A possible minimal gauge-Higgs unification

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A possible minimal model of the gauge-Higgs unification based on the higher dimensional spacetime $M^4 \otimes (S^1/Z_2)$ and the bulk gauge symmetry $SU(3)_C \otimes SU(3)_W \otimes U(1)_X$ is constructed in some details. We argue that the Weinberg angle and the electromagnetic current can be correctly identified if one introduces the extra $U(1)_X$ above and a bulk scalar triplet. The VEV of this scalar as well as the orbifold boundary conditions will break the bulk gauge symmetry down to that of the standard model. A new neutral zero-mode gauge boson $Z'$ exists that gains mass via this VEV. We propose a simple fermion content that is free from all the anomalies when the extra brane-localized chiral fermions are taken into account as well. The issues on recovering a standard model chiral-fermion spectrum with the masses and flavor mixing are also discussed, where we need to introduce the two other brane scalars which also contribute to the $Z'$ mass in the similar way as the scalar triplet. The neutrinos can get small masses via a type I seesaw mechanism. In this model, the mass of the $Z'$ boson and the compactification scale are very constrained as respectively given in the ranges: $2.7 \text{ TeV} < m_{Z'} < 13.6 \text{ TeV}$ and $40 \text{ TeV} < 1/R < 200 \text{ TeV}$.

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I. INTRODUCTION

The standard model of fundamental particles and interactions has been very successful in describing observed phenomena. However, a serious problem exists if it tries to match a certain new physics at much higher energy scales. In deed, the squared-mass parameter of
the Higgs field in the standard model receives quadratically divergent radiative corrections. These divergences imply that the low-energy parameter is sensitive to contributions of heavy fields with masses lying at the cut-off scale, which in principle can reach the Planck scale. The physics at the weak scale is strongly disturbed that requires a striking cancellation between the various contributions and/or the bare parameter. However, this introduces an unjustifiably huge fine-tuning from the Planck scale down to the weak scale of the order $M_{\text{Pl}}/M_{\text{weak}} \sim 10^{16}$, known as the hierarchy problem. The scalar sector of the standard model is thus not natural. This indicates that there should be underlining principles that prevent finite Higgs masses from the radiative corrections. A typical example is supersymmetry, where such divergences can be removed by a symmetry relating boson and fermion \[1\]. Another example is the Randall-Sundrum model where the hierarchy can be understood via a warped factor associated with the bulk dimension \[2\]. For other approaches, let us call the reader’s attention to Refs. \[3\].

An alternative to solve the hierarchy problem is the gauge-Higgs unification \[4\], which is recently called for much attention \[5-9\]. In this scenario, the Higgs fields are identified as extra dimensional components of the gauge fields, that results in a bulk gauge vector transforming as adjoint representations under a higher dimensional gauge symmetry. The gauge symmetry will protect a finite Higgs mass against radiative corrections. In fact, the gauge symmetry actually preventing the Higgs fields from obtaining a higher dimensional mass is spontaneously broken by the compactification; As a result, a finite effective mass term is allowed and gets naturally generated as the sum of all radiative corrections necessarily independent of the cut-off scale \[5\]. A nice feature of the theory is that the Yukawa interactions become universal with an unique coupling constant as combined with the gauge interactions in higher dimensional spacetime. The low energy chiral fermions and residual gauge symmetry can be recognized if the theory is compactified on orbifolds, for example, $S^1/Z_2$ (which will be used throughout the work) \[6, 10\]. The hierarchical smallness of the Yukawa couplings at the low energy may arise from wave-function overlap profiles associated with extra space. And, a consistent fermion content with flavor mixing may be original from couplings with heavy brane-localized chiral fermions \[7\].

Because the Higgs and gauge bosons of the standard model lie in the adjoint representations, the smallest gauge symmetry of which must be $SU(3)_C \otimes SU(3)_W$. This simplest version has been extensively studied since the birth of the gauge-Higgs unification until now.
The problems with it are that (i) the Weinberg angle $s^2_W = 3/4$ is not correct, (ii) because the generators of non-Abelian Lie group $SU(3)_W$ is normalized such as $\text{Tr}[T_a T_b] = \delta_{ab}/2$, the electromagnetic interactions are wrong. For examples, the electromagnetic coupling of the neutrinos is non-zero that equals to that of up quarks; similarly the coupling of electron, muon or tau is the same as that of down quarks. In this work we will provide a solution to these problems by adding a group factor $U(1)_X$ to the gauge symmetry and imposing a bulk scalar triplet. The nature of this scalar as well as introductory of the other brane scalars will be discussed accordingly in the text.

In the literature, the bulk fermions have usually been assigned with large rank $SU(3)_W$ representations such as sextets, octets, and even 10-plets. In this work we propose a simpler fermion content which contains only bulk triplets or antitriplets. The heavy brane-localized chiral fermions are also introduced to make unwanted fermions heavy and producing flavor mixing. The presence of these exotic chiral fermions have another effect that all the chiral anomalies at branes are necessarily cancelled out [11].

The rest of this work is organized as follows. In Sec. II we introduce the model with stressing on the gauge symmetry, orbifold boundary conditions, zero mode fields, proposal of bulk scalar triplet, and symmetry breakings. In Sec. III we diagonalize the mass matrix of zero mode neutral gauge bosons, identifying physical fields and matching of gauge coupling constants and Weinberg angle. Constraints on the new physics are also given. Section IV is devoted to a fermion content of the model, presenting the way to identify the standard model fermions, mass generations, and flavor mixings. The small masses of neutrinos are also obtained. Section V presents anomaly cancellations. Finally, we summarize our results, and make conclusions and some remarks on effective potential in the last section–Sec. VI.

II. THE MODEL

The five dimensional (5D) spacetime is supposed to be a direct product of the ordinary four dimensional (4D) Minkowski spacetime $M^4$ and an orbifold $S^1/Z_2$ with a radius $R$ of $S^1$, namely $M^4 \otimes (S^1/Z_2)$. The 5D coordinate is generally denoted as $x^M = (x^\mu, y)$ in which $\mu = 0, 1, 2, 3$ and $y = x^5$. The symmetry of the orbifold, namely the symmetries on the fifth dimension due to $S^1 : y \rightarrow y + 2\pi R$ and $Z_2 : y \rightarrow -y$, implies the following two basic transformations $Z_i : y_i + y \rightarrow y_i - y$ around the orbifold fixed points $y_i$ ($i = 0, \pi$, and $y_0 =$
0, \ y_\pi = \pi R), \ also \ called \ boundaries \ or \ branes. \ The \ orbifold \ is \ therefore \ exact \ an \ interval \ y \in [0, \pi R] \ of \ the \ length \ \pi R.

The 5D gauge symmetry is considered as $SU(3)_C \otimes SU(3)_W \otimes U(1)_X$. The $SU(3)_W$ is the most minimal group that contains electroweak gauge bosons and a Higgs doublet. The $U(1)_X$ is needed to recover correct Weinberg angle and electromagnetic current. Assuming $G_M, g_s \in SU(3)_C$, $A_M, g \in SU(3)_W$ and $B_M, g_x \in U(1)_X$ as their corresponding 5D gauge bosons and gauge coupling constants, and $\psi$ as a general 5D fermion multiplet, we have the following Lagrangian (up to the gauge fixing and ghost terms):

$$
\mathcal{L} = -\frac{1}{2} \text{Tr} G_{MN} G^{MN} - \frac{1}{2} \text{Tr} A_{MN} A^{MN} - \frac{1}{4} B_{MN} B^{MN} + \bar{\psi} (i \slashed{D} - M_\psi \epsilon(y)) \psi,
$$

where

$$
B_{MN} = \partial_M B_N - \partial_N B_M,
$$

$$
A_{MN} = \partial_M A_N - \partial_N A_M + ig [A_M, A_N],
$$

$$
G_{MN} = \partial_M G_N - \partial_N G_M + ig_s [G_M, G_N],
$$

$$
\slashed{D} = \Gamma^M (\partial_M + ig_s G_M + ig A_M + ig_x X B_M),
$$

where $X$ is the charge of $U(1)_X$, $A_M \equiv T_a A^a_M$ with $T_a (a = 1, 2, ..., 8)$ being the generators of $SU(3)_W$ and satisfying $\text{Tr} [T_a T_b] = \frac{1}{2} \delta_{ab}$, similarly for $G_M$, $\Gamma^M \equiv (\gamma^\mu, i\gamma^5)$ so that $\{\Gamma^M, \Gamma^N\} = 2g^{MN} = 2 \text{diag}(1, -1, -1, -1, -1)$ defining a Clifford algebra of the 5D spacetime symmetry, $\epsilon(y) = |y|/y$ is the sign function of $y$, and $M_\psi$ is the bulk kink mass term for $\psi$.

Under the orbifold symmetries, the Lagrangian is invariant and we have boundary conditions on the fields as follows:

$$
\begin{bmatrix}
G_\mu(x, y_i - y) \\
G_5(x, y_i - y)
\end{bmatrix} = P_i \begin{bmatrix}
G_\mu(x, y_i + y) \\
-G_5(x, y_i + y)
\end{bmatrix} P_i^{-1},
$$

where $P_i$ is the representation of $Z_i$ (which including all the following similar ones have properties unitary and hermitian), acting on 5D gluons and thus identified as $P_i = \text{diag}(1, 1, 1)$. In terms of parity-value pair $(P_0 \ P_\pi)$, we can explicitly write

$$
G_\mu = \begin{bmatrix}
(+) \ (+) \ (+) \\
(+) \ (+) \ (+) \\
(+) \ (+) \ (+)
\end{bmatrix}, \quad G_5 = \begin{bmatrix}
(-) \ (-) \ (-) \\
(-) \ (-) \ (-) \\
(-) \ (-) \ (-)
\end{bmatrix},
$$
which imply that the zero modes of $G_\mu$ are exact ordinary 4D gluons of the standard model, while $G_5$ does not have any zero mode.

For electroweak gauge bosons, we have also

$$
\begin{bmatrix}
A_\mu(x, y_i - y) \\
A_5(x, y_i - y)
\end{bmatrix} = P_i \begin{bmatrix}
A_\mu(x, y_i + y) \\
-A_5(x, y_i + y)
\end{bmatrix} P_i^{-1},
$$

(8)

$$
\begin{bmatrix}
B_\mu(x, y_i - y) \\
B_5(x, y_i - y)
\end{bmatrix} = \begin{bmatrix}
B_\mu(x, y_i + y) \\
-B_5(x, y_i + y)
\end{bmatrix},
$$

(9)

where $P_i$ is another representation of $Z_i$ that acts on electroweak bosons, and chosen as $P_i = \text{diag}(-1, -1, 1)$. Explicitly we write

$$
A_\mu = \begin{bmatrix}
(++) & (++) & (- -) \\
(++) & (++) & (- -) \\
(- -) & (- -) & (+ +)
\end{bmatrix}, \quad A_5 = \begin{bmatrix}
(- -) & (- -) & (+ +) \\
(- -) & (- -) & (+ +) \\
(+ +) & (+ +) & (- -)
\end{bmatrix}.
$$

(10)

The zero modes of $A_\mu$ and $B_\mu$ will contain the standard model electroweak gauge bosons and a new neutral gauge boson, i.e. $A_\mu^1, A_\mu^2 \sim W$ and $A_\mu^3, A_\mu^8, B_\mu \sim \gamma, Z, Z'$ (new). The zero modes of $A_5$ will contain the standard model Higgs doublet: $(A_5^4 - i A_5^5, A_5^6 - i A_5^7)^T \sim H$.

Here all the superscripts are the indices of SU(3)$_W$ adjoint representation. The remaining gauge bosons including $B_5$ do not have zero mode. They as well as $G_5$ and all the Kaluza-Klein (KK) excitations, which have masses equal to or larger than compactification or KK scale $1/R$, must be typically heavy.

The fermion $\psi$, that may be a triplet or an antitriplet of SU(3)$_W$, will satisfy the following boundary condition:

$$
\psi(x, y_i - y) = P_i \gamma^5 \psi(x, y_i + y).
$$

(11)

Writing $\psi = \psi_L + \psi_R$, the components will have simpler transformational rules:

$$
\psi(x, y_i - y)_L = -P_i \psi(x, y_i + y)_L, \quad \psi(x, y_i - y)_R = P_i \psi(x, y_i + y)_R,
$$

(12)

which yield

$$
\psi_L = \begin{bmatrix}
(++) \\
(++) \\
(- -)
\end{bmatrix}_L, \quad \psi_R = \begin{bmatrix}
(- -) \\
(- -) \\
(+ +)
\end{bmatrix}_R.
$$

(13)

The zero modes are just a left-handed doublet or antidioublet and a right-handed singlet under the standard model symmetry, responsible for ordinary leptons and quarks. All other
generators of SU never broken is the electric charge operator. It can be obtained by a combination of diagonal T

mode if M are written as V

have been done. For a general gauge field V, putting Mn = n/R we have

Vµ,5(x,y) = \frac{1}{\sqrt{2\pi R}} V_{µ,5}(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{∞} V_{µ,5}^{(n)}(x) \cos(M_n y) \quad (even),

(14)

Vµ,5(x,y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{∞} V_{µ,5}^{(n)}(x) \sin(M_n y) \quad (odd).

(15)

The mode expansion for the fermion is quite different from those of gauge fields due to the presence of the bulk kink mass term:

ψ(x,y) = \left[ \psi_L(x)f_L(y) + \sum_{n=1}^{∞} \left\{ \psi_L^{(n)}(x) f_L^{(n)}(y) + \psi_R^{(n)}(x) S_n(y) \right\} \right], \quad \psi_R(x)f_R(y) + \sum_{n=1}^{∞} \left\{ \psi_R^{(n)}(x) f_R^{(n)}(y) + \psi_L^{(n)}(x) S_n(y) \right\},

(16)

where ψL,R = (ψ₁, ψ₂)T

are written as Sn(y) = 1/√πR sin(M_n y) and

f_L(y) = \sqrt{M_ψ \over 1 - e^{-2πRM_ψ} e^{-M_ψ |y|}}, \quad f_R(y) = \sqrt{M_ψ \over e^{2πRM_ψ} - 1} e^{M_ψ |y|},

(17)

f_L^{(n)}(y) = \frac{M_n}{\sqrt{πRM_ψ}} \left[ \cos(M_n y) - \frac{M_ψ}{M_n} e(y) \sin(M_n y) \right], \quad f_R^{(n)}(y) = \frac{M_n}{\sqrt{πRM_ψ}} \left[ \cos(M_n y) + \frac{M_ψ}{M_n} e(y) \sin(M_n y) \right].

(18)

(19)

If M_ψ is positive, the zero mode f_L in (17) is concentrated at y = y₀ = 0 while the zero mode f_R is concentrated at y = y_π = πR. Vice versa, if it is negative the f_L is concentrated at y = y_π while the f_R is concentrated at y = y₀. This can therefore give a very simple realization that although the Yukawa couplings in the gauge-Higgs unification original from the gauge coupling, they could be small and hierarchical (due to difference in the bulk masses), without fine tuning.

At the boundaries (or branes), the 5D gauge symmetry is broken (by orbifolding) into

SU(3)_C ⊗ SU(3)_W ⊗ U(1)_X → SU(3)_C ⊗ SU(2)_L ⊗ U(1)_r_s ⊗ U(1)_X,

(20)

where T_s is the generator of SU(3)_W. On the other hand, an exact residual symmetry that is never broken is the electric charge operator. It can be obtained by a combination of diagonal generators of SU(3)_W ⊗ U(1)_X due to electric charge conservation, which is given by

Q = T_3 + \frac{1}{\sqrt{3}} T_s + X.

(21)
In this case the second component of $H$ is electrically neutral. Otherwise, if the first component of $H$ considered is electrically neutral, the coefficient of $T_8$ will change sign. Both the cases are equivalent, therefore we can consider the first case. It is also noted that

$$Y = \frac{1}{\sqrt{3}}T_8 + X$$

is just hypercharge of the standard model.

Because of the 5D gauge symmetry all the gauge fields including the Higgs cannot have any tree level explicit mass term. The Higgs field in the framework cannot get at this level any nonzero vacuum expectation value (VEV). The electroweak symmetry can only be dynamically broken by the Wilson line phase, $e^{i(\lambda_\alpha/2)\alpha} \equiv Pe^{ig\oint dyA_5}$, through Hosotani mechanics. We therefore suppose that

$$\langle A_5 \rangle = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i\frac{\alpha}{4\pi g R} \\ 0 & i\frac{\alpha}{4\pi g R} & 0 \end{bmatrix},$$

where $A_5$ is arranged to develop VEV in the direction of $A_0^7$ (in the matrix we have put $\alpha^7 = \alpha$ for a simplicity). Notice that the zero mode field has not been normalized yet; otherwise, $g$ should be replaced by that of 4D and the equation remaining unchanged. In the following, the usages of such similar ones should be flexibly understood.

With this VEV, in the neutral gauge sector the $B_\mu$ remains massless and does not mix with $A_\mu^3$ and $A_\mu^8$. It is natural to identify the $B_\mu$ as a new physical gauge boson but encountered with the inconsistent zero-mass (because any mixing between $B_\mu$ and the photon in this case is unreasonable). In addition, we can verify that $s^2_W = 3/4$ and the electromagnetic current are not correct, which happen in the same with the simplest gauge-Higgs unification version. Let us next deal with such issues. A suggestion is that the $A_\mu^8$ has to be directly mixed with $B_\mu$ by some source so that the resulting gauge boson associated with the hypercharge is correctly identified (instead of $A_\mu^8$), and the resulting new gauge boson (instead of $B_\mu$) can get heavy. This is possible if we introduce a heavy scalar field charged under both the $U(1)_X$ and $SU(3)_W$, so that the VEV of this scalar will at least break $U(1)_{T_8} \otimes U(1)_X$ symmetry into $U(1)_Y$ of the standard model and simultaneously providing the mass for the new gauge boson. This breaking probably lies at the same stage with the orbifolding breaking.
Denoting the quantum numbers by $(SU(3)_C, SU(3)_W, U(1)_X)$, the scalar transforms as

$$\chi = \begin{bmatrix} \chi^+ \\ \chi^0 \\ \chi'^0 \end{bmatrix} \sim (1, 3, 1/3),$$

(24)

satisfying the boundary condition

$$\chi(x, y_i - y) = P_i \chi(x, y_i + y).$$

(25)

The parity is explicitly written as

$$\chi = \begin{bmatrix} (- -) \\ (- -) \\ (+ +) \end{bmatrix}.$$

(26)

Due to parity conservation, only the third component can develop VEV, which satisfies our minimal requirement for the symmetry breaking,

$$\langle \chi \rangle = \begin{bmatrix} 0 \\ 0 \\ \frac{\beta}{4\pi g R} \end{bmatrix},$$

(27)

with $\beta \gg \alpha$, i.e. $k = \frac{\beta}{\alpha} \gg 1$. Here, notice that this VEV always conserves the residual standard model symmetry. There exist two simultaneous symmetry breaking processes of the same stage: the symmetry breaking of $SU(3)_W \rightarrow SU(2)_L$ is a result of orbifolding characterized by $1/R$ scale and the symmetry breaking of $U(1)_T \otimes U(1)_X \rightarrow U(1)_Y$ is due to $\beta$. The VEV $\beta$ can be naturally taken in the same order with $1/R$, i.e. $\beta \sim \mathcal{O}(1)$ and thus $\alpha$ close to zero (this $\alpha$ should be provided from the effective potential for the Higgs field). This is dynamical because as any ordinary scalar field theory the brane field $\chi'^0$ is unstable under radiative corrections. The VEV $\beta$ that is obtained from the resulting effective potential for $\chi$ can get naturally generated in the cut-off scale $1/R$; and the $\chi'^0$ has also a heavy mass in this scale. With the $\chi$ scalar, all the issues as stated above are solved (shown explicitly below). In addition, to make a consistent fermion spectrum the other brane scalars will be introduced, that provide large masses for exotic fermions such as right-handed neutrinos and extra chiral leptons and quarks. The VEVs of these scalars like $\beta$ have similar contributions to the gauge boson spectrum as shown in the last section (IV), therefore in the following we consider only the case with the $\chi$ scalar and its VEV $\beta$. 
A summary on the symmetry breaking is as follows. The first stage of symmetry breaking down to that of the standard model is due to the orbifold boundary conditions and the $\chi$ scalar VEV $\beta$. The second stage of symmetry breaking from the standard model symmetry down to that of QCD and QED is due to the Wilson line phase characterized by $\alpha$. It is

$$SU(3)_C \otimes SU(3)_W \otimes U(1)_X \xrightarrow{1/R_3^\beta} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{\alpha} SU(3)_C \otimes U(1)_Q,$$

where the gauge hierarchy is

$$\frac{\beta}{R} \sim \frac{1}{R} \gg \frac{\alpha}{R}.$$  

(29)

III. GAUGE BOSONS

The mass Lagrangian for the zero mode gauge bosons is

$$L_{\text{mass}}^{\text{gauge}} = \int_{-\pi R}^{\pi R} dy \left\{ g^2 \text{Tr}[A_\mu, (A_5)^\dagger] + (D_\mu(\chi))^\dagger(D^\mu(\chi)) \right\}$$

$$= \frac{1}{2} \left( \frac{\alpha}{4\pi R} \right)^2 W^- W^+ + \frac{1}{4} \left( \frac{\alpha}{4\pi R} \right)^2 (A^3 - \sqrt{3} A^8)^2 + \frac{1}{3} \left( \frac{\beta}{4\pi R} \right)^2 (A^8 - \frac{t}{\sqrt{3}} B)^2,$$

(30)

where $t \equiv \frac{\alpha}{g}$ and $W^\pm \equiv \frac{A^1 \pm A^2}{\sqrt{2}}$. From the first term, we obtain the mass of $W$ boson:

$$m_W = \frac{\alpha}{4\sqrt{2}\pi R}.$$  

(31)

The remaining terms provide the mass Lagrangian for the neutral gauge bosons that can be rewritten as $\frac{1}{2}(A^3 A^8 B)M^2(A^3 A^8 B)^T$, with

$$M^2 = \left( \frac{\alpha}{4\pi R} \right)^2 \begin{bmatrix}
\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\
-\frac{\sqrt{3}}{2} & \frac{3}{2} + \frac{2}{3} k^2 & -\frac{2}{3\sqrt{3}} k^2 \\
0 & -\frac{2}{3\sqrt{3}} k^2 & \frac{2}{9} k^2
\end{bmatrix}.$$  

(32)

The straightforward procedure for diagonalizing this mass matrix as well as identifying the Weinberg angle and physical gauge bosons can be found in [12]. In details, one can check that the mass matrix always has a non-degenerate zero-eigenvalue, $\text{det}M^2 = 0$, that corresponds to the photon. The photon field as associated with the electric charge operator $Q$ is the corresponding eigenstate obtained by

$$A_\gamma = \frac{\sqrt{3} t}{\sqrt{3} + 4t^2} A^3 + \frac{t}{\sqrt{3} + 4t^2} A^8 + \frac{\sqrt{3}}{\sqrt{3} + 4t^2} B,$$

(33)
which is independent of the VEVs. All these are natural consequences of the electromagnetic gauge invariance [12].

Now, with the help of (33) we can obtain electromagnetic interactions, e.g. choosing a lepton triplet \( \psi = (\nu e e')^T \sim (1, 3, -2/3) \) will yield matching condition of the gauge coupling constants \( e = g s_W \), with

\[
s_W \equiv \frac{\sqrt{3} t}{\sqrt{3 + 4t^2}}
\]

that defines the Weinberg angle. We can evaluate \( t = g_x/g \) so that \( s_W \) gets the correct value. Namely, \( t = \sqrt{3s_W/\sqrt{3 - 4s_W^2}} \simeq 0.58 \) provided that \( s_W^2 \simeq 0.231 \) [13]. The photon field (33) can be rewritten as

\[
A_\gamma = s_W A^3 + c_W \left( \frac{t_W}{\sqrt{3}} A^8 + \sqrt{1 - \frac{t_W^2}{3}} B \right).
\]

The standard model Z boson is orthogonal to \( A_\gamma \) as usual:

\[
Z = c_W A^3 - s_W \left( \frac{t_W}{\sqrt{3}} A^8 + \sqrt{1 - \frac{t_W^2}{3}} B \right).
\]

Notice that the one in parentheses is just ordinary gauge field as associated with the hypercharge \( Y \) given above. It is a mixing of \( A^8 \) and \( B \). A field that is orthogonal to it, i.e. to \( A_\gamma \) and \( Z \), will be a new gauge boson:

\[
Z' = \sqrt{1 - \frac{t_W^2}{3}} A^8 - \frac{t_W}{\sqrt{3}} B.
\]

In the new basis \((A_\gamma Z Z')\), the mass matrix (32) becomes

\[
M^2 \rightarrow M'^2 = \begin{bmatrix}
0 & 0 & 0 \\
0 & m_Z^2 & m_{ZZ'}^2 \\
0 & m_{ZZ'}^2 & m_{Z'}^2
\end{bmatrix},
\]

where

\[
m_Z^2 = \frac{3 + 4t^2}{2(3 + t^2)} \left( \frac{\alpha}{4\pi R} \right)^2,
\]

\[
m_{ZZ'}^2 = \frac{81 + 4(3 + t^2)^2k^2}{18(3 + t^2)} \left( \frac{\alpha}{4\pi R} \right)^2,
\]

\[
m_{Z'}^2 = \frac{3\sqrt{3} + 4t^2}{2(3 + t^2)} \left( \frac{\alpha}{4\pi R} \right)^2,
\]

that defines the mixing of \( Z \) and \( Z' \). The mixing angle of \( Z - Z' \) is given by

\[
\tan(2\phi) = \frac{2m_{ZZ'}^2}{m_{Z'}^2 - m_Z^2}.
\]
We thus obtain the following physical gauge bosons

\[ Z = c_\phi Z - s_\phi Z', \quad Z' = s_\phi Z + c_\phi Z', \]  

(42)

with masses

\[ m_Z^2 = \frac{1}{2}(m^2_Z + m^2_{Z'} - \sqrt{(m^2_Z - m^2_{Z'})^2 + 4m^4_{ZZ'}}), \]  

(43)

\[ m_{Z'}^2 = \frac{1}{2}(m^2_Z + m^2_{Z'} + \sqrt{(m^2_Z - m^2_{Z'})^2 + 4m^4_{ZZ'}}). \]  

(44)

Because of \( k = \beta/\alpha \gg 1 \), from (39) and (40) we have \( m_{Z'} \gg m_Z, m_{ZZ'} \). This implies that the \( Z' \) is heavy and the mixing angle \( \phi \) is small. In details, the approximations can be calculated as follows:

\[ \phi \simeq -\frac{(3 - 4s^2_W)^{3/2}}{4c^2_W} \left( \frac{\alpha}{\beta} \right)^2 \simeq -\left( \frac{\alpha}{\beta} \right)^2, \quad m_{Z'} \simeq \frac{\sqrt{2c_W}}{\sqrt{3 - 4s^2_W}} \frac{\beta}{4\pi R} \simeq 0.068 \times \frac{\beta}{R}. \]  

(45)

Consequently, \( Z \simeq Z \) is the standard model \( Z \) like boson, and \( Z' \simeq Z' \) being a new gauge boson. We have also

\[ m_Z^2 \simeq m_{Z'}^2 = \frac{m^2_W}{c^2_W}, \]  

(46)

which can see from (43), (39), (34) and (31). Strictly, we evaluate the tree level \( \rho \) parameter:

\[ \rho = \frac{m^2_W}{c^2_W m_Z^2} \simeq 1 + \frac{(3 - 4s^2_W)(13 - 16s^2_W)}{16c^4_W} \left( \frac{\alpha}{\beta} \right)^2 \simeq 1 + 2 \left( \frac{\alpha}{\beta} \right)^2, \]  

(47)

which is absolutely close to one since \( \alpha \ll \beta \), in good agreement with the data \[13\]. Anyway, the \( \rho \) modifies the standard model expressions for observables by \( m_Z \to m_Z/\sqrt{\rho}, \Gamma_Z \to \rho \Gamma_Z \), and \( \mathcal{L}_Z \to \rho \mathcal{L}_Z \), where \( \mathcal{L}_Z \) is an effective four-fermion neutral current operator. Detailed analyses can be found in, for example, \[14\]. Furthermore, from the global fit \[13\] we have \( 1.0001 < \rho < 1.0025 \), thus \( 0.7 \times 10^{-2} < \frac{\alpha}{\beta} < 3.5 \times 10^{-2} \). Combined with (31) and taking \( \beta = 1 \), we derive the range of Kaluza-Klein scale:

\[ 40 \text{ TeV} < \frac{1}{R} < 200 \text{ TeV}. \]  

(48)

It is also easy to derive the range of the \( Z - Z' \) mixing angle and \( Z' \) mass:

\[ -12.5 \times 10^{-4} < \phi < -0.5 \times 10^{-4}, \]  

(49)

\[ 2.7 \text{ TeV} < m_{Z'} < 13.6 \text{ TeV}. \]  

(50)
In summary, $Z'$ is the heavy neutral gauge boson with mass in the TeV range. This zero mode, including $\chi^0$ and all the excitations of the theory, can be integrated out. The physics below TeV scale, which is localized at the branes, is the standard model symmetry, the ordinary gauge bosons and Higgs doublet, with a perfect consistency of the Weinberg angle, of the $\rho$ parameter and of all the currents including electromagnetic current as in our useful standard model. To confirm the last points, we first notice that $W^\pm = (A^1 \mp iA^2)/\sqrt{2}$ and the mixing matrix of the neutral gauge bosons (neglect the $Z-Z'$ mixing):

$$
\begin{bmatrix}
A^3 \\
A^8 \\
B
\end{bmatrix}
= \begin{bmatrix}
s_W & c_W & 0 \\
\frac{s_W}{\sqrt{3}} & -\frac{2w_\mu}{\sqrt{3}} & \sqrt{1 - \frac{t_\mu^2}{3}} \\
c_W \sqrt{1 - \frac{t_\mu^2}{3}} & -s_W \sqrt{1 - \frac{t_\mu^2}{3}} & -\frac{t_\mu}{\sqrt{3}}
\end{bmatrix}
\begin{bmatrix}
A_\gamma \\
Z \\
Z'
\end{bmatrix}.
$$

(51)

Substituting them into the covariant derivative with notation that $e = g s_W$, $g_x = g t = g \sqrt{3} s_W/\sqrt{3 - 4 t_W^2}$ and the form of the electric charge operator, we get the desirable result:

$$
D_\mu \supset \partial_\mu + ig (T_1 A^1_\mu + T_2 A^2_\mu) + ig (T_3 A^3_\mu + T_8 A^8_\mu + t X_B_\mu) \\
\supset \partial_\mu + \frac{ig}{\sqrt{2}} (T^+ W^+_\mu + T^- W^-_\mu) + ie Q A_\gamma_\mu + \frac{ig}{c_W} (T_3 - s_W^2 Q) Z_\mu,
$$

(52)

where $T^\pm = T_1 \pm iT_2$ is the weak isospin raising or lowering operator.

IV. FERMIONS

For each lepton family and each quark family, we introduce two 5D multiplets:

$$
\psi^1_a = \begin{bmatrix}
\nu^1_a \\
\nu^2_a \\
\nu^3_a
\end{bmatrix} \sim (1, 3, -2/3),
\quad
\psi^2_a = \begin{bmatrix}
\nu^1_a \\
\nu^2_a \\
\nu^3_a
\end{bmatrix} \sim (1, 3^*, -1/3),
$$

(53)

$$
Q^1_a = \begin{bmatrix}
u^1_a \\
\nu^2_a \\
\nu^3_a
\end{bmatrix} \sim (3, 3, 0),
\quad
Q^2_a = \begin{bmatrix}
u^1_a \\
\nu^2_a \\
\nu^3_a
\end{bmatrix} \sim (3, 3^*, 1/3),
$$

(54)

where $a = 1, 2, 3$ is family index. With the parity as given before, the zero modes are

$$
\psi^1_a = \begin{bmatrix}
\nu^1_{aL} \\
\nu^1_{aR}
\end{bmatrix} \oplus \nu^1_{aL},
\psi^2_a = \begin{bmatrix}
\nu^2_{aL} \\
\nu^2_{aR}
\end{bmatrix} \oplus \nu^2_{aR},
$$

(55)
\[ Q^1_a = \begin{bmatrix} u^1_{aL} \\ d^1_{aL} \end{bmatrix} \oplus d^1_{aR}, \quad Q^2_a = \begin{bmatrix} d^2_{aL} \\ -u^2_{aL} \end{bmatrix} \oplus u_{aR}, \]  

under the standard model symmetry at the branes.

We thus have the desirable lepton and quark singlets \( e_{aR}, \nu_{aR}, d_{aR} \) and \( u_{aR} \). But this rises to the unwanted light lepton and quark doublets (three for leptons and three for quarks). Such doublets have to be removed by some mechanism. As in the literature, we can integrate them out by coupling them to heavy chiral fermions localized at the same brane they concentrate: \((N_{aR}, E_{aR})^T \sim (1, 2, -1/2)\) and \((U_{aR}, D_{aR})^T \sim (3, 2, 1/6)\) under the standard model symmetry. Here the quantum numbers are given by \((SU(3)_C, SU(2)_L, U(1)_Y)\). The 4D bare couplings are therefore:

\[
- \langle \bar{N}_{aR}, \bar{E}_{aR} \rangle \left\{ m_{ab}^{l1} \begin{bmatrix} \nu_{bL}^1 \\ e_{bL}^1 \end{bmatrix} + m_{ab}^{l2} \begin{bmatrix} \nu_{bL}^2 \\ e_{bL}^2 \end{bmatrix} \right\} \\
- \langle \bar{U}_{aR}, \bar{D}_{aR} \rangle \left\{ m_{ab}^{q1} \begin{bmatrix} \nu_{bL}^1 \\ d_{bL}^1 \end{bmatrix} + m_{ab}^{q2} \begin{bmatrix} \nu_{bL}^2 \\ d_{bL}^2 \end{bmatrix} \right\} \\
+ \text{h.c.},
\]

where the mass parameters \( m_{ab}^{l1,2} \) and \( m_{ab}^{q1,2} \) are in general not diagonal in \( a \) and \( b \) which could lead to flavor changing profiles for both the lepton and quark sectors despite the fact that the theory is original from the gauge principle. The heavy doublets integrated away are exact the combinations as appearing in the above parentheses; the remaining ones orthogonal to them are just the expected doublets of the standard model:

\[
\psi_{aL} = \begin{bmatrix} \nu_{aL} \\ e_{aL} \end{bmatrix}, \quad Q_{aL} = \begin{bmatrix} u_{aL} \\ d_{aL} \end{bmatrix},
\]

where \( \psi_{aL}^{1,2} = U_{ab}^{1,2} \psi_{bL} + \cdots \) and \( Q_{aL}^{1,2} = V_{ab}^{1,2} Q_{bL} + \cdots \) For a detailed analysis, see [1].

Now, the Yukawa interactions come from

\[
\int_{-\pi R}^{\pi R} dy \left[ \bar{\psi}_a^1 \Gamma^5 (igA^5) \psi_a^1 \bar{\psi}_a^2 \Gamma^5 (-igA^5) \psi_a^2 + \bar{Q}_a^1 \Gamma^5 (igA^5) Q_a^1 + \bar{Q}_a^2 \Gamma^5 (-igA^5) Q_a^2 \right] \\
\quad - g O_{aLR}^{\nu} (\bar{\nu}_a^1, \bar{e}_a^1) iHe_{aR} + g O_{aLR}^{\nu} (\bar{e}_a^2, -\bar{\nu}_a^2) iH^* \nu_{aR} \\
\quad - g O_{aLR}^{d} (\bar{u}_a^1, \bar{d}_a^1) iHd_{aR} + g O_{aLR}^{u} (\bar{d}_a^2, -\bar{u}_a^2) iH^* u_{aR} \\
+ \text{h.c.}
\]

\[(59)\]
where \( H = (1/2)(A_5^4 - iA_5^5, A_5^4 - iA_5^5)^T \) and the integrals of wavefunction overlaps given by

\[
O^e_{aLR} = \int_{-\pi R}^{\pi R} dy f_L(\psi^1_a) f_R(\psi^1_a) \simeq 2\pi R M_{\psi^1_a} e^{-\pi R M_{\psi^1_a}},
\]

\[
O^\nu_{aLR} = \int_{-\pi R}^{\pi R} dy f_L(\psi^2_a) f_R(\psi^2_a) \simeq 2\pi R M_{\psi^2_{\bar{a}}} e^{-\pi R M_{\psi^2_{\bar{a}}}},
\]

\[
O^d_{aLR} = \int_{-\pi R}^{\pi R} dy f_L(Q^1_a) f_R(Q^1_a) \simeq 2\pi R M_{Q^1_{\bar{a}}} e^{-\pi R M_{Q^1_{\bar{a}}}},
\]

\[
O^u_{aLR} = \int_{-\pi R}^{\pi R} dy f_L(Q^2_a) f_R(Q^2_a) \simeq 2\pi R M_{Q^2_{\bar{a}}} e^{-\pi R M_{Q^2_{\bar{a}}}},
\]

provided that \( RM_{\psi, Q} \) is around or larger than 1. Noting that \( \langle H \rangle = (0, -i\frac{\alpha}{4\pi g R})^T \), we obtain the following mass terms

\[
- \frac{1}{2} m_{ab}^{\nu} \bar{\nu}_a R R_b + \frac{1}{2} m_{ab}^{\nu} \bar{\nu}_a R R_b + h.c. \]

\[
- \frac{1}{2} m_{ab}^{\nu} \bar{\nu}_a R R_b + \frac{1}{2} m_{ab}^{\nu} \bar{\nu}_a R R_b + h.c. \]

which realize hierarchical and small masses due to the exponent suppressions of the different bulk masses. We see that the mass of \( W \) boson enters because in the framework the Yukawa couplings are just the gauge coupling \( h = g \). At this stage we could have a consistent quark sector. However, the neutrinos have only Dirac masses, the right-handed neutrinos should also be made heavy. This is possible because the zero mode \( \nu_R \) is singlet under the standard model symmetry that can be self-coupled at the brane \( y = y_\pi \) to perform a mass term:

\[
- \frac{1}{2} m_{ab}^{\nu} \bar{\nu}_a R R_b + h.c. \]

The \( \nu_R \) thus have large Majorana masses \( m_{ab}^{R} \sim \beta/R \) (as shown below). Hence in the model, the masses of the (effective) light neutrinos are generated via a type I seesaw mechanism

\[
m_{\nu}^{\text{eff}} = -m^{\nu}(m^{R})^{-1}m^{R \nu T} \sim \left( \frac{m^{\nu}}{10 \text{ MeV}} \right)^2 \times \text{eV},
\]

provided \( R^{-1} \sim 100 \text{ TeV} \). The neutrino masses are in sub eV if the Dirac ones are around values of the first generation quark and lepton masses.

In the following, charges of the extra brane fermions under the surviving brane \( U(1)_{T_5} \) and \( U(1)_X \) will respectively be assigned by \( (N_R, E_R)^T \sim (0, -1/2) \) and \( (U_R, D_R)^T \sim (0, 1/6) \) to
cancel anomalies. In addition, such charges for the other chiral fermions are easily obtained that can be founded in the next section, and the scalar $\chi^0 \sim (T_8, 1/3)$. We can check that the mass terms in (57) and (67) cannot be lift to Yukawa couplings with $\chi^0$ due to those surviving gauge symmetries. We therefore introduce other scalars (which must be singlet under the standard model symmetry):

1. $\eta_1 \sim (T_8, -1/6)$ coupled to all terms in (57), thus $m_{ab}^{\eta_1} \sim \langle \eta_1 \rangle$ and $m_{ab}^{\eta_2} \sim \langle \eta_1 \rangle$, where the $T_8$ is that of $SU(2)_L$ doublet (as given below). Let us denote $\langle \eta_1 \rangle \equiv \frac{\beta_1}{4\pi g_R}$.

2. $\eta_2 \sim (2T_8, 2/3)$ coupled to (67), thus $m_{ab}^{\eta_2} \sim \langle \eta_2 \rangle \equiv \frac{\beta_2}{4\pi g_R}$. In this case the $T_8$ is that of $SU(2)_L$ singlet.

These scalars will also contribute to the mass of the extra gauge boson and the mixing, as given in the following terms:

$$
\frac{1}{3} \left( \frac{\beta_1/2}{4\pi R} \right)^2 \left( A^8 - \frac{t}{\sqrt{3}} B \right)^2 + \frac{1}{3} \left( \frac{2\beta_2}{4\pi R} \right)^2 \left( A^8 - \frac{t}{\sqrt{3}} B \right)^2,
$$

(69)

respectively, to be added to the mass Lagrangian (30). We see that these contributions (69) are similar to the last term in (30). This is because the charges of $\eta_1, \eta_2$ and $\chi^0$ by themselves are aligned in the same direction. Let us denote $k_1 \equiv \frac{\beta_1}{2\alpha}$ and $k_2 \equiv \frac{2\beta_2}{\alpha}$. The mass matrix of the neutral gauge bosons (32) remains unchanged with the replacement: $k^2 \rightarrow k_1^2 + k_2^2$ (or, in the other words $\beta^2 \rightarrow \beta^2 + \beta_1^2/4 + 4\beta_2^2$). Hence, the presence of $\eta_1$ and $\eta_2$ does not change our conclusions. Also, the number values retain if we take, for example, $\beta = \beta_1/2 = 2\beta_2 = 1/\sqrt{3}$.

V. ANOMALY CANCELLATION

A. Bulk anomalies are absent

Because we have introduced the whole fermions transforming on bulk as vectorlikes under any bulk gauge group, all the anomalies including the mixed ones are canceled on bulk [11].

Concretely, the vectorlikes mean that the left and right components of fermions transform similarly under the gauge groups, i.e. on bulk, the generators of $\psi_L$ and $\psi_R$ respectively satisfy $T_{aL} = U^{-1} T_{aR} U$ for some unitary matrix $U$. Here $T_a$ includes the $U(1)_X$ one as well. It deduces that the following general anomaly vanishes

$$
A_{abc} = \text{Tr}[\{T_{aL}, T_{bL}\}T_{cL} - \{T_{aR}, T_{bR}\}T_{cR}] = 0.
$$

(70)
B. Brane anomalies are also absent due to exotic chiral fermions

At the branes, due to the orbifold boundary conditions the survival (residual) gauge symmetry is now

\[ SU(3)_C \otimes SU(2)_L \otimes U(1)_{T_8} \otimes U(1)_X, \]

as associated with the standard model gauge bosons and a new gauge boson \( Z' \). For convenience we will work on the basis (71). The basis in which the two \( U(1) \)s above are changed to the \( U(1)_Y \) and the other one orthogonal to \( Y \) as coupled to \( Z' \) as mentioned is equivalent.

The zero mode fermions transform under \( (SU(3)_C, SU(2)_L, U(1)_{T_8}, U(1)_X) \) as follows:

- \( \psi^1_{aL} \sim (1, 2, T_8, -2/3) \)
- \( \psi^2_{aL} \sim (1, 2^*, -T_8, -1/3) \)
- \( Q^1_{aL} \sim (3, 2, T_8, 0) \)
- \( Q^2_{aL} \sim (3, 2^*, -T_8, 1/3) \)
- \( e_{aR} \sim (1, 1, T_8, -2/3) \)
- \( \nu_{aR} \sim (1, 1, -T_8, -1/3) \)
- \( d_{aR} \sim (3, 1, T_8, 0) \)
- \( u_{aR} \sim (3, 1, -T_8, 1/3) \)

Notice that the charge \( T_8 \) is the one embedded into the corresponding \( SU(2)_L \) representation. For examples, if it is a \( SU(2)_L \) doublet, its \( T_8 \) is the first 2 \times 2 block diagonal matrix in the 3 \times 3 Gell-Mann matrix \( \frac{1}{2} \lambda_8 \); if it is a \( SU(2)_L \) singlet, the \( T_8 \) is the 33 component of \( \frac{1}{2} \lambda_8 \).

It is to be noted that the above zero mode fields have their history as being originally born from the corresponding bulk fields under the maximal 5D gauge symmetry. Hence, the charges of \( U(1)_{T_8} \) and \( U(1)_X \) are not arbitrary. Also, their hypercharges must be constrained by (22). But, the status of the exotic chiral fermions is different because they do not have any prehistory. They are not controlled by the higher gauge symmetries, therefore the condition (22) with the component charges resulting from a decomposition is not applied. The two charges of such \( U(1) \)s for the exotic chiral fermions are somewhat arbitrary, however their hypercharges necessarily get the correct values and given in a similar form as (22). This is the important point to cancel the chiral anomalies on brane. Namely, let us put \((U_{aR}, D_{aR})^T \sim (3, 2, 0, 1/6)\) and \((N_{aR}, E_{aR})^T \sim (1, 2, 0, -1/2)\). It is easily checked that all the brane anomalies are removed, when taking into account all the fermions as mentioned.

Concretely, let us take some examples as follows. \([SU(3)_C]^3\) anomaly: looking at the particles colored such as \( u^1_L, d^1_L, u^2_L, d^2_L, u_R, d_R, U_R \) and \( D_R \) we see that the number of left chiral components equals to the number of right chiral ones, therefore the anomaly is canceled out on every generation. It is well known that the \([SU(2)_L]^3\) anomaly or mixed anomalies between \( SU(2)_L \) and \( SU(3)_C \) are always absent. The gravity anomalies that are potentially troublesome are \([gravity]^2U(1)_{T_8}\) and \([gravity]^2U(1)_X\). The first one is actually canceled because \( T_8 \) and \(-T_8\) always appear in pair respective to every two fermions of the
same chiral kind left or right. The second one over every fermion generation is as follows:

\[ \sim \sum_{\text{lepton, quark, chiral-fermion}} (X_L - X_R) = 2(-2/3) - (-2/3) + 2(-1/3) - (-1/3) \]
\[ + 3(2)(0) - 3(0) + 3.2.1(3)/3 - 3(1/3) \]
\[ - 2(-1/2) - 3.2.1(6)/3 = 0. \] (72)

It is no hard to check that \[ [SU(3)_C]^2U(1)_{T_8} \] and \[ [SU(3)_C]^2U(1)_{X} \] anomalies vanish. The latter is since \( 2(1/3) - 1/3 - 2(1/6) = 0 \) for every generation. The \[ [SU(2)_L]^2U(1)_{T_8} \] anomaly is also absent. Now we consider \[ [SU(2)_L]^2U(1)_{X} \] anomaly which for each generation is proportional to

\[ \sim \sum_{\text{doublets, antidoublets}} (X_L - X_R) = -2/3 - 1/3 + 3(1/3) - (-1/2) - 3(1/6) = 0. \] (73)

Finally, we check the \[ [U(1)_{T_8}]^2U(1)_{X} \] anomaly:

\[ \frac{1}{12}2(-2/3) + \frac{1}{12}2(-1/3) + 3.\frac{1}{12}2(1/3) - \frac{1}{3}(-2/3) - \frac{1}{3}(-1/3) - 3.\frac{1}{3}(1/3) = 0. \] (74)

The remaining anomalies with the \( U(1)s \) such as \[ [U(1)_X]^2U(1)_{T_8}, [U(1)_{T_8}]^3, \] and \[ [U(1)_X]^3 \] also vanish for every generation.

It is noteworthy that the presence of the exotic chiral fermions in the model is more natural because it makes the fermion content consistent and the model calculable (as such it makes the useless fermions heavy, provides the realistic fermion flavor mixings, and makes the model free from all the anomalies to ensure the consistency of the theory).

**VI. CONCLUSIONS**

In this work, we have shown that on the view of the electromagnetic current, the possible minimal bulk gauge symmetry responsible for the gauge-Higgs unification could be the one based on \( SU(3)_C \times SU(3)_W \times U(1)_X \). A bulk scalar triplet has been introduced to interpret the natural consequences of the model. First, the zero mode field of this scalar must be heavy because as any other 4D scalar theories the mass parameter is unstable under radiative corrections, it is natural to take this parameter in the cut-off scale \( 1/R \). The VEV of this scalar field is thus in the same order, which with the orbifold boundary conditions breaks the bulk gauge symmetry into that of the standard model, providing the new neutral zero-mode gauge boson \( Z' \) with a corresponding large mass. Second, the explicit mixings among
the zero-mode neutral gauge bosons have been achieved. Thereby, the identifications of the
standard model gauge bosons and the new $Z'$ are obvious. By the electromagnetic coupling,
we have naturally identified the Weinberg angle as in the case of the ordinary standard
model. The correct value of the Weinberg angle has been obtained. Since the extra scalar
fields $\eta_{1,2}$ similar to the zero mode field of the scalar triplet can live in the branes, the right-
handed neutrinos and exotic chiral leptons and quarks could be made heavy by coupling to
these brane scalars. The contributions of all the scalars as mentioned to the gauge boson
spectrum are the same. Our general conclusions and number values have thus been obtained
with considering just the zero mode of the scalar triplet.

A minimal bulk fermion content for quarks and leptons that includes only the triplets and
antitriplets has been introduced. For every fermion generation, we have also assumed the
two brane-localized chiral-fermion doublets. The consequences are (i) the unwanted light
fermions due to the orbifold projection can be made heavy, the rest is the standard model
fermion spectrum plus the three right-handed neutrinos; (ii) the fermion masses and flavor
mixings could be obtained through the interplay of the bulk kink mass terms and couplings
to the exotic chiral fermions (since the fermion mass matrices are given as a product of these
contributions); (iii) The theory is free from any anomalies, despite the fact that the extra
$U(1)_X$ is included. On the other hand, the small masses of the neutrinos have been generated
through the type I seesaw mechanism. A evaluation has shown that if the observed neutrino
masses are in sub eV, their Dirac masses are around the mass values of the first generation
particles of the standard model.

The model we proposed is very constrained. From the global fit data on the $\rho$ parameter,
we obtain that the compactification scale is in the range $40 \text{ GeV} < 1/R < 200 \text{ GeV}$ which
strongly coincides with the previous studies (see, for example, Y. Adachi et al. in [7]). The
$Z - Z'$ mixing angle and the mass of $Z'$ are also given as $-12.5 \times 10^{-4} < \phi < -0.5 \times 10^{-4}$
and $2.7 \text{ GeV} < m_{Z'} < 13.6 \text{ GeV}$, which are in good agreement with models other than
containing a new neutral gauge boson $Z'$ [13, 14].

Finally, also from the experimental data on the $\rho$ parameter and the mass of $W$ boson,
we have obtained the $\alpha$ parameter characterizing for electroweak symmetry breaking in the
range: $0.7 \times 10^{-2} < \alpha < 3.5 \times 10^{-2}$ (provided that $\beta = 1$). In practice, the value of $\alpha$
should be compared with that obtained from the minimization condition of the effective potential
of the model. Unfortunately, in the theories of gauge-Higgs unification based on the flat
space like ours, such a small value of $\alpha$ is not easy to derive. There have often been two options to enhance the electroweak symmetry breaking parameter: (i) tune matter content appropriately, for example, see G. Cacciapaglia et al. in [8]; (ii) construct the model based on the warped spacetime [9]. These issues of the current model are worth exploring to be devoted for future studies.

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