Flux Stabilization of D-branes in a non-threshold bound state background

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ABSTRACT

We study some configurations of brane probes which are partially wrapped on spheres transverse to a stack of non-threshold bound states. The latter are represented by the corresponding supergravity background. Two cases are studied: D(10-p)-branes in the background of (D(p-2), Dp) bound states and D(8-p)-branes in the (NS5, Dp) geometry. By using suitable flux quantization rules of the worldvolume gauge field, we determine the stable configurations of the probe. The analysis of the energy and supersymmetry of these configurations reveals that they can be interpreted as bound states of lower dimensional objects polarized into a D-brane.
1 Introduction

A brane probe wrapped on a sphere in such a way that it captures some flux of a background
gauge field may be stable against shrinking if it is located at a discrete set of positions
determined by a flux quantization rule. This flux stabilization phenomenon, discovered in
refs. [1, 2] for the Neveu-Schwarz (NS) gauge field, was generalized in refs. [3, 4] for Ramond-
Ramond (RR) gauge field fluxes. In these papers the brane probe is (partially) wrapped on a
sphere $S^d$, which is defined as the set of points of a $(d+1)$-dimensional sphere which have the
same latitude, i.e. with the same polar angle. The flux quantization rules and the minimal
energy condition fix this angle, whose value must belong to a finite set. In particular, in ref.
[4] a generalization of the flux quantization rule of [1] is proposed, and new sets of angles
and energies are obtained. These brane configurations admit the interpretation of bound
states of strings polarized by the background fields by means of the Myers mechanism [5].

In this paper we shall study the flux stabilization in backgrounds created by stacks of
non-threshold bound states. Two cases will be analyzed: a D(10-p)-brane probe in the
background of (D(p-2), Dp) bound states and a D(8-p)-brane moving in the (NS5, Dp)
geometry. In these two cases we will characterize the stable configurations and we will
determine their energy. From these results we will conclude that our configurations can
be regarded as bound states of fundamental strings or (F, D(6-p))-branes in the (D(p-2),
Dp) or (NS5, Dp) case respectively. We shall confirm this conclusion by determining the
supersymmetry preserved by our solutions.

2 Flux quantization in the (D(p-2), Dp) background

The string frame metric $ds^2$ and the dilaton $\phi$ generated by a stack of (D(p-2), Dp) bound
states ($p \geq 2$) are [3]:

$$ds^2 = f_p^{-1/2} \left[ - (dx^0)^2 + \cdots + (dx^{p-2})^2 + h_p \left( (dx^{p-1})^2 + (dx^p)^2 \right) \right] +$$

$$+ f_p^{1/2} \left[ dr^2 + r^2 d\Omega^2_{8-p} \right],$$

$$e^{\tilde{\phi}} = f_p^{\frac{3-p}{2}} h_p^{1/2},$$

(2.1)

where $\tilde{\phi} = \phi - \phi(r \to \infty)$ and $d\Omega^2_{8-p}$ and $r$ are, respectively, the line element of a unit
$(8-p)$-sphere and a radial coordinate which measures the distance to the bound state. The
functions $f_p$ and $h_p$ in eq. (2.1), in the near-horizon region of the metric, are:

$$f_p = \frac{R^{7-p}}{r^{7-p}} , \quad h_p^{-1} = \sin^2 \varphi f^{-1} + \cos^2 \varphi ,$$

(2.2)

with $\varphi$ being a constant angle characteristic of the bound state and, if $N$ denotes the number
of branes of the stack, $R$ is given by:

$$R^{7-p} \cos \varphi = N g_s 2^{5-p} \pi^{\frac{3-p}{2}} \left( \alpha' \right)^{\frac{3-p}{2}} \Gamma \left( \frac{7-p}{2} \right).$$

(2.3)
In eq. (2.3) $g_s$ is the string coupling constant ($g_s = e^{\phi(r \to \infty)}$) and $\alpha'$ is the Regge slope. It is clear from the form of the metric (2.1) that the Dp-brane of the background extends along the directions $x^0 \cdots x^p$, whereas the D(p-2)-brane component lies along $x^0 \cdots x^{p-2}$. This supergravity solution also contains a NSNS two-form potential $B$:

$$B = \tan \varphi \, h_p \, f_p^{-1} \, dx^{p-1} \wedge dx^p , \quad (2.4)$$

and is charged under two RR field strengths $F^{(p)}$ and $F^{(p+2)}$, whose components along the directions parallel to the bound state are:

$$F^{(p)}_{x^0, x^1, \ldots, x^{p-2}, r} = \sin \varphi \, \partial_r \, f_p^{-1} , \quad F^{(p+2)}_{x^p, x^1, \ldots, x^{p+2}, r} = \cos \varphi \, h_p \partial_r \, f_p^{-1} . \quad (2.5)$$

From the components of the RR fields displayed in eq. (2.3) one can compute the components of the Hodge dual fields $* F^{(p)}$ and $* F^{(p+2)}$ along the directions transverse to the bound state. Clearly $* F^{(p)}$ is a $(10-p)$-form whereas $* F^{(p+2)}$ is a $(8-p)$-form. Then, they can be represented by means of two RR potentials $C^{(9-p)}$ and $C^{(7-p)}$ which are, respectively, a $(9-p)$-form and a $(7-p)$-form. In order to write the relevant components of these potentials, let us parametrize the $S^{8-p}$ transverse sphere by means of the spherical angles $\theta^1, \theta^2, \ldots, \theta^{8-p}$ and let $\theta \equiv \theta^{8-p}$ be the polar angle measured from one of the poles of the sphere ($0 \leq \theta \leq \pi$). Then, the $S^{8-p}$ line element $d\Omega^2_{8-p}$ can be decomposed as: $d\Omega^2_{8-p} = d\theta^2 + (\sin \theta)^2 \, d\Omega^2_{7-p}$, where $d\Omega^2_{7-p}$ is the metric of the constant latitude $(7-p)$-sphere. Let us now define the functions $C_p(\theta)$ as the solutions of the initial value problems:

$$\frac{d}{d\theta} \, C_p(\theta) = -(7-p) \, (\sin \theta)^{7-p} , \quad C_p(0) = 0 , \quad (2.6)$$

which can be straightforwardly solved by elementary integration. In terms of the $C_p(\theta)$'s, the components of the RR potentials in which we are interested in are:

$$C^{(7-p)}_{\theta^1, \ldots, \theta^{7-p}} = - \cos \varphi \, R^{7-p} \, C_p(\theta) \, \sqrt{\hat{g}^{(7-p)}} , \quad C^{(9-p)}_{x^{p-1}, x_p, \theta^1, \ldots, \theta^{7-p}} = - \sin \varphi \, R^{7-p} \, h_p \, f_p^{-1} \, C_p(\theta) \, \sqrt{\hat{g}^{(7-p)}} . \quad (2.7)$$

In eq. (2.7) $\hat{g}^{(7-p)}$ is the determinant of the metric of the unit $S^{7-p}$ sphere.

Let us now place a D(10-p)-brane probe in the (D(p-2), Dp) geometry. The action of such a brane probe is the sum of a Dirac-Born-Infeld and a Wess-Zumino term, namely [1]:

$$S = -T_{10-p} \int d^{11-p} \xi \, e^{-\hat{\phi}} \sqrt{-\text{det} \, (g + \mathcal{F})} + T_{10-p} \int \left[ C^{(9-p)} \wedge \mathcal{F} + \frac{1}{2} C^{(7-p)} \wedge \mathcal{F} \wedge \mathcal{F} \right] , \quad (2.8)$$

where $g$ is the induced metric on the worldvolume of the brane probe, $T_{10-p}$ is the tension of the D(10-p)-brane and $\mathcal{F} = F - B$, with $F$ being a $U(1)$ worldvolume gauge field and $B$ the NSNS gauge potential (actually its pullback to the probe worldvolume). We want to find stable configurations in which the probe is partially wrapped on the $S^{7-p}$ constant latitude sphere. From the analysis performed in ref. [1] for RR backgrounds, it follows that we must extend the probe along the radial coordinate and switch on an electric worldvolume.
field along this direction. Moreover, our background has a $B$ field with non-zero components along the $x^{p-1}x^p$ plane. Then, in order to capture the flux of the $B$ field, we must also extend our D(10-p)-brane probe along the $x^{p-1}x^p$ directions. Therefore, the natural set of worldvolume coordinates $\xi^\alpha$ ($\alpha = 0, \ldots, 10-p$) is $\xi^\alpha = (t, x^{p-1}, x^p, r, \theta^1, \ldots, \theta^{7-p})$, where $t \equiv x^0$. Moreover, we will adopt the following ansatz for the field $\mathcal{F}$:

$$\mathcal{F} = F_{0,r} dt \wedge dr + \mathcal{F}_{p-1,p} dx^{p-1} \wedge dx^p .$$

(2.9)

Notice that $\mathcal{F}_{p-1,p}$ gets a contribution from the pullback of $B$, namely:

$$\mathcal{F}_{p-1,p} = F_{p-1,p} - h_p f_p^{-1} \tan \varphi .$$

(2.10)

It is interesting to point out that the components of $\mathcal{F}$ in eq. (2.9) are precisely those which couple to the RR potentials (2.7) in the Wess-Zumino term of the action. With the election of worldvolume coordinates we have made above, the embedding of the brane probe in the transverse space is encoded in the dependence of the polar angle $\theta$ on the $\xi^\alpha$’s. Although we are interested in configurations in which $\theta$ is constant, we will consider first the more general situation in which $\theta = \theta(r)$. It is easy to compute the lagrangian in this case. One gets:

$$L = \int dx^{p-1}dx^p \int_{S^{7-p}} d^{7-p} \sqrt{\tilde{g}} \int dr dt \mathcal{L}(\theta, F) ,$$

(2.11)

where $\tilde{g} \equiv \tilde{g}^{(7-p)}$ and the lagrangian density $\mathcal{L}(\theta, F)$ is:

$$\mathcal{L}(\theta, F) = -T_{0-10-p} R^{7-p} \left[ (\sin \theta)^{7-p} \sqrt{h_p f_p^{-1} + h_p^{-1} \mathcal{F}_{p-1,p}^2} \sqrt{1 + r^2 \theta'^2} - F_{0,r}^2 + \right.$$

$$+ \cos \varphi F_{p-1,p} F_{0,r} C_p(\theta) \right] .$$

(2.12)

We want to find solutions of the equations of motion derived from $\mathcal{L}(\theta, F)$ in which both the angle $\theta$ and the worldvolume gauge field are constant. The equation of motion for $\theta$ with $\theta = \bar{\theta}$ is constant reduces to:

$$\cos \bar{\theta} \sqrt{1 - F_{0,r}^2} \sqrt{h_p f_p^{-1} + h_p^{-1} \mathcal{F}_{p-1,p}^2} - \sin \bar{\theta} \cos \varphi F_{0,r} F_{p-1,p} = 0 .$$

(2.13)

If $F_{0,r}$ and $F_{p-1,p}$ are constant, eq. (2.13) is only consistent when its left-hand side is independent of $r$. However, the square root involving $\mathcal{F}_{p-1,p}$ does depend on $r$ in general. Actually, after a simple calculation one can verify that:

$$h_p f_p^{-1} + h_p^{-1} \mathcal{F}_{p-1,p}^2 = \cos^2 \varphi \mathcal{F}_{p-1,p}^2 + f_p^{-1} (F_{p-1,p} \sin \varphi - \frac{1}{\cos \varphi})^2 .$$

(2.14)

By inspecting the right-hand side of eq. (2.14) one immediately concludes that it is only independent of $r$ when $F_{p-1,p}$ takes the value:

$$F_{p-1,p} = \frac{1}{\sin \varphi \cos \varphi} = 2 \csc(2\varphi) .$$

(2.15)

Plugging back this value of $F_{p-1,p}$ into eq. (2.13), one gets that $F_{0,r} = \cos \bar{\theta}$. In order to determine the allowed values of $\theta$, and therefore of $F_{0,r}$, we need to impose a quantization
condition. Let us consider again a configuration in which $\theta = \theta(r)$ and assume that $F_{p-1,p}$ is given by eq. (2.13). Moreover, let us introduce a quantization volume $\mathcal{V}$ in the $x^{p-1}x^p$ plane which corresponds to one unit of flux, namely:

$$\int_{\mathcal{V}} dx^{p-1} dx^p F_{p-1,p} = \frac{2\pi}{T_f},$$

where $T_f = (2\pi \alpha')^{-1}$. By using the constant value of $F_{p-1,p}$ written in eq. (2.13), one gets that $\mathcal{V}$ is given by:

$$\mathcal{V} = 2\pi^2 \alpha' \sin(2\varphi).$$

(2.17)

We now determine $F_{0,r}$ by imposing the quantization condition of ref. [4] on the volume, i.e.:

$$\int_{\mathcal{V}} dx^{p-1} dx^p \int_{S^{7-p}} d^{7-p} \theta \sqrt{g} \frac{\partial \mathcal{L}}{\partial F_{0,r}} = n T_f,$$

(2.18)

with $n \in \mathbb{Z}$. By using the explicit form of $\mathcal{L}$ and $F_{p-1,p}$, one can easily compute the left-hand side of the quantization condition (2.18):

$$\int_{\mathcal{V}} dx^{p-1} dx^p \int_{S^{7-p}} d^{7-p} \theta \sqrt{g} \frac{\partial \mathcal{L}}{\partial F_{0,r}} = \frac{T_{10-p} \Omega_{7-p} R^{7-p} \mathcal{V}}{\sin \varphi} \left[ \frac{F_{0,r} \sin \theta}{\sqrt{1 + r^2 \theta'^2 - F_{0,r}^2}} - C_p(\theta) \right],$$

(2.19)

where $\Omega_{7-p}$ is the volume of the unit $(7-p)$-sphere and we have assumed that $F_{0,r}$ does not depend on $\theta^1, \cdots \theta^{7-p}$. It is not difficult now to find $F_{0,r}$ as a function of $\theta(r)$ and the quantization integer $n$. First of all, let us notice that the global coefficient of the right-hand side of (2.19) is:

$$\frac{T_{10-p} \Omega_{7-p} R^{7-p} \mathcal{V}}{\sin \varphi} = \frac{NT_f}{2\sqrt{\pi}} \frac{\Gamma\left(\frac{7-p}{2}\right)}{\Gamma\left(\frac{8-p}{2}\right)}.$$

(2.20)

Secondly, let us define a new function $C_{p,n}(\theta)$ as:

$$C_{p,n}(\theta) = C_p(\theta) + 2\sqrt{\pi} \frac{\Gamma\left(\frac{8-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)} n.$$

(2.21)

Then, the electric field is given by:

$$F_{0,r} = \sqrt{\frac{1 + r^2 \theta'^2}{C_{p,n}(\theta)^2 + (\sin \theta)^2 (7-p)}} \times C_{p,n}(\theta).$$

(2.22)

Once $F_{0,r}$ is known, we can obtain the hamiltonian $H$ by means of a Legendre transformation:

$$H = \int dx^{p-1} dx^p \int_{S^{7-p}} d^{7-p} \theta \sqrt{g} \int dr \left[ F_{0,r} \frac{\partial \mathcal{L}}{\partial F_{0,r}} - \mathcal{L} \right].$$

(2.23)

By using eqs. (2.12) and (2.22), one easily obtains the following expression of $H$:

$$H = \frac{T_{10-p} \Omega_{7-p} R^{7-p} \mathcal{V}}{\sin \varphi} \int dx^{p-1} dx^p \int dr \sqrt{1 + r^2 \theta'^2} \sqrt{(\sin \theta)^2 (7-p)} + (C_{p,n}(\theta))^2.$$
The constant $\theta$ solutions of the equation of motion are those which minimize $H$ for $\theta' = 0$. In order to characterize these solutions, let us define the functions:

$$\Lambda_{p,n}(\theta) \equiv (\sin \theta)^{6-p} \cos \theta - C_{p,n}(\theta) .$$

(2.25)

Then, the vanishing of $\partial H/\partial \theta$ for $\theta' = 0$ occurs when $\theta = \bar{\theta}_{p,n}$, where $\bar{\theta}_{p,n}$ is determined by the condition:

$$\Lambda_{p,n}(\bar{\theta}_{p,n}) = 0 .$$

(2.26)

The properties of the functions $\Lambda_{p,n}(\bar{\theta}_{p,n})$ and the solutions of eq. (2.26) have been studied in ref. [4], where it was proved that there exists a unique solution $\bar{\theta}_{p,n}$ in the interval $[0, \pi]$ for $p \leq 5$ and $0 \leq n \leq N$. The values of the angles $\bar{\theta}_{p,n}$ for $p = 4, 5$ can be given in analytic form, namely $\bar{\theta}_{5,n} = n\pi/N$ and $\bar{\theta}_{4,n} = \arccos[1 - 2n/N]$. Moreover, for all values of $p \leq 5$, $\bar{\theta}_{p,0} = 0$ and $\bar{\theta}_{p,N} = \pi$, which correspond to singular configurations in which the brane probe collapses at one of the poles of the $S^{7-p}$ sphere. Excluding these points, there are exactly $N - 1$ angles which minimize the energy. The corresponding electric field is $F_{0,r} = \cos \bar{\theta}_{p,n}$.

Furthermore, if we integrate $x^{p-1}$ and $x^p$ in eq. (2.24) over the quantization volume $V$, we obtain the energy $H_{p,n}$ of these solutions on the volume $V$, which can be written as:

$$H_{p,n} = \int dr \mathcal{E}_{p,n} ,$$

(2.27)

where the constant energy density $\mathcal{E}_{p,n}$ is given by:

$$\mathcal{E}_{p,n} = \frac{NT_f}{2\sqrt{\pi}} \Gamma\left(\frac{7-p}{2}\right) \Gamma\left(\frac{8-p}{2}\right) (\sin \bar{\theta}_{p,n})^{6-p} .$$

(2.28)

The expression of $\mathcal{E}_{p,n}$ in (2.28) is the same as that found in ref. [4] for a Dp-brane background. It was argued in [4] that $\mathcal{E}_{p,n}$ can be interpreted as the energy density of a bound state of $n$ fundamental strings. Actually, one can verify from (2.28) that $\mathcal{E}_{p,n} \leq nT_f$ and that $\mathcal{E}_{p,n} \to nT_f$ in the semiclassical limit $N \to \infty$. Thus, we are led to propose that the states we have found are, in fact, a bound state of polarized fundamental strings stretched along the radial direction and distributed over the $x^{p-1}x^p$ plane in such a way that there are $n$ fundamental strings in the volume $V$. Notice that $V \to 0$ when $\varphi \to 0$ and, thus, the bound state becomes point-like in the $x^{p-1}x^p$ directions as $\varphi \to 0$. This fact is in agreement with ref. [4], since, in this limit, the (D(p-2), Dp) background becomes the Dp-brane geometry.

In order to confirm the interpretation of our results given above, let us study the supersymmetry preserved by our brane probe configurations. In general, the number of supersymmetries preserved by a D-brane is the number of independent solutions of the equation $\Gamma_\kappa \epsilon = \epsilon$, where $\epsilon$ is a Killing spinor of the background and $\Gamma_\kappa$ is the so-called $\kappa$-symmetry matrix [8]. For simplicity, we shall restrict ourselves to the analysis of the $p = 3$ case, i.e. for the (D1, D3) background. The Killing spinors in this case have the form:

$$\epsilon = e^{\frac{\alpha}{2}} \Gamma_{x^{M_1}x^{M_2}...} \bar{\epsilon} ,$$

(2.29)

where $\Gamma_{x^{M_1}x^{M_2}...}$ are antisymmetric products of ten-dimensional constant gamma matrices, $\bar{\epsilon}$ is a spinor which satisfies $(i\sigma_2) \Gamma_{x^{M_1}x^{M_2}...} \bar{\epsilon} = \bar{\epsilon}$ and $\alpha$ is given by:

$$\sin \alpha = f_{3}^{\frac{3}{2}} h_3^{\frac{1}{2}} \sin \varphi , \quad \cos \alpha = h_3^{\frac{3}{2}} \cos \varphi .$$

(2.30)
Moreover, the $\kappa$-symmetry matrix of the D7-brane probe can be put as:

$$\Gamma_\kappa = \frac{i\sigma_2}{\sqrt{1 - F_{0,r}^2}} \left[ F_{0,r} - \Gamma_{\alpha_0 r} \sigma_3 \right] e^{-\eta \Gamma_{\alpha_3 r} \sigma_3} \Gamma_* ,$$

(2.31)

where $\Gamma_* = \Gamma_{\theta^1...\theta^4}$ and $\eta$ is:

$$\sin \eta = \frac{f_3 h_3^{\frac{1}{2}}}{\sqrt{h_3 f_3 - h_3^{-1} \mathcal{F}_{2,3}^2}} , \quad \cos \eta = \frac{\mathcal{F}_{2,3} h_3^{-\frac{1}{2}}}{\sqrt{h_3 f_3 - h_3^{-1} \mathcal{F}_{2,3}^2}} .$$

(2.32)

For our configurations, in which $F_{2,3}$ is given by eq. (2.15), the angles $\alpha$ and $\eta$ of eqs. (2.30) and (2.32) are equal, and the $\Gamma_\kappa \epsilon = \epsilon$ condition becomes:

$$\frac{1}{\sin \theta} \left[ \cos \theta + \Gamma_{\alpha_0 r} \sigma_3 \right] \Gamma_{\theta^0} \bar{\epsilon} = \bar{\epsilon} .$$

(2.33)

Notice that, in order to derive (2.33), we have used that $F_{0,r} = \cos \theta$ in eq. (2.31). Moreover, introducing the $\theta$-dependence of the spinors, i.e. $\bar{\epsilon} = \exp\left[-\frac{\theta}{2} \Gamma_{\theta^0} \right] \hat{\epsilon}$, with $\hat{\epsilon}$ independent of $\theta$, we get the following condition on $\hat{\epsilon}$:

$$\Gamma_{\alpha_0 r} \sigma_3 \hat{\epsilon} = \hat{\epsilon} ,$$

(2.34)

which can be rewritten as:

$$\Gamma_{\alpha_0 r} \sigma_3 \epsilon |_{\theta=0} = \epsilon |_{\theta=0} ,$$

(2.35)

which certainly corresponds to a system of fundamental strings in the radial direction. Notice that the point $\theta = 0$ can be regarded as the “center of mass” of the expanded fundamental strings.

### 3 Flux quantization in the (NS5, Dp) background

We will now consider a background generated by a stack of $N$ bound states of NS5-branes and Dp-branes for $1 \leq p \leq 5$. The bound state is characterized by two coprime integers $l$ and $m$ which, respectively, determine the number of NS5-branes and Dp-branes which form the bound state. We shall combine $l$ and $m$ to form the quantity $\mu_{(l,m)} = l^2 + m^2 g_s^2$. Moreover, for a stack of $N$ (NS5, Dp) bound states we define $R_{(l,m)}^2 = N \left[ \mu_{(l,m)} \right]^{\frac{1}{2}} \alpha'$, in terms of which the near-horizon harmonic function $H_{(l,m)}(r)$ is defined as:

$$H_{(l,m)}(r) = \frac{R_{(l,m)}^2}{r^2} .$$

(3.1)

The metric of this background in the string frame is:

$$ds^2 = \left[ h_{(l,m)}(r) \right]^{-\frac{1}{2}} \left[ H_{(l,m)}(r) \right]^{\frac{1}{2}} \left( -dt^2 + (dx^1)^2 + \cdots + (dx^p)^2 \right) +
\left. h_{(l,m)}(r) \left[ H_{(l,m)}(r) \right]^{\frac{1}{2}} \left( (dx^{p+1})^2 + \cdots + (dx^5)^2 \right) + \left[ H_{(l,m)}(r) \right]^{\frac{3}{2}} (dr^2 + r^2 d\Omega_3^2) \right] ,$$

(3.2)
where the function $h_{l,m}(r)$ is given by:

$$h_{l,m}(r) = \frac{\mu_{l,m}}{l^2 \left[H_{l,m}(r)\right]^{\frac{1}{2}} + m^2 g_s^2 \left[H_{l,m}(r)\right]^{-\frac{1}{2}}} . \quad (3.3)$$

The NS5-branes of this background extend along the $tx^1 \cdots x^5$ coordinates, whereas the Dp-branes lie along $tx^1 \cdots x^p$ and are smeared in the $x^{p+1} \cdots x^5$ coordinates. The integers $l$ and $m$ represent, respectively, the number of NS5-branes in the bound state and the number of Dp-branes in a $(5-p)$-dimensional volume $V_p = (2\pi \alpha')^{5-p}$ in the $x^{p+1} \cdots x^5$ directions. We shall choose, as in section 2, spherical coordinates, and we will represent the determinant of its metric. Moreover, to simplify the equations that follow we shall take from $g_s = 1$ (the dependence on $g_s$ can be easily restored).

By inspecting the form of the NSNS and RR potentials in eqs. (3.5) and (3.6) one easily realizes that, in order to get flux-stabilized configurations, one must consider a D(8-p)-brane probe wrapping the $S^2$ and extended along $r, x^{p+1}, \cdots, x^5$. The action of such a probe is:

$$S = -T_{8-p} \int d^{9-p} \xi e^{-\phi} \sqrt{-\text{det}(g + \mathcal{F})} + T_{8-p} \int \left[ C^{(7-p)} \wedge \mathcal{F} + \frac{1}{2} C^{(5-p)} \wedge \mathcal{F} \wedge \mathcal{F} \right] . \quad (3.7)$$

We shall take in (3.7) the following set of worldvolume coordinates $\xi^\alpha = (t, x^{p+1}, \cdots, x^5, r, \theta^1, \theta^2)$ and we will consider configurations of the brane probe in which $\theta$ is a function of $r$. To determine the worldvolume gauge field $F$, we first impose the flux quantization condition:

$$\int_{S^2} F = \frac{2\pi n_1}{T_f} , \quad n_1 \in \mathbb{Z} . \quad (3.8)$$

Eq. (3.8) can be easily solved, and its solution fixes the magnetic components of $F$. Actually, if we assume that the electric worldvolume field has only components along the radial direction, one can write the solution of (3.8) as $F = \pi n_1 \alpha' \epsilon(2) + F_{0,r} dt \wedge dr$, which is equivalent to the following expression of $\mathcal{F}$:

$$\mathcal{F} = f_{12}(\theta) \epsilon(2) + F_{0,r} dt \wedge dr , \quad (3.9)$$
with \( f_{12}(\theta) \) being:

\[
f_{12}(\theta) \equiv lN\alpha' C_5(\theta) + \pi n_1\alpha'.
\] (3.10)

Using eq. (3.9) in (3.7) one finds that the lagrangian of the system is:

\[
L = \int dx^{p+1} \cdots dx^5 \int_{S^2} d^2\theta \sqrt{\hat{g}} \int dr dt \mathcal{L}(\theta, F),
\] (3.11)

where \( \hat{g} \equiv \hat{g}^{(2)} \) and the lagrangian density is given by:

\[
\mathcal{L}(\theta, F) = -T_{8-p} \left[ \sqrt{r^4 \left[ H_{(l,m)}(r) \right]^2 (\sin \theta)^4 + h_{(l,m)}(r) f_{12}(\theta)^2} \times \right.
\]
\[
\times \sqrt{\left[ H_{(l,m)}(r) \right]^2 (1 + r^2\theta^2) - h_{(l,m)}(r) F_{0,r}^2 +}
\]
\[
\left. + (mN\alpha' C_5(\theta) - C^{(5-p)} f_{12}(\theta)) F_{0,r} \right].
\] (3.12)

In eq. (3.12) we have suppressed the indices of the RR potential \( C^{(5-p)} \). We now impose the following quantization condition \([4]\) (see eq. (2.18)):

\[
\int_{V_{p}} dx^{p+1} \cdots dx^5 \int_{S^2} d^2\theta \sqrt{\hat{g}} \frac{\partial \mathcal{L}}{\partial F_{0,r}} = n_2 T_f,
\] (3.13)

where \( n_2 \in \mathbb{Z} \) and \( V_p \) is the quantization volume defined after eq. (3.3) \((i.e. V_p = (2\pi \sqrt{\alpha'})^{5-p})\). Eq. (3.13) allows to obtain \( F_{0,r} \) as a function of \( \theta(r) \) and of the quantization integers \( n_1 \) and \( n_2 \). By means of a Legendre transformation one can get the form of the hamiltonian of the system. After some calculation one arrives at:

\[
H = T_{8-p} \Omega_2 \int dx^{p+1} \cdots dx^5 \int dr \sqrt{1 + r^2\theta^2} \times
\]
\[
\times \left[ R^4_{(l,m)} (\sin \theta)^4 + [\mu_{(l,m)}]^{-1} \left[ (l f_{12}(\theta) + m \Pi(\theta))^2 + H_{(l,m)}(r)(mf_{12}(\theta) - l\Pi(\theta))^2 \right] \right],
\] (3.14)

where \( \Pi(\theta) \) is the function:

\[
\Pi(\theta) \equiv mN\alpha' C_5(\theta) + \pi n_2\alpha'.
\] (3.15)

By inspecting the right-hand side of eq. (3.14) one immediately reaches the conclusion that there exist configurations with constant \( \theta \) which minimize the energy only when \( mf_{12}(\theta) = l\Pi(\theta) \). By looking at eqs. (3.10) and (3.15) it is immediate to verify that this condition is equivalent to \( mn_1 = ln_2 \). Since \( l \) and \( m \) are coprime, one must have \( n_1 = ln \), \( n_2 = mn \) with \( n \in \mathbb{Z} \). Then, our two quantization integers \( n_1 \) and \( n_2 \) are not independent and \( f_{12}(\theta) \) and \( \Pi(\theta) \) are given in terms of \( n \) by:

\[
f_{12}(\theta) = lN\alpha' C_{5,n}(\theta), \quad \Pi(\theta) = mN\alpha' C_{5,n}(\theta),
\] (3.16)
where $C_{5,n}(\theta)$ is the function defined in eq. (2.21) for $p = 5$. By using eq. (3.16) in eq. (3.14), one gets the following expression of $H$:

$$H = T_{8-p} \Omega_2 R^2_{(l,m)} \int_0^{\infty} \int \sqrt{1 + r^2 \theta^2} \sqrt{(\sin \theta)^4 + \left( C_{5,n}(\theta) \right)^2}. \quad (3.17)$$

By comparing the right-hand side of eq. (3.17) with that of eq. (2.24) one immediately realizes that the constant angles which minimize the energy are the solutions of eq. (2.26) for $p = 5$, i.e. $\theta = \bar{\theta}_{5,n} = \frac{n}{N} \pi$ with $0 \leq n \leq N$. The electric field $F_{0,r}$ which we must have in the worldvolume in order to wrap the D(8-p)-brane at $\theta = \bar{\theta}_{5,n}$ is easily obtained from eq. (3.13). After a short calculation one gets that, for a general value of $g_s$, $F_{0,r}$ is given by:

$$F_{0,r} = \frac{mg_s}{\sqrt{l^2 + m^2 g_s^2}} \cos \left[ \frac{n}{N} \pi \right]. \quad (3.18)$$

Let $H_n^{(l,m)}$ be the energy of our configurations and $\mathcal{E}_n^{(l,m)}$ the corresponding energy density, whose integral over $x^{p+1} \cdots x^5$ gives $H_n^{(l,m)}$. After a short calculation one easily proves that:

$$\mathcal{E}_n^{(l,m)} = \frac{N T_{6-p}(m,l)}{\pi} \sin \left[ \frac{n}{N} \pi \right], \quad (3.19)$$

where, for an arbitrary value of $g_s$, $T_{6-p}(m,l)$ is given by:

$$T_{6-p}(m,l) = \frac{1}{(2\pi)^{6-p} (\alpha')^{7-p} g_s \sqrt{l^2 + m^2 g_s^2}}. \quad (3.20)$$

$T_{6-p}(m,l)$ is the tension of a bound state of fundamental strings and D(6-p)-branes [10]. In such a (F, D(6-p))-brane state, $l$ is the number of D(6-p)-branes, whereas $m$ parametrizes the number of fundamental strings. Indeed, one can check that $T_{6-p}(0,l) = l T_{6-p}$ and, on the other hand, $T_{6-p}(m,0) = m T_{6-p}$, which means that there are $m$ fundamental strings in the $(5 - p)$-dimensional volume $V_{6-p}$. These strings are stretched in the radial direction and smeared in the $x^{p+1} \cdots x^5$ coordinates. This interpretation of $T_{6-p}(m,l)$ suggests that our configurations with $\theta = \bar{\theta}_{5,n}$ are bound states of (F, D(6-p))-branes. Indeed, since $\mathcal{E}_n^{(l,m)} \to n T_{6-p}(m,l)$ as $N \to \infty$, the number of (F, D(6-p))-branes which form our bound state is precisely the quantization integer $n$. Moreover, we can determine the supersymmetry preserved by our configuration. This analysis is similar to the one carried out at the end of section 2. Let us present the result of this study for the (NS5, D3) background, which corresponds to taking $p = 3$ in our general expressions. If $\epsilon$ denotes a Killing spinor of the background, only those $\epsilon$ which satisfy:

$$\left[ \cos \alpha \Gamma_{\varphi} \sigma_3 + \sin \alpha \Gamma_{\varphi} \mathcal{G}_{\varphi} (i \sigma_2) \right] \epsilon|_{\theta = 0} = \epsilon|_{\theta = 0}, \quad (3.21)$$

generate a supersymmetry transformation which leaves our configuration invariant. In eq. (3.21) $\alpha$ is given by:

$$\sin \alpha = \frac{l}{\mu_{(l,m)}^{1/2}} H_{(l,m)}^{1/4} h_{(l,m)}^{1/2}, \quad \cos \alpha = \frac{mg_s}{\mu_{(l,m)}^{1/2}} H_{(l,m)}^{-1/4} h_{(l,m)}^{1/2}. \quad (3.22)$$

The supersymmetry projection (3.21) certainly corresponds to that of a (F, D3) bound state of the type described above, with $\alpha$ being the mixing angle.
4 Discussion

In order to check the stability of our configurations one can study their behaviour under small fluctuations. This analysis, which we will not detail here, is similar to the one performed in refs. [1]-[4] and shows that our configurations are indeed stable. On the other hand, following ref. [4], one can verify that our solutions saturate a BPS bound on the energy, which shows that they certainly minimize the energy.

As compared to the cases studied in ref. [4], it seems that the general rule to find flux-stabilized configurations in a non-threshold bound state background is to consider probes which are also extended in the directions parallel to the bound state in such a way that the probe could capture the flux of the background gauge fields. However, nothing guarantees that the corresponding configurations are free of pathologies. To illustrate this fact, let us consider the case of the background generated by a (F,Dp) bound state. The string frame metric and dilaton for this bound state are [10]:

\[ ds^2 = f_p^{-1/2} h_p^{-1/2} \left[ - (dx^0)^2 + (dx^1)^2 + h_p \left( (dx^2)^2 + \cdots + (dx^p)^2 \right) \right] + \\
+ f_p^{1/2} h_p^{-1/2} \left[ dr^2 + r^2 d\Omega_{8-p}^2 \right] , \]

\[ e^{\tilde{\phi}} = f_p^{3-p} h_p^{p-5} , \] (4.1)

while the $B$ field is $B = \sin \varphi \, f_p^{-1} \, dx^0 \wedge dx^1$ and the RR potentials are:

\[ C_{\theta^1, \ldots, \theta^{7-p}}^{(7-p)} = - \cos \varphi \, R^{7-p} \, C_p(\theta) \sqrt{\hat{g}^{(7-p)}} , \]

\[ C_{x^0, x^1, \theta^1, \ldots, \theta^{7-p}}^{(9-p)} = - \sin \varphi \, \cos \varphi \, R^{7-p} \, f_p^{-1} \, C_p(\theta) \sqrt{\hat{g}^{(7-p)}} . \] (4.2)

According to our rule we should place a D(10-p)-brane probe extended along $(t, x^1, x^2, r, \theta^1, \ldots, \theta^{7-p})$. Moreover, we will adopt the ansatz $F = F_{0,1} \, dt \wedge dx^1 + F_{2,r} \, dx^2 \wedge dx^r$ for the gauge field, with $F_{0,1} = F_{0,1} - f_p^{-1} \sin \varphi$ with constant values of $F_{0,1}$ and $F_{2,r}$. Following the same steps as in our previous examples, we obtain that there exist constant $\theta$ configurations if $F_{0,1} = - \cos^2 \varphi / \sin \varphi$. This is an overcritical field which makes negative the argument of the square root of the Born-Infeld term of the action and, as a consequence, the corresponding value of $F_{2,r}$ is imaginary, namely $F_{2,r} = -i \cos \theta$. These configurations are clearly unacceptable.

5 Acknowledgments

We are grateful to A. Paredes and J. M. Sanchez de Santos for discussions. This work was supported in part by DGICYT under grant PB96-0960, by CICYT under grant AEN99-0589-CO2-02 and by Xunta de Galicia under grant PGIDT00-PXI-20609.
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