Fast, ultra-luminous X-ray bursts from tidal stripping of White Dwarfs by Intermediate-Mass Black Holes

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ABSTRACT

Two X-ray sources were recently discovered by Irwin et al. in compact companions to elliptical galaxies to show ultra-luminous flares with fast rise (∼ minute) and decay (∼ hour), and with a peak luminosity ∼ 10^{40-42} erg s^{-1}. Together with other 5 sources found earlier, they comprise a new type of fast transients which cannot be attributed to neutron stars but might be to intermediate-mass black holes (IMBHs; 10^{2-4} M_{\odot}). The flaring behaviour is recurrent for at least 3 sources. If the flare represents a short period of accretion onto the IMBH during the periastron passage of a donor star on an eccentric (i.e., repeating) or parabolic orbit (non-repeating), we argue that the flare’s rise time corresponds to the duration during which the donor’s tidally stripped mass joins a residual disk at the pericenter. This duration is in turn equal to the three time scales: the duration of the stripping, the sound crossing time of the donor, and the circular orbit time at the pericenter radius. Only can a white dwarf have a sound crossing time as short as one minute. Therefore, the donor must be a white dwarf and it was stripped of ∼ 10^{-10} M_{\odot} upon each passage at several to tens of Schwarzschild radii from the IMBH. The flux decay corresponds to the viscous drainage of the supplied mass toward the hole. Aided with long-term X-ray monitoring, this type of fast transients would be an ideal target for next-generation gravitational wave detectors.

Keywords: accretion disks — hydrodynamics — stars: black holes — white dwarfs — X-rays: bursts

1. INTRODUCTION

There are much evidence for the existence of stellar-mass (∼ 10 M_{\odot}) and supermassive (∼ 10^6 − 10^9 M_{\odot}) black holes (BHs), but still no firm evidence for existence of intermediate-mass black holes (IMBHs; ∼ 10^{2-4} M_{\odot}) (Portegies Zwart & McMillan 2002; Baumgardt et al. 2003; Tremou et al. 2018), which fill a gap of the mass range in between. Nevertheless, the search and identification of them has great impact on understanding of the seeds and growth history of SMBHs (Volonteri, Haardt & Madau 2003; Greene 2012).

Recently, Irwin et al. (2016) found two luminous fast flaring sources in nearby galaxies from an archival search. One source is located in a globular cluster in the galaxy NGC4636. It brightens within 22 seconds by a factor of 100 to reach a peak luminosity of 0.9 × 10^{41} erg s^{-1}, then decays in 1,400 s. The persistent emission before and after the flare is at 0.8 × 10^{39} erg s^{-1}.

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The second source is in the elliptical galaxy NGC 5128. It flared five times during a total observation time of 790 ks, yielding an approximate recurrent time of 1.8 days. The flares show very similar light curves, rising rapidly within 30 s by a factor of 200 to a peak luminosity of 0.8 × 10^{40} erg s^{-1}, staying in a roughly steady ultra-luminous state for ∼ 200 s, then decaying over 4,000 s to the pre-flare level of ∼ 4 × 10^{37} erg s^{-1}. The optical counterpart is either a massive globular cluster, or a ultra-compact dwarf companion galaxy of NGC 5128 (Irwin et al. 2016).

The pre-flare and post-flare emission are found to be persistent during all the non-flare observation periods for the two sources. Absorbed power-law fits to the spectra of persistent emission give photon indices Γ ∼ 1.6 ± 0.5. The spectra of the two sources are more poorly constrained due to low photon statistics. They can be fit either by an absorbed power law (photon indices Γ ∼ 1.3 − 1.6) or by a disk blackbody (kT ∼ 1.3 − 2.2 keV). No significant spectral evolution is found, either between the persistent and the flare periods, or during the flares.
Sivukoff, Sarazin & Jordán (2005) reported two fast flares from an off-center source CXOU J124839.0–054750 in the elliptical galaxy NGC 4697, from two of five Chandra observations (∼40 ks each) of the galaxy. The flares have a peak luminosity of ∼6 × 10^{39} \text{erg s}^{-1}, a duration ∼70 s and a count rate ratio of the flare to the persistent emission ∼90. The photon counts are few (2–3 for each flare) so the statistical significance is not as good as those in Irwin et al. (2016). The small number of photons likely underestimates the real duration of flares. Similar flares from other four sources were reported (Jonker et al. 2013; Sun, Shu & Wang 2013; Glennie et al. 2015; Bauer et al. 2017). Following Irwin et al. (2016), we refer them collectively as fast, ultra-luminous X-ray bursts (UXBs), and summarize them in Table 1.

If the peak luminosity of the flare is limited by the Eddington luminosity \( L_{\text{Edd}} \) of the black hole (though one exception is a beamed emission), then the black hole mass mass \( M \sim 10^{2−4} M_\odot \) is implied for most of the sources. Each flare emits a total energy of \( 10^{42−43} \text{erg} \) for the two flaring sources reported in Irwin et al. (2016), which translates to a total accreted mass of \( 10^{−11−10} \eta_{0.1} M_\odot \), where \( \eta = \eta_{0.1} \times 10\% \) is the radiative efficiency. Note that this mass estimate is a lower limit because the radiative luminosity is probably Eddington-limited so the efficiency could be much lower.

Under the condition of a pure black body, the loosely inferred temperatures could also hint at the size of the emission region: \( R_{\text{BB}} = [L/\left(4\pi\sigma T^4\right)]^{1/2} \). For source 1, \( R_{\text{BB}} \approx 5 \times 10^7 \text{cm} \), and for source 2, \( R_{\text{BB}} \approx 5 \times 10^6 \text{cm} \), implying very compact emission regions.

If the flare represents a short period of accretion onto the IMBH during the periastron passage of a donor star on an eccentric or parabolic orbit, we argue that the flare’s rise time corresponds to the duration during which the donor’s tidally stripped mass returns to the pericenter. This duration is in turn equal to the other three time scales: the duration of the stripping, the sound crossing time of the donor, and the circularly orbital time of the transient disk formed at the pericenter radius. Only can a white dwarf (WD) have a sound crossing time as short as one minute.

In §2 we describe the tidal stripping and relevant physical time scales. We investigate the causes of the flare’s flux decay and rise in §3 and 4, respectively, particularly the interaction of the stripped stream with a residual disk, and the subsequent accretion. §5 summaries and gives further discussion.

2. PERIASTRON TIDAL STRIPPING OF THE SECONDARY

Consider an IMBH with a companion star orbiting around it on an elliptical orbit. Let \( M \) and \( M_\ast \) be the masses of the black hole and the secondary star, respectively. The orbital period \( P \) and the semimajor axis \( a \) is related as \( GM P^2 = 4\pi^2 a^3 \), thus, \( a = 3 \times 10^{12} M_\odot^{1/3} P_{\text{d}}^{2/3} \text{cm} \) where \( M_\odot = M/10^3 M_\odot \) and \( P_{\text{d}} = P/1 \text{day} \). Expressed in units of the Schwarzschild radius, \( a = 10^4 (P_d/M_\odot)^{2/3} R_S \).

The secondary provides the mass supply to the BH each time it moves to the pericenter whose distance from the BH is \( R_p \), at which the secondary just fills its Roche lobe, i.e., the star’s radius \( R_\ast \) is about its Roche lobe size. Therefore (Paczyński 1971; Eggleton 1983; Sepinsky, Willems & Kalogera 2007),

\[
R_p \approx 2R_\ast \left( \frac{M}{M_\ast} \right)^{1/3} \simeq 24 R_\ast \left( \frac{M_3}{M_\ast} \right)^{1/3} \ . \tag{1}
\]

In terms of the Schwarzschild radius,

\[
R_p = 55 M_\odot^{2/3} \frac{R_\ast}{0.01 R_\odot} \left( \frac{0.6M_\odot}{M_\ast} \right)^{1/3} R_S \ . \tag{2}
\]

Though here and after we normalize the secondary by typical numbers for a WD, the equations are valid for all types of stars.

The duration of the Roche lobe overflow (stripping) is \( t_{\text{of}} \simeq R_p/v_\ast \), where \( v_\ast \simeq (2GM/R_p)^{1/2} \) is the secondary’s orbital speed at \( R_p \). From equation (1), it is easy to see that the duration of the stripping is roughly the internal dynamical time scale of the secondary star \( t_{\text{dyn}} \simeq (G\rho_\ast)^{-1/2} \simeq [R_p/(GM_\ast)]^{1/2} \), i.e.,

\[
t_{\text{of}} \simeq 2 t_{\text{dyn}} \simeq 6 \left( \frac{R_\ast}{0.01 R_\odot} \right)^{3/2} \left( \frac{0.6M_\odot}{M_\ast} \right)^{1/2} \text{s} \ . \tag{3}
\]

After the periastron passage, since the stripped matter has a binding energy (with respect to the black hole) of \( E_{\text{min}} = -GM R_\ast/R_p^2 \), it follows an elliptical trajectory with a semimajor axis of \( a_{\text{min}} = 2R_\ast(M/M_\ast)^{2/3} \), and a fallback time of

\[
t_{\text{fb}} = 2\pi \sqrt{\frac{a_{\text{min}}^3}{GM}} \simeq 18 \left( \frac{M}{M_\ast} \right)^{1/2} t_{\text{dyn}} \ . \tag{4}
\]

The above is valid under the assumption that the binding energy of the center of mass of the star is close to zero. This condition is satisfied as long as \( a_{\text{min}} \ll a \). Within the stripped mass, the spread of binding energy is small, therefore, the spread of \( t_{\text{fb}} \) is also small (see Appendix).

The fourth time scale is the local circularly orbital time scale at \( R_p \):

\[
t_{\text{cir}}(R_p) = 2\pi \sqrt{\frac{R_p^3}{GM}} \simeq 18 t_{\text{dyn}} \ . \tag{5}
\]
3. CAUSE OF FLUX DECAY OF BURSTS

The duration of the mass supply at $R_p$ is approximately equal to the duration of the stripping, and again is approximately equal to the internal dynamical time scale of the donor, $t_{\text{dyn}}$ (see Eqs. 1-3). The closeness of numbers for $t_{\text{of}}$ and $t_{\text{cir}}(R_p)$ for a WD to the observed fast rise time ($\lesssim$ minute) of the X-ray flares already hints at a WD being the donor. But here let us consider the decay time of the flare firstly, regardless of this prior. The disk's viscous time scale (representing the time that each mass element spends on its way to BH) at any radius $R$ is

$$t_{\text{vis}}(R) = t_{\text{cir}}(R) \left( \frac{H}{R} \right)^{-2}, \quad (6)$$

where $\alpha$ is the Shakura & Sunyaev viscosity parameter, and $H/R$ is the disk height-to-radius ratio. From Eqs. (5-6) one gets $t_{\text{vis}}(R_p) \simeq 28 \, \alpha^{-1} (H/R)^{-2} \, t_{\text{dyn}}$. This is the time scale over which the subsequent accretion rate (also the radiative luminosity) from a suddenly supplied mass at $R_p$ decays self-similarly (e.g., Lynden-Bell & Pringle (1974)).

If the disk is in the radiatively efficient, geometrically thin regime (Shakura & Sunyaev disk), then $H/R \approx 0.02 \, (\alpha m)^{-1/10} m^{1/5} r^{1/20}$. Here $m$, $\dot{m}$ and $r$ are black hole mass, accretion rate and radius, normalized by $M_\odot$, $L_{\text{Edd}}/(0.1c^2)$ and $R_S$, respectively. In the advective cooling dominated, geometrically thick regime (slim disk), $H/R \approx 1$. The border line between the two regimes is $\dot{m} \sim r/10$ (e.g., Kato, Fukue & Mineshige (2008)). Since the accretion rate near the flare peak is around the Eddington rate, the real $H/R$ is likely between the two limiting values: $0.02 < H/R < 1$.

Therefore, we see that $t_{\text{vis}}(R_p) \gg t_{\text{dyn}}$ since $H/R < 1$. This suggests that the flare decay time is more likely determined by $t_{\text{vis}}(R_p)$, rather than by $t_{\text{dyn}}$. The observed decay time of $10^3 \sim 10^4$ s means a rather short internal dynamical time scale of the donor $t_{\text{dyn}} \sim$
100 $\alpha_{-1}(H/R)^2$ s. Therefore, a main-sequence donor is unlikely, for instance, the Sun has $t_{\text{dyn}} \approx 1.6 \times 10^3$ s; but it is consistent with a WD being the donor.

4. CAUSES OF FLUX RISE OF BURSTS

Now back to the rise time. Here we consider two independent scenarios.

4.1. Onset of accretion near ISCO

The disk surface temperature typically drops with radius as $T(R) \propto R^{-p}$ where $p > 0$ (e.g., Kato, Fukue & Mineshige (2008)). Suppose $R_X$ is a radius in the disk within which the disk is hot enough to be X-ray bright. So the rise time corresponds to the time scale over which the “head” of the supplied mass accretes from $R_X$ to the BH within the disk, i.e., the viscous time scale at $R_X$:

$$t_{\text{vis}}(R_X) = 0.14 \frac{M_3}{\alpha_{-1}} \left( \frac{R_X}{R_S} \right)^{3/2} \left( \frac{H}{R} \right)^{-2} \text{ s.}$$

(7)

So if $R_X \approx 6.6R_S$ and $H/R \approx 0.2$, then $t_{\text{vis}}(R_X) \sim 60$ s, consistent with the observed rise time. That is to say, most of the emission during the flare is radiated from close to the innermost stable circular orbit (ISCO) of the BH.

4.2. Stream–disk interaction

After returning to the pericenter, the stripped material needs to dissipate its kinetic energy in order to circularize and form a disk. The specific energy to be dissipated is $\sim GM/R_p$. The dissipation is efficient when there is a residual accretion disk left from the previous episode of tidal stripping and mass replenishment. The existence of such a residual disk is supported by the fact that those bursting sources show ‘persistent’ emission before and after the flares, and during all the ‘non-flare’ observation periods (e.g., Irwin et al. (2016)).

The outer radius of the freshly formed disk is $\sim 2R_p$ if the bound material carries the same specific angular momentum as that of the star. Therefore, the returning stripped stream will collide with the outer disk at $R_p$, with a relative speed $\simeq 0.4(GM/R_p)^{1/2}$ between the colliding material. The colliding disk mass in situ can be comparable to the stream’s mass, while the total disk mass might easily exceed the latter because it is a cumulative residual from many previous rounds of mass replenishing and accretion. The collision efficiently dissipates a large portion of the stream’s orbital energy, with an equivalent efficiency (converting the rest-mass energy of the stream) of $\eta \sim 0.001$ (cf. Eq. 2).

The dissipation heats the interaction site, whose size $R_s$ would be slightly larger than the width of the returning stream, but be smaller than the WD itself (because only its surface layer was stripped). A reasonable estimate would be $R_s \sim 10^7$ cm. This agrees with what were inferred from the spectral data of the two sources (see §1).

The returning WD collides with the disk at $R_p$ as well. Because of the star’s strong gravity and larger cross section (compared with the returning stream), this interaction probably scoops away a large chunk of the outer disk material through an extended bow shock in front of the moving star. These material heated at the shock might produce a bright optical flare via the Bremsstrahlung radiation.

Once the stripped stream joins and replenishes the disk, the disk material drains into BH on the viscous time scale $t_{\text{vis}}(R_p)$. The enhanced accretion rate (thus, the disk radiative luminosity) also subsides self-similarly on the same time scale (see §3).

5. SUMMARY AND DISCUSSION

For a handful of ultra-luminous X-ray flaring sources, we identify the flux decay of each flare with the viscous drainage of a suddenly supplied mass tidally stripped from a donor by the central IMBH; the rapidness ($\sim$ hrs) of the decay suggests that the donor can only be a WD. The fast rise ($\sim$ minute) can be interpreted either as the onset of emission from the innermost region of the disk, or as due to the collision between the stripped stream and the outer disk when the former joins the latter at $R_p$.

The interval between two recurrent flares must be the eccentric orbital period $P$ of the donor. Independent from the type of the donor and the BH mass, it is straightforward to show that (see §2)

$$1 - e = \frac{R_p}{a} = 2 \left( \frac{\pi t_{\text{of}}}{P} \right)^{2/3},$$

(8)

where $e$ is the orbital eccentricity. If we identify the flare rise time with the duration of the stripping (§4.2), then for $t_{\text{of}} \approx 1$ minute and $P \approx 1$ day, we get $e \approx 0.97$. However, if instead the flux rise corresponds to the onset of X-ray emission from the innermost region of the disk (§4.1 and Eq. 7), then $t_{\text{of}}$ can be larger and the above constraint on $e$ relaxes. Subsequent X-ray monitoring of those flaring sources would be key to constraining the binary parameters and verifying the tidal stripping scenario.

Tidal disruptions of WDs by IMBHs have been widely studied (Krolik & Piran 2011; MacLeod et al. 2014, 2016; Law-Smith et al. 2017), which differ from the tidal stripping considered here in several ways. First, for parabolic orbits, the event rate of strippings is slightly higher than the disruptions since $R_p \approx 2R_t$. Second,
for eccentric orbits, the system spends several orders of magnitudes longer time in the life stage during which tidal stripping repeatedly occurs than the disruption does (see below).

Third and the most important, in a WD disruption event, the debris fallback mass supply rate is extremely super-Eddington ($\sim 10^7 \times$ at peak) and it remains above the Eddington rate for $\sim 1$ year. One would expect enormous mass ejection in forms of quasi-spherical outflow during this long period due to energy dissipation from debris stream collision and (later) central accretion. Any high-energy emission from near the BH would be reprocessed by the outflow and the photospheric emission of the latter dominates the observation, similar to a main-sequence stellar TDE by IMBHs (Chen & Shen 2018). Therefore, the emission of a WD TDE will be at lower photon energies (UV to soft X-rays) and last much longer (months to a year). Although the fallback time of the most bound debris (which falls back earliest) is relatively short ($\sim 10$ minutes), it is still very hard to produce a fast transient which brightens to $L_{\text{edd}}$ and shines within 1 minute and then decays. Moreover, these detected flares lack any sign of significant spectral or absorption evolution, which disfavours a scenario in which the flare had appeared in the earliest minute of a WD TDE and it was quickly obscured by the launching of an outflow.

Zalamea et al. (2010) have studied the impact of gravitational wave (GW) emission on the orbit of a WD stripping binary with an IMBH of $10^5 M_\odot$. They show that the slow decrease of $R_p$ (thus, the increase of stripped mass $\delta M$ in each passage; cf. Appendix) experiences two stages. Stage (i) is controlled by the gentle GW emission. The fractional change of $R_p$ during each orbit is $\gamma = -\dot{P} R_p / R_p \sim 10^{-5} M_5^{2/3}$. Stage (ii) is controlled by the WD mass loss, i.e., the increasing $R_*$. Stage (i) lasts longer (roughly $\gamma^{-3/5} \sim 10^3$ orbits) and mass loss is gentle, but the mass loss accelerates in stage (ii) until it reaches total disruption ($\sim 200$ orbits). To scale down to $M = 10^3 M_\odot$, stage (i) would be even longer, $\sim 10^4$ orbits. Such systems are ideal targets for next-generation GW detectors, e.g., DECIGO and Einstein Telescope, with the aid of long-term X-ray monitoring.

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Here we take a closer look at the tidal stripping, in order to characterize the mass, depth, and specific energy distribution of the stripped layer. Let \( x \ll R^* \) be the depth of the stripped surface layer, as is illustrated in Figure A1. Consider a ring of radius \( y \) within the layer at depth \( x \). Each point on the right is at distance \( r = \sqrt{y^2 + (R^* - x)^2} \) from the center of the star. At any fixed depth \( x \), the mass per unit depth is

\[
\frac{dM}{dx} = 2\pi \int_0^{\sqrt{R^2 - (R^* - x)^2}} \rho(x, y) y dy. \tag{A1}
\]

Changing the variable from \( y \) to \( r \) by \( ydy = rdr \), the integral becomes

\[
\frac{dM}{dx} = 2\pi \int_{R_*}^{R} \rho(r) rdr. \tag{A2}
\]

Since \( x \ll R_* \), we can approximate \( r \approx R_* \) and move it out of the integrand. Then we use another depth variable \( x' = R_* - r \) to rewrite the above to

\[
\frac{dM}{dx} = 2\pi R_* \int_0^{x} \rho(x') dx'. \tag{A3}
\]

![Figure A1. The white dwarf donor at the pericenter when its surface layer of a depth \( x \) facing the black hole is tidally stripped. Here we neglect tidal deformation.](image)

For simplicity, we assume the surface structure of the white dwarf is described by a polytrope of \( P = K\rho^\Gamma \) with \( \Gamma = 5/3 \), the same as in the deeper region where electrons are degenerate and non-relativistic. Once \( M_* \) and \( R_* \) are given, the value of \( K(M_*, R_*) \) is known from numerically solving the Lane-Emden equation. The hydrostatic equilibrium at the surface \( dP/dx = GM_*\rho/R_*^2 \) gives the density structure there

\[
\rho(x) = A\bar{\rho} \left( \frac{x}{R_*} \right)^{3/2}, \tag{A4}
\]

where \( A \approx 3.8 \) and \( \bar{\rho} = M_*/(4\pi R_*^3/3) \) is the average density. Therefore,

\[
\frac{dM}{dx} = \frac{3}{5} A \frac{M_*}{R_*} \left( \frac{x}{R_*} \right)^{5/2}. \tag{A5}
\]

The fraction of the total stripped mass is

\[
\frac{\delta M}{M_*} = \frac{6}{35} A \left( \frac{x}{R_*} \right)^{7/2}. \tag{A6}
\]

Equation (A6) provides a relation between \( \delta M \) and \( x \), so that one could estimate the depth ratio \( x/R_* \) from \( \delta M/M \). The observed fluence of the two flaring sources suggest about \( \sim 10^{-10} \) \( M_\odot \) of mass is accreted in each case. This
implies the depth of the stripped layer is \( x/R_\ast \sim 10^{-3} \). An alternative version of equation (A6) is \( \delta M/M_\ast \propto (x/R_\ast)^{5/2} \) as was given by Zalamea et al. (2010) who adopted a spherical-shell shape of the stripped layer. There, the stripped layer depth ratio is even smaller, \( x/R_\ast \sim 10^{-5} \), for the same \( \delta M/M_\ast \).

Within the stripped mass, the spread of the binding energy relative to the BH is very small, \( \delta E/E \simeq x/R_\ast \), which means a very small spread of the returning time \( \delta t_{fb}/t_{fb} \simeq 3\delta E/(2E) \simeq 3x/(2R_\ast) \ll 1 \). Since \( t_{fb} \sim (M/M_\ast)^{1/2}t_{of} \) (Eq. 4), it means a small ratio of the spread of returning time over the “length” of the stream, \( \delta t_{fb}/t_{of} \sim (M/M_\ast)^{1/2}x/R_\ast \sim 0.1 \) for \( M/M_\ast = 1 \) and \( x/R_\ast = 10^{-4} \). Therefore, the duration over which the stream of the stripped matter returns to the pericenter and collides with the residual disk is set by the duration of stripping \( t_{of} \).