Two-step Calibration Method for Triaxial Magnetometers

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Abstract. In this paper, two important sources of the non-orthogonal errors of triaxial magnetometers were analysed. The non-orthogonal error mainly come from the transverse sensitivity of uniaxial sensor or non-orthogonality between the three axes of magnetic magnetometer. These two error sources were separated by mathematical analysis. Then a two-step calibration method of triaxial magnetometers was designed. It showed that the peak-to-peak value of the calculated result of the scalar of magnetic field could be reduced to 56nT from 350nT within the range of 50000nT.

1. Introduction

Magnetometers are broadly applied in the fields of geomagnetic navigation, ferromagnetic object detection, geophysical exploration and mineral exploration [1-3]. The commonly used magnetometers can be divided into scalar magnetometers and vector magnetometers. Scalar magnetometers have several types, such as proton magnetometer, optical pump magnetometer and Superconducting Quantum Interference Devices(SQUID), etc. There are also two-axis or three-axis vector magnetometers also, such as fluxgate magnetometers, AMR, etc. Compared with scalar magnetometers, vector magnetometers, especially fluxgate magnetometers, have advantages of lower cost, smaller size, lower power consumption and higher response frequency, and can work at room temperature with good measurement accuracy.

Like other sensors, triaxial magnetometers have a variety of internal errors, such as bias deviation, sensitivity coefficient deviation, nonlinear error, hysteresis error, transverse sensitivity error, and non-orthogonal error [4,5]. These parameters are also temperature dependent and change slowly over time. In particular, bias, sensitivity coefficient and non-orthogonality are the most important factors affecting the final measurement results.

Calibration of the triaxial magnetometers can be performed by using a triaxial Helmholtz coil and a current source to generate standard magnetic field. However, the non-orthogonal error of the triaxial Helmholtz coil limit the calibration accuracy of the triaxial magnetometer. Rong Zhu et al. aligned an orthogonal optical system and calculated the non-orthogonal error of the triaxial magnetometer by the least square estimation principle [6]. This method also had the disadvantage of complex operation.

Scalar calibration methods have attracted more and more attention in recent years. [7-19]. These methods need to be carried out in a stable magnetic field environment, and the magnetic field is assumed to be uniform and constant. A high-precision scalar magnetometer, such as the optical pump magnetometer, is used to measure the scalar of magnetic field, which is considered to be an accurate and reliable truth value of the ambient magnetic field. In this environment, the triaxial magnetometers...
are rotated randomly or according to certain rules, and the outputs of the magnetometers are recorded at the same time. Next, different algorithms are adopted to solve parameter values in a model, such as Nonlinearity Suppression [11], differential evolution algorithm [12], nonlinear least square method [14], Gauss-Newton, unscented Kalman filtering (UKF), Levenberg-Marquardt algorithm [15], particle swarm optimization(PSO) [16,17], etc. The bias, sensitivity and non-orthogonality of the triaxial magnetometers are calculated and fitted. After the outputs of the magnetometers are compensated, the scalar of the magnetic field are calculated.

A two-step calibration method was studied in this paper, which could improve the calibration accuracy, especially reduce the influence of non-orthogonal errors. This is of great significance for improving the accuracy of magnetic anomaly detection and geomagnetic navigation.

2. Model and analysis

Previous studies were all based on the following mathematical model, or a more simplified model, for analysis and calculation [5-17]:

\[ V = R \cdot B + b + \sigma \]  

(1)

Where \( V \) represents the output of the triaxial magnetometer. \( R \) represents the sensitivity coefficient matrix of the triaxial magnetometer, including the influence of sensitivity coefficient and non-orthogonality. \( B \) represents the vector of external measured magnetic field. \( b \) represents the bias vector of the triaxial magnetometer. \( \sigma \) represents the random noise of each axial output. \( V, B, b \) and \( \sigma \) are \( 3 \times 1 \) vectors. \( R \) is a \( 3 \times 3 \) matrix. In matrix form, formula (1) can be expressed as:

\[
\begin{bmatrix}
V_x \\
V_y \\
V_z
\end{bmatrix} =
\begin{bmatrix}
r_{xx} & r_{xy} & r_{xz} \\
r_{yx} & r_{yy} & r_{yz} \\
r_{zx} & r_{zy} & r_{zz}
\end{bmatrix}
\begin{bmatrix}
B_x \\
B_y \\
B_z
\end{bmatrix} +
\begin{bmatrix}
b_x \\
b_y \\
b_z
\end{bmatrix} +
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z
\end{bmatrix}
\]  

(2)

From this mathematical model, it can be seen that the purpose of magnetometer calibration is to calculate \( R \) and \( b \), a total of 12 unknown coefficients. This problem can be transformed into the parametric optimization problem of linear equation or nonlinear equation. Previous studies had used different methods to solve these coefficients.

However, it can be seen from this mathematical model that the sensitivity coefficient matrix \( R \) contains the combined influence of the sensitivity coefficient and non-orthogonality. The non-orthogonality mainly comes from the transverse sensitivity of uniaxial sensor and non-orthogonality between the three axes of magnetic magnetometer. The separation of these error sources is beneficial to the better calibration of the triaxial magnetometer.

Uniaxial magnetic sensor not only has response to the magnetic field parallel to the sensitive direction, but also has response to the magnetic field perpendicular to the sensitive direction. This is called transverse sensitivity. The sensitivity and transverse sensitivity of each uniaxial sensor are independent of each other. Considering only the transverse sensitivity, bias and random noise of the sensors, the outputs of the uniaxial sensors can generally be expressed as follows:

\[ V_i = (k_{ix} \ k_{iy} \ k_{iz}) \cdot (B_x \ B_y \ B_z)^T + b_i + \sigma_i \]  

(3)

Where, \( V_i \) represents the output of the \( i \)th uniaxial sensor; \( k_{ix}, k_{iy} \) and \( k_{iz} \) represent respectively the sensitivity coefficients of the sensor to different axial magnetic fields; \( B_x, B_y \) and \( B_z \) represent the vectors of external measured magnetic field. \( b_i \) represents the bias of the output of the sensor. \( \sigma_i \) represents the random noise of the sensor.

For the triaxial magnetometer, the sensitivity coefficients of each independent axial sensor \( k_{ij} \) is independent of each other. And \( k_{ij} (\ i \neq j) \) is generally much smaller than that of \( k_{ii} \). \( k_{ii} \) can be directly measured in magnetic shielding barrel and uniaxial standard magnetic field generator. The measurement accuracy is only affected by the errors of the measuring devices and the sensor. Generally, this accuracy can be better than 1nT.
Affected by machining and installation errors, the three axes of the triaxial magnetometers are difficult to be orthogonal to each other. This phenomenon leads to a kind of error, called triaxial non-orthogonal error. Taking Bartington's MAG-03MS triaxial fluxgate magnetometers as an example, they noise density are lower than 6pT(RMS)/√Hz@1Hz, but the non-orthogonal errors are about 0.1°. Without calibration and compensation, the errors of calculated scalar of the magnetic fields may over 200nT within the full range (1Gs).

Non-orthogonal error and transverse sensitivity have same form. They are coupled together and hard to separate. In order to establish a better error model, a detailed analysis of these two errors was performed.

Figure 1 The relationship between the sensitive axial direction and the reference coordinate system of the triaxial magnetometer

Figure 2 the independent reference coordinate system of the ysen sensor and the reference coordinate system

Suppose that the three sensitive axes of the magnetometer are $\mathbf{x}_{\text{sen}}, \mathbf{y}_{\text{sen}}$ and $\mathbf{z}_{\text{sen}}$ respectively, which are not necessarily orthogonal. Construct the orthogonal reference frame $\mathbf{Oxyz}$ according to the right hand Cartesian coordinate system rule. Let the $z$-axis of the reference frame be parallel to the $\mathbf{z}_{\text{sen}}$ axis. Take the plane of the $\mathbf{z}_{\text{sen}}$ axis and $\mathbf{y}_{\text{sen}}$ axis as the $\mathbf{Oyz}$ plane of the reference coordinate system. Let $\mathbf{y}$-axis and $\mathbf{z}$-axis perpendicular, in the $\mathbf{Oyz}$ plane. Sets the angle of $\mathbf{y}_{\text{sen}}$ and $\mathbf{y}$-axis as $\Phi_1$. Let the $x$-axis direction of the reference coordinate system be the vector product direction of the $\mathbf{y}$-axis and $\mathbf{z}$-axis. Set the angle between the projection of the $\mathbf{x}_{\text{sen}}$ onto $\mathbf{Oyz}$ plane and the $x$-axis as $\Phi_2$. The relationship between the sensitive axial direction and the reference coordinate system of the triaxial magnetometers is shown in figure 1.

Assume that the coordinate system of the external measured magnetic field corresponds to the reference coordinate system. At this time, the output of the direction of the $\mathbf{z}_{\text{sen}}$ sensor is:

$$V_z = (k_{zx} \ k_{zy} \ k_{zz}) \cdot (B_x \ B_y \ B_z)^T + b_z + \sigma_z$$  \hspace{1cm} (4)

Let the reference coordinate system $\Phi_1$ rotate around the $\mathbf{x}$-axis, and let $\mathbf{y}_{\text{sen}}$ parallel to the $\mathbf{y}$-axis. The $\mathbf{Oxy}_1\mathbf{z}_1$ coordinate system is defined as the independent reference coordinate system of the $\mathbf{y}_{\text{sen}}$ sensor, as shown in figure 2. Then the transformation matrix from $\mathbf{Oxyz}$ coordinate system to $\mathbf{Oxy}_1\mathbf{z}_1$ coordinate system is:

$$R_x(\Phi_1) = 
\begin{pmatrix}
1 & 0 & 0 \\
0 & \cos(\Phi_1) & \sin(\Phi_1) \\
0 & -\sin(\Phi_1) & \cos(\Phi_1)
\end{pmatrix}$$  \hspace{1cm} (5)

The output of the ysen sensor could be obtained as follows:

$$V_y = (k_{yx} \ k_{yy} \ k_{yz}) \cdot R_x(\Phi_1) \cdot (B_x \ B_y \ B_z)^T + b_y + \sigma_y$$  \hspace{1cm} (6)
Figure 3 the independent reference coordinate system of the xsen sensor and the reference coordinate system

The reference coordinate system $Oxyz$ (system $\Phi_3$) rotate around $z$-axis, with $x$-axis parallel to the projection of $xsen$ onto $Oxy$ plane. Then reference frame $\Phi_2$ rotation around the $y$-axis, lets $x$-axis parallel with $xsen$. The coordinate system obtained after 2 rotations is defined as the independent reference coordinate system of the $xsen$ sensor, $Ox_2y_2z_2$. See figure 3. Transformation matrix from $Oxyz$ coordinate system to $Ox_2y_2z_2$ coordinate is

$$R_y(\Phi_2) \cdot R_z(\Phi_3)$$

And

$$R_y(\Phi_2) = \begin{pmatrix}
\cos(\Phi_2) & 0 & -\sin(\Phi_2) \\
0 & 1 & 0 \\
\sin(\Phi_2) & 0 & \cos(\Phi_2)
\end{pmatrix}$$

$$R_z(\Phi_3) = \begin{pmatrix}
\cos(\Phi_3) & -\sin(\Phi_3) & 0 \\
\sin(\Phi_3) & \cos(\Phi_3) & 0 \\
0 & 0 & 1
\end{pmatrix}$$

The output of the $xsen$ sensor can be obtained as follows:

$$V_x = (k_{xx} \quad k_{xy} \quad k_{xz}) \cdot R_y(\Phi_2) \cdot R_z(\Phi_3) \cdot (B_x \quad B_y \quad B_z)^T + b_x + \sigma_x$$

After finishing, the outputs of the magnetometers are written as a matrix in the same form as (2), but the sensitivity coefficient matrix $R$ in it could be expressed by $k_{ij}$.

$$r_{xx} = k_{xx} \cdot \cos(\Phi_2) \cdot \cos(\Phi_3) + k_{xy} \cdot \sin(\Phi_3) + k_{xz} \cdot \sin(\Phi_2) \cdot \cos(\Phi_3)$$
$$r_{xy} = -k_{xx} \cdot \cos(\Phi_2) \cdot \sin(\Phi_3) + k_{xy} \cdot \cos(\Phi_3) - k_{xz} \cdot \sin(\Phi_2) \cdot \sin(\Phi_3)$$
$$r_{xz} = -k_{xx} \cdot \sin(\Phi_2) + k_{xz} \cdot \cos(\Phi_2)$$
$$r_{yx} = k_{yx}$$
$$r_{yy} = k_{yy} \cdot \cos(\Phi_1) - k_{yz} \cdot \sin(\Phi_1)$$
$$r_{yz} = k_{yy} \cdot \sin(\Phi_1) + k_{yz} \cdot \cos(\Phi_1)$$
$$r_{zx} = k_{xz}$$
$$r_{zy} = k_{zy}$$
$$r_{zz} = k_{zz}$$

Ignoring the random noise, the estimated magnetic induction intensity of the measured magnetic field can be deduced from the following equation:

$$\begin{pmatrix}
\bar{B}_x \\
\bar{B}_y \\
\bar{B}_z
\end{pmatrix} = \begin{pmatrix}
r_{xx} & r_{xy} & r_{xz} \\
r_{yx} & r_{yy} & r_{yz} \\
r_{zx} & r_{zy} & r_{zz}
\end{pmatrix}^{-1} \begin{pmatrix}
V_x \\
V_y \\
V_z
\end{pmatrix} - \begin{pmatrix}
b_x \\
b_y \\
b_z
\end{pmatrix}$$

The estimated scalar of the measured magnetic field is:
\[ B_t = \sqrt{B_x^2 + B_y^2 + B_z^2} \]  

(13)

3. Particle Swarm Optimization

If the scalar of the measured magnetic field can be measured with high precision by other methods, the unknown coefficients in the calculation model can be fitted by using the least square principle by comparing the difference between the estimated total field and the real value. In this study, PSO algorithm was used to optimize the parameters.

PSO is a common method for parameter optimization of nonlinear equations [20,21], which was derived from the study of predation behavior of birds. The basic idea of PSO algorithm is to find the optimal solution through the cooperation and information sharing among individuals in the group.

With the PSO algorithm, different estimates of unknowns are assumed to be different particles. Then, the set of estimates with the least error between the estimated value and the actual value are found as the optimal estimate. To determine which set of estimates have the smaller error, an fitness value function is used to calculate the adaptive value for each set of estimates.

In the problem of calibration for triaxial magnetometers, the fitness value function is set as:

\[ f(x) = \sum_{i=1}^{N} (B_t - B)^2 \]  

(14)

The goal of the PSO algorithm is to find an optimal set of estimates that minimize the value of \( f(x) \).

4. Calibration method

4.1. The First Step: Calibration Method for \( k_{ll} \) and \( b \)

The accurate measurement of the bias and sensitivity coefficient of the magnetic sensor usually needs to be carried out in the magnetic shielding tube. A solenoid and a current source were used as the standard magnetic field generator. The device requires the use of a high-precision magnetometer to calibrate the linear relationship between the excitation current and the generated magnetic field.

In general, magnetic shielding tube cannot completely shield the geomagnetic field, there are trace remanence. The bias of the magnetometer is measured firstly. The sensor is placed in the magnetic shielding tube to find the angle with the maximum output, and the output at this time is recorded as \( V_{0+} \). Then turned the sensor 180° and recorded the output as \( V_{0-} \). The bias \( b \) of the axial sensor can be obtained as follows:

\[ b = (V_{0+} + V_{0-})/2 \]  

(15)

The remanence of magnetic shielding tube is:

\[ B_r = (V_{0+} - V_{0-})/2 \]  

(16)

In order to measure the sensitivity coefficient \( k_{ll} \), the sensor needs to be carefully aligned with the direction of the standard magnetic field, which can be determined by monitoring the extreme value of the output value. After alignment, the axial sensitivity coefficient \( k_{ll} \) can be measured by applying different standard magnetic fields.

\[ k_{ll} = \Delta V/\Delta B \]  

(17)

4.2. The Second Step: Calibration Method for \( k_{ij} \) and \( \Phi \)

Place the triaxial magnetometer in a uniform geomagnetic field with little fluctuation, such as a suburban environment with no artificial equipment or power lines nearby. Use a high-precision scalar magnetometer, such as an optical pump magnetometer, to measure magnetic induction intensity \( B \). And consider this value to be true. In this environment, rotate the triaxial magnetometer randomly and synchronously sample a series of output in each axial direction, recorded as \( V_i \) and \( i \) as the serial number of the sample.
The fitness value function is calculated according to (13). \( k_{ij}, b \) and \( V_i \) are replaced by measurements, and the rest of the nine unknown \( k_{ij}, \Phi \) are assigned an estimate. PSO algorithm is used to calculate the optimal estimate of these 9 parameters.

5. Experiment and results

The experiment was conducted in a magnetic shielding tube. The remanence in the magnetic shielding tube was about 22nT, and the fluctuation was within 0.05nT. This magnetic shielding tube meets the requirements of the experiment. The measurement accuracy of the optical pump magnetometer is 0.05nT. The solenoid, excitation current source and 24-bit 100kSPS data acquisition card used in the experiment are all self-made. And the triaxial magnetometer used in this experiment was Bartington's MAG-03MSL. The mean square deviation of the uncertainty that under the condition of generating 1Gs magnetic field was less than 0.1nT.

In order to reduce the influence of temperature change on the experiment, the experiment was conducted in a temperature controlled environment. The temperature range was 25°C ± 2°C. And the formal experiment did not start until 60 minutes after the equipment was switched on.

In the first step of the experiment, \( k_{xx}, k_{yy}, k_{zz}, b_x, b_y, b_z \) of the triaxial magnetometer were measured according to the method in chapter IV.A. The measurement results are shown in table 1.

| mean  | 0.9987 | 0.9994 | 0.998 | -86.3 | 80.2 | 49.9 |

The second experiment was conducted in the suburbs. The scalar of the magnetic field was 49650nT. The triaxial magnetometer was rotated randomly in this magnetic field. 200 sets of data were sampled. The first 100 sets of data were used to calibrate and the last 100 sets of data were used to verify.

Using the first 100 sets of data, PSO method was used for fitting. Calculation was carried out by using the commercial software MATLAB. After calibration, the calibration equation of the triaxial magnetometer was calculated as

\[
\begin{pmatrix}
\hat{B}_x \\
\hat{B}_y \\
\hat{B}_z
\end{pmatrix} = \begin{pmatrix}
0.9985 & 0.0016 & 0.0023 \\
0.0014 & 0.9996 & 0.0002 \\
0.0022 & 0.0002 & 0.9982
\end{pmatrix}^{-1} \begin{pmatrix}
V_x \\
V_y \\
V_z
\end{pmatrix} - \begin{pmatrix}
-86.3 \\
80.2 \\
49.9
\end{pmatrix}
\]

Then the last 100 sets of data were used to calculate the estimated value of the scalar of the magnetic field. The uncalibrated and calibrated results are shown in figure 4. The mean value before calibration was 49637nT, peak-to-peak value was 353nT, and the mean variance was 82nT. After calibration, the mean value was 49647nT, peak-to-peak value was 56nT, and mean variance was 11nT.
6. Discussion and conclusion
In this paper, two important sources of the non-orthogonal errors of triaxial magnetometers were analyzed. The non-orthogonal errors mainly come from the transverse sensitivity of uniaxial sensor and non-orthogonal between each axial sensors. These two types of error sources were separated from the original model by mathematical analysis.

And then, a two-step calibration method of triaxial magnetometers was designed. In the first step, some parameters were measured, which reduces the uncertainty and equation dimension in the subsequent parameter optimization process, in order to make the process of parameter optimization solution more rapid and accurate. In the second step, PSO algorithm was used to optimize the remaining parameters.

A triaxial magnetometer with uncertainty of 0.1nT was used in the experiment, the peak-to-peak value of the calculated result of the scalar of magnetic field was 56nT within the range of 49650nT after calibration. It show that this calibration method was effective and could be extended to other calibration applications of triaxial magnetometers.

References
[1] C.B Wan, M.C Pan, Q Zhang, F.H. Wu, L. Pan, X.Y. Sun 2018 Sensors and Actuators A 278 11–17
[2] Q. Zhang, X. Li, H.L. Pan, J.T. Wang, Z.J. Zhao 2018 Sensors and Actuators A 276 83–90
[3] Z. Zalevsky, Y. Bregman, N. Salomonski, H. Zafir 2012 J. Appl.Geophys 84 70–76
[4] H.F. Pang, J. Li, D.X. Chen, M.C. Pan, S.T. Luo, Q. Zhang, F.L. Luo 2013 Measurement 46, 1600–1606
[5] J M G Merayo, P Brauer, F Primdahl, J R Petersen, O V Nielsen 2000 Meas. Sci. Technol. 11 120–132
[6] Rong Zhu, Zhaoying Zhou 2006 Sensors and Actuators A 127 340–344.
[7] J.M.G. Merayo, F. Primdahl, P. Brauer, T. Risbo, N. Olsen, T. Sabaka 2001 Sensors and Actuators A 89 185–196
[8] Jan Včelák 2006 J. Appl. Phys. 99 08D913
[9] C. C. FOSTER, G. H. ELKAIM 2008 IEEE TRANSACTIONS ON AEROSPACE AND ELECTRONIC SYSTEMS 44 3 1070-1078
[10] H Ghanbarpour Asl, S H Pourtakdoust, M Samani 2009 Proc. IMechE 223 Part G: J. Aerospace Engineering 729-739
[11] H.F. Pang, D.X. Chen, M.C. Pan, S.T. Luo, Q. Zhang, J. Li, F.L. Luo 2013 IEEE TRANSACTIONS ON MAGNETICS 49 9 5011-5015
[12] H.F. Pang, Q. Zhang, W. Wang, J.Y. Wang, J. Li, S.T. Luo, et al. 2013 Journal of Magnetism and Magnetic Materials 346 5-10
[13] S.T. Luo, H.F. Pang, J. li, Q. Zhang, D.X. Chen, M.C. Pan, F.L. Luo 2013 Measurement 46 3918–3923
[14] H.F. Pang, D.X. Chen, M.C. Pan, S.T. Luo, Q. Zhang, J. li, F.L. Luo 2014 IEEJ Trans 9 324–328
[15] H.F. Pang, M.C. Pan, J.F. Chen, J. Li, Q. Zhang, S.T. Luo 2016 Measurement 93 409–413
[16] X.N. Zhu, T. Zhao, D.F. Cheng, Z.J. Zhou 2017 Meas. Sci. Technol 28 055106 (7pp)
[17] Z.C. Yang, S.G. Yan, B. Li 2017 IEEE MAGNETICS LETTERS 8 6508505
[18] Z.Y. Wu, Y.X. Wu, X.P. Hu, M.P. Wu 2011 IEEE International Symposium on Robotic & Sensors Environments
[19] BAGUS ADIWILUHUNG RIWANTO, TUOMAS TIKKA, ANTTI KESTILA”, JAAN PRAKS 2017 IEEE TRANSACTIONS ON AEROSPACE AND ELECTRONIC SYSTEMS, 53 2
[20] J. Kennedy, R. Eberhart 1995 Proceedings of the IEEE International Conference on Neural Networks 1942-1945
[21] E. Mezura-Montes, C.A. Coello 2011 Swarm and Evolutionary Computation 1 173–194