Clock synchronization with correlated photons

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Event synchronisation is a ubiquitous task, with applications ranging from 5G technology to industrial automation and smart power grids. The emergence of quantum communication networks will further increase the demand for precise synchronization in the optical and electronic domains, which implies significant resource overhead, such as the requirement for ultrastable clocks or additional synchronization lasers. Here we show how temporal correlations of energy-time entangled photons may be harnessed for synchronisation in quantum networks. We achieve stable synchronisation jitter \( < 68 \text{ ps} \) with as few as 44 correlated detection events per 100-ms data package and demonstrate feasibility in realistic emulated high-loss link scenarios, including atmospheric turbulence. In contrast to previous work, this is accomplished without any external timing reference and only simple crystal oscillators. Our approach replaces the optical and electronic transmission of timing signals with classical communication and computer-aided postprocessing. It can be easily integrated into a wide range of quantum communication networks and could pave the way to future applications in entanglement-based secure time transmission.

I. INTRODUCTION

Secure transmission of information and reliable event synchronisation are key requirements in critical infrastructures [1, 2], especially in power grids [3], financial networks [4], and cloud database services [5]. Regarding information security, quantum key distribution (QKD) offers the unique proposition of encryption keys whose confidentiality can be lower bounded by the laws of physics [6–9]. Quantum communication is thus poised to become a backbone of a secure information infrastructure, with networks of many hundreds of fibre links established, and integration of ground-space relays over more than 1,000 kilometres already underway[10].

Quantum communication networks, which involve tight timing budgets in the optical and electronic domains, are also a prime example for the need for accurate synchronisation of remote parties. In classical communication net-
Quantum clock synchronization protocols have been proposed to tackle synchronizing distant clocks [31–33] in multi-partite network settings [34, 35] and with quantum enhancement beyond the possibilities of classical physics [36]. To this end, investigations over the last years have focused on the exploitation of the temporal correlations of time-energy entangled photons [37–40]. When photon pairs originate from a common creation event, this gives rise to a narrow correlation peak in their arrival time at remote receivers. In other words, the location of this correlation peak indicates the average time delay between the receivers and can thus be used to extract the time and frequency differences between remote clocks [41, 42]. The recent past has seen further advancement with experiments achieving exceptionally high precision with respect to a common frequency reference [43]: 600 fs with correlated photons [37], or down to 60 fs using Hong–Ou–Mandel interference [38–40]. In deployed long-distance links, synchronization jitters as low as a few tens of picoseconds or GPS-disciplined clocks [19, 21] have been demonstrated using ultraprecise rubidium clocks or GPS-disciplined clocks [19, 21, 44]. Without atomic frequency references, the timing jitter is orders of magnitude higher (e.g., the frequency offset for standard crystal oscillators may be up to 8 orders of magnitude higher [45, 46]). This frequency difference results in a temporal drift of the correlation peak and makes it impossible to find the initial timing offset for synchronization. This has limited the applicability of synchronization based purely on quantum correlations [42]. Additional rubidium clocks, GPS-disciplined clocks, or other common time references have long been a necessary requirement for deploying long-distance and high-bit rate quantum communication systems [16, 18, 19, 21].

In this article, we build on the ground-breaking work of Ho et al. [42] and Valencia et al. [41] and establish the feasibility of picosecond-level synchronization using the correlations of photon pairs in realistic link scenarios. Unlike state-of-the-art field experiments, which employ active electro-optic modulation to actively encode synchronization sequences [47–49], we accomplish this purely in postprocessing, without any requirement for auxiliary hardware. Our protocol (section II A) estimates the clock frequency difference and compensates for the resulting broadening of correlation peaks (section II B 1). This allows us to find the correlation peak, even in low signal-to-noise scenarios. Tracking the position of the correlation peak during the communication session allows us to correct for residual clock instabilities and achieve synchronization with root-mean-square (RMS) jitters < 68 ps (section II B 2). What is remarkable is that the approach also works when correlated detection events are as low as 440 ± 200 cps, as is expected in real lossy links scenarios. Even more remarkably, these values are completely comparable to 30–50 ps jitters reported for systems that employ high-performance GPS-disciplined rubidium clocks [19, 21], Finally, we establish the feasibility of the protocol for communication links through turbulent atmosphere, where channel fades have a major impact on the detected photons statistics (section II C) [50].

The results of our proof-of-concept experiment show that time-correlated photon pairs can be a valuable resource for synchronization with minimal hardware overhead. The methods work not only for laboratory experiments, but also for deployed QKD systems in high-loss link scenarios. In this approach, synchronization is a by-product of the key exchange without compromising the secure key rate. It provides a simple way of enhancing the timing resolution in distributed quantum information processing tasks. The core of the algorithm is not limited to correlated photon pairs and applies universally to any correlation features. In particular, it is readily extended to prepare-and-measure approaches [6]. Our method for software-based synchronization in postprocessing can thus be implemented straightforwardly in state-of-the-art QKD systems and paves the way towards quantum networks [51] with improved synchronization performance, as well as entirely new applications such as secure time transfer [52, 53].

II. RESULTS

A. Synchronization protocol

The performance of remote clocks used in communication networks has a great impact on the most appropriate method for their synchronization. High performance oscillators (e.g. Rubidium clocks) provide a precise frequency, but also exhibit small but constant difference frequencies (or clock frequency skew). In analogy to archery, the archer hits a spot on average, but off-center (see Fig. 1). These oscillators provide high accuracy, i.e., small clock drift, as well – the frequency variation over time is small (the archer hits an aimed spot with small scatter). Both a small clock drift and clock frequency skew are necessary to maintain low synchronization timing jitters (the archer always hits the same spot that corresponds to the mark). Such precise and stable references relax the requirements on synchronization protocols and are still commonly employed in experiments [19, 21, 44]. The focus of the proposed work are general oscillators, which are widely used but exhibit a frequency skew that is
orders of magnitude higher. Hence, any synchronization protocol for such oscillators will rely crucially on an initial compensation of frequency skew - especially in real link scenarios with low signal-to-noise ratio. Their strong and nonlinear drifts also make live tracking of the correlation with short feedback loop times necessary. Furthermore, the optical link characteristics play a crucial role, since it is the only mediator for timing information.

To this end, we have established a synchronization protocol, which consists of four main steps, where each step reduces the timing uncertainty further - from coarse millisecond timing down to a picosecond level or smaller (Fig. 2a). In our experimental implementation, we use entangled photon pairs from an untrusted source as a mediator of timing signals between distant parties (Fig. 2b). The photons are produced at 810 nm via spontaneous parametric down-conversion with a 405-nm continuous-wave pump laser, and exhibit an intrinsic timing correlation jitter of < 2.5 ps (spectral bandwidth 0.4 ± 0.17 nm). The first part of the correlation-based synchronization protocol is to align the data packages at the two receivers. The electronic time taggers generate time tags for single-photon detection events relative to their respective internal clocks. These time tags are stored on a personal computer in the form of data packages, each with an acquisition time of approximately 100 ms. To detect the photons, we employ Si-SPAD detectors with a photon detection efficiency of approximately 60%, dark count rates of 300 cps and a timing jitter of approximately 140-180 ps (RMS). As a very first step in the initialization, we use classical network pinging through the NTP [11]. This establishes coarse millisecond synchronization and ensures that the data packages to be compared carry correlated detection events - this procedure is not part of the paper. The next step is to identify the correlation peak in these data packages. However, due to the clock frequency skew, the position of the correlation peak is itself a function of time, which results in significant spreading of the correlation peak over the 100 ms integration time. The clock frequency skew could be computed by observing a moving correlation peak, which is a typical chicken-and-egg problem, as it implies that the peak has already been identified [42]. This becomes especially critical in low-signal scenarios, where it is impossible to recover correlation features (see Methods IV A for impact of signal and clock frequency skew on correlation peaks). To address this issue, we introduce a coarse clock frequency skew compensation before identifying the correlation peak with high visibility in the second step. This reduces the frequency skew of typical quartz oscillators from approx. 20 µs/s to ≤1 µs/s, and thereby reduces the spreading of the correlation peak to ≤ 100 ns over the typical acquisition time of 100 ms. By squeezing the correlation peak, it also enhances the signal-to-noise ratio, which makes synchronization in high-loss communication scenarios possible. In the third step,

FIG. 1. Clock performance impacts. The frequency of real, imperfect clocks differs from a nominal value and varies over time. The effect is much stronger for general oscillators (e.g. quartz oscillators) than precise and stable references (e.g. rubidium clocks). The compensation of frequency skew (i.e. frequency difference of clocks) increases the accuracy, and further enhancements to the precision are made through live tracking of the clock drift (i.e. time-varying clock frequencies). The correlation peak in time-domain shows significant timing jitter and may even be asymmetric, because of strong frequency variation over time in general oscillators. Clock frequency skew compensation and live tracking can reduce the additional timing jitter from poor-performing clocks to a minimum.
we cross-correlate the time tags of matched data packages using the fast Fourier transform (FFT) convolution and derive the precise time difference between the master and slave, i.e., the timing offset. Optionally, we fine-tune the clock frequency skew through calculating the correlation peak over the Start-Stop method [54–56]: a photon arriving at Alice (Bob) starts the counter and the second photon at Bob (Alice) stops it. The collection of time differences in a histogram represents the cross-correlation. This is computationally substantially more efficient than FFTs because it considers only two subsequent and neighboring time tags (as opposed to all, in the case of FFTs, see Fig. 2c). The downside of this cross-correlation method is that it requires a good initial estimate of the timing offset. Specifically, the precision of the timing offset must be smaller than the inverse single photon count rate (see Methods IV D for more information). In summary, after data package synchronization (step 1), we match the clock frequencies in step 2 and estimate the timing offset in step 3. These are the requirements to start the quantum communication session. During a long communication session, the frequency of unstable clocks will change unpredictably. It follows a frequency mismatch between the two clocks that enormously increases the total system jitter. Live tracking of the correlation peak during the communication session and fast feedback loops in step 4 mitigates against fast variations of the clock frequencies.

B. Implementation of the synchronization protocol

With the basic workflow established, we now consider the major challenges associated with its implementation in real link scenarios. Low signal-to-noise ratio and large clock frequency skews have a severe impact during synchronization initialization (section II B 1). In worst-case scenarios, it is not possible to synchronize the system, as the correlation features stay hidden under noise. Initial sweeps of the clock frequency counteract this and enable successful initialization. Having taken this first hurdle, it is easy to keep the slave clock frequency locked to the master during the communication session. Here we take advantage of filtering the data in time by narrowing down the observation window of correlation features. In contrast to time windows as large as the acquisition time of 100 ms during the initialization, it is narrowed down to a few tens of nanoseconds that reduces the noise drastically. However, clock drifts create a frequency mismatch between the master and the slave clock after the initialization of synchronization procedure. This
increases the timing jitter during the communication session, which in turn increases the quantum bit error rate. All of this is circumvented by tracking of the clock frequency and immediate compensation as described in section II B 2.

1. Initialization

Accumulation of simultaneous photon detection events from matched data packages results in a correlation peak that is located at the timing offset between two communicating parties. Distinguishing the peak correlation that corresponds to this offset from the noisy background is a major challenge. For that, we introduce the statistical significance as the peak height normalized to the noisy background standard deviation $[42]$. Strong spread of the correlation peak in time reduces the significance. This spread is equivalent and characterized by the total system timing jitter, including detection, time tagging, and synchronization jitter. Typical root-mean-squared (RMS) timing jitters of single photon detection systems are smaller than 500 ps and also time tagging units achieve jitters smaller than a few tens of picoseconds with ease. Poor synchronization on the other hand, with its corresponding RMS synchronization jitter $\sigma_{\text{sync}}$, has the strongest impact on the total system jitter. It may amount to 1 µs, with uncorrected clock frequency skew of 20 μs/s and an acquisition time of 100 ms. The synchronization jitter is time-dependent and increases with acquisition time $T_a$ and uncorrected clock frequency skew $\Delta u$,

$$\sigma_{\text{sync}} = \frac{1}{2} \Delta u T_a. \quad (1)$$

Frequency skew reduces the height of the correlation peak and thereby decreases the probability to correctly identify it. Systems with stable oscillators, like rubidium clocks, do not suffer from a reduction of significance (Fig. 3a) without correction. General crystal oscillators, on the other hand, lower the signal close to the noise level and thus make it impossible to find the correlation peak. To counteract this, we introduce the compensation of skew, so that the $i^{th}$ time tag on Bob’s side $t_i$ is corrected to $t^*_i$,

$$t^*_i = t_i - \Delta u \cdot (t_i - t_0). \quad (2)$$

For different values of corrections $\Delta u$, we calculate the cross-correlation under the constraint of computation effort, limiting us to the computation of approximately 280 cross-correlation functions (see Methods IV B for the algorithm and computational requirements). This is sufficient to search in a clock frequency skew range from -20 to +20 μs/s (decided according to our quartz crystal clock). The step size of 0.14 μs/s provides clock frequency skews with an uncertainty $< 0.14 \mu s/s$. Optimum clock frequency skew is indicated by the maximum significance of the correlation peak in the search window (Fig. 3a). The maximum significance of 142 allows to create helpful trend lines with the coincidence rate 7 kcps, single rate on Alice’s side 244 kcps, Bob’s side 232 kcps and acquisition time of 0.1 seconds (see Methods IV C 1). The experimental timing jitter is derived by fitting the correlation peak with a flat-top Gaussian function. Note that the significance enhances by adapting the bin size to the correlation peak spread (see Methods IV A) and reaches values of up to 6.8 (see Methods IV A, equation 4) without clock frequency skew compensation. However, the correlation peak spread is unknown at this point of time, as the true clock frequency is uncertain. Such a random selection of the bin size reduces the significance again. In contrast to previous work $[42]$, we could increase the significance by a factor of $\sqrt{\Delta u/\delta u} \approx 20$, thanks to our new method of clock frequency skew compensation. Further reduction of the residual clock frequency skew is made by a more computation-efficient fine-tuning (see Methods IV D) or by observing moving correlation features $[42]$. Whereas we cannot avoid noise, clock frequency skew compensation sharpens the correlation features and creates higher visibility of them (more information on the signal-to-noise ratio dependence in Methods IV A, Fig. 6). This provides noise resistance and makes it feasible to initialize low signal-to-noise ratio links.

2. Live tracking

At this point in the protocol, the sender and receiver have matched clock frequencies and the corresponding timing offset. The next challenge is to maintain this common time basis over extended periods via live tracking of the correlation peak. In the case of rubidium oscillators or GPS-disciplined clocks, which do not exhibit significant clock frequency drifts, this step is not necessary. Typical quartz oscillators, however, require the initial estimate for the clock frequency skew to be adapted in real time. In other words, we adjust the time-varying slave clock frequency to match the changed master clock frequency. These drifts result in a measurable change of the position of the correlation peak. The location is continuously tracked, and the slave clock frequency is afterwards adapted by the peak displacement over the elapsed time (see detailed algorithm in Methods IV E). In our experiment, the acquisition time is 100 ms and feedback loop time for adapting the instantaneous clock frequency is 600 ms. Figure 3b compares the total timing jitter with and without live tracking to Bob’s reference that indicates zero synchronization jitter (see Methods IV F for the single count rate). Over a time window of 5 min we find maximum clock frequency skew variations of up to ±11 ns/s that show the importance of live-correcting these clock drifts of 320 ps/s² (see Methods IV G). Without live tracking, the timing jitter would increase from 200 ps to almost 700 ps with 100 ms acquisition time. Stable frequency references (here: rubidium clocks) introduce very low timing jitters of smaller than 1 ps as consequence of live tracking (see Methods IV E). Quartz oscillators require more rigorous treatment and optimization of the algorithms. With a stronger drift of the clock $\partial (\Delta u)/\partial t$, there is timing jitter $\sigma_{\text{drift}}$ introduced with longer feedback loop times $T_{\text{feed}}$ and acquisition time $T_a$,

$$\sigma_{\text{drift}} = \frac{1}{2} \frac{\partial (\Delta u)}{\partial t} T_{\text{feed}} T_a. \quad (3)$$
FIG. 3. Clock frequency skew compensation and live tracking. a The ratio of peak height to background standard deviation (significance) increases and the correlation peak spread reduces for correctly compensated clock frequency skew. Rubidium clocks do not require any compensation of the clock frequency skew and result in strong correlation peaks. The significance can be increased by adapting the bin width to the current clock frequency skew (see Methods IV A). The analytical trend lines (significance equation 20 and timing jitter equation 19 in Methods IV C1) are derived from the experimental maximum significance of 141. It provides the smallest residual clock frequency skew of 48 ns/s and smallest timing jitter of 2.4 ns over the acquisition time of 0.1 seconds. b Live tracking of the correlation peak reduces the drift of clock frequency skew over time. The clock frequency skew is adjusted every 600 ms in a feedback loop and result in much smaller total timing jitter that is almost identical to the reference with same clocks. The synchronization jitter contribution is as small as 35 ± 8 ps with acquisition time of 100 ms. Other experimental parameters in Methods IV F. c The timing offset from the cross-correlation peak location is stored for every incoming data package up to the feedback loop time. The instantaneous clock frequency difference is equal to dividing the average timing offset by the passed time. Whereas the feedback loop time does not affect much the performance of stable rubidium oscillators, it heavily affects the synchronization jitters achieved with quartz oscillators. Here we characterize the performance by accumulating the occurrence probability for all jitter values during a 5-minutes session. The 1σ (2σ, 3σ) timing jitter represents the maximum jitter in 68.3 % (95.5 %, 99.7 %) of time during the session. The synchronization jitter increases linearly with the feedback loop time (equation 3). Grey data points correspond to measurements that are not displayed in the cumulative probability plot.

TABLE I. Residual synchronization jitter root-mean-square (RMS) for different link rates and coincidence to-accidentals ratios (CAR) during live tracking. Considering the synchronization initialization, see Methods IV A, Fig. 7 for comparison of the different setups. The clocks are external crystal oscillators (XO) or clocks locked by the Global Navigation Satellite System (GNSS). a Low signal experiment, including satellite up-link turbulence (see Methods IV H), b Moderate signal experiment, including satellite up-link turbulence (Fig. 4), c 1.2-km free-space and high-dimensional link experiment across Vienna by Steinlechner et. al. [19] - synchronization jitter estimated from timing offset variation, d 143-km free-space link experiment between the Canary Islands La Palma and Tenerife by Ecker et. al [21] - synchronization jitter estimated from timing offset variation. The coincidence window (half-width) depend on the detection system and is a-b 0.27 ns c 1 ns, d 0.5 ns. *Only live tracking – synchronization initialization at stronger signal

| Parameter                      | a Low signal | b Moderate signal | c 1.2-km link in Vienna | d 143-km link between Canary islands |
|-------------------------------|-------------|------------------|------------------------|-------------------------------------|
| Clock type                    | External XO| External XO      | GNSS                   | GNSS                                |
| Count rate Alice (kcps)       | 165 ± 3    | 195 ± 3          | 400                    | 13 300                              |
| Count rate Bob (kcps)         | 437 ± 6    | 15 ± 5           | 100                    | 10                                   |
| Correlation event rate (cps)  | 430 ± 160  | 440 ± 200        | 20 000                 | < 5                                 |
| CAR                           | 10 ± 4     | 84 ± 13          | 250                    | < 5                                 |
| Synchronization jitter RMS (ps)| 98 ± 6^{*} | 68 ± 8           | > 300                  | ≈ 50                                |
| Clock drift RMS (ps/s^2)      | 320        | 320              | < 5 (typical)          | < 5 (typical)                       |
| Clock frequency skew (ppm)    | 19         | 19               | 10^{-6} (typical)      | 10^{-6} (typical)                   |

Large feedback loop times compensate the fast changing clock frequencies not early enough and results in a linear increase of the overall timing jitter (Fig. 3c). Low signal and other link conditions reduce the synchronization quality further. This is part of the following section II C.

C. Experiment under emulated link conditions

To confirm the feasibility of our approach in real-world link conditions, we perform a series of experiments with added background noise, and signal fades, as is expected in free-space links with atmospheric turbulence. To this
FIG. 4. **In-lab emulated free-space link experiment.** At different times we add additional noise at Bob’s receiver, block the optical link or add turbulence (Fried parameter 1 mm, $1/e^2$ full-width beam waist of 3.4 mm). a Single count rate at the receiver Bob. The mean single count rate is $15 \pm 5$ kcps with coincidence rate of $440 \pm 200$ cps (coincidence window is equal to RMS timing jitter of 269 ps) in the time window from minute 1 to 3.5. The count rate at Alice lies constantly at $195 \pm 3$ kcps. b Introduction of a noise source increases Bob’s count rate to 6 Mcps. Low signal-to-noise ratio stops the correlation peak tracking and keeps the instantaneous clock frequency skew for correction constant. c The time-dependent total timing jitter root-mean-squared (RMS) remains constant (d) even after switching on the turbulence at minute 1. e The timing jitter probability distribution describes a Gaussian with its fitted center at 268.7 $\pm$ 0.7 ps (minute 1 to 3.5). Setups with the same clock (perfect synchronization) show timing jitters of $260 \pm 2$ ps from a fit to 10 s of data acquisition, estimating the synchronization jitter RMS to $68 \pm 8$ ps. f The tracking algorithm is stopped in regions with low signal-to-noise ratios (blocked optical link or additional noise) and results in constant clock frequency skew for correction there. g The timing offset jumps after low signal-to-noise ratios time intervals, as the correlation peak is not tracked and the clock frequency not adapted. This phenomenon is stronger for longer time of blocked links. h The timing offset varies within a window of $\pm 1$ ns on time scales of 30 s. The feedback loop time and data package size amounts to 200 ms and 100 ms, respectively. Summary of this figure in table I.

end, we emulate several link scenarios and introduce turbulence via a rotating phase plate, designed to represent an atmospheric uplink to a satellite from the Mt. Teide optical ground station (Fried parameter is 1 mm with $1/e^2$ full-width beam waist of 3.4 mm [50]). This turbulence results in strong variations of the detected count rate of Bob (Fig. 4a). The rate at Alice, who is assumed to be co-located with the quantum source, remains constant. To test the algorithm’s stability under elevated noise levels (for example, stray light), we introduced background counts via a light-emitting diode (see Fig. 4b). Even under these unfavourable conditions, live tracking with short feedback loops (200 ms) ensures synchronization RMS timing jitters of $68 \pm 8$. This is comparable to 50 ps synchronization jitter from systems with GPS-disciplined clocks [21]. Our timing jitter is estimated with the same-clock timing jitter RMS of $260 \pm 2$ ps and the average timing jitter of 268.7 $\pm$ 0.7 ps after a Gauss fit to the jitter values from minute 1-3.5 (Fig. 4c-e). The correlation peak tracking algorithm stops when reaching a signal-to-noise ratio threshold that is set to 5. Setting any threshold is required, as it avoids that noise wrongly identifies as a correlation feature. Both the increased noise
and interruptions of the optical link (e.g., due to objects crossing the beam path) stop the tracking, as the signal-to-noise threshold is not reached and leave the instantaneous clock frequency skew constant (Fig. 4f). This impacts only very lossy links with an increased synchronization timing jitter RMS to 98 ps, as described in Methods IV H. When the clock frequency skew for correction is not adapted in low signal regions, the correlation peak will start to move in time. As a consequence, the timing offset jumps to the value of the current location, as soon as the signal is back. Long periods of broken links follow in large timing offset jumps and depend on the clock’s stability. These timing offset jumps are unique to unstable clocks. They require sufficiently large observation windows to not lose the correlation peak that is essential for synchronization. The timing offset is bound to fluctuations in a window of ±1 ns (Fig. 4g-h) in conditions of good signal (coincidence-to-accidentals ratio > 5). In the Methods IV H we present stable live tracking for much lower coincidence-to-accidentals ratios of only 10 (Table I), comparable to high-loss link scenarios. The synchronization jitter RMS amounts to 98 ± 6 ps with only 43 correlation events in a 100 ns data package and 200 ms feedback loop time. Note that the correlation peak significance during synchronization initialization is extremely low with the rates given, similar to [21] (see Methods IV A, Fig. 7 for comparison of different setups in literature). Our synchronization initialization algorithms fail here, due to limited computation time and power that could not compensate the clock frequency skew enough for sufficient significant correlation features. Therefore, we shortly increase the signal-to-noise ratio during initialization and reduce it again later. In conclusion, the combination of a) adaptive correlation observation window, b) introduction of a coincidence-to-accidental ratio threshold, and c) short feedback loop times enable very stable operation and only a minor increase of the total timing jitters.

III. DISCUSSION

These results clearly show that the compensation of clock frequency skew and live tracking of the frequency are suitable for application in a variety of real-world link scenarios. We could show that very few correlation events are already sufficient to establish and keep a synchronized quantum communication session with timing jitter in the range of tens of picoseconds over time scales of 10 minutes. Recently, we also confirmed these results on a 1.7 km intra-city link. Reference [42] predicts that significant significance of larger than 7 can only be achieved by clock frequency skews of < 1.3 µs/s in our realistic link experiment (Table Ib, Equation 4 in Methods IV C 1). This means that the synchronization initialization would fail as our clock frequency skew amounts to 18.5 µs/s (see Methods IV A, Fig. 7 for comparison of different setups from other publications). It is a consequence of the poor visibility of the correlation peak. Furthermore, previous treatments have been limited to high signal-to-noise ratios. Here we demonstrate successful synchronization for much lower signal-to-noise ratios thanks to compensation with an effective frequency skew of smaller than 48 ns/s that enables the enhancement of the significance by a factor of 20 as part of the initialization (Fig. 3a). This brings correlated photons and quartz oscillators in a much better position - not only for today’s typical quantum communication schemes [19], but also for tomorrow’s long-distance and low-signal scenarios. The intrinsic timing relations of correlated photons clearly have the potential to replace bulky external high-precision synchronization schemes and move resources from hardware to software.

Finding the timing offset is crucial to start communication, but the low signal-to-noise ratio will reduce the correlation peak significance. Especially high-loss scenarios of > 46.9 dB [21] (Table Id) call for very low clock frequency differences for sufficient correction peak visibility. This typically requires highly precise oscillators, like rubidium or GPS-disciplined clocks. Quartz clocks are acceptable in principle, but demand a precise slave clock frequency sweep to achieve a frequency match with the master. Within our presented search window −20 µs/s to +20 µs/s it is impossible to find this precise frequency to the required accuracy of < 14 ns/s. High computation times give rise to limitations (see Methods IV B). The solution is the prior knowledge of the approximate clock frequencies to narrow down the search window - ideally there will be just a single initial cross-correlation necessary to find the correlation peak. We propose to minimize the search window by a) more accurate frequency specifications from the clock manufacturer, b) calibration of clocks with high signal rates, or c) finding the clock frequency at a time, when computation limits do not constrain. High-performance computers with high parallel processing speeds, for example, with a graphical processing unit, reduce the computation time as well. Note that finding the initial clock frequency skew is a single-time issue, as it will not occur during high-duty quantum communication sessions. The algorithm tracks the aging of the clocks and adjusts the clock frequency continuously. Starting with large frequency uncertainty, the estimations improve over time. This provides a narrow clock frequency search range before every session, where the computation effort reduces drastically down to a few seconds – even for lossy long distant links.

Systems with very high losses or accelerating clocks suffer from increased synchronization jitters during communication. Low number of correlation events increase the uncertainty of the clock frequency skew for compensation and unlock live-tracking feedback loops. Low clock stability further increases the synchronization timing jitter. High precision clocks clearly have an advantage here that allow for much lower synchronization jitters < 1 ps during communication sessions, due to low drifts through their high stability. Apart from signal-to-noise ratio and clock stability, the detector jitter is another factor for high synchronization performance. Particularly, high loss scenarios benefit from low detection jitters that increase the signal-to-noise ratio. We demonstrated that even 44 correlation events in 100 ms data packages are sufficient for a synchronization jitter of a few tens of picoseconds. The event rate and signal-to-noise ratios are comparable to the required accuracy of < 14 ns/s. High computation times give rise to limitations (see Methods IV B). The solution is the prior knowledge of the approximate clock frequencies to narrow down the search window - ideally there will be just a single initial cross-correlation necessary to find the correlation peak.
to extremely high loss scenarios [21] (Table 1d), but are enough to enable correlation peak tracking and keep the system locked. As nanowire detectors already have timing jitters of as low as tens of picoseconds, the synchronization performance with correlated photons will only increase in future.

To set our approach into a broader perspective, we also shortly discuss limits to moving objects, like satellite-[14, 15] or emerging drone-based [57, 58] quantum communication. Quantum sources in space provide a great platform to test and measure space-time effects on quantum communication protocols [59, 60] or perform high precision metrology [61, 62]. Satellites introduce an effective clock frequency skew, due to the Doppler Effect caused by varying distance to the observer. The normalized Doppler shift $\frac{\nu(t)}{c}$ for low earth orbit satellites increases from 0 to $2 \times 10^{-5}$ over a time scale of 6 min, where $\nu(t)$ is the satellite’s time-dependent relative velocity and $c$ is the speed of light [63]. Equivalently, this creates a clock frequency skew that varies from 0 to a maximum of $\Delta u = 20 \mu s/s$. Whereas the maximum clock frequency skew is comparable to our crystal oscillator, the clock acceleration $\partial^2(\Delta u)/\partial t^2$ is orders of magnitude bigger and amounts to approximately $55 ns/s^2$ ($20 \mu s/s$ divided by 6 min). Thus, it is desirable to reduce the feedback and acquisition time to 100 ms or smaller that could result in clock drift jitters of 265 ps (equation 3). However, with only a few correlation events per second available [14, 64], it is impossible to choose acquisition times of < 1 second and still have sufficient signal-to-noise ratio. Using only correlated photons, synchronization is not possible without knowledge of the satellite’s orbit. Sources with kcps rates [52], on the other hand, give the opportunity to select small feedback cycles of 100 ns and still have correlation events comparable to our experiment (Table Ib) that could be used to synchronize clocks with our method. Drones move much slower than satellites and may also be suitable for synchronization – at least classically [65]. The speed is up to 30 m/s, introducing clock frequency skews of up to 100 ns/s ($30 m/s$ divided by the speed of light), being much smaller than the clock frequency skew of our crystal oscillators. The main concern is acceleration of the drone, reaching up to 7 g (gravitational constant) and translating to $228 ns/s^2$ clock drift. Sufficient kcps coincidence rates [57] give chance to select short feedback cycles for the compensation of high drifts during drone acceleration and make synchronization feasible. With small feedback loops, correlation events open doors for live remote detection of velocity and acceleration of moving objects.

In conclusion, correlated photons are great timing carriers, come to quantum communication systems naturally, and are easy to recycle for high performance synchronization down to a few tens of picoseconds. Today’s point-to-point or lab-to-lab communication sessions will be integrated in a network with multiple users tomorrow, as can be found in action already [10]. High scalability, integration and fewer resources characterize the networks, where highly stable but bulky clocks should be an exception. Where correlated photons used to be inappropriate for synchronizing clocks with high skew and strong drifts [42], we showed stable operation by new synchronization methods. Clock frequency skew compensation and correlation peak live tracking allows for a wider range of cases - especially in terms of scalability. Resistance to high losses, as would be common in large networks, still enables synchronization RMS jitters of < 68 ps and presents feasibility for application in real-life communication scenarios. Single photons are not copyable and bit-errors during communication are easy to detect due to the quantum origin of the single photon detection events [6, 7], indicating furthermore the potential for quantum secured time transfer [52, 53, 66].

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AUTHOR CONTRIBUTIONS

C.S. designed the experiments with guidance from D.R.. C.S. performed the experiments. S.T. and S.S. developed essential hardware and software components with support and guidance from D.R.: S.S. and U.C. developed the entanglement source. C.S. wrote the main part of the Python processing script with assistance from S.T.. A.K. developed the turbulence testing setup with assistance from N.L.D.. F.S. proposed and directed the research. The first draft of the manuscript was written by C.S., F.S. and D.R. with assistance by M.C.P.. All authors discussed the results and reviewed the manuscript.

COMPETING INTERESTS STATEMENT

The authors declare no competing interest.
IV. METHODS

A. Correlation peak significance and comparison to previous work

The peak significance $S_p$ of correlation features reduces with high clock frequency differences $\Delta u$, as described in [42],

$$S_p = \sqrt{\frac{r_C^2 N}{r_A r_B}} = \sqrt{\frac{r_C^2}{r_A r_B \Delta u}}, \quad (4)$$

with coincidence (or signal) count rates $r_C$ and single rates from Alice $r_A$ and Bob $r_B$. This, however, only considers the situation that the bin width $\delta t = T_a/N$, with the number of bins $N$, has been perfectly adapted to the spread of the correlation peak $\delta t_{\text{spread}}$ over the acquisition time $T_a$,

$$\delta t_{\text{spread}} = T_a \Delta u. \quad (5)$$

The clock frequency skew $\Delta u$ is not know usually, so that it will be hard to guess the bin size correctly. This means reduction of significance by $\sqrt{N}$ for too large bin widths (equation 4). Similarly, if the bin width was too small, signal would be distributed over several bins. The number of coincidences per bin will reduce by the ratio $\delta t_{\text{spread}}/\delta t$,

$$S(N) = \frac{1}{\Delta u N} \sqrt{\frac{r_C^2 N}{r_A r_B}}, \quad (6)$$

with $\Delta u N \geq 1$. Figure 5a depicts this behavior for the photon pair source from [42]. With their high signal-to-noise ratio, it is possible to find the very first correlation peak and determine the clock frequency skew from the correlation peak displacement over time. In this paper, we have tested lower signal-to-noise ratios that would result in a significance of barely 2.5 with a clock frequency skew of $\Delta u = 2 \times 10^{-4}$ (Fig. 5b). As a consequence, it would not even be possible to start with the algorithms as described in [42]. In this work, we propose a clock frequency skew compensation for this crucial initial step in low signal-to-noise environments. We find that improvements of the clock frequency skew uncertainty by a factor of 140 are feasible, depending on the fast Fourier transform - run times (see Methods IV B for computational requirements). This provides an improvement of the significance by a factor $\sqrt{140} \approx 12$ to a value close to 30 and enables reliable identification of the correlation peak even under low signal-to-noise ratios. Note that we reduce the residual clock frequency skew to $\delta u = 48 \text{ ns/s}$ (with the rate given and its final peak significance) that even provide a significance improvement by a factor of 20.

More detailed relations between significance and the signal-to-noise ratio are depicted in Fig. 6. Starting from single count rates at Alice $r_A$ and Bob’s side $r_B$, the single rate at Bob’s reduces as the link transmission $T$ reduces by means of a variable attenuator. Furthermore reduces the coincidence rate $r_C$ similarly. Following Refs. [67–69], the coincidence-to-accidentals ratio ($\text{CAR}$) is defined as the number of true coincidence counts $r_{C\text{True}}$ over the accidental coincidence counts $r_{C\text{Acc}}$.

$$\text{CAR} = \frac{r_{C\text{True}}}{r_{C\text{Acc}}} = \frac{r_C - r_{C\text{Acc}}}{r_{C\text{Acc}}}. \quad (7)$$

The accidental correlation events, depending on the link transmission $T$, within the root-mean-squared coincidence window $\sigma$ is,

$$r_{C\text{Acc}}(T) = 2r_A [(r_B - r_{\text{back}}) T + r_{\text{back}}] \sigma. \quad (8)$$

Here we also introduce the background rate $r_{\text{back}}$ that can not be further reduced by higher channel losses. This includes factors like detector dark counts, not sufficiently filtered daylight, or other other noise sources in the system. The transmission-dependent $\text{CAR}$ summarizes as

$$\text{CAR}(T) = \frac{r_C T - 2r_A [(r_B - r_{\text{back}}) T + r_{\text{back}}] \sigma}{2r_A [(r_B - r_{\text{back}}) T + r_{\text{back}}] \sigma}. \quad (9)$$

The transmission-dependent peak significance can be derived from equation 4 as,

$$S_p(T) = \sqrt{\frac{(r_C T)^2}{r_A [(r_B - r_{\text{back}}) T + r_{\text{back}}] \Delta u}}. \quad (10)$$

The analytical trend, as described by equation 10, has been confirmed experimentally (Fig. 6) after compensation of the clock frequency skew by different amounts. The peak significance is achieved by matching the cross-correlation...
FIG. 6. Correlation peak significance for different loss scenarios. 

(a) Estimation of correlation peak significance under different coincidence-to-accidentals ratios (loss) and clock frequency skews $\Delta u$. The experimental data points base on initial lossless rates of $R_A = 200$ kcps, $R_B = 800$ kcps, $R_C = 14$ kcps, $r_{\text{back}} = 5$ kcps and maximum number of bins in the cross-correlation $N = 10^8$. The number of bins are adapted to the correlation peak spread, caused by the residual clock frequency skew, with $\Delta u = 1/N$. Error bars indicate standard deviation after slight variation of bin sizes by ±5%. Threshold for recovering the correlation peak is significance of 7, providing a probability of $\approx 10^{-12}$ of a wrongly found peak [42]. clock frequency skew compensation increases the visibility of the correlation peak and thus enables higher noise resistance in this work. 

(b) Coincidence-to-accidentals ratio depending on the transmission from Alice to Bob (Methods IV A, equation 9). 

Bin size with the peak spread from the residual clock frequency skew, as described earlier. However, for clock frequency skews smaller than 10 ns/s is the number of bins already $N = 1/(10 \times 10^{-9}) = 10^8$ that results in immense computation effort (more information in Methods IV B). It is hardly feasible to adapt the bin size to its optimum for clock frequency skews of $\leq 10$ ns/s. It follows a non-optimized cross-correlation that does not provide maximum significance. Nevertheless, Fig. 6 shows impressively how the significance of correlation peaks can be increased by compensation of the clock frequency skew.

Reduction of the clock frequency skew by our algorithm can help to recover the correlation features under very low signal-to-noise ratios. High signal rates do not demand any kind of compensation. On the other hand, it is crucial in high-loss link scenarios (Fig. 7 with data from table I). Here we limited to a clock frequency skew compensation to 140 ns/s that allows for synchronization initialization of our emulated link experiment (Fig. 4 and table Ib). Even lower rates, as in Methods IV H (table Ia) or in [21] causes too small correlation peak significance for synchronization initialization today (Fig. 7). Higher correlation peak significance is expected to reach with more computation power or reduction of the clock frequency skew search window.

B. Synchronization initialization algorithms and computation efforts

The correlation peak is found by a convolution of the time tag stream from Alice and Bob. Without precise knowledge of the clock frequency skew, it will be difficult to find correlation peaks with sufficient significance. First, the clock frequency skew search window is created (Fig. 8). From our quartz oscillator data sheet, we know the approximate range of clock frequency skews, starting from -20 to +20 µs/s (see Methods IV G). The time tags from Bob are compensated by different clock frequency skews every loop analogue to equation 2 and then convoluted with time tags from Alice. It is either based on fast Fourier transform, in the case of determining the clock frequency skew coarsely (Fig. 3a) without timing offset knowledge, or it is based on the Start-Stop method. In the case of fast Fourier transform, it is required to arrange the time tags to the bins and then apply the cross-correlation. In the Start-Stop method time differences binned after the correlation.

Full cross-correlations over all time tags are time-consuming and limit the feasible number of calculations. The correlations are performed through a Python environment with the NumPy module on the personal computer (Table II). The time for a single fast Fourier transform-based cross-correlation may amount up to approximately 40 seconds for $N = 10^7$ bins (Fig. 9a). Optimum significance is reached by choosing enough number of bins $N$, so that the correlation peak spread is equal to the bin size (see Methods IV A). This gives the achievable clock frequency
Correlation peak significance for various experimental setups. The clock frequency skew of quartz oscillator is up to 20 µs/s, providing the clock frequency skew before compensation. Even without compensation it is easy to find the correlation peaks with sufficient signal, as in Ho2009 [42] and Steinlechner2017 [19]. Threshold for recovering the correlation peak is significance of 7, giving a probability of \( \approx 10^{-12} \) of a wrongly found peak [42]. Low coincidence-to-accidentals ratios, as in our low signal (Methods IV H and Table Ia) or moderate signal experiment (Fig. 4 and Table Ib) or in Ecker2021 [21], require compensation of the clock frequency skew. Under our limited computation power we can reduce and compensate the clock frequency skew to \(< 140 \text{ ns/s} \). This is not sufficient to reach a significance of 7 in our low signal experiment or in Ecker2021 [21]. Solution would be to reduce the initial clock frequency skew search window.

| Parameter          | Value                  |
|--------------------|------------------------|
| Processor          | Intel(R) Core(TM) i5-8250U CPU |
| Speed              | 1.60 GHz (1.80 GHz)    |
| RAM                | 16 GB                  |
| System             | 64-bit based processor |
| Environment        | Python 3.7.0 64 bit, NumPy 1.19.4 |

C. Estimation of synchronization jitter and trend lines

1. Description of total system jitter

The total system jitter of a quantum communication system determines the overall performance - from signal to noise ratio to secure bit rate. The total jitter can be easily calculated analytically by knowledge of the clock frequency skew. Here we provide important scaling laws to estimate the final signal-to-noise ratios, like significance or coincidence to accidentals ratio. As all jitter contributions originate from a random source, we consider a Gaussian function \( g \) with RMS jitter \( \sigma \) that is normalized to have an area of 1,

\[
g(t) = \frac{1}{\sqrt{2\pi \sigma^2}} \exp\left(-\frac{t^2}{2\sigma^2}\right). \tag{13}\]

The total jitter from the source coherence time \( \sigma_{coh} \), time tagger \( \sigma_{tt} \), detector \( \sigma_{det} \) and synchronization \( \sigma_{sync} \),

\[
\sigma = \sqrt{\sigma_{coh}^2 + \sigma_{tt}^2 + \sigma_{det}^2 + \sigma_{sync}^2}, \tag{14}\]

reduces to,

\[
\sigma \approx \sqrt{\sigma_{det}^2 + \sigma_{sync}^2}. \tag{15}\]

skew accuracy \( \delta u \),

\[
\delta u = \frac{1}{N}. \tag{11}\]

The total computation time \( T_{tot} \) from a single cross-correlation \( T(\delta u) \) for a range of clock frequency skews \( \Delta U \) is then,

\[
T_{tot} = \frac{\Delta U}{\delta u} \times T(\delta u). \tag{12}\]

With a time limit of slightly above 2 hours (7500 seconds) and a clock frequency skew range of \( \Delta U = 40 \mu s/s \) (from -20 to +20 µs/s) is the maximum affordable time for a single cross-correlation approximately 26 seconds and clock frequency skew accuracy 0.14 µs/s (\( N = 7.14 \times 10^9 \)). The speed of 26 seconds for a single cross-correlation may be drastically increased by using a dedicated higher-performing computer, instead of a laptop here. More specifically, more processing cores and higher processing speed could improve the situation. Graphical processing units may also perform much better by parallelization of the processes and for-loops.
sleep(integration time = 0.1 sec)  
$t_A, t_B = \text{read timetags}()$  
skew vector $= -20 \mu s/s \cdots + 20 \mu s/s$, step: 140 ns/s  
bins $= 0 \ldots 0.1 s$, step: 14 ns  
$P = \text{zeros}(\text{skew vector size})$  
i = 0  
for $\Delta u_{\text{corr}}$ in skew vector do  
$\Delta u_{\text{corr}} = t_B - (t_B - t_B[0]) \times \Delta u_{\text{corr}}$  
c = convolution($t_A, t_{B, \text{corr}}$, bins)  
$P[i] = \text{max}(c)$  
i += 1  
end

Output: optimum clock frequency skew = skew vector(argmax($P$))

FIG. 8. Representation of the function coarse clock frequency skew() and accurate clock frequency skew() in Methods IV E, Fig. 12 The convolution is fast Fourier transform (FFT)-based and Start-Stop method-based, respectively. A FFT-based approach has to be taken if the precise timing offset is not known. The algorithm includes the following variables, time tags from Alice and Bob $t_{A/B}$, peak values of the cross-correlations $P$, counter $i$, clock frequency skew for correction $\Delta u_{\text{corr}}$, clock-skew correct time tags from Bob $t_{B, \text{corr}}$, cross-correlation output $c$. Note that the bin step size of 14 ns corresponds to integration time $\times$ skew vector step size. Smaller bin and skew vector step sizes require considerably higher computation effort. If there is knowledge of the approximate clock frequency skew, the range of clock frequency skews can be reduced as the jitter contribution from the time tagger (around 30 ps root-mean-squared) and entangled photon source coherence time (around 2.5 ps root-mean-squared) is much smaller than the detector jitter. After a few steps of editing, we get

$$\sigma = \sigma_{\text{det}} \sqrt{1 + \frac{\sigma_{\text{sync}}^2}{\sigma_{\text{det}}^2}}, \quad (16)$$

resulting in the simulated jitter curves in Fig. 3a. Due to the residual clock frequency skew after compensation is the smallest jitter not equal to the detector jitter. The highest correlation peak value $P$ (replaceable by coincidence to accidental-ratios $\text{CAR}$ or significance $S$), given by the smallest timing jitter $\sigma_{\text{min}}$, reduces, due to the stronger synchronization jitter. With equation 15 is the correlation peak value described as,

$$P = \frac{P_0}{\sqrt{1 + \left(\frac{2\sigma_{\text{sync}}}{\sqrt{2}\pi\sigma_{\text{min}}}\right)^2}} \quad (17)$$

The prefactor of $\sqrt{2/\pi}$ originates from the area of a Gaussian being equal to $\sqrt{2/\pi}\sigma$ and its changing shape to a flat-top Gaussian by the synchronization jitter. Figure 10 indicates a smooth trend of $\text{CAR}$ and total timing jitter RMS for rubidium clocks. The smallest experimental timing jitter amounts to $\sigma_{\text{min}} = 310$ ps with maximum coincidence-to-accidental-ratios $\text{CAR}_0$ of 91 and acquisition time $T_a$ of 10 seconds. Both together enable an analytical trend line from equations 1, 16 and 17 with acquisition time $T_a = 10$ s (Fig. 10),

$$\text{CAR}(\Delta u) = \frac{\text{CAR}_0}{\sqrt{1 + \frac{2}{\pi}\left(\frac{1/2\Delta u T_a}{\sigma_{\text{min}}}\right)^2}}, \quad (18)$$

$$\sigma(\Delta u) = \sigma_{\text{min}}\sqrt{1 + \left(\frac{1/2\Delta u T_a}{\sigma_{\text{min}}}\right)^2}. \quad (19)$$

The equation 17 may also be transformed to be dependent on the residual clock frequency skew $\Delta u$ and smallest residual clock frequency skew $\delta u$, as the smallest timing jitter $\sigma_{\text{min}}$ and smallest residual clock frequency skew $\delta u$ are directly related through equation 1,

$$S(\Delta u, S_p) = \frac{S_p}{\sqrt{1 + \frac{2}{\pi}\left(\frac{\Delta u}{\delta u(S_p)}\right)^2}}, \quad (20)$$

with the smallest residual clock frequency skew $\delta u$ defined by[42],

$$\delta u(S_p) = \frac{r_C^2}{r_{A/B}^2S_p^2}. \quad (21)$$

The maximum significance of 142 allows to create helpful trend lines with the coincidence rate $r_C = 7$ kcps, single rate on Alice’s side $r_A = 244$ kcps, Bob’s side $2r_B =$
FIG. 10. Coincidence-to-accidentals ratio and total timing jitter for corrected clock frequency skew of rubidium oscillators. The smooth trend indicates a stable reference. The analytical trend lines are derived from the experimental acquisition time of $T_a = 10$ seconds, smallest timing jitter of 320 ps and maximum coincidence-to-accidentals ratio of 91.

232 kcps and acquisition time of 0.1 seconds (see Methods IV C 1). The analytical trend of the significance directly derives from the maximum experimental significance $S_p$ and follows equation 20 in Methods IV C 1. The smallest residual clock frequency skew at the peak significance can be calculated simultaneously to $\delta u = 48$ ns/s with $S_p = 142$ (see Methods IV C 1 equation 21), and leads to the smallest timing jitter $\sigma_{\text{min}} = 2.4$ ns with equation 1. Furthermore, the peak significance $S_p$ enables analytical trend lines of the total timing jitter $\sigma$ as $\sigma^2 = \sigma_{\text{min}}^2 + \sigma_{\text{sync}}^2$, providing a close match with the experimental timing jitters (Fig. 3a).

2. Achievable synchronization timing jitter during live tracking

The final synchronization jitter after live correction during a communication session is decided by the clock drift and the number of correlation events. Here we present analytical estimations of the synchronization limits that might be helpful for easy transfer to any communication scenario. The most important parameter is the instantaneous clock frequency skew during a communication session. If it is nonzero, due to insufficient correction, the synchronization jitter will be nonzero. The clock frequency skew $\Delta u$ is calculated from two or more detected peak locations $\tau_1$ and $\tau_2$ with temporal separation of $T_{\text{meas}}$,

$$\Delta u = \frac{\tau_2 - \tau_1}{T_{\text{meas}}}. \quad (22)$$

However, low numbers of $n$ correlation events give rise to an uncertainty of the correlation peak location $\delta \tau$ via the total timing jitter $\sigma$ [42],

$$\delta \tau = \frac{\sigma}{\sqrt{n - 1}}. \quad (23)$$

Via error propagation, we get the uncertainty of the clock frequency skew from measurement $\Delta u_{\text{meas}}$,

$$\Delta u_{\text{meas}} = \sqrt{2} \frac{\delta \tau}{T_{\text{meas}}}. \quad (24)$$

Together with equations 1 and 23, coincidence rate $r_C$ and acquisition time $T_a$, is the synchronization jitter due to the uncertainty of peak position measurement,

$$\sigma_{\text{meas}} = \frac{1}{2} \sqrt{2} \frac{\sigma}{\sqrt{r_C T_a - T_{\text{meas}}}} T_a. \quad (25)$$

The second jitter contribution comes from the clock drift $\frac{\partial (\Delta u)}{\partial t}$. clock frequency skew, that has not been foreseen previously, may accumulate over the feedback loop time $T_{\text{feed}}$,

$$\Delta u_{\text{drift}} = \frac{\partial (\Delta u)}{\partial t} T_{\text{feed}}. \quad (26)$$

Together with equation 1 is the resulting synchronization jitter, due to the drifting clock,

$$\sigma_{\text{drift}} = \frac{1}{2} \frac{\partial (\Delta u)}{\partial t} T_{\text{feed}} T_a. \quad (27)$$

Both measurement and synchronization jitter limit the total achievable synchronization jitter during live tracking (Fig. 3c),

$$\sigma^2_{\text{sync}} = \sigma^2_{\text{meas}} + \sigma^2_{\text{drift}}. \quad (28)$$
FIG. 11. **Fine tuning of clock frequency skew through cross-correlation with Start-Stop method**

a) Correlation peaks for various timing offset accuracy. The noise values are highest at small time delay with -20 dB and reduce exponentially until approximately the inverse count rate at Bob is reached. The peak noise level is larger than the correlation peak for poor timing offset precision > 2.5 µs. Peak search algorithms may still recover the peak here, instead of simple search of maximum at timing offset precision < 2.5 µs. b) Magnified view of timing offset-corrected cross-correlation peaks for various clock frequency skew compensation values. The shape of correlation peaks depends on the compensated clock frequency skew and may be asymmetrical from clock instabilities. c) The correlation peak coincidence to accidentals ratio or its timing jitter is ideal indicator for optimum clock frequency skew compensation, as they become maximum and minimum, respectively. The analytical trend lines are derived from the experimental acquisition time of 1.4 seconds, smallest timing jitter of 258 ps and maximum coincidence-to-accidentals ratio of 204.

**D. Clock frequency skew fine tuning and Start-Stop method**

Careful fine tuning of the optimum clock frequency skew for compensation reduces the synchronization jitter to a minimum. As the timing offset is calculated with nanosecond accuracy in the synchronization initialization, it is feasible to shrink down the observation window to < 100 ns to reduce the computation effort. Cross-correlations are now derived by the computation-efficient Start-Stop method [54–56]. In contrast to the FFT cross-correlation that correlates all timetags from Alice with all Bob timetags from Bob, the Start-Stop method calculates time differences between neighboring timetags. If the timing offset was not calculated with a precision smaller than Min{1/rA, 1/rB}, it rises the probability of wrongly calculated time differences and thus reduces the number of correlation events. The correlation peak height reduces and finally becomes smaller than the peak noise values (Fig. 3a). The correlation peak compresses to the smallest width σmin = 258 ps by compensation of the residual clock frequency skew. In addition maximizes the coincidence-to-accidentals ratio at CAR0 = 204 over a coincidence window that is determined by the total timing jitter. The experimental correlation peak is fit by a stretched Gaussian function and then the timing jitter derived from it. The experimental total jitter and peak value behaves perfectly as analytically given (see Methods IV C 1). Clocks with high stability, i.e. a constant frequency difference between two clocks, create a jitter envelope with a plateau. Rubidum oscillators are known for their high stability of 10^{-12} over 1 second [70, 71] and create a jitter envelope with a plateau, as the frequency difference is almost constant over time. Quartz crystal oscillators on the other hand may show weak stability of only 10^{-11} to 10^{-9} over 1 second [45] that result in asymmetry and give rise to deviations between analytical and experimental trends in a region between -1 to 2 ns/s. The method for compensation of the clock frequency skew by a simple peak search (Fig. 3c) may be done with arbitrary resolution. Step sizes of < 1 ns/s work greatly to start live tracking of clock drifts in the last step of synchronization.
FIG. 12. **Initialisation and live tracking algorithm**  

**a** Coarse search of the clock frequency skew.  
**b** Fine-tuning of the clock frequency skew over a feedback time of approximately 1 minute with two rubidium clocks. The residual clock frequency skew over the last 4 minutes amounts to 6.4 ps/s, resulting in synchronization jitters of approximately 0.32 ps over acquisition time of 100 ms.  
**c** The algorithm for synchronization initialization and live tracking includes the following variables, time tags from Alice and Bob $t_{A/B}$, first estimated clock frequency skew $\Delta u_{\text{est}}$, estimated timing offset $\Delta T_{\text{est}}$, accurate timing offset $\Delta T$, and clock frequency skew $\Delta u$. The location of the correlation peak is found through an efficient Start-Stop cross-correlation.  
**d** Input: timetags Alice $t_A$ and Bob $t_B$  
Functions: read timetags(), coarse clock skew(), accurate clock skew(), sleep(), location of correlation peak()  
/* Synchronization initialization */  
$t_{A,B} =$ read timetags()  
/* FFT-based cross-correlation */  
$\Delta u_{\text{est}}, \Delta T_{\text{est}} =$ coarse clock skew($t_A, t_B$)  
/* Start-Stop method cross-correlation */  
$\Delta u, \Delta T =$ accurate clock skew($t_A, t_B, \Delta T_{\text{est}}, \Delta u_{\text{est}}$)  
$\Delta u_{\text{drift}} = 0$
$i_{\text{buff}} = 0$
$\Delta T_{\text{local}} =$ location of correlation peak($t_A, t_{B_{\text{corr}}}, \Delta T$)$
\Delta T = \Delta T + \Delta T_{\text{local}}$
\Delta T_{\text{corr}} = \Delta T_{\text{local}} + \Delta T_{\text{local}}$
$dt_{\text{buff}}[i_{\text{buff}}] = \Delta t$
$\Delta T_{\text{buff}}[i_{\text{buff}}] = \Delta T_{\text{local}}$
save parameters
$i_{\text{buff}} = i_{\text{buff}} + 1$
if $i_{\text{buff}} =$ buffer size then
$\Delta u_{\text{drift}} = \Delta u_{\text{drift}} - \text{mean}(\Delta T_{\text{buff}} / dt_{\text{buff}})$
$\Delta T_{\text{buff}}[i_{\text{buff}}] = \Delta T_{\text{local}}$
$dt_{\text{buff}}[i_{\text{buff}}] = \text{zeros}(\text{buffer size})$
$i_{\text{buff}} = \text{zeros}(\text{buffer size})$
$i_{\text{buff}} = 0$
end
end

E. Live tracking algorithm

After synchronization initialization, including the coarse clock frequency skew search (Fig. 12a) and its fine-tuning (Fig. 12b), residual and time-dependent clock frequency skews increase the synchronization jitter and call for compensation during communication sessions. Fig. 12c represents the effect of correct compensation for rubidium clocks by the algorithm in Fig. 12d. The correlation peak timing offset is tracked over $n$ data points. Any change of timing offset would be caused by an instantaneous drift. Here, the timing offset changes by approximately 8 ns in 1 minute. By applying the measured residual clock frequency skew to future time tags, we can reduce the timing offset change and subsequently the synchronization jitter. Long averaging times work especially well for highly stable clocks, such
as rubidium clocks. The timing offset changes by less than 1.5 ns over 4 minutes after the first correction, already providing residual clock frequency skews of < 6.4 ps/s and thus easily synchronization jitters < 0.32 ps in typical 100 ms data package sizes.

**F. Photon pair rates**

![Photon pair rates](image)

FIG. 13. Photon pair rates according to Fig. 3b

The experimentally detected photon pair rates determine achievable signal-to-noise ratios and limit the synchronization jitter under high losses. Figure 13 represents the rates from experiment (Fig. 3b).

**G. Experimental clock drift**

![Clock drift](image)

FIG. 14. a The instantaneous clock frequency skew $\Delta u$ drifts without live tracking. b The time-dependent clock drift $\partial(\Delta u)/\partial t$ is zero on average with standard deviation of 320 ps/s².

Knowledge of the clock stability provides important measures about the limiting synchronization jitters. To estimate the clock stability, we first compare the timing jitter without tracking to the reference (Fig. 4b). Any increase from the reference is caused by synchronization jitters through the residual clock frequency skews $\Delta u$ (equation 1). The residual, not compensated clock frequency skew is plotted in Fig. 14a and amounts to more than 10 ns/s. Almost ideal synchronization may be achieved if the residual clock frequency skew is constant. However, crystal oscillators may have time-dependent frequencies that result in variation of the residual clock frequency skew. Fig. 14b indicates the acceleration of the clocks with time with a mean value of zero and standard deviation of 320 ps/s². High clock drifts require small feedback times for compensation that simultaneously demands many correlation events.

The time taggers are equipped with external crystal oscillators without temperature control. The data sheet accuracy is ±20 ppm, aging ±3 ppm/first year, ±1 ppm/year and temperature dependence ±0.125 ppm (25°C ... 85°C). The rubidium clocks are temperature controlled with accuracy ±10⁻⁴ ppm (ambient temperature 0°C ... 40°C), aging < 5 × 10⁻⁵ ppm/month, stability over 1s is 10⁻⁵ ppm.

**H. Low signal live tracking**
FIG. 15. Low correlation rate live tracking under turbulence. Turbulence parameter: Fried parameter $1\text{ mm, } 1/e^2$ full-width beam waist of $3.4\text{ mm}$. 

- The count rate at the receiver Bob of $437 \pm 6 \text{ kcps}$ is increased artificially by a noise source. Alice’s count rate amounts to $165 \pm 3 \text{ kcps}$. 
- The average coincidence rate is $430 \pm 160 \text{ cps}$ (coincidence window is equal to RMS timing jitter of $267 \text{ ps}$) and indicates periodic fluctuations from the turbulence disk. 
- The mean coincidence-to-accidentals ratio is $10 \pm 4$ with a correlation peak live tracking threshold of 5. 
- The total timing jitter probability distribution describes a Gaussian with its fitted center at $277.8 \pm 0.8 \text{ ps}$. Together with same-clock total timing jitter of $260 \pm 2 \text{ ps}$ is the synchronization jitter approximately $98 \pm 6 \text{ ps}$. 
- The tracked clock frequency skew displayed over time. 
- The timing offset varies more than in high-signal environment (Fig. 4), because of partially stopped tracking when the coincidence-to-accidentals ratio falls below 5. This gives rise to an increase of the total timing jitter. The feedback loop time and data package size amounts to $200 \text{ ms}$ and $100 \text{ ms}$, respectively. Summary of this figure in table I.

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