Shape coexistence in $^{153}$Ho

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The high-spin states in $^{153}$Ho, have been studied by $^{139}$La($^{20}$Ne, 6n) reaction at a projectile energy of 139 MeV at Variable Energy Cyclotron Centre (VECC), Kolkata, India, utilizing an earlier campaign of Indian National gamma Array (INGA) setup. Data from gamma-gamma coincidence, directional correlation and polarization measurements have been analyzed to assign and confirm the spins and parities of the levels. We have suggested a few additions and revisions of the reported level scheme of $^{153}$Ho. The RF-gamma time difference spectra have been useful to confirm the half-life of an isomer in this nucleus. From the comparison of experimental and theoretical results, it is found that there are definite indications of shape coexistence in this nucleus. The experimental and calculated lifetimes of several isomers have been compared to follow the coexistence and evolution of shape with increasing spin.

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I. INTRODUCTION

The neutron deficient rare-earth isotopes near the magic nucleus $^{146}$Gd have shown multitude of structural features as functions of neutron number as well as spin [1, 2]. For isotones with N = 86, excitation spectra show single particle nature associated with non-collective modes. For N = 88, strong collectivity in terms of appreciable prolate deformation is manifested in the low-lying spectra. However, even for the nuclei which are very close to $^{146}$Gd, although low spin excitations are usually irregular and complex indicating spherical shape with single or multi-particle excitations, at relatively higher energies superdeformed (SD) bands [1, 3, 4] are observed. The observations indicate that these nuclei are very soft against shape changes. The features have been interpreted theoretically using microscopic shell-model and mean-field descriptions [2]. One of the distinguishing features of this mass region is the existence of an island of high spin isomers [1, 3, 8], which are excited in heavy ion reactions. These isomers can also indicate a sharp change of structural configurations within the same nucleus [5, 6].

This mass region has been investigated extensively particularly for even - even nuclei [1]. It has been possible to observe structural changes and shape coexistence effects for them. Some nuclei in this region, like $^{152}$Dy$_{86}$ [3, 4], are found to be near-spherical oblate in shape at low spins. However, they exhibit coexisting collective prolate shapes like superdeformation at higher spins. On the other hand there are evidences of excitation pattern in $^{154}$Dy$_{88}$ similar to collective prolate rotors at low spins which evolve to non collective oblate shape at high spins. Nuclei with higher neutron numbers (N > 90), behave like collective prolate rotors throughout their entire excitation pattern. For odd Z nuclei, like Holmium (Ho), although individual studies of different isotopes [5, 11] exist, systematic analysis of the structural evolution of this element with variations in neutron number is scarce. Ho (Z=67) is the nearest odd-Z neighbor of most extensively studied Dy (Z=66). A systematic study [12-18] of the isotopes of Ho element has been initiated to understand how nuclear structure differs due to the addition of a single unpaired proton. In the present work, $^{153}$Ho isotope has been studied.

The neutron deficient isotope of Ho, $^{153}$Ho has an unpaired proton coupled to the even-even core $^{152}$Dy. $^{153}$Ho with neutron number N=86 has been studied previously [10, 13, 26] to high spins (J $\simeq$ 81/2) [19]. Its 11/2$^+$ ground state quadrupole moment measured using Laser resonance ionization mass spectroscopy (LRIMS) method
II. EXPERIMENTAL DETAILS AND DATA ANALYSIS

High spin states of $^{153}$Ho were populated by bombarding 139 MeV $^{20}$Ne beam on a La$_2$O$_3$ target at Variable Energy Cyclotron Centre (VECC), Kolkata. The target of thickness 3mg/cm$^2$ was prepared by centrifuge method on Al backing of 2.2mg/cm$^2$. $^{153}$Ho was populated most strongly, along with population of $^{151,152,154}$Ho, $^{152−153}$Dy nuclei. Results from preliminary data analysis from this experiment have been reported earlier [12, 14–16]. The experiment has been carried out using one of the earlier campaigns [22] of Indian National Gamma Array (INGA) setup, which comprised of six Compton-suppressed Clover detectors. In this setup, the detectors were placed at 40°(2), 90°(2), 125°(2) with respect to the beam direction. Data were acquired in LIST mode. At least two correlated gamma energies from the Clovers, their time information as well as corresponding RF time information with respect to the master gate have been included in the LIST data.

The energy and efficiency calibration of the Clover detectors have been done using radioactive $^{133}$Ba and $^{152}$Eu sources. The coincidence events were sorted into two symmetric $\gamma$-$\gamma$ matrices with time gates of 800 ns and 200 ns. These matrices are used to generate various background subtracted gated spectra. The matrices were analyzed using the codes INGASORT [23] and RADWARE [24]. To generate background subtracted gated spectra, INGASORT program has been used. In this program, the background is eliminated, on peak by peak basis.

A. Angular Correlation Data Analysis

Angle dependent asymmetric $\gamma$-$\gamma$ matrices have been generated to determine the multipolarity of $\gamma$-ray transitions from directional correlation of $\gamma$-rays emitted from excited oriented states (DCO) measurements. The DCO ratio ($R_{DCO}$) of a $\gamma$ transition ($\gamma_1$) is defined as the ratio of intensities of that $\gamma$-ray ($I_{\gamma_1}$) for two different angles in coincidence with another $\gamma$-ray ($\gamma_2$) of known multipolarity. It is given by the ratio, defined as

$$R_{DCO} = \frac{I_{\gamma_1} \text{ observed at } 40^\circ, \text{ gated by } \gamma_2 \text{ at } 90^\circ}{I_{\gamma_1} \text{ observed at } 90^\circ, \text{ gated by } \gamma_2 \text{ at } 40^\circ}$$

The DCO ratio of each $\gamma$ has been obtained by putting a gate on a $\gamma$ transition of known multipolarity with zero or very small mixing ratio (Fig. 1). For the stretched transitions with same multipolarity as the gating transition, $R_{DCO}$ value should be very close to unity. For different multiplicities of the gating and projected transitions, the $R_{DCO}$ value depends on the angle between the detectors and the amount of mixing present in the mixed multipolarity transition. For the assignment of spins and the $\gamma$-ray multipole mixing ratios ($\delta$), the experimental DCO values were compared with the theoretical values calculated by using the computer code ANGCOR [25]. Spin alignment parameter $\sigma / J = 0.3$ was used for this calculation. This choice was guided by several earlier works [26–29]. However, the choice has been also tested further. The experimental mixing ratio of 140 keV gamma determined in the present work has been used to calculate the
B. Polarization Data Analysis

Integrated polarization asymmetry measurements (IPDCO) have been done to determine the electric or magnetic nature of the $\gamma$-ray transitions. Two asymmetric IPDCO matrices were constructed from the data. The first (second) matrix named as parallel (perpendicular) was constructed having on first axis the simultaneous events recorded in the two crystals of the $90^\circ$ Clover detector which are parallel (perpendicular) to the emission plane and on the second axis the coincident $\gamma$ ray registered in any other detector. The polarization asymmetry is defined as

$$\Delta_{IPDCO} = \frac{a(E_\gamma)N_\perp - N_\parallel}{a(E_\gamma)N_\perp + N_\parallel}$$  \hspace{1cm} (1)$$

where $N_\perp$ and $N_\parallel$ are the intensities of the full energy peaks observed in the perpendicular and parallel matrices, respectively. The correction term $a(E_\gamma)$ is introduced due to asymmetry in the response of the different crystals of the Clover detector at $90^\circ$. It is defined as

$$a(E_\gamma) = \frac{N_\parallel(unpolarized)}{N_\perp(unpolarized)}.$$  \hspace{1cm} (2)$$

In the present experiment $a$ is measured as a function of energy of unpolarized $\gamma$-rays from radioactive $^{152}$Eu source. Fig.2 shows the variation of $a$ with $E_\gamma$ and it was fitted with the expression $a(E_\gamma) = a_0 + a_1 E_\gamma$ resulting into $a_0 = 1.03$ and $a_1 = -1.93 \times 10^{-5}$, where $E_\gamma$ is in keV.

For determination of experimental polarization asymmetry from each of the IPDCO matrices, we have put gates on $\gamma$s on the second axis and observed the projected parallel and perpendicular spectra of the $90^\circ$ Clover detectors (Fig.3). A positive (negative) value of $\Delta_{IPDCO}$ indicates a pure electric (magnetic) transition. But for mixed transitions, usually this value is close to zero and the sign varies depending on the extent of mixing.

C. Lifetime data analysis

In this experiment, the RF frequency is $\approx 5.56$ MHz, so the time difference between two consecutive RF pulses from the cyclotron is around 180 ns. Fig.4 shows the RF-$\gamma$ time-difference to amplitude converted (TAC) spectrum with $\gamma$ energies ranging from $\approx 100$ keV to 4000 keV. This TAC spectrum has been taken within a range of 200 ns. In this condition the resolution of the prompt time spectrum comes out to be $\approx 27.7$ ns without any restriction in energy.

III. RESULTS

A. Level Scheme

The level scheme of $^{153}$Ho, shown in Fig.5, has been established using the coincidence relationship, relative intensities, $R_{DCO}$ and $\Delta_{IPDCO}$ ratios of $\gamma$ rays. All transitions above 2772 keV ($31/2^+$) up to $E_\gamma \approx 12$ MeV and a tentative spin of $81/2^-$ reported in the earlier work by Radford et al. [19], have been observed in the present experiment. The transitions below the long-lived iso-

FIG. 2: (Color Online) Asymmetry parameter $a(E_\gamma)$ plotted as a function of energy. The fitted straight line is shown as dotted line in the figure.

K-shell internal conversion coefficient from BrIcc v2.3S Conversion Coefficient Calculator [20] which agrees reasonably well with the experimental K-shell electron conversion coefficient for this transition as reported in Fig. 2 of the reference [19] as well as in [10].

FIG. 3: (Color Online) Polarization symmetry for a few transitions in $^{153}$Ho. The dotted line separates the regions corresponding to pure Electric and Magnetic transitions.
mer at 2772 keV have also been observed and confirmed in this work (Fig. 6). However, in most of the cases their intensities could not be determined unambiguously. Thus the figure (Fig. 6) does not contain the intensities of these transitions.

Symmetric $\gamma - \gamma$ matrix has been used to place different gamma transitions in the level scheme. To assign the spins and parities of the levels, the conventional DCO and polarization measurements have been performed. The relative intensities of these transitions have been obtained from 913-keV gated spectrum and relative intensities are normalized with respect to the intensity of 363 keV transition. The relative intensities, experimental $R_{DCO}$ values, mixing ratios (for mixed transitions) and experimental polarization asymmetry values are listed in Table I. For transitions parallel to 913 keV transition, alternate gating transition has been considered to get intensities.

The intensities of transitions below the long-lived isomer at 2772 keV, are also determined from 200 ns time gated matrix. However, the ratios provided in Table I indicate that their intensities should increase by $\simeq 30\%$ if they were measured from a matrix generated with 800 ns time gate. As expected, the angular correlation data for transitions below the isomer have been found to be isotropic with gates on transitions above the isomer. The intensities of gamma rays below the isomer in asymmetric matrix were quite low. Moreover, the excitation pattern is fragmented and contains many gamma rays of similar energies as those above the isomer. So it was also not possible to do correlation measurements of these transitions with gates on transitions below the isomer.

The relative intensities of most of the gamma rays in $^{153}$Ho up to that of 466 keV gamma ray emitted from 7403 keV level, determined from the spectra gated by 913 keV gamma ray emitted from 2772 keV level, show good agreement with placements shown in Ref. [19]. However, the positions of 335 and 1001 keV gammas have been interchanged in the level scheme (Fig. 5) based on their intensities (Table I). The suggestion for reversal of the ordering of the 761- and 1042-keV transitions in Ref. [20] is not supported by the intensities of these transitions obtained in the present work. For low energy gammas like, 195 - , 140- keV transitions inclusion of internal conversion (IC) corrections have been important for proper placement. We have utilized IC coefficients obtained from online ICC calculator of BRICC [30].

Several isomers have been reported in $^{153}$Ho in the earlier works [10, 19]. The relative intensities in the present work also provide information regarding the existing isomers. As two gamma transitions of 631 and 633 keV exist in the level scheme above and below the 2772 keV isomeric level, the intensities in Table I have been quoted with respect to that of 363 keV transition. Therefore any fall in intensity of gammas decaying from levels above the previously reported $\simeq 500$ ps isomer at 4679 keV level could not be observed. Moreover, this half life is also not comparable to 200ns prompt window selected for the time-gated symmetric matrix to be manifested in the decrease of intensity. For the reported 3 ns isomer [19] at 7598 keV, the relative intensity of 195 keV gamma transition decaying from it is $\simeq 75\%$ compared to $\simeq 52\%$ intensity of the 1001 keV gamma transition feeding this level, supporting the possibility of an isomer. Similarly, it is found that (Table I) the feeding to the 9074 keV state is highly fragmented. The several weak gamma transitions (796 keV, 1126 keV, 1185 keV and 1528 keV) feed this level. The level sequence above this state is also highly irregular. In the earlier work a $\simeq 300$ ps isomer was reported at this level. This sudden change in excitation pattern can also indicate presence of a structure isomer.
FIG. 5: Partial level scheme of $^{153}$Ho.

at this level.

For most of the gamma rays, the DCO and polarization results satisfy earlier assignments [1, 19]. However, for a few, some differences have been noted. In Ref. [19], 455 keV transition has been indicated as a M1 transition. The DCO measurement in the present work (Fig. 1) results to a value of $R_{DCO} = 1.07(3)$ from 90° vs 40° asymmetric matrix when gated by an E2 transition. The theoretical $R_{DCO}$ values have been calculated by varying the mixing ratio to reproduce the experimental DCO ratio. This comparison indicated a large mixing ratio (E2/M1) (δ) 0.33(3) for this transition (Table I).

According to the present data analysis (Table I), 637 keV gamma transition is M1 in nature and the mixing ratio (δ) is 0.08, whereas it was previously mentioned [1, 19] as an E1 transition. The spin and parity assignments were not mentioned [1, 19] for 701, 376, 441 keV transitions. In the present work, it has been found that 701 and 376 keV transitions are of E2 character while 441 keV transition is M1 in nature. Previously, nature of 734 and 711 keV transitions were not specified [19]. Present measurement indicates that 734 keV and 711 keV transitions have E1 and E2 character, respectively (Table I).

B. Lifetime measurement

In Fig 7, RF-γ TACs correspond to, (i) 913 keV, a prompt gamma which decays from a state above the isomer at 2772 keV and (ii) the 533 keV gamma, which is emitted from a state just below the isomer. The decay curves clearly distinguish the two (Fig. 7).

The lifetimes of the isomers have been determined by comparing a sequence of gamma gated TAC spectra and fitting them with one, two or three component exponential decay curves. The prompt RF- TAC spectrum generated with gate on 913 keV transition corresponds to a resolution of 15.23 ± 0.23 ns. For 533 keV, decaying from the isomer through 36 keV, the half-life comes out to be $251^{+54}_{-38}$ ns (Fig. 7).

A few other isomers, apart from this long lived isomer, have been reported in the earlier work [16, 19]. It has
been discussed already that the relative intensities of the transitions in the coincidence spectra gated by a transition below the isomers have been utilized to reconfirm their presence qualitatively.

For confirming these isomers further [10, 19] and to search for new isomers, another technique has been adopted. It has been mentioned earlier that two time-gated symmetric matrices with 200 ns and 800 ns gates in the $\gamma$–$\gamma$ time spectra have been generated. In the Table II ratios of intensities of different $\gamma$s ($I_\gamma$) in 913 keV gated spectra from 200 ns and 800 ns time matrices are tabulated. It is demonstrated that for the $\gamma$’s below the long-lived isomer ($T_{1/2} > 200$ ns), this ratio is $\approx 0.66-0.78$ indicating their increased yield in the spectra generated from 800 ns matrix. On the other hand for prompt gammas, this ratio ranges from 0.94-0.98. However, at several parts of the level scheme, gamma rays of similar energies exist above and below the $\approx 200$ ns isomer. For example, the intensity ratios of 363 and 631 keV gammas can not be used for validating isomers as at least two gamma rays of similar energies are emitted from levels above and below the long-lived isomer. Systematic comparison of these ratios also indicates existence of a relatively longer lived isomer ($>300$ ps as reported in [19] at 9074 keV). For

FIG. 6: Partial level scheme of $^{153}$Ho below the isomer $31/2^+$ at 2772 keV.

FIG. 7: (Color Online) Determination of (a) (top) prompt resolution of the TAC spectrum by fitting the decay curve of prompt 913 keV transition decaying from 3685 keV level, and (b) (bottom) the lifetime of the isomeric level at 2772 keV from the RF–$\gamma$ TAC spectra gated by 533 keV transition. The logarithmic (In) values of counts (to the base e) are plotted against time (ns) for ease of fitting.
TABLE I: Relative Intensity ($I_{rel}$), $R_{DCO}$, $\Delta IPDCO$ and the mixing ratio ($\delta$) of the $\gamma$ transitions in $^{153}$Ho.

| $E_{\gamma}$(keV) | $J_i^*$ | $J_f^*$ | $I_{rel}$ | $E_{gate}$(keV) | $R_{DCO}$ | $\delta$ | $\Delta IPDCO$ |
|-----------------|---------|---------|-----------|----------------|-----------|---------|---------------|
| 140             | 67/2−   | 65/2−   | 32.72     | 913[E2]       | 1.21(9)   | 0.20$^{+0.09}_{-0.05}$ |               |
| 195             | 61/2+   | 57/2+   | 75.38     | 913[E2]       | 0.96(3)   | E2      | 0.11(1)      |
| 233a            | 25/2−   | 23/2−   | 6.80      |               |           |         |               |
| 252a            | 23/2−   | 23/2−   | 4.66      |               |           |         |               |
| 287a            | 19/2+   | (19/2)− | 4.82      |               |           |         |               |
| 305             | 49/2(+) | 47/2(+) | 7.97      | 913[E2]       | 1.38(8)   | 0.11$^{+0.11}_{-0.05}$ | -0.15(1)      |
| 335             | 65/2−   | 63/2−   | 48.45     | 913[E2]       | 1.39(6)   | 0.10(3) | -0.03(3)     |
| 343             | 71/2−   | 69/2−   | 11.05     | 761[E2]       | 1.24(3)   | 0.19$^{+0.09}_{-0.08}$ | -0.07(3)      |
| 363             | 43/2+   | 39/2+   | 100       | 913[E2]       | 0.86(2)   | E2      | 0.18(3)      |
| 376             | 79/2(−) | 75/2(−) | 19.12     | 913[E2]       | 0.89(8)   | E2      | 0.04(8)      |
| 378a            | 27/2+   | 25/2−   | 22.17     |               |           |         |               |
| 389             | 69/2−   | 67/2−   | 9.68      | 466[E2]       | 1.27(4)   | 0.17$^{+0.11}_{-0.08}$ | -0.02(3)      |
| 424a            | 27/2−   | 23/2−   | 5.91      |               |           |         |               |
| 441a            | 81/2(−) | 79/2(−) | 18.04     | 913[E2]       | 1.59(3)   | -0.01(1) | -0.05(1)     |
| 455             | 45/2+   | 43/2+   | 88.68     | 913[E2]       | 1.07(3)   | 0.33(3) | 0.09(2)      |
| 466             | 57/2+   | 53/2+   | 71.06     | 913[E2]       | 0.84(3)   | E2      | 0.08(2)      |
| 497             | 51/2(+) | 49/2(+) | 2.44      | 637[M1]       | 0.79(4)   | 0.3(5)  | -0.01(5)     |
| 515a            | 15/2+   | 15/2−   | 13.87     |               |           |         |               |
| 533a            | 27/2+   | 23/2+   | 33.12     |               |           |         |               |
| 557a            | 23/2+   | 19/2+   | 52.35     |               |           |         |               |
| 576a            | 15/2−   | 11/2+   | 36.15     |               |           |         |               |
| 631             | 39/2+   | 35/2+   | 113.08    | 913[E2]       | 0.91(3)   | E2      | 0.02(2)      |
| 637             | 47/2(+) | 45/2+   | 14.13     | 913[E2]       | 1.43(1)   | 0.08$^{+0.12}_{-0.09}$ | -0.17(8)      |
| 666a            | 23/2−   | 19/2−   | 8.31      |               |           |         |               |
| 701             | 75/2(−) | 71/2−   | 12.03     | 913[E2]       | 1.03(1)   | E2      | 0.10(8)      |
| 711             | 73/2−   | 69/2−   | 7.87      | 1001[E1]      | 0.62(7)   | E2      | 0.03(8)      |
| 727a            | 15/2−   | 11/2−   | 11.99     |               |           |         |               |
| 732             | 71/2−   | 67/2−   | 3.65      | 1001[E1]      | 0.31(2)   | E2      |               |
| 734             | 53/2(−) | 51/2(+) | 2.93      | 637[M1]       | 0.85(3)   | E1      | 0.42(4)      |
| 761             | 53/2+   | 49/2+   | 76.77     | 913[E2]       | 0.89(3)   | E2      | 0.01(2)      |
| 783a            | (19/2)− | 15/2−   | 3.86      |               |           |         |               |
| 796             | 67/2−   | 67/2−   | 9.68      | 913[E2]       | 1.11(8)   | -0.03(2) |               |
| 918a            | 23/2−   | 19/2−   | 8.26      |               |           |         |               |
| 1001            | 63/2−   | 61/2+   | 51.79     | 913[E2]       | 1.24(7)   | 0.19$^{+0.05}_{-0.04}$ | 0.05(2)      |
| 1042            | 49/2+   | 45/2+   | 77.27     | 913[E2]       | 0.99(3)   | E2      | 0.02(3)      |
| 1126            | 69/2−   | 67/2−   | 9.02      | 1001[E1]      | 1.02(4)   | 0.17     | -0.11(1)     |
| 1185            | 69/2−   | 67/2−   | 1.89      |               |           |         |               |
| 1325            | 69/2−   | 65/2−   | 4.60      | 1001[E1]      | 0.56(9)   | E2      | 0.02(8)      |
| 1476            | 67/2−   | 61/2+   | 5.74      |               |           |         |               |
| 1528            | 71/2−   | 67/2−   | 6.09      | 1001[E1]      | 0.42(7)   | E2      |               |

*aBelow the isomer

bMay contain contribution from 442 keV (53/2(+) → 49/2(+) )

As reported in [19].
TABLE II: Ratio of intensity ($I_γ$) in 913 keV gated spectra from 200ns and 800ns time matrices for $γ$ - ray transitions in $^{153}$Ho. Gammas below and above the long -lived isomer have been indicated as $L$ and $U$, respectively. The 195 keV emitted from the 3 ns isomer at 7598 keV is indicated by $I$.

The 140 keV gamma from the new isomer has been marked as $N$. $U_{140}$ indicates gammas emitted from states above 9074 keV state in coincidence with 140keV gamma decaying from this state.

| $E_γ$(keV) | Intensity($I_γ$) | Ratio of $I_γ$s ($200$ns/$800$ns) | Comment |
|------------|------------------|-----------------------------------|---------|
| 378        | 8566             | 10352                             | 0.83    | L+U          |
| 701        | 6946             | 7564                              | 0.92    | U140         |
| 711        | 5074             | 5973                              | 0.85    | SU140        |
| 343        | 5163             | 5541                              | 0.93    | U140         |
| 389        | 4045             | 4609                              | 0.88    | U140         |
| 734        | 1986             | 2260                              | 0.88    | 732 at U140  |
| 1528       | 1691             | 1941                              | 0.87    | U140         |
| 796        | 6005             | 6453                              | 0.83    | U140         |
| 1185       | 1392             | 1716                              | 0.81    | U140         |
| 1126       | 5578             | 6230                              | 0.89    | U140         |
| 1326       | 2614             | 2771                              | 0.94    | U140 but bypass 140 |
| 1476       | 1845             | 2240                              | 0.84    | U140         |
| 140        | 9164             | 10457                             | 0.87    | U            |
| 1001       | 31392            | 32872                             | 0.95    | U            |
| 335        | 24849            | 26834                             | 0.93    | U            |
| 195        | 35258            | 37697                             | 0.96    | I            |
| 497        | 1912             | 2210                              | 0.89    | U            |
| 305        | 5512             | 5629                              | 0.98    | U            |
| 442        | 8671             | 9848                              | 0.88    | U+U140       |
| 637        | 6597             | 7019                              | 0.94    | U            |
| 466        | 38900            | 41260                             | 0.94    | U            |
| 761        | 41609            | 43533                             | 0.95    | U            |
| 1042       | 46346            | 49211                             | 0.94    | U            |
| 455        | 48853            | 52600                             | 0.93    | U            |
| 363        | 56562            | 62031                             | 0.91    | U+L          |
| 631        | 56652            | 62900                             | 0.90    | L+U          |
| 475        | 1377             | 1964                              | 0.70    | L            |
| 533        | 10269            | 13303                             | 0.77    | L            |
| 557        | 15669            | 21518                             | 0.73    | L            |
| 515        | 3988             | 5409                              | 0.74    | L            |
| 576        | 11913            | 17014                             | 0.70    | L            |
| 439        | 9268             | 11931                             | 0.78    | L            |
| 424        | 2394             | 3152                              | 0.76    | L            |
| 233        | 3001             | 4509                              | 0.66    | L            |
| 252        | 1706             | 7043                              | 0.72    | L            |
| 918        | 2449             | 3402                              | 0.72    | L            |
| 666        | 3017             | 4246                              | 0.71    | L            |
| 727        | 4109             | 5426                              | 0.76    | L            |

FIG. 8: (Color Online) The alignment plots for (a) positive and (b) negative parity states in $^{153}$Ho

IV. DISCUSSIONS AND THEORETICAL CALCULATIONS

The structure of this nucleus poses a few interesting features as discussed below.

- Several isomers have been reported earlier in the level scheme. They are similar to the yrast traps observed frequently in the nuclei in this mass region. The study of these isomers will be useful to follow the evolution of the structure of this nucleus with increasing excitation energy and spin.

- The negative parity states are distributed unevenly in the scheme. After 27/2$^-$, the next negative parity states are observed at spin 53/2$^-$ and then at spin 63/2$^-$. These large gaps in spins for negative parity states are interesting and need special attention.

- For the positive parity 49/2$^+$ and 53/2$^+$ levels, the preferred deexcitation path includes the non-yrast
49/2^+ and 53/2^+, which contradicts normal expectation. This needs special attention and theoretical interpretation.

- The level scheme apparently is not regular and it has been emphasized earlier that the non-collective character of the motion manifests itself in this irregularity. However, in neighboring Dy isotopes, non-yrast collective states have been found to coexist with non-collective yrast states. So whether Ho also follows this trend or not needs to be understood phenomenologically as well as utilizing reliable theory.

The alignment plot of the positive and negative parity bands are shown in Fig. 5. Here \( J_x \) has been calculated using the expression \( J_x = \sqrt{(J(J+1) - K^2)} \). Positive parity band has been plotted taking \( K=7/2 \), whereas for negative parity band, \( K=5/2 \) has been considered. These figures are useful to reveal the structural features.

- For the positive parity states, up to 45/2^+ state the excitation pattern consists of several twists and turns indicating several changes in structure. Interestingly most of the bends are associated with isomers. The longest life isomer is at the first bend (\( J_x \approx 15.5 \)).

- At 45/2^+, the excitation scheme bifurcates, the yrast branch is irregular indicating single particle alignment, whereas the non-yrast one is more regular and probably signify collective structure.

- The negative parity states show smooth increase in alignment till 23/2^-\( \). Beyond that the alignment is irregular, indicating single particle mode of angular momentum generation.

- A large discontinuity in alignment is observed beyond 27/2^-\( \) state. The next negative parity state has a spin 53/2^(-), about 13 units of angular momentum is gained.

The structure of this nucleus has been studied theoretically using two models. The issue of searching for energetically most stable shape at each angular momentum has been understood by calculating total Routhian surfaces \[31\] at different angular momenta. Moreover, a version of Particle Rotor Model \[32, 33\] has been utilized to calculate energies and transition probabilities of specific states.

### A. TRS Calculations

The Total Routhian Surface (TRS) calculations \[31\] with a Woods - Saxon potential and monopole pairing predict a near-prolate deformation of \( \beta_2 \approx 0.20 \) for this nucleus. As observed in Figs. \[9, 10\] at low frequencies with \( \hbar \omega = 0.20 - 0.24 \) MeV, both the negative and positive parity states favor prolate deformation. However, at \( \hbar \omega = 0.25 \) MeV, shown for positive parity states (Fig. \[9\]), there is a sudden transition to oblate shape as the most favored one. At higher rotational frequencies, the oblate shape remains energetically favored for both the parities (Figs. \[9, 10\]). This can be associated with the rotation alignment of a pair of \( 1_{13/2} \) neutrons, increasing the angular momentum from \( J \approx 9 \) to 21, over a small rotational frequency range of 0.25 to 0.28 MeV, contributing 12 units of angular momentum. These observations have been important while extending our studies using PRM.

The presence of several positive parity single particle states near the Fermi level smoothens shape transition and shape coexistence is observed (Fig. \[9\]). However, for negative parity, single particle orbitals arising only from intruder \( 1h_{11/2} \) state are available. The shape transition therefore is drastic here and it also possibly leads to large gap in the available negative parity states in the spectrum in the band crossing region as predicted by TRS calculations (Fig. \[10\]). There may be other negative parity states in the spin gap which has almost no overlap with higher spin negative parity states. Possibly they are not populated in the present heavy-ion fusion evaporation reaction for this reason.

### B. PRM Calculations

To calculate specific energies and transition probabilities of different levels, PRM calculations have been done. The particular version of the model \[32, 33\] is discussed below.

#### 1. Formalism

The model is based on the assumption that the nucleus under consideration is axially symmetric. In this model, the motion of an unpaired quasiparticle in a Nilsson deformed orbit is coupled to the rotational motion of the core through Coriolis interaction. We have used a version of the PRM \[32, 33\] in which the experimental core energies can be fed directly as input parameters.

The Hamiltonian of the odd-A system can be written as

\[
H = H_{0p}^o + cR.j + E_c(R) \tag{3}
\]

The first term is the Hamiltonian of a single quasiparticle. The quasiparticle (quasiproton, in the present case) is assumed to be moving in an axially symmetric Nilsson potential under the influence of BCS pairing. The pairing gap and the Fermi level are represented by \( \Delta \) and \( \lambda \) respectively.

The total Hamiltonian is then diagonalize, giving the energy eigenvalues and the wave functions of the final
states $|JM>$ in terms of the Coriolis mixing amplitudes $f_{JK}$ and the basis states $|JMK>$:

$$|JM> = \sum_K f_{JK} |JMK>$$

(4)

In the present version of the model [33], to identify the rotational composition of the final state $|JM>$, these states are expanded in terms of states with sharp $R$ and $j$:

$$|JM> = \sum_{jR} \sum_K f_{JK} \alpha_{jR}^{(K)} |JMjR>$$

(5)
So to calculate a state with total angular momentum $J$, where the single-particle angular momentum involved is $j$ (say), the experimental core energies required will be given by the following range of $R$ values:

$$R_{\text{max}} = J + j,$$  \hspace{1cm} (7)  

$$R_{\text{min}} = J - j.$$  \hspace{1cm} (8)

where

$$\alpha_{J,R}^{(K)} = \sqrt{2} \begin{bmatrix} J & j & R \\ K & -K & 0 \end{bmatrix}$$ \hspace{1cm} (6)
2. Parameter choice

There are several parameters involved in the PRM calculations. The single-particle Nilsson parameters $\mu$ and $\kappa$ (=$0.5920$ and $0.065$, respectively), have been deduced from the expression provided by Nilsson et al. \cite{31}. The deformation parameter for the odd nucleus is chosen from the systematics of the experimentally deduced values in the neighboring even isotopes \cite{32}, which agrees quite well with the deformation ($\beta_2$) obtained from the TRS calculations (Figs. 9 and 10). In our calculations, $\delta(=0.95/\beta_2)=\pm0.14$. The pairing gap $\Delta$ is deduced from the experimental odd-even mass difference ($\Delta_{o-e}$) calculated from mass data \cite{35}. In our case, the experimental odd-even mass difference $\Delta_{o-e}=1.5$ MeV. For positive (negative) parity states, all Nilsson single quasiparticle states from $N=4$ ($N=5$) have been considered. We have selected the Fermi level $\lambda$ in such a way that it remains consistent with the observed ground state spin. We have calculated the energy spectra with the Fermi level at $\lambda=43.8$ MeV, for both positive and negative parity states. For positive parity, it lies close to $3/2^+[411]$ and $3/2^+[422]$ orbitals originating primarily from $2d_{5/2}$ and $1g_{7/2}$ spherical states, respectively. The Fermi level is near $5/2^-[532]$ Nilsson orbital originating from $1h_{11/2}$ for negative parity states.

Here, the only adjustable parameter is Coriolis attenuation coefficient $\alpha$. For $^{153}$Ho nucleus, $\alpha$ is taken to be 0.90 and 0.99 for positive and negative parity bands, respectively. While calculating transition probabilities, for electric transitions effective charge $e_{eff}=1.0$ have been used. Intrinsic quadrupole moment ($Q_0$) is necessary for these calculations has been provided from experimental quadrupole moment of $^{152}$Dy \cite{32}. For magnetic transitions, free values of $g_l$ and $g_s$ have been used.

3. Choice of Core Energies

The core energies are usually taken as the excitation energies of the yrast band of the neighboring even isotope. The $^{154}$Ho nucleus is represented as an odd proton coupled to a $^{152}$Dy core. In the present work, the level energies of the yrast spectra of $^{152}$Dy are used as core energies.

While choosing the yrast spectra of Dy shown in Fig. 11 it is found that it has an interesting feature, the core possesses rotational as well as vibrational character \cite{36}. Although at certain angular momentum the vibrational core states are yrast, the states populated via heavy ion fusion evaporation reaction also deexcites via nonyrast states which are members of the rotational band \cite{37}. Two parallel branches of excitation scheme are observed, mildly collective and non-collective states are prevalent at low spins, at higher spins the coexisting non yrast states show collective behavior \cite{36}. This is similar to the parallel branches observed in $^{153}$Ho (Fig. 5) for spins $49/2^+$ and $53/2^+$ (Fig. 8).

The TRS calculations have been useful in this regard suggesting shape coexistence at certain angular frequency in $^{153}$Ho. These results suggested that the alignment of $i_{13/2}$ neutrons are responsible for shape changes in $^{153}$Ho. So the shape coexistence observed in even- proton nucleus $^{152}$Dy \cite{36} caused by neutron alignment is also manifested in neighboring odd-proton $^{153}$Ho. Usually, the PRM calculations are not supposed to reproduce alignment effects. However, it has been demonstrated earlier \cite{36} that, if the backbending (bb) is observed in the core for neutron (proton) alignment, for neighboring odd-pro-
ton (neutron) nucleus, the backbending is reproduced using the present version of PRM provided the input core states included the backbending. This shape transition can be reproduced theoretically by this version of PRM, provided the proper core states are included in the calculation.

Therefore, spectra for $^{153}$Ho have been calculated considering only yrast states of the core (indicated as yrast) and both the yrast states and non yrast rotational states of the core, indicated as newcore (Fig. 11). Calculations were performed with prolate as well as oblate deformation for the single particle Nilsson potential (Figs. 12, 13). In Table II, Table IV and Table V, the results for the single particle Nilsson potential (Figs. 12, 13) have been compared with the experimental yrast states including non yrast 49/2$^+_2$ and 53/2$^+_2$.

1. **Excitation energies as a function of spin (with normalization at 33/2$^+$) and transition probabilities**

- For yrast core, the strongly populated 49/2$^+_2$ and 53/2$^+_2$ states (the decay out gamma transition energies are 761 and 1042 keV, respectively) are non yrast positive parity states (Fig. 5). It has been found that the high spin yrast positive parity states up to 49/2$^+_1$ state have better agreement with oblate structure (Fig. 13a). Although the prolate option also reproduces the results well. Theoretical energies for 53/2$^+_2$, 59/2$^+_1$ and 61/2$^+_1$ states deviate from experimental values. However, the $B(E2)$ value extracted from the lifetime of 61/2$^+_1$ state decaying by 195 keV gamma is reproduced well by the prolate option of this calculation (Table III).

- In new-core calculation (Fig. 13b), in which some of the non-yrast core states are taken as input, theoretical energies for 49/2$^+_1$ and 53/2$^+_1$ states have been compared with the experimental 49/2$^+_2$ and 53/2$^+_2$ states. The agreement is reasonably better till 39/2$^+$ with oblate deformation. For 43/2$^+$, 45/2$^+$ and 47/2$^+$ states, prolate deformation is favored. Both prolate and oblate deformation give almost similar agreement for 49/2$^+_2$ and 53/2$^+_2$. The calculated energy for 59/2$^+_1$ with prolate deformation agrees quite well to experiment. The lifetimes of the 31/2$^+$ and 43/2$^+$ states decaying by 36 keV and 363 keV gammas, respectively, are reproduced best by the prolate and oblate deformation (Table III). So the theoretical results also (Table III) indicate that the 31/2$^+$ state is a long-lived isomer. For 61/2$^+_1$ state decaying by 195 keV gamma, results (energies as well as lifetimes) for both deformation worsens further compared to yrast core (Fig. 13).

C. **Positive-parity states**

We have shown the results for the positive parity states in Fig. 12. It is noted that above 23/2$^+$, all states are somewhat over predicted with both the cores. For lower spin states, results for both prolate and oblate structure are almost same for both the cores.

However, we found that only if the states above the isomeric state 31/2$^+$ are isolated from the set below, and the results are normalized to the energy of 33/2$^+$ state, the normalized results match reasonably well with the experimental data (Fig. 13). This indicates structural differences between states above and below the isomer. Specific deformations which give best agreement at different spins have been discussed below in detail.

![Energy vs angular momentum plots for positive parity states](attachment:image.png)

**FIG. 13:** (Color Online) The energy vs angular momentum plots for the positive parity states for different shape options. a) Results from calculations with yrast core compared with experimental yrast states. (b) Results from calculations with newcore compared with experimental states including nonyrast 49/2$^+_2$ and 53/2$^+_2$.
TABLE III: Comparison of experimental and theoretical reduced transition probabilities (B(E2)s in (e^2b^2)). For experimental quantities, corresponding errors are included within parentheses. The average core angular momentum (R) corresponding to each state has also been tabulated for different options.

| E_x (keV) | J_i | J_f | E_γ (keV) | T_{1/2} (ns) | B(E2) \( (e^2b^2) \) | Prolate | Oblate | Prolate | Oblate |
|-----------|-----|-----|-----------|-------------|-----------------|--------|--------|--------|--------|
| 2772      | 31/2^+ | 27/2^+ | 36        | 251 ± 34    | 0.015(3)        | 0.0003 | 16.1(12.7) | 0.0002 | 17.0(11.6) |
| 4679      | 43/2^+ | 39/2^+ | 363       | 0.5(2)      | 0.02(1)         | 0.106  | 21.9(20.1) | 0.007  | 22.9(20.4) |
| 7598      | 61/2^+ | 57/2^+ | 195       | 2.95(15)    | 0.052(3)        | 0.038  | 31.4(28.7) | 0.001  | 30.7(30.5) |
| 9074      | 67/2^- | 65/2^- | 140^c     | 0.3(1)      | 0.07(2)         | 0.001  | 33(32)    | 0.002  | 33(32)    |

*extracted from experimental half-lives.

From present work.

M1+E2 transition, mixing ratio =0.2

TABLE IV: Comparison of experimental and theoretical branching ratios from calculations with newcore for 57/2^+ state to investigate the reason behind strong decay path through non-yrast states. Note that in this set, as some of the core states are non-yrast, experimental non-yrast 53/2 state is equivalent to theoretical yrast state. See text for details.

| E_x (keV) | J_i | J_f | E_γ | Exp | New core |
|-----------|-----|-----|-----|-----|----------|
| 7403 57/2^+ | 53/2^+ \((\text{theory}) (53/2^+ \text{\textit{exp}})) | 466 | 100 | ≃100 | ≃100 |
| 7403 57/2^+ | 53/2^+ \((\text{theory}) (53/2^+ \text{\textit{exp}})) | 885 | Not observed | 2.07×10^{-5} | 7.13×10^{-5} |

- The contribution of the different core states in the composition of the states of interest in \(^{153}\text{Ho}\) are tabulated in (Table III). The average core angular momentum value gives an idea about the most relevant core states for a particular state in the odd-A nucleus. As expected, unlike that for negative parity states (which only originate from intruder orbital \(1h_{11/2}\) of \(N=5\)), for positive parity, this value changes for different options of deformation. This is due to the fact that the most important lowest energy positive parity single quasiparticle Nilsson states originate from all states of \(N=4\), except, \(1g_{9/2}\).

- While calculating the branching of the decay of 7403 keV \((57/2^+ \text{\textit{exp}})\) level in the calculation with new core, the branching to the 6937 keV \((53/2^+ \text{\textit{exp}})\) is 100% with extremely small branch populating 6518 keV \((53/2^+ \text{\textit{exp}})\) state (Table IV). This justifies the decay of the yrast state at 7403 keV to the non-yrast state 6937 keV \((53/2^+ \text{\textit{exp}})\).

- For the branch parallel to the main excitation scheme, consisting of \(49/2^+_1\), \(47/2^+_1\), \(51/2^+_1\), \(53/2^+_1\) etc the yrast states in the calculations with yrast core, reproduce the mixing ratios reasonably well (Table V). Interestingly, the large mixing of 455 keV gamma decaying from \(45/2^+\) state has been reproduced by the prolate deformation option of the yrast core calculations.

FIG. 14: (Color Online) Comparison of experimental and theoretical energies of negative parity states in \(^{153}\text{Ho}\).
D. Excitation energies and transition probabilities of Negative-parity states

For negative parity levels, it is found that for lower spins the nucleus is oblate, which changes to an oblate structure at higher spins (Fig. 14) as expected from TRS calculations. For the PRM calculations, the choices of yrast or non-yrast cores do not make much difference in the results for these states. Only at the highest spins (81/2), prolate option appears to reproduce the energy better. For negative parity states, single quasiparticle Nilsson states near Fermi level originate only from intruder $1h_{11/2}$ ($N=5$), leading to relatively less fluctuations in core angular momentum for different options (Table III). Thereby making the results less sensitive to the change in the choice of core.

In the earlier work [19], a $\approx 300$ ps isomer has been reported at 9074 keV state decaying by 140 keV gamma, corresponding to a B(E2) value of 0.07 (2) $e^2 fm^4$ ($\approx 659$ $e^2 fm^4$) (Table III) extracted utilizing the experimental mixing ratio determined in the present work (Table I). None of the calculations reproduce this B(E2) value. The predicted values are orders of magnitude smaller than the experimental value. These theoretical values correspond to longer half life for this state, ranging from around 10 ns (B(E2) = 19.8 $e^2 fm^4$) to 30 ns (B(E2) = 6.6 $e^2 fm^4$). In the present work, definite experimental evidences have been discussed (Section IIIIB) to indicate a longer half life ($\approx 50$ ns) for this state. The theoretical results also support this estimation from experimental observations.

V. SUMMARY AND CONCLUSION

The high-spin states in $^{153}$Ho, have been populated by $^{139}$La($^{20}$Ne, 6n) reaction at a projectile energy of 139 MeV at Variable Energy Cyclotron Centre (VECC), Kolkata, India. An earlier campaign of Indian National Gamma Array (INGA) setup has been utilized in this experiment. Data from gamma-gamma coincidence, directional correlation and polarization measurements have been analyzed to assign and confirm the spins and parities of the levels. We have also utilized the RF-gamma time spectrum to investigate the isomers in the excitation spectrum. A few additions and revisions of the reported level scheme of $^{153}$Ho have been suggested. Lifetime of a high spin isomer has been suggested to be longer than the earlier result.

The alignment plots of the excitation spectra have been useful to understand the different modes of excitation phenomenologically. Theoretical calculations, both TRS and PRM have provided additional microscopic insight.

The regularity in the increase in alignment with angular frequency for low energies for both positive and negative parity states indicate mild collectivity. This is also supported by TRS results showing prolate minimum for low angular frequency. Although PRM calculations indicate similar agreement with energy values for both prolate and oblate options, the isomer lifetime at 31/2$^+$ is best reproduced by a prolate deformation.

For positive parity this regularity is repeated till relatively higher spins with turns in between which may indicate intrinsic configuration changes. The PRM calculations reproduce 43/2$^+$ lifetime by an oblate option and the 61/2$^+$ lifetime by a prolate option.

The shape coexistence is clearly manifested in positive parity alignment plot with a smooth branch (non-yrast) and a irregular branch (yrast) beyond 45/2$^+$. This is supported by PRM. The results for prolate deformation with newcore agree well with the non-yrast branch. This shape coexistence is also observed in TRS plots.

The higher spin negative parity states are erratic as shown in alignment plot. It also agrees well with PRM results for oblate deformation and supported by oblate minimum in TRS plot.

Future investigations are needed to improve the lifetime measurements to confirm the conclusions from this work on firmer ground.

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