ON p*gp-LOCALLY CLOSED SETS IN TOPOLOGICAL SPACES

M. JEYACHITRA1,2,* K. BAGEERATHI3

1Department of Mathematics, Sri Muthukumaran Arts and Science College, Chennai, India
2Manonmaniam Sundaranar University, Tirunelveli, India
3Department of Mathematics, Aditanar College of Arts and Science, Tiruchendur, Tamilnadu, India

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Abstract: In this article, we consider a new class of sets which are called p*gp-locally closed sets and obtain some of their properties and also their relationships with some other classes of topological spaces. In addition, we found P*GPLC continuous function and P*GPLC irresolute function. Moreover, several examples are providing to illustrate the behavior of these new classes of sets.

Keywords: p*gp-locally closed; P*GPLC continuous; P*GPLC irresolute.

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1. INTRODUCTION

Kuratowski and Sierpinski [7] have been studied the notion of a locally closed sets in a topological space. Bourbaki [1] defined by locally closed sets in topological spaces. Ganster and Reilly [4] used locally closed sets to define LC-continuity and LC-irresoluteness. The concept of generalized closed sets was considered by Levine [8] plays a significant role in general topology.
Noiri, Maki, and Umehara [10] provided the class of pre generalized closed sets and used them to obtain properties of pre-$T_{1/2}$ spaces. Selvi [11] further investigated pre*closed sets using the g-closure operator due to Dunham [2, 3]. The notion of pre open set was discovered by Mashhour [9]. This characterization paved a new direction.

The authors [5, 6] brings out the p*gp-closed sets and p*gp-open sets in topological spaces and established their relationships with some generalized sets in topological spaces. The purpose of this paper is to discuss about the concept of p*gp-locally closed sets in topological spaces and study their basic properties. Also, we provide P*GPLC continuous function, P*GPLC* continuous function and P*GPLC** continuous function and discuss P*GPLC irresolute function. We obtain many interesting results, to substantiate these result, suitable examples are given at the respective places.

This paper is organized as follows. In the second section, a brief survey of basic concepts and results in topological spaces which are essentially needed are given Section 3, we consider the properties of p*gp-locally closed sets and some basic results, while section 4, introduces the classes of P*GPLC continuous function, P*GPLC* continuous function, P*GPLC** continuous function and P*GPLC irresolute function and some of the properties of these functions. Last section, we provide a brief summary of work done in this paper.

2. PRELIMINARIES

Throughout this paper $(X, \tau)$ represents a topological space on which no separation axiom is assumed unless otherwise mentioned. $(X, \tau)$ will be replaced by $X$ if there are no changes of confusion. For a subset $A$ of a topological space $X$, $\text{cl}(A)$, $\text{int}(A)$ and $X \setminus A$ denote the closure of $A$, the interior of $A$ and the complement of $A$ respectively. Further, we denote the collection of all locally closed subsets of $(X, \tau)$ by $\text{LC}(X, \tau)$. We recall the following definitions and results which are prerequisites for our present work.

**Definition 2.1.** [8] Let $(X, \tau)$ be a topological space. Then the subset $A$ of $X$ is said to be

(i) generalized closed (briefly g-closed) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an open in $(X, \tau)$.

(ii) generalized open (briefly g-open) if its complement, $X \setminus A$ is g-closed.

**Definition 2.2.** Let $(X, \tau)$ be a topological space and $A \subseteq X$. The generalized closure of $A$ [2], denoted by $\text{cl}^*(A)$ and is defined by the intersection of all g-closed sets containing $A$ and
generalized interior of A [3], denoted by \( \text{int}^*(A) \) and is defined by union of all g-open sets contained in A.

**Definition 2.3.** Let \((X, \tau)\) be a topological space and \(A \subseteq X\). Then

(i). \(A\) is pre open if \(A \subseteq \text{int}(\text{cl}(A))\) and pre closed if \(\text{cl}(\text{int}(A)) \subseteq A\) [9].

(ii). \(A\) is pre*open if \(A \subseteq \text{int}^*(\text{cl}(A))\) and pre*closed if \(\text{cl}^*(\text{int}(A)) \subseteq A\) [11].

**Definition 2.4.** [9] Let \((X, \tau)\) be a topological space and \(A \subseteq X\). The pre closure of \(A\) denoted by \(\text{pcl}(A)\) and is defined by the intersection of all pre closed sets containing \(A\).

**Definition 2.5.** [5] A subset \(A\) of a topological space \((X, \tau)\) is said to be pre*generalized pre closed set (briefly \(p^*\text{gp}-\text{closed}\)) if \(\text{pcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is pre*open in \((X, \tau)\). The collection of all \(p^*\text{gp}\)-closed sets of \(X\) is denoted by \(p^*\text{gp}-\text{C}(X)\).

**Lemma 2.6.** [5] Let \((X, \tau)\) be a topological space. Then

(i). Every closed set is \(p^*\text{gp}\)-closed.

(ii). Intersection of any two \(p^*\text{gp}\)-closed sets is \(p^*\text{gp}\)-closed.

**Definition 2.7.** [6] A subset \(A\) of a topological space \((X, \tau)\) is said to be \(p^*\text{gp}\)-open if \(X\setminus A\) is \(p^*\text{gp}\)-closed. The collection of all \(p^*\text{gp}\)-open sets of \(X\) is denoted by \(p^*\text{gp}-\text{O}(X)\).

**Lemma 2.8.** [6] Let \((X, \tau)\) be a topological space. Then

(i). Every open set is \(p^*\text{gp}\)-open.

(ii). Union of any two \(p^*\text{gp}\)-open sets is \(p^*\text{gp}\)-open.

**Definition 2.9.** A subset \(A\) of a topological space \((X, \tau)\) is called a locally closed (briefly \(\text{lc}\)) set [4] if \(A = U \cap V\) where \(U\) is open and \(V\) is closed in \((X, \tau)\).

**Definition 2.10.** A function \(f : (X, \tau) \rightarrow (Y, \sigma)\) is called \(\text{LC}\)-continuous [4] if \(f^{-1}(F)\) is locally closed set in \((X, \tau)\) for each closed set \(F\) of \((Y, \sigma)\).

**Definition 2.11.** A function \(f : (X, \tau) \rightarrow (Y, \sigma)\) is called \(\text{LC}\)-irresolute [4] if \(f^{-1}(F)\) is locally closed set in \((X, \tau)\) for locally closed set \(F\) of \((Y, \sigma)\).

### 3. \textbf{Pre*Generalized Pre Locally Closed Sets}

In this section, \(p^*\text{gp}\)-locally closed sets are introduced to obtain some of their properties and their relationships with other existing sets.

**Definition 3.1.** A subset \(A\) of a topological space \((X, \tau)\) is said to be a \(p^*\text{gp}\)-locally closed (briefly \(p^*\text{gp}\text{lc}\)) set if \(A=V \cap F\) where \(V\) is \(p^*\text{gp}\)-open and \(F\) is \(p^*\text{gp}\)-closed.
The class of all p*gp-locally closed sets in \((X, \tau)\) is denoted by \(P^*\text{GPLC}(X, \tau)\).

**Definition 3.2.** A subset \(A\) of a topological space \((X, \tau)\) is said to be p*gplc* if there exist a p*gp-open set \(V\) and a closed set \(F\) of \((X, \tau)\) such that \(A = V \cap F\).

The class of all p*gplc* sets in \((X, \tau)\) is denoted by \(P^*\text{GPLC}^*(X, \tau)\).

**Definition 3.3.** A subset \(A\) of a topological space \((X, \tau)\) is said to be p*gplc** if there exist an open set \(V\) and a p*gp-closed set \(F\) of \((X, \tau)\) such that \(A = V \cap F\).

The class of all p*gplc** sets in \((X, \tau)\) is denoted by \(P^*\text{GPLC}^{**}(X, \tau)\).

**Theorem 3.4.** If a subset \(A\) of \((X, \tau)\) is locally closed then it is a p*gplc set, p*gplc* set and p*gplc** set.

**Proof.** Let \(A\) be a locally closed subset of \(X\). Then \(A = V \cap F\), where \(V\) is open and \(F\) is closed in \((X, \tau)\). By Lemma 2.6 and Lemma 2.8, \(A\) is a p*gplc set, p*gplc* set and p*gplc** set.

**Remark 3.5.** The converse of the above theorem need not be true as seen from the following example.

**Example 3.6.** Let \(X = \{a, b, c\}\) and \(\tau = \{\phi, \{c\}, X\}\). Then the locally closed sets are \(\{\phi, \{c\}, \{a, b\}, X\}\), \(P^*\text{GPLC}(X, \tau) = P^*\text{GPLC}^*(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}\) and \(P^*\text{GPLC}^{**}(X, \tau) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, X\}\). Here, \(\{a\}\) is p*gplc, p*gplc* and p*gplc** but not locally closed.

**Theorem 3.7.** If a subset \(A\) of \((X, \tau)\) is p*gplc** then it is a p*gplc set.

**Proof.** Let \(A\) be a p*gplc** set. Then by Definition 3.3, \(A = V \cap F\), where \(V\) is an open set in \((X, \tau)\) and \(F\) is a p*gp-closed set in \((X, \tau)\). By Lemma 2.8, \(A\) is p*gplc set.

**Remark 3.8.** The converse of the above theorem need not be true as shown in the following example.

**Example 3.9.** Let \(X = \{a, b, c\}\) and \(\tau = \{\phi, \{a\}, \{a, b\}, X\}\). Let \(A = \{a, c\}\). Then \(\{a, c\}\) is p*gplc set but not p*gplc** set.

**Theorem 3.10.** If \(A \in P^*\text{GPLC}(X, \tau)\) and \(B\) is p*gp-closed in \((X, \tau)\), then \(A \cap B \in P^*\text{GPLC}(X, \tau)\).

**Proof.** Since \(A \in P^*\text{GPLC}(X, \tau)\), there exist a p*gp-open set \(V\) and a p*gp-closed set \(F\) such that \(A = V \cap F\). Now \(A \cap B = (V \cap F) \cap B = V \cap (F \cap B)\). Since \(V\) is p*gp-open and \(F \cap B\) is p*gp-closed, \(A \cap B \in P^*\text{GPLC}(X, \tau)\).

**Theorem 3.11.** If \(A \in P^*\text{GPLC}^*(X, \tau)\) and \(B\) is closed in \((X, \tau)\), then \(A \cap B \in P^*\text{GPLC}^*(X, \tau)\).
Proof. Since \( A \in P^{*}GPLC^{*}(X, \tau) \), there exist a \( p^{*}gp \)-open set \( V \) and a closed set \( F \) such that \( A = V \cap F \). Since \( B \) is a closed set, we have \( A \cap B = (V \cap F) \cap B = V \cap (F \cap B) \). Since \( V \) is \( p^{*}gp \)-open and \( F \cap B \) is closed, \( A \cap B \in P^{*}GPLC^{*}(X, \tau) \).

**Theorem 3.12.** If \( A \in P^{*}GPLC^{**}(X, \tau) \) and \( B \) is \( p^{*}gp \)-closed (resp. open) in \((X, \tau)\), then \( A \cap B \in P^{*}GPLC^{**}(X, \tau) \).

Proof. Since \( A \in P^{*}GPLC^{**}(X, \tau) \), there exist an open set \( V \) and a \( p^{*}gp \)-closed set \( F \) such that \( A = V \cap F \). Now \( A \cap B = (V \cap F) \cap B = V \cap (F \cap B) \). Since \( V \) is open and \( F \cap B \) is \( p^{*}gp \)-closed, \( A \cap B \in P^{*}GPLC^{**}(X, \tau) \).

In this case \( B \) being an open set, we have \( A \cap B = (V \cap F) \cap B = (V \cap B) \cap F \). Since \( V \cap B \) is open and \( F \) is \( p^{*}gp \)-closed, \( A \cap B \in P^{*}GPLC^{**}(X, \tau) \).

**Theorem 3.13.** Let \((X, \tau)\) and \((Y, \sigma)\) be topological spaces. Then

(i) If \( A \in P^{*}GPLC(X, \tau) \) and \( B \in P^{*}GPLC(Y, \sigma) \), then
\[
A \times B \in P^{*}GPLC(X \times Y, \tau \times \sigma).
\]

(ii) If \( A \in P^{*}GPLC^{*}(X, \tau) \) and \( B \in P^{*}GPLC^{*}(Y, \sigma) \), then
\[
A \times B \in P^{*}GPLC^{*}(X \times Y, \tau \times \sigma).
\]

(iii) If \( A \in P^{*}GPLC^{**}(X, \tau) \) and \( B \in P^{*}GPLC^{**}(Y, \sigma) \), then
\[
A \times B \in P^{*}GPLC^{**}(X \times Y, \tau \times \sigma).
\]

Proof. Let \( A \in P^{*}GPLC(X, \tau) \) and \( B \in P^{*}GPLC(Y, \sigma) \). Then there exist \( p^{*}gp \)-open sets \( V \) and \( V_{1} \) of \((X, \tau)\) and \((Y, \sigma)\) and \( p^{*}gp \)-closed sets \( F \) and \( F_{1} \) of \((X, \tau)\) and \((Y, \sigma)\) respectively such that \( A = V \cap F \) and \( B = V_{1} \cap F_{1} \). Then \( A \times B = (V \times V_{1}) \cap (F \times F_{1}) \) holds. Hence \( A \times B \in P^{*}GPLC(X \times Y, \tau \times \sigma) \). This proves (i).

Let \( A \in P^{*}GPLC^{*}(X, \tau) \) and \( B \in P^{*}GPLC^{*}(Y, \sigma) \). Then there exist \( p^{*}gp \)-open sets \( V \) and \( V_{1} \) of \((X, \tau)\) and \((Y, \sigma)\) and closed sets \( F \) and \( F_{1} \) of \((X, \tau)\) and \((Y, \sigma)\) respectively such that \( A = V \cap F \) and \( B = V_{1} \cap F_{1} \). Then \( A \times B = (V \times V_{1}) \cap (F \times F_{1}) \) holds. Hence \( A \times B \in P^{*}GPLC^{*}(X \times Y, \tau \times \sigma) \). This proves (ii).

Let \( A \in P^{*}GPLC^{**}(X, \tau) \) and \( B \in P^{*}GPLC^{**}(Y, \sigma) \). Then there exist open sets \( V \) and \( V_{1} \) of \((X, \tau)\) and \((Y, \sigma)\) and \( p^{*}gp \)-closed sets \( F \) and \( F_{1} \) of \((X, \tau)\) and \((Y, \sigma)\) respectively such that \( A = V \cap F \) and \( B = V_{1} \cap F_{1} \). Then \( A \times B = (V \times V_{1}) \cap (F \times F_{1}) \) holds. Hence \( A \times B \in P^{*}GPLC^{**}(X \times Y, \tau \times \sigma) \). This proves (iii).
4. Functions via Pre*-Generalized Pre Locally Closed Sets

In this section, we introduce the concept of P*-GPLC continuous function, P*-GPLC* continuous function and P*-GPLC** continuous function in topological spaces and study some of their properties. Also, we describe P*-GPLC irresolute function, P*-GPLC* irresolute function and P*-GPLC** irresolute function in topological spaces and study some of their properties.

Definition 4.1. A function \( f : (X, \tau) \to (Y, \sigma) \) is said to be P*-GLC continuous (resp. P*-GPLC* continuous, P*-GPLC** continuous) if \( f^{-1}(V) \in P*gplc(X, \tau) \) (resp. \( f^{-1}(V) \in P*GPLC^*(X, \tau) \), \( f^{-1}(V) \in P*GPLC**(X, \tau) \)) for each closed set \( V \) of \( (Y, \sigma) \).

Example 4.2. Let \( X = Y = \{a, b, c\}, \tau = \{\emptyset, \{c\}, X\} \) and \( \sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, Y\} \). Define \( f : (X, \tau) \to (Y, \sigma) \) by the identity function. Then, \( f \) is P*-GPLC continuous, P*-GPLC* continuous and P*-GPLC** continuous.

Theorem 4.3. Let \( f : (X, \tau) \to (Y, \sigma) \) be a function. Then we have the following:

(i) If \( f \) is LC continuous, then \( f \) is P*-GPLC continuous, P*-GPLC* continuous and P*-GPLC** continuous.

(ii) If \( f \) is P*-GPLC** continuous function, then \( f \) is P*-GPLC continuous.

Proof. Suppose that \( f : (X, \tau) \to (Y, \sigma) \) is LC continuous. Let \( V \) be a closed set of \( (X, \tau) \). Then \( f^{-1}(V) \) is a locally closed set in \( (X, \tau) \). By Theorem 3.4, it follows that \( f \) is P*-GPLC continuous (resp. P*-GPLC* continuous and P*-GPLC** continuous). This proves (i).

Let \( f : (X, \tau) \to (Y, \sigma) \) be a P*-GPLC** continuous function. Let \( V \) be a closed set of \( (X, \tau) \). Then \( f^{-1}(V) \) is p*gplc** set in \( (X, \tau) \). By Theorem 3.7, it follows that \( f \) is P*-GPLC** continuous is P*-GPLC continuous. This proves (ii).

Remark 4.4. The converse of the above theorem need not be true as seen from the following example.

Example 4.5. Let \( X = Y = \{a, b, c\}, \tau = \{\emptyset, \{a\}, \{a, b\}, X\} \) and \( \sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, Y\} \). Define \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = b, f(b) = c \) and \( f(c) = a \). Then, \( f \) is P*-GPLC continuous, P*-GPLC* continuous and P*-GPLC** continuous. It can be proved that, \( f^{-1}([a, b]) = \{a, c\} \) is not a locally closed set in \( X \). Hence \( f \) is not LC continuous.

Example 4.6. Let \( X = Y = \{a, b, c\}, \tau = \{\emptyset, \{a\}, \{a, b\}, X\} \) and \( \sigma = \{\emptyset, \{c\}, \{a, b\}, Y\} \). Define \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = a, f(b) = c \) and \( f(c) = b \). Then, \( f \) is P*-GPLC continuous. It can be found that, \( f^{-1}([a, b]) = \{a, c\} \) is not p*gplc** set in \( X \). Hence \( f \) is not P*-GPLC** continuous.
Theorem 4.7. If \( f : (X, \tau) \rightarrow (Y, \sigma) \) and \( g : (Y, \sigma) \rightarrow (Z, \eta) \) are any two functions. Then

(i) \( g \circ f \) is P*GPLC continuous if \( f \) is P*GPLC continuous and \( g \) is continuous.

(ii) \( g \circ f \) is P*GPLC* continuous if \( f \) is P*GPLC* continuous and \( g \) is continuous.

(iii) \( g \circ f \) is P*GPLC** continuous if \( f \) is P*GPLC** continuous and \( g \) is continuous.

**Proof.** Let \( F \) be a closed set in \((Z, \eta)\). Since \( g \) is continuous, \( g^{-1}(F) \) is closed set in \((Y, \sigma)\). Again, since \( f \) is P*GPLC continuous, \( f^{-1}(g^{-1}(F)) \) is p*gplc in \((X, \tau)\). Thus \( g \circ f \) is P*GPLC continuous function. This proves (i).

Let \( F \) be a closed set in \((Z, \eta)\). Since \( g \) is continuous, \( g^{-1}(F) \) is closed in \((Y, \sigma)\). Since \( f \) is P*GPLC* continuous, \( f^{-1}(g^{-1}(F)) \) is p*gplc* in \((X, \tau)\). Thus \( g \circ f \) is P*GPLC* continuous function. This proves (ii).

Let \( F \) be a closed set in \((Z, \eta)\). Since \( g \) is continuous, \( g^{-1}(F) \) is closed in \((Y, \sigma)\). Since \( f \) is P*GPLC** continuous, \( f^{-1}(g^{-1}(F)) \) is p*gplc** in \((X, \tau)\). Thus \( g \circ f \) is P*GPLC** continuous function. This proves (iii).

**Definition 4.8.** A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is said to be P*GPLC irresolute (resp. P*GPLC* irresolute, P*GPLC** irresolute) if \( f^{-1}(V) \in P^*\text{GPLC}(X, \tau) \) (resp. \( f^{-1}(V) \in P^*\text{GPLC}^*(X, \tau) \), \( f^{-1}(V) \in P^*\text{GPLC}**(X, \tau) \)) for each \( V \in P^*\text{GPLC}(Y, \sigma) \) (resp. \( V \in P^*\text{GPLC}^*(Y, \sigma) \), \( V \in P^*\text{GPLC}**(Y, \sigma) \)).

**Example 4.9.** Let \( X = Y = \{a, b, c\} \), \( \tau = \{\emptyset, \{a\}, \{a, b\}, X\} \) and \( \sigma = \{\emptyset, \{c\}, \{a, b\}, Y\} \). Define \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = b, f(b) = a \) and \( f(c) = c \). Then, \( f \) is P*GPLC irresolute, P*GPLC* irresolute and P*GPLC** irresolute.

**Theorem 4.10.** If a function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is LC irresolute, then \( f \) is P*GPLC irresolute (resp. P*GPLC* irresolute and P*GPLC** irresolute).

**Proof.** Suppose that \( f \) is LC irresolute. Let \( V \) be a locally closed set of \((X, \tau)\). Then \( f^{-1}(V) \) is a locally closed set in \((X, \tau)\). By Theorem 3.4, it follows that \( f \) is P*GPLC irresolute (resp. P*GPLC* irresolute and P*GPLC** irresolute).

**Remark 4.11.** The converse of the above theorem need not be true as seen from the following example.

**Example 4.12.** Let \( X = Y = \{a, b, c\} \), \( \tau = \{\emptyset, \{c\}, X\} \) and \( \sigma = \{\emptyset, \{c\}, \{a, b\}, Y\} \). Define \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = c, f(b) = b \) and \( f(c) = a \). Then, \( f \) is P*GPLC irresolute, P*GPLC** irresolute.
irresolute and \( P^*GPLC^{**} \) irresolute. It can be verified that, \( f^{-1}(\{a, c\}) = \{a, c\} \) is not locally closed in \( X \). Hence \( f \) is not LC irresolute.

**Theorem 4.13.** Let \( f: (X, \tau) \to (Y, \sigma) \) and \( g: (Y, \sigma) \to (Z, \eta) \) be any two functions. Then

(i) \( g \circ f: (X, \tau) \to (Z, \eta) \) is \( P^*GPLC \) irresolute if \( g \) is \( P^*GPLC \) irresolute and \( f \) is \( P^*GPLC \) irresolute.

(ii) \( g \circ f: (X, \tau) \to (Z, \eta) \) is \( P^*GPLC \) continuous if \( g \) is \( P^*GPLC \) continuous and \( f \) is \( P^*GPLC \) irresolute.

**Proof.** Let \( F \in P^*GPLC(Z, \eta) \). Since \( g \) is \( P^*GPLC \) irresolute, \( g^{-1}(F) \) is \( p^*gplc \) in \( (Y, \sigma) \). As \( f \) is \( P^*GPLC \) irresolute, \( f^{-1}(g^{-1}(F)) \) is \( p^*gplc \) in \( (X, \tau) \). That is \( (g \circ f)^{-1}(F) \in P^* GPLC(X, \tau) \). Thus \( g \circ f \) is \( P^*GPLC \) irresolute. This proves (i).

Let \( F \) be a closed set in \( (Z, \eta) \). Since \( g \) is \( P^*GPLC \) continuous, \( g^{-1}(F) \) is \( p^*gplc \) in \( (Y, \sigma) \). Again, since \( f \) is \( P^*GPLC \) irresolute, \( f^{-1}(g^{-1}(F)) \) is \( p^*gplc \) in \( (X, \tau) \). Thus \( g \circ f \) is \( P^*GPLC \) continuous. This proves (ii).

**Theorem 4.14.** Let \( f: (X, \tau) \to (Y, \sigma) \) and \( g: (Y, \sigma) \to (Z, \eta) \) be any two functions. Then

(i) \( g \circ f \) is \( P^*GPLC^* \) irresolute if \( f \) and \( g \) are \( P^*GPLC^* \) irresolute.

(ii) \( g \circ f \) is \( P^*GPLC^{**} \) irresolute if \( f \) and \( g \) are \( P^*GPLC^{**} \) irresolute.

(iii) \( g \circ f \) is \( P^*GPLC^* \) continuous if \( f \) is \( P^*GPLC^* \) irresolute and \( g \) is \( P^*GPLC^* \) continuous.

(iv) \( g \circ f \) is \( P^*GPLC^{**} \) continuous if \( f \) is \( P^*GPLC^{**} \) irresolute and \( g \) is \( P^*GPLC^{**} \) continuous.

**Proof.** Let \( F \in P^*GPLC^*(Z, \eta) \). Since \( g \) is \( P^*GPLC^* \) irresolute, \( g^{-1}(F) \) is \( p^*gplc^* \) in \( (Y, \sigma) \). As \( f \) is \( P^*GPLC^* \) irresolute, \( f^{-1}(g^{-1}(F)) \) is \( p^*gplc^* \) in \( (X, \tau) \). That is \( (g \circ f)^{-1}(F) \in P^* GPLC^*(X, \tau) \). Thus \( g \circ f \) is \( P^*GPLC^* \) irresolute. This proves (i).

Let \( F \in P^*GPLC^{**}(Z, \eta) \). Since \( g \) is \( P^*GPLC^{**} \) irresolute, \( g^{-1}(F) \) is \( p^*gplc^{**} \) in \( (Y, \sigma) \). As \( f \) is \( P^*GPLC^{**} \) irresolute, \( f^{-1}(g^{-1}(F)) \) is \( p^*gplc^{**} \) in \( (X, \tau) \). That is \( (g \circ f)^{-1}(F) \in P^* GPLC^{**}(X, \tau) \). Thus \( g \circ f \) is \( P^*GPLC^{**} \) irresolute. This proves (ii).

Let \( F \) be a closed set in \( (Z, \eta) \). Since \( g \) is \( P^*GPLC^* \) continuous, \( g^{-1}(F) \) is \( p^*gplc^* \) in \( (Y, \sigma) \). Again, since \( f \) is \( P^*GPLC^* \) irresolute, \( f^{-1}(g^{-1}(F)) \) is \( p^*gplc^* \) in \( (X, \tau) \). Thus \( g \circ f \) is \( P^*GPLC^* \) continuous. This proves (iii).
Let $F$ be a closed set in $(Z, \eta)$. Since $g$ is $P^*\text{GPLC}^*$ continuous, $g^{-1}(F)$ is $p^*\text{GPLC}^*$ in $(Y, \sigma)$. Since $f$ is $P^*\text{GPLC}^*$ irresolute, $f^{-1}(g^{-1}(F))$ is $p^*\text{GPLC}^*$ in $(X, \tau)$. Thus $g \circ f$ is $P^*\text{GPLC}^*$ continuous. This proves (iv).

5. CONCLUSION

In this paper, $p^*\text{gp}$-locally closed sets in topological spaces are projected. Also $P^*\text{GPLC}$ continuous function and $P^*\text{GPLC}$ irresolute function are found.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests.

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