Conformal gravity: Newton’s constant is not universal

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Newton’s gravitational constant $G$ has been measured to high accuracy in a number of independent experiments. For currently unresolved reasons, indicated values from different well-designed and thoroughly analyzed experiments differ by more than the sum of estimated errors. It has recently been shown that requiring both Einstein general relativity and the Higgs scalar field model to satisfy conformal symmetry (local Weyl scaling covariance) introduces gravitational effects that explain anomalous galactic rotation, currently accelerating Hubble expansion, and dark galactic halos, without invoking dark matter. This implies different values $G_n$ and $G_p$ for neutron and proton, respectively, but retains the Einstein equivalence principle for test objects accelerated by a given gravitational field. Isotopic mass defect $\mu$ per nucleon determines independent $G_m$. Thus $G$ differs for each nuclear isotope. Several recent measurements are used here to estimate $G_n = 6.60216$, $G_p = 6.38926$, and $G_m = -11.60684$ in units $10^{-11} m^3 kg^{-1} s^{-2}$.

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I. INTRODUCTION

Newton’s law $F = Gm_am_b/r_{ab}^2$, for the attractive gravitational force between two nonoverlapping massive objects at sufficiently large center of mass separation $r_{ab}$, is valid for weak interactions in general relativity and is the basis for most contemporary models of cosmology. Despite a continuing sequence of progressively more accurate measurements of Newton’s constant $G$, individual measured values differ by a collective spread ten times larger than the uncertainty of a typical measurement.

A possible resolution of this paradox is considered here. Newton’s law follows from analysis of gravitation in the Schwarzschild metric, valid for a spherically symmetric static mass/energy source in general relativity. Bare fermion and gauge fields satisfy local Weyl scaling (conformal) symmetry, but the Einstein metric tensor and Higgs scalar field do not. If conformal symmetry is imposed on general relativity, an external Schwarzschild solution is retained, but acquires an acceleration term that becomes dominant on a galactic scale, implying anomalously large orbital rotation velocities, as observed. Conformal gravity preserves subgalactic phenomenology while describing observed excessive rotational velocities in the outer reaches of galaxies, without invoking dark matter. The equivalence principle is retained, but $G$ is no longer a universal constant.

Possible formal difficulties with the conformal version of the Schwarzschild field equation have been discussed and resolved in detail. $G_n$ and $G_p$, for neutron and proton respectively, have distinct universal values. Hence any material mass-energy source produces a gravitational field and effective $G$ that depends on its neutron/proton ratio. Taking nuclear binding energy density into account, with parameter $G_m$, each nuclear isotope has a unique value of $G$.

Although the equivalence principle is retained for test objects in a fixed gravitational field, the interaction between two gravitating masses depends on atomic composition, governed by neutron/proton ratios and nuclear binding energy. The difference of $G$ for source mass densities with different atomic composition may account for the continuing failure of accurate measurements to converge to a unique common value.

The present paper summarizes this theoretical argument and applies it to deduce approximate values of $G_n, G_p, G_m$ from selected experimental data. The inferred values are clearly not equal. Several current experimental results are found to be consistent.

II. SUMMARY OF THEORY

Physical field equations are determined by requiring that a spacetime action integral, invariant under symmetry operations such as Lorentz transformations, should be stationary under infinitesimal variations of all elementary fields, including the metric tensor field of Riemann and Einstein. Each distinct physical field contributes a Lagrangian density to the integrand of the action integral. Variation of the metric tensor determines a gravitational field equation with a uniquely defined metric functional derivative tensor for each field.

Multiplying all distinct field amplitudes by specified powers of a real differentiable function $\Omega(x)$ defines local Weyl (conformal) scaling. Invariance of the action integral under local Weyl scaling defines conformal symmetry. Fermion and gauge boson fields satisfy conformal symmetry, but Einstein gravity and the standard Higgs model do not. Postulated universal con-
formal symmetry\textsuperscript{[20]} removes implied inconsistency by modifying the Lagrangian densities of both the metric tensor\textsuperscript{[5, 8]} and the Higgs scalar field\textsuperscript{[13, 21]}.  
Conformal Higgs theory\textsuperscript{[21, 22]} is a uniquely defined version of standard scalar field theory\textsuperscript{[8]}. Mass and dark energy must be dynamical quantities resulting from field interactions\textsuperscript{[20]}. Conformal symmetry causes the Weyl gravitational field to drop out of the theory of cosmic evolution, but introduces a gravitational term for the Higgs field which has been shown to account for observed Hubble expansion, including an explanation of dark energy\textsuperscript{[20, 22]}. The universal conformal symmetry postulate has been shown to explain excessive galactic rotation velocities\textsuperscript{[23, 26]}, Hubble expansion with positive acceleration\textsuperscript{[21, 22]}, and dark galactic halos\textsuperscript{[27]}, without evoking dark matter, while deriving dark energy\textsuperscript{[13]}.  
The conformal Lagrangian density combines two gravitational terms: conformal gravity (CG) $L_g$\textsuperscript{[5]} and the conformal Higgs model (CHM) $L_\phi$\textsuperscript{[13, 14]}. The field equation adds the two metric fundamental derivatives $X^{\mu\nu}_g + X^{\mu\nu}_\phi = \frac{1}{2} \Theta^{\mu\nu}_m$, for source energy-momentum $\Theta^{\mu\nu}_m$. Defining mean source density $\delta$ and residual density $\delta = d - d$, and assuming $\Theta^{\mu\nu}_m(d) \approx \Theta^{\mu\nu}_m(\delta) + \Theta^{\mu\nu}_m(\delta)$, decoupled solutions for $r \leq r_0$ of the two equations

$$X^{\mu\nu}_g = \frac{1}{2} \Theta^{\mu\nu}_m(\delta), X^{\mu\nu}_\phi = \frac{1}{2} \Theta^{\mu\nu}_m(\delta)$$

imply a solution of the full equation\textsuperscript{[13]}. This requires a composite hybrid metric\textsuperscript{[13]} such as

$$ds^2 = -B(r)dt^2 + a^2(t)\left(\frac{dr^2}{B(r)} + r^2 d\omega^2\right),$$

combining Schwarzschild potential $B(r)$ and Friedmann scale factor $a(t)$. Solutions are made consistent by fitting parameters to boundary conditions\textsuperscript{[13]} and setting cosmic curvature constant $k = 0$, justified by currently observed data.  
The classical solution $B(r) = 1 - 2\beta/r$ determines $\beta = Gm/c^2$, for $m = \int_0^r q^2 d(q) dq$, which defines gravitational constant $G$. The conformal $q^4$ integral for $\beta$\textsuperscript{[10]} depends on residual mass density\textsuperscript{[13]}, whose $q^2$ integral vanishes by definition, consistent with vanishing of the Weyl tensor for a uniform source. It cannot be interpreted to define a universal constant $G$.  

III. STATIC, SPHERICALLY SYMMETRIC SOURCE DENSITY

In the relativistic Schwarzschild model, valid for an isolated spherical source density confined inside radius $\bar{r}$, the standard gravitational potential in the source-free exterior space $r \geq \bar{r}$ is $B(r) = 1 - 2\beta/r$. For integrated source mass $m$, $\beta = Gm/c^2$ where $G$ is Newton’s constant. Mannheim and Kazanas\textsuperscript{[9, 10]} showed that conformal gravity replaces this by $B(r) = \alpha - 2\beta/r + \gamma r - \kappa r^2$. A solution of the full tensorial field equation requires $\alpha^2 = 1 - 6\beta\gamma\textsuperscript{[15, 28]}$. The $q^2$ integral for parameter $\beta$ is replaced by $2\beta = \frac{1}{6} \int_0^r q^4 d(q) dq\textsuperscript{[8, 10]}$, no longer strictly proportional to integrated mass $m$. $G = \beta c^2/m$ depends on residual mass density\textsuperscript{[13]}. It may be reduced by orders of magnitude from $G$ calculated from the full mass density.  
For orbital velocity $v$, such that $v^2 = \frac{1}{2}\gamma r - \kappa r^2$, circular orbit of a test particle is stabilized by centripetal radial acceleration $v^2/r$. The implied conformal extension of Kepler’s law,

$$v^2 = \beta/r + \frac{1}{2} \gamma r - \kappa r^2,$$

has been fitted to orbital velocity data for 138 galaxies\textsuperscript{[23, 26]}. This determines parameters $\beta = N^*\beta^*, \gamma = \gamma_0 + N^*\gamma^*, \kappa$, for each galactic mass $M = N^*M_\odot$ in solar units, defining four parameters $\beta^*, \gamma^*, \gamma_0, \kappa$. More recently, galactic rotation data has been found to define a function of classical Newtonian acceleration\textsuperscript{[29]} that requires $\gamma$ to be independent of galactic mass\textsuperscript{[30]}, as predicted by the conformal Higgs model\textsuperscript{[13, 22]}.  

IV. CONFORMAL GRAVITATIONAL CONSTANT

In standard theory, the differential equation for $B(r)$ is of second order, so that the integral solution for $\beta$ of the form $\int q^2 d(q) dq$ is strictly proportional to the integrated mass. However, conformal theory replaces this by an integral of the form $\int q^4 d(q) dq$, for a fourth order differential equation\textsuperscript{[10]}. This depends on residual density $\rho$\textsuperscript{[13]}, which integrates to zero over a closed sphere if weighted by $q^2$. Residual density is determined by mass-energy density but the $q^4$ integral is not integral to integrated mass. The residual mass integral may be reduced by orders of magnitude from $\beta$ for the full source mass density, accounting for the long-questioned small value of implied $G$. However, the equivalence principle, due to full mass following a geometrical geodesic, is maintained.  
As argued by Mannheim\textsuperscript{[8, 12]}, the requirement for Newton’s formalism to be valid on a macroscopic scale is that each atom should produce a gravitational potential proportional to $1/r$, since only the sum of such terms at very large distance (in atomic units) is measured. In fact, the true basic scale is nanoscopic, since all stable matter observed in natural conditions is composed of neutrons and protons. Assuming global charge neutrality, electron mass should be added to each proton mass, and nuclear binding energy should also be included. Chemical energy may be relevant for absolute precision.  
Analysis at the nanoscopic level\textsuperscript{[13]} does not imply identical neutron and proton internal source densities $d_n(r)$ and $d_p(r)$. There does not appear yet to be an accurate prediction of these densities from QCD. Proton and neutron masses are different, and there is no reason to assume the $\beta$ integral $\int q^4 d(q) dq$ to be explicitly
proportional to the mass integral$^{10}$. Hence definition $G = \beta c^2/m$ can be expected to produce two different fundamental constants $G_n$ and $G_p$, for neutron and proton respectively$^{10}$.$^{13}$.

V. RECENT ACCURATE MEASUREMENTS

Conformal gravity retains the mathematical formalism of standard general relativity. A test particle moving in a fixed gravitational field follows a geodesic of that field, which ensures the Einstein equivalence principle$^8$.$^{12}$. More generally, an experiment that measures acceleration of a negligibly small mass $m_n$ in the far gravitational field of a much larger mass $m_b$ is a specific measurement of $G_b$ only. This is appropriate to a recent atomic-physics experiment in which two isotopes of strontium are accelerated by the earth’s gravitational field. The equivalence principle is verified to one part in $10^8$.$^{31}$. This Sr isotope experiment has been reconfigured as a measurement of $G$ using rubidium atoms accelerated by a source mass of heavy tungsten alloy cylinders.$^{32}$ Error analysis gives $G = 6.67191(99)$ in units $10^{-11}m^3kg^{-1}s^{-2}$.

Another recent experiment uses laser interferometry to measure deflection of two free-hanging pendulum masses by a much larger tungsten alloy source mass.$^{33}$ The gravitational constant is found to be $G = 6.67234(14)$. These two experiments, with overlapping error bars, are characterized by a massive field source, with negligible deflected test mass. Their results will be considered here as measurements of $G$ for the source tungsten alloy. For element W with the natural isotopic abundance, the neutron and proton fractions are $f_n = 0.598$, $f_p = 0.402$.

A somewhat earlier torsion-balance experiment was designed to eliminate or greatly reduce uncertainty due to the suspending torsion fiber, to the mass and geometry of the pendulum probe, and to background noise.$^{34}$ The 1.5mm thick pyrex pendulum mass is negligible compared with field source mass due to four 8.14kg stainless steel alloy (SS316) spheres. Reported $G = 6.674215(929)$ is considered here to measure $G$ for the SS316 alloy. Averaged over alloy composition, $f_n = 0.535$, $f_p = 0.465$.

An experiment using the same material for source and test masses measures $G$ for that material. Such an experiment was reported in 2001$^{35}$, using a torsion-strip balance in two different measurement modes. Both source and test masses were made from the same Cu alloy. The apparatus was rebuilt and error analysis refined to give a more recent result$^{36}$.$^{37}$, $G = 6.67554(16)$. Averaged $f_n = 0.544$, $f_p = 0.456$.

Another experiment, using a compensated torsion balance designed to reduce or eliminate systematic error, used a suspended mass probe of 500g Cu, with two large cylindrical source masses, approximately 27kg each.$^{38}$ Results were obtained for two different source materials. Measured $G(SS316) = 6.67392(49)$ is consistent with $G = 6.674215(929)$.$^{34}$. Measured $G(Cu) = 6.67385(20)$.$^{38}$ is inconsistent with $G = 6.67554(16)$.$^{37}$.

Measurements of $G$ cited here are shown in Figure I, including the 2010 CODATA value 6.67384(80)$^{39}$. Evaluation of $G$ requires computing Schwarzschild parameter $\beta$ using accurate intra-nuclei energy density$^{13}$. Then $G = \beta c^2/M$. Empirical data used here is neutron mass $m_n$, proton plus electron mass $m_p$, and mass defect $\Delta$ per nucleon $\mu = \Delta/Ac^2$. Constants are $N_p = Z$, $N_n = N$, $A = N_p + N_n$, and $M = N_p m_n + N_n m_p - \Delta\mu$. $M$ and $N_n$ here are summed over stable isotopes for each element, using known abundance fractions. Neutron and proton fractions are computed for weighted $N_n$. Assuming additive contributions to $\beta = MG/c^2$,

$$\beta c^2/A = f_n m_n G_n + f_p m_p G_p - \mu G_m = MG/A$$

for each element. Using data for Fe$^{34}$, Cu$^{37}$, W$^{38}$ in units $10^{-11}m^3kg^{-1}s^{-2}$, Eq.$^4$ implies

Fe : $0.53960G_n + 0.46867G_p - 0.009434G_m = 6.66648$

Cu : $0.54886G_n + 0.45942G_p - 0.009397G_m = 6.66810$

W : $0.60277G_n + 0.40556G_p - 0.008595G_m = 6.67055$.$^5$

Hence $G_n = 6.60216$, $G_p = 6.38926$, $G_m = -11.60684$. Since $G_m$ depends on residual energy density, which integrates to zero, its sign is not predetermined.

Figure I. Newton G in units $10^{-11} m^3 kg^{-1} s^{-2}$
VI. CONCLUSIONS

Conformal theory indicates that Newton’s constant $G$ must depend on the atomic composition of any massive gravitational field source. Neutron and proton masses and nuclear binding energy contribute separately. This appears to resolve some but not all of the apparent inconsistencies among recent experimental measurements of assumed universal constant $G$. The situation might be clarified by measuring $G$ in the same apparatus for different source mass materials. Definitive results are more likely to be achieved when the test mass is negligible compared with the gravitational field source.

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