A clustering-based self-calibration of the richness-to-mass relation of CAMIRA galaxy clusters out to \( z \approx 1.1 \) in the Hyper Suprime-Cam survey

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ABSTRACT

We perform a self-calibration of the richness-to-mass (\(N-M\)) relation of CAMIRA galaxy clusters with richness \(N \geq 15\) at redshift \(0.2 \leq z < 1.1\) by modeling redshift-space two-point correlation functions, namely, the auto-correlation function \(\xi_{cc}\) of CAMIRA clusters, the auto-correlation function \(\xi_{gg}\) of the CMASS galaxies spectroscopically observed in the BOSS survey, and the cross-correlation function \(\xi_{cg}\) between these two samples. We focus on constraining the normalization \(A_N\) of the \(N-M\) relation in a forward-modeling approach, carefully accounting for the redshift-space distortion, the Finger-of-God effect, and the uncertainty in photometric redshifts of CAMIRA clusters. The modeling also takes into account the projection effect on the halo bias of CAMIRA clusters. The parameter constraints are shown to be unbiased according to validation tests using a large set of mock catalogs constructed from N-body simulations. At the pivot mass \(M_{500} = 10^{14}h^{-1}\text{M}_\odot\), and the pivot redshift \(z_{\text{pivot}} = 0.6\), the resulting normalization \(A_N\) is constrained as \(13.8^{+5.8}_{-4.2}\), \(13.2^{+3.4}_{-2.7}\), and \(11.9^{+3.0}_{-1.9}\) by modeling \(\xi_{cc}\), \(\xi_{cc}+\xi_{cg}\), and \(\xi_{cc}+\xi_{cg}+\xi_{gg}\), with average uncertainties at levels of 36\%, 23\%, and 21\%, respectively. We find that the resulting \(A_N\) is statistically consistent with those independently obtained from weak-lensing magnification and from a joint analysis of shear and cluster abundance, with a preference for a lower value at a level of \(\lesssim 1.9\sigma\). This implies that the absolute mass scale of CAMIRA clusters inferred from clustering is mildly higher than those from the independent methods. We discuss the impact of the selection bias introduced by the cluster finding algorithm, which is suggested to be a subdominant factor in this work.

Key words: galaxies: clusters: general, galaxies: clusters: distances and redshifts, cosmology: large-scale structure of Universe, cosmology: observations, cosmology: cosmological parameters

1 INTRODUCTION

Galaxy clusters are powerful cosmological tools because they provide a representative view of large-scale structures of the Universe. Therefore, galaxy clusters enable independent tests to examine viable cosmological models with strong constraints on fundamental properties, such as the degree of inhomogeneity in cosmic density fields and the equation of state of dark energy (e.g., Wang & Steinhardt 1998; Holder et al. 2001). With the progress in utilizing the technique of weak gravitational lensing to calibrate the mass of clusters (Umetsu et al. 2014; von der Linden et al. 2014b,a; Hoekstra et al. 2015; Schrabback et al. 2018; Dietrich et al. 2019; McClintock et al. 2019), there has been a wide success in constraining cosmology by using the abundance of galaxy clusters identified in the millimeter wavelength (Planck Collaboration et al. 2015; Bocquet et al. 2015; de Haan et al. 2016; Bocquet et al. 2019), in X-ray (Mantz et al. 2015), and in optical (Costanzi et al. 2019b). The recent development of cluster cosmology has promised a competitive cosmological tool that is complementary to and independent of other probes, especially distance-based methods relying on the temperature anisotropy of Cosmic Microwave Background (CMB).
Despite the success of constraining cosmology by using cluster abundance, there have been relatively less efforts in utilizing the halo clustering of galaxy clusters in a cosmological analysis. This was mainly due to the fact that galaxy clusters, as peaks of cosmic density fields, are rare, which inevitably results in insufficient constraining power in terms of two-point or higher-order statistics. For example, the baryon acoustic oscillation (BAO) signature of galaxy clustering power in terms of two-point or higher-order statistics. For example, the baryon acoustic oscillation (BAO) signature of galaxy clustering was only marginally detected at a level of $\approx 2\sigma$ by using the largest cluster catalog by then (Estrada et al. 2009; Hütsi 2010). This situation of lacking a sizable sample of clusters will be rapidly improved with upcoming large and deep surveys, such as the Large Synoptic Survey Telescope (Ivezic et al. 2008), the Euclid mission (Laureijs et al. 2011), and the eROSITA X-ray all-sky survey (Merloni et al. 2012). Thus, it is essential and imminent to study the clustering of galaxy clusters, paving a way for upcoming data sets.

Apart from the signature of BAO, few pilot studies were carried out to measure the large-scale clustering of galaxy clusters identified in the existing or ongoing surveys, in which some of them were further used to infer cosmology: The correlation functions of optically selected clusters were measured and compared to simulations in Balagall et al. (2003). Collins et al. (2000) measured the correlation function of a X-ray flux-limited sample of $\approx 450$ clusters at redshift $z \lesssim 0.3$, for which the measurements together with those of cluster abundance were used to constrain cosmological parameters (Schuecker et al. 2003). Later, Mana et al. (2013) demonstrated that a joint analysis of cluster abundance and clustering could significantly improve the constraints on the amplitude of the density fluctuation $\sigma_8$ and the matter density $\Omega_M$ by an amount of $\approx 50\%$.

Meanwhile, the development of the advanced cluster finding algorithm, redMaPPer (Rykoff et al. 2014), significantly improved the size and quality of cluster samples constructed in the Sloan Digital Sky Survey (hereafter SDSS; York et al. 2000), which further bolstered the study of cluster clustering. By using a sample of $\approx 120k$ redMaPPer clusters at redshift $z \lesssim 0.3$, Sereno et al. (2015) presented a joint analysis of weak lensing and clustering, for the first time, on the cluster-scale. A similar work was done in Jimeno et al. (2015), where they secured the redshift determination of redMaPPer clusters by utilizing the spectra from the Baryon Oscillation Spectroscopic Survey (hereafter BOSS; Dawson et al. 2013) and combined the measurements of cluster abundance and clustering to infer cosmology. Baxter et al. (2016) constrained the observable-to-mass scaling relation of redMaPPer clusters based on angular clustering alone. In addition, the angular cross-correlation between redMaPPer clusters and a sample of photometrically selected galaxies was also studied in Paech et al. (2017).

To achieve the goal of precision cosmology, it is absolutely necessary to combine the information from both cluster abundance and clustering to tighten the constraint on parameters in order to discover possible failures of the concordance $\Lambda$CDM cosmological model, and/or to identify new systematics to explain tensions among different probes. For example, a combined analysis of cluster abundance and clustering sheds light on properties of cosmological neutrinos (Marulli et al. 2011; Emami et al. 2017). Moreover, there are distinct advantages in using galaxy clusters as tracers of large-scale structures. As galaxy clusters are the most massive and gravitationally dominated objects in the Universe, their halo bias, which describes the strength of clustering on large scales with respect to the underlying dark matter, is less sensitive to baryonic properties and environmental effects, which are often referred to as the "astrophysical bias". This results in a cleaner connection between the underlying matter density and galaxy clusters, of which the halo bias is relatively easier to be characterized through N-body simulations (e.g., Tinker et al. 2010) than on the galaxy-scale. Meanwhile, it is of critical importance to combine different surveys, especially those observed spectroscopically. This is because the clustering strength of a photometrically selected sample on small scales would significantly diminish due to the uncertainty of redshift estimates (Sereno et al. 2015). Thus, the inclusion of spectroscopic surveys would significantly improve the accuracy and precision of clustering measurements (e.g., as done in Jimeno et al. 2015).

In this work, we aim to study the clustering properties of the galaxy clusters optically selected in the Hyper Suprime-Cam (HSC) survey (Aihara et al. 2018a) and their cross-correlation with the CMASS galaxies, which are spectroscopically observed in the BOSS (see Section 3.3 for more details). Specifically, we will perform a self-calibration of the observable-to-mass relation based on these clustering measurements alone. As the deepest optical imaging survey at the achieved depth to date, the combination of the depth and area of the HSC survey enables a construction of a sizable sample of clusters to study their clustering properties out to high redshift ($z \approx 1.1$), for the first time. Although the clustering signal of galaxy clusters detected in the HSC survey is distorted on small scales because of lacking secure redshifts, the precision of their clustering measurements is significantly improved by cross-correlating with the sample of CMASS galaxies. The uniqueness of this work is that we perform the mass calibration of galaxy clusters based on halo clustering alone by using a joint data set of the largest cluster sample out to high redshift ($z \approx 1.1$) to date and the spectroscopic sample in the common footprint of the BOSS. It is worth mentioning that a similar analysis is difficult to be achieved in the southern hemisphere using the Dark Energy Survey (The Dark Energy Survey Collaboration 2005; The Dark Energy Survey Collaboration et al. 2016), due to the lack of a large spectroscopic sample in a common footprint. With the upcoming era of large spectroscopic surveys, such as the Dark Energy Spectroscopic Instrument (DESI Collaboration et al. 2016) and the Subaru Prime Focus Spectrograph surveys (Takada et al. 2014), the synergy with imaging and spectroscopic surveys will be common and essential to study galaxy clusters. In this regard, this work serves a pilot study in this topic.

This paper is organized as follows. In Section 2, a brief overview of structure formation in the context of halo clustering is provided. The data products used in this work are described in Section 3. The detailed methodology used to measure the correlation functions are presented in Section 4. The modeling of these correlation functions is presented in Section 5. We discuss the results in Section 6. The discussions of the selection bias are in Section 7. The conclusions are given in Section 8. Throughout this paper, we assume a flat $\Lambda$CDM cosmology with $\Omega_M = 0.3$, the mean baryon density $\Omega_b = 0.05$, the Hubble expansion rate $H_0 = h \times 100 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1}$ with $h = 0.7$, $\sigma_8 = 0.8$, and the spectral index of the primordial power spectrum $n_s = 0.95$. The mass $M_{500}$ of a cluster is defined by a sphere with the radius $R_{500}$, in which the enclosed mass density is equal to 500 times the critical density $\rho_c(z)$ of the Universe at the cluster redshift. Unless otherwise stated, all quoted errors represent 68% confidence levels (i.e., 1σ). The notation $\mathcal{N}^\theta(x,y)$ ($\mathcal{W}(x,y)$) stands for a normal distribution with the mean $x$ and the standard deviation $y$ (a uniform distribution between $x$ and $y$).
2 THEORY

An overview of structure formation in the context of halo clustering is given in this section. We refer interested readers to Mandelbaum et al. (2013) and Okumura et al. (2016) for more details.

The two-point statistics of matter distributions is one of the most straightforward ways to describe cosmic structures. For instance, the correlation function \( \xi_{mm} (r) \) of matter, which is an inverse transform of the matter power spectrum \( P_{mm} (k) \) in the Fourier-space, describes the excess of matter density fields separated by a distance \( r \) in the comoving coordinate with respect to a random distribution. That is,

\[
\xi_{mm} (r) \equiv \langle \delta_m (x) \delta_m (x + r) \rangle , \tag{1}
\]

where \( \delta_m (x) \) is the matter overdensity at \( x \). The bracket \( \langle \rangle \) stands for the ensemble average over \( x \).

Halos form via gravitational collapse and result in biased tracers of the overall density field. In the linear perturbation theory, the overdensity \( \delta_h \) of a halo population can be linked to the matter overdensity by a halo bias \( b_h \) as follows:

\[
\delta_h (x) = b_h \delta_m (x) , \tag{2}
\]

such that the clustering strength of halos reads

\[
\xi_{hh} (r) = b_h^2 \xi_{mm} (r) , \tag{3}
\]

where \( \xi_{hh} \) is the correlation (or auto-correlation) function of halos, and the halo bias \( b_h \) mainly depends on the halo mass and redshift. As an analogy to the auto-correlation function, the cross-correlation function between two populations of halos is expressed as

\[
\xi_{XY} (r) = b_X b_Y \xi_{mm} (r) , \tag{4}
\]

where \( b_X \) and \( b_Y \) are the halo biases of the halo population X and Y, respectively. In a regime where the linear perturbation theory fails, e.g., on small scales, the halo bias \( b_h \) could be scale-dependent. If the halo bias is known, one can determine the correlation function \( \xi_{hh} \) of a halo population to further unveil the underlying matter distribution.

However, it is challenging to accurately determine three-dimensional correlation functions in observations, because the line-of-sight distance is unknown and must be inferred from observables. In the context of redshift surveys, the line-of-sight distance to each object is usually inferred from the observed redshift \( z_{obs} \). Using the inferred distance, the resulting correlation function \( \xi_{hh} (s) \) in the “redshift-space”, denoted as \( s \), is modulated with respect to that in the real-space \( \xi_{hh} (r) \). This is because the observed redshift \( z_{obs} \) is deviated from the cosmological redshift \( z_c \) due to the presence of the peculiar velocity \( v_{pec} \) of halos and the measurement uncertainty \( \Delta z_c \):

\[
z_{obs} = z_c + \frac{v_{pec}}{c} (1 + z_c) + \Delta z_c , \tag{5}
\]

where the subtitle \( \parallel \) denotes the component along the line of sight, and \( c \) is the speed of light.

On large scales, halos are experiencing a coherent movement toward the potential center of cosmic structures, as a result of gravitational collapse. This leads to a squash in the distribution of the line-of-sight distance that is inferred by the observed redshift. Consequently, the redshift-space correlation function is distorted, as known as the redshift-space distortion (RSD: Kaiser 1987, or the Kaiser effect). On small scales, halos act as particles with a random motion due to the presence of peculiar velocity, resulting in a stretch in the distance distribution along the line of sight. This is a nonlinear RSD effect known as the Fingers-of-God (FoG) effect (Jackson 1972).

It is important to note that measurement uncertainties of redshift play an important role in determining redshift-space correlation functions. In an imaging survey, as used in this work, the redshift is usually estimated by the photometry redshift (or photo-z) with a typical uncertainty \( \Delta z_c \). Because the line-of-sight comoving distance to a halo at redshift \( z \) is

\[
D_{\text{LoS}} = \frac{r}{H(z)} ,
\]

where \( H(z) \) is the Hubble constant, a dispersion \( \sigma_{\Delta z_c} \) in the redshift uncertainty would result in a characteristic scale,

\[
\sigma_{\text{LoS}} = \frac{c \sigma_{\Delta z_c}}{H(z)} , \tag{6}
\]

such that the line-of-sight clustering signature is significantly smeared out on the scale \( \lesssim \sigma_{\text{LoS}} \). Taking a typical value of \( \sigma_{\Delta z_c} = 0.01 \) for optically selected clusters, this corresponds to \( \sigma_{\text{LoS}} \approx 20 ~\text{Mpc}/h \) at \( z \approx 0.3 \). That is, the power spectrum would be largely suppressed at \( k \lesssim 1/\sigma_{\text{LoS}} \) due to the photo-z uncertainty. Moreover, this effect is significantly larger than the FoG effect: the typical line-of-sight velocity dispersion \( \sigma_v \) for halos is at the order of \( \approx 300 ~\text{km/sec} \), which only leads to \( \approx \sigma_v (1 + z)/c \approx 0.0013 \) at \( z \approx 0.3 \), i.e., a factor of \( \approx 10 \) smaller than the dispersion of the photo-z uncertainty. Therefore, the uncertainty of photo-z is the dominant factor over the peculiar velocity of halos in determining redshift-space correlation functions in imaging surveys and needs to be modeled (Sereno et al. 2015). We refer readers to Section 5 for the detailed modeling of observed redshift-space correlation functions.

In this work, we measure (1) the correlation function of the cluster sample in the HSC survey, (2) the correlation function of CMASS galaxies, which are spectroscopically observed in the BOSS, and (3) their cross-correlation function. The goal is to calibrate the observable-to-mass relation, i.e., the richness-to-mass relation, of the galaxy clusters detected in the HSC survey by using these clustering measurements in a joint analysis.

3 DATA

A brief overview of the HSC survey is given in Section 3.1. In this work, we make use of the cluster sample from the HSC Survey and the CMASS galaxy sample from the BOSS survey, as described in Section 3.2 and Section 3.3, respectively. Meanwhile, we construct the mock catalogs for both samples of clusters and CMASS galaxies using a large set of N-body simulations, as detailed in Section 3.4.

3.1 The HSC survey

The HSC survey is an imaging survey in the framework of a Subaru Strategic Program to image a sky area of 1400 deg^2 in five broadband filters (grizy). The imaging is carried out using the wide-field camera Hyper Suprime-Cam (Miyazaki 2015; Miyazaki et al. 2018) installed on the 8.2 m Subaru Telescope. There are three layers in the HSC survey: WIDE, DEEP and UltraDEEP. In the interest of a large and uniform coverage on the sky, we only use the data from the WIDE layer for constructing the cluster catalog (see Section 3.2). The imaging reduction and catalog construction are
processed by the 

processed by the hscPipe (Bosch et al. 2018), for which the performance of photometric measurements is fully verified in Huang et al. (2018).

In this work, we make use of the S18A data set from the HSC survey to construct the cluster catalog. We have applied the bright star masks modified from Coupon et al. (2018) to the footprint, because a different scheme of background subtraction is used in cataloging the S18A data (for more details, see Aihara et al. 2019a).

3.2 Cluster sample

In this work, we make use of the cluster sample constructed by the CAMIRA algorithm (Cluster finding Algorithm based on Multi-band Identification of Red-sequence gAlaxies; Oguri 2014) in the HSC Survey. We refer interested readers to Oguri (2014) and Oguri et al. (2018) for more details of the CAMIRA cluster finder. In what follows, a brief overview of the CAMIRA algorithm is given.

CAMIRA is a matched-filter and red-sequence based cluster finder that relies on the stellar population synthesis model with the aid of calibration using spectroscopically confirmed galaxies. After identifying a galaxy cluster, CAMIRA assigns a photometric redshift estimate and a richness \( N \), as the cluster mass proxy, to the system. The center of each cluster is identified as the location of the Brightest Cluster Galaxy (BCG), which is suggested to be a good representative of the cluster center given the small offset \( \lesssim 0.1 \) Mpc/h in the physical coordinate) between the BCGs and the X-ray peaks (Oguri et al. 2018). Since the purpose of this work is to investigate the correlation functions in the comoving coordinate on a scale \( \gtrsim 10 \) Mpc/h, this level of mis-centering is negligible.

In this work, we use the CAMIRA cluster catalog in the Full-Depth-Full-Color (FDFC) footprint of the HSC WIDE layer with area of \( \approx 427 \) deg\(^2\) (after applying the star mask). In this cluster catalog, we further select the clusters with richness \( N \gtrsim 15 \) at redshift \( 0.2 \leq z < 1.1 \) in the interest of consistency with previous work. Specifically, the cluster sample selected in this criteria was previously studied using weak-lensing shearing (Murata et al. 2019) and magnification (Chiu et al. 2020). Therefore, this choice of the cluster selection enables a direct comparison with the results from gravitational lensing. After the selection in richness and redshift, we further apply the mask of the CMASS galaxy sample (see Section 3.3) that will cross-correlate with, such that the footprints of the cluster and CMASS samples are identical. This reduces the area to \( \approx 403 \) deg\(^2\). As a result, the final cluster sample consists of 3057 systems with \( N \gtrsim 15 \) at \( 0.2 \leq z < 1.1 \), which are shown as the color-coded points in Figure 1.

Given the quality of the HSC data sets, the performance of the photometric redshift \( z_{\mathrm{cl}} \) estimation of CAMIRA clusters is remarkable out to redshift \( z \lesssim 1.2 \). Specifically, the bias and scatter in terms of \( (z_{\mathrm{cl}} - z_{\mathrm{BCG}})/(1 + z_{\mathrm{BCG}}) \) are quantified to be \(-0.0013\) and \(0.0081\), respectively, where \( z_{\mathrm{BCG}} \) is the observed spectroscopic redshift of the BCGs. In the interest of uniformity, we use the photometric redshift \( z_{\mathrm{cl}} \) for each cluster, regardless of available spectroscopic redshifts.

We note that a detailed modeling of photo-z uncertainties is needed to correctly interpret observed correlation functions in redshift-space (see Section 2), even with this sub-percent level of precision in the redshift estimation.

Cluster random catalogs

To calculate correlation functions from a given survey, we need to build random catalogs with the same survey geometry and redshifts randomly distributed against the data. To construct the random catalog for CAMIRA clusters, we follow a similar procedure as described in Baxter et al. (2016) to take into account the survey geometry, bright star masks, and the mask due to the CAMIRA cluster finder. Specifically, for a given redshift we randomly draw a point in the survey footprint, excluding the regions indicated by the star mask, to investigate whether a CAMIRA cluster could be detected at this point. If the cluster could be detected, we add this point into the random catalog. We repeat this process until the size of the random catalog reaches 40 times larger than the CAMIRA cluster catalog. This procedure guarantees a random angular distribution while taking into account the masks due to bright stars and the cluster finder.

We stress that the spatial filter of the CAMIRA algorithm is independent of richness (Oguri 2014), so the mask of the CAMIRA finder only depends on the cluster redshift. For the default analysis, we create the random catalog at redshift of 0.56, which is approximately the mean redshift of the CAMIRA cluster sample. We also create two cluster random catalogs at lower and higher redshifts (0.26 and 0.84 respectively), and find that the change to the default analysis is negligible compared to the statistical uncertainty (see Appendix B for more details). This is expected, because the spatial filter of the CAMIRA algorithm has only mild redshift dependence (Oguri 2014; Oguri et al. 2018).

After randomizing the angular distribution of the random catalog, we then sample a random distribution along the line-of-sight direction based on the data. Specifically, we bootstrap the observed redshifts of CAMIRA clusters (with replacements) and assign it to each point in the random catalog. This is referred to as the “shuffled” redshift in Section 6 of Ross et al. (2012). This ensures that the redshift distribution of the random catalog includes not only the redshift uncertainty of observed clusters, but also the systematics introduced by the cluster finding algorithm (e.g., the filter transition effect; Rykoff et al. 2014; Soergel et al. 2016). As a result, the
random catalog with a density of 1000 points per square degree (or ≈ 40 times larger than the data) is obtained.

We find that sampling the redshift estimates to the random points following the smoothed redshift distribution of CAMIRA clusters results in negligible difference compared to the current statistical uncertainty. We refer readers to Appendix B for more details.

3.3 The CMASS sample

We use the spectroscopic sample of galaxies from the Baryon Oscillation Spectroscopic Survey (BOSS; Dawson et al. 2013), which is the largest survey in the Sloan Digital Sky Survey-III (SDSS-III; Eisenstein et al. 2011) program. In the BOSS, there are two galaxy samples: LOWZ and CMASS. The LOWZ sample targets the low-redshift galaxy population at $z \lesssim 0.4$, mainly dominated by Luminous Red Galaxies (Eisenstein et al. 2001). On the other hand, the CMASS sample targets galaxies at high redshift of $0.4 < z < 0.7$, which are pre-selected by using the imaging of the SDSS-II program (Aihara et al. 2011). The target selection in the CMASS sample is based on a combination of customized color and magnitude cuts, such that a sample of galaxies with approximately “constant stellar mass” is expected. We refer readers to Rodríguez-Torres et al. (2016) for more details of the selection of CMASS galaxies.

In this work, we focus on the CMASS sample (both north and south Galactic Caps) from the Data Release 12 (DR12; Alam et al. 2015), as the final data release of the SDSS-III. We only use the regions that are overlapping the S18A FDFC footprint of the HSC survey. Moreover, we carefully apply the bright star mask of the HSC survey to the CMASS catalog, such that the footprints of the CMASS and cluster samples share the same geometry on the sky. The common footprint between the CAMIRA and CMASS samples has area of ≈ 403 deg², in which about 37k CMASS galaxies are present with a median redshift of $z_{\text{BOSS}} = 0.57$.

For the random catalog of observed CMASS galaxies, we make use of the random catalogs from both the north and south Galactic Caps that are publicly available from the DR12. These random catalogs already account for the angular mask of the BOSS. For the random catalogs from both northern and southern hemispheres, we first select the common footprint between the BOSS and HSC survey and then exclude the regions indicated by the HSC masks due to bright stars. Then, we randomly draw a redshift estimate from the observed CMASS sample (with replacements) and assign it to each point in the random catalog, separately for both hemispheres. This is identical to the construction of the CMASS random catalog as in Ross et al. (2012), for which the random redshift is referred to as the “shuffled” redshift. In the end, the final random catalog for the observed CMASS galaxies is obtained by combining the random catalogs from both northern and southern hemispheres. This process naturally accounts for the difference in the observed redshift distributions between the north and south Galactic Caps.

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1 https://www.sdss.org/dr15/spectro/lss/#BOSS

2 https://www.sdss.org/dr15/spectro/lss/#BOSS
3.4 Mock catalogs

In this work, we make use of the mock halo catalogs from the N-body simulations in Takahashi et al. (2017) for the tasks of (1) the construction of covariance matrices of the correlation functions, and (2) the end-to-end validation of the codes. We will detail these two tasks in Section 4.2 and Section 5, respectively. In what follows, a brief summary of the mock catalogs is given. We refer readers to Takahashi et al. (2017) for more details of the mock halo catalogs.

A number of 108 full-sky cosmological N-body simulations with high resolution is presented in Takahashi et al. (2017) in a framework of the standard flat \( \Lambda \)CDM cosmology with \( \Omega_M = 0.279, \Omega_{\Lambda} = 0.721, \sigma_8 = 0.82, \) and \( H_0 = 70 \text{km/s/Mpc,} \) which is consistent with the WMAP result (Hinshaw et al. 2009). Each set of the N-body simulations is performed using the GADGET2 code (Springel et al. 2001, 2005) in the order of nested cubic boxes around an observer with different box sizes, ranging from 450h\(^{-1}\)Mpc to 6300h\(^{-1}\)Mpc with a step of 450h\(^{-1}\)Mpc. Each box contains 2048\(^3\) particles, which corresponds to a particle mass ranging from \( \approx 10^6h^{-1}M_{\odot} \) to \( \approx 10^{13}h^{-1}M_{\odot} \), depending on the box side. The resulting matter power spectra of the N-body simulations are fully resolved on the scale of \( k \approx 50 \text{Mpc}^{-1} \) at \( z < 1 \) and are in good agreement with the theoretical model predicted by the Halofit (Smith et al. 2003; Takahashi et al. 2012). The dark matter halos are identified by the ROCKSTAR halo finder (Behroozi et al. 2013) with a criterion that the minimal halo mass must be at least 50 times of the particle mass. In this configuration, the resulting mock halos effectively cover the halo mass range of CMASS galaxies and CAMIRA clusters, by design. In the mass and redshift range of interested in this work, the mass function and the linear halo bias of mock halos are verified to agree with those predicted by the Tinker et al. (2008) and Tinker et al. (2010) formulas within 12% and 10%, respectively. In other words, we expect a systematic uncertainty in the linear halo bias of mock halos at a level of 10%.

With proper rotations of the sky, from each mock catalog we further tail four non-overlapping footprints with the same geometry of the common footprint between the BOSS and HSC survey. This ensures that the four catalogs from the same full-sky simulation are nearly independent with each other on the scale smaller than the size of the survey, which is studied in this work. As a result, we make use of a total number of 432 (\( = 108 \times 4 \)) mock catalogs that are able to represent the realistic properties, e.g., the mass function and halo clustering, of CMASS galaxies and CAMIRA clusters. From each mock halo catalog among these 432, we further construct a mock catalog of CAMIRA clusters and a mock catalog of CMASS galaxies, as detailed in Section 3.4.1 and Section 3.4.2, respectively.

3.4.1 Mock clusters

To utilize these mock catalogs in the same way as we do for CAMIRA clusters, we need to assign the mass proxy, i.e., richness \( N \), for each mock halo. Specifically, we assign a richness estimate to each mock halo following the \( N-M \) relation defined in equation (19) with the parameters constrained by gravitational lensing. We use the parameter obtained in Chiu et al. (2020), in which they constrained the \( N-M \) relation of CAMIRA clusters in the same richness and redshift ranges using weak-lensing magnification (flux magnification bias) alone. To be exact, for each mock catalog we sample a set of the \( N-M \) parameters from the chain of the lensing magnification constraint\(^3\). Then, we assign a richness estimate to each mock halo in this catalog accounting for the intrinsic scatter and measurement uncertainty of richness given the true halo mass. We repeat this procedure for 432 mock halo catalogs. The mean values of the \( N-M \) relation used in the richness assignment are

\[
(A_B, B_B, C_B, \sigma_B) = (17.72, 0.92, -0.48, 0.15). \]

We stress that the \( N-M \) relation from lensing magnification is statistically consistent \((\lesssim 1.8\sigma)\) with the result independently obtained by combining weak shear and cluster abundance (Murata et al. 2019). By marginalizing over the parameter constraint from lensing magnification, we effectively takes into account the uncertainty of the \( N-M \) relation in the richness assignment.

After assigning the estimate of richness, we further perturb the redshift \( z_{\text{mock}} \) with respect to the true redshift \( z_{\text{true}} \) of mock halos. This is done by following a Gaussian distribution with a zero mean and the scatter with a form of \( \sigma_{z_{\text{mock}}-z_{\text{true}}}/(1+z_{\text{true}}) = 0.0093 \times (N/N_{piv})^{-0.18} \), depending on the cluster richness. Note that this richness-dependent scatter in the redshift estimate is directly constrained by CAMIRA clusters (for more details, see Section 5.2), and that we specifically apply this to mock catalogs in order to mimic the observed photo-\( \zeta \) uncertainty.

Finally, we apply the richness and redshift cuts \((N \geq 15 \text{ and } 0.2 \leq z < 1.1)\) to mock catalogs, as we do in the analysis of CAMIRA clusters. This leads to 432 mock cluster catalogs.

We construct a random catalog for 432 mock cluster catalogs in a similar way as done in constructing the random catalog for CAMIRA clusters. Specifically, we follow the same randomization of the angular distribution to account for various masks in the survey footprint, and then assign a redshift to each random point by bootstrapping the redshifts from the joint catalog of 432 mock cluster samples. That is, the only difference to the random catalog of observed CAMIRA clusters is that the random redshift estimate is “shuffled” from mock clusters. It is important to note that we do not bootstrap the redshift from observed CAMIRA clusters, because (1) the mock clusters are constructed by using a different cluster finder algorithm rather than that based on cluster red-sequence (see Section 3.2), and (2) the redshift estimates of mock clusters only take into account measurement uncertainties but not the systematics that is subject to the CAMIRA cluster finder. Meanwhile, we ensure that the resulting random catalog is at least 40 times larger than each mock cluster catalog.

3.4.2 Mock CMASS galaxies

In this study, we also construct a mock catalog of CMASS galaxies from each out of the 432 realizations. First, we assume that the observed redshift \( z_{\text{mock}} \) of mock halos is identical to the true redshift \( z_{\text{true}} \), ignoring measurement uncertainties. This is a reasonable assumption, because the redshifts of CMASS galaxies are secured by spectroscopic data with negligible uncertainties. Note that the redshift \( z_{\text{mock}} \) still includes the contribution from the peculiar velocity of halos. That is, the RSD and FoG effects are present in the mock catalogs.

Second, we randomly select mock halos following the observed redshift distribution (denoted as \( P_{\text{CMASS}}(z) \)) of CMASS galaxies. In practice, this is done by sampling a random variable uniformly distributed between 0 and \( \max \{P_{\text{CMASS}}(z)\} \) to each mock halo, for which it is selected if the random variable is smaller than

\[
(A_N, B_N, C_N, \sigma_N) = (17.72, 0.92, -0.48, 0.15). \]

\[^3\] The “Joint” constraints in the fifth column of Table 1 in Chiu et al. (2020)
than $P_{\text{CMASS}}(z)$ at the mock halo redshift $z_{\text{mock}}$. We derive $P_{\text{CMASS}}(z)$ using an interpolation over the observed redshift distribution in a redshift interval of $[0, 1.1]$ with a step of 0.001. This ensures that the selected mock CMASS galaxies have the same redshift distribution as the data. Despite different redshift distributions of observed CMASS galaxies between the north and south Galactic Caps (Ross et al. 2012), we note that $P_{\text{CMASS}}(z)$ is derived based on the joint sample of CMASS galaxies from both hemispheres in the common footprint of the BOSS and the HSC survey. That is, $P_{\text{CMASS}}(z)$ represents the effective redshift distribution of the CMASS sample in this work, as a whole.

Third, we assign a probability of hosting a central galaxy to each mock halo by applying the prescription of halo occupation distribution (HOD) in Manera et al. (2013), which is specifically designed for creating CMASS-like mock galaxies. Specifically, the mean number of central galaxies for each mock halo reads

$$\langle N_{\text{cen}} \rangle = \left\{ \begin{array}{ll} 1 & \text{if } \left( \log M_{200m} - 13.09 \right) / 0.596 > 1 \text{,} \\ 0 & \text{otherwise.} \end{array} \right.$$ 

Then, each halo is selected with a probability equal to $\langle N_{\text{cen}} \rangle$. For example, a halo with $\langle N_{\text{cen}} \rangle = 0.5$ has a probability of 50% to be selected into the mock CMASS sample. This selection using the HOD modeling effectively leads to a mock galaxy sample with a mass distribution that is consistent with that of observed CMASS galaxies, by design.

Last, only the halos satisfying the selection criteria of redshift and HOD modeling are selected into the final mock CMASS samples. This procedure is repeated for the 432 mock realizations, resulting in the same number of mock CMASS catalogs. Note that we assume no correlation between the mass and redshift in selecting mock CMASS galaxies in this approach. The resulting mock CMASS galaxies have a mass range of $11.9 < \log (M_{200m}/\text{M}_\odot) < 13.9$ with a median value of $\approx 12.8$. Using the formula in Tinker et al. (2010), this implies that the halo bias has a range between $\approx 1.1$ and $\approx 4.2$ with a mean (median) value of $\approx 1.9$ ($\approx 1.7$), which is in good agreement with the observational result of CMASS galaxies ($\approx 1.93 \pm 0.17$; Chiang et al. 2013). Given the systematic uncertainty at a level of 10% in the halo bias of mock halos (see Section 3.4), this implies good consistency between the resulting mock and observed CMASS galaxies. Additionally, we also extract the information of the line-of-sight velocity dispersion $\sigma_v$ from these mock CMASS catalogs; it is estimated as $\sigma_v \approx 310 \text{ km/sec}$ in the physical space.

We cannot directly use the random catalog constructed for observed CMASS galaxies in calculating the correlation functions of mock CMASS galaxies, because (1) the redshift distributions of observed CMASS galaxies are different between the north and south Galactic Caps (Ross et al. 2012), and (2) we select the mock galaxies following the effective redshift distribution $P_{\text{CMASS}}(z)$ of observed CMASS sample across the footprint (see Section 3.4.2). That is, the random catalog of observed CMASS galaxies contains a redshift distribution with spatial dependence, which is not true for mock CMASS galaxies. Therefore, for each mock CMASS catalog, we build a random catalog by first sampling random points on the sky with the identical footprint of the data (after applying the star mask), then followed by the redshift assignment to each random point according to the effective redshift distribution $P_{\text{CMASS}}(z)$. We ensure that the size of the resulting random catalog is at $\approx 30$ times larger than each mock CMASS catalog.

4 MEASUREMENTS

In this work, we determine three correlation functions in the redshift space: auto-correlations of CAMIRA clusters and CMASS galaxies, and their cross-correlations. In what follows, we detail our procedure to obtain the measurements, which will be used to calibrate the $N-M$ relation of CAMIRA clusters in a self-calibration manner, i.e., solely based on halo clustering, in Section 5.

4.1 Correlation Functions

The two-point auto-correlation function of a tracer $X$, $\xi_{XX}(s)$, is derived using the Landy & Szalay (1993) estimator, namely

$$\xi_{XX}(s) = \frac{DD - 2DR + RR}{RR},$$

where $X = \{c, g\}$ and $c$ and $g$ stand for clusters and galaxies, respectively. Here, $DD = DD(s)$, $DR = DR(s)$, and $RR = RR(s)$ are the normalized numbers of pairs with a separation of the redshift-space distance $s$ between the data-data, data-random, and random-random catalogs, respectively. This estimator given by equation (7) can be generalized for the cross-correlation $\xi_{XY}(s)$ as

$$\xi_{XY}(s) = \frac{D_XD_Y - D_XR_Y - R_XD_Y + R_XR_Y}{R_XR_Y},$$

where $D_XD_Y$, $D_XR_Y$, $R_XD_Y$, and $R_XR_Y$ are, respectively, the normalized numbers of data-data, data-random, random-data, and random-random pairs found at a separation of $s$ for tracers $X$ and $Y$. All tasks of pair counting in this work is done by using TreeCorr (Jarvis et al. 2004).

Using equation (7) and (8), we derive the following correlation functions in this work:

(i) $\xi_{cc}$: the auto-correlation of CAMIRA clusters,
(ii) $\xi_{cg}$: the auto-correlation of CMASS galaxies, and
(iii) $\xi_{gg}$: the cross-correlation between CAMIRA clusters and CMASS galaxies.

The correlation functions of $\xi_{cc}$, $\xi_{cg}$, and $\xi_{gg}$ are estimated in the redshift-space distance $s$ between $10 h^{-1}\text{Mpc}$ and $60 h^{-1}\text{Mpc}$ with logarithmic binning of 7 steps.

Except treating the CAMIRA clusters as a whole in calculating (i) and (iii), we also perform a “subsample” analysis to further investigate the clustering properties as functions of richness and redshift. Namely, we split the cluster sample into two redshift bins ($0.2 \leq z < 0.7$ and $0.7 \leq z < 1.1$) and two richness bins (15 $\leq N < 25$ and $N \geq 25$), with four subsamples in total. Then, we re-measure the clustering functions (i) and (iii) independently for each subsample.

4.2 Construction of covariance matrices

Covariance matrices are needed for statistical analyses because different bins of the observed correlation functions are strongly correlated. By taking the advantage of a large size of N-body simulations, we derive covariance matrices based on the 432 mock halo catalogs, which are described in Section 3.4.

We first repeat the measurements ($\xi_{cc}$, $\xi_{cg}$, and $\xi_{gg}$) on each
mock catalogs. In this way, we generate 432 sets of the measurements in the identical configuration as the data. Then, for any combination of a data vector, denoted as $\mathbf{D}$, we can find the corresponding measurements $\mathcal{D}_{(i)}$ from the $i$-th mock catalog and derive the covariance matrix as

$$\mathcal{C} = \frac{1}{N_{\text{mock}} - 1} \sum_{i=1}^{N_{\text{mock}}} (\mathcal{D}_{(i)} - \mathcal{D}_{(i)}) \cdot (\mathcal{D}_{(i)} - \mathcal{D}_{(i)})^T,$$

where $\mathcal{D}_{(i)} = \frac{1}{N_{\text{mock}}} \sum_{m=1}^{N_{\text{mock}}} \mathcal{D}_{(i,m)}$, and $N_{\text{mock}} = 432$. The resulting covariance matrix normalized by the diagonal elements, $\mathcal{C}_{ij}/(\mathcal{C}_{ii} \cdot \mathcal{C}_{jj})^{1/2}$, is shown in Figure 3. We have verified that the covariance matrices are converged given this amount of realizations.

We further multiply a factor of $N_{\text{mock}}^{-1} - N_D^{-1}$, where $N_D$ is the length of a data vector, to equation (9) to account for the underestimation of the uncertainty because of a finite number of realizations used in estimating the covariance matrix (Hartlap et al. 2007). In this work, this correction factor ranges from $\approx 1.9\%$ (for the modeling of a correlation function of the whole sample; $N_D = 7$) to $\approx 17\%$ (for a joint modeling of the auto- and cross-correlation functions in the “subsample” analysis; $N_D = 7 \times 4$ subsamples $\times$ (1 CAMIRA auto + 1 cross) + 7 $\times$ (1 CMASS auto) = 63).

5 MODELLING

We use the approach as described in Sereno et al. (2015) to model the correlation functions of $\xi_{cc}$, $\xi_{cg}$, and $\xi_{gg}$. In what follows, the details are given.

In this paper, we consider only the angle-averaged monopole component of a correlation function,

$$\xi_{XY}(s) = \int \frac{k^2 dk}{2\pi^2} h_0(k)s P_{XY}(k),$$

in which $XY = \{cc, cg, gg\}$, $j_0$ is a zero-order spherical Bessel function, and $P_{XY}(k)$ is the angle-averaged power spectrum.

In the presence of only the redshift-smearing (or FoG) and Kaiser effects, a redshift-space correlation function depends on the two directions that are perpendicular to and along the line of sight, respectively. Therefore, it is common to re-express the power spectrum in a polar coordinate of $k = (k, \mu)$, where $\mu$ is the cosine angle of the vector $k$ with respect to the line of sight. In this way, the term $P_{XY}$ has a generic form (e.g., Park et al. 1994; Peacock & Dodds 1994; Okumura et al. 2015),

$$P_{XY}(k) = P_{m}(k) \times \left( b_X + f_m(\mu) \right) \left( b_Y + f_m(\mu) \right) \left( b_Y + f_m(\mu) \right) \times G_X(k\mu \sigma_{\text{LoS},X}) G_Y(k\mu \sigma_{\text{LoS},Y}) G_X(k\mu \sigma_{\text{LoS},Y}),$$

where $P_{m}(k)$ is the matter power spectrum; the term $(b_X + f_m(\mu))^2$ contains the Kaiser (1987) term describing the linear RSD effect; $b_X$ is the linear bias of halos which host a tracer; $f_m(\mu)$ is the growth rate evaluated at the redshift $\chi_X$ of a tracer $X$; the functions $G_X$ and $G_Y$ are the damping functions caused by the nonlinear velocity dispersion ($\sigma_{\text{LoS}}$) and photo-$\gamma$ uncertainties ($\sigma_{\text{LoS}}$), respectively. We describe each term as follows.

5.1 Modeling of the Finger-of-God effect

In this study, we assume a Gaussian function for the nonlinear smearing effect due to the line-of-sight velocity dispersion, $\sigma_v$, as

$$G_X(k\mu \sigma_{\text{LoS}}) = \exp \left( -\frac{k^2 \mu^2 \sigma_{\text{LoS}}^2}{2} \right).$$
We model the dispersion $\Delta z$ of photo-$z$ uncertainties in equation (11) by a Gaussian distribution, described as

$$\sigma_{\delta, \text{LoS}}(k \mu, \sigma_{\delta, \text{LoS}}) = \exp \left( -\frac{k^2 \mu^2 \sigma^2_{\delta, \text{LoS}}}{2} \right),$$

where

$$\sigma_{\delta, \text{LoS}} = \frac{c}{H(z)} \times \sigma_{\Delta z},$$

following equation (6). This is a reasonable assumption, given that the measurement uncertainty of photo-$z$ is indeed distributed as a Gaussian in our work. Thus, $G_x$ and $G_z$ eventually have the same functional form.

We derive the dispersion $\sigma_{\Delta z}$ of photo-$z$ uncertainties $\Delta z$ for CAMIRA clusters as follows. We assume that cosmological redshift, $z_{\mathrm{c}}$, is identical to the redshift of the BCG, $z_{\mathrm{BCG}}$, ignoring peculiar velocity as a subdominant factor. That is, $\Delta z \equiv z_{\mathrm{c}} - z_{\mathrm{BCG}} = z_{\mathrm{c}} - z_{\mathrm{BCG}}$. Next, we model $\sigma_{\Delta z/(1+z_{\mathrm{BCG}})}$ using a power law of cluster richness:

$$\sigma_{\Delta z/(1+z_{\mathrm{BCG}})}(N) \equiv \frac{\sigma_{\Delta z}}{1+z_{\mathrm{BCG}}} = \delta \left( \frac{N}{N_{\mathrm{mean}}} \right)^{\Gamma},$$

with two free parameters ($\delta$, $\Gamma$) that can be constrained from the data. Specifically, we bin CAMIRA clusters in seven richness bins, in which the photo-$z$ uncertainty in terms of $\Delta z/(1+z_{\mathrm{BCG}})$ in the richness bin $N_i$ is modeled by a Gaussian distribution with the dispersion of $\sigma_{\Delta z/(1+z_{\mathrm{BCG}})}(N)$ where $i = 1, \cdots, 7$. Then, we fit equation (16) to the derived data points of $\sigma_{\Delta z/(1+z_{\mathrm{BCG}})}(N)$ over all richness bins. We show the best fit of equation (16) together with the data points in the left panel of Figure 3, with the best-fit parameters,

$$(\delta, \Gamma) = (0.0093 \pm 0.0002, -0.18 \pm 0.05).$$

Note that we use 1165, out of the 3057 CAMIRA clusters, that have BCGs with available spectroscopic redshifts $z_{\mathrm{BCG}}$ to determine equation (16).

On the other hand, we do not observe a monotonic redshift dependence in $\sigma_{\Delta z/(1+z_{\mathrm{BCG}})}$, as seen in the right panel of Figure 5. Rather, the value is roughly a constant, $\sigma_{\Delta z/(1+z_{\mathrm{BCG}})} \approx 0.009$, and shows a dip at $z \approx 0.4$. This is in great agreement with Murata et al. (2019), where larger dispersion at both low ($z \lesssim 0.4$) and high ($z \gtrsim 0.6$) redshifts was seen than that at $z \approx 0.45$. The larger dispersion at high redshift is expected, because photometry measurements of distant galaxies are noisier. Meanwhile, larger dispersion at low redshift is mainly due to the lack of $u$-band data, as well as a difficulty in estimating the accurate color of close galaxies that are sometimes too bright for the HSC survey (Murata et al. 2019). Additionally, high-richness clusters are more abundant at low redshift than at high redshift (see Figure 2), which could result in a redshift dependence in the best-fit parameters of $(\delta, \Gamma)$. Ideally, the photo-$z$ dispersion should be modeled as a function of both richness and redshift. In this work, however, we cannot simultaneously constrain the richness- and redshift-dependence of the photo-$z$ dispersion, due to the lack of a large spec-$z$ sample. We thus ignore the redshift dependence of $\sigma_{\Delta z/(1+z_{\mathrm{BCG}})}$ in this work. The number-weighted average of the richness- and redshift-dependent $\sigma_{\Delta z/(1+z_{\mathrm{BCG}})}$ over the whole sample is 0.0092 and 0.0098, respectively. To first-order approximation, this corresponds to an increase at a level of 6% if accounting for the redshift dependence in $\sigma_{\Delta z/(1+z_{\mathrm{BCG}})}$. A larger spec-$z$ sample is clearly warranted for future work with a detailed modeling of $\sigma_{\Delta z/(1+z_{\mathrm{BCG}})}$.

For the CAMIRA clusters, the term $\sigma_{\Delta z}$ is a subdominant factor, given that their redshifts are secured by spectroscopic observations with negligible measurement uncertainties. We thus ignore the measurement uncertainty of the redshift of CAMIRA galaxies.

5.3 Modeling of Halo Bias

In this work, we model the halo bias of CAMIRA galaxies by a free parameter, $b_h = b_{h, \text{CAMIRA}}$. Meanwhile, the halo bias of CAMIRA clusters, $b_c$, is linked to their cluster mass based on the Tinker et al. (2010) fitting formula. Additionally, we account for the projection effect on the halo bias of CAMIRA clusters by following the method introduced in Baxter et al. (2016). In what follows, we briefly describe the modeling of the projection effect and refer interested readers to Baxter et al. (2016) for more details.

The projection effect is referred to that a single cluster is due to the nature of the survey.
Figure 5. Distribution and the dispersion of the photo-z uncertainty, which is characterized in terms of $\Delta z/(1 + z_{BCG})$ with $\Delta z = z_{cl} - z_{BCG}$, as a function of the cluster richness (left) and redshift (right). In the left panel, the distribution of the photo-z uncertainty is shown as the grey points in the upper plot. The black circles represent the distributions of the photo-z uncertainty in seven richness bins, assuming Gaussian distributions, with their errorbars as the size of dispersion. The best-fit values and uncertainties of the Gaussian dispersion $\sigma_{\Delta z/(1 + z_{BCG})}$ in seven richness bins are shown in the lower plot, for which we model them as a function of cluster richness by a power-law function (the dashed line). In the right panel, we show the distribution of the photo-z uncertainty (upper) and the sizes of their Gaussian dispersion (lower) as functions of cluster redshift, following the same configuration as in the left panel. We observe a non-monotonic behavior of $\sigma_{\Delta z/(1 + z_{BCG})}$ in cluster redshift (see more discussions in Section 5.2).

The parameter $f$ needs to be distinguished from the growth rate parameter $f_{\text{m}}$. 

$\sigma_{\Delta z/(1 + z_{BCG})}$

\[
\sigma_{\Delta z/(1 + z_{BCG})} = \sigma_{\text{Gauss}}(z_{BCG}) 
\]

\[
\sigma_{\text{Gauss}}(z_{BCG}) = \exp\left(-\frac{(1 + z_{BCG})^2}{2\sigma_z^2}\right) 
\]

where $\sigma_z = 0.0093 \pm 0.0002$.

Assuming that the projected system is observed with a richness $N_1$, in which the sub-halos with mass $M_1$ and $M_2$ have the richness of $N_1 = qN$ and $N_2 = (1 - q)N = \frac{1 - q}{q}N_1$ with $0.5 \leq q \leq 1$, the projected halo bias $b_{\text{proj}}$ is given by

\[
b_{\text{proj}}(N, z) = \frac{1}{f}b_{\text{non-proj}}(N, z) + f b_{\text{proj}}(N, z),
\]

where $f$ is the probability that the cluster is a product of a projection effect. The non-projected bias $b_{\text{non-proj}}(N, z)$ is expressed as

\[
b_{\text{non-proj}}(N, z) = \frac{\int b_{\text{T10}}(M_{500}, z) P(N|M_{500}, z; p_r) N_M(M_{500}, z) dM_{500}}{\int P(q|M_{500}, z; p_r) N_M(M_{500}, z) dM_{500}},
\]

where $b_{\text{T10}}$ is the Tinker et al. (2010) halo bias. $N_M(M_{500}, z)$ is the mass function evaluated by using the fitting formula in Bocquet et al. (2010), and the term $P(N|M_{500}, z; p_r)$ with the N–M parameters $p_r$ describes the probability of observing the richness $N$ given the true mass $M_{500}$ at the redshift $z$. We stress that the form of equation (18) already accounts for the Eddington and Malmquist bias, as proved and widely used in previous work (e.g., Liu et al. 2015; Chiu et al. 2016; Chiu et al. 2018; Bulbul et al. 2019; Chiu et al. 2020).

It is important to note that the information of the $N$–$M$ scaling relation is fully contained in the term $P(N|M_{500}, z; p_r)$, which includes both measurement uncertainties and intrinsic scatter of richness at fixed cluster mass. The $N$–$M$ scaling relation is parameterized by the parameters $p_{\text{eg}} = (A_N, B_N, C_N, \sigma_N)$ as

\[
(\ln N|M_{500}) = \ln A_N + B_N \ln \left(\frac{M_{500}}{M_{\text{proj}}(z)}\right) + C_N \ln \left(\frac{1 + z}{1 + z_{\text{proj}}(z)}\right),
\]

with log-normal intrinsic scatter $\sigma_N$ at fixed mass, where $B_N$ and $C_N$ are the mass and redshift power-law indices, respectively; $A_N$ is the normalization at the pivot mass $M_{\text{proj}} = 10^{15} h^{-1} M_{\odot}$ and the pivot redshift $z_{\text{proj}} = 0.6$.

As for the projected halo bias $b_{\text{proj}}$, it reads

\[
b_{\text{proj}}(N, z) = \frac{\int b_{\text{T10}}(M_{500}, z) P(q|M_{500}, z; p_r) N_M(M_{500}, z) dM_{500}}{\int P(q|M_{500}, z; p_r) N_M(M_{500}, z) dM_{500}},
\]

in which tilde put on mass $M$ stands for

\[
\tilde{M} = M + g \left(\frac{1 - q}{q}\right)^{1/\bar{n}}
\]

where $g$ and $q$ are two nuisance parameters with criteria of $0 \leq g \leq 1$ and $0.5 \leq q \leq 1$, respectively.

The scenario described by equation (21) is as follows. In a projected system consisting of two halos with mass $M_1$ and $M_2$, respectively, the halo bias of the projected system is confined to be between those inferred by $M_1$ and $M_1 + M_2$ with a definition of $M_1 > M_2$. Then, it is easy to write the halo bias of a projected system as

\[
b_{\text{T10}}(M_{500}, z) = b_{\text{T10}}(M_1 + g M_2, z).
\]

Assuming that the projected system is observed with a richness $N_1$, in which the sub-halos with mass $M_1$ and $M_2$ have the richness of $N_1 = qN$ and $N_2 = (1 - q)N = \frac{1 - q}{q}N_1$ with $0.5 \leq q \leq 1$, the projected halo bias $b_{\text{proj}}$ is expressed as
respectively, we can obtain equation (21) by substituting $M_2 = \left(\frac{1 - q}{q}\right)^2 M_1$ in equation (22) using the relation of $N \sim M^{qs}$ or $\xi \sim N^{0.5}$. In a limit of either no projection effect ($q = 1$), or zero separation of two halos in a projected system ($q < 1$ and $g = 1$), equation (20) reduces to equation (18).

Despite quite complex forms of equation (20) and (21), the physical interpretation is rather straightforward: In a two-halo projected system with an observed richness $N$, the projection effect leads to an increasing halo bias of the main sub-halo with respect to that without projection. As a result, this is equivalent to a shift by a factor of $\left(\frac{1 - q}{q}\right)^2$ in the redshift binning and thus is negligible (Baxter et al. 2016). The modeling of the projection effect in a greater depth requires an end-to-end validation based on large simulations (see e.g., Costanzi et al. 2019a; Sunayama et al. 2020), which is not available for our CAMIRA cluster sample. However, the recent study of Sunayama et al. (2020) suggests that a red-sequence based cluster finder could result in a cluster sample that preferentially selects systems located at filaments aligning along the line of sight. This selection bias changes the underlying mass distribution of optically selected cluster samples and ultimately bias their clustering measurements. This selection bias is not included in our current modeling of the projection effect, which only accounts for the mis-match between observed richness and underlying true halo mass. We refer readers to Section 7 for more discussions about the selection bias suggested by Sunayama et al. (2020).

5.4 Modeling of Correlation Functions

Based on Sections 5.1 to 5.3, we further express the three power spectra in equation (11) for modeling $\xi \nu v$, $\xi v z$, and $\xi z z$. They are explicitly given by

\[ P_{\nu v} (k) = P_{\nu v} (k) \left( b_\nu + f_{\nu} (z) \right)^2 \left( b_\nu + f_{\nu} (z) \right)^2 \exp \left( -k^2 \mu^2 \sigma_{\nu v}^2 \right), \]  

\[ P_{v z} (k) = P_{v z} (k) \left( b_v + f_{v} (z) \right)^2 \left( b_v + f_{v} (z) \right)^2 \exp \left( -k^2 \mu^2 \sigma_{v z}^2 \right), \]  

\[ P_{z z} (k) = P_{z z} (k) \left( b_z + f_{z} (z) \right)^2 \left( b_z + f_{z} (z) \right)^2 \exp \left( -k^2 \mu^2 \sigma_{z z}^2 \right), \]  

where the nonlinear velocity dispersion of CAMIRA clusters and the photo-$z$ uncertainty of CMASS galaxies are ignored (see sections 5.1 and 5.2, respectively). The growth rate $f_{\nu,v,z}$ is evaluated at the median redshift of a given sample, $z_\nu$ and $z_{v,z}$ for CAMIRA clusters and CMASS galaxies, respectively.

The nonlinear velocity dispersion of CMASS galaxies, $\sigma_{\nu v}$, is computed as

\[ \sigma_{v z} = \sigma_{v z} \times \frac{1 + z_v}{H(z_v)}, \]  

where the line-of-sight velocity dispersion $\sigma_{v z}$ of CMASS galaxies is fixed to 310 km/sec, as suggested by our mocks (see Section 5.1). Note that we fix $z_v$ to 0.57, as the median redshift of the CMASS sample.

Given a sample of CAMIRA clusters with a set of observed richness $N$, $\sigma_{v z}$ is computed as in equation (15),

\[ \sigma_{v z} = \frac{c}{H(z_v)} \sigma_{v z}, \]  

where $\sigma_{v z}$ is evaluated as the mean value of $\sigma_{v z} (N)$ (i.e., equation (16)) among the clusters in the sample, given a set of parameters $(\delta, \Gamma)$. We use the cluster photo-$z$ in evaluating $\sigma_{v z}$.

The linear halo bias of CMASS galaxies, $b_v = b_{\nu v}$, is the only free parameter to model the amplitude of $\xi \nu v$. The remaining quantity is the halo bias of CAMIRA clusters, $b_\nu$, which is linked to the cluster mass and further connected to the observable (i.e., richness) by the $N-$ scaling relation. In this way, one can calibrate the $N-$ parameters by forward-modeling to an observed correlation function. This process is referred to as the "self-calibration" of the $N-$ relation based on clustering alone. The halo bias of CAMIRA clusters is modeled as the mean value of the cluster sample, namely, $b_\nu = b_{\nu model}$, where $b_{\nu model}$ is the mean value of $b_{\nu model} (N, z)$ given by equation (27) over the cluster sample, given a set of parameters $(\delta, \Gamma)$. To sum up, we will have nine free parameters, $\mathbf{p} = (\delta, \Gamma, N_0, f, g, q, \sigma_{v z})$, in modeling $\xi \nu v$. The first four parameters characterize the $N-$ relation. The parameters of $f$, $g$, and $q$ are used to model the projection effect, while the last two $(\delta, \Gamma)$ describe the redshift-smearing effect due to the photo-$z$ uncertainty of CAMIRA clusters. The self-calibration of the $N-$ relation is performed by modeling $\xi \nu v$ with an additional parameter of the CMASS halo bias $b_{\nu v}$ on top of the nine free parameters above, and thus we have ten free parameters for the modeling of the cross-correlation function.

5.5 Statistical Inference

In this subsection, we describe the forward-modeling approach to calibrate the $N-$ relation by modeling the measurements of auto- and/or cross-correlation functions in a framework of fixed cosmology.

We explore the parameter space using encee (Foreman-Mackey et al. 2013; Foreman-Mackey et al. 2019), which implements the Affine Invariant Markov Chain Monte Carlo (MCMC) algorithm. For a given data vector $\mathbb{D}$ and the parameter vector $\mathbf{p}$, the posterior $\mathbb{P} (\mathbf{p} | \mathbb{D})$ of $\mathbf{p}$ is expressed as

\[ \mathbb{P} (\mathbf{p} | \mathbb{D}) = \mathcal{L} (\mathbb{D} | \mathbf{p}) \times \mathcal{P} (\mathbf{p}), \]  

where $\mathcal{P} (\mathbf{p})$ is the prior on $\mathbf{p}$, and $\mathcal{L} (\mathbb{D} | \mathbf{p})$ is the likelihood of the model evaluated with the parameter vector $\mathbf{p}$. The log-likelihood $\ln \mathcal{L} (\mathbb{D} | \mathbf{p})$ reads

\[ \ln \mathcal{L} (\mathbb{D} | \mathbf{p}) = -\frac{1}{2} (\mathbb{M} (\mathbf{p}) - \mathbb{D})^T \mathbb{C}^{-1} (\mathbb{M} (\mathbf{p}) - \mathbb{D}), \]  

where $\mathbb{C}$ is the covariance matrix defined in Section 4.2, and $\mathbb{M}$ is the model of auto- and/or cross-correlation functions corresponding to the data vector $\mathbb{D}$ (see Section 5). The correlation functions...
of clusters in the model \( \Psi \) are evaluated at the mean redshift of CAMIRA clusters in the sample (or the subsample).

Note that \( \mathcal{D} \) represents a generic term of data vectors, which can be a combination of various auto- and cross-correlation functions measured in the whole sample or different subsamples of richness and redshift. In this work, we perform the modeling of \( \xi_{\text{cc}}, \xi_{\text{cg}}, \) and \( \xi_{\text{gg}} \) separately, and the joint modeling of \( \xi_{\text{cc}} + \xi_{\text{cg}} \) and \( \xi_{\text{cc}} + \xi_{\text{cg}} + \xi_{\text{gg}} \). For fitting the \( \xi_{\text{cc}} \) alone, we have nine free parameters, \( p = \{A_N, B_N, C_N, \sigma_N, f, g, q, \delta, \Gamma_i\} \). For modeling \( \xi_{\text{gg}} \) alone, as the simplest case, there is only one free parameter, the halo bias \( b_{\text{cMASS}} \). If a cross-correlation function (\( \xi_{\text{cg}} \)) is included in the modeling, then we have ten free parameters, \( p = \{A_N, B_N, C_N, \sigma_N, f, g, q, \delta, \Gamma_i, b_{\text{cMASS}}\} \).

In this work, we cannot meaningfully constrain all parameters at the same time without informative priors, given current measurement uncertainties as well as the degeneracy among parameters. Therefore, we focus on constraining the normalization \( A_N \) of the \( N-M \) relation while applying the informative priors on other parameters. Specifically, we apply Gaussian priors of \( \mathcal{N}(0.92, 0.13^2) \), \( \mathcal{N}(-0.48, 0.69^2) \), and \( \mathcal{N}(0.15, 0.07^2) \) on \( B_N, C_N, \) and \( \sigma_N \), respectively. These priors are suggested by the posterior independently constrained by lensing magnification (Chiu et al. 2020) and are statistically consistent with those obtained by a joint analysis of weak shear and cluster abundance (Murata et al. 2019). Only a uniform prior between 0 and 100 is applied on \( A_N \).

A Gaussian prior \( \mathcal{N}(0, 0.05^2) \) is applied on the parameter \( f \) with an additional requirement of \( 0 \leq f \leq 1 \) to describe the percentage of projected systems in the sample. This value is suggested by another optically selected cluster sample in the SDSS (Baxter et al. 2016; Simet et al. 2017). Flat priors of \( \mathcal{U}(0, 1) \) and \( \mathcal{U}(0.5, 1) \) are applied on \( g \) and \( q \), respectively. Note that we cannot well constrain the parameters of \( f, g, \) and \( q \) based on cluster clustering alone, for which a dedicated effort using large simulations is needed (e.g., Costanzi et al. 2019a) and is currently not available for our sample. By applying these priors in a MCMC framework, we effectively marginalize these parameters over the range of the parameter space with a minimal requirement of informative knowledge. For the parameters of \( \delta_i \) and \( \Gamma_i \), we apply Gaussian priors of \( \mathcal{N}(0.0093, 0.0002^2) \) and \( \mathcal{N}(-0.18, 0.05^2) \), which are suggested by our data (see Section 5.2), respectively.

The halo bias \( b_{\text{cMASS}} \) is varied with a Gaussian prior \( \mathcal{N}(1.93, 0.17^2) \), which was the posterior independently constrained by RSD and BAO together with the WMAP9 CMB data (Chuang et al. 2013). Although different cosmological parameters are used and fixed in this work, we note that the constraint of \( b_{\text{cMASS}} \) in Chuang et al. (2013) takes into account the variation of cosmological parameters and serves an adequate prior here. A summary of the adopted priors is given in Table 1.

### Table 1.

| Parameter | Priors | Reference |
|-----------|--------|-----------|
| \( A_N \) | \( \mathcal{N}(0, 100) \) | Section 5.5 |
| \( B_N \) | \( \mathcal{N}(0.92, 0.13^2) \) | Chiu et al. (2020) |
| \( C_N \) | \( \mathcal{N}(-0.48, 0.69^2) \) | Chiu et al. (2020) |
| \( \sigma_N \) | \( \mathcal{N}(0.15, 0.07^2) \) | Chiu et al. (2020) |

### 5.6 Validations using mock catalogs

By using the mock catalogs, we perform end-to-end validation tests of our codes and the assumptions made in the modeling. For example, in this work we assume that the power spectrum can be simply described by equation (11), which only models the effects of the RSD, FoG, and photo-z smearing without accounting for, e.g., the assembly bias (Lin et al. 2016; Zu et al. 2017). By carrying out the modeling on mock measurements in the identical way as on the data, our goal is to ensure that we can recover the input parameters \( \{A_N, B_N, C_N, \sigma_N\} \) of the \( N-M \) scaling relation without significant bias.

To do so, we randomly draw 10 different sets of mock data, enlarge the sample size in mock modeling, such that the bias (if exists) would not be hidden by statistical uncertainties, which are \( 3.2 \approx \sqrt{10} \) times smaller than those of the observations. For a combination of data vectors in equation (28), the posteriors of the parameters in the joint modeling of the mock measurements are

\[
\Psi(p; \{\mathcal{D}_1, \mathcal{D}_2, \ldots, \mathcal{D}_{10}\}) = \prod_{i=1}^{10} \mathcal{L}(\mathcal{D}_i | p) \propto \mathcal{P}(p),
\]

where \( \mathcal{D}_i \) is the \( i \)-th mock measurement.

Despite the efforts in carefully mimicking observational properties of CAMIRA clusters and CMASS galaxies in the mock catalogs, we note following remarks in the mock modeling. First, the redshift estimates of mock catalogs only contain measurement uncertainties, which are \( 3.2 \approx \sqrt{10} \) times smaller than those of the observations. For a combination of data vectors in equation (28), the posteriors of the parameters in the joint modeling of the mock measurements are

\[
\Psi(p; \{\mathcal{D}_1, \mathcal{D}_2, \ldots, \mathcal{D}_{10}\}) = \prod_{i=1}^{10} \mathcal{L}(\mathcal{D}_i | p) \propto \mathcal{P}(p),
\]

where \( \mathcal{D}_i \) is the \( i \)-th mock measurement.
ing of $\xi_{\text{CC}}$, $\xi_{\text{CG}}$, and $\xi_{\text{GG}}$, independently, and the combination among them. As seen in Figure 6, our modeling can recover the input parameters (within 1σ), indicated by the dashed lines. This suggests that (1) our modeling approach can deliver an unbiased result, and that (2) the assumptions made in constructing the models are valid. We further show the best-fit profiles and the mock measurements in Figure 7, demonstrating that our models provide a good description of the measurements. Note that the mock validations on the subsample analysis deliver the same picture as the default analysis, we thus only present the results of mock validations using the cluster sample as a whole.

It is interesting to note that the constraint on the normalization $A_M$ in modeling $\xi_{\text{CG}}$ alone is weaker than that in modeling $\xi_{\text{CC}}$ alone. This is due to a strong degeneracy between the normalization $A_M$ and the CMASS halo bias $b_{\text{CMASS}}$, which is completely dominated by the prior. Consequently, the constraint on $A_M$ largely depends on the prior on $b_{\text{CMASS}}$ and becomes weaker than that from modeling $\xi_{\text{CC}}$ alone. On the other hand, the joint modeling of $\xi_{\text{CC}}$ and $\xi_{\text{CG}}$...
mentioning that the correlation patterns among the parameters in Figure 8 are in great agreement with those based on the mock validations (see Figure 6), suggesting that the mock catalogs indeed well describe the observed properties of the CAMIRA and CMASS samples.

We then show the measurements (black points) and the best-fit profiles\(^6\) (red regions) of the CAMIRA auto-correlation functions in the subsample analysis in the right panel of Figure 9. In addition, the 68\% confidence regions of the mean of the correlation functions among the 432 mock catalogs are indicated by the grey shaded area. Although the errorbars are large, it is seen that (1) the best-fit models provide a good description for the observed correlation functions, and that (2) the observed correlation functions show a hint for slightly higher amplitudes than those measured from the mocks (grey regions).

The discrepancy can be seen more clearly in the right panel of Figure 9, where we present the more precise measurements of the cross-correlation functions between the CAMIRA and the CMASS samples. In this case, the best-fit models (red regions) are produced based on the joint modeling of \(\xi_{cc}\), \(\xi_{cg}\), and \(\xi_{gg}\) in the subsample analysis\(^7\). There indeed exists a discrepancy between the measured \(\xi_{cg}\) (black circles) and those estimated from the mocks (grey regions), especially the low-redshift sample at 0.2 < \(z\) < 0.7. In addition, this discrepancy is higher for high-richness clusters (at the level of 2.2\(\sigma\)) than the low-richness samples (at the level of 1\(\sigma\))\(^8\). We expect that this discrepancy could be mitigated by accounting for the redshift dependence in the photo-z dispersion of CAMIRA clusters (see Section 5). This is because the redshift distribution of the CMASS sample is peaked at the redshift of \(z \approx 0.5\), such that the resulting \(\xi_{cg}\) is weighted at this redshift, which is approximately the minimum of the photo-z dispersion of CAMIRA clusters. A smaller dispersion in the redshift uncertainty results in a higher amplitude of a correlation function, which is consistent with the observed \(\xi_{cg}\) as opposed to the mocks. Taking into account the photo-z dispersion as a decreasing function of richness, this discrepancy is expected to be larger in the high-richness bin than the low-richness bin, as also seen in our results. Currently, this discrepancy is not significant in this work (i.e., at a level of 2.2\(\sigma\) for the high-richness and low-redshift bin, and \(\approx 1.2\sigma\) in the whole-sample analysis). This implies that modeling the photo-z dispersion as a function of richness and redshift is required in improving the mock catalogs in the future.

With the precision in \(\xi_{cg}\) in the subsample analysis, we can constrain the mass- and redshift-trend power-law indices of the N–M relation without the Gaussian priors. Specifically, we replace the Gaussian priors \(\mathcal{N} (0.92, 0.13^2)\) and \(\mathcal{N} (-0.48, 0.69^2)\) on \(B_N\) and \(C_N\) with the uniform priors \(\mathcal{U}(0, 2)\) and \(\mathcal{U}(-5, 5)\), respectively, and then repeat the whole modeling. The resulting constraints are shown in Figure A2 and are tabulated in Table 2. Except the poor constraints obtained from the modeling based on \(\xi_{cg}\) alone, it can be seen that the resulting constraints on \(A_N\), \(B_N\), and \(C_N\) are all statistically consistent with those from the default analysis. This suggests that the constraint on \(A_N\) from the default analysis is not sensitive to the adopted Gaussian priors on \(B_N\) and \(C_N\).

In Figure 10, we show the results of the default analysis (i.e.,

\(6\) These profiles are evaluated using the best-fit parameters in the sixth row in Table 2

\(7\) The ninth row in Table 2

\(8\) Note that we take into account the correlation among the radial bins in calculating the significance of these discrepancies.
respectively. In terms of $\xi$ except that the results are obtained by modeling the observed $\xi$ without binning the clusters in richness and redshift). The observed $\xi_{cc}$ (blue circles) also shows a hint for a higher amplitude than the mean value of the 432 mocks (blue shaded region), although they are consistent with each other given the errorbars. On the other hand, the observed $\xi_{gg}$ (red circles) clearly shows a higher amplitude compared to the mocks (red shaded regions). This enhancement is at a level of $1.2\sigma$, accounting for the correlation among the radial bins. Last, the observed $\xi_{gg}$ (green circles) is also higher than the mean value of the 432 mocks (green shaded regions) at a level of $2\sigma$.

To sum up, the observed correlation functions show higher amplitudes than those estimated from the mocks. These discrepancies are at levels of $\leq 0.5\sigma$, $1.2\sigma$ and $2\sigma$ for $\xi_{cc}$, $\xi_{gg}$, and $\xi_{gg}$, respectively. In terms of $\xi_{gg}$, the discrepancy is mainly attributed to the low-redshift sample, especially for the high-richness clusters. In addition, Figures 9 and 10 show that these discrepancies are nearly independent of the scale, suggesting that the difference is due to the linear halo bias which changes the overall normalization.

While our constraint on the linear halo bias of the CMASS galaxies ($b_{\text{CMASS}} = 1.838 \pm 0.032$) obtained from the modeling of $\xi_{gg}$ alone is in good agreement with the independent result of Chuang et al. (2013), $b_{\text{CMASS}} = 1.93 \pm 0.17$, the observed amplitude of $\xi_{gg}$ is higher than that from the mocks at a level of $\approx 10\%$ (or $\approx 2\sigma$). This corresponds to a higher linear halo bias for the observed CMASS galaxies at a level of $\approx 5\%$. Note that this comparison only accounts for the statistical uncertainty but not the systematics between the simulations and the observation. Recalling that the distribution of the linear halo bias of mock CMASS galaxies is
suggested to have a median (mean) value of 1.7 (1.9) with a systematic discrepancy to the linear prediction at a level of 10% (see Section 3.4), our constraint of \( b_{\text{CMASS}} \approx 1.84 \pm 0.03 \) is broadly consistent with that inferred from the N-body simulations (\( \approx 1.7 \sim 1.9 \)) for accounting for the systematic uncertainty. That is, the consistency in \( \xi_{\text{CC}} \) between the mocks and the observation is largely limited by the systematics, given the precision of the measured \( \xi_{\text{CC}} \). By adopting the informative prior from the result of the BOSS collaboration (Chuang et al. 2013), we effectively marginalize the systematic uncertainty of \( b_{\text{CMASS}} \) in the modeling.

Moreover, the difference between the mock and observed \( \xi_{\text{CC}} \) reflects an offset in the normalization \( A_N \) of the \( N-M \) relation between the mock and observed cluster samples, additionally to the systematics in the N-body simulations. For CAMIRA clusters, we evaluate \( b_N \) following the Tinker et al. (2010) formula as a function of cluster mass, which is mainly determined by the normalization \( A_N \) of the \( N-M \) relation in the forward-modeling of halo clustering. Conversely, the mock clusters are selected by the richness that is assigned by the \( N-M \) relation with the normalization \( A_N \) calibrated against lensing magnification. Thus, the difference in the amplitude of \( \xi_{\text{CC}} \) between the mocks and the observation reflects the offset in the absolute mass scale of CAMIRA clusters inferred between lensing magnification and halo clustering.

It is worth mentioning that degeneracy between \( A_N \) and \( b_{\text{CMASS}} \) seen in the modeling of \( \xi_{\text{CC}} \) alone (red in Figure 8) is broken by including the modeling of \( \xi_{\text{CC}} \) (green contours). On the other hand, the inclusion of the CMASS auto-correlation (brown in Figure 8) does not significantly improve the constraint on \( A_N \) but only on \( b_{\text{CMASS}} \), as opposed to the case of \( \xi_{\text{CC}} + \xi_{\text{gg}} \) (green contours).

This picture can be highlighted in Figure 11, where we show the constraints of \( A_N \) and \( b_{\text{CMASS}} \) in the default analysis obtained from the modeling of \( \xi_{\text{CC}} \) (blue), \( \xi_{\text{CC}} + \xi_{\text{gg}} \) (red), and \( \xi_{\text{CC}} + \xi_{\text{gg}} + \xi_{\text{gg}} \) (green). We find that the uncertainty of \( A_N \) is decreased by \( \approx 30\% \) by including \( \xi_{\text{gg}} \) into the modeling of \( \xi_{\text{CC}} \). However, including the auto-correlation of the CMASS sample into the joint modeling of \( \xi_{\text{CC}} + \xi_{\text{gg}} \) does not significantly improve the constraint on \( A_N \). Based on the joint modeling of \( \xi_{\text{CC}} + \xi_{\text{gg}} + \xi_{\text{gg}} \), we obtain the constraint on \( A_N \) as \( 11.9 \pm 3.0 \) with an average precision at a level of \( \approx 21\% \). This constraining power on \( A_N \) is comparable to that from lensing magnification alone (Chiu et al. 2020), which has an uncertainty of \( \approx 15\% \) on \( A_N \). In Figure 11, we additionally show the constraint on \( A_N \) inferred from lensing magnification (\( A_N = 17.72 \), dashed line; Chiu et al. 2020) and from the results of Murata et al. (2019) using a joint analysis of cluster abundance and weak shear in the cosmology fixed to that anchored by WMAP9 (\( A_N = 17.40 \), dotted line) and Planck (\( A_N = 13.70 \), dotted-dashed line). We stress that both Chiu et al. (2020) and Murata et al. (2019) studied CAMIRA clusters with the same selection (i.e., \( 0.2 \leq z_{\text{d}} \leq 1.1 \) and \( N \geq 15 \)), which thus enables a direct comparison in this work. We find that the constraint on \( A_N \) using halo clustering are broadly lower than, but statistically consistent with, those inferred from lensing magnification and from a joint analysis of weak shear and cluster abundance at a level of \( \lesssim 1.9\sigma \), with slight preference for the latter in the Planck cosmology. That is, the clustering-inferred mass scale at fixed richness is higher than those inferred from the independent methods of gravitational lensing and cluster abundance, but not at a statistically significant level (\( \lesssim 1.9\sigma \)).

It is worth noting that the projection effect arising from optical cluster finding algorithms could result in biased lensing signals in the one-halo regime, as suggested by Sunayama et al. (2020). However, the bias in lensing signals of the one-halo term has a monotonic trend from \( \lesssim -5\% \) to \( \lesssim 5\% \) with increasing richness (see Figure 4 in Sunayama et al. 2020). Therefore, to first-order approximation, this bias over all clusters in the one-halo term would be averaged out, which is not expected to significantly affect the comparison between the weak-lensing and clustering results. We
will continue to discuss the impact of the projection effect on large-scale clustering in Section 7.

We further note that our constraints on \( A_N \) depend on cosmological parameters, especially \( \sigma_8 \). This is because the amplitude of clustering strength is proportional to \( (b_0 \sigma_8)^2 \), in which \( \sigma_8 \) is fixed to the default value of 0.8 in this work. This value is different from that used in the joint analysis of cluster abundance and weak shear in Murata et al. (2019), where \( \sigma_8 = 0.82 \) (\( \sigma_8 = 0.831 \)) is used in the WMAP (Planck) cosmology. Changing \( \sigma_8 \) to 0.82 (0.831) anchored by the WMAP (Planck) cosmology results in a reduction of the halo bias at a level of 1 − 0.80/0.82 ≈ 2.4% (1 − 0.80/0.831 ≈ 3.7%), implying a mass scale smaller by 5.7% (8.7%) at the pivotal mass \( M_{\text{piv}} = 10^{14} h^{-1} \text{M}_\odot \), and the pivotal redshift \( z_{\text{piv}} = 0.6 \) assuming the Tinker et al. (2010) relation. Since \( A_N = M_{500}^{Bz} \) gives an observed richness, the mass scale smaller by 5.7% (8.7%) corresponds to an increase in the inferred \( A_N \) by (1 − 5.7%)^−0.9 ≈ 5.4% ((1 − 8.7%)^−0.9 ≈ 8.5%) if changing \( \sigma_8 \) to the value anchored by the WMAP (Planck) cosmology. That is, our results would be in better agreement with those from Murata et al. (2019) if accounting for the different \( \sigma_8 \) used in both analysis. We therefore conclude that the self-calibration of CAMIRA clusters based on halo clustering infers an absolute mass scale that is consistent with those estimated from lensing magnification, weak shear and cluster abundance (within \( \lesssim 1.9 \sigma \)).

7 COMMENTS ON THE SELECTION OF THE CAMIRA CLUSTERS

The recent work Sunayama et al. (2020) has demonstrated that red-sequence based cluster finders introduce the selection bias to preferentially select galaxy clusters locating at filaments aligning along the line of sight. This selection bias results in a strong anisotropic pattern in the underlying mass distribution of optically detected clusters on large scale, which ultimately gives boosts to observed lensing signals and the strength of halo clustering compared to the theoretical prediction assuming an isotropic distribution.

In terms of halo clustering, this selection bias leads to a significant quadrupole moment in the 2D correlation function. In comparison to the halo clustering assuming an isotropic distribution, halos with this selection bias tend to over-cluster (under-cluster) in the direction along (perpendicular to) the line of sight, as seen in Figure 13 of Sunayama et al. (2020). As a result, there exists a scale-dependent bias in a projected correlation function, in which enhancements of \( \approx 60\% \) and \( \approx 10\% \) are expected at the projected radius of \( R \approx 10 h^{-1} \text{Mpc} \) and \( R \approx 50 h^{-1} \text{Mpc} \) respectively, compared to the case without the selection bias. A similar picture is implied for lensing signals, where an enhancement up to \( \approx 20\% \) is expected in the two-halo term regime.

Since the CAMIRA clusters studied in this work are selected based on a red-sequence finding algorithm, we do expect that such a selection bias exists in our sample. However, in this work we measured the monopole moment of 3D correlation functions of halos, instead of projected correlation functions as investigated in Sunayama et al. (2020). Thus, the selection bias is expected to be less significant on our results. This is because the 3D correlation function is an azimuthal average of the 2D correlation function, such that the effect arising from the quadrupole pattern is significantly alleviated for the 3D correlation function (see a more quantified discussion below).

Motivated by Sunayama et al. (2020), we construct a toy model to quantify the effect raised from this selection bias on the 3D correlation function. We assume that the quadrupole pattern of the 2D correlation function only depends on the cosine angle \( \mu_c \),
such that the logarithmic ratio of the clustering strength between the observed clusters and the isotropic prediction, \( \xi_{\text{obs}}/\xi_{\text{true}} \), follows the relation, \( \log_{10}(\xi_{\text{obs}}/\xi_{\text{true}}) = 0.5 \times P_2(\mu_b) \), where \( P_2(\mu_b) \) is the 2-th order Legendre polynomial. This normalization of 0.5 is roughly consistent with what is implied in Sunayama et al. (2020, see their Figure 13). Then, the integration of \( 10^{0.5P_2(\mu_b)} \) from \( \mu_b = 0 \) to \( \mu_b = 1 \) gives the ratio of \( \xi_{\text{obs}}/\xi_{\text{true}} \approx 1.15 \). That is, the clustering strength of the 3D correlation function of CAMIRA clusters could be biased high at a level of \( \approx 15\% \) compared to that from an isotropic prediction. This leads to an overestimated halo bias at a level of \( \approx \sqrt{1.15} \approx \approx 7\% \). To first-order approximation, this result in the cluster mass biased high by \( \approx 17\% \) at the pivotal mass of \( M_{\text{pivot}} = 10^{15} h^{-1} M_\odot \), assuming that the linear halo bias follows the Tinker et al. (2010) relation at the pivotal redshift \( z_{\text{pivot}} = 0.6 \). Because of \( A_N \propto M_{500} - B_N \approx M_{500} - 0.9 \) given an observed richness, this corresponds to a normalization \( A_N \) that is biased low by \( \approx 1\% \). If we change the normalization of \( \log(\xi_{\text{obs}}/\xi_{\text{true}}) \) to 0.3, then this results in a normalization \( A_N \) biased low by \( \approx 6\% \). With this toy model to characterize the effect from the selection bias on the 2D correlation function, the normalization \( A_N \) is suggested be biased low by an amount smaller than the current statistical uncertainty, which is at a level of \( \approx 36\% \) \(( \approx 21\% \) for the modeling of \( \xi_{cc} \) \( (\xi_{cg} + \xi_{gg}) \)). Therefore, we conclude that the selection bias would not significantly alter the interpretation of this work. However, this selection bias needs to be further quantified in detail if a larger sample of optically detected clusters is studied in future work.

In what follows, we provide two final remarks. First, in this work we are measuring the 3D correlation function in the redshift-space, in which the photo-z uncertainty of CAMIRA clusters would significantly smear out the clustering strength along the line of sight. This is a distinct difference to Sunayama et al. (2020), where the interpretation is based on projected correlation functions, in which the effects of FoG and RSD are canceled out by integrating along the line of sight. Moreover, there is no photo-z uncertainty present in Sunayama et al. (2020). The toy model above does not account for the photo-z uncertainty, which would mitigate the anisotropic pattern in the 2D correlation function and thus result in an even less impact on \( A_N \). Therefore, a quantitative comparison between Sunayama et al. (2020) and this work is not trivial. Second, the interpretation of Sunayama et al. (2020) is based on the redMaPPer cluster finder that uses an algorithm different from our CAMIRA finder. Specifically, the redMaPPer algorithm uses a richness-dependent cutout radius to calculate the observed richness for each cluster in an iterative manner (Rykoff et al. 2014), while the cutout radius in the CAMIRA algorithm is fixed at a given redshift (Oguri 2014). It is also important to note that the redMaPPer cluster finder conducts the global background subtraction in calculating the observed richness, while the local background subtraction is used in the CAMIRA algorithm to account for local variations in large-scale structures around clusters. These differences introduce a difficulty in quantifying the selection bias of CAMIRA clusters based on the redMaPPer results (Murata et al. 2020). A dedicated simulation to quantify the selection bias of CAMIRA clusters is warranted for future work.

![Figure 10. Auto- and cross-correlation functions in the default analysis. The observed correlation functions (best-fit models) of \( \xi_{cc}, \xi_{cg}, \) and \( \xi_{gg} \) are shown by the blue, red, and green circles (solid, dashed, and dotted lines), respectively. The mean values of \( \xi_{cc}, \xi_{cg}, \) and \( \xi_{gg} \) among the 432 mocks are marked by the blue, red, and green shade regions, respectively.](image-url)

![Figure 11. Constraints on the normalization \( A_N \) of the N–M relation and the linear halo bias \( b_{\text{CMASS}} \) of CMASS galaxies. We show the constraints obtained in the default analysis using the modeling of \( \xi_{cc}, \xi_{cg}, \) and \( \xi_{gg} \) in blue, red, and green, respectively. These modeling are carried out with an informative prior \( \phi \{1.93, 0.175\} \) on \( b_{\text{CMASS}} \), which is adopted from the BAO result (Chuang et al. 2013), as shown by the black solid line. This effectively marginalizes over the systematic uncertainty of \( b_{\text{CMASS}} \) in the modeling. Additionally, we show the constraints on \( A_N \) that are independently obtained from lensing magnification (Chiu et al. 2020, dashed line) and from a joint analysis of cluster abundance and weak shear from Murata et al. (2019) with the cosmological parameters fixed to those anchored by the WMAP9 (dotted line) and the Planck (dotted-dashed line) results.](image-url)
8 CONCLUSIONS

In this work, we have measured (1) the auto-correlation function of CAMIRA clusters, \( \xi_{cc} \), with richness \( N \geq 15 \) at \( 0.2 < z < 1.1 \), which are constructed using the HSC survey, (2) the auto-correlation function of CMASS galaxies, \( \xi_{gg} \), which are spectroscopically observed in the BOSS, and (3) the cross-correlation function of these samples, \( \xi_{cg} \). These correlation functions are measured in redshift space. Based on these clustering measurements, we carried out a forward-modeling approach to calibrate the \( N-M \) relation of CAMIRA clusters, accounting for the effects of the RSD, FoG, and the photo-\( z \) uncertainty of CAMIRA clusters. We also take into account the projection effect of the CAMIRA sample on the cluster halo bias. The modeling is shown to deliver unbiased constraints on the parameters by validation tests against a large set of the mock catalogs, which are carefully constructed from N-body simulations to mimic the observed properties of the CAMIRA and CMASS samples.

We focus on constraining the normalization \( A_N \) of the \( N-M \) relation, as an absolute calibration of the cluster mass scale, while applying informative priors on other parameters. In the modeling of \( \xi_{cc} (\xi_{cc} + \xi_{cg} + \xi_{cg}) \), we obtain the constraint of \( A_N = 13.8^{+5.8}_{-4.5} (13.2^{+3.4}_{-2.7}, 11.9^{+3.0}_{-1.9}) \) with an average uncertainty at a level of \( 36\% \) (23\%, 21\%). We also carry out a subsample analysis to model the correlation functions in different richness and redshift bins, returning results consistent with the default analysis, given the errorbars. The self-calibration based on halo clustering alone results in an uncertainty in \( A_N \) that is comparable to that independently obtained by lensing magnification (Chiu et al. 2020), which has an uncertainty at a level of \( \approx 15\% \).

We compare the resulting constraints on \( A_N \) to those inferred from lensing magnification (Chiu et al. 2020) and from a joint analysis of cluster abundance and weak shear (Murata et al. 2019). We find that \( A_N \) constrained by halo clustering alone is statistically consistent with the results inferred from those independent methods, with a preference for a lower \( A_N \) (or a higher cluster mass scale) at a level of \( \lesssim 1.9\sigma \). Meanwhile, the constraint on the linear halo bias \( b_{\text{CMSS}} \) of the CMASS sample is in agreement with the N-body simulations and the observational constraint independently obtained from the BAO collaboration (Chuang et al. 2013), given the uncertainty.

We discuss the effect arising from the selection bias of CAMIRA clusters, in light of the recent work of Sunayama et al. (2020). We use a simple model to characterize the anisotropic distribution of halo clustering introduced by the selection bias, and assess the potential systematics on redshift-space correlation functions, as studied in this work. According to this model, the normalization \( A_N \) is suggested to be underestimated by an amount of \( \approx 13\% \), to first-order approximation. This amount is subdominant compared to the current statistical uncertainty at a level between 21\% and 36\%, depending on the data sets used in the modeling. A detailed investigation specifically for the CAMIRA sample is needed if a larger sample is studied in future work. We also investigate the systematics raised from the cluster random catalog, which is a subdominant factor to our interpretation in this work.

To sum up, we have shown that the halo clustering of galaxy clusters provides a competitive method in self-calibrating the cluster mass. By modeling the photo-\( z \) uncertainty, the redshift-space correlation functions result in a more precise and accurate measurement than the projected or angular correlation functions. In this work, the clustering-based self-calibration delivers the constraint on the normalization of the \( N-M \) relation with a competitive uncertainty of \( \approx 36\% \), by only using \( \approx 3k \) clusters over a footprint with area of \( \approx 400 \text{ deg}^2 \). Including a spectroscopic sample in a joint analysis of halo clustering is able to improve the uncertainty to \( \approx 21\% \). It is also worth mentioning that clustering analysis is less sensitive to the incompleteness of a tracer sample (Guo et al. 2018). Therefore, this paper provides an attractive method of mass calibration for cluster cosmology, paving a way forward with the upcoming large and uniform imaging and spectroscopic surveys (e.g., LSST, DESI, and PFS).

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APPENDIX A: THE PARAMETER CONSTRAINTS

We repeat the analysis in our subsample analysis, in which we bin the parameter constraints obtained in the subsample analysis in Figure A1 and those without the Gaussian priors (see Section 5) on $B_N$ and $c_N$ in the subsample analysis in Figure A2.

APPENDIX B: SYSTEMATICS DUE TO THE CLUSTER RANDOM CATALOG

In this section, we assess the systematics introduced by the random catalog of the CAMIRA clusters. The construction of the random catalog for the CAMIRA clusters consists of two phases: the randomization of (1) the angular distribution and (2) the redshift distribution. For the former, the default analysis is done using the random catalog based on the aperture with the fixed angular size. For the latter, the default analysis is done using the random catalog at the fixed redshift, we re-measured $\xi_{cc}$ of all the CAMIRA clusters with the random cluster catalogs produced at redshift of 0.26 and 0.89, as shown by the red and blue circles in Figure B1, respectively. As seen in Figure B1, the resulting $\xi_{cc}$ is insensitive to the choice of redshift where the random catalog is produced, given the errorbars. We therefore conclude that the chosen redshift in generating the angular distribution of the cluster random catalog is a subdominant factor in our analysis. For randomization of the redshift distribution, we use the redshifts “shuffled” from the observed clusters (i.e., bootstrapping the redshift estimates from the CAMIRA cluster catalog). We also assess the systematics raised from the random distribution of the redshifts in the similar way as done in Section 6 of Ross et al. (2012). Specifically, we alternatively assign the redshift estimate to each point in the random catalog following the observed redshift distribution of the CAMIRA clusters after the smoothing using a Gaussian kernel. We use a Gaussian kernel with a dispersion of 0.009, which is the observed dispersion in the photo-$\gamma$ uncertainty (see Section 5), to convolve the redshift distribution of the CAMIRA clusters with $N \geq 15$ derived using a redshift step of 0.002. The size of the redshift step is chosen in order to have enough sampling to resolve one interval of the redshift dispersion. The resulting $\xi_{cc}$ is shown by the blue circles in Figure B1. Given the errorbars, the difference to the default analysis (black circles) is negligible. To sum up, the interpretation of this work thus remains intact from the systematics introduced from the random cluster catalogs.

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Figure A1. Parameter constraints obtained in the subsample analysis. This plot is generated in the same configuration as in Figure 6.
Figure A2. Parameter constraints obtained in the subsample analysis without the Gaussian priors on the mass- and redshift-trend power-law indices of the $N$–$M$ relation. This plot is generated in the same configuration as in Figure 6.
Figure B1. Comparison of auto-correlation functions $\xi_{cc}$ of CAMIRA clusters with $N \geq 15$ at $0.2 \leq z < 1.1$ among different random catalogs. The black circles are the result of the default analysis, with the random catalog generated at the redshift of 0.56. The results using the cluster random catalogs generated at redshift of 0.26 and 0.84 are shown as the red and blue circles, respectively. The resulting $\xi_{cc}$ using the random catalog with the smoothed redshift distribution of observed CAMIRA clusters is indicated by the purple circles (see Section B for more details). The difference among these results is negligible compared to the size of current errorbars.