Time of dynamic impact to elements of RC frame at column buckling

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Abstract. A method for calculating the time of the dynamic additional impact to the elements of a reinforced concrete framing structural system under special impact caused by a sudden loss of stability of a compressed bearing element is presented. Using the discrete-linear moment-curvature diagram for reinforced concrete compressed-bended element, the analytical relationships for different stages of deformation of a reinforced concrete element before and after cracking are carried out. The obtained relationships for time of the additional dynamic impact to the elements makes it possible to determine the dynamic strength of concrete and reinforcement and the stiffness of the sections of reinforced concrete elements of structural systems under such impacts more accurately and consequently ensure more exact computational analysis to progressive collapse.

1. Introduction

Since the 1970s, when 16 May 1968 22-storey Ronan Point building in East London had been destroyed after initial partial failure of several load-bearing walls caused by the explosion of domestic gas, a significant amount of scientific researches around the world has been devoted to assessing the resistance of buildings and structures to the progressive collapse. An advanced analysis of these studies was provided in the papers [1,2]. However, the increasing number of new publications on this topic all around the world demonstrates the continuing relevance of this problem and existence of a large quantity of important unsolved tasks. Analysis of recent scientific publications allows identifying some trends and common approaches that are used in the solution to problems of assessing the resistance of structural systems of buildings and structures to the progressive collapse and in the design of protection against such phenomena. Almost all the work performed in this direction can be divided into two large groups: 1) investigations related to the assessment of the risk of progressive collapse [3,4] and 2) analytical [5-8], numerical [9-16] and experimental [17-22] studies of loads' dynamic increment in the elements of structural systems under accidental impacts caused by the sudden removal one of the load-bearing elements of a system. Within the second direction there are three main approaches to the assessment of such loads. It is dynamic calculation [8-16], an approach based on the use of the limit equilibrium method [5] considering the behavior of structures when their material reaches the plastic stage of operation (formation of plastic hinges) and a quasi-static method [6,7,9], which is based on the analysis of the potential energy of deformation of the structure during its active loading on the first half-wave of oscillations.
A significant number of works assessing the resistance of structural systems to progressive collapse is reduced to the selection of substructure containing the removed element and the adjacent in the plan and through the height structural elements [9,17]. In this case, the stiffness of the entire structural system is taken into account by introducing elastic-springs restricting linear displacements and rotations in the substructure [7-12]. The validity of such approach was confirmed by experimental investigations [9,17-22], which showed that amplitudes of dynamic forces, that are caused by accidental impacts, quickly decrease through the distance from removed element. And in the case of local destruction, the dynamic splashes of the forces are mainly distributed to adjacent structural elements. Investigation of forces' dynamic increment in the elements of structural systems is usually carried out, taking into account the specific types and scenarios of impacts that causes local destruction, for example, explosions, high temperatures, mechanical shocks, etc. However, in a number of works the defined reasons that cause damage or destruction of a structural element were not considered. It should be noted that in most works concerning the assessment of structures' resistance to progressive collapse, the changing of strength and deformation properties of construction materials under such accidental impacts were also not considered. However, as the experimental studies' results [9] showed, these properties of construction materials significantly depend on the rate of loads' application and, as a rule, increase quantitatively as the time of impact's application decreases. In turn, the time of force's dynamic increment depends on the mechanism of local destruction. For reinforced concrete structures, the destruction by strength criterion is usually considered as such a scenario [2-7,13-23]. However, as a number of studies shown, as such a scenario can be the buckling caused by the accumulation of a critical level of corrosion damages or sudden changes of the length factors of compressed-bent elements [10-12]. And for high strength reinforced concrete structures, it can be decreasing of the cross sections' sizes and changing of restraints in the nodes. In this regard, the purpose of this work is determining of the time of force's dynamic increment \( t_d \) for structural element under accidental impact caused by a sudden loss of stability of a load-bearing element.

2. Method

In the scientific literature on the issue to assessment of resistance to progressive collapse is known a number of works [9], in which for destruction of RC building frames by strength criterion it is obtained dependence, linking the time of force's dynamic increment \( t_d \) with the frequency of natural oscillations of the system with one degree of freedom (mass, applied instead removed structural element or restraint).

\[
 t_d = T/4 = \frac{\pi}{(2\omega)} = \left(\frac{\pi}{2}\right)\left(\frac{y_{st}}{g}\right)^{1/2},
\]

where \( y_{st} \) is static deflection of the structural element due to the static load attached instead removed element in the \((n-1)\)-level system (with local damage) and determined as axial static force in restraint or construction chosen as removed element at operational stage in the \(n\)-level system (without any damage); \( g \) is gravitational acceleration; \( T \) is natural vibration’s period of considering structural element.

Let us consider the structural element's buckling as oscillatory process at first half-wave, then fibers' deformations (or curvature) reach the ultimate values at the time \( t_d = T/4 \), that is caused acting of the critical force \( N_{cr} \). In the case of buckling of reinforced concrete rod it should be considered as oscillation of system with infinite number of degrees of freedom. Taking into account the similarity of eigen flexural vibrations' mode and buckling mode, we use the known solution to the problem of free vibrations of the rod with an infinite number of degrees of freedom and arbitrary restraints at the ends [9] to find the limit time of dynamic impact at buckling. Let us write this solution in the general form:

\[
 \omega = k_{\omega}\sqrt{\frac{EI}{(m\cdot l^4)}}
\]
where \( k_\omega \) is dimensionless parameter of the natural frequency depending on restraints on the ends of the rod, \( m \) is rod's linear mass density, \( l \) is the length of the rod, \( E \) is the modulus of deformation, \( I \) is the inertial moment of cross section.

Let us make a substitution

\[
m = q/g, \quad y_{st} = k_\omega q^2/(EI),
\]

where \( q \) is the evenly distributed transverse load to the rod, \( k_\omega \) is dimensionless parameter of the maximum deflection, which depends on the boundary conditions, \( y_{st} \) is the maximum static deflection at buckling. Then we obtain in general form:

\[
\omega = k_\omega k_{wq}^{1/2} \sqrt{g/y_{st}},
\]

To find \( y_{st} \), we use the known solution of the stability equation in dimensionless parameters and coordinates for the case when one end of the rod is rigidly clamped, and the other has arbitrary restraint [23]:

\[
\overline{w} = \overline{w}_0^* (1 - \cos k_\xi)/k^2 + \overline{w}_0^* (\xi - (\sin k_\xi)/k)/k^2;
\]

\[
\overline{w}' = \overline{w}_0^* (\sin k_\xi)/k + \overline{w}_0^* (1 - \cos k_\xi)/k^2;
\]

\[
\overline{w}'' = \overline{w}_0^* \cos k_\xi + \overline{w}_0^* (\sin k_\xi)/k;
\]

\[
\overline{w}''' = -k\overline{w}_0^* \sin k_\xi + \overline{w}_0^* \cos k_\xi.
\]

The following dimensionless variables and parameters are accepted in the expressions (5): \( \xi = x/l \), \( \overline{w} = w/l \), \( k^2 = Pl^2/(E_h I_{red, min}) \).

In the case, when the lower end of the rod is rigidly clamped, and the upper one is fixed by elastic spring with stiffness \( C_1 \) and \( C_2 \) from displacement and rotation, respectively (fig. 1, a), the stability equations (5) for \( \xi = 1 \) are reduced to the system [23]:

\[
\begin{align*}
\overline{w}_0^* C_1 (1 - \cos k)/k^2 + \overline{w}_0^* C_1 (1 - (\sin k)/k)/k^2 - 1 &= 0, \\
\overline{w}_0^* C_2 (\sin k)/k^2 - \overline{w}_0^* C_2 (1 - \cos k)/k^2 - (\sin k)/k &= 0.
\end{align*}
\]

Using a two-component model of G.A. Geniev [24] for concrete and reinforced concrete subjected short-term dynamic load, we assume that the integral coefficient of reinforced concrete viscosity \( \omega_0 \) equals to the parameter of concrete viscosity \( \omega = 3.14 \cdot 10^{-2} \) in the first approximation. In this case, taking in account dynamic strengthening of the concrete, the limit dynamic moment takes the value:

\[
M_{ult}^d = \phi_0^* M_{ult, b}^* + \phi_{sc}^* M_{ult, sc}^*,
\]

where \( M_{ult, b}^* \), \( \phi_0^* \) are limit static moment and concrete dynamic strengthening coefficient respectively, \( M_{ult, sc}^* \), \( \phi_{sc}^* \) are the same parameters for reinforcement.
The equations' system (6) has a nontrivial solution if the determinant of the matrix composed of coefficients at $\ddot{w}_0$ and $\dddot{w}_0$ equals to zero. Equating the determinant to zero, we find the critical force parameter $k_{cr}$. Substituting the value of $k_{cr}$ in the first equation of the system (6), we express $\dddot{w}_0$ through $\ddot{w}_0$, and then substituting the expression for $\dddot{w}_0$ in the second equation of the system (5), and equating $\dddot{w}$ to zero, we find the coordinate $\xi$ with the largest deflection value at considered boundary conditions.

Using a simplified three-line discrete diagram "moment-curvature" proposed in the paper [25], which is used to determine the curvature of the reinforced concrete element with cracks, we impose a restriction on the maximum value of the curvature $\chi_{ult}$ (Fig. 2) to find the values of the deflection $y_{st}$ at the point with coordinate $\xi$:

$$\chi_{ult} = \left( M_{ult} - \phi_2 b h^2 R_{ult,cr}^d \right) / \left( \phi_1 E_x' A_x h_0^2 \right), \quad (8)$$

where

$$\phi_2 b h^2 R_{ult,cr}^d = M_0, \quad \phi_1 E_x' A_x h_0^2 = B_1, \quad (9)$$

$$M_{ult}^d = R_0^d \cdot b \cdot x (h_0 - 0.5 x) + R_0^d A_x' (h_0 - a'), \quad (10)$$

In these expressions and Fig. 2 the following notation are accepted. $R_0^d$ is the design resistance of compressed concrete under dynamic loading (dynamic strength of concrete), $b$ is the width of the cross section, $h_0$ is the height of the working area of concrete, $R_{ult,cr}^d$ is the design resistance of the compressed reinforcement under dynamic loading, $A_x'$ is the area of compressed reinforcement, $a'$ is the distance from the most compressed fiber to the center of gravity of the compressed reinforcement, $\phi_1$, $\phi_2$ are coefficients, defined by [25], $M_{ult}$ is the bending moment in the cross section of the reinforced concrete element when it loss stability, $M_{cr}$ is the moment of cracking, $B_0$, $B_1$, $B_1(t)$ is the rigidity of the concrete rod without cracks, after the formation of cracks and after buckling respectively.

Equation (8) is the basic computational dependence allows to determine the limit time $t^d$ of the force's dynamic increment in the reinforced concrete structural element under special accidental impact using equality $M = M_{ult}^d$ or by the known value of $t^d$ to determine the limit dynamic moment $M_{ult}^d$ in cross-section.
Figure 2. Diagram "Moment-Curvature" for the case of compressed-bent reinforced concrete rod structural element.

Substituting the expression (8) for the limit value of curvature \( \chi_{ult} \) in the third equation of the system (5), we find the values of dimensionless bending moment and transverse force in the initial section (\( \xi = 0 \)): \( \bar{\omega} \) and \( \bar{w} \). And inputting these ones in the first equation of the system (5), simultaneously taking into account \( k_{cr} \) and \( \xi \), we obtain the value of the maximum deflection \( y_{st} \) at buckling.

Then the equation for the limit time of load's dynamic increment in the considered element (fig. 1, a) at buckling takes the form:

\[
t^d = T/4 = \pi/(2\omega) = \left( \pi/(2)k_1k_{cr}^{1/2} \right)(y_{st}/g)^{1/2}.
\]

Investigations represented in the papers [26] show that for rods of constant cross-section regardless of the type of boundary conditions it is obtained \( k^2 \omega = 4/\pi \).

3. Results and discussion

To illustrate the proposed technique, we consider the first floor column (figure 1, b) of the precast-monolithic frame of a multi-story building, for which, as shown in [10-12], at violation of operating conditions or due to errors at the stage of construction, a buckling is possible. In the first approximation, we assume that the lower end of the column is rigidly clamped, and the upper end has restraints for \( Y \) and \( U_z \) (Fig. 1, b). Column material is concrete of C70 compressive strength class; reinforcement is steel rebar of A500S class. The dimensions of the cross section of the column are 200 x 200 mm, the percentage of reinforcement is \( \mu = 1\% \).

There is known solution to the buckling problem for considered boundary conditions [23]: \( k = 2\pi \). In this case, the maximum transverse deflection under axial compression is achieved in the section with a dimensionless coordinate \( \xi = 0.5 \). In this section, the curvature calculated by the formula (8) taking into account (10) is \( \bar{\omega} = 0.5 \). \( l = \bar{\omega}/0.5 \cdot l = \chi_{ult} = 0.0526 \ m^3 \). Substituting the found values \( k \), \( \bar{\omega} \), \( \chi \), into the first equation from the system (5) and multiplying the result by \( l \), we obtain \( y_{st} = 0.0549 \ m \). For this value of the deflection from the formula (11), we find the limit time of the force dynamic increment \( t^d = 0.09 \ sec \), that corresponds to the natural oscillation frequency of the considered element on the first half-wave \( \omega = 17.02 \ sec^{-1} \).

4. Conclusions

A method for calculating the limit time of force's dynamic increment \( (t^d) \) in reinforced concrete element of the structural system under special accidental impact caused by a sudden buckling one of the load-bearing elements is proposed. Calculated value of the limit time of force's dynamic increment
allows assessing the resistance of reinforced concrete structural system to progressive collapse since it gives possibility taking into account the values of the dynamic strength of concrete and reinforcement as well as dynamic cross section stiffness at different stages of deformation.

References
[1] Karpenko N I, Kolchunov V I 2007 J. Stroitel'naya Mekhanika i Raschet Sooruzheniy 1 4
[2] Jose M Adam, Fulvio Parisi, Juan Sagaseta, Xinzeng Lu 2018 Eng. Struct. 173 122
[3] Yang Ding, Xiaoran Song, Hai-Tao Zhu 2017 J. Construct. Steel Research 129 129
[4] Emanuele Brunesi, Fulvio Parisi 2017 Eng. Struct. 152 579
[5] Zenin S A, Sharipov R Sh, Kudinov O V, Shapiro G I, Gasanov A A 2016 Academia. Arkhitettura i stroitel'stvo 4 109
[6] Elisa Livingston, Mehrdad Sasani, Marlon Bazan, Serkan Sagiroglu 2015 Eng. Struct. 95 61
[7] Fedorova N V, Androsova N B 2018 Stroitel'stvo i Rekonstruktsiya 75 73
[8] Kolchunov V I, Savin S Yu 2017 J. Appl. Eng. Sci. 15 325
[9] Kolchunov V I, Androsova N B, Klyueva N V, Bukhtiyarova A S 2014 Survivability of buildings and structures during beyond-design impacts (Moscow: Publishing house ASV)
[10] Kolchunov V I, Prasolov N O, Bukhtiyarova A S 2003 Industrial and civil engineering 12 42
[11] Kolchunov V I, Savin S Yu 2018 MCE 80 73
[12] Fedorova N V, Savin S Yu 2018 IOP Conf. Ser.: MSE 365 052018
[13] Weng J, Tan K H, Lee C K 2017 Eng. Struct. 151 136
[14] Setareh Amiri, Hamed Saffari, Javad Mashhadi 2018 Eng. Failure Analysis 84 300
[15] Bo Yang, Yong Yang, Xu-Hong Zhou, Qiang-Fu Jiang, Shao-Bo Kang 2018 J. Constr. Steel Research 151 25
[16] Al-Salloum Y A, Abbas H, Almusallam T H, Ngo T, Mendis P 2017 Eng. Sci. 29 313
[17] Pascal Forquin, Wen Chen 2017 Constr. and Build. Mater. 152 1068
[18] Peng Feng, Hanlin Qiang, Weihong Qin, Meng Gao 2017 Eng. Struct. 147 752
[19] Limin Tian, Jianpeng Wei, Jiping Hao 2018 J. Constr. Steel Research. 144 270
[20] Shao-Bo Kang, Kang Hai Tan, Hui-Yuan Liua, Xu-Hong Zhou, Bo Yang 2017 J. Constr. Steel Research 138 150
[21] Waleed Mohamed Elsayed, Mohamed A N Abdel Moaty, Mohamed E Issa 2016 HBRC J. 12 242
[22] Xianzhong Zhao, Shen Yan, Yiyi Chen, Zhenyu Xu, Yong Lu 2017 Eng. Struct. 135 104
[23] Gordon V A, Kolchunov V I 2006 J. Construction Mechanics and Structuring 4 33
[24] Geniev G A, Kolchunov V I, Klyueva N V, Nikulin A I, Pyatikrestovsky K P 2004 Strength and deformability of reinforced concrete structures under impacts exceeding design values (Moscow: Publishing ASV)
[25] Veryuzhsky Yu V, Kolchunov V I, Barabash M S, Genzersky Yu V 2006 Computer technologies of RC structures’ design (Kyiv: Book Publishing NAU)
[26] Korobko V I 1988 News of universities. Construction and architecture 3 41

Acknowledgements
Authors wishing to acknowledge financial support from Russian Academy of Architecture and Construction Science by the direction 7 “Development of the theoretical basis of construction science”, topic 7.4.12 “Criteria of states beyond the limit for reinforced concrete structural systems of buildings and structures, subjected emergency impacts, caused by buckling of one of the bearing elements”.