**1. INTRODUCTION**

Many of the features of quantum chromodynamics long expected to emerge in the nonperturbative regime of the theory have been realised in a range of phenomenological models as well as demonstrated quantitatively in lattice gauge theory. Confinement of quarks, spontaneously broken flavour chiral symmetry with a nonzero quark condensate and resolution of $U_A(1)$ problem are believed to be intrinsically connected to each other. Models of the QCD vacuum, which can give insight into what is going right with lattice simulations of QCD, can invariably connect two of these phenomena, but rarely all three simultaneously. In this paper, we pursue one step further the exploration of the "domain model" for the vacuum, originally proposed in [1], as a scenario for simultaneous appearance of all three phenomena: confinement, spontaneous chiral symmetry breaking via the appearance of a quark condensate and a continuous $SU(N_f)_L \times SU(N_f)_R$ degeneracy of the vacuum for $N_f$ massless quarks, but without a $U_A(1)$ continuous degeneracy of ground states that would be indicative of an unwanted Goldstone boson.

The model under discussion is defined by a partition function describing an ensemble of hyperspherical domains. Each domain is characterised by a background covariantly constant self-dual or anti-self-dual gluon field of random orientation. Summing over all orientations and dualities guarantees a Lorentz and CP invariant ensemble. Confinement of quarks is manifested in the model, as demonstrated in the original work [1]. On the boundaries of each hypersphere, fermion fluctuations satisfy a chirality violating boundary condition

$$i \eta(\mu)e^{i\alpha\gamma_5}\psi(x) = \psi(x)$$  \hspace{1cm} (1)

which is $2\pi$ periodic in the chiral angle $\alpha$. Here $\eta_\mu$ is a unit radial vector at the boundary. Integrating over all such chiral angles guarantees chiral invariance of the ensemble. As a consequence of Eq. (1) the spectrum of eigenvalues $\lambda$ of the Dirac
operator in a single domain is asymmetric under \( \lambda \to -\lambda \). Such asymmetries have been studied in other contexts, for example by [2]. In the case of the domain model, the above boundary conditions are combined with the (anti-)self-dual gluon field which leads to a strong correlation between the local chirality of quark modes at the centres of domains with the duality of the background gluon field [3]. In this paper we study how these aspects contribute to quark condensate formation and the pattern of chiral symmetry breaking in the vacuum.

The vacua of the quantum problem associated with an ensemble of domains are the minima of the free energy determined from the partition function. The problem of the quark contributions to the free energy requires calculation of the determinant of the Dirac operator in the presence of chirality violating boundary conditions. For slightly different choice of boundary condition (with \( \alpha \to -i\theta - \pi/2 \)) this problem has been addressed in [4] where the parity odd part of the logarithm of the determinant was identified as \( \ln \det(iD) \sim 2q\theta \) with \( q \) the topological charge (not necessarily integer) of the underlying gluon field. This is basically the chiral anomaly. For the specific gluon field relevant to the domain model we have obtained a similar result for the parity odd part [5, 6]

\[
\ln \det(iD) \sim 2iq(\alpha \mod \pi),
\](2)

consistent with [4]. We study the consequences of this result in the context of the domain model.

A nonzero quark condensate is generated in the model without there being a continuous chiral \( U_A(1) \) degeneracy of minima of the ensemble free energy. There are two factors which interplay to achieve this result. Firstly, when self-dual and anti-self-dual configurations are summed, the anomaly Eq. [2] leads to a contribution to the free energy of the form \( -\ln \cos(2q\arctan(\tan \alpha)) \) which vanishes when \( \alpha = 0, \pi \). The vacuum is degenerate with respect to \( Z_2 \) chiral transformations. The second factor is the spectral asymmetry itself which in the presence of an infinitesimally small quark mass leads to an addition to the free energy linear in the mass, which removes degeneracy between two discrete minima, and thus generates the nonzero quark condensate. This gives a model with the chiral \( Z_2 \) discrete subgroup of \( U_A(1) \) being spontaneously broken, and not the continuous \( U_A(1) \) itself. In the absence of the mass term the ensemble average of \( \bar{\psi}\psi \) correctly vanishes.

Moreover, the form of Eq. [2] means that the free energy does not depend on flavour nonsinglet chiral angles when more than one massless quark flavours are introduced. This allows for the correct degeneracy of vacua with respect to continuous \( SU(N_f)_L \times SU(N_f)_R \) chiral transformations. This vacuum structure implies the existence of Goldstone bosons in the flavour nonsinglet pseudoscalar channel but not in the singlet channel, which a study of the structure of pseudoscalar correlation functions unveils.

2. THE DOMAIN MODEL

For motivation and a detailed description of the model we refer the reader to [1]. The essential definition of the model is given in terms of the following partition function for \( N \to \infty \) domains of radius \( R \)

\[
\mathcal{Z} = N \lim_{V,N \to \infty} \prod_{i=1}^{N} \int d\sigma_i \int_{\mathcal{F}^i} D\psi^{(i)} D\bar{\psi}^{(i)}
\times \int_{\mathcal{F}_Q} DQ^{i} [D(\bar{B}^{(i)})Q^{(i)}] \Delta_{FP}[\bar{B}^{(i)}, Q^{(i)}]
\times e^{-S_{\text{QCD}}[Q^{(i)} + B^{(i)}, \psi^{(i)}, \bar{\psi}^{(i)}]} \tag{3}
\]

where the functional spaces of integration \( \mathcal{F}_Q \) and \( \mathcal{F}^i \) are specified by the boundary conditions \( (x - z_i)^2 = R^2 \)

\[
\tilde{n}_i Q^{(i)}(x) = 0, \tag{4}
\]

\[
i \bar{\psi}^{(i)}(x) e^{i\alpha_i \gamma_5} \psi^{(i)}(x) = \psi^{(i)}(x), \tag{5}
\]

\[
\bar{\psi}^{(i)}(x) e^{i\alpha_i \gamma_5} i \gamma_5 \psi^{(i)}(x) = -\bar{\psi}^{(i)}(x). \tag{6}
\]

Here \( \tilde{n}_i = n^a t^a \) with the generators \( t^a \) of \( SU_c(3) \) in the adjoint representation and the \( \alpha_i \) are chiral angles associated with for the boundary condition Eq. [3] with different values randomly assigned to domains. We shall discuss this constraint in detail in later sections. The thermodynamic limit assumes \( V, N \to \infty \) but with the
density $v^{-1} = N/V$ taken fixed and finite. The partition function is formulated in a background field gauge with respect to the domain mean field, which is approximated inside and on the boundaries of the domains by a covariantly constant (anti-)self-dual gluon field with the field-strength tensor of the form

$$ F_{\mu\nu}^a(x) = \sum_{j=1}^{N} a^{(j)a} B_{\mu\nu}^{(j)} \theta(1 - (x - z_j)^2/R^2), $$

$$ B_{\mu\nu}^{(j)} B^{(j)\mu\nu} = B^2 \delta_{\nu\rho}. $$

Here $z_j$ are the positions of the centres of domains in Euclidean space.

The measure of integration over parameters characterising domains is

$$ \int d\sigma_1 \ldots = \frac{1}{48\pi^2} \int_V \frac{d^4 z_j}{V} \int_0^{2\pi} d\alpha_i \times \int_0^{2\pi} d\varphi_i \int_0^\pi d\sin \theta_i \int_0^{2\pi} d\xi_i \times \sum_{l=0,1,2} \delta(\xi_l - (2l + 1)\pi/6) \times \int_0^{2\pi} d\omega_i \sum_{k=0,1} \delta(\omega_i - \pi k) \ldots, $$

where $(\theta_i, \varphi_i)$ are the spherical angles of the chromomagnetic field, $\omega_i$ is the angle between chromoelectric and chromomagnetic fields and $\xi_i$ is an angle parametrising the colour orientation.

This partition function describes a statistical system of these domain-like structures, of density $v^{-1}$ where the volume of a domain is $v = \pi^2 R^4/2$, and where each domain is characterised by a set of internal parameters and whose internal dynamics are represented by fluctuation fields. It respects all the symmetries of the QCD Lagrangian, since the statistical ensemble is invariant under spacetime and colour gauge symmetries. For the same reason, if the quarks are massless then the chiral invariance is respected.

Thus the model involves only two free parameters: the mean field strength $B$ and the mean domain radius $R$. Within this framework the gluon condensate to lowest order in fluctuations is $4B^2$ and the topological charge per domain is $q = B^2 R^4/16$. More significantly, an area law is obtained for static quarks. Computation of the Wilson loop for a circular contour of a large radius $L \gg R$ gives a string tension $\sigma = B f(\pi BR^2)$ where $f$ is given for colour $SU(2)$ and $SU(3)$ in [1] and has a purely geometrical meaning in terms of overlaps of hyperspheres. Estimations of the values of these quantities are known from lattice calculation or phenomenological approaches and can be used to fit $B$ and $R$. As described in [1] these parameters are fixed to be $\sqrt{\eta} = 947\text{MeV}, R = (760\text{MeV})^{-1} = 0.26\text{fm}$ with the average absolute value of topological charge per domain turning out to be $q \approx 0.15$ and the density of domains $v^{-1} = 42\text{fm}^{-4}$. The topological susceptibility then turns out to be $\chi \approx (197\text{MeV})^4$, comparable to the Witten-Veneziano value [2]. This fixing of the parameters of the model remains unchanged in this investigation of the quark sector. The quark condensate at the origin of a domain where angular dependence drops out was estimated in paper [1] with a result $-(228\text{MeV})^3$.

3. DIRAC OPERATOR AND SPECTRUM

The eigenvalue problem

$$ D\psi(x) = \lambda \psi(x), $$

$$ i \gamma(\alpha) e^{i x_0 \sigma_0} \psi(x) = \psi(x), \quad x^2 = R^2 $$

was studied in [3]. Dirac matrices are in anti-hermitean representation. For $\alpha$ assumed to be real a bi-orthogonal basis has to be constructed. Solutions can be labelled via the Casimirs and eigenvalues

$$ K_1^2 = K_2^2 \rightarrow \frac{k}{2} \left( \frac{k}{2} + 1 \right), \quad k = 0, 1, \ldots, \infty $$

$$ K_{1,2}^3 \rightarrow m_{1,2}, $$

$$ m_{1,2} = -k/2, -k/2 + 1, \ldots, k/2 - 1, k/2, $$

corresponding to the angular momentum operators

$$ K_{1,2} = \frac{1}{2}(L \pm M) $$

with $L$ the usual three-dimensional angular momentum operator and $M$ the Euclidean version.
of the boost operator. The solutions for the self-dual background field are then

$$\psi_{km}^{-\kappa} = i \not\chi_{km}^{-\kappa} + \varphi_{km}^{-\kappa},$$

(10)

where $\chi$ and $\varphi$ must both have negative chirality in the self-dual field and $\kappa$ related to the polarisation of the field defined via the projector

$$O_\kappa = N_+ \Sigma_\kappa + N_- \Sigma_{-\kappa}$$

(11)

with

$$N_- = \frac{1}{2}(1 \pm \hat{n}/|\hat{n}|), \quad \Sigma_\pm = \frac{1}{2}(1 \pm \Sigma B/B)$$

being respectively separate projectors for colour and spin polarizations. Significantly, the negative chirality for $\chi$ and $\varphi$ is the only choice for which the boundary condition Eq. (3) can be implemented for the self-dual background. The explicit form of the spinors $\chi$ and $\varphi$ can be found in [3], where it is demonstrated that the eigenspinor Eq. (10) has definite chirality at the centre of domain correlated with the duality of the gluon field. The boundary condition reduces to

$$\chi = -e^{\mp i\alpha}\varphi, \quad \bar{\chi} = \bar{\varphi}e^{\mp i\alpha}, \quad x^2 = R^2,$$

(12)

where upper (lower) signs correspond to $\varphi$ and $\chi$ with chirality $\mp$, which, using the solutions, amounts to equations for the two possible polarisations, for $\Lambda_k^\mp$:

$$e^{-i\alpha}M \left( k + 2 - \Lambda^2, k + 2, z_0 \right) - \sqrt{z_0} \frac{i\Lambda}{i\Lambda}$$

$$\times \left[ M \left( k + 2 - \Lambda^2, k + 2, z_0 \right) - \frac{k + 2 - \Lambda^2}{k + 2}M \left( k + 3 - \Lambda^2, k + 3, z_0 \right) \right] = 0,$$

(13)

and for $\Lambda_k^-$:

$$e^{-i\alpha}M \left( -\Lambda^2, k + 2, z_0 \right)$$

$$+ \frac{k + 2 - \Lambda^2}{k + 2}M \left( 1 - \Lambda^2, k + 3, z_0 \right) = 0,$$

(14)

where $z_0 = B R^2/2$ and $\Lambda = \lambda/\sqrt{2B}$. For the present work Eqs. (13,14) are the starting point, from which we see by inspection that a discrete spectrum of complex eigenvalues emerges for which there is no symmetry of the form $\lambda \rightarrow -\lambda$. For given chirality and polarisation and angular momentum $k$, an infinite set of discrete $\Delta$ are obtained labelled by a “principal quantum number” $n$.

4. QUARK DETERMINANT AND FREE ENERGY FOR A SINGLE DOMAIN

We consider the one-loop contribution of the quarks to the free energy density $F(B, R|\alpha)$ of a single (anti-)self-dual domain of volume $v = \pi^2 R^4/2$

$$\exp \{-vF(B, R|\alpha)\} = \det_{\alpha} \left( \frac{iD}{i\phi} \right)$$

$$= \prod_{\kappa, \kappa, m, n} \left( \frac{\lambda_{\kappa, n}^m(B)}{\lambda_{\kappa, n}^m(0)} \right)$$

$$= \exp \{-\zeta(s)|_{s=0} \}.$$ (15)

The normalization is chosen such that $\lim_{B \rightarrow 0} F(B, R|\alpha) = 0$. The free energy is then $F = v^{-1}\zeta'(0)$. In [5,6], we discuss in more detail the technical aspects of the computation of this quantity in zeta function regularisation, starting with a purely imaginary choice for $\alpha$ which provides for a real spectrum, and analytically continuing the final result to real values of $\alpha$. It is known that $\zeta(s)$ has two contributions: from $\varphi_2(s)$, which is insensitive to the spectral asymmetry, and from the spectral function $\eta(s)$ which is odd under $\lambda \rightarrow -\lambda$. These two functions are defined by

$$\varphi_2(s) = \sum_{k, n, \kappa} \left( k + 1 \right) \left( \frac{\mu^{2s}}{|\lambda_{kn}^\kappa(B)|^{2s}} \right)$$

$$- \frac{\mu^{2s}}{|\lambda_{kn}^\kappa(0)|^{2s}},$$

(16)

$$\eta(s) = \mu^s \sum_{k, n, \kappa} \left( k + 1 \right) \left( \frac{\sgn(|\lambda_{kn}^\kappa(B)|)}{|\lambda_{kn}^\kappa(B)|^s} \right)$$

$$- \frac{\sgn(|\lambda_{kn}^\kappa(0)|)}{|\lambda_{kn}^\kappa(0)|^s}.$$ (17)

In terms of these, the free energy density for a given parameter $\alpha$ is:

$$F = v^{-1} \left( \frac{1}{2} \varphi_2(0) \pm \frac{i}{2} \varphi_2(0) \mp \frac{i}{2} \eta(0) \right),$$

(18)
with \( \mu \) – arbitrary scale. The final result for \( F \) is complex with the imaginary part of the form
\[
\Im F = \pm 2q \arctan(\tan(\alpha)) \tag{19}
\]
where \( q \) is the absolute value of topological charge in a domain. This charge is not integer here in general but the anomalous term is \( \pi n \) periodic in \( \alpha \). This is the Abelian anomaly as observed within the context of bag-like boundary conditions by [4]. Its appearance here is in the spirit of the derivation by Fujikawa [8], where the phase appears as an extra contribution under a chiral transformation on the fermionic measure of integration. However, in our calculations [5,6], an \( \alpha \) dependent real part also appears. To explain its possible fate some technical details of the calculation are necessary. The sum over modes \( n \) is converted into a contour integral via the relation
\[
\sum_{\lambda} \frac{1}{\lambda^s} = \frac{1}{2\pi i} \oint_{C} \frac{d\xi}{\xi^s} \ln f(\xi) \tag{20}
\]
where the zeroes of \( f \) define the eigenvalues. Thus the left hand side of the constraint equations Eqs. (13,14) appear in these expressions. One then performs a Debye-like expansion in \( 1/k \) of the Kummer functions. Exchanging this summation with the sum over \( k \) one can read off each order in \( 1/k \) via an ordinary Riemann function whose analytic behaviour is known. However in principle the order of summations does not commute - there can be extra terms, which are difficult for computation even in the simplest examples [9]. Taking into account such contributions is expected to remove the \( \alpha \) dependence from the real part, leaving only the anomaly contribution from the imaginary part, consistent with [4]. Thus for the purpose of the following investigation, we will assume that the anomaly Eq. (19) provides the entire result for the \( \alpha \)-dependent part of the free energy of massless fermions in a domain.

The chiral condensate is computed via the presence in the free energy of a term linear in an infinitesimal mass, namely [2]
\[
F = F_{m=0} + i \frac{m}{\mu v} \eta(1). \tag{21}
\]
Using the analogue of the representation Eq. (20) the summed contributions of both polarisations in the self-dual domain to the asymmetry function can be written,
\[
\eta(s) = i\mu Re^{i\alpha} \frac{\cos(\pi s/2)}{\pi(1-s)} \sum_{k=1}^{\infty} \frac{k^{1-s}}{k+1} \times \left[ 1 + M(1,k+2,z) \right. \\
- \frac{z}{k+2} M(1,k+3,-z) \left. \right] \tag{22}
\]
We next evaluate the asymptotic behaviour in \( k \). A singular term as \( s \to 1 \) can be extracted, which turns out to be field (that is, \( B \)) independent and is canceled by the normalization. The final expression for the term linear in mass in the free energy density for a self-dual domain is
\[
\delta F^{(sd)} = -e^{i\alpha} m \left\langle \bar{\psi} \psi \right\rangle,
\]
where we have used the suggestive notation
\[
\left\langle \bar{\psi} \psi \right\rangle = \frac{1}{\pi^2 R^3} \sum_{k=1}^{\infty} \frac{k}{k+1} \times \left[ M(1,k+2,z) \right. \\
- \frac{z}{k+2} M(1,k+3,-z) - 1 \left. \right] \tag{23}
\]
coming from \( \eta(1) \) with the sum over \( z \) corresponding to a color trace. The \( \alpha \) dependent part of the free energy of a self-dual domain for massive quarks is complex with the following real and imaginary parts:
\[
\Re F = \Re F + i\Im F \\
\Re F = -m \cos \alpha \left\langle \bar{\psi} \psi \right\rangle \tag{24} \\
\Im F = 2 \frac{q}{v} \arctan(\tan(\alpha)) \\
- m \sin \alpha \left\langle \bar{\psi} \psi \right\rangle \tag{25}
\]
The free energy of an anti-self-dual domain is obtained via complex conjugation.

5. GROUND STATE AND CONDENSATE

Under the assumption that only the anomalous term depends on the chiral angle, the part of the free energy density \( F \) relevant for the present consideration of an ensemble of \( N \to \infty \) domains
with both self-dual and anti-self-dual configurations takes the form
\[
e^{-vNF} = N \prod_j^{N} \int_0^{2\pi} d\alpha_j \times \frac{1}{2} \left[ e^{iv\Im F(\alpha_j)} + e^{-iv\Im F(\alpha_j)} \right]
\]
\[
= N \prod_j^{N} \int_0^{2\pi} d\alpha_j e^{\ln(\cos(v\Im F(\alpha_j)))}
\]
\[
= N \exp \left( N \max_\alpha \ln(\cos(v\Im F(\alpha))) \right).
\]

The maxima (minima of the free energy density) are achieved at \(\alpha_1 = \ldots = \alpha_N = \pi n\).

In the absence of a quark mass, only the anomaly contribution in the imaginary part, \(\Im F\), of the free energy of a single domain appears under the logarithm of the cosine and defines the minima of the free energy density, \(\ln(\cos(v\Im F(\alpha))) = 0\). Thus for massless quarks there is no continuous \(U_A(1)\) symmetry in the ground states rather a discrete \(Z_2\) chiral symmetry. The anomaly plays a peculiar role here: selecting out those chiral angles which minimise the free energy so that the full \(U_A(1)\) group is no longer reflected in the vacuum degeneracy. It should be stressed here that this residual discrete degeneracy is sufficient to ensure zero value for the quark condensate in the absence of mass term or some other external chirality violating sources.

In a different and more general context the idea of dynamical relaxation of the effective \(\theta\)-parameter based on an energy minimization criterion was discussed in great detail in [10].

Now switching on the quark mass, we see this discrete symmetry spontaneously broken, and one of the two vacua selected in the infinite volume limit according to the sign of the mass. In this case (for these conventions of boundary condition and mass term), it is the minimum at \(\alpha = 0\). The quark condensate can be now extracted from the free energy via
\[
\langle \bar{\psi}(x)\psi(x) \rangle = - \lim_{m \to 0} \lim_{N \to \infty} (vN)^{-1} \times \frac{d}{dm} e^{-vNF(m)}.
\]

Taking the thermodynamic limit \(N \to \infty\) and then \(m \to 0\) gives a nonzero condensate
\[
\langle \bar{\psi}(x)\psi(x) \rangle = - \langle \bar{\psi}\psi \rangle.
\]

According to Eq. (26) the condensate is equal to
\[
\langle \bar{\psi}(x)\psi(x) \rangle = -(237.8 \text{ MeV})^3
\]

for the values of field strength \(B\) and domain radius \(R\) fixed earlier by consideration of the pure gluonic characteristics of the vacuum – string tension, topological susceptibility and gluon condensate. A nonzero condensate is generated without a continuous degeneracy of the ground states of the system.

6. MULTIFLAVOUR CASE

The question remains though whether any continuous directions in the vacua to be expected when the full flavour chiral symmetry is brought into play. For this we must generalise the analysis. We consider \(N_f\) massless quark flavours. Firstly, we observe that the fermion boundary condition in Eq. (13) explicitly breaks all chiral symmetries, flavour singlet and non-singlet (see also [14]). Thus the procedure we have used here of integrating over all \(\alpha\) does not suffice to restore the full chiral symmetry of the massless QCD action. Rather, the boundary condition must be generalised to include flavour non-singlet angles,
\[
\alpha \to \alpha + \beta^a T^a,
\]
with \(T^a\) the \(N_f^2 - 1\) generators of \(SU(N_f)\). Then integration over \(1 + N_f^2 - 1 = N_f^2\) angles \(\alpha, \beta^a \in [0, 2\pi]\) must be performed for a fully chiral symmetric ensemble. The spectrum of the Dirac problem now proceeds quite analogously, except that the boundary condition mixes flavour components, thus an additional projection into flavour sectors is required in order to extract the eigenvalue equation analogous to Eq. (12).

For \(N_f = 2\) the projection is easy to write down, \((1 \pm \beta \cdot \vec{\sigma})/2\), which will have the effect that \(\alpha\) in the dependence of eigenvalues and spectral functions is replaced by \(\alpha \pm |\beta|/2\) for the two \(N_f = 2\) isospin projections. Thus for a given domain, the imaginary part of the free energy will be the sum of two terms involving the imaginary
part determined for a single flavour but evaluated at the two shifted angles:

\[ 3F(\alpha + \theta) + 3F(\alpha - \theta) = 3F(\alpha) \]  \hspace{1cm} (27)

since \( 3F(\alpha) = 2q \arctan(\tan(\alpha)) = 2q(\alpha + n\pi) \).

This expresses the known fact that the anomaly only appears in the flavour singlet sector or is Abelian. Thus for an ensemble of domains the free energy is identical to that for one massless flavour, namely it depends only on the Abelian angle \( \alpha \). Thus for \( N_f = 2 \), the \( U_A(1) \) direction remains fixed by energy minimisation while the \( SU(2)_L \times SU(2)_R \) directions represent degeneracies in the space of ground states in the thermodynamic limit.

The generalisation to \( SU(N_f)_L \times SU(N_f)_R \) can be summarised as follows: the functions \( \pm |\tilde{\beta}| \) in Eq. (27) become functions \( B^i(\beta^a) \) which correspond to the number of diagonal generators of the flavour group. The cancellation in Eq. (27) generalises reflecting the tracelessness of the \( SU(N_f) \) generators, \( \sum_i B^i(\beta^a) = 0 \). Thus for \( N_f = 3 \) one expects eight not nine continuous directions in the space of vacua.

The consequence of this realisation of chiral symmetries in the meson spectrum can be seen in the general structure of correlators of the flavour nonsinglet \( J^a_P(x) \) and singlet \( J^\mu_P(x) \) pseudoscalar quark currents as they appear in the domain model,

\[
\langle J^a_P(x)J^b_P(y) \rangle = \frac{\langle \langle J^a_P(x)J^b_P(y) \rangle \rangle}{\langle \langle J^\mu_P(x)J^\mu_P(y) \rangle \rangle} - \frac{\langle \langle J^\mu_P(x)J^a_P(y) \rangle \rangle}{\langle \langle J^\mu_P(x) \rangle \rangle \langle \langle J^a_P(y) \rangle \rangle}.
\]

Here double brackets denote integration over quantum fluctuation fields and the overline means integration over all configurations in the domain ensemble. The second line in the right hand side of the flavour singlet correlator reflects the inhomogeneity of the background field for a particular domain configuration, translation invariance being restored only for the ensemble average. The second term is entirely determined by the correlation function of the background gluon field \( B \) in the ensemble [1]. The analogous “disconnected term” in the flavour nonsinglet correlator is equal to zero due to the trace over flavour indices. It should be added that the pseudoscalar condensate, \( \langle \bar{\psi}\gamma^5\psi \rangle \), naturally vanishes since parity is not broken in the ensemble of domains. Thus massless modes can be expected in the nonsinglet channel, but not in the flavour singlet due to the additional term in the correlator. This general structure of correlators is exactly the same as in the instanton liquid model [11] but manifests the mechanism for eta-prime mass generation proposed by Witten in [1] and appreciated in chiral perturbation theory by [12].

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