Crossover from Jamming to Clogging Behaviors in Heterogeneous Environments

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Jamming describes a transition from a flowing or liquid state to a solid or rigid state in a loose assembly of particles such as grains or bubbles. In contrast, clogging describes the ceasing of the flow of particulate matter through a bottleneck. It is not clear how to distinguish jamming from clogging, nor is it known whether they are distinct phenomena or fundamentally the same. We examine an assembly of disks moving through a random obstacle array and identify a transition from clogging to jamming behavior as the disk density increases. The clogging transition has characteristics of an absorbing phase transition, with the disks evolving into a heterogeneous phase-separated clogged state after a critical diverging transient time. In contrast, jamming is a rapid process in which the disks form a homogeneous motionless packing, with a rigidity length scale that diverges as the jamming density is approached.

The concept of jamming is used in loose assemblies of particles such as grains or bubbles to describe the transition from an easily flowing fluidlike state to a rigid jammed or solidlike state \cite{14}. Liu and Nagel proposed a generalized jamming phase diagram combining temperature, load, and density, where a particularly important point is the density $\phi_j$ at which jamming occurs \cite{1}. Jamming has been extensively studied in a variety of systems \cite{3,5}, and there is evidence that in certain cases, the jamming transition has the properties of a critical point, such as a correlation length that diverges as the jamming density is approached \cite{2,4}. A related phenomenon is the clogging that occurs for particles flowing through a hopper, where as a function of time there is a probability for arch structures to form that block the flow \cite{10,13}. Clogging is associated with the motion of particulate matter past a physical constraint such as wells, barriers, obstacles, or bottlenecks \cite{14,13}; however, it has not been established whether jamming and clogging are two forms of the same phenomenon or whether there are key features that distinguish jamming from clogging.

Here we show for frictionless disks moving through a random obstacle array that jamming and clogging are distinct phenomena and that a transition from clogging to jamming occurs as a function of increasing disk density. We identify the number of obstacles required to stop the flow and the transient times needed to reach a stationary clogged or jammed state. In the jamming regime, the obstacle density $\phi_j^c$ at which flow ceases decreases with increasing disk density, and the system forms a homogeneous jammed state when the rigidity correlation length associated with $\phi_j$ becomes larger than the average distance between obstacles. In contrast, during clogging the system organizes over time into a heterogeneous or phase-separated state, and the transient time diverges at a critical obstacle density $\phi^c_p$ that is independent of the disk density. The phase-separated state consists of regions with a density near $\phi_j$ coexisting with low density regions.

RESULTS

Time evolution to a jammed or clogged state. We numerically examine disks driven through a two-dimensional array of obstacles in the form of immobile disks. The total area density of the system is $\phi_{\text{tot}} = \phi_p + \phi_{ps}$, where $\phi_p$ is the area density of the moving disks and $\phi_{ps}$ is the area density of the obstacles. Starting from a uniformly dense sample, we apply a driving force and find that over time the system evolves either to a steady free flowing state or to a motionless clogged or jammed state. In Fig. 1a,b,c we illustrate the time evolution of a system with a disk density of $\phi_p = 0.2186$ and an obstacle density of $\phi_{ps} = 0.175$, beginning with the uniform density initial state in Fig. 1a. Upon application of a drive, we find a transient flowing state as shown in Fig. 1b, which gradually evolves into the final motionless phase-separated or clustered clogged state in Fig. 1c. For a higher disk density of $\phi_p = 0.436$, Fig. 1d,e,f shows that the same evolution from uniform initial state to transient flowing state to static clogged state occurs, but the dense clusters in the clogged state are larger. At much higher disk densities of $\phi_p = 0.785$, we find jamming behavior when the obstacle density is larger than a critical value $\phi_j^c$. Below $\phi_j^c$, the system quickly settles into steady state flow, as shown in Fig. 1g, for $\phi_{ps} = 0.043$ and in Fig. 1h for $\phi_{ps} = 0.065$. The magnitude of the flow decreases with increasing $\phi_{ps}$. Above $\phi_j^c$, the disks quickly form a jammed state when driven, as illustrated in Fig. 1i for $\phi_{ps} = 0.0872$. In contrast to the density phase-separated clogged states that form at lower $\phi_p$, the jammed states are homogeneously dense.

Velocity measurement of the transition from clogging to jamming. To characterize the system we perform a series of simulations with varied $\phi_p$ and $\phi_{ps}$.
FIG. 1: Clogging and jamming in obstacle arrays. Images of (a) initial, (b) transient flowing, and (c) final clogged state for mobile disks (dark blue open circles) driven in the positive $x$ direction through obstacles (red filled circles) in a sample with disk density $\phi_p = 0.2186$ and obstacle density $\phi_{ps} = 0.175$. Light blue lines indicate the disk trajectories over a fixed time period. The disks are initially in a flowing state and evolve into a phase-separated clogged state. (d) Initial, (e) transient flowing, and (f) final clogged state images for a sample with higher disk density $\phi_p = 0.436$ and $\phi_{ps} = 0.175$, which also begins in a uniform flowing state and evolves toward a phase-separated clogged state. The dense regions have a local disk density of $\phi_l = 0.84$. (g, h, i) Jamming under increasing obstacle density in a sample with $\phi_p = 0.785$. (g) At an obstacle density of $\phi_{ps} = 0.043$, we find steady state flow. (h) At $\phi_{ps} = 0.065$ the steady state flow is reduced but still present. (i) At $\phi_{ps} = 0.0872$, the system jams. The jammed state is much more uniform in density than the clogged state, and jamming occurs rapidly with almost no transient flow above a critical obstacle density $\phi^c_c$.

We measure the final velocity $V_0$ of the mobile disks after a fixed time interval, and average over ten disorder realizations. In Fig. 2 we plot a velocity heat map as a function of $\phi_{ps}$ versus $\phi_p$. The dashed white line indicates the density $\phi_{ps}$ above which $\phi_{tot} = \phi_j = \pi/2\sqrt{3} \approx 0.9069$, where a dense or jammed hexagonal disk packing would form. We find a flowing regime at small $\phi_{ps}$, a clogged regime for $\phi_p < 0.67$, and a jammed regime for $\phi_p > 0.67$. The critical obstacle density $\phi^c_c$ above which the velocity $V_0$ drops to zero in the clogging regime remains roughly constant at $\phi^c_c \approx 0.153$, independent of the value of $\phi_p$. In the jamming regime, the critical obstacle density $\phi^c_j$ separating flowing from jammed states decreases linearly with increasing $\phi_p$ and reaches $\phi^c_j = 0$ for $\phi_p \approx 0.9069$. This indicates that the transition to a clogged state is controlled by the average spacing $l_{ps} = 1/\sqrt{\phi_{ps}}$ between obstacles, similar to the manner in which hopper clogging is controlled by the aperture size. In contrast, the transition to a jammed state is controlled by a growing correlation length $\xi$ associated with the jamming or crystallization point $\phi_j$. We argue that the system jams when $\xi = l_{ps}$. If we assume that near jamming in a clean system, the correlation length grows as $\xi \propto (\phi_j - \phi_p)^{-\nu}$, then the transition to the jammed state varies with obstacle density according to $\phi^c_j \propto (\phi_j - \phi_p)^{2\nu}$. In Fig. 2 $\phi^c_j \propto \phi_p$, implying that $\nu = 1/2$, consistent with the exponent
FIG. 2: Clogging-jamming phase diagram. The heat map of the disk velocity $V_0$ after $10^6$ simulation time steps as a function of obstacle density $\phi_{ps}$ vs disk density $\phi_p$. Yellow indicates high $V_0$ and blue indicates zero $V_0$. The white dashed line is the density $\phi_{ps} = \pi/2\sqrt{3} \approx 0.9069$ at which the disks would form a hexagonal solid. Clogging occurs for $\phi_p < 0.67$, and the critical obstacle density for clogging is nearly independent of $\phi_p$, $\phi_c^c \approx 0.153$. Jamming occurs for $\phi_p > 0.67$, as indicated by the red vertical dashed line, and $\phi_c^j$, the critical obstacle density for jamming, decreases linearly with increasing $\phi_p$. The dots along $\phi_p = 0.234$ indicate the values of $\phi_{ps}$ shown in the time series of Fig. 3, while the dots along $\phi_p = 0.785$ indicate the values of $\phi_{ps}$ shown in the time series of Fig. 3. Above the dashed white line is a region in which no data can be taken.

$\nu = 1/2$ proposed for jamming in Refs. [20] [21], as well as with simulation measurements giving $\nu$ in the range 0.6 to 0.7 for two-dimensional bidisperse disks [6] [8]. The exponent we find is also in agreement with that observed for the shift in the jamming point in bidisperse disks on random pinning arrays [22]. Studies of bidisperse disk jamming with dilute obstacles very near $\phi_j$ also show that $\phi_j$ decreases linearly with obstacle density, giving $\nu = 1/2$ [23].

Previous simulations of bidisperse disks of radius $R_a = 0.5$ and $R_t = 0.7$ flowing through a periodic array of obstacles with radius $R_a = 0.5$ showed that clogging is strongly enhanced when $l_{ps} \lesssim 2.35$ [24]. This is because in order for a pair of disks, one large and one small, to fit between two obstacles, the lattice constant $a$ of the obstacle array must be at least $4R_a + 2R_t = 2.4$. In our monodisperse disk system, the obstacles are placed randomly, but one can obtain an estimate of the $l_{ps}$ for the onset of clogging by considering the holes in the obstacle array [25]. The minimum number $n$ of holes of size $R$ per obstacle required to prevent the obstacle arrange-ment from becoming anisotropic is $n = 5$. For a pair of disks to pass between two obstacles, the obstacle spacing must be at least $l_{ps} = 6R = 3.0$. This spacing can be achieved by placing the obstacles such that the holes cannot overlap, giving an effective obstacle radius of $3R$ and an obstacle density of $\phi_{ps} = (1/6)(\pi/(2\sqrt{3})) = 0.15$. In Fig. 2 the onset of clogging, $\phi_c^c \approx 0.153$, is close to this density. When $\phi_p \lesssim 0.15$, $\phi_c^j$ is no longer constant but decreases with decreasing $\phi_p$. At these low disk densities, mobile disks are trapped independently, so at least one additional obstacle must be added for every additional mobile disk, giving $\phi_c^j \propto \phi_p$.

Transient velocities near the clogging and jamming transitions. In Fig. 3, we show representative time series of the average velocity $V$ per mobile disk in the clogging regime at $\phi_p = 0.234$ for $\phi_{ps}$ ranging from $\phi_{ps} = 0.022$ to 0.175. At low obstacle densities such as $\phi_{ps} = 0.022$ and $\phi_{ps} = 0.044$, the disks reach a steady state flow after a very short transient time $\tau$. As $\phi_{ps}$ increases, $\tau$ increases, showing a divergence at the critical obstacle density $\phi_c^c$ where clogging first occurs, while for $\phi_{ps} > \phi_c^c$, $\tau$ decreases with increasing $\phi_{ps}$ and the disks reach a completely clogged state with $V = 0$. We fit $V(t) \propto A \exp(-t/\tau) + V_0$, and plot the resulting values of $\tau$ in Fig. 3 as a function of $\phi_{ps}$ for $\phi_p = 0.234$ to 0.349. In each case, $\tau$ diverges near $\phi_{ps} = 0.15$. We fit this divergence for $\phi_{ps} > \phi_c^c$ to a power law, $\tau \propto (\phi_{ps} - \phi_c^c)^\gamma$, as shown in the inset of Fig. 3 for $\phi_p = 0.234$, where $\gamma = -1.29 \pm 0.1$. The plot of $\gamma$ versus $\phi_p$ in the main panel of Fig. 3(c) indicates that $\gamma$ has a constant value in the range $-1.25$ to $-1.35$. The transient time and exponent values are similar to those found for the diverging time scales that appear near the irreversible-reversible transition in systems exhibiting random organization [26] [28] and near the depinning transition for colloids [29] and vortices [30] [31] driven over random pinning arrays. The power law exponents are also close to the value $\gamma = -1.295$ expected for the universality class of two-dimensional directed percolation [32], and we find similar values of $\gamma$ for $\phi_p < 0.67$ throughout the clogging regime. Directed percolation is often used to describe nonequilibrium absorbing phase transitions [32], and in our case the steady state flow corresponds to a fluctuating state, while the clogged state is the non-fluctuating or absorbed state.

In the jamming regime the transient times are much shorter, as shown by the plot of $V(t)$ in Fig. 3 for $\phi_p = 0.785$. In Fig. 3, $\tau$ versus $\phi_{ps}$ in the range $\phi_p = 0.785$ to 0.872 has a value that is an average of 20 times smaller than in the clogging regime from Fig. 3. The peak in $\tau$ shifts to lower $\phi_{ps}$ with increasing $\phi_p$, reflecting the behavior of the critical jamming density $\phi_c^j$. By fitting the curves in Fig. 3 to $\tau \propto (\phi_{ps} - \phi_c^j)^\gamma$, as demonstrated in the inset of Fig. 3, for $\phi_p = 0.872$, we obtain $\gamma \approx -0.66$, as shown in the plot of $\gamma$ versus $\phi_p$ in the main panel of Fig. 3. This indicates that there is a pronounced
Transient times for clogging and jamming. (a) The average velocity $V$ per mobile disk vs time in simulation time steps for samples with mobile disk density $\phi_p = 0.234$ at varied obstacle density $\phi_p = 0.022$ to 0.175, from top to bottom. The transient time decreases with increasing $\phi_p$, and the disks reach a clogged state for $\phi_p > 0.153$. (b) Transient times $\tau$ vs $\phi_p$ obtained from the curves in (a) by fitting $V \propto A \exp( -t/\tau ) + V_0$ for $\phi_p = 0.234$ to 0.349, from top to bottom. There is a divergence in $\tau$ near the clogging density of $\phi_c^p = 0.153$. (c) The value of the exponent $\gamma$ vs $\phi_p$ obtained by fitting the curves in (b) to $\tau \propto (\phi_p - \phi_c^p)^\gamma$. Inset: $\tau$ vs $\phi_p - \phi_c^p$ at $\phi_p = 0.234$. The pink line indicates a power law fit with $\gamma = -1.29 \pm 0.1$. (d) $V$ vs time for samples with $\phi_p = 0.785$ in the jamming regime for varied $\phi_p = 0.022$ to 0.087 from top to bottom. (e) $\tau$ vs $\phi_p$ obtained from the curves in (d) for $\phi_p = 0.785$ to 0.872, from top to bottom. The transient times are much shorter than those in the clogging regime in panel b. (f) Exponent $\gamma$ vs $\phi_p$ obtained by fitting the curves in e to $\tau \propto (\phi_p - \phi_c^p)^\gamma$. Inset: $\tau$ vs $\phi_p - \phi_c^p$ at $\phi_p = 0.872$. The pink line indicates a power law fit with $\gamma = -0.66$. 

In Fig. 3a we show a heat map of the transient time $\tau$ obtained by fitting $V(t) = A \exp( -t/\tau ) + V_0$. The transient times become large near the crossover from flowing to clogging for $\phi_p < 0.67$, while in the jamming regime for $\phi_p > 0.67$, the transient times are strongly reduced. This provides further evidence that in the clogging regime it is necessary for the system to organize over time into a clogged state, gradually forming phase-separated regions of high and low density as illustrated in Fig. 3b-f. In contrast, the jammed system has strong spatial correlations, and once the correlation length associated with $\phi_j$ is larger than the distance $l_{ps}$ between defects, very few disk rearrangements are needed to bring the system into a stationary, nonflowing state.

In Fig. 3a we show the transition from the flowing to the clogged or jammed state as a function of $\phi_c$ versus $\phi_p$ by identifying the points from Fig. 3a for which $V = 0.01$. Figure 3b shows the transient times $\tau$ along this transition line, and in Fig. 3c we plot the transient exponent $\gamma$. The dashed vertical line at $\phi_p = 0.67$ indicates a transition from clogging to jamming behavior, correlated with a change from $\gamma \approx -1.29$ in the clogging regime to $\gamma \approx -0.66$ in the jamming regime, as well as with a drop in $\phi_p$ and $\tau$. The point $\phi_p = 0.67$ can be interpreted as the density at which the correlation length associated with the jamming or crystallization density drops below $4R_d$, corresponding to two disk diameters. We find a third value of $\gamma$ for $\phi_p < 0.07$ in a density regime where the value of $\phi_p$ at which a clogged state appears decreases with decreasing $\phi_p$. This regime is dominated by the trapping of single disks rather than collective clogging dynamics.

Local disk densities in clogged and jammed states. The clogged and jammed systems can also be distinguished by examining the local disk density $\phi_{loc}$ measured in areas $6R_d \times 6R_d$ in size. In Fig. 3d we plot the local density distribution $P(\phi_{loc})$ averaged over ten realizations of the final clogged state for a system with $\phi_p = 0.5$ and $\phi_p = 0.175$. As shown in the inset of Fig. 3d, the disks phase separate into low density regions associated with the peak at $\phi_{loc} = 0.1$ and high density regions which produce a second peak at $\phi_{loc} = 0.85$. The local density of the dense regions is lower than the value of $\phi_{loc} = 0.9069$ for a dense ordered hexagonal disk ar-
FIG. 4: Transient time behavior. The heat map of the transient times $\tau$ obtained from fitting $V(t) = A \exp(-t/\tau) + V_0$ as a function of $\phi_{ps}$ vs $\phi_p$. Yellow indicates large $\tau$ and blue indicates small $\tau$. The dark dashed line is a guide to the eye marking the crossover from a flowing state to a clogged state, while the lower dashed white line indicates the transition from a flowing state to a jammed state. The upper dashed line is the crystallization density $\phi_{tot} = \pi/2\sqrt{3}$, and no data was obtained above this line. The dots along $\phi_p = 0.234$ indicate the values of $\phi_{ps}$ shown in the time series of Fig. 3(b), while the dots along $\phi_p = 0.785$ indicate the values of $\phi_{ps}$ shown in the time series of Fig. 3(c). The system must organize into a clogged state, giving large transient times in the clogging regime, but can quickly enter a jammed state, giving small transient times in the jamming regime.

arrangement due to the considerable disorder introduced in the packing by the randomly placed obstacles. In Fig. 6(a), $P(\phi_{loc})$ for a system with $\phi_p = 0.8$ and $\phi_{ps} = 0.06$ that reaches a jammed state has a single peak near $\phi_{loc} = 0.9$, reflecting the uniform disk density at jamming that is illustrated in the inset of Fig. 6(b).

**DISCUSSION**

Our results suggest that clogging and jamming processes have different dynamics. Clogging in the presence of random obstacles has signatures of an absorbing transition falling in a directed percolation universality class, and its dynamics are controlled by the average spacing of the obstacles. In the jamming that occurs for higher $\phi_{tot}$, the dynamics are controlled by the growing correlation length associated with $\phi_p$, the jamming or crystallization density of an obstacle-free system. These results show that jamming and clogging in obstacles are indeed different phenomena. Jamming is associated with an equilibrium critical point, the formation of a homogeneous rigid state, and short transient times to reach this state, while clogging is a nonequilibrium dynamical phenomenon in which the system evolves over an extended time into a strongly spatially heterogeneous state. Our results have implications for flow through heterogeneous media [33], erosion [34], depinning transitions in particle assemblies [35], and active matter in disordered environments [36, 37]. Experimentally our results could be tested using colloidal particles at low flow rates to reduce hydrodynamic effects. It would also be interesting to examine the effects of adding frictional contacts between the disks, since these can change the characteristics of the jamming transition [38, 39], or to replace the disks by elongated particles [40] or chains [41, 42].

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FIG. 6: Local density distributions in clogged and jammed systems. (a) The local density distribution \( P(\phi_{\text{loc}}) \) averaged over 10 realizations for a system with \( \phi_p = 0.5 \) and \( \phi_{ps} = 0.175 \) that reaches a clogged state in which the local density is bimodally distributed. Inset: Image of a clogged configuration from one of the realizations, showing the mobile disks (dark blue open circles) trapped by the obstacles (red filled circles). (b) \( P(\phi_{\text{loc}}) \) averaged over ten realizations for a jammed system with \( \phi_p = 0.8 \) and \( \phi_{ps} = 0.06 \) shows a single peak at \( \phi_{\text{tot}} \). Inset: Image of a jammed configuration from one of the realizations.

METHODS

Numerical simulation. We conduct simulations of nonoverlapping disks and obstacles confined to a two-dimensional plane. The system size is \( L \times L \) with \( L = 60 \), and we use periodic boundary conditions in both the \( x \) and \( y \) directions. We introduce \( N_p \) mobile disks of radius \( R_d = 0.5 \) along with \( N_{ps} \) obstacles represented by disks of radius \( R_o \) that are not allowed to move. The area coverage of the mobile disks is \( \phi_p = N_p \pi R_d^2 / L^2 \), the area coverage of the obstacles is \( \phi_{ps} = N_{ps} \pi R_o^2 / L^2 \), and the total area coverage is \( \phi_{\text{tot}} = \phi_p + \phi_{ps} \).

The disk dynamics are given by the overdamped equation of motion

\[
\frac{1}{\eta} \frac{\Delta r_i}{\Delta t} = F_{pp}^i + F_{\text{obs}}^i + F_d
\]

where \( \eta = 1 \) is the viscosity. The interaction between two disks at \( r_i \) and \( r_j \) is a short range harmonic repulsion, \( F_{dd}^i = k(r_{ij} - 2R_d) \Theta(r_{ij} - 2R_d) \hat{r}_{ij} \), where \( r_{ij} = |r_i - r_j| \), \( \hat{r}_{ij} = (r_i - r_j) / r_{ij} \), and \( \Theta \) is the Heaviside step function. We set \( k = 200 \), which is large enough that overlap between disks does not exceed 0.01\( R_d \), placing us in the hard disk limit. The interactions with mobile disks are given by \( F_{pp}^i = \sum_{j \neq i} N_p F_{ij}^d \), while the interactions with obstacles are given by \( F_{\text{obs}}^i = \sum_{p} N_{ps} F_{ij}^d \). We apply a uniform driving force \( F_d = F_0 \hat{x} \) to all mobile disks, with \( F_0 = 0.5 \). We initialize the system by placing \( N_p + N_{ps} \) disks of reduced radius in randomly chosen nonoverlapping positions, and then gradually expanding the radii to size \( R_d \) while allowing all disks to move. This produces a randomized packing of homogeneous density with no internal tensions. We then randomly assign \( N_{ps} \) of the disks to be obstacles, and apply an external driving force. After a fixed simulation time of \( 1 \times 10^9 \) simulation time steps, we determine whether the system has reached a clogged or jammed state based on whether the average disk velocity \( V \) has dropped to zero.

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