Relative Coordinates Generality of Gradient Stability Zones and Force Factor between Hemispherical Polar Tips (in Faraday Magnetometer)

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Abstract. Coordinate dependencies of the field induction, its gradient, and the magnetic force factor in the region between hemispherical polar tips proposed for use (in the Faraday magnetometer) are given. Dependencies (and their functional form) are determined by the influence of the distance between the polar tips on the coordinates of the gradient extrema and the force factor, and also on the extreme values of these parameters: they are close to logarithmic and power-like ones, respectively. It has been shown that using hemispherical polar tips of one or another diameter, it is entirely possible to operate with more universal, namely relative (referred to the diameter or radius) values of the distance between the polar tips and coordinates of the gradient extremum or the force factor.

1. Introduction
To solve many technical and fundamental problems Faraday magnetometers of various types continue to stay in demand for a long time. In particular, they are magnetometers based on electromagnetic system use (a block consisting of magnetizing coils, cores and polar tips) [1-14], short (including superconducting) coils [15-23], permanent high-energy magnets (for example, Nd-Fe-B) [24,25].

As for the magnetometers of the first type (the most numerous), an original decision was made in [26-28] on the expediency of using hemispherical polar tips in the Faraday magnetometer. Such a solution is argued by the fact that the specially obtained coordinate (in the radial direction of the interpolar region symmetry plane) characteristic of induction \( B \) is tortuous. From a mathematical point of view, this guarantees obtaining an extreme coordinate characteristic of the induction gradient (practically identical with the partial derivative \( \frac{dB}{dx} \)), while in the vicinity of the \( x_{\text{extr}} \) extremum the \( \frac{dB}{dx} \) values are almost stable. The coordinate characteristic of the so-called [29] magnetic force factor \( B \frac{dB}{dx} \) - with the individual coordinate of the extremum \( x_{\text{extr}} \), whose vicinity values \( B \frac{dB}{dx} \) are almost stable, is also extreme.

The \( x_{\text{extr}} \) coordinates (which turned out to be independent of the current load \( I \) and dependent on the mutual removal of the hemispherical tips \( b \)) were obtained in [26-28] for the example of hemispherical tips with a diameter \( D = 100 \, \text{mm} \), which, as it is, indicates their particular character. Therefore, development of researches [26-28] with use hemispherical tips of other diameter is required. In this connection, a comparative analysis of the corresponding coordinates of the stability zones of the gradient and the force factor for different diameters of the hemispherical tips \( D \) is of scientific and practical interest. In this case, it is possible to determine the eligibility of operating with relative (as more universal) parameters \( b/D \) and \( x_{\text{extr}} / D \).
2. Research results and their analysis
With use of polar hemispherical tips \( D = 135 \text{mm} \) in diameter there are received and shown on the Figure 1 the coordinate dependencies of the field \( B \) induction in the region between the (alternatively, the dependencies for \( D = 100 \text{mm} \) in [26-28]) for their various mutual distances \( b \) - from 4.7 mm to 17.6 mm and different values of the winding supply current \( I \) is from 4A to 30A. It can be seen that each of the curves \( B \) has an inflection (in the neighborhood of which, as before [26-28], its section can be linearized), which indicates the presence of an extremum of the coordinate characteristic of the gradient \( dB/dx \) (figure 2), determined, as in [26-28], on the basis of the fourth degree polynomial with the corresponding coordinates of the extremum \( x_{extr} \) (table 1). There is also a corresponding extremum of one or another coordinate dependence of the magnetic force factor \( BdB/dx \) (figure 3) - with the corresponding values of the coordinates of the extremum \( x_{extr} \) (table 2).

It should be noted that, similarly to the results obtained earlier [26-28] (using hemispherical polar tips \( D = 100 \text{mm} \) in diameter), it is also characteristic here for hemispherical polar tips with a diameter \( D = 135 \text{mm} \) that the coordinate of the extremum \( x_{extr} \) of both the gradient and the force factor does not depend from the current load of the winding (figure 1-3, Table 1, 2).

![Figure 1](image1.png)

**Figure 1.** Coordinate characteristics of field induction between hemispherical polar tips, points - experiment, lines - calculation using a polynomial; a) \( b = 4.7 \text{mm} \), b) \( b = 8.1 \text{mm} \), c) \( b = 10.8 \text{mm} \), d) \( b = 13.5 \text{mm} \), e) \( b = 15.5 \text{mm} \), f) \( b = 17.6 \text{mm} \); 1 - \( I = 4 \text{A} \), 2 - \( I = 8 \text{A} \), 3 - \( I = 16 \text{A} \), 4 - \( I = 30 \text{A} \).

**Table 1.** The coordinates of the extrema of the dependencies shown in figure 2.

| \( I, \text{A} \) | \( x_{extr}, \text{mm} \) |
|---------------|---------------------|
| \( b=4.7\text{mm} \) | \( b=8, \text{mm} \) | \( b=10.8\text{mm} \) | \( b=13.5\text{mm} \) | \( b=15, \text{mm} \) | \( b=17, \text{mm} \) |
**Figure 2.** The coordinate characteristics of the gradient field obtained from the data in figure 1.

**Table 2.** The coordinates of the extrema of the dependencies shown in figure 3.

| I, A | $x_{extr}$, mm |
|------|-----------------|
| 4    | 8.559           |
| 8    | 8.458           |
| 16   | 8.458           |
| 30   | 8.391           |
| 4    | 11.05           |
| 8    | 11.01           |
| 16   | 11.16           |
| 30   | 11.10           |
| 4    | 12.77           |
| 8    | 12.18           |
| 16   | 12.11           |
| 30   | 12.07           |
| 4    | 14.00           |
| 8    | 14.57           |
| 16   | 14.56           |
| 30   | 15.43           |
| 4    | 16.91           |
| 8    | 15.42           |
| 16   | 15.38           |
| 30   | 15.07           |
| 4    | 18.17           |
| 8    | 16.15           |
| 16   | 16.37           |
| 30   | 16.63           |
| 4    | 18.39           |
| 8    | 16.83           |
| 16   | 16.83           |
| 30   | 16.83           |

| $I$, A | $x_{extr}$, mm | $\frac{dB}{dx}$, mT/mm |
|--------|----------------|------------------------|
| 4      | 8.559          | 11.05                  |
| 8      | 8.458          | 11.01                  |
| 16     | 8.458          | 11.16                  |
| 30     | 8.391          | 11.10                  |
| 4      | 12.77          | 14.00                  |
| 8      | 12.18          | 14.57                  |
| 16     | 12.11          | 14.56                  |
| 30     | 12.07          | 15.43                  |
| 4      | 16.91          | 18.17                  |
| 8      | 15.42          | 16.15                  |
| 16     | 15.38          | 16.37                  |
| 30     | 15.07          | 16.63                  |
Figure 3. The coordinate characteristics of the magnetic field force factor obtained from the data in figure 1 and figure 2.

As for the influence on $x_{extr}$ of the mutual removal $b$ of the hemispherical tips, then, for greater values $x_{extr}$ (in comparison with the values of $x_{extr}$ for $D = 100$mm [26-28]), figure 4, the realizable data generalization in relative coordinates is noticeable: $x_{extr}/D$ from $b/D$ (figure 5). This testifies to the universality of the obtained results on the identification of the stability zones (gradient and force factor) in the case of using polar tips of just such a (spherical) shape.

Figure 4. Influence of mutual removal of hemispherical polar tips ($D = 135$mm) on: a) the coordinate of the induction gradient extremum (the conditional center of its stability zone), b) coordinate of the extremum of the magnetic force factor (the conditional center of its stability zone).
Figure 5. Influence of mutual removal of hemispherical polar tips on: a) relative coordinate of the induction gradient, b) the relative the coordinate of the extremum of the magnetic force factor; ○ - $D = 100\text{mm}$ (data [26]), ● - $D = 135\text{mm}$; In semilogarithmic coordinates, the data are quasilinearizable, so that indicates their logarithmic form.

Of course, what has been said fully applies to the size of the zones themselves, i.e. $\Delta x, \Delta y$ and $\Delta z$, the more universal representation of which is in relative form: $\Delta x/D, \Delta y/D$ and $\Delta z/D$ (see data in figures 1-3 in comparison with the data [26-28]). Thus, for example, if the dimensions of the zones described below are about $5x5x5\text{mm}$ for hemispherical polar tips of diameter $D = 100\text{mm}$, then these dimensions are almost 1.35 times larger, in practice, for hemispherical polar tips of diameter $D = 135\text{mm}$ approximately $7x7x7\text{mm}$.

We also note that, judging by the quasilinearization of the $x_{ind}/D$ data from $b/D$ in semilogarithmic coordinates (figure 5), in the selected $b/D$ range they correspond to the logarithmic function. In this case, the extremes of the force factor are located 30-40% closer to the axial line of the poles than for the gradient. In addition, for polar tips of hemispheres of a different diameter (in this case, $D = 135\text{mm}$), as before [26-28] ($D = 100\text{mm}$), the dependence of the induction gradient abscissas is practically the same (figure 6a, exponent: -1, 3) and the force factor (figure 6b, exponent: -2.2) from the mutual removal of hemispherical polar tips.
Figure 6. Illustration of the power-law dependence of the induction gradient (a) and magnetic force factor (b) from the mutual removal of hemispherical polar tips (after the quasi-linearization of these data in logarithmic coordinates); 1 - I = 4A, 2 - I = 8A, 3 - I = 16A, 4 - I = 30A.

3. Conclusion
The families of the coordinate characteristics $B$, $dB/dx$, $dB/dx$ are obtained and analytically analyzed, depending on the basic parameters of the unit functioning responsible for creating a non-uniform field in the Faraday magnetometer: current load $I$, mutual removal $b$ between the hemispherical tips proposed for use, their diameter $D$. The $x_{extr}$ extremum coordinates for the parameters $dB/dx$ and $dB/dx$ are found and compared (including on the basis of the calculated dependence obtained). It is shown that for each of the values of $b$, the $x_{extr}$ data for both the gradient and the force factor remain practically unchanged irrespective of $I$, while the extrema of the force factor are 30-40% closer to the axial line of the poles than to the gradient. The dependencies (and their functional form) are determined by the influence of $b$ on the $x_{extr}$ values for $dB/dx$ and $dB/dx$, and on the extreme values of these parameters; they turned out to be close to the logarithmic and power-law dependencies, respectively. It is shown that using hemispherical polar tips it is quite possible to operate with more universal ones, namely the relative parameters $b/D$ and $x_{extr}/D$.

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