Disordered hyperuniform materials with vanishing long-wavelength density fluctuations are attracting attention due to their unique physical properties. In these systems, the large-scale density fluctuations are strongly suppressed as in a perfect crystal, even though the system can be disordered like a liquid. Yet, hyperuniformity can be affected by the different types of quenched disorder unavoidably present in the host medium where constituents are nucleated. Here, we use vortex matter in superconductors as a model elastic system to study how planar correlated disorder impacts the otherwise hyperuniform structure nucleated in samples with weak point disorder. Planes of defects suppress hyperuniformity in an anisotropic fashion: while in the transverse direction to defects the long-wavelength density fluctuations are non-vanishing, in the longitudinal direction they are smaller and the system can eventually recover hyperuniformity for sufficiently thick samples. Our findings stress the need of considering the nature of disorder and thickness-dependent dimensional crossovers in the search for novel hyperuniform materials.
A great number of disordered physical and biological systems are endowed with a universal hidden order characterized by a macroscopically uniform density of constituents. This hidden order is the structural property of hyperuniformity, characterized by an anomalous suppression of large-scale density fluctuations in the system. This property is naturally expected in a crystal, but it is also observed in a wide variety of disordered systems, such as two-dimensional material structures, jammed particles, bubbles in foam, vortex matter in type-II superconductors, patterns of photoreceptors in avian retinas, biological tissues, and even the distribution of the density fluctuations in the early Universe. Hyperuniform systems present a vanishing structure factor in the infinite-wavelength or small wavenumber limit, namely \( \langle q \rangle \to 0 \) as \( q \to 0 \). This magnitude can be directly measured via different diffraction techniques and provides information on the fluctuations of the density of constituents of the system at different wavenumbers. Since hyperuniformity is a property defined in an asymptotic limit, strict hyperuniformity is difficult to ascertain in experimental as well as computer-simulated systems. Thus, most works show that the systems are effectively hyperuniform.

This exceptional but ubiquitous state of matter presents a phenomenology that goes against the conventional wisdom on the effect of disorder in the physical properties of systems of interacting objects. For instance, the disorder typically lowers the electrical conductivity of metallic materials. However, a recent work reports that disordered hyperuniform systems present a closing of bandgaps resulting in an enhanced conductivity. Also, periodic or quasiperiodic order was assumed as a prerequisite for a material to present photonic bandgap properties. Strikingly, disordered hyperuniform-engineered materials possess complete photonic bandgaps blocking all directions and polarizations for short wavelengths. In addition, hyperuniform patterns can be very useful in practical technological applications. For example, hyperuniform patterns of defects can pin with high efficiency the vortex structure nucleated in superconductors, avoiding the undesirable dissipation that can occur in superconducting devices.

Theoretically, due to the fluctuation-compressibility theorem, hyperuniformity may naturally emerge at thermal equilibrium in incompressible systems with long-range repulsive interactions between the constituents. Nevertheless, a hyperuniform point pattern within a higher dimensional system presenting only short-range interactions or gradient terms at equilibrium may also exist. Indeed, a three-dimensional vortex lattice model with short-range repulsions and local elasticity may present hyperuniform two-dimensional point patterns at every plane perpendicular to the vortex lines. In general terms, this road to hyperuniformity results from bulk-mediated effective long-range interactions between the points in the hyperuniform pattern.

Vortex matter in superconductors is a model system to study the occurrence of hyperuniformity in media with different types of disorder. Vortices are elastic objects that nucleate in type-II superconductors when applying a magnetic field. They are string-like zones of the material that concentrate a quantized amount of magnetic flux and interact repulsively between each other. The competition between this repulsion and the pressure exerted by the field results in vortices forming a structure with lattice spacing \( a \propto B^{-1/2} \). The vortex structure stabilizes in solid, glassy and liquid phases, depending on temperature, applied field, the particular material and the nature of the disorder in the samples. The nucleation of quasi-ordered and disordered hyperuniform vortex structures has been first reported experimentally in samples of the high-\( T_c \) \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta} \) with respectively weak point and strong columnar disorder. Later, disordered hyperuniform vortex structures have been observed at high fields in several superconductors presenting weak and strong point disorder. Hyperuniform vortex structures are theoretically expected for media with weak point disorder since in this case the effective interaction between vortex tips at the sample surface is long ranged. Theoretically, a suppression of hyperuniformity is expected for media with columnar correlated disorder. However, in the latter case an algebraic decay of \( S(q) \) in the \( q \to 0 \) limit is detected experimentally. This apparent discrepancy between theory and experiment is quite likely due to the viscous freezing of the system when field-cooling from the hyperuniform vortex liquid phase towards the low-temperature glassy vortex phase. In contrast, in the case of a type-II superconductor with planar correlated defects, strong fluctuations of the vortex density have been proposed as the fingerprint of a disordered gel of vortices. Thus, the nature of the disorder in the host medium plays a dominant role on the magnitude of density fluctuations, and thus their effect on the nucleation of hyperuniform materials deserves further investigation.

Here we address the question of whether planar correlated quenched disorder, even if present in a reduced region of the sample, can ultimately affect the hyperuniform hidden order. We use vortex matter in two different high-\( T_c \) superconductors as model systems. In order to be reliable, these studies on long-range vortex density fluctuations require high-resolution direct imaging of individual vortices in extended fields-of-view with thousands of vortices or more. Our experimental data with such a resolution and extension are contrasted with numerical simulations of a system of interacting elastic strings nucleated in media with the planar disorder. We show that planes of crystal defects running all the way through the sample thickness produce a suppression of hyperuniformity in an anisotropic fashion. Furthermore, we discuss how finite size effects are relevant for these observations and how its removal in a sufficiently-thick sample can produce a recovery of the hyperuniform hidden order in the direction longitudinal to planar defects.

**Results**

**Density fluctuations on large length scales in media with planar defects.** A practical way to image vortex density fluctuations on large length scales is to apply the magnetic decoration technique to take snapshots of thousands of vortices at the sample surface. The magnetic decoration consists in producing nanometer-size Fe clusters that are attracted towards the local field gradient entailed by vortices on the surface of the sample. We decorate vortex positions at 4.2 K after field-cooling; then the sample is warmed up to room temperature and the Fe clusters are imaged by means of scanning electron microscopy. We investigate \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta} \) and \( \text{YBa}_2\text{Cu}_3\text{O}_7 \) high-\( T_c \) superconducting samples as model media with planar correlated quenched disorder, namely domains with enhanced pinning. In the first case we study samples with few and many planar defects separating zones of the sample with slightly different orientations of their c-axes; in the second case we consider samples with twin-boundaries that act as planar defects. For comparison, we study samples of both materials with point disorder only, namely with no planar defects as revealed by means of magnetic decoration. See Methods for further details on the experimental techniques and sample characterization.

Figure 1 shows images of the vortex structures nucleated in some of the studied samples. In the case of point disorder the structure is hexagonal whereas the planar defects induce the formation of vortex rows oriented along the direction of defects \( s_x \) or correspondingly \( q_{||} \) in reciprocal space. These vortex rows have generally a larger density than the average, in agreement with evidence from different imaging and dynamic techniques that indicate that planar defects in...
both materials act as strong pinning centers for vortices. In Bi$_2$Sr$_2$CaCu$_2$O$_{8+d}$ samples with few planar defects, vortex rows are observed in a micron-sized region, and the polygonal structure is recovered elsewhere, see Fig. 1(b). The sample with many planar defects was specially chosen since most vortices are aligned in vortex rows in the whole crystal, see Fig. 1(c, d). In this sample, as also reported in samples with few planar defects, the alignment of the rows is not altered by surface steps resulting from cleaving, indicating the defects extend towards the bulk of the crystals. In the case of YBa$_2$Cu$_3$O$_7$, the structure is hexagonal in the untwinned sample and vortex rows are also observed in a heavily twinned sample, see Fig. 1(e, f).

In order to characterize the vortex density fluctuations on extended fields-of-view, we analyze the structure factor $S(q) \equiv S(q_x, q_y)$ from snapshots of the vortex arrangements taken at the surface of these samples. Figure 2 shows $S(q) = |\tilde{\rho}(q_x, q_y, z = 0)|^2$, with $\tilde{\rho}$ the Fourier transform of the local vortex density modulation at the surface of the studied samples with typical thickness $t \sim 5-30$ μm. A strong anisotropy is evident for samples with planar defects: Fig. 2(b), (c) and (e) show lines of local maxima in $S(q)$ extended along the $q_x$ direction (angle $\theta = 0$) corresponding to vortex density fluctuations transversal to planar defects. At first sight, the intensity seems to faint on going towards $q \to 0$. In the case of media with point disorder, $S(q)$ decays algebraically when $q \to 0$, as reported previously, and also shown in Fig. 3(a, b). We wonder whether in samples with planar defects this fainting is produced by an algebraic decay in the $q \to 0$ limit as expected for disordered hyperuniform systems.

Figure 3 shows one of the main findings of this paper: The suppression of effective hyperuniformity induced by the addition of correlated planar disorder to the host medium. In samples with planar defects, the angularly-averaged structure factor $\langle S(q) \rangle$ for $q \to 0$ is larger than that for samples with point defects (for definition of this magnitude, see Fig. 2(f) and Methods). More significantly, $\langle S(q) \rangle$ tends to saturate in the low-$q$ limit. This phenomenon is observed for the two studied compounds and for crystals presenting few or many planar defects. In contrast, in samples of the same compounds but with point disorder, $\langle S(q) \rangle \sim q^\beta$ with $\beta \approx 1.2$ when $q \to 0$, a signature of effective disordered hyperuniformity.

Anisotropy in the density fluctuations on large length scales in samples with planar defects. Here we show that in samples with planar defects the suppression of hyperuniformity is anisotropic, with vortex density fluctuations of greater magnitude in the $q_y$ than the $q_x$ direction. First, we show that the saturation of the structure factor in the $q \to 0$ limit is anisotropic for vortices nucleated in samples with planar defects. Figure 3(c, d) show data of the angular structure factor $S_\theta(q)$, for different reciprocal space directions. Curves with black (white with black edge) points correspond to $S_\theta(q)$ data in the transverse direction $q_y$ (parallel direction $q_x$), whereas color points are data for intermediate angles. Irrespective of the direction, at low $q$ all curves tend to saturate, but while the color and white points form a pack of data around $-0.02-0.03$, the black curves corresponding to the transverse modes $S_{\theta=0}(q)$ stand out and saturate at a value between 2 and 10 times larger. This is better depicted in the inserts to Fig. 3(c, d). In addition, the peaks in $S_\theta(q)$ are detected at smaller $q$ for $\theta = 0$ than for 90 degree, indicating that the average vortex spacing is smaller in the longitudinal than in the transverse direction to planar defects.

Second, in order to better characterize this anisotropy, we consider the one-dimensional structure factor of individual vortex rows that is sensitive to vortex density fluctuations along the direction of planar defects. In order to compute this magnitude, the experimental positions of vortices in individual rows are mapped in a straight line such that adjacent vortices are spaced a distance $a_i$ and the coordinate of vortex $i + 1$ is $s_i$, see the schematic representation of Fig. 4(a). The one-dimensional structure factor of a given row is then computed as $|\tilde{\rho}_i(q)|^2$, with $\tilde{\rho}_i$ the Fourier transform of the vortex density modulation along the line. Then, for each vortex row we calculate the average lattice spacing in a row, $a_{\parallel} \equiv \langle a_i \rangle$, and the wavenumber $q_{\parallel}$ is normalized by $q_{\parallel} \equiv 2\pi/a_{\parallel}$ Finally, we average the one-dimensional structure factor over many rows to obtain the...
direction. Figure 4(c) shows the dependence of $W$ with $s_{||}$ averaged over tens of vortex rows and panel (d) shows the evolution with $s_{\perp}$ averaged along tens of vortex lines. For the two studied compounds, the data are reasonably well fitted with an algebraic growth with exponent $2\gamma \approx 1$ in both directions, at odds with expectations for a hyperuniform system. The fits yield a multiplicative factor $A$ roughly three and a half times larger for the $s_{\perp}$ than for the $s_{||}$ direction. This is another proof that from experimental evidence vortex density fluctuations are anisotropic in media with planar defects.

Simulations of a structure of interacting elastic vortex lines in media with planar defects. Here we gain insight on the origin of the anisotropic vortex density fluctuations in media with planar defects by means of Langevin dynamics simulations of vortex lines in three dimensions with an applied field in the $z$-direction. We consider a media with randomly-spaced parallel planar defects oriented with their normal pointing along the $s_{\perp}$-axis in an orthogonal coordinate system $(s_{\perp}, s_{||}, z)$. We model $N_v$ vortices as elastic lines discretized in the $z$-direction, such that $\mathbf{r}_i(z) \equiv (s_{\perp}(z_i), s_{||}(z))$ describe the two-dimensional coordinate of vortex $i$ at the layer $z = 1, \ldots, L_z$ with $L_z$ the total number of layers. Periodic boundary conditions are taken in all directions in a system of size $L_x \times L_y \times L_z$. The total energy per unit length of the structure of elastic lines is $E_i[\{\mathbf{r}_i(z)\}] = E_r + E_{\text{rep}}$. Each line has an elastic tension energy given by Hook couplings of strength $k$

$$E_r[\{\mathbf{r}_i(z)\}] \equiv \frac{N_v}{\sum_{i=1}^{N_v} \left(\frac{\lambda_{\|}}{2}\right)^2} \mathbf{r}_i(z) \cdot \mathbf{r}_i(z) - \frac{1}{2} \mathbf{r}_i(z) \cdot \mathbf{S}(\mathbf{q}),$$

with $k = \epsilon_0/\lambda_{\|}^2$ a local harmonic approximation for the single vortex elastic tension, $\epsilon_0 \equiv \phi_0^2/(8\pi^2\lambda_{\|}^2)$ the interaction energy-scale per unit length, and $\lambda_{\|}$ the in-plane penetration length. The repulsive interaction energy between three-dimensional vortex lines derived from the London model is

$$E_{\text{rep}}[\{\mathbf{r}_i(z)\}] = \sum_{i \neq j} \sum_{l=1}^{N_v} \sum_{s=1}^{L_z} \epsilon_{ij}(z) \left(\frac{\mathbf{r}_i(z) - \mathbf{r}_j(z)}{\lambda_{\|}}\right)^2,$$

with $K_0(x)$ the nth-order modified Bessel function of the second kind. The pinning energy due to $N_d$ defects is modeled as Gaussian-well channels

$$E_{\text{PD}}[\{\mathbf{r}_i(z)\}] = -A_{\text{pin}} \sum_{n=1}^{N_d} \sum_{l=1}^{N_v} \sum_{l=1}^{L_z} \epsilon_{ij}(z) \left(\frac{\mathbf{r}_i(z) - \mathbf{r}_j(z)}{\xi_{\|}}\right)^2,$$

where $\xi_{\|}$ is the in-plane coherence length, $A_{\text{pin}}$ the pinning strength of the planar defects, and $N_d$ the random positions of the planar defects uniformly sampled along $L_z$. Finite-temperature Langevin dynamics simulations of the system are performed to obtain equilibrated low-temperature configurations (see Methods). A snapshot of a configuration is shown in Fig. 5(a).

Figure 5(a, b) show the main results of the simulations that are in accordance with experimental observations: (i) The $S_{\parallel}(q)$ is anisotropic and displays similar density fluctuations for all $\theta \neq 0$ directions; (ii) fluctuations in the $q_\perp$ ($\theta = 0$) transverse direction are orders of magnitude larger, particularly at low $q$. The peak in the transverse direction is detected at a smaller wavenumber than in other directions, and $a_{\perp} > a_{\|}$ is also found in the simulations. Thus, this model of a structure of interacting elastic vortex lines nucleated in planar defects that are in controlled positions allow us to ascertain that the pinning generated by defects is strong enough as to increase the vortex density inside defects above the average. Furthermore, this model reveals that the anisotropic suppression of hyperuniformity has origin in the interactions allowing important vortex density fluctuations at large wavelengths for vortices caged in defects but also allowing for a
rarefied distribution of vortices in between defects. As discussed in detail in the next section, by performing simulations and analytical calculations of a simplified version of the model of Eqs. (1), (2) and (3) we can go further in the comparison between experiments and theory, and show that the number of layers $l_z$ (proportional to the sample thickness), plays a very relevant role in assessing hyperuniformity.

**Discussion**

The suppression of disordered hyperuniformity in media with planar defects can be discussed in a broader context than that of its implications for the synthesis of hyperuniform materials. This issue is connected to the related problem of the structural phases stabilized in media with different types of disorder. In the case of planar defects oriented in the direction of the magnetic flux as we study here, the stabilization of a robust planar-glass phase is expected. In this phase, the positional correlation function is expected to decay exponentially, implying both, a displacement correlator function $W \sim \delta_z$, and a structure factor behaving as $S_{q=0}(q \to 0) = \text{const} \neq 0$. These theoretical implications are consistent with our experimental and theoretical findings on the suppression of hyperuniformity in the direction transverse to planar defects. Nevertheless, these theoretical works do not study the vortex density fluctuations in the direction longitudinal to planar defects nor the experimentally relevant size effects. The saturation at a finite value in the longitudinal direction $S_{q=0}(q \to 0)$, appreciably smaller than $S_{q=0}(q \to 0)$, is a subtle issue. Indeed, we argue below that at low densities, the vortex structure confined in a planar defect can be disordered hyperuniform provided the sample is thick enough and the confinement is strong.

In order to sustain these claims, we start by highlighting some relevant findings. First, both in experiments and simulations, the vortex structure in samples with planar defects presents well-defined vortex rows. Simulations also show that at low temperatures most of vortex rows are parallel to planar defects. Second, the average vortex spacing along a row is appreciably smaller than in between rows, for instance $a_{l_z} \approx 0.7a_{l_z}$ in the experiments. This indicates that intra-row vortex-vortex interactions are stronger than inter-row ones, motivating a single-row-based mean-field-like phenomenological approach.

Then, to further sustain our claims based on an analytical insight of the problem, we now consider a simplified model that captures the essential physical ingredients of the problem. We neglect the interaction between vortex rows as well as transverse vortex fluctuations, and model the system as a non-interacting collection of single vortex rows with strongly localized vortices inside a planar defect. We also neglect for the moment the effect of quenched point disorder since it is expected to be weaker than the planar defect pinning. The thermally-equilibrated configuration of the elastic system can then be obtained analytically in the elastic approximation by using the displacement field $u_l(q_y, z)$. This field describes the mismatch of the planar vortex row with respect to a perfectly periodic chain of straight vortices aligned in the $q_{l_z}$-direction, see Fig. 5(c). Within this simple model, as detailed in Methods, the large-wavelength density fluctuations at a single layer $z$ give a structure factor

$$S_l(q_z) \sim q_z^2 \langle |\tilde{u}_l(q_z, z)|^2 \rangle \approx \begin{cases} \frac{a_{l_z}}{\sqrt{c_{11} c_{44}}} q_{l_z} & q_{l_z} > \frac{\pi}{l_z} \\ \frac{a_{l_z}}{c_{11}} & q_{l_z} < \frac{\pi}{l_z} \end{cases}$$

where $\tilde{u}_l(q_z, z)$ is the Fourier transform of the displacement field, $c_{11}$ and $c_{44}$ are the compression and tilt elastic moduli of the planar vortex system, $t$ is the sample thickness, and $l_z$ is a relevant crossover length. Assuming translation symmetry along $z$, Eq. (4) implies that in real space the displacement correlator in

**Fig. 3 Evidence supporting the suppression of hyperuniformity in media with planar defects.** Angularly-averaged structure factor of the vortex structures nucleated in (a) $Bi_2Sr_2CaCu_2O_{8+δ}$ and (b) $YBa_2Cu_3O_7$ samples with point (full symbols) and planar (open symbols) defects. The wavenumber is normalized by the Bragg wavenumber $q_0 = 2\pi/\alpha$. Full red lines are fits to the data in samples with point disorder considering $S(q_z) \sim (q_z/\beta)^{\beta}$ when $q_z/q_0 \to 0$ yielding $\beta = 1.2 \pm 0.2$. Angular $S(q_z)$ structure factor of vortex matter nucleated in (c) $Bi_2Sr_2CaCu_2O_{8+δ}$ and (d) $YBa_2Cu_3O_7$ samples with planar defects. Curves obtained averaging the $S(q_z, q_y)$ data shown in the inserts considering ±0.01. This indicates that intra-row vortex-vortex interactions are stronger than inter-row ones, motivating a single-row-based mean-field-like phenomenological approach.
perfect vortex chain with lattice spacing correlator computation of the top-left insert of this planar defect for different thicknesses, see Fig. 5(d). Furthermore, thickness, Eq. (4) predicts a crossover towards a non-l

Bi$_2$Sr$_2$CaCu$_2$O$_8$ that to explain the experimental observations it is necessary to composed by rigid vortices. Interestingly, this phenomenology is one-dimensional elastic system equivalent to an elastic chain dimensional crossover from a two-dimensional to an effective

dots) in a region of the Bi$_2$Sr$_2$CaCu$_2$O$_8$ (YBa$_2$Cu$_3$O$_7$). Fits considering an algebraic decay of $S(q_\parallel)$ for $q_\parallel \rightarrow 0$ yield $\beta = 0$ within the error, for both materials. $\delta$ Displacement correlator $W/\alpha_0^2$ calculated along the $q_\parallel$ direction of vortex rows (statistics for 1,500 vortices in Bi$_2$Sr$_2$CaCu$_2$O$_{8-\delta}$ and 1,300 in YBa$_2$Cu$_3$O$_7$). Insert: Schematic representation of the magnitudes considered for the calculation of $W$: $u(q_\parallel) = q_\parallel - i \cdot q_\perp$ is the displacement of the $i$-th vortex from the site of a perfect vortex chain with lattice spacing $q_\parallel = (a)$, the average in a row. $\delta$ Displacement correlator $W/\alpha_0^2$ calculated along the transversal direction $s_\perp$ (statistics for 1,300 vortices in Bi$_2$Sr$_2$CaCu$_2$O$_{8-\delta}$ and 1,200 in YBa$_2$Cu$_3$O$_7$). Insert: Schematic representation on the magnitudes considered for the computation of $W$ with $q_\parallel = (a)$, the average in a line. Fits of the displacement correlators with algebraic functions $A - x^{2s_\parallel}t$ (full lines) yield the roughening exponents $2s_\parallel$ and factors $A$ indicated in the legends. Error bars represent the standard deviation of data when averaging at a given $q$.

the longitudinal direction to planar defects scales as

$$W(q_\parallel) \sim \begin{cases} 
\log(s_\parallel) & s_\parallel < l_{FS}, \\
1 & s_\parallel > l_{FS}.
\end{cases}$$

(5)

see Methods for further details. The result of a log($s_\parallel$) roughness is not a surprise since the system is essentially a thermally-fluctuating two-dimensional elastic lattice. The novelty here is that to explain the experimental observations it is necessary to consider the finite-thickness induced crossover distance

$$l_{FS} \approx \frac{t}{2\pi} \sqrt{\frac{c_{11}}{c_{44}}}.$$ 

(6)

This crossover behaviour in $S(q_\parallel)$ and $W(q_\parallel)$ is also confirmed in numerical simulations of a single vortex row confined in a planar defect for different thicknesses, see Fig. 5(d). Furthermore, the top-left insert of this figure shows that all $S(q_\parallel)t$ vs. $q_\parallel t$ curves collapse into a master-curve, confirming quantitatively that $l_{FS} \propto t$, Eq. (6). According to Eq. (4), in the thermodynamic limit $l_{FS} \rightarrow \infty$, and the vortex row at a constant-$z$ cross section is class II hyperuniform since $S(q_\parallel) \sim q_\parallel^\alpha$ with $\alpha = 1$. This hyperuniformity class contrasts with more ordered class I hyperuniform systems where $\alpha > 1$. However, in systems with finite thickness, Eq. (4) predicts a crossover towards a non-hyperuniform behaviour for $s_\parallel > l_{FS}$. This corresponds to a dimensional crossover from a two-dimensional to an effective one-dimensional elastic system equivalent to an elastic chain composed by rigid vortices. Interestingly, this phenomenology is closely related to the crossover predicted for Luttinger liquids at a characteristic thermal length$^{15}$.

A caveat in our model might be that we ignore that real samples have weak point disorder. However, as shown in Methods, if this disorder is considered, the main results of Eqs. (4), (5) and (6) remain qualitatively valid. Namely, for $s_\parallel > l_{FS}$, $S(q_\parallel) \rightarrow 0$ = const and $W(s_\parallel) \rightarrow s_\parallel$ while for $s_\parallel < l_{FS}$, the structure is disordered hyperuniform but class III ($\alpha < 1$) instead of class II ($\alpha = 1$)$^2$.

Finally, we argue that a finite size effect is a plausible explanation for the suppression of hyperuniformity observed in experiments and simulations. Considering the vortex-vortex interaction potential per unit length $U(x) = e_0K_0(x/\lambda_l)$, with $e_l \sim e_0$ the single vortex elastic tension, the elastic constants of the planar vortex system can be estimated as $c_{11} = aU''(a)$ and $c_{44} = c_{11}/a^2$. Thus, using these approximations in Eq. (6), the crossover length can be estimated by considering only the sample thickness since we get $l_{FS} \approx (t/2\pi)(a/\lambda_l)\sqrt{(K_0(a/\lambda_l) + K_1(a/\lambda_l))/2}$. Provided $a \sim \lambda_l$ in the experiments, $l_{FS}$ shortens with either decreasing $t$ or $1/a$. In the studied samples with $t \sim 5-30$ μm$^{41}$, $l_{FS} \approx t/10 \sim a$, and then this dimensional crossover is quite likely at the origin of the observation of a nonvanishing structure factor for large wavelengths. For thick enough samples and/or smaller $a$ such that $l_{FS} \gtrsim 100a$, the crossover to disordered hyperuniform vortex density fluctuations might be observed experimentally. This would be a state with directional hyperuniformity$^{47}$. In other words, vortex matter nucleated in thin samples with a dense distribution of planar defects effectively behave as a collection of one-dimensional elastic manifolds. The suppression of hyperuniformity in elastic structures nucleated in media with the
Fig. 5 Simulations of a structure of interacting elastic vortex lines in samples with planar defects. a) Snapshot of a quenched configuration obtained from three dimensional Langevin dynamics simulations of interacting vortices (red lines) in presence of parallel randomly-located PD (gray planes). Circles highlight vortex tips at the sample surface. The coordinate system and the thickness are indicated. b) Angular structure factor $S_\theta(q)$ of the vortex positions at the sample surface for different angles $\theta$ indicated in the legend. The insert shows the $S(q_x, q_z)$ data considered to calculate the curves in the main panel. c) Schematics of the planar vortex row model indicating the displacement field $u(q_x, z)$, the average spacing $a_t$ for a perfect row with the same vortex density, and the characteristic thickness-dependent crossover length $l_{FS}$. d) Results for the configurations of the planar vortex row at the surface of the sample $(z = 0)$ for different sample thicknesses proportional to the number of layers in the simulation $L_z$. Main panel: $S(q_x)$ structure factor for different sample thicknesses. The dashed black line is a linear function. Arrows indicate the crossover behavior at $q_x = 1/l_{FS}$. Bottom-right insert: Displacement correlator as a function of the distance along the row $s_z$. The full line is a linear function that reasonably describes the data in the large wavelength limit of thin samples whereas the dashed line is a logarithmic growth that follows the data at short wavelengths for thick enough samples. Top-left insert: Structure factor data $S(q_x)t$ vs. $q_x t$ showing a scaling collapse and the two regimes separated by the crossover wavevector, $2\pi/l_{FS} \propto 2k_BT_c$. 

Methods

**Sample preparation and characterization.** We studied Bi2Sr2CaCu2O8+δ and YBa2Cu3O6+δ samples with point disorder and few planar defects were grown following the travelling-solvent-floating zone method using an image furnace with two ellipsoidal mirrors and has a critical temperature of $\approx 87$ K. The YBa2Cu3O6 single crystals were obtained following a growth from the melt technique and are fully oxygenated with $T_c \approx 90$ K, see Ref. for further details on the growing method.

**Vortex imaging by means of magnetic decoration experiments.** We image individual vortex positions at the sample surface in large fields-of-view ranging from 1000 to 35,000 vortices by means of magnetic decoration experiments. For all the data presented here, the magnetic field is applied above $T_c$ and the sample is cooled down to 4.2 K. At this temperature Fe particles are evaporated in a pressure-controlled helium chamber and clusters of these particles land on the sample surface decorating the positions of vortices. Even though the snapshots of the structure are taken at 4.2 K, during the field-cooling process the vortex structure gets frozen, at length-scales of the lattice parameter $a_0$, at a temperature $T_{freez} - T_{cryo}$ the irreversibility temperature at which pinning (sample disorder) sets in. On further cooling down to 4.2 K, vortices can move but in length scales of the order of coherence length, 200 times smaller than the typical size of a vortex as detected by magnetic decoration. Therefore the structure image by magnetic decoration at 4.2 K corresponds to the equilibrium one at $T_{freez}$.

**Structure factors.** In order to calculate the structure factors we start considering the vortex density modulation

$$\rho(x, y, z) = \frac{1}{N} \sum_{j=1}^{N} \delta(x - x_j(z))\delta(y - y(z)) - \rho_0,$$

(7)

where $\rho_0$ is the average density and $N$ the number of vortices. In magnetic decoration experiments we have access to the vortex structure at the surface, namely the vortex density $\rho(x, y, z = 0)$. The structure factor is obtained from the two-dimensional Fourier transform of the density, $\rho$, as

$$S(q_x, q_z = 0) = |\hat{\rho}(q_x, q_z = 0)|^2.$$

(8)

In the same token, the one-dimensional structure factor in a vortex row, $S_1$, is obtained from the vortex density modulation along a line $\rho_z$. The angular structure factor $S_\theta(q)$ is the polar-coordinate representation of $S(q_x, q_z = 0)$, see Fig. 2(f) for schematics.

The angularly-averaged $S_\theta(q)$ has to be calculated carefully when studying the low-$q$ density modes. Due to finite size effects, the borders and shape of the experimental field-of-view hinder the study of $S_\theta(q)$ in the low-$q$ range due to the annoying windowing effect. In rectangular fields-of-view as we study here, this artifact produces an excess in $S_\theta(q_x, q_z)$ in a cross-shaped region centered at $q_x = q_z = 0$. When analyzing our experimental data, in order to get rid of this effect we neglect the contribution from this cross. In simulations, this effect is avoided considering in-plane periodic boundary conditions.
Planar elastic vortex array model: Analytical details. The simple model of a planar elastic vortex array considered in the discussion is the continuum in the by the scalar longitudinal displacement field \(u_i(\not x, z)\) with hamiltonian
\[
H \approx \frac{1}{2} \int d\not x \int d\not y \left( u_i(\not x) \cdot \nabla \right) \left( u_i(\not x) \cdot \nabla \right) + \frac{1}{2} \int d\not x \left( \nabla \cdot u_i(\not x) \right)^2 + \epsilon \int d\not x \left( \nabla \cdot u_i(\not x) \right)^2.
\]
where \(u_i(\not x, z)\) is the Fourier transform of \(u_i(s_1, z)\). By assuming translational invariance along \(z\) and evaluating the elastic constants at \(q_0 = 2\pi/n\), the correlation function
\[
S_i(q_0, z_1 - z_2) = \frac{n_0}{2}\int d\not x \left( \nabla \cdot u_i(\not x) \right)^2
\]
leads to
\[
S_i(q_0, z_1 - z_2) = \frac{n_0}{2}\int d\not x \left( \nabla \cdot u_i(\not x) \right)^2 = \frac{1}{2} \frac{n_0}{\sqrt{1 + 4\pi^2 q_0^2}}.
\]
with
\[
\xi(q_0) = q_0^{-1} \sqrt{c_{44}(q_0, 2\pi/n) c_{11}(q_0, 2\pi/n)}.
\]

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Author contributions

Y.F. and A.B.K. designed research and discussed the general method to analyze the data, Y.F. and R.C.M. performed measurements, G.N. and P.P. grew samples, F.E. and A.B.K. performed simulations and theoretical calculations, J.R.P., F.E., J.A.S., G.R., A.B.K., and Y.F. analyzed data; all authors discussed the data analysis and interpretation; Y.F. and A.B.K. wrote the paper.

Competing interests

The authors declare no competing interests.

Additional information

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