The radiative decay of a neutrino with a magnetic moment in a strong magnetic field with taking account of the positronium influence on the photon dispersion

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Abstract. We study the process of the radiative decay of the neutrino with a magnetic moment in a strong magnetic field, with taking account of the influence of the positronium on the photon dispersion. The positronium contribution into the photon polarization operator leads to an essential modification of the photon dispersion law, and of the neutrino radiative decay amplitude. It has been shown that the probability of the neutrino radiative decay essentially increases under an influence of the positronium on the photon dispersion.

1. Introduction
Neutrino processes such as radiative decay of the neutrino $\nu \rightarrow \nu \gamma$ also called Cherenkov neutrino process, are of great interest for a long time. The process $\nu \rightarrow \nu \gamma$ in a magnetic field has been investigated in the case of a relatively weak field [1], of a strong field [2] and for the field of arbitrary intensity [3]. In all these papers, the situation was analysed of low-energy neutrinos compared to the mass of the electron. The case of more significant neutrino energies should be more interesting in the light of possible astrophysical applications. In this case, the radiative corrections become substantially enhanced, leading to a renormalization of the photon wave function and a significant deviation from the vacuum dispersion law. These factors were taken into account for the first time in [4]. In the most of the previous papers, the analyses of the neutrino radiative decay were performed with taking account only of the contribution of free electrons and positrons in the loop correction to the photon polarization operator. The influence of a bounded electron-positron pair on the photon polarization operator was taken into account in [5] for the case of the standard electromagnetic interaction of a neutrino, arising at the loop level and thus being strongly suppressed. In this paper, we study the process of the radiative decay $\nu \rightarrow \nu \gamma$ of the neutrino with a magnetic moment, with taking account of the positronium influence on the photon polarization operator.
2. The amplitude of the process $\nu \rightarrow \nu\gamma$

The interaction Lagrangian corresponding to the process $\nu \rightarrow \nu\gamma$ for a neutrino with a magnetic moment has the form:

$$L = -\frac{i\mu_{\nu}}{2} \left( \bar{\nu}(x)\sigma_{\alpha\beta}\nu(x) \right) F^{\alpha\beta},$$

(1)

where $\sigma_{\alpha\beta} = \frac{1}{2}(\gamma_{\alpha}\gamma_{\beta} - \gamma_{\beta}\gamma_{\alpha})$, $F^{\alpha\beta} = \partial^{\alpha}A^{\beta} - \partial^{\beta}A^{\alpha}$, $\mu_{\nu}$ is the magnetic moment of a neutrino. The 4-momentum conservation law for this process takes the form: $p = p' + q$. The neutrino field operator $\bar{\nu}(x)$ and the photon field operator $A^{\alpha}$ are defined as an expansion over the de Broglie plane waves. The amplitude of the radiative decay of a neutrino with a magnetic moment has the form:

$$M = -\frac{i\mu_{\nu}}{2} \left( \bar{\nu}(p')\sigma_{\alpha\beta}\nu(p) \right) \left( q^\alpha \varepsilon^{*\beta}_\chi(q) - q^\beta \varepsilon^{*\alpha}_\chi(q) \right).$$

(2)

The photon polarization operator in a magnetic field has four eigenvectors, two of which correspond to the physical photon polarization vectors [6]. It is known that in the limit of a strong magnetic field, only photons of the so-called second mode can be born in a radiative decay of a neutrino. The polarization vector of the second mode photon has the form:

$$\varepsilon^{(2)}_\alpha = \sqrt{Z(q^2)} \varepsilon_\alpha \sqrt{q^\parallel}. \quad (3)$$

Here, the factor $Z = \left(1 - \partial\Pi/\partial q^2_{\parallel}\right)^{-1}$ is caused by the effect of the renormalization of the photon wave function, where $\Pi$ is the eigenvalue of the photon polarization operator of the second mode, $\varepsilon^{\alpha\beta} = F^{\alpha\beta}/B$ is the dimensionless tensor of the external electromagnetic field, and $\varepsilon^{\alpha\beta} = (1/2)\varepsilon^{\alpha\beta\mu\nu}F_{\mu\nu}$ is the dual tensor. It is convenient to split the 4-momentum of the photon into two components. In the frame where the $z$ axis is directed along the external magnetic field, these are the vector in the Minkowski subspace $(0; 3)$: $q^\parallel_\alpha = (\omega; 0; 0; k_3)$ and the vector in a plane perpendicular to the external magnetic field $(1; 2)$: $q^\perp_\alpha = (0; k_1; k_2; 0)$.

3. The probability of the process $\nu \rightarrow \nu\gamma$

The probability of the neutrino radiative decay is determined by the integral over the phase space:

$$W = \int \frac{(2\pi)^4|M|^2\delta^4(p - p' - q)}{8E} \cdot \frac{d^3k}{(2\pi)^3\omega} \cdot \frac{d^3p'}{(2\pi)^3E'}.$$ \quad (4)

After simple transformations, the expression for the amplitude squared of this process can be reduced to the form:

$$|M|^2 = -4\mu_{\nu}^2 Z q^2 \varepsilon_\alpha \varepsilon_\alpha (p\varphi p').$$ \quad (5)

After substitution the amplitude square (5) into (4) and integration, the decay probability can be reduced to a single integral:

$$W = \frac{\mu_{\nu}^2 E^3}{\pi^2 m^2} \int_0^1 \frac{x^2(-\Pi)}{1 - \partial\Pi/\partial q^2_{\parallel}} \left( \sqrt{1 - x^2} - x \arccos x \right) dx, \quad (6)$$

where $x = |q^\perp|/(2E)$. The eigenvalue of the photon polarization operator and its derivative depend on $q^2_{\perp}$ and $q^2_{\parallel}$ that are linked together by the dispersion law:

$$q^2_{\parallel} - q^2_{\perp} = \Pi(q^2_{\perp}, q^2_{\parallel}). \quad (7)$$

Thus the eigenvalue of the photon polarization operator and its derivative can be expressed through the value $q^2_{\perp}$. 

2
4. The photon polarization operator with taking account of a positronium

It is necessary to calculate the contribution of the free and bounded electron-positron pair into the photon polarization operator, in order to calculate the probability of the radiative decay of a neutrino with a magnetic moment, with taking account of the positronium influence on the photon dispersion. It is convenient to use the relation between the photon decay probability into some final state of \( X \), and the imaginary part of the polarization operator [7]:

\[
Im(\Pi) = -\omega_{\gamma\rightarrow X}.
\] (8)

Under the state of \( X \), we mean here the free and bounded electron-positron pair. The photon polarization operator with taking account of the positronium can be written as:

\[
\Pi = -2\alpha(\varepsilon) e^{-\frac{q^2}{2m}} \left[ \frac{1}{\pi} H(z) + \frac{\lambda z}{1 - \lambda^2 - z} \right],
\] (9)

where \( H(z) = \frac{1}{\pi} \int_0^1 dx / [1 - z(1 - x^2) - i0] - 1 \), \( z = q^2/(4m_e^2) \), \( \lambda = \alpha/(2\nu) \), \( \alpha \) is a fine structure constant, the parameter \( \nu \) determines the energy levels of the positronium in a strong magnetic field: \( \varepsilon = \alpha^2 m_e/(4\nu^2) \). The expression for the probability of the radiative decay of the neutrino with a magnetic moment can be written as:

\[
\frac{W}{W_0} = \xi^3 \int_0^1 dx x^2 e^{-2\xi x^2} \left( \frac{1}{\pi} H(z) + \frac{\lambda z}{1 - \lambda^2 - z} \right) \left( \sqrt{1 - x^2} - x \arccos x \right).
\] (10)

The variables \( x \) and \( z \) are linked by the dispersion equation:

\[
z - \xi \beta x^2 = -\frac{\alpha \beta}{2} e^{-2\xi x^2} \left[ \frac{1}{\pi} H(z) + \frac{\lambda z}{1 - \lambda^2 - z} \right],
\] (11)

where \( \xi = E^2/(eB) \), \( \beta = B/B_e \), \( B_e = 4.41 \times 10^{13} G \), \( W_0 = 2\alpha\beta^2 \mu^2 m_e^3/\pi^2 \).

Figure 1. The probability of the radiative decay of a neutrino.

In figure 1, the plot is shown of the probability of the radiative decay of a neutrino with a magnetic moment as a function of the field parameter. The lower curve describes the decay probability without taking account of the positronium influence on the photon dispersion, while in the upper curve the positronium influence is taken into account.
5. Conclusions
In this paper, we have studied the process of the radiative decay of a neutrino with a magnetic moment in a strong magnetic field with taking account of the positronium influence on the photon dispersion. The result of a numerical calculation of the decay probability is presented in Fig. 1 which shows that taking account of the positronium influence on the photon dispersion leads to a significant change in the behaviour of the dependence $W(\xi)$ and its essential increase.

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References
[1] Galtsov D V and Nikitina N S 1972 Sov. Phys. JETP 35 1047 (Original Russian text: 1971 ZhETF 62 2008)
[2] Skobelev V V 1976 Sov. Phys. JETP 44 660 (Original Russian text: ZhETF 71 1263)
[3] Ioannisian A N and Raffelt G G 1997 Phys. Rev. D 55 7038
[4] Gvozdev A A, Mikheev N V and Vasilevskaya L A 1977 Phys. Lett. B 410 211
[5] Anikin R A and Mikheev N V 2013 Phys. At. Nucl. 76 1541 (Original Russian text: Yad. Fiz. 76 1610)
[6] Batalin I A and Shabad A E 1971 JETP 33 483 (Original Russian text: ZhETF 60 894)
[7] Weldon H A 1983 Phys. Rev. D 28 2007