Towards the definition of a reference ideal radiator for the assessment of heat emission efficiency in buildings

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Abstract. Radiators, as the primary devices used in space heating, cover a fundamental role in the energy efficient use of buildings. In the search of an optimal configuration, from both energy consumption and thermal comfort viewpoints, comparison of different types and emitter size is crucial. However, an ideal heater with the highest maximum heat emission efficiency that can be used as a baseline still needs to be defined. This study contributes to the development of the heat emission standard EN 15316-2-1:2007. We perform an analysis of heat transfer in enclosures with a 21- and 10-type panel radiators, using an ideal convector as a reference heater. Accounting for room side effects of thermal radiation, we compute the induced operative temperature both analytically and numerically. The numerical simulations are carried out for a European reference room specified in the CEN TC130 standardisation committee, introducing a number of assumptions for defining the benchmark radiator. For instance, back wall losses are neglected for ideally isolated emitters.

First we find that, contrary to what might be expected, the convector is the worst performing heater. On the other hand, studying analytically the operative temperature as a function of radiator type and size, we identify the most performing heater in the 10-type.

Furthermore, our analytical solution highlights a simple predictive method for computing the operative temperature for any panel size, in complete generality. As an example, we provide a series of analytical formulas for calculating the operative temperature of the radiators addressed in this study. The results of this work therefore provide a useful background for a precise and rigorous definition of an ideal radiator.

1. Introduction

Space heating constitutes the primary source of energy consumption in the building sector [1, 2, 3]. Heating can be provided by several different types of emitters, such as panel radiators, convectors, underfloor, ceiling and air heating. It is generally understood that each type of system can impact the energy demand very differently [4, 5, 6, 7, 8].

In particular, panel radiators are a common solution to heating demands in residential buildings. Assessment of their performance on both the view points of energy consumption and thermal comfort has produced a rich literature, mostly concentrated on experimental or numerical methods [9, 10, 11, 12, 13]. Such investigations are necessarily involved, as a large number of factors determine energy consumption and thermal comfort. These range from heater design to their specific type and location in the room (e.g. close to a window or slightly detached from a wall) [4, 13, 14, 15]. For example, laboratory measurements [16] favour low temperature panel radiators instead of conventional heaters with a higher heating curve. Also, serial and parallel connected radiators provide different results; for
instance, recent experimental tests found a better performance of serial-connected 11-type and 22-type
[17].

In this paper we address the issue of defining an ideal radiator with the highest maximum heat
emission efficiency, which should then be used as a reference baseline. To our knowledge, such a
benchmark heater is not yet present in the literature [18]. Specifically, we examine the performance
of 10- and 21- type panel radiators with heat output $\sim 134$ W, in an average enclosure provided by the
Standard CEN TC130. Keeping the heat output constant, we study how different panel sizes affect the
thermal comfort in the room; this is a non-trivial effect, as the surface temperature decreases with size.
This is accomplished by means of the operative temperature $t_{op}$, defined as the uniform temperature of
an enclosure in which an occupant would exchange the same amount of heat by radiation and convection
as in the existing non-uniform environment [19]. Such quantity is here computed directly from the mean
radiant temperature [15, 19, 20] in two ways: numerically, by performing simulations in steady state
with the software IDA ICE [21], and analytically, following the standard ISO 7726 [19].

With these two methods, we obtain operative temperature curves in function of the panel size for both
the 21- and 10-type. Comparison of our $t_{op}$ values with the one of an ideal convector (100% convection)
with the same heat output shows that, contrary to what might be expect, the ideal heater cannot be
identified with the convector, which is outclassed by both the 21- and 10-type panel radiators. Rather, we
find the 10-type to be the most performing. This restricts the range of candidates and sets some ground
for future developments.

Additionally, our analytical study introduces a simple, yet general and predictive method for
computing the operative temperature for any panel size and radiator type. For the case at hand, we
list a collection of analytical formulas for calculating $t_{op}$ for radiators of 10- or 21- type, with panel size
in the range considered and no back wall losses.

The paper is structured as follows: in Section 2 we describe our setup and how IDA ICE and
the analytical method compute the operative temperature, then in Section 3 we provide plots and
performance curves for each radiator, identifying the most performing emitter and describing the
predictive method for $t_{op}$. Our results are finally summarized in Section 4.

2. Method

In this study we consider a room with a single external wall and adiabatic internal walls, floor and ceiling
under steady-state conditions (Table 1). Thermal layer properties and room dimensions are chosen
according to CEN TC130 European Committee specification, namely the U-value of external wall is
0.25 W/m2K. A 21- and 10-type panel radiators with the same heat output $\sim 134$ W are then used in IDA
ICE simulations to offset the heat loss through the external wall.

We consider panel radiators of varying size (figure 2) and type (10 and 21-type radiators) installed in
the middle of the external (cold) wall, at 15cm from the floor. Catalogue values of a well-known panel
radiator manufacturer are used as input values for the characteristic equation in the IDA ICE radiator
model, namely the nominal heat output $\phi_n$ and power law exponent $n$. A supply temperature of 40°C is
used for both types of the radiator.

The wall portion directly behind the radiator is modelled as an ideal insulator to eliminate back wall
losses. Accordingly, the $U$-value of the external wall is modified as the size of the radiator changes, so
that the overall $U.A$-value of the wall and according heat loss stay constant.

The software calculates numerically, with the finite difference method, the surface temperatures of
radiator and enclosure walls, together with the operative temperature for a person sitting in the centre
of the room (figure 1). We locate the calculation point at 0.6m above the floor in the room centre (i.e.
at 2m distance from each wall) [19]. The most performing heater configuration will then be the one for
which the operative temperature is maximal, with the same heater nominal output.

The surface temperatures obtained with IDA ICE are then implemented into an analytical computation
of the operative temperature $t_{op}$, which follows the ISO 7726 standard. For a fixed radiator height $h$, we
obtain an expression for $t_{op} = t_{op}(w)$ that is a function of the width $w$, then study its behaviour in the
Table 1. Boundary conditions for steady-state simulation.

| Parameter                      | Value      |
|--------------------------------|------------|
| Indoor air temperature         | 20 °C      |
| External air temperature       | -15°C      |
| External relative humidity     | 85%        |
| Direct normal irradiance       | 0 W/m²     |
| Diffuse horizontal irradiance  | 0 W/m²     |

**Figure 1.** Room layout.

**Figure 2.** Panel radiator setup on the external wall.

range considered by the simulations.

The numerical and analytical methods compute the operative temperature differently. IDA ICE evaluates it as the arithmetic average of air temperature $t_{air}$ and mean radiant temperature $\bar{t}_r$ [21],

$$t_{op} = \frac{t_{air} + \bar{t}_r}{2},$$

(throughout this paper, $[t_i]=[°C]$ and $[T_i]=[K]$). This differs from the exact definition given in the ISO 7726 [19],

$$t_{op} = \frac{h_c t_{air} + h_r \bar{t}_r}{h_c + h_r} \equiv A t_{air} + (1 - A) \bar{t}_r,$$

where the average is weighted by the radiation and convection heat transfer coefficients $h_r$ and $h_c$ at the calculation point, as explained in e.g. [15].

Another difference between the analytical solution (that follows the ISO standard) and IDA ICE consists of a unique way of the latter for computing the mean radiant temperature $t_r$. The numerical software considers the radiator heat flux only on the surfaces that are parallel to the principal calculation surface; $\bar{t}_r$ is then obtained as the average of mean radiant temperatures from the six principal directions, weighted by the respective view factors.

In contrast, the ISO 7726 prescribes that for each direction one considers the radiator heat flux on
both parallel and perpendicular surfaces, calculating the mean radiant temperature [19],

\[
\bar{t}_r = 4 \sqrt{\frac{6}{\sum_{i=1}^{6} \beta_i T_{pr}^{(i)}}} - 273.15, \tag{3}
\]

from the plane radiant temperatures \(T_{pr}^{(i)}\), which are computed from the six surface temperatures \(T^{(i)}_{sj}\) and the view factors in all the six directions. \(\bar{t}_r\) is therefore a weighted average over the projected area factors of a person \(\beta_i = 0.18, 0.22, 0.30\) in the vertical, lateral and frontal directions respectively [19].

Throughout this paper, all the analytical results are calculated with Eqs. (2) and (3), and follow the ISO standard completely. We will show that \(t_{op}\) values computed with both methods do not differ remarkably; however, our conclusions are drawn mostly from the analytical results, to fully comply with ISO 7726 and to account for the full physical processes. The only point in common with IDA ICE consists of the surface temperatures \(T^{(i)}_{sj}\), which are written as polynomial interpolations from the data provided by the software. Since both the definitions of \(\bar{t}_r\) and \(t_{op}\) are different, the analytical operative temperatures are independent of the numerical ones: the interpolations are implemented for having a more general form of the operative temperature than by using the raw data for the surface temperatures. This way, instead of calculating \(t_{op}\) for each point, we can write \(t_{op} = t_{op}(x)\) and obtain more general and predictive results, which are described in the next section.

3. Results

We first notice, as illustrated in figures 3 and 4, that it is not advisable to assess the performance in relation to the area only. The \(t_{op}\) values show that the efficiency varies with height, given the same area.

More importantly, a reference ideal convector (100% convection and 0% radiation) returns the lowest operative temperature out of the three. It therefore cannot be identified with a hypothetical "ideal" heater.

Operative temperatures for fixed heights are plotted in figures 5 and 6 for a 10-type and a 21-type panel radiator, respectively (some values for the 10-type are missing, as it could not reach the required power output of about 134W). Overall, the numerical and analytical solution do not differ appreciably. This is mostly evident for the 21-type radiator, while for the 10-type we see a slightly larger difference. Remarkably, the 10-type is the most performing heater: \(t_{op}\) values are constantly larger by \(\sim 0.1^\circ\text{C}\), approaching for \(h = 0.9\)m the air temperature 20°C. Namely, for the study at hand, the 10-type can be regarded as our "ideal" radiator.

Furthermore, it comes clearly that for a given width, the \(t_{op}\) values are linearly distributed along
different heights \( h \). This can be proven by using an analytical method first introduced in [22]; interpolating by least squares the operative temperature versus the height for a fixed width, we obtain the curves listed in Table 2 for both radiators. These can be formally written as

\[
t_{\text{op}}(h, w) = A(w)h + B(w),
\]

which returns the operative temperature for any desired height in the range \( 0.3 \text{m} \leq h \leq 0.9 \text{m} \).

On the other hand, for a given height the \( t_{\text{op}} \) values are not linearly distributed along different widths \( w \): they actually take the quadratic form

\[
t_{\text{op}}(h, w) = A(h)w^2 + B(h)w + C(h).
\]

Specifically, as it is shown in Table 3, \( A(h) < 0 \) for any height. Since instead \( t_{\text{op}} \) grows linearly as the width increases \( (A(w) > 0) \), this proves rigorously that the operative temperature is more dependent on the height than on the width, as expected for this situation on physical grounds.

Finally, one can prove that Eqs.(4) and (5) are equivalent, namely by substituting one value for \( h \) and \( w \) they return the same \( t_{\text{op}} \) (discrepancy of order \( \sim 0.001 \), around 0.02%).

Further conclusions can be made by means of the analytical solution. As an example, consider the 21-type radiator. In general, the explicit form of the operative temperature is going to be highly non linear, but plotting the first derivative \( D_{t_{\text{op}}} \equiv dt_{\text{op}}/dw \) with respect to the width provides with additional useful information. Figure 7 shows that while the increase of \( t_{\text{op}} \) with \( w \) is generally dropping, this happens first linearly, until a "plateau" starts at \( w \sim 1 \text{m} \), ending at about \( 2 \text{m} \). Here the decrease with \( w \) is less steep, providing a range \( \Delta w \) where \( t_{\text{op}} \) is optimised with respect to width increase.

In other words, there exist widths which are most advantageous. Investigating the second and third derivatives \( D^2t_{\text{op}} \equiv d^2t_{\text{op}}/dw^2 \) and \( D^3t_{\text{op}} \equiv d^3t_{\text{op}}/dw^3 \), we locate the beginning and ending of the plateau precisely at \( w = 0.87 \text{m} \) and \( w = 1.86 \text{m} \), respectively. The latter point corresponds to a minimum of \( D^3t_{\text{op}} \), which identifies a change of concavity in \( D^2t_{\text{op}} \). Moreover, the second derivative also gives the exact point for which the increase is minimal, at \( w = 2.736 \text{m} \). Notice how a similar value holds for \( h = 0.9 \text{m} \), figure 8.
Figure 7. First derivative, analytical $t_{op}$ for a 21-type radiator, $h = 0.3$ m.

Figure 8. First derivative, analytical $t_{op}$ for a 21-type radiator, $h = 0.9$ m.

Table 2. Operative temperature $t_{op}$(°C) in function of panel width and height.

| Height (m) | 10-type | 21-type |
|-----------|---------|---------|
| 0.30      | $-0.0046w^2 + 0.0357w + 19.833$ | $-0.0031w^2 + 0.0216w + 19.766$ |
| 0.45      | $-0.0066w^2 + 0.0494w + 19.839$ | $-0.0049w^2 + 0.0355w + 19.773$ |
| 0.60      | $-0.0069w^2 + 0.053w + 19.851$ | $-0.0054w^2 + 0.042w + 19.778$ |
| 0.90      | $-0.0111w^2 + 0.0786w + 19.859$ | $-0.007w^2 + 0.0573w + 19.788$ |

| Width (m) | 10-type | 21-type |
|-----------|---------|---------|
| 0.60      | $0.0593h + 19.765$ | -       |
| 0.80      | $0.0773h + 19.761$ | -       |
| 1.20      | $0.1079h + 19.84$  | $0.0983h + 19.763$ |
| 1.60      | $0.1268h + 19.842$ | $0.1101h + 19.764$ |
| 2.00      | $0.1404h + 19.846$ | $0.125h + 19.764$  |
| 2.60      | $0.1489h + 19.853$ | $0.1408h + 19.764$ |
| 3.00      | $0.1403h + 19.86$  | $0.1509h + 19.763$ |

Table 3. Formulas for the coefficients entering Eqs.(4) and (5).

|                | 10-type | 21-type |
|----------------|---------|---------|
| $A(h)$        | $-0.1074h^3 + 0.1828h^2 - 0.1045h + 0.0132$ | $-0.0061h - 0.0017$ |
| $B(h)$        | $0.6012h^3 - 1.0361h^2 + 0.6114h - 0.0707$ | $0.3025h^3 - 0.5728h^2 + 0.3929h - 0.0529$ |
| $C(h)$        | 19.846  | $0.0741h^3 - 0.1444h^2 + 0.1233h + 19.74$ |
| $A(w)$        | $-0.0234w^2 + 0.1174w + 3 \times 10^{-5}$ | $-0.0084w^2 + 0.0667w + 0.0254$ |
| $B(w)$        | $0.0039w^2 - 0.0054w + 19.841$ | 19.763 |

4. Conclusions

In the present study we addressed the performance of 10- and 21-type panel radiators using both an analytical and a numerical approach, in the search for identification of an ideal radiator. We performed
a rigorous investigation on the operative temperature sensed by a user sitting in the middle of a test room of average size, in compliance with the standard CEN TC130. The overall behaviour of operative temperature \( t_{op} \) as a function of radiator size and type was determined quantitatively with two different methods: the numerical simulation software IDA ICE and an analytical computation based on the standard ISO 7726.

Comparison of the two radiators with an ideal convector providing the same output \( \sim 134 \text{W} \) shows that the most performing emitter is the 10-type. For instance, a common panel size with \( h = 0.6 \text{m} \), \( w = 1.6 \text{m} \) returns \( t_{op} = 19.83\text{°C} \) and \( t_{op} = 19.92\text{°C} \) resp. for 21- and 10-type, while \( t_{op} = 19.69\text{°C} \) for the convector.

For larger sizes, the 10-type radiator even reaches 20°C operative temperature, which is equal to the air temperature. In other words, it constitutes our ”ideal” radiator for the setup considered.

Overall, we found that larger panels provide a higher \( t_{op} \), as naturally expected; nevertheless, since the surface temperature of the panel, with same power output, decreases with size, the operative temperature grows linearly only with height \( h \). The increment along the width \( w \) is instead less pronounced, due to a negative quadratic term \( \propto -w^2 \) in \( t_{op}(h,w) \). In other words, the performance of radiators is more sensitive to the height than to the width. The formulas listed in Tables 2 and 3 allow to quantify this behaviour precisely.

Furthermore, the analytical study highlights various features. The performance increment with larger panels is evident mostly with 10-type panels, where it approaches a linear behaviour. For the 21-type radiator we found instead that for small \( h < 0.6 \text{m} \) there is no substantial increase in the operative temperature for \( w \gtrsim 2 \text{m} \). Accordingly, an optimised 21-type panel radiator should lie within this range. We have thus shown that for smaller \( h \) there exists a width interval for which adopting larger panels is most profitable.

More generally, through our analytical calculation we could highlight a simple predictive method for computing the operative temperature for any panel size and room configuration, in complete generality. In particular, this paper provides precise analytical formulas for calculating \( t_{op} \) for radiators of 10- or 21-type with dimensions \( 0.3 \text{m} \leq h \leq 0.9 \text{m} \) and \( 1.2 \text{m} \leq w \leq 3 \text{m} \), assuming no back wall losses, in a room corresponding to the Standard CEN TC130. It would then be very interesting to extend the above results and method to a more standard situation, e.g. adding back wall losses to assess their impact on our quantitative results.

Finally, we should remark that this study constitutes only an early attempt in formalizing the heating performance studies of radiators; it can and should be extended to other types of emitters and to parametric studies on the more general impact of view factors, room and heater geometry. These should eventually aim to identify a more general pattern, for practical utility and towards a rigorous definition of an ideal or benchmark emitter for each relevant case.

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References
[1] Serrano S, Ürge Vorsatz D, Barreneche C, Palacios A and Cabera L F 2017 Energy 119 425 – 434 ISSN 0360-5442
[2] D’Agostino D, Cuniberti B and Bertoldi P 2017 Data in Brief 14 759 – 762 ISSN 2352-3409
[3] Yao R and Steemers K 2005 Energy and Buildings 37 663 – 671 ISSN 0378-7788
[4] Olesen B W, Mortensen E, Thorshauge J and Berg-Munch B 1980 ASHRAE transactions 86 34–48
[5] Inard C, Meslem A and Depecker P 1998 Building and Environment 33 279 – 291 ISSN 0360-1323
[6] Olesen B W and de Carli M 2011 Energy and Buildings 43 1040 – 1050 ISSN 0378-7788 tackling building energy consumption challenges - Special Issue of ISHVAC 2009, Nanjing, China
[7] Léger J, Rousse D R, Borgne K L and Lassue S 2018 Building and Environment 128 161 – 169 ISSN 0360-1323
[8] Maivel M, Ferrantelli A and Kurnitski J 2018 Energy and Buildings 166 220 – 228 ISSN 0378-7788
[9] Hasan A, Kurnitski J and Jokiranta K 2009 Energy and Buildings 41 470 – 479 ISSN 0378-7788
[10] Shati A, Blakey S and Beck S 2011 Energy and Buildings 43 400 – 406 ISSN 0378-7788
[11] Munaretto F, Recht T, Schalbart P and Peuportier B 2017 Journal of Building Performance Simulation 0 1–22
[12] Sevilgen G and Kılıç M 2011 Energy and Buildings 43 137 – 146 ISSN 0378-7788
[13] Jahanbin A and Zanchini E 2016 Applied Thermal Engineering 105 467 – 473 ISSN 1359-4311
[14] Ali A H H and Gaber Morsy M 2010 Energy Efficiency 3 283–301 ISSN 1570-6478
[15] Kalmár F and Kalmár T 2012 Energy and Buildings 55 414 – 421 ISSN 0378-7788
[16] Maivel M and Kurnitski J 2014 Energy and Buildings 69 224 – 236 ISSN 0378-7788
[17] Vösa K V, Ferrantelli A, Kull T M and Kurnitski J 2018 Applied Thermal Engineering 132 531 – 544 ISSN 1359-4311
[18] EN 442-1:2014 Radiators and convectors. Technical specifications and requirements. Standard. CEN. Bruxelles, BE.
[19] ISO 7726:1998 Ergonomics of the thermal environment - Instruments for measuring physical quantities. Standard. International Organization for Standardization. Geneva, CH.
[20] d’Ambrosio Alfano F R, Dell’Isola M, Palella B I, Riccio G and Russi A 2013 Building and Environment 63 79 – 88 ISSN 0360-1323
[21] IDA ICE - Indoor Climate and Energy. Tech. rep. EQUA Stockholm, Sweden, 2013.
[22] Ferrantelli A, Ahmed K, Pylsy P and Kurnitski J 2017 Energy and Buildings 143 53 – 60 ISSN 0378-7788