Non-singular solutions in multidimensional model with scalar fields and exponential potential

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Abstract

Using developed earlier our methods for multidimensional models \textsuperscript{[1,2,3]} a family of cosmological-type solutions in $D$-dimensional model with two sets of scalar fields $\vec{\varphi}$ and $\vec{\psi}$ and exponential potential depending upon $\vec{\varphi}$ is considered. The solutions are defined on a product of $n$ Ricci-flat spaces. The fields from $\vec{\varphi}$ have positive kinetic terms and $\vec{\psi}$ are "phantom" fields with negative kinetic terms. For vector coupling constant obeying $0 < \vec{\lambda}^2 < (D-1)/(D-2)$ a subclass of non-singular solutions is singled out. The solutions from this subclass are regular for all values of synchronous "time" $\tau \in (-\infty, +\infty)$. For $\vec{\lambda}^2 < 1/(D-2)$ we get an asymptotically accelerated and isotropic expansion for large values of $\tau$.

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1 Introduction

Recently, the discovery of the cosmic acceleration \[4, 5\] stimulated a lot of papers, on multidimensional cosmology in particular, with the aim to explain this phenomenon using certain multidimensional models, e.g. those of superstring or supergravity origin (see \[10, 12, 13, 14, 15, 16\], and refs. therein).

At present rather popular models are those with multiple exponential potential of the scalar fields (see \[6, 7, 8, 9\] and refs. therein). Potentials of such type arise naturally in certain supergravitational models \[10\] and in sigma-models \[17, 18\], related to configurations with \(p\)-branes (for a review see also \[26, 27, 28\]).

Here we are interested in non-singular solutions (e.g. with a bounce) that appear in certain cosmological models, see \[29, 30\] and refs. therein. Such non-singular solutions are not new ones. For pioneering papers with non-singular solutions based on scalar fields see \[31, 36\] - for conformal massless scalar field and cosmological constant, \[32\] - for phantom scalar fields, \[33\] - for conformal Higgs-type scalar field with spontaneous symmetry breaking (SSB) back reaction, \[34\] - for a scalar field with polarization of vacuum (PV) effect, \[35\] - for a scalar field with SSB and PV etc. See all these results also in \[36\].

In this paper we consider the \(D\)-dimensional model governed by the action

\[
S = \int_M d^Dz \sqrt{|g|} \{ R[g] - g^{MN} \partial_M \vec{\varphi} \partial_N \vec{\varphi} \\
+ g^{MN} \partial_M \vec{\psi} \partial_N \vec{\psi} - 2V(\varphi) \},
\]

with the scalar potential

\[
V(\varphi) = \Lambda \exp(2\vec{\lambda} \vec{\varphi}).
\]

Here \(\vec{\varphi} = (\varphi^1, \ldots, \varphi^k)\) is a set (vector) of scalar fields, \(\Lambda\) is a constant, \(\vec{\lambda} = (\lambda_1, \ldots, \lambda_k) \in \mathbb{R}^k\) is a vector of dilatonic couplings, \(\vec{\psi} = (\psi^1, \ldots, \psi^m)\) is a set (vector) of ”phantom” scalar fields, \(g = g_{MN} dx^M \otimes dx^N\) is metric, \(|g| = |\det(g_{MN})|\).

We note that phantom scalar fields (with negative kinetic terms) appear in various multidimensional models, e.g. in a field model for \(F\)-theory \[19\].
in a chain of $B_D$-models in dimensions $D \geq 11$ \cite{25} ($B_{11}$ corresponds to $M$-theory \cite{20} and $B_{12}$ to $F$-theory \cite{21}).

The article is devoted mainly to cosmological type solutions with the vector coupling constant obeying

$$\tilde{\lambda}^2 \neq b \equiv \frac{D-1}{D-2},$$

which follow from condition of non-zero scalar product for certain $U$-vector \cite{12}.

The paper is organized as follows. In Section 2 a class of solutions on a product of $n$ Ricci-flat spaces for the model under consideration is described. In Section 3 non-singular solutions (e.g. with simple bounce of volume scale factor) are singled out.

## 2 Solutions on product of Ricci-flat spaces

We deal with the cosmological solution to field equations in the model (1.1)–(1.3) that is the special case of solutions obtained in \cite{12}. Our main interest here will be the role of phantom scalar fields in combination with the usual matter fields.

These solutions are defined on the manifold

$$M = (u_-, u_+) \times M_1 \times \ldots \times M_n$$

and are described by the metric

$$g = f^{2h}[-e^{2\phi t+2\phi} w du \otimes du + f^{-2h} \sum_{i=1}^{n} e^{2c_i t+2\bar{c}_i} g^i].$$

and scalar fields: material

$$\phi = -h \tilde{\lambda} \ln |f| + \bar{c}_\phi t + \bar{c},$$

and phantom

$$\bar{\psi} = \bar{c}_\psi t + \bar{c}_\psi.$$  

Here and in what follows we denote

$$h \equiv (\tilde{\lambda}^2 - b)^{-1}$$

\[3\]
and \( w = \pm 1 \). The manifolds \((M_i, g^i)\) are Ricci-flat,

\[
\text{Ric}[g^i] = 0,
\]

and \( d_i = \dim M_i, \ i = 1, \ldots, n. \)

The function \( f \) reads

\[
f = R \sinh(\sqrt{|C|(u - u_0)}), \ C > 0, \ \epsilon < 0; \quad (2.7)
\]

\[
R \sin(\sqrt{|C|(u - u_0)}), \ C < 0, \ \epsilon < 0; \quad (2.8)
\]

\[
R \cosh(\sqrt{|C|(u - u_0)}), \ C > 0, \ \epsilon > 0; \quad (2.9)
\]

\[
|2\Lambda/h|^{1/2}(u - u_0), \ C = 0, \ \epsilon < 0, \quad (2.10)
\]

where \( u_0 \) and \( C \) are constants and

\[
R = |2\Lambda/(hC)|^{1/2},
\]

\[
\epsilon = \text{sign}(\Lambda w h). \quad (2.12)
\]

satisfy the following constraint relations

\[
\sum_{i=1}^{n} d_i c^i + \vec{\lambda} \vec{c} \phi = 0,
\]

\[
\sum_{i=1}^{n} d_i \bar{c}^i + \vec{\lambda} \bar{c} \phi = 0,
\]

and

\[
Ch + (\vec{c}_\phi)^2 - (\vec{c}_\psi)^2 + \sum_{i=1}^{n} d_i (c^i)^2 - \left( \sum_{i=1}^{n} d_i c^i \right)^2 = 0. \quad (2.14)
\]

Relations (2.13) means that integration constants are orthogonal to a certain U-vector and relation (2.14) is equivalent to the zero-energy constraint, see [22] [23] [24] [25].

In (2.2)

\[
c^0 = \sum_{i=1}^{n} d_i c^i, \quad \bar{c}^0 = \sum_{i=1}^{n} d_i \bar{c}^i. \quad (2.15)
\]

The solutions under consideration are general \( D \)-dimensional cosmological type solutions defined on product of \( n \) Ricci-flat spaces \( D = \sum_{i=1}^{n} d_i \) with generalized (i.e. multidimensional) Bianchi-I metrics in the model (1.1)- (1.3). Here \( u \) is the harmonic time variable. In what follows we put \( u > u_0. \)
3 Non-singular solutions

Here we restrict our consideration to small dilatonic couplings when

\[ w\Lambda < 0, \quad 0 < \lambda^2 < b = \frac{D - 1}{D - 2}, \]  

(3.1)

see (1.2). Thus, in (2.12) \( \epsilon = -1 \). As we shall see below the inequalities (3.1) guarantee the existence of non-singular solutions.

In what follows we investigate the behaviour of solutions from Section 2 for parameters from (3.1) using the so-called "synchronous" variable.

We introduce scale factors

\[ a_i = \sqrt{h/\alpha}, \quad e^{\varepsilon_1 t + \varepsilon_i}, \]  

(3.2)

for \( i = 1, \ldots, n \). The relation between synchronous and harmonic variables reads

\[ \tau = \int_{u}^{u_1} du' v(u'), \]  

(3.3)

where \( u_1 > u_0 \) and

\[ v = a_1 \ldots a_n = f^{bh} e^{\varepsilon_0 u + \varepsilon_0}, \]  

(3.4)

is a volume scale factor.

From (3.3) for \( u \to u_0 + 0 \) we get \( \tau \to +\infty \) for all values of parameters and

\[ v \sim \tau^{(D-1)\nu}. \]  

(3.5)

It follows from (3.3) that the solution is a non-singular one for \( \tau \in (-\infty, +\infty) \) only in such three cases

\[ (A_+) \quad \epsilon^0 > b|h|\sqrt{C}, \quad C \geq 0; \]  

(3.6)

\[ (A_0) \quad \epsilon^0 = b|h|\sqrt{C}, \quad C > 0; \]  

(3.7)

\[ (B) \quad C < 0. \]  

(3.8)

In the cases \((A_+)\) and \((B)\) we get a bouncing behaviour of volume scale factor \( v(\tau) \) at some point \( \tau_b \), i.e. the function \( v(\tau) \) is monotonically decreasing in the interval \(( -\infty, \tau_b )\) and monotonically increasing in the interval \(( \tau_b, +\infty)\).

In the case \((A_0)\) the solution is non-singular, the function \( v(\tau) \) is monotonically increasing to infinity from a non-zero value.
For other values of parameters the solution is singular in general position (excepting certain special cases), the function \( v(\tau) \) is monotonically increasing to infinity from zero value. This takes place when either (i) \( c^0 < b|h|\sqrt{C} \), \( C > 0 \) or (ii) \( C = 0 \), \( c^0 \leq 0 \). The solution for these values of parameters is defined in the semi-infinite interval \((\tau_0, +\infty)\).

The relations (3.1) lead to isotropisation of scale factors for large values of synchronous variable \( \tau \). Namely, for \( \bar{\lambda}^2 \neq 0 \) we get the following asymptotical relations in the limit \( \tau \to +\infty \) [12]

\[
g_{as} = w d\tau \otimes d\tau + \sum_{i=1}^{n} A_i \tau^{2\nu} g^i, \tag{3.9}
\]
\[
\bar{\varphi}_{as} = -\frac{\bar{\lambda}}{\lambda^2} \ln \tau + \bar{\varphi}_0, \tag{3.10}
\]
\[
\bar{\psi}_{as} = \bar{\psi}_0, \tag{3.11}
\]

where

\[
\nu = \frac{1}{(D - 2)\bar{\lambda}^2}, \tag{3.12}
\]

\( A_i > 0 \) are constant, and \( \bar{\varphi}_0, \bar{\psi}_0 \) are constant vectors obeying

\[
2|\Lambda| \exp(2\bar{\lambda}\bar{\varphi}_0) = |\bar{\lambda}^2 - b|/(\bar{\lambda}^2)^2. \tag{3.13}
\]

For \( \bar{\lambda} = 0 \) we get for \( \tau \to +\infty \) [12]

\[
g_{as} = w d\tau \otimes d\tau + \sum_{i=1}^{n} A_i \exp(2M\tau) g^i, \tag{3.14}
\]
\[
\bar{\varphi}_{as} = \bar{\varphi}_0, \quad \bar{\psi}_{as} = \bar{\psi}_0 \tag{3.15}
\]

where \( |2\Lambda| = (D - 2)(D - 1)M^2 \) (in agreement with the case of one scalar field from [37, 38].)

Accelerated expansion in the limit \( \tau \to +\infty \) takes place for

\[
\lambda^2 < \frac{1}{D - 2} \tag{3.16}
\]

(In (3.14) should be \( M > 0 \) for \( \lambda = 0 \).)
4 The role of scalar charges

Here we show that non-singular solutions defined for all $\tau \in \mathbb{R}$ exist only for large enough value of (vector) phantom charge squared $(\vec{c}_\psi)^2$.

Let us introduce anisotropy parameters $\tilde{c}^i$ by the following relations

$$c^i = -\frac{\bar{\lambda}\vec{c}_\psi}{D-1} + \tilde{c}^i,$$

(4.1)

$i = 1, \ldots, n$. We get from (2.15)

$$\sum_{i=1}^{n} d_i \tilde{c}^i = 0.$$

(4.2)

In terms of these parameters the main relations on parameters (2.14) may be rewritten as following

$$Ch + (\vec{c}_\psi \text{par})^2 - \frac{1}{b}(\bar{\lambda}\vec{c}_\psi)^2$$

$$= (\vec{c}_\psi)^2 - (\vec{c}_\psi \text{ort})^2 - \sum_{i=1}^{n} d_i (\tilde{c}^i)^2 \equiv Q,$$

(4.3)

It follows also from (3.6) and (3.8) that

$$c^0 = -\bar{\lambda}\vec{c}_\psi \geq bh\sqrt{C}, \text{ for } C \geq 0$$

(4.4)

and, hence,

$$(c^0)^2 \geq b^2 h^2 C$$

(4.5)

for all non-singular cases $(A_+), (A_0)$ and $(B)$.

Here we use decomposition of the vector of scalar charges into a sum of two terms orthogonal and parallel to $\bar{\lambda}$, respectively,

$$\vec{c}_\psi = \vec{c}_\psi \text{ort} + \vec{c}_\psi \text{par},$$

(4.6)

$$\vec{c}_\psi \text{par} = (\vec{c}_\psi \bar{\lambda})\frac{\bar{\lambda}}{\lambda^2},$$

(4.7)

$$\vec{c}_\psi \text{ort} = \vec{c}_\psi - (\vec{c}_\psi \bar{\lambda})\frac{\bar{\lambda}}{\lambda^2}$$

(4.8)
It is clear that

\[ \vec{c}_\phi^2 = (\vec{c}_\phi^{\text{ort}})^2 + (\vec{c}_\phi^{\text{par}})^2. \]  

(4.9)

We call \( Q \) the "main" parameter of the model. We show that the necessary condition for the existence of non-singular solutions (defined for \( \tau \in (-\infty, +\infty) \) ) is the following one

\[ Q \geq 0, \]  

(4.10)

that means

\[ (\vec{c}_\psi)^2 \geq (\vec{c}_\phi^{\text{ort}})^2 + \sum_{i=1}^{n} d_i (\vec{c}_i)^2. \]  

(4.11)

Indeed, relations (4.3) and (4.4) imply either

\[ (A) \quad Q_- \leq Q \leq Q_+, \quad 0 \leq C \leq \bar{\lambda}^2 Q_-, \]  

(4.12)

or

\[ (B) \quad Q > Q_+, \quad C < 0, \]  

(4.13)

where

\[ Q_- = \frac{(b - \bar{\lambda}^2)}{b^2 \bar{\lambda}^2} (\vec{c}_\phi \bar{\lambda})^2, \quad Q_+ = \frac{(b - \bar{\lambda}^2)}{b \bar{\lambda}^2} (\vec{c}_\phi \bar{\lambda})^2 \]  

(4.14)

\((\bar{\lambda} \neq 0)\). Relation \( Q_- \geq 0 \) implies \( Q \geq 0 \).

Subclasses (A) and (B) differ by asymptotical relations for \( \tau \to -\infty \):

\[ (A) \quad v \sim |\tau|. \]  

(4.15)

and

\[ (B) \quad v \sim |\tau|^{(D-1)\nu}. \]  

(4.16)

with \( \nu \) defined in (3.12).

The first asymptotics corresponds to Kasner-like behaviour of scale factors in agreement with the billiard approach of (40, 39). The second one is just the same as in the limit \( \tau \to +\infty \) (see 3.5). This follows from the relations \( \sin x = \sin(\pi - x) \sim x \) for small \( x = (u - u_0) \sqrt{|C|} \) and (3.3).

Non-singular solutions defined for \( \tau \in (-\infty, +\infty) \) are absent for zero phantom charges \( \vec{c}_\psi = 0 \) even for isotropic case with \( \vec{c}_i = 0 \) (in this case
$\vec{c}_\varphi = C = 0$ and we get a solution with a power-law dependent scale factors, see also [12].

For $\vec{c}_\psi \neq 0$ the anisotropy parameters $\vec{c}_i$ and orthogonal component $\vec{c}_{i\text{ort}}$ should be small enough (in comparison with absolute value of phantom vector charge, see (4.11)) for the existence of non-singular solutions defined for all values of synchronous variable $\tau \in \mathbb{R}$.

## 5 Conclusions

In this paper we have considered a family of generalized Bianchi-I cosmological type solutions defined on product of Ricci-flat spaces in $D$-dimensional model (1.1)-(1.3) with $k$ ordinary scalar fields $\varphi^i$ and $m$ phantom scalar fields $\psi^j$. These solutions with phantom scalar fields are the special case of the solutions obtained earlier in [12].

For the coupling constant obeying $0 < \lambda^2 < (D - 1)/(D - 2)$ we have singled out a subclass of non-singular solutions (e.g. with a simple bounce of the volume scale factor). Solutions from this subclass are regular for all values of synchronous time $\tau \in (-\infty, +\infty)$ and have an isotropization behaviour for $\tau \to +\infty$.

We have found a necessary condition on scalar charges and anisotropy parameters for the existence of such non-singular solutions, see (4.11). For $\lambda^2 < 1/(D - 2)$ this expansion is asymptotically accelerated one for large values of $\tau$.

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**References**

[1] V.N. Melnikov, Multidimensional Classical and Quantum Cosmology and Gravitation. Exact Solutions and Variations of Constants. CBPF-NF-051/93, Rio de Janeiro, 1993, 93pp.;
V.N. Melnikov, in: “Cosmology and Gravitation”, ed. M. Novello, Editions Frontieres, Singapore, 1994, p. 147.
[2] V.N. Melnikov, Multidimensional Cosmology and Gravitation, CBPF-MO-002/95, Rio de Janeiro, 1995, 210 pp.; V.N. Melnikov. In: *Cosmology and Gravitation. II*, ed. M. Novello, Editions Frontières, Singapore, 1996, p. 465.

[3] V.N. Melnikov, Exact Solutions in Multidimensional Gravity and Cosmology III. CBPF-MO-03/02, Rio de Janeiro, 2002, 297 pp.

[4] A.G. Riess et al, *AJ*, 116, 1009 (1998).

[5] S. Perlmutter et al, *ApJ*, 517, 565 (1999).

[6] B. Ratra and P.J.E. Peebles, Cosmological consequences of a rolling homogeneous scalar field, *Phys. Rev. D* 37, 3406 (1998).

[7] E.J. Copeland, A. R. Liddle and D. Wands, Exponential potentials and cosmological scaling solutions, *Phys. Rev. D* 57, 4686 (1998); *gr-qc/9805085*

[8] T. Bareiro, E.I. Copeland and N.J. Nunes, Quintessence arising from exponential potentials, astro-ph/9910214

[9] V. Sahni, A.A. Starobinsky et al. *JETP Lett.* 77 (2003) 201; *JCAP* 0304 (2003) 002; *MNRAS* 344 (2003) 1057.

[10] P.K. Townsend, Quintessence from $M$-theory, *JHEP* 0111 (2001) 042; hep-th/0110072

[11] H. Dehnen, V.R. Gavrilov and V.N. Melnikov, General solutions for flat Friedmann universe filled by perfect fluid and scalar field with exponential potential, *Grav. Cosmol.*, 9, No 3 (35), 189 (2003); *gr-qc/0212107*

[12] V.D. Ivashchuk, V.N. Melnikov and A.B. Selivanov, Cosmological solutions in multidimensional model with multiple exponential potential, *JHEP* 0309 (2003) 059; hep-th/0309027

[13] C.-M. Chen, P.-M. Ho, I. P. Neupane, N. Ohta and J. E. Wang, Hyperbolic Space Cosmologies, *JHEP* 0310 (2003) 058; hep-th/0306291

[14] N. Ohta, Accelerating Cosmologies and Inflation from M/superstring Theories, *Int. J. Mod. Phys. A* 20, 1 (2005); hep-th/0411230

[15] V.D. Ivashchuk, V.N. Melnikov and A.B. Selivanov, Composite S-brane solutions on product of Ricci-flat spaces, *GRG* 36, 1593-1602 (2004); hep-th/0404113
[16] V.D. Ivashchuk, V.N. Melnikov and S.-W. Kim, S-brane solutions with acceleration, in models with forms and multiple exponential potential, *Grav. Cosmol.* **10**, 141-148 (2004); hep-th/0405009.

[17] V.D. Ivashchuk and V.N. Melnikov, Intersecting p-brane Solutions in Multidimensional Gravity and M-Theory, *Grav. Cosmol.*, **2**, No 4 (8), 297-305 (1996), hep-th/9612089. Generalized Intersecting P-brane Solutions from Sigma-model, *Phys. Lett.*, **B 403**, 23-30 (1997).

[18] V.D. Ivashchuk and V.N. Melnikov, Sigma-model for the Generalized Composite p-branes, *Class. Quantum Grav.*, **14**, 3001-3029, (1997); hep-th/9705036.

[19] N. Khviengia, Z. Khviengia, H. Lü and C.N. Pope, Toward Field Theory of F-Theory, *Class. Quantum Grav.*, **15**, 759-773, (1998); hep-th/9703012.

[20] C. Hull and P. Townsend, Unity of Superstring Dualities, *Nucl. Phys.*, **B 438**, 109 (1995); P. Horava and E. Witten, *Nucl. Phys.* **B 460**, 506 (1996).

[21] C. Vafa, Evidence for F-Theory, *Nucl. Phys.* B 469, 403 (1996).

[22] V.R. Gavrilov, V.D. Ivashchuk and V.N. Melnikov, Integrable pseudo-Euclidean Toda-like systems in multidimensional cosmology with multicomponent perfect fluid., *J. Math. Phys.*, **36**, No 10, 5829-5847 (1995).

[23] K.A. Bronnikov, M.A. Grebeniuk, V.D. Ivashchuk, V.N. Melnikov, Integrable multidimensional cosmology for intersecting p-branes, *Grav. Cosmol.* **3**, 2 (10), 105-112 (1997).

[24] M.A. Grebeniuk, V.D. Ivashchuk and V.N. Melnikov, Integrable multidimensional quantum cosmology for intersecting p-branes, *Grav. Cosmol.* **3**, No 3 (11), 243-249 (1997); gr-qc/9708031.

[25] V.D. Ivashchuk and V.N. Melnikov, Multidimensional Classical and Quantum Cosmology with Intersecting p-Branes, *J. Math. Phys.*, **39**, 2866-2889 (1998); hep-th/9708157.

[26] V.D. Ivashchuk and V.N. Melnikov, Multidimensional cosmological and spherically symmetric solutions with intersecting p-branes. In: Lecture Notes in Physics, Vol. 537, “Mathematical and Quantum Aspects of Relativity and Cosmology. Proceedings of the Second Samos Meeting on Cosmology, Geometry and Relativity held at Pythagoreon, Samos, Greece, 1998, eds: S. Cotsakis, G.W. Gibbons., Berlin, Springer, 2000, pp. 214-148.
[27] V.D. Ivashchuk and V.N. Melnikov, Exact solutions in multidimensional gravity with antisymmetric forms, topical review, *Class. Quantum Grav.* **18**, R82-R157 (2001); hep-th/0110274.

[28] V.D. Ivashchuk, On exact solutions in multidimensional gravity with antisymmetric forms, In: Proceedings of the 18th Course of the School on Cosmology and Gravitation: The Gravitational Constant. Generalized Gravitational Theories and Experiments (30 April-10 May 2003, Erice). Ed. by G.T. Gillies, V.N. Melnikov and V. de Sabbata, Kluwer Academic Publishers, Dordrecht, 2004, pp. 39-64; gr-qc/0310114.

[29] L.E. Allen and D. Wands, Cosmological perturbations through a simple bounce, *Phys. Rev.* **D 70** (2004) 063515; astro-ph/0404441.

[30] K.A. Bronnikov and J.C. Fabris, Nonsingular multidimensional cosmologies without fine tuning, *JHEP* **0209** (2002) 062; hep-th/0207213.

[31] K.A. Bronnikov and V.N. Melnikov, Abstr. IV Russian Geom. Conf., 1969, p. 167-168 (in Russian).

[32] N.A. Zaitsev, S.M. Kolesnikov and A.G. Radynov. Preprint ITP-72-21P, Kiev, 1972.

[33] V.N. Melnikov, *Doklady Acad. Nauk*, **246**, No 6, 1351 (1979) (in Russian); V.N. Melnikov and S.V. Orlov, *Phys. Lett.*, **A 70**, No 4, 263 (1979).

[34] V.Ts. Gurovich and A.A. Starobinsky, *JETP*, **77**, 1689 (1979) (in Russian).

[35] K.A. Bronnikov and V.N. Melnikov, A non-singular model of preinflationary Universe, Abstr. of USSR Conf. GR-6, MGPI Press, Moscow, 1984, p. 46-47 (in Russian).

K.A. Bronnikov and V.N. Melnikov, A nonsingular vacuum model for pre-inflationary universe. In: Proc. A. Eddington Cent. Symp., V.3, Gravitational Radiation and Relativity, Eds. J. Weber and T.M. Karade, WS, Singapore, 1986, p. 386.

[36] K.P. Stanyukovich and V.N. Melnikov, Hydrodynamics, Fields and Constants in the Theory of Gravitation, Energoatomizdat, Moscow, 1983 (in Russian). English translation of first 5 chapters see in: V.N. Melnikov, Fields and Constants in the Theory of Gravitation. CBPF-MO-02/02, Rio de Janeiro, 2002, 134pp.
[37] V.D. Ivashchuk and V.N. Melnikov, Exact Solutions in Multidimensional Cosmology with Cosmological Constant, *Teor. Mat. Fiz.* **98**, 312 (1994) (in Russian); *Theor. Mat. Phys.* **98**, 212 (1994).

[38] U. Bleyer, V.D. Ivashchuk, V.N. Melnikov and A.I. Zhuk, Multidimensional Classical and Quantum Wormholes in Models with Cosmological Constant, *Nucl. Phys.*, **B 429**, 177-204 (1994).

[39] V.D. Ivashchuk and V.N. Melnikov, Billiard representation for multidimensional cosmology with intersecting p-branes near the singularity. *J. Math. Phys.*, **41**, No 9, 6341-6363 (2000); [hep-th/9904077](http://arxiv.org/abs/hep-th/9904077)

[40] H. Dehnen, V.D. Ivashchuk and V.N. Melnikov, Billiard representation for multidimensional multi-scalar cosmological model with exponential potentials, *GRG*, **36**, N 7, 1563-1578 (2004); [hep-th/0312317](http://arxiv.org/abs/hep-th/0312317)