Quantum Criticality and Yang-Mills Gauge Theory

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ABSTRACT: We present a family of nonrelativistic Yang-Mills gauge theories in $D+1$ dimensions whose free-field limit exhibits quantum critical behavior with gapless excitations and dynamical critical exponent $z = 2$. The ground state wavefunction is intimately related to the partition function of relativistic Yang-Mills in $D$ dimensions. The gauge couplings exhibit logarithmic scaling and asymptotic freedom in the upper critical spacetime dimension, equal to $4 + 1$. The theories can be deformed in the infrared by a relevant operator that restores Poincaré invariance as an accidental symmetry. In the large-$N$ limit, our nonrelativistic gauge theories can be expected to have weakly curved gravity duals.
We present a class of nonrelativistic Yang-Mills gauge theories which exhibit anisotropic scaling between space and time. Our motivation originates from several different areas of physics, which have been experiencing a stimulating confluence of theoretical ideas recently: condensed matter theory, in particular quantum critical phenomena, string theory, and gauge-gravity duality.

In our study, we consider the case of $D+1$ spacetime dimensions, continuing the viewpoint advocated in [1] that the interface of condensed matter and string theory is best studied from the vantage point of arbitrary number of dimensions, even though practical applications to condensed matter are likely to be expected only for $D \leq 3$.

The theories presented here are candidates for the description of new universality classes of quantum critical phenomena in various dimensions. They combine the idea of non-Abelian gauge symmetry, mostly popular in relativistic high-energy physics, with the concept of scaling with non-relativistic values of the dynamical critical exponent, $z \neq 1$. In combination, this anisotropic scaling together with Yang-Mills symmetry opens up a new perspective on gauge theories, changing some of the basic features of relativistic Yang-Mills such as the critical dimension in which the theory exhibits logarithmic scaling.

Since our theories can be constructed for any compact gauge group, the choice of the $SU(N)$, $SO(N)$ or $Sp(N)$ series yields a class of theories with anisotropic scaling and a natural large-$N$ expansion parameter. These theories can then be expected to have weakly-curved gravitational duals, perhaps leading to new realizations of the AdS/nonrelativistic CFT correspondence which has attracted considerable attention recently [3–6]. Finally, it turns out that our theories are intimately related to relativistic theories in one fewer dimension, and therefore can shed some new light on the dynamics of the relativistic models.

\footnote{A different proposal for a quantum critical gauge theory, in $2 + 1$ dimensions and with $G = SU(2)$, was made in [2].}
1. Theories of the Lifshitz Type

We work on a spacetime of the form $\mathbb{R} \times \mathbb{R}^D$, with coordinates $t$ and $\mathbf{x} \equiv (x^i)$, $i = 1, \ldots D$, equipped with the flat spatial metric $\delta_{ij}$ (and the metric $g_{tt} = 1$ on the time dimension). The theories proposed here are of the Lifshitz type, and exhibit fixed points with anisotropic scaling characterized by dynamical critical exponent $z$ (see, e.g., [7]),

$$x \to b x, \quad t \to b^z t. \quad (1.1)$$

We will measure dimensions of operators in the units of spatial momenta, defining

$$[x^i] = -1, \quad [t] = -z. \quad (1.2)$$

The prototype of a quantum field theory with nontrivial dynamical exponent $z$ is the theory of a single Lifshitz scalar $\phi(x, t)$. In its simplest incarnation, this theory is described by the following action,

$$S = \frac{1}{2} \int dt \, d^D x \left\{ (\dot{\phi})^2 - \frac{1}{4\kappa^2} (\Delta \phi)^2 \right\}, \quad (1.3)$$

where $\Delta \equiv \partial_i \partial_i$ is the spatial Laplacian. Throughout most of the paper we adhere to the nonrelativistic notation, and denote the time derivative by “$\dot{\cdot}$.”

The Lifshitz scalar is a free-field fixed point with $z = 2$. The engineering dimension of $\phi$ is $[\phi] = D/2 - 1$, i.e., the same as the dimension of the relativistic scalar in $D$ spacetime dimensions, implying an interesting shift in the critical dimensions of the $z = 2$ system compared to its relativistic cousin.

Note that the potential term in the Lifshitz action (1.3) is of the form

$$\left( \frac{\delta W[\phi]}{\delta \phi} \right)^2, \quad (1.4)$$

where $W$ is the Euclidean action of a massless relativistic scalar in $D$ dimensions,

$$W[\phi] = \frac{1}{2\kappa} \int d^D x (\partial_i \phi \partial_i \phi). \quad (1.5)$$

When Wick rotated to imaginary time $\tau = it$, the action can be written as a perfect square,

$$S = \frac{i}{2} \int d\tau \, d^D x \left\{ \left( \partial_\tau \phi + \frac{1}{2\kappa} \Delta \phi \right)^2 \right\}, \quad (1.6)$$

because the cross-term in (1.6) is a total derivative, $\dot{\phi} \Delta \phi/\kappa = -\dot{W}$, and can be dropped.

The coupling $\kappa \in [0, \infty)$ parametrizes a line of fixed points. If we wish, we can absorb $\kappa$ into the rescaling of the time coordinate and a rescaling of $\phi$.

In the original condensed-matter applications [8–10], the anisotropy is between different spatial dimensions, and the Lifshitz scalar is designed to describe the tricritical point at the juncture of the phases with a zero, homogeneous and spatially modulated condensate.
2. Yang-Mills Theory and Quantum Criticality

Our nonrelativistic gauge theory in \( D + 1 \) dimensions will be similarly associated with with relativistic Yang-Mills in \( D \) dimensions.

Our gauge field is a one-form on spacetime, with spatial components \( A_i = A^a_i(x^j, t)T_a \) and a time component \( A_0 = A^0_i(x^j, t)T_a \). The Lie algebra generators \( T_a \) of the gauge group \( G \) (which we take to be compact and simple or a \( U(1) \)) satisfy commutation relations \( [T_a, T_b] = if_{abc}T_c \). We normalize the trace on the Lie algebra of \( G \) by \( \text{Tr}(T_a T_b) = \frac{1}{2} \delta_{ab} \).

Our theory will be invariant under gauge symmetries

\[
\delta_\epsilon A_i = \left( \partial_i \epsilon^a + f_{bc}^a A^b_i \epsilon^c \right) T_a \equiv D_i \epsilon, \quad \delta_\epsilon A_0 = \dot{\epsilon} - i[A_0, \epsilon]. \tag{2.1}
\]

Gauge-invariant Lagrangians will be constructed from the field strengths

\[
E_i = \left( \dot{A}_i^a - \partial_i A_0^a + f_{bc}^a A_i^b A_0^c \right) T_a = \dot{A}_i - \partial_i A_0 - i[A_i, A_0],
\]
\[
F_{ij} = \left( \partial_i A_j^a - \partial_j A_i^a + f_{bc}^a A_i^b A_j^c \right) T_a = \partial_i A_j - \partial_j A_i - i[A_i, A_j]. \tag{2.2}
\]

We will now construct a theory which has \( z = 2 \) in the free field limit. The engineering dimensions of the gauge field components at the corresponding Gaussian fixed point will be

\[
[A_i] = 1, \quad [A_0] = 2. \tag{2.3}
\]

The Lagrangian should contain a kinetic term which is quadratic in first time derivatives, and gauge invariant. The unique candidate for this kinetic term is \( \text{Tr}(E_i E_i) \), of dimension \( \text{Tr}(E_i E_i) = 6 \). One can then follow the strategy of effective field theory, and add all possible terms with dimensions \( \leq 6 \) to the Lagrangian. This would allow terms such as \( \text{Tr}(F_{ij} F_{kl} F_{ik}) \), \( \text{Tr}(D_i F_{jk} D_j F_{ik}) \), \( \text{Tr}(D_i F_{ik} D_j F_{j k}) \), (all of dimension six), a term \( \text{Tr}(F_{ij} F_{ij}) \) of dimension four, etc. One could indeed define the theory in this fashion, study the renormalization-group (RG) behavior in the space of all the couplings, and look for possible fixed points. This interesting problem is beyond the scope of the present paper. Instead, we pursue a different strategy, and limit the number of independent couplings in a way compatible with renormalization. The trick that we will use is familiar from a variety of areas of physics, such as dynamical critical systems [11, 12], stochastic quantization [13, 14], and nonequilibrium statistical mechanics.

Inspired by the structure of the Lifshitz scalar theory, we take our action to be

\[
S = \frac{1}{2} \int dt d^D x \left\{ \frac{1}{e^2} \text{Tr}(E_i E_i) - \frac{1}{g^2} \text{Tr} \left( (D_i F_{ik})(D_j F_{jk}) \right) \right\}. \tag{2.4}
\]

This is a Lagrangian with \( z = 2 \) and no Galilean invariance. As a result, there is no symmetry relating the kinetic term and the potential term, and therefore no \textit{a priori} relation between the renormalization of the two couplings \( e \) and \( g \). The potential term is again the square of the equation of motion that follow from an action: the relativistic Yang-Mills in \( D \) Euclidean dimensions. When a theory in \( D + 1 \) dimensions is so constructed from the action of a theory in \( D \) dimensions, we will say that it \textit{satisfies the detailed balance condition}, borrowing the terminology common in nonequilibrium dynamics.
3. At the Free-Field Fixed Point with \( z = 2 \)

The free-field fixed point will be obtained from (2.4) by taking \( e \) and \( g \) simultaneously to zero. Keeping both the kinetic and the potential term finite in this limit requires rescaling the gauge field, \( \tilde{A}_i^a \equiv A_i^a / \sqrt{eg} \), and keeping \( \tilde{A}_i^a \) finite as we take \( e \) and \( g \) to zero. This gives

\[
S = \frac{1}{2} \int dt d^D x \left\{ \frac{g}{e} \text{Tr}(\tilde{E}_i \tilde{E}_i) - \frac{e}{g} \text{Tr} \left( (\partial_i \tilde{F}_{ik})(\partial_j \tilde{F}_{jk}) \right) \right\},
\]

(3.1)

where \( \tilde{E}_i \) and \( \tilde{F}_{ij} \) are the linearized field strengths of \( \tilde{A}_i \).

We see that there is actually a line of free fixed points, parametrized by the dimensionless ratio

\[
\lambda = \frac{g}{e}.
\]

(3.2)

As in the Lifshitz scalar theory, if we wish we can absorb \( \lambda \) into a rescaling of time, \( t_{\text{new}} = t / \lambda \).

The special properties of the Lifshitz scalar make it possible to determine the exact ground-state wavefunction [9],

\[
\Psi[\phi(x)] = \exp \left\{ -\frac{1}{4\kappa} \int d^D x (\partial_i \phi \partial_i \phi) \right\}.
\]

(3.3)

This \( \Psi \) is equal to \( \exp(-W[\phi]/2) \), where \( W[\phi] \) is the action (1.5) of the relativistic scalar in \( D \) dimensions. The norm \( \int D\phi(x) \Psi^* \Psi \) equals the partition function of this relativistic theory.

Similarly, we can relate the ground-state wavefunction of our \( z = 2 \) gauge theory to the partition function of relativistic Yang-Mills. The momenta and the Hamiltonian are

\[
\tilde{P}_i^a = \frac{\lambda}{2} \tilde{E}_i^a, \quad H = \frac{2}{\lambda} \int dt d^D x \text{Tr} \left\{ \tilde{P}_i \tilde{P}_i + \frac{1}{4}(\partial_i \tilde{F}_{ik})(\partial_j \tilde{F}_{jk}) \right\}.
\]

(3.4)

The preferred role of time suggests a natural gauge-fixing condition,

\[
A_0 = 0.
\]

(3.5)

This does not fix all gauge symmetries, leaving the subgroup of time-independent gauge transformations unfixed. We can eliminate the residual gauge invariance by setting

\[
\partial_i A_i = 0
\]

(3.6)

at some fixed time slice with \( t = t_0 \). However, since the equations of motion obtained from varying \( A_0 \) yield \( \partial_i \tilde{A}_i - D_i D_i A_0 = 0 \), once we adopt the gauge (3.5) and select (3.6) at \( t = t_0 \), this condition will continue to hold for all \( t \). First we consider the linearized theory describing the free-field fixed point. The Hamiltonian operator can be written as

\[
H = \frac{1}{\lambda} \int dt d^D x \text{Tr}(Q^\dagger Q + QQ^\dagger) = \frac{2}{\lambda} \int dt d^D x \text{Tr}(Q^\dagger Q) + E_0,
\]

(3.7)
where we have defined

$$Q^a_i = -\frac{\delta}{\delta A^a_i} + \frac{1}{2} \partial_k \bar{F}^a_{ki}. \quad (3.8)$$

The vacuum energy $E_0$ is a field-independent normal-ordering constant. The energy will be minimized by solutions of $Q\Psi[A_i] = 0$. Thus, we obtain the ground-state wavefunction

$$\Psi[A_i(x)] = \exp \left\{ -\frac{1}{4} \int \text{Tr}(\bar{F}^i_{ij} \bar{F}^i_{ij}) \right\} = \exp \left\{ -\frac{1}{4eg} \int \text{Tr}(F^i_{ij} F^i_{ij}) \right\}, \quad (3.9)$$

where we have restored the original normalization of the linearized gauge field. The exponent is one half of the quadratic part of the Euclidean Yang-Mills action

$$W[A_i] = \frac{1}{2g_{YM}^2} \int d^Dx \text{Tr}(F^i_{ij} F^i_{ij}) \quad (3.10)$$

in Lorenz gauge, if we identify the Yang-Mills coupling as

$$g_{YM}^2 = eg. \quad (3.11)$$

A closer inspection shows that (3.9) is indeed the correct ground-state wavefunction of the linearized theory. In particular, the energy is bounded from below, and the spectrum of excitations at the free-field fixed point consists of $D - 1$ polarizations of gauge bosons with the gapless nonrelativistic dispersion relation

$$\omega^2 = \left(\frac{k^2}{\lambda} \right)^2, \quad (3.12)$$

Note the presence of both positive and negative frequency modes: The concept of particles and antiparticles in our model is similar to that of a relativistic theory.

The form of the ground-state wavefunction (3.9) should be contrasted with that of relativistic Yang-Mills theory in 3+1 dimensions, which also obeys the detailed balance condition, with the Chern-Simons action

$$W_{CS} = \int \omega_3(A), \quad \omega_3(A) = \text{Tr} \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right). \quad (3.13)$$

This makes $\Psi \sim \exp(-W_{CS}/2)$ a candidate solution of the Schrödinger equation. This solution is unphysical, for many reasons (see [15] for a detailed discussion), while in our case the solution (3.9) of the Schrödinger equation is the physical ground-state wavefunction.

The anisotropic scaling with $z = 2$ shifts the critical dimension of the system: The engineering dimensions of the couplings at our free fixed point are

$$[e^2] = [g^2] = 4 - D. \quad (3.14)$$

The system is at its upper critical dimension when $D = 4$, i.e., in 4+1 spacetime dimensions.
4. Quantization

The quantum theory is formally defined by the path-integral

\[ \int \mathcal{D}A_i \exp(iS[A]), \]

where \( \mathcal{D}A_i \) is the gauge-invariant measure on the space of gauge orbits. As in relativistic quantum field theory, it will be convenient to perform the Wick rotation to imaginary time, \( \tau = it \). After the Wick rotation, our action can be rewritten in the following form,

\[ S = \frac{i}{2} \int d\tau d^Dx \text{Tr} \left( \frac{1}{e^2} E_k E_k + \frac{1}{g^2} D_i F_{ik} D_j F_{jk} \right) \]

\[ = \frac{i}{2} \int d\tau d^Dx \text{Tr} \left\{ \left( \frac{1}{e^2} E_k + \frac{1}{g} D_i F_{ik} \right) \left( \frac{1}{e^2} E_k + \frac{1}{g} D_j F_{jk} \right) \right\}, \]

i.e., as a sum of squares, one for each field \( A_i^a \). The cross-terms are a combination of a total derivative, \( \dot{A}_i D_k F_{ik}/(2eg) = \dot{W}[A_j] \), and a gauge transformation, \( (D_i A_0)(D_k F_{ik})/(2eg) = \delta_{A_0} W[A_j] = 0 \), of the \( D \)-dimensional Euclidean Yang-Mills action \( W[A_i] \). The imaginary-time action (4.2) is formally equivalent to the action that appears in stochastic quantization [13] of the relativistic Yang-Mills theory in \( D \) dimensions, if we identify \( \tau \) as the “fictitious time,” and \( A_0 \) as the field introduced in the process of “stochastic gauge fixing” [16].

Using an auxiliary field \( B_j^a \), we can rewrite

\[ \int \mathcal{D}A_i \exp(iS) = \int \mathcal{D}A_i \mathcal{D}B_i \exp \left\{ -\int d\tau d^Dx \left( B_i \left( \frac{1}{e^2} E_i + \frac{1}{g} D_k F_{ik} \right) - \frac{1}{2} B_i B_i \right) \right\}. \]

The key to the renormalizability of this setup [16] is to show that any field-dependent counterterm generated is at least linear in \( B_i^a \). As a result, the theory inherits the renormalization properties from the associated theory in \( D \) dimensions, with the only new renormalization to be performed being that of the relative normalization of time and space.

In the critical dimension \( 4 + 1 \), this “quantum inheritance principle” suggests that our nonrelativistic gauge theory could be asymptotically free. We define the RG beta functions \( \beta_e = \mu (d/d\mu) e(\mu) \), \( \beta_g = \mu (d/d\mu) g(\mu) \), with \( e(\mu) \) and \( g(\mu) \) the renormalized couplings and \( \mu \) the RG scale. Borrowing results from stochastic quantization, we get at one loop in \( g_{YM} [14, 17, 18] \)

\[ \beta_e = -\frac{3}{2} C_2 e^2 g + \ldots, \quad \beta_g = -\frac{35}{6} C_2 eg^2 + \ldots, \]

where \( C_2 \equiv c_2(\mathcal{G})/(4\pi)^2 \), with \( c_2(\mathcal{G}) \) the quadratic Casimir of the adjoint of \( \mathcal{G} \) (for example, \( c_2(SU(N)) = N \)). This RG flow pattern implied by [14] can be disentangled by switching

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\(^2\)An important part of the argument is to show that the Jacobian of the change of variables from \( A_i \) to \( \dot{A}_i + D_k F_{ik}/\lambda \) is independent of \( A_k \), see [16] for details. One can also represent the Jacobian by integrating in a pair of fermions \( \eta_i^a, \bar{\eta}_i^a \). Together with \( A_i^a \) and \( B_j^a \), these fields can be interpreted as members of a supermultiplet of Parisi-Sourlas supersymmetry. We will not use such a supersymmetric framework in this paper.
from \((e,g)\) to \((g_{YM},\lambda)\), which yields
\[
\mu \frac{dg_{YM}}{d\mu} = -\frac{11}{3} C_2 g_{YM}^3 + \mathcal{O}(g_{YM}^5), \quad \mu \frac{d\lambda}{d\mu} = -\frac{13}{3} C_2 g_{YM}^2 \lambda + \mathcal{O}(g_{YM}^4 \lambda). \tag{4.5}
\]
This is solved by
\[
\frac{1}{g_{YM}^2(\mu)} = \frac{1}{g_{YM}^2(\mu_0)} + \frac{22}{3} C_2 \log \left(\frac{\mu}{\mu_0}\right), \quad \lambda(\mu) = \lambda(\mu_0) \left(\frac{g_{YM}(\mu)}{g_{YM}(\mu_0)}\right)^{13/11}. \tag{4.6}
\]
The theory is indeed asymptotically free. Moreover, the one-loop beta function for \(g_{YM}\) coincides with that of relativistic Yang-Mills in four dimensions. The theory exhibits dimensional transmutation, with a dynamically generated scale \(\Lambda\). The line of Gaussian fixed points parametrized by \(\lambda\) is lifted: Nonzero \(g_{YM}\) induces a flow of \(\lambda\) towards smaller values.

In lower dimensions \(D < 4\), the theory superrenormalizable by power counting, and we expect it to become strongly coupled in the infrared (IR). It would be interesting to investigate this regime further, with possible applications to quantum critical systems in mind. Gauge-gravity duality might be the right technique how to understand such strongly coupled theories, especially at large \(N\).

5. Relevant Deformations: Softly Broken Detailed Balance

The asymptotic freedom of our theory in \(4 + 1\) dimensions suggests the possibility of using it as a UV completion of otherwise nonrenormalizable effective theories, and in particular, of relativistic Yang-Mills theory.

In theories that satisfy the detailed balance condition, we could add relevant deformations to \(W\), preserving the detailed balance condition. For example, adding \(m_0^2 \phi^2\) to \(W[\phi]\) of the Lifshitz scalar theory is such a deformation: the theory flows under this deformation to a \(z = 1\) theory with accidental relativistic invariance in the infrared. Our Yang-Mills theory has no relevant deformations of this type.

Alternatively, we can add relevant deformations directly to \(S\). This will represent a soft violation of the detailed balance condition. There is one relevant term that can so be added to \(S\),
\[
\Delta S = -m^{D-2} \int dt d^Dx \text{Tr}(F_{ij} F_{ij}). \tag{5.1}
\]
If \(m^{D-2} > 0\), this will lead naturally to a \(z = 1\) relativistic theory at long distances. No other, lower-dimensional nontrivial gauge-invariant scalar operators exist.

In order to make the relativistic symmetry in the infrared manifest, it is natural to rescale the time dimension,
\[
x^0 = ct. \tag{5.2}
\]
In terms of the UV variables, the speed of light is given by
\[
c = 2 m^{D/2-1} e. \tag{5.3}
\]
Until we couple matter to the Yang-Mills sector, the relativistic symmetry is protected by the gauge invariance if the gauge group is simple. In terms of the relativistic notation for the fields, $A_{\mu} = (A_0/c, A_i)$, our deformed classical action is

$$S + \Delta S = -\frac{1}{2g_{\text{eff}}^2} \int d^{D+1}x \left\{ \text{Tr}(F_{\mu\nu}F^{\mu\nu}) + \frac{m^{2-D}}{2g^2} \text{Tr} \left( (D_kF_{ki})(D_jF_{ji}) \right) \right\},$$ \hspace{0.5cm} (5.4)

The dimensionful effective Yang-Mills coupling of the infrared theory is given by

$$g_{\text{eff}}^2 = m^{1-D/2}e.$$ \hspace{0.5cm} (5.5)

From the perspective of the infrared free-field fixed point, the $(DF)^2$ term in (5.4) represents an irrelevant deformation, which modifies the relativistic massless dispersion relation $k_\mu k^\mu \equiv -k_0^2 + k^2 = 0$ to

$$k_\mu k^\mu = -\frac{m^{2-D}}{2g^2} (k_i k_i)^2.$$ \hspace{0.5cm} (5.6)

This correction only becomes important at high energies, confirming that microscopically there is no speed limit in this theory. In the relativistic spacetime coordinates, the dispersion relation asymptotes at large $k$ to $|k_0| = k^2/(2\mathcal{M})$, with $\mathcal{M} = m^{D/2-1}g$.

In the free-field limit $g_{\text{YM}} = 0$, this flow from the $z = 2$ UV fixed point to the relativistic $z = 1$ theory in the infrared is exact. Turning on $g_{\text{YM}}$ in $4+1$ dimensions leads to dimensional transmutation, implying a competition of scales: If $m \gg \Lambda$, the theory starts flowing towards the $z = 1$ IR fixed point before reaching strong coupling, while for $m \ll \Lambda$ it is driven to strong coupling first.

Another interesting possibility is to start with $m^{D-2} < 0$, which would be analogous to the spatially modulated phases of the Lifshitz scalar theory.

6. Conclusions

In this paper, we focused on the basic features of the simplest, bosonic version of Yang-Mills gauge theory with anisotropic scaling in $D+1$ dimensions. Clearly, many interesting questions remain open for further study.

One immediate question is whether this framework can be supersymmetrized, at least in the $D$ dimensional sense: One can consider replacing the relativistic $D$ dimensional theory in our construction with one of its supersymmetric extensions. If the quantum inheritance principle continues to hold in such cases, our construction might lead to new nontrivial non-relativistic RG fixed points in $D+1$ dimensions. One particularly tempting question to ask is how the $\mathcal{N} = 4$ super Yang-Mills CFT fits into this framework.

It should also be interesting to see whether the class of nonrelativistic gauge theories presented in this paper can be engineered from string theory, in particular from D-brane configurations. Such a construction would allow a more systematic study of these theories and their anticipated gravitational duals.
Acknowledgments

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