Abstract

We show that a hierarchical $CPT$ violating neutrino spectrum can simultaneously accommodate all published neutrino data, including the LSND result, with a total $\chi^2$ which is almost identical to the $CPT$ conserving best fit. In our scenario the oscillation signal seen by the KamLAND experiment in antineutrinos is independent of the LMA solar oscillation signal seen in neutrinos. A larger antineutrino mass splitting accounts for the LSND signal and also contributes to atmospheric oscillations. Because of this feature, there is some tension between our model and certain Super-K atmospheric results. Thus, if LSND did not exist, our model would survive only at the 99% confidence level; with LSND, our model is (essentially) statistically equivalent to the $CPT$ conserving best fit.
1 Introduction

CPT violating neutrino masses allow the possibility [1] - [4] of reconciling the LSND [5], atmospheric [6], and solar oscillation [7, 8] data without resorting to sterile neutrinos. As argued in [2], there are good reasons to imagine that CPT violating dynamics couples directly to the neutrino sector, but not to other Standard Model degrees of freedom. An explicit CPT violating model of this type was presented in [4].

KamLAND [9], a medium baseline reactor antineutrino disappearance experiment, is sensitive to antineutrino mass-squared splittings in the $10^{-4}$ eV$^2$ range characteristic of the large mixing angle (LMA) solar neutrino scenario. The KamLAND collaboration has recently reported [10] an electron antineutrino survival probability which is significantly less than one:

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 0.611 \pm 0.085 \pm 0.041 . \quad (1.1)$$

If the neutrino mass spectrum conserves CPT, then this result is consistent with the LMA interpretation of solar neutrino oscillations. If the neutrino mass spectrum violates CPT, however, the KamLAND result provides no information about solar oscillations, but rather constrains the splittings in the antineutrino spectrum.

In this paper we show that a hierarchical CPT violating neutrino spectrum can simultaneously accommodate the oscillation data from LSND, atmospheric, solar and KamLAND, as well as the nonobservation of antineutrino disappearance in short baseline reactor experiments. Comparing our model to the global neutrino data set we obtain a total $\chi^2 = 201$ for 228 degrees of freedom. This can be compared to the usual CPT conserving best fit, which has a total $\chi^2 = 199$ for 232 degrees of freedom [11].

While the total $\chi^2$ are about the same, there are striking differences when we make a more detailed comparison to the data. The CPT conserving best fit parameter set (two mass differences and 3 mixing angles) matches remarkably well to the global data set, with the glaring exception of the LSND result. This single data point contributes a $\Delta \chi^2$ of about +12, leading to widespread speculation that the LSND result is simply wrong, and will be disconfirmed by MiniBooNE.
As we will see, the hypothesis of a $CPT$ violating neutrino sector (four mass differences and 6 mixing angles) leads to a completely different conclusion. In this framework our (non-optimized) parameter choices give an excellent fit to LSND. The largest single discrepancy with the global data set instead occurs with the atmospheric Super-K multi-GeV muons and thru-going muons. In either case the $\Delta \chi^2$ is no worse than +4. Thus instead of casting a skeptical eye upon LSND one is led to speculate that the forthcoming re-analysis of the Super-K atmospheric data will improve this discrepancy. Indeed the new fluxes announced for the Super-K re-analysis definitely work in this direction.

We also make a dramatic prediction for the observation of atmospheric neutrinos using the MINOS detector \cite{12}. Because the MINOS detector discriminates positive and negative charge, this experiment can disentangle the neutrino and antineutrino components of atmospheric oscillations in a straightforward way. As the mass differences in the atmospheric sectors differ by orders of magnitude in our scenario, MINOS will be able to tell them apart easily.

\section{The spectrum}

To analyze all the possible $CPT$ violating spectra is not an easy job. With four mass differences and six mixing angles (not taking into account the two $CP$ violating phases which participate in oscillations) a complete scan of the whole parameter space is impractical. However, thanks to the available experimental data, it is possible to reduce the allowed regions to two sets of well-differentiated spectra with (quasi) orthogonal experimental signatures.

The easiest way to make contact with the experimental results is in terms of the neutrino survival and transition probabilities, which are given by

$$
P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j=1}^{3} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2 \left[ \frac{\Delta m^2_{ij} L}{4E} \right]
$$

(2.1)
for neutrinos and

\[ P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j=1}^{3} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2 \left( \frac{\Delta m_{ij}^2 L}{4E} \right) \] (2.2)

for antineutrinos. Here the matrix \( U = \{ U_{\alpha i} \} \) \( (\overline{U} = \{ \overline{U}_{\alpha i} \}) \) describes the weak interaction neutrino (antineutrino) states, \( \nu_\alpha \), in terms of the neutrino (antineutrino) mass eigenstates, \( \nu_i \). That is,

\[ \nu_\alpha = \sum_i U_{\alpha i} \nu_i \quad \text{and} \quad \overline{\nu}_\alpha = \sum_i \overline{U}_{\alpha i} \overline{\nu}_i \] (2.3)

where we have ignored the possible CP violation phases in both matrices and took them to be real. The matrices can be parametrized as follows:

\[
U = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}s_{13} \\
-s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}s_{13}
\end{pmatrix}
\] (2.4)

and similarly for \( \overline{U} \). In Eq. (2.1) \( L \) denotes the neutrino flight path, \( i.e. \) the distance between the neutrino source and the detector, and \( E \) is the energy of the neutrino in the laboratory system.

Regarding the mass spectrum of the three neutrinos we assume that it is hierarchical and thus characterized by two different squared masses

\[ \Delta m_{12}^2 = m_2^2 - m_1^2 \quad \text{and} \quad \Delta m_{13}^2 = m_3^2 - m_1^2 \]

whose numerical values are rather different, \( i.e. \Delta m_{13}^2 \gg \Delta m_{12}^2 \) and similarly for the antineutrinos. Having said that, it becomes apparent that the larger mass-squared difference in the neutrino sector will be related to the atmospheric neutrino signal observed by Super-Kamiokande, while the smaller one will drive the solar neutrino oscillations. In the antineutrino sector, the largest mass difference will provide an explanation to the signal observed in LSND, while the smaller one is the one which might have been (mis)identified by KamLAND as a confirmation of LMA.

The key ingredient to sort out the antineutrino spectra are reactor experiments. Their results indicate [13, 14] that electron antineutrinos produced in reactors remain electron
antineutrinos on short baselines. As the distance traveled by our antineutrinos is small we can forget about the smallest mass difference and average the other two, thus the survival probability can be expressed as

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - 2U^2_{e3}(1 - U^2_{e3}).$$  \hspace{1cm} (2.5)

It is clear that there are two possible ways to achieve a survival probability close to one, i.e. $U_{e3}$ can be almost one or almost zero. Physically this means that we can choose between having almost all the antielectron flavor in the heavy state (or in the furthest away state) or just leave in this state almost no antielectron flavor. The first possibility (which is depicted in Fig. 1) is the one we explored in our previous works. This spectrum predicts for KamLAND a survival probability consistent with one. Since this is strongly disfavored by the KamLAND result (1.1), we instead pursue the second possibility, which is represented by the spectrum shown in Figure 2.

This second family of spectra is characterized by a strong violation of $CPT$ in the mass
Figure 2: *Possible neutrino mass spectrum with almost no electron content in the heavy state.* Although the figure shows an explicit mixing pattern, there is a whole family of mixing matrices that can do an equally good job. The flavor content is distributed as follows: electron flavor (red), muon flavor (brown) and tau flavor (yellow).

differences but a much slighter effect in the mixing matrix. This is seen in Fig. 2 where the flavor distribution in the neutrino and antineutrino spectra is rather similar. The most distinctive feature of this family of solutions is its $\theta_{23}$, which lives far away from maximal mixing, or in other words which has a large component of antitau neutrino in the heavy state. The small antimuon neutrino component in the heavy state is not bounded by the non observation of muon neutrino disappearance over short baselines in the CDHS experiment [15], as the antineutrino component in this experiment was minimal.

KamLAND could have observed an oscillation signal driven by the smaller antineutrino mass splitting and interpreted it as LMA oscillations. To explicitly see how this might have happened, we will choose two sample points in our parameter space and calculate the transition probabilities for it. Let us emphasize that we have not performed a chi-squared fit and therefore the points we are selecting (by eye and not by chi) are not optimized to give the best fit to the existing data. Instead, they must be regarded as two among the many equally good sons in this family of solutions.
The point we have chosen has $\theta_{13} = .08$, $\theta_{23} = .5$, $\theta_{12} = .6$, $\Delta m^2_{12} = 5 \cdot 10^{-4}$ eV$^2$ and $\Delta m^2_{13} = \mathcal{O}(1)$ eV$^2$. Since we are dealing with an antineutrino signal, we do not need to identify either the flavor distribution or the mass eigenstates of the neutrino sector. We will do it later, when showing the zenith angle dependence this model predicts for Super-Kamiokande atmospheric neutrinos.

The survival probability measured by KamLAND is given by

$$P_{\text{KamLAND}} = 1 - 4 U_{e3}^2 (1 - U_{e3}^2) \sin^2 \left[ \frac{\Delta m^2_{13} L}{4E} \right] - 4 U_{e1}^2 U_{e2}^2 \sin^2 \left[ \frac{\Delta m^2_{12} L}{4E} \right], \quad (2.6)$$

where the second term (proportional to $U_{e3}^2$) is negligible. Plugging our numbers in, it is straightforward to see that $P_{\text{KamLAND}} \approx .6$ regardless of whether the mass difference that drives the solar neutrino oscillations belongs to the LMA region.

By the same token, we can calculate the probability associated with the LSND signal. It is given by

$$P_{\text{LSND}} = 4 U_{\mu 3}^2 U_{e3}^2 \sin^2 \left[ \frac{\Delta m^2_{13} L}{4E} \right], \quad (2.7)$$

where we have neglected terms proportional to $\Delta m^2_{12}$ which are irrelevant for such small distances. As the reader can easily verify, we predict a $P_{\text{LSND}} \simeq .0022$ in excellent agreement with the LSND final analysis:

$$P_{\text{LSND-final}} = 0.00264 \pm .00081. \quad (2.8)$$

The only piece of experimental evidence involving antineutrinos which remains to be checked is the signal found for Super-Kamiokande atmospheric neutrinos. As we are introducing an antineutrino mass difference roughly two orders of magnitude larger than the Super-K best fit point (for an analysis with two generations and conserving CPT), there is cause for concern. In fact we pass this test as successfully as we did the others. To see this, we have first to state the parameters in the neutrino sector. Once more they have been chosen almost randomly from the different analyses available in the literature and are given by $\theta_{13} = .08$, $\theta_{23} = .78$, $\theta_{12} = .52$, $\Delta m^2_{12} = 1 \cdot 10^{-4}$ eV$^2$ and $\Delta m^2_{13} = 2.8 \cdot 10^{-3}$.
eV^2. We stress that although we have chosen a point in the LMA region, the particular election of both $\Delta m_{12}^2$ and $\theta_{12}$ does not affect the quality of the agreement with the data.

With these parameters we have calculated the zenith angle dependence of the ratio (observed/expected in the no oscillation case) for muon and electron neutrinos for the sub-GeV and multi-GeV energy ranges (remember that since Super-K is a water Cherenkov detector it does not distinguish neutrinos from antineutrinos and washes out any possible difference between the conjugated channels). The results are shown in Fig. 3 where we have also included the experimental data for the sake of comparison. As we have closely followed the spirit of the calculation in [16, 3], we refer the reader to this article for details and skip the technicalities. We worked in a complete three generation framework and included matter effects.

In Fig. 4 we show the comparison to Super-K for our second example point. For this point we have chosen $\theta_{13} = 0.08$, $\theta_{23} = 0.5$, $\theta_{12} = 0.785$, $\Delta m_{12}^2 = 7 \cdot 10^{-5}$ eV$^2$ and $\Delta m_{13}^2 = O(1)$ eV$^2$. Note that this point is consistent with the best-fit point of KamLAND [10].

3 Comparison with other analyses

It is clear from Figs. 3 and 4 that our CPT violating spectrum does pretty well, with a moderate discrepancy apparent only for the multi-GeV muons. In order to understand the results it is important to remember that due to production and cross section effects Super-K is dominated by neutrinos, with antineutrinos a minor (but not negligible) contribution. This is only part of the story, however, since an analysis done by the Super-K collaboration allowing for CPT violation did not allow (at 99% C.L.) a mass difference in the antineutrino sector as drastically different as the one we are proposing. The difference is due to the fact that the Super-K analysis was done in a two generation context and, most importantly, forcing the two mixing angles to be maximal. This latter fact indeed maximizes the antineutrino contribution and compels the antineutrino mass difference to take the closest possible value to the neutrino one. By the same token, if we want a large antineutrino mass difference, we expect to improve the fit to Super-K data by combining
this with a non-maximal mixing angle, which suppresses the antineutrino contribution to the Super-K oscillation signal.

The two vs three generation analysis has also an impact, as is seen by inspecting the transition probability for muon antineutrinos into tau antineutrinos, which is given by,

\[ P(\nu_\mu \rightarrow \nu_\tau) = 4U_{\mu 3}^2U_{\tau 3}^2\sin^2\left(\frac{\Delta m^2_{23}L}{4E}\right) - 4U_{\mu 2}U_{\tau 2}U_{\mu 1}U_{\tau 1}\sin^2\left(\frac{\Delta m^2_{12}L}{4E}\right) \]  

(3.1)

From this formula it becomes apparent that for neutrinos coming from above only the largest mass difference contributes. However, for those neutrinos which have travelled through sizeable portions of the Earth and have covered distances of the order of \(10^4\) km, the second mass difference also plays a role. This contribution (which does affect the final result, especially for sub-GeV neutrinos) is neglected if only one mass difference is taken into account.

Our analysis agrees with the spirit of the findings in Ref [17] where a two generation approximation that didn’t include matter effects was used. Also a simplified analysis based only on the up/down asymmetry in the number of multi-GeV events (in the CPT violating case) is available in the literature [18], which used an older Super-K data set. If one uses (as we do) the result from the full 1490 day of SK-I data, \(i.e. A_\mu = -0.288 \pm 0.030\) [19] the CPT violating case (which gives for the sample points we have been using \(A_\mu = -0.27\) turns out to be as acceptable as the CPT conserving one (with \(A_\mu = -0.33\) for maximal mixing but where smaller values can be obtained by departures of maximal mixing). In all the cases the electron neutrino asymmetry is consistent (within experimental errors) with zero.

In order to get a quantitative estimate of how well our CPT violating spectrum fits the global data set not including LSND, we have compared (for both of our sample points) the total \(\chi^2\) to the minimum total \(\chi^2\) which is obtained for different values of the 10 input parameters. Fixing \(\theta_{13} = \bar{\theta}_{13} = 0.08\) our (admittedly crude) program finds that the minimum total \(\chi^2\) occurs for \(\theta_{23} = \overline{\theta}_{23} = 0.8, \theta_{12} = \overline{\theta}_{12} = 0.6, \Delta m^2_{12} = \Delta m^2_{13} = 6 \cdot 10^{-5}\) eV\(^2\), and \(\Delta m^2_{13} = \Delta m^2_{13} = 3 \cdot 10^{-3}\) eV\(^2\). This agrees rather well with more sophisticated best fit analyses. We find the \(\Delta \chi^2\) for either of our sample points to be smaller than 9.6.
Making the conservative observation that variations in $\theta_{13}$, $\bar{\theta}_{13}$ don’t affect the total $\chi^2$ as long as the central values are small, we can interpret $\Delta \chi^2 \geq 9.6$ for 8 parameters as occurring with probability 29% due to Gaussian fluctuations in the global data set.

However this result is questionable since, as it happens, the minimum total $\chi^2$ is obtained for values of the 8 input parameters which are very close to $CPT$ conserving. Thus one can argue that we are only interested in contributions to $\Delta \chi^2$ arising from fluctuations in the data which simulate variation in the 4 independent $CPT$ violating parameters. In that case we are dealing with $\Delta \chi^2 \geq 9.6$ for 4 parameters, which occurs with probability 5%.

After our model was proposed it was pointed out by Gonzalez-Garcia, Maltoni and Schwetz [11] that the comparison with Super-K atmospheric data needs to include the zenith angle dependence of the thru-going muons. Because the data contains too many events near the horizon compared to the prediction of our model, this turns out to give a large contribution to the $\Delta \chi^2$ in an analysis like that described above. Using a $CPT$ violating spectrum very close to our sample points, these authors obtain a $\Delta \chi^2 = 12.7$, which for 4 parameters implies a fluctuation with probability 1.3%. This is not especially encouraging, although it is worth pointing out that the Standard Model fit to the global electroweak data set has a confidence level of only $CL = 0.02$ [20].

Thus, if the LSND experiment did not exist, we would have concluded that our model has a confidence level of $CL = 0.01$. The fact that it competes well with the $CPT$ conserving scenario once LSND is included is an indication that LSND is not statistically very compatible with the rest of the global data set. Indeed Gonzalez-Garcia, Maltoni and Schwetz have estimated, using their own customized goodness-of-fit test, that the LSND and all-but-LSND data sets are only statistically compatible at confidence level $CL = 7.5 \times 10^{-4}$. It is important to note that this stringent result, although it employs the $CPT$ violating parameter space, does not represent the confidence level for our model. As seen above, the confidence level for our model if LSND did not exist would be $CL = 0.01$. With the entire global data set including LSND, the confidence level is $CL = 0.74$ ($\Delta \chi^2 \geq 2$ for 4 parameters).
4 Outlook

Once we have established that a \( CPT \) violating mass spectrum as the one shown in Fig.
2 can account for all the available experimental evidence data, it is time to ask how we
might confirm \( CPT \) violation in future data.

The most straightforward answer is through experiments able to run in both modes (neu-
trino and antineutrino), by simple comparison of the conjugated channels. The first of
them is MiniBooNE, which is meant to close the discussion about LSND one way or the
other. MiniBooNE is taking data and is expected to give a definite answer to the \( CPT \)
question after some years of running. Needless to say we expect MiniBooNE to confirm
LSND only when running in the antineutrino mode.

For our type of spectrum, the observation of atmospheric neutrinos using the MINOS
detector \([12]\) is also ideal. Because the MINOS detector discriminates positive and neg-
ative charge, this experiment can disentangle the neutrino and antineutrino components
of atmospheric oscillations in a straightforward way. As the mass differences in the atmo-
spheric sectors differ by orders of magnitude in our scenario, MINOS will be able to tell
them apart easily.

A positive oscillation signal at KamLAND (which could be a misidentification of a \( CPT \)
violating spectrum as LMA) and Borexino \([21]\) finding a day/night asymmetry (evidence
of a LOW solution \([22]\) ) or a seasonal variation (an indication of VAC \([22]\) ) will point
towards \( CPT \) violation. Indeed a conflict between KamLAND and Borexino results would
constitute strong evidence for \( CPT \) violation even if LSND is disconfirmed by MiniBooNE.

It is worth mentioning that, despite the accumulation of strong evidence for the LMA
solution, some interesting discrepancies remain \([23]\).

All in all, \( CPT \) violation has the potential to explain all the existing evidence about
neutrinos with oscillations to active flavors. Such a scenario makes distinctive predictions
that will be tested in the present round of neutrino experiments. One should always bear
in mind that so far we have no evidence of \( CPT \) conservation in the neutrino sector. The
true status of \( CPT \) in the neutrino sector might be established by the combined results of
KamLAND, Borexino and SNO, and certainly by MiniBooNE. In the atmospheric sector MINOS is the ideal experiment for such a test.

Acknowledgments

We are grateful to André de Gouvêa, Bill Louis, Michele Maltoni, and Steve Mrenna for comments and assistance. This research was supported by the U.S. Department of Energy Grant DE-AC02-76CHO3000.

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Figure 3: *SK zenith angle distributions normalized to no-oscillations expectations, for our first CPT violating example. Circles with error bars correspond to SK data.*
Figure 4: *SK* zenith angle distributions normalized to no-oscillations expectations, for our second CPT violating example. Circles with error bars correspond to *SK* data.