Leading Electroweak Corrections to the Production of Heavy Top Quarks at Hadron Colliders

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ABSTRACT

We calculate the electroweak corrections of the order $O(m_t^2/v^2)$ to the QCD production of $t\bar{t}$ pairs via $q\bar{q} \rightarrow t\bar{t}$ at hadron colliders and show that these corrections to the total production rate are small. This correction can be characterized as increasing the cross section most near the threshold region, where top quark signals are important, while the corrections become negative at higher $t\bar{t}$ energies where the top quark is a background for heavy Higgs boson searches or investigations involving the strongly interacting longitudinal $W$ system. The polarization of the $t\bar{t}$ pair is also discussed, including the effect that this has on proposed techniques for measuring the top quark mass.
1. Introduction

The mass of the top quark is one of the yet-to-be-measured parameters in the Standard Model (SM). To test the SM and probe new physics, we need to know the mass ($m_t$) of the top quark well, since at present, we still know neither the mechanism of electroweak symmetry breaking nor the mechanism for the generation of fermion masses. For instance, one might be able to determine the mass of the SM Higgs boson from the precision test of electroweak radiative corrections if $m_t$ is known. From another perspective, at the SSC (Superconducting Super Collider) and the LHC (Large Hadron Collider) the top quark production rate is large enough that it can potentially become a serious background when searching for new physics. Therefore, we need to know the production rate of the top quark as precisely as possible. Because the decay products of the top quark can mimic the signature of signal events, such as those involved with Higgs boson searches, it is also useful to know the polarization of the top quark which in turn controls the kinematics of the particles created by the top quark decays.

In this paper, we present the results for the leading electroweak corrections of the order $O\left(\frac{m_t^2}{v^2}\right)$ to the production of $t\bar{t}$ pairs as computed for the QCD subprocess $q\bar{q} \rightarrow t\bar{t}$. We find that the corrections to the total rate are small with a few percent increase in the cross section near the threshold region for a light (around 100 GeV) Higgs boson and about the same percent decrease in cross section at high subprocess energies. These corrections can be taken most seriously for higher top quark masses, where not only is the $O\left(\frac{m_t^2}{v^2}\right)$ approximation most valid, but also the nonperturbative effects which can modify the threshold behavior become less significant since the faster decay time for the top quark prevents bound states from forming.$^{[1]}$

The higher order corrections were applied using helicity amplitude techniques in the computation of $K$–factors for both polarized and unpolarized final states. Though the QCD corrections to the $q\bar{q} \rightarrow t\bar{t}$ production rates$^{[2]}$ are larger than the leading electroweak results presented here, the parity violation manifest in the
electroweak interactions produces effects unobtainable by theories like QCD that maintain parity ($P$) and charge conjugation ($C$) symmetries separately. These effects are realized, for example, in the difference between the $K$–factors describing the higher order corrections for final state polarizations related by parity transformations. Such differences are often largest as the invariant mass of the $t\bar{t}$ pair gets large, making the polarization effects most relevant when considering the backgrounds to signal events like those discussed for probing the electroweak symmetry breaking mechanism in the TeV region.

The effect that polarization has on the observed kinematics of the $q\bar{q} \to t\bar{t}$ events is reflected in the decay products of the top quarks. Consequently, there may be a change in the efficiency of experimental cuts used to remove the top quark background or to observe the top quark signal depending upon the spin asymmetries in top quark production. For example, the charged lepton produced from the decay of a top quark with a right–handed helicity via $t \to b l^+ \nu$ will preferentially receive a greater boost along the direction of motion of the top quark than the charged lepton would from top quark with a left–handed helicity. An enhancement of top quarks with a right–handed (left–handed) polarization will then produce charged leptons with more (less) energy in the laboratory. Realizing this, it is plausible that a fixed cut on lepton energies determined from leading order top quark production may automatically produce a different efficiency in removing background or collecting signals than would be obtained from the production rate that contained electroweak corrections. Perhaps even more pertinent to present interests is the effect that the polarization asymmetry has on the distribution of $M(eb)$, the mass of the charged lepton and bottom quark system, in $t \to e^+ b \nu$, since it is through a related mass distribution (invariant mass of the $e^+$ and $\mu^-$, $M(e\mu)$, where the muon comes from fragmentation of the bottom quark) that the best techniques for determining the top quark mass are derived.

The remainder of this paper is organized as follows. In section 2, we present the analytical results of our calculations in terms of form factors. Using these form factors, we then give the numerical results on the production rate and the degree of
polarization of the top quark in section 3. In section 4 we examine the polarization effects on the $M(\ell\ell)$ distribution and discuss how this relates to measurements of the top quark mass. Section 5 contains our conclusion.

2. The Loop Corrections

The effective theory considered in this paper is obtained by taking the limit of the electroweak coupling $g \to 0$ after replacing the mass of the $W^+$ boson ($M_W$) with $gv/2$ in the SM Lagrangian. The neutral and charged Goldstone bosons that remain ($\phi^0$ and $\phi^\pm$) are massless. The parameter $v \approx 246$ GeV characterizes the scale of the electroweak symmetry breaking and corresponds to the vacuum expectation value (VEV) of the Higgs field in the SM.

We show that the form factors are infrared-safe, and it is not necessary to include real diagrams with an additional $\phi^0$ or $\phi^\pm$ associated with the $t\bar{t}$ production in our calculations. The Landau gauge has been chosen to evaluate the loop diagrams, and the ultraviolet divergences are regularized by dimensional regularization with the regulator $\Delta \equiv 2/(4 - N) - \gamma_E + ln(4\pi)$, where $N$ is the spacetime dimension and $\gamma_E$ is the Euler constant.

2.1. Wave Function Renormalization

There are three diagrams, as shown in Fig. 1(a), contributing to the self energy of the top quark and its wave function renormalization. The wave function renormalization constant $Z_t$ can be written as

$$Z_t = 1 + \frac{1}{16\pi^2} \frac{m^2}{v^2} [\delta Z_t^V - \delta Z_t^A \gamma_5].$$  \hspace{1cm} (2.1)

Hereafter, we use $m$ and $m_t$ interchangeable. Employing the on-shell renormalization-
tion scheme, we obtain \[^5\]

\[
\delta Z^V_t = -\frac{1}{2} [3\Delta - 3 \ln\left(\frac{m_2^2}{\mu^2}\right) + 1 - 2I(r) + 2J(r) + 4L(r) + i\pi],
\]

\[
\delta Z^A_t = \frac{1}{2} [\Delta - \ln\left(\frac{m_2^2}{\mu^2}\right) + 2 + i\pi],
\]

where $\mu$ is the 't Hooft mass parameter, $r = m_H^2/m^2$, and $m_H$ is the mass of the Higgs boson. The integrals $I(r), J(r)$ and $L(r)$ are defined as

\[
I(r) = \int_0^1 \ln[x^2 + r(1 - x) - i\epsilon] \, dx,
\]

\[
J(r) = \int_0^1 x\ln[x^2 + r(1 - x) - i\epsilon] \, dx,
\]

\[
L(r) = \int_0^1 \frac{x(1 - x^2)}{(1 - x)^2 + rx - i\epsilon} \, dx.
\]

2.2. Vertex Corrections

The $g\bar{t}t$ vertex can be expressed as

\[
ig_s\bar{u}(p)T^a\Gamma_\mu v(q),
\]

where $g_s$ is the strong coupling and the $T^a$ are the $SU(3)$ matrices with $Tr(T^a T^b) = \frac{1}{2}\delta^{ab}$. The $u(p)$ and $v(q)$ are the Dirac spinors of the $t$ and $\bar{t}$ with momenta $p$ and $q$, respectively.

The tree level vertex function is $\Gamma_\mu^{\text{tree}} = \gamma_\mu$. At the 1-loop level, as shown in
Fig. 1(b), the vertex function can be written as

\[ \Gamma_{\mu}^{\text{loop}} = \frac{1}{16\pi^2} \frac{m^2}{v^2} \bar{u}(p) \Lambda_{\mu} v(q), \]

\[ \Lambda_{\mu} = \gamma_{\mu} (A - B \gamma_5) + \frac{1}{2} (p_{\mu} - q_{\mu}) (C - D \gamma_5) + \frac{1}{2} (p_{\mu} + q_{\mu}) (E - F \gamma_5), \] (2.5)

where

\[ A = \frac{3}{2} [\Delta - \ln\left(\frac{m^2}{\mu^2}\right) + 1] + \frac{1}{4} \left(1 - \frac{3}{\beta^2}\right) \ln\left(\frac{s}{m^2}\right) - \beta \ln\left(\frac{1 + \beta}{1 - \beta}\right) + \frac{m^2_H}{s \beta^2} [-I(r) - 2 + \beta \ln\left(\frac{1 + \beta}{1 - \beta}\right)] - \left(\frac{4 m^2}{s \beta^2} + 4 m^2\right) C_0^H - \frac{s}{16} \left(\frac{1}{\beta^2} + 2 - 3 \beta^2\right) C_0^{\phi^+} + i \pi \left(-\frac{m^2_H}{s \beta} + \frac{1}{2} + \beta\right) \]

\[ B = -\frac{1}{2} [\Delta - \ln\left(\frac{m^2}{\mu^2}\right) + 1] + \frac{1}{4} \left(\frac{3}{\beta^2} - 1\right) \ln\left(\frac{s}{m^2}\right) + \frac{s}{16} \left(-\frac{3}{\beta^2} + 2 + \beta^2\right) C_0^{\phi^+} - i \left(\frac{\pi}{2}\right), \] (2.6)

\[ C = \frac{m}{s \beta^2} \left(\frac{3}{\beta^2} - 1\right) \ln\left(\frac{s}{m^2}\right) - 18 - 8 I(r) + 4 \beta \ln\left(\frac{1 + \beta}{1 - \beta}\right) + \left(\frac{12 m^2_H}{s \beta^2}\right) [-I(r) - 2 + \beta \ln\left(\frac{1 + \beta}{1 - \beta}\right)] + 2 r [I(r) - \ln(r) + 1] - 12 m^2_H \left(\frac{m_H^2}{s \beta^2} + 1\right) C_0^H - \frac{s}{4} \left(\frac{3}{\beta^2} - 2 - \beta^2\right) C_0^{\phi^+} - 4 i \pi \left(\frac{3 m^2_H}{s \beta} + \beta\right), \]

\[ F = \frac{m}{s} \left[\left(1 - \frac{3}{\beta^2}\right) \ln\left(\frac{s}{m^2}\right) - 2 + \frac{s}{4} \left(\frac{3}{\beta^2} - 2 - \beta^2\right) C_0^{\phi^+}\right], \]

\[ D = E = 0, \]

and

\[ C_0^H = C_0(m^2, m^2, s, m^2, m_H^2, m^2), \]

\[ C_0^{\phi^+} = C_0(m^2, m^2, s, 0, 0, 0), \] (2.7)

given that \( s = (p + q)^2 \) is the squared of the \( t\bar{t} \) center-of-mass energy and \( \beta = \)
\[\sqrt{1 - 4m^2/s}.\] The form factor \(D\) is zero because this theory is CP invariant. We note from Eq. (2.9) that vector current conservation demands that \(\delta Z_t^A + B = -sF/4m.\)

The 3-point function \(C_0\) is defined as

\[
C_0(p_1^2, p_2^2, p_5^2, m_1, m_2, m_3) \equiv \frac{1}{i\pi^2} \int d^N q \frac{1}{[q^2 - m_1^2][q + p_1]^2 - m_2^2][(q + p_1 + p_2)^2 - m_3^2]}
\]

where \(p_5 = p_1 + p_2.\) The loop integrals in the form factors have been evaluated with the code LOOP. \(^{[6,7]}\) For simplicity, the mass of the bottom quark \((m_b)\) is taken to be zero. \(^\dagger\)

The renormalized vertex function becomes

\[
\Gamma_\mu^R = \gamma_\mu + \frac{1}{16\pi^2} \frac{m^2}{e^2} \{\gamma_\mu[(\delta Z_t^V + A) - (\delta Z_t^A + B)\gamma_5] \}
\]

\[
+ \frac{1}{2}(p_\mu - q_\mu)(C - D\gamma_5) + \frac{1}{2}(p_\mu + q_\mu)(E - F\gamma_5)\}.
\]

It can be seen explicitly that the terms with the regulator, \(\Delta,\) and the mass parameter, \(\mu,\) cancel exactly among themselves, therefore the renormalized vertex function is free of ultraviolet divergence and independent of the \(\mu\) parameter as expected. In addition, it has been checked that \(\Gamma_\mu^R\) is free of infrared divergence.

\(^\dagger\) We checked that the difference between using \(m_b = 5\) GeV and zero is less than 0.1\% in the numerical results of the form factors.
When the Higgs mass is very large, i.e., $m_H^2 \gg s > m^2$,

\[
\begin{align*}
    rI(r) &\to -\frac{1}{2} + r \ln(r) + \ln(r) - r, \\
    J(r) &\to -\frac{3}{4} + \frac{1}{2} \ln(r), \\
    L(r) &\to 0, \\
    C_{0}^{\sigma^+} &\to 0, \\
    C_{0}^{H} &\to -\frac{1}{m_H^2}[\ln(r) + 1 - \beta \ln(\frac{1 + \beta}{1 - \beta}) + i\pi \beta] + \frac{1}{m_H^4} \ln(r)(\frac{s}{2} - 3m^2).
\end{align*}
\]

\[ (2.10) \]

It is straightforward to check that the dependence of the renormalized vertex function $\Gamma_R^R$ on Higgs mass vanishes as $m_H \to \infty$. (Recall that $r = m_H^2/m^2$.) Therefore, $m_H$ decouples in this case for the heavy Higgs mass limit. This is in contrast to the usual one loop SM electroweak corrections which grow like $\ln(m_H)$ in the heavy Higgs mass limit.\[8\]

### 3. Numerical Results

#### 3.1. General

It has been demonstrated that the modifications of the $g-t-\bar{t}$ vertex due to the electroweak corrections given by Eq. (2.9) appear as finite modifications to the form factors. In particular, the values of $E$ and $F$ are irrelevant for the $q\bar{q} \to tt$ process considered, as these terms will vanish when the momentum factor in front of them couples to the annihilation vertex for the massless quarks in the initial state. With the corrections written in the manner of Eq. (2.9), it becomes possible to use helicity amplitude techniques for computing the loop effects in $tt \bar{t}$ production.\[3\] In particular, this allows us to preserve the polarization information.

In Fig. 2 and Fig. 3 we present the variation of the hadron level distribution with the subprocess center–of–mass energy for the Fermilab Tevatron, the LHC, and the SSC for $m_t = 180$ GeV and $m_t = 140$ GeV, respectively. We use the parton
distribution functions of Morfin and Tung [9] (set SL) using a scale of \( Q = \sqrt{s} \). This same scale, which is the center–of–mass energy of the \( t\bar{t} \) pair, is also used in evaluating the strong coupling constant. We note that at the SSC and the LHC, the dominant production mechanism for the \( t\bar{t} \) pairs is \( gg \to t\bar{t} \), yet in the very high invariant mass region (about or above 1 TeV) \( q\bar{q} \) fusion can be important as a background to the study of the electroweak symmetry breaking.\[10\]

**Definition of \( K \)–Factor**

Denoting the higher order cross section which includes the leading electroweak corrections up to \( O(\alpha_s^2 m_t^2/v^2) \) at the parton level as \( \hat{\sigma}^{H.O.} \), we define the \( K \)–factor,

\[
\hat{K}(s, m_t, m_H) \equiv \frac{d\hat{\sigma}^{H.O.}}{ds}/\frac{d\hat{\sigma}^{Born}}{ds}, \tag{3.1}
\]

which quantifies the leading electroweak corrections to the parton cross section at the Born level, \( \hat{\sigma}^{Born} \). For processes like the \( s \)–channel \( q\bar{q} \to t\bar{t} \) considered here, this \( K \)–factor is valid also at the hadron level for all system rapidities,

\[
y = \frac{1}{2} \ln \frac{x_a}{x_b}, \tag{3.2}
\]

given our choice of scale, where \( x_a, x_b \) are the fractions of momenta that the \( q \) and \( \bar{q} \) take from their parent hadrons.

The hadron level \( K \)–factor is defined as

\[
K(s, m_t, m_H) \equiv \frac{d\sigma^{H.O.}}{ds}/\frac{d\sigma^{Born}}{ds}. \tag{3.3}
\]

Eq. (3.3) differs from Eq. (3.1) in that the ratio is with regards to the hadronic differential cross sections, which are simply the parton differential cross sections convoluted with the parton distribution functions,

\[
\sigma = \sum_{a,b} \int d\hat{x}_a dx_b f_{a/A}(x_a, Q)f_{b/B}(x_b, Q)d\hat{\sigma}(a + b \to t + \bar{t}), \tag{3.4}
\]

where \( f_{a/A}(x_a, Q) \) provides the density for partons of flavor \( a \) carrying momentum fraction \( x_a \) of the total momentum of hadron \( A \). As with the strong coupling, the
scale $Q$ in the parton densities has been set to $\sqrt{s}$. After converting the $dx_a dx_b$ integrals over the parton momentum fractions to $dy ds/S$ integrals in Eq. (3.4), where $S = s/x_a x_b$ is the center–of–mass energy of the hadron–hadron system, the parton cross section factors outside the $dy$ integration because the parton level cross section depends only on $s$ and masses. (It is implied that the integration over top polar angle has been performed.) Computing the integral over $dy$ yields a cancellation of the contribution from the parton distribution functions between the numerator and the denominator in Eq. (3.3), such that the parton level $K$–factor of Eq. (3.1) is the same as the analogous hadron level $K$–factor of Eq. (3.3).

This is independent of whether the initial hadrons are protons or antiprotons. So, the parton level $K$–factors presented may be considered as the hadron level $K$–factors at the Fermilab Tevatron,

$$\hat{K}(s, m_t, m_H) = K_{FNAL}(s, m_t, m_H) \equiv \frac{d\sigma^{H.O.}_{FNAL}}{ds}/\frac{d\sigma^{Born}_{FNAL}}{ds}, \quad (3.5)$$

and at the SSC and LHC,

$$\hat{K}(s, m_t, m_H) = K_{SSC/LHC}(s, m_t, m_H) \equiv \frac{d\sigma^{H.O.}_{SSC/LHC}}{ds}/\frac{d\sigma^{Born}_{SSC/LHC}}{ds}, \quad (3.6)$$

where $\sigma^{H.O.}$ and $\sigma^{Born}$ respectively represent the hadron level cross sections for the production of $t\bar{t}$ pairs through quark–antiquark annihilation at the one loop level and at the tree level. Note that Eq. (3.5) and Eq. (3.6) are true provided that no kinematic cuts are applied to the $t$ or $\bar{t}$.

One of the advantages of computing the higher order correction through modification of the form factors is that we can conveniently implement the corrections at the amplitude level and examine the consequent changes in the production of polarized top quarks. The higher order effects vary somewhat when we compare the production of unpolarized $t\bar{t}$ pairs to the production of polarized final states. The results for polarized top quarks are given in Figs. 4–6.
Magnitude of Results

In general the higher order electroweak effects in $q\bar{q} \to t\bar{t}$ yield only a small correction of a few percent to the cross section. In Fig. 4 and Fig. 5 the $K$–factors that describe these corrections are greater than unity near the threshold region for a light Higgs boson, reaching magnitudes around 1.08 (1.03) for $m_t = 180$ (140) GeV and $m_H = 100$ GeV; a drop in value occurs as we go to subprocess energies of 3 TeV providing a negative correction to the Born level rates of no greater than a ten percent reduction. For the production of $t\bar{t}$ pairs in the threshold region given a heavy Higgs boson (see Fig. 6), we find a small decrease in rate yielding $K$–factors just under unity. Despite the large size of the $K$–factor for the lighter Higgs boson when $s$ is extremely close to the mass threshold of producing the $t\bar{t}$ pair, the effect on the total cross section is small because of the suppression of the phase space indicated in Figs. 2 and 3. At subprocess energies far from threshold, the event rate for top quark production is much smaller; nevertheless, it is useful to know that the electroweak corrections cause a decrease in the event rate for large invariant masses, $M(t\bar{t})$, of the $t\bar{t}$ pairs because it is in the high invariant mass region at the SSC/LHC that the top quark is a background to signals needed to study the electroweak symmetry breaking sector (given that no light Higgs boson is found).\[10\]

The $K$–factor has a dependence on both the mass of the top quark and the mass of the Higgs boson, both unknown quantities at this time. The general outcome of an increase in top quark mass from 140 GeV to 180 GeV is that $|1 - K|$ is slightly larger for the heavier quark near threshold and at high $M(t\bar{t})$. If we fix $m_H$ at either 100 GeV or 1 TeV while changing the top quark mass from 140 GeV to 180 GeV, we find the $K$–factor deviation from unity is about a factor of two or three greater for the lighter Higgs boson mass. From the perspective of fixed $m_t$, we also see the variation in the $K$–factor between $m_t = 100$ GeV and $m_H = 1$ TeV is larger for the larger top quark mass. As mentioned previously, in the limit that mass of the Higgs boson is taken to infinity, the renormalized vertex function $\Gamma^R_\mu$ in Eq. (2.9) loses its dependence on $m_H$.  

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3.2. **New Effects That Did Not Appear at the Tree Level**

Though the effect of the electroweak corrections are small when compared to the total cross section, there are conditions where the form factor modifications can yield a relatively significant change in a particular production mode. In particular, because the form factors become complex, there are polarization asymmetries which develop nonzero values in contradistinction to their Born level counterparts. Such is the case when considering inclusive cross sections for top quark production where the polarization of the observed top quark is transverse to the scatter plane.

As was discussed in Ref. 3, the Born level amplitudes for $q\bar{q} \rightarrow t\bar{t}$ are all real. For this reason, a single particle asymmetry is zero when considering the transverse polarization perpendicular to the scatter plane. In computing the higher order corrections, however, an imaginary portion is generated in the form factors that takes the polarization perpendicular to the scatter plane to nonzero values. These nonzero values can in principle be used to test for CP violation effects.

The radiative corrections computed here make the helicity amplitudes complex in value. This produces a nonzero value for the polarization of single top quark spins directed perpendicular to the scatter plane, $P_\perp$. Define $P_\perp$ by

$$
\frac{d\hat{\sigma}^\uparrow/d\cos\theta - d\hat{\sigma}^\downarrow/d\cos\theta}{d\hat{\sigma}^\uparrow/d\cos\theta + d\hat{\sigma}^\downarrow/d\cos\theta}, \tag{3.7}
$$

where $\hat{\sigma}^\uparrow (\hat{\sigma}^\downarrow)$ describes the parton level cross section when the transverse spin of the top quark is pointing “up” (“down”) with respect to the scatter plane. (Think of “up” as the $+Y$ direction given that the $q\bar{q} \rightarrow t\bar{t}$ hard scattering occurs in the $X-Z$ plane with the initial quark moving in the $+Z$ direction and the top quark carrying a positive value for its $X$ component of momentum.) Though $P_\perp$ carries nonzero values, numerical results indicate that the polarization for single top quark spins directed perpendicular to the scatter plane are small, yielding $P_\perp$ values not much larger than $10^{-3}$ for $m_t = 180$ GeV. In Fig. 7, we present curves of $P_\perp$ vs. $\cos\theta$, where $\theta$ is the angle the top quark subtends with the incoming
quark in the center–of–mass frame of the subprocess. We found an interesting qualitative change in the plots as the Higgs boson mass increases. For the lower Higgs boson masses (around 100 GeV) the polarization is positive for \( \cos \theta < 0 \) while for the higher Higgs boson masses (around 1 TeV) the polarization for \( \cos \theta < 0 \) is negative. Such quantities are mainly relevant for proton–antiproton collisions, where a convenient means is available for determining “up” and “down” directions with respect to the scatter plane by defining the scatter plane with the proton and antiproton beams as opposed to the annihilating quark and antiquark. Although the \( P_\perp \) plots exhibit this interesting feature, it might be extremely difficult to measure this polarization because of its small value.

Another new effect which is absent at the tree level is the double polarization asymmetry \( P_\perp (in, out) \) which is also sensitive to higher order corrections.\(^{[12]}\) Specifically, \( P_\perp (in, out) \) refers to the asymmetry produced when the top quark spin is perpendicular to the scatter plane while the transverse spin of the top antiquark is in the scatter plane. This quantity, analogous to Eq. (3.7), is zero at the Born level and only achieves its nonzero value because of the imaginary correction to the form factors. Analogous to the single spin asymmetry presented for \( P_\perp \), \( P_\perp (in, out) \sim 10^{-3} \) for \( m_t = 180 \text{ GeV} \).

So, though we can say we have an effect with \( K = \infty \), the statistics are too poor for any reasonable study.

3.3. \( K \)-Factors for Spin Effects Present at the Born Level

Because QCD is C (charge conjugation) and P (parity) invariant, the single particle polarization of the top quark has to vanish at the tree level for the process \( q\bar{q} \rightarrow t\bar{t} \). Nevertheless, the top quark can have a single particle polarization if weak effects are present in their production. The effects of top quark polarization for the Born level electroweak reaction \( q\bar{q} \rightarrow (\gamma, Z) \rightarrow t\bar{t} \) were discussed in Ref. 13. The contribution of this process to the total cross section for the production of \( t\bar{t} \) pairs is small (about a percent at the Tevatron), so any spin effects present in
\( q\bar{q} \rightarrow (\gamma, Z) \rightarrow t\bar{t} \) are diluted by the QCD production of \( t\bar{t} \) pairs. Considering this larger rate for the QCD production of \( t\bar{t} \) pairs, similar spin effects that appear when considering the degree of polarization due to the leading electroweak corrections to \( q\bar{q} \rightarrow g \rightarrow t\bar{t} \) at the loop level are more significant. The degree of polarization for a single top quark due to leading electroweak corrections can be obtained from the curves (a) in Figs. 4–6. Typically, this effect is of the order of a few percent for large \( M(t\bar{t}) \).

Besides examining single particle polarizations, there are also double particle asymmetries in the spin dependence which can be investigated. In the following, we consider the longitudinal and transverse spins of the top quark and top antiquark.

**Longitudinal Spins**

When considering the longitudinal polarizations for the top quarks, we classify the states as carrying either right–handed (R) or left–handed (L) helicity. In the figures and the text the correlated spin states for the \( t\bar{t} \) pairs will be labelled either RR,RL,LR,LL, where the first letter is the top helicity and the second letter is the top antiquark helicity. Since the interactions described by Eq. (2.9) conserve CP, the higher order corrections make no distinction between the RR and LL states because they are CP transforms of each other. For all cases of \( m_H \) and \( m_t \) considered, the \(|1 - K|\) value for the LR state is larger than that for the RL state when the \( K \)–factors for both of these helicity combinations are below unity. The reverse is true when these \( K \)–factors are above unity. When \( m_H \) becomes larger, the \(|1 - K|\) values of the \( RR, LL \) spin states become smaller, as shown in Figs. 4 and 6.

**Transverse Spins**

Though we do not present plots of the \( K \)–factors when both the \( t \) and \( \bar{t} \) quarks are polarized transverse to their direction of motion, we discuss some of the results here. We consider transverse spins for the top quark and antiquark to be either perpendicular to the scatter plane or within it. Since both quarks are con-
considered simultaneously, the $t, \bar{t}$ spins are further classified as being either aligned or antialigned.

For the case where both $t, \bar{t}$ spins are perpendicular to the scatter plane, we found the $K$–factor covering the widest range of all our plots as we move through values of $s$. As guided by the unpolarized results, for $m_t = 180$ GeV and $m_H = 100$ GeV the threshold effect nears $K = 1.08$, while as we move to larger $\sqrt{s}$, the $K$–factor decreases to about $K = 0.94$. Throughout the kinematic region considered, the $K$–factor for when the $t$ and $\bar{t}$ spins are aligned or antialigned remains near the $K$–factor curve for the unpolarized final state. For the lighter top quark ($m_t = 140$ GeV), the $K$–factor is not as large and the difference between aligned and antialigned top spins is just as indistinct. We also note that for the lighter top quark the decrease in the $K$–factor near threshold is not as steep as when the top quark was heavier.

Consider the case where the transverse spins for the two top quarks live in the scatter plane. For the higher $m_t$ the $K$ factor drops to around $K = 0.94$ at large $\sqrt{s}$ (around 1 TeV), but no matter whether $m_t = 140$ GeV or $m_t = 180$ GeV, the configuration where the transverse spins are aligned receives more suppression from the high order corrections than the configuration where the two top spins are antialigned in the TeV region. Here the $K$ factor doesn’t stray from the unpolarized result by more than about ±0.01.

4. Polarization and Top Quark Mass Measurements

In Ref. 4 the most effective method considered for measuring the mass of the top quark concentrated on the analysis of the invariant mass distribution, $M(e\mu)$, which is determined from the combined momentum of the charged $e^+$ lepton from the decay $t \rightarrow b e^+ \nu$ and the muon from the fragmentation of the bottom quark. An error of about 1.6% was estimated in the determination of the top quark mass for $m_t = 150, 250$ GeV using a series of kinematic cuts on the unpolarized production of top quarks. It is known, however, that if a polarization
asymmetry were to develop in the top quark production that the kinematics of
the observed particles would change. If there were no kinematic cuts, this would
be of no consequence since integrating out the angular dependence washes out the
polarization effects on this measurement; however, with kinematic cuts, as required
in reality, a polarization asymmetry can affect the $M(e\mu)$ mass spectrum. With
this observation it becomes necessary to investigate the effects such an asymmetry
may produce and whether it interferes with the precision of the mass measurement.

We proceed by examining an analogous quantity, namely, the invariant mass of
the bottom quark and charged lepton from the top decay. Respectively denoting
the momenta of the $e^+$, $\nu$, $b$ quark, $W$–boson, and $t$ quark as $p_e$, $p_\nu$, $p_b$, $p_W$, $p_t$,
the amplitude squared for the three–body decay $t \rightarrow bW^+ \rightarrow be^+\nu$ is given by

$$|M|^2 = \frac{64G_F^2m_W^4}{(p_W^2 - m_W^2)^2 + m_W^2\Gamma_W^2}(p_\nu \cdot p_b)[(p_e \cdot p_t) - m_t(p_e \cdot s)], \quad (4.1)$$

where $G_F$ is the Fermi coupling constant and $s$ describes the polarization of the top
quark.\textsuperscript{[13]} The neutrino and positron have been taken as massless and the masses
of the top quark, $W$–boson and bottom quark are given by $m_t$, $m_W$, $m_b$. With the
conventions chosen, the top decay rate is given by

$$d\Gamma_t = \frac{1}{2m_t}|M|^2d\Phi_3, \quad (4.2)$$

where the three–body phase space is

$$d\Phi_3 = \frac{1}{32m_t^2(2\pi)^5}dM(eb)^2dm_W^2d\Omega_b d\phi^*_e \Theta[-G(M(eb)^2, p_W^2, m_t^2, 0, m_b^2, 0)] \quad (4.3)$$

with

$$G(M(eb)^2, p_W^2, m_t^2, 0, m_b^2, 0) = 2p_W^2(m_b^2M(eb)^2 - m(eb)^4 - M(eb)^2p_W^2 - m_t^2m_b^2 + M(eb)^2m_t^2). \quad (4.4)$$

We choose to perform the calculation in the rest frame of the top decay. The
angular dependence in the phase space factor of Eq. (4.3) is comprised of the
differential for the solid angle of the bottom quark, \( d\Omega_b = d\cos \theta_b d\phi_b \), and \( d\phi^*_\nu \), which is the azimuthal angle of the neutrino measured from the coordinate system that is rotated such that the bottom quark momentum defines the \( z \)-axis.

For the decay of unpolarized top quarks, Eq. (4.1) indicates that there is no angular dependence to the \( M(eb) \) distribution. The phase space integration may be easily performed in the narrow width approximation yielding the unpolarized decay distribution,

\[
\frac{d\Gamma_t}{dM(eb)^2} = \frac{G_F^2}{16\pi^2\Gamma_W} \left( \frac{m_W}{m_t} \right)^3 (m_t^2 - m_W^2 - M(eb)^2)(m_W^2 - m_b^2 + M(eb)^2). \quad (4.5)
\]

It is also clear from Eq. (4.1) that for polarized top quark decay there is a spin component that contributes to the behavior of the \( M(eb) \) distribution, and this term does have an angular dependence. To demonstrate the effect of the spin–dependent term in Eq. (4.1), we plot the \( M(eb) \) distribution for \( q\bar{q} \rightarrow t\bar{t} \rightarrow b^+\nu\bar{b}q_1\bar{q}_2 \) at the Tevatron in Fig. 8, separating the contributions for left–handed and right–handed helicities of the top quark. These curves were created for top quarks of mass 140 GeV by restricting the rapidities of the \( e^+ \) and \( b \) quark to within 2.5 in magnitude and insisting that the transverse momentum for each of these two particles be greater than 20 GeV for top quark production via \( q\bar{q} \rightarrow g \rightarrow t\bar{t} \) at the Tevatron. (In our result we also impose the same rapidity and transverse momentum cuts for the \( \bar{b}, q_1, \bar{q}_2 \).) A difference in the two curves for pure helicity states, created purely by the kinematic constraints, is realized mainly in the low \( M(eb) \) region. To understand how this affects the \( M(e\mu) \) mass distribution and the measurement of the top quark mass, one has to convolute our results with the hadronization of the bottom quark to produce the muon. This is beyond the scope of this paper.
5. Conclusion

We have computed the leading electroweak corrections, of the order $O(\frac{m_t^2}{v^2})$, for $q\bar{q} \rightarrow t\bar{t}$ and found the corrections to provide an increase in the total cross section of no more than a few percent, which is smaller than the typical uncertainty in the prediction of the top quark event rate in the usual QCD processes.

A decrease appeared in $d\hat{\sigma}/ds$ of no greater than ten percent compared to the lowest order result for subprocess center–of–mass energies from around 1 TeV. The perturbative results indicate an increase in the $K$–factor just under 10% near the threshold region for the $m_t = 180$ GeV and $m_H = 100$ GeV values considered here, but this does not include any relevant nonperturbative physics. Small transverse polarizations were obtained from the imaginary contributions to the form factors (generated by the loop corrections), as we found that the polarization when considering solely the spin of the top quark perpendicular to the scatter plane was about $10^{-3}$.

Given that our results are valid for high top quark masses and large center–of–mass energies, we find our $K$–factors in disparity with the results presented for the LHC in Ref. 15, where the full electroweak corrections were shown to produce a large reduction around 40% in the Born level rate for $q\bar{q} \rightarrow t\bar{t}$ at large $\sqrt{s}$ values. In Fig. 9 the $K$ factors are shown for $m_t = 150, 200, 250$ GeV using $m_H = 100$ GeV. As $\sqrt{s}$ enters the TeV regime, the variation in the $K$ factor becomes very flat, never indicating a change in the Born level rate of more than 20%.

The parity violation due to the electroweak couplings appears in the $K$–factors for the production of polarized $t\bar{t}$ pairs, where for $m_H = 100$ GeV the RL states generally received a larger $K$–factor enhancement near threshold than that for the production of LR states, while the $K$–factor suppression the LR states received in the TeV region was greater than the suppression for the production of RL states. With $m_H = 1$ TeV we saw that all helicity combinations for the final state were suppressed, though the $K$–factor was close to unity near threshold. We also showed that the invariant mass spectrum of $M(e\mu)$ depends on the polarization of the top
quark. To ensure that $M(e\mu)$ is a good variable for measuring the mass of the top quark, one has to take the effect of the top quark polarization into consideration when performing the analysis.

While this paper was being completed, we became aware of similar research by Stange and Willenbrock \cite{16} which overlaps in part with our work. Our results agree with theirs in the total event rate.

**Acknowledgements**

We would like to thank Jiang Liu for discussions. C.K. was supported in part by the U. S. Department of Energy under contract number DE-FG05-87ER40319. The work of G.A.L. and C.P.Y. was supported in part by TNRLC grant #RGFY9240.
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FIGURE CAPTIONS

1) Diagrams contributing to (a) the top quark self energy and wave function renormalization and (b) the electroweak corrections to the $gt\bar{t}$ vertex.

2) Taking $m_t = 180$ GeV, we plot the distribution $d\sigma/ds$ vs. $s$ from the Born level QCD subprocess $q\bar{q} \rightarrow t\bar{t}$ in (a) proton–antiproton collisions at Tevatron energies of 1.8 TeV, (b) proton–proton collisions at an LHC energy of 16 TeV and (c) proton–proton collisions at an SSC energy of 40 TeV.

3) These figures (a)–(c) are the same as Fig. 2, except that $m_t = 140$ GeV.

4) Taking $m_t = 180$ GeV and $m_H = 100$ GeV, we plot the $K$–factor of Eqs.(3.5) and (3.6) against the subprocess center–of–mass energy $M(t\bar{t})$ considering (a) $t$ helicity states with an unpolarized $\bar{t}$ (R indicates right–handed $t$, L indicates a left–handed $t$); (b) $t$ and $\bar{t}$ helicity states (RL indicates right–handed $t$, left–handed $\bar{t}$, etc.)

5) These figures (a)–(b) are the same as Fig. 4, except that $m_t = 140$ GeV.

6) These figures (a)–(b) are the same as Fig. 4, except that $m_H = 1$ TeV.

7) Taking $m_t = 180$ GeV and $\sqrt{s} = 500$ GeV, we plot the single particle asymmetry $P_\perp$ as described by Eq. (3.7) when top quark spin is perpendicular to the scatter plane against $\cos \theta$. The two curves represent the asymmetry for two different values of the Higgs boson mass.

8) These two curves represent the distribution $d\sigma/dM(\ell b)$ vs. $M(\ell b)$ at the Tevatron for right–handed and left–handed top quark helicities using $m_t = 180$ GeV and $m_b = 0$. Kinematic constraints in the lab frame on the rapidity ($|\eta| < 2.5$) and transverse momentum ($p_T > 20$ GeV) were imposed on the $e^+$ and $b$ quark.

9) This plot shows the variation of the $K$ factor with the mass of the $t\bar{t}$ pair ($M(t\bar{t})$) using $m_t = 150, 200, 250$ GeV and $m_H = 100$ GeV in the unpolarized production rates.
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