Flip-flopping binary black holes

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We perform a full numerical simulation of binary spinning black holes to display the long term spin dynamics. We start the simulation at an initial proper separation between holes of \( d \approx 25M \) and evolve them down to merger for nearly 48 orbits, 3 precession cycles and half of a flip-flop cycle. The simulation lasts for \( t = 20000M \) and displays a change in the orientation of the spin of the secondary black hole from initially aligned with the orbital angular momentum to a complete anti-alignment after half of a flip-flop cycle. This process continuously flip-flops the spin during the lifetime of the binary. We discuss the consequences of this oscillation mode for accreting binaries, in particular for the spin growth and binary dynamics as well as the observational consequences for galactic and supermassive black holes.

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Introduction: Numerical relativity techniques are now able to directly simulate binary black holes near merger [1,8]. In particular one can follow up the dynamical effects on black hole spins in an inspiral orbit down to the formation of the final remnant black hole [1]. One of the most striking results of those studies has been the discovery of very large recoil velocities [5] acquired by the merger remnant, up to 50000 km/s [6].

It has been pointed out [7] that the presence of accreting matter can align (or counter-align) spins with the orbital angular momentum thus reducing the recoil velocities to a few hundred km/s [8]. Recent studies of the tidal effects on tilted accretion disks around spinning black holes find almost perfect alignments of the spin with the orbital angular momentum [9,10] on a relatively shorter time scale than that of gravitational radiation (for black hole separations above a thousand gravitational radii).

While those studies have been performed on individual black holes, we revisit this scenario to study the precession dynamics of black hole spins in a binary system. In particular we are interested in the leading conservative dynamics of partial precession of each individual spin. We find a flip-flop mode with periods shorter than the gravitational radiation scale and with relatively high probability to occur given generic (but equal mass) initial configurations. We briefly discuss the effects that this flip-flopping spin could have on the inner accretion disk dynamics and its potential observational consequences.

Full Numerical Evolution: In order to verify the long term dynamics of spinning binary black holes we start a numerical simulation at a proper separation \( d \approx 25M \) (in a regime where the post-Newtonian expansion should provide a good approximation). We study a binary with equal masses but different spin magnitudes and orientations. In particular, we choose one of the black holes to have its spin \( \vec{S}_1 \) exactly aligned with the orbital angular momentum \( \vec{L} \) initially, while the second black hole spin \( \vec{S}_2 \) lies mostly along the orbital plane, but slightly anti-aligned with \( \vec{L} \), such that the total spin \( \vec{S} \) lies in the orbital plane, i.e. \( \vec{S} \cdot \vec{L} = 0 \). These choices (See Table I) are for the sake of simplicity of the analysis, and also provide a scenario where accretion has proceeded to align one of the black holes with \( \vec{L} \) and led to comparable masses by preferably accreting onto the initially smaller hole [11]. We also choose the magnitude of the first black hole to be smaller than that of the second, foreseeing (as discussed later in this paper) that the flip-flopping spin neutralizes the accretion growth of intrinsic spin magnitudes.

We use the TwoPunctures thorn [12] to generate ini-

| Parameter | Value |
|-----------|-------|
| \( x_1/m \) | 10.73983 |
| \( x_2/m \) | -10.76016 |
| \( z/m \) | -0.01968 |
| \( P/m \) | 0.05909 |
| \( d/m \) | 25.37 |
| \( m_1^0/m \) | 0.48543 |
| \( m_2^0/m \) | 0.30697 |
| \( S_{1z}/m^2 \) | 0.05 |
| \( S_{2z}/m^2 \) | 0.0322 |
| \( S_{2x}/m^2 \) | 0.0006 |
| \( N \) | 48.5 |

| \( \alpha_1 \) | 0.50000 |
| \( \delta\alpha_1 \) | 0.00002 |
| \( \alpha_2 \) | 0.49974 |
| \( \delta\alpha_2 \) | 0.0001 |

| \( \alpha_1 \) | 0.20003 |
| \( \delta\alpha_1 \) | 0.00056 |
| \( \alpha_2 \) | 0.80088 |
| \( \delta\alpha_2 \) | 0.00066 |
Had we started the binary further separated apart this merger both display an almost total flip, around 160° momentum $\vec{S}$ hole spin components of the black holes represented over a sphere in to merger.

Fig. 1. Directional evolutions of the spins and angular momentum in the initial coordinate frame (left) and in the non-inertial $\hat{L}$ frame (right). Color Keys: red $\hat{L}$, blue $\hat{S}_1$, green $J$.

The full evolution required 2.5 million service units on 25 to 30 nodes of our local cluster “Blue Sky” with dual Intel Xeon E5-2680 processors nearing 100 $M$ of evolution per day. Our evolution is free and we verify its accuracy by the satisfaction of the Hamiltonian and Momentum constraints. All four $L_2$-norm quantities remain well below $10^{-8}$ until merger. Individual horizon masses $m_1$ and $m_2$ are preserved to a level of 2 and 1.4 parts in 10$^5$ respectively until merger. Spins grow linearly with time until merger by a total increase of $1.5 \times 10^{-4}$. Thus the total increase of the intrinsic spin magnitudes $\alpha_i = S_i/m_i^2$ are $\delta \alpha_1 = 6 \times 10^{-4}$ and $\delta \alpha_2 = 6 \times 10^{-4}$ from initial data to merger.

The azimuthal precessional effect and polar flip-flop can be directly seen in the evolution of the spin components of the black holes represented over a sphere in Fig. 1. The effect is apparent in the frame of the orbital plane as well as the fixed initial set of coordinates.

Fig. 2 displays the angles that the secondary black hole spin $\hat{S}_1$ forms with the precessing orbital angular momentum $\hat{L}$ or with the fixed $\hat{z}$-axis as a function of time. Both start originally aligned and by the time of merger both display an almost total flip, around 160°. Had we started the binary further separated apart this spin would continue to flip-flop between complete alignment and counter alignment as described in the next section. We also compare our results with the corresponding 3.5 post-Newtonian (PN) integration of the equations of motion and spin evolution [21, 22]. We observe a long initial superposition of the PN and full numerical precession curves corresponding to the early 15000$M$ of evolution of the binary, up to separations above around 15$M$. As the merger proceeds and the evolution becomes more dynamical we begin to observe the expected deviations from each other, with the full numerical solution to general relativity presenting a stronger flip-flop effect.

Fig. 3 displays the leading waveform modes for the strain. In the top panel is the characteristic chirp in the $(\ell, m) = (2, 2)$ mode, with an increasing amplitude slightly modulated at around the orbital frequency due to the nutation of $\hat{L}$ around the total angular momentum $\hat{J}$ (See Fig. 1 in Ref. [23]). The lower panel shows the azimuthal precessional effect of $\hat{L}$ on the amplitude of the $(2, 1)$ mode, showing that we evolved for nearly three precessional cycles (See Ref. [24] for a first discussion relating this mode to precession in full numerical simulations).

Table II displays the properties of the final remnant formed after merger. Notably, the recoil reaches 1500 km/s, and the orientation of the final spin changes by only 1.62 degrees with respect to the initial direction of the total angular momentum, as expected for comparable mass binaries [25].

TABLE II. Remnant properties and recoil velocity. The final mass and spin are measured from the horizon, and the recoil velocity is calculated from the gravitational waveforms. The error in the mass and spin is determined by the drift in those quantities after the remnant settles down. The error in the recoil velocity is the difference between first and second order polynomial extrapolation to infinity.

| $M_{\text{rem}}/M$ | $|\alpha_{\text{rem}}|$ | $V_{\text{recoil}}$ [km/s] |
|-----------------|------------------|-----------------|
| 0.94904 ± 0.00000 | 0.70377 ± 0.00002 | 1508.49 ± 16.08 |
| $\alpha_{\text{rem}}$ | $\alpha_{\text{rem}}^2$ | $\alpha_{\text{rem}}^3$ |
| 0.10815 ± 0.00003 | -0.01986 ± 0.00000 | 0.69513 ± 0.00002 |

FIG. 2. The angle between the spin of the secondary black hole $\hat{S}_1$ with respect to the orbital angular momentum $\hat{L}$ (left) and with respect to the fixed $z$-axis (right). For comparison we also plot the 3.5PN prediction.

FIG. 3. The leading waveform modes for the strain. In the top panel is the characteristic chirp in the $(\ell, m) = (2, 2)$ mode, with an increasing amplitude slightly modulated at around the orbital frequency due to the nutation of $\hat{L}$ around the total angular momentum $\hat{J}$ (See Fig. 1 in Ref. [23]). The lower panel shows the azimuthal precessional effect of $\hat{L}$ on the amplitude of the $(2, 1)$ mode, showing that we evolved for nearly three precessional cycles (See Ref. [24] for a first discussion relating this mode to precession in full numerical simulations).
follows that the following quantities are conserved: in full nonlinear simulations of binary black holes solving general relativity field equations numerically [26]). It is the spin of the primary identified with the larger spin magnitude \( S \) is the spin of the secondary identified with the smaller spin magnitude \( q \). From Eqs. (1) the magnitude of the individual spins \( S_1 \) and \( S_2 \) are conserved as well as the magnitude of their sum, \( S \) (This has been observed to be approximately true in full nonlinear simulations of binary black holes solving general relativity field equations numerically [26]). It follows that the following quantities are conserved:

\[
\hat{S} \cdot \vec{L} = S^2 = S_1^2 + S_2^2 + 2S_1S_2 \cos \beta = \text{constant}, \quad (3)
\]

\[
\hat{S} \cdot \hat{S}_1 = SS_1 \cos \gamma = S_1^2 + S_2S_1 \cos \beta = \text{constant}. \quad (4)
\]

**Post Newtonian spin dynamics:** In order to provide an analytic understanding of the flip-flop spin mode, we look at the precession equations for the spins \( \hat{S}_1 \) and \( \hat{S}_2 \) with a mass ratio \( q = m_1/m_2 \) to leading spin-orbit and spin-spin couplings in the (2PN) post-Newtonian expansion [22]

\[
d\hat{S}_1 = \frac{1}{r^3} \left[ \left( 1 + \frac{1}{q} \right) \hat{S}_0 + \left( 1 + q \right) \hat{S}_2 \right] \times \hat{S}_1 ,\]

\[
d\hat{S}_2 = \frac{1}{r^3} \left[ \left( 2 + \frac{3q}{2} \right) \hat{S}_0 - \frac{3q}{1+q} \hat{n} \right] \times \hat{S}_2 ,\]

where \( \hat{n} = \vec{r}_1 - \vec{r}_2 \) and

\[
\hat{S}_0 = \left( \frac{1}{q} \right) \hat{S}_1 + \left( 1 + q \right) \hat{S}_2 . \quad (2)
\]

For the sake of simplicity and to match the full numerical simulation above we will consider here the equal mass case, i.e. \( q = 1 \) and conservative 2PN dynamics. We will consider a generic configuration of binary black holes with arbitrary spins \( \hat{S}_1 \) and \( \hat{S}_2 \) at an angle \( \beta \) with respect to each other and adding up to the vector \( \hat{S} \). For definiteness \( \hat{S}_1 \) is the spin of the secondary black hole at an angle \( \gamma \) with respect to \( \hat{S} \) as shown in Fig. 4 and \( \hat{S}_2 \) is the spin of the primary identified with the larger spin magnitude \( S_2 \).

From Eqs. (1) the magnitude of the individual spins \( S_1 \) and \( S_2 \) are conserved as well as the magnitude of their sum, \( S \) (This has been observed to be approximately true in full nonlinear simulations of binary black holes solving general relativity field equations numerically [26]). It follows that the following quantities are conserved:

\[
\hat{S} \cdot \hat{S} = S^2 = S_1^2 + S_2^2 + 2S_1S_2 \cos \beta = \text{constant}, \quad (3)
\]

\[
\hat{S} \cdot \hat{S}_1 = SS_1 \cos \gamma = S_1^2 + S_2S_1 \cos \beta = \text{constant}. \quad (4)
\]

In turn, this leads to the conservation of \( \beta \) and \( \gamma \) during the evolution of the binary. In particular we find that \( \hat{S}_1 \) oscillates between angles \( \gamma \) and \( -\gamma \) (when it is both coplanar to \( \hat{S} \) and \( \hat{L} \)). We call this the flip-flop angle

\[
\theta_{ff} = \theta_{max} - \theta_{min} = 2\gamma , \quad (5)
\]

where

\[
\cos \gamma = \frac{S_1 + S_2 \cos \beta}{\sqrt{S_1^2 + S_2^2 + 2S_1S_2 \cos \beta}} = \frac{S^2 + S_1^2 - S_2^2}{2SS_1} . \quad (6)
\]

By decomposing the spin evolution equations (1) along \( \hat{L} \) and perpendicular to it, in the fashion of [23], we can read-off the polar and azimuthal oscillations frequencies of the secondary spin \( \hat{S}_1 \) (See also [27])

\[
\Omega_{ff} = 3S \left[ 1 - \frac{1}{M^{1/3} r^{1/2}} \right] , \quad (7)
\]

\[
\Omega_p = \frac{7L}{2r^3} + \frac{2}{r^3} (\hat{S} \cdot \hat{L}) . \quad (8)
\]

that we identify with the flip-flop and precession frequencies respectively.

Note that the primary black hole also oscillates at this \( \Omega_{ff} \) frequency, but with a smaller flip-flop angle given by \( 2(\beta - \gamma) \) where

\[
\cos(\beta - \gamma) = \frac{S_2^2 + S_1^2 - S_2^2}{2SS_2} . \quad (9)
\]

This oscillation of the spins represent a genuine spin-flip in the sense that it is the same object that completely changes its spin orientation. This is different from the case where the final remnant spin has flipped direction when compared to the spin of one of the individual black holes [28].

**Discussion:** In the scenario of binary black holes carrying individual accretion disks and a common circumbinary disk, spins changing their orientation can generate dramatic dynamical effects on the accreting matter around them. For definiteness, we focus on the secondary black hole, with spin \( \hat{S}_1 \) undergoing direction changes,
which when viewed in the orbital frame, resembles the peeling of an orange (See Fig. 1). Due to the relatively short time scale of flip-flop at close separations, the accreting matter increases the black hole spin during half the flip-flop period, but decreases it during the other half. On the other hand, mass is always added to the black holes during both the up and down states. The resulting net effect is to lower the intrinsic spin, $S_i/m_i^2$.

From Eqs. (6) and (7), requiring a flip-flop angle of 180°, implies $\gamma = \pi/2$ and $\Omega_{ff} = 3\sqrt{S_2^2 - S_1^2}/r^2$. For a maximally spinning primary and a secondary with a relatively small spin, at 1000M of separation we obtain a flip-flop period

$$T_{ff} = \frac{2\pi}{\Omega_{ff}} = 32,700 \text{ yr} \left( \frac{r}{1000M} \right)^3 \left( \frac{M}{10^8M_\odot} \right),$$

(10)

which is shorter than the gravitational radiation periods reported in [9] used to compare with the accretion-driven alignment mechanisms [29]. We thus conclude that such alignment processes can be less effective than expected when the flip-flop of spins is taken into account.

These flip-flop configurations can be very effective at disrupting the inner accretion disk dynamics and at circumventing the spin alignment (and growth) process by accretion, thus leading to important observational consequences. For instance, the change of the location of the internal rim of the disk due to the flip of the spin will change the high frequencies of fluctuations and the electromagnetic spectrum due to changes in the efficiency of the conversion of the accreting flow, i.e. proportional to $E_{ISCO}(\pm a)$. Flip-flopping spins may also generate turbulent accretion by changing the stirring leading to increase/decrease of the radiation (See [30]). These examples provide rough estimates of the disrupting effects of a flip-flopping spin and a more accurate evaluation requires a full numerical magnetohydrodynamic simulation of such binary black hole configurations. Our full numerical run proves that, although demanding, these simulations are currently possible and they can be performed using a magnetohydrodynamic description of the matter on a dynamical binary black hole background [31].

The change in the spin orientation at the latest stage of the merger could be followed through detailed observation of the gas jets in X-shaped galaxies [22]. The time scale for the phenomena, for instance, for the ∼ 25000M semiperiod we observe for the flip-flop in our full numerical simulation, corresponds to 1.2 seconds for 10$^8M_\odot$ binaries and 142 days for 10$^9M_\odot$ binaries. Note that according to Eq. (7) frequencies can be even higher if the primary would be closer to maximally spinning.

It is important to determine the likelihood of these flip-flop configurations out of all possible generic binary black hole merger processes. In order to evaluate this we can consider an initial random angle $\beta$ between the directions of the primary and secondary black hole spins (See Fig. 1). This could be the scenario where the accretion process succeeded in aligning the spin of the secondary black hole, while the spin of the primary still has a random orientation due to its much larger time scale for alignment.

One can then obtain from (6) that the probability of an angle $\gamma$ given a random distribution of $\cos \beta$ is

$$P(\cos \gamma) = 1 + \left( \frac{S_1}{S_2} \right) (2 \cos \gamma - 1),$$

(11)

which upon assuming random distribution of spin magnitudes $0 \leq (S_1/S_2) \leq 1$, leads to the probability for a flip-flop angle larger than $x$

$$P(\theta_{ff} > x) = \frac{1}{2} \cos \left( \frac{x}{2} \right) \left( \cos \left( \frac{x}{2} \right) + 1 \right).$$

(12)

This probability distribution shows, for instance, that (for equal mass binaries) there is nearly a 60% probability of flip-flops of more than 90°.

Accretion tends to bring the mass ratio towards 1 because the smaller black hole is further away from the center of mass of the system and can sweep out more mass from the internal parts of the circumbinary accretion disk [11]. However we need to evaluate to what extent this phenomena occurs for unequal mass binaries and if it applies, for instance, also to black hole - neutron stars binaries.

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