When joined the unified gauge picture of fundamental interactions, the gravitation theory leads to geometry of a space-time which is far from simplicity of pseudo-Riemannian geometry of Einstein’s General Relativity. This is geometry of the affine-metric composite dislocated manifolds. The goal is modification of the familiar equations of a gravitational field and entirely the new equations of its deviations. In the present brief, we do not detail the mathematics, but discuss the reasons why it is just this geometry. The major physical underlying reason lies in spontaneous symmetry breaking when the fermion matter admits only the Lorentz subgroup of world symmetries of the geometric arena.

I.

Gauge theory is well-known to call into play the differential geometric methods in order to describe interaction of fields possessing a certain symmetry group. Moreover, it is this geometric approach phrased in terms of fibred manifolds which provides the adequate mathematical formulation of classical field theory.

Note that the conventional gauge principle of gauge invariance of Lagrangian densities losts its validity since symmetries of realistic field models are almost always broken. We follow the geometric modification of the gauge principle which is formulated as follows.

Classical fields can be identified with sections of fibred manifolds

\[ \pi : Y \rightarrow X \]  \hspace{1cm} (1)

whose n-dimensional base X is treated a parameter space, in particular, a world manifold. A locally trivial fibred manifold is called the fibre bundle (or simply the bundle). We further provide Y with an atlas of fibred coordinates \((x^\mu, y^i)\) where \((x^\mu)\) are coordinates of its base \(X\).

Classical field theory meets the following tree types of fields as a rule:

- matter fields represented by sections of a vector bundle

\[ Y = (P \times V)/G \]

associated with a certain principal bundle \(P \rightarrow X\) whose structure group is \(G\);
• gauge potentials identified to sections of the bundle

\[ C := J^1 P / G \to X \]  \hspace{1cm} (2)

of principal connections on \( P \) (where \( J^1 P \) denotes the first order jet manifold of the bundle \( P \to X \));

• classical Higgs fields described by global sections of the quotient bundle

\[ \Sigma_K := P / K \to X \]

where \( K \) is the exact symmetry closed subgroup of the Lie group \( G \).

Dynamics of fields represented by sections of the fibred manifold (1) is phrased in terms of jet manifolds \([2, 3, 9, 10, 20, 22]\).

Recall that the \( k \)-order jet manifold \( J^k Y \) of a fibred manifold \( Y \to X \) comprises the equivalence classes \( j^k_x s, \ x \in X \), of sections \( s \) of \( Y \) identified by the \((k + 1)\) terms of their Taylor series at \( x \). It is important that \( J^k Y \) is a finite-dimensional manifold which meets all conditions usually required of manifolds in field theory. Jet manifolds have been widely used in the theory of differential operators. Their application to differential geometry is based on the 1:1 correspondence between the connections on a fibred manifold \( Y \to X \) and the global sections

\[ \Gamma = dx^\lambda \otimes (\partial_\lambda + \Gamma^i_\lambda(y)\partial_i) \]  \hspace{1cm} (3)

of the jet bundle \( J^1 Y \to Y \) \([11, 21, 27]\). The jet bundle \( J^1 Y \to Y \) is an affine bundle modelled on the vector bundle

\[ J^1 Y = T^* X \otimes V Y \]

where \( V Y \) denotes the vertical tangent bundle of the fibred manifold \( Y \to X \). It follows that connections on a fibred manifold \( Y \to X \) constitute an affine space modelled on the vector space of soldering forms \( Y \to J^1 Y \) on \( Y \).

In the first order Lagrangian formalism, the jet manifold \( J^1 Y \) plays the role of a finite-dimensional configuration space of fields. Given fibred coordinates \((x^\lambda, y^i)\) of \( Y \to X \), it is endowed with the adapted coordinates \((x^\lambda, y^i, y^i_\lambda)\) where coordinates \( y^i_\lambda \) make the sense of values of first order partial derivatives \( \partial_\lambda y^i(x) \) of field functions \( y^i(x) \). In jet terms, a first order Lagrangian density of fields is represented by an exterior horizontal density

\[ L = \mathcal{L}(x^\mu, y^i, y^i_\mu)\omega, \quad \omega = dx^1 \wedge \ldots \wedge dx^n, \]  \hspace{1cm} (4)

on \( J^1 Y \to X \).

In field theory, all Lagrangian densities are polynomial forms with respect to velocities \( y^i_\lambda \). Since the jet bundle \( J^1 Y \to Y \) is affine, polynomial forms on \( J^1 Y \) are factorized by morphisms \( J^1 Y \to J^1 Y \). It follows that every Lagrangian density of field theory is represented by composition

\[ L : J^1 Y \overset{D}{\to} T^* X \otimes V Y \to \wedge^n T^* X \]  \hspace{1cm} (5)

2
where $D$ is the covariant differential
\[
D = (y^i_y - \Gamma^i_{\lambda})dx^\lambda \otimes \partial_i
\]
with respect to some connection (3) on $Y \rightarrow X$. It is the fact why the gauge principle and the variation principle involve connections on fibred manifolds in order to describe field systems. In case of the standard gauge theory, they are principal connections treated the mediators of interaction of fields.

Note that several equivalent definitions of connections on fibred manifolds are utilized. We follow the general notion of connections as sections of jet bundles, without appealing to transformation groups. This general approach is suitable to formulate the classical concept of principal connections as sections of the jet bundle $J^1P \rightarrow P$ which are equivariant under the canonical action of the structure group $G$ of $P$ on $P$ on the right. Then, we get description of principal connections as sections of the bundle (2).

\section{II.}

In comparison with case of internal symmetries, the gauge gravitation theory meets two objects. These are the fermion matter and the geometric arena. They might arise owing to \textit{sui generis} primary phase transition which had separated prematter and pregeometry. One can think of the well-known possibility of describing a space-time in spinor coordinates as being the relic of that phase transition.

Here, we are not concerned with these questions, and by a world manifold is meant a 4-dimensional oriented manifold $X^4$ coordinatized in the standard manner. By world geometry is called differential geometry of the tangent bundle $TX$ and the cotangent bundle $T^*X$ of $X^4$. The structure group of these bundles is the general linear group
\[
GL_4 = GL(4, \mathbb{R}).
\]
The associated principal bundle is the bundle $LX \rightarrow X$ of linear frames in tangent spaces to $X^4$.

Note that, on the physical level, a basis of the tangent space $T_x$ to $X^4$ at $x \in X^4$ is usually interpreted as the local reference frame at a point. However, realization of such a reference frame by physical devices remains under discussion. A family $\{z_\xi\}$ of local sections of the principal bundle $LX$ sets up an atlas of $LX$ (and so the associated atlases of $TX$ and $T^*X$) which we, following the gauge theory tradition, treat \textit{sui generis} the world reference frame. In gauge gravitation theory in comparison with case of internal symmetries, there exists the special subclass of holonomic atlases $\{\psi_T^\xi\}$ whose trivialization morphisms are the tangent morphisms
\[
\psi^T_\xi = T\chi_\xi
\]
to trivialization morphisms $\chi_\xi$ of coordinate atlases of $X^4$. The associated bases of $TX$ and $T^*X$ are the holonomic bases $\{\partial_\lambda\}$ and $\{dx^\lambda\}$ respectively. Note that Einstein’s General Relativity was formulated just with respect to holonomic reference frames, so that
many mixed up reference frames and coordinate systems. The mathematical reason of separating holonomic reference frames consists in the fact that the jet manifold formalism is phrased only in terms of holonomic atlases. However, holonomic atlases are not compatible generally with spinor bundles whose sections describe the fermion fields.

Different spinor models of the fermion matter have been suggested. But all observable fermion particles are the Dirac fermions. There are several ways in order to introduce Dirac fermion fields. We do this as follows.

Given a Minkowski space $M$ with the Minkowski metric $\eta$, let $\mathbb{C}_{1,3}$ be the complex Clifford algebra generated by elements of $M$. A spinor space $V$ is defined to be a minimal left ideal of $\mathbb{C}_{1,3}$ on which this algebra acts on the left. We have the representation

$$
\gamma : M \otimes V \to V \quad (6)
$$

of elements of the Minkowski space $M$ by Dirac’s matrices $\gamma$ on $V$. Let us consider the transformations preserving the representation (6). These are pairs $(l, l_s)$ of Lorentz transformations $l$ of the Minkowski space $M$ and invertible elements $l_s$ of $\mathbb{C}_{1,3}$ such that

$$
\gamma(lM \otimes l_sV) = l_s\gamma(M \otimes V).
$$

Elements $l_s$ form the Clifford group whose action on $M$ however is not effective, therefore we restrict ourselves to its spinor subgroup $L_s = SL(2, \mathbb{C})$.

Let us consider a bundle of complex Clifford algebras $\mathbb{C}_{3,1}$ over $X^4$. Its subbundles are both a spinor bundle $S_M \to X^4$ and the bundle $Y_M \to X^4$ of Minkowski spaces of generating elements of $\mathbb{C}_{3,1}$. To describe Dirac fermion fields on a world manifold, one must require that $Y_M$ is isomorphic to the cotangent bundle $T^*X$ of a world manifold $X^4$. It takes place if the structure group of $LX$ is reducible to the Lorentz group $L = SO(3, 1)$ and $LX$ contains a reduced $L$ subbundle $L^hX$ such that

$$
Y_M = (L^hX \times M)/L.
$$

In this case, the spinor bundle $S_M$ is associated with the $L_s$-lift $P_h$ of $L^hX$:

$$
S_M = S_h = (P_h \times V)/L_s. \quad (7)
$$

In accordance with the well-known theorem, there is the 1:1 correspondence between the reduced subbubdles $L^hX$ of $LX$ and the tetrad gravitational fields $h$ identified with global sections of the quotient bundle

$$
\Sigma := LX/L \to X^4.
$$

This bundle is the 2-fold cover of the bundle of pseudo-Riemannian bilinear forms in cotangent spaces to $X^4$. Global sections of the latter are pseudo-Riemannian metrics $g$ on $X^4$. Thus, existence of a gravitational field is necessary condition in order that Dirac fermion fields live on a world manifold.
Note that a world manifold $X^4$ must satisfy the well-known global topological conditions in order that gravitational fields, space-time structure and spinor structure can exist. To summarize these conditions, one may assume that $X^4$ is not compact and the linear frame bundle $LX$ over $X^4$ is trivial.

Given a tetrad field $h$, let $\Psi^h$ be an atlas of $LX$ such that the corresponding local sections $z^h_\xi$ of $LX$ take their values into the reduced subbundle $L^hX$. This atlas has $L$-valued transition functions. It fails to be holonomic in general. Moreover, relative to $\Psi^h$, the pseudo-Riemannian metric $g$ corresponding to $h$ comes to the Minkowski metric and $h$ takes its values in the center $\sigma_0$ of the quotient space $GL_4/L$. With respect to an atlas $\Psi^h$ and a holonomic atlas $\Psi^T = \{\psi^T_\xi\}$ of $LX$, the tetrad field $h$ can be represented by a family of $GL_4$-valued tetrad functions

$$h_\xi = \psi^T_\xi \circ z^h_\xi,$$

$$dx^\lambda = h_\lambda^a(x)h^a,$$

which carry atlas (gauge) transformations between fibre bases $\{dx^\lambda\}$ and $\{h^a\}$ of $T^*X$ associated with $\Psi^T$ and $\Psi^h$ respectively. The well-known relation

$$g^{\mu\nu} = h_\mu^a h_\nu^b \eta^{ab} \quad (8)$$

takes place.

It is the feature only of a tetrad gravitational field that it itself determines reference frames.

Given a tetrad field $h$, one can define the representation

$$\gamma_h : T^*X \otimes S_h = (P_h \times (M \otimes V))/L_s \to (P_h \times \gamma(M \otimes V))/L_s = S_h, \quad (9)$$

$$\hat{dx}^\lambda = \gamma_h(dx^\lambda) = h_\lambda^a(x)\gamma^a,$$

of cotangent vectors to a world manifold $X^4$ by Dirac’s $\gamma$-matrices on elements of the spinor bundle $S_h$.

Let $A_h$ be a connection on $S_h$ associated with a principal connection on $L^hX$ and $D$ the corresponding covariant differential. Given the representation (9), one can construct the Dirac operator

$$D_h = \gamma_h \circ D : J^1S_h \to T^*X \otimes VS_h \to VS_h$$

on $S_h$. Then, we can say that sections of the spinor bundle $S_h$ describe Dirac fermion fields in the presence of the tetrad gravitational field $h$ and the gauge potential $A_h$.

Thus, the geometry of the gauge gravitation theory is the metric-affine geometry characterized by the pair $(h, A_h)$ of a tetrad field $h$ and a reduced Lorentz connection $A_h$ treated as the gauge gravitational potential. Note that the metric-affine geometry is an attribute of all gauge approaches to gravitation theory [4, 12]. Moreover, it is also the Klein-Chern geometry of Lorentz invariants.
III.

If to forget fermion fields for a moment, one can derive the gauge gravitation theory from the equivalence principle reformulated in geometric terms \[7, 18\].

In Einstein’s General Relativity, the equivalence principle is called to provide transition to Special Relativity with respect to some reference frames. In the spirit of F.Klein’s Erlanger program, the Minkowski space geometry can be characterized as geometry of Lorentz invariants. The geometric equivalence principle then postulates that there exist reference frames with respect to which Lorentz invariants can be defined everywhere on a world manifold \(X^4\). This principle has the adequate mathematical formulation in terms of fibre bundles. It requires that the linear frame bundle \(LX\) is reducible to a Lorentz subbundle \(L^hX\) whose structure group is \(L\). They are atlases of \(L^hX\) with respect to which Lorentz invariants can be defined, and the pseudo-Riemannian metric \(g\) corresponding to \(h\) exemplifies such a Lorentz invariant.

Thereby, the geometric equivalence principle provides a world manifold with the so-called \(L\)-structure \[26\]. From the physical point of view, it singles out the Lorentz group as the exact symmetry subgroup of world symmetries broken spontaneously \[7\]. The associated classical Higgs field is a tetrad gravitational field.

Note that the geometric equivalence principle sets also a space-time structure on a world manifold \(X^4\). In virtue of the well-known theorems, if the structure group of \(LX\) is reducible to the structure Lorentz group, the latter, in turn, is reducible to its maximal compact subgroup \(SO(3)\). Moreover, we have the commutative diagram

\[
\begin{array}{ccc}
GL_4 & \longrightarrow & SO(4) \\
\downarrow & & \downarrow \\
L & \longrightarrow & SO(3)
\end{array}
\]

of structure groups of \(LX\). It follows that, for every reduced subbundle \(L^hX\), there exist a reduced subbundle \(FX\) of \(LX\) with the structure group \(SO(3)\) and the corresponding \((3+1)\) space-time decomposition

\[
TX = FX \oplus T^0X
\]

of the tangent bundle of \(X^4\) into a 3-dimensional spatial distribution \(FX\) and its time-like orthocomplement \(T^0X\).

In other words, we have also the Klein-Chern geometry of spatial invariants on a world manifold. One can always choose an atlas of \(LX\) whose transition functions are \(SO(3)\)-valued, and spatial invariants are exemplified by the global field of local time directions \(T^0X\).

Recall that there is the 1:1 correspondence

\[
FX|\Omega = 0
\]
between the nonvanishing 1-forms $\Omega$ on a manifold $X$ and the 1-codimensional distributions on $X$.

Then, we get the following modification of the well-known theorem [18, 23].

- For every gravitational field $g$ on a world manifold $X^4$, there exists an associated pair $(FX, g^R)$ of a space-time distribution $FX$ generated by a tetrad 1-form

$$h^0 = h^0_\mu dx^\mu$$

and a Riemannian metric $g^R$, so that

$$g^R = 2h^0 \otimes h^0 - g.$$  \hspace{1cm} (11)

Conversely, given a Riemannian metric $g^R$, every oriented smooth 3-dimensional distribution $FX$ with a generating form $\Omega$ is a space-time distribution compatible with the gravitational field $g$ given by expression (11) where

$$h^0 = \frac{\Omega}{|\Omega|^2}, \quad |\Omega|^2 = g^R(\Omega, \Omega) = g(\Omega, \Omega).$$

The triple $(g, FX, g^R)$ (11) sets up uniquely a space-time structure on a world manifold. In particular, if the generating form of a space-time distribution $FX$ is exact, we have the causal space-time foliation of $X^4$ what corresponds exactly to the stable causality by Hawking.

A Riemannian metric $g^R$ in the triple (11) defines a $g$-compatible distance function on a world manifold $X^4$. Such a function turns $X^4$ into a metric space whose locally Euclidean topology is equivalent to the manifold topology on $X^4$. Given a gravitational field $g$, the $g$-compatible Riemannian metrics and the corresponding distance functions are different for different space-time distributions $FX$. It follows that physical observers associated with different distributions perceive the same world manifold as different Riemannian spaces. The well-known relativistic changes of sizes of moving bodies exemplify this phenomenon.

One often loses sight of the fact that a certain Riemannian metric and, consequently, a metric topology can be associated with a gravitational field [18]. For instance, one attempts to derive a world topology directly from pseudo-Riemannian structure of a space-time. These are path topology etc. [5]. If a space-time obeys the strong causality condition, such topologies coincide with the familiar manifold topology of $X$. In general case, they however are rather extraordinary.

IV.

A glance on the diagramm (10) shows that, in the gravitation theory, we have a collection of spontaneous symmetry breakings:

- $GL_4 \rightarrow L$ where the corresponding Higgs field is a gravitational field;
• $L \to SO(3)$ where the Higgs-like field is represented by the tetrad 1-form $h^0$ as a global section of the quotient bundle

$$L^hX/\text{SO}(3) \to X^4;$$

• $GL_4 \to SO(4)$ where the Higgs-like field is the Riemannian metric $\Omega$ from expression (11).

Spontaneous symmetry breaking is quantum phenomenon modelled by a Higgs field. In the algebraic quantum field theory, Higgs fields characterize nonequivalent Gaussian states of algebras of quantum fields. They are *sui generis* fictitious fields describing collective phenomena. In the gravitation theory, spontaneous symmetry breaking displays on the classical level and the feature of a gravitational field is that it is a dynamic Higgs field. Indeed, the splitting (8) of the metric field looks like the standard decomposition of a Higgs field where the Minkowski metric $\eta$ and the tetrad functions play the role of the $L$-stable vacuum Higgs field and the Goldstone fields respectively. However, in contrast with the internal symmetry case, the Goldstone components of a gravitational field can not be removed by gauge transformations because the reference frames $\Psi^h$ fail to be holonomic in general and, roughly speaking, the associated basis elements $h_a = h^\mu_a \partial_\mu$ contain tetrad functions.

For the first time, the conception of a graviton as a Goldstone particle corresponding to violation of Lorentz symmetries in a curved space-time had been advanced in mid 60s by Heisenberg and Ivanenko in discussion on cosmological and vacuum asymmetries. This idea was revived in connection with constructing the induced representations of the group $GL_4$ and then in the framework of the approach to gravitation theory as a nonlinear $\sigma$-model [6, 13, 14]. In geometric terms, the fact that a pseudo-Riemannian metric is similar to a Higgs field has been pointed out by A.Trautman and by us [7].

The Higgs character of classical gravity is founded on the fact that, for different tetrad fields $h$ and $h'$, Dirac fermion fields are described by sections of spinor bundles associated with different reduced $L$-principal subbundles of $LX$ and so, the representations $\gamma_h$ and $\gamma_{h'}$ (8) are not equivalent [18]. It follows that e Dirac fermion field must be regarded only in a pair with a certain tetrad gravitational field $h$. These pairs constitute the so-called fermion-gravitation complex [13]. They can not be represented by sections of any product $S \times \Sigma$ where $S \to X^4$ is some standard spinor bundle. At the same time, there is the 1:1 correspondence between these pairs and the sections of the composite spinor bundle

$$S \to \Sigma \to X^4$$

where $S \to \Sigma$ is a spinor bundle associated with the $L$ principal bundle $LX \to \Sigma$ [18, 24]. In particular, every spinor bundle $S_h$ (7) is isomorphic to restriction of $S$ to $h(X^4) \subset \Sigma$.
By a composite manifold is meant the composition
\[ Y \to \Sigma \to X \] (13)
where \( Y \to \Sigma \) is a bundle denoted by \( Y_\Sigma \) and \( \Sigma \to X \) is a fibred manifold.

In analytical mechanics, composite manifolds
\[ Y \to \Sigma \to \mathbb{R} \]
classify systems with variable parameters, e.g. the classical Berry’s oscillator \[19\]. In gauge theory, composite manifolds
\[ P \to \Sigma_K \to X \]
describe spontaneous symmetry breaking \[17, 20\].

Application of composite manifolds to field theory is founded on the following speculations. Given a global section \( h \) of \( \Sigma \), the restriction \( Y_h \) of \( Y_\Sigma \) to \( h(X) \) is a fibred submanifold of \( Y \to X \). There is the 1:1 correspondence between the global sections \( s_h \) of \( Y_h \) and the global sections of the composite manifold (13) which cover \( h \). Therefore, one can say that sections \( s_h \) of \( Y_h \) describe fields in the presence of a background parameter field \( h \), whereas sections of the composite manifold \( Y \) describe all pairs \((s_h, h)\). It is important when the bundles \( Y_h \) and \( Y_{h' \neq h} \) fail to be equivalent in a sense. The configuration space of these pairs is the first order jet manifold \( J^1Y \) of the composite manifold \( Y \).

The feature of the dynamics of field systems on composite manifolds consists in the following \[20, 25\].

Let \( Y \) be a composite manifold (13) provided with the fibred coordinates \((x^\lambda, \sigma^m, y^i)\) where \((x^\lambda, \sigma^m)\) are fibred coordinates of \( \Sigma \). Every connection
\[ A_\Sigma = dx^\lambda \otimes (\partial_\lambda + \bar{A}_i^\lambda \partial_i) + d\sigma^m \otimes (\partial_m + A_i^m \partial_i) \]
on \( Y \to \Sigma \) yields splitting
\[ VY = VY_\Sigma \oplus (Y \times V\Sigma) \]
and, as a consequence, the first order differential operator
\[ \bar{D} : J^1Y \to T^*X \otimes VY_\Sigma, \]
\[ \bar{D} = dx^\lambda \otimes (y^i_\lambda - \bar{A}_i^\lambda - A_i^m \sigma^m_\lambda) \partial_i, \]
on \( Y \). Let \( h \) be a global section of \( \Sigma \) and \( Y_h \) the restriction of the bundle \( Y_\Sigma \) to \( h(X) \). The restriction of \( \bar{D} \) to \( J^1Y_h \subset J^1Y \) comes to the familiar covariant differential relative to a certain connection \( A_h \) on \( Y_h \). Thus, it is \( \bar{D} \) that one may utilize in order to construct a Lagrangian density \(5\)
\[ L : J^1Y \to T^*X \otimes VY_\Sigma \to \wedge^n T^*X \]
for sections of a composite manifold. It should be noted that such a Lagrangian density is never regular because of the constraint conditions

$$A^i_m \partial^\mu_i \mathcal{L} = \partial^\mu_m \mathcal{L}.$$ 

Recall that, if a Lagrangian density is degenerate, the corresponding Euler-Lagrange equations are underdetermined and need supplementary gauge-type conditions which remain elusive in general. Therefore, to describe constraint field systems, we utilize the multimomentum Hamiltonian formalism where canonical momenta correspond to derivatives of fields with respect to all world coordinates, not only the temporal one [1, 4, 19, 20, 21, 22]. Note that application of the conventional Hamiltonian formalism to field theory fails to be successful. In the straightforward manner, it leads to infinite-dimensional phase spaces. In the framework of the multimomentum approach, the phase space of fields is the Legendre bundle

$$\Pi = \wedge^n T^* X \otimes T^* X \otimes V^* Y \quad (14)$$

over \(Y\). It is provided with the fibred coordinates \((x^\lambda, y^i, p^\lambda_i)\). Note that every Lagrangian density \(L\) on \(J^1 Y\) determines the Legendre morphism

$$\hat{L} : J^1 Y \rightarrow \Pi,$$

$$(x^\mu, y^i, p^\mu_i) \circ \hat{L} = (x^\mu, y^i, \partial^\mu_i \mathcal{L}).$$

The Legendre bundle (14) carries the multisymplectic form

$$\Omega = dp^\lambda_i \wedge dy^i \wedge \omega \otimes \partial^\lambda.$$ 

We say that a connection \(\gamma\) on the fibred Legendre manifold \(\Pi \rightarrow X\) is a Hamiltonian connection if the form \(\gamma \rfloor \Omega\) is closed. Then, a Hamiltonian form \(H\) on \(\Pi\) is defined to be an exterior form such that

$$dH = \gamma \rfloor \Omega \quad (15)$$

for some Hamiltonian connection \(\gamma\). The key point consists in the fact that every Hamiltonian form admits splitting

$$H = p^\lambda_i dy^i \wedge \omega_{\lambda} - p^\lambda_i \Gamma^i_\lambda \omega - \mathcal{H} \omega = p^\lambda_i dy^i \wedge \omega_{\lambda} - \mathcal{H} \omega, \quad \omega_{\lambda} = \partial^\lambda \rfloor \omega, \quad (16)$$

where \(\Gamma\) is a connection on the fibred manifold \(Y\) and \(\mathcal{H} \gamma \omega\) is a horizontal density on \(\Pi \rightarrow X\). Given the Hamiltonian form (16), the equality (17) comes to the Hamilton equations

$$\partial_{\lambda} r^i = \partial^\lambda \mathcal{H},$$

$$\partial_{\lambda} r^\lambda_i = -\partial_i \mathcal{H} \quad (17)$$

for sections \(r\) of the fibred Legendre manifold \(\Pi \rightarrow X\).
We thus observe that the multimomentum Hamiltonian formalism exemplifies the generalized Hamiltonian dynamics which is not merely a time evolution directed by the Poisson bracket, but it is governed by partial differential equations \((17)\) where temporal and spatial coordinates enter on equal footing. Maintaining covariance has the principal advantages of describing field theories, for any preliminary space-time splitting shades the covariant picture of field constraints. Contemporary field models are almost always the constraint ones.

We shall say that the Hamiltonian form \(H\) \((16)\) is associated with a Lagrangian density \(L\) \((4)\) if \(H\) satisfies the relations which take the coordinate form

\[
\mathcal{H}(y^j, p^\mu_j) = p^\lambda_i \partial^i \mathcal{H} - L(y^j, \partial^j H).
\]

If a Lagrangian density \(L\) is regular, there exists the unique Hamiltonian form \(H\) such that the first order Euler-Lagrange equations and the Hamilton equations are equivalent, otherwise in general case. One must consider a family of different Hamiltonian forms \(H\) associated with the same degenerate Lagrangian density \(L\) in order to exhaust solutions of the Euler-Lagrange equations. Lagrangian densities of field models are almost always quadratic and affine in derivative coordinates \(y^\mu_i\). In this case, given an associated Hamiltonian form \(H\), every solution of the corresponding Hamilton equations which lives on \(\hat{L}(J^1 Y) \subset \Pi\) yields a solution of the Euler-Lagrange equations. Conversely, for any solution of the Euler-Lagrange equations, there exists the corresponding solution of the Hamilton equations for some associated Hamiltonian form. Obviously, it lives on \(\hat{L}(J^1 Y)\) which makes the sense of the Lagrangian constraint space.

The feature of Hamiltonian systems on composite manifolds \((13)\) lies in the fact that the Lagrangian constraint space is

\[
p^\lambda_m + A^i_m p^\lambda_i = 0
\]

\[20, 21, 25\]. Moreover, if \(h\) is a global section of \(\Sigma \rightarrow X\), the submanifold \(\Pi_h\) of \(\Pi\) given by the coordinate relations

\[
\sigma^m = h^m(x), \quad p^\lambda_m + A^i_m p^\lambda_i = 0
\]

is isomorphic to the Legendre bundle over the restriction \(Y_h\) of \(Y_\Sigma\) to \(h(X)\). The Legendre bundle \(\Pi_h\) is the phase space of fields in the presence of the background parameter field \(h\).

VI.

In gravitation theory, we have the composite manifold

\[LX \rightarrow \Sigma \rightarrow X^4\]

and the associated composite spinor bundle \((12)\). Roughly speaking, values of tetrad gravitational fields play the role of coordinate parameters, besides the familiar world coordinates.
In the multimomentum Hamiltonian gravitation theory, the constraint condition (18) takes the form

\[ p^c_\lambda + \frac{1}{8} \eta^{cb} \sigma^a_\mu (y^B [\gamma_a, \gamma_b]^A B p^A_\lambda + p^{A\lambda}_+ [\gamma_a, \gamma_b]^+ B y^+_B) = 0 \]  \hspace{1cm} (19)

where \((\sigma^\mu_c, y^A)\) are tetrad and spinor coordinates of the composite spinor bundle (12), and \(p^c_\lambda\) and \(p^A_\lambda\) are the corresponding momenta [24, 25]. The condition (19) replaces the standard gravitational constraints

\[ p^c_\lambda = 0. \]  \hspace{1cm} (20)

The crucial point is that, when restricted to the constraint space (20), the Hamilton equations (17) of gravitation theory come to the familiar gravitational equations

\[ C^a_\mu + T^a_\mu = 0 \]

where \(T\) denotes the energy-momentum tensor of fermion fields, otherwise on the modified constraint space (19). In the latter case, we have the modified gravitational equations of the total system of fermion fields and gravity:

\[ D_\lambda p^{a\lambda}_\mu = C^a_\mu + T^a_\mu \]

where \(D_\lambda\) denotes the covariant derivative with respect to the Levi-Civita connection which acts on the indices \(^a_\mu\).

VII.

Since, for different tetrad fields \(h\) and \(h'\), the representations \(\gamma_h\) and \(\gamma_{h'}\) (9) are not equivalent, even weak gravitational fields, unlike matter fields and gauge potentials, fail to form an affine space modelled on a linear space of deviations of some background field. They thereby do not satisfy the superposition principle and cannot be quantized by usual methods, for in accordance with the algebraic quantum field theory quantized fields must constitute a linear space. This is the common feature of Higgs fields. In algebraic quantum field theory, different Higgs fields correspond to nonequivalent Gaussian states of a quantum field algebra. Quantized deviations of a Higgs field cannot change a state of this algebra and so, they fail to generate a new Higgs field.

At the same time, one can examine superposable deviations \(\sigma\) of a tetrad gravitational field \(h\) such that \(h + \sigma\) is not a tetrad gravitational field [16, 18]. In the coordinate form, such deviations read

\[ \bar{h}^a_\mu = H^b_\alpha h^a_b = (\delta^a_\alpha + \sigma^a_\alpha) h^a_\mu = H^\mu_\nu h^\nu_\alpha = (\delta^\mu_\nu + \sigma^\mu_\nu) h^\nu_\alpha = h^\mu_\alpha + \sigma^\mu_\alpha, \] 
\[ \bar{h}^a_\mu = g_{\mu\nu} \eta^{ab} h^b_\nu = H^{a\alpha}_\mu h^a_\alpha, \]
\[ \delta^\alpha_\beta \bar{h}^a_\mu \neq \delta^\alpha_\beta, \quad \delta^\mu_\nu \bar{h}^a_\mu \neq \delta^a_\nu. \]  \hspace{1cm} (21)

Note that the similar factors have been investigated by R.Percacci [15].
In bundle terms, we can describe the deviations (21) as the special morphism \( \Phi_1 \) of the cotangent bundle \([14, 18]\). Given a gravitational field \( h \) and the corresponding representation morphism \( \gamma_h \), the morphism \( \Phi_1 \) yields another \( \gamma \)-matrix representation

\[
\begin{align*}
\tilde{\gamma}_h &= \gamma_h \circ \Phi_1, \\
\tilde{\gamma}_h(h^a) &= H^a_{\ b}\gamma_h(h^b) = H^a_{\ b}\gamma^b,
\end{align*}
\]

of cotangent vectors, but on the same spinor bundle \( S_h \). Therefore, deviations (21) and their superposition \( \sigma + \sigma' \) can be defined.

Let us note that, to construct a Lagrangian density of deviations \( \epsilon \) of a gravitational field, one usually utilize a familiar Lagrangian density of a gravitational field \( h' = h + \epsilon \) where \( h \) is treated as a background field. In case of the deviations (21), one can not follow this method, for quantities \( \tilde{h} \) fail to be true tetrad fields. To overcome this difficulty, we use the fact that the morphisms \( \Phi_1 \) appears also in the dislocation gauge theory of the translation group. We therefore may apply the Lagrangian densities of this theory in order to describe deviations \( \sigma \) (21).

Let the tangent bundle \( TX \) be provided with the canonical structure of the affine tangent bundle. It is coordinatized by \((x^\mu, u^\lambda)\) where \( u^\lambda \neq \dot{x}^\lambda \) are the affine coordinates.

Every affine connection \( A \) on \( TX \) is brought into the sum

\[
A = \Gamma + \sigma
\]

of a linear connection \( \Gamma \) and a soldering form

\[
\sigma = \sigma^\lambda (x) \partial_\lambda \otimes dx^\mu
\]

which plays the role of a gauge translation potential.

In the conventional gauge theory of the affine group, one faces the problem of physical interpretation of both gauge translation potentials and sections \( u(x) \) of the affine tangent bundle \( TX \). In field theory, no fields possess the transformation law

\[
u(x) \rightarrow u(x) + a
\]

under the Poincaré translations.

At the same time, one observes such fields in the gauge theory of dislocations [8] which is based on the fact that, in the presence of dislocations, displacement vectors \( u^k, k = 1, 2, 3 \), of small deformations are determined only with accuracy to gauge translations

\[
u^k \rightarrow u^k + a^k(x)
\]

In this theory, gauge translation potentials \( \sigma^k_i \) describe the plastic distortion, the covariant derivatives

\[
D_i u^k = \partial_i u^k - \sigma^k_i
\]
consist with the elastic distortion, and the strength
\[ \mathcal{F}_k^{ij} = \partial_i \sigma_k^j - \partial_j \sigma^k_i \]
the dislocation density. Equations of the dislocation theory are derived from the gauge
invariant Lagrangian density
\[ \mathcal{L} = \mu D_i u^k D^i u_k + \frac{\lambda}{2} D_i u^i D_m u^m - \epsilon \mathcal{F}_k^{ij} \mathcal{F}_k^{ij} \] (23)
where \( \mu \) and \( \lambda \) are the Lame coefficients of isotropic media. These equations however are
not independent of each other since a displacement field \( u^k(x) \) can be removed by gauge
translations and, thereby, it fails to be a dynamic variable.

In the spirit of the gauge dislocation theory, we have suggested that gauge potentials of
the Poincaré translations may describe new geometric structure (\textit{sui generis} dislocations)
of a world manifold [16, 18].

Let the tangent bundle \( TX \) be provided with an affine connection \( \Gamma \). By dislocation
morphism of a world manifold \( X \) is meant the special bundle isomorphism of \( TX \) over \( X \)
which takes the coordinate form
\[ \rho : \tau^\mu \frac{\partial}{\partial x^\mu} \rightarrow \frac{\partial}{\partial \tau^\lambda} + \left( \Gamma^\alpha_{\beta\mu} u^\beta + \sigma^\alpha_{\mu} \right) \frac{\partial}{\partial \tau^\alpha} \rightarrow \tau^\mu H^\alpha_{\mu} \frac{\partial}{\partial \tau^\alpha}, \] (24)
where
\[ \sigma^\alpha_{\mu} = D_\mu u^\alpha|_{u=0} = (\partial_\mu u^\alpha + \Gamma^\alpha_{\beta\mu} u^\beta + \sigma^\alpha_{\mu})|_{u=0} \] (25)
is the covariant derivatives of a displacement field \( u \).

Let \( Y \) be a bundle over \( X \) and \( J^1 Y \) the jet manifold of \( Y \). The deformation mor-
phism \( \Phi_1 \) has the jet prolongation
\[ j^1 \rho : y^i_\lambda \rightarrow H^\alpha_{\lambda}(x) y^i_\alpha \]
over \( Y \). Then, given a Lagrangian density \( L \) of fields on a world manifold, one can think
of the composition
\[ \widetilde{L} = L \circ \rho \]
as being the corresponding Lagrangian density of fields on the dislocated manifold. If the
above-mentioned morphism \( \Phi_1 \) consists with the dual to the morphism \( \Phi_1 \), one can apply
these Lagrangian densities in order to describe fields in the presence of the deviations \( \sigma \).
Moreover, we may assume that the deviations \( \sigma \) have the dislocation nature \( \sigma \).

Note that a Lagrangian density \( L(\sigma) \) of translation gauge potentials \( \sigma^\lambda_{\mu} \) cannot be
built in the standard Yang-Mills form since the Lie algebra of the affine group does not
admit an invariant nondegenerate bilinear form. To construct \( L(\sigma) \), one can utilize the
torsion
\[ \mathcal{F}^{\alpha}_{\nu\mu} = D_\nu \sigma^\alpha_{\mu} - D_\mu \sigma^\alpha_{\nu} \]
of the linear connection $\Gamma$ with respect to the soldering form $\sigma$. The general form of a Lagrangian density $L_\sigma$ is given by the expression

$$L_\sigma = \frac{1}{2} [a_1 F_{\mu \nu} F_{\alpha \nu}^\alpha + a_2 F_{\mu \rho} F^\mu_{\rho \sigma} + a_3 F_{\mu \rho} F^\nu_{\rho \sigma} + a_4 \epsilon^\mu_{\nu \sigma \gamma} F_{\mu \nu} F_{\sigma \nu} - \mu \sigma^\nu_{\nu} + \lambda \sigma^\mu_{\mu} \sigma^\nu_{\nu}] \sqrt{-g}.$$ 

This Lagrangian density differs from the familiar gravitational Lagrangian densities. In particular, it contains the mass-like term originated from the Lagrangian density (23) for displacement fields $u$ under the gauge condition $u = 0$. Solutions of the corresponding field equations show that fields $\sigma$ make contribution to the standard gravitational effects. In particular, they lead to the "Yukawa type" modification of Newton’s gravitational potential.

### References

1. J. Cariñena, M. Crampin and L. Ibort, Diff. Geom. Appl., 1, 345 (1991).
2. G. Giachetta and L. Mangiarotti, Int. J. Theor. Phys., 29, 789 (1990).
3. M. Gotay, in Mechanics, Analysis and Geometry: 200 Years after Lagrange, ed. M.Francaviglia (Elseiver Science Publishers B.V., 1991) p. 203.
4. C. Günther, J. Diff. Geom., 25, 23 (1987).
5. S. Hawking and G. Ellis, The Large Scale Structure of a Space-Time (Cambrifge Univ. Press, Cambridge, 1973).
6. F. Hehl, J. McCrea, E. Mielke and Y. Ne’eman: Metric-affine gauge theory of gravity, Physics Reports (1995) (to appear).
7. D. Ivanenko and G. Sardanashvily, Physics Reports, 94, 1 (1983).
8. A. Kadic and D. Edelen, A Gauge Theory of Dislocations and Disclinations (Springer, New York, 1983).
9. I. Kolář, P.W. Michor, J. Slovák, Natural operations in differential geometry, (Springer-Verlag, Berlin etc. 1993).
10. B. Kupershmidt, Geometry of Jet Bundles and the Structure of Lagrangian and Hamiltonian Formalisms, Lect. Notes in Math., 775, 162 (1980).
11. L. Mangiarotti and M. Modugno, in Geometry and Physics, ed. M.Modugno (Pitagora Editrice, Bologna, 1982) p. 135.
12. E. Mielke: *Geometrodynamics of Gauge Fields - On the Geometry of Yang-Mills and Gravitational Gauge Theories* (Akademic-Verlag, Berlin, 1987).

13. J. Ne’eman and Dj. Šijački, *Annals of Physics*, **120**, 292 (1979).

14. R. Percacci, *Geometry of Nonlinear Field Theories* (World Scientific, Singapore, 1986).

15. R. Percacci, *Nuclear Physics*, **B353**, 271 (1991).

16. G. Sardanashvily, *Acta Physica Polonica*, **B21**, 583 (1990).

17. G. Sardanashvily, *J. Math. Phys.*, **33**, 1546 (1992).

18. G. Sardanashvily and O. Zakharov, *Gauge Gravitation Theory* (World Scientific, Singapore, 1992).

19. G. Sardanashvily and O. Zakharov, *Diff. Geom. Appl.*, **3**, 245 (1993).

20. G. Sardanashvily, *Gauge Theory in Jet Manifolds* (Hadronic Press, Palm Harbor, 1993).

21. G. Sardanashvily, *Generalized Hamiltonian Formalism for Field Theory. Constraint Systems* (World Scientific, Singapore, 1994).

22. G. Sardanashvily, Multimomentum Hamiltonian Formalism in Field Theory, E-print: hp-th/9403172, 9405040.

23. G. Sardanashvily, Gravitation Singularities of the Caustic Type, E-print: gr-qc/9404024.

24. G. Sardanashvily, Gravity as a Higgs Field, E-print: gr-qc/9405013, 9407032.

25. G. Sardanashvily, Hamiltonian field systems on composite manifolds, E-print: hep-th/9409159.

26. R. Sulanke and P. Wintgen, *Differentialgeometrie und Faserbündel* (Veb Deutscher Verlag der Wissenschaften, Berlin, 1972).

27. D. Saunders, *The Geometry of Jet Bundles*, (Cambridge Univ. Press, Cambridge, 1989).