Stability analysis of articulated dump truck based on eigenvalue method

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Abstract. The lateral stiffness of articulated dump truck is relatively weak. When articulated dump truck is running, the driving stability is deteriorated and it is prone to occur "snake shape" instability phenomena. Taking the force and its mechanical characteristics of articulated dump truck with three axles into account, the dynamic mathematical model of the 3-DOF was established using the Lagrange equation. Based on the Lyapunov stability principle, the driving stability of articulated dump truck was analyzed using the eigenvalue method. The dynamic mathematical model can be used to analyze the influence of the following parameters on driving stability, such as different speed, centroid position, steering parameters, wheelbase, mass etc. It can provide theoretical basis for parameters optimization in stability control and stability design.

1. Introduction

Compared with the rigid body truck, articulated dump truck has better passing capacity, small turning radius, and can work freely under the conditions of narrow space and soft ground [1-4]. However, the middle hinged point increases the freedom of truck and reduces its lateral stability. When driving at a higher speed, articulated dump truck is prone to "snaking" instability [5-9]. Taking 60t six-wheel articulated dump truck as the research object, the three-degree-of-freedom (the 3-DOF) dynamic mathematical model of articulated dump truck was established. And the influencing factors of driving stability of articulated dump truck can be analyzed by using the eigenvalue method which can provide theoretical basis for parameters optimization in stability control and stability design. Mathematical model of articulated dump truck.

1.1. Vertical force analysis of articulated dump truck

In order to study the basic characteristics of articulated dump truck’s stability, the dynamic mathematical model was simplified as follows: (1) The cylinder angle was the input of the steering system; (2) Articulated dump truck only moved parallel to the ground, so its displacement along the Z-axis was zero; (3) The forward velocity along the X-axis was constant; (4) The pitch angle around the Y-axis and the side inclination angle around the X-axis were zero with the 3-DOF of lateral and yaw motion; (5) When articulated dump truck was only subjected to small disturbance near the equilibrium state, the swing angle and the movement amount were small. So, it could be considered that the motion equation of the truck was linear [10]. The vertical forces on articulated dump truck referred with Figure 1.
According to the statics balance equation, the forces on the front and rear axles were as follows respectively:

\[ N_1 = \frac{m_1g(b+c+d+e)+m_2g(d+e)}{a+b+c+d+e} \]  
\[ N_r = (m_1+m_2)g - N_1 \]  

Where, 
- \( m_1 \) - front body mass; 
- \( m_2 \) - rear body mass; 
- \( a \) - the distance from the front axle to the center of mass of the front body; 
- \( b \) - distance from the hinged point to the center of mass of the front body; 
- \( c \) - distance from the center of mass of the rear body to the hinged point; 
- \( d \) - distance from the rear axle to the center of mass of the rear body; 
- \( e \) - distance between two rear axles.

Since the distance between two rear axles was equal, the load \( N_2 \) and \( N_3 \) undertaken by the middle wheel and the rear wheel were as follows:

\[ N_2 = N_3 = \frac{N_r}{2} \]  

1.2. tire model

The empirical formulas of tire model were as follows:

\[ F_y = NC_a\alpha \]  
\[ M_z = NC_{Ma}\alpha \]  

Where, 
- \( F_y \) - lateral force; 
- \( N \) - the vertical pressure acting on the tire; 
- \( C_a \) - tire lateral force coefficient; 
- \( \alpha \) - tire lateral angle 
- \( M_z \) - tire aligning moment.

1.3. Establishment of dynamic mathematical equation of articulated dump truck

The 3-DOF model of articulated dump truck referred with Figure 2. \( X-Y \) was the absolute coordinate system. And \( x_1-y_1 \) was the follow-up coordinate of the origin fixed on the front body. \( X_1 \) and \( Y_1 \) were the absolute coordinates of the center of mass in front.
The 3-DOF model of articulated dump truck.

The speed relation of the front center of mass was expressed by the following formulas:

\[ u = \dot{X}_1 \cos \psi + \dot{Y}_1 \sin \psi \]  
(6)

\[ v = -\dot{X}_1 \sin \psi + \dot{Y}_1 \cos \psi \]  
(7)

Global variable \( q \) was adopted to describe the whole system, as shown in the following formula:

\[ q = [X_1, Y_1, \psi, \theta] \]  
(8)

In generalized coordinates, the general equation of Lagrange dynamics was as follows:

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial \dot{q}_i} - \frac{\partial R}{\partial \dot{q}_i} = Q_i \]  
(9)

Where, \( V \) - system potential energy; 
\( R \) - energy dissipation function; 
\( q \) - global variables describing the system;

The kinetic energy \( T, T_1, T_2 \) of the truck were as follows:

\[ T = T_1 + T_2 \]  
(10)

\[ T_1 = \frac{1}{2} m_1 (\dot{X}_1^2 + \dot{Y}_1^2) + \frac{1}{2} I_1 \dot{\psi}^2 \]  
(11)

\[ T_2 = \frac{1}{2} m_2 (\dot{X}_1^2 + (\dot{Y}_1 - b\dot{\psi} - c\dot{\theta})^2) + \frac{1}{2} I_2 \dot{\theta}^2 \]  
(12)

Where, \( T_1, T_2 \) - kinetic energy of the front and rear body respectively; 
\( I_1, I_2 \) - moment of inertia of front and rear body respectively.

The storage energy of hydraulic cylinder stiffness was equivalent to system potential energy \( V \) was as follows:

\[ V = \frac{1}{2} K_R (\psi - \theta)^2 \]  
(13)

The damping energy consumption of the hydraulic cylinder was equivalent to the dissipated energy of the system [11] \( R \) was as follows:

\[ R = \frac{1}{2} C_R (\dot{\psi} - \dot{\theta})^2 \]  
(14)

As the system was affected by lateral force and tire aligning moment, the virtual work done by external force and external moment for small rotation angle \( \psi \) and \( \theta \) was as follows:

\[ \delta W = F_{y1}\delta(Y_1 + A\psi) + F_{y2}\delta(Y_1 - b\psi - (c + d)\theta) + F_{y3}\delta(Y_2 - b\psi - (c + d + e)\theta) + M_{z1}\delta\psi + (M_{z2} + M_{z3})\delta\theta \]  
(15)

Therefore, generalized force and generalized moment of the system were as follows respectively:
\[ Q_1 = F_{y1} + F_{y2} + F_{y3} \]  (16)
\[ Q_\psi = aF_{y1} - b(F_{y2} + F_{y3}) + M_{z1} \]  (17)
\[ Q_\theta = -(c + d)F_{y2} - (c + d + e)F_{y3} + M_{z2} + M_{z3} \]  (18)

Assume that the forward speed of articulated dump truck was constant. \( X \) and \( Y \) were as follows for small angles:
\[ \dot{X} \approx u \]  (19)
\[ \dot{Y} \approx uw\psi + v \]  (20)

The articulated angle was as follows:
\[ \phi = \psi - \theta \]  (21)

The average lateral deflection angles of the front axle and rear axle tires were as follows:
\[ \alpha_1 = \frac{v + u\psi}{u} \]  (22)
\[ \alpha_2 = \frac{v - b\psi - (c + d)(\psi - \phi) + \phi}{u} \]  (23)
\[ \alpha_3 = \frac{v - b\psi - (c + d + e)(\psi - \phi) + \phi}{u} \]  (24)

By substituting equations (10) - (24) into equations (9), the system dynamic state equation could be obtained as follows:
\[ \dot{X} = AX \]  (25)
\[ X = [v, \psi, \phi, \dot{\phi}]^{T} \]  (26)

Where, \( A \) represents the coefficient of equation of state.

The characteristic equation of the system was as follows:
\[ \det(\lambda E - A) = 0 \]  (27)
\[ A = -M^{-1}C \]  (28)

2. Stability determination based on eigenvalue

According to the truck parameters, MATLAB was used to solve the four characteristic roots \( \lambda_1, \lambda_2, \lambda_3 \) and \( \lambda_4 \) of the above characteristic equation. The driving stability of articulated dump truck was determined by the eigenvalue’s sign. According to the Lyapunov stability principle, when all the real parts of the eigenvalue of the characteristic equation are negative, the motion is stable. If the eigenvalue of the system has at least one positive real part, the motion is unstable, that is, the so-called "snaking" instability. When the real part of the complex eigenvalue is zero, it is the critical state of motion with the critical speed. Curves of characteristic equations with truck speed as the horizontal coordinate referred with Figure 3 and Figure 4.
Table 1. The parameters of each part of articulated dump truck.

| The name of the structure                                | Parameter | Unit   | Value   |
|----------------------------------------------------------|-----------|--------|---------|
| Front body mass                                          | $m_1$     | Kg     | 13779   |
| Moment of inertia of the front body                       | $I_1$     | Kg m²  | 81560   |
| Rear body mass                                           | $m_2$     | Kg     | 24010   |
| Moment of inertia of the rear body                        | $I_2$     | Kg m²  | 501005  |
| The distance from the front axle to the front center of mass | $a$       | m      | -0.159  |
| The distance from the front center of mass to the hinged  | $b$       | m      | 1.762   |
| The distance from the rear center of mass to the hinged point | $c$       | m      | 2.954   |
| The distance from the rear center of mass to the second axle | $d$       | m      | -0.3    |
| The distance between the second axle and the third axle   | $e$       | m      | 2       |
| Lateral stiffness of front axle tire                      | $K_{a1}$  | N/rad  | 60000   |
| Lateral stiffness of the second axle tire                 | $K_{a2}$  | N/rad  | 60000   |
| Lateral stiffness of the third axle tire                  | $K_{a3}$  | N/rad  | 60000   |
| Tire aligning moment coefficient                          | $C_{Ma}$  | m/rad  | 0       |
| Equivalent stiffness coefficient                          | $K_R$     | N·m/rad| 122803  |
| Equivalent damping coefficient                            | $C_R$     | N·m/rad| 0       |

In Figure 3, when the truck speed was less than 7 m/s, the real parts of the eigenvalue $\lambda_1$, $\lambda_2$ were all negative, the truck was stable. When the truck speed was more than 7 m/s, that was, 25.2 km/h, the real parts of the two eigenvalues became positive. The system started to be unstable. And the snaking instability phenomenon started to appear.

3. Conclusion
Aiming at the structural characteristics of 60t articulated dump truck, the 3-DOF mathematical model of articulated dump truck was established. The driving stability of articulated dump truck could be determined by the eigenvalue method of the differential equations. When the eigenvalue's real part is negative, the movement is stable. On the contrary, when the eigenvalue's real part is positive, the movement is unstable. So, when the eigenvalue's real part is zero, it is the critical state of movement and the corresponding velocity is critical velocity or the top speed limit of the designed maximum speed. In this study, the maximum stable driving speed of articulated dump truck was 25.2 km/h. At this point, the eigenvalue of the characteristic equation was zero, it was the critical state of driving stability and instability. When the truck speed was higher than 25.2 km/h, the snaking instability started to appear. And as the speed increased, the instability became more obvious. The dynamic
A mathematical model can be used to analyze the influence of the following parameters on driving stability, such as different speed, centroid position, steering parameters, wheelbase, mass etc. It can provide theoretical basis for parameters optimization in stability control and stability design.

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