Lepton flavor violating processes $l_i \rightarrow l_j \nu_l \bar{\nu}_l$ in topcolor-assisted technicolor models

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Abstract

We study the lepton flavor violating (LFV) processes $l_i \rightarrow l_j \nu_l \bar{\nu}_l$ in the context of the topcolor-assisted technicolor (TC2) models. We find that the branching ratios $B_r(\tau \rightarrow l_j \nu_l \bar{\nu}_l)$ are larger than the branching ratios $B_r(\tau \rightarrow l_j \nu_l \bar{\nu}_l)$ in all of the parameter space. Over a wide range of parameter space, we have $B_r(\tau \rightarrow l_j \nu_l \bar{\nu}_l) \sim 10^{-6}$ and $B_r(\tau \rightarrow l_j \nu_l \bar{\nu}_l) \sim 10^{-9}(l = \mu \text{ or } e)$. Taking into account the bounds given by the experimental upper limit $Br^{exp}(\mu \rightarrow 3e) \leq 1 \times 10^{-12}$ on the free parameters of TC2 models, we further give the upper limits of the LFV processes $l_i \rightarrow l_j \nu_l \bar{\nu}_l$. We hope that the results may be useful to partly explain the data of the neutrino oscillations and the future neutrino experimental data might be used to test TC2 models.

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It is well known that the individual lepton numbers $L_e$, $L_\mu$ and $L_\tau$ are automatically conserved and the tree-level lepton flavor violating (LFV) processes are absent in the standard model (SM). However, the solar neutrino experiments [1], the data on atmospheric neutrinos obtained by the Super-Kamiokande Collaboration [2], and the results from the KamLAND reactor antineutrino experiments [3] provide very strong evidence for mixing and oscillation of the flavor neutrinos, which imply that the separated lepton number are not conserved. Thus, the SM requires some modification to account for the pattern of neutrino mixing suggested by the data and the LFV processes like $l_i \rightarrow l_j \gamma$ and $l_i \rightarrow l_j l_k l_l$ are allowed. The observation of these LFV processes would be a clear signature of new physics beyond the SM, which has been widely studied in different scenarios such as Two Higgs Doublet Models, Supersymmetry, Grand Unification [4, 5] and topcolor models [6].

On the other hand, neutrino oscillations imply that there are solar $\nu_e \rightarrow \nu_\mu, \nu_\tau$ transitions and there are atmospheric $\nu_\mu \rightarrow \nu_\tau$ transitions. The standard tau decays $\tau \rightarrow \mu \nu_\mu \bar{\nu}_\tau, e\nu_e \bar{\nu}_\tau$ and the standard muon decay $\mu \rightarrow e\nu_e \bar{\nu}_\mu$ can not explain the experimental fact. However, the LFV processes $l_i \rightarrow l_j \nu_l \bar{\nu}_l$, where $l_i = \tau$ or $\mu$, $l_j = \mu$ or $e$ and $l = \tau, \mu$ or $e$, might explain the neutrino oscillation data. With these motivations in mind, we study the LFV processes $l_i \rightarrow l_j \nu_l \bar{\nu}_l$ in the context of topcolor-assisted technicolor (TC2) models [7]. These models predict the existence of the extra $U(1)$ gauge boson $Z'$, which can induce the tree-level FC coupling vertices $Z' l_i l_j$. The effects of the gauge boson $Z'$ on the LFV processes $l_i \rightarrow l_j \gamma$, $l_i \rightarrow l_j l_k l_l$ and $Z \rightarrow l_i l_j$ have been studied in Ref.[6, 8]. They have shown that the contributions of $Z'$ to these processes are significantly large, which may be detected in the future experiments. In this letter, we show that the $Z'$ can generate large contributions to the LFV processes $\tau \rightarrow \mu \nu_\tau \bar{\nu}_\tau, e\nu_\tau \bar{\nu}_\tau$ and $\mu \rightarrow e\nu_\tau \bar{\nu}_\tau$, which may be used to partly explain the data of the neutrino oscillations. Furthermore, considering the constraints of the present experimental bound on the LFV process $\mu \rightarrow 3e$ on the free parameters of TC2 models, we give the upper bounds on the branching ratios $Br(l_i \rightarrow l_j \nu_l \bar{\nu}_l)$, which arise from $Z'$ exchange.

For TC2 models, the underlying interactions, topcolor interaction, are non-universal.
This is an essential feature of TC2 models, due to the need to single out the top quark for condensate. Therefore, TC2 models predict the existence of the non-universal $U(1)$ gauge boson $Z'$. The new particle treats the third generation fermions differently from those in the first and second generations and can lead to the tree-level FC couplings. The flavor-diagonal couplings of $Z'$ to leptons can be written as [7, 9]:

\[
\mathcal{L}_{Z'}^{FD} = \frac{1}{2} g_1 c_{\theta'} Z'_{\mu} (\bar{\tau}_L \gamma^\mu \tau_L + 2 \bar{\tau}_R \gamma^\mu \tau_R - \bar{\nu}_L \gamma^\mu \nu_L) - \frac{1}{2} g_1 t_{\theta'} Z'_{\mu} (\bar{\mu}_L \gamma^\mu \mu_L
\]

\[+ 2 \bar{\mu}_R \gamma^\mu \mu_R + \bar{\nu}_L \gamma^\mu \nu_L + \bar{e}_L \gamma^\mu e_L + 2 \bar{e}_R \gamma^\mu e_R + \bar{\nu}_e \gamma^\mu \nu_e), \tag{1}
\]

where $g_1$ is the ordinary hypercharge gauge coupling constant, $\theta'$ is the mixing angle with $\tan \theta' = g_1 / \sqrt{4\pi k_1}$. The flavor changing couplings of $Z'$ to leptons can be written as:

\[
\mathcal{L}_{Z'}^{FC} = - \frac{1}{2} g_1 Z'_{\mu} [k_{\tau\mu} (\bar{\tau}_L \gamma^\mu \tau_L + 2 \bar{\tau}_R \gamma^\mu \tau_R) + k_{\tau e} (\bar{\tau}_L \gamma^\mu e_L + 2 \bar{\tau}_R \gamma^\mu e_R)
\]

\[+ k_{\mu e} \tan^2 \theta' (\bar{\mu}_L \gamma^\mu e_L + 2 \bar{\mu}_R \gamma^\mu e_R)], \tag{2}
\]

where $k_{ij}$ are the flavor mixing factors. For the sake of simplicity, we consider the case where all three generations of leptons mix with a universal constant $k$, i.e. $k_{\tau\mu} = k_{\tau e} = k_{\mu e} = k$ in this letter.

![Feynman diagram](image)

Fig.1 Feynman diagram for the LFV processes $l_i \rightarrow l_j \nu_l \bar{\nu}_l$ induced by $Z'$ exchange.

From Eq.(1) and Eq.(2), one can see that the LFV processes $l_i \rightarrow l_j \nu_l \bar{\nu}_l$ can be generated via gauge boson $Z'$ exchange at tree-level. The relevant Feynman diagrams are depicted in Fig.1. The partial widths can be written as:

\[
\Gamma_1 = \Gamma(\tau \rightarrow \mu \nu_l \bar{\nu}_l) = \Gamma(\tau \rightarrow e \nu_l \bar{\nu}_l) = \frac{5 k_1 \alpha_e}{384 \pi C_W^2 M_W^4} \frac{m_\tau^5}{k^2},
\]

\[3\]
\[ \Gamma_2 = \Gamma(\tau \rightarrow \mu \nu_\mu \bar{\nu}_\mu) = \Gamma(\tau \rightarrow \mu \nu_e \bar{\nu}_e) = \Gamma(\tau \rightarrow e \nu_\mu \bar{\nu}_\mu) = \Gamma(\tau \rightarrow e \nu_e \bar{\nu}_e) = \frac{5\alpha_e^3}{384\pi k_1 C_W^3 M_{Z'}^2} m_e^5 k^2; \]

\[ \Gamma_3 = \Gamma(\mu \rightarrow e \nu_\mu \bar{\nu}_\mu) = \frac{5\alpha_e^5}{384\pi k_1 C_W M_{Z'}^2} m_\mu^5 k^2; \]

\[ \Gamma_4 = \Gamma(\mu \rightarrow e \nu_e \bar{\nu}_e) = \frac{5\alpha_e^5}{384\pi k_1 C_W M_{Z'}^2} m_\mu^5 k^2. \]

Where \( C_W^2 = \cos^2 \theta_W, \theta_W \) is the Weinberg angle, \( M_{Z'} \) is the mass of the non-universal \( U(1) \) gauge boson \( Z' \) predicted by TC2 models. In above equations, we have assumed \( m_\mu \approx 0, m_e \approx 0 \) for the processes \( \tau \rightarrow l_j \nu_\mu \bar{\nu}_\mu \) and \( m_\mu \approx 0, m_e \approx 0 \) for the processes \( \mu \rightarrow e \nu_\mu \bar{\nu}_\mu \).

The widths of the processes \( l_i \rightarrow l_j \nu_\mu \bar{\nu}_\mu \) are equal to those of the processes \( l_i \rightarrow l_j e \nu_e \bar{\nu}_e \).

This is because the gauge boson \( Z' \) only treats the fermions in the third generation differently from those in the first and second generations and treats the fermions in the first generation same as those in the second generation.

The corresponding branching ratios can be written as:

\[ Br_1 = Br^{exp}(\tau \rightarrow e \nu_e \bar{\nu}_e) \frac{\Gamma_1}{\Gamma(\tau \rightarrow e \nu_e \bar{\nu}_e)}; \]
\[ Br_2 = Br^{exp}(\tau \rightarrow e \nu_e \bar{\nu}_e) \frac{\Gamma_2}{\Gamma(\tau \rightarrow e \nu_e \bar{\nu}_e)}; \]
\[ Br_3 = \frac{\Gamma_3}{\Gamma(\mu \rightarrow e \nu_\mu \bar{\nu}_\mu)}; \]
\[ Br_4 = \frac{\Gamma_4}{\Gamma(\mu \rightarrow e \nu_e \bar{\nu}_e)}; \]

with

\[ \Gamma(\tau \rightarrow e \nu_e \bar{\nu}_e) = \frac{m_e^5 G_F^2}{192\pi^3}; \]

\[ \Gamma(\mu \rightarrow e \nu_e \bar{\nu}_e) = \frac{m_\mu^5 G_F^2}{192\pi^3}. \]

Here the Fermi coupling constant \( G_F = 1.16639 \times 10^{-5} \text{GeV}^{-2} \) and the branching ratio \( Br^{exp}(\tau \rightarrow e \nu_e \bar{\nu}_e) = (17.83 \pm 0.06)\% \) \cite{10}.

To obtain numerical results, we take the SM parameters as \( C_W^2 = 0.7685, \alpha_e = \frac{1}{128.8}, \)
\[ m_\tau = 1.78 \text{GeV}, \; m_\mu = 0.106 \text{GeV} \] \cite{10}. It has been shown that vacuum tilting and the constraints from Z-pole physics and \( U(1) \) triviality require \( k_1 \leq 1 \) \cite{11}. The limits on the \( Z' \) mass \( M_{Z'} \) can be obtained via studying its effects on various experimental observables \cite{9}. For example, Ref.\cite{12} has been shown that to fit the electroweak measurement data, the \( Z' \) mass \( M_{Z'} \) must be larger than \( 1 \text{TeV} \). As numerical estimation, we take the \( M_{Z'} \) and \( k_1 \) as free parameters.

The branching ratios \( Br_1 \) and \( Br_2 \) are plotted in Fig.2 and Fig.3 as functions of \( M_{Z'} \) for \( k = \lambda = 0.22 \) (\( \lambda \) is the Wolfenstein parameter \cite{13}) and three values of the parameter
$k_1$: $k_1 = 0.2$(solid line), 0.5(dotted line), 0.8(dashed line). One can see that the value of $Br_1$ is larger than that of $Br_2$ in all of the parameter space of TC2 models. This is because the extra $U(1)$ gauge boson $Z'$ couple preferentially to the third generation fermions. The value of the branching ratio $Br_1$ increases from $3.09 \times 10^{-8}$ to $7.91 \times 10^{-6}$ as $M_{Z'}$ decreasing from $4\text{TeV}$ to $1\text{TeV}$ for $k_1 = 0.5$ and the value of branching ratio $Br_2$ increases from $1.26 \times 10^{-11}$ to $3.23 \times 10^{-9}$. For $k_1 = 1$, $M_{Z'} = 1\text{TeV}$, the branching ratio $Br_1$ can reach $1.6 \times 10^{-5}$. Certainly, the numerical results are changed by the value of the flavor mixing parameter $k$. If we take the maximum values of the parameters i.e. $(k_1)_{\text{max}} = 1$ and $(k)_{\text{max}} = 1/\sqrt{2}$, then we have $Br_1 = 1.63 \times 10^{-4}$, $Br_2 = 1.67 \times 10^{-8}$ and $Br_1 = 1.02 \times 10^{-5}$, $Br_2 = 1.04 \times 10^{-9}$ for $M_{Z'} = 1\text{TeV}$ and $2\text{TeV}$, respectively.

The extra $U(1)$ gauge boson $Z'$ can also contribute to the LFV process $\mu \rightarrow 3e$. The relevant decay width arisen from the $Z'$ exchange can be written as:

$$\Gamma(\mu \rightarrow 3e) = \frac{25\alpha e}{384\pi k_1^3 C_W^{10}} \frac{m_\mu^5}{M_{Z'}^4} k^2.$$  

The current experimental upper limit is $Br^{\exp}(\mu \rightarrow 3e) \leq 1 \times 10^{-12}$ [11]. Therefore, the present experimental bound on the LFV process $\mu \rightarrow 3e$ can give severe constraints on

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Fig.2: Branching ratio $Br_1$ as a function of the $Z'$ mass $M_{Z'}$ for the flavor mixing factor $k = 0.22$ and $k_1 = 0.2$(solid line), 0.5(dotted line), 0.8(dashed line).
the free parameters of TC2 models. Then the branching ratios $Br_1$, $Br_2$, $Br_3$ and $Br_4$ can be written as:

$$Br_1 \leq \frac{k_1^4 C_8^8}{5 \alpha^4_e} Br^{exp}(\tau \rightarrow e\nu_e\bar{\nu}_\tau) Br^{exp}(\mu \rightarrow 3e),$$

$$Br_2 \leq \frac{k_1^4 C_4^4}{5 \alpha^2_e} Br^{exp}(\tau \rightarrow e\nu_e\bar{\nu}_\tau) Br^{exp}(\mu \rightarrow 3e),$$

$$Br_3 \leq \frac{k_1^4 C_W^4}{5 \alpha^2_e} Br^{exp}(\mu \rightarrow 3e),$$

$$Br_4 \leq \frac{1}{5} Br^{exp}(\mu \rightarrow 3e).$$

Observably, the maximum values of these branching ratios are only dependent on the free parameter $k_1$. For $k_1 \leq 1$, we have $Br_1 \leq 3.42 \times 10^{-6}$, $Br_2 \leq 3.49 \times 10^{-10}$, $Br_3 \leq 1.96 \times 10^{-9}$ and $Br_4 \leq 2 \times 10^{-13}$.

Fig.3: Same as Fig.2 but for $Br_2$.

Extra gauge bosons $Z'$ are the best motivated extensions of the SM. If discovered they would represent irrefutable proof of new physics, most likely that the SM gauge groups must be extended \cite{15}. If these extensions are associated with flavor symmetry breaking, the gauge interactions will not be flavor-universal \cite{12}, which predict the existence of non-universal gauge bosons $Z'$. After the mass diagonalization from the flavor eigenbasis
into the mass eigenbasis, the non-universal gauge interactions result in the tree-level FC couplings. Thus, the $Z'$ may have significant contributions to some FCNC processes. In this letter, we study the contributions of the non-universal gauge bosons $Z'$ predicted by TC2 models to the LFV processes $l_i \rightarrow l_j \nu_l \bar{\nu}_l$. We find that the branching ratios $B_r(\tau \rightarrow l_j \nu_l \bar{\nu}_\tau)$ are larger than the branching ratios $B_r(\tau \rightarrow l_j \nu_l \bar{\nu}_l)(l = \mu$ or $e)$ in all of the parameter space. Over a wide range of parameter space, we have $B_r(\tau \rightarrow l_j \nu_l \bar{\nu}_\tau) \sim 10^{-6}$ and $B_r(\tau \rightarrow l_j \nu_l \bar{\nu}_l) \sim 10^{-9}$. For $k_1 = 1$, $M_{Z'} = 1 TeV$ and $k = 1/\sqrt{2}$, the value of the branching ratio $B_r(\tau \rightarrow l_j \nu_l \bar{\nu}_\tau)$ can reach $1.63 \times 10^{-4}$. Considering the bounds given by the experimental upper limit $Br^{\text{exp}}(\mu \rightarrow 3e) \leq 1 \times 10^{-12}$ on the free parameters of TC2 models, we further give the upper limits of the LFV processes $l_i \rightarrow l_j \nu_l \bar{\nu}_l$. The results are $Br(\tau \rightarrow l_j \nu_l \bar{\nu}_\tau) \leq 3.42 \times 10^{-6}$, $Br(\tau \rightarrow l_j \nu_l \bar{\nu}_l) \leq 3.49 \times 10^{-10}$, $Br(\mu \rightarrow e\nu_l \bar{\nu}_\tau) \leq 1.96 \times 10^{-9}$ and $Br(\mu \rightarrow e\nu_l \bar{\nu}_l) \leq 2 \times 10^{-13}$ ($l = \mu$ or $e$). We hope that the results may be useful to partly explain the data neutrino oscillations. The future neutrino experiment data might be used to test TC2 models.

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