Assessment of the actual depth of cutting with cylindrical external grinding

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Abstract. Article presents the correlations that make it possible to use the results of measurements of coordinates in the process of machining a part in order to build estimates of the actual depth of cut while machining the part. Such estimates are optimal assuming the gaussianity of the noise of measurements and excitations, and can be used directly to solve process control problems.

High quality of product processing can only be ensured by using reliable data on the actual cutting depth and trends of its change during workpiece processing. These values characterize the output parameters of grinding process and, therefore, are subject to control and evaluation [1]. However, direct and indirect measurements of these parameters during processing are quite complicated due to relative smallness of the changes in the geometrical characteristics of the part associated with them, impossibility of measuring such characteristics directly in the contact zone and presence of significant errors in the measurements of each individual parameter.

In the course of solving this problem, it is necessary to take into account both the dynamic properties of the machining process and the statistical properties of measurement errors (Fig.1).

![Equivalent grinding process design](image)

Figure 1. Equivalent grinding process design

There is a possibility of measuring coordinates $y_1, y_2$ (coordinates of the wheel’s and part’s centers) with errors $\nu_1, \nu_2$ that are directly determined by the parameters of the meters and can be characterized by independent Gaussian white noises with zero averages and covariance matrix $\Psi_v$. 
\[
\begin{align*}
\ddot{y}_1 &= y_1 + v_1, \\
\ddot{y}_2 &= y_2 + v_2.
\end{align*}
\] (1)

Moreover, the cutting depth is defined as
\[
T_f = \ddot{y}_2 - \ddot{y}_1
\] (2)

The system of equations representing a description of the dynamic system behavior, characterizing the cutting process, in deviations from the nominal mode has the following form [2]

\[
\begin{align*}
m_1 \ddot{y}_1 + (h_1 + h_3) \dot{y}_1 + (c_1 + c_3) y_1 - c_3 y_2 - h_3 \dot{y}_2 + h_3 (\Delta \dot{r}_0 + \Delta \dot{r}_k) + c_3 (\Delta r_0 + \Delta r_k) &= 0, \\
m_2 \ddot{y}_2 + (h_2 + h_3) \dot{y}_2 + (c_2 + c_3) y_2 - c_3 y_1 - h_3 \dot{y}_1 - h_3 (\Delta \dot{r}_0 + \Delta \dot{r}_k) - c_3 (\Delta r_0 + \Delta r_k) + c_2 s + h_2 s &= 0,
\end{align*}
\] (3)

where \( y_1, y_2 \) — are the deviations of the coordinates of the part’s and the wheel’s center; \( \Delta \dot{r}_0, \Delta \dot{r}_k \)
and \( \Delta \dot{r}_0, \Delta \dot{r}_k \) are the deviations of the shape of the part and the wheel from the nominal and the derivatives of the specified parameters; \( s, \dot{s} \) – movements and motion that compensate deviations; \( c_1, h_1, m_1 \) – unit stiffness, damping and mass of the part; \( c_2, h_2, m_2 \) – unit stiffness, damping and mass of the wheel; \( c_3, h_3 \) – equivalent stiffness and damping for the grinding process model.

In the state space, the matrix form of system (1), (2), (3) takes the following form:

\[
\begin{align*}
\dot{X}_o &= A_o \cdot X_o + B_o \cdot U_o + E_o \cdot R_o, \\
Y_o &= C_o \cdot X_o + F_o \cdot V_o, \\
T_o &= Q_o \cdot Y_o
\end{align*}
\] (4)

where

\[
X_o = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad A_o = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -c_1 -c_3 & -h_1 -h_3 & c_3 & h_3 \\ m_1 & m_1 & m_1 & m_1 \\ 0 & 0 & 1 & 0 \end{bmatrix}
\]

- the matrix of derivatives of the state of the dynamic system (1), the matrix of the state of the system and the matrix that determines the dynamic properties of the same system,

\( x_1 = y_1; \quad x_2 = y_2 \) – coordinates of the deviations of the part and wheel centers,

\( x_3, x_4 \) – derivatives of the specified coordinates, respectively;

\[
B_o = \begin{bmatrix} B_{10} \\ B_{20} \end{bmatrix}, \quad B_{10} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ m_2 & m_2 \end{bmatrix}, \quad B_{20} = \begin{bmatrix} c_2 \\ h_2 \\ m_2 \\ m_2 \end{bmatrix}
\]

- the block matrix, the matrix of blocks of control coefficients; the matrix of controls of a dynamic system; the matrix of impact coefficients due to deviation of the shape of the part from the nominal shape, and the matrix of the specified deviations of the form, respectively;

\[
C_o = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad F_o = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}, \quad V_o = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \quad T_o = [\mathbf{f} \cdot Q_o = \begin{bmatrix} 1 & 1 \end{bmatrix}]
\]
the matrix of measurements, the matrix of intensities of measuring instruments noise; the matrix of independent Gaussian white noises of unit intensity measuring instruments, the matrix of the depth of cut, and the matrix of aggregate measurements conversion.

However, the depth of cut so estimated includes some errors determined by the quality of the measurement process through the noise of the measuring instruments (1), and from the influence of random components deviating the shapes of the part and the wheel from the nominal ones, which complicates the practical application of the constructed dependencies. Thus, obtaining the speed of the cutting process \( t_f \) via the differentiation of the depth of cut \( t_f \) determined in accordance with the relations (2), is impractical due to the presence of additive noise in the aggregate measurements. To solve the problem of determining the speed of the cutting process, it is advisable to construct an observation system in the form of a Kalman filter [3].

However, in this case as well, the direct application of the model (4) to construct an observer is impractical due to the necessity to differentiate the control signal \( s \), which, with the noise in the control channel and computational errors during differentiation included, can lead to significant difficulties in the application of this approach. To eliminate this drawback, it is possible to represent the product of the \( B_o \), \( U_o \) matrices of system (4) in the following form:

\[
B_o \cdot U_o = B_o \cdot s + B_o \cdot s = \begin{bmatrix} O \\ O \\ c_2 \\ m_2 \end{bmatrix} \cdot s + \begin{bmatrix} O \\ O \\ h_2 \\ h_2 \end{bmatrix} \cdot s
\]

(5)

and to reorganize the system model (4) in such a form that does not require any differentiation of the control signal. However, to get the state reconstruction for (1), (2) that are to be controlled, it is also necessary to transform the equations of the system observation. Such transformations can be performed, for instance, with the use of the methodology [4], which allows obtaining the following:

\[
\begin{align*}
\dot{Z}_o &= A_o \cdot Z_o + B_o \cdot s + E_o \cdot R_o \\
Y_o &= C_o \cdot Z_o + D_o \cdot s + F_o \cdot V_o \\
T_o &= Q_o \cdot Y_o
\end{align*}
\]

(6)

where the matrices \( A_o, E_o, R_o, Y_o, C_o, F_o, V_o, T_o, Q_o \) coincide with the same matrices in correlations (4),

\[
\begin{align*}
Z_m &= \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}, & \dot{Z}_m &= \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix}, & B_m &= \begin{bmatrix} 0 \\ \frac{h_3 h_2}{m_1 m_2} \\ \frac{m_1 m_2}{(h_3 + h_2) h_2} - c_2 \\ \frac{m_2^2}{m^2} \end{bmatrix}, & D_m &= \begin{bmatrix} 0 \\ \frac{h_2}{m_2} \end{bmatrix}
\end{align*}
\]

Each of the deviation of the form from the nominal \( \Delta r_o, \Delta r_i \) and their derivatives are random in nature and can be characterized by the second-order Gaussian-Markov random processes with the correlation functions of the following form:

\[
K_i(v, \tau) = K_{0i} \cdot \exp(\alpha_i v|\tau|) \cdot \cos(\beta_i v, \tau),
\]

(7)
where \( v_r \) - peripheral speed of the wheel or the part, respectively, \( K_0, \alpha, \beta_i \) - parameters of the correlation function which can be determined in an experimental way from the partial fraction expansion of the power spectral density.

It can also be shown that in any important practical cases, the effects caused by the cross-correlation functions of the presented processes and the corresponding power cross-spectral densities are small in relation to the power spectral characteristics of the wheel and the part.

For such random processes, the expression of a forming filter in [5] allows to represent the third addend of the first equation of system (6) with equivalent parameters and variables of the state equations of the generating filters, as follows:

\[
G = A_f \cdot G + B_f \cdot W
\]

\[
R = C_f \cdot G
\]

\[
H_f = Q_f \cdot R
\]

where

\[
G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix},
G = \begin{bmatrix} G_1' \\ G_2' \end{bmatrix},
A_f = \begin{bmatrix} A_{f1} \\ A_{f2} \end{bmatrix},
B_f = \begin{bmatrix} B_{f1} \\ B_{f2} \end{bmatrix},
W = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix},
C_f = \begin{bmatrix} C_{f1} & 0 \\ 0 & C_{f2} \end{bmatrix},
\]

\[
\bar{G}_f = \begin{bmatrix} \bar{g}_{1f} \\ \bar{g}_{2f} \end{bmatrix},
G_f = \begin{bmatrix} \bar{g}_{1f}' \\ \bar{g}_{2f}' \end{bmatrix},
A_{fi} = \begin{bmatrix} 0 & T_{fi} \\ T_{fi} & 0 \end{bmatrix},
B_{fi} = \begin{bmatrix} k_{fi} \cdot T_{3i} \\ -k_{fi} \cdot T_{3i} \cdot T_{2i} \end{bmatrix},
C_{fi} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},
i \in \{\kappa, \delta\},
\]

\[
H_f = \begin{bmatrix} \Delta \bar{g}_{1f} + \Delta \bar{g}_{2f} \\ \Delta \bar{g}_{1f} + \Delta \bar{g}_{2f} \end{bmatrix},
Q_f = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix},
\Psi_w = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},
\]

\[
T_u = \frac{1}{v_i \left( \alpha_i^2 + \beta_i^2 \right)},
T_{ul} = \frac{2\alpha_i}{v_i \left( \alpha_i^2 + \beta_i^2 \right)},
T_u = \frac{1}{v_i \left( \alpha_i^2 + \beta_i^2 \right)},
k_f = \frac{2K_0 \alpha_i}{v_i \left( \alpha_i^2 + \beta_i^2 \right)}.
\]

\( g_{1f}, g_{2f}, g_{1u}, g_{2u} = \bar{g}_{2f} \) - auxiliary states of the generating filter; \( w_1, w_2 \) - independent of each other and measurement noise; \( V_o \) - white noise of unit intensity.

With the consideration of (8), the model of system (6) takes the following form:

\[
\begin{bmatrix} Z_m \\ \bar{G}_f \end{bmatrix} = \begin{bmatrix} A_o & E_o \cdot Q_f \\ 0 & A_f \end{bmatrix} \cdot \begin{bmatrix} Z_m \\ G_f \end{bmatrix} + \begin{bmatrix} B_m \\ 0 \end{bmatrix} \cdot s + \begin{bmatrix} 0 \\ B_f \end{bmatrix} \cdot W
\]

\[
Y_o = C_o \cdot Q_o \cdot \begin{bmatrix} Z_m \\ G_f \end{bmatrix} + D_m \cdot s + F_o \cdot V_o
\]

\[
T_o = Q_o \cdot Y_o
\]

where, in addition \( Q_f = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \).

It is advisable to show the first and second equations of system (9) in the following form:
\[ X = A \cdot X + B \cdot U + E \cdot W \]
\[ Z = C \cdot X + F \cdot V \]

(10)

The type and structure of matrices in correlations (10) are univalently determined by system (9).

For system (10), it is possible to construct the system state estimates, that are optimal in the mean square, in the form of a Kalman filter [3].

The minimum achievable variance of the system (10) state estimates can be obtained by solving the Riccati matrix equation of the following form:

\[ \hat{V} = A \cdot \hat{V} + \hat{V} \cdot A + E \cdot \Psi_w \cdot E^T - \hat{V} \cdot C^T \cdot \Psi_v^{-1} \cdot C \cdot \hat{V}, \]

(11)

which is solved before the start of the processing of a specific part due to the fact that it does not contain any results of observations of a dynamic system.

The Kalman filter gain matrix is determined by the following system of equations:

\[ K = \hat{V} \cdot C \cdot \Psi_v^{-1} \]

(12)

With consideration of (11), (12), the observation filtering algorithm is determined by the following matrix equations:

\[
\begin{bmatrix}
\hat{Y} \\
\hat{t_f}
\end{bmatrix}
= \begin{bmatrix}
-1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\hat{Y} \\
\hat{t_f}
\end{bmatrix}
\]

(13)

Correlations (12), (13) make it possible to use the results of measurements of coordinates in the process of machining a part in order to build estimates of the actual depth of cut while machining the part. Such estimates are optimal assuming the gaussianity of the noise of measurements and excitations, and can be used directly to solve process control problems.

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