Spin-waves in antiferromagnetic single crystal LiFePO₄

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Spin-wave dispersions in the antiferromagnetic state of single crystal LiFePO₄ were determined by inelastic neutron scattering measurements. The dispersion curves measured from the (010) reflection along both a' and b' reciprocal-space directions reflect the anisotropic coupling of the layered Fe²⁺ (S = 2) spin-system. The spin-wave dispersion curves were theoretically modeled using linear spin-wave theory by including in the spin-Hamiltonian in-plane nearest- and next-nearest-neighbor interactions (J₁ and J₂) and inter-plane nearest-neighbor interactions (J⊥) and a single-ion anisotropy (D). A weak (010) magnetic peak was observed in elastic neutron scattering studies of the same crystal indicating that the ground state of the staggered iron moments is not along (010) direction, as previously reported from polycrystalline samples studies, but slightly rotated away from this axis.

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I. INTRODUCTION

Lithium-orthophosphates LiMPO₄ (M = Mn, Fe, Co, Ni) have attracted a renewed interest in recent years, both for their relatively high lithium ionic-conductivity that can potentially be applied in rechargeable battery technology [1] and for their intriguing magnetic properties, in particular, the strong magneto-electric (ME) effect they exhibit [2]. In this regard, of particular importance is LiFePO₄ as it has already been tested as a high-potential cathode in secondary Li-ion rechargeable battery [1, 2, 3, 4]. Other members of the lithium-orthophosphates, LiFePO₄ is an insulator adopting the Pnma space group [1, 2, 7, 8]. In this structure, the Fe²⁺ ion occupies the center of a slightly distorted MO₆ octahedron that shares oxygen anions with a PO₄ tetrahedron forming a closely packed oxygen framework. The Fe²⁺ ions (S = 2) form corrugated layers that are stacked along the [100] crystallographic axis, as shown in Fig. 1(a). Nearest neighbors in the b-c plane are coupled magnetically by a relatively strong exchange interaction, J₁ through an Fe-O-Fe oxygen-bond, whereas in-plane next-nearest-neighbors are coupled (J₂) via Fe-O-O-Fe [9, 10] (see Fig. 1(b) for definition of exchange couplings). Interlayer magnetic coupling is mediated by a phosphate ion via an Fe-O-P-Fe bonding [11, 12]. Thus, the olivine family of LiMPO₄ exhibits highly anisotropic properties which are between those of two- (2D) and three-dimensional (3D) systems [11, 12].

The magnetic properties of LiMPO₄ systems have been studied since the early 1960s [13, 14, 15]. They all undergo an antiferromagnetic transition at low temperatures to a similar magnetic arrangement differing only in the orientation of the staggered spins. Nearest-neighbor (NN) spins in the b-c plane are anti-parallel and the stacking along the a-axis is such that ferromagnetic sheets perpendicular to the b-axis are formed; nearest neighbor sheets are anti-parallel giving rise to (010) fundamental magnetic reflection that, depending on the direction of the magnetic moment, can be intense or extinct. Earlier neutron diffraction studies of polycrystalline samples showed that the magnetic space group of LiFePO₄ is Pnma with the spins oriented along b crystallographic direction (i.e., the (010) reflection is absent), and Pnma a magnetic space group for LiNiPO₄ and LiMnPO₄ with the spins aligned parallel to the c-axis (i.e., strong intensity at the (010) reflection) [11, 12, 13, 16, 17, 18]. Recent neutron diffraction measurements of single crystal LiCoPO₄ reported finite intensity at the (010) reflection, interpreted in terms of a ground state with a spin direction that is slightly rotated from the b-axis [18]. Weak ferromagnetism has been reported for LiMnPO₄ [18] and LiNiPO₄ [19] at a temperature below Tₙ.

Recently, the magnetic structure and properties of lithium orthophosphates have been reexamined theoretically and experimentally [20, 21]. Rousse et al. [21] reported neutron diffraction results from polycrystalline samples confirming the collinear structure below Tₙ = 52 K. Magnetic properties of LiFePO₄ investigated by Mössbauer spectroscopy and magnetization measurement determined that Tₙ = 50 K [22]. Theoretical estimations of the spin exchange coupling by spin-dimer analysis, while neglecting the single ion-anisotropy, yield the following values J₁ = 1.08 meV, J₂ = -0.4 meV and J⊥ = -0.92 meV [10]. However, so far there has been no experimental determination of the exchange coupling among Fe²⁺ spins and of the single-ion anisotropy in LiFePO₄, for comparison with the theoretical predictions. Knowledge of exchange couplings and single-ion-anisotropy is also important for understanding the origin of the strong magneto-electric (ME) effect in LiFePO₄ [13]. All lithium orthophosphates exhibit a strong yet anomalous linear magneto-electric (ME) effect with respect to the observed ME tensor components, αₓᵧ, αᵧz, for LiFePO₄ and LiCoPO₄, and αₓz, αᵧx for LiNiPO₄, as expected with their respective antiferromagnetic point
FIG. 1: (color online) (a) Atomic structure of LiFePO$_4$. The Fe$^{2+}$ ions form buckled layers stacked perpendicular to the [100] crystallographic direction. The ground state of LiFePO$_4$ is collinear antiferromagnetic with the average moment along $b$ direction. (b) Spin arrangement of the two Fe$^{2+}$ layers, the in-plane nearest and next-nearest neighbor interactions $J_1$ and $J_2$ and inter-plane nearest neighbor interaction $J_\perp$ are labeled.

In particular, the ME effect measurements of LiFePO$_4$ as a function of temperature reveal a decrease of the ME coefficient along one direction $\alpha_{xx}(T)$ below a maximum close to $T_N$ [23, 24]. Detailed determination of the magnetic structure using neutron diffraction from single crystals can shed light on the origin of these anomalies.

Herein, we report measurements of spin-wave dispersion curves of single crystal LiFePO$_4$ by inelastic neutron scattering measurements. Spin wave dispersion curves can provide the values of exchange interactions and the single ion anisotropy. The measured dispersion curves were modeled using linear spin wave theory by including the in-plane nearest-neighbors (NN) and next-nearest-neighbors (NNN) spin couplings the inter-plane nearest-neighbor spin coupling and the single-ion anisotropy. We have also employed single-crystal elastic neutron diffraction techniques to investigate whether there are subtle deviations from the previously reported magnetic structure determined from neutron diffraction measurements of polycrystalline samples.

II. EXPERIMENTAL DETAILS

LiFePO$_4$ single crystals were grown by standard flux growth technique (LiCl was used as the flux) from a stoichiometric mixture of high purity FeCl$_2$ (99.999% Aldrich) and Li$_3$PO$_4$ (99.999% Aldrich) [27]. The grown single crystals have a dark greenish color. The composition and structure were confirmed by carrying out Rietveld analysis of the X-ray powder diffraction (XRD) data, using the GSAS software package [28]. No extra peaks from impurities were detected in the XRD pattern. Powder, for the XRD, was produced by crushing typical isolated single crystals from the melt. The lattice parameters yielded from the refinement ($a = 10.337$ Å, $b = 6.011$ Å, and $c = 4.695$ Å) are in good agreement with literature values [15, 20, 21].

Neutron scattering measurements were carried out on the HB1A triple axis spectrometer at High Flux Isotope Reactor at Oak Ridge National Laboratory. A monochromatic neutron beam of wavelength $\lambda = 2.366$ Å (14.6125 meV, $k_0 = 2\pi/\lambda = 2.656\text{Å}^{-1}$) was selected by a double monochromator system, using the (0 0 2) Bragg reflection of highly oriented pyrolytic graphite (HOPG) crystals. HOPG crystal was also used as analyzer for both the elastic and the inelastic studies. The $\lambda/2$ component in the beam was removed by two HOPG filters located before and after the second monochromator. The collimating configuration 40$'$ - 40$'$ - Sample - 34$'$ 68$'$ was used throughout the experiments. Temperature measurements and control were achieved by a conduct LT20 using Lake Shore silicon-diode temperature sensors. An irregular shaped single crystal (weight $\sim 0.4$ g) was folded in aluminum foil and mounted on a thin aluminum post. It was then sealed in an aluminum can under helium atmosphere and loaded onto the tip of a closed-cycle helium refrigerator (Displex). Two temperature sensors were mounted on the cold-tip of the Displex and on the sample can. The temperature was controlled using the cold-tip sensor. The temperature difference between the two sensors was about 0.2 K over the temperature range investigated.
III. RESULTS AND DISCUSSION

A. Elastic neutron scattering

The LiFePO$_4$ crystal was oriented such that the $a$-$b$ plane coincided with the horizontal scattering plane of the spectrometer. The elastic measurements confirmed that the magnetic structure of LiFePO$_4$ is antiferromagnetic with the main direction of the moment oriented along the $b$-axis. However, contrary to the previous neutron diffraction experiments performed on powder samples, we have detected the presence of the (0 1 0) reflection. The intensity of this peak is relatively weak but its intensity follows a similar temperature dependence as that of a stronger magnetic peak (210) (see Fig. 2). It is worth noting that the (0 1 0) peak is forbidden by the symmetry of the $Pnma$ magnetic space group previously proposed to describe the spin arrangement in LiFePO$_4$ [13, 20]. In the $Pnma$ crystal symmetry, the point group of the Fe 4c site is $m_y$ and the only allowable magnetic point groups are, therefore, $m_y$ with the Fe magnetic moments along the $b$-axis (perpendicular to the mirror (010) plane) and $m''_y$ with the magnetic moment lying in the mirror (010) plane. The magnetic contribution to the (010) peak indicates that the ordered moment is not strictly oriented along the $b$-axis and a small component perpendicular to this axis is present. This implies a lowering of the symmetry of the magnetic space group where both magnetic components (along and perpendicular to $b$-axis) are allowed. From the intensity ratio of the two reflections $I_{(010)}/I_{(210)}$ at low temperatures, we can estimate the angle of the staggered moment with respect to the $b$-axis, by using the following relation,

$$\frac{F_{(010)} \sin(\alpha_{(010)})}{F_{(210)} \sin(\alpha_{(210)})} = \sqrt{\frac{I_{(010)} \sin(2\theta_{(010)})}{I_{(210)} \sin(2\theta_{(010)})}} \frac{f_{(010)}}{f_{(210)}}$$

(1)

where, $F_{(210)}$ and $F_{(010)}$ are the magnetic structure factors of peaks (210) and (010), $\alpha_{(010)}$, $\alpha_{(210)}$ are the angles between the scattering vector of reflections (210), (010) and the magnetic moment, and $f_i$ are the corresponding form factors [12]. Using $f_{(210)}/f_{(010)} \approx 0.85$, we estimate the moments are rotated by $7.5 \pm 0.5$ deg toward $c$-axis or $3 \pm 0.5$ deg toward $a$-axis. The ratio between the magnetic and nuclear contributions to the peak intensities of reflections ($I_{mag}/I_{nuc}$) can be used to determine the average magnetic moment, $\mu$ from

$$\mu = \sqrt{\frac{I_{mag}}{I_{nuc}} \frac{|F_{nuc}|^2}{|F_{mag}|^2} \frac{1}{f^2(Q) \sin^2 \alpha}}$$

(2)

where for the reflection in question, $F_{nuc}$, and $F_{mag}$ are the nuclear and magnetic structure factors, $I_{nuc}$ and $I_{mag}$ are the nuclear and magnetic intensities and $f(Q)$ is the magnetic form factor of Fe$^{2+}$ at momentum transfer $Q = 2q_0 \sin \theta$. $I_{mag}$ can be calculated from the peak intensity difference at temperatures above and below $T_N$. Using the peak intensities of (210) at 300 K and 10 K to calculate the $I_{mag}$ and $I_{nuc}$, and using $f_{(210)} = 0.85$, the calculated average magnetic moment $\mu$ for Fe is $3.93 \pm 0.05 \mu_B$, which is very close to the values of 3.99 $\mu_B$ and 3.8 $\mu_B$ obtained in Refs. [29] and [30].

To determine the temperature dependence of the order parameter, the (010) and (210) reflections were monitored as a function of temperature in the range 10 to 60 K. Figure 2(a) and (b) show the square root of the integrated intensity ($\sqrt{I}$) versus temperature. The $\sqrt{I}$ quantity is proportional to the antiferromagnetic staggered magnetization, i.e., the order parameter. It was fitted to the following power law function near the transition temperature,

$$\sqrt{I} \propto M_{\parallel}^0 M^\beta (t, T_N)$$

(3)

where, $M_{\parallel}^0$ is the sub-lattice magnetization at $T = 0$ K, $t = (1-T/T_N)$ is the reduced temperature, and $\beta$ is the critical exponent. For the two magnetic peaks (010) and (210) the obtained transition temperatures are the same, $T_N = 50 \pm 0.5$ K and the critical exponent $\beta$ is $0.27 \pm 0.03$. The transition temperature is very close to the values reported in the literature $50 \pm 2$ K [13, 24, 22]. The
critical exponent $\beta$ is slightly smaller than that calculated for the 3D Ising model ($\beta = 0.32$) \cite{31}. The temperature dependent background-like scattering above $T_N$ and below $\sim 60$ K indicates some kind of critical scattering due to short-range order formation or due to a dimensionality cross-over.

B. Inelastic neutron scattering

The spin wave excitations were measured at 10 K along the $(\xi, 1, 0)$ and $(0, 1+\xi, 0)$ reciprocal space directions, for energy transfers (energy loss mode) ranging from 1 to 8.5 meV. As illustrated in Fig. 3(a, b, c), well defined dispersive magnetic-modes of resolution-limited energy-width were observed at all wave vectors. A typical constant-q scan, performed at the zone center (010), is shown in Fig. 3(a), indicating a single excitation at an energy transfer of 5.86 meV, above the transition temperature ($T_N = 50$ K), at approximately 55 K, the inelastic peak at 5.86 meV disappears, confirming its magnetic origin. Such an energy gap in the dispersion curve is usually driven by single ion anisotropy. A similar energy gap of 2 meV at 2 K, was also observed in the LiNiPO$_4$, and it was found to decrease with increasing the temperature \cite{32}. In the case of LiFePO$_4$, measurements performed at different temperatures indicate that the energy gap is temperature independent. The inelastic scattering signal measured at different constant wave-vectors $\xi$ along the (100) and (010) reciprocal-space directions, at 10 K, are shown in Figure 4(b, c). The data were fitted to Gaussian functions (solid line in Fig. 4(a, b, c)) where the background was assumed to be constant.

The spin-wave dispersion branches deduced from these fits, for both $b^*$ and $a^*$ reciprocal-space directions, are plotted in Figure 4. It is shown that the dispersion curves monotonically increase in energy with $\xi$, and that the spin-waves propagating in the plane along the (010) direction have higher frequencies than those propagating transversally along the (100). Qualitatively, this behavior reflects the anisotropy in the strength of exchange couplings in the system; as expected the in-plane exchange couplings are much stronger than those between planes.

To construct a Hamiltonian for the spin system, we recall that in LiMPO$_4$ olivine family the in-plane super-exchange or super-super exchange interactions between nearest and next-nearest neighboring Fe$^{2+}$ ions ($J_1$ and $J_2$) are expected to be much stronger than that between the nearest inter-plane neighbors ($J_\perp$) \cite{13, 33}. Therefore, we propose the following Hamiltonian,

$$
\mathcal{H} = -J_1 \sum_{i,\delta} (S_i S_{i+\delta}) - J_2 \sum_{i,\xi} (S_i S_{i+\xi}) \\
-J_\perp \sum_{i,\delta_\perp} (S_i S_{\delta_\perp}) + D \sum_i (S^z)^2
$$

where, $J_1$ and $J_2$ are the in-plane NN and NNN coupling constants, respectively, and $J_\perp$ is the inter-plane NN coupling constant. The illustrations of $J_1$, $J_2$ and $J_\perp$ are shown in Figure 3(b). $D$ is the single-ion anisotropy constant quantifying the tendency of the spins to align along the easy axis (the $S^z$ component is defined to be along the direction of the moment at the ground state - $b^*$). The Ising like ground state of the system is be-
lieved to be invoked by the single-ion anisotropy term which comes about from crystal field effects and spin-orbit coupling [34]. Using the antiferromagnetic spin-wave theory [35, 36], the lattice with N sites was divided into two sublattices A and B, where nearest-neighbors of an Fe$^{2+}$ site in one sublattice are all sites in the other sublattice. The next-nearest neighbors of an Fe$^{2+}$ site are in the same sublattice. The magnon dispersion curves were calculated using the Holstein-Primakoff spin operator transformation to linear approximation (i.e., linear spin-wave theory [37]). The resulting spin-wave dispersion is given by

$$\hbar \omega = \sqrt{A^2 - F^2}$$

where $A = (2J_1 Z S - 2J_2 Z S - 2J_3 Z S + 2J_4 Z S \gamma_{3N} + 2J_5 Z S \gamma_{\perp} + 2DS)$ and $F = 2J_1 Z S \gamma_{2N}$, in which Z is the number of the nearest neighbors $Z = 4$, $S = 2$ for Fe$^{2+}$. $\gamma_{2N}$, $\gamma_{3N}$ and $\gamma_{\perp}$ are calculated using the following equation:

$$\gamma(2N,3N,\perp) = \frac{1}{Z} \sum_i e^{iQ\cdot r}$$

where $r = (\delta, \xi, \delta_{\perp})$ are the components of vectors to the intra-plane nearest, next-nearest neighbors and to the inter-plane nearest-neighbor. We get

$$\gamma_{2N} = \cos(\pi k_y) \cos(\pi k_z)$$

$$\gamma_{3N} = \frac{1}{2} (\cos(2\pi k_y) + \cos(2\pi k_z))$$

$$\gamma_{\perp} = \cos(\pi k_x) \cos(\pi k_y)$$

The experimental data along the $(0, 1+\delta, 0)$ and $(\delta, 1, 0)$ directions in Fig. 4 were fitted to Eq. 6 yielding the following values: $J_1 = -0.662 \pm 0.02$ meV, $J_2 = -0.27 \pm 0.02$ meV, $J_3 = 0.021 \pm 0.001$ meV and $D = -0.37 \pm 0.01$ meV.

The in-plane nearest- and next-nearest-neighbor coupling constants quantitatively agree with theoretical calculations [4], $J_1 = -1.08$ meV and $J_2 = -0.4$ meV [3]. The two spin couplings, $J_1 < 0$ and $J_2 < 0$, compete oppositely over the alignment of in-plane NNN spins; whereas $J_1$ leads to parallel alignment of NNN spins $J_2$ favors their antiparallel alignment. Such competing interactions can lead to incommensurate phases [3] which were not found in this system. However, incommensurate phases have been reported for the isostructural LiNiPO$_4$ [26]. The inter-plane coupling $J_3 = 0.021$ meV determined in this study is significantly smaller than the theoretical one $J_3 = -0.92$ meV [4]. It should be noted that single-ion anisotropy was not considered in the theoretical calculations [3].

To summarize, we have measured spin-wave dispersions and determined spin exchange couplings in LiFePO$_4$. Our results show that although there are competing interactions between NN and NNN spins in LiFePO$_4$, they do not lead to more complicated, incommensurate or non-collinear, magnetic structures. This is in contrast to the observation of incommensurate magnetic phases in LiNiPO$_4$ [26]. These competing interactions may explain the observation of weak ferromagnetism in LiMPO$_4$ systems [13]. They may also be related to the observation, in this study, that the staggered magnetic moment is not aligned along a principal direction. From the gap in the spin wave dispersion curve, we have been able to extract the single-ion anisotropy in LiFePO$_4$ using linear spin-wave theory.

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