From inversion to enhancement of the Josephson current by an exchange field in S/F-I-F/S tunnel structures

V. N. Krivoruchko, E.A.Koshina

Donetsk Physics & Technology Institute, Donetsk, Ukraine

(January 7, 2022)
Abstract

We study theoretically the dc Josephson tunnel current for a junction of two proximity S/F bilayers of a massive superconductor (S) / a thin ferromagnet (F) separated by an insulating (I) barrier. The dependence of the critical current on the relative orientation of the F layers magnetization is analyzed within the microscopic theory of the proximity effect for an S/F bilayer. We demonstrate that for the S/F-I-F/S contact critical current can reverse its sign (\(\pi\)-state of the junction) for the parallel orientation of the F layers magnetization, while for antiparallel alignment an enhancement of the critical current takes place. The results provide a new effect of the interplay between superconductivity and ferromagnetism in hybrid structures.

PACS: 74.50.+r, 74.80.Dm, 75.70.Cn

Progress in nanotechnology in the last few years made it possible to produce nanostructures in which new physical phenomena have been observed. Specifically, hybrid systems consisting of superconductors (S) and ferromagnets (F) have been created, which opens a possibility to explore various mesoscopic effects in their superconducting and magnetic properties. Particularly, the transport properties of S/F structures with artificial geometry have turned out to be quite unusual. E.g., for SFS weak links a crossover from 0-phase to \(\pi\)-phase superconductivity function of the thickness \(d_F\) of ferromagnet, or the exchange field \(H_{\text{exc}}\), have been theoretically described [1,2] and experimentally observed [3], as well as oscillation of the S/F multilayers transition temperatures [4-7]. Recently Bergeret et al. [8] predicted that in the case of an antiparallel orientation of the exchange field of the bilayers, critical current of the S/F-I-F/S junction increases at low temperatures with increasing \(H_{\text{exc}}\) and at zero temperature has a singularity when \(H_{\text{exc}}\) equals the superconducting gap. This behavior contrasts common knowledge that exchange field reduces the Josephson critical current. The authors consider the model, when the influence of the F layers on superconductivity is equivalent to inclusion of a homogeneous exchange field with a reduced value, and come to
limit of effective values of the superconducting order parameter, the coupling constant, and the effective magnetic moment. However, as is well known, any quantitative calculation of the properties of inhomogeneous superconductors in contact with magnetic interfaces must start with an accurate boundary conditions and calculation of the superconducting properties near the S/F interface [9,10]. In real systems the exchange field of the F layer leads to such decrease of the interface transparency that allows a jump of the pairing amplitude near the interface, while theory [8] assumes that the pairing function is continuous across the interface. As a matter of fact, the results of Ref.8 should not be considered as conclusive ones and it is reasonable to analyze a generalization of the model to a more realistic case. In the present report, on the basis of microscopic theory of the proximity effect for an S/F bilayer [11], the critical current of an S/F-I-F/S tunnel junction is analyzed as a function of the parameters of the S/F bilayer, such as the proximity-effect magnitude, the transparency of the S/F interface, the exchange-field strength, and relative magnetizations orientation of the ferromagnetic layers.

We consider the bilayer of a massive superconductor and a thin ferromagnet, where (singlet) superconducting and ferromagnetic metals are assumed to be dirty. It is shown that by changing the relative magnetizations orientation one can for the same symmetric S/F-I-F/S contact turn the tunnel current from 0-phase state to π-phase state (inversion of the critical current), if there is parallel orientation of the F layers magnetization, or enhance the tunnel current, if there is antiparallel orientation of the F layers. The feature important for practical application is that the conditions for critical current inversion or enhancement can be changed by varying the parameters of the S/F bilayer and external magnetic field orientation. We also show that critical current singularity predicted in Ref.8 is the result of the oversimplified model the authors considered.

To be specific, let us consider the proximity coupled S/F bilayer of a massive superconductor and a thin ferromagnet, with the thickness of the F layer much less than its superconducting coherence length $\xi_F \gg d_F$ ($\xi_F^{-} \sqrt{\hbar D_F/\mu_B H_{exc}}$ where $D_F$ is a diffusion coefficient in the F metal). One can expect a kind of tunneling interaction in the S/F-sandwich
that is similar to the superconducting proximity effect. Actually, when diffusing into the thin ferromagnetic layer, superconducting electrons are subject to an interaction with the local exchange field. E.g., an electron with a spin, e.g. "up", has an extra energy $\mu_B H_{exc}$, while an electron with a spin "down" has lower energy $\mu_B H_{exc}$, caused by the intrinsic magnetic field $H_{exc}$ into F layer (see, e.g. [12]). After tunneling back to S layer the Cooper pair quickly loses its extra energy during the time $\tau^{-1} = \hbar/\mu_B H_{exc}$ being related to a small range $\lambda_F \sim \hbar / \mu_B H_{exc} (v_F$ - Fermi velocity). Such an equilibrium process leads to a modification of the electron spectrum of superconductor on the nanoscale length $\lambda_F$ (in most cases $\lambda_F \sim 10^{-8} m$). So, we can speak about induced exchange magnetic correlation into the S layer that affects the Cooper pairs and characterize the S/F bilayer as a unified system with strong superconducting-ferromagnetic correlation. Owing to this magnetic proximity effect the S/F-I-F/S junction can get the $\pi$-phase superconductivity even if the thickness of the F layer is much less than the superconducting coherence length of the F metal. This mechanism should not be confused with the $\pi$-junction behavior induced by magnetic impurities [13,14], or resulting from the symmetry of the order parameter [15], or due to direct access to the microscopic current-carrying electronic state inside the link [16,17]. Our case also differs from the situation in S/F sandwiches with thick F layer, where spatially dependent phase in the F layer causes an exchange field dependent oscillation in the critical current of the SFS weak links and in the $T_C$ of the multilayers [1-7] (for details see Refs. 11, 18).

The critical current of the (S/F)$_L$-I-(F/S)$_R$ tunnel contact can be represented in the form (Ref.18)

$$ j_C = (eR_N/2\pi T_C) I_C = (T/T_C) \text{Re} \sum_{\omega > 0} \{G_{SL} \Phi_{SL} G_{SR} \Phi_{SR}/\omega^2\} \times $$

$$(1 + 2\varpi G_S (\gamma_B/\pi T_C) + \varpi^2 (\gamma_B/\pi T_C)^2)_L \times (1 + 2\varpi G_S (\gamma_B/\pi T_C) + \varpi^2 (\gamma_B/\pi T_C)^2)_R}^{-1/2}$$

where $R_N$ is the resistance of the contact in the normal state; $\varpi = \omega + i(\pm H_{exc})$ and the sign of the exchange field depends on mutual orientation of the bank magnetizations; $\omega = \pi T(2n+1)$ are Matsubara frequencies; and the subscript L (R) labels quantities referring to the left (right) bank. Henceforth, we have taken the system of units in with $\hbar = \mu_B = \ldots$
\( k_B = 1 \), and have also used the modified [19] Usadel function \( \Phi_{S,F} \) defined by relations
\[ G_S = \omega/(\omega^2 + \Phi_S^2)^{1/2}, \quad F_S = G_S\Phi_S/\omega, \quad G_F = \omega/(\omega^2 + \Phi_F^2)^{1/2}, \quad F_F = G_F\Phi_F/\omega. \]
So, we have taken explicitly the normalization condition \( G^2 + |F|^2 = 1 \) on usual Usadel functions \( G_{S,F}, F_{S,F} \).

Presented below are the results obtained on the basis of microscopic theory of proximity effect for an S/F bilayer characterized by arbitrary values of the exchange field \( H_{exc} \), S/F interface boundary transparency \( \gamma_B \) and for the cases of a weak \( (\gamma_M << 1) \) or a strong \( (\gamma_M >> 1) \) proximity effect (Ref.11). Namely, for a weak proximity effect we have for the function \( \Phi_S(\omega) : \)
\[ \Phi_S(\omega) = \Delta_0 \left( 1 - \frac{\gamma_M \beta \omega}{\gamma_M \beta \omega + \omega A} \right) \quad (2) \]
where \( \Delta_0 \) is the absolute value of the BCS order parameter in the bulk of the S layer,
\[ \beta^2 = (\omega^2 + \Delta_0^2)^{1/2}/\pi T_C, \quad \text{and} \quad A = [1 + \gamma_B \omega (\gamma_B \omega + 2\omega/\beta^2)]/((\pi T_C)^2]^{1/2}. \]
For a strong proximity effect our calculations yield
\[ \Phi_S(\omega) = B(T) \left( \pi T_C + \gamma_B \omega \right)/\gamma_M \omega, \quad (3) \]
where \( B(T) = 2T_C[1 - (T/T_C)^2][7\zeta(3)]^{-1/2} \) and \( \zeta(3) \) is the Riemann \( \zeta \) function. As is seen from the expressions (2), (3) ferromagnetic correlations are induced into S layer and Green’s functions of the S layer now depend upon \( H_{exc} \). Using these results, we calculate the dependence of the amplitude of S/F-I-F/S junction critical current on the orientation of the magnetization in the F layers.

**Parallel orientation of the layer’s magnetizations.** Let us present here an analytical consideration for the case of a vanishing interface resistance, \( \gamma_B = 0 \). Using the expressions (1) and (2) with \( \omega_L = \omega_R \), we have obtained for a weak influence of the F layer on superconducting properties of the S metal, \( \gamma_M << 1 \):
\[ j_C^p = \Delta_0^2 \sum_{\omega > 0} \frac{\Delta_0^2 + \omega^2 - (\gamma_M \beta H_{exc})^2}{[\Delta_0^2 + \omega^2 - (\gamma_M \beta H_{exc})^2]^2 + (2\omega \gamma_M \beta H_{exc})^2} \quad (4) \]
One can see, that if the exchange field is strong enough, namely $\gamma_M H_{\text{exc}} > \pi T_C \beta$, the critical current changes its sign; that is the phase difference between the superconducting order parameters on banks of the junction changes by $\pi$.

If there is a strong suppression of the order parameter near SF boundary, $\gamma_M >> 1$, critical current of the contact can be presented in the form:

$$j_C^p \approx \left( T/T_C \right) B_M^2(T) 2 \sum_{\omega>0} \omega^2 / \left\{ \omega^2 (\omega^2 + H_{\text{exc}}^2) \right\}$$  \hspace{1cm} (5)$$

where $B_M(T) = B(T) \pi T_C / \gamma_M$, and in proceeding these relations, we have taken into account that value of $\Phi_S$ is small, $\Phi_S \sim \gamma_M^{-1}$. For $H_{\text{exc}} \to 0$ expressions (4) and (5) restore the result for S/N-I-N/S junction (see, e.g., Ref.19). In the opposite case of an increasing magnetic energy the critical current changes its sign for large enough $H_{\text{exc}}$, $H_{\text{exc}} >> \pi T_C$, or, in other words, the crossover of the junction from 0-phase state to the $\pi$-phase state takes place.

**Antiparallel orientation of the layer’s magnetizations.** To be definite, we took $\omega_L = \omega + i H_{\text{exc}}$, $\omega_R = \omega - i H_{\text{exc}}$. After simple transformations we have for the case $\gamma_B = 0$ and $\gamma_M << 1$:

$$j_C^a = T/T_C \text{Re} \sum_{\omega>0} \frac{\Phi_S}{\sqrt{\omega^2 + \Phi_S^2}} |_{L} \frac{\Phi_S}{\sqrt{\omega^2 + \Phi_S^2}} |_{R} \approx$$

$$2T/T_C \sum_{\omega>0} \frac{\Delta_0^2}{\omega^2 + \Delta_0^2} \left[ 1 - 2(\gamma_M \beta H_{\text{exc}})^2 \frac{\Delta_0^2 - \omega^2}{(\omega^2 + \Delta_0^2)^2} + \frac{(\gamma_M \beta H_{\text{exc}})^4}{(\omega^2 + \Delta_0^2)^2} \right]^{-1/2}$$

In proceeding these relations, we have taken into account that $\gamma_M \beta$ is small, $\gamma_M \beta << 1$. One can see that for $\omega < \Delta_0$ the expression in square brackets is lower than 1 for $H_{\text{exc}}$ from a broad region $0 < H_{\text{exc}} < \sqrt{2(\Delta_0^2 - \omega^2)/\gamma_M \beta}$. As the result, for some values of $H_{\text{exc}}$ one can obtain the enhancement of the tunnel current, $j_C^a(H_{\text{exc}}) > j_C^a(0)$, in contrast to its suppression by the magnetic moments aligned in parallel.

For the case of a strong proximity effect $\gamma_M >> 1$ we obtain:

$$j_C^s \sim B_M^2(T)T/T_C \sum_{\omega>0} \omega^{-2} \left[ (\omega^2 - H_{\text{exc}}^2 + B_M^2(T)/\omega^2)^2 + (2\omega H_{\text{exc}})^2 \right]^{-1/2}$$
Now the exchange field region where critical current increase is much less \( 0 < H_{\text{exc}} \leq \sqrt{B_{\text{M}}^2(T) - \omega^4/\omega} \), however at low temperature there is also the region where \( j_{\text{C}}(H_{\text{exc}}) > j_{\text{C}}(0) \).

For a general configuration, when the magnetizations of the banks are at an angle \( \theta \), the conductivity for parallel channel is proportional to \( \cos^2(\theta/2) \), while the conductivity for antiparallel channel is proportional to \( \sin^2(\theta/2) \). So, the critical current can be written in the form [21, 8]:

\[
j(\theta) = j_{\text{pC}} \cos^2(\theta/2) + j_{\text{aC}} \sin^2(\theta/2)
\]

As it was before, we suppose that the Hamiltonian involved with the tunneling process is spin independent.

On Figs. 1 and 2 we show the results of numerical calculations of the amplitude of the Josephson current for the case of a weak, \( \gamma_M << 1 \) (Fig.1), and strong \( \gamma_M >> 1 \) (Fig.2) proximity effect and different quality of the S and F metals electrical contact versus the exchange field strength for parallel (solid curves) and antiparallel (dashed curves) mutual orientation of the electrodes’ magnetizations. It can be seen, that a state of the junction depends greatly on the bilayer parameters \( H_{\text{exc}}, \gamma_M \) and \( \gamma_B \), and relative orientation of the left and right magnetizations. For the case of parallel orientation (solid curves on Figs. 1, 2) the critical current drops down to zero and then acquires negative values. So, in some interval of exchange field strengths, a state with \( \pi \) phase shift across the contact is formed. At weak proximity effect and high boundary transparency, varying the bilayer parameters, we can change sizably the conditions under which the state characterized by a \( \pi \) phase difference at the banks of the junction is realized (see Fig.1). At strong proximity effect there is a reduction of the absolute value of \( j_{\text{C}} \), and the point of the crossover is not so sensitive to the S/F boundary conditions (see Fig.2).

For the case of antiparallel orientation (dashed curves on Figs. 1 and 2) a state with 0 phase shift across the contact is formed, but in some interval of exchange field strengths the enhancement of dc Josephson current takes place. As in the case of parallel orientation, if
\(\gamma_B << 1\) and \(\gamma_M << 1\) by varying the bilayer parameters we can change sizably the range of exchange field where the current enhancement is observed. If \(\gamma_M >> 1\) the role of the interface is reduced. As is seen on Figs. 1 and 2, for antiparallel geometry there is not singularity predicted in Ref.8.

Figure 3 shows the dc tunnel current versus exchange field at different temperature. One interesting point to note is that the Josephson current enhancement holds only for low enough temperatures, while at \(T > 0.5T_C\) the phenomenon disappears - see results presented on Fig.3 by dashed curves. On the contrary, a spontaneous \(\pi\) shift of superconducting wave functions phase of the banks holds for all the superconducting state temperature range.

The physics behind both striking behaviors of dc Josephson current in question is the induced magnetic properties of the S layers. Namely, the S metal in good electric contact with the F one acquires some magnetic properties and one can characterize the S/F bilayer as unified system with strong superconducting–ferromagnetic correlation. As a result, the superconducting order parameter at the F/S boundary acquires phase shift depending on the orientation and value of the exchange field in the F layers [18]. Due to the Cooper pair amplitude fluctuation as a function of \(H_{exc}\), there is a possibility at some value of the exchange field to arrange the minimum of the pair’s amplitude exactly on S/F interface. As a result, the relative S/F boundary influence on superconducting order will be even lower than in a case of S/N boundary with the same interface parameters \(\gamma_B\) and \(\gamma_M\).

The enhancement of the Josephson current by an exchange field in superconductor was recently discussed by Bergeret et. al.(Ref.8) considering a toy model. However, the effects of inversion at parallel and enhancement at antiparallel configurations for S/F-I-F/S junction found in our work have not been discussed yet. The results of our report are based on microscopic theory of the proximity effect for S/F bilayer and accurate calculation of the superconducting properties near the S/F interface. By using general expressions, obtained in [18], the amplitude of the critical current of symmetric S/F-I-F/S tunnel contact has been calculated as function of the F layers magnetization orientation and the S/F interface parameters such as the proximity-effect strength, the transparency of the S/F interface, and
the strength of the exchange field in the F metal, and temperature. Our results show that
the superconducting properties of S/F-I-F/S junctions based on S/F bilayers of a massive
superconductor and a thin ferromagnet can be varied from a state with 0-phase supercon-
ductivity with enhancement of Josephson critical current to a $\pi$-phase superconductivity
with reversal of tunnel current by simply changing the relative orientation of the left and
right banks magnetization. These results provide new effects of the proximity coupled su-
perconductor/ferromagnet hybrid structures.

We are grateful to V.V.Ryazanov who has turned our attention to the work [8].
REFERENCES

[1] 1. A. I. Buzdin, L. N. Bulaevskii, S. V. Panjukov. Letters to JETP .v35, p.147 (1982).
2. A. I. Buzdin, M. Yu. Kuprijanov, Letters to JETP n.53, p.308 (1991).
3. V. V. Ryazanov, V. A. Oboznov, A. Yu. Rusanov, A. V. Veretennikov, A.A.Golobov, J. Aarts. cond-mat/0008364.
4. Z. Radovic, M. Ledvij, L. Dobrosavljevic-Grujic, A. I. Buzdin, J. R. Clem. Phys.Rev. B 44, 759 (1991).
5. L. Lazar, K. Westerholt, H. Zabel, L. R. Tagirov, Yu. V. Goryunov, N. N. Garif’yanov, I. A. Garifullin. Phys. Rev. B 61, 3711 (2000).
6. J.S.Jiang, D. Davidovic, D.H.Reich, C. L. Cheien. Phys.Rev. Lett. 74, 314 (1995).
7. M. G. Khusainov Yu. N. Proshin. Phys. Rev. B 22, R14283 (1997).
8. F. S. Bergeret, A. F. Volkov, K. E. Efetov. cond-mat/0102012.
9. J.Aarts, J.M.Geers, E.Bruck, A.A.Golobov, R.Coehorn. Phys.Rev. B 56, p.2779 (1997).
10. L.Lazar, K.Westerhold, H.Zabel, L.R.Tagirov, Yu.V.Gorynov, N.N.Garif’yanov, I.A.Garifullin. Phys.Rev. B 61, p.3711 (2000).
11. E. A. Koshina, V. N. Krivoruchko. Low. Tem. Phys.v.26, 115 (2000);
12. E. A. Demler, G. B. Arnold, M. R. Beasley, Phys.Rev. B 55, 15174 (1997).
13. L. N. Bulaevskii, V. V. Kuzii, A. A. Sobyainin. Pis’ma Zh. Eksp. Teor. Fiz. v.25, p.314 (1977).
14. A. V. Andreev, A. I. Buzdin, R. M. Osgood. Phys. Rev. B 43, 10124 (1991).
15. M. Siegrist, T. M. Rice. J. Phys. Soc. Jpn. v.61, p.4283 (1992).
16. A. F. Volkov. Phys. Rev. Lett. v.74, p.4730 (1995).
17. J.J.A. Baselmans, A.F.Morpurgo, B. J. van Wees,T. Klapwijk. Nature 397, 43 (1999).
18. E. A. Koshina, V. N. Krivoruchko. JETP Letters v.71, 123 (2000); Phys.Rev. B 2001
(to be published)

19. A. A. Golubov, M. Yu. Kupriyanov, JETP 96, 1420 (1989).

20. M. Yu. Kupriyanov, V. F. Lukichev. Sov. J. Low Tem. Phys. v.8, 1045 (1982).

21. P. Raychaudhuri, K. Sheshadri, P. Taneja, S. Banyopadhyay, P. Ayyub, A. K. Nigam. R. Pinto. Phys.Rev. B 59, 13919 (1999).
Figure 1: Critical current of an S/F-I-F/S tunnel junction for $T << T_C$ versus exchange energy at high S/F interface transparency, $\gamma_B = 0.1$ and weak proximity effect $\gamma_M = 0, 0.1, \text{and } 0.2$ (curves 1, 2 and 3, respectively). Solid curves illustrate the case of parallel orientation of F layers magnetization; dashed curves illustrate the case of antiparallel orientation of F layers magnetization.
Figure 2: Critical current of an S/F-I-F/S tunnel junction for $T \ll T_C$ versus exchange energy at strong proximity effect $\gamma_M = 10$, and high $\gamma_B = 0$ and low $\gamma_B = 2$ and 5 S/F interface transparency (curves 1, 2 and 3, respectively). As on Fig. 1, solid curves illustrate the case of parallel, while dashed curves illustrate the case of antiparallel orientation of F layers magnetization.
Figure 3: Temperature influence on the critical current of an S/F-I-F/S tunnel junction at high S/F interface transparency, $\gamma_B = 0.1$ and weak proximity effect $\gamma_M = 01$. $T/T_C = 0.1, 0.35, 0.5$ and $0.7$ (curves 1, 2, 3 and 4, respectively). Solid curves illustrate the case of parallel orientation and dashed curves illustrate the case of antiparallel orientation of F layers magnetization.