Probing scalar–pseudoscalar mixing in the CP violating MSSM at high–energy $e^+e^-$ colliders

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Abstract

We study the production processes $e^+e^- \rightarrow H_i^0Z$, $H_i^0H_j^0$ and $H_i^0\nu_e\bar{\nu}_e$ in the context of the CP violating MSSM. In a given channel we show that the cross-section for all $i (= 1, 2, 3)$ can be above 0.1 fb provided $M_{H_{2,3}} \lesssim 300$ GeV. This should be detectable at a Next Linear Collider and would provide evidence for scalar–pseudoscalar mixing.

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1. INTRODUCTION

The search for the scalar particles referred to as “Higgs bosons” \cite{1} is one of the major goals of present and future colliders. Such particles break the electroweak symmetry and are responsible for the masses of the fermions and bosons. The Standard Model (SM) \cite{2} predicts one neutral scalar ($\phi^0$) while extensions of the SM often predict several scalars, both neutral and charged \cite{3}. Since 1989 the $e^+e^-$ collider LEP has searched for $\phi^0$. In the final run at energies around $\sqrt{s} = 208$ GeV two of the four experiments found excesses in the search for a Higgs boson in association with a Z boson \cite{4}. Although it was agreed that a further run with about 200 pb$^{-1}$ per experiment at a centre–of–mass energy of 208.2 GeV would enable the four experiments to establish a 5$\sigma$ discovery (assuming the signal is genuine) \cite{5}, the extended run was not approved and LEP was consequently shut down. The search for Higgs bosons will continue with Run II at the Tevatron \cite{6} which has a chance of confirming the existence of the Higgs boson in the mass range hinted at by LEP ($\approx 115$ GeV). This region is fairly problematic for the LHC \cite{7} and would require several years searching in the channel $h \rightarrow \gamma\gamma$ to confirm such a light Higgs. A higher–energy $e^+e^-$ collider (NLC) operating at energies $\sqrt{s} \geq 500$ GeV would be an ideal place to perform precision measurements of a Higgs boson \cite{8}, \cite{9}.

Supersymmetric (SUSY) theories, in particular the Minimal Supersymmetric Standard Model (MSSM) \cite{10}, are currently considered as the most theoretically well motivated extensions of the SM. Such models predict a rich Higgs phenomenology. The MSSM contains 2 neutral CP–even scalars ($h^0$ and $H^0$), a CP–odd neutral scalar $A^0$, and a pair of charged scalars ($H^+, H^-$). High–energy $e^+e^-$ colliders offer a very clean environment in which to search for Higgs bosons, and the simplest way to produce a CP–even scalar is in the Higgsstrahlung process $e^+e^- \rightarrow Z^* \rightarrow H^0Z$ and the $W$ boson fusion process $e^+e^- \rightarrow H^0\nu\bar{\nu}e$ \cite{11,12}. The CP–odd $A^0$ possesses no tree–level coupling $A^0VV$ (where $V = Z, W^\pm$) and the other tree–level diagrams contributing to $e^+e^- \rightarrow A^0Z$ are
proportional to the electron mass and consequently negligible\(^1\). Therefore the dominant contribution to \(e^+e^- \rightarrow A^0Z\) is from higher order diagrams. In previous works we calculated the 1–loop induced rate in the context of the CP–conserving THDM \([14]\) and the CP–conserving MSSM \([15]\), finding maximum values of order 0.01 fb and 0.1 fb respectively if \(\tan \beta \geq 2\). The Higgsstrahlung, vector boson fusion, and Higgs pair production mechanisms \((e^+e^- \rightarrow A^0h^0/H^0\) and \(e^+e^- \rightarrow h^0H^0\)), have all been extensively studied (including radiative corrections \([16]–[18]\)) at NLC energies in the context of the MSSM.

In recent years the phenomenology of the MSSM with complex SUSY parameters has received growing attention \([19]–[21], [22]\). Such phases may allow baryogenesis \([23]\), and do not necessarily violate the stringent bound from the non–observation of Electric Dipole Moments (EDMs) \([24], [25]\). The presence of SUSY phases induces mixing between the CP–even and CP–odd scalars, resulting in the mass eigenstates \(H^0_1, H^0_2\) and \(H^0_3\) which are mixed states of CP. This mixing affects their phenomenology at present and future colliders, both in production mechanisms and decay partial widths \([26]\). In this paper we will study the processes \(e^+e^- \rightarrow Z^* \rightarrow H^0_1Z\) and \(e^+e^- \rightarrow H^0_1\nu_e\bar{\nu}_e\) in the context of a NLC. Both mechanisms are mediated by the tree–level couplings \(H^0_iVV\), but their cross–sections have different phase space and \(\sqrt{s}\) dependence. In the CP conserving case one of these couplings would be zero, corresponding to the absence of the coupling \(A^0VV\). We will also study the production of neutral Higgs pairs, \(e^+e^- \rightarrow H^0_iH^0_j\) \((i \neq j)\). In the CP conserving MSSM, only the vertices \(Zh^0A^0\) and \(ZH^0A^0\) exist at tree–level while in the CP violating scenario all three couplings \(ZH^0_1H^0_2, ZH^0_1H^0_3\) and \(ZH^0_2H^0_3\) are generated at tree–level. Therefore an observable signal for all \(i(= 1, 2, 3)\) in a given mechanism \((e^+e^- \rightarrow ZH^0_1, e^+e^- \rightarrow H^0_1H^0_j\) or \(e^+e^- \rightarrow H^0_1\nu_e\bar{\nu}_e\)), would be a way of probing CP violation in the Higgs sector. Such an approach was used in the context of the THDM in \([27]\), with related analyses in \([28]\). We will calculate the tree–level rates of the above mechanisms in the

\(^1\)Note that at a muon collider \([13]\), the tree–level diagrams for \(e^+e^- \rightarrow A^0Z\) cannot be discarded anymore and higher order diagrams would induce corrections to the tree-level rate.
context of the CP violating MSSM, showing that in the most favourable scenarios this way of probing scalar–pseudoscalar mixing can be effective if $M_{H_{2,3}} \lesssim 300$ GeV.

Our work is organized as follows. In section 2 we outline our approach for evaluating the cross–sections for the above production mechanisms in the CP violating MSSM. In section 3 we present our numerical results and section 4 contains our conclusions.

2. SCALAR-PSEUDOSCALAR MIXING IN THE MSSM

The tree–level Higgs potential of the MSSM conserves CP, which ensures that the three neutral Higgs eigenstates can be divided into the CP–even $h^0$ and $H^0$ and CP–odd $A^0$. Recent studies \cite{19,20,22} have shown that the 1–loop effective potential may violate CP resulting in three Higgs mass eigenstates which cannot be assigned a definite CP quantum number, denoted by $H^0_1, H^0_2$ and $H^0_3$ (in ascending order of mass). In the above studies it is shown that the CP violation is generated by complex phases which reside in the $\mu$ term and the soft SUSY breaking parameters $A_t$ and $A_b$. These phases generate terms $\mathcal{M}^2_{SP}$ in the $3 \times 3$ neutral Higgs mass squared matrix $\mathcal{M}_{ij}^2$ which mix the CP–odd and CP–even scalar fields. These may be given approximately by \cite{19}

$$\mathcal{M}^2_{SP} \approx \mathcal{O} \left( \frac{m_t^4 |\mu||A_t|}{v^2 2\pi^2 M_{SUSY}^2} \right) \sin \phi_{CP} \times \left[ 6, \frac{|A_t|^2}{M_{SUSY}^2}, \frac{|\mu|^2}{\tan \beta M_{SUSY}^2}, \frac{\sin 2\phi_{CP} |A_t||\mu|}{\sin \phi_{CP} M_{SUSY}^2} \right]$$

where $\phi_{CP} = \arg (A_t \mu)$, and we have only displayed the contributions from the top squarks, $\tilde{t}_{1,2}$, which are dominant for small $\tan \beta$. Sizeable scalar–pseudoscalar mixing is possible for large $|\mu|, |A_t| > M_{SUSY}$. In \cite{22} the mass matrix $\mathcal{M}_{ij}^2$ is evaluated to one–loop order using the effective potential techniques and includes large two–loop non–logarithmic corrections induced by one-loop threshold effects on the top and bottom quark Yukawa coupling. The public code which we will employ in our numerical analysis can be found in \cite{23}. If the SUSY phases are set to zero, the mass eigenstates become definite eigenstates of CP. A phenomenological consequence of the scalar–pseudoscalar mixing is that all the eigenstates $H^0_i$ possess a tree–level coupling $VVH_i^0$ with respective strength $C_i$ (normalized to SM strength). These couplings can be easily obtained from the covariant
derivative of the Higgs fields in which the non–physical Higgs fields are expressed in terms of the physical mass eigenstates. Following the convention of \[22\], the \( C_i \) are given by:

\[
C_i = O_{1i} \cos \beta + O_{2i} \sin \beta
\]  

(4.2)

Here \( O_{ij} \) is the orthogonal matrix which diagonalizes \( M^2_{ij} \). One can easily show that

\[
C_1^2 + C_2^2 + C_3^2 = 1
\]  

(4.3)

Note that this sum rule applies to the tree–level vertices, although it holds to a very good approximation if higher order corrections to the vertices are included. We will present results using the tree–level values for \( C_i \) which we will generate by use of the program cph.f [29].

In the CP conserving MSSM scalar–pseudoscalar mixing is absent. In this case \( C_1 = \sin(\beta - \alpha) \), while one of \( C_2, C_3 \) is identified as \( \cos(\beta - \alpha) \), and the other is identically zero. \( C_i \equiv 0 \) corresponds to the couplings \( A^0 ZZ \) and \( A^0 WW \), which will take on a non–zero value at 1-loop. Hence to lowest order in the MSSM only \( H^0_1 \) and one of \( H^0_2, H^0_3 \) can be produced in the Higgsstrahlung and WW fusion mechanisms, \( e^+e^- \rightarrow Z^* \rightarrow H^0_1Z \) and \( e^+e^- \rightarrow H^0_1\nu_e\overline{\nu}_e \). In the presence of SUSY phases all \( H^0_i \) may be produced at tree–level via \( e^+e^- \rightarrow Z^* \rightarrow H^0_iZ \), and an observable signal for all three \( H^0_i \) would be evidence for CP violation in the Higgs sector. We stress here that the smallest of \( \sigma(e^+e^- \rightarrow Z^* \rightarrow H^0_iZ) \) should exceed the maximum rate for \( e^+e^- \rightarrow A^0Z \) in the context of the CP conserving MSSM [15], since the latter would constitute a “background” to any interpretation of scalar–pseudoscalar mixing. Note that the process \( e^+e^- \rightarrow H^0_i\nu_e\overline{\nu}_e \) proceeds via the same couplings \( C_i \), but possesses a different phase space and \( \sqrt{s} \) dependence. This mechanism is competitive with the Higgsstrahlung process, and becomes the dominant one as \( \sqrt{s} \) increases. We will also consider the mechanism \( e^+e^- \rightarrow H^0_jH^0_k \), which proceeds via the coupling \( C_{jk} \), where \( C_i = C_{jk} \) for \( i \neq j \neq k \).

In the MSSM (with or without SUSY phases), the properties of the lightest eigenstate \( H^0_1 \) become very similar to that of the SM Higgs boson in the decoupling region of \( M_{H^\pm} \geq 200 \) GeV. In this region, \( C_1 \) is very close to 1, and so the sum \( C_2^2 + C_3^2 \) is constrained
to be small (Eq. (4.3)). Therefore we expect that $M_{H^\pm} \leq 200$ GeV will allow larger values for the sum $C_2^2 + C_3^2$, and thus observable rates for both $H^0_2$ and $H^0_3$ in the above mechanisms. Following the approach of [28] we will take the threshold of observability as $\sigma_{obs} = 0.1$ fb. This would give 50 raw events (before cuts) for the assumed luminosities of 500 fb$^{-1}$. Distinct signals for all three $H^0_i$ in a given channel would be evidence for scalar–pseudoscalar mixing. Note that [15] found maximum values of $\sigma(e^+e^- \rightarrow A^0Z) = 0.1$ fb in the context of the CP conserving MSSM.

A caveat here is that extended Higgs sectors with more than two doublets or extra Higgs singlets (e.g. the NMSSM) would also predict multiple signals in these mechanisms [28]. We will show that the CP violating MSSM can only produce multiple signals below a certain mass for $M_{H_2}$ and $M_{H_3}$, and thus any such signal for a larger Higgs mass would be evidence against the CP violating MSSM. Therefore the measurement of the Higgs mass may act as a discriminator among the models.

3. NUMERICAL RESULTS AND DISCUSSION

We now present our numerical results which we will generate with the fortran program cph.f [29]. We note that this program does not include $\chi^+W-H^\pm$ contributions to the 1–loop neutral Higgs mass matrix, which have been shown to be sizeable in some regions of parameter space [30]. The analytic expressions for the various cross–sections are given in the literature [8]. For the $WW$ fusion process we will use the exact expression given in [31].

Graphs showing the numerical values of the couplings $C_i$ and $C_{ij}$ have appeared in Refs [19,22]. However, these papers were more concerned with light $M_{H_2}$ and $M_{H_3}$ of interest at LEP2 and the Tevatron. Detection at these colliders would require quite sizeable values for $C_2^2$ and $C_3^2$. We are concerned with a NLC collider which has the ability to probe $\sigma(e^+e^- \rightarrow H^0_iZ, H^0_iH^0_j, H^0_i\nu_\tau \tau) \geq 0.1$ fb, and so we are also interested in smaller values for $C_2^2, C_3^2$ and larger $M_{H_i}$. An earlier discussion of the potential of a NLC to probe scalar–pseudoscalar mixing can be found in [32]. However this study only addressed the
pair production process $e^+e^- \rightarrow H_i^0H_j^0$ and numerical results were only presented for the couplings $ZH_i^0H_j^0$. We shall be presenting results for the production cross–sections of all the above mechanisms, in contrast to [19,21,22] which were more concerned with the numerical values of $C_i$ and $C_{ij}$ and LEP2 phenomenology. Since the $WW$ fusion process is considerably more important at NLC energies than at LEP2 energies, our analysis is complementary to that in [21] and extends that of [32]. For alternative ways of probing CP violating SUSY phases at $e^+e^-$ colliders, see [33].

As noted in the introduction, the cross–sections for the processes $e^+e^- \rightarrow Zh^0/H^0$, $e^+e^- \rightarrow Ah^0/H^0$, $e^+e^- \rightarrow ZA^0$ and $e^+e^- \rightarrow \nu_\ell \nu_\ell h^0/H^0$ in the CP conserving MSSM are accurately known [14,16–18]. Deviations from these rates would be evidence for scalar–pseudoscalar mixing.

The presence of large SUSY phases can give contributions to the EDM which exceed the experimental upper bound. To avoid conflict with experiment one may assume that the masses of first two generation of squarks are well above the TeV scale while the third generation may be relatively light ($\leq 1$ TeV) [24]. A recent paper [34] suggests that sizeable scalar–pseudoscalar mixing would prefer the cancellation mechanism [25] over the above mechanism as the solution to keep the value of EDM within the experimental limits. Another option is to adopt a non–universal scenario for the tri–linear couplings $A_f$ [35]. In particular, one may require $\arg(\mu) \leq 10^{-2}$ and $A_f = (0,0,1)A$, with $A_t,A_b$ and $A_\tau$ taking maximal phases. Such a scenario comfortably satisfies the EDM constraints. Since the scalar–pseudoscalar mixing $\sim \phi_{CP} = \arg(A_t\mu)$, it is sufficient to have maximal phase in $A_t$ to maximize $\phi_{CP}$. However, two–loop Barr-Zee type diagrams [36] can violate the EDM constraints for large $\tan \beta$ ($\geq 30$). Therefore we will restrict ourselves to low to intermediate values of $\tan \beta$.

In our numerical analysis we will choose the CP violating benchmark scenario (CPX) which was introduced in [21] and maximizes the CP violating effects. The CPX scenario is as follows:

$$\tilde{M}_Q = \tilde{M}_t = \tilde{M}_b = M_{SUSY} = 0.5 \rightarrow 1\text{TeV} \ , \ \mu = 4M_{SUSY}$$
\[ |A_t| = |A_b| = 2M_{SUSY}, \quad |m_{\tilde{g}}| = 1\text{TeV} \quad \text{and} \quad |m_{\tilde{B}}| = |m_{\tilde{W}}| = 0.3\text{TeV} \]  

Note that \( \mu \) will be taken real while we allow a CP phase in the soft tri-linear parameters \( A_t \) and \( A_b \) and in \( m_{\tilde{g}} \). The CP phases of \( A_t \) and \( A_b \) are chosen to be equal and may be maximal. In addition, we choose the charged Higgs mass and \( \tan \beta \) as free parameters.

Our strategy to probe the scalar–pseudoscalar mixing requires the identification of the Higgs signals as distinct resonances. The inclusion of the phases in \( A_t \) and \( A_b \) breaks the near degeneracy among \( M_{H_2} \) and \( M_{H_3} \) \cite{22}, and gives sufficient splittings to allow identification of separate resonances for \( H_0^0 \) and \( H_3^0 \). These splittings may be \( > 10 \text{ GeV} \), which is sufficiently large for a NLC \cite{8} to resolve the separate peaks. This will lead to three different peaks in the Higgsstrahlung and \( WW \) fusion processes and motivates us to present the individual cross–sections for \( e^+e^- \rightarrow ZH_0^0 \) and \( e^+e^- \rightarrow \nu\bar{\nu}H_0^0 \) for \( i = 1, 2, 3 \).

This is in contrast to \cite{37} where the study was devoted to LEPII energies and the cross–sections were summed over the three Higgs states. It has been shown in \cite{19, 21, 22} that the inclusion of SUSY phases may drastically change the size of the couplings \( ZZH_1^0 \) and \( ZH_1^0H_2^0 \) for low and intermediate \( \tan \beta \). In such cases the bound on the light Higgs boson obtained at LEPII may be weakened to \( \lesssim 60 \text{ GeV} \) for large CP violation in the MSSM Higgs sector. We study the potential of a NLC to discover such a weakly coupled Higgs.

In Fig. 1 the left (right) plots depict regions of \( \sigma (e^+e^- \rightarrow ZH_0^0) \) in the plane \( (M_{H_i}, \arg(A_t)) \) for \( \sqrt{s} = 500 \text{ GeV}, \ \tan \beta = 6(15), \ \text{and} \ M_{SUSY} = 1000(500) \text{ GeV} \). In all plots the charged Higgs mass has been varied in increments from 140 \( \rightarrow 400 \text{ GeV} \), which determines the values of \( M_{H_i} \). Comparing the left and right plots it is clear that lower \( \tan \beta \) provides larger cross–sections for \( H_2^0 \) and \( H_3^0 \) over a wider region of the plane, corresponding to the fact that the scalar–pseudoscalar mixing is enhanced. For \( H_1^0 \) discovery is possible over most of the \( (M_{H_i}, \arg(A_t)) \) plane, with small unobservable regions where \( \sigma (e^+e^- \rightarrow ZH_1^0) < 0.1 \text{ fb} \) which occur for \( \arg(A_t) \approx 1.5(2) \) for \( \tan \beta = 6(15) \) and \( M_{H_1} \lesssim 105(115) \text{ GeV} \). The smallness of \( \sigma (e^+e^- \rightarrow ZH_1^0) \) is due to the suppression of \( C_1 \). This suppression arises when \( O_{21} \) changes sign, which induces destructive interference in Eq. (4.2). Note that \( C_1 \) is dominated by \( O_{21} \) and so \( C_1 \) also flips sign for \( \tan \beta = 6(15) \)
and \(\arg(A_t) \approx 1.5 \, (\approx 2)\). \(C_2\) is positive in both cases \(\tan \beta = 6\) and \(\tan \beta = 15\), and is maximized for \(M_{H_2} \lesssim 150\) GeV; it is minimized for \(\arg(A_t) \approx 1.5 \, (\gtrsim 2)\) and \(M_{H_2} \gtrsim 150\) GeV for \(\tan \beta = 6(15)\). For \(H_{2}^{0}\) and \(H_{3}^{0}\), both \(\sigma(e^+e^- \to ZH_{2,3}^{0})\) can be observable over a wide region of the plane, even up to relatively large mass values \(e.g.\) for \(\tan \beta = 6\) and \(\arg(A_t) = 1\), \(\sigma(e^+e^- \to ZH_{2,3}^{0}) \gtrsim 0.1\) fb for \(M_{H_2} \leq 250\) GeV and \(M_{H_3} \leq 270\) GeV. Note that the scalar–pseudoscalar composition of \(H_{2}^{0}\) and \(H_{3}^{0}\) can change with increasing \(M_{H_i}\) \(e.g.\) for the \(\tan \beta = 6\) plot with low \(\arg(A_t)\) \(i.e.\) small scalar–pseudoscalar mixing one can see that \(H_{3}^{0}\) is dominantly scalar for low masses, and has a much larger cross–section than for that for \(H_{2}^{0}\). As \(M_{H_{2,3}}\) increases, \(H_{2}^{0}\) has the larger scalar component and may be produced with an observable rate for \(M_{H_2} \leq 300\) GeV. The coverage for \(e^+e^- \to H_{i}^{0}\nu_e\bar{\nu}_e\) at the same \(\sqrt{s}\) is comparable to that in Fig.1.

Fig. 2 shows \(\sigma(e^+e^- \to ZH_{i}^{0})\) for \(\sqrt{s} = 800\) GeV, \(\tan \beta = 6\), and \(M_{SUSY} = 1000\) GeV. Ones sees that the coverage is inferior to that in Fig.1 since \(\sigma(e^+e^- \to ZH_{i}^{0})\) is reduced for larger \(\sqrt{s}\).

Fig. 3 is analogous to Fig.1 and shows \(\sigma(e^+e^- \to H_{i}^{0}\nu_e\bar{\nu}_e)\) for \(\sqrt{s} = 800\) GeV. Here we find improved coverage compared to that in Figs. 1 and 2, since the cross–section for this process is enhanced with increasing \(\sqrt{s}\). For the \(\tan \beta = 6\) plot with \(\arg(A_t) = 0.5\), both \(\sigma(e^+e^- \to H_{2,3}^{0}\nu_e\bar{\nu}_e) \gtrsim 0.1\) fb for \(M_{H_{2,3}} \leq 300\) GeV. The window of unobservability for \(H_{i}^{0}\) has essentially been closed.

In Fig. 4 we show \(\sigma(e^+e^- \to H_{i}^{0}H_{j}^{0})\) in the plane \((M_{H_i}, \arg(A_t))\) for \(\tan \beta = 6\), \(M_{SUSY} = 1000\) GeV, and \(\sqrt{s} = 500\) GeV. This mechanism offers comparable cross–sections to those for the Higgsstrahlung and \(WW\) fusion processes, and consequently is also effective at probing scalar–pseudoscalar mixing. The best coverage is obtained at \(\arg(A_t) \approx 0.5\). Using the fact that \(C_i^2 = C_j^2\) for \(i \neq j \neq k\), the behaviour of pair production \(e^+e^- \to H_{i}^{0}H_{j}^{0}\) can be roughly understood from the rate of the Higgsstrahlung process \(e^+e^- \to ZH_{2}^{0}\). As can be seen from the plots, there are some similarities between \(e^+e^- \to H_{i}^{0}H_{j}^{0}\) and \(e^+e^- \to ZH_{k}^{0}\), for \(i \neq j \neq k\). Note that \(\sigma(e^+e^- \to H_{i}^{0}H_{2,3}^{0})\) have large cross–sections \((\gtrsim 5\) fb\) in the region where the Higgsstrahlung and \(WW\) fusion processes have very suppressed rates. This situation corresponds to strong scalar–pseudoscalar
mixing, since in the absence of SUSY phases one of \( \sigma(e^+e^- \rightarrow H_1^0 H_{2,3}^0) \) would be zero.

4. CONCLUSIONS

We have studied the production processes \( e^+e^- \rightarrow H_i^0 Z, H_i^0 H_j^0 \) and \( H_i^0 \nu_e \bar{\nu}_e \) in the context of the CP violating MSSM. We showed that in a given channel the cross-section for all \( H_i^0 \) \((i = 1, 2, 3)\) can be observable at a Next Linear Collider and would provide evidence for scalar–pseudoscalar mixing. At \( \sqrt{s} = 500 \text{ GeV} \) the coverage of \( e^+e^- \rightarrow H_i^0 Z \) and \( H_i^0 \nu_e \bar{\nu}_e \) are comparable, with observable cross-sections for \( M_{H_2} \leq 250 \text{ GeV} \) and \( M_{H_3} \leq 270 \text{ GeV} \) for the most favourable choice of \( \text{arg}(A_t) \). At \( \sqrt{s} = 800 \text{ GeV} \), the process \( e^+e^- \rightarrow H_i^0 \nu_e \bar{\nu}_e \) offers superior coverage, with a reach up to \( M_{H_{2,3}} \leq 300 \text{ GeV} \) in the most favourable cases. The scalar–pseudoscalar mixing causes a mass splitting between \( H_2^0 \) and \( H_3^0 \) which should be sufficient for separate peaks to be resolved at a NLC. The problematic region of a light \( H_1^0 \) with a very suppressed coupling to vector bosons \((VVH_1^0)\) has a window of unobservability at \( \sqrt{s} = 500 \text{ GeV} \) and \( \text{arg}(A_t) \approx \pi/2 \). This is almost closed at \( \sqrt{s} = 800 \text{ GeV} \) in the \( H_i^0 \nu_e \bar{\nu}_e \) channel. The mechanism \( e^+e^- \rightarrow H_i^0 H_j^0 \) is competitive with the above mechanisms for probing scalar–pseudoscalar mixing at \( \sqrt{s} = 500 \text{ GeV} \), and can comfortably detect \( H_1^0 \) in the region of suppressed coupling \( VVH_1^0 \).

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FIG. 1. $\sigma(e^+e^- \rightarrow ZH_i^0)$ at $\sqrt{s} = 500$ GeV in ($M_{H_i}$, Arg($A_t$) plane; $M_{SUSY} = 1$ TeV, $\tan\beta = 6$ (left panels) and $M_{SUSY} = 500$ GeV, $\tan\beta = 15$ (right panels)
FIG. 2. $\sigma(e^+e^- \rightarrow ZH^0)$ at $\sqrt{s} = 800$ GeV in $(M_{H_1}, \text{Arg}(A_t))$ plane for $M_{SUSY} = 1$ TeV, $\tan \beta = 6$.
FIG. 3. $\sigma(e^+e^- \rightarrow H^0\nu\bar{\nu})$ at $\sqrt{s} = 800$ GeV in ($M_{H_1}$, Arg($A_t$) plane, $M_{SUSY} = 1$ TeV;

$\tan\beta = 6$ (left panels) and $M_{SUSY} = 500$ GeV, $\tan\beta = 15$ (right panels)
FIG. 4. $\sigma(e^+e^- \to H_i^0H_j^0)$ at $\sqrt{s} = 500$ in ($M_{H_{i,j}}, \text{Arg}(A_t)$) plane, $M_{SUSY} = 1$ TeV, $\tan\beta = 6$