Magnetoresistance of double layer hybrid system in tilted magnetic field

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I. INTRODUCTION

Magnetoresistance [1], the change of a material’s resistivity in the presence of external magnetic field, has been of interest both as a tool to prob the fundamental properties of an electronic material and for technological applications. Classically, the magnetoresistivity effect depends on both the strength of the magnetic field and the relative direction of the magnetic field with respect to the current due to the Lorentz force. For non-magnetic metals, magnetoresistivity effects at low magnetic fields are very small, although the effect can become quite large for high magnetic fields.

The effect of a parallel magnetic field on the conductivity of a gated two-dimensional electron gas (2DEG) has been investigated [2] and a huge rise of resistivity close to metal-to-insulator transition and the positive differential magnetoresistance have been reported. It has been shown that the metallic-like conductivity of 2DEG first decreases with increase of in-plane magnetic field and then saturates to a new constant value when electrons become fully polarized. This effect can be understood by the Zeeman coupling and the reduction of screening of charge impurities in a polarized Fermi liquid system.

Transport properties of graphene, a one atom thick layer of carbon atoms arranging in a honeycomb lattice [3, 4], in the presence of a magnetic field is one of the interesting subjects in graphene physics both from academic studies and potential applications. Under a perpendicular component of a magnetic field, the low-energy dispersion relation in graphene leads to unique Landau levels [5] which the filling factor \( \nu \) appears at the zero-energy Landau level, presents something new in quantum Hall physics. Magnetoresistance of the insulating state that forms at the charge neutrality point in the presence of tilted magnetic field has been measured and data indicated that the \( \nu = 0 \) quantum Hall state in single layer graphene is not spin-polarized [7].

Magnetoresistance of graphene in the presence of parallel magnetic field has been studied by Hwang and Das Sarma within the Boltzmann equation [8]. In the presence of the in plane magnetic field, charge carriers of graphene will spin polarized and the effect of magnetic vector potential is negligible owing to one atom thickness of graphene. The authors showed that the applied magnetic field gives rise to an increasing of the resistivity of graphene up to a saturation field where all electrons of the conduction band are spin polarized. The magnetic field beyond saturation field excites electrons from the valence band to the conduction band and leads to a negative differential magnetoresistance. These behaviors are in contrast to those obtained in conventional 2DEG systems in which the resistivity increases up to a certain magnetic field and then saturates [9–11]. Practically, it is difficult to reach to the saturation magnetic field of graphene. The saturate field is indeed proportional to the electron density, \( B_S \sim 140 \sqrt{n} \) where \( n \) is in units of \( 10^{10} \text{cm}^{-2} \) and \( B_S \) is scaled by units of Tesla. Although, decreasing the density of carriers of graphene is feasible in experiment [12], one has to use an extremely clean and pristine sample to meet the required necessary condition in which impurity density is being much less than the charge carrier density to avoid any localization regime.

If the magnetic field has a small deviation from sample’s plane, in addition to the spin polarization of conduction electrons, it gives rise to a Hall effect. Tilting the magnetic field has been shown to be a straightforward tool to disentangled spin and orbit effects in 2DEG systems [13]. If the deviation angle of the magnetic field from sample’s plane consider to be very small, the perpendicular part of magnetic field will be so small and as a result one can assume the dispersion doesn’t change. In such a situation, there is an ordinary Hall effect in contrast to a strong perpendicular magnetic field which generates the Landau levels. The longitudinal resistivity and Hall coefficient of conventional 2DEG have been investigated experimentally [14] and theoretically [11] in a slightly tilted magnetic field.

Assembling graphene, on the other hand, with various 2D layers into artificial heterostructures to explore novel
or tailed properties has been proposed [15] and realized in tunneling effect transistor [16]. Hybridizing a gapless graphene layer with another gapless graphene layer makes the system decoupled layer graphene (DLG) [17] which both layers are chiral and differs with a situation in which gapless graphene is assembled by a 2DEG sample (G-2DEG) [18] which is a chiral-non chiral hybrid system. This structure makes it possible to study transport properties of the graphene layer according to an interlayer electron-electron interaction.

In this paper, we consider a single layer graphene interacting with either another layer of graphene (DLG) or 2DEG (G-2DEG) to explore the effect of the interlayer interaction on the magnetoresistances. The system is addressed by the presence of the tilted magnetic field, assuming the magnetic field is slightly off the electronic planes so that there is a weak perpendicular component. The longitudinal resistivity and Hall coefficient are explored and the results are compared with that of a single layer graphene. We also analyze the dependence of the dielectric material which is filled the space between two layers as well as the distance between layers on the magnetoresistance and Hall coefficient and show that the interlayer interaction plays a vital role even at longer distances by using slightly a strong dielectric material between two layers. The angle dependence of the Hall coefficient is obtained and we show that a quite large Hall resistivity occurs in graphene for certain values of the carrier density and screened interaction. This particular result is in contrast to that obtained in a 2DEG system where the Hall coefficient increases up to \( \simeq 30\% \).

The paper is organized as follows. In section II we describe a model Hamiltonian of the double layer system, including inter- and intralayer interactions and then derive the conductivity and Hall coefficient in a tilted magnetic field by using a semiclassical approach. We also discuss the Boltzmann conductivity by using long-range charged impurities within Random Phase Approximation (RPA) dielectric function. Section III is devoted to our numerical results of the longitudinal conductivity and Hall coefficient of double graphene-graphene or graphene-2DEG structures in the presence of the tilted magnetic field where we show explicitly the difference of the two assembled structure. Finally, a brief summary of results is given in Section IV.

II. MODEL AND THEORY

We consider a double layer structure incorporating a graphene layer placed on another two-dimensional layer with a distance \( d \). A schematic structure is shown in Fig. 1, where the second layer can be chiral (another graphene layer) or non-chiral (2DEG) layer. We assume each layer being zero thickness in the direction normal to the plane of the system at zero temperature. The layers are separated by a dielectric material (shown in Fig. 1) with a dielectric constant \( \epsilon_2 \) and we suppose that the tunneling of electrons between layers are negligible, however the Coulomb interlayer interaction plays role in the system. The Hamiltonian of such a system can be written as [17]

\[
\hat{H} = \hbar v_F \sum_{k,\alpha,\beta} \psi_{k,\alpha}^\dagger (\sigma_{\alpha\beta}k) \psi_{k,\beta} + \hat{T}_2 \\
+ \frac{1}{2\varepsilon} \sum_{q\neq0,l,l'} V_{l\ell}(q) \hat{\rho}_l(q) \hat{\rho}_l(-q)
\]

where \( \alpha, \beta \) are pseudospin indexes in the \( x \) and \( y \) directions, \( l = 1(2) \) is the layer index, \( v_F \simeq 10^6 m/s \) is the Fermi velocity of graphene and \( \hat{\rho}_l(q) \) denotes the density operator in layer \( l \) with the momenta \( q \). \( \hat{T}_2 \) denotes the kinetic energy of the second layer which is the same as the Dirac equation for a DLG system, while it is \( \psi_{k,\beta}^\dagger \hbar^2 k^2 / 2m^* \psi_{k,\beta} \) in the case of a G-2DEG hybrid [18]

where \( m^* \) is the electron band mass. The last term \( V_{l\ell}(q) \), refers to the inter- and intralayer electron-electron Coulomb interactions can be obtained by using electrostatic relations for a two parallel conducting system [17]. The intralayer interaction is given by

\[
V_{11}(q) = \frac{4\pi e^2}{qD(q)} [(\epsilon_2 + \epsilon_3) e^{qd} + (\epsilon_2 - \epsilon_3) e^{-qd}] 
\]

where

\[
D(q) = \sqrt{[(\epsilon_1 + \epsilon_2)(\epsilon_2 + \epsilon_3) e^{qd} + (\epsilon_1 - \epsilon_2)(\epsilon_2 - \epsilon_3) e^{-qd}]}
\]

and the interlayer interaction is defined as

\[
V_{12}(q) = V_{21}(q) = \frac{8\pi e^2}{qD(q)} \epsilon_2
\]
where $\epsilon_i$ denotes the dielectric constant of materials as illustrated in Fig. [1]. Furthermore, the interaction in the second layer can be obtained by replacing $\epsilon_1 \leftrightarrow \epsilon_3$ in $V_{11}(q)$. Notice that, we are interested in transport properties of the graphene layer located as the layer one in the presence of the second layer.

### A. Conductivity and Hall coefficient in the tilted magnetic field

Applying a tilted magnetic field with a small deviation from layers’ plane $\theta << 1$, in which $\theta$ is the angle between plane and the applied magnetic field, has two impacts. First of all, the parallel component of the magnetic field gives rise to the spin polarization of carriers in the system. In the fully polarized system, the spin degeneracy is lifted. Second, the perpendicular component of the magnetic field is assumed to be weak and it results in an ordinary Hall conductivity.

In plane component of the magnetic field, $B_\parallel = B \cos(\theta)$, leads to a spin polarization of the conduction electrons and changes the chemical potential by $\pm \Delta/2$ where $\Delta = g^* \mu_B B$ with $g^*$ is the effective Landé $g$-factor and $\mu_B$ is the magnetic bohr. An enhanced $g$-factor, $g^* = 2.7 \pm 0.2$ for a single and bilayer graphene has been measured [19]. This enhancement is due to the impact of the electron-electron interaction in electronic systems [20]. Notice that for $B = 0$, the spin degeneracy is $g_s = 2$ while for large magnetic field the degeneracy factor is given by $g_s = 1$ and for the intermediate fields, the system is partially spin polarized. Furthermore, the electron density for spin up and spin down are given by

$$n_{\pm} = \frac{g_\nu}{4\pi \gamma^2} (\mu \pm \Delta/2)^2$$

where $\mu$ is the chemical potential, $g_\nu$ is the valley degeneracy and $\gamma$ is $\hbar v_F$. By increasing the magnetic field, the number of electrons with spin up increases and therefore the number of electrons with spin down decreases upon to a saturation field, $B_{S1}$ in which $n = 0$. At this field $\mu = \Delta/2$, thus the saturation field is $B_{S1} = \sqrt{2}E_F/g^* \mu_B$ in which we use $n = n_+ + n_- = E_F^2/\pi \gamma^2$ where $E_F$ is the Fermi energy. Increasing the magnetic field beyond $B_{S1}$ excites electrons from the valence band to the conduction band owing to the gapless dispersion relation of graphene and makes them to be spin polarized. Therefore, charge densities in term of the magnetic field can be changed and we thus have

$$n_+ = \begin{cases} \frac{n}{2}(\sqrt{1 - B^2/2} - \sqrt{1/2})^2 & \text{if } B' < 1 \\ n(1 + \frac{1}{\pi \gamma^2} (B^2 - 1)^2) & \text{if } B' > 1 \end{cases}$$

$$n_- = \begin{cases} \frac{n}{2}(\sqrt{1 - B^2/2} - \sqrt{1/2})^2 & \text{if } B' < 1 \\ n(\frac{1}{\pi \gamma^2} (B^2 - 1)^2) & \text{if } B' > 1 \end{cases}$$

where we define $B' = B/B_{S1}$. Accordingly, the parallel component of the magnetic field leads to decomposing the conductivity into two different spin dependence, namely $\sigma_+$ and $\sigma_-$. We might sum them in order to obtain the total longitudinal conductivity since the contributions of the spin up and down channels are parallel. Notice that in our study here, there is no mechanism to change spin direction.

It is worth mentioning that the magnetic field also affects on the second layer transport properties and leads to change it’s electron density distribution. For a DLG system, the aforementioned formula remains the same for the second layer although the saturation field of the layer can be differ from the top layer since in quite general case, two layers can have different electron densities. On the other hand, a 2DEG layer in the presence of the tilted magnetic field has been studied experimentally [14] and theoretically [11]. If we consider a 2DEG system as the second layer, magnetic field polarizes the spin of the conduction electron up to a saturation field which is $B_{S2} = 2\varepsilon_F/\mu_B$ where $\varepsilon_F$ is the Fermi energy of the 2DEG. Increasing the magnetic field can not excite electrons from the valence band owing to the existence of a large band gap in the dispersion relation of 2DEG system. Therefore, electron densities in such a system is

$$n^\tau = \begin{cases} \frac{n_{B1}}{2} (1 + \tau B/B_{S2}) & \text{for } B < B_S \\ n_{B2} \delta_{\tau, \tau} & \text{for } B > B_S \end{cases}$$

where $\tau = \pm$.

The second consequence of the tilted magnetic field belongs to it’s perpendicular component in which $B_\perp = B \sin(\theta)$. We assume that it is not strong enough to change the dispersion relation into the Landau levels producing a quantum Hall conductivity [21] and therefore it only results in an ordinary Hall conductivity, $\sigma_{xy}$. A perpendicular magnetic field gives rise to the Lorentz force on the electron and the electromagnetic fields, using semiclassical relations [22], are given by

$$E = \frac{1}{\sigma_+}(J_+ + \beta_+ B \times J_+) = \frac{1}{\sigma_-}(J_- + \beta_- B \times J_-)$$

where $\beta_i = e\tau_i v_F / h k_{F,i}$, $i = \pm$ indicates spin up or spin down charge carrier and $k_{F,i}$ denotes the Fermi wave vector of $i$th spin component. Using the current vector as $J_i = \alpha_i^1 E + \alpha_i^2 B \times E$ and plucking it in the above equation to solve the equation for $\alpha_i^1, \alpha_i^2$, we thus get

$$J = \begin{cases} \frac{\alpha_+^1}{1 + \beta_+^2 B_\perp^2} + \frac{\alpha_-^1}{1 + \beta_-^2 B_\perp^2} E & \text{if } B' < 1 \\ \frac{\sigma_+^\tau B_\perp^2}{1 + \beta_+^2 B_\perp^2} + \frac{\sigma_-^\tau B_\perp^2}{1 + \beta_-^2 B_\perp^2} B \times E & \text{if } B' > 1 \end{cases}$$

where $J = J_+ + J_-$. Now, by replacing back the electric field, $E = R_{xx} J + R_{xy} B \times J$, we get the magnetoresistance and Hall coefficient expressions as $R_{xx} = a/(a^2 + b^2 B_\perp^2)$ and $R_{xy} = b/(a^2 + b^2 B_\perp^2)$ in which

$$a = \sum_{i=+,-} \frac{\sigma_i}{1 + \beta_i^2 B_\perp^2}, b = \sum_{i=+,-} \frac{\sigma_i \beta_i}{1 + \beta_i^2 B_\perp^2}$$
\( R_{zz} \) is the longitudinal resistivity and \( R_H = R_{xy} / B_L \) is the Hall coefficient which are depend on the conductivities of spin up and spin down, \( \sigma \), and they will be explored in the next subsection within the Boltzmann approximation.

Before describing the Boltzmann equation, it is worth mentioning that the conductivity of a two-layer system can be written as, \( \sigma = \begin{pmatrix} \sigma_1 & \sigma_D \\ \sigma_D & \sigma_2 \end{pmatrix} \) where \( \sigma_{1(2)} \) is the intralayer conductivity and \( \sigma_D \) is the drag conductivity. To calculate the resistivity, one should take the inverse of the conductivity matrix, therefore we have

\[
\rho = \begin{pmatrix} \rho_1 & \rho_D \\ \rho_D & \rho_2 \end{pmatrix} = \frac{1}{\sigma_1 \sigma_2 - \sigma_D^2} \begin{pmatrix} \sigma_2 & -\sigma_D \\ -\sigma_D & \sigma_1 \end{pmatrix} \tag{11}
\]

Using the fact that the drag conductivity is much smaller than the intralayer conductivities \( \sigma_D^2 \ll \sigma_1 \sigma_2 \) and assuming that the resistivity of each layer is \( \rho_i = 1/\sigma_i \), therefore, we can define the inverse of the conductivity as the resistivity in both longitudinal and Hall conductivities.

### B. Boltzmann approach for the conductivity

To calculate the conductivity of a system in which the number of conducting carriers \( n \) is much larger than the number of impurities \( n_i \) and electrons are uniformly distributed, we therefore can use the Boltzmann approach which is based on the relaxation time approximation. Our theory is formulated for weak disorder and we consider also charged impurity scattering without spin-flip processes. In this approach, the conductivity of the system is given by [23]

\[
\sigma^\pm = \frac{e^2}{2} \hbar^2 v_F^3 \int d\varepsilon D^\pm(\varepsilon) \tau(\varepsilon^\pm) \left| \frac{\partial n_F(\varepsilon)}{\partial \varepsilon} \right| \tag{12}
\]

At zero temperature, the above equation reduces [8] [23] to \( \sigma^\pm = \frac{e^2 v_F^2 D(E_F^\pm) \tau(E_F^\pm)}{2} \), where \( D(E_F^\pm) \) is the density of states at the Fermi energy and in the case of graphene it is equal to \( g_s g_v E_F/2\pi \gamma^2 \). \( \tau(E_F) \) is the relaxation time at the Fermi energy which is, in general, given by [24]

\[
\frac{1}{\tau(\varepsilon_k)} = \frac{\pi}{\hbar} \sum_{k'} n_i |\psi(q)|^2 (1 - \cos^2(\theta)) \delta(\varepsilon_k - \varepsilon_{k'}) \tag{13}
\]

for graphene layer where \( q = |k - k'| \), \( \theta = \theta_k - \theta_{k'} \), \( n_i \) represents the concentration of charged impurities, and \( \psi(q) \) represents the interaction between impurities and electrons. Short-range disorder plays a role in resistive scattering at higher carrier density and also high mobility [23], however for mid-range densities, the major scattering source is the charged impurity disorder [23] [25]. We thus neglect all disorder mechanisms other than random charged impurities in the system and assume the charged impurities to be of random negative sign of units of the electron charge. We assume that the charged disorder with density \( n_i \) located in the plane of graphene.

Changing the variable \( \sin(\theta/2) = y \) in Eq. [13] and writing the relation for different doping \( n^\pm \) separately, the relaxation time reads

\[
\frac{1}{\tau^\pm} = \frac{8 n_i E_F^\pm}{2 \pi \hbar \gamma^2} \int_0^1 dy W(2 k_F^0 y)^2 y^2 \sqrt{1 - y^2} \tag{14}
\]

where \( W(q) \) is the effective electron-impurity interaction and the relaxation time is evaluated at the Fermi energy of the graphene layer. We use a minimal theory to describe the effects of the parallel magnetic field. Screening effects are taken into account within the RPA many-body effects. For this purpose, the effective interaction can be written as [27]

\[
W(q) = \varepsilon(q)^{-1} \Psi_{ei}(q) = (1 + W(q) \chi(q))^{-1} \Psi_{ei}(q) \tag{15}
\]

where \( W(q) \) is a 2 \times 2 matrix for electron-electron interactions, and \( \varepsilon(q) \) is the same matrix for electron-impurity interaction. Using \( \chi(q)^{-1} = \chi_0(q)^{-1} - W(q) \) we have

\[
W(q) = (\Psi_{ei}(q)^{-1} - \varepsilon(q)^{-1} W(q) \chi_0(q))^{-1} \tag{16}
\]

where

\[
W_{ii}(q) = V_{11}(q) + [V_{12}^2(q) - V_{11}(q) V_{22}(q)] \Pi_0^2(q) \varepsilon(q) \tag{17}
\]

In the case of 2DEG, we need to modify the dielectric function by using the \( \Pi_i^0 \) of 2DEG as \( \Pi_i^0(q) = \Pi_i^+(q) - \Pi_i^-(q) \), where

\[
\Pi_i^\pm(q) = D[1 - \sqrt{1 - (2 k_F^0 \pm 2 q)^2 \theta(q - 2 k_F^0 \pm q)}] \tag{18}
\]

the density of states is \( D = g_s m/2 \pi \hbar^2 \) and the Fermi wave vector is \( k_F^0 = \sqrt{4 \pi n^\pm / g_s g_v} \). In the case of 2DEG, we consider \( g_v = 1 \).

Using Eq. [14], we can write the magnetoresistance of the graphene layer in the presence of another layer as

\[
\frac{\sigma}{\sigma_0} = \frac{1}{2} \left( \frac{I^0}{I^+} + \frac{I^0}{I^-} \right) \tag{22}
\]
The second layer can be either a gapless graphene sheet that is coupled to another doped graphene within the interlayer Coulomb interaction and they are separated by a dielectric material, as schematically illustrated in Fig. 1. We consider a hybrid double-layer system composed by a doped graphene sheet that is placed on the surface of the layer in close proximity to the considered graphene when \( n_1 \) is small. After saturating electrons in the second layer, the number of

\[ I^\pm = \frac{\sqrt{\sigma_0(n_1 + n_2)}}{4\pi e^2} \int_0^1 dy y^2 \sqrt{1 - y^2} W_{11}^2(2k_F^\pm y) \]

where \( I^\pm \) is the conductivity of the system in the absence of the magnetic field. \( I^0 \) is the same as \( I^+ \) when \( B = 0 \).

### III. NUMERICAL RESULT AND DISCUSSION

In this section, we present our main numerical results based on the theory presented in the previous section. Our aim is to explore the impact of correlation between the second layer on the magnetoresistance and Hall coefficient of graphene layer which is labeled by layer number one. The second layer can be either a gapless graphene layer or a 2DEG system. We therefore study the structure by considering both parallel and tilted magnetic fields. In all numerical results we use \( e^2/\gamma = 2.2 \) and \( g^* = 2 \).

#### A. Decoupled layer graphene in a parallel magnetic field

We consider a hybrid double-layer graphene system (DLG) composed by a doped graphene sheet that is coupled to another doped graphene within the interlayer Coulomb interaction and they are separated by a dielectric material, as schematically illustrated in Fig. 1. We show the resistivity as a function of the parallel magnetic field in Fig. 2 for \( n \)-doped double-layer graphene with electron densities \( n_1 = 10^{11} \text{cm}^{-2} \) and \( n_2 = 10^{10} \text{cm}^{-2} \), respectively. The resistivity is scaled by its value at zero magnetic field denoted by \( R_0 \). The total resistivity increases by increasing magnetic field owing to the suppression of screening and increases of effective interaction screened within RPA dielectric function. As the magnetic field increases beyond its saturation field value, \( B_{S1} = \hbar v_F \sqrt{2\pi n_1/(g^* \mu_B)} \) in which the system is fully spin polarized, the hole density creates in the valence band and results in the sharp decreasing in the resistivity which demonstrates the negative differential magnetoresistance. This feature is totally different from that of conventional 2DEG where the resistivity saturates for \( B > B_S \). We analyze the resistivity for the different layer distances, \( d \) and the media dielectric constants, \( \varepsilon_2 \) and the results are shown in Fig 2(a) and 2(b) respectively. Interestingly enough, Fig 2(a) shows that the resistivity of graphene layer is significantly modified when a graphene sheet is placed on the surface of the layer in close proximity to the considered graphene when \( n_2 < n_1 \). After saturating electrons in the second layer, the number of

![FIG. 2. (Color online) Longitudinal resistivity of a hybrid double-layer system composed by a doped graphene sheet that is Coulomb coupled to another doped graphene in which they are separated by a dielectric material as a function of the magnetic field for (a) different distance between layers in units of nm for \( \varepsilon_2 = 12.53 \) and (b) different value of \( \varepsilon_2 \) with constant distance \( d = 1.0 \text{nm} \). We also consider a plot with \( d = 100 \text{nm} \) to suppress the interlayer effect. We choose \( \epsilon_1 = 1.0, \epsilon_3 = 12.53 \) and the electron densities are \( n_1 = 10^{11} \text{cm}^{-2} \) and \( n_2 = 10^{10} \text{cm}^{-2} \), respectively.](image)

![FIG. 3. Longitudinal resistivity at saturation field versus the distance between layers in units of nm in a hybrid double-layer graphene system for different values of \( \varepsilon_2 \). We choose \( \epsilon_1 = 1.0, \varepsilon_3 = 12.53 \) and the electron densities are \( n_1 = 10^{11} \text{cm}^{-2} \) and \( n_2 = 10^{10} \text{cm}^{-2} \), respectively.](image)
carriers in the bottom layer increases according to the excitation of its electrons from valence band and the effective interaction which is screened by RPA dielectric function decreases which gives rise to the decreasing of the resistivity. Our numerical results show that the resistivity illustrates a negative differential magnetoresistance for small \( d \) value and for a region in which \( B \) is much smaller than \( B_{S1} \). The region depends on the electron density in the second layer, \( n_2 \) and smaller \( n_2 \) leads to change of the slope of \( R_{xx} \) at smaller \( B \). Thus we can have a larger conductivity of the graphene layer by using the electron density of the second graphene layer smaller than \( n_1 \). It is obvious that we also get the resistivity of a single layer by considering a very large \( d \) value.

It has been shown that \([28]\) the dielectric constant between two layers in double-layer graphene systems has a significant role in transport properties. Although increasing the dielectric constant screens the interlayer interaction, it has a significant role on the intralayer screening of the considered layer as well. Accordingly, a competition between the effect of inter- and intralayer interactions on the magnetoresistance of graphene is vital when dielectric materials with higher \( \epsilon_2 \) between layers is used. We also obtain the reduction of the resistivity when the dielectric constant increases. We choose three different materials, namely h-BN, Al\(_2\)O\(_3\) and HfO\(_2\) for \( d = 1 \)nm to explore the effect of interlayer interaction on the magnetoresistance of the DLG system. In the same manner, we obtain the negative differential magnetoresistance in area which \( B < B_{S1} \) due to the correlation effects induced on the graphene layer from another layer.

We analyze the resistivity at the saturation magnetic field for the graphene layer with electron density \( n_1 \) as a function of \( d \) in Fig. 3. We obtain the enhanced resistivity about 20\% by increasing the layer distance for the case that we have larger dielectric constant between layers owing to the long-ranged interlayer interaction since \( V_{12}(q) \) is proportional to the \( \epsilon_2 \). \( R_{xx}(B_{S1}) \) significantly changes for small \( d \) and \( \epsilon_2 \) values.

**B. Graphene-2DEG hybrid in a parallel magnetic field**

The parabolic band dispersion of 2DEG systems results in a saturation magnetic field under the in plane magnetic field, \( B_{S2} = \frac{2\pi h^2 \pi D}{g^* m^* q^* \mu_B} \) which is smaller than the corresponding one in graphene system with the same density. The behavior of the magnetoresistance in 2DEG is totally different from graphene one and it would be interesting to explore transport properties in a hybrid system combined with graphene and 2DEG systems [18]. We consider a graphene and 2DEG hybrid system (G-2DEG) composed by a doped graphene sheet that is coupled to a 2DEG in which a dielectric material is filled the space between the two layers.

Figure 4 shows the longitudinal magnetoresistance of graphene in the presence of 2DEG layer for different materials [29]. The electron density of both layers is the same and equal to \( 1 \times 10^{11} \) cm\(^{-2} \), \( \epsilon_1 = 1 \), \( \epsilon_2 = \epsilon_3 \) has chosen to be the same as dielectric constant of 2DEG for \( d = 10 \)nm. This figure shows that as 2DEG carriers fully polarized, the magnetoresistance of the graphene layer exhibits a sharp jump at \( B_{S2} \) which occurs at very earlier \( B_{S1} \) value. The longitudinal resistivity increases by the dielectric constant of the 2DEG material due to
the increasing of the screening. Importantly, this feature differs with that in DLG structure. Meanwhile, since $B_{S2} \sim m^*, \text{ thus the position of the first peak of } R_{xx} \text{ decreases by increasing the carrier effective mass.}$

In order to instance the difference between G-2DEG and DLG, we plot the magnetoresistance of the graphene layer at the saturation magnetic field of graphene, $B_{S1}$ in Fig. 5 when a 2DEG layer is placed near the graphene layer. The results show that the $R_{xx}$ decreases at small $d$, behaves non-monotonic as a function of $d$ and results are in different trend with respect to Fig. 3 for DLG systems. The difference can be understood by the chirality effect in the graphene system which appears in the charge-charge response function \[30\].

C. Longitudinal resistivity in a tilted magnetic field

In the presence of a tilted magnetic field where $\theta \neq 0$, we should use the full expression given in Eq. \[10\] and it can be rewrite as

$$B_{\bot} \beta_{\pm} = \frac{B I_0 n_1}{2B_{S1} I_{\pm} n_{1\pm}} x = \alpha_{\pm} x \quad (23)$$

where $I_{\pm}$ is the function given under Eq. \[22\], and $n_1$ is the total density of two layers ($n_1 + n_2$) in the system. Here, we define a new parameter $x_1$, $x = 0.895 \sin(\theta) / \sqrt{n_1/\bar{n}_{1\text{imp}}}$, ($\bar{n}_1$ and $\bar{n}_{1\text{imp}}$ are in units of $10^{12} \text{ cm}^{-2}$). Note that $x$ depends on both the angle as well as $\bar{n}_{1\text{imp}}$, therefore larger values of $x$ can be obtained by using clean samples. Figure 6(a) shows the effect of the tilted magnetic field on the longitudinal resistivity of the graphene layer for different values of $x$. The magnetoresistance of graphene layer is restored by setting $x \to 0$. The longitudinal resistivity increases by increasing the value of $x$ and thus the value of differential magnetoresistance changes accordingly. We choose $x = 100$ for an extremely clean sample and this figure shows the $R_{xx}$ increases slightly by increasing $x$ however the maximum at $B_{S1}$ does not change anymore and it has a finite value even at $x \to \infty$.

In Fig. 6(b) we consider a DLG system. This figure exhibits that interlayer interaction has a more significant effect for lower values of the tilting parameter $x$. In Figure 6(c) we place graphene in the presence of 2DEG. A sharp change in the diagram at saturation field of 2DEG are still appeared.

D. Hall resistivity in a tilted magnetic field

Finally, by applying the tilted magnetic field in which there is a weak perpendicular component $B_{\bot}$ and using two-species conductivities $\sigma_+$ and $\sigma_-$, we can straightforwardly calculate the Hall coefficient $R_H = R_{xy}/B_{\bot}$ form Eq. \[10\]. Normalizing the Hall coefficient to it’s
zero magnetic field value, $R_{H0} = R_H(B \to 0)$ one gets

$$R_H/R_{H0} = \frac{b}{a^2 + b^2B_0^2} \frac{\sigma_0}{\beta_0}$$  \hspace{1cm} (24)$$

Note that $\beta_+ = \beta_-$ for the limit of $B \to 0$ and $\beta_0 = \beta_+(B = 0)$. The Hall coefficient of graphene differs with the Hall coefficient of 2DEG system. First of all, in the region of fields which is higher than the saturation field $B_{S1}$, the conductivities $\sigma_+ \text{ and } \sigma_-$ change according to gapless graphene and thus the Hall coefficient changes above the $B_{S1}$ value in contrast to that of 2DEG where it is a constant after the saturation field. However, the Hall coefficient of 2DEG system shows a peak before the saturation magnetic field point and gets constant at the saturation magnetic field $B_0$. Moreover, the Hall coefficient of 2DEG system increases monotonically with decreasing $x$ value, however our numerical results show that the Hall coefficient of graphene increases very largely by decreasing the $x$ values.

In Fig. 7 the behavior of the Hall coefficient of graphene as a function of the magnetic field amplitude is shown in the presence of 2DEG InAs for different values of $x$. Same qualitatively results can be obtained for a single graphene layer. As it is obvious in this figure, the two peaks occur at finite $x$. Importantly, if we interpretable the Hall resistivity as an effective density of charge carriers, the results will show that the effective density of carriers change more sharply around $B_{S1}$. Nevertheless, the Hall coefficient in graphene has two peaks at finite $x$ values in which one peak occurs just below the $B_{S1}$ value and another takes place above that. These peaks become sharper at smaller $x$, accordingly a giant like Hall coefficient diagram of graphene can be reached at $x$ tends to a tiny value. Notice that at $B \sim B_{S1}$ the Hall coefficient decreases and eventually reaches to its zero field value.

FIG. 7. (Color online) Hall coefficient of graphene as a function of magnetic field amplitude in the presence of 2DEG InAs for different values of $x$ at given constant $\theta = 4^\circ$. We consider $\epsilon_3 = \epsilon_2 = 12.2$, $m^* = 0.026m_e$ and $n_1 = 0.1 \times 10^{12}$, $n_2 = 0.01 \times 10^{12}$cm$^{-2}$. The layers are separated with distance $d = 10$nm.

FIG. 8. (Color online) Hall coefficient of a single layer graphene (SLG), double layer graphene (DLG) and graphene-2DEG hybrid (G-2DEG) systems as a function of magnetic field amplitude at given constant $\theta = 4^\circ$ for (a) $x = 0.2$ (b) $x = 0.5$ and (c) $x = 1.5$. We consider $\epsilon_3 = 12.53$ and 2DEG in InAs with $\epsilon_3 = 12.2$, $n_1 = 0.1 \times 10^{12}$, $n_2 = 0.01 \times 10^{12}$cm$^{-2}$ and the layers are separated with distance $d = 10$nm.
\[ R_{H0} \text{ at } B = B_{S1}. \] To understand these features, we analytically calculate the \( R_H \) in different regions. Let us start off by using the definition of the Hall coefficient

\[
\frac{R_H}{R_{H0}} = \frac{\sigma_0}{\beta_0} \frac{b}{a^2 + b^2 B_0^2} = D\frac{\sigma_0}{\beta_0} \frac{\tau_{+\beta} + \tau_{-\beta}}{(\tau_{+\alpha} + \tau_{-\alpha})^2 + x^2(\tau_{+\alpha} + \tau_{-\alpha})^2) + \tau_{-\alpha}(1 + x^2 a^2_2)^2}
\]

(25)

where \( D = (1 + x^2 a^2_2)(1 + x^2 a^2_2). \) Notice that, in the above equation, there is a competition between \( x \) and \( \alpha_- \) since \( \alpha_- \) is proportional to the inverse of \( n_- \) and it diverges at \( B = B_{S1} \) where \( n_- = 0 \), thus the zero limit of \( x \) can not prevent this singular behavior. We can take the \( x \alpha_+ \rightarrow 0 \) limit around \( B_{S1} \). Notice that \( \alpha_- \) has a finite nonzero value even at \( B = B_{S1} \) where there is no spin down electron [23].

By considering a case that \( x \rightarrow 0 \) and \( \alpha_- \rightarrow \infty \) however \( x \alpha_- \rightarrow 0 \) in which the Hall resistivity is noticeably large, we have

\[
\frac{R_H}{R_{H0}} = D\frac{\sigma_0}{\beta_0} \frac{\tau_{+\beta} + \tau_{-\beta}}{(\tau_{+\alpha} + \tau_{-\alpha})^2 + x^2(\tau_{+\alpha} + \tau_{-\alpha})^2) + \tau_{-\alpha}(1 + x^2 a^2_2)^2}
\]

(26)

which has a large value at \( B \) close to \( B_{S1} \) in which \( n_- \) becomes very small. It should be noted that such a huge resistivity does not appear in 2DEG since the Hall coefficient of 2DEG at \( x = 0 \) is given by [11]

\[
R_H/R_{H0} = \frac{2 \sum_{\pm} n_{\pm}(I_{0}/I_{\pm})^2}{(\sum_{\pm} n_{\pm} I_{0}/I_{\pm})^2}
\]

(27)

which is constant at \( x \rightarrow 0 \). If we consider the case that \( \alpha_- \rightarrow \infty \) and \( x \alpha_- \rightarrow \infty \) as well, the \( R_H \) will be independent of the \( x \) value. In this limit, Eq. (25) takes the following form

\[
\frac{R_H}{R_{H0}} = D\frac{\sigma_0}{\beta_0} \frac{\tau_{+\beta} + \tau_{-\beta}(1 + x^2 a^2_2)}{(\tau_{+\alpha} + \tau_{-\alpha})^2 + x^2(\tau_{+\alpha} + \tau_{-\alpha})^2) + \tau_{-\alpha}(1 + x^2 a^2_2)^2}
\]

(28)

Using an approximation that \( \beta_- \sim \alpha_- \) in this limit, the term with \( a^2_2 \) dominates and also \( D \rightarrow (1 + x^2 a^2_2)2a^2_2 \) and thus we get

\[
\frac{R_H}{R_{H0}} = \frac{\sigma_0}{\beta_0} \frac{\tau_{+\beta}}{\tau_{+\alpha}} = \frac{n_1}{n_+}
\]

(29)

Interestingly, it becomes unity when \( B = B_{S1} \). Figure 8 shows the behavior of the normalized \( R_H/R_{H0} \) as a function of the magnetic field for different values of \( x \) for different systems. As it is shown in these figures, the Hall coefficients have a peak in the region \( B < B_{S1} \) same as 2DEGs but it changes for larger magnetic field and the results depend on the \( x \) value. The peak at \( B > B_{S1} \) becomes larger by reducing \( n_2 \) and namely increasing the interlayer correlation. By increasing \( x \) value, the shape of the Hall coefficient changes significantly and decreases sensitively with increasing \( x \) values. Contrary to \( R_{Hx} \) where interaction with graphene (2DEG) makes lower (higher) values of the magnetoresistance, the interlayer interaction always makes \( R_H/R_{H0} \) increase comparing to it’s single layer values. In addition, 2DEG layer has most impact on the Hall coefficient value.

**IV. CONCLUSION**

In summary, We have calculated the longitudinal resistivity and Hall coefficient of gapless graphene in hybrid structure and compared results with those of a single layer graphene in the presence of the tilted magnetic field. We have considered a single layer graphene interacting with either another layer of gapless graphene (DLG) or 2DEG (G-2DEG) in the presence of the tilted magnetic field. We have also assumed that the magnetic field is slightly off the electronic planes so that there is a weak perpendicular component.

In the parallel magnetic field, we have analyzed the dependence of the dielectric material which is filled the space between two layers as well as the distance between layers on the magnetoresistances and shown that the interlayer interaction plays an important role even at longer distances by using slightly strong dielectric materials in DLG systems. Our numerical results show that the resistivity of graphene layer is significantly modified when a graphene sheet is placed on the surface of the layer in close proximity to the considered graphene when \( n_2 < n_1 \). After saturating electrons in the second layer, the number of carriers in another layer increases according to the excitation of it’s electrons from valence band and the effective interaction which is screened by RPA dielectric function decreases which gives rise to the decreasing of the resistivity. Moreover, the resistivity shows a negative differential magnetoresistance for small \( d \) value and for a region in which \( B \) is much smaller than the saturation magnetic field. We have also obtained the reduction of the resistivity when the dielectric constant between two layers increases and obtained the negative
differential magnetoresistance in area which $B < B_{S1}$ due to the correlation effects induced on the graphene layer from another layer. We have shown that the $R_{xx}$ in a G-2DEG structure decreases at small $d$, behaves non-monotonic as a function of $d$ and results are in different trend with respect to those in DLG systems. The difference can be explained by the chirality effect in the graphene system which appears in the charge-charge response function.

In the presence of the tilted magnetic filed, the Hall coefficient of the graphene layer assembled by a 2D layer shows two peaks at finite $x$ values (for definition of $x$, see the text) in which one peak occurs just below the $B_{S1}$ value and another takes place above that. These peaks become sharper at very small $x$ values and small the electron density of the second layer as well. Accordingly, a giant like Hall coefficient diagram of graphene can be reached in a hybrid graphene structure. We have shown that the magnetoresistance and the Hall coefficient of a single graphene can be tuned by the electron density and dielectric constants of the materials in the system. Our obtained results should be verified by experiments.

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