Neutrinoless double beta decay ($\beta\beta_{0\nu}$) occurs through the magnetic coupling of dimension five, $\lambda_\nu^{(*)}/m_{\nu*}$, among the excited electron neutrino $\nu^*$, electron, and $W$ boson if $\nu^*$ is a massive Majorana neutrino. If the coupling is not small, i.e., $\lambda_\nu^{(*)}>1$, the mass of the excited neutrino must not be less than the $Z$ boson mass, $m_Z$. Since $\nu^*$ contributes in the ($\beta\beta_{0\nu}$) decay as a virtual state, this decay will give an opportunity to explore the much heavier mass region of $\nu^*$.

In this paper, we present the decay formula of ($\beta\beta_{0\nu}$) decay through the $\nu^*$ exchange and discuss the constraint on the coupling constant and the mass of the excited neutrino.

We start from the interaction in Eq. (1) which leads to the effective four point interaction

$$L_{\text{int}} = g \frac{\lambda_\nu^{(*)}}{m_{\nu*}} \bar{\nu}_e \gamma^\mu (\eta_L \gamma^5 R + \eta_R \gamma^5 L) \nu^* \gamma_\mu W^- + \text{h.c.}, \quad (1)$$

where $\nu^*$ is a heavy excited electron neutrino, $L = (1 - \gamma_5)/2$, $R = (1 + \gamma_5)/2$ and $m_{\nu*}$ is the mass dimension, which is on the order of the mass of $\nu^*$. This interaction is derived by the SU(2)$\times$U(1) gauge invariant form, and parameters $\eta_L$ and $\eta_R$ satisfy the following normalization:

$$|\eta_L|^2 + |\eta_R|^2 = 1. \quad (2)$$

Extensive searches for $\nu^*$ have been made by many groups, and it has been found that $m_{\nu*} > 91$ GeV by assuming that $\lambda_\nu > 1$, which is the coupling for $\nu^* \to \nu Z$ decay similarly defined for $\lambda_\nu^{(*)}$. So far, the severe mass bound in the region $m_{\nu*} > m_Z$ has not been obtained.

The purpose of this paper is to explore the mass range $m_{\nu*} > m_Z$ by using the neutrinoless double beta decay ($\beta\beta_{0\nu}$) and assuming that $\nu^*$ is a massive Majorana neutrino. Then the ($\beta\beta_{0\nu}$) decay occurs through $\nu^*$ exchange. Since $\nu^*$ enters as a virtual state, we can investigate heavy $\nu^*$. Panella and Srivastava were the first to investigate ($\beta\beta_{0\nu}$) decay, but unfortunately their formula is incorrect, as we shall see later. Here we shall present the correct expression for the decay.

We start from the interaction in Eq. (1) which leads to the effective four point...
interaction between leptons and hadrons as

$$L_{\text{eff}} = - G_{\text{eff}} \bar{e} \sigma^{\mu \nu} (\eta_1^* R + \eta_2^* L) \nu^* \partial_\mu J'_\nu + \text{h.c.},$$

where $J'_\nu$ is the hadronic current, and

$$G_{\text{eff}} = 2 G_F \frac{q \lambda_{\nu}(\nu^*)}{m_\nu}.$$  

§ 2. Decay formula of neutrinoless double beta decay

In the second order perturbation of the effective interaction in Eq. (3), the $(\beta \beta)_{0\nu}$ decay takes place, and the $S$-matrix for this decay is given by

$$S = i \frac{G_{\text{eff}}^2}{2(2\pi)^3} \int \! dx dy \int \! dq \frac{m_N}{q^2} e^{-iq(x-y)}$$

$$\times e(x) \sigma^{\mu \nu} \sigma^{\rho \sigma} (\eta_1^* R + \eta_2^* L) e^c(y) \partial_\mu \partial_\nu T(J'_\nu(x) J'_\rho(y)),$$

where $e^c$ is the charge conjugation of $e$, i.e., $e^c = c \bar{e}^T$. In the following, we take the $S$-wave of the electron wave function which is given by

$$\langle 0 | e(x) | p \rangle = \psi_5(e) e^{-i\xi e}, \quad \psi_5(e) = \sqrt{\frac{\xi}{2\epsilon}} \left( \frac{\epsilon + m}{\epsilon + m_e} \right) F_0(Z, \epsilon),$$

where $\epsilon$ is the energy of electron, and $F_0(Z, \epsilon)$ is the relativistic Coulomb factor defined in Eq. (3.1.25) in Ref. 5). The $S$-wave function is independent of the space coordinate. With this wave function, we obtain

$$S_N = i \frac{G_{\text{eff}}^2}{2(2\pi)^3} \int \! dx dy \int \! dq \frac{m_N}{q^2} \langle N_f | T(J'_\nu(x) J'_\rho(y)) | N_i \rangle$$

$$\times \left[ t^{\nu \sigma}(e_1, e_2, q, q) e^{i(e_2 x^0 + e_1 y^0)} - (e_1 \leftrightarrow e_2) \right],$$

where

$$t^{\nu \sigma}(e_1, e_2, q, q) = \bar{\psi}_5(e_1) \sigma^{\mu \nu} \sigma^{\rho \sigma} (\eta_1^* R + \eta_2^* L) \psi_5(e_1) (q_\mu - e_2 g_{\mu 0}) (q_\rho - e_1 g_{\rho 0}),$$

and $m_N$ is the mass of the excited Majorana neutrino $\nu^*$. This formula may be compared with Eq. (9) in the paper by Panella and Srivastava. Our formula is quite different from theirs, and this causes an important difference, as shown later. Now we perform the $q^0$ integration first, and then do the $x^0$ and $y^0$ integration. We find

$$R_n = - \frac{G_{\text{eff}}^2}{\sqrt{2} 12(2\pi)^3} \int \! dx dy \int \! dq \frac{m_N}{q^2} e^{-i\omega(x-y)}$$

$$\times \frac{1}{\omega + E_n - E_i + \epsilon_i} \left[ \sum_{n} \langle N_f | J'_\nu(x) | n \rangle \langle n | J'_\rho(y) | N_i \rangle t^{\nu \sigma}(e_1, e_2, \omega, q) - (e_1 \leftrightarrow e_2) \right],$$

where $\sqrt{21}$ in the denominator is a statistical factor. In this expression, $q^0 = \omega = \sqrt{m_N^2 + q^2}$ with $q = |q|$ which comes from the pole at $q^0 = \omega$ of the neutrino propaga-
tor. The contribution from the $q^0 = -\omega$ pole turns out to be equal to the contribution from the $q^0 = \omega$ pole in the approximation to keep $S$-wave of electron wave function.

So far our formula is exact up to the $S$-wave approximation of the electron wave function. Next, we make the following approximations: (1) We take the closure approximation, where $E_n$ is replaced by the average value $\langle E_n \rangle$ so that the sum of the intermediate states can be taken. (2) $\varepsilon_1$ and $\varepsilon_2$ in the numerator are neglected, $(q_\mu - \varepsilon_2 q_\mu)(q_\mu + \varepsilon_1 q_\mu) \approx q_\mu q_\mu$. (3) The energy denominator $\omega + \langle E_n \rangle - E_i + \varepsilon_i = \omega + \langle E_n \rangle - (E_i + E_f)/2 + (\varepsilon_1 - \varepsilon_2)/2$ is replaced by $\mu_0 m_e = \langle E_n \rangle - (M_i + M_f)/2$. Here we replaced $E_i$ and $E_f$ with their masses $M_i$ and $M_f$. The approximations (2) and (3) are valid because $\varepsilon_1$ and $\varepsilon_2$ are of order 1 MeV, which are much smaller than $\omega = \sqrt{m_N^2 + q^2}$, where the average value of $q$ is of order $1/R$, with $R$ being the nuclear radius.

With these approximations, we get

$$R_{fi} = \frac{G_{\text{eff}}^2}{\sqrt{2}! \pi^3} \int dx dy \int dq \frac{m_N}{\omega} e^{i q \cdot (x-y)} \frac{\langle N_f | \bar{J}_i^+(x) J_i^+(y) | N_i \rangle}{\omega + \mu_0 m_e} \times q_\mu q_\mu \bar{\psi}_S(\varepsilon_1)(\sigma^{\mu \nu} \sigma^{\nu \sigma})(\eta^2 R + \eta \eta^2 L) \psi_S(\varepsilon_1).$$

(10)

Now we use the identity $(\sigma^{\mu \nu} \sigma^{\nu \sigma}) = 2(g^{\mu \nu} g^{\sigma \rho} - g^{\mu \rho} g^{\nu \sigma} - i e^{\mu \nu \rho} \gamma_5)$ and the non-relativistic approximation of the hadronic current as

$$J_i^+(x) = \sum_n J_i^+(n) \delta(x - r_n), \quad J_i^+(n) = \tau_n^+(g_{\nu \rho} q_\mu + g_A \delta_{\mu \rho} \sigma_5) F(q^2),$$

(11)

where $r_n$ is the position of the $n$-th nucleon in the nucleus, and $F(q^2)$ is the form factor defined by

$$F(q^2) = \left(1 + \frac{1}{4(q^2/m_A^2)} \right)^2$$

(12)

with the value $m_A = 0.85$ MeV. We obtain

$$R_{fi} = \frac{G_{\text{eff}}^2}{\sqrt{2}! \pi^3} \sum_n \int dq \frac{m_N}{\omega(\omega + \mu_0 m_e)} \langle N_f | \bar{J}_i^+(n) J_i^+(m) | N_i \rangle \times m_N(m_A^2 g^{\mu \nu} - q^\mu q^\nu) \bar{\psi}_S(\varepsilon_1)(\eta^2 R + \eta \eta^2 L) \psi_S(\varepsilon_1),$$

(13)

where $r_{nm} = r_n - r_m$. It should be noted that the $m_A^2 g^{\mu \nu}, q^0 q^0 = \omega^2$, and $q^0 q^i$ terms contribute only to $0^+ \to 0^+$ transition, while the $q^i q^i = \omega q^i$ term contributes to $0^+ \to 0^-$ transition. The $0^+ \to 2^+$ transition does not occur because we keep the $S$-wave of electron wave function.

In the following, we concentrate on the $0^+ \to 0^+$ transition. Since we assume that the heavy composite neutrino mass $m_N$ is much greater than $m_A$, we expand $\omega$ and $\omega + \mu_0 m_e$ in the $q$ integration in the power of $1/m_N$. We find

$$\frac{m_N}{\omega(\omega + \mu_0 m_e)} m_N \langle m_N | \bar{J}_i^+(n) J_i^+(m) \rangle \approx m_N^2 \tau_n^+ \tau_m^+ \left[ \left( - \frac{m_N}{\mu_0 m_e} + \frac{(\mu_0 m_e)^2}{m_N^2} \right) \sigma_n \cdot \sigma_m + \left( \frac{g_A}{g_A^2} - \sigma_n \cdot \sigma_m \right) \frac{q^i q^j}{m_N^2 + \sigma_n \cdot \sigma_m \cdot q^i q^j m_N^2} \right] F^2(q^2).$$

(14)

Thus we obtain
where

\[ M_{GT,N} = \langle N_f | \sum_{n,m} \tau_n^{(+)\dagger} \tau_m^{(+)} \sigma_n \cdot \sigma_m \left( \frac{R}{\gamma_{nm}} \right) F_N(x_A) | N_i \rangle, \]

\[ M_f = \langle N_f | \sum_{n,m} \tau_n^{(+)\dagger} \tau_m^{(+)} \left( \frac{R}{\gamma_{nm}} \right) F_4(x_A) | N_i \rangle, \]

\[ M_{GT} = \langle N_f | \sum_{n,m} \tau_n^{(+)\dagger} \tau_m^{(+)} \sigma_n \cdot \sigma_m \left( \frac{R}{\gamma_{nm}} \right) F_5(x_A) | N_i \rangle, \]

\[ M_i = \langle N_f | \sum_{n,m} \tau_n^{(+)\dagger} \tau_m^{(+)} \left( 3(\sigma_n \cdot r_{nm})(\sigma_m \cdot r_{nm}) - \sigma_n \cdot \sigma_m \right) \left( \frac{R}{\gamma_{nm}} \right) F_0(x_A) | N_i \rangle, \]

where \( x_A = m_A \gamma_{nm} \), and neutrino potentials are

\[ F_N(x) = \frac{x}{48} (3 + 3x + x^2) e^{-x}, \quad F_4(x) = \frac{x}{48} (3 + 3x - x^2) e^{-x}, \quad F_5(x) = \frac{x^3}{48} e^{-x}. \]

After taking the spin sum and performing the phase-space integration, we find the half-life of the transition of the neutrinoless double beta decay due to the heavy composite neutrino to be

\[ T^{-1}(0^+ \rightarrow 0^+) = 4 \left( \frac{\lambda m_A}{m_{\nu^a}} \right)^4 \left( |\eta_L|^4 + |\eta_R|^4 \right) (\frac{m_A}{m_e})^2 G_0 \left( \frac{m_N}{m_A} - \frac{\mu_0 m_e}{m_N} + (\frac{\mu_0 m_e}{m_N})^2 \right) M_{GT,N} \]

\[ + \frac{m_A}{m_N} \left[ (\frac{\eta_N}{\eta_A}) \left( \frac{M_i}{m_{GT}} - \frac{2}{3} M_{GT} - \frac{1}{3} M_i \right) \right]^2, \]

where \( G_0 \) is the phase space factor defined in Eq. (3.17a) in Ref. 5). This is our final result for the half-life formula. By comparing this result with Eq. (20) in Ref. 4), we see the difference of the excited neutrino mass \( m_N \) dependence. The transition rate is proportional to \( m_N^2 \) for our case, while to \( m_N^{-2} \) for the Panella and Srivastava formula.\(^4\) This difference gives qualitatively different bounds on the coupling and the mass. In addition, since \( m_N \) is very heavy, this difference is substantial.

\section*{3. Constraint from neutrinoless double beta decay}

In the following, we analyze the constraint on the coupling by using the experimental half-life limit of \(^{76}\text{Ge}\), which was measured by the Heidelberg-Moscow collaboration.\(^6\)

\[ T(0^+ \rightarrow 0^+: ^{76}\text{Ge}) > 5.6 \cdot 10^{24} \text{ yr} \quad 90 \% \text{ c.l.} \]

We derive the constraint on composite parameters from this data. By using the values of nuclear matrix elements obtained by Hirsch, Klapdor-Kleingrothaus and
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Kovalenko,\textsuperscript{7} M_{GT,N} = 0.113, \ M'_{\tau} = 3.06 \cdot 10^{-3}, \ M'_{\tau} = -7.70 \cdot 10^{-3}, \ M'_{\tau} = -3.09 \cdot 10^{-3}, \ (23)

and the phase space factor $G_{v} = 6.4 \cdot 10^{-15}/\text{yr}$ given in Ref. 6, we find the half-life of the $0^+ \rightarrow 0^+$ transition for $^{76}\text{Ge}$ is given by

$$T^{-1} = \left(\frac{\lambda m_A}{m_{\nu^*}}\right)^4 \left(\frac{m_N}{m_A} - \frac{\mu_0 m_e}{m_A} + \frac{(\mu_0 m_e)^2}{m_{N m_A}} + 7.2 \cdot 10^{-2} \frac{m_A}{m_N}\right)^2 \cdot 9.1 \cdot 10^{-10}/\text{yr}, \ (24)$$

where we assumed $|\eta_\ell|^2 + |\eta_R|^2 = 1$ for simplicity. This is valid for the chirality conserving cases\textsuperscript{8} where $(\eta_\ell, \eta_R) = (1, 0)$ or $(0, 1)$.

By comparing this formula with the data, we find

$$\left(\frac{\lambda m_A}{m_{\nu^*}}\right)^2 \left[\left(\frac{m_N}{m_A} - \frac{\mu_0 m_e}{m_A} + \frac{(\mu_0 m_e)^2}{m_{N m_A}} + 7.2 \cdot 10^{-2} \frac{m_A}{m_N}\right)^2 < 1.4 \cdot 10^{-3}\right]. \ (25)$$

Our formula in Eq. (21) is valid for the case where the excited neutrino is a Majorana neutrino which is much heavier than $m_A = 0.85\text{GeV}$. In this case, the dominant term in Eq. (25) is that which is proportional to the excited neutrino mass $m_N$, because $\mu_0 m_e$ is of order $10\text{MeV}$. Thus the constraint reduces to the simple form

$$\lambda \left(\frac{1\text{TeV}}{m_{\nu^*}}\right) \cdot \left(\frac{m_N}{1\text{TeV}}\right)^{1/2} < 4.1 \cdot 10^{-3}. \ (26)$$

\section*{4. Limit on the excited neutrino mass and the composite scale}

First, we discuss the excited neutrino mass. If we assume that $m_N = m_{\nu^*}$, we obtain

$$m_{\nu^*} > 5.9 \cdot 10^4 \text{TeV}, \ (\lambda w^{*} > 1). \ (27)$$

This is the most stringent bound so far.

Secondly we discuss the bound on the composite scale. Since the relation between the coupling constant and the composite scale is model dependent, we have to confine some specific models.\textsuperscript{8}

1. Sequential type model

We consider the sequential type model, where the excited leptons are assigned as

$$L^t = \left(\begin{array}{c} \nu^* \\ e^* \end{array}\right)_L, \ [\nu^*_R, e^*_R]. \ (28)$$

and $[\nu^*_R]$ indicates that there are two cases, that with and that without $\nu^*_R$. There are two kinds of gauge invariant interactions. If $\nu^*_R$ exists, the interaction is given by

$$L_{\mu} = -\frac{1}{2A} \bar{L} e^{\nu} \left(gf' \frac{Y}{2} W^a_{\mu} + g' f' \frac{Y}{2} B_{\mu}\right) \phi \nu^*_R + \text{h.c.}, \ (29)$$
where $l_L$ is the left-handed ground state lepton doublet, and $\phi$ is the doublet Higgs boson. This interaction yields that in Eq. (1) with

$$\frac{\lambda W^{(\nu)}}{m_{\nu^*}} = \frac{f v}{\sqrt{2} A^2} \text{ with } (\eta_L, \eta_R) = (1, 0), \quad (30)$$

where $v$ is the vacuum expectation value of $\sqrt{2}\phi$ which is about 250 GeV, and $\nu_{R}^{*}$ mediates the decay.

If $\nu_{R}^{*}$ does not exist, the other type of interaction will be present:

$$L_{\text{int}} = \frac{1}{2A} \bar{e}_{\nu'} \sigma^{\mu\nu} \left( g' f^a W^a_{\mu\nu} + g' f^B_{\mu\nu} \right) L_L + \text{h.c.} \quad (31)$$

This interaction gives

$$\frac{\lambda W^{(\nu)}}{m_{\nu^*}} = \frac{f v}{\sqrt{2} A^2} \text{ with } (\eta_L, \eta_R) = (0, 1), \quad (32)$$

and $\nu_{L}^{*}$ mediates.

For both cases, we find

$$\Lambda > 6.6 f^{1/2} \left( \frac{m_N}{1 \text{ TeV}} \right)^{1/4} \text{ TeV.} \quad (33)$$

2. Mirror type

In this model, the multiplet of the excited leptons are

$$[\nu_{L}^{*}, e_{L}^{*}, L_{R}^{*} = \begin{pmatrix} \nu_{L}^{*} \\ e_{L}^{*} \end{pmatrix}, \quad (34)$$

The gauge invariant interaction is

$$L_{\text{int}} = \frac{1}{2A} T_{L} \sigma^{\mu\nu} \left( g' f^a W^a_{\mu\nu} + g' f^B_{\mu\nu} \right) L_{R}^{*} + \text{h.c.} \quad (35)$$

This leads to the coupling

$$\frac{\lambda W^{(\nu)}}{m_{\nu^*}} = \frac{f}{\sqrt{2} A} \text{ with } (\eta_L, \eta_R) = (1, 0), \quad (36)$$

and $\nu_{R}^{*}$ mediates. In this case, we find

$$\Lambda > 170 f \left( \frac{m_N}{1 \text{ TeV}} \right)^{1/2} \text{ TeV.} \quad (37)$$

3. Homodoublet type

In this model, the multiplets of excited leptons are

$$L_{L}^{*} = \begin{pmatrix} \nu_{L}^{*} \\ e_{L}^{*} \end{pmatrix}, \quad L_{R}^{*} = \begin{pmatrix} \nu_{R}^{*} \\ e_{R}^{*} \end{pmatrix}. \quad (38)$$

In this case, the gauge invariant interaction is the same as in the Mirror case,
so that we have the same coupling in Eq. (36), and \( \nu^* \) mediates. Thus, we obtain
\[
\Lambda > 170 f \left( \frac{m_N}{1 \text{TeV}} \right)^{1/2} \text{TeV}.
\] (39)

In the above, we obtain rather strong constraints on the mass of \( \nu^* \) and the composite scale. These constraints may be modified if there is a mixing of the excited neutrinos. There are two types of mixing, flavor mixing and \( \nu_L - \nu_R \) mixing. If such mixings are present, the excited neutrino mass \( m_N \) should be replaced by an effective mass according to
\[
m_N \Rightarrow \langle m_{\nu^*} \rangle = \sum_j (U_{ej}^2 \text{ or } V_{ej}^2) m_j,
\] (40)
as the mass appearing in the neutrino mass contribution to the ordinary neutrinoless double beta decay. The mixing parameters are defined by
\[
\nu^* = \sum_j U_{ej} N^* j,
\nu_R = \sum_j V_{ej} N^* j,
\] (41)
where \( N^*_j \) is the mass-eigenstate Majorana neutrino with mass \( m_j \). The effective mass \( \langle m_{\nu^*} \rangle \) in essence comes from the Majorana mass of \( \nu^*_L \) or \( \nu^*_R \). Thus, if the main part or the \( \nu^* \) mass comes from the Dirac mass, then the constraint is not valid. In particular, if \( \nu^* \) is a Dirac particle, there is no constraint from the neutrinoless double beta decay.

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References

1) N. Cabbibo, L. Maiani and Y. Srivastava, Phys. Lett. 139B (1984), 459.
2) K. Hagiwara, S. Komamiya and D. Zeppenfeld, Z. Phys. C29 (1985), 115.
3) L3 Collaboration, Phys. Rep. C236 (1993), 1.
ALEPH Collaboration, Phys. Rep. C216 (1992), 253.
See references in Particle Data Group.
4) O. Panella and Y. N. Srivastava, College de France Preprint, LPC 94 39.
5) M. Doi, T. Kotani and E. Takasugi, Prog. Theor. Phys. Suppl. No. 83 (1985), 1.
6) HEIDELBERG-MOSCOW Collaboration: A. Balysh et al., Proceedings of the 27th Int. Conf. on High Energy Physics, 20th-27th July 1994, Glasgow.
7) M. Hirsh, H. V. Klapdor-Kleingrothaus and S. G. Kovalenko, Max Planck Institute Preprint, MPI-H-V 6-1995.
8) Particle Data Group, Review of Particle Properties, Phys. Rev. D50 (1994), 1173.