The goal of high-energy nuclear collisions is to identify and study the equation of state of the Quark-Gluon Plasma (QGP) which is believed to exist at the early stage of our universe [1]. In $\sqrt{s_{NN}} = 200$ GeV Au + Au collisions, the observations of jet-quenching [2] and collective flow have demonstrated the formation of hot and dense matter with partonic collectivity [3, 4]. Local thermalization of the system created in heavy ion collisions is yet to be tested. Heavy flavors including charm and bottom quarks are powerful tools [5] for studying the equation of state of the QGP itself. Both regenerated and initially produced $J/\psi$s are simultaneously taken into account. Including both processes is important for RHIC since, unlike the collisions at the LHC, not all the initially produced $J/\psi$s are destroyed.

The massive $J/\psi$s are unlikely fully thermalized with the medium. Thus their phase space distribution should be governed by transport rather than the kinetic equation of Refs. [23, 24]. To comprehensively treat the $J/\psi$ distribution, we determine the $J/\psi$ transport equation including both initial production and anomalous suppression as well as regeneration. The transport equation is then solved together with the hydrodynamic equation which characterizes the space-time evolution of the QGP. In order to compare with the experimental measurements, we calculate the average transverse momentum squared, $\langle p_T^2 \rangle$, elliptic flow, $v_2$, and nuclear modification factor, $R_{AA}$, for $J/\psi$ at both RHIC and LHC.

In p + p collisions, about 30-40% of final state $J/\psi$s are from the feed-down [25] of $\psi'$ and $\chi_c$. In order to simplify the numerical calculation, we neglect the $\psi'$ contribution and assume 40% of the final state $J/\psi$s come from $\chi_c$ [16]. Since the $\Psi (=J/\psi, \chi_c)$ is heavy, we use a classical Boltzmann-type transport equation to describe its evolution. The distribution function $f_{\Psi}(p_t, x_t, \tau | b)$ in the central rapidity region and in the transverse phase space $(p_t, x_t)$ at fixed impact parameter $b$ is controlled by the equation

$$\frac{\partial f_{\Psi}}{\partial \tau} + v_\Psi \cdot \nabla f_{\Psi} = -\alpha_{\Psi} f_{\Psi} + \beta_{\Psi}. \quad (1)$$

The second term on the left-hand side arises from free-streaming of $\Psi$ with transverse velocity $v_\Psi = p_t / \sqrt{p_T^2 + m_\Psi^2}$, which leads to the “leakage” effect and is needed to explain the averaged transverse momentum squared at the SPS [15]. The anomalous suppression and regeneration mechanisms are reflected in the loss term $\alpha_{\Psi}$ and gain term $\beta_{\Psi}$, respectively.

Suppose the medium locally equilibrates at time $\tau_0$, when nuclear absorption of the initially produced $J/\psi$ has ceased. Absorption effects can be included in the initial distribution $\int_{p_T} f_0(p_t, x_t, \tau_0 | b)$, of the transport equation. To determine the initial condition at RHIC, we take $\tau_0 = 0.6$ fm, the initial $J/\psi$ production cross section $\sigma_{pp}^\Psi = 2.61 \pm 0.20 \, \mu b$ [26] and nuclear absorption cross section $\sigma_{abs} = 3 \, \mu b$ [27]. We also assume that $a_{pN} = 0.076$ GeV$^2$/fm and $\langle p_T^2 \rangle_{PP} = 4.31$ GeV$^2$ [26] to describe the initial $p_t$ broadening due to multiple gluon scattering.
When the loss and gain terms $\alpha_\Psi$ and $\beta_\Psi$ are known, the transport equation (11) can be solved analytically with the result

$$f_\Psi(p_t, x_t, \tau | b) = f_\Psi(p_t, x_t - v_\Psi(\tau - \tau_0), \tau_0 | b) \times e^{-\int_{\tau_0}^{\tau} dt' \alpha_\Psi(p_t, x_t - v_\Psi(\tau - t'), \tau' | b)} + \int_{\tau_0}^{\tau} dt' \beta_\Psi(p_t, x_t - v_\Psi(\tau - t'), \tau' | b) \times e^{-\int_{\tau_0}^{\tau'} dt'' \alpha_\Psi(p_t, x_t - v_\Psi(\tau - \tau'), \tau'' | b)}.$$  (2)

The first and second terms on the right-hand side indicate the contributions from the initial production and continuous regeneration, respectively. Both suffer anomalous suppression. The coordinate shift $x_t \rightarrow x_t - v_\Psi \Delta \tau$ reflects the leakage effect during the time period $\Delta \tau$.

We determine now the $J/\psi$ suppression and regeneration in a QGP. We consider only the gluon dissociation process, $g + \Psi \rightarrow c + \bar{c}$, for the loss term (16). Its inverse process is the gain term. Then $\alpha_\Psi$ and $\beta_\Psi$ are

$$\alpha_\Psi(p_t, x_t, \tau | b) = \frac{1}{2E_p} \int \frac{d^3p_g}{(2\pi)^32E_g} W_{g\Psi}^c(s) f_g(p_g, x_t, \tau) \times \Theta(T(x_t, \tau | b) - T_c),$$

$$\beta_\Psi(p_t, x_t, \tau | b) = \frac{1}{2E_p} \int \frac{d^3p_g}{(2\pi)^32E_g} \frac{d^3p_c}{(2\pi)^32E_c} \frac{d^3p_{\bar{c}}}{(2\pi)^32E_{\bar{c}}} \times W_{g\Psi}^c(s) f_c(p_c, x_t, \tau | b) f_{\bar{c}}(p_{\bar{c}}, x_t, \tau | b) \times (2\pi)^4 \delta^4(p - p_g - p_c - p_{\bar{c}}) \times \Theta(T(x_t, \tau | b) - T_c),$$  (3)

where $E_g, E_\Psi, E_c$ and $E_{\bar{c}}$ are the gluon, charmion, $c$ and $\bar{c}$ energies. The step function $\Theta$ ensures that anomalous suppression and regeneration occur only in the QGP phase. The gluon thermal distribution is $f_g$, $W_{g\Psi}^c(s)$ [28] is transition probability of the gluon dissociation as a function of $s = (p + p_g)^2$ calculated in pQCD [28], and $W_{g\Psi}^c_c$ is $c$ and $\bar{c}$ recombination transition probability. Note that $W_{g\Psi}^c$ can be obtained from $W_{g\Psi}^{cc\bar{c}}$ using detailed balance. In numerical calculations, we take $m_c = 1.87$ GeV, including in-medium effects [16, 22], $m_{J/\psi} = 3.1$ GeV, $m_{\chi_c} = 3.51$ GeV and critical temperature $T_c = 165$ MeV.

Unlike gluons that are constituents of a QGP, charm quarks are heavy and may not be thermalized in the QGP. In principle, a self-consistent treatment of charm quark motion in the QGP should be described by a transport equation similar to Eq. (11). For simplicity, we consider the $c$ and $\bar{c}$ distribution functions, $f_c$ and $f_{\bar{c}}$, in two extreme scenarios. In the weak interaction limit, the charm quarks in QGP are assumed to keep their original momentum distribution, $g_c(q)$, calculated in pQCD [24] and their initial space distribution determined by nuclear geometry,

$$f_{c, \bar{c}}(q, x_t | b) = \sigma_{pp}^{c\bar{c}} T_A(x_t) T_B(x_t - b) g_c(q),$$  (4)

where $\sigma_{pp}^{c\bar{c}} = 622 \pm 57 \mu b$ [24] is the charm production cross section at RHIC. In the opposite limit, when the $c$ and $\bar{c}$ are strongly correlated to the medium, they are thermalized and distributed statistically. Both the pQCD and thermal distributions are normalized to the initial charm quark number.

Since the hadronic phase occurs later in the evolution of heavy ion collisions when the density of the system is lower compared to the early hot and dense period, we have neglected the hadronic dissociation. The suppression and regeneration region controlled by the step function in the loss and gain terms and the gluon and thermal charm quark distributions are determined by the QGP evolution. With the Hubble-like longitudinal expansion and boost invariant initial condition, the transverse hydrodynamical equations [16] can be solved numerically to determine the evolution of temperature and transverse fluid velocity.

The final state $J/\psi$ distributions at fixed centrality can be found by integrating the distribution function, Eq. (2), over $p_t$ and $x_t$ up to $\tau \rightarrow \infty$. We first examine the mid-rapidity nuclear modification factor $R_{AA}$. The numerical result, as a function of participant number, $N_{part}$, is compared to the mid-rapidity RHIC data in Fig. 1. Plots (a) and (b) correspond to the pQCD and thermal charm quark distributions, respectively. The bands are theoretical results including the uncertainty in the initial charm quark and $\Psi$ production cross sections $\sigma_{pp}^{c\bar{c}}$ and $\sigma_{pp}^{\Psi\Psi}$. The solid and dot-dashed curves indicate calculations with only initial production (without regeneration, $\beta_\Psi = 0$) and only regeneration (without initial production, $\sigma_{abs} = \infty$). Almost all the initial 40% of $J/\psi$ from $\chi_c$ decay is lost in semi-central collisions [31]. In central collisions, only the directly produced $J/\psi$ suffer anomalous suppression. When nuclear absorption effect is reduced by decreasing the initial time $\tau_0$ or the absorption cross section $\sigma_{abs}$, the initial $J/\psi$ yield is less suppressed. Due to the strong regeneration in central collisions, the full $J/\psi$ yield is no longer a monotonically decreasing function of centrality. The data seem to show a flat region at $50 \leq N_{part} \leq 150$. This feature can not be reproduced in the present model calculations.

![FIG. 1: The nuclear modification factor $R_{AA}$ as a function of participant number $N_{part}$ in pQCD (a) and thermal (b) charm quark distributions. The solid and dot-dashed curves are the calculations with only initial production and only regeneration. The bands are the full result, including the uncertainty in initial charm quark and $\Psi$ production. The data are from PHENIX [30].](image-url)

The momentum spectra are expected to be more sensitive to the production mechanism than the integrated yields. We show the $J/\psi$ average squared transverse momentum, $\langle p_T^2 \rangle$, as a function of centrality in Fig. 2. It is well known that the multiple gluon scattering in the initial state leads to transverse momentum broad-
ening. The leakage effect due to the anomalous suppression also results in $p_{T}$ broadening in central collisions. These two broadening effects on the initial $J/\psi$ are reflected in the solid curves in Fig. 2. For the regenerated $J/\psi$, the charm quarks in the pQCD scenario have no scattering in the initial state and in the QGP, and they satisfy the statistic distribution in the thermal scenario. The $\langle p_{T}^{2} \rangle$ of the regenerated $J/\psi$ in these two limits looks independent of centrality, see dot-dashed curves in Fig. 2. Since the high momentum charm quarks lose energy during thermalization, the average squared momentum ($\sim 2$ GeV$^{2}$) in the thermal scenario is smaller than the one ($\sim 2.5$ GeV$^{2}$) in the pQCD scenario. The collective flow develops with time, the $\langle p_{T}^{2} \rangle$ for the sudden-produced $J/\psi$ on the hadronization hypersurface and the continuously regenerated $J/\psi$ in the whole volume of QGP. Because the initial production decreases with centrality, due to absorption and anomalous suppression, and regeneration increases with centrality, shown in Fig. 1, the $\langle p_{T}^{2} \rangle$ starts at the initial result and approaches the regeneration result as centrality increases.

The $J/\psi$ elliptic flow, $v_{2}$, is a useful tool for studying charm production mechanism in nuclear collisions. When no regeneration is considered, the leakage effect gives the lower limit of $J/\psi$ $v_{2}$, the solid curve in Fig. 3. In the pQCD scenario, charm quarks do not interact with the medium and the regenerated $J/\psi$ do not carry any collective property of the system, so we only discuss $v_{2}$ in the thermal scenario. If the regeneration occurs only at hadronization of the QGP, the well-developed azimuthal anisotropic flow leads to a large $J/\psi$ $v_{2}$, see discussions in Ref. [22]. However, for the continuous regeneration in the QGP volume the average elliptic flow becomes much smaller since the asymmetric flow is very small in the early stage. At impact parameter $b = 7.8$ fm, the continuously regenerated $J/\psi$ $v_{2}$, the dot-dashed line in Fig. 3 is only one half of the suddenly regenerated $J/\psi$ $v_{2}$ in the coalescence model [22]. To check the difference between the continuous and sudden regeneration mechanisms, we force the regeneration to occur in a thin spherical shell by adjusting the regeneration temperature region to $T_{c} \leq T \leq T_{c} + \delta T$. Indeed, the regenerated $v_{2}$ approaches the value of Ref. [22] for $\delta T \rightarrow 0$. Since the $J/\psi$ yield in semi-central collisions is still dominated by the initial production, see Fig. 1, there is no sizable difference in $v_{2}$ between the total (dashed line) and initial-produced $J/\psi$ (solid line), see Fig. 3.

What is the case for nuclear collisions at the LHC? Due to the extremely high center of mass energy, the QGP formed at LHC energy will have much higher temperatures, longer lifetimes and larger sizes. Most of the initial $J/\psi$ will be suppressed by stronger gluon dissociation in QGP. On the other hand, more charmonia will be created by the regeneration. The ratio $N^{reg}/N^{ini}$ of regenerated to initial-produced $J/\psi$ is shown as a function of centrality in Fig. 4. While at RHIC, the ratio can reach unity only in the most central collisions, it is much larger than one at almost any centrality bin at the LHC. Therefore, regeneration will dominate the observed $J/\psi$. As a consequence, the $J/\psi$ $R_{AA}$, $v_{2}$ and $\langle p_{T}^{2} \rangle$ at the LHC will follow the regeneration calculations in Figs. 1, 2 and 3.

![FIG. 2: The $J/\psi$ averaged transverse momentum square as a function of the number of binary collision, $N_{coll}$. The calculations are the same as in Fig. 1. The PHENIX data are shown.](image)

![FIG. 3: The $J/\psi$ elliptic flow in $\sqrt{s_{NN}} = 200$ GeV Au + Au collisions at impact parameter $b = 7.8$ fm. The calculations are the same as in Fig. 1.](image)

![FIG. 4: The ratio of regenerated to initially produced $J/\psi$ yields at RHIC and LHC energies in pQCD and thermal scenarios. At the LHC, we choose $\sigma_{pp}^{\psi} = 18.9$ $\mu$b, $\sigma_{pp}^{\psi} = 5740$ $\mu$b, $\tau_{0} = 0.3$ fm and initial temperature 680 MeV.](image)
are important at RHIC. The average squared transverse momentum with thermalized charm quarks fits the RHIC data reasonably well. In contrast to previous calculations [22] with only sudden regeneration at hadronization, the dominant initial production in semi-central collisions and the continuous regeneration in the whole QGP volume lead to a rather small $J/\psi$ elliptic flow at RHIC.

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