Di-vector boson production in association with a Higgs boson at hadron colliders

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\textbf{Abstract:} We consider the production of a Higgs boson in association with two electroweak vector bosons at hadron colliders. In particular, we examine $\gamma\gamma H$, $\gammaZH$, $ZZH$, and $W^+W^- H$ production at the LHC (14 TeV), HE-LHC (27 TeV), and FCC-hh (100 TeV) colliders. Our main focus is to estimate the gluon-gluon ($gg$) channel ($gg \to VV'H$) contributions to $pp \to VV'H$ ($V, V' = \gamma, Z, W$) and compare them with corresponding contributions arising from the quark-quark ($qq$) channel ($q\bar{q} \to VV'H$). Technically, the leading order $gg$ channel contribution to $pp \to VV'H$ cross section is a next-to-next-to-leading order correction in strong coupling parameter, $\alpha_s$. In the processes under consideration, we find that in the $gg$ channel, $W^+W^- H$ has the largest cross section. However, relative contribution of the $qq$ channel is more important for the $pp \to ZZH$ production. At the FCC-hh, $gg \to ZZH$ contribution is comparable with the next-to-leading order QCD correction to $qq \to ZZH$. We also compute the cross sections when $W$ and $Z$-bosons are polarized. In the production of $W^+W^- H$ and $ZZH$, we find that the $gg$ channel contributes more significantly when the vector bosons are longitudinally polarized. By examining such events, one can increase the fraction of the $gg$ channel contribution to these processes. Further, we have studied beyond-the-standard-model effects in these processes using the $\kappa$-framework parameters $\kappa_t, \kappa_V$, and $\kappa_\lambda$. We find that the $gg$ channel processes $ZZH$ and $WWH$ have very mild dependence on $\kappa_\lambda$, but strong dependence on $\kappa_t$ and $\kappa_V$. The $qq$ channel processes mainly depend on $\kappa_V$. Dependence of the $gg$ channel contribution on $\kappa_V$ is stronger than that of the $qq$ channel contribution. Therefore focusing on events with longitudinally polarized $W$ and $Z$-bosons, one can find stronger dependence on $\kappa_V$ that can help us measure this parameter.

\textbf{Keywords:} Electroweak, Higgs boson, Polarization, LHC, Anomalous couplings

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1 Introduction

After the discovery of a Higgs-like resonance, with a mass of 125 GeV, at the Large Hadron Collider (LHC) in 2012, various properties of this new particle have been studied. The spin and parity measurements have established it as a $0^+$ state at 99.9% CL against alternative scenarios [1]. Couplings of this new particle with the fermions and gauge bosons predicted in the standard model are getting constrained as more and more data are being analyzed by the LHC experiments [2–4]. To this end, the vector-boson fusion production of the Higgs boson, associated production of $VH (V = Z, W)$, and Higgs boson’s decay into vector bosons set limits on the $HVV$ couplings [5, 6]. The gluon-gluon ($gg$) channel production of the Higgs boson helps in constraining the $Ht\bar{t}$ coupling [6]. In addition, the evidence for the associated production of Higgs boson with a top-quark pair [7, 8] will provide the direct measurement of $Ht\bar{t}$ coupling. We still need to measure the trilinear and quartic Higgs self-couplings in order to know the form of the Higgs potential which will in turn reveal the exact symmetry breaking mechanism. The Higgs self-couplings can be probed directly in multi-Higgs production processes [9–11]. Recently, indirect methods of probing them at hadron and lepton colliders have also been proposed [12–14]. Similarly, the quartic couplings involving Higgs and vector bosons $HHVV$ are also not constrained independently. This coupling can be probed in the vector-boson fusion production of a Higgs boson pair [15, 16]. In order to find the signals of new physics, it is important that we improve our theoretical predictions for the processes involving Higgs boson at current and future colliders.

Loop-induced decay and scattering processes can play an important role in searching for new physics. In the presence of new physics (new particles and/or interactions), the rates for such processes can differ significantly from their standard model predictions. In this regard, many $gg$ channel scattering processes in $2 \to 2$ and $2 \to 3$ category have been studied [11,17–41]. In the present work, we are interested in loop-induced $gg$ channel contribution to $VVH$ ($\gamma\gamma H$, $\gamma ZH$, $ZZH$, and $W^+W^- H$) production. In QCD perturbation theory, the leading order $gg$ channel contribution to $pp \to VVH$ is an NNLO contribution at the cross section level. Due to many electroweak couplings involved and loop-induced nature of $gg \to VVH$ processes, their cross sections are expected to be small. However, they can be important at high energy hadron colliders like 100 TeV $pp$ collider such as proposed hadronic Future Circular Collider (FCC-hh) facility at CERN [42] and Super Proton-Proton Collider (SPPC) facility in China [43]. At such energy scale, the gluon flux inside the proton becomes very large. In fact, for $\gamma\gamma H$, the $gg$ channel gives the dominant contribution.

Unlike the quark-quark contributions, which are mainly sensitive to $HVV$ couplings, the gluon-gluon contribution allows access to $Ht\bar{t}, HHH$, and $HHVV$ couplings as well. Note that the processes under consideration are background to $pp \to HH$ when one of the Higgs bosons decays into $\gamma\gamma/\gamma Z/ZZ^*$ or $WW^*$ final states. The process $pp \to ZZH$ is also a background to $pp \to HHH$ when two of the three Higgs bosons decay into $b\bar{b}$ final states. In this work, we present a detailed study of $gg \to \gamma\gamma H$ and $\gamma ZH$ for the first time in the SM. The $gg$ channel contribution to $ZZH$ and $WWH$ in the SM have been studied in the past [26,44,45]. We have presented the $ZZH$ and $WWH$ calculations in detail and have proposed methods to enhance the relative contribution of gluon-gluon channel over quark-quark channel. Since loop-induced processes are sensitive to new physics, we also study the effect of new physics in all $VVH$ processes using a common BSM framework — the $\kappa$-framework. Going beyond the $\kappa$-framework, we have treated the $HHVV$ coupling independently and emphasized its effect in $ZZH$ and $WWH$ processes. BSM study in a more sophisticated framework is desirable but it is beyond the scope of the present work.

Experimentally, $W$ and $Z$-boson polarizations have been measured at hadronic colliders [46–48]. We also compute the cross sections for the processes when these bosons are polarized. For each process, the different production channels contribute predominantly to specific polarization configurations.
This can help in enhancing the contribution of the $gg$ channel, as compared to the $qq$ channel. The $gg$ channel have sometimes stronger dependence on the kappa parameters, in particular on $\kappa_V$. Therefore, an event sample with larger $gg$ channel contribution can be helpful.

The paper is organized as follows. In Sec. 2, we discuss the Feynman diagrams which contribute to $gg \rightarrow VVH$ amplitudes. The model independent framework to study new physics effects is outlined in Sec. 3. In Sec. 4, we provide details on the calculation techniques and various checks that we have performed in order to ensure the correctness of our calculation. In Sec. 5, we present numerical results in SM and BSM scenarios for all the $VVH$ processes. Finally, we summarize our results and conclude in Sec. 6.

2 Gluon fusion Contribution to $VVH$

The $gg$ channel contribution to $pp \rightarrow VVH$ is due to a loop-induced scattering process mediated by a quark-loop. The classes of diagrams contributing to $gg \rightarrow VVH$ processes are shown in Fig. 1\(^1\). For convenience, the diagrams contributing to $gg \rightarrow WWH$ process are shown separately in Fig. 2. The $gg \rightarrow \gamma\gamma H$ process receives contribution only from the pentagon diagrams, while, $\gamma ZH$ receives contribution from both pentagon and box class of diagrams. In case of $gg \rightarrow ZZH$, $WWH$ processes, triangle class of diagrams also contribute. We have taken all quarks but the top-quark as massless. Therefore, the top-quark contribution is relevant in diagrams where Higgs boson is directly attached to the quark loop. In the diagrams where Higgs boson does not directly couple to the quark loop, light quarks can also contribute. The complete set of diagrams for each process can be obtained by permuting external legs. These permutations imply that there are 24 diagrams in pentagon topology, 6 diagrams in each box topology and 2 diagrams in each triangle topology. The diagrams in which only one type of quark flavor runs in the loop, are not independent. Due to Furry’s theorem only half of them are independent \([50]\). This observation leads to a significant simplification in the overall calculation. This simplification, however, is not applicable to the $WWH$ case, where flavor changing interaction is involved in the quark loop. For example, see (a) and (b) in Fig. 2.

Thus, there are 12 independent pentagon diagrams (Fig. 1(a)) due to top-quark loop contributing to $gg \rightarrow \gamma\gamma H$ process. Similarly, the $gg \rightarrow \gamma ZH$ process receives contribution from 12 independent pentagon diagrams (Fig. 1(a)) due to top-quark loop and 3 independent box diagrams (Fig. 1(b)) for each quark flavor. In principle, 5 light quarks ($u,d,c,s,b$) and 1 heavy quark ($t$) contribute. The box class of diagrams arise due to $ZZH$ coupling and has effective box topology of $gg \rightarrow \gamma Z^* \gamma H$ amplitude. Furry’s theorem, in this case, implies that the axial vector coupling of $Z$ boson with quark does not contribute to $gg \rightarrow \gamma ZH$ amplitude.

Like the $gg \rightarrow \gamma ZH$ process, the $gg \rightarrow ZZH$ amplitude receives contribution from 12 independent pentagon diagrams with top-quark in the loop (Fig. 1(a)). However, there are 6 independent box diagrams with effective box topology of $gg \rightarrow ZZ^* \gamma H$ amplitude for each quark flavor which covers the possibilities of $H$ coupling with any of the two external $Z$ bosons (Fig. 1(b)). Further, a new box type contribution arises which has effective box topology of $gg \rightarrow HH^* \gamma H$ amplitude (Fig. 1(c)). Once again there are 3 such independent diagrams with only top-quark in the loop. In addition to that, there are 4 independent triangle diagrams with top-quark in the loop and having effective triangle topology of $gg \rightarrow H^* \gamma H$ amplitude (Fig. 1 (d), (e), (f)). In $gg \rightarrow ZZH$ amplitude, the Furry’s theorem implies that the vector and axial vector coupling of $Z$ boson with quarks can contribute at quadratic level only.

\(^1\)Feynman diagrams have been made using Jaxodraw \([49]\).
Figure 1: Different classes of diagrams for $gg \rightarrow VVH$, $V = \gamma, Z$. In diagram (b), $q$ represents all quark flavors. Process $gg \rightarrow \gamma\gamma H$ receives contribution only from (a) type diagrams, while $gg \rightarrow \gamma Z H$ gets contribution from both (a) and (b) type diagrams. In the case of $ZZ H$, all the diagrams contribute; the diagrams (b) and (f) cover the situation in which $H$ is attached to a $Z$ boson.

Figure 2: Different classes of diagrams contributing to $gg \rightarrow WW H$ process. With respect to $ZZH$, new classes of box and triangle diagrams appear due to $ZWW$ coupling. In (a) and (b), due to the flavor changing interaction of $W$ with quarks, both the quark flavors of a given generation enter in the loop. The diagrams (b), (g) and (i) cover the case when $H$ is attached to a $W$ boson.

Among all $VVH$ amplitudes, the structure of $gg \rightarrow WW H$ amplitude is the most complex. Due to the involvement of flavor changing interactions in Fig. 2 (a) and (b), the Furry’s theorem is not applicable to these diagrams. Therefore, 24 independent pentagon diagrams contribute to $gg \rightarrow WW H$ process for each generation of quarks. However, since we neglect Higgs coupling with light quarks including the $b$ quark, there are only 12 non-zero independent pentagon diagrams. In Fig. 2 (b), all the three quark generations contribute. Taking into account the possibility of Higgs boson coupling with any of the two external $W$ bosons, there are total 12 independent box diagrams of type (b) for each generation. In diagrams (a) and (b), the axial vector coupling of $W$ with quarks contributes at quadratic as well as at linear level. Like in the $gg \rightarrow ZZ H$ process, there are 3 independent box diagrams of type (c). Due to $ZWW$ coupling, a new box contribution of type (d) having effective box
topology of $gg \rightarrow HZ^*$ amplitude appears. Furry’s theorem for diagram (d) implies that the vector coupling of $Z$ with quarks does not contribute to the amplitude. The same explains the absence of similar box diagram due to $\gamma WW$ coupling. Further, there are 4 independent triangle diagrams with top-quark loop (Fig. 2 (e), (f) (g)) as in case of the $gg \rightarrow ZZH$ process. A new type of 3 independent triangle diagrams for each quark flavor with effective triangle topology of $gg \rightarrow Z^*$ amplitude appears, once again due to $ZWW$ coupling (Fig. 2 (h), (i)). These triangle diagrams are anomalous and they can receive contribution only from the third generation quarks as the bottom and top-quarks have very different masses. This is indeed the case for (h) type diagrams. However, we find that (i) type diagrams do not contribute. This is explained in the appendix A.

3 BSM Parametrization

Measuring the couplings of the Higgs boson with fermions, gauge bosons and with itself is an important aspect of finding the signatures of new physics at colliders. With the help of the data collected so far at the LHC, we now know couplings of the Higgs boson with the top quark with an accuracy of 10-20% and with vector bosons with an accuracy of 10% at 1$\sigma$ [51]. The Higgs self-couplings, on the other hand, are practically unconstrained [52].

To study the new physics effects in $VVH$ processes, we take the simplest approach of modifying the SM like couplings only, also known as the kappa framework for the parametrization of new physics [53,54]. In this framework, no new Lorentz structures and no new interaction vertices appear. The LHC experiments have interpreted the data using this framework so far. The couplings of our interest are $Ht\bar{t}$, $HVV$, $HHH$ and $HHVV$. Out of these couplings, $gg \rightarrow \gamma\gamma H$ is sensitive to only $Ht\bar{t}$ coupling. The $HVV$ coupling affects all other processes. The couplings $HHH$ and $HHVV$ affect only $gg \rightarrow VVH$, $V = Z, W$ processes.

The modification in these couplings due to new physics is implemented through scale factor $\kappa_i$ for various couplings of the Higgs boson in the SM. In kappa framework, there are three such scale factors namely $\kappa_t$ for Higgs coupling with top-quark, $\kappa_V$ for Higgs coupling with vector bosons ($\kappa_{HZZ} = \kappa_{HWV} = \kappa_V$) \(^2\) and $\kappa_\lambda$ for Higgs coupling with itself. Since in the SM both $HVV$ and $HHVV$ couplings are related, the scaling of $HHVV$ coupling is also parametrized by $\kappa_V$. In a more generic BSM framework, the $HHVV$ coupling, in principle, can be independent of $HVV$ coupling.

In the presence of BSM effects, the amplitudes for the $gg$ channel processes depend on $\kappa_t$, $\kappa_V$, and $\kappa_\lambda$ as follows.

\[
\mathcal{M}_{BSM}^{SM}(gg \rightarrow \gamma\gamma H) = \kappa_t \mathcal{M}_{SM}^{PEN} \\
\mathcal{M}_{BSM}^{SM}(gg \rightarrow \gamma ZH) = \kappa_t \mathcal{M}_{PEN}^{SM} + \kappa_V \mathcal{M}_{BX_1}^{SM} \\
\mathcal{M}_{BSM}^{SM}(gg \rightarrow ZZH) = \kappa_t \mathcal{M}_{PEN}^{SM} + \kappa_V \mathcal{M}_{BX_3}^{SM} + \kappa_t^2 \kappa_V \mathcal{M}_{BX_2}^{SM} + \kappa_t \kappa_V \kappa_\lambda \mathcal{M}_{TR_1}^{SM} + \kappa_t \kappa_V \mathcal{M}_{TR_2}^{SM} + \kappa_t^2 \kappa_V \mathcal{M}_{TR_3}^{SM} \\
\mathcal{M}_{BSM}^{SM}(gg \rightarrow WWH) = \kappa_t \mathcal{M}_{PEN}^{SM} + \kappa_V \mathcal{M}_{BX_1}^{SM} + \kappa_t^2 \kappa_V \mathcal{M}_{BX_2}^{SM} + \kappa_t \kappa_V \mathcal{M}_{BX_3}^{SM} + \kappa_t \kappa_V \kappa_\lambda \mathcal{M}_{TR_1}^{SM} + \kappa_t \kappa_V \mathcal{M}_{TR_2}^{SM} + \kappa_t^2 \kappa_V \mathcal{M}_{TR_3}^{SM} + \kappa_t^2 \kappa_V \mathcal{M}_{TR_4}^{SM}
\]

In the above, the amplitude $\mathcal{M}_{i}^{SM}$ is related to one of the diagram classes displayed in Fig. 1 (Fig. 2 for $WWH$). This can be easily identified by looking at $\kappa$-factors in front of the amplitude. Note that in $WWH$ amplitude, $\mathcal{M}_{TR_4}^{SM}$ includes both (h) and (i) type diagrams of Fig. 2. This parametrization

\(^2\)Note that in the SM, the tree level interaction vertices $H\gamma\gamma$ and $H\gamma Z$ do not exist.
does not affect the gauge invariance of the amplitudes with respect to the gluons as it will become clear in the next section. The standard model prediction can be obtained by setting $\kappa_t = \kappa_V = \kappa_\lambda = 1$. Thus, except in $gg \rightarrow \gamma\gamma H$, we can expect nontrivial interference effects on total and differential cross sections for $gg \rightarrow VVH$ processes due to new physics in $\kappa$-framework.

4 Calculation and Checks

The calculation of quark-loop diagrams is carried out using a semi-automated in-house package OVReduce [55] which allows the calculation of any one-loop amplitude with maximum five propagators in the loop. The main steps involved in our calculation are: quark-loop trace evaluation, one-loop tensor reduction to master integrals and evaluation of master integrals. Trace calculation and simplification of the amplitude are done using symbolic manipulation software FORM [56]. Tensor reduction of one-loop amplitudes into one-loop master integrals is done numerically following the method of Oldenborgh-Vermaseren [57]. Further, the one-loop master integrals are also calculated numerically using the OneLoop package [58].

There are a number of checks that we have performed in order to ensure the correctness of the amplitudes. We have checked that the amplitudes are separately UV and IR finite. In $4 - 2\epsilon$ dimensions, these divergences appear as poles in $1/\epsilon$ (for UV and IR) and $1/\epsilon^2$ (for IR only). Each pentagon diagram is UV finite. This we expect from the naive power counting. The individual box diagram is not UV finite, however, the full box amplitude, in each class, is UV finite. The UV finiteness of triangle amplitudes holds for each diagram. One-loop diagrams with all massive internal lines are IR finite, as expected. Thus, IR finiteness check is relevant to the diagrams with massless quarks in the loop. This includes box class of diagrams of Fig. 1(b) in $gg \rightarrow \gamma ZH$ and $ZZH$. In $gg \rightarrow WWH$ case, potentially IR divergent diagrams include Fig. 2(a), (b), (h) and (i). Unlike UV, the IR finiteness holds for each diagram [23].

We have also checked the gauge invariance of the amplitudes with respect to the external gluons. For that we numerically replace the gluon polarization vector $\epsilon^\mu(p)$ by its four momentum $p^\mu$ and expect a gauge invariant amplitude to vanish. We find that the gauge invariance check holds for each class of diagrams. This is expected because different box and triangle topologies for each process arise due to the existence of various electroweak couplings. This is a very strong check on our calculation which is organized using only prototype amplitudes. However, this check cannot verify relative signs between different classes of diagrams. In order to verify such relative signs, one needs to perform gauge invariance check in electroweak theory which is a non-trivial task [3]. We rather rely on cross-checking

\[3\text{A wrong relative sign between different class of diagrams may lead to violation of unitarity in certain processes [59].}\]
the calculation using different methods and tools. We have compared our matrix element for each process with those calculated using MadLoop [60] and have found an excellent agreement. Being process specific, our code is efficient and provides greater flexibility when producing phenomenological result.

Numerical predictions for cross section and kinematic distributions are obtained using Monte Carlo techniques for phase space integration. We use AMC [61] package for Monte Carlo phase space integration which is based on VEGAS [62] algorithm and allows parallelization of phase space point generation and matrix-element computation using PVM software [63].

5 Numerical Results

The cross section and kinematic distributions for $pp \rightarrow VVH$ processes in SM and in BSM constitute the main results of this section. The numerical results are produced using following basic selection cuts unless stated otherwise,

$$p_T^\gamma > 50 \text{ GeV}, \mid \eta_\gamma \mid < 2.5, \Delta R_{\gamma\gamma} > 0.4, \mid y_{H,Z,W} \mid < 5.$$ (5)

The results for the $gg$ channel processes are calculated using CT14LO [64] parton distribution function (PDF) and partonic center-of-mass energy ($\sqrt{s}$) is chosen as common scale for renormalization ($\mu_R$) and factorization ($\mu_F$). The results are obtained for three different choices of collider energies: $\sqrt{s} = 14, 27, \text{ and } 100 \text{ TeV}$. From phenomenological point of view we will focus on $p_T(H)$ and $M(VV)$ distributions.

We compare the $gg$ channel contribution to $pp \rightarrow VVH$ with contribution arising from the $qq$ channels. The $qq$ channel contribution at LO and NLO (QCD) is calculated using MadGraph5_aMC@NLO [60] in five flavor scheme for all but WWH production. The $qq$ channel contribution to WWH production is instead calculated in four flavor scheme$^4$. The LO $qq$ channel contributions are pure electroweak processes and they do not depend on $\alpha_s$. For LO and NLO (QCD) results, we use CTEQ14LO and CT14NLO PDFs, respectively [64]. The scale choice is same as in the $gg$ channel calculation. In both $gg$ and $qq$ channel calculations, the scale uncertainties are estimated by varying $\mu_R$ and $\mu_F$ independently by a factor of two. We quote only minimum and maximum uncertainties thus obtained.

To quantify the relative importance of the $gg$ channel contribution in processes dominated by the $qq$ channel, we define following ratio,

$$R_1 = \frac{\sigma_{VVH,LO}^{gg}}{\sigma_{VVH,NLO}^{qq} - \sigma_{VVH,LO}^{qq}}.$$ (6)

This ratio compares the leading order $gg$ channel contribution with NLO QCD correction in the $qq$ channel. Recall that technically $gg$ channels contribute at NNLO. Similarly, at differential level we define another ratio,

$$R_2 = \frac{\frac{d\sigma}{dX}^{VVH,LO}_{gg}}{\frac{d\sigma}{dX}^{VVH,NLO}_{qq}},$$ (7)

where, $X$ denotes a kinematic variable.

As mentioned in section 3, the BSM effects are parametrized in terms of scale factors $\kappa_t$, $\kappa_V$ and

$^4$For WWH production, currently MadGraph5_aMC@NLO cannot produce NLO correction to the $bb$ channel.
\(\kappa_\lambda\). In order to compare their relative importance, we vary them independently by 10\% about their SM values. Further, we comment on the effect of \(\kappa_\lambda\) and \(\kappa_{HHVV}\) (the scale factor for the \(HHVV\) coupling\(^5\)) which are least constrained at present, in \(ZZH\) and \(WWH\) processes.

### 5.1 Predictions for the \(pp \rightarrow \gamma\gamma H\) process

| \(\sqrt{s}\) (TeV) | \(\sigma_{gg}^{\gamma\gamma H, \text{LO}}\) [ab] | \(\sigma_{qq}^{\gamma\gamma H, \text{LO}}\) [ab] | \(\sigma_{qq}^{\gamma\gamma H, \text{NLO}}\) [ab] |
|---------------------|---------------------------------|---------------------------------|---------------------------------|
| 14                  | 5.36_{-20\%}^{+28\%}           | 0.033_{-14\%}^{+13\%}         | 0.046_{-6\%}^{+5\%}            |
| 27                  | 22.0_{-19\%}^{+22\%}           | 0.153_{-17\%}^{+15\%}        | 0.234_{-7\%}^{+5\%}           |
| 100                 | 220.1_{-21\%}^{+27\%}          | 1.4_{-20\%}^{+20\%}          | 2.25_{-8\%}^{+5\%}           |

Table 1: A comparison of different perturbative orders in QCD coupling contributing to \(pp \rightarrow \gamma\gamma H\) hadronic cross section at \(\sqrt{s} = 14, 27,\) and 100 TeV.

The cross section for this process is dominated by the \(gg\) channel. In the \(qq\) channel, only bottom-quark initiated subprocess contribute to \(\gamma\gamma H\) production. However, this cross section is quite small, owing to small bottom Yukawa coupling. In Tab. 1, we compare the \(gg\) and \(qq\) channel contributions to the hadronic cross section at 14, 27 and 100 TeV colliders. The results are with minimum 50 GeV transverse momentum of photons. We find that the \(gg\) channel contribution increases 40 times as the collider center-of-mass energy goes from 14 TeV to 100 TeV. Due to a small cross-section, this process cannot be observed at the HL-LHC; FCC-pp will be more suitable. The \(gg\) channel contribution becomes important at higher center-of-mass energy collider, as in this case smaller partonic momentum fractions \(x\) are accessible, where gluon flux is significantly large. The scale uncertainties on the cross sections for the \(gg\) channel are in the range of 20-30\%. It is clear from the table that the \(qq\) channel contribution is negligible compared to the \(gg\) channel contribution. It is merely 1\% of the \(gg\) channel contribution even after including the NLO-QCD corrections.

In Fig. 3, we have plotted \(p_T\) distributions for hardest photon, next-to-hardest photon, and Higgs in the left figure, and diphoton invariant mass distribution \((M(\gamma\gamma))\) in the right figure for the 100 TeV collider (FCC-hh). The \(p_T\) distributions for them peak around 150 GeV, 90 GeV, and 70 GeV, respectively. We find that the tail of \(p_T(H)\) is softer than that of photons. The \(M(\gamma\gamma)\) distribution shows an interesting feature – it has two peaks. The right peak occurs at around 350 GeV, exhibiting the \(t\bar{t}\) threshold effect in the distribution. To verify that the second peak is indeed due to \(t\bar{t}\) threshold effect, we changed in our code the value of \(m_t\) from 173 GeV to 200 GeV and the second peak was found to get shifted to 400 GeV.

As mentioned before, this process is a background to double Higgs production process when one of the Higgs bosons decays into a photon pair. To manage the background one usually looks at \('\gamma\gamma b\bar{b}'\) final state, instead of \('bb\bar{b}\bar{b}'\), as the signature of the double Higgs boson production. At a 100 TeV collider, while the cross section for the \(gg \rightarrow \gamma\gamma H\) production, with the cuts in Eq. 5, is about 220 ab, the cross section for \(gg \rightarrow HH \rightarrow \gamma\gamma H\), with the same set of cuts, is about 2600 ab. From the right

\(^5\)Note this is different from \(k_\nu\), which scales both \(HV\) and \(HHVV\) couplings at the same time.
Panel of Fig. 3, it can be seen that the cross section for $\gamma\gamma H$ production in the bin from 120 GeV to 140 GeV is about 3 ab. On the other hand, all the cross section for $gg \rightarrow HH \rightarrow \gamma\gamma H$ is concentrated in a very narrow width around the mass of Higgs, 125 GeV \(^6\). As a result, $gg \rightarrow \gamma\gamma H$ is an insignificant background to the process $gg \rightarrow HH \rightarrow \gamma\gamma H$.

Regarding anomalous coupling contributions, we note that as only pentagon diagrams contribute to the process $gg \rightarrow \gamma\gamma H$, its cross section scales as $\kappa_t^2$. So a 10% change in $\kappa_t$ will change the cross section and distributions by about 20%. For the $qq$ channel process, the cross section is too small. It depends on $\kappa_b$, which we do not change from the standard model value.

5.2 Predictions for the $pp \rightarrow \gamma ZH$ process

Unlike $\gamma\gamma H$ case, the $\gamma ZH$ production receives dominat contribution from the $qq$ channel. With $p_T^\gamma > 50$ GeV, the $gg$ channel contributions to $\gamma ZH$ production at 14, 27, and 100 TeV colliders are 4 ab, 16 ab, and 168 ab, respectively. The corresponding values for the LO $qq$ channel contribution are 689 ab, 1733 ab, and 7498 ab, respectively. From Tab. 2, it can be seen that $R_1$, which is the ratio of the $gg$ channel contribution to NLO correction in the $qq$ channel, is as small as 0.06 for 100 TeV collider, and even smaller for HE-LHC (27 TeV) and LHC (14 TeV). The scale uncertainties for the $gg$ channel are around 20% while those for the $qq$ channel at NLO are in the range of 2 − 3%. A larger scale dependence in the $gg$ channel contribution can be attributed to the presence of higher power of $\alpha_s$ factor in the $gg$ amplitudes.

\(^6\)In the right panel of Fig. 3, at 125 GeV, rather than showing a very narrow Breit-Wigner distribution, we have shown the total cross section for $gg \rightarrow HH \rightarrow \gamma\gamma H$ by a single vertical line.
Table 2: A comparison of different perturbative orders in QCD coupling contributing to $pp \rightarrow \gamma Z H$ hadronic cross section at $\sqrt{s} = 14$, 27, and 100 TeV. $R_1$ compares the $gg$ channel contribution with correction at NLO and it is defined in Eq 14.

| $\sqrt{s}$ (TeV) | $\sigma_{gg}^{\gamma Z H, \text{LO}}$ [ab] | $\sigma_{qq}^{\gamma Z H, \text{LO}}$ [ab] | $\sigma_{qq}^{\gamma Z H, \text{NLO}}$ [ab] | $R_1$ |
|-----------------|-----------------|-----------------|-----------------|-----|
| 14              | $4.0^{+26\%}_{-20\%}$ | $689 + 0\% - 0.2\%$ | $909 + 1.7\% - 1.3\%$ | 0.02 |
| 27              | $16^{+22\%}_{-17\%}$ | $1773 + 3.0\% - 3.6\%$ | $2349 + 1.7\% - 2.1\%$ | 0.03 |
| 100             | $168^{+21\%}_{-19\%}$ | $7498 + 8.8\% - 9.4\%$ | $10430 + 2.2\% - 3.8\%$ | 0.06 |

Table 3: Effect of $p_T^\gamma$ cuts on the cross section of $pp \rightarrow \gamma Z H$ production at the 100 TeV collider (FCC-hh).

| $p_{T,\text{min}}$ (GeV) | $gg \rightarrow \gamma Z H$ [ab] | $qq \rightarrow \gamma Z H(\text{LO})$ [ab] | $qq \rightarrow \gamma Z H(\text{NLO})$ [ab] |
|--------------------------|-----------------|-----------------|-----------------|
| 50                       | 168             | 7498            | 10430           |
| 100                      | 95              | 2812            | 4072            |
| 150                      | 47              | 1366            | 2069            |
| 200                      | 28              | 765             | 1190            |

In Tab. 3, the effect of various $p_T^\gamma$ cuts in $gg$ and $qq$ channels has been shown. As the cut on $p_T^\gamma$ increases, the $qq$ channel cross section decreases faster than the $gg$ channel. In going from 50 GeV to 200 GeV cut, the cross section of the $gg$ channel decreases roughly by a factor of 6, while that of the $qq$ channel decreases by a factor of 9. Thus relative contribution from the $gg$ channel can be enhanced with the help of harder $p_T^\gamma$ cut. We find that the $p_T(H)$ cuts have opposite effect i.e. the $gg$ channel is favored at low $p_T(H)$.

In Fig. 4, we have displayed $p_T$ distributions for the final state particles on the left, and $\gamma Z$ pair invariant mass distribution on the right for the 100 TeV collider. The $p_T$ distributions peak around 100 GeV while the $M(\gamma Z)$ distribution peaks around 200 GeV. Like the case of $gg \rightarrow \gamma \gamma H$ process as a background to $gg \rightarrow HH \rightarrow \gamma \gamma H$, the $gg \rightarrow \gamma Z H$ process is also an insignificant background to $gg \rightarrow HH \rightarrow \gamma Z H$. This is because at a 100 TeV collider, with the cuts in Eq. 5, the cross section for $gg \rightarrow HH \rightarrow \gamma Z H$ is about 2000 ab, while the cross section for $gg \rightarrow \gamma Z H$ process is about 170 ab. Moreover, all the cross section for the $gg \rightarrow HH \rightarrow \gamma Z H$ process congregates around the mass of the decaying Higgs boson, 125 GeV, while, as can be seen from the right panel of the Fig. 4, the cross section for the $gg \rightarrow \gamma Z H$ process in the bin from 120 GeV to 140 GeV is about 3 ab. However, the $qq$ channel for $\gamma Z H$ production may act as an important background for the $gg \rightarrow HH \rightarrow \gamma Z H$ process.

In Fig. 5, we show $p_T(H)$ distributions for different classes of diagrams – pentagon, box, and sum of their individual contributions, their interference, and total at the 100 TeV collider. The contribution of the box diagrams is more than the pentagon diagrams mainly because of the light quark contributions.

\footnote{However, instead of showing a very narrow Breit-Wigner distribution for Higgs' decay, we have depicted the total cross section at 125 GeV by a single vertical line.}
The interference effect between the pentagon and box diagrams has kinematic dependence. We find that in the region of our kinematic interest, it is always destructive and, near the peak, its effect is close to -30%.

Figure 4: Kinematic distributions for $gg \rightarrow \gamma ZH$ in the SM at 100 TeV. The purple vertical line in the right plot at 125 GeV shows the total cross section for the process $gg \rightarrow HH \rightarrow \gamma ZH$. “×200” means that the height of the purple vertical line needs to be scaled by a factor of 200 to get the correct cross section for the $gg \rightarrow HH \rightarrow \gamma ZH$ process.

Figure 5: Left: The contribution of pentagon (blue) and box (green) diagrams, as well as their squared sum, interference, and total contribution to $p_T(H)$ distributions for the $gg \rightarrow \gamma ZH$ process at the 100 TeV FCC-hh collider. Right: The effect of excluding top-quark contribution from the diagrams in Fig. 1(b) to the full amplitude.

Since the $gg\gamma Z^*$ type box amplitude does not depend on the axial-vector coupling of the off-shell longitudinal $Z$-boson with the quarks, the top-quark contribution is not very significant at the level of total cross section. This is shown in the right panel of the Fig. 5. We can see that in the tail where top quark is effectively light, the cross section increases by about 20%.
We have noted that the relative importance of gluon fusion channel can be enhanced by applying higher $p_T(\gamma)$ cuts. To distinguish the $gg$ channel contribution from the dominant $qq$ channel, one can use the polarized cross sections and distributions. In Fig. 6, we have displayed the LO cross sections for various helicity states of the final state particles, $\gamma$ and $Z$ boson. The figure also shows the contribution of various polarization states of the initial state particles. We cannot measure the polarization of the initial state particles that are in a bound state, proton. However, experimentally, one can measure the $Z$-boson polarization [46–48]. The polarization of photon has been measured by the LHCb collaboration in $b$-baryon’s decay [65–71]. At a 100 TeV collider, the contribution of the $gg$ channel process to the production of $\gamma Z H$ is only 2.2%. However, if we look at those final states where photon and $Z$-boson have the same transverse polarization, then this ratio increases to $10−11\%$. (The $qq$ channel makes largest contribution when the $Z$ boson is longitudinally polarized.) This is a non-trivial contribution, and can be measured, if enough integrated luminosity is available. In Fig. 7, we have plotted the Higgs boson and $Z$-boson $p_T$ distributions. By making appropriate cuts on the small and large $p_T$ of these
particles, we can further enhance the \( gg \) channel contribution.

Turning to the effect of anomalous couplings, we find that the \( gg \) channel shows very small dependence on the \( \kappa_t \), as it is present only in pentagon diagrams whose contribution is small (see Fig. 5). However, it strongly depends on \( \kappa_V \), as the box-diagrams contribution is much more than the pentagon-diagram contribution. We find that the change in cross section for \( \kappa_t = 1.1(0.9) \) is 5.4% (-1.2%). On the other hand, for \( \kappa_V = 1.1(0.9) \) the cross section changes by 18% (15%). We do not show the effect of anomalous couplings on the distribution. It can be understood qualitatively from Eq. 2 and Fig. 5 in the \( gg \) channel. The \( qq \) channel is sensitive to \( \kappa_V \) only. The amplitude has overall linear dependence on \( \kappa_V \) due to which the effect of anomalous coupling \( k_V \) is flat for both total and differential cross sections.

### 5.3 Predictions for \( pp \to ZZH \)

The cross sections for \( ZZH \) production via various channels have been tabulated in Table 4 along with the corresponding scale uncertainties. The total cross section for \( gg \to ZZH \) is significantly larger than that of \( gg \to \gamma ZH \). This increase is mainly due to the contribution from axial-vector coupling of \( Z \) with quarks. The \( gg \) channel contributions to \( ZZH \) production at 14, 27, and 100 TeV colliders are 124 ab, 579 ab, and 7408 ab, respectively. The corresponding values of the LO \( qq \) channel contributions are 2184, 5997, and 36830 ab, respectively. The ratio, \( R_1 \), is found to be 0.25, 0.4, and 1.05, respectively. Thus at 100 TeV, the \( gg \) channel contribution is as important as the QCD NLO correction. As has already been discussed, this increase in ratio \( R_1 \) with collider energy is due to the large gluon flux.

| \( \sqrt{s} \) (TeV) | \( \sigma_{gg,\,LO} \) [ab] | \( \sigma_{qg,\,LO} \) [ab] | \( \sigma_{qg,\,NLO} \) [ab] | \( R_1 \) |
|-----------------|-----------------|-----------------|-----------------|-------|
| 14              | 124\(^{+28.2\%}\)\(^{-21.0\%}\) | 2184\(^{+0.2\%}\)\(^{-0.6\%}\) | 2710\(^{+1.4\%}\)\(^{-1.0\%}\) | 0.24  |
| 27              | 579\(^{+23.3\%}\)\(^{-18.5\%}\) | 5997\(^{+2.4\%}\)\(^{-3.0\%}\) | 7396\(^{+1.3\%}\)\(^{-1.6\%}\) | 0.41  |
| 100             | 7408\(^{+22\%}\)\(^{-18\%}\) | 36830\(^{+8.0\%}\)\(^{-8.7\%}\) | 43940\(^{+1.2\%}\)\(^{-2.6\%}\) | 1.04  |

Table 4: A comparison of different perturbative orders in QCD coupling contributing to \( pp \to ZZH \) cross section at \( \sqrt{s} = 14, 27, \) and 100 TeV. The ratio \( R_1 \), defined in Eq. 6, quantifies the \( gg \) channel contribution with respect to the NLO correction in \( qq \) channel process.

In the \( gg \) channel, the scale uncertainties of the total cross sections are in the range of 20-30% which is similar to the scale uncertainties observed for \( \gamma \gamma H \) and \( \gamma ZH \). We find that the uncertainty due to the renormalization scale variation is more than that due to the factorization scale variation. While the change in the renormalization scale mainly changes \( \alpha_s \), the change in the factorization scale changes the parton distribution function. The uncertainty for the renormalization scale variation is nearly same at all the collider energies. This happens as the contribution to the total cross section comes from nearly same region of partonic center of mass energy of the process and in every bin of this region, \( \alpha_s \) changes by nearly same factor for the change in the renormalization scale. However, uncertainty for the factorization scale variation is different for different colliders. This happens as for different collider energies, different \( x \) regions contribute to the process and for different \( x \) regions change
in parton distribution function with the factorization scale is different, where $x$ is partonic momentum fraction. We have also observed that with an increase in the factorization scale, for 14 and 27 TeV colliders, the cross-section decreases; however for 100 TeV collider the cross-section increases.

![gg → ZZH](image1.png)

**Figure 8:** Kinematic distributions for $gg \rightarrow ZZH$ in the SM at the 100 TeV collider. $Z_1$ and $Z_2$ refer to the hardest, and second hardest in $p_T$, respectively.

In the tree level $qq$ channel, there is no QCD vertex. So here change in the renormalization scale does not affect the cross section. But, the change in the factorization scale can affect the cross section, and uncertainty increases with collider energy. However, when NLO QCD correction is considered, change in either of renormalization and factorization scales changes the cross section. The uncertainty in the cross section due to the renormalization scale variation is small as NLO QCD correction is much smaller than the LO results. The overall uncertainty in this case is smaller than the LO case, which is expected for higher order calculation.

![gg → ZZH](image2.png)

**Figure 9:** Left: SM contribution of pentagon (blue), box (green), triangle (gray) diagrams, as well as their squared sum (black), interference (orange) and total (red) contribution to $p_T(H)$ distributions in $gg \rightarrow ZZH$ at 100 TeV collider (FCC-hh). Right: The effect of excluding top-quark contribution from Fig. 1(b) to full amplitude.
Figure 10: The left figure shows the normalized distribution for $p_T(H)$ in $gg$ and $qq$ channel processes. In the top panel of the right figure, we show the distribution of $qq$ (NLO) + $gg$ (LO) and $qq$ (NLO) production with $p_T(H)$. The lower panel shows the ratio of them.

In Fig. 8, we have plotted $p_T$ distributions for leading $p_T(Z_1)$, next-to-leading $p_T(Z_2)$, and Higgs boson in the left figure, and $Z$-pair invariant mass distribution in the right figure for the 100 TeV collider. The $p_T$ distributions peak around 100 GeV, 60 GeV, and 80 GeV, respectively. The $M(ZZ)$ distribution peaks around $Z$ pair threshold.

Interference of various diagrams plays a major role in $gg \rightarrow ZZH$ production. In Fig. 9, we have shown the $p_T(H)$ distributions for penta, box, triangle, sum of their individual contributions, interference, and total at the 100 TeV collider (FCC-hh). As can be seen, the box diagrams give the largest contribution, then comes the triangle contribution and the penta contributes the least. Like in $γZH$ case, the large box contribution is due to the light quarks in the loop. Further, because of large destructive interference, the total contribution is smaller by about a factor of five than the box contribution.

Figure 11: LO cross section for $ZZH$ production in different helicity configurations in $gg$ (left) and $qq$ (right) channels. Legends correspond to different helicities of initial states.

We have found that the top-quark contribution in $ggZZ^*$-type box diagram is quite significant despite the propagators suppression. This is due to the coupling of off-shell longitudinal $Z$ boson (e-
effectively the Goldstone boson) with top-quark and it is proportional to $m_t^8$. We show the effect of excluding the top-quark contribution in $ggZZ^*$-type box diagram (Fig.1(b)) on $p_T(H)$ distribution in the right panel of Fig. 9. As we expect, excluding top-quark contribution in $ggZZ^*$-type box diagram leads to non-unitary behavior in the full amplitude.

In the left figure of Fig. 10, we see that the shape of $p_T$ distribution for Higgs boson in the $gg$ and $qq$ channel processes is nearly same at 100 TeV collider (FCC-hh). The relative importance of the $gg$ channel over the $qq$ channel is visible in the tail. In the right plot, we give $p_T(H)$ distribution combining $gg$ and $qq$ (NLO) contributions as the best prediction from our calculations. In the bottom panel of the plot, $R_2$ signifies the ratio of differential cross section from the $gg$ channel to that from NLO $qq$ channel process. The dashed line shows the ratio of corresponding total cross sections, which is 0.17. At the tail of the distribution, we see the $gg$ channel contribution becomes further important, but there differential cross section itself is quite small.

Once again we find that if we categorize events based on the helicity states of the two $Z$ bosons, the relative importance of the $gg$ channel contribution over the $qq$ channel contribution can be increased. From Fig. 11, we see that in the $gg$ channel the longitudinal $Z$ bosons contribute the most, while in the $qq$ channel their transverse helicity states give dominant contribution. The relative cross section of the $gg$ channel with respect to the $qq$ channel is about 20%. However, if we restrict ourselves to the case when both $Z$-bosons are longitudinally polarized, then this ratio almost doubles. Since the cross section for these polarized states for the $gg$ channel is about 2000 ab, there will be enough events to observe this process at a 100 TeV machine. At the distribution level, from the Fig. 12, we observe that if we restrict ourselves to the contributions from the longitudinal $Z$ bosons with $p_T(H)$ beyond 150 GeV, the relative contribution of the $gg$ channel increases significantly. Experimentally, one may look at the signature $l^+l^-l'^+l'^-$ and for $H \to b\bar{b}$. Taking into account the branching ratios, and $b$-tagging efficiency, one may expect about 75 events at the FCC-hh collider (with 30 ab$^{-1}$ integrated luminosity) from $gg$ channel and about 210 events from $8$The results for $ZZH$ process presented in the conference proceeding [34] did not include top-quark contribution. We also fixed a bug in the code, numerical impact of which has been found to be small.

Figure 12: The kinematic distributions from $gg$ and $qq$ channels when the final state $Z$-bosons are longitudinal. The ratio of the distributions from the two channels have been shown in the lower panel of each figure. In the right figure, $Z_1$ denotes the harder of two $Z$-bosons in $p_T$.
$qq$ channel. This is when both $Z$ bosons are longitudinally polarized. This number will go down when detection and kinematic-cut efficiency factors are included. However, if in future, one could use hadronic decay modes of a $Z$ boson to measure its polarization, then the number of events would increase.

As can be seen from Eq. 3, the $gg$ channel depends on $\kappa_t$, $\kappa_V$, and $\kappa_\lambda$. We vary these $\kappa$’s by 10% from their SM values. The $gg$ channel strongly depends on both $\kappa_t$ and $\kappa_V$. In the $gg$ channel, $\pm10\%$ change in $\kappa_t$ causes 68% and -18% variations in the cross section, respectively. And $\pm10\%$ change in $\kappa_V$ causes 45% and -28% changes in the cross section, respectively. Similar variation in $\kappa_\lambda$ does not lead to much variation in the total cross section. Since this coupling is not yet well constrained, we will discuss it in detail in subsection 5.5.

\begin{align*}
\kappa_t &= 1 \\
\kappa_t &= 1.1 \\
\kappa_t &= 0.9
\end{align*}

\begin{align*}
\kappa_V &= 1 \\
\kappa_V &= 1.1 \\
\kappa_V &= 0.9
\end{align*}

In Fig. 13, we display the effect of anomalous $\kappa_t$ and $\kappa_V$ on $p_T(H)$ distribution. We show the absolute distribution in the top panel, while in the bottom panel we show the ratio of distribution with anomalous coupling to that with the SM coupling. We can see that in the presence of anomalous $\kappa_t$ and $\kappa_V$, the shape of the distribution remains more or less same. However, due to non-trivial interference effects, the modifications in presence of anomalous couplings are not same in all the bins. We see that for $\kappa_t = 1.1$ the cross section in the bins near tail of the distribution increases by a factor of 2. On the other hand for $\kappa_V = 1.1$, the maximum change in the cross section is around 1.5. Thus tail of the distributions are more sensitive to modifications in couplings due to high scale new physics. The $qq$ channel depends mainly on $\kappa_V$. However, as we have considered bottom quark contribution also, the $qq$ channel depends on $\kappa_\lambda$ as well. In the $qq$ channel, $\kappa_V$ comes as an overall factor both for LO and NLO amplitude, and so the effect of 10% change in $\kappa_V$ causes around 20% change in the cross section, both at total and differential levels. We find a very mild dependence on $\kappa_\lambda$.

5.4 Predictions for $pp \rightarrow WW H$

The cross section for this process is the largest among all the $VVH$ processes considered in this paper. In Tab. 5, we report the cross section predictions for $WWH$ process at different collider center-of-mass energies. The $gg$ channel contributions to $WWH$ production at 14, 27, and 100 TeV colliders are 290 ab, 1344 ab, and 17403 ab, respectively. These numbers are roughly 2.3 times higher than $ZZH$ cross sections. As regards scale uncertainties, the $gg \rightarrow WW H$ cross sections follow the same pattern as observed in $gg \rightarrow ZZH$. The corresponding values of the LO $qq$ channel cross sections are 8658, 23040,
and 128000 ab, respectively\(^9\). The ratio, \(R_1\), is found to be 0.15, 0.19, and 0.43, respectively. Unlike \(ZZH\) production, the contribution of the \(gg\) channel is relatively smaller.

| \(\sqrt{s}\) (TeV) | \(\sigma_{gg}^{WWH, LO}\) [ab] | \(\sigma_{qq}^{WWH, LO}\) [ab] | \(\sigma_{qq}^{WWH, NLO}\) [ab] | \(R_1\) |
|-------------------|------------------|------------------|------------------|--------|
| 14                | 290\(^{+27.6\%}_{-21.0\%}\) | 8658\(^{+0.3\%}_{-0.7\%}\) | 11220\(^{+1.5\%}_{-1.1\%}\) | 0.11   |
| 27                | 1344\(^{+22.5\%}_{-18.8\%}\) | 23040\(^{+2.1\%}_{-2.7\%}\) | 30090\(^{+1.7\%}_{-1.8\%}\) | 0.19   |
| 100               | 17403\(^{+20.6\%}_{-17.8\%}\) | 128000\(^{+7.5\%}_{-8.1\%}\) | 167300\(^{+2.0\%}_{-3.3\%}\) | 0.44   |

Table 5: A comparison of different perturbative orders in QCD coupling contributing to \(pp \rightarrow WWH\) hadronic cross section at \(\sqrt{s} = 14, 27,\) and 100 TeV. The ratio \(R_1\) defined in Eq. 6 quantifies the \(gg\) channel contribution with respect to the \(qq\) (NLO) correction. The \(qq\) results are reported in four flavor scheme.

In the left figure of Fig. 14, we can see that the \(p_T\) distribution of \(W^+\) and \(W^-\) overlap with each other, which is expected in the case of the \(gg\) channel. The \(p_T(H)\) distribution peaks around 100 GeV, and its fall in the tail is slower than that of \(p_T(W^\pm)\) distributions. In the right of Fig. 14, the distribution for invariant mass of \(W^+\) and \(W^-\) has been shown, which peaks around 200 GeV.

Figure 14: \(p_T\) and \(M(WW)\) distributions for \(gg \rightarrow WWH\) in the SM at the 100 TeV collider (FCC-hh).

Like \(gg \rightarrow ZZH\) production case, in \(gg \rightarrow WWH\) production also, interference of various diagrams plays a major role. On the left of Fig. 15, we have shown \(p_T(H)\) distributions for individual topologies as well as for their interference at a 100 TeV collider. The box contribution is the largest in all the bins while the pentagon contribution is the lowest beyond \(p_T > 100\) GeV. The total contribution is much smaller than the box contribution because of strong destructive interference effect which is shown by orange line in the figure.

\(^9\)Due to technical reasons in the NLO calculation using \textsc{mg5amcnlo}, the \(qq\) results are provided in 4 flavor scheme.
Due to the presence of top quark propagators in $ggWW^*$ type box diagram, one may naively think of a suppressed contribution from the third generation quarks at low $p_T(H)$. In Fig. 15, we show the effect of excluding the third generation quark contribution from the $ggWW^*$ type box diagram, on the $p_T(H)$ distribution. Like in $gg \rightarrow ZZH$, the third generation quark contribution in $ggWW^*$ type box diagram is necessary for the unitarization of the full amplitude.

In the left plot of Fig. 16, the normalized $p_T$ distributions for Higgs boson in the $gg$ and $qq$ channel processes have been shown for 100 TeV collider (FCC-hh). The $p_T(H)$ distribution in the $gg$ channel peaks slightly on the harder side making the channel more relevant in higher $p_T(H)$ bins. To quantify it better we also plot the the ratio of distributions due to $qq$ (NLO) + $gg$ (LO) and $qq$ (NLO). At

![Figure 15](image1.png)

**Figure 15:** Left : SM contribution of pentagon (blue), box(green), triangle (gray) diagrams, as well as their square sum, interference, and total contribution to $p_T(H)$ distributions in $gg \rightarrow WWH$ at 100 TeV FCC-hh collider. Right: The effect of excluding third generation quark contribution from Fig. 2(b) to full amplitude.

![Figure 16](image2.png)

**Figure 16:** The left figure shows the normalized distribution for $p_T(H)$ in the $gg$ and $qq$ channel process. In the top panel of the right figure, we show the distribution due to $qq$ (NLO) + $gg$ (LO) and $qq$ (NLO) production with $p_T(H)$. The lower panel shows their ratio. Results do not include contribution of the $bb$ channel process.
differential level the ratio varies between 1.05 and 1.18 compared to its value (1.1) for the total cross section. Once again, we find that the $gg$ channel contribution is more relevant at higher $p_T$ where its contribution reaches 18%.

Figure 17: LO cross section for $WWH$ production in different helicity configurations in $gg$ (left) and $qq$ (right) channels. Legends correspond to different helicities of initial states.

Figure 18: Comparing the $gg$ and $qq$ channel contributions to $W^+W^-H$ for events with longitudinal $W$ bosons.

Similar to the case of $ZZH$, for this process also, the cross section in the $gg$ channel is dominated by longitudinally polarized $W$-bosons (Fig. 17). The relative contribution of this channel is about 13%, with respect to the $qq$ channel. However, when both $W$-bosons are longitudinally polarized, then this ratio increases to 32%. There will also be enough events at a 100 TeV collider to observe the $gg$ channel contribution. The relative contribution of the $gg$ channel over the $qq$ channel can be further increased by requiring the $p_T(W)$ to be beyond a certain value between 50 and 100 GeV, see Fig. 18. Here also one may consider leptonic decay channel for $W$ bosons, as that will help in the measurement of its polarization. We consider $l^+\nu l^-\bar{\nu}bb$ final state as the signature. Here, as before $l = e/\mu$. In the literature, various techniques, including Neural Network methods have been discussed to measure the $W$ boson momentum [72]. Taking into account the branching ratios and the $b$-tagging efficiency, one
may expect about 1750 events from $gg$ channel and 5900 events from the $qq$ channel at the FCC-hh collider with 30 ab$^{-1}$ integrated luminosity. The number of these events would change depending on the detector and kinematic-cut efficiency factors.

![Figure 19: Effect of anomalous values of $\kappa_t$ and $\kappa_V$ on $WWH$ production via the $gg$ channel. The upper panel shows absolute distribution, and the lower panel shows the ratio of the BSM and SM distributions.](image)

Next, we focus on the effect of anomalous couplings on the total and differential cross sections. The $gg$ channel depends on $\kappa_t$, $\kappa_\lambda$, and $\kappa_V$ (see Eq. 4). We find that the channel is mostly sensitive to $\kappa_V$ and $\kappa_t$. For $\kappa_V = 1.1(0.9)$ the cross section changes by about 38%(-26%). While, for $\kappa_t = 1.1(0.9)$ the cross section changes by about 54% (-3%). The dependence on $\kappa_\lambda$ is found to be relatively small. In Fig. 19, we show the effect of $\kappa_t$ and $\kappa_V$ on the $p_T(H)$ distribution for the $gg$ channel. We do not show the distribution for anomalous $\kappa_\lambda$ as its effect on cross section is very small for 10% variation. We see that the shape remains more or less same in presence of anomalous couplings. We see that in the bins around 400 GeV, this ratio is around 1.5 for $\kappa_t = 1.1$ and $\kappa_V = 1.1$. For $\kappa_t = 0.9$, the ratio remains close to 1 throughout all the bins and for $\kappa_V = 0.9$, it is in the range 0.7–0.8. Similar to the case for $qq \rightarrow ZZH$, the $qq \rightarrow WWH$ cross section is also proportional to $\kappa_V^2$ at LO and NLO(QCD). So here as well, a 10% change in $\kappa_V$ gives around 20% obvious change in cross section, both at the total and differential levels.

5.5 Remarks on anomalous $HHH$ and $HHVV$ couplings

We have seen that the gluon fusion $ZZH$ and $WWH$ processes are most relevant for BSM physics due to their large cross sections. We found that their cross sections do not change much for a 10% variation in $\kappa_\lambda$. However, we know that this coupling is presently unconstrained by the experimental data. According to the future projections for HL-LHC, only values $\kappa_\lambda \lesssim -2$ and $\kappa_\lambda \gtrsim 8$ can be ruled out [9]. In this range the cross section for $ZZH$ and $WWH$ processes in the $gg$ channel varies significantly. In fact, it can change maximum by a factor of 3. This is shown in the left panel of Fig. 20. Notice that the $WWH$ process is more affected by anomalous $HHH$ coupling than $ZZH$ process.

Although in SM model, the $HHVV (V = Z, W)$ coupling is correlated to the $HHV$ coupling, in presence of new physics this correlation may not exist. Keeping this possibility in mind, we have varied the $HHVV$ coupling independently\(^{10}\) and we find that the cross section changes very strongly. This is shown in the right panel of Fig. 20. We can see that the effect of the $HHVV$ coupling is relatively

\(^{10}\)It should be noted that independent variation of $HHV$ and $HHVV$ couplings can be done systematically in an EFT framework which is beyond the scope of the present work.
Figure 20: Dependence of $gg \rightarrow ZZH, WWH$ cross sections on $HHH$ (left) and $HHVV$ (right) couplings at 14 TeV. The vertical lines in the left plot represent projected sensitivity on $\kappa_\lambda$ at HL-LHC and those on the right represent current sensitivity on $\kappa_{HHVV}$ at the LHC.

Table 6: $c_1$ and $c_2$ that appear in the definition of signal strengths for $gg \rightarrow ZZH, WWH$ processes at 14 TeV LHC and 100 TeV collider.

| Collider | $gg$ process | $c_1^{k_\lambda}$ | $c_2^{k_\lambda}$ | $c_1^{k_{HHVV}}$ | $c_2^{k_{HHVV}}$ |
|----------|--------------|--------------------|-------------------|------------------|------------------|
| 14 TeV   | $ZZH$        | -0.275             | 0.053             | -0.458           | 0.335            |
|          | $WWH$        | -0.318             | 0.071             | -0.440           | 0.301            |
| 100 TeV  | $ZZH$        | -0.256             | 0.046             | -0.563           | 0.772            |
|          | $WWH$        | -0.281             | 0.057             | -0.524           | 0.672            |

The quantity plotted in Fig. 20 is known as signal strength ($\mu$) which has been utilized by experimentalists as observable for data analyses. The signal strength for each process can be parametrized as

$$\mu = \frac{\sigma^{BSM}}{\sigma^{SM}} = 1 + c_1^i (\kappa_i - 1) + c_2^i (\kappa_i - 1)^2,$$

(8)

where $\kappa_i = \kappa_{\lambda, HHVV}$. In table 6, we have provided the values of $c_1^i$ and $c_2^i$ for $ZZH$ and $WWH$ processes for the 14 TeV LHC and a 100 TeV pp collider. We note that $c_2^{k_\lambda}$ is smaller by an order of magnitude than $c_1^{k_\lambda}$, suggesting a strong interference effect mentioned before. Therefore, $c_2^{k_\lambda}$ is relevant mostly for large values of $\kappa_i$. On the other hand, $c_2^{k_{HHVV}}$ is of the same order as $c_1^{k_{HHVV}}$. Since $c_1^i$ is negative, the cross section increase observed in the figures for $\kappa_i < 1$ is quite significant, which causes the (negative) lower bound on $\kappa$ to be tighter than the (positive) upper one. At a 100 TeV pp collider, while the other $c_i$s remain more or less same as that in 14 TeV collider, $c_2^{k_{HHVV}}$ increases by around a factor of two, implying the possibility of a far more stringent bound on the $HHVV$ couplings.

Since the $gg$ fusion channel contribution to $ZZH$ and $WWH$ processes cannot be fully separated from the corresponding contributions from the $qq$ channel, the above result should be interpreted care-
fully. A realistic estimate of the BSM effects discussed above must include the contributions from $qq$ channel. Since $qq$ channel contributions are insensitive to $\kappa_\lambda$ and $\kappa_{HHVV}$, they can be seen as one of the major backgrounds to the gluon fusion processes. As we have pointed out, the measurement of the polarization of the W/Z boson can help in reducing this background. A systematic signal-background analysis is beyond the scope of the present work. For the benefit of the reader, in Fig 21, we present the ratio $\sigma/\sigma_{SM}$ for $pp \to ZZH, WWH$ which includes both $qq$ and $gg$ channel contributions as functions of $\kappa_\lambda$ and $\kappa_{HHVV}$. In obtaining these results only standard cuts mentioned in the previous sections have been applied. We can see that at the 14 TeV, the ratio of BSM and SM cross sections due to $qq + gg$ channels is significantly smaller than that due to $gg$ channel alone. Moreover, the $ZZH$ process turns out to be more affected by $\kappa_\lambda$ and $\kappa_{HHVV}$ than the $WWH$.

![Figure 21](image_url)

Figure 21: Dependence of $pp \to ZZH, WWH$ cross sections on $HHH$ (left) and $HHVV$ (right) couplings at 14 TeV. The vertical lines in the left plot represent projected sensitivity on $\kappa_\lambda$ at HL-LHC and those on the right represent current sensitivity on $\kappa_{HHVV}$ at the LHC.

To be more precise, we find that by changing $\kappa_\lambda$ in the range ($-2,8$), the cross section for $ZZH$ process changes in the range $7 - 4\%$ at the 14 TeV. The corresponding change at the 100 TeV falls in the range of $20 - 8\%$. On the other hand, when changing $\kappa_{HHVV}$ in the range ($-0.56, 2.89$), the maximum cross section change in $ZZH$ process is found to be $\sim 8\%$ and $\sim 46\%$ at the 14 TeV and the 100 TeV, respectively. Again, we may mention that the polarization measurements would increase the fraction of $gg$ channel events, thus increasing the dependence on $\kappa_{HHVV}$.

6 Conclusions

In this paper, we have considered production of $VV'H$ ($\gamma\gamma H$, $\gamma ZH$, $ZZH$, and $WWH$) at proton-proton colliders. We investigated the sensitivity of these processes to various couplings of the Higgs boson, in particular to $HHH$ and $HHVV$ couplings which are practically unconstrained. Our main focus was the $gg$ channel contribution, which occurs at NNLO in $\alpha_s$. The scale uncertainties on the total cross sections are found to be of the order of $20\%$. A number of checks like UV and IR finiteness and gauge invariance of the amplitudes with respect to the gluons have been performed to ensure the correctness of the calculation. At a 100 TeV collider, the cross sections for these processes via the $gg$ channel range from $0.2$ fb to $17$ fb, $gg \to WWH$ being the dominant channel among all. We have seen the $gg \to \gamma\gamma H$ and $gg \to \gamma ZH$ processes are insignificant background to $gg \to HH \to \gamma\gamma H$ and $gg \to HH \to \gamma ZH$, respectively.

We have also compared the $gg$ channel contribution with the fixed order NLO QCD correction to
$pp \rightarrow VV'H$ in order to emphasize their relative importance. For $γγH$ production, the $gg$ channel can be said to be the only production channel, as the $bb$ channel process contribution is negligibly small. At a 100 TeV collider, the $gg \rightarrow γZH$ channel contribution is around 6% of the NLO QCD correction in the $qq$ channel. The $γZH$ production shows one interesting feature: with an increase in the $p_T$ cut on photon, the $qq$ channel contribution decreases faster than the $gg$ channel contribution. At this collider, the contribution of the $gg$ channel to $ZZH$ production is as important as the fixed order QCD NLO correction to the $qq$ channel. On the other hand, the $gg \rightarrow WHH$ channel cross section is around half the fixed order NLO QCD correction to the $qq$ channel. We have observed strong destructive interference effects among various classes of diagrams in $gg \rightarrow γZH, ZZH, WHH$. Besides total cross sections at the LHC, HE-LHC, and FCC-hh, we have obtained relevant kinematic distributions at FCC-hh in the $gg$ channel. We find that the $p_T(H)$ spectrum from the $gg$ channel is harder than that from the $qq$ channel for $ZZH$ and $WHH$ productions. We have also shown that by selecting events based on the polarization of final state vector bosons, the relative contribution of the $gg$ channel over the $qq$ channel can be enhanced.

In addition to the SM results, effect of anomalous couplings ($κ_t$, $κ_V$, and $κ_λ$) for $Ht\bar{t}$, $HVV$, $HHVV$, and $HHH$ vertices have been studied in the kappa framework. We find that the new physics effects are quite important in $gg \rightarrow ZZH, WHH$ processes due to non-trivial interference effects in these processes. A 10% change in $κ_t$ on the higher side can enhance the $gg \rightarrow ZZH$ and $WHH$ cross sections by 68% and 54%, respectively. Similar change in $κ_V$ enhances these cross sections by about 40%. Unlike in $qq$ channels, the kinematic distributions in $gg$ channels display non-trivial changes in presence of new physics. The dependence of the $gg$ channel on the $κ_V$ is stronger than that of the $qq$ channel. By considering events with longitudinally polarized vector bosons for the processes $pp \rightarrow ZZH, WHH$, we can enhance the fraction of the $gg$ channel contribution. This event sample will have even stronger dependence on $κ_V$. Since the $HHH$ and $HHVV$ couplings are not well constrained, we have also considered larger independent variations in $κ_λ$ and $κ_{HHVV}$. We find that the effect of $κ_{HHVV}$ on the cross section is much stronger than that of $κ_λ$. Therefore the process $pp \rightarrow ZZH, WHH$ with longitudinally polarized $Z$ and $W$ bosons can help in determining the $HHVV$ coupling.
A Comment on $Z$ mediated triangle diagrams in $gg \to WWH$

It is a well known theorem due to Landau and Yang that a massive spin-1 particle cannot decay into two on-shell spin-1 massless particles [73, 74]. The same theorem can be applied to argue that the $gg \to Z^*$ amplitude vanishes for on-shell $Z$ boson. This can be easily verified at LO using the on-shell conditions for the gluons and the $Z$ boson. In the past, we have shown that even if the $Z$ boson is off-shell, the LO $gg \to Z^*$ can vanish provided the off-shell $Z$ boson is linked to a conserved current [23]. This is so because $M_0 \propto (p_2 - p_1)\nu$. This result is useful for many $gg$ channel processes which receive contribution from such triangle topology. See Fig. 2 (h) and (i). We will explicitly show that Fig. 2(i) does not contribute to the $gg \to WWH$ amplitude. For this we need to just prove that the sum of the currents shown in Fig. 22 when contracted with the momentum $(p_1 + p_2)\nu$ vanishes.

![Figure 22: Currents attached to $Z^*$ in Fig. 2(i). All the momenta are incoming.](image)

In the following derivation we use, $p_1 + p_2 = p_{12}$, $p_3 + p_4 = p_{35}$ and $p_4 + p_5 = p_{15}$. The polarization vectors of $W^-$ and $W^+$ are denoted by $e_3^\alpha(p_3)$ and $e_4^\alpha(p_4)$, respectively. We first calculate the contraction of current $J_1$ with $p_{12}$.

$$M_1 = p_{12}^\nu J_1 \nu = p_{12}^\nu \left(\bar{g}_{\nu\alpha\beta}(p_{12} - p_{35})\beta + g_{\alpha1\beta}(p_{35} - p_4)\nu + g_{\beta\nu}(p_4 - p_{12})\alpha\right) \times$$

$$\frac{-g_{\alpha1\beta} + p_{35}^{\alpha1} p_{35}^{\beta\beta}/M_W^2}{p_{35}^2 - M_W^2} g_{\alpha2\beta} e_3^\beta(p_3) e_4^\alpha(p_4)$$

$$= \left(p_{12} \alpha_1 (p_{12} - p_{35}).e_4 + e_4 \alpha_1 p_{12}.(p_{35} - p_4) + p_{12} e_4 (p_4 - p_{12})\alpha_1\right) \times$$

$$\frac{-e_3^{\alpha1} + p_{35}^{\alpha1} p_{35}.e_3/M_W^2}{p_{35}^2 - M_W^2}$$

$$= \left( - p_{12}.e_3 (p_{12} - p_{35}).e_4 - e_3.e_4 p_{12}.(p_{35} - p_4) - p_{12} e_4 (p_4 - p_{12}).e_3 \right)/\left(p_{35}^2 - M_W^2\right)$$

$$+ \frac{p_{35}.e_3}{M_W^2 (p_{35}^2 - M_W^2)} \left(p_{12}.p_{35} (p_{12} - p_{35}).e_4 + p_{35}.e_4 p_{12}.(p_{35} - p_4) - p_{12}.e_4 (p_4 - p_{12}).p_{35}\right)$$

$$= \left( - p_{12}.e_3 (p_{12} - p_{35}).e_4 - e_3.e_4 p_{12}.(p_{35} - p_4) - p_{12} e_4 (p_4 - p_{12}).e_3 \right)/\left(p_{35}^2 - M_W^2\right)$$

$$+ \frac{p_{35}.e_3}{M_W^2 (p_{35}^2 - M_W^2)} \left(p_{12}.p_{35} (p_{12} - p_{35}).e_4 + p_{35}.e_4 p_{12}.(p_{35} - p_4) - p_{12}.e_4 (p_4 - p_{12}).p_{35}\right)$$

$$= \left( - p_{12}.e_3 (p_{12} - p_{35}).e_4 - e_3.e_4 p_{12}.(p_{35} - p_4) - p_{12} e_4 (p_4 - p_{12}).e_3 \right)/\left(p_{35}^2 - M_W^2\right)$$

$$+ \frac{p_{35}.e_3}{M_W^2 (p_{35}^2 - M_W^2)} \left(p_{12}.p_{35} (p_{12} - p_{35}).e_4 + p_{35}.e_4 p_{12}.(p_{35} - p_4) - p_{12}.e_4 (p_4 - p_{12}).p_{35}\right)$$

$$= \left( - p_{12}.e_3 (p_{12} - p_{35}).e_4 - e_3.e_4 p_{12}.(p_{35} - p_4) - p_{12} e_4 (p_4 - p_{12}).e_3 \right)/\left(p_{35}^2 - M_W^2\right)$$

$$+ \frac{p_{35}.e_3}{M_W^2 (p_{35}^2 - M_W^2)} \left(p_{12}.p_{35} (p_{12} - p_{35}).e_4 + p_{35}.e_4 p_{12}.(p_{35} - p_4) - p_{12}.e_4 (p_4 - p_{12}).p_{35}\right)$$
Using momentum conservation $p_{12} = -p_{35} - p_4$ and transversality conditions $e_3.p_3 = e_4.p_4 = 0$, we get

$$\mathcal{M}_1 = \left( -2p_{35}.e_4p_{45}.e_3 + e_3.e_4(p_{35}^2 - p_4^2) + p_{35}.e_4(p_4 + p_{45}).e_3 \right) / (p_{35}^2 - M_W^2)$$

$$+ \frac{p_{35}.e_3}{M_W^2(p_{35}^2 - M_W^2)} \left( 2p_{35}.e_4(p_{35} + p_4).p_{35} - p_{35}.e_4(p_{35}^2 - p_4^2) - p_{35}.e_4(2p_4 + p_{35}).p_{35} \right)$$

$$= \frac{-p_{35}.e_3p_{35}.e_4 + e_3.e_4(p_{35}^2 - p_4^2)}{(p_{35}^2 - M_W^2)} + \frac{p_{35}.e_3}{M_W^2(p_{35}^2 - M_W^2)}p_{35}.e_4p_4^2$$

(12)

(13)

Using on-shell condition $p_4^2 = M_W^2$, we arrive at

$$\mathcal{M}_1 = e_3.e_4.$$  

(14)

Following similar steps, it can be shown that contraction of current $J_2$ with $p_{12}$ leads to,

$$\mathcal{M}_2 = p_{12}^\nu J_2^\nu = -e_3.e_4.$$  

(15)

Combining equations 14 and 15 we obtain the desired result: $\mathcal{M}_1 + \mathcal{M}_2 = 0$. Thus we have proved that indeed the current associated with $Z^*$ in Fig. 22 is a conserved current and therefore the triangle amplitude for Fig. 2(i) vanishes for each quark flavor in the loop. It can be verified explicitly that the current associated with $Z^*$ in Fig. 2(h) is not a conserved current and therefore it does give non-vanishing contribution to $gg \rightarrow WWH$ amplitude.
References

[1] ATLAS Collaboration, G. Aad et al., “Study of the spin and parity of the Higgs boson in diboson decays with the ATLAS detector,” Eur. Phys. J. C75 no. 10, (2015) 476, arXiv:1506.05669 [hep-ex]. [Erratum: Eur. Phys. J.C76,no.3,152(2016)].

[2] ATLAS, CMS Collaboration, G. Aad et al., “Measurements of the Higgs boson production and decay rates and constraints on its couplings from a combined ATLAS and CMS analysis of the LHC pp collision data at $\sqrt{s} = 7$ and 8 TeV,” JHEP 08 (2016) 045, arXiv:1606.02266 [hep-ex].

[3] CMS Collaboration, A. M. Sirunyan et al., “Combined measurements of Higgs boson couplings in proton–proton collisions at $\sqrt{s} = 13$ TeV,” Eur. Phys. J. C79 no. 5, (2019) 421, arXiv:1809.10733 [hep-ex].

[4] ATLAS Collaboration, “Combined measurements of Higgs boson production and decay using up to 80 fb$^{-1}$ of proton–proton collision data at $\sqrt{s} = 13$ TeV collected with the ATLAS experiment,” http://cds.cern.ch/record/2668375.

[5] I. Anderson et al., “Constraining Anomalous HVV Interactions at Proton and Lepton Colliders,” Phys. Rev. D 89 no. 3, (2014) 035007, arXiv:1309.4819 [hep-ph].

[6] CMS Collaboration, “Constraints on anomalous Higgs boson couplings to vector bosons and fermions in production and decay in the $H \rightarrow 4\ell$ channel,” http://cds.cern.ch/record/2725543.

[7] ATLAS Collaboration, M. Aaboud et al., “Evidence for the associated production of the Higgs boson and a top quark pair with the ATLAS detector,” Phys. Rev. D97 no. 7, (2018) 072003, arXiv:1712.08891 [hep-ex].

[8] CMS Collaboration, A. M. Sirunyan et al., “Observation of $t\bar{t}H$ production,” Phys. Rev. Lett. 120 no. 23, (2018) 231801, arXiv:1804.02610 [hep-ex].

[9] ATLAS Collaboration, “Prospects for measuring Higgs pair production in the channel $H(\rightarrow \gamma\gamma)H(\rightarrow b\bar{b})$ using the ATLAS detector at the HL-LHC,” http://cds.cern.ch/record/1956733.

[10] C.-Y. Chen, Q.-S. Yan, X. Zhao, Y.-M. Zhong, and Z. Zhao, “Probing triple-Higgs productions via 4b2$\gamma$ decay channel at a 100 TeV hadron collider,” Phys. Rev. D 93 no. 1, (2016) 013007, arXiv:1510.04013 [hep-ph].

[11] B. Fuks, J. H. Kim, and S. J. Lee, “Scrutinizing the Higgs quartic coupling at a future 100 TeV proton–proton collider with taus and b-jets,” Phys. Lett. B771 (2017) 354–358, arXiv:1704.04298 [hep-ph].

[12] E. Rossi, “Measurement of Higgs-boson self-coupling with single-Higgs and double-Higgs production channels,” arXiv:2010.05252 [hep-ex].

[13] M. McCullough, “An Indirect Model-Dependent Probe of the Higgs Self-Coupling,” Phys. Rev. D 90 no. 1, (2014) 015001, arXiv:1312.3322 [hep-ph]. [Erratum: Phys.Rev.D 92, 039903 (2015)].

[14] S. Borowka, C. Duhr, F. Maltoni, D. Pagani, A. Shivaji, and X. Zhao, “Probing the scalar potential via double Higgs boson production at hadron colliders,” JHEP 04 (2019) 016, arXiv:1811.12366 [hep-ph].
[15] F. Bishara, R. Contino, and J. Rojo, “Higgs pair production in vector-boson fusion at the LHC and beyond,” *Eur. Phys. J.* C 77 no. 7, (2017) 481, arXiv:1611.03860 [hep-ph].

[16] ATLAS Collaboration, G. Aad et al., “Search for the $HH \to b\bar{b}b\bar{b}$ process via vector-boson fusion production using proton-proton collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector,” *JHEP* 07 (2020) 108, arXiv:2001.05178 [hep-ex].

[17] D. de Florian and Z. Kunszt, “Two photons plus jet at LHC: The NNLO contribution from the $gg$ initiated process,” *Phys. Lett.* B460 (1999) 184–188, arXiv:hep-ph/9905283 [hep-ph].

[18] T. Melia, K. Melnikov, R. Rontsch, M. Schulze, and G. Zanderighi, “Gluon fusion contribution to $W^+W^-$ + jet production,” *JHEP* 08 (2012) 115, arXiv:1205.6987 [hep-ph].

[19] P. Agrawal and A. Shivaji, “Di-Vector Boson + Jet Production via Gluon Fusion at Hadron Colliders,” *Phys. Rev.* D86 (2012) 073013, arXiv:1207.2927 [hep-ph].

[20] F. Campanario, “Towards $pp \to VVjj$ at NLO QCD: Bosonic contributions to triple vector boson production plus jet,” *JHEP* 10 (2011) 070, arXiv:1105.0920 [hep-ph].

[21] P. Agrawal and A. Shivaji, “Production of $\gamma Zg$ and associated processes via gluon fusion at hadron colliders,” *JHEP* 01 (2013) 069, arXiv:1211.5429 [hep-ph].

[22] F. Campanario, Q. Li, M. Rauch, and M. Spira, “$ZZ+$jet production via gluon fusion at the LHC,” *JHEP* 06 (2013) 069, arXiv:1211.5429 [hep-ph].

[23] A. K. Shivaji, Gluon Fusion Processes at One-loop within the Standard Model and Beyond. PhD thesis, Bhubaneswar, Inst. Phys., 2013. arXiv:1305.4926 [hep-ph].

[24] J. M. Campbell, R. K. Ellis, E. Furlan, and R. Röntsch, “Interference effects for Higgs boson mediated $Z$-pair plus jet production,” *Phys. Rev.* D90 no. 9, (2014) 093008, arXiv:1409.1897 [hep-ph].

[25] P. Agrawal and A. Shivaji, “Gluon Fusion Contribution to $VHj$ Production at Hadron Colliders,” *Phys. Lett.* B741 (2015) 111–116, arXiv:1409.8059 [hep-ph].

[26] M. Song, W.-G. Ma, R.-Y. Zhang, L. Guo, S.-M. Wang, and L. Han, “QCD corrections to associated Higgs boson production with a $W$ boson pair at the LHC,” *Phys. Rev.* D79 (2009) 054016, arXiv:0903.2885 [hep-ph].

[27] B. Hespel, F. Maltoni, and E. Vryonidou, “Higgs and $Z$ boson associated production via gluon fusion in the SM and the 2HDM,” *JHEP* 06 (2015) 065, arXiv:1503.01656 [hep-ph].

[28] E. Gabrielli, B. Mele, F. Piccinini, and R. Pittau, “Asking for an extra photon in Higgs production at the LHC and beyond,” *JHEP* 07 (2016) 003, arXiv:1601.03635 [hep-ph].

[29] F. Caola, K. Melnikov, R. Röntsch, and L. Tancredi, “QCD corrections to $ZZ$ production in gluon fusion at the LHC,” *Phys. Rev.* D92 no. 9, (2015) 094028, arXiv:1509.06734 [hep-ph].

[30] F. Caola, K. Melnikov, R. Röntsch, and L. Tancredi, “QCD corrections to $W^+W^-$ production through gluon fusion,” *Phys. Lett.* B754 (2016) 275–280, arXiv:1511.08617 [hep-ph].

[31] J. M. Campbell, R. K. Ellis, Y. Li, and C. Williams, “Predictions for diphoton production at the LHC through NNLO in QCD,” *JHEP* 07 (2016) 148, arXiv:1603.02663 [hep-ph].

[32] F. Caola, M. Dowling, K. Melnikov, R. Röntsch, and L. Tancredi, “QCD corrections to vector boson pair production in gluon fusion including interference effects with off-shell Higgs at the LHC,” *JHEP* 07 (2016) 087, arXiv:1605.04610 [hep-ph].
[33] F. Granata, J. M. Lindert, C. Oleari, and S. Pozzorini, “NLO QCD+EW predictions for HV and HV +jet production including parton-shower effects,” JHEP 09 (2017) 012, arXiv:1706.03522 [hep-ph].

[34] A. Shivaji, P. Agrawal, and D. Saha, “Gluon fusion contribution to HBB (B = H, γ, Z) at the LHC,” EPJ Web Conf. 129 (2016) 00005, arXiv:1609.04790 [hep-ph].

[35] T. Plehn and M. Rauch, “The quartic higgs coupling at hadron colliders,” Phys. Rev. D72 (2005) 053008, arXiv:hep-ph/0507321 [hep-ph].

[36] T. Binoth, S. Karg, N. Kauer, and R. Ruckl, “Multi-Higgs boson production in the Standard Model and beyond,” Phys. Rev. D74 (2006) 113008, arXiv:hep-ph/0608057 [hep-ph].

[37] F. Maltoni, E. Vryonidou, and M. Zaro, “Top-quark mass effects in double and triple Higgs production in gluon-gluon fusion at NLO,” JHEP 11 (2014) 079, arXiv:1408.6542 [hep-ph].

[38] A. Papaefstathiou and K. Sakurai, “Triple Higgs boson production at a 100 TeV proton-proton collider,” JHEP 02 (2016) 006, arXiv:1508.06524 [hep-ph].

[39] W. Kilian, S. Sun, Q.-S. Yan, X. Zhao, and Z. Zhao, “New Physics in multi-Higgs boson final states,” JHEP 06 (2017) 145, arXiv:1702.03554 [hep-ph].

[40] V. Hirschi and O. Mattelaer, “Automated event generation for loop-induced processes,” JHEP 10 (2015) 146, arXiv:1507.00020 [hep-ph].

[41] P. Agrawal, D. Saha, and A. Shivaji, “Production of HHH and HVV (V = γ, Z) at the hadron colliders,” Phys. Rev. D97 no. 3, (2018) 036006, arXiv:1708.01650 [hep-ph].

[42] M. Mangano and M. Mangano, Physics at the FCC-hh, a 100 TeV pp collider, vol. 3 of CERN Yellow Reports: Monographs. CERN, Geneva, Oct, 2017. https://cds.cern.ch/record/2270978.

[43] M. Ahmad et al., “CEPC-SPPC Preliminary Conceptual Design Report. 1. Physics and Detector,”. http://cepc.ihep.ac.cn/preCDR/main_preCDR.pdf.

[44] J. Baglio, “Next-To-Leading Order QCD Corrections to Associated Production of a SM Higgs Boson with a Pair of Weak Bosons in the POWHEG-BOX,” Phys. Rev. D93 no. 5, (2016) 054010, arXiv:1512.05787 [hep-ph].

[45] J. Baglio, “Gluon fusion and bb corrections to HW+W−/HZZ production in the POWHEG-BOX,” Phys. Lett. B764 (2017) 54–59, arXiv:1609.05907 [hep-ph].

[46] CMS Collaboration, S. Chatrchyan et al., “Measurement of the Polarization of W Bosons with Large Transverse Momenta in W+Jets Events at the LHC,” Phys. Rev. Lett. 107 (2011) 021802, arXiv:1104.3829 [hep-ex].

[47] ATLAS Collaboration, G. Aad et al., “Measurement of the W boson polarization in top quark decays with the ATLAS detector,” JHEP 06 (2012) 088, arXiv:1205.2484 [hep-ex].

[48] ATLAS Collaboration, M. Aaboud et al., “Measurement of W±Z production cross sections and gauge boson polarisation in pp collisions at √s = 13 TeV with the ATLAS detector,” Eur. Phys. J. C 79 no. 6, (2019) 535, arXiv:1902.05759 [hep-ex].

[49] D. Binosi, J. Collins, C. Kaufhold, and L. Theussl, “JaxoDraw: A Graphical user interface for drawing Feynman diagrams. Version 2.0 release notes,” Comput. Phys. Commun. 180 (2009) 1709–1715, arXiv:0811.4113 [hep-ph].
[50] K. Nishijima, “Generalized Furry’s Theorem for Closed Loops,” *Progress of Theoretical Physics* **6** no. 4, (08, 1951) 614–615.

[51] **Particle Data Group** Collaboration, M. Tanabashi et al., “Review of Particle Physics,” *Phys. Rev. D* **98** no. 3, (2018) 030001.

[52] **ATLAS** Collaboration, G. Aad et al., “Combination of searches for Higgs boson pairs in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector,” arXiv:1906.02025 [hep-ex].

[53] **LHC Higgs Cross Section Working Group** Collaboration, A. David, A. Denner, M. Duerrssen, M. Grazzini, C. Grojean, G. Passarino, M. Schumacher, M. Spira, G. Weiglein, and M. Zanetti, “LHC HXSWG interim recommendations to explore the coupling structure of a Higgs-like particle,” arXiv:1209.0040 [hep-ph].

[54] M. Ghezzi, R. Gomez-Ambrosio, G. Passarino, and S. Uccirati, “NLO Higgs effective field theory and $\kappa$-framework,” *JHEP* **07** (2015) 175, arXiv:1505.03706 [hep-ph].

[55] P. Agrawal and G. Ladinsky, “Production of two photons and a jet through gluon fusion,” *Phys. Rev. D* **63** (2001) 117504, arXiv:hep-ph/0011346 [hep-ph].

[56] J. A. M. Vermaseren, “New features of FORM,” arXiv:math-ph/0010025 [math-ph].

[57] G. J. van Oldenborgh and J. A. M. Vermaseren, “New Algorithms for One Loop Integrals,” *Z. Phys. C* **46** (1990) 425–438.

[58] A. van Hameren, “OneLoop: For the evaluation of one-loop scalar functions,” *Comput. Phys. Commun.* **182** (2011) 2427–2438, arXiv:1007.4716 [hep-ph].

[59] F. Maltoni, K. Paul, T. Stelzer, and S. Willenbrock, “Associated production of Higgs and single top at hadron colliders,” *Phys. Rev. D* **64** (2001) 094023, arXiv:hep-ph/0106293.

[60] J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, H. S. Shao, T. Stelzer, P. Torrielli, and M. Zaro, “The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations,” *JHEP* **07** (2014) 079, arXiv:1405.0301 [hep-ph].

[61] S. Veseli, “Multidimensional integration in a heterogeneous network environment,” *Comput. Phys. Commun.* **108** (1998) 9–19, arXiv:physics/9710017 [physics.comp-ph].

[62] G. P. Lepage, “VEGAS - an adaptive multi-dimensional integration program,” Tech. Rep. CLNS-447, Cornell Univ. Lab. Nucl. Stud., Ithaca, NY, Mar, 1980.

http://cds.cern.ch/record/123074.

[63] A. Geist, A. Beguelin, J. Dongarra, W. Jiang, R. Manchek, and V. Sunderam, *PVM: Parallel Virtual Machine: A Users’ Guide and Tutorial for Networked Parallel Computing.* MIT Press, Cambridge, MA, USA, 1994.

[64] S. Dulat, T.-J. Hou, J. Gao, M. Guzzi, J. Huston, P. Nadolsky, J. Pumplin, C. Schmidt, D. Stump, and C. P. Yuan, “New parton distribution functions from a global analysis of quantum chromodynamics,” *Phys. Rev. D* **93** no. 3, (2016) 033006, arXiv:1506.07443 [hep-ph].

[65] **LHCb** Collaboration, R. Aaij et al., “First experimental study of photon polarization in radiative $B^0_s$ decays,” *Phys. Rev. Lett.* **118** no. 2, (2017) 021801, arXiv:1609.02032 [hep-ex]. [Addendum: Phys.Rev.Lett. 118, 109901 (2017)].
[66] F. Legger and T. Schietinger, “Photon helicity in $\Lambda_b \rightarrow pK\gamma$ decays,” Phys. Lett. B 645 (2007) 204–212, arXiv:hep-ph/0605245. [Erratum: Phys.Lett.B 647, 527–528 (2007)].

[67] G. Hiller, M. Knecht, F. Legger, and T. Schietinger, “Photon polarization from helicity suppression in radiative decays of polarized Lambda(b) to spin-3/2 baryons,” Phys. Lett. B 649 (2007) 152–158, arXiv:hep-ph/0702191.

[68] V. D. Orlovsky and V. I. Shevchenko, “On the photon polarization in radiative $B \rightarrow \phi K$ gamma decay,” Phys. Rev. D 77 (2008) 093003, arXiv:0708.4302 [hep-ph].

[69] L. Shchutska, Y. Xie, A. Golutvin, V. Egorychev, V. Shevchenko, and I. Belyaev, “Probing the photon polarization in $B_s \rightarrow \phi$ gamma at LHCb,” http://cds.cern.ch/record/1099116.

[70] L. Oliver, J. C. Raynal, and R. Sinha, “Note on new interesting baryon channels to measure the photon polarization in $b \rightarrow s$ gamma,” Phys. Rev. D 82 (2010) 117502, arXiv:1007.3632 [hep-ph].

[71] L. M. García Martín, B. Jashal, F. Martínez Vidal, A. Oyanguren, S. Roy, R. Sain, and R. Sinha, “Radiative $b$-baryon decays to measure the photon and $b$-baryon polarization,” Eur. Phys. J. C 79 no. 7, (2019) 634, arXiv:1902.04870 [hep-ph].

[72] M. Grossi, J. Novak, B. Kersevan, and D. Rebuzzi, “Comparing traditional and deep-learning techniques of kinematic reconstruction for polarization discrimination in vector boson scattering,” Eur. Phys. J. C 80 no. 12, (2020) 1144, arXiv:2008.05316 [hep-ph].

[73] L. D. Landau, “On the angular momentum of a system of two photons,” Dokl. Akad. Nauk SSSR 60 no. 2, (1948) 207–209.

[74] C.-N. Yang, “Selection Rules for the Dematerialization of a Particle Into Two Photons,” Phys. Rev. 77 (1950) 242–245.

[75] T. Binoth, M. Ciccolini, N. Kauer, and M. Kramer, “Gluon-induced WW background to Higgs boson searches at the LHC,” JHEP 03 (2005) 065, arXiv:hep-ph/0503094 [hep-ph].

[76] J. M. Campbell, R. K. Ellis, and C. Williams, “Gluon-Gluon Contributions to $W^+ W^-$ Production and Higgs Interference Effects,” JHEP 10 (2011) 005, arXiv:1107.5569 [hep-ph].