Chaplygin inflation in loop quantum cosmology

Xin Zhang, Jingfei Zhang, Jinglei Cui, and Li Zhang

Department of Physics, College of Sciences, Northeastern University, Shenyang 110004, China
School of Physics and Optoelectronic Technology, Dalian University of Technology, Dalian 116024, China

In this paper we discuss the inflationary universe in the context of a Chaplygin gas equation of state within the framework of the effective theory of loop quantum cosmology. Under the slow-roll approximation, we calculate the primordial perturbations for this model. We give the general expressions of the scalar spectral index, its running, and the tensor-to-scalar ratio, etc. For the chaotic inflation with a quadratic potential, using the WMAP 5-year results, we determine the parameters of the Chaplygin inflation model in loop quantum cosmology. The results are consistent with the WMAP observations.

I. INTRODUCTION

The inflation paradigm provides a compelling solution for many long-standing problems of the cosmological standard model (such as flatness, horizon, monopoles, etc.) by positing an epoch of accelerated expansion in the early universe [1–5]. This accelerated period of expansion also generates superhorizon fluctuations and thus predicts an almost scale-invariant density perturbation power spectrum, which has received strong observational support from the measurement of the temperature fluctuation in the cosmic microwave background (CMB) radiation [6–12]. Conceptually, however, the inflationary scenario is incomplete due to the existence of the big bang singularity [13]. Einstein’s classical theory of general relativity (GR) breaks down near such a singularity since quantum effects are expected to be important at very high energies in the early universe. Thus, the classical theory of GR has to be replaced by some theoretical framework of quantum gravity which should remain well defined even at very high curvatures.

Loop quantum gravity (LQG) is a leading nonperturbative background independent approach to quantizing gravity [14–16]. The underlying geometry in LQG is discrete and the continuum spacetime can be obtained from the quantum geometry in a large eigenvalue limit. Loop quantum cosmology (LQC) focuses on symmetry reduced models (with homogeneous and isotropic space) but inherits quantization scheme and techniques from LQG [17]. Within the framework of LQC, some long-standing issues concerning the quantum nature of the big-bang are resolved in the context of homogeneous and isotropic universe with a scalar field. Using extensive analytical and numerical methods, the analysis of the evolution of the semiclassical states for a spatially flat universe has shown that the universe has a pre-big-bang branch, joined deterministically to the post-big-bang branch by a quantum bounce in the deep Planck regime through the LQC evolution [18, 20, 21]. Thanks to the nonperturbative background independent methods of LQC, the idea of the nonsingular bounce can be realized in a natural fashion.

Recently, it has been shown that the discrete quantum dynamics can be well approximated by an effective modified Friedmann dynamics [20–22]. The modified Friedmann equation can be obtained from the effective Hamiltonian constraint, which can be used to investigate the role of nonperturbative quantum correction conveniently. It is remarkable that the quantum geometric effects lead to a \( \rho^2 \) modification to the Friedmann equation at the scales where \( \rho \) becomes comparable to a critical density \( \rho_c \), which is close to the Planck density \( (\rho_c = 0.82G^{-2}) \) [20, 21, 23–25]. Within the framework of LQC, the inflationary universe model has been studied in detail [26].

On the other hand, it is well known that the generalized Chaplygin gas (GCG) is an alternative candidate for explaining the acceleration of the universe. The GCG is described by an exotic equation of state in the form [27]

\[
p_{ch} = -\frac{A}{\rho^{\alpha}_{ch}},
\]

where \( p_{ch} \) and \( \rho_{ch} \) are, respectively, the energy density and pressure of the GCG, \( \alpha \) is a constant in the range \( 0 < \alpha \leq 1 \), and \( A \) is a positive constant. It should also be mentioned, however, that “superluminal” values \( \alpha > 1 \) are well possible too, as was recently shown in Ref. [28]; causality requires only some change in Eq. (1) for \( \rho_{ch} \) very close to \( A^{1/\alpha} \). The case \( \alpha = 1 \) corresponds to the original Chaplygin gas [29, 30]. Inserting this equation of state into the stress-energy conservation equation leads to the following energy density

\[
\rho_{ch} = \left[ A + \frac{B}{a^{3(1+\alpha)}} \right]^{\frac{1}{1+\alpha}},
\]

where \( a \) is the scale factor of the universe and \( B \) is a positive integration constant. The model of GCG as well as its further generalization have been extensively studied in the literature [31–42]. In addition, it should be pointed out that the GCG model may be viewed as a modification of gravity [43].

Recently, it was considered that the GCG model can also be used to describe the early universe. In this consideration, Eq. (2) is not viewed as a consequence of the equation of state (1), but rather, as arising due to a modification of gravity itself. Furthermore, one can also
assume that during inflation the gravity dynamics may give rise to a modified Friedmann equation [44]

\[ 3M_p^2 H^2 = \left[ A + \rho_m^{(1+\alpha)} \right]^{\frac{1}{1+\alpha}}, \tag{3} \]

where \( M_p \) is the reduced Planck mass, \( \rho_m \) is the energy density of the inflaton field. This is the so-called Chaplygin inspired inflation scenario [44], since the modification in Eq. (3) is realized from an extrapolation of Eq. (2):

\[ \rho_{ch} = \left[ A + \rho_m^{(1+\alpha)} \right]^{\frac{1}{1+\alpha}} \rightarrow \left[ A + \rho_p^{(1+\alpha)} \right]^{\frac{1}{1+\alpha}}, \tag{4} \]

where \( \rho_m \sim a^{-3} \) corresponds to the matter energy density. Following this idea, lately, some work has been done, this involves tachyon-Chaplygin inflation [45], warm-Chaplygin inflation [46], Chaplygin inflation on a brane [47], and so forth [48]. In this paper, we shall study the Chaplygin inspired inflationary model in loop quantum cosmology.

This paper is organized as follows. In the next section we briefly review the effective theory of LQC. In section II, we describe the Chaplygin inflationary universe in the framework of LQC. In section IV, we study the primordial perturbations and give the expressions of scalar and tensor perturbations for this model. Also, we shall compare our results to the WMAP five-year observations. Conclusion is given in section V.

II. EFFECTIVE DYNAMICS IN LOOP QUANTUM COSMOLOGY

LQG is a canonical quantization of gravity based upon Ashtekar-Barbero connection variables. The phase space of classical GR in LQG is spanned by SU(2) connection \( A_i^a \) and the triad \( E_i^a \) on a 3-manifold \( M \) (labels \( a \) and \( i \) denote space and internal indices respectively), which are two conjugate variables encoding curvature and spatial geometry, respectively. Likewise, LQC is a canonical quantization of homogeneous spacetimes based upon techniques used in LQG. In LQC, due to the symmetries of the homogeneous and isotropic spacetime, the phase space structure is simplified, i.e., the connection is determined by a single quantity labeled \( \gamma \) and likewise the triad is determined by a parameter \( p \). The variables \( \gamma \) and \( p \) are canonically conjugate with Poisson bracket \( \{ \gamma, p \} = \gamma / 3 \), here we have used the unit \( M_p = 1 \) for convenience, and \( \gamma \) is the dimensionless Barbero-Immirzi parameter which is set to be \( \gamma \approx 0.2375 \) by the black hole thermodynamics in LQG. For the spatially flat model of cosmology, the new variables have the relations with the metric components of the Friedmann-Robertson-Walker (FRW) universe as

\[ \gamma = \gamma \tilde{a}, \quad p = a^2, \tag{5} \]

where \( a \) is the scale factor of the universe. Classically in terms of the connection-triads variables the Hamiltonian constraint is given by

\[ H_{\text{cl}} = -\frac{3}{\gamma^2} p \gamma^2 (\gamma \tilde{a}^2) + H_M, \tag{6} \]

where \( H_M \) is the matter Hamiltonian.

The elementary variables used for quantization in LQC are the triads and holonomies of the connection. The holonomy over an edge of a loop is defined as \( h_i(\mu) = \cos(\mu) \tilde{a}^2 + 2 \sin(\mu) \tau_i \), where \( \gamma \) is related to Pauli spin matrices as \( \gamma = -i \sigma_i / 2 \) and dimensionless \( \mu \) is related to the physical length of the edge over which holonomy is evaluated (note that \( \mu \) is also the eigenvalue of the triad operator \( \tilde{p} \)). In the Hamiltonian formulation for homogeneous and isotropic spacetime, the dynamical equations can be determined completely by the Hamiltonian constraint. Under quantization, the Hamiltonian constraint gets promoted to an operator and the quantum wave functions are annihilated by the operator of the Hamiltonian constraint. In LQC, it is expected that modifications due to LQC effects will appear in the Hamiltonian constraint, and from the modified Hamiltonian constraint the effective Friedmann constraint will be derived. In quantization the Hamiltonian constraint operator is obtained by promoting the holonomies and the triads to the corresponding operators. Consequently, this leads to a discrete quantum difference equation, which indicates that the underlying geometry in LQC is discrete [19]. Interestingly, the solutions of this difference equation are nonsingular.

So far we see that the underlying dynamics in LQC is governed by a discrete quantum difference equation in quantum geometry. However, an effective Hamiltonian description on a continuum spacetime can be constructed by using semiclassical states, which has been shown to very well approximate the quantum dynamics [20, 21]. This analysis reveals that on backward evolution of our expanding phase of the universe, the universe bounces at a critical density (near the big bang singularity) into a contracting branch [18, 25]. Thus the classical singular problem can be successfully overcome within the context of LQC by a nonsingular bounce. In addition, the effective equations for the modified Friedmann dynamics can be derived from the effective Hamiltonian constraint with loop quantum modifications, which can be used to investigate the role of nonperturbative quantum corrections. An important feature for the modified dynamics is that a \( p^2 \) term which is relevant in the high energy regime is included in the classical Friedmann equation. The modified term is negative definite implying a bounce when the energy density reaches a critical value on the order of the Planck density.

The effective Hamiltonian constraint, to leading order, is given by [22]

\[ H_{\text{eff}} = -\frac{3}{\gamma^2} p \gamma^2 a \sin^2(\mu \tilde{a}) + H_M, \tag{7} \]

where \( \mu \tilde{a} \) is the kinematical length of the edge of a square loop which has the area given by the minimum eigenvalue
of the area operator in LQG; the area is $A = \mu a^2 = \alpha l_p^2$, where $\alpha$ is of the order unity and $l_p$ is the Planck length.

The modified Friedmann equation can then be derived by using the Hamilton's equation for $p$,

$$\dot{p} = \{p, \mathcal{H}_{\text{eff}}\} = -\frac{\gamma}{3} \frac{\partial \mathcal{H}_{\text{eff}}}{\partial \dot{a}} = \frac{2a}{\gamma \mu} \sin(\mu \dot{a}) \cos(\mu \dot{a}), \quad (8)$$

which combined with Eq. (5) yields the rate of change of the scale factor

$$\dot{a} = \frac{1}{\gamma \mu} \sin(\mu \dot{a}) \cos(\mu \dot{a}). \quad (9)$$

Furthermore, the vanishing of the Hamiltonian constraint, $\mathcal{H}_{\text{eff}} \approx 0$, implies

$$\sin^2(\mu \dot{a}) = \frac{\gamma^2 \mu^2}{3a} \mathcal{H}_M. \quad (10)$$

Combining Eqs. (9) and (10) yields the effective Friedmann equation for the Hubble rate $H = \dot{a}/a$,

$$H^2 = \frac{1}{3} \rho \left( 1 - \frac{\rho}{\rho_c} \right), \quad (11)$$

with the critical density given by

$$\rho_c = 4\sqrt{3} \gamma^{-3}. \quad (12)$$

The modified Friedmann equation provides an effective description for LQC which very well approximates the underlying discrete quantum dynamics. The nonperturbative quantum geometric effects are manifested in the modified Friedmann equation with a $\rho^2$ correction term. The negative definition of the $\rho^2$ term implies that the Hubble parameter vanishes when $\rho = \rho_c$ and the universe experiences a turnaround in the scale factor. When $\rho \ll \rho_c$, the modifications to the Friedmann equation become negligible, and the standard Friedmann equation is recovered. In addition, it should be noted that, interestingly, $\rho^2$ modifications also appear in string inspired braneworld scenarios and it has been shown that there exist interesting dualities between the two frameworks [23]. Such modifications in braneworlds, however, are usually positive definite so that a bounce is absent, unless the existence of two timelike extra dimensions is assumed [49]. For extensive studies in this framework, see e.g. [50–57]. Other mechanism of realizing a bouncing (or oscillating) universe can be found in e.g. [58–63].

### III. Chaplygin Inflationary Universe in Loop Quantum Cosmology

In this section, we shall investigate Chaplygin inspired inflation within the framework of the effective theory of LQC. In this case the dynamics of the inflationary universe is governed by the following equation,

$$3H^2 = \left( A + \rho_0 (1+\alpha) \right) \frac{1}{\rho_0} \left[ 1 - \frac{(A + \rho_0 (1+\alpha))^{1/2}}{\rho_c} \right]. \quad (13)$$

Here, $\rho_0$ is the canonical inflaton, $\rho_0 = \dot{\phi}/2 + V(\phi)$, and $V(\phi)$ is its potential. It should be noted that in the low-energy regime, $[A + \rho_0 (1+\alpha)]^{1/2} \ll \rho_c$, the standard Chaplygin inflation model is recovered; while in the high-energy regime, the quantum gravity effects leads to a super-inflation that is in the “primary inflation”. However, what is of interest for us is the “observable inflation” with the last 60 $e$-foldings. We thus only discuss the subsequent normal inflation stage (after the super-inflation phase) in what follows. Also, in the following we will take $\alpha = 1$ for simplicity, which means the usual Chaplygin gas.

For the inflaton field, its equation of motion is of the standard form,

$$\ddot{\phi} + 3H \dot{\phi} + V' = 0, \quad (14)$$

where dots denote the derivatives with respect to the cosmic time and $V' = dV(\phi)/d\phi$, and the Hubble parameter $H$ is given by Eq. (13). If $\dot{\phi}^2 \ll V(\phi)$ and $\dot{\phi} \ll 3H \dot{\phi}$, the scalar field will slowly roll down its potential, and the exact evolution equation (14) can be replaced by the slow-roll approximation

$$\dot{\phi} = -V'/3H. \quad (15)$$

Under the slow-roll approximation, the energy density of the scalar field approximates as $\rho \sim V(\phi)$, the Friedmann equation (13) consequently can be written as

$$H^2 = \frac{1}{3} \sqrt{A + V^2} \left( 1 - \frac{\sqrt{A + V^2}}{\rho_c} \right). \quad (16)$$

We can also define the slow-roll parameters

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{2} VV'' 2 (1 - \frac{2\sqrt{A + V^2}}{\rho_c}) \quad (17)$$

$$\eta = \epsilon + \delta = \frac{V''}{(A + V^2)^{1/2}} \left( 1 - \frac{\sqrt{A + V^2}}{\rho_c} \right)^{-1}, \quad (18)$$

where $\delta = -\dot{\phi}/H \dot{\phi}$, by definition. The slow-roll condition can be expressed as $\epsilon, |\eta| \ll 1$.

Note that in the limit $A \to 0$, the slow-roll parameters $\epsilon$ and $\eta$ coincide with the case in inflation model in LQC [26]. Also, in the low-energy limit, $\sqrt{A + \rho_0^2} \ll \rho_c$, the slow-roll parameters reduce to the standard form of Chaplygin inflation.

The parameter $\epsilon$ quantifies how much the Hubble rate $H$ changes with time during inflation. Inflation can be attained only if $\epsilon < 1$. This condition could be written explicitly in terms of the scalar potential $V$ and its derivative $V''$

$$VV'' \left[ 1 - \frac{2(A + V^2)^{1/2}}{\rho_c} \right] < (A + V^2)^{3/2} \left[ 1 - \frac{A + V^2}{\rho_c} \right]^2. \quad (19)$$
Inflation ends when $\epsilon < 1$ ceases to be satisfied, which implies

$$V_j V' \left[ 1 - \frac{2(A + V^2)^{1/2}}{\rho_c} \right]$$

$$= (A + V^2)^{3/2} \left[ 1 - \frac{(A + V^2)^{1/2}}{\rho_c} \right]^2,$$  \hspace{1cm} (20)

where the subscript $f$ is used to mark the end of inflation.

The small change of $e$-foldings satisfies $dN \equiv -H dt$; using Eqs. (15) and (16), one obtains the number of $e$-foldings of slow-roll inflation remaining at a given epoch,

$$N(\phi_*) = \int_{\phi_f}^{\phi_*} \frac{\sqrt{A + V^2}}{V'} \left[ 1 - \frac{\sqrt{A + V^2}}{\rho_c} \right] d\phi,$$  \hspace{1cm} (21)

where the subscript $*$ marks the epoch when the cosmological scales exit the horizon.

**IV. PRIMORDIAL PERTURBATIONS AND WMAP FIVE-YEAR RESULTS**

In this section we shall study the primordial perturbations of this inflationary model. We shall give the expressions of scalar and tensor perturbations for the Chaplygin inflation in LQC.

The perturbation $\delta \phi$ can be treated as a massless free field, and its vacuum fluctuation can be regarded as a classical quantity a few Hubble times after horizon exit. The spectrum of the field perturbation is

$$\Delta_{\phi}^2 = (H/2\pi)^2.$$  \hspace{1cm} (22)

The corresponding curvature perturbation is given by $\Delta = (H/\phi)\delta \phi$. Using Eqs. (15), (16) and (22), we get the amplitude of scalar perturbation as

$$\Delta_s^2 = \left( \frac{H^2}{2\pi} \right)^2 \left( \frac{A + V^2}{12\pi^2 V^2} \right)^{3/2} \left[ 1 - \frac{\sqrt{A + V^2}}{\rho_c} \right]^3$$

which is evaluated at the Hubble radius crossing $k = aH$. Note that in the limit $A \rightarrow 0$, the amplitude of the scalar perturbation given by Eq. (23) coincides with Ref. [26].

Since $H$ is slowly varying, we have the relation $d \ln k(\phi) = -dN(\phi)$. Then, from Eq. (23) we get the scalar spectral index, $n_s - 1 = d \ln \Delta_s^2 / d \ln k = -6\epsilon + 2\eta$, or equivalently,

$$n_s = 1 - (A + V^2)^{-1/2} \left[ 1 - \frac{(A + V^2)^{1/2}}{\rho_c} \right]^{-1}$$

$$\times \left( \frac{3V^2}{(A + V^2)} \right) \left[ 1 - \frac{2(A + V^2)^{1/2}}{\rho_c} \right].$$  \hspace{1cm} (24)

Once again, we notice that in the limit $A \rightarrow 0$ the scalar spectral index coincides with that in Ref. [26].

Another important quantity for inflationary model is the running of the scalar spectral index $\alpha_s = d\alpha_s/d \ln k$. From Eq. (24), we can get the running of the scalar spectral index for this model

$$\alpha_s = \left( \frac{4(A + V^2)}{V^2} \right) \left[ 1 - \frac{(A + V^2)^{1/2}}{\rho_c} \right] \left[ 3\epsilon,_{\phi} - \eta,_{\phi} \right] \epsilon.$$  \hspace{1cm} (25)

where $\epsilon$ and $\eta$ are given by Eqs. (17) and (18), and the subscript $_{\phi}$ labels the derivative with respect to the inflaton field $\phi$.

Inflation also generates gravitational waves with two independent components $h_{+,x}$ which have the same action as a massless scalar field. Likewise, the amplitude of tensor perturbation can also be given,

$$\Delta_t^2 = 8 \left( \frac{H}{2\pi} \right)^2 \left( \frac{2(A + V^2)^{1/2}}{3\pi^2} \right) \left[ 1 - \frac{\sqrt{A + V^2}}{\rho_c} \right]$$

which is also evaluated at the horizon exit $k = aH$. The spectral index of tensor perturbation is given by $n_t = d\Delta_t^2 / d \ln k$ that will not be written explicitly here. Then, from the expressions (23) and (26), we can write the tensor-to-scalar ratio, $k = aH$, as

$$r = \frac{\Delta_t^2}{\Delta_s^2} = \frac{8V^2}{(A + V^2)^2(1 - (A + V^2)^{1/2}/\rho_c)}.$$  \hspace{1cm} (27)

Recently, the Wilkinson Microwave Anisotropy Probe (WMAP) five-year data were released [12]. The WMAP 5-year data provide stringent constraints on inflationary models. If we assume that the primordial fluctuations are adiabatic with a power law spectrum, the WMAP 5-year data (WMAP only) give: $n_s = 0.963^{+0.015}_{-0.015}$; while the WMAP data combined with the distance measurements from the Type Ia supernovae (SN) and Baryon Acoustic Oscillations (BAO) in the distribution of galaxies (WMAP+BAO+SN) give: $n_s = 0.960^{+0.014}_{-0.010}$. The amplitude of curvature perturbation is $\Delta_k^2 = 2.4 \times 10^{-9}$, measured by WMAP at $k_s = 0.002$ Mpc$^{-1}$. With the WMAP data combined with BAO and SN, it is found that the limit on the tensor-to-scalar ratio is $r < 0.20$ (95% CL) and $n_s > 1$ is disfavored even when gravitational waves are included. We will make use of the WMAP 5-year results to determine the parameters of the model.

Consider now an inflaton scalar field $\phi$ with a chaotic potential. We focus on a quadratic potential $V(\phi) = m^2\phi^2/2$, where $m$ is the mass of the scalar field. First, we should evaluate $\phi_*$ (that corresponds to the time of horizon-crossing) by using Eq. (21). In view of that the LQC effect is not prominent (for convenience we define $\nu \equiv (A + V^2)/\rho_c$, and we have $\nu < 10^{-9}$, see [26]), we neglect $\nu$ in this evaluation. Then, the integral in Eq. (21) can be given directly (see Eq. (19) in [44]). Furthermore, considering $\phi_f \sim 0$ and the unimportance of logarithm term, one obtains $\phi_0 = 2\sqrt{N}$ [44].
Next, using the WMAP 5-year results (WMAP+BAO+SN), $\Delta^2_\theta = 2.4 \times 10^{-9}$ and $n_s = 0.960$, we can determine the parameters of the LQC Chaplygin inflation model. Taking $N = 55$, we derive $A = 3.36 \times 10^{-18}$ and $m = 5.9 \times 10^{-6}$, note that here the unit $M_p = 1$ has been used. Based on these values of parameters, we can further evaluate the other observational quantities. For example, we give the running of the scalar spectral index $\alpha_s = -6.8 \times 10^{-5}$ and the tensor-to-scalar ratio $r = 0.1$, which are in good consistency with the WMAP observations. Also, we verified the fact that the LQC effect can only leads to a very tiny imprint in the primordial power spectrum, $\nu = 8.5 \times 10^{-10}$, which is in good accordance with the result in Ref. [26].

V. CONCLUSION

In this paper we have studied the Chaplygin inflation model in the framework of the effective theory of LQC. In LQC, the nonperturbative quantum geometry effects lead to a $p^2$ term with a negative sign in the modified Friedmann equation. On the other hand, inspired by the Chaplygin equation of state, a phenomenological modification of gravity is also considered in the Friedmann dynamics. The LQC Chaplygin inflationary model combines the above both features.

Under the slow-roll approximation, we calculated the primordial perturbations for this model. We gave the general expressions of the scalar spectral index $n_s$, its running $\alpha_s$, and the tensor-to-scalar ratio $r$, etc. For the chaotic inflation with a quadratic potential, using the WMAP 5-year results, $\Delta^2_\theta(k_*) = 2.4 \times 10^{-9}$ and $n_s(k_*) = 0.960$, we determined the parameters of the LQC Chaplygin inflation model, $A = 3.36 \times 10^{-18}$ and $m = 5.9 \times 10^{-6}$. This leads to the results of the running of the scalar spectral index $\alpha_s(k_*) = -6.8 \times 10^{-5}$ and the tensor-to-scalar ratio $r(k_*) = 0.1$, which are consistent with the WMAP observations.

The theory of LQC gives rise to a quantum bounce in the high energy regime when the loop quantum effects are dominative power. The classical big-bang singularity is thus replaced by the quantum bounce. After the bounce, a super-inflation phase was emergent in a natural way. Then the universe underwent a normal inflation stage. In this paper, we restrict that the Chaplygin-inflation happens in the normal inflation phase. This leads to that the imprint of the LQC effect in CMB sky is too weak to be observed by present observational data (the LQC parameter $\nu < 10^{-9}$, see Ref. [26]). For the Chaplygin inflation case with quadratic potential, we derived $\nu = 8.5 \times 10^{-10}$ that is in agreement with the conclusion of Ref. [26]. If we set the Chaplygin inflation to happen in the super-inflation phase, it is believed that the LQC effect should be prominent and the imprint of LQC effect may be detected in the CMB sky. We hope to return to this point in the near future.

Acknowledgments

This work was supported in part by the Natural Science Foundation of China.
[27] M. C. Bento, O. Bertolami and A. A. Sen, Phys. Rev. D 66, 043507 (2002) [arXiv:gr-qc/0202064].
[28] V. Gorini, A. Y. Kamenshchik, U. Moschella, O. F. Piattella and A. A. Starobinsky, JCAP 0802, 016 (2008) [arXiv:0711.4242 [astro-ph]].
[29] A. Y. Kamenshchik, U. Moschella and V. Pasquier, Phys. Lett. B 511, 265 (2001) [arXiv:gr-qc/0103004].
[30] N. Bilic, G. B. Tupper and R. D. Viollier, Phys. Lett. B 535, 17 (2002) [arXiv:astro-ph/0111325].
[31] M. C. Bento, O. Bertolami and A. A. Sen, Phys. Rev. D 67, 063503 (2003) [arXiv:gr-qc/0103004].
[32] M. C. Bento, O. Bertolami and A. A. Sen, Phys. Rev. D 70, 083519 (2004) [arXiv:astro-ph/0407239].
[33] M. C. Bento, O. Bertolami, N. M. C. Santos and A. A. Sen, Phys. Rev. D 71, 063501 (2005) [arXiv:astro-ph/0412638].
[34] Z. H. Zhu, Astron. Astrophys. 423, 421 (2004) [arXiv:astro-ph/0411039].
[35] Y. G. Gong, JCAP 0503, 007 (2005) [arXiv:astro-ph/0411253].
[36] R. Banerjee, S. Ghosh and S. Kulkarni, Phys. Rev. D 75, 025008 (2007) [arXiv:gr-qc/0611311].
[37] P. Wu and H. W. Yu, Phys. Lett. B 644, 16 (2007) [arXiv:astro-ph/0612055].
[38] X. Zhang, F. Q. Wu and J. Zhang, JCAP 0601, 003 (2006) [arXiv:astro-ph/0411221].
[39] H. B. Benaoum, arXiv:hep-th/0205140.
[40] L. P. Chimento and R. Lazkoz, Phys. Lett. B 615, 146 (2005) [arXiv:astro-ph/0411068].
[41] S. Chattopadhyay and U. Debnath, arXiv:0805.0070 [gr-qc].
[42] J. Lu, L. Xu, J. Li, B. Chang, Y. Gui and H. Liu, Phys. Lett. B 662, 87 (2008).
[43] T. Barreiro and A. A. Sen, Phys. Rev. D 70, 124013 (2004) [arXiv:astro-ph/0408185].
[44] O. Bertolami and V. Duvvuri, Phys. Lett. B 640, 121 (2006) [arXiv:astro-ph/0603366].
[45] S. del Campo and R. Herrera, Phys. Lett. B 660, 282 (2008) [arXiv:0801.3251 [astro-ph]].
[46] S. del Campo and R. Herrera, arXiv:0806.0575 [astro-ph].
[47] R. Herrera, arXiv:0805.1005 [gr-qc].
[48] A. Moneret et al., Phys. Rev. D 76, 024017 (2007) [arXiv:0704.2585 [gr-qc]].
[49] Y. Shnirnov and V. Sahni, Phys. Lett. B 557, 1 (2003) [arXiv:astro-ph/0208047].
[50] D. Samart and B. Gumjudpai, Phys. Rev. D 76, 043514 (2007) [arXiv:0704.3414 [gr-qc]].
[51] H. Wei and S. N. Zhang, Phys. Rev. D 76, 063005 (2007) [arXiv:0705.4002 [gr-qc]].
[52] J. Zhang, X. Zhang and H. Liu, Eur. Phys. J. C 52, 693 (2007) [arXiv:0708.3121 [hep-th]].
[53] X. Zhang, arXiv:0708.1408 [gr-qc].
[54] X. Zhang, arXiv:0711.0667 [hep-th].
[55] Y. S. Piao, Phys. Rev. D 70, 101302 (2004) [arXiv:hep-th/0407258].
[56] H. H. Xiong, T. Qiu, Y. F. Cai and X. Zhang, arXiv:0711.4469 [hep-th].
[57] P. Wu and S. N. Zhang, JCAP 0806, 007 (2008) [arXiv:0805.2255 [astro-ph]].
[58] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, Phys. Rev. D 64, 123522 (2001) [hep-th/0103239].
[59] P. J. Steinhardt and N. Turok, Science 296, 1436 (2002).
[60] Y. F. Cai, T. Qiu, Y. S. Piao, M. Li and X. Zhang, JHEP 0710, 071 (2007) [arXiv:0704.1090 [gr-qc]].
[61] Y. F. Cai, T. Qiu, R. Brandenberger, Y. S. Piao and X. Zhang, JCAP 0803, 013 (2008) [arXiv:0711.2187 [hep-th]].
[62] H. H. Xiong, Y. F. Cai, T. Qiu, R. Brandenberger, Y. S. Piao and X. Zhang, JCAP 0803, 013 (2008) [arXiv:0711.2187 [hep-th]].
[63] N. Kanekar, V. Sahni and Y. Shtanov, Phys. Rev. D 63, 083520 (2001) [arXiv:astro-ph/0101448].