Nonlinear dynamics and bifurcation for a Jeffcott rotor with seal aerodynamic excitations

Wang Yuefang¹, Lü Lefeng and Li Yong

Department of Engineering Mechanics, Dalian University of Technology, Dalian 116024, China

E-mail: yfwang@dlut.edu.cn

Abstract. The nonlinear vibration and bifurcation characteristics of a Jeffcott rotor aerodynamically excited by gas seals are presented in this paper. The Muszynska’s model is adopted to express the seal forces as the nonlinear function of rotor displacement, velocity and acceleration. The Runge-Kutta method is used to obtain the displacement and bifurcation diagrams with parameters of the pressure drop and the length of the seal. Various period-multiple bifurcations are found showing complexity of the rotor’s motion under the aerodynamic excitation of the seal.

1. Introduction

It is well known that the dynamical characteristics of modern rotary machines depend on external excitations that act on the systems’ boundary regions. Practically, these excitations include oil-film forces of journal bearings, fluid-induced excitations inside seals, impact and dry frictions, etc. Among them the aerodynamic excitation generated in the clearance of seals plays an important role determining the nonlinear response and stability of gas turbines and air compressors. The investigation on the aerodynamic force inside the seal firstly begun in 1960’s and was greatly improved in 1980’s when the rotordynamic coefficients of annular and labyrinth seals were experimentally by Childs and Nelson, Scharrer and Childs [1-3]. The influence of the aerodynamic force created by the disturbing flow in the seal clearance on the motion stability was systematically studied by Muszynska, Muszynska and Bently [4-7]. By assuming the gas rotates at a speed less than half of the shaft speed, the seal force is modeled as a nonlinear function of the displacement, velocity and acceleration of the rotor center, as follows:

\[
\begin{bmatrix}
F_x \\
F_y
\end{bmatrix} =
\begin{bmatrix}
K - m_f \tau^2 \omega^2 & \tau_0 D \\
-\tau_0 D & K - m_f \tau^2 \omega^2
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix} -
\begin{bmatrix}
D & 2\tau m_f \omega \\
-2\tau m_f \omega & D
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix}
\begin{bmatrix}
m_f & 0 \\
0 & m_f
\end{bmatrix}
\begin{bmatrix}
\ddot{x} \\
\ddot{y}
\end{bmatrix}
\] (1)

where \(x\) and \(y\) are transversal displacements of the rotor center, respectively; \(F_x\) and \(F_y\) are the components of the seal force, respectively; \(K, D\) and \(m_f\) are equivalent stiffness, damping and mass of the gas with an arbitrary eccentricity of the rotor; \(\omega\) is the shaft speed of the rotor and \(\tau < 0.5\) is the circumferential velocity ratio of the gas revolving with the rotor. The above Muszynska’s model has

¹ To whom any correspondence should be addressed.
been applied successfully by Ding et al. [8], Li et al. [9], Zhang et al. [10] and Hua et al.[11], among others, to study the nonlinear vibration and Hopf bifurcation behavior of gas sealed rotary machines.

The nonlinear vibration and bifurcation of a Jeffcott rotor is investigated in this paper. Like previous authors we use the Muszynska’s model and numerically solve the equation of motion. The bifurcation diagrams are obtained along with the Poincaré’s map and time history of the rotor’s response with varying parameters of shaft speed, pressure drop and the seal length.

2. Mathematical Model
Following [10], the governing equation of the Jeffcott rotor is expressed as

\[
\begin{bmatrix}
\ddot{x} \\
\ddot{y}
\end{bmatrix} + \begin{bmatrix}
D_x & 0 \\
0 & D_e
\end{bmatrix} \begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix} + \begin{bmatrix}
K_x & 0 \\
0 & K_e
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix} = \begin{bmatrix}
F_x \\
F_y
\end{bmatrix} + \begin{bmatrix}
0 \\
-mg
\end{bmatrix} + m \omega^2 e_m \begin{bmatrix}
\cos \omega t \\
\sin \omega t
\end{bmatrix}
\]

(2)

where \( m \) is the mass of the rotor; \( D_e \) and \( K_e \) are mechanical damping and stiffness of the system, respectively; \( e_m \) is the distance between the mass centroid and the rotor center. As aforementioned the seal force can be expressed by (1), where coefficients are defined by

\[
K = K_0 \left(1 - \frac{x^2 + y^2}{c^2}\right)^b, D = D_0 \left(1 - \frac{x^2 + y^2}{c^2}\right)^b, \tau = \tau_0 \left(1 - \sqrt{x^2 + y^2}/c\right)\]

(3)

where \( c \) is width of the seal clearance; \( 0.5 < n < 3 \) and \( 0 < b < 1 \) are empirical coefficients; \( \tau_0 < 0.5 \) is the circumferential velocity ratio of the flow for the undisturbed rotor. The rotodynamic coefficients \( K_0, D_0 \) and \( m_f \) are expressed through the short bearing model by Childs [12]:

\[
K_0 = \mu_0 \mu_1, D_0 = \mu \mu_1 T, m_f = \mu \mu_1 T^2
\]

(4)

where parameters \( \mu_0, T \) and \( m_f \) are defined in [12] and are related to other characteristics including pressure drop, loss at entrance, etc. Carrying out the non-dimensionalization of (2), one obtains the equation of motion (see also [10])

\[
\begin{align*}
\dot{X}_1 &= X_2 \\
\dot{X}_2 &= -\frac{D_e + D}{M \omega^2} X_2 - \frac{2 \tau m_f}{M} Y_2 - \frac{K_x + K - \tau^2 \omega^2 m_f}{M \omega^2} X_1 + \frac{\tau D}{M \omega} Y_1 + \frac{m e}{M c} \cos t \\
\dot{Y}_1 &= Y_2 \\
\dot{Y}_2 &= -\frac{D_e + D}{M \omega^2} Y_2 + \frac{2 \tau m_f}{M} X_2 - \frac{K_x + K - \tau^2 \omega^2 m_f}{M \omega^2} Y_1 + \frac{\tau D}{M \omega} X_1 - \frac{m g}{M \omega^2 c} + \frac{m e}{M c} \sin t
\end{align*}
\]

(5)

3. Nonlinear vibration and bifurcation results
Equation (5) can be solved by the Runge-Kutta method to obtain the nonlinear vibration and bifurcation behavior of the system. Assign the following parameters

\[
m = 50kg, K_x = 7.2762 \times 10^6 N \cdot m^{-1}, D_e = 2000N \cdot s \cdot m^{-1}, e = 0.2mm
\]

\[
R = 0.067m, l = 0.102m, n = 2.5, b = 0.45, \tau_0 = 0.4, c = 0.3mm
\]

where \( R \) and \( l \) are the radius and the length of the seal, respectively. Let the pressure drop be 0.2MPa and \( s = \omega \sqrt{K_x/m} \) be the non-dimensional shaft speed. The bifurcation diagram of the displacement \( x \) is illustrated in Fig.1 by the swept frequency method. It can be found that the rotor’s motion is period-1 when \( s \) is less than 1.3631 (\( \omega = 520 \text{rad/s} \)).
Figure 1. Bifurcation diagram of displacement $x$.

Table 1. Bifurcation points of $s$ with a 0.2Mpa pressure drop

| $s$   | Type of bifurcation |
|-------|---------------------|
| 1.3631| Period-4            |
| 3.4078| Period-8            |
| 3.6961| Period-10           |
| 4.4039| Period-20           |

Figure 2. Poincaré map of the period-8 motion.    Figure 3. The period-8 motion.

The system experiences various period-multiple bifurcations as the shaft speed increases. Table 1 presents the bifurcation points of $s$ and the corresponding type of periodic motion. It is worth pointing out that each of these periodic motions appears in a very narrow window of shaft speed separated by infinite numbers of quasi-periodic motions. Figure 2 shows the Poincaré return map of the period-8 motion in the displacement $x$ when $s=3.4078$ ($\omega=1300\text{rad/s}$). Figure 4 shows the time history of the displacement. The 1:8 sub-harmonic whirling motion can be observed from the orbit of the rotor center in figure 4. The quasi-periodic motion of $x$ is illustrated in figure 5 with $s=3.932$ ($\omega=1500\text{rad/s}$), while the Poincaré map and the orbit plot are presented in figure 6 and figure 7, respectively.
The numerical analysis is carried out again with the pressure-drop of the seal increased to 0.4MPa. Similarly, for low shaft speed there exist period-1 motions followed by a series of period-multiple bifurcations. Figure 8 plots the bifurcation diagram of displacement $x$ with increment of $s$. The bifurcation points of $s$ and the type of bifurcation are shown in table 2. It is noticed that both the critical values of $s$ and the type of bifurcation are different from their counterparts in table 1. Between the periodic motions the displacement responses are all quasi-periodic. Figure 9 shows the bifurcation diagram of the displacement with a pressure drop of 0.8MPa. New series of bifurcation can be found in table 3.
Table 2. Bifurcation points of $s$ with a 0.4MPa pressure drop

| $s$  | Type of bifurcation |
|------|---------------------|
| 1.9136 | Period-4          |
| 2.1233 | Period-15         |
| 3.5127 | Period-17         |
| 4.3253 | Period-10         |

Let $s=3.1457$, the bifurcation diagram is generated by using the pressure drop as the bifurcation parameter, shown in figure 10. It can be seen that the average amplitude of the response decrease with advancing pressure drops. This shows that the self-excitation motion is more likely to happen with small pressure drops. Based on the results, the pressure drop plays a key role in stabilizing the rotor, keeping it away from excessively disturbed motion in the seal clearance. Nevertheless, the motion is complicated due to the existence of quasi-periodic displacement response.

![Bifurcation diagram](image)

Figure 9. Bifurcation of $x$ with a 0.8MPa pressure drop.

Table 3. Bifurcation points of $s$ with a 0.8MPa pressure drop

| $s$  | Type of bifurcation |
|------|---------------------|
| 2.7263 | Period-4          |
| 2.9622 | Period-13         |
| 3.5782 | Period-4          |
In practice, the length of the seal is treated as a parameter for improvement of the sealing performance. The numerical analysis is repeated carrying different lengths of seal to obtain the bifurcation diagram of displacement $x$ shown in figure 11. It can be seen that the average amplitude of the motion is reduced with increment of the length. This can be explained by that, with a larger seal length, the relative pressure drop between the exit and the entrance of the seal becomes bigger. Same as in figure 10, the average displacement drops as a result of that.

4. Conclusions
The aerodynamically induced, self-excitation of rotary systems has been studied extensively in the past several decades. However, few previous publications focused on bifurcation behavior with varying pressure drop and length of seal, which are important in improving both the sealing effect and the stability of rotor vibration. The present results show complicated period-multiple bifurcations in the transversal vibration. The type of the bifurcation changes with the shaft speed, the pressure drop and the length of the seal. It is pointed out that the amplitude of the response decreases with increasing pressure drop and length of seal. The periodic motions are separated by vast numbers of quasi-periodic motions.

Acknowledgement
The financial support by the National Science Foundation of China (Projects 10472021, 10421002) is gratefully acknowledged.

References
[1] Childs D W and Nelson C E 1986 J. Tribol. 108 426
[2] Childs D W and Nelson C E 1986 J. Tribol. 108 433
[3] Childs D W and Scharrer J K 1988 J. Vib. Acoust. Stress. Reliab. Des. 110 281
[4] Muszynska A 1986 J. Sound. Vib. 110 443
[5] Muszynska A 1988 J. Vib. Acoust. Stress. Reliab. Des. 110 129
[6] Muszynska A and Bently D E 1990 J. Sound. Vib. 143 103
[7] Muszynska A 1986 Fluid-Related Rotor/Bearing/Seal Instability Problems (Bently Rotor Dynamics Research Corp. BRDRC Report No. 2 Minden, NV)
[8] Ding Q, Cooper J E and Leung A Y T 2002 J. Sound. Vib. 252 817
[9] Li S T, Xu Q Y, Wan F Y and Zhang X L 2003 Appl. Math. Mech. 24 1290
[10] Zhang Y, Hua J and Xu Q 2005 Chin J. Comput. Mech. 22 541
[11] Hua J, Liu Z S and Xu Q Y 2005 J. Sound. Vib. 283 525
[12] Childs D W 1983 J. Lubric. Technol. 105 429