Boosting hierarchical structure formation with scalar-interacting dark matter

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ABSTRACT

We investigate the effect of long-range scalar interactions in dark matter (DM) models of cosmic structure formation with a particular focus on the formation times of haloes. Utilizing N-body simulations with 512^3 DM particles we show that in our models DM haloes form substantially earlier: tracing objects up to redshift $z \sim 6$ we find that the formation time, as characterized by the redshift $z_{1/2}$ at which the halo has assembled half of its final mass, is gradually shifted from $z_{1/2} \approx 1.83$ in the fiducial Λ cold dark matter (ΛCDM) model to $z_{1/2} \approx 2.54$ in the most extreme scalar-interaction model. This is accompanied by a shift of the redshift that marks the transition between merger and steady accretion epochs from $z_* \approx 4.32$ in the ΛCDM haloes to $z_* \approx 6.39$ in our strongest interaction model. In other words, the scalar-interacting model employed in this work produces more structures at high redshifts, prolonging at the same time the steady accretion phases. These effects taken together can help the ΛCDM model to account for a high-redshift reionization as indicated by the Wilkinson Microwave Anisotropy Probe (WMAP) data and can alleviate issues related to the survival of the thin-disc-dominated galaxies at low redshifts.

Key words: galaxies; evolution – galaxies: haloes – cosmology: theory – dark matter – methods: n-body simulations.

1 INTRODUCTION

The Λ cold dark matter (ΛCDM) model has proven to be capable of explaining a tremendous amount of observational data. As we entered the era of precision cosmology – both observationally and from a modeller’s perspective – we are left with a more and more detailed picture of what actually took place when the Universe cooled down and formed structures starting from $z \sim 1100$ to the present day. But there are still issues related to the structure formation on galactic scales to be resolved. For instance, observations of high-redshift quasars (Willott et al. 2010) and galaxies (Ouchi et al. 2009) imply that the epoch of the reionization of the Universe ended at $z \sim 6$. On the other hand observations of the polarization of the cosmic microwave background radiation (Kogut et al. 2003; Benson et al. 2006; Komatsu et al. 2010) suggest that the Universe was already reionized at $z \gtrsim 10$–11. However, galaxy formation within the ΛCDM model using the suggested moderate normalization of the power spectrum of primordial density perturbations $\sigma_8 = 0.8$ [as favoured by recent Wilkinson Microwave Anisotropy Probe (WMAP) observations; Komatsu et al. 2010] may not happen early enough to account for such a high redshift of reionization. Furthermore, the astonishing array of recent observations of ultrafaint distant objects (Bouwens et al. 2009, 2010) as well as advancements in the understanding of the high-z physics (Volonteri, Haardt & Madau 2003; Haiman 2004; González et al. 2010) have shown that the young Universe was a busy and already structure-rich place. In addition, there are issues related to the survival of thin-disc-dominated galaxies. It was shown that in a pure CDM model of galaxy-halo formation, the majority of Milky Way sized haloes with $M_h \sim 10^{12} h^{-1} M_\odot$ have accreted at least one object with a greater total mass than the Milky Way disc ($M > 5 \times 10^{10} h^{-1} M_\odot$) over the redshifts $z \leq 3$ (Stewart et al. 2008; Driver 2010). It has been shown recently (Governato et al. 2009; Moster et al. 2010) that the disc may reform after mergers or even survive minor mergers, somewhat easing the problem of disc survival. Still, late-time bombardment of Milky Way like haloes, predicted within the ΛCDM model, could still be considered as a concern about the survival of thin-disc-dominated galaxies and the existence of galaxies with unusually quiet merger history like our Galaxy. Moreover, recent observations of the distant Universe suggest that much of the stellar mass of bright galaxies was already present in place at $z > 1$. This presents the ‘downsizing’ challenge for models of galaxy formation because massive haloes are assembled late in the hierarchical

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clustering process intrinsic to the CDM cosmology (Cowie et al. 2003; Bower et al. 2006; Fontanot et al. 2009). In addition, there were reports on massive superclusters found at intermediate redshifts (z \sim 1), which seem to be very rare and extreme objects within standard ΛCDM model (Colless et al. 2003; Swinbank et al. 2007; Yamila Yaryura, Baugh & Angulo 2010). Finally, there is a recent claim of possible observational evidence for a ‘fifth force’ in the dark matter (DM) sector (Lee 2010).

Even though we would not go so far as to call this a ‘galaxy formation crisis’, there are noticeable tensions between what observations suggest and what (cosmological simulations of) the ΛCDM model seems to predict. The daRk Breaking Equivalence principle (ReBEL) model discussed in recent years in the literature (Farrar & Peebles 2004; Gubser & Peebles 2004a,b; Farrar & Rosen 2007; Peebles & Nusser 2010) has the potential to alleviate these tensions and to help the theoretical ΛCDM model to better match the observations. The ReBEL model is a slight modification of the standard CDM model. It introduces an additional long-range scalar mediated force only in the dark sector. This has been studied in numerical simulations (Nusser, Gubser & Peebles 2005; Hellwing & Juszkiewicz 2009; Hellwing 2010; Hellwing, Juszkiewicz, van de Weygaert 2010; Keselman, Nusser & Peebles 2010) and these studies have shown that the model can match observational properties of the large-scale structures [like one the one-dimensional Lyra power spectrum and the power spectrum of the Sloan Digital Sky Survey (SDSS)] as the standard ΛCDM does, providing at the same time interesting features with respect to the formation of galactic structures. We also acknowledge that recently other authors performed structure formation tests in similar scalar-interacting DM scenarios (Baldi 2009; Baldi et al. 2010), but they focused on models containing scalar-field-like dark energy interacting with DM. Elementary considerations related to the ReBEL model and the reionization mechanism showed that scalar-interacting DM could accommodate early reionization of the intergalactic medium (IGM: Cen 2006). The low-resolution simulations presented in the ReBEL literature have indicated that this model can foster a picture in which structure formation starts at higher redshifts. In this Letter we will show that this is indeed the case. We will present, for the first time in the literature, results of the analysis of the high-resolution N-body simulations of hierarchical DM halo formation in the ReBEL framework.

2 SELF-INTERACTING DM: THE REBEL MODEL

The theoretical ground of the ReBEL model was formed in two papers by Gubser & Peebles (2004a,b). We follow the phenomenological description of this scalar-interacting DM model presented in Nusser et al. (2005) and Hellwing & Juszkiewicz (2009). We consider a modified effective potential and force law between DM particles of the forms

\[ \Phi(r) = -\frac{Gm}{r}(1 + \beta e^{-r/s}), \]  

\[ F_{\text{DM}} = \frac{Gm^2}{r^2} \left[ 1 + \beta \left( 1 + \frac{r}{r_s} \right) e^{-r/s} \right], \]

where the two free parameters of the model are \( \beta \) and \( r_s \), the measure of the strength of the scalar force compared to usual Newtonian gravity and \( r_s \), the screening length of the additional force that is constant in a comoving frame. Previous considerations in the literature of the subject provide a crude estimate of the values of these free parameters to be of orders \( \beta \sim 1 \) and \( r_s \sim 1 h^{-1}\text{Mpc} \). In this Letter we explore two possible values of the \( \beta = 0.5, 1 \) and two values of the screening length parameter \( r_s = 0.5, 1 h^{-1}\text{Mpc} \). We label the corresponding simulation runs as ΛCDM, b05rs05, b05rs1 and b1rs1, where b stands for \( \beta \) and rs marks the \( r_s \) parameter. In the ΛCDM run we use \( \beta = 0 \) (i.e. no scalar forces). Equations (1) and (2) yield a simple phenomenological description of the ReBEL addition to the CDM model.

3 NUMERICAL MODELLING

To follow the formation of structures within the ReBEL framework we use an adapted version of the Gadget2 code (Springel 2005). For the detailed descriptions of the modifications made to the code we refer the reader to our previous paper on this subject (Hellwing & Juszkiewicz 2009). We have conducted a series of high-resolution DM only N-body simulations containing 512\(^3\) particles within periodic box of \( 32 h^{-1}\text{Mpc} \) comoving width. The cosmology used in the simulations was the canonical ΛCDM with \( \Omega_0 = 0.3, \Omega_\Lambda = 0.7, \sigma_8 = 0.8 \) and \( h = 0.7 \). Thus our mass resolution is \( m_p \simeq 2.033 \times 10^7 \text{M}_\odot \). The force softening parameter was set to be \( \epsilon = 6 h^{-1}\text{kpc} \). All simulations were started with the same initial conditions at \( z = 50 \), and varied only in the ReBEL parameters. For each run we saved 30 snapshots equally spaced in the logarithmic time-scale starting from \( z = 6.092 \) to 0.

We used the MPI+OpenMP hybrid AMiga halo finder (AHF) to identify haloes and subhaloes in our simulation,\(^2\) AHF is the successor of the MHI halo finder by Gill, Knebe & Gibson (2004); a detailed description of AHF is given in the code paper (Knollmann & Knebe 2009). We note that we have adjusted the code slightly to take the modified gravity of the ReBEL models into account. Starting with the halo catalogues at \( z = 0 \) we construct the merger trees for each simulation by cross-correlating the particles constituting the haloes in consecutive time-steps. For each halo we record all progenitors and select the main progenitor as the halo that maximizes the merit function \( N_1 N_2 / N^2 \), where \( N_1, 2 \) and \( N \) are the number of particles of the haloes and the number of shared particles, respectively.

4 THE RESULTS

In this Letter we focus primarily on the mass accretion histories (MAHs) of DM haloes and their masses, respectively. A more in-depth study of the effects of the ReBEL model on their internal properties will be presented in a companion paper (Hellwing, Knollmann & Knebe, in preparation).

4.1 Mass accretion histories

Following the scheme presented in Wechsler et al. (2002, hereafter W02), we fit each halo’s MAH to the exponential law of the form

\[ \dot{M}(a) = \exp[a(1 - a^{-1})], \quad \tilde{M}(a) \equiv \frac{M(a)}{M_0}, \]

where \( \tilde{M} \) is the halo mass for a given cosmic scalefactor \( a \) expressed in terms of its final mass \( M_0 \) at \( z = 0 \) and \( a \) is a free parameter to be determined via fitting.

\(^2\) AHF is freely available from http://popia.ft.uam.es/AMIGA.
4.2 Formation and characteristic redshifts

The fit parameter can be used to express a characteristic redshift of formation

\[ z_* = \frac{S}{\alpha} - 1, \] (4)

defined as the time when the logarithmic slope of the accretion rate \( d \log M / d \log a \) falls below some specified value \( S \). In our studies we adopt the value \( S = 2 \) advocated in W02. Therefore our \( z_* \) is the redshift indicating the time when a DM halo has entered the steady accretion phase of its mass accumulation (i.e. the last major merger should have taken place at \( z > z_* \)). The formation redshift – usually defined as the time when an object constituted half of its present-day mass (e.g. Lacey & Cole 1993) – can now be expressed in terms of \( z_* \)

\[ z_{1/2} = -\log \frac{0.5}{S}(z_* + 1) = -\log \frac{0.5}{\alpha}. \] (5)

We will use \( z_* \) and \( z_{1/2} \) as obtained by fitting our numerical MAHs to equation (3) as estimators of the steady accretion transitions and the formation times of our haloes. We acknowledge that our derivation of both these redshifts is obviously not independent. To this extent, we also derived the formation redshift \( z_{1/2} \) by solely using the numerical MAH of each halo finding the point where the mass drops below \( M_0/2 \); the cross-correlation between these two \( z_{1/2} \) values clearly follows a straight 1:1 relation with a scatter of less than 5 per cent. We therefore decided to stick to the value obtained by equation (5).

In general, equation (3) was shown to be a universal good fit to the halo’s MAHs (e.g. W02); however, Tasitsiomi et al. (2004) pointed out that the simple exponential one parameter form of W02 is not a good fit to the MAH of cluster haloes as well as haloes that experienced recent major mergers. To not be biased by including such objects in our study we removed all haloes whose \( x^2 \) value deviated from the mean by more than 1σ and whose most massive progenitor (MMP) was not present in at least 24 (out of 30) snapshots, thus eliminating of the order of \( \sim 25 \) per cent of all haloes in the original catalogues. We further limit the analysis to objects containing in excess of 500 particles (at redshift \( z = 0 \)) corresponding to a lower mass cut of \( \sim 10^{10} h^{-1} M_{\odot} \). This leaves us with \( \sim 7000 \) objects in the ΛCDM sample and \( \sim 4700 \) objects in the b1rs1 ReBEL run.

Figure 1. The probability distribution of the formation redshift \( z_{1/2} \) for all haloes containing at least 500 particles (\( M_0 > 10^{10} h^{-1} M_{\odot} \)).

### Table 1.

The mean and the 1σ scatter of the \( z_*/z_* \) distributions.

| Model (\( \beta/r_s \)) | \( \langle z_{1/2} \rangle \) | \( \sigma_{z_{1/2}} \) | \( \langle z_* \rangle \) | \( \sigma_z \) |
|-------------------------|-----------------|-----------------|-----------------|-----------------|
| ΛCDM                    | 1.83            | 0.8             | 4.32            | 2.3             |
| 0.5/0.5 \( h^{-1} \) Mpc | 2.12            | 0.9             | 5.16            | 2.51            |
| 0.5/1.0 \( h^{-1} \) Mpc | 2.21            | 0.9             | 5.4             | 2.7             |
| 1.0/1.0 \( h^{-1} \) Mpc | 2.54            | 1.0             | 6.39            | 3.0             |

4.2 Formation and characteristic redshifts

When plotting the distribution function of formation times \( z_{1/2} \) in Fig. 1 we observe a clear trend for the mean of \( z_{1/2} \) to shift to higher redshifts in the ReBEL models. We like to note in passing that for the ΛCDM model our results agree with the ones presented in Lin, Jing & Lin (2003, their figs 2 and 3), when using the same mass bins as in the said work.

We calculated the same kind of distribution for the characteristic redshift \( z_* \) (though not explicitly shown here) and list for both distributions the respective means and variances in Table 1. We note that for the strongest ReBEL model considered in this Letter the mean value of \( z_{1/2} \) is 39 per cent larger than the corresponding ΛCDM value. This effect is even more pronounced for the mean values of the \( z_* \) distributions, the corresponding shift is more than 48 per cent there. The corresponding 1σ scatters are also larger. This is not surprising as these two quantities are linked with linear relation of equation (5). We also note that the magnitude of changes of \( z_{1/2} \) and \( z_* \) in models b05rs05 and b05rs1 are of comparable order. The reason for this is the following: our sample is dominated by objects with masses \( M_0 < 10^{12} h^{-1} M_{\odot} \) whose virial radii are \( r_{vir} < 200 h^{-1} \) kpc; therefore for majority of our objects the screening length of the ReBEL force is much greater than their radii, hence for this class of objects only the \( \beta \) parameter measures overall enhancement of scalar attraction. To conclude this paragraph we denote that for the parameter space probed by our simulations the \( \beta \) parameter plays a more important role on the examined quantities than the screening length.

Viewing the shifts for formation and characteristic redshift in combination we determine that, on average, haloes in the ReBEL model form earlier, thus arriving at moderate and low redshifts with higher masses. Moreover, haloes with exceptionally long quiet accretion epochs are much more abundant in the populations of our ReBEL models. We will return to the issue of larger masses in Section 4.3 where we study the halo mass function at various redshifts.

To check whether the hierarchical structure formation scenario is preserved in the ReBEL models and how the effect of the earlier structure formation depends on the halo mass we show in Fig. 2 the average \( z_{1/2} \) binned in halo mass. We see that the hierarchical character of the structure formation is preserved in all the simulation runs, i.e. small-mass objects form first, with higher mass objects forming later. In all the ReBEL runs the formation redshifts are higher for all considered masses, but the effect is strongest for less massive haloes. However, we also note a clear dependence on the parameters of the ReBEL model. For small \( \beta \) and small \( r_s \) the effect of shifting structure formation to the higher redshifts is less pronounced. This is of course expected, since the screening length acts here as the effective radius of an enhanced accretion and \( \beta \) measures the strength of this enhancement. It is interesting to note that for the b05rs05 and b05rs1 runs the aforementioned effect disappears at some fixed mass scale. In other words, we see that the averaged ReBEL \( z_{1/2} \) values start to agree with the ΛCDM values starting from mass \( M \sim 5 \times 10^{12} h^{-1} M_{\odot} \). It is only the b1rs1 model that deviates throughout the whole mass range probed by our simulations. Nevertheless, we observe that in the b05rs05 and b05rs1...
up with ReBEL at lower redshifts. This can again be seen in Fig. 3: the discrepancies of the amplitudes at redshift $z = 0$ are much suppressed in comparison to the high-redshift CMFs. In addition, we note an interesting feature at the final redshift: while the CMFs have a higher amplitude in the ReBEL models for redshifts $z = 6$ and 3 across the whole mass range, at $z = 0$ only the high-mass end of the ReBEL mass functions shows an excess relative to the fiducial $\Lambda$CDM model. The low-mass end of the ReBEL CMFs at this redshift shows deficiency of haloes compared to the $\Lambda$CDM. The latter effect indicates that in the ReBEL model some of the low-mass haloes were already used in structure formation processes as bricks for more massive objects. All this taken together clearly points out that the process of rapid non-linear structure formation takes place at later times in the $\Lambda$CDM model compared to models with the ReBEL forces.

5 DISCUSSION

The main conclusion of this Letter is that a slight modification to the CDM model in the form of scalar interactions can accommodate more gravitationally bound structures at high redshifts (e.g. Fig. 3) which in turn promotes an early reionization of the Universe. This can be expressed by examining the $\eta$ factor that describes the efficiency of reionization of the IGM (Cen, 2006, 2003):

$$ \eta \equiv \frac{c_\star f_{\text{esc}}(d_{\text{FF}}/dt)e_{\text{UV}}}{C(1 + z)^3}, $$

where $c_\star$ is the star formation factor efficiency, $f_{\text{esc}}$ is the ionizing photon escape fraction, $d_{\text{FF}}/dt$ is the halo formation rate, $e_{\text{UV}}$ is the ionizing photon production efficiency and $C$ is the gas clumping factor. Here the numerator reflects the ionizing photon production rate while the denominator is the ionizing photon destruction rate. When $\eta$ exceeds a certain threshold, the IGM becomes ionized. As the ReBEL models exhibit a richer structure at $z = 6$, their formation rate must have been higher than the one of the $\Lambda$CDM runs. This would make it easier to reach the threshold of reionization at high redshifts in the ReBEL framework.

We further have to acknowledge that earlier formation of the DM haloes fostered by the ReBEL model results in shifting a significant part of the violent merger events to higher redshifts. This indicates that there are many more haloes with extended steady accretion histories than expected in the standard $\Lambda$CDM structure formation scenarios. This would help thin-disc-dominated galaxies to preserve their discs at low redshifts and would also imply that the exceptionally long period of peace experienced by the Milky Way is no more extraordinary (Hammer et al. 2007).

Finally, we have also shown that long-range scalar interactions between DM particles increase to some extent the masses of DM haloes. That is also an important effect that may help to understand the existence of objects like massive superclusters at moderate redshifts. The simulations presented in this Letter however do not encompass a large enough volume to study the formation of these massive objects in detail.

In conclusion, we have shown that the ReBEL model allows us to alleviate some of the tension between predictions of the $\Lambda$CDM and observations (Kogut et al. 2003; Benson et al. 2006; Stewart et al. 2008; Driver 2010; Yamila Yaryura et al. 2010); however, as we only focused on the issue of formation time, it still needs to be investigated if and how internal properties of DM haloes are affected by the ReBEL formalism. One interesting quantity is the halo concentration parameter as it is correlated with the halo formation time (e.g. W02). Preliminary work shows this to indeed

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{Hierarchical structure formation for all models. The relation between the formation redshift ($z_f$) of objects and their masses $M$. The error bars represent 1$\sigma$ scatter within the binned distributions. The error bars of the ReBEL model were slightly shifted to the right-hand side for clarity.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3}
\caption{The CMFs for all haloes containing 20 and more particles calculated at redshifts $z = 6$, 3 and 0. Mass functions for redshifts $z = 6$ ($z = 3$) were scaled down by $10^{-3}$ ($10^{-2}$) for clarity.}
\end{figure}

4.3 The evolution of the mass function

As speculated before, haloes in the ReBEL models not only form earlier but also carry a larger mass. To actually confirm and quantify this previously made claim (cf. Section 4.2) we plot in Fig. 3 the cumulative mass functions (CMFs) at the redshifts $z = 6$, 3 and 0.

We note that for $z = 6$ and 3, all ReBEL’s CMFs have higher amplitudes than the fiducial $\Lambda$CDM CMF in the whole probed mass range: at high redshifts the ReBEL model predicts more non-linear structures compared to the standard CDM model. However, structure formation in the $\Lambda$CDM paradigm seems to be ‘catching'
be the case, with the mean concentration \( \langle c_{\text{vir}} \rangle \) being 9 (11, 11, 13) for \( \Lambda \)CDM (b05rs05, b05rs1, b1rs1) for haloes with masses \( M \sim 2 \times 10^{12} h^{-1} M_{\odot} \) respectively. However, the issue of the halo concentration parameters requires deeper studies and, since a more detailed study of the haloes’ internal properties is in progress, we will cover the concentration parameter issue in a forthcoming work (Hellwing et al., in preparation).

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