Associated production of $A^0$ and $Z^0$ bosons and
Rare Pseudoscalar Higgs Decays

B. Field

C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, New York 11794-3840, USA and
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973, USA

(Dated: February 21, 2005)

We study the production of a pseudoscalar Higgs boson $A^0$ in association with a $Z^0$ boson at a future international linear collider (ILC). We consider the contributions to this process at the one loop level in the Minimal Supersymmetric Standard Model (MSSM) from top and bottom quarks as well as stop and sbottom squarks. We also study the squark contributions to the decay widths of the pseudoscalar Higgs boson for the decays $A^0 \to \gamma Z^0$ and $A^0 \to Z^0 Z^0$. The contribution from the supersymmetric loops are found to be directly proportional to the squark mixing and potentially large due to the massive pseudoscalar Higgs coupling to squarks.

PACS numbers: 13.66.Fg, 14.70.Hp, 14.80.Cp

I. INTRODUCTION

The Higgs mechanism is the means by which the electroweak symmetry is broken in the Standard Model (SM) and in the Minimally Supersymmetric Standard Model (MSSM) (see \[1,2,3\] for review). The MSSM has two Higgs doublets which are used to generate the masses of the up- and down-type quarks. This leads to five physical Higgs bosons. These consist of two CP-even neutral scalar bosons ($h^0$, $H^0$), one CP-odd neutral pseudoscalar ($A^0$), and two charged bosons ($H^\pm$). The extended Higgs sector has several new parameters that can be determined with the input of two parameters, the mass of the pseudoscalar $M_{A^0}$ and $\tan \beta$ given reasonable assumptions as to the size of the other supersymmetric parameters in the theory. The parameter $\tan \beta \equiv v_u/v_d$ is the ratio of the vacuum expectation values (VEVs) of the up and down sectors. We are interested in the phenomenology of the pseudoscalar Higgs boson at a future international linear collider, in particular, what the heavy squark contributions can tell us about the phenomenology.

There have been many studies of pseudoscalar Higgs boson phenomenology\[5,6,7,8,9,10,11\]. The phenomenology of the pseudoscalar Higgs produced in association with a $Z^0$ boson at a hadron collider (such as the Tevatron and the CERN LHC) has also been studied\[12,13,14,15,16\], as well as at a future international linear collider (ILC)\[17,18,19,20,21\]. There have been some studies of the decays of Higgs bosons into squarks\[22,23\] that show how large these processes can become, but squark contributions are rarely taken into account for pseudoscalar Higgs processes. In this paper we will discuss squark contributions to processes involving the production and decay of a pseudoscalar Higgs boson which have been missing from the literature thus far.

The construction of an ILC would greatly enhance our ability to measure the parameters in many new physics scenarios\[24,25\] including the MSSM. The process $e^+ e^- \rightarrow Z^0 A^0$ at an ILC is interesting because it has the possibility of four bottom quarks in the final state allowing for precise measurements of the process as well as many other excellent final states. This process is also interesting because it could compete with the $e^+ e^- \rightarrow Z^0 h^0$ process. The $e^+ e^- \rightarrow Z^0 A^0$ process occurs at the one loop level because the pseudoscalar Higgs boson does not couple to vector bosons at tree level. This process also allows for the exploration of squark mixing for the heaviest two generations of squarks.

We find that the dominant contributions to the $e^+ e^- \rightarrow Z^0 A^0$ process does not come from top and bottom quark loops. We find that the squark contributions to the $e^+ e^- \rightarrow Z^0 A^0$ process are relevant at all values of $\tan \beta$ and although they depend on the mixing in the stop and sbottom squark sectors, they dominate over the standard model field contributions due to the large coupling of the pseudoscalar to squarks. We find that the pseudoscalar decay $A^0 \rightarrow Z^0 Z^0$ has significant contributions from the squark sector and that the $A^0 \rightarrow Z^0 Z^0$ branching ratio becomes more important for large values of the mass of the pseudoscalar. We also find that the $A^0 \rightarrow \gamma Z^0$ decay squark has contributions on the order of the quark contributions for a light pseudoscalar Higgs at large values of $\tan \beta$. Overall, squark contributions need to be added to processes involving neutral Higgs bosons to complete our understanding of their phenomenology.
II. SQUARK CONTRIBUTIONS

In the SM, quarks generically couple to the pseudoscalar Higgs boson as $\gamma^5 m_q/v$. Beyond this base coupling, the up- and down-type quarks couple to the pseudoscalar differently. Up-type quarks couple as $\cot \beta$ and down-type quarks couple as $\tan \beta$, leading to the well-known conclusion that bottom quarks become more important at large values of $\tan \beta$. This is also true in the stop/sbottom sectors. The sbottom squarks become more important as $\tan \beta$ becomes large.

In the MSSM, right and left handed quarks, $q_{R,L}$, have scalar super-partners, $\tilde{q}_{L,R}$. We are interested in the stop and sbottom squarks ($\tilde{t}_R, \tilde{t}_L, \tilde{b}_R, \tilde{b}_L$). The squark sector of the MSSM has the possibility for squarks to mix into mass eigenstates that are different than the usual left-right basis. We can introduce a mixing angle that diagonalizes the squarks into their mass eigenstates. This can be written generically for the new squark eigenstates $\tilde{q}_{1,2}$ as

$$\begin{align*}
\tilde{q}_1 &= \tilde{q}_L c_q + \tilde{q}_R s_q, \\
\tilde{q}_2 &= -\tilde{q}_L s_q + \tilde{q}_R c_q,
\end{align*}$$

(1)

where $c_q \equiv \cos \theta_q$ and $s_q \equiv \sin \theta_q$ are the mixing angles in each of the squark sectors. Squark mixing is particularly interesting when it comes to the squark loop contributions for pseudoscalar Higgs production. In the left-right basis, the pseudoscalar Higgs couples only to squarks that change their handedness at the vertex leaving only vertices such as $A^0 \tilde{q}_R \tilde{q}_L$. However it is now easy to see that in the presence of squark mixing, vertices of the type $A^0 \tilde{q}_i \tilde{q}_j$ for all values of $(i,j)$ are present and retain the size of the $A^0 \tilde{q}_R \tilde{q}_L$ vertex times a function of the new mixing angle. This is also true of other neutral Higgs bosons in the MSSM, but the analysis is more complicated as there are more vertices present for the $h^0$ and $H^0$ Higgs bosons.

In this model, all of the pseudoscalar Higgs vertices in the squark sector are proportional to the squark mixing angles. Thus the mixing angles regulate the squark contribution to any pseudoscalar Higgs process. It is possible to tune the mixing angles $s_q$ and $c_q$ to eliminate all but one of these vertices. For instance if $\sin \theta_q = 0$, then only the $A^0 \tilde{t}_1 \tilde{t}_2$ vertex remains and it is maximal. It is easy to see in Eqn. 1 that this would correspond to being back in the left-right basis.

The mixing angles themselves are not completely independent from the other parameters in the MSSM. We can write the mixing angles as

$$\begin{align*}
\sin(2\theta_t) &= \frac{2 m_{\text{top}} (A_t - \mu \cot \beta)}{m_{t_1}^2 - m_{t_2}^2}, \\
\sin(2\theta_b) &= \frac{2 m_{\text{bot}} (A_b - \mu \tan \beta)}{m_{b_1}^2 - m_{b_2}^2}.
\end{align*}$$

(2)

(3)

Here $A_t,b$ are the tri-linear scalar couplings for soft supersymmetry breaking in the MSSM Lagrangian and $\mu$ is the supersymmetry breaking Higgs mass parameter in the super potential.

We can see that there is more splitting in the stop sector as it is proportional to the top quark mass which is much larger than the bottom quark mass. This is a general feature of squark mixing. In our analysis we picked $m_1 \equiv 1$ TeV in both the stop and sbottom sectors and generated $m_2$ using the other parameters in the theory for an arbitrary mixing angle allowing us to study the mixing in the squark sector. Thus we always have $m_2 > m_1$ in the stop sector. This is the standard ordering for squark masses. Our choices for squark masses are well beyond current experimental limits and we have chosen such heavy squarks to make our results more conservative. The other parameters in the theory were chosen to be $\mu = 300$ GeV and $A_{t,b} = 1500$ GeV. Of course, the squark contributions to the process depend on the mass of the squarks and lighter squarks lead to larger contributions to any process involving them. In choosing $m_1 = 1$ TeV we find maximal mixing in the squark sector leads to the lightest masses for the second quark. Although, our results show a very complicated dependence on these mixing angles, the dominant effect of the supersymmetric contributions are seen when the squarks are the lightest is as to be expected.

These mass relations are tree level relations and the corrections to the mixing angles are known in the literature. However, since we are interested in studying the effects of the squark mixing on our observables, it is unclear how to incorporate these corrections to produce squark masses that obey specific mixing angles. The effects of these higher order corrections are the most pronounced in the sbottom sector and future studies should be completed on this subject. We have used the MS running-mass of the bottom quark in our generation of the second sbottom squark mass.
Squark mixing imposes new Feynman rules for the vertices in our processes. The results of the application of the squark mixing to the pseudoscalar vertices can be found in Table I. The coupling of the squarks to the pseudoscalar can be written as

\[ 
\tilde{A}_i = -\frac{m_{\text{top}}}{v} (\mu - A_i \cot \beta), \tag{4} 
\]

\[ 
\tilde{A}_b = -\frac{m_{\text{top}}}{v} (\mu - A_b \tan \beta), \tag{5} 
\]

where \( v = \sqrt{v_1^2 + v_2^2} = 246 \text{ GeV} \) is the Higgs VEV and we have chosen \( \text{sign}(\mu) < 0 \) implicitly in our choice of couplings.

The squark contributions to pseudoscalar Higgs processes are often missing in the literature and were believed to be quite small because they are loop suppressed and the squark would be very heavy if they exist in nature. Although this is certainly a possibility given the right parameter choice, the pseudoscalar coupling to squarks is quite large and is not entirely suppressed by the heavy squark loops. This is due to the fact that the pseudoscalar couples to right- and left-handed squarks only in a non-diagonal way. In the MSSM, the non-diagonal squark vertices with any neutral Higgs boson (\( \Phi = \{ h^0, H^0, A^0 \} \)), written here as \( \Phi_{iR}\tilde{q}_L \), is proportional to the tri-linear soft-supersymmetry breaking parameter multiplied by the same factor as the coupling in the quark sector modulo any mixing angles that may also be present at the vertex. For the pseudoscalar Higgs, there are no mixing angles present, just \( \tan \beta \) or \( \cot \beta \) depending on the nature of the squark. This could also increase this effect in the sbottom sector once the difference in the masses between the top and bottom squarks is overcome. Therefore the squark-squark-Higgs coupling is potentially greatly enhanced compared to the quark-quark-Higgs coupling, so much so as to overcome the loop suppression and the heavy squark masses to become the dominant contribution.

If we consider the top/stop sector as an illustration, the importance of the squark contributions will become clear. The vertex \( A^0 \tilde{t} \tilde{t} \sim m_{\tilde{t}}/v \) and the dominant contribution to the vertex \( A^0 \tilde{t} \tilde{t} \sim m_{\tilde{t}}/v \times A_t \cot \beta \). Thus, the squark vertex is as large as the quark vertex times a large factor, in our case 1500 GeV for \( \tan \beta = 1 \). This factor of 10^5 is squared in a cross-section or width and gives an overall factor of 10^{10}. We have picked squarks with masses on the order of a TeV (about five times the mass of the top quark) which decreases the cross-section or width by an order of magnitude when compared to a squark with a mass lowered to that of the top quark. So naively we would consider the squark contributions to be larger than the quark contributions by a factor of 10^5. This is exactly what we have found for the \( e^+e^- \rightarrow Z^0 A^0 \) process and the \( A^0 \rightarrow Z^0 Z^0 \) width. There are some cancellations in the \( A^0 \rightarrow \gamma Z^0 \) width leading to a result that is on the same order as the contributions from the SM fields for some values of \( M_{A^0} \) and \( \tan \beta \).

We would like to emphasize that this is the case for the non-diagonal squark vertices in the entire neutral Higgs sector of the MSSM. The \( h^0 \tilde{u}_R\tilde{d}_L \) and \( H^0 \tilde{u}_R\tilde{d}_L \) should show similar enhancements with some additional dependence on the mixing angles \( (\alpha, \beta) \). The squark loop processes involving the production of these particles could very well be larger than the tree-level production processes and could lead to better bounds on squark masses or trilinear soft-supersymmetry breaking terms from the LEP2 data for the production of the other neutral Higgs bosons in the MSSM.

It should be noted that this is partially an arbitrary enhancement because we have chosen the soft-supersymmetry breaking parameters to be large. However, if the soft breaking terms exist in nature, they will enhance the squark-squark-Higgs vertex over that of the quark-quark-Higgs vertex proportionately to their size. With this information we are ready to construct the matrix elements for our decays \( A^0 \rightarrow \gamma Z^0 \) and \( A^0 \rightarrow Z^0 Z^0 \) and our process \( e^+e^- \rightarrow Z^0 A^0 \).
IV. MATRIX ELEMENTS AND RESULTS

To understand the $e^+e^- \rightarrow Z^0A^0$ process, we calculated the three-point functions $\gamma Z^0A^0$ and $Z^0Z^0A^0$ with both top and bottom quarks and stop and sbottom squarks. A generic representation of the three-point functions can be seen in Fig. 1. The contributions from the top and bottom quarks were checked against known results. We also checked our calculation with the known partial width for $\Gamma(A^0 \rightarrow \gamma Z^0)$ as given by HDECAY. We found excellent agreement. We also found that due to the tensor structure of the quark and squark contributions, the two processes do not interfere with each other. The exact form of the contributions is worked out in the appendix and it is shown explicitly that the two contributions do not mix.

The squark contributions to $\Gamma(A^0 \rightarrow \gamma Z^0)$ become less and less important as $M_{A^0}$ becomes large. This can be easily seen in our parameterization of the matrix elements squared below

$$|\mathcal{M}_{\text{SUSY}}^{\gamma Z^0}|^2 = \frac{\alpha^2 N_c^2}{16 \pi^2} \left\{ \frac{1}{(16 \pi)^2} \left\{ 3 \sum_q A_q^q \right\}^2 - \frac{(M_{A^0}^2 - M_Z^2)^2}{4} \left\{ \sum_q E_q^q \right\}^2 \right\}.$$

where the superscript susy here implies that this is the contribution from the squark fields (not the SM fields) and $N_c = 3$ is the number of colors.

The functional form of the $A_q^q, E_q^q$ functions can be found in the appendix. However, it is simple to see in Eqn. 5 that the second term makes the amplitude smaller and smaller as $M_{A^0} \gg M_Z$. The functional form of the $A_q^q, E_q^q$ functions guarantee the matrix elements are positive definite. What cannot be immediately seen is that the two pieces of the squark contribution almost cancel. Even though the pseudoscalar squark vertices are greatly enhanced compared to their standard model counterparts due to the trilinear term, the supersymmetric contribution to the width is only slightly larger than the SM contribution in some of the parameter space.

We also needed to calculate the three-point function for $Z^0Z^0A^0$ for the $e^+e^- \rightarrow Z^0A^0$ process. Although HDECAY does not have this final state for the pseudoscalar Higgs for comparison, we did calculate the branching ratio $\Gamma(A^0 \rightarrow Z^0Z^0)$ and added it to our analysis. The matrix elements squared, listed below, have the opposite effect from the $\gamma Z^0$ channel as they become more important as the mass of the pseudoscalar grows. The matrix elements grow like positive powers of $(M_{A^0} - 2M_Z)^n$. This can be seen in the positive powers of this mass difference in the first and third terms of the matrix elements,

$$|\mathcal{M}_{\text{SUSY}}^{Z^0 Z^0}|^2 = \frac{\alpha^2 N_c^2}{32 \pi^2} \frac{1}{(16 \pi)^2} \left\{ \left( 2 + \frac{(M_{A^0}^2 - 2M_Z^2)^2}{4M_Z^2} \right) \sum_q A_q^q \right\}^2$$

$$+ \left( M_Z^4 - \frac{1}{2}(M_{A^0}^2 - 2M_Z^2)^2 \right) \left\{ \sum_q E_q^q \right\}^2$$

$$+ \left( \frac{(M_{A^0}^2 - 2M_Z^2)^3}{8M_Z^2} - \frac{M_{A^0}^4 - 2M_Z^4}{2} \right) 2 \text{Re} \left( \sum_q A_q^q (E_q^q)^* \right).$$

The results of these partial widths can be seen in Fig. 2. These graphs were created using the output of HDECAY and adjusted to include the new contributions from the squark loops in the $A^0 \rightarrow \gamma Z^0$ channel as well as the entirely new $A^0 \rightarrow Z^0Z^0$ channel.

FIG. 1: The three-point functions needed for the $e^+e^- \rightarrow Z^0A^0$ calculation. Top and bottom quarks as well as stop and sbottom squarks were allowed to run inside the blob. The momentum assignment was chosen to make the $e^+e^- \rightarrow Z^0A^0$ calculation simpler.
FIG. 2: Improved branching ratios for the pseudoscalar Higgs boson (including the $Z^0 Z^0$ channel). The top and bottom quark loops as well as stop and sbottom squark loops have been included in the $A^0 \rightarrow \gamma Z^0$ and $A^0 \rightarrow Z^0 Z^0$ channels. The other branching ratios have been taken from hdecay and have been adjusted to allow for the new channel. We see that the $b\bar{b}$ channel dominates at large $\tan\beta$, but the $Z^0 Z^0$ channel also plays an important role. The dip in the $Z^0 Z^0$ channel is not a kinematic one, nor is it due to the opening of a new decay channel in this case (the $W^+ W^-$ channel has not been included). It is due to a value of the pseudoscalar Higgs mass that allows for large cancellations to occur.

FIG. 3: Diagrams contributing to the $e^+ e^- \rightarrow Z^0 A^0$ process. The graphs with the top and bottom quarks in the loop and the box graph are referred to as the standard model contributions in the text. The squark loop graphs are the dominant contribution to the processes.
Using our expressions for the two three-point functions with squark loops, the squark contributions to the matrix elements for the $e^+e^- \to Z^0 A^0$ process can be written

$$|\mathcal{M}|^2 (e^+e^- \to Z^0 A^0) = 32\alpha^3\pi^3 N_c^2 \frac{(tu - M_Z^2 M_A^2)}{M_Z^2} \left\{ \left( 1 + \frac{2sM_Z^2}{(tu - M_Z^2 M_A^2)} \right) \left| x_1 \right|^2 + \left| x_3 \right|^2 \right\} + \left( \frac{(t + u)^2}{4} - M_Z^2 M_A^2 \right) \left| x_2 \right|^2 + \left| x_4 \right|^2 + \left( \frac{(t + u)}{2} - M_Z^2 \right) 2 \text{Re} \left( x_1 x_2^* + x_3 x_4^* \right),$$

(8)

where,

$$x_1 = \sum_q \frac{A_q^u}{s} + \frac{1}{4c_w s} \frac{1 - 4s^2}{s - M_Z^2} A_q^Z,$$

(9)

$$x_2 = \sum_q \frac{E_q^u}{s} + \frac{1}{4c_w s} \frac{1 - 4s^2}{s - M_Z^2} E_q^Z,$$

(10)

$$x_3 = -\sum_q \frac{1}{4c_w s} \frac{1}{s - M_Z^2} A_q^Z,$$

(11)

$$x_4 = -\sum_q \frac{1}{4c_w s} \frac{1}{s - M_Z^2} E_q^Z.$$

(12)

The diagrams contributing to the $e^+e^- \to Z^0 A^0$ process are shown in Fig. 4. The process was written as $e^+(p_1)e^-(p_2) \to Z^0(-p_3)A^0(-p_5)$ and we have employed the usual kinematic variables $s = (p_1 + p_2)^2$, $t = (p_2 + p_3)^2$, and $u = (p_3 + p_1)^2$. We took the electrons to be massless and set $p_1^2 = p_2^2 = 0$, $p_3^2 = M_Z^2$, and $p_5^2 = M_A^2$.

The differential cross-sections for a future ILC are shown in Fig. 4. The squark contributions can be seen for the two different center-of-mass energies (500 GeV and 1 TeV). The quark contributions are not shown because they are several orders of magnitude smaller than the squark contributions which are on the order of 0.2 fb for $M_A^2 = 120$ GeV and $\tan \beta = 1$. The contributions from the SM fields and the MSSM fields would not have mixed in any of the processes in this paper due to their tensor structure, as described in the appendix. The differential cross-section is symmetric with respect to the scattering angle $\cos \theta$ and with the interchange of the kinematic variables $t \leftrightarrow u$.

V. CONCLUSIONS

In conclusion, we have found that the dominant contributions to the $e^+e^- \to Z^0 A^0$ process does not come from top and bottom quark loops. The extreme enhancement of the pseudoscalar Higgs-squark-squark vertices due to the trilinear soft-supersymmetry breaking terms make the squark loops the dominant contribution to this process. This result is generically true in the entire neutral Higgs sector of the MSSM. Based on these results, squark loops appear to play a very large role in Higgs phenomenology in the neutral sector.

Acknowledgments

The author is supported in part by the National Science Foundation grant PHY-0098527 and under DOE Contract No. DE-AC02-98CH10886. The author would like to thank S. Dawson and J. Smith for all their help and comments as well as A. Field-Pollatou and N. Christensen for many other helpful comments and suggestions.

APPENDIX A: MATRIX ELEMENTS

The matrix elements for the process $e^+e^- \to Z^0 A^0$ has implicit in it two one-loop three-point functions $\gamma Z^0 A^0$ and $Z^0 Z^0 A^0$. We constructed these two three-point functions with quark and squark contributions for the collider process and to understand the squark contributions to our decay widths. There are also four box-type quark diagrams that have not been discussed so far and are not related to the three-point functions analyzed in this paper. These were
FIG. 4: Differential cross-sections for different parameters and center-of-mass energies at a future international linear collider for the process $e^+e^- \rightarrow Z^0 A^0$. All are for the case of maximal mixing in each of the squark sectors ($\sin(2\theta_{t,b}) = 1$) which leads to the largest contributions. Only the contributions from the squark loops are shown as they are several orders of magnitude greater than the contributions from the SM fields which are on the order of 0.2 fb for $M_{A^0} = 120$ GeV and $\tan \beta = 1$. Increasing the mass of the pseudoscalar drops the differential cross-section by about 30% and increasing the center-of-mass energy to 1 TeV decreases the differential cross-section by about 75% for $\tan \beta = 1$.

Included in our analysis but they are not part of the squark sectors for which explicit results will be presented and are negligible in size to the SM contributions.

The matrix elements for the rare pseudoscalar decays $A^0 \rightarrow \gamma Z^0$ and $A^0 \rightarrow Z^0 Z^0$ can be written as the sum of two parts. These are one loop decays that can have standard model fields and supersymmetric fields in the loops. The standard model and supersymmetric contributions do not mix due to their tensor structure. The supersymmetric diagrams can be broken into two form factors based on gauge invariance, for the $A^0(p_5) \rightarrow \gamma(-Q^\mu)Z^0(-p_5)$ decay we can write the one-loop $\gamma Z^0 A^0$ vertex with squark loops as

$$i \Gamma^{\mu\nu}_{\gamma,\text{susy}} = \eta^{\mu\nu} A^0_{\gamma} + Q^\nu p_5^\mu E^q. \tag{A1}$$

The $Z^0 Z^0 A^0$ one-loop vertex will take the same form, but will have different coefficients ($A^0_Z$ and $E^3_Z$). There is no term that is proportional to the $\epsilon$ tensor in the squark loops because the pseudoscalar does not couple to squarks with a $\gamma^5$. However, this is the case when there are standard model fields in the loop. Therefore, the same $\gamma Z^0 A^0$ vertex with quark loops can be written as

$$i \Gamma^{\mu\nu}_{\gamma,\text{SM}} = \epsilon^{\mu\nu\alpha\beta} p_3^\alpha Q_{\beta}. \tag{A2}$$

When we try to interfere these two vertices it is now easy to see that we find zero due to a repeated index in the $\epsilon$ tensor in the first term and a term anti-symmetric in the indices multiplied by one symmetric in the indices in the second term. Therefore, the decay widths will be additive for these processes. This is also true for the differential cross-section. Thus we can write

$$\Gamma_{\text{tot}} = \Gamma_{\text{SM}} + \Gamma_{\text{susy}}, \tag{A3}$$

$$d\sigma_{\text{tot}} = d\sigma_{\text{SM}} + d\sigma_{\text{susy}}. \tag{A4}$$
This leads to some interesting phenomenology. It needs to be noted that the pseudoscalar Higgs boson does not appear in the Standard Model, thus the standard model contributions listed here are the contributions from the standard model fields, in this case, the top and bottom quarks. Thus deviations from the SM contributions do not tell us about the existence of supersymmetry in nature, but they do tell us about the mixing in the squark sector directly.

When we add all the squark loop diagrams we can determine the unknown coefficients in our vertices. Thus, we can write

$$A'_q = \frac{4\epsilon^2 \tilde{A}_q s_q c_q}{s_w c_w} \left\{ s_q^2 T_\frac{3}{q} [C_{24}(112) + C_{24}(221) - 2C_{24}(111)] - 2Q_q [B_0(21) - B_0(22)] \right\} - \left\{ s_q \leftrightarrow c_q; 1 \leftrightarrow 2 \right\},$$

(A5)

$$E_x^Z = A_q^q (B_0 \rightarrow 0; C_{24} \rightarrow C_{12} + C_{23}),$$

(A6)

for the $\gamma Z^0 A^0$ vertex and

$$A_Z^Z = \frac{-4\epsilon^2 \tilde{A}_Z s_Z c_Z}{s_w^2 c_w^2} \left\{ c_q^2 (T_\frac{3}{q})^2 [C_{24}(121) + C_{24}(211) - 2C_{24}(111)] - c_q^2 T_\frac{3}{q} Q_q s_w^2 [C_{24}(122) + C_{24}(121) + C_{24}(211) + C_{24}(221) - 4C_{24}(111)] + 4\epsilon^2 T_\frac{3}{q} [B_0(12) - B_0(22)] - 2Q_q s_w^2 [B_0(22) - B_0(11)] + Q_q s_w^2 C_{24}(111) - s_q^2 c_q (T_\frac{3}{q})^2 [C_{24}(122) + C_{24}(121) + C_{24}(112) + C_{24}(221)] \right\} - \left\{ s_q \leftrightarrow c_q; 1 \leftrightarrow 2 \right\},$$

(A7)

$$E_Z^Z = A^q_Z (B_0 \rightarrow 0; C_{24} \rightarrow C_{12} + C_{23}),$$

(A8)

for the $Z^0 Z^0 A^0$ vertex. The $C_{ij}$ functions are the usual functions that appear in the Passarino-Veltman reduction prescription $[31]$. It is easy to see that these expressions are finite. The $B_0$ functions appear in pairs with opposite signs, canceling the $1/\epsilon$ poles which are independent of their arguments. The $C_{24}$ functions also have argument independent $1/\epsilon$ poles that all cancel in the expressions. The same is true of the $C_{12}$ and $C_{13}$ functions. These functions can be written out fully as

$$C_{ij}(123) \equiv C_{ij}(M_Z^2, Q^2, M_{\tilde{q}^2}, m_{\tilde{q}_1}^2, m_{\tilde{q}_2}^2, m_{\tilde{q}_3}^2),$$

(A9)

$$B_0(12) \equiv B_0(M_Z^2, m_{\tilde{q}_1}^2, m_{\tilde{q}_2}^2),$$

(A10)

where $Q^2 = \{0, M_Z^2, s\}$ for the $A^0 \rightarrow \gamma Z^0, A^0 \rightarrow Z^0 Z^0$, and $e^+ e^- \rightarrow Z^0 A^0$ processes respectively. This completes the missing squark contributions to the decay widths and the differential cross-section for the $e^+ e^- \rightarrow Z^0 A^0$ process.

[1] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, “The Higgs Hunter’s Guide”, (Addison-Wesley, Reading, MA, 1990), Erratum ibid. [arXiv:hep-ph/09302272].
[2] M. Carena and H. E. Haber, Prog. Part. Nucl. Phys. 50, 63 (2003) [arXiv:hep-ph/0208209].
[3] S. Heinemeyer, W. Hollik and G. Weiglein, [arXiv:hep-ph/0412214].
[4] H. E. Haber and G. L. Kane, Phys. Rept. 117, 75 (1985).
[5] R. V. Harlander and W. B. Kilgore, JHEP 0210, 017 (2002) [arXiv:hep-ph/0208096].
[6] B. Field, Phys. Rev. D 66 (2002) 114007 [arXiv:hep-ph/0208262].
[7] B. Field, J. Smith, M. E. Tjejed-Yeomans and W. L. van Neerven, Phys. Lett. B 551, 137 (2003) [arXiv:hep-ph/0210369].
[8] V. Ravindran, J. Smith and W. L. Van Neerven, Nucl. Phys. B 634, 247 (2002) [arXiv:hep-ph/0201114].
[9] B. Field, S. Dawson and J. Smith, Phys. Rev. D 69, 074013 (2004) [arXiv:hep-ph/0311199].
[10] B. Field, Phys. Rev. D 70, 054008 (2004) [arXiv:hep-ph/0405219].
[11] B. Field, [arXiv:hep-ph/0407254].
[12] C. Kao, Phys. Rev. D 36, 4907 (1992).
[13] J. Yin, W. G. Ma, R. Y. Zhang and H. S. Hou, Phys. Rev. D 66, 095008 (2002).
[14] C. Kao, G. Lovelace and L. H. Orr, Phys. Lett. B 567, 259 (2003) [arXiv:hep-ph/0305028].
[15] C. Kao and S. Sachithanandam, [arXiv:hep-ph/0411331].
[16] Q. Li, C. S. Li, J. J. Liu, L. G. Jin and C. P. Yuan, arXiv:hep-ph/0501070.
[17] V. D. Barger, K. M. Cheung, A. Djouadi, B. A. Kniehl and P. M. Zerwas, Phys. Rev. D 49, 79 (1994) arXiv:hep-ph/9306270.
[18] A. G. Akeroyd, A. Arhrib and M. Capdequi Peyranere, Mod. Phys. Lett. A 14, 2093 (1999) [Erratum-ibid. A 17, 373 (2002)] arXiv:hep-ph/9907542.
[19] A. G. Akeroyd, A. Arhrib and M. Capdequi Peyranere, Phys. Rev. D 64, 075007 (2001) [Erratum-ibid. D 65, 099903 (2002)] arXiv:hep-ph/0104243.
[20] T. Farris, J. F. Gunion and H. E. Logan, in Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001) ed. N. Graf, eConf C010630, P121 (2001) arXiv:hep-ph/0202087.
[21] A. Arhrib, Phys. Rev. D 67, 015003 (2003) arXiv:hep-ph/0207330.
[22] A. Bartl, H. Eberl, K. Hidaka, T. Kon, W. Majerotto and Y. Yamada, Phys. Lett. B 402, 303 (1997) arXiv:hep-ph/9701398.
[23] H. Eberl, K. Hidaka, S. Kraml, W. Majerotto and Y. Yamada, Phys. Rev. D 62, 055006 (2000) arXiv:hep-ph/9912463.
[24] E. Accomando et al. [ECFA/DESY LC Physics Working Group], Phys. Rept. 299, 1 (1998) arXiv:hep-ph/9705442.
[25] S. Dawson and M. Oreglia, arXiv:hep-ph/0403015.
[26] S. Eidelman et al. [Particle Data Group], Phys. Lett. B 592, 1 (2004).
[27] M. Carena, D. Garcia, U. Nierste and C. E. M. Wagner, Nucl. Phys. B 577, 88 (2000) arXiv:hep-ph/9912510.
[28] J. F. Gunion, H. E. Haber and C. Kao, Phys. Rev. D 46, 2907 (1992).
[29] A. Djouadi, J. Kalinowski and M. Spira, Comput. Phys. Commun. 108, 56 (1998) arXiv:hep-ph/9704448.
[30] G. Passarino and M. J. G. Veltman, Nucl. Phys. B 160, 151 (1979).