Variational secure cloud quantum computing

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Variational quantum algorithms (VQAs) have been considered to be useful applications of noisy intermediate-scale quantum (NISQ) devices. Typically, in the VQAs, a parametrized ansatz circuit is used to generate a trial wave function, and the parameters are optimized to minimize a cost function. On the other hand, blind quantum computing (BQC) has been studied in order to provide the quantum algorithm with security by using cloud networks. A client with a limited ability to perform quantum operations hopes to have access to a quantum computer of a server, and BQC allows the client to use the server’s computer without leakage of the client’s information (such as input, running quantum algorithms, and output) to the server. However, BQC is designed for fault-tolerant quantum computing, and this requires many ancillary qubits, which may not be suitable for NISQ devices. Here, we propose an efficient way to implement the NISQ computing with guaranteed security for the client. In our architecture, only \(N+1\) qubits are required, under an assumption that the form of ansatzes is known to the server, where \(N\) denotes the necessary number of the qubits in the original NISQ algorithms. The client only performs single-qubit measurements on an ancillary qubit sent from the server, and the measurement angles can specify the parameters for the ansatzes of the NISQ algorithms. No-signaling principle guarantees that neither parameters chosen by the client nor the outputs of the algorithm are leaked to the server. This work paves the way for new applications of NISQ devices.

I. INTRODUCTION

Quantum devices have the potential to offer significant advantages over classical devices. Entanglement and superposition play an essential role in the quantum advantage. Especially, quantum computation, quantum cryptography, and quantum metrology are considered promising applications of quantum devices [1–13]. Quantum computation provides faster calculations than the classical one [14,15]. Quantum cryptography ensures information-theoretic security for the communication between distant sites [16,17]. Quantum metrology aims to create a superior sensor to a classical one by using entanglement [18,19].

Recently, great efforts have been devoted to the hybridization between quantum computation, quantum cryptography, and quantum metrology [20,21]. A technique of quantum computation such as error correction or phase estimation algorithm has been used in quantum sensing to improve sensitivity [20,22] and/or dynamic range [23,24]. Quantum network can be combined with quantum sensing to detect global information of the target, get fields [25,26], and to add security about the sensing target [27,28].

Especially, blind quantum computation (BQC) is an idea to combine quantum computation and quantum cryptography [29,30]. Suppose that a client who does not have a sophisticated quantum device hopes to access a server that has a scalable fault-tolerant quantum computer. The BQC provides a client with a way to access the server’s quantum computer in a secure way where the client’s information such as input, output, and algorithm is not leaked to the server. The key idea of the BQC is to use measurement-based quantum computation (MBQC) [31,32]. In the MBQC, a cluster state is generated as a resource of the entanglement, and then a sequence of single-qubit measurements is performed. Depending on the algorithm, one needs to change angles of the single-qubit measurements, while the form of the cluster state does not depend on the choice of the algorithm. If the server sends a cluster state to the client and the client performs the single-qubit measurements, the server does not obtain any information of either the details or output of the algorithm set by the client. The no-signaling principle guarantees the security of the protocol [33,34].

Recently many theoretical and experimental works have been devoted to developing quantum devices in the noisy intermediate-scale quantum (NISQ) era. The NISQ device could involve tens to thousands of qubits with a gate error rate of around \(10^{-3}\) [17]. The NISQ computing typically requires only a shallow circuit to imple-
ment quantum algorithms. Variational quantum algorithms (VQAs) are the typical application of the NISQ computing [48–54]. In the VQA, one generates a trial wave function from a parametrized ansatz circuit that is typically shallow. In order to optimize a cost function tailored to a problem, one updates the parameters with classical computation to generate a new trial wave function. One can search exponentially large Hilbert space with the parametrized quantum circuit via the repetition of such hybrid quantum-classical operations, and thus could find a solution to a given problem.

A natural question is whether one can implement the NISQ computing in the blind architecture. If one adopts the BQC with the MBQC, one can in principle perform any gate-type quantum computation including NISQ computing. However, to implement the BQC with the MBQC on the cluster state, the necessary number of the qubits is around $3^N$ [42–44], where $N$ is the number of the qubits required in the original NISQ algorithm. Since the number of the qubits in the blind architecture with the MBQC is much larger than that in the original algorithm without blind properties [37–41], such a scheme may not be implementable with the NISQ device with a limited number of qubits.

Here, we propose an efficient scheme to implement the variational secure cloud quantum computing. The purpose of our scheme is that the client accesses the quantum computer of the server to implement the NISQ computing in a secure way where the information of the ansatz circuit’s parameters and output of the algorithm are not leaked to the server. This is essential for security, because the ansatz circuit’s parameters could contain important information such as private data especially when we perform machine learning with NISQ devices [55–59]. Importantly, our scheme requires only $N + 1$ qubits while MBQC on the cluster state requires around $3^N$ qubits. The key idea of our scheme is to use an ancillary qubit for the implementation of the quantum gates on register qubits of the server. The server performs only a limited set of gate operations with fixed angles, namely, Hadamard operations and controlled-$Z$ gates on the register qubits, while the client performs arbitrary single-qubit measurements on the ancillary qubit.

A key idea of our scheme is the use of ancilla-driven quantum computation (ADQC) [60–63]. The ADQC was originally discussed as one of the novel ways to perform gate-type quantum computation including NISQ computing [60–63]. We discuss, for the first time to our best knowledge, the use of the ancilla-driven architecture for NISQ computing with security in-built. In our architecture, the server couples an ancillary qubit to a register qubit via a fixed two-qubit gate at the server side, and the ancillary qubit is sent to the client. Then the client implements single-qubit measurements on the ancillary qubit. By repeating this process, the client can specify the parameters for the NISQ computing by the angles of the single-qubit measurements, and also can obtain the output of the algorithm from the read-out of the ancillary qubit. Importantly, in this scheme, the client does not send any qubits nor classical signals to the server, and thus both client’s operations and measurement results are unknown to the server. Therefore, the information about the parameters and output of the NISQ algorithm cannot be leaked to the server due to the no-signaling principle [45–46].

The paper is structured as follows. In Secs. II and III, we review the ADQC and NISQ algorithm, respectively. In Sec. IV, we describe our architecture of the NISQ computing with security in-built. In Sec. V, we conclude our results.

## II. ANCILLA-DRIVEN QUANTUM COMPUTATION

In the ADQC [60], we define register qubits to execute algorithms, and also define an ancillary qubit that can be spatially transferred from one place to another. The basic idea of the ADQC is to entangle the register qubit and ancillary qubit, and the ancillary qubit is sent to another place for the measurement at a specific angle. These operations allow one to perform a universal set of operations. For the implementation with the physical systems, register qubits can be solid-state systems that can interact with photons, and the ancillary qubit can be an optical photon that is transmitted to a distant place.

### A. Single-qubit rotation on a register qubit

We explain a realization of single-qubit rotation along $z$-axis as follows (see Fig. 1).

1. We prepare a state $|+\rangle_A \equiv \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ of an ancilla qubit (which we call qubit A) and any state $|\psi\rangle$ of register qubits (which we call qubits R).
2. The ancillary qubit A is coupled with one of the register qubits R via a controlled-$Z$ gate $CZ_{AR}$, and subsequently, we implement two Hadamard gates $H_A$ and $H_R$ to the qubit A and the qubit R, respectively. Thus, we have a unitary operation of $E_{AR} \equiv H_AH_RCZ_{AR}$.
3. A rotation about the $z$-axis $R_z(\beta)$ and a Hadamard gate are implemented on the ancillary qubit, where $\beta$ is an arbitrary rotation angle.
4. Measuring the ancillary qubit in the $z$-basis projects the state of the register qubit onto $X^{j_A}H_RR_z(\beta)|\psi\rangle$, where $j_A = 0$ or 1 is the result of the measurement on the ancillary qubit.

The third and the last steps can be unified into a single measurement step if an arbitrary-angle single-qubit measurement can be implemented on the ancillary qubit. The details of performing an arbitrary single-qubit rotation and two-qubit gates with ADQC are explained in Appendix A.
In variational algorithms, we should implement not only the original quantum circuit but also variant types of the original circuit. For example, in many variational algorithms, derivatives of quantum states, i.e., \( \frac{\partial \psi(\vec{\theta})}{\partial \theta_k} \) are used. They are generated from a different quantum circuit from the original ansatz circuit. To discuss these cases in a general form, we denote the set of variational quantum circuits used in the algorithm as \( \{U_{AN}(\vec{\theta})\}_{i=1}^G \equiv \{V_L^{(i)}(\theta_L) V_{L-1}^{(i)}(\theta_{L-1}) \ldots U_1^{(i)}(\theta_1) V_1^{(i)}(\theta_1)\}_{i=1}^G \), where \( G \) is the number of variational quantum circuits in these cases. Accordingly, we denote the set of the observables measured in these quantum circuits as \( \{A_j^{(i)}, A_2^{(i)}, \ldots, A_K^{(i)}\}_{i=1}^G \), where \( A_i(\vec{\theta}) \) is a Pauli matrix (or an operator made up of tensor products of the Pauli matrices) and \( K^{(i)} \) is the number of observables measured in the \( i \)-th quantum circuits. We will use these notation throughout this paper. We show a prescription about how to implement the conventional variational algorithms with these notation in Appendix \( \text{B} \).

\section*{IV. VARIATIONAL SECURE CLOUD QUANTUM COMPUTING}

We explain our protocol of the variational secure cloud quantum computing. Suppose that a client who has the ability to perform only single-qubit measurements hopes to access the NISQ computer of the server in a secure way. The main purpose of our scheme is to hide the information of the ansatz parameters \( \vec{\theta} \) set by the client and output of the algorithm. In our scheme, the ansatz circuit to be implemented by the server is publicly announced beforehand. Our scheme is efficient for the NISQ device that has a limited resource, because our scheme requires only a single ancillary qubit independently of the number of qubits needed in the original NISQ algorithm. These are in stark contrast with the original BQC. In the BQC, every information of the choice of the client is hidden \([37][41]\), while 3N qubits are required to execute an algorithm using N qubits.

Throughout our paper, we assume that the client has his/her own private space, and any information in the private space is not leaked to the outside. This is the standard assumption in the quantum key distribution \([64]\).

The key of our protocol is to use the concept of the ADQC when the server runs the NISQ computing algorithm. We assume that the server has register qubits, and an ancillary qubit can be sent from the server to the client. When the server needs to implement a single-qubit operation based on the ansatz, the server uses the single-qubit rotation scheme of the ADQC as shown in Fig. 2.

More specifically, the server performs a two-qubit gate \( E_{\text{AR}} \) between the register qubit (that we want to perform the single-qubit rotation) and the ancillary qubit, and sends the client the ancillary qubit to be measured.
by the client side. The angle and axis of the single-qubit rotation are determined by the client. With three sets of the rotation, an arbitrary single-qubit rotation can be achieved in a register qubit (see Appendix A).

Moreover, by performing a single-qubit rotation on every register qubit in this way, we have byproduct operators of $X^{\pm1}, Z^{\pm1}$ on every register qubit as shown in Eq. (A1). It is known that, when Pauli matrices or an identity operator are randomly implemented on a quantum state (see Sec. 8.3.4 in Ref. [65]), the state becomes completely mixed. This means that the byproduct operators make the state completely mixed for the server. Due to this property, any measurements on the server side provide random outcomes if the server side does not have any information of the client’s dataset, which is helpful for the client to hide the output of the algorithm. The security of our scheme can be also interpreted as follows. During the implementations of these gates in our scheme, the gate operations executed by the server do not depend on the ansatz parameters. Moreover, the client does not send any information to the server during our protocol. Therefore, the server cannot find the parameters of the ansatz circuit set by the client. Such security is guaranteed by the no-signaling principle [45, 46].

When the server needs to perform a two-qubit gate based on the ansatz with a specific angle, we adopt a quantum circuit shown in Fig. 3b. The point is that an arbitrary two-qubit gate can be decomposed by arbitrary single-qubit gates and controlled-$Z$ gates. We combine the single-qubit rotations in the ADQC with two controlled-$Z$ gates as shown in Fig. 3b. In this case, the angles of the two-qubit gates can be determined by the client because the angle of the single-qubit gate can be specified just by the client. Similar to the case of the single-qubit gates, the no-signaling principle guarantees that the server does not obtain any information about the ansatz parameters during the implementation of the two-qubit gates.

The combinations of the single-qubit gates and two-qubit gates in our architecture are shown in Fig. 4. The server performs only Hadamard gates, phase gates, and two-qubit gates. We do not need to prepare ancillary qubits to generate the trial wave functions. When the observables $\{\hat{X}, \hat{Z}\}$ are measured, the effect of the byproduct operators can be canceled out by the client (see Appendix A).

1. Adopting the quantum circuits of $\{U_{AN}^{(i)}\}_{i=1}^{G}$, the server and client implement these unitary operations to generate the trial wave functions $\{|\psi^{(i)}(\hat{\theta}[1])\rangle\}_{i=1}^{G}$. Here, parametrized single- and two-qubit gates should be implemented in the specific ways as described in Figs. 2 and 3b, respectively. More specifically, the server performs operations, such as the Hadamard, the controlled-$Z$, and the phase gates (1.a in Fig. 4), while the client specifies the measurement angles (1.b of Fig. 4). We do not need to prepare $\{|\psi^{(i)}(\hat{\theta}[1])\rangle\}_{i=1}^{G}$ simultaneously by using G quantum computers, but we can prepare and measure these in sequence by using a single quantum computer, similar to the standard VQA for NISQ devices (see Appendix B).

2. The server measures the states of the register qubits with $\{\hat{A}_{1}^{(i)}, \hat{A}_{2}^{(i)}, \cdots, \hat{A}_{K(i)}^{(i)}\}_{i=1}^{G}$, and sends the results to the client with classical communications.

3. For the sampling, the server and client repeat the first and the second steps with $\{N^{(i)}\}_{i=1}^{G}$ times for each state $\{|\psi^{(i)}(\hat{\theta}[1])\rangle\}_{i=1}^{G}$ so that the client should obtain the expectation values of $\{\hat{A}_{1}^{(i)}, \hat{A}_{2}^{(i)}, \cdots, \hat{A}_{K(i)}^{(i)}\}_{i=1}^{G}$. When the observables are measured, the effect of the byproduct operators can be canceled out by the client (see Appendix A).

4. By processing the measurement results with a
be the solid-state systems that interact with a photon, phase gate. The Hadamard, the controlled-Z circuit is the same as that in (a), where the server in our scheme. The basic structure of the client while the rotation parameters are hidden to δ and for the equivalence. (b) A quantum circuit to operation on the server. parameters (α, β, γ) we need to choose appropriate parameters of δ, and δ for the equivalence. (b) A quantum circuit to implement an arbitrary controlled-gate operation by the server while the rotation parameters are hidden to the server in our scheme. The basic structure of the circuit is the same as that in (a), where S denotes a phase gate. The Hadamard, the controlled-Z, and the phase gates are implemented by the server in the register qubits. An important point is that every single-qubit rotation in the circuit should be performed by the client in the same way as described in Fig. 3. In this case, no-signaling principle guarantees that the rotation parameters (α, β, γ, and δ) cannot be inferred by any operation on the server.

As a physical implementation, the register qubits can be the solid-state systems that interact with a photon, and the ancillary qubit can be an optical photon that transmits to a distant place. We implicitly assumed that the photon loss would be negligible during the transmission in the discussion above.

Finally, we discuss the effect of photon loss on our scheme. When the server sends the client an ancillary qubit that corresponds to an optical photon, there is a possibility that the photon can be lost during the transmission. In principle, if the server and the client have quantum memories, they can share a Bell pair under the effect of photon loss by repeating the entanglement generation process until success [66], and they can use the Bell pairs to perform our gate operations in a deterministic way. In this case, the client needs to ask the server to send the photons again and again, depending on how many times the photon is lost [66]. However, in order to apply the no-signaling principle, the client is not allowed to send the server any information. This means that the client cannot ask the server to send the photon again. So we cannot adopt the repeat-until-success strategy with quantum memories.

Thus, we assume that the client adopts the observation results of the readout of the register qubits by the server only when all photons are successfully transmitted to the client during the computation. In this case, the probability of the no photon loss during the computation exponentially decreases as the number of sending photons increases. The number of required photons sent to the client can be determined by the number of the tunable parameters used in the ansatz circuit. When \( U_{AN}^{(i)} \) is composed of \( n_{\text{single}}^{(i)} \) single-qubit operations and \( n_{\text{two}}^{(i)} = L - n_{\text{single}}^{(i)} \) two-qubit operations, the necessary number \( N_{\text{ph}}^{(i)} \) of the photons to send the client is at most

\[
N_{\text{ph}}^{(i)} = 3n_{\text{single}}^{(i)} + 6n_{\text{two}}^{(i)}
\]

as shown in Eq. A1 and Figs. 2 and 3. The probability for all the photons to be detected by the client is \((1 - p_{\text{loss}})^{N_{\text{ph}}^{(i)}}\), where \( p_{\text{loss}} \) is a photon loss probability for a single transmission. Therefore, the repetition number \( N^{(i)} \) with the photon loss should be set to be much larger than \( N^{(i)}_{\text{ideal}}/(1 - p_{\text{loss}})^{N_{\text{ph}}^{(i)}} \), where \( N^{(i)}_{\text{ideal}} \) denotes the required number of repetition with no photon loss. To keep \( N^{(i)} \) within a reasonable amount, \( p_{\text{loss}} \) should be smaller than 1% under the assumption that \( N_{\text{ph}}^{(i)} \) is around a few hundreds.

We could overcome such a problem due to the recent experimental and theoretical developments of quantum repeating technology. The best single-photon detector in optics has 99% efficiency [67–69]. Microwave quantum repeater with a short distance such as 100 m has been proposed [70], and a qubit can catch a microwave photon with 99.4% absorption efficiency in the microwave regime [71]. Also, there are proposals to physically move the solid-state qubit [72, 73] for distributed quantum computation or a quantum repeater. Through the combination of these protocols and a long-lived quantum memory such as a nuclear spin [74, 75], the ancillary solid-state qubits might be carried to the client without the problems of the photon loss.

![Diagram](image-url)
FIG. 4: A quantum circuit to implement our variational secure cloud quantum computing. The NISQ algorithm requires the parameters \( \{\theta_j\}_{j=1}^L \) to change the ansatz circuit in a variational way. The server implements gate operations that do not depend on the parameters, and sends the ancillary qubit to the client. On the other hand, the client can specify the parameters by changing the measurement angles on the ancillary qubits sent from the server. Importantly, in our scheme, the client does not send any signal to the server, and thus the server does not know the parameters set by the client, due to the no-signaling principle.

FIG. 5: Before the client starts the protocol, the server broadcasts the information about their quantum circuit. This includes the set of unitary operations \( \{U_{AN}^{(i)}\}_{i=1}^G \), the set of the observables \( \{\hat{A}_1^{(i)}, \hat{A}_2^{(i)}, \ldots, \hat{A}_K^{(i)}\}_{i=1}^G \) to be measured, the repetition numbers \( \{N(i)\}_{i=1}^G \) for the quantum circuits, initial states \( \{\psi(\vec{\theta}(0))\}_{i=1}^G \), and the total number \( M \) of iteration steps to update the parameters. The client implements the NISQ algorithm based on this information.

V. CONCLUSION

In conclusion, we proposed a noisy intermediate-scale quantum (NISQ) computing with security inbuilt. The main targets of our scheme are variational quantum algorithms (VQAs), which involve parameters of an ansatz to be optimized by minimizing a cost function. We considered a circumstance that a client with a limited ability to perform quantum operations hopes to access a NISQ device possessed by a server and the client tries to avoid leakage of the information about the quantum algorithm that he/she runs. Importantly, the naive application of the previously known blind quantum computation (BQC) \(^{40}\) requires around \( 3N \) qubits \(^{42}44\), where \( N \) denotes the number of the qubits to run the quantum algorithm in the original architecture. That may not be suitable for the NISQ devices with the limited number of qubits. Our proposal is more efficient in the sense that we use a single ancillary qubit and \( N \) register qubits required in the original NISQ algorithm. In VQAs, we use a parametrized trial wave function, and our scheme prevents the information about the parameters from the leakage to the server. We rely on the no-signaling principle to guarantee security. Our scheme paves the way for new applications of the NISQ devices.

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1. Generate a parametrized wave function

1.a. Perform $V_n^{(i)}$ by the server

![Diagram of quantum circuit]

1.b. Specify the parameter by the measurement angle with the client

![Diagram of quantum circuit]

2. Readout

![Diagram of quantum circuit]

3. Repeat 1 and 2 with $N^{(i)}$ times

4. Update the parameters

![Diagram of quantum circuit]

5. Repeat 1-4 with parameter updates from $\theta[j]$ to $\theta[j + 1]$ for $j = 2, 3, \ldots, M - 1$

FIG. 6: The sequence of our scheme to implement a NISQ algorithm with a parameter set of $\vec{\theta}$ in a variational secure cloud quantum computing. (1) The server sequentially performs the unitary operations $\{U_{AN}^{(i)}\}_{i = 1}^G$ for the register qubits, where $G$ denotes the number of the quantum circuits to be performed. (1.a) The server implements a unitary (non-parameterized) operation $V_n^{(i)}$ for $n = 1, 2, \ldots, L + 1$; a Hadamard or a controlled-Z, or a phase gate, on the register qubits. (1.b) The server entangles a register qubit with an ancillary qubit and sends the ancillary qubit to the client in the same way as Fig. 2. The client measures the ancillary qubit sent from the server, where the client specifies a measurement angle based on the initial parameters $\vec{\theta}[1]$. (2) The server measures $A_1^{(i)}, A_2^{(i)}, \ldots$, and $A_{K(i)}^{(i)}$ for $i = 1, 2, \ldots, G$ and sends the results to the client by the classical communication. (3) For each $\{U_{AN}^{(i)}\}_{i = 1}^G$, the server and the client repeat these two steps $\{N^{(i)} \}_{i = 1}^G$ times, and then the client obtains expectation values of $\{A_1^{(i)}, A_2^{(i)}, \ldots, A_{K(i)}^{(i)}\}_{i = 1}^G$ with $\{|\psi^{(i)}(\vec{\theta}[1])\rangle\}_{i = 1}^G$. (4) The client updates the parameters as $\vec{\theta}[2]$ by processing the measurement data with classical computation. (5) The server and the client repeat these four steps with $M - 2$ times updating the parameters from $\vec{\theta}[j]$ to $\vec{\theta}[j + 1]$ for $j = 2, 3, \ldots, M - 1$, and the client obtains the output. Since the client does not send any signals to the server during the computation, the server cannot obtain any information about $\vec{\theta}[1], \vec{\theta}[2], \ldots, \vec{\theta}[M]$, because of the no-signaling principle.

Appendix A: Detailed ancilla-driven quantum computation

1. Arbitrary single-qubit rotation

We describe a way to implement an arbitrary single-qubit rotation. Any single-qubit rotation $U$ can be represented by $U = R_x(\beta')R_y(\gamma')R_z(\delta')$, where $R_x$ denotes a rotation about the $x$-axis, and $\beta', \gamma'$, and $\delta'$ denote the rotation angles about the corresponding axis. Defining $J(\beta) = HR_z(\beta)$, one can rewrite $U$ as $U = J(\beta)J(\gamma)J(\delta)$, where we choose $\beta$, $\gamma$, and $\delta$ to satisfy $R_z(\beta)R_y(\gamma)R_z(\delta) = HU$. As we explained, one can implement the single-qubit rotation of $X^JH_R(\beta)|\psi\rangle$ on the register qubit by the coupling with an ancillary qubit and a subsequent measurement. Therefore, three sequential operations of this type of the single-qubit rotation provide us with the following operation

$$
\begin{align*}
J(\beta)J(\gamma)J(\delta) & = \left( X^JH_R(\beta) \right) \left( X^JH_R(\gamma) \right) \left( X^JH_R(\delta) \right) \\
& = X^JH_R((-1)^{J\beta} \gamma) X^JH_R((-1)^{J\gamma} \delta) \\
& = (-1)^{J\beta} X^JH_R((-1)^{J\gamma} \delta) \\
& = (-1)^{J\beta} X^JH_R((-1)^{J\gamma} \delta) = J(\beta)J(\gamma)J(\delta)
\end{align*}
$$

(A1)
where $j_i$ denotes the result of the $i$-th measurement on the ancillary qubits. For the implementation of this operation, we change the rotation angle of the ancillary qubit depending on the previous measurement results. Equation (A1) involves the byproduct operator $X^{j_1}Z^{j_2}$. However, as long as we measure the qubit in a computational basis for the readout, the byproduct operators just flip the measurement result from 0 to 1 or vice versa, and so we can effectively remove the byproduct operators from the states by changing the interpretation of the measurement results.

2. Two-qubit gate between the register qubits

We explain a way to perform the controlled-Z gate on the two register qubits $R$ and $R'$ in the ADQC. Firstly, we implement $E_{AR}$ on the ancillary qubit (prepared in the state $|+\rangle_A$) and the register qubit $R$, and subsequently perform $E_{AR'}$ on the ancillary qubit and the register qubit $R'$. Secondly, one measures the ancillary qubit in the $y$-basis. These operations are equivalent to the controlled-Z gate, up to local operations. When we perform several single-qubit gates and two-qubit gates, the byproduct operators are applied as $U_S U_{\text{ideal}} (|0\rangle)$, where $U_S$ denotes the total byproduct operators and $U_{\text{ideal}}$ denotes the unitary operations that we aim to implement. Again, when one measures observables of Pauli matrices (or a tensor product of Pauli matrices), one can effectively remove the byproduct operators from the states by changing the interpretation of the measurement results.

Appendix B: VQA for NISQ devices

We show a prescription about how to implement the conventional variational algorithms with our notation. We prepare a parametrized wave function on a quantum circuit $|\psi(\vec{\theta})\rangle$ with the variational parameters $\vec{\theta}$ to be optimized by minimizing a cost function $C(\vec{\theta})$ tailored to a problem. Firstly, with the quantum circuits of $\{U_{AN}^{(i)}\}_{i=1}^M$, we realize parametrized wave functions of $N$-qubits $\{|\psi^{(i)}(\vec{\theta}[i])\rangle\}_{i=1}^G$, where $|\psi^{(i)}(\vec{\theta}[1])\rangle \equiv V_N^{(i)} U_N^{(i)} (\theta_L[1]) V_N^{(i)} \cdots \tau_{i=1}^M U_N^{(i)} (\theta_i[1]) V_N^{(i)} \rangle_0$ with $|0\rangle \equiv \bigotimes_{i=1}^M |0\rangle$ denotes the wave function, $\vec{\theta}[1] = (\theta_1[1], \cdots, \theta_L[1])$ is a vector of the parameters and $\{|\psi^{(i)}(\vec{\theta}[0])\rangle\}_{i=1}^G$ are initial states, and we measure the state of the wave function with observables of $\{\hat{A}_1^{(i)}, \hat{A}_2^{(i)}, \cdots, \hat{A}_K^{(i)}\}_{i=1}^G$.

Secondly, for the sampling, we repeat the first step to obtain expectation values of $\{\hat{A}_1^{(i)}, \hat{A}_2^{(i)}, \cdots, \hat{A}_K^{(i)}\}_{i=1}^G$ with $\{|\psi^{(i)}(\vec{\theta}[1])\rangle\}_{i=1}^G$. Thirdly, based on the expectation values, we implement a classical algorithm so that we can obtain updated parameters $\vec{\theta}[2]$ for the next quantum circuits, where we typically use a gradient method to make the cost function smaller. For example, we use $\vec{\theta}[j+1] = \vec{\theta}[j] - \alpha \nabla C(\vec{\theta}[j])$ for the gradient method.

Finally, we repeat the first, second, and third steps $M - 2$ times with $\{U_{AN}^{(i)}\}_{i=1}^M$ and $\vec{\theta}[k]$, where classical computation based on the results at the $k$-th step provides the updated parameters of $\vec{\theta}[k+1]$ for $k = 2, 3, \cdots, M - 1$. These processes provide us with an output of the algorithm.

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