A New Spin-Orbit Induced Universality Class in the Quantum Hall Regime?

Yshai Avishai and Yigal Meir

Department of Physics, Ben Gurion University, Beer Sheva 84105, ISRAEL
and
The Ilse Katz Center for Meso- and Nanoscale Science and Technology

Using heuristic arguments and numerical simulations it is argued that the critical exponent $\nu$ describing the localization length divergence at the quantum Hall transition is modified in the presence of spin-orbit scattering with short range correlations. The exponent is very close to $\nu = 4/3$, the percolation correlation length exponent, the prediction of a semi-classical argument. In addition, a region of weakly localized regime, where the localization length is exponentially large, is conjectured.

72.20.My, 73.40.Kp, 73.50.Jt

Spin-orbit scattering (SOS) is known to give rise to pronounced effects in disordered systems. In three dimensions, the presence of SOS changes the universality class of the metal-insulator transition [1]. More dramatically, in two dimensions, SOS of non-interacting electrons leads to a metal-insulator transition, which does not exist in its absence [2]. In the weakly localized regime, SOS changes localization into anti-localization [3], reversing the sign of the magnetoresistance, while in the strongly localized regime, SOS increases the localization length (e.g. by a factor of 4 compared to its value in the absence of SOS in quasi one dimensional systems), thus affecting the resistance by orders of magnitude [4]. The change in the universality class manifested in the level statistics also attenuates the conductance fluctuations in the weakly localized regime (again by a factor of 4) [3].

In spite of these remarkable effects, there have not been many studies of the effects of SOS in the quantum Hall regime [1]. One possible reason is that in the presence of a strong magnetic field, SOS does not change the symmetry of the Hamiltonian, and thus may not be expected to change the underlying universality class. A counter example, however, exists in the strongly localized regime, where even in the presence of a strong magnetic field, SOS increases the localization length (again by a factor of 4 in quasi-one dimensional system, as in the absence of a field). Spin mixing induced by random magnetic field was studied by Lee [4], Hanna et al. [5] (as a specific model for SOS), and Kagalovsky et al. [6]. The main conclusion is that random Zeeman term causes splitting of the spin-degenerate quantum Hall transition, but does not change its universality class. The critical exponent for this kind of disorder then remains about 2.35 $\pm$ 0.02, the accepted numerical value for the quantum Hall transition [1]. Hikami et al. [7] studied an electron interacting with a two-dimensional random magnetic field with white noise correlations, and demonstrated the existence of a different universality class at $E = 0$ (apparently related to non-analyticity of the density of states).

In this work we discuss a two-dimensional electron system in the quantum Hall regime, subject to a random potential and random SOS, manifested in a spin-dependent random magnetic field (that couples to the z-component of the electron spin), and random spin-flip processes. The Zeeman splitting is assumed negligible (a specific Hamiltonian will be addressed below). We give heuristic arguments why one expects a change of critical behavior of the quantum Hall transition, a change that becomes more evident for potentials with short-range correlations. In addition we argue that when the potential correlation length decreases, there exists a quasi-metallic region, where the localization length is exponentially large. These arguments compare very well with numerical calculations.

The heuristic arguments are based on the semi-classical approach of Mil’nikov and Sokolov [8], which was later applied to the quantum Hall effect in layered three dimensional systems [9]. In the semi-classical description, the electron follows skipping orbit trajectories around potential hills or valleys, and there is a critical energy $E_c$ where the trajectory percolates through the system [10]. Thus, away from the critical energy the electron is confined to a percolation cluster of typical size $\xi_p$, the percolation correlation length. Near threshold $\xi_p \sim |E_c - E|^{-\nu_p}$, where $\nu_p = 4/3$ is the two-dimensional percolation exponent. As one approaches the transition the clusters approach each other near saddle points of the potential energy landscape. While classically the electron cannot move from one cluster to another, quantum mechanically it can tunnel through the potential barrier. If the electron energy $E$ is close enough to the transition, the potential barrier is close to parabolic and the tunneling probability through such a saddle point is proportional to $\exp[-(E_c - E)]$. The number of such saddle points through which tunneling occurs in a system of length $L$ is typically $L/\xi_p$. Since the transmission coefficient is multiplicative, the conductance $\sigma$ (or the tunneling probability) through the whole system is

$$\sigma \sim \left[ e^{-(E_c - E)} \right]^{L/\xi_p} \equiv e^{-L/\xi},$$

with $\xi \sim (E_c - E)^{-\nu}$ and $\nu = \nu_p + 1 = 7/3$. The numerical estimate $\nu = 2.35 \pm 0.02$ [10], which is somewhat supported by experimental data [11], has a surprisingly excellent agreement with the result of the above argument, especially in view of its crudeness.

Following [14], this argument can be generalized to include SOS. If the spin-dependent part of the Hamiltonian is slowly varying, one can carry out a local gauge tran-
formation, so that the local spin points in the direction of the local effective random magnetic field (generated by the SOS potential) \( \mathcal{H}_1 \). In the adiabatic limit, where the spin-dependent potentials vary slowly in space, the problem separates into two independent ones with different critical energies, split by twice the typical magnetic field \( H_{\text{eff}} \). Nonadiabaticity (short-range correlations) leads to mixing between these two effective spin directions. Consequently, one may repeat the above argument taking into account the fact that the critical energy \( E_c \) in this case is not equal to the potential energy of the saddle-point, but is \( H_{\text{eff}} \) away from it \( \xi_\sigma \). Thus, the conductance \( \sigma_{\text{so}} \) is

\[
\sigma_{\text{so}} \sim \left[ e^{-H_{\text{eff}}} \right]^{L/\xi_\sigma} \equiv e^{L/\xi_\sigma},
\]

with \( \xi_{\sigma} \sim (E - E)^{-\nu} \) and \( \nu = \nu_p = 4/3 \).

So the semi-classical argument predicts that the localization length critical exponent is equal to the two-dimensional classical percolation exponent. The physical picture behind the reduction in the localization exponent is simple: since the potential landscape for the opposite spin directions is different, then, due to the random effective magnetic field, an electron approaching a saddle point may “prefer” to flip its spin (rather than tunnel through the saddle point), and then continue to propagate semiclassically. The probability for such a Zener tunneling depends on the potential gradients at the point, and is exponentially close to unity for rapidly changing potentials \( \xi_0 \). In fact, since the tunneling probability at the saddle point energy is equal to \( 1/2 \) \( \xi_0 \), one may expect that for rapidly changing potentials there will be a region in energy where the electron will always “prefer” to flip its spin as it approaches the saddle point, and thus may cross the system classically. Quantum effects in two dimensions will localize the electron, but the localization length in this anomalous regime is expected to be exponentially large.

To check these predictions we use a specific, physically relevant model. Consider the lowest two Landau levels, and denote the states in these levels by \( |n_\sigma, k > \), where \( n \) is the Landau level index (0 or 1), \( \sigma \) is the spin and \( k \) is the momentum. We consider the case where, in the absence of disorder, the spin-down state in the lower Landau level is degenerate with the spin-up state in the upper one. This may happen for electrons with a magnetic field dependent g-factor, and seems to be relevant for composite fermions, e.g. with filling factors around \( \nu = 3/2 \) \( \xi_0 \), or at filling factors \( \nu = 2/3 \) and \( \nu = 4/5 \) \( \xi_0 \). We consider only the subspace of these two degenerate Landau levels (i.e. assume that the Landau level splitting is much larger than the disorder potential and the SOS potential). The disorder Hamiltonian \( \mathcal{H}_1 \) is of the form

\[
\mathcal{H}_1 = V(x, y, z) + \alpha \sigma \cdot \nabla V(x, y, z) \times \Pi,
\]

where \( V(x, y, z) \) is the three-dimensional disorder potential, \( \Pi \equiv p - eA/c \) is the electron kinetic momentum, and \( \alpha \) determines the strength of SOS. As the momentum is constrained to two-dimensions, taking the limit \( z \rightarrow 0 \) yields,

\[
\mathcal{H}_1 = V(x, y) + \alpha V_z(x, y) (\sigma_x \Pi_x - \sigma_y \Pi_y),
\]

with \( V_z \equiv \partial V/\partial z \). Since the operator \( \Pi \) operating on a state \( |n_\sigma, k > \) yields the state \( |n_\sigma, k > \), the matrix form of above Hamiltonian, \( < n_\sigma, k|\mathcal{H}_1|n_\sigma', k' > \) reads,

\[
\left( < 0k|V|0k' > < 1k|V_{\text{so}}|1k' > < 1k'|V_{\text{so}}|1k > < 1k|V|1k' > \right),
\]

where the Landau level index now implicitly carries also the spin quantum number, and \( V_{\text{so}} \equiv \alpha V_z \). The regular random potential then plays the role of an effective random magnetic field (which couples to the \( z \)-component of the generalized spin), while the SOS term allows random spin-flips. In order to investigate the effects of the latter term, the random potentials \( V \) and \( V_{\text{so}} \) are assumed independent. We fix the parameters of \( V \) and vary those of \( V_{\text{so}} \), namely, its strength \( V_0 \) (relative to the strength of the disorder potential which defines the energy unit), and its correlation distance \( \lambda \),

\[
< V_{\text{so}}(x, y)V_{\text{so}}(x', y') > = V_0^2 f(x - x')f(y - y'),
\]

with \( f(x) = (2\pi \lambda)^{-1/2}e^{-x^2/2\lambda} \). The corresponding correlation distance for the disorder potential \( V(x, y) \) was taken to be unity (all lengths are expressed in units of the magnetic length).

Fig. 1. The finite-size localization length \( \xi_L \) scaled by the system size \( L \), for \( L = 80 \), as a function of energy, for different values of \( \lambda \), the correlation length of the SOS potential. SOS causes a splitting of the quantum Hall transition, an increase in the localization length and, for small \( \lambda \), a quasi-metallic (weakly localized) regime between the two critical points (the graph is symmetric around \( E = 0 \) and thus only positive \( E \)’s are plotted). In the inset we show \( \xi_L/L \) in the absence of SOS in the lowest two Landau levels. \( \xi_L \) is maximal at \( E = 0 \), the critical energy in this case.

Given the parameters \( V_0 \) and \( \lambda \) we generate and diagonalize an ensemble of random Hamiltonians of the form
in the space of the lowest two Landau levels, using
the Landau gauge and periodic boundary conditions in
the x-direction, for squares of different sizes \( L = 40, 60 \)
and 80. The localization length of a specific eigenstate \( \Psi \)
is determined by \[20\]
\[
\xi_L^2 [\Psi] \propto \int y^2 |\Psi(x, y)|^2 dxy - \left( \int y|\Psi(x, y)|^2 dxy \right)^2.
\]
(7)

By dividing the energy spectrum into bins and averaging
over many disorder realizations, we are able to obtain the
energy dependence of \( \xi_L(E) \). In Fig. 1 we plot \( \xi_L(E)/L \)
for several values of \( \lambda \), for \( L = 80 \) and \( V_0 = 4 \). The
immediate conclusions one can draw from the figure are
the following: (1) The localization length, which in the
absence of SOS was maximal at \( E = 0 \) (see inset), now
has a maximum at two energies, \( E = \pm E_c \) (we show only
positive energies - the graph is symmetric around \( E = 0 \)).
This leads to a splitting of the quantum Hall transition.
(2) The localization length increases with decreasing \( \lambda \)
on both sides of the critical point, in accordance with
the above arguments. (3) At small \( \lambda \), the localization
length in the region between the two critical points is
constant and of the order of the size of the system, again
in accordance with the semi-classical arguments.

Fig. 2. The inverse finite-size localization length \( \xi_L \) scaled by
the system size \( L \), as a function of energy for different values
of \( \lambda \) for \( \lambda = 1 \). The value of \( E \) where the curves meet determines the critical energy. By scaling the curves near the
critical energy (inset), one finds \( \xi(E) \), from which the critical
exponent is determined.

In order to determine whether the change in the local-
ization length is simply due to a numerical prefactor,
or rather, to a different critical behavior, we carry out
the usual scaling analysis - evaluate \( \xi_L(E) \) for different
\( L \)'s, and collapse all the data onto a single plot after
scaling the system size by \( \xi \), (the \( L \to \infty \) localization
length), and set \( L/\xi_L(E) = F[L/\xi(E)] \). In Fig. 2 we plot
\( L/\xi_L(E) \) for different \( L \)'s, for \( \lambda = 1 \). At the critical
energy \( E = E_c \), \( L/\xi_L(E) \) does not change, and thus \( \xi \)
diverges as \( E \to E_c \). The scaling of all curves onto a
single plot (Fig. 2, inset) determines \( \xi(E) \), and by fitting
\( \xi(E) \sim (E - E_c)^{-\nu} \), we obtain the critical exponent \( \nu \).

Fig. 3 depicts the derived best values of \( \nu \) as a function
of \( \lambda \). We find that for small \( \lambda \) the critical exponent
is indeed very close to the expected value from the semi-
classical argument, \( \nu = \nu_p = 4/3 \). As expected, when \( \lambda \)
increases \( \nu \) eventually increases and approaches its regular
quantum Hall value. We find that for very large \( \lambda \) (not plotted)
the critical exponent is very close to the regular quantum Hall exponent. Interestingly, for small \( \lambda \), if we try to scale \( \xi_L(E) \) for a large range of \( E \) around \( E_c \), we
find a larger critical exponent, closer to the quantum Hall
one. This may indicate that the system flows towards its
low energy critical behavior by passing close to the quan-
tum Hall critical point. We cannot, however, based on our
numerical procedure, determine the full phase dia-
gram of the quantum Hall effect in the presence of SOS.

It is interesting to compare our results with previous
works. Lee \[7\] and Hanna et al. \[8\] studied an Hamilton-
ian with a spin-dependent term \( H(r) \cdot S \), in which \( H(r) \)
is a random field that couples to the electron spin \( S \).
Their conclusion is that, at least for random field which varies
smoothly in space, the quantum Hall transition splits,
but the critical behavior remains unchanged. Hanna et al.
also noted that for random field with short-range
(white noise) correlations, the conductance is peaked at
\( E = 0 \), similarly to our curve for \( \xi_L/L \) for \( \lambda = 0.5 \) in
Fig. 1. Their interpretation is that the critical energy may have shifted close to \( E = 0 \), but the critical behav-
ior remains that of the regular quantum Hall effect. If
one assumes, however, a single critical point at \( E = 0 \),
one finds a critical exponent larger than the quantum
Hall one (which may explain the experimental observa-
tions for spin-degenerate Landau levels \[13\]). Indeed we
checked that within the \( H(r) \cdot S \) model, even if the corre-
lation length of the random magnetic field decreases,
the critical behavior remains quantum-Hall-like (\( \nu \approx 2.35 \)).
The difference between that model and ours might seem
surprising, since the model we use looks very much like a random field. However, the distinction becomes apparent in Fig. 4, where we plot the density of states and $\xi_L(E)/L$ for the two models. While for the $\mathbf{H}(r) \cdot \mathbf{S}$ model the density of states splits into two peaks, indicating a splitting of the spin-degenerate Landau level into two independent Landau levels, in our model the density of states remains peaked at $E = 0$ (even though the critical points move away from it), and thus the two effective spin-directions are still strongly mixed. The motivation to study the $\mathbf{H}(r) \cdot \mathbf{S}$ model stems from the fact that the magnetic field breaks the simplectic symmetry of the spin-orbit Hamiltonian, and thus it was assumed enough to study an Hamiltonian with a unitary symmetry. Our model, however, still obeys the full symmetry one expects for SOS in the presence of a magnetic field – time-reversal symmetry followed by reversing the (uniform) magnetic field. Thus, it is expected that the two models belong to different universality classes. A similar argument has been successfully applied to the change in localization length in the strongly localized regime where it was shown that an application of SOS changes the scaling of the localization length, even in the presence of a strong magnetic field, though the universality class might have been expected to remain the same.

To conclude, we have presented arguments and demonstrated numerically that spin-orbit scattering in the quantum Hall regime may alter the critical behavior for potentials with short range correlations. The calculated critical exponent agrees very well with the percolation correlation length exponent, a value predicted by semiclassical arguments.

This work is supported by DIP, BSF and ISF funds.

For a review, see, e.g., Metal-Insulator Transitions, N. F. Mott (Taylor & Francis, London, 1990).
[2] S. N. Evangelou and T. Ziman, J. Phys. C 20, L235 (1987); A. Kawabata, J. Phys. Soc. Japan 57, 1717 (1988).
[3] S. Hikami, A. I. Larkin and Y. Nagaoka, Prog. Theor. Phys. 63, 707 (1980); G. Bergmann, Phys. Rev. Lett. 48, 1046 (1982); See also Y. Meir, Y. Gefen and O. Entin-Wohlman, Phys. Rev. Lett. 63, 798 (1989).
[4] J. L. Pichard et al., Phys. Rev. Lett. 65, 1812 (1990); Y. Meir et al., Phys. Rev. Lett. 66, 1517 (1991); E. Medina and M. Kardar, Phys. Rev. Lett. 66, 3187 (1991); Y. Meir and O. Entin-Wohlman, Phys. Rev. Lett. 70, 1988 (1993).
[5] B. L. Altshuler and B. I. Shklovskii, Zh. Eksp. Teor. Fiz. 91, 220 (1986) [ Sov. Phys. JETP 64, 127 (1986) ]; N. Zanon and J.-L. Pichard, J. Phys. (Paris) 49, 907 (1988).
[6] For a review see, e.g., M. Stone, The Quantum Hall Effect (World Scientific, Singapore, 1992).
[7] D. K. K. Lee, Phys. Rev. B 50, 7743 (1994).
[8] C. B. Hanna et al., Phys. Rev. B 52, 5221 (1995).
[9] V. Kagalovsky et al, Phys. Rev. B 55, 7761 (1997).
[10] For a review of the numerical work, see B. Huckestein, Rev. Mod. Phys. 67, 357 (1995).
[11] S. Hikami, M. Shirai and F. Wegner, Nucl. Phys. B408, 415 (1993); K. Minakuchi and S. Hikami, Phys. Rev. B53, 10898 (1996).
[12] G. V. Mil’nikov and I. M. Sokolov, JETP Lett. 48, 536 (1988).
[13] Y. Meir, Phys. Rev. B 58, R1762 (1998).
[14] S. Luryi and R. F. Kazarinov, Phys. Rev. B 27, 1386 (1983); S. A. Trugman, Phys. Rev. B 27, 7539 (1983); R. Mehr and A. Aharony, Phys. Rev. B 37, 6349 (1988).
[15] H. P. Wei et al., Phys. Rev. Lett. 61, 1294 (1988); H. P. Wei et al., Surf. Sci. 229, 34 (1990); S. Koch et al., Phys. Rev. B 43, 6828 (1991); F. Hohls, U. Zeitler and R. J. Haug, cond-mat/0107412; See, however, V. T. Dolgopolov and G. V. Kravchenko and A. A. Shashkin, Phys. Rev. B 46, 13303 (1992) and A. A. Shashkin, V. T. Dolgopolov and G. V. Kravchenko, Phys. Rev. B 49, 14486 (1994), who report $\nu \simeq 1$, and D. Shahar et al., Solid Stat. Comm. 107, 479 (1998), who see no critical behavior.
[16] A. Gramada and M. E. Raikh, Phys. Rev. B 56, 3965 (1997).
[17] H. A. Fertig and B. I. Halperin, Phys. Rev. B 36, 7969 (1987).
[18] R. R. Du et al., Phys. Rev. Lett. 75, 3926 (1995).
[19] S. Kronmüller et al., Phys. Rev. Lett. 81, 2526 (1998);82, 4070 (1999); J. H. Smet et al., Phys. Rev. Lett. 86, 2412 (2000).
[20] J. L. Cardy (ed.), Finite Size Scaling (North Holland, Amsterdam, 1988); V. Privman (ed.), Finite Size Scaling and Numerical Simulations of Statistical Systems (World Scientific, Singapore, 1990).