the column, the effluent solution is continuously sampled and monitored, and the pressure drops are recorded simultaneously until the packed column is completely plugged. The reproducibility of the measurements is checked for most of the experiments reported herein. The pressure drops are measured with $\pm 10\%$ in repeated experiments. Additional experimental details can be found in Refs[24, 26, 30-31].
3. Network model description

Network models can simulate the dynamic and static properties of porous media, such as pressure drop, particle concentration of effluent, and relative permeability curves, which are similar to experimental observations. The network, which consists of pore bodies (also referred to as nodes), is connected by a set of pore throats (also referred to as bonds).

A 2D square network is used in this study to represent porous media, and this network consists of interconnected circular tubes that represent the pores and globes that represent the nodes. Periodic boundary conditions are applied to avoid surface effects, as shown in Fig. 3. The number of nodes in the 2D square network is 80 (flow direction) × 60. To evaluate the effects of node, pore, and particle size distributions on filtration in porous media, a node size distribution is assigned to the nodes in the network and a pore size distribution is assigned to the bonds in the network. The method used to estimate the pore body (node) and pore throat (bond) size distributions from packing glass beads is the Monte Carlo procedure with Latin-hypercube sampling based on solving 3D and 2D Apollonius’ problem[32] (the 2D problem follows Descartes’ theorem [30]). As shown in Fig. 4, each pore body consists of four sphere grains. The pore throat consists of three grains with two mutually tangent in cross section. If the statistical distribution is lognormal, then the node and pore size distribution parameters μ and σ can be estimated from grain size distribution and are listed in Table 1.

![Fig. 3 Structure of the 2D square network](image)

![Fig. 4 Formation of the node by four tangent spheres and pore throat by three tangent circles in cross section (Descartes’ theorem)](image)

| distribution parameter | Node | bond |
|------------------------|------|------|
| Mean (μm)              | 31.66| 15.88|
| Std (μm)               | 10.07| 2.53 |
| μ                      | 3.45 | 2.76 |
| σ                      | 0.10 | 0.10 |

Table 1. Pore body (node) and pore throat (bond) size distribution parameters estimated from packing glass beads

| distribution parameter | Node (lognormal) | bond (lognormal) | colloid particle (normal) |
|------------------------|------------------|------------------|---------------------------|
| Mean (μm)              | 31.66            | 15.88            | 9.20, 9.50, 9.80          |
|                        |                  |                  | 9.20, 8.60                |

Table 2. Pore body (node), pore throat (bond), and particle size distribution parameters used in the sensitivity analysis.
The bonds (pore throat) of the network in this study are set with a constant length. The volumes of each bond and node are given by: $V_b = \pi r_p^2 l$, and $V_n = \frac{4}{3}\pi r_n^3$, where $V_b$ and $V_n$ are the volumes of each bond and node, respectively; $r_n$ and $r_p$ are the radii of the node and bond, respectively; and $l$ is the bond length. The combined volumes of the bonds and nodes consist of the total void space of the porous media and can be used to calculate the porosity of network.

The flux that passes through each bond can be obtained by applying a mass balance to each node. In the current network model, a node is linked to two bonds. The hydraulic conductance of bond $C_b$ can be expressed as follows:

$$C_b = \frac{\pi r_p^4}{8}.$$  \hfill (3)

The flow rate in a bond is determined by the Poiseuille law as follows:

$$q = \frac{C_b \Delta p}{\eta l},$$  \hfill (4)

where $r_p$ is the radius of the bond, $q$ is the flow in the bond, $\Delta p$ is the pressure loss in each bond, and $l$ is the length of the bond. According to mass conservation, the fluxes on the bonds connected to each node on the network are zero except the nodes at the entrance and outlet. The pressure of each node can be obtained by solving the linear system, and the flow rate can be calculated using Eq. (4).

![Computation algorithm for the network model](image)

Fig. 5 Computation algorithm for the network model

Several colloid particles are sent into the network and are flow-biased randomly walk[33] until they are captured or penetrate through the network. In this model, the attachment of colloid particles to the grain can be neglected because net repulsion occurs between the particles and grain surfaces in the experiment performed under high pH and low salinity. A colloid particle moves from one node to another by passing through the pore throat (bond). As a result, a particle may either pass through a bond or be captured in a node. The capture probability is counted whereas the passing time is not. Fig. 5 shows the computation algorithm for the network model. We assume that the particles are deposited only in nodes. When a bond is plugged, accessible and inaccessible fractions of pore cross section can be calculated using the method described in Ref[34].

| Std($\mu$m) | 10.07 | 2.53 | 0.00 | 0.20 | 1.20 |
|-------------|-------|------|------|------|------|
| $\mu$       | 3.45  | 2.76 | 0.00 | 9.20 | 8.60 |
| $\sigma$    | 0.1   | 0.1  | 0.00 | 0.20 | 1.20 |
4. Modeling results and discussion

The effects of simulation parameters, such as particle concentration of influent and node, pore, and particle size distributions, on straining-dominant deep bed filtration in porous media are investigated. The simulation results are compared with the experimental results in terms of normalized effluent concentrations (Ce/C0) and pressure drop.

4.1 Effect of particle concentration of influent

As particle concentration increases, the simultaneous approach of many particles to a bond causes plugging of the pore body. The fluid then flows to unplugged pores with high interstitial velocities. The multiple particles are straining at plenty of constriction space (nodes in the network) while the convective jamming phenomenon can occur nearly simultaneously. Accordingly, a rapid and drastic rise in pressure drop can occur in the experiment and simulation. In the model, we use particle numbers generated at each node of the entrance to represent the particle concentration of influent. Fig. 1 in Ref[31] and Fig. 6 in the current study show that pressure drop increases with the increase of the particle concentration of influent in the experiment and simulation. The simulation results show that the normalized effluent particle concentration at steady state decreases with the increase in influent particle concentration.

4.2 Effect of node size distribution

In the model, the colloid particle can be captured in a node only. The node size distribution parameters will significantly affect the normalized particle concentration of effluent and pressure drop. A sensitivity analysis on two node size distribution parameters is performed to confirm this effect. The experimental data from the medium with node and size distribution parameters estimated by the Monte Carlo method (μn = 3.45, σ n= 0.10, μb = 2.76, and σ b= 0.10) are considered. The results are shown in Figs. 8 and 9. Fig. 8 shows that node size distribution parameter μn exhibits a high sensitivity to Ce/C0 and pressure drop and that the increase in parameter μn decreases pressure drop whereas increases Ce/C0. However, node size distribution parameter σn is lowly sensitive to Ce/C0 and pressure drop. As shown in Fig. 9, the 5% variation in node size distribution parameter σn insignificantly changes Ce/C0 and pressure drop. The reason is that the 5% variation in node size distribution parameter μn exerts a greater impact on the distribution than does the variation in σn, as shown in Fig. 7. The increase in the pore body (node size) provides the particle with a large activity space, thereby bringing difficulty in being captured. Each particle deposited in node also increases the flow resistance. These factors decrease pressure drop whereas increase Ce/C0.

4.3 Effect of pore size distribution

Pore plugging is usually caused by straining, and straining occurs when a particle encounters a pore throat (bond) with a diameter smaller than the particle size. Pore size will affect the particle capture...
probability, thereby affecting pressure drop and increasing \( \frac{C_e}{C_0} \). The parameters used in the analysis on pore size distribution sensitivity are listed in Table 2. The simulation results are shown in Figs. 10 and 11. Fig. 10 shows that pore size distribution parameter \( \mu_b \) is highly sensitive to pressure drop and increases \( \frac{C_e}{C_0} \). The increase in \( \mu_b \) significantly changes the viable pathway for particles to penetrate through the entire network when the particle size is fixed at 9.20 \( \mu m \). As a result, the particle capture probability is decreased and the time for the network to be fully clogged is delayed. The trends of the simulated results are consistent with those of the experimental data (Fig. 1) reported by Alexis A. Porubcan[20]. Pore size distribution parameter \( \sigma_b \) is lowly sensitive to \( \frac{C_e}{C_0} \) and pressure drop. As shown in Fig. 11, pressure drop and \( \frac{C_e}{C_0} \) insignificantly change with the increase in \( \sigma_b \). Similar to node size distribution, the 5% variation in pore size distribution parameter \( \mu_b \) insignificantly changes the distribution curve, as shown Fig. 7.

4.4 Effect of particle size distribution
The effects of particle size for straining-dominant DBF network model are presented in Figs.12-15. Three sizes (9.20, 9.50, and 9.80 \( \mu m \)) are used in the simulation for investigating the effects of mono-disperse colloid particle, and the other parameters are listed in Table 2. The results in Fig. 12 show
that the increase in particle size increases pressure drop drastically and increases Ce/C0. The reason is that, when particle size increases, the number of bonds in the network decreases, thereby allowing particles to pass through and thus easing their being captured.

In investigating the effects of poly-disperse colloid particle, the simulation is conducted under normal distribution with location parameter $\mu_r = 9.20$ and standard deviation $\sigma_r = 0.20$. The 5% variation in particle size distribution parameters $\mu_r$ and $\sigma_r$ is used for the sensitivity analysis. The results are shown in Fig. 13. The effects caused by the variation in $\mu_r$ are similar to those of mono-disperse colloid particle size. Specifically, the increase in $\mu_r$ decreases Ce/C0 whereas increases pressure drop. As shown in Fig. 14, given that the selected $\sigma_r$ is small, the change in pressure drop and Ce/C0 caused by the variation in $\sigma_r$ is not obvious. The particle size distribution of effluent is presented in Fig.S1. When influent particle size distribution location parameter $\mu_r$ increase 5%, there is a slight decrease in effluent particle size distribution location parameter $\mu_r$ (from 9.66 to 9.657) and $\sigma_r$ (from 0.20 to 0.195). Considering that the selected $\sigma_r$ is small, the particle size distribution of effluent particles is nearly the same as that of influent particles.
Rege S D et al.[33] reported that the overlap of particle and pore size distributions affects the permeability ratio and particle size of effluent. Hence, we choose other particle size distribution parameters ($\mu_r = 8.60$, $\sigma_r = 1.20$) and a 50% variation for $\sigma_r$ sensitivity analysis. The results are presented in Fig. 15. The increase in $\sigma_r$ increases pressure drop whereas reduces $C_e/C_0$. The 50% variation in $\sigma_r$ yields a considerable overlap of particle and pore size distributions. The variation also decreases the fraction of bonds in network that allow particles to pass through, thereby increasing particle strain rate and advancing the time for the network to be fully clogged. As shown in Fig. S2, the particle size distribution of effluent presents a certain extent of change compared with the particle size distribution of influent. The values of particle size distribution parameters for influent are $\mu_{rin} = 8.60$ and $\sigma_{rin} = 1.80$, and the values of particle size distribution parameters for effluent are $\mu_{reff} = 8.51$ and $\sigma_{reff} = 1.75$.

Fig. 16 Simulated $C_e/C_0$ and pressure drop compared with the experimental results

4.5 Comparison with experimental results

The experimental data sets used for model validation are based on the long-term injection of mono-sized particles performed in packing glass granules. $C_e/C_0$ and pressure drop can be obtained using the experimental condition applied in the straining-dominant DBF network model simulation. In comparing the experimental results on the variation tendency of $C_e/C_0$ and pressure drop, the initial point of simulated pressure drop is adjusted. The results are shown in Fig. 16. The trends of simulated pressure drops are similar to those of the experimental data. The simulated $C_e/C_0$ agrees with the experimental data. The simulation is performed under a square network of $80 \times 60$; node size distribution parameters $\mu_n = 3.45$, $\sigma_n = 0.10$; pore size distribution parameters $\mu_b = 2.76$, $\sigma_b = 0.10$; mono-disperse colloid particle size $r_s = 8.6 \mu$m; and porosity of 0.26.

5. Conclusions

In this work, a straining-dominant filtration model based on a 2D square network is proposed. The network is composed of interconnected circular tubes that represent the pore throat (bond) and globes that represent the pore body (node). The colloid particle can be captured in a node only. On the basis of the model, the effects of network parameter (node and pore throat size distribution) and particle size distribution on straining-dominant deep bed filtration in porous media are investigated.

With the experimental condition applied in the simulation, the sensitivity analysis (a 5% variation in parameter) reveals that, among the six parameters (node size distribution parameters $\mu_n$ and $\sigma_n$, pore size distribution parameters $\mu_b$ and $\sigma_b$, and particle size distribution parameters $\mu_r$ and $\sigma_r$), three parameters ($\mu_n$, $\mu_b$, and $\mu_r$) significantly influence the simulated pressure drop and $C_e/C_0$. When the variation in $\sigma_r$ is sufficiently large, the simulated pressure drop and $C_e/C_0$ significantly change.
The simulation results are consistent with the laboratory test data under proper simulation conditions.

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