Amplitude analysis of resonant production in three pions

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Joint Physics Analysis Center (JPAC)

- The Joint Physics Analysis Center (JPAC) formed in October 2013
- We support physics analysis of experimental data for accelerator facilities (JLab12, COMPASS, . . . )
- http://www.indiana.edu/~jpac/
- JPAC Talks
  - Vladislav Pauk (Today 17:55 in Parallel B)
  - Adam Szczepaniak (Friday 9:00 Plenary)
  - Emilie Passemar (Friday 15:25 in Parallel A)
  - Alessandro Pilloni (Monday 17:15 in Parallel B)
  - Vincent Mathieu (Poster Session)
Introduction

$3\pi$ at COMPASS

- Study peripheral resonance production of $3\pi$ systems at COMPASS.
  - High statistics, high purity data allows for detailed analysis
  - JPAC affiliated with COMPASS to perform analysis on data
- Construct analytic amplitudes to extract resonance information
  - Amplitude satisfy S-matrix principles
  - Emphasize production process and unitarization of amplitude

[ C. Adolph et al. [COMPASS Collaboration], arXiv:1509.00992]
Production Mechanisms

- Peripheral production is advantageous - Effective $2 \rightarrow 2$, $2 \rightarrow 3$, etc. meson scattering
  - By effective we mean particle-reggeon scattering
- Production mechanisms dictate physics
  - Expect exchange mechanism dominated by pomeron at high-energies
  - Effective $2 \rightarrow 2$, $2 \rightarrow 3$, etc. meson scattering production by particle exchange

$$\pi_{\text{beam}} \rightarrow \pi^+ \pi^- \pi^- \Rightarrow \rho/f_2 \rightarrow \pi^+ \pi^-$$
PWA of $3\pi$ final state

- Develop method of analysis satisfying S-matrix principles, study $J^{PC}$ resonances in $3\pi$
- In this presentation, we focus on $2^{-+}$,
  - long standing puzzle about $\pi_2(1670)-\pi_2(1880)$ interplay,
  - 17 waves out of 88 have $J^{PC} = 2^{-+}$,

\[ S \rightarrow [C. Adolph et al. [COMPASS Collaboration], arXiv:1509.00992] \]
The Model

- Partial wave analysis of $3\pi$ system in $\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$
- Use isobar model, with first approximation of stable isobars in $(\pi^- \pi^+)$
- Pomeron phenomenologically approximated by vector particle, $\alpha_\mathbb{P} \approx 1$
  - Factorize $N \rightarrow \mathbb{P}N$ vertex from rest of amplitude

- For $J^{PC} = 2^{-+}$, focus on high event intensities
  - e.g. $\rho \pi$ F-wave, $f_2(1270)\pi$
  - $S$- and $D$-waves, ...

- Coupled channel analysis for partial wave amplitudes $F_i(s)$, with channel index
  - $i = \{\rho \pi (F), f_2 \pi (S), f_2 \pi (D), \ldots\}$
Unitarity and Analyticity

- Partial wave unitarity of $\pi^-p \rightarrow (\pi^-\pi^+)\pi^-$ amplitude

\[
\text{Disc } F_i(s) = 2i \sum_j t_{ij}^*(s) \rho_j(s) F_j(s)
\]

- Rescattering amplitude satisfies its own unitarity equation

\[
\text{Im } t_{ij}(s) = \sum_k t_{ik}^*(s) \rho_k(s) t_{kj}(s)
\]

- One can separate $F_i$ into LHC and RHC terms, and write dispersive integral equation for $F_i$, with solution given by Omnes

\[
F_i(s) = b_i(s) + \sum_j t_{ij}(s)c_j + \frac{1}{\pi} \sum_j t_{ij}(s) \int_{s_j}^{\infty} ds' \frac{\rho_j(s')b_j(s')}{s' - s}
\]
K-Matrix Parameterization

- To preserve unitarity, rescattering amplitude $t_{ij}(s)$ is parameterized by $K$-matrix

\[
[t^{-1}]_{ij}(s) = [K^{-1}]_{ij}(s) - l_i(s)\delta_{ij}
\]

where $l_i(s)$ is Chew-Mandelstam phase space factor, with $\text{Im } l_i(s) = \rho_i(s)$

- The real $K$-matrix is parameterized by resonant and non-resonant contributions

\[
K_{ij}(s) = \sum_r \frac{g^r_i g^r_j}{m^2_r - s} + \sum_n \gamma^n_{ij}s^n
\]

- Fit $K$-matrix parameters to data and extract resonance information
For the production amplitude $b_i(s)$, we model with Deck amplitude.

Consider $\pi$ exchange:
- Closest LHC to physics region $\implies$ Expected to be significant contribution
- Ignoring subtleties of $\pi$-exchange (May need absorption corrections)

Model:
$$A_{\text{Deck}}(s, \Omega) = \frac{g_{\rho\pi\pi} g_{P\pi\pi}}{t(s, \theta) - m_\pi^2} \epsilon_\lambda \cdot p_2 \epsilon_{\lambda'}^* \cdot \{p_a\}$$

$b_i(s)$ is partial wave projection of $A_{\text{Deck}}$ in definite $J$, $M$, and $L$ states.
Fit Attempts

As first attempt, we consider a more simplified model, where the production amplitude is conformal expansion

$$F_i(s) = \sum_j t_{ij}(s) \alpha_j(s)$$

$\alpha_i$ contains no RHCs and has free parameters

Also, consider only $f_2 \pi$ in $S$- and $D$-wave
Simple Production Model Fit

\[ \chi^2 / dof = 12.2 \]

\[ m_{R_1} = 1.820 \text{ GeV} \]
\[ \Gamma_{R_1} = 0.214 \text{ GeV} \]

\[ m_{R_2} = 1.612 \text{ GeV} \]
\[ \Gamma_{R_2} = 0.194 \text{ GeV} \]
Unitarized Deck Fits

- The fits for a general production term $\alpha_i$ seem too flexible in the current approach.
- Now use unitarized Deck amplitude developed for this analysis.

$$F_i(s) = b_i(s) + \sum_j t_{ij}(s) c_j + \frac{1}{\pi} \sum_j t_{ij}(s) \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_j(s')}{s' - s}$$

- Fit Intensities and phase differences of three channel case.
Unitarized Deck Fits

Data: three main waves at low $|t'|$ (0.1 GeV$^2$-0.113 GeV$^2$):

$$2^{-+}0^+ f_{2\pi} S, \quad 2^{-+}0^+ f_{2\pi} D, \quad 2^{-+}0^+ (\pi\pi)_S\pi D.$$

Figure: Fit model: 3 channel K-matrix with two poles and unitarized "Deck".

- K-matrix assumes elasticity, so simultaneous fit of all decay channels are needed (all $3\pi$ waves),
- data for 11 $|t'|$ intervals are available. $|t'|$-dependence of non-resonance component is fixed by “Deck” model.
Future Developments for COMPASS Analysis

- Develop Framework to analyze $3\pi$ resonances satisfying S-matrix principles
- Will investigate Finite Energy Sum Rules to constrain amplitudes
- We are fitting data based on COMPASS model. Will extend to 4-vectors and for GlueX at JLab

- Want to describe entire $3\pi$ spectrum, but some interesting cases along the way ($2^{-+}$ and $1^{++}$)
  - Will continue the work on COMPASS in $2^{-+}$ sector
  - Perform analysis on $1^{++}$ sector, $a_1(1420)$ puzzle

[C. Adolph et al. [COMPASS Collaboration], Phys. Rev. Lett. 115, 082001 (2015)]
We have developed the analysis formalism to analyze $3\pi$ systems for peripheral reactions.

- Formalism satisfies S-matrix principles.
- Applying formalism to COMPASS and extracting resonances:
  - Focus on $J^{PC} = 2^{-+}$ first, then apply to all $3\pi$ $J^{PC}$.
- Extend formalism for photon beams (JLab12 physics).
Backup
Phase Space Factors

- In stable isobar limit, phase space factor is 2-body: \( \rho_i \sim \sqrt{(s - s_i)/s} \)
- Decaying isobar introduces \( \pi^+\pi^- \) scattering amplitude \( f(s) \)
- Phase space factors change to quasi-two body phase space factors

\[
\rho_{\text{Quasi}}(s) \sim \int_{4m_{\pi}^2}^{\sqrt{s - m_{\pi}}} ds' \rho_{\text{Isobar}}(s') \text{Im } f(s')
\]

- Affects how we continue to unphysical sheets, new (Woolly) cut introduced
Resonance Extraction

- Analytically continue amplitudes to unphysical sheets to search for poles
- Stable isobars involve only two-body phase space factors (simple square-roots)
- For decaying isobars, Woolly cut may hide pole onto a deeper sheet
Summary of the project

Unitarity condition:
- two body unitarity and quasi-two-body, isobar+pion
- consideration of various solutions, \( \chi N/D \) (deadlock), \( K \)-matrix
- generalisation for multi-channel case,
- incorporation of threshold behaviour.

Analytical continuation of amplitude:
- additional isobar structure
  “Woolly” cut [Aitchison]
- pole search

Production mechanism
- \( P \)-vector solution (deadlock),
- short-long range approximation, explicit incorporation of “Deck” amplitude [Basdevant-Berger]
- \( PW \) projection of scalar “Deck”, threshold behaviour check
- \( PW \) projection of spin-“Deck”, threshold behaviour check [Ascoli, Griss-Fox]

Fit and systematics
- Implementation of the fit procedure, \( C++ \), Mathematica, Fortran
- MC studies of \( \chi^2 \)-function