Method for generating mesostructure of soil-rock mixture based on Minkowski sum

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Abstract. The soil-rock mixture is widely distributed in nature, and the mechanical properties of it is a key concern in geotechnical engineering field. The mechanical response of soil-rock mixture can be obtained using numerical approaches. As the precondition for numeric simulation, the mesostructure of soil-rock mixture is indispensable. The random rock packing is the main step in the construction of the mesostructure. The classical trial packing method are inefficient when the rock area fraction is high. Permissible positions to pack new rock can be predicted in the algorithm using entrance block and background mesh, resulting in dramatic improvement in efficiency and robustness. Entrance block and Minkowski sum can be transformed into each other, and there are more algorithms for Minkowski sum. Therefore, a random rock packing algorithm based on Minkowski sum is constructed, each rock can be packed successfully at one time, avoiding multiple trials in the traditional trial packing method and improving the efficiency of packing.

1. Introduction

The soil-rock mixture consists of rock and soil, which is not only widely distributed in nature but also used as engineering materials. The mechanical response of soil-rock mixture can be obtained using numerical approaches. The mesostructure of soil-rock mixture, as the precondition for numeric simulation, is indispensable. The random rock packing is the main step in the construction of the mesostructure model of soil-rock mixture, in which the rock packed later should not penetrate with the previously packed rock. The trial packing method[1-2] first randomly determines a location to try to pack rock, and then judges whether the rock overlaps with the previously packed rock. If there is a penetration, reselect a new location and try again. Repeated trials are inefficient when the rock area fraction is high. The layering disposition method[3] tends to produce more blank areas, thus reducing the maximum rock area fraction.

An algorithm using entrance block[4] and background mesh is proposed by Chen et al[5-6], in which permissible positions to pack new rock can be predicted, resulting in dramatic improvement in efficiency and robustness. Entrance block and Minkowski sum can be transformed into each other[7]. The concept of entrance block is relatively new and there are fewer algorithms for it, while there are more algorithms for Minkowski sum. Therefore, this paper introduces Minkowski sum into the random rock packing and uses the existing algorithm for Minkowski sum. A random rock packing algorithm based on Minkowski sum is constructed.
2. Polygon penetration judgment

2.1. Polygon penetration judgment based on entrance block

In order to accurately judge the contact between two blocks, Shi proposed the concept of the entrance block and formed the complete contact theory[4]. Therefore, the position relationship between two blocks is reduced to the position relationship between a reference point and the entrance block, the contact judgment of two blocks is converted into the inclusion judgment of a reference point and the entrance block.

Select any point $a_0$ in block $A$ as the reference point, the entrance block $E(A, B)$ of block $A$ and block $B$ is defined as

$$E(A, B) = B - A + a_0 = \{b - a + a_0 | a \in A, b \in B\}.$$  \hspace{1cm} (1)

where $a \pm b$ represents the addition and subtraction between the position vectors corresponding to point $a$ and point $b$. The entrance block $E(A, B)$ of block $A$ and block $B$ is illustrated in figure 1. In this paper, convex polygon is used to represent blocks (rock) in two-dimension.

![Figure 1. The entrance block $E(A, B)$ of block $A$ and block $B$.](image)

When the reference point $a_0$ is on the boundary $\partial E(A, B)$ of the entrance block $E(A, B)$, block $A$ and block $B$ contact with each other (Figure 1 shows this state); when the reference point $a_0$ is located outside the entrance block $E(A, B)$, block $A$ and block $B$ are separated with each other (Figure 2(a) shows this state); when the reference point $a_0$ is located inside the entrance block $E(A, B)$, block $A$ and block $B$ penetrate each other. The positional relationship between the reference point $a_0$ and the entrance block $E(A, B)$ is the direct and clear expression of the positional relationship between block $A$ and block $B$.

2.2. Polygon penetration judgment based on Minkowski sum

The Minkowski sum of two objects $A$ and $B$ in an Euclidian space was defined by the German mathematician Hermann Minkowski (1864-1909) as the position vector addition of each point $a$ in $A$ and each point $b$ in $B$

$$A \oplus B = \{a + b | a \in A, b \in B\}.$$  \hspace{1cm} (2)

By comparing equations (1) and (2), we can immediately see that the entrance block $E(A, B)$ is actually the Minkowski sum $B \oplus \bar{A}$, $E(A, B) = B \oplus \bar{A} = \bar{A} \oplus B$, where $\bar{A}$ is obtained through translating the centrally symmetric counterpart of $A$ by vector $a_0$, $\bar{A} = -A + a_0$, which is illustrated in figure 2.
Therefore, $\hat{A}$ is firstly obtained by changing $A$, and then the positional relationship between block $A$ and block $B$ can be determined by using the position relationship between the reference point $a_0$ and $B \oplus \hat{A}$. When the reference point $a_0$ is on the boundary $\partial (B \oplus \hat{A})$ of $B \oplus \hat{A}$, block $A$ and block $B$ contact with each other; when the reference point $a_0$ is located outside $B \oplus \hat{A}$, block $A$ and block $B$ are separated with each other (Figure 2(b) shows this state); when the reference point $a_0$ is located inside $B \oplus \hat{A}$, block $A$ and block $B$ penetrate each other.

Before the position relationship judgment, Minkowski sum $B \oplus \hat{A}$ should be formed firstly. Minkowski sum algorithms include slope diagram algorithm\[8\], contributing vertices method\[9\] and so on. The concept in contributing vertices method is intuitive and simple, and it will be utilized to form Minkowski sum in this paper.

3. Random rock packing

3.1. Target region discretizing and boundary expanding

In order to quantitatively represent the position of rock in the target region, selecting a reference point in the rock, and the position of the rock is represented by the coordinate of the reference point. If the reference point is located at a certain point in the target region, the position of the rock is determined. A rock to be packed can be anywhere within the target region, and the reference point can be anywhere on the rock, so the reference point can be located at any point in the target region. There are an infinite number of such potential points. In order to facilitate the packing operation, the number of potential points needs to be limited. A background grid covering the target region is arranged to discretize it, and the background grid nodes within it are taken as the potential points. The red points in figure 3(b) are the potential points after the discretization of the target region in figure 3(a), that is, the initial potential points for the first rock.

The Minkowski sum algorithm based on contributed vertices is applicable to two convex polygons, so it is necessary to expand each boundary line of the target region into a convex polygon outside the
target region. In this paper, we chose to expand it into a rectangle. The four blue rectangles in figure 3(b) are the expansion of the four boundary lines of the target region in figure 3(a) to the outside of the region.

3.2. The first rock packing
According to the particle size requirement, the first rock polygon $A_1$ is generated randomly, and any point in it is selected as the reference point $a_{01}$. The yellow polygon on the right of figure 4 is $A_1$, and the blue point in it is $a_{01}$. Calculate $\bar{A}_1$ from $A_1$ and $a_{01}$, and then calculate the Minkowski sum of $\bar{A}_1$ and each boundary polygon. The four light blue convex polygons in figure 4 are the Minkowski sum polygons formed by $\bar{A}_1$ and the four boundary rectangles. According to section 2.2, if the selected point is located inside Minkowski sum, the packed rock and boundary block penetrate each other, so the potential points inside Minkowski sum must be deleted, so that the potential points are located outside Minkowski sum. The red points in figure 4 are the potential points for the first rock after the boundary Minkowski sum modification.

3.3. The other rock packing
Randomly generate the $i$-th rock polygon $A_i$ ($i > 1$), and select any point in it as the reference point $a_{0i}$, and calculate $\bar{A}_i$ from $A_i$ and $a_{0i}$. The yellow polygon on the right of figure 6(b) is $A_2$, and the blue point in it is $a_{02}$. The reference point of the $i$-th rock can be placed anywhere in the target region except for the previously packed $i$-1 rock, so the initial potential points of the $i$-th rock needs to be reset. The potential points within the previously packed $i$-1 rock should be deducted at this time. The red points in figure 6(a) are the potential points for the second rock in theory. But the Minkowski sum of $\bar{A}_i$ and the previously packed $i$-1 rock should be calculated later, and the potential points within the Minkowski sum will be deducted. The $k$-th rock $A_k$ is located in the Minkowski sum of $\bar{A}_i$ and $A_k$ ($0 < k < i$), so the subsequent deduction of the potential points within the Minkowski sum can ensure that the potential points in previously packed $i$-1 rock are deducted. In order to facilitate the operation, the potential points in previously packed $i$-1 rock are no longer deducted at this time. The red points in figure 6 are the potential points for the second rock in the algorithm.
Figure 6. The initial potential points for the second rock (a) in theory (b) in the algorithm.

Calculate the Minkowski sum of $A_i$ and each boundary polygon and the previously packed rock, delete the potential points in the Minkowski sum, and the potential points for the $i$-th rock can be obtained. The five light blue convex polygons in figure 7 are the Minkowski sum polygons formed by $A_2$ and the four boundary rectangles and the packed first rock, and the red points are the potential points for the second rock after the Minkowski sum modification.

Figure 7. The potential points for the second rock.

Figure 8. The result of the second packed rock.

Randomly select a point among the potential points of the $i$-th rock to locate the reference point $a_{0i}$, and then the $i$-th rock is successfully packed. The big blue point in figure 8 is the selected point, and the yellow polygon containing the big blue point is the second packed rock. It can be seen from the figure that it is completely within the target region and does not overlap with the boundary and the first packed rock.

Pack the other rock according to the method, and stop packing when the packing conditions are met (such as reaching the specified rock area fraction or there is nowhere to pack).

4. Implementation

Based on the algorithm described above, using C++, a system for generating a mesostructure of soil-rock mixture based on Minkowski sum was compiled. Using the system, a two-dimensional soil-rock mixture slope model and a two-dimensional soil-rock mixture dam model were generated respectively, as shown in figure 9 and figure 10 respectively.

The total length of the two-dimensional soil-rock mixture slope is 105m, the height is 40m, the step length is 30m, the step height is 20m, the slope gradient is 45°, the diameter of the circumscribed circle of the rock is 10m, the background grid size is 0.25m, and the rock area fraction is 0.20. Figure 9 shows three packed results. It can be seen from the figure that the size and shape of the generated rock and the distribution of rock are random.

Figure 9. Three packed results of a two-dimensional soil-rock mixture slope.
The bottom length of the two-dimensional soil-rock mixture dam is 5m, the top length is 2m, and the height is 2m. The slope on the left is 60°, and the slope on the right is 45°. The diameter of the circumscribed circle of the rock is 0.25m, and the background grid size is 0.0125m. Figure 10(a-b) shows the packed results of three kinds of rock area fraction of 0.10, 0.30. Figure 10(c) shows the packed result of the maximum rock area fraction, which is 0.56.

Figure 10. Three packed results of a two-dimensional soil-rock mixture dam of rock area fraction of (a) 0.10, (b) 0.30, (c) 0.56.

5. Conclusion
The relationship between the entrance block and the Minkowski sum is described in this paper. The Minkowski sum is introduced into the random packing algorithm of the soil-rock mixture. The existing algorithm of Minkowski sum can be fully utilized to promote the study of soil-rock mixture. In this paper, a random packing algorithm based on Minkowski sum is proposed for soil-rock mixture, which can obtain the potential points for each rock, so that each rock can be packed successfully at one time, avoiding multiple trials in the traditional trial packing method and improving the efficiency of packing.

In addition, Minkowski sum is not only suitable for two-dimensional rock penetration judgment, but also for three-dimensional rock penetration judgment. The random packing algorithm for three-dimensional soil-rock mixture based on Minkowski sum will be studied in the future.

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