Confronting predictive texture zeros in lepton mass matrices with current data

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(Dated: April 27, 2015)

Several popular Ansätze of lepton mass matrices that contain texture zeros are confronted with current neutrino observational data. We perform a systematic χ²-analysis in a wide class of schemes, considering arbitrary Hermitian charged lepton mass matrices and symmetric mass matrices for Majorana neutrinos or Hermitian mass matrices for Dirac neutrinos. Our study reveals that several patterns are still consistent with all the observations at 68.27% confidence level, while some others are disfavored or excluded by the experimental data. The well-known Frampton-Glashow-Marfatia two-zero textures, hybrid textures and parallel structures, among others, are considered.

I. INTRODUCTION

In the absence of a convincing theory to explain the origin of the lepton flavor structure, different approaches have been pursued to address this question. Among them, the imposition of texture zeros in the lepton mass matrices has been quite popular. The reason is two-fold. The vanishing of some matrix elements obviously reduces the number of free parameters, thus increasing, in some cases, the predictive power of the flavor patterns. Furthermore, texture zeros can naturally appear in theories with an extended scalar sector in the presence of Abelian symmetries [1,2]. Thus, the study of the phenomenological implications of lepton mass matrices with vanishing elements is well motivated on theoretical grounds.

During the last years, our knowledge of neutrino masses and leptonic mixing has been enriched thanks to the data accumulated from several solar, atmospheric, reactor and accelerator neutrino experiments [3–5], as well as to cosmological observations [6]. Furthermore, an improved sensitivity to the Dirac CP phase has emerged from the complementarity of accelerator and reactor neutrino data. It is conceivable that leptonic CP violation is observed in current and next-generation neutrino oscillation experiments, which makes the search for such effects one of the main goals of the future research in neutrino physics [7].

It has been known for some time that, in the flavor basis where the charged lepton mass matrix is diagonal, neutrino mass matrices with more than two independent zero entries are not compatible with neutrino oscillation data, while seven patterns with two zeros are viable, as shown by Frampton, Glashow and Marfatia (FGM) in Ref. [3]. The latter contain four complex parameters, from which nine physical quantities should be determined (three neutrino masses, three mixing angles, one Dirac CP phase and two Majorana phases), assuming that light neutrinos are Majorana particles. More recently, the aforementioned two-zero textures have been scrutinized (see e.g. Refs. [9,10]). Other predictive textures can be envisaged as well in the flavor basis. The so-called hybrid textures [11], having one texture zero and two equal nonzero elements, contain the same number of physical parameters as the FGM textures. A systematic analysis of such hybrid textures has been presented in Ref. [12], in which the authors concluded that 39 patterns for Majorana neutrinos are compatible with current neutrino oscillation data at the 3σ confidence level (C.L.).

Restrictive patterns for the lepton mass matrices can also be constructed when the charged lepton mass matrix is not diagonal. For instance, one can consider scenarios in which both matrices exhibit a “parallel” structure [13,14] with the vanishing matrix elements located at the same positions [15,16] (see also Ref. [17] and references therein). Recently, a detailed survey of texture zeros in lepton mass matrices has been performed, for both Dirac and Majorana neutrinos, considering parallel and non-parallel matrix structures [18]. In the latter study, however, the Dirac phase was not included in the numerical χ²-analysis, which was carried out at the 5σ C.L..

In this work, we perform a detailed χ²-analysis of several popular Ansätze for lepton mass matrices that contain texture zeros. We aim at determining whether such patterns are consistent or not with current neutrino oscillation data at the 1σ (68.27%) C.L.. In particular, the well-known FGM two-zero textures, the hybrid textures, as well as parallel structures will be analyzed. In our fitting procedure, we take into account six neutrino observables, namely, the two mass-squared differences, the three mixing angles, and the Dirac CP-violating phase. We also impose the recent cosmological bound on the sum of the neutrino masses [6].

The paper is organized as follows. In Sec. [19] we briefly explain our strategy for the numerical analysis and minimization procedure. Then, we proceed in Sec. [11] to revisit the FGM two-zero textures for Majorana neutrinos.
Two-zero textures for the lepton mass matrices in the case of Dirac neutrinos are also considered. Section IV is devoted to the systematic $\chi^2$-analysis of hybrid textures containing one-zero texture and two equal nonzero elements, for both Majorana and Dirac neutrinos. Parallel structures with two and three zeros are studied in Sec. V.

Finally, our concluding remarks are given in Sec. VI.

II. STRATEGY FOR THE NUMERICAL ANALYSIS

Leptonic mixing is described by the Pontecorvo, Maki, Nakagawa and Sakata (PMNS) matrix [19], which, in the standard parametrization, can be written as [20]

$$U = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
    -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
    s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix} \cdot \text{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2}),$$

where $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$, with all the angles $\theta_{ij}$ in the first quadrant, $\delta$ is the Dirac CP phase, and $\alpha_{21}, \alpha_{31}$ are two Majorana phases. The unitary matrix $U$ in Eq. (1) relates the mass eigenstate neutrinos $\nu_i$ ($i = 1, 2, 3$) to the flavor eigenstate neutrinos $\nu_f$ ($f = e, \mu, \tau$).

For Majorana neutrinos, the neutrino mass matrix $m_\nu$ is a $3 \times 3$ complex symmetric matrix, which can be diagonalized by the unitary transformation $U^\dagger_{\nu L} m_\nu U_{\nu L} = \text{diag}(m_1, m_2, m_3)$, with $U_{\nu L}$ a unitary matrix and the neutrino masses $m_i$ real and positive. If neutrinos are Dirac particles, then the corresponding unitary transformation is $U^\dagger_{\nu R} m_\nu U_{\nu R} = \text{diag}(m_1, m_2, m_3)$, in analogy to the charged leptons, for which the mass matrix $m_\ell$ is diagonalized by $U^\dagger_{\ell L} m_\ell U_{\ell L} = \text{diag}(m_e, m_\mu, m_\tau)$. The leptonic mixing matrix $U$ is then given by $U = U^\dagger_{\ell L} U_{\nu L}$, which can always be parametrized in the form of Eq. (1).

The absolute scale of neutrino masses is not yet known and there are two possible orderings of the light neutrino masses: normal ordering (NO) with $m_1 < m_2 < m_3$ or inverted ordering (IO) with $m_3 < m_1 < m_2$. The spectrum may vary from hierarchical to quasi-degenerate masses. Nevertheless, cosmological observations place a stringent upper bound on the sum of the masses. Assuming three species of degenerate massive neutrinos and a $\Lambda$CDM model, the Planck collaboration has released the bound [3]

$$\sum_i m_i < 0.23 \text{ eV (95\% C.L.)},$$

obtained from a combined analysis of data [4]. Although this bound is not definite and requires confirmation by forthcoming experiments, its inclusion in the analysis of neutrino mass models may lead to important conclusions about the viability of a given model.

In this work, we shall perform a $\chi^2$-analysis using the standard $\chi^2$-function

$$\chi^2(x) = \sum_i \frac{(P_i(x) - \overline{O}_i)^2}{\sigma_i^2},$$

where $x$ denotes the physical input parameters (in our case, the matrix elements of the lepton mass matrices), $P_i(x)$ are the predictions of the Ansätze for the observables $O_i$, $\overline{O}_i$ are the best-fit values of $O_i$, and $\sigma_i$ are their corresponding $1\sigma$ errors. In our study, we make use of the current neutrino parameters at $1\sigma$, obtained in Ref. [4] from the global fit of neutrino oscillation data. Furthermore, we impose the cosmological constraint on the sum of the neutrino masses given in Eq. (2).

We shall fit the zero-textures of lepton mass matrices taking into account six observables: the mass-squared differences $\Delta m^2_{21}$, $\Delta m^2_{31}$, the mixing angles $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$, and the Dirac CP phase $\delta$. Since the Majorana phases are presently not constrained, we do not include them in the analysis. A given texture is considered to agree well with the experimental data if the model predictions for the physical observables in Eq. (3) are within the $1\sigma$ interval given in Table II. Thus, $\chi^2_{\text{min}} < 6$ is a necessary condition for a pattern to be consistent with all observations.

We remark that our approach to the determination of the charged lepton masses slightly differs from that of Ref. [13]. In our search for viable charged-lepton mass matrices, we always require that the eigenvalues of the input mass matrix correctly reproduce the central values of the charged lepton masses [20], i.e.

$$m_e = 0.510998928 \text{ MeV},$$
$$m_\mu = 105.6583715 \text{ MeV},$$
$$m_\tau = 1776.82 \text{ MeV}.$$
TABLE I. Neutrino oscillation parameters at 68% C.L. taken from Ref. [4]. The upper and lower rows in $\Delta m^2_{31}$, $\sin^2 \theta_{23}$, $\sin^2 \theta_{13}$, and $\delta$ correspond to normal (NO) and inverted (IO) neutrino mass ordering, respectively.

| Parameter | Best fit $\pm 1\sigma$ |
|-----------|----------------------|
| $\Delta m^2_{21}$ [$10^{-5}$ eV$^2$] | $7.60^{+0.19}_{-0.18}$ |
| $|\Delta m^2_{31}|$ [$10^{-3}$ eV$^2$] (NO) | $2.48^{+0.05}_{-0.07}$ |
| (IO) | $2.38^{+0.05}_{-0.06}$ |
| $\sin^2 \theta_{12}$/10$^{-1}$ | $3.23\pm0.16$ |
| $\sin^2 \theta_{23}$/10$^{-1}$ (NO) | $5.67^{+0.32}_{-1.24}$ |
| (IO) | $5.73^{+0.25}_{-0.39}$ |
| $\sin^2 \theta_{13}$/10$^{-2}$ (NO) | $2.26\pm0.12$ |
| (IO) | $2.29\pm0.12$ |
| $\delta/\pi$ (NO) | $1.41^{+0.55}_{-0.40}$ |
| (IO) | $1.48\pm0.31$ |

III. FGM TEXTURES

In this section we revisit the well-known FGM patterns for lepton mass matrices $\mathbf{m}_e$ consisting of $3 \times 3$ Majorana neutrino mass matrices $\mathbf{m}_\nu$ with two zero elements in the charged lepton flavor basis with $\mathbf{m}_e = \text{diag}(m_e, m_\mu, m_\tau)$ and thus this matrix is fixed. Moreover, one can easily show that the absolute value of any matrix element of $\mathbf{m}_\nu$ is always smaller than the largest neutrino mass, i.e. $|\langle \mathbf{m}_\nu \rangle_{ij}| < \max_k (m_k)$. Therefore, the cosmological bound in Eq. (2) implies $|\langle \mathbf{m}_\nu \rangle_{ij}| \lesssim 0.08$ eV.

Here, the symbol “∗” stands for arbitrary nonzero matrix elements. Clearly, the matrices $\mathbf{F}_i$ can be straightforwardly excluded since they lead to the decoupling of one generation and thus are not experimentally viable.

Our results are presented in Table II in which the minimum of $\chi^2$ for each FGM texture with a normal or inverted neutrino mass ordering is given. The results are obtained using the current neutrino oscillation data of Table I and imposing the upper bound on the sum of neutrino masses given in Eq. (3). We indicate with a check mark or a cross whether the texture predictions are or not within the $1\sigma$ interval given in Table II. Note that, in order to ease the reading of the table, whenever a given observable is simultaneously compatible (or incompatible) with data for NO and IO, we just indicate it with a single symbol, i.e. with a check mark (or a cross).

From Table II we conclude that patterns $\mathbf{A}_{1,2}$ and $\mathbf{B}_{1,2,3,4}$ are allowed for NO, while only patterns $\mathbf{B}_{1,3}$ and $\mathbf{C}$ are compatible with neutrino oscillation data for an IO mass spectrum at the $1\sigma$ level. We remark that, if the stringent upper bound on the sum of neutrino masses given in Eq. (2) is relaxed, pattern $\mathbf{C}$ is also allowed for a NO neutrino mass spectrum. In the latter case, we obtain $\chi^2_{\text{min}} \approx 0.32$ with $\sum_i m_i < 1$ eV.

For completeness, in Figs. [17] of Appendix A we present the probability distribution of the six neutrino

$^2$ The seven matrices were previously found to be compatible with neutrino oscillation data at the $1\sigma$ level for NO and IO mass spectrum [9].
TABLE II. The minimum of $\chi^2$ for the FGM zero-textures of the neutrino mass matrix with a normal (inverted) mass ordering. We use the data given in Table I and impose the upper bound on the sum of neutrino masses of Eq. (2). In all cases, the charged lepton mass matrix is $m_e = \text{diag}(m_e, m_\mu, m_\tau)$. We also indicate with a check mark or a cross whether the predictions are or not within the $1\sigma$ interval given in Table I.

| Majorana $m_\nu$ | $\chi^2_{\text{min}}$ NO (IO) | $\Delta m^2_{21}$ | $\Delta m^2_{31}$ | $\theta_{12}$ | $\theta_{23}$ | $\theta_{13}$ | $\delta$ |
|------------------|--------------------------------|-------------------|-------------------|----------------|----------------|----------------|------|
| A1               | $2.92 \times 10^{-1}$ (3.81 $\times 10^2$) | $\checkmark$ | $\checkmark$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark(\times)$ |
| A2               | $1.23 \times 10^{-2}$ (3.14 $\times 10^3$) | $\checkmark$ | $\checkmark$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark(\times)$ |
| B1               | $8.39 \times 10^{-1}$ (4.04 $\times 10^3$) | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| B2               | $3.39 \times 10^{-2}$ (1.02 $\times 10^4$) | $\checkmark$ | $\checkmark$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark$ | $\checkmark$ |
| B3               | $9.12 \times 10^{-1}$ (3.45 $\times 10^3$) | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| B4               | $2.10 \times 10^{-2}$ (1.11 $\times 10^4$) | $\checkmark$ | $\checkmark$ | $\checkmark(\times)$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| C                | $6.20 \times 10^2$ (1.04 $\times 10^{-1}$) | $\times(\checkmark)$ | $\checkmark$ | $\checkmark$ | $\checkmark(\times)$ | $\checkmark$ | $\checkmark(\times)$ |
| D1               | $1.33 \times 10^2$ (3.43 $\times 10^5$) | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| D2               | $2.82 \times 10^2$ (4.88 $\times 10^5$) | $\checkmark$ | $\checkmark$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark(\times)$ |
| E1               | $1.40 \times 10^1$ (1.15 $\times 10^2$) | $\checkmark$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark(\times)$ |
| E2               | $1.03 \times 10^2$ (1.14 $\times 10^2$) | $\checkmark$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark(\times)$ |
| E3               | $2.09 \times 10^1$ (1.17 $\times 10^2$) | $\checkmark$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark(\times)$ |

TABLE III. As in Table II, for the case of Dirac neutrinos. We present only the patterns for which the Dirac phase $\delta$ is different from 0 or $\pi$, leading to leptonic CP violation.

| Dirac $m_\nu$ | $\chi^2_{\text{min}}$ NO (IO) | $\Delta m^2_{21}$ | $\Delta m^2_{31}$ | $\theta_{12}$ | $\theta_{23}$ | $\theta_{13}$ | $\delta$ |
|---------------|--------------------------------|-------------------|-------------------|----------------|----------------|----------------|------|
| C             | $6.19 \times 10^2$ (1.04 $\times 10^{-1}$) | $\checkmark$ | $\times(\checkmark)$ | $\checkmark$ | $\checkmark$ | $\checkmark(\times)$ | $\checkmark(\times)$ |
| E1            | $1.40 \times 10^1$ (1.15 $\times 10^2$) | $\checkmark$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark$ | $\checkmark$ |
| E2            | $1.03 \times 10^2$ (1.14 $\times 10^2$) | $\checkmark$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark(\times)$ | $\checkmark(\times)$ |

Observables, obtained for the seven viable FGM textures $A_{1,2}, B_{1,2,3,4}$ and $C$, for both NO and IO mass spectra. We notice that textures in the same category lead in general to similar physical predictions for the observables.

We now consider the case of Dirac neutrinos. We analyze again the two-zero textures given in Eq. (5). These patterns have been recently studied for Dirac neutrinos in Ref. [24], where the authors concluded that only the patterns $A_{1,2}$ and $C$ are compatible with the oscillation data at the $2\sigma$ level.

First we note that by redefining the right-handed neutrino fields we can assume, without loss of generality, that the mass matrix $m_\nu$ is Hermitian. Furthermore, it is straightforward to show that if one off-diagonal matrix element is zero, then the invariant quantity $J_{\text{CP}} = \text{Im} \left[ U_{12} U_{23} U_{13}^\ast U_{22} \right]$ vanishes, and thus CP is conserved in the lepton sector. Therefore, only patterns $C$, $E_1$, and $E_2$ can lead to leptonic CP violation, while $\delta = 0$ or $\pi$ for the remaining two-zero patterns.

In view of the above, we shall only present the results for patterns $C$, $E_1$, and $E_2$. The minimum of $\chi^2$ is given in Table III. As can be seen from the table, there is essentially no difference with respect to the results obtained for Majorana neutrinos. Only pattern $C$ with an inverted hierarchy is allowed by current data. Relaxing the cosmological bound on the sum of the neutrino masses, we conclude that a normal hierarchical neutrino spectrum is also allowed for pattern $C$, with $\chi^2_{\text{min}} \simeq 0.29$ for $\sum_i m_i < 1$ eV. Notice also that the parameter counting for Hermitian Dirac matrices differs from that of symmetric Majorana matrices, since in the former case the counting depends on the position of the zeros. For two vanishing diagonal matrix entries, the matrix $m_\nu$ contains at most seven real parameters.

IV. HYBRID TEXTURES

Hybrid textures are particular cases of one-zero textures of the Majorana neutrino mass matrix, which additionally have two equal nonzero elements, and are defined in the flavor basis. There are $(6!/5! \times 5!/(2! \times 3!)) = 60$ possible hybrid textures. Among them, it has been shown that only 39 textures are compatible with current neutrino oscillation data at the $3\sigma$ level [12]. To keep a coherent notation, without the need of introducing any new classification scheme, we shall label these matrices as follows. We associate to each FGM matrix $M$ given in Eq. (5) a hybrid-type matrix $\hat{M}$, in which the two zeros in $M$ are replaced by equal nonvanishing elements in $\hat{M}$. Then, the position of the zero element in the hybrid matrix $\hat{M}$ is indicated with a subscript in parenthesis. Consider, for instance, the hybrid texture

$$
\begin{pmatrix}
X & X & \ast \\
X & \ast & \ast \\
\ast & \ast & 0
\end{pmatrix},
$$

(6)
where “X” stands for equal nonzero elements. Following the definition of the matrix $A_1$ given in Eq. [4], the hybrid matrix [6] would be represented as $A_{(13)}$ in our notation. Obviously, for each FGM texture in Eq. (5), the definition of the matrix $A_{(13)}$ would be represented as $B_{(11)}$, $B_{(12)}$, $B_{(13)}$, $B_{(21)}$, $B_{(22)}$, $B_{(23)}$, $B_{(31)}$, $B_{(32)}$, $B_{(33)}$, $B_{(41)}$, $B_{(42)}$, $B_{(43)}$, $D_{(11)}$, $D_{(12)}$, $D_{(13)}$, $D_{(21)}$, $D_{(22)}$, $D_{(23)}$, $D_{(31)}$, $D_{(32)}$, $D_{(33)}$, $D_{(41)}$, $D_{(42)}$, $D_{(43)}$, $F_{(11)}$, $F_{(12)}$, $F_{(13)}$, $F_{(21)}$, $F_{(22)}$, $F_{(23)}$, $F_{(31)}$, $F_{(32)}$, $F_{(33)}$, $F_{(41)}$, $F_{(42)}$, $F_{(43)}$, where we have indicated, inside curly brackets, the possible choices for the texture-zero position.

The results of the $\chi^2$-minimization are summarized in Tables IV and V. First we note that all textures given in Eq. (7) are compatible with data at the 1σ level either for NO, IO or both types of neutrino mass spectrum. In particular, the patterns $A_{(12)}$, $A_{(23)}$, $B_{(11)}$, $B_{(12)}$, $B_{(22)}$, $B_{(13)}$, $B_{(23)}$, $D_{(12)}$, $D_{(13)}$, $D_{(22)}$, $D_{(23)}$, $D_{(32)}$, and $F_{(12)}$ turn out to be compatible with experimental data for NO and IO mass spectrum.

Among the sixty possible hybrid patterns for Majorana neutrinos, it turns out that only six can be completely excluded, since they fail in reproducing the data for any hierarchy. These are the matrices $B_{(11)}$, $B_{(21)}$, $B_{(31)}$, $B_{(41)}$, $F_{(21)}$, and $F_{(31)}$, all having the texture-zero in the (1,1) position.

In the case of Dirac neutrinos, thirty patterns were considered, which are listed in Table VI. We include all the Hermitian patterns that do not have any off-diagonal zero element, and thus may lead to Dirac-type CP violation. For the remaining patterns, the Dirac phase $\delta$ is always 0 or $\pi$ and CP is conserved in the lepton sector. Looking at the table we note that only twelve textures are consistent with data either for NO or IO neutrino mass spectrum. These are the matrices $B_{(33)}$, $B_{(22)}$, $B_{(33)}$, $B_{(22)}$, $B_{(33)}$, $B_{(22)}$,
to textures with the same physical content. Indeed, they
are not allowed in a scheme with a diagonal and ordered
mass spectrum.

V. PARALLEL TEXTURES

In this section, we perform a systematic $\chi^2$-analysis
of lepton mass matrices that exhibit the same texture,
i.e. with $m_l$ and $m_\nu$ having their zeros located at the
same positions. Besides the possibility of implementing a
universal flavor structure in the context of grand unified
models, there is an additional theoretical motivation for
considering parallel textures. It is well known that an
attractive and economical framework to generate small
neutrino masses is the seesaw mechanism. In its sim-
plest type-I realization, three right-handed neutrinos are
added to the standard model particle content. It is then
conceivable that the presence of family symmetries en-
forces texture-zero structures in the Dirac neutrino mass
matrix $m_D$ and the heavy Majorana mass matrix $M_R$,
which, in some cases, could be preserved by the effective
neutrino mass matrix $m_\nu = -m_D M_R^{-1} m_D^T$.3

It is worth noticing that any permutation transformation
acting on parallel patterns is allowed, since it leads to
textures with the same physical content. Indeed, they
can be related by a weak basis transformation, performed
by a permutation matrix $P$,

$$m'_l = P^T m_l P, \quad m'_\nu = P^T m_\nu P \quad (8)$$

which automatically preserves the parallel structure, but
changes the position of the zeros. The matrix $P$ belongs
to the group of six permutations matrices, which are isomor-
phic to the symmetry group $S_3$.

### A. Two-zero textures

The FGM-type Ansätze can be classified into four weak
basis equivalent classes (or permutation sets) $[(11)]:$

$$\text{Class I:} \quad A_1, A_2, B_3, B_4, D_1, D_2;$$

$$\text{Class II:} \quad B_1, B_2, E_3;$$

$$\text{Class III:} \quad C, E_1, E_2;$$

$$\text{Class IV:} \quad F_1, F_2, F_3.$$

It is clear that class IV is not experimentally viable, since
it always leads to the decoupling of one generation. Note
also that the weak basis transformations given in Eq. 8
are not allowed in a scheme with a diagonal and ordered
charged lepton mass matrix, as in the texture schemes
discussed in previous section.

In our $\chi^2$-analysis, all parallel FGM textures with ar-
bitrary complex Hermitian (or real symmetric) $m_\ell$
and complex symmetric $m_\nu$ were found to be viable for both
normal and inverted neutrino mass ordering. Similar re-
results were obtained for Dirac neutrinos with Hermitian
neutrino mass matrices.

We have also considered the feasibility of arbitrary
complex Hermitian $m_\ell$ and real symmetric $m_\nu$. In
this case, the number of physical parameters is equal
to 10 for classes I and II, while for class III there are
11 parameters since, in general, the invariant quantity
$\text{arg}(m_\ell_{12} m_\ell_{13} (m_\ell_{23})^*)$ does not vanish. As far as the
analysis of the neutrino oscillation data is concerned,
there is no distinction between Majorana or Dirac neutrinos. The minimum of $\chi^2$ was always found to be much smaller than one, so that all patterns in classes I, II, and III are consistent with neutrino data for any mass hierarchy.

### B. Three-zero textures

There are only 6 possible three-zero parallel textures that can be constructed for both the charged-lepton and Majorana neutrino mass matrices. Since these matrices are related by weak basis transformations (permutations), they all have the same physical content and thus lead to the same predictions. We denote them by

\[
\begin{align*}
A_{(12)} & , A_{(22)} , B_{(13)} , B_{(22)} , D_{(11)} , D_{(21)} ,
\end{align*}
\]

where the subscript in parenthesis refers to the position of the additional zero in the corresponding two-zero texture given in Eq. (10). Note that the matrix $A_{(11)}$ is not included in the above list since it is traceless and, therefore, incompatible with the lepton masses. Furthermore, textures with null determinant or those leading to the decoupling of one generation have also been excluded.

For an arbitrary complex Hermitian $m_\nu$ and a complex symmetric $m_\nu$ (Majorana neutrinos), the above parallel 3-zero textures contain 9 physical parameters. No viable solution was found either for NO ($\chi^2_{min} \simeq 74$) or IO ($\chi^2_{min} \simeq 182$) neutrino mass spectrum. For a normal ordering of neutrino masses, all the textures fail in reproducing the three mixing angles, while for an inverted spectrum the mixing angles $\theta_{12}$ and $\theta_{23}$, and the phase $\delta$ did not satisfy the required $\chi^2$ criteria.

In Fig. 8 of Appendix B we present the probability distribution of the neutrino observables, obtained for the textures given in Eq. (10), for NO and IO mass spectrum, respectively. For Dirac neutrinos, with the matrix $m_\nu$ being Hermitian, similar results were found, thus excluding these patterns for both NO and IO mass spectra.

\[^4\text{Our conclusions do not agree with the result of Ref. 25, in which the parallel texture } A_{(22)} \text{ is found to be compatible with the experimental data.}\]
| $\text{Dirac } m_\nu$ | $\chi^2_{\text{min}}$ (IO) | $\Delta m^{2}_{21}$ | $\Delta m^{2}_{31}$ | $\theta_{12}$ | $\theta_{13}$ | $\theta_{23}$ | $\delta$ |
|----------------|----------------|-----------------|----------------|-----------|------------|-----------|-----|
| $\mathcal{A}_{1(2)}$ | $1.16 \times 10^4$ ($3.15 \times 10^3$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{A}_{1(33)}$ | $1.31 \times 10^4$ ($3.09 \times 10^3$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{B}_{2(22)}$ | $5.65$ ($4.35 \times 10^3$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{A}_{2(33)}$ | $7.65 \times 10^4$ ($5.01 \times 10^4$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{B}_{1(11)}$ | $6.91$ ($3.15 \times 10^2$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{B}_{1(33)}$ | $1.72 \times 10^{-2}$ ($5.40 \times 10^1$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{B}_{2(21)}$ | $3.07 \times 10^4$ ($3.80 \times 10^2$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{B}_{2(22)}$ | $9.05 \times 10^{-1}$ ($4.16 \times 10^1$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{B}_{2(31)}$ | $6.37$ ($1.08 \times 10^2$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{B}_{3(33)}$ | $5.65 \times 10^{-3}$ ($1.41 \times 10^1$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{B}_{3(4)}$ | $2.62 \times 10^4$ ($1.18 \times 10^2$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{C}_{2(2)}$ | $9.89 \times 10^{-1}$ ($3.43 \times 10^1$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{C}_{3(11)}$ | $4.51 \times 10^{-1}$ ($1.08 \times 10^2$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{D}_{3(11)}$ | $1.60 \times 10^4$ (2.62) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{D}_{2(11)}$ | $1.49$ ($1.08 \times 10^2$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{E}_{3(33)}$ | $7.76 \times 10^4$ ($7.34 \times 10^2$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{E}_{3(4)}$ | $5.79 \times 10^1$ (8.78) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{E}_{4(22)}$ | $1.33 \times 10^2$ ($1.92 \times 10^{-3}$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{E}_{3(4)}$ | $2.80 \times 10^2$ ($4.02 \times 10^{-1}$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{F}_{1(11)}$ | $2.94 \times 10^{-2}$ ($1.08 \times 10^2$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{F}_{1(2)}$ | $1.37 \times 10^4$ (2.12) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{F}_{1(33)}$ | $9.53 \times 10^1$ (3.50) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{F}_{2(11)}$ | $2.00 \times 10^4$ ($3.68 \times 10^2$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{F}_{2(22)}$ | $9.31$ ($2.40 \times 10^1$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{F}_{2(33)}$ | $2.50 \times 10^2$ (1.34) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{F}_{3(11)}$ | $1.86 \times 10^4$ ($3.07 \times 10^2$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{F}_{3(22)}$ | $1.04 \times 10^4$ ($1.22 \times 10^4$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\mathcal{F}_{3(33)}$ | $1.95 \times 10^4$ ($2.37 \times 10^1$) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

TABLE X. The minimum of $\chi^2$ for Dirac-type hybrid textures. We present only the patterns for which the Dirac phase $\delta$ is different from 0 or $\pi$ and CP is violated.

VI. CONCLUSIONS

There has been lately a revival of the interest in texture-zero models that aim at explaining the flavor structure observed in lepton mass matrices. In this work, we have confronted various popular texture-zero Ansätze of lepton mass matrices with current neutrino data. We have performed a thorough $\chi^2$-analysis in a wide class of schemes, considering Hermitian charged lepton mass matrices in combination with symmetric Majorana or Hermitian Dirac neutrino mass matrices. In our study we included the well-known FGM textures, the so-called hybrid textures, as well as parallel patterns. We concluded that while a significant number of these patterns is still consistent with all the observations at 68.27% C.L., there are several textures that can be excluded or are marginally allowed.

It is well known that texture-zero models have in general a weak predictive power. We have not addressed here the question of the predictability of a given texture. This issue is beyond the scope of the present work. The reader is referred, e.g., to Ref. [18], in which the authors attempted to identify predictive classes of texture zeros by defining numerical measures of predictability. For instance, maximally restrictive Majorana textures can predict, in most cases, the effective neutrino mass parameter $m_{\nu} = |\sum_{i} U_{ei}^2 m_i|$, relevant in neutrinoless double beta decays.

The precise measurements of neutrino oscillation parameters in upcoming experiments (including the determination of the absolute neutrino mass scale and the Dirac CP phase, and the improvement of the bounds on the sum of neutrino masses and the effective mass in $0\nu\beta\beta$ decays) are expected to shed some light on the flavor structure of the neutrino sector. This in turn would allow us to determine, among the plethora of texture-zero patterns, what are the most predictive textures capable of explaining the experimental data, as well as those that...
are disfavored or excluded at a high confidence level.

ACKNOWLEDGEMENTS

We are grateful to M. Nebot and S. Palomares-Ruiz for useful discussions and comments. The work of D.E.C. was supported by Associação do Instituto Superior Técnico para a Investigação e Desenvolvimento (IST-ID) and by Fundação para a Ciência e a Tecnologia (FCT) through the project CERN/FP/123580/2011. D.E.C. and R.G.F. acknowledge support from FCT through the project PTDC/FIS-NUC/0548/2012 and thank CERN Theoretical Physics Unit for hospitality and financial support.

Appendix A: Neutrino observables for the FGM two-zero textures

For completeness, in this appendix we present the probability distribution of the neutrino observables for the viable FGM patterns $A_{1,2}$, $B_{1,2,3,4}$ and $C$ for NO and IO mass spectra.

Appendix B: Neutrino observables for three-zero parallel textures

Here we present the probability distribution of neutrino observables for the three-zero parallel patterns given in Eq. (10), in the case of Majorana neutrinos. As in the case of the FGM textures, $10^4$ random input mass matrices for charged leptons and neutrinos were generated. The results are presented in Fig. 8. Similar distributions are obtained if neutrinos are Dirac particles.
FIG. 1. The probability distribution of neutrino observables for pattern $A_1$ in the case of Majorana neutrinos. The vertical dashed line denotes the best-fit value of the observable in the case of a normal ordering of the neutrino mass spectrum.

FIG. 2. The probability distribution of neutrino observables for pattern $A_2$ in the case of Majorana neutrinos.
FIG. 3. The probability distribution of neutrino observables for pattern $B_1$ in the case of Majorana neutrinos.

FIG. 4. The probability distribution of neutrino observables for pattern $B_2$ in the case of Majorana neutrinos.
FIG. 5. The probability distribution of neutrino observables for pattern $B_3$ in the case of Majorana neutrinos.

FIG. 6. The probability distribution of neutrino observables for pattern $B_4$ in the case of Majorana neutrinos.
FIG. 7. The probability distribution of neutrino observables for pattern $C$ in the case of Majorana neutrinos.

FIG. 8. The probability distribution of neutrino observables for the three-zero parallel patterns given in Eq. (10), in the case of Majorana neutrinos. Similar results are obtained for Dirac neutrinos.