A spectral model of stably stratified surface-layer turbulence

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Abstract. A new model to determine the spectral velocity tensor in a stably stratified flow is proposed. This model is complementary to the Mann model as it solves the stratified inviscid Rapid Distortion Theory equations analytically, allowing for the determination of the single and two-point velocity spectra as well as the temperature-velocity cross-spectra. The model has been here calibrated and validated against field measurements conducted over a forested area with measurements up to 140 m, therefore covering a region of interest for wind-energy applications.

1. Introduction

The knowledge of the instantaneous velocity field characteristics of the atmospheric boundary layer has crucial importance for an efficient wind-energy utilisation. However, the flow over the rotor is highly turbulent and depends on a large number of parameters, like the boundary-layer height, the geostrophic-wind variation and the surface inhomogeneities, to mention some. Wind turbines must be designed to operate in and withstand such a turbulent environment. The current physical understanding of turbulent motions comes from experiments (in real fields or wind tunnels) and numerical simulations. In the latter case the high Reynolds number of the flow becomes challenging because of the increasing scale separation between large and small eddies both inside the domain and at the upstream domain boundaries. In order to obtain a realistic turbulent velocity field, we must force a realistic turbulent field at the inlet of the computational domain with prescribed characteristics or, alternatively, use a precursor simulation that serves as input for the main simulation. The latter approach provides a realistic field but it has the cost of additional computational power that is needed to provide the input for the main simulation. The former method, on the other hand, is quite efficient but necessitates of some information about the boundary-layer velocity field and its temporal characteristics. The model developed by Mann [1], for instance, provides a description of the spectral velocity tensor ($\Phi_{ij}$, where $i$ and $j$ correspond to the fluctuating velocity components $u'$, $v'$ and $w'$, indicating the streamwise, lateral and vertical velocity components, respectively) that can be used to determine the single- and two-point spectra of velocity time series inside and at the boundaries of the numerical domain. This approach provides some information about the frequency distribution of the energy so that a synthetic velocity field can be created that has the same one- and two-point
statistics of the observed one. Since 1994, the model has become the standard tool to synthesise turbulence time series at the inlet of many numerical simulations aimed at the characterisation of unsteady loads and aeroelastic effects.

However, the Mann model provides results in absence of density stratification and empirical approaches must be used to extend its application to non-neutral conditions (see for instance \[2\]). The absence of a density equation in the Mann model limits its use as an in-flow tool since, in addition to the initial velocity spectrum, models with non-neutral stratification need a description of the initial temperature spectrum.

The Mann model is based on Rapid Distortion Theory (RDT) where the turbulence is driven by the mean-flow distortion rather than by the non-linear distortions by turbulence itself. As the theory is time-dependent, a characteristic time scale is introduced to stop the structures evolution at a time related to their expected lifetime. In order to extend the model to stratified conditions, a solution of the inviscid stratified RDT equations with homogeneous shear is proposed, extending the solution of Hanazaki and Hunt \[3\] to the two horizontal velocity components. The analytical solution allows for the determination of the spectra at any given time, starting from a prescribed initial condition. Following the same approach adopted by Mann \[1\], a model for the spectral velocity tensor in the atmospheric surface layer is obtained where the spectral tensor, assumed to be isotropic at the initial time, evolves until the break-up time, where the spectral tensor is supposed to achieve its equilibrium state observed in surface-layer turbulence. A similar approach was developed by Chougule \[4\] by numerically integrating the stratified RDT equations.

The present paper aims at assessing whether or not the new extended model (discussed in section 2) can be used to simulate realistic atmospheric turbulence (as the Mann model does) with the additional information that regards the heat fluxes and, in general, the temperature-velocity correlations for different stability classes. Here only stably stratified conditions will be discussed, although the model can be extended to unstable stratification as well. Data from a measurement campaign over a forested site (section 3) will be used to calibrate and validate the model (section 4). Section 5 will conclude the paper with some final remarks.

2. Mathematical model

Let us consider a homogeneously stratified flow (with constant $d\rho_0/dz$) with a uniform mean velocity shear in the $z$-direction. By normalising physical variables with the characteristic length and velocity scale $L$ and $U_s$ (for the moment left unspecified), the dimensionless inviscid RDT governing equations can be written in Fourier space (where the $\cdot$ indicates the Fourier transform operator) as

$$\frac{d\hat{u}}{dt} = \alpha \left( \frac{2k_x^2}{k^2} \right) \hat{w} + \frac{k_x k_z}{k^2} \hat{\rho}, \quad (1)$$

$$\frac{d\hat{v}}{dt} = \alpha \frac{2k_y k_z}{k^2} \hat{w} + \frac{k_y k_z}{k^2} \hat{\rho}, \quad (2)$$

$$\frac{d\hat{w}}{dt} = \alpha \frac{2k_x k_z}{k^2} \hat{w} + \left( \frac{k_z^2}{k^2} - 1 \right) \hat{\rho}, \quad (3)$$

$$\frac{d\hat{\rho}}{dt} = N^2 \hat{w}, \quad (4)$$

where $\alpha = dU/dz L/U_s$ is the dimensionless mean wind shear, $N = \left[ -\left( g/\rho_0 \right) \left( d\rho_0/dz \right) \right]^{1/2} L/U_s$ is the scaled Brunt-Väisälä frequency, $g$ is the gravitational acceleration, $k = (k_x^2 + k_y^2 + k_z^2)^{1/2}$ is the wavenumber magnitude and the density is normalised as $\rho/\left( \rho_0 \text{Fr}^2 \right)$ where $\text{Fr} = U_s/\left( gL \right)^{1/2}$ is the characteristic Froude number.
The equation for the evolution of the wavenumber vector, $\mathbf{k}(t) = (k_x, k_y, k_z)$, is [5]

$$\frac{\mathrm{d}k_x}{\mathrm{d}t} = \frac{\mathrm{d}k_y}{\mathrm{d}t} = 0 \quad , \quad \frac{\mathrm{d}k_z}{\mathrm{d}t} + \alpha k_z = 0 \quad \rightarrow \quad \mathbf{k} = (k_{x0}, k_{y0}, k_{z0} - \alpha k_{z0}t) ,$$

(5)

where $(k_{x0}, k_{y0}, k_{z0})$ is the initial wavenumber vector with magnitude $k_0$. As a notation, the horizontal wavenumber $k_h = (k_x^2 + k_y^2)^{1/2}$ is introduced, a quantity that, according to equation (5), is independent of time.

As discussed in the introduction, Hanazaki & Hunt [3] provided an analytical solution of the stratified RDT equations for equations (3) and (4). The solution is based on the introduction of the variable $\zeta = ik_z/k_h$ in the equation which results from the combination of the density and vertical-velocity equations, leading to the differential equation

$$(1 - \zeta^2) \frac{d^2\rho}{d\zeta^2} - 2\zeta \frac{d\rho}{d\zeta} + \lambda (1 + \lambda) \dot{\rho} = 0 ,$$

(6)

where

$$\lambda (1 + \lambda) = - \frac{N^2 k_h^2}{\alpha^2 k_x^2} .$$

(7)

Equation (6) can be solved in terms of Legendre functions of first and second order as

$$\dot{\rho} = A P_\lambda (\zeta) + B Q_\lambda (\zeta) ,$$

(8)

while the vertical-velocity Fourier transform is

$$\tilde{\dot{w}} = - \frac{i\alpha k_z}{N^2 k_h} \left[ A P'_\lambda (\zeta) + B Q'_\lambda (\zeta) \right] ,$$

(9)

in which the primes indicate the derivative of the Legendre functions with respect to their argument $\zeta$.

After some algebra [6], it is possible to get the analytical solution for the other two velocity components as

$$\dot{u} = \dot{u}_0 + \frac{\alpha}{N^2 k_h} \left\{ A \left[ k_x^2 \zeta P'_\lambda (\zeta) - k_y^2 P_\lambda (\zeta) \right] \zeta |_{\zeta_0} + B \left[ k_x^2 \zeta Q'_\lambda (\zeta) - k_y^2 Q_\lambda (\zeta) \right] \zeta |_{\zeta_0} \right\} ,$$

(10)

$$\dot{v} = \dot{v}_0 + \frac{\alpha k_x k_y}{N^2 k_h^2} \left\{ A \left[ \zeta P'_\lambda (\zeta) + P_\lambda (\zeta) \right] \zeta |_{\zeta_0} + B \left[ \zeta Q'_\lambda (\zeta) + Q_\lambda (\zeta) \right] \zeta |_{\zeta_0} \right\} ,$$

(11)

where $\zeta_0 = ik_{z0}/k_h$ indicates the value of $\zeta$ at the initial time.

The constants $A$ and $B$ can be determined through the initial-density and vertical-velocity fluctuations Fourier transforms by solving the linear system

$$\begin{bmatrix} P_\lambda (\zeta_0) & Q_\lambda (\zeta_0) \\ P'_\lambda (\zeta_0) & Q'_\lambda (\zeta_0) \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} \dot{\rho}_0 \\ \left( iN^2 k_h \dot{\tilde{w}} \right) / (\alpha k_x) \end{bmatrix} .$$

(12)

The time evolution of the velocity and density perturbation vector $\mathbf{q} = [\dot{u}, \dot{v}, \tilde{w}, \dot{\rho}]^T$ from its initial state $\mathbf{q}_0 = [\dot{u}_0, \dot{v}_0, \tilde{w}_0, \dot{\rho}_0]^T$ can be written as $\mathbf{q} = \mathbf{M} \mathbf{q}_0$ where the matrix $\mathbf{M}$ is given by

$$\mathbf{M} (t; \mathbf{k}_0) = \begin{bmatrix} 1 & 0 & F_{uw0} & F_{up0} \\ 0 & 1 & F_{vw0} & F_{vp0} \\ 0 & 0 & F_{uw0} & F_{wp0} \\ 0 & 0 & F_{uw0} & F_{pp0} \end{bmatrix} ,$$

(13)
with the coefficients obtained from equations (10), (11), (9) and (8) and reported in [6].

The generic time-dependent spectral matrix \( \Phi(t; k) \) is now introduced as

\[
\Phi(t; k) = \Re \left[ \mathbf{q}(t) \mathbf{q}^\ast(t) \right] = \begin{bmatrix}
\Phi_{xx} & \Phi_{xy} & \Phi_{xz} & \Phi_{xp} \\
\Phi_{xy} & \Phi_{yy} & \Phi_{yz} & \Phi_{yp} \\
\Phi_{xz} & \Phi_{yz} & \Phi_{zz} & \Phi_{zp} \\
\Phi_{xp} & \Phi_{yp} & \Phi_{zp} & \Phi_{pp}
\end{bmatrix},
\]

where the \( \ast \) superscript indicates the complex conjugate operator. The evolution of the spectral tensor (14) is given by

\[
\Phi(t; k) = \mathbf{M}(t; k_0) \Phi(0; k_0) \mathbf{M}^T(t; k_0).
\]

Equation (15) is the key result of this work since it determines the evolution of the spectral tensor of velocity and density fluctuations starting from a generic initial condition \( \Phi(0; k_0) \) after a generic time \( t \).

The dimensionless initial velocity field can be assumed to be isotropic and characterised by means of the von-Kármán spectrum [1] that, according to the present scaling, becomes

\[
\Phi_{ij}(0; k_0) = \frac{Q_u}{4\pi} \frac{\delta_{ij}k_0^3 - k_{i,0}k_{j,0}}{(1 + k_0^2)^{17/6}} \quad \text{with} \quad Q_u = \frac{\alpha_1 \epsilon U_s^2}{\nu},
\]

where \( \epsilon \) is the turbulent kinetic energy dissipation rate and \( \alpha_1 \) is the Kolmogorov constant [7]. The corresponding isotropic dimensionless density spectrum can be also written as a power law [8] as

\[
\Phi_{\rho\rho}(0; k_0) = \frac{Q_\rho}{(1 + k_0^2)^{11/6}} \quad \text{with} \quad Q_\rho = \frac{5\beta_1}{6\pi\alpha_1} \frac{1}{\nu} \frac{\epsilon_u U_s^2}{\rho_0 \sigma_u^2} Q_u,
\]

where \( \epsilon_\rho \) is the dissipation rate of half the density variance and \( \beta_1 \) is a universal constant [7]. Equations (16) and (17) characterise the initial spectral tensor and allow some flexibility by means of the parameters \( Q_u \) and \( Q_\rho \) that represent two important properties of the initial isotropic field: \( Q_u \) is the normalised turbulent kinetic energy dissipation of the velocity field, while \( Q_\rho/Q_u \) is the ratio between the density and turbulent kinetic energy dissipation. Starting with isotropic conditions brings the important property that the initial density-velocity covariances are all zero, which reduces the addition in non-neutral stratification to the density spectrum alone.

Following [1], the dimensionless eddy break-up time scale under neutral stratification can be written in terms of the hypergeometric function

\[
\tau_{\text{M94}} = \frac{\Gamma}{\alpha} k^{-2/3} \left[ \, \text{2F1} \left( \frac{1}{3}, \frac{17}{6}; \frac{4}{3}; -k^{-2} \right) \right]^{-1/2},
\]

where \( \Gamma \) is the anisotropy parameter. The presence of the oscillation time scale introduced by the Brunt-Väisälä frequency suggests a new time scale, \( \tau \), here assumed to be of the form

\[
\frac{1}{\tau} = \frac{1}{\tau_{\text{M94}}} + cN \rightarrow \tau = \frac{\tau_{\text{M94}}}{1 + cN \tau_{\text{M94}}},
\]

The advantage of this assumption is that the neutral time scale is kept exactly as in [1] and the stability dependence comes from an independent parallel coupling of \( \tau \) with \( N \) by the constant \( c \). Besides the scaling quantities \( U_s \) and \( L \) and the initial spectral tensor parameters \( Q_u \) and \( Q_\rho \), two other parameters are therefore needed to complete the model by determining the break-up time scale, namely \( \Gamma \) and \( c \). By assuming that \( U_s \) is equal to the streamwise velocity standard deviation \( \sigma_u \), and according to the observations done in Ryningsnäs, some of these parameters can be assumed to be constant (\( \Gamma \) and \( c \)) or function of the Richardson number (\( Q_\rho/Q_u \) and \( z/L \)), as discussed in [6].
3. Experimental setup
The experiment took close to the inland mansion of Ryningsnäs in Sweden, located 30 km inland from the Swedish south-eastern coast. A complete description of the site and the equipment has been reported by Arnqvist et al. [9] and here only some brief details are recalled. A 138 m tall meteorological tower was located in the northwestern corner of a 200 m by 250 m large clearing surrounded by a forested area consisting of predominantly Scots Pine trees (with a mean canopy height of $h_c = 20$ m). Two wind turbines, labeled T1 and T2 in figure 1, are located approximately 200 m (corresponding to 2.2 rotor diameters) from the tower in the southern and north-eastern directions, respectively. The hub height is $z_{\text{hub},T1} = 100$ m and $z_{\text{hub},T2} = 80$ m, for the turbine T1 and T2, respectively.

The experiment ran between November 2010 and February 2012, yielding a total of 10560 hours of available measurements. 6 tri-axial Metek sonic anemometers were placed at $z = 40, 59, 80, 98, 120, 137.7$ m complemented by additional Risø PT-100 temperature sensors at $z = 40, 98, 120, 137.7$ m. The data were first sorted into 30-min blocks and, for each 30-min time series, statistics were computed. At each measurement height the velocity components were also rotated to a Cartesian reference frame $xyz$ where the average lateral velocity, $V$, was zero. The data were sorted according to the velocity at $z = 100$ m (in bins of 1 m/s) and to the ratio $h_c/L_o$, where $L_o$ is the Obukhov length, evaluated by means of the conventional definition

$$L_o = -\frac{u_*^2 \Theta_0}{\kappa g \langle w' \theta' \rangle},$$ (20)

where $\kappa = 0.4$ is the von Kármán constant, $\langle w' \theta' \rangle$ is the vertical temperature flux determined from the sonic anemometer at the lowest height, $\Theta_0$ is the reference temperature taken from the temperature sensor at the lowest height, and $u_*$ is the friction velocity that is consistently evaluated at the lowest height ($z = 40$ m), by $u_* = \left[ \langle u' u' \rangle^2 + \langle v' w' \rangle^2 \right]^{1/4}$.

The western sector (with direction within $[240^\circ, 280^\circ]$) was defined as the sector where the influence from the two turbines and the clearing was minimal (see [9] for a discussion regarding the nearby clearing effects). Luckily, the western sector was also coincident with the most probable mean wind direction as well as being the direction with the longest upwind forest cover.

![Figure 1. Landscape of the Ryningsnäs site.](image)
4. Results
The model parameters have been calibrated against single-point spectra measured at Ryningsnäs
in the velocity bin where $U(z = 100 \text{ m}) \approx 8 \text{ ms}^{-1}$. Figure 2 shows a comparison between
the measured and modelled pre-multiplied spectra. Obviously the agreement here is a result
of the fitting method (which determines the parameters $L$, $Q_u$, $Q_r$, $\Gamma$ and $\epsilon$ by maximising
the agreement between the measured and modelled pre-multiplied single-point velocity and
temperature spectra at some frequencies), although it is worth to notice that the $u\theta$ spectrum
was not used in the parameters determination, which can be considered here as an indicator
of the quality of the model. The single-point temperature spectra have not been shown as the
measured spectra for near-neutral to moderately stable conditions shows limited quality with
large high frequency scatter. This is a known problem of sonic anemometers and does not affect
the co-variances [10]. Due to the high frequency scatter causing upward tails of the spectra
and the subsequent problem with model comparison the temperature spectra have not been
considered in the present analysis.

At this point, it is worth computing the integral of the spectra providing the measured velocity
and temperature covariances, shown in figure 3. It is visible that the model is able to compute
the velocity variances and the shear stress $\langle u' w' \rangle$ as well as the heat fluxes $\langle u' \theta' \rangle$ and $\langle w' \theta' \rangle$.
All these integral quantities appear to be well estimated, underlining the fact that the present
model complements [1, 2, 4]. The only information going in to the model is the background
stratification and shear as well as horizontal velocity variance. It seems thus that the mean
distortion by temperature and velocity gradients is enough to reproduce the same distribution
of energy between different Reynold stress components and heat fluxes that is observed in nature.

Both single-point spectra and associated integrals are however a direct result of the fitting
of the spectra and their agreement with the measurements is expected. The fitting process
was performed by using only single-point data with no information about two-point spectra.
Therefore a comparison of the model prediction with two-point spectra represents a way to
validate the proposed model. Figures 4 and 5 show two-point data that assess how well the model
performs. Figure 4 shows the covariance between two different heights for the $uu$, $vv$, $ww$ cross
spectra in three different stability conditions (from nearly neutral to stable). The covariances are
here normalised with the product of the standard deviations of the relative velocity components
at the heights $z_1$ and $z_2$, and plotted against the height difference $\Delta z = |z_2 - z_1|$ normalised by
the average wind speed $\bar{U} = [U(z_1) + U(z_2)]/2$. The advantage of this normalisation is discussed
in [11] where it is demonstrated that the data collapse well. As visible in the figure, the modelled
covariances have the same qualitative behaviour of the measured data, although with a slight
overestimation of the covariances (especially in $uu$). The stability trend is correctly predicted
for all the velocity components.

Figure 5 shows the comparison between the phase of the cross-spectra between the two
different heights (for a frequency of 0.01 Hz) against the variable $\Delta z / \bar{U}$ where a monotonic trend
is visible (except for $ww$ that is scattered around a zero-phase shift). Here the model is able to
qualitatively follow the data trend but the phase shift is slightly underestimated. No significant
stability trend can be discerned in both measurements and modelled phases. Noteworthy, the
phases follow the inequality $\phi_w < \phi_u < \phi_v$ proposed by [12] for neutral stratification.

5. Conclusions
In the present paper a new model to estimate the velocity and temperature spectral tensor
of the atmospheric boundary layer is proposed. The model is based on an analytical solution
of the stratified RDT equations for a homogeneous turbulent flow subjected to homogeneous
shear and stable stratification. This approach allows to determine the time-wise evolution of
the Fourier transform of the velocity and density fluctuations starting from an arbitrary initial
Figure 2. Comparison of the pre-multiplied spectra at \( z \approx 138 \text{ m} \) and \( U(z = 100 \text{ m}) \approx 8 \text{ ms}^{-1} \) for (a – b) Neutral conditions (\( \text{Ri} \approx 0.07 \)), (c – d) Near-Neutral Stable conditions (\( \text{Ri} \approx 0.10 \)) and (e – f) Stable conditions (\( \text{Ri} \approx 0.14 \)). Experimental data: (◦) \( F_{u u} \), (+) \( F_{v v} \), (∗) \( F_{w w} \), (□) \( F_{u w} \), (▽) \( F_{u \theta} \), (△) \( F_{w \theta} \). Model: (solid line) \( F_{u u} \), \( F_{u w} \), \( F_{u \theta} \), \( F_{w \theta} \). (dashed line) \( F_{v v} \). (dash-dotted line) \( F_{w w} \).
state, as long as the rapid-distortion closure is valid. By following the same approach of Mann [1], the equilibrium state of the time-dependent spectral tensor is determined by introducing a wavenumber-dependent time scale.

The model is based upon a number of scaling quantities and parameters. These have all been determined by optimising the comparison between model outputs and measurements over a forested area [2, 9]. According to our observations, the proposed model is able to determine reasonably well the single-point velocity spectra and the single-point velocity and temperature cross-spectra (including $F_{u\theta}$, which was purposefully kept out of the optimisation approach) for different Richardson numbers. Although the model is a function of only one initial velocity scale and one initial temperature scale it predicts the same distribution of energy between all the different covariances as is observed in the measurements. This suggests that in turbulent flows at the height span and velocity range that is interesting to wind energy the rapid distortion equations provide a valid approximation to the flow in a statistical sense. When evaluated against vertically separated two-point spectra, the model showed qualitative agreement with the available data both in terms of decay with separation distance and the tendency of decreasing correlation with increasing stratification.

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Figure 4. Comparison of the integral of the two-point cross-spectra when $U(z = 100 \, \text{m}) \approx 8 \, \text{ms}^{-1}$ for different stability conditions (black $R_i \approx 0.07$, blue $R_i \approx 0.10$ and cyan $R_i \approx 0.14$) between different points at heights $z_1$ and $z_2$ between the $u$-component (a), $v$-component (b) and $w$-component (c). Experimental data: (○) $\langle u'(z_1)u'(z_2) \rangle$, (+) $\langle v'(z_1)v'(z_2) \rangle$, (∗) $\langle w'(z_1)w'(z_2) \rangle$. Model prediction: (solid line) $\langle u'^2 \rangle$, (dashed line) $\langle v'^2 \rangle$, (dash-dotted line) $\langle w'^2 \rangle$.

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Figure 5. Comparison of the spectral phase when $U(z = 100 \text{ m}) \approx 8 \text{ ms}^{-1}$ and $f = 0.01 \text{ Hz}$ for different stability conditions (black $\text{Ri} \approx 0.07$, blue $\text{Ri} \approx 0.10$ and cyan $\text{Ri} \approx 0.14$) between different points at heights $z_1$ and $z_2$ between the $u$-component $(a)$, $v$-component $(b)$ and $w$-component $(c)$. Experimental data: $(\circ) \varphi_u(z_1, z_2)$, $(+) \varphi_v(z_1, z_2)$, $(\ast) \varphi_w(z_1, z_2)$. Model prediction: $(\text{solid line}) \langle u'^2 \rangle$, $(\text{dashed line}) \langle v'^2 \rangle$, $(\text{dash-dotted line}) \langle w'^2 \rangle$. 

