An alternative design approach for Fractional Order Internal Model Controllers for time delay systems

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Introduction: Fractional Order Internal Model Control (FO-IMC) extends the capabilities of the classical IMC approach into the generalized domain of fractional calculus. When dealing with processes that exhibit time delays, implementation of such controllers in a classical feedback loop requires the approximation of the fractional order terms, as well as of the corresponding time delays.

Objectives: The present study proposes an alternative design procedure of FO-IMC controllers based on a novel approximation method of the process time delay, proving the efficiency of the proposed method and its suitability for time delay systems.

Methods: The generalized IMC control laws are obtained analytically, based on a novel approximation method of the process time delay, proving the efficiency of the proposed method and its suitability for time delay systems.

Results: Several numerical examples are chosen to illustrate the efficiency of the proposed approach. In addition, a vertical take-off and landing unit exhibiting second order plus time delay dynamics is chosen to experimentally validate the proposed control strategy. The obtained results are used to compare the...
The IMC scheme is proposed in [7]. The analytical tuning rules are the parameters of a fractional order PI controller based on the fractional order IMC controller are determined such that a desired order model of the process is presented also in [8,9] and [10]. The process' model are used to assess the robustness of the closed loop systems. Several tests considering disturbance uncertainties in the developed to enhance the performance in disturbance rejection time-domain identification of One Noninteger Order Plus Time Delay with one pole model from step response measurements. The parameters of the resulting controller are obtained using the IMC approach, by inverting the process model. Thus, in the case of a fractional order model, a corresponding FO-IMC controller is obtained. The fractional order model is obtained using an optimization technique for reduced order modeling. A model reduction technique for approximating a complex fractional order system is also used in [4]. The parameters of the fractional order PI and PID controllers are computed analytically based on previously obtained reduced models. The numerical results presented prove the efficiency of the proposed tuning algorithm. In [5], the authors propose a time-domain identification of One Noninteger Order Plus Time Delay with one pole model from step response measurements. The resulting model can later be used as the internal model for designing the FO-IMC controller. In [6], the parameters of the fractional order IMC controller are determined such that a desired bandwidth specification is met. A unified method to determine the parameters of a fractional order PI controller based on the IMC scheme is proposed in [7]. The analytical tuning rules are developed to enhance the performance in disturbance rejection and setpoint tracking scenarios. The authors show that the method works well for integer, as well as fractional order, time delay systems. Several tests considering disturbance uncertainties in the process' model are used to assess the robustness of the closed loop system. A FO-IMC controller obtained by inverting the fractional order model of the process is presented also in [8,9] and [10].

FO-IMC controllers are also defined based on a fractional order filter, rather than being the result of inverting a fractional order model. The use of a fractional order filter with an integer order PID controller, in an IMC structure is presented in [11]. Ziegler-Nichols rules are used to tune the parameters in [12] or using Taylor series as in [13]. Both a fractional order filter and a fractional order model are used to determine a FO-IMC controller in [14]. In [2] the tuning procedure is based on inverting the process model and adding a fractional order filter. The design is presented for single-input–single-output systems, as well as multivariable ones. A tuning approach based on the equivalency between the IMC and the Smith Predictor structure is also included. The tuning rules are presented for both first and second order processes with time delay. The tuning procedure consists in imposing gain crossover and phase margin constraints leading to a certain settling time and overshoot of the closed loop system. Several simulation results are included to show that the FO-IMC controllers provide for an improved closed loop performance, as well as robustness, when compared to the classical integer order IMC. A multivariable system is also the case study in [15]. First the multivariable system is decoupled and approximated into First Order Plus Dead Time (FOPDT) model. The tuning of the FO-IMCcontrollers is based on inverting these FOPDT models and adding a suitable fractional order filter. Controller parameters are determined mathematically using Novel Bat Optimization Algorithm. The simulation results show the effectiveness of the proposed controller. In [16], a FO-IMC controller is designed based on a two degrees of freedom paradigm for a multivariable system with time delays. With a suitable pairing of the input–output signals, a setpoint controller is first calculated, followed by the design and tuning of a controller that reduces the effect of the interactions between the loops. The design of this latter controller is based on defining a suitable complementary sensitivity function. Two illustrative examples are presented to show the merits of the proposed method, as well as the robustness to process variations. A FO-IMC controller for a multivariable non-square system is proposed in [17], where a Smith predictor control structure is used. The system is firstly decoupled. Particle Swarm Optimization algorithm is then used to reduce the high order model to a first order plus time delay model. Finally, a fractional order controller is designed according to the IMC methodology. Simulations validate the proposed methodology in terms of closed loop performance and robustness, also focusing on the method's simplicity.

A fractional order filter is also used in [18], where a FO-IMC controller is designed for a non-minimum phase system to satisfy desired phase at a desired gain-crossover frequency. Several simulation studies are performed and comparisons with other well-known techniques for tuning IMC based controllers are included. In [19], the concept behind CRONE control is used to tune the two parameters of a fractional order low pass filter by imposing gain crossover frequency and phase margin. Comparisons with existing control techniques are presented to demonstrate the efficacy of the FO-IMC controller. Simulation results show that using the FO-IMC control strategy leads to a better transient response as well as improved disturbance rejection performance. Ultimately, parametric uncertainties are handled by an excellent robustness of the closed loop system. A tuning procedure for a FO-IMC, designed based on a fractional order filter, is presented in [20], solely for the case of second order plus dead time processes. Simulations on several lag/delay dominant processes are presented. The robustness is verified using variations in the process parameters and analyzed according to sensitivity functions. The closed loop results show that the proposed controller leads to an enhanced control performance, as well as robustness. An additional study is included where the effect of measurement noise is tested for setpoint tracking and load disturbance variations. A few papers regarding FO-IMC control deal also with experimental results, either for single-input–single-output processes [3,11], or for multivariable systems [2].
One of the core problems related to fractional order controllers, including FO-IMC types, is their discrete-time implementation. To produce a digital implementation of the equivalent controller from an FO-IMC closed loop scheme, not only the fractional order terms of the controller have to be approximated, but also the process time delays. These approximations affect the final closed loop performance of the overall system. Quite frequently, time delays are approximated using the series \([2,11,12,18]\) or Padé method \([11,15,20]\). Consequently, the tuning equations are modified according to the approximation method used. Apart from the time delay approximation, the fractional order terms in the equivalent controller of an FO-IMC control structure have to be approximated as well, using either a direct or indirect approach \([21]\). The indirect approach consists of two steps: first, a continuous time approximation of the fractional order system is determined, followed by the discretization of this approximated transfer function using some well-known discretization techniques. The Oustaloup Recursive Approximation \([22]\), the Carlson method \([23]\) and the Modified Oustaloup Filter \([24]\) are the most popular continuous-time approximation methods.

In direct discretization methods, the Laplace variable is replaced with a generating function that converts the continuous-time operator to the discrete-time operator. Existing direct discretization methods focus on the approximation of the fractional order differentiator/integrator \([21,25-28]\). Some methods are also available for approximating first/second order low-pass fractional order filters \([29-31]\). A new direct discrete time approximation method has been developed in \([32]\). The procedure, defined as the Non-Rational Transfer Function (NRTF) approximation method, is suitable for all types of fractional order systems. A detailed description of this approach is included in this manuscript.

In \([33]\), a comparison for several time delay processes controlled in a FO-IMC methodology is presented. Tuning details are not presented and it is assumed that regardless of the chosen approximation method, the same FO-IMC parameters are obtained. The equivalent fractional order controllers are tested with respect to the approximation accuracy. The Taylor series, Padé and the novel NRTF approximations are used to approximate the time delay. The fractional order part of the equivalent fractional order controller is approximated in all cases using the NRTF approach \([32]\). It is shown that the NRTF approximation leads to better closed loop results, compared to the other approximation methods. Only simulation results are presented, whereas the present study validates the NRTF strategy on a real process using experimental tests. The chosen process presents considerable non-linear dynamics as well as time delay, increasing the difficulty of the control task.

In the current paper, a similar approach to \([2]\) for the tuning of FO-IMC controllers for time delay systems is developed. The tuning is based on specifying a gain crossover frequency and a phase margin for the overall loop transfer function. The design in \([2]\) is altered to include the specifics of the NRTF approximation method. Comparative simulations show that the proposed FO-IMC based on the NRTF time delay approximation method outperforms the FO-IMC controller computed using the series approach as presented in \([2]\), in the case of delay dominant systems. For lag dominant systems, the proposed FO-IMC tuning approach using the NRTF approximation method provides similar closed loop performance to the FO-IMC tuning approach based on the series approximation. To validate the proposed tuning approach and closed loop performance, a vertical take-off and landing (VTOL) unit is used as a case study. Experimental results are included to demonstrate the efficiency of the NRTF approximation method used in the novel tuning approach for FO-IMC controllers.

The paper is structured into six sections. The next section reviews the tuning method in \([2]\), for both first and second order systems, which will be used for comparative purposes. The details of the NRTF approximation method are given in the following section. The novel tuning method using the NRTF approach is detailed next and describes the new tuning steps for the FO-IMC for both first and second order systems. A numerical results section is included next, followed by the experimental validation on a case study. The numerical results cover first order lag/delay dominant systems along with comparative simulations. The experimental validation is exemplified on a VTOL unit. Practical implementation details are also presented. The last section contains some concluding remarks.

The novelty of the paper lies in the development of new IMC strategies based on the NRTF approximation method. The novel procedure targets both fractional and integer order IMC controllers. The obtained control laws are validated through numerical simulations for both lag/delay dominant processes. Apart from showing once more the superiority of the fractional order approach, the present paper also demonstrates that in the case of delay dominant processes, the importance of an accurate approximation of the model time delay in computing the equivalent controller in an IMC methodology is important. Comparisons with a previously developed tuning method \([2]\) are presented to validate the results. The novel method for tuning FO-IMC controllers is also experimentally validated. The case study consists in a considerable non-linear and poorly damped process with time delay. The experimental results show that the proposed tuning method for FO-IMC controllers based on the NRTF approximation leads to excellent closed loop dynamics. Further research includes an in-depth analysis of the efficiency of the approach based on a mathematical proof, as well as the stability and robustness analysis \([34-36]\).

**Review of existing tuning method for FO-IMC controllers. The series approach for time delay approximation**

The proposed tuning procedure for FO-IMC controllers has been thoroughly detailed in \([2]\) for both first and second order processes with time delays. In what follows, the basic tuning equations are revised for both first and second order dead time processes. The implementation of the equivalent fractional order controller is based on a series approximation of the process time delay.

The following transfer functions describe the dynamics of a first order plus dead time (FOPDT) system, as well as that of a second order plus dead time (SOPDT):

\[
H_{nt1}(s) = \frac{k}{Ts + 1} e^{-ts} \quad \text{and} \quad H_{nt2}(s) = \frac{k}{as^2 + bs + c} e^{-ts} \quad (1)
\]

The closed loop is given in Fig. 1, where \(H_{FO-IMC}(s)\) stands for the FO-IMC controller, \(H_n(s)\) is the process, \(H_{nt}(s)\) is the process model defined as either the FOPDT in (1) or the SOPDT in (2), \(H_{c}(s)\) is the equivalent controller.

![Fig. 1. FO-IMC closed loop control scheme.](image-url)
If the first order Taylor series expansion of exponential functions is used in the approximation of the time delay, with $e^{-\tau s} = (1 - \tau s)$, the model of the process is computed as:

$$H_{M1}(s) = \frac{k_1}{s + \tau}$$

for the FOPDT case or $H_{M2}(s) = \frac{k_1}{s^3 + \tau s}$ for the SOPDT case. Then, the FO-IMC controller is given by:

$$\begin{align*}
H_{FO-IMC1}(s) &= \frac{Ts + 1}{k_1} \\ H_{FO-IMC2}(s) &= \frac{as^2 + bs + c}{k_1} \left(1 + \frac{1}{s^2 + \tau s}\right)
\end{align*}$$

(3)

(4)

where $s \in [0A \cdot 2]$ is the fractional order. The equivalent controller in Fig. 1 is computed as follows, considering the series approximation of the time delay, as mentioned above:

$$H_{C1}(s) = \frac{H_{FO-IMC1}(s)}{1 - H_{FO-IMC1}(s)H_{M1}(s)} = \frac{Ts + 1}{k_1(s^2 + \tau s)}$$

(5)

for the FOPDT case, and:

$$H_{C2}(s) = \frac{H_{FO-IMC2}(s)}{1 - H_{FO-IMC2}(s)H_{M2}(s)} = \frac{as^2 + bs + c}{k_1(s^2 + \tau s)}$$

(6)

for the SOPDT process.

The open loop transfer functions for the two cases are computed as $H_C(s) = H_{C1}(s)H_{M1}(s)$ for the FOPDT process, and $H_C(s) = H_{C2}(s)H_{M2}(s)$ for the SOPDT process. The result is given by:

$$H_C(s) = H_{C1}(s) = H_{C2}(s) = \frac{1}{k_1s^2 + \tau s}$$

(7)

Notice from (7) that regardless of the process, the same open loop transfer function is obtained, which will be used for the design of the controller, as indicated next. When comparing the FO-IMC controller structure from (3) and (4) to the traditional IO-IMC controller, one can observe an additional tuning parameter consisting of the fractional order $\alpha$. The time constant represented by $\lambda$ is common for both types of controllers. With traditional IMC controllers, the only tuning parameter $\lambda$ is used to specify a desired settling time. However, in the case of the FO-IMC controller, two performance criteria can be addressed in the frequency domain, through a settling time and the closed loop overshoot. These two performance criteria can be addressed in the frequency domain, through a shaping knob $\alpha$. The system of equations that needs to be solved is composed by two complex transcendental equations, hence several nonlinear approaches can be engaged in order to determine the parameters. Optimization methods are usually employed when dealing with more complex fractional order controllers, such as the fractional order PID, with more tuning parameters. In this case, the fmincon() MATLAB function, based on algorithms such as interior-point, active set, etc., is widely used in solving the nonlinear equations [1,34]. Particle Swarm Optimization is also a viable option for solving the nonlinear equations required for the tuning [37]. The approach preferred in this manuscript is based on a graphical representation of the FO-IMC time constant as a function of the fractional order $\alpha$, i.e., $\lambda = f(\alpha)$. To plot the resulting functions, $\alpha$ is taken in small 0.001 increments and the corresponding $\lambda$ values are computed according to (10) and (12), respectively. The final solution is determined at the intersection point of the two functions.

In a classical feedback loop, the equivalent controllers in (5) and (6) can be implemented as integer order PI/PID controllers with fractional order filters of the form

$$H_{F0}(s) = \frac{1}{s^{2\alpha} + \tau s}$$

(13)

### The NRTF approximation method

The NRTF method has been proposed as a solution to approximate and implement fractional order systems [32]. However, the method is suitable for a wide variety of processes, including any non-rational transfer function as well as fractional order elements and time delays [33]. The NRTF method consists in four steps, as described below.

**Step 1:** Compute a discrete time approximation of the fractional order system, by replacing the Laplace operator $s$ with the following new generating function:

$$w(z^{-1}) = \frac{1 + \delta z^{-1} - z^{-1}}{1 + \delta z^{-1}}$$

(14)

with the shaping knob $\delta \in [0 \cdot 1]$, as shown in [32], and $T_i$ being the sampling period. Larger values of $\delta$ decrease the phase error, whereas lower values decrease magnitude error. This first step leads to a discrete time fractional order system, $G(z^{-1})$. The NRTF approximation of the FO-IMC controller is realized in a bounded interval defined by $\omega = (0, \omega_0)$. The maximum frequency denoted by $\omega_0$ is computed with respect to the Nyquist sampling theorem, as $\omega_0 = \frac{\pi}{T_i}$.

**Step 2:** Calculate the frequency response of the discrete time fractional order system obtained at Step 1. This is achieved by simply replacing the operator $z$ with $e^{i\omega t}$, where $\omega = \frac{2\pi}{T_i} [0 \ 1 \ 2 \ \cdots \ \frac{N_s}{2}]$. The parameter $N_s$ is also a tuning knob. An increased value of $N_s$ leads to better approximations of the discrete time frequency response for lower frequencies. The result of this step is a unidimensional array of frequency values corresponding to the discrete time fractional order transfer function.

**Step 3:** Use the inverse Fast Fourier Transform (FFT) algorithm to compute the impulse response of the discrete time system. Similarly to Step 2, this leads to a vector of $N_s$ values representing the impulse response of the transfer function.
g[n] = \sum_{k=0}^{N_i-1} G[k] e^{j2\pi kn/N_i}, n = 0, 1, 2, \ldots, N_i - 1 \tag{15}

Step 4: The data obtained at Step 3 is used to determine a rational discrete time model that has a similar impulse response as the inverse FFT. This can be realized using the Steiglitz-McBride algorithm, as shown in [38]. The result of the final step is the approximation of the original fractional order transfer function:

\[ G(z^{-1}) = \frac{C_0 + C_1 z^{-1} + \cdots + C_N z^{-N}}{D_0 + D_1 z^{-1} + \cdots + D_N z^{-N}}. \tag{16} \]

where \( N \) is the order of the approximation.

### Novel tuning method for FO-IMC controllers. The NRTF approximation approach

As it has been demonstrated for both IO-IMC and FO-IMC controllers, designed for time-delay systems, the NRTF method allows for a more accurate approximation of the process time delay, thus ensuring better closed loop performance [33,39]. However, so far, the NRTF method has been used solely in the approximation step, without considering the effect it might have upon the tuning of the IMC parameters. In this section, a novel tuning of the FO-IMC controller for FOPDT/SOPDT processes based on the NRTF approximation method is introduced.

For the FOPDT and SOPDT processes defined in (1) and (2), respectively, the proposed fractional order IMC (FO-IMC) controllers are given by (3) and (4). The equivalent controller structure from Fig. 1 can be written as:

\[ H_{c1}(s) = \frac{H_{IO-IMC1}(s)}{1 - H_{IO-IMC1}(s)H_{m1}(s)} = \frac{T_s + 1}{k(s^2 - e^{-Ts})} \tag{17} \]

for the FOPDT case, with the model defined as \( H_{m1}(s) = \frac{k}{\tau_m} e^{-\frac{s}{\tau_m}} \) and:

\[ H_{c2}(s) = \frac{H_{IO-IMC2}(s)}{1 - H_{IO-IMC2}(s)H_{m2}(s)} = \frac{a s^2 + b s + c}{k(s^2 - e^{-Ts})} \tag{18} \]

for the SOPDT process, with the model defined as \( H_{m2}(s) = -\frac{1}{s + \tau_m} e^{-\frac{s}{\tau_m}} \) for the SOPDT case. The resulting loop transfer functions are given as:

\[ H_{L1}(s) = H_{u1}(s) = H_{u2}(s) = \frac{1}{s^2 + \tau_s^2} e^{-\frac{s}{\tau_s}} \tag{19} \]

Eq. (19) clearly shows that the tuning procedures of fractional order IMC controller for FOPDT and SOPDT processes will be similar, since the open loop transfer function is the same. Similarly to the case of the FO-IMC controller tuned using the series approximation of the time delay, in this case also, two performance criteria can be addressed through the two tuning parameters. To meet the gain crossover frequency and phase margin requirements, the phase and modulus equations in (8) and (11) are also used here to determine the parameters of the FO-IMC controller. The following result is obtained using the phase condition in (8) applied to the loop transfer function in (19):

\[ \tan^{-1} \left( \frac{\lambda \omega_c^2 \sin \left( \frac{\lambda \omega_c}{2} \right) + \sin \left( \tau \omega_c \right)}{\lambda \omega_c \cos \left( \frac{\lambda \omega_c}{2} \right) + 1 - \cos \left( \tau \omega_c \right)} \right) = \pi - \frac{PM}{\tau \omega_c} \tag{20} \]

leading to:

\[ \lambda = \frac{\tan \left( \pi - \frac{PM}{\tau \omega_c} - \tan \left( \pi - \frac{PM}{\tau \omega_c} \right) \cos \left( \tau \omega_c \right) - \sin \left( \tau \omega_c \right) \right)}{\omega_c^2 \sin \left( \frac{\lambda \omega_c}{2} \right) - \omega_c^2 \tan \left( \pi - \frac{PM}{\tau \omega_c} \right) \cos \left( \frac{\lambda \omega_c}{2} \right)} \tag{21} \]

Applying the modulus equation in (11) to the loop transfer function from (19) yields the following result:

\[ \lambda^2 \omega_c^2 + 2 \lambda \omega_c \left( \cos \left( \frac{\lambda \omega_c}{2} \right) - \cos \left( \frac{\lambda \omega_c}{2} + \tau \omega_c \right) \right) - 2 \cos \left( \tau \omega_c \right) + 1 = 0. \tag{22} \]

The tuning of the FO-IMC controller consists of imposing a set of frequency domain specification targeting the gain crossover frequency and phase margin \((\omega_{gc}, PM)\), leading to a system of nonlinear Eqs. (21) and (22). Solving the system leads to the parameters describing the FO-IMC controller, \((\lambda, \alpha)\), with \( \lambda > 0 \). Comparing the tuning equations in (21) and (22), with those obtained using the series approximation (10) and (12), it is obvious that for the same set of performance specifications, different parameters will be obtained. The numerical and the experimental validation examples included in this manuscript are intended to demonstrate this aspect and conclude upon the advantages of using the NRTF in the tuning step, rather than just in the approximation/implementation step.

A simplified step-by-step algorithm for the tuning of fractional order IMC controllers is given next.

**Step 1**: For a process described as (1) or (2), select the performance specifications \((\omega_{gc}, PM)\) for the loop transfer function.

**Step 2**: For \( \alpha = 0 : 0.001 : 2 \),

Compute \( \lambda \) according to (21)

Compute \( \lambda_{12} \) according to (22)

Fig. 1: plot(\( \alpha, \lambda \)) and plot (\( \lambda, \lambda_{12} \))

Fig. 2. Comparative result for FOPDT lag dominant system with FO controllers a) output signal b) input signal.
Fig. 2: plot(\(x, \lambda\)) and plot(\(x, \lambda^2\)) Check intersection point in either Fig. 1 or Fig. 2 to determine final solution (\(x^*, \lambda^*\))

Step 3: For \(x^*, \lambda^*\) determine equivalent controller as in (17)/(28) for processes as (1)/(2)

Step 4: Choose \(\omega_n\), \(N\), and \(\delta\) according to [32] and determine the discrete-time approximation of (17)/(18)

Step 5: Implement the discrete-time approximation of the controller on a dedicated device and test its functionality

Numerical results

In this section, numerical results are presented for two cases: a lag dominant FOPDT process and a delay dominant FOPDT process. Comparisons between the FO-IMC and the Integer Order-IMC (IO-IMC) controllers are presented, for the cases when the equivalent controllers are computed based on the NRTF or the series approach.

The IO-IMC is a particular case of the FO-IMC that is obtained by replacing the parameter \(\alpha = 1\) in the controller tuning Eq. (21). The tuning procedure of the IO-IMC controller consists in tuning a single parameter, \(\lambda\), such that the modulus condition is met, referring to certain gain crossover frequency (settling time). A certain gain crossover frequency \(\omega_c\) is specified and the filter time constant, with \(\lambda > 0\), is determined.

Real world control system applications are characterized by unpredictable and continuously changing conditions. From this perspective, disturbance rejection is a key factor in robust control performance. Any improvement in disturbance rejection performance is considered as a benefit for the overall control system [40]. At the same time, load disturbance rejection is the main issue with IMC controllers and for this reason, disturbance rejection and robustness issues will be tackled in this section.

The lag dominant FOPDT process

Consider a FOPDT process described by the following transfer function:

\[
H_m(s) = \frac{1}{4s + 1} e^{-s} \tag{23}
\]

Imposing a phase margin \(PM = 85^\circ\) and a gain crossover frequency \(\omega_c = 0.3\) rad/s allows the design of the FO-IMC controller with regard to the new tuning method. These values indirectly ensure a certain overshoot and settling time for the closed loop system. Selection of the performance criteria values starts from approximate relations between time and frequency domains specifications [41], with a constraint regarding the controller parameter existence conditions [42]. Then, based on trial and error the optimal values are selected in order to ensure a low overshoot and fast settling time. The solution of (21) and (22) gives the FO-IMC parameters as \(\lambda = 1.95\) and \(\alpha = 0.85\), leading to the FO-IMC controller described by:

\[
H_{\text{FO-IMC-NRTF}}(s) = \frac{4s + 1}{1.95s + 1} \tag{24}
\]

while the equivalent controller is computed as:

\[
H_{\text{EO-NRTF}}(s) = \frac{4s + 1}{1.95s + 1 - e^{-s}} \tag{25}
\]

The NRTF approximation method is used to implement the controller from (25) with the following design parameters: \(N = 7\), \(\delta = 1\) and \(\omega_n = 10\pi\) (\(T_s = 0.1\) s). To tune the IO-IMC controller, the same gain crossover frequency \(\omega_c = 0.3\) rad/s is imposed. Solving (21) for \(\alpha = 1\), leads to a filter time constant \(\lambda = 2.35\) and an IO-IMC controller of the form:

\[
H_{\text{IO-IMC-NRTF}}(s) = \frac{4s + 1}{1} \frac{1}{2.35s + 1} \tag{26}
\]

In this case, the equivalent controller for a classical feedback loop is computed as:

\[
H_{\text{EO-NRTF}}(s) = \frac{4s + 1}{2.35s + 1 - e^{-s}} \tag{27}
\]

To implement the controller in (27), the NRTF approximation method is used with the same design parameters as in the case of the FO-IMC controller: \(N = 7\), \(\delta = 1\) and \(\omega_n = 10\pi\) (\(T_s = 0.1\) s).

For comparison purposes, an FO-IMC controller is designed according to the tuning rules in (10) and (12), based on the same performance specifications regarding the phase margin (PM = 85°) and gain crossover frequency (\(\omega_c = 0.3\) rad/s), but considering a series approximation of the time delay. The solution of (10) and (12) yields \(\lambda = 1.87\) and \(\alpha = 0.81\), with the FO-IMC controller given as:

\[
H_{\text{FO-IMC-SERIES}}(s) = \frac{4s + 1}{1} \frac{1}{1.875s + 1} \tag{28}
\]

while the equivalent controller is computed as:

\[
H_{\text{EO-SERIES}}(s) = \frac{4s + 1}{1.875s + 1 - e^{-s}} \tag{29}
\]

Fig. 3. Comparative result for FOPDT lag dominant system with IO controllers a) output signal b) input signal.
To implement the controller in (29), the Oustaloup Recursive Approximation method is used [22], as the most widely accepted continuous-time approximation of fractional order systems, with order $N = 7$ and the low and high frequency approximation bounds taken as $(0.05–50)\text{rad/s}$. Next, the continuous-time approximated integer order transfer function is discretized using Tustin method and the same sampling period as in the controllers above, $T_s = 0.1 \text{s}$.

To tune the IO-IMC controller, based on a series approximation of the delay, Eq. (10) is solved for $\alpha = 1$, by considering the same gain crossover frequency $\omega_c = 0.3 \text{ rad/s}$. The resulting filter time constant is $\lambda = 2.33$ and the IO-IMC controller has the form:

$$H_{IO-IMC-SERIES}(s) = \frac{4s + 1}{3.33s + 1}$$  \hspace{1cm} (30)$$

In this case, the equivalent controller for a classical feedback loop is computed as:

$$H_{C-IO\text{-SERIES}}(s) = \frac{4s + 1}{3.33s}$$  \hspace{1cm} (31)$$

To implement the equivalent controller in (31), the transfer function is discretized using Tustin method and the same sampling period as in the controllers above, $T_s = 0.1 \text{s}$.

Fig. 2 presents the comparative results for this situation. The equivalent FO controllers in (25) and (29) are compared. The results in Fig. 2 show that the NRTF approximation method is a viable alternative, since the closed loop responses are slightly better compared to the series approach.

Fig. 3 presents the comparative results for the equivalent integer order controllers in (27) and (31). Again, similarly to the results in Fig. 2, the NRTF approximation method is a viable alternative, since the closed loop responses are slightly better compared to the series approach.
The corresponding Bode diagrams for the loop transfer functions with either FO or IO controllers are given in Fig. 4. Notice that all performance specifications are met. Also notice that in the case of IO controllers, the phase margin is lower compared to the FO controllers. Additionally, the resulting gain margin is larger with controllers computed based on the NRTF approximation, compared to those obtained based on a series approach.

The delay dominant FOPDT process

Consider a FOPDT process described by the following transfer function:

\[ H_m(s) = \frac{2}{s + 1} e^{-2s} \]  

(32)

A phase margin PM = 80° and a gain crossover frequency \( \omega_c \) = 0.3 rad/s are imposed in order to design the FO-IMC controller based on the new tuning method. Similarly to the previous case study, these values are chosen based on a trial and error procedure considering the equivalence between time and frequency domains specifications [41], as well as constraints regarding the controller parameter existence conditions [42]. The final values for the performance specifications are selected in order to ensure a fast settling time and low overshoot. The solution of (21) and (22) yields \( \lambda = 0.88 \) and \( \alpha = 0.62 \), with the FO-IMC controller given as:

\[ H_{\text{FO-IMC-NRTF}}(s) = \frac{s + 1}{2} \frac{1}{0.88^{0.62} + 1} \]  

(33)

while the equivalent controller is computed as:

\[ H_{\text{FO-IMC-NRTF}}(s) = \frac{s + 1}{2(0.88^{0.62} + 1 - e^{-2s})} \]  

(34)

The implementation of the the controller in (34) is done based on the NRTF approximation method with the following design parameters: \( N = 5, \delta = 0.5 \) and \( \omega_b = 10\pi \) (\( T_s = 0.1 \) s).

The same gain crossover frequency \( \omega_c \) = 0.3 rad/s is imposed for the tuning of the IO-IMC controller. Solving (21) for \( \alpha = 1 \), leads to a filter time constant \( \lambda = 1.40 \) and the following transfer function of the controller:

\[ H_{\text{IO-IMC-NRTF}}(s) = \frac{s + 1}{2} \frac{1}{1.40s + 1} \]  

(35)

The equivalent controller for a classical feedback loop is computed as:

\[ H_{\text{IO-IMC-NRTF}}(s) = \frac{s + 1}{2(1.40s + 1 - e^{-2s})} \]  

(36)

The NRTF approximation method is used to implement the controller in (36). The same same design parameters are used as in the case of the FO-IMC controller: \( N = 5, \delta = 0.5 \) and \( \omega_b = 10\pi \) (\( T_s = 0.1 \) s).

A FO-IMC controller, as well as an IO-IMC one, are designed for comparison purposes based on the series approach. The FO-IMC controller is designed based on (10) and (12), using the same performance specifications regarding the phase margin (PM = 80°) and gain crossover frequency (\( \omega_c \) = 0.3 rad/s). The solution of (10) and...
(12) yields $\lambda = 0.85$ and $\alpha = 0.41$. The resulting FO-IMC controller given as:

$$H_{FO-IMC-SERIES}(s) = \frac{s + 1}{2 \cdot 0.85^s + 1}$$

while the equivalent controller is computed as:

$$H_{c-FO-SERIES}(s) = \frac{s + 1}{2(0.85^s + 1)}$$

The Oustaloup Recursive Approximation method [22] is used to implement the controller in (38), with order $N = 5$ and the low and high frequency approximation bounds taken as $(0.05–50)$ rad/s. Next, the continuous-time approximated integer order transfer function is discretized using Tustin method and the same sampling period as in the controllers above, $T_s = 0.1$ s. This indirect approach of obtaining the discrete time approximation of a fractional order controller can lead to numerical errors. However, an adequate choice of the approximation parameters and of the sampling period can significantly reduce the possibility of such errors to occur.

The next numerical example is used to validate the results also when a direct approximation method is used.

To tune the IO-IMC controller, based on a series approximation of the delay, Eq. (10) is solved for $\alpha = 1$, by considering the same gain crossover frequency $\omega_c = 0.3$ rad/s. This results in a filter time constant $\lambda = 1.33$. The IO-IMC controller has the form:

$$H_{IO-IMC-SERIES}(s) = \frac{s + 1}{2 \cdot 1.33s + 1}$$

while the equivalent controller is:

$$H_{c-IO-SERIES}(s) = \frac{s + 1}{6.67s}$$

Fig. 7. Comparative Bode diagrams for FOPDT delay dominant system with a) IO controllers and b) FO controllers.
To implement the equivalent controller in (40), the transfer function is discretized using Tustin method and the same sampling period as in the controllers above, $T_s = 0.1 \text{s}$.

Fig. 5 presents the comparative results for the load disturbance situation, considering the two equivalent FO controllers in (34) and (38). The results show that the NRTF approximation method is a viable alternative, since the closed loop responses are better compared to the series approach. Fig. 6 presents the comparative results for the equivalent integer order controllers in (36) and (40). Again, similarly to the results in Fig. 5, the NRTF approximation method is a viable alternative, since the closed loop responses are better compared to the series approach. The corresponding Bode diagrams for the loop transfer functions with either FO or IO controllers are given in Fig. 7. Notice that all performance specifications are met. Additionally, the resulting gain margin is larger with controllers computed based on the NRTF approximation, compared to those obtained based on a series approach.

As a general remark, notice that the NRTF can be used as an alternative way of approximating the model time delays when implementing either FO or IO controllers in an IMC methodology. As the final form of the equivalent controller is greatly influenced by the time delay, it is obvious that for delay dominant systems, a more accurate approximation should be considered. As the results presented here for a delay dominant system indicate, the NRTF approximation ensures better closed loop dynamics of the overall system, compared to the series approach. The settling times are faster and the system does not exhibit oscillatory behavior. This is valid both for the FO and IO controllers. Additionally, comparing the closed loop dynamics of FO and IO controllers, it is obvious that the FO controllers ensure a better performance. This is consistent with previous findings related solely to the series approach [2]. The Bode diagrams also show that the FO controllers have a greater potential in achieving an increased degree of robustness, with larger phase and gain margins, compared to the IO controllers.

### Robustness evaluation of the proposed approach

In this section, a third numerical example is considered, in the form of a delay dominant FOPDT process:  

$$H_m(s) = \frac{2}{0.1s + 1} e^{-0.4s}$$  \hspace{1cm} (41)

A phase margin PM = 80° and a gain crossover frequency $\omega_c = 1.4 \text{rad/s}$ are imposed in order to design the FO-IMC controller based on the new tuning method. The solution of (21) and (22) yields $\lambda = 0.36$ and $\alpha = 0.67$, with the FO-IMC controller given as:  

$$H_{FO-IMC-NRTF}(s) = \frac{0.1s + 1}{2 - \frac{1}{0.36e^{0.67} + 1}}$$  \hspace{1cm} (42)$$

while the equivalent controller is computed as:

$$H_{c_{FO-NRTF}}(s) = \frac{0.1s + 1}{2(0.36e^{0.67} + 1 - e^{-0.4s})}$$  \hspace{1cm} (43)$$
A FO-IMC controller is now designed for comparison purposes based on the series approach. The FO-IMC controller is designed based on (10) and (12), using the same performance specifications regarding the phase margin (PM = 80°) and gain crossover frequency (ωc = 1.4 rad/s). The solution of (10) and (12) yields λ = 0.45 and α = 0.49. The resulting FO-IMC controller given as:

$$H_{FO-IMC}\text{-SERIES}(s) = \frac{0.1s + 1}{20.45s^{0.49} + 1}$$

while the equivalent controller is computed as:

$$H_{eq\text{-SERIES}}(s) = \frac{0.1s + 1}{2(0.45s^{0.49} + s)}$$

The implementation of the controllers in (43) and (45) is done based on the NRTF approximation method with the following design parameters: N = 4, δ = 1 and ωn = 100π (Ts = 0.01 s). For this particular numerical example, the same direct discretization method is used for the series approach, as for the NRTF approach. This is chosen in order to demonstrate that the results for the series approach are independent upon the used discretization method.

Comparative results for the load disturbance situation, considering the two equivalent FO controllers in (43) and (45), are indicated in Fig. 8. In this case also, the simulation results clearly show that the new FO-IMC tuning approach leads to better closed loop performance, with significant reduction in the setting time (see Fig. 9).

To check the robustness of the method, significant delay variations were considered. Such variations have a serious effect upon the stability of the closed loop system. The results obtained with the two controllers are indicated in Fig. 10 and show that the FO-IMC controller tuned according to the proposed method leads to improved robustness, compared to the standard FO-IMC controller.

**Experimental validation on a VTOL unit**

The VTOL experimental platform used as a case study in this research is depicted in Fig. 10. Data acquisition for system identification, as well as controller implementation is done using the NI Elvis board from National Instruments. The VTOL is an add-on board dedicated to the NI Elvis platform, that provides all the necessary electronics for data acquisition and control. The NI Elvis platform is a real-time microcontroller that can be easily programmed using the graphical programming language LabVIEW. The discrete-time controller is implemented directly on the NI Elvis board, the data being read with a constant sampling time and the control value is computed with respect to the implemented FO-IMC algorithm. The VTOL platform consists in a cantilever beam. A balancing weight is placed to the left end of the beam, while a fan enclosed by safety guards is mounted to the right. A DC motor is used to control the angular velocity of the fan. The beam is fixed at one point along the length of cantilever at 1/3 near the weight, which allows it to rotate in an interval [-26°, 60°]. The parallel position of the beam to the base of the platform is considered as having 0°.

The output that needs to be controlled is the angular displacement of the beam around the fixed point. This is done by varying the voltage applied to the DC motor. The angular displacement is measured by an encoder with respect to a horizontal axis in the fixed point.

Strong nonlinearities affect the process due to several reasons. The motion of the fan, creates a circular shape, while balancing weight also moves in a circular pattern. The base platform creates a nonlinear characteristic by influencing the air propelled by the fan. A sampling time of Ts = 0.005 s is used to acquire the experimental data for system identification [43].

The control task is related to the position of the cantilever beam. A reference value is given for the pitch, the angle between the platform and the moving part, and the platform is expected to smoothly navigate to the setpoint value. A reduced overshoot and settling time, as well as zero steady state error and robustness are desired in terms of closed loop performance.

The identified SOPDT model of the VTOL unit has been determined to be [43]:

$$H_{VTOL}(s) = \frac{22.24}{s^2 + 0.69s + 4.25} \exp(-0.8t)$$

For this process, the FO-IMC controller is designed using the proposed method, where the NRTF approximation is chosen for the time delay. To tune the controller, a phase margin PM = 75° and a gain crossover frequency ωc = 0.5 rad/s are requested [43]. Previous research has shown that for these performance specifications, an improved closed loop dynamics is obtained, in terms of minimum overshoot and settling time, combined with zero steady state error.

For the NRTF approximation, using (21) and (22), the FO-IMC parameters are obtained as: λ = 1.15 and α = 0.94. The FO-IMC controller is given by:

$$H_{FO-IMC\text{-NRTF}}(s) = \frac{s^2 + 0.69s + 4.25}{22.24(1.15s^{0.94} + 1)}$$

while the equivalent controller is computed as:

$$H_{eq\text{-NRTF}}(s) = \frac{s^2 + 0.69s + 4.25}{22.24(1.15s^{0.94} + 1 - \exp(-0.8t))}$$

The controller in (48) is implemented using the NRTF method, with N = 5, δ = 0.9 and ωn = 200π (Ts = 0.005 s). A filter to filter out noisy signals is added, with a filter time constant T = 0.1 s. The discrete time approximation of the equivalent fractional order controller follows the recurring relation for the control signal c(k), which is implemented on the VTOL unit:

$$c(k) = 5.79 + c(k - 1) - 13.99 + c(k - 2) + 18.04 + c(k - 3) - 13.1 + c(k - 4) + 5.1 + c(k - 5) - 0.84 + c(k - 6) + 0.01 + c(k - 7) + 0.53 + e(k) - 3.20 + e(k - 1) + 7.96 + e(k - 2) - 10.65 + e(k - 3) + 8.15 + e(k - 4) - 0.01 + c(k - 7) + 0.53 + e(k) - 3.20 + e(k - 1) + 7.96 + e(k - 2) - 10.65 + e(k - 3) + 8.15 + e(k - 4) - 3.45 + e(k - 5) + 0.69 + e(k - 6) - 0.04 + e(k - 7),$$

![Fig. 10. Vertical Take-Off and Landing unit used as a case study.](image-url)
where \( c(k) \) is the current control signal value, \( c(k-1), c(k-2), \ldots \) represent previous control signal values, \( e(k) \) is the current error signal and \( e(k-1), e(k-2), \ldots \) represent previous error signal values. The control law from (49) is implemented on the NI Elvis platform using LabVIEW. The recurrence formula is programmed using graphical tools.

The experimental results, considering staircase reference tracking and disturbance rejection, are given in Figs. 11 and 12, for the output and input signals, respectively. Notice in this case also the reduced overshoot and fast settling time. Notice also that the overall output dynamics is quite smooth with little oscillations. As a remark, as indicated in Figs. 11 and 12, the actual applied control signal is saturated [0–10]V. However, for the computation of the control signal applied to the VTOL unit, according to (49), the original values are used for the previous command values and not the saturated ones.

Conclusions

Time delays are frequently encountered in many industrial applications. The IMC approach is among the most popular control schemes for dealing with time delays. Its extension to fractional order can lead to improved closed loop dynamics, due to the supplementary tuning parameter involved. The resulting FO-IMC controller needs to be approximated to fit an equivalent controller in a classical feedback control loop. For this reason, the process time delay must be approximated, as well.

The proposed tuning procedure presented in this paper is based on a novel approximation (NRTF) of the whole non-rational transfer function. It is compared to the existing approach of series approximation of the time delay. The tuning, in both cases, is based on specifying a certain phase margin and gain crossover frequency for the loop transfer function. The numerical results show that similar closed loop performance can be obtained when considering the NRTF approximation in the tuning of the fractional order controllers of an IMC scheme, compared to the series approach. This is valid for lag dominant processes, with the process time constant significantly larger than the process time delay. Improved closed loop performance can be obtained using the proposed approach for delay dominant processes, where time delays are larger than the process time constants. In this particular case, the approximation of the time delays becomes important.

Previous results have demonstrated that the FO-IMC controller outperforms the IO-IMC controller, when using the series approximation of the time delay. The numerical results in this paper show that the same conclusion is valid when using the NRTF approach for approximating the process time delay. Further, experimental results are provided to validate experimentally the proposed tuning approach. Overall, it can be concluded that the proposed method is a viable alternative for designing FO-IMC controllers.

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