A New $\mathcal{N} = 1$ AdS$_4$ Vacuum of Maximal Supergravity

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Abstract

The recent comprehensive numerical study of critical points of the scalar potential of four-dimensional $\mathcal{N} = 8$, SO(8) gauged supergravity using Machine Learning software in [1] has led to a discovery of a new $\mathcal{N} = 1$ vacuum with a triality-invariant SO(3) symmetry. Guided by the numerical data for that point, we obtain a consistent SO(3) $\times$ $\mathbb{Z}_2$-invariant truncation of the $\mathcal{N} = 8$ theory to an $\mathcal{N} = 1$ supergravity with three chiral multiplets. Critical points of the truncated scalar potential include both the $\mathcal{N} = 1$ point as well as two new non-supersymmetric and perturbatively unstable points not found by previous searches. Studying the structure of the submanifold of SO(3) $\times$ $\mathbb{Z}_2$-invariant supergravity scalars, we find that it has a simple interpretation as a submanifold of the 14-dimensional $\mathbb{Z}_2^3$-invariant scalar manifold (SU(1, 1)/U(1))$^7$, for which we find a rather remarkable superpotential whose structure matches the single bit error correcting (7, 4) Hamming code. This 14-dimensional scalar manifold contains approximately one quarter of the known critical points. We also show that there exists a smooth supersymmetric domain wall which interpolates between the new $\mathcal{N} = 1$ AdS$_4$ solution and the maximally supersymmetric AdS$_4$ vacuum. Using holography, this result indicates the existence of an $\mathcal{N} = 1$ RG flow from the ABJM SCFT to a new strongly interacting conformal fixed point in the IR.
1 Introduction

The four-dimensional gauged supergravity of de Wit and Nicolai [2] has proven to be a remarkably rich theory with a plethora of applications. It has an SO(8) gauge group and the maximal $\mathcal{N} = 8$ supersymmetry. Much of the interesting physics in this theory arises from a highly nontrivial potential for the 70 scalar fields. It is perhaps fair to say that unlocking that physics is tantamount to understanding the structure of the potential. In particular, it has been a long standing problem to determine all of its critical points, which lead to AdS$_4$ vacuum solutions of the theory.

A systematic study of the nontrivial critical points, that is other than the maximally supersymmetric SO(8)-invariant one with vanishing scalar fields, was initiated in [3, 4], where several
AdS$_4$ vacua were found by imposing certain symmetry constraints and thus effectively reducing the 70-dimensional scalar manifold to a smaller one, which could be fully analyzed.

As shown in [5–7], the four-dimensional SO(8) gauged supergravity is a consistent truncation of the eleven-dimensional supergravity on $S^7$. Therefore, classifying the critical points of the four-dimensional theory amounts to finding a large class of AdS$_4$ vacua of eleven-dimensional supergravity with internal space that is topologically $S^7$.

Those AdS$_4$ vacua of the SO(8) gauged supergravity and their uplifts to eleven dimensions are also interesting from the point of view of holography. Indeed, such backgrounds are dual to conformal field theories arising on the worldvolume of coincident M2-branes. The most studied and well-understood example is the ABJM theory [8], and its BLG version [9, 10], which has maximal supersymmetry. Another example is the so called mABJM SCFT which has $\mathcal{N} = 2$ supersymmetry and arises as a particular mass deformation of ABJM [11, 12].

The vacuum structure of the SO(8) gauged supergravity suggests that there are also conformal phases of M2-branes with $\mathcal{N} = 1$ and $\mathcal{N} = 0$ supersymmetry. In particular, the $G_2$-invariant and the $U(1) \times U(1)$-invariant critical points found in [4] and [14], respectively, preserve $\mathcal{N} = 1$ supersymmetry. The $SO(3) \times SO(3)$-invariant critical point found in [3] has no supersymmetry but it is perturbatively stable [14]. There is a conjecture that all non-supersymmetric AdS$_4$ vacua are unstable [15]. However, it has not been explicitly shown how the non-perturbative instability of the $SO(3) \times SO(3)$-invariant critical point might arise.

Due to the low amount of supersymmetry not much is known about these $\mathcal{N} = 0$ and $\mathcal{N} = 1$ strongly interacting CFTs. Nevertheless, it is interesting to understand whether there are any other supersymmetric or non-supersymmetric perturbatively stable critical points of the SO(8) gauged supergravity since this will amount to non-trivial predictions for the IR phases of the ABJM theory.

The consistent truncation of the maximal supergravity using a suitable symmetry as introduced by Warner [3] has remained the state of art for the analytic studies of the potential since 1982. However, more recently there has been also a considerable progress in developing numerical techniques to search for the critical points in the full 70-parameter space. Those methods were used by one of us to explore the vacuum structure of maximal gauged supergravity theories in three dimensions [16, 17] and then ported to four dimensions in [18–20]. In particular, a new $\mathcal{N} = 1$ supersymmetric critical point $S1200000^1$ was discovered in [18] and, using the numerical data as a guide, subsequently confirmed analytically in [14].

Recently, a new numerical approach based on Machine Learning (ML) software libraries, such as Google’s TensorFlow [21], was employed in [1] to simplify the analysis of the potential resulting in the total of 192 critical points together with a precise information about those points that

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1Following [20], we label the critical points by the first 7 digits of the critical value of the potential.
includes the mass spectra of small fluctuations and unbroken (super)symmetries. It is expected that this list of critical points should be nearly complete.

Perhaps the most interesting result of the search in [1] is a discovery of yet another $\mathcal{N} = 1$ supersymmetric critical point, $S1384096$, which is invariant under a triality symmetric $SO(3)$ subgroup of the $SO(8)$ gauge group. Moreover, this point gives rise to the only new AdS$_4$ solution that is perturbatively stable. Therefore, it is most interesting to understand how to construct it using a more analytic approach. This is our goal in this paper.

The numerical data for the $\mathcal{N} = 1$ critical point, $S1384096$, in [1] point towards additional symmetry, which we identify as a discrete $Z_2$ subgroup of the $SO(8)$ gauge group. The resulting $SO(3) \times Z_2$-invariant truncation of the $\mathcal{N} = 8$ supergravity can be constructed analytically. Its bosonic sector consists of the metric and three complex scalar fields. Despite the small scalar sector, the potential in this truncation has 15 inequivalent critical points with $S1384096$ amongst them. Surprisingly, two of those 15 critical points, $S2096313$ and $S2443607$, were not found by the numerical search in [1] and thus they represent new AdS$_4$ vacua. However, they are not supersymmetric and are perturbatively unstable.

We also study some of the properties of the supersymmetric point $S1384096$, in more detail. In particular, we compute the mass spectrum of excitations for all bosonic and fermionic fields of the $\mathcal{N} = 8$ supergravity around this point. Using holography, we map it to the spectrum of operators in the dual $\mathcal{N} = 1$ three-dimensional SCFT, which are then organized into multiplets of $\mathcal{N} = 1$ superconformal symmetry. Our explicit analytic construction of the truncation with three complex scalar fields also allows us to initiate the study of the web of holographic RG flows connecting the four supersymmetric critical points in this model. We find explicit domain wall solutions which interpolate between $S1384096$ and the maximally supersymmetric critical point of the $\mathcal{N} = 8$ supergravity.

In the next section we discuss two $SO(3)$- and $SO(3) \times Z_2$-invariant truncations of the maximal supergravity. In Section 3 we show that the new $\mathcal{N} = 1$ AdS$_4$ solution found in [1] corresponds to a critical point in the $SO(3) \times Z_2$-invariant truncation. In Section 4 we apply the same numerical technique as in [1] to the potential in the $SO(3) \times Z_2$-invariant truncation and find the total of 15 critical points that also include two non-supersymmetric and perturbatively unstable ones that were missed by previous searches. We show how other well-known critical points arise in our truncation. In Section 6 we perform a preliminary study of the holographic RG flows to the new $\mathcal{N} = 1$ point. In Section 7 we present an $\mathcal{N} = 1$ supergravity truncation of the maximal supergravity with 7 complex scalar fields which contains all perturbatively stable critical points. We conclude with some comments in Section 8. Some group theory details, the full spectrum of four-dimensional supergravity fields around the new $\mathcal{N} = 1$ AdS$_4$ vacuum, as well as more details on the two new non-supersymmetric critical points are given in the appendices.
2 A consistent truncation

The starting point of our analytic search for the new $\mathcal{N} = 1$ critical point, $S_{1384096}$, is the precise information about its symmetry that is given, together with the numerical data for the position of the point and the spectrum of supergravity fluctuations, in [1]. Specifically, we know that $S_{1384096}$ lies within an SO(3)-invariant sector of the 70-dimensional scalar manifold, $E_7(7)/SU(8)/Z_2$, of the $\mathcal{N} = 8$ supergravity. The particular SO(3) symmetry is identified as the triality invariant subgroup of the SO(8) gauge group specified by the following branchings of the three fundamental representations:

$$8_{v,s,c} \rightarrow 3 \oplus 3 \oplus 1 \oplus 1.$$  \hfill (2.1)

Using the standard group theory summarized in Appendix A, this completely determines the embedding of that SO(3) into both SO(8) and $E_7(7)$ at the level of Lie algebras:

$$\mathfrak{so}(8) \supset \mathfrak{so}(3) \times \mathfrak{u}(1) \times \mathfrak{u}(1) \quad \text{and} \quad \mathfrak{e}_{7(7)} \supset \mathfrak{so}(3) \times \mathfrak{g}_2(2) \times \mathfrak{su}(1,1).$$  \hfill (2.2)

Indeed, those embeddings are confirmed by the U(1) $\times$ U(1) unbroken gauge symmetry and the presence of $8 + 2 = 10$ scalar fluctuations that are SO(3) singlets in the $\mathcal{N} = 8$ supergravity spectrum around that point.

It follows from (2.2) that the scalar manifold spanned by the SO(3)-invariant scalars is the coset

$$\frac{G_2(2)}{SO(4)} \times \frac{SU(1,1)}{U(1)}.$$  \hfill (2.3)

In fact, keeping track of all invariant fields in the $\mathcal{N} = 8$ supergravity, one finds that the (consistent) SO(3)-invariant truncation is to a four-dimensional $\mathcal{N} = 2$ gauged supergravity coupled to an Abelian vector multiplet and two hypermultiplets. The scalars in the hyper and the vector multiplets parametrize the first and second factor in (2.3), respectively. While considerably simpler than the full $\mathcal{N} = 8$ theory, this truncation is still too complicated to effectively work with.

The crucial hint that allows us to further simplify the analytic search for $S_{1384096}$ comes from the numerical values of the scalar fields at that point. It has been observed in [1] that one can specify the position of this point in terms of 6 independent scalars when using a certain parametrization of the $E_7(7)/SU(8)$ coset. This suggests the presence of an additional discrete symmetry that might allow further truncation of the scalar manifold to a smaller subspace.

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2See, Table B.1 in Appendix B.

3Incidentally, the first factor in the coset has appeared in the SU(3)-invariant truncation of the type IIB supergravity [22, 23] and in the SO(3)-invariant consistent truncation of the maximal five-dimensional SO(6) supergravity [24].
To identify that discrete symmetry we will use the standard parametrization of the scalar manifold of the $\mathcal{N} = 8$ theory in which the scalar 56-bein is given by

$$V = \left( \begin{array}{cc} u_{ijIJ} & v_{ijIJ} \\ v_{kIJ} & u^{kL} \end{array} \right) = \exp \left( \begin{array}{cc} 0 & -\frac{1}{4} \sqrt{2} \phi_{ijkl} \\ -\frac{1}{4} \sqrt{2} \bar{\phi}^{ijkl} & 0 \end{array} \right) \in E_{7(7)},$$

where the scalar fields, $\phi_{ijkl}$, are components of a complex selfdual 4-form in $\mathbb{R}^8$,

$$\Phi = \frac{1}{24} \sqrt{2} \phi_{ijkl} dx^i \wedge dx^j \wedge dx^k \wedge dx^l,$$

and the indices $i, j, \ldots$ transform in the $8_v$ representation of $SO(8)$. To simplify the notation, the wedge product above will be denoted by $dx^{ijkl}$.

We take the $SO(3)$ symmetry of the truncation to act diagonally on the indices $(123)$ and $(456)$ with the singlets $(7)$ and $(8)$. The invariant forms under this action are spanned by:

$$\Phi^{(1)} = \omega_1 \left( dx^{1267} - dx^{1357} + dx^{2347} \right) + \bar{\omega}_1 \left( dx^{1568} - dx^{2468} + dx^{3458} \right),$$

$$\Phi^{(2)} = \omega_2 dx^{4567} + \bar{\omega}_2 dx^{1238},$$

$$\Phi^{(3)} = \omega_3 \left( dx^{1245} + dx^{1346} + dx^{2356} \right) + \bar{\omega}_3 \left( dx^{1478} + dx^{2578} + dx^{3678} \right),$$

and

$$\Phi^{(4)} = \vartheta_1 \left( dx^{1268} - dx^{1358} + dx^{2348} \right) - \bar{\vartheta}_1 \left( dx^{1567} - dx^{2467} + dx^{3457} \right),$$

$$\Phi^{(5)} = \vartheta_2 dx^{4568} - \bar{\vartheta}_2 dx^{1237},$$

where

$$\omega_a = s_{2a-1} + i s_{2a}, \quad a = 1, 2, 3, \quad \vartheta_j = t_{2j-1} + i t_{2j}, \quad j = 1, 2.$$

Each $\Phi^{(\alpha)}$ contains a scalar and a pseudoscalar that parametrize an $SU(1, 1)/U(1)$ coset. The $SU(1, 1)$ subalgebras of $E_{7(7)}$ generated by $\Phi^{(1)}$, $\Phi^{(2)}$ and $\Phi^{(3)}$ mutually commute as do the two subalgebras corresponding to $\Phi^{(4)}$ and $\Phi^{(5)}$. The $\Phi^{(1)}$, $\Phi^{(2)}$, $\Phi^{(4)}$ and $\Phi^{(5)}$ correspond to the noncompact generators of $G_{2(2)}$ and $\Phi^{(3)}$ to the $SU(1, 1)$ that commutes with that $G_{2(2)}$.

This truncation contains the smaller $SU(3)$-invariant truncation used by Warner [4] and more recently discussed in [20]. It is obtained by setting $\omega_1 = -\omega_2$ and $\vartheta_1 = -\vartheta_2$, which results in the noncompact group $SU(2, 1) \times SU(1, 1)$.

Using the numerical data for $S1384096$ in [11], we find that, modulo a suitable $SO(8)$ gauge rotation, the critical point of the potential lies within the 6-dimensional subspace spanned by $\Phi^{(1)}$, $\Phi^{(2)}$ and $\Phi^{(3)}$. We also find that the fluctuation of the gravitino field, $\psi_\mu^8$, remains massless as required by the unbroken $\mathcal{N} = 1$ supersymmetry.

Consider the following discrete symmetry

$$g_S: \quad (x^1, x^2, x^3, x^4, x^5, x^6, x^7, x^8) \rightarrow (x^1, x^2, x^3, -x^4, -x^5, -x^6, -x^7, x^8).$$

(2.9)
Clearly, \( g_S \) is an SO(8) rotation that does not belong to the SO(3) symmetry subgroup. Under the action of \( g_S \), the forms in (2.6) are even while the ones in (2.7) are odd, and the correct supersymmetry is preserved. This provides us with an additional discrete symmetry for the truncation to 6 scalar fields, where the truncation simply amounts to setting

\[
t_1 = t_2 = t_3 = t_4 = 0. \tag{2.10}
\]

In the next section we will confirm directly that the critical point \( S_{1384096} \) indeed lies in the \( \text{SO}(3) \times \mathbb{Z}_2 \) invariant sector, where the \( \mathbb{Z}_2 \) is generated by the SO(8) rotation \( g_S \).

To summarize, we have been led by the group theory and numerical data to consider a \( \text{SO}(3) \times \mathbb{Z}_2 \)-invariant truncation of the \( \mathcal{N} = 8 \ d = 4 \) supergravity with the scalar coset

\[
\frac{\text{SU}(1,1)}{U(1)} \times \frac{\text{SU}(1,1)}{U(1)} \times \frac{\text{SU}(1,1)}{U(1)}. \tag{2.11}
\]

The first two factors above are embedded in the first factor in (2.3), and the last factor above corresponds to the second factor in (2.3). The resulting theory is an \( \mathcal{N} = 1 \ d = 4 \) supergravity coupled to 3 scalar multiplets. The \( \mathcal{N} = 8 \) fields that remain in this truncation are indicated by the star in Tables B.1-B.4.

To recast the bosonic sector of this \( \mathcal{N} = 1 \ d = 4 \) supergravity in a canonical form [27], we use the usual complex coordinates, \( z_a \), on each \( \text{SU}(1,1)/U(1) \). First set

\[
s_{2a-1} + is_{2a} = \lambda_a e^{i\varphi_a}, \quad a = 1, 2, 3, \tag{2.12}
\]

with the \( s_a \) given in (2.8), and then define

\[
z_a = \tanh(\frac{1}{2}\lambda_a) e^{i\varphi_a}, \quad a = 1, 2, 3. \tag{2.13}
\]

In this parametrization of the scalar fields, the bosonic Lagrangian of the truncated four-dimensional supergravity is given by

\[
\mathcal{L} = \frac{1}{2} R - K^{ab} \partial_\mu z_a \partial^\mu z_b - g^2 \mathcal{P}. \tag{2.14}
\]

The scalar kinetic term is determined by the Kähler metric of the coset (2.11) with Kähler potential

\[
K = -3 \log(1 - z_1 \bar{z}_1) - \log(1 - z_2 \bar{z}_2) - 3 \log(1 - z_3 \bar{z}_3), \tag{2.15}
\]

where the integer coefficients of the logarithms are the embedding indices of the numerator SU(1,1)’s in (2.11) in \( E_7/\mathbb{Z}_7 \). The Kähler metric and its inverse are defined by

\[
K_{a\bar{b}} = \partial_a \partial_{\bar{b}} K, \quad K^{a\bar{b}} = (K_{a\bar{b}})^{-1}. \tag{2.16}
\]
The potential can be succinctly written as
\[ P = 2e^K \left( K^{ab} \nabla_a W \nabla_b \bar{W} - 3W \bar{W} \right), \] \hspace{1cm} (2.17)
where the holomorphic superpotential is\footnote{The holomorphic superpotential can be read-off from the component of the $A_{ij}$-tensor of the $\mathcal{N} = 8$ supergravity along the unbroken supersymmetry.}
\[ W = (z_3 - 1)(z_1^3z_2z_3^2 + z_1^3z_2z_3 + z_1^3z_2^2 - 3z_1z_2z_3 - z_3^2 - z_3 - 1), \] \hspace{1cm} (2.18)
and the Kähler covariant derivative is given by
\[ \nabla_a (\cdot) = \partial_a (\cdot) + (\cdot) \partial_a K. \] \hspace{1cm} (2.19)

### 3 The new $\mathcal{N} = 1$ critical point

In this section we will look first for those critical points of the potential (2.17) that preserve the $\mathcal{N} = 1$ (or more) supersymmetry of our truncation. The expectation is that those points should include the new supersymmetric point, $S1384096$.

Supersymmetric critical points correspond to the “covariant extrema” of the holographic superpotential (2.18) satisfying
\[ \nabla_a W = 0, \quad \nabla_b \bar{W} = 0, \quad a, b = 1, 2, 3. \] \hspace{1cm} (3.1)

It is easy to check that any solution to (3.1) is also a critical point of the potential (2.17).

In the domain $|z_a| < 1$, (3.1) unpack to the following system of septic polynomial equations
\[ z_1^2z_2 + 2z_1z_3 - z_2z_3 + z_1^2z_2^2 + z_1z_2z_3^2 - z_1(1 + z_3 - z_1^2z_3 + 2z_1z_2z_3 + z_3^2) = 0, \]
\[ z_1^3 - 3z_1z_3 - 3z_1^2z_2^2 - z_1^3z_3^2 - z_2(1 - 3z_1^2z_3 + 3z_1^2z_3^2 - z_3^3) = 0, \]
\[ z_1^2 - z_1z_2 - 2z_1^2z_3 + 2z_1z_2z_3 + z_3^2 - z_1z_2z_3^2 - z_2z_3^2 - z_1z_2z_3^2 = 0, \] \hspace{1cm} (3.2)
plus the three complex conjugate equations.

Finding all solutions to the system (3.2) analytically, if feasible at all, will require techniques that go beyond what is employed in the present article. However, one can fully analyze (3.2) using standard numerical routines such as NSolve\footnote{\texttt{NSolve[]}} in Mathematica \cite{28}. In this way we recover three known supersymmetric critical points with the $SO(8)$, $G_2$ and $SU(3) \times U(1)$ invariance, respectively. As we discuss in Section 4.2 each can be found analytically by performing a further truncation that reduces (3.2) to a simpler system.
We also find four numerical solutions at approximately
\begin{align*}
  z_1 &= 0.1696360 \pm 0.1415740 i, \\
  z_2 &= 0.4833214 \pm 0.3864058 i, \\
  z_3 &= -0.3162021 \pm 0.5162839 i. \\
\end{align*}
(3.3)

The value of the potential at these points is
\begin{equation}
P = -13.840964, \tag{3.4}
\end{equation}
which is the same as at $S_{1384096}$ in $[1]$.

As we discuss in more detail in Appendix C, there is a residual action of the SO(8) gauge symmetry on the coset (2.11). Indeed, solutions in the two columns in (3.3) are related by the rotation, $g_H$, defined in (C.3). In turn, solutions within each column are related by complex conjugation. This degeneracy is expected given the corresponding invariance of the system of equations in (3.1) or, equivalently, (3.2). However, those complex conjugate solutions are not related by an SO(8) rotation and thus represent two distinct critical points of the potential in the $N = 8$ supergravity.

To complete the identification of the solutions (3.3) with the new supersymmetric point, $S_{1384096}$, and its complex conjugate, $\bar{S}_{1384096}$, in $[1]$, one can also perform an explicit change of variables (D.1) accompanied by an SO(8) rotation.

The critical values (3.3) of the coordinates, $z_a$, $a = 1, 2, 3$, can be efficiently determined to an arbitrary precision from the roots of the following system of integer coefficient polynomials:
\begin{align}
P_{\text{Re} z_1}(x) &= 256 x^{24} + 5632 x^{22} + 78592 x^{20} - 2135808 x^{18} - 543360 x^{16} - 4684032 x^{14} \\
  &- 12045600 x^{12} - 15419808 x^{10} - 6033744 x^8 - 1553904 x^6 - 222264 x^4 \\
  &- 17496 x^2 + 729, \\
\end{align}
(3.5)
\begin{align}
P_{\text{Im} z_1}(y) &= 16 y^{12} - 96 y^{11} + 144 y^{10} + 848 y^9 - 944 y^8 - 2000 y^7 - 3504 y^6 + 3456 y^5 \\
  &- 2028 y^4 + 740 y^3 - 164 y^2 + 20 y - 1. \\
\end{align}
(3.6)
\begin{align}
P_{\text{Re} z_2}(x) &= 60715264 x^{24} - 256862720 x^{22} + 937708288 x^{20} - 2138845440 x^{18} \\
  &+ 3067400064 x^{16} - 2992061952 x^{14} + 1409137632 x^{12} - 388837152 x^{10} \\
  &+ 229269744 x^8 - 95418000 x^6 + 4147848 x^4 + 1627128 x^2 + 59049, \\
\end{align}
(3.6)
\begin{align}
P_{\text{Im} z_2}(y) &= 7792 y^{12} + 4320 y^{11} + 26256 y^{10} + 16832 y^9 + 62032 y^8 + 107504 y^7 + 70872 y^6 \\
  &+ 37872 y^5 + 14172 y^4 + 19880 y^3 + 4900 y^2 - 1372 y - 2401, \\
\end{align}

\footnote{Since such “conjugate” critical points have the same values of the potential and the same mass spectra of fluctuations around them, they were identified as a single point in the numerical searches $[20, 1]$.}
\footnote{See, Section 5.}
\[ P_{\text{Re} z_3}(x) = 16x^{17} + 96x^{16} + 496x^{15} + 672x^{14} + 456x^{13} - 1584x^{12} - 1384x^{11} - 816x^{10} \\
+ 1388x^9 - 1512x^8 + 462x^7 + 2028x^6 + 537x^5 + 810x^4 - 819x^3 - 639x^2 \\
- 180x - 27, \\
\]
\[ P_{\text{Im} z_3}(y) = 768y^{24} - 48128y^{22} + 1018112y^{20} - 2517248y^{18} + 4496192y^{16} - 8476736y^{14} \\
+ 7496864y^{12} - 8223008y^{10} + 4957568y^8 - 1487120y^6 + 233460y^4 \\
- 18900y^2 + 675. \]

Polynomials \( P_{\text{Re} z_a}(x) \) and \( P_{\text{Im} z_a}(y) \), \( a = 1, 2 \), have precisely two real roots, \( \pm x_a^* \), and one real root, \( y_a^* \), respectively, such that \( z_a = x_a^* + iy_a^* \), \( x_a^*, y_a^* > 0 \), lie in the unit disk. Similarly, \( P_{\text{Re} z_3}(x) \) and \( P_{\text{Im} z_3}(y) \) have one real root, \( x_3^* \), and two real roots, \( \pm y_3^* \), \( y_3^* > 0 \), with \( |z_3^*| < 1 \). Then

\[
(z_1^*, z_2^*, z_3^*), \quad (-z_1^*, -z_2^*, z_3^*), \quad (\bar{z}_1^*, \bar{z}_2^*, \bar{z}_3^*), \quad (-\bar{z}_1^*, -\bar{z}_2^*, \bar{z}_3^*),
\]

are the four solutions to (3.1) that we found above in (3.3). By acting on these solutions with the SO(8) rotation \( g_C \) defined in (C.3), we obtain 4 additional solutions that exhaust the critical points of the potential (2.17) with this critical value. Those 8 critical points of (2.17) represent two different critical points of the \( \mathcal{N} = 8 \) supergravity.

The exact value of the potential at these critical points is given by the negative real root of the following polynomial [1],

\[
5^{15}v^{12} - (2^8 \cdot 3^4 \cdot 7 \cdot 53 \cdot 107 \cdot 887 \cdot 1567)v^8 + (2^{15} \cdot 3^{17} \cdot 210719)v^4 - 2^{20} \cdot 3^{30} = 0. \quad (3.9)
\]

The mass spectra of the fermion and scalar fluctuations of \( \mathcal{N} = 8 \) supergravity are the same around each of the points and have been obtained as part of the numerical search in [1]. Those results are summarized in Tables B.2, B.4 and B.1 in Appendix B. In particular, the presence of a single unit mass gravitino mode in the spectrum shows that there is \( \mathcal{N} = 1 \) unbroken supersymmetry. In Appendix B we also find the masses of the fluctuations of the vector fields, which allows us to verify explicitly that the entire spectrum of operators in the dual three-dimensional superconformal field theory can be arranged into multiplets of the \( \mathcal{N} = 1 \) superconformal algebra, \( \text{osp}(1|4) \). This provides a nontrivial consistency test for the calculation of the spectra and of the unbroken supersymmetry.

4 All critical points of the truncated potential

Although the truncated potential (2.17) can be written in a closed analytic form, to determine all of its critical points we have to resort to numerical methods outlined in Section 5. Here, we summarize the results of that search, which yielded critical points with 15 different values of the cosmological constant, including the two new ones that were not captured by the search in [1].
Given a critical point at \((z_1, z_2, z_3)\), the reality of the potential implies that there is another (conjugate) critical point at \((\bar{z}_1, \bar{z}_2, \bar{z}_3)\). In Appendix C we argue that unless \(z_3\) is real, the two points are not related by an SO(8) rotation and thus represent two distinct critical points of the potential in the \(\mathcal{N} = 8\) supergravity. In addition, we show that each point in the coset (2.11) lies on an orbit of the discrete subgroup of SO(8) that preserves the coset. This discrete subgroup is generated by the rotations \(g_H\) and \(g_C\) defined in (C.3). Generically, the corresponding orbit through a point \((z_1, z_2, z_3)\) consists of 4 points:

\[
(z_1, z_2, z_3), \quad (-z_1, -z_2, z_3), \quad (\bar{z}_1, \bar{z}_2, z_3), \quad (-\bar{z}_1, -\bar{z}_2, z_3),
\]

obtained by applying the rotations \(1, g_H, g_C\) and \(g_H g_C\), respectively. Clearly, when \(z_1\) and \(z_2\) \((z_1 z_2 \neq 0)\) are both either real or imaginary, that orbit degenerates to just two points. For some points we also find additional discrete SO(8) rotations (C.1) that preserve the coset (2.11) at that particular point giving rise to additional critical points of the potential (2.17).

The end result is that for each critical value of the potential (2.17) at \((z_1, z_2, z_3)\), there are either two orbits or a single orbit of critical points, namely (4.1) and its complex conjugate, or just (4.1) when \(z_3\) is real. In \(\mathcal{N} = 8\) supergravity those SO(8) orbits correspond two “conjugate” critical points, \(S_{n_1 \ldots n_7}\) and \(\bar{S}_{n_1 \ldots n_7}\), or a single point, \(S_{n_1 \ldots n_7}\), respectively.

All points in this section have at least an SO(3) symmetry, which is the continuous symmetry of the truncation. We should note that there are other critical points in [1] that are SO(3)-invariant: \(S_{0847213}, S_{1075828}, S_{1195898}, S_{1271622}, S_{2503105}\). However, their symmetry is incompatible with the symmetry of our truncation since it corresponds to a different embedding of SO(3) in SO(8).

In the following we list all the critical points of the potential (2.17). For each point we give its location, the value of the potential, the continuous symmetry, and the SO(8) rotations for the orbit(s). All but two of those points were discovered in previous searches as indicated by the references where they first appeared. For all but two points the location and the critical value of the potential are known in either a closed analytic form or via a minimal polynomial. In some cases where the explicit analytic form is too involved, we omit it.

The mass spectra of scalar fluctuations around the known points can be found in [1]. For the two new points, the mass spectra are given in Appendix D. Perhaps unsurprisingly, the only perturbatively stable points, that is with the scalar masses satisfying the Breitenlohner-Freedman bound [29], are the supersymmetric ones and the non-supersymmetric SO(3) × SO(3)-invariant point, \(S_{1400000}\). In particular, both new points are perturbatively unstable. Overall, for the solutions \(S_{0668740}, S_{0698771}, S_{0800000}, \) and \(S_{1424025}\), all instabilities in this truncation are due to modes that are not SO(3) singlets and thus can be seen only within the full \(\mathcal{N} = 8\) supergravity. For the SU(4)-invariant solution \(S_{0800000}\) this observed previously in [26].

\[\text{Also, see earlier work referred to in [1].}\]
4.1 The critical points

**S0600000** [2]

\[ z_1 = z_2 = z_3 = 0, \quad P = -6. \quad (4.2) \]

Symmetry: SO(8), \( N = 8 \). Orbit: \( \langle 1 \rangle \).

**S0668740** [30]

\[ z_1 = -z_2 = -z_3 = \frac{1}{2} \left( 3 + \sqrt{5} - \sqrt{10 + 6\sqrt{5}} \right) \approx 0.1985088, \quad (4.4) \]

\[ P = -2 \cdot 5^{3/4} \approx -6.687403. \quad (4.5) \]

Symmetry: SO(7)\(^+\), \( N = 0 \). Orbit: \( \langle 1, g_H \rangle \).

**S0698771 & S0698771** [31, 32]

\[ z_1 = -z_2 = -z_3 = -i(2 - \sqrt{5}) \approx 0.2360680i, \quad (4.7) \]

\[ P = -\frac{5^{5/2}}{8} \approx -6.987712. \quad (4.8) \]

Symmetry: SO(7)\(^-\), \( N = 0 \). Orbits: \( \langle 1, g_H \rangle \).

**S0719157 & S0719157** [3]

\[ z_1 = -z_2 = -z_3 = \frac{1}{4} \left( 3 + \sqrt{3} - 3^{1/4} \sqrt{10} \right) \left( 1 - i 3^{-1/4} \sqrt{2 + \sqrt{3}} \right) \approx 0.1425648 + 0.2092695i, \quad (4.10) \]

\[ P = -\frac{2^{7/2} \cdot 3^{13/4}}{5^{5/2}} \approx -7.191576. \quad (4.11) \]

Symmetry: G_2, \( N = 1 \). Orbits: \( \langle 1, g_H, g_C, g_H g_C \rangle \).

\[ P_{\text{Re}z_1,2,3}(x) = 8x^4 + 24x^3 + 24x^2 + 24x + 3, \]
\[ P_{\text{Im}z_1,2,3}(x) = 64x^8 - 704x^6 + 240x^4 - 32x^2 + 1. \]
\[ z_1 = -z_2 = i \sqrt{5 - 2\sqrt{6}} \approx 0.3178372i, \quad z_3 = \sqrt{3} - 2 \approx -0.2679492, \quad (4.13) \]
\[ \mathcal{P} = -\frac{9\sqrt{3}}{2} \approx -7.794229. \quad (4.14) \]
Symmetry: SU(3) × U(1), \quad \mathcal{N} = 2. \quad \text{Orbit: } \langle 1, g_H \rangle.
\[ P_{\text{Im} z_1,z_2}(x) = x^4 - 10x^2 + 1, \quad P_{\text{Re} z_3}(x) = x^2 + 4x + 1. \quad (4.15) \]

**S0800000** \[4\]
\[ z_1 = -z_2 = i (\sqrt{2} - 1) \approx 0.4142136i, \quad z_3 = \bar{z}_3 = 0, \quad (4.16) \]
\[ \mathcal{P} = -8. \quad (4.17) \]
Symmetry: SU(4), \quad \mathcal{N} = 0. \quad \text{Orbit: } \langle 1, g_H \rangle.
\[ P_{\text{Im} z_1,z_2}(x) = x^2 + 2x - 1. \quad (4.18) \]

**S0869597** \[20\]
\[ z_1 = i \sqrt{9 + 2\sqrt{21} - 2\sqrt{41 + 9\sqrt{21}}} \approx 0.1659702i, \]
\[ z_2 = \frac{i}{67} \sqrt{7521 + 738\sqrt{21} - 2\sqrt{11962961 + 2775249\sqrt{21}}} \approx 0.4641278i, \quad (4.19) \]
\[ z_3 = \frac{1}{4} \left( -1 - \sqrt{21} + \sqrt{2 \left( 3 + \sqrt{21} \right)} \right) \approx -0.4220824, \]
\[ \mathcal{P} = -\frac{4}{5} \sqrt{54 + 14\sqrt{21}} \approx -8.695969. \quad (4.20) \]
Symmetry: SO(3) × U(1), \quad \mathcal{N} = 0. \quad \text{Orbit } \langle 1, g_H \rangle.
\[ P_{\text{Im} z_1}(x) = x^8 - 36x^6 - 10x^4 - 36x^2 + 1, \]
\[ P_{\text{Im} z_2}(x) = 4489x^8 - 30084x^6 + 49190x^4 - 30084x^2 + 4489, \quad (4.21) \]
\[ P_{\text{Re} z_3}(x) = x^4 + x^3 - 3x^2 + x + 1. \]

**S0880733** \[14\]
\[ z_1 = z_2 = i \sqrt{3 + 2\sqrt{3} - 2\sqrt{5 + 3\sqrt{3}}} \approx 0.2789600i, \quad (4.22) \]
\[ z_3 = -\frac{1}{2} + \frac{3^{1/4}}{\sqrt{2}} - \frac{\sqrt{3}}{2} \approx -0.435421, \]
\[ P = -2\sqrt{9 + 6\sqrt{3}} \approx -8.807339. \]  
(4.23)

Symmetry: \( \text{SO}(4) \), \( \mathcal{N} = 0 \).\ Orbit: \( \langle 1, g_{R}, g_{H}g_{R} \rangle \).

\[ P_{\text{Im} z_{1,2}}(x) = x^8 - 12x^6 - 10x^4 - 12x^2 + 1, \]
\[ P_{\text{Re} z_{3}}(x) = x^4 + 2x^3 + 2x + 1. \]  
(4.24)

**S0983994 & \overline{S0983994} [20]**

\[ z_{1} \approx 0.184246, \quad z_{2} \approx 0.5073269, \quad z_{3} \approx 0.1331835 + 0.4676097i, \]  
(4.25)

\[ \mathcal{P} = -5 \cdot 15^{1/4} \approx -9.839948. \]  
(4.26)

Symmetry: \( \text{SO}(3) \times \text{U}(1) \), \( \mathcal{N} = 0 \). \ Orbits: \( \langle 1, g_{H} \rangle \).

\[ P_{\text{Re} z_{1}}(x) = x^8 - 28x^6 - 42x^4 - 28x^2 + 1, \]
\[ P_{\text{Re} z_{2}}(x) = 289x^8 - 1372x^6 + 1302x^4 - 1372x^2 + 289, \]
\[ P_{\text{Re} z_{3}}(x) = 1397x^4 - 2380x^3 + 4350x^2 - 2380x + 245, \]
\[ P_{\text{Im} z_{3}}(x) = 1951609x^8 - 12150144x^6 + 22045824x^4 - 17915904x^2 + 2985984. \]  
(4.27)

**S1039230 & \overline{S1039230} [33]**

\[ z_{1} = z_{2} = \sqrt{5 - 2\sqrt{6}} \approx 0.3178372, \quad z_{3} = -i\sqrt{2 - \sqrt{3}} \approx -0.517638i, \]  
(4.28)

\[ \mathcal{P} = -6\sqrt{3} \approx -10.39230. \]  
(4.29)

Symmetry: \( \text{SO}(4) \), \( \mathcal{N} = 0 \). \ Orbits: \( \langle 1, g_{H}, g'_{R}, g_{H}g'_{R} \rangle \).

\[ P_{\text{Re} z_{1,2}}(x) = x^4 - 10x^2 + 1, \quad P_{\text{Im} z_{3}}(x) = x^4 - 4x^2 + 1. \]  
(4.30)

**S1384096 & \overline{S1384096} [1]**

\[ z_{1} \approx 0.1696360 + 0.1415740i, \]
\[ z_{2} \approx 0.4833214 + 0.3864058i, \]
\[ z_{3} \approx -0.3162021 - 0.5162839i, \]  
(4.31)

\[ \mathcal{P} = -13.840964. \]  
(4.32)

Symmetry: \( \text{SO}(3) \), \( \mathcal{N} = 1 \). \ Orbit: \( \langle 1, g_{H}, g_{C}, g_{H}g_{C} \rangle \).

Comment: See, Section 3.
\[ z_1 = z_2 = \frac{1}{2} (1 + i) \sqrt{3 - \sqrt{5}} \approx 0.4370160 (1 + i), \quad z_3 = \bar{z}_3 = 0, \quad (4.33) \]

\[ P = -14. \quad (4.34) \]

Symmetry: SO(3) \( \times \) SO(3), \( \mathcal{N} = 0 \). Orbit: \( \langle 1, g_H, g_C, g_H g_C \rangle \).

\[ P_{\text{Re} z_1, 2}(x) = P_{\text{Im} z_1, 2}(x) = 4x^4 - 6x^2 + 1. \quad (4.35) \]

Comment: This point is non-supersymmetric, but perturbatively stable [14].

\[ \bar{S}2096313 \text{ & } S2096313 \]

\[ z_1 \approx 0.4490422 + 0.4843455 i, \]
\[ z_2 \approx 0.3750597 + 0.2850151 i, \]
\[ z_3 \approx -0.0453920, \quad \mathcal{P} \approx -14.24026. \quad (4.36) \]

Symmetry: SO(3), \( \mathcal{N} = 0 \). Orbit: \( \langle 1, g_H, g_C, g_H g_C \rangle \).

Comment: Determined numerically, \( |\nabla \mathcal{P}| \sim 10^{-115} \).

\[ \bar{S}2443607 \text{ & } S2443607 \]

\[ z_1 \approx -i \left( 2 - \sqrt{5} \right) \approx 0.2360680 i, \]
\[ z_2 \approx -\frac{i}{2} \left( 1 - \sqrt{5} \right) \approx 0.6180340 i, \]
\[ z_3 \approx \frac{2}{\sqrt{21}} \left( 15 - 2\sqrt{5} \right) - \frac{i}{\sqrt{21}} \left( 49 - 12\sqrt{5} \right) \approx 0.5135543 - 0.5262366 i, \]
\[ \mathcal{P} = -\frac{75}{8} \sqrt{5} \approx -20.96314. \quad (4.38) \]

Symmetry: SO(3) \( \times \) U(1), \( \mathcal{N} = 0 \). Orbits: \( \langle 1, g_H \rangle \).

\[ P_{\text{Im} z_1}(x) = x^2 - 4x - 1, \quad P_{\text{Im} z_2}(x) = x^2 - x - 1, \]
\[ P_{\text{Re} z_3}(x) = 41x^2 - 60x + 20, \quad P_{\text{Im} z_3}(x) = 1681x^4 - 2058x^2 + 441. \quad (4.40) \]

\[ \bar{S}2443607 \text{ & } S2443607 \]

\[ z_1 \approx 0.2187103 + 0.1800635 i, \]
\[ z_2 \approx -0.2046730 + 0.4973759 i, \]
\[ z_3 \approx 0.4188443 - 0.6668735 i, \]
\[ \mathcal{P} \approx -24.43607. \quad (4.41) \]

Symmetry: SO(3), \( \mathcal{N} = 0 \). Orbits: \( \langle 1, g_H, g_C, g_H g_C \rangle \).

Comment: Determined numerically, \( |\nabla \mathcal{P}| \sim 10^{-12} \).
4.2 Subtruncations

The locations of the critical points above suggest a number of further truncations to simpler subsectors. In particular we have the $G_2$-invariant truncation, which in the parametrization used in [34] is obtained by setting

$$z_1 = -z_2 = -z_3 = z.$$  \hfill (4.43)

The superpotential and the Kähler potential reduce then to

$$W_{G_2} = z^7 + 7z^4 + 7z^3 + 1, \quad K_{G_2} = -7 \log(1 - z \bar{z}).$$ \hfill (4.44)

Within this truncation one finds 6 critical points: the $SO(8)$ point, $S0600000$, the $SO(7)^+$ point, $S0668740$, the $SO(7)^-$ points, $S0698771$ and $\bar{S}0698771$, and the $G_2$ points, $S0719157$ and $\bar{S}0719157$.

Other simple truncations to one complex scalar field are the $SO(4) \times SO(4)$-invariant truncation obtained by setting

$$z_1 = z_3 = 0, \quad z_2 = z,$$ \hfill (4.45)

with

$$W_{SO(4) \times SO(4)} = 1, \quad K_{SO(4) \times SO(4)} = -\log(1 - z \bar{z}),$$ \hfill (4.46)

and the $SU(3) \times U(1)^2$-invariant truncation

$$z_1 = z_2 = 0, \quad z_3 = -z,$$ \hfill (4.47)

with

$$W_{SU(3) \times U(1)^2} = z^3 + 1, \quad K_{SU(3) \times U(1)^2} = -3 \log(1 - z \bar{z}).$$ \hfill (4.48)

Although there are no critical points other than the maximally supersymmetric $S0600000$ one, those truncations admit nontrivial generalizations of the RG flows dual to three dimensional field theories with interfaces [34].

5 Numerical searches - an outline of the method

The TensorFlow code that was published alongside [1] is readily adapted to search for critical points not on the full 70-dimensional scalar manifold but on submanifolds that are invariant under some residual symmetry, such as the six-dimensional space studied here. As for the unconstrained problem, one starts from some random linear combination of the six $E_7(7)$ generators that is sufficiently close to the origin for the numerical value of the potential to still be reliable, and then numerically minimizes the violation of the (un-truncated) stationarity-condition.
This way, one manages to discover all the critical points on the scalar manifold listed in Section 4.1 after about 10,000 such iterations. We observe that some tweaks to the code as published can improve search efficiency further. In particular, it turns out to be beneficial to not use a second order numerical optimization method (such as BFGS) directly, but to first perform a few hundred gradient descent steps per iteration before switching to such a more advanced method. Intuitively, if a second order optimization method gets to see from the start a sum of stationarity-violation contributions having very different scale, it will tend to be mostly sensitive to the most important contribution’s second order approximation and hence in its first few steps move to very similar positions on the manifold, counteracting the need for good exploration.

Even with such tricks, the TensorFlow based search is fundamentally only a probabilistic method that converges to the various critical points with very uneven likelihood. So, it might be conceivable that, even with much computational effort, some critical points remain undiscovered. It hence makes sense to look for alternative approaches to the problem of finding critical points of algebraic functions (or, equivalently, intersections of algebraic varieties) on spaces of moderate dimension.

While simple techniques based on interval arithmetic or affine arithmetic appear too limited to conveniently study the restricted six-dimensional potential at hand, this problem is still well within reach of modern computational algebraic geometry.

For a task like this, one will typically want to first employ a modern computational algebraic geometry package such as \([35]\) to reduce/factorize the problem, and then use an adaptive-precision homotopy continuation solver such as Bertini2 \([36]\) that uses the algorithm described in \([37]\) to systematically find solutions of the generalized problem with complex coordinates. These solutions then have to be filtered, discarding all those with non-real coordinates. Depending on the difficulty of the task, computations may take hours to days with Bertini2, and while this approach might hypothetically still miss some solutions, this is not observed to happen in practice.

These numerical algebraic geometry methods found the same list of critical points for our six-scalar model as the TensorFlow based search. We also note that for the eight-dimensional scalar manifold studied in \([38]\), the same numerical methods manage to reproduce the list of critical points presented in that publication without uncovering additional ones\(^8\).

For all critical points listed in Section 4, with two exceptions, one can obtain algebraic expressions for their location, the potential, and other physical properties via inverse symbolic computation (i.e. employing the PSLQ algorithm). The exceptions are \(S_{1424025}\) and \(S_{2443607}\). In both cases, there does not seem to be a polynomial of degree no more than 36 with integer coefficients each of magnitude no larger than \(10^{60}\) that would have a zero at the value of the

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\(^8\)We thank Jonathan Hauenstein for performing these calculations.
potential computed to high accuracy.

6 Holographic RG flows

The truncation derived in Section 2 makes it feasible to study explicitly supersymmetric holographic RG flows to the new $\mathcal{N} = 1$ critical point. For other supersymmetric critical points that lie within our truncation such flows have been constructed previously in [39,43].

The RG flows we are interested in are given by domain wall solutions of the BPS equations in $\mathcal{N} = 8$ $d = 4$ supergravity with the metric of the form

$$ds^2 = e^{2A(r)} ds_{1,2}^2 + dr^2,$$

(6.1)

where $ds_{1,2}^2$ is the metric on the three-dimensional Minkowski space. Setting the supersymmetry variations of the supergravity fermion fields to zero, the standard analysis, see e.g. [43], yields the following system of BPS equations:

$$z'_a(r) = \mp \sqrt{2} g e^{K/2} \frac{W}{|W|} \nabla_b \bar{W}, \quad \bar{z}'_b(r) = \mp \sqrt{2} g e^{K/2} \frac{W}{|W|} \nabla_a W, \quad (6.2)$$

$$A'(r) = \pm \sqrt{2} g |W|, \quad (6.3)$$

for the dependence of the scalar fields, $z_a$ and $\bar{z}_a$, and the metric function, $A$, on the radial coordinate.

In an $\mathcal{N} = 1$ theory these equations are completely determined by the Kähler potential and the superpotential of the truncated model as indeed they are in (6.2) and (6.3). The choice of sign in (6.2) reflects the freedom to choose the sign of the radial coordinate $r$ in (6.1). In the calculations below we choose the upper sign in (6.2).

It follows from the discussion in Sections 3 and 4 that the system of ODEs (6.2) has critical points at the $\text{SO}(8)$ vacuum $S0600000$, the $G_2$ vacua $S0719157$ and $S0779422$ as well as the new $\mathcal{N} = 1 \text{SO}(3)$ vacuum $S1384096$. The expectation is that there should be a web of domain wall solutions corresponding to RG flows between the superconformal fixed points of the dual ABJM theory. Indeed, in a simpler setting that included the first three points only, families of such flows were constructed explicitly in [43].

To study these domain wall solutions it proves convenient to split the complex scalar fields, $z_a$, into their real and imaginary parts, $z_a = x_a + i y_a$, $a = 1, 2, 3$. At each critical point, the real fields, $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$, collectively denoted by $\phi_\alpha$, $\alpha = 1, \ldots, 6$, have the asymptotic expansions

$$\phi_\alpha(r) = \sum_{\beta=1}^{6} A_{\alpha\beta} e^{-\delta_{\beta} r/L} + \ldots, \quad L^2 = -\frac{3}{P}, \quad (6.4)$$

As usual, a prime denotes a derivative with respect to $r$. 

17
where $L$ is the radius of the corresponding AdS$_4$ solution determined by the value of the potential $\mathcal{P}$ at a given critical point. The exponents $\delta_\alpha$ are related to the scaling dimensions, $\Delta_\alpha$, of the dual operators by

$$
\delta_\alpha = \Delta_\alpha \quad \text{or} \quad \delta_\alpha = 3 - \Delta_\alpha.
$$

(6.5)

The BPS equations (6.2) can be integrated numerically and, as expected, we find families of domain wall solutions interpolating between the SO(8) vacuum and the new SO(3) vacuum. Examples of such solutions are shown in the middle column in Figure 6.1. There are also finely tuned solutions that realize holographically a “triangular RG flow” starting from the SO(8) vacuum in the UV, approaching one of the two $G_2$ vacua and then ultimately ending in the SO(3) vacuum in the deep IR. Plots of those flows are shown in the left and right columns in Figure 6.1. Similarly, there are supersymmetric triangular RG flows, see Figure 6.2, interpolating between the SO(8), the SU(3) $\times$ U(1), and the $G_2$ critical points. Those solutions were first studied in [43] and are also present within the consistent truncation here.

However, using a simple shooting method we were not able to find similar triangular RG flows involving the SO(8), the SU(3) $\times$ U(1), and the SO(3) points or for that matter all four supersymmetric points. Indeed, a more exhaustive numerical search using Machine Learning, to be discussed elsewhere, strongly suggests that such RG flows do not exist within our 6-scalar SO(3) $\times Z_2$-invariant truncation. Still, we suspect that those flows might exist within a larger truncation, perhaps the $Z_2 \times Z_2 \times Z_2$-invariant truncation discussed in Section 7, whose remarkable properties make it a compelling candidate to look at.

To interpret this web of RG flows in the dual ABJM SCFT, it is convenient to employ the $N = 1$ superspace language and, using the same notation as in [43], consider the following combinations of the eight chiral superfields,

$$
\tilde{Z}_a = \Phi_{2a-1} + i\Phi_{2a}, \quad a = 1, 2, 3, 4.
$$

(6.6)

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
Point & $\delta_0$ & $\delta_1$ & $\delta_2$ & $\delta_3$ & $\delta_4$ & $\delta_5$ \\
\hline
S0600000 & 1.00000 & 1.00000 & 1.00000 & 1.00000 & 1.00000 & 1.00000 \\
S0719157 & 3.44949 & 1.40825 & 1.40825 & 0.591752 & 0.591752 & -1.44949 \\
S0779422 & 3.56155 & 2.56155 & 1.33333 & 0.666667 & -0.561553 & -1.56155 \\
S1384096 & 4.80254 & 3.71632 & 1.25126 & 0.748735 & -1.71632 & -2.80254 \\
\hline
\end{tabular}
\caption{Asymptotic exponents, $\delta_\alpha$, at the four supersymmetric critical points.}
\end{table}
Then the deformation of the ABJM superpotential,

$$\Delta W = \frac{1}{2}m_3(\tilde{Z}_1^2 + \tilde{Z}_2^2 + \tilde{Z}_3^2) + \frac{1}{2}m_7\Phi_7^2 + \frac{1}{2}m_8\Phi_8^2. \quad (6.7)$$

breaks the conformal invariance of the ABJM theory, but preserves $\mathcal{N} = 1$ supersymmetry. For general values of the mass parameters $m_3$, $m_7$ and $m_8$, the deformation preserves the SO(3) symmetry. The structure of the holographic RG flows above suggests that the IR dynamics of this model is controlled by a new interacting $\mathcal{N} = 1$ SCFT, which is the field theory dual of the

Figure 6.1: Numerical solutions to the BPS equations for RG flows from the SO(8) point to the SO(3) point. The generic flows (middle column) asymptote to flows through the G$_2$ point (side columns). The top row shows the flows in the superimposed $z_1$, $z_2$ and $z_3$-planes. The colored dots represent the supersymmetric critical points: SO(8) (black), G$_2$ (green), SU(3) × U(1) (orange), and SO(3) (red). The middle row gives the radial dependence of the real scalars, $x_1, \ldots, y_3$. The bottom row gives $A'$ along the flows, which asymptotes to a constant that depends on the radius of the AdS$_4$ vacuum.
new $\mathcal{N} = 1$ vacuum $S_{1384096}$. Note that for $m_3 = 0$ we recover the RG flows to the $G_2$ and $SU(3) \times U(1)$ critical points discussed in [43].

7 A $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$-invariant truncation

A lesson one should draw from the construction of the critical point $S_{1384096}$ above, as well as from a similar construction in [14] of the $\mathcal{N} = 1$ supersymmetric critical point, $S_{1200000}$, which is not captured by our truncation, is that discrete symmetries may lead to simple, explicit truncations of the $\mathcal{N} = 8$ supergravity that are accessible analytically. In both of these constructions, the scalar manifold of the truncated theory is simply a product of three Poincaré disks, $SU(1,1)/U(1)$. One may note that the same coset arises in the so-called STU-model with a particularly simple superpotential [44]. Observing that in our construction, two of the three $SU(1,1)$ factors are embedded non-regularly into $E_7(7)$, it seems suggestive to try interpreting the subspace studied here as originating from a collapsing of roots, starting from the maximal number of regularly embedded $SU(1,1)$’s. Both the enlargement of $SU(1,1) \times SU(1,1)$ to $G_2(2)$ when dropping the $\mathbb{Z}_2$ symmetry (which is obtained from $SO(8)$ by collapsing three roots) as well as the coefficients in the Kähler potential (2.15) suggest that one should look for a regular embedding of $SU(1,1)^{\times 7}$ into $E_{7(7)}$. The remarkable properties of this subgroup have been discussed in the context of qubit entanglement and black holes, cf. [45] and subsequent research, with a comprehensive review in [46].

It turns out that an $\mathcal{N} = 1$ supersymmetric truncation with the scalar manifold given by the product of 7 Poincaré disks,

$$\left[ \frac{SU(1,1)}{U(1)} \right]^{\times 7},$$

(7.1)
can be obtained using a discrete $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \subset SO(8)$ symmetry. It can be constructed explicitly as follows.
Consider the $S \equiv \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ group generated by the following SO(8) rotations:

$$
g_1 = \text{diag}(1, 1, 1, -1, -1, -1, -1),
\quad g_2 = \text{diag}(1, 1, -1, -1, 1, 1, -1),
\quad g_3 = \text{diag}(1, -1, 1, -1, 1, -1, -1),
$$

in $8_v$. The non-identity elements of this group are naturally labelled by the 7 points on the Fano plane (see, e.g., [47])

$$
h_1 = g_1, \quad h_2 = g_2, \quad h_3 = g_3, \quad h_4 = g_1 g_2, \quad h_5 = g_2 g_3, \quad h_6 = g_3 g_1, \quad h_7 = g_1 g_2 g_3,
$$

such that the product along each line is the identity. It is straightforward to verify explicitly that the Lie subalgebra of $\mathfrak{e}_{7(7)}$ invariant under $S$ is precisely $\mathfrak{su}(1,1)^{\oplus 7}$, with the compact generators, $h_a$, of $\mathfrak{u}(1)^{\oplus 7}$ given by the matrices $h_a = i h_a$, $a = 1, \ldots, 7$ in (7.3). Note that the generators $h_j$ are orthogonal to $\mathfrak{so}(8)$ in $\mathfrak{su}(8)$. Since each $\mathfrak{su}(1,1)$ corresponds to a complex scalar field in the truncation, there is a natural identification of the resulting 7 complex scalars, $\zeta_a$, with the points on the Fano plane,

$$
\zeta_a \longleftrightarrow h_a = i h_a, \quad a = 1, \ldots, 7.
$$

As in Section 2, the superpotential can be read off from the eigenvalue of the $A_{ij}$ tensor along the unbroken supersymmetry. A direct calculation yields the result

$$
W_{7^2} = \zeta_1 \zeta_2 \zeta_3 \zeta_4 \zeta_5 \zeta_6 \zeta_7 
+ \zeta_1 \zeta_3 \zeta_4 \zeta_5 + \zeta_1 \zeta_2 \zeta_6 \zeta_5 + \zeta_2 \zeta_4 \zeta_7 \zeta_5 + \zeta_3 \zeta_6 \zeta_7 \zeta_5 + \zeta_2 \zeta_3 \zeta_4 \zeta_6 + \zeta_1 \zeta_2 \zeta_3 \zeta_7 + \zeta_1 \zeta_4 \zeta_6 \zeta_7
$$

(7.5)

where each cubic term, together with the complementary quartic term\footnote{Complementary terms are defined by their product given by $\zeta_1 \zeta_2 \ldots \zeta_7$.} corresponds to one of the 7 lines in the Fano plane. Even more remarkably, the terms in this polynomial match the 16 code words in the single-error-correcting (7,4) Hamming code [48].

From the kinetic terms, or equivalently from the same embedding indices of $\mathfrak{su}(1,1)$’s in $\mathfrak{e}_{7(7)}$, we find the canonical Kähler potential

$$
K = - \sum_{a=1}^{7} \log(1 - \zeta_a \bar{\zeta}_a).
$$

(7.6)

This completely specifies the truncation with the scalar potential in this sector given by (2.17).

One can check that all truncations of interest with fewer SU(1,1)/U(1) factors can be obtained by imposing additional continuous symmetry with respect to some subgroup of SO(8). This
amounts to setting some scalars equal (up to a sign) and/or setting them to zero. For example, the truncation discussed in this paper can obtained by setting

\[
\zeta_1 = -z_2, \quad \zeta_2 = \zeta_6 = \zeta_7 = -z_3, \quad \zeta_3 = \zeta_4 = \zeta_5 = z_1, \tag{7.7}
\]

upon which \(\mathcal{W}_{\mathbb{Z}_2^3}\) reduces to (2.18) and the Kahler potential (7.6) to (2.15). Similarly, the \(\text{SO}(2) \times \text{SO}(2) \times \mathbb{Z}_2 \times \mathbb{Z}_2\)-invariant truncation in [14] is obtained by setting

\[
\zeta_1 = \zeta_3 = i\xi_1, \quad \zeta_2 = \zeta_7 = 0, \quad \zeta_4 = \zeta_5 = \xi_2, \quad \zeta_6 = \xi_0, \tag{7.8}
\]

where \(\xi_i\) are the scalar fields in [14]. Finally, the superpotential of the STU model is obtained by keeping just one cubic term, for example by setting

\[
\zeta_3 = \zeta_5 = \zeta_6 = \zeta_7 = 0. \tag{7.9}
\]

A preliminary numerical search has revealed 48 critical points which, as expected, include all 5 supersymmetric points: \(\text{S}0600000, \text{S}0719157, \text{S}0779422, \text{S}1200000, \text{S}1384096\), and the non-supersymmetric stable point, \(\text{S}1400000\). It is clear that this truncation should be the natural arena to study the holographic RG flows between the supersymmetric critical points and to look for the interplay between the structure of the critical points and the underlying octonion structure in the truncation. For further details we refer the reader to the follow up publication [49].

8 Conclusions

In this paper we presented an explicit construction of a new AdS\(_4\) vacuum of \(\mathcal{N} = 8\) supergravity which preserves \(\mathcal{N} = 1\) supersymmetry. We also discussed the spectrum of all supergravity fields around this vacuum and its relation to operators in the holographically dual CFT. Moreover, we constructed numerical holographic RG flow solutions which interpolate between this new vacuum and other supersymmetric vacua of the \(\mathcal{N} = 8\) supergravity.

One interesting outcome of combining an explicit analytic truncation with the numerical methods using TensorFlow or Bertini2 is a discovery of two additional non-supersymmetric and perturbatively unstable AdS\(_4\) vacua that were not identified in the numerical search in [1]. A possible explanation is that the numerical algorithms in [1] were applied to the full 70-dimensional scalar manifold of the \(\mathcal{N} = 8\) supergravity whereas here the search could be restricted to a simpler and explicitly known potential that depends on 6 scalars only. This also points to a possible strategy for refining the numerical search by restricting it to scalar submanifolds that are invariant under continuous and/or discrete symmetries of the points that had been found already.
Out of all known AdS$_4$ vacua in the $\mathcal{N} = 8$ supergravity there are only 6 that are perturbatively stable. The SO(3) \times Z_2 supergravity truncation discussed in this paper contains 5 of these vacua. The one not included is the U(1) \times U(1) $\mathcal{N} = 1$ vacuum studied in [19, 14]. The larger truncation with 14 scalar fields presented in Section 7 contains all 6 perturbatively stable AdS$_4$ vacua and therefore is a natural starting point for the study of explicit holographic RG flows between them. In fact, by imposing additional U(1) symmetry, one may further truncate to 10 scalar fields, while preserving all the interesting critical points. At the end such an analysis will elucidate the phase structure of the ABJM SCFT and will provide a rich testing ground for the “$\mu$-theorem” discussed in [50].

The results presented here and in [1] suggest that one should apply similar techniques to investigate the vacuum structure of other maximal supergravity theories using an amalgam of analytic and numerical methods. Two particularly interesting examples which can be embedded in string theory and have well-understood holographic duals are the $\mathcal{N} = 8$ ISO(7) gauged four-dimensional supergravity [51] and the maximal five-dimensional SO(6) gauged supergravity. We expect to report some preliminary results shortly [52].

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A Some group theory

We use the convention in which the gravitino, $\psi^i_{\mu}$, transforms in $8_v$, the spin-1/2 fermions are in the $\mathbf{56}_v$ while the scalars and the pseudoscalars in $\mathbf{35}^+ = \mathbf{35}_s$ and $\mathbf{35}^- = \mathbf{35}_c$ representations of $\mathfrak{so}(8)$. The metric is neutral under $\mathfrak{so}(8)$ and the gauge field is in the adjoint.

The commutant of the $\mathfrak{so}(3) \simeq \mathfrak{su}(2)$ symmetry algebra in $\mathfrak{so}(8)$ is $\mathfrak{u}(1) \times \mathfrak{u}(1)$. It arises from the following chain of maximal subalgebras:

$$\mathfrak{so}(8) \supset \mathfrak{su}(4) \times \mathfrak{u}(1)_2 \supset \mathfrak{su}(3) \times \mathfrak{u}(1)_1 \times \mathfrak{u}(1)_2 \supset \mathfrak{su}(2) \times \mathfrak{u}(1)_1 \times \mathfrak{u}(1)_2 .$$  \hspace{1cm} (A.1)

The corresponding branchings of the $\mathfrak{so}(8)$ representations relevant for our analysis are as follows\footnote{We use the same group theory conventions as in [9].}

$$8_v \rightarrow 4_1 + \bar{4}_{-1} \rightarrow 3_{1,1} + 1_{-3,1} + \bar{3}_{-1,-1} + 1_{3,-1} \rightarrow 3_{1,1} + 1_{-3,1} + \bar{3}_{-1,-1} + 1_{3,-1} .$$  \hspace{1cm} (A.2)

For the vectors, the branching is

$$28 \rightarrow 15_0 + 6_2 + 6_{-2} + 1_0$$
$$\rightarrow 8_{0,0} + 3_{4,0} + 3_{-4,0} + 1_{0,0} + 3_{-2,2} + \bar{3}_{2,2} + 3_{-2,-2} + \bar{3}_{2,-2} + 1_{0,0}$$
$$\rightarrow 5_{0,0} + 3_{0,0} + 3_{4,0} + 3_{-4,0} + 1_{0,0} + 3_{-2,2} + \bar{3}_{2,2} + 3_{-2,-2} + \bar{3}_{2,-2} + 1_{0,0} .$$  \hspace{1cm} (A.3)

For the scalars we have

$$35_s \rightarrow 20'_0 + 6_2 + 6_{-2} + 1_4 + 1_0 + 1_{-4}$$
$$\rightarrow 6_{-4,0} + 8_{0,0} + 6_{4,0} + 3_{-2,2} + 3_{2,2} + 3_{-2,-2} + \bar{3}_{2,-2} + 1_{0,4} + 1_{0,0} + 1_{0,-4}$$
$$\rightarrow 5_{-4,0} + 1_{-4,0} + 5_{0,0} + 3_{0,0} + 5_{4,0} + 1_{4,0} + 3_{-2,2} + 3_{2,2}$$
$$+ 3_{-2,-2} + 3_{2,-2} + 1_{0,4} + 1_{0,0} + 1_{0,-4} ,$$  \hspace{1cm} (A.4)

and for the pseudoscalars we find

$$35_c \rightarrow 10_{-2} + 15_0 + \bar{10}_{-2}$$
$$\rightarrow 6_{2,-2} + 3_{-2,-2} + 1_{-6,-2} + 8_{0,0} + 3_{4,0} + 3_{-4,0} + 1_{0,0} + 6_{-2,2} + 3_{2,2} + 1_{6,2}$$
$$\rightarrow 5_{2,-2} + 1_{2,-2} + 3_{-2,-2} + 1_{-6,-2} + 5_{0,0} + 3_{0,0} + 3_{4,0} + 3_{-4,0}$$
$$+ 1_{0,0} + 5_{-2,2} + 1_{-2,2} + 3_{2,2} + 1_{6,2} .$$  \hspace{1cm} (A.5)

To determine the commutant of $\mathfrak{so}(3)$ in $\mathfrak{e}(7)$ in $\mathfrak{e}(3)$, we also need

$$35_v \rightarrow 10_{2} + 15_0 + \bar{10}_{-2}$$
$$\rightarrow 6_{2,2} + 3_{-2,2} + 1_{-6,2} + 8_{0,0} + 3_{4,0} + 3_{-4,0} + 1_{0,0} + 6_{-2,2} + \bar{3}_{2,-2} + 1_{6,-2}$$
$$\rightarrow 5_{2,2} + 1_{2,2} + 3_{-2,2} + 1_{-6,2} + 5_{0,0} + 3_{0,0} + 3_{4,0} + 3_{-4,0} + 1_{0,0}$$
$$+ 5_{-2,-2} + 1_{-2,-2} + 3_{2,-2} + 1_{6,-2} .$$  \hspace{1cm} (A.6)
Finally for the spin-1/2 fields we have

$$56_e \rightarrow 20_{-1} + 20_1 + 4_1 + 4_{-1} + 4_{-3} + 4_3$$

$$\rightarrow 6_{-1,-1} + 8_{3,-1} + 3_{-5,-1} + 3_{-1,-1} + 6_{1,1} + 8_{-3,1} + 3_{5,1} + 3_{1,1}$$

$$+ 3_{1,1} + 1_{-3,1} + 3_{-1,-1} + 1_{3,-1} + 3_{1,-3} + 1_{-3,-3} + 3_{-1,3} + 1_{3,3}$$

$$\rightarrow 5_{-1,-1} + 1_{-1,-1} + 5_{3,-1} + 3_{-5,-1} + 3_{-1,-1} + 5_{1,1} + 1_{1,1} + 5_{-3,1} + 3_{-3,1}$$

$$+ 3_{5,1} + 3_{1,1} + 3_{1,1} + 1_{-3,1} + 3_{-1,-1} + 1_{3,-1} + 3_{1,-3} + 1_{-3,-3} + 3_{-1,3} + 1_{3,3}.$$  \(\text{(A.7)}\)

The singlets under \(so(3)\) are the metric, two Abelian gauge fields, 2 spin-3/2 fields, 10 scalars, and 6 spin-1/2 fields. This is precisely the matter contents of a four-dimensional \(\mathcal{N} = 2\) gauged supergravity coupled to 1 vector multiplet and 2 full hypermultiplets.

**B The full spectrum of \(\mathcal{N} = 8\) supergravity**

In this appendix we present the masses of all bosonic and fermionic fields of the four-dimensional \(\mathcal{N} = 8\) supergravity around the SO(3) invariant \(\mathcal{N} = 1\) AdS vacuum, S1384096, studied in the main text. The spectrum of the spin-0, spin-1/2, and spin-3/2 fields and their SO(3) representations were already presented in \([1]\) and is summarized in Table B.1, Table B.2, and Table B.4, respectively. The spin-2 graviton is of course massless and not charged under SO(3) and the spectrum and SO(3) representations of the spin-1 vector fields are presented in Table B.3.

Using the standard AdS/CFT dictionary, see \([54]\) for a review, the spectrum of these fields is mapped to the spectrum of operators in the dual \(\mathcal{N} = 1\) SCFT. The conformal dimensions of the dual operators of spin \(s\) can be computed using formulae in Table B.5. We use the dimensionless mass \(mL\) of the supergravity fields, where \(L\) is the AdS scale. We have also indicated a reference where the derivation of each of the formulae can be found.

To understand how the spectrum of supergravity excitations maps to operators in the dual three-dimensional \(\mathcal{N} = 1\) SCFT it is useful to recall some aspects of the representation theory of the \(\mathcal{N} = 1\) superconformal algebra, see \([55]\) for a recent discussion. Operators in the \(\mathcal{N} = 1\) SCFT are labelled by their conformal dimension \(\Delta\) and spin \(s\) and will be denoted by \(|\Delta, s\rangle\).\(^{12}\)

These operators belong to one of the superconformal multiplets summarized in Table B.6.

Using the information in Tables B.1-B.4 we can organize the spectrum of operators in the \(\mathcal{N} = 1\) SCFT dual to the SO(3) AdS vacuum in the following superconformal multiplets:

\(^{12}\)The authors of \([55]\) label the operators with \(j = 2s\).
Table B.1: Masses of the 70 supergravity scalars at the $\mathcal{N} = 1$ SO(3)-invariant point, the corresponding SO(3) representations, and the conformal dimensions of the dual operators. The conformal dimensions are obtained using the standard AdS/CFT formula $m^2 L^2 = \Delta(\Delta - 3)$ and choosing the root of this quadratic equation which obeys the unitarity bound $\Delta \geq 1/2$. When $-9/4 \leq m^2 L^2 < 5/4$ one has a choice of alternate quantization, see [56], and both possible conformal dimensions are presented.

| #   | $m^2 L^2$   | SO(3) irreps | $\Delta$   |
|-----|-------------|--------------|------------|
| 1*  | 16.26186    | 1            | 5.802541   |
| 1   | 16.09544    | 1            | 5.783158   |
| 1*  | 8.656777    | 1            | 4.802541   |
| 1   | 8.529126    | 1            | 4.783158   |
| 1*  | 8.094691    | 1            | 4.716316   |
| 3   | 5.322114    | 3            | 4.251747   |
| 3   | 5.182218    | 3            | 4.226210   |
| 5   | 3.817573    | 5            | 3.963244   |
| 1*  | 2.662058    | 1            | 3.716316   |
| 25  | 0           | $5 \oplus 6 \times 3 \oplus 2 \times 1$ | 3          |
| 5   | -0.108916   | 5            | 2.963244   |
| 5   | -1.099493   | 5            | 2.572617   |
| 8   | -1.396494   | $5 \oplus 3$ | 0.5761462 or 2.423854 |
| 1*  | -1.685601   | 1            | 0.7487353 or 2.251265 |
| 1*  | -2.188131   | 1            | 1.251265 or 1.748735 |
| 3   | -2.244202   | 3            | 1.423854 or 1.576146 |
| 5   | -2.244727   | 5            | 1.427383 or 1.572617 |

- **Short spin-3/2:** This is simply the energy momentum multiplet which is neutral under $\mathfrak{so}(3)$ and contains the following operators

$$\begin{align*}
| \frac{5}{2}, \frac{3}{2} \rangle & | 3, 2 \rangle.
\end{align*}$$

The supergravity modes corresponding to these operators are the spin-3/2 mode in the last line of Table B.4 and the metric.
Table B.2: Masses of the 56 spin-1/2 supergravity fermions at the $\mathcal{N} = 1 \text{ SO}(3)$-invariant point, the corresponding SO(3) representations, and the conformal dimensions of the dual operators.

- **Short spin-1/2:** This is the conserved $\mathfrak{so}(3)$ current and contains the following operators

\[
\begin{align*}
|\frac{3}{2}, \frac{1}{2}\rangle & \quad |2, 1\rangle.
\end{align*}
\]

The supergravity modes corresponding to these operators are in the $3$ of $\mathfrak{so}(3)$ and correspond to the massless vector and spin-1/2 modes in Table B.3 and Table B.2 respectively.

- **Long spin-1:** There are three such multiplets. We present all operators in them below and indicate also the $\mathfrak{so}(3)$ representations

\[
\begin{align*}
|2.704760, 1\rangle & \quad |3.204759, \frac{1}{2}\rangle & \quad |3.204759, \frac{3}{2}\rangle & \quad |3.704760, 1\rangle, \quad 3, \\
|2.783158, 1\rangle & \quad |3.283158, \frac{1}{2}\rangle & \quad |3.283158, \frac{3}{2}\rangle & \quad |3.783158, 1\rangle, \quad 1, \quad (B.3) \\
|2.847013, 1\rangle & \quad |3.347013, \frac{1}{2}\rangle & \quad |3.347013, \frac{3}{2}\rangle & \quad |3.847013, 1\rangle, \quad 3.
\end{align*}
\]

27
| #  | $m^2L^2$     | SO(3) irreps | $\Delta$  |
|----|-------------|--------------|-----------|
| 3  | 7.322116    | 3            | 4.251748  |
| 3  | 7.182220    | 3            | 4.226210  |
| 3  | 5.258471    | 3            | 3.847013  |
| 1  | 4.962811    | 1            | 3.783158  |
| 3  | 4.610963    | 3            | 3.704760  |
| 3  | 1.564444    | 3            | 2.847013  |
| 1  | 1.396495    | 1            | 2.783158  |
| 3  | 1.201444    | 3            | 2.704760  |
| 5  | 0.603506    | 5            | 2.423854  |
| 3  | 0           | 3            | 2         |

Table B.3: Masses of the 28 supergravity vector fields at the $\mathcal{N} = 1$ SO(3)-invariant point, the corresponding SO(3) representations, and the conformal dimensions of the dual operators.

The supergravity modes corresponding to these operators are the spin-$\frac{1}{2}$, spin-1, and spin-$\frac{3}{2}$ modes of the corresponding dimension in Table B.2, Table B.3 and Table B.4, respectively.

- **Long spin-1/2:** There are three such multiplets. We present all operators in them below and indicate also the $\mathfrak{so}(3)$ representations

  \[
  |3.751747, \frac{1}{2}\rangle, |4.251748, 1\rangle, |4.251747, 0\rangle, |4.751747, \frac{1}{2}\rangle, 3, \quad \text{(B.4)}
  \]

The supergravity modes corresponding to these operators are the spin-0, spin-$\frac{1}{2}$, and spin-1 modes of the corresponding dimension in Table B.1, Table B.2 and Table B.3, respectively.

- **Long scalar:** There are seven such multiplets. We present all operators in them along with their $\mathfrak{so}(3)$ representations below

  \[
  |4.802541, 0\rangle, |5.302541, \frac{1}{2}\rangle, |5.802541, 0\rangle, 1, \quad \text{(B.5)}
  \]

28
The supergravity modes corresponding to these operators are the spin-0 and spin-$\frac{1}{2}$ modes of the corresponding dimension in Table B.1 and Table B.2 respectively.

In addition to the modes discussed above it should be noted that due to the spontaneous breaking of the $\mathcal{N} = 8$ supersymmetry and the $\mathfrak{so}(8)$ gauge symmetry of the supergravity theory there are spin-0 and spin-$\frac{1}{2}$ modes that are “eaten” by the usual (super)Higgs mechanism. These are the spin-$\frac{1}{2}$ modes in the 3rd, 4th and 5th line of Table B.2 as well as the spin-0 mode in the 10th line of Table B.1.

It should also be noted that for spin-0 modes with mass in the range $-\frac{9}{4} \leq m^2 L^2 < -\frac{5}{4}$ there is an ambiguity in assigning a conformal dimension of the dual CFT operator. This happens because for both choices of sign in Table B.5 $\Delta$ obeys the unitarity bound. Invoking supersymmetry however uniquely fixes the choice of sign in Table B.5 and we have chosen the only possible sign that allows for organizing the bottom five entries in Table B.1 into $\mathcal{N} = 1$ superconformal multiplets.
**C Discrete SO(8) rotations**

Consider the SO(3) subgroup of SO(8) introduced in Section 2. It is straightforward to check that any SO(8) rotation that commutes with this subgroup must be of the form

\[
\begin{pmatrix}
\cos \alpha & 0 & 0 & \sin \alpha & 0 & 0 & 0 & 0 \\
0 & \cos \alpha & 0 & 0 & \sin \alpha & 0 & 0 & 0 \\
0 & 0 & \cos \alpha & 0 & 0 & \sin \alpha & 0 & 0 \\
\mp \sin \alpha & 0 & 0 & \pm \cos \alpha & 0 & 0 & 0 & 0 \\
0 & \mp \sin \alpha & 0 & 0 & \pm \cos \alpha & 0 & 0 & 0 \\
0 & 0 & \mp \sin \alpha & 0 & 0 & \pm \cos \alpha & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \cos \beta & \sin \beta \\
0 & 0 & 0 & 0 & 0 & 0 & \mp \sin \beta & \pm \cos \beta
\end{pmatrix}.
\]

(C.1)

Choosing the upper sign, we obtain the two parameter family of rotations, \(g(\alpha, \beta)\), corresponding to the U(1) \(\times\) U(1) gauge group of the \(\mathcal{N} = 2\) supergravity of the SO(3)-invariant truncation. For the lower sign, the rotations are given by

\[
g(-\alpha, -\beta + \pi) g_S,
\]

(C.2)

where \(g_S\) is the generator (2.9) of the discrete \(\mathbb{Z}_2\) symmetry that defines our truncation.

Since the scalar fields \(s_a, a = 1, \ldots, 6\), are by construction invariant under \(g_S\), any residual nontrivial action of the SO(8) subgroup given in (C.1) on the coset (2.11) must come from the U(1) \(\times\) U(1) transformations that preserve (2.10). At a generic point in the coset, this leaves a discrete \(\mathbb{Z}_2 \times \mathbb{Z}_4\) subgroup of SO(8) generated by \(g_H \equiv g(0, \pi)\) and \(g_C \equiv g(\pi/2, \pi/2)\), which are elements of order 2 and 4, respectively. However, since \(g_C^2 = g(\pi, \pi)\) acts trivially on the coset,
we end up with only \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) worth of \( \text{SO}(8) \) rotations that preserve the scalar manifold in the \( \text{SO}(3) \times \mathbb{Z}_2 \)-invariant truncation. Those are generated by the transformations

\[
\begin{align*}
g_H &: (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, z_3), \\
g_C &: (z_1, z_2, z_3) \rightarrow (-\bar{z}_1, -\bar{z}_2, z_3).
\end{align*}
\]

(C.3)

Hence any point in the coset lies on an \( \text{SO}(8) \) orbit obtained by acting on that point with 1, \( g_H \), \( g_C \) and \( g_H g_C \). When both \( z_1 \) and \( z_2 \) are either real or imaginary, the orbit degenerates to two points.

At special points of the coset there might be additional rotations, \( g(\alpha, \beta) \), that map it onto another point on the coset. In particular, this happens for two critical points, \( \text{S0880733} \) and \( \text{S1039230} \), in Section 4, with the special rotations given by \( g_R \equiv g(-3\pi/4, \pi/4) \) and \( g'_R \equiv g(-\pi/4, \pi/4) \), respectively.

While both rotations in (C.3) are obviously symmetries of the potential (2.17), only \( g_H \) preserves the superpotential (2.18). This is just the reflection of the fact that the second transformation acts nontrivially on the supersymmetry by mapping the \( \mathcal{N} = 1 \) supergravity given by the \( \text{SO}(3) \times \mathbb{Z}_2 \)-invariant truncation to an equivalent one obtained by a different discrete \( \mathbb{Z}_2 \) symmetry.

Finally, we note that 4-form \( \Phi^{(3)} \) in (2.6) is invariant under the transformations (C.1). In particular, this implies that \( z_3 \) is invariant under the discrete symmetries above.

## D New critical points S2096313 and S2443607

In this appendix we present numerical data for the two new critical points, \( \text{S2096313} \) and \( \text{S2443607} \), in the same format as in [1]. The parametrization of the scalar coset (2.11) in terms of \( s_a, a = 1, \ldots, 6 \) introduced in Section 2 is related to the one in [1] and in the tables below by:

\[
\begin{align*}
A &= \frac{1}{8} (2s_1 + s_5), \\
B &= \frac{1}{8} (s_1 - s_3 + 2s_5), \\
C &= \frac{1}{8} (s_5 - 2s_1), \\
D &= \frac{1}{8} (s_1 - s_3 - 2s_5), \\
E &= -\frac{s_5}{8}, \\
F &= \frac{1}{8} (-s_1 - s_3), \\
G &= \frac{1}{8} (s_2 + s_4 - s_6), \\
H &= \frac{1}{8} (-s_2 - s_4 - s_6), \\
I &= \frac{1}{8} (-3s_2 + s_4 + 3s_6), \\
J &= \frac{1}{8} (3s_2 - s_4 + 3s_6),
\end{align*}
\]

(D.1)

where the parameters \( A, \ldots, J \) satisfy

\[
A - B - E + F = 0, \quad B - D + 4E = 0, \quad C + D - E - F = 0, \quad 3G + 3H + I + J = 0. \quad (D.2)
\]

### D.1 Point S2096313


\[ S2096313 : \mathfrak{so}(8) \to \mathfrak{so}(3) + \mathfrak{u}(1) \]
\[ V/g^2 \approx -20.9631372891 \]
\[ 8_c \to 2 \times 3 + 1^{++} + 1^{--}, \ 8_{v,s} \to 3^+ + 3^- + 1^+ + 1^- \]
\[ 28 \to 5 + 2 \times 3^{++} + 3 \times 3 + 2 \times 3^{--} + 2 \times 1 \]
\[ m^2/m_{0,[\psi]}^2 : 7.750_{1^{++}+1^{--}}, 2.550_{3^+3^-} \]
\[ m^2/m_{0,[\chi]}^2 : 15.500_{1^{++}+1^{--}}, 7.875_{3^+3^-}, 5.137_{3^+3^-}, 5.100_{3^+3^-}, 4.596_{5^{++}5^{--}}, 2.475_{1^{++}+1^{--}}, 0.875_{1^{+}1^{-}}, 0.754_{5^{++}5^{--}}, 0.727_{3^+3^-}, 0.160_{3^+3^-} \]
\[ m^2/m_{0,[\phi]}^2 : 18.400_{1^{+++}+1^{---}}, 10.000_{1^{++}+1^{--}}, 9.600_{5^{++}5^{--}}, 8.728_{1^m}, 8.596_{5^m}, 0.400_{3^m}, 0.000_{3^+3^-}, 0.000_{5^+2\times3^{++}+2\times3^{--}+1}, 0.000_{3^m}, -0.596_{5^m}, -0.612_{1^m}, -1.200_{1^{++}+1^{--}}, -2.000_{5^m}, -2.400_{m^m}, -2.516_{m^m} \]
\[ M_{\alpha \beta} = \text{diag} \ (-3A, -3A, A, A, A, A, A, A) \]
\[ M_{\alpha \beta} = \text{diag} \ (C, D, B, B, C, D, D, C) \]
\[ A \approx -0.1641598793, \ B \approx -0.5046414602, \ C \approx -0.0723920925, \]
\[ D \approx 0.4088197326 \]

**D.2** Point S2443607

\[ S2443607 : \mathfrak{so}(8) \to \mathfrak{so}(3) \]
\[ V/g^2 \approx -24.4360747652 \]
\[ 8_{v,s,c} \to 2 \times 3 + 2 \times 1, \ 28 \to 5 + 7 \times 3 + 2 \times 1 \]
\[ m^2/m_{0,[\psi]}^2 : 11.568_{1^m}, 11.463_{1^m}, 4.354_{3^m}, 3.726_{3^m} \]
\[ m^2/m_{0,[\chi]}^2 : 23.137_{1^m}, 22.926_{1^m}, 18.908_{3^m}, 18.908_{3^m}, 10.714_{1^m}, 10.711_{1^m}, 8.708_{3^m}, 7.451_{1^m}, 6.299_{3^m}, 6.141_{3^m}, 5.757_{5^m}, 5.744_{5^m}, 2.122_{3^m}, 2.064_{1^m}, 1.624_{3^m}, 1.417_{3^m}, 1.257_{5^m}, 1.251_{5^m}, 0.201_{3^m}, 0.082_{3^m} \]
\[ m^2/m_{0,[\phi]}^2 : 55.474_{1^m}, 55.474_{1^m}, 20.600_{1^m}, 20.600_{1^m}, 19.887_{3^m}, 19.876_{5^m}, 8.040_{3^m}, 7.864_{3^m}, 7.464_{3^m}, 5.996_{5^m}, 4.125_{5^m}, 0.000_{3^m}, 0.000_{3^m}, -0.023_{3^m}, -0.562_{1^m}, -0.743_{5^m}, -2.513_{m^m}, -2.836_{m^m}, -3.205_{m^m} \]
\[
M_{\alpha \beta} = \begin{pmatrix}
A & 0 & 0 & 0 & 0 & B & 0 & 0 \\
0 & E & 0 & 0 & C & 0 & 0 & 0 \\
0 & 0 & F & 0 & 0 & 0 & 0 & D \\
0 & 0 & 0 & F & 0 & 0 & -D & 0 \\
0 & 0 & C & 0 & 0 & E & 0 & 0 \\
B & 0 & 0 & 0 & 0 & A & 0 & 0 \\
0 & 0 & 0 & -D & 0 & 0 & F & 0 \\
0 & 0 & D & 0 & 0 & 0 & 0 & F
\end{pmatrix}
\]
\[ M_{\alpha\beta} = \text{diag} \left( G, G, G, H, H, I, J \right) \]

\[ A \approx 0.2540142811, \ B \approx 0.3965710947, \ C \approx -0.1697655242, \]

\[ D \approx 0.0009726589, \ E \approx 0.0291540284, \ F \approx -0.1415841547, \]

\[ G \approx 0.4106803872, \ H \approx 0.0401731985, \ I \approx -0.6761535338, \]

\[ J \approx -0.6764072234 \]

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