21 cm angular power spectrum from minihalos as a probe of primordial spectral runnings

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Abstract. Measurements of 21 cm line fluctuations from minihalos have been discussed as a powerful probe of a wide range of cosmological models. However, previous studies have taken into account only the pixel variance, where contributions from different scales are integrated. In order to sort out information from different scales, we formulate the angular power spectrum of 21 cm line fluctuations from minihalos at different redshifts, which can enhance the constraining power enormously. By adopting this formalism, we investigate expected constraints on parameters characterizing the primordial power spectrum, particularly focusing on the spectral index $n_s$ and its runnings $\alpha_s$ and $\beta_s$. We show that future observations of 21 cm line fluctuations from minihalos, in combination with cosmic microwave background, can potentially probe these runnings as $\alpha_s \sim \mathcal{O}(10^{-3})$ and $\beta_s \sim \mathcal{O}(10^{-4})$. Its implications to the test of inflationary models are also discussed.

Keywords: cosmological parameters from LSS, power spectrum, cosmological parameters from CMBR, inflation

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1 Introduction

Current precise measurements of the cosmic microwave background (CMB) from Planck [1, 2] and other cosmological observations, such as baryon acoustic oscillation (BAO) (see e.g., [3]), type Ia supernovae (SNe) (see e.g., [4]) and so on, have unveiled various aspects of the Universe from the very early time to the present. The very early Universe can be probed by investigating primordial fluctuations whose properties can be measured by observing the CMB anisotropies and large scale structures of the Universe. Since the primordial fluctuations are considered to be generated during inflation, by studying the properties of the primordial fluctuations, we can test inflationary models and generation mechanisms of these fluctuations.

The measurement of the CMB by Planck in combination with other observations has provided strong constraints on inflationary parameters describing the nature of primordial fluctuations such as their amplitude $A_s$, the spectral index $n_s$, the tensor-to-scalar ratio $r$ and the non-linearity parameter $f_{NL}$ [5]. However, we are still far from specifying the model of inflation and more observational information is required to fully understand the very early epoch of the Universe.

Among the inflationary parameters, the tensor-to-scalar ratio is one of the promising parameters which, in the near future, can be probed more accurately from on-going and...
planned CMB B-mode polarization experiments (see, e.g., [6–10]). Since the tensor-to-scalar ratio is directly related to the energy scale of inflation, its information would give important implications to the models of inflation. Another property of primordial fluctuations worth investigating is their scale-dependence, which is usually represented by the spectral index $n_s$. Although $n_s$ at a reference wavenumber on large scales is accurately determined by Planck data [5], it also depends on the scale in general. The scale-dependence of $n_s$ is commonly denoted as the running $\alpha_s$, which is defined as $\alpha_s = dn_s/d\ln k$. Furthermore, $\alpha_s$ also generally depends on the scale, which is referred as the running of the running $\beta_s = d^2n_s/d\ln^2 k$. Compared to $n_s$, the current constraints on these runnings are not so severe even with the Planck data [5]. However, these can be probed more precisely by future observations on smaller scales such as 21 cm line fluctuations from the intergalactic medium (IGM) [11, 12], the CMB spectral $\mu$ distortion [13, 14] and galaxy surveys [12].

We in this paper focus on future observations of angular power spectrum of 21 cm line fluctuations from minihalos, which are virialized objects with the virial temperature $T < 10^4$ K. Since the virial temperature of minihalos is not high enough to cause the effective collisional ionization, the inside of a minihalo is filled with dense neutral gas. Therefore, the existence of minihalos can induce the additional 21 cm line signals. The abundance and the clustering of minihalos depend on the matter density fluctuations at $20 \text{Mpc}^{-1} < k < 500 \text{Mpc}^{-1}$. Hence the 21 cm signatures from minihalos have been discussed as a probe of various cosmological models, particularly on small scales.

The important observable effect of minihalos is the enhancement of the fluctuations of 21 cm line signals related to the matter density fluctuations by the clustering of minihalos. In other words, minihalos are biased tracers of the matter density fluctuations. Once we measure how much the 21 cm line fluctuations are biased by minihalos, we can probe matter fluctuations on much smaller scales compared with CMB or large scale structure observations.

The clustering of minihalos has also been studied in the context of warm dark matter [17], isocurvature fluctuations [18, 19], primordial non-Gaussianity [20], cosmic strings [21] and so on. In this paper we show how the 21 cm line fluctuations due to minihalos are sensitive to the running parameters describing primordial power spectrum.\footnote{Ref. [22] also has discussed the potential of minihalos as a probe of small-scale primordial power spectrum. They have focused on the absorption features produced by minihalos in the continuum spectrum of the background radio sources at high redshifts. In contrast, we in this paper choose the CMB as backlight and focus on the minihalo signals in the differential brightness temperature.}

Moreover, the aim of this paper is not only to present the sensitivity to the running parameters, but also to formulate the angular power spectrum of 21 cm line fluctuations due to minihalos. In the previous studies so far related to 21 cm line fluctuations with minihalos, only the pixel variance of the fluctuations has been focused on to investigate the feasibility of future 21 cm line observations. Although the pixel variance would be easy to obtain from actual observational data, it cannot tell us the scale dependence of the fluctuations. Hence, instead of the pixel variance, we discuss the cosmological application of the angular power spectrum of the 21 cm line signals for the first time. We show that the next generation 21 cm survey such as Square Kilometer Array (SKA) [23] and Fast Fourier Transform Telescope (FFTT) [24] can measure the scale-dependence very accurately, which will be very helpful to differentiate models of inflation.

Let us comment on the differences of our work and other previous ones which have also studied prospective constraints on the primordial spectral parameters by using future 21 cm line observations. Refs. [11, 12] have investigated 21 cm line signals from IGM at the
epoch of reionization. Their analyses of 21 cm line signals are based on the perturbation theory. However, nonlinear structures and collapsed objects have also evolved by this epoch. Minihalos in particular are expected to be non-negligible sources of 21 cm line signals [15, 16] (see also figure 2). Ref. [25] has studied 21 cm line signals from neutral hydrogens in the post-reionization era, which exist as residual clumps in IGM or clouds in galaxies. They are useful tracers of matter fluctuations at high redshifts. However, as many astrophysical processes are involved in the formation of these structures, hence it is difficult to relate their bias to the primordial power spectrum.

In contrast to those previous works, we in this paper focus on 21 cm line signals from minihalos. The formation of minihalos can be modeled at the basis of the hierarchical structure formation in the ΛCDM model, though refinement by numerical studies is awaited yet. This allows us to quantify the abundance and bias of the minihalos once the primordial power spectrum is given. These quantities are sensitive to the small scale fluctuations, where the impact of the spectral parameters such as $\alpha_s$ and $\beta_s$ is significant. In this work, we will demonstrate that minihalo signals in the 21cm line fluctuations can be a novel probe of the primordial spectrum with unique sensitivity to the higher order spectral parameters.

The structure of this paper is as follows. In the next section, we describe the formalism to calculate the angular power spectrum of 21 cm line fluctuations from minihalos. Then in section 3, we discuss the Fisher matrix for the angular power spectrum of 21 cm line fluctuations as well as that for CMB. In section 4, we estimate expected constraints on the power spectrum of primordial fluctuations, focusing on the runnings of the spectral index, assuming future 21 cm surveys including SKA and FFTT in combination with CMB observations. Implications for specific models for inflation are also discussed here. We conclude in the final section. In appendix A, we present general expressions for the spectral parameters for single-field inflation models and multi-field ones with the inflaton and spectator fields. In appendix B, constraints on higher order spectral runnings are given. Some details of the derivation of eq. (2.1) is given in appendix C.

2 Angular power spectrum of 21 cm line fluctuations from minihalos

Minihalos are neutral virial objects which are dense enough to decouple the spin temperature of neutral hydrogen from the CMB temperature. Therefore, minihalos are important sources which can produce the observable 21 cm signal. Since minihalos are biased objects of the matter density fluctuations, the spatial abundance of minihalos also fluctuates. As a result, the fluctuations of 21 cm signals due to minihalos arise. Here, extending the treatment of the previous works [15–21], we derive the angular power spectrum of 21 cm line fluctuations from minihalos.

The observable of 21 cm measurements is written in terms of the differential brightness temperature, which represents the deviation of the brightness temperature from the CMB one. The mean differential brightness temperature due to minihalos at redshift $z$ can be obtained from [15]

$$\Delta T_b(z) = \frac{c(1+z)^4}{H(z) \nu_0} \int_{M_{\text{min}}}^{M_{\text{max}}} \frac{dn}{dM} \Delta \nu_{\text{eff}} A \langle \delta T_b \rangle,$$

(2.1)

where $\nu_0$ is the frequency corresponding to 21 cm, $\Delta \nu_{\text{eff}} = [\phi(\nu_0)(1+z)]^{-1}$ with the intrinsic line profile $\phi(\nu_0)$ of a minihalo, $dn/dM$ is a mass function of minihalos. We consider 21 cm line fluctuations on scales which are much larger than the typical formation scale of minihalos. In
other words, the angular resolution of an observation is assumed to be much larger than the size of an individual minihalo. Therefore we introduce \( \langle \delta T_b \rangle \), which is the typical brightness temperature of a minihalo averaged over the cross-section \( A \). We consider that minihalos are in the mass range between \( M_{\text{max}} \) and \( M_{\text{min}} \), which are set to be the virial mass with the virial temperature \( T_{\text{vir}} = 10^4 \text{ K} \) and the Jeans mass, respectively. We provide a detailed derivation of eq. (2.1) in appendix C.

We adopt the Press-Schechter mass function for \( \frac{dn}{dM} \). The nature of minihalos, e.g., the profiles of the gas density and pressure, determines \( A, \phi(\nu_0) \) and \( \langle \delta T_b \rangle \). Here we use a truncated isothermal sphere as the model of a minihalo, which depends on the minihalo mass \( M \) [26]. Since the distribution of minihalos traces the underlying density fluctuations, 21cm line fluctuations from minihalos at a redshift \( z \) in the line-of-sight direction \( \hat{n} \) should be written as (neglecting the redshift space distortions) [15, 17, 19]

\[
\delta \Delta T_b(\hat{n}, z) = \Delta T_b(z) \beta(z) \delta(\vec{x} = r(z) \hat{n}, z),
\]

where \( \beta(z) \) is the effective bias of minihalos, \( \delta(\vec{x}, z) \) represents the matter density fluctuations at the comoving coordinates \( \vec{x} \) and redshift \( z \), and \( r(z) \) is the comoving distance from us to the redshift \( z \). The effective bias \( \beta(z) \) is given as

\[
\beta(z) = \frac{\int_{M_{\text{min}}}^{M_{\text{max}}} dM \frac{dn}{dM} F(z, M) b(M, z)}{\int_{M_{\text{min}}}^{M_{\text{max}}} dM \frac{dn}{dM} F(z, M)},
\]

where \( F = \langle \delta T_b \rangle A \sigma_V \) with the velocity dispersion of a minihalo \( \sigma_V \) and \( b(M, z) \) being the bias of minihalos with mass \( M \) for which we adopt one obtained in ref. [27]. We refer the readers to refs. [15, 17, 19] for the detailed derivation of \( \beta(z) \).

Minihalos discussed in this paper are collapsed objects and the corresponding scales of matter density fluctuations are at \( 20 \text{ Mpc}^{-1} < k < 500 \text{ Mpc}^{-1} \) [17]. Therefore, the bias (clustering) as well as the mean number density (abundance) of minihalos depends on the fluctuations at these scales. As shown in eq. (2.2) with eqs. (2.1) and (2.3), the abundance and clustering can enhance the signal of 21 cm fluctuations over all scales. As a result, the measurement of the 21 cm line fluctuation amplitude provides useful information about the matter density fluctuations on small scales where minihalos can form.

Given a redshift \( z \), we consider the maps of the 21 cm line fluctuations \( \delta \Delta T_b(\hat{n}, z) \) and matter ones \( \delta(\vec{r}(z) \hat{n}, z) \) on the sky. They can be transformed into the spherical harmonic coefficients as

\[
a^{(21\text{cm})}_{lm}(z) = \int d\hat{n} \delta \Delta T_b(\hat{n}, z) Y_{lm}^*(\hat{n}),
\]

\[
a^{(\text{matter})}_{lm}(z) = \int d\hat{n} \delta(\vec{r}(z) \hat{n}, z) Y_{lm}^*(\hat{n}).
\]

Now we can obtain the angular power spectrum. As expected from eq. (2.2), the angular power spectrum depends on the matter power spectrum due to minihalos. Since we are interested in the scales which are much larger than the typical size of minihalos, we take into account the two-halo term contribution, neglecting the one-halo (Poisson) term. With this assumption, due to the statistical isotropy of the matter fluctuations, the angular power spectrum of the 21 cm line fluctuations from minihalos can be given in the following form:

\[
C^{(21\text{cm})}_{l}(z, z') = \Delta T_b(z) \Delta T_b(z') \beta(z) \beta(z') C^{(\text{matter})}_{l}(z, z'),
\]
where $C_l^{(21\text{cm})}(z,z')$ and $C_l^{(\text{matter})}(z,z')$ are respectively the angular power spectra of 21cm line fluctuations and the matter ones between redshifts $z$ and $z'$, whose definitions are

\begin{equation}
\langle a_l^{(21\text{cm})}(z) a_{l'}^{(21\text{cm})\ast}(z') \rangle = C_l^{(21\text{cm})}(z,z') \delta_{ll'} \delta_{mm'}, \quad (2.7)
\end{equation}

\begin{equation}
\langle a_l^{(\text{matter})}(z) a_{l'}^{(\text{matter})\ast}(z') \rangle = C_l^{(\text{matter})}(z,z') \delta_{ll'} \delta_{mm'}. \quad (2.8)
\end{equation}

The angular power spectrum of matter fluctuations $C_l^{(\text{matter})}$ can be related to the matter power spectrum $P(k)$ as

\begin{equation}
C_l^{(\text{matter})}(z,z') = D(z)D(z') \int \frac{k^2dk}{2\pi^2} P(k) j_l(kr(z)) j_l(kr(z')), \quad (2.9)
\end{equation}

where $D(z)$ is the growth factor relative to a reference redshift $z_{\text{ref}}$ (i.e., $\delta(k,z) = D(z)\delta(k,z_{\text{ref}})$ with $\delta(k,z)$ being the matter fluctuations in the $k$-space) and the linear matter power spectrum at a reference redshift $z_{\text{ref}}$ is denoted as $P(k)$. $j_l$ is the spherical Bessel function. In the redshift range we consider in the following analysis, the Universe is well approximated as matter-dominated, in which $D(z)$ is given by $D(z) \propto a(z)$, with $a(z)$ being the scale factor at $z$.

So far we have neglected the redshift space distortions due to the peculiar velocity of minihalos. Although this effect has not been taken into account in previous studies [15–21], it can be dominant especially in correlations between different redshifts. The redshift space distortions are classified largely into two types. One is the linear effect known as the Kaiser effect while the other is the nonlinear effect called as the Fingers-of-God (FoG) effect [31]. Since we are focusing on the Universe at high redshifts, we expect that the redshift space distortions are weak and can be well-approximated within the framework of the linear perturbation theory. Thus we neglect the FoG effect. Including the Kaiser effect [31], we can rewrite eq. (2.2) as

\begin{equation}
\delta \Delta T_b(\hat{n}, z) = \Delta T_b(z) \left[ \beta(z) + f(z) \mu^2 \right] \delta(\vec{x} = r(z)\hat{n}, z), \quad (2.10)
\end{equation}

where $\mu = \hat{k} \cdot \hat{n}$ is the cosine between the wave vector of perturbations $\vec{k}$ and line-of-sight direction $\hat{n}$, and $f = d\ln D/d\ln a$ is the growth rate. Note that in the matter domination epoch, $f = 1$. After taking account of the redshift space distortions, eq. (2.9) should then be replaced with

\begin{equation}
C_l^{(\text{matter})}(z,z') = D(z)D(z') \int \frac{k^2dk}{2\pi^2} P(k) \left[ j_l(kr(z)) - \frac{f(z)}{\beta(z)} j''_l(kr(z)) \right] \left[ j_l(kr(z')) - \frac{f(z')}{\beta(z')} j''_l(kr(z')) \right], \quad (2.11)
\end{equation}

where $j''_l$ denotes a second derivative with respect to its argument and it should not be confused with a prime attached to $z$.

In figure 1, we plot the angular power spectra $C_l^{(21\text{cm})}(z,z')$ around a central redshift $z = 5$ as an example. The cosmological parameters we assumed to calculate $C_l^{(21\text{cm})}(z,z')$ are given in the caption of figure 1. From the figure, one can see that the amplitude of the power spectra becomes maximum when the redshift difference, $\Delta z \equiv |z - z'|$, vanishes. Then the amplitude drops as $\Delta z$ increases, with $l$ being fixed. The correlation at the same redshift, $C_l^{(21\text{cm})}(z,z)$, is almost constant at large angular scales $l \lesssim 100$. In other words,
Figure 1. Shown are the angular power spectra of minihalo 21cm line fluctuations $C_l^{(21\text{cm})}(z + \Delta z/2, z - \Delta z/2)$ with the central redshift $z = 5$. Thick and thin lines show positive and negative values, respectively. The cosmological parameters are taken to be the mean values of the analysis for a power-law $\Lambda$CDM model from Planck 2015 TT,TE,EE+lowP+lensing+ext [2]: CDM density $\omega_c(= \Omega_bh^2) = 0.1188$, baryon density $\omega_b(= \Omega_ch^2) = 0.0223$, the reduced Hubble parameter $h = 0.6774$, the reionization optical depth $\tau_{\text{reion}} = 0.066$, the amplitude and the spectral index of primordial power spectrum (at $k_0 = 0.05 \text{Mpc}^{-1}$) $A_s = 2.141 \times 10^{-9}$ and $n_s = 0.9667$, respectively. For the runnings, we assumed $\alpha_s = \beta_s = 0$. 

the correlations between different redshifts and different pixels are not very large. This result supports the previous analysis of 21 cm signals from minihalos in which only pixel variance at the same redshift bins are taken into account while the covariance is neglected. However, at smaller angular scales $l \gtrsim 100$, the angular spectrum deviates from the white spectrum $C_l = \text{const.}$, which indicates nonzero covariance between different pixels. In addition, with small but nonzero redshift differences $\Delta z = O(0.1)$, the correlations between different redshifts give non-negligible amplitude. This suggests that, in order to optimally exploit cosmological information in 21 cm line fluctuations from minihalos, we need to take into account the correlation of their fluctuations at different redshifts, too.

2.1 Comparison with IGM contributions to 21 cm fluctuations

Although we focus on the 21 cm signals from minihalos in this paper, the IGM also contributes to the signal, which can be written as

$$\Delta T_{\text{IGM}} \approx 28 \text{mK} \sqrt{\frac{z + 1}{10}} x_{\text{HI}}(z)(1 + \delta)(1 - T_{\text{CMB}}/T_s), \quad (2.12)$$

where $x_{\text{HI}}(z)$ and $T_s$ are the neutral fraction and the spin temperature of the IGM, respectively [28]. Whether the contribution from minihalos or the IGM dominates depends on the thermal and ionization history of the IGM. In general, the reionization process proceeds faster in the IGM than in minihalos [29]. This is because minihalos are $O(100)$ times denser than the IGM. Accordingly much more ionizing background photons are required to ionize minihalos in comparison with the IGM. In this paper, for simplicity, we assume that minihalos are not ionized at all until the completion of reionization. Under this assumption, figure 2
Figure 2. Comparison of the signals in 21 cm line fluctuations from minihalos (red solid line) and the IGM (green dashed line) as functions of redshift $z$. For the IGM signal, we have assumed a rapid reionization with the optical depth $\tau_{\text{reion}} = 0.066$ and the width of the duration $\Delta z = 1$ in accordance with the recent result [2].

compares the signal of the 21 cm line fluctuations from minihalos, $\overline{\Delta T_b(z)}\beta$, with that of the IGM $\overline{\Delta T_{\text{IGM}}}$ given in eq. (2.12). In the figure, we assume a rapid reionization with the optical depth $\tau_{\text{reion}} = 0.066$ and the width of the duration to be $\Delta z = 1$ in accordance with the recent Planck result [2] for $x_{\text{HI}}(z)$ to calculate the signal from the IGM. The spin temperature can be obtained by taking the simple model of the IGM gas temperature evolution in which the gas temperature is proportional to $x_{\text{HI}}$ during the reionization and reaches $10^{4} \text{ K}$ when the reionization completes [30]. The figure shows that the signal from minihalos can surpass that from IGM in almost overall redshifts. Although, as mentioned above, the signal from the IGM depends on the reionization history, we have also checked other several models and found that, even if we change the reionization history, the signal from minihalos dominates for some redshift range. Therefore, in the following analysis, we neglect the contribution from the IGM. We also note that the contribution from minihalos is dominant in the end of the dark ages ($15 < z < 40$). At this epoch, the temperature of the IGM is so low that the spin temperature is almost same as the CMB temperature. Therefore, the signals from the IGM are suppressed. However, as we will discuss later, the observational noise becomes large as the observation redshift increases. Hence we consider the signals only from $z < 20$ in our analysis.

3 Fisher matrix analysis

Now we discuss the Fisher matrix for $C_l^{(21\text{cm})}(z, z')$. In the analysis, we take the correlations for all combinations of observed redshift slices. According to ref. [34], the Fisher matrix based on a measurement of 21 cm line power spectrum should be given by

$$F_{ab}^{(\text{power})} = \sum_l \frac{2l+1}{2} \text{Tr} \left[ C_l^{(21\text{cm})^{-1}} \frac{\partial C_l^{(21\text{cm})}}{\partial p_a} C_l^{(21\text{cm})^{-1}} \frac{\partial C_l^{(21\text{cm})}}{\partial p_b} \right]. \quad (3.1)$$

Here $p_a$ ($p_b$) indicates a cosmological parameter, $[C_l]^{(21\text{cm})}_{ij}(z_i, z_j) \equiv C_l^{(21\text{cm})}(z_i, z_j) + N_l^{(21\text{cm})}(z_i, z_j)$ is the covariance matrix for 21 cm observations where $N_l^{(21\text{cm})}(z_i, z_j)$ is the noise power spectrum.
and $i$ and $j$ respectively denote the $i$-th and $j$-th redshift slices. Assuming the isotropic Gaussian beam profile, we can write the noise power spectrum as [32]

$$N_l^{(21cm)}(z_i, z_j) = \delta_{ij} \varsigma(z_i)^2 \Delta \theta^2 \exp \left[ l(l+1) \frac{\Delta \theta^2}{8 \ln 2} \right],$$  \tag{3.2}

where $\Delta \theta$ is the beam width and $\varsigma(z_i)$ is the noise root-mean-squared per pixel at observed redshift $z_i$, which depends on the instrumental and the foreground noises.

In redshifted 21 cm observations, dominant noise comes from the foregrounds, which primarily consist of the synchrotron radiation in the Milky Way. In our analysis, we assume that foregrounds are removed appropriately and the residual component can be modeled as noise that is white both in the angular and frequency domains with the typical sky temperature at high galactic latitude, $T_{sys} = 180(\nu/180 \text{ MHz})^{-2.6}$ K. In this case, ref. [33] provides the approximation form of $\varsigma(z_i)$ as

$$\varsigma(z_i) = 20 \text{ mK} \left( \frac{A_{tot}}{10^4 \text{ m}^2} \right)^{-1} \left( \frac{\Delta \theta}{10'} \right)^{-2} \left( \frac{1 + z_i}{10} \right)^{4.6} \left( \frac{\Delta \nu}{\text{ MHz}} \right)^{1/2} \left( \frac{t}{100 \text{ h}} \right)^{1/2},$$  \tag{3.3}

where $A_{tot}$ is total effective area, $\Delta \nu$ is a bandwidth of an observation and $t$ is the integrated observation time. Eq. (3.3) tells us that the noise increases as the observation redshift becomes higher.

We again note that we in this paper neglect the contribution from IGM in order to focus on the potential of the measurement of 21 cm signals from minihalos. This treatment is also motivated from the fact that, although the IGM component is also expected to contribute to the covariance matrix in eq. (3.1), its contribution would be subdominant in broad redshift range as shown in figure 2.

In a similar fashion, we can define the Fisher matrix from CMB power spectrum measurements:

$$F_{ab}^{(\text{CMB})} = \sum_l \frac{2l+1}{2} \text{Tr} \left[ C_l^{(\text{CMB})} \frac{\partial C_l^{(\text{CMB})}}{\partial p_a} C_l^{(\text{CMB})} \frac{\partial C_l^{(\text{CMB})}}{\partial p_b} \right],$$  \tag{3.4}

where $[C_l^{(\text{CMB})}]_{PQ} = C_l^{(\text{CMB})} + N_l^{(\text{CMB})}$ is the covariance matrix of measured CMB anisotropies, with the subscript $P(Q)$ indicating the temperature or E-mode polarization. Following ref. [32], we approximate the CMB noise power spectrum $N_l^{(\text{CMB})}$ as

$$N_l,PQ = \delta_{PQ} \theta_{\text{FWHM}}^2 \sigma_P^2 \exp \left[ l(l+1) \frac{\theta_{\text{FWHM}}^2}{8 \ln 2} \right],$$  \tag{3.5}

where $\theta_{\text{FWHM}}$ are the full width at half maximum of the Gaussian beam, and $\sigma_P$ is the root-mean-square of the instrumental noise per pixel.

## 4 Constraints on spectral runnings of primordial power spectrum

In order to demonstrate the potential power of the angular power spectrum of 21 cm line fluctuations from minihalos, here we present expected constraints on the power spectrum of primordial curvature perturbation, particularly focusing on the spectral index $n_s$ and its runnings $\alpha_s, \beta_s$. 

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4.1 Runnings of the spectral index

Conventionally the scale-dependence of the primordial power spectrum is given by the power law form with the spectral index $n_s$, which is often assumed to be constant in scale (wavenumber). However, the spectral index can generally depend on the scale and such scale-dependence could give us detailed information on the primordial power spectrum $P_s(k) := k^3 P(k)/2\pi^2$, where $P(k)$ is defined as

$$
(\zeta(k)\zeta(k')) = (2\pi)^3 \delta^{(3)}(k + k') P(k),
$$

with $\zeta$ being the primordial curvature perturbations. Here, $\delta^{(3)}(k + k')$ is a 3-dimensional Dirac’s delta function and $P(k)$ determines the initial condition of the linear matter power spectrum through the Poisson equation. By taking into account the scale-dependence of the spectral index, we can perturbatively write $P_s$ as

$$
P_s(k) = A_s \left( \frac{k}{k_0} \right)^{n_s-1 + \frac{1}{2} n_s \ln(k/k_0) + \frac{1}{3} \beta_s \ln^2(k/k_0)},
$$

where $A_s$, $k_0$ and $n_s$ are respectively the amplitude, the pivot scale and the spectral index, and we have expanded $P_s$ in terms of $\ln k$ up to the 2nd order. The expansion coefficients in the 1st and 2nd orders are denoted as $\alpha_s(\equiv dn_s/d\ln k)$ and $\beta_s(\equiv d^2 n_s/d\ln^2 k)$, which we call the running and the quadratic running of $n_s$, respectively. In the framework of the slow-roll inflation, the runnings such as $\alpha_s$ and $\beta_s$ can be explicitly written down by using the so-called slow-roll parameters. We provide those expressions in appendix A. In principle, we can expand $P_s$ up to arbitrarily higher orders and hence we also, in appendix A, give expressions for higher order runnings in terms of the slow-roll parameters not only for the single-field case but also for the multi-field case.

4.2 Forecasts based on the Fisher matrix analysis

Let us investigate the expected constraints on primordial power spectrum in future 21cm line observations, especially focusing on the parameters $n_s$, $\alpha_s$ and $\beta_s$. The determination of these parameters requires precise measurements of cosmological perturbations over a wide range of scales. Therefore, we consider the observations combining the CMB and 21 cm line fluctuations. The CMB observation can probe large scales upto the horizon scale, while the 21 cm observation is sensitive to the small scales through the minihalo abundance as mentioned above.

For 21 cm line observations, we adopt the specifications of SKA [23] and FFTT [24], which are summarized in table 1. One of the advantages in 21 cm line observations is redshift tomography. Here we consider observed redshift slices with a redshift thickness corresponding to the frequency width $\Delta \nu$ in table 1 between the minimum redshift $z_{min}$ and the maximum one $z_{max}$. Therefore the number of the observed redshift slices is $\nu_0 (1/(1 + z_{min}) - 1/(1 + z_{max})) / \Delta \nu$. As default, we adopt $z_{min} = 6$ and $z_{max} = 20$ in deriving constraints, but we also present constraints with different $z_{min}$. We note that our results do not depend on $z_{max}$ much because the noise dominates over the signal at high redshifts. We take the correlation $C^{(21\text{cm})}_l$ for all combinations of the observed redshift slices. For CMB observations, we set the sensitivities of Planck [35] and COrE [36] as shown in table 2.

\footnote{While the survey parameters we adopt here are somewhat different from those in the most recent proposal of the COrE mission, our results do not differ significantly.}
### Table 1. Specification of 21 cm surveys.

|                      | SKA   | FFTT  |
|----------------------|-------|-------|
| total effective area $A_{\text{tot}}$ [m$^2$] | $10^5$ | $10^7$ |
| bandwidth $\Delta \nu$ [MHz]            | 1     |       |
| beam width $\Delta \theta$ [arcmin]    | 9     |       |
| integration time $t$ [hour]            | 1000  |       |

### Table 2. Specification of CMB surveys.

|                      | Planck | COreE |
|----------------------|--------|-------|
| band frequency [GHz] | 100    | 147   |
| beam width $\Delta \theta$ [arcmin] | 9.9    | 7.2   |
| Temperature noise $\Delta T$ [$\mu$K arcmin] | 31.3   | 20.1  |
| Polarization noise $\Delta P$ [$\mu$K arcmin] | 44.2   | 33.3  |

In our analysis, we assume a flat $\Lambda$CDM model and the pivot scale $k_0$ is fixed to 0.05 Mpc$^{-1}$ as in the Planck analysis. In addition to $n_s$, $\alpha_s$ and $\beta_s$, we also include the following parameters in the Fisher matrix: the reduced Hubble parameter $h$, baryon and CDM densities $\omega_b$ and $\omega_c$, the reionization optical depth $\tau_{\text{reion}}$, and the amplitude of the primordial power spectrum $A_s$. The fiducial parameters are assumed to be the same as the ones used in figure 1.

Before demonstrating the prospective constraints on the primordial fluctuations, we show the dependence of $C_l^{(21\text{cm})}$ on the running parameters of the primordial fluctuations. Figure 3 shows the derivatives of $C_l^{(21\text{cm})}$ with respect to parameters $n_s$ (red), $\alpha_s$ (green), $\beta_s$ (blue). These derivatives capture the dependence of $C_l^{(21\text{cm})}$ on each parameter. When $C_l^{(21\text{cm})}$ is highly dependent on a parameter, the derivative of $C_l^{(21\text{cm})}$ with respect to this parameter becomes large in the figure. In order to see the scale-dependence easily, the fiducial $C_l^{(21\text{cm})}$ is plotted (purple). If the change of a running parameter just modifies the overall amplitude of $C_l^{(21\text{cm})}$, the derivative with this parameter has the same shape as the fiducial $C_l^{(21\text{cm})}$. On the contrary, if the parameter affects the scale-dependence of $C_l^{(21\text{cm})}$, the shape of the derivative is different from the purple line. In the figure, we also show the dependence of $C_l^{(21\text{cm})}(z, z')$ on the redshift difference $\Delta z = z - z'$. Fixing the central redshift $(z + z')/2 = 5$, we take $z - z' = 0$ (top-left), 0.1 (top-right), 0.2 (bottom-left), and 0.3 (bottom-right).

From the figure, we can see that the derivatives with respect to $n_s$ (red lines) represent the different shapes from the purple lines. In contrast, the shapes of the derivatives with respect to $\beta_s$ (green lines) are same as the purple lines. This fact means that the modification of $n_s$ changes the shape of $C_l^{(21\text{cm})}$ and $\beta_s$ just modifies the amplitude without changing the shape. As shown in eq. (2.6) with eq. (2.9), the effect of the primordial power spectrum on $C_l^{(21\text{cm})}$ arises through $P(k)$ directly and $dn/dM$. The modification of $n_s$ changes the scale-dependence of $C_l^{(21\text{cm})}$ directly through $P(k)$. On the other hand, the higher-order running parameters affect the primordial power spectrum on small scales, because the pivot scale is on a large scale. Therefore, the effect of $\alpha_s$ and $\beta_s$ arises through the minihalo abundance $dn/dM$, which changes the overall amplitude of $C_l^{(21\text{cm})}$. 

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**Table 1. Specification of 21 cm surveys.**

|                      | SKA   | FFTT  |
|----------------------|-------|-------|
| total effective area $A_{\text{tot}}$ [m$^2$] | $10^5$ | $10^7$ |
| bandwidth $\Delta \nu$ [MHz]            | 1     |       |
| beam width $\Delta \theta$ [arcmin]    | 9     |       |
| integration time $t$ [hour]            | 1000  |       |

**Table 2. Specification of CMB surveys.**

|                      | Planck | COreE |
|----------------------|--------|-------|
| band frequency [GHz] | 100    | 147   |
| beam width $\Delta \theta$ [arcmin] | 9.9    | 7.2   |
| Temperature noise $\Delta T$ [$\mu$K arcmin] | 31.3   | 20.1  |
| Polarization noise $\Delta P$ [$\mu$K arcmin] | 44.2   | 33.3  |
Since $\alpha_s$ and $\beta_s$ only change the overall amplitude, it seems that these parameters have the degeneracy with $A_s$. However, as shown in figure 3, the response of the overall amplitude to the higher order running parameters depends on the difference of the observed redshifts, $\Delta z$. Combined with the CMB measurements, the correlation $C_l^{(21\text{cm})}$ between the different redshift slices enables us to disentangle these parameters.

Now we show the constraints on the primordial power spectrum from 21 cm, CMB, and combinations of these observations in figure 4. In the figure, we have assumed that 21 cm line observations can measure the signal from minihalos for all redshift slices of the corresponding frequency width $\Delta \nu$ between $z_{\text{min}} = 6$ and $z_{\text{max}} = 20$. The number of redshift slices is then 135. In table 3, we summarize expected 1$\sigma$ constraints on $n_s$, $\alpha_s$ and $\beta_s$ from different combinations of observations.

First of all, it is remarkable in the figure that 21 cm line power spectrum from minihalos (i.e. SKA and FFTT) is competitive or even more powerful in comparison with the CMB observations (i.e. Planck and COrE) as a probe of the primordial power spectrum. In particular, $C_l^{(21\text{cm})}$ can measure $\beta_s$ more tightly than $C_l^{(\text{CMB})}$. This is because the abundance of minihalos is sensitive to the linear matter fluctuations at very small scales (in wavenumber...
Figure 4. Expected constraints on the primordial power spectrum from observations of 21 cm signals from minihalos in combination with the CMB. Contours depict 1σ errors with other cosmological parameters being marginalized. We assume 21 cm signals from minihalos can be measured down to $z_{\text{min}} = 6$.

between 20 Mpc$^{-1}$ and 500 Mpc$^{-1}$), which is difficult to be measured by CMB observations. Although the constraints on $n_s$ and $\alpha_s$ are relatively weak compared with $\beta_s$, $C_l^{(21\text{cm})}$ can also probe $n_s$ and $\alpha_s$ on large scales.

Once observations of 21 cm line fluctuations from minihalos and the CMB are combined, we can probe matter fluctuations over a wide range of scales, i.e., from the horizon scale to 0.01 Mpc. Combinations of these two different observations offer a great advantage by the lever-arm effect when we try to constrain the primordial power spectrum that is close to the power-law as in eq. (4.2). In other words, combinations of these observations can break parameter degeneracies that each observation suffers from by itself. This is most notable in figure 4 when Planck and SKA are combined.

On the other hand, as can be read off by eq. (3.3), the signal-to-noise ratio in observations of the 21cm signal from minihalos rapidly increases at low redshifts. Therefore, the resultant constraints are expected to be dependent of $z_{\text{min}}$, the minimum redshift until when minihalos can be observed. There is a theoretical uncertainty on the determination of $z_{\text{min}}$, because the existence of minihalos depends on the cosmological reionization process [37, 38]. Hence, in order to examine the dependence on $z_{\text{min}}$, we evaluate the expected (marginalized) 1σ uncertainties for the determination of $n_s$, $\alpha_s$ and $\beta_s$ from two data sets, Planck+SKA and COoE+FFTT for $z_{\text{min}} = 4$, 6 (baseline), 8 and 10. We summarize the results in table 4. As expected, the constraints become less tight as $z_{\text{min}}$ is increased although the changes are not so significant. For example, the change of $z_{\text{min}}$ from 6 to 8 degrades the constraint on
Table 3. Constraints on parameters for the primordial power spectrum. For 21 cm observations, $z_{\text{min}} = 6$ is assumed.

|                | $10^{-3} \Delta n_s$ | $10^{-3} \Delta \alpha_s$ | $10^{-3} \Delta \beta_s$ |
|----------------|------------------------|-----------------------------|---------------------------|
| Planck         | 7.7                    | 10.7                        | 15.1                      |
| COre           | 3.2                    | 2.9                         | 6.5                       |
| SKA            | 4.8                    | 2.9                         | 1.0                       |
| FFTT           | 2.6                    | 1.6                         | 0.54                      |
| Planck+SKA     | 1.9                    | 2.1                         | 0.59                      |
| Planck+FFTT    | 1.4                    | 1.3                         | 0.38                      |
| COre+SKA       | 1.4                    | 1.6                         | 0.37                      |
| COre+FFTT      | 0.95                   | 1.1                         | 0.28                      |

Table 4. Dependence of the constraints on $z_{\text{min}}$ for the data sets, Planck+SKA and COre+FFTT.

|                | $z_{\text{min}}$ | $10^{-3} \Delta n_s$ | $10^{-3} \Delta \alpha_s$ | $10^{-3} \Delta \beta_s$ |
|----------------|------------------|------------------------|-----------------------------|---------------------------|
| Planck+SKA     | 4                | 1.5                    | 1.4                         | 0.38                      |
|                | 6                | 1.9                    | 2.1                         | 0.59                      |
|                | 8                | 2.8                    | 3.0                         | 0.85                      |
|                | 10               | 3.8                    | 4.6                         | 1.3                       |
| COre+FFTT      | 4                | 0.83                   | 0.98                        | 0.24                      |
|                | 6                | 0.95                   | 1.1                         | 0.28                      |
|                | 8                | 1.0                    | 1.2                         | 0.31                      |
|                | 10               | 1.2                    | 1.3                         | 0.33                      |

Each parameter by around 50% for Planck+SKA. On the other hand, for the combination of COre+FFTT, the degradation becomes modest; between $z_{\text{min}} = 6$ and 10, the constraint changes only by 20%.

4.3 Implication for the constraint on the inflationary models

Now let us consider the implication of our results for the constraint on the inflationary models. From Planck data [5], many inflation models now are excluded, which can be read off by looking at the constraint in the spectral index and the tensor-to-scalar ratio ($n_s-r$) plane (For the predictions of various inflation models, see [39]). In fact, the simple chaotic inflationary models with the inflaton’s potential $V \propto \phi^n$ are almost ruled out for $n \gtrsim 2$ and the so-called $R^2$-inflation model [40–42] seems to be favored. However, once the multi-field inflationary models are taken into account such as the curvaton model [43–45], modulated reheating model [46, 47] and so on, where a light scalar field other than inflaton exists and its fluctuations also contribute to primordial fluctuations, not only $R^2$-inflation but also several inflationary models are well inside the allowed region on the $n_s-r$ plane [48–59].

It is well-known that one of the powerful tools to distinguish single-field inflationary models from multi-field inflationary models is warm inflation. Warm inflation can also make some inflation models viable due to the modification to the inflationary prediction as in multi-field models [60]. It has also been argued that in some cases the runnings can be large in the context of warm inflation [60].
the multi-field ones is the non-Gaussianity of the primordial fluctuations, especially the so-called local-type non-Gaussianity. In general, the single-field models predict small local-type non-Gaussianity, while multi-field models could generate relatively larger one. However, at the level of current constraint on non-Gaussianity from Planck [61], we cannot differentiate between single-field and multi-field models.

Now we discuss the potential of the runnings of the spectral index to distinguish among inflation models, having our Fisher analysis results given in the previous section in mind. In the following, we choose some representatives of single-field and multi-field models and apply the expressions for the spectral parameters provided in appendix A. As examples for single-field models, we consider the $R^2$-inflation and brane-inflation. For multi-field models, the natural- and inverse monomial-spectator models are investigated. We take the model parameters in such a way that these models are consistent with the current observations and hardly distinguished from one another to date (see section 4.3.5).

4.3.1 $R^2$-inflation

$R^2$-inflation was proposed in refs. [40–42], where the inflationary phase can be realized by the higher order curvature term, $R^2$. It has been known that this model corresponds to the single-field inflation in Einstein frame with the potential of

$$V(\phi) = \Lambda^4 \left( 1 - e^{-\sqrt{2/3 \phi/M_{pl}}} \right)^2.$$  \hfill (4.3)

Based on the slow-roll approximation, the $e$-folding number measured from the time when the pivot scale $k_0$ left the Hubble radius during the inflation to the end of inflation can be estimated as

$$N_0 = \int_{t_0}^{t_e} H dt \simeq \frac{1}{M_{pl}^2} \int_{\phi_0}^{\phi_e} \frac{V}{V'} d\phi,$$  \hfill (4.4)

where the index $e$ and $0$ respectively represent the time when the inflation ends and the pivot scale $k_0$ left the Hubble radius during the inflation. For the $R^2$-inflation model, this $e$-folding number can be obtained as

$$N_0 \simeq \frac{3}{4} e^{2/3 \phi_0/M_{pl}},$$  \hfill (4.5)

where we keep only the leading term. The slow-roll parameters are also given in terms of $N_0$ as

$$\epsilon \simeq \frac{3}{4} \frac{1}{N_0^2}, \quad \eta \simeq -\frac{1}{N_0}, \quad \xi^{(2)} \simeq \frac{1}{N_0^2}, \quad \sigma^{(3)} \simeq -\frac{1}{N_0^3}, \quad \cdots,$$  \hfill (4.6)

where we have assumed $N_0 \gg 1$. For the single-field models, since the tensor-to-scalar ratio and the spectral index are respectively given by

$$n_s - 1 = -6 \epsilon + 2 \eta, \quad r = 16 \epsilon,$$  \hfill (4.7)

we can estimate $n_s$ and $r$ for the $R^2$-inflation as

$$n_s - 1 \simeq -\frac{2}{N_0} \left( \simeq -(3.3-4) \times 10^{-2} \right), \quad r = \frac{12}{N_0^2} \left( \simeq (3.3-4.8) \times 10^{-3} \right) \quad \text{(for } N_0 = 50-60),$$  \hfill (4.8)

at the leading order in $N_0$. 

Figure 5. The power spectrum of the primordial curvature perturbations generated from the $R^2$-inflation. The red line is obtained by numerically calculating the background dynamics and evaluating $H^2/\dot{\phi}_0^2$. The blue dotted line corresponds to the one calculated from the power-law form given by (A.1) including up to $n_s$, and the green dashed line corresponds to that up to $\alpha_s$.

The scales of the primordial fluctuations on which we focus here are around $20 \text{ Mpc}^{-1} < k < 500 \text{ Mpc}^{-1}$ as we have discussed. In figure 5, we plot the power spectrum of the primordial curvature perturbations generated from $R^2$-inflation, numerically calculated one and approximated by the power-law expansion form. From this figure, one can find that at minihalos’ scales the power spectrum only including $n_s$ deviates from the numerically-evaluated power spectrum. On the other hand, the power spectrum including up to $\alpha_s$ seems to be in good agreement with the numerical result. Hence we need to take account of a higher order running to test inflationary models using observations of small scales such as minihalos. Based on appendix A, $\alpha_s$ and $\beta_s$ in the $R^2$-inflation can be respectively obtained as

$$\alpha_s \simeq -\frac{2}{N_0} \left( \simeq -(5.5-8) \times 10^{-4} \right), \quad \beta_s \simeq -\frac{4}{N_0} \left( \simeq -(1.9-3.2) \times 10^{-5} \right) \quad \text{for} \quad N_0 = 50-60, \quad (4.9)$$

at the leading order in $N_0$.

4.3.2 Brane inflation

The potential of the brane inflation models is phenomenologically given as (see [39, 62] and references therein)

$$V(\phi) = V_0 \left[ 1 - \left( \frac{\phi}{\mu} \right)^{-p} \right], \quad (4.10)$$

where $V_0$ represents the energy scale of the model and $p$ and $\mu$ are also model parameters.

The slow-roll parameters in this model are given as

$$\epsilon = \frac{p^2}{2\mu^2\phi_0^2(\phi_0^p - 1)^2}, \quad \eta = -\frac{p(p+1)}{\mu^2\phi_0^2(\phi_0^p - 1)}, \quad \xi^{(2)} = \frac{p^2(p+1)(p+2)}{\mu^2\phi_0^4(\phi_0^p - 1)^2}, \quad \sigma^{(3)} = -\frac{p^3(p+1)(p+2)(p+3)}{\mu^6\phi_0^6(\phi_0^p - 1)^3}, \quad (4.11)$$

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where we have defined $\tilde{\mu}$ and $\tilde{\phi}_0$ as $\tilde{\mu} \equiv \mu/M_{\text{pl}}$ and $\tilde{\phi}_0 \equiv \phi_0/\mu$. The number of e-folds can be expressed as

$$N_0 = \frac{\tilde{\mu}^2}{2p} \left[ \frac{2}{p+2} \left( \tilde{\phi}_0^{p+2} - \tilde{\phi}_e^{p+2} \right) - \left( \tilde{\phi}_e^{2} - \tilde{\phi}_0^{2} \right) \right],$$

(4.12)

with $\phi_e$ being the value of $\phi$ at the end of inflation, at which one of the slow-roll parameters exceeds unity. Assuming that $\tilde{\phi}_0 \gg 1$ and $\tilde{\phi}_0 \gg \tilde{\phi}_e$ in eq. (4.12), $\tilde{\phi}_0$ can be approximated by

$$\tilde{\phi}_0 \simeq \left[ p(p+2) \frac{N_0}{\tilde{\mu}^2} \right]^{1/(p+2)}.$$

(4.13)

The spectral index $n_s$, the tensor-to-scalar ratio $r$, the running parameters $\alpha_s$ and $\beta_s$ can be calculated in the same way by putting the slow-roll parameters given in eq. (4.12) into the formulas eqs. (A.5) and (A.6).

### 4.3.3 Natural-spectator model

Next we consider a multi-field model. To predict the spectral index, its higher order runnings and the tensor-to-scalar ratio, we do not have to specify the spectator field model itself, but need to specify the potential for the spectator field $\chi$. Here we take a quadratic potential for $\chi$ as $V(\chi) = \frac{1}{2} m_{\chi}^2 \chi^2$ with $m_{\chi}$ being the mass of the spectator field and assume that $m_{\chi}$ is much smaller than the Hubble parameter during inflation, which gives $\eta_{\chi} \simeq 0$ and $\xi_{\chi}^{(2)} = \sigma_{\chi}^{(3)} = 0$. Furthermore, we also have to specify the inflaton potential.

A simplest natural inflation model is characterized by the potential [63, 64]:

$$V(\phi) = \Lambda^4 \left[ 1 - \cos \left( \frac{\phi}{f} \right) \right], \quad (0 \leq \phi \leq \pi f),$$

(4.14)

where $\Lambda$ denotes the energy scale of the model and $f$ corresponds to some breaking scale which determines the curvature of the potential. Based on the slow-roll approximation, the e-folding number for the natural inflation is expressed as

$$N_0 = f^2 \frac{M_{\text{pl}}^2}{M_{\text{pl}}^2} \ln \left| \frac{\cos^2(\phi_e/2f)}{\cos^2(\phi_0/2f)} \right|,$$

(4.15)

where $\phi_e$ denotes the field value at the end of inflation. The slow-roll parameters are respectively given by

$$\epsilon = \frac{M_{\text{pl}}^2 \cos^2(\phi_0/2f)}{2f^2 \sin^2(\phi_0/2f)}, \quad \eta = \frac{2M_{\text{pl}}^2}{f^2} \frac{1 - 2 \sin^2(\phi_0/2f)}{2 \sin^2(\phi_0/2f)},$$

$$\xi^{(2)} = -\frac{M_{\text{pl}}^4 \cos^2(\phi_0/2f)}{f^4 \sin^2(\phi_0/2f)}, \quad \sigma^{(3)} = -\frac{M_{\text{pl}}^6 \cos^2(\phi_0/2f)(1 - 2 \sin^2(\phi_0/2f))}{f^6 \sin^2(\phi_0/2f)}.$$

(4.16)

The end of inflation is defined by $\epsilon = 1$, and it gives

$$\cos^2 \frac{\phi_e}{2f} = \frac{2f^2/M_{\text{pl}}^2}{1 + 2f^2/M_{\text{pl}}^2}.$$

(4.17)

The prediction for the spectral index, its higher order runnings and the tensor-to-scalar ratio can be calculated by putting the slow-roll parameters eqs. (4.16) into the formulas eqs. (A.20), (A.21), (A.22) and (A.26) given in appendix A.
4.3.4 Inverse monomial-spectator model

The potential of the inverse monomial inflation model is given by

$$V(\phi) = V_0 \left( \frac{\phi}{M_{pl}} \right)^{-p},$$

(4.18)

where $V_0$ represents the energy scale of the model and $p$ is assumed to be positive. In the slow-roll approximation, the number of e-folds is written as

$$N_0 = \frac{1}{2p} \left[ \left( \frac{\phi_e}{M_{pl}} \right)^{2} - \left( \frac{\phi_0}{M_{pl}} \right)^{2} \right].$$

(4.19)

Notice that the inflation does not end by the slow-roll violation in this model, hence it needs some mechanism to exit from the inflationary era. Here we just assume some mechanism works to stop the inflation. The slow-roll parameters are given by

$$\epsilon = \frac{1}{2} p^2 \left( \frac{\phi_0}{M_{pl}} \right)^{-2}, \quad \eta = p(p+1) \left( \frac{\phi_0}{M_{pl}} \right)^{-2},$$

$$\xi^{(2)} = p^2(p+1)(p+2) \left( \frac{\phi_0}{M_{pl}} \right)^{-4}, \quad \sigma^{(3)} = p^3(p+1)(p+2)(p+3) \left( \frac{\phi_0}{M_{pl}} \right)^{-6}.$$  (4.20)

From eq. (4.19), one can express $\phi_0$ as

$$\left( \frac{\phi_0}{M_{pl}} \right)^{2} = \left( \frac{\phi_e}{M_{pl}} \right)^{2} - 2pN_0.$$  (4.21)

As mentioned above, the inflation should end by some mechanism (not by the slow-roll violation), $\phi_e$ depends on the mechanism. Since varying $\phi_e$ corresponds to changing $\phi_0$ as seen from eq. (4.21), we take $\phi_0$ as a phenomenological parameter which is chosen to obtain $n_s$ and $r$ in accordance with observational constraint.

4.3.5 Predictions of the models

In figure 6, we plot the predictions for $r$, $n_s$, $\alpha_s$ and $\beta_s$ for the $R^2$-inflation (yellow), brane-inflation (magenta), natural-spectator (green) and inverse monomial-spectator (purple) models. The upper left, upper right and lower panels show the predictions in the $n_s$-$r$, $n_s$-$\alpha_s$ and $\alpha_s$-$\beta_s$ planes, respectively.

For the $R^2$ model, we take $50 < N_0 < 60$, which gives a very good fit to the current Planck observations [5]. For other models, we choose the values of the parameters such that the prediction for $n_s$ and $r$ become almost the same with those for $R^2$ inflation, which can be seen in the upper left panel of figure 6. Also some of the parameters are chosen to give a correct amplitude for the primordial power spectrum. For the brane inflation, we consider the case with $p = 4$ and $\mu = 2.6M_{pl}$, and assume $43 < N_0 < 52$. For the natural-spectator model, we take $f = 3.5M_{pl}$ and $51 < N_0 < 54$ for the inflaton sector and assume the fractional contribution of the spectator, $Q_\chi$ (defined in eq. (A.18)), to be $0.5 < Q_\chi < 0.6$. For the inverse monomial-spectator model, we consider the case with $p = 6$ and take $Q_\chi \simeq 0.98$ (or precisely speaking, by using $R$ defined in eq. (A.19), we take $75 < R < 80$.) The field value at the end of inflation $\phi_e$ is varied to be tuned to give almost the degenerate prediction for $n_s$ and $r$ with $R^2$ inflation as depicted in the upper left panel of figure 6.
Figure 6. Predictions for $n_s$, $\alpha_s$, $\beta_s$ and $r$ for the $R^2$-inflation (yellow), brane inflation (orange), the natural-spectator model (green) and the inverse monomial-spectator model (purple). The upper left panel shows the predictions in the $n_s$-$r$ plane, and the upper right one shows those in the $n_s$-$\alpha_s$ plane. The lower panel shows the predictions in the $\alpha_s$-$\beta_s$ plane.

As seen in the upper left in figure 6, models discussed here can give almost degenerate predictions for $n_s$ and $r$, however, when we compare the predictions of these models in the $n_s$-$\alpha_s$ and the $\alpha_s$-$\beta_s$ planes, we can see that those models give different runnings, which would be helpful to distinguish the model. It should be noted here that some models could be easily differentiated by using the expected constraint from future minihalo observations on the $n_s$-$\alpha_s$ and $\alpha_s$-$\beta_s$ planes shown in figure 4. On the other hand, some models are still difficult to be distinguished from the expected constraint investigated in this paper.

5 Conclusion

In this paper, we have developed the formalism to use the angular power spectrum of 21 cm line fluctuations from minihalos to constrain cosmological models, instead of adopting the analysis based on the pixel variance, which have been used in previous studies. Our formulation can take account of cross-correlations not only in angular domains, but also in redshift ones. The advantage in considering such cross-correlations over the simple pixel variance is that we can decode the scale-dependence on top of the increase in the statistics.

By making use of the formalism, we have shown that the measurement of 21 cm line fluctuations from minihalos is sensitive to the shape of the power spectrum of primordial fluctuations, in particular, with respect to the spectral index $n_s$ and its runnings $\alpha_s$ and $\beta_s$. The effects of $\alpha_s$ and $\beta_s$ are prominent on small scales, while the shape of the spectrum on
large scales is almost determined by $n_s$. We have demonstrated that, while the measurement of 21 cm line signals from minihalos can probe $n_s$ from the scale-dependence of its angular spectrum, its overall amplitude is sensitive to $\alpha_s$ and $\beta_s$ through the abundance of minihalos, which reflects primordial fluctuations on very small scales.

Our results exhibit the potential of the angular power spectrum of 21 cm line fluctuations as a promising probe of primordial power spectrum. Particularly, the synergy in combination with the CMB is remarkable due to the lever-arm effect since 21 cm fluctuations from minihalos are sensitive to those on small scales, while CMB observations probe those on large scales. For any combinations of CMB and 21 cm observations (Planck+SKA, Planck+FFTT, COrE+SKA, COrE+FFTT), the spectral runnings can be probed down to the level of $\alpha_s \sim \mathcal{O}(10^{-3})$ and $\beta_s \sim \mathcal{O}(10^{-4})$ (see table 3).

We have also discussed implications of future constraints on the runnings for differentiating inflationary models. For the purpose, we considered several representative models of single- and multi-field models including $R^2$-inflation. We have shown that future sensitivities of 21 cm signals from minihalos on the runnings $\alpha_s$ and $\beta_s$ would be helpful to differentiate inflationary models in some cases. Although we have just looked at some representative models, a more exhaustive study of inflationary models from the viewpoint of the spectral runnings would give more insight to really understand the mechanism of inflation.

In this paper, we have demonstrated that measurements of 21 cm signals from minihalos is useful to probe the primordial fluctuations on very small scales, showing that minihalo signals dominates the IGM signals in our model with a simple reionization model (see figure 2). However, there are theoretical uncertainties on the prediction of both signals, related to the modeling of cosmological reionization. The IGM signals can be enhanced due to the patchy reionization in some reionization models, and the efficiency of the reionization process modifies the abundance of minihalos \[37, 38\]. Therefore the further quantitative analysis for the sensitivity of minihalo signals to the primordial fluctuations requires the detailed simulations about cosmological reionization. However it is worth emphasizing again that, thanks to the distinct sensitivity of minihalos to the primordial fluctuations at very small scales, 21 cm signals from minihalos can be powerful especially in constraining higher order spectral parameters (e.g., $\alpha_s$ and $\beta_s$). This is contrastive to other sources of 21 cm line signals, such as smooth IGM components at epoch of reionization \[11, 12\] and post-reionization hydrogen clumps \[25\], which are sensitive to lower spectral parameters (e.g., $n_s$). This indicates that 21 cm line signals from different sources at different epochs can play complementary roles in probing the primordial power spectrum. It would be very interesting to combine all these different sources of 21 cm signals as well as other observations such as galaxy surveys to probe the primordial fluctuations, which we leave for a future work.

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Expressions for higher order runnings with slow-roll parameters

In eq. (4.2), we have expanded the primordial power spectrum $P_s$ up to the 2nd order. However, in principle, the primordial power spectrum can be expanded up to arbitrarily higher orders. Here we truncate the expansion at the 4th order, in which $P_s$ is given by

$$P_s(k) = A_s \left( \frac{k}{k_0} \right)^{n_s - 1 + \frac{1}{2} \alpha_s \ln \left( \frac{k}{k_0} \right) + \frac{1}{3!} \beta_s \ln^2 \left( \frac{k}{k_0} \right) + \frac{1}{4!} \gamma_s \ln^3 \left( \frac{k}{k_0} \right) + \frac{1}{5!} \delta_s \ln^4 \left( \frac{k}{k_0} \right)},$$

where the runnings $\alpha_s, \beta_s, \gamma_s$ and $\delta_s$ are defined as

$$\alpha_s \equiv \frac{d^2 \ln P_s(k)}{(d \ln k)^2} \bigg|_{k=k_0}, \quad \beta_s \equiv \frac{d^3 \ln P_s(k)}{(d \ln k)^3} \bigg|_{k=k_0},$$

$$\gamma_s \equiv \frac{d^4 \ln P_s(k)}{(d \ln k)^4} \bigg|_{k=k_0}, \quad \delta_s \equiv \frac{d^5 \ln P_s(k)}{(d \ln k)^5} \bigg|_{k=k_0}. \quad (A.1)$$

Below we give explicit expressions for these runnings using the slow-roll parameters for the single-field and multi-field models.

### A.1 Single-field case

Assuming a slow-roll single-field inflation model with a canonical kinetic term, $n_s$ and the running parameters can be explicitly written down with the slow-roll parameters, which are defined using the inflaton potential $V(\phi)$ as

$$\epsilon \equiv \frac{1}{2} M_{pl}^2 \frac{V'(V)}{V}^2, \quad \eta \equiv M_{pl}^2 \frac{V''}{V}, \quad \xi^{(2)} \equiv M_{pl}^4 \frac{V'V'''}{V^2}, \quad \sigma^{(3)} \equiv M_{pl}^6 \frac{(V')^2V^{(4)}}{V^3}, \quad \tau^{(4)} \equiv M_{pl}^8 \frac{(V')^3V^{(5)}}{V^4}, \quad \zeta^{(5)} \equiv M_{pl}^{10} \frac{(V')^4V^{(6)}}{V^5}, \quad (A.3)$$

where a prime denotes the derivative with respect to $\phi$. Using these slow-roll parameters, the spectral index and the runnings are given by:

$$n_s - 1 = -6 \epsilon + 2 \eta, \quad (A.4)$$

$$\alpha_s = -24 \epsilon^2 + 16 \epsilon \eta - 2 \xi^{(2)}, \quad (A.5)$$

$$\beta_s = -192 \epsilon^3 + 192 \epsilon^2 \eta - 32 \epsilon \eta^2 - 24 \epsilon \xi^{(2)} + 2 \eta \xi^{(2)} + 2 \sigma^{(3)}, \quad (A.6)$$

$$\gamma_s = -2304 \epsilon^4 + 3072 \epsilon^3 \eta - 1024 \epsilon^2 \eta^2 - 384 \epsilon \eta^2 \xi^{(2)} + 64 \eta^3 \xi^{(2)} + 148 \epsilon \eta \xi^{(2)} + 36 \sigma \xi^{(3)} - 6 \eta \sigma^{(3)} - 2 \eta^2 \xi^{(2)} - 2(\xi^{(2)})^2 - 2 \tau^{(4)}, \quad (A.7)$$

$$\delta_s = -36864 \epsilon^5 + 61440 \epsilon^4 \eta - 30720 \epsilon^3 \eta^2 - 7680 \epsilon^2 \eta^3 - 7680 \epsilon \eta^3 \xi^{(2)} + 4736 \epsilon \eta^3 + 5448 \epsilon \eta \xi^{(2)} + 744 \epsilon \sigma (\xi^{(2)}) - 128 \epsilon \eta \xi^{(2)} + 652 \epsilon \eta^2 \xi^{(2)} - 164 \epsilon (\xi^{(2)})^2 - 340 \epsilon \eta \sigma (\xi^{(2)}) - 52 \epsilon \tau^{(4)} + 2 \eta^2 \xi^{(2)} + 14 \eta^2 \sigma^{(3)} + 8 \eta (\xi^{(2)})^2 + 12 \eta \tau^{(4)} + 10 (\xi^{(2)})^2 \sigma^{(3)} + 2 \zeta^{(5)}. \quad (A.8)$$

The tensor-to-scalar ratio is given by

$$r = 16 \epsilon. \quad (A.9)$$
A.2 Multi-field case

When a light scalar field $\chi$ other than the inflaton $\phi$ exists during the inflationary epoch,$^4$ such a scalar field can also acquire primordial fluctuations and affect the present-day density fluctuations as in the curvaton model [43–45], the modulated reheating model [46, 47] and so on. In general, fluctuations from the inflaton can also contribute to the primordial fluctuations, and therefore the power spectrum can be given by the sum of those contributions.$^5$

\[
P_s(k) = P_s^{(\phi)}(k) + P_s^{(\chi)}(k), \tag{A.10}
\]

where $P_s^{(\phi)}(k)$ and $P_s^{(\chi)}(k)$ are the primordial power spectra generated by the inflaton $\phi$ and another spectator field $\chi$. Due to the fact that the energy density of $\chi$ is subdominant during inflation, the spectral index and its runnings for $P_s^{(\chi)}$ are different from those for $P_s^{(\phi)}$ which are given in eqs. (A.4)–(A.8). To write down the spectral index and its runnings for $P_s^{(\chi)}$, we also need to define the slow-parameters for $\chi$:

\[
\eta_\chi \equiv \frac{U''}{3H^2_s}, \quad \xi^{(2)}_\chi \equiv \frac{U'U'''}{(3H^2_s)^2}, \quad \sigma^{(3)}_\chi \equiv \frac{(U''^2U^{(4)})(3H^2_s)^3}, \tag{A.11}
\]

where $U$ is a potential for $\chi$ field and a prime indicates the derivative with respect to the $\chi$ field. A superscript $(i)$ denotes the $i$-th derivative. $H_s$ is the Hubble parameter at the horizon exit during inflation. By using these slow-roll parameters along with those defined for $\phi$ provided in eqs. (A.3), the spectral index and its runnings for $\chi$ are given as$^6$

\[
n_s^{(\chi)} - 1 = -2\epsilon + 2\eta_\chi, \tag{A.12}
\]

\[
\alpha^{(\chi)} = -8\epsilon^2 + 4\epsilon\eta + 4\epsilon\eta_\chi - 2\xi^{(2)}_\chi, \tag{A.13}
\]

\[
\beta^{(\chi)} = -64\epsilon^3 + 56\epsilon^2\eta + 24\epsilon^2\eta_\chi - 8\epsilon\eta^2 - 8\epsilon\eta_\chi - 4\epsilon\xi^{(2)}_\chi - 12\epsilon\xi^{(2)}_\chi - 12\epsilon\xi^{(2)}_\chi + 2\eta_\chi \xi^{(2)}_\chi + 2\sigma^{(3)}_\chi, \tag{A.14}
\]

\[
\gamma^{(\chi)} = -768\epsilon^4 + 944\epsilon^3\eta + 240\epsilon^3\eta_\chi - 288\epsilon^2\eta^2 - 160\epsilon^2\eta\eta_\chi - 88\epsilon^2\xi^{(2)}_\chi - 120\epsilon^2\xi^{(2)}_\chi + 16\epsilon\eta^3 + 16\epsilon\eta^2\eta + 28\epsilon\eta_\chi \xi^{(2)}_\chi + 32\epsilon\eta_\chi \xi^{(2)}_\chi + 24\epsilon\eta_\chi \xi^{(2)}_\chi + 4\epsilon\sigma^{(3)}_\chi + 24\epsilon\sigma^{(3)}_\chi - 2\eta_\chi \sigma^{(3)}_\chi + 2\xi^{(3)}_\chi - 2\sigma^{(4)}_\chi, \tag{A.15}
\]

\[
\delta^{(\chi)} = -12288\epsilon^5 + 19360\epsilon^4\eta + 3360\epsilon^4\eta_\chi - 9120\epsilon^3\eta^2 - 3360\epsilon^3\eta_\chi - 2000\epsilon^3\xi^{(2)}_\chi - 1680\epsilon^3\xi^{(2)}_\chi + 1312\epsilon^2\eta^3 + 800\epsilon^2\eta^2\eta_\chi + 1296\epsilon^2\eta^2\eta_\chi + 240\epsilon^2\eta\eta_\chi \xi^{(2)}_\chi + 960\epsilon^2\eta_\chi \xi^{(2)}_\chi + 360\epsilon^2 \eta_\chi \xi^{(2)}_\chi + 128\epsilon^2\sigma^{(3)}_\chi + 360\epsilon^2\sigma^{(3)}_\chi - 32\epsilon\eta^3 - 32\epsilon\eta^2\eta - 132\epsilon\eta_\chi \xi^{(2)}_\chi - 56\epsilon\eta_\chi \xi^{(2)}_\chi + 28\epsilon\xi^{(2)}_\chi - 80\epsilon\eta^2\xi^{(2)}_\chi - 80\epsilon\eta_\chi \xi^{(2)}_\chi - 40\epsilon\eta_\chi \xi^{(2)}_\chi - 40\epsilon_\chi \xi^{(2)}_\chi - 40\epsilon_\chi \xi^{(2)}_\chi - 44\epsilon\sigma^{(3)}_\chi + 8\epsilon_\chi \sigma^{(3)}_\chi - 80\epsilon_\chi \sigma^{(3)}_\chi - 120\epsilon_\chi \sigma^{(3)}_\chi + 4\epsilon_\chi \sigma^{(4)}_\chi + 2\eta_\chi \sigma^{(3)}_\chi + 14\eta_\chi \sigma^{(3)}_\chi + 8\eta_\chi \xi^{(2)}_\chi + 12\eta_\chi \xi^{(2)}_\chi + 10\xi^{(2)}_\chi - 2\xi^{(5)}_\chi. \tag{A.16}
\]

$^4$Such another (light) scalar field is sometimes called a spectator field since it does not affect the inflationary dynamics.

$^5$This kind of model is called mixed inflaton and spectator field models, which has been studied in the context of the curvaton and modulated reheating models [48–59].

$^6$Here we assume that there is no coupling between the inflaton and the spectator fields.
Since $P_s^{(\phi)}$ and $P_s^{(x)}$ have different scale dependences, they should be treated separately. However, we can define the effective spectral index and its runnings by using the total power spectrum as

$$n_s^{(\text{eff})} - 1 = \frac{d\ln(P_s^{(\phi)}(k) + P_s^{(x)}(k))}{d\ln k},$$

(A.17)

with which we can describe the power spectrum as if there is only one power spectrum.

To explicitly express the effective spectral index and its runnings with the slow-roll parameters, we also need to define the fraction of the contribution to the (total) power spectrum as

$$Q_\phi = \frac{P_s^{(\phi)}(k_0)}{P_s^{(\phi)}(k_0) + P_s^{(x)}(k_0)}; \quad Q_x = \frac{P_s^{(x)}(k_0)}{P_s^{(\phi)}(k_0) + P_s^{(x)}(k_0)},$$

(A.18)

where these quantities are to be evaluated at the pivot scale. In some literature, the ratio $R$ between $P_s^{(x)}$ and $P_s^{(\phi)}$ is also used to characterize the contribution from the spectator

$$R = \frac{P_s^{(x)}(k_0)}{P_s^{(\phi)}(k_0)} = \frac{Q_x}{1 - Q_x},$$

(A.19)

which is again defined at the pivot scale. With these variables, the spectral index and its runnings are given by

$$n_s^{(\text{eff})} - 1 = Q_\phi(n_s^{(\phi)} - 1) + Q_x(n_s^{(x)} - 1),$$

(A.20)

$$\alpha_s^{(\text{eff})} = Q_\phi\alpha_s^{(\phi)} + Q_x\alpha_s^{(x)} + Q_\phi Q_x(\Delta n_s)^2,$$

(A.21)

$$\beta_s^{(\text{eff})} = Q_\phi\beta_s^{(\phi)} + Q_x\beta_s^{(x)} + 3Q_\phi Q_x(\Delta n_s)\Delta s - 6Q_\phi Q_x(\Delta n_s)^3,$$

(A.22)

$$\gamma_s^{(\text{eff})} = Q_\phi\gamma_s^{(\phi)} + Q_x\gamma_s^{(x)} + Q_\phi Q_x(10\Delta n_s^2\Delta s + 15\Delta n_s^2\Delta s^2 + 6\Delta n_s^2\Delta s^3)$$

$$- 6Q_\phi Q_x(Q_X - Q_\phi)(\Delta n_s^2\Delta s + \{Q_\phi Q_x(Q_X - Q_\phi)^2 - 2Q_\phi^2 Q_x^2\}(\Delta n_s)^4,$$

(A.23)

$$\delta_s^{(\text{eff})} = Q_\phi\delta_s^{(\phi)} + Q_x\delta_s^{(x)} + Q_\phi Q_x(10\Delta n_s\Delta s + 5\Delta n_s^3)$$

$$- Q_\phi Q_x(Q_X - Q_\phi)^2(10\Delta n_s^2\Delta s + 15\Delta n_s^2\Delta s^2)$$

$$+ \{10Q_\phi Q_x(Q_X - Q_\phi)^2 - 20Q_\phi^2 Q_x^2\}(\Delta n_s)^3\Delta s$$

$$- \{Q_\phi Q_x(Q_X - Q_\phi)^3 - 8Q_\phi^2 Q_x^2(Q_X - Q_\phi)^2\}(\Delta n_s)^5,$$

(A.24)

where

$$\Delta n_s = n_s^{(\phi)} - n_s^{(x)}, \quad \Delta s = \alpha_s^{(\phi)} - \alpha_s^{(x)}, \quad \Delta \beta_s = \beta_s^{(\phi)} - \beta_s^{(x)}, \quad \Delta \gamma_s = \gamma_s^{(\phi)} - \gamma_s^{(x)}, \quad \Delta \delta_s = \delta_s^{(\phi)} - \delta_s^{(x)}.$$

(A.25)

In the limits where $Q_\phi \to 0$ and $Q_x \to 0$, the expressions become the same as the pure spectator and inflaton cases, respectively.

The tensor-to-scalar ratio in multi-field models is given by

$$r^{(\text{multi})} = 16\epsilon Q_X,$$

(A.26)

from which one can see that the tensor-to-scalar ratio is generally suppressed in multi-field models.
Figure 7. Constraints on the spectral index $n_s$ and its runnings up to the cubic order. For 21 cm line observations, $z_{\text{min}} = 6$ is assumed.

B Constraints on higher order spectral runnings

In the main text, we have truncated the expansion at the quadratic order running $\beta_s$. However, minihalos can probe small scale fluctuations at $20 \text{ Mpc}^{-1} < k < 500 \text{ Mpc}^{-1}$. Therefore, we can also probe the higher order runnings such as the cubic and quadratic runnings, $\gamma_s$ and $\delta_s$. In figure 7, the expected constraints on $n_s$ and the runnings up to $\gamma_s$ for the case with $z_{\text{min}} = 6$ are given and their $1\sigma$ sensitivities are summarized in table 5. Also, as discussed in the text, the constraints depend on the minimum redshift. In table 6, the dependence of $1\sigma$ errors is also summarized for the combinations of Planck+SKA and COrE+FFTT.

C 21 cm fluctuations from minihalos

In this section, we derive 21 cm line fluctuations from minihalos, eq. (2.1). First we consider a single minihalo with mass $M$ at a redshift $z$. The 21 cm brightness temperature at the distance (impact parameter) $\alpha$ from the center of the halo on the sky can be provided by

$$T_b(z, \alpha, M) = T_{\text{CMB}}(z) e^{-\tau_{21\text{cm}}} + \int dR T_S(l) e^{-\tau_{21\text{cm}}(R)} \frac{d\tau_{21\text{cm}}(R)}{dR},$$

(C.1)

where $l$ is the radial distance from the center of the minihalo and $R$ is the distance along the line-of-sight direction from the center. These distances satisfy the relation, $l^2 = R^2 + \alpha^2$. We
\[
\begin{array}{|c|c|c|c|c|}
\hline
 & 10^3 \Delta n_s & 10^3 \Delta \alpha_s & 10^3 \Delta \beta_s & 10^3 \Delta \gamma_s \\
\hline
\text{Planck} & 11.3 & 10.8 & 37.4 & 32.1 \\
\text{CORe} & 3.5 & 5.0 & 9.4 & 14.2 \\
\text{SKA} & 5.3 & 3.3 & 7.4 & 2.7 \\
\text{FFTT} & 2.6 & 1.7 & 2.3 & 0.84 \\
\text{Planck+Ska} & 2.1 & 2.2 & 2.8 & 1.0 \\
\text{Planck+FFTT} & 1.5 & 1.4 & 1.7 & 0.65 \\
\text{CORe+Ska} & 1.5 & 1.8 & 1.7 & 0.61 \\
\text{CORe+FFTT} & 0.97 & 1.2 & 1.2 & 0.43 \\
\hline
\end{array}
\]

Table 5. Expected 1σ sensitivities for the spectral index \(n_s\) and its runnings up to the cubic order. For 21 cm line observations, \(z_{\text{min}} = 6\) is assumed.

\[
\begin{array}{|c|c|c|c|c|}
\hline
z_{\text{min}} & 10^3 \Delta n_s & 10^3 \Delta \alpha_s & 10^3 \Delta \beta_s & 10^3 \Delta \gamma_s \\
\hline
\text{Planck+Ska} & & & & \\
4 & 1.6 & 1.6 & 2.3 & 0.82 \\
6 & 2.1 & 2.2 & 2.8 & 1.0 \\
8 & 3.0 & 3.3 & 3.3 & 1.2 \\
10 & 3.9 & 4.6 & 4.3 & 1.8 \\
\hline
\text{CORe+FFTT} & & & & \\
4 & 0.84 & 1.1 & 1.1 & 0.40 \\
6 & 0.97 & 1.2 & 1.2 & 0.43 \\
8 & 1.0 & 1.4 & 1.3 & 0.45 \\
10 & 1.2 & 1.6 & 1.4 & 0.50 \\
\hline
\end{array}
\]

Table 6. Dependence of the constraints on \(z_{\text{min}}\) for Planck+Ska and CORe+FFTT.

Note that the first term of eq. (C.1) represents the 21 cm line absorption of the background radiation while the second term gives the 21 cm line emission from the minihalo.

In eq. (C.1), \(T_S(l)\) represents the radial profile of the spin temperature in the minihalo and \(\tau_{21\text{cm}}(R)\) is the 21 cm optical depth along the line-of-sight direction at the distance \(R\). The optical depth \(\tau_{21\text{cm}}(R)\) can be written with the radial profile of neutral hydrogen \(n_{\text{HI}}\) as

\[
\tau_{21\text{cm}}(R) = \frac{2e^2 A_{10} T_s}{32 \pi \nu_0^2} \int_{-\infty}^{R} \frac{n_{\text{HI}}(l') \phi(\nu_0)}{T_S(l')} dR',
\]

where \(l' = \sqrt{R'^2 + r^2}\), \(A_{10} = 2.85 \times 10^{-15} \text{ s}^{-1}\) is the spontaneous decay rate, \(k_B T_s = h \nu_0 = 5.9 \times 10^{-6} \text{ eV}\) is the energy of the 21 cm hyperfine transition, and \(\phi(\nu)\) is the line profile. The total optical depth \(\tau_{21\text{cm}}\) in eq. (C.1) is given as \(\tau_{21\text{cm}} = \tau_{21\text{cm}}(s \to \infty)\). The spin temperature in a minihalo can be obtained by

\[
T_S(l) = \frac{T_{\text{CMB}} + y_K T_K(l)}{1 + y_K},
\]

where \(y_K\) is the kinetic coupling term which depends on the density and the temperature. Here we adopt the fitting formula given in ref. [65].

For the radial profiles of the hydrogen density and temperature, we adopt the model of a minihalo which represents a non-singular, truncated isothermal sphere, “TIS”, of dark matter.
and baryons in hydrostatic equilibrium [26]. In the TIS model, the gas temperature and the profiles of the density are given by functions of the redshift and the mass of a minihalo.

The observable value in redshift 21 cm observations can be written in terms of the differential brightness temperature

\[ \delta T_b(z, \alpha, M) = \frac{T_b(z, \alpha, M)}{1 + z} - T_{CMB}(0). \]  

(C.4)

Moreover, in actual observations, since the size of a minihalo is much smaller than the observation beam, we cannot resolve the structure of an individual minihalo. Therefore, it is convenient to introduce the mean surface brightness temperature as

\[ \langle \delta T_b \rangle(z, M) = \frac{1}{A(z, M)} \int d\alpha \frac{\delta T_b(z, \alpha, M)}{2\pi \alpha}, \]  

(C.5)

where \( A(z, M) \) gives the geometric cross-section of a minihalo with mass \( M \) at \( z \), \( A = \pi \alpha_t^2 \) with the radial size of a minihalo \( \alpha_t \) given by the TIS model.

The number of minihalos intersecting the line of sight per redshift can be obtained by

\[ \frac{dN}{dz} = \int_{M_{\text{min}}(z)}^{M_{\text{max}}(z)} dM \frac{dn}{dM} \frac{\delta T_b(z, \alpha, M)}{A(z, M)}, \]  

(C.6)

where \( \frac{dn}{dM} \) is the comoving mass function. Here, \( M_{\text{min}}(z) \) and \( M_{\text{max}}(z) \) are respectively the minimum and maximum masses of minihalos. We assume that \( M_{\text{min}}(z) \) corresponds to the Jeans mass

\[ M_J(z) = 5.73 \times 10^3 \left( \frac{\Omega_m h^2}{0.15} \right)^{-1} \left( \frac{\Omega_b h^2}{0.022} \right)^{-3/5} \left( \frac{1 + z}{10} \right)^{3/2} M_\odot. \]  

(C.7)

We set \( M_{\text{max}}(z) \) to the virial mass with virial temperature \( 10^4 \text{K} \) which corresponds to the critical temperature for hydrogen atomic cooling. When its virial temperature is larger than \( 10^4 \text{K} \), the dark matter halo can further collapse to form stars. According to ref. [15], the maximum mass of a minihalo can be approximately given as

\[ M_{\text{max}}(z) = 3.95 \times 10^7 \left( \frac{\Omega_m h^2}{0.15} \right)^{-1} \left( \frac{1 + z}{10} \right)^{-3/2} M_\odot. \]  

(C.8)

Now we can calculate the mean differential brightness temperature per unit observation frequency

\[ \overline{\delta T_b}(\nu) \approx \frac{d\nu}{d\nu_{\text{obs}}} \frac{d\nu(z)}{dz} \int_{M_{\text{min}}(z)}^{M_{\text{max}}(z)} dM \frac{dn}{dM}(M, z) \Delta \nu_{\text{eff}}(z) A(M, z) \langle \delta T_b \rangle(z, M), \]  

(C.9)

where \( \Delta \nu_{\text{eff}}(z) \) is the redshifted line profile, \( \Delta \nu_{\text{eff}}(z) = \phi(\nu_0)/(1 + z) \). For the intrinsic line profile of a minihalo \( \phi(\nu) \), we adopt the thermal Doppler-broadening model given by

\[ \phi(\nu) = \left( \frac{\pi \Delta \nu}{\nu_0} \right)^{-1} \exp[-(\nu - \nu_0)^2/\Delta \nu^2] \]  

with \( \Delta \nu = (\nu_0/c) \sqrt{2k_B T_K/m_H} \). Substituting \( d\nu_{\text{obs}}/dz = \nu_0/(1 + z)^2 \) and \( d\nu(z)/dz = c/H(z) \) into eq. (C.9) yields eq. (2.1).
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