Lepton-flavour violating $B$ decays in generic $Z'$ models

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LHCb has reported deviations from the Standard-Model expectations in $B \to K^* \mu^+ \mu^-$ angular observables, in the $B_s \to \phi \mu^+ \mu^-$ decay rate and in the ratio $R(K) = \text{Br}[B \to K \mu^+ \mu^-]/\text{Br}[B \to K e^+ e^-]$. For all three decays, a heavy neutral gauge boson mediating $b \to s \mu^+ \mu^-$ transitions is a prime candidate for explaining the observed tensions. As $R(K)$ measures violation of lepton-flavour universality, it is interesting to examine the possibility that also lepton flavour is violated, especially in the light of the CMS excess in $h \to \tau^+ \tau^-$. In this article, we investigate the perspectives to discover the lepton-flavour violating modes $B \to K^{(*)} \tau^+ \tau^-$, $B_s \to \tau^+ \tau^-$ and $B \to K^{(*)} \mu^+ \mu^-$. For this purpose we consider a simplified model in which new-physics effects originate from an additional neutral gauge boson ($Z'$) with generic couplings to quarks and leptons. The constraints from $\tau \to 3\mu$, $\tau \to \mu \nu \nu$, $\mu \to e\gamma$, $g_\mu - 2$, neutrino trident production, $Z \to \ell^+ \ell^-$, semi-leptonic $b \to s \mu^+ \mu^-$ decays and $B_s - \bar{B}_s$ mixing are examined. Taking into account these limits, we determine upper bounds on the decay rates of $B \to K^{(*)} \tau^+ \tau^-$, $B_s \to \tau^+ \tau^-$, $B \to K^{(*)} \mu^+ \mu^-$ and $B_s \to \mu^+ \mu^-$ as a function of fine-tuning in $B_s - \bar{B}_s$ mixing. We find that for a significant degree of fine-tuning the effect in all considered decays with $\tau \mu$ final states can be sizable (leading to branching ratios of the order of $10^{-6}$). For $\mu \mu$ final states, $B_s \to \mu^+ \mu^-$ is strongly suppressed and only $B \to K^{(*)} \mu^+ \mu^-$ can be relevant (of the order of $10^{-7}$). However, this is only possible in a region of the parameter space disfavoured by the current LHCb data on $b \to s \mu^+ \mu^-$ transitions.

I. INTRODUCTION

While most flavour observables agree very well with their Standard-Model (SM) predictions, there are some exceptions in semi-leptonic $B$ decays (see for example [1] for a recent review). LHCb [2] recently found indications for the violation of lepton-flavour universality in the ratio

$$R(K) = \frac{\text{Br}[B \to K \mu^+ \mu^-]}{\text{Br}[B \to K e^+ e^-]} = 0.749_{-0.074}^{+0.090} \pm 0.036,$$

which deviates from the theoretically clean SM prediction $R_{K}^{\text{SM}} = 1.0003 \pm 0.0001$ [4] by 2.6$\sigma$. In addition, LHCb has reported deviations from the SM predictions [4,7] in the decay $B \to K^{(*)} \mu^+ \mu^-$ (mainly in an angular observable called $P_{K}^*$ [3]) with a significance of about 3$\sigma$ [1,10]. Furthermore, also the measurement of $\text{Br}[B_s \to \phi \tau^+ \tau^-]$ disagrees with the SM prediction [11,12] by about 3$\sigma$ [6].

Interestingly, three discrepancies can be explained in a model-independent approach by a rather large new-physics (NP) contribution $C_9^{\mu\mu}$ to the Wilson coefficient of the operator $G_9^{\mu\mu}$ (the component of the usual SM operator $O_9$ that couples to muons, see eq. [3]) [13,19]. It is encouraging that the value for $C_9^{\mu\mu}$ required to explain $R(K)$ (with $C_9^{ee} = 0$) is of the same order as the one needed for $B \to K^{(*)} \mu^+ \mu^-$ and $B_s \to \phi \mu^+ \mu^-$ [6,20]. Taking into account the 3$\sigma$ data for $B \to K^{(*)} \mu^+ \mu^-$ recently released by the LHCb collaboration [10], the global significance for a scenario with a non-vanishing NP contribution to $C_9$ is found to be 3.7$\sigma$ (or even 4.3$\sigma$ for NP contributing to $C_9^{\mu\mu}$ only), and 3.1$\sigma$ in a scenario with $C_9^{\mu\mu} = -C_{10}^{\mu\mu}$ [18].

Many models proposed to explain the $s \to \mu^+ \mu^-$ data contain a heavy neutral gauge boson ($Z'$) which generates a tree-level contribution to $C_9^{\mu\mu}$ [13,21,25]. In case the $Z'$ couples differently to muons than to electrons, lepton-flavour universality is violated and $R(K)$ can be explained simultaneously [25,29].

Since $R(K)$ is a measure of lepton-flavour-universality violation, it has been proposed to search for lepton-flavour violating (LFV) $B$ decay modes as well [30]. This is also motivated by the CMS excess in $\text{Br}[h \to \mu \tau]$ [31] which can be explained simultaneously together with $\text{Br}[h \to \mu \tau]$, $R(K)$, $\text{Br}[B_s \to \phi \mu^+ \mu^-]$ and $\text{Br}[B \to K^{(*)} \mu^+ \mu^-]$ within a single model [26,27].

While the specific model of Refs. [26,27] predicts only small effects in LFV $B$ decays, the situation could be different in a generic model. In this article we examine the LFV decays $B \to K^{(*)} \tau^+ \tau^-$, $B_s \to \tau^+ \tau^-$ (and the corresponding $\mu^+ \mu^-$ channels) studying a simplified model in which the NP effects originate from a heavy new gauge boson $Z'$ with generic couplings to quarks and leptons [53]. We thus introduce a $Z'$ boson with mass $M_{Z'}$ and arbitrary couplings to $s\bar{b}$ and charged lepton pairs $\ell, \ell'$ = $\tau, \mu, e$:  

$$\mathcal{L}_{Z'} \supset \Gamma_{\ell\ell'}^{\mu \nu} \ell^\mu \ell'^\nu + \Gamma_{\ell\ell}^{s \bar{s}} s^\mu s'^\nu P_L b + L \leftrightarrow R. \quad (2)$$

As the $Z'$ is assumed to be much heavier than the scale of electroweak symmetry breaking, its couplings must respect $SU(2)_L$ gauge invariance. This implies that the couplings to neutrinos are the same as the ones to left-handed charged leptons: $\Gamma_{\ell\ell}^{\mu \nu} = \Gamma_{\nu \nu}^{\mu \nu}$. To study bounds on the LFV $B$ decay modes, we perform the following steps:
FIG. 1: Feynman diagrams illustrating the steps 1-4 of our analysis (see text). The diagrams display the dominant $Z'$ contribution to $B_s - \bar{B}_s$ mixing, $\bar{B} \to K^{(*)} \mu^+ \mu^-$, $B_s \to \phi \mu^+ \mu^-$, $\tau \to 3\mu$, $\tau \to \mu \nu \bar{\nu}$ and $\bar{B} \to K^{(*)} \tau^+ \mu^-$. 

1. From $B_s - \bar{B}_s$ mixing we obtain upper limits on $\Gamma_{sb}^L$ as a function of an imposed fine-tuning measure.

2. Motivated by model-independent analyses and using fits to $B \to K^\ast \mu^+ \mu^-$, $B_s \to \phi \mu^+ \mu^-$ and $R(K)$, we consider two scenarios for the $Z'$ couplings to leptons: Scenario 1 assumes vectorial couplings, i.e. $\Gamma_{\ell\ell}^V = \Gamma_{\ell\ell}^R \equiv \Gamma_{\ell\ell}^V$, corresponding to $C_{10}^{(\ast)} = C_{10}^{(\ast)} = 0$. Scenario 2 allows left-handed couplings, i.e. $\Gamma_{\ell\ell}^R = 0$, corresponding to $C_{9}^{(\ast)} = -C_{10}^{(\ast)}$.

3. In the lepton sector the $Z'$ couplings can be constrained by $\tau \to 3\mu$, $\tau \to \mu \nu \bar{\nu}$, $Z \to \ell\ell^{(\ast)}$ and trident neutrino production.

4. Taking into account these constraints we derive upper limits on the branching ratios of the LFV decays $B_s \to \tau^+ \mu^-$, $B \to K^{(*)} \tau^+ \mu^-$. 

In Fig. 1 we show the Feynman diagrams for the dominant $Z'$ contribution corresponding to the steps 1-4 of our analysis. We apply a similar procedure to $\mu^+ e^- \bar{\nu}$ final states. In this case with the best bounds on the lepton couplings are coming from $\mu \to e\gamma$ and $\mu \to e\nu \bar{\nu}$.

II. PROCESSES AND OBSERVABLES

In this section we collect formulates for the LFV $B$ decays and for the processes relevant to constrain these decays, before we study in the next section the phenomenological impact.

A. $B_s - \bar{B}_s$ mixing

$B_s - \bar{B}_s$ mixing is governed by the effective Hamiltonian

$$H_{eff}^{\Delta F=2} = \sum_{j=1}^{5} C_j O_j + \sum_{j=1}^{3} C'_j O'_j ,$$

with the full set of $\Delta F = 2$ operators $O^{(i)}$ given e.g. in Refs. [33, 35]. In a $Z'$ model with generic couplings the operators with non-vanishing Wilson coefficients are

$$O_1 = [\bar{s}_\alpha \gamma^\mu P_L b_\beta] [\bar{s}_\alpha \gamma^\mu P_L b_\beta] ,$$

$$O_5 = [\bar{s}_\alpha P_L b_\beta] [\bar{s}_\beta P_R b_\alpha] ,$$

with $O'_j$ obtained from $O_j$ by interchanging $L \leftrightarrow R$. The coefficients are given by

$$C_1^{(i)} = \left( \frac{L_{sb}^{(R)}}{2M_{Z'}^2} \right)^2 , \quad C_5 = -\frac{2L_{sb}^{(R)}M_{Z'}}{M_{Z'}^2} .$$

They enter physical observables, i.e. mass differences and CP asymmetries, via the calculation of matrix elements involving decay constants and bag factors calculated with lattice QCD (see for example [36] for a review). In addition, QCD renormalization group effects must be taken into account. To this end we use the next-to-leading order equations calculated in Refs. [31, 35].

B. $B \to K^\ast \mu^+ \mu^-$, $B_s \to \phi \mu^+ \mu^-$ and $R(K)$

$b \to s\ell^+ \ell^-$ transitions are governed at leading order in $\alpha_s$ by the effective Hamiltonian

$$H_{eff}^{\ell\ell} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^{\ast} \sum_{j=9}^{10} \left( C_i^{(\ell\ell)} O_i^{(\ell\ell)} + C_i^{(\ell\ell)} O'_i^{(\ell\ell)} \right) ,$$

$$O_9^{\ell\ell} = \frac{\alpha}{4\pi} [\bar{s}_\gamma^\mu P_L b_\ell] [\bar{\ell}_\gamma^\mu \ell] ,$$

$$O_{10}^{\ell\ell} = \frac{\alpha}{4\pi} [\bar{s}_\gamma^\mu P_L b_\ell] [\bar{\ell}_\gamma^\mu 5\ell] ,$$

where again for the primed Wilson coefficients one should replace $P_L \leftrightarrow P_R$. We have omitted the magnetic operator $O_7$ contributing to $b \to s\ell^+ \ell^-$ transitions through photon exchange. In our model, contributions to $C_7$ are loop-suppressed, while the coefficients $C_9^{(\ell\ell)}$ and $C_{10}^{(\ell\ell)}$ are induced at tree-level and read

$$C_9^{(\ell\ell)} = -\frac{\pi}{\sqrt{2}M_{Z'}^2} \frac{1}{\alpha G_F V_{tb} V_{ts}^{\ast}} L_{sb}^{(R)} \left( \Gamma_{\ell\ell}^{R} + \Gamma_{\ell\ell}^{L} \right) ,$$

$$C_{10}^{(\ell\ell)} = -\frac{\pi}{\sqrt{2}M_{Z'}^2} \frac{1}{\alpha G_F V_{tb} V_{ts}^{\ast}} L_{sb}^{(R)} \left( \Gamma_{\ell\ell}^{L} - \Gamma_{\ell\ell}^{R} \right) .$$

As first noted in Ref. [33, 37], $C_9^{(\mu\mu)} < 0$ with $C_9^{(\mu\mu)}, C_{10}^{(\mu\mu)} \sim 0$ gives a good fit to $B \to K^\ast \mu^+ \mu^-$.
Another interesting solution is given by $C_9^{\mu\mu} = -C_{10}^{\mu\mu}$ [6, 18]. As outlined in the introduction, we thus study the two scenarios of vectorial $Z'$ couplings $\Gamma_{Z'}^{L} = \Gamma_{Z'}^{R} = \Gamma_{Z'}^{V}$ inducing only a non-vanishing $C_9^{\mu\mu}$ (possibly accompanied by a small $C_{10}^{\mu\mu}$), and of left-handed $Z'$ couplings inducing $C_9^{\mu\mu} = -C_{10}^{\mu\mu}$ (possibly accompanied by a small $C_{10}^{\mu\mu} = -C_{10}^{\mu\mu}$). In our analysis we use the global fit of Ref. [25], resulting for the two scenarios under consideration in

\[ -0.53 (-0.81) \geq C_9^{\mu\mu} \geq (-1.32) - 1.54, (7) \]
\[ -0.18 (-0.35) \geq C_{10}^{\mu\mu} = -C_{10}^{\mu\mu} \geq (-0.71) - 0.91, (8) \]

at the (1 $\sigma$) 2 $\sigma$ level, respectively. The quoted ranges are in good agreement with preliminary results of Ref. [19].

Note that here we have neglected such $Z'$ loop corrections to the SM diagram of $W$ exchange that would generate a neutrino flavour configuration different from $\nu_{\tau}\bar{\nu}_{\mu}$ as those terms would be both loop- and $1/m_Z^2$ suppressed. Following Ref. [25] we use the PDG value [38]

\[ \text{BR}(\tau \to \mu \nu \bar{\nu})_{\text{exp}} = (17.41 \pm 0.04)\%. \quad (10) \]

This should be compared to the SM prediction [39]

\[ \text{BR}(\tau \to \mu \nu \bar{\nu})_{\text{SM}} = \tau_{\tau} (5.956 \pm 0.002) \times 10^{11} / \text{s}. \quad (11) \]

The dominant uncertainty in the SM prediction for the branching ratio comes from the $\tau$ lifetime $\tau_{\tau}$. Combining the result on the $\tau$ lifetime from Belle [40] with previous LEP [41, 42] and CLEO [43] measurements, gives $\tau_{\tau} = (290.29 \pm 0.53) \times 10^{-15}$ s. Using this value in the SM prediction for $\text{BR}(\tau \to \mu \nu \bar{\nu})$, we find that the experimental value in Eq. (10) is more than 2$\sigma$ above the SM prediction. Translated into the variable $\Delta_{\tau \to \mu \nu \bar{\nu}}$, one obtains

\[ \Delta_{\tau \to \mu \nu \bar{\nu}} = \text{BR}(\tau \to \mu \nu \bar{\nu})_{\text{SM}} - \text{BR}(\tau \to \mu \nu \bar{\nu})_{\text{exp}} = (-7.0 \pm 3.0) \times 10^{-3}. \quad (12) \]

In the case of non-zero values of $\Gamma_{\mu e}$, in which similar contributions to the muon decay $\mu \to e\nu\bar{\nu}$ are generated, we require

\[ |\Delta_{\mu \to e\nu\bar{\nu}}| \leq 4 \times 10^{-5}. \quad (13) \]

This choice restricts corrections to the Fermi-constant, defined through the decay $\mu \to e\nu\bar{\nu}$, to the sub per-mille level and avoids in this way conflicts with electroweak precision data.

### D. $Z'$ couplings to leptons

The $Z'$ boson also generates loop corrections to the $Z\ell\ell$ vertex of the SM $Z$ boson, possibly inducing LFV $Z\ell\ell'$ couplings. Generalizing the results of Ref. [25] to the case of chiral and LFV couplings, we find for the relevant branching ratios
\[
\frac{\text{Br}[Z \to \mu^+\mu^-]}{\text{Br}[Z \to \mu^+\mu^-]_{\text{SM}}} = 1 + \sum_{\ell=e,\mu,\tau} \frac{g_L^2 |\Gamma_{\mu\ell}|^2 + g_R^2 |\Gamma_{\ell\mu}|^2}{(4\pi)^4 (g_L^2 + g_R^2)} |K (m^2_{Z^0}/m^2_{Z^0})|^2, \quad (14)
\]
\[
\frac{\text{Br}[Z \to \mu^+\tau^-]}{\text{Br}[Z \to \mu^+\mu^-]_{\text{SM}}} = 2 \sum_{\ell=e,\mu,\tau} \frac{g_L^2 |\Gamma_{\mu\ell}\Gamma_{\ell\mu}|^2 + g_R^2 |\Gamma_{\mu\ell}\Gamma_{\mu\ell}|^2}{(4\pi)^4 (g_L^2 + g_R^2)} |K (m^2_{Z^0}/m^2_{Z^0})|^2, \quad (15)
\]

where \( g_L = 1 - 2s_W^2 \), \( g_R = -2s_W^2 \) (with \( s_W = \sin \theta_W \) and \( \theta_W \) denoting the weak mixing angle), and
\[
K(x) = -\frac{4 + 7x}{2x} + \frac{2 + 3x}{x} \log(x) - \frac{2(1 + x)^2}{x^2} \left[ \ln(x) \ln(1 + x) + \text{Li}_2(-x) \right] + i\pi \left[ -\frac{2 + 3x}{x} + \frac{2(1 + x)^2}{x^2} \ln(1 + x) \right]. \quad (16)
\]

Comparing the SM prediction
\[
\text{Br}[Z \to \mu^+\mu^-]_{\text{SM}} = 3.366\% \quad (17)
\]
with the experimental results \[38\]
\[
\text{Br}[Z \to \mu^+\mu^-]_{\text{exp}} = (3.366 \pm 0.007)\%, \quad (18)
\]
\[
\text{Br}[Z \to \tau^+\tau^-]_{\text{exp}} \leq 1.2 \times 10^{-5}, \quad (19)
\]
one obtains bounds on the \( Z' \) couplings to muons and taus.

E. \( \tau \to 3\mu \) and \( \mu \to 3e \)

The \( Z' \) boson mediates at tree-level the LFV three body decay \( \tau \to 3\mu \), with the branching ratio given by \( \text{Br}[\tau \to 3\mu] \) (compare e.g. \[46, 47\]).

\[
\text{Br}[\tau \to 3\mu] = \frac{m_{\tau}^5}{1536 \pi^3 M_{Z'}^2} \left( 2 \left( |\Gamma_{\mu\tau}\Gamma_{\mu\mu}|^2 + |\Gamma_{\mu\mu}\Gamma_{\mu\mu}|^2 \right) + |\Gamma_{\mu\tau}\Gamma_{\mu\mu}|^2 + |\Gamma_{\mu\mu}\Gamma_{\mu\mu}|^2 \right). \quad (20)
\]

F. \( \mu \to e\gamma \) and \( a_\mu \)

The current experimental bound on \( \text{Br}[\tau \to 3\mu] \) obtained by Belle is \( 2.1 \times 10^{-8} \) at 90\% C.L. \[18\]. A combination with data from BaBar \[49\] gives the even stronger limit of \( 1.2 \times 10^{-8} \) at 90\% C.L. \[50\].

The corresponding expression for \( \mu \to 3e \) can be directly inferred from eq. \[20\], with the experimental limit \( \text{Br}[\mu \to 3e]_{\text{exp}} \leq 1.0 \times 10^{-12} \[51\]. Note that \( \text{Br}[\mu \to 3e] \) involves \( \Gamma_{ee} \) which we set to zero to comply with the \( R(K) \) measurement, and thus \( \mu \to 3e \) does not affect our phenomenology.

\[
\text{Br}[\mu \to e\gamma] = \frac{1}{(4\pi)^4} \frac{\alpha m_{\mu}^5}{g_{\mu} M_{Z'}^4} \left( \sum_{\ell=e,\mu,\tau} \left( \Gamma_{\mu\ell} \Gamma_{\ell\tau} - \frac{3m_{\ell}}{m_{\mu}} \Gamma_{\mu\ell} \Gamma_{\mu\ell} \right) \right)^2 + (L \leftrightarrow R). \quad (21)
\]

\[
\Delta a_\mu = \frac{1}{12\pi^2} \frac{m_{\mu}^2}{M_{Z'}^2} \sum_{\ell=e,\mu,\tau} \text{Re} \left[ \frac{3m_{\ell}}{m_{\mu}} \Gamma_{\mu\ell} \Gamma_{\ell\mu} - \Gamma_{\mu\ell} \Gamma_{\mu\ell} - \Gamma_{\mu\ell} \Gamma_{\mu\ell} \right]. \quad (22)
\]
For $\mu \to e\gamma$ transitions, the current bound is given by $\text{Br}[\mu \to e\gamma] < 1.2 \times 10^{-14}$ [52]. Concerning $a_\mu$, there is the longstanding discrepancy with the SM predictions encoded in $\Delta a_\mu \equiv a_{\mu,\exp} - a_{\mu,\text{SM}} = (2.9\pm 0.9) \times 10^{-9}$ [53].

### G. Trident Neutrino Production

Bounds on flavour-diagonal $Z'$ couplings to muons can also arise from neutrino trident production (NTP), where a muon pair is created by scattering a muon-neutrino with a nucleon: $\nu_\mu N \to \nu N \mu^+ \mu^-$. Note that as the flavour of the neutrino in the final state is not detected, one must sum over all three generations in the case of flavour-violating interactions. Generalizing the formula of Ref. [54] to the flavour-violating case, we obtain for the cross section of NTP

$$
\frac{\sigma}{\sigma_{\text{SM}}} = \sum_{\ell=e,\mu,\tau} \frac{\Delta^2 \delta_{\ell\mu} - V_{\ell\mu}^{\text{NP}}}{2G_F^2 (1 - 4s_W^2 + 1)},
$$

(23)

with

$$
V_{\ell\mu}^{\text{NP}} = \frac{1}{m_{Z'}^2} (\Gamma^{L}_{\mu\mu} + \Gamma^{R}_{\mu\mu}) \Gamma^{L}_{\nu\nu},
$$

(24)

$$
A_{\ell\mu}^{\text{NP}} = \frac{1}{m_{Z'}^2} (\Gamma^{L}_{\mu\mu} - \Gamma^{R}_{\mu\mu}) \Gamma^{L}_{\nu\nu}.
$$

(25)

However, as we will see in the phenomenology section, even if we use combined bounds from CHARM-II, CCFR and NuTeV,

$$
\sigma_{\text{exp}}/\sigma_{\text{SM}} = 0.83 \pm 0.18,
$$

(26)

the resulting constraints on the couplings $\Gamma^{L,R}_{\mu\mu}$ are not very relevant for our analysis. The reason for this is that we are mainly interested in the region of parameter space with small $\Gamma^{L,R}_{\mu\mu}$ such that $\Gamma^{L,R}_{\tau\mu}$ can be sizable without violating the bounds from $\tau \to 3\mu$ [20].

### H. Lepton-flavour violating $B$ decays

Here we give formulas for the branching ratios of LFV $B$ decays, taking into account only contributions from the operators $O_9^{(t\ell\nu)}$ and $O_{10}^{(t\ell\nu)}$ while neglecting contributions from operators with scalar currents not relevant in our model. For $B_s \to \ell^+\ell^-$ (with $\ell \neq \ell'$) we use the results of Ref. [55] neglecting the mass of the lighter lepton. The branching ratios for $B \to K^{(*)} \tau^\pm \mu^\mp$, $B \to K^{(*)} \mu^\mp e^\mp$ are computed using form-factors obtained from lattice QCD in Ref. [56] (see also Refs. [12, 57]). The final results read

$$
\text{Br}[B_s \to \ell^+\ell^-] = \frac{\tau_{B_s} m_{B_s}^2 f_{B_s}^2 f_{B_s}^2}{32\pi^3} \alpha^2 G_F^2 |V_{ts} V_{ts}|^2 \left(1 - \frac{\text{Max}[m_9^2, m_9^2]}{M_{B_s}^2}\right)^2 \left(\left|C_9^{t\ell} + C_9^{t\ell'}\right|^2 + \left|C_{10}^{t\ell} - C_{10}^{t\ell'}\right|^2\right),
$$

(27)

$$
\text{Br}[B \to K^{+} \ell^-] = 10^{-9} \left(a_{K^{t\ell'}} - a_{K^{t\ell'}}\right),
$$

(28)

$$
\text{Br}[B \to K^{-} \ell^+] = 10^{-9} \left(a_{K^{'t\ell'}} - a_{K^{'t\ell'}}\right),
$$

(29)

with

| $\ell^\prime$ | $a_{K^{t\ell'}}$ | $b_{K^{t\ell'}}$ | $a_{K^{'t\ell'}}$ | $b_{K^{'t\ell'}}$ | $c_{K^{'t\ell'}}$ | $d_{K^{'t\ell'}}$ |
|---------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $\tau\mu$     | $9.6 \pm 1.0$  | $10.0 \pm 1.3$ | $3.0 \pm 0.8$  | $2.7 \pm 0.7$  | $16.4 \pm 2.1$ | $15.4 \pm 1.9$ |
| $\mu\mu$     | $15.4 \pm 3.1$ | $15.7 \pm 3.1$ | $5.6 \pm 1.9$  | $5.6 \pm 1.9$  | $29.1 \pm 4.9$ | $29.1 \pm 4.9$ |
FIG. 2: Left: Allowed regions in the $\Gamma_{\mu\mu} - \Gamma_{\tau\tau}$ plane from $\tau \to \mu\nu\bar{\nu}$ (at 3$\sigma$ level) for $\Gamma_{\mu\nu}^\tau = 0$ (blue), $\Gamma_{\mu\nu}^\tau = -2$ (yellow), $\Gamma_{\mu\nu}^\tau = 2$ (green), $\tau \to 3\mu$ (red) and $a_{\mu}$ (light gray) for $m_{Z'} = 1$ TeV. The 1$\sigma$ region allowed from NTP lies between the magenta dashed lines. Although NP effects move $a_{\mu}$ to the right direction, it cannot be explained within our model and we do not impose it as a constraint later on in our analysis. Right: Allowed regions in the $\Gamma_{LL}^\tau - \Gamma_{LR}^\tau$ plane: from $\tau \to \mu\nu\bar{\nu}$ for $\Gamma_{LL}^\tau = 0$ (blue), $\Gamma_{LL}^\tau = -2$ (yellow), $\Gamma_{LL}^\tau = 2$ (green), $\tau \to 3\mu$ (red) for $m_{Z'} = 1$ TeV. The contour lines denote the shift in $a_{\mu}$ in units of $10^{-10}$. For regions compatible with $\tau \to \mu\nu\bar{\nu}$, the NP effects in $a_{\mu}$ are rather small. Therefore, we do not impose it as a constraint later on. Bounds from NTP lie outside the plotted range and are not shown.

The formula for the branching ratio of $B_s \to \ell^+\ell^-$ is symmetric with respect to the exchange of $C_9^{(\ell\ell')} \leftrightarrow C_{10}^{(\ell\ell')}$, while in the case of $B \to K^{(*)}\ell^+\ell^-$ this symmetry is broken by lepton-mass effects. There is a small difference between the theoretical prediction for the charged mode $B^+ \to K^{(*)}\ell^+\ell^-$ and the neutral one $B^0 \to K^{(*)0}\ell^+\ell^-$ due to the different $B$-meson lifetime $\tau_B$ which we neglected fixing the numerical value of $\tau_B$ to the one of the neutral meson. Note that the results above are given for $\ell^-\ell^+$ final states and not for the sum $\ell^+\ell^+\ell^-\ell^-$ to which the experimental constraints apply [30]:

\[
\begin{align*}
\text{Br} \left[ B^+ \to K^{+}\ell^+\ell^- \right]_{\exp} & \leq 4.8 \times 10^{-5}, \\
\text{Br} \left[ B^+ \to K^{+}\mu^+\mu^- \right]_{\exp} & \leq 9.1 \times 10^{-8}, \\
\text{Br} \left[ B \to K^\tau\ell^+\ell^- \right]_{\exp} & \leq \cdots, \\
\text{Br} \left[ B \to K^\tau\mu^+\mu^- \right]_{\exp} & \leq 1.4 \times 10^{-6}, \\
\text{Br} \left[ B_s \to \ell^+\ell^- \right]_{\exp} & \leq \cdots, \\
\text{Br} \left[ B_s \to \mu^+\mu^- \right]_{\exp} & \leq 1.2 \times 10^{-8}.
\end{align*}
\] (30) (31) (32) (33) (34) (35)

III. PHENOMENOLOGICAL ANALYSIS

Having identified the processes relevant to constrain our $Z'$ model and having specified our treatment of them, we will in this section study their phenomenological impact and quantify the possible size of the LFV $B$ decays.

We start with the lepton sector, where we consider, as mentioned before, the two scenarios with vectorial (scenario 1) and with left-handed couplings (scenario 2). We examine the numerical impact of the leptonic constraints given in Sec. II C-II G. Fig. 2 shows the allowed regions in the plane of the couplings $\Gamma_{\mu\mu}$ and $\Gamma_{\tau\tau}$ from $\tau \to \mu\nu\bar{\nu}$ for different values of $\Gamma_{\tau\tau}$, as well as the bounds from $\tau \to 3\mu$ and the anomalous magnetic moment of the muon. Note that the experimental value of $\text{Br}[\tau \to \mu\nu\bar{\nu}]$ is already above the SM prediction by more than 2$\sigma$, and since in addition the tree-level $Z'$ contribution interferes destructively with the SM terms, we show the allowed 3$\sigma$ regions for this decay. Interestingly, in scenario 1, $\tau \to \mu\nu\bar{\nu}$ rules out an explanation of $a_\mu$ via a non-vanishing $\Gamma_{\mu\nu}^\tau$ (contrary to claims in Ref. [35] where the constraints from $\tau \to \mu\nu\bar{\nu}$ were not considered), while in scenario 2 the interference with the SM terms in $a_\mu$ is always destructive, even though its contribution is small. The constraints from $Z \to \mu^+\mu^-$ and $Z \to \tau^+\tau^-$ are subleading compared to the bounds from NTP and $\tau \to 3\mu$ for the $Z'$ masses under consideration (at the TeV scale or above) and thus not visible in Fig. 2.

In the quark sector we are only interested in the couplings $\Gamma_{bs}^{L,H}$ mediating $b \to s$ transitions. The most stringent constraints on these couplings stem from $B_s - \bar{B}_s$ mixing. Using the 95% CL results on $\Delta m_{B_s}$ of the UFTfit
The bounds resulting from eqs. \ref{eq:bound1} and \ref{eq:bound2} are shown by the blue contour of Fig. \ref{fig:Fig3} as can be seen, the constraints are weakened if \( \Gamma^L_{sb} \) and \( \Gamma^R_{sb} \) have the same sign with \( |\Gamma^R_{sb}| < |\Gamma^L_{sb}| \) or \( |\Gamma^R_{sb}| > |\Gamma^L_{sb}| \), as a consequence of cancellations in eq. \ref{eq:bound3}. We note that \( R(K) \) and \( B \to K^\ast \mu^+ \mu^- \) require at the 2\( \sigma \) level a substantial non-zero contribution to \( C^\mu\mu \) eliminating the option \( |\Gamma^R_{sb}| > |\Gamma^L_{sb}| \). As an illustration, we show in Fig. \ref{fig:Fig3} the combined constraints from \( R(K) \) and \( B \to K^\ast \mu^+ \mu^- \) for different values of a vectorial \( Z' \) coupling \( \Gamma^V_{\mu\mu} \) to muons (scenario 1). Note that in principle there is no upper limit on \( |\Gamma^L_{sb}| \) as long as \( R(K) \) and \( B \to K^\ast \mu^+ \mu^- \) allow for small but non-vanishing contributions to the primed operators \( C^\mu \) and/or \( C_{10} \). Therefore, we define the following measure of fine tuning in the \( B_s \) system

\[
X_{B_s} = \frac{(\Gamma^L_{sb})^2 + (\Gamma^R_{sb})^2 + b_{B_s}\Gamma^L_{sb}\Gamma^R_{sb}}{M^2_{Z'}} \leq c_{B_s} \sqrt{1 + X_{B_s}}. \quad (39)
\]

quantifying the degree of cancellation encountered in eq. \ref{eq:bound3}. Restricting \( X_{B_s} \) to an acceptable value limits the maximal size of the coupling \( \Gamma^L_{sb} \).

As we are interested exclusively in scenarios with \( C^\mu\mu \gg C^\mu\mu \), we can neglect the \( (\Gamma^R_{sb})^2 \) term in Eq. \ref{eq:bound3} and express \( \Gamma^L_{sb} \) in terms of the fine-tuning measure \( X_{B_s} \) and \( \Delta R_{B_s} \) as

\[
|\Gamma^L_{sb}| = \frac{\sqrt{\Delta R_{B_s}(1 + X_{B_s})}}{2a_{B_s}} \leq c_{B_s} \sqrt{1 + X_{B_s}}. \quad (40)
\]

Note that here and in the following we assume that all couplings \( \Gamma^L_{ij} \) are real. Using the maximally allowed value for \( \Delta R_{B_s} \) from eq. \ref{eq:bound3}, we find

\[
c_{B_s} = \max \left[ \sqrt{\Delta R_{B_s}/2a_{B_s}} \right] \approx 0.0045 \text{ TeV}^{-1}. \quad (41)
\]

Inserting the bound on \( \Gamma^L_{sb} \) into the formulae for the branching ratios of \( \tau \to 3\mu \) and \( \tau \to \mu\nu\bar{\nu} \) from Sec. \ref{sec:Br_3mu} and \ref{sec:Br_mu} we derive upper limits for the coefficient \( C^\mu \):

\[
|C^\mu|^2 \leq \frac{64\pi^7 \Gamma^L_{sb} \Gamma^L_{sb}}{m^2_{\tau} \alpha^2 G^4_F |V_{tb}V_{ts}|} \times A_{3\mu} \max \{ \text{Br}[\tau \to 3\mu] \text{exp} \} \times \frac{(1 + X_{B_s})^2}{|C^\mu\mu|^2}, \quad (42)
\]

\[
|C^\mu|^2 \leq \frac{96\sqrt{2}\pi^7 \Gamma^L_{sb} \Gamma^L_{sb}}{\alpha^2 G^4_F m^2_{\tau} |V_{tb}V_{ts}|} \times A_{\mu\nu} \max \{ \Delta_{\tau \to \mu\nu} \} \times (1 + X_{B_s}) \cdot \quad (43)
\]

Note that in the contrast from \( \tau \to \mu\nu\bar{\nu} \) we have neglected terms decoupling with \( 1/m^4_{Z'} \), as they are sub-
FIG. 4: Left: Maximal value of $\text{Br}[B \to K^*\tau^\pm\mu^\mp]$ (red), $\text{Br}[B \to K^\tau\mu\mp]$ (blue) and $\text{Br}[B_s \to \tau^\pm\mu^\mp]$ (green) in scenario 1 as a function of $c_{9\mu}^{\mu\mu}$ for a fine-tuning of $X_{B_s} = 100$ (solid lines) and $X_{B_s} = 20$ (dashed lines). Right: Same as the left plot for scenario 2. The white area represents the 2$\sigma$-allowed range for $c_{9\mu}^{\mu\mu}$ from the fits of Ref. [5, 13].

leading for the range of $Z'$ masses we are considering (at the order of TeV or above). For scenario 1 with vectorial lepton couplings we obtain $A_{3\mu}^{(1)} = 16$ and $A_{1\mu\nu}^{(1)} = 4$, while for scenario 2 with left-handed lepton couplings (so that $C_{9\mu}^{(1)} = -C_{10}^{(1)}$) we get $A_{3\mu}^{(2)} = 3$ and $A_{1\mu\nu}^{(2)} = 1$.

The bounds from $\tau \to \mu\nu\bar{\nu}$ only depend on the fine-tuning measure $X_{B_s}$, while the bounds from $\tau \to 3\mu$ also depend on the value of $C_{9\mu}^{\mu\mu}$ (and $C_{10}^{\mu\mu}$ in scenario 2) determined from the fit to $B \to K^*\mu^+\mu^-$, $B_s \to \phi\mu^+\mu^-$ and $R(K)$ data. The latter bounds disappear in the limit $c_{9\mu}^{\mu\mu} \to 0$, as in this case the $Z'\mu\mu$ couplings may vanish so that the $\tau \to 3\mu$ decay does not receive contributions from $Z'$ exchange.

For $\mu\epsilon$ final states we get from $\mu \to e\gamma$ and $\mu \to e\nu\bar{\nu}$ decays (see Sec. [11C] and [11F])

\begin{equation}
|c_{9\mu}^{\mu\mu}|^2 \leq \frac{576\pi^5\Gamma_\mu(c_{B_s})^4}{m_\mu^2\alpha^5 G_F^4 |V_{tb}V_{ts}^\dagger|^2} \times A_{c\gamma} \max\{|\text{Br}[\mu \to e\gamma]_{\text{exp}}|\} \times \left(1 + X_{B_s}\right)^2, \tag{44}
\end{equation}

\begin{equation}
|c_{9\mu}^{\epsilon\mu}|^2 \leq \frac{96\sqrt{2}\pi^5\Gamma_\mu c_{2\nu}^{\nu\mu}}{\alpha^2 G_F^3 m_\mu^3 |V_{tb}V_{ts}^\dagger|^2} \times A_{\epsilon\nu\bar{\nu}} \max\{|\Delta_{\mu\to e\nu\bar{\nu}}|\} \times \left(1 + X_{B_s}\right), \tag{45}
\end{equation}

where $A_{c\gamma}^{(1)} = 2$, $A_{c\nu\epsilon}^{(1)} = 4$ for scenario 1, and $A_{c\gamma}^{(2)} = A_{c\nu\epsilon}^{(2)} = 1$ for scenario 2, respectively.

From the upper bounds on $C_{9,10}^{\mu\mu}$ and $C_{9,10}^{\epsilon\mu}$, we can now determine the maximally allowed branching ratios for the LFV $B$ decays. For $B \to K^*(\tau^\pm\mu^\mp)$ and $B_s \to \tau^\pm\mu^\mp$ the maximal values are shown in Fig. [4] for a fine-tuning in $B_s - \overline{B}_s$ mixing of $X_{B_s} = 20$ and $X_{B_s} = 100$. The kink in the curves occurs at the point where the $C_{9,10}^{\mu\mu}$-independent constraint from $\tau \to \mu\nu\bar{\nu}$ becomes stronger than the constraint from $\tau \to 3\mu$. Comparing these results to the experimental upper limits in Eq. [32] we see that the current experimental sensitivity is still two orders of magnitude weaker. However, LHCb will be able to achieve significant improvements in these channels.

As the branching ratio for the LFV $B$ decays with $\mu\epsilon$ final states turn out to be quite suppressed, we confine ourselves to displaying the resulting upper limits for scenario 1 (Fig. [5]). As we have seen in the case of $\tau\mu$ final states, the quantitative behavior for scenario 2 is very similar. Due to the stringent bounds from $\mu \to e\gamma$ the allowed values are very small and unobservable for the currently favoured $c_{9\mu}^{\mu\mu}$ range. The kink is located at the
point where the bounds from $\mu \rightarrow e\gamma$ and $\mu \rightarrow e\nu\bar{\nu}$ are equal.

\[ \begin{align*}
\text{FIG. 5: Maximal value of } &\text{Br}[B \rightarrow K^{*}e^{\pm}\mu^{\mp}] \text{ (red), Br}[B \rightarrow K^{*}\mu^{\pm}\mu^{\mp}] \text{ (blue) and Br}[B_{s} \rightarrow e^{\pm}\mu^{\mp}] \text{ (green) in scenario 1 as a function of } C_{9}^{\mu\mu} \text{ for a fine-tuning of } X_{B_{s}} = 100 \text{ (solid lines) and } X_{B_{s}} = 20 \text{ (dashed lines). Note that the limit on } \\
\text{Br}[B_{s} \rightarrow e^{\pm}\mu^{\mp}] \text{ is so stringent that it cannot be resolved in the plot. The white area represents the } 2\sigma \text{-allowed range for } C_{9}^{\mu\mu} \text{ from the fits of Ref. [6].}}
\end{align*} \]

\[ \begin{align*}
\text{IV. CONCLUSIONS}
\end{align*} \]

In this article we have investigated the possible size of the branching ratios of the lepton-flavour violating $B$ decays $B_{s} \rightarrow \tau^{\pm}\mu^{\mp}$, $B_{s} \rightarrow \mu^{\pm}\nu\bar{\nu}$, $B \rightarrow K^{(*)}\tau^{\pm}\mu^{\mp}$ and $B \rightarrow K^{(*)}e^{\pm}\mu^{\mp}$ in generic $Z'$ models. To this purpose we have focused on two different scenarios motivated by the model-independent fit to $b \rightarrow s$ transitions: in scenario 1 we assumed vectorial couplings of the $Z'$ to leptons corresponding to NP in the Wilson coefficients $C_{9}^{(i)\ell\ell'}$ only (with $|C_{9}^{(i)\ell\ell'}| \ll |C_{9}^{ij}|$), whereas in scenario 2 we considered a $Z'$ with purely left-handed couplings to leptons corresponding to NP contributions fulfilling $C_{9}^{(i)\ell\ell'} = -C_{10}^{(i)\ell\ell'}$ (with $|C_{9}^{(i)\ell\ell'}| \ll |C_{9}^{ij}|$). We have found that in both scenarios the branching ratios with $\mu\tau$ final states can be sizable (of the order of $5 \times 10^{-6}$) if we allow for a significant amount of fine-tuning between the different terms contributing to $B_{s} - \bar{B}_{s}$ mixing. However, for degrees of fine-tuning below $X_{B_{s}} = 100$, the branching ratios for $\mu\mu$ final states can only reach $10^{-7}$, and this only in a region of parameter space disfavoured by the data on $B \rightarrow K^{*}\mu^{\pm}\mu^{-}$, $B_{s} \rightarrow \phi\mu^{+}\mu^{-}$ and $R(K)$.

The cancellations in $B_{s} - \bar{B}_{s}$ mixing that are necessary to permit sizable effects in lepton-flavour violating $B$ decays can only occur if $\Gamma_{sb}^{L}$ does not vanish but is small compared to $\Gamma_{sb}^{R}$ (or vice versa) and of the same sign. If the $Z'$ couples flavour-diagonal to muons, given the present data on $b \rightarrow s\mu^{+}\mu^{-}$ transitions, this implies non-vanishing $C_{9}^{\mu\mu}$ with $|C_{9}^{\mu\mu}| \ll |C_{9}^{ij}|$. Future data on $b \rightarrow s\mu^{+}\mu^{-}$ transitions constraining the $C_{9}^{\mu\mu}$ and $C_{10}^{\mu\mu}$ Wilson coefficients can thus rule out the possibility of large lepton-flavour violating $B$ decay rates in such models.

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