The Interplay of Landau Level Broadening and Temperature on Two-Dimensional Electron Systems

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Abstract

This work investigates the influence of low temperature and broadened Landau levels on the thermodynamic properties of two-dimensional electron systems. The interplay between these two physical parameters on the magnetic field dependence of the chemical potential, the specific heat and the magnetization is calculated. In the absence of a complete theory that explains the Landau level broadening, experimental and theoretical studies in literature perform different model calculations of this parameter. Here it is presented that different broadening parameters of Gaussian-shaped Landau levels cause width variations in their contributions to interlevel and intralevel excitations. Below a characteristic temperature, the interlevel excitations become negligible. Likewise, at this temperature range, the effect of the Landau level broadening vanishes.

Key words: two-dimensional electron systems, Landau levels, heat capacity, magnetization

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1. Introduction

Two-dimensional electron systems (2DES) are widely investigated due to their intriguing non-bulk like properties and their possible applications. One example of interest is the oscillating behavior of their thermodynamic properties with respect to a strong perpendicular magnetic field \( B \) at low temperature \( T \) \[2, 3, 4, 5\]. This general trend is attributed to the presence of Landau levels – the discretized energy spectrum of a 2DES subject to a strong magnetic field applied normal to the 2D plane \[1\]. Landau levels are represented in the density of energy states (DOS) of a non-interacting 2DES as a series of Dirac-delta functions with gaps that are empty of electronic states \[6\].

On the side of theoretical investigations, earlier studies have obtained Landau level broadening, in the extreme quantum limit, when self-consistent treatment of electron-impurity scattering is taken into account \[7, 8\]. Another work, employing perturbative treatment of disorder and Coulomb interaction, obtained a DOS with sharp edges in the limit where Landau level mixing is ignored \[9\]. Nevertheless, reports on the DOS extracted from experiments indicate broadened Landau levels. This is true whether the DOS is inferred from magnetization \[2, 3, 10\], heat capacity \[1, 5\] or capacitance measurements \[11, 12\]. Moreover, depending on the applied magnetic field strength, overlaps of Landau levels may occur which indicate existence of electronic states in between the ideal DOS peaks. These overlaps have been analytically confirmed in the presence of a weak disorder \[13\]. But this derivation has no localization effects and it contains parameters related to the impurity configuration, namely, its density and distance from the 2DES plane. Such conditions make quantitative comparison with experimental results difficult.

Others resorted to phenomenological calculations employing either of these DOS shapes: Gaussian \[10\], Lorentzian \[3, 14\], exponential \[14\] or Gaussian with a background \[2, 3, 5\]. These depend on a single broadening parameter \( \Gamma \), which according to the short range model is inversely proportional to \( B^{1/2} \) \[1\]. Several researchers have made data fits to obtain the DOS using this \( B \) dependence on \( \Gamma \) \[2, 3, 11\]. However, the DOS obtained using \( \Gamma \propto B^{1/2} \) is found to be four times smaller than measurements in 2DES in GaAs/AlGaAs heterostructures \[10\]. Furthermore, the Landau level widths are predicted to oscillate due to the fluctuations in the screening of the disorder by the electrons in the 2DES \[15, 16\]. Experimental data from
multiple-quantum-well structures are consistent with a \( \Gamma \) oscillating as a function of \( 1/B \). Although there is yet no consensus on the \( B \)-dependence of \( \Gamma \), it is still advantageous to use a broadening parameter as it contains all the collective temperature- and magnetic field-dependence of the 2DES. This then makes comparison with experimental data of macroscopic properties easier.

Despite the lack of a complete theory for the Landau level broadening caused by impurity disorder, information on the effects of the broadening on the thermodynamic quantities will provide further insights on the scattering mechanisms present. In this work, the behavior of the specific heat and the magnetization as functions of \( \Gamma \) and \( T \) is obtained through a phenomenological approach. Here a Gaussian-shaped DOS is employed. This form is utilized since it is a commonly used model for experimental data and has been found to fit better than other semi-elliptical functions and long-ranged models \([2]\). Using various broadening parameters, the above mentioned physical quantities are calculated for a single sub-band in temperatures in the order of \( 0.3 \) K to \( 4.2 \) K and magnetic fields in the order of \( 0.5 \) T to \( 15 \) T.

2. The Simulation Model

The specific heat capacity \( C_V \) and the magnetization \( M \) of two-dimensional electron systems are both functions of the magnetic field \( B \) and temperature \( T \). The specific heat at constant volume of an electron gas is given as

\[
C_V(B,T) = \frac{\partial}{\partial T} \int_0^{\infty} f(E,\mu,T)(E-\mu)D(B,E)dE, \quad (1)
\]

where \( f(E,\mu,T) = 1/(1+\exp[(E-\mu)/k_BT]) \) is the Fermi-Dirac distribution function, \( \mu = \mu(B,T) \) is the chemical potential, \( k_B \) is the Boltzmann’s constant, and \( D(B,E) \) is the density of states.

Similarly, the magnetization for an electron gas having no spin splitting can be obtained from the free energy \( F \),

\[
M(B,T) = -\frac{\partial F}{\partial B} \bigg|_{N=\text{constant}}, \quad (2)
\]

where

\[
F = \mu N - k_BT \int_0^{\infty} D(B,E) \ln \left(1 + \exp \left[\frac{\mu - E}{k_BT}\right]\right) dE, \quad (3)
\]

where \( N \) is the electron concentration. The behavior of these magneto-thermodynamic properties can be determined once the chemical potential and the density of states are known.

The chemical potential \( \mu \) can be obtained via a root-finding method for a fixed value of \( N \). This condition is not a problem since experiments on 2DES usually keep \( N \) constant. For a given \( T \) and \( B \), the value of \( \mu \) is sought that provides the least percentage error of not more than 0.001\% from the set value of the electron concentration in

\[
N = \int_0^{\infty} f(E,\mu,T)D(B,E)dE. \quad (4)
\]

In this work, \( N \) is set equal to \( 3.6 \times 10^{11} \) cm\(^{-2}\). This value is chosen to be of the same order of magnitude of high mobility 2DES samples as found in experimental reports \([2, 3, 4, 5, 10, 12]\).

A number of experiments support a Gaussian density of states \( D(B,E) \) model for two-dimensional electron systems \([2, 3, 4, 5]\). The density of states at an energy \( E \) in its Gaussian form is given as

\[
D(B,E) = \frac{eB}{\pi \hbar} \sum_n \frac{1}{\sqrt{2\pi}\Gamma} \exp \left[\frac{-\left(E - E_n\right)^2}{2\Gamma^2}\right], \quad (5)
\]

where \( e \) is the electron charge, \( \hbar = h/2\pi \) and \( h \) is Planck’s constant. The \( n \)th Landau level is given as \( E_n = (n + \frac{1}{2})\hbar \omega_c \), where \( \omega_c = eB/m^* \) is the cyclotron frequency. The effective mass \( m^* \) used here is that of carriers in a GaAs/AlGaAs heterostructure which is equal to 0.0667 \( m_e \), where \( m_e \) is the mass of the electron. Some experimental data fits use Eq. (5) with an additional constant background obtained from a fraction of the zero-field DOS \([2, 3]\). This fraction provides an adjustable parameter dependent on experimental factors. However, depending on \( B \), Eq. (5) is found to produce Gaussian peaks with overlap between Landau levels. This overlap acts as a sort of a uniform background. Therefore, there is no need to introduce an additional term to Eq. (5).

In our simulations, we investigate the case when the broadening parameter \( \Gamma \) is set to (i) fixed values, (ii) a function of \( \gamma B^{1/2} \) where \( \gamma = 0.01 \) meV/T\(^{1/2} \), and (iii) an oscillating function of the filling factor \( \nu \) similar to Ref. \([4]\) such that

\[
\Gamma(B) = 0.704B^{1/2} + 0.296B^{3/2}[1.8\cos^2(2\pi\nu) - 1], \quad (6)
\]

where \( \nu = hN/eB \). For constant \( \Gamma \), only the results for \( \Gamma = 0.2 \) meV and 1.0 meV will be shown.

3. Results and Discussion

In the absence of Landau level broadening, the chemical potential exhibits oscillations as a function of \( B \) with sharp peaks occurring at even filling factors \( \nu \). Since the last occupied Landau level is fully filled for even \( \nu \), this implies that one needs to go to the next higher level to add one more electron. On the contrary, an odd \( \nu \) corresponds to a last occupied level filled to half its degeneracy. Adding an electron will not change \( \mu \). Therefore, at odd \( \nu \) the chemical potential is a constant.

The sharp oscillations of \( \mu(B,T) \) are reduced and softened upon the introduction of broadening as shown in Fig. [1] for \( T = 4.2 \) K at \( \Gamma = 0.2 \) meV. A similar effect was ob-
The chemical potential with broadened levels using $\Gamma = 0.2$ meV at different temperatures. The Landau levels are located on the diagonal slopes of the saw-tooth like oscillations.

Fig. 1. The chemical potential with broadened levels using $\Gamma = 0$ meV at different temperatures. The Landau levels are located on the diagonal slopes of the saw-tooth like oscillations.

The Landau level broadening is reflected in the specific heat as spikes at the low field region. At $\nu > 4$, the narrow peaks occur at magnetic fields when the chemical potential crosses from one Landau level to the next. These crossings are known as interlevel excitations. As $B$ increases, the Landau levels are farther apart and it becomes more difficult to have interlevel excitations. This is reflected in the diminishing peak heights of $C_V$ at $\nu = 8$ and $\nu = 6$ in Fig. 2. Until finally, $C_V$ vanishes at $\nu = 2$. This is comparable with the results in Ref. 15 where interlevel excitations appear as small structures near the minima in the $C_V$ oscillations at high $B$.

At high $B$ ($\nu \leq 4$), Landau levels are far apart and the degree of degeneracy within a broadened level is high. Excitations brought about by an increase in $B$ are limited within the last occupied level. In this $B$ region the intralevel contributions dominate [14, 17]. For example, $C_V$ exhibits a wide peak at $2 < \nu < 4$.

This general trend of competing dominance between interlevel and intralevel contributions of $C_V$ with $B$ presented in Fig. 2 is conserved regardless of the broadening parameter used. An increase in magnitude of the constant $\Gamma$ appears in $C_V$ as widening of the interlevel contributions and narrowing of the intralevel excitations. The latter is easily observed in the region $2 < \nu < 4$ wherein the change of $C_V$ has a slower slope with $B$. This is consistent with the fact that a wider Landau level corresponds to a slower variation of the DOS with energy.

When $\Gamma$ oscillates with $B$ one obtains a similar trend for $C_V$ except for slight variations in the region where intralevel contributions dominate. Although not shown here, the $C_V$ result for $\Gamma = 0.2$ meV at $T = 4.2$ K.

The influence of the temperature on the specific heat as $T \rightarrow 0$ is illustrated in Fig. 3. For clarity, we will only show here the result for $T = 4.2$ K and $T = 0.3$ K. Reducing $T$ from 4.2 K minimizes the interlevel contributions and, consequently, the magnitude of the sharp peaks at $B < 4$ T diminishes. Until at $T = 0.3$ K, the effect of $\Gamma$ vanishes. This shows that despite the presence of broadened Landau levels excitations between them are no longer possible. Only intralevel excitations contribute to $C_V$ which is a characteristic of an ideal 2DES. At even $\nu$, where $\nu \geq 6$ for $\Gamma = 0.2$ meV, $C_V$ changes from a maximum when $T = 4.2$ K to a minimum when $T = 0.3$ K. This crossover is likewise found here for $\Gamma = 1.0$ meV at $\nu \geq 4$ and in Ref. 9 for the case when $\Gamma \propto B^{1/2}$.

The corresponding influence of $T$ on $M$ is shown in Fig. 4. Orbital magnetization in 2DES with electron-impurity scattering do not have sharp spikes with $B$ according to a self-consistent theory. Moreover, experimental evidence of the de Haas-van Alphen effect displays rounding at the extrema of the $M$ oscillations which are attributed
is observed at ent fractional filling factors is illustrated in Fig. 5. A pea k broadening are felt.

regime where interlevel excitations and the Landau level ture signifies the shift from an ideal 2DES behavior to the form of the broadening, even as large as $\Gamma = 1$ is that these sharp oscillations are observed regardless of the form of $\Gamma$. The broadening only plays a role in the width of the oscillations with $B$ for each interlevel and intralevel contribution. A characteristic temperature is observed such that the 2DES approaches its ideal electron gas behavior. Below this characteristic temperature, the Landau level broadening no longer affects the behavior of the thermodynamics properties. This indicates that scattering mechanisms present in 2DES are suppressed below $T_p$. The magnitude of $T_p$ is obtained here using the electron concentration and effective mass value for a GaAs system. It would be interesting to know if experiments are able to measure $T_p$. If not, this should be a ground for theoretical studies to find out why $T_p$ is masked in experiments.

4. Conclusions

The effect of Landau level broadening and temperature is considered in two-dimensional electron systems using a Gaussian-shaped density of states. The broadening parameters considered took different forms such as different constant values, a square-root dependence on the magnetic field and an oscillating function with respect to the filling factor. The observed $B$ behavior of the chemical potential, specific heat and magnetization show the same trend regardless of the form of $\Gamma$. The broadening only plays a role in the width of the oscillations with $B$ for each interlevel and intralevel contribution. A characteristic temperature is observed such that the 2DES approaches its ideal electron gas behavior. Below this characteristic temperature, the Landau level broadening no longer affects the behavior of the thermodynamics properties. This indicates that scattering mechanisms present in 2DES are suppressed below $T_p$. The magnitude of $T_p$ is obtained here using the electron concentration and effective mass value for a GaAs system. It would be interesting to know if experiments are able to measure $T_p$. If not, this should be a ground for theoretical studies to find out why $T_p$ is masked in experiments.

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References
[1] T. Ando, A.B. Fowler, and F. Stern. *Rev. Mod. Phys.*, 54 (1982) 437–672.
[2] M.A. Wilde, M.P. Schwarz, Ch. Heyn, D. Heitmann, D. Grundler, D. Reuter, and A.D. Wieck. *Phys. Rev. B*, 73 (2006) 125325.
[3] M. Zhu, A. Usher, A. Matthews, A. Potts, M. Elliott, W. Herrenden-Harker, D. Ritchie, and M. Simmon. *Phys. Rev. B*, 67 (2003) 155329.
[4] J.K. Wang, D.C. Tsui, M. Santos, and M. Shayegan. *Phys. Rev. B*, 45 (1992) 4384–4389.
[5] E. Gornik, R. Lassnig, G. Strasser, H.L. Störmer, A.C. Gossard, and W. Wiegmann. *Phys. Rev. Lett.*, 54 (1985) 1820–1823.
[6] Q. Li, X.C. Xie, and S. Das Sarma. *Phys. Rev. B*, 40 (1989) 1381–1384.
[7] S. Das Sarma. *Phys. Rev. B*, 23 (1981) 4592–4596.
[8] S. Das Sarma. *Solid State Commun.*, 36 (1980) 357–360.
[9] A.H. MacDonald, H.C.A. Oji, and K.L. Liu. *Phys. Rev. B*, 34 (1986) 2681–2689.
[10] J.P. Eisenstein, H.L. Stormer, V. Narayanamurti, A.Y. Cho, A.C. Gossard, and C.W. Tu. *Phys. Rev. B*, 55 (1985) 875–878.
[11] V. Mosser, D. Weiss, K. von Klitzing, K. Ploog, and G. Weimann. *Solid State Commun.*, 58 (1986) 5–7.
[12] T.P. Smith, P.J. Stiles B.B. Goldberg, and M. Heiblum. *Phys. Rev. B*, 32 (1985) 2696–2699.
[13] S. Das Sarma and X.C. Xie. *Phys. Rev. Lett.*, 61 (1988) 738–741.
[14] B. Özdemir, Z. Yakar, and M. Özdemir. *Turk. J. Phys.*, 28 (2004) 1–15.
[15] X.C. Xie, Q.P. Li and S. Das Sarma. *Phys. Rev. B*, 42 (1990) 7132–7147.
[16] K. Esfarjani, H.R. Glyde and V. Sa-yakanit. *Phys. Rev. B*, 41 (1990) 1042–1053.
[17] W. Zawadzki and R. Lassnig. *Solid State Commun.*, 50 (1984) 537–539.
[18] N.W. Ashcroft and N.D. Mermin. *Solid State Physics*. Saunders College Publishing, Fort Worth, 1976.
[19] T. Chakraborty and P. Pietiläinen. *Phys. Rev. B*, 55 (1997) R1954–R1957.