Identification and compensation of stiffness and damping between short and long stroke in wafer stage

Xu Yang¹, Ming Zhang², Rong Cheng, Rong Cheng³*, Yu Zhu⁴

¹Department of Mechanical Engineering, Tsinghua University Beijing, China
²Department of Mechanical Engineering, Tsinghua University Beijing, China
³Department of Mechanical Engineering, Tsinghua University Beijing, China
⁴Department of Mechanical Engineering, Tsinghua University Beijing, China

*Corresponding author: Rong Cheng
E-mail: chengr@tsinghua.edu.cn

Abstract. The disturbance of cable force is an important aspect that should be taken into account for precision motion systems involved in CNC machine tools, wafer scanners, etc. In this paper, unlike paying attention to the cable force loaded on long stroke in previous works, the disturbance between long and short stroke had been analyzed in details. To minimize the disturbance force loaded on short stroke of motion system, a disturbance model identification and the corresponding compensation approach method was proposed. A two-step identification in frequency domain was developed such that the parameters of stiffness and damping between long and short stroke can be estimated. Then a state feedback method using ESO and estimated parameters had been applied to compensate the disturbance, which improved the servo performance of short stroke significantly. Finally, the effectiveness of the proposed method is illustrated through numerical simulation.

1. Introduction

The wafer stage is the most critical part of the step scan lithography, which is a kind of ultra-precise motion system. The wafer stage would move according to the predetermined trajectory when the wafer was exposed under the lens, and the exposure accuracy was dependent on the stage motion accuracy. In a current day lithographic tool, moving average (MA) error must be limited to about 1 nm while moving standard deviation MSD must be limited to about 7 nm[1]. According to the research of Hans, the disturbance of 60 mN in feedforward force would cause the increasing of 1 nm in the motion error when the control bandwidth of the wafer stage was 200 Hz[2]. Thus the motion accuracy of wafer stage is very sensitive to the unkown disturbance during process of movement. The structure of the wafer stage also developed from the initial rolling guide scheme to the air bearing supported and magnetic suspension, which is to reduce the influence of disturbance on moving accuracy. So during the motion process of wafer stage, beside loading accurate feedforward force of the profile on the stage to eliminate the trajectory error, some other disturbance should be taken into account to guarantee the accuracy, e.g. the stiffness of motor, dynamic link force, magnetic interaction, hysteresis, etc. All these kinds of disturbance mentioned above shows different effects on the stage. Some previous researches had been done to compensate certain disturbance source. Hans provided a method of compensating the position-dependent disturbance force introduced by the magnet interaction with reticle stage[1]. The force was
estimated from the control signals by moving the stage in both directions and be recorded in a table. Then the force in the table would be added in the control loop as feedforward to the counteract the disturbance. Xia proposed a method of estimating and compensating the position dependent disturbance of wafer stage caused by cable link and magnet interaction, which also use a table to record the distinguished disturbance force from control loop[3]. While these kinds of compensation method are strongly dependent on the current reference profile, and the calibration of the table should be done again when the trajectory had been changed. M. Hoogerkamp proposed a compensation method of the disturbance of cable force loaded on the long stroke of wafer stage[4]. In his work, the disturbance was modeled by stiffness and damping matrix, and the disturbance force would be estimated by multiply the stiffness and damping with the estimated states of stage. However, both of the accuracy of estimated disturbance and states relays on the model. In addition, the cable shows strongly nonlinear property, but only linear model was applied in this work. The method of identifying the model parameter had not been mentioned. Though M. Hoogerkamp[5] had proposed a method of modeling the cable of long stroke by nonlinear finite element method in another research paper, the attenuation results of the disturbance according to the nonlinear model had not been confirmed. In addition, the parameters of finite element model usually should be updated by system identification.

Figure 1 Overview of the stages of a lithography machine

The disturbance mentioned above can be classified into two different kinds. The first is introduced by magnet interaction and the other is introduced by cable. The wafer stage was consist of short and long stroke. The magnet interaction disturbance affects accuracy of short stroke directly, while the cable force was loaded on the long stroke and it would not affect the short stroke when there is no stiffness and damping between short and long stroke. However, there are signal cables and hoses containing air and liquid which have stiffness and damping as well as the actuators and gravity balance module. Then an additional force would be loaded on the short stroke when there is relative displacement between short and long stroke. So the cable force must be modeled and attenuated to assure the following accuracy of long stroke. Compared to the disturbance of the long stroke, the disturbance compensation of short stroke was more important. However, there is no research about modeling and compensating the disturbance between short and long stroke had been reported.

According to the discussion above, this paper has three main contributions:

- The disturbance model between long and short stroke was analyzed and the effects on control loop was discussed in details.
- The identification method of the disturbance model had been developed.
- The method of identifying parameters of disturbance model and the corresponding compensation method using extended state observer (ESO) had been proposed.

2. Modeling of the disturbance force

This section first discussed the effect of disturbance on the long stroke motion system and illustrated the general attenuation method of the disturbance. In section II.B, the unavailability of the general attenuation method for motion system with long and short stroke was discussed by analyzing the control loop in details.
2.1. Disturbance force of motion system with only long stroke

The assumption in this part is that there is only long stroke in the motion system. The mover attached with cable schlepp was approximated as a linear time-invariant (LTI) system shown in Fig. 2. The mass \( m \) representing a stage in a lithography machine, and it is controlled in the degrees of freedom \( y_1 \) to \( y_n \) using a feedback system which has a force input \( f_1 \) to \( f_n \). The stage is connected with the fixed world by stiffness and damping. M. Hoogerkamp[4] had taken vibration freedom into account when establishing the disturbance model. However, the displacement of the vibrating freedom can be hardly detected and its coupled stiffness and damping on rigid freedom can not identified directly. So in this paper, the wafer stage was be modeled as the rigid body and only the stiffness and damping of rigid body freedom were considered.

![Figure 2](image)

Figure 2 Simple example of the modeling of long stroke with mass-spring systems. The stage mass \( m \) stage moving in directions \( y_n \) is controlled by control forces \( f_n \). The stage has rigid body stiffness and damping \( k_n \) and \( c_n \).

Then the equations of motion of the system in Fig. 1 are

\[
M \ddot{q} + C \dot{q} + K q = f
\]

where \( q = [y_1, y_2, \ldots, y_n]^T \) is the vector of degrees of freedom, \( M \) the mass matrix, \( C \) the damping matrix and \( K \) the stiffness matrix. \( f = [f_1, f_2, \ldots, f_n]^T \) is the control force input acted on the degrees of freedom. Then the disturbance force \( f_d \) on the stage denote the position and velocity dependent force induced by cable schlepp of long stroke and it can be computed from the equations of motion

\[
f_d = C \dot{q} + K q
\]

Without loss of generality, the motion system can be simplified as a mass-spring-damping system with only one DOF. Then the equation (1) can be reformulated as

\[
m_1 \ddot{y}_1 + c_1 \dot{y}_1 + k_1 y_1 = f_1
\]

The transfer function form of the differential equation is

\[
P_i(s) = \frac{1}{m_1 s^2 + c_1 s + k_1}
\]

The stiffness and damping lead to the resonance of the system in low frequency range. The position of \( P_i(s) \) was usually controlled by a feedback controller \( C_{fb1}(s) \) which can produce driving force \( f_{fb} \) according to the displacement relative to the reference set point and an additional feedforward compensation \( F_{ff1}(s) \) is needed to mitigate tracking error when a reference trajectory was loaded into the system, as illustrated in Fig. 3. Then the tracking error can be characterized as
Some design methods of the feedforward controller had been developed in previous works, and the main purpose of the methods was to make the feedforward equal the inverse of plant $P_1(s)$ as

$$F_{ff1}(s) = P_1(s)^{-1}$$

then the tracking error would be zero. For the system in equation (4), the feedforward signal would have the following form like

$$f_{ff1} = \theta_0 a + \theta_1 v + \theta_2 p = m_1 \ddot{r} + c_1 \dot{r} + k_1 r = m_1 \ddot{r} + f_d$$

where $p$, $v$ and $a$ correspond to respectively position, velocity and acceleration, i.e., the 0, 1st and 2nd derivative of the trajectory $r$, and the $\theta_0$, $\theta_1$, and $\theta_2$ are the corresponding parameters. The position and velocity dependent part in $f_{ff1}$ is equal to the disturbance $f_d$ induced by stiffness and damping. Generally, system identification was the most common method of estimating the accurate feedforward parameters, which is not only suitable for SISO system formulated equation (4), but also for the MIMO system shown in Fig. 1. Even for the system containing nonlinear behavior, many nonlinear system identification method could usually be used to estimate accurate feedforward parameters. For instance, when taking the nonlinear property of cable schlepp of long stroke into account, the parameters of stiffness and damping would be variables dependent on the scheduling parameter $r$, as $k_1(r)$ and $c_1(r)$.

2.2. Disturbance force of motion system with short and long stroke

However, the disturbance compensation method using feedforward signal would not suitable for the motion system containing long and short stroke, when the stiffness and damping between the short and long stroke were taken into account. Only single degree of freedom is considered, and the short-long stroke of motion system can be simplified as a two mass-spring-damping system as shown in Fig. 4. The $m_2$ represents the mass of long stroke connected to the fixed world with the cable schlepp, which also had been simplified as a linear stiffness $k_2$ and damping $c_2$. The $m_1$ denotes the mass of short stroke which is connected to the $m_2$ with stiffness $k_1$ and damping $c_1$. $y_1$ denote the displacement of $m_1$ relative to the fixed world, $y_2$ the displacement of $m_2$ relative to the fixed world, and the $y_3$ denote the relative displacement between $m_1$ and $m_2$. The differential equations of the system in Fig. 4 can be formulated as

$$m_1 \ddot{y}_1 + k_1 (y_1 - y_2) + c_1 (\dot{y}_1 - \dot{y}_2) = f_1$$
$$m_2 \ddot{y}_2 - k_1 (y_1 - y_2) - c_1 (\dot{y}_1 - \dot{y}_2) + k_2 y_2 + c_2 \ddot{y}_2 = f_2$$

where the parameters all correspond to that in Fig. 4.
Differing from the modeling method in M.Hoogerkamp[4], the nominal plant had been defined as two dependent rigid body models with mass of $m_1$ and $m_2$, respectively.

\[
\begin{align*}
    x_{a1} &= A_{a1}x_{a1} + B_{a1}u_{a1} \\
    y_{a1} &= C_{a1}x_{a1} \\
    x_{a2} &= A_{a2}x_{a2} + B_{a2}u_{a2} \\
    y_{a2} &= C_{a2}x_{a2}
\end{align*}
\]  

(9)  

(10)

with $A_{a1} = A_{a2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $C_{a1} = C_{a2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $B_{a1} = \begin{bmatrix} 1/m_1 \\ 1/m_2 \end{bmatrix}$, $B_{a2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $x_{a1} = \begin{bmatrix} y_1 \\ \dot{y}_1 \end{bmatrix}$, $x_{a2} = \begin{bmatrix} y_2 \\ \dot{y}_2 \end{bmatrix}$, $u_{a1} = f_1$ and $u_{a2} = f_2$. Then the disturbance terms containing stiffness and damping can be considered as the state feedback with $f_{d1} = K_{k1}(x_{a1} \cdot x_{a2}) = [k_1 \ c_1] (x_{a1} \cdot x_{a2})$ and $f_{d2} = K_{k2}x_{a2}\dot{f}_{d1} = [k_2 \ c_2]x_{a2}\dot{f}_{d1}$. The position $y_1$ of short stroke ($m_1$) relative to the fixed world would be measured as the feedback of position control loop. The long stroke ($m_2$) would operate in the following mode so that the relative displacement $y_3$ would be sensed directly. So the plant of the equation (8) can be illustrated as in Fig. 5. Ideally, when the plant shown in Fig. 5 is controlled by a continues controller, the close loop of the system can be transformed to the structure shown in Fig. 6, where $P_1(s)$ and $P_2(s)$ are the nominal rigid body model

\[
\begin{align*}
    P_1(s) &= C_{a1}(sI - A_{a1})^{-1}B_{a1} \\
    P_2(s) &= C_{a2}(sI - A_{a2})^{-1}B_{a2}
\end{align*}
\]  

(11)

and $H_1(s)$ and $H_2(s)$ are the disturbance model of state feedback loop in Fig. 5, as

\[
\begin{align*}
    H_1(s) &= k_1 + c_1 \cdot s \\
    H_2(s) &= k_2 + c_2 \cdot s
\end{align*}
\]  

(12)
Figure 6 equivalent control loop of simplified wafer stage model

The structure in this Fig. 6 is an equivalent control loop assuming the reaction force produced by actuators of short stroke that loaded on long stroke had been compensated exactly. Then transfer function from reference $r$ to short stroke output $y_1$ would have the form with

$$
y_1 = \frac{1+G(s)H_1(s)+G(s)C_{\beta_2}(s))P_1(s)C_{\beta_1}(s)}{1+G(s)H_1(s)+G(s)C_{\beta_2}(s))1+P_1(s)C_{\beta_1}(s)+P_1(s)H_1(s)}r
$$

$$+ \frac{1+G(s)H_1(s)+G(s)C_{\beta_2}(s))P_1(s)F_{f1}(s)}{1+G(s)H_1(s)+G(s)C_{\beta_2}(s))1+P_1(s)C_{\beta_1}(s)+P_1(s)H_1(s)}r
$$

$$+ \frac{G(s)P_1(s)H_1(s)F_{f2}(s)}{1+G(s)H_1(s)+G(s)C_{\beta_2}(s))1+P_1(s)C_{\beta_1}(s)+P_1(s)H_1(s)}r
$$

(13)

where

$$G(s) = \frac{P_1(s)}{1+P_1(s)H_2(s)}
$$

(14)

In case of the system get accuracy feedforward controller $F_{f1}(s)$ and $F_{f2}(s)$ which is equal to the $P_1(s)$ and $P_2(s)$, respectively, the equation would be reformulated as

$$
y_1 = \frac{1+G(s)H_1(s)+G(s)C_{\beta_2}(s))P_1(s)C_{\beta_1}(s)}{1+G(s)H_1(s)+G(s)C_{\beta_2}(s))1+P_1(s)C_{\beta_1}(s)+P_1(s)H_1(s)}r
$$

$$+ \frac{1+G(s)H_1(s)+G(s)C_{\beta_2}(s))P_1(s)F_{f1}(s)}{1+G(s)H_1(s)+G(s)C_{\beta_2}(s))1+P_1(s)C_{\beta_1}(s)+P_1(s)H_1(s)}r
$$

$$+ \frac{G(s)P_1(s)H_1(s)P_2(s)^{-1}}{1+G(s)H_1(s)+G(s)C_{\beta_2}(s))1+P_1(s)C_{\beta_1}(s)+P_1(s)H_1(s)}r
$$

(15)

The expression of output $y_1$ is really complex and it is related to the disturbance loop of $H_1(s)$ and $H_2(s)$. However, $y_1$ would be identical to the reference $r$ when either of $H_1(s)$ and $H_2(s)$ has been compensated, thus the position error would be equal to zero.

Generally, the cable schlep of the long stroke often contains non-linear behaviors and it can hardly be modeled accurately. Comparing with the long stroke, the short stroke is to maintain the positioning accuracy of the stage. So attenuation of the disturbance loaded on short stroke is a dominant work to ensure accuracy. In addition, the relative maximum relative displacement of short stroke is less than 2mm, which determines that the disturbance model $H_1(s)$ would be obtained more easily than $H_2(s)$. When the system identification method was applied on the short stroke in Fig. 6, only close loop identification method can be chosen because the feedback is required to ensure stability of motion.
system. The open-loop estimation of $P_1(s)$ can be obtained from the closed-loop frequency response of sensitivity function $S_{ud}(s)$ and process sensitivity function $S_{yd}(s)$:

$$
\hat{P}_1(s) = S_{ud}^{-1}(s) S_{yd}(s)
$$

where $k$ is used to denote the discrete frequency at which the FRF is defined. Because of the effect of long stroke, the process sensitivity function would be shown as (The reference of short stroke was set to zero)

$$
S_{yd}(s) = \frac{1+G(s)H_1(s)+G(s)C_{\beta_2}(s)}{P(s)[1+G(s)H_1(s)+G(s)C_{\beta_2}(s)]} + P(s)H_1(s)
$$

and the sensitivity function

$$
S_{ud}(s) = \frac{1+G(s)H_1(s)+G(s)C_{\beta_2}(s)}{P(s)[1+G(s)H_1(s)+G(s)C_{\beta_2}(s)]} + P(s)H_1(s)
$$

So the identified model of the short stroke would be formulated as

$$
\hat{P}_1(s) = \frac{1+G(s)H_1(s)+G(s)C_{\beta_2}(s)}{P(s)[1+G(s)H_1(s)+G(s)C_{\beta_2}(s)]} + P(s)H_1(s)
$$

which is not only related to the model of short stroke $P_1(s)$, but also the model of long stroke and the disturbance model $H_1(s)$ and $H_2(s)$. The cable schlep would always contain non-linear parts which are not be modeled using $H_2(s)$. In addition, the identified nominal model will change when the parameters of feedback controller $C_{\beta_2}(s)$ was replaced. So the identified transfer function can not be used to represent the real plant, and it means that the feedforward signal could not be defined dependent on model.

However, when the disturbance model $H_1(s)$ in equation (19) had been eliminated, the identified nominal model $\hat{P}_1(s)$ would be equal to the real system $P_1(s)$. Though the method of disturbance observers had been developed to attenuate the effect of the $H_1(s)$, the effectiveness of the observer is also dependent on the accuracy of the nominal model $\hat{P}_1(s)$. So a modeling and compensation method of the disturbance $H_1(s)$ was proposed in section III and it is the main contribution of this paper.

According to the analysis above, the disturbance between short and long stroke would cause trajectory error inevitably. In addition, the nominal model of short stroke obtained by system identification could not be used to design feedforward controller or disturbance observer. So there are two key problems that need to be solved in this work.

a. An identification method should be developed to acquire the disturbance $H_1(s)$.

b. Compensating the disturbance $H_1(s)$ using a state observer which is not sensitive to the modeling error of $P_1(s)$.

The solution would be proposed in next section.

3. Estimation of stiffness and damping

To identify of the disturbance model $H_1(s)$ in Fig. 6, the system also should operate on close loop mode to keep the stability. In addition, the position of short stroke ($m_1$) is measured relative to the fixed coordinate system and the long stroke ($m_2$) needs to be operated in following mode, thus the displacement of $m_2$ relative to $m_1$ is measured.

Step1
Loading a random noise signal on long stroke \((m_2)\), which would create a relative displacement between \(m_1\) and \(m_2\). Then the relationship between \(y_1\) and \(y_3\) can be formulated in frequency domain as

\[
S_{p1}(s_k)H_i(s_k) = \frac{Y_i(s_k)}{Y_3(s_k)}
\]

where \(S_{p1}(s_k)\) denotes the frequency response of process sensitivity function and \(H_i(s_k)\) the frequency response of the disturbance model.

Step2

Loading a random excitation signal \(f_1\) on short stroke \((m_1)\), and the displacement \(y_1'\) of \(m_1\) and the relative displacement \(y_3'\) of \(m_2\) should be recorded. Then the following relationship would be obtained:

\[
Y_i'(s_k) = S_{p1}(s_k)H(s_k)Y_3'(s_k) + G_i(s_k)F_i(s_k)
\]

Then the frequency response of sensitivity function of \(m_1\) can be estimated:

\[
S_{p1}(s_k) = \left[ Y_i'(s_k) - \frac{Y_i(s_k)}{Y_3(s_k)}Y_3'(s_k) \right]F_i(s_k)
\]

Combining equation (19) with the equation (17), the FRFs of \(H_i(s_k)\) can be calculated:

\[
H_i(s_k) = S_{p1}^{-1}(s_k)\frac{Y_i(s_k)}{Y_3(s_k)}
\]

Generally, the discrete control system would be applied to the system. So the excitation signal \(f_1\) would be loaded on \(m_1\) with a time delay, relative to the recorded displacement \(y_1'\) and \(y_2'\) in equation (19). So the delay time should be compensated when calculating the FRFs of \(S_{p1}(s_k)\) and \(H_i(s_k)\) with correct phase. Then a parameter identification procedure should be conducted to fit the stiffness and damping value. The parameter identification method had been reported in a lot of precious works, and the details would not be introduced in this paper.

4. Compensation of stiffness and damping

According to the system scheme shown in Fig. 5, the disturbance force of would be formulated as a state feedback with the form with \(f_d = cx + kx\). So it also could be offset using a new state feedback loop. The parameters of \(c\) and \(k\) can be acquired using the method proposed in section III, Then the key problem to be resolved is to get the velocity value of the stage because the position value can be measured directly by sensors. Two methods had usually be used to get velocity value. One is from the derivative of measured position signal which would cause a delay of half a period and introduce loud noise. The other is estimate it from a state observer, which may be sensitive to modeling error of plant. According to the analysis of section II.B, accurate model of plant may not be obtained only if the stiffness and damping had been compensated.

The extended state observer (ESO) is a critical part of the active disturbance rejection control (ADRC) technology which had been applied widely in recent years[6, 7]. Apart from the state of system, the extended term of ESO can giving the estimated total disturbance which including the disturbance force and the model error. The quick tuning method through pole placement of LESO also had been proposed by Gao[8], which is the foundation that it can be applied widely. What’s more, only the relative order of the system is necessary when designing an ESO, and the estimated state would be accurate when the bandwidth is high enough. In addition, the extended state can be used as a supplement of the disturbance compensation.
In this paper, the black box approach would be used in design the ESO because the accurate model of plant may not be obtained. Define the extended state of plant $m_1$ or $m_2$:

$$x = \begin{bmatrix} y \\ y \\ f \end{bmatrix}$$

(24)

The state space description of the system is

$$\dot{x} = Ax + Bu + Eh$$
$$y = Cx$$

(25)

with

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

and $h = \dot{f}$

for which the ESO in the form of the Luenberger observer is given as

$$\dot{z} = Ax + Bu + L(y - \hat{y})$$
$$y = Cz$$

(26)

For the sake of simplicity and easy tuning, the observer gain

$$L = \begin{bmatrix} 3\omega_0 & 3\omega_0^2 & \omega_0 \end{bmatrix}^T$$

(27)

is chosen where all eigenvalues of $A-LC$ are placed at $\omega_0$, denoted as the observer bandwidth. Then the control scheme for compensation is presented in Fig. 7.

Figure 7 Simple example of the modeling of long stroke with mass-spring systems. The stage mass $m$ stage moving in directions $y_n$ is controlled by control forces $f_n$. The stage has rigid body stiffness and damping $k_n$ and $c_n$.

5. Simulation results

In this section, the simulation results of the proposed methods using MATLAB Simulink are presented.

The proposed method was tested using a one DOF system illustrated in Fig. 4 by simulation using a set of parameters given in Table 1. The control loop of the system was be set as the same as the scheme shown in Fig. 7. The short stroke ($m_1$) was controlled relative to the fixed world and the long stroke ($m_2$) was controlled relative to $m_1$. A noise with the same variance with the real sensor signal should be added to the displacement output to simulate the real measurement system.
Table 1 The parameters of system in simulation

| $m_1$ | $m_2$ | $k_1$ | $c_1$ | $k_2$ | $c_2$ |
|-------|-------|-------|-------|-------|-------|
| 20 kg | 30 kg | $2 \times 10^3$ N/m | $1 \times 10^2$ N/(m/s) | $2 \times 10^4$ N/m | $1 \times 10^2$ N/(m/s) |

The disturbance model of $H_1(s)$ would be identified using the method proposed in section III and corresponding bode diagram was illustrated in Fig. 8(a). The black line ($d1$) in Fig. 8(a) was bode plot of the real model that added into the simulation system, and the red one ($d2$) is the calculated FRFs from the recorded outputs, and the green line ($d3$) is the fitted model. Though there is large noise in FRFs in high frequency, the error of fitted result is also small enough. Two ESOs was designed to estimate the states of $m_1$ and $m_2$, and the disturbance introduced by $H_1(s)$ was compensated using the method shown in Fig. 7. The linearized nominal model of $m_1$ was illustrated in Fig. 8(b). The $L1$ in Fig. 8(b) is the bode diagram of the real short stroke model with form of double integrator. The $L2$ and $L3$ are the linearized model with different feedback controller of $m_2$. The $L4$ is the model obtained from the control scheme with compensation of $H_1(s)$. The $L2$, $L3$ and $L4$ can be approximated as the transfer function with form of $1/20s(s+\omega_c)$, where $\omega_c$ is the corner frequency. The high frequency behavior of $1/20s(s+\omega_c)$ would be similar as $1/20s^2$, while the low frequency as the $1/20\omega_c$. The value of $\omega_c$ would move when the feedback controller of $m_2$ was replaced, which confirmed the analysis of equation (16). If the low frequency behavior of $L2$, $L3$ and $L4$ was ignored during system identification, the model would be fitted as double integrator $1/20s^2$. Then only the acceleration feedforward signals would be added on the plant when $m_1$ moves with reference trajectory. The uncompensated disturbance force can be roughly estimated using $\omega_c \times \nu_{max}/20$, where the $\nu_{max}$ denote maximum velocity of the reference. The theoretical maximum value of disturbance would be about 125N for $L2$ and $L3$, and be 12.5mN for $L4$, when the maximum velocity was 1m/s. The disturbance force for $L4$ was really small enough that it can be attenuated by the feedback controller. So only $L4$ can be fitted as $1/20s^2$ approximately.

Figure 9 presents the bode diagram of equivalent sensitivity function from the force input of long stroke to the tracking error of short stroke. The red line denote the sensitivity function without offset $H_1(s)$ and the blue dash line the compensated one. The results shows that the magnitude of sensitivity function had been reduced by $150db$ using the proposed compensation method. Thus the identification problem of $m_1$ mentioned in equation (16) can be solved effectively using the disturbance compensation method proposed in section III.
Figure 8  (a) Bode plot of the stiffness and damping model between long and short stroke; (b) Bode plot of linearized nominal model of short stroke in control loop

Figure 9  Bode plot of equivalent sensitivity function: s1 is the sensitivity function before compensation, s2 is the one after compensation

A 4th order reference trajectories are considered as depicted in Fig. 10. The acceleration feedforward signals respect to the mass would be loaded on $m_1$ and $m_2$, respectively. The disturbance of $H_2(s)$ would not be compensated and the feedforward signal loaded on $m_1$ will contain a 5% error to create a relative poor control condition for $m_1$. The time-domain error signals of $m_1$ would be shown in Fig. 11, where the dash curve represents the scaled jerk setpoints. The peak value of error is decreased from above 10μm to 10nm by state feedback proposed in section IV.
Figure 10  The 4th order reference trajectories. “Position”, “Velocity” and “Acceleration” are abbreviated as “Pos”, “Vel” and “Acc”, respectively. Maximum snap = 1280000 m/s^4, maximum jerk = 3200 m/s^3, maximum acc 32 m/s^2, cruising velocity = 1 m/s, and range = 0.1 m

Figure 11  Time-domain error signals of short stroke

6. Conclusion
In this paper, to minimize the effect of the disturbance between long and short stroke and improve the servo performance, an identification approach was proposed to estimate and compensate the stiffness and damping between long and short stroke for the motion system. The stiffness and damping between long and short stroke would cause model error between identified nominal and real plant model. With the aim at minimizing the effect of this model error, a two-step excited method had been utilized to identify the disturbance model. Accordingly, the parameters of stiffness and damping can be fitted from the FRFs. The disturbance introduced by the stiffness and damping can be compensated using state feedback and the state was estimated by an ESO. Comparative simulation results confirmed that the proposed approach achieved the unbiased estimate of the optimal parameters combined with a small estimate variance and the compensation method can improve the performance of motion system significantly.
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