Attitude dynamics of nanosatellite with a module on retractable beams

V S Aslanov1, A V Doroshin2, A V Eremenko3
Samara National Research University, 34, Moscovskoe shosse, Samara, 443086, Russia
1E-mail: aslanov_vs@mail.ru, aslanov.ssau.ru
2E-mail: doran@inbox.ru, doroshin.ssau.ru
3E-mail: huntergalaxy@bk.ru

Abstract. Attitude dynamics of nanosatellites with a module on retractable beams is considered. The movable module is attached to the main body of the nanosatellite with the help of retractable beams of variable length. The attitude control is fulfilling by the beams length changing that translates the position of the mass center and creates the torque from the jet-engine with constant fixation relative the main body. The movable module can represent the multifunctional element, e.g. telescope, radiometer etc. Such control scheme allows to use the functional module as the simple actuator of the control system. So in this case the control don’t need the presence of special executive bodies.

1. Introduction
Research of the attitude dynamics of the spacecraft still is the important task of the modern space flight dynamics [1], which topicality increases due to developing the modern schemes of the nanosatellites with simplest control systems, including the control systems with the movable internal masses [2-6].
In this paper the attitude motion of the nanosatellite (NS) with one movable module on retractable beams of variable length (figure 1). The attitude control is fulfilling by the beams length changing that translates the position of the mass center and creates the torque from the jet-engine with constant fixation relative the main body. The movable module can represent the multifunctional element, e.g. telescope, radiometer etc. Such control scheme allows to use the functional module as the simple actuator of the control system. So in this case the control don’t need the presence of special executive bodies.

2. The mathematical model
Let us consider the compound structure of the nanosatellite with one movable module (Figure 1), which is connected by the retractable beams with variable length. The sequence of rotation angles of the main body of the nanosatellite relative the inertial space is following: $X \rightarrow Y \rightarrow Z$, by angles $\alpha_1, \beta_1, \gamma_1$. 
The coordinates systems are:
1. OXYZ – the coordinate frame with the origin in the mass center, which axes are parallel to the main axes of the main body;
2. O1X1Y1Z1 – the frame with the origin in mass center of the main body, which axes are parallel to the main axes of the main body;
3. O2X2Y2Z2 – the main connected frame of the movable module.

The changing length of the retractable beams allows to change the position of the mass center of system and, therefore, it allows to change the torque from a piecewise-constant jet-engine force \( P \).

At small angles of the rotation of the movable module it is possible to neglect the terms from the curvature of retractable beams. In this case we can obtain the mathematical model with the help of the angular momentum law:

\[
\frac{d\mathbf{K}}{dt} = \omega_1 \times \mathbf{K} + \mathbf{M},
\]

where \( \mathbf{K} \) – is the angular momentum of the system, \( \omega_1 \) – the angular velocity of the NS main body, \( \mathbf{M} \) – the external torque. The angular momentum is the sum of the angular momentum of the main body \( \mathbf{K}_1 \) and the moment on movable module \( \mathbf{K}_2 \):

\[
\mathbf{K} = \mathbf{K}_1 + (\sigma_1 \mathbf{K}_2),
\]

where \( \sigma_1 \) – is the translation matrix into the frame \( \mathbb{C}_2X_2Y_2Z_2 \) from the frame \( \mathbb{O}_1X_1Y_1Z_1 \). The angular momentum of the main body is:

\[
\mathbf{K}_1 = \mathbf{I}_1 \omega_1 + \mathbf{C}_1 \times M_1 \mathbf{V}_1,
\]

where \( \mathbf{I}_1 \) – the tensor of inertia of the main body, \( \mathbf{C}_1 \) – the radius-vector of the mass center of the main body in \( \mathbb{C}_1X_1Y_1Z_1 \), \( M_1 \) – the mass of main body, \( \mathbf{V}_1 \) – the linear velocity of the mass center of the main body in projections on axes of the frame \( \mathbb{C}_1X_1Y_1Z_1 \).

The angular momentum of the movable module is, analogously:
\[ K_2 = I_2 \omega_2 + CC_2 \times M_2 V_2, \]  

where all parameters are calculated in the frame \( C_2X_2Y_2Z_2 \).

Let us calculate the linear velocity of the bodies:

\[ V_1 = \frac{dCC_1}{dt} + \omega_1 \times CC_1. \]  

\[ V_2 = \frac{dCC_2}{dt} + \omega_2 \times CC_2. \]

At the assumptions that the angular deviations of the movable body from the main body, we can write:

\[ \frac{dCC_1}{dt} \cong 0; \quad \frac{dCC_2}{dt} \cong 0 \]  

The radius-vector of the mass center of the SN is (in the frame \( C_1X_1Y_1Z_1 \)):

\[ R_c = \frac{M_1}{(M_1 + M_2)} OC_1 + \frac{M_2}{(M_1 + M_2)} (\sigma_1 OC_2), \]

And

\[ CC_1 = OC_1 - R_c. \]  

\[ CC_2 = OC_2 - (\sigma_2 R_c), \]

where \( \sigma_2 \) – the translation matrix from \( C_1X_1Y_1Z_1 \) into \( C_2X_2Y_2Z_2 \).

The components of the angular velocity of the main body in its own connected frame are:

\[ \omega_1 = \begin{bmatrix} p_1 \\ q_1 \\ r_1 \end{bmatrix}, \]

The movable modus has two degrees of freedom, which correspond to two relative rotations around axes \( X, Y \) on angles \( \alpha_2 \) and \( \beta_2 \).

\[ \omega_2 = \begin{bmatrix} p_2 \\ q_2 \\ r_2 \end{bmatrix} = \begin{bmatrix} \alpha_2 \cos \beta_2 \\ \hat{\beta}_2 \\ 0 \end{bmatrix} + (\sigma_2 \omega_1), \]

where \( p_2, q_2, r_2 \) – are components of the movable module’s velocity. Assume the general tensors of inertia:

\[ I_1 = \text{diag}(A_1, B_1, C_1), \]
\( \mathbf{I}_2 = \text{diag}(A_2, A_2, C_2) \) \hspace{1cm} (14)

The torque from the jet-force is:

\[ \mathbf{M} = \mathbf{C}_1 \times \mathbf{F}, \] \hspace{1cm} (15)

where \( \mathbf{F} \) – is the vector of the jet-force:

\[ \mathbf{F} = \begin{bmatrix} 0 \\ 0 \\ P \end{bmatrix}, \] \hspace{1cm} (16)

and \( P = \text{const} \). Then the torque (15) has the simple shape:

\[ \mathbf{M} = \begin{bmatrix} mP \sin \alpha z_2 \\ mP \cos \alpha \sin \beta z_2 \\ 0 \end{bmatrix} \] \hspace{1cm} (17)

where \( m = \frac{M_2}{(M_2 + M_1)} \), \( z_2 \) – the distance between the point \( O \) and the mass center of the movable module.

The main kinematical equations are:

\[
\begin{align*}
\ddot{\alpha}_1 &= \frac{p_1 \cos \gamma_1 - q_1 \sin \gamma_1}{\cos \beta_1} \\
\dot{\beta}_1 &= q_1 \cos \gamma_1 + p_1 \sin \gamma_1 \\
\ddot{\gamma}_1 &= q_1 \sin \gamma_1 \sin \beta_1 - p_1 \cos \gamma_1 \sin \beta_1 + r_1 \cos \beta_1
\end{align*}
\] \hspace{1cm} (18)

The complete equations of the angular motion of the NS as the result will be very complex, and therefore we don’t present the explicit form. To simplification of the consideration we assume the smallest of the angles \( \alpha_2 \) и \( \beta_2 \). Moreover, in this paper, we present only simplest form of the planar motion \( (p_1 = \text{const} , \ q_1 = 0 \ , \ r_1 = 0) \) to show the most important properties of the dynamics:

\[
\begin{align*}
\dot{\gamma}_1 &= 2M_1M_2 \sin \alpha_2 z_2 z_2 \left( \dot{\alpha}_2 p_1 + \dot{\alpha}_2 \right) + M_2 (M_1 + M_2) \\
&\quad + M_1 \left( A_1 + A_2 \right) + M_1M_2 \left( z_2^2 + z_2^2 + z_2^2 \cos \alpha_2 \right) \\
\ddot{\gamma}_1 &= p_1
\end{align*}
\] \hspace{1cm} (19)

where \( z_1 \) – the distance between the mass center of the main body and the point \( O \) (Figure 1) in the frame \( C_1X_1Y_1Z_1 \), \( z_2 \) – the distance between the mass center of the movable module and the point \( O \) in \( C_2X_2Y_2Z_2 \), \( M_x \) – the component of the jet-torque along \( X_1 \).

\[ M_x = \frac{M_2 z_2 P \sin \alpha_2}{M_1 + M_2} \] \hspace{1cm} (20)

3. Modeling dynamics of NS
Let us demonstrate the motion dynamics of the NS in the planar case. The parameters of NS are presented in the table 1.

**Table 1. Parameters for the calculation**

| Parameters                           | Value       |
|--------------------------------------|-------------|
| Mass of the main body [kg]           | 3           |
| Mass of the movable module [kg]      | 2           |
| \(A_1\) [kg*m^2]                     | 0.013       |
| \(B_1\) [kg*m^2]                     | 0.01        |
| \(C_1\) [kg*m^2]                     | 0.005       |
| \(A_2\) [kg*m^2]                     | 0.0025      |
| \(C_2\) [kg*m^2]                     | 0.0025      |
| \(OC_1\) [m]                         | 0.1         |
| \(OC_2\) [m]                         | 0.1         |

3.1. The reorientation of NS from the quiescence.

The algorithm of the reorientation is composed from the following steps:

- The definition of the plane of the reorientation.
- The definition of the angle value.
- The calculation of the direction of the relative deviation of the movable module, time of the action of the jet-force and time of passive part of motion.
- The fulfillment of the relative deviation of the movable module, and initiation of the jet-force.
- The termination of the jet-force and the realization of the passive part of the rotation.
- The initiation of back-jet-force.
- The termination of the back-jet-force and the fulfillment of the back-deviation of the movable module relative the main body.

The law of relative deviation:

\[
\alpha_2 = \begin{cases} 
0 \leq t \leq t_1 : & 0 \\
t_1 \leq t \leq t_2 : & c(t - t_1) \\
t_2 \leq t \leq t_3 : & ct_r \\
t_3 \leq t \leq t_4 : & t_c - 2c(t - t_3)t_r/t_p \\
t_4 \leq t \leq t_5 : & -ct_r \\
t_5 \leq t \leq t_6 : & -c(t_r + t - t_5) \\
t \leq t_6 : & 0 
\end{cases},
\]

where \(t_i\) – time-points of steps of the algorithm, \(c\) – the constant of control, \(t_p\) – the time of passive part. The law of the jet-force:
$P = \begin{cases} 
0 < t < t_2 : & 0 \\
 t_2 < t < t_3 : & c_p \\
 t_3 < t < t_4 : & 0 \\
 t_4 < t < t_5 : & c_p \\
 t > t_5 : & 0 
\end{cases}, \quad (22)$

where $c_p$ – the value of the jet-force. The time-points are:

\begin{align*}
t_2 &= t_1 + t_r \\
t_3 &= t_2 + t_r \\
t_4 &= t_3 + t_p \\
t_5 &= t_4 + t_r \\
t_6 &= t_5 + t_r 
\end{align*} \quad (23)

where $t_r$ – is the time of working the jet-engine.

The initial conditions are: $p_1 = 0 \, [s^{-1}], \; q_1 = 0 \, [s^{-1}], \; r_1 = 0 \, [s^{-1}], \; \alpha_1 = 0 \, [\text{rad}], \; \alpha_2 = 0 \, [\text{rad}], \; c_p = 2 \, [\text{N}], \; t_1 = 1 \, [s], \; t_r = 2 \, [s], \; t_p = 2 \, [s]$, the time of modeling corresponds to $12 \, [s]$. The results are presented at the figures 2-3.

**Figure 2.** The time-evolution of $p_1(t)$.

**Figure 3.** The time-evolution of $\alpha_1(t)$. 
As we can see from figure 3, the proposed algorithm can be applied to control of the motion, and to reorientation of the NS.

4. The analytical estimation of the angle of the reorientation of NS

Assume the absence of the change of the main body position at the small passive rotation of the movable module, then we can write equation (19) in simplified form:

\[
\begin{align*}
\dot{p}_1 &= \frac{M_1 (M_1 + M_2)}{(M_1 + M_2)(A_1 + A_2) + M_1 M_2 (z_1^2 + z_2^2 + z_1 z_2 \cos \alpha_2)} \\
\dot{\alpha}_1 &= p_1
\end{align*}
\] (24)

After substituting (20), (21) we take as the result:

\[
\begin{align*}
\dot{p}_1 &= \frac{PM_2 \sin(ct_\tau)z_2}{(M_1 + M_2)(A_1 + A_2) + M_1 M_2 (z_1^2 + z_2^2 + z_1 z_2 \cos(ct_\tau))} = \rho \\
\dot{\alpha}_1 &= p_1
\end{align*}
\] (25)

And then the angle of reorientation \( \alpha_1 \) can be calculated as:

\[
\alpha_1 = \rho t_r^2 + \rho t_r t_p
\] (26)

If we use the values from the table 1, then the following result is actual \( \alpha_1 = 1.504 \), that quite corresponds to the direct integration (Figure 3).

5. Conclusion

In this paper the modeling of the angular reorientation of NS is fulfilled in the case of the simplest planar motion. The reorientation can be realized with the help of changing the length of the retractable beams, which rotate the movable module relative the main body. Other schemes of the constrains of retracted beams, and control laws can be used – this is the next topic of our research.

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