Determination of Relaxation Time in Warming (Cooling) Processes of Atmosphere-Surface Thermal Interaction Based on A Serial RC Circuit Model

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Determination of Relaxation Time in Warming (Cooling) Processes of Atmosphere-Surface Thermal Interaction Based on A Serial RC Circuit Model

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Abstract. An inverse calculation method of a simple serial RC circuit model (RSRC model) has been implemented to construct a more appropriate model in order to explain the process of warming (cooling) of the atmosphere and the surface in a period of day and night, as well as to determine the relaxation time related to it. Testing of the model involving only radiation mechanisms lead to the conclusion that the better values of relaxation time can only be obtained when the interaction between the atmosphere and the surface are taken into account. Meanwhile, with a few flaws that still encountered, the results of the study recommends the need for the development of a model with a mechanism involving convection and phase transformation, while also increase a physical dimension of the depth of the surface layer.

1. Introduction

There are great many similarities between the variations of air temperature (and surface temperature) within a period of day and night with the variation of charges inside the capacitor in an RC circuit under the influence of the periodic voltage battery. In simple terms the things above happens because the air and the soil (as well as the water) have the ability to store the thermal energy that they have been received, which partly expressed as an increase in temperature, then it can be released back when the energy supply from the outside (especially the solar radiation) stopped, which partially of them are represented in the form of a decrease in it’s temperature.

For the majority of the university students of science and engineering, they have also known that the charge inside capacitor, as a function of time, $Q(t)$ on a simple serial RC circuits can be expressed as [1], [2]

$$\frac{dQ}{dt} = -\frac{1}{RC}(Q - Ce)$$ (1)

with $R$ and $C$ denote the resistance and capacitance of the system, while $\varepsilon$ stated the voltage source. In this case $\tau = RC$ is the time constant which, for a discussion here, because its value can change with time, we call it as the relaxation time.

Equation (1) is a mathematical formula of the Serial RC Circuit Model (RSRC Model). They are in the form of issue related to standards problem in order to the determination of $Q(t)$ as the solution of differential equations [1], [3], but in many applications it often appears as an inverse problem [4].

Heating and cooling processes have been studied empirically for a long time, which generally started by Newton (1701), Dulong and Petit (1817), and later by Stefan (1879). Afterwards, the theoretical study on this subject conducted by Boltzmann who then made publicly in 1884. However, in its development, various studies on the laws of heating and cooling experiencing a critical appraisal and revision, given that the issue actually difficult to separate the mechanisms of heat transfer [5], [6].

The simplest concept of heating/cooling is the transfer or exchange of thermal energy between the two objects (systems) as well as between the system with the environment, either through the mechanism
of radiation [7] [8] [9], convection [8], conduction [9], and the phase transformation. In the latter case the process also involves an exchange (transfer) of mass.

By limiting the process that only involves radiation mechanism, the formula to express changes in air temperature and surface temperature, which is identical to the RSRC model, can be approximated based on the following case (model):

Case#1: The Self Heating/Cooling Model (The PDM Model)

In this case the warming or cooling process is considered takes place separately, no interaction between the atmosphere and the surface so the statement that is identical to equation (1) can be written as

\[
\frac{dT_x}{dt} = \frac{-e_x \sigma T_x^4}{\rho_x c_x z_x} \left( T_x - \frac{\gamma_x}{e_x \sigma T_x^4} I_G \right)
\]  

(2)

the index \(x\) expresses the atmosphere (\(a\)) or the surface (\(s\)); \(\sigma\) is the Stefan-Boltzmann constant \( (5.6704 \times 10^{-8} \text{ W/m}^2\text{K}^4)\); \(e\) is emissivity; \(\rho\) is density; \(c\) is specific heat; and \(z\) is the layer thickness, the zero point on the surface. \(T\) and \(I_G\) are the temperature and global radiation intensity respectively; and \(\gamma\) is the absorption coefficient of the \(I_G\).

When \(T_x\) and \(I_G\) in equation (2) is identical with \(Q\) and \(\epsilon\) in equation (1) respectively, the relaxation time, equivalent resistance and equivalent capacitance can be expressed as

\[
\tau_x' = \frac{\rho_x c_x z_x}{e_x \sigma T_x^4} 
\]  

(3a)

\[
R_x^{eq} = \frac{\rho_x c_x z_x}{\gamma_x} \]  

(3b)

\[
C_x^{eq} = \frac{\gamma_x}{e_x \sigma T_x^4} 
\]  

(3c)

Case#2: The Mutual Warming/Cooling Model (PDB Model)

If, in the reality, there is interaction (thermal energy exchange) between the atmosphere and the surface, equation (2) should be formulated as

\[
\frac{dT_x}{dt} = \frac{-\sigma}{\rho_x c_x z_x} e_{xy} \left( T_x - T_y - \frac{\gamma_x}{\sigma e_{xy}} I_G \right)
\]  

(4)

the indices \(x\) and \(y\) express the atmosphere (\(a\)) and the surface (\(s\)) or vice versa; while \(e_{xy}\) expressed by

\[
e_{as} = \frac{e_a T_a^4 - a_a a_s T_s^4}{T_a - T_s}
\]  

(5a)

for the atmosphere, and

\[
e_{sa} = \frac{e_s T_s^4 - a_s a_a T_a^4}{T_s - T_a}
\]  

(5b)

for the surface. Meanwhile \(\alpha_{xy}\) is the absorption coefficient of \(x\) in relation to the radiation of \(y\). In this case, the relaxation time, equivalent resistance and equivalent capacitance can be expressed as

\[
\tau_x'' = \frac{\rho_x c_x z_x}{\sigma e_{xy}} 
\]  

(6a)

\[
R_x^{eq} = \frac{\rho_x c_x z_x I_G}{\sigma T_y e_{xy} - \gamma_x I_G} 
\]  

(6b)

\[
C_x^{eq} = \frac{\sigma T_y e_{xy} - \gamma_x I_G}{\sigma e_{xy} I_G} 
\]  

(6c)

The fact that the problem faced is the inverse problem [10], the scope and purpose of this paper is how to get \(\tau\), through the time series data of \(T\) and \(I_G\), for the atmosphere and the surface respectively.

2. Material and Method

2.1. Data

The main data of this study are the time series data of air temperature, surface temperature, and global radiation at intervals of every 3 minutes recording on the observation of Mauna Loa Observatory (MLO), Hawaii [11]. Because of this study is still at an early stage, the data used is chosen arbitrarily, limited only from observations during the month of February 2014 alone.
Although they have been averaging for as much as 28 days of observation, in order to be processed further the preparation to every kind of the data above was done by the "5-stage running mean", each for the five data before and after. Furthermore, in order that the data can be treated as like as a truly daily cycle events, the running mean calculation is first done by connecting the beginning and the end of the data in question.

2.2. Method

As already stated above, this study is limited only to the two cases (models) that involve only the mechanism of heat exchange by radiation. The first step, the determination of $C_{eq}$ is done by using the information about the extreme condition of $T$ and implementation or the Inverse Calculation Method (MPT) that have been studied and tested on RSRC Model [10]. The difference here is that the function of the C-Test has been adapted to the shape function $C_{eq}$ as stated in the equation (3c) and (6c), and by setting the values of physical parameters of the calculation resulted by the other methods [12] [13] and/or by other rational approach.

The next step, the relaxation time $\tau$ can be directly determined by the equation (2) and/or (4) without calculating first the $R_{eq}$, by equation (3b) and/or (6b). This can be done because if $C_{eq}$ has been obtained then all the information needed to get the $\tau$ has been fulfilled by equation (2) and (4).

3. Results and Discussion

The graph in Figure 1 shows the pattern of global radiation, air temperature, and surface temperature resulted by preparation of the related data from observation in the MLO during February 2014. Note that the pattern of $T$-$I_G$ relationships are very similar to the pattern relations of $Q$-$\varepsilon$ in the RSRC Model [10]. Figure 1 reveal solar radiation began to emerge ($I_G \geq 0.1$ Wm$^{-2}$) at ± 5:30 LST (Local Standard Time), peaked at ± 12:00 LST to the intensity of 788.74 Wm$^{-2}$ and sinks ($I_G \leq 0.1$ Wm$^{-2}$) at ± 19:45 LST. The existence of points representing the extreme conditions of $T_a$ (6:04 LST, 276.49 K & 12:21 LST, 281.77 K), $T_s$ (5:54 LST, 270.87 K & 12:31 LST, 302.41 K), and the points of intersection of $T_a$ and $T_s$ (8:11 LST, 278.62 K & 18:25 LST, 279.13 K) was decisive in the calculation of $C_{eq}$ and $\tau$ [11].

Case#1: The PDM Model Results

Figure 2 shows the pattern of $\tau$ for the Case # 1 (PDM model) by setting $e_a = 0.7$ and $e_s = 0.9$. Rational values of $\tau$ obtained mainly in the region $I_G = 0$, generally with $\tau_a > \tau_s$. This can be understood by remembering that the surface (soil) have Diurnal Temperature Range (DTR) greater than atmosphere’s DTR that obviously have to experience rate (changes in) temperature faster.

However, Figure 2 also shows the negative values and/or too large values of $\tau$ in the region $I_G \neq 0$, which indicates that the model in this case failed for region in question. Some critical point in the case where the values of $\tau$ deviate from rational estimate is precisely at points of extreme and points of intersection of $T_a$ and $T_s$.

![Figure 1. Typical form of atmosphere and surface temperature in relation to global solar radiation represented after data preparation (Data taken from MLO, 2014).](image)
Figure 2. Relaxation time for Case-1 by setting $e_a = 0.7$ and $e_s = 0.9$. Asymptotic values are not shown. The negative and very high values of $\tau$ at $I_G \neq 0$ indicate the model is failed for that region.

Case#2: The PDB Model Results

The $\tau$ chart for Case # 2 (PDB Model), with $e_a = 0.7$ and $e_s = 0.9$, the same as in Case # 1, is shown in Figure 3. Note that the pattern of $\tau$ in the the region with the $I_G = 0$, are similar in both cases, but with values of $\tau$ in the Case #1 are 50 times greater than the values of the corresponding $\tau$ in the Case # 2.

Figure 3. Relaxation time for Case-2 with $e_a = 0.7$ and $e_s = 0.9$. $\tau_a$ is reasonable, except in the second half of region with $I_G \neq 0$, whereas the $\tau_s$ is not.

Figure 3 also shows the pattern and value of $\tau_s$ that fairly rational in the half of the region with $I_G \neq 0$, but still led to a weakness when the values of $\tau_s$ are negative in the other half of region in question. Theoretically these problems arise due to the sameness signs regarding the value of the component parts of the left and right inside the brackets of equation (4). The solution of this problem is to add the mechanism of conduction, convection, and/or phase transformation on the part inside the brackets.

Another weakness of Case # 2 (PDB Model) is shown by the $\tau_s$ graph which in most of the region is worth negative. This weakness has been identified when $T_a$ is still down on the condition that $I_G = 0$ because when $T_a > T_s$, in accordance with the PDB Model, $T_s$ should be increase. Physically $T_s$ can still go down if we add the 'depth component', with a lower temperature than the surface temperature, to the surface sub-system.

4. Conclusion

Although based on a model that is very simple but, as generally the inverse problem, it is very difficult to determine exactly just one parameter associated with a process or characteristic of the system. On the one hand this is because the parameter is changed in value at any time, but in the other hand due to the process or the mechanisms associated with many components that are intertwined and with only
involves the mechanism of heat transfer by radiation, the relaxation time in the warming/cooling of the atmosphere and surface still can’t be determined well except for the condition without external radiation (specially the solar radiation), in which case the PDM Model and PDB Model produce similar pattern of the relaxation time but with a degree of rationality better value on the PDB Model.

From better calculation result of the PDB Model we also concluded that the process of warming (cooling) must involve the interaction, as well as mutual influences between the atmosphere and the surface. Meanwhile, the failure of the PDB Model in determining the relaxation time related to the atmosphere in the second half of the region with \( I_G \neq 0 \) and the failure in determining the relaxation time associated to the surface on most of the region suggests the need for the development of this model through inclusion the role of the other heat transfer mechanism, and put a layer of depth surface as an integral part of the system.

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**References**

1. Giancoli, D.C., *PHYSICS, Principles with Applications, 6th* Edition, Pearson Prentice Hall, Pearson Education Inc. Upper Saddle River NJ, 07458, 2005.
2. https://en.wikipedia.org/wiki/RC_circuit
3. Boas, M.L., *Mathematical Methods in the Physical Sciences, 3rd* Edition, John Wiley & Sons, New Jersey, USA, 2006.
4. Tarantola, A., *Inverse Problem Theory and Methods for Model Parameter Estimation*, SIAM, 2005.
5. O’Sullivan, C.T., “Newton’s Law of Cooling-A Critical Assessment”, *Am. J. Phys. 58 (10): 956-960*, 1990.
6. Vollmer, M., “Newton’s Law of Cooling Revisited”, *Eur. J. Phys. 30: 1063-1084*, doi: 10.1088/0143-0807/30/5/013, 2009.
7. Twomey P, C. O’Sullivan & J. O’Riordan, “An Experimental Investigation of The Role of Radiation in Laboratory Bench-Top Experiments in Thermal Physics”, *Eur. J. Phys.* 30 559–66, 2009.
8. Besson, U., “Cooling and Warming Laws: An Exact Analytical Solution”, *Eur. J. Phys. 31: 1107-1121*, doi: 10.1088/0143-0807/31/5/013, 2010.
9. Battaglia O.R., C. Fazio, N. Pizzolato, & R.M.Sperandeo-Mineo, “An Investigation of Environmental Temperature Effects on Energy Exchange by Thermal Radiation”, *Am. J. Phys.* 81 (12), 923-928, 2013.
10. Arsali, O.C. Satya, & Supardi, “Determination of System Dynamic Characteristics Based on A Serial RC Circuit Model”, paper on the *International Conference of Mathematics, Science, Education, and Technology*, Padang State University, Padang, 2015.
11. http://www1.ncdc.noaa.gov/pub/data/uscrn/products/subhourly01
12. Arsali, O.C. Satya, & S. Hamdi, “Uji Formula Brutsaert-Crawford pada Perhitungan Radiasi Gelombang Panjang Atmosfer”, Proceeding of *Simposium Fisika Nasional (SFN) XXVII*, Universitas Udayana, Denpasar-Bali, 2014
13. Arsali, O.C. Satya, & S. Hamdi, “Perlakuan Mirip Koreksi Cosinus pada Model Parametrik Permukaan untuk Perbaikan Estimasi Radiasi Balik Gelombang Panjang Atmosfer”, Paper and Poster on *Seminar Nasional Sains Atmosfer (SNSA) 2015*, LAPAN, Bandung, 2015.