Dynamical coupled-channels model of $K^-p$ reactions (I):
Determination of partial-wave amplitudes

H. Kamano,1 S. X. Nakamura,2 T.-S. H. Lee,3 and T. Sato2

1Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567-0047, Japan
2Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan
3Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA

Abstract
We develop a dynamical coupled-channels model of $K^-p$ reactions, aiming at extracting the parameters associated with hyperon resonances and providing the elementary anti-kaon-nucleon scattering amplitudes that can be used for investigating various phenomena in the strangeness sector such as the production of hypernuclei from kaon-nucleus reactions. The model consists of (a) meson-baryon (MB) potentials $v_{MB,MB}$ derived from the phenomenological SU(3) Lagrangian, and (b) vertex interactions $\Gamma_{MB,Y^*}$ for describing the decays of the bare excited hyperon states ($Y^*$) into $MB$ states. The model is defined in a channel space spanned by the two-body $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, and $K\Xi$ states and also the three-body $\pi\pi\Lambda$ and $\pi\bar{K}N$ states that have the resonant $\pi\Sigma^*$ and $\bar{K}^*N$ components, respectively. The resulting coupled-channels scattering equations satisfy the two- and three-body unitarity conditions and account for the dynamical effects arising from the off-shell rescattering processes. The model parameters are determined by fitting the available data of the unpolarized and polarized observables of the $K^-p \to \bar{K}N, \pi\Sigma, \pi\Lambda, K\Xi$ reactions in the energy region from the threshold to invariant mass $W = 2.1$ GeV. Two models with equally good $\chi^2$-fits to the data have been constructed. The partial-wave amplitudes obtained from the constructed models are compared with the results from a recent partial-wave analysis by the Kent State University group. We discuss the differences between these three analysis results.

PACS numbers: 14.20.Jn, 13.75.Jz, 13.60.Le, 13.30.Eg
I. INTRODUCTION

The spectrum and structure of baryons with non-vanishing strangeness \((S)\) quantum number, the hyperons \((Y^\ast)\), are currently much less understood than the \(N^\ast\) and \(\Delta^\ast\) excited states of the nucleon. As pointed out in Ref. \[1\], the past partial-wave analyses \[2–8\] for investigating \(Y^\ast\) were mostly performed using the Breit-Wigner parametrization and did not extract the resonance parameters defined by the poles and residues of the scattering amplitudes. In fact, the values of poles and residues for the \(Y^\ast\) resonances are not given by the Particle Data Group (PDG) \[9\], unlike the \(N^\ast\) and \(\Delta^\ast\) resonances. To establish the \(Y^\ast\) mass spectrum, more extensive investigations are needed theoretically and experimentally.

In this work, we develop a model for anti-kaon-nucleon \((\bar{K}N)\) reactions within a coupled-channels formulation developed in Refs. \[10–18\]. The \(\bar{K}N\) reactions are particularly suitable for studying \(Y^\ast\) with \(S = -1\), \(\Lambda^\ast\) and \(\Sigma^\ast\), since those appear as direct \(s\)-channel processes in the reactions. Following the formulation of Ref. \[10\], we assume that the model Hamiltonian for \(\bar{K}N\) reactions is defined in a channel space spanned by the two-body \(\bar{K}N, \pi\Sigma, \pi\Lambda\) and \(K\Xi\) states and also the three-body \(\pi\pi\Lambda\) and \(\pi\bar{K}N\) states that have the resonant \(\pi\Sigma^\ast\) and \(K^\ast N\) components, respectively. The interaction Hamiltonian consists of (a) meson-baryon (\(MB\)) potentials \(v_{MB,B^\ast}MB\) derived from the phenomenological SU(3) Lagrangian, and (b) vertex interactions \(\Gamma_{MB,Y^\ast}\) for describing the decays of bare excited hyperon states \((Y^\ast = \Lambda^\ast, \Sigma^\ast)\) into \(MB\) states. The resulting coupled-channels scattering equations satisfy the two- and three-body unitarity conditions and account for the dynamical effects arising from the off-shell rescattering processes. We will apply the model to analyze all of the available data of the unpolarized and polarized observables of the \(K^- p \rightarrow \bar{K}N, \pi\Sigma, \pi\Lambda, K\Xi\) reactions from the threshold up to \(W = 2.1\) GeV, where \(W\) is the total scattering energy in the center of mass frame.

The \(\bar{K}N\) reaction data included in our analysis are similar to those used in the partial-wave analysis by the Kent State University (KSU) group \[19\]. It is useful to note here that there are some connections and differences between our dynamical approach and the KSU analysis. In their single-energy partial-wave analysis, a multi-channel \(K\)-matrix model developed by Manley \[20, 21\] was used to guide/constrain their determinations of partial-wave amplitudes from fitting the data. It was pointed out in Ref. \[22\] that this \(K\)-matrix model can be derived from a dynamical model based on a Hamiltonian, such as the one employed in this work, by taking the on-shell approximation to evaluate the meson-exchange potentials \(v_{MB,B^\ast}MB\). In the KSU approach, the needed background scattering matrix \(\omega_{bg}\) defined by the on-shell matrix elements of \(t_{bg}\) are parametrized in terms of unitary matrices and their parameters in each partial wave are adjusted independently from each other in fitting the data. This difference makes the KSU approach more efficient in fitting the data. Perhaps mainly because the amount and the quality of the data in each energy bin (20 MeV) could be very different, the partial-wave amplitudes determined in the KSU’s single-energy analysis could be not smooth in energy. Thus they impose “smoothness” as an additional condition in finalizing
their results. This is a reasonable approach since they also verify their final results by showing that the observables calculated from their partial-wave amplitudes are in agreement with the data. In our dynamical approach, the parameters associated with the potential \( v^{\text{mes}} \) and the vertices \( f_{MB,B^*} \) are adjusted to fit the data of observables in all considered energies. Thus the determined partial-wave amplitudes in all partial waves depend on the same set of the parameters of the constructed meson-baryon potentials. This makes the fits to the data of the observables of \( \bar{K}N \) reactions more difficult than the KSU analysis. Furthermore, solving the coupled-channels equations in a dynamical approach is rather time consuming.

As discussed previously [17], the purpose of taking a much more complicated dynamical model to analyze the meson-baryon reaction data is not only to determine the partial-wave amplitudes for resonance extractions, but also to provide an understanding of the dynamical content of the extracted baryon resonances. In addition, the dynamical model constructed in this work accounts for the off-shell effects due to the \( \bar{K}N \) rescattering processes. Those effects are known to be important for a quantitative understanding of the production of hypernuclei and kaonic nuclei in kaon-induced nuclear reactions [23]. Our dynamical model thus has a great advantage also in the applications to various reaction systems in the strangeness sector that are relevant to the recent experimental efforts at J-PARC [24].

Here we also note that most of the previous investigations of \( Y^* \) based on coupled-channels models (e.g., Refs. [25–27]) focus on studying the resonances extracted from the s-wave amplitudes of \( \bar{K}N \) scattering at low energies. Higher partial waves were also considered in Ref. [28], but the channels and the data considered in this analysis are much more limited than what we will present in this paper. There also exist model studies based on tree-diagrams (e.g., Ref. [29]) of \( Y^* \) which are obviously different from the coupled-channels approaches.

Our first task is to determine the model parameters by fitting the available data of \( K^-p \) reactions from the threshold to \( W = 2.1 \) GeV. The partial-wave amplitudes of the \( \bar{K}N \) reactions obtained from the constructed models are then compared with the results from the recent single-energy partial-wave analysis [13] of the KSU group. These two results will be presented in this paper. The \( Y^* \) resonance parameters, which are extracted from our partial-wave amplitudes by using the analytic continuation method developed in Ref. [30], will be presented and also compared with the KSU results [1] in a separated paper [31].

In Sec. II we recall the coupled-channels formulation of Ref. [10] to write down the scattering equations for investigating \( \bar{K}N \) reactions. The fits to the data are presented in Sec. III. In Sec. IV we present the partial-wave amplitudes obtained from our models and compare them with the KSU results. The threshold parameters (scattering lengths and effective ranges) and the predicted \( K^-p \) reaction total cross section are also presented. Summary and discussions on necessary future works are given in Sec. V.

II. DYNAMICAL COUPLED-CHANNELS MODEL

Following the formulation of Ref. [10], we assume that the interaction Hamiltonian \( H_I \) for \( \bar{K}N \) reactions can be written as

\[
H_I = \sum_{M'B',MB} v_{M'B',MB} + \sum_{Y^*,MB} \left( \Gamma_{MB,Y^*} + \Gamma_{Y^*,MB} \right),
\]

where \( MB = \bar{K}N, \pi\Sigma, \pi\Lambda, K\Xi, \pi\Sigma^*, \bar{K}^*N ; \) \( v_{M'B',MB} \) is the meson-baryon exchange potentials derived from the phenomenological SU(3) Lagrangian; and \( \Gamma_{MB,Y^*} \) are the vertex in-
teractions describing the decays of bare excited hyperon states ($Y^* = \Lambda^*, \Sigma^*$) to $MB$ states. As shown in Fig. 1, the meson-baryon exchange potentials $v_{M'B',MB}$ consist of the tree diagrams of $s$-channel and $u$-channel baryon exchanges, $t$-channel meson exchanges, and contact terms. We consider the ground state baryons belonging to the flavor SU(3) octet and decuplet representations for the $u$-channel exchange baryons, while only the ground state octet baryons are considered for the $s$-channel exchange baryons. This is because the $s$-channel decuplet baryon exchanges are taken into account via the $Y^*$-excitation term as described below. For the $t$-channel processes, however, the octet vector and scalar mesons are considered as exchanged particles. We list the Lagrangian used in our derivations and summarizes the exchanged hadrons included in $v_{T}$ describing the $T$-nents of the resulting equations that are used in the calculations.

The driving terms of Eq. (3) are

$$ G_{MB}(k; W) = \frac{1}{W - E_{M}(k) - E_{B}(k) + i\epsilon}, $$

for the stable $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, and $K\Xi$ channels, and

$$ G_{MB}(k; W) = \frac{1}{W - E_{M}(k) - E_{B}(k) - \Sigma_{MB}(k; W)}, $$

for the unstable $\pi\Sigma^*$ and $\bar{K}^*N$ channels. The details of the self energy, $\Sigma_{MB}(k; W)$ in Eq. (6), are given in Appendix A.

The driving terms of Eq. (3) are

$$ V_{M'B',MB}(k', k; W) = v_{M'B',MB}(k', k) + Z_{M'B',MB}^{(E)}(k', k; W). $$

Following the steps in Ref. [10], we can derive a set of coupled-channels equations for describing the $T$-matrix elements $T_{M'B',MB}$ for the $MB \to M'B'$ reactions. Because this coupled-channels formulation has been given in detail in Refs. [10–18], here we only present concisely the resulting equations that are used in the calculations.

By applying the projection operator method [32], we can cast the partial-wave compo-nents of the $T$ matrix elements of the meson-baryon reactions, $M(\vec{k}) + B(-\vec{k}) \to M'\bar{k}') + B'(-\vec{k})$, into the following form

$$ T_{M'B',MB}(k', k; W) = t_{M'B',MB}(k', k; W) + t_{M'B',MB}^{R}(k', k; W), $$

(2)

where $W$ is the total energy, $k$ and $k'$ are the meson-baryon relative momenta in the center of mass frame. [The label “MB” also specifies quantum numbers (spin, parity, isospin etc) associated with the channel MB.] The “non-resonant” amplitudes $t_{M'B',MB}(k', k; W)$ in Eq. (2) are defined by a set of coupled-channels integral equations,

$$ t_{M'B',MB}(k', k; W) = V_{M'B',MB}(k', k; W) + \sum_{M''B''} \int_{C_{M'B'}} k'^{R}dk''V_{M'B',M''B''}(k', k''; W) \times G_{M''B''}(k''; W)t_{M''B''MB}(k'', k; W). $$

(3)

Here $C_{M'B''}$ is the integration path, which is taken from 0 to $\infty$ for the physical $W$; the summation $\sum_{M''B''}$ runs over the orbital angular momentum and total spin indices for all $M''B''$ channels allowed in a given partial wave; $G_{M''B''}(k''; W)$ are the meson-baryon Green’s functions. Defining $E_{\alpha}(k) = [m_{\alpha}^{2} + k^{2}]^{1/2}$ with $m_{\alpha}$ being the mass of a particle $\alpha$, the meson-baryon Green’s functions in the above equations are

$$ G_{MB}(k; W) = \frac{1}{W - E_{M}(k) - E_{B}(k) + i\epsilon}, $$

for the stable $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, and $K\Xi$ channels, and

$$ G_{MB}(k; W) = \frac{1}{W - E_{M}(k) - E_{B}(k) - \Sigma_{MB}(k; W)}, $$

(5)

for the unstable $\pi\Sigma^*$ and $\bar{K}^*N$ channels. The details of the self energy, $\Sigma_{MB}(k; W)$ in Eq. (6), are given in Appendix A.

The driving terms of Eq. (3) are

$$ V_{M'B',MB}(k', k; W) = v_{M'B',MB}(k', k) + Z_{M'B',MB}^{(E)}(k', k; W). $$

(6)
Here the potentials $v_{MB,MB}(k', k)$ are the partial-wave components of $v_{MB,MB}$ in Eq. (1). Within the unitary transformation method [33,34] used in the derivation, those potentials are energy independent. The energy-dependent $Z_{MB,MB}^{(E)}(k', k; W)$ terms in Eq. (6) contain the singularities owing to the three-body $\pi \pi \Lambda, \pi K \Lambda$ cuts. The procedures for evaluating the partial-wave matrix elements of $Z_{MB,MB}^{(E)}(k', k; W)$ are explained in detail in Appendix E of Ref. [10]. In our previous calculations [17] for $\pi N$ reactions, $Z_{MB,MB}^{(E)}(k', k; W)$ have only few percent effects on the total cross sections. For simplicity, we thus neglect this term in this first attempt to construct a $\bar{K}N$ model. This of course needs to be improved along with other necessary tasks in the future, as will be discussed in Sec. V.

The second term on the right-hand-side of Eq. (2) is the $Y^*$-excitation term defined by

$$
t_{MB,MB}^R(k', k; W) = \sum_{Y_\alpha,Y_m} \bar{\Gamma}_{MB,Y}(k'; W)[D(W)]_{n,m} \bar{\Gamma}_{Y_m,MB}(k; W).$$

Here the dressed $Y^* \rightarrow MB$ and $MB \rightarrow Y^*$ vertices are, respectively, defined by

$$
\bar{\Gamma}_{MB,Y^*}(k; W) = \Gamma_{MB,Y^*}(k) + \sum_{M'B'} \int_{C_{M'B'}} q^2 dq t_{MB,M'B'}(k, q; W) G_{M'B'}(q, W) \Gamma_{M'B',Y^*}(q),
$$

$$
\bar{\Gamma}_{Y^*,MB}(k; W) = \Gamma_{Y^*,MB}(k) + \sum_{M'B'} \int_{C_{M'B'}} q^2 dq \Gamma_{Y^*,M'B'}(q) G_{M'B'}(q, W) t_{M'B',MB}(q, k; W),
$$

with $\Gamma_{MB,Y^*}(k)$ being the bare $Y^* \rightarrow MB$ decay vertex. The inverse of the dressed $Y^*$ propagator is defined by

$$[D^{-1}(W)]_{n,m} = (W - M_{Y^*_m}^0) \delta_{n,m} - [\Sigma_{Y^*}(W)]_{n,m},$$

where $M_{Y^*_m}^0$ is the mass of the bare $Y^*$ state and the $Y^*$ self-energies $\Sigma_{Y^*}(W)$ are given by

$$[\Sigma_{Y^*}(W)]_{n,m} = \sum_{MB} \int_{C_{MB}} k^2 dk \Gamma_{Y^*,MB}(k) G_{MB}(k; W) \Gamma_{MB,Y_m}(k; W).$$

We emphasize here that in general the $Y^*$ propagator $D(W)$ becomes non-diagonal and multivalued in complex $W$ owing to the meson-baryon interactions in the coupled-channels system. This makes the relation between bare states and physical resonances highly non-trivial (see, e.g., Ref. [35]).

Equations (2)-(11) define the DCC model used in our analysis. In the absence of theoretical input, the DCC model, as well as all hadron reaction models, has parameters that can only be determined phenomenologically from fitting the data. The exchange potentials $v_{M'B',MB}$ depend on the coupling constants and the cutoffs of form factors that qualitatively characterize the finite sizes of hadrons. While the values of some of the model parameters can be estimated from the flavor SU(3) relations, we allow most of them to vary in the fits. The $s$-channel and $u$-channel mechanisms of $v_{M'B',MB}$ ($v^s$ and $v^u$ in Fig. 1) include at each meson-baryon-baryon vertex a form factor of the form

$$F(k, \Lambda) = \left(\frac{\Lambda^2}{k^2 + \Lambda^2}\right)^2,$$
with $\vec{k}$ being the meson momentum. For the meson-meson-meson vertex of $t$-channel mechanism (of $v^t$), Eq. (12) is also used with $\vec{k}$ being the momentum of the exchanged meson. For the contact term ($v^c$) we regularize it by $F(\vec{k}', \Lambda')F(\vec{k}, \Lambda)$. The bare vertex functions in Eqs. (8) and (9) are parametrized as
\[
\Gamma_{MB(LS),Y^*}(k) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{m_N}} C_{MB(LS),Y^*} \left( \frac{\Lambda_{Y^*}^2}{\Lambda_{Y^*}^2 + k^2} \right)^{(2+L/2)} \left( \frac{k}{m_\pi} \right)^L,
\]
where $L$ and $S$ denote the orbital angular momentum and spin of the $MB$ state, respectively [note that $\Gamma_{Y^*,MB}(k) = \Gamma_{MB,Y^*}(k)$]. All of the possible $(L, S)$ states in each partial wave included in our coupled-channels calculations are listed in Table. The vertex function (13) behaves as $k^L$ at $k \sim 0$ and $k^{-4}$ for $k \to \infty$. The coupling constant $C_{MB(LS),Y^*}$ and the cutoff $\Lambda_{Y^*}$ are adjusted along with the bare masses $M_{0,Y^*}$ in the fits.

### III. RESULTS OF THE FIT

As already mentioned in the previous sections, we determine the model parameters by fitting the available data of unpolarized and polarized observables of $K^-p \to \bar{K}N, \pi \Sigma, \pi \Lambda, K\Xi$ from the threshold up to $W = 2.1$ GeV. The procedure and strategy for the fitting, e.g., criteria how many bare $Y^*$ states are included in each partial wave, are essentially the same as those employed in our coupled-channels analysis of $N^*$ resonances [17], and we will not repeat it here. The number of the data of each observable included in our fits is listed in Table II. Our database is similar to what were used in the KSU single-energy partial-wave analysis [19]. It is known that for the considered pseudoscalar-meson-baryon scattering, the complete data for determining partial-wave amplitudes need to include spin-rotation observables ($\beta$, $R$, or $A$). As seen in Table II there exists no data for such spin-rotation observables that can be included in our fits. We thus have enough uncertainties of the constraints by the data to construct two models, called Model A and Model B. As mentioned in the introduction, solving the coupled-channels equations is rather time consuming compared to the on-shell approaches. As a result, it is quite difficult to accomplish a detailed error estimation of the partial-wave analyses within an acceptable time. Instead, here we shall regard the discrepancies between the partial-wave amplitudes from Models A and B as a measure of the “error” of the determined amplitudes, resulting from the incompleteness of the data. Here we also note that Models A and B have not only different sets of model parameters, but also different forms for the vector-meson-exchange processes in $v_{M'B',MB}$ with $MB, M'B' = \bar{K}N, \pi \Sigma, \pi \Lambda, K\Xi$: familiar vector-meson-exchange diagrams are used in Model A, while in Model B a hybrid of the so-called Weinberg-Tomozawa (WT) terms and modified vector-meson-exchange diagrams is employed. The latter is intended to make a clear comparison with recent studies on the near-threshold phenomena in $s$-wave such as $\Lambda(1405)$ (see e.g., Ref. [27]). The details are explained in Appendix C.1.

In our fits, we have also made an effort to find the well established decuplet baryon $\Sigma^*(1385)$ with $S = -1$, $J^P = 3^+/2$ and $I = 1$. However the corresponding resonance parameters can not be constrained directly by the $K^-p$ reaction data included in our fits since $\Sigma^*(1385)$ is below the $\bar{K}N$ threshold. We therefore take the pole mass of $\Sigma^*(1385)$, 1381$-i20$ MeV [34], as “data” and determine the model parameters such that this resonance pole is reproduced.
The $\chi^2$ values from the fits are also listed in Table II. We see that these values from Models A and B are equally acceptable in determining the model parameters, as presented in Appendix D. Since the data included in the fits are far from complete, as discussed above, the $\chi^2$ values do not give accurate assessments of the constructed models. It is therefore necessary to show that our fits are indeed very good.

We begin by showing in Fig. 2 that we are able to give very good fits to the total cross section data of the considered $K^- p \rightarrow K^- p, \bar{K}^0 n, \pi^0 \Lambda$ (upper row), $K^- p \rightarrow \pi^- \Sigma^+, \pi^0 \Sigma^0, \pi^+ \Sigma^-$ (middle row), and also $K^- p \rightarrow K^0 \Xi^0, K^+ \Xi^-$ (bottom row) reactions. The differences between Models A and B are significant only in the fits to the data of $K^- p \rightarrow K^0 \Xi^0$ in the low $W$ region where the data are poor.

In the next few subsections, we will show in more detail the quality of our fits to the differential cross sections ($d\sigma/d\Omega$), polarizations ($P$), and their product ($P \times d\sigma/d\Omega$) for each of the considered reactions.

A. $K^- p \rightarrow \bar{K}N$

Our fits to the data of $d\sigma/d\Omega$ are shown in Figs. 3 and 4 for the elastic $K^- p \rightarrow K^- p$, and in Figs. 5 and 6 for the charge-exchange $K^- p \rightarrow \bar{K}^0 n$. We see that the data for the elastic scattering $K^- p \rightarrow K^- p$ are rather extensive and accurate. The data for $K^- p \rightarrow \bar{K}^0 n$ are a little less accurate, but are sufficient for playing an important role in the coupled-channels fits. In Figs. 3-6, we see that both Models A and B can fit the data equally well. We note that the data near the threshold at $W = 1464-1469$ MeV have rather large errors, which must be further improved for extracting accurately the $K^- p$ scattering length, as we will also present later.

The data for the polarization $P$ are very limited for $K^- p \rightarrow K^- p$. In fact, we could not find any data at $W \sim 1.7$ GeV. Both models can describe these data well, as shown in Fig. 7. There is no polarization data for $K^- p \rightarrow \bar{K}^0 n$.

B. $K^- p \rightarrow \pi \Sigma$

Our fits to the differential cross sections are shown in Fig. 8 for $K^- p \rightarrow \pi^- \Sigma^+$, Fig. 9 for $K^- p \rightarrow \pi^+ \Sigma^-$, and Fig. 10 for $K^- p \rightarrow \pi^0 \Sigma^0$. Both models can give good fits to the data. The data for $K^- p \rightarrow \pi^0 \Sigma^0$ are available only up to $W = 1763$ MeV. Here the differences between the constructed models are more visible in the high $W > 1683$ MeV region where the data clearly need improvements.

The high precision data of $P \times d\sigma/d\Omega$ for $K^- p \rightarrow \pi^- \Sigma^+$ in the low energy region can be fitted very well, as shown in Fig. 11. In the higher $W$ region, the data of $P$ for $K^- p \rightarrow \pi^- \Sigma^+$ are very qualitative, as seen in Fig. 12. The differences between Models A and B are rather significant at some energies, but both follow well the general trend of the data.

The polarization observables, $P \times d\sigma/d\Omega$ and $P$, for $K^- p \rightarrow \pi^0 \Sigma^0$ are presented in Figs. 13 and 14 respectively. Although the data of $P$ are very limited, there are some from the recent Crystal Ball experiment \cite{39, 40}. As seen in Fig. 14 the data from Ref. 39 and Ref. 40 seem inconsistent in low-energies. As explained in Ref. 40, this inconsistency could be from differences in analysis methods taken by the two analysis groups, even though they used the same data sample. Here we included both in our dataset, and fitted them...
along with other data simultaneously. Our fits shown in Fig. 14 are closer to the data of Ref. [39], not necessarily supporting them.

C. $K^- p \rightarrow \pi^0 \Lambda$

Our fits to the differential cross section data of $K^- p \rightarrow \pi^0 \Lambda$ are shown in Figs. 15 and 16. The fits from Models A and B are equally good. The high precision data of $P \times d\sigma/d\Omega$ at low $W$ can also be fitted well, as shown in Fig. 17. The data for $P$ at higher $W$ are very qualitative. It is seen in Fig. 18 that our fits can reproduce the general trend of the data and it is hard to judge our two models with the quality of the current data, even though there are visible differences in $P$ between them at most energies. The more precise and extensive data of polarization observables of this reaction would be helpful to establish the $\Sigma^*$ mass spectrum since only the $\Sigma^*$ resonances with $I = 1$ can contribute to the $s$-channel processes.

D. $K^- p \rightarrow K\Xi$

As for the $K^- p \rightarrow K^0 \Xi^0$ and $K^- p \rightarrow K^+ \Xi^−$ reactions, currently only the total cross section data are available in the considered energy region. The results of the fits are shown in the lower panels of Fig. 2. There is a clear difference between Model A and Model B in the $K^- p \rightarrow K^0 \Xi^0$ total cross section at $W \sim 1.85$ GeV, although both are within the error of the available data. We find that the enhancement of the $K^- p \rightarrow K^0 \Xi^0$ cross section near the threshold seen in Model A arises from the constructive interference between $S_{01}$ and $S_{11}$ waves, while such interference is not seen in Model B. More precise data near the threshold are desirable to eliminate the difference.

IV. DISCUSSIONS

A. Comparison of partial-wave amplitudes

With the good fits to the available data of $K^- p$ reactions, as shown in the previous section, the partial-wave amplitudes from the models (Models A and B) can be used to extract the $S = −1$ hyperon resonance parameters. The partial-wave amplitudes are also essential in theoretical calculations of the production of hypernuclei from kaon-induced nuclear reactions within the well-studied multiple scattering theory. It is therefore interesting to compare our resulting partial-wave amplitudes with those determined in the recent single-energy partial-wave analysis performed by the KSU group [19]. There exist several previous partial-wave analyses [2–8] of $\bar{K}N$ reactions. However, these earlier works only account for limited data and are based on simple Breit-Wigner parametrizations that do not account for the complex coupled-channels effects as done in the KSU analysis and in this work. We thus will not include those earlier partial-wave analyses in the discussions. In our notation, the partial-wave amplitudes $F_{M'B',MB}(W)$ are given by

$$F_{M'B',MB}(W) = -\frac{1}{2}T_{M'B',MB}(k'_\text{on}, k_\text{on}; W),$$

where $\rho_{MB}(k, W) = \pi k E_M(k) E_B(k)/W$, and $k_\text{on} [k'_\text{on}]$ is the on-shell momentum defined by $W = E_M(k_\text{on}) + E_B(k_\text{on}) [W = E_{M'}(k'_\text{on}) + E_{B'}(k'_\text{on})]$. 

8
In Figs. 19-23, the $KN \rightarrow KN, \pi\Sigma, \pi\Lambda$ partial-wave amplitudes obtained from our two models (solid red for Model A and dashed blue for Model B) are compared with those (solid circles with errors) determined by the single-energy partial-wave analysis of KSU [19]. Overall, the results from the three analyses agree qualitatively. In particular, some of the amplitudes, e.g., $D_{03}$, $D_{15}$, and $F_{05}$ of $KN \rightarrow KN$, $D_{13}$, $D_{15}$, and $F_{17}$ of $KN \rightarrow \pi\Sigma$, and $D_{15}$ and $F_{17}$ of $KN \rightarrow \pi\Lambda$, show good agreements between the three analyses. It is interesting to see that all of these “stable” amplitudes show a clear resonance behavior. On the other hand, large discrepancies can be seen in several partial waves, such as in $P_{01}$ and $P_{11}$ of $KN \rightarrow KN$, $P_{11}$ of $KN \rightarrow \pi\Sigma$, and $P_{13}$ of $KN \rightarrow \pi\Lambda$. Such discrepancies are not surprising since the database used in the analyses is far from complete. It is known that for the considered pseudoscalar-baryon scattering, the complete data should include three observables, such as the differential cross section ($d\sigma/d\Omega$), polarization ($P$), and one of the spin rotations. From Table 11 and the fits presented in Sec. III, we see that no data of spin rotation is available. Furthermore, the number of data points for the polarization $P$ are not sufficiently large. The discrepancies seen here will lead to the differences of the hyperon resonances extracted from the three partial-wave amplitudes displayed in Figs. 19-23, as will be presented in our separated paper [31]. For example, the rapid changes in the $P_{01}$ amplitudes at $W \sim 1.52$ GeV and in the $P_{11}$ amplitudes at $W \sim 1.68$ GeV in Model A, which are not seen in Model B and the KSU analysis, are found to be due to the existence of $Y^*$ resonances with a quite narrow width.

It is therefore important to obtain more high precision data of the $K^-p$ reactions from hadron beam facilities such as J-PARC, in particular for the polarization $P$ and spin rotations. To motivate future experimental efforts, we compare in Fig. 24 the spin-rotation angle $\beta$ for $K^-p \rightarrow KN, \pi\Sigma, \pi\Lambda$, calculated from the considered three partial-wave amplitudes. We observe that the results agree qualitatively at low energies $W \lesssim 1600$ MeV. However, the discrepancy becomes visible at $W \sim 1700$ MeV and sizable at higher energies. This trend of the discrepancy is consistent with that of partial-wave amplitudes shown in Figs. 19-23. We expect that the discrepancy in the predicted $\beta$ can be distinguished by experiments and thus the spin-rotation data can play a crucial role for eliminating the discrepancies in the determined partial-wave amplitudes at higher energies.

In Figs. 25 and 26, we only present the partial-wave amplitudes for $KN \rightarrow K\Xi$ obtained from Models A and B, since this channel was not taken into account in the KSU analysis. We find that in the near threshold region $S_{01}$ and $S_{11}$ amplitudes show a clear difference between the two models and this is responsible for the discrepancy in $K^-p \rightarrow K^0\Xi^0, K^+\Xi^-$ total cross sections around $W \sim 1.85$ GeV (Fig. 2). Within the current models, the $F$-wave amplitudes are found to be almost zero in the considered energy region $W \leq 2.1$ GeV. Thus the $F$-wave amplitudes give just a negligible contribution to the total cross sections at the corresponding energies. Obviously, the total cross sections as well as indirect information through the coupled channels are not enough to determine the amplitudes, and the differential cross section and polarization data are highly desirable.

We should emphasize here that it may be unlikely that the high precision data from complete experiments can be realized in practice. Furthermore, it is not clear that one really can determine partial-wave amplitudes model-independently even if the data of complete experiments are available. This was examined [11] carefully for the pion photoproduction data. Thus it is advantageous to determine the partial-wave amplitudes using a model within which the well-established physics is used to extrapolate the available data to the region where the measurements are difficult. This was done [42, 43] for determining the
partial-wave amplitudes of $\pi N$ scattering using the dispersion relations. Also, the very well-established $NN$ amplitudes at low energies were obtained by imposing one-pion-exchange tails in all partial-wave analyses. Here we follow the same approach by also making use of the hadron-exchange mechanisms. The purely phenomenological $K$-matrix analysis of the KSU group does not have such theoretical constraints, and the accuracy of their partial-wave amplitudes totally depend on the amount and quality of the data. On the other hand, they are much more flexible in fitting the data, while the dynamical model may lead to large errors in the region where the hadron-exchange picture of reactions is not valid. Therefore the cross checks of results from two different approaches are essential to pin down the resonance parameters.

B. Threshold parameters

Threshold parameters such as the scattering length and effective range are important quantities that characterize interacting systems. Those parameters can also provide information on resonances existing near the threshold (see, e.g., Ref. [44]). It is therefore interesting to see the threshold parameters given by our constructed models. In Table III, we present the scattering lengths ($a_{MB}$) and effective ranges ($r_{MB}$) for the $MB \to MB$ scattering ($MB = \bar{K}N, K\Xi$) above $\bar{K}N$ threshold. The results are listed in the isospin basis. With the relation $a_{K-p} = (a_{\bar{K}N}^{I=0} + a_{\bar{K}N}^{I=1})/2$, we obtain the scattering length of $K^-p$ scattering as $a_{K^-p} = -0.600 + i0.080$ fm for Model A ($a_{K^-p} = -0.628 + i1.169$ fm for Model B). This value of the scattering length is fairly consistent with those obtained in previous works (see, e.g., Refs. [45–47]). Here it is noted that we extracted the scattering lengths from the $K^-p$ reaction data above the $\bar{K}N$ threshold through the analysis in which the isospin symmetry is assumed. For a more precise determination, we would need to take into account the isospin breaking effect, and also need to use data that provide accurate threshold information such as the kaonic hydrogen spectrum [47]. Although we have not found a previous work that gave the effective range, we presented it for a future reference.

Regarding the $K\Xi$ channel, we can see a rather large model-dependence of the threshold parameters for $I = 0$, while a relatively small model-dependence is observed for $I = 1$. These quantities are constrained indirectly by the data of $K^-p$ reactions, in particular by those of $K^-p \to K\Xi$, through coupled-channels effects. Because the quality of the data for $K^-p \to K\Xi$ is still poor, it is difficult to strongly constrain the threshold parameters for $K\Xi$.

The threshold parameters for channels below the $\bar{K}N$ threshold ($\pi\Sigma$, $\pi\Lambda$) are difficult to extract unambiguously because of the lack of data below the $\bar{K}N$ threshold. Analyses including only data above the $\bar{K}N$ threshold, as done in this work, would result in obtaining rather model-dependent threshold parameters for the subthreshold channels. This was demonstrated in Ref. [44] within various chiral unitary models and phenomenological potential approaches. Thus, at this moment we refrain from presenting the threshold parameters for the subthreshold channels.

C. $K^-p$ reaction total cross section

Finally, we present the $K^-p$ reaction total cross sections predicted from our models. Within our current framework, the contributions from the considered reaction channels,
i.e., $K^-p \to MB$ with $MB = KN, \pi \Sigma, \pi \Lambda, K\Xi, \pi \Sigma^*, K^*N$, to the $K^-p$ reaction total cross section are expressed as

$$\sigma^{\text{tot}}_{K^-p}(W) = \sum_{M'B' = KN, \pi \Sigma, \pi \Lambda, K\Xi} \sigma_{K^-p \to M'B'}(W) + \sum_{M'B' = \pi \Sigma^*, K^*N} \bar{\sigma}_{K^-p \to M'B'}(W). \quad (15)$$

Here, the first (second) term in the right hand side represents the contribution from the two-body $KN, \pi \Sigma, \pi \Lambda,$ and $K\Xi$ channels (the quasi-two-body $\pi \Sigma^*$ and $K^*N$ channels) to the cross section.

The contribution from a stable two-body channel $M'B'$, $\sigma_{K^-p \to M'B'}$, is calculated with

$$\sigma_{K^-p \to M'B'}(W) = \frac{4\pi}{k^2} \rho_{M'B'}(k'; W) \rho_{KN}(k; W) \times \frac{1}{2} \sum_{LS,JI} (2J + 1) |C^{I}_{K^-p} \times T^{IJ}_{M'B'(LS),KN(LS)}(k', k; W)|^2, \quad (16)$$

where the explicit form of $\rho_{MB}(k; W)$ is given just below Eq. (13); $k$ and $k'$ are defined by $W = E_K(k) + E_N(k) = E_{M'}(k') + E_{B'}(k')$; the factor 1/2 comes from the spin average of the initial proton; $C^{I}_{K^-p}$ is the isospin factor with $C^{I=0}_{K^-p} = C^{I=1}_{K^-p} = 1/2$; and all indices of $T^{IJ}_{M'B',KN}(k', k; W)$ are explicitly shown. In this formula, all allowed charge states of the final $M'B'$ channel are summed up. Multiplying Eq. (16) by an appropriate isospin factor for the final $M'B'$ channel, we can have the $K^-p \to M'B'$ total cross section for a specific charge state, which has been shown in Fig. 2.

The explicit form of $\bar{\sigma}_{K^-p \to \pi \Sigma^*}(W)$ is given by

$$\bar{\sigma}_{K^-p \to \pi \Sigma^*}(W) = \int_{m_\pi + m_\Lambda}^{W-m_\pi} dM_{\pi \Lambda} \frac{M_{\pi \Lambda}}{E_{\Sigma^*}(k')} \times \frac{1}{2\pi} \left| W - E_\pi(k') - E_{\Sigma^*}(k') - \Sigma_{\pi \Sigma^*}(k'; W) \right|^2 \times \sigma_{K^-p \to \pi \Sigma^*}(k'; W), \quad (17)$$

where $k'$ is defined by $W = E_\pi(k') + E_{\pi \Lambda}(k')$ with $E_{\pi \Lambda}(k') = \sqrt{M_{\pi \Lambda}^2 + (k')^2}$; $\Sigma_{\pi \Sigma^*}(k'; W)$ is the self-energy of the $\pi \Sigma^*$ Green’s function given in Eq. (A1); $\Gamma_{\pi \Sigma^*}(k'; W) = -2\text{Im}[\Sigma_{\pi \Sigma^*}(k'; W)]$; and $\sigma_{K^-p \to \pi \Sigma^*}(k'; W)$ is the total cross section for the half-off-shell $K^-p \to \pi \Sigma^*$ reaction,

$$\sigma_{K^-p \to \pi \Sigma^*}(k'; W) = \frac{4\pi}{k^2} \rho_{\pi \Sigma^*}(k'; W) \rho_{KN}(k; W) \times \frac{1}{2} \sum_{L'S'LS,JI} (2J + 1) |C^{I}_{K^-p} \times T^{IJ}_{\pi \Sigma^*(L'S'),KN(LS)}(k', k; W)|^2, \quad (18)$$

with $k$ defined by $W = E_K(k) + E_N(k)$. Comparing Eqs. (16) and (18), we see that in Eq. (18) the summation of total spin ($S$) and angular momentum ($L$) is taken independently for the initial $KN$ and final $\pi \Sigma^*$ channels. This is because $\Sigma^*$ has the spin 3/2 and allowed $LS$ quantum numbers for a given total $J^P$ are different between the $KN$ and $\pi \Sigma^*$ channels as shown in Table II. Also, it should be emphasized that the decay of $\Sigma^*$ to the $\pi \Lambda$ state, which subsequently occurs after the $K^-p \to \pi \Sigma^*$ process, is appropriately taken into account in Eq. (17). The corresponding expression for $\bar{\sigma}_{K^-p \to K^*N}(W)$ can be obtained from Eqs. (17) and (18) by changing the channel labels.
The resulting $K^-p$ reaction total cross sections are shown in Fig. 27. In the left panels, the comparison with the experimental data is presented. We see that our results agree well with the data up to $W \sim 1.9$ GeV, though some deviation is seen for Model B in the region $1.7 \lesssim W \lesssim 1.8$ GeV. Since the contributions from the two-body $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$ and $K\Xi$ channels are well fixed by the data as shown in Fig. 2, the difference in $\sigma_{K^-p}^{tot}$ between Models A and B arises from the predicted contributions from the $\pi\Sigma^*$ and $\bar{K}^*N$ channels. On the other hand, both Models A and B start to underestimate the data above $W \sim 1.9$ GeV, which is expected to be mainly because other inelastic channels that are not included in this work also become relevant at those energies.

From the right panels of Fig. 27, we see that the $\bar{K}N$ channel has the largest contribution in the considered energy region. The $\pi\Sigma$ channel also gives a sizable contribution at low energies, while it becomes very small above $W \sim 1.85$ GeV. The contributions from the $\pi\Lambda$ and $K\Xi$ channels are rather small in the entire energy region considered. In particular, the $K\Xi$ channel is found to give just a negligible contribution. The contributions from the quasi-two-body $\pi\Sigma^*$ and $\bar{K}^*N$ channels become visible at $W \sim 1.65$ GeV and at $W \sim 1.85$ GeV, respectively, and those become comparable with the two-body $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$ and $K\Xi$ contributions above $W \sim 1.9$ GeV.

V. SUMMARY AND FUTURE DEVELOPMENTS

In this work, we have constructed a coupled-channels model of $K^-p$ reactions within the Hamiltonian formulation developed in Refs. [10–18]. The model consists of meson-baryon potentials $v_{M'B',MB}$ derived from the phenomenological SU(3) Lagrangian, and vertex interactions $\Gamma_{MB,Y^*}$ describing the decays of the bare excited hyperon states $Y^*$ into $MB$ states. The parameters of the model are determined by fitting the data of the unpolarized and polarized observables of the $K^-p \to \bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, $K\Xi$ reactions from the threshold up to $W = 2.1$ GeV. Practically, we have constructed two models, Models A and B, for which we used different vector-meson-exchange mechanisms in $v_{M'B',MB}$ yet both reproduce equally well the currently available data of $K^-p$ reactions within their uncertainties. Once a model is constructed, we can extract various physical parameters associated with $Y^*$ resonances, e.g., complex resonance masses and coupling constants defined by poles and residues of the scattering amplitudes, from the model by performing the analytic continuation to the complex energy plane. The extracted $Y^*$ resonance parameters will be presented in detail in a separated paper [31]. Although we have employed different vector-meson-exchange mechanisms for Models A and B, it seems difficult to figure out its consequence on the dynamical content of the determined partial-wave amplitudes, given the incompleteness of the available data. However, such a difference could become more visible when one investigates the role of reaction dynamics in understanding $Y^*$ resonances.

We found that the determined partial-wave amplitudes depend rather strongly on the analysis methods, owing to the fact that the available data of $K^-p$ reactions are far from complete. With comparable quality of the fits to the data, two models constructed in our fits give rather different results in several partial waves. Similar large differences are also found with the results from the recent single-energy analysis by the KSU group [19]. More high precision data on $K^-p$ reactions, in particular for the spin-dependent observables, $P$ and spin rotations ($\beta$, $A$, or $R$), from J-PARC will be highly desirable to pin down the partial-wave amplitudes for high precision extractions of hyperon resonances.

The anti-kaon-nucleon scattering amplitudes obtained in this work can be used to inves-
tigate various phenomena in the strangeness sector such as hypernuclei and kaonic nuclei production reactions, as being actively pursued at J-PARC. Also, the amplitudes set a basis to explore the dynamics below the $\bar{K}N$ threshold where $\Lambda(1405)$ is expected to play an important role. Previously, this interesting region and $\Lambda(1405)$ have been studied with an assumption that an s-wave interaction dominates the $\bar{K}N$-$\pi\Sigma$ coupled-channels system \[27\]. However, $\Lambda(1405)$ mass is above the $\pi\Sigma$ threshold by \(\sim 100\) MeV, and there is no reason to ignore the dynamics in partial waves higher than the s-wave. Our DCC model treats all relevant partial waves on the same footing, and can be used to analyze data not only above the $\bar{K}N$ threshold but also below that. Such combined analysis, if done, will be much more comprehensive than what has been done so far for the $S = -1$ meson-baryon system, and would give a clear picture of the dynamics in the $\Lambda(1405)$ region. A J-PARC experiment \[48\] is expected to provide useful information for this interesting future prospect.

The formulae presented in Sec. II satisfy the two- and three-body unitarity conditions. However, at this stage we have neglected the $Z$-diagram term, $Z_{M'B'MB}'(k', k; W)$ in Eq. (6), in solving the coupled-channels equations. While we expect from our previous investigation \[17\] of $\pi N$ reactions that the effects of $Z_{M'B'MB}'(k', k; W)$ have only a few percent effects on the total cross sections, we need to include this in our next investigations. Also, we have not included the direct $\pi \Sigma^* \rightarrow \pi \Sigma^*$, $\pi \Sigma^* \leftrightarrow \bar{K}^* N$, and $K^* N \rightarrow \bar{K}^* N$ mechanisms in the meson-baryon potentials $v_{M'B'MB}$. (In the current work, those transitions occur indirectly via the processes such as $\pi \Sigma^* \rightarrow \bar{K}N \rightarrow \pi \Sigma^*.$) While in the $K^- p$ reactions the direct quasi-two-body to quasi-two-body transition mechanisms are expected to be less important than the two-body to two-body or two-body to quasi-two-body processes, we must also include them in the future.

ACKNOWLEDGMENTS

H.K. thanks Y. Ikeda for useful discussions on the scattering lengths and for providing the data of the $K^- p$ reaction total cross sections near the threshold. This work was supported by the JSPS KAKENHI Grant No. 25800149 (H.K.) and Nos. 24540273 and 25105010 (T.S.), and by the U.S. Department of Energy, Office of Nuclear Physics Division, under Contract No. DE-AC02-06CH11357. H.K. acknowledges the support of the HPCI Strategic Program (Field 5 “The Origin of Matter and the Universe”) of Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan. This research used resources of the National Energy Research Scientific Computing Center, which is supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231, and resources provided on Blues and/or Fusion, high-performance computing cluster operated by the Laboratory Computing Resource Center at Argonne National Laboratory.

Appendix A: Self-energies in meson-baryon Green functions

In this appendix, we give an expression of the self-energy $\Sigma_{MB}(k; W)$ appearing in the meson-baryon Green’s function [Eq. (5)] for the unstable channels $MB = \pi \Sigma^*, \bar{K}^* N$. The self-energies are explicitly given by

$$\Sigma_{\pi\Sigma^*}(k; W) = \frac{m_{\pi\Sigma^*}}{E_{\pi\Sigma^*}(k)} \int_{C_3} q^2 dq \frac{M_{\pi\Lambda}(q)}{[M_{\pi\Lambda}^2(q) + k^2]^{1/2}} \frac{|f_{\pi\Lambda,\Sigma^*}(q)|^2}{W - E_{\pi}(k) - [M_{\pi\Lambda}^2(q) + k^2]^{1/2} + i\epsilon},$$

(A1)
\[ \Sigma_{K^*N}(k; W) = \frac{m_{K^*}}{E_{K^*}(k)} \int_{C_3} q^2 dq \frac{M_{\pi K}(q)}{[M^2_{\pi K}(q) + k^2]^{1/2}} W - E_N(k) - [M^2_{\pi K}(q) + k^2]^{1/2} + i\epsilon, \]

where \( M_{MB}(q) = E_M(q) + E_B(q) \), and the momentum integral path \( C_3 \) is chosen appropriately when one makes an analytic continuation of the scattering amplitudes.

The form factors \( f_{\pi\Lambda\Sigma^*}(q) \) and \( f_{\pi K K^*}(q) \) are for describing the \( \Sigma^* \to \pi\Lambda \) and \( K^* \to \pi\bar{K} \) decays in the \( \Sigma^* \) and \( K^* \) rest frames, respectively. Those are parametrized as

\[ f_{\pi\Lambda\Sigma^*}(q) = -i \frac{\tilde{g}_{\pi\Lambda\Sigma^*}}{(2\pi)^{3/2}} \sqrt{\frac{1}{2E_{\pi}(q)}} \sqrt{\frac{E_{\Lambda}(q) + m_{\Lambda}}{2E_{\Lambda}(q)}} \left( \frac{q}{m_{\pi}} \right) \left( \frac{\Lambda_{\pi\Lambda\Sigma}^2}{\Lambda_{\pi\Lambda\Sigma}^2 + q^2} \right)^2 \sqrt{\frac{4\pi}{3}}. \]  

\[ f_{\pi K K^*}(q) = \frac{\tilde{g}_{\pi K K^*} \sqrt{m_{\pi}}}{\sqrt{m_{\pi}}} \left( \frac{q}{m_{\pi}} \right) \left( \frac{\Lambda_{\pi K K^*}^2}{\Lambda_{\pi K K^*}^2 + q^2} \right)^{3/2}. \]

The parameters associated with \( f_{\pi\Lambda\Sigma^*}(q) \) are determined such that the pole mass of the decuplet \( \Sigma^* \) baryon, 1381 \(-i\)20 MeV \[38\], is reproduced. The resulting value for the parameters are \( m_{\Sigma^*} = 1435.2 \) MeV, \( \tilde{g}_{\pi\Lambda\Sigma^*} = 1.753 \), and \( \Lambda_{\pi\Lambda\Sigma} = 650 \) MeV (fixed). The parameters associated with \( f_{\pi K K^*}(q) \) are determined by fitting to the \( \pi K \) scattering phase shift \[49\] for the isospin 1/2 and \( P \) wave. We then obtain \( m_{K^*} = 930.4 \) MeV, \( \tilde{g}_{\pi K K^*} = -0.152 \), and \( \Lambda_{\pi\Lambda\Sigma^*} = 341 \) MeV. With these parameters, we find the \( K^* \) pole mass becomes 899.3 \(-i\)29.7 MeV.

Here it is noted that the decuplet \( \Sigma^* \) baryon can decay also to \( \pi\Sigma \) channel via the strong interaction, although its decay ratio is known to be much smaller than the dominant \( \pi\Lambda \) channel \[39\]. As a first step, we only consider the \( \Sigma^* \to \pi\Lambda \) decay in this work, and the contribution of the \( \Sigma^* \to \pi\Sigma \) process will be taken into account in our future development.

Appendix B: Model Lagrangian

Here we present the effective Lagrangian used in our model. In this appendix and Appendix \[40\] the symbols \( P, V, S, B, \) and \( D \) denote pseudoscalar-octet meson, vector-octet meson, scalar-octet meson, spin-\( \frac{3}{2} \) octet baryon, and spin-\( \frac{5}{2} \) decuplet baryon, respectively.

1. \( PBB' \) interaction

The Lagrangian for the \( PBB' \) interaction is expressed as

\[ L_{PBB'} = L_{PBB'}^L \times L_{PBB'}^F + [H.c. \ for \ B \neq B'], \]

where the superscripts \( L \) and \( F \) indicate the Lorentz and flavor parts of the Lagrangian, respectively. The Lorentz part is explicitly given by

\[ L_{PBB'}^L = -\bar{B} \gamma_\mu \gamma_5 B' \partial^\mu P. \]

The flavor part of the \( PBB' \) Lagrangian is derived from the following SU(3) singlet form \[50\],

\[ L_{PBB'B'B'}^F = g_1 [[B_8^\dagger \otimes B_8'^\dagger](8i) \otimes P_8]^4 + g_2 [[B_8^\dagger \otimes B_8'^\dagger](8j) \otimes P_8]^4, \]

14
where $B_8$, $B_3'$, and $P_8$ denote the SU(3) octet representations, to which the $B$, $B'$, and $P$ hadrons belong, respectively. The necessary information for our calculation is then the following flavor matrix elements,

$$
\langle B|L_{PBB'}^F|PB'\rangle = \langle B|L_{PBB'}^F|PB'\rangle = G_{B,PB'} \times (I_B I_{P}^{*}, I_B^{*} I_{B'}^{*}|I_B I_{B'}^{*}),
$$

(B4)

$$
\langle PB|L_{PBB'}^F|B'\rangle = \langle PB|L_{PBB'}^F|B'\rangle = G_{PBB'} \times (I_P I_B^{*}, I_B I_{B'}^{*}|I_B I_{B'}^{*}).
$$

(B5)

With the isoscalar factors\footnote{\cite{50}}, $G_{B,PB'}$ and $G_{PBB'}$ are given by

$$
G_{B,PB'} = (-1)^{-(Y_B/2)+I_B^{-1}} \frac{1}{\sqrt{8}} \sqrt{\frac{2I_B+1}{2I_B+1}} \left[ \sum_{\gamma=1,2} g_\gamma \left( \begin{array}{ccc} 8 & 8 & 8 \\ I_B-Y_B & I_B^{*} Y_B & I_P - Y_P \end{array} \right) \right],
$$

(B6)

and $G_{PBB'} = G_{B,PB'}$. In Eqs. (B4)-(B6), $I_P$, $I_B^{*}$, and $Y_P$ denote the isospin, its $z$-component, and the hypercharge of a hadron $P$, respectively. Owing to the SU(3) symmetric construction of the Lagrangian, all of the $PBB'$ coupling constants are expressed with just two parameters, $g_1$ and $g_2$. Introducing the notation $g_p = (\sqrt{30}/40)g_1 + (\sqrt{6}/24)g_2$ and $\alpha_p^D = (\sqrt{30}/40)(g_1/g_p)$, we can relate these to $g_{\pi NN} = f_{\pi NN}/m_\pi$ and $\alpha$ appearing in Eqs. (B28)-(B37) of Ref.\footnote{\cite{17}} as follows,

$$
g_p = -g_{\pi NN}, \quad \alpha_p^D = \alpha.
$$

(B7)

In this work, we fix the value of $g_p$ by $f_{\pi NN} = \sqrt{4\pi \times 0.08}$ as in Ref.\footnotemark[17], while $\alpha_p^D$ is varied freely in the fits.

2. VBB' interaction

The Lagrangian for the $VBB'$ interaction is expressed as

$$
L_{VBB'} = L_{VBB'}^L \times L_{VBB'}^F + [\text{H.c. for } B \neq B'].
$$

(B8)

The Lorentz part is given by

$$
L_{VBB'}^L = \bar{B} \left[ \gamma - \frac{\kappa_{VBB'}}{m_B + m_B'} \sigma^{\mu \nu}(\partial_\nu V_\mu) \right] B'.
$$

(B9)

The flavor part of the $VBB'$ Lagrangian has the exactly same structure as that of $PBB'$, and it is obtained by making the replacement of $\pi \rightarrow \rho$, $K \rightarrow K^*$, $\bar{K} \rightarrow \bar{K^*}$, $\eta \rightarrow \omega_8$, $g_p \rightarrow g_v$, and $\alpha_p^D \rightarrow \alpha_v^D$. Here $\omega_8$ is the eighth component of the octet representation of the vector mesons, and $g_v$ and $\alpha_v^D$ are the counterpart of $g_p$ and $\alpha_p^D$ in the $PBB'$ interaction, respectively. Assuming the ideal mixing, $\omega_8$ is related to the physical $\omega$ and $\phi$ mesons as

$$
\omega_8 = \frac{1}{\sqrt{3}} \omega - \sqrt{\frac{2}{3}} \phi.
$$

(B10)

In this work, $g_v$, $\alpha_v^D$ and $\kappa_{VBB'}$ with $VBB' = \rho NN$, $\omega NN$, $\phi NN$, $\rho \Xi \Xi$, $\omega \Xi \Xi$, $\phi \Xi \Xi$, $\bar{K}^* \Lambda \Lambda$, $K^* \Xi \Lambda$, $K^* \Xi \Sigma$ are parameters determined by the fits. The other $\kappa_{VBB'}$ are fixed as $\kappa_{VBB'}/(m_B + m_B') = \kappa_{\rho NN}/(2m_N)$. Note that $g_v$ is related to the $\rho NN$ coupling constant, $g_{\rho NN}$ in Ref.\footnotemark[17], as $g_v = -g_{\rho NN}$.
3. $SBB'$ interaction

The Lagrangian for the $SBB'$ interaction is expressed as

$$L_{SBB'} = L^L_{SBB'} \times L^F_{SBB'} + [\text{H.c. for } B \neq B'] .$$

The Lorentz part is given by

$$L^L_{SBB'} = \bar{B}BS.$$  \hspace{1cm} (B12)

As is the case in the $VBB'$ interaction, the flavor part of the $SBB'$ Lagrangian also has the same structure as that of $PBB'$, and it is obtained by making the replacement of $\pi \to S_{3,4,5}$, $K \to \kappa$, $\bar{K} \to \bar{\kappa}$, $\eta \to S_8$, $g_p \to g_s$, and $\alpha_s^D \to \alpha_s^D$. Assuming again the ideal mixing, the eighth component of the scalar-octet meson, $S_8$, is related to the $\sigma$ and $f_0$ mesons as

$$S_8 \approx \frac{1}{\sqrt{3}} \sigma - \sqrt{\frac{2}{3}} f_0.$$  \hspace{1cm} (B13)

The value of $g_s$ and $\alpha_s^D$, which are the counterpart of $g_p$ and $\alpha_p^D$ in the $PBB'$ interaction, are determined by the fits. With the above definition of $\sigma$ meson, the $\sigma NN$ coupling constant, $g_{\sigma NN}$ in Ref. [17], is related to $g_s$ and $\alpha_s^D$ as $g_{\sigma NN} = g_{s\pi NN}/\sqrt{3} = -g_s(4\alpha_s^D - 1)/3$.

4. $PBD$ interaction

The Lagrangian for the $PBD$ interaction is expressed as

$$L_{PBD} = L^L_{PBD} \times L^F_{PBD} + \text{H.c.}.$$  \hspace{1cm} (B14)

The Lorentz part is given by

$$L^L_{PBD} = -\bar{D}^\mu B \partial_{\mu} P.$$  \hspace{1cm} (B15)

In the same manner as the $PBB'$ interaction, the flavor part of the $PBD$ Lagrangian is derived from the following SU(3) singlet form,

$$L^F_{PBD_{10}} = g[[D^I_{10} \otimes B_s^{(8)}] \otimes P_s^{(1)}],$$  \hspace{1cm} (B16)

The necessary information for our calculation is then the following flavor matrix elements,

$$\langle D | L^F_{PBD} | PB \rangle = \langle D | L^F_{PBD} | B \rangle = G_{D,PB} \times (I_D I_p, I_B I_B | I_D I_D),$$  \hspace{1cm} (B17)

$$\langle PB | L^F_{PBD} | D \rangle = \langle PB | L^F_{PBD} | B \rangle = G_{PBD} \times (I_P I_p, I_B I_B | I_D I_D),$$  \hspace{1cm} (B18)

$$\langle PD | L^F_{PBD} | B \rangle = \langle PD | L^F_{PBD} | B \rangle = G_{PD,B} \times (I_D I_p, I_B I_B | I_D I_D),$$  \hspace{1cm} (B19)

$$\langle B | L^F_{PBD} | PD \rangle = \langle B | L^F_{PBD} | PD \rangle = G_{B,PD} \times (I_D I_p, I_D I_B | I_B I_B).$$  \hspace{1cm} (B20)

with

$$G_{D,PB} = (-1)^{-Y_B/2} Y_B^{-1} 8 \sqrt{2 I_p + 1} 2 I_D + 1 \begin{pmatrix} I_D - Y_D & 8 \\ 8 & I_D - Y_p \end{pmatrix},$$  \hspace{1cm} (B21)

$$G_{PBD} = (-1)^{-Y_D/2} Y_D^{-1} 8 \sqrt{2 I_p + 1} 2 I_D + 1 \begin{pmatrix} I_D - Y_D & 8 \\ 8 & I_D - Y_p \end{pmatrix},$$  \hspace{1cm} (B22)

$$G_{PBD} = G_{D,PB}, \hspace{1cm} G_{B,PD} = G_{PBD}. $$  \hspace{1cm} (B23)

Within the above SU(3) symmetric Lagrangian, all of the $PBD$ coupling constants are specified by the single parameter $g$. In this work, the value of $g_{pdb} \equiv -g/(2\sqrt{3})$, which corresponds to $f_\pi N/A/m_\pi$ in Ref. [17], is determined by the fits.
5. **VBD interaction**

The Lagrangian for the \( VBD \) interaction is expressed as

\[
L_{VBD} = L_{VBD}^L \times L_{VBD}^F + \text{H.c.} 
\]  

(B24)

The Lorentz part is given by

\[
L_{VBD}^L = -i \bar{D}^\mu \gamma^\nu \gamma_5 B (\partial_\mu V_\nu - \partial_\nu V_\mu). 
\]  

(B25)

As in the case of \( PBB' \) and \( VBB' \) interactions, the flavor part of the \( VBD \) Lagrangian is given by that of \( PBD \) with the replacement of \( \pi \to \rho, K \to K^*, \bar{K} \to \bar{K}^*, \eta \to \omega_8, \) and \( g \to \bar{g}. \) In this work, the value of \( g_{vbd} \equiv -\bar{g}/(2\sqrt{5}) \), which corresponds to \( f_{\rho N\Delta}/m_\rho \) in Ref. [17], is determined by the fits.

6. **PDD' interaction**

The Lagrangian for the \( PDD' \) interaction is expressed as

\[
L_{PDD'} = L_{PDD'}^L \times L_{PDD'}^F + [\text{H.c. for } D \neq D']. 
\]  

(B26)

The Lorentz part is given by

\[
L_{PDD'}^L = + \bar{D}^\mu \gamma^\nu \gamma_5 D'_\mu \partial_\nu P. 
\]  

(B27)

The flavor part of the \( PDD' \) Lagrangian is derived from the following SU(3) singlet form,

\[
L_{P8D10D'10}^F = g' \left[ [D_{10}^\dagger \otimes D'_{10}]^8 \otimes P_8 \right]. 
\]  

(B28)

The necessary information for our calculation is then the following flavor matrix elements,

\[
\langle D|L_{PDD'}^F|PD' \rangle = \langle D|L_{P8D10D'10}^F|PD' \rangle = G_{D,PD'} \times (I_P I_P^*, I_D I_D'|I_D' I_D') \), 
\]  

(B29)

\[
\langle PD|L_{PDD'}^F|D' \rangle = \langle PD|L_{P8D10D'10}^F|D' \rangle = G_{PD,D'} \times (I_P I_P^*, I_D I_D'|I_D' I_D') \), 
\]  

(B30)

with

\[
G_{D,PD'} = (-1)^{(Y_{D'}/2)+1} \left( \frac{2I_P + 1}{2I_D + 1} \right)^{1/2} \left( \begin{array}{c|c}
10^* & 10 \\
I_D - Y_D & I_{D'} Y_{D'} \\
\hline
8 & I_P - Y_P \\
\end{array} \right), 
\]  

(B31)

and \( G_{PD,D'} = G_{D,PD'} \). In this work, we freely vary \( g_{pdd} \), which is defined by \( g_{pdd} = -2\sqrt{15} g' \) and corresponds to \( f_{\pi\Delta\Delta}/m_\pi \) in Ref. [17], in the fits.

7. **VPP' interaction**

The Lagrangian for the \( VPP' \) interaction is expressed as

\[
L_{VPP'} = L_{VPP'}^L \times L_{VPP'}^F. 
\]  

(B32)
The Lorentz part is given by
\[ L_{VPP'}^L = iP(\partial_\mu P')V^\mu. \] (B33)

The flavor part of the $VPP'$ Lagrangian is derived from the following SU(3) singlet form,
\[ L_{VaPP'}^F = g''[\{P_8 \otimes P_8'(8_2) \otimes V_8\}^{(1)}. \] (B34)

Let us evaluate the matrix elements for the flavor part that are needed for our calculation. We first consider the $VP_1 \rightarrow P_2$ and $P_1 \rightarrow VP_2$ transitions. For the case that the operator $P$ ($P'$) contracts with the $P_1$ ($P_2$) meson in the ket (bra) state, we have
\[ \langle P_2| L_{VPP'}^F|VP_1 \rangle = \langle P_2| L_{VaPP'}^F|VP_1 \rangle = G_{P_2,VP_1} \times (I_VI_V, I_P, I_P, I_P, I_P, I_P, I_P), \] (B35)
\[ \langle VP_2| L_{VPP'}^F|P_1 \rangle = \langle VPP_2| L_{VaPP'}^F|P_1 \rangle = G_{VPP_2, P_1} \times (I_VI_V, I_P, I_P, I_P, I_P, I_P, I_P), \] (B36)
with
\[ G_{P_2,VP_1} = \left(-1\right)^{-(\nu_2 + 1/2)} + 1 \frac{1}{\sqrt{3}} \sqrt{\left(\nu_2 + 1 \right)} \frac{1}{\left(\nu_2 + 1 \right)} \left(\begin{array}{ccc} 8 & 8 & 8 \end{array} \right), \] (B37)
and $G_{VPP_2,P_1} = G_{P_2,VP_1}$. On the other hand, for the case that the operator $P'$ ($P$) contracts with the $P_1$ ($P_2$) meson in the ket (bra) state, we have
\[ \langle P_2| L_{VPP'}^F|VP_1 \rangle = \langle P_2| L_{VaPP'}^F|VP_1 \rangle = G'_{P_2,VP_1} \times (I_VI_V, I_P, I_P, I_P, I_P, I_P, I_P), \] (B38)
\[ \langle VP_2| L_{VPP'}^F|P_1 \rangle = \langle VPP_2| L_{VaPP'}^F|P_1 \rangle = G'_{VPP_2, P_1} \times (I_VI_V, I_P, I_P, I_P, I_P, I_P, I_P), \] (B39)
with $G'_{P_2,VP_1} = -G_{P_2,VP_1}$ and $G'_{VPP_2,P_1} = -G_{VPP_2,P_1}$.

Next consider the $V \rightarrow P_1 P_2$ and $P_1 P_2 \rightarrow V$ transitions. For the case that the operator $P$ ($P'$) contracts with the $P_1$ ($P_2$) meson, we have
\[ \langle V| L_{VPP'}^F|P_1 P_2 \rangle = \langle V| L_{VaPP'}^F|P_1 P_2 \rangle = G_{V,P_1 P_2} \times (I_P, I_P, I_P, I_P, I_P, I_P, I_P), \] (B40)
\[ \langle P_1 P_2| L_{VPP'}^F|V \rangle = \langle P_1 P_2| L_{VaPP'}^F|V \rangle = G_{P_1 P_2, V} \times (I_P, I_P, I_P, I_P, I_P, I_P, I_P), \] (B41)
with
\[ G_{V,P_1 P_2} = -\frac{1}{\sqrt{3}} \frac{g''}{8} \left(\begin{array}{ccc} 8 & 8 & 8 \end{array} \right), \] (B42)
and $G_{V,P_1 P_2} = G_{V,P_1 P_2}$. For the case that the operator $P$ ($P'$) contracts with the $P_2$ ($P_1$) meson, however, we have
\[ \langle V| L_{VPP'}^F|P_1 P_2 \rangle = \langle V| L_{VaPP'}^F|P_1 P_2 \rangle = G'_{V,P_1 P_2} \times (I_P, I_P, I_P, I_P, I_P, I_P, I_P), \] (B43)
\[ \langle P_1 P_2| L_{VPP'}^F|V \rangle = \langle P_1 P_2| L_{VaPP'}^F|V \rangle = G'_{P_1 P_2, V} \times (I_P, I_P, I_P, I_P, I_P, I_P, I_P), \] (B44)
with $G'_{V,P_1 P_2} = -G_{V,P_1 P_2}$ and $G'_{P_1 P_2, V} = -G_{P_1 P_2, V}$. The parameter $g_{VPP} = (-2) \times (\sqrt{3}/24)g''$, which corresponds to $g_{\rho\pi\pi}$ in Ref. [17], is varied freely and determined by the fits.
8. **SPP′ interaction**

The Lagrangian for the SPP′ interaction is expressed as

\[ L_{SPP'} = L_{SPP'}^L \times L_{SPP'}^F. \]  
(B45)

The Lorentz part is given by

\[ L_{SPP'}^L = - (\partial^\mu P)(\partial_\mu P') S. \]  
(B46)

The flavor part of the SPP′ Lagrangian is derived from the following SU(3) singlet form,

\[ L_{SPP'}^F = g''' \left[ (P_8 \otimes P_8')^{(s_1)} \otimes S_8 \right]^{(1)}. \]  
(B47)

The necessary information for our calculation is the following matrix elements:

\[ \langle P_2 | L_{SPP'}^F | S P_1 \rangle = \langle P_2 | L_{SPP'}^F | S P_1 \rangle = G_{P_2; S P_1} \times (8 S z P_1 | F_1 P_1 | F_2 P_2 | F_z P_2), \]  
(B48)

\[ \langle S P_2 | L_{SPP'}^F | P_1 \rangle = \langle S P_2 | L_{SPP'}^F | P_1 \rangle = G_{S P_2; P_1} \times (8 S z P_1 | F_1 P_1 | F_2 P_2 | F_z P_2), \]  
(B49)

with

\[ G_{P_2; S P_1} = 2 \times (-1)^{(y P_1/2)+I P_1} \frac{1}{\sqrt{8}} \sqrt{\frac{2 I S + 1}{2 I P_2 + 1}} g''' \left( \begin{array}{ccc} 8 & 8 & 8 \\ I P_2 - Y P_2 & I P_1 Y P_1 & I S - Y S \end{array} \right), \]  
(B50)

and \( G_{S P_2; P_1} = G_{P_2; S P_1} \). Note that the factor 2 appears in the right hand side of Eq. (B50).

For \( P_1 \neq P_2 \), it arises from the fact that the full flavor-part-Lagrangian (B47) contains two terms that can contract with a given \( S, P_1, \) and \( P_2 \). For \( P_1 = P_2 \), however, only one term in Eq. (B47) can contract with a given \( S, P_1, \) and \( P_2 \), but there are two ways of contractions with \( P_1 \) and \( P_2 \). The parameter \( g_{spp} \equiv (-2/3) \times (\sqrt{30}/40) g''' \), which corresponds to \( g_{\pi\pi}/(2m_\pi) \) in Ref. [17], is varied freely and determined by the fits.

9. **Weinberg-Tomozawa (WT) interaction**

Finally, we present the so-called WT interaction for the \( PP'BB' \) four-point vertex, which is used for Model B. The Lagrangian is expressed as

\[ L_{WT} = L_{WT}^L \times L_{WT}^F. \]  
(B51)

The Lorentz part is given by

\[ L_{WT}^L = i P' (\partial_\mu P') \bar{B}' \gamma^\mu B, \]  
(B52)

and the flavor part is derived from the following SU(3) singlet form,

\[ L_{WT; P_8 P_8' B_8 B_8}^F = g''' \left[ (P_8' \otimes P_8)^{(s_2)} \otimes [B_8^{(s)} \otimes B_8]^{(s_2)} \right]^{(1)}. \]  
(B53)

Here it is noted that the SU(3)×SU(3) chiral symmetry does not allow one to have the contribution from the combination of \( B_8^{(s)} \otimes B_8 \rightarrow s_1 \). For the case that the operator \( P' \) \( (P) \) contracts with \( P_2 \) \( (P_1) \) meson in the bra (ket) state, we obtain the following matrix element,

\[ \langle P_2 B'| L_{WT}^F | P_1 B \rangle = \langle P_2 B' | L_{WT; P_8 P_8' B_8 B_8}^F | P_1 B \rangle = \sum_T G_{P_2 B'; P_1 B}^{WT(T)} \times (I P_1, I B' I B'|T T') (I P_1, I P_1, I B I B'|T T'), \]  
(B54)
with
\[ G_{P_2B',P_1B}^{WT(T)} = \left( \frac{\sqrt{2}}{48} g'' \right) \lambda_{P_2B',P_1B}^T, \]
and
\[ \lambda_{P_2B',P_1B}^T = (-12)(-1)^{-\frac{(Y_{B'}+Y_{P_1})+Y_{B'}+Y_P}{2}} \]
\[ \times \sum_{I} (2I+1) \left( \begin{array}{ccc} 8 & 8 & 8 \end{array} \right) \left( \begin{array}{ccc} 8_Y & 8_Y & 8_Y \end{array} \right) \]
\[ \times W(I_{P_1}I_{P_2}I_{B}I_{B'}; IT), \quad \text{(B56)} \]
where \( T^z = I_{P_1} + I_{P_2} - I_B - I_{B'} \), \( Y = Y_B - Y_{B'} = Y_{P_2} - Y_{P_1} \), and \( W(abcd;ef) \) is the Racah coefficients. On the other hand, for the case that the operator \( P (P') \) contracts with \( P_2 (P_1) \) meson in the bra (ket) state, we have
\[ \langle P_2B'|L^F_{WT}|P_1B \rangle = \langle P_2B'|L^F_{WT;P_2PB_1P_1B_1}|P_1B \rangle \]
\[ = \sum_T G_{P_2B';P_1B}^{WT(T)} \times \langle I_{P_2}I_{P_2}I_{B}I_{B'}|TT^z\rangle(I_{P_1}I_{P_1}I_{B}I_{B}|TT^z), \quad \text{(B57)} \]
with \( G_{P_2B';P_1B}^{WT(T)} = -G_{P_2B';P_1B}^{WT(T)} \).

Comparing with the leading order term of the SU(3)×SU(3) chiral Lagrangian, we find that the coupling constant \( g'' \) in Eq. (B53) can be related to the low energy constant \( f \),
\[ -\frac{1}{8f^2} = \frac{\sqrt{2}}{48} g''. \quad \text{(B58)} \]
The constant \( f \) is known as the decay constant of the pseudoscalar Nambu-Goldstone bosons in the chiral limit, and in this work it is taken to be \( f = 92.4 \) MeV. Also, we multiply \(-1/(8f^2)\) by a factor \( \gamma_{WT} \) and vary the factor in the fits.

**Appendix C: Matrix elements of meson-baryon potentials**

The plane-wave matrix elements for the meson-baryon exchange potentials \( v_{M'B',MB} \) can be expressed as
\[ \langle M'(k'), B'(p')|v_{M'B',MB}|M(k), B(p) \rangle = \frac{1}{(2\pi)^3} \sqrt{\frac{m_{B'}}{E_{B'}(p')}} \sqrt{\frac{m_B}{2E_M(k')}} \sqrt{\frac{m_B}{2E_B(p)}} \]
\[ \times \sum_T (I_{M'}I_{M}, I_{B'}I_{B}|TT^z\rangle(I_{M}I_{M}, I_{B}I_{B}|TT^z) \]
\[ \times V^{(T)}. \quad \text{(C1)} \]
The partial-wave decomposition of Eq. (C1) is explained in detail in Refs. \([10, 17]\) and is not presented here. In the following, the explicit expressions of \( V^{(T)} \) for \( P + B \to P' + B' \), \( P + B \to P' + D \), and \( P + B \to V + B' \) are presented, while we omit those for \( P' + D \to P + B \) and \( V + B' \to P + B \) since those can be deduced from the corresponding inverse processes.
1.  \( P(k) + B(p) \to P'(k') + B'(p') \)

a.  \textit{s-channel} \( B \) exchange

\[
V_{sB_{ex}}^{1(T)} = \sum_{B_{ex}} C_{sB_{ex}}^{1(T)} G_{P'B',B_{ex}} G_{B_{ex},PB} \bar{u}_B(p') \not{k'} \gamma_5 S_{B_{ex}}(p + k) \not{k} \gamma_5 u_B(p),
\]  

where \( u_B(p) \) is the Dirac spinor for the baryon \( B \). In evaluating the time component of the propagators, \( S_B(p) = 1/(\not{p} - m_B) \) in the above as well as in the following, we follow the definite procedures defined by the unitary transformation method \cite{33, 34}. For more detail, see Appendix C of Ref. \cite{10}. Hereafter the particles exchanged are indicated with the subscript “ex.” The summation in Eq. (C2) runs over the spin-\( \frac{3}{2} \) octet \( B_{ex} \) states listed in Table IV.

b.  \textit{u-channel} \( B \) exchange

\[
V_{uB_{ex}}^{1(T)} = \sum_{B_{ex}} C_{uB_{ex}}^{1(T)} G_{B',PB_{ex}} G_{P'B_{ex},B} \bar{u}_B(p') \not{k'} \gamma_5 S_{B_{ex}}(p - k') \not{k} \gamma_5 u_B(p),
\]  

\[
C_{uB_{ex}}^{1(T)} = \sqrt{2I_B + 1} \sqrt{2I_{B'}} + 1 \sqrt{2I_{B'} + 1} W(I_P I_{B'} I_{B'} I_{B'}; I_{B_{ex}} T).
\]  

(5)

c.  \textit{u-channel} \( D \) exchange

\[
V_{uD_{ex}}^{1(T)} = \sum_{D_{ex}} C_{uD_{ex}}^{1(T)} G_{B',PD_{ex}} G_{P'D_{ex},B} \bar{u}_B(p') \not{k}_\alpha S_{D_{ex}}^{\alpha\beta}(p - k') \not{k}_\beta u_B(p),
\]  

\[
C_{uD_{ex}}^{1(T)} = \sqrt{2I_B + 1} \sqrt{2I_{B'} + 1} W(I_P I_{B'} I_{B'} I_{D_{ex}}; I_{D_{ex}} T),
\]  

(7)

where \( S_{D_{ex}}^{\alpha\beta}(p - k') \) is the propagator for the spin-\( \frac{3}{2} \) Rarita-Schwinger field \cite{17}.

d.  \textit{t-channel} \( V \) exchange

\[
V_{tV_{ex}}^{1(T)} = \sum_{V_{ex}} C_{tV_{ex}}^{1(T)} G_{P',V_{ex}P'} G_{V_{ex}B'} G_{B_{ex}B'} -1 \frac{1}{q^2 - m_{V_{ex}}^2} \not{q}_B \not{k'} \gamma_5 S_{V_{ex}}(p + k) \not{k} \gamma_5 u_B(p),
\]  

\[
C_{tV_{ex}}^{1(T)} = (-1)^{I_B + I_P - T} \sqrt{2I_{P'} + 1} \sqrt{2I_B + 1} W(I_P I_{B'} I_{B'} I_{V_{ex}}; I_{V_{ex}} T),
\]  

(9)

where the momentum transfer \( q \) is defined by \( q = k' - k \) or \( q = p - p' \).
e. $t$-channel $S$ exchange

$$V_{tS_{ex}}^{1(T)} = \sum_{s_{ex}} C_{tS_{ex}}^{1(T)} G_{P',s_{ex}p} G_{B's_{ex}B} \frac{-k \cdot k'}{q^2 - m_{s_{ex}}^2} \bar{u}_{B'}(p') u_B(p),$$  \hspace{1cm} (C10)

$$C_{tS_{ex}}^{1(T)} = (-1)^I_B + I_{P'} - T \sqrt{2I_{P'} + 1} \sqrt{2I_B + 1} W(I_P I_{P'} I_B'; I_{S_{ex}} T).$$  \hspace{1cm} (C11)

f. Modified $t$-channel $V$ exchange used for Model B

It is known that in the $q \to 0$ limit the Lorentz structure of the vector-meson-exchange potentials (C8) reduces to the one known as the WT interaction. Recent studies suggest that this contact interaction plays an important role for understanding the near-threshold phenomena in $s$-wave (see, e.g., Ref. [27]). To make a clear connection with such studies, in Model B we employ a modified vector-meson exchange potentials, instead of using the one described in Appendix (C1d). The explicit form is,

$$V_{tV,mod}^{1(T)} = V_{WT}^{1(T)} + V_{tV}^{1(T)}.$$  \hspace{1cm} (C12)

Here, $V_{WT}^{1(T)}$ is the contribution from the WT term described in Appendix (B9).

$$V_{WT}^{1(T)} = C_{WT}^{(T)} \left( -\frac{\gamma_{WT}}{8f^2} \right) \bar{u}_{B'}(p')(k + k') u_B(p),$$  \hspace{1cm} (C13)

$$C_{WT}^{(T)} = \lambda_{P'B';PB}^T.$$  \hspace{1cm} (C14)

The second term in the right hand side of Eq. (C12), $V_{tV}^{1(T)}$, is same as $V_{tV}^{1(T)}$ but the following term is subtracted,

$$V_{tV}^{1(T)} \big{|}_{q \to 0} = \sum_{V_{ex}} C_{tV_{ex}}^{1(T)} G_{P',V_{ex}p} G_{B'V_{ex}B} \left( \frac{1}{m_{V_{ex}}^2} \right) \bar{u}_{B'}(p')(k + k') u_B(p).$$  \hspace{1cm} (C15)

This term corresponds to the WT term in terms of the resonance saturation, and thus should be subtracted to avoid the double counting. Here we note that for $V_{WT}^{1(T)}$ we attach the following combination of the form factors:

$$c_{P'B',PB}^{WT} F(\bar{q}, \Lambda_{P'B',PB}^{WT}) F(q, \Lambda_{P'B',PB}^{WT}) + (1 - c_{P'B',PB}^{WT}) F(\bar{k}', \Lambda_{P'B',PB}^{WT}) F(k, \Lambda_{P'B',PB}^{WT}),$$  \hspace{1cm} (C16)

where $\bar{q} = \bar{k}' - k$, $F(k, \Lambda)$ is defined in Eq. (12), and the coefficients $c_{P'B',PB}^{WT}$ and cutoffs $\Lambda_{P'B',PB}^{WT}$, which satisfy $c_{P'B',P'B'}^{WT} = c_{P'B',P'B'}^{WT}$ and $\Lambda_{P'B',P'B'}^{WT} = \Lambda_{P'B',P'B'}^{WT}$, respectively, are determined by the fits.

2. $P(k) + B(p) \to P'(k') + D(p')$

a. $s$-channel $B$ exchange

$$V_{sB_{ex}}^{2(T)} = \sum_{B_{ex}} C_{sB_{ex}}^{2(T)} G_{P'D,B_{ex}} G_{B_{ex}PB} \bar{U}_D(\mu)(p') k' \gamma_5 u_B(p),$$  \hspace{1cm} (C17)
$$C_{sB_{ex}}^{2(T)} = \delta_{T I_B},$$  \hspace{1cm} (C18)

where $U_D^\mu(p)$ is the Rarita-Schwinger vector-spinor for spin-$3/2$ baryon $D$ with the momentum $p$.

b. u-channel $B$ exchange

$$V_{uB_{ex}}^{2(T)} = \sum_{B_{ex}} C_{uB_{ex}}^{2(T)} G_{D, PB_{ex}} G_{P'B_{ex}, B} \bar{U}_{D}^\mu(p') k'_\mu S_{B_{ex}}(p - k') \not{k}' \gamma_5 u_B(p),$$  \hspace{1cm} (C19)

$$C_{uB_{ex}}^{2(T)} = \sqrt{2I_B + 1}\sqrt{2I_D + 1} W(I_P I_B I_{P'}; I_{B_{ex} T}).$$  \hspace{1cm} (C20)

c. u-channel $D$ exchange

$$V_{uD_{ex}}^{2(T)} = \sum_{D_{ex}} C_{uD_{ex}}^{2(T)} G_{D, PD_{ex}} G_{P'D_{ex}, B} (\not{k} \gamma_5 S_{D_{ex}}^{mu} (p - k') k'_\mu u_B(p),$$  \hspace{1cm} (C21)

$$C_{uD_{ex}}^{2(T)} = \sqrt{2I_B + 1}\sqrt{2I_D + 1} W(I_P I_B I_{P'}; I_{D_{ex} T}).$$  \hspace{1cm} (C22)

d. t-channel $V$ exchange

$$V_{tV_{ex}}^{2(T)} = \sum_{V_{ex}} C_{tV_{ex}}^{2(T)} G_{P', V_{ex} P} G_{V_{ex} D, B} \frac{1}{q^2 - m_{V_{ex}}^2} \bar{U}_{D}^\mu(p') [q_\mu (\not{k}' + \not{k} - (k + k')_\mu \not{\gamma}_5) u_B(p),$$  \hspace{1cm} (C23)

$$C_{tV_{ex}}^{2(T)} = (-1)^{I_B + I_{P'} - T} \sqrt{2I_{P'} + 1}\sqrt{2I_B + 1} W(I_P I_B I_{D}; I_{V_{ex} T}).$$  \hspace{1cm} (C24)

3. $P(k) + B(p) \rightarrow V(k') + B'(p')$

a. s-channel $B$ exchange

$$V_{sB_{ex}}^{3(T)} = \sum_{B_{ex}} C_{sB_{ex}}^{3(T)} i G_{V B', B_{ex}} G_{B_{ex}, PB} u_B(p') \gamma_5 u_B(p),$$  \hspace{1cm} (C25)

$$C_{sB_{ex}}^{3(T)} = \delta_{T I_B}. \hspace{1cm} (C26)$$

Here we have introduced

$$\Gamma_{V B_{ex} B'} = \left[ \not{\epsilon}_V^* + \frac{\kappa_{V B_{ex} B'}}{2(m_{B_{ex}} + m_{B'}) (\not{\epsilon}_V^* \not{k}' - \not{k}' \not{\epsilon}_V^*)} \right],$$  \hspace{1cm} (C27)

and $\epsilon_V^\mu$ is the polarization vector of the vector meson $V$.  

23
b. $u$-channel $B$ exchange

\[ V_{uB_{ex}}^{3(T)} = \sum_{B_{ex}} C_{uB_{ex}}^{3(T)} i G_{B',p_{B_{ex}}B} G_{V_{B_{ex},B}B'}(p') \gamma_5 S_{B_{ex}}(p - k') \Gamma_{V_{B_{ex}}B} u_B(p), \]  
\[ C_{uB_{ex}}^{3(T)} = \sqrt{2I_B + 1 + 2I_{B'}} + 1W(I_P I_B I_B; I_{B_{ex}} T). \] (C28)

(c) $t$-channel $P$ exchange

\[ V_{tP_{ex}}^{3(T)} = \sum_{P_{ex}} C_{tP_{ex}}^{3(T)} i G_{V,P_{ex}P} G_{P_{ex}B',B} \frac{1}{q^2 - m_{P_{ex}}^2} \bar{u}_{B'}(p')(q - k) \gamma_5 \Gamma_{V P} u_B(p), \]  
\[ C_{tP_{ex}}^{3(T)} = (-1)^{I_B + I_{P_{ex}} - T} \sqrt{2I_V + 1 + 2I_B + 1W(I_P I_V I_B; I_{P_{ex}} T).} \] (C30)

Appendix D: Model parameters

In this appendix, we list the values of model parameters determined via our analysis of the unpolarized and polarized observables of $K^- p \to K^0 N, \pi \Sigma, \pi \Lambda, K \Xi$ up to $W = 2.1$ GeV. The channel masses are presented in Table V. The parameters associated with the exchange potentials are listed in Tables VI-X while those associated with the bare $Y^*$ states are listed in Tables XI-XII.

[1] H. Zhang, J. Tulpan, M. Shrestha, and D. M. Manley, Phys. Rev. C 88, 035205 (2013).
[2] R. Armenteros et al., Nucl. Phys. B14, 91 (1969).
[3] B. Conforto et al., Nucl. Phys. B8, 265 (1968); B34, 41 (1971).
[4] E. Burkhardt et al., Nucl. Phys. B14, 106 (1969).
[5] A. J. Van Horn, Nucl. Phys. B87, 145 (1975).
[6] R. J. Hemingway, J. Eades, D. M. Harmsen, J. O. Petersen, A. Putzer, C. Kiesling, D. E. Plane, and W. Wittek, Nucl. Phys. B91, 12 (1975).
[7] P. Baillon and P. J. Litchfield, Nucl. Phys. B94, 39 (1975).
[8] G. P. Gopal, R. T. Ross, A. J. Van Horn, A. C. McPherson, E. F. Clayton, T. C. Bacon, and I. Butterworth, Nucl. Phys. B119, 362 (1977).
[9] J. Beringer et al. (Particle Data Group), Phys. Rev. D 86, 010001 (2012).
[10] A. Matsuyama, T. Sato, and T.-S. H. Lee, Phys. Rep. 439, 193 (2007).
[11] B. Juliá-Díaz, T.-S. H. Lee, A. Matsuyama, and T. Sato, Phys. Rev. C 76, 065201 (2007).
[12] B. Juliá-Díaz, T.-S. H. Lee, A. Matsuyama, T. Sato, and L. C. Smith, Phys. Rev. C 77, 045205 (2008).
[13] H. Kamano, B. Juliá-Díaz, T.-S. H. Lee, A. Matsuyama, and T. Sato, Phys. Rev. C 79, 025206 (2009).
[14] B. Juliá-Díaz, H. Kamano, T.-S. H. Lee, A. Matsuyama, T. Sato, and N. Suzuki, Phys. Rev. C 80, 025207 (2009).
[15] H. Kamano, B. Juliá-Díaz, T.-S. H. Lee, A. Matsuyama, and T. Sato, Phys. Rev. C 80, 065203 (2009).
[16] H. Kamano, S. X. Nakamura, T.-S. H. Lee, and T. Sato, Phys. Rev. C 81, 065207 (2010).
[17] H. Kamano, S. X. Nakamura, T.-S. H. Lee, and T. Sato, Phys. Rev. C 88, 035209 (2013).
[18] H. Kamano, Phys. Rev. C 88, 045203 (2013).
[19] H. Zhang, J. Tulpan, M. Shrestha, and D. M. Manley, Phys. Rev. C 88, 035204 (2013).
[20] D. M. Manley, Int. J. Mod. Phys. A 18, 441 (2003).
[21] M. Shrestha and D. M. Manley, Phys. Rev. C 86, 055203 (2012).
[22] T.-S. H. Lee, NSTAR2005: Proceedings of the Workshop on the Physics of Excited Nucleons, edited by S. Capstick, V. Crede, and P. Eugenio (World Scientific, 2006), p. 1.
[23] T. Motoba, D. E. Lanskoy, D. J. Millener, and Y. Yamamoto, Nucl. Phys. A804, 99 (2008); references therein.
[24] T. Nagae et al., Spectroscopic Study of Ξ-Hypernucleus, 12 Be, via the 12 C(K−, K+) Reaction (J-PARC E05), http://j-parc.jp/researcher/Hadron/en/pac_0606/pdf/p05-Nagae.pdf.
[25] J. Schnick and R. H. Landau, Phys. Rev. Lett. 58, 1719 (1987).
[26] S. Shinmura, M. Wada, M. Obu, and Y. Akaishi, Prog. Theor. Phys. 124, 1255 (2010); references therein.
[27] T. Hyodo and D. Jido, Prog. Part. Nucl. Phys. 67, 55 (2012); references therein.
[28] G.-L. He and R. H. Landau, Phys. Rev. C 48, 3047 (1993).
[29] J. K. S. Man, Y. Oh, and K. Nakayama, Phys. Rev. C 83, 055201 (2011).
[30] N. Suzuki, T. Sato, and T.-S. H. Lee, Phys. Rev. C 79, 025205 (2009); 82, 045206 (2010).
[31] H. Kamano, S. X. Nakamura, T.-S. H. Lee, and T. Sato, in preparation.
[32] H. Feshbach, Theoretical Nuclear Physics, Nuclear Reactions (Wiley, New York, 1992).
[33] M. Kobayashi, T. Sato, and H. Ohtsubo, Prog. Theor. Phys. 98, 927 (1997).
[34] T. Sato and T.-S. H. Lee, Phys. Rev. C 54, 2660 (1996).
[35] N. Suzuki, B. Juliá-Díaz, H. Kamano, T.-S. H. Lee, A. Matsuyama, and T. Sato, Phys. Rev. Lett. 104, 042302 (2010).
[36] D. B. Lichtenberg, Phys. Rev. D 10, 3865 (1974).
[37] A. Baldini, V. Flaminio, W. G. Moorhead, and D. R. O. Morrison, Total Cross-Sections for Reactions of High Energy Particles, Landolt-Börnstein Numerical Data and Functional Relationships in Science and Technology, Vol. 12 (Springer-Verlag, Berlin, 1988).
[38] Y. Ikeda and T. Sato, Phys. Rev. C 76, 035203 (2007); references therein.
[39] R. W. Manweiler et al., Phys. Rev. C 77, 015205 (2008).
[40] S. Prakhov et al., Phys. Rev. C 80, 025204 (2009).
[41] A. M. Sandorfi, S. Hoblit, H. Kamano, and T.-S. H. Lee, J. Phys. G 38, 053001 (2011).
[42] G. Höhler, F. Kaiser, R. Koch, and E. Pietarinen, Handbook of Pion Nucleon Scattering, Physics Data Vol.12 (Karlsruhe, 1979); G. Höhler, Pion-nucleon scattering (Springer-Verlag, Berlin 1983), Vol. I/92; G. Höhler and A. Schulte, πN Newsletter 7, 94 (1992); G. Höhler, πN Newsletter 9, 1 (1993).
[43] R. E. Cutkosky, C. P. Forsyth, R. E. Hendrick, and R. L. Kelly, Phys. Rev. D 20, 2839 (1979).
[44] Y. Ikeda, T. Hyodo, D. Jido, H. Kamano, T. Sato, and K. Yazaki, Prog. Theor. Phys. 125, 1205 (2011).
[45] B. Borasoy, U.-G. Meißner, and R. Nißler, Phys. Rev. C 74, 055201 (2006).
[46] M. Döring and U.-G. Meißner, Phys. Lett. B704, 663 (2011).
[47] Y. Ikeda, T. Hyodo, and W. Weise, Nucl. Phys. A881, 98 (2012).
[48] H. Noumi et al., Proposal for experiment at J-PARC for Spectroscopic study of hyperon resonances below $\bar{K}N$ threshold via the $(K^-,n)$ reaction on Deuteron (J-PARC E31), http://j-parc.jp/researcher/Hadron/en/pac_1207/pdf/E31_2012-9.pdf.
[49] D. Aston et al., Nucl. Phys. B296, 493 (1988).
[50] J. J. de Swart, Rev. Mod. Phys. 35, 916 (1963).
FIGURES
FIG. 1. Exchange mechanisms in $v_{M'B',MB}$
FIG. 2. (Color online) Total cross sections of the $K^-p$ reactions up to $W = 2.1$ GeV. The red solid curves (blues dashed curves) are the fitted results of Model A (Model B). The same applies to Figs. 3-18 below.
FIG. 3. (Color online) $d\sigma/d\Omega$ of $K^-p \rightarrow K^-p$. 
FIG. 4. (Color online) $d\sigma/d\Omega$ of $K^- p \rightarrow K^- p$ (continued).
FIG. 5. (Color online) $d\sigma/d\Omega$ of $K^{-}p \to K^{0}n$. 

$1466 \text{ MeV}$ to $1796 \text{ MeV}$
FIG. 6. (Color online) $d\sigma/d\Omega$ of $K^-p \rightarrow \bar{K}^0n$ (continued).
FIG. 7. (Color online) $P$ of $K^- p \rightarrow K^- p$. 
FIG. 8. (Color online) $d\sigma/d\Omega$ of $K^-p \rightarrow \pi^-\Sigma^+$. 
FIG. 9. (Color online) $d\sigma/d\Omega$ of $K^-p \rightarrow \pi^+\Sigma^-$. 
FIG. 10. (Color online) $d\sigma/d\Omega$ of $K^- p \rightarrow \pi^0 \Sigma^0$. 
FIG. 11. (Color online) $P \times d\sigma/d\Omega$ of $K^- p \rightarrow \pi^- \Sigma^+$. 
FIG. 12. (Color online) $P$ of $K^- p \rightarrow \pi^- \Sigma^+$. 
FIG. 13. (Color online) $P \times d\sigma/d\Omega$ of $K^- p \rightarrow \pi^0 \Sigma^0$. 
FIG. 14. (Color online) $P$ of $K^-p \rightarrow \pi^0 \Sigma^0$. Filled (open) circles are the data from Ref. [39] (Ref. [40]).
FIG. 15. (Color online) $d\sigma/d\Omega$ of $K^- p \rightarrow \pi^0 \Lambda$. 

42
FIG. 16. (Color online) $d\sigma/d\Omega$ of $K^-p \rightarrow \pi^0\Lambda$ (continued).
FIG. 17. (Color online) $P \times d\sigma/d\Omega$ of $K^- p \to \pi^0 \Lambda$. 

-50 0 50 -50 0 50 -50 0 50
FIG. 18. (Color online) $P$ of $K^-p \rightarrow \pi^0\Lambda$. 
FIG. 19. (Color online) Determined partial-wave amplitudes of $\bar{K}N \rightarrow \bar{K}N$ with isospin $I = 0$. Upper (lower) panels are for real (imaginary) parts of the amplitudes. Results of Model A (Model B) are shown in red solid (blue dashed) curves. Our results are compared with the single-energy solution (filled circles) given in Ref. [19].
FIG. 20. (Color online) Determined partial-wave amplitudes of $\bar{K}N \to \bar{K}N$ with isospin $I = 1$. See the caption of Fig. 19 for the description of the figure.
FIG. 21. (Color online) Determined partial-wave amplitudes of $\bar{K}N \rightarrow \pi\Sigma$ with isospin $I = 0$. See the caption of Fig. 19 for the description of the figure.
FIG. 22. (Color online) Determined partial-wave amplitudes of $\bar{K}N \to \pi \Sigma$ with isospin $I = 1$. See the caption of Fig. 19 for the description of the figure.
FIG. 23. (Color online) Determined partial-wave amplitudes of $\bar{K}N \rightarrow \pi\Lambda$. See the caption of Fig. [19] for the description of the figure.
FIG. 24. (Color online) Spin-rotation angle $\beta$ predicted from Models A (red solid curves) and Model B (blue dashed curves). The results are shown for the $K^- p \rightarrow \bar{K} N, \pi \Sigma, \pi \Lambda$ reactions. Our predictions are compared with the $\beta$ calculated by using the partial-wave amplitudes of the KSU single-energy solution [19] (black dotted curves). Note that $\beta$ is modulo $2\pi$. 
FIG. 25. (Color online) Determined partial-wave amplitudes of $\bar{K}N \to K\Xi$ with isospin $I = 0$. See the caption of Fig. 19 for the description of the figure.
FIG. 26. (Color online) Determined partial-wave amplitudes of $\bar{K}N \rightarrow K\Xi$ with isospin $I = 1$. See the caption of Fig. 19 for the description of the figure.
FIG. 27. (Color online) Predicted $K^-p$ reaction total cross section. The upper (lower) row is the results of Model A (Model B). (Left) Comparison of our predicted $\sigma_{K^-p}^{\text{tot}}$ (solid curve) with the data (open circles). The data are taken from Ref. [9]. (Right) The curves showing how the predicted contributions from each channel are added up to the total $\sigma_{K^-p}^{\text{tot}}$. 
TABLES
TABLE I. The orbital angular momentum \((L)\) and total spin \((S)\) of each \(MB\) channel allowed in a given partial wave. In the first column, partial waves are denoted with the conventional notation \(l_{J2J}\) as well as \((I,J^P)\).

| \(l_{J2J}\) \((I,J^P)\) | \(KN\) | \(\pi\Sigma\) | \(\pi\Lambda\) | \(\bar{K}\bar{\Xi}\) | \(\pi\Sigma^*\) \((\pi\Sigma^*)_1\) | \(\pi\Sigma^*\) \((\pi\Sigma^*)_2\) | \(\bar{K}^*N\) \((\bar{K}^*N)_1\) | \(\bar{K}^*N\) \((\bar{K}^*N)_2\) | \(\bar{K}^*N\) \((\bar{K}^*N)_3\) |
|---------------------------|-------|-----------|-----------|----------------|-----------------|----------------|--------------------|--------------------|--------------------|
| \(S_0\) \((0,\frac{1}{2}^-)\)   | \((0,\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) |
| \(S_1\) \((1,\frac{1}{2}^-)\)   | \((0,\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) |
| \(P_0\) \((0,\frac{3}{2}^-)\)   | \((0,\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) |
| \(P_{01}\) \((0,\frac{3}{2}^-)\)  | \((0,\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) |
| \(P_{11}\) \((1,\frac{3}{2}^-)\)  | \((0,\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) |
| \(P_{13}\) \((1,\frac{3}{2}^-)\)  | \((0,\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) |
| \(D_{03}\) \((0,\frac{5}{2}^-)\)  | \((0,\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) |
| \(D_{05}\) \((0,\frac{5}{2}^-)\)  | \((0,\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) |
| \(D_{13}\) \((1,\frac{5}{2}^-)\)  | \((0,\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) |
| \(D_{15}\) \((1,\frac{5}{2}^-)\)  | \((0,\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) |
| \(F_{05}\) \((0,\frac{7}{2}^+)\)  | \((0,\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) |
| \(F_{07}\) \((0,\frac{7}{2}^+)\)  | \((0,\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) |
| \(F_{15}\) \((1,\frac{7}{2}^+)\)  | \((0,\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) |
| \(F_{17}\) \((1,\frac{7}{2}^+)\)  | \((0,\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((0,\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) | \((-\frac{1}{2})\) |
TABLE II. Observables and number of the data considered in this coupled-channels analysis. See Ref. [19] for the data references for $d\sigma/d\Omega$, $P$, and $P \times d\sigma/d\Omega$; for total cross sections ($\sigma$), we took data from Refs. [37, 38]. Resulting “$\chi^2$/data” values for Model A (Model B) are listed in the forth (fifth) column.

| Reactions     | Observables | Number of data | $\chi^2$/data |
|---------------|-------------|----------------|---------------|
|               |             | Model A | Model B | Model A | Model B |
| $K^-p \rightarrow K^-p$ | $d\sigma/d\Omega$ | 3962   | 4.26   | 4.18   |
|               | $P$         | 510    | 2.26   | 2.17   |
|               | $\sigma$    | 253    | 2.50   | 2.72   |
| $K^-p \rightarrow \bar{K}^0n$ | $d\sigma/d\Omega$ | 2950   | 3.79   | 3.15   |
|               | $\sigma$    | 260    | 1.93   | 2.98   |
| $K^-p \rightarrow \pi^-\Sigma^+$ | $d\sigma/d\Omega$ | 1792   | 5.83   | 5.55   |
|               | $P$         | 418    | 2.37   | 2.22   |
|               | $P \times d\sigma/d\Omega$ | 177    | 2.21   | 2.80   |
|               | $\sigma$    | 173    | 2.75   | 4.86   |
| $K^-p \rightarrow \pi^0\Sigma^0$ | $d\sigma/d\Omega$ | 580    | 6.24   | 8.15   |
|               | $P$         | 196    | 5.43   | 5.13   |
|               | $P \times d\sigma/d\Omega$ | 189    | 1.26   | 1.30   |
|               | $\sigma$    | 125    | 1.90   | 1.86   |
| $K^-p \rightarrow \pi^+\Sigma^-$ | $d\sigma/d\Omega$ | 1786   | 3.46   | 4.06   |
|               | $\sigma$    | 181    | 1.37   | 1.71   |
| $K^-p \rightarrow \pi^0\Lambda$ | $d\sigma/d\Omega$ | 2178   | 2.68   | 4.12   |
|               | $P$         | 693    | 1.46   | 1.66   |
|               | $P \times d\sigma/d\Omega$ | 176    | 1.43   | 1.38   |
|               | $\sigma$    | 207    | 2.23   | 2.51   |
| $K^-p \rightarrow K^0\Xi^0$ | $\sigma$ | 15    | 0.63   | 0.61   |
| $K^-p \rightarrow K^+\Xi^-$ | $\sigma$ | 27    | 1.60   | 1.52   |
| Total         |             | 16848  | 3.67   | 3.81   |
**TABLE III.** Scattering length \((a_{MB})\) and effective range \((r_{MB})\) extracted from our analysis. The results are shown in the isospin basis. The sign convention of these threshold parameters is taken to be the same as that used in Ref. [44].

|                  | Model A          | Model B          |
|------------------|------------------|------------------|
|                  | \(I = 0\)       | \(I = 1\)       | \(I = 0\)       | \(I = 1\)       |
| \(a_{KN} \) (fm)| \(-1.75 + i1.46\) | \(0.55 + i0.56\) | \(-1.59 + i1.79\) | \(0.34 + i0.55\) |
| \(a_{K\Xi} \) (fm)| \(0.31 + i0.10\) | \(-0.38 + i0.07\) | \(-0.27 + i0.05\) | \(-0.77 + i0.04\) |
| \(r_{KN} \) (fm)| \(1.12 - i0.24\) | \(1.35 - i0.61\) | \(0.88 - i0.15\) | \(1.27 + i0.17\) |
| \(r_{K\Xi} \) (fm)| \(0.10 + i0.73\) | \(-0.51 - i0.39\) | \(-17.2 - i6.42\) | \(-0.87 - i0.31\) |
TABLE IV. Exchanged particles considered in the potentials $v_{M'B',MB}$.

|       | $\bar{K}N$ | $\pi\Sigma$ | $\pi\Lambda$ | $K\Xi$ | $\pi\Sigma^*$ | $K^*N$ |
|-------|------------|-------------|--------------|--------|--------------|--------|
| $\bar{K}N$ | s          | $\Lambda, \Sigma$ | $\Lambda, \Sigma$ | $\Lambda, \Sigma$ | $\Lambda, \Sigma$ | $\Lambda, \Sigma$ |
|        | u          | $N, \Delta$ | $N$ | $\Lambda, \Sigma, \Sigma^*$ | $N, \Delta$ | $-$ |
|        | t          | $\rho, \omega, \phi, \sigma, f_0$ | $K^*, \kappa$ | $K^*, \kappa$ | $-$ | $K^*$ |
| $\pi\Sigma$ | s          | $\Lambda, \Sigma$ | $\Sigma$ | $\Lambda, \Sigma$ | $\Lambda, \Sigma$ | $\Lambda, \Sigma$ |
|        | u          | $\Lambda, \Sigma, \Sigma^*$ | $\Sigma, \Sigma^*$ | $\Xi, \Xi^*$ | $\Lambda, \Sigma, \Sigma^*$ | $N, \Delta$ |
|        | t          | $\rho, \sigma, f_0$ | $\rho$ | $K^*, \kappa$ | $\rho$ | $K$ |
| $\pi\Lambda$ | s          | $\Sigma$ | $\Sigma$ | $\Sigma$ | $\Sigma$ | $\Sigma$ |
|        | u          | $\Sigma, \Sigma^*$ | $\Xi, \Xi^*$ | $\Sigma, \Sigma^*$ | $N$ | $-$ |
|        | t          | $\sigma, f_0$ | $\kappa$ | $\rho$ | $K$ | $-$ |
| $K\Xi$ | s          | $\Lambda, \Sigma$ | $\Lambda, \Sigma$ | $\Lambda, \Sigma$ | $\Lambda, \Sigma$ | $\Lambda, \Sigma$ |
|        | u          | $\Omega$ | $\Xi^*$ | $-$ | $-$ | $-$ |
|        | t          | $\rho, \omega, \phi, \sigma, f_0$ | $K^*$ | $-$ | $-$ | $-$ |
| $\pi\Sigma^*$ | | | | | | $-$ |
| $K^*N$ | | | | | | $-$ |
TABLE V. Channel masses that are used for the meson-baryon Green’s functions (4) and (5), and for external particles in the exchange potentials $v_{MB',MB}$.

| Masses | (MeV)   |
|--------|---------|
| $m_N$  | 938.5   |
| $m_{\Sigma}$ | 1193.2 |
| $m_{\Lambda}$ | 1115.7 |
| $m_{\Xi}$ | 1318.3 |
| $m_{\Sigma}^*$ | 1435.2 |
| $m_{\pi}$ | 138.5   |
| $m_{\bar{K}}$ | 493.7   |
| $m_{\bar{K}}^*$ | 930.4   |
TABLE VI. Masses used for the exchange particles in $v_{M'B'MB}$. All masses are kept constant during the fits except for the $\sigma$ ($m_\sigma$), $f_0$ ($m_{f0}$), and $\kappa$ ($m_\kappa$) masses.

| Masses | Model A (MeV) | Model B (MeV) |
|--------|---------------|---------------|
| $m_N$  | 938.5         | 478.9         |
| $m_\Lambda$ | 1115.7       | 1114.6        |
| $m_\Sigma$ | 1193.2        | 804.1         |
| $m_\Xi$  | 1318.3        | 871.3         |
| $m_\Delta$ | 1211.0        |
| $m_\Sigma^*$ | 1384.5        |
| $m_\Xi^*$ | 1533.4        |
| $m_\Omega$ | 1672.5        |
| $m_K$  | 495.6         |
| $m_\rho$ | 775.2         |
| $m_\omega$ | 782.7         |
| $m_\phi$ | 1019.5        |
| $m_{K^*}$ | 893.8         |
TABLE VII. Fitted values of coupling constants associated with the exchange potentials $v_{M'B',MB}$.

| Couplings                  | Model A      | Model B      |
|----------------------------|--------------|--------------|
| $g_v$                      | -2.279       | -2.508       |
| $g_s$                      | 12.192       | 11.050       |
| $\alpha_p^D$              | 0.616        | 0.606        |
| $\alpha_v^D$              | 0.225        | -0.07        |
| $\alpha_s^D$              | 0.710        | 0.743        |
| $g_{pbd} \times m_\pi$    | 1.446        | 1.620        |
| $g_{vbd} \times m_\rho$   | 10.230       | 10.561       |
| $g_{pdd} \times m_\pi$    | 0.053        | 0.040        |
| $g_{vpp}$                  | 13.304       | 13.710       |
| $g_{spp} \times 2m_\pi$   | 0.044        | 0.015        |
| $\kappa_{\rho NN}$        | 2.649 $\times \kappa_{\rho NN}$ | 0.216 $\times \kappa_{\rho NN}$ |
| $\kappa_{\omega NN}$      | 1.247 $\times \kappa_{\rho NN}$ | 0.302 $\times \kappa_{\rho NN}$ |
| $\kappa_{\phi NN}$        | 0.065 $\times \kappa_{\rho NN} / (2m_N)$ | 2.753 $\times \kappa_{\rho NN} / (2m_N)$ |
| $\kappa_{\phi NN}$        | 0.092 $\times \kappa_{\rho NN} / (2m_N)$ | 2.955 $\times \kappa_{\rho NN} / (2m_N)$ |
| $\kappa_{\phi NN}$        | 0.689 $\times \kappa_{\rho NN} / (2m_N)$ | 2.648 $\times \kappa_{\rho NN} / (2m_N)$ |
| $\kappa_{\rho NN} / (m_N + m_\Lambda)$ | 2.221 $\times \kappa_{\rho NN} / (2m_N)$ | 2.385 $\times \kappa_{\rho NN} / (2m_N)$ |
| $\kappa_{\rho NN} / (m_N + m_\Lambda)$ | 0.002 $\times \kappa_{\rho NN} / (2m_N)$ | 1.384 $\times \kappa_{\rho NN} / (2m_N)$ |
| $\kappa_{\rho NN} / (m_\Xi + m_\Sigma)$ | 2.057 $\times \kappa_{\rho NN} / (2m_N)$ | 2.311 $\times \kappa_{\rho NN} / (2m_N)$ |
| Cutoffs           | Model A (MeV) | Model B (MeV) |
|------------------|---------------|---------------|
| $\Lambda_{\pi NN}$ | 576           | 642           |
| $\Lambda_{\pi \Sigma}$ | 1735          | 1670          |
| $\Lambda_{\pi \Sigma\Sigma}$ | 795           | 1141          |
| $\Lambda_{K^* NA}$ | 1400          | 1437          |
| $\Lambda_{K^* N\Sigma}$ | 637           | 886           |
| $\Lambda_{\rho NN}$ | 900           | 551           |
| $\Lambda_{\rho \Lambda\Sigma}$ | 529           | 1207          |
| $\Lambda_{\rho \Sigma\Sigma}$ | 651           | 1396          |
| $\Lambda_{K^* N\Lambda}$ | 1696          | 1569          |
| $\Lambda_{K^* N\Sigma}$ | 1200          | 594           |
| $\Lambda_{K^* \Xi\Lambda}$ | 1111          | 500           |
| $\Lambda_{K^* \Xi\Sigma}$ | 590           | 502           |
| $\Lambda_{\omega NN}$ | 915           | 950           |
| $\Lambda_{\phi NN}$ | 902           | 632           |
| $\Lambda_{\sigma NN}$ | 1308          | 1690          |
| $\Lambda_{f_0 NN}$ | 539           | 1188          |
| $\Lambda_{\kappa NN}$ | 1048          | 826           |
| $\Lambda_{\kappa N\Sigma}$ | 1172          | 948           |
| $\Lambda_{\pi N\Delta}$ | 1263          | 1216          |
| $\Lambda_{\pi \Sigma\Sigma^*}$ | 1698          | 1595          |
| $\Lambda_{\pi \Lambda\Sigma^*}$ | 1749          | 1794          |
| $\Lambda_{K\Sigma\Delta}$ | 744           | 730           |
| $\Lambda_{K^* N\Sigma^*}$ | 1255          | 1318          |
| $\Lambda_{\rho N\Delta}$ | 1297          | 1410          |
| $\Lambda_{\rho \Sigma\Sigma^*}$ | 1735          | 1636          |
| $\Lambda_{K^* \Sigma\Delta}$ | 1294          | 1384          |
| $\Lambda_{\pi \Delta\Delta}$ | 1209          | 988           |
| $\Lambda_{K^* \Delta\Sigma^*}$ | 852           | 893           |
| $\Lambda_{\rho \pi\pi}$ | 1795          | 1765          |
| $\Lambda_{\rho K\bar{K}}$ | 915           | 959           |
| $\Lambda_{K^* K\pi}$ | 995           | 1149          |
| $\Lambda_{\omega K\bar{K}}$ | 734           | 1001          |
| $\Lambda_{\phi K\bar{K}}$ | 692           | 518           |
| $\Lambda_{\pi\pi\pi}$ | 601           | 1800          |
| $\Lambda_{\sigma K\bar{K}}$ | 848           | 524           |
| $\Lambda_{f_0\pi\pi}$ | 611           | 1588          |
| $\Lambda_{f_0 K\bar{K}}$ | 611           | 1494          |
| $\Lambda_{\kappa K\pi}$ | 868           | 1537          |
TABLE IX. The parameters associated with the modified $t$-channel potentials in Appendix C 1 f.
These are only relevant to Model B.

| Cutoffs                          | (MeV) | Parameters               |
|----------------------------------|-------|--------------------------|
| $\Lambda_{(K_N,\pi\Sigma,\pi\Lambda),K_N}$ | 1009  | $c_{K_N,K_N}^{WT}$       |
| $\Lambda_{(\pi\Sigma,K\Xi),\pi\Sigma}$    | 1241  | $c_{(K_N,\pi\Sigma),\pi\Sigma}^{WT}$ |
| $\Lambda_{K\Xi,\pi\Lambda}$           | 1195  | $c_{K_N,\pi\Lambda}^{WT}$ |
| $\Lambda_{K\Xi,K\Xi}$                | 648   | $c_{(\pi\Sigma,\pi\Lambda,K\Xi),K\Xi}^{WT}$, $\gamma^{WT}$ |
TABLE X. Fitted values of bare mass $M_{Y^*}^0$ of the $Y^*$ states. The numbers ($i = 1, 2$) in parentheses in the first column indicate the $i$-th bare state in a given partial wave.

| $l_{2J}$ | $M_{Y^*}^0$ (MeV) | Model A | Model B |
|----------|-------------------|---------|---------|
| $S_{01}$ (1) | 2109 | 1800 |
| $S_{01}$ (2) | 2381 | 2530 |
| $P_{01}$ (1) | 1880 | 1880 |
| $P_{01}$ (2) | 2100 | 2028 |
| $P_{03}$ | 2078 | 2169 |
| $D_{03}$ (1) | 1900 | 1962 |
| $D_{03}$ (2) | 2100 | 2003 |
| $D_{05}$ (1) | 2330 | 2400 |
| $D_{05}$ (2) | 2484 | 2565 |
| $F_{05}$ | 2295 | 2216 |
| $F_{07}$ | 2586 | 2455 |
| $S_{11}$ (1) | 1974 | 2116 |
| $S_{11}$ (2) | 2301 | 2280 |
| $P_{11}$ (1) | 1860 | 1995 |
| $P_{11}$ (2) | 1970 | 2316 |
| $P_{13}$ (1) | 1600 | 1602 |
| $P_{13}$ (2) | 2437 | 2146 |
| $D_{13}$ (1) | 1913 | 1926 |
| $D_{13}$ (2) | 2062 | 1987 |
| $D_{15}$ | 2321 | 2174 |
| $F_{15}$ | 2253 | 2300 |
| $F_{17}$ | 2484 | 2466 |
TABLE XI. Fitted values of cutoffs and coupling constants of the bare $Y^* \to MB$ vertices ($MB = KN, \pi\Sigma, \pi\Lambda, K\Xi, \pi\Sigma^*, K^*N$) for Model A. The corresponding $(LS)$ quantum numbers of each $MB$ state are shown in Table II. The cutoff $\Lambda_{Y^*}$ is listed in the unit of MeV. The numbers ($i = 1, 2$) in parentheses in the first column indicate the $i$-th bare state in a given partial wave.

| $l$ | $I$ | $J$ | $\Lambda_{Y^*}$ | $K\Sigma$ | $\pi\Sigma$ | $\pi\Lambda$ | $K\Xi$ | $C_{MB(LS),Y^*}$ | $(\pi\Sigma^*)_1$ | $(\pi\Sigma^*)_2$ | $(K^*N)_1$ | $(K^*N)_2$ | $(K^*N)_3$ |
|-----|-----|-----|-----------------|-----------|------------|------------|--------|----------------|----------------|----------------|-------------|-------------|-------------|
| $S_{01}$ (1) | 763 | 10.000 | 8.483 | -1.066 | -1.742 | - | 5.291 | -0.001 | - | |
| $S_{01}$ (2) | 1913 | 10.000 | -0.856 | -1.667 | 0.197 | - | 2.213 | -0.112 | - | |
| $P_{01}$ (1) | 762 | 1.333 | 5.086 | 3.723 | -6.968 | - | 3.716 | -8.019 | - | |
| $P_{01}$ (2) | 1671 | 2.376 | -0.341 | -1.460 | -1.985 | - | 0.993 | 0.214 | - | |
| $P_{03}$ | 643 | 0.000 | 10.000 | -8.000 | 5.079 | -0.185 | 3.999 | 10.000 | 0.150 | |
| $D_{03}$ (1) | 998 | 0.417 | -0.131 | -0.099 | 7.253 | 0.047 | -0.019 | -10.000 | 0.263 | |
| $D_{03}$ (2) | 1194 | 0.578 | 0.189 | 0.143 | 0.470 | 0.316 | 0.044 | 2.513 | -0.618 | |
| $D_{05}$ (1) | 760 | 0.196 | -1.024 | -0.486 | 1.272 | -0.018 | 0.821 | -0.561 | 0.045 | |
| $D_{05}$ (2) | 1127 | 0.131 | -0.654 | 0.288 | 0.858 | -0.001 | -0.195 | 0.146 | 0.002 | |
| $F_{05}$ | 1294 | 0.063 | -0.009 | 0.001 | 2.065 | -0.006 | 0.005 | -1.539 | 0.003 | |
| $F_{07}$ | 825 | 0.000 | 0.150 | 0.020 | 0.047 | -0.003 | -0.021 | 0.089 | 0.004 | |
| $S_{11}$ (1) | 1418 | 3.972 | 3.274 | 3.803 | -6.611 | -0.179 | - | 7.387 | 0.178 | |
| $S_{11}$ (2) | 1119 | 9.042 | -2.110 | -2.191 | 6.050 | 0.183 | - | 0.711 | -0.976 | |
| $P_{11}$ (1) | 1094 | 2.240 | -0.578 | 1.075 | 2.569 | -5.986 | - | 1.306 | 2.049 | |
| $P_{11}$ (2) | 631 | 1.313 | 1.105 | -2.472 | 9.289 | 4.991 | - | 4.565 | 6.859 | |
| $P_{11}$ (2) | 680 | 2.117 | -4.241 | -0.816 | 3.601 | 2.466 | 0.287 | 6.777 | 3.465 | 0.638 | |
| $P_{13}$ (2) | 720 | 3.125 | -1.013 | 9.945 | -0.438 | 6.296 | 0.539 | 4.846 | 4.455 | -0.191 | |
| $D_{13}$ (1) | 626 | 0.000 | 0.512 | -1.184 | 0.268 | 10.000 | 0.547 | -0.051 | -6.000 | 3.338 | |
| $D_{13}$ (2) | 895 | 0.404 | 0.670 | 1.055 | -0.060 | 8.000 | -1.095 | -0.063 | -0.724 | 0.868 | |
| $D_{15}$ | 783 | 0.647 | 0.568 | 0.459 | 0.009 | -1.718 | 0.021 | 0.077 | -1.942 | 0.003 | |
| $F_{15}$ | 644 | 0.060 | -0.375 | 0.521 | 0.012 | -0.244 | 0.999 | -0.023 | 1.190 | 0.021 | |
| $F_{17}$ | 944 | 0.044 | -0.010 | 0.181 | -0.016 | -0.254 | 0.006 | 0.024 | -0.015 | -0.002 | |

66
TABLE XII. Fitted values of cutoffs and coupling constants of the bare $Y^* \to MB$ vertices ($MB = KN, \pi\Sigma, \pi\Lambda, K\Xi, \pi\Sigma^*, K^*N$) for Model B. The corresponding ($LS$) quantum numbers of each $MB$ state are shown in Table I. The cutoff $\Lambda_{Y^*}$ is listed in the unit of MeV. The numbers ($i = 1, 2$) in parentheses in the first column indicate the $i$-th bare state in a given partial wave.

| $l_{12J}$ | $\Lambda_{Y^*}$ | $C_{MB(LS), Y^*}$ |
|----------|-----------------|-------------------|
|          | $K N$ | $\pi \Sigma$ | $\pi \Lambda$ | $K \Xi$ | $(\pi \Sigma^*)_1$ | $(\pi \Sigma^*)_2$ | $(K^*N)_1$ | $(K^*N)_2$ | $(K^*N)_3$ |
| $S_{01}$ (1) | 2000 | 2.649 | -2.451 | - | 1.882 | -0.026 | - | 0.216 | 0.215 | - |
| $S_{01}$ (2) | 1788 | 6.958 | -0.432 | - | -1.383 | -0.080 | - | 10.000 | 0.078 | - |
| $P_{01}$ (1) | 844  | 8.208 | 7.992 | - | 7.954 | -1.989 | - | -3.826 | -6.086 | - |
| $P_{01}$ (2) | 1419 | 4.150 | 1.378 | - | -1.017 | -1.968 | - | 0.245 | -0.302 | - |
| $P_{03}$ | 575  | 2.160 | 10.000 | - | -7.124 | 8.810 | -0.175 | -8.686 | 10.000 | 0.610 |
| $D_{03}$ (1) | 1024 | 0.849 | -0.086 | - | -0.138 | 0.455 | 0.872 | -0.450 | 0.899 | -0.080 |
| $D_{03}$ (2) | 600  | 0.000 | -0.133 | - | -1.304 | -5.993 | 2.837 | 3.454 | -9.230 | 0.450 |
| $D_{05}$ (1) | 760  | 0.196 | -1.024 | - | -0.486 | -1.272 | -0.018 | 0.821 | -0.561 | 0.045 |
| $D_{05}$ (2) | 1080 | 0.239 | -0.964 | - | 0.123 | 0.570 | -0.002 | -0.038 | 0.069 | 0.011 |
| $F_{05}$ | 1234 | 0.069 | 0.001 | - | 0.003 | 2.086 | -0.006 | 0.011 | -1.575 | 0.006 |
| $F_{07}$ | 825  | 0.000 | -0.073 | - | 0.016 | 0.511 | -0.005 | -0.001 | 0.126 | 0.009 |
| $S_{11}$ (1) | 2000 | 4.322 | 4.382 | 0.482 | 6.231 | 0.107 | - | -1.065 | -0.016 | - |
| $S_{11}$ (2) | 895  | 0.000 | -1.513 | 10.000 | -10.000 | -2.198 | - | -7.404 | -0.471 | - |
| $P_{11}$ (1) | 677  | 0.000 | -4.646 | -2.090 | -0.810 | 6.000 | - | -5.075 | 3.464 | - |
| $P_{11}$ (2) | 1572 | 0.159 | -3.056 | -0.642 | -2.813 | 0.463 | - | -1.708 | 1.284 | - |
| $P_{13}$ (1) | 558  | 3.336 | -7.898 | -1.363 | -2.302 | 10.000 | -0.965 | 4.867 | 2.165 | 0.216 |
| $P_{13}$ (2) | 525  | 5.530 | -7.133 | -4.361 | -0.102 | 0.510 | -2.943 | 4.620 | 0.202 | 0.164 |
| $D_{13}$ (1) | 1248 | 0.211 | 0.311 | 0.068 | 0.059 | -9.316 | 0.026 | 0.073 | -4.300 | -0.024 |
| $D_{13}$ (2) | 758  | 0.014 | 0.152 | 2.588 | -0.783 | 1.360 | -1.067 | -1.714 | 4.806 | -0.615 |
| $D_{15}$ | 821  | 0.383 | 0.524 | 0.159 | 0.178 | -1.109 | 0.016 | -0.084 | -0.283 | 0.087 |
| $F_{15}$ | 780  | 0.070 | -0.164 | 0.360 | -0.034 | -0.803 | 0.245 | -0.011 | 2.329 | 0.028 |
| $F_{17}$ | 840  | 0.053 | 0.008 | 0.151 | -0.020 | -0.205 | 0.020 | 0.042 | 0.081 | -0.004 |

67