Critical Level Statistics in Two-dimensional Disordered Electron Systems

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(Received )

Abstract. The level statistics in the two dimensional disordered electron systems in magnetic fields (unitary ensemble) or in the presence of strong spin-orbit scattering (symplectic ensemble) are investigated at the Anderson transition points. The level spacing distribution functions $P(s)$’s are found to be independent of the system size or of the type of the potential distribution, suggesting the universality. They behave as $s^2$ in the small $s$ region in the former case, while $s^4$ rise is seen in the latter.

KEYWORDS: level statistics, level spacing distribution, random matrix theory, Gaussian unitary ensemble, Guassian symplectic ensemble, Anderson transition, universality

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The random matrix theory, first developed by Wigner in 1950’s in the field of nuclear physics, is now widely applied to many problems in solid state physics such as the energy spectrum of fine particles, conductance fluctuations in mesoscopic systems and the persistent current in mesoscopic rings [1]. In metals where the electronic states are extended, the level repulsion is strong and the distribution function $P(s)$ for the level spacing $s$ is well approximated by the Wigner surmise. On the other hand, in the insulating regime no level repulsion is expected since the electrons are localized and $P(s)$ becomes Poissonian. Natural question that arises from the above facts is how the properties of the level statistics are changed when the system undergoes the localization-delocalization transition, i.e., Anderson transition.

To answer this question, much numerical effort has been devoted to the three dimensional (3D) Anderson model. Shklovskii et al. are the first to notice that $P(s)$ at the Anderson transition becomes size-independent [2], and the critical exponent $\nu$ for the localization length can be extracted from the behavior of the size dependence of $P(s)$ near the Anderson transition. Evangelou et al. [3] as well as Hofstetter and his coworkers [4] made detailed analyses on the level statistics at the critical point, and $P(s)$ is again shown to be size independent and to be different from the Wigner surmise or Poissonian.

The analytical calculation of $P(s)$ by Kravtsov et al. [5] has suggested the possibility to obtain the critical exponent $\nu$ from the asymptotic behavior of $P(s)$ at the Anderson transition. They predicted that $P(s)$ falls off as

$$P(s) \sim \exp(-Bs^{2-\gamma}),$$

with $\gamma = 1 - 1/d\nu$, $d$ being the dimensionality of the system. It is further suggested that the whole behavior of $P(s)$ might well be described by

$$P(s) = As^\beta \exp(-Bs^{2-\gamma}),$$

with $A$ and $B$ determined by the normalization condition, and $\beta$ by the symmetry of the system. Setting $\nu \approx 1.4$ [6], we can estimate $\gamma$ at the three dimensional Anderson transition to be 0.76, and the exponent of the large $s$ behavior of $\ln P(s)$ becomes 1.24. The overall behavior of $P(s)$ has been claimed to be well fitted to [3, 4]

$$A \exp(-Bs^{1.24}).$$
It is further claimed that the same \( P(s) \) has been obtained in 3D disordered system without time reversal symmetry [7] and that it is consistent with the recent study on the critical exponent in magnetic fields [8].

The linear rise of \( P(s) \) even without time reversal symmetry is controversial, since it is believed that the behavior of \( P(s) \) in the small \( s \) region is determined by the symmetry of the system, and in the absence of time reversal symmetry, \( P(s) \) should rise as \( \sim s^2 \). The large \( s \) behavior of \( P(s) \) is recently questioned by Zharekeshev and Kramer [9], who have performed the large size diagonalization and asserted Poissonian behavior \( P(s) \propto \exp(-\kappa s) \) in large \( s \) limit, with \( \kappa < 1 \).

Thus the existence of the critical level statistics at the Anderson transition is well established, but it is fair to say that the functional form of \( P(s) \) is still controversial.

In this confusing situation, it is very important to study the critical level statistics in other universality classes in the different dimensionality where the extensive studies on the Anderson transition have been performed. One of the examples to show the Anderson transition is the quantum Hall (QH) regime, where the Anderson transition takes place at the Landau band center with the divergence of the localization length as \( 1/|E|^\nu \), \( E \) being the energy measured from the Landau band center and \( \nu \approx 2.4 \) [10, 11]. Independently of the above mentioned work on 3D transition, the level statistics in the quantum Hall regime has been studied by the present authors [12, 13]. It has been shown that the level statistics obeys the scaling behavior, and the size independent level distribution function \( P(s) \) has been obtained at the Landau band center. Another example is the two dimensional (2D) system with spin-orbit interactions which derive the localized states to the extended ones due to the anti-localization effect.

In this paper, we discuss the critical level statistics in two dimensional systems, especially \( P(s) \), at the Anderson transition in QH regime (unitary case) as well as in the presence of strong spin-orbit scattering (symplectic case). We have obtained the energy spectrum by diagonalizing the Hamiltonian with randomness for different systems and different sizes. The energy eigenvalues are then unfolded in order to make the average spacing unity [12, 13]. From the whole spectrum we take out levels in a central region with a width of one tenth of the total width, and discuss their statistics. The number of samples for each system size is chosen so that the total number of levels for which the statistics are considered should not be less than 25,000.
in the case of QH regime and 50,000 in the symplectic case, respectively.

First we discuss the QH case. The disordered potential is assumed to be \( \delta \)-correlated. Here we have adopted the random matrix Hamiltonian which describes the energy spectrum in the QH regime \([11, 12]\). The area of the sample is varied from 200 to 800 in units of \( 2\pi \ell^2 \) with \( \ell \) the magnetic length. In Fig. 1, the \( P(s) \) at the Landau band center is shown in both linear (Fig. 1(a)) and logarithmic scales (Fig. 1(b)). Different marks represent different system sizes. These figures indicate that the critical distribution is size independent, which is consistent with the scaling argument \([13]\).

Now we apply (2) using the value \( \nu \approx 2.4 \) which results in \( \gamma = 0.79 \). The result is shown by the broken lines. We see considerable deviation from the numerical data. Regarding \( \gamma \) as a fitting parameter, very good agreement is obtained with \( \gamma = 0.35 \), but this, together with \( \gamma = 1 - 1/d\nu \), gives absurd value of \( \nu(= 0.77) \) that violates the inequality \( \nu > 2/d \) \([14]\). It should be noted that the small \( s \) behavior of \( P(s) \) is proportional to \( s^2 \), consistent with the symmetry argument.

A similar difficulty also appears in the analysis of \( P(s) \) at the Anderson transition in the presence of spin-orbit interaction. Using the Ando model \([15]\) and estimating the critical disorder by the finite size scaling method \([6, 15, 16]\), we have obtained the critical level statistics. In the numerical simulation, the ratio between the spin flip transfer \( V_2 \) and the total transfer \( V = \sqrt{V_1^2 + V_2^2} \) (\( V_1 \) the non-spin-flip transfer) is set to 0.5, and the critical disorder \( W_c \) is found to be 5.75\( V \) and 6.7\( V \) for box distribution and Gaussian distribution of diagonal disorder, respectively. In the case of the box distribution the parameter \( W \) describing the strength of disorder is defined as the width of the box, while in the Gaussian distribution it is defined as \( 2\sqrt{3} \) times the r.m.s. of the fluctuation of the site energy. The factor \( 2\sqrt{3} \) is introduced so that both distributions may have the same variance for the same value of \( W \). The linear system size is 8, 12, 16 and 20 in units of the lattice constant.

The results can be plotted in a similar way as in Fig. 1. In Fig. 2, \( P(s) \) at the Anderson transition in the presence of strong spin-orbit scatterings is plotted in the logarithmic scale. It shows that the critical distribution is independent of the system size and the type of disorder. The behavior of the \( P(s) \) is similar to the results obtained recently by Schweitzer and Zharekeshev \([17]\) who have diagonalized very large systems in the case of box
type potential distribution. The critical distribution function $P(s)$ cannot be fitted to the theoretically suggested form $P(s) \propto s^4 \exp(-Bs^{2-\gamma})$ with $\gamma = 0.82$ (broken line) which corresponds to the critical exponent $\nu = 2.8$. The best fit in the linear scale is obtained by putting $\gamma = 0.35$. Though this value of $\gamma$ does not reproduce the critical exponent $\nu$, good agreement in the small $s$ region supports $s^4$-rise of $P(s)$.

In summary, we have obtained the level spacing distribution function in 2D quantum Hall regime as well as in the presence of the spin-orbit interaction. The level spacing functions $P(s)$'s at the Anderson transition are shown to be independent of the system size or of the type of the potential distributions. In the QH case, $P(s)$ at the Anderson transition rises as $s^2$ while it rises as $s^4$ in the symplectic case.

The numerically obtained data can not be fitted to the theoretically proposed functional form (2) with $\gamma = 1 - 1/d\nu$. This should not be taken seriously, since the argument of ref. is valid only in the large $s$ region, and the overall functional form, though very successful in 3D case, is only a conjecture. The numerical data in the large $s$ region qualitatively support the asymptotic behavior predicted by them. From the practical point of view, however, it is very difficult to determine quantitatively the behavior of $P(s)$ in this region, since the number of data corresponding to large spacing is extremely small.

In 3D, $P(s)$ at the critical points has been reported to rise linearly in $s$ irrespectively of the symmetry of the system. In ref. the time reversal symmetry is broken by gauge fields which can be absorbed into boundary conditions, and the system involves no real magnetic field. A study of more realistic systems will be necessary to clarify the behavior of the critical statistics in 3D unitary ensemble.

The authors would like to thank Professor V.E. Kravtsov and Dr. T. Kawarabayashi for fruitful discussions.

References

[1] For recent review on the application of the random matrix theory to various topics, see, for example, Quantum chemistry and technology in the mesoscopic level, ed. H. Hasegawa. J. Phys. Soc. Jpn. Supplement A 63 (1994).
[2] B.I. Shklovskii, B. Shapiro, B.R. Sears, P. Lambrianides and H.B. Shore: Phys. Rev. B47 (1993) 11487.

[3] S.N. Evangelou: Phys. Rev. B49 (1994) 16805.

[4] E. Hofstetter and M. Schreiber: Phys. Rev. B49 (1994) 14726; I. Varga, E. Hofstetter, M. Schreiber and J. Pipek: Phys. Rev. B, to be published.

[5] V.E. Kravtsov, I.V. Lerner, B.L. Altshuler and A.G. Aronov: Phys. Rev. Lett. 72 (1994) 888, A.G. Aronov, V.E. Kravtsov and I.V. Lerner: JETP Letters 59 (1994) 39, Phys. Rev. Lett. 74 (1994) 1174.

[6] A. MacKinnon and B. Kramer: Phys. Rev. Lett. 47 (1981) 1546, Z. Phys. B 53 (1983) 1; E. Hofstetter and M. Schreiber: Europhys. Lett. 21 (1993) 933.

[7] E. Hofstetter and M. Schreiber: Phys. Rev. Lett. 73 (1995) 3137.

[8] T. Ohtsuki, B. Kramer and Y. Ono: J. Phys. Soc. Jpn. 62 (1993) 223; M. Henneke, B. Kramer and T. Ohtsuki: Europhys. Lett. 27 (1994) 389.

[9] I. Kh. Zharekeshev and B. Kramer: Phys. Rev. B51 (1995) 17239, to appear in the special issue “Mesoscopic Physics & Electronics” of Jpn. J. Appl. Phys.

[10] H. Aoki and T. Ando: Phys. Rev. Lett. 54 (1985) 831, T. Ando and H. Aoki: J. Phys. Soc. Jpn. 54 (1985) 2238.

[11] B. Huckestein and B. Kramer: Phys. Rev. Lett. 64 (1990) 1437.

[12] Y. Ono, H. Kuwano, K. Slevin, T. Ohtsuki and B. Kramer: J. Phys. Soc. Jpn. 62 (1993) 2762.

[13] Y. Ono and T. Ohtsuki: J. Phys. Soc. Jpn. 62 (1993) 3813.

[14] J.T. Chayes, L. Chayes, D.S. Fisher and T. Spencer: Phys. Rev. Lett. 57 (1986) 2999.

[15] T. Ando: Phys. Rev. B40 (1989) 5325.

[16] U. Fastenrath, G. Adams, R. Bundschuh, T. Hermes, B. Raab, I. Schlosser, T. Wehner and T. Wichmann: Physica A172 (1991) 302.
[17] L. Schweitzer and I.Kh. Zharekeshev: J. Phys. Condensed Matter 7 (1995) L377.
Figure Caption

Fig. 1 $P(s)$ in the quantum Hall regime in the linear (a) and logarithmic (b) scales. The circles (○), squares (□), diamonds (◇), and triangles (△) correspond to the system area 200, 400, 600, and 800 in units of $2\pi\ell^2$. It can be very well fitted to the formula $A s^2 \exp(-B s^{1.65})$ indicated by the solid line. Applying (2) with $\gamma = 0.79$, we end up with poor fitting (broken line).

Fig. 2 $P(s)$ in the presence of strong spin-orbit scatterings in the logarithmic scale. The circles (○), squares (□), diamonds (◇), and triangles (△) correspond to the system size 8, 12, 16 and 20 in units of the lattice constant with the box type potential distribution, while the crosses (×) and pluses (+) to the system size 16 and 20 with the Gaussian type potential distribution. The application of (2) with $\nu = 2.8$ and $\gamma = 1 - 1/d\nu$ is indicated by the broken line, which shows considerable deviation in the small $s$ region. The best fit in the linear scale is obtained by setting $\gamma = 0.35$ (solid line).