Cosmological perturbations in warm inflationary models with viscous pressure

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Abstract

Scalar and tensorial cosmological perturbations generated in warm inflationary scenarios whose matter-radiation fluid is endowed with a viscous pressure are considered. Recent observational data from the WMAP experiment are employed to restrict the parameters of the model. Although the effect of this pressure on the matter power spectrum is of the order of a few percent, it may be detected in future experiments.

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I. INTRODUCTION

As is well known, warm inflation -as opposed to the conventional “cool” inflation \[1\]- has the attractive feature of not necessitating a reheating phase at the end of the accelerated expansion thanks to the decay of the inflaton into radiation and particles during the slow-roll \[2\]. Thus, the temperature of the Universe does not drop dramatically and the Universe can smoothly proceed into the decelerated, radiation-dominated era essential for a successful big-bang nucleosynthesis \[3\]. This scenario has further advantages, namely: (i) the slow-roll condition \( \dot{\phi}^2 \ll V(\phi) \) can be satisfied for steeper potentials, (ii) the density perturbations originated by thermal fluctuations may be larger than those of quantum origin \[4\], and (iii) it may provide a very interesting mechanism for baryogenesis \[5\].

Warm inflation was criticized on the basis that the inflaton cannot decay during the slow-roll \[6\]. However, in the recent years, it has been demonstrated that the inflaton can indeed decay during the slow-roll phase -see \[7\] and references therein- whereby it now rests on solid theoretical grounds.

Usually, for the sake of simplicity, when studying the dynamics of warm inflation the particles created in the decay of the inflaton are treated as radiation thereby ignoring altogether the existence of particles with mass in the fluid thus generated. However, the very existence of these particles necessarily alters the dynamics as they modify the fluid pressure in two important ways: (i) its hydrodynamic, equilibrium, pressure is no longer \( p = \rho/3 \), with \( \rho \) the energy density of the matter-radiation fluid, but the slightly more general expression \( p = (\gamma - 1)\rho \) where the adiabatic index, \( \gamma \), is bounded by \( 1 \leq \gamma \leq 2 \). (ii) It naturally arises a non-equilibrium, viscous, pressure \( \Pi \), via two different mechanisms: (a) the inter-particle interactions \[8\], and (b) the decay of particles within the fluid \[9\].

A radiative fluid (e.g., a mixture of photons and electrons) is a well-known example of mechanism (a). Actually, it plays a significant role in the description of the matter-radiation decoupling in the standard cosmological model \[3,10\]. Likewise, a hefty viscous pressure arises in mixtures of different particles species, or of identical species but with different energies -a case in point is the Maxwell-Boltzmann gas \[11\].

Concerning mechanism (ii), it is well known that the decay of particles within a fluid can be formally described by a bulk viscous pressure, \( \Pi \). This is so because the decay is an entropy-producing scalar phenomenon linked to the spontaneous widening of the phase
space and the bulk viscous pressure is also an scalar entropy-producing agent. In the case of warm inflation it has been proposed that the inflaton can excite a heavy field and trigger the decay of the latter into light fields [12].

Recently, a detailed analysis of the dynamics of warm inflation with viscous pressure showed that when $\Pi \neq 0$ the inflationary region takes a larger portion of the phase space associated to the autonomous system of differential equations than otherwise [13]. It then follows that the viscous pressure facilitates inflation and lends support to the warm inflationary scenario.

Our main target is to study the scalar and tensorial cosmological perturbations associated to this scenario and contrast them with the data gathered by the three-year Wilkinson Microwave Anisotropy Probe (WMAP) [14]. We shall follow the method used in a recent paper by two of us in which the cosmological perturbations generated by warm inflation driven by a tachyon field were considered [15]. The main differences between the latter work and the present one are: (i) here the inflaton is a scalar field, not a tachyon field; (ii) in Ref. [15] no bulk dissipative pressure was considered. For the viscous pressure we shall assume the usual fluid dynamics expression $\Pi = -3\zeta H$ [8], where $\zeta$ denotes the phenomenological coefficient of bulk viscosity and $H$ the Hubble function. This coefficient is a positive-definite quantity (a restriction imposed by the second law of thermodynamics) and in general it is expected to depend on the energy density of the fluid. We shall resort to the WMAP data to restrict the aforesaid coefficient.

The outline of the paper is as follows. Next Section presents our model. Sections III and IV deal with the scalar and tensor perturbations, respectively. Section V specifies to a chaotic potential and studies the limit of high dissipation. Finally, Section VI summarizes our findings. We choose units so that $c = \hbar = 8\pi G = 1$.

II. WARM INFLATIONARY MODEL WITH VISCOUS PRESSURE

We consider a spatially flat Friedmann-Robertson-Walker (FRW) universe filled with a self-interacting scalar field $\phi$ (the inflaton), of energy density, $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$, and an imperfect fluid, of energy density $\rho$ and total pressure $p + \Pi$, consisting into a mixture of matter and radiation of adiabatic index $\gamma$. 


The corresponding Friedmann equation reads
\[ 3H^2 = \frac{\dot{\phi}^2}{2} + V(\phi) + \rho. \] (1)

Because of the inflaton decays with rate \( \Gamma \) into the imperfect fluid the conservation equations for the inflaton and the fluid generalize to
\[ \dot{\rho} + 3H(\rho + p_\phi) = -\Gamma \dot{\phi}^2 \implies \ddot{\phi} + (3H + \Gamma)\dot{\phi} = -V_\phi, \] (2)

and
\[ \dot{\rho} + 3H(\rho + p + \Pi) = \dot{\rho} + 3H(\gamma\rho + \Pi) = \Gamma \dot{\phi}^2, \] (3)

respectively. In general, \( \Gamma \) depends on \( \phi \) and as a consequence of the second law of thermodynamics one has that \( \Gamma = \Gamma(\phi) > 0 \). As is customary, an upper dot indicates temporal derivative, and \( V_{\phi} = \partial V(\phi)/\partial \phi \).

During the inflationary phase \( V(\phi) \) dominates over any other form of energy, therefore Friedmann’s equation reduces to
\[ 3H^2 = V(\phi). \] (4)

Likewise, imposing the slow-roll conditions, \( \dot{\phi}^2 \ll V(\phi) \), and \( \ddot{\phi} \ll (3H + \Gamma)\dot{\phi} \) [1], Eq. (2) becomes
\[ 3H [1 + r] \dot{\phi} = -V_{\phi}, \] (5)

where the quantity \( r \equiv \Gamma/(3H) \) quantifies the dissipation of the model. Throughout this paper we shall restrict our analysis to the high dissipation regime, i.e., \( r \gg 1 \). The reason for this limitation is the following. Outside this regime radiation and particles produced both by the decay of the inflaton and the decay of the heavy fields will be much dispersed by the inflationary expansion, whence they will have little chance to interact and give rise to a
non-negligible bulk viscosity. Likewise, because a much lower number of heavy fields will be excited the number of decays of heavy fields into lighter ones will diminish accordingly. (The weak dissipation regime \( r \leq 1 \) has been considered by Berera and Fang (second reference of [2]) and Moss [16]). Further, if \( r \) is not big, the fluid will be largely diluted and the mean free path of the particles will become comparable or even larger than the Hubble horizon. Hence, the regime will no longer be hydrodynamic but Knudsen’s and the hydrodynamic expression \( \Pi = -3\zeta H \) we are using for the viscous pressure will become invalid. In such a situation, a consistent analysis should make use of Boltzmann’s equation but this would complicate matters tremendously. This lies beyond the scope of this paper.

Assuming that the decay of the inflaton is quasi-stable during inflation it follows that \( \dot{\rho} \ll 3H(\gamma \rho + \Pi) \) and \( \dot{\rho} \ll \Gamma \dot{\phi}^2 \) whence, from Eq. (3), we get

\[
\rho \simeq \frac{r \dot{\phi}^2 - \Pi}{\gamma} \tag{6}
\]

for the energy density of the fluid. It becomes apparent that the viscous pressure (being necessarily negative) augments the energy density of the matter-radiation fluid.

With the help of the dimensionless slow-roll parameter

\[
\varepsilon \equiv -\frac{\dot{H}}{H^2} = \frac{1}{2(1+r)} \left[ \frac{V_{,\phi}}{V} \right]^2, \tag{7}
\]

a useful relation follows between the energy densities, namely,

\[
\rho = \frac{1}{\gamma} \left[ \frac{2r}{3(1+r)} \varepsilon \rho_\phi - \Pi \right]. \tag{8}
\]

By definition, inflation lasts so long as \( \ddot{a} > 0 \). This amounts to the condition \( \varepsilon < 1 \), which with the help of last equation can be expressed as

\[
\rho_\phi > 3\frac{(1+r)}{2r} [\gamma \rho + \Pi]. \tag{9}
\]

Consequently, warm inflation with viscous pressure comes to a close when

\[
\rho_\phi \simeq 3\frac{(1+r)}{2r} [\gamma \rho + \Pi]. \tag{10}
\]
The number of e-folds at the end of inflation is given by

$$N(\phi) = -\int_{\phi_i}^{\phi_f} \frac{V}{V', \phi} (1 + r) d\phi',$$

(11)

where the subscripts $i$ and $f$ stand for the beginning and end of inflation, respectively.

For later purpose we introduce here the second slow-roll parameter, $\eta \equiv -\frac{\ddot{H}}{H \dot{H}}$, as a function of the potential and its two first derivatives,

$$\eta \approx \frac{1}{(1 + r)} \left[ \frac{V_{,\phi\phi}}{V} - \frac{1}{2} \left( \frac{V_{,\phi}}{V} \right)^2 \right].$$

(12)

III. SCALAR PERTURBATIONS

In terms of the longitudinal gauge, the perturbed FRW metric can be written as

$$ds^2 = (1 + 2\Phi) dt^2 - a(t)^2 (1 - 2\Psi) \delta_{ij} dx^i dx^j,$$

(13)

where the functions $\Phi = \Phi(t, x)$ and $\Psi = \Psi(t, x)$ denote the gauge-invariant variables of Bardeen [17]. Introducing the Fourier components $e^{ikx}$, with $k$ the wave number, the following set of equations, in the momentum space, follow from the perturbed Einstein field equations -to simplify the writing we omit the subscript $k$-

$$\Phi = \Psi,$$

(14)

$$\dot{\Phi} + H\Phi = \frac{1}{2} \left[ -\frac{(\gamma\rho + \Pi)}{k} a \frac{v}{v} + \dot{\phi} \delta\phi \right],$$

(15)

$$\left( \delta\phi \right) + [3H + \Gamma] (\delta\phi) + \left[ \frac{k^2}{a^2} + V_{,\phi\phi} + \dot{\phi}\Gamma_{,\phi} \right] \delta\phi = 4\dot{\phi} \Phi - \left[ \phi \Gamma + 2V_{,\phi} \right] \Phi,$$

(16)

$$\left( \delta\rho \right) + 3\gamma H \delta\rho + ka(\gamma\rho + \Pi)v + 3(\gamma\rho + \Pi)\dot{\Phi} - \phi^2 \Gamma_{,\phi} \delta\phi - \Gamma \dot{\phi} [2(\delta\phi) + \dot{\phi}\Phi] = 0,$$

(17)
and

\[ \dot{v} + 4Hv + \frac{k}{a} \left[ \Phi + \frac{\delta \rho}{(\rho + p)} + \frac{\Gamma \dot{\Phi}}{(\rho + p)} \delta \phi \right] = 0, \quad (18) \]

where

\[ \delta p = (\gamma - 1)\delta \rho + \delta \Pi, \quad \delta \Pi = \Pi \left[ \frac{\zeta \delta \rho}{\zeta} \delta \phi + \Phi + \frac{\dot{\phi}}{H} \right], \quad (19) \]

and the quantity \( v \) arises upon splitting the velocity field as \( \delta u_j = -\frac{ik_j}{k} v \ e^{ikx} \ (j = 1, 2, 3) \) [17].

Since the inflaton and the matter-radiation fluid interact with each other isocurvature (i.e., entropy) perturbations emerge alongside the adiabatic ones. This occurs because warm inflation can be understood as an inflationary model with two basics fields [18, 19]. In this context, dissipative effects themselves can produce a variety of spectral ranging from red to blue [4, 19, 20], thus producing the running blue to red spectral suggested by WMAP three-year data [14]. We will come back to this point below.

When looking for non-decreasing adiabatic and isocurvature modes on large scales, \( k \ll aH \) (which depend only weakly on time), it is permissible to neglect \( \dot{\Phi} \) and those terms with two-times derivatives. Upon doing this and using the slow-roll conditions, the above equations simplify to

\[ \Phi \simeq \frac{1}{2H} \left[ -\frac{(\gamma \rho + \Pi)}{k} a \ v \ \delta \phi \right], \quad (20) \]

\[ [3H + \Gamma] (\delta \phi) + \left[ V,_{\phi \phi} + \dot{\phi} \Gamma,_{\phi} \right] \delta \phi \simeq - \left[ \dot{\phi} \Gamma + 2V,_{\phi} \right] \Phi, \quad (21) \]

\[ \delta \rho \simeq \frac{\dot{\phi}^2}{3\gamma H} \left[ \Gamma,_{\phi} \delta \phi + \Gamma \Phi \right], \quad (22) \]

and
\[ v \simeq -\frac{k}{4aH} \left[ \Phi + \frac{(\gamma - 1)\delta \rho + \delta \Pi}{\gamma \rho + \Pi} + \frac{\Gamma \dot{\phi}}{\gamma \rho + \Pi} \delta \phi \right]. \tag{23} \]

By combining of Eqs. (22) and (23) with (20), the latter becomes

\[ \Phi \simeq \frac{\dot{\phi}}{2H G(\phi)} \left[ 1 + \frac{\Gamma}{4H} + \left( \frac{\gamma - 1}{\gamma} + \frac{\Pi \frac{\zeta}{\rho}}{\Pi} \right) \frac{\dot{\phi}}{\Gamma, \phi} \right], \tag{24} \]

where

\[ G(\phi) = 1 - \frac{1}{8H^2} \left[ 2\gamma \rho + 3\Pi + \gamma \rho + \Pi \left( \frac{\zeta}{\rho} - 1 \right) \right]. \tag{25} \]

In the non-viscous limit, i.e., \( \Pi \rightarrow 0 \) when \( G(\phi) \rightarrow 1 \) (since \( 1 \gg \rho/H^2 \)) and \( \gamma = 4/3 \), Eq. (24) reduces to the expression obtained for the conventional warm inflation [18].

With the help of Eq. (24) and substituting \( \phi \) by \( t \) as independent variable, Eq. (21) can be solved. Upon using Eq. (5) we get

\[ (3H + \Gamma) \frac{d}{dt} = (3H + \Gamma) \frac{\dot{\phi}}{\phi} \frac{d}{d\phi} = -V, \phi \frac{d}{d\phi}, \tag{26} \]

and introducing the ancillary function

\[ \varphi = \frac{\delta \phi}{V, \phi} \exp \left[ \int \frac{1}{(3H + \Gamma)} \Gamma, \phi \ d\phi \right], \tag{27} \]

we are led to the following differential equation for \( \varphi \),

\[ \frac{\varphi, \phi}{\varphi} = -\frac{3}{8 G(\phi)} \left( \frac{\Gamma + 6H}{(\Gamma + 3H)^2} \right) \left[ \Gamma + 4H - \left( \frac{\gamma - 1}{\gamma} + \frac{\Pi \frac{\zeta}{\rho}}{\Pi} \right) \frac{\Gamma, \phi V, \phi}{3\gamma H(3H + \Gamma)} \right] V, \phi. \tag{28} \]

Upon integrating it and resorting to Eq. (27) we get

\[ \delta \phi = CV, \phi \exp[\Im(\phi)], \tag{29} \]

where \( \Im(\phi) \) is given by

\[ \Im(\phi) = -\int \left[ \frac{1}{(3H + \Gamma) \Gamma, \phi} + \frac{3}{8G(\phi)} \left( \frac{\Gamma + 6H}{(\Gamma + 3H)^2} \right) \left( \Gamma + 4H - \right. \right. \]

\[ \left. \left. \left[ (\gamma - 1) + \frac{\Pi \frac{\zeta}{\rho}}{\Pi} \right] \frac{\Gamma, \phi V, \phi}{3\gamma H(3H + \Gamma)} \right) \frac{V, \phi}{V} \right] d\phi. \tag{30} \]
and $C$ is an integration constant.

Thus, the density perturbations reads

$$\delta_H = \frac{16\pi}{5} \exp[-3(\phi)] \frac{V_{,\phi}}{\delta\phi}.$$  \hspace{1cm} (31)

It is seen that in the absence of inflaton decay ($\Gamma = 0$, and therefore, $\Pi = 0$), Eq. (31) reduces to the typical expression for scalar perturbations, $\delta_H \sim V \delta\phi/(H \dot{\phi}) \sim H \delta\phi/\dot{\phi}$.

For high dissipation ($r \gg 1$), Eq. (31) simplifies to

$$\delta^2_H = \left(\frac{16\pi}{15}\right)^2 \exp[-2 \tilde{3}(\phi)] \frac{V_{,\phi}}{H^2 r^2 \phi^2} \delta\phi^2,$$ \hspace{1cm} (32)

where now $\tilde{3}(\phi) := 3(\phi) \mid_{r \gg 1}$ becomes

$$\tilde{3}(\phi) = -\int \left\{ \frac{1}{3Hr} \Gamma_{,\phi} + \frac{3}{8G(\phi)} \left[ 1 - \left( (\gamma - 1) + \Pi \frac{\zeta_{,\phi}}{\zeta} \right) \frac{\Gamma_{,\phi} V_{,\phi}}{9\gamma r H^2} \right] \right\} d\phi.$$ \hspace{1cm} (33)

During the slow-roll phase, the total density fluctuation, $\delta\rho_T = \delta\rho_{\phi} + \delta\rho$, and the metric perturbation, $\Phi$, are related -at first approximation- by

$$\delta\rho_T \simeq V_{,\phi} \delta\phi \simeq -2[1 + r] V G(\phi) \left[ 1 + \frac{\Gamma}{4H} + \left( [\gamma - 1] + \Pi \frac{\zeta_{,\phi}}{\zeta} \right) \frac{\dot{\phi}}{12\gamma H^2} \right]^{-1} \Phi,$$ \hspace{1cm} (34)

where use of Eq. (24) has been made. Again, for $\Gamma = \Pi = 0$, we recover the usual relation $\delta\rho_T/\rho_T \simeq -2\Phi$, typical of cool inflation, in which $\rho_T = \rho_{\phi} + \rho \simeq V(\phi)$.

In warm inflation, the fluctuations of the scalar field arise mainly from thermal interaction with the matter-radiation field. Therefore, following Taylor and Berera [21], for $r \gg 1$ we can write

$$\langle \delta\phi \rangle^2 \simeq \frac{k_F T_r}{2\pi^2},$$ \hspace{1cm} (35)

where $T_r$ stands for the temperature of the thermal bath and the wave number $k_F$ is given by $k_F = \sqrt{\Gamma H} = H \sqrt{3r} \geq H$, and corresponds to the freeze-out scale at which dissipation damps out the thermally excited fluctuations. The freeze-out wave number $k_F$ is defined at the point where the inequality $V_{,\phi \phi} < \Gamma H$, holds [21].
From Eqs. (31), (32) and (35) it follows that

$$\delta_H^2 \approx \frac{2}{25 \pi^2} \exp[-2\tilde{\Omega}(\phi)] \left[ \frac{T_r}{\tilde{\epsilon}^{1/2} V^{3/2}} \right],$$  \hspace{1cm} (36)$$

where $\tilde{\epsilon} \approx \frac{1}{2r} \left[ \frac{V_{,\phi}}{V} \right]^2$ denotes the dimensionless slow-roll parameter in the high dissipation phase.

The scalar spectral index $n_s$ is given by

$$n_s - 1 = \frac{d \ln \delta_H^2}{d \ln k},$$  \hspace{1cm} (37)$$

where the interval in wavenumber and the number of e-folds are related by $d \ln k(\phi) = -dN(\phi)$. Upon using Eqs. (36) and (37), it can be written as

$$n_s \approx 1 - \left[ \tilde{\epsilon} + 2 \tilde{\eta} + \left( \frac{2\tilde{\epsilon}}{r} \right)^{1/2} \left[ 2\tilde{\Omega},\phi - \frac{r,\phi}{2r} \right] \right],$$  \hspace{1cm} (38)$$

where

$$\tilde{\eta} \approx \frac{1}{r} \left[ \frac{V_{,\phi}\phi}{V} - \frac{1}{2} \left( \frac{V_{,\phi}}{V} \right)^2 \right]$$  \hspace{1cm} (39)$$

stands for the second slow-roll parameter, $\eta$, when $r \gg 1$.

One interesting feature of the three-year data gathered by the WMAP experiment is a significant running in the scalar spectral index $dn_s/d\ln k = \alpha_s$ [14]. Dissipative effects can lead to a rich variety of spectral from red to blue [19, 20]. From Eq. (38) it is seen that in our model the running of the scalar spectral index is given by

$$\alpha_s \approx -\sqrt{\frac{2}{r} \left[ \tilde{\epsilon},\phi + 2\tilde{\eta},\phi \right]} - \frac{\tilde{\epsilon}}{r} \left[ \left( \frac{\tilde{\epsilon},\phi}{\tilde{\epsilon}} - \frac{r,\phi}{r} \right) \left[ 2\tilde{\Omega},\phi - \frac{r,\phi}{2r} \right] + \left[ 4\tilde{\Omega},\phi - (\ln(r)),\phi \right] \right].$$  \hspace{1cm} (40)$$

In models with only scalar fluctuations, the marginalized value of the derivative of the spectral index can be approximated by $dn_s/d\ln k = \alpha_s \sim -0.05$ for WMAP-3 only [14].
IV. TENSOR PERTURBATIONS

As argued by Bhattacharya et al. [22], the generation of tensor perturbations during inflation gives rise to stimulated emission in the thermal background of gravitational waves. As a consequence, an extra temperature dependent factor, \( \coth(k/2T) \), enters the spectrum, \( A_g^2 \propto k^{n_g} \). Thus it now reads,

\[
A_g^2 = 2 \left( \frac{H}{2\pi} \right)^2 \coth \left( \frac{k}{2T} \right) \simeq \frac{V}{6 \pi^2} \coth \left( \frac{k}{2T} \right), \tag{41}
\]

the spectral index being

\[
n_g = \frac{d}{d \ln k} \ln \left( \frac{A_g^2}{\coth[k/2T]} \right) = -2 \varepsilon, \tag{42}
\]

where we have used Eq.(7).

A quantity of prime interest is the tensor-scalar ratio, defined as \( R(k_0) = \left( \frac{A_g^2}{P_R} \right) \bigg|_{k=k_0} \) where \( P_R \equiv 25 \delta_H^2/4 \) and \( k_0 \) is known as the pivot point. Its expression in the high dissipation limit, \( r \gg 1 \), follows from using Eqs. (36) and (41),

\[
R(k_0) = \left( \frac{A_g^2}{P_R} \right) \bigg|_{k=k_0} = \frac{2}{3} \left[ \left( \frac{\bar{e}_r^{1/2} V^{5/2}}{T_r} \right) \exp[2 \tilde{S}(\phi)] \coth \left( \frac{k}{2T} \right) \right] \bigg|_{k=k_0}. \tag{43}
\]

Combining the WMAP three-year data [14] with the SDSS large scale structure surveys [23], yields the upper bound \( R(k_0)=0.002 \text{ Mpc}^{-1} < 0.28 \) at 95% confidence level, where \( k_0 = 0.002 \text{ Mpc}^{-1} \) corresponds to \( \tau_0 k_0 \simeq 30 \), with the distance to the decoupling surface being \( \tau_0 = 14.4 \text{ Mpc} \). The SDSS is a measure of the galaxy distributions at red-shifts \( a \sim 0.1 \) and probes \( k \) in the range \( 0.016 h \text{ Mpc}^{-1} < k < 0.011 h \text{ Mpc}^{-1} \). The recent WMAP three-year data imply for the scalar curvature spectrum the values \( P_R(k_0) \equiv 25 \delta_H^2(k_0)/4 \simeq 2.3 \times 10^{-9} \) and the tensor-scalar ratio \( R(k_0) = 0.095 \). We shall make use of these values to set constraints on the parameters of our model.
V. THE HIGH DISSIPATION LIMIT WITH CHAOTIC POTENTIAL

In this section we assume for the inflaton the chaotic potential

\[ V(\phi) = \frac{1}{2} m^2 \phi^2, \quad (44) \]

where \( m > 0 \) is a free parameter, and (as mentioned above) we restrict ourselves to study the high dissipation regime \((r \gg 1)\).

A. The case \( \Gamma = \Gamma_0, \zeta = \zeta_0 \)

When the inflaton decay rate and the bulk viscosity coefficient are given by \( \Gamma = \Gamma_0 = \) constant \( > 0 \) and \( \zeta = \zeta_0 = \) constant, respectively, the first slow roll parameter reduces to

\[ \tilde{\varepsilon} = \frac{\sqrt{6} m \Gamma_0}{\phi}, \quad (45) \]

Likewise, the Hubble factor is given by

\[ H(\phi) = \frac{m \phi}{\sqrt{6}}, \quad (46) \]

and the parameter \( r \) becomes

\[ r = \frac{\sqrt{6} \Gamma_0}{3 m \phi} \gg 1. \quad (47) \]

Because in this scenario

\[ \dot{\phi} = -\frac{V_{,\phi}}{3rH}, \quad (48) \]

the inflaton depends on time as

\[ \phi(t) = \phi_i \exp \left[ -\frac{m^2 t}{\Gamma_0} \right] \approx \phi_i \left[ 1 - \frac{m^2}{\Gamma_0} t \right], \quad (49) \]

where \( \phi_i = \phi(t = t_i = 0) \). This entirely agrees with Taylor and Berera findings [21].
As for the energy density of the matter-radiation fluid we have the expression

\[
\rho = \frac{m \phi}{\gamma} \left[ \frac{\sqrt{6} m^2}{3 \Gamma_0} + \frac{3 \zeta_0}{\sqrt{6}} \right],
\]

(50)

or in terms of \( \rho_\phi \),

\[
\rho = \frac{\sqrt{3}}{\gamma} \left[ \frac{2 m^2}{3 \Gamma_0} + 3 \zeta_0 \right] \rho_\phi^{1/2}.
\]

(51)

By means of Eq. (11), the number of e-folds at the end of warm inflation is found to be

\[
N_{\text{total}} = - \int_{\phi_i}^{\phi_f} \frac{V}{V_\phi} \, r \, d\phi = \frac{\sqrt{6} \Gamma_0}{3 m} [\phi_i - \phi_f],
\]

(52)

where, because of \( V_i > V_f \), the initial and final values of the scalar field satisfy \( \phi_i > \phi_f \).

It is readily seen that \( \epsilon_f \approx 1 \) (i.e., \( \ddot{a}(t = t_f) \approx 0 \)), and that the scalar field at the end of inflation reads

\[
\phi_f = \frac{\sqrt{6} m}{\Gamma_0}.
\]

(53)

Rewriting the total number of e-folds in terms of \( \phi_i \) and \( \phi_f \) and using Eq. (53), we get

\[
\phi_i = \frac{1}{2} [N_{\text{total}} + 2] \phi_f = \frac{\sqrt{6} m}{2} \frac{\Gamma_0}{\Gamma_0} [N_{\text{total}} + 2] = \frac{\sqrt{6} m}{\Gamma_0} [N_{\text{total}} + 2].
\]

(54)

To comfortably solve the problems of the standard big-bang cosmology we must have \( N_{\text{total}} \approx 60 \) e-folds. This implies \( \phi_i \approx 76 m/\Gamma_0 \).

At the beginning of inflation

\[
\begin{align*}
\epsilon_r &= r(\phi = \phi_i) = r_i = \frac{1}{93} \left( \frac{\Gamma_0}{m} \right)^2 > 1,
\end{align*}
\]

(55)

which implies that \( \Gamma_0 \gg \sqrt{93} m \).
From Eq. (36), the scalar power spectrum results to be

\[
P_R(k_0) \approx \frac{1}{2\pi^2} \left[ 8\gamma_0 V(\phi_0) \right]^{1/2} + 2\sqrt{3}m^2(1 - 2\gamma) + 3\sqrt{3}\zeta_0 \Gamma_0(2 - 3\gamma)]^{3/2} \left[ \frac{\Gamma_0^{1/2} T_r}{3^{1/4}m^2 V(\phi_0)^{3/4}} \right],
\]

Likewise, Eq. (43) provides us with the tensor-scalar ratio

\[
R(k_0) \approx \frac{2}{3} \left[ 8\gamma_0 V(\phi_0) \right]^{1/2} + 2\sqrt{3}m^2(1 - 2\gamma) + 3\sqrt{3}\zeta_0 \Gamma_0(2 - 3\gamma)]^{-3/2} \\
\times \left[ \frac{3^{1/4}m^2 V(\phi_0)^{7/4}}{\Gamma_0^{1/2} T_r} \right] \coth \left( \frac{k}{2T} \right),
\]

where \( V(\phi_0) \) and \( \phi_0 \) stand for the potential and the scalar field, respectively, when the perturbation, of scale \( k_0 = 0.002 \) Mpc\(^{-1} \), was leaving the horizon.

By resorting to the WMAP three-year data, \( P_R(k_0) \approx 2.3 \times 10^{-9} \) and \( R(k_0) = 0.095 \), and choosing the parameters \( \gamma = 1.5, m = 10^{-6} m_p, T \simeq T_r \simeq 0.24 \times 10^{16} \) GeV and \( k_0 = 0.002 \) Mpc\(^{-1} \), it follows from Eqs. (56) and (57) that \( V(\phi_0) \approx 1.5 \times 10^{-11} m_p^4 \), and \( \zeta_0 \approx 3 \times 10^{-6} m_p^3 \). When the scale \( k_0 \) was leaving the horizon the inflaton decay rate \( \Gamma_0 \) is seen to be of the order of \( 10^{-3} m_p \). Thus Eq. (40) tells us that one must augment \( \zeta_0 \) by two orders of magnitude to have a running spectral index \( \alpha_s \) close to the observed value [14].

B. The case \( \Gamma = \Gamma(\phi), \zeta = \zeta(\rho) \)

Here we assume, \( \Gamma = \Gamma(\phi) = \alpha V(\phi) = \alpha m^2 \phi^2/2 \), and \( \zeta = \zeta(\rho) = \zeta_1 \rho \), where \( \alpha \) and \( \zeta_1 \) are positive-definite constants. Obviously, \( \Pi = -3 \zeta_1 \rho H \).

Using the chaotic potential, Eq. (44), we find that the slow roll parameter, the Hubble factor and \( r \) can be written as

\[
\tilde{\varepsilon} = \frac{2\sqrt{6}}{m \alpha \phi^3}, \quad H(\phi) = \frac{m \phi}{\sqrt[6]{6}}, \quad \text{and} \quad r = \frac{m \alpha \phi}{\sqrt[6]{6}} \gg 1,
\]

respectively. By combining the middle equation of (58) with Eq. (48) we find that

\[
\phi^2(t) = \phi_i^2 - \frac{4}{\alpha} t.
\]
The energy density of the matter-radiation fluid reads

$$\rho = \frac{4\sqrt{3} \, m}{3\phi} \left[ \frac{1}{\sqrt{2} \, \gamma - 3 \, \zeta_1 \, m \, \phi} \right],$$  \hspace{1cm} (60)

and, in terms of $\rho_\phi$,

$$\rho = \frac{\sqrt{12} \, m^2}{3\rho_\phi^{1/2}} \left[ \frac{1}{\gamma - 3 \, \zeta_1 \, \rho_\phi^{1/2}} \right].$$  \hspace{1cm} (61)

Using Eq. (11), the number of e-folds at the end of warm inflation results to be,

$$N_{\text{total}} = - \int_{\phi_i}^{\phi_f} \frac{V}{V_\phi} \, r \, d\phi = \frac{\alpha \, m \, \sqrt{6}}{36} \left[ \phi_i^3 - \phi_f^3 \right],$$ \hspace{1cm} (62)

For $\epsilon_f \simeq 1$ it is seen that the scalar field at the end of inflation reduces to

$$\phi_f = \left[ \frac{2\sqrt{6}}{\alpha \, m} \right]^{1/3}.$$ \hspace{1cm} (63)

Rewriting the total number of e-folds in terms of $\phi_f$ and $\phi_i$, and using last equation, we obtain

$$\phi = [3 \, N_{\text{total}} + 1]^{1/3} \phi_f = [3 \, N_{\text{total}} + 1]^{1/3} \left[ \frac{2\sqrt{6}}{\alpha \, m} \right]^{1/3}.$$ \hspace{1cm} (64)

To get $N_{\text{total}} \approx 60$ e-folds we must have $\phi_i \approx [362\sqrt{6}/(\alpha \, m)]^{1/3}$.

At the beginning of inflation the $r$ parameter becomes

$$r(\phi = \phi_i) = r_i = \left[ \frac{181}{3} \right]^{1/3} \alpha^{2/3} \, m^{2/3},$$ \hspace{1cm} (65)

resulting in the requirement that $\alpha \gg \sqrt{\frac{3}{181 \, m^2}}$ in the high dissipation regime.

From Eq.(66), the scalar power spectrum is seen to obey

$$P_R(k_0) \approx \frac{1}{2\pi^2} \Re_0 \left[ \frac{\alpha^{1/2} \, T_r}{3^{1/4} \, m^2 \, V(\phi_0)^{1/4}} \right],$$  \hspace{1cm} (66)

with

$$\Re_0 = \alpha^2 \, V(\phi_0)^2 + \beta_1^{16(\gamma + 2m^2\zeta_1)/(9\zeta_1 - 8\gamma)} + \left( 1 + \frac{\beta_2}{\beta_3} \right)^{\beta_4} - \left( 1 - \frac{\beta_2}{\beta_3} \right)^{\beta_4},$$

being $\zeta_1 \neq 8\gamma/9$, and where

$$\beta_1 = -8\gamma \, V(\phi_0) + 6\gamma^2 - 12\sqrt{3} \gamma \zeta_1 \sqrt{V(\phi_0)} - 3\gamma + 3\sqrt{3} \zeta_1 \sqrt{V(\phi_0)} + 9\zeta_1^2 V(\phi_0).$$
\[
\beta_2 = 2\sqrt{V(\phi_0)(8\gamma - 9\zeta_1^2)} + 3\sqrt{3}\zeta_1(4\gamma - 1),
\]
\[
\beta_3 = (8\gamma - 9\zeta_1^2)\sqrt{192\gamma^3 + 216\gamma^2\zeta_1^2 - 96\gamma^2 - 108\gamma\zeta_1^2 + 27\zeta_1^2},
\]
and
\[
\beta_4 = \left(\frac{8\sqrt{3}}{3\beta_3}\right)[64m^2\gamma^2 + 27m^2\gamma\zeta_1^2 - 64m^2\gamma + 36m^2\zeta_1 + 72\zeta_1\gamma^2 - 18\zeta_1\gamma].
\]

From Eq. (43) the tensor-scalar ratio can be obtained as
\[
R(k_0) \approx 2\left[\frac{3^{1/4}m^2V(\phi_0)^{5/4}}{\alpha^{1/2}R_0T_r}\right]\coth\left(\frac{k}{2T}\right).
\]

Resorting again to the WMAP three-year data \(P_{\mathcal{R}}(k_0) \approx 2.3 \times 10^{-9}, R(k_0) = 0.095\) and choosing the parameters \(\gamma = 1.5, \alpha \approx 10^7 \text{m}_P^3, V(\phi_0) \approx 1.5 \times 10^{-11} \text{m}_P^4, T \approx T_r \approx 0.24 \times 10^{14} \text{GeV}\) and \(k_0 = 0.002 \text{Mpc}^{-1}\), we find from Eqs. (66) and (67) that \(m \approx 10^3 \text{m}_P\), and \(\zeta_1 \approx 10^{-8} \text{m}_P^{-1}\). We note that the dissipation coefficient when the scale \(k_0\) was leaving the horizon is of the order of \(\Gamma(\phi_0) = \alpha V(\phi_0) \sim 10^{-4} \text{m}_P\). It follows from Eq. (40) that one must decrease \(\zeta_1\) by three orders of magnitude to have a running spectral index \(\alpha_s\) close to the WMAP observed value.

**VI. CONCLUDING REMARKS**

In this paper we considered a warm inflationary scenario in which a viscous pressure is present in the matter-radiation fluid. This pressure arises on very general grounds, either as a hydrodynamical effect [8] or as a consequence of the decay of massive fields -previously excited by the inflaton- into light fields, or both. We investigated the corresponding scalar and tensor perturbations. The contributions of the adiabatic and entropy modes were obtained explicitly. Specifically, a general relation for the density perturbations is given in Eq. (31). The tensor perturbations are generated via stimulated emission into the existing thermal background, Eq. (41), and the tensor-scalar ratio—as well as the dissipation parameter—is modified by a temperature dependent factor.

The effect of the viscous pressure reveals itself at the e-folding level. Indeed, in the first case, i.e., when \(\Gamma = \Gamma_0 = \text{constant}\) and \(\zeta = \zeta_0\), as Eq. (52) shows, the total number of e-folds depends on the difference of the inflaton field evaluated at the beginning and at the end of inflation, i.e., \(\phi_i - \phi_f\), where the initial value of the inflaton field is \(\phi_i^2 = \ldots\)
Because $\Pi_i < 0$ one has that $\phi_i$ is lower than when the viscous pressure is not considered, thereby the viscous pressure shortens the inflationary phase.

Also, in this first case, we constrained the parameters of our model with the help of the WMAP three-year data [14]. Thus, we have found that $\zeta_0$ should be the order of $10^{-6} m_{P}^3$, and the potential, $V(\phi_0)$, of the order of $10^{-11} m_{P}^4$ when the perturbation exits the horizon at the scale of $k_0 = 0.002 \text{ Mpc}^{-1}$. Likewise, we have found that the contribution of the viscous pressure to the scalar power spectrum, i.e., $\chi \equiv \frac{P_{\delta} - P_{\delta}(\Pi=0)}{P_{\delta}}$, is the order of two percent.

Similar results follow in the second case, i.e., when $\Gamma = \Gamma(\phi)$ and $\zeta = \zeta(\rho)$. In this instance, the ratio $\chi$ is about four percent. While the deviation in both cases is small (but not negligible), we think it is still big enough to be detected by future measurements of the large scale structure of the Universe.

We have implicitly assumed that $\gamma$ is a constant which is not really the case since matter and radiation redshift at different rates, and that the bulk viscosity pressure is given by the conventional stationary formula $\Pi = -3\zeta H$ while it is well known that it does not respect relativistic causality and should be generalized to the Israel-Stewart expression $\tau \dot{\Pi} + \Pi = -3\zeta H$, where $\tau$ is the relaxation time of the viscous process [24]. However, it is to be expected that these generalizations, when properly incorporated, will not substantially alter our findings.

We have not studied the effect of the viscous pressure on the bispectrum of density perturbations -though there are good reasons to do it- as it lies a bit outside the main target of our paper. While cool inflation typically predicts a nearly vanishing bispectrum, and hence a small (just a few per cent) deviation from Gaussianity in density fluctuations -see e.g. [25]-, warm inflation clearly predicts a non-vanishing bispectrum. The latter effect arises from the non-linear coupling between the the fluctuations of the inflaton and those of the radiation. This can produce a moderate non-Gaussianity [26] or even a stronger one -likely to be detected by the Planck satellite [27]- if the aforesaid non-linear coupling is extended to subhorizon scales [28]. Because $\Pi$ implies an additional coupling between the radiation and density fluctuations it is to be expected that non-Gaussianity will be further enhanced. Perhaps, this could serve to observationally constrain $\Pi$ by future experiments. We plan to consider this question in a subsequent paper.

In summary, our model presents two interesting features: (i) Related to the fact that the dissipative effects plays a crucial role in producing the entropy mode, they can themselves
produce a rich variety of spectral ranging from red to blue. The possibility of a spectrum which does run so is particularly interesting because it is not commonly seen in inflationary models which typically predict red spectral. (ii) The viscous pressure may tell us about how the matter-radiation component behaves during warm inflation. Specifically, it will be very interesting to know how the viscosity contributes to the large scale structure of the Universe. In this respect, we anticipate that the Planck mission \[27\] will significantly enhance our understanding of the large scale structure by providing us with high quality measurements of the fundamental power spectrum over an larger wavelength range than the WMAP experiment.

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