Wigner’s infinite spin representations and inert matter
dedicated to the memory of Robert Schrader

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Abstract

Positive energy ray representations of the Poincaré group are natu-
urally subdivided into three classes according to their mass and spin con-
tent: $m > 0$, $m = 0$ finite helicity and $m = 0$ infinite helicity. For a long time
the localization properties of the massless infinite spin class remained un-
known before it became clear that such matter does not permit compact
spacetime localization and its generating covariant fields are localized on
semi-infinite spacelike strings.

Using a new perturbation theory for higher spin fields we show that
infinite spin matter cannot interact with normal matter and we formulate
condition under which this also could happen for finite spin $s > 1$ fields.
This raises the question of a possible connection between inert matter and
dark matter.

1 Wigner’s infinite spin representation and string-localization

Wigner’s famous 1939 theory of unitary representations of the Poincaré group $\mathcal{P}$
was the first systematic and successful attempt to classify relativistic particles
according to the intrinsic principles of relativistic quantum theory [1]. As we
know nowadays, his massive and massless spin/helicity class of positive energy
ray representations of $\mathcal{P}$ does not only cover all known particles, but their
"covariantization" [2] leads also to a complete description of all covariant point-
local free fields. For each with physical spin or helicity compatible covariant
transformation property there exists a point-local (pl) field.
The only presently known way to describe interactions in four-dimensional Minkowski space is to start from a scalar interaction density in terms of Wick-products of free fields with the lowest short distance dimension and use it as the starting point of the cutoff- and regularization-free causal perturbation theory \[3\]. These free fields do not have to be Euler-Lagrange fields; perturbative QFT can be fully accounted for in terms of interaction densities defined in terms of free fields obtained from Wigner’s representation theory without referring to any classical parallelism.

All positive energy representations are "induced" from irreducible representations of the "little group". This subgroup of the Lorentz group is the stability group of a conveniently chosen reference momentum on the forward mass shell \[H^+\] respectively the forward surface of the light cone \[V^+\]. For \(m > 0\) this is a rotation subgroup of the Lorentz group and for \(m = 0\) the noncompact Euclidean subgroup \(E(2)\). Whereas the massive representation class \((m > 0, s = \frac{n}{2})\), of particles with mass \(m\) and spin \(s\) covers all known massive particles (the first Wigner class), the massless representations split into two quite different classes.

For the finite helicity representations the \(E(2)\) subgroup of Lorentz-"translations" are trivially represented ("degenerate" representations), so that only the abelian rotation subgroup \(U(1) \subset E(2)\) remains; this accounts for the semi-integer helicity \(|\hbar|, |\hbar| = \frac{n}{2}\) representations (the second Wigner class). The third Wigner class consists of faithful unitary representations of \(E(2)\). Being a noncompact group, they are necessarily infinite dimensional and their irreducible components are characterized in terms of a continuous Pauli-Lubanski invariant \(\kappa\).

Since this invariant for massive representations is related to the spin as \(\kappa^2 = m^2s(s+1)\) one may at first think that the properties of this infinite spin matter can be studied by considering it as a limit \(m \to 0, s \to \infty\) with \(\kappa\) fixed. However it turns out that the \((m, s)\) spinorial fields do not possess such an infinite spin limit. Our main result concerning the impossibility of quantum field theoretical interactions between WS with normal matter depends among other things on the absence of such an approximation; for this reason we will refer to these representations briefly as the "Wigner stuff" (WS). This terminology is also intended to highlight some of the mystery which surrounded this class for the more than 6 decades after its discovery and which also the present paper does not fully remove.

For a long time the WS representation class did not reveal its quantum field theoretic localization properties. The standard group theoretical covariantization method to construct intertwiners \[2\] which convert Wigner's unitary representations into their associated pl quantum fields does not work for the WS representations. Hence it is not surprising that attempts in \[3\] (and more recently in \[5\]) which aim at the construction of covariant wave functions and associated Lagrangians fell short of solving the issue of localization. In fact an important theorem \[6\] dating back to the 70s proved that it is not possible to associate pl fields (Wightman fields) with these representations.

Using the concept of modular localization, Brunetti, Guido and Longo showed that WS representations permit to construct subspaces which are "modular localized" in arbitrary narrow spacelike cones \[7\] whose core is a semi-infinite
string. In subsequent work \[8\] \[9\] such generating string-local covariant fields were constructed in terms of modular localization concepts. In the same paper attempts were undertaken to show that such string-local fields cannot have pl composites. These considerations were strengthened in \[10\]. A rigorous proof which excludes the possibility of finding compact localized subalgebras (related to testfunction-smeared pl fields) was finally presented in an seminal paper by Longo, Morinelli and Rehren \[11\].

Being a positive energy representation, WS shares its stability property and its ability to couple to gravity through its energy-momentum tensor with the other two positive energy classes; hence it cannot be dismissed from the outset as being unphysical. It will be shown that this form of matter is inert which means that it cannot interact with normal matter. There are reason to believe that higher spin fields \(s \geq 2\) may share this lack of reactivity, but the additional calculations which would be necessary to resolve this problem are outside the scope of this paper.

Absence or at least reduced reactivity is also a property of the ubiquitous dark matter; only additional measurement in underground counters will be able to resolve this problem.

The aim of the present paper is to convert the question of whether nature uses WS matter into a problem of particle theory. Since the task of local quantum physics (LQP) is to explain properties of matter in terms of the causal localization principles, one must show that the lack of reactivity of WS is a consequence of its intrinsically noncompact localization.

In the context of quantum theory these principles is much more powerful than their classical counterpart. The concept of modular localization permits to address structural problems of QFT in a completely intrinsic way which avoids the use of "field-coordinatizations". An illustration of the power of this relatively new concept is the proof of existence of a certain class of two-dimensional models starting from the observations that certain algebraic structures in integrable \(d=1+1\) models can be used to construct modular localized wedge algebras \[14\]. In the work of Lechner and others this led to existence proofs for integrable models with nontrivial short distance behavior together with a wealth of new concepts (see the recent reviews \[15\] \[16\] and literature cited therein). Even in renormalized perturbation theory modular localization has become useful in attempts to replace local gauge theory in Krein space by a positivity preserving string-local fields in Hilbert space \[17\] \[18\].

In \[7\] it was essential to extract localization properties directly in the form of modular localized subspaces since Weinberg’s group theoretic method of constructing the intertwiners of local field via group theoretical covariance requirements does not work for WS.

In an unpublished previous note \[19\] I tried to address the problem of a possible connection between WS and dark matter. But the recent gain of knowledge from modular localization regarding attempts to unite WS with normal matter under the conceptual roof of AQFT in \[11\], as well as new insights coming from perturbative studies of couplings involving string-local fields \[17\] \[18\], led to a revision of my previous ideas.
In [11] it was shown that the attempt to unite normal matter together with WS in a nontrivial way under the conceptual roof of algebraic QFT (AQFT) leads to an unexpected (suspicious looking) loss of the so-called "Reeh-Schlieder property" for compact localized observable algebra. The R-S property states that the set of state vectors obtained by the application of operators from a compact localized subalgebra of local observables to the vacuum is "total" in the vacuum Hilbert space. The possibility to manipulate large distance properties of states in the vacuum sector by applying operators localized in a compact spacetime region $\mathcal{O}$ to the vacuum is considered to be a universal manifestation of vacuum polarization.

The R-S property plays an important role in the Doplicher-Haag-Roberts (DHR) superselection theory [20] which leads to the concept of inner symmetries and Bose/Fermi statistics (absence of parastatistics) and its absence in the presence of WS asks for further clarification which in the present work is obtained from a new positivity preserving string-local perturbation theory (SLFT) for $s \geq 1$ fields. It turns out that, different from the pl case, the pcb requirement on the first order interaction density $d_{\text{int}}(L) \leq 4$ is the only restriction for the perturbative existence of a model; string-local (sl) interactions must also fulfill quite restrictive additional conditions which prevent total delocalization and maintain renormalizability in higher orders.

The main result of the present paper is that these conditions cannot be fulfilled in couplings of WS to normal matter. The reason is that the class of WS remains completely isolated; its fields are not the massless limit of spin $s$ massive fields for fixed Pauli-Lubanski invariant $\kappa^2 = m^2 s (s + 1)$.

This leaves only the conclusion that, apart from interactions with gravity as a consequence of the positive energy property and the existence of an energy-momentum tensor, WS cannot interact with normal matter. In mathematical terminology: WS tensor-factorizes with normal matter and the Reeh-Schlieder property holds only in the tensor factor of normal matter.

A world in which the WS matter only reacts with gravity may be hard to accept from a philosophical viewpoint. But after we got used to chargeless leptons which only couple to the rest of the world via weak and gravitational interactions, the step to envisage a form of only gravitationally interacting kind of matter is not as weird as it looks at first sight.

The paper is organized as follows.

The next section presents a "crash course" on Wigner’s theory of positive energy representations of the Poincaré group including the explicit construction of sl WS free fields and their two-point functions.

The third section highlights an important restriction on renormalizable couplings involving sl fields which is necessary to avoid higher order total delocalization.

In section 4 it is explained why this perturbative restriction is violated for WS which is the cause of its inertness.

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1 Excluding the trivial possibility of a tensor product of WS with the world of ordinary matter (total inertness except with respect to classical gravity).
Section 5 addresses the problem of the role of sl localization in the construction of the correct energy-momentum tensor for higher spin quantum (i.e. acting in Hilbert space) fields which is the prerequisite for the Einstein-Hilbert gravitational coupling.

The concluding remarks point at problems arising from the identification of WS with dark matter.

2 Matter as we (think we) know it and Wigner’s infinite spin ”stuff”

The possible physical manifestations of WS matter can only be understood in comparison to normal matter. Hence before addressing its peculiarities it is necessary to recall the localization properties of free massive and finite helicity zero mass fields.

It is well known that all pl massive free fields can be described in terms of matrix-valued functions \( u(p) \) which intertwine between the creation/annihilation operators of Wigner particles [2]. Their associated covariant fields are of the form

\[
\psi^{A,B}(x) = \frac{1}{(2\pi)^{3/2}} \int (e^{ipx} u^{A,B}(p) \cdot a^*(p) + e^{-ipx} v^{A,B}(p) \cdot b(p)) \frac{d^3p}{2p_0} \tag{1}
\]

The intertwiners \( u(p) \) and their charge-conjugate counterpart \( v(p) \) are rectangular \((2A+1)(2B+1) \otimes (2s+1)\) matrices which intertwine between the unitary \((2s+1)\)-component Wigner representation and the covariant \((2A+1)(2B+1)\) dimensional spinorial representation labeled by the semi-integer \(A, B\) which characterize the finite dimensional representations of the covering of the Lorentz group \(SL(2, \mathbb{C})\). The \(a^\#(p), b^\#(p)\) refer to the Wigner particle and antiparticle creation/annihilation operators and the dot denotes the scalar product in the \(2s+1\) dimensional spin space.

For a given physical spin \(s\) there are infinitely many spinorial representation indices of the homogeneous Lorentz group; their range is restricted by [2]

\[
|A - B| \leq s \leq A + B, \quad m > 0 \tag{2}
\]

For explanatory simplicity we restrict our subsequent presentation to integer spin \(s\); for half-integer spin there are similar results.

All fields associated with integer spin \(s\) representation can be written in terms of derivatives acting on symmetric tensor potentials \((A = B)\) of degree \(s\) with lowest short distance dimension \(d^s_{sd} = s + 1\). For \(s = 1\) one obtains the divergenceless \((\partial \cdot A^\mu = 0)\) Proca vector potential \(A^\mu_{PROCA}\) with \(d_{sd} = 2\), whereas for \(s = 2\) the result is a divergence- and trace-less symmetric tensor \(g_{\mu\nu}\) with \(d_{sd} = 3\).
Free fields can also be characterized in terms of their two-point functions whose Fourier transformation are tensors in momenta instead of intertwiners. For $s = 1$ on obtains

$$\langle A^P(x)A^P(x') \rangle = \frac{1}{(2\pi)^3} \int e^{-ip(x-x')} M^P_{\mu\nu}(p) \frac{d^3p}{2p^0}, \quad M^P_{\mu\nu}(p) = -g_{\mu\nu} + \frac{p_{\mu}p_{\nu}}{m^2} \quad (3)$$

and for higher spin the $M’s$ are symmetric tensors formed from products of $g_{\mu\nu}$ and of $p’s$ (P stands interchangeably for “Proca” or “point-like”)

For $m = 0$ and finite integer helicity $h$ the two dimensional $\pm |h|$ helicity representation replaces the $2s + 1$ component spin. Despite this difference, the covariant fields turn out to be of the same form (1), except that (2) is now replaced by the more restrictive relation

$$|A - \hat{B}| = |h|, \quad m = 0 \quad (4)$$

which excludes all the previous tensor potentials but preserves their field strengths (which are tensors of degree $|h|$ and $d_{st} = |h| + 1$ with mixed symmetry properties). This is well-known in case of $|h| = 1$ where there exist no massless pl vector potential $A = 1/2 = \hat{B}$ who’s curl is associated to the electromagnetic field strength.

The absence of pl tensor potentials in (1) results from a clash between pl spin s tensor potentials and Hilbert space positivity. Gauge theory substitutes the non-existent pl Hilbert space vector potential by pl potentials in an indefinite Krein space: symmetry unde gauge transformations prescriptions by which one extracts a physical subtheory from a Krein space lead to gauge theory. This is also a clash between the classical Lagrange formalism (positivity has no place in classical physics) and the most basic Hilbert space positivity on which quantum theory’s probability hinges. It shows the limitation of that parallelism to classical theory called Lagrangian quantization.

The problem can be resolved in two ways; either one sacrifices positivity or one gives the Hilbert space a chance to determine the tightest localization which is consistent with positivity, which turns out to be localization on semi-infinite spacelike strings $x + \mathbb{R}_+ e, \quad e^2 = -1$. Beware that there is no relation between sl fields and string theory. Whereas the change from pl to sl fields for $s \geq 1$ is required in order to uphold Hilbert space positivity, ST has no conceptual compass, it is the result of a playful spirit to extend the game of QFT.

The first solution leads to a physically restricted theory in Krein space in which all gauge dependent fields are physically void. The advantage is only computational since pl fields are computationally easier (however this does not apply to the explicit extraction of the physical data with the help of the BRST ghost formalism which remains involved). The perturbation theory of sl fields turns out to be more demanding, but as a reward one obtains a full QFT in

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2Not to be confused with “second quantization” which is an unfortunate terminology for a functorial relation between Wigner’s representation theory of particles and the associated quantum free fields acting in a Wigner-Fock Hilbert space.
which all fields are physical (though, as for pl $s < 1$ interactions in Hilbert space, not all operators represent local observables).

sI tensor potentials also exist for massive fields. The sI counterpart of the pl massive two-point functions $(3)$ turn out to be

$$M^s_{\mu\nu}(p; e, e') = -g_{\mu\nu} - \frac{p_{\mu}p_{\nu}e \cdot e'}{(p \cdot e - i\varepsilon)(p \cdot e' + i\varepsilon)} + \frac{p_{\mu}e_{\nu}}{p \cdot e - i\varepsilon} + \frac{p_{\nu}e'_{\mu}}{p \cdot e' + i\varepsilon}$$

(5)

Its massless limit is of the same form, except that the momentum $p$ is on the boundary of the positive lightlike surface $H^+_0$ of the forward light cone $H^+_m$. This has to be taken into account in the Fourier transformation to $x$-space.

The more complicated form as compared to the simpler $-g_{\mu\nu}$ in the Feynman gauge setting is the prize to pay for improving the high energy behavior while preserving positivity and securing the existence of a massless limit. The only way I know which secures positivity is the intertwiner representation of covariant fields as linear combinations of Wigner creation/annihilation operators. Lagrangian quantization account for positivity only for $s < 1$ interactions; in all higher spin cases it leads to indefinite metric which only permits a partial return to positivity in case of a gauge formalism in Krein spaces. I am not aware that physical (Hilbert space) $s > 1$ energy-momentum tensors have been constructed in the existing literature.

The best description of the interacting massless theory is to first calculate the renormalized massive correlation functions and then take their massless limit. This has the advantage of performing perturbation theory in the simple Wigner Fock particle space and leaving the reconstruction of the massless limit (in which this physical description of the Hilbert space in terms of particle states is lost) to the application of Wightman’s reconstruction theorem to the limiting correlation functions.

Massive theories are simpler from a conceptual viewpoint because the presence of a mass gap permit to use the tools of scattering theory and the identification of the Hilbert space with a Wigner Fock space. Whereas it is plausible that the asymptotic short distance behavior of the Hilbert space setting is correctly accounted for in terms of the asymptotic freedom properties of gauge theories, the problems related to long-distance properties as confinement remain outside the physical range of the gauge setting.

There is a very efficient way to derive the relation between the pl Proca potential and its sI counterpart. Integrating the latter along the space-like direction $e$, one obtains a sI scalar field $\phi(x, e)$

$$\phi(x, e) := \int_0^\infty d\lambda e^\mu A^\mu_\lambda(x + \lambda e) = \frac{1}{(2\pi)^{3/2}} \int (e^\nu p x u(p, e) \cdot a(p) + h.c.) \frac{d^3 p}{2p_0}$$

(6)

$$u(p, e) := u(p) \cdot e \frac{1}{ip \cdot e} \quad M^{s, \phi} = \frac{1}{m^2} - \frac{e \cdot e'}{(p \cdot e - i\varepsilon)(p \cdot e' + i\varepsilon)}$$

(7)

where the inner product in the first line refers to the 3-dim. spin space and the denominator is simply the Fourier transform of the Heavyside function. The $\phi$
two-point function can either be computed from carrying out the line integrals on the Proca two-point function or by using the intertwiner. The sl vector is defined as

\[
A_\mu(x, e) := \int_0^\infty e^\nu F_{\mu\nu}(x + \lambda e)ds, \quad F_{\mu\nu} := \partial_\mu A_\nu^P - \partial_\nu A_\mu^P
\]

which leads to the two-point function (5).

The three fields turn out to be linearly related

\[
A_\mu(x, e) = A_\mu^P(x) + \partial_\mu \phi(x, e)
\]

which either can be derived from the previous definition or by defining the three fields in terms of their intertwiners in which case (9) is a numerical relation between the numerical intertwiner functions. A similar looking relation in which the sl vector potential is replaced by the Gupta-Bleuler pl gauge potential and the \( \phi \) by the negative metric Stückelberg field whose two-point function is the negative of a scalar pl field [12]. In the presence of interactions one needs additional ghost degrees of freedom. In a somewhat metaphoric terminology one may say that the positivity preserving sl field theory (SLFT) is the result of applying Okhams razor to Gauge Theory (GT).

We will refer to fields which mediate between pl potentials and their sl counterparts as escorts. When passing from positivity preserving \( m = 0 \) potentials to massive Proca potentials it is not enough to "turn on the mass" but one also needs the intervention of the escorts. They play exactly the role which is erroneously attributed to the Higgs field ("fattening the photons").

By changing the \( \lambda \)-measure \( d\lambda \to \kappa(\lambda)d\lambda \) one can improve the short distance behavior and get arbitrarily close to \( d_{sd} = 0 \), but this will be of no avail since this would destroy the linear relation (9) which permits to use the lower \( d_{sd} \) to achieve renormalizability (the \( L, V_\mu \) condition in the next section).

The relation (9) looks like a gauge transformation; indeed the extension to interactions with matter fields suggests a formal connection between pl \( \psi(x) \) and its sl counterpart which has the expected exponential form \( \psi(x, e) = \psi(x) \exp ig\phi(x, e) \)

But in contrast to gauge theory these relations intertwine between sl fields and their more singular pl siblings within the same sl relative localization class; in fact this formula, after making it precise in terms of normal products, may be seen as the definition of an \( e \)-independent \( d_{sd} = \infty \) singular pointlike counterpart of a polynomially bounded sl field. The singular nature of the pl "field coordinatization" leads to the typical with perturbative order increasing number of counter-term parameters whereas in the sl coordinatization the number of parameters remains finite, just as in renormalizable pl \( s < 1 \) interactions.

This construction can be extended to all integer spin fields [13] (and with appropriate modifications also to Fermi-fields). The divergence-free Proca potential is replaced by divergence- and trace-free symmetric tensor potentials

\footnote{In particular the lowering of \( d_{sd} \) is of no help for \( s < 1 \) fields. There is no Elko trick which improves the short distance properties of \( s = 1/2 \) fields. The proposal in (21), formula (1)) shows a total misunderstanding of what QFT is about.}
\(A_{\mu_1, \ldots, \mu_s}\) of tensor rank \(s\). Iterated integration along a space-like direction \(\nu\) leads to \(s\) sl \(\phi\) tensor fields of lower rank

\[
\phi_{\mu_1 \ldots \mu_s}(x, e) = \int d\lambda_1 \ldots d\lambda_{s-k} e^{\nu_1 \ldots \nu_{s-k}} A_{\nu_1 \ldots \nu_{s-k}, \mu_1 \ldots \mu_k}(x + \lambda_1 e + \ldots \lambda_{s-k} e)
\]

(10)

Again one can construct the \(\nu\)-dependent intertwiners from these relations.

The extension of (9) spin \(s\) relates the pl tensor-potential \(A^P\) to its sl counterpart and the symmetrized contributions from the derivatives of the sl tensor escorts \(\phi\ldots(x, e)\) of \(A\ldots(x, e)\).

\[
A_{\mu_1 \ldots \mu_s}(x, e) = A^P_{\mu_1 \ldots \mu_s}(x) + \text{sym} \sum_{k=1}^s \partial_{\mu_1} \ldots \partial_{\mu_k} \phi_{\mu_k+1 \ldots \mu_s}
\]

(11)

\[
g_{\mu\nu}(x, e) = g^P_{\mu\nu}(x) + \text{sym} \partial_{\mu} \phi_{\nu} + \partial_{\nu} \phi
\]

(12)

where the second line is the special case of the connection between the trace- and divergence-less \(s = 2\) pl symmetric tensor and its sl counterpart including the two sl \(s < 2\) escorts \(\phi_{\mu}\) and \(\phi\).

The appearance of these lower spin \(\phi\)-escorts is characteristic for the change of massive pl fields into their sl siblings acting in a Hilbert space. They are important new fields which depend on the same degrees of freedom (the Wigner creation/annihilation operators) as the other two operators.

For \(s = 1\) the scalar escort field \(\phi\) may be seen as the QFT analog of the bosonic Cooper pairs which are the result of a reorganization of the condensed matter degrees of freedom in the superconducting phase. Without the formation of Cooper pairs from existing condensed matter degrees of freedom it is not possible to convert the long-range classical vector potentials into its short range counterparts within the superconductor (as anticipated by London).

The relation between long range massless and short range pl Proca potentials requires the presence of the \(\phi\); in fact it is not possible to formulate massive QED as a renormalizable theory in Hilbert space without the presence of these scalar escorts, and where there is need of additional \(H\) fields, as for massive self-interacting vector mesons, it is for entirely different physical reasons than spontaneous symmetry breaking.

The reason why additional degrees of freedom in the form of \(H\)-fields are indispensable in the case of self-interacting massive vector mesons is quite deep but bears no relation to physical spontaneous symmetry breaking.

The lower spin escort fields in (11) have no massless limit, but together with the Proca tensor potentials their presence is necessary for the construction of the massive sl potential; all these fields are relatively local and act in the same Wigner-Fock Hilbert space. Only the correlation functions of the degree \(s\) sl tensor potential possess a massless limit.

The sl \(A\ldots\) in (11) is related to the pl field strength

\[
\mathcal{F}_{\mu_1 \ldots \mu_s, \nu_1 \ldots \nu_s} = g_{\mu
u} \{\partial_{\mu_1} \ldots \partial_{\mu_s} A_{\nu_1 \ldots \nu_s}\}
\]

(13)
where the $as$ imposes antisymmetry between the $\mu - \nu$ pairs. The antisymmetrization effects the $d_{sd}$ of the pl tensor potential is the same as that of field strengths namely $d_{sd} = s + 1$. As in the previous case of the vector potential $[3]$, the sl tensor potentials $[10]$ can be obtained in terms of iterated integrations along $e$ starting from the field strength. The field strength tensor is the lowest rank pl field which has a pl massless limit$^4$. With appropriate changes these results have analogs for semi-integral spin.

Before passing to the sl fields of the WS class it may be helpful to collect those properties which turn out to be important for higher spin sl fields.

- Whereas pl massive tensor potentials have short distance dimension $d_{sd} = s + 1$, their sl counterparts have $d_{sd} = 1$ independent of spin. Hence there are always first order sl interaction densities within the power-counting limit $d_{int}^{sd} \leq 4$, but whether they can be used in a consistent perturbative renormalization setting is another story.

- Sl tensor potentials are smooth $m \to 0$ limits of their sl massive counterpart; they inherit in particular the $d_{sd} = 1$ from their massive counterpart. The lowest pl fields in the same representation class are field strengths (tensors of rank $2s$ and $d_{sd} = s + 1$ with mixed symmetry properties).

- The pl $d_{K}^{K} = 1$ zero mass vector potentials $A_{K}^{K}$ of local gauge theory act in an indefinite metric Krein space. The physical price for resolving the clash between point-like localization and Hilbert space positivity is the is the loss of both the positivity and the correct physical localization whose validity is restricted to gauge invariant observables. What makes gauge theory useful for particle theory (the Standard Model) is the fact that the perturbative unitary on-shell $S$-operator is gauge invariant. The absence of local observables excludes the use of gauge theory in WS models.

Whereas the two-point functions of pl massive free fields are polynomial in $p$, their sl counterparts have a rational $p$-dependence $[4] [9]$. The family of all WS intertwiners for a given Pauli-Lubanski invariant $\kappa$ has been computed in $[8]$, their two point-functions are transcendental functions of $p, e$ which are boundary values of from $Im(e) \in V^+$.

A particular simple WS intertwiner with optimal small and large momentum space behavior (corresponding to the minimal choice for sl massless $s \geq 1$) has been given in terms of an exponential function in $[10]$

$$u(p, e)(k) = \exp\frac{\vec{k}(\vec{e} - \frac{p - \vec{p}}{e} \vec{p}) - k}{p \cdot e}$$

(14)

here $k$ is a two-component vector of length $\kappa$; the Hilbert space on which Wigner’s little group $E(2)$ acts consists of square integrable functions $L^2(k, d\mu(k) = \delta(k^2 - \kappa^2)dk)$ on a circle of radius $\kappa$. The vector arrow on $e$ and $p$ refer to the

$^4$ For $s = 2$ this tensor has the same mixed symmetry property as the linearized Riemann tensor whereas the symmetric second rank tensor deserves to be denoted as $g_{\mu\nu}$. 

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projection into the 1-2 plane, and the and $e_-, p_-$ refer to the difference between the third and zeroth component. The most general solution of the intertwiner relation differs from this special one by a function $F(p\cdot e)$ which is the boundary value of a function which is analytic in the upper half-plane $\mathbb{R}$. 

The two-point function is clearly a $J_0$ Bessel function. The calculation in [10] was done in a special system. Writing its argument in a covariant form one obtains:

$$
M^{WS}(p, e) \sim J_0(\kappa|w(p, e)|) \exp -ik\left(\frac{1}{p\cdot e - i\varepsilon} - \frac{1}{p\cdot e' + i\varepsilon}\right) \quad (15)
$$

with

$$
w^2(p, e) = -\left(\frac{e}{e\cdot p - i\varepsilon} - \frac{e'}{e'\cdot p + i\varepsilon}\right)^2
$$

The exponential factor compensates the singularity of $J_0$ at $e \cdot p = 0$. Note that the Pauli-Lubanski invariant $\kappa$ has the dimension of a mass so that the argument of the two-point function of the $\text{sl}$ field has the correct engineering dimension $d_{en} = 1$ of a quantum field.

The main purpose of this calculation is to convince the reader that there are explicitly known transcendental $WS$ intertwiner and two-point functions whose associated propagators have a well behaved ultraviolet and infrared behavior. As already mentioned, the physical reason why these fields are nevertheless excluded from appearing in interaction densities is that higher orders lead to a complete delocalization; this will be explained in the next section.

3 The problem of maintaining higher order sl localization

It is well known that the only restriction for pl interaction densities is the power-counting inequality $d_{int}^{pl} \leq 4$. Since the minimal short distance dimension of pl spin $s$ fields is $s + 1$, there are no pl renormalizable interactions involving $s \geq 1$ fields. Sl free fields on the other hand have an $s$-independent short distance behavior $d_{sd} = 1$, so that one always can find polynomials of maximal degree 4 which represent interaction densities within the power-counting limitation.

Point- and sl fields represent two different descriptions of the same spin $s$ quantum matter, just like two different coordinatization in differential geometry. For $s \geq 1$ the use of the pl coordinatization in interaction densities becomes too singular; the breakdown of the power-counting bound $d_{int}^{pl} \leq 4$ leads to singular interacting fields (unbounded increase of $d_{sd}$ with perturbative order). The bad aspect of such a singular (non Wightman) behavior is not primarily the polynomial unboundedness in momentum space, but rather the fact that the perturbative counterterm formalism leads to an ever increasing number of undetermined counterterm parameters which destroys the predictive power. 

5I am indebted to Henning Rehren for showing me the covariantization of Köhler’s result.

6Fields (without the added specification “gauge”) are always acting in Hilbert space.
There are two explanations for the cause of this situation, either the interaction density is incompatible with the principles of QFT or the model is consistent but the pl coordinatization is too singular for the application of the rules of renormalized perturbation theory. In the latter case the use of sl field coordinatization may lead to reduction of the short distance singularity within \( d_{sd}^{int} \leq 4 \) and in this way save the model. If this fails one falls back to square one, this time without remedy.

Fields which even in their sl coordinatization do not lead to renormalizable couplings cannot be used for defining interaction densities of renormalizable model will be referred to as inert. The main claim of the present work is that WS fields fall into this category whereas for higher finite spin fields \( s \geq 2 \) the question whether they are ”reactive” or remain inert remains unsettled (see next section).

Formally renormalizable sl interaction densities within the power-counting bound come with a physical hitch. Unless they fulfill an additional requirement it is not possible to maintain the sl localization in higher orders. In that case the result will be a complete delocalization and hence the principles of QFT exclude such interactions. In the next section it will be shown that the presence of WS fields in a interaction density leads to such a situation.

On the other hand free WS fields fulfill all general localization- and stability-requirements (energy-positivity) of QFT and consequently cannot be excluded as being unphysical [11]. This justifies the terminology ”inert matter” used in the next section. The remainder of this section will address the problem of upholding sl localization in higher orders which will be taken as the defining property of ”reactive (or dynamic) quantum matter”.

As a simple nontrivial illustration of this additional delocalization preventing requirement we start with a sl interaction density \( L \) of massive QED

\[
L = A_\mu(x,e)j^\mu(x)
\]

Here \( A_\mu(x,e) \) is a massive sl vector potential \( \mathbb{S} \) and \( j^\mu \) is the conserved current of a massive complex scalar field. These fields act in a Wigner-Fock Hilbert space of the corresponding Wigner particles, and since \( d_{sd}(A_\mu) = 1 \) and hence the short distance dimension \( d_{sd}^{int}(L) = 4 \), \( L \) in (16) stays within the power-counting bound \( d_{sd}^{int} = 4 \) of renormalizability. The sl \( L \) is related to its \( d_{sd} = 5 \) pl counterpart \( L^P \) as

\[
L^P = A^P \cdot j = L - \partial^\mu V_\mu, \quad V_\mu(x,e) := \partial_\mu \phi j^\mu \\
\int L^P d^4x = \int L d^4x, \quad \text{i.e.} \quad S^{(1)} = S^{(1)}_P = S^{(1)}_S
\]

where the second line follows since in the presence of a mass gap the divergence of \( V_\mu \) does not contribute to the adiabatic limit which represents the first order S-matrix. In other words one splits the \( d_{sd} = 5 \) pl density into its sl \( d_{sd} = 4 \) counterpart and a \( d_{sd} = 5 \) divergence term which can be disposed of in the adiabatic (on-shell) S-matrix limit.
In this way one solves two problems in one stroke, on the one hand one expresses the (first order) S-matrix in terms of a $d_{sd} = 4$ interaction density, and at the same time the $e$-dependence disappears in the first order on-shell S-matrix. The linear relation between $L$ and its pl counterpart $L^\mu$ is a consequence of the linear relation between the massive pl Proca potential and its sl $d_{sd} = 1$ counterpart (9). The lowering to $d_{sd} < 1$ by using instead of $d\lambda$ another measure $\mu(\lambda)d\lambda$ would be possible, but this would destroy the linear relation (17) and as a result also the relation (18) which is the basis of the $e$-independence of $S^7$.

For the following it is convenient to formulate the $e$-independence in terms of a differential calculus on the $d = 1+2$ dimensional directional de Sitter space. The differential form of the relation (9) reads

$$d_a A_\mu = \partial_\mu u, \quad u = d_e \phi$$

(19)

$$d_e (L - \partial^\mu V_\mu) = d_e L - \partial^\mu Q_\mu = 0, \quad Q_\mu := d_e V_\mu$$

(20)

Hence $A, \phi, L, V$ are $d_{sd} = 1$ zero-forms whereas $u, Q$ are exact $d_{sd} = 1$ one-forms; together with the exact two-form $\hat u$ (10) they exhaust the linear with $A^\mu$ linear related relatively local $d_{sd} = 1$ forms.

We will refer to the relation (20), which expresses the $e$-independence in terms of a closed zero form, as the "$L, V$ (or $L, Q$) relation" (20). It is a necessary condition for the $e$-independence of $S$. Its extension to vacuum expectation values of fields requires that they depend only on those $e'$s of the fields and not on the $e'$s of inner propagators in Feynman diagrams which contribute to these correlations.

As the independence of $S$ from gauge-fixing parameters in gauge theory, this $e$-independence in the Hilbert space setting results from cancellations between different contributions in the same order; but different from unphysical gauge dependent correlation functions, correlations of charge-carrying sl fields are expectation values of physical fields in an extended Wightman setting (endpoint $x$- and directional $e$-smearing).

In order to secure the $e$ independence in higher orders we must extend the $L, Q_\mu$ relation (20) to higher order time-ordered products. The second order $L, Q_\mu$ pair requirement reads

$$(d_e + d_{e'}) TL'L' - \partial^\mu TQ_\mu L' - \partial^\mu' T' LQ'_\mu = 0$$

(21)

If it were not for the distributional singularities of $T$-products at coalescent points, this would follow from (20). For the second order S-matrix we only need the one particle contraction component ("tree approximation").

---

1Recent suggestions [21] that the mere lowering of $d_{sd} = 3/2$ to 1 for $s = 1/2$ (probably to lower the power-counting bound of the 4-Fermi interaction) reveal a misunderstanding of QFT.

2The $Q_\mu$ formalism is somewhat simpler than its $V_\mu$ counterpart. For massive QED and couplings of massive vector mesons to Hermitian matter ("Hermitian QED", the Higgs abelian model) it is easy to see their equivalence.
For massive spinor QED the relation is fulfilled in term of the standard free field propagator. The more singular scalar QED contains $d_{sd} = 2$ derivatives $\partial \varphi$ which according to the minimal scaling rules of the divergence and regularization free Epstein-Glaser renormalization theory lead to a delta counterterm

$$\langle T \partial_\mu \varphi^\ast \partial'_\nu \varphi' \rangle = \partial_\mu \partial'_\nu \langle T \varphi^\ast \varphi' \rangle + c g_{\mu\nu} \delta(x - x')$$  \hspace{1cm} (22)

The imposition of the relation (21) fixes the parameter $c$ with the expected result of an induced second order term $g^2 g \delta(x - x') A_\mu A'^\mu \varphi^\ast \varphi$.

Note that no arguments of classical gauge theory (as the replacement $\partial \rightarrow D = \partial + ig A$) has been used; the result is solely a consequence of the causal localization principles and Hilbert space positivity.

There are some interesting foundational aspects of this otherwise trivial calculation. The independent fluctuation in $e$ and $e'$ do not allow to set $e = e'$ in off-shell correlations (5): the different $i \varepsilon$ prescriptions for $e$ and $e'$ in the off-shell propagator prevent this; apart from the fact that unlike the dependence in $x$ the two-point function in $e$ does not depend on the difference $e - e'$ the less singular behavior at coinciding direction is similar to those at coalescent points. The remedy in both cases is to use Wick-products.

The on-shell $e$-independence corresponds to the second order gauge invariance of the scattering amplitude; individual contributions are generally $e$-dependent and upon setting $e = e'$ lead to infinite fluctuations.

The "magic" of $L, V_\mu$ pairs with (20) is that on the one hand they permit to use the lower short distance dimension of sl fields (and in this way lower the power-counting bound of renormalizability) and on the other hand they also guaranty the $e$-independence of the S-matrix since the derivative contributions in (21) disappear in the adiabatic S-matrix limit

$$(d_e + d_{e'}) S^{(2)} = 0, \quad S^{(2)} \sim \int TLL'$$  \hspace{1cm} (23)

The extension of the adiabatic limit of the Bogoliubov $S(g)$ operator functionals to quantum fields leads to correlation functions of interacting sl fields. As the S-matrix is independent of the $e'$s of the inner propagators (after summing over sufficiently many contributions in a fixed perturbative order), the correlation functions of interacting fields only depend on the $e'$s of those fields.

A new phenomenon is that the higher order interactions spread the $e$-dependence also to those fields which entered the first order interaction density as $s < 0$ pl fields\[9.\] In fact the interacting matter fields in the new sl setting of renormalization theory are sl in a stronger sense than the vector potentials which remain linearly related with their pl field strengths.

In the limit of massless sl vector mesons the correlation functions change their physical properties define a very different theory; the particle setting in a Wigner-Fock Hilbert space disappears and the strings of the charge-carrying

\[9\]The perturbation theory of interacting string-local fields is still in its beginnings. A mathematically rigorous presentation will be the subject of forthcoming work by Jens Mund.
fields become more "stiff" and cause a spontaneously breaking of Lorentz invariance in charged sectors. Little is known about the spacetime aspects of these physical changes (particles→infraparticles) apart from prescription in momentum space for photon-inclusive cross sections.

The $L, V_\mu$ (or $L, Q_\mu$) pair property is a necessary condition for maintaining sl localization; it permits to sail between Scilla of nonrenormalizability and the Charybdis of total delocalization. Heuristically speaking it provides a compensatory mechanism between contributions to the same order which prevents the total delocalization resulting from the integration over $x$ in inner strings $x + R + e$ in individual Feynman diagrams. The main point of the present work is the argument that in the presence of WS fields in $L$ it is not possible to fulfill the $L, V_\mu$ condition so that WS matter can only exist in the interaction-free form. We will refer to such matter as inert (next section).

There is another important physical aspect of the $L, V_\mu$ pair property in which the escort field $\phi$ plays an essential physical role even though it does not add new degrees of freedom. Heuristically speaking the transition from long range massless sl vector potentials to their short range massive counterpart is not possible without the appearance of the $\phi$ escort. The escort appears explicitly in the pair condition; in some models they already appear in the first order interaction density (see below).

This is somewhat reminiscent of the presence of the bosonic Cooper pairs in the BCS description of superconductivity; without their presence it is not possible to convert long range classical vector potentials into their short ranged counterparts inside the superconductor. As the $\phi$ in massive QED they are not the result of additional degrees of freedom, they rather arise from rearrangements of existing condensed matter degrees of freedom in the low temperature phase.

The QFT analog of the BCS or the Anderson screening mechanism is the screened "Maxwell charge" i.e.

$$j_\mu := \partial^\nu F_{\nu\mu}, \quad Q_{scr} = \int j_0(x) d^3x = 0$$

$$\partial^\mu j_\mu = 0, \quad Q_{SSB} = \int j_0(x) d^3x = \infty, \text{ long dist. divergence}$$

The screening property (first line) only depends on the massive field strength and not on the kind of matter to which it couples (which may be complex or Hermitian matter). This includes non-interacting massive vector mesons for which $j_\mu \sim A_\mu^P$. Spontaneous symmetry breaking on the other hand reveals itself in form of a conserved current whose charge diverge instead of being zero (second line).

Renormalizable models are generally uniquely specified in terms of their field content. In the above case of massive scalar QED the form of the first order sl coupling is uniquely fixed by the $L, Q_\mu$ pair condition (the preservation of sl localization). The second order pair condition is a normalization requirement which induces the $A \cdot A |\phi|^2$ term.
This induction which in classical QED results from the fibre-bundle structure $\partial \mu \rightarrow D \mu = \partial \mu - ig A_\mu$ is in (positivity preserving) QFT a structural consequence of the causal localization principle. Gauge theory hides this important fact by preserving the classical fibre bundle interpretation at the price of indefinite metric (which does not only violate the probability interpretation but also denaturalizes the physical localization of QFT). The role of gauge symmetry and gauge invariance is to recover part of the lost physical properties for a more restricted gauge invariant subtheory. The induction of the quadratic $A \cdot A |\varphi|^2$ is a result of the imposed invariance under "gauge symmetry" which (apart from the fact that this is not a physical symmetry) is is equivalent to the classical fibre bundle requirement.

The new SLFT setting retains all physical properties by replacing the mute global gauge fixing parameters by individually fluctuating local space-like string directions. In this way all the unphysical "dead wood" of indefinite metric Stückelberg fields and ghosts will not be allowed to enter in the first place. The idea that quantum fields should follow the pl localization of classical field theory is too restrictive for constructing interacting higher spin quantum fields.

The sl localization preserving $L, V_\mu$ induction becomes much richer if the interaction of $A_\mu$ with complex matter is replaced by Hermitian matter (the abelian Higgs model). The application of the $L, V$ requirement to the coupling of a massive vector meson to a Hermitian $H$ field proceeds as follows. The pl interaction with the lowest short distance dimension $d_{\text{P, int}} = 5$ is $L = mA P \cdot A P H$. Converting it into a sl $L, V_\mu$ pair, one obtains (easy to check by the use of the free Klein-Gordon equation for $H$ and relation (9)):

$$L = m \left\{ A \cdot (AH + \phi \partial H) - \frac{m^2_H}{2} \phi^2 H \right\}, \quad V_\mu = m \left\{ A_\mu \phi H + \frac{1}{2} \phi^2 \partial_\mu H \right\}$$

(26)

$$L - \partial V = L = mA P \cdot A P H, \quad d_\epsilon (L - \partial V) = 0$$

In this case the on-shell e-independence requirement (21) in second and third order tree approximation leads to a much richer collection of induced terms than that of scalar massive QED [17] (for a gauge-theoretic derivation of the induction see [25] section 4.1).

Whereas in the massive scalar QED model this requirement induces only the $A \cdot A \varphi^* \varphi$ term, the induction in case of an interaction with a Hermitian field leads besides the expected $A \cdot AH^2, A \cdot A\varphi^2$ terms (which as in scalar QED can be absorbed into a changed time-ordered product) also to second order induced $H^4, \varphi^4, H^2 \varphi^2$ terms (from second order $A-A$ contractions in (21) as well as to an additional first order $H^3$ term [17] [18]. The coupling strengths of these second order induced terms are fixed in terms of the 3 physical parameters of the elementary model-defining $A_\mu, H$ fields, namely the coupling strength and (ratios of) the two masses $m, m_H$.

\[^{10}\]In higher ($4^{th}$) order one also expects the appearance of new counterterm parameters as known from point-local interactions.
The result corresponds to the terms induced by gauge invariance of the S-matrix in \[25\]. It is also the same as that of the formal calculation based on the SSB Higgs mechanism, except that in that case one postulates a Mexican hat potential instead of inducing it from gauge invariance or from causal localization in a positivity preserving sl setting. As soon as vector potentials enter one has to follow the rules of either GT or SLFT.

QFT is a foundational quantum theory in which all physical properties of a model are intrinsic; for the physical interpretation of its content one is not forced to rely on prescriptions. In many cases it is the field content alone which determines the form of the interaction density. For the case at hand the \(A,H\) field content and the \(L,Q_\mu\) renormalization requirement fix the first and second order interaction density including the \(H\) self-interaction which is erroneously attributed to a SSB.

The shift in field space on a Mexican hat potential is a useful trick whenever the model permits a SSB i.e. whenever a conserved current leads to a (long distance) divergent charge \(Q_{ssb} = \infty\) (the definition of SSB). This is the case as long as the Mexican hat potential is not coupled to a vector potential (or any other \(s \geq 1\) potential). The \(A_\mu\) coupling changes this since the only conserved current of interactions of abelian massive vector mesons with any matter (massive QED, \(H\)-matter) is the identically conserved Maxwell current of a massive \(F_{\mu \nu}\) field which always leads to a screened charge \(Q = 0\). In other words the "Mexican hat + shift in field space trick" leads to a SSB \(L\) only if the field content permits a SSB (\(\partial j = 0, Q = \infty\)). In that case it is useful to determine an \(L_{SSB}\) from a \(L_{SYM}\) (maintaining the symmetry in the quadrilinear terms while causing a current conservation preserving change in the lower degree contributions).

To refer to classical gauge symmetry as a "local symmetry" makes perfect physical sense which is however lost in QFT where gauge invariance is a formal device to extract a physical subtheory (local observables, S-matrix) from an unphysical indefinite metric setting. The full QFT in which all fields are physical can be obtained by fighting the increase \(d_{sd} = s + 1\) of short distance singularities by using the only physical resource of lowering \(d_{sd}\) namely passing from pl Wigner fields. The correct sl fields are those which lead to the \(L,Q_\mu\) renormalization theory. Neither the gauge prescription nor the SLFT setting offer a physical arena for SSB.

One reason why in GT the BRST invariance of the on-shell S-matrix which leads to the correct induced \(H\) self-interactions is easily confused with the off-shell Mexican hat prescription is that in functional Feynman graph representation it is difficult to distinguish between relations which only hold on-shell from off-shell relations. For this reason it was important to use the causal gauge invariant (CGI) operator (Epstein-Glaser) formulation of the BRST formalism \[25, 26\].

There is no problem to extend the construction of the \(L,Q_\mu\) pair to self-interacting massive vector mesons \(A_\mu\) and calculate the second order induced terms. \textit{One finds that there is an uncompensated \(d_{sd} = 5\) induced term}. Such a nonrenormalizable second order contribution is deadly if there would be no
possibility to extend the field content of the model in such a way that the interaction of $A_\mu$ with the new field leads to a compensating second order $d_{sd} = 5$ contribution. The new field should have a lower spin (in order not to worsen the short distance situation) and the same Hermiticity property as $A_\mu$ i.e. it must be a $H$-field.

The compensation against another second order induced term from a first order $AAH$ interaction works and converts the extended model into a renormalizable sl QFT [17] [18] (or [25] in gauge theoretical setting in Krein space). It attributes a fundamental role to the $H$ coupling which is consistent with the principles of QFT. This compensating field is the Higgs field and the compensation is its raison d’être.

This higher order compensation is a new phenomenon of $s \geq 1$ sl interactions which has no analog in $s < 1$ pl interactions. Both the $L,Q_\mu$ pair condition as well as the higher order compensation mechanism are the prerequisites for the concepts of reactive and inert $s > 1$ matter in the next section.

4 Reactive and inert fields for $s \geq 1$

There are good reasons to believe that problems of lack of convergence of its power series in coupling parameters are related to the singular nature of the quantum fields (which are the objects which one expands). This view is supported by recent existence proofs for two-dimensional integrable models. These proofs are based on top-to-bottom constructions i.e. they start from on-shell objects as the S-matrix and pass to operator algebras with the help of modular localization [16] and avoid the use of objects as fields (whose singular behavior in the presence of interactions is caused by vacuum polarization clouds which result from their application to the vacuum).

Leaving the problem of the possible cause of divergence of perturbative series aside we will follow the standard parlance of particle theory and identify the existence and physical properties of a model with those of its renormalized perturbation theory.

The fact that positivity-preserving pl renormalizable models exist only for $s < 1$ interactions and that the use of $s \geq 1$ fields in renormalizable interaction densities requires to use them in their sl form leads however to peculiarities which play an important role in the division into reactive and inert fields. Here inert fields are fields which do not admit any renormalizable couplings with themselves nor with other fields. They only exist as free fields and since particle counters can only register particles which interact they remain invisible.

Their ”darkness” does however not impede their coupling to gravity and their ability to make their presence felt in the form of gravitational back-reactions through Einstein-Hilbert couplings of the energy momentum tensor. The claim that all positive energy matter possesses an energy-momentum tensor will be addressed in the next section.

\[\text{Note that } d_{sd} = 5 \text{ contributions which have to be compensated do not occur in SSB models.}\]
For the division into reactive and inert matter the sl localization preserving $L,Q_{\mu}$ property and the higher order renormalizability-preserving $d_{sd} \leq 5$ compensation mechanism are indispensable. Without the existence of the compensatory $H$ field, massive vector mesons could not be "self-reactive". It is immediately clear that $s < 1$ fields are reactive since they admit renormalizable pl interactions.

It is not possible to decide whether a $s \geq 1$ field is reactive or inert without looking into the details of sl renormalization theory. The $L,Q_{\mu}$ pair condition with $d_{sd}(L) \leq 4$ encodes $s$-dependent renormalizability violating short distance contributions into the $Q_{\mu}$ which becomes disposed in the adiabatic limit. From second and higher order tree approximations one learns that there are induced terms which lead to a modification of $L$. For $s > 1$ these induced terms may lead to second order modification of the original $L$ (see remarks after (26)).

In case the $d_{sd}$ of these induced terms is larger than 4 on has to look for compensating extensions by enlarging the model’s field content. Since the coupling to spin $\geq s$ fields worsens the renormalizability properties only additional couplings with fields of lower spin can save the model.

**Definition 1** A massive spin $s \geq 1$ field is called reactive if it possesses sl renormalizable interactions with lower spin fields. Fields which are not reactive will be referred to as inert.

We have seen that $s = 1$ fields are not only reactive with respect to $s < 1$ fields but they are also self-reactive provided that their self-interaction is accompanied by short distance compensating $A-H$ interactions. On the other hand WS matter is inert since massless sl WS fields cannot be obtained as limits of massive fields. Whereas for each finite $s > 1$ the problem of whether the pair condition obeying couplings to lower spin fields are possible remains open, it is easy to see that the pair condition cannot be fulfilled in the presence of WS matter since the $d_e$ its sl field $\Psi^{WS}(x,e)$ (15) is not of the form of a divergence $\partial^\mu \phi^{WS}_\mu$ which is the prerequisite for the existence of a $L,Q_{\mu}$ pair.

The difficulties of finding reactive spin $s$ fields increase with increasing spin. The most likely scenario is that there exists a $s_{max}$ above which all fields are inert. Without further calculations this $s_{max}$ may be any half-integer or integer between 1 and $\infty$, only future calculations can resolve this problem.

The new SLFT setting of models involving $s \geq 1$ fields leads also to new problems which affect our understanding of symmetries. The standard view is that inner symmetries encode the existence of superselection sectors of observables [20]. This works in both ways; if one starts from a model with an inner symmetry one may construct its observable algebra as the fixed point algebra under the action of the symmetry; vice versa one may recover the field algebra from the smaller observable algebra by looking for all local equivalence classes of its localizable representations.

With any such QFT with inner symmetries one may associate another one with the same type of fields but changed couplings between them and possibly different masses. In this way a symmetric QFT can be associated with models
with the same field content but broken symmetries. This conceptual situation changes in the presence of \( s \geq 1 \). In the positivity maintaining SLFT setting the presence of a second order induced \( A \cdot A|\varphi|^2 \) term in scalar QED is the result of the causal localization principle of QFT and does not require any gauge theoretical \( \partial \rightarrow D = \partial - iA \partial \). Similarly the Lie structure of the \( f_{abc} \) in \( L = f_{abc}F_{\mu \nu}A_{\mu}^b A_{\nu}^c + \ldots \) is a consequence of the second order pair condition which in turn results from the implementation of causal localization in a positivity-maintaining Hilbert space setting.

Whereas inner symmetries can be (spontaneous or explicitly) broken, a QFT of self-interaction vector mesons without the Lie structure of its couplings would contradict the causal localization principles. One can also paraphrase this situation by saying that there exists a quantum fibre bundle-like structure which is totally intrinsic (i.e. not the consequence of quantizing a classical structure).

This remains somewhat hidden in GT where the BRST gauge symmetry formalism results from a quantum adjustment of the fibre bundle properties of classical gauge theory (Stora). Gauge theory accounts for gauge invariant local observables and the S-matrix, but it misses to explain the properties of interacting \( s \geq 1 \) QFT as consequences of the causal localization principles. A "gauge principle" bears no relation to the foundational concepts of QFT, one only needs it in order to extract a physical subtheory from a description which violates quantum theory’s positivity property (which insures its probability interpretation).

In the next section we will address the important problem of higher spin energy-momentum tensors. Even if fields turn out to be inert, they still interact with classical gravity and modify the gravitational field through Einstein-Hilbert back-reactions.

### 5 The problem of the \( s \geq 2 \) E-M tensor, coupling to gravity

The classical energy-momentum tensor \( T_{\mu \nu}^{cl} \) is a trace- and divergence-less quadratic expression in terms of classical fields which can conveniently be obtained within the Lagrangian formalism. For low spin \( s \leq 1 \) fields the Lagrangian quantization leads to the same free fields and energy momentum tensor as that obtained in Wigner’s representation theoretic quantum setting.

However for \( s \geq 2 \) the quantum free fields start to differ from their classical counterparts; pl tensor potentials and their fermionic counterparts are not solutions of Euler-Lagrange equation and the quantization leads to a formalism which requires the use of indefinite metric Krein spaces. The classical gauge symmetry ("local symmetry" in difference to global inner symmetries) looses its physical content and becomes a formal device which filters a physical subtheory (local observables, S-matrix). The form of the energy-momentum (E-M) tensor in the positivity preserving description based on Wigner’s unitary representation theory turns out to be the same as that in the indefinite metric setting; but
this ceases to be the case for massless $s \geq 2$ fields.

For $m = 0$, $s = 1$ the E-M tensor is the well-known expression

$$T_{\mu\nu}^P \simeq F_{\mu\kappa}F_{\nu}^{\kappa} - \frac{1}{4}\delta_{\mu\nu}F_{\kappa\lambda}F^{\kappa\lambda}$$  \hspace{1cm} (27)

It is easy to see that its massive counterpart is different by additional $m^2$ contributions

$$T_{\mu\nu}^P \simeq F_{\mu\kappa}^{P}F_{\nu}^{\kappa} - \frac{1}{4}\delta_{\mu\nu}F_{\kappa\lambda}^{P}F^{\kappa\lambda} + g_{\mu\nu}m^2aA_{\kappa}^{P}A_{\kappa}^{P} + m^2bA_{\mu}^{P}A_{\nu}^{P}$$  \hspace{1cm} (28)

where the superscript $P$ refers to the pl Proca potentials. The conservation of the E-M tensor uses the "massive Maxwell" equation

$$\partial^{\nu}T_{\mu\nu}^P = F_{\mu\kappa}^{P}\partial^{\nu}F_{\nu\kappa}^P$$  \hspace{1cm} (29)

$$\partial^{\nu}F_{\kappa\nu}^{P} = m^2A_{\kappa}^{P} \to 0 \text{ for } m \to 0$$

The remaining terms are compensated with the last 2 terms in (28) if one chooses $b = -2a$ and $a = 1/2$; all this is well-known.

Naively one would expect that in the massless limit these $m^2$ terms drop out so that the massive tensor converges against the Maxwell E-M tensor. But this does not happen, rather these terms contribute the E-M tensor of a pl scalar massless field so that the degrees of freedom... In fact besides the zero mass limit of sl vector potential which describes a helicity $h = 1$ situation which accounts for two $h = \pm 1$ degrees of freedom there is a massless pl scalar field $\phi$ defined as $\lim m\phi(x,e) = \phi(x)$ which takes care of the remaining of the three $s = 1$ degrees of freedom.

This means that, different from the gauge formalism were the massless indefinite metric vector potential accounts for only the two physical (helicity) degrees of freedom, the positivity preserving SLFT also maintains the degrees of freedom. The important role of the degrees of freedom preserving escort field in the "fattening of the photon" (the "switching on the mass" as the inverse of taking the massless limit) was already mentioned in section 3.

There is no problem to extend this construction of conserved tensors quadratic in the massive spin $s = n > 1$ fields. For convenience we use the following condensed index notation

$$F_{\mu\kappa,x}^{P} = \text{as } \partial_{\mu}A_{\kappa,x}^{P}, x = \kappa_1,..\kappa_{n-1}$$  \hspace{1cm} (30)

where $A_{\kappa,x}^{P}$ is a point-local totally symmetric $s = n$ tensor potential and the anti-symmetrization refers to $\mu,\kappa$. Note that apart from the case $s = 1$ the $F$ is not the pl field strength; the latter corresponds to the tensor $F$ of degree $2s$ in (13). The dimension in mass units ("engineering" dimension) is $d_{e_n} = 1$ for the potentials $A^P$ and 2 for the $F^P$, whereas their short distance dimension increases

\footnote{This interesting remark I owe to K.-H. Rehren.}
as $d_{sd} = s + 1$ for the $A^P$ as well as for the $F^P$ (the anti-symmetrization undoes
the gain from the differentiation). The differentiations which convert $A^P$ into
the field strength $F$ (13) increase the engineering dimension $d_{en} = d_{sd} = 2s$.
The $F$ are the only $s \geq 1$ pl fields which have a massless limit.

The construction of the E-M tensor $T_{\mu\nu}^P$ in terms of the $F^P$ (30) for general
massive $s > 1$ fields proceeds in the same way as (28). We again view $A_{\kappa,\times}^P$ as
a $\times$-indexed 1-form, so that the exact 2-form $F_{\mu\kappa,\times}$ satisfies the cyclic relation
which implied the kinematic relation (29). The divergence of $F$ can be computed
using the properties of the symmetric trace- and a divergence-less
$A_{\kappa,\times}^P$ tensor.

A conserved $s > 1$ tensor which extends (28) is

$$T_{\mu\nu}^P = F_{\mu\kappa,\times}^P F_{\nu,\kappa,\times}^P - \frac{1}{4} g_{\mu\nu} F_{\lambda,\kappa,\times}^P F_{\kappa,\lambda,\times}^P + g_{\mu\nu} m^2 \frac{1}{2} A_{\kappa,\times}^P A_{\kappa,\times}^P - m^2 A_{\mu,\times}^P A_{\mu,\times}^P$$

(31)

Note that the $s$-dependent short distance dimension $d_{sd} = 2 + 2s$ of $T^P$ is larger
than its mass- ("engineering"-) dimension which remains at $d = 4$. But since in
the calculation of the global Poincaré "charges" the spatial integration together
with the summation over the $\times$ indices leads to cancellations of $m^2$-factors in
the denominator against $p^2$-factors in the numerator this discrepancy can be
shown to disappear in the infinite volume limit.

The construction of the string-local E-M tensors follows similar formal
arguments. Instead of $A^P$ one starts from the string-local potential $A_{\mu,\times}$ with
$d_{sd} = d_{en} = 1$ which was obtained by successively "peeling off" leading short
distance dimensions (11). The differential form arguments (i.e. the use of the
homogenous Maxwell relation) is analogous. The divergence of $F$ is again of the
form of $m^2 B_{\mu}$ where $d_{sd}(B) = 2$ and $B$ can be expressed in terms of $A$ and the
$\phi$ (11). Hence the compensating quadratic $B$ terms are of a similar algebraic
form as before i.e.

$$T_{\mu\nu}(x,e) = F_{\mu\kappa,\times}^P F_{\nu,\kappa,\times}^P - \frac{1}{4} g_{\mu\nu} F_{\lambda,\kappa,\times}^P F_{\kappa,\lambda,\times}^P + g_{\mu\nu} m^2 \frac{1}{2} B_{\mu,\times}^P B_{\mu,\times}^P - m^2 B_{\mu,\times}^P B_{\mu,\times}^P$$

(32)

the only formal difference is the lower $d_{sd} = 2$ which is a result of the weakening
from pl to sl localization.

The claim is now that the pl and its sl counterpart have the same conserved
global charges. Although the charges densities $T_{0\nu}^P$ and $T_{0\nu}$ are different they
only differ by boundary contributions which vanish in the infinite volume limit.
The proof would render this article somewhat unbalanced and will be presented
together with the construction of a consered $T_{\mu\nu}$ for infinite spin in a separate paper.

Here we will be satisfied with a presentation of the ideas in the context of the
simpler case of a conserved free electromagnetic current. The conserved sl
current of a charge-carrying $s = 1$ field is related to its pl counterpart as $(A^P, \phi}$
complex, $A = A^P + \partial \phi$, $\partial \cdot A^P = 0$

$$j_{\mu}(x,e) = iA_\mu^*(x,e)\overrightarrow{\partial}_{\mu}A^P(x,e) = j^P_{\mu}(x) + \partial_\alpha G^P_{\mu}(x,e) + m^2i\phi^*\overrightarrow{\partial}_{\mu}\phi$$

(33)

with

$$\partial^\kappa G_{\kappa,\mu} = iA_\mu^P\overrightarrow{\partial}^\kappa \phi + h.c. = i\partial^\kappa (A_\mu^P\overrightarrow{\partial}_{\mu}\phi) + h.c.$$ and since a spatial divergence does not contribute to the charge

$$\int j_0(x,e)d^3x = \int j^P_0(x)d^3x + \int \partial^\mu G_{0,0}(x,e)d^3x - m^2\int j^\phi_0d^3x$$

(34)

The last step consists in realizing that one can use $\partial \cdot A^P = 0$ to convert the middle term into spatial divergence

$$\partial^\mu G_{0,0}(x,e) = i\partial^\mu A^P_0\overrightarrow{\partial}_{\mu}\phi + h.c. = boundary\; term$$

(35)

$$j^P_0(x) = j_0(x,e) + m^2j^\phi_0(x,e) + boundary\; term$$

(36)

whereupon it can be disposed of in the infinite volume limit. The scalar contribution has a massless limit since $\lim_{m \to 0} m\phi(x,e) = \phi(x)$ exists as a massless pl field. The massless sl $j_0(x,e)$ current lives in the Wigner-Fock $h = \pm 1$ helicity space whereas the massless scalar $\phi$ current accounts for one degree of freedom. Hence the $2+1$ $h = 1$ helicity+scalar degrees of freedom precisely match the $3$ spin $s = 1$ degrees of freedom.

This highlights the important role of the escort field $\phi$ in a positivity maintaining setting. This property is lost in the gauge theory; the indefinite metric and ghost degrees of freedom are no substitute for the degrees of freedom maintaining escort fields.

The expected result of the extension to $s > 1$ should be obvious. In addition to the massless pl scalar field from $\lim_{m \to 0} m\phi(x,e) = \varphi(x)$ there are sl helicity $|h| = s - k$ potentials of degree $k$ from $\lim_{m \to 0} m^{s-k}\varphi_{\mu_1...\mu_k}(x,e) = \varphi_{\mu_1...\mu_k}(x,e)$ fields which account for the preservation of degrees of freedom in the massless limit.

The relation between $T_{\mu\nu}(x,e)$ and $T_{\mu\nu}(x)$ is analogous but somewhat more involved. One of the complications is that there are several conserved $T_{\mu\nu}$. One may use the Noether theorem for translations or for the full Poincaré symmetry. The one which one would use for gravitational backreactions is the one which is defined in terms of the $g_{\mu\nu}$ metric variation of the action. We conjecture that the equivalence of their pl with their sl form up to boundary terms holds for each one.

These results have an interesting connection with the No Go theorem by Weinberg and Witten [30] [31]. These authors claim that there exist no electric charge-measuring conserved currents for $s \geq 1$ and no E-M tensors for $s \geq 1$. It one adds pl than the theorem hold for the rather trivial reason that their existence would violate positivity as a consequence of the nonexistence of zero mass $s \geq 1$ pl potentials. In those cases the correponding sl tensors perfectly

\[13\] In fact it was the $s = 1 T_{\mu\nu}$ for which Rehren pointed out to me that the $m^2$ terms converge to a finite nontrivial massless contribution.
exist and are even more natural in the massive case in the sense that their operator- and engineering- dimensions agree.

Massless infinite spin representations are described in terms of scalar string-local fields with transcendent two-point functions as in (15); the most general form of intertwiners can be found in [8]. In this case there are no tensor potentials from which one can form $T_{\mu\nu}$; one must be content with the unitary Wigner description in terms of $2s + 1$-component wave functions and operators.

Formally the infinite spin representation may be viewed as the $s \to \infty$ limit at fixed Pauli-Lubanski invariant $\kappa^2 = m^2 s(s + 1)$ and since our string-local fields guaranty the existence of smooth massless limits of correlation functions, the following strategy suggests itself. The first step consists in carrying out the $2s$ tensor contractions which results in the following representation

$$T_{\mu\nu}(x, e) = \sum_{\sigma, \kappa} \int \int e^{i(p-q)x} u^{(+,-)}(p, q; \sigma, \kappa) a^*(p, \sigma) a(q, \kappa) \frac{d^3p}{2p_0} \frac{d^3q}{2q_0} + \text{fluct. terms} + \text{h.c.}$$

where the $T_{\mu\nu}$ intertwiners $u_{+,-}, u_{+,+}$ and their conjugates are contracted sums over products of spin $s$ tensor intertwiners.

The existence of a Pauli-Lubanski limit is connected with the limiting behavior of these mass dependent composite $u(p, q)$ intertwiners for $s \to \infty$ at varying mass $m^2 = \kappa^2 / s(s + 1)$. One expects that the infinite spin E-M tensor has the correct equal time commutation relation with fields as (14) or with the general class of such fields in [8]; since a computational check depends on explicit formulas for the intertwiner in (37) we defer the necessary calculation to a separate paper.

The construction of an E-M tensor is the sine qua non for a coupling of WS to gravity; merely pointing to the energy positivity of the WS representation does not address the important problem of backreaction [32].

The idea of a local generation of global spacetime symmetries receives support from the algebraic setting of QFT where it can be shown that under very general conditions ("split property") there exist a local implementation of Poincaré transformations [20]. In the point-local case this means that the Poincaré transformation in a small parameter range around zero acting on an operator localized in a compact double cone spacetime region can be implemented by (highly non-unique) unitary operators which are localized in a slightly larger region. For the noncompact localizable WS matter the region would be a narrow spacelike cone whose core is a spacelike string.

6 Concluding remarks

The existence of forms of inert matter enriches the discussion about dark matter in an interesting way. There are two problems with identifying both. The inert WS matter seems to run into contradictions with astrophysical data which show that dark matter hovers around galaxies in the form of a halo which excludes
"fleeting" massless matter. Inert massive matter can presently not be excluded by astrophysical observations.

There is presently no serious problem with more conventional proposals which identify dark matter with matter of low reactivity as WIMPS or Cold Dark Matter. Such proposals become however increasingly problematic if refined earthly detection attempts remain inconclusive and the failure of counter-registering dark matter in terms of interactions with ordinary matter forces darkness increasingly towards inertness (≡total darkness).

With refinements of astrophysical observations and the failure to see the direct effects of dark matter in particle counters one is entering a "catch 22 situation": inert dark matter is consistent with its apparently exclusive gravitational manifestations but gets into conflict with the role which cosmologists attribute to it in the formation of ordinary matter in the Big Bang standard model of cosmology.

Any particle counter observation of a new form of matter for which there are good reasons to interpret it as a manifestation of the ubiquitous galactic dark matter will eliminate inert matter from the list of dark matter candidates. But this would not diminish Wigner's important role as a catalyzer of new ideas concerning the interplay between Hilbert space positivity, localization and short distance behavior for interactions involving higher spin $s \geq 1$ fields.

Independent of the problem of inert matter and its possible relation with dark matter, Wigner's 1939 discovery of the WS representation class of the Poincaré group and the more than 7 decades lasting attempts to understand its causal localization properties are an important achievement in unravelling unknown regions of QFT which remains our most successful theory about Nature's physical properties.

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