Can the Heston Model Forecast Energy Generation? A Systematic Literature Review

Bianca Reichert1*, Adriano Mendonça Souza2

1Post-Graduation Program in Industrial Engineering, Federal University of Santa Maria, Santa Maria, Brazil, 2Department of Statistics, Federal University of Santa Maria, Santa Maria, Brazil. *E-mail: bianca.reichert@hotmail.com

Received: 03 September 2021 Accepted: 10 December 2021 DOI: https://doi.org/10.32479/ijeep.11975

ABSTRACT

The ability to predict the price of stock exchange assets has attracted the attention of economists and physicists around the world, as physical models are useful to predict volatility behaviors. Knowing that volatility is crucial for energy sector planning, the research aim was to investigate whether the Heston pricing model is useful to predict energy generation, through the steps established by the systematic review protocol. In a corpus of 25 documents, it was possible to identify: Lots of financial studies, energy and demography researches; a low level of interaction among universities; the largest number of publications from Australia and China; the most important journal; and the advantages of applying Econophysics models to solve volatility problems. In conclusion, the Heston model can be applied to predict energy generation, since it is a closed-form model and capable of modeling the stochastic volatility, reversing it to the predicted value of average energy generation.

Keywords: Electricity, Stock Exchange, Stochastic Volatility, Systematic Review
JEL Classifications: A12, C10, C53

1. INTRODUCTION

The field of Econophysics emerged in the 90s from the fusion of two different areas to understand economic phenomena through the application of the laws of physics (Mantegna and Stanley, 2008). Since then, many studies have already been developed and financial events explained by physical theories, such as the power law that is widely applied by economists and used as a mathematical basis in the development of new pricing asset models (Gabaix, 2016; Singer, 2020). The popularity of Econophysics increased after successful attempts to price stock exchange assets, as physical models are able to explain high frequency data and randomness patterns (Ferreira et al., 2020; Mainardi, 2020). Volatility is an example of random variable and the statistical metric most analyzed in financial modeling, due to its relevance in portfolio risk management (Cheng, 2020; Stilger et al., 2021). The main pricing models, that take into account the volatility of variables, are: the Merton model used for risk analysis; the Bates model that incorporates jump diffusion; the most applied Black-Scholes model for considering a stochastic price series with constant volatility; and the Heston model, which describes volatility as a stochastic process (Balajewicz and Toivanen, 2017; Russo et al., 2020; Salvador and Oosterlee, 2021).

In addition to the problems of the financial sector, the need to understand volatility is also observed in others subjects, as quality control in industries, forecast of power generation and effects of climate change (Babar et al., 2020; Mehmood et al., 2020; Qiu et al., 2020; Ramos-Meza et al., 2021; Schilling and Holzgrabe, 2020). The successful pricing models could contribute to solving these problems, such as the study that extended the used of the Heston model to forecast vehicle collision rates and another that applied the Black–Scholes model to analyze the evolution of a brain tumor under health treatment (Shannon and Fountas, 2021; Sinkala and Nkalashe, 2020).
Based on these studies results, the possibility of using pricing models to predict energy generation is now considered; because, with the increase of renewable generation, the energy supply becomes more unstable and difficult to predict and control (Park and Baldick, 2020; Reichert and Souza, 2021). For this reason, the aim of this study was to verify whether the Heston model can be used to predict energy generation, through a systematic literature review. This review protocol was chosen to analyze the state of art on the subject, the main characteristics of the Heston model, the variables and areas of application, theoretical justifications and emerging topics.

The content of this paper is divided into five sections: the first one addresses the topic of interest and defines the research aims; the second one presents a brief theoretical review of the Heston model; the third one identifies the materials and methodological steps performed; the fourth one describes the analyzes and the main results; and the last one reports the findings of the study.

2. HESTON MODEL

The Heston model was developed with the purpose of filling the gaps of the Black-Scholes pricing model, such as returns skewness (Heston, 1993). The result was a closed-form model based on a geometric Brownian motion, to represent the asset price behavior, and on the Cox et al. (CIR) equations, which describes the stochastic volatility of the variables (Cox et al., 1985; Mantegna and Stanley, 2008). Due to this, Heston modeling is indicated for series with fat tails returns, non-constant volatility and implied volatility smiles (Karlsson et al., 2017; Schwartz, 1997).

A typical Heston model is defined by the following Equations (Zhong et al., 2019; Zhou et al., 2019).

\[ dS_t = \mu S_t dt + \sqrt{v_t} S_t dW^S_t \]  
(1)

\[ dv_t = \kappa (\theta - v_t) dt + \sigma \sqrt{v_t} dW^v_t \]  
(2)

Where: \( S_t \) is the asset price, \( v_t \) characterizes the volatility/standard deviation of the asset price, \( \theta \) is the long-term variance, \( \kappa \) represents the reversion rate from \( v_t \) to \( \theta \), \( \sigma \) is considered the volatility of volatility, \( W^S_t \) and \( W^v_t \) are the Wiener process, i.e., random walks.

For a \( v_t \) process strictly positive or equal to zero, the parameters must follow the Feller condition: \( 2\kappa \theta - \sigma^2 \) (Teng et al., 2018).

In Equations 1 and 2, the Wiener terms \( W^S_t \) and \( W^v_t \) have a correlation \( \rho \), which is responsible for the link between the asset price and its own volatility, making possible to introduce volatility in the forecast of the average price (Karel in ’t Hout and Toivanen, 2018; Teng et al., 2016). Since the Wiener stochastic process is obtained by an integral of the Gaussian noise, it can be interpreted as the white noise from Time Series analysis (Heston, 1993; Psaros et al., 2020).

When comparing with other volatility models, as the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, the Heston model has a more complex application; however, the single-lag GARCH model converges to the Heston model as the frequency of observations increases (Heston and Nandi, 2000). In relation to other pricing models, the Heston model is superior for its flexibility and for covering the variable stochastic volatility. The main advantage of Heston modeling is the non-assumption of the series normality, which is crucial for the application of the Black and Scholes pricing model (Daniel et al., 2005; Mota, 2012).

3. MATERIALS AND METHODS

A systematic literature review was performed to assess the state of art in studies that applied the Heston model in the energy sector. The review was based on research protocol, which it was possible to define inclusion and exclusion strategies for documents in textual corpus (Pan et al., 2021; Tranfield et al., 2003).

First, the search string that satisfies the study aims was established: ALL (“Heston model*”AND energ*). Subsequently, the referential bases were defined according to the level of coverage of its collection and its importance in the academic area under study; therefore, the referential bases Scopus and Web of Science were chosen (Xiao and Watson, 2017). The search for the string was performed concomitantly in the two bases on March 22, 2021.

The strategy of only including papers and reviews was applied, followed by the exclusion strategies: duplicated documents and Journals without the Scimago Journal & Country Rank (SJR) indicator. After this refinement, adherence to the theme was verified by reading and analyzing title, abstract, keywords, methodology and results of the papers. In this analysis, even papers not applied to the energy sector were included in textual corpus, because there are not many Econophysics studies involving energy generation and it would be a loss for the study not to include them. Still at this stage, papers were excluded from the corpus for not using or just citing the methodology under study. A detailed summary of the steps used in the systematic review protocol can be seen in Figure 1.

The analysis of final corpus was performed in two stages: the first dedicated to main information, as number of documents and authors, annual scientific production, global distribution of the scientific production and the most important sources and authors; the second stage related to meta-analysis, which made it possible to identify the main application areas, the type of variables, the most used methodologies, the best methods of parameters estimation and trending topics. The corpus were refined and tabulated in an electronic spreadsheet, while the main information was extracted using the Bibliometrix R package (Aria and Cuccurullo, 2017).

4. RESULTS AND DISCUSSION

This section is divided into two subsections; the first one summarizes the main information about the corpus; the second one describes the results of meta-analysis.

4.1. Main Information of the Corpus

A textual corpus of 25 papers was obtained using the systematic review protocol applied to the studies that applied the Heston
model in the energy sector, which represents a sample of the scientific production of the last two decades. These and other general information are shown in Table 1.

According to Table 1, it can be concluded that the average number of co-authors per paper is close to two, which represents a high level of scientific contribution by the authors in their own research field and a low level of interaction among different research groups. Other conclusions refer to the good theoretical foundation of the corpus, since the average number of references per article is equal to 40, and the relevance and quality of the papers included in the corpus, due to the average citations per document (17.88 citations). The limited timeline of the publications means that the subject under study is recent in the academic area; but, even with a low number of papers, there has been an increase in publications in the last years, as it can be seen in Figure 2.

From 2002 to 2010, the application of the Heston model was restricted to the financial sector and focused on forecasting or analyzing the price of stock exchange assets (Drăgulescu and Yakovenko, 2002; Fatone et al., 2009; Masoliver and Perelló, 2007). The first paper that used variables from the energy sector was published in 2011, which aimed to predict the gas price in England (Benth, 2011). Since then, the use of energy variables has increased significantly, but still with studies focused only on forecasting the price of energy sources, such as crude oil and gas (Dong et al., 2020; Hsu et al., 2017; Leng and Li, 2020).

The sources of publications diversified in the fields of physics and finance, as the most relevant sources were Physica A (ISSN 0378-4371) and Quantitative Finance (ISSN 1469-7688) with Cite Score equal to 5.6 and 2.8, respectively. According to the Bradford’s law, these sources represent the Zone 1, which accounts for at least a third of the publications in the corpus (Bradford, 1934; Desai et al., 2018).

On a global scale, studies involving the Heston model were basically published in two countries, China (16 papers) and Australia (7 papers), as shown in Figure 3. Other countries that present at least one publication were: Ireland, Canada, Finland, Italy, Norway, Spain, United Kingdom and United States.

When it comes to Heston modeling, the spatial distribution of publications confirms the low level of interaction among universities and different research groups (Figure 3). The main universities in the corpus were the University of Wollongong in Australia and the Yunnan University of Finance and Economics in China. In Wollongong, the studies were developed by the school of mathematics and applied statistics to predict stock exchange assets and evaluate variance swaps (He and Chen, 2021; He and Zhu, 2016; Rujivan and Zhu, 2012; Zhu and Lian, 2011; 2012; 2015). In China, most studies focused on application of Econophysics models to predict the price of energy-related assets (Ding et al., 2019; Dong et al., 2020; Leng and Li, 2020; Li et al., 2016; Zhong et al., 2019).

Table 1: General information of the corpus

| General information     | Number of authors | Average citations | References |
|-------------------------|-------------------|-------------------|------------|
| Timeline                | 2002:2021         |                   |            |
| Number of documents     | 25                | 17.88             |            |
| Sources (Journals)      | 16                |                   | 1001       |

Figure 1: Flowchart of systematic literature review protocol steps

Figure 2: Annual scientific production
The most relevant authors are also from these countries: Song-Ping Zhu from Australia and Jiang-Cheng LI from China. Zhu was the author of 6 papers and, for the most part, appears as the main author. Otherwise, Li participated as a co-author in the 5 papers. The scientific production of these and other authors can be analyzed over time in Figure 4.

The temporal analysis of Figure 4 allows us to verify the most relevant authors published their papers in the last decade (2011-2021), proving that the theme of interest is recent and presents an opportunity for expansion. In addition to the large number of publications, these authors share a collaboration network, for example, the link formed by Li, Mei and Tang, and another formed by Zhu, Lian and Rujivan, according to the Louvain clustering algorithm (Wang and Koopman, 2017).

4.2. Meta-analysis

After analyzing adherence to the research subject, the corpus was submitted to a critical review of the main elements, such as objective, methodology and the results. All papers aimed to predict or fit a forecasting model based on different methods, except for one study that analyzed the stability of electricity price market (Ding et al., 2019). The studies were summarized in three areas: financial, energy and demography; the proportion of each is shown in Figure 5.

Most studies applied financial variables due to volatility inherent in these data and because it was the area where the Heston model emerged (Heston, 1993; Heston and Nandi, 2000). The energy sector appears in the second position, since it also has volatile and unpredictable data (Dong et al., 2020). The main variables used were: option prices (European and American options, energy prices and simulated data); stock exchange indexes (S&P 500, CSI 300 and DJIA); personal income; and mean residence time (Balajewicz and Toivanen, 2017; Bi et al., 2016; Kyriakou et al., 2016; Li et al., 2016; Masoliver and Perelló, 2007; Richmond and Sabatelli, 2004). The application format of these variables were spot data (48%), variance swaps (28%) and series of returns (24%).

The Heston model was used in all papers, either in its pure version, in hybrid modeling or in comparison with other pricing model, as can be seen in Figure 6. The main variations applied to the Heston model were:

- The pure two-factor stochastic model (Cao and Fang, 2017; Rujivan and Zhu, 2014)
- Regime-switching model based on a Markov chain (Elliott and Lian, 2013; Lin and He, 2021)
- Periodic or delayed model with Fourier and Laplace transforms (Ding et al., 2019; Dong et al., 2020).
Comparing with other pricing models, such as Black–Scholes, Merton and Bates, the Heston model showed superior results (Balajewicz and Toivanen, 2017; Kyriakou et al., 2016). Some stochastic volatility models also had their results compared to the Heston model, which, in most cases, had the best performance. However, some models were superior, such as Barndorff-Nielsen and Shephard model, mean reversion–stochastic volatility–jump diffusion–seasonality model (MRSVJS), and regime-switching Heston–Cox–Ingersoll–Ross hybrid model (Benth, 2011; He and Chen, 2021; Hsu et al., 2017). This superiority may be related to more flexible structures, incorporation of regime changes and stylized effects, as seasonality.

Another point that differentiated the studies was the estimation of parameters, either through mathematical methods or through values established by the authors. The frequency of appearance of each parameter estimation method can be seen in Figure 7.

The parameters established by the authors were used in studies that simulated comparisons between different models using the Monte Carlo simulation (Cao and Fang, 2017; Lin and He, 2021). The second most used estimation method was the Least Squares, since it is a simple mathematical optimization tool and widely applied in regression analysis (Du et al., 2020; Miller, 2017). Even in a low proportion, Bayesian inference shows itself as a resourceful tool for solving problems in econophysics (Leng and Li, 2020; Zhong et al., 2019).

When analyzing emerging topics, Econophysics is considered as the most recent theme with great opportunities for expansion. The temporal evolution of this and other trending topics can be seen in Figure 8, according to the Zipf’s law applied to the authors’ keywords (Zipf, 1945).

In Figure 8, “stochastic volatility” is the most quoted term, as the Heston model is a very successful closed-form model based on stochastic volatility (Zhu and Lian, 2012). Other trend topics are related to the justifications of the studies, since the Heston model is able to describe the dynamics of assets pricing and variance swaps (Balajewicz and Toivanen, 2017; Bi et al., 2016; Cao and...
Fang, 2017; Dong et al., 2020). In summary, the Heston model is a very well evaluated and widely used method in the financial area. However, its application in other areas would facilitate the planning of the volatile dynamics of variables, as energy generation, industrial quality control and climate forecasting.

5. CONCLUSION

The Heston model, originated in Econophysics, is a recent theme and has a great opportunity for expansion. Its main application refers to variables with stochastic volatility characteristics and, knowing that energy sector variables are volatile, the aim of this study was to verify whether the Heston modeling could be used to predict energy generation. Therefore, a systematic literature review protocol was carried to identify the state of art of studies that used the Heston model in energy variables.

From the review protocol, a textual corpus of 25 papers was obtained, written by 49 authors and published in 16 different journals. The corpus production was basically concentrated in two countries – Australia and China – showing that there is a low level of contribution among different research groups around the world. In meta-analysis, the Heston model was the most used and had superior results, even being compared with other stochastic volatility and asset pricing models. Furthermore, the Bayesian parameter estimation proved to be a successful methodology in the field of Econophysics.

Based on the results, it was concluded that the Heston model can be applied in the energy sector, especially to predict energy generation, as it is a closed-form model and capable of understanding the stochastic volatility, reversing it to the forecast of the mean value.

The main limitation of the study was having to include in the systematic review papers that applied the Heston model in other sectors, due to the lack of studies in the area of energy generation, since the Heston model was only applied to predict the price of the energy sector.

It is suggested for future work to extend the application of the Heston model to other areas beyond the financial sector, in order to increase the collaboration among research groups from different institutions and to expand the potential of this and other Econophysics models to predict and plan volatile behaviors.

6. ACKNOWLEDGMENTS

We thank the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) for financial support and the Statistical Analysis and Modeling Laboratory (LAME) of the Federal University of Santa Maria for technical support.

REFERENCES

Aria, M., Cuccurullo, C. (2017), Bibliometrix: An R-tool for comprehensive science mapping analysis. Journal of Informetrics, 11(4), 959-975.

Babar, Z.B., Ashraf, F., Park, J.H., Lim, H.J. (2020), Volatility parameters of secondary organic aerosol components determined using a thermal denuder. Atmospheric Environment, 226, 117405.

Balajewicz, M., Toivanen, J. (2017), Reduced order models for pricing European and American options under stochastic volatility and jump-diffusion models. Journal of Computational Science, 20, 198-204.

Benth, F.E. (2011), The stochastic volatility model of Barndorff-Nielsen and Shephard in commodity markets. Mathematical Finance, 21(4), 595-625.

Bi, M., Escobar, M., Goetz, B., Zagst, R. (2016), Principal component models with stochastic mean-reverting levels. Pricing and covariance surface improvements. Applied Stochastic Models in Business and Industry, 32(5), 585-606.

Bradford, S.C. (1934), Sources of information on specific subjects. Engineering, 137, 85-86.

Cao, J.P., Fang, Y.B. (2017), An analytical approach for variance swaps with an ornstein-uhlenbeck process. ANZIAM Journal, 59(1), 83-102.

Cheng, I.H. (2020), Volatility markets underreacted to the early stages of the COVID-19 pandemic. The Review of Asset Pricing Studies, 10(4), 635-668.

Cox, J.C., Ingersoll, J.E., Ross, S.A. (1985), An intertemporal general equilibrium model of asset prices. Econometrica, 53(2), 363-384.

Daniel, G., Joseph, N.L., Brée, D.S. (2005), Stochastic volatility and the goodness-of-fit of the Heston model. Quantitative Finance, 5(2), 199-211.

Desai, N., Veras, L., Gosain, A. (2018), Using Bradford’s law of scattering to identify the core journals of pediatric surgery. Journal of Surgical Research, 229, 90-95.

Ding, W., Wang, B., Xing, Y., Li, J.C. (2019), Correlation noise and delay time enhanced stability of electricity futures market. Modern Physics Letters B, 33(30), 3.

Dong, Y., Wen, S.H., Hu, X.B., Li, J.C. (2020), Stochastic resonance of drawdown risk in energy market prices. Physica A: Statistical Mechanics and its Applications, 540, 123098.

Drăgulescu, A.A., Yakovenko, V.M. (2002), Probability distribution of returns in the heston model with stochastic volatility. Quantitative Finance, 2(6), 443-453.

Du, J., Lai, S., Lai, K.K., Zhou, S. (2020), A novel term structure stochastic model with adaptive correlation for trend analysis. International Journal of Finance and Economics, 26(4), 5485-5498.

Elliott, R.J., Lian, G.H. (2013), Pricing variance and volatility swaps in a stochastic volatility model with regime switching: Discrete observations case. Quantitative Finance, 13(5), 687-698.

Fatone, L., Mariani, F., Recchioni, M.C., Zirilli, F. (2009), An explicitly solvable multi-scale stochastic volatility model: Option pricing and calibration problems. Journal of Futures Markets, 29(9), 862-893.

Ferreira, P., Pereira, É.J.A., Pereira, H.B.B. (2020), From big data to econophysics and its use to explain complex phenomena. Journal of Risk and Financial Management, 13(7), 153.

Gabaix, X. (2016), Power laws in economics: An introduction. Journal of Economic Perspectives, 30(1), 185-206.

He, X.J., Shen, W. (2021), A semianalytical formula for European options under a hybrid heston-cox-ingersoll-ross model with regime switching. International Journal of Finance and Economics, 26(1), 343-352.

He, X.J., Zhu, S.P. (2016), An analytical approximation formula for European option pricing under a new stochastic volatility model with regime-switching. Journal of Economic Dynamics and Control, 71, 77-85.

Heston, S.L. (1993), A closed-form solution for options with stochastic volatility with applications to bond and currency options. The Review of Financial Studies, 6(2), 327-343.
