Inflation driven by Einstein-Gauss-Bonnet gravity

Sumanta Chakraborty *, Tanmoy Paul † and Soumitra SenGupta ‡

*1Department of Theoretical Physics, Indian Association for the Cultivation of Science, Kolkata-700032, India

April 10, 2018

Abstract

We have explicitly demonstrated that scalar coupled Gauss-Bonnet gravity in four dimension can have non-trivial effects on the early inflationary stage of our universe. In particular, we have shown that the scalar coupled Gauss-Bonnet term alone is capable of driving the inflationary stages of the universe without incorporating slow roll approximation, while remaining compatible with the current observations. Subsequently, to avoid the instability of the tensor perturbation modes we have introduced a self-interacting potential for the inflaton field and have shown that in this context as well it is possible to have inflationary scenario. Moreover it turns out that presence of the Gauss-Bonnet term is incompatible with the slow roll approximation and hence one must work with the field equations in the most general context. Finally, we have shown that the scalar coupled Gauss-Bonnet term attains smaller and smaller values as the universe exits from inflation. Thus at the end of the inflation the universe makes a smooth transition to Einstein gravity.

1 Introduction

Einstein’s general relativity describes the gravitational interaction in its simplest form. Since viability of any theory is based on its falsifiable predictions and consistency with existing observations, one can safely argue that general relativity is the most viable theory of gravitation till date. This is mainly due to the fact that so far general relativity has passed the experimental tests with flying colours [1–4]. However, as it is necessary for advancement of theoretical sciences, despite its enormous successes, general relativity is also riddled with many open questions. These are scattered across various length scales and include — (a) Behaviour of gravity at small length scales and possible existence of extra dimensions [5–9], (b) The strong gravity regime and the breakdown of predictability near the black hole singularity [10, 11] and finally (c) The accelerated expansion of the universe as well as early inflationary epoch and big bang singularity in the context of cosmology [12–21]. Each of these scenarios are interesting, with broader implications, in their own respect. However in this work we will concentrate on cosmological dynamics in the early stages of the universe. In this particular context there exists several issues among which, flatness of the universe at a large scale, uniformity of the temperature of Cosmic Microwave Background in super-horizon scales are some of the important ones. These problems are believed to be answered in one way or another by the introduction of various inflationary models of our universe [12–15, 22, 23]. According to the standard
inflationary paradigm, in the very early stages the universe went through an exponentially accelerating expansion, which later on starts to decelerate and makes path for the standard cosmological epochs. One of the most popular attempt to achieve the same is by considering a scalar field with a self-interacting potential sourcing gravity and assuming that the scalar field satisfies the “slow-roll” condition (i.e., kinetic energy of the scalar field is very much less than the potential energy) [24–31] (however also see [32, 33]). Therefore most of the inflationary paradigms are driven by a scalar field with a non-trivial self-interacting potential.

Apart from the above there can be several alternative scenarios as well. One of the most important and natural alternative corresponds to introduction of higher curvature terms. This is expected, since at the early stages of the universe the spacetime curvature is expected to be at the Planck Scale and hence general relativity must be supplemented by additional higher curvature terms. Currently these terms are highly suppressed and therefore cannot affect our local physics. Such a higher curvature generalization may correspond to introduction of $f(R)$ gravity [34–42], which is closely related to Einstein gravity with a scalar field [43–50] and has the ability to explain the inflationary paradigm as well as late time acceleration while remaining consistent with observations [51–58]. Besides the above success story of $f(R)$ gravity, it is also riddled with the Ostrogradsky instability [59]. This is because the gravitational field equations involve more than second derivatives of the metric. It is therefore legitimate to ask whether there exist any other viable higher curvature theories that can provide a consistent description of inflationary paradigm, while avoiding the Ostrogradsky instability. One of the most natural choice corresponds to the Lanczos-Lovelock theories of gravity and in particular the Gauss-Bonnet term. The most useful feature associated with any Lanczos-Lovelock theory is that, even though the action involves higher curvatures, the field equations are still of second order, which ensures that these theories are intrinsically ghost free [60–66]. The Gauss-Bonnet term is also of no exception and results into a second order field equation for gravity. Thus these Lanczos-Lovelock theories are very natural higher curvature corrections to the gravitational Lagrangian and can potentially affect the early universe inflationary scenario [67–70].

Unfortunately, in the context of four dimensional physics the Gauss-Bonnet term itself is a total divergence and hence cannot affect the dynamics of the universe in a non-trivial manner. To overcome this difficulty, one often introduces a scalar coupling to the Gauss-Bonnet invariant, since the scalar coupling makes the Gauss-Bonnet term (and hence the field equations) non-trivial [69–82]. This opens up new interesting possibilities in the early inflationary scenario of the universe and hence there have been several works where the coupling of the Gauss-Bonnet term with the scalar field has been explored. However in all these approaches either the inflationary paradigm is being explored only in the context of Gauss-Bonnet gravity excluding the Einstein term or the self-interacting potential itself governs the inflation, while the Gauss-Bonnet term contributes in the late time behaviour. For example, it has been demonstrated in [69] that a possible solution of inflationary de-Sitter phase and a natural exit mechanism through a linearly expanding Milne phase can be achieved by assuming that the Ricci scalar is sub-dominant with respect to the scalar coupled Gauss-Bonnet term at very early stages of the universe. Similarly, in [81] Einstein-Gauss-Bonnet gravity in presence of a scalar field has been explored, where the self-interacting potential drives the exponential inflation as well as exit with no reference to the Gauss-Bonnet term, which becomes effective only at late times resulting into an accelerated expansion of the universe. Thus non-trivial effects of the Gauss-Bonnet term along with the Ricci scalar on the inflationary paradigm has not been explored before.

In this paper, we will like to fill this gap by keeping the Ricci scalar, as well as the Gauss-Bonnet term coupled with a scalar field while describing the inflationary paradigm. We will also try to answer the question, whether such a scalar coupled Gauss-Bonnet term alone (of course, in presence of the Ricci scalar) is capable of driving the exponential expansion of the early universe, while remaining consistent
with the current observations. After answering the above in affirmative, we demonstrate that the scalar coupled Gauss-Bonnet gravity in absence of a self-interacting potential for the scalar field is plagued with the instability of the tensor perturbations. This forces us to introduce the self-interacting potential for the scalar field. In this context we have kept both the Gauss-Bonnet term as well as the self-interacting potential and have demonstrated that the theory supports two different sets of analytic solutions for different choices of scalar field potential and coupling function of scalar field with the Gauss-Bonnet term. One set of solution corresponds to accelerating expansion of the universe and the other one corresponds to deceleration. Interpolating these two sets of self-interacting potential as well as coupling function with the Gauss-Bonnet term, we construct the full form of the scalar field potential and coupling function of the early universe numerically. Using the constructed form of potential as well as the coupling function to the Gauss-Bonnet term, we have finally solved the field equations numerically to arrive at a solution for the Hubble parameter and the scalar field. From these solutions, it is possible to trace over the whole inflationary epoch, which shows an initial de Sitter phase and a final deceleration phase effecting exit from inflation.

This paper is organized as follows — In Section 2 we preview the scalar coupled Einstein-Gauss-Bonnet gravity and the field equations thereof in the context of cosmology. We have also commented on the feasibility of the model without any self-interacting potential in the inflationary paradigm. Interestingly, in Section 3 we have explicitly demonstrated that it is indeed possible to have inflationary scenario without the self-interacting potential and also consistent with observations. While a possible source of instability of this model has been discussed in Section 4. In Section 5 we have introduced a scalar potential and have demonstrated that the theory supports two different sets of analytic solutions for different choices of scalar field potential and coupling function of scalar field with the Gauss-Bonnet term. These results have been used later to present numerical solutions of the Hubble parameter and the scalar field necessary for our purpose. Finally we finish the paper by providing some concluding remarks and future directions of exploration.

Notations and Conventions — Throughout this paper Greek indices have been used to represent four-dimensional quantities. The fundamental constants $c$ and $\hbar$ have been set to unity, while the Newton’s constant $G$ has been kept throughout. We have adopted the mostly positive signature.

2 Scalar coupled Einstein-Gauss-Bonnet gravity

We consider a scalar coupled theory of modified gravity, in which the scalar field is non-minimally coupled to the Gauss-Bonnet invariant $\mathcal{G} = R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R^{\alpha\beta\rho\sigma} R_{\alpha\beta\rho\sigma}$ in four dimensional spacetime. Therefore in the most general setting, the action for the scalar coupled Einstein-Gauss-Bonnet gravity consists of four terms — (a) The Ricci scalar, (b) The Gauss-Bonnet invariant coupled to an arbitrary function of the scalar field, (c) kinetic term of the scalar field and finally (d) a self-interaction term for the scalar field, such that

$$A = \int d^4 x \sqrt{-g} \left[ \frac{1}{16\pi G} \left( R - \xi(\Phi) \mathcal{G} \right) - \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - V(\Phi) \right],$$

where $R$ is the Ricci scalar obtained from the metric $g_{\mu\nu}$, $\Phi$ is the scalar field under consideration and $\mathcal{G}$, defined earlier, is the Gauss-Bonnet invariant. The non-topological character of the Gauss-Bonnet term in the above action is ensured by the coupling function between the scalar field and the Gauss-Bonnet term, symbolized by $\xi(\Phi)$. One possible origin of the term $\xi(\Phi) \mathcal{G}$ is from the compactification of a higher dimensional spacetime to an effective four dimensional description, where $\Phi$ plays the role of the
radion field. The above can be argued along the following lines, in the high energy limit and in higher
spacetime dimensions, the Gauss-Bonnet term is a natural candidate besides the Einstein-Hilbert term in
the gravitational action. Given the higher dimensional action it is customary to derive an effective four
dimensional action by integrating out the contributions from extra dimensions. This generically results into
potential for the inter-brane separation (known as the radion field), which in presence of the Gauss-Bonnet
term must couple to the four dimensional Gauss-Bonnet invariant \( G \) in the Einstein frame, resulting into
the desired term in the four dimensional gravitational action.

Variation of the above action, presented in Eq. (1), with respect to the metric and the scalar field
results into the following field equations for gravity and the scalar field individually,

\[
G_{\mu\nu} + \frac{1}{2} g_{\mu\nu} \xi(\Phi) G - 2 \xi(\Phi) \left[ RR_{\mu\nu} - 2 R_{\mu\rho} R^\rho_{\nu} + R_{\mu}^{\rho\sigma\tau} R_{\nu\rho\sigma\tau} - 2 R_{\mu\rho\sigma\tau} R^{\rho\sigma\tau}\right] \\
+ 2 \{ \nabla_{\mu} \nabla_{\nu} \xi(\Phi) \} R - 2 g_{\mu\nu} \{ \nabla^2 \xi(\Phi) \} R - 4 \{ \nabla_{\rho} \nabla_{\mu} \xi(\Phi) \} R^\rho_{\nu} - 4 \{ \nabla_{\rho} \nabla_{\nu} \xi(\Phi) \} R^\rho_{\mu} \\
+ 4 \{ \nabla^2 \xi(\Phi) \} R_{\mu\nu} + 4 g_{\mu\nu} \{ R^{\rho\sigma} \nabla_{\rho} \nabla_{\sigma} \xi(\Phi) \} + 4 \{ \nabla^\rho \nabla^\sigma \xi(\Phi) \} R_{\mu\rho\sigma} \\
= 8\pi G \left[ \nabla_{\mu} \Phi \nabla_{\nu} \Phi - g_{\mu\nu} \left\{ \frac{1}{2} \nabla_{\rho} \Phi \nabla^{\rho} \Phi + V(\Phi) \right\} \right].
\]

\( (2) \)

\[
\Box \Phi - \left( \frac{\partial \xi}{\partial \Phi} \right) \frac{G}{16\pi G} - \frac{\partial V}{\partial \Phi} = 0.
\]

\( (3) \)

As expected, the gravitational field equations do not contain more than second order derivatives of the
metric and hence is intrinsically ghost free, resulting into stability of the model. At this stage, there are
two possibilities where one may apply the above general analysis. The first one corresponds to the effect of
the \( \xi(\phi)G \) term in the black hole physics, where higher curvature terms may become important. This has
been studied extensively in the literature in the past few decades in various contexts, starting from hairy
black hole solutions to their observational implications in various astrophysical processes [83–88]. The
second arena to search for the effect of scalar coupled Einstein-Gauss-Bonnet gravity is in the inflationary
paradigm, where also the higher curvature effects are supposed to be important [67–70].

In this work we will concentrate on the inflationary paradigm and hope to understand the same in the
context of scalar coupled Gauss-Bonnet gravity. Even though there exists several works in this territory,
however we will try to address the same from a different perspective altogether. In particular, we will first
try to understand the inflationary paradigm in the premise of \( \xi(\Phi)G \) term alone, without the potential
term \( V(\Phi) \) for the scalar field (also known as the inflaton field) in the action. In particular, we will
examine whether the inflationary era of the early universe can be triggered due to the presence of the
non-trivial coupling of the scalar field with the Gauss-Bonnet term alone, without any self-interacting
inflaton potential. Keeping this mind, we note that the inflationary paradigm requires the background
spacetime to be described by a homogeneous and isotropic metric, which takes the following form,

\[
ds^2 = -dt^2 + a^2(t) \{ dx^2 + dy^2 + dz^2 \},
\]

\( (4) \)

where the scale factor \( a(t) \) solely governs evolution of the spacetime structure. For such a metric, the
expression for the Ricci scalar \( R \) and the Gauss-Bonnet invariant \( G \) can be easily computed, which results
into,

\[
R = 6 \left( 2H^2 + \dot{H} \right); \quad G = 24H^2 \left( H^2 + \dot{H} \right),
\]

\( (5) \)

with \( H = \dot{a}/a \) and ‘dot’ denotes derivative of the respective quantity with respect to time. In order to
be consistent with the symmetry of the background spacetime it is necessary that the inflaton field be
dependent on the time coordinate alone, i.e., $\Phi = \Phi(t)$. Finally, using the expressions for the Ricci scalar and the Gauss-Bonnet invariant from Eq. (5), along with the Ricci and Riemann tensor for the spacetime metric presented in Eq. (4), the field equations can be simplified and they turn out to be,

$$3H^2 - 12H^3 \dot{\xi} = 8\pi G \left( \frac{1}{2} \dot{H}^2 \right) ;$$  \hspace{1cm} (6)

$$2\dot{H} - 4H^2 \left[ \ddot{\xi} - H \dot{\xi} + 2 \frac{\dot{H}}{H} \dot{\xi} \right] = -8\pi G \dot{\Phi}^2 ;$$  \hspace{1cm} (7)

$$\ddot{\Phi} + 3H \dot{\Phi} + \frac{12H^2}{8\pi G} \left( H^2 + \dot{H} \right) \frac{\partial \xi}{\partial \Phi} = 0 .$$  \hspace{1cm} (8)

It is evident that due to the presence of the Gauss-Bonnet term, cubic as well as quartic powers of $H(t)$ appear in the above field equations. Further due to Bianchi identity and conservation of matter energy momentum tensor, all the three field equations presented above are not independent, but one of them can be derived from the other two. For example, one can derive Eq. (7) by differentiating Eq. (6) with respect to the time coordinate and then using Eq. (8) to replace $\ddot{\Phi}$. Similarly, using Eq. (6) and Eq. (7) it is possible to derive Eq. (8) as well.

Moreover, it is well known that Einstein-Gauss-Bonnet gravity in 4-dimensions reduces to standard Einstein gravity, the additional terms being topological in nature. In the present case as well, if the function $\xi(\Phi)$ is assigned a constant value (essentially no coupling), the above field equations would immediately reduce to the corresponding field equations in the Einstein gravity since the contribution from the Gauss-Bonnet term becomes trivial. This shows the internal consistency of this model.

The best way to describe the inflationary paradigm is through the slow-roll approximation imposed on the scalar field, which requires $\dot{\varphi}^2 \ll \dot{\varphi}$ and $\ddot{\varphi} \ll \dot{\varphi}$. Under these approximations the gravitational field equation for the scale factor $a(t)$, presented in Eq. (6), simplifies considerably and it becomes possible to solve for $\dot{\varphi}$, yielding

$$\dot{\varphi} = \frac{1}{4H} \left( \frac{\partial \xi}{\partial \varphi} \right)^{-1} .$$  \hspace{1cm} (9)

On the other hand, the field equation for the scalar field, as in Eq. (8), under the slow-roll approximation equates $3H \dot{\Phi}$ to $\dot{H} + H^2$. Therefore by substituting the expression for $\dot{\varphi}$ from Eq. (9) one immediately obtains the following result for $\dot{H} + H^2$,

$$\dot{H} + H^2 = -\frac{\pi G}{2H^2} \left( \frac{\partial \xi}{\partial \varphi} \right)^{-2} .$$  \hspace{1cm} (10)

The above expression explicitly shows that $\ddot{a}/a = \dot{H} + H^2$ is a negative quantity, since neither $H$, nor $(\partial \xi / \partial \Phi)$ are imaginary. The above result ensures that under slow-roll approximation, it is impossible to arrive at an inflationary solution for our universe in this context. One would therefore tend to introduce a self-interacting potential term to achieve the desired slow-roll inflation. However, we will show that even in the absence of such a self-interacting potential one can still have inflationary solutions compatible with current observations without going into the slow-roll approximation. This is what we will elaborate in the next section.
3 Inflation without a self-interacting potential

This section is devoted to the study of inflationary paradigm in the absence of self-interacting potential, but with a non-minimal coupling of the scalar field with Gauss-Bonnet invariant. As we have argued before, the slow-roll approximation can not lead to an inflationary paradigm and hence we would now like to go beyond this approximation. To set the stage, let us first ask whether it is possible to have any solution for $\xi(\Phi)$ with constant Hubble parameter in absence of potential term for the inflaton field. If this can be achieved then only one can proceed further and try to obtain a complete inflationary scenario which is compatible with the current observational constraints.

3.1 Possibility for constant Hubble parameter

In this section we will concentrate on the possibility of having constant Hubble parameter (i.e., $H(t) = H_0 = \text{constant}$), which is consistent even without the potential term for the inflaton field. In other words, we have to use the fact that Hubble parameter is constant, in the field equations for gravity as well as the scalar field and then inspect whether a non-trivial solution for $\xi(\Phi)$ can be obtained. Keeping this in mind, we rewrite Eq. (6) and Eq. (7) in the following manner,

\begin{align}
3H_0^2 - 12H_0^3 \dot{\xi} &= 8\pi G \left( \frac{1}{2} \dot{\Phi}^2 \right) ; \\
4H_0^2 \left( \ddot{\xi} - H_0 \dot{\xi} \right) &= 8\pi G \dot{\Phi}^2 .
\end{align}

Given the above equations one can eliminate the $\dot{\Phi}^2$ term from both of them and obtain the following second order differential equation for $\xi(t)$,

$$2 \ddot{\xi} + 10H_0 \dot{\xi} - 3 = 0.$$  

It is straightforward to solve for $\xi(t)$ given the above equation and it turns out to be,

$$\xi(t) = \frac{1}{5H_0} \left[ \frac{3}{2} t + Ae^{-5H_0 t} \right] + B ,$$

where $A$ and $B$ are constants of integration. The above solution for $\xi(t)$ when substituted in Eq. (11) immediately leads to the following first order differential equation for $\Phi(t)$,

$$8\pi G \ddot{\Phi}^2 = 24AH_0^3 e^{-5H_0 t} - \frac{6H_0^2}{5} .$$

The above equation can be readily integrated yielding the following solution for the inflaton field $\Phi(t)$ as,

$$\sqrt{8\pi G} \Phi(t) = \frac{2\sqrt{6}}{5\sqrt{3}} \tan^{-1} \left( \sqrt{20AH_0 e^{-5H_0 t} - 1} - \sqrt{20AH_0 e^{-5H_0 t} - 1} \right) .$$

Note that in order to have a real solution it is of utmost importance to have $A > 0$, otherwise the term within the square root will turn negative. For $A > 0$ one will have non-trivial time evolution for the inflaton field as well as for the coupling $\xi(\Phi)$ as evident from Eq. (15). Therefore, the scalar coupled Einstein-Gauss-Bonnet gravity without any self-interaction term for the scalar field is capable of producing exponential expansion of the universe. However there is one major shortcoming of the above result, namely it does not predict when the inflation will end. It is easy to determine from Eq. (15) that after a time
\( t \equiv t_f = (1/5H_0) \ln(20AH_0) \) the \( H = H_0 \) = constant solution is no longer valid. However the model can not explain any natural mechanism to exit from the inflation before \( t = t_f \). Therefore, in order to describe the inflationary era of the early universe consistently it is necessary for the inflation to end and the duration of inflation, represented by the number of e-foldings, must be in consonance with the recent Planck observations.

3.2 Inflation with an exit

In this section, we will demonstrate that it is indeed possible to have a proper inflationary phase in the early universe described by the scalar coupled Einstein-Gauss-Bonnet gravity without any self-interacting scalar potential. For this purpose, we first consider the simpler scenario presented in Section 3.1. As evident from Eq. (13) and Eq. (15) it is not possible to write \( \xi = \xi(\Phi) \) in a closed form, since the solution for \( \Phi(t) \) is a transcendental equation. Therefore, in the more general context we should not expect a simple closed form expression for the coupling function \( \xi(\Phi) \).

Given this difficulty, we will employ the well known reconstruction scheme in order to arrive at a viable inflationary model in the present context [46,89–91]. For completeness we provide here a brief description of the same. In the reconstruction scheme one normally assumes that it is possible to learn about the expansion history of the universe in an exact manner and subsequently one inverts the field equations to deduce what class of modified theories can give rise to the desired cosmological epochs. The reconstruction scheme has been successfully applied in \( f(R) \) theories of gravity to determine the functional form for \( f(R) \), which can give rise to viable late time as well as inflationary cosmologies. The above method has also been used in the Galileon as well as Horndeski theories to determine unknown parameters, which will make these theories consistent with observations in the cosmological background [92–94]. Taking a cue from the above we have also employed the reconstruction scheme in order to determine viable choices for \( \xi(\Phi) \) which can provide a complete inflationary paradigm within the context of scalar coupled Einstein-Gauss-Bonnet gravity.

Let us work out the general methodology of the reconstruction method, which we will subsequently apply to some specific situations. As a first step, one starts with a particular ansatz for the time dependence of the Hubble parameter \( H(t) \) and ensures that it is consistent with the observational constraints, i.e., it predicts correct value of the scalar to tensor ratio and the power spectrum. Given the Hubble parameter, one can immediately eliminate \( \dot{\Phi} \) between Eq. (6) and Eq. (7) respectively. This results into the following second order differential equation for \( \xi(t) \)

\[
\ddot{\xi} + \left( 5H + 2\frac{\dot{H}}{H} \right) \dot{\xi} - \left( \frac{\dot{H}}{2H^2} + \frac{3}{2} \right) = 0 \tag{16}
\]

One can integrate the above equation by multiplying both sides by the integrating factor, which reads,

\[
\text{Integrating Factor} = \exp \left[ \int dt \left( 2\frac{\dot{H}}{H} + 5H \right) \right] \equiv e^P \tag{17}
\]

Therefore multiplying both sides of Eq. (16) by the integrating factor \( e^P \) one can immediately integrate the above second order differential equation for \( \xi(t) \) yielding,

\[
\dot{\xi}(t) = e^{-P(t)} \int dt' e^{P(t')} \left\{ \frac{\dot{H}(t')}{2H(t')^2} + \frac{3}{2} \right\} + C_1 e^{-P(t)} \tag{18}
\]
Finally integrating the above differential equation once again we arrived at,

$$\xi(t) = \int dt e^{-P} \int dt' e^{P(t')} \left\{ \frac{\dot{H}(t')}{2H(t')^2} + \frac{3}{2} \right\} + C_1 \int dt e^{-P(t)} + C_2$$

(19)

where $C_1$ and $C_2$ are constants of integration. Thus having derived the coupling function $\xi(t)$ the time evolution of the scalar field follows from the following differential equation

$$4\pi G \dot{\Phi}^2 = 3H^2 - 12H^3 \left[ e^{-P(t)} \int dt' e^{P(t')} \left\{ \frac{\dot{H}(t')}{2H(t')^2} + \frac{3}{2} \right\} + C_1 e^{-P(t)} \right]$$

(20)

At this stage, it deserves mentioning that at initial stages of the inflation, the Hubble parameter is almost constant and hence one may assume $H = H_0 = \text{constant}$. This situation has already been discussed in Section 3.1 and one may derive the relevant results by setting $\dot{H} = 0$ in Eq. (19) and Eq. (20) respectively.

So far, we have kept our discussion completely general and have not specified any particular choice for the Hubble parameter $H(t)$. The choice for the Hubble parameter cannot be arbitrary, as it must satisfy the following condition: at the onset of inflation the Hubble parameter must take nearly constant values. Further keeping in mind that a natural exit from the inflationary dynamics is necessary, here we propose a time dependent ansatz for the Hubble parameter as follows:

$$H(t) = \left[ c - d(t - t_*) \right]^\alpha,$$

(21)

where $c$, $d$ and $\alpha$ are the free parameters of the theory. The time scale $t_*$ is assumed to represent the onset of inflation and as evident from the above ansatz, for $t \sim t_*$ the Hubble parameter is almost constant with $H \sim e^\alpha$. Therefore at the beginning of inflation we have a very small value for $\dot{H}$ which will subsequently grow and will be order of the Hubble parameter requiring the inflation to end. Therefore, we may introduce a dimensionless variable $\epsilon(t)$ as $-\dot{H}/H^2$. From the previous discussion it is clear that $\epsilon \ll 1$ at the onset of inflation, while $\epsilon \sim 1$ as the inflation ends. This ensures that $\dot{H} + H^2 > 0$ throughout the course of inflation. Using the explicit form of the Hubble parameter $H(t)$ from Eq. (21), the parameter $\epsilon(t)$ can be computed such that,

$$\epsilon(t) = \alpha d \left\{ c - d(t - t_*) \right\}^{-\alpha - 1}.$$

(22)

Since the Hubble parameter and hence $e^\alpha$ is much larger than unity it follows that for $t \sim t_*$, $\epsilon$ is much smaller compared to unity. The above expression of $\epsilon(t)$ can also be used to determine the end of inflation as well. For this we assume that the exit time of inflation, i.e., $t_f$ is being determined by the equation $\epsilon(t_f) = 1$. This immediately leads to the following expression for $\Delta t = t_f - t_*$, corresponding to the duration of inflation as,

$$\Delta t \equiv t_f - t_* = \frac{1}{d} \left\{ c - (\alpha d)^{1(1+\alpha)} \right\}.$$

(23)

Moreover, Eq. (22) clearly reveals that $\epsilon(t)$ remains less than unity for $t_* < t < t_f$. Therefore the above ansatz for Hubble parameter can describe the evolution of the universe during inflationary epoch quite well. The parameter $\epsilon$ starts from a small value at $t \sim t_*$ and then grows to become order unity as $t \sim t_f$ and then the universe exits from inflation.
In order to be compatible with precision observations associated with the inflationary paradigm \[95,96\], it is crucial to compute various parameters of experimental interest, for which the number of e-foldings in the present context reads

\[ N \equiv \int_{t_*}^{t_f} H(t) dt = \frac{e^{\alpha+1}}{d(\alpha+1)} - \frac{(c - d(t - t_*))^{\alpha+1}}{d(\alpha+1)} . \] (24)

In order to arrive at the last line, the solution for \( H(t) \) from Eq. (21) has been used in order to perform the integral in the definition of the number of e-foldings. Substitution of the time span for inflation from Eq. (23) further simplifies the above expression and one finally obtains the number of e-foldings as follows:

\[ N = \frac{e^{\alpha+1}}{d(\alpha+1)} - \frac{\alpha}{\alpha + 1} . \] (25)

It turns out that the associated observables, namely the tensor to scalar ratio \( r \) and the spectral index of curvature perturbation \( n_s \) can be determined using the number of e-foldings and parameter \( \alpha \) appearing in the expression for Hubble parameter. Therefore, using Eq. (25) and Eq. (21) it is straightforward to determine the tensor to scalar ratio and the spectral index of curvature perturbations as,

\[ r = 16\epsilon(t_*) = 16 \left[ N \left( \frac{\alpha+1}{\alpha} + 1 \right) \right]^{-1} ; \] (26)

\[ n_s = 1 - 2\epsilon(t_*) - \frac{\dot{\epsilon}}{H\epsilon} \bigg|_{t_*} = 1 - \frac{(3\alpha + 1)}{(\alpha + 1)} \left\{ N + \frac{\alpha}{\alpha + 1} \right\}^{-1} . \] (27)

In order to derive Eq. (26) and Eq. (27) respectively, we have used the expression for the number of e-foldings that has been obtained in Eq. (25). From current precession cosmology one has the following bounds on the tensor to scalar ratio \( r \) and the spectral index of curvature perturbation \( n_s \): \( n_s = 0.968 \pm 0.006 \) and \( r < 0.14 \) respectively. The above constraints essentially originate from the joint analysis of temperature cross correlations in the Cosmic Microwave Background and the weak gravitational lensing obtained from Planck satellite \[95,96\]. Using Eq. (26) and Eq. (27), it can be easily shown that in order to have the theoretical estimates to be consistent with the observational results, \( N \) and \( \alpha \) should be equal to 60 and \( \frac{3}{5} \) respectively. Putting these values of \( N, \alpha \) into Eq. (26) and Eq. (27), we obtain the following numerical estimates for \( r \) and \( n_s \) such that, \( r = 0.10 \) and \( n_s = 0.970 \), which are well within the experimental bounds. Therefore the Hubble parameter as presented in Eq. (21) is indeed compatible with current observational bounds, provided the parameter \( \alpha \simeq 3/5 \). Thus using the reconstruction scheme we have been able to determine a suitable Hubble parameter, which we will use subsequently to determine the coupling function \( \xi(\Phi) \).

Given the Hubble parameter it is straightforward to obtain the differential equation determining the time evolution of the coupling function \( \xi(\Phi) \) using Eq. (16). The computation of individual coefficients of \( \xi \) and the \( \xi \) independent term requires \( H \), which for the Hubble parameter presented in Eq. (21) with \( \alpha \simeq 3/5 \) becomes, \( H = (-3d/5)\{c - d(t - t_*)\}^{-2/5} \). Therefore the differential equation satisfied by the potential \( \xi(\phi) \) becomes,

\[ 2\dddot{\xi} + 2\left[ 5 \{c - d(t - t_*)\}^{3/5} - \frac{6d}{5 \{c - d(t - t_*)\}} \right] \ddot{\xi} + \left[ \frac{3d}{5 \{c - d(t - t_*)\}^{8/5}} - 3 \right] \xi = 0 . \] (28)
The above second order linear differential equation can be solved by evaluating the associated integrating factor, which in this scenario reads,

\[
\text{Integrating Factor } \equiv e^P = \exp\left\{ \int dt \left[ 5 \{ c - d(t - t_*) \}^{3/5} - \frac{6d}{5 \{ c - d(t - t_*) \}} \right] \right\}
\]

\[
= \exp \left\{ - \frac{25}{d} \frac{\{ c - d(t - t_*) \}^{8/5}}{8} + \frac{6}{5} \ln \{ c - d(t - t_*) \} \right\}
\]

\[
= \{ c - d(t - t_*) \}^{6/5} \exp \left[ - \frac{25}{8d} \{ c - d(t - t_*) \}^{8/5} \right]. \tag{29}
\]

Therefore, multiplying the second order differential equation for the coupling function \( \xi(t) \), presented in Eq. (28), by the integrating factor it can be integrated once, resulting into,

\[
\dot{\xi} = e^{-P(t)} \int_{t_*}^{t} dt \left[ \frac{3}{2} - \frac{3d}{10 \{ c - d(t - t_*) \}^{8/5}} \right] \{ c - d(t - t_*) \}^{6/5} \exp \left[ - \frac{25}{8d} \{ c - d(t - t_*) \}^{8/5} \right]. \tag{30}
\]

which will result into incomplete Gamma functions. This in turn provides the expression for \( \dot{\Phi} \) from Eq. (6). However, due to the complicated nature of the differential equations for \( \xi(t) \) and \( \Phi(t) \), as evident from Eq. (30), it is not possible to obtain an analytic solution, unlike the case of constant Hubble parameter. Therefore, we have solved both the differential equations for \( \xi(t) \) and \( \Phi(t) \) using numerical techniques and have presented the results in Fig. 1.

As evident from Fig. 1, the scalar field decreases with time, which is expected, since at the beginning of the inflationary paradigm the scalar field was at the Planck scale, while as the inflation progresses the scalar field attains lower and lower values. An identical scenario also takes place for the coupling function \( \xi(\Phi) \), which also shows a decreasing nature with time. Furthermore, if the Gauss-Bonnet invariant is taken into account, the object \( \xi(\Phi) G \) starts decreasing with time. This is partly due to the decrease of \( \xi(\Phi) \) but also due to the rapid fall of the Gauss-Bonnet invariant with time, since the curvatures decreases rapidly with time as the inflation comes to an end. Therefore it turns out that the Gauss-Bonnet invariant alone is capable of driving the inflation.

4 Instability of the Model

It would have been really interesting if this becomes the end of the story. However unfortunately it turns out that despite having such intriguing features the above model faces a serious difficulty, namely stability of this model against perturbations. In particular, the tensor perturbations in the above spacetime grow rapidly [97]. To see the above, one of the natural possibility is to analyse the sound speed derived from the evolution equation for tensor perturbations. The expression for the sound speed tailored to our present context with arbitrary choice for \( \xi(\Phi) \) reads,

\[
\epsilon_s^2 = \frac{1 - 4\ddot{\xi}}{1 - 4H \ddot{\xi}}, \tag{31}
\]

where \( \ddot{\xi} = \left( \frac{\partial \xi}{\partial \Phi} \right) \dot{\Phi} \). The expression for \( \ddot{\xi} \) can also be derived from Eq. (16) and can be used to obtain,

\[
1 - 4\ddot{\xi} = 1 - 4 \left( 5H + 2 \frac{\dot{H}}{H} \right) \dot{\xi} - \left( \frac{\dot{H}}{2H^2} + \frac{3}{2} \right) = (2\epsilon - 5) \left( 1 - 4H \xi \right), \tag{32}
\]
where, $\epsilon$ is the slow-roll parameter $-\dot{H}/H^2$. The above expression when substituted in Eq. (31) for sound speed yields,

$$c_s^2 = 2\epsilon - 5$$

Therefore throughout the inflationary epoch, we have $\epsilon \ll 1$ and hence $c_s^2$ is negative. Thus irrespective of the choice of the Gauss-Bonnet coupling function $\xi(\Phi)$ there is an instability in the tensor perturbation. As a consequence the fluctuations in the tensor modes will grow rapidly and hence the above model without a self-interacting potential for the inflaton field can not lead to a viable inflationary scenario. Thus it is necessary to include a self-interacting term in the Lagrangian in order to explain the behaviour of the perturbations in a consistent manner.
5 Inflation with a self-interacting potential

We have just described the instability of the tensor perturbation in absence of a self-interacting potential for the scalar field, this being a strong motivation towards introduction of such a self-interacting potential, even though the scalar coupled Gauss-Bonnet term alone can provide a consistent inflationary scenario (keeping aside the perturbations). Thus in this section we will explore the possible solutions of the field equations consistent with inflationary paradigm in presence of such a self-interacting potential. This will result into modifications of the gravitational field equations, which in turn will modify Eq. (6) and Eq. (8) respectively, while Eq. (7) will remain unchanged. In particular the right hand side of Eq. (6) will get modified by the introduction of $8\pi G V(\phi)$ term, while the left hand side of Eq. (8) will inherit an additional $\partial V/\partial \phi$ term. Given these modifications we are now in a position to study effect of both these terms on the inflationary epoch. Alike the previous scenario with the Gauss-Bonnet term alone, in the present context as well the inflationary paradigm and slow-roll approximation for the scalar field are incompatible with each other as we will demonstrate below. In the slow-roll approximation we neglect $\ddot{\Phi}$ and $\dot{\Phi}^2$ terms in comparison with $\Phi$ and hence the field equation as in Eq. (6) yields,

$$\dot{\Phi} = \frac{3H^2 - 8\pi GV(\Phi)}{12H^3(\partial \xi/\partial \Phi)}.$$  (34)

Note that this is in complete contrast with the normal situation without the Gauss-Bonnet term, where the same equation would yield $H^2 \sim V(\Phi)$. On the other hand, $\dot{H}$ can be obtained from Eq. (7), such that the slow-roll parameter becomes,

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \approx \frac{2H(\partial \xi/\partial \Phi)\dot{\Phi}}{1 - 4H\dot{\Phi}(\partial \xi/\partial \Phi)} = \frac{3H^2}{16\pi GV} - \frac{1}{2}.$$  (35)

Thus if we neglect the Gauss-Bonnet term then of course this is a very small quantity and the normal inflationary paradigm would follow. But in presence of the Gauss-Bonnet term the above slow-roll parameter is always $\sim O(1)$ and hence it is not possible to have accelerated expansion of the universe while respecting slow-roll approximation. Thus one must abandon the slow-roll approximation if the non-trivial effects of the Gauss-Bonnet term in the early universe cosmology is asked for. This suggests to take an identical route as in the previous scenario. However due to the complicated nature of the field equations, unlike the previous situation here we will not employ the reconstruction scheme, rather should provide viable choices for the potential $V(\Phi)$ as well as the coupling function $\xi(\Phi)$ for which analytical solutions can be obtained. We would again like to emphasize that we are not neglecting the Gauss-Bonnet term while considering inflationary paradigm, rather we are keeping both the self-interacting potential and the Gauss-Bonnet term to have an initial accelerated expansion of the universe as well as a final deceleration signifying end of the inflationary epoch.

5.1 Accelerated expansion with a quadratic potential

As a first choice it is convenient to consider a quadratic potential for the scalar field, i.e., the potential function $V(\Phi)$ involves a constant contribution and a quadratic part proportional to $\Phi^2$. A similar form for the coupling function $\xi(\Phi)$ is also suggestive. However the field equations involves derivative of $\xi(\Phi)$ and hence the constant term in $\xi(\Phi)$ plays no role. This implies the following form of the scalar field
potential and the coupling function,

\[ V_1(\Phi) = V_0^{(1)} + V_1^{(1)} \Phi^2; \quad (36) \]

\[ \xi_1(\Phi) = \xi_0^{(1)} \Phi^2; \quad (37) \]

where the subscript ‘1’ denotes that the above corresponds to the first set of solutions. Furthermore, \( V_0^{(1)}, V_1^{(1)} \) and \( \xi_0^{(1)} \) stands for arbitrary parameters in the theory, which needs to be determined later. Substituting the above form of the potential function \( V_1(\Phi) \) and \( \xi_1(\Phi) \) into the field equations, one easily obtains the following solutions of the scalar field and the Hubble parameter as,

\[ H(t) = H_0 \equiv \sqrt{\frac{8\pi G V_0^{(1)}}{3}} = \text{constant}; \quad (38) \]

\[ \Phi(t) = \Phi_0 \exp(-\lambda t). \quad (39) \]

Here, the unknown parameters namely \( \lambda \) and \( V_1^{(1)} \) can be obtained in terms of the constant Hubble parameter \( H_0 \) as well as \( \xi_0^{(1)} \) as,

\[ \lambda = \frac{8H_0^3 \xi_0^{(1)}}{8\pi G - 16H_0^2 \xi_0^{(1)}}, \quad V_1^{(1)} = \frac{24H_0^3 \lambda \xi_0^{(1)}}{8\pi G} - \frac{\lambda^2}{2}, \quad (40) \]

while the parameter \( \Phi_0 \) remains undetermined. As evident, constant value for the Hubble parameter ensures that the scale factor scales exponentially with time, i.e., \( a(t) = \exp(H_0 t) \). Thus the solution corresponds to accelerating phase of the universe. Furthermore it is straightforward to determine the time evolution of the self-interacting potential \( V_1(\Phi) \) as well as the coupling function \( \xi_1(\Phi) \) using the time evolution of the scalar field. This ensures that \( V(\Phi) \) has a constant piece and the rest of the part decays exponentially with time, while \( \xi(\Phi) \) also decays exponentially. Thus at later stages of inflation these potentials must be replaced with some other scalar potentials, allowing for decelerated expansion of the universe, which we consider in the subsequent section.

### 5.2 Power law expansion and deceleration

In this section we will discuss another set of solutions for the scalar field and the scale factor, given some appropriate form for the scalar potential as well as the coupling function. We assume that the potential is an exponentially decaying function of the scalar field, while the coupling function is an exponentially growing one. The growing behaviour is necessary since we would like to keep the Gauss-Bonnet term relevant even at the end stages of inflation. (Note that the Gauss-Bonnet term alone should have negligible contribution at the end of inflation as the curvatures has become quite small.) Thus for our purpose we consider a different form of the scalar field potential and the coupling function,

\[ V_2(\Phi) = V_0^{(2)} \exp[-2\Phi(t)/\Phi_0]; \quad (41) \]

\[ \xi_2(\Phi) = \xi_0^{(2)} \exp[2\Phi(t)/\Phi_0], \quad (42) \]

where the subscript ‘2’ is just to remind us that this corresponds to the second set of solutions. In the above expression \( V_0^{(2)}, \Phi_0 \) and \( \xi_0^{(2)} \) are the model parameters. It can be easily verified that the field
equations for gravity plus scalar field is satisfied provided the time dependence of the scale factor and the scalar field corresponds to

\[ \Phi(t) = \Phi_0 \ln \left( \frac{t}{t_0} \right); \quad H(t) = \frac{n}{t}, \]

where \( n < 1 \). One can easily check that \( \dot{H} + H^2 \) for this particular case is negative and thus corresponds to the decelerating scenario at the end of the inflation. Since it is normally believed that the end of inflation results into a radiation dominated universe, it is legitimate to assume \( n = 1/2 \). However for the moment we will keep \( n \) arbitrary. The field equations also result into several constraints connecting the free parameters present in the model. In particular, the parameter \( \xi_0^{(2)} \) and \( V_0^{(2)} \) gets determined in terms of the other free parameters as,

\[ \frac{\xi_0}{t_0^2} = \frac{8\pi G}{24n^3(n-1)} [(1-3n)\Phi_0^2 t_0 + 2V_0 t_0^2]; \quad V_0 t_0^3 = \frac{(n-1)}{2} \left[ \frac{3n^2}{8\pi G} - \frac{\Phi_0^2 t_0^2}{2(n-1)} (1-5n) \right]. \]

Finally plugging the solution for the time evolution of the scalar field into the expressions for the self-interacting potential as well as coupling function one gets both of them as a function of time:

\[ V_2[\Phi(t)] = V_0^{(2)} \left( \frac{t_0^2}{t^2} \right); \quad \xi_2[\Phi(t)] = \xi_0^{(2)} \left( \frac{t_0^2}{t^2} \right). \]

Thus as in the previous scenario here also the scalar field potential decays with time but as a power law, while the interaction potential depicts a growth with time.

### 5.3 Estimation of parameters associated with the inflationary scenario

Having described the two situations, one depicting accelerated expansion of the universe at the early stages of inflation and the other providing a decelerating phase marking the exit from inflationary paradigm, we concentrate on estimation of various parameters in the model. The inflationary paradigm comes into existence at very early stages of the universe and it lasted from \( t_\text{in} \sim 10^{-11} \text{ GeV}^{-1} \) to \( t_\text{end} \sim 6 \times 10^{-8} \text{ GeV}^{-1} \). Thus we assume that the potential \( V_1(\Phi) \) existed for an initial phase of the inflationary epoch which we choose to be in the range \( 10^{-11} \text{ GeV}^{-1} < t < 10^{-8} \text{ GeV}^{-1} \), while the other potential \( V_2(\Phi) \) appeared in the end stages of the inflationary scenario and was effective for \( t > 6 \times 10^{-8} \text{ GeV}^{-1} \). During the regime \( 10^{-8} \text{ GeV}^{-1} < t < 6 \times 10^{-8} \text{ GeV}^{-1} \), there must be an intermediate potential interpolating between these two regimes, which we will determine later using numerical techniques. Along identical lines the coupling potential \( \xi(\Phi) \) also has two different behaviour in the two distinct regimes. We will have \( \xi(\Phi) = \xi_1(\Phi) \) for \( 10^{-11} \text{ GeV}^{-1} < t < 10^{-8} \text{ GeV}^{-1} \), while the coupling function becomes, \( \xi(\Phi) = \xi_2(\Phi) \) for \( t > 6 \times 10^{-8} \text{ GeV}^{-1} \). In the intermediate region we will numerically construct an interpolating coupling function that matches with both \( \xi_1(\Phi) \) and \( \xi_2(\Phi) \) appropriately at both ends.

The above process of interpolation requires appropriate choices for the values of the free parameters present in our model. As far as the first situation is considered, the relevant parameters are the Hubble parameter \( H_0 \) and the decaying parameter \( \lambda \) in the solution of the scalar field (see Eq. (39) for a detailed description), both having mass dimension one. The choice of these parameters are also connected with the observational viability of this model and hence it must have number of e-foldings \( \sim 60 \). Since the number of e-foldings correspond to integration of Hubble parameter over the entire duration of inflation, it follows that \( H_0 \approx 6 \times 10^9 \text{ GeV} \).

Using the scalar field solution presented in Eq. (39), one can verify that the energy density \( \rho \) of the scalar field \( \Phi \) varies as \( \rho \sim \exp(-2\lambda t) \) with time. Again the energy density of the scalar field can be
written in terms of the scale factor yielding, $\rho \sim a^{-3} = \exp(-3H_0 t)$, if we assume that the scalar field behaves like ordinary matter, since for the same the energy density is proportional to cubic inverse of the scale factor. Thus consistency of the above two relations for the energy density of the scalar field demands $\lambda$ to be equal to $3H_0/2$. Thus the parameter $\lambda$ becomes, $\simeq 9 \times 10^9 \text{ GeV}$. Similarly using Eq. (40), we immediately obtain both $\xi_0^{(1)}$ and $V_1^{(1)}$ in terms of the other parameters, namely $H_0$ and $\lambda$.

Returning to the post inflationary scenario we concentrate on the second set of solution given by the the potential $V_2(\Phi)$ and $\xi_2(\Phi)$ respectively, presented in Eq. (41) and Eq. (42). As evident we can choose the initial time instant to be located at $t_0 \sim 10^{-8} \text{ GeV}$ and hence the parameter $V_0^{(2)}$ gets determined from Eq. (44) as $V_0^{(2)} t_0^3 \simeq (1/8\pi G)$. The rest of the parameters can also be accordingly determined. As a consequence we can interpolate both the potential and the coupling function in the intermediate region.

5.4 Numerical solutions in the interpolating region

Given the structure of the potential as well as the coupling function in the initial and final stages of inflation, we would like to provide a complete picture by interpolating between these regions. Due to complicated nature of the equations governing the evolution of the scalar field and the scale factor in a general context, we will determine the interpolating function using numerical techniques and shall illustrate the same. In particular, taking the Planck mass to be $M_{pl} = 10^{19} \text{ GeV}$ and the expressions for potential in the early and late stages of inflation, we interpolate the potential function for $10^{-11} < t < 6 \times 10^{-8} \text{ GeV}^{-1}$, which has been presented in Fig. 2. Note that the axes in Fig. 2 are rescaled according to convenience, namely

![Figure 2: The self-interacting scalar Potential $V(\Phi)$ is being plotted against time $t$ for the complete duration of inflation. The initial and final portions are determined analytically, while the intermediate region is obtained by interpolation. The curve explicitly shows the decreasing behaviour of the scalar potential with time.](image-url)

x-axis corresponds to a “rescaled” time coordinate obtained as $\sim 10^9 t$ which is in $\text{GeV}^{-1}$ unit, while the y axis corresponds to “rescaled” potential, which is in $\text{GeV}^4$ unit. It is evident that the potential function is smooth everywhere and decays with time.

Similarly substituting the values of various parameters presented into Eq. (37) and Eq. (42), one gets the coupling $\xi(\Phi)$ within the two time scales, $10^{-11} < t < 10^{-8} \text{ GeV}^{-1}$ as well as for $\xi(\Phi)$ with $t > 6 \times 10^{-8} \text{ GeV}^{-1}$ respectively. Using the above two expressions, the time variation of the coupling function for the intermediate region can also be determined by interpolation. However rather than the
coupling function, the combination $\xi(\Phi)G$, where $G$ is the Gauss-Bonnet invariant is of more importance and has been presented in Fig. 3, where the x axis correspond to “rescaled” time. As evident from Fig. 3 there exist an intermediate region where the effect of the coupling function times the Gauss-Bonnet invariant attains a maximum value. Thus during the inflationary epoch it is not at all justified to ignore the effect of the Gauss-Bonnet term. On the other hand, as the universe exits from the inflationary epoch, the combination attains a fairly constant value and thus one may use it in the context of quintessential inflation. By using these forms of the scalar field potential and the coupling function, we are next going to solve the field equations for the Hubble parameter (or, equivalently the scale factor) as well as the scalar field numerically to understand their behaviour.

Given the gravitational field equations involving only first order time derivatives of the Hubble parameter $H(t)$, a numerical solution of the same requires one boundary condition. Choosing the initial value of the Hubble parameter $H(0)$ as the inverse of the duration of the inflationary epoch i.e., $H(0) \sim 0.6 \times 10^9$ GeV, we obtain the required solution as depicted in Fig. 4. As in the earlier plots, in Fig. 4 as well the x and y axes are rescaled such that the “rescaled” Hubble parameter $\sim 10^{-9}H$ in GeV unit. The figure explicitly demonstrates that the Hubble parameter at the initial stages remained almost constant, signifying a very small value for the parameter $\epsilon(t)$, while at the later stages the Hubble parameter decreases with time and finally results into deceleration signifying an exit from inflationary paradigm. Thus we can safely argue that the numerical solutions obtained above indeed matches with the analytic one both at the beginning and at the end of the intermediate region.

The above numerical solution of the Hubble parameter can be immediately integrated providing the evolution of scale factor $a(t)$ with respect to time. However in the context of inflation it is more convenient to depict the solution for $\ddot{a}/a$, the acceleration parameter of the universe, which has been presented in Fig. 5. Here the y-axis of Fig. 5 corresponds to $\ddot{a}(t)/a(t)$ associated with the “rescaled” Hubble parameter. From the above figure, one can easily conclude that the inflation ends near about $t \sim 6 \times 10^{-8}$ GeV$^{-1}$ or, equivalently $t \simeq 6 \times 10^{-32}$ sec, after which $\ddot{a}/a$ becomes negative. To get a better view of what is happening near the end of the inflationary epoch, we provide in Fig. 6 a zoomed-in version of Fig. 5 near $t \sim 6 \times 10^{-8}$ GeV$^{-1}$.

Using the form of scalar potential, coupling function and the Hubble parameter one can easily solve
Figure 4: Numerical solution of the Hubble parameter $H$ is being presented with time $t$. As evident at the onset of inflation, the Hubble parameter was fixed at a constant value, signifying initial exponential expansion of the universe, which then give way to final power law expansion. The behaviour of the Hubble parameter in the intermediate regime has been obtained by appropriate interpolation of the initial and final phases.

Figure 5: The above figure presents the variation of the acceleration parameter $\ddot{a}/a$ with time. As evident in the initial stages of inflation, the acceleration was almost constant, while the acceleration decreases as time passes by and finally it turns negative around $t \sim 6 \times 10^{-8}$ GeV$^{-1}$. This presents the exit from inflation.

for the only remaining bit, i.e., the scalar field equation numerically. Given the scalar field potential as a function of time as well as the scalar field as a function of time one can eliminate time from the two and hence plot the potential as a function of the scalar field. This is what we have presented in Fig. 7, where the scalar field as well as the potential have been “rescaled” in an appropriate manner. In order to match the numerical solution for the scalar field with the analytic ones, we use suitable boundary conditions on $\Phi$ and $\dot{\Phi}$ respectively. From Fig. 7, it is clear that the scalar field rolls down the scalar potential $V(\Phi)$ in a rapid manner and hence it is completely consistent with our earlier findings that slow-roll approximations will not work here. Finally for $t > 6 \times 10^{-8}$ GeV$^{-1}$, the potential becomes flat and the field exits from inflation. This is completely consistent with our analytical estimates as well. Thus from Eq. (39) and Eq. (43), one can easily conclude that just like the Hubble parameter, the numerical solution of scalar field
Figure 6: A magnified plot depicting $\dot{a}/a$ turning negative near the end of the inflationary paradigm, where a transition from acceleration to deceleration takes place. In this context the exit from inflation happened roughly when $t \sim 6 \times 10^{-8} \text{ GeV}^{-1}$.

Figure 7: Scalar potential $V$ has been depicted against the scalar field $\phi$. The potential decreases steeply with time and hence the slow-roll approximation will not work in this context. As the inflation ends the potential becomes flat and hence having little influence on dynamics of the universe.

also matches with the analytic one near about the beginning and the end stages of inflation.

Expression for the Ricci scalar $R = 6(2H^2 + \dot{H})$ implies that it varies as $1/t^2$ at the end of the inflation and consequently the ratio of $\xi(t)G/8\pi GR \sim O(10^{-27})$ just after the end of inflation. Thus once the universe exits from inflationary period, the Gauss-Bonnet term (coupled with the scalar field) can be safely ignored with respect to the Ricci scalar and hence the universe is dominated only by Einstein’s gravity.

6 Concluding Remarks
In this work we set out to explore the influence of the Gauss-Bonnet term on the inflationary paradigm. In particular, even though the Gauss-Bonnet term alone in four dimension is topological in nature, a non-trivial coupling of the same with the inflaton field can influence the evolution of the universe. To
understand the effect of the coupling of the Gauss-Bonnet term in some detail we consider a particular scenario in which the self-interacting potential for the inflaton field is absent. By solving the associated field equations we could explicitly show that the above model indeed exhibits an exponential expansion of the universe. Subsequently, using the powerful reconstruction technique, we have been able to argue that the Gauss-Bonnet term coupled with a scalar field can indeed drive the inflation of the universe, while also providing an exit. The above model also turned out to be consistent with current observations. However, the scalar coupled Gauss-Bonnet term encounters difficulty when one considers evolution of tensor perturbations and in general circumstances we have been able to demonstrate that it will always be unstable. This motivates us to introduce the self-interacting potential for the scalar field. Unlike the results derived in earlier literatures, here we have considered the effect of the Gauss-Bonnet invariant as well as the scalar potential on the inflationary paradigm. Having derived the initial accelerating phase and the final decelerating phase we have interpolated the behaviour of the Hubble parameter, the scalar field and the potential between these two phases numerically. It turns out that in both these contexts, with or without the potential, the scalar coupling to the Gauss-Bonnet term gradually decreases to small and constant value. This essentially makes the Gauss-Bonnet term having negligible influence and hence it goes out of the dynamical picture, thereby the evolution of the universe is governed by the Einstein term alone.

Acknowledgements

Research of S.C. is supported by the SERB-NPDF grant (No. PDF/2016/001589) from DST, Government of India.

References

[1] C. M. Will, *Theory and experiment in gravitational physics*. Cambridge University Press, 1993.

[2] S. M. Carroll, *Spacetime and geometry. An introduction to general relativity*, vol. 1. 2004.

[3] T. Padmanabhan, *Gravitation: Foundations and Frontiers*. Cambridge University Press, Cambridge, UK, 2010.

[4] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation*. W. H. Freeman and Company, 3 ed., 1973.

[5] P. Horava and E. Witten, “Eleven-dimensional supergravity on a manifold with boundary,” *Nucl.Phys.* **B475** (1996) 94–114, arXiv:hep-th/9603142 [hep-th].

[6] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, “The Hierarchy problem and new dimensions at a millimeter,” *Phys.Lett.* **B429** (1998) 263–272, arXiv:hep-ph/9803315 [hep-ph].

[7] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, “New dimensions at a millimeter to a Fermi and superstrings at a TeV,” *Phys.Lett.* **B436** (1998) 257–263, arXiv:hep-ph/9804398 [hep-ph].

[8] L. Randall and R. Sundrum, “An Alternative to compactification,” *Phys.Rev.Lett.* **83** (1999) 4690–4693, arXiv:hep-th/9906064 [hep-th].

19
[9] L. Randall and R. Sundrum, “A Large mass hierarchy from a small extra dimension,” Phys.Rev.Lett. 83 (1999) 3370–3373, arXiv:hep-ph/9905221 [hep-ph].
[10] S. W. Hawking, “Breakdown of Predictability in Gravitational Collapse,” Phys. Rev. D14 (1976) 2460–2473.
[11] S. W. Hawking and G. F. R. Ellis, The Large Scale Structure of Space-Time. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2011.
[12] A. H. Guth, “The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems,” Phys. Rev. D23 (1981) 347–356.
[13] A. A. Starobinsky, “A New Type of Isotropic Cosmological Models Without Singularity,” Phys. Lett. 91B (1980) 99–102.
[14] A. D. Linde, “A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems,” Phys. Lett. 108B (1982) 389–393.
[15] A. D. Linde, “Coleman-Weinberg Theory and a New Inflationary Universe Scenario,” Phys. Lett. 114B (1982) 431–435.
[16] Supernova Search Team Collaboration, A. G. Riess et al., “Observational evidence from supernovae for an accelerating universe and a cosmological constant,” Astron. J. 116 (1998) 1009–1038, arXiv:astro-ph/9805201 [astro-ph].
[17] Supernova Cosmology Project Collaboration, S. Perlmutter et al., “Measurements of Omega and Lambda from 42 high redshift supernovae,” Astrophys. J. 517 (1999) 565–586, arXiv:astro-ph/9812133 [astro-ph].
[18] V. Rubakov and M. Shaposhnikov, “Extra Space-Time Dimensions: Towards a Solution to the Cosmological Constant Problem,” Phys.Lett. B125 (1983) 139.
[19] P. J. E. Peebles and B. Ratra, “The Cosmological constant and dark energy,” Rev. Mod. Phys. 75 (2003) 559–606, arXiv:astro-ph/0207347 [astro-ph].
[20] S. M. Carroll, “The Cosmological constant,” Living Rev. Rel. 4 (2001) 1, arXiv:astro-ph/0004075 [astro-ph].
[21] T. Padmanabhan, “Cosmological constant: The Weight of the vacuum,” Phys. Rept. 380 (2003) 235–320, arXiv:hep-th/0212290 [hep-th].
[22] L. F. Abbott, E. Farhi, and M. B. Wise, “Particle Production in the New Inflationary Cosmology,” Phys. Lett. 117B (1982) 29.
[23] A. D. Linde, “Chaotic Inflation,” Phys. Lett. 129B (1983) 177–181.
[24] S. Dodelson, Modern Cosmology. Academic Press, Amsterdam, 2003. http://www.slac.stanford.edu/spires/find/books/www?c1=QB981:D62:2003.
[25] N. Turok, “A critical review of inflation,” Class. Quant. Grav. 19 (2002) 3449–3467.
[26] D. H. Lyth, “Introduction to cosmology,” in *Proceedings, Summer School in High-energy physics and cosmology: Trieste, Italy, June 14-July 30, 1993*, pp. 0069–136. 1993. arXiv:astro-ph/9312022 [astro-ph].

[27] A. R. Liddle, “An Introduction to cosmological inflation,” in *Proceedings, Summer School in High-energy physics and cosmology: Trieste, Italy, June 29-July 17, 1998*, pp. 260–295. 1999. arXiv:astro-ph/9901124 [astro-ph].

[28] R. H. Brandenberger, “Inflationary cosmology: Progress and problems,” in *IPM School on Cosmology 1999: Large Scale Structure Formation Tehran, Iran, January 23-February 4, 1999*. 1999. arXiv:hep-ph/9910410 [hep-ph].

[29] A. H. Guth, “Inflation and eternal inflation,” *Phys. Rept.* **333** (2000) 555–574, arXiv:astro-ph/0002156 [astro-ph].

[30] J. E. Lidsey, A. R. Liddle, E. W. Kolb, E. J. Copeland, T. Barreiro, and M. Abney, “Reconstructing the inflation potential: An overview,” *Rev. Mod. Phys.* **69** (1997) 373–410, arXiv:astro-ph/9508078 [astro-ph].

[31] C. P. Burgess, P. Martineau, F. Quevedo, G. Rajesh, and R. J. Zhang, “Brane - anti-brane inflation in orbifold and orientifold models,” *JHEP* **03** (2002) 052, arXiv:hep-th/0111025 [hep-th].

[32] T. Padmanabhan and T. R. Seshadri, “Does inflation solve the horizon problem?,” *Class. Quant. Grav.* **5** (1988) 221–224.

[33] T. Padmanabhan, T. R. Seshadri, and T. P. Singh, “Making Inflation Work: Damping of Density Perturbations Due to Planck Energy Cutoff,” *Phys. Rev.* **D39** (1989) 2100.

[34] S. Nojiri and S. D. Odintsov, “Unified cosmic history in modified gravity: from F(R) theory to Lorentz non-invariant models,” *Phys.Rept.* **505** (2011) 59–144, arXiv:1011.0544 [gr-qc].

[35] T. P. Sotiriou and V. Faraoni, “f(R) Theories Of Gravity,” *Rev.Mod.Phys.* **82** (2010) 451–497, arXiv:0805.1726 [gr-qc].

[36] A. De Felice and S. Tsujikawa, “f(R) theories,” *Living Rev.Rel.* **13** (2010) 3, arXiv:1002.4928 [gr-qc].

[37] S. Nojiri and S. D. Odintsov, “Unifying inflation with LambdaCDM epoch in modified f(R) gravity consistent with Solar System tests,” *Phys. Lett.* **B657** (2007) 238–245, arXiv:0707.1941 [hep-th].

[38] S. Nojiri and S. D. Odintsov, “Modified gravity with negative and positive powers of the curvature: Unification of the inflation and of the cosmic acceleration,” *Phys. Rev.* **D68** (2003) 123512, arXiv:hep-th/0307288 [hep-th].

[39] S. Nojiri, S. D. Odintsov, and V. K. Oikonomou, “Modified Gravity Theories on a Nutshell: Inflation, Bounce and Late-time Evolution,” arXiv:1705.11098 [gr-qc].

[40] V. K. Oikonomou, “Exponential Inflation with F(R) Gravity,” *Phys. Rev.* **D97** no. 6, (2018) 064001, arXiv:1801.03426 [gr-qc].
[41] S. Chakraborty and S. SenGupta, “Spherically symmetric brane spacetime with bulk $f(R)$ gravity,” *Eur.Phys.J.* C75 no. 1, (2015) 11, arXiv:1409.4115 [gr-qc].

[42] S. Chakraborty and S. SenGupta, “Effective gravitational field equations on m-brane embedded in n-dimensional bulk of Einstein and f(R) gravity,” *Eur. Phys. J.* C75 no. 11, (2015) 538, arXiv:1504.07519 [gr-qc].

[43] S. Capozziello, R. de Ritis, and A. A. Marino, “Some aspects of the cosmological conformal equivalence between ‘Jordan frame’ and ‘Einstein frame’,” *Class. Quant. Grav.* 14 (1997) 3243–3258, arXiv:gr-qc/9612053 [gr-qc].

[44] T. P. Sotiriou, “f(R) gravity and scalar-tensor theory,” *Class. Quant. Grav.* 23 (2006) 5117–5128, arXiv:gr-qc/0604028 [gr-qc].

[45] R. Catena, M. Pietroni, and L. Scarabello, “Einstein and Jordan reconciled: a frame-invariant approach to scalar-tensor cosmology,” *Phys. Rev.* D76 (2007) 084039, arXiv:astro-ph/0604492 [astro-ph].

[46] S. Chakraborty and S. SenGupta, “Solving higher curvature gravity theories,” *Eur. Phys. J.* C76 no. 10, (2016) 552, arXiv:1604.05301 [gr-qc].

[47] S. Chakraborty and S. SenGupta, “Gravity stabilizes itself,” *Eur. Phys. J.* C77 (2017) 573, arXiv:1701.01032 [gr-qc].

[48] T. Paul and S. Sengupta, “Radion tunneling in higher curvature gravity,” arXiv:1801.05027 [hep-th].

[49] A. Karam, A. Lykkas, and K. Tamvakis, “Frame-invariant approach to higher-dimensional scalar-tensor gravity,” arXiv:1803.04960 [gr-qc].

[50] H. Sami, J. Ntahompagaze, and A. Abebe, “Inflationary $f(R)$ Cosmologies,” *Universe* 3 no. 4, (2017) 73, arXiv:1709.04860 [gr-qc].

[51] J. D. Barrow and S. Cotsakis, “Inflation and the conformal structure of higher-order gravity theories,” *Physics Letters B* 214 no. 4, (1988) 515 – 518.

[52] G. F. R. Ellis and M. S. Madsen, “Exact scalar field cosmologies,” *Classical and Quantum Gravity* 8 no. 4, (1991) 667. http://stacks.iop.org/0264-9381/8/i=4/a=012.

[53] G. Cognola, E. Elizalde, S. Nojiri, S. D. Odintsov, and S. Zerbini, “One-loop $f(R)$ gravity in de Sitter universe,” *JCAP* 0502 (2005) 010, arXiv:hep-th/0501096 [hep-th].

[54] K. Bamba and S. D. Odintsov, “Inflation and late-time cosmic acceleration in non-minimal Maxwell-$F(R)$ gravity and the generation of large-scale magnetic fields,” *JCAP* 0804 (2008) 024, arXiv:0801.0954 [astro-ph].

[55] S. A. Appleby, R. A. Battye, and A. A. Starobinsky, “Curing singularities in cosmological evolution of $F(R)$ gravity,” *JCAP* 1006 (2010) 005, arXiv:0909.1737 [astro-ph.CO].

[56] L. Sebastiani and R. Myrzakulov, “$F(R)$ gravity and inflation,” *Int. J. Geom. Meth. Mod. Phys.* 12 no. 9, (2015) 1530003, arXiv:1506.05330 [gr-qc].
[57] N. Banerjee and T. Paul, “Inflationary scenario from higher curvature warped spacetime,” *Eur. Phys. J.* C77 no. 10, (2017) 672, arXiv:1706.05964 [hep-th].

[58] A. Das, D. Maity, T. Paul, and S. SenGupta, “Bouncing cosmology from warped extra dimensional scenario,” *Eur. Phys. J.* C77 no. 12, (2017) 813, arXiv:1706.00950 [hep-th].

[59] R. P. Woodard, “Ostrogradsky’s theorem on Hamiltonian instability,” *Scholarpedia* 10 no. 8, (2015) 32243, arXiv:1506.02210 [hep-th].

[60] B. Zwiebach, “Curvature Squared Terms and String Theories,” *Phys. Lett.* 156B (1985) 315–317.

[61] D. J. Gross and J. H. Sloan, “The Quartic Effective Action for the Heterotic String,” *Nucl. Phys.* B291 (1987) 41–89.

[62] N. Dadhich, “Characterization of the Lovelock gravity by Bianchi derivative,” *Pramana* 74 (2010) 875–882, arXiv:0802.3034 [gr-qc].

[63] T. Padmanabhan and D. Kothawala, “Lanczos-Lovelock models of gravity,” *Phys.Rept.* 531 (2013) 115–171, arXiv:1302.2151 [gr-qc].

[64] S. Chakraborty, “Lanczos-Lovelock gravity from a thermodynamic perspective,” *JHEP* 08 (2015) 029, arXiv:1505.07272 [gr-qc].

[65] S. Chakraborty and S. SenGupta, “Spherically symmetric brane in a bulk of f(R) and Gauss-Bonnet Gravity,” *Class. Quant. Grav.* 33 no. 22, (2016) 225001, arXiv:1510.01953 [gr-qc].

[66] S. Chakraborty, K. Parattu, and T. Padmanabhan, “A Novel Derivation of the Boundary Term for the Action in Lanczos-Lovelock Gravity,” *Gen. Rel. Grav.* 49 no. 9, (2017) 121, arXiv:1703.00624 [gr-qc].

[67] G. Cognola, E. Elizalde, S. Nojiri, S. D. Odintsov, and S. Zerbini, “Dark energy in modified Gauss-Bonnet gravity: Late-time acceleration and the hierarchy problem,” *Phys. Rev.* D73 (2006) 084007, arXiv:hep-th/0601008 [hep-th].

[68] S. Nojiri and S. D. Odintsov, “Modified Gauss-Bonnet theory as gravitational alternative for dark energy,” *Phys. Lett.* B631 (2005) 1–6, arXiv:hep-th/0508049 [hep-th].

[69] P. Kanti, R. Gambouji, and N. Dadhich, “Gauss-Bonnet Inflation,” *Phys. Rev.* D92 no. 4, (2015) 041302, arXiv:1503.01579 [hep-th].

[70] C. van de Bruck and L. E. Paduraru, “Simplest extension of Starobinsky inflation,” *Phys. Rev.* D92 (2015) 083513, arXiv:1505.01727 [hep-th].

[71] C. Charmousis and J.-F. Dufaux, “General Gauss-Bonnet brane cosmology,” *Class. Quant. Grav.* 19 (2002) 4671–4682, arXiv:hep-th/0202107 [hep-th].

[72] S. Nojiri, S. D. Odintsov, and O. G. Gorbunova, “Dark energy problem: From phantom theory to modified Gauss-Bonnet gravity,” *J. Phys.* A39 (2006) 6627–6634, arXiv:hep-th/0510183 [hep-th].
B. M. Leith and I. P. Neupane, “Gauss-Bonnet cosmologies: Crossing the phantom divide and the transition from matter dominance to dark energy,” *JCAP* **0705** (2007) 019, arXiv:hep-th/0702002 [hep-th].

S. Deser and R. P. Woodard, “Nonlocal Cosmology,” *Phys. Rev. Lett.* **99** (2007) 111301, arXiv:0706.2151 [astro-ph].

Z.-K. Guo and D. J. Schwarz, “Slow-roll inflation with a Gauss-Bonnet correction,” *Phys. Rev. D* **81** (2010) 123520, arXiv:1001.1897 [hep-th].

P.-X. Jiang, J.-W. Hu, and Z.-K. Guo, “Inflation coupled to a Gauss-Bonnet term,” *Phys. Rev. D* **88** (2013) 123508, arXiv:1310.5579 [hep-th].

P. Kanti, R. Gannouji, and N. Dadhich, “Early-time cosmological solutions in Einstein-scalar-Gauss-Bonnet theory,” *Phys. Rev. D* **92** no. 8, (2015) 083524, arXiv:1506.04667 [hep-th].

L. Sberna and P. Pani, “Nonsingular solutions and instabilities in Einstein-scalar-Gauss-Bonnet cosmology,” *Phys. Rev. D* **96** no. 12, (2017) 124022, arXiv:1708.06371 [gr-qc].

I. V. Fomin and S. V. Chervon, “Exact inflation in Einstein-Gauss-Bonnet gravity,” *Grav. Cosmol.* **23** no. 4, (2017) 367–374, arXiv:1704.03634 [gr-qc].

S. Lahiri, “Anisotropic inflation in Gauss-Bonnet gravity,” *JCAP* **1609** no. 09, (2016) 025, arXiv:1605.09247 [hep-th].

C. van de Bruck, K. Dimopoulos, C. Longden, and C. Owen, “Gauss-Bonnet-coupled Quintessential Inflation,” arXiv:1707.06839 [astro-ph.CO].

C. van de Bruck, K. Dimopoulos, and C. Longden, “Reheating in Gauss-Bonnet-coupled inflation,” *Phys. Rev. D* **94** no. 2, (2016) 023506, arXiv:1605.06350 [astro-ph.CO].

T. P. Sotiriou and S.-Y. Zhou, “Black hole hair in generalized scalar-tensor gravity,” *Phys. Rev. Lett.* **112** (2014) 251102, arXiv:1312.3622 [gr-qc].

A. Hees *et al.*, “Testing General Relativity with stellar orbits around the supermassive black hole in our Galactic center,” *Phys. Rev. Lett.* **118** no. 21, (2017) 211101, arXiv:1705.07902 [astro-ph.GA].

G. Antoniou, A. Bakopoulos, and P. Kanti, “Evasion of No-Hair Theorems and Novel Black-Hole Solutions in Gauss-Bonnet Theories,” *Phys. Rev. Lett.* **120** no. 13, (2018) 131102, arXiv:1711.03390 [hep-th].

C. Charmousis, “From Lovelock to Horndeski’s Generalized Scalar Tensor Theory,” *Lect. Notes Phys.* **892** (2015) 25–56, arXiv:1405.1612 [gr-qc].

I. Banerjee, S. Chakraborty, and S. SenGupta, “Excavating black hole continuum spectrum: Possible signatures of scalar hairs and of higher dimensions,” *Phys. Rev. D* **96** no. 8, (2017) 084035, arXiv:1707.04494 [gr-qc].
[88] S. Mukherjee and S. Chakraborty, “Horndeski theories confront Gravity Probe B,” arXiv:1712.00562 [gr-qc].

[89] S. Carloni, R. Goswami, and P. K. S. Dunsby, “A new approach to reconstruction methods in $f(R)$ gravity,” Class. Quant. Grav. 29 (2012) 135012, arXiv:1005.1840 [gr-qc].

[90] S. Nojiri, S. D. Odintsov, A. Toporensky, and P. Tretyakov, “Reconstruction and deceleration-acceleration transitions in modified gravity,” Gen. Rel. Grav. 42 (2010) 1997–2008, arXiv:0912.2488 [hep-th].

[91] S. Nojiri, S. D. Odintsov, and D. Saez-Gomez, “Cosmological reconstruction of realistic modified F(R) gravities,” Phys. Lett. B681 (2009) 74–80, arXiv:0908.1269 [hep-th].

[92] D. Pirtskhalava, L. Santoni, E. Trincherini, and F. Vernizzi, “Weakly Broken Galileon Symmetry,” JCAP 1509 no. 09, (2015) 007, arXiv:1505.00007 [hep-th].

[93] S. Banerjee and E. N. Saridakis, “Bounce and cyclic cosmology in weakly broken galileon theories,” Phys. Rev. D95 no. 6, (2017) 063523, arXiv:1604.06932 [gr-qc].

[94] R. Banerjee, S. Chakraborty, A. Mitra, and P. Mukherjee, “Cosmological implications of shift symmetric Galileon field,” Phys. Rev. D96 no. 6, (2017) 064023, arXiv:1705.06941 [gr-qc].

[95] Planck Collaboration, N. Aghanim et al., “Planck 2015 results. XI. CMB power spectra, likelihoods, and robustness of parameters,” Astron. Astrophys. 594 (2016) A11, arXiv:1507.02704 [astro-ph.CO].

[96] Planck Collaboration, P. A. R. Ade et al., “Planck 2015 results. XIII. Cosmological parameters,” Astron. Astrophys. 594 (2016) A13, arXiv:1502.01589 [astro-ph.CO].

[97] G. Hikmawan, J. Soda, A. Suroso, and F. P. Zen, “Comment on Gauss-Bonnet inflation,” Phys. Rev. D93 no. 6, (2016) 068301, arXiv:1512.00222 [hep-th].