Investigation of the rupture of a synthetic tape within the framework of the percolation theory

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Abstract. Within the framework of the percolation theory (bond problem), a new model of breaking a complex synthetic tape is proposed as a continuous-type phase transition when the state jump is zero. The percolation threshold and accompanying characteristics are calculated for the model of rupture of a synthetic reinforced tape when flowing along the first and second neighbours. The knots of the tape form a strip of a square lattice, the width of which is fixed. All nodes are intact and cannot be damaged, links (tape threads) can be intact and broken (blocked). The dependences of the percolation threshold in the bond problem and the relative deviation of the threshold from the ribbon length are calculated. It is proved that for the simplest model of one-dimensional percolation with percolation along the nearest neighbours (the problem of nodes), the percolation threshold in the thermodynamic limit is equal to unity. It is shown that, with an accuracy of 10%, the percolation threshold for a sufficiently long ribbon is equal to unity. This indicates that the system is quasi-one-dimensional. Thus, using the method of computer simulation, the percolation threshold, root-mean-square and relative threshold deviations were calculated. The critical susceptibility index was also calculated. In contrast to the usual percolation problem, in the proposed model it makes sense to consider only the region above the percolation threshold. The proposed model can be generalized to the case when nodes are also damaged (blocked), then we come to a mixed percolation model, which is supposed to be considered in the future.

1. Introduction
The issues of tearing of fabrics and other products play an important role in the design of garments, as these products must be sufficiently durable. To simulate the rupture of a complex chemical tape, it is promising to use the theory of percolation [1, 2]. The theory of percolation is a relatively young theory, the applications of which have long gone far beyond the scope of physics, computer science and chemistry [3-11].

Within the framework of the theory of percolation (the problem of bonds [1, 10, 11]), the rupture of a complex synthetic tape [4-8] is considered as a continuous-type phase transition [1-5, 8]. Purpose of the work: to calculate the percolation threshold and accompanying characteristics for the model of rupture of a synthetic reinforced tape in the framework of the theory of percolation when flowing along the first and second neighbours.

2. Model
Let's introduce the basic concepts. Nodes are the points at which two or more threads (wires) intersect and are fixed [1, 3-5, 8]. The tape consists of several filaments that are connected to each other. Further,
an elementary (simple) thread is simply called a thread. Several threads emerge from one node. The threads that connect the nodes are called ties. Nodes (links) can be whole or blocked [1, 8, 11]. A thread (link) is intact if it is not broken and connected with two nodes. A broken thread is called a blocked thread.

In the problem of nodes, a node is blocked if all threads (links) leaving this node are cut [1, 8]. All links are intact, except for those that leave the blocked node. In the node problem, nodes are randomly blocked. In the problem of links, all nodes are intact, except for those for which all links outgoing from this node are blocked. In the link task, individual links are blocked (individual threads are torn). \(N\) — the total number of nodes (links). \(N_1\) — the number of whole links (nodes). \(N_0\) — the number of blocked links (nodes). These quantities are related

\[N = N_1 + N_0.\]

All variables are non-negative. The main variable of the linkage problem is

\[P = \frac{N_1}{N}.\] (1)

Stretch the tape. The elementary threads are gradually torn and \(P\) is reduced. The sequence in which the filaments (bonds) will break cannot be predicted due to the presence of random defects. As soon as the tape is completely broken, we are on the verge of a percolation \(P = P_c\). Before that, we were above the threshold. On the threshold in the problem of connections

\[P = P_c = \frac{N_1}{N}.\]

Here \(N_1\) is the number of whole links on the threshold (it can be different!), \(N\) is the total number of links.

The simplest case is one elementary thread (the system is one-dimensional), but there are nodes on it. Such a thread will break if any one connection between the nodes breaks. And we will be on the doorstep:

\[N_0 = 1, \ N_1 = N - 1, \ P_c = \frac{N - 1}{N} = 1 - \frac{1}{N}.\] (2)

If the links connect only the nearest neighbouring nodes, then the radius of percolation is equal to one \(R = 1\) Second neighbours are nodes that are at the minimum (smallest) distance from this node, but farther than the nearest neighbours. If the bonds connect the first and second neighbours, then the percolation radius is equal to two \(R = 2\) [1]. We enumerate the connections between the nodes with natural numbers.

Figure 1 shows a model of stretching the tape (the force acts downward on the lower side). For the calculation, let us set a simplifying condition that the horizontal ties are inextensible and absolutely strong (threads 9, 10, 11 in figure 1). The lattice nodes are located only at the vertices of the squares, that is, in the centres of the squares, the bonds do not intersect. The number of blocks (squares) of the tape (\(N_b\)) can be arbitrary (in figure 1, the number of blocks \(N_b = 2\)). The number of experiments was taken to be equal \(Q = N_b \times 100\).
3. Results

The simulation results are presented in figures 2 and 3.

\[ \Delta = \frac{\delta}{P_c} \]

Figure 1. A tape with 11 links percolating through first and second neighbours \( R = 2 \).

Figure 2 – Average value of the percolation threshold depending on the size of the tape.

Next, we find the relative deviation of the threshold \( \Delta \) by the formula

\[ \Delta = \frac{\delta}{P_c} \]

where \( \delta \) is the standard deviation of the percolation threshold.

Figure 3 shows the obtained values of the relative deviation depending on the number of tape blocks.

Figure 3 – Relative percolation threshold deviation as a function of the number of blocks in the tape.
The system has a small size, so the relative deviation of the percolation threshold turned out to be quite large. For a more realistic model, the relative deviation of the threshold will be significantly less.

To determine the leakage threshold in the case of a sufficiently large system, you need to change the number of blocks in the tape and get the dependence of the threshold on the size of the system $P_c(N_b)$. This dependence is well described by the following analytical expression [1] ($N_b = N$)

$$ y = P_c(N_b) = P_c(\infty) + \frac{D}{N^\gamma} = P_c + \frac{D}{N^\gamma}. $$

Here $P_c(\infty) = P_c$ is the percolation threshold for a system of infinite size, $D$ and $\gamma$ are some constants, the index $\gamma$ is called the critical susceptibility index [1, 2]. Note that expression (2) for one-dimensional percolation along the nearest neighbours has the form (4), the critical susceptibility index $\gamma$ being equal to one and the percolation threshold for an infinite system is also equal to one. This value of the percolation threshold ($P_c(\infty) = 1$) is the specificity of the one-dimensional problem with a finite radius of flow.

For two sizes of the system, we obtain

$$ y_1 = P_c + \frac{D}{(N_1)^\gamma}, $$

$$ y_2 = P_c + \frac{D}{(N_2)^\gamma}. $$

Subtract from (5) equality (6):

$$ y_1 - y_2 = D\left(\frac{1}{(N_1)^\gamma} - \frac{1}{(N_2)^\gamma}\right) = Df_{12}. $$

Same as

$$ y_1 - y_3 = D\left(\frac{1}{(N_1)^\gamma} - \frac{1}{(N_3)^\gamma}\right) = Df_{13}. $$

Divide (7) by (8)

$$ L(123) = \frac{y_1 - y_2}{y_1 - y_3} = \frac{f_{12}}{f_{13}} = f_{123}(\gamma). $$

(9) can be written as

$$ L(123) - f_{123}(\gamma) = 0. $$

Changing the index $\gamma$, we find solutions to this equation, that is, the values of the index satisfying (10). Next, we can find the percolation threshold for an infinite system and the value of the constant $D$. The results are as follows

$$ P_c(N_b) = P_c(\infty) + \frac{D}{(N_b)^\gamma} \approx 1 - \frac{0.7}{(N_b)^{0.25}}, $$
where $D \approx -0.7$, $\gamma \approx 0.25$ (the error in the susceptibility index is about 10%). The percolation threshold for a system of infinite size turned out to be close to unity, which is natural (see (2) and (4)), since the considered model is one-dimensional (quasi-one-dimensional). The results on the susceptibility index are nontrivial, since for two-dimensional and three-dimensional space this index is greater than one (albeit when percolating along the nearest neighbours), and it grows with decreasing space dimension [1].

4. Conclusions
For the first time, for the simplest case, a new model of rupture of a complex synthetic (chemical) tape is analysed within the framework of the percolation theory with a radius of percolation equal to two. Consideration is carried out within the framework of the problem of connections. The percolation threshold, root-mean-square and relative threshold deviations were calculated by the method of computer simulation. The critical susceptibility index was also calculated. In contrast to the usual percolation problem, in the proposed model it makes sense to consider only the region above the percolation threshold.

If, in addition to broken bonds, some of the nodes are damaged (blocked), then we arrive at a mixed percolation problem, which is more consistent with the real situation [11]. The rupture of the tape in the framework of the mixed problem is supposed to be considered in the future. Within the framework of the proposed approach, it is possible to calculate other critical indices and check the scaling (similarity) relations [1, 2, 7, 8].

The considered complex reinforced chemical tapes, apparently, can be used to create fabrics and materials for special purposes for operation in extreme conditions: in space, in the polar regions, to create climbing equipment and parachutes, for underwater work.

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