Rare B decays in a single Universal Extra Dimension scenario

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Exclusive rare $B \to K^{(*)}\ell^+\ell^-$, $B \to K^{(*)}\nu\bar{\nu}$ and $B \to K^*\gamma$ decays are studied within the Applequist-Cheng-Dobrescu model, an extension of the Standard Model in presence of universal extra dimensions. In the case of a single universal extra dimension compactified on a circle of radius $R$, we study the dependence of several observables on $1/R$, and discuss whether the hadronic uncertainty due to the form factors obscures or not such a dependence. We find that, using present data, it is possible in many cases to put a sensible lower bound to $1/R$, the most stringent one coming from $B \to K^*\gamma$.

1. INTRODUCTION

Among the various new Physics scenarios, those with extra dimensions are particularly attracting [1]. A special case is the Applequist-Cheng-Dobrescu (ACD) model [2] in which universal extra dimensions are considered, which means that all the fields can propagate in all available dimensions. In the case of a single extra dimension compactified on a circle of radius $R$, Tevatron run I data allow to put the bound $1/R \geq 300$ GeV. Constraints can be put studying other processes, namely rare B decays induced by $b \to s$ transition [3] which are induced at loop level and hence suppressed in the Standard Model (SM) [4].

In [5] the effective Hamiltonian inducing $b \to s$ decays was computed in the ACD model. Here, we summarize the results obtained in [7] for exclusive $b \to s$-induced modes. In this case, the uncertainty in the form factors must be considered, since it can overshadow the sensitivity to $1/R$. Indeed, we find that computing the branching ratios of $B \to K^{(*)}\ell^+\ell^-$ and the forward-backward lepton asymmetry in $B \to K^*\ell^+\ell^-$ for a representative set of form factors, a bound can be put. Other interesting observables are the lepton polarization asymmetries in the case of the modes $B \to K^{(*)}\tau^+\tau^-$. Finally, we discuss how the branching ratio of $B \to K^*\gamma$ depends on $1/R$, from which we can establish the most stringent bound on this parameter.

2. THE ACD MODEL WITH A SINGLE UED

The ACD model [2] is a minimal extension of the SM in $4 + \delta$ dimensions; we consider $\delta = 1$. The fifth dimension $x_5 = y$ is compactified to the orbifold $S^1/Z_2$, i.e. on a circle of radius $R$ and runs from 0 to $2\pi R$ with $y = 0, y = \pi R$ fixed points of the orbifold. Hence a field $F(x, y)$ (x denoting the usual 3+1 coordinates) would be a periodic function of $y$, and it could be expressed as

$$F(x, y) = \sum_{n=-\infty}^{n=+\infty} F_n(x) e^{i n y/R}.$$  

If $F$ is a massless boson field, the KK modes $F_n$ obey the equation $(\partial^2 + n^2/R^2) F_n(x) = 0$, $\mu = 0, 1, 2, 3$ so that, apart the zero mode, they behave in four dimensions as massive particles with $m_n^2 = (n/R)^2$. Under the parity transformation $P : y \to -y$ fields having a correspondent in the 4-d SM should be even, so that their zero mode in the expansion is interpreted as the ordinary SM field. On the other hand, fields having no SM partner should be odd, so that they do not have zero modes.

Important features of the ACD model are: i) a single additional free parameter with respect to the SM: the compactification radius $R$; ii) conservation of KK parity, so that there is no tree-level contribution of KK modes in low energy processes and no production of single KK excitation in ordinary particle interactions. A detailed description of this model is provided in [5].
3. DECAYS $B \to K^{(*)}\ell^+\ell^-$

In the Standard Model the effective $\Delta B = -1$, $\Delta S = 1$ Hamiltonian governing the transition $b \to s\ell^+\ell^-$ is $H_W = 4G_F\sqrt{2}V_{ts}^*V_{tb}\sum_{i=1}^{10}C_i(\mu)O_i(\mu)$. $G_F$ is the Fermi constant and $V_{ij}$ are elements of the Cabibbo-Kobayashi-Maskawa mixing matrix; we neglect terms proportional to $V_{ub}V_{us}^*$. $O_1$, $O_2$ are current-current operators, $O_3$, ..., $O_9$ QCD penguins, $O_7$, $O_8$ magnetic penguins, $O_{10}$ semileptonic electroweak penguins. We do not consider the contribution to $B \to K^{(*)}\ell^+\ell^-$ with the lepton pair coming from $c\bar{c}$ resonances, mainly due to $O_1$, $O_2$. We also neglect QCD penguins whose coefficients are very small compared to the others. Therefore, in the case of the modes $B \to K^{(*)}\ell^+\ell^-$, the relevant operators are: $
O_7 = \frac{\alpha}{16\pi^2}m_b(\bar{s}_L\gamma^\mu b_R)F_{\mu\nu}, \nO_9 = \frac{\alpha^2}{16\pi^2}m_b(\bar{s}_L\gamma^\mu b_L)\ell_\mu\ell, \nO_{10} = \frac{\alpha^2}{16\pi^2}(\bar{s}_L\gamma^\mu b_L)\ell_\mu\ell_5\ell. \n$ Their coefficients have been computed at NLO in the Standard Model [5,6]. For large $\alpha$, $\theta_1$, $\theta_2$ and $\theta_3$, and at NLO for the ACD model [5,6], we use these results in our study. No new operators are found in ACD, while the coefficients are modified because particles not present in SM can contribute as intermediate states in loop diagrams.

As a consequence, they are expressed in terms of functions $F(x_1, 1/R)$, $x_1 = m_b^2/M_W^2$, generalizing the corresponding SM functions $F_0(x_1)$ according

to

$F(x_t, 1/R) = F_0(x_1) + \sum_{n=1}^{\infty} F_n(x_t, x_n)$, where $x_n = m_n^2/M_W^2$ and $m_n = n/R$ [5,6]. For large values of $1/R$ the SM phenomenology should be recovered, since the new states, being more and more massive, decouple from the low-energy theory.

The exclusive $B \to K^{(*)}\ell^+\ell^-$ modes involve the matrix elements of the operators in the effective Hamiltonian between the $B$ and $K$ or $K^*$ mesons, for which we use the standard parametrization in terms of form factors. We use two sets of form factors: the first one (set A) obtained by three-point QCD sum rules based on the short-distance expansion [9]; the second one (set B) obtained by light-cone QCD sum rules [10]. For both sets we include in the numerical analysis the errors on the parameters.

In Fig. 1 we plot, for the two sets of form factors, the branching fractions relative to $B \to K^{(*)}\ell^+\ell^-$ versus $1/R$ and compare them with the experimental data provided by BaBar [11], $BR(B \to K^{(*)}\ell^+\ell^-) = (3.4 \pm 0.7 \pm 0.3) \times 10^{-7}$, $BR(B \to K^{*\ell^+\ell^-}) = (7.8 \pm 1.9 \pm 1.2) \times 10^{-7}$, and Belle [12]: $BR(B \to K^{*\ell^+\ell^-}) = (5.50 \pm 0.75 \pm 0.27 \pm 0.02) \times 10^{-7}$, $BR(B \to K^{*\ell^+\ell^-}) = (16.5 \pm 3.3 \pm 0.9 \pm 0.4) \times 10^{-7}$.

Set $B$ excludes $1/R \lesssim 200$ GeV. Improved data will resolve the discrepancy between the experiments and increase the lower bound for $1/R$.

In the case of $B \to K^{*\ell^+\ell^-}$ the investigation of the forward-backward asymmetry $A_{fB}$ in the dilepton angular distribution may also reveal effects beyond the SM. In particular, in SM, due to the opposite sign of the coefficients $C_7$ and $C_9$, $A_{fB}$ has a zero that is almost independent of the model for the form factors [13]. Let $\theta_1$ be the angle between the $\ell^+$ direction and the $B$ direction in the rest frame of the lepton pair (we consider massless leptons). We define ($z = \cos\theta_1$):

$$A_{fB}(q^2) = \frac{\int_0^1 d^2\Gamma dq^2dz - \int_1^0 d^2\Gamma dq^2dz}{\int_0^1 d^2\Gamma dq^2dz + \int_1^0 d^2\Gamma dq^2dz}. \quad (1)$$

We show in Fig. 2 the predictions for the SM, $1/R = 250$ GeV and $1/R = 200$ GeV. The zero of $A_{fB}$ is sensitive to the compactification parameter, so that its experimental determination would constrain $1/R$. At present, the analysis performed by Belle Collaboration indicates that the relative sign of $C_9$ and $C_7$ is negative, confirming that $A_{fB}$ should have a zero [13].

We considered also the case of the modes $B \to K^{(*)}\tau^+\tau^-$, i.e. with a massive lepton. These modes have not been observed yet, so that it is not possible to compare their branching ratios with data. However their analysis in the ACD model shows that an eventual measurement of a branching ratio larger than $2 \cdot 10^{-7}$ would be incompatible with the SM, independently of the set of form factors used.\[1\] See [9] for a detailed discussion.
Figure 2. Forward-backward lepton asymmetry in $B \to K^{*+} \ell^-$ versus $1/R$ using set A (left) and B (right). The dark (blue) bands correspond to the SM results, the intermediate (red) band to $1/R = 250$ GeV, the light (yellow) one to $1/R = 200$ GeV.

It is also interesting to consider the asymmetry in the $\tau^-$ polarization, defined as:

$$A_A(q^2) = \frac{d\Gamma(s_A) - d\Gamma(-s_A)}{d\Gamma(s_A) + d\Gamma(-s_A)}$$

with $A = L, T, N$ and $s_L = \frac{1}{m^2}([k_1^2, 0, 0, k_1^0]), s_T = (0, 0, -1, 0)$, $s_N = (0, 1, 0, 0)$ being the $\tau^-$ longitudinal, transverse and normal polarization vectors, and $k_1$ its momentum in the lepton pair rest frame. The longitudinal asymmetry $A_L$ shows a mild dependence on $1/R$, while the transverse asymmetry $A_T$, is a more sensitive observable as displayed in Fig. 3 for the representative case of $B \to K\tau^-\tau^+$. 

Figure 3. Transverse $\tau^-$ polarization asymmetry in $B \to K\tau^+\tau^-$ obtained using set A (left) and B (right) of form factors. The dark (blue) region is obtained in SM, the intermediate (red) one for $1/R = 500$ GeV, the light (yellow) one for $1/R = 200$ GeV.

4. THE DECAYS $B \to K^{(*)}\nu\bar{\nu}$

In the SM the effective Hamiltonian governing $b \to s\nu\bar{\nu}$ induced decays is

$$H_{e f f} = \frac{G_F^2}{\sqrt{2} 2\pi \sin^2(\theta_W)} V_{ts} V_{tb}^* \eta_X X(x_t)$$

$$\bar{b} \gamma^\mu (1 - \gamma_5) s \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu$$

obtained from $Z^0$ penguin and box diagrams dominated by the intermediate top quark. In $\theta_W$ is the Weinberg angle. The function $X$ was computed in $[15, 16]$ in the SM and in the ACD model in $[14]$. We put to unity the QCD factor $\eta_X = 1.01.7$.

$B \to K^{(*)}\nu\bar{\nu}$ decays have been studied within the SM $[18]$. However, only an experimental upper bound exists for $B \to K\nu\bar{\nu}$. $BR(B^- \to K^-\nu\bar{\nu}) < 3.6 \times 10^{-5}$ (90% CL) $[19]$, $BR(B^- \to K^-\nu\bar{\nu}) < 5.2 \times 10^{-5}$ (90% CL) $[20]$. Furthermore the $1/R$ dependence, studied in $[7]$ turns out to be too mild for distinguishing values above $1/R \geq 200$ GeV.

5. THE DECAY $B \to K^{*}\gamma$

The transition $b \to s\gamma$ is described by the operator $O_7$. The most recent measurements for the exclusive branching fractions are provided by Belle $[21]$: $BR(B^0 \to K^{*0}\gamma) = (4.01 \pm 0.21 \pm 0.17) \times 10^{-5}$, $BR(B^- \to K^{*-}\gamma) = (4.25 \pm 0.31 \pm 0.24) \times 10^{-5}$ and BaBar $[22]$: $BR(B^0 \to K^{*0}\gamma) = (3.92 \pm 0.20 \pm 0.24) \times 10^{-5}$, $BR(B^- \to K^{*-}\gamma) = (3.87 \pm 0.28 \pm 0.26) \times 10^{-5}$.

In Fig. 4 the branching ratio computed in the ACD model is plotted versus $1/R$: the sensitivity to the this parameter is evident; a lower bound of
$1/R \geq 250$ GeV can be put adopting set A, and a stronger bound of $1/R \geq 400$ GeV using set B, which is the most stringent bound that can be currently put on this parameter from the $B$ decay modes we have considered.

Figure 4. $BR(B \to K^{*}\gamma)$ versus $1/R$ using set A (left) and B (right) of form factors. The horizontal band corresponds to the experimental result.

6. CONCLUSIONS AND PERSPECTIVES

We have shown how the predictions for $B \to K^{(*)}\ell^{+}\ell^{-}$, $B \to K^{(*)}\nu\bar{\nu}$, $B \to K^{*}\gamma$ decays are modified within the ACD scenario. The constraints on $1/R$ are slightly model dependent, being different using different sets of form factors. Nevertheless, various distributions, together with the lepton forward-backward asymmetry in $B \to K^{(*)}\ell^{+}\ell^{-}$ are very promising in order to constrain $1/R$, the most stringent lower bound coming from $B \to K^{*}\gamma$. Improvements in the experimental data, expected in the near future, will allow to establish more stringent constraints for the compactification radius.

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