Calculation of evaporation length of a liquid bridge flowing between inclined hot tubes

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Abstract. The bridge consists of liquid held by surface tension forces between two inclined tubes in an LNG heat exchanger. The shape of the bridge is calculated by the hydrostatic equation, which is reduced to a nonlinear integral equation and resolved by the Newton method. The velocity and temperature fields in the bridge are described by the Navier-Stokes and energy equations, respectively. They are reduced to the boundary integral equations and calculated by the method of boundary elements. Heat transfer coefficient is calculated for evaporating bridge and the length of total bridge evaporation is estimated.

1. Bridge shape and velocity of liquid in flowing down bridge
The cooling mixture flows down the spiral tubes of the LNG heat exchanger first as a continuous film of liquid, but since the film evaporates, at the bottom of the column the film disappears and the liquid starts to flow in the form of bridges sandwiched between the tubes. Due to the inclination of the tubes to the horizon at an angle $\gamma$, the liquid in the bridge flows along the axis of the tubes. The paper considers heat transfer coefficient in such flow and length of bridge flow zone. Various bridge shapes and liquid velocities in flowing down bridges were calculated in our paper [1]. Here we shortly describe the algorithm of these calculations.

Let us introduce the dimensionless values for the bridge problem: the coordinates $x$ and $y$, the radius of tubes $r$, the free surface coordinate $\delta$, and the gap between the tubes $d$:

$$x = \frac{X}{\Lambda}, \quad y = \frac{Y}{\Lambda}, \quad r = \frac{R}{\Lambda}, \quad \delta = \frac{\Delta}{\Lambda}, \quad d = \frac{D}{\Lambda},$$

where $\Lambda = \sqrt{\sigma / \rho g}$ is the characteristic Laplace scale ($\sigma$ is the surface tension). It is convenient to set the contact points by angles $\alpha$ and $\beta$ (as it is shown in Fig.1 for parameters $r = 1.5$, $d = 1$, $\theta = 20^\circ$): $x_a = r \sin \alpha$, $y_a = \frac{d}{2} + 2r \sin \frac{\alpha}{2}$, $y_\beta = -\frac{d}{2} - 2r \sin \frac{\alpha}{2}$.

Figure 1. Bridge. In the limited formulation and without heat exchange analysis, similar bridge flows have been considered in [2, 3].

From hydrostatics equation after integration we obtain the differential equation describing the free surface of liquid:
\[
\delta = x_\beta + \int_{y_\alpha}^{y_\beta} \frac{\sin \phi(y) \, dy}{\sqrt{1 - \sin^2 \phi(y)}}
\]

\[\sin \phi(y) = \frac{(y - y_\alpha) \cos(\alpha + \theta) + (y - y_\alpha) \cos(\beta + \theta)}{h} + \frac{(y_\alpha - y)(y - y_\beta)}{2},\]

\[h = d + 2r\left[\sin^2(\alpha/2) + \sin^2(\beta/2)\right].\]

The complete integral (3) gives an additional equation, connecting unknown values

\[x_\alpha - x_\beta = \int_{y_\alpha}^{y_\beta} \frac{\sin \phi(y) \, dy}{\sqrt{1 - \sin^2 \phi(y)}},\]

where all coordinates of the contact points are determined by formulas (2).

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**Figure 2.** Examples of free surfaces of bridges: \(d = 1, \ \theta = 20^\circ;\) a) \(r = 1\) and b) \(r = 2.\)

Equation (4) connects two angular coordinates of contact points \(\alpha\) and \(\beta\) with the parameters given in the problem: contact angle \(\theta\), inter-tube distance \(d\), and tube radius \(r\). Choosing some angle \(\alpha\) and solving equation (4) by the Newton method, we obtain the second angle \(\beta\) as a function of \(\alpha\) and above-listed parameters. Using the calculated value \(\beta(\alpha)\), we calculate the free surface by equation (3). In Fig. 2 several free surfaces of bridges are shown. In our paper [1] we have demonstrated bridges of various shapes for several sets of parameters \(\theta\), \(d\) and \(r\).

The dimensionless velocity is introduced as \(w = W/W_r\), where \(W_r = \sigma \sin \gamma / \eta \cos \gamma\) (\(\eta\) is the liquid viscosity, \(\gamma\) is the angle between tube axes and horizon). This velocity is described by the dimensionless Navier-Stokes equation, taking the form of the Poisson equation. The dimensionless velocity field is presented as sum \(w = v + \psi\), where \(v\) is the general solution to Laplace equation and function \(\psi = -(x^2 + y^2)/4\) is a partial solution of Poisson equation (see [1]). Let us introduce the fundamental solution of the Poisson equation \(G = -(2\pi)^{-1} \ln(|\xi - \xi'|/2)\). We apply the Green theorem to the Laplace equation and obtain for value \(v\) the boundary integral equation [1, 4]:

\[\frac{1}{2}v(\xi) = \int_\Gamma \left(G(\xi - \xi') \frac{\partial v(\xi')}{\partial n_{\xi'}} - v(\xi') \frac{\partial G(\xi - \xi')}{\partial n_{\xi'}}\right) \, dl_{\xi'}\]

where \(\Gamma\) is the bridge contour; coordinates \(\xi\) (observation points) and coordinates \(\xi\) (integration points) are laying on the contour.

From the boundary conditions for velocity \(w\) and its normal derivative, we obtain the boundary conditions on the surfaces of tubes and on the free surface of the liquid (\(\Sigma\)), respectively, \(v|_{\Sigma} = -\psi|_{\Sigma}\).
\[ \frac{\partial v}{\partial n} = -\frac{\partial v}{\partial n} . \] These quantities are substituted into integral equation (5). Unknown values of normal derivative \( \frac{\partial v}{\partial n} \) on the surfaces of tubes and function \( v \) itself on the free surface of the liquid \( (\Sigma) \) are determined from the system of linear equations resolved by boundary element method (BEM). Equation (5) is the basic boundary integral equation in the Boundary Element Method [4].

2. Temperature field in the bridge

Bridge evaporation is a weak process. It will be shown later that the length of total bridge evaporation is very large. Therefore we can ignore the convection term in the equation for temperature field, and describe temperature \( T \) only by heat conductivity effects via the Laplace equation

\[ \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) T = 0 \]  

(6)

with boundary conditions: a) \( T = T_s \) on the free surface, contacting with vapor; b) \( \partial T / \partial n = 0 \) at the axis of symmetry \( x = 0 \); c) \( -\lambda \partial T / \partial n = \lambda_s T_s / \delta_s \) on the tube surface, where \( T_s \) is the saturation temperature, \( T_s \) is the temperature of the flow inside the tube, \( \delta_s \) is a thickness of tube wall, \( \lambda \) is the thermal conductivity of liquid in the bridge and \( \lambda_s \) is the thermal conductivity of metallic tube. The boundary condition of the third kind is used here on the tube surface to avoid the divergence problem in heat flux density at the three-phase contact point.

Equation (6) is transformed to a boundary integral equation like it was done above for the velocity problem. Let us introduce the dimensionless temperature \( t = (T_s - T) / (T_t - T_s) \) and dimensionless coordinates \( x, y \). For temperature \( t \), the following boundary integral equation can be derived

\[ \frac{1}{2} t(\xi) = \int_{\Gamma} \left( G(\xi - \bar{X}) \frac{\partial t(\bar{X})}{\partial n} - t(\bar{X}) \frac{\partial G(\xi - \bar{X})}{\partial n} \right) d\bar{l} , \]

(7)

where the observation \( (\xi) \) and integration points \( (\bar{X}) \) are both placed on the bridge contour \( \Gamma \).

Boundary conditions in the dimensionless form are as follows: a) on the free boundary of the bridge \( t = 1 \); b) on the symmetry axis \( x = 0 \) \( \partial t / \partial n = 0 \); c) on the tube surface \( \partial t / \partial n = C \cdot t \), where the parameter of conjugate heat transfer is \( C = \Lambda / \delta_s \lambda \).
Boundary integral equations (5) and (7) can be resolved by the Boundary Element Method [4], which is briefly described below. The boundary integral equations contain the functions and their normal derivatives, which are partially specified on the contour $\Gamma$ via boundary conditions a), b), c). These known quantities are substituted into integral equations (5), (7), and unknown values of normal derivatives and functions are determined from the system of linear equations. We obtain this system of equations by dividing contour $\Gamma$ into $N$ small linear segments and integrating integrals (5), (7) over them. In our calculations, $N$ was up to $N=1000$. During integration, the unknown values are considered as constant along the elements. Integrals along the elements of function $G$ and its normal derivative can be calculated analytically. The resulting system of equations is solved by the Gauss method. BEM belongs to a class of direct (not iterative) methods of solution to the elliptic problems of a potential theory.

In Fig. 3 for some bridge contour (a), velocity $w$ (b), temperature $t$ (c), and normal derivative of temperature $t'$ (d) versus the dimensionless length of the path along the contour.

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In Fig. 3 for some bridge contour (a), velocity $w$ (b), temperature $t$ (c), and normal derivative of temperature $t'$ (d) are shown vs. $l$, the length of the path along the contour. The path goes through points A, B, C, D, and along the free surface returns to point A (see Fig.3a): the arcs A-B and C-D are the boundaries of bridge contacting with tube walls. Parameters are: $d=1$; $r=2$; $\theta=20^\circ$. Note the very large values of the dimensionless temperature gradient at contact points A and D. Therefore we use very small steps along the contour and it leads to a large number of control points on the contour $N=1000$. But the built BEM-based algorithm is so effective that calculation of solution for the velocity and temperature fields on the contour takes only a few seconds. Therefore we can calculate the effect of all parameters in detail.
3. The bridge evaporation length and Nusselt criterion

While moving along z axis, the liquid bridge is evaporating permanently. The balance equation for the heat of liquid mass, evaporated from the free surface, supplied through a wall, for length $dz$ takes the form $dQ = \frac{F dz}{\Delta h}$, where $Q$ is a mass flow rate of liquid in the bridge, $F$ is a heat flux through the bridge (per a unit of a tube length), $\Delta h$ is the specific heat of liquid evaporation (difference between vapor and liquid enthalpies). The heat flux $F$ is calculated by the integral over wetted surfaces of tubes and contains two parts $F = -\lambda \Delta TR \left\{ \alpha \int_0^t \alpha + \frac{\beta^t}{\gamma} \right\}$ where the temperature gradient $t = \partial t / \partial n$ is calculated by BEM.

Since the local values in any cross-section of the bridge are calculated, the length of bridge evaporation can be obtained as $Z_\ast = \Delta h \sqrt{\frac{dQ}{F}}$, where $Q_\ast$ is the dimensional limiting mass flow rate, for the bridge that is formed just after the disappearing of the film flow. At a small amount of liquid flowing in the film, the film flow stops and only bridge flow can exist. At the transition from film to bridge flow the shape of liquid is transformed from an absolutely wetting bridge (with $\theta = 0^\circ$) to a bridge with a given contact angle (f.e. $\theta = 20^\circ$). Therefore we can calculate the limiting value $Q_\ast$ as the maximal flow rate for the case $\theta = 0^\circ$. Then we can find the limiting angle $\alpha_\ast$, corresponding to this dimensionless rate $q_\ast$.

In the dimensionless form, we obtain

$$ \frac{Z_\ast}{A} = \frac{\alpha \gamma \nu R K a}{\nu R K a} f(d, R, \theta),$$

where $\gamma$ is the angle of tube inclination relative to the horizon; Prandtl, Kutateladze, and Kapitza criteria are also introduced as: $Pr = \frac{\eta C_p}{\lambda}$; $K_u = \frac{\Delta h}{C_p \Delta T}$; $Ka = \frac{\sigma^2 \rho}{\eta^4 \rho \cos \gamma}$, where $\eta, \lambda, \sigma, C_p, \Delta h$ are viscosity, heat conductivity, surface tension, specific heat capacity, and latent heat of vaporization, respectively. Function $f$ is the dimensionless integral, calculated numerically

$$ f(d, R, \theta) = \int_0^{q_\ast} \left( \int_0^t \alpha + \frac{\beta^t}{\gamma} \right) dq.$$  

Calculated values of this function are small: $f = (1..5) \times 10^{-3}$. The inner integrals are calculated by the surfaces of both tubes wetted by the bridge (with angles $\alpha$ and $\beta$), value $q_\ast$ is the characteristic flow rate of liquid, explained above.

The total heat, evaporating in the bridge is $\Delta h Q_\ast$. The total heat transfer coefficient is $H_{tot} = \frac{\Delta h Q_\ast}{Z_\ast R \Delta T}$ (heat flux is not divided here to the wetted surface, but is referred to full tube surface $\pi R Z_\ast$). Nusselt criterion is defined as $Nu = H_{tot} 2R / \lambda$ and after transformation one can obtain

$$ Nu = \frac{2 q_\ast}{\pi f(d, R, \theta)}.$$  

No reliable data about contact angles can be found in the literature for cryogenic temperatures. Therefore we will calculate results for the several contact angles $\theta$ ($1^\circ, 5^\circ, 10^\circ, 20^\circ, 30^\circ$).
Figure 4. Nusselt criterion vs. distance \( d \): a) Al tubes of radii \( r=1 \) and 4; b) for two contact angles and \( r=2.5 \): Red lines with triangles show results for Al tubes (\( \delta_i = 0.8 \) mm); black lines demonstrate \( \text{Nu} \) for stainless steel tubes (\( \delta_i = 0.5 \) mm). Dashed lines show \( \text{Nu}_{\text{theor}} \) calculated according to formula (12).

In Fig.4, a Nusselt criterion is shown vs. gap distance \( d \) for two radii \( r \) and four contact angles (Al tubes and \( \delta_i = 0.8 \) mm are assumed). Note that from Fig.4 it is seen that the Nusselt criterion is almost independent on radius \( r \) and gap \( d \) and has a strong dependence on the contact angle \( \theta \) and on the parameter of conjugate heat transfer \( C = \Lambda \lambda_j / \delta_i \lambda \). Therefore a simple model for calculating the heat flux between feed flow with temperature \( T_f \) and ambient vapor temperature \( T_v \) can be developed and the final approximating formula for the Nusselt criterion was obtained

\[
\text{Nu}_{\text{theor}} = \frac{4}{\pi \theta} \left( 1 + \frac{d^2}{10} \right) \ln \left\{ 1 + \theta \left[ \frac{\theta}{C^{\frac{1}{3}}} + 0.08 \right] \right\} .
\]

In Fig. 4,b the Nusselt criterion calculated according to this simple model is compared with directly calculated values of \( \text{Nu} \) for parameters \( r=2.5 \) and \( \theta = 1^\circ, 20^\circ \). One can conclude, that this formula can catch the main features of the conjugate heat transfer. The large deviation is seen only at very small contact angles (\( \theta = 1^\circ \)) for tubes made of aluminum with very high parameter \( C > 2000 \).

4. Results of calculations

For cooling ethane-methane mixture with a composition of 50\%+50\%, the physical properties at low cryogenic temperatures (-150\...-60 \degree C) can be estimated according to [5] as follow: density is \( \rho = 550 \text{ kg/m}^3 \), viscosity is \( \eta = 2 \cdot 10^{-4} \text{ N} \cdot \text{sec/m}^2 \), surface tension coefficient is \( \sigma = 0.018 \text{ N/m} \), thermal conductivity is \( \lambda = 0.17 \text{ W/(m K)} \), the specific heat of vaporization is \( \Delta h = 400 \text{ kJ/kg} \), specific heat capacity is \( c_p = 2.9 \text{ kJ/(kg K)} \). This gives the capillary length \( \Lambda = 1.81 \text{ mm} \).
The tubes in the LNG heat exchanger according to [6, 7] are made of aluminum with high heat conductivity $\lambda_a = 200 \text{ W/(m K)}$. The wall thickness of the tube is $\delta_t = 0.8 \text{ mm}$. Therefore, the parameter $C$ becomes very large: $C = A \lambda a / \delta C = 2650$. In a new LNG column designed by Linde the spiral tubes are made of stainless steel. In this case, the parameter $C$ takes the value of 336 ($\lambda_s = 16 \text{ W/m K}$, $\delta_s = 0.5 \text{ mm}$). The characteristic temperature difference between feed flow in tube and saturation temperature of the vapor in the shell of LNG column is $\Delta T = T_y - T_f$. According to [6] this temperature difference is about $5^\circ \text{C}$ and the angle of inclination of tubes relative to horizon is $\gamma = 7^\circ$. The criteria in formula (8) have the following values: $Pr=3.3$; $Ku=27$; $Ka^{1/2}=4.5 \times 10^5$. According to paper [6] in the LNG column, the aluminum tubes with a tube diameter of 9.5 mm are used. The dimensionless radius is $r = R / \Lambda = 2.5$. In Fig. 5 the calculated length of bridge evaporation $Z_*$ vs. gap distance $d$ for three tube radii $r$: a) 1; b) 2.5; c) 4 are shown. One can see that the characteristic length $Z_*$ can reach several tens meters. Along the column axes, the bridge domain takes length $Z_* \sin \gamma = 2...3 \text{ m}$ (for aluminum tubes, $r = 2.5$, $\theta = 20^\circ$, $d = 1...3 \text{ mm}$).

In Fig. 5 evaporation lengths for Al and stainless-steel tubes are compared. For stainless steel tubes (solid lines) the length $Z_*$ is larger 1.5 times than for the case of aluminum tubes (dashed lines). Therefore, the domain in the LNG column, occupied by bridge flow in the case of stainless-steel tubing is about 3...4.5 m, which seems relatively large. It is not very good due to the lower total heat transfer coefficient in the bridge flow (in comparison with film flow, [6]).

5. Conclusion

The nonlinear equation for the shape of the bridge is solved by Newton's method, and free surface coordinate is calculated for fixed parameters of the lower contact point, tube radius, tube spacing, and contact angle. The velocity and temperature fields are described in the laminar flow regime by elliptical equations (Navier-Stokes, Laplace), which are reduced to boundary integral equations and calculated by the BEM. It is assumed that the liquid evaporates from the free surface of the bridge into the surrounding vapor. At the surface of the bridge, the temperature is assumed to be equal to the temperature of saturated steam. Heat transfer from hot flow in tubes through the bridge is calculated taking into account the thermal resistance of the tube wall. In the vicinity of contact points, heat flux density becomes very large especially for the case of aluminum tubes with high thermal conductivity. Heat transfer coefficients referring to the wetted surface turn out to be rather high: 1000...4000 (W/m K). However, since the area wetted by the bridge is not large, the heat transfer coefficient referred to the whole surface of the tube is much smaller, 300...800 (W/m K). The estimate for the heat transfer coefficient of the LNG column made in paper [6] is 3...5 times higher than our values. The reason for this discrepancy is that the flow regime in the paper [6] was not laminar bridge flow, but turbulent film flow.

The formula for the bridge evaporation length follows from the balance equation of heat flux and flow rate of evaporating liquid. The evaporation length turns out to be large for cases with large tube spacing ($d>1$). For stainless steel tubes, this evaporation length is 1.5...2.5 times longer than for aluminum tubes, which is due to the lower thermal conductivity of steel in comparison to aluminum. According to the column's height, the size of the area occupied by bridge flow is obtained by multiplication of bridge evaporation length by the sine of slope angle, which in our case (7 Deg.) gives a multiplier of 0.12. We have obtained the estimate of the height of bridge mode area in the column of 3...6 m (for steel tubes), which is quite significant taking into account the total length of the column of 15...20 m [6].

References

[1] Geshev P I. Interfacial Phenomena and Heat Transfer, vol. 5, no. 4, pp. 273-286, 2017.
[2] Alekseenko S V, Kuibin P A. *Bridge flow between inclined cylinders*, Proc. XXV Int. Siberian Seminar, 1996, Novosibirsk, Russia, 19-31.

[3] Maltsev L I, Houghton P A and Kulikov D V. 2016 *Interfacial Phenomena and Heat Transfer* 4, N.4, 269-277.

[4] Brebbia C A, Telles J C F and Wrobel L C. *Boundary element techniques. Theory and applications in engineering*, Berlin: Springer-Verlag, 1984.

[5] Vargaftik N B, Vinogradov Y K and Yargin V S. *Handbook of Physical Properties of Liquids and Gases*. Begell House, 1996. 1358 P.

[6] Fredheim A O, Jorstad O, Owren G, Vist S, Neeraas B O. *Coil, a model for simulation of spiral wound LNG heat exchangers*. Proceedings from World Gas Conference 2000, Nice, June 2000.

[7] Neeraas B O, Fredheim A O, Aunan B. *Int. J. Heat and Mass Transfer* 2004 47 3565–3572.