Magnetic impurity resonance states for different pairing symmetries in twisted bilayer graphene

Liang Chen, Hui-Zhen Li and Rong-Sheng Han

Mathematics and Physics Department, North China Electric Power University, Beijing 102206, People’s Republic of China

E-mail: slchern@ncepu.edu.cn

Received 22 September 2018, revised 30 November 2018
Accepted for publication 4 December 2018
Published 21 December 2018

Abstract
In this work, we study the magnetic impurity resonance states in the superconducting phase of 'magic' angle twisted bilayer graphene for different pairing symmetries. Using a two-orbital model on the emergent honeycomb lattice, we find that the resonance states are dramatically different for s-wave pairing and topological nontrivial pairings. When the magnetic impurity is located at one site of the emergent honeycomb lattice, i.e. the center of the AB spot of the moiré pattern, the spacial distributions of the resonance states will break both the threefold and twofold rotation symmetries of \( D_3 \) point group for pairing symmetries which belong to the irreducible representations of this point group. When the magnetic impurity is located at the center of the emergent honeycomb lattice i.e. the center of the AA spot of the moiré pattern, the appearance of resonance peak at the position close to the impurity can be considered as a strong evidence of non-s-wave pairing.

Keywords: pairing symmetry, magnetic impurity, twisted bilayer graphene

(Some figures may appear in colour only in the online journal)
wave pairing has been derived using the random phase approximation perturbation theory. It is demonstrated that the $d_{x^2-y^2}$ and $d_{xy}$-wave pairings dominate near the half-filling points. The unbiased quantum Monte Carlo method is applied to study the dominated pairing symmetry of this material in [10] and [12]. By calculating the effective pairing susceptibility near the half-filling points, both [10] and [12] find that the $d_{x^2-y^2}$ and $d_{xy}$-wave pairings dominate over the $s$-wave and $p$-wave pairings. In addition, the $p$-wave pairing [14, 15], $s^{\pm}$ and $s^{++}$ pairings [16], and $f$-wave pairing [17, 18] are also potential candidates according to different conditions. Some other theoretical investigations show that the TwBG exhibits strong electron-phonon coupling for some special twisted angles and electron densities, which leads to a remarkable high critical temperature of magnitude $1 \sim 10$ K [19–21]. As demonstrated in these works, the pairing potential mediated by electron-phonon coupling reveals the $s$-wave symmetry. Further investigations show that [19], if the electron-electron interaction is sufficient large, the $d$-wave symmetric pairing is also possible for the electron-phonon coupling mechanism. Among the comprehensive investigations on the nature of superconductivity in this material, the determination of pairing symmetry is of fundamental importance.

In this paper, we study the magnetic impurity resonance states for typical pairing symmetries in the superconducting phases of TwBG. Like the famous Kondo resonance peak for magnetic doping in normal conductor [22], the magnetic impurity in superconductor can induce some (typically two) resonance peaks in the superconducting gap. These resonance states are well known as the Yu–Shiba–Rusinov quasiparticle states [23–25]. Experimentally, the phase-sensitive tetracrytal measurements [26, 27], quasiparticle interference pattern [28–31] and the local density of states (LDOS) near a magnetic (nonmagnetic) impurity are the most frequently used methods to uncover the pairing symmetries of unconventional superconductors [32]. The magnetic impurity resonance state is widely studied in the high-$T_c$ cuprate superconductors [33–42], iron-based superconductors [43–45], chiral $p$-wave superconductors and topological superconductors [46–49], etc. Theoretically, the quantum Monte Carlo simulation [50–55] and numerical renormalization group calculation [56–65] are the most commonly-used methods to study effects of magnetic impurity on superconductors. In this paper, we use the slave-boson mean field theory [66, 67] to study the magnetic impurity resonance states. It is expected to capture the qualitative physics in the strong correlation regime at zero temperature [57].

The paper is organized as follows: in section 2, we construct the total Hamiltonian with three parts, (1) the Bogoliubov–de Gennes (BdG) Hamiltonian of superconducting TwBG for four kinds of different pairing symmetries, the $s$-wave, extended $s$-wave, chiral $p$-wave, and $d + id$-wave pairings, (2) the impurity Hamiltonian with strong Coulomb interaction, and (3) the hybridization between the magnetic impurity and TwBG. We give a theoretical derivation of the LDOS at the end of this section. In section 3, we analyze the numerical results for different pairing symmetries and different filling ratios. According to the three different irreducible representations of the $D_3$ point group, the LDOS for three kinds of different hybridizations are presented. A conclusion is given in section 4.

2. Theoretical model

2.1. Tight binding model

Figure 1 shows the geometry of the model Hamiltonian studied in this work. Figure 1(a) presents the moiré pattern of TwBG, a small twisted angle between the two graphene layers.
makes the unit cell been enlarged significantly. In figure 1(a), we chose the twisted angle $\theta$ to be $6.01^\circ$ for viewing, for the practical situation, the twisted angle is tuned to the first ‘magic’ angle, $\theta \approx 1.05^\circ$, there are more than 1000 atoms in the supercell. Previous theoretical investigations show that the low energy bands near the Fermi surface can be described by a two-orbital model on the emergent honeycomb lattice located at the AB and BA spots of the moiré pattern [68–71]. Figure 1(b) shows the primitive vectors, unit cell, and the $p_{x, y}$-orbitals of this emergent honeycomb lattice. We use the two-orbital tight-binding model proposed by Yuan and Fu [68] with a few modifications to describe the low-energy physics of TwBG, which is expressed in the following form,

$$H_0 = \sum_{\mathbf{k}} \psi^\dagger \mathbf{h}(\mathbf{k}) \psi,$$

(1)

$$\mathbf{h}(\mathbf{k}) = \sum_{ij} \epsilon_{ij}(\mathbf{k}) \tau_i \otimes \chi_j,$$

(2)

where $i, j = 0, x, y, z$, $\tau_0$ and $\chi_0$ are the 2 $\times$ 2 identity matrices in the emergent AB-BA sublattice space and $\{\tau_i, \tau_j\}$ orbital space, respectively. $\tau_{xy}$ and $\chi_{xy}$ are the corresponding Pauli matrices in these spaces. $\psi_\mathbf{k} = (\psi_{\mathbf{A}B, \mathbf{a}, \mathbf{b}}, \psi_{\mathbf{B}A, \mathbf{a}, \mathbf{b}}, \psi_{\mathbf{A}B, \mathbf{c}, \mathbf{a}}, \psi_{\mathbf{B}A, \mathbf{c}, \mathbf{a}}) \mathbf{T}$, $c_{\mathbf{a}, \mathbf{b}, \mathbf{k}}$ is the annihilation operator of electron state for specified sublattice $\mathbf{a} = \mathbf{A} \mathbf{B}$, orbital $\mathbf{o} = \mathbf{p}_x, \mathbf{p}_y$, and wave-vector $\mathbf{k}$ (the spin index is suppressed here). The superscript $\mathbf{T}$ means matrix transpose. The summation of wave-vector $\mathbf{k}$ in equation (1) is taken over the MBZ of TwBG. The explicit expressions of $\epsilon_{ij}(\mathbf{k})$ are written as,

$$\epsilon_{x,0}(\mathbf{k}) = t_1 \left( \cos \frac{k_x}{\sqrt{3}} + 2 \cos \frac{k_x}{2\sqrt{3}} \cos \frac{k_y}{2} \right),$$

(3)

$$\epsilon_{y,0}(\mathbf{k}) = t_1 \left( \sin \frac{k_x}{\sqrt{3}} - 2 \cos \frac{k_x}{2\sqrt{3}} \cos \frac{k_y}{2} \right),$$

(4)

$$\epsilon_{0,0}(\mathbf{k}) = 2t_2 \left( \cos \frac{\sqrt{3}k_x}{2} \cos \frac{k_y}{2} + \cos k_y \right)$$

$$+ t_1' \left( \cos \frac{\sqrt{3}k_x}{2} \cos \frac{k_y}{2} + \cos k_y \right) - \mu,$$

(5)

$$\epsilon_{x,\pm}(\mathbf{k}) = t_1' \left( \cos \frac{k_x}{\sqrt{3}} - \cos \frac{k_x}{2\sqrt{3}} \cos \frac{k_y}{2} \right),$$

(6)

$$\epsilon_{y,\pm}(\mathbf{k}) = t_1' \left( \sin \frac{k_x}{\sqrt{3}} + \sin \frac{k_x}{2\sqrt{3}} \cos \frac{k_y}{2} \right),$$

(7)

$$\epsilon_{0,\pm}(\mathbf{k}) = \sqrt{3}t_1' \sin \frac{k_x}{2\sqrt{3}} \sin \frac{k_y}{2},$$

(8)

$$\epsilon_{x,\mp}(\mathbf{k}) = -\sqrt{3}t_1' \cos \frac{k_x}{2\sqrt{3}} \sin \frac{k_y}{2},$$

(9)

$$\epsilon_{y,\mp}(\mathbf{k}) = -\sqrt{3}t_1' \cos \frac{k_x}{2\sqrt{3}} \sin \frac{k_y}{2},$$

(10)

$$\epsilon_{0,\mp}(\mathbf{k}) = t_1' \left( \cos \frac{\sqrt{3}k_x}{2} \cos \frac{k_y}{2} - \cos k_y \right),$$

(11)

where $\mu$ is the Fermi energy, $t_1$ is the normal hopping amplitude between nearest neighbors within different sublattices, $t_2$ denotes the normal hopping amplitude between next nearest neighbors within the same sublattice. The term with $t_2'$ contained, equation (12), represents the inter-sublattice hopping amplitude between the fifth nearest neighbors with definite chirality. As shown in [68], this term, equation (12), breaks the emergent $SU(4)$ symmetry and hence the fourfold degeneracy (orbital and spin) along the $\Gamma M$ line in the MBZ is split.

The hopping amplitudes with a single prime in the superscript, i.e. $t_1'$ and $t_2'$, refer to the special hopping amplitudes with inter-orbital scatterings included. For this special case, $t_1'$ and $t_2'$ represent the hopping amplitudes between nearest and next nearest neighbors, respectively. The terms with $t_1''$ and $t_2''$ contained, i.e. equations (6)–(10) and the second term in equation (5), further break the orbital $U(1)$ symmetry and split the fourfold degeneracy along the $K\Gamma$ and $MK$ lines in the MBZ [68]. It is easy to check that this Hamiltonian preserves all the symmetries proposed in [68]. By comparing the model Hamiltonian (1) with that given in [68], one can find that the only difference is the terms proportional to $t_2''$ are presented in our model Hamiltonian. These terms with $t_2''$ contained are straightforward generalizations of the $t_1''$ terms from nearest neighbor hopping to next nearest neighbor hopping. Furthermore, it is easy to check that, in the special case $t_2'' = 0$, our model Hamiltonian is consistent with the tight-binding Hamiltonian proposed in [9] via the following relationships, $t_1 = \frac{1}{2}(t_1^{(1)} + t_1^{(1)}), t_2 = t_2^{(2)}, t_1' = \frac{1}{2}(t_1^{(1)} - t_1^{(1)}),$, and $t_2' = t_2^{(2)} - t_2^{(2)}$. As defined in [9], $t_\sigma$ and $t_\pi$ are the Slater-Koster parameters referring to the hopping integrals contributed by $\sigma$ and $\pi$ orbitals, respectively. The superscript (1) and (2) refer to the nearest and next nearest neighbor hopping amplitudes.

Previous theoretical investigations show that the TwBG structures may belong to one of two different point groups, $D_3$ and $D_6$ [68–71]. Detailed analysis shows that the rotation symmetry is dependent on the location of the rotation center. In this work, we assume that the rotation center is located at the corner of the honeycomb lattice [4], in this case, the $D_3$ point group is identified. Table 1 shows the group elements and irreducible representations of $D_3$ point group. For the Hamiltonian given in equation (1), the specified representations of the group elements are presented in the caption of table 1. The parameters $t_1, t_2, t_1'$ and $t_2'$ for numerical calculation are chosen to fit the band structure given in [9]. $t_2''$ has been set to be zero, the effect of equation (12) with $t_2''$ contained will be discussed at the end of section 3. Figure 2 shows the band structure and density of states (DOS) of the model Hamiltonian with the chosen parameters. The three dashed lines in figure 2(b) from top to bottom denote the typical
Table 1. Character tab for point group $D_3$, $A_1$, $A_2$ and $E$ in the first column represent the three irreducible representations. $E$ in the first row refers to the identity element of the $D_3$ point group. $2\gamma_d$ means the two group elements related to threefold rotation operations along the $z$-axis, under the basis shown in the context below equation (2), these group elements are given by, $e^{i \gamma_d} = \gamma_0 \otimes (\frac{1}{\sqrt{2}} x_0 - i \frac{1}{\sqrt{2}} x_1)$ and the square of $e^{i \gamma_d}$, which is also the inverse of $e^{i \gamma_d}$, denoted as $e^{2 i \gamma_d}$. $3\gamma_d'$ in the first row means the three group elements related to the twofold rotation operations along the three symmetric axes, under the basis shown in the context below equation (2), these group elements are given by $e^{i \gamma_d'} = \gamma_0 \otimes \gamma_1 \otimes \gamma_2$, $e^{i \gamma_d'} \otimes \gamma_2$, and $e^{i \gamma_d'} \otimes \gamma_1$. The last three columns show the exemplified polynomial representations of $D_3$ point group.

| $D_3$ | $E$ | $2\gamma_d$ | Linear | Quadratic | Cubic |
|-------|-----|-------------|--------|-----------|-------|
| $A_1$ | $+1$ | $+1$ | $+1$ | $x^2 + y^2$ | $x(x^2 - 3y^2)$ |
| $A_2$ | $+1$ | $+1$ | $-1$ | $y(3x^2 - y^2)$ | |
| $E$   | $+2$ | $-1$ | $0$ | $(x, y)$ | $(x^2 - y^2, xy)$ | $(x^3 + xy^2, x^2y + y^3)$ |

Figure 2. (a) Band structure of the tight-binding model Hamiltonian (1) for $t_1 = 1$, $t_2 = t_1/9$, $t_1' = t_1/4$, $t_1'' = t_1/4$, $t_2' = 0$, and $\mu = 0$. (b) The corresponding DOS. The dashed lines refer to the chemical potentials corresponding to different charge densities, i.e., $+2e$, $0$ and $-2e$ per unit cell from top to bottom.

charge densities $+2e$, $0$ and $-2e$ per unit cell of the superlattice, respectively. One can find that charge density per unit cell equals to $2e$ and $-2e$ are close to two Van Hove singularities of the band structure. The Van Hove singularity in TwBG [72–75] plays an important role in the optical properties of this material [76, 77], e.g., the enhanced optical absorption [78], the selectively enhanced photocurrent generation [79], etc. It is also suggested that the superconductivity of ‘magic’ angle TwBG is induced by the Van Hove singularity [72, 75]. Figure 3 shows the Fermi surfaces of these half-filled bands (charge density $= \pm 2e$ per unit cell) and the near-by Van Hove singularities. When the relatively strong electron-electron interactions are presented, TwBG can be driven into the Mott insulator phase at half-filling. The experiment [1] shows that there is a superconducting phase when the electron density is doped slightly away from $\pm 2e$ per unit cell. In the following subsection, we consider the superconducting phase.

Figure 3. Fermi surfaces for different chemical potentials. (a) $\mu = -1.7$ (charge density $= -2e$ per unit cell). (b) $\mu = 1.5$ (charge density $= +2e$ per unit cell). (c) $\mu = -1.6945$ (the Van Hove singularity close to $\mu = -1.7$). (d) $\mu = 1.3058$ (the other Van Hove singularity close to $\mu = 1.5$). The dashed line represents the MBZ. The colors for different pockets of the Fermi surfaces are specified to correspond to the colors of band structure shown in figure 2(a).

2.2. Pairing symmetries

Generally, the pairing potential can be written as,

$$H_{\text{pairing}} = \sum_{k, s, s'} \psi_{k, s}^T \Delta_{\tau, \sigma'}(k) \psi_{k, s'} + h.c.,$$

(13)

where $s, s' = \uparrow, \downarrow$ refer to the spin of electron states, h.c. means Hermitian conjugate, $\Delta_{\tau, \sigma'}(k)$ is the pairing potential matrix. In this work, we consider only the spin-singlet pairing (potential spin-triplet pairings are discussed in [11, 18, 80]), so that the summation over spin indices contains only one situation, $s = \uparrow$ and $s' = \downarrow$. Without causing confusion, we suppress the superscript of pairing potential. In this case, $\Delta(k)$ is a $4 \times 4$ matrix in the orbital and sublattice spaces. Following the previous studies [8–14, 16, 19–21], here we consider four different pairing potentials. All of them belong to the intra-orbital pairing. The first one is the on-site $s$-wave pairing,

$$\Delta_s(k) = \Delta_s \gamma_0 \otimes \chi_0,$$

(14)

the second one is the nearest neighbor (extended) $s$-wave pairing,

$$\Delta_{s'}(k) = \Delta_{s'} \left[ \left( \cos \frac{k_x}{\sqrt{3}} + 2 \cos \frac{k_x}{2\sqrt{3}} \cos \frac{k_y}{2} \right) \tau_x \otimes \chi_0 + \left( \sin \frac{k_x}{\sqrt{3}} - 2 \cos \frac{k_x}{2\sqrt{3}} \sin \frac{k_y}{2} \right) \tau_y \otimes \chi_0 \right],$$

(15)
the third one is the nearest neighbor \textit{p}-wave pairing,
\begin{equation}
\Delta_p(k) = \Delta_p \left[ \cos \frac{k_x}{\sqrt{3}} - \cos \frac{k_y}{2\sqrt{3}} \cos \frac{k_z}{2} + i \sqrt{3} \sin \frac{k_x}{2 \sqrt{3}} \sin \frac{k_y}{2} \sin \frac{k_z}{2} \right] \tau_y \otimes \chi_0 + \left( \sin \frac{k_x}{\sqrt{3}} + i \sqrt{3} \cos \frac{k_x}{2 \sqrt{3}} \cos \frac{k_y}{2} \sin \frac{k_z}{2} \right) \tau_y \otimes \chi_0 ,
\end{equation}
and the last one is the \textit{d} + \textit{id}-wave pairing between next nearest neighbor sites,
\begin{equation}
\Delta_{d+\text{id}}(k) = 2\Delta_{d+\text{id}} \left[ \cos k_y - \cos \frac{k_x}{2} \cos \frac{\sqrt{3} k_z}{2} + i \sqrt{3} \sin \frac{k_y}{2} \sin \frac{\sqrt{3} k_z}{2} \right] \tau_0 \otimes \chi_0 ,
\end{equation}
where the constants \Delta_\textit{p}, \Delta_\textit{s}', \Delta_\textit{p} and \Delta_{d+\text{id}} represent the strength of the pairing potentials for different symmetries. It is easy to check that the \textit{s}-wave and extended \textit{s}-wave pairing potentials belong to the one-dimensional \textit{A}_1 representation of the \textit{D}_3 point group, the \textit{p}-wave and \textit{d} + \textit{id}-wave pairings belong to the two-dimensional representation (the \textit{E} representation shown in table 1) of the \textit{D}_3 point group. Figures 1(c) and (d) show the nearest neighbor and the next nearest neighbor pairing potentials in superlattice space.

2.3. Hybridizations

The total Hamiltonian for a magnetic impurity coupled to the superconducting TwBG is consisted of four terms,
\begin{equation}
H = H_{\text{imp}} + H_{\text{hyb}} + H_0 + H_{\text{pairing}},
\end{equation}
\begin{equation}
H_{\text{imp}} = \sum_x \epsilon_x d_x^\dagger d_x + U d_x^\dagger d_x^\dagger d_x d_x^\dagger ,
\end{equation}
\begin{equation}
H_{\text{hyb}} = \frac{1}{\sqrt{N}} \sum_{k,x} \left( \psi_k^\dagger V_k d_x + \text{h.c.} \right) ,
\end{equation}
where \(\epsilon_x\) is the impurity energy, \(U\) is the on-site Coulomb interaction, \(d_x\) and \(d_x^\dagger\) are the creation and annihilation operators of spin-\textit{s} impurity state, respectively, \(N\) is the total number of wave-vectors in the MBZ. \(V_k\) is the \(4 \times 1\) hybridization matrix between the impurity state and the conduction state with wave-vector \(k\). This term has to be considered very carefully. Generally, the impurity Hamiltonian (19) belongs to the \textit{A}_1 representation of the \textit{D}_3 point group, and the two Wannier orbitals belong to the two-dimensional representation of the \textit{D}_3 point group. If the impurity is coupled to only one site of the emergent honeycomb lattice, i.e. shown by equation (23) in the following context, the hybridization equation (20) will break both the threefold and twofold rotation symmetries. We study the \textit{E}_1 rotation symmetric hybridizations first. We consider that the impurity is located at the center of the emergent honeycomb lattice, i.e. the center of an \textit{AA} spot of the moiré pattern. The impurity is hybridized to the six nearest neighbors symmetrically, detailed analysis shows that there are only two different hybridizations belong to the two-dimensional representation of \textit{D}_3 point group, figure 4 shows the form of these hybridizations in real space. Their explicit expressions in wave-vector space are given by,
\begin{equation}
V_k^{(1)} = V_0 \begin{pmatrix} 2 \left( e^{i \frac{\sqrt{3} \pi}{6}} - e^{-i \frac{\sqrt{3} \pi}{6}} \cos \frac{k_x}{2} \right) \\ 2 \sqrt{3} e^{i \frac{\sqrt{3} \pi}{6}} \sin \frac{k_x}{2} \\ -2 \left( e^{-i \frac{\sqrt{3} \pi}{6}} - e^{i \frac{\sqrt{3} \pi}{6}} \cos \frac{k_x}{2} \right) \\ -2 \sqrt{3} e^{-i \frac{\sqrt{3} \pi}{6}} \sin \frac{k_x}{2} \end{pmatrix} ,
\end{equation}
\begin{equation}
V_k^{(2)} = V_0 \begin{pmatrix} 2 \sqrt{3} e^{-i \frac{\sqrt{3} \pi}{6}} \sin \frac{k_x}{2} \\ -2 \left( e^{i \frac{\sqrt{3} \pi}{6}} - e^{-i \frac{\sqrt{3} \pi}{6}} \cos \frac{k_x}{2} \right) \\ -2 \sqrt{3} e^{i \frac{\sqrt{3} \pi}{6}} \sin \frac{k_x}{2} \\ -2 \sqrt{3} e^{-i \frac{\sqrt{3} \pi}{6}} \sin \frac{k_x}{2} \end{pmatrix} ,
\end{equation}
where \(V_0\) represents the strength of the hybridizations. Next, we study the \textit{E}_1 rotation symmetry broken hybridization. We consider that the impurity is located at the center of an \textit{AB} spot and hybridized to the \textit{p}_1- and \textit{p}_2-orbitals equally at this emergent site. In this case, the hybridization matrix in equation (20) can be written as,
\begin{equation}
V_k^{(3)} = (V_0, V_0, 0, 0)^T .
\end{equation}
We need to emphasize that, in this work, the original point is set to be located at the impurity site, which means that, for the hybridizations \(V_k^{(1)}\) and \(V_k^{(2)}\), the original point is located at the center of the emergent honeycomb lattice, for the hybridization \(V_k^{(3)}\), the original point is located at the center of one of the \textit{AB} spot of the moiré pattern.

2.4. LDOS

Now we consider the solution of the total Hamiltonian (18). In the strong Coulomb interaction limit, \(U \rightarrow \infty\), the double
occupation state on the impurity site can be excluded. This condition may be represented by introducing the slave-boson operators \( b \) and \( b^\dagger \), the fermionic operators can be recast as \( d_s = f_s b^\dagger \), \( d_s^\dagger = bf_s^\dagger \). The extra degrees of freedom can be ruled out by the no-double occupation condition, \( Q = b^\dagger b + \sum_i f_s f_s^\dagger = 1 \). In the mean-field approximation, the slave-boson operators can be replaced by the expectation value \( \langle b \rangle = \langle b^\dagger \rangle = b_0 \), the constraint condition can be approximated by introducing a Lagrangian multiplier term to the Hamiltonian, \( \lambda_0 (b_0^2 + \sum_i f_s f_s^\dagger - 1) \). The parameters \( b_0 \) and \( \lambda_0 \) can be determined by minimizing the free energy of the mean-field Hamiltonian. Following these operations, we get the mean-field Hamiltonian,

\[
H_{\text{MF}} = H_{\text{imp}}^{\text{MF}} + H_{\text{hyb}} + H_0 + H_{\text{pairing}} + \lambda_0 (b_0^2 - 1),
\]

where \( \tilde{c}_d = c_d + \lambda_0 \) is the renormalized impurity energy, \( \tilde{V}_d = b_0 V_d \) is the renormalized hybridization matrix. In the BdG formalism, the mean-field Hamiltonian can be recast as,

\[
H_{\text{BdG}} = \lambda_0 (b_0^2 - 1) + \tilde{c}_d + \Phi^\dagger \Lambda \Phi + \sum_k \psi_k^\dagger \chi_k \Psi_k + V_{\text{pairing}} \Psi_k^\dagger \Phi + \text{h.c.},
\]

where the Nambu spinor is defined as, \( \Psi_k = (\psi_{k,\uparrow}, \psi_{k,\downarrow}^\dagger) \), \( \Phi = (f_{\uparrow}, f_{\downarrow}^\dagger) \), \( \Lambda = \tilde{c}_d \gamma_0 \otimes \chi_0 \otimes \varsigma_y \),

\[
h_{\text{BdG}}(k) = \left[ \begin{array}{cc}
\Delta(k) & -h(k) \\
-h(k) & \Delta^\ast(k)
\end{array} \right], \quad \chi_k = \left[ \begin{array}{cc}
\tilde{V}_k & 0 \\
0 & -\tilde{V}_k
\end{array} \right],
\]

\( \varsigma_y \) is the third Pauli matrix in the Nambu spinor space.

Using the standard functional integration method in quantum field theory, we find that the finite temperature free energy of the magnetic impurity can be written as,

\[
\mathcal{F} = \lambda_0 (b_0^2 - 1) + \tilde{c}_d + k_B T \sum_n \text{Tr} \ln [G_f (i\omega_n)],
\]

where \( k_B \) is the Boltzmann constant and \( T \) refers to temperature, \( \omega_n = (2n + 1) \pi k_B T \) is the Matsubara frequency of fermion, \( G_f (i\omega_n) = \omega_n - \Lambda - \Sigma_f (i\omega_n) \) is the Green’s function of impurity state expressed in imaginary frequency representation, \( \Sigma_f (i\omega_n) \) is the self-energy of the impurity state. It is given by, \( \Sigma_f (i\omega_n) = \frac{1}{N} \sum_k \chi_k^\dagger G_f^{(0)} (i\omega_n, k) \chi_k \), where \( G_f^{(0)} (i\omega_n, k) = [i\omega_n - h_{\text{BdG}}(k)]^{-1} \) is the unperturbed Green’s function of the conduction states in superconducting TwBG. The parameters \( \lambda_0 \) and \( b_0 \) are determined by minimizing the free energy (29), which gives,

\[
b_0^2 + k_B T \sum_n \text{Tr} [G_f (i\omega_n) \varsigma_y] = 0.
\]

The LDOS near the magnetic impurity is obtained by analytic continuation of the imaginary-time Green’s function, \( i\omega_n \rightarrow E + i\epsilon_0 \),

\[
\rho_\psi (E, R) = -\frac{1}{\pi} \text{Im} \text{Tr} \left[ G_{\psi} (E; R, R) \left( 1 + \varsigma_y \otimes \gamma_0 \otimes \chi_0 \right)^2 \right],
\]

where \( G_{\psi} (E; R, R) \) is the full Green’s function of the conduction states, which is given by [36],

\[
G_{\psi} (E; R, R) = \frac{1}{N} \sum_{k k'} e^{i(k-k')R} G_{\psi} (E; k, k'),
\]

\( G_{\psi} (E; k, k') = G_{\psi}^{(0)} (E; k) \delta_{k k'} + T_{k k'} (E) G_{\psi}^{(0)} (E; k') \),

where \( T_{k k'} (E) = V_k G_f (E) V_{k'}^\dagger / N \) is the \( T \)-matrix.

3. Numerical results

Before the detailed discussion of the numerical results, we need to clarify that, as demonstrated in previous works [36, 81–86], there are two distinct phases for magnetic doping in superconductor and marginal Fermi liquid. When the hybridization between the impurity and host material is not strong enough, the impurity behaviors as an isolated impurity in superconductor and marginal Fermi liquid. When the \( \Delta_{\text{BdG}} \) is strong enough, the impurity hybridization between the impurity and host material is not strong enough, the impurity behaves as an isolated impurity in superconductor and marginal Fermi liquid. When the \( \Delta_{\text{BdG}} \) is strong enough, the impurity behaves as an isolated impurity in superconductor and marginal Fermi liquid. When the \( \Delta_{\text{BdG}} \) is strong enough, the impurity behaves as an isolated impurity in superconductor and marginal Fermi liquid. When the \( \Delta_{\text{BdG}} \) is strong enough, the impurity behaves as an isolated impurity in superconductor and marginal Fermi liquid. When the \( \Delta_{\text{BdG}} \) is strong enough, the impurity behaves as an isolated impurity in superconductor and marginal Fermi liquid. When the \( \Delta_{\text{BdG}} \) is strong enough, the impurity behaves as an isolated impurity in superconductor and marginal Fermi liquid.
The parameters are chosen as follows: $\Delta_3 = \Delta_y = \Delta_p = 0.08$, $\Delta_{d, id} = 0.02$, $V_0 = \Delta_x$, $\epsilon_y = -\Delta_y/4$.

These observations lead to a central conclusion in this work. Here we demonstrate from symmetry analysis that the non-vanishing resonance peak at the center of the AA spot (the center of the emergent honeycomb lattice) can be considered as a strong evidence of the unconventional pairings belonging to the two-dimensional representation or the $A_2$ representation of the $D_3$ point group. Generally, the in-gap resonance peak is induced by the impurity scattering in equation (34). If the pairing potential belongs to the $A_1$ representation, the BdG Hamiltonian (27) will be invariant under the $D_3$ point group operations, i.e. \( \rho_3 H_{\text{BdG}} \rho_3^{-1} = H_{\text{BdG}} \), and \( \rho_6 H_{\text{BdG}} \rho_6^{-1} = H_{\text{BdG}} \), so that $\rho_3 G_\psi^{(0)}(E,k) \rho_3^{-1} = G_\psi^{(0)}(E,\rho_3^{-1}k\rho_3)$, $\rho_6 G_\psi^{(0)}(E,k) \rho_6^{-1} = G_\psi^{(0)}(E,\rho_6^{-1}k\rho_6)$. Here $\rho_3$ and $\rho_6$ denote the threefold and twofold rotation operations given in table 1. The hybridizations $V_k^{(1)}$ and $V_k^{(2)}$ preserve the desired symmetries, $\rho_3 V_k^{(1,2)} = V_k^{(1,2)}$, \( \rho_6 V_k^{(1,2)} = V_k^{(1,2)} \rho_6^{-1} \), so we get,

\[
\rho_3 \sum_k G_\psi^{(0)}(E,k) V_k \rho_3^{-1} = \sum_k G_\psi^{(0)}(E,k) V_k,
\]

\[
\rho_6 \sum_k G_\psi^{(0)}(E,k) V_k \rho_6^{-1} = \sum_k G_\psi^{(0)}(E,k) V_k.
\]

It is easy to check that the only solution of these equations is $\sum_k G_\psi^{(0)}(E,k) V_k = 0$. Substituting this result into equations (34) and (33), one can find that, at the original point, $R = (0,0)$, the scattering term has no contribution. This analysis is evident for the $s$-wave pairing and extended $s$-wave pairing which belong to the $A_1$ representation. For the topological nontrivial $p$-wave and $d + id$-wave pairings, one can check that $\rho_3 \Delta_p(k) \rho_3^{-1} = e^{2\pi i/3} \Delta_p(c_{e_z}^{-1}k\epsilon_i)$,

\[
\rho_3 \Delta_{d, id}(k)c_{e_z}^{-1} = e^{2\pi i/3} \Delta_{d, id}(c_{e_z}^{-1}k\epsilon_i),
\]

so that the in-gap resonance peaks may be non-vanishing, as evident in figures 5(c) and (d), there does exhibit one resonance peak below the Fermi energy for each case (see the red lines with ‘×’ in these figures).

Previous theoretical investigations show that, both the quantum fluxes [87] and magnetic impurities [88] can induce Majorana bound states in topological superconductors [89–91]. We need to emphasize that the unique in-gap resonance peak presented here can not be regarded as the Majorana bound state, because the resonance peak is not located at $E = 0$. In addition, the numerical results show that the other resonance peak with opposite resonance energy appears at $R \neq (0,0)$. For example, the blue lines in figure 5(c) and (d) give the results for LDOS versus energy at the nearest neighbor site away from the impurity, $R = (-1/\sqrt{3},0)$. One can find that there are two resonance peaks in the superconducting gap. These results demonstrate that there do exist two in-gap resonance energies and they are not protected by particle-hole symmetry. The other related issue we need to emphasize is that the absence of resonance peak at $R = (0,0)$ does not mean that the resonance state is absent at other positions. The blue line in figure 5(b) shows LDOS versus energy at $R = (-1/\sqrt{3},0)$. One can find that there are two in-gap resonance peaks located at the two sides of the Fermi energy symmetrically. Figures 5(e) and (f) show the corresponding spacial distributions of these two resonance states. These two patterns are significantly different from each other, which demonstrates that the particle-hole symmetry is broken. Figures 5(g), (i) and (h), (j) show the spacial distributions of the resonance states near the magnetic impurity for the $p$-wave and $d + id$-wave pairings, respectively. One can find that all of these patterns reveal the sixfold rotation symmetry.
The maximum intensity is located at $R = (0, 0)$ in figures 5(g) and (h). For the other cases, figures 5(i) and (j), the maximum intensity is located at the bonds linking the nearest and next nearest neighbors away from the impurity.

Figure 6 shows the results for the second hybridization given in equation (22). For both these two kinds of hybridizations, one can find that the results for $p$-wave pairing and $d + id$-wave pairing have only quantitative differences. Here we summarize the similarities between $p$-wave and $d + id$-wave pairings. Firstly, at $R = (0, 0)$, as shown in figures 5(c), (d) and 6(c), (d), there is only one resonance peak below the Fermi surface (see the red lines with ‘+’ in these figures). Secondly, at the nearest neighbor site away from the impurity, $R = (-1/\sqrt{3}, 0)$, there are two resonance peaks located at the two sides of the Fermi energy (see the blue lines in the figures). Thirdly, the spacial distributions of LDOS are very similar to each other, see, e.g. figures 6(e) and (f). However, when we compare the results for different hybridizations, we find that $V_k^{(1)}$ and $V_k^{(2)}$ can be differentiated for $p$-wave and $d + id$-wave pairings. For example, the maximum intensity of the negative resonance energy is located at $R = (0, 0)$ for $V_k^{(1)}$, it is located on the bonds linking the nearest neighbors away from the impurity for $V_k^{(2)}$.

Now we analyze the results for the third hybridization given in equation (23). The first column in figure 7 shows the LDOSs versus energy for different pairing symmetries and at different positions. We find that, in this case, all of the four kinds of pairing symmetries exhibit two in-gap resonance peaks at $R = (0, 0)$ (see the red lines with ‘+’ in figures 7(a)–(d) and inserts therein). Another important difference is that, as shown in figures 7(e)–(l), the spatial distributions of the LDOSs break both the threefold rotation symmetry along the $z$-axis and the twofold rotation symmetry along the $y$-axis. Like the first two cases, the spatial distributions for the $p$-wave and $d + id$-wave pairings are very similar to each other, which demonstrates that the $p$-wave and $d + id$-wave pairings are difficult to be distinguished via the LDOS of magnetic impurity resonance states. However, the spatial distributions of $p$-wave and $d + id$-wave pairings at the negative resonance energy are significant different from the $s$-wave and the extended $s$-wave pairings. For example, the maximum intensity points for the latter are located at $R = (0, 0)$, for the former, they are located at $R \approx (0.25, -1.0)$.

Figure 8 gives another formulation of the threefold rotation symmetry breaking. We calculate the LDOS versus energy at three rotation symmetric points for the four kinds of pairing potentials. One can find that the intensities of the resonance peaks are significant different for the three different locations.

We have also calculated LDOSs for different chemical potentials, $\mu = -1.6945$ (the Van Hove singularity close to charge density $= -2e$ per unit cell), $\mu = 1.5$ (charge density $= +2e$ per unit cell), and $\mu = 1.3058$ (the Van Hove singularity close to charge density $= +2e$ per unit cell). We find that the results (i.e. the LDOSs versus energy and spatial distributions of the two in-gap resonance states) for $\mu = -1.6945$ are very close to that for $\mu = -1.7$, and the results for $\mu = 1.3058$ are qualitatively identical to that for $\mu = 1.5$. These results demonstrate that the Van Hove singularities do not have significant influence on the LDOS of magnetic impurity resonance states. Figure 9 gives an integrated presentation of LDOSs for $\mu = 1.3058$. We find...
Figure 7. LDOS for the third hybridization $\bar{V}^{(3)}$, the impurity is located at the center of one of the AB spot, $R = (0, 0)$. (a)–(d) LDOS as a function of energy for $s$-wave, extended $s$-wave, $p$-wave, and $d + id$-wave pairings, respectively. The gray lines with ‘+’ in (a)–(d) show the clean LDOS without impurity. The red lines with ‘×’ in (a)–(d) show the LDOS near the impurity $R = (0, 0)$. The blue lines in (a)–(d) show the LDOS at the nearest neighbor site $R = (−1/\sqrt{3}, 0)$. The second and third columns show the spacial distributions of LDOS corresponding to the two in-gap resonance peaks shown in the first column for each row. The inserts in (c) and (d) show the details of the resonance peaks. The silvery lines in (e)–(l) show the emergent honeycomb lattice. The parameters are chosen as follows: $\Delta_s = \Delta_p = \Delta_d = 0.08$, $\Delta_d + id = 0.02$, $\epsilon_d = −\Delta_s / 4$, $V_0 = \Delta_s$ for the $s$-wave (a) and extended $s$-wave (b) pairings, $V_0 = \Delta_p / 4$ for the $p$-wave pairing (c), and $V_0 = 2\Delta_d$ for the $d + id$-wave pairing (d).
that the central results obtained for \( \mu = -1.7 \) still hold: (1) when the impurity is hybridized to the two orbitals at one site of the emergent honeycomb lattice, the spacial distributions shown in the last two rows in figure 9 break both the threefold rotation symmetry along the \( z \)-axis and the twofold rotation symmetry along the \( y \)-axis, (2) when the impurity is located at the center of the emergent honeycomb lattice and hybridized to the nearest neighbors symmetrically, the spacial distributions of the resonance states exhibit sixfold rotation symmetry, (3) for the impurity location given in (2), the LDOS versus energy for \( s \)-wave pairing and extended \( s \)-wave pairing at \( \mathbf{R} = (0, 0) \) do not have in-gap resonance peak, and (4) for the impurity location given in (2), the LDOS versus energy for \( p \)-wave and \( d + id \)-wave pairings at \( \mathbf{R} = (0, 0) \) have only one in-gap resonance peak below the Fermi energy.

In addition, the maximum points in the spacial distributions of the LDOS need to be emphasized here. For the third hybridization \( \tilde{V}_k^{(3)} \), the spacial distributions of the LDOSs show similar characteristics for different chemical potentials, see, i.e. figure 7 and the last two rows of figure 9 for details. For the other two hybridizations, however, the results are different. Comparing all the contour plots in figures 5 and 6, one can find that there are only two subfigures where the maximum points are located at \( \mathbf{R} = (0, 0) \), figures 5(g) and (h). Both of them correspond to the \( \tilde{V}_k^{(1)} \) hybridization. When the chemical potential is doped to close to the other half-filling point, i.e. \( \mu = 1.308 \) shown in figure 9, we find that in which the maximum points located at \( \mathbf{R} = (0, 0) \) are figures 9(42) and (45). Both of them correspond to the \( \tilde{V}_k^{(2)} \) hybridization. Experimentally, when the impurity is located at the center of the emergent honeycomb lattice, the maximum point located at \( \mathbf{R} = (0, 0) \) is a strong evidence for unconventional pairing symmetry. However, which one of the two hybridizations is preferred for a specified magnetic impurity is difficult to discriminate. Our calculations suggest that, if the pairing symmetry is \( p \)-wave or \( d + id \)-wave, by tuning the chemical potential close to the two half-filling points, there must be a situation where the maximum point is located at \( \mathbf{R} = (0, 0) \) for the negative resonance energy.

Now we consider the influence of the term proportional to \( t''_G \) given in equation (12). We calculate the LDOS versus energy at \( \mathbf{R} = (0, 0) \) for (1) three different values of \( t''_G, t''_G = 0, t_2, \) and \( 2t_2 \), (2) two different topological nontrivial pairings, the \( p \)-wave and \( d + id \)-wave pairings, (3) the three kind of hybridizations, and (4) the four different chemical potentials: charge density = \( \pm 2e \) per unit cell and the Van Hove singularities nearby (the plots are highly similar to each other, not shown here). For all the cases, we find that the LDOSs of the resonance states are almost not changed by the variation of \( t''_G \), so we suspect that the effect of the \( t''_G \) term is limited.

Figure 8. LDOS versus energy \( E \) for the third hybridization \( \tilde{V}_k^{(3)} \). (a) \( s \)-wave pairing, (b) extended \( s \)-wave pairing, (c) \( p \)-wave pairing, (d) \( d + id \)-wave pairing. The green lines, red lines with ‘+’, and the blue dashed lines present the LDOS for \( \mathbf{R} = (-1/\sqrt{3}, 0) \), \( \mathbf{R} = (1/2\sqrt{3}, 1/2) \), and \( \mathbf{R} = (1/2\sqrt{3}, -1/2) \). The parameters for numerical calculation are given in figure 7.
(11), (31) and (51) show LDOSs versus energy for the topography of the two resonance states, e.g. (12) and (13) show the spacial distributions of the two in-gap resonance peaks in (11) at \( \bar{E} \). The blue lines show LDOSs at the nearest neighbor, \( \bar{R} \). The red lines with \( \bar{X} \) show clean LDOSs without impurity. The red lines with \( \bar{X} \) show LDOSs at \( \bar{R} = (0, 0) \). The blue lines show LDOSs at the nearest neighbor, \( \bar{R} = (-1/\sqrt{3}, 0) \). The contour plots close to each line plot show the topography of the two resonance states, e.g. (12) and (13) show the spacial distributions of the two in-gap resonance peaks in (11) at \( E = -0.023 \) and \( E = 0.023 \), respectively. The inserts in line plots show the details of the resonance peaks.

4. Conclusions

Based on the two-orbital model proposed in [68], we give a systematic study of the resonance states near a magnetic impurity in superconducting TwBG for typical pairing symmetries. We need to emphasize that, though the parameters chosen in this work is different from those given in [68] (especially for the case \( t_0^2 \neq 0 \) and \( t_1 = t_2 = 0 \) used to fit the band structure given in [92]), our results are general and available for the parameters given in [68]. Here are our main results from both symmetry analysis and numerical calculations:

1. When the impurity is hybridized to the two orbitals at one site of the emergent honeycomb lattice, i.e. the impurity is located at the center of the AB or BA spot, for any pairing symmetry belongs to the irreducible representations of \( D_3 \) point group, the spacial distribution of the resonance states will break both the threefold rotation symmetry around the \( z \)-axis and the twofold rotation symmetry around the \( y \)-axis.

2. When the impurity is located at the center of the emergent honeycomb lattice, i.e. at the center of the AA spot, and hybridized to the six nearest neighbors symmetrically, there are only two kinds of hybridizations which belong to the two-dimensional irreducible representations of the \( D_3 \) point group. For each hybridization and pairing symmetry studied, the spacial distribution of the resonance states reveals sixfold rotation symmetry.
(3) For the hybridizations given in point (2), the in-gap resonance peak at the position close to the impurity must be vanishing for the $s$-wave pairing, extended $s$-wave pairing, and any other pairing symmetry belongs to the $A_1$ representation of $D_3$ point group.

(4) For the hybridizations given in point (2), the unique resonance peak with negative resonance energy at $R = (0, 0)$ indicates that the pairing potential may be the topological nontrivial $p$-wave or $d + id$-wave pairing.

Here we give a discussion about these results, focusing on further investigations. Firstly, these conclusions, especially point (1) and point (2), are essentially dependent on the assumption given in [68] that the two orbitals belong to the two-dimensional irreducible representation of $D_3$ point group. Further investigations based on other models [3, 4, 71, 93] will be important reference to find the pairing symmetry of superconducting materials.

Secondly, here we consider only the intra-orbital pairing potentials. When the inter-orbital pairings are considered, there will be lots of possible pairing potentials. For the on-site pairing, it is easy to check that there are 2 kinds of pairing potentials for each irreducible representation. For the nearest neighbor pairing and next nearest neighbor pairing, there are numerous pairing potentials for each irreducible representation, a complete analysis for each pairing potential will be straightforward. Thirdly, as shown in the inserts of figures 6(c), (d) and 9(21), (24), even if the pairing potential is $p$-wave or $d + id$-wave symmetric, the in-gap resonance peak at $R = (0, 0)$ may be too weak to be detected. Fortunately, the in-gap resonance peak is significant for the charge density close to the other half-filling, see the inserts of figures 9(41), (44) and S(c), (d), correspondingly. This is experimental accessible by tuning the gate voltage. Fourthly, when the impurity is located at the center of the emergent honeycomb lattice, the two hybridizations are obtained from symmetry analysis, for a practical adatom like manganese or chromium, the hybridization may break the rotation symmetry beforehand. Systematic theoretical investigations based on first-principle calculations should be helpful in finding the proper adatoms with desired hybridizations.

Acknowledgments

We appreciate the support from NSFC under Grants No. 11504106 and No. 11447167 and the Fundamental Research Funds for the Central Universities under Grant No. 2018MS049.

ORCID iDs

Liang Chen https://orcid.org/0000-0001-6057-5867

References

[1] Cao Y, Fatemi V, Fang S, Watanabe K, Taniguchi T, Kaxiras E and Jarillo-Herrero P 2018 Nature 556 43
[2] Cao Y et al 2018 Nature 556 80
[3] Lopes dos Santos J M B, Peres N M R and Castro Neto A H 2007 Phys. Rev. Lett. 99 256802
[4] Bistritzer R and MacDonald A H 2011 Proc. Natl Acad. Sci. 108 12233
[5] Suárez Morell E, Correa J D, Vargas P, Pacheco M and Bartieciecz Z 2010 Phys. Rev. B 82 121407
[6] Trambly de Laissardière G, Mayou D and Magaud L 2012 Phys. Rev. B 86 125413
[7] Fang S and Kaxiras E 2016 Phys. Rev. B 93 235153
[8] Xu C and Balents L 2018 Phys. Rev. Lett. 121 087001
[9] Liu C C, Zhang L D, Chen W Q and Yang F 2018 Phys. Rev. Lett. 121 217001
[10] Guo H, Zhu X, Feng S and Scalatter R T 2018 Phys. Rev. B 97 235453
[11] Fidrysiak M, Zegrodnik M and Spalek J 2018 Phys. Rev. B 98 085436
[12] Huang T, Zhang L and Ma T 2018 (arXiv:1804.06096)
[13] Kennes D M, Lischner J and Karrasch C 2018 (arXiv:1805.06310)
[14] Roy B and Juričič V 2018 (arXiv:1803.11190)
[15] González J and Stauber T 2018 (arXiv:1807.01275)
[16] Sherkunov Y and Betoures J J 2018 Phys. Rev. B 98 205151
[17] González J 2013 Phys. Rev. B 88 125434
[18] Tang Q K, Yang L, Wang D, Zhang F C and Wang Q H 2018 (arXiv:1809.06772)
[19] Wu F, MacDonald A H and Martin I 2018 Phys. Rev. Lett. accepted
[20] Lian B, Wang Z and Bernevig B A 2018 (arXiv:1807.04382)
[21] Peltonen T J, Ojajärvi J, Mast H, Mendels P and Potemski M 2018 Phys. Rev. B 97 134512
[22] Lian B, Wang Z and Bernevig B A 2018 (arXiv:1809.06772)
[23] Lian B, Wang Z and Bernevig B A 2018 (arXiv:1810.01309)
[24] Hewson A C 1997 The Kondo Problem to Heavy Fermions (Cambridge: Cambridge University Press)
[25] Yu L 1965 Acta Phys. Sin. 21 75
[26] Shiba H 1968 Prog. Theor. Phys. 40 435
[27] Rusinov A I 1969 JETP Lett. 9 85
[28] Tsuei C C, Kirtley J R, Chi C C, Yu-Jahnnes L S, Gupta A, Shaw T, Sun J Z and Ketchen M B 1994 Phys. Rev. Lett. 73 593
[29] Tsuei C C, Kirtley J R, Ren Z F, Wang J H, Raffy H and Li Z Z 1997 Nature 387 481
[30] Hoffman J E, McElroy K, Lee D H, Lang K M, Eisaki H, Uchida S and Davis J C 2002 Science 297 1148
[31] Wang Q H and Lee D H 2003 Phys. Rev. B 67 020511
[32] Hanaguri T, Kohsaka Y, Davis J C, Lupien C, Yamada I, Azuma M, Takano M, Ohishi K, Ono M and Takagi H 2007 Nat. Phys. 3 865
[33] Hanaguri T, Kohsaka Y, Ono M, Maltseva M, Coleman P, Yamada I, Azuma M, Takano M, Ohishi K and Takagi H 2009 Science. 323 923
[34] Balatsky A V, Vekhter I and Zuz J X 2006 Rev. Mod. Phys. 78 373
[35] Pan S H, Hudson E W, Lang K M, Eisaki H, Uchida S and Davis J C 2000 Nature 403 746
[36] Hudson E W, Lang K M, Madhavan V, Pan S H, Eisaki H, Uchida S and Davis J C 2001 Nature 411 920
[37] Bobroff J, Alloul H, MacFarlane W A, Mendels P, Blanchard N, Collin G and Marucco J F 2001 Phys. Rev. Lett. 86 4116
[38] Zhang G M, Hu H and Yu L 2001 Phys. Rev. Lett. 86 704
[39] Polkovnikov A, Sachdev S and Vojta M 2001 Phys. Rev. Lett. 86 296
[40] Zhu J X and Ting C S 2000 Phys. Rev. B 63 020506
[41] Vojta M, Zitzler R, Bulla R and Pruschke T 2002 Phys. Rev. B 66 134527
[42] Polkovnikov A 2002 Phys. Rev. B 65 064503
[43] Dai X and Wang Z 2003 Phys. Rev. B 67 180507
[44] Baer S et al 2016 J. Supercond. Novel Magn. 29 659
[45] Tsai W F, Zhang Y Y, Fang C and Hu J 2009 Phys. Rev. B 80 064513
