A fuzzy inventory model with acceptable shortage using graded mean integration value method

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Abstract: In many inventory models uncertainty is due to fuzziness and fuzziness is the closest possible approach to reality. In this paper, we proposed a fuzzy inventory model with acceptable shortage which is completely backlogged. We fuzzily the carrying cost, backorder cost and ordering cost using Triangular and Trapezoidal fuzzy numbers to obtain the fuzzy total cost. The purpose of our study is to defuzzify the total profit function by Graded Mean Integration Value Method. Further a numerical example is also given to demonstrate the developed crisp and fuzzy models.

1. Introduction

Inventory control is the stock of any item or resource used in organization. An inventory system is the set of policies and controls that monitor levels of inventory and determine what levels should be maintained, when replenished and how large orders should be. Manufacturing inventory is typically classified into raw materials, finished products, component parts, supplies, and work-in-progress. There are many more reason reasoning of inventories. The problem of inventory control is broadly associated with answering two questions. 1. When to order. 2. How much to order. Inventory control is the means by which materials of the correct quality and in correct quantity are made available as and when required with due regards to economic storage and ordering cost. Here the desired level of inventory can either be high or low because high level inventory will lead to increase in carrying cost while low level of inventory will lead to increase the ordering cost.

The fuzzy set theory in inventory modeling is the closest possible approach to reality, as reality is not exact and can only be calculated to some extent. Same way, fuzzy theory helps one to incorporate unpredictability in the design of the model, thus bringing it closure to reality.

N Kamezi et al. (2010) \textsuperscript{[1]} developed an inventory with backorder with fuzzy parameters and decision variables. J K Syed and L A Aziz (2007) \textsuperscript{[2]} studied a fuzzy inventory model without shortage using signed distance method. P Parvathi and S Gajalakshmi (2013) \textsuperscript{[3]} Applied Trapezoidal fuzzy number with acceptable shortage with an inventory model. R M Rajalakshmi and G Michael Rosario (2017) \textsuperscript{[4]} Investigate a fuzzy inventory model with allowable shortage using signed distance method. Nabendu Sen and Sanjukta Malakar (2015) \textsuperscript{[14]} considered inventory problems without shortage in fuzzy environment.
and they also considered different costs as fuzzy numbers and defuzzified by using signed distance method. F Harris (1915) [9] developed an Operations and Cost. J S Yao et al. (2000) [10] introduced an Fuzzy inventory without backorder For fuzzy order quantity and fuzzy total demand quantity. L A Zadeh (1965) [7] developed a Fuzzy set.

2. Definition and Preliminaries

2.1. Fuzzy set

A fuzzy set is a where the members allowed to have partial membership and hence the degree of membership varies 0 to 1.

2.2. Fuzzy number

Fuzzy number is expressed as a fuzzy set defining a fuzzy interval in the real number R. Fuzzy number should be normalized and convex.

2.3. Triangular fuzzy number

Let \( A = (a, b, c), a < b < c \), be a fuzzy set on \( R = (-\infty, \infty) \). It is called a triangular fuzzy number, if its membership function is

\[
\mu_A(x) = \begin{cases} 
\frac{x-a}{b-a}, & \text{if } a \leq x \leq b \\
\frac{c-x}{c-b}, & \text{if } b \leq x \leq c \\
0, & \text{otherwise}
\end{cases}
\]

2.4. Graded means integration for triangular fuzzy number

Let \( A = (k, l, m) \) be a triangular fuzzy number and the graded mean integration of A measured

\[
\int_A = \frac{1}{6}(k + 4l + m)
\]

2.5. Trapezoidal fuzzy number

A trapezoidal fuzzy number \( \tilde{A} = (a, b, c, d), 0 \leq \alpha \leq 1 \) is represented with membership function

\[
\mu_A(x)
\]
The $\alpha$-cut of $A = (a, b, c, d)$, $0 \leq \alpha \leq 1$ is $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$.

Where $A_L(\alpha) = k + (l - k)\alpha$ and $A_R(\alpha) = n - (n - m)\alpha$ are the left and right endpoints of $A(\alpha)$.

### 2.6. Graded mean integration method for trapezoidal fuzzy number

$$
= \frac{1}{2} \int_0^1 \alpha [A_L(\alpha) + A_R(\alpha)] \, d\alpha
$$

$$
= \frac{1}{6} [k + 2l + 2m + n]
$$

### 3. Model description

#### 3.1. Notations

For a crisp inventory model with backorder, we the following notations and related parameters.

- $D$ - Length of plan (days).
- $q$ - Order quantity per cycle.
- $s$ - Shortage quantity per cycle.
r - Total demand over the planning time period [0,D].
k - Storing cost for one unit per day.
l - Backorder cost for one unit per day.
m - Cost of placing an order.
\( t_q \) - Length of a cycle.

3.2. Assumptions
In my project, the following assumptions are considered.
1. Total demand is considered as a constant.
2. Time plan is constant.
3. Shortage cost is allowed.
4. To fuzzily holding cost, ordering cost and shortage cost only.

4. Proposed inventory model in crisp sense
The crisp total cost on the planning period [0,D] is given by

\[
F(q, s) = \left[ kt_1 \frac{q - s}{2} + lt_2 \frac{s}{2} + m \frac{r}{q} \right] q
\]

\[
= k (q - s)^2 \frac{D}{2q} + ls^2 \frac{D}{2q} + mr \frac{r}{q}, (0 < s < q)
\]

Here \( t_1 \) denotes ordering time of length.
And \( t_2 \) denotes shortage time of length so that

\[
\frac{q - s}{t_1} = \frac{q}{t_q} = \frac{s}{t_2} = \frac{r}{D}
\]

The crisp total cost on the planning period [0,D] is given by

\[
F(q, s) = \left[ kt_1 \frac{q - s}{t_q} + lt_2 \frac{s}{2} + m \frac{r}{q} \right] q
\]

\[
= \frac{k (q - s)^2 D}{2q} + \frac{ls^2 D}{2q} + \frac{mr r}{q}, (0 < s < q)
\] (4.1)

The crisp optimal solutions are

Optimal order quantity \( q_* = \sqrt{\frac{2(k + l)mr}{kLD}} \) (4.2)

Optimal backorder quantity \( s_* = \sqrt{\frac{2kmr}{l(k + l)D}} \) (4.3)

Minimal total cost \( F(q_*s_*) = \sqrt{\frac{2klmrD}{k + l}} \) (4.4)
5. Diagrammatic representation:

6. Proposed inventory model in fuzzy environment

6.1. Triangular fuzzy total cost

In this model, we consider the ordering cost, holding cost and shortage cost as imprecise and are represented as triangular fuzzy numbers.

Let us denote the above mentioned costs respectively using LR-form as:

\[ \tilde{k} = (k_1, k_2, k_3) \quad \tilde{l} = (l_1, l_2, l_3) \quad \tilde{m} = (m_1, m_2, m_3) \]

\[ FTC = \frac{\tilde{k}(q - s)^2 D}{2q} + \frac{\tilde{l} s^2 D}{2q} + \frac{\tilde{m} r}{q} \]

\[ FTC = \frac{(k_1, k_2, k_3) \otimes (q - s)^2 \otimes D}{2 \otimes q}, \frac{(l_1, l_2, l_3) \otimes s^2 \otimes D}{2 \otimes q}, \frac{(m_1, m_2, m_3) \otimes r}{q} \]

\[ = \frac{k_1(q - s)^2 D}{2q} + \frac{k_2(q - s)^2 D}{2q} + \frac{k_3(q - s)^2 D}{2q} + \frac{l_1 s^2 D}{2q} + \frac{l_2 s^2 D}{2q} + \frac{l_3 s^2 D}{2q} + \frac{m_1 r}{q} + \frac{m_2 r}{q} + \frac{m_3 r}{q} \]

\[ = \frac{k_1(q - s)^2 D}{2q} + \frac{l_1 s^2 D}{2q} + \frac{m_1 r}{q} \]

The Graded mean integration method for defuzzification is given by the following equation,

\[ d \left( F_{\tilde{q}, \tilde{s}} (\tilde{k}, \tilde{l}, \tilde{m}) \right) = \frac{1}{6} (k + 4l + m) \]
Applying defuzzification,

\[
= \frac{1}{6} \left[ \frac{k_1(q-s)^2D}{2q} + \frac{l_1s^2D}{2q} + \frac{m_1r}{q} + 4 \left( \frac{k_2(q-s)^2D}{2q} + \frac{l_3s^2D}{2q} + \frac{m_3r}{q} \right) \right] \\
+ \frac{k_3(q-s)^2D}{2q} + \frac{l_3s^2D}{2q} + \frac{m_3r}{q} \\
= \frac{1}{6} \left[ \frac{(q-s)^2D}{2q} (k_1 + 4k_2 + k_3) + \frac{s^2D}{2q} (l_1 + 4l_2 + l_3) + \frac{r}{q} (m_1 + 4m_2 + m_3) \right]
\]

\[= F_d(q,s) \quad (6.1.1)\]

Computation of minimum value of \( q_* \) and \( s_* \).

\( F_d(q,s) \) is minimum at \( F_d'(q,s) = 0 \), where \( F_d''(q,s) = 0 \) is positive

\[
F_d'(q,s) = \frac{1}{6} \left[ \frac{D}{2} (k_1 + 4k_2 + k_3)(1 - \frac{s^2}{q^2}) - \frac{s^2D}{2q^2} (l_1 + 4l_2 + l_3) - \frac{r}{q} (m_1 + 4m_2 + m_3) \right]
\]

\[
= \frac{D}{2} (k_1 + 4k_2 + k_3)(1 - \frac{s^2}{q^2}) - \frac{s^2D}{2q^2} (l_1 + 4l_2 + l_3) - \frac{r}{q} (m_1 + 4m_2 + m_3) - 0
\]

After simplification we get,

\[
q_* = \sqrt{\frac{2r(m_1 + 4m_2 + m_3)(k_1 + 4k_2 + k_3 + l_1 + 4l_2 + l_3)}{D(l_1 + 4l_2 + l_3)(k_1 + 4k_2 + k_3)}} \quad (6.1.2)
\]

\[
s_* = \sqrt{\frac{2r(k_1 + 4k_2 + k_3)(m_1 + 4m_2 + m_3)}{D(l_1 + 4l_2 + l_3)(k_1 + 4k_2 + k_3 + l_1 + 4l_2 + l_3)}} \quad (6.1.3)
\]

This shows that \( F_d(q,s) \) is minimum at \( q_* \) and \( s_* \).

6.2. Trapezoidal fuzzy total cost

In this model, we consider the ordering cost, holding cost and shortage cost as imprecise and are represented as trapezoidal fuzzy numbers.

Let us denote the above mentioned costs respectively using LR-form as:

\[
\tilde{k} = (k_1, k_2, k_3, k_4) \quad \tilde{l} = (l_1, l_2, l_3, l_4)
\]

\[
FTC = \frac{\tilde{k}(q-s)^2D}{2q} + \frac{\tilde{l}s^2D}{2q} + \frac{\tilde{m}r}{q}
\]
FTC = \left[ \frac{(k_1, k_2, k_3, k_4) \otimes (q-s)^2 \otimes D}{2 \otimes q}, \frac{(l_1, l_2, l_3, l_4) \otimes s^2 \otimes D}{2 \otimes q}, \frac{(m_1, m_2, m_3, m_4) \otimes r}{q} \right]

= \left[ \frac{k_1 \otimes (q-s)^2 D}{2q}, \frac{k_2 \otimes (q-s)^2 D}{2q}, \frac{k_3 \otimes (q-s)^2 D}{2q}, \frac{k_4 \otimes (q-s)^2 D}{2q} \right]

\oplus \left[ \frac{l_1 \otimes s^2 D}{2q}, \frac{l_2 \otimes s^2 D}{2q}, \frac{l_3 \otimes s^2 D}{2q}, \frac{l_4 \otimes s^2 D}{2q} \right]

\oplus \left[ \frac{m_1 \otimes r}{q}, \frac{m_2 \otimes r}{q}, \frac{m_3 \otimes r}{q}, \frac{m_4 \otimes r}{q} \right]

= \left[ \frac{k_1 (q-s)^2 D}{2q} + \frac{l_1 s^2 D}{2q} + \frac{m_1 r}{q}, \frac{k_2 (q-s)^2 D}{2q} + \frac{l_2 s^2 D}{2q} + \frac{m_2 r}{q}, \frac{k_3 (q-s)^2 D}{2q} + \frac{l_3 s^2 D}{2q} + \frac{m_3 r}{q}, \frac{k_4 (q-s)^2 D}{2q} + \frac{l_4 s^2 D}{2q} + \frac{m_4 r}{q} \right]

The Graded mean integration for defuzzification is given by the following equation,

\[ d(F_{(q,s)}(k, l, m), 0) = \frac{1}{6} (k + 2l + 2m + n) \]

Applying defuzzification

\[ d(F_{(q,s)}(k, l, m), 0) = \frac{1}{6} \left[ \frac{(q-s)^2 D}{2q} \left( k_1 + 2k_2 + 2k_3 + k_4 \right) + \frac{s^2 D}{2q} \left( l_1 + 2l_2 + 2l_3 + l_4 \right) \right] \]

\[ + \frac{r}{q} \left( m_1 + 2m_2 + 2m_3 + m_4 \right) \]

\[ = F_d(q, s) \] (6.2.1)

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After simplification we get,

\[ q_* = \sqrt{\frac{2r(k_1 + 2k_2 + 2k_3 + k_4 + l_1 + 2l_2 + 2l_3 + l_4)(m_1 + 2m_2 + 2m_3 + m_4)}{D(l_1 + 2l_2 + 2l_3 + l_4)(k_1 + 2k_2 + 2k_3 + k_4)}} \]  \quad (6.2.2)

\[ s_* = \sqrt{\frac{2r(k_1 + 2k_2 + 2k_3 + k_4)(m_1 + 2m_2 + 2m_3 + m_4)}{D(l_1 + 2l_2 + 2l_3 + l_4)(k_1 + 2k_2 + 2k_3 + k_4 + l_1 + 2l_2 + 2l_3 + l_4)}} \]  \quad (6.2.3)

This shows that \( F_d(q, s) \) is minimum at \( q_* \) and \( s_* \).

7. Numerical analysis

7.1. Crisp model

Let \( D = 6, k = 4, l = 10, m = 20 \)

Using Equations (4.1),(4.2)&(4.3) and we get the order quantity \( q^* \), shortage quantity \( s^* \) and minimum total cost (TC).

Table 1.

| Demand | \( q^* \)   | \( s^* \)   | TC      |
|--------|------------|------------|---------|
| 1000   | 48.3045    | 13.8013    | 828.0786|
| 1100   | 50.6622    | 14.4749    | 868.4962|
| 1200   | 52.9150    | 15.1185    | 907.1147|
| 1300   | 55.075     | 15.7359    | 944.1549|
| 1400   | 57.1557    | 16.3299    | 979.7959|
| 1500   | 59.1608    | 16.9030    | 1014.1851|
7.2. Triangular fuzzy model

Let $D=6, \ k=(1,4,7), \ l=(8,10,12), \ m=(15,20,25)$

Using Equations (4.1),(4.2)&(4.3) and we get the order quantity $q^*$, shortage quantity $s^*$ and minimum fuzzy total cost (FTC).

Table 2.

\begin{array}{|c|c|c|c|}
\hline
\text{Demand} & q^* & s^* & \text{FTC} \\
\hline
1000 & 48.3045 & 13.8013 & 829.0189 \\
1100 & 50.6622 & 14.4749 & 868.4962 \\
1200 & 52.9150 & 15.1185 & 907.1148 \\
1300 & 55.0757 & 15.7359 & 944.1549 \\
1400 & 57.1540 & 16.3299 & 979.7958 \\
1500 & 59.1608 & 16.9030 & 1014.171 \\
\hline
\end{array}

7.3. Trapezoidal fuzzy model

Let $D=6, \ k=(1,3,5,6), \ l=(8,9,11,12), \ m=(15,18,22,25)$

Using Equations (5.2.1),(5.2.2)&(5.2.3) and we get the order quantity $q^*$, shortage quantity $s^*$ and minimum fuzzy total cost (FTC).

Table 3.

\begin{array}{|c|c|c|c|}
\hline
\text{Demand} & q^* & s^* & \text{FTC} \\
\hline
1000 & 49.0489 & 13.5918 & 815.5123 \\
1100 & 51.4429 & 14.2552 & 855.3163 \\
1200 & 53.7304 & 14.8891 & 893.3488 \\
1300 & 55.9243 & 15.4971 & 929.8270 \\
1400 & 58.0354 & 16.0821 & 964.9269 \\
1500 & 60.0724 & 16.6465 & 998.7944 \\
\hline
\end{array}
8. Conclusion
In this paper, we studied fuzzy inventory model with acceptable shortage and constant demand is considered and carrying cost, ordering cost are fuzzified using Triangular, Trapezoidal fuzzy numbers. By comparing the results of fuzzy model and crisp model and triangular number are provide minimum total cost when compared to the trapezoidal number. Numerical examples are illustrates the analysis.

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