Systematic uncertainties of artificial neural-network pulse-shape discrimination for $0\nu\beta\beta$-decay searches using true-coaxial HPGe detectors

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Abstract

A pulse-shape discrimination method based on artificial neural networks was applied to pulses simulated for different background, signal and signal-like interactions inside a germanium detector. The simulated pulses were used to investigate the systematic uncertainties of the method. It is verified that neural networks are well-suited to identify background pulses in true-coaxial high-purity germanium detectors. The systematic uncertainty on the signal recognition efficiency derived using signal-like samples from calibration measurements is estimated to be 5%. This uncertainty is due to differences between signal and calibration samples.

1 Introduction

Experiments searching for neutrinoless double beta ($0\nu\beta\beta$) decay require an extremely low background level in the region of interest around a few MeV. Compton scattered $\gamma$-particles, originating from radioactive decays in the proximity of the detectors, are an important background contribution at such energies.

In high-purity germanium (HPGe) experiments, these interactions are often identified and removed from the signal data set through pulse-shape analysis (PSA). In order to extract a half-life limit, the signal recognition
efficiency has to be known. Usually, experimentally obtained pulse-shape libraries with signal-like events are used to obtain the signal recognition efficiency. However, these libraries can have energy-deposition topologies and event-location distributions different to those of the signal searched for. Efficiencies obtained like this can be systematically different from the recognition efficiency for the real signal. Furthermore, the libraries used to derive the recognition efficiencies often contain events of the wrong type, making a direct determination of the efficiencies impossible.

This paper presents investigations of the reproducibility and systematic uncertainties of the efficiencies of pulse-shape discrimination (PSD) using artificial neural networks (ANNs) with libraries of simulated pulses. The general idea of PSD using ANNs is introduced and the sources of possible systematic effects are discussed. The simulations and the libraries used for the analysis are described as well as the ANNs and the procedures used to train them. The stability of the method against initial conditions and ANN topologies is investigated. The focus is on the differences obtained in recognition efficiencies using different test libraries and the associated systematic uncertainties.

2 Pulse-shape discrimination for HPGe detectors using artificial neural-networks

The detection principle of semiconductor detectors is based on the creation and detection of electron–hole pairs, i.e. charge carriers, when radiation interacts with the detector material. Charge-sensitive preamplifiers are commonly used to detect the drifting charge carriers in large volume HPGe detectors. The time structure of an event, the pulse-shape, is defined by the mirror charge signal induced on the electrodes as a function of time. The pulse length is given by the time needed to fully collect the charges on the electrodes. See e.g. [1, 2] for a detailed description of the pulse creation process.

For photons in the MeV range, the dominant interaction process is Compton scattering. A photon with an energy of one MeV has a mean free path of $\simeq 3\text{ cm}$ in germanium. Thus, photon-induced events with energies of about 2 MeV are mostly composed of several energy deposits within a HPGe detector, separated by a few centimetres. These background-like events are referred to as multi-site events (MSE). In contrast, electrons with the same energy have a range of the order of millimetres and deposit their kinetic energy “locally”. Signal-like events of this kind are referred to as
single-site events (SSE). The two electrons emitted in $0\nu\beta\beta$ decay result predominantly in SSEs. Due to Bremsstrahlung, a fraction of a few % of the $0\nu\beta\beta$-decay events become MSEs [3]. Events identified as MSE in the energy region of interest are rejected as background.

Methods to distinguish between SSEs and MSEs in HPGe detectors using ANNs were developed previously [4, 5, 6]. In most previous works, events from double escape peaks (DEP) and full absorption peaks (FAP) were used to create training libraries of signal-like and background-like events, respectively. These were obtained from calibration data for which sources such as $^{228}$Th or $^{56}$Co were used. The ANN efficiencies to correctly identify events are also typically evaluated using libraries from calibration measurements.

The efficiencies of PSD methods are not necessarily homogeneous throughout the detector volume. For a realistic evaluation, the spatial distribution of the events in a given test library has to be taken into account. Especially, DEP events will exhibit a non-uniformity in event location distribution due to the topology of the events. If pair production occurs in a coaxial HPGe at high radii, $r$, and height, $z$, i.e. close to the extreme boundaries, the probability for the two 511 keV $\gamma$-particles to escape is the highest. Hence, libraries of DEP events have a higher event location density in these parts of the detector. On the other hand, signal events due to $0\nu\beta\beta$ decay are expected to be homogeneously distributed. Using a library with an event location distribution different from the one expected for the signal can lead to systematic biases.

3 Strategy

In order to quantify the uncertainties on the ANN recognition efficiencies, simulations are used. The signal (background) recognition efficiency $\eta$ ($\rho$) of an ANN is defined as the probability that an ANN correctly identifies an SSE (MSE) from an event-library containing only SSEs (MSEs).

Realistic SSE and MSE pulse-shape libraries always contain events of both classes. Hence, the ANN method applied to a library of predominantly SSE or MSE pulses will result in a survival probability $E$ or rejection probability $R$, defined as the fraction of pulses in the library that are categorized as SSE or MSE, respectively:

$$\begin{align*}
E &= \eta \cdot S_{SSE} + (1 - \rho) \cdot S_{MSE} , \\
R &= \rho \cdot B_{MSE} + (1 - \eta) \cdot B_{SSE} ,
\end{align*}$$

(1)

where $S_{SSE}$ and $S_{MSE}$ are the fraction of SSEs and MSEs in the SSE-library,
respectively, and $B_{MSE}$ and $B_{SSE}$ are the fraction of MSEs and SSEs in the MSE-library, respectively. When obtaining $\eta$ and $\rho$ from simulated pulses, clean SSE and MSE libraries with $S_{SSE} = 1$ and $B_{SSE} = 0$ are used.

Different libraries with different percentages of the wrong type of events are used to train individual ANNs. By comparing $\eta$ of these individual ANNs, the effect of the presence of wrong type of events in the training libraries ($S_{MSE} > 0$ or $B_{SSE} > 0$) on the ANN signal-recognition efficiency is quantified.

In order to quantify the effect of non-homogeneous event-location distributions, $\eta$ for ANNs obtained with libraries with realistic event-location distributions are compared to those obtained from libraries with a homogeneous event-location distribution.

The stability of the method is verified by training and evaluating a set of ANNs with different initial weights of the ANN synapses and with training libraries of different sizes. Finally, the influence of the number of hidden layers and the number of neurons in the ANN on $\eta$, is investigated.

True-coaxial HPGe detectors are considered in this paper. They have a simple radial electric field and, thus, have relatively simple pulse shapes. Consequently, pulse-shapes of this type of detectors have lower systematic uncertainty due to smaller uncertainties in the field calculations compared to detectors with more complex geometries. This makes this type of detectors interesting for this analysis.

4 Libraries of simulated pulse shapes

HPGe detectors for low background experiments typically have a radius, $r_{\text{max}}$, and a height of a few cm\(^1\). The simulated n-type true-coaxial germanium detector has a height of 70 mm and $r_{\text{max}}=37.5$ mm with the diameter of the borehole being 10 mm. The simulated geometry describes an existing true-coaxial 18-fold segmented n-type HPGe developed as a prototype detector \cite{7} for the GERDA experiment \cite{8}.

Photon and electron interactions for different libraries were simulated within the Mage framework \cite{9}, based on GEANT4 \cite{10, 11}. Pulse shapes were simulated for the core electrode. For each event, individual energy deposits inside the detector were combined whenever the distance between subsequent hits was less than 0.1 mm. Pulse shapes for the combined energy

\(^1\)A polar coordinate system is used with the origin at the centre of the crystal and the $z$ axis pointing upwards. In Cartesian coordinates, the $x$- and $y$- axes coincide with the crystallographic $\langle 110 \rangle$ axes, while the $z$ axis coincides with the crystallographic $\langle 001 \rangle$ axis.
Figure 1: Typical pulse shapes for (a) SSE and (b) MSE. The dashed vertical lines denote the times at which the pulses reach 10% and 90% of their amplitude.

Deposits were simulated using pre-calculated electric and weighting fields using the pulse-shape simulation package described in [2].

The number of grid points for the electric- and weighting-field calculations was $33(r) \times 181(\phi) \times 71(z)$. The electrically active impurities were assumed to be homogeneous within the detector, with a density of $0.63 \times 10^{10}$ cm$^{-3}$. The length of the simulated pulses is 1 μs. The step frequency of the simulation is 1125 MHz, a multiple of 75 MHz to which the pulses were resampled to take the effects of a typical DAQ into account. Above 1 GHz, the step frequency is sufficient to correctly describe trajectories [12, 13].

The amplifier RC-integration constant was set to 20 ns, corresponding to a bandwidth of about 10 MHz, while the amplifier decay time was set to 50 μs. Each individual pulse shape was convoluted with Gaussian noise, $\sigma = 6$ keV. Typical SSE and MSE pulses are shown in Fig. 1. The pulse length, $t^{10-90}$, is between 160 and 500 ns [5, 14], where $t^{10-90}$ is defined as the time in which the pulse increases from 10% to 90% of its amplitude. This part of the pulse contains the relevant information regarding the event topology.

The simulated libraries are listed in Table 1. The DEP, $2\nu\beta\beta$ and $0\nu\beta\beta$ event-libraries were created with and without a realistic admixture of MSEs due to Bremsstrahlung and Compton-scattered $\gamma$-particles. All MSE libraries were simulated for the 1620 keV FAP, corresponding to a $^{228}$Th
Table 1: MSE and SSE libraries used to evaluate the recognition efficiencies of ANNs. The energy range of the events contained in the libraries are given in the second column. The third column describes the selection criteria for the individual libraries, while information on the location of the simulated source, influencing the event location distribution, is listed in the fourth column. For details on the notation, see the text.

| Library        | Energy          | Processes          | Source location |
|----------------|-----------------|--------------------|-----------------|
| SSE - Single Site Event Libraries | | | |
| DEP top        | (1593 ± 5) keV  | No Comp & Brems    | Top             |
| DEP side       | (1593 ± 5) keV  | No Comp & Brems    | Side            |
| DEP real       | (1593 ± 5) keV  | All processes      | Side            |
| DEP clean      | (1593 ± 5) keV  | No Comp & Brems    | Homog.          |
| 0νββ real      | (2039 ± 5) keV  | All processes      | Homog.          |
| 0νββ clean     | (2039 ± 5) keV  | No Comp & Brems    | Homog.          |
| 2νββ real      | 450 keV – 540 keV | All processes      | Homog.          |
| 2νββ clean     | 1000 keV – 1450 keV | No Comp & Brems | Homog.          |
| MSE - Multi Site Event Libraries | | | |
| FAP top        | (1620 ± 5) keV  | Comp & Brems only  | Top             |
| FAP all        | (1620 ± 5) keV  | All processes      | Top             |
| FAP side       | (1620 ± 5) keV  | Comp & Brems only  | Side            |
| FAP clean      | (1620 ± 5) keV  | R$_{90} >$2 mm, Comp & Brems only | Top |

source, typically used for calibration. The notation of *No Comp & Brems* is used for SSE libraries in which all events with Compton scattering or hard Bremsstrahlung were removed. MSE libraries that contain only events which have at least one energy deposition due to Compton scattering or hard Bremsstrahlung in the detector are marked as *Comp & Brems only*. Additionally, in one case, it was required that R$_{90}$, the radius within which 90% of the deposited energy was contained [5], is larger than 2 mm.

To indicate the origin of incoming photons, the last column in Table 1 lists either “Top” or “Side”. This means that the photons were simulated to come from either the $xy$- or $xz$-plane, respectively. Their origins are homogeneously distributed on the planes with their momentum perpendicular to the plane of origin. The planes are located 17.5 cm from the centre of the detector and their area is sufficiently large to cover the detector. Libraries with homogeneous event location distributions are listed as “Homog”. The training libraries contain between 7,000 and 20,000 simulated pulses.

The radial distributions of the energy barycenters, defined as the energy-weighted mean radial position of the energy deposit, of individual events for SSE training libraries containing no MSEs are shown in Fig. 2. Top, middle and bottom refer to events contained in the upper, middle and lower
Table 2: Sets of libraries used for ANN training. Hom and inhom are used to indicate sets where a SSE library with homogeneous and inhomogeneous event location distribution was used.

| SSE library       | MSE library      |
|-------------------|------------------|
| set I - inhom DEP | DEP side         |
| set II - real $2\nu\beta\beta$ | $2\nu\beta\beta$ real | FAP side |
| set III - hom DEP | DEP clean        |
| set IV - top DEP  | DEP top          |
| set V - ideal $0\nu\beta\beta$ | $0\nu\beta\beta$ clean | FAP top |

third of the detector, respectively. These three volumes are equal. The barycenter of an individual event corresponds approximately to the position of the interaction/decay. For the DEP clean library, where clean is used here and below to identify libraries with no Compton or Bremsstrahlung interactions, it is flat as a function of $r$ and equivalent to the distribution of the $2\nu\beta\beta$ real library. Real is used to indicate libraries where all processes, i.e. including Compton scattering and Bremsstrahlung, are present. For DEP clean, the 2.6 MeV photons were forced to interact homogeneously in the detector. The DEP side and DEP top libraries have inhomogeneous event-location distributions, events being located with a higher probability at high $r$ close to the bottom and top of the detector. Side and top indicate the location of the source with respect to the detector.

5 ANN training and efficiencies

The libraries listed in Table 1 were used to create five different ANN training sets. They are listed in Table 2 showing the combinations of SSE and MSE libraries.

The ANNs used in this analysis were built using the TMultiLayerPerceptron (TMLP) within the ROOT framework [15]. Only the part of the pulse containing the relevant information on the event topology is used by the ANNs. The pulses in the considered detector are maximally around 500 ns long. In total, 40 time steps, corresponding to 530 ns, were used. The center of the resulting trace was chosen to be the point where the pulse reaches 50% of its amplitude. The amplitude of each pulse was normalized to unity.

The ANNs are composed of 40 input neurons, one hidden layer with the same number of neurons and an output layer with only one neuron. The
ANNs were trained by iteratively adjusting the weights using the Broyden-Fletcher-Goldfarb-Shanno learning method [16, 17, 18, 19]. During the training procedure, background-like MSEs were assigned an ANN output, $NN$, of 0 and signal-like SSEs were assigned an $NN$ of 1. The input samples were separated into two equal subsets, one for training and one for testing.
Figure 3: Simulated spectra of events contained in the libraries *DEP real* and *FAP all* (see Table 1) in the energy region around the 1593 keV DEP and the 1620 keV FAP before (solid line) and after (dashed line) MSE rejection using the ANN trained with *set I*.

Libraries of the same size for MSEs and SSEs were used. For a trained network, $NN$ should be close to 1 for SSEs and close to 0 for MSEs.

The number of iterations sufficient for training was established. For the training *set II*, ANNs were trained fifteen times adopting each time an increased number of training iterations. After each ANN training, the survival probabilities $E$ and $(1 - R)$ were evaluated (see Eq. 1). For up to 1500 iterations, $E$ and $(1 - R)$ were increasing. After more than 1500 iterations, no significant changes were observed. Accordingly, the number of iterations used during the ANN training was set to 1500 for all further tests.

Events are classified as signal-like if $NN > \overline{NN}$, where $\overline{NN}$ is a parameter that has to be optimized. The rejection probability $R(\overline{NN})$ represents the fraction of events from an MSE dominated library, FAP in this case, rejected by the cut $NN \leq \overline{NN}$. The survival probability $E(\overline{NN})$ represents the fraction of events from a SSE dominated library (DEP, $2\nu\beta\beta$ or $0\nu\beta\beta$) kept with $NN > \overline{NN}$. The cut value $NN_{\text{max}}$ is chosen to maximize the quantity $\varepsilon = \sqrt{R \cdot E}$.

The solid histogram in Fig. 3 shows the simulated energy spectrum for
Table 3: Summary of the variables used to evaluate the performance of the ANNs.

| Variable                      | Expression                        |
|-------------------------------|-----------------------------------|
| Survival probability          | \( E_{max} = E(NN_{max}) \)      |
| Rejection probability         | \( R_{max} = R(NN_{max}) \)      |
| Signal recognition efficiency | \( \eta_{max} = \eta(NN_{max}) \) |
| Background recognition efficiency | \( \rho_{max} = \rho(NN_{max}) \) |
| Background reduction power    | \( \varepsilon_{max} = \varepsilon(NN_{max}) \) |

events contained in the DEP real and FAP all libraries (Table 1). The FAP is significantly reduced while the DEP remains almost untouched.

The survival probability \( E \) for 0νββ and DEP events is given by the ratio of the peak areas after and before the ANN rejection. The areas are determined by fitting a Gaussian plus constant background to the spectra.

In Figures 4(a) and 4(b) the \( NN \) distribution for MSEs and SSEs for training sets I and II are shown. A clear separation between the \( NN \) distributions of MSE and SSE libraries is visible. Figures 4(c) and 4(d) show \( E(NN), R(NN) \) and \( \varepsilon(NN) \) for training sets I and II, respectively. The vertical line represents \( NN_{max} \). The \( E, R, \varepsilon \) and \( \eta \) values determined for \( NN_{max} \) are called \( E_{max}, R_{max}, \varepsilon_{max} \) and \( \eta_{max} \), respectively, in the following. These variables are summarized in Table 3. For libraries with purely SSE or MSE events, for which \( S_{SSE} = 1 \) and \( B_{MSE} = 1 \), \( E_{max} \) and \( R_{max} \) coincide with \( \eta_{max} \) and \( \rho_{max} \), respectively (see Eq. 1).

Statistical uncertainties quoted in the following are derived from the statistical fluctuations expected due to the limited number of simulated events and events surviving the selection.

6 Influence of initial conditions, ANN topology and training libraries on recognition efficiencies

6.1 Initial conditions and topologies

The reproducibility of \( \eta_{max} \) was investigated by training five ANNs with the same ANN topology. The same training samples were used but the initial weights of the individual synapses of the untrained ANN were different in each case. Also the order in which individual pulses from the training sets were chosen for the iterative training was different for each ANN. The fluctuations of \( \eta_{max} \) between the different ANNs are of the order of 1% of the
Figure 4: NN output distributions for SSEs and MSEs in (a) training set I and (b) training set II. Distributions of $E(\text{NN})$, $R(\text{NN})$ and $\varepsilon(\text{NN})$ derived from (c) training set I and (d) training set II. The vertical lines represent the cut $NN_{\text{max}}$ to obtain $\varepsilon_{\text{max}}$.

value of $\eta_{\text{max}}$, the RMS of the distribution of $\eta_{\text{max}}$ is taken as its systematic uncertainty.

Two groups of five ANNs, each with a different number of neurons in the hidden layer, were trained using the same training set (set II). The value of $\eta_{\text{max}}$ for the default ANN with 40 hidden neurons was $0.976^{+0.002}_{-0.001}(\text{stat.})\pm0.005(\text{syst.})$, while an ANN with 40 input neurons and one hidden layer with 10 neurons had a recognition efficiency $\eta_{\text{max}}=0.962^{+0.001}_{-0.002}(\text{stat.})\pm0.010(\text{syst.})$. This is not significantly worse. Five ANNs with three hidden layers with 40 neurons each were trained and have $\eta_{\text{max}}=0.979^{+0.001}_{-0.001}(\text{stat.})\pm0.008(\text{syst.})$. This is not a significant improvement with respect to the default ANN. The corresponding $\varepsilon_{\text{max}}$ values are $(0.917\pm0.013)$, $(0.905\pm0.037)$ and $(0.933\pm0.020)$ for the default network, the network with one hidden layer of 10 neurons and the network with three hidden layers, respectively. In summary, the variation of $\eta_{\text{max}}$ due to the choice of the topology of the network is $^{+0.003}_{-0.014}$. 

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In the following, the default network with one hidden layer with 40 neurons is used and the variation due to the topology is not considered in the following uncertainties.

6.2 Recognition efficiencies as a function of training sample

ANNs were trained with the training sets listed in Table 2. The trained ANNs were applied to the libraries $0\nu\beta\beta$ clean and FAP clean, containing purely SSEs and MSEs. In this case, $S_{SSE} = 1$ and $B_{MSE} = 1$, respectively. Hence, $E_{max}(S_{SSE} = 1) = \eta_{max}$ and $R_{max}(B_{MSE} = 1) = \rho_{max}$ for a clean library (see Eq. 1). The resulting $\eta_{max}$, $\rho_{max}$ and $\varepsilon_{max}$ values are given in Table 4.

Table 4: Signal recognition efficiency $\eta_{max}$, background recognition efficiency $\rho_{max}$ and $\varepsilon_{max}$ for ANNs trained with library sets having different SSE samples. Only the systematic uncertainties are quoted. The statistical uncertainties are for all numbers less than $\pm 0.003$.

| Training set            | $\eta_{max}$ ($0\nu\beta\beta$ peak) | $\rho_{max}$ (FAP at 1620 keV) | $\varepsilon_{max}$ |
|-------------------------|--------------------------------------|---------------------------------|---------------------|
| set I – inhom DEP       | 0.915±0.017                          | 0.893±0.014                     | 0.904               |
| set II – real $2\nu\beta\beta$ | 0.976±0.005                          | 0.862±0.008                     | 0.917               |
| set III – hom DEP       | 0.964±0.009                          | 0.887±0.006                     | 0.924               |
| set IV – top DEP        | 0.921±0.012                          | 0.888±0.006                     | 0.904               |
| set V – ideal $0\nu\beta\beta$ | 0.958±0.008                          | 0.888±0.008                     | 0.922               |

The highest values for $\varepsilon_{max}$ were obtained with ANNs trained with SSE samples with homogeneous event-location distributions. The $\varepsilon_{max}$ values for ANNs trained with inhomogeneous samples are by approximately 0.02 lower. The variation on $\eta_{max}$ is up to 0.06 and hence more pronounced than on $\rho_{max}$ ($\approx 0.03$). The variations due to SSE libraries with different event-location distributions used for the ANN training are significantly bigger than the fluctuations due to changes of the ANN initial conditions.

7 $0\nu\beta\beta$ detection efficiencies

7.1 Survival probabilities for realistic $0\nu\beta\beta$ and DEP samples

The $E_{max}$ obtained with the trained ANNs were evaluated on the SSE sets $2\nu\beta\beta$ real, $0\nu\beta\beta$ real and DEP real according to the method explained in
The results are listed in Table 5 for training sets I, II and V.

Table 5: $E_{\text{max}}$ for the DEP real, $2\nu\beta\beta$ real and $0\nu\beta\beta$ real libraries from ANNs trained with sets I, II and V. The differences of $E_{\text{max}}$ evaluated with the DEP real and $2\nu\beta\beta$ real samples to $E_{\text{max}}$ evaluated using the $0\nu\beta\beta$ real sample are also listed. Statistical and systematic uncertainties are quoted separately.

|            | set I          | set II         | set V          |
|------------|----------------|----------------|----------------|
| DEP real   | $0.898^{+0.011}_{-0.016} \pm 0.012$ | $0.936^{+0.003}_{-0.001} \pm 0.003$ | $0.914^{+0.011}_{-0.016} \pm 0.007$ |
| $2\nu\beta\beta$ real | $0.885^{+0.007}_{-0.005} \pm 0.017$ | $0.944^{+0.004}_{-0.005} \pm 0.005$ | $0.915^{+0.004}_{-0.006} \pm 0.008$ |
| ($1000\text{ keV} < E < 1450\text{ keV}$) | $0.867^{+0.002}_{-0.003} \pm 0.018$ | $0.937^{+0.003}_{-0.004} \pm 0.005$ | $0.916^{+0.003}_{-0.004} \pm 0.009$ |
| $0\nu\beta\beta$ real |                      |                |                |
| $\Delta E_{\text{max}}(\text{DEP} - 0\nu\beta\beta)$ | $(3.5^{+1.3}_{-1.9} \pm 1.1)\%$ | $(-0.1^{+1.0}_{-1.6} \pm 0.7)\%$ | $(-0.2^{+1.2}_{-1.4} \pm 0.6)\%$ |
| $\Delta E_{\text{max}}(2\nu\beta\beta - 0\nu\beta\beta)$ | $(2.1^{+0.9}_{-0.8} \pm 0.6)\%$ | $(0.8^{+0.5}_{-0.7} \pm 0.1)\%$ | $(0.3^{+0.5}_{-0.8} \pm 0.4)\%$ |

For $0\nu\beta\beta$ events in the energy interval $(2039 \pm 5)\text{ keV}$, $E_{\text{max}}$ values of $(0.937 \pm 0.006)$ and $(0.867 \pm 0.018)$ were obtained with the ANN trained with set II and set I, respectively, where the statistical and systematic uncertainties were added in quadrature. For the ANN trained on set I, $E_{\text{max}}$ is lower than for sets II and V, as expected from the lower $\eta_{\text{max}}$ for this training set (see Table 4). The realistic signal-like libraries also contain a significant amount of MSEs. This explains why the obtained $E_{\text{max}}$ values for $0\nu\beta\beta$ are significantly different from the $\eta_{\text{max}}$ listed in Table 4 and implies that $E_{\text{max}}$ by itself is not a useful quantity to compare PSD methods.

### 7.2 Inhomogeneity of signal recognition efficiency

The position distribution of the rejected events inside the detector, i.e. the position dependence of the signal recognition efficiency was studied. In Fig. 5, the location dependence of the mean value of the $NN$ output inside the detector is depicted for the SSEs from the $0\nu\beta\beta$ clean library.

Regions where the average $NN$ output is lower than $NN_{\text{max}} = 0.55$ are seen as blue areas. In these regions, SSEs are systematically rejected. The fraction of the volume where the SSEs of the $0\nu\beta\beta$ clean library are more likely to be rejected than to be accepted is $$(8.0 \pm 1.7)\%$$ for an ANN trained with the SSE sample with inhomogeneous event-location distribution set I. For the ANNs trained with sets II and V, the affected volume is reduced to $(2.2 \pm 0.5)\%$ and $(3.7 \pm 0.8)\%$, respectively [14]. Using an ANN training set with similar event-location distribution as for the evaluation
Figure 5: Average NN output value (color palette) for events from the $0\nu\beta\beta$ 
\textit{clean} library at different positions within the detector for ANNs trained with 
training (a) set \textit{V} and (b) set \textit{I}.

set decreases the effect of the systematic volume cut, however, it does not 
completely remove it.

The symmetry in the patterns observed in Fig. 5 is connected to the 
crystallographic symmetry of the detector. The mechanism of pattern for-
formation is, however, not fully understood. Affected zones appear close to 
the inner detector surface and in the middle of the bulk around $r \approx 18$ mm. 
The pulses in these regions are especially featureless. For events close to the 
inner (outer) surface, the pulse is determined mostly by the drift of holes 
(electrons). For energy depositions in the center of the bulk, the induced 
electrons and holes will reach the contacts at the same time. The absence 
of the “kink” in the pulse shape, corresponding to the time at which one 
type of the charge carriers reached the corresponding contact, could be the 
reason for the poor SSE recognition capability of the ANN in these regions. 
The axis dependence of the effect might be due to the dependence of the 
electron to hole mobility ratio on the position of the charge carriers with 
respect to the crystal axes (see Fig. 2 in [2]).

7.3 Consequences for $0\nu\beta\beta$ analyses

The different event-location distributions for DEP samples from calibration 
and $0\nu\beta\beta$ signal events (see Fig. 2) was identified as the major source of 
systematic uncertainty for the approach of ANNs trained with DEP sets. For
2νββ training samples, the different energy distribution leads to a different signal-to-noise ratio. The η_{max} obtained for different SSE libraries with ANNs trained with different training sets are listed in Table 6.

The signal recognition efficiencies η_{max} of the different ANNs are within uncertainties the same for the different evaluation libraries with homogeneous event-location distribution. This demonstrates that the normalization of the input to the ANN makes the influence of the lower energy of events, down to 1 MeV, insignificant. However, when η_{max} is derived using the DEP side set with realistic event-location distribution it is systematically overestimated. There is a Δη_{max} [set I](DEP side − 0νββ) = (4.2 ± 0.6 ± 0.8)% effect for the ANNs trained with an independent DEP side SSE training set. For ANNs trained with homogeneous samples the effect is reduced but nevertheless significant with Δη_{max} [set II](DEP side − 0νββ) = (1.1 ± 0.3 ± 0.7)% and Δη_{max} [set V](DEP side − 0νββ) = (1.4 ± 0.7 ± 0.9)%. Note that the efficiencies obtained when training the network with the 2νββ set are higher than the ones obtained using the 0νββ set.

Comparing the resulting η_{max} with E_{max} quoted in Table 5 shows that the additional admixture of MSEs to the evaluation libraries slightly reduces E_{max} with respect to η_{max}.

Table 6: Values of η_{max} for different libraries for ANNs trained with different sets (see Table 2). The difference in efficiency between evaluation sets is also listed.

| Evaluation set       | Training set | set I inhom DEP | set II real 2νββ | set V ideal 0νββ |
|----------------------|--------------|-----------------|------------------|------------------|
| DEP side             | 0.954 ± 0.004 ± 0.007 ± 0.011 | 0.987 ± 0.004 ± 0.007 ± 0.006 | 0.971 ± 0.004 ± 0.006 ± 0.003 |
| DEP clean            | 0.917 ± 0.004 ± 0.006 ± 0.018 | 0.976 ± 0.002 ± 0.005 ± 0.008 | 0.960 ± 0.003 ± 0.009 ± 0.002 |
| 2νββ clean           | 0.911 ± 0.005 ± 0.007 ± 0.018 | 0.970 ± 0.003 ± 0.007 ± 0.008 | 0.956 ± 0.004 ± 0.010 ± 0.003 |
| 0νββ clean           | 0.915 ± 0.004 ± 0.007 ± 0.017 | 0.976 ± 0.001 ± 0.005 ± 0.008 | 0.958 ± 0.003 ± 0.003 ± 0.008 |
| Δη_{max}(DEP side − 0νββ) | (4.2 ± 0.9 ± 0.8)% | (1.1 ± 0.3 ± 0.2)% | (1.4 ± 0.7 ± 0.4)% |
| Δη_{max}(DEP clean − 0νββ) | (0.1 ± 0.8 ± 0.2)% | (0.0 ± 0.2 ± 0.03)% | (0.2 ± 0.4 ± 0.1)% |
| Δη_{max}(2νββ − 0νββ) | (-0.4 ± 0.9 ± 0.1)% | (-0.6 ± 0.3 ± 0.3)% | (-0.2 ± 0.7 ± 0.2)% |

8 Summary and conclusions

Systematic effects on the determination of the efficiency of pulse-shape-analysis using ANNs were investigated. The most important effect was found to be due to the event-location distribution of the training sets. In contrast, the energy distribution of events in the training sample was found
to be irrelevant within reasonable limits.

The signal recognition efficiencies of ANNs determined from DEP libraries were found to be up to 5% too high. Differences in the energy distribution of the events of the evaluation samples do not have a significant effect. The different event-location distributions resulting from different positions of the calibration sources may result in variations of the ANN signal recognition efficiency by up to 6% and the background discrimination power by 2%.

The signal detection efficiency of an ANN depends on the location of the events inside a true-coaxial detector. The efficiency is above 90% in most parts of the detector. However, SSEs in the inner regions and in the centre of the bulk are systematically misidentified. About 2% to 8% of the volume is affected, depending on the homogeneity of the event-location distribution of the training set used. Using training sets with homogeneous SSE location distribution reduces the affected regions but does not eliminate them completely.

The true-coaxial detectors assumed for these studies have particularly simple field configurations. The effects on detectors with more complex field configurations will have to be studied very carefully.

Pulse-shape discrimination with artificial neural networks is a useful tool to identify multi-site events. It potentially increases the sensitivity of $0\nu\beta\beta$ experiments like GERDA [8, 20]. The usage of $2\nu\beta\beta$ events for training and efficiency evaluation of the artificial neural networks is recommended.

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