Heterodyne Holography with full control of both signal and reference arms

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Heterodyne holography is a variant of phase shifting holography in which reference and signal arms are controlled by acousto optic modulators (AOM). In this review paper, we will briefly describe the method and its properties, and we will illustrate its advantages in experimental applications.

I. INTRODUCTION

Heterodyne holography \cite{1,2} is a technique firstly introduced 15 years ago realized by modifying a phase shifting holography setup \cite{3}. The main advantage of heterodyne holography is that the frequency, phase and amplitude of both reference and signal arms are controlled by acousto optic modulators (AOM). By shifting the frequency $\omega_{LO}$ of the reference beam, called here local oscillator, with respect to the frequency $\omega$ of illumination, heterodyne holography is able to detect the light scattered by the object at any frequency $\omega$ close to $\omega_{LO}$, which can be equal or different than the frequency $\omega_I$ of illumination of the object.

If the light is scattered by the object over a broad continuous frequency spectrum, heterodyne holography must be combined with off-axis holography in order to separate the images corresponding to the $+1$ and $-1$ holographic grating orders. Indeed, in that case, the $+1$ and $-1$ holographic images are both non-zero, since they correspond to the detection of signals at frequencies that are different but very close and within the object broad frequency spectrum \cite{3}. This point will be discussed in section II.C. Heterodyne holography in off axis configuration has been used since 2003 \cite{4}.

Heterodyne holography performs heterodyne detection with a slow 2D multi pixels detector (CCD or CMOS camera). The bandwidth BW of the detection is narrow (10 to 1000 Hz depending on the camera), but the beat frequency $\omega - \omega_{LO}$ can be large (up to several MHz \cite{2}). Heterodyne holography must be distinguished from optical scanning holography that uses a fast mono pixel detector (photodiode), and that makes mechanical scans along the x and y axis to get holograms \cite{5}.

We will briefly describe the heterodyne holographic technique and illustrate its advantages on experimental examples.

II. PRINCIPLE AND PROPERTIES OF HETERODYNE HOLOGRAPHY

A. Typical setup

Figure 1 shows a typical heterodyne holography setup in transmission geometry. BS: beam splitter; AOM1, AOM2: acousto-optic modulators; $E_{LO}$ and $E$: reference (i.e. local oscillator LO) and object fields whose optical frequencies are $\omega_{LO}$ and $\omega$; $\omega_{AOM1/2}$: driving frequencies (\(\approx 80\) MHz) of the acousto optics modulators AOM1 and AOM2.

Figure 1 shows a typical heterodyne holography setup. The frequency and the amplitude of both the illumination and the reference arms are controlled by the two acousto optics modulator (Bragg cells) AOM1 and AOM2. AOM1, which is driven by a Radio Frequency (RF) signals at $\omega_{AOM1}$ (with $\omega_{AOM1} \approx 80$ MHz), shifts the frequency $\omega_I$ of illumination so that

$$\omega_I = \omega_L + \omega_{AOM1}$$  \hspace{1cm} (1)

where $\omega_L$ is the optical frequency of the main laser. Similarly AOM2 shift the frequency $\omega_{LO}$ of the local oscillator arm so that

$$\omega_{LO} = \omega_L + \omega_{AOM2}$$  \hspace{1cm} (2)

An angularly tilted beam splitter BS mixes the signal field scattered by the sample $E(t) = E_{E(t)}$ and reference field $E_{LO}(t) = E_{LO}\exp(i\omega_{LO}t)$. Thus, the interference pattern $I(t)$ that is recorded by the CCD camera is:

$$|E(t) + E_{LO}(t)|^2 = |E|^2 + |E_{LO}|^2 + EE^* \exp(-j(\omega_{LO} - \omega)t) + c.c.$$  \hspace{1cm} (3)

where c.c. is the complex conjugate of the $EE^*_{LO}$ term i.e. $c.c. = E^*E_{LO} \exp(j(\omega_{LO} - \omega)t)$. Sequence of frames $I_n$
are recorded by the camera, with:

\[ I_n = \frac{1}{T} \int_{t=nT_{CCD}-T/2}^{nT_{CCD}+T/2} dt |E(t) + \mathcal{E}_{LO}(t)|^2 \]

(4)

\[ = |E|^2 + |E_{LO}|^2 \]

\[ + c.c. \]

\[ + EE_{LO}^* \left[ \frac{1}{T} \int_{t=nT_{CCD}-T/2}^{nT_{CCD}+T/2} dt \ e^{-j(\omega_{LO} - \omega)t} \right] + c.c. \]

where \( t_n = nT_{CCD} \) with \( n \) integer, \( T_{CCD} = 2\pi/\omega_{CCD} \), and \( \omega_{CCD}/2\pi \) is the camera frame frequency. The sinc factor of Eq. 4 corresponds to the integration of the beat signal of frequency \( \omega_{LO} - \omega \) over the camera exposure time \( T \).

Note that a microscope objective can added to the setup to perform heterodyne holographic microscopy [6, 8].

**B. Different number of frames detection**

\[ I_n = \frac{1}{T} \int_{t=nT_{CCD}-T/2}^{nT_{CCD}+T/2} dt |E(t) + \mathcal{E}_{LO}(t)|^2 \]

(4)

\[ = |E|^2 + |E_{LO}|^2 \]

\[ + EE_{LO}^* \left[ \frac{1}{T} \int_{t=nT_{CCD}-T/2}^{nT_{CCD}+T/2} dt \ e^{-j(\omega_{LO} - \omega)t} \right] + c.c. \]

where \( t_n = nT_{CCD} \) with \( n \) integer, \( T_{CCD} = 2\pi/\omega_{CCD} \), and \( \omega_{CCD}/2\pi \) is the camera frame frequency. The sinc factor of Eq. 4 corresponds to the integration of the beat signal of frequency \( \omega_{LO} - \omega \) over the camera exposure time \( T \).

Note that a microscope objective can added to the setup to perform heterodyne holographic microscopy [6, 8].

**1. One frame off axis holography**

One frame off axis holography is made by choosing \( \omega_{AOM2} = \omega_{AOM1} \) (so that \( \omega_{LO} = \omega_{I} = \omega \)), and \( H = 1 \) for the hologram. We get then:

\[ F = I_1 = |E|^2 + |E_{LO}|^2 \]

\[ + \text{sinc}((\omega - \omega_{LO})T/2)(EE_{LO}^* + E^*E_{LO}) \]

Figure 2 (a) shows the reconstructed image obtained in that case. The image exhibits 3 bright zones. The bright square in the center of the reconstructed image is the zero grating order. It corresponds to the \( |E_{LO}|^2 + |E|^2 \) terms. The blurred bright zone in the upper right side of the image is the +1 grating order. It corresponds to \( E_{LO}E^* \). Lastly, the USAF image that is sharp in the lower left side of the reconstructed image is the +1 grating order that corresponds to \( E_{LO}^*E \).

**2. Two frames phase shifting holography**

Phases shifting holography with 2 frames is made by tuning the local oscillator frequency to have \( \omega_{AOM2} = \omega_{AOM1} = \omega_{CCD}/2 \) (so that \( \omega_{LO} - \omega_{I} = \omega_{CCD}/2 \)). For detection at the illumination frequency (i.e. for \( \omega = \omega_{I} \)), the phase factor \( e^{-j(\omega_{LO} - \omega)t_n} \) of Eq. 4 becomes thus equal to \(-1^{n-1}\). We get for \( I_1 \) and \( I_2 \):

\[ I_1 = |E|^2 + |E_{LO}|^2 \]

\[ + \text{sinc}((\omega - \omega_{LO})T/2)(EE_{LO}^* + E^*E_{LO}) \]

\[ I_2 = |E|^2 + |E_{LO}|^2 \]

\[ + \text{sinc}((\omega - \omega_{LO})T/2)(-EE_{LO}^* - E^*E_{LO}) \]

By choosing \( H = I_1 - I_2 \), we get:

\[ H = I_1 - I_2 \]

\[ = 2 \text{sinc}((\omega - \omega_{LO}T/2))(EE_{LO}^* + E^*E_{LO}) \]

Figure 2 (b) shows the 2 frames reconstructed image. This image exhibits only 2 bright zones, which correspond to the orders +1 and −1 i.e. to \( EE_{LO}^* \) and \( E^*E_{LO} \).

**3. Four frames phase shifting holography**

Four phases phase shifting holography is made with \( \omega_{AOM2} = \omega_{AOM1} = \omega_{CCD}/4 \) so that \( \omega_{LO} = \omega_{I} = \omega_{CCD}/4 \). For \( t = t_n \), the phase factor \( e^{-j(\omega_{LO} - \omega)t} \) is thus equal to \((-j)^{n-1}\). We get:

\[ I_1 = |E|^2 + |E_{LO}|^2 \]

\[ + \text{sinc}((\omega - \omega_{LO}T/2))(EE_{LO}^* + E^*E_{LO}) \]

\[ I_2 = |E|^2 + |E_{LO}|^2 \]

\[ + \text{sinc}((\omega - \omega_{LO}T/2)(-jEE_{LO}^* + jE^*E_{LO}) \]

\[ I_3 = |E|^2 + |E_{LO}|^2 \]

\[ + \text{sinc}((\omega - \omega_{LO}T/2)(-EE_{LO}^* - E^*E_{LO}) \]

\[ I_4 = |E|^2 + |E_{LO}|^2 \]

\[ + \text{sinc}((\omega - \omega_{LO}T/2)(+jEE_{LO}^* - jE^*E_{LO}) \]

Different types of holographic detection schemes can be implemented with the setup of Fig. 1. To illustrate this point, we have considered a USAF target sample. Since the target does not move, the light is scattered by the target at the illumination frequency \( \omega = \omega_{I} \).
By choosing \( H = (I_1 - I_3) + j(I_2 - I_4) \), we get:
\[
H = (I_1 - I_3) + j(I_2 - I_4) = 4 \text{sinc}((\omega - \omega_{LO}T/2))E^{*}_{LO}
\]  
(9)

Figure 3(c) shows the 4 frames reconstructed image. This image exhibits only the +1 grating order \( EE^{*}_{LO} \), which gives a sharp image of the USAF target.

C. Frequency response \( \eta(\omega) \) of the holographic heterodyne detection

To determine \( \eta \) an holographic control experiment illustrated by Fig. 3 has been performed. The experiment is made by recording sequence of frames \( I_n \) for different frequency \( \omega = \omega_j \) of the object beam, the LO beam frequency \( \omega_{LO} \) being kept constant. The four phases hologram \( H \) is calculated and the +1 grating order signal \( EE^{*}_{LO} \) is selected in the Fourier space [10].

For each signal frequency \( \omega \), a sequence of frames \( I_n \) is recorded, and four phases holograms \( H \) and \( \tilde{H} \) are calculated in real and Fourier space:
\[
H(x, y) = (I_1(x, y) - I_3(x, y)) + j(I_2(x, y) - I_4(x, y))
\]
(10)
\[
\tilde{H}(k_x, k_y) = \text{FFT} [H(x, y)]
\]
where FFT is the 2D Fourier transform operator.

Figure 3(a) displays \( |\tilde{H}|^2 \) that is obtained with \( \omega_{AOM2} - \omega_{AOM1} = +\omega_{CCD}/4 \). The brighter point on the left (within circle 1) corresponds to the +1 grating order signal \( |EE^{*}_{LO}|^2 \), while the point on the right (within circle 2) is the -1 grating order signal \( |E^{*}E_{LO}|^2 \), whose intensity is much lower. Here in Fig. 3(a), the 4 phase detection made with \( \omega_{LO} - \omega = +\omega_{CCD}/4 \), allows one to select the +1 grating order and to reject the -1 order.

\[ W_{\pm 1} = \sum_{\pm 1} |\tilde{H}(k_x, k_y)|^2 \]  
(11)

where \( \sum_{\pm 1} \) is the sum over a 10 x 10 pixel region centered on the \( \pm 1 \) peaks. By sweeping the AOM1 frequency, we have studied how \( W_{\pm 1} \) varies with \( \omega \). Figure 3(c,d) shows \( W_{\pm 1} \) as a function of \( \omega_{LO} - \omega \). When the four-phase condition is fulfilled, i.e., when \( \omega_{LO} - \omega = +\omega_{CCD}/4 \), the weight \( W_{+1} \) of +1 signal is maximum (i.e. \( W_{+1} \sim 1 \)) and the twin signal is nearly zero (i.e. \( W_{-1} \sim 10^{-4} \)). Conversely, when \( \omega_{LO} - \omega = -\omega_{CCD}/4 \), \( W_{-1} \) is large and \( W_{+1} \) nearly zero.

The weight \( W_{+1} \) of \( |EE^{*}_{LO}|^2 \) term is proportional to \( |\eta|^2 \), where \( \eta \) is the \( EE^{*}_{LO} \) detection efficiency. For four phases detection with \( 4N \) frames i.e. for
\[ H = (1/4N) \sum_{n=1}^{4N} j^{n-1}I_n \]  
(12)
we have [11]:
\[
\eta(x) = \frac{1}{4NT} \sum_{n=0}^{4N-1} j^n \int_{t=NT_{CCD}-T/2}^{nt_{CCD}+T/2} e^{2\pi x t} dt
\]  
(13)

In figure 4(b), in contrast, with \( \omega_{LO} - \omega = -\omega_{CCD}/4 \) one selects the -1 grating order and rejects the +1 order, since the -1 image is bright, while the +1 image is dark.

We have measured the weight of the \( \pm 1 \) signals \( W_{\pm 1} \) defined by

\[ W_{\pm 1} = \sum_{\pm 1} |\tilde{H}(k_x, k_y)|^2 \]  
(11)
where \( x = (\omega - \omega_{LO})/2\pi \) is the heterodyne beat frequency in Hz, \( T \) the frame exposure time and \( T_{CCD} = 2\pi/\omega_{CCD} \) the frame to frame time.

In Eq. (13) the factor \( \text{sinc}(2\pi T) \), which has been already introduced in Eq. (4), corresponds to the integration of the beat signal, whose frequency is non-zero, over the camera exposure time \( T \). Because of Eq. (12) the summation over the \( 4N \) frames of Eq. (10) is made with a phase factor \( \eta^j \).

To the end, the factor \( 1/4N \) is a normalization factor that is the inverse of the number of terms within the summation. With this normalization factor the maximum of \( \eta(x) \) is slightly lower than 1.

As depicted in Fig. 4 (c), the experimental results for \( W_{\pm 1} \) (points) agree with \( \eta^2 \), where \( \eta \) is given by Eq. (13) with \( T = T_{CCD} = 0.1 \) s and \( 4N = 4 \) (solid gray curves). This means that heterodyne holography is able to perform phase shifting with very accurate phase \( \pi \).

The hologram \( H \) can thus be calculated without having to take account for the phase errors of the experimental setup.

We must notice also that the +1 grating order image corresponds to the \( \omega_{LO} + \omega_{CCD}/4 \) signal, while the −1 image corresponds to \( \omega_{LO} - \omega_{CCD}/4 \). If the frequency spectrum of the signal field \( E \) is broad and cover both \( \omega_{LO} \pm \omega_{CCD}/4 \) frequency components, the ±1 images both exist. One must thus work off axis in order to separate them, and to avoid image alias.

The purpose of Fig. 4 experiment is to measure the frequency response of the holographic detection. It turns out that the detection bandwidth BW (that is the width of the frequency response) is very narrow (about \( \pm 2.5 \) Hz). It is nevertheless possible to detect signals with Doppler shift or Doppler broadening much larger than BW.

To illustrate this point let us consider an example. Digital holography is made with an 8 bits camera, whose full well capacity is \( 2 \times 10^4 \) e. The camera signal \( I \) varies from 0 to 255 Digital Counts (DC) and the camera "gain" is \( G = 78 \text{ e}/\text{DC} \). In a typical situation, the local oscillator is adjusted to half saturation, and the local oscillator is \( |E_{LO}| = 10^4 \text{ e} = 128 \text{ DC} \).

The shot noise, whose standard deviation is \( \sigma = \sqrt{T} = 100 \text{ e} \), is much larger than the camera read noise (\( \sigma < 30 \text{ e} \)), and than the quantization noise of the camera Analog Digital converter (\( \sigma = 78/\sqrt{T} = 6.5 \text{ e} \)). On the other hand, the shot noise equivalent signal is 1 e per pixel. Indeed, if we consider a very low signal : \( |E| = 1 \text{ e} \), the +1 grating order holographic term \( |EE_{LO}| = 100 \text{ e} \) is equal to the shot noise variance \( \sigma = 100 \text{ e} \).

This result, which is valid for 1 frame, remains valid whatever the number of frames \( 4N \) is. Indeed, shot noise is a broadband white noise. The noise that is detected is thus proportional to the product of the total exposure time \( 4NT \) with the detection bandwidth BW, which is proportional to \( (4NT)^{-1} \). The shot noise equivalent signal does not depend on the number of frames \( 4N \). The shot noise is thus always equal to 1 e per pixel.

So, at this point a question arose : how it is possible to reach this shot noise limit in real time holographic experiment ? As discussed above, the read noise and the quantization noise are lower than shot noise. The last extra noise that must be considered is the technical noise.

![Figure 5. Detection efficiency \( |\eta|^2 \) for four phases heterodyne detection with \( 4N \) frames: \( 4N = 4 \) (heavy grey line), \( 4N = 8 \) (solid black line), and \( 4N = 16 \) (dashed black line). Calculation is done for \( T = T_{CCD} = 0.1 \) s. Vertical axis is \( |\eta|^2 \) in linear (a) and logarithmic (b) scales. Horizontal axis is \( (\omega_{LO} - \omega)/2\pi \) in Hz.]

### E. Double filtering of the zero order signal and shot noise

Since laser emission and photodetection on a camera pixel are random processes, the signal \( I \) that is detected on each pixel exhibits shot noise. This noise is gaussian and its standard deviation \( \sigma = \sqrt{T} \), where \( I \) is the pixel signal expressed in photo electron Units (e). When performing heterodyne holography in dim light condition, the detection sensitivity is generally limited by the shot noise on the local oscillator, since \( I \approx |E_{LO}|^2 \).

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So, at this point a question arose : how it is possible to reach this shot noise limit in real time holographic experiment ? As discussed above, the read noise and the quantization noise are lower than shot noise. The last extra noise that must be considered is the technical noise.
on the LO beam, because the LO signal is dominant since $|E_{LO}|^2 \gg |E|^2$. This technical noise is the sum of the noises that differ from shot noise and affect the LO beam signal. The noise on the power supply of main laser, or on the RF signal that drives the AOMs contribute to technical noise. Whatever its origin, the LO technical noise is highly correlated from pixel to the next, since to technical noise. Whatever its origin, the LO technical noise is much larger than the shot-noise limit, which is equal, within a few percent as experimentally verified, to the four frames noise floor seen on Fig. 6(d).

Here, in the control holographic experiment made without signal, the shot-noise limit can be reached with four frames detection, because the LO signal $|E_{LO}|^2$ (and the LO technical noise) are filtered off by a double filtering procedure in space and time.

- Filtering in space is made by selecting, in the Fourier space, the off axis region where the $+1$ grating order $EE^*_1$ is located. This region is far from the center of the Fourier space (see for example Fig. 2(a)), where the LO beam signal $|E_{LO}|^2$ is located.

- Filtering in time is made by the demodulation equation $H = (I_1 - I_3) + j(I_2 - I_4)$, which calculate $H$ from difference of frames recorded at different times. Since $|E_{LO}|^2$ does not vary with time, $|E_{LO}|^2$ is filter off by the demodulation equation, as seen on Eq. [8]

III. HETERODYNE HOLOGRAPHY

EXAMPLES OF EXPERIMENTS

To illustrate the advantages of heterodyne holography, we will present here two examples of experiment.

A. Sideband holography

The main advantage of heterodyne holography is its ability to detect the holographic signal at a frequency that is different from that of illumination. To illustrate this point, we will first consider the detection of the light scattered by a vibrating object.

1. Optical signal scattered by a vibrating object

![Image of experimental setup](image)

Figure 7. Heterodyne holography setup applied to analyse vibration of a clarinet reed. L: main laser; AOM1, AOM2: acousto-optic modulators; M: mirror; BS: beam splitter; BE: beam expander; CCD: camera; LS: loud-speaker exciting the vibrating clarinet reed at frequency $\omega_A/2\pi$. 

In the one frame case (Fig. 6(c)), the peak is much larger and much broader, than in the four frames case (Fig. 6(c)). It results that most of the Fourier space is polluted by the one frame LO parasitic signal, which is several orders of magnitude larger than its four frames counterpart. The one frame LO parasitic signal is much larger than the shot-noise limit, which is equal, within a few percent as experimentally verified, to the four frames noise floor seen on Fig. 6(d).

We have made diagonal cuts in order to explore zones of the Fourier space that are far from regions with $k_x$ or $k_y$ axis of the Fourier space. The Fourier space is thus $2048 \times 2048$ pixels, whose size is $2\pi/(2048D_{pix})$, where $D_{pix} = 6.7\mu m$ is the camera pixel size. Images and cut are calculated with one frame (a,c) and four frames holograms (b,d). See [12].

![Fourier holograms](image)

Figure 6. (a,b) Images of the Fourier holograms $\tilde{H}^2$ obtained without signal (i.e. with $|E|^2 = 0$). (c,d) Cut along the white dashed diagonal lines of the (a,b) images. Vertical axis is $\tilde{H}$ along the cut averaged over 11 pixels: $\tilde{H}^2$ is plotted in log scale. Horizontal axis is the pixel index ($0...2047$) along the $k_x$ and $k_y$ axis of the Fourier space. The $1280 \times 1024$ recorded hologram is padded into $2048 \times 2048$ calculation grid. The Fourier space is thus $2048 \times 2048$ pixels, whose size is $2\pi/(2048D_{pix})$, where $D_{pix} = 6.7\mu m$ is the camera pixel size. Images and cut are calculated with one frame (a,c) and four frames holograms (b,d). See [12].
Figure 7 shows a typical example of vibration holographic experiment setup in reflection geometry. The object is a clarinet reed, whose vibration is excited by a loudspeaker LS. The vibration is studied by heterodyne holography with a reference local oscillator beam (field $E_{LO}$) that can be frequency shifted with respect to the illumination beam (field $E_I$).

Let us consider that the objet (the clarinet reed) that vibrates at frequency $\omega_A$ with amplitude $z_{max}$. The displacement $z$ along the out of plane direction is

$$z(t) = z_{max} \sin \omega_A t$$

(14)

In backscattering geometry, this corresponds to a phase modulation $\varphi(t)$ of the signal:

$$\varphi(t) = 4\pi z(t)/\lambda = 4\pi z_{max} / \lambda$$

(15)

where $\lambda$ is the optical wavelength and $\Phi$ the amplitude of the phase modulation of the signal at angular frequency $\omega_A$:

$$\Phi = 4\pi z_{max} / \lambda$$

(16)

Let us define the slowly varying complex amplitude $E(t)$ of the field $E(t)$ scattered by the vibrating object. We have:

$$E(t) = E(t) e^{j\omega t} + \text{c.c.}$$

(17)

Because of the Jacobi-Anger expansion, we get:

$$E(t) = E(t) e^{j\varphi(t)} = E(t) e^{j\Phi \sin \omega_A t}$$

$$= E(t) \sum_m J_m(\Phi) e^{jm\omega_A t}$$

where $E$ is the complex amplitude without vibration, and $J_m$ the mth-order Bessel function of the first kind, with $J_{-m}(z) = (-1)^m J_m(z)$ for integer $m$ and real $z$. The scattered field $E(t)$ is then the sum of the carrier and sideband field components $E_m(t)$ of frequency $\omega_m$, where $m$ is the sideband index with:

$$E(t) = \sum_{m=-\infty}^{+\infty} E_m(t)$$

(18)

$$E_m(t) = E_m e^{j\omega_m t} + E_m^* e^{-j\omega_m t}$$

$$\omega_m = \omega_I + m\omega_A$$

where $E_m$ is the complex amplitude of the field component $E_m(t)$. Note that $\omega_0 = \omega_I$ is the illumination optical frequency. Equation (18) yields:

$$E_m = J_m(\Phi) E$$

(19)

Figure 8 presents the distribution of the field energy on the sidebands components $|E_m|^2$ for $\Phi = 0.3$ and $\Phi = 3$. If the amplitude of modulation $\Phi$ is low (see Fig. 8 (a)), most of the energy is on the carrier: $|E_0|/|E| \simeq 1$, and energy $|E_m|^2$ decreases rapidly with the sideband index $m$. If the amplitude $\Phi$ is large (see Fig. 8 (b)), the energy of the carrier is low: $|E_0|/|E| \ll 1$, while energy is distributed over many sidebands $|E_m|^2$.

B. Selective detection of the sideband components $E_m$ : by sideband holography [13]

Heterodyne holography is well suited to detect the vibration sideband components $E_m$. To selectively detect by four phase demodulation the sideband $m$ of frequency $\omega_m$, the frequency, $\omega_{LO}$ must be adjusted to fulfill the condition:

$$\omega_{LO} = \omega_I + m\omega_A + \omega_{CCD} / 4$$

(20)

$$\omega_m = \omega_I + m\omega_A$$

Figure 9 shows images obtained by detecting different sideband $m$ of a clarinet reed [13]. The clarinet reed is attached to a clarinet mouthpiece and its vibration is driven by a sound wave propagating inside the mouthpiece, as in playing conditions, but the sound wave is created by a loudspeaker excited at frequency $\omega_A$ and has a lower intensity than inside a clarinet. The excitation frequency is adjusted to be resonant with the first flexion mode (2143 Hz) of the reed.

Figure 9 (a) is obtained at the unshifted carrier frequency $\omega_0$. It corresponds to an image obtained by time
Sideband heterodyne holography is a very powerful technique, which has been used to image objects whose vibration amplitude are both large [22] and small [23–27]. Combined with stroboscopic illumination/detection [28] or with sideband correlation analysis [29], sideband heterodyne holography has been made sensitive to the phase of the vibration.

C. Analyse of Doppler spectrum of light that travel through the breast [30]

The purpose of the second example of experiment is to measure the spectral broadening of the light that travel through a woman breast in vivo. This experiment has been made in the context of ultrasonic photon tagging [31–35], whose purpose is to detect breast cancer. In that context the measure of the light spectral broadening, which is related to the breast transmitted light correlation time is important, since light decorrelation is important limitation of the performance of the ultrasonic photon tagging method.

Figure 11 shows the heterodyne holography experimental setup. The object, whose image is reconstructed by holography, is a rectangular slit that is back illuminated by laser beam (70 mW, 780 nm) that travel through the breast. The position of the lens L is adjusted so that the LO beam reach the camera off axis with respect to the object (the slit). Moreover, the slit to camera distance is made equal to the radius of curvature of the LO beam that reach the camera. By this way, the reconstruction of the +1 and -1 grating order images of the slit are made by a simple Fourier transform.

Figure 12 (a) shows the four frames Fourier space hologram $|H|^2$ that was obtained for $\omega_{AOM1} = \omega_{AOM2}$ i.e. sideband $m = 1$ by sweeping the frequency $\omega_A$ from 1.4 kHz up to 20 kHz by steps of 25 per cents (factor 1, 1.25, (1.25)$^2$, (1.25)$^3$...). The amplitude of the excitation signal is exponentially increased in the range 1.4 to 4 kHz, from 0.5 to 16 V, then kept constant at 16 V up to 20 kHz. This crescendo limits the amplitude of vibration of the first two resonances of the reed. The different vibration modes of the reed can be easily recognized on the reconstructed reed images of Fig. 10.

Figure 10. Clarinet reed reconstructed images obtained on sideband $m = 1$. Frequency $\omega_A$ is sweep from 1.4 KHz up to 20 KHz, and images are displayed from left to right and top to bottom ($26 \times 7$ images). The display is made with arbitrary grey scale for the intensity $|E_0(x, y)|^2$. See [21].
for $\omega_{LO} = \omega_1$. The two brighter vertical bands are the +1 and -1 grating order images of the slit, which are both on focus because of the peculiar location of lens L. The +1 image corresponds to the $|E|^2$ signal at frequency $\omega_{LO} + \omega_{CCD}/4$, while the -1 image corresponds to $\omega_{LO} - \omega_{CCD}/4$.

Because the breast inner motions are random, the frequency spectrum of the light scattered by breast is continuously broadened. In that spectrum the 4 phases detection that is made here selects the signal at frequency $\omega_{LO} - \omega_1/2\pi$ (curve 1) and $\omega_{LO} - \omega_1/2\pi + 100$ kHz (curve 2).

To explore the Doppler profile of the scattered photons, sequences of four frames. We have then measured the averaged signal within both the +1 and -1 images of the slit, and compared this signal with the averaged signal in a quite zone of the Fourier space, like the zone within the white circle in the right of Fig. 12 (a). The signal, within the quite zone, corresponds to shot noise i.e. to $1/e$ per pixel.

We have plotted, on Fig. 12 (b), (curve 1), the ratio of the averaged +1 and -1 signal $|\tilde{H}|^2$, versus the averaged signal within the quite zone, as a function of $(\omega_{LO} - \omega_1)/2\pi$ (curve 1). Because of the ratio, the vertical scale is the averaged signal $|E|^2$ in photon electron Units. In order to better visualise the background noise, we have also plotted on Fig. 12 (b) (curve 2), the ratio versus $(\omega_{LO} - \omega_1)/2\pi + 100$ KHz. Since 100 KHz is much larger than the half width of Doppler spectrum (about 1.5 kHz), curve 2 corresponds to the background noise, that is effectively equal to shot noise, i.e., $1/e$.

This last experiment shows how it is possible to explore a Doppler spectrum by heterodyne holography. It illustrates also the sensitivity of Doppler holography, since the signal that is analyzed is very low : $|E|^2$ very less than 0.05 $e$ at maximum.

The Doppler heterodyne holography method, that is illustrated here, has been used in many other contexts. The method has been used to detect the tagged photons in ultrasonic photon tagging experiments [4], to study brownian motion effects in coherent back scattering [36], and to image blood flow, in mouse crania [37, 38], rat eye [39] and fish embryo [8].

IV. CONCLUSION

In this paper, we have presented the digital heterodyne holography technique that is able to fully control the amplitude, phase and frequency of both illumination and reference beams. This control is made by acousto optic modulator, which are driven electronically by RF generators. Heterodyne holography is an extremely versatile and powerful tool, able to perform automatic data acquisition in any holographic configuration with accurate phase and shot noise sensitivity.

Heterodyne holography is also able to perform the holographic detection at frequencies that are different than the illumination one. This capability is especially useful for vibration analysis, and for laser Doppler full field measurement.

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