Measuring properties of a Heavy Higgs boson

in the $H \to t\bar{t} \to bW^+\bar{b}W^-$ decay

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Abstract

Suppose a heavy neutral Higgs or scalar boson $H$ is discovered at the LHC, it is important to investigate its couplings to the standard model particles as much as possible. Here in this work we attempt to probe the CP-even and CP-odd couplings of the heavy Higgs boson to a pair of top quarks, through the decay $H \to t\bar{t} \to bW^+\bar{b}W^-$. We use the helicity-amplitude method to write down the most general form for the angular distributions of the final-state $b$ quarks and $W$ bosons. We figure out that there are 6 types of angular observables and, under CPT$^T$ conservation, one-dimensional angular distributions can only reveal two of them. Nevertheless, the $H$ couplings to the $t\bar{t}$ pair can be fully determined by exploiting the one-dimensional angular distributions. A Higgs-boson mass of 380 GeV not too far above the $t\bar{t}$ threshold is illustrated with full details. With a total of $10^4$ events of $H \to t\bar{t} \to bW^+\bar{b}W^+$, one can determine the couplings up to 10-20% uncertainties.
I. INTRODUCTION

The scalar boson that was discovered at the LHC in 2012 [1, 2] turned out to be best described by the Standard Model (SM) Higgs boson [3], which is remarkable confirmation of the Higgs mechanism proposed in 1960s [4]. Among the Higgs boson couplings to the SM particles, the most constrained one is its coupling to the massive gauge bosons that is very close to the corresponding SM value with about 10% uncertainty [5]. Nevertheless, the couplings to fermions are much less constrained, especially for the first two generations. The coupling to the top-quark pair from global fits has about 20–30% uncertainty [5]. There was also direct measurements of the top Yukawa coupling in $pp \rightarrow t\bar{t}h$ production [6], which still needs more data to have more precise measurements than the global fitting.

Even though the SM has achieved a great success in accounting for the interactions among the basic building blocks of matter, however extra particles and new interactions are required to explain the experimental observations of dark matter, non-vanishing neutrino mass, the baryon asymmetry of our Universe, inflation, etc. In most extensions beyond the SM, the Higgs sector is enlarged to include more than one Higgs doublet resulting in charged Higgs bosons and several neutral Higgs bosons in addition to the one discovered at the LHC. For example, the minimal supersymmetric extension of the SM, aka MSSM [7], requires two Higgs doublet fields, thus leading to a pair of charged Higgs bosons and 3 neutral ones. In the next-to-minimal supersymmetric standard model, there are two additional neutral Higgs bosons [8].

Suppose that in future experiments a neutral Higgs boson $H$ heavier than the SM 125 GeV Higgs boson (denoted by $h$) is discovered. It is generally expected that the decays of the heavy Higgs boson $H$ into gauge bosons would be suppressed as it becomes heavier and heavier, because the measurement of the gauge-Higgs coupling of the SM-like 125 GeV boson allows only a little room for the $H$ couplings to massive vector bosons. However, its couplings to fermions have no reasons to be small. Indeed, once it is above the $t\bar{t}$ threshold, the decay into $t\bar{t}$ pair could be dominant.

In this work, we assume that the heavy Higgs boson $H$ is not too far above the $t\bar{t}$ threshold, say 380 GeV, and the dominant decay mode is $t\bar{t}$. Without requiring $H$ to carry any definite CP-parity, we consider the possibility to probe its CP nature via the decay $H \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^-$. We employ the helicity-amplitude method [9] to calculate the decay
amplitude taking into account all spin correlations in the decay chain. By measuring various angular distributions, in particular the angle between the decay planes of the top and anti-top quarks, one can discern the CP-even and CP-odd couplings of the Higgs boson. This is the main goal of this work. Other fermionic modes, in general, are either too small or suffer tremendously from SM backgrounds. The top quark also has the advantage that it decays before hadronization, in contrast to the bottom or charm quarks, and therefore the spin information is retained in the decay products of the top quark. Thus, the spin and CP properties of the parent Higgs boson can be determined by studying several kinematical distributions of the decay products of the top and anti-top quarks.

On the other hand, when the heavy Higgs boson \( H \) is below the \( t\bar{t} \) threshold, its bosonic decay modes \( ZZ, hh, \) and \( hZ \) are useful for probing the CP nature of it. By taking account of the spin-0 nature of \( H \), only the \( ZZ \) mode may lead to nontrivial angular correlations among the decay products of the \( Z \) bosons through the interferences among various helicity states of the two intermediate \( Z \) bosons before their decays [10]. This bosonic mode was suggested to determine the spin and parity of the Higgs boson [11] quite a number of years ago. After the 125 GeV Higgs-boson discovery, the method was practically applied to determine the spin and CP properties of the observed Higgs boson [12, 13]. We shall not concern with the bosonic modes in this work.

Under the current experimental status, in which active searches for heavy resonances decaying into a \( t\bar{t} \) pair have been continually performed [14], our study may show how well one can determine the properties of such a heavy scalar Higgs boson at the LHC and/or High Luminosity LHC (HL-LHC). We refer to, for example, Ref. [15] for some previous studies at \( e^+e^- \) or \( \mu^+\mu^- \) colliders.

The remainder of this article is organized as follows. In Sec. II, based on the helicity amplitude method [9], we present a formalism for the study of angular distributions in the decay \( H \to t\bar{t} \to bW^+\bar{b}W^- \). We point out that there are 6 types of angular observables in general and we can classify them according to the CP and CPT parities of each observable. In Sec. III, we illustrate how well one can measure the couplings of the heavy Higgs boson by exploiting the angular observables introduced in Sec. II. Finally, Sec. IV is devoted to a brief summary, some prospects for future work and conclusions.
II. FORMALISM

Without loss of generality, the Lagrangian describing the interactions of the Higgs boson $H$ with top quarks can be written as \cite{16}

$$\mathcal{L}_{Htt} = -g_t H \bar{t}(g^S + i\gamma_5 g^P) t = -g_t \sum_{A=L,R} (g^S + iA g^P) H \bar{t} P_A t$$

(1)

where $P_A = (1 + A \gamma_5)/2$ with $A = -(L), +(R)$. $g_t$ is the overall strength of the $H-t-\bar{t}$ coupling and $g^{P(S)} = 0$ when $H$ is the pure CP-even (odd) state. If $H$ is a CP-mixed state, both $g^S$ and $g^P$ are non-vanishing. For the SM Higgs boson, $g_t = g m_t/(2 M_W)$, $g^S = 1$, and $g^P = 0$. On the other hand, the Lagrangian describing the interactions of the top quarks with bottom quarks and $W$ bosons is

$$\mathcal{L} = -\frac{g}{\sqrt{2}} \left[ W^- \bar{b} \gamma^\mu (f_L P_L + f_R P_R) t + W^+ \bar{t} \gamma^\mu (f_L^* P_L + f_R^* P_R) b \right]$$

$$= -\frac{g}{\sqrt{2}} \left[ (W^+)^* \bar{b} \gamma^\mu (f_L P_L + f_R P_R) t + (W^-)^* \bar{t} \gamma^\mu (f_L^* P_L + f_R^* P_R) b \right].$$

(2)

In the SM, $f_L = 1$ and $f_R = 0$.

A. Helicity Amplitudes

We first present the helicity amplitude for the process

$$H \to t(p_t, \sigma_t) \bar{t}(\bar{p}_t, \bar{\sigma}_t) \to b(p_b, \sigma_b) W^+(k_1, \epsilon_1) \bar{b}(\bar{p}_b, \bar{\sigma}_b) W^-(k_2, \epsilon_2).$$

Here $p_{t,b}$ and $\bar{p}_{t,b}$ are four-momenta of the quarks ($t, b$) and antiquarks ($\bar{t}, \bar{b}$), respectively, and $\sigma_{t,b}$ and $\bar{\sigma}_{t,b}$ denote their helicities. The four-momenta of $W^+$ and $W^-$ are denoted by $k_1$ and $k_2$, respectively, with $p_t = p_b + k_1$ and $\bar{p}_t = \bar{p}_b + k_2$ and $\epsilon_1(\lambda_1)$ and $\epsilon_2(\lambda_2)$ are the polarization 4-vectors of $W$ bosons. Depending on the helicities of the final-state particles, the amplitude can be cast into the form

$$i \mathcal{M}_{\sigma_b \lambda_1; \bar{\sigma}_b \lambda_2} = \sum_{A,B,C=L,R} \left\{ \bar{u}_b(p_b, \sigma_b) \left[ -i g \frac{g_1^C}{2} (k_1) f_A P_A \right] \frac{i}{\not{p}_t - m_t} \left[ -i g_t (g^S + i B g^P) P_B \right] \right\}$$

$$\times \frac{i}{\not{\bar{p}}_t - m_t} \left[ -i g \frac{g_2^B}{2} (k_2) f_C P_C \right] v_b(\bar{p}_b, \bar{\sigma}_b)$$

$$= -i \frac{1}{p_t^2 - m_t^2 + im_t \not{p}_t} \frac{1}{\not{\bar{p}}_t^2 - m_b^2 + im_b \not{\bar{p}}_t} \sum_{\sigma, \bar{\sigma}} \mathcal{M}_{t \to \bar{b} W^+} \mathcal{M}_{H \to \bar{t} t} \mathcal{M}_{\bar{t} \to \bar{b} W^-} (3)$$
using
\[\sum_{\sigma_t} u(p_t, \sigma_t) \bar{u}(p_t, \sigma_t) = \gamma_t + m_t, \quad \sum_{\bar{\sigma}_t} v(\bar{p}_t, \bar{\sigma}_t) \bar{v}(ar{p}_t, \bar{\sigma}_t) = \bar{\gamma}_t - m_t.\] (4)

The helicity amplitude for the decay $H(q) \to t(p_t, \sigma_t) \bar{t}(\bar{p}_t, \bar{\sigma}_t)$ in the rest frame of $H$ is given by
\[\mathcal{M}_{\sigma_t \bar{\sigma}_t}^H = g_t \sqrt{s} \langle \sigma_t \rangle \bar{\delta}_{\sigma_t \bar{\sigma}_t} e^{-i\phi_t},\] (5)
where $s = q^2$, $\phi_t$ is the azimuthal angle of the $t$ momentum, and
\[\langle + \rangle = X g^S_t - i Y g^P_t, \quad \langle - \rangle = -X g^S_t - i Y g^P_t.\] (6)
The momentum-dependent $X$ and $Y$ are given by
\[X = \sum_{\lambda} \left[ \frac{(1 + \lambda_H^{1/2})^2 - (\alpha_t - \bar{\alpha}_t)^2}{2} - \frac{(1 - \lambda_H^{1/2})^2 - (\alpha_t - \bar{\alpha}_t)^2}{2} \right],\]
\[Y = \sum_{\lambda} \left[ \frac{(1 + \lambda_H^{1/2})^2 - (\alpha_t - \bar{\alpha}_t)^2}{2} + \frac{(1 - \lambda_H^{1/2})^2 - (\alpha_t - \bar{\alpha}_t)^2}{2} \right],\] (7)
where $\lambda_H = (1 + \alpha_t^2 + \bar{\alpha}_t^2 - 2\alpha_t - 2\bar{\alpha}_t - 2\alpha_t\bar{\alpha}_t)$ with $\alpha_t = p_t^2/s$ and $\bar{\alpha}_t = \bar{p}_t^2/s$. When the top quarks are on-shell, $X = \beta_t = (1 - 4m_t^2/s)^{1/2}$ and $Y = 1$. One may take $\phi_t = 0$ without loss of generality.

The helicity amplitude for the decay $t(p_t, \sigma_t) \to b(p_b, \sigma_b) W^+(k_1, \epsilon_1)$ in the $t$ rest frame is given by
\[\mathcal{M}_{\sigma_t \sigma_b \lambda}^{t \to bW^+} = -\frac{g}{\sqrt{2}} \sqrt{2} \sqrt{p_t^2 E_b} \langle \sigma_t : \sigma_b \lambda_1 \rangle_t,\] (8)
where $2\sqrt{p_t^2 E_b} = p_t^2 + m_b^2 - M_W^2$. The reduced helicity amplitudes $\langle \sigma_t : \sigma_b \lambda_1 \rangle_t$ are given by
\[\langle \sigma_t : \sigma_b \lambda_1 \rangle_t = \left\{ \begin{array}{ll}
\sum_{A=L,R} & \left[ -A(\sqrt{2} f_A c_{\theta_1/2}) (1 + A \sigma_1 \bar{\sigma}_b)^{1/2} \delta_{\sigma_t \lambda_1} \delta_{\sigma_t \sigma_b} \\
& + A \sigma_t (\sqrt{2} f_A s_{\theta_1/2} e^{+i\phi_1}) (1 - A \sigma_1 \bar{\sigma}_b)^{1/2} \delta_{\sigma_t - \lambda_1} \delta_{\sigma_t - \sigma_b} \right] \text{ for } \lambda_1 = \pm \\
\sum_{A=L,R} & \left[ (f_A s_{\theta_1/2}) \frac{\partial_b E_b + A \sigma_1 E_W}{M_W} (1 + A \sigma_1 \bar{\sigma}_b)^{1/2} \delta_{\sigma_t \sigma_b} \\
& + \sigma_t (f_A c_{\theta_1/2} e^{+i\phi_1}) \frac{\partial_b E_b - A \sigma_1 E_W}{M_W} (1 - A \sigma_1 \bar{\sigma}_b)^{1/2} \delta_{\sigma_t - \sigma_b} \right] \text{ for } \lambda_1 = 0 \\
\end{array} \right.\] (9)
where $\theta_1$ and $\phi_1$ are the polar and azimuthal angles of the $W^+$ momentum in the $t$ rest frame and $c_{\theta_1/2} = \cos(\theta_1/2)$ and $s_{\theta_1/2} = \sin(\theta_1/2)$. We note that $\sigma_b = \lambda_1$ when $\lambda_1 = \pm$ and...
the 4 amplitudes of $\langle \pm : ++ \rangle_t$ and $\langle \pm : + - \rangle_t$ are identically vanishing. In the $m_b \to 0$ limit, the reduced amplitudes simplify and we have the following non-vanishing 8 amplitudes$^1$

\[
\begin{align*}
\langle - : - - \rangle_t &= + \sqrt{2} f_L c_{\theta_1/2}, \\
\langle - : 0 \rangle_t &= + \frac{\sqrt{p_t^2}}{m_W} f_L s_{\theta_1/2}, \\
\langle + : - - \rangle_t &= - \sqrt{2} f_L s_{\theta_1/2} e^{+i\phi_1}, \\
\langle + : 0 \rangle_t &= + \frac{\sqrt{p_t^2}}{m_W} f_L c_{\theta_1/2} e^{+i\phi_1}, \\
\langle + : ++ \rangle_t &= - \sqrt{2} f_R c_{\theta_1/2}, \\
\langle + : +0 \rangle_t &= + \frac{\sqrt{p_t^2}}{m_W} f_R s_{\theta_1/2}, \\
\langle - : ++ \rangle_t &= - \sqrt{2} f_R s_{\theta_1/2} e^{-i\phi_1}, \\
\langle - : +0 \rangle_t &= - \frac{\sqrt{p_t^2}}{m_W} f_R c_{\theta_1/2} e^{-i\phi_1}.
\end{align*}
\]  

(10)

We note that $A = \sigma_b$ in the $m_b = 0$ limit.

The helicity amplitude for the decay $\bar{t}(p_t, \sigma_t) \to \bar{b}(p_b, \sigma_b)W^-(k_2, \epsilon_2)$ in the $\bar{t}$ rest frame is similarly given by

\[
\mathcal{M}_{\sigma_t : \sigma_b \lambda_2}^{\bar{t} \to \bar{b}W^-} \equiv - \frac{g}{\sqrt{2}} \sqrt{2 \sqrt{p_t^2} E_b} \langle \sigma_t : \sigma_b \lambda_2 \rangle_{\bar{t}}
\]  

(11)

where $2 \sqrt{p_t^2} E_b = p_t^2 + m_b^2 - M_W^2$ and the reduced amplitudes $\langle \sigma_t : \sigma_b \lambda_2 \rangle_{\bar{t}}$ can be obtained by replacing $f_A$ with $f_A^*$ in Eq. (9) together with $\sigma_{t,b} \to \sigma_{t,b}$, $\lambda_1 \to \lambda_2$, etc$^2$ Further we note the relations

\[
\begin{align*}
\langle - : - - \rangle_{\bar{t}} &= - \langle + : ++ \rangle_{\bar{t}}^*, \\
\langle - : 0 \rangle_{\bar{t}} &= + \langle + : +0 \rangle_{\bar{t}}^*, \\
\langle + : - - \rangle_{\bar{t}} &= + \langle - : ++ \rangle_{\bar{t}}^*, \\
\langle + : 0 \rangle_{\bar{t}} &= - \langle - : -0 \rangle_{\bar{t}}^*, \\
\langle + : ++ \rangle_{\bar{t}} &= - \langle - : - - \rangle_{\bar{t}}^*, \\
\langle + : +0 \rangle_{\bar{t}} &= + \langle - : -0 \rangle_{\bar{t}}^*, \\
\langle - : ++ \rangle_{\bar{t}} &= + \langle + : - - \rangle_{\bar{t}}^*, \\
\langle - : +0 \rangle_{\bar{t}} &= - \langle + : -0 \rangle_{\bar{t}}^*.
\end{align*}
\]  

(12)

Collecting all the sub-amplitudes we obtain

\[
\mathcal{M}_{\sigma_b \lambda_1 : \sigma_b \lambda_2} \equiv - g^2 g_t \sqrt{s} \sqrt{\sqrt{p_t^2 + m_b^2 - M_W^2} \sqrt{p_t^2 + m_b^2 - M_W^2}} \langle \sigma_b \lambda_1 : \sigma_b \lambda_2 \rangle_{\bar{t}}
\]  

(13)

where

\[
\langle \sigma_b \lambda_1 : \sigma_b \lambda_2 \rangle = \sum_{\sigma_t = \pm} \langle \sigma_t \rangle_{\bar{t}} \langle \sigma_t : \sigma_b \lambda_1 \rangle_{\bar{t}} \langle \sigma_t : \sigma_b \lambda_2 \rangle_{\bar{t}}.
\]  

(14)

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$^1$ In Ref. [17], the authors presented the helicity amplitudes in the $m_b \to 0$ limit. We find a minor discrepancy in four of the amplitudes with $\lambda_1 = \pm$ by an overall factor of $e^{-i\lambda_1 \phi_1}$, which does not affect the full amplitude squared for the process $H \to \bar{t} \to bW^+ \bar{b}W^-$.  

$^2$ For details of the relation between the helicity amplitudes for the $t$ and $\bar{t}$ decays, see Appendix A.
Explicitly, from Table I, we have only four non-vanishing amplitudes:

$$
\langle - - : ++ \rangle, \langle -- : +0 \rangle, \langle -0 : ++ \rangle, \text{ and } \langle -0 : +0 \rangle.
$$

| $\sigma_t \sigma_b \lambda_1$ | $\langle \sigma_t : \sigma_b \lambda_1 \rangle_t$ | $\sigma_t \sigma_b \lambda_2$ | $\langle \sigma_t : \sigma_b \lambda_2 \rangle_t$ |
|-----------------------------|---------------------------------|-----------------------------|---------------------------------|
| $+ +$                       | $- \sqrt{2} s_{\theta_1/2} e^{i\phi_1}$ | $+ +$                       | $- \sqrt{2} c_{\theta_2/2}$     |
| $+ 0$                       | $\sqrt{p_t^2 M_W^2 c_{\theta_1/2} e^{i\phi_1}}$ | $+ 0$                       | $\sqrt{p_t^2 M_W^2 s_{\theta_2/2}}$ |
| $- -$                       | $\sqrt{2} c_{\theta_1/2}$ | $- +$                       | $- \sqrt{2} s_{\theta_2/2} e^{-i\phi_2}$ |
| $- 0$                       | $\sqrt{p_t^2 M_W^2 s_{\theta_1/2}}$ | $- 0$                       | $- \sqrt{p_t^2 M_W^2 c_{\theta_2/2} e^{-i\phi_2}}$ |

### B. Angular coefficients and observables

The partial decay width of the process $H \to t\bar{t} \to bW^+\bar{b}W^-$ is given by\(^3\)

$$
d\Gamma = \frac{N_C}{2M_H} \left( \sum_{\sigma_b,\lambda_2} |M_{\sigma_b \lambda_1 : \sigma_b \lambda_2}|^2 \right) d\Phi_4
$$

$$
= \frac{N_C}{2^{13} \pi^6 M_H} \lambda_t \lambda_1 \frac{1}{\sqrt{p_t^2 \sqrt{p_t^2}}} \left( \sum |M|^2 \right) d\sqrt{p_t^2} d\sqrt{p_t^2} dc_{\theta_1} dc_{\theta_2} d\Phi
$$

where $N_C = 3$ and

$$
\lambda_t = \left( 1 - \frac{m_b^2}{p_t^2} - \frac{M_W^2}{p_t^2} \right)^2 - 4 \frac{m_b^2}{p_t^2} \frac{M_W^2}{p_t^2}, \quad \lambda_1 = \left( 1 - \frac{m_b^2}{p_t^2} - \frac{M_W^2}{p_t^2} \right)^2 - 4 \frac{m_b^2}{p_t^2} \frac{M_W^2}{p_t^2}.
$$

For any values of $f_L$ and $f_R$, taking also account of finite $m_b$ effects, the precise differential angular distribution $\frac{d\Gamma}{dc_{\theta_1} dc_{\theta_2} d\Phi}$ can be obtained numerically by integrating Eq. (15) over $\sqrt{p_t^2}$ and $\sqrt{p_t^2}$ and using Eqs. (13), (14), (6), and (9).

On the other hand, in the $m_b \to 0$ limit, the amplitudes take much simpler forms and one can derive analytic expressions for the differential angular distributions in terms of physically meaningful angular coefficients and observables. When $f_L = 1$ and $f_R = 0$, there are only four non-vanishing amplitudes: $\langle -- : ++ \rangle, \langle -- : +0 \rangle, \langle -0 : ++ \rangle, \text{ and } \langle -0 : +0 \rangle$. Explicitly, from Table I we have

$$
\langle -- : ++ \rangle = 2 \left( \langle + \rangle s_{\theta_1/2} c_{\theta_2/2} e^{i\phi_1} - \langle - \rangle c_{\theta_1/2} s_{\theta_2/2} e^{-i\phi_2} \right)
$$

$$
\langle -- : +0 \rangle = -\sqrt{2} \frac{\sqrt{p_t^2}}{M_W} \left( \langle + \rangle s_{\theta_1/2} c_{\theta_2/2} e^{i\phi_1} + \langle - \rangle c_{\theta_1/2} s_{\theta_2/2} e^{-i\phi_2} \right)
$$

\(^3\) For the four-body phase space, see Appendix B.
\begin{align}
\langle -0 : ++ \rangle &= -\sqrt{2} \frac{p_t^2}{M_W^2} (\langle + \rangle c_{\theta_1 / 2} c_{\theta_2 / 2} e^{i\phi_1} + \langle - \rangle s_{\theta_1 / 2} s_{\theta_2 / 2} e^{-i\phi_2}) \\
\langle -0 : +0 \rangle &= \sqrt{2} \frac{p_t^2}{M_W^2} (\langle + \rangle c_{\theta_1 / 2} c_{\theta_2 / 2} e^{i\phi_1} - \langle - \rangle s_{\theta_1 / 2} c_{\theta_2 / 2} e^{-i\phi_2})
\end{align}
(17)

where \( \theta_{1(2)} \) and \( \phi_{1(2)} \) denote the direction of \( W^{+(-)} \) in the \( t(\bar{t}) \) rest frame. And then, the sum of the amplitudes squared can be organized as

\[
\sum_{\sigma_b, \bar{\lambda}_1, \bar{\sigma}_1, \lambda_2} |\langle \sigma_b \lambda_1 : \bar{\sigma}_b \lambda_2 \rangle|^2 = C_1 \left[ 1 + \frac{p_t^2 p_{\bar{t}}^2}{4M_W^4} \right] (1 - c_{\theta_1} c_{\theta_2}) + \frac{p_t^2 + p_{\bar{t}}^2}{2M_W^2} \left( 1 + c_{\theta_1} c_{\theta_2} \right)
\]
\[
+ C_2 \left[ -1 + \frac{p_t^2 p_{\bar{t}}^2}{4M_W^4} \right] (c_{\theta_1} - c_{\theta_2}) + \frac{p_t^2 - p_{\bar{t}}^2}{2M_W^2} (c_{\theta_1} + c_{\theta_2})
\]
\[
+ C_3 \left( \frac{p_t^2}{2M_W^2} - 1 \right) \left( \frac{p_{\bar{t}}^2}{2M_W^2} - 1 \right) (-s_{\theta_1} s_{\theta_2} c_{\Phi})
\]
\[
+ C_4 \left( \frac{p_t^2}{2M_W^2} - 1 \right) \left( \frac{p_{\bar{t}}^2}{2M_W^2} - 1 \right) s_{\theta_1} s_{\theta_2} s_{\Phi}
\]
(18)

with \( \Phi = \phi_1 + \phi_2 \) denoting the angle between the two decay planes and the 4 angular coefficients are given by

\[
C_1 \equiv |\langle + \rangle|^2 + |\langle - \rangle|^2 = 2 \left[ |X|^2 \left( g^S \right)^2 + |Y|^2 \left( g^P \right)^2 \right],
\]
\[
C_2 \equiv |\langle + \rangle|^2 - |\langle - \rangle|^2 = 4 \Re m(X^*Y) g^S g^P,
\]
\[
C_3 \equiv 2 \Re \left[ \langle + \rangle \langle - \rangle^* \right] = 2 \left[ -|X|^2 \left( g^S \right)^2 + |Y|^2 \left( g^P \right)^2 \right],
\]
\[
C_4 \equiv 2 \Im m \left[ \langle + \rangle \langle - \rangle^* \right] = 4 \Re \left( X^*Y \right) g^S g^P.
\]
(19)

Under CP and CPT\(^4\) transformations, the reduced \( H-t-\bar{t} \) helicity amplitudes transform as follows:

\[
\langle \pm \rangle \xrightarrow{\text{CP}} \langle \mp \rangle, \quad \langle \pm \rangle \xrightarrow{\text{CPT}} \langle \mp \rangle^*.
\]
(20)

We note that the CP parities of \( C_1, C_2, C_3 \) and \( C_4 \) are \( +, -, +, \) and \( - \), respectively, implying that \( C_2 \) and \( C_4 \) are non-vanishing only when \( g^S \) and \( g^P \) exist simultaneously. Furthermore, the angular coefficient \( C_2 \) is CPT\(^4\) odd and it can be induced only in the presence of non-vanishing absorptive (or imaginary) parts of \( X \) and \( Y \).

\(^4\) T\(^*\) denotes the naive time-reversal transformation under which the matrix element gets complex conjugated.
By integrating Eq. (15) over $\sqrt{p_t^2}$ and $\sqrt{p_t^2}$, we have
\[
\frac{d\Gamma}{d\theta_1 d\theta_2 d\Phi} = N_G \frac{g^4 g^2 \sqrt{s}}{2^{15} \pi^6} \sum_i F_i f_i(\theta_1, \theta_2, \Phi)
\]
where $\sqrt{s} = M_H$ and $i = 11, 12, 21, 22, 3, 4$. The angular functions are
\[
\begin{align*}
    f_{11}(\theta_1, \theta_2, \Phi) &= 1 - c_\theta c_\theta, \quad f_{12}(\theta_1, \theta_2, \Phi) = 1 + c_\theta c_\theta, \\
    f_{21}(\theta_1, \theta_2, \Phi) &= c_\theta - c_\theta, \quad f_{22}(\theta_1, \theta_2, \Phi) = c_\theta + c_\theta, \\
    f_3(\theta_1, \theta_2, \Phi) &= -s_\theta s_\theta c_\Phi, \\
    f_4(\theta_1, \theta_2, \Phi) &= s_\theta s_\theta s_\Phi,
\end{align*}
\]
and the numerical factors $F_i$ are given by
\[
F_i = \int \lambda_H^{1/2} \lambda_i^{1/2} \lambda_i^{1/2} \sqrt{p_t^2} \sqrt{p_t^2} \tilde{C}_i(p_t^2, \bar{p}_t^2) \\
\times \frac{p_t^2 + m_b^2 - M_W^2}{(p_t^2 - m_t^2)^2 + m_t^2 \Gamma_t^2} \frac{\bar{p}_t^2 + m_b^2 - M_W^2}{(\bar{p}_t^2 - m_t^2)^2 + m_t^2 \Gamma_t^2} \ d\sqrt{p_t^2} \ d\sqrt{\bar{p}_t^2}
\]
in which the tilded 6 angular coefficients $\tilde{C}_i$ are related to $C_i$ as follows:
\[
\begin{align*}
    \tilde{C}_{11} &= C_1 \left( 1 + \frac{p_t^2 \bar{p}_t^2}{4 M_W^2} \right), \quad \tilde{C}_{12} = C_1 \frac{p_t^2 + \bar{p}_t^2}{2 M_W^2}, \\
    \tilde{C}_{21} &= C_2 \left( -1 + \frac{p_t^2 \bar{p}_t^2}{4 M_W^2} \right), \quad \tilde{C}_{22} = C_2 \frac{p_t^2 - \bar{p}_t^2}{2 M_W^2}, \\
    \tilde{C}_3 &= C_3 \left( \frac{p_t^2}{2 M_W^2} - 1 \right) \left( \frac{\bar{p}_t^2}{2 M_W^2} - 1 \right), \\
    \tilde{C}_4 &= C_4 \left( \frac{p_t^2}{2 M_W^2} - 1 \right) \left( \frac{\bar{p}_t^2}{2 M_W^2} - 1 \right).
\end{align*}
\]
To proceed further, we have introduced weight factor $w_i$'s which are defined by
\[
w_i \equiv \frac{F_i}{\mathcal{F} \tilde{C}_i}
\]
where
\[
\mathcal{F} = \int \lambda_H^{1/2} \lambda_i^{1/2} \lambda_i^{1/2} \sqrt{p_t^2} \sqrt{\bar{p}_t^2} \\
\times \frac{p_t^2 + m_b^2 - M_W^2}{(p_t^2 - m_t^2)^2 + m_t^2 \Gamma_t^2} \frac{\bar{p}_t^2 + m_b^2 - M_W^2}{(\bar{p}_t^2 - m_t^2)^2 + m_t^2 \Gamma_t^2} \ d\sqrt{p_t^2} \ d\sqrt{\bar{p}_t^2}
\]
and
\[
\overline{C}_i = \tilde{C}_i(p_t^2 = m_t^2, \bar{p}_t^2 = m_t^2)
\]
are the constant tilded angular coefficients at the $t$ pole. Explicitly,
\[
\bar{C}_{11} = 2 \left(1 + \frac{m_t^4}{4M_W^4}\right) \left[\beta_t^2 (g_s)^2 + (g_P)^2\right],
\bar{C}_{12} = 2 \left(\frac{m_t^2}{M_W^2}\right) \left[\beta_t^2 (g_s)^2 + (g_P)^2\right],
\]
\[
\bar{C}_{21,22} = 0,
\]
\[
\bar{C}_3 = 2 \left(\frac{m_t^2}{2M_W^2} - 1\right)^2 \left[-\beta_t^2 (g_s)^2 + (g_P)^2\right],
\bar{C}_4 = 4 \left(\frac{m_t^2}{2M_W^2} - 1\right)^2 \beta_t g_s g_P.
\] (28)

We observe $\bar{C}_{21,22}$ are identically vanishing because $X = \beta_t$ and $Y = 1$ are real when $p_t^2 = \bar{p}_t^2 = m_t^2$. We also note that
\[
\bar{C}_{11} + \bar{C}_{12} = 2 \left(1 + \frac{m_t^2}{2M_W^2}\right)^2 \left[\beta_t^2 (g_s)^2 + (g_P)^2\right].
\] (29)

Finally, we have obtained the normalized differential angular distribution
\[
\frac{1}{\Gamma} \frac{d\Gamma}{dc_{\theta_1} dc_{\theta_2} d\Phi} = \sum_i \bar{R}_i \left(\frac{f_i(\theta_1, \theta_2, \Phi)}{8\pi}\right),
\] (30)

with the 6 angular observables defined by
\[
\bar{R}_i \equiv \omega_i \bar{C}_i / (\omega_{11} \bar{C}_{11} + \omega_{12} \bar{C}_{12}).
\] (31)

After integrating over any two of the angles $\theta_1$, $\theta_2$, and $\Phi$, one can obtain the following one-dimensional angular distributions in terms of the constant $t$-pole angular observables $\bar{R}_i$'s:
\[
\frac{1}{\Gamma} \frac{d\Gamma}{dc_{\theta_1}} = \frac{1}{2} + \frac{1}{2} \left(\bar{R}_{21} + \bar{R}_{22}\right) c_{\theta_1},
\frac{1}{\Gamma} \frac{d\Gamma}{dc_{\theta_2}} = \frac{1}{2} + \frac{1}{2} \left(-\bar{R}_{21} + \bar{R}_{22}\right) c_{\theta_2},
\]
\[
\frac{1}{\Gamma} \frac{d\Gamma}{d\Phi} = \frac{1}{2\pi} + \frac{\pi}{32} \left(-\bar{R}_{3} c_{\Phi} + \bar{R}_{4} s_{\Phi}\right),
\] (32)

where the decay width is given by
\[
\Gamma = N_C \frac{g^4 g_t^2 \sqrt{s}}{2^{12} \pi^5} \left(\omega_{11} \bar{C}_{11} + \omega_{12} \bar{C}_{12}\right) \mathcal{F}.
\] (33)

When $M_H > 2m_t$ and the top quarks are on-shell, we have found that the weight factors do not deviate from unity by more than 1% and one may safely take $\bar{C}_i = \bar{C}_i$ in Eq. (23) by adopting the narrow-width approximation (NWA) for the intermediate top quarks.

---

5 In Appendix C, we show $\Gamma = \Gamma(H \rightarrow t\bar{t}) [B(t \rightarrow bW)]^2$ in the leading order (LO) by adopting the narrow width approximation for the intermediate top quarks.
that the only non-trivial one-dimensional angular distribution is $1/\Gamma \, d\Gamma/d\Phi$ since $\tilde{R}_{21,22} = 0$. Again, we note that the analytic expressions for the normalized angular distributions, Eq. (32), have been obtained in the $m_b = 0$ limit taking $f_L = 1$ and $f_R = 0$. We find that finite $m_b$ effects are negligible as we shall show in Fig. 1 by comparing the distributions obtained by using the exact expression Eq. (15) with finite $m_b$ and the expression Eq. (32) with $m_b = 0$. We then use Eq. (32) to analyze angular distributions in our numerical analysis.

III. NUMERICAL ANALYSIS

For our numerical analysis we are taking $M_H = 380$ GeV. This choice of $M_H$ ensures that the top and anti-top quarks are both on-shell and thus the decay channel $H \to t\bar{t}$ would be the dominant decay mode of the heavy Higgs boson $H$. Also, the production cross section of $H$ would be substantially larger than the heavier Higgs boson with, for example, $M_H \gtrsim 500$ GeV. For the $t\bar{b}-W$ interaction, we assume the SM couplings: $f_L = 1$ and $f_R = 0$. These input values simplify our numerical analysis and there remain only 2 real input parameters of $g^S$ and $g^P$ to vary\footnote{In passing, we note that the expressions given in the last section are very general and indeed can allow a more general analysis.} In this case, we find that the total decay width is given by

$$\Gamma \simeq 22 \, g_t^2 \left[ \beta_t^2 \left( g^S \right)^2 + \left( g^P \right)^2 \right] \text{ GeV}$$

with $F \simeq 6100$. Note $\beta_t = 0.412$ for our choice of $M_H$ close to the $2m_t$ threshold and $m_t = 173.1$ GeV and therefore the $g^S$ contribution in the above equation is suppressed by the $\beta_t^2$ factor.

For the $H$ couplings to top quarks, we consider the following 6 representative scenarios:

- **S1** : $(g^S, g^P) = (1, 0)$
- **S2** : $(g^S, g^P) = (0, 1)$
- **S3** : $(g^S, g^P) = (1, 1)$
- **S4** : $(g^S, g^P) = (1, -1)$
- **S5** : $(g^S, g^P) = (1, 0.42)$
TABLE II. The values of $\overline{C}_{11} + \overline{C}_{12}$ and the 6 angular observables $\overline{R}_i = \overline{C}_i/(\overline{C}_1 + \overline{C}_3)$ with $i = 11, 12, 21, 22, 3, 4$ taking $w_i = 1$ for the 6 scenarios under consideration. The CP and CPT parities of each observable are shown in the square brackets.

| $g^S$ | $g^P$ | $(\overline{C}_{11} + \overline{C}_{12})[++]$ | $\overline{R}_{11}[++]$ | $\overline{R}_{12}[++]$ | $\overline{R}_{21}[-]$ | $\overline{R}_{22}[-]$ | $\overline{R}_{3}[++]$ | $\overline{R}_{4}[-+]$
|-------|-------|-------------------------------|-----------------|-----------------|------------------|------------------|-----------------|------------------|
| S1    | 1     | 0                              | 3.76            | 0.580           | 0.420            | 0                | 0               | -0.159           |
| S2    | 0     | 1                              | 22.1            | 0.580           | 0.420            | 0                | 0               | 0.159            |
| S3    | 1     | 1                              | 25.9            | 0.580           | 0.420            | 0                | 0               | 0.113            |
| S4    | 1     | -1                             | 25.9            | 0.580           | 0.420            | 0                | 0               | 0.113            |
| S5    | 1     | 0.42                           | 7.67            | 0.580           | 0.420            | 0                | 0               | 0.00295          |
| S6    | 1     | -0.42                          | 7.67            | 0.580           | 0.420            | 0                | 0               | -0.159           |

- **S6**: $(g^S, g^P) = (1, -0.42)$

In the first two scenarios of S1 and S2, only one of the couplings is non-vanishing and $H$ is supposed to be a pure CP-even (odd) state in the S1 (S2) scenario. In the other scenarios, CP is violated and the couplings $g^S$ and $g^P$ take on a relative phase. In the scenarios S5 and S6, in particular, the relative sizes of the couplings are chosen such that $|g^P/g^S| \sim \beta_i$ in order that the two couplings contribute more or less equally to the amplitude squared.

In Table II, we show the values of $\overline{C}_{11} + \overline{C}_{12}$ and the 6 angular observables $\overline{R}_i = \overline{C}_i/(\overline{C}_1 + \overline{C}_3)$ for the 6 scenarios under consideration with $i = 11, 12, 21, 22, 3, 4$. We have taken $w_i = 1$. First of all, we observe that $\overline{R}_{11} = (1 + m_t^4/4M_W^4)/(1 + m_t^2/2M_W^2)^2$ and $\overline{R}_{12} = (m_t^2/M_W^2)/(1 + m_t^2/2M_W^2)^2$ independent of the scenario and the CPT-odd $\overline{R}_{21}$ and $\overline{R}_{22}$ are identically vanishing in all the scenarios. This leaves only $\overline{R}_3$ and $\overline{R}_4$ as non-trivial angular observables which can be probed by studying the $d\Gamma/d\Phi$ distribution. The CP-odd $\overline{R}_4$ observable is vanishing in the CP-conserving S1 and S2 scenarios and, if it is not vanishing, its sign is determined by the sign of the product of $g^S$ and $g^P$. Further we note that $\overline{R}_3$ is very suppressed in S5 and S6 because it is proportional to $-\beta_i^2 \left(g^S\right)^2 + \left(g^P\right)^2$.

In Fig. 1 we show the normalized angular distributions (solid dots) generated according to Eq. (15) and compare them with those (dashed lines) obtained using the analytic expressions Eq. (32) obtained in the $m_b = 0$ limit taking $w_i = 1$. Without any noticeable differences.
FIG. 1. The normalized angular distributions (solid dots) generated according to Eq. (15) integrating over $m_t - 5\Gamma_t \leq \sqrt{p_t^2} \leq m_t + 5\Gamma_t$ and $m_t - 5\Gamma_t \leq \sqrt{\bar{p}_t^2} \leq m_t + 5\Gamma_t$. We have taken $f_L = 1$ and $f_R = 0$. In each frame, we are taking $(g^S, g^P) = (1, 0)$ (S1: upper-left), $(g^S, g^P) = (0, 1)$ (S2: upper-right), $(g^S, g^P) = (1, 1)$ (S3: middle-left), $(g^S, g^P) = (1, -1)$ (S4: middle-right), $(g^S, g^P) = (1, 0.42)$ (S5: lower-left), and $(g^S, g^P) = (1, -0.42)$ (S6: lower-right). The dashed lines are drawn using the expressions in Eq. (32), which are obtained in the $m_b = 0$ limit, taking $w_i = 1$.

Numerically, we find that the absolute difference is smaller than $5 \times 10^{-3}$ for the 6 scenarios under...
m_b effects are negligible and the NWA for the angular coefficients and observables works excellently. Thus, we conclude that the analytic expressions in Eq. (32) provide a sufficient theoretical framework to analyze the angular distributions and to extract the g^S and g^P couplings when M_H > 2m_t. Incidentally, the behavior of the angular distribution in Φ can be easily understood as it varies according to $-\frac{\tilde{R}_3 c_3}{0.1} + \frac{\tilde{R}_3 s_3}{0.1}$: see Eq. (32) and Table II.

Now we are going to illustrate how well one can measure the properties of the 380 GeV Higgs by taking the examples of scenarios S4 and S6. For this purpose, we assume that one top quark decays hadronically and the other one leptonically with $\ell = e, \mu$. Then, the expected number of events would be

$$N_{\text{evt}} \sim (7 \times 10^3) \left[ \frac{\sigma(H) \times B(H \rightarrow t\bar{t})}{10 \text{ pb}} \right] \left[ \frac{B(t \rightarrow bW)}{1} \right]^2 \left( \frac{\epsilon_b}{0.7} \right)^2$$

(35)

where $\sigma(H)$ denotes the production cross section of the heavy Higgs boson $H$. When $M_H \lesssim 500$ GeV, the experimental constraint on $\sigma(H)$ may come from the search for narrow scalar resonances in the $b$-tagged dijet mass spectrum: $\sigma(pp \rightarrow H) \cdot B(H \rightarrow b\bar{b}) \lesssim 10$ pb at 95% confidence level (CL) [18]. Taking $B(H \rightarrow b\bar{b}) \sim 0.1$, we have $\sigma(H) \lesssim 100$ pb at the LHC. Another 95% CL limit may be derived from $\sigma(gg \rightarrow H) \cdot B(H \rightarrow ZZ) \lesssim 0.1$ pb [19], which leads to $\sigma(H) \lesssim 10$ pb with $B(H \rightarrow ZZ) \sim 0.01$. In Eq. (35), $\epsilon_b$ denotes the $b$-tagging efficiency and $\epsilon_{\text{rec}}$ stands for a collective efficiency of reconstructing the $H \rightarrow t\bar{t}$ system. Note that $\epsilon_{\text{rec}}$ includes the efficiency of fully reconstructing the four momenta of the $t$ and $\bar{t}$ quarks which are necessary to measure the $\Phi$ distribution. We observe that $\epsilon_{\text{rec}}$ may also account for the dilutions due to interference with irreducible backgrounds and incorrect reconstruction of the neutrino momentum. One may achieve $\epsilon_{\text{rec}} \sim 1$ at the future $e^+e^-$ colliders but the production cross sections would be much suppressed compared to $pp$ colliders. In our analysis, we are taking $N_{\text{evt}} = 10^4$ as reference.

In Fig. 2, we show the normalized angular distributions for the S4 (left) and S6 (right) scenarios with $N_{\text{evt}} = 10^4$ [8]. The histograms represent the pseudo-data of a total of $N_{\text{event}} = 10^4$ generated according to Eq. (15). The (red) solid lines present the results of fitting to consideration.

8 Using the pseudo-top algorithm, for example, the missing neutrino momentum can be reconstructed with a two-fold ambiguity at the LHC [20]. See also, for example, Ref. [21] for more sophisticated algorithms for top-quark pairs.

9 In practice, we have generated 10 pseudo datasets with each set having $10^4$ events and take average of them to obtain the histograms.
FIG. 2. S4 (left) S6 (right): The normalized angular distributions from the pseudo dataset generated with $\sqrt{p_t^2} , \sqrt{p_t^2} = m_t \pm 5 \Gamma_t$ and $\Delta \Phi = \pi / 9$. The results of fitting to the angular distributions with Eq. (32) are shown in the (red) solid lines.

the angular distributions using Eq. (32). With $N_{\text{evt}} = 10^4$ and $\Delta \Phi = \pi / 9$, we find that the absolute size of 1-$\sigma$ errors of the output angular observables of $\overline{R}_{3,4}$ are about 0.02, see Table III. The input values are the same as in Table II. The output values together with parabolic errors have been obtained by fitting to the normalized angular distributions $1 / \Gamma d \Gamma / d \Phi$ in Fig. 2.

Now we are ready to carry out a $\chi^2$ analysis to achieve our ultimate goal of extracting the couplings $g^S$ and $g^P$ from the angular observables $\overline{R}_3$ and $\overline{R}_4$. To implement the analysis, we further need $\overline{C}_{11} + \overline{C}_{12}$. Using Eqs. (29) and (34), we have

$$\overline{C}_{11} + \overline{C}_{12} \simeq \left(1 + \frac{m_t^2}{2 M_W^2} \right)^2 B(H \to t \bar{t}) \frac{\Gamma_{H_{\text{tot}}}}{\text{GeV}}.$$  \hfill (36)

Assuming information on $B(H \to t \bar{t})$ and the coupling $g_t$ can be eventually extracted from $\sigma \cdot B$ measurements by considering several $H$ production and decay processes, together with an independent measurement of the total decay width, one may determine the combination of $\overline{C}_{11} + \overline{C}_{12}$. In our analysis, similar to the angular observables $\overline{R}_{3,4}$, we simply assume 20% error in $\overline{C}_{11} + \overline{C}_{12}$.

In the upper-left frame of Fig. 3 we show the confidence-level regions of the $\chi^2$ analysis by varying $g^S$ and $g^P$ simultaneously for the scenario S4. We have found that $\chi^2_{\text{min}} / \text{d.o.f} = \ldots$
TABLE III. Summary of the results obtained with $N_{\text{evt}} = 10^4$ and $\Delta \Phi = \pi/9$. The input values are the same as in Table II. The output values of $\overline{R}_{3,4}$ have been obtained by fitting to the normalized angular distributions in Fig. 2. For $\overline{C}_{11} + \overline{C}_{12}$, we simply assume 20% error. Implementing $\chi^2$ analysis gives the best-fit values of $g^S$ and $g^P$, see Figs. 3 and 4. Also shown are the best-fit values for $\overline{C}_{11} + \overline{C}_{12}$ and $\overline{R}_{3,4}$ calculated using the best-fit values of $g^S$ and $g^P$.

| $S4$ | Input       | Output       | Best-fit       |
|------|-------------|--------------|----------------|
| $\chi^2_{\text{min}} = 0.728$ | value | value | value |
| $\overline{C}_{11} + \overline{C}_{12}$ | 25.9 | $25.9 \pm 5.18$ | 25.8 |
| $\overline{R}_{3}$ | +0.113 | +0.0985 ± 0.0228 | +0.0877 |
| $\overline{R}_{4}$ | −0.112 | −0.149 ± 0.0229 | −0.133 |
| $g^S$ | +1 | N/A | +1.25 ± 0.22 |
| $g^P$ | −1 | N/A | −0.950$^{+0.11}_{-0.10}$ |

| $S6$ | Input       | Output       | Best-fit       |
|------|-------------|--------------|----------------|
| $\chi^2_{\text{min}} = 2.42$ | value | value | value |
| $\overline{C}_{11} + \overline{C}_{12}$ | 7.67 | $7.67 \pm 1.53$ | 7.73 |
| $\overline{R}_{3}$ | +0.00295 | −0.0102 ± 0.0228 | −0.00842 |
| $\overline{R}_{4}$ | −0.159 | −0.195 ± 0.0229 | −0.159 |
| $g^S$ | +1 | N/A | +1.04 ± 0.13 |
| $g^P$ | −0.42 | N/A | −0.405 ± 0.050 |

$0.728/(3-2) = 0.728$ and the minimum occurs at

$$g^S = 1.25 \pm 0.22 ; \quad g^P = -0.950^{+0.11}_{-0.10} ,$$

(37)

which are consistent with the input values $(g^S, g^P) = (+1, -1)$ within $\sim 1$-\(\sigma\) range. Therefore, we conclude that the two couplings of $H$ to the top-quark pair can be determined with about 10-20% errors when $N_{\text{evt}} = 10^4$ for the scenario $S4$. For the scenario $S6$, the confidence-level regions are shown in Fig. 4. The minimum occurs at

$$g^S = 1.04 \pm 0.13 ; \quad g^P = -0.405 \pm 0.050 ,$$

(38)

with $\chi^2_{\text{min}}/d.o.f = 2.42$. We again note that the fitted values are consistent with the input values $(g^S, g^P) = (+1, -0.42)$ safely within 1-\(\sigma\) range. Also, we conclude that the two couplings can be determined with about 13% errors in scenario $S6$. The results are also summarized in Table III.
FIG. 3. Upper-left: The confidence-level (CL) regions for scenario $S_4$: $(g^S, g^P) = (1, -1)$ with $\Delta \chi^2 = 2.3$ (red), 5.99 (green), and 11.83 (blue) above the minimum, which correspond to confidence levels of 68.3%, 95%, and 99.7%, respectively. The vertical and horizontal lines show the best-fit values of $(g^S, g^P)$. The others: The scatter plots for $\Delta \chi^2$ versus $g^S$ (upper-right), $\Delta \chi^2$ versus $g^P$ (lower-left). The horizontal lines are for the 68.3% (red), 95% (green), and 99.7% (blue) CL regions.
FIG. 4. The same as in Fig. 3 but taking scenario S6: \((g^S, g^P) = (1, -0.42)\).

IV. CONCLUSIONS

We have performed a comprehensive study of the renormalizable CP-even and CP-odd couplings of a spin-0 heavy Higgs boson to a pair of top quarks, using the angular distributions in the decay \(H \to t\bar{t} \to bW^+\bar{b}W^-\). Based on the helicity amplitude method, we figure out there are 6 types of angular observables \(R_i\) \((i = 11, 12, 21, 22, 3, 4)\) according to their CP and CP\(\tilde{T}\) parities. We found that \(R_{21, 22}\) are identically zero unless CP\(\tilde{T}\) is violated through the presence of the absorptive part in the loop correction of the \(Ht\bar{t}\) vertex. Furthermore,
we find that among the 6 observables only the $R_3$ and $R_4$ observables can be probed by the one-dimensional angular distribution $d\Gamma/d\Phi$. This is our novel strategy for analyzing the decay $H \rightarrow t\bar{t} \rightarrow bW^+\bar{W}^-$ to measure the properties of a heavy Higgs boson $H$.

We have illustrated with $10^4$ events for $H \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^-$ that the parameters $g^S$ and $g^P$ can be determined with about 10-20% uncertainties through the one-dimensional distribution $d\Gamma/d\Phi$. This is the major numerical result of this work.

We offer the further comments in our findings:

1. As long as the heavy Higgs boson is above the $t\bar{t}$ threshold and, at least, as long as the angular distributions are concerned, the narrow-width approximation is always a good one: the weight factors deviate from unity less than 1%.

2. The angular coefficient $C_2$ is CP odd and CPT odd which implies that it is only nonzero in presence of non-vanishing absorptive (or imaginary) parts of the $t\bar{t}H$ vertex and in the simultaneous existence of $g^S$ and $g^P$.

3. The numerical analysis presented here is only limited to the left-handed decay of the top quark, i.e., $f_L = 1, f_R = 0$. Nevertheless, the formalism here is very general and can be applied to general studies.

4. In the current study, while respecting the present LHC upper limits on $H$ production, we have used $10^4$ events of $H \rightarrow t\bar{t} \rightarrow 2b2j\ell\nu$ with a luminosity of 100 $fb^{-1}$. The high-luminosity run of the LHC is supposed to collect 3000 $fb^{-1}$. The uncertainty in extracting the Yukawa couplings from the heavy Higgs boson decay would go down substantially.

5. When the heavy Higgs boson becomes heavier, of order 1 TeV, the decay products of the top and anti-top quarks become more boosted. In such a case, the angular analysis is more challenging and thus deserves a new study.

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APPENDIX

Appendix A: Relation between the $t$ and $\bar{t}$ helicity amplitudes

The helicity amplitude for the decay $\bar{t}(\bar{p}_t, \bar{\sigma}_t) \rightarrow \bar{b}(\bar{p}_b, \bar{\sigma}_b)W^{-}(k_2, \epsilon_2)$ can be obtained from that for the decay $t(p_t, \sigma_t) \rightarrow b(p_b, \sigma_b)W^{+}(k_1, \epsilon_1)$ by replacing $f_A$ with $f_A^*$ together with $\sigma_{t,b} \rightarrow \bar{\sigma}_{t,b}$, $\lambda_1 \rightarrow \lambda_2$, etc. This can be easily understood through the relation

$$\bar{v}_t \gamma^{\mu}P_A v_b = \bar{u}_b \gamma^{\mu}P_{-A} u_t.$$  \hspace{1cm} (A.1)

The above relation can be shown by calculating both sides explicitly or by observing

$$C = i\gamma^2 \gamma^0 ; \quad C = -C^{-1} = -C^T = -C^\dagger$$  \hspace{1cm} (A.2)

where $C$ denotes charge conjugation and

$$u = C\bar{v}^T , \quad \bar{u} = v^TC = -v^TC^{-1} \quad \text{(or} \quad v = C\bar{u}^T)$$  \hspace{1cm} (A.3)

together with

$$C(\gamma^\mu)^TC^{-1} = -\gamma^\mu , \quad C(\gamma^\mu\gamma_5)^TC^{-1} = +\gamma^\mu\gamma_5.$$  \hspace{1cm} (A.4)

Incidentally, $C\gamma_5^T C^{-1} = +\gamma_5$.

Appendix B: The four-body phase space

For the $H \rightarrow tt \rightarrow bW^+\bar{b}W^-$ decay, the phase space can be factorized into

$$d\Phi_4(q \rightarrow p_t\bar{p}_t \rightarrow p_b k_1 \bar{p}_b k_2) = \frac{dp_t^2}{2\pi} \frac{dp_b^2}{2\pi} \frac{\lambda_H^{1/2}(1, p_t^2/s, p_b^2/s)}{32\pi^2} d\cos\varTheta d\Phi^* \times \frac{\lambda_t^{1/2}(1, m_b^2/p_t^2, M_W^2/p_t^2)}{32\pi^2} d\cos\varTheta_1 d\phi_1 \times \frac{\lambda_{\bar{t}}^{1/2}(1, m_b^2/p_b^2, M_W^2/p_b^2)}{32\pi^2} d\cos\varTheta_2 d\phi_2 \hspace{1cm} (B.1)$$

where $s = q^2$ and $\lambda(1, a, b) = (1 - a - b)^2 - 4ab$. For our purpose, we may be able to take

$$d\Phi_4(q \rightarrow p_t\bar{p}_t \rightarrow p_b k_1 \bar{p}_b k_2) = \frac{\lambda_H^{1/2}\lambda_t^{1/2}\lambda_{\bar{t}}^{1/2}}{2\pi} \frac{dp_t^2}{2\pi} \frac{dp_b^2}{8\pi} \frac{1}{32\pi^2} \frac{d\cos\varTheta_1 d\Phi}{16\pi} \frac{d\cos\varTheta_2}{16\pi}.$$  \hspace{1cm} (B.2)
Appendix C: Narrow width approximation

Using
\[
\delta(p^2 - m^2) = \lim_{\Gamma \to 0} \frac{m \Gamma}{\pi} \frac{1}{(p^2 - m^2)^2 + m^2 \Gamma^2}
\]  
(C.1)
and taking \( \omega_i = 1 \), we note that Eq. (33) together with Eq. (26) can be factorized into

\[
\Gamma = \Gamma(H \to t\bar{t}) \left( \frac{\Gamma^{\text{LO}}(t \to bW)}{\Gamma_t} \right)^2
\]  
(C.2)
where, taking \( m_b = 0 \),

\[
\Gamma(H \to t\bar{t}) = N_C \frac{\beta_t g_t^2 M_H}{8\pi} \left[ \beta_t^2 (g_s)^2 + (g_P)^2 \right],
\]
\[
\Gamma^{\text{LO}}(t \to bW) = \frac{g^2 m_t}{2^6 \pi} \left( 1 - \frac{M_W^2}{m_t^2} \right)^2 \left( 2 + \frac{m_t^2}{M_W^2} \right)
\]
\[
= \frac{G_F m_t^3}{8\pi \sqrt{2}} \left( 1 - \frac{M_W^2}{m_t^2} \right)^2 \left( 1 + 2 \frac{M_W^2}{m_t^2} \right)
\]  
(C.3)
with \( G_F = \sqrt{2} g^2 / 8 M_W^2 \). In our analysis, we are taking \( \Gamma^{\text{NLO}}(t \to bW) \) for \( \Gamma_t \) or \( \Gamma_t = \Gamma^{\text{LO}}(t \to bW) \left[ 1 - \frac{2\alpha_s}{3\pi} \left( \frac{2\pi^2}{3} - \frac{5}{2} \right) \right] \) which is about 1.35 GeV.
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