Research Article

MIMO Radar Adaptive Waveform Design for Extended Target Recognition

Lulu Wang,1 Kai-Kit Wong,2 Hongqiang Wang,1 and Yuliang Qin1

1Department of Electronic Science and Engineering, National University of Defense Technology, Changsha 410073, China
2Department of Electronic and Electrical Engineering, University College London, London WC1E 7JE, UK

Correspondence should be addressed to Lulu Wang; wanglulunudt@163.com

Received 6 January 2015; Revised 7 June 2015; Accepted 10 June 2015

The problems of multiple-input multiple-output (MIMO) radar adaptive waveform design in additive white Gaussian noise channels and multitarget recognition based on sequential likelihood ratio test are jointly addressed in this paper. Two information-theoretic waveform design strategies, namely, the optimal waveform for maximizing the mutual information (MI) between the extended target impulse response and the target echoes and the optimal waveform for maximizing the Kullback-Leibler (KL) divergence (or relative entropy), are applied in the multitarget recognition application. For multitarget case, two adaptive waveform design methods for all possible targets based on the current knowledge of each hypothesis are proposed. Method 1 is the probability weighted waveform method. Method 2 is the probability weighted target signature method. The optimal waveform is transmitted and adaptively changed such that a decision is made based on the likelihood ratio after several illuminations. Numerical results demonstrate that the best waveform is the KL divergence-based optimal waveform using Method 1 as it has the lowest average illumination number and the highest correct decision rate for target recognition. By optimally designing and adaptively changing the transmitted waveform, the average number of illuminations required for multitarget recognition can be much reduced.

1. Introduction

Unlike the traditional phased-array radar that transmits a scaled version of a single waveform, multiple-input multiple-output (MIMO) radar can transmit different waveforms through its antennas and therefore provide more degrees of freedom in the radar system which lead to many advantages such as improved spatial resolution, better parametric identifiability, and greater flexibility to achieve the desired transmit beam pattern [1–6]. MIMO radar systems can be classified into two categories: statistical MIMO radar with widely spaced antennas [3] and colocated MIMO radar [5] which has close enough transmit antennas such that the target radar cross sections (RCS) observed by all the antennas are the same.

Optimal waveform design is of paramount importance for many kinds of active sensing systems such as radar, sonar, and communication. MIMO radar waveform design has attracted much attention for several years [6–20]. According to whether the waveform is designed directly or not, the MIMO radar waveform design problem can be divided into two categories. One is the optimal waveform design based on the ambiguity function [14–16]. In this case, the waveform is optimized directly to have a good autocorrelation or cross-correlation property which sharpens the ambiguity function. The other optimal waveform design problem optimizes the waveform covariance matrix instead of the waveform itself [17–20]. Mutual information (MI) was firstly introduced by Bell [21] in radar waveform design. Information-theoretic criteria have been widely used in waveform design from then on [8–11, 22]. In [9], it has been proved that the optimal waveform which maximizes the MI also minimizes the minimum mean-square error (MMSE) for target estimation in white Gaussian noise. The optimal solution performs a water filling operation [23] which allocates more power into the frequencies with large target power and low noise power. Kullback-Leibler (KL) divergence (or relative entropy) is also used in waveform design for target classification and detection [11, 24–28]. In [26–28], KL divergence was employed in sequential MIMO
detection which reduces the average number of required samples. In this paper, MI and KL divergence are used as the criteria for extended target waveform optimization in white Gaussian noise channels.

Most of the above mentioned waveform design methods are nonadaptive. However, real environment is complicated and may change rapidly owing to the target movement, nonstationary interference, vast usage of various electronic devices, and so on. To make the radar more flexible to the changing environment, the transmitted waveform should change adaptively. For this purpose, cognitive or knowledge-aided radar system [29–31] was proposed, in which the radar is capable of adaptively and intelligently interrogating the environment using many different kinds of available knowledge, such as the a priori knowledge as well as the knowledge from previous measurements. Sequential hypothesis test enables the radar to adjust its transmitted waveform based on its previous observations [28, 32–34] and is thus suitable for a cognitive radar framework. In this paper, sequential hypothesis testing for multitarget recognition is considered. The closed-loop operation is illustrated in Figure 1. One of $K$ possible targets is assumed to be present. At each illumination, the MIMO radar transmits a set of signals to interrogate the environment. The radar echoes are contaminated by ambient noise. The radar knowledge of the environment includes the noise power and the probability of each hypothesis. Based on the radar echoes, the optimal waveform for the next transmission is designed according to the current knowledge of the environment. As the understanding of the environment grows iteratively, the transmitted waveform tends to be the optimal signal for the target and the corresponding environment.

In this paper, the waveform optimization methods based on MI and KL divergence for the extended target case are used in the aforementioned closed-loop MIMO radar system. We assume that only the probability of each hypothesis is changed during the illumination. The optimal waveforms for the $K$ hypotheses are generated and stored in advance. During the illumination, the probabilities of the $K$ hypotheses are updated according to the Bayes’ rule. Two methods are proposed to form the optimal waveform for all the $K$ possible hypotheses. In Method 1, the optimal waveform is supposed to be the weighted sum of the $K$ optimal waveforms for the corresponding hypothesis, where the weights are the probability of each hypothesis. In Method 2, the optimal waveform is the MI-based optimal waveform or KL divergence-based optimal waveform for the ensemble target signature. Therefore, the transmitted waveform is changing during the illumination based on the current knowledge of the probabilities. The combination of the two methods together with the MI-based optimal waveform and KL divergence-based optimal waveform will be compared to find out which one is more suitable in the multitarget recognition case. The main contributions of our work are as follows. On one hand, optimal waveform design for MIMO radar for one known target is extended to the case of multitarget case by using likelihood ratio test. Two adaptive waveform design methods, namely, the probability weighted waveform method and the probability weighted target signature method, are proposed to form the optimal waveform of multitarget. On the other hand, the optimal waveform for SISO radar multitarget classification problem is extended to MIMO radar. The result is consistent with that of the SISO radar, that is, the optimal detection waveform (maximum Kullback-Leibler divergence waveform for MIMO radar, eigenvalue optimal waveform for SISO radar) using probability weighted waveform method is more suitable for multitarget classification than the other methods as the former results in a smaller probability of error.

The remainder of the paper is organized as follows. The common MIMO radar signal model is presented in Section 2. Section 3 describes the sequential likelihood ratio test for the multitarget recognition problem. The MI-based and KL divergence-based optimal waveforms for MIMO radar are provided in Section 4, together with the two optimal adaptive waveform methods for the multitarget recognition problem. Finally, simulation results are presented in Section 5 to validate the optimal waveform strategy and their usage in the sequential likelihood ratio testing problem. We conclude the paper in Section 6.

2. Signal Model

In this section, we review the general discrete-time baseband signal model for the case of extended target [9]. The MIMO radar is equipped with $P$ transmit antennas and $Q$ receiving antennas. The extended target impulse response from the $p$th transmitter to the $q$th receiver is modeled as a finite impulse response (FIR) linear filter with order $v$ and denoted by $g^{(p,q)}(l)$, for $l \in [0,v]$. Therefore, the echo at the $q$th receiver can be expressed as

$$y_q(s) = \sum_{p=1}^{P} \sum_{l=0}^{v} g^{(p,q)}(l) x_p(s-l) + n_q(s), \quad (1)$$

where $x_p(s)$ denotes the transmitted waveform from the $p$th transmitter at the sample time instant $s$ and $n_q(s)$ is
the complex-valued additive white Gaussian noise (AWGN) at the $q$th receiver at the corresponding time instant. In addition, the second summation denotes the echo from the $p$th transmitter to the $q$th receiver which is the convolution of the $p$th transmitted signal with the target impulse response from the $p$th transmitter to the $q$th receiver.

Suppose that the length of the observation signal is $L$, and assume that $L > n$. We let

$$\mathbf{g}^{(p,q)} = [g^{(p,q)}(0), \ldots, g^{(p,q)}(y)]^T,$$

$$\mathbf{y}_q = [y_q(s), \ldots, y_q(s + L - 1)]^T,$$

$$\mathbf{n}_q = [n_q(s), \ldots, n_q(s + L - 1)]^T.$$  (2)

Then (1) can be written in the matrix-vector form as

$$\mathbf{y}_q = \sum_{p=1}^{P} \mathbf{X}_p \mathbf{g}^{(p,q)} + \mathbf{n}_q,$$  (3)

where $\mathbf{X}_p$ is an $L \times M$ ($M = n + 1$) waveform convolution matrix, which is of the form

$$\mathbf{X}_p = \begin{bmatrix}
x_p(s) & \cdots & x_p(s - v) \\
\vdots & \ddots & \vdots \\
x_p(s + L - 1) & \cdots & x_p(s + L - 1 - v)
\end{bmatrix}.  \tag{4}$$

For all the $P$ transmitted waveforms, we define \( \mathbf{X} = [\mathbf{X}_1, \ldots, \mathbf{X}_P] \) and \( \mathbf{g} = [\mathbf{g}^{(1,q)}]^T, \ldots, [\mathbf{g}^{(P,q)}]^T]^T \). As a consequence, the received signal at the $q$th receiver can be further reformulated into

$$\mathbf{y}_q = \mathbf{X} \mathbf{g}_q + \mathbf{n}_q.$$  (5)

Combining all the received waveforms from the receiving antennas to create \( \mathbf{y} = [\mathbf{y}_1^T, \ldots, \mathbf{y}_Q^T]^T \) and defining \( \mathcal{X} \equiv \mathbf{I}_Q \otimes \mathbf{X} \) in which \( \mathbf{I}_Q \) is a $Q \times Q$ identity matrix and \( \otimes \) is the Kronecker product, we get [9]

$$\mathbf{y} = \mathcal{X} \mathbf{g} + \mathbf{n},$$  (6)

where \( \mathbf{g} = [\mathbf{g}_1^T, \ldots, \mathbf{g}_Q^T]^T \), and \( \mathbf{n} = [\mathbf{n}_1^T, \ldots, \mathbf{n}_Q^T]^T \).

The target impulse response vector $\mathbf{g}$ is assumed to be a complex-valued Gaussian random vector with zero mean and full rank covariance matrix $\Sigma_{g}$. The noise vector $\mathbf{n}$ is assumed to be a complex-valued Gaussian random vector with zero mean and covariance matrix $\sigma_n^2 \mathbf{I}_{QL}$; that is, the components of the noise vector are independently and identically distributed (i.i.d).

The eigenvalue decomposition of the target covariance matrix is given by

$$\Sigma_{g} = \mathbf{U}_g \Lambda_g \mathbf{U}_g^H,$$  (7)

where $\mathbf{U}_g$ is the unitary matrix whose columns are eigenvectors and $\Lambda_g = \text{diag} \{\Lambda_{g,1}, \ldots, \Lambda_{g, PQM}\}$ is a diagonal matrix with the diagonal entries being the eigenvalue in an increasing order.

3. Multitarget Recognition and Sequential Likelihood Ratio Test

The multiple extended targets recognition problem for MIMO radar is considered in this paper. Suppose that one of the $K$ possible targets is known to be present and each is characterized as a complex-valued Gaussian random vector with zero mean and known covariance matrix $\Sigma_{g,k}$, for $k = 1, 2, \ldots, K$. The target impulse response for each hypothesis is denoted by $\mathbf{g}_k$. The basic problem is to decide on one of the $K$ possible hypotheses using as few observations as possible, while maintaining the error rate under a tolerable level [32]. According to the MIMO radar signal model, the $k$th hypothesis is written as

$$\mathcal{H}_k : \mathbf{y} = \mathcal{X} \mathbf{g}_k + \mathbf{n}. \tag{8}$$

The probability density function (pdf) of the echo $\mathbf{y}$ under the $k$th hypothesis is

$$p_k(\mathbf{y}) = \frac{1}{\pi^{QL}} \det \left( \mathcal{X} \Sigma_{g,k} \mathcal{X}^H + \sigma_n^2 \mathbf{I}_{QL} \right) \cdot \exp \left[ -\mathbf{y}^H \left( \mathcal{X} \Sigma_{g,k} \mathcal{X}^H + \sigma_n^2 \mathbf{I}_{QL} \right)^{-1} \mathbf{y} \right],$$  (9)

for $k = 1, 2, \ldots, K$,

where the superscript $H$ denotes the conjugate transpose of a vector or matrix.

In a sequential probability ratio testing problem, a sequence of observations is obtained and at each time, a decision is made to either accept one of the hypotheses or to continue the test by making another observation [32]. The decision is based on the likelihood ratio. The likelihood ratio after the $t$th observation from hypothesis $\mathcal{H}_k$ to $\mathcal{H}_{k'}$ where $k \neq k'$ is given by

$$l_{k,k'}^t = \frac{p_{k1}(\mathbf{Y}_1) p_{k2}(\mathbf{Y}_2) \cdots p_{kt}(\mathbf{Y}_t) P_k}{p_{k1}(\mathbf{Y}_1) p_{k2}(\mathbf{Y}_2) \cdots p_{kt}(\mathbf{Y}_t) P_{k'}},$$  (10)

where $p_{k}(\mathbf{Y}_t)$ denotes the pdf of the $t$th observation under the $k$th hypothesis, $\mathbf{Y}_t$ denotes the echo of the $t$th illumination, and $P_k$ is the prior probability of the $k$th hypothesis. Because the waveform is adaptively designed, and the noise is a random process, the echo and the pdf are different at each illumination. At each time, the following condition is checked to decide whether to terminate the experiment or not:

$$l_{k,k'}^t > \frac{1 - \alpha_{k,k'}}{\alpha_{k,k'}}, \quad \forall k' \neq k,$$  (11)

where $\alpha_{k,k'}$ denotes the tolerable error rate of deciding $\mathcal{H}_{k'}$ provided that $\mathcal{H}_k$, $k \neq k'$ is true. If (11) is satisfied for some $k$, then $\mathcal{H}_k$ is deemed to be true; otherwise the experiment continues.

The pdf of the $k$th hypothesis after the $t$th illumination can be obtained by (9). Note that the pdf of each illumination is different due to its dependence on the transmitted waveform. Therefore, this sequential testing problem is non-i.i.d. [32].
4. Waveform Optimization for Target Estimation and Detection

Two waveform optimization schemes are used in this paper, namely, the MI-based optimal waveform which was initially proposed in [9] and the KL divergence-based optimal waveform which is an application of the method proposed in [11] to the extended target case in AWGN channels. We simply show the results of the two optimal waveforms for a specific extended target case and propose the optimal waveform for the multitarget recognition problem based on the two optimal waveforms in this section.

4.1. Waveform Optimization for Target Estimation. The optimal waveform which maximizes the MI between the echo and the target impulse response will provide as much target information as possible for the radar system and thus lead to better estimation performance. In [9], the waveform which maximizes the MI is equal to the waveform which minimizes the minimum mean-square error (MMSE) in AWGN from the viewpoint of estimation. The optimization problem under total transmitted power constraint is given by [9]

\[
\begin{align*}
\max_{\mathbf{\mathcal{X}}} \, & \quad I(\mathbf{y}, \mathbf{g} | \mathbf{\mathcal{X}}) \\
\text{s.t.} \quad & \quad \text{tr}(\mathbf{\mathcal{X}}^H \mathbf{\mathcal{X}}) \leq QLP_0,
\end{align*}
\]

where \( I(\mathbf{y}, \mathbf{g} | \mathbf{\mathcal{X}}) \) is the conditional MI. In this case, the optimal solution is

\[
\mathbf{\mathcal{X}} = \Psi \left( \text{diag} \left( \left( \eta - \frac{\sigma_n^2}{\Lambda_{g1}} \right)^\dagger, \ldots, \left( \eta - \frac{\sigma_n^2}{\Lambda_{gPQM}} \right)^\dagger \right) \right)^{1/2} \mathbf{U}_\mathbf{g}^H,
\]

where \(\Psi\) is a \(QL \times PQM\) matrix with orthonormal columns, and the constant \(\eta\) is chosen to satisfy

\[
\sum_{i=1}^{PQM} \left( \eta - \frac{\sigma_n^2}{\Lambda_{gj}} \right)^\dagger = QLP_0.
\]

Note that the eigenvalues of the target covariance matrix do not need to be sorted here. The optimal waveform performs a water filling operation on the noise-to-target power, which allocates more power to the eigenmodes of larger target power.

4.2. Waveform Optimization for Target Detection. In this subsection, the method in [11] based on KL divergence is modified to the case for extended target in AWGN channels. Target detection can be modelled as a binary hypothesis testing problem according to the signal model in (6) as

\[
\begin{align*}
\mathcal{H}_0 : \mathbf{y} = \mathbf{n}, & \quad (\text{target absent}), \\
\mathcal{H}_1 : \mathbf{y} = \mathbf{\mathcal{G}} \mathbf{g} + \mathbf{n}, & \quad (\text{target present}).
\end{align*}
\]

Based on Stein’s lemma [11], the larger the relative entropy (KL divergence) between the two distributions of the hypotheses, the better the detection performance. The KL divergence from \(\mathcal{H}_0\) to \(\mathcal{H}_1\) is denoted by \(D(p_0(\mathbf{y}) \parallel p_1(\mathbf{y}))\), where \(p_0(\mathbf{y})\) and \(p_1(\mathbf{y})\) are the pdfs of the received echo under \(\mathcal{H}_0\) and \(\mathcal{H}_1\).

Using the signal model and the assumptions above, the distributions of the received echo are \(\mathbf{y} | \mathcal{H}_0 \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_{QL})\) and \(\mathbf{y} | \mathcal{H}_1 \sim \mathcal{CN}(\mathbf{\mathcal{G}} \mathbf{g}, \sigma_n^2 \mathbf{I}_{QL})\), respectively. Thus the pdfs are expressed as

\[
\begin{align*}
p_0(\mathbf{y}) &= \frac{1}{\pi^{QL}} \det(\sigma_n^2 \mathbf{I}_{QL}) \exp\left[-\frac{1}{\sigma_n^2} \mathbf{y}^H \mathbf{y} \right], \\
p_1(\mathbf{y}) &= \frac{1}{\pi^{QL}} \det(\mathbf{\mathcal{G}} \mathbf{g}^H \mathbf{\mathcal{G}} + \sigma_n^2 \mathbf{I}_{QL}) \exp\left[-\frac{1}{\sigma_n^2} \mathbf{y}^H (\mathbf{\mathcal{G}} \mathbf{g}^H + \sigma_n^2 \mathbf{I}_{QL})^{-1} \mathbf{y} \right],
\end{align*}
\]

respectively.

The optimal waveform design which maximizes the KL divergence from the pdf of \(\mathcal{H}_0\) to that of \(\mathcal{H}_1\) is

\[
\begin{align*}
\max_{\mathbf{\mathcal{X}}} \, & \quad D(p_0(\mathbf{y}) \parallel p_1(\mathbf{y})) \\
\text{s.t.} \quad & \quad \text{tr}(\mathbf{\mathcal{X}}^H \mathbf{\mathcal{X}}) \leq QLP_0,
\end{align*}
\]

Note that the KL divergence measures the “distance” from one pdf to another. However, it is asymmetric, so not a true distance. The total power constraint of the transmitted waveform is considered; that is,

\[
\text{tr}(\mathbf{\mathcal{X}}^H \mathbf{\mathcal{X}}) \leq QLP_0.
\]

Note that \(\mathbf{\mathcal{X}} = \mathbf{I}_Q \otimes \mathbf{X}\) and \(\text{tr}(\mathbf{\mathcal{X}}^H \mathbf{\mathcal{X}}) \leq QLP_0\) is the same as \(\text{tr}[\mathbf{X}^H \mathbf{X}] \leq LP_0\).

Substituting the pdfs in (16) and (17) into the KL divergence yields

\[
\begin{align*}
D(p_0(\mathbf{y}) \parallel p_1(\mathbf{y})) &= \int p_0(\mathbf{y}) \log \frac{p_0(\mathbf{y})}{p_1(\mathbf{y})} d\mathbf{y} \\
&= \log \left[ \det (\sigma_n^2 \mathbf{\mathcal{G}} \mathbf{g}^H \mathbf{\mathcal{G}} + \mathbf{I}_{QL}) \right] + \text{tr} \left( (\sigma_n^2 \mathbf{\mathcal{G}} \mathbf{g}^H + \mathbf{I}_{QL})^{-1} \mathbf{I}_{QL} \right).
\end{align*}
\]

Therefore, the optimization problem based on maximizing the KL divergence under the energy constraint can be expressed as (after omitting the constant QL)

\[
\begin{align*}
\max_{\mathbf{\mathcal{X}}} \, & \quad \log \left[ \det (\sigma_n^2 \mathbf{\mathcal{G}} \mathbf{g}^H + \mathbf{I}_{QL}) \right] \\
& \quad + \text{tr} \left( (\sigma_n^2 \mathbf{\mathcal{G}} \mathbf{g}^H + \mathbf{I}_{QL})^{-1} \right), \\
\text{s.t.} \quad & \quad \text{tr}(\mathbf{\mathcal{X}}^H \mathbf{\mathcal{X}}) \leq QLP_0,
\end{align*}
\]

Obviously, \(\mathbf{\mathcal{G}} \mathbf{g}^H \mathbf{\mathcal{G}}\) is a positive semidefinite Hermitian matrix and has the eigenvalue decomposition

\[
\mathbf{\mathcal{G}} \mathbf{g}^H \mathbf{\mathcal{G}} = \mathbf{U}_1 \Lambda_1 \mathbf{U}_1^H,
\]
where $U_i$ is the $QL \times QL$ unitary matrix, $A_1 = \text{diag}[A_{1,1}, \ldots, A_{1,QL}]$, and $A_{1,1} \leq A_{1,2} \leq \cdots \leq A_{1,QL}$. Note that $\text{rank}(X \Sigma_g X^H) = \text{min}(QL, PQM)$. Assume that $QL > PQM$ without loss of generality. Therefore, $A_{1,i} = 0$, for $i = 1, \ldots, QL - PQM$. Letting $X = U_1 X_1$, we have $X_1 \Sigma_g^{-1} X_1^H = A_1$.

According to the proof in the appendix, the optimal waveform is found as

$$X_{\text{opt}} = \left[0_{(QL - PQM) \times PQM} \begin{pmatrix} \Sigma_g^{-1/2} \Lambda_g & \Lambda_g \end{pmatrix} \right] U_{g}^{H},$$

where $D_{1}^{\text{opt}} = \text{diag}[\Lambda_{1,QL - PQM + 1}^{\text{opt}}, \ldots, \Lambda_{1,QL}^{\text{opt}}]$, where

$$\Lambda_{1,QL - PQM + k}^{\text{opt}} = \begin{pmatrix} -\sigma_n^2 & \Lambda_{g,k} + \sqrt{\Lambda_{g,k}^2 + 4 \lambda \sigma_n^2 A_{g,k}} \end{pmatrix}^+, \quad \text{for } k = 1, \ldots, PQM.$$

The constant $\lambda$ is obtained by

$$\sum_{k=1}^{PQM} \left( -\sigma_n^2 - \Lambda_{g,k} + \sqrt{\Lambda_{g,k}^2 + 4 \lambda \sigma_n^2 A_{g,k}} \right)^+ \Lambda_{g,k}^{-1} = QLP_0.$$

4.3. Adaptive Waveform Optimization for Multitarget Recognition. The former waveform optimizations are both for the specific extended target case. In the multitarget recognition problem addressed in this paper, which target is present is unknown at the beginning. We use two different waveform design methods for the multitarget recognition problem. Method 1 is the probability weighted waveform method. The $K$ hypotheses are assigned to some probability. The optimal waveform is supposed to be the weighted sum of the optimal waveform for each hypothesis where the weights are the probabilities [33]; that is,

$$\mathcal{X}_{\text{ensemble}}(t) = \sum_{k=1}^{K} P(\mathcal{H}_k | y_i) \mathcal{X}_k^{\text{opt}},$$

where $P(\mathcal{H}_k | y_i)$ denotes the probability for the $k$th hypothesis at time instant $t$, $\mathcal{X}_k^{\text{opt}}$ denotes the optimal waveform for the $k$th hypothesis, and $\mathcal{X}_{\text{ensemble}}(t)$ is the optimal waveform for the ensemble at time instant $t$, that is, the multitarget recognition case. At each hypothesis, the target is known. Therefore, the optimal waveform $\mathcal{X}_k^{\text{opt}}$ can be obtained using the aforementioned waveform optimization method based on either mutual information or KL divergence criterion. Method 2 is the probability weighted target signature method.

The knowledge of the multitarget is summarized as the ensemble covariance matrix [32],

$$\Sigma_{\text{ensemble}} = U_g \left( \sum_{k=1}^{K} P(\mathcal{H}_k | y_i) A_g \right) - \left( \sum_{k=1}^{K} P(\mathcal{H}_k | y_i) \sqrt{A_g} \right)^2 U_g^{H},$$

where $\sqrt{\cdot}$ is the elementwise square root of the matrix and $| \cdot |$ is also elementwise. Using the ensemble covariance matrix as the new target, the optimal waveform can be obtained based on either mutual information or KL divergence criterion. Note that the ensemble covariance matrix can be considered as the effective target covariance matrix of the current ensemble.

At each illumination, the probabilities of the $K$ hypotheses are updated according to the Bayes' rule as

$$P(\mathcal{H}_k | y_i) \propto p_{y_i}(y_i) P(\mathcal{H}_k | y_{i-1}),$$

where $P(\mathcal{H}_k | y_i)$ is the conditional probability of $\mathcal{H}_k$ after the $i$th observation. When $t = 0$, $P(\mathcal{H}_k | y_0) = P_k$ is the prior probability. Therefore, the optimal waveform for the ensemble is adaptively changed at each illumination due to the changes of the probability of each hypothesis.

For Method 1, the optimal waveform of multitarget is the weighted sum of the individual optimal waveforms for each hypotheses scaled by their corresponding update probabilities. The optimal waveforms for the $K$ hypotheses can be designed in advance which reduces the time consumption of calculating the optimal waveform for the ensemble during the illumination process. Therefore, this method is more computationally efficient than Method 2, which is good for a closed-loop radar. For Method 2, the ensemble covariance matrix can be regarded as the current knowledge of the environment. All the hypotheses are integrated with the Bayesian representation of the target covariance matrix. The ensemble covariance matrix can be substituted into waveform design methods maximizing either mutual information or KL divergence.

5. Numerical Results

In this section, numerical results are presented to illustrate the performance of the two optimal waveform design methods applied to multitarget recognition problem. The MIMO radar system parameters are set to $P = 2$, $Q = 1$, $M = 10$, and $L = 20$. Suppose that $K = 4$. The four targets' power spectral density (PSD) $V_{ii}$, $i = 1, \ldots, PQM$, is shown in Figure 2, where the target power is assumed to be 100J and the noise power is $\sigma_n^2 = 1$. For simplicity, we set $A_{g,i} = V_{ii}$, $i = 1, \ldots, PQM$, for each target.

5.1. Waveform Optimization for a Specific Target. The MI-based optimal waveform for the first target is illustrated in Figure 3(a), where the power constraint is $LP_0 = 10$ dB. Figure 3(a) shows the noise-to-target power, while
the bottom shows the optimal waveform power \( \ell^i_{ij} \), where \( \ell^i_{ij} = (\eta - \sigma_n^2 / \Lambda_{g_i})^+, i = 1, \ldots, PQM \). The optimal waveform performs a water filling operation on the noise-to-target power. Clearly, the optimal MI-based waveform allocates the power into the eigenmodes of smaller noise-to-target power. The optimal waveform for the first target based on KL divergence is also illustrated in Figure 3(b) under the same power constraint \( LQ_P = 10 \) dB. For the KL divergence-based optimal waveform, the eigenvalues of the target covariance matrix should be sorted to have an increasing order. The top panel of Figure 3(b) shows the eigenvalues of the first target covariance matrix in increasing order. The optimal waveform power allocation strategy is illustrated at the bottom of Figure 3(b), where \( \ell^i_{ij} = D^\text{opt}_{LQ} \Lambda_{g_i}^{-1}, i = 1, \ldots, PQM \). Clearly, the two optimal waveforms have different power allocation strategies for the same target covariance matrix under the same power constraint.

For different transmitted power \( LQ_P \in [2, 20] \) dB, the MI of using the MI-based optimal waveform for the first target is calculated and compared with that of using the equal power allocation waveform. The MI can be calculated by [9]

\[
I = \sum_{i=1}^{PQM} \log \left( \sigma_n^{-2} \Lambda_{g_i} \ell^i_{ij} + 1 \right),
\]

where \( \ell^i_{ij} = (\eta - \sigma_n^2 / \Lambda_{g_i})^+, \) for \( i = 1, \ldots, PQM \), for the MI-based optimal waveform. For the equal power allocation waveform, \( \ell^i_{ij} = LQ_P / PQM, \) for \( i = 1, \ldots, PQM \). The MI comparison of using the optimal MI-based waveform and the equal power allocation waveform is provided in Figure 4(a). Similarly, the KL divergence comparison of using the optimal KL divergence-based optimal waveform and the equal power allocation waveform is shown in Figure 4(b). The KL divergence is calculated by [11]

\[
D = \sum_{i=1}^{PQM} \left[ \log \left( 1 + \frac{\ell^i_{ij} \Lambda_{g_i}}{\sigma_n^2} \right) + \left( 1 + \frac{\ell^i_{ij} \Lambda_{g_i}}{\sigma_n^2} \right)^{-1} \right],
\]

where \( \ell^i_{ij} = D^\text{opt}_{LQ} \Lambda_{g_i}^{-1}, \) for \( i = 1, \ldots, PQM, \) for the KL divergence-based optimal waveform. From Figures 4(a) and 4(b), the two optimal strategies improve the corresponding criteria. When the transmitted power is larger, the MI and KL divergence difference between the optimal waveform and the equal power allocation waveform is smaller, which indicates that the optimal waveform design method is especially useful in power limited environment.

Using the same method, the optimal waveforms with different transmitted power constraints for the other three targets based on either mutual information or KL divergence can be obtained. In the subsequent simulations, the waveforms are applied to the sequential likelihood ratio testing problem to recognize the true target in the four hypotheses.

5.2. Adaptive Waveform Optimization and Sequential Hypothesis Testing for Multitarget Recognition. In this subsection, Method 1 and Method 2 are used to adaptively design the optimal waveform for multitarget recognition using sequential hypothesis testing. Both the mutual information and KL divergence criteria are used in the waveform design stage. The tolerable error rate for the sequential hypothesis testing problem is assumed to be \( \alpha_{k,k'} = 0.01 \) for all \( k \) and \( k' \). The prior probability of the four hypotheses are assumed to be \( P_1 = P_2 = P_3 = P_4 = 0.25 \). Suppose that the first hypothesis is true. During the illumination, the probability of the four hypotheses is changed. Figure 5 shows such a process. The probability of the first hypothesis tends to 1 at the end of the illumination while that of the others is all small. Thus the likelihood ratio of the first hypothesis with respect to the other three exceeds the threshold. The illumination terminates and a decision is made.

For different transmitted power \( LQ_P \in [-5, 5] \) dB, the average illumination number is calculated using Monte Carlo methods, with 1000 simulation runs. The average illumination numbers for the MI-based optimal waveform using Method 1 and Method 2, the KL divergence-based optimal waveform using Method 1 as well as Method 2, and the equal power allocation waveform are compared in Figure 6(a). We conclude that the average illumination numbers of all the five methods gradually decrease with respect to the transmitted power. The best optimal waveform for the multitarget recognition problem is the KL divergence-based waveform using Method 1, that is, the probability weighted target signature method. The next one is the MI-based optimal waveform using Method 1, which is followed by the MI-based optimal waveform using Method 2, that is, the probability weighted waveform method. The next one is the equal power allocation waveform. The worst one is the KL divergence-based waveform using Method 2. When the transmitted energy is large, the average illumination numbers of all the five waveforms are rather similar. Note that Method 1 is always better than Method 2 no matter which optimization criterion is used. Besides, Method 2 is more time consuming than Method 1, because, in Method 2, waveform optimization has to be performed at each illumination when the target signature is updated. But in Method 1, the optimal waveform for each hypotheses can be calculated and stored first.
We also compare the probability of correct decision during the simulations. The probability is calculated by dividing the correct decision number by the total number of Monte Carlo simulations. The results are shown in Figure 6(b). The KL divergence-based optimal waveform using Method 1 has the best performance. Therefore, the KL divergence-based optimal waveform using probability weighted waveform method is more suitable to be used in the multitarget recognition problem which has the lowest average illumination number and the highest probability of correct decision compared with the others. This result is consistent with the result in [33], where single-input single-output (SISO) radar adaptive waveform design for target recognition was considered. For SISO radar, the best waveform for multitarget recognition is the SNR-based optimal waveform using probability weighted energy (PWE) technique, that is, Method 1 in this paper. Maximizing SNR for SISO radar is equivalent to maximizing KL divergence. Therefore, the optimal waveform for SISO radar is KL divergence-based optimal waveform using Method 1. In this paper, similar conclusions are obtained for MIMO radar.

6. Conclusion

In this paper, the MI-based optimal waveform and the KL divergence-based optimal waveform for MIMO radar for
a specific extended target are studied. Sequential likelihood ratio test is used to make a decision in the multitarget recognition problem. Two methods are used to form the optimal waveform for multitarget recognition. In Method 1, the optimal waveform for multitarget is the weighted sum of the optimal waveform for each of the hypothesis, where the weights are the probabilities of the hypotheses. In Method 2, the knowledge of the multitarget is first calculated as the ensemble covariance matrix, where the probabilities of the hypotheses are also included. The ensemble covariance matrix denotes the current characteristic of the environment, which is used to design the optimal waveform. During the illumination, the probabilities change and thus the waveform adaptively changes. Simulation results show that the KL divergence-based optimal waveform using Method 1 is the best waveform for the multitarget recognition as it has the lowest average illumination number in order to make a decision with the highest probability of correct decision.

Note that the optimization constraint considered in this paper is only the transmitted power. More practical constraints like constant envelope, good range resolution, and so on will be discussed in future.
Appendix

The optimization of (21) is very similar to that in [11]. Problem (21) can be rewritten as

$$\max_{X_1} \log \left[ \det \left( \sigma_n^{-2} U_1 \Sigma_g^{-1} X_1^H U_1^H + I_{QL} \right) \right]$$

$$+ \text{tr} \left[ \left( \sigma_n^{-2} U_1 \Sigma_g^{-1} X_1^H U_1^H + I_{QL} \right)^{-1} \right],$$

s.t. \( \text{tr} \left[ X_1^H X_1 \right] \leq QLP_0 \)

(A.1)

According to [11, Lemma 2], given a \( QL \times PQM \) matrix \( X_1 \) and a positive semidefinite \( PQM \times PQM \) matrix \( \Sigma_g \) such that \( \Sigma_g^{-1} X_1^H X_1 \) is a diagonal matrix with diagonal elements in increasing order, it is always possible to find another matrix \( \Sigma_g^{-1} X_1^H X_1 \) which satisfies \( \Sigma_g^{-1} X_1^H X_1 = \alpha \Sigma_g^{-1} X_1^H X_1 \), where \( \alpha \geq 1 \) and \( \text{tr} \left[ \Sigma_g^{-1} X_1^H X_1 \right] = \text{tr} \left[ \Sigma_g^{-1} X_1^H X_1 \right] \). The matrix \( \Sigma_g^{-1} X_1^H X_1 \) has the form

$$\Sigma_g^{-1} X_1^H X_1 = \begin{bmatrix} 0_{(QL-PQM) \times PQM} & \Lambda_{1/2} \\ \Lambda_{1/2} & I \end{bmatrix} \begin{bmatrix} U_g^H \\ \Lambda_{1/2} \end{bmatrix},$$

(A.2)

where \( \Lambda_{1/2} \) is a \( PQM \times PQM \) diagonal matrix and \( U_g \) is the unitary matrix whose columns are the eigenvectors of \( \Sigma_g \) corresponding to the eigenvalues in increasing order. Furthermore, note that \( \log(\det(I + R)) + \text{tr}(I + R)^{-1} \) is a monotonic increasing function of positive semidefinite matrix \( R \), the optimal solution of the problem in (A.1) has the form

$$X_1^{\text{opt}} = \begin{bmatrix} 0_{(QL-PQM) \times PQM} & \Lambda_{1/2} \\ \Lambda_{1/2} & I \end{bmatrix} \begin{bmatrix} U_g^H \\ \Lambda_{1/2} \end{bmatrix},$$

(A.3)

If \( U_1 \) is known, and \( X_1^{\text{opt}} \) is found, the optimal waveform is given by

$$X^{\text{opt}} = U_1 \begin{bmatrix} 0_{(QL-PQM) \times PQM} & \Lambda_{1/2} \\ \Lambda_{1/2} & I \end{bmatrix} \begin{bmatrix} U_g^H \\ \Lambda_{1/2} \end{bmatrix},$$

(A.4)

Using the properties of determinant and trace, we have

$$\text{det} \left( \sigma_n^{-2} AA^H + I_{QL} \right) = \text{det} \left( \sigma_n^{-2} A^H A + I_{PQM} \right),$$

$$\text{tr} \left[ \left( \sigma_n^{-2} AA^H + I_{QL} \right)^{-1} \right]$$

(A.5)

$$= QL - PQM + \text{tr} \left[ \left( \sigma_n^{-2} A^H A + I_{PQM} \right)^{-1} \right],$$

where \( A \) is a \( QL \times PQM \) matrix. Letting \( A = X \Sigma_g^{1/2} \), we then have

$$\log \left[ \det \left( \sigma_n^{-2} X \Sigma_g^{1/2} X^H + I_{QL} \right) \right]$$

$$+ \text{tr} \left[ \left( \sigma_n^{-2} X \Sigma_g^{1/2} X^H + I_{QL} \right)^{-1} \right]$$

$$= \log \left[ \det \left( \sigma_n^{-2} \Sigma_g^{1/2} X^H X \Sigma_g^{1/2} + I_{PQM} \right) \right]$$

$$+ \text{tr} \left[ \left( \sigma_n^{-2} \Sigma_g^{1/2} X^H X \Sigma_g^{1/2} + I_{PQM} \right)^{-1} \right]$$

$$+ \text{tr} \left[ \left( \sigma_n^{-2} \Sigma_g^{1/2} X^H X \Sigma_g^{1/2} + I_{PQM} \right)^{-1} \right]$$

(A.6)

Hence, the optimization problem in (21) can be expressed as

$$\max_{X} \log \left[ \det \left( \sigma_n^{-2} \Sigma_g^{1/2} X^H X \Sigma_g^{1/2} + I_{PQM} \right) \right]$$

$$+ \text{tr} \left[ \left( \sigma_n^{-2} \Sigma_g^{1/2} X^H X \Sigma_g^{1/2} + I_{PQM} \right)^{-1} \right],$$

(A.7)

s.t. \( \text{tr} \left[ X^H X \right] \leq QLP_0 \)

Problems (A.1) and (A.7) are almost the same. Therefore, we know that \( U_1 \) is an arbitrary permutation matrix \( U_1 = P_1 \).

The optimal waveform in (A.4) is then given by

$$X^{\text{opt}} = P_1 \begin{bmatrix} 0_{(QL-PQM) \times PQM} & \Lambda_{1/2} \\ \Lambda_{1/2} & I \end{bmatrix} \begin{bmatrix} U_g^H \\ \Lambda_{1/2} \end{bmatrix},$$

(A.8)

where \( P_1 \) is an arbitrary permutation matrix. Substituting the optimal waveform expression in (A.8) into (21), the optimization problem can be expressed as

$$\max_{D_1, P_1} \log \left[ \det \left( \sigma_n^{-2} P_1 \begin{bmatrix} 0 & 0 \\ 0 & D_1 \end{bmatrix} P_1^T + I_{QL} \right) \right]$$

$$+ \text{tr} \left[ \left( \sigma_n^{-2} P_1 \begin{bmatrix} 0 & 0 \\ 0 & D_1 \end{bmatrix} P_1^T + I_{QL} \right)^{-1} \right],$$

(A.9)

s.t. \( \sum_{k=1}^{PQM} \Lambda_{1,QL-PQM+k} \Lambda_{g,k}^{-1} \leq QLP_0 \),

where \( D_1 = \text{diag}(\Lambda_{1,QL-PQM+1}, \ldots, \Lambda_{1,QL}) \) is a \( PQM \times PQM \) diagonal matrix and \( D_1 = \Lambda_{1,QL} \Lambda_{g} \) according to \( X_1 \Sigma_g^{-1} X_1^H = \Lambda_1 \).

To solve the optimization in (A.9), we first investigate the choice of the permutation matrix \( P_1 \). According to [11, Lemma 5], if \( a > c > 0, b > d > 0 \), then we have

$$\log \left( 1 + \frac{a}{b} \right) \leq \log \left( 1 + \frac{a}{b} \right) + \frac{1}{1 + a/b} + \frac{1}{1 + c/d}$$

$$\leq \log \left( 1 + \frac{a}{b} \right) \leq \frac{1}{1 + a/d}$$

(A.10)

Note that the diagonal elements of \( D_1 \) are in increasing order. By (A.10), we know that to maximize the objection function in (A.9) \( P_1 \) should be \( I_{QL} \) if \( D_1 \) is given. Therefore, the optimization problem becomes
\[
\max_{\Lambda, k} \sum_{k=1}^{PQM} \left[ \log \left( 1 + \frac{\Lambda^{1\text{QL}-PQM} + k}{\sigma_n^2} \right) \right] + \left( 1 + \frac{\Lambda^{1\text{QL}-PQM} + k}{\sigma_n^2} \right)^{-1}
\]
\]
\[\text{s.t. } \sum_{k=1}^{PQM} \Lambda^{1\text{QL}-PQM} + k \Lambda^{-1}_{g,k} \leq QLP_0.\]

We use the Lagrange multiplier method to solve (A.11). Let
\[
J(\lambda) = \sum_{k=1}^{PQM} \left[ \log \left( 1 + \frac{\Lambda^{1\text{QL}-PQM} + k}{\sigma_n^2} \right) \right] + \left( 1 + \frac{\Lambda^{1\text{QL}-PQM} + k}{\sigma_n^2} \right)^{-1} + \lambda \left( \sum_{k=1}^{PQM} \Lambda^{1\text{QL}-PQM} + k \Lambda^{-1}_{g,k} - QLP_0 \right).
\]
The first order differentiation of \( J(\lambda) \) with respect to \( \Lambda^{1\text{QL}-PQM} + k \) yields
\[
\frac{\partial J}{\partial \Lambda^{1\text{QL}-PQM} + k} = \frac{\lambda \Lambda^{2\text{QL}-PQM} + k (2\sigma_n^2 \lambda + \Lambda_{g,k}) \Lambda^{1\text{QL}-PQM} + k / \sigma_n^2}{\sigma_n^2 (1 + \Lambda^{1\text{QL}-PQM} + k / \sigma_n^2)} + \lambda (2\Lambda^{1\text{QL}-PQM} + k + \Lambda_{g,k}) \Lambda^{2\text{QL}-PQM} + k / \sigma_n^2 - \frac{\lambda \sigma_n^2}{\sigma_n^2 (1 + \Lambda^{1\text{QL}-PQM} + k / \sigma_n^2)} \Lambda_{g,k} + \Lambda^{1\text{QL}-PQM} + k + \lambda \sigma_n^2 \Lambda_{g,k} - \Lambda^{1\text{QL}-PQM} + k \Lambda_{g,k}.
\]
Noting that \( \partial J(\lambda) / \partial \Lambda^{1\text{QL}-PQM} + k = 0 \) is a quadratic equation of \( \Lambda_{1,k} \), the two solutions are
\[
\Lambda^{(1)}_{1\text{QL}-PQM} + k = -\sigma_n^2 - \frac{1}{2\lambda} \left[ \Lambda_{g,k} + \sqrt{\Lambda_{g,k}^2 + 4\lambda \sigma_n^2 \Lambda_{g,k}} \right],
\]
\[
\Lambda^{(2)}_{1\text{QL}-PQM} + k = -\sigma_n^2 - \frac{1}{2\lambda} \left[ \Lambda_{g,k} - \sqrt{\Lambda_{g,k}^2 + 4\lambda \sigma_n^2 \Lambda_{g,k}} \right].
\]
Since \( \lambda < 0, \Lambda^{(1)}_{1\text{QL}-PQM} + k \geq \Lambda^{(2)}_{1\text{QL}-PQM} + k \) and the objective function in (A.11) is a monotonic increasing function of \( \Lambda_{1,k} \); the optimal solution can be found as
\[
\Lambda^{\text{opt}}_{1\text{QL}-PQM} + k = \Lambda^{(1)}_{1\text{QL}-PQM} + k^+, \quad \text{for } k = 1, \ldots, PQM.
\]
According to the waveform power constraint. Therefore, the optimal waveform is given by
\[
\mathcal{X}^\text{opt} = \left[ \theta_{(QLM)-PQM} \times \text{PQM} \right] \left[ (D_{1}^\text{opt})^{1/2} \Lambda_{g,k}^{1/2} \right]_{g,k},
\]
where \( D_{1}^\text{opt} = \text{diag}(\Lambda_{1\text{QL}-PQM}^{\text{opt}} , \ldots, \Lambda_{1\text{QL}-PQM}^{\text{opt}}) \), where
\[
\Lambda_{1\text{QL}-PQM}^{\text{opt}} = -\sigma_n^2 - \frac{\sqrt{\Lambda_{g,k}^2 + 4\lambda \sigma_n^2 \Lambda_{g,k}}}{2\lambda},
\]
for \( k = 1, \ldots, PQM \).

The constant \( \lambda \) is obtained by
\[
\sum_{k=1}^{PQM} \left( -\sigma_n^2 - \frac{\sqrt{\Lambda_{g,k}^2 + 4\lambda \sigma_n^2 \Lambda_{g,k}}}{2\lambda} \right) \Lambda_{g,k}^{-1} = QLP_0.
\]

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

**Acknowledgments**

The authors would like to thank the editor and the anonymous reviewers for their comments leading to improvement of this paper.

**References**

[1] J. Li and P. Stoica, *MIMO Radar Signal Processing*, Wiley, Hoboken, NJ, USA, 2009.

[2] E. Fishler, A. Haimovich, R. S. Blum, L. J. Cimini Jr., D. Chizhik, and R. A. Valenzuela, “Spatial diversity in radars—models and detection performance,” *IEEE Transactions on Signal Processing*, vol. 54, no. 3, pp. 823–838, 2006.

[3] A. M. Haimovich, R. S. Blum, and L. J. Cimini Jr., “MIMO radar with widely separated antennas,” *IEEE Signal Processing Magazine*, vol. 25, no. 1, pp. 116–129, 2008.

[4] K. Forsythe, D. Bliss, and G. Fawcett, “Multiple-input multiple-output (MIMO) radar: performance issues,” in *Proceedings of...*
the 38th Asilomar Conference on Signals, Systems and Computers, vol. 1, pp. 310–315, Pacific Grove, Calif, USA, November 2004.

[5] J. Li and P. Stoica, “MIMO radar with colocated antennas,” IEEE Signal Processing Magazine, vol. 24, no. 5, pp. 106–114, 2007.

[6] S. Ahmed and M.-S. Alouini, “MIMO-radar waveform covariance matrix for high SINR and low side-lobe levels,” IEEE Transactions on Signal Processing, vol. 62, no. 8, pp. 2056–2065, 2014.

[7] S. Ahmed and M.-S. Alouini, “MIMO radar transmit beampattern design without synthesizing the covariance matrix,” IEEE Transactions on Signal Processing, vol. 62, no. 9, pp. 2278–2289, 2014.

[8] Y. Yang and R. S. Blum, “Minimax robust MIMO radar waveform design for MIMO radar systems using signal cross-correlation,” IEEE Transactions on Signal Processing, vol. 56, no. 8, pp. 3959–3968, 2008.

[9] Y. Yang and R. S. Blum, “Minimax robust MIMO radar waveform design,” IEEE Journal on Selected Topics in Signal Processing, vol. 1, no. 1, pp. 147–155, 2007.

[10] B. Tang, J. Tang, and Y. Peng, “MIMO radar waveform design in colored noise based on information theory,” IEEE Transactions on Signal Processing, vol. 58, no. 9, pp. 4684–4697, 2010.

[11] B. Tang, J. Tang, and Y. Peng, “Waveform optimization for MIMO radar in colored noise: further results for estimation-oriented criteria,” IEEE Transactions on Signal Processing, vol. 60, no. 3, pp. 1517–1522, 2012.

[12] W. Zhang and L. Yang, “Communications-inspired sensing: a case study waveform design,” IEEE Transactions on Signal Processing, vol. 58, no. 2, pp. 792–803, 2010.

[13] C.-Y. Chen and P. P. Vaidyanathan, “MIMO radar ambiguity properties and optimization using frequency-hopping waveforms,” IEEE Transactions on Signal Processing, vol. 56, no. 12, pp. 5926–5936, 2008.

[14] J. Li, P. Stoica, and X. Zheng, “Signal synthesis and receiver design for MIMO radar imaging,” IEEE Transactions on Signal Processing, vol. 56, no. 8, pp. 3959–3968, 2008.

[15] W.-Q. Wang, “MIMO SAR chirp modulation diversity waveform design,” IEEE Geoscience and Remote Sensing Letters, vol. 11, no. 9, pp. 1644–1648, 2014.

[16] D. R. Fuhrmann and G. San Antonio, “Transmit beamforming for MIMO radar systems using signal cross-correlation,” IEEE Transactions on Aerospace and Electronic Systems, vol. 44, no. 1, pp. 171–186, 2008.

[17] P. Stoica, J. Li, and Y. Xie, “On probing signal design for MIMO radar,” IEEE Transactions on Signal Processing, vol. 55, no. 8, pp. 4151–4161, 2007.

[18] J. Li, L. Xu, P. Stoica, K. Forsythe, and D. W. Bliss, “Range compression and waveform optimization for MIMO radar: a Cramer-Rao bound based study,” IEEE Transactions on Signal Processing, vol. 56, no. 1, pp. 218–232, 2008.

[19] P. Stoica, J. Li, and X. Zhu, “Waveform synthesis for diversity-based transmit beampattern design,” IEEE Transactions on Signal Processing, vol. 56, no. 6, pp. 2593–2598, 2008.

[20] M. R. Bell, “Information theory and radar waveform design,” IEEE Transactions on Information Theory, vol. 39, no. 5, pp. 1578–1597, 1993.

[21] A. Leshem, O. Naparstek, and A. Nehorai, “Information theoretic adaptive radar waveform design for multiple extended targets,” IEEE Journal on Selected Topics in Signal Processing, vol. 1, no. 1, pp. 42–55, 2007.

[22] T. M. Cover and J. A. Thomas, Elements of Information Theory, Wiley-Interscience, New York, NY, USA, 2nd edition, 2006.

[23] L. Wang, H. Wang, Y. Cheng, Y. Qin, and P. V. Brennan, “Adaptive waveform design for maximizing resolvability of targets,” in Proceedings of the 18th International Conference on Digital Signal Processing (DSP’13), pp. 1–6, July 2013.

[24] E. Grossi and M. Lops, “Space-time code design for MIMO detection based on Kullback-Leibler divergence,” IEEE Transactions on Information Theory, vol. 58, no. 6, pp. 3989–4004, 2012.

[25] E. Grossi and M. Lops, “MIMO radar waveform design: a divergence-based approach for sequential and fixed-sample size tests,” in Proceedings of the 3rd IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP ’09), pp. 165–168, IEEE, Oranjestad, Aruba, December 2009.

[26] E. Grossi and M. Lops, “Waveform design for sequential MIMO detection,” in Proceedings of the IEEE Sensor Array and Multichannel Signal Processing Workshop (SAM ’10), pp. 61–64, IEEE, Jerusalem, Israel, October 2010.

[27] M. Teitel and J. Tabrikian, “Waveform design for sequential detection with subspace interference,” in Proceedings of the IEEE 7th Sensor Array and Multichannel Signal Processing Workshop (SAM ’12), pp. 401–404, Hoboken, NJ, USA, June 2012.

[28] S. Haykin, “Cognitive radar: a way of the future,” IEEE Signal Processing Magazine, vol. 23, no. 1, pp. 30–40, 2006.

[29] A. Aubry, A. Demiao, A. Farina, and M. Wicks, “Knowledge-aided (potentially cognitive) transmit signal and receive filter design in signal-dependent clutter,” IEEE Transactions on Aerospace and Electronic Systems, vol. 49, no. 1, pp. 93–117, 2013.

[30] H. Griffiths and C. J. Baker, “Towards the intelligent adaptive radar network,” in Proceedings of the IEEE Radar Conference (RadarCon ’13), pp. 1–5, Ottawa, Ontario, Canada, May 2013.

[31] N. A. Goodman, P. R. Venkata, and M. A. Neifeld, “Adaptive waveform design and sequential hypothesis testing for target recognition with active sensors,” IEEE Journal on Selected Topics in Signal Processing, vol. 1, no. 1, pp. 105–113, 2007.

[32] R. Romero and N. A. Goodman, “Improved waveform design for target recognition with multiple transmissions,” in Proceedings of the International Waveform Diversity and Design Conference (WDD ’09), pp. 26–30, Kissimmee, Fla, USA, February 2009.

[33] L. Wang, H. Wang, and Y. Qin, “Adaptive waveform design for multi-target classification in signal-dependent interference,” in Proceedings of the 19th IEEE International Conference on Digital Signal Processing (DSP ’14), pp. 167–172, IEEE, Hong Kong, August 2014.