Final state interaction effect in pure annihilation $B_s \rightarrow \rho \rho$ decay

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Abstract
We analyzed the process of $\rho \rho \rightarrow B_s$ decay in quantum chromodynamics factorization (QCDF) and final state interaction (FSI) effects. In QCDF, for this decay, we have only the annihilation graph and we expected a small branching ratio. Then we considered the FSI effect as a sizable correction where the intermediate states are $\pi \pi^0$, $\pi^+\pi^-$, $K^0\bar{K}^0$, and $K^+K^-$ mesons. To consider the amplitudes of these intermediate states, the QCDF approach was used. The experimental branching ratio of $\rho \rho \rightarrow B_s$ is less than $3.20 \times 10^{-4}$ and our result is $1.08 \times 10^{-9}$ and $3.29 \times 10^{-4}$ from QCDF and FSI, respectively.

Keywords: B meson, final state interaction, QCD factorization

1. Introduction
Final state interaction (FSI) effects in $B$ decay were expected to play only the role of a small correction to the standard description in short distance amplitude. In the factorization approach, the amplitude of a $B$ decay mode, which is described as the short distance contributions, consists of (1) the usual factorization amplitude of color-allowed and color-suppression topology and (2) the annihilation topology (w-exchanged or w-annihilation) [1]. In pure annihilation $B$ decay mode, the theoretical amplitude is often too small in comparison to the expected date. In this decay mode, FSI effects may play an important role. Following a weak decay, the intermediate state particles rescatter into the final particles through a nonperturbative strong interaction. The nonperturbative nature of FSI effects makes it difficult to study them in a systematic way, so different mechanisms of the rescattering effects have been considered [2, 3]. To analyze a $B$-meson decay through FSI, it is important to understand the structure of the intermediate multiparticle states. One can treat FSI as the soft rescattering processes of intermediate two-body hadronic states e.g., $B_s \rightarrow K^0\bar{K}^0 \rightarrow \rho \rho$, and omit the other intermediate multi-body states, where after weak decay of $B$-meson to two light mesons, they rescattered to two new mesons through nonperturbative strong interaction. The hadronic loop level (HLL) is used in the strong interaction process where it is obtained from the effective chiral Lagrangian [2, 3].

Our result for the QCDF approach was $1.08 \times 10^{-9}$, where the leading order (LO) results for coefficients $C_i$ were used and the correction terms were omitted. The experimental result is less than $3.20 \times 10^{-4}$ [13]. Because the results from the QCDF approach are very small, the FSI effect may give a sizable correction where the intermediate states are $\pi \pi^0$, $\pi^+\pi^-$, $K^0\bar{K}^0$, and $K^+K^-$ mesons. We calculated the $B_s \rightarrow pp$ decay according to the HLL method. In this case, the branching ratio is $3.29 \times 10^{-4}$.

This paper is organized as follows. In section 2, we present the QCDF approach and calculated the amplitudes of the main decay and the intermediate states using this approach. Then, in section 3, we present the FSI effects and calculated the amplitude of $B_s \rightarrow pp$ decay from three possible intermediate states. In section 4, we give the numerical results, and in the last section, we offer a summary.

2. Weak amplitude of the pure annihilation $B$ decays
To calculate the amplitudes of the pure annihilation $B$ decay modes, we use the QCD factorization method, in which we only consider the annihilation topology. We consider $b$-quark decay and use the convention that the $M_1(M_2)$ meson contains an anti-quark (quark) from the weak vertex with longitudinal momentum fraction $\bar{y}(x)$ where $M_1$ and $M_2$ are the final mesons [4]. The weak annihilation contributions to the decay $B \rightarrow M_1M_2$ can be described in terms of the building blocks $b_i$.
Where \( \lambda_p = V_{pd}V_{p\gamma}^{\ast} \) with \( q = d, s \) and the building blocks have the expressions [5]

\[
\begin{align*}
&b_1 = \frac{C_F}{N_C} C_A A_i^i, \\
&b_2 = \frac{C_F}{N_C^2} C_A A_i^i, \\
&b_3 = \frac{C_F}{N_C^2} \left[ C_A A_i^i + C_S \left( A_i^i + A_f^f \right) + N_C C_8 A_i^f \right], \\
&b_4 = \frac{C_F}{N_C^2} \left[ C_A A_i^i + C_8 A_i^f \right], \\
&b_{3,EW} = \frac{C_F}{N_C^2} \left[ C_A A_i^i + C_8 A_i^f \right], \\
&b_{4,EW} = \frac{C_F}{N_C^2} \left[ C_{10} A_i^i + C_8 A_i^f \right],
\end{align*}
\]  

(2)

The subscripts 1, 2, and 3 of \( A_i^{i,f} \) denote the annihilation amplitudes induced from \((V - A)(V - A)\), \((V - A)(V + A)\), and \((S - P)(S + P)\) operators, respectively, and the superscripts \( i \) and \( f \) refer to gluon emission from the initial and final-state quarks, respectively, which are shown in figure 1 and given by [5]

\[
\begin{align*}
A_i^i &= \pi \alpha_s \int_0^1 dy dx \left( \Phi_M^f(x) \Phi_M^i(y) \frac{1}{y (1 - xy)} + \frac{1}{x y} \right), \\
&+ r_{\rho A}^{M_i} r_{\rho A}^{M_i} \Phi_{M_i}^f(x) \Phi_{M_i}^i(y) \frac{2}{x y}, \\
A_i^f &= 0, \\
A_i^i &= \pi \alpha_s \int_0^1 dy dx \left( \Phi_M^f(x) \Phi_M^i(y) \frac{1}{x (1 - xy)} + \frac{1}{x y} \right), \\
&+ r_{\rho A}^{M_i} r_{\rho A}^{M_i} \Phi_{M_i}^f(x) \Phi_{M_i}^i(y) \frac{2}{x y}, \\
A_i^f &= 0.
\end{align*}
\]  

(3)

When all the basic blocks equations are solved, we found that weak annihilation kernels exhibit endpoint divergent [5]:

\[
X_A = \int_0^1 \frac{dy}{y},
\]  

(4)

Because the treatment of this logarithmically divergence is model-dependent, sub-leading power corrections generally can be studied only in a phenomenological way. Although the endpoint divergence is regulated in the perturbative QCD approach by introducing the parton’s transverse momentum, it is parametrized in QCDF by modifying \( y \rightarrow y + \epsilon \) with \( \epsilon = O(\alpha_{QCD}/m_{\rho}) \) [5, 6], so we replace equation (4) with:

\[
X_A = \int_0^1 \frac{dy}{y + \epsilon} = \ln \frac{m_{\rho}}{\lambda_h} \left( 1 + \rho_A e^{\lambda_h} \right).
\]  

(5)

Different \( X_A \) are allowed for four cases: PP, PV, VP, and VV where P(V) is a final meson by pseudoscalar (vector) polarization. For the VV case, in [7], by evaluating the convolution integrals with asymptotic distribution amplitudes \( \Phi(x) = \Phi_0(x) = 6x \xi, \Phi_2 = 1, \) and \( \Phi(x) = 3(x - \bar{x}) \), we find the simple expressions:

\[
\begin{align*}
A_i^i &\simeq A_f^f = 2 \pi \alpha_s \left( 9 \left( X_A - 4 + \frac{x_i^2}{3} \right) + \left( \frac{x_i}{\lambda} \right)^2 (X_A - 2)^2 \right), \\
A_i^f &= 0, \\
A_i^i &\simeq -36 \pi \alpha_s r_{\rho A}^{M_i} (2X_A^2 - 5X_A + 2).
\end{align*}
\]  

(6)

Now we can calculate the weak amplitude of the \( B_s \rightarrow \rho \rho \) decay and the intermediate state. According to the annihilation diagrams of the \( B_s \rightarrow \rho \rho \) decay, which is given in figure 2, the pure annihilation amplitude is given by

\[
M (B \rightarrow \rho \rho) = -i \frac{G_F}{\sqrt{2}} f_{\rho d} f_{\rho s} V_{ub} V_{ts}^\ast \left( 2b_4 - b_{4,EW} \right).
\]  

(7)

2.1. Weak amplitude of the intermediate states

To consider the FSI effects in \( B_s \rightarrow \rho \rho \) decay, we must extract the accessible intermediate states and calculate their weak amplitude. According to figure 3, by considering the utη part of the \( \rho \) mesons while two intermediate mesons and final state mesons exchange the same quark (u-quark), \( \pi^0 \) and \( \pi^0 \) mesons can be produced for the intermediate state via an exchange \( \pi^0(\eta) \) meson. Likewise, when two intermediate mesons exchange a d-quark (s-quark) and two final state
mesons a exchange u-quark, \(\pi^+\pi^- (K^{(*)}+K^{(*)})\), mesons can be produced for the intermediate state via exchange \(\pi^0 (K^{(*)})\) mesons. By considering the \(dd\) part of the \(\rho\) mesons while two intermediate mesons exchange a d-quark (u-quark or s-quark) and two final state mesons exchange d-quark, \(\pi^0\) or \(\pi^0 K^{(*)}\) mesons can be produced for the intermediate state via exchange \(\pi^0 (\pi^0)\) meson. Now that the intermediate states have been obtained, we can calculate the weak amplitude of the intermediate states that were produced in \(B_s \rightarrow m_1m_2\) decay modes where \(m_1\) and \(m_2\) are the intermediate state mesons. In the two case, \(\pi^0 \pi^0\) and \(\pi^+\pi^-\), we calculate the amplitude is similar to the \(B \rightarrow \rho\rho\) decay mode since these decay modes are pure annihilation. So according to section 2, we have

\[
M(B_s \rightarrow \pi^0 \pi^0 (\pi^+\pi^-)) = -iG_F f_{\rho B} f_{\rho B}^2 \left\{ V_{ub} V_{ud}^* b_1 + 2 V_{ub} V_{ud}^* b_2 \right\}. \tag{8}
\]

However, in the other cases, the color-allowed and color-suppression topologies are allowed and we must consider the usual factorization approach [7, 9] to calculate the amplitude. Therefore, we obtain

\[
M(B_s \rightarrow K^{0*} K^{0*}) = -iG_F V_{ub} V_{ub}^* \left\{ f_{K^0 B^0} f_{K^0 B^0}^2 \left( M_{K^0} - M_{B^0} \right) a_4 + f_{f_{B^0 K^0}} f_{f_{B^0 K^0}}^2 (2b_4) \right\}, \tag{9}
\]

where the coefficients \(a_4\) correspond to the penguin topology and is defined as:

\[
a_4 = C_4 + \frac{1}{N_c} C_3, \tag{10}
\]

In the QCDF amplitude, all terms are not expected to be equally large. The color-allowed and color-suppression topology (\(a_4\) terms) which involve form factors are dominate and the annihilation topology (\(b_4\) terms) can be neglected [10]. Likewise we have:

\[
M(B_s \rightarrow K^{0*} K^{0*}) = - \frac{G_F}{\sqrt{2}} \left\{ f_{K^0 B^0} f_{K^0 B^0}^2 \left[ (e_1^2 + e_2^2) \left( m_{B^0}^2 + m_{K^0}^2 \right) A_{11}^{KK*} \left( m_{K^0}^2 \right) \right] \\
- (e_1 \cdot p_{B^0}) (e_2 \cdot p_{B^0}) \frac{2 A_{22}^{KK*} (m_{K^0})}{\left( m_{B^0}^2 + m_{K^0}^2 \right)} \right\} \times a_4 V_{ub} V_{ud}^* - f_{K^0 B^0} f_{K^0 B^0}^2 \left\{ V_{ub} V_{ud}^* b_4 + b_{1,EW} \right\}, \tag{11}
\]

\[
M(B_s \rightarrow K^{+} K^-) = - i G_F \left\{ f_{K^0 B^0} \left[ f_{f_{B^0 K^0}} f_{f_{B^0 K^0}}^2 \left( M_{K^0} - M_{B^0} \right) a_1 \right] \right\} \\
+ i G_F f_{f_{B^0 K^0}} f_{f_{B^0 K^0}}^2 \left\{ V_{ub} V_{ud}^* b_1 + V_{ub} V_{ud}^* \left( b_4 + b_{1,EW} \right) \right\}, \tag{12}
\]

3. The one particle exchange method for FSI

At the quark level, final state rescattering can occur through quark exchange and quark annihilation. The quark level diagram for \(B \rightarrow \rho\rho\) decay is shown in figure 3. This decay has only a quark annihilation mode because the final mesons (\(\rho\)) have the same flavor quark-antiquark. In practice, it is extremely difficult to calculate the FSI effects, but at the hadronic level formulated as rescattering processes with s-channel resonances and one particle exchange in the t-channel. S-channel resonant FSI effects in \(B \rightarrow \rho\rho\) decay are expected to vanish because of the lack of the existence of resonances. Therefore, one can model FSI effects as rescattering processes of a two-body intermediate state with one particle exchange in the t-channel and compute the absorptive part via the optical theorem [2].

According to the HLL diagram, shown in figure 4, the absorptive part of the amplitude is calculated with the following formula

\[
Abs(B_s (p_B) \rightarrow \pi(p_1) \pi(p_2) \rightarrow \rho(p_3) \rho(p_4)) \\
= \frac{1}{16 \pi m_B} A(B_s \rightarrow \pi(p_1) \pi(p_2)) \times G(\pi(p_1) \pi(p_2)) \rightarrow \rho(p_3) \rho(p_4)), \tag{14}
\]

where the \(A(B_s \rightarrow \pi\pi)\) is the amplitude of the decay of the B...
\[ F(q^2, m_\omega^2) = \left( \frac{A^2 - m_\omega^2}{A^2 - q^2} \right). \] (18)

The parameter \( A \) is the off-shellness compensating in function \( F(q^2, m_\omega^2) \), which is not an universal parameter, but should be near the mass of the mesons involved in the effective coupling. Likewise, for diagram (b) in figure 4, the absorptive part for the \( B \to \pi\pi \to \rho\rho \) process where \( \omega \) meson is exchanged particle at t-channel is given by

\[ A_{F(4b)} = \int_{-1}^{1} \left[ \frac{d(cos \theta)}{16\pi m_B} \right] M(B_s \to \pi\pi) \times (-i) g_{\omega\rho\pi} e^{\mu\nu} e^{\alpha\beta} p_1^\mu p_2^\nu p_3^\alpha p_4^\beta \]

\[ \times \int_{-1}^{1} \left| \frac{d(cos \theta)}{q^2 - m_\omega^2} \right| F_2(q^2, m_\omega^2) H_2, \] (19)

where

\[ H_2 = -2 \left[ (p_1 \cdot p_2) (p_3 \cdot p_4) - (p_1 \cdot p_3) (p_2 \cdot p_4) \right] \]

\[ + \rho^3 \left[ 0 \right] \]

\[ - \left( p_2^0 - \left| p_2^0 \right| \right) (p_1 \cdot p_4) \]

\[ + \left( p_1^0 - \left| p_1^0 \right| \right) \]

\[ \left( p_2^0 - \left| p_2^0 \right| \right) (p_3 \cdot p_4) - \left| p_1^0 \right| \left| p_2^0 \right| (p_2 \cdot p_3). \] (20)

As the bridge between the dispersive part of the FSI amplitude and the absorptive part, the dispersion relation is

\[ \text{Dis}(m_B^2) = \frac{1}{\pi} \int_{s} A_{F(4a)}(s') + A_{F(4b)}(s') \frac{ds'}{s' - m_B^2}. \] (21)

where \( s' \) is the square of the momentum carried by the exchanged particle and \( s \) is the threshold of intermediate states, in this case \( s \sim m_B^2 \).

Finally, the amplitude of the \( B_s \to \pi\pi \to \rho\rho \) decay via the HLL diagram is

\[ A(B_s \to \pi\pi \to \rho\rho) = A_{F(4a)} + A_{F(4b)} + \text{Dis}(4). \] (22)

Likewise, we can calculate that the FSI effects comes from other intermediate states \((K^{(*)\pi^0}K^{(*)0}\) and \((K^{(*)\pi^0}K^{(*)0}\) by replacing \( \pi\pi \) with these new intermediate states given in the preceding formula.
4. Numerical results

In this paper, we used Wilson coefficients $c_i$ in LO at $\mu = m_B$, which are given by [9]

$$
c_1 = 1.114, \quad c_2 = -0.308, \\
c_3 = 0.014, \quad c_4 = -0.030, \\
c_5 = 0.009, \quad c_6 = -0.038, \\
c_7 = -3.4 \times 10^{-4}, \quad c_8 = 3.7 \times 10^{-4}, \\
c_9 = -0.01, \quad c_{10} = 0.002.
$$

The elements of the CKM matrix can be parametrized by three mixing angles $\Lambda, \lambda, \rho$ [12] and a CP-violating phase $\eta$

$$
V_{ud} = 1 - \lambda^2/2, \quad V_{us} = \lambda, \quad V_{ub} = \Lambda \lambda (\rho - i \eta), \\
V_{cd} = -\lambda, \quad V_{cs} = 1 - \lambda^2/2, \quad V_{cb} = \Lambda \lambda^2, \\
V_{td} = \Lambda \lambda (1 - \rho - i \eta), \quad V_{ts} = -\Lambda \lambda^2, \quad V_{tb} = 1.
$$

The results for the Wolfenstein parameters are

$$
\lambda = 0.2257 \pm 0.0001, \quad A = 0.814 \pm 0.02, \\
\rho = 0.135 \pm 0.023, \quad \eta = 0.349 \pm 0.016,
$$

and we use the central values of the Wolfenstein parameters and obtain

$$
V_{ud} = 0.9745, \quad V_{us} = 0.2257, \quad V_{ub} = 0.0013 - 0.0033i, \\
V_{cd} = -0.2257, \quad V_{cs} = 0.9745, \quad V_{cb} = 0.0415, \\
V_{td} = 0.0081 - 0.0033i, \quad V_{ts} = 0.0415, \quad V_{tb} = 1.
$$

For endpoint parametrizing in QCDF approach according to the polarization of the final mesons, we give: $\rho = 1$, $\Lambda = 0.5$, $\Phi_A = -40^\circ (VV)$, $\Phi_A = 20^\circ (PV)$, $\Phi_A = -55^\circ (PP)$ [6].

The mass of the mesons and decay constants are given in units of GeV:

$$
m_B = 5.28, \quad m_K = 0.49, \quad m_{K^*} = 0.89, \quad m_{\rho} = 0.775, \\
m_{\omega} = 0.139, \quad m_{\phi} = 0.783, \quad f_{B_K} = 0.230, \quad f_{K} = 0.16, \\
f_{K^*} = 0.214, \quad f_{\omega} = 0.133, \quad f_{\rho} = 0.216, \quad f_{K^*}^\perp = 0.175, \\
F_K^{B,K} = 0.26, \quad A_1^{B,K} = 0.29, \quad A_2^{B,K} = 0.26, [1,14].
$$

The other input parameters used are given by:

$$
g_{KK\rho} = g_{KK^*\rho} = 3.025, \quad g_{\omega\rho} = 5.89, \quad r_{K}^\rho = 1.09, \quad r_{K^*}^\rho = 0.29, [14-16]
$$

We calculated the branching ratio for the QCDF method as $1.08 \times 10^{-9}$, which is very small compared to the experimental result. Within FSI, the branching ratio is shown in table 1 and if $\eta = 1$ selected, the branching ratio is $3.22 \times 10^{-4}$, which is near the upper bound of the experimental value, which is $3.20 \times 10^{-4}$ [13]. The main phenomenological parameter in the FSI effects is $\eta$, which is determined from the measured ratios. Its value in form factor is expected to be of the order of unity. In this work, we have considered $\eta = 0.5 \sim 1$ and the best result obtained by $\eta = 1$.

5. Summary

We analyzed the $B \rightarrow \rho \rho$ decay in QCDF approach and then we added the FSI effects. In the QCDF approach, we have only weak annihilation topology and, as expected, we obtained a small branching ratio of $1.08 \times 10^{-9}$, whereas after considering the FSI effect, we obtained $3.29 \times 10^{-4}$, which is near the upper bound of the experimental value, which is $3.20 \times 10^{-4}$ [13]. The main phenomenological parameter in the FSI effects is $\eta$, which is determined from the measured ratios. Its value in form factor is expected to be of the order of unity. In this work, we have considered $\eta = 0.5 \sim 1$ and the best result obtained by $\eta = 1$.

Table 1. The branching ratio of $B \rightarrow \phi \phi$ decay with $\eta = 0.5 \sim 1$ (in units of $10^{-9}$).

| $\eta$ | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | EXP. [13] |
|-------|-----|-----|-----|-----|-----|----------|
| BR    | 0.23| 0.47| 0.88| 1.42| 2.22| 3.29     |

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