The mass of charmonium in nuclear matter

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Abstract. The masses of charmonium states immersed in nuclear matter are calculated in LO QCD and in QCD sum rules. While the mass shift for $J/\psi$ are found to be less than -10 MeV, those for the $\chi_{c0,1,2}$ and $\psi(3686)$ and $\psi(3770)$ are found to be more than -40 MeV. We investigate the feasibility of observing such mass shifts in the future accelerator project at GSI.

INTRODUCTION

Understanding hadron mass changes in nuclear medium and/or at finite temperature can provide valuable information about the QCD vacuum[1,2,3]. While the mass shifts for hadrons made of light quarks are sensitive to the restoration of the spontaneously broken chiral symmetry breaking[2,4,5,6], those for the heavy quark systems are sensitive to the changes of the non-perturbative gluon fields in nuclear matter. For the $J/\psi$, which consists of a charm and anticharm quark pair, both the QCD sum rules analysis[7,8] and the LO perturbative QCD calculation[9,10] show that its mass is reduced slightly in the nuclear matter mainly due to the reduction of the gluon condensate ($\langle \alpha_s G^2 \rangle$) in nuclear matter, which is expected to decrease by 6% at normal nuclear matter density. However, the changes are much larger for excited charmonium states, due mainly to larger color dipole size of these excited states.

In this report, we summarize the expected mass shift for charmonium states in nuclear matter and study the feasibility of observing such mass shift in the future accelerator project at GSI[11].

The lowest dimensional QCD operators that characterizes the non-perturbative nature of the QCD vacuum are the quark and gluon condensate. These condensate are estimated to have the following large non-perturbative expectation values in the vacuum[12],

$$
\langle \frac{\alpha_s}{\pi} F_{\mu\nu}^2 \rangle \sim 1.5 \text{ GeV/fm}^3,
\langle \bar{q}q \rangle \sim 2 \text{ fm}^{-3}.
$$

(1)

The gluon condensate can be written as the difference between the magnetic $B^2 = F_{ij}^2$ and electric $E^2 = \frac{1}{2} F_{0i}^2$ condensate, which respectively contribute to half of the zero temperature gluon condensate,

$$
\langle \frac{\alpha_s}{\pi} B^2 \rangle = -\langle \frac{\alpha_s}{\pi} E^2 \rangle = \frac{1}{2} \langle \frac{\alpha_s}{\pi} F_{\mu\nu}^2 \rangle.
$$

(2)
The above relation follows naturally from the Euclidean formulation of QCD at zero
temperature, such as in the lattice QCD, where the Euclidean space electric (mag-
etic) condensate is defined with a minus (plus) sign relative to its Minkowski space
counterpart[12]. Due to the symmetry in the time and space directions in the 4 di-
mensional Euclidean space, the Euclidean space electric condensate is expected to have
the same expectation value as the magnetic one[13, 14]. Hence the relation among the
Minkowski space condensate in Eq.(2) follows.

At nuclear matter, the non perturbative quark and gluon field configuration are ex-
pected to change appreciably, such that the average gluon and quark condensate values
decrease by 6% and 30%, respectively. These model independent results are obtained
from the linear density approximation and the nucleon expectation values of the quark
and gluon condensate, which are respectively known from the experimentally measured
pi-N sigma term and from taking the nucleon expectation value of the trace anomaly
relation[15]. The electric and magnetic part of the gluon condensate at nuclear matter
can be estimated separately, by using the twist-2 gluon operator,

$$\langle N(p) | \mathcal{D} F_{\mu}^{\alpha} F_{\alpha \nu} | N(p) \rangle = \left( p_{\mu} p_{\nu} - \frac{1}{4} m_N^2 \delta_{\mu \nu} \right) 2A_2(g),$$

(3)

where $A_2(g)$ is the second moment of the gluon distribution in the nucleon and is around
0.45, when the renormalization scale is in the of order 1 to 2 GeV. Using this and the
linear density approximation, we have,

$$\langle \frac{\alpha_s}{\pi} E^2 \rangle_{n.m.} = \left( \frac{4}{9} m_N m_0^0 + \frac{3}{2} m_N^2 \frac{\alpha_s}{\pi} A_2 \right) \frac{\rho_{n.m.}}{2m_N},$$

$$\langle \frac{\alpha_s}{\pi} B^2 \rangle_{n.m.} = -\left( \frac{4}{9} m_N m_0^0 - \frac{3}{2} m_N^2 \frac{\alpha_s}{\pi} A_2 \right) \frac{\rho_{n.m.}}{2m_N}.$$  

(4)

Here, $m_N^0 \sim 0.75$ GeV is the mass of the nucleon in the chiral limit[16], which comes
from taking the nucleon expectation value of the trace anomaly relation $T_\mu^\mu = -\frac{9}{8} \alpha_s \pi F_{\mu \nu}^2$. As can be seen from Eq.(4), due to the additional factor of $\frac{\alpha_s}{\pi}$ in the second terms, the
changes are dominated by the contribution from the first terms.

CHARMONIUM MASS SHIFT FROM QCD

The mass shift of charmonium states in nuclear medium can be evaluated in the pertur-
bative QCD when the charm quark mass is large, i.e., $m_c \to \infty$. In this limit, one can
perform a systematic operator product expansion (OPE) of the charm quark-antiquark
current-current correlation function between the heavy bound states by taking the sep-
aration scale ($\mu$) to be the binding energy of the charmonium[9, 17, 18]. The forward
scattering matrix element of the charm quark bound state with a nucleon then has the
following form:

$$T(q^2 = m_T^2) = \sum_n \frac{C_n}{(\mu)^n} \langle \mathcal{O}_n \rangle_N.$$  

(5)
Here, $C_n$ is the Wilson coefficient evaluated with the charm quark bound state wave function and $\langle \mathcal{O}_n \rangle_N$ is the nucleon expectation value of local operators of dimension $n$.

For heavy quark systems, there are only two independent lowest dimension operators; the gluon condensate ($\langle \frac{\alpha_s}{\pi} G^2 \rangle$) and the condensate of twist-2 gluon operator multiplied by $\alpha_s$ ($\langle \frac{\alpha_s}{\pi} G_{\mu \nu} G_{\mu \nu}^a \rangle$). These operators can be rewritten in terms of the color electric and magnetic fields: $\langle \frac{\alpha_s}{\pi} E^2 \rangle$ and $\langle \frac{\alpha_s}{\pi} B^2 \rangle$. Since the Wilson coefficient for $\langle \frac{\alpha_s}{\pi} B^2 \rangle$ vanishes in the non-relativistic limit, the only contribution is thus proportional to $\langle \frac{\alpha_s}{\pi} E^2 \rangle$, similar to the usual second-order Stark effect. We shall thus calculate the mass shift of charmonium states due to change of the gluon condensate in nuclear medium by the QCD second-order Stark effect [10].

The mass shift of charmonium states to leading order in density is obtained by multiplying the leading term in Eq. (5), by the nuclear density $\rho_N$. This gives,

$$\Delta m_\psi(\varepsilon) = -\frac{1}{9} \int dk^2 \frac{\partial \psi(k)}{\partial k} \left(\frac{k}{k^2/m_c + \varepsilon}\right) \times \frac{\alpha_s E^2}{\pi} \left(\frac{\rho_N}{2m_N}\right). \tag{6}$$

In the above, $m_N$ and $\rho_N$ are the nucleon mass and the nuclear density, respectively; $\langle \frac{\alpha_s}{\pi} E^2 \rangle_N \sim 0.5 \text{ GeV}^2$ is the nucleon expectation value of the color electric field obtained from Eq. (4) and $\varepsilon = 2m_c - m_\psi$. In Ref. [9], the LO mass shift formula was derived in the large charm quark mass limit. As a result, the wave function $\psi(k)$ is Coulombic and the mass shift is expressed in terms of the Bohr radius $a_0$ and the binding energy $\varepsilon_0 = 2m_c - m_\psi$. This might be a good approximation for $J/\psi$ but is not realistic for the excited charmonium states as Eq. (6) involves the derivative of the wave function, which measures the dipole size of the system. We have thus rewritten in the above the LO formula for charmonium mass shift in terms of the QCD parameters $\alpha_s = 0.84$ and $m_c = 1.95$, which are fixed by the energy splitting between $J/\psi$ and $\psi(3686)$ in free space [9]. Furthermore, we take wave functions of the charmonium state to be Gaussian with the oscillator constant $\beta$ determined by their squared radii $\langle r^2 \rangle = 0.47^2, 0.74^2, 0.96^2$, and 1 fm$^2$ for $J/\psi$, $\chi_{c0,1,2}$, $\psi(3686)$, and $\psi(3770)$, respectively, as obtained from the potential models [12]. This gives $\beta = 0.52, 0.43, 0.39$, and 0.37 GeV if we assume that these charmonium states are in the $1S$, $1P$, $2S$, and $1D$ states, respectively. Using these parameters, we find that the mass shifts at normal nuclear matter density obtained from the LO QCD formula Eq. (5) are -8, -40, -100, and -140 MeV for $J/\psi$, $\chi_{c0,1,2}$, $\psi(3686)$, and $\psi(3770)$, respectively [20].

Although the higher twist effects on the charmonium masses are expected to be nontrivial, the result for $J/\psi$ is consistent with those from other non-perturbative QCD studies, such as the QCD sum rules [7, 8] and the effective potential model [21, 22, 23], which are all based on the dipole interactions between quarks in the charmonium and those in the nuclear matter. The QCD sum rule results can also be applied for the $\chi_{c0,1,2}$ states, and the results from the leading order gluon condensate is summarized in Table [1].

Higher twist effects can be estimated in some calculations. For QCD sum rules for $J/\psi$, the corrections coming from dimension 6 operators are less than 30% of the
leading order results\cite{8}. The contributions from the $D\bar{D}$ meson loops in the $\psi(3686)$ and $\psi(3770)$ are also found to be less than 30\% of the LO QCD result\cite{20}.

All the results are summarized in table 1.

| Charmonium | $J^{PC}$ | QCD 2nd order Stark Effect | QCD sum rules | Effects of $D\bar{D}$ loop |
|------------|---------|---------------------------|---------------|--------------------------|
| $\eta_{c}$ | $0^{-+}$ | -$8$ MeV                  | -$5$ MeV      | No effect                |
| $J/\psi$   | $1^{--}$ | -$8$ MeV                  | -$7$ MeV      | $< 2$ MeV                |
| $\chi_{0,1,2}$ | $0,1,2^{++}$ | -$40$ MeV                | -$60$ MeV     | No effect on $\chi_{1}$ |
| $\psi(3686)$ | $1^{--}$ | -$100$ MeV               | $< 30$ MeV    |                          |
| $\psi(3770)$ | $1^{--}$ | -$140$ MeV               | $< 40$ MeV    |                          |

**OBSERVABILITY**

Since the mass shift of the heavy quark system reflects the changes of the Gluon field configuration in the vacuum, it would be interesting to observe such effects in experiment.

Consider an anti-proton with incoming four momentum $(\omega,0,k)$ annihilating a proton at rest $(m_{N},0,0)$ and creating a charmonium moving with velocity $v$. The required incoming momentum $k$ to create a charmonium state are summarized in Table 1.

| Charmonium | $J^{PC}$ | QCD 2nd order Stark Effect | QCD sum rules | Effects of $D\bar{D}$ loop |
|------------|---------|---------------------------|---------------|--------------------------|
| $\eta_{c}$ | $0^{-+}$ | -$8$ MeV                  | -$5$ MeV      | No effect                |
| $J/\psi$   | $1^{--}$ | -$8$ MeV                  | -$7$ MeV      | $< 2$ MeV                |
| $\chi_{0,1,2}$ | $0,1,2^{++}$ | -$40$ MeV                | -$60$ MeV     | No effect on $\chi_{1}$ |
| $\psi(3686)$ | $1^{--}$ | -$100$ MeV               | $< 30$ MeV    |                          |
| $\psi(3770)$ | $1^{--}$ | -$140$ MeV               | $< 40$ MeV    |                          |

Hence, the required incoming energy of the anti-proton to produce the charmonium state range from 4 to 6 GeV. In these energy region, the absorption cross section $\sigma_{\bar{p}-p} \sim 50$ mb. Hence, the anti-proton would be absorbed after travelling less than 1 fm in the nuclear matter. Moreover, once a charmonium state is created, their speed would be less than 0.9 c, which means that it will have to travel more than 10 fm/c to pass the diameter of a nucleus of $A=125$. Hence, considering the increased width of charmonium due to nuclear absorption\cite{24}, the charmonium is expected to decay inside the nucleus.

The cross section for the production of charmonium states and its subsequent decay into dileptons or $J/\psi + \gamma$ states are given by the following Breit-Wigner formula

$$\sigma_{BW}(E) = \frac{2J+1}{(2s_1+1)(2s_2+1)} \pi \frac{B_{in}B_{out}\Gamma_{Total}^2}{k^2(\Gamma_{Total}^2 + \Gamma_{Total-\text{medium}}^2/4)^4},$$

(7)

where $k$ is the c.m. momentum, $E$ is the c.m. energy, $B_{in}$ and $B_{out}$ are the branching fractions of the resonance into the entrance and exit channels. The $2s+1$ are the spin multiplicities of the incident spin states and $J$ the spin of the charmonium. Also $\Gamma_{Total-\text{medium}} = \Gamma_{Total} + \Gamma_{\text{medium}}$. 
If we substitute the medium mass shift and increase in width (due mainly to collision broadening), the cross sections are in the order of one to few hundred pbarn. The expected luminosity at the antiproton project at GSI is \(2 \times 10^{32} \text{cm}^{-2}\text{s}^{-1}\). Therefore if the cross section is 1 pb, it would correspond to about 17 events per day.

**TABLE 3.** Measurable decay channel and expected event rate at GSI future accelerator.

| Charmonium | \(\Gamma_{\text{Total} + \Gamma_{\text{medium}}}\) | Final state | cross-section to final state | events per day |
|------------|---------------------------------|-------------|-----------------------------|---------------|
| \(J/\psi(3097)\) | 87 KeV + 20 MeV | \(e^+ + e^-\) | 6 pb | 100 |
| \(\psi(3686)\) | 300 KeV + 20 MeV | \(e^+ + e^-\) | 0.6 pb | 10 |
| \(\psi(3770)\) | 23.6 MeV + 20 MeV | \(e^+ + e^-\) | 1 pb | 17 |
| \(\chi_{c0}(3417)\) | 16.2 MeV + 20 MeV | \(J/\psi + \gamma\) | 200 pb | 3400 |
| \(\chi_{c1}(3510)\) | 0.92 MeV + 20 MeV | \(J/\psi + \gamma\) | 80 pb | 1360 |
| \(\chi_{c2}(3556)\) | 2.08 MeV + 20 MeV | \(J/\psi + \gamma\) | 350 pb | 5950 |

Hence, the mass shift will be observable in the anti-proton project at the future accelerator facility at GSI.

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