Electron viscosity, current vortices and negative nonlocal resistance in graphene

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Quantum-critical strongly correlated electron systems are predicted to feature universal collision-dominated transport resembling that of viscous fluids⁴–⁸. However, investigation of these phenomena has been hampered by the lack of known macroscopic signatures of electron viscosity⁵–⁹. Here we identify vorticity as such a signature and link it with a readily verifiable striking macroscopic d.c. transport behaviour. Produced by the viscous flow, vorticity can drive electric current against an applied field, resulting in a negative nonlocal voltage. We argue that the latter may play the same role for the viscous regime as zero electrical resistance does for superconductivity. Besides offering a diagnostic that distinguishes viscous transport from ohmic currents, the sign-changing electrical response affords a robust tool for directly measuring the viscosity-to-resistivity ratio. A strongly interacting electron–hole plasma in high-mobility graphene¹⁰–¹² affords a unique link between quantum-critical electron transport and the wealth of fluid mechanics phenomena.

Symmetries and respective conservation laws play a central role in developing our understanding of strongly interacting states of matter. This is the case, in particular, for many systems of current interest, ranging from quantum-critical states in solids and ultracold atomic gases to quark–gluon plasmas¹³–¹⁴, which share common long-wavelength behaviour originating from the fundamental symmetries of spacetime. The ensuing energy and momentum conservation laws take the centre stage in these developments, defining hydrodynamics that reveals the universal collective behaviour. Powerful approaches based on conformal field theory and anti-de Sitter/conformal field theory (AdS/CFT) duality grant the well-established notions of fluid mechanics, such as viscosity and vorticity, an entirely new dimension.

Despite their prominence and new paradigmatic role, viscous flows in strongly correlated systems have so far lacked directly verifiable macroscopic transport signatures. Surprisingly, this has been the case even for condensed matter systems where a wide variety of experimental techniques is available to probe collective behaviours. Identifying a signature that would do to viscous flows what zero electrical resistance did to superconductivity has remained an outstanding problem. The goal of this article is to point out that vorticity generated in viscous flows leads to a unique macroscopic transport behaviour that can serve as an unambiguous diagnostic of the viscous regime. Namely, we predict that vorticity of the shear flows generated by viscosity can result in a backflow of electrical current that can run against the applied field (see Fig. 1). The resulting negative nonlocal voltage therefore provides a clear signature of the collective viscous behaviour. Associated with it are characteristic sign-changing spatial patterns of electric potential (see Figs 1 and 2) which can be used to directly identify vorticity and shear flows with modern scanning capacitance microscopy techniques¹⁵.

The negative electrical response, which is illustrated in Fig. 1, originates from the basic properties of shear flows. We recall that the collective behaviour of viscous systems results from momenta rapidly exchanged in carrier collisions while maintaining the net momentum conserved. Because momentum remains a conserved quantity collectively, it gives rise to a hydrodynamic momentum transport mode. Namely, momentum flows in space, diffusing transversely to the source–drain current flow and away from the nominal current path. A shear flow established as a result of this process generates vorticity and (for an incompressible fluid) a backflow in the direction reverse to the applied field. Such a complex and manifestly non-potential flow pattern has a direct impact on the electrical response, producing a reverse electric field acting opposite to the field driving the source–drain current (see Fig. 2). This results in a negative nonlocal resistance which persists even in the presence of fairly significant ohmic currents (see Fig. 2).

Attempts to connect electron theory with fluid mechanics have a long and interesting history, partially summarized in refs 8,16,17. Early work on the viscosity of Fermi liquids made a connection with ultrasound damping¹⁶. Subsequently, Gurzhi introduced an electronic analogue of Poiseuille flow¹⁷. Related temperature-dependent phenomena in nonlinear transport were observed by de Jong and Molenkamp¹⁸. Recent developments started with the theory of a hydrodynamic, collision-dominated quantum-critical regime advanced by Damle and Sachdev¹⁹. Andreev et al. argued that hydrodynamic contributions can dominate resistivity in systems with a large disorder correlation length. Forcella et al. predicted that electron viscosity can impact electromagnetic field penetration in a striking way. Davison et al. linked electron viscosity to linear resistivity of the normal state of the copper oxides²⁰.

As a parallel development, recently there has been a surge of interest in the electron viscosity of graphene²¹–²³. Quantum-critical behaviour is predicted to be particularly prominent in graphene¹⁰–¹². Electron interactions in graphene are strengthened near charge neutrality owing to the lack of screening at low carrier densities²¹,²². As a result, carrier collisions are expected to dominate transport in pristine graphene for a wide range of temperatures and dopings²⁴. Furthermore, estimates of electronic viscosity near charge neutrality yield one of the lowest known viscosity-to-entropy ratios, which approaches the universal AdS/CFT bound²⁵. Despite the general agreement that graphene holds the key to electron viscosity, experimental progress has been hampered by the lack of easily discernible signatures in macroscopic transport. Several striking effects have been predicted, such as vortex shedding in the pre-turbulent regime induced by a strong current, as well as nonstationary flow in a ‘viscometer’ comprised of an a.c.-driven image vorticity and shear flows with modern scanning capacitance microscopy techniques²⁶.

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Nonlocal response (a.u.), which leads to large graphene. According to Fermi-liquid theory, this rate behaves as the momentum diffusion coefficient and associated collective transport. Second, although carrier symmetry. This ensures the prominence of momentum conserving Umklapp processes are forbidden by several orders of magnitude by tuning the carrier density to the degeneracy as compared to its value at zero temperature. However, the rate \( \gamma_\text{ee} \) grows and \( v \) approaches the AdS/CFT limit—namely \( sh/4\pi k_B s \), where \( s \) is entropy density. Refs 12,23 estimate this rate as \( \gamma_\text{ee} \approx Aa^2 T/\hbar \), where \( A \) is the interaction strength. For \( T = 100 \text{K} \), assuming \( E_F = 0 \) and approximating the pre-factor as \( A \approx 1 \) (refs 12,23), this predicts characteristic times as short as \( \gamma_\text{ee}^{-1} \approx 80 \) fs. Disorder scattering can be estimated from the measured mean free path values, which reach a few microns at large doping\(^9\). Using the momentum relaxation rate square-root dependence on doping, \( \gamma_\text{ee} \propto n^{-1/2} \), and estimating it near charge neutrality, \( n < 10^{10} \text{ cm}^{-2} \), gives times \( \gamma_\text{ee}^{-1} \approx 0.5 \) ps which are longer than the values \( \gamma_\text{ee}^{-1} \) estimated above. The inequality \( \gamma_\text{ee} \ll \gamma_\text{ee} \) justifies our hydrodynamical description of transport.

Momentum transport in the hydrodynamic regime is described by the continuity equation for momentum density,

\[
\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \vec{v}_i) = -\gamma_\text{ee} \rho_i, \quad T_{ij} = P \delta_{ij} + \mu \nu_i \nu_j + T_{ij}^{(\nu)}
\]

(1)

where \( T_{ij} \) is the momentum flux tensor, \( P \) and \( \mu \) are pressure and mass density, and \( \nu \) is the carrier drift velocity. The quantity \( \gamma_\text{ee} \), introduced above, describes electron-lattice momentum relaxation due to disorder or phonons, which we will assume to be small compared to the carrier scattering rate. We can relate \( \gamma_\text{ee} \) to the electrochemical potential \( \Phi \) through \( P = e \int_0^\infty \Phi(n) \, dn' \). Here we work at degeneracy, \( E_F \gg k_B T \), ignoring the entropic/thermal contributions, and approximating \( P \approx e(n - n_0) \Phi \), with \( n \) the particle number density. While carrier scattering is suppressed at degeneracy as compared to its value at \( E_F = 0 \), here we assume that the carrier-carrier scattering remains faster than the disorder scattering, as required for the validity of hydrodynamics. Viscosity contributes to the momentum flux tensor through

\[
T_{ij}^{(\nu)} = \eta (\partial \nu_i + \partial \nu_j) + (\zeta - \eta) \partial \nu_i \partial \nu_j
\]

(2)

where \( \eta \) and \( \zeta \) are the first and second viscosity coefficients. For drift velocities smaller than plasmonic velocities, transport in charged systems is described by an incompressible flow with a viscous contribution.
divergenceless velocity field, \( \partial_x v_x = 0 \). In this work, we consider the limit of low Reynolds number, \( \nu v_x \ll \eta (\partial_y v_x + \partial_x v_y) \), such that the role of viscosity is most prominent. At linear order in \( \nu \), we obtain an electronic Navier–Stokes equation

\[
\partial_t \eta \nabla^2 v_x + \gamma_x \partial_y p = -\partial_y P
\]

This equation describes momentum transport: imparted by the external field \( f = -\nabla P \), momentum flows to the system boundary where dissipation takes place. It is therefore important to endow equation (3) with suitable boundary conditions. In fluid mechanics this is described by the no-slip boundary condition \( v_x = 0 \). We use a slightly more general boundary condition

\[
v_x = 0, \quad v_y = -\alpha \partial_y P
\]

where the subscripts \( \perp \) and \( \parallel \) indicate the velocity and derivative components normal and tangential to the boundary. The second relation in equation (4) generalizes the no-slip condition to account for non-hydrodynamical effects in the boundary layer on the scales \( \gtrsim l = \sqrt{\nu \gamma_x} \). The model (4), equipped with the parameter \( \alpha \), provides a convenient way to assess the robustness of our predictions.

It is instructive to consider a current flowing down a long strip of a finite width. A steady viscous flow features a non-uniform profile in the strip cross-section governed by the momentum flow to the boundary. Equation (3), applied to a strip \( 0 < y < w \), yields

\[
(\nabla^2 + \gamma_y \partial_x) v_x = \eta E
\]

where \( \gamma_y \) and \( E \) are the drift velocity and electric field directed along the strip (and \( m \) is an effective mass defined through the relation \( p = m v_x \)). Setting \( \gamma_x \) to zero for simplicity, we find a parabolic profile \( v_y = A y \), where \( A = \eta E / 2\eta \). The zeroth shear \( \partial_x v_x = A = (2\eta / \nu)^{1/2} \nu / \eta \) describes momentum flow to the boundary. The net current \( I = \int_0^w \nu v_x d y = (2\eta / \nu)^{1/2} w^2 E \), scaling as a cube of the strip width, is the electronic analogue of the Poiseuille law. Being distinguish from the linear scaling \( I \propto \nu \) in the ohmic regime, the cubic scaling can in principle be used to identify the viscous regime. It is interesting to put the current–field relation in a ‘Drude’ form using kinematic viscosity: \( I = (2\eta / \nu)^{1/2} w^2 E \), with \( \tau_e = w^2 / 12 \nu \) being an effective scattering time. Evaluating the latter as \( \tau_e \approx (1/6)(w/\eta)^{1/2} \), we find values that, for realistic system parameters, can greatly exceed the naive estimate \( \gamma_x^{-1} = w/\gamma_x \) based on the ballistic transport picture. This remarkable observation was first made by Guzzi, see ref. 19.

Next, we proceed to analyse the nonlocal response in a strip with transverse current injected and drained through a pair of contacts, as pictured in Fig. 1. Unlike the above case of longitudinal current, here the potential profile is not set externally but must be obtained from equation (3). The analysis is facilitated by introducing a stream function \( \psi = \psi_x \nabla \psi_x \), which solves the incompressibility condition. At first we will completely ignore the ohmic effects, setting \( \alpha \) and \( \gamma_x \) to zero as above, which leads to a biharmonic equation \( \partial_x^2 \partial_y^2 + \gamma_x \partial_y \psi_x = 0 \) with the boundary conditions \( v_x = 0 \), \( \nu v_y = \partial_x \psi_x \) for \( y = 0, w \). Using a Fourier transform in \( x \), we write \( \psi(x,y) = (2\pi)^{-1} \int d k e^{i k x} \psi(x,y) \) and then determine \( \psi(x,y) \) separately for each \( k \) (see Supplementary Information). Inverting the Fourier transform gives the stream function

\[
\psi(x,y) = \frac{1}{2\pi k} \int d k e^{i k x} \left[ \frac{e^{i \alpha k (w-y)} + e^{i \alpha k (y-w)}}{e^{i \alpha k} + 1} \right]
\]

where we defined \( a_k = k \tanh(k w/2)/(k w + \sinh k w) \). Contours (isolines) of \( \psi \) give the streamlines for the flow shown in Fig. 1. Although most of them are open lines connecting source and drain, some streamlines form loops. The latter define vortices occurring on both sides of the current path. Numerically, we find that vortex centres are positioned very close to \( x = \pm w \) (see Supplementary Information).

We can now explore the electrical potential of the viscous flow. The latter can be found directly from \( \psi(x,y) \), giving

\[
\phi(x,y) = \frac{\beta I}{2} \int d k e^{i k x} a_k \left[ \sinh k (y-w) + \sinh k y \right]
\]

where \( \beta = 2\eta / (\pi n^2 e^2) \) (see Supplementary Information). As illustrated in Fig. 1, the spatial dependence expected in the ohmic regime. This, as well as multiple sign changes, provides a clear signature of a viscous flow.

The nonlocal voltage measured at a finite distance from the current leads (see schematic in Fig. 2 inset) can be evaluated as \( V(x) = \phi(x,w) - \phi(x,0) \). From equation (7) we predict a voltage that is falling off as \( x^{-3} \) and is of a negative sign:

\[
V(x) \approx -\frac{\beta I}{x^3}
\]

\([(x),|y| \ll w]\). Microscopically, negative voltage originates from a viscous shear flow which creates vorticity and backflow on both sides of the current path. Similar behaviour is found near the other contact, \( \phi(x,y) \approx -(\beta ((w-y)^2 - x^2) / ((w-y)^2 + x^2)^2) I \). We note that the \( r^{-3} \) power law dependence is much stronger than the \( \ln r \) dependence expected in the ohmic regime. This, as well as multiple sign changes, provides a clear signature of a viscous flow.

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double-peak structure. The peaks roll off at $|k| \sim \tilde{a}^{-1/2}$, which sets an ultraviolet cutoff for the integral similar to that above for a finite-size contact model. Our numerical analysis shows that this is indeed the case, with a finite $\alpha$ translating into an effective contact size $\alpha \sim \tilde{a}^{-1/2}$. In other words, the modified boundary conditions can alter the response in the proximity of the contact while rendering it unaffected at larger distances. So far we have ignored the bulk momentum relaxation, setting $\nu_r = 0$ in equation (3). We now proceed to show that the signatures of viscous flow identified above are robust in the presence of a background ohmic resistance $\rho = m\nu_r/e^2$ so long as it is not too strong. The dimensionless parameter which governs the respective roles of resistivity versus viscosity is

$$\epsilon = \rho(\text{en})^2/\eta \approx 2\nu_r\lambda_r(w/\text{en})^3,$$  \hspace{1cm} (11)

For the values $\nu_r$ and $\lambda_r$ quoted above, and taking $w = 1 \mu m$, one obtains $\epsilon \sim 10$. Incorporating finite resistivity in the calculation is uneventful, yielding a response

$$V(x) = I \int \frac{dk}{\pi k} \frac{\rho e^{ik(\text{en} - 1)}(e^{ik} - 1)}{q \epsilon^{q} - e^{ik} - 1} + e^{ik}$$(12)

where $q^2 = k^2 + \epsilon w^2$, $q = \pm k$ (see Supplementary Information). For $\epsilon = 0$ we recover the pure viscous result, which is negative, whereas for $\epsilon \to \infty$, equation (12) gives the well-known ohmic result $V(x) = (\rho/\pi) \ln \left[ \coth(\pi x/2w) \right]$, which is positive. With both $\eta$ and $\rho$ nonzero, the resistance given by equation (12) is positive at large $x$ but remains negative close to the contact. The dimensionless threshold that determines the possibility of negative electric response depends on the actual contact size. For the values used in Fig. 2 the negative response occurs even at fairly high resistivity values corresponding to $\epsilon \lesssim 100$.

The robustness of the negative response can be understood by noting that viscosity is the coefficient of the highest derivative term in equation (3) and thus dominates at short distances $x \lesssim x_c = \sqrt{\eta/(\rho \text{en})^2}$. The pervasive character of the negative response, manifest in Fig. 2, will facilitate experimental detection of viscous transport. The positions of the nodes, marked by arrows in Fig. 2, vary with the ratio $\rho/\eta$, which provides a convenient way to directly measure the electronic viscosity.

The hydrodynamic transport regime also features interesting thermal effects. At leading order in the flow velocity those are dominated by convective heat transfer, described by a proportionality relation between entropy flux and flow velocity $v$ (to be discussed elsewhere). At higher order in $v$, besides the conventional Joule heating, vorticity of a viscous flow manifests itself in heat production

$$W = \eta (\partial_v v^i + \partial_v v^j)^2 = 2\eta (\partial_i + i\partial_j)^2 \psi(x,y)^2$$ \hspace{1cm} (13)

The vorticity-induced heating pattern, shown in Fig. 3, features hot spots near contacts and cold arc-shaped elongated patches in vorticity regions. In contrast, for an ohmic flow the pattern is essentially featureless. This makes viscous flows an interesting system to explore with the nanoscale temperature scanning techniques\textsuperscript{18}.

We finally note that there are tantalizing parallels—both conceptual and quantitative—between electronic viscous flows and current microfluidics systems. Namely, a model essentially identical to equation (3) describes the low-Reynolds (microfluidic) flow between two plates separated by a distance $h$, where $\nu_r = 12\eta h^2$ (it also describes viscous electron flow in a 3D slab, see Supplementary Information). A new research area at the frontier of nanoscience and fluid mechanics, microfluidics aims to manipulate and control fluids at a nanoscale with the ultimate goal of developing new lab-on-a-chip microtechnologies. Graphene, which can be easily patterned into any shapes without compromising its excellent qualities, can become a basis of electronic microfluidics, with multiple applications in information processing and nanoscale charge and energy transport that remain to be explored.

Note added in proof: After submitting our work, it came to our attention that negative resistance has recently been observed in graphene (see refs 31, 32).

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Author contributions
All authors contributed to all aspects of this work.

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Competing financial interests
The authors declare no competing financial interests.