Strength analysis of warship hull's bottom loaded by the pressure wave from a non-contact explosion of sea mine

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Abstract. The study was based on the analysis of stamina of steel flat bottom section of transport warships, burdened by the spherical pressure wave from the non-contact explosion of TNT at a distance of 20 m under the keel. This study aims to determine the TNT mass required to break the hull. The task was solved by finite element method (FEM) explicite using CAE program [1], in which the hull’s bottom was modelled as thin shell space. The hull’s burden with pressure wave was modelled as a pressure impulse specified by the formula introduced by T.L. Geers, K.S. Hunter and R.S. Price [2]. To describe the material properties, considering high-speed strain, the Johnson-Cook model was used [3]. Therefore, the main goal of the hereby paper is to present how to correctly model the impact of large, concentrated masses of the ship’s equipment on its hull. The study presents the results of the calculated stress and strain states of the analysed section of the construction of the hull.

1. Introduction

Warships due to their intended purpose, are directly exposed to the impact of shocks such as the pressure wave caused by an underwater non-contact explosion of sea mine or torpedo. To such impacts are exposed all vessels, including commercial and passenger ships sailing in the areas of military operations. The stage of the designing process of warship’s hulls should provide them with adequate stamina to facilitate surviving the pressure wave burdens on the assumed parameters caused by the non-contact explosion in the water. The study analysed the stamina of the transport ship’s flat bottom which is not a real construction because the aim of this study is to present the methodology of solving similar problems. Because considered is the only fragment of the ship’s hull, analysed is the stamina of the structure itself. Due to the lack of whole ship’s hull and equipment received results in this work are only illustrative and cannot be used, for instance, to determine forces and accelerations acting on the equipment mounted on this construction because of the absence of interactions of mass forces omitted ship’s parts and the surrounding water. The main aim of the work is to present how to correctly model the impact of large, concentrated masses of the ship’s equipment on its hull. Despite these restrictions, the results of this analysis shall allow locating the weakest nodes of the construction even in the project phase, to re-design and re-submit verification calculations. This study aims to determine the pressure pulse values which destroys the newly designed hull’s bottom construction of the transport ship and check if the proposed hull’s bottom construction can withstand the impact of the
pressure wave from the explosion of a load approx. 100 ÷ 200 kg of TNT at a distance of 15÷20 m (Figure 1).

When mapping the geometry of the ship, many simplifications are used, related to its equipment. The ship consists of thousands of devices of smaller and larger masses. To keep the total mass of the ship, simplifications are made, consisting in increasing the density of the hull material. Such simplifications are related to the neglect of the inertia forces that cause concentrated masses. They destroy the ship’s hull.

![Figure 1. Illustration of a mine explosion under the warship keel.](image)

The presented problem is a quick-change task. In terms of the finite element method (FEM) – _explicit_ implemented in the CAE program this task comes down to solve motion equations in matrix form [4, 5].

\[
\mathbf{M}(U)\dddot{U} + \mathbf{C}(U)\ddot{U} + \mathbf{K}(U)U = \mathbf{F}(t, R, m_{TNT})
\]

(1)

where:

- \( \mathbf{C} = \alpha \mathbf{M} + \beta \mathbf{K} \) damping matrix, where \( \alpha \) and \( \beta \) are constant coefficients [6];
- \( \mathbf{U}, \dot{\mathbf{U}}, \ddot{\mathbf{U}} \) – vector of displacement, velocity and acceleration;
- \( \mathbf{K} \) – stiffness matrix structure;
- \( \mathbf{M} \) – inertia matrix;
- \( \mathbf{F} \) – burden vector;
- \( m_{TNT} \) – mass of TNT;
- \( R \) – distance from explosion centre;
- \( t \) – time.

![Figure 2. Section of the warship’s flat bottom made in the Abaqus program as a 3D thin coat.](image)
2. Research of object’s geometry
The analysed section of the flat warship’s bottom has dimensions $12 \times 12 \times 1$ m. It is burdened by the spherical pressure wave from the non-contact explosion of TNT detonated at a distance $R_0 = 20$ m under the bottom centre. The space around the warship is unlimited. A section is made of metal sheet thickness 6 mm, frames 8 mm and stringers and bottom transverses “T” shape with thickness 6 mm. The geometry of the section has been mapped in the Abaqus program in creating parts module (Figure 2).

3. Material characteristics
In the task, Johnson-Cook’s constitutive model [7] has been used to describe the material, where the plastic stresses $\sigma_{pl}$ reduced HMH $^1$ are described by the equation:

$$\sigma_{pl} = (A + B\varepsilon_{pl}^n) \left[ 1 + C \ln \left( \frac{\varepsilon_{pl}}{\varepsilon_0} \right) \right] \left[ 1 - \left( \frac{\theta - \theta_0}{\theta_{top} - \theta_0} \right)^m \right]$$  \hspace{1cm} (2)

Assumed for calculations that the entire section is made of higher quality steel, with properties of which are [3] (Figure 3):

- $A = 553.1$ MPa
- $B = 600.8$ MPa
- $n = 0.234$
- $m = 0.75 \pm 1.0$
- $\theta_{top} = 1733$ K
- $\theta_0 = 293.15$ K
- $C = 0.0134$
- $\dot{\varepsilon}_0 = 0.0001 \text{ s}^{-1}$

![Figure 3. Steel characteristic in Johnson-Cook’s model.](image)

Other values are:
- density $- \rho = 7850 \text{ kg/m}^3$;
- Young’s modulus $- E = 2.09 \times 10^5$ MPa;
- Poisson’s ratio $- \nu = 0.3$;
- yield strength $- R_y = 430$ MPa;
- tensile strength $- R_m = 800$ MPa.

To describe the material destruction the Johnson-Cook’s model was used [8, 9] as

$$\epsilon_{failure} = [D_1 + D_2 \exp(D_3 \sigma_{triax})] \left[ 1 + D_4 \ln \left( \frac{\varepsilon_{pl}}{\varepsilon_0} \right) \right] \left[ 1 + D_5 \left( \frac{\theta - \theta_0}{\theta_{top} - \theta_0} \right) \right]$$  \hspace{1cm} (3)

The coefficients of the analysed steel in the Johnson-Cook’s model are:

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$^1$ reduced stress by Huber-Mises-Hencky’s hypothesis
$D_1 = 0.0112$
$D_2 = 0.24$
$D_3 = 0.75$
$D_4 = 0.14$
$D_5 = 3.87$
$\theta_{\text{ep}} = 1733 \text{ K}$
$\theta_0 = 293.15 \text{ K}$
$\dot{\varepsilon}_0 = 0.0001 \text{ s}^{-1}$

\[ \text{Figure 4. Steel destruction characteristic in Johnson–Cook’s model.} \]

4. CONSTRUCTION’S BURDEN

The hull’s bottom construction is burdened with pressure pulse from the detonation of TNT (a sea mine) at a distance of 20 m under the bottom centre. The pressure value falling to the hull’s bottom at a given point is a function of the time and distance of this point to the epicentre of the explosion. The distribution of this burden is presented in the scheme (Figure 5) and the graph (Figure 6). Pressure pulses incident on each partition section is shifted in time depending on the distance from the epicentre. Values of the various pressure pulses were calculated based on empirical formulas by T.L. Geers, K.S. Hunter and R.S. Price [2] (Fig. 6). In 2002, T. L. Geers and K. S. Hunter in [2] presented an empirical description of a pressure wave in the form of:

\[ p_{\text{max}} = 1000 \cdot K_1 \left( \frac{a_c}{R} \right)^{1+A_1}, \text{ MPa} \]  
\[ p(t) = p_{\text{max}} e^{-t/C}, \text{ for } C \leq 1 \]  
\[ p(t) = p_{\text{max}} \left( 0.8251 e^{-1.3367 t/C} + 0.1749 e^{-0.1805 t/C} \right), \text{ for } C \leq 7 \]  
\[ C = 10^{-3} \cdot K_2 \left( \frac{a_c}{R} \right)^{A_2}, s \]  
\[ a_c = \frac{3}{4\pi \rho} \sqrt{\frac{m}{3}} \]  

where:
- $t$ – time, s
- $R$ – distance from the charge, m
- $m$ – mass of charge, kg
- $a_c$ – explosion radius
- $\rho$ – density of the explosive charge
- $K_1, A_1, K_2, A_2$ – experimental constants (Table 1)

The description of the pressure wave by the formulas by T. L. Geers and K. S. Hunter has become extremely popular and is implemented in some commercial CAE programs as a ready-made procedure, in which it is enough to enter the mass of the charge, density and $K_1, A_1, K_2, A_2$ constants. Table 1 presents the constant values for TNT, Pentolite and HBX-1.
Table 1. Values of constants $K_1$, $A_1$, $K_2$, $A_2$ for selected explosives [2,12,13].

| Explosive material | source (author) | $K_1$ GPa | $K_2$ km/s | $A_1$ | $A_2$ |
|--------------------|-----------------|----------|-----------|------|------|
| TNT, $\rho \approx$1520 kg/m$^3$ | R. H. Cole, 1948 | 1.42 | 0.992 | 0.13 | 0.18 |
| TNT, $\rho \approx$1600 kg/m$^3$ | T.E. Farley & H.G. Snay, 1978 | 1.45 | 1.24 | 0.13 | 0.23 |
| TNT, $\rho \approx$1600 kg/m$^3$ | R.S. Price, 1979 | 1.67 | 1.01 | 0.18 | 0.185 |
| Pentolite, $\rho \approx$1710 kg/m$^3$ | M.A. Thiel, 1961 | 1.65 | 1.22 | 0.14 | 0.23 |
| Pentolite, $\rho \approx$1710 kg/m$^3$ | R.S. Price, 1979 | 1.92 | 1.29 | 0.194 | 0.257 |
| Pentolite, $\rho \approx$1710 kg/m$^3$ | R.S. Price, 1979 | 1.67 | 1.22 | 0.14 | 0.23 |
| Pentolite, $\rho \approx$1710 kg/m$^3$ | R.S. Price, 1979 | 1.65 | 1.22 | 0.14 | 0.23 |
| Pentolite, $\rho \approx$1710 kg/m$^3$ | R.S. Price, 1979 | 1.65 | 1.22 | 0.14 | 0.23 |
| $\text{HBX}–1$, $\rho \approx$1720 kg/m$^3$ | M.M. Swisdak, 1978 | 1.71 | 1.47 | 0.15 | 0.29 |
| $\text{HBX}–1$, $\rho \approx$1720 kg/m$^3$ | R.S. Price, 1979 | 1.58 | 1.17 | 0.144 | 0.247 |

Figure 5. Scheme task.

Figure 6. Distribution of pressure pulses incident on the hull’s bottom calculated based on formulas by T.L. Geers’a, K.S. Hunter’s and R.S. Price’s [2] for 100 kg of TNT at a distance of 20 m under the keel.
5. FEM simulation results
Calculations of the analysed construction were made in the Abaqus program by solving task *explicit* described by equation (1). Calculations were performed for different values of the maximum pressure falling on the bottom constructions in the range of 4÷20 MPa, which corresponds to the explosion of the TNT mass 10 ÷ 660 kg detonated at a distance of 20 m under the keel. Example results of FEM simulations are presented in Figures 7 ÷ 8.

![FEM Simulation Results](image)

**Figure 7.** Reduced stresses H MH in the hull’s bottom caused by the explosion of 64 kg of TNT at a distance of 20 m under keel ($p_{\text{max}} = 8$ Mpa), $t_1 = 0.0001$ s, $t_2 = 0.0002$ s, $t_3 = 0.0003$ s, $t_4 = 0.0005$ s, $t_5 = 0.0006$ s.

![FEM Simulation Results](image)

**Figure 8.** H MH effective stresses in the bottom of the hull caused by the explosion of 350 kg of TNT at a distance of 20 m under the keel ($p_{\text{max}}=16$ MPa), $t = 0.0005$ s, linear cracks along with the braced structures.

5.1. Modelling of devices and equipment
During the strength analysis of the ship’s hull and its behaviour during a non-contact explosion, it is necessary to consider the impact of the masses of all devices as well as all of the equipment mounted on it. Analysis of the hull strength without reflecting the mass forces of the equipment is considered to be pointless because the inertia forces of the equipment are the cause of the greatest deformation and
disruption of the hull. In addition, devices and equipment, their foundations, fixings, etc. significantly change the rigidity of the entire structure. It is also necessary to consider the mass of all media in tanks. The mass of the analysed ship’s structure must be consistent with its buoyancy, which is a necessary condition for a correct dynamic analysis.

\[ V \rho_{\text{water}} = M \]  \hspace{1cm} (7)

where:

- \( V \) – volume of immersed hull part
- \( \rho_{\text{water}} \) – fluid mass density
- \( M \) – the total mass of the ship’s structure with equipment and devices

Figure 9 shows the impact effect of the device having dimensions of the ship’s engine on the hull plating, which breaks the plating and practically pierces the hull, in comparison with the same plating without the device for identical explosion parameters.

When analysing the strength of the hull structure, all devices should be simplified in the form of simple solids, shells, beams and even points to which the mass of a given device is assigned. Devices with large dimensions and masses are mounted on foundations. In such a case, it is enough to reflect the foundation itself in the form of a shell or beam-rod elements and the mass of such device should be assigned to this foundation in the form of increased density. If there is more computing power, the equipment can be replaced by a plate, shell or solid figure, which shape (geometric properties) and mechanical properties reflect the actual stiffness, main dimensions, mass moments of inertia and centre of gravity of the device. This problem is presented by the example of the mechanical and geometrical properties of the ZU-23-2MR cannon. A relatively accurate reflection of its geometry in a discrete model would consume several thousand degrees of freedom (Figure 10).

![Figure 9. Deformations of the bottom of the hull after a non-contact explosion, a) fragment of plating without devices, b) fragment of plating with device mass of \( m \)](image-url)
In the most economical simplification, the weight of the cannon should be assigned to the upper ring of the foundation on which it rests by increasing the density of the ring or by assigning the weight of the cannon to its nodes (Figure 11).

In both cases, the stiffness of the foundation can be increased by previously calculating the equivalent stiffness of the cannon. The advantage of this type of simplification is that the degrees of freedom of the hull structure are not increased. However, there are also disadvantages. In this case, only the mass of the device and its location as well as the approximate stiffness are reflected, but geometrical properties such as mass moments of inertia are omitted, and the centre of gravity is reduced to the level of the foundation. Of course, the impact of these disadvantages on the analysis may be negligible, however, this problem can be solved by modelling a given device in the form of a cylindrical or rectangular shell or in the form of simple solids whose external dimensions will reflect the external dimensions of a given device. By appropriate selection of the density and stiffness of the materials of these shells or solids, one can reflect the rigidity of the device, the mass moments of inertia and the centre of gravity. In many CAE programs, it is possible to assign mass and geometric characteristics to one node located, for example, at the end of a rod of appropriate height, which can be perfectly rigid or deformable (Figure 12).
Figure 12. A replacement model of the ZU-23-2MR cannon.

On every ship and vessel, a significant part of the equipment is linear, e.g., all types of installations or shaft tunnels. Such devices should be reflected with beam-rod elements and point masses for the appropriate nodes, in which mass moments of inertia of the represented devices can be also ascribed (Figure 13).

Figure 13. Shaft tunnels in a beam-rod model [15].

6. Conclusions
Conducted stamina analysis of the hull fragment is a considerable simplification. Lack of whole warship’s hull and equipment leads to skipping impacts the cut parts on the relevant passage. For this reason, the results obtained can not be used to determine the burdens of construction’s equipment pieces mounted on the hull’s bottom fragment because of the absence of interactions inertial abscissa (omitted) parts of the ship and surrounding water weight. However, conducted analysis estimate the stamina of the hull’s fragment construction, determine the pressure values, and explosion parameters necessary to the hull unsealing. Based on the analysis of the received states of deformations, stresses and destructions it is concluded that the proposed hull’s bottom construction is plastically deformed even at a pressure wave, which maximum value is 4 MPa, at a value of 12 MPa appear first crack point. With pressure values, 13÷15 MPa cracks take linearity along reinforcements. For pressures values, 16÷20 MPa cracks are transformed into the breach plating between reinforcements. Received results are consistent with the results of polygon experiments described in works [10,11]. Analysed hull’s bottom construction keep tight with burdens caused by the non-contact explosion of TNT mass approx. 100÷200 kg at a distance of 15÷20 m.
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