Evidence for weak collective pinning and $\delta l$ pinning in topological superconductor Cu$_x$Bi$_2$Se$_3$

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Abstract

We investigated the vortex pinning behavior in the single crystal topological superconductor Cu$_{0.10}$Bi$_2$Se$_3$ with a pronounced anisotropic peak effect. A weak collective pinning regime is clarified from the power-law behavior in $J_c (B)$ and the small critical current density ratio of $J_c/J_0 \sim 10^{-5}$ ($J_c$ is the critical current density, $J_0$ is the depairing current density). The spatial variation of the charge-carrier mean free path induced pinning is evidenced and probably results from the well-defined atomic defects. Within the framework of collective pinning theory, we computed the values of the correlated length and volume at 1.8 K, which start declining prior to the onset field of the peak effect $B_{\text{onset}}$, demonstrating the vortex lattices already suffered a preferential collapse ahead of the peak effect turns up. Thus, the peak effect can be understood by elastic moduli softening near the upper critical field $B_{c2}$. We suggest Cu$_x$Bi$_2$Se$_3$ is a prototype topological material for investigating the vortex pinning dynamics associated with the peak effect phenomenon.

Keywords: topological superconductor, peak effect, vortex pinning regime, collective pinning analysis

Supplementary material for this article is available online
(Some figures may appear in colour only in the online journal)
demonstrated by NMR and high-resolution calorimetry measurements [5, 6], inspiring the search for MFs in the vortices core in this topological superconductor [11–13].

Due to strong spin–orbit coupling (SOC), the odd-parity pairing state of Cu₄Bi₂Se₃ may still be robust against disorder [9, 12, 14], though the bulk materials composed of a lot of intrinsic atomic defects in the nanoscale [15–17]. These defects are not only physically interesting but also technically relevant for their interaction with vortices to immobilize vortex lines and achieve non-dissipative supercurrents. Particularly, the vortex pinning effect can be used to control and manipulate the vortices and MFs in topological superconductors [13]. In most cases, the effect of quenched disorders, thermal fluctuations and quantum fluctuations on vortex lattices can result in a complex vortex phase diagram of type-II superconductors [18]. While in odd-parity spin-triplet superconductors, the vortex matter might hold exotic states due to the equal-spin pairing phase, e.g. the half-quantum vortices and vortex coalescence phenomenon observed in p-wave pairing Sr₂RuO₄ [19, 20]. Theoretically, the odd-parity pairing symmetry is a key requirement for topological superconductivity in inversion-symmetric systems. As argued in Cu₀.₁₅Bi₂Se₃ [21], the vortex current induces a non-uniform superconductivity in inversion-symmetric systems. As argued, the effect of quenched disorders, thermal fluctuations and quantum fluctuations on vortex lattices can result in a complex vortex phase diagram of type-II superconductors [18].

However, only a little work focusing on the vortex pinning regimes has been reported so far. Previous theoretical and dc magnetization studies showed evidence of the spin-triplet vortex state as well as the flexible motion of the vortices in Cu₀.₁₀Bi₂Se₃ [21]. Very recently, the peak effect, characterized by a large anomalous increase of magnetization near the upper critical field was revealed, which implies intrinsic vortex dynamics in Cu₀.₁₀Bi₂Se₃ (see figure 1(a)). Motivated by the pursuit of these exotic vortex states, this work aims to lay out a fundamental understanding of the vortex pinning mechanism in Cu₀.₁₀Bi₂Se₃ topological superconductor. We demonstrate that this material shows weak collective pinning behavior that is probably related to atomic defects, causing an electron mean free path fluctuation induced vortex pinning. Based on the collective pinning theory [22, 23], we calculated the longitudinal correlation length (Lₗ) and transverse correlation length (Rₜ) of a Larkin domain caused by random pinning, within which the vortex lines interact elastically and retain a short-range order. Throughout our collective pinning analysis, we found that the correlation length and volume start to decrease in the field of the critical current density minimum and collapse fast between B_p^{onset} and B_p. These results indicate the peak effect can be ascribed to the softening of the elastic moduli near B_c2.

2. Experiments

Cu₀.₁₀Bi₂Se₃ has a rhombohedral structure with a R₃m space group (No. 166) like parent Bi₂Se₃, and the Cu atoms are assumed to be intercalated in the van der Waals gap between the Se₁–Bi–Se₂–Bi–Se₃ quintuple layers (QtLs), partially occupying the octahedrally-coordinated 3b (0, 0, ½) sites [15]. Single crystals of nominal Cu₀.₂₀Bi₂Se₃ were grown using a modified Bridgman method. The sample growth was carried out by slowly cooling the stoichiometric melts of Cu (5N), Bi (5N) and Se (5N) in evacuated quartz ampoules, which were flushed with pure argon gas three times before melt sealing, from 1148 K to 853 K at a rate of 2.5 K h⁻¹ in a vertical tube furnace. After growth, the crystals were annealed at 853 K for more than 24 h before quenching in an ice water bath. The actual composition was determined to be Cu₀.₁₀Bi₂Se₃ by inductively-coupled plasma-atomic emission spectroscopy. A piece of the 3.35 × 2.45 × 0.66 mm³ rectangular shaped specimen was used for the isothermal magnetization hysteresis loops (MHLs) measurements on a commercial SQUID magnetometer (Quantum Design). The sample was cooled down to target temperatures in the zero field cooling (ZFC) mode and subsequently, the isothermal MHL data was dynamically collected with different sweeping rates of dB/dt = 30, 50, 100, and 200 Oe s⁻¹ to evaluate the dynamical vortex relaxation effect. Additionally, magnetic relaxation measurements were also performed at temperatures ranging from 2.0 K to 3.5 K with the magnetic field parallel to c axis under 500 Oe, which is sufficiently larger than the first full penetration field [14].

The following procedure was carried out: (i) the sample was ZFC cooled down to the desired temperature. (ii) the magnetic field was increased to 500 Oe with 200 Oe s⁻¹. (iii) the magnetization as a function of time was immediately recorded for 3600 s after the field was ramped up to the desired value.

3. Results and discussion

3.1 Weak collective pinning behavior

Figure 1(a) shows the isothermal MHLs at 1.8 K for B || ab and B || c. When the magnetic field approaches the upper critical field, a peak effect can be observed in both field configurations, but it is more remarkable for B || c than B || ab due to the superconducting anisotropy caused by its layered-structure. The observation of the peak effect close to B_c2 indicates a very weak pinning in Cu₀.₁₀Bi₂Se₃, that can be regarded as a nearly clean crystal. In cuprates, the peak effect can only be seen in ultra-clean crystals with a low level of pinning centers, for instance, in YBa₂Cu₃O₇−δ [24]. With increasing weak point disorders in clean YBa₂Cu₃O₇−δ, by electron irradiation [24], the Jc(B) peak broadens accompanying with the shift of the peak position towards the intermediate fields away from B_c2, which is often assigned to the second magnetization effect or fishtail effect that distinguishes the peak effect. The field dependence of Jc was estimated according to the extended Bean critical model, as shown in figures 1(b) and (c). Below 3.0 K, a series of pronounced Jc peaks appear in the high field region under B || c. The magnitude of Jc is suppressed by the external field until a critical field of B* = 0.31 T, above which Jc rises again. By further increasing the field, the peak effect emerges at the B_p^{onset} and reaches a cusp at B_p. The same
The feature also exists for $B \parallel ab$ as shown in figure 1(c), but the $B^*$, $B_p^{\text{onset}}$, and $B_p$ shift to higher field locations. Below 0.14 T, the $J_c(B)$ curves obey a power-law relationship $J_c \propto B^{-n}$ at 1.8 K, which is a signature of collective pinning [25, 26]. Above 0.14 T, the $J_c$ deviates from the power law scaling, and the peak effect occurs at the $B_p^{\text{onset}}$. In a superconducting crystal, the quenched disorder can destroy the long-range order of the Abrikosov vortex lattices, forming a quasi-long range ordered Bragg glass in the case of weak disorders [22, 27]. The power-law behavior of $J_c(B)$ implies the existence of a fairly dense distribution of weak pins in Cu$_{0.10}$Bi$_2$Se$_3$ [23]. In a weakly pinned state, the power-law behavior actually reflects the increase of the vortex lattice rigidity with the increasing field and is typical for a weakly-disordered vortex lattice in Larkin–Ovchinnikov’ collective pinning theory. In high-$T_c$ cuprates, it should be noted that a similar power-law dependence with $n = 1/2$ follows from the single-vortex pinning on large normal particles (dimension $> \xi$) [28] while $n = -1$ from the thermodynamic analysis of MHL scaling in a TmBa$_2$Cu$_3$O$_{6.8}$ crystal [29]. Based on the strong pinning theory [30], it is predicted that there is a crossover field $B_c$ in the $J_c(B)$ curve. For $B < B_c$, the $J_c(B)$ is nearly constant, while $J_c(B) \propto B^{-5/8}$ for $B > B_c$ showing a temperature independent exponent. However, we observed a considerable change of $n$ as a function of temperature and no plateau was observed in the low field region, which is inconsistent with the power-law behavior of cuprates.

Moreover, the exponent $n$ tends to decrease with an increase in the temperature, e.g. $n$ changes from 0.88 to 0.40 within the temperature span of 1.8 K–3.0 K, which may be related to the flux creep effect. To elucidate this, the normalized relaxation/creep rate is determined either in the form of the dynamical relaxation rate [18], $Q = \frac{d \ln(J_s)}{d \ln(\frac{dB}{dt})}$, or the conventional magnetic relaxation rate, $S = \frac{d \ln(M)}{d \ln(t)}$, where $J_s$ is the superconducting current density (normally $J_s \leq J_c$) and $t$ is time. Both relaxation rates are fundamentally identical, i.e. $Q = S$, meaning the dependence of the MHL amplitude on the sweep rate contains the same information of the magnetization versus time during relaxation. As seen in figure SM2 (see supplementary materials for details, available at stacks.iop.org/JPhysCM/30/31LT01/mmedia), the temperature dependence of $S$ was obtained within the order of $10^{-2}$–$10^{-3}$ for Cu$_{0.10}$Bi$_3$Se$_6$, which is much smaller than that of high-$T_c$ cuprates [18, 31]. This is understandable since the Ginzburg

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Figure 1. (a) Isothermal MHLs at 1.8 K for $B \parallel ab$ and $B \parallel c$. (b) The $J_c(B)$ curves at $T = 1.8$ K, 2.4 K and 3.0 K for $B \parallel ab$. The inset shows the $n$ as a function of temperature and the solid line is a guide to the eyes. (c) The $J_c(B)$ curves at $T = 1.8$ K for $B \parallel c$. The isothermal MHLs were collected with sweeping rate of $dB/dt = 50$ Oe s$^{-1}$. 

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number $G_c$, quantifying the effect of thermal fluctuations, is estimated to be $\sim 10^{-5}$ for Cu$_{0.10}$Bi$_2$Se$_3$ using the reported penetration depth value [14], which is three orders of magnitude smaller than that of high-\textit{T}_c cuprates ($10^{-2}$) [18], indicating quite a narrow critical fluctuation region. Meanwhile, the $S$ firstly tends to decrease up to 3.3 K before a small increase is observed. This seems to be consistent with the decrease of $n$ when increasing the temperature, which may be caused by a diminution of flux creep.

Figure 2 shows the temperature dependence of $J_c$ at different temperatures. We fitted the $J_c(T)$ by a widely adopted phenomenological formula $J_c(T)/J_c(0) = (1 - T/T_c)^n$ with the exponent $n = 1.5-2$ [32]. The best fitting result yields $J_c(0) = 1.89 \times 10^5$ A cm$^{-2}$ at 0 K and $n = 1.70 \pm 0.10$. Note that the fitting exponent is very close to $n = 3/2$ as predicted by single band GL theory [33]. As a result, this provides the critical current ratio of Cu$_{0.10}$Bi$_2$Se$_3$ in an order of magnitude of $J_c/J_{0.1} \sim 10^{-5}$ in the limit of $B \rightarrow 0$ T, which further demonstrates a very weak pinning regime in Cu$_{0.10}$Bi$_2$Se$_3$. Compared with other superconductors, the ratio of $J_c/J_0$ for Cu$_{0.10}$Bi$_2$Se$_3$ is $3-4$ orders of magnitude smaller than high-$\text{T}_c$ cuprates ($10^{-3}-10^{-2}$ for YBa$_2$Cu$_3$O$_{6.6}$ and low-$\text{T}_c$ superconductors ($10^{-2}-10^{-1}$) with strong pinning [18]. According to the relation $J_c \propto J_0(\xi/L_c)^2$, the length scale of $L_c$ should be much larger than the coherence length $\xi$, ensuring the validity of elasticity theory. This allows us to discuss the influence of quenched disorder in terms of the weak collective pinning theory [23].

3.2. Pinning regimes: $\delta l$ or $\delta \text{T}_c$ pinning

In fact, the quenched disorder for vortex pinning can arise from various types of atomic defects in Cu$_{0.10}$Bi$_2$Se$_3$, as identified by scanning tunneling microscopy (STM) measurements [15–17]. Besides Se vacancies and Bi$_{16c}$ antisites in pristine Bi$_2$Se$_3$, a number of new defects can be introduced by Cu doping [17], including type A: Cu$_{16c}$ substitution sites (in the second atomic layer in QTL), type B and C: Cu intercalation sites (H$_3$&T$_1$, T$_4$ sites between QtLs), and type D: Cu$_{16d}$/Cu$_{16e}$ substitution sites (the fourth/fifth atomic layer in QTL). Since the defects counts of type B and C determined by high-resolution topographs are significantly larger than the $D$ defect count, they are more likely to occur in Cu$_{0.10}$Bi$_2$Se$_3$. For superconducting Cu$_3$Bi$_2$Se$_3$, the distance between the internal atomic defects may be within the superconducting coherence length (e.g. $\xi_{ab} = 13.96$ nm for Cu$_{0.10}$Bi$_2$Se$_3$), hinting that a group of related defects is required to pin one single flux-lattice line collectively. On the other hand, these atomic defects mainly act as point pins because they have smaller dimensions than the vortex lattice constant $a_0 = 1.07/(\Phi_0/\text{B})^{1/2}$, e.g. $a_0 = 37.46$ nm at $\text{B}^{1/2}_{c2}(0) = 1.69$ T. Importantly, the atomic defects are typically below $1$ nm size, which is far less than $\xi_{ab}(0)$ and also confines them as weak point pins.

Since the Cu$_{16}$Bi$_2$Se$_3$ is classified as one of the extremely large $\kappa$ superconductors [15], one can safely assume that its core interaction is dominant in vortex pinning dynamics. The source of vortex pinning is usually divided into two categories, $\delta l$ pinning resulting from the spatial variations of charge carrier mean free path near normal particles or lattice defects and $\delta \text{T}_c$ pinning from the spatial variations of the GL coefficient associated with fluctuations of the $\text{T}_c$. On the basis of collective pinning theory [34], the temperature dependence of normalized $J_c$ in a single vortex pinning regime can be described by $J_c^{\delta l}(t)/J_c^{\delta l}(0) = (1 - r^2)^{3/2}(1 + r^2)^{-1/2}$ for $\delta l$ pinning, and $J_c^{\delta \text{T}_c}(t)/J_c^{\delta \text{T}_c}(0) = (1 - r^2)^{5/6}(1 + r^2)^{5/6}$ for $\delta \text{T}_c$ pinning, where $t = T/T_c$. Correspondingly, the $g(t) = U(t)/U(0)$ functions characterizing the temperature dependence of the pinning energy are given by $g^{\delta l}(t) = 1 - r^4$ for $\delta l$ pinning, and $g^{\delta \text{T}_c}(t) = (1 - r^2)^{1/3}(1 + r^2)^{5/3}$ for $\delta \text{T}_c$ pinning. Due to the presence of the giant flux creep in high-$\text{T}_c$ cuprates [31], the measured $J_c$ values can differ markedly from the true $J_c$ corresponding to that without thermally activated flux flow. Therefore, the $J_c(T)$ should be first derived by the general inversion scheme (GIS) before applying the pinning regime analysis [31, 34]. Following this procedure, the randomly distributed weak pinning centers of the oxygen vacancies induced $\delta l$ pinning in YBa$_2$Cu$_3$O$_7 \&$ YBa$_2$Cu$_3$O$_6$ films [34] and a DyBa$_2$Cu$_3$O$_7$–$\delta$ single crystal [31]. Accordingly, to evaluate the relaxation effect on $J_c$, we attempted to derive the dynamical relaxation rate $Q$ by measuring MHLs with different sweeping rates at 2 K. The $J_c(B)$ curves are plotted in figure SM3. However, all the curves overlapped and meaningful field dependence of $Q$ was not obtained, implying a negligible relaxation effect on the true $J_c$ values, i.e. $J_c \approx J_0$ satisfies in the experimental temperature region at least. This also assures that the previously yielded small $J_c/J_0$ ratio sounds reasonable given the small relaxation effect in this material.

In figures 3(a) and (b), we plot the normalized $J_c(t)$ and $g(t)$ data obtained at $B = 0.005$ T, 0.01 T, 0.02 T, and 0.03 T, together with the theoretical curves of the $\delta l$ and $\delta \text{T}_c$ pinning regimes. The experimental data follows the theoretical $\delta l$ pinning well, evidencing that the vortex pinning in Cu$_{0.10}$Bi$_2$Se$_3$ mainly originates from the spatial variations in the charge-carrier mean free path by lattice defects. As the most abundant atomic defects [15–17], the intercalated Cu atoms are

![Figure 2](image-url)
 probably more relevant to the weak pins. Here, the pinning regime in Cu_{0.10}Bi_{2}Se_{3} is distinguished from the Co/Mn intercalated 2H-NbSe_{2} [35], for which the effective pinning is suggested to come from the fluctuations of local T_{c} caused by Co/Mn magnetic impurities. However, no direct evidence of the magnetic order of the doped Cu atoms in Cu_{0.10}Bi_{2}Se_{3} has been reported yet [36].

3.3. Collective pinning analysis

The collective-pinning theory has been successfully applied for pinning analysis in low T_{c} type-II superconductors with randomly distributed weak pinning centers [25, 37, 38]. According to collective pinning theory [23], the volume pinning force F_{p} is written as:

\[ F_{p} = |I_{c} \times B| = (W/V_{c})^{1/2} \] (1)

where W = n_{p} \langle f_{p}^{2} \rangle is the collective pinning parameter as a measure of the pinning and V_{c} = R_{c}^{2}L_{c} is the correlation volume (n_{p} is the volume density of the pinning centers, f_{p} is the elementary pinning interaction with an interaction range r_{f}). The longitudinal correlation length (L_{c}) and transverse correlation length (R_{c}) are defined by L_{c} = (c_{44}/c_{66})^{1/2}R_{c} and R_{c} = (c_{44}/c_{66}r_{f}^{2})/W, where c_{44} is the tilt modulus, and c_{66} is the shear modulus. To characterize the configuration of the vortex matter [18], it is convenient to use the ratio L_{c}/d (where L_{c} = \xi_{ab}(J_{0}/J_{c})^{1/2}/\Gamma is the Larkin scale, \Gamma the anisotropy ratio, and d sample thickness), i.e. an isotropic vortex structure is expected for L_{c}/d \gg 1 with 2D collective pinning, whereas an anisotropic structure is expected for L_{c}/d \ll 1 with 3D collective pinning [18, 23]. For Cu_{0.10}Bi_{2}Se_{3}, the Larkin scale is estimated to be L_{c} \approx 3.40 \mu m at 1.8 K, which is much smaller than the sample thickness. According to weak collective pinning theory [23], no peak effect appears for a 2D collective pinning case unless a dimensional crossover occurs to trigger it [37, 38]. So far, 2D collective pinning has only been demonstrated in micrometer-thin amorphous films or crystals [37, 38]. Hence, it is reasonable to suppose that 3D collective pinning applies in Cu_{0.10}Bi_{2}Se_{3}, which means it has an anisotropic vortex structure with flexible flux-lattice line configurations that favors a rapid relaxation of diamagnetic magnetization [21].

Calculating the values of L_{c} and R_{c} as a function of the magnetic field gives a deeper insight into vortex pinning dynamics. For anisotropic superconductors with B \parallel c [39], the shear modulus is given by:

\[ c_{66} = \frac{\mu_{0}}{\mu_{0}} \frac{b(1-b)^{2}}{4} \left[ 1 - 0.29b \right] \exp \left( \frac{b}{3+6b} \right) \left( \frac{2c_{2}}{2c_{2} - 1} \right) \left( \frac{1}{(2c_{2} - 1) + 1} \right) \] with 

b = B/B_{c2}, where B_{c}(T) = B_{c}(0) (1 - \tau^{2}) is thermodynamic critical field and \beta_{0} = 1.16 for the triangular lattice. The dispersive tilt modulus is \tilde{c}_{44} = \tilde{c}_{44} = \frac{\mu_{0}}{\mu_{0}} \frac{1}{(1+B_{0}^{x^{2}} + B_{0}^{y^{2}} + B_{0}^{z^{2}})} \] [40], where k_{\perp} = (k_{x}^{2} + k_{y}^{2})^{1/2} \approx \pi/R_{c}, and k_{c} \approx \pi/L_{c}, describe the deformation field normal and parallel to the magnetic field direction, k_{\perp}^{\kappa_{B}Z} = \kappa_{B}Z/\lambda_{ab}^{z}/(1 - b)^{1/2} is the magnetic penetration depth, and \Gamma^{2} = M_{c}/M is the mass ratio where M_{c} is the quasiparticle effective mass along the c-axis and M is in the ab-plane. For extreme type-II superconductors, where k_{\perp}^{\kappa_{B}Z} \lambda^{2} = 2h\nu^{2} + \kappa_{B}Z/\lambda_{ab}^{z}/(1 - b) \gg 1, the shear c_{66} and tilt \tilde{c}_{44} moduli can be further simplified as:

\[ c_{66} \approx \frac{\mu_{0}}{\mu_{0}} \frac{b(1-b)^{2}}{8\kappa_{B}^{2}} (1 - 0.29b) \] (2)
equation (1) with equation (4) to obtain the field dependence of \( L_c \) and \( R_c \).

We present the computed results of the field dependence of \( L_c \) and \( R_c \) and the ratios of \( L_c/d \), \( R_c/a_0 \) in figures 4(a) and (b). Both \( L_c \) and \( R_c \) first tend to increase with the rising field. However, once reaching a maximum at \( B^* = 0.31 \) T, a rapid decrease of \( L_c \) and \( R_c \) subsequently follows with the further increasing field. We note that a curvature change of \( L_c(B) \) and \( R_c(B) \) appears at the \( B^\text{onset} \). Above \( B^* \), the \( V_c \) start to collapse dramatically while the \( W \) actually tends to increase in a broad range of \( 0-B_p \) (see supplementary materials for details). A simultaneous faster decay of the elastic energy \( (E_d) \) than the pinning energy \( (E_p) \) appears, which results in a significant increase of \( F_p \) and \( J_c \) near \( B_{c2} \), creating the peak effect. In general, the \( J_c \) value of weak pinning superconductors decreases with a field increase up to the \( B^\text{onset} \), above which \( J_c \) shows a significant increase due to the enhanced pinning force. Surprisingly, the collapse of \( L_c \) and \( R_c \) starts at \( B^* \), corresponding to the minimum of \( J_c(B) \), instead of the \( B^\text{onset} \). This differs from the behavior of \( L_c \) and \( R_c \) in \( Cu_0.10Bi_2Se_3 \) and \( 2H-NbSe_2 \), for which the drastic decline of \( L_c \) and \( R_c \) occurs at the \( B^\text{onset} \). Figure 4(b) shows the magnetic field dependence of \( L_c/d \) and \( R_c/a_0 \) at 1.8 K. Both \( L_c/d \) and \( R_c/a_0 \) begin to decrease between \( B^* \) and \( B_p \), and \( R_c/a_0 \) \( \approx \) 1.85 when it approaches \( B_p \). It should be highlighted that \( L_c/d \) starts to decline exactly at \( B^* \) since \( d \) is constant. Nevertheless, due to the field dependence of \( a_0 \), \( R_c/a_0 \) starts decreasing at a slightly higher field than \( B^* \). Our results indicate that the peak effect of \( Cu_{0.10}Bi_2Se_3 \) can be interpreted as the softening of the elastic moduli in the framework of 3D collective pinning theory.

Finally, we will discuss some technical application implications drawn from our first observation of the peak effect in the topological superconductor \( Cu_0.10Bi_2Se_3 \). Firstly, the normal core of vortices in \( Cu_0.10Bi_2Se_3 \) can host MFs applied in fault-tolerant quantum computing [2, 3, 11]. To detect the surface MFs in \( Cu_0.10Bi_2Se_3 \), a sample with a Cu doping content \( x_c < 0.12 \) is preferred as it can shift the chemical potential below a critical \( \mu_c \approx 0.24 eV \) for \( B \parallel c \). This makes the present \( Cu_{0.10}Bi_2Se_3 \) single crystal a promising host candidate for MFs in the vortices core when the external field is fixed to or tilted from the \( B \parallel c \) configuration. However, the complex vortex pinning dynamics reported here put constraints on the detection of MFs in the various vortex phases controlled by external fields [42]. In this context, whether the MFs remain robust in various magnetic fields strengths needs further theoretical and experimental exploration. Secondly, as a weakly pinned type-II superconductor \( Cu_0.10Bi_2Se_3 \), like \( 2H-NbSe_2 \), can in principle provide the basis of a binary device used in our computation, the parameters of \( \zeta_{ab} = 19.61 \text{nm}, \Gamma = 2.25 \), \( J_c(0) = 2.33 \text{mK}, J_c(0) = 5.95 \times 10^3 \text{A/m}^2 \), and \( B^c_{c2} = 1.04 \text{T} \) are adopted.

In our computation, the parameters of \( \zeta_{ab} = 19.61 \text{nm}, \Gamma = 2.25 \), \( J_c(0) = 2.33 \text{mK}, J_c(0) = 5.95 \times 10^3 \text{A/m}^2 \), and \( B^c_{c2} = 1.04 \text{T} \) are adopted.

Next, we consider the derived expressions of the pinning parameter \( W \), as defined by \( W = W_0(T) b^2 (1 - b)^2 \) [41], where \( n = 1 \) for \( \delta T_c \) pinning, while \( n = 3 \) for \( \delta \delta T_c \) pinning. In a single-vortex pinning regime [38], the parameter \( W_0(T) \) at \( T = 1.8 \text{K} \) can be determined by \( W_0(T) \approx L_c(0) J^2_c(0) \Phi_0 B_{c2} = 1.77 \times 10^{-9} \text{N}^2 \text{m}^{-2} \) for \( Cu_{0.10}Bi_2Se_3 \). In general, we compute \( L_c \) and \( R_c \) by combining

\[
\begin{align*}
\bar{c}_{44} & \approx \frac{B^2}{\mu_0} \left[ 1 + \left( \frac{\pi n \zeta_{ab}}{L_c} \right)^2 \left( \frac{1}{1 - b^2} \right) + \left( \frac{\pi n \zeta_{ab}}{R_c} \right)^2 \left( \frac{1}{1 - b^2} \right) \right] \\
L_c & \approx \left( \frac{\bar{c}_{44}}{c_{66}} \right)^{1/2} R_c = \left\{ 1 + \left( \frac{\pi n \zeta_{ab}}{L_c} \right)^2 \left( \frac{1}{1 - b^2} \right) + \left( \frac{\pi n \zeta_{ab}}{R_c} \right)^2 \left( \frac{1}{1 - b^2} \right) \right\}^{1/2} R_c.
\end{align*}
\]
as a magnetic memory cell. Thirdly, our work indicates that the atomic defects (e.g. Cu intercalants) result in a $B^\text{onset}_{c}$ pinning regime. The atomic defects can induce a local point potential to pin vortices at sites, and this vortex pinning effect is both theoretically and practically interesting. Alternatively, one can utilize the movable atomic-scale tip of an atomic force microscope to generate the point potential [13]. When the tip approaches the vortex core and subsequently moves slowly away from the vortex center, the vortex can be dragged along with the point potential, which offers an effective way of controlling vortices in a topological superconductor. This implies a possible means of braiding MFs by manipulating the pinning centers. Thus, Cu$_x$Bi$_2$Se$_3$ is a promising candidate material for examining the theoretical scheme.

4. Conclusions

In summary, we report the analysis results of vortex pinning regimes in single crystal Cu$_{0.10}$Bi$_2$Se$_3$. The observation of power-law behavior in $J_c(B)$ and the small $J_c/J_0$ ratio indicates a weak collective pinning regime. We provide experimental evidence of the $B^\text{onset}_{c}$ pinning origin of vortex pinning, which may be associated with the intercalated Cu atomic defects. Based on collective pinning theory, the values of $L_c$, $R_c$, and $V_c$ at 1.8 K were computed and begin to decline before the $B^\text{onset}_{c}$, indicating a preferential collapse of the vortex lattices before the peak effect sets in. The peak effect can thus be understood by the softening of the elastic moduli near $B_{c2}$. Our results not only facilitate the understanding of complex vortex pinning dynamics in the topological superconductor Cu$_x$Bi$_2$Se$_3$ but also provide a candidate topological material platform for detecting and braiding MFs for future superconducting electronic applications.

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