Improving the ‘self-tuning’ mechanism with a Gauss-Bonnet term

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Abstract. The effects of higher order gravity terms on a dilatonic brane world model are discussed [1]. For a single positive tension flat 3-brane, and one infinite extra dimension, we present a particular class of solutions with finite 4-dimensional Planck scale and no naked singularities. A ‘self-tuning’ mechanism for relaxing the cosmological constant on the brane, without a drastic fine tuning of parameters, is discussed in this context.

1 Dilatonic brane worlds

There has been a great deal of interest in ‘brane world’ models, in which matter and fundamental gauge interactions are localized on a four-dimensional spacetime surface or 3-brane, while gravity is free to propagate in the higher dimensional bulk spacetime [2]. In such models, a fine tuned relation between the bulk curvature and the brane tension has to be specified in order to switch off the effective cosmological constant on the brane. Without any specific dynamical mechanism to justify it, this fine-tuning may be seen as a new version of the cosmological constant problem in this context.

This was the starting point for a series of efforts aimed at resolving this fine tuning problem by means of a dynamical mechanism, called ‘self-tuning’ [3]. A static scalar field, φ, which loosely models the dilaton and moduli fields of string theory, is added to the bulk. The extra degree of freedom is then used to ensure the existence of a solution of the dynamical equations with a zero effective brane cosmological constant, whatever the value of the brane tension. Typically a conformally flat solution of the form

$$ds^2 = e^{2A(z)}\eta_{\mu\nu}dx^\mu dx^\nu + dz^2,$$

with

$$A(z) \propto \ln \left( 1 + \frac{|z|}{z_*} \right), \quad \phi(z) \propto \pm \ln \left( 1 + \frac{|z|}{z_*} \right),$$

is used (we have assumed a $\mathbb{Z}_2$-symmetric bulk). Significantly, the constant $z_*$ is undetermined by the bulk field equations. The boundary conditions relate $z_*$ to the brane tension, $T$. Since $z_*$ is arbitrary, these can be satisfied for any value of $T$, and so the brane tension does not need to be fine tuned.

There are several problems with this mechanism. For example, why should one value of $z_*$ be favoured over another, and is the solution stable? A dynamical analysis of the system is required to address these issues.
However, the model has a far more serious flaw. By integrating over the fifth dimension we obtain an effective four dimensional theory on the brane. The effective Planck mass is then

\[ M_{\text{Pl}}^2 = M_s^3 \int_0^{z_{\text{max}}} dz e^{2A}, \]  

where where \( z_{\text{max}} \) is the maximum value of \( z \), so \( z_{\text{max}} = \infty \) if \( z_\ast > 0 \), and \( z_{\text{max}} = |z_\ast| \) if \( z_\ast < 0 \). For \( z_\ast > 0 \) it is obvious that \( M_{\text{Pl}} \) is never finite. Alternatively if we choose \( z_\ast < 0 \), \( M_{\text{Pl}} \) is finite, but the curvature diverges as \( z \to \pm z_\ast \). Thus no solution of the form (2) is acceptable. A similar problem occurs in a wide range of conformally flat dilatonic brane world models [4].

2 A higher order gravity tensor

In four dimensions, the vacuum field equations for gravity are taken to be \( G_{ab} + \Lambda g_{ab} = 0 \) since this is the most general tensor which (a) is symmetric, (b) depends only on the metric and its first two derivatives, (c) is divergence free, and (d) is linear in second derivatives of the metric.

But in five dimensions there is another possibility. Variation of an action containing the Gauss-Bonnet term,

\[ L_{\text{GB}} = R^2 - 4R_{ab}R^{ab} + R^{abcd}R_{abcd}, \]  

(4)

gives the second order Lovelock tensor [5]

\[ H_{ab} = (RR_{ab} - 2R_{ac}R^c_b - 2R^{cd}R_{acbd} + R_a^{cde}R_{bcde}) - \frac{1}{4}g_{ab}L_{\text{GB}}, \]  

(5)

which also satisfies the above four conditions.

Thus, in the absence of any evidence to the contrary, we should take the five dimensional vacuum field equations to be \( G_{ab} + 2\alpha H_{ab} + \Lambda g_{ab} = 0 \), where \( \alpha \) and \( \Lambda \) are constants.

A further motivation for higher order curvature terms is that they also appear in the low energy effective field equations arising from most string theories [6]. Since brane worlds are motivated by string theories, it is particularly natural to include the extra terms in the five-dimensional field equations. In this case we expect \( \alpha \sim \Lambda^{-1} \sim M_s^{-2}. \)

We will consider a string theory inspired model with the bulk action

\[ S_5 = \frac{M_5^3}{2} \int d^5x \sqrt{-g} \left\{ R - \frac{4}{3}(\nabla \phi)^2 + \alpha e^{-4\phi/3} \left[ L_{\text{GB}} + \frac{16}{27} (\nabla \phi)^4 \right] - 2\Lambda e^{4\phi/3} \right\}, \]  

(6)

The corresponding action in the string frame, which is related to (6) by a conformal transformation, is

\[ S_{\text{string}} = \frac{M_5^3}{2} \int d^5x \sqrt{-\bar{g}} e^{-2\phi} \left\{ \bar{R} + 4(\nabla \phi)^2 + \alpha \left[ \bar{L}_{\text{GB}} + \cdots \right] - \bar{\Lambda} \right\}. \]  

(7)
Note that the $R$, $\mathcal{L}_{\text{GB}}$, and $\Lambda$ terms have the same couplings to $\phi$ in the string frame. However, since they have different dependencies on $g_{ab}$, this is no longer true in the Einstein frame.

The brane contribution to the action, including boundary terms \cite{7, 8} is

$$S_4 = -M_s^4 \int d^4 x \sqrt{-h} \left\{ [K] + \alpha e^{-4\phi/3}[\mathcal{L}_B] + T e^{2\phi/3} \right\}, \quad (8)$$

where

$$\mathcal{L}_B = 2K K_{ac} K^{ac} - \frac{4}{3} K_{ac} K^{cb} K^a_b - \frac{2}{3} K^3 - 4G^{(4)}_{ab} K^{ab}, \quad (9)$$

$h_{ab}$ is the induced metric on the brane, and $K_{ab}$ is the extrinsic curvature.

### 3 A non-singular solution

Variation of generalised boundary term (8) gives the junction conditions on brane \cite{8}. Note that the resulting expression has no dependence on the thickness of the brane. This would not be the case for any other second order combination of $R_{abcd}$.

The bulk field equations are rather complicated, but for a conformally flat solution (1) with the logarithmic ansatz \cite{1, 9}

$$A(z) = x \ln \left( 1 + \frac{|z|}{z_*} \right), \quad \phi(z) = \phi_0 - \frac{3}{2} \ln \left( 1 + \frac{|z|}{z_*} \right), \quad (10)$$

they all reduce to algebraic equations

$$\frac{2\alpha}{z_*^2} = \frac{1 - x}{1 - 6x^3 - 4x^2 e^{4\phi_0/3}} > 0 \quad (11)$$

$$\Lambda\alpha = \frac{-3(-40x^5 + 24x^4 - 52x^3 + 16x^2 + 3x - 1)(1 - x)}{8(1 - 6x^3 - 4x^2)^2}. \quad (12)$$

The expression for the effective four dimensional Planck mass now includes additional $\alpha$ dependent corrections, but it is qualitatively similar to expression (3). To obtain a finite $M_{\text{Pl}}$ when $z_* > 0$ (i.e. no singularities) we need to find solutions with $x < -1/2$.

Using figure 1, which is a plot of (12), we see that non-singular solutions with localised gravity exist if $-22.2 \lesssim \Lambda\alpha < -5/12$ \cite{1}. We expect $\Lambda\alpha \sim -1$, so this is quite natural. If we set $\alpha = 0$, we see that the only solution is $x = 1$, which has infinite $M_{\text{Pl}}$. Thus the Gauss-Bonnet term has not removed the singularity from the $\alpha = 0$ solutions, but has instead produced a new branch of solutions.

Using the junction conditions we also obtain an algebraic expression for the brane tension

$$T = \frac{(-x)(3 - 12x - 2x^2 - 16x^3)\sqrt{1 - x}}{\sqrt{2\alpha(1 - 4x^2 - 6x^3)^{3/2}}} \text{sgn}(z_*) \quad (13).$$
Thus $T > 0$ for $x < 0$ when $z_\ast > 0$, so our non-singular, finite $M_{Pl}$ solutions do work for models with positive tension branes. Unfortunately these solutions are not suitable for the ‘self-tuning’ mechanism, since the value of $T$ is uniquely determined by the value of $\Lambda$, and so the solution requires fine-tuning after all. It may be that we need to use a different potential, or it could be that the mechanism has some flaw which was previously obscured by the singularity. Further work is required to determine the precise nature of the problem.

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