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 terse theory on the Gilbert damping due to spin
 pumping effects in the magnetic multi-layer system

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 Abstract. We have calculated the Gilbert damping coefficient \( \alpha \) in the magnetic multi-layer structures consisted of ferromagnetic (FM) and non-magnetic (NM) layers which are FM/NM and FM/NM/FM from microscopic theory. In this study, our model Hamiltonian doesn’t include the spin-orbit interaction term, but the kinetic electron hopping term. For obtaining \( \alpha \) in theory, we used Kubo formula and Projection method. We found \( \alpha \) is dependent on structure of systems and both thickness of the FM and NM layer. Spin pumping effect which is predicted by experimental results is confirmed by this calculation in theory. We conclude that transferring conduction electrons carrying angular momentum to adjacent layers are the origin of the Gilbert damping for multi-layer systems.

 1. Introduction
 In the field of spintronics, there is rapidly growing interest in controlling the damping behavior of dynamical spins in the magnetic devices such as magnetic random access memory (MRAM) and so on, and then the microscopic foundation on the origin of the Gilbert damping coefficient (\( \alpha \)) is strongly desired to understand completely. Phenomenologically, the low-temperature magnetization \( M \) dynamics in ferromagnets is well described by the following Landau-Lifshitz-Gilbert (LLG) equation [1, 2, 3].

 \[
 \frac{dM}{dt} = -\gamma M \times H_{\text{eff}} + \frac{\alpha M_s}{M_s} \times \frac{dM}{dt}.
\]

 Here, \( H_{\text{eff}} \) is the "effective" magnetic field (including external, exchange and anisotropy fields), \( \gamma (>0) \) is the gyromagnetic ratio, \( M_s \) is the saturation magnetization, and \( \alpha \) is the Gilbert damping coefficient, often measured in terms of the Landau-Lifshitz damping rate [4] \( \lambda = \alpha \gamma M_s \) (in cgs units). As regards MRAM, \( \alpha \) has a close relation to speed of switching a bit of information encoded by the magnetization direction of a ferromagnetic grain and energy demands of switching [5, 6]. A detailed understanding of the Gilbert damping mechanisms in metallic ferromagnets and precise calculation of damping constant would greatly facilitate the design of new materials appropriate for a variety of applications.

 Kamberský [7] and Gilmore et al. [8] have recently proposed a convenient way to calculate the Gilbert damping coefficient for the ferromagnetic bulks by using the first principles technique for the electronic structure calculations. This approach is based upon the theory called torque correlation model, which gives the Gilbert damping coefficient as

 \[
 \alpha = \lim_{\omega \to 0} \frac{1}{\hbar \omega} \text{Im} \left( \frac{1}{\langle M^- | M^- \rangle} \left( \frac{L M^-}{\hbar \omega + L + i\delta} \right) \right),
\]

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where \( \langle A | B | C \rangle \equiv \left\langle \left[ B, C \right] \right\rangle \) for any operator \( A, B, C \), \( \delta(>0) \) is the infinitesimal quantity and \( \mathcal{L}A \equiv [\mathcal{H}, A] \) is Liouville superoperator that commutes with the Hamiltonian \( \mathcal{H} \) for any operator \( A \). Kamberský and Gilmore \textit{et al} chose the spin-orbit interaction \( \mathcal{H}_{SO} \) for the \( \mathcal{H} \) and obtained quantitative results for \( \alpha \) of transition metals such as Fe, Co and Ni metals.

On the other hand, the Gilbert damping mechanisms in the magnetic inhomogeneous systems are different from that in the magnetic homogeneous bulk structures. Mizukami \cite{9, 10} have experimentally measured the ferromagnetic resonance (FMR) linewidth for NM (non-magnetic)/FM (ferromagnetic) /NM structures and found that \( \alpha \) is strongly related to thickness of the NM and FM layers. This phenomenon is due to spin current pumped into NM layer by the localized spins in FM layer, which is so-called spin pumping effect. In theory, we consider that not only spin-orbit interaction but also the kinetic (electron hopping) term \( \mathcal{H}_t \) contribute to the origin of \( \alpha \) in magnetic inhomogeneous systems.

From these viewpoints, we have calculated \( \alpha \) for the multi-layer structures consisted of FM and NM layers from microscopic theory using Hamiltonian which includes \( \mathcal{H}_t \).

2. Model
Our model is one dimension and described by the total of the tight-binding model Hamiltonian (electron hopping term) \( \mathcal{H}_t \) and \( s\text{-}d \) model Hamiltonian \( \mathcal{H}_{sd} \):

\[
\mathcal{H} = \mathcal{H}_t + \mathcal{H}_{sd} = -\sum_{i,j,\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} - J_{sd} \sum_i \mathbf{S}_i \cdot \mathbf{\sigma}_i ,
\]

where \( c_{i\sigma}^\dagger (c_{i\sigma}) \) create (annihilate) a electron of \( \sigma \) spin on \( i \) site, \( t_{ij} \) is the hopping rates, \( \mathbf{S} \) denotes localized \( d \) spins, \( \mathbf{\sigma} \) is a vector of Pauli spin matrices) denotes conducting \( s \) electrons, \( J_{sd}(>0) \) is \( s\text{-}d \) exchanging coupling constant, \( \sum_{i,j} \) means summing when \( i \) and \( j \) sites are adjacent and \( \sum_i^{FM} \) means summing when \( i \) sites are in FM layer.

3. Theory
Linear response to a harmonic field \((h_x - i\hbar \dot{t}) \exp(-i\omega t)\) is described by a single susceptibility function \( \chi_m(\omega) \). One can obtain the macroscopic susceptibility:

\[
\chi_m(\omega) = \frac{\gamma M_s}{\gamma H_{eff} - \omega - i\alpha \omega}
\]

from LLG equation and \( \mathbf{M} \equiv \chi_m \mathbf{H}_{eff} \). On the other hand, in microscopic theory, the transverse spin susceptibility can be written as

\[
\chi_{\perp}(\omega) = -\frac{1}{2\hbar} \int_0^\infty dt \exp(i\omega t) \left\langle \left[ \mathcal{M}^+(t), \mathcal{M}^- \right] \right\rangle ,
\]

from formal first-order response theory (Kubo formula) \cite{11, 12}. Here, \( \left\langle \ldots \right\rangle \) denotes the statistics average in adiabatic approximation and \( \mathcal{M}^+(t) \) denotes the Heisenberg representation of \( \mathcal{M}^+ = \sum_i -2s\sigma_i^+ + \sigma_i^- \sigma_i^+ + \mu_B \mathbf{M}^+ \) (\( \mu_B \) is Boltzmann constant), which may be written as \( \mathcal{M}^+(t) = \exp(i\mathcal{L}t/\hbar)\mathcal{M}^- \). The projection formalism \cite{13} allows us to rewrite Eq.(5)

\[
\chi_{\perp}(\omega) = -\frac{1}{2} \frac{\left\langle \mathcal{M}^- | \mathcal{M}^- \right\rangle}{\hbar \omega - \Omega + \Gamma(\hbar \omega + i\delta)} ,
\]

\[
\Omega = -\frac{1}{\left\langle \mathcal{M}^- | \mathcal{M}^- \right\rangle} \left\langle \mathcal{M}^- | \mathcal{L} | \mathcal{M}^- \right\rangle ,
\]

\[
\Gamma(\hbar \omega + i\delta) = -\frac{1}{\left\langle \mathcal{M}^- | \mathcal{M}^- \right\rangle} \left( \mathcal{L} \mathcal{M}^- \right) \left[ \frac{1}{\hbar \omega + \mathcal{L} + i\delta} \mathcal{L} \mathcal{M}^- \right] \left( \frac{1}{\mathcal{L} \mathcal{M}^-} \right) .
\]
and we obtain Eq.(2) from comparing Eqs.(4) with (6). Substituting Eq.(3) for Eq.(2) gives the average of $\alpha$ in FM layer:

$$\alpha = -\lim_{\omega \to 0} \frac{1}{\hbar \omega} \text{Im} \Gamma(\hbar \omega + i\delta)$$

$$= \frac{\pi}{4\mu_B} \sum_i \langle M_i \downarrow \rangle \sum_{m,n} \left| \langle m, \downarrow | [H, M^\dagger] | n, \uparrow \rangle \right|^2 \delta(\epsilon_F - \epsilon_m \downarrow) \delta(\epsilon_F - \epsilon_n \uparrow)$$

(9)

where $\epsilon_{m\downarrow(\uparrow)}$ is a one-electron eigenvalue, $\epsilon_F$ is the Fermi energy and $|m(n), \downarrow(\uparrow)\rangle$ denotes an eigenstate.

We introduce a lifetime of conduction electrons and use the Lorentzian instead of delta-function.

4. Results and Discussion

Figure 1 shows NM layer thickness dependence of $\alpha$ in FM layer for FM(100 sites)/NM. We found that $\alpha$ increases with increasing of the thickness of NM layer. The oscillating behavior is an interference effect appearing in the finite size system. We consider this result is due to transferring of conduction electrons having a spin. Concretely, current of conduction electrons carrying spin angular momentum from FM layer to NM layer increase because of increasing the thickness of NM layer, and then more angular momentums in FM layer go through the interface to NM layer. It can be considered that this phenomenon is occurred by spin-current induced by the magnetizations rotation in FM layer, which is spin pumping effect.

![Figure 1. NM layer thickness dependence of $\alpha$ for FM(100 sites)/NM. ($t$ is the hopping rate and $J_{sd}$ is the s-d exchanging coupling constant)](image1)

![Figure 2. FM layer thickness dependence of $\alpha$ for FM/NM(100 sites).](image2)

Figure 2 shows FM layer thickness dependence of $\alpha$ in the FM layer for FM/NM(100 sites). We confirmed that $\alpha$ tends to decrease by increasing the FM layer. This may reflect the situation that the fraction of interface layer relative to the FM layer region decreases with increasing FM layer, leading to depress of the spin pumping efficiency.

After obtaining $\alpha$ for FM/NM systems, we calculated $\alpha$ for three layers system of FM1($J_{sd} = 0.2$, 100 sites)/NM(10 sites)/FM2 (Figure 3). In the system, we suppose that magnetizations in FM1 layer are rotating while magnetizations in FM2 layer are motionless. Figure 4 shows FM2 layer thickness dependence of $\alpha$ in FM1 layer for (red line). (NM layer thickness dependence of $\alpha$ in FM1 layer for FM1/NM (blue line) is shown for comparison with red line on Figure 3.) Comparing red and blue lines, we found that $\alpha$ becomes larger by substituting FM2 layer for a part of NM layer in FM1/NM. This suggests a possibility that conduction electrons carrying angular momentum from FM1 layer to NM
layer go to FM2 layer to rotate the magnetizations in FM2 layer. We conclude that since magnetizations in FM2 layer need more spin torques to rotate itself, spin-current is larger and that causes $\alpha$ to increase.

5. Summary
We have calculated Gilbert damping coefficient $\alpha$ in the magnetic multi-layer systems consisted of ferromagnetic (FM) and non-magnetic (NM) layers from microscopic theory using Hamiltonian which includes the kinetic term $H_t$. We found $\alpha$ is dependent on structure of systems and both thickness of the FM and NM layer. We conclude that transferring conduction electrons carrying angular momentum to adjacent layers are the origin of the Gilbert damping for multi-layer systems. In other words, we could confirm that spin pumping effect in theory.

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