Effects of turbine’s selection on hydraulic transients in the long pressurized water conveyance system

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Abstract. For a hydropower station with longer water conveyance system, an optimum turbine’s selection will be beneficial to its reliable and stable operation. Different optional turbines will result in possible differences of the hydraulic characteristics in the hydro-mechanical system, and have different effects on the hydraulic transients’ analysis and control. Therefore, the premise for turbine’s selection is to fully understand the properties of the optional turbines and their effects on the hydraulic transients. After a brief introduction of the simulation models for hydraulic transients’ computation and stability analysis, the effects of hydraulic turbine’s characteristics at different operating points on the hydro-mechanical system’s free vibration analysis were theoretically investigated with the hydraulic impedance analysis of the hydraulic turbine. For a hydropower station with long water conveyance system, based on the detailed hydraulic transients’ computation respectively for two different optional turbines, the effects of the turbine’s selection on hydraulic transients were analyzed. Furthermore, considering different operating conditions for each turbine and the similar operating conditions for these two turbines, free vibration analysis was comprehensively carried out to reveal the effects of turbine’s impedance on system’s vibration characteristics. The results indicate that, respectively with two different turbines, most of the controlling parameters under the worst cases have marginal difference, and few shows obvious differences; the turbine’s impedances under different operating conditions have less effect on the natural angular frequencies; different turbine’s characteristics and different operating points have obvious effects on system’s vibration stability; for the similar operating conditions of these two turbines, system’s vibration characteristics are basically consistent with each other.

1. Introduction
Turbine selection is an important and key technology in the engineering design of a hydropower station. A reasonable type and corresponding parameters are the premises for the perfect energy economy indexes, stable and reliable operation of the hydropower station. Generally, in the turbine section for a hydropower station, its development type, energy parameters, layout of hydraulic structures are mainly considered in detail, and the parameters of the produced turbines and the technological capability of the producers in the world are further consulted. According to the main influencing factors in turbine selection, further analysis and investigation was carried out in recent years. Husain et al. [1] introduced different types of hydraulic turbine and the basic considerations in turbine selection for the hydropower stations. Based on different types of turbine configurations available for low-head small hydro developments, Gordon [2] outlined the steps used in conjunction with a computer program to arrive at the types of units available for the site. MATLAB was applied to
realize the digital image processing of the characteristic curves of the model turbine for the requirements of the type selection of hydraulic turbine by Wang et al. [3]. Fang et al. [4] pointed out the importance of operation stability in the choice of water turbine type. A robust method was proposed based on data envelopment analysis (DEA) to solve the problem of interval valued weights in selection of water turbine type by Liu et al. [5]. Ma [6] put forward a method for selection of water turbine type based on fuzzy-AHP to overcome the difficulty in quantitative analysis and evaluation. Williamson et al. [7] emphasized that turbine types suit specific ranges of head, flow rate and shaft speed, and are usually categorized by specific speed. Therefore, with the known unit’s parameters mainly including the maximum, minimum, rated operating head and unit’s capacity etc., several turbines should be selected for further comprehensive comparison in technology and economy.

For the optimal turbines, the detailed hydraulic transients computation and analysis should be carried out under different worst cases including the combined cases, and the controlling values of the guaranteed regulation parameters are obtained, which will be the basis of the final selection of the hydraulic turbines. During the final selection of hydraulic turbines, their differences in hydraulic vibration are less concerned, while hydraulic vibration analysis is an important and necessary aspect in hydraulic transient analysis and the theory of hydraulic vibration has achieved great development in recent years. Suo et al. [8] comprehensively discussed how to evaluate the hydraulic vibration characteristics in the pressurized water conveyance systems, and to predict the possible hydraulic resonance. Zhou et al. [9] revealed a complex hydraulic vibration problem in a hydropower station which resulted in unit’s instable operation and local structural damage. Kim [10] greatly improved the computational efficiency of the impedance matrix method for large pipe networks with various dimensions and complexity. Based on the development in the theory of hydraulic vibration and further hydraulic vibration analysis, Zhou [11] comprehensively investigated the self-excited vibration, unit’s operating stability and coupling vibration in the hydro-mechanical-electrical system. Hydraulic turbine is an important element in the hydro-mechanical system, and it is also a special boundary in the hydraulic vibration analysis because of the variety and complexity of its characteristics. Feng et al. [12] and Huang et al. [13] systematically discussed turbines’ hydraulic impedance characteristics by the combination of theoretical derivation and free vibration analysis. Therefore, different optional turbines will result in possible differences of the hydraulic characteristics in the hydro-mechanical system, and have different effects on the hydraulic transients’ analysis, and hydraulic vibration analysis. Therefore, the premise for turbine’s selection is to fully understand the properties of the optional turbines and their effects on the hydraulic transients. After a brief introduction of the simulation models for hydraulic transients’ computation and stability analysis, the effects of hydraulic turbine’s characteristics at different operating points on the hydro-mechanical system’s free vibration analysis were theoretically investigated along with the hydraulic impedance analysis of the hydraulic turbine. For a hydropower station with longer water conveyance system, based on the detailed hydraulic transients’ computation respectively for two different optional turbines, the effects of the turbine’s selection on hydraulic transients were analyzed. Furthermore, considering different operating conditions for each turbine and the similar operating points for these two turbines, free vibration analysis was comprehensively carried out, which revealed the effects of turbine’s impedance on system’s vibration characteristics.

2. Mathematical models for hydraulic transients’ analysis in turbine selection
For the optional turbines according to the basic considerations, further hydraulic transients analysis should be complemented for the hydro-mechanical system and detailed comparative analysis will help us have a final decision. Hydraulic transients’ analysis mainly includes large fluctuations analysis, stability analysis under small fluctuations and hydraulic disturbance, and hydraulic vibration analysis.

2.1. Large fluctuations analysis
The method of characteristics [14] is the most important and convenient method to simulate the hydraulic transient under large fluctuations of the pressured piping system. Based on the method of
characteristics, all the boundary conditions can easily be simulated, including the reservoir, the series pipelines, the surge tank, the bifurcation pipes, the turbines and the tail water etc., and then the mathematical model for the whole hydro-mechanical system is built by the combination of units’ motion equations and state equations for governors. Finally, hydraulic transients’ analysis under large fluctuations can be conducted in detail and the controlling variables and their differences for the selected turbines can be evaluated, such as the maximum pressure head at the inlet of spiral case, the minimum pressure head at the inlet of draft tube and the rising ratio of turbine’s rotational speed.

2.2. Stability analysis
In the stability analysis of hydraulic disturbance (the governor involved in) and small fluctuations for the hydro-mechanical system [15], the state equations are introduced to describe the dynamic characteristics of the governor and unit’s rotational speed. Therefore, based on the method of characteristics and analysis of state equations, an integrated algorithm is recommended in which the strong nonlinear characteristics of unit’s discharge and efficiency are also involved. In this algorithm, the time-step is also determined by the stability conditions of the method of characteristics (Courant condition). After the detailed stability analysis of hydraulic disturbance and small fluctuations, the dynamic characteristics of unit’s rotational speed and output can be computed out. Based on the obtained time histories of unit’s rotational speed, main indexes for regulation performance will be calculated out and further analyzed, including regulation time $T_p$, oscillation time $x$ within regulation time $T_p$, maximum deviation $\Delta n_{max}$, overshoot $\delta$ and attenuation degree $\psi$ [16].

2.3. Hydraulic vibration analysis
Hydraulic vibration analysis mainly includes free vibration analysis and forced vibration analysis [14]. Hydraulic impedance method and transfer matrix method are basically used to solve any hydraulic vibration problem with periodical pressure and discharge oscillation. Based on the basic equations for hydraulic vibration in the pressurized pipeline, the total impedance model by the combination of different transfer matrixes for the pipelines and hydraulic elements can be built for further detailed hydraulic vibration analysis.

For the method of hydraulic impedance, the hydraulic impedance is defined as the ratio of complex oscillatory head and complex oscillatory discharge. With the known boundary conditions, the hydraulic impedance of each section can be obtained with an amplitude and a phase which depends on the numerical results of complex head and complex discharge.

For a complex pressurized water conveyance system, in order to obtain all the hydraulic impedance at different sections, a lot of united equations must be built and solved, and it is relatively complex and difficult to reach a solution. Therefore, the method of hydraulic impedance is often used to simulate the series pipelines, bifurcation pipes and other boundary conditions, and it is inconvenient to solve the complex pipelines, such as the loop systems.

The method of transfer matrix is more systematical than the method of hydraulic impedance. Based on the great development in matrix computation, the whole mathematical model for the hydro-mechanical system is easily built with different transfer matrixes for the hydraulic elements, and that is, a total transfer matrix is formed for further hydraulic vibration analysis.

3. Turbine’s hydraulic impedance analysis
The turbine in a hydropower station has various operating conditions with different speed characteristics and discharge characteristics, so for each operating condition, the turbine has different characteristics of hydraulic impedance which affects the hydraulic vibration characteristics of the hydro-mechanical system.

3.1. Hydraulic impedance calculation
With the turbine’s model characteristics hill curves, introducing the oscillating head $h'$ and oscillating discharge $q'$, considering turbine’s head balance condition, turbine’s flow continuity condition and the
equation for turbine’s unit parameters, the formula to calculate turbine’s hydraulic impedance \( Z_T \) was derived [11,14].

\[
Z_T = \frac{2n}{n_1D_1 \left( Q_{11} - \frac{\partial Q_{11}}{\partial n_{11}} n_{11} \right)}
\]

(1)

in which \( n \) is unit’s rotational speed; \( D_1 \) is the diameter of turbine’s runner; \( n_{11} \) and \( Q_{11} \) are unit speed and unit discharge respectively.

Wylie and Streeter [14] defined the hydraulic impedance as the gradient of the pressure and discharge relation curve at the corresponding operating point. Considering that the gradient of the pressure and discharge relation curve can be seen as a minor increment, turbine’s hydraulic impedance can be described as below.

\[
Z_T = \frac{\partial H}{\partial Q} = \frac{\Delta H}{\Delta Q}
\]

(2)

in which \( \Delta H \) is the minor increment of unit’s head; \( \Delta Q \) is the minor increment of unit’s discharge.

In the turbine-regulation system, both turbine’s discharge and head is represented by the form of relative increment, so the equations to describe turbine’s dynamic characteristics can be used to discuss turbine’s hydraulic impedance. Therefore, based on the turbine’s dynamic characteristic equation, and the introduction of turbine’s synthetic characteristic coefficient \( e \) [17] with the premise of turbine’s constant opening and constant rotational speed, the corresponding hydraulic impedance of the turbine was deduced [12].

\[
Z_T = -\frac{1}{e} \frac{H_0}{Q_0}
\]

(3)

The definition of \( e \) is

\[
e = \frac{e_q \cdot e_h - e_{qh}}{1 - 0.5 \left( \frac{\partial (\eta / \eta_0)}{\partial (Q_{11} / Q_{110})} + \frac{\partial (\eta / \eta_0)}{\partial (n_{11} / n_{110})} \right)}
\]

(4)

in which \( e_q, e_y, e_{qh}, e_h \) are turbine’s characteristic coefficients at the specified operating condition on the basis of the steady state; \( \eta \) is turbine’s operating efficiency; \( \eta_0, n_{110} \) and \( Q_{110} \) are turbines’ initial efficiency, unit speed and unit discharge.

Furthermore, with the premise of turbine’s constant output and constant rotational speed, the hydraulic impedance of the turbine was derived, which is consistent with Eq. (1).

3.2. Comparative analysis of different calculating formulas

Equations (1) and (3) show that turbine’s hydraulic impedance is strong relevant with its efficient characteristics and discharge characteristics at the operating condition. In Eq. (1), the hydraulic impedance is calculated with the known turbine’s unit parameters, the gradient \( \partial Q_{11}/\partial n_{11} \) along the equal-efficient curve and rotational speed. Therefore, both these two formulas reflect turbine’s operating characteristics at the specified operating condition.

For the analysis on hydraulic vibration characteristics of the longer pressurized water conveyance system in the hydropower station, turbine’s hydraulic impedance is an important boundary in system’s free vibration analysis and it directly affects system’s vibration characteristics and even operating stability. Therefore, the effect of turbine’s characteristics should be fully considered in the free vibration analysis, and that is, in the analysis of turbine’s hydraulic impedance, turbine’s hydraulic characteristics at the specified operating condition should reasonably be included. Considering the
operating characteristics and regulation performance of the hydro-mechanical system in the hydropower station, and \( \frac{\partial Q_{11}}{\partial n_{11}} \) in Eq. (1) intuitively reflecting turbine’s hydraulic characteristics in different operating regions, turbine’s hydraulic impedance at different operating conditions is calculated by use of Eq. (1) for the optional turbines, and then the attenuation factors and natural frequencies are analyzed in the free vibration analysis, and the operating stability is evaluated at different operating conditions.

### 3.3. Evaluation of turbine’s operating characteristics

For a given hydraulic system in a hydropower station, at a certain operating condition, the system has specified vibration characteristics, including a series of natural frequencies and corresponding vibration modes. Therefore, for each operating point in turbine’s model characteristics hill curves, it is easy to obtain the concerned complex frequencies and vibration modes to reveal the vibration characteristics of the hydro-mechanical system.

For \( \frac{\partial Q_{11}}{\partial n_{11}} \) in Eq. (1), it is defined as the multiplicative inverse of the gradient at each operating point along the characteristic curves, therefore, turbine’s characteristic hill chart can be divided into four regions based on the sign of \( \frac{\partial Q_{11}}{\partial n_{11}} \), and the sign of \( (Q_{11} - \frac{\partial Q_{11}}{\partial n_{11}} \times n_{11}) \) in Eq. (1) directly reflects the sign of turbine’s hydraulic impedance. Starting from the origin of the hill chart, two tangent lines are drawn to the equal-efficiency curve in the regions II and IV with two tangency points A and B, see Figure 1 below. In Figure 1, the curve ab is the connecting line for the points with \( \frac{\partial Q_{11}}{\partial n_{11}}=\infty \) along all the equal-efficiency curves and the curve cd is the connecting line for the points with \( \frac{\partial Q_{11}}{\partial n_{11}}=0 \) along all the equal-efficiency curves.

**Figure 1.** Division of turbine’s characteristics hill chart.

The sign analysis for each region is shown in Table 1.

**Table 1.** Sign analysis of turbine’s hydraulic impedance.

| Parameters | Regions | II | III | IV |
|------------|---------|----|-----|-----|
| \( \frac{\partial Q_{11}}{\partial n_{11}} \) | I | 1  | 2  | 1  | 2  |
|            | <0       | >0 | >0  | <0 | >0  |
| \( Q_{11} - \frac{\partial Q_{11}}{\partial n_{11}} \times n_{11} \) | >0 | <0 | >0  | >0 | <0  |
| \( Z_T \) (Equation (1)) | >0 | <0 | >0  | >0 | <0  |

It is analyzed that, because \( \frac{\partial Q_{11}}{\partial n_{11}} \) is negative in regions I and III, \( (Q_{11} - \frac{\partial Q_{11}}{\partial n_{11}} \times n_{11}) \) is positive and then turbine’s hydraulic impedance is positive. \( \frac{\partial Q_{11}}{\partial n_{11}} \) is positive in regions II and IV, but the sign of \( (Q_{11} - \frac{\partial Q_{11}}{\partial n_{11}} \times n_{11}) \) and turbine’s hydraulic impedance should be further analyzed. Considering that two tangency points in regions II and IV satisfy \( Q_{11} - \frac{\partial Q_{11}}{\partial n_{11}} \times n_{11} = 0 \), \( (Q_{11} - \frac{\partial Q_{11}}{\partial n_{11}} \times n_{11}) \) both in the upper region ① above tangency point A of II and the lower region ①
below tangency point B of IV is negative and then turbine’s hydraulic impedance is negative, while
\( Q_{11} - \frac{\partial Q_{11}}{\partial n_{11}} \times n_{11} \) both in the lower region below tangency point A of II and the upper region above
tangency point B of IV is positive and then turbine’s hydraulic impedance is positive. Therefore, based on Figure 1 and Table 1, combined with turbine’s characteristics hill chart, it is convenient to judge the sign of turbine’s hydraulic impedance at different operating conditions.

4. Comparative analysis of two optional turbines

4.1. System description

Figure 2 shows a typical layout of a hydropower station with longer pressurized water conveyance system and two turbines sharing a tail tunnel. The pressurized pipelines and hydraulic elements are partitioned and numbered successively.

![Figure 2. Layout of a water-diversion hydropower station.](image)

For the hydropower station shown in Figure 2, according to its operating parameters, two optional turbines are selected and both are satisfied with the requirements of turbine’s designing parameters. These two optional turbines are defined as turbine A and turbine B. The main parameters for the pipes, turbines and downstream surge tank are listed in Table 2.

| Pipe no. | Length (m) | Area (m²) | Roughness | Note |
|----------|------------|-----------|-----------|------|
| 1        | 283.90     | 95.756    | 0.014     | Turbine A: Rated speed 107.1 r/min; inlet diameter 8.6 m; rated head 202 m; rated discharge 547.8 m³/s; unit’s inertia \( GD^2 = 360000 \) t.m². |
| 2        | 263.00     | 75.657    | 0.012     | |
| 3        | 242.14     | 165.137   | 0.014     | 547.8 m³/s; unit’s inertia \( GD^2 = 360000 \) t.m². |
| 4        | 277.00     | 95.774    | 0.014     | Turbine B: Rated speed 111.1 r/min; inlet diameter 8.45 m; rated head 202 m; rated discharge 547.8 m³/s; unit’s inertia \( GD^2 = 300000 \) t.m². |
| 5        | 263.00     | 75.657    | 0.012     | |
| 6        | 258.13     | 168.561   | 0.014     | 547.8 m³/s; unit’s inertia \( GD^2 = 300000 \) t.m². |
| 7        | 657.08     | 246.370   | 0.014     | Area of surge tank 1385.4 m². |
| 8        | 586.94     | 357.920   | 0.014     | Wicket gates: Linear closing law with \( T_s = 12.0 \) s. |

For this hydropower station with two optional turbines, large fluctuations analysis and stability analysis are computed and investigated in detail. Furthermore, considering turbine’s hydraulic impedance characteristics at different operating conditions, and combined with the analysis of all the natural frequencies obtained from free vibration analysis, hydraulic vibration characteristics are systematically analyzed.

4.2. Large fluctuations analysis

For the hydropower station shown in Figure 2, with given parameters for the pipes, turbines and downstream surge tank, and with the same wicket closing pattern for turbine A and turbine B, hydraulic transients under large fluctuations are analyzed in detail, and some main controlling parameters are computed out for further comparative analysis listed in Table 3. The time histories of the pressure head at the inlet of spiral case and draft tube are shown in Figure 3 and Figure 4.

It can be known from Table 3, Figure 3 and Figure 4, that for turbine A and turbine B with the same wicket closing pattern, there are less effect on surge analysis of downstream surge tank, while obvious differences appear in the analysis of water hammer. For turbine A, the minimum pressure at the end of upper horizontal penstock has smaller margin, the maximum pressure at the inlet of spiral
case is relatively large, and minimum pressure at the inlet of draft tube is obviously small. Therefore, it is necessary to fully understand the effect of turbines’ selection on large fluctuations analysis.

Table 3. Hydraulic transient’s analysis under large fluctuations.

| Controlling parameters                          | Turbine A | Turbine B |
|------------------------------------------------|-----------|-----------|
| Min. pressure at the end of upper horizontal penstock (m) | 3.34      | 6.85      |
| Max. pressure at the inlet of spiral case (m)       | 330.42    | 307.98    |
| Max. rising ratio of turbine rotational speed (%)   | 49.66     | 50.89     |
| Min. pressure at the inlet of draft tube (m)        | -5.34     | -1.02     |
| Max. surge of surge tank (m)                       | 638.55    | 638.67    |
| Min. surge of surge tank (m)                       | 566.56    | 566.39    |
| Max. downwards pressure difference at the bottom of surge tank (m) | 10.91 | 11.15 |
| Max. upwards pressure difference at the bottom of surge tank (m) | 8.71 | 8.53 |

4.3. Stability analysis

Based on the setting analysis of governor’s parameters, for both turbine A and turbine B, the governor’s parameters are, proportional gain $K_p=2.5$, integrating gain $K_i=0.385s^{-1}$, differential gain $K_d=2.5s$ and permanent droop $b_f=0.4$. For stable analysis under hydraulic disturbance, the controlling cases include:

Case D1: two units are operating with rated loads at rated operating head and then one unit rejects full load and another unit is in normal operation.

Case D2: one unit is operating with rated loads at rated operating head and another unit accepts rated load.

Case D3: two units are operating with rated loads at maximum operating head and then one unit rejects full load and another unit is in normal operation.

For stable analysis under small fluctuations, the controlling case is Case S1: two units are operating with full loads at minimum operating head and then reject 10% load simultaneously.

With the defined technical parameters and worst cases listed above, the regulation performance under hydraulic disturb or small fluctuation can completely be analyzed and evaluated in Table 4 and Table 5. In Table 4, the number in the parentheses is oscillation time.

It can be analyzed, that for turbine A and turbine B under hydraulic disturbance or small fluctuations, their regulation performance is basically the same and there is less difference in some evaluation indexes. For turbine A, the regulation performance is relative good in total, with smaller $\Delta n_{max}$ and output swing while it has relatively small attenuation degree $\psi$ in some cases. Therefore, it
is also necessary to clarify the effect of turbines’ selection on stability analysis and have a best choice to ensure system’s stable operation.

**Table 4.** Regulation performance analysis under hydraulic disturbance.

| Case no. | Turbine | $T_p$(s) | $\Delta n_{max}$(rpm) | $\delta$ | $\psi$ (%) | Max. output $N_{max}$(MW) | Output swing $\Delta N$(MW) |
|----------|---------|---------|------------------------|---------|-----------|---------------------------|---------------------------|
| D1       | A       | $>500$ | 1.80                   | 5.14    | 48.89     | 1057.0                    | 69.5                      |
|          | B       | $>500$ | 1.90                   | 6.79    | 60.92     | 1058.2                    | 70.5                      |
| D2       | A       | 335    | -2.84                  | 8.11    | 60.92     | 1043.4                    | 68.2                      |
|          | B       | 334(2.0) | -3.00                  | 7.89    | 60.67     | 1043.8                    | 69.0                      |
| D3       | A       | 0(0.0) | 0.61                   | 12.20   | 86.89     | 1034.7                    | 26.4                      |
|          | B       | 0(0.0) | 0.74                   | 14.80   | 90.54     | 1038.9                    | 38.9                      |

**Table 5.** Regulation performance analysis under small fluctuations.

| Case no. | Turbine | $T_p$(s) | $x$ | $\Delta n_{max}$(rpm) | $\psi$ (%) |
|----------|---------|---------|----|------------------------|-----------|
| S1       | A       | 31.0    | 0.5| 3.93                   | 91.09     |
|          | B       | 23.0    | 0.5| 4.42                   | 95.25     |

**4.4. Hydraulic vibration analysis**

**4.4.1. Built of mathematical model**

Based on the method of transfer matrix, the pressure increment $H(s)$ and discharge increment $Q(s)$ in frequency domain at each section are defined as state variables, and then the state vector at a given section $i$ is $\{V\}_i = [H(s), Q(s)]^T$, and all the state vectors are united by field and point transfer matrices. For the hydro-mechanical system shown in Figure 2, the transfer matrices for all the characteristic pipes and hydraulic elements are built, and then the total transfer matrix is constructed for further hydraulic vibration analysis. By defining the inlet section of pipe 1 as the first section and the outlet section of pipe 8 as the last section, based on the field transfer matrices for all the characteristic pipes and the point transfer matrix for the hydraulic elements, the total transfer matrix $[U]$ is derived

\[
[V]_{d8} = [U] [V]_{u1} = [F]_8 [F]_7 [P]_{st} [F]_3 [P]_{st} [F]_2 [F]_1 [V]_{u1}
\]

in which $[F]_i$ is a field transfer matrix for pipe $i$; $[P]_{st}$ is point transfer matrix for turbine 1#; $[P]_{st}$ is point transfer matrix for the downstream surge tank. The simplified equation is

\[
\begin{bmatrix}
H_{d8} \\
Q_{d8}
\end{bmatrix} = \begin{bmatrix}
[u_{11} & u_{12} \\
[u_{21} & u_{22}]
\end{bmatrix} \begin{bmatrix}
H_{u1} \\
Q_{u1}
\end{bmatrix}
\]

By assuming the constant water level for the reservoir and the tail water, their corresponding pressure increment in frequency domain is zero, that is $H_{u1}=0$ and $H_{d8}=0$, or their hydraulic impedance is zero, and satisfies

\[
Z_{d8}(s) = \frac{H_{d8}}{Q_{d8}} = \frac{u_{12}}{u_{22}} = 0
\]

$Z_{d8}$ is a function of complex frequency $s$ and Equation (7) is a transcendental equation, so it is difficult to obtain all the complex frequency $s$ directly. Take the possible angular frequencies based on direct search in full frequency domain as the initial values for iteration in the free vibration analysis, all the actual complex frequencies $s = \sigma + i\omega$ satisfying Eq. (7) are computed out.

**4.4.2. Free vibration analysis**

For both turbine A and B, some similar operating points are selected in different operating regions. At the similar operating points, the unit speed and unit discharge of turbine A and B are approximately equal. Then, free vibration analysis is carried out in detail considering the similar operating points in different operating regions I-IV for turbine A and B. Table 6 gives the hydraulic impedance and operating parameters at different operating points for turbine A and B. All the complex frequencies for the hydro-mechanical system with turbine A or turbine B are computed out. Considering the
uncertainty of wave speed, it is not necessary to include the high-order natural frequencies, and Table 7 and Table 8 list the 1st-10th complex frequencies for turbine A and B.

Table 6. Operating parameters for two optional turbines.

| Case no. | Region | \( \frac{\partial Q_{11}}{\partial n_{11}} \) | \( Q_{11} - \frac{\partial Q_{11}}{\partial n_{11}} \times n_{11} \) | Hydraulic impedance |
|---------|--------|------------------|-----------------|------------------|
|         |        | Turbine A        | Turbine B        | Turbine A        | Turbine B        |
| 1       | I      | -0.0129          | -0.0195          | 1.3453           | 1.8732           | 0.2857           | 0.2125           |
| 2       | II     | 0.0106           | 0.0082           | -0.4142          | -0.1385          | -0.9279          | -2.8750          |
| 3       | III    | -0.0152          | -0.0126          | 1.2730           | 1.1041           | 0.3019           | 0.3606           |
| 4       | IV \( \frac{Q_{11}}{n_{11}} \) | 0.0046           | 0.0047           | 0.1721           | 0.1519           | 2.2328           | 2.6215           |

Table 7. Complex frequencies under different cases with turbine A.

| Vibration modes | Case 1 | Case 2 | Case 3 | Case 4 |
|-----------------|--------|--------|--------|--------|
| 1st             | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) |
| 2nd             | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) |
| 3rd             | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) |
| 4th             | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) |
| 5th             | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) |
| 6th             | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) |
| 7th             | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) |
| 8th             | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) |
| 9th             | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) |
| 10th            | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) |

Table 8. Complex frequencies under different cases with turbine B.

| Vibration modes | Case 1 | Case 2 | Case 3 | Case 4 |
|-----------------|--------|--------|--------|--------|
| 1st             | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) |
| 2nd             | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) |
| 3rd             | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) |
| 4th             | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) |
| 5th             | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) |
| 6th             | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) |
| 7th             | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) |
| 8th             | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) |
| 9th             | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) |
| 10th            | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) | \( \sigma \) | \( \omega \) |

Considering that turbine’s hydraulic impedance is varied continuously, a series of operating points with continuous hydraulic impedance are reasonably selected for further free vibration analysis, and then the effect of turbine’s hydraulic impedance on system’s hydraulic vibration characteristics is revealed with the analysis of the complex frequencies and attenuation factors. Take turbine A as an example, enough operating points are set along the equal-efficiency curve in regions II and IV to realize the gradual increase of turbine’s hydraulic impedance, and then free vibration analysis is complemented with the varied turbine’s hydraulic impedance. Figure 5 illustrates the variation trend of attenuation factor \( \sigma \) in the 3rd and 10th natural frequencies with the increase of turbine’s hydraulic impedance.

It can be analyzed that, for turbine A and turbine B under the similar operating points, the difference of turbine’s discharge, operating head and hydraulic impedance is relatively small, and system’s hydraulic vibration characteristics is basically similar with the approximately equal attenuation factors and angular frequencies in any complex frequencies. Therefore, for these two optional turbines, the effect of turbine’s hydraulic impedance characteristics on system’s hydraulic vibration characteristics is
consistent under the similar operating conditions. Therefore, for hydraulic vibration analysis, both of
these two optional turbines are satisfied with requirements of this hydropower station.

If turbine’s hydraulic impedance is negative, the attenuation factor of some vibration modes is
positive or equal to zero and this will be possible to lead to self-excited vibration. If turbine’s
hydraulic impedance is positive, the attenuation factor of all the vibration modes is negative and the
system is stable. For these two optional turbines under case 2, their hydraulic impedance is negative
while it is positive in other cases. It can be seen that, the attenuation factor $\sigma$ of all the vibration modes
is negative except case 2, and this can also be concluded from Figure 5.

![Figure 5. Relation of turbine’s hydraulic impedance and attenuation factor of free vibration.](image)

5. Conclusions
For a hydropower station with longer water conveyance system, different optional turbines have
different effects on the hydraulic transients’ analysis in its hydro-mechanical system. In turbine
selection, it is necessary to fully understand the effects of the optional turbines on the hydraulic
transients, including large fluctuations, operating stability and hydraulic vibration. After the analysis
of the hydraulic impedance characteristics of the hydraulic turbine, based on a case study with two
optional turbines, the effects of hydraulic turbine’s characteristics on the hydraulic transients’ analysis
were comprehensively investigated. The results indicate that,

For different optional turbines, obvious differences often appear in the analysis of water hammer,
and then it is necessary to fully understand the effect of turbines’ selection on large fluctuations
analysis. The final selection should meet the requirements of large fluctuations analysis.

For hydraulic disturbance or small fluctuations, though stability analysis shows that the regulation
performance has less difference for the optional turbines in this case study, it is also necessary to
clarify the effect of turbines’ selection on stability analysis and then have a best choice to ensure
system’s stable operation.

For these two optional turbines at different operating points, the attenuation factors in the complex
frequencies have relatively obvious difference while less difference exists for the angular frequencies.
The angular frequencies obtained from free vibration analysis are system’s inherent characteristics,
and less influenced by different operating points of the turbines.

If turbine’s hydraulic impedance is positive, the attenuation factors of all the vibration modes are
negative and the system is stable, otherwise, if turbine’s hydraulic impedance is negative, the
attenuation factors of some vibration modes are positive and this will be possible to lead to self-
excited vibration.

For these two optional turbines, the effect of turbine’s hydraulic impedance characteristics on
system’s hydraulic vibration characteristics is basically consistent under the similar operating points
with the approximately equal attenuation factors and angular frequencies in any complex frequencies.
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