LANDAU GINZBURG MODEL AND DECONFINEMENT TRANSITION FOR EXTENDED $SU(2)$ WILSON ACTION

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ABSTRACT

We compute the effective action in terms of the Polyakov loop for the 3-dimensional pure fundamental-adjoint $SU(2)$ lattice gauge theory at non-zero temperatures using the strong coupling expansion. In the extended coupling plane we show the existence of a tricritical point where the nature of the deconfinement transition undergoes a change from second to first order. The resulting phase structure is in excellent agreement with the Monte Carlo results both in the fundamental and adjoint directions. The possible consequences of our results on universality are discussed.

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1. INTRODUCTION

The deconfinement transition in gauge theories is well known to be intimately connected to the spontaneous symmetry breaking of the corresponding center symmetry with the Polyakov loop ($L$) as the order parameter. As a consequence of universality, various gauge models in $(d + 1)$ dimensional Euclidean space get related to much simpler $(d)$ dimensional scalar or spin models near their critical fixed point [1, 2]. The confining and deconfining phases in the language of spin models correspond to disordered ($< L > = 0$) and ordered phases ($< L > \neq 0$), respectively. The effective action written in terms of the Polyakov loop, after integrating out all the other degrees of freedom, contains all the relevant information about the phase structure of the theory. These effective actions for SU(N) gauge theories and their universality with spin models have been well studied in the past using analytical tools like large N [3, 4, 5], mean field [6, 7], strong coupling [8, 9] and weak coupling [10] approximations as well as by Monte Carlo techniques [6, 11].

On the other hand, regulating the theory on a lattice, it is expected that the various choices of gauge invariant lattice actions, which in the naive classical continuum limit (i.e. $\hbar \to 0, a \to 0$) reduce to the Yang Mills theory also correspond to working within the same universality class. However, in the absence of full non-perturbative renormalization group equations, different lattice models expected to be in the same universality class have been explicitly tested for universality and their scaling behaviours explored both analytically and by Monte Carlo simulations. A class of lattice models, with a rich phase structure and therefore well studied in the literature is the pure fundamental-adjoint SU(N) Wilson action defined by the partition function

$$ Z = \int \prod_{n,\mu} dU_{\mu}(n) \exp \sum_{P} \left[ \frac{\beta_F}{2} Tr F_U P + \frac{\beta_A}{3} Tr A_U P \right] $$

(1)

Here $F, A$ denote fundamental and adjoint representations, respectively, and $U_P$ is the ordered product of the four directed link variables $U_{\mu}(n)(\in SU(N))$ which form an elementary plaquette. The sum over $P$ denotes the sum over all independent plaquettes of the lattice. $\int \prod_{n,\mu} dU_{\mu}(n)$ is the product of group invariant Haar measure over all the links of the lattice. This model represents a particular choice of lattice action from a more general
class defined by summing the above single plaquette action over all the representations of the gauge group with corresponding couplings. As in the naive classical continuum limit, the theory is only sensitive to the Lie algebra, thus the above class is believed to be in the same universality class as that of Wilson action. In fact, the universality between (1) and the Wilson action \( i.e, \beta_A = 0 \) has been explicitly demonstrated in perturbation theory near the critical fixed point \([12]\), and, as expected, the two differ only in their scale factors \( (\Lambda) \).

In the special case of SU(2), the phase structure and the scaling properties of the above model have been extensively studied by Monte Carlo simulations \([13, 14, 15]\) as well as by analytical methods \([12, 16, 17, 18, 19]\). Along the \( \beta_F = 0 \) axis, it describes the SO(3) gauge theory with a first order phase transition at \( \beta_A^c \sim 2.5 \). At \( \beta_A = \infty \), it corresponds to the Z\(_2\) lattice gauge theory with a first order phase transition at \( \beta_F^c = \frac{1}{2} \ln(1 + \sqrt{2}) \sim 0.44 \). From the \( \beta_F = 0 \) and \( \beta_A = \infty \) axes, the above two bulk transitions extend into the \((\beta_F, \beta_A)\) plane, meet at a point and then continue as another line of bulk first order phase transitions. The fact that the latter line ends at a point in the phase diagram allows one to bypass all the bulk singularities without losing confinement in SU(2) gauge theory. This model therefore retains its strong coupling confining property also in the continuum and is expected to be in the same universality class as that of the Wilson action \( (\beta_A = 0) \).

However, as the above phase diagram was mainly established using Monte Carlo simulations on relatively small lattices, we recently argued \([20, 21, 22]\) that it is incomplete and besides the bulk transitions discussed above, one must also worry about deconfinement phase boundary in the extended coupling \((\beta_F, \beta_A)\) plane. Monitoring the Polyakov loop expectation values \( < |L| > \) and its susceptibility by Monte Carlo techniques, it was observed that the known second order deconfinement transition at \( \beta_A = 0 \) moves into the phase diagram as \( \beta_A \) is increased and eventually joins the first order bulk transition line at its endpoint around \( \beta_A = 1.0 \). The above merging together of the deconfinement transition line with the previously claimed bulk transition line lead to a paradox as their nature as well as scaling behaviours are completely different. Even more surprisingly, it was observed that

1. For \( \beta_A \leq 1 \), the model was in the universality class of the 3-dimensional Ising model as predicted by the universality hypothesis \([1, 2, 23]\). However, the deconfinement transition was found to be \textit{discontinuous} above
\( \beta_A \approx 1 \) showing a definite qualitative change in the properties of the theory and the existence of a tricritical point.

2. Absence of the previously claimed first order bulk transition. There was good evidence that the above transition was misidentified in the Monte Carlo simulations and is, in fact, first order deconfinement transition mentioned above. It was found that the first order portion of the deconfinement transition lies exactly along the previously known bulk transition line and there was no evidence of two transitions. The plaquette susceptibility does show a peak along the above transition on smaller lattices, however, on going to larger lattices, this peak was found to decrease considerably with the total 4-volume of the lattice.

To understand the above unexpected and puzzling lattice Monte-Carlo results and their possible implications to the continuum physics and universality, one needs to study and derive these results analytically. Such a study can yield further insight and a global understanding of such phenomena for \( SU(N) \) lattice gauge theories in general. Analytical results also have the advantage of being free from finite size artifacts. With this motivation, in this paper we study the deconfinement transition and its nature in the above extended \( SU(2) \) model in the strong coupling expansion. We explicitly compute the Landau Ginzburg effective action in terms of the Polyakov loop and show the existence of the tricritical point in the extended \( (\beta_F, \beta_A) \) plane where the deconfinement transition changes its order (Fig. 2). Moreover, the above effective potential reproduces the Monte Carlo phase diagram of [20, 21] with good accuracy (Table 1). It is also found that within the strong coupling regime the tricritical point moves in the direction of \( (\beta_F = \infty, \beta_A = \infty) \) as \( N_\tau \) increases. This possibly indicates that the continuum limit of the theory may not be affected by the existence of the tricritical point. Surprisingly, we find that the predictions of the critical couplings along the Wilson line \( \beta_A = 0 \) are quite close to their corresponding Monte Carlo values (Table 2), even well inside the scaling region established by numerical simulations [24]. In the above case of pure Wilson action, the deconfinement transition has also been studied by Polonyi and Szlachanyi [8] using strong coupling expansion. However, our techniques differ from theirs and our results in this special case \( (\beta_A = 0) \) are different and closer to the Monte-Carlo findings. Following the work [20, 21], a more recent paper
studied the phase diagram of the fundamental-adjoint model for \( SU(3) \) lattice gauge theory at nonzero temperature by Monte Carlo simulations. They reported results consistent with the usual universality picture. We will briefly summarise and discuss their results at the end.

1. THE LANDAU GINZBURG MODEL

We want to compute the partition function in terms of the Polyakov loop after integrating out all the spatial link degrees of freedom in (2). As usual, we impose periodic boundary conditions in the temperature direction and assume its length to be \( N_\tau \). Defining \( U^\tau(\vec{n}) = \prod_{n_0=1}^{N_\tau} U_0(n_0, \vec{n}) \), the Polyakov loop can be written as,

\[
L(\vec{n}) = \frac{1}{2} \text{Tr} U^\tau(\vec{n})
\]  

The partition function (1) can be trivially written as product over the plaquettes,

\[
Z = \left[ \prod_{\text{Links}} \int dU_\mu(n) \right] \prod_{\text{Plaquettes}} \left( 1 + G^F_p \right) \left( 1 + G^A_p \right),
\]  

where,

\[
G^F_p = \sum_{m_p=1}^{\infty} \frac{1}{m_p} \left( \frac{\beta F}{2} S_p \right)^{m_p}
\]  

and

\[
G^A_p = \sum_{n_p=1}^{\infty} \frac{1}{n_p} \left( \frac{\beta A}{3} \right)^{n_p}.
\]  

Here \( S_p \) denotes the standard one plaquette action for the gauge group \( SU(2) \).

The computation of the effective action in terms of the Polyakov loops get considerably simplified by noticing that after integration over the spatial links in an arbitrary cluster of plaquettes in (3),
1. Due to gauge invariance of the partition function and the periodic boundary conditions, the non-trivial dependence on $U_0(n_0, \vec{n})$ will come only if the cluster contains at least one set of plaquettes forming closed loops in the temporal direction. Moreover, the above dependence can come only implicitly through the Polyakov loops $\tilde{U}$.

2. The rest of the cluster’s contribution to the partition function is either zero or proportional to some powers of fundamental or adjoint couplings.

Therefore, after the above spatial link integrals are performed the Landau Ginzburg effective action $S_{\text{eff}}$ can be defined as,

$$Z = \int \prod_{\vec{n}} d\mu(L(\vec{n})) \exp - S_{\text{eff}}(L).$$

(6)

Here $d\mu(L(\vec{n}))$ is the effective measure in terms of the Polyakov loop and is computed below. For matter of convenience and clarity, we also define,

$$S_{\text{eff}} \equiv - \sum_N \sum_{M_F} \sum_{M_A} S_{[N,M_F,M_A]}(\beta_F, \beta_A, L)$$

(7)

Here $N$ is the number of the closed loops in the temporal direction (e.g., in Fig. 1 $N=2, 4$ and $6$ respectively). For $SU(2)$, it is an even integer implying the invariance under the $Z_2$ center symmetry. $S_{[N,M_F,M_A]}(\beta_F, \beta_A, L)$ is a functional of the Polyakov loop $L(\vec{n})$. $M_F$ and $M_A$ are integers and denote the highest powers of $\beta_F$ and $\beta_A$ in $S_{[N,M_F,M_A,L]}$. As we are interested in investigating the presence of a tricritical point in the above theory, we compute the polynomial $S_{[N,M_F,M_A]}(\beta_F, \beta_A)$ for $N=2,4,6$ in the strong coupling expansion. The higher order terms in $L$ are found to be negligibly small in the region of interest in the extended coupling plane and do not play any significant role in determining the phase boundary and the nature of the deconfining transition and therefore will be ignored here onwards.

$\dagger$ A generic gauge invariant term constructed out of $U_0(n_0, \vec{n})$ at spatial site $(\vec{n})$ is $Tr(U^\tau(\vec{n}))^m$ for some integer $m$. After making a gauge transformation to diagonalise $U^\tau(\vec{n})$, the above term can be written in terms of $L(\vec{n})$. 

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As mentioned earlier, the measure over the Polyakov loop can be fixed by the gauge invariance and periodic boundary conditions at finite temperature. We define,

\[ U^\tau (\vec{n}) = U^\tau_0 (\vec{n}) \sigma_0 + i \sum_{i=1}^{3} U^\tau_i (\vec{n}) \sigma_i \]  

(8)

In the above equation, \( \sigma_0 \) and \( \sigma_i \) are identity and the Pauli matrices respectively. Now at a particular spatial site \( (\vec{n}) \), the measure over temporal links, \( \prod_{n_0=1}^{N_\tau} dU_0(n_0, \vec{n}) \), can be written as \( \left[ \prod_{n_0=2}^{N_\tau} dU_0(n_0, \vec{n}) \right] / dU^\tau(\vec{n}) \). As the integrand at site \( (\vec{n}) \) depends only on \( U^\tau_0(\vec{n}) \), the first \( (N_\tau - 1) \) Haar integrals can be trivially done and the integrals over \( U^\tau_i(\vec{n}) \) give the Jacobian \( J(\vec{n}) \)

\[ J(\vec{n}) = \left( \frac{\pi}{2} \right) \left( 1 - L^2(\vec{n}) \right)^{\frac{1}{2}} \]

(9)

Here the factor \( \frac{\pi}{2} \) in the measure is due to the normalisation \( (f dU = 1) \).

The leading strong coupling diagrams contributing to the effective action are shown in Fig. 1-a,b,c. We define \( \gamma_F \equiv \frac{1}{2!} \left( \frac{\beta_F}{2} \right)^2 \) and \( \gamma_A \equiv \frac{4A}{3} \). After some extensive computations\(^\dagger\) for \( N_\tau > 2 \), the leading local contributions to the effective action are:

\[ S_{[0,4,2]} = \left( \gamma_F + \gamma_A + \gamma_F \gamma_A - \frac{1}{3!} (\gamma_F)^2 + \frac{1}{2!} (\gamma_A)^2 \right) N_\rho \]  

(10)

\[ S_{[2, N_\tau, 0]} = 4 \left( \frac{\beta_F}{4} \right)^{N_\tau} \sum_{\vec{n}, i} L(\vec{n}) L(\vec{n} + i) \]  

(11)

\[ S_{[4, 2N_\tau, N_\tau]} = (\gamma_F + \gamma_A)^{N_\tau} Y (L(\vec{n}), L(\vec{n} + i)) \]  

(12)

\[ S_{[6, 3N_\tau, N_\tau]} = \left( \frac{1}{3!} \right)^3 \left( \frac{\beta_F}{2} \right)^3 + \frac{1}{2} \gamma_A \beta_F \right)^{N_\tau} Z (L(\vec{n}), L(\vec{n} + i)) \]  

(13)

\(^\dagger\)The details will be presented elsewhere.
In the above equations we have used the notation:

\[ Y(L(\vec{n}), L(\vec{n} + i)) \equiv \left( \frac{1}{3} \right)^{N_\tau} \sum_{\vec{n},i} \left( 16L^2(\vec{n})L^2(\vec{n} + i) - 8L(\vec{n}) + 4 \right) \]  

(14)

\[ Z(L(\vec{n}), L(\vec{n} + i)) \equiv \left( \frac{1}{3} \right)^{N_\tau} \sum_{\vec{n},i} \left( 32L^3(\vec{n})L^3(\vec{n} + i) + 8L(\vec{n})L(\vec{n} + i) \right) \]

\[ \left( 1 - 2 \left( L^2(\vec{n}) + L^2(\vec{n} + i) \right) \right) + 4 \]  

(15)

The local higher order corrections to the above leading effective action come from filling the diagrams in Fig. 1. by extra plaquettes. The two and four plaquettes corrections to \( L^2 \) and \( L^4 \) terms are,

\[ S_{[2,N_\tau+4,2]} = \left( -\frac{1}{3} \gamma_F + \gamma_A \right) - \frac{(N_\tau + 2)}{3} \gamma_F \gamma_A + \frac{1}{2!} \gamma_F \gamma_A \]  

+ \( \frac{1}{N_\tau} \left[ N_\tau S_{[2,N_\tau,0]} \right] \)  

(16)

\[ S_{[4,2N_\tau+4,2]} = \frac{1}{4} C_2 \left( (\gamma_F + \gamma_A)^2 - \left( \frac{2}{3} \right) (\gamma_F)^2 \right)^2 (\gamma_F + \gamma_A)^{N_\tau-2} \]  

+ \( \frac{3}{2} \gamma_F \gamma_A \)  

(17)

\[ S_{[4,2N_\tau+4,2]} = \left[ \frac{1}{4} C_2 \left( (\gamma_F + \gamma_A)^2 - \left( \frac{2}{3} \right) (\gamma_F)^2 \right)^2 (\gamma_F + \gamma_A)^{N_\tau-2} \right] \]

+ \( \frac{3}{2} \gamma_F \gamma_A \)  

(18)

\[ \text{In [8], } \beta_A = 0 \text{ and the plaquettes correction } L^2 \text{ term in the effective action is computed up to order } \gamma_F, \text{ i.e. the first term in (16). However, its coefficient is } \gamma_F \text{ instead of } -\frac{1}{3} \gamma_F \text{ in our case.} \]}
For $N \geq 4$ in (7), the strong coupling diagrams also give non-local contributions to $S$. However, these contributions are proportional to much higher powers of the fundamental and adjoint couplings and are small near the investigated phase boundaries and thus have been ignored. The Landau Ginzburg effective potential can be trivially obtained by putting $L(\vec{n}) = L = \text{constant}$ in the above effective action. The effective potentials for $\beta_A = 0.5, 0.75, 0.9, 1.4, 1.5$ and 1.75 at $N_r = 4$ are plotted in Fig. 2-a,b,c,d,e,f. In Fig. 2-a,b,c corresponding to a second order deconfining transition, the three effective potential curves are at $\beta_F^{\text{critical}}$ and $\beta_F^{\text{critical}} \pm 0.05$. In the Fig. 2-d,e,f, where the transition becomes first order, the potentials are at $\beta_F^{\text{critical}}$ and $\beta_F^{\text{critical}} \pm 0.008$. They clearly show a dramatic change in the physical properties of the theory for large values of the adjoint couplings. Both in the second and first order regions the sharpness of the transition increases with increasing values of the adjoint coupling as found earlier by Monte Carlo simulations [20]. Moreover, in the first order transition region, the discontinuity in the Polyakov loop is large, leading to strong first order transitions. In fact, at $\beta_A = 1.5$ the Monte Carlo value of $<|L|>$ is 0.5 [20], in close agreement with its analytical value 0.55 from Fig. 2-e. The predicted $\beta_F^{\text{critical}}$ for the various values of the adjoint couplings $\beta_A$ at $N_r = 4$ along with their corresponding Monte-Carlo values [20, 21] are given in Table 1. In all cases good agreement is found between the two. The tricritical points for $N_r = 4$ and $N_r = 6$ are found to be at $[\beta_F = 1.352, \beta_A = 1.296]$ and $[\beta_F = 1.6711, \beta_A = 1.864]$, respectively. For $N_r = 2$, the above contributions to the effective potential are slightly different and are presently under investigation. However, both Monte Carlo simulations and lower order effective action indicate that the tricritical point is in the vicinity of $\beta_A \approx 0.7-0.8$. The above strong coupling shifts in the direction $(\beta_F = \infty, \beta_A = \infty)$ is probably an indication that the continuum limit is not affected by the existence of the tricritical point. However, such a constrained movement of the tricritical point with $N_r$, if also true in the intermediate and weak coupling regions, requires analytical explanation. The more interesting possibility of the existence of a new continuum theory in the extended coupling plane is still an open question. It will also be interesting to investigate other quantitative properties (e.g., $T_{\sqrt{\sigma}}$) of this model for higher values of $\beta_A$ by Monte Carlo simulations [20]. This requires very large lattices because of the known strong violation of the scaling relation in the large $\beta_A$ region [21, 43]. However, the more important problem is
to understand the origin for such a drastic change in the qualitative properties of the theory on the lattice itself and to identify the degrees of freedom and the mechanism which make the deconfining transition first order. An obvious guess for the above phenomenon is the different global properties of the $SU(2)$ and $SO(3)$ groups. This difference can be formally described by writing the $SO(3)$ part of the action \footnote{1} in its Villain form \footnote{18}. In the latter model the above difference corresponds to the $\mathbb{Z}_2$ vortices associated with plaquette fields, $\sigma_p = \pm 1$ in the action. In the extreme case, when all the vortices are suppressed the above Villain action reduces to the Wilson action with modified coupling with only a second order transition. Therefore, it is expected that the above change in the qualitative properties is due to the condensation of the vortices. A more careful study both via strong coupling as well as Monte-Carlo simulations is possible by controlling these objects with a potential term $\lambda \sum_{\text{plaquettes}} \sigma_p$ in the action and studying its effect on the deconfinement transition. It will be interesting to see the dependence of the tricritical point on $\lambda$. Work in this direction is in progress and will be reported elsewhere.

Table 2 gives the predicted critical couplings and their Monte Carlo values \footnote{24} along the Wilson line ($\beta_A = 0$) at different $N_\tau$. It clearly shows that the predictions of the strong coupling analysis for the deconfining transition are very close to the corresponding Monte Carlo values up to $N_\tau = 16$. Further comparisons in the region even closer to the asymptotic scaling region could not be made because of the lack of Monte Carlo data. The above validity of the strong coupling expansion in the scaling region ($N_\tau \geq 4$) found by Monte-Carlo simulations \footnote{24} and beyond $\beta_F = 2.2$, where the crossover from strong coupling to weak coupling is expected, is again a surprising phenomenon. However, it needs further verifications with other lattice observables and a careful study of the crossover region at $T \neq 0$.

In the forementioned recent paper \footnote{25} studying $SU(3)$ deconfinement transition at non-zero temperature, no evidence for the change in the order of the deconfining transition was reported. Also by extrapolating the Polyakov loop data to infinite volume, the deconfinement transition was separated from the discontinuity in the plaquette corresponding to the bulk transition. However, scaling laws for the plaquette discontinuity remained untested. The discrepancies in the qualitative aspects of the phase diagrams of $SU(2)$ and $SU(3)$ lattice gauge theories need an explanation. One should again check the role of $\mathbb{Z}(3)$ vortices in $SU(3)$ deconfinement transition. In \footnote{21} the $SU(2)$
plaquette susceptibility peak was found to decrease on going to relatively large volume. Therefore, it is important to verify the scaling behaviour of the bulk transition in the case of $SU(3)$ also.

Finally we summarize the findings of this paper. We have analytically confirmed the earlier Monte Carlo evidence for the existence of the tricritical point in the extended $SU(2)$ model. All the qualitative as well as quantitative features of the deconfinement transition in the extended coupling plane found by Monte Carlo simulations are reproduced to a good accuracy. As discussed above, this study also opens some new analytical as well as numerical avenues for understanding and analysing the new features associated with the extended model. Surprisingly, it is found that the strong coupling predictions for the critical couplings are valid well within the scaling region.

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Table 1

Table 1: The values of the critical couplings ($N_\tau = 4$) at various $\beta_A$.

| $\beta_A$ | $\beta^\text{critical}_{F}$ (Strong Coupling) | $\beta^\text{critical}_{F}$ (Monte Carlo$^{[1]}$) |
|-----------|---------------------------------------------|---------------------------------------------|
| 0.5       | 1.793                                       | 1.83                                        |
| 0.75      | 1.619                                       | 1.610                                       |
| 0.9       | 1.531                                       | 1.489                                       |
| 1.1       | 1.432                                       | 1.327                                       |
| 1.4       | 1.314                                       | –                                           |
| 1.5       | 1.28                                        | 1.05                                        |
| 1.75      | 1.147                                       | –                                           |

Table 2

Table 2: The scaling of the critical coupling with $N_\tau$ at $\beta_A = 0$.

| $N_\tau$ | $\beta^\text{critical}_{F}$ (Strong Coupling) | $\beta^\text{critical}_{F}$ (Monte Carlo$^{[2]}$) |
|----------|---------------------------------------------|---------------------------------------------|
| 4        | 2.00                                        | 2.2986(6)                                   |
| 5        | 2.25766                                     | 2.3726(45)                                  |
| 6        | 2.38535                                     | 2.4265(30)                                  |
| 8        | 2.51942                                     | 2.5115(40)                                  |
| 16       | 2.81613                                     | 2.7395(100)                                 |

$[1]$ The data taken from $^{[20, 21]}$.  
$[2]$ The data taken from $^{[24]}$. 

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Figure 1: The leading SU(2) strong coupling diagrams contributing to $L^2$, $L^4$ and $L^6$ terms in the effective action, respectively.
Figure 2: The effective potentials at different adjoint couplings and $N_f = 4$. The figures [a] $\beta_A = 0.5$, [b] $\beta_A = 0.75$, [c] $\beta_A = 0.9$ and [d] $\beta_A = 1.4$, [e] $\beta_A = 1.5$, [f] $\beta_A = 1.75$, correspond to second order and first order deconfinement transitions, respectively. The three second order (first order) effective potentials in each figure are at $\beta_F^{\text{critical}}$, $\beta_F^{\text{critical}} \pm 0.05$ (0.008).
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