RECENT PROGRESS IN QCD FACTORIZATION FOR $B \to M_1 M_2^*$

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After briefly introducing the framework of QCD factorization for $B \to M_1 M_2$ in the language of the Soft-Collinear Effective Theory, we firstly address the recent efforts on higher-order radiative corrections in QCD factorization. Then we discuss some phenomenologies in $B \to VV$ within the framework of QCD factorization.

Keywords: $B$ decays; QCD factorization; effective theory.

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1. Introduction

$B$ non-leptonic two-body decays provide an abundant sources of information about the CKM matrix elements as well as possible new physics effects. However, the complicated strong interaction makes it difficult to extract the CKM parameters or identify the new physics signals from the experimental data straightforwardly. Thus, the more we know about the QCD effects in these decays, the better we understand the CP violations and new physics.

With the help of the weak effective Hamiltonian, the essential task to calculate the decay amplitude of $B$ decays is left to computation of the matrix elements of the effective operators $Q_i$.

Beneke et al proposed a factorization formula for $\langle M_1 M_2 | Q_i | \bar{B} \rangle$ in the limit $m_b \to \infty$ based on Feynman diagrams expansion in $\Lambda_{\text{QCD}}/m_b$. This is so-called QCD factorization method (also known as BBNS method) for $B \to M_1 M_2$. Here we present this factorization formula in the language of the Soft-Collinear Effective Theory (SCET) which is also one of recent major theoretical development of $B$ physics.

$$ \langle M_1 M_2 | Q_i | \bar{B} \rangle = F^{B \to M_1} C_i^{I} \ast f_{M_2} \Phi_{M_2} $$
$$ + C_i^{II} \ast J \ast f_B \Phi_B \ast f_{M_1} \Phi_{M_1} \ast f_{M_2} \Phi_{M_2}$$

$$ + O(\Lambda_{\text{QCD}}/m_b) \quad (1) $$

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where the perturbatively calculable hard function $C_{I,II}^i$ and the jet function $J$ are defined at the hard scale $m_b$ and hard-collinear scale $\sqrt{m_b \Lambda_{\text{QCD}}}$ respectively, and the form-factors $F^{B \to M}$ and light-cone distribution amplitudes $\Phi_M$ are non-perturbative quantities. The second term of (1) is from the so-called hard spectator interaction (HSI) which is missing in the literature before.

QCD factorization method has become one of major theoretical tools for $B$ decays, and already been implemented into many phenomenological studies. Here I would like to review some recent developments in this field by mainly focusing on efforts towards the higher-order radiative corrections and phenomenological studies of the polarization puzzles in $B$ decays to two light vector mesons. Besides QCD factorization, PQCD is another common-used method in this community. One can refer to the C.D. Li’s paper in this proceeding for the comparison between these two approaches.

2. Developments on higher-order radiative corrections

The branching ratios of $B \to \pi\pi$ and $\pi K$ have been measured very well. The direct CP asymmetries in these decays are also observed. However, due to the graphical analysis, the large enhancement to the ratio $|C/T|$ is needed to meet the data, where $T$, $C$ stand for the color-allowed and color-suppressed tree-amplitude respectively. This is strongly against the usual expectation. It leads to so-called $B \to \pi\pi$ and $\pi K$ puzzles.

In QCD factorization, these color-suppressed tree-amplitudes are related to the QCD coefficient $\alpha_2(M_1 M_2)$. For illustration, 

$$\alpha_2(\pi\pi) = 0.17_{\text{LO}} - [0.17 + 0.08i]v_2 + [0.46]_{\text{LOHSI}}$$ (2)

The accidental cancelation between the leading order (LO) and next-to-leading order (NLO) vertex corrections ($v_2$) makes the hard spectator interaction (HSI) very important. The NLO corrections to the HSI are recently studied by Beneke, Jager and Yang. In their papers, the next-to-leading order corrections to the jet function $J$ and hard coefficient $C_{I,II}^i$ are calculated in 4 and 5 respectively. Both of these corrections enhance the color-suppressed amplitude effectively.

In Table 1 the predictions in QCDF with NLO HSI in a certain parameter setting ($G_4$) is shown. The agreement between the prediction and experimental data is very good except for the direct CP asymmetries $A_{CP}$. It means that the strong phases still need further study. Recently, the efforts towards the NLO corrections to the imaginary part of the amplitude has started.

3. $B \to VV$ and polarization puzzles

The fact of the $V - A$ dominance in the Standard Model (SM) shows the hierarchy of the helicity amplitudes for $B \to VV$

$$\mathcal{A}_0 : \mathcal{A}_- : \mathcal{A}_+ = 1 : \frac{\Lambda}{m_b} : \left(\frac{\Lambda}{m_b}\right)^2$$ (3)
Table 1. Predictions in QCDF with NLO HSI vs. experimental data.

| \( \text{Br} \times 10^6 \) | \( G_{\bar{A}} \) | Exp. | \( A_{CP} \) | \( G_{\bar{A}} \) | Exp. |
|-----------------|--------|------|--------|--------|------|
| \( \pi^+\pi^- \) | 5.6    | 5.7  ± 0.4 | \( \pi^0\pi^0 \) | 0.00    | 0.04  ± 0.05 |
| \( \pi^0\pi^0 \) | 0.81   | 1.31 ± 0.21 | \( \pi^0\pi^0 \) | −0.38   | 0.36  ± 0.33 |
| \( \pi^+K^- \)  | 22.6   | 23.1 ± 1.0  | \( \pi^+K^- \)  | 0.00    | 0.008 ± 0.0025 |
| \( \pi^0K^- \)  | 12.9   | 12.8 ± 0.6  | \( \pi^+K^- \)  | −0.05   | 0.047 ± 0.026 |
| \( \pi^0K^0 \)  | 20.6   | 19.4 ± 0.6  | \( \pi^+K^- \)  | −0.02   | −0.095 ± 0.013 |
| \( \pi^0K^0 \)  | 9.1    | 10.0 ± 0.6  | \( \pi^0K^0 \)  | 0.04    | −0.12 ± 0.11 |

with simple estimation by the naive factorization. However, experimentally, such hierarchy is obeyed in the tree-dominated decays (\( B \rightarrow \rho \rho \) and \( \omega \rho \)), but violated in penguin dominated \( B \rightarrow \phi K^* \) system[8]. The more puzzling case happens in \( B \rightarrow \rho K^* \) system which are penguin-dominated, in which both transverse polarization is enhanced in \( B^− \rightarrow \rho^− K^{*0} \) but suppressed in \( B^− \rightarrow \rho^0 K^{*-} \).

Analogue to \( B \rightarrow PP \) and \( PV \), we can derive the QCD factorization formula for each helicity-amplitude of \( B \rightarrow VV \) [9] which obeys the hierarchy in (3) for most of case. The annihilation contribution brings the large uncertainties to the QCD factorization prediction in \( B \rightarrow PP \) and \( PV \) as well as \( B \rightarrow VV \) due to the severe endpoint singularity. However, the penguin weak annihilation in longitudinal helicity amplitude is predicted to be small and with small uncertainty, perhaps due to an accidental cancelation; in negative-helicity amplitude, the penguin weak annihilation could be very large. This leads to two consequences immediately:

1) For the tree-dominated decays which are dominated by the longitudinal polarization, the penguin amplitude gives small contribution to the branching ratios. As a result, we could have a better determination of \( \sin 2\alpha \) or \( \alpha \) from \( B \rightarrow \rho \rho \) than \( B \rightarrow \pi \pi \) and \( \pi \rho \). In [9], the authors get \( \alpha = (85.6^{+7.4}_{-7.3})^\circ \).

2) For the penguin dominated decays, the negative-helicity amplitude could be large. This could be an solution for the \( B \rightarrow \phi K^* \) polarization puzzle. However, in this case, QCD factorization loses its predictive power.

When the final states involve the neutral vectors \( \rho^0 \), \( \omega \), and \( \phi \), the electromagnetic dipole operator will lead to a violation of (3). This results in an enhancement for electro-weak penguin in the negative-helicity amplitude[10]

\[
\Delta P^{EW}(V_1V_2) \propto - \frac{2\alpha^\text{em}}{3\pi} C_{\gamma\gamma,\text{eff}} \frac{m_B m_h}{m_{V_2}^2}.
\]

This effects can be tested in \( B \rightarrow \rho K^* \) system because graphically the decay amplitudes for \( B \rightarrow \rho K^* \) is dominated by QCD penguin amplitude \( P \) and electro-weak penguin amplitude \( P^{EW} \). One has

\[
A_h(\rho^- K^{*0}) : A_h(\rho^0 K^{*-}) \simeq P_h : P_h^{EW}
\]

where \( h \) denotes for the helicities 0, \( \mp \).

In Table 2, the predictions for \( B \rightarrow \phi K^* \) and \( \rho K^* \) by QCD factorization are listed. ("\( \hat{\alpha}_{\rho}^{\phi} \) from data" means the penguin of negative-helicity amplitude is extracted from the experiments.) One can see that the predictions agree with the
Table 2. QCD factorization predictions for $B \to \phi K^*$ and $\rho K^*$ vs. experimental data.

| Observable       | Theory | Experiment |
|------------------|--------|------------|
|                  | default $\hat{\alpha_s}$ from data |            |
| $\text{Br}_{Av}/10^{-6}$ | $\phi K^*$ | $10.1^{+0.6+1.2}_{-0.5-1.1}$ | $10.4^{+0.9+0.2}_{-0.5-0.9}$. $9.7 \pm 1.5$ |
|                  | $\phi K^*$ | $9.3^{+0.5+1.7}_{-0.5-1.6}$ | $9.6^{+0.5+0.6}_{-0.5-0.6}$. $9.5 \pm 0.8$ |
| $K^{\ast0} \rho^0$ | $5.9^{+0.6+0.9}_{-0.3-1.7}$ | $5.8^{+0.4+0.1}_{-0.3-0.9}$. $9.2 \pm 1.5$ |
| $f_L/\%$        | $\phi K^*$ | $45^{+0.9}_{-0.8-0.4}$ | $44^{+0.9}_{-0.8-0.4}$. $50 \pm 7$ |
|                  | $\phi K^*$ | $44^{+0.9}_{-0.8-0.4}$ | $43^{+0.9}_{-0.8-0.4}$. $49 \pm 3$ |
| $K^{\ast0} \rho^0$ | $56^{+0.9}_{-0.8-0.4}$ | $57^{+0.9}_{-0.8-0.4}$. $48.0 \pm 8.0$ |
| $K^{\ast-} \rho^0$ | $84^{+2.9}_{-1.8-0.4}$ | $85^{+2.9}_{-1.8-0.4}$. $96^{+16}_{-16}$ |

experimental data very well. It reproduces the polarization pattern of the penguin-dominated $B \to VV$ decays as well.

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