Determination of displacements in cross-sections of four-bar mechanism links from distributed dynamic loads and their animation using MAPLE

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The links of high-speed mechanisms and manipulators are deformed under the action of inertia forces and external loads. These deformations have significantly influence on the accuracy of execution of the required law of motion by the operating point of the mechanism and the positioning of the manipulator grip. Accordingly, longitudinal and transverse displacements, angles of rotation of cross-sections of links under the action of distributed dynamic and external loads are investigated in this paper. The developed technique allows defining deformations of links of mechanisms and manipulators and can be applied at their designing. To determine the transverse displacements, the angles of rotation of the cross-sections of the links – the basic differential equation of the elastic line of the beam, to determine the longitudinal displacements of the points of the links – Hooke’s law and the boundary conditions of the computed scheme of the investigated linkages for elastic computation are used. The bending moment in the basic differential equation of the elastic line of the beam and the longitudinal force in Hooke’s law were determined by the theory developed by the authors of the analytical definition of internal forces in the links of planar linkages with statically determinate structures, taking into account the distributed dynamic loads from the masses of links, dead weight and from the acting external loads. According to the developed technique, programs are created in the MAPLE system and animations of the movement of mechanisms are received, with the construction on the links the diagrams of transverse, longitudinal displacements and angles of rotation of the link cross-sections. The developed analytical technique for determining deformations in the cross-sections of links is used to calculate the strength and stiffness of elements of movable linkages.

Key words: Mechanisms, movable linkages, displacements, distributed dynamic loads.
В высокоскоростных механизмах и манипуляторах звенья деформируются под воздействием сил инерции и внешних нагрузок. Эти деформации существенно влияют на точность исполнения требуемого закона движения рабочей точки механизма и на позиционирование схватов манипулятора. В связи с этим, в данной работе исследуются продольные, поперечные перемещения, углы поворота сечений звеньев находящихся под воздействием распределенных динамических и внешних нагрузок. Разработанный метод позволяет определить деформаций звеньев механизмов и манипуляторов и может применяться при их проектировании.

Для определения поперечных перемещений, углов поворота сечений звеньев использовано основное дифференциальное уравнение упругой линии балки, для определения продольных перемещений точек звеньев – закон Гука и граничные условия расчетной схемы исследуемых стержневых систем для упругой линии. Изгибающий момент, входящий в основное дифференциальное уравнение упругой линии балки и продольная сила, входящая в закон Гука были определены на основе разработанной теории аналитического определения внутренних силовых полагательных стержневых механизмов и манипуляторов со статически определямыми структурами с учетом распределенных динамических нагрузок от масс звеньев, собственного веса и от действующих внешних нагрузок.
Introduction

Dynamic analysis of high-speed mechanisms and manipulators received considerable attention in the last two decades. Every frame structure is deformed under the action of large static and dynamic loads. Whenever such a load occurs, several problems persist, for instance: the problems of failure, caused by large forces of inertia; elastic deformations of the mechanism can be significant, as a consequence, the mechanism become unusable; the mechanism cannot satisfy the kinematic requirements because of the large deformations of links. When designing high-speed mechanisms, the designer must either reduce the elastic deformations of the mechanism, or take them into account in computation. To test on stiffness and stability of the structure, it is necessary to be able to determine the displacements caused by the deformation of its elements. The techniques for determining these displacements are very diverse. They mainly differ from each other by the degree of complexity and scope of application.

Literature review

The method of direct integration of differential equation of elastic beam line is an earlier one for determination of displacements. However, in the case of beams with a large number of cross-sections, the implementation of this method involves considerable difficulties, which are not in the integration of differential equations, but in the technique of determining the arbitrary integration constants – drafting and solving of systems of linear algebraic equations (Jindal, 2012 : 294), (Timoshenko, 1948 : 134 – 135), (Darkov, 1975 : 289).

When computing by the displacement method, the main sought values are the displacements of the nodal points caused by the deformation of the system. Knowledge of these displacements is necessary and sufficient to determine all internal forces that arise in the cross-sections of the elements of a given system (Kaveti, 2014 : 412), (Tschiras, 1989 : 111), (Pisarenko, 1979 : 85). In the works of Sadler and Sandor (Sandor, 1973 : 497 – 516), the lateral bending vibrations of the elements of mechanisms, which can be considered as pin-ended beams making planar motions, are investigated. The normal dynamic stresses caused by the concerted actions of bending and axial loads are studied. A scheme is given for minimizing the maximum stresses in the flexible linkages of a given length without increasing the total mass. This is done using an iterative method of finding a full-strength form seeking method. The study is limited to the case of a rectangular cross-section, where the only variable is the width. Longitudinal deformations are considered negligible and, therefore, are not considered here. Abe proposed an accurate mathematical model of the flexible link in two-link rigid-flexible manipulators by taking the axial displacement and nonlinear curvature arising from large bending deformation into consideration for suppressing residual vibrations in optimal trajectory planning (Akira Abe, 2009 : 1627 – 1639). Mingxiang et al. presented...
a kinetostatic modeling method for flexure-hinge-based compliant mechanisms with hybrid serial-parallel substructures to provide accurate and concise solutions by combining the matrix displacement method with the transfer matrix method. This work established a general kinetostatic model of the whole compliant mechanisms based on the equilibrium equation of the nodal force (Mingxiang, 2018).

Finite element method (FEM) is used to structure the system into single finite elements and the stiffness matrix of element and of the whole system provides connection between displacement of nodes of element and system, as well as forces therein (Hutton, 2007 : 387), (Gokhale, 2008 : 416). Du and Ling have developed a general non-linear finite element model for dynamic analysis of three dimensional beam-like mechanisms undergoing both large rigid body motion and large elastic deflections. They adopted the non-linear strain-displacement relationship taking into account the axial strain and the shear strains due to the pre-twist in the beams (Hejun Du, 1995 : 56). Absy and Shabana show that the consideration of longitudinal displacement caused by bending would eliminate the third and higher order terms from the strain-energy expression, if the strain energy is written in terms of axial deformation. This leads to nonlinear inertia terms and a constant stiffness matrix (El-Abisy, 1997 : 207). Zhaocai studied the dynamic stress of the flexible beam element of planar flexible manipulators. Considering the effects of bending-shearing strain and tensile compression strain, the dynamic stress of the links and its position are derived by using the Kineto-Elastodynamics theory and the Timoshenko beam theory (Ding Zhaocai, 2006 : 17-20). Yue computed the maximum payload of kinematically redundant manipulators using a finite element method for describing the dynamics of a system (Shigang Yue, 2001 : 36). Korayem et al. considered a complete dynamic model to characterize the motion of a compliant link capable of large deflection (Moharam H. Korayem, 2010 : 17).

In this paper the longitudinal and transverse displacements, the angle of rotation of link cross-sections under the action of distributed dynamic loads and external forces are studied. The developed analytical technique makes it possible to accurately and quickly determine the deformations of links of mechanisms and manipulators and can be used in their design. Earlier the authors have developed a new analytical technique for determination of internal forces in the links of planar mechanisms and manipulators under the action of distributed dynamical loads and it was described in the work (Utenov, 2016 : 5-10).

3 Materials and methods

The main differential equation of elastic beam line (for the element $k$) has the form (Darkov, 1975 : 289), (Kaveti, 2014 : 412), (Tschiras, 1989 : 111), (Pisarenko, 1979 : 85):

$$
\frac{d^2 y_k}{d(x'_k)^2} = \frac{M_k(x'_k)}{E_k I_k}
$$

When a transverse distributed trapezoidal load acts on the element, the bending moment in the cross-sections of the element is determined by (3) from the previous work (Utenov, 2016 : 5-10). Substituting the values of $M_k(x'_k)$ from (3) into (1) and integrating one time, we will have expression to the rotation angles of cross-sections of the element $k$:
\[ \hat{O}_k(x_k') = \frac{dy_k'}{dx_k'} = \frac{1}{E_k I_k} \int \left\{ 1 - \frac{11}{2l_k} x_k' + \frac{9}{l_k^2} (x_k')^2 - \frac{9}{2l_k^3} (x_k')^3 \right\} M_{k1} + \left[ \frac{9}{l_k} x_k' - \frac{45}{2l_k^2} (x_k')^2 + \frac{27}{2l_k^3} (x_k')^3 \right] M_{k2} + \left[ -\frac{9}{2l_k} x_k' + \frac{18}{l_k^2} (x_k')^2 - \frac{27}{2l_k^3} (x_k')^3 \right] M_{k3} + \left[ \frac{1}{l_k} x_k' - \frac{9}{2l_k^2} (x_k')^2 + \frac{9}{2l_k^3} (x_k')^3 \right] M_{k4} \right\} dx_k' = \frac{1}{E_k I_k} \left\{ \left[ x_k' - \frac{11}{4l_k} (x_k')^2 + \frac{9}{3l_k^2} (x_k')^3 - \frac{9}{8l_k^3} (x_k')^4 \right] M_{k1} + \left[ \frac{1}{l_k} x_k' - \frac{45}{6l_k^2} (x_k')^2 + \frac{27}{8l_k^3} (x_k')^3 \right] M_{k2} + \left[ -\frac{9}{4l_k} (x_k')^2 + \frac{18}{3l_k^2} (x_k')^3 - \frac{27}{8l_k^3} (x_k')^4 \right] M_{k3} + \left[ \frac{1}{2l_k} (x_k')^2 - \frac{9}{6l_k^2} (x_k')^3 + \frac{9}{8l_k^3} (x_k')^4 \right] M_{k4} \right\} + C_{k1} \] (2)

which contains one arbitrary constant \( C_{k1} \). By integrating second time, we find the expression for beam deflection \( y_k(x_k') \)

\[ w_k(x_k') = y_k(x_k') = \frac{1}{E_k I_k} \left\{ \left[ \frac{1}{2} (x_k')^2 - \frac{11}{12l_k} (x_k')^3 + \frac{9}{12l_k^2} (x_k')^4 - \frac{9}{40l_k^3} (x_k')^5 \right] M_{k1} + \left[ \frac{9}{6l_k} (x_k')^3 - \frac{45}{24l_k^2} (x_k')^4 + \frac{27}{40l_k^3} (x_k')^5 \right] M_{k2} + \left[ -\frac{9}{12l_k} (x_k')^3 + \frac{18}{12l_k^2} (x_k')^4 - \frac{27}{40l_k^3} (x_k')^5 \right] M_{k3} + \left[ \frac{1}{6l_k} (x_k')^3 - \frac{9}{24l_k^2} (x_k')^4 + \frac{9}{40l_k^3} (x_k')^5 \right] M_{k4} \right\} + C_{k1} x_k' + C_{k2} \] (3)

which contains two arbitrary constants \( C_{k1} \) and \( C_{k2} \). The values of these arbitrary constants \( C_{k1} \) and \( C_{k2} \) are defined from consideration of two boundary conditions, i.e. from the conditions of end restraint.

The element aspect ratio \( dx_k' \) from the longitudinal force \( N_k(x_k') \) by Hooke’s law would be:

\[ \Delta dx_k' = \frac{N_k(x_k') dx_k'}{E_h I_k} \] (4)

When a distributed trapezoidal load is applied to the element, the longitudinal force is given by (5) in the work (Moharam H. Korayem, 2010 : 17). Substituting it into (4) and integrating one time, we find the following expression for longitudinal displacements of the element points:

\[ u_k(x_k') = \frac{1}{E_k A_k} \int N_k(x_k') dx_k' = \frac{1}{E_k A_k} \int \left\{ \left[ 1 - \frac{3}{l_k} x_k' + \frac{2}{l_k^2} (x_k')^2 \right] N_{k1} + \left[ \frac{4}{l_k} x_k' - \frac{4}{l_k^2} (x_k')^2 \right] N_{k2} + \left[ -\frac{1}{l_k} x_k' + \frac{2}{l_k^2} (x_k')^2 \right] N_{k3} \right\} dx_k' = \frac{1}{E_k A_k} \left\{ \left[ x_k' - \frac{3}{2l_k} (x_k')^2 + \frac{2}{3l_k^3} (x_k')^3 \right] N_{k1} + \left[ x_k' - \frac{3}{2l_k} (x_k')^2 + \frac{2}{3l_k^3} (x_k')^3 \right] N_{k1} \right\} \]
\[ + \left[ \frac{4}{2l_k} (x_k')^2 - \frac{4}{3l_k^2} (x_k')^3 \right] N_{k2} + \left[ - \frac{1}{2l_k} (x_k')^2 + \frac{2}{3l_k^2} (x_k')^3 \right] N_{k3} \right] + C_{kn} \]  

(5)

The arbitrary constant \( C_{kn} \) is determined from the conditions of end restraint.

Let us consider the determination of displacements in the links of four-bar mechanism (Figure 1). As the displacements of the link 1 cross-section \( O \) are known (\( \Phi h_{1}(0) = 0 \) – the angle of rotation of cross-section \( O \), \( uy_{10} = w_{1}(0) = 0 \) – the displacement that is perpendicular to the axis of the rod of the same cross-section, \( ux_{10} = w_{1n}(0) = 0 \) – the displacement along the axis of the rod of the same cross-section, that it is possible to define constants \( C_{11}, C_{12} \) and \( C_{1n} \):

Substituting into (2), (3) and (5) the value of \( x_k' = 0 \) and taking into account above said three boundary conditions, we will receive that \( C_{11}, C_{12} \) and \( C_{1n} \) are equal to zero. It allows defining transverse and longitudinal displacements in any cross-section of the link 1. Let us introduce three Cartesian coordinate systems \( BX_1'Y_1', BX_2'Y_2' \) and \( BX_2Y_2 \) at the point \( B \) (Figure 1), where \( X_1' \) is directed along the axis of the first link, \( X_2' \) is directed along the axis of the second link, \( X_2 \) is directed parallel to the axis \( X \). Let us determine the coordinates of the point \( B \) of the first link \( B' \) (a new position of the point \( B \) after deformation) \( ux_{1l_1}, uy_{1l_1} \) with respect to the coordinate system \( BX_1'Y_1' \). For this we substitute into (5) and (3) the value of \( x_k' = 0 \) and we get:

\[ ux_{1l} = \frac{l_1}{E_1 A_1} \left( \frac{1}{6} N_{11} + \frac{2}{3} N_{12} + \frac{1}{6} N_{13} \right) \]  

(6)

\[ uy_{1l} = \frac{l_1^2}{E_1 I_1} \left( \frac{13}{120} M_{11} + \frac{3}{10} M_{12} + \frac{3}{40} M_{13} \right) \]  

(7)

We will denote the coordinates of the point \( B' \) in the coordinate system \( BX_2'Y_2' \) through \( ux_{20}, uy_{20} \). Then position of the point \( B' \) with respect to the coordinate system \( BX_2Y_2 \), using the coordinates of the point \( B' \) in the coordinate system \( BX_1'Y_1' \), will be equal to:

\[
\begin{bmatrix}
ux_{20} \\
uy_{20}
\end{bmatrix} = \begin{bmatrix}
\cos \theta_1 & - \sin \theta_1 \\
\sin \theta_1 & \cos \theta_1
\end{bmatrix} \begin{bmatrix}
ux_{1l_1} \\
uy_{1l_1}
\end{bmatrix}
\]  

(8)

and using the coordinates of the point \( B' \) in the coordinate system \( BX_2'Y_2' \), we get:

\[
\begin{bmatrix}
x_{20} \\
y_{20}
\end{bmatrix} = \begin{bmatrix}
\cos \theta_2 & - \sin \theta_2 \\
\sin \theta_2 & \cos \theta_2
\end{bmatrix} \begin{bmatrix}
ux_{20} \\
uy_{20}
\end{bmatrix}
\]  

(9)

Equating the equalities (8) and (9) we receive two equations with two unknowns \( ux_{20} \) and \( uy_{20} \):

\[ ux_{20} \cos \theta_2 - uy_{20} \sin \theta_2 = ux_{1l_1} \cos \theta_1 - uy_{1l_1} \sin \theta_1 \]
Determination of displacements in cross-sections of four-bar mechanism links . . .

Figure 1 – The displacements in the links of four-bar mechanism

\[
ux_{20} \sin \theta_2 + uy_{20} \cos \theta_2 = ux_1 l_1 \sin \theta_1 + uy_1 l_1 \cos \theta_2
\]

Solving the resulting system of equations, we have:

\[
ux_{20} = (ux_1 l_1 \cos \theta_1 - uy_1 l_1 \sin \theta_1) \cos \theta_2 + (ux_1 l_1 \sin \theta_1 + uy_1 l_1 \cos \theta_1) \sin \theta_2 \tag{10}
\]

\[
uy_{20} = (ux_1 l_1 \sin \theta_1 + uy_1 l_1 \cos \theta_1) \cos \theta_2 - (ux_1 l_1 \sin \theta_1 + uy_1 l_1 \cos \theta_1) \sin \theta_2 \tag{11}
\]

Now, let us introduce three Cartesian coordinate systems \( CX_2'Y_2' \), \( CX_3'Y_3' \) and \( CX_3Y_3 \) in the point \( C \) where the axis \( X_2' \) is directed along the axis of the second link, the axis \( X_3' \) is directed along the axis of the third link, the axis \( X_3 \) is directed parallel to the axis \( X \). Then position of the point \( C' \) with respect to the coordinate system \( CX_3Y_3 \), using the coordinates of the point \( C' \) in the coordinate system \( CX_2'Y_2' \) will be equal to:

\[
\begin{bmatrix}
    x_{3w} \\
    y_{3w}
\end{bmatrix} =
\begin{bmatrix}
    \cos \theta_2 & -\sin \theta_2 \\
    \sin \theta_2 & \cos \theta_2
\end{bmatrix}
\begin{bmatrix}
    ux_{2l} \\
    uy_{2l}
\end{bmatrix},
\]

using the coordinates of the point \( C' \) in the coordinate system \( CX_2'Y_3' \) will be equal to:
\[
\begin{align*}
\begin{pmatrix} x_{3y} \\ y_{3y} \end{pmatrix} &= \begin{bmatrix} \cos \theta_3' & -\sin \theta_3' \\ \sin \theta_3' & \cos \theta_3' \end{bmatrix} \begin{pmatrix} ux3l \\ uy3l \end{pmatrix},
\end{align*}
\]

where \( \theta_3' = \arctg \frac{y_c - y_b}{x_c - x_b} \).

Equating the equalities (12) and (13) we receive:

\[
-uuy2l \sin \theta + uy3l \sin \theta'_3 = -ux2l \cos \theta + ux3l \cos \theta'_3
\]

\[
uuy2l \cos \theta - uy3l \cos \theta'_3 = -ux2l \sin \theta + ux3l \sin \theta'_3
\]

As the point \( D \) is hingedly fixed, that we have following boundary conditions:

\[
\begin{align*}
\begin{cases}
ux30 &= 0 \\
uy30 &= 0
\end{cases}
\end{align*}
\]

Substituting the value of \( X'_2 = l_2 \) into (5) we get:

\[
ux2l = ux20 + \frac{l_2}{E_2 A_2} \left( \frac{1}{6} N_{21} + \frac{2}{3} N_{22} + \frac{1}{6} N_{23} \right)
\]

Using the first boundary condition (15), and substituting \( x'_3 = l_3 \) into (5) with respect to the coordinate system \( CX'_3Y'_3 \) we have:

\[
ux3l = -\frac{l_3}{E_3 l_3} \left( \frac{1}{6} N_{31} + \frac{2}{3} N_{32} + \frac{1}{6} N_{33} \right)
\]

Substituting the found values \( ux2l \) and \( ux3l \) into (14) and (15), solving in common, we get:

\[
uy2l = \frac{-(-ux2l \cos \theta + ux3l \cos \theta'_3) \cos \theta'_3 - (-ux2l \sin \theta + ux3l \sin \theta'_3) \sin \theta_3}{\sin \theta_2 \cos \theta'_3 - \sin \theta'_3 \cos \theta_2}
\]

\[
yy3l = \frac{-(-ux2l \sin \theta + ux3l \sin \theta'_3) \sin \theta'_3 - (-ux2l \cos \theta + ux3l \cos \theta'_3) \cos \theta_2}{\sin \theta_2 \cos \theta'_3 - \sin \theta'_3 \cos \theta_2}
\]

Substituting the found values \( uy20 \) and \( uy2l \) into (2), conducting some simple transformations, we find:
\[ \Phi_{20} = \frac{uy2l - uy20 - \frac{l_2^2}{E_2I_2} \left( \frac{3}{10} M_{21} + \frac{3}{40} M_{23} \right)}{l_2} \]

Using the second boundary condition (16), we get:

\[ \Phi_{30} = \frac{-uy3l - \frac{l_3^2}{E_3I_3} \left( \frac{3}{10} M_{32} + \frac{3}{40} M_{33} \right)}{l_3} \]

4 Results and discussion

For the first time the authors have developed the technique for analytical determination of longitudinal and transverse displacements and the angles of rotation of cross-sections of links of the four-bar mechanism under the action of distributed dynamical loads. According to the given algorithm the programs in the Maple system were created and animations of the movement of mechanisms with the construction on the links the diagrams of transverse and longitudinal displacements and angles of rotation of the link cross-sections were received (Figure 2 – 6).

Figure 2 – The investigated mechanism with the construction on the links the diagrams of longitudinal inertial loads
**Figure 3** – The investigated mechanism with the construction on the links the diagrams of transverse inertial loads

![Diagram of transverse inertial loads](image1)

**Figure 4** – The investigated mechanism with the construction on the links the diagrams of the angles of rotation of the link cross-sections

![Diagram of angles of rotation](image2)

**Figure 5** – The investigated mechanism with the construction on the links the diagrams of the longitudinal displacements of the link cross-sections

![Diagram of longitudinal displacements](image3)
5 Conclusion

The developed technique allows determining the deformations in the links of mechanisms and manipulators under the action of distributed dynamical loads and can be used in the study of stress-strain state of the projected and existing movable and fixed linkages (planar linkages, manipulators, frames, etc.).

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References

[1] U.C. Jindal, Strength of Materials (India: Pearson Education, 2012), 294.

[2] Stephen Timoshenko, Strength of Materials: Part I. Elementary Theory and Problems (New York: D. Van Nostrand Company Inc., 1948), 134 – 135.

[3] Anatoly Darkov and Hayman Shpiro, Strength of Materials (Moscow: High School, 1975), 289.

[4] Seetharamulu Kaveti, "Displacement Method", in Dynamic Analysis of Skeletal Structures: Force and Displacement Methods and Iterative Techniques (India: McGraw-Hill Education, 2014), 412.

[5] Alexander Tschiras, Structural Mechanics: Theory and Algorithms (Moscow: Stroyizdat, 1989), 111.

[6] Georgy Pisarenko et al., Strength of Materials (Kiev: Vischa shkola, 1979), 85.

[7] George N. Sandor and Imad Imam, "A general method of kineto-elastodynamic design of high speed mechanisms", Mech. Mach. Theory 8 (1973), pp. 497 – 516, doi.org/10.1016/0094-114X(73)90023-2.

[8] Akira Abe, "Trajectory planning for residual vibration suppression of a two-link rigid-flexible manipulator considering large deformation", Mech. Mach. Theory 44 (2009), pp. 1627 – 1639, doi.org/10.1016/j.mechmachtheory.2009.01.009.

[9] Ling Mingxiang et al., "Kinetostatic modeling of complex compliant mechanisms with serial-parallel substructures: a semi-analytical matrix displacement method", Mech. Mach. Theory In Press 2018, doi.org/10.1016/j.mechmachtheory.2018.03.014.
[10] David V. Hutton, *Fundamentals of finite element analysis* (New Delhi: Tata McGraw-Hill Publishing Company, 2007), 387.

[11] Nitin S. Gokhale, *Practical Finite Element Analysis* (India: Finite To Infinite, 2008), 416.

[12] Hejun Du and Shihfu Ling, "A nonlinear dynamic model for Three-Dimensional Flexible Linkages", *Computers and Structures* 56 (1995), doi.org/10.1016/0045-7949(94)00529-C.

[13] H. El-Abey and Ahmed A. Shabana, "Geometric stiffness and stability of rigid body modes", *Journal of Sound and Vibration* 207 (1997), doi.org/10.1006/javi.1997.1051.

[14] Ding Zhaocai, "Analysis of dynamic stress and fatigue property of flexible robot", (paper presented at the IEEE International Conference on Robotics and Biometrics, Kunming, China, December 17-20, 2006).

[15] Shigang Yue, Shiu Kit Tso, Weiliang Xu, "Maximum-dynamic-payload trajectory for flexible robot manipulators with kinematic redundancy", *Mech. Mach. Theory* 36 (2001), doi.org/10.1016/S0094-114X(00)00059-8.

[16] Moharam H. Korayem, Mohammad Haghpanahi, Hamidreza Heidari, "Maximum allowable dynamic load of flexible manipulators undergoing large deformation", *Transaction B: Mechanical Engineering* 17 (2010), https://pdfs.semanticscholar.org/c481/b62788e8b3ffbc6ba3a1af07b05e849439d8.pdf.

[17] Muratulla Utenov et. al. "Computational method of determination of internal efforts in links of mechanisms and robot manipulators with statically definable structures considering the distributed dynamically loadings", (paper presented at the *European Congress on Computational Methods in Applied Sciences and Engineering Biometrics*, Crete, Greece, June 5-10, 2016).