Thermodynamic properties of the exactly solvable transverse Ising model on decorated planar lattices

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Abstract

The generalized mapping transformation technique is used to obtain the exact solution for the transverse Ising model on decorated planar lattices. Within this scheme, the basic thermodynamic quantities are calculated for different planar lattices with arbitrary spins of decorating atoms. The particular attention has been paid to the investigation of transverse-field effects on magnetic properties of the system under investigation. The most interesting numerical results for the phase diagrams, compensation temperatures and several thermodynamic quantities are discussed in detail for the ferrimagnetic version of the model.

Keywords: exact solution; mapping transformation; transverse field, Ising model

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1 Introduction

For many years magnetic properties of decorated Ising spin systems consisting of spin-1/2 and spin-$S$ ($S \geq 1/2$) atoms, have been very intensively studied both theoretically and experimentally. In particular, the behaviour of various decorated models has been explored by variety of mathematical techniques, with some exact results [1], even in the presence of the single-ion anisotropy [2]. The strong interest in these models arises partly on account of the rich critical phenomena they display and partly due to the fact that they represent more complicated but simultaneously exactly solvable systems than the simple, undecorated ones. Moreover, the decorated planar models belong to the simplest exactly solvable theoretical models of ferrimagnetism. In this respect, they may exhibit under certain conditions the compensation phenomenon, i.e. the compensation point at which the resultant magnetization vanishes below the critical temperature. All these properties of decorated systems make them very interesting also from the experimental point of view, first of all in connection with many possible technological application of ferrimagnets to practice. On the other hand, much work is currently being done in the area of molecular-based magnetic materials and so molecular magnetism has become an important topic of scientific interest. In fact, a rapid progress in molecular engineering brought the possibility to control and to design the magnetic structure and properties of molecular systems. Thus, a number of bimetallic network assemblies have been synthesized which perfectly fulfil the claim of sufficiently small interplane coupling. Nevertheless, the molecular magnets which would possess the decorated network structure have been prepared only a decade ago. From this family, the most frequently prepared compounds are the bimetallic assemblies having the decorated honeycomb sheet structure [3] or the decorated square sheet structure [4] (see Fig. 1). Obviously, the experimental discovery of this wide class of compounds has stimulated the renewed interest in studying the mixed spin-1/2 and spin-$S$ ($S \geq 1/2$) decorated planar models theoretically.

Owing to these facts, the present article will be devoted to the investigation one of the simplest mixed-spin quantum model. Namely, we will study the decorated model in the presence of a transverse field that can potentially be useful for understanding some of the above mentioned
experimental systems. The transverse Ising model has been originally introduced by de Gennes as a pseudo-spin model of hydrogen bonded ferroelectrics \[5\], however, during the last decade it has found a wide application in the description of diverse physical systems, for instance, cooperative Jahn-Teller systems \[6\], strongly anisotropic magnetic materials in the transverse field \[7\], or many other systems \[8\]. Although the transverse Ising model is one of the simplest quantum models, the complete exact solution has been obtained in the one-dimensional case only \[9\]. For two-dimensional systems there are exactly known only the initial transverse susceptibilities on regular lattices \[10\]. On the other hand, over the last few years a simple straightforward method has been developed for obtaining exact results to the transverse Ising model, by assuming that it is composed of quantal and ‘classical’ (Ising-type) spins \[11, 12\]. The method is based on the generalized mapping transformation (the decoration-iteration or star-triangle one) introduced into the system with the transverse field affecting only one kind of spins (quantal).

The primary purpose of this work is to provide further extension of the exact solution from the reference \[11\], by the use of the same approach. Adopting the basic ideas of the transformation techniques, we examine the influence of applied transverse field on the thermodynamic properties of decorated lattices with different planar topology and arbitrary spin values of decorating atoms.

The outline of this paper is as follows. In Section 2, the fundamental framework of the transformation method as applied to the present model, are briefly reviewed. This is followed by a presentation of the numerical results for the spin-\(\mu\) (\(\mu = 1/2\)) and spin-\(S_B\) (\(S_B \geq 1/2\)) decorated transverse Ising ferrimagnet on several planar lattices, in Section 3. Finally, some concluding remarks are given in Section 4.

2 Formulation

In this work we will study the transverse Ising model on decorated planar lattices. A typical example of the system under investigation is depicted for the case of the square lattice in Fig. 1. As one can see, the system consists of two interpenetrating sublattices and we will further assume that the sites of the original lattice that constitute the sublattice \(A\) are occupied by atoms with the fixed spin \(\mu = 1/2\). The second sublattice \(B\) is occupied by decorating atoms with an arbitrary spin value \(S_B\). Then the system is described by the Hamiltonian

\[
\hat{H}_d = -\frac{1}{2} \sum_{i,k} J \hat{S}_i^z \hat{\mu}_k^z - H_A \sum_{k=1}^N \hat{\mu}_k^z - H_B \sum_{i=1}^{Nq/2} \hat{S}_i^z - \Omega \sum_{i=1}^{Nq/2} \hat{S}_i^x,
\]

where \(\hat{\mu}_k^z\) and \(\hat{S}_i^z\) and \(\hat{S}_i^x\) represent the relevant components of standard spin operators and \(J\) is the exchange integral that specifies the exchange interaction between nearest-neighboring atoms. The last three terms describe the interaction of \(A\) atoms with an external longitudinal magnetic field \(H_A\), as well as the interaction of \(B\) atoms with an external longitudinal (\(H_B\)) and transverse (\(\Omega\)) magnetic field, respectively. Furthermore, \(N\) and \(q\) denote the total number of atoms and the coordination number of the original lattice, respectively. As we have already mentioned, the transverse Ising model in which all the atoms interact with the transverse field is not exactly solvable for two- and three-dimensional lattices. This fact is closely related to the mathematical complexities appearing due to the noncommutability of the spin operators in the relevant Hamiltonian. However, in our recent work \[12\] we have succeeded to solve exactly the simplified version of this model (described by the Hamiltonian \(\hat{H}_d\)) in which only the decorating (\(B\)) atoms interact with the transverse field. In this section, we at first briefly repeat the main formulae derived in the Ref. \[11\] and then we further extend the formalism for the calculation of several interesting thermodynamic quantities of this model.

Following the same steps as in Ref. \[11\], one obtains a simple relation between the partition function of the decorated transverse Ising model (\(Z_d\)) and that of the spin-1/2 Ising model on the original lattice (\(Z_0\)). Namely,

\[
Z_d(\beta, J, \Omega, H_A, H_B) = A(\beta, J, \Omega, H_B)^{Nq/2} Z_0(\beta, R, H),
\]

where \(A\) is a constant.
where $\beta = 1/k_B T$, $k_B$ is Boltzmann constant and $T$ is the absolute temperature. The parameters $A, R$ and $H$ represent so-called decoration-iteration parameters that specify the corresponding original (undecorated) system as well as decorating system. All these parameters can be obtained from the decoration-iteration transformation (see Ref. [1]) and they are, respectively, given by

$$A = (V_1 V_2 V_3^2)^{1/4},$$

$$\beta R = \ln \frac{V_1 V_2}{V_3^2},$$

$$\beta H = \beta H_A + \frac{q}{2} \ln \frac{V_1}{V_2},$$

(3)

where we have defined the functions $V_1, V_2$ and $V_3$ as follows:

$$V_1 = \sum_{n=-S_B}^{S_B} \cosh \left( \beta n \sqrt{(J + H B)^2 + \Omega^2} \right),$$

$$V_2 = \sum_{n=-S_B}^{S_B} \cosh \left( \beta n \sqrt{(J - H B)^2 + \Omega^2} \right),$$

$$V_3 = \sum_{n=-S_B}^{S_B} \cosh \left( \beta n \sqrt{H_B^2 + \Omega^2} \right).$$

(4)

It is worth noticing that the foregoing equations express the exact mapping relationship between the decorated model under investigation and the spin-1/2 model on the corresponding original lattice. One should also notice that this transformation is valid for arbitrary values of the decorating spin $S_B$ in the presence of the external longitudinal fields. Thus, the relevant thermodynamic quantities can be calculated even for the nonzero longitudinal fields. Unfortunately, after deriving the final equations, we have to set $H_A = H_B = 0$ since the original planar Ising models are exactly solvable for $H = 0$ only.

In order to investigate the magnetic properties of the system, we employ the well-known relations from the statistical mechanics and thermodynamics. Indeed, after a simple calculation one derives the following equations for the Gibbs free energy, internal energy, enthalpy, entropy and specific heat:

$$G_d = G_0 - \frac{N q k_B T}{2} \ln A,$$

(5)

$$U_d = \left( \frac{2U_0}{R} - \frac{N q}{4} \right) \sqrt{J^2 + \Omega^2} K_0,$$

(6)

$$H_d = \frac{2U_0}{R} \left( \sqrt{J^2 + \Omega^2 K_0 - \Omega K_1} \right) - \frac{N q}{4} \left( \sqrt{J^2 + \Omega^2 K_0 + \Omega K_1} \right),$$

(7)

$$S_d = S_0 + \frac{N q k_B}{2} \ln A - \frac{N q}{4T} \left( \sqrt{J^2 + \Omega^2 K_0 + \Omega K_1} \right),$$

$$S_0 = k_B \ln Z_0 + \frac{2U_0}{RT} \left( \sqrt{J^2 + \Omega^2 K_0 - \Omega K_1} \right),$$

(8)

and

$$C_\Omega = C_0 + \frac{N q}{4k_B T^2} \left[ (J^2 + \Omega^2)(K_2 - K_0^2) + \Omega^2(K_3 - K_1^2) \right],$$

(9)

where $G_d, U_d, H_d, S_d$ and $C_\Omega$ denote, respectively, the Gibbs free energy, internal energy, enthalpy, entropy and specific heat of the decorated lattice. Similarly, $G_0, U_0, S_0$ and $C_0$ ($C_0 = T(\partial S_0/\partial T)\Omega$) represent the relevant quantities of the original lattice that are well-known for two-dimensional
Finally, the coefficients $K_0 - K_3$ depend on the temperature and transverse field and they are given by

$$K_0 = F_1(J, \Omega), \quad K_1 = F_1(0, \Omega), \quad K_2 = F_2(J, \Omega), \quad K_3 = F_2(0, \Omega),$$  \hspace{1cm} (10)

where

$$F_1(x, y) = \frac{\sum_{n=-S_B}^{S_B} n \sinh(\beta n \sqrt{x^2 + y^2})}{\sum_{n=-S_B}^{S_B} \cosh(\beta n \sqrt{x^2 + y^2})},$$  \hspace{1cm} (11)

$$F_2(x, y) = \frac{\sum_{n=-S_B}^{S_B} n^2 \cosh(\beta n \sqrt{x^2 + y^2})}{\sum_{n=-S_B}^{S_B} \cosh(\beta n \sqrt{x^2 + y^2})}. \hspace{1cm} (12)$$

Next, we turn to the calculation of the magnetization. The spontaneous longitudinal magnetization, as well as transverse one can be directly obtained by the differentiation of the Gibbs free energy (5) with respect to the relevant longitudinal and transverse magnetic fields. Consequently, the spontaneous longitudinal magnetization per spin can be written for both sublattices in the following compact form:

$$m^z_A = m_0,$$

$$m^z_B = m_0 \frac{2J}{\sqrt{J^2 + \Omega^2}} K_0,$$  \hspace{1cm} (13)

where $m^z_A$ ($m^z_B$) means the spontaneous longitudinal magnetization of sublattice $A$ ($B$) and $m_0$ stands for the spontaneous magnetization of the original lattice ($m_0$ depends only on the exchange interaction $R$ and temperature). Similarly, the same procedure leads to the simple relation for the reduced transverse magnetization of sublattice $B$. Namely,

$$m^x_B = \frac{1}{2} \left( \frac{\Omega}{\sqrt{J^2 + \Omega^2}} K_0 + K_1 \right) + 2\varepsilon_0 \left( \frac{\Omega}{\sqrt{J^2 + \Omega^2}} K_0 - K_1 \right).$$  \hspace{1cm} (14)

Here, $\varepsilon_0$ denotes the two-spin correlation function between nearest-neighboring spins of the original lattice (depending again only on the exchange parameter $R$ and temperature). To complete the analysis of the system under investigation, one has to investigate the phase boundaries, as well as a possibility of compensation phenomena in the case of the ferrimagnetic ordering ($J < 0$). The structure of equations for the spontaneous sublattice magnetization implies that the phase transition temperature $T_c$ (or $\beta_c = 1/(k_B T_c)$) can be directly found from the transformation formula after setting $H_A = H_B = H = 0$ and substituting the inverse critical temperature $\beta_c R$ of the relevant original lattice into (4). In this way one obtains the relation

$$\beta_c R = 2 \ln \frac{\sum_{n=-S_B}^{S_B} \cosh(\beta_c n \sqrt{J^2 + \Omega^2})}{\sum_{n=-S_B}^{S_B} \cosh(\beta_c n \Omega)},$$  \hspace{1cm} (15)

which is valid for any planar lattice with the decorating spin $S_B$ and for the special case of $S_B = 1/2$ naturally recovers Eq.(13) in the Ref. [11].

Moreover, in the case of ferrimagnetic system the compensation temperature $T_k$ (or $\beta_k = 1/(k_B T_k)$) can be found from the condition that the total magnetization $M$ of the system vanishes
bellow the critical temperature. Since in our case we have \( M = N(m_A + qm_B/2) \), then from the condition \( M = 0 \) one easily finds the following relation for the compensation temperature

\[
\frac{q|J|}{\sqrt{J^2 + \Omega^2}} K_0(\beta_k) = 1, 
\]

(16)

where the inverse compensation temperature \( \beta_k \) is also included in the coefficient \( K_0 \).

Finally, we determine the transverse susceptibility of the decorated transverse Ising system. For this aim, we differentiate the formula (14) for the transverse magnetization with respect to the transverse field and after a straightforward but a little bit tedious algebra, we can write the transverse susceptibility \( \chi_T \) in the form

\[
\chi_T = \frac{1}{2} \left\{ \frac{J^2}{(J^2 + \Omega^2)^{3/2}} K_0 + \frac{\beta \Omega^2}{J^2 + \Omega^2} (K_2 - K_0^2) + \beta (K_3 - K_1^2) \right\} 
+ 2\varepsilon_0 \left\{ \frac{J^2}{(J^2 + \Omega^2)^{3/2}} K_0 + \frac{\beta \Omega^2}{J^2 + \Omega^2} (K_2 - K_0^2) - \beta (K_3 - K_1^2) \right\} 
+ 4\varepsilon_0 \frac{\partial}{\partial J} \left\{ \frac{\Omega}{\sqrt{J^2 + \Omega^2}} (K_0 - K_1) \right\}^2. 
\]

(17)

In above, \( \varepsilon_0 \) means the two-spin correlation function between nearest-neighboring atoms of the original lattice.

### 3 Numerical results

In this section we will illustrate the effect of the transverse field, as well as the influence of the decorating spin \( S_B \) on magnetic properties of the system under investigation. Moreover, the role of the lattice topology will be also examined. Although we will restrict our numerical calculation to the ferimagnetic case (\( J < 0 \)) only, the detailed investigation reveals that all dependences (excepting those for the longitudinal magnetization) remain unchanged also for the ferromagnetic version of the model. This observation follows from the fact that the relevant equations are independent under transformation \( J \to -J \).

We start our discussion with the analysis of the critical and compensation temperatures. At first, the variations of the critical (dashed lines) and compensation temperatures (solid lines) with the transverse field are shown in Fig. 2. Here, we have selected the system with \( q = 4 \) (i.e. the decorated square lattice), taking different spin values of decorating atoms (\( S_B \)). On the other hand, in Fig. 3 we have illustrated the influence of the lattice topology (of different \( q \)) on the critical and compensation temperature for the system with the fixed decorating spin (\( S_B = 1 \)). In both figures, the ordered ferrimagnetic phase is stable bellow dashed lines, and the disordered paramagnetic one becomes stable above the relevant boundary. A closer mathematical analysis reveals that the phase transition between these two phases is of the second order and belongs to the same universality class as that of the usual spin-1/2 Ising model. As one can see, the qualitative features of the results do not significantly depend neither on the lattice topology nor the spin value of atoms of sublattice \( B \). In fact, the critical temperature monotonically decreases with increasing in the transverse field, but only in the limit of the infinity strong transverse field tends to zero (due to the fact that the transverse field affects only one sublattice). One also observes here that the value of transition point increases with the coordination number of the original lattice, as well as with the spin value of decorating atoms. Contrary to this behavior, the compensation temperature seems to be independent of the transverse field strength (with the accuracy of twelve orders), although it changes both with the coordination number \( q \) and the spin value \( S_B \). In general, on basis of our numerical calculation one can state that for arbitrary but fixed \( q \) and \( S_B \) the compensation points appear only for \( \Omega_k/|J| = \sqrt{q^2 S_B^2 - 1} \). It is easy to find that this characteristic value of \( \Omega \) can be obtained from the Eq. (16) by taking the limit \( T_k \to 0 \) (or \( \beta_k \to \infty \)). Physically, the independence of the compensation temperature on the transverse field comes from the fact that the compensation effects appear at relatively strong transverse fields.
where the relevant transition temperature is rather low and this fact significantly influences the behavior of sublattice magnetization.

Now, we turn to the discussion of the internal energy and enthalpy. Owing to the fact, that all decorated lattices behave similarly, we will further present numerical results for one representative lattice, namely, the decorated square lattice. In Fig. 4 we have depicted the thermal variations of the internal energy and enthalpy for the spin case $S_B = 1/2$ ($N_t$ denotes a total number of atoms). From these dependences one finds that both quantities tend monotonically to zero with increasing the temperature. Nevertheless, if we compare both dependences, we can conclude that the enthalpy is more sensitive (changes the shape of the curve more rapidly) to the transverse field than the internal energy. It is also clear that the thermal dependences of the internal energy (as well enthalpy) exhibit a typical weak energy-type singularity behaviour irrespective of the strength of transverse field.

Further, we have also examined the temperature and transverse field dependences of the spontaneous longitudinal magnetization. In Figs. 5 and 6 we report some typical results for the total spontaneous longitudinal magnetization, when the value of the transverse field is changed (we choose the magnetization curves at temperatures $k_B T/J = 0.1$ and $k_B T/J = 0.2$ and different spin values of decorating atoms from 1/2 until 2). It follows from these dependences that the total magnetization may exhibit two possible shapes of the magnetization curves. Namely, the magnetization curve with one compensation point (at lower temperatures) and the downward magnetization curvature without any compensation points (at higher temperatures). Both types of the magnetization curves are closely related to the fact that the transverse field does not directly act on the atoms of original lattice. Hence, the spontaneous magnetization of sublattice $A$ varies very smoothly with the transverse field, whereas the spontaneous magnetization of sublattice $B$ is rapidly destroyed with the transverse field increasing. On the other hand, the temperature dependences of the spontaneous magnetization of both sublattices are standard, regardless of the transverse field strength. The only exceptional case arises as the transverse field reaches the value $\Omega_c$ at which the considered system behaves similarly as an ordinary antiferromagnet.

In contrast to the standard temperature variations of the longitudinal magnetization, the transverse magnetization may display very interesting and unexpected thermal behaviour. To illustrate the case, we have depicted in Figs. 7 and 8 some typical temperature dependences of the transverse magnetization. As shown in Fig. 7 for the spin case $S_B = 2$, the transverse magnetization at relatively small transverse field ($\Omega/J = 1.5$) firstly gradually decreases to its local minimum value and then nearby the transition point increases in the narrow temperature region. However, the transverse magnetization by the stronger transverse fields ($\Omega/J = 2.0$ and 3.0) remains almost constant and again in the vicinity of the Curie point gradually increases, too. Far beyond the transition temperature, the transverse magnetization monotonically decreases with increasing in temperature, regardless of the transverse field strength. The possible explanation of the temperature-induced increase of the transverse magnetization can be related to the spin release from the spontaneous magnetization direction and spin-ordering towards the transverse field direction. In fact, the stronger the transverse field, the smaller the transition temperature and therefore, by sufficiently strong transverse field the transverse magnetization increases from its initial value, since the thermal reshuffling is relatively small in this temperature region. On the other hand, when the transverse field is smaller, the relevant spin reorientation takes place at higher temperatures, thus the transverse magnetization firstly decreases to its local minimum due to the strong thermal fluctuations in this temperature region. Apparently, also the increase in the transverse magnetization which is connected with the spin reorientation is then smaller, since it is overlapped with the stronger thermal fluctuation. Before proceeding further, we have depicted in Fig. 8 the transverse magnetization against the temperature that illustrate the influence of the different values of spin variable $S_B$.

Furthermore, let us now look more closely at the thermal variations of the specific heat For this purpose, we have studied the specific heat of the simplest spin case $S_B = 1/2$. As one can see from Fig. 9, the logarithmical singularity in the specific heat dependence indicates the second order phase transition towards the paramagnetic state. Moreover, it can be also clearly seen that the broadening of the maximum in the paramagnetic region of the specific heat arises due to the
transverse field effect. It turns out that the observed maximum may be thought as a Schottky-type maximum, which has its origin in the thermal excitation of the paramagnetic spins inserted into the transverse field. Next, in order to illustrate the effect of increasing spin $S_B$ we have shown in Fig. 10 the thermal dependences of specific heat for several values of the spin variable $S_B$. As one can ascertain, the lower the spin value $S_B$ of decorating atoms, the stronger the influence of the transverse field on the excitation of these spins in the paramagnetic region. To complete our analysis of the specific heat, we have plotted in Fig. 11 the transverse-field dependences of the specific-heat for $S_B = 1/2$. As one can expect, the depicted behaviour strongly depends on the temperature and above the critical point, the dependence changes suddenly due to the paramagnetic character of the system.

Moreover, in Fig. 12 we display the transverse susceptibility against the transverse field for the same spin case as in Fig. 11. Here one observes, that for sufficiently high temperature (see the dashed line), the transverse susceptibility falls down monotonically with the increasing transverse field. Contrary to this behavior, for the lower temperature (see the solid line), the transverse susceptibility exhibits the standard singularity due to the second order phase transition from the ferrimagnetic state to the paramagnetic one. For completeness, in the insert of Fig. 12, we compare the variations of the transverse susceptibilities for different spin values of decorating atoms. Apparently, there is no essential difference in the behaviour of the transverse susceptibilities, although it is shifted to higher temperatures, as the spin value of the decorating atom is increased. Finally, in Fig. 13 we have depicted the temperature dependence of transverse susceptibility for some selected values of the transverse field. Here one can see, that for the smaller transverse fields, for instance $(\Omega/J = 0.5)$, the transverse susceptibility decreases with increasing the temperature, then diverges at $T_c$ and repeatedly decreases. However, in the case of the stronger transverse fields $(\Omega/J = 1.0 \text{ and } 1.5)$, the transverse susceptibility remains at its initial value, exhibits a divergence at phase transition and afterwards whether gradually decreases (the case $\Omega/J = 1.0$) or exhibits a broad maximum (the case $\Omega/J = 1.5$). The existence of this broad maximum in the paramagnetic region arises evidently on account of the thermal excitation of the paramagnetic spins inserted into the transverse field and therefore, can be observed for relatively strong transverse fields only.

4 Conclusion

In this work we have presented the exact results (the phase diagrams, compensation temperatures, spontaneous longitudinal magnetization, transverse magnetization, internal and free energy, enthalpy, entropy, specific heat and transverse susceptibility) for the ferrimagnetic transverse Ising model on decorated planar lattices. We have illustrated that the magnetic properties of the system under investigation exhibit the characteristic behaviour depending on the strength of the applied transverse field and the spin of the decorating atoms. In particular, we have found that the considered ferrimagnetic system does not exhibit more than one compensation temperature that is surprisingly completely independent of the transverse field, though it depends on the coordination number of the original lattice and also on the spin of the decorating atoms. As far as we know, such a finding has not been reported in the literature before. Perhaps, the most interesting result to emerge here is the temperature-induced increase of the transverse magnetization in the vicinity of the transition temperature. We have found a strong evidence that this increase arises due to the spin release from the spontaneous magnetization direction, since in the vicinity of the transition temperature spins tending to align into the transverse field direction.

Finally, we would like to emphasize that the presented method can be applied to more complex and realistic models, for example, the transverse Ising models with a crystal field anisotropy, or those with next-nearest-neighbor and multispin interactions. Further generalizations are possible by increasing the spin of atoms on the sublattice $A$ or introducing more realistic Heisenberg interactions. In addition to the above mentioned generalizations, one can also obtain very accurate results for this model on three-dimensional lattice. This can be done by combining the present method with other accurate methods, such as series expansion technique, Monte Carlo simulations or renormalization group methods.

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Figure captions

Fig. 1  Part of the decorated square lattice. The black circles denote the spin-1/2 atoms of sublattice A (referred to as the atoms of original lattice) and gray circles represent the spin-$S_B$ atoms of sublattice B (decorating atoms).

Fig. 2  Phase boundaries (dashed lines) and the compensation temperatures (solid lines) in the $\Omega - T$ plane for the decorated square lattice ($q = 4$) and different spin values of the decorating atoms.

Fig. 3  Critical (dashed lines) and compensation (solid lines) temperatures as a function of the transverse field for various decorated planar lattices with the fixed decorating spin $S_B = 1$.

Fig. 4  Thermal variations of the internal energy (dashed lines) and enthalpy (solid lines) when the transverse field is changed.

Fig. 5  Total longitudinal magnetization per one site versus transverse field for $k_B T/|J| = 0.1$ and different spins of decorating atoms.

Fig. 6  Total longitudinal magnetization versus transverse field for $k_B T/|J| = 0.2$ and different spins of decorating atoms.

Fig. 7  Transverse magnetization as a function of temperature for $S_B = 2$ and different transverse fields.

Fig. 8  Thermal dependences of the transverse magnetization for several spin values of decorating atoms, when the transverse field is fixed ($\Omega/|J| = 0.5$).

Fig. 9  Specific-heat variations with the temperature for selected values of the transverse field.

Fig. 10  Thermal variations of the specific heat for different spin cases of decorating atoms and the fixed transverse field value ($\Omega/|J| = 1.5$).

Fig. 11  Specific heat dependence on the transverse field when the temperature is changed.

Fig. 12  Transverse susceptibility against the transverse field for selected temperatures. In the insert, the comparison between different spin cases is shown.

Fig. 13  Transverse susceptibility dependence on the temperature for the spin $S_B = 2$, when the transverse field is changed.
Fig. 1, Strecka et al
Fig. 2, Strecka et al

\[ k_B T_k / |J| \text{ and } k_B T_c / |J| \]

\[ \Omega / |J| \]

\[ q = 4 \]

\[ S_B = 2 \]

\[ 3/2 \]

\[ 1 \]

\[ 1/2 \]
$S_B = 1$

Fig. 3, Strecka et al.
\[ \Omega / |J| = 2.0 \]

\[ S_B = 1/2 \]

\[ q = 4 \]

Fig. 4, Strecka et al
$S_B = 2 \quad k_B T / |J| = 0.1 \quad q = 4$
Fig. 6, Strecka et al.

The graph shows the total magnetization as a function of \(\frac{\Omega}{|J|} \) for different values of quantum numbers:

- \(S_B = 2\)
- \(S_B = 3/2\)
- \(S_B = 1\)
- \(S_B = 1/2\)

The condition is given as \(k_B T / |J| = 0.2 \) and \(q = 4\).
transverse magnetization

Ω / |J| = 3.0

SB = 2
q = 4

Fig. 7, Strecka et al.
\[ \Omega / |J| = 0.5 \]

\[ q = 4 \]

\[ S_B = 3/2 \]

\[ \frac{k_B T}{|J|} \]

\[ \text{transverse magnetization} \]

Fig. 8, Strecka et al.
$S_B = 1/2$
$q = 4$

$\Omega / |J| = 0.5$

$C_{\Omega} / N_t k_B$

$k_B T / |J|$

Fig. 9, Strecka et al.
\[ C_{\Omega} / N_t k_B = 1.5 \]

\[ \Omega / |J| = 1.5 \]

\[ q = 4 \]

\[ S_B = 2 \]

\[ k_B T / |J| \]
\[ S_B = \frac{1}{2} \]
\[ q = 4 \]
\[ k_B T / |J| = 0.1 \]
\[
\chi_T / |J| = \frac{k_B T}{|J|} = 0.5
\]

\[
S_B = \frac{1}{2}, \quad q = 4
\]

Fig. 12, Strecka et al.
\[ S_B = 2 \]
\[ q = 4 \]

\[ \chi_T \frac{|J|}{k_B T} = 1.5 \]