Chaotic Behavior in Warm Deformed Nuclei Induced by Residual Two-body Interactions

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Abstract

Band mixing calculations in rapidly rotating well-deformed nuclei are presented, investigating the properties of energy levels and rotational transitions as a function of excitation energy. Substantial fragmentation of E2 transitions is found for $E_x \sim 800$ keV above yrast, which represents the onset of rotational damping. Above $E_x \approx 2$ MeV, energy levels and E2 strengths display fluctuations typical of quantum chaotic systems, which are determined by the high multipole components of the two-body residual interaction.

1. INTRODUCTION

It is a widely accepted conjecture [1] that quantum chaotic systems exhibit generic fluctuations governed by a random matrix theory such as the Gaussian orthogonal ensemble (GOE). The fluctuations refer not only to the energy levels (nearest neighbor level spacing distribution, $\Delta_3$ statistics, etc) but also to amplitudes of wave functions and strengths. A typical example is provided by the strength fluctuations in the widths of neutron resonances, which display a Porter-Thomas distribution [2]. This shows that the nuclear system at an excitation energy around $E_x \approx 8$ MeV and angular momentum $\approx 1\hbar$ is chaotic. Recently, many attempts have been made to determine regions of nuclear chaos varying parameters such as excitation energy, spin, mass numbers, etc. Rapidly rotating warm deformed nuclei, characterized by $I \gtrsim 30\hbar$, $E_x \lesssim$ a few MeV above yrast line in rare earth region are also investigated because the rotational E2 transitions in well deformed nuclei can be used as a measure of quantum chaos [3,4]. It is argued that the rotational band structures correspond to regularity while the rotational damping [5](disappearance of the band structure associated with fragmentation of E2 strength) is related to chaos.

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through complex mixing of rotational bands. The rapidly rotating deformed nuclei at high excitation energy \( (E_x \gtrsim \text{several hundred keV above yrast}) \) have high level density and the detectors in experiments have a finite resolution. Therefore, at the present stage of the development of experimental techniques, a statistical treatment of the experimental data by a fluctuation analysis of \( E_\gamma - E_\gamma \) spectra \([6]\) becomes more informative than discrete spectroscopy aiming at identifying individual states. If the observed E2 gamma-rays are emitted from a chaotic region, the fluctuations of the \( E_\gamma - E_\gamma \) spectra are governed mainly by the fluctuations of E2 strengths while the fluctuations of energy levels play a relatively minor role due to their rigidity. Hence, a study of the strength fluctuation in stretched E2 transitions will be useful in characterizing rapidly rotating deformed nuclei at high excitation energies.

2. CRANKING MODEL WITH RESIDUAL TWO-BODY INTERACTION

The cranking model is a simple and realistic model of the rapidly rotating warm nuclei, especially if many-particle many-hole excitations as well as residual interactions among them are taken into account. In this way thermal internal excitation in the rotating nuclei is represented. In this work we adopt the two-body surface delta interaction \([7]\) which will cause mixing of the unperturbed rotational bands with np-nh configurations. The residual interaction is diagonalized to give wave function contents of the mixed bands (See ref.\([8]\) for detail). The procedure is similar to that followed by S. Aberg \([4]\), who assumes however constant two-body matrix elements with a random sign irrespective of the cranked single-particle orbits involved.

Figure 1. The calculated rotational E2 strength distribution as a function of the \( \gamma \)-ray energy \( E_\gamma \) for four initial levels with different excitation energies above yrast line \( (E_x \) indicated in figure).
The following results are based on the diagonalization for $^{168}\text{Yb}$ at a rotational frequency $\hbar \omega = 0.5$ MeV, corresponding to an average angular momentum $I \approx 40$. The diagonalization is carried out for each parity and signature, with the lowest 1000 np-nh basis states yielding stable solutions for the lowest 300 levels ($E_x \lesssim 2.3$ MeV) above yrast. The stretched E2 strengths are calculated between levels at the rotational frequency $\hbar \omega$ and those at $\hbar \omega - 2\hbar^2/\mathcal{J} = 0.47$ MeV, corresponding to a moment of inertia $\mathcal{J} = 67\hbar^2$/MeV.

Figure 1 illustrates how the rotational E2 transitions change the structure with increasing excitation energy. The E2 transition from the yrast level shows a single peak in fig.1(a), populating one final state belonging to the same rotational band. With increasing internal excitation energy of the rotating nuclei, the rotational transition branches into several fragments due to the band mixing caused by the surface delta interaction. Actually, fig.1(b) illustrates how the rotational transition strength starts to become fragmented around an excitation energy of $E_x \approx 800$ keV above yrast. Further up in excitation energy (fig.1(c,d)) the rotational transitions exhibits not only fragmentation but also a smooth profile as a function of gamma-ray energy $E_\gamma$, from which a damping width (FWHM) may be extracted.

3. THE ONSET OF ROTATIONAL DAMPING

The structure change in the rotational E2 transitions illustrated in fig.1 is represented by characteristic excitation energies above yrast line. As the excitation energy increases we first see the onset of rotational damping as fragmentation of E2 strength from individual levels. To quantitatively define the fragmentation of the E2 strength, we introduce the branching number of level $i$:

$$n_{\text{branch}}(i) = \left( \sum_j W_{ij}^2 \right)^{-1}$$

where $W_{ij}$ denotes the rotational transition probability from level $i$ to $j$ ($\propto$ E2 strength $S_{ij}$). This expression is equal to the number of final states in the case of equiprobable branching $W_{ij} = 1/n_{\text{branch}}$. The branching number is shown as a function of the excitation energy in fig.2. If the onset of rotational damping is defined by a condition $n_{\text{branch}} > 2$, the characteristic excitation energy $U_0$ for the onset of rotational damping is

**Figure 2** The branching number averaged over 100 keV intervals in the excitation energy, and plotted as a function of the energy above yrast. The onset of rotational damping is marked by a borderline $n_{\text{branch}} = 2$. 


is extracted from fig. 2 as

$$U_0 \approx 800 \text{ keV} .$$

(2)

The corresponding number of non-fragmented rotational bands, defined by the condition $n_{\text{branch}} < 2$, is 36. This number is in reasonable agreement with the recent result of the fluctuation analysis [6] of the part of the $E_\gamma - E_\gamma$ spectrum which displays rotational energy correlations (the first ridge). The fluctuations of that part of the spectrum are in accordance with 25 undamped bands.

4. CHAOTIC FLUCTUATIONS IN ROTATIONAL DAMPING

As the excitation energy increases further, strong band mixing is expected to take place. If the mixing amplitudes become complex, the E2 strengths in fragmented transitions exhibit large fluctuations. Figure 1(c) and (d) show sizable fluctuation in the heights of individual peaks. If the strength fluctuations of the E2 transitions obey the Porter-Thomas distribution above a certain excitation energy $U_1$, one may take this energy as the threshold for the realization of quantum chaos in the system.

A quantitative analysis of the E2 strength fluctuation is made by taking statistics (probability distribution) of the E2 strengths. The result is shown in fig. 3. It should be noted that the normalized strengths $s_{ij} = S_{ij}/\langle S_{ij} \rangle$ are analyzed here instead of the bare

Figure 3. The probability distribution of the normalized rotational E2 strengths $s_{ij}$ for gamma-ray energies in the interval $0.90 < E_\gamma < 1.05$ MeV. The dashed curve represents the Porter-Thomas distribution. To display the dependence on the excitation energy above yrast, the lowest 300 levels for each parity and signature are grouped into four bins, containing 1-st to 50-th levels, 51- to 100-th levels, 101- to 200-th levels, and 201- to 300-th, and depicted in (a),(b),(c),and (d), respectively. Excitation energies above yrast of the bins are indicated in the figures.
E2 strength $S_{ij}$ in order to take into account the smooth dependence $\langle S_{ij} \rangle$ of the E2 strength on $E_\gamma$ and $E_x$.

Fluctuations in the energy levels are also analyzed by means of the $\Delta_3$ statistics [2], with the result shown in fig.5(a). Figure 3 and fig.5(a) indicate that fluctuations in both the E2 strengths and the energy levels reach the GOE limits as the excitation energy above yrast exceeds about 2 MeV. Thus the model predicts a characteristic excitation energy of

$$U_1 \approx 2 \text{ MeV}$$

where the chaotic fluctuations set in.

5. MICROSCOPIC ORIGIN OF CHAOTIC BEHAVIOR

Let us investigate the origin of the chaotic fluctuations in the rotational damping in connection with the residual interactions among the unperturbed rotational bands [9]. It is useful to decompose the residual two-body interaction in terms of multipolarities of interacting pair of nucleons, since much of our knowledge of the residual two-body interaction has been acquired by studying the nuclear response to probes with specific multipolarity. For example, the pairing plus quadrupole force has been quite successful in accounting for many systematic features of nuclear levels [10], but, it contains only low multipolarities ($L = 0, 2$) by definition. The surface delta interaction [7] (SDI), on the other hand, contains all the possible multipolarities as displayed in its expression:

$$V(1,2) = 4\pi V_0 \delta(r_1 - R_\alpha)\delta(r_2 - R_\alpha)$$

$$\times \sum_{\lambda,\mu} Y^*_{\lambda\mu}(\hat{r}_1)Y_{\lambda\mu}(\hat{r}_2) ,$$

where the sum of spherical harmonics denotes the delta function of the angle between interacting pair. Thus the two interactions differ in the multipolarities they contain.

A consequence of the difference between the two forces is seen (fig.4) in the probability distribution of the two-body matrix elements $V_{\alpha\beta\gamma\delta}$ for the cranked-Nilsson s.p. orbits, denoted by $\alpha, \beta, \gamma, \delta$. The strong peak at $V_{\alpha\beta\gamma\delta} = 0$ for the pairing plus quadrupole force indicates the presence of strong selection rules while the distribution for the surface delta force resembles much more a Gaussian shape, which represents the

Figure 4 Probability distribution of the off-diagonal two-body matrix elements $V_{\alpha\beta\gamma\delta}$ of the surface delta interaction (SDI) and the pairing plus quadrupole force (P+QQ). The dashed curve represents a Gaussian distribution which gives the same r.m.s. as SDI.
limit of no selection rule. In other words, the higher multipole terms contained in the SDI show considerably less selectivity to specific orbits than the low multipole terms common to the SDI as well as to the pairing plus quadrupole interaction.

A comparison between the two forces is made in fig.5 both for E2 strength and energy level fluctuations. For fairness, a pairing plus quadrupole force whose strength gives the same root-mean-square value of the off-diagonal two-body matrix elements as that of SDI (the selfconsistent values scaled by 1.5) is used here. It is seen that the pairing and quadrupole force does neither give rise to the Porter-Thomas distribution in E2 strength fluctuations nor the GOE limit in the energy level fluctuations in the relevant energy region \( E_x \approx 2 \text{ MeV} \), in contrast to the surface delta interaction, which reaches to the chaotic limits (GOE).

Thus, it is a specific result of the present model of the rotational damping that the high multipole components present in the residual interaction are responsible for the mixing of rotational bands and for the chaotic fluctuations (Porter-Thomas fluctuation in the E2 strength and the energy level fluctuations of GOE) in the energy and spin region investigated. This is because the high multipole components of the two-body interaction allow the unperturbed rotational bands to interact more democratically irrespective of the participating single-particle orbits than low multipole components.

**Figure 5.** Comparison between the surface delta interaction and the paring plus quadrupole force for the \( \Delta_3 \) statistics of energy level fluctuation (a) and for the normalized E2 strength distribution (b). The three curves in (a), marked by a, b, c, are the results for first to 100-th levels, 101- to 200-th levels, and 201- to 300-th levels for each parity and signature, respectively. The normalized strength distribution is shown for the 201- to 300-th levels.
6. CONCLUSIONS

It is shown that the band mixing caused by two-body residual interactions brings about the rotational damping in rapidly rotating warm deformed nuclei. With the surface delta interaction acting among unperturbed rotational bands of np-nh configurations, the rotational damping gradually sets in at around excitation energy $U_0 \approx 800$ keV above yrast line, at which the rotational E2 strength from individual level breaks into more than two fragments. At higher excitation energy $E_x \gtrsim 2$ MeV, the generic fluctuations proper of quantum chaos (GOE) are found both in energy levels and in E2 strengths (Porter-Thomas strength fluctuation). From the comparison between the pairing plus quadrupole force representing low multipole ($L = 0, 2$) components of the residual two-body interaction and the surface delta force including high multipole components as well, it is found that high multipole components in the two-body residual interaction are responsible for the chaotic behavior of the rotational damping.

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