Signals of Axion Like Dark Matter in Time Dependent Polarization of Light

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Abstract

We consider the search for axion-like particles (ALPs) by using time series data of the polarization angle of the light. If the condensation of an ALP plays the role of dark matter, the polarization plane of the light oscillates as a function of time and we may be able to detect the signal of the ALP by continuously observing the polarization. In particular, we discuss that the analysis of the Fourier-transformed data of the time-dependent polarization angle is powerful to find the signal of the ALP dark matter. We pay particular attention to the light coming from astrophysical sources such as protoplanetary disks, supernova remnants, the foreground emission of the cosmic microwave background, and so on. We show that, for the ALP mass of \( \sim 10^{-22} - 10^{-19} \) eV, ALP searches in the Fourier space may reach the parameter region which is unexplored by other searches yet.
1 Introduction

The existence of axion-like particles (ALPs) may be ubiquitous in string theory \cite{1,3} and they may have a wide range of masses and decay constants. In particular, a very light ALP is a candidate for dark matter (DM) in the present universe. The ALP field, denoted by $a$, begins coherent oscillation when the Hubble parameter becomes comparable to the ALP mass and it behaves as a non-relativistic matter. Thus finding the evidence of such ALP DM would be a probe of physics beyond the Standard Model. Actually, several ideas are proposed to search for ALP DM through the (extremely) weak interaction between the ALP and the Standard Model particles \cite{4,12}.

One important effect of ALP is on the polarization plane of the light propagating through the ALP condensation \cite{13}. If the ALP amplitude depends on time, which is the case for the ALP DM, the polarization plane of the light becomes also time-dependent. Thus, if the ALP plays the role of DM, we have a chance to observe the effects of ALP condensation by precisely observing the polarization of the light. In particular, if we consider the light from astrophysical sources, which travels a significant amount of distance before being observed, the effects of the ALP condensation may be accumulated in the polarization plane of the light; such an effect may be experimentally detectable. The ALP search using the polarization of light from astrophysical sources have been considered in literatures, using the light from radio galaxies \cite{14}, protoplanetary disks \cite{15}, jets in active galaxies \cite{16}, pulsars \cite{17,18}, and the cosmic microwave background (CMB) \cite{19,24}. Even if the ALP is not DM, the axion cloud may be formed around rotating black holes through the superradiance \cite{25}, and the effect on the polarization of light passing through such axion cloud was discussed in \cite{26,27}. In particular, in \cite{16,18,20,24,26,27}, possibilities of using the time dependence of the polarization of light were discussed.

In this letter, we consider how we can extract information about the ALP DM from the time dependence of the polarization of the light from astrophysical sources. Assuming that the ALP is the dominant component of cold DM, the ALP potential should be well approximated by a parabolic one in the present universe. In such a case, the time dependence of the polarization plane becomes also harmonic-oscillator-like with the angular frequency of $m_a$ (with $m_a$ being the mass of the ALP). With such knowledge about the time dependence of the polarization plane, we can extract information about the ALP from the behavior of the polarization plane by Fourier transforming the time-dependent data. While the ALP search in the Fourier space has already been done by using jets from active galaxies \cite{16} and radio pulsar \cite{18}, we discuss that we may use polarized light from a variety of astrophysical light sources for the ALP search, like polarized light from protoplanetary disk, astrophysical radio sources like supernova remnant (SNR), foreground emission of the CMB, and so on. We show that the analysis with the Fourier transformation applies to these light sources. We estimate the possible discovery reaches for the ALPs using these astrophysical polarized light; our formulation is generally applicable to various sources and can be used as a guideline for future ALP search. We will argue that, with the use of the information about the time dependence of the polarization plane of the light, the discovery reach for the ALPs can be
significantly enlarged. We also show that the ALP search of our proposal may explore the parameter region which has not been reached by the experiments in the past.

The organization of this letter is as follows. In Section 2 we give the setup of our analysis and explain how the information about the ALP DM can be extracted from the data. In Section 3 we estimate the sensitivity of the ALP search by observing the time dependence of the polarization plane of astrophysical light. We give a general formula which gives the reach to the strength of the ALP-photon coupling as a function of the ALP mass, observation time, angular resolution for the measurement of the polarization plane, and so on. Then, we estimate the expected sensitivities of ALP search using lights from various sources, such as protoplanetary disks, radio sources like SNRs, CMB, and so on. Section 4 is devoted to conclusions and discussion.

2 Setup and Methodology

We first explain the methodology of how we can perform the ALP search by observing the polarization of lights from astrophysical sources. We adopt the following Lagrangian:

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \partial_\mu a \partial^\mu a - \frac{1}{2} m_a^2 a^2 + \frac{1}{8} g \epsilon^{\mu\nu\rho\sigma} a F_{\mu\nu} F_{\rho\sigma}, \]

(2.1)

where \( F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \) is the field strength for the electromagnetic gauge boson \( A_\mu \), \( a \) is the ALP field, and \( g \) is the photon-ALP coupling constant. (Here, we neglect terms irrelevant for our discussion.)

We assume that the universe is filled with the ALP oscillation that is assumed to be DM. Because the interaction of the ALP is extremely weak, it obeys the equation of motion of free bosons. We approximate the ALP amplitude at the position \( \vec{x} \) as

\[ a_{\vec{x}}(t) \simeq \tilde{a}_{\vec{x}} \sin (m_a t + \delta_{\vec{x}}). \]

(2.2)

We will come back to the validity of the above expression later. The amplitude of the oscillation is related to the (local) DM density as

\[ \rho_{\vec{x}}^{(ALP)} = \frac{1}{2} m_a^2 \tilde{a}_{\vec{x}}^2. \]

(2.3)

We can find

\[ \tilde{a}_{\vec{x}} \simeq 2.1 \times 10^9 \text{ GeV} \times \left( \frac{m_a}{10^{-21} \text{ eV}} \right)^{-1} \left( \frac{\rho_{\vec{x}}^{(ALP)}}{0.3 \text{ GeV/cm}^3} \right)^{1/2}. \]

(2.4)

Now we discuss how the light propagates under the influence of the ALP oscillation. The equation of motion of \( A_\mu \) is given by

\[ \Box A^\mu - \partial^\mu \partial_\nu A^\nu + g \epsilon^{\mu\nu\rho\sigma} (\partial_\nu a)(\partial_\rho A_\sigma) = 0. \]

(2.5)
Because we are interested in the polarization of the light, we concentrate on the plane-wave light and pay particular attention to the electric field \( \vec{E} \) perpendicular to the direction of the propagation. We take \( z \) (i.e., \( x^3 \)) direction as the propagation direction, and consider the behaviors of \( E_x = \partial_t A_x - \partial_x A_0 \) and \( E_y = \partial_t A_y - \partial_y A_0 \).

Hereafter, we take the Lorentz gauge,
\[
\partial_\mu A^\mu = 0, \tag{2.6}
\]
and discuss how the solutions of Eq. (2.5) behave. Because the effect of the ALP is so weak that it can be treated as a perturbation, we discuss the effect of the ALP on \( A_\mu \) up to the linear order in \( g \). Then, \( A_\mu \) can be expressed as
\[
A_\mu = (0, A_x, A_y, 0) + \delta A_\mu, \tag{2.7}
\]
with \( \delta A_\mu \sim O(g) \), where we have used the fact that, in the vacuum, we can take a gauge in which both \( A_0 = 0 \) and \( \partial_i A_i = 0 \) hold. Besides, we concentrate on the case where
\[
\omega_\gamma \gg m_a, \tag{2.8}
\]
with \( \omega_\gamma \) being the angular frequency of the light of our interest.

For our discussion, it is convenient to use the fact that the following equation holds in the Lorentz gauge (with neglecting terms of \( O(g^2) \) or higher):
\[
\Box (A_x + i A_y) = -ig \left[ (\partial_t a) \partial_z - (\partial_z a) \partial_t \right] (A_x + i A_y). \tag{2.9}
\]
Because \( \omega_\gamma \gg m_a \), \( \partial_t a \) and \( \partial_z a \) can be approximately treated as constants in solving Eq. (2.9). Postulating
\[
A_x + i A_y = \tilde{A}_- \exp \left[ -i \{ \omega_\gamma t - k_- (\omega_\gamma) z \} \right] + \tilde{A}_+ \exp \left[ i \{ \omega_\gamma t - k_+ (\omega_\gamma) z \} \right], \tag{2.10}
\]
with \( \tilde{A}_\pm \) being constants, we can find the following dispersion relation:
\[
k_\pm (\omega_\gamma) = \omega_\gamma \mp \frac{1}{2} g \frac{da}{d\ell}, \tag{2.11}
\]
where
\[
\frac{da}{d\ell} \equiv \partial_t a + \partial_z a. \tag{2.12}
\]
Then, integrating over the path of the light, parameterized as \( (t_S + \ell, 0, 0, z_S + \ell) \), we obtain
\[
(A_x + i A_y) (t, \vec{x}) = \tilde{A}_- e^{-i \Phi_- (t, \vec{x}; t_S, \vec{x}_S)} + \tilde{A}_+ e^{i \Phi_+ (t, \vec{x}; t_S, \vec{x}_S)} \tag{2.13}
\]
with
\[
\Phi_\pm (t, \vec{x}; t_S, \vec{x}_S) = \pm \frac{1}{2} g \left[ a(t, \vec{x}) - a(t_S, \vec{x}_S) \right], \tag{2.14}
\]
where $t_S$ is the time when the light is emitted while $\vec{x}_S$ is the position of the source. Thus, $A_\mu$ is affected by the change of the phase velocity of the light; we can see $\delta A_{x,y} \sim O(gaA_{x,y})$.

We can also estimate the size of $A_0$. In the Lorentz gauge, $A_0$ obeys

$$\Box A_0 = -g e^{0ijk}(\partial_i a)(\partial_j A_k) \simeq \pm i\omega_\gamma g e^{0ijk}(\partial_i a)A_k, \quad (2.15)$$

where, in the second equality, we used the fact that $A_k$ is approximately proportional to $e^{\pm i\omega_\gamma(t-z)}$. Notably, $A_0 = 0$ if the ALP field is homogeneous (in other words, $v \to 0$ with $v$ being the velocity of the ALP). Because of the oscillatory behavior of the ALP, we can postulate as $A_0 \propto e^{\pm i(\omega_\gamma \pm m_\alpha)t \mp iz}$ and estimate the size of $A_0$. Using the relation $\partial_i a \sim O(m_\alpha v a)$, $A_0$ is found to be of $O(gavA_{x,y})$ and hence, compared to $\delta A_{x,y}$, $A_0$ is suppressed by the factor of $\sim v$. Thus, we can neglect the effects of $A_0$ on $E_x$ and $E_y$ as far as the ALP is non-relativistic.

If the light is linearly polarized when it is emitted, the polarization plane becomes tilted after the propagation by the angle $\Phi = \Phi_+ - \Phi_-$.

$$\Phi_\odot(t) = g [\tilde{a}_\odot \sin(m_\alpha t + \delta_\odot) - \tilde{a}_S \sin \{m_\alpha(t-L) + \delta_S\}] \quad : \quad \text{small source}, \quad (2.16)$$

where $L$ is the distance to the source. (Here and hereafter, the subscript “$\odot$” is used for quantities in the solar system unless otherwise mentioned.) We emphasize that the polarization plane oscillates with the angular frequency of $m_\alpha$.

- If the size of the source (which we call $\Delta L$) is non-negligible, lights with various $L$ contribute to the signal. In such a case, the second term of the right-hand side of Eq. (2.16) should be replaced by the average, $\langle a(t_S, \vec{x}_S) \rangle_L$. In particular, if $\Delta L \gg m_\alpha^{-1}$, we expect $\langle a(t_S, \vec{x}_S) \rangle_L \sim 0$.

Notice that the above expression is also applicable to the case when the DM density at the source is much smaller than that at the observer.

We consider the situation in which we measure the polarization of the light coming from astrophysical sources. We assume that an area of the sky can be observed or scanned within the time $\tau$. The observation is assumed to be continuously performed during $0 < t < T$. The observed area is divided into smaller independent patches; the number of sky patches is denoted as $n_{\text{patch}}$, and the direction to the patch $\alpha$ is indicated as $\vec{e}_\alpha$. Then, each sky patch is observed $n_{\text{obs}}$ times, where

$$n_{\text{obs}} = \frac{T}{\tau}, \quad (2.19)$$
and each observation time is represented by

\[ t_I \equiv \tau I \quad (I = 1, 2, \cdots, n_{\text{obs}}). \] (2.20)

For each patch \( \alpha \), we assume that the direction of the polarization on the plane perpendicular to \( \vec{e}_\alpha \) is observed as a function of time; we denote the observed value of the polarization angle as \( \Theta_\alpha(t) \). We define the angle relative to the time-averaged value as

\[ \theta_\alpha(t_I) \equiv \Theta_\alpha(t_I) - \frac{1}{n_{\text{obs}}} \sum_{J=1}^{n_{\text{obs}}} \Theta_\alpha(t_J). \] (2.21)

It is expected to be expressed as

\[ \theta_\alpha(t_I) = \left[ \Phi_\odot(t_I) - \frac{1}{n_{\text{obs}}} \sum_{J=1}^{n_{\text{obs}}} \Phi_\odot(t_J) \right] + N_\alpha(t_I), \] (2.22)

where \( N_\alpha(t_I) \) is the effect unrelated to the ALP oscillation (which we call “noise”), like the detector noise and observational errors. We also define \( N_\alpha(t_I) \) such that its averaged value vanishes (i.e., \( \lim_{T \to \infty} \frac{1}{T} \sum_I N_\alpha(t_I) = 0 \)). The variance of \( N_\alpha \) is given by

\[ \sigma_N^2 = \frac{1}{n_{\text{obs}}} \sum_{I=1}^{n_{\text{obs}}} N^2_\alpha(t_I). \] (2.23)

Here and hereafter, we assume that (i) the variance does not depend on the patch, (ii) there is no correlation between \( N_\alpha(t_I) \) and \( N_\beta(t_J) \) if \( \alpha \neq \beta \) or \( t_I \neq t_J \), and (iii) \( N_\alpha \) is uncorrelated with \( \Phi_\alpha \). In Eq. (2.22), the effect of the time dependence of the source, which may add an extra contribution to \( \theta_\alpha \), is not included. We assume that such an effect is negligible and study the reach for the case that the sensitivity is noise limited. The effect of the time dependence of the source should strongly depend on the choice of the source, and its detailed study is beyond the scope of this letter. We expect, however, that the effect of the time dependence of the source is distinguishable from the signal unless it is characterized by a frequency with a very narrow bandwidth. This is because, as we see below, the signal is given by a very sharp peak in the Fourier space.

For the time interval \( T \) shorter than \( \sim 1/(m_a v^2) \), \( \theta_\alpha(t_I) \) approximately behaves as

\[ \theta_\alpha(t_I) = A_* \sin(m_a t_I + \delta_*) + N_\alpha(t_I) - \frac{A_*}{n_{\text{obs}}} \sum_{J=1}^{n_{\text{obs}}} \sin(m_a t_J + \delta_*) \] (2.24)

where \( A_* \) and \( \delta_* \) are constants. Taking \( \tilde{a}_S = \tilde{a}_\odot \) and \( \delta_S = \delta_\odot \) in Eq. (2.17), for example, \( A_* = 2g\tilde{a}_\odot \sin \frac{m_a L}{2} \) and \( \delta_* = \delta_\odot - \frac{1}{2} m_a L + \frac{1}{2} \pi \).

To obtain information about the ALP oscillation, we introduce the following quantity:

\[ S(\omega, \delta) \equiv \sum_{\alpha=1}^{n_{\text{patch}}} \sum_{I=1}^{n_{\text{obs}}} \theta_\alpha(t_I) \sin(\omega t_I + \delta). \] (2.25)
We require $\omega$ to satisfy

$$\frac{2\pi}{T} \lesssim \omega \lesssim \frac{2\pi}{\tau},$$

(2.26)

with which the constant term in the right-hand side of Eq. (2.24) can be made unimportant in the calculation of $S(\omega, \delta)$. Then, we obtain

$$S(\omega, \delta) = \frac{1}{2} n_{\text{patch}} A_s \sum_{I=1}^{n_{\text{obs}}} \left[ \cos \{(\omega - m_a) t_I + \delta - \delta_s\} - \cos \{(\omega + m_a) t_I + \delta + \delta_s\} \right]$$

$$+ \sum_{\alpha=1}^{n_{\text{patch}}} \sum_{I=1}^{n_{\text{obs}}} N_\alpha(t_I) \sin(\omega t_I + \delta).$$

(2.27)

When $\tau \ll m_a^{-1}$, the first summation can be approximately replaced by the integration over time, i.e.,

$$\sum_{I=1}^{n_{\text{obs}}} \cos \{(\omega \pm m_a) t_I + \delta \pm \delta_s\} \simeq \frac{\sin\{(\omega \pm m_a) T\}}{(\omega \pm m_a) T} n_{\text{obs}} \cos(\delta \pm \delta_s)$$

$$- \frac{1 - \cos\{(\omega \pm m_a) T\}}{(\omega \pm m_a) T} n_{\text{obs}} \sin(\delta \pm \delta_s).$$

(2.28)

The function $S(\omega, \delta)$ fluctuates as we vary $\omega$ because of the noise. Using the fact that $N_\alpha(t_I) \sin(\omega t_I + \delta)$ $(I = 1, \cdots, n_{\text{obs}})$ are random variables with the variance of $1/2 \sigma_N^2$, the typical size of $S$ is estimated as

$$S(\omega, \delta) \sim \begin{cases} 
\frac{n_{\text{tot}}}{2} A_s \cos(\delta - \delta_s) + \sqrt{\frac{n_{\text{tot}}}{2} \sigma_N} & : |\omega - m_a| \ll \frac{1}{T}, \\
\sqrt{\frac{n_{\text{tot}}}{2} \sigma_N} & : |\omega - m_a| \gg \frac{1}{T},
\end{cases}$$

(2.29)

where

$$n_{\text{tot}} = n_{\text{obs}} n_{\text{patch}}.$$  

(2.30)

With choosing $\delta \sim \delta_s$, the function $S(\omega, \delta)$ has a peak at $\omega = m_a$ when $\sqrt{\frac{n_{\text{tot}}}{2} |A_s|}$ is substantially larger than $\sigma_N$. Thus, with a large enough number of data, we may be able to observe the effect of the ALP even if $\sigma_N \gg |A_s|$.

For a demonstration purpose, we generate a set of Monte-Carlo (MC) sample data of $\theta_\alpha(t_I)$, taking $\sigma_N = 1$, $n_{\text{obs}} = n_{\text{patch}} = 1000$, $A_s = 5\sqrt{2/n_{\text{tot}}}$, and $m_a = 100\Delta \omega$ with

$$\Delta \omega = \frac{2\pi}{T}.$$

(2.31)
Figure 1: Behavior of $|S(\omega, \delta_*)|$ based on MC data generated with $\sigma_N = 1$, $n_{\text{obs}} = n_{\text{patch}} = 1000$, $A_* = 5\sqrt{2/n_{\text{tot}}}$, and $m_a = 100\Delta \omega$.

Here, the noise $N_\alpha(t_I)$ is assumed to be $(0, 1)$ Gaussian random variable. Then, we calculate the signal function $S$ with $\delta = \delta_*$; the behavior of $|S(\omega, \delta_*)|$ around $\omega \sim m_a$ is shown in Fig. 1. We can see that $|S|$ is sharply peaked at $\omega = m_a$; the width of the peak is $\sim \Delta \omega/2$.

In order to understand how large $T$ can be, we consider the validity of Eq. (2.2). Because the ALP evolves under the influence of the gravitational potential and also because its amplitude has fluctuations, the ALP field is given by the superposition of the modes with various frequencies. We expect that the ALP amplitude is *locally* expressed as

$$a_x(t) = \int d\omega_a \tilde{a}_x(\omega_a) \sin[\omega_a t + \delta_x(\omega_a)].$$

(2.32)

We also expect that the ALP oscillation is dominated by the modes with $\omega_a - m_a \lesssim O(m_a v_x^2)$, where $v_x$ is the infall velocity, and hence $\tilde{a}_x(\omega_a \gg m_a + m_a v_x^2) \sim 0$. (Near the solar system, the infall velocity is $v_\odot \sim 10^{-3}$. Thus, for the time period shorter than $\sim 1/(m_a v_x^2)$, we may use Eq. (2.2); for $|t| \lesssim 1/(m_a v_x^2)$, $\tilde{a}_x$ and $\delta_x$ in Eq. (2.2) are given by

$$\tilde{a}_x \simeq \sqrt{s_x^2 + c_x^2}, \quad \delta_x \simeq \cos^{-1} \frac{c_x}{\sqrt{s_x^2 + c_x^2}},$$

(2.33)

where

$$s_x = \int d\omega_a \tilde{a}_x(\omega_a) \sin \delta_x(\omega_a), \quad c_x = \int d\omega_a \tilde{a}_x(\omega_a) \cos \delta_x(\omega_a).$$

(2.34)
For the time period longer than $\sim 1/(m_a v_x^2)$, on the contrary, the coherence of the modes with different frequencies breaks down; in such a case, Eq. (2.2) is not applicable. Thus, our estimation of $S(\omega, \delta)$ given in Eq. (2.29) is valid only when $T \lesssim 1/(m_a v_x^2)$.

### 3 Sensitivity

We have seen that the signal of the ALP may be imprinted in the polarization of the lights from astrophysical sources, and that it may be extracted if the information about the time dependence of the polarization angle is available. Hereafter, we estimate the sensitivity of the ALP search using various polarized lights from various sources.

For $m_a \gtrsim T^{-1}$, the sensitivity using the polarized light can be obtained by solving

$$\sqrt{\frac{\text{Max}}{2}} |A_*| \gtrsim Z \sigma_N,$$

where $Z$ is the significance. Taking $|A_*| \sim g\tilde{a}_\odot$, the reach for the case of $m_a \gtrsim T^{-1}$ is found to be

$$g \gtrsim 2.1 \times 10^{-15} \text{ GeV}^{-1} \times Z \left( \frac{m_a}{10^{-21} \text{ eV}} \right) \left( \frac{T}{1 \text{ year}} \right)^{-1/2} \left( \frac{\tau}{n_{\text{patch}}/1 \text{ sec}} \right)^{1/2} \left( \frac{\sigma_N}{1^\circ} \right), \quad (3.1)$$

where Eq. (2.4) is used to estimate the ALP amplitude with taking $\rho^{(\text{ALP})}_\odot = 0.3 \text{ GeV/cm}^3$. For $m_a \lesssim T^{-1}$, on the contrary, the observation period $T$ is too short to observe the oscillatory behavior of the signal and hence we expect no sensitivity. Note that $\tau$ cannot be made arbitrarily small since the error $\sigma_N$ may increase for smaller $\tau$. There is an optimal choice of $\tau$ in a realistic experimental setup.

In Fig. 2, we show the $5\sigma$ discovery reach calculated with Eq. (3.1) by black dotted lines, taking $(T/1 \text{ year})^{-1/2}(\tau/n_{\text{patch}}/1 \text{ sec})^{1/2}(\sigma_N/1^\circ) = 1 - 10^4$. In the same figure, we also show the region already excluded by the CERN Axion Solar Telescope (CAST) experiment [28] and the astronomical observation of the supernova SN1987A [29], as well as by the theoretical requirement for the galaxy-scale structure [30]. Furthermore, the green lines show the prospects of the experimental reaches by the Any Light Particle Search (ALPS) II [31, 32] and International Axion Observatory (IAXO) [33]. Although not shown in the Figure, the X-ray observation of sources in or behind galaxy clusters also give stringent upper bound [34–37].

In the following, we discuss several examples of astrophysical sources of polarized light for the ALP search. In particular, we estimate the reach for each example.

### Protoplanetary disks (PPDs)

PPDs are often formed around young stars. The light emitted from the young star and scattered in the PPD can be highly polarized, which may be used for the ALP search of our proposal. (For another attempt of the ALP search using a PPD without relying on the time dependence, see [15].)

One of the precise measurements of the polarized intensity (PI) of PPD can be found in [38] for the Herbig Ae star AB Aur, which is about 144 pc away from the earth. In [38],
Figure 2: Expected $5\sigma$ discovery reaches on $m_a$ vs. $g$ plane; the regions above the lines will be explored with the observations of the time dependence of the polarization plane of astrophysical light. The black dotted lines are for $(T/1\ \text{year})^{-1/2}(\tau/n_{\text{patch}}/1\ \text{sec})^{1/2}(\sigma_N/1^\circ) = 1$ to $10^4$. The solid lines show the expected $5\sigma$ reaches using a PPD with $T = 4$ months (blue), an SNR with $T = 4$ months (orange), and by the CMB-S4 with $T = 7$ years (red), with our choices of detector parameters. The end points of these lines are determined by the requirement (2.26). The current and the future constraints on the plane are also shown. The gray shaded regions are existing bounds from the CAST experiment [28] and the observation of the SN1987A [29], while the blue shaded region are the theoretical bound from the galaxy-scale structure [30]. The green lines are the prospects by the ALPS II [31,32] and IAXO [33] experiments. The purple lines indicate bounds from the observation of parsec-scale jets [16] and the pulsar PSR J0437-4715 [18].

Linear polarization images of AB Aur were obtained by using Subaru / HiCIAO; the field of view was $10'' \times 20''$ with the pixel scale of 9.3 mas/pixel, and the total integration time of the PI image was 189.6 sec. The field of view was divided into the “sky patches” of the size of $6 \times 6$ pixel binning, and the polarization vector of each sky patch was determined; the direction of the polarization vector of each sky patch was defined as the relative angle between the polarization vector and the line from AB Aur to the sky patch. Using $\sim 1400$ sky patches, each of which has the PI larger than 50$\sigma$, the distribution of the polarization direction was obtained. With the Gaussian fitting, the distribution of the direction of the polarization vector has the central position of $90^\circ \pm 0.2^\circ$ and its FWHM is $4.3^\circ \pm 0.4^\circ$.

Based on the result of [38] as well as those of other observations of PPDs [39-42], we expect that precise measurements of the PI of PPDs are possible. Adopting the performance
given in [38] as an example, let us estimate the sensitivity of the ALP search with the use of polarized light from PPDs. For this purpose, we first estimate the size of the noise, $\sigma_N$. Here, conservatively, we assume that the FWHM of the distribution of the polarization angle obtained in [38] is fully due to the effects of the noise, although the polarization angle may receive other effects; we use $\sigma_N \approx 1.8^{\circ}$ for the measurements of the polarization angle of the light from PPDs. Taking $\tau = 189.6$ sec and $n_{\text{patch}} = 1400$, a 4-month observation of a PPD similar to AB Aur gives a 5\sigma discovery of the ALP signal if $g \gtrsim 1.1 \times 10^{-14}$ GeV$^{-1} \times (m_a/10^{-21}$ eV)$^2$. The 5\sigma reach for the setup described above is shown in Fig. 2 by the blue line. According to the requirements Eq. (2.26), the line extends within the range (#2)

$$\frac{2\pi}{T} \sim 4 \times 10^{-22} \text{eV} \lesssim m_a \lesssim \frac{2\pi}{\tau} \sim 2 \times 10^{-17} \text{eV}. \quad (3.2)$$

We can see that, for the ALP mass $m_a \lesssim 10^{-19} \text{eV}$, we may reach a parameter region which is not constrained by any existing bound.

**Radio sources**

Synchrotron emission from the electron motion in the magnetic field is necessarily polarized. There are many astrophysical sources of such polarized emission: radio galaxies, pulsar, SNR, pulsar wind nebula, and so on. In the radio frequency band, the linearly polarized light usually experience birefringence while it is propagating through the intergalactic plasma and magnetic field (i.e., Faraday effect). It can be distinguished from the ALP-induced birefringence by using the multi-wavelength observation since the former depends on the wavelength while the latter does not. Since we are interested in the periodic change of the polarization angle, the Faraday effect does not much affect our argument.

Crab nebula is one of the bright SNRs seen in a broad frequency range from radio to gamma-ray. The polarization angle and its distribution is measured in optical band [43] and also in radio band [44, 45]. For example, the NIKA instrument at the IRAM telescope [45] measured the polarization spatial distribution of the Crab nebula at the 150 GHz frequency. With the observation time of 2.4 hours, the number of patches is about $n_{\text{patch}} \sim 150$ with an angular resolution of 18" and the typical error of the polarization angle at each patch is about 1°. In Fig. 2, we show the corresponding bound with an orange line, assuming the same total observation time as the PPD observation mentioned above. Compared with the case of PPD observation by the Subaru / HiCIAO, an expected constraint on $g$ is an order of magnitude weaker. Still we can get a meaningful constraint on $g$ around the mass range $m_a \sim 10^{-21} \text{–} 10^{-20}$ eV.

Other polarized radio sources include jets in active galaxies [16] and pulsars [17, 18] as we mentioned in Section 1. They already have time series observational data of the polarization

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#1 The reach here is obtained with the assumption of 24-hour observation of the PPD each day. Such an assumption may be unrealistic in particular for observation in the optical band that is possible only at night. If the daily observation time is reduced down to 8 hours, for example, the reach is worsened by the factor of $\sqrt{3}$.

#2 The upper bound on $m_a$ given in (3.2) is comparable to that from the coherent time.
angle and derived constraint on $g$. The observational data of the pulsar PSR J0437-4715 [46] was used in [15]. During the whole 1609 days of observation, there were 393 data points of the polarization angle with a typical variance of a few degrees, which resulted in the constraint $g \lesssim 10^{-12} \text{GeV}^{-1} \times (m_a/10^{-21} \text{eV})$. The constraint is roughly consistent with Eq. (3.1).

**Galactic center**

Polarized light from the galactic center may be also used for the ALP search. In particular, linearly polarized radio waves are observed from Sgr A*, where a supermassive black hole (BH) is expected to exist. The BH is associated with magnetic fields, which cause the synchrotron radiation of the polarized light. The BH mass is measured to be $M_{\text{BH}} \sim 4.1 \times 10^6 M_\odot$ (with $M_\odot$ being the solar mass), while the position of the radiation source is $\sim 10 R_S$ away from the center of the BH [47], with $R_S \sim 10^{-6} \text{pc}$ being the Schwarzschild radius.

For the ALP search, one advantage of using the polarized light from the galactic center is the large DM density. In the case where a very light scalar field (i.e., the ALP) plays the role of DM, we should take into account the fact that the DM distribution cannot have a non-trivial structure for the scale smaller than its de Broglie length. Thus, at the galactic center, there exists a cut-off length $\lambda$ below which the DM profile should become (almost) flat. The cut-off length $\lambda$ can be estimated by requiring that the de Broglie length for $r > \lambda$ be shorter than $r$; we estimate $\lambda$ by solving $\lambda = 1/(m_a v_{\text{vir}}(\lambda))$ with $v_{\text{vir}}(r)$ being the virialized velocity at the position of $r$. If the BH mass dominates over the total DM mass within $r < \lambda$, which is the case of our interest, the virial velocity is estimated as

$$v_{\text{vir}}(r) = \sqrt{\frac{G M_{\text{BH}}}{r}}, \quad (3.3)$$

where $G$ is the Newton constant and $M_{\text{BH}}$ is the mass of the BH, which results in

$$\lambda \sim 2.1 \times 10^2 \text{pc} \times \left(\frac{m_a}{10^{-21} \text{eV}}\right)^{-2}. \quad (3.4)$$

Note that the source position $\sim 10^{-5} \text{pc}$ is well within the cut-off length for $m_a \lesssim 10^{-18} \text{eV}$. Then, the DM density at the position of the light source can be evaluated by $\rho_{\text{DM}}(r \sim \lambda)$, where $\rho_{\text{DM}}$ is the density profile of DM. When we use the Einasto profile [48,49]:

$$\rho_{\text{Einasto}}(r) = \rho_s \exp \left[-\frac{2}{\alpha} \left\{ \left(\frac{r}{r_s}\right)^{\alpha} - 1 \right\} \right], \quad (3.5)$$

with $\rho_s = 9 \times 10^6 M_\odot \text{kpc}^{-3}$, $\alpha = 0.17$, and $r_s = 20 \text{kpc}$ [50,51] for our galaxy, the ALP amplitude at the light source is about 0–4 orders of magnitude larger than that around the earth for $m_a = 10^{-22}–10^{-18} \text{eV}$. When we use the Navarro-Frenk-White (NFW) profile [52,53]:

$$\rho_{\text{NFW}}(r) = \frac{\rho_H}{r H \left(1 + \frac{r}{R_H}\right)^2}, \quad (3.6)$$
with $\rho_R = 4 \times 10^7 M_\odot \text{ kpc}^{-3}$ and $R_H = 20 \text{ kpc}$ [54], the enhancement of the ALP amplitude becomes more significant. Such an enhancement of the ALP amplitude is advantageous for the ALP search.

In using the polarized light from the galactic center (in particular, from the region near Sgr A*), we should also take account of the strong gravitational field at the source. If the light source of our concern is near the BH, the gravitational potential at the source position should be sizable, resulting in a significant redshift of the frequency of the ALP oscillation. Besides, if the polarized light is emitted from a region where the gravitational potential significantly varies, the peak of $S(\omega, \delta)$ is broadened, which makes ALP search more difficult. Thus, although it may be interesting to use the polarized light from Sgr A* for the ALP search, more careful study is needed. We leave it as a future task.

**CMB foreground**

Now let us consider the possibility to use the data taken by CMB polarization experiments, whose primary purpose is to find the inflationary $B$-mode signals, to find ALPs. We consider the Fourier-space analysis of the time dependence of the polarization of the CMB; for earlier discussion about the time dependence of the CMB polarization due to the ALP DM, see [20][24]. We focus on the polarized foreground emission below, although the polarized CMB (induced by the Thomson scattering) may also be used as a probe of ALP as well.

It is well known that the CMB foreground emission from sources like synchrotron radiation and Galactic dust is polarized. The polarization amplitude of such foreground emission is sizable and the understandings of foregrounds are essential in the CMB experiments searching for the inflationary $B$-mode signals. The foreground amplitude is strongly dependent on the CMB frequency, and many of the CMB detectors have several frequency bands, some of which are (mainly) dedicated to measuring the foreground amplitude. Even though the foreground emission is harmful for the $B$-mode detection, it may be useful for the ALP search. Hereafter, we estimate the sensitivity of the CMB experiments to the signal of the ALP with the analysis using the foreground polarization.

In the analysis, for simplicity, we assume that the CMB experiment scans the sky region with the total solid angle of $\Omega$ within the period of $\tau$. The total solid angle $\Omega$ is divided into sky patches each of which has a solid angle $\Delta \Omega$. Then,

$$n_{\text{patch}} = \frac{\Omega}{\Delta \Omega},$$

(3.7)

and the time available for scanning a single sky patch (within the period of $\tau$) is

$$\Delta \tau \equiv \frac{\tau}{n_{\text{patch}}},$$

(3.8)

Then, using the relation between the polarization angle $\theta$ and the Stokes parameters $Q$ and $U$, i.e., $\theta = \frac{1}{2} \tan^{-1}(U/Q)$, $\sigma_N$, the uncertainty in the measurement of the polarization angle,
is estimated as
\[ \sigma_N = \frac{\text{NET}_{\text{arr}}}{\sqrt{2} P}, \quad (3.9) \]
where \( \text{NET}_{\text{arr}} \) is the noise-equivalent-temperature (NET) of the detector array (which is given by \( \text{NET}_{\text{arr}} = \text{NET}_{\text{det}}/\sqrt{n_{\text{det}}} \), with \( \text{NET}_{\text{det}} \) and \( n_{\text{det}} \) being the NET of single detector and the number of detectors, respectively), and \( P \equiv \sqrt{Q^2 + U^2} \) is the polarization amplitude \[56\].

Here, the uncertainties of both \( Q \) and \( U \) are estimated to be \( \sqrt{2} \text{NET}_{\text{arr}}/\sqrt{\Delta \tau} \), taking into account the factor of \( \sqrt{2} \) to convert the temperature noise to the polarization noise. Although the polarization amplitude \( P \) should depend on each sky patch, for simplicity, we consider the case where the polarization amplitudes for the patches of our interest are typically of the same size. Then, the reach is estimated to be
\[ g \gtrsim 8.3 \times 10^{-14} \text{ GeV}^{-1} 	imes Z \left( \frac{m_a}{10^{-21} \text{ eV}} \right) \left( \frac{T}{1 \text{ year}} \right)^{-1/2} \left( \frac{\text{NET}_{\text{arr}}}{10 \mu \text{K}\sqrt{\text{sec}}} \right) \left( \frac{P}{10 \mu \text{K}} \right)^{-1}. \quad (3.10) \]

There are on-going and future CMB experiments looking for the \( B \)-mode signals with \( T \sim \) a few years and \( \text{NET}_{\text{arr}} \) of \( O(10) \mu \text{K}\sqrt{\text{sec}} \) or smaller. The data available from those experiments can be used for the ALP search. The sensitivity of each experiment depends on the details of the detector parameters, and can be evaluated by using (3.10). For example, the 27 GHz detector of the small area telescope of Simons Observatory, which will observe 10% of the sky, will realize the angular resolution of 25 \( \mu \text{K-arcmin} \) with the (effective) observation time of 1 year \[55\]. Then, \( \text{NET}_{\text{arr}} \) is estimated to be 36 \( \mu \text{K}\sqrt{\text{sec}} \). Using the RMS polarization amplitude of \( P(30 \text{ GHz}) \approx 7 \mu \text{K} \) \[56\], Simons Observatory may observe the ALP signal at 5\( \sigma \) level if \( g \gtrsim 2.1 \times 10^{-12} \text{ GeV}^{-1} \times (m_a/10^{-21} \text{ eV}) \). In more future, CMB-S4 experiment may improve the sensitivity. For the CMB-S4 experiment, the NET of single detector (for the frequency of 30 GHz) is expected to be 177 \( \mu \text{K}\sqrt{\text{sec}} \), while 576 detectors may become available for such a frequency \[57\], resulting in \( \text{NET}_{\text{arr}} \approx 7.4 \mu \text{K}\sqrt{\text{sec}} \). With 7 years of observation, CMB-S4 may realize the 5\( \sigma \) discovery of the ALP signal if \( g \gtrsim 1.7 \times 10^{-13} \text{ GeV}^{-1} \times (m_a/10^{-21} \text{ eV}) \). The expected 5\( \sigma \) reach of CMB-S4 experiment after 7-year survey is shown in Fig. 2 by the red line, taking \( \tau \sim 1 \text{ day} \). We comment here that the sensitivity depends on the polarization amplitude \( P \). In particular, if the observation is performed with a larger \( P \), a better discovery reach can be realized.

### 4 Conclusions and Discussion

We have discussed the possibility to search for the signal of ALP DM by observing the time dependence of the polarization of the light. We have proposed the Fourier-space analysis using the light from various astrophysical sources. We concentrated on the light from astrophysical sources, but a similar analysis may be performed in the proposed ALP search using
optical cavity \[58,59]\) or gravitational wave detector \[60]\). The discovery reach depends on the performance of the detector (i.e., the time required to observe the polarization of the light from a single sky patch and the size of the noise in the observation) as well as the period available for the observation. If observed, the signal not only provides strong evidence of ALP DM but also gives information about the ALP mass.

We listed several possible sources of polarized light from optical to radio wave bands. PPDs are good polarized sources in optical or infrared band and it will give strong constraint on the axion coupling constant if the long-time observation will be performed for \(m_a \simeq 10^{-22} - 10^{-19}\) eV. Some radio sources such as SNRs, radio galaxies and pulsars are also possible candidates for such a purpose and hence radio telescopes will also give comparable (or a bit weaker) constraint if the observation time is long enough. We also estimated the reach of CMB polarization experiments, paying particular attention to the possibility to use the foreground polarized emission. The last one is economical in the sense that the ALP search is possible without affecting the operation of the mission to observe the inflationary \(B\)-mode.

Our essential idea is to use time series data of the polarization angle and find a peak in the Fourier space. It has several advantages as well as some cautions compared to that in the real time space. In the following, we list them in order:

- It may be possible to combine the results from different observations to improve the sensitivity because the position of the peak in the Fourier space is uniquely determined by the ALP mass and is independent of the observation (as far as the effect of the redshift is negligible). In addition, if two (or more) experiments are done simultaneously, we may use the information about \(\delta_\ast\) to check the consistency of the signal.

- For the case where the source has a time dependence, the analysis in the Fourier space may help to discriminate the effects of the time dependence of the source from the signal. We expect that the former does not mimic the latter unless the time dependence of the source is characterized by a very particular angular frequency with a width narrower than \(\sim \pi/T\). This is because, if the ALP DM exists, the signal function \(S\) should have a very sharp peak at \(\omega = m_a\) with the width of \(\sim \pi/T\).

- In our analysis, we assumed that the observation of the polarization angle is continuously performed during the observation period. Such an assumption is just for simplicity and the qualitative behavior of the signal function \(S\) holds in other cases. The analysis using the signal function given in Eq. (2.25) applies to observations with unequal time steps.

- Even if there is no ALP DM, the signal function \(S\) fluctuates because of the noise, and \(|S|\) has many local peaks. The heights of some of the peaks may become accidentally much larger than the expected value, i.e., \(\sqrt{n_{\text{tot}}/2}\). In deriving information about the ALP parameters from actual experiments, one should take into account look-elsewhere effects, whose study is beyond the scope of this letter.
In summary, Fourier-space analysis of the polarization of the light from various sources may shed light on the properties of the ALP DM.

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