ABSOLUTE ANTICOMMUTATIVITY OF THE NILPOTENT SYMMETRIES IN THE HAMILTONIAN FORMALISM: FREE ABELIAN 2-FORM GAUGE THEORY

R. P. Malik\(^{(a,b)}\)\(^1\), B. P. Mandal\(^{(a)}\)\(^2\), S. K. Rai\(^{(a)}\)\(^3\)

\(^{(a)}\)Physics Department, Centre of Advanced Studies, Banaras Hindu University, Varanasi - 221 005, India

\(^{(b)}\)DST Centre for Interdisciplinary Mathematical Sciences, Faculty of Science, Banaras Hindu University, Varanasi - 221 005, India

Abstract: The celebrated Curci-Ferrari (CF) type of restrictions are invoked to obtain the off-shell nilpotent and absolutely anticommuting (anti-) BRST as well as (anti-) co-BRST symmetry transformations in the context of the Lagrangian description of the physical four \((3 + 1)\)-dimensional \((4D)\) free Abelian 2-form gauge theory. We show that the above CF type conditions, which turn out to be the secondary constraints of the theory, remain invariant with respect to the time-evolution of the above 2-form gauge system in the Hamiltonian formulation. This time-evolution invariance (i) physically ensures the linear independence of the BRST versus anti-BRST as well as co-BRST versus anti-co-BRST symmetry transformations, and (ii) provides a logical reason behind the imposition of the above CF type restrictions in the proof of the absolute anticommutativity of the off-shell nilpotent (anti-) BRST as well as (anti-) co-BRST symmetry transformations.

PACS numbers: 11.15.-q, 03.70.+k

Keywords: 4D free Abelian 2-form gauge theory, Hamiltonian formulation, anticommutativity, nilpotent (anti-) BRST and (anti-) co-BRST symmetries, CF type restrictions

\(^1\)e-mail: malik@bhu.ac.in, rudra.prakash@hotmail.com

\(^2\)e-mail: bhabani.mandal@gmail.com

\(^3\)e-mail: sumitssc@gmail.com
1 Introduction

The principle of local gauge invariance, in the context of the (non-) Abelian 1-form gauge theories, has played a key role in providing a successful theoretical description of the strong, weak and electromagnetic interactions of nature. The existence of the first-class constraints, in the language of Dirac's prescription for the classification scheme [1,2], is at the heart of the above (non-) Abelian 1-form \( A^{(1)} = dx^\mu A_{\mu} \) gauge theories which provide the cornerstones for the beautiful edifice of the standard model of theoretical high energy physics. It is now a common folklore in theoretical physics that any arbitrary \( p \)-form \((p = 1, 2, 3...)\) gauge theory should always be endowed with the first-class constraints. These constraints, in fact, generate precisely the local gauge symmetry transformations of any specific \( p \)-form gauge theory in any arbitrary \( D \)-dimension of spacetime [1,2].

In the recent past, the 4D free Abelian 2-form \( B^{(2)} = [(dx^\mu \wedge dx^\nu)/2!] B_{\mu\nu} \) gauge field \( B_{\mu\nu} \) [3,4] has become quite popular mainly due to its appearance in the supergravity multiplet [5] and excited states of the (super)string theories [6,7]. It has played, furthermore, a crucial role in providing a noncommutative structure in the context of string theory [8]. We have shown, moreover, in our earlier works [9-11], that this theory provides a tractable field theoretical model for the Hodge theory and a model for the quasi-topological field theory [12]. One of the most interesting observations, connected with the above theory, has come out from its discussion in the framework of superfield formulation proposed in [13,14]. This has led to the existence of a Curci-Ferrari (CF) type restriction\(^4\) [15] which happens to be the hallmark of a 4D non-Abelian 1-form gauge theory (see, e.g., [16]).

It is well-known that, for the absolute anticommutativity and existence of the off-shell nilpotent Becchi-Rouet-Stora-Tyutin (BRST) and anti-BRST symmetry transformations, one invokes the CF restriction [16] in the case of the description of the 4D non-Abelian 1-form gauge theory [17-20]. For the first time, however, it has been shown that the replication of this CF type restriction is required in the context of the 4D Abelian 2-form gauge theory [15] so that one could obtain (i) the absolute anticommutativity\(^5\) of the (anti-)BRST symmetry transformations, and (ii) an independent identity of the anti-BRST symmetry transformations (and corresponding anti-BRST charge) [21,22]. It has been possible to obtain a set of coupled Lagrangian densities that incorporates the above CF type restriction to demonstrate that the (anti-) BRST symmetry transformations (and their generators) have their own independent identity [21,22]. This CF type restriction has also been shown to have connection with the geometrical objects called gerbes [21].

The existence of the above CF type restriction has so far been shown in the framework of (i) the superfield formalism [15], and (ii) the Lagrangian formulations [21-23,9]. Physically, it has not been made clear as to why this type of restrictions should be imposed

\(^4\)The appearance of the CF type restriction in the context of the Abelian gauge theory is first of its kind. In fact, the superfield formulation of [13,14] has been applied, for the first time, to the Abelian 2-form gauge theory in [15]. Its application in the context of 1-form gauge theories is quite well-known.

\(^5\)The nilpotent (anti-) BRST symmetry transformations have been shown to be anticommuting only up to a vector gauge transformation in the context of Abelian 2-form gauge theory (see, e.g., [10]).
in the dynamical description of the Abelian 2-form gauge theory within the framework of BRST formalism. The purpose of our present endeavour is to answer the above query in the framework of the Hamiltonian formulation. We demonstrate that the above CF type restrictions are the secondary constraints which are derived by requiring the time-evolution invariance of the primary constraints of the theory. Furthermore, we show that the above CF type restrictions remain invariant with respect to the time-evolution of the Abelian 2-form gauge system (within the framework of the Hamiltonian formulation). This key result of our present investigation physically ensures the imposition of the CF type restrictions, for the absolute anticommutativity of the (anti-) BRST and (anti-) co-BRST symmetry transformations, at any arbitrary moment of the time-evolution.

In our earlier works (see, e.g. [9,23]), we have derived the CF type restrictions from the coupled, equivalent and (anti-) BRST as well as (anti-) co-BRST invariant Lagrangian densities in two steps\(^6\) by exploiting the Euler-Lagrange equations of motion. It would be economical as well as aesthetically beautiful to derive the same restrictions from a single Lagrangian density and corresponding Hamiltonian density. We accomplish this goal in our present paper where we derive the CF type restrictions in a single stroke and show their time-evolution invariance from a single Hamiltonian density. The latter property, in the context of the dynamical evolution of the Abelian 2-form system, has been established in a convincing manner. This analysis has been performed explicitly so that the anticommutativity of the (anti-) BRST and (anti-) co-BRST symmetries could be ensured at each moment of the time-evolution of our present 2-form gauge system.

Our present investigation has been motivated by the following factors. First and foremost, the time-evolution invariance of the CF type restrictions cannot be demonstrated within the framework of either superfield or Lagrangian formulation. Thus, it is essential for us to describe the Abelian 2-form gauge system within the framework of the Hamiltonian approach. Second, for aesthetic reasons, it is always desirable to obtain the CF type restrictions from a single Lagrangian density (and corresponding Hamiltonian density). We have accomplished this goal in our present endeavour. Finally, our present attempt is a modest step in the direction to provide the physical reasons behind the appearance of the CF type restrictions in the context of the higher \(p\)-form (\(p > 2\)) gauge theories within the framework of BRST formalism. Thus, our present study might have relevance in the description of the higher-form fields (associated with string and other extended objects).

The outline of our present paper is as follows. To set up the conventions and notations, we briefly mention in Sec. 2, the (anti-)BRST symmetries in the Lagrangian formulation. Our Sec. 3 is devoted to the discussion of the time-evolution invariance of the CF-type restriction that is invoked for the proof of anticommutativity of the off-shell nilpotent (anti-) BRST symmetries in the Hamiltonian formulation. For the paper to be self-contained, in Sec. 4, we provide a brief synopsis of the (anti-) co-BRST symmetries within the framework of Lagrangian formalism. Our Sec. 5 deals with the time-evolution invariance of the CF type restriction, in the framework of Hamiltonian formulation, that is

\(^6\)First of all the Euler-Lagrange equations of motion are derived from the coupled Lagrangian density. This is followed, then, by the subtraction and addition of the above equations of motion.
required in the proof of the absolute anticommutativity of the off-shell nilpotent (anti-)co-
BRST symmetry transformations. Finally, in Sec. 6, we make some concluding remarks
and point out a few new directions for future investigations.

2 Preliminaries: Off-shell Nilpotent (Anti-) BRST
Symmetries in Lagrangian Formulation

We begin with the following Lagrangian densities for the 4D free abelian 2-form gauge
theory 7 within the framework of the BRST formalism (see, e.g. [9])

\[
\mathcal{L}^{(1)} = \frac{1}{12} H^{\mu\nu\kappa} H_{\mu\nu\kappa} + B^\mu \left( \partial^\nu B_{\nu\mu} + \frac{1}{2} \partial_\mu \varphi_1 \right) - \frac{1}{2} B^\mu B_\mu + \partial_\mu \bar{\beta} \partial^\mu \beta
+ \left( \partial_\mu \bar{C}_\nu - \partial_\nu \bar{C}_\mu \right) \partial^\mu C^\nu + \left( \partial \cdot C - \lambda \right) \rho + \left( \partial \cdot \bar{C} + \rho \right) \lambda,
\]

(1)

\[
\mathcal{L}^{(2)} = \frac{1}{12} H^{\mu\nu\kappa} H_{\mu\nu\kappa} + B^\mu \left( \partial^\nu B_{\nu\mu} - \frac{1}{2} \partial_\mu \varphi_1 \right) - \frac{1}{2} B^\mu B_\mu + \partial_\mu \bar{\beta} \partial^\mu \beta
+ \left( \partial_\mu \bar{C}_\nu - \partial_\nu \bar{C}_\mu \right) \partial^\mu C^\nu + \left( \partial \cdot C - \lambda \right) \rho + \left( \partial \cdot \bar{C} + \rho \right) \lambda,
\]

(2)

where the totally antisymmetric curvature tensor \( H_{\mu\nu\kappa} = \partial_\mu B_{\nu\kappa} + \partial_\nu B_{\kappa\mu} + \partial_\kappa B_{\mu\nu} \) is
derived from the 3-form \( H^{(3)} = dB^{(2)} = [(dx^\mu \wedge dx^\nu \wedge dx^\kappa)/3!]H_{\mu\nu\kappa} \) constructed with the
help of the exterior derivative \( d = dx^\mu \partial_\mu \) (with \( d^2 = 0 \)) and the Abelian 2-form connection
\( B^{(2)} = [(dx^\mu \wedge dx^\nu)/2!] B_{\mu\nu} \) which defines the antisymmetric \( B_{\mu\nu} = -B_{\nu\mu} \) gauge potential
\( B_{\mu\nu} \) of the present Abelian 2-form gauge theory.

The BRST invariance in the theory requires the fermionic \( (C_\mu \bar{C}_\nu + \bar{C}_\mu C_\nu = 0, C_\mu^2 = \bar{C}_\mu^2 = 0, \bar{C}_\mu = 0, \mbox{etc.}) \) Lorentz vector (anti-) ghost \( (\bar{C}_\mu) \) \( C_\mu \) fields, fermionic \( (\rho^2 = \lambda^2 = 0, \rho \lambda + \lambda \rho = 0) \) auxiliary (anti-) ghost fields \( \rho \) and \( \lambda \) and bosonic \( (\beta^2 \neq 0, \bar{\beta}^2 \neq 0, \beta \bar{\beta} = \bar{\beta} \beta) \) (anti-) ghost fields \( (\beta) \). In the above, \( B_\mu \) and \( \bar{B}_\mu \) are the Nakanishi-Lautrup
type of auxiliary fields that are invoked for the linearization of the gauge fixing terms
\( \left[ \frac{1}{2} (\partial^\nu B_{\nu\mu} + \frac{1}{2} \partial_\mu \varphi_1)^2 \right] \) and \( \left[ \frac{1}{2} (\partial^\nu B_{\nu\mu} - \frac{1}{2} \partial_\mu \varphi_1)^2 \right] \) where \( \varphi_1 \) is the massless \( (\Box \varphi_1 = 0) \) scalar
field required for the stage-one reducibility in the theory. The gauge-fixing term \( (\partial^\nu B_{\nu\mu}) \)
obeys its origin to the co-exterior derivative \( \delta = - \ast \delta \ast \) because \( \delta B^{(2)} = (\partial^\nu B_{\nu\mu}) dx^\mu \) where
\( \ast \) is the Hodge duality operator on the 4D spacetime manifold.

The following off-shell nilpotent \( (s_b^2 = 0) \) BRST transformations \( (s_b) \)

\[
s_b B_{\mu\nu} = -(\partial_\mu C_\nu - \partial_\nu C_\mu), \quad s_b C_\mu = -\partial_\mu \beta, \quad s_b \bar{C}_\mu = -B_\mu, \quad s_b \varphi_1 = -2\lambda, \quad s_b \bar{\beta} = -\rho, \quad s_b \bar{B}_\mu = -\partial_\mu \lambda, \quad s_b [\rho, \lambda, \beta, B_\mu, H_{\mu\nu\kappa}] = 0,
\]

(3)

\footnote{We adopt here the conventions and notations such that the flat 4D Minkowski metric \( \eta_{\mu\nu} \) is with
signature \((+1, -1, -1, -1)\). The 4D totally antisymmetric Levi-Civita tensor is chosen to obey \( \varepsilon_{\mu\nu\rho\kappa} \varepsilon^{\mu\nu\rho\kappa} = -4!, \varepsilon_{\mu\nu\rho\kappa} \varepsilon^{\mu\nu\rho\kappa} = -3\delta_\kappa^\rho \), etc., and \( \varepsilon_{0123} = +1 = -\varepsilon^{0123} \). The 3D Levi-Civita tensor is defined as:
\( \varepsilon_{0ijk} = \varepsilon_{ijk} \). Here the Greek indices \( \mu, \nu, \eta, \kappa \ldots = 0, 1, 2, 3 \) correspond to the spacetime directions of
the 4D Minkowski spacetime manifold and Latin indices \( i, j, k \ldots = 1, 2, 3 \) stand for space directions only.}
and the off-shell nilpotent \((s^2_{ab} = 0)\) anti-BRST transformations \((s_{ab})\)

\[
s_{ab}B_{\mu\nu} = -(\partial_\mu \bar{C}_\nu - \partial_\nu \bar{C}_\mu), \quad s_{ab}\bar{C}_\mu = -\partial_\mu \bar{\beta}, \quad s_{ab}C_\mu = \bar{B}_\mu, \\
s_{ab}\varphi_1 = -2\rho, \quad s_{ab}\beta = -\lambda, \quad s_{ab}B_\mu = \partial_\mu \rho, \quad s_{ab}[\rho, \lambda, \bar{\beta}, \bar{B}_\mu, H_{\mu\nu\kappa}] = 0, \tag{4}
\]

are

(i) the symmetry transformations for the Lagrangian densities \((1)\) and \((2)\) \([9]\), and

(ii) absolutely anticommuting \((s_b s_{ab} + s_{ab} s_b = 0)\) in nature because their absolute anticommutativity property \((e.g. \{s_b, s_{ab}\} B_{\mu\nu} = 0)\) is ensured due to the following Curci-Ferrari (CF) type of restriction

\[
B_{\mu} - \bar{B}_{\mu} - \partial_\mu \varphi_1 = 0. \tag{5}
\]

The above condition emerges from \((1)\) and \((2)\) due to the equations of motion

\[
B_{\mu} = \partial^\nu B_{\nu\mu} + \frac{1}{2} \partial_\mu \varphi_1, \quad \bar{B}_{\mu} = \partial^\nu \bar{B}_{\nu\mu} - \frac{1}{2} \partial_\mu \varphi_1.
\]

The key points that ought to be noted, at this stage, are as follows. First, it can be seen that the CF type restriction \((5)\) is derived in two steps from the Lagrangian densities \((1)\) and \((2)\). Second, unlike in the context of the 4D non-Abelian 1-form gauge theory where the (anti-) ghosts fields also participate in the CF condition \([16]\), for the Abelian 2-form gauge theory only the bosonic fields contribute to its existence. Finally, the time evolution invariance of the CF type condition \((5)\) is not guaranteed in the Lagrangian description of the free 4D Abelian 2-form gauge theory. Thus, the logical reason behind the imposition of the CF type restriction \((5)\), for the above anticommutativity property, is not clear within the framework of the Lagrangian formalism. This is why, in the next section, we resort to the Hamiltonian formalism.

### 3 Time-Evolution Invariance of the Curci-Ferrari Type Condition: Hamiltonian Approach

It can be noted that the ghost part of the Lagrangian densities \((1)\) and \((2)\) is same. The corresponding Hamiltonian density \((\mathcal{H}_{(g)})\) can be expressed as

\[
\mathcal{H}_{(g)} = \Pi^{(\beta)} \Pi^{(\beta)} + \Pi^{(c)} \Pi^{(c)} + \Pi^{(c)} (\partial_i C_0) + \Pi^{(c)} (\partial_i \bar{C}_0) + \partial_i \bar{\beta} \partial_i \beta \\
- \left( \partial_i \bar{C}_j - \partial_j \bar{C}_i \right) \partial_i C_j + (\partial_i C_i) \Pi^{(ca)}_0 + (\partial_i \bar{C}_i) \Pi^{(ca)}_0 + 2 \Pi^{(ca)}_0 \Pi^{(ca)}_0, \tag{6}
\]

where the canonical momenta, corresponding to the (anti-) ghost fields, are:

\[
\Pi^{(\beta)} \equiv \frac{\partial \mathcal{L}^{(1,2)}}{\partial (\partial_0 \beta)} = \dot{\beta}, \quad \Pi^{(\beta)} \equiv \frac{\partial \mathcal{L}^{(1,2)}}{\partial (\bar{\partial}_0 \beta)} = \dot{\beta}, \\
\Pi^{(ca)}_0 \equiv \frac{\partial \mathcal{L}^{(1,2)}}{\partial (\partial_0 C_0)} = \rho, \quad \Pi^{(ca)}_0 \equiv \frac{\partial \mathcal{L}^{(1,2)}}{\partial (\bar{\partial}_0 C_0)} = \lambda,
\]
\[ \Pi^{(c)}_i \equiv \frac{\partial L^{(1,2)}}{\partial (\partial \bar{B}^i)} = (\partial_0 C_i - \partial_i C_0), \]
\[ \Pi^{(c)}_i \equiv \frac{\partial L^{(1,2)}}{\partial (\partial \bar{B}^i)} = -(\partial_0 \bar{C}_i - \partial_i \bar{C}_0). \]

It is worthwhile to mention that, in the operation of the derivative w.r.t the fermionic ghost fields, we have adopted the convention of the left derivative.

The following Heisenberg equations of motion for the generic field \( \Psi \)
\[ \dot{\Psi} = \pm i \left[ \Psi, H_{(g)} \right], \quad H_{(g)} = \int d^3x \mathcal{H}_{(g)}, \quad \dot{\Psi} = \frac{\partial \Psi}{\partial t}, \]
(where \([+\text{-}]\) signs correspond to the (fermionic) bosonic nature of the generic field \( \Psi \)) lead to the dynamical equations of motion for momenta as well as basic fields. It can be checked that the Euler Lagrange equations of motion
\[ \square \beta = \square \bar{\beta} = 0, \quad \square C_0 = -\partial_0 \rho, \quad \square C_0 = \partial_0 \lambda, \]
\[ \square \bar{C}_i = -\partial_i \rho, \quad \square C_i = \partial_i \lambda, \quad \lambda = \frac{1}{2} (\partial \cdot C), \quad \rho = -\frac{1}{2} (\partial \cdot \bar{C}), \]
for the (anti-)ghost fields, derived from the Lagrangian densities \( L^{(1,2)} \), also emerge from equation (8) when \( \Psi = \Pi^{(\beta)}, \Pi^{(\bar{\beta})}, \Pi^{(c_0)}, \Pi^{(\bar{c}_0)}, \Pi^{(c)}_i, \Pi^{(\bar{c})}_i \). On the other hand, for \( \Psi = \beta, \bar{\beta}, C_0, \bar{C}_0, C_i, \bar{C}_i \), we obtain the definition of the canonical momenta (7). In our Appendix A, these explicit computations are illustrated in a detailed fashion.

The non-ghost parts of the Lagrangian density (1) and (2) lead to the following pair of the canonical Hamiltonian densities in terms of canonical momenta and fields:
\[ \mathcal{H}_{(0)}^{(1)} = (\Pi_{ij})^2 + 2 (\bar{\Pi}_{\varphi_1})^2 - \frac{1}{2} (\Pi^{(1)}_{0i})^2 - 2 \Pi_{ij} (\partial_i B_{j0}) + \frac{1}{2} \left( \Pi^{(1)}_{0i} \right) \partial_i \varphi_1 \]
\[ - (\Pi^{(1)}_{0j}) \partial_i B_{ij} - 2 (\bar{\Pi}^{(1)}_{\varphi_1}) \partial_i B_{0i} + \frac{1}{12} H_{ijk} H_{ijk}, \]
\[ \mathcal{H}_{(0)}^{(2)} = (\Pi_{ij})^2 + 2 (\bar{\Pi}_{\varphi_1})^2 - \frac{1}{2} (\Pi^{(2)}_{0i})^2 - 2 \Pi_{ij} (\partial_i B_{j0}) - \frac{1}{2} \left( \Pi^{(2)}_{0i} \right) \partial_i \varphi_1 \]
\[ - (\Pi^{(2)}_{0j}) \partial_i B_{ij} - 2 (\bar{\Pi}^{(2)}_{\varphi_1}) \partial_i B_{0i} + \frac{1}{12} H_{ijk} H_{ijk}, \]
where the canonical momenta are defined as follows
\[ \Pi_{ij} \equiv \frac{\partial L^{(1,2)}}{\partial (\partial \bar{B}^j)} = \frac{1}{2} H_{0ij}, \]
\[ \Pi^{(1)}_{0i} \equiv \frac{\partial L^{(1)}}{\partial (\partial \bar{B}^0)} = B_i, \quad \Pi^{(2)}_{0i} \equiv \frac{\partial L^{(2)}}{\partial (\partial \bar{B}^0)} = \bar{B}_i, \]
\[ \Pi^{(1)}_{\varphi_1} \equiv \frac{\partial L^{(1)}}{\partial (\partial \bar{B}^0)} = \frac{B_0}{2}, \quad \Pi^{(2)}_{\varphi_1} \equiv \frac{\partial L^{(2)}}{\partial (\partial \bar{B}^0)} = -\frac{\bar{B}_0}{2}. \]
Exploiting the appropriate form of the Heisenberg equation (8) with the Hamiltonian densities (10) and (11) and using the following canonical brackets \(^8\) (with \(\hbar = c = 1\))

\[
\begin{align*}
[B_{ij}(x, t), \Pi_{ij}(y, t)] &= \frac{i}{2}(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk})\delta^{(3)}(x - y), \\
[B_{0i}(x, t), \Pi_{0j}^{(1,2)}(y, t)] &= -i\delta_{ij}\delta^{(3)}(x - y), \\
[\varphi_1(x, t), \Pi_{\varphi_1}^{(1,2)}(y, t)] &= i\delta^{(3)}(x - y),
\end{align*}
\]

(13)

it can be checked that the Hamiltonian densities (10) and (11) produce all the Euler Lagrange equations of motion derived from the Lagrangian densities (1) and (2). These derivations are clearly illustrated in our Appendix B. It will be noted that the CF condition (5) is still not derivable from a single Hamiltonian density (10) and/or (11). The CF condition (5) can be derived in one stroke, however. Towards this goal in mind, we define the following Lagrangian density \(^9\) that is constructed from (1) and (2), namely;

\[
\mathcal{L}^{(3)} = \frac{1}{2}(\mathcal{L}^{(1)} + \mathcal{L}^{(2)}) \equiv \frac{1}{12}H_{\mu\nu\kappa}H_{\mu\nu\kappa} + \frac{1}{2}(B_{\mu} + \bar{B}_{\mu}) \partial^\nu B_{\nu\mu} \\
+ \frac{1}{4}(B_{\mu} - \bar{B}_{\mu}) \partial_\mu \varphi_1 - \frac{1}{4}(B \cdot B + \bar{B} \cdot \bar{B}) + \mathcal{L}_g,
\]

(14)

where

\[
\mathcal{L}_g = \partial_\mu \bar{\beta} \partial^\mu \beta + \left(\partial_\mu \bar{C}_\nu - \partial_\nu \bar{C}_\mu\right) \partial^\mu C^\nu + (\partial \cdot C - \lambda) \rho + (\partial \cdot \bar{C} + \rho) \lambda,
\]

(15)

is the ghost part of the Lagrangian densities (1) and/or (2). It can be checked that, even from the Lagrangian density (14), the CF type of restriction (5) can be derived only in two steps. To obtain the same condition (i.e. (5)) in a single stroke, one has to redefine the following pair of auxiliary fields:

\[
b_{\mu} = \frac{1}{2}(B_{\mu} + \bar{B}_{\mu}), \quad \bar{b}_{\mu} = \frac{1}{2}(B_{\mu} - \bar{B}_{\mu}).
\]

(16)

As a result of the above re-definitions, it can be shown that the following equality

\[
B \cdot B + \bar{B} \cdot \bar{B} = 2 (b \cdot b + \bar{b} \cdot \bar{b}),
\]

(17)

leads to a different looking form of (14), namely,

\[
\mathcal{L}^{(3)} = \frac{1}{12}H_{\mu\nu\kappa}H_{\mu\nu\kappa} + b^\mu \partial^\nu B_{\nu\mu} + \frac{1}{2}\bar{b}^\mu \partial_\mu \varphi_1 - \frac{1}{2}(b \cdot b + \bar{b} \cdot \bar{b}) + \mathcal{L}_g.
\]

(18)

From the very outset, it is clear that

\[
\Pi_{\mu}^{(b)}(b) = 0, \quad \Pi_{\bar{b}}^{(b)} = 0,
\]

(19)

\(^8\)All the rest of the brackets are zero.

\(^9\)It will be noted that the other linearly independent combination \(\frac{1}{2}[\mathcal{L}^{(1)} - \mathcal{L}^{(2)}]\) is not interesting because the kinetic term of the gauge field and the ghost part of the Lagrangian densities cancel out in this combination. Thus, this combination is not useful from the point of view of our present discussions.
are the primary constraints on the theory. The canonical Hamiltonian density, derived
from the Lagrangian density (18), is

\[ \mathcal{H}^{(3)}_{(b,b)} = \Pi_{ij}^2 + 2 \Pi_{ij}^2 - \frac{1}{2} \Pi_{0i}^2 - 2 \Pi_{ij} \partial_j B_{0i} - \Pi_{0j} \partial_i B_{ij} - b_0 \partial_i B_{0i} \]

\[ + \frac{1}{2} \bar{b}_i \partial_i \varphi_1 + \frac{1}{2} \left( b_0 b_0 - \bar{b}_i \bar{b}_i \right) + \frac{1}{12} H_{ijk} H_{ijk} + \mathcal{H}_{(g)}, \]

(20)

where the other canonical momenta, besides (19) for the Lagrangian density (18), are

\[ \Pi_{\varphi_1} = \frac{\bar{b}_0}{2}, \quad \Pi_{0i} = b_i, \quad \Pi_{ij} = \frac{1}{2} H_{0ij}. \]

(21)

It is trivial to note that the auxiliary fields \( b_0 \) and \( \bar{b}_i \) appear in the above Hamiltonian
density but corresponding momenta are not present. The latter happen to be the primary
constraints on the theory as is evident from (19). These can be added to the canonical
Hamiltonian (20) in the following manner (see, e.g., [1, 2])

\[ \mathcal{H}^{(3)}_{(b,b)} = \Pi^{(b)}_0 \partial_0 b_0 - \Pi^{(b)}_i \partial_i \bar{b}_i + \Pi_{ij}^2 + 2 \Pi_{ij} - \frac{1}{2} \Pi_{0i}^2 - 2 \Pi_{ij} \partial_j B_{0i} - \Pi_{0j} \partial_i B_{ij} \]

\[ - b_0 \partial_i B_{0i} + \frac{1}{2} \bar{b}_i \partial_i \varphi_1 + \frac{1}{2} \left( b_0 b_0 - \bar{b}_i \bar{b}_i \right) + \frac{1}{12} H_{ijk} H_{ijk} + \mathcal{H}_{(g)}, \]

(22)

where \( \mathcal{H}_{(g)} \) is the usual ghost part of the Hamiltonian (cf. (6)) and \( \Pi^{(b)}_0 \) \( \Pi^{(b)}_i \) are the
momenta corresponding to the co-ordinate fields \( b_0 \) and \( \bar{b}_i \) (cf. (19)). It will be noted that
one can also add \( \Pi^{(b)}_0 \partial_0 b_0 - \Pi^{(b)}_i \partial_i b_i \) in the Hamiltonian density (22) but these do not
play any significant role as: \( \dot{\Pi}^{(b)}_0 = 0, \dot{\Pi}^{(b)}_i = 0, \dot{\bar{b}}_0 = \bar{b}_0, \dot{\bar{b}}_i = \bar{b}_i. \)

With the help of the canonical brackets (13) and the following

\[ [b_0(x, t), \Pi^{(b)}_0(y, t)] = i\delta^{(3)}(x - y), \]

\[ [\bar{b}_i(x, t), \Pi^{(b)}_i(y, t)] = -i\delta^{(3)} \delta_i^j (x - y), \]

(23)

we obtain the equations of motion as given below

\[ \bar{b}_\mu = \frac{1}{2} \partial_\mu \varphi_1, \quad \partial \cdot \bar{b} = 0, \quad \square \varphi_1 = 0, \]

\[ b_\mu = \partial^\nu B_{\nu\mu}, \quad \partial \cdot b = 0, \quad \Pi_{ij} = \frac{1}{2} H_{0ij}, \]

\[ \partial_\mu H^{\mu\nu\kappa} + (\partial^\nu b^\kappa - \partial^\kappa b^\nu) = 0. \]

(24)

It is worth emphasizing that \( \bar{b}_\mu = \frac{1}{2} \partial_\mu \varphi_1 \) (which leads to \( B_\mu - \bar{B}_\mu - \partial_\mu \varphi_1 = 0 \) ) and
\( b_\mu = \partial^\nu B_{\nu\mu} \) (i.e. \( B_\mu + \bar{B}_\mu = 2 \partial^\nu B_{\nu\mu} \)) are obtained from the Hamiltonian density \( \mathcal{H}^{(3)}_{(b,b)} \) by
exploiting the Heisenberg equation of motion \( \dot{\Pi}^{(b)}_0 = 0, \dot{\Pi}^{(b)}_i = 0, \dot{\varphi}_1 = -i \left[ \varphi_1, \mathcal{H}^{(3)}_{(b,b)} \right] \),
and \( \dot{B}_{0i} = -i \left[ B_{0i}, \mathcal{H}^{(3)}_{(b,b)} \right] \) where \( \mathcal{H}^{(3)}_{(b,b)} = \int d^3 x \mathcal{H}^{(3)}_{(b,b)}. \) This establishes the fact that

\[ \bar{b}_\mu = \frac{1}{2} \partial_\mu \varphi_1 \Rightarrow B_\mu - \bar{B}_\mu - \partial_\mu \varphi_1 = 0, \]

\[ b_\mu = \partial^\nu B_{\nu\mu} \Rightarrow B_\mu + \bar{B}_\mu - 2 \partial^\nu B_{\nu\mu} = 0. \]

(25)
are the secondary constraints on the theory.

The time-evolution invariance of the above constraints (i.e. $\bar{b}_\mu = \frac{1}{2} \partial_\mu \varphi_1$, $b_\mu = \partial^\nu B_{\nu \mu}$) can be seen to be true as:

$$\left[ 2\bar{b}_0 - \partial_0 \varphi_1, H_{(b,b)}^{(3)} \right] = 0, \quad \left[ 2\bar{b}_i - \partial_i \varphi_1, H_{(b,b)}^{(3)} \right] = 0,$$

$$\left[ b_0 - \partial_0 B_{0i}, H_{(b,b)}^{(3)} \right] = 0, \quad \left[ b_i - \partial_0 B_{0i} - \partial_j B_{ij}, H_{(b,b)}^{(3)} \right] = 0. \quad (26)$$

This establishes the time-evolution invariance of the CF type conditions which are invoked in the proof of the anticommutativity of the nilpotent (anti-) BRST symmetries.

### 4 (Anti-) Dual BRST Symmetries in Lagrangian Formulation: A Brief Sketch

The kinetic term $(\frac{1}{12} H^{\mu \nu \kappa} H_{\mu \nu \kappa})$ of the Lagrangian densities (1) and (2) can be linearized by introducing the Nakanishi-Lautrup type of auxiliary fields $B_\mu$ and $\bar{B}_\mu$ and a massless $(\Box \varphi_2 = 0)$ field $\varphi_2$ as given below (see, e.g. [9]):

$$L^{(4)} = \frac{1}{2} B_\mu B_\mu - B_\mu \left( \frac{1}{2} \epsilon_{\mu \nu \eta \kappa} \partial^\nu B^{\eta \kappa} + \frac{1}{2} \partial_\mu \varphi_2 \right) + B_\mu \left( \partial^\nu B_{\nu \mu} + \frac{1}{2} \partial_\mu \varphi_1 \right)$$

$$- \frac{1}{2} B_\mu B_\mu + L_{(g)}, \quad (27)$$

$$L^{(5)} = \frac{1}{2} \bar{B}_\mu \bar{B}_\mu - \bar{B}_\mu \left( \frac{1}{2} \epsilon_{\mu \nu \eta \kappa} \partial^\nu B^{\eta \kappa} - \frac{1}{2} \partial_\mu \varphi_2 \right) + \bar{B}_\mu \left( \partial^\nu B_{\nu \mu} - \frac{1}{2} \partial_\mu \varphi_1 \right)$$

$$- \frac{1}{2} \bar{B}_\mu \bar{B}_\mu + L_{(g)}, \quad (28)$$

where $L_{(g)}$ is same as the ghost part of the Lagrangian densities (1) and (2) and $\varphi_2$, $B_\mu$ and $\bar{B}_\mu$ satisfy the following equations of motion

$$\Box \varphi_2 = 0, \quad B_\mu = \frac{1}{2} \epsilon_{\mu \nu \eta \kappa} \partial^\nu B^{\eta \kappa} + \frac{1}{2} \partial_\mu \varphi_2, \quad \bar{B}_\mu = \frac{1}{2} \epsilon_{\mu \nu \eta \kappa} \partial^\nu B^{\eta \kappa} - \frac{1}{2} \partial_\mu \varphi_2, \quad (29)$$

which lead to a set of CF type restrictions

$$B_\mu + \bar{B}_\mu = \epsilon_{\mu \nu \eta \kappa} \partial^\nu B^{\eta \kappa}, \quad B_\mu - \bar{B}_\mu = \partial_\mu \varphi_2. \quad (30)$$

It is clear that the derivation of (30), from (27) and (28), is a two step process.

It has been demonstrated in our earlier works (see, e.g. [9]) that the Lagrangian densities (27) and (28) are endowed with (anti-) BRST symmetry transformation as well as absolutely anticommuting ($s_d s_{ad} + s_{ad} s_d = 0$) (anti-)co-BRST symmetry transformations ($s_{(a)d}$). The latter symmetry transformations are [9, 10]

$$s_d B_{\mu \nu} = -\epsilon_{\mu \nu \eta \kappa} \partial^\eta \bar{C}_\kappa, \quad s_d \bar{C}_\mu = -\partial_\mu \bar{\beta}, \quad s_d C_\mu = -B_\mu, \quad s_d \varphi_2 = 2 \rho, \quad s_d \beta = -\lambda, \quad s_d \left[ \rho, \lambda, \bar{\beta}, \varphi_1, B_\mu, B_\mu, \partial^\nu B_{\nu \mu} \right] = 0, \quad (31)$$
where

(i) the off-shell nilpotent \( s_{(a)d} \) (anti-)co-BRST symmetry transformations \( (s_{(a)d}) \) leave the gauge fixing terms \( (\partial^n B_{\nu\mu} \pm \frac{1}{2} \partial_\mu \varphi_1) \) invariant,

(ii) the co-BRST symmetry transformations \( (s_{d}) \) absolutely anticommute with the anti-co-BRST symmetry transformations \( (s_{ad}) \) (i.e. \( s_d s_{ad} + s_{ad} s_d = 0 \)), and

(iii) the absolute anticommutativity property is ensured if and only if the condition

\[
B_\mu - \bar{B}_\mu - \partial_\mu \varphi_2 = 0 \quad \text{(cf. (30) is imposed (i.e.} \{s_d, s_{ad}\}B_{\mu\nu} = 0).
\]

The time-evolution invariance of the above condition cannot be proven within the framework of the Lagrangian description. Thus, in the next section, we discuss the time-evolution invariance of \( B_\mu - \bar{B}_\mu - \partial_\mu \varphi_2 = 0 \) in the framework of Hamiltonian formalism.

5 Anticommutativity of the (Anti-) Dual BRST symmetries: Hamiltonian Formalism

It is clear from our previous section that, for the absolute anticommutativity of the co-BRST and anti-co-BRST symmetry transformations, one has to invoke a CF type restriction (i.e. \( B_\mu - \bar{B}_\mu - \partial_\mu \varphi_2 = 0 \)). For this condition, to persist with respect to the time-evolution of our gauge system, it is essential requirement that it should remain time invariant quantity. To this goal in mind, it can be seen that the following canonical Hamiltonian densities emerge from the Lagrangian densities (27) and (28):

\[
\mathcal{H}^{(4)} = (\Pi_{ij}^{(4)})^2 + 2 (\Pi_{\varphi_1}^{(4)})^2 - 2 (\Pi_{\varphi_2}^{(4)})^2 - \frac{1}{2} (\Pi_{0i}^{(4)})^2 + 2 \Pi_{ij}^{(4)} \partial_0 B_{ij} + \frac{1}{2} (\Pi_{0i}^{(4)}) \partial_i \varphi_1
\]

\[
-\Pi_{0i}^{(4)} \partial_i B_{ij} + 2 (\Pi_{\varphi_1}^{(4)}) (\partial_i B_{i0}) + (\Pi_{\varphi_2}^{(4)}) \epsilon_{ijk} \partial_j B_{jk} + \frac{1}{2} \epsilon_{ijk} \Pi_{jk}^{(4)} \partial_i \varphi_2 + \mathcal{H}_{(g)}, \quad (33)
\]

\[
\mathcal{H}^{(5)} = (\Pi_{ij}^{(5)})^2 + 2 (\Pi_{\varphi_1}^{(5)})^2 - 2 (\Pi_{\varphi_2}^{(5)})^2 - \frac{1}{2} (\Pi_{0i}^{(5)})^2 + 2 \Pi_{ij}^{(5)} \partial_0 B_{ij} + \frac{1}{2} (\Pi_{0i}^{(5)}) \partial_i \varphi_1
\]

\[
-\Pi_{0i}^{(5)} \partial_i B_{ij} - 2 (\Pi_{\varphi_1}^{(5)}) (\partial_i B_{i0}) - (\Pi_{\varphi_2}^{(5)}) \epsilon_{ijk} \partial_j B_{jk} + \frac{1}{2} \epsilon_{ijk} \Pi_{jk}^{(5)} \partial_i \varphi_2 + \mathcal{H}_{(g)}, \quad (34)
\]

where the canonical momenta are defined as:

\[
\Pi_{\varphi_1}^{(4)} = \frac{\partial \mathcal{L}^{(4)}}{\partial (\partial_0 \varphi_1)} = \frac{B_0}{2}, \quad \Pi_{\varphi_1}^{(5)} = \frac{\partial \mathcal{L}^{(5)}}{\partial (\partial_0 \varphi_1)} = \frac{-\bar{B}_0}{2},
\]

\[
\Pi_{\varphi_2}^{(4)} = \frac{\partial \mathcal{L}^{(4)}}{\partial (\partial_0 \varphi_2)} = \frac{-\bar{B}_0}{2}, \quad \Pi_{\varphi_2}^{(5)} = \frac{\partial \mathcal{L}^{(5)}}{\partial (\partial_0 \varphi_2)} = \frac{\bar{B}_0}{2},
\]
\[
\Pi_{(i)}^{(4)} \equiv \frac{\partial \mathcal{L}^{(4)}}{\partial (\partial_0 B_{(i)})} = B_i, \quad \Pi_{(i)}^{(5)} \equiv \frac{\partial \mathcal{L}^{(5)}}{\partial (\partial_0 B_{(i)})} = \bar{B}_i,
\]
\[
\Pi_{ij}^{(4)} \equiv \frac{\partial \mathcal{L}^{(4)}}{\partial (\partial_0 B_{ij})} = -\frac{1}{2} \epsilon_{ijk} B_k, \quad \Pi_{ij}^{(5)} \equiv \frac{\partial \mathcal{L}^{(5)}}{\partial (\partial_0 B_{ij})} = -\frac{1}{2} \epsilon_{ijk} \bar{B}_k. \quad (35)
\]

It will be noted that the superscripts ("(4) and (5)"") on the Hamiltonian densities and momenta correspond to such superscripts on the Lagrangian densities (27) and (28). The equations of motion, derived from the Heisenberg’s equation of motion (with \(H^{(4,5)} = \int d^3x \, \mathcal{H}^{(4,5)}\)), are found to be exactly same as the following juxtaposed Euler-Lagrange equation of motion derived from the Lagrangian densities (27) and (28), namely
\[
B_\mu = \partial^\nu B_{\nu\mu} + \frac{1}{2} \partial_\mu \varphi_1, \quad \bar{B}_\mu = \partial^\nu B_{\nu\mu} - \frac{1}{2} \partial_\mu \bar{\varphi}_1,
\]
\[
\mathcal{B}_\mu = \frac{1}{2} \varepsilon_{\mu\nu\kappa} \partial^\nu B^{\kappa} + \frac{1}{2} \partial_\mu \varphi_2, \quad \bar{\mathcal{B}}_\mu = \frac{1}{2} \varepsilon_{\mu\nu\kappa} \partial^\nu \bar{B}^{\kappa} - \frac{1}{2} \partial_\mu \bar{\varphi}_2,
\]
\[
\partial_\mu B_\nu - \partial_\nu B_\mu - \varepsilon_{\mu\nu\kappa} \partial^\kappa B^\kappa = 0, \quad \partial_\mu \bar{B}_\nu - \partial_\nu \bar{B}_\mu - \varepsilon_{\mu\nu\kappa} \partial^\kappa \bar{B}^\kappa = 0, \quad (36)
\]
where the left set of equations are from (27) and that of the right are from (28). Exactly the above set of equations can be derived from the Hamiltonian densities (33) and (34) which are explicitly given in our Appendix C.

It is worthwhile to mention that the CF type restrictions (\(\mathcal{B}_\mu - \mathcal{B}_\mu - \partial_\mu \varphi_2 = 0, \quad \mathcal{B}_\mu + \bar{\mathcal{B}}_\mu - \varepsilon_{\mu\nu\kappa} \partial^\nu \bar{B}^{\kappa} = 0\)) invoked for the proof of the absolute anticommutativity of the (anti-) dual-BRST symmetry transformations, are derived in two steps and they cannot emerge from a single Lagrangian and/or Hamiltonian densities. We achieve this goal below and show that a single Lagrangian density (and the corresponding Hamiltonian density) can produce the CF type restrictions in one step.

Besides the re-definitions in (16), we re-define the following auxiliary fields
\[
h_\mu = \frac{1}{2} (\mathcal{B}_\mu + \bar{\mathcal{B}}_\mu), \quad \bar{h}_\mu = \frac{1}{2} (\mathcal{B}_\mu - \bar{\mathcal{B}}_\mu), \quad (37)
\]
to express the following Lagrangian density (cf. (27) and (28)) as:
\[
\mathcal{L}^{(6)} = \frac{1}{2} \left( \mathcal{L}^{(4)} + \mathcal{L}^{(5)} \right) \equiv \frac{1}{2} \left( h \cdot h + \bar{h} \cdot \bar{h} \right) - \frac{1}{2} h^\mu \varepsilon_{\mu\nu\kappa} \partial^\nu \bar{B}^{\kappa} - \frac{1}{2} \bar{h}_\mu \partial_\mu \varphi_2 + \bar{b}^\mu \left( \partial^\nu B_{\nu\mu} \right) + \frac{1}{2} \bar{b}^\nu \partial_\nu \varphi_1 - \frac{1}{2} \left( b \cdot b + \bar{b} \cdot \bar{b} \right) + \mathcal{L}_0, \quad (38)
\]
where we have used
\[
(\mathcal{B} \cdot \mathcal{B} + \bar{\mathcal{B}} \cdot \bar{\mathcal{B}}) = 2 (h \cdot h + \bar{h} \cdot \bar{h}). \quad (39)
\]
The following Euler-Lagrange equations of motion emerge from (38):
\[
\bar{b}_\mu = \frac{1}{2} \partial_\mu \varphi_1, \quad \bar{h}_\mu = \frac{1}{2} \partial_\mu \varphi_2, \quad b_\mu = \partial^\nu B_{\nu\mu}, \quad h_\mu = \frac{1}{2} \varepsilon_{\mu\nu\kappa} \partial^\nu B^{\kappa}, \quad (40)
\]
\[
\partial \cdot b = 0, \quad \partial \cdot \bar{b} = 0, \quad \Box \varphi_1 = 0, \quad \Box \varphi_2 = 0, \quad \partial \cdot h = 0, \quad \partial \cdot \bar{h} = 0, \quad \varepsilon_{\mu\nu\kappa} \partial^\nu h^\kappa + (\partial^\nu b^\kappa - \partial^\kappa b^\nu) = 0,
\]
besides the ghost field equations that are derived from $\mathcal{L}_{(g)}$. The canonical momenta, from (38), are:

$$
\Pi_{\varphi_1} = \frac{\bar{b}_0}{2}, \quad \Pi_{\varphi_2} = -\frac{\bar{h}_0}{2}, \quad \Pi_{0i} = b_i, \quad \Pi_{ij} = -\frac{1}{2} \epsilon_{ijk} h_k.
$$

(41)

It is evident that $\Pi_{\mu}^{(b)} = 0, \Pi_{\mu}^{(h)} = 0, \Pi_{\mu}^{(\bar{b})} = 0, \Pi_{\mu}^{(\bar{h})} = 0$, because $b_\mu, \bar{b}_\mu, h_\mu, \bar{h}_\mu$ are the auxiliary fields of the theory.

At this juncture, it can be seen that $\bar{h}_\mu = \frac{1}{2} \partial_\mu \varphi_2$, and $h_\mu = \frac{1}{2} \epsilon_{\mu\nu\eta\kappa} \partial^\nu B^{\nu\eta\kappa}$ lead to the CF type of restrictions: $B_\mu - \bar{B}_\mu - \partial_\mu \varphi_2 = 0$ and $B_\mu + \bar{B}_\mu - \epsilon_{\mu\nu\eta\kappa} \partial^\nu B^{\nu\eta\kappa} = 0$ in a single step and they are derived from a single Lagrangian density (i.e. $\mathcal{L}(6)$) that is obtained from the linear combination of $\mathcal{L}(4)$ and $\mathcal{L}(5)$. It will be noted that the other linear combination [$\mathcal{L}(4) - \mathcal{L}(5)$] does not lead to an interesting Lagrangian density because the ghost parts of the Lagrangian densities $\mathcal{L}(4,5)$ cancel out with each other in this combination.

The canonical Hamiltonian density, emerging from the Lagrangian density $\mathcal{L}(6)$, is

$$
\mathcal{H}(6) = \Pi_{ij}^2 - 2 \Pi_{\varphi_2}^2 - \frac{1}{2} (\Pi_{0i})^2 + 2 \Pi_{\varphi_1}^2 + \frac{1}{2} (b_0 b_0 - \bar{b}_i \bar{b}_i) - \frac{1}{2} (h_0 h_0 - \bar{h}_i \bar{h}_i) + \frac{1}{2} \bar{b}_i \partial_i \varphi_1 - \frac{1}{2} \bar{h}_i \partial_i \varphi_2 - b_0 \partial_i B_{0i} - \Pi_{0j} \partial_i B_{ij} + 2 \Pi_{jk} \partial_j B_{0k} - \frac{\bar{h}_0}{2} \epsilon_{ijk} \partial_i B_{jk} + \mathcal{H}(g).
$$

(42)

It will be noted that, corresponding to the auxiliary fields $b_0, h_0, \bar{b}_i, \bar{h}_i$, there are no momenta in the above expression because these are the primary constraints on the theory (i.e. $\Pi_0^{(b)} \approx 0, \Pi_0^{(\bar{b})} \approx 0, \Pi_0^{(h)} \approx 0, \Pi_0^{(\bar{h})} \approx 0$). It is straightforward to check that the time evolution invariance of these constraints (with $H(6) = \int d^3x \mathcal{H}(6)$):

$$
\dot{\Pi}_0^{(b)} = -i [\Pi_0^{(b)}, H(6)] = 0 \Rightarrow b_0 = \partial^\nu B_{\nu 0},
$$

$$
\dot{\Pi}_0^{(\bar{b})} = -i [\Pi_0^{(\bar{b})}, H(6)] = 0 \Rightarrow \bar{b}_i = \frac{1}{2} \partial_i \varphi_1,
$$

$$
\dot{\Pi}_0^{(h)} = -i [\Pi_0^{(h)}, H(6)] = 0 \Rightarrow h_0 = -\frac{1}{2} \epsilon_{ijk} \partial_i B_{jk},
$$

$$
\dot{\Pi}_0^{(\bar{h})} = -i [\Pi_0^{(\bar{h})}, H(6)] = 0 \Rightarrow \bar{h}_i = \frac{1}{2} \partial_i \varphi_2,
$$

(43)

leads to the CF type restrictions $B_0 + \bar{B}_0 - 2 \partial^\nu B_{\nu 0} = 0, B_i - \bar{B}_i - \partial_\mu \varphi_1 = 0, B_0 + \bar{B}_0 + \epsilon_{ijk} \partial_i B_{jk} = 0, B_i + \bar{B}_i - \partial_\mu \varphi_2 = 0$ which are like the secondary constraints on the theory.

The full set of CF type restrictions (i.e. $B_\mu - \bar{B}_\mu - \partial_\mu \varphi_1 = 0, B_\mu + \bar{B}_\mu - \epsilon_{\mu\nu\eta\kappa} \partial^\nu B^{\nu\eta\kappa} = 0$) can be obtained from the Hamiltonian ($H(6)$) if we invoke the time-evolution of the following basic fields:

$$
\dot{\varphi}_1 = -i [\varphi_1, H(6)] \Rightarrow \bar{b}_i = \frac{1}{2} \partial_0 \varphi_1,
$$

$$
\dot{\varphi}_2 = -i [\varphi_2, H(6)] \Rightarrow \bar{h}_0 = \frac{1}{2} \partial_0 \varphi_2,
$$

$$
\dot{B}_{0i} = -i [B_{0i}, H(6)] \Rightarrow b_i = \partial^\mu B_{\mu i},
$$

$$
\dot{B}_{ij} = -i [B_{ij}, H(6)] \Rightarrow h_i = -\frac{1}{2} \epsilon_{ijk} \partial_0 B_{jk} - \epsilon_{ijk} \partial_j B_{k0},
$$

(44)
in addition to the expression obtained in (43). Thus, we note that it is the combination of (44) and (43) that yields all the components of the CF type restriction that are invoked in the proof of the absolute anticommutativity of the nilpotent symmetry transformations.

In its full glory, the total Hamiltonian density is the sum of the canonical Hamiltonian density (42) and the primary constraints on the theory as given below.

\[
H_T^{(6)} = \Pi_0^{(b_0)} \partial_0 b_0 + \Pi_0^{(h_0)} \partial_0 h_0 - \Pi_i^{(b)} \partial_0 b_i - \Pi_i^{(h)} \partial_0 h_i + H^{(6)}. \tag{45}
\]

Time-evolution invariance of the CF type restrictions (\(B_\mu - \bar{B}_\mu - \partial_\mu \varphi_2 = 0, B_\mu + \bar{B}_\mu - \varepsilon_{\mu\nu\eta\kappa} \partial^\nu B^{\kappa} = 0\)) can be now checked to be true with the total Hamiltonian density \(H_T^{(6)}\). Infact, using the canonical brackets, it is quite straightforward to check that

\[
\begin{align*}
[B_0 - \bar{B}_0 - \partial_0 \varphi_2, H_T^{(6)}] &= 0, \\
[B_i - \bar{B}_i - \partial_i \varphi_2, H_T^{(6)}] &= 0, \\
[B_0 + \bar{B}_0 - \varepsilon_{ijk} \partial_i B_{jk}, H_T^{(6)}] &= 0, \\
[B_i + \bar{B}_i + \varepsilon_{ijk} (\partial_0 B_{jk} + 2 \partial_j B_{k0}), H_T^{(6)}] &= 0. \tag{46}
\end{align*}
\]

The above relations show that the CF type restrictions remain the same during the full time-evolution of the 2-form Abelian gauge system. As a consequence, it is proper to impose these conditions for the proof of the absolute anticommutativity of the dual-BRST and anti-dual BRST symmetries during the full dynamical evolution of our present free Abelian 2-form gauge theory in physical four dimensions of spacetime.

6 Conclusions

In our present investigation, we have concentrated on the dynamical aspects of the 4D free Abelian 2-form gauge theory in the framework of the Hamiltonian formulation. This field theoretic model happens to be the off-shell nilpotent (anti-) BRST as well as (anti-) co-BRST invariant model of a 4D gauge theory. We have derived the dynamical equations of the theory with the help of the Heisenberg equations of motion where the Hamiltonian (of the (anti-) BRST as well as (anti-)co-BRST invariant system) plays a central role.

Our earlier works [9-11,21-23], devoted to the discussion of the Abelian 2-form gauge theory, have been carried out in the Lagrangian formulation where the CF type restrictions have been derived as the Euler-Lagrange equations of motion from the coupled Lagrangian densities. These CF type restrictions are required for the proof of an absolute anticommutativity between the off-shell nilpotent

(i) BRST and anti-BRST symmetry transformations, and

(ii) co-BRST and anti-co-BRST symmetries transformations.

However, the Lagrangian formulation does not shed any light on the time-evolution invariance of the above CF type restrictions.

We have chosen, in our present endeavour, the Hamiltonian formalism so that we can clearly demonstrate that the CF type restrictions remain invariant w.r.t. time-evolution of the Abelian 2-form gauge system. This result provides a logical reason behind the
imposition of the CF type restrictions which are valid at any moment of time for the full time-evolution of our physical 2-form Abelian gauge system in 4D spacetime.

The key difference between our present endeavour and our earlier attempts [9, 10] is the fact that CF type restrictions, that are at the heart of the absolute anticommutativity of the (anti-) BRST and (anti-) co-BRST symmetry transformations, are derived from a single Lagrangian density (and corresponding Hamiltonian density) in a single step. This should be contrasted with our earlier Lagrangian formulation where a set of coupled Lagrangian densities led to the derivations of the CF type restrictions in two steps as the Euler-Lagrange equations of motion and their subtraction/addition.

The absolute anticommutativity of the nilpotent (anti-) BRST and (anti-) co-BRST symmetry transformations is an essential requirement because it ensures the linear independence of the (i) BRST versus anti-BRST and (ii) co-BRST versus anti-co-BRST symmetries. Furthermore, it confirms physically the independent roles of the anti-BRST symmetries and anti-co-BRST symmetries in the context of the 4D Abelian 2-form gauge theory. It will be recalled that the anti-BRST and anti-co-BRST symmetries do not play any independent role vis-à-vis the BRST and co-BRST symmetries in the context of the 4D Abelian 1-form⁹ gauge theory (see, e.g. [24]). These points are consistent with the results of our work on superfield formulation of the Abelian 2-form gauge theory [15].

One of us has studied the gauge theories in BRST superspace [25-29], which is slightly different from the usual approach of the superspace formulation (see, e.g. [15]). The main features of this BRST superspace are (i) the whole action, including the source terms for the composite operators, is accommodated in a single compact superspace action, (ii) theory has generalized gauge invariance and WT identities which are realised in a simple way, and (iii) operation like super-rotation and super-translation, in anticommuting variable, can be carried out in a completely unrestricted manner. Such superspace formulation is very useful in studying the renormalization problem in gauge theories. It would be nice endeavour to apply this approach to study the 2-form [25] and higher-form gauge theories.

To generalize our present work and earlier works [9-11,21-23] to 4D non-Abelian 2-form and higher p-form (p > 2) gauge theories is one of the challenging future endeavour. We expect that even the higher-form (p > 2) Abelian gauge theories would lead to some very interesting observations in the framework of BRST formalism. A thorough constraint analysis of our current theory¹¹ and higher-form gauge theories is also on our future agenda. Discussion of the above theories in the framework of superfield formulations [15,25-29] is yet another direction for future investigation. Currently these problems are under investigation and our results would be reported in our future publications [30].

¹⁰In the case of 4D Abelian 1-form gauge theory, the operator form of the first-class constraints annihilate the physical states of the theory due to the physicality criteria \((Q_{(a)b}|_{phys} >= 0)\) with the (anti-) BRST charges \(Q_{(a)b}\). In other words, the BRST and anti-BRST charges lead to the same conditions through \(Q_{(a)b}|_{phys} >= 0\). Thus, the anti-BRST charge does not play an independent role here.

¹¹Only a few comments have been made by us on the constraints of our present theory. However, an elaborate discussion on the classification of these constraints and their specific roles, in the context of our present theory, would be taken up in our future endeavour [30].
Acknowledgements:

One of us (RPM) thankfully acknowledges the financial support from the Department of Science and Technology (DST), Government of India, under the SERC project sanction grant No. SR/S2/HEP-23/2006.

Appendix A

We explicitly demonstrate that the Hamiltonian $H_{(g)} = \int d^3x \mathcal{H}_{(g)}$, corresponding to the ghost part $\mathcal{L}_{(g)}$ (cf. (15)) of the Lagrangian densities (1) and (2), yields all the equations of motion (cf. (9)) for the (anti-) ghost fields. For this purpose, we have to exploit the following canonical (anti-) commutators (with $\hbar = c = 1$):

\[
\begin{align*}
\{ \beta(x,t), \Pi^{(\beta)}(y,t) \} &= i\delta^{(3)}(x - y), \\
\{ \bar{\beta}(x,t), \Pi^{(\beta)}(y,t) \} &= i\delta^{(3)}(x - y), \\
\{ C_0(x,t), \Pi^{(c_0)}(y,t) \} &= i\delta^{(3)}(x - y), \\
\{ \bar{C}_0(x,t), \Pi^{(c_0)}(y,t) \} &= i\delta^{(3)}(x - y), \\
\{ C_i(x,t), \Pi^{(c_i)}(y,t) \} &= -i\delta_{ij}\delta^{(3)}(x - y), \\
\{ \bar{C}_i(x,t), \Pi^{(c_i)}(y,t) \} &= -i\delta_{ij}\delta^{(3)}(x - y),
\end{align*}
\]

and all the other (anti-) commutators are zero.

Using (47), it can be checked that the time-evolution of the canonical momenta

\[
\begin{align*}
\dot{\Pi}^{(\beta)} &= -i \left[ \Pi^{(\beta)}, H_{(g)} \right] \Rightarrow \Box \beta = 0, \\
\dot{\Pi}^{(\bar{\beta})} &= -i \left[ \Pi^{(\bar{\beta})}, H_{(g)} \right] \Rightarrow \Box \bar{\beta} = 0, \\
\dot{\Pi}^{(c_0)}_0 &= +i \left[ \Pi^{(c_0)}_0, H_{(g)} \right] \Rightarrow \Box C_0 = -\partial_0 \rho, \\
\dot{\Pi}^{(c_0)}_0 &= +i \left[ \Pi^{(c_0)}_0, H_{(g)} \right] \Rightarrow \Box C_0 = \partial_0 \lambda, \\
\dot{\Pi}^{(c)}_i &= +i \left[ \Pi^{(c)}_i, H_{(g)} \right] \Rightarrow \Box C_i = -\partial_i \rho, \\
\dot{\Pi}^{(\bar{c})}_i &= +i \left[ \Pi^{(\bar{c})}_i, H_{(g)} \right] \Rightarrow \Box C_i = \partial_i \lambda,
\end{align*}
\]

lead to the Euler-Lagrange equation of motion derived from the Lagrangian densities (1) and/or (2) for the basic (fermionic) bosonic (anti-) ghost fields of the theory.

On the other hand, it is interesting that the time-evolution of the (anti-) ghost fields

\[
\dot{\beta} = -i \left[ \beta, H_{(g)} \right] \Rightarrow \dot{\beta} = \Pi^{(\beta)},
\]
\[ \dot{\beta} = -i \left[ \beta, H_{(g)} \right] \quad \Rightarrow \quad \dot{\beta} = \Pi^{(\beta)}, \]
\[ \dot{C}_0 = +i \left[ C_0, H_{(g)} \right] \quad \Rightarrow \quad \Pi^{(c)}_0 = \frac{1}{2} (\partial \cdot C) = \lambda, \]
\[ \dot{C}_0 = +i \left[ C_0, H_{(g)} \right] \quad \Rightarrow \quad \Pi^{(c)}_0 = -\frac{1}{2} (\partial \cdot C) = \rho, \]
\[ \dot{C}_i = +i \left[ C_i, H_{(g)} \right] \quad \Rightarrow \quad \Pi^{(c)}_i = (\partial_0 C_i - \partial_i C_0), \]
\[ \dot{C}_i = +i \left[ C_i, H_{(g)} \right] \quad \Rightarrow \quad \Pi^{(c)}_i = -(\partial_0 C_i - \partial_i C_0), \] (49)

leads to the definition of the canonical momenta corresponding to the bosonic and fermionic (anti-) ghost fields. This establishes the consistency and equivalence between the Lagrangian and Hamiltonian descriptions of the Abelian 2-form gauge theory.

**Appendix B**

Dynamics of the non-ghost part of the Lagrangian densities (1) and (2) remain unaffected due to their description in the framework of Lagrangian and Hamiltonian formalism. To establish this fact, it can be checked that the Hamiltonian \( H_{(b)}^{(1)} = \int d^3x \, \mathcal{H}_{(b)}^{(1)} \), produces the following Hesisenberg dynamical equations of motion for the basic fields:

\[ \dot{\varphi}_1 = -i \left[ \varphi_1, H_{(b)}^{(1)} \right] \quad \Rightarrow \quad B_0 = \frac{1}{2} \partial_0 \varphi_1 + \partial_i B_{0i}, \]
\[ \dot{B}_{0i} = -i \left[ B_{0i}, H_{(b)}^{(1)} \right] \quad \Rightarrow \quad B_1 = \partial^\nu B_{\nu i} + \frac{1}{2} \partial_i \varphi_1, \]
\[ \dot{B}_{ij} = -i \left[ B_{ij}, H_{(b)}^{(1)} \right] \quad \Rightarrow \quad \Pi_{ij} = \frac{1}{2} H_{0ij}, \] (50)

where we have exploited the canonical brackets (13).

On the other hand, the time evolution of the canonical momenta, namely;

\[ \dot{\Pi}_{\varphi_1}^{(1)} = -i \left[ \Pi^{(1)}_{\varphi_1}, H_{(b)}^{(1)} \right] \quad \Rightarrow \quad \partial \cdot B = 0, \]
\[ \dot{\Pi}_{\nu}^{(1)} = -i \left[ \Pi^{(1)}_{\nu}, H_{(b)}^{(1)} \right] \quad \Rightarrow \quad \partial_k H^{k\nu i} + \partial^\nu B^i - \partial^i B^\nu = 0, \]
\[ \dot{\Pi}_{ij} = -i \left[ \Pi^{(1)}_{ij}, H_{(b)}^{(1)} \right] \quad \Rightarrow \quad \partial_\mu H^{\mu ij} + (\partial^i B^j - \partial^j B^i) = 0, \] (51)

produces the dynamical equations of motion. It will be noted that the top two equations of (50) and bottom two equations of (51) can be combined together as: 
\( B_\mu = \partial^\nu B_{\nu \mu} + \frac{1}{2} \partial_\mu \varphi_1, \) 
\( \partial_\mu H^{\mu \nu \kappa} + \partial^\nu B^\kappa - \partial^\kappa B^\nu = 0. \) These finally lead to the simple equations of motion: \( \Box \varphi_1 = 0 \) (due to \( \partial \cdot B = 0 \)) and \( \Box B_{\nu \mu} = 0 \) as well as \( \Box B_\mu = 0. \)

Similarly, the Hamiltonian \( H_{(b)}^{(2)} = \int d^3x \, \mathcal{H}_{(b)}^{(2)} \) leads to the following equations of motion (that are different from \( H_{(b)}^{(1)} \)), namely;

\[ \dot{\varphi}_1 = -i \left[ \varphi_1, H_{(b)}^{(2)} \right] \quad \Rightarrow \quad \dot{B}_0 = \partial_i B_{i0} - \frac{1}{2} \partial_0 \varphi_1, \]
\begin{align*}
\dot{B}_{oi} &= -i \left[B_{oi}, H^{(2)}_{(b)}\right] \quad \Rightarrow \quad B_{i} = \partial^\nu B_{\nu i} - \frac{1}{2} \partial_i \varphi_1, \\
\dot{\Pi}^{(2)}_{0i} &= -i \left[\Pi^{(1)}_{0i}, H^{(2)}_{(b)}\right] \quad \Rightarrow \quad \partial_k H^{ki0} + \partial^0 \overline{B}^i - \partial^i \overline{B}^0 = 0, \\
\dot{\Pi}_{ij} &= -i \left[\Pi_{ij}, H^{(2)}_{(b)}\right] \quad \Rightarrow \quad \partial_{\mu} H^{\mu ij} + (\partial^i \overline{B}^j - \partial^j \overline{B}^i) = 0, \\
\dot{\Pi}^{(2)}_{\varphi_1} &= -i \left[\Pi^{(2)}_{\varphi_1}, H^{(2)}_{(b)}\right] \quad \Rightarrow \quad \partial \cdot \overline{B} = 0.
\end{align*}

Ultimately, the above equation imply that \(\Box \varphi_1 = 0, \Box B_{\mu \nu} = 0\,\text{ and } \Box \overline{B}_\mu = 0\). These equations primarily emerge from \(\overline{B}_\mu = \partial^\nu B_{\nu \mu} - \frac{1}{2} \partial_\mu \varphi_1\) and \(\partial_\mu H^{\mu \nu \kappa} + \partial^\nu \overline{B}^\kappa - \partial^\kappa \overline{B}_\nu = 0\).

**Appendix C**

The Euler-Lagrange equations of motion (36) can be re-derived from the Hamiltonians \(H^{(4,5)} = \int d^3 x \, \mathcal{H}^{(4,5)}\) as illustrated below:

\begin{align*}
\dot{\varphi}_1 &= -i \left[\varphi_1, H^{(4)}\right] \quad \Rightarrow \quad \varphi_0 = \frac{1}{2} \partial_0 \varphi_1 - \partial_i B_{i0}, \\
\dot{\varphi}_2 &= -i \left[\varphi_2, H^{(4)}\right] \quad \Rightarrow \quad \varphi_0 = \frac{1}{2} \partial_0 \varphi_2 - \frac{1}{2} \epsilon_{ijk} \partial_i B_{jk}, \\
\dot{\Pi}^{(4)}_{\varphi_1} &= -i \left[\Pi^{(4)}_{\varphi_1}, H^{(4)}\right] \quad \Rightarrow \quad \partial \cdot \mathcal{B} = 0, \\
\dot{\Pi}^{(4)}_{\varphi_2} &= -i \left[\Pi^{(4)}_{\varphi_2}, H^{(4)}\right] \quad \Rightarrow \quad \partial \cdot \mathcal{B} = 0, \\
\dot{B}_{oi} &= -i \left[B_{0i}, H^{(4)}\right] \quad \Rightarrow \quad B_i = \partial_0 B_{0i} - \partial_k B_{ki} + \frac{1}{2} \partial_i \varphi_1, \\
\dot{B}_{ij} &= -i \left[B_{ij}, H^{(4)}\right] \quad \Rightarrow \quad B_i = \frac{1}{2} \epsilon_{ijk} (\partial_j B_{0k} - \partial_k B_{0j} - \partial_0 B_{jk}) + \frac{1}{2} \partial_i \varphi_2, \\
\dot{\Pi}^{(4)}_{ij} &= -i \left[\Pi^{(4)}_{ij}, H^{(4)}\right] \quad \Rightarrow \quad \partial_0 \mathcal{B}_i - \partial_i \mathcal{B}_0 - \epsilon_{ijk} \partial_j B_k = 0, \\
\dot{\Pi}^{(4)}_{0i} &= -i \left[\Pi^{(4)}_{0i}, H^{(4)}\right] \quad \Rightarrow \quad \partial_0 B_i - \partial_i B_0 + \epsilon_{ijk} \partial_j B_k = 0,
\end{align*}

and the Hamiltonians \(H^{(5)} = \int d^3 x \, \mathcal{H}^{(5)}\) leads to:

\begin{align*}
\dot{\varphi}_1 &= -i \left[\varphi_1, H^{(5)}\right] \quad \Rightarrow \quad \varphi_0 = \partial_i B_{i0} - \frac{1}{2} \partial_0 \varphi_1, \\
\dot{\varphi}_2 &= -i \left[\varphi_2, H^{(5)}\right] \quad \Rightarrow \quad \varphi_0 = -\frac{1}{2} \partial_0 \varphi_2 - \frac{1}{2} \epsilon_{ijk} \partial_i B_{jk}, \\
\dot{\Pi}^{(5)}_{\varphi_1} &= -i \left[\Pi^{(5)}_{\varphi_1}, H^{(5)}\right] \quad \Rightarrow \quad \partial \cdot \mathcal{B} = 0, \\
\dot{\Pi}^{(5)}_{\varphi_2} &= -i \left[\Pi^{(5)}_{\varphi_2}, H^{(5)}\right] \quad \Rightarrow \quad \partial \cdot \mathcal{B} = 0, \\
\dot{B}_{oi} &= -i \left[B_{0i}, H^{(5)}\right] \quad \Rightarrow \quad B_i = \partial_0 B_{0i} - \partial_k B_{ki} - \frac{1}{2} \partial_i \varphi_1, \\
\dot{B}_{ij} &= -i \left[B_{ij}, H^{(5)}\right] \quad \Rightarrow \quad B_i = \frac{1}{2} \epsilon_{ijk} (\partial_j B_{0k} - \partial_k B_{0j} - \partial_0 B_{jk}) - \frac{1}{2} \partial_i \varphi_2, \\
\dot{\Pi}^{(5)}_{ij} &= -i \left[\Pi^{(5)}_{ij}, H^{(5)}\right] \quad \Rightarrow \quad \partial_0 \mathcal{B}_i - \partial_i \mathcal{B}_0 - \epsilon_{ijk} \partial_j B_k = 0, \\
\dot{\Pi}^{(5)}_{0i} &= -i \left[\Pi^{(5)}_{0i}, H^{(5)}\right] \quad \Rightarrow \quad \partial_0 B_i - \partial_i B_0 + \epsilon_{ijk} \partial_j B_k = 0.
\end{align*}
It is elementary to check that finally we obtain the following simple equations of motion

\[ \Box B_{\mu \nu} = 0, \quad \Box B_{\mu} = 0, \quad \Box \bar{B}_{\mu} = 0, \]
\[ \Box \varphi_1 = 0, \quad \Box \varphi_2 = 0, \quad \Box \mathcal{B}_{\mu} = 0, \quad \Box \bar{\mathcal{B}}_{\mu} = 0, \]
\[ \partial \cdot B = 0, \quad \partial \cdot \bar{B} = 0, \quad \partial \cdot \mathcal{B} = 0, \quad \partial \cdot \bar{\mathcal{B}} = 0, \]

from the above Hamiltonians \( H^{(4,5)} \).

References

[1] P. A. M. Dirac, *Lectures on Quantum Mechanics, Belfer Graduate School of Science* (Yeshiva University Press, New York, 1964).

[2] K. Sundermeyer, *Constrained Dynamics: Lecture Notes in Physics*, Vol. 169 (Springer-Verlag, Berlin, 1982).

[3] V. I. Ogievetsky and I. V. Palubarinov, * Yad. Fiz.* 4, 216 (1966).

[4] V. I. Ogievetsky and I. V. Palubarinov, *Sov. J. Nucl. Phys.* 4, 156 (1967).

[5] See, e.g., A. Salam and E. Sezgin, *Supergravities in Diverse Dimensions* (World Scientific Publications, Singapore, 1989).

[6] See, e.g., M. B. Green, J. H. Schwarz and E. Witten, *Superstring Theory* (Cambridge University Press, Cambridge, 1987).

[7] See, e.g., J. Polchinski, *String Theory* (Cambridge University Press, Cambridge, 1998).

[8] N. Seiberg and E. Witten, *JHEP 9909*, 032 (1999).

[9] Saurabh Gupta and R. P. Malik, *Eur. Phys. J. C* 58, 517 (2008), arXiv: 0807.23606 [hep-th]).

[10] E. Harikumar, R. P. Malik and M. Sivakumar, *J. Phys. A : Math. Gen.* 33, 7149 (2000), hep-th/0004145.

[11] R. P. Malik, Notoph gauge theory as the Hodge theory in *Proc. of the International Workshop on Supersymmetries and Quantum Symmetries* (SQS’03), BLTP, JINR, Dubna, 24-29 July 2003, pp 321-326, hep-th/0309245.

[12] R. P. Malik, *J. Phys. A: Math. Gen.* 36, 5095 (2003), hep-th/0209136.

[13] See, e.g., L. Bonora and M. Tonin, *Phys. Lett. B* 98, 48 (1981).

[14] See, e.g., L. Bonora, P. Pasti and M. Tonin, *Nuovo Cimento A* 63, 353 (1981).
[15] R. P. Malik, *Eur. Phys. J. C* **60**, 457 (2009), hep-th/0702039.

[16] G. Curci and R. Ferrari, *Phys. Lett.* B **63**, 91 (1976).

[17] N. Nakanishi and I. Ojima, *Covariant Operator Formalism of Gauge Theory and Quantum Gravity* (World scientific Publications, Singapore, 1990).

[18] See, e.g., K. Nishijima, *Czech. J. Phys.* **46**, 01 (1996).

[19] D. M. Gitman and I. V. Tyutin, *Quantization of Fields with Constraints* (Springer-Verlag, Berlin, 1990).

[20] M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems* (Princeton University Press, Princeton, 1992).

[21] L. Bonora and R. P. Malik, *Phys. Lett.* B **655**, 75 (2007), arXiv: 0707.3922 [hep-th].

[22] R. P. Malik, *Eur. Phys. J. C* **55**, 687 (2008), arXiv: 0802.4129 [hep-th].

[23] R. P. Malik, *Euro. Phys. Lett.* **84**, 31001 (2008), arXiv: 0805.4470 [hep-th].

[24] R. P. Malik, *Int. J. Mod. Phys.* A **22**, 3521 (2007), hep-th/0609201.

[25] S. Deguchi and B. P. Mandal, *Mod. Phys. Lett.* A **15**, 965 (2000).

[26] S. D. Joglekar and B. P. Mandal, *Phys. Rev.* D **55** 5038 (1997).

[27] S. D. Joglekar and B. P. Mandal, *Phys. Rev.* D **52** 7129 (1995).

[28] S. D. Joglekar and B. P. Mandal, *Phys. Rev.* D **49**, 5617 (1994).

[29] S. D. Joglekar and B. P. Mandal, *Z. Phys.* C **70**, 673 (1996).

[30] R. P. Malik *etal*, in preparation.