Dynamic model of the robotic vehicle motion on a deformable irregular terrain

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Abstract. The paper presents a system approach to the estimation of the off-road performance of the wheeled vehicles operating on deformable terrains. A problem-solving approach to the development of the model for the wheeled vehicle – irregular deformable terrain system is presented. The model was used for the simulation of the vehicle motion on different terrains. The presented method for the calculation of the wheel sinkage (calculation of the resistance to the motion of a wheeled vehicle) is based on the Y.S. Ageykin model. The paper presents the analysis of the research results.

Introduction

It is known that deformation and adhesion properties of the terrain surface is one of the main factors defining the type of the running gear, arrangement of the axles, type and properties of the suspension of a robotic wheeled vehicle. Relative deformations of the tire and the terrain affect the main aspect of the off-road performance – the wheel sinkage. On the other hand, the wheel sinkage significantly depends on the vertical load on the wheel axis. As the evidence from practice shows, additional loads can exceed the static loads by more than 100 % due to the dynamic loads caused by the terrain irregularities and time-varying deformation properties of the terrain. There is no ready method, algorithm or software for the analysis of the off-road performance parameters of the vehicle with dynamic loads on its systems, primarily, of the “wheeled vehicle – deformable irregular terrain” system.

Therefore, the problem of development, selection and analysis of the dynamic models of the real load conditions of the components and units of the vehicle sprung and unsprung parts taking into account conditions of the vehicle motion over the irregular deformable terrain is of engineering and scientific interest.

Research objective: development of spatial dynamic models of the wheeled vehicles moving on the deformable irregular terrain defined by its physical and mechanical non-linear properties.

Statement of the problem

The following assumptions based on the research in the field of modeling and simulation of the loading modes of the vehicle chassis and on the analysis of the dynamical models of the all-wheel-drive vehicles [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11] can be made:

1. Terrain deformation properties are estimated with the use of the method developed by Y.S. Ageykin [12, 13]. Vertical and shear deformations of the terrain are defined by its physical and mechanical state and depend on its type, moisture \( W \) and density \( \rho_C \). Depending on the physical state of the terrain,
the following parameters are calculated: stress-stain modulus $E$, angle $\varphi_0$ of internal shearing resistance, terrain cohesion $c_0$; the depth $H_G$ of the soft layer is set by the researcher.

2. Tire interaction with the terrain is described by a simulation model presenting the terrain as an elastoviscoplastic or viscoplastic body (Shvedov and Bingam models).

3. The vehicle is presented as a dynamic model consisting of the oscillating sprung and unsprung parts $M, m$ which are connected by means of elastic elements $c_{\text{susp}}, c_{\text{tyre}}$ and damping elements $k_{\text{susp}}, k_{\text{tyre}}$.

4. Stochastic road microprofile is defined by its spectral density $S_q(\theta)$. It is a random function based on the following assumptions: it is time independent and ergodic, microprofile ordinates $q$ are distributed normally.

$$S_q(\theta) = A \cdot \theta^{-b}, \quad (1)$$

where $A$ and $b$ - spectral density coefficients defining the microprofile type, $\theta$ - track frequency.

Ride quality criterion for the selection of the suspension concept is the level of the equivalent root mean square of the vibrational accelerations at the chassis points located above the wheels.

Research objectives:

1. Development of dynamical models of a two-axle vehicle. The models must represent irregularities and varying physical and mechanical state of the deformable terrain interacting with the tire.

2. Analysis of the mutual influence of the vehicle sprung and unsprung part oscillations and deformations of the irregular terrain.

3. Development of a calculation method for the evaluation of the parameters of the tire interaction with the deformable terrain. The method must take into account dynamic loading, terrain relaxation properties and cyclic loading.

4. To use the developed spatial dynamic model of the two-axle all-wheel-drive vehicle for the estimation of the optimality of the selected design parameters defining the dynamic deflection of the suspension and the size of the tire of the ZIL 432720 vehicle.

Research results

In the presented research, the structure of five dynamic models of a two-axle vehicle (fig. 1) on the deformable irregular terrain has been developed. In contrast to the known models (fig. 1, b, c, d, e), the developed models take into account relaxation of the terrain according to the Shvedov and Bingam models (fig. 2). The most realistic dynamical model of the vehicle is spatial model 1, a. The model has eight degrees of freedom. For every dynamic model, a system of differential equations is generated. The solution of these systems of non-linear equations is carried out with the use of Laplace transformations. In standard form, oscillation equation for a spatial system can be presented as:
where: \( \mathbf{A}(\mathbf{p}) \) – matrix of vehicle parameters; \( \mathbf{B}(\mathbf{p}) \) – perturbation matrix; \( z_c \) – vertical displacement of the sprung part; \( \alpha \) – pitch angle of the vehicle sprung part; \( \beta_1 \) and \( \beta_2 \) – roll angles of the vehicle sprung part under front and rear axles respectively; \( \xi_{i,j} \) – displacement of the unsprung parts; \( q_0 \) – half of the microprofile height; \( q_{i,j} \) – displacement of the wheel contact patch relative to the microprofile at the \( i \)-th axle at the \( j \)-th side of the vehicle.

\[
\begin{align*}
A(p) & = \begin{pmatrix}
A_1 & A_2 \\
A_3 & A_4
\end{pmatrix} \\
B(p) & = \begin{pmatrix}
B_1 & B_2 \\
B_3 & B_4
\end{pmatrix}
\end{align*}
\]
The developed models allow a researcher to:
1) find the influence of the ratio of the sprung and unsprung parts \((M,m)\), suspension and tire stiffnesses, shock absorber damping \(c_{\text{susp}}, c_{\text{tyre}}, k_{\text{susp}}\), type and design of the suspension, tire type and size, tire inflation pressure \(p_{\text{tire}}\), axle base, track width and lateral distance between the springs on the vehicle off-road performance;
2) evaluate numerical values of the vehicle off-road performance parameters: additional dynamic load caused by the vehicle oscillations propagated through the tires into the terrain \(\Delta P\); wheel sinkage and tire deformation \(z, h\); terrain compaction after each wheel pass \(\rho_{\text{C},i}\); coefficient of rolling resistance of the terrain and tire deflection \(f\); coefficient of the free tractive force \(\Psi_{\text{T}}\); probability of loss of contact between the wheels and the terrain; probability of contact between the vehicle body (vehicle axles) and the terrain and etc.;
3) evaluate numerical values of the criteria of the ride quality and component strength of the vehicle: root mean square vibrational accelerations and displacements at every point of the vehicle body \(\tilde{z}_{A_{i,j}}, z_{A_{i,j}}\); ratio of the microprofile change after the first wheel pass \(S_{\text{q}}(\nu)_{2,j}\); lateral and longitudinal angular accelerations \(\ddot{\alpha}_2, \dot{\beta}_2\); deflection of the springs and the probability of the suspension maximum deflection \(f_{\text{max}}\); dynamic loads of vehicle frame \(\Delta P_{\text{dyn}}\); crossing of the front and rear axles; frame torsion angle and frame rail distortion angle \(\beta_{\text{fu}}, \alpha_{2,j}\).
Model efficiency and its scope in the tire – terrain interaction domain

Figures 3-9 show examples of calculation results for the five models. The following parameters were estimated:

1) influence of the vehicle velocity on the value of additional dynamic loads on the deformable irregular terrain \( A = 10^{-3} \) with different moisture (fig. 3). For the velocities within the range 10…50 km/h for the left and right vehicle wheels, minimal and maximal additional dynamic loads for models 2 and 5 differ by a factor of 4.5. At the same time, \( \Delta P \) for model 2 increases by 25 % and 100 % in cases of the terrain moisture \( W=70 \% \) and \( W=90 \% \) respectively, when \( V = 50 \text{ km/h} \), the maximal values of \( \Delta P \) for model 5 differ by about 13 %.
Figure 3. Influence of the vehicle velocity on the values of additional dynamic loads on the deformable irregular terrain with different moisture content $(A=10^{-3}; 1...5 – numbers of the dynamic models).

2) Figure 4 illustrates the influence of the terrain irregularities with $A = 10^{-3}$ or $A = 10^{-4}$ on the additional dynamic loads in the same interval of varying velocities of the vehicle. Depending on the choice of the dynamic model within the analyzed interval of the vehicle velocities on the deformable terrains with different degree of irregularity, additional dynamic loads obtained with simple model 2 and complex spatial model 5 can differ by a factor of 9, relative increase in the load being 230 % and 100 % respectively;

3) Figure 5 shows influence of the dynamical model type on the wheel sinkage $z$ which determines rolling resistance on the given terrain with different physical and mechanical properties depending on its moisture. When the soil moisture $W=70\%$, different dynamical models give conflicting results: in model 2, when the the wheel velocity increases, the wheel sinkage decreases down to zero. For model 5, in case of low velocity, sinkage $z = 0.15 \, \text{m}$ and it decreases insignificantly (by 10 %) when the velocity increases.

4) Figure 6 shows the wheel sinkage $z$ for the front left wheel as a function of the vehicle velocity on plain and irregular deformable terrains with the same soil. The maximal value of $z$ ($z = 0.32 \, \text{m}$) is obtained on the overmoisted terrain with $W = 90\%$ in case of minimal velocity. On a plain terrain, during motion without oscillations, the sinkage is less, than in case of dynamic loads ($z=0.2 \, \text{m}$ on an average);
Figure 4. Influence of the terrain irregularity degree on the additional dynamic load.

Figure 5. Influence of the soil moisture on the wheel sinkage on a deformable terrain ($A=10^{-3}$).
5) figure 7 shows ride quality parameters of the vehicle during motion on the irregular deformable terrain;

6) figure 8 shows ride quality parameters for firm terrain and deformable terrain with different moisture content. The parameters have been obtained with dynamic model 5. These parameters define maximum allowable velocities of the vehicle. Allowable root mean square values of the vibrational acceleration during short-period motion is 4 m/s^2, which means that the allowable velocity on firm irregularities with $A = 10^{-3}$ is 8 km/h (fig. 8), on soft terrain – 10 ± 22 km/h according to model 1, 17 ± 19 km/h according to model 4 and 16 ± 18 km/h according to model 5 (fig. 8);

7) for estimation of the cross-country capability, which is limited by the road clearance, figure 9 shows dependencies of sinkage $z$ on terrain moisture, they are estimated for 4 models. The calculation results show, that motion is possible if the terrain moisture $W$ is no more then 85% and road clearance is 0.3 m. Models 1-4 have shown that the vehicle can move without any body – terrain contact on all the
analyzed deformable terrains. Model 5 shows that body–terrain contact might occur on a terrain with the limit moisture content $W=90\%$.

Analysis of the calculation results shows, that oscillations of unsprung parts have the most significant influence on the dynamic loads.

The analysis has shown that:
- it is necessary to take into account additional dynamic loads $\Delta P$, while determining parameters of the tire interaction with irregular deformable terrain;
- transversal angular oscillations have a significant influence on $\Delta P$ values, so it is important to take them into account, especially if vehicle is moving on an overmoisted terrain;
- if spectral density coefficient $A$ of the the terrain profile is larger then 10$^{-3}$, it is necessary to take into account transversal and lateral angular accelerations (model 3 and model 5);
- weight of unsprung parts has the most significant influence (more than 50\%) on wheel dynamic loads of the terrain and on the wheel sinkage.

The paper describes a method for estimation of the and off-road performance of an vehicle. A dynamic model of the wheel interaction with an irregular deformable terrain was used.

For estimation of the wheel sinkage $z$ and tire deflection $h$, the following equations system should be solved:

$$
q_{\text{dwy}} = \frac{\pi \cdot h \cdot (p_0 + p_W) \cdot (\frac{B}{H} + \frac{3}{2}\cdot \frac{H}{B}) \cdot \left(1 - \frac{h}{B}\right)}{1 + \frac{q_{\text{dwy}}}{l^2 n \cdot \phi}};
$$

$$
q_{\text{dwy}} = \frac{E \cdot z}{0.5 \cdot a \cdot (b + b_k) \cdot \arctg \left[\frac{H_G - z}{0.5 \cdot a \cdot (b + b_k)}\right] + \frac{2 \cdot I \cdot E \cdot z \cdot \arctg \left(\frac{\pi \cdot (H_G - z)}{b + b_k}\right)}{\pi \cdot \left(I_1 \cdot X_1 \cdot k_{\beta_1} \cdot b + I_2 \cdot X_2 \cdot k_{\beta_2} \cdot c_0 + X_3 \cdot z\right)}};
$$

$$
P_z = 0.25 \cdot \pi \cdot q_{\text{dwy}} \cdot \left[\left(1 - 0.5 \xi\right) \cdot 2 \cdot b \cdot \sqrt{D \cdot h - h^2} + \xi \cdot b_k \cdot \sqrt{D \cdot (h + z) - (h + z)^2}\right],
$$

where $q_{\text{dwy}}$ - contact patch pressure, expressed as a function of the tire deflection, MPa; $D$ - wheel diameter, m; $B$ and $H$ - width and height of the tire profile, m; $p_0$ - terrain pressure caused by the tire carcass stiffness, MPa; $p_W$ - tire inflation pressure, MPa; $P_z$ - wheel load, N; $b_k$ - track width, m; $I_1, I_2, I_3$ - coefficients of influence of the contact surface size and shape on the terrain deformation; $V_a$ - velocity of the vehicle, m/s.

First, sinkage and tire deflection for the vehicle front and rear axles are calculated. Then, on the assumption of the constant sinkage, the same parameters of the tire-terrain interaction are calculated for the new physical and mechanical properties of the terrain. The new terrain properties are recalculated for each axle passing over the already deformed terrain by the following dependencies:

$$\rho_{\text{n}} = f \left(n; q_{\text{dwy}}; \rho_1\right); E, \varphi_0, c_0 = f \left(\rho_n; W_1; W\right).$$

After evaluation of $h$ and $z$ for each wheel of the two-axle vehicle we calculate the following parameters: tire rolling resistance, $f_{\text{tyre}}$; terrain rolling resistance, $f_{\text{gr}}$; total rolling resistance, $f$; longitudinal slip, $\xi$; free tractive force, $\psi_T$.

The above mentioned method has been used for the calculation of the relative deformations of the tire and terrain with provision for the additional dynamic load. Assessment of the additional dynamic load $\Delta P_c$ is based on the following dependency:
\[ \Delta P_{dyn} = \ddot{z}_C \cdot M + \ddot{\xi}_C \cdot m, \]  

(4)

where \( \ddot{z}_C \) and \( \ddot{\xi}_C \) – root mean-square vibrational accelerations of the sprung and unsprung parts \( m \) and \( M \) of each wheel of the vehicle.

For estimation of \( \ddot{z}_C \) and \( \ddot{\xi}_C \), terrain damping and stiffness properties are taken into account (fig. 2). Calculations of the floatation and tractive effort for a single wheel and for the wheels of the front and rear axles of a two-axle vehicle (plain model) on loam terrain with different degree of irregularity and different physical and mechanical states have shown that the vehicle oscillation parameters have significant influence on the obtained results.

The experimental study of the vehicle ride quality and off-road performance on a deformable irregular terrain has been carried out.

For estimation of the adequacy of the developed model of the vehicle motion and for validation of the method for the terrain damping properties assessment, a series of experiments was performed on a test rig and on a real vehicle on a natural deformable terrain.

Difference between calculated and experimental root mean-square values of the vibrational accelerations does not exceed 6%.

The authors have used the results of the presented research for estimating optimality of the chosen parameters of the given two-axle vehicle on given deformable terrains.

The developed spatial dynamic model of the vehicle was used for estimation of the vehicle ride quality and off-road performance on a standard route. The route consists of 4 segments shown in table 1.

The parameters of the ride quality and off-road performance were calculated for the two-dimensional model and spatial models (4), (5) on the standard route. Parameters \( Z, \psi, \), which define ability to move in off-road conditions, for the spatial and two-dimensional model differ by 20 % on the average.

| Segment | Type of the segment surface | Initial data |
|---------|-----------------------------|--------------|
| 1       | Irregular road with firm surface | \( H_G = 0 \text{ m}, A = 10^{-4} \) |
| 2       | Uneven soil road             | \( H_G = 0 \text{ m}, A = 10^{-3} \) |
| 3       | Smooth deformable terrain     | \( H_G = 0, 4 \text{ m}, A = 0 \) |
| 4       | Irregular deformable terrain  | \( H_G = 0, 4 \text{ m}, A = 10^{-3} \) |

Table 1. Surface properties of the segments of the standard route.

The significant influence of the unsprung part on the vehicle ride quality and off-road performance has been confirmed. Unsprung part oscillation loads amount to 50-60 % of the additional dynamic loads on the irregular deformable terrain.

General results and conclusions

1. Dynamic models of the vehicle oscillation have been developed. Vehicle sprung and unsprung parts are connected with elastic and damping elements. Oscillations of this parts are related to the deformations of the deformable irregular terrain with non-linear mechanical properties depending on its physical state. Numerical disarrangement of theoretical and experimental data is within the range of 6-12 %.

2. Stiffness and damping properties of the deformable irregular terrain have significant influence on the oscillation parameters of the vehicle sprung and unsprung parts. Estimations of additional dynamic loads for firm and deformable irregular terrain (\( A=10^{-3} \)) have disarrangement up to 100 %. Increase in terrain moisture by 30 % relative to the basic value \( W=70 \% \) leads to decrease in additional dynamic loads by 10 %.
3. Application areas of the developed dynamic models have been defined. If sprung and unsprung parts masses are in the following relation: \( \frac{M}{m} > 10 \), it is possible to disregard the unsprung part for simplicity and use dynamic model 2. It is recommended to select dynamic systems taking into account transversal angular oscillations (models 3 and 5) for estimation of additional dynamic loads on microprofile with irregularities higher than 50 mm. Calculations with the use of the spatial dynamic model (model 5) lead to a 25\% increase of dynamic loads in relation to the two-dimensional model (model 4).

4. As a performance criterion for the two-axle vehicle, its average velocity on the standard route is chosen. The route consists of segments with different deformation properties. Spatial model 5 is recommended as the most realistic one for the simulation of the vehicle – deformable terrain system.

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