Non-commutative Holographic QCD
and
Jet Quenching Parameter

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Abstract

Using gauge/gravity duality, we compute jet quenching parameter in confined and deconfined phases of noncommutative Sakai-Sugimoto model. In the confined phase jet quenching parameter is zero and noncommutativity does not affect it. In deconfined phase we find that the leading correction is negative i.e. it reduces the magnitude of the jet quenching parameter as compared to its value in commutative background. Moreover it is seen that the effect of leading correction is more pronounced at high temperatures.
1 Introduction

At the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory [1], quark-gluon plasma (QGP) has been produced by Au-Au collision at an ultra-relativistic center of mass $\sqrt{s} = 200$ Gev. QGP is a low viscosity, hot strongly coupled fluid. When a very energetic quark or gluon, with transverse momenta about a few tens Gev is moving through the strongly coupled plasma it interacts strongly with the plasma and loses energy via medium-induced radiation so that its transverse momenta becomes at most about 20 Gev [2]. The energy loss of the quark or gluon in the plasma is described by the jet quenching parameter $\hat{q}$ which is a property of the strongly coupled QGP. Jet quenching parameter decreases as the hot plasma expands and cools and the system re-enters to the confined phase and hadronization takes place. The time-averaged jet quenching parameter determined in comparison with RHIC data was found to be around 5-15 Gev$^2$/fm [3, 4].

Quantum chromodynamic (QCD) is a theory describing strong interaction between quarks and gluons. The coupling constant of QCD is large and the interaction is then strong at low energy or large distance. In other words, at low energy quarks or gluons are not free and we see colorless combinations of them known as mesons or hadrons. This is confinement phase where perturbative analysis does not work. However at high energy the coupling constant is small and quarks or gluon can freely move known as deconfinement phase. QGP is a new phase of QCD. In this new phase the quarks and gluons are neither confined nor free but instead form some new kind of strongly interacting fluid. In fact QGP produced at RHIC must be described by QCD in a regime of strong, and hence nonperturbative, interactions. As a result, due to strong interactions, QCD perturbative calculation is not reliable in this phase and it seems that a new theoretical tools is needed. One of the best candidates is Anti-de Sitter/Conformal Field Theory (AdS/CFT) correspondence [5].

According to the AdS/CFT correspondence, IIB string theory on $AdS_5 \times S^5$ background is dual to $D = 4 \; \mathcal{N} = 4 \; SU(N_c)$ super Yang-Mills theory (SYM) [5, 6]. In the large $N_c$ and large ’t Hooft coupling limit, the SYM theory is dual to Iib supergravity which is low energy effective theory of superstring theory. In this setting the strongly coupled SYM at non-zero temperature corresponds to supergravity in an AdS black brane background where the SYM theory temperature is identified with the Hawking temperature of the AdS black hole [7]. The AdS/CFT correspondence has been applied to study different quantities in condense matter [8] or in strongly
coupled plasma such as shear viscosity, jet quenching parameter \cite{9} and potential between quark-antiquark system \cite{10}. On the gauge theory side, the value of jet quenching parameter is related to thermal expectation value of Wilson loop \cite{11}. On the gravity side, the related calculation was originally introduced in \cite{9} and it was then elaborated in \cite{12}. The result of \cite{12} shows that there is only one extremized string world sheet that is bounded by light-like Wilson loop and it is the one which stretches from horizon to boundary. Our aim in this paper is to study jet quenching parameter, by employing AdS/CFT tools, in the noncommutative Sakai-Sugimoto model which will be reviewed in next section. Some properties of this background have been studied in \cite{13}.

Non-commutative gauge theories naturally appear on the D-branes with a background NSNS $B$-field on them. Explicitly consider a system of $D_p$-branes with a constant NSNS $B$-field along their worldvolume directions. By taking a certain low energy limit, closed strings decouple and the resulting action for open strings is the non-commutative gauge theory \cite{14}. It is possible to extend the AdS/CFT dictionary to the cases involving background $B$-field in the gravity side and noncommutative gauge theory in the CFT side \cite{15,16,17}. In addition non-commutativity can be considered as a way to model the magnetic fields in real systems like heavy-ion collisions at RHIC \cite{18} or in QCD \cite{19}.

In section 2 the noncommutative Sakai-Sugimoto model will be reviewed. In the next section we will then introduce and compute jet quenching parameter at low and high temperature noncommutative backgrounds. The last section is devoted to summary. In appendix we will calculate jet quenching parameter in commutative Sakai-Sugimoto model for a more general string configuration.

## 2 Noncommutative Sakai-Sugimoto Model

Holographic QCD background (Sakai-Sugimoto model), at low and high temperatures, is introduced in \cite{21,20}. This model is constructed from the near horizon limit of a set of $N_c$ D4-branes compactified on a circle with an anti-periodic boundary condition for the adjoint fermions. This makes the adjoint fermions massive and breaks supersymmetry. In the probe limit, namely $N_f \ll N_c$ where flavour branes do not backreact on the background, fermions in (anti-)fundamental representation are introduced by $N_f$ (anti-)D8-branes intersecting the D4-brane at a 3+1-dimensional defect. There is thus a global $U(N_f) \times U(N_f)$ chiral symmetry from the worldvolume of
D4-brane point of view.

It was shown in [21] that this theory undergoes a confinement-deconfinement phase transition at a temperature $T_d = 1/2\pi R$ where $R$ is radius of compactification. For quark separation obeying $R > 0.97 R$, the chiral symmetry is restored at this temperature but for $R < 0.97 R$ there is an intermediate phase which is deconfined with broken chiral symmetry and the chiral symmetry is restored at $T_{\chi_{SB}} = 0.154 R$. In the next two subsections, noncommutative low and high temperature sakai-Sugimoto model are introduced.

### 2.1 Noncommutative low temperature background

The low temperature noncommutative background is given by [17]

$$ds^2 = \left(\frac{u}{R}\right)^{3/2} \left(h(dt_E^2 + dx_1^2) + dx_2^2 + dx_3^2 + f dx_4^2\right) + \left(\frac{R}{u}\right)^{3/2} \left(\frac{du^2}{f} + u^2 d\Omega_4^2\right),$$

$$f = 1 - \frac{u^3}{w^3}, \quad h = \frac{1}{1 + \theta^3 u^3}, \quad e^\phi = g_s \left(\frac{u}{R}\right)^{3/4} \sqrt{h}, \quad B \equiv B_{11} = \left(\frac{\theta}{R}\right)^{3/2} u^3 h,$$

$$F_4 = dC_3 = \frac{2\pi N_c}{V_4} \epsilon_4,$$

(1)

where $t_E$ (Euclidean time), $x^i (i = 1, 2, 3)$ and $x_4$ are the directions along which the D4-branes are extended. $dt_E^2$, $d\Omega_4^2$, $\epsilon_4$ and $V_4 = 8\pi^2/3$ are the line element, the volume form and the volume of a unit $S^4$, respectively. $R$ and $u_k$ are constant parameters. $R$ is related to the string coupling $g_s$ and string length $l_s$ as $R^3 = \pi g_s N_c l_s^3$. $\theta$ is noncommutative parameter.

The coordinate $u$ is bounded from below by condition $u \geq u_k$. In order to avoid singularity at $u = u_k$, $x_4$ must be a periodic variable with period $R$ i.e.

$$2\pi R = \frac{4\pi}{3} \sqrt{\frac{R^3}{u_k}}.$$

(2)

### 2.2 Noncommutative high temperature background

The high temperature noncommutative background, in units where $R$ is one, is given by

$$ds^2 = u^{3/2} \left(h(f dt_E^2 + dx_1^2) + dx_2^2 + dx_3^2 + dx_4^2\right) + u^{-3/2} \left(\frac{du^2}{f} + u^2 d\Omega_4^2\right),$$

$$f = 1 - \frac{u^3}{w^3}.$$

(3)
In this case we have a black hole solution where \( u_h \) shows its horizon. As in low temperature case, at \( u = u_h \) we must have

\[
  u_h = \left(\frac{4\pi T}{3}\right)^2,
\]

where \( T \) is background temperature.

### 3 Jet Quenching parameter

The collision of high energy particles can produce jets of elementary particles. In the initial stage of nucleus-nucleus collision such as Au-Au at RHIC or Pb-Pb at large hadron collider (LHC) create QGP and these jets interact with the strongly coupled medium. Jets then lose energy by radiating gluons as they interact with the medium (this is similar to bremsstrahlung phenomena in quantum electrodynamics). This energy loss is called ”jet quenching” phenomenon.

As it was argued in [22, 23, 24], the jet quenching can be quantified through the ”jet quenching parameter” \( \hat{q} \), which is related to the thermal expectation value of Wilson loop as follows

\[
  \langle W^A(C_{\text{Light-Like}}) \rangle \approx e^{-\frac{1}{\sqrt{2}} \hat{q} L^2}.
\]

The closed rectangular loop includes two light-like Wilson lines connected by two short transverse segments of length \( L \) (see Fig. 1). Quark (gluon) propagates along the light-cone through the medium of length \( L^- \) and it is assumed to be large but finite i.e. \( L^- \gg L \). From AdS/CFT corresponding, in the large 't Hooft coupling and large \( N_c \) limits the thermal expectation value of \( \langle W^F(C) \rangle \) can be related to the extremal string action in AdS black hole background. In fact what we have is

\[
  \langle W^F(C) \rangle = e^{-S(C)}.
\]

It is clearly seen that, in order to compute jet quenching parameter, we need to find the value of the string action in our specific background. We will do this computation in the next two following subsections.

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\(^1\)In (5) and (6), \( A \) and \( F \) denote adjoint and fundamental representation. For \( SU(N_c) \), the Wilson line in adjoint representation can be obtained using the identity \( \text{Tr} W = \text{tr} W W^\dagger \) where \( \text{Tr} \) and \( \text{tr} \) denote trace in the adjoint and fundamental representation respectively.
Figure 1: Schematic picture of an open string (describing quark-antiquark system) moving along $x^-$. 

3.1 Jet quenching parameter in noncommutative low temperature

The noncommutative background (1) in the light-cone coordinate is given by

$$ds^2 = u^{3/2} \left( - h dx^+ dx^- + dx_2^2 + dx_3^2 + f dx_4^2 \right) + u^{-3/2} \left( \frac{du^2}{f} + u^2 d\Omega_4^2 \right), \quad (7)$$

where $x^\pm = ix_1 \pm t_E$. In the new coordinate, the $B$-field in the background becomes

$$B = B_{+-} = \theta^{3/2} u^3 h. \quad (8)$$

The action of string in an arbitrary background with a nontrivial $B$-field is

$$S = \frac{1}{2\pi \alpha'} \int d\tau d\sigma \left( \sqrt{-\det g_{\mu\nu}} + \epsilon^{\mu\nu} B_{MN} \partial_{\mu} x^{M} \partial_{\nu} x^{N} \right), \quad (9)$$

where $g_{\mu\nu} = G_{MN} \partial_{\mu} x^{M} \partial_{\nu} x^{N}$. $G_{MN}$ is background metric and $M$ index runs from 0 to 9. $\mu(=\tau, \sigma)$ labels string worldsheet where in static gauge we set $\tau = x^-$ and $\sigma = x_2$. By choosing $u(\sigma)$ and constant values for other
diagram.

\footnote{Note that by setting $R = 1$ we have $\alpha' = (\frac{1}{\pi g_s N_c})^{2/3}$.}
coordinates, in static gauge, we have
\[ g_{\tau\tau} = 0, \quad g_{\tau\sigma} = 0, \] (10)
and hence the first term in (9) clearly becomes zero. The second term in (9) does not contribute in the action because \( x^+ \) has been supposed to be a constant. As a result at low temperature jet quenching parameter vanishes.

By setting \( \theta \) equal to zero, (7) reduces to commutative background describing confining phase of QCD [21]. At commutative low energy jet quenching parameter was computed in [25] and reported that it is zero. Then our result means that noncommutativity does not change the value of jet quenching parameter at low temperature and it is still zero. Since jet quenching parameter describes interaction between test quark (gluon) and strongly coupled medium, vanishing jet quenching parameter tells us that the medium is transparent to test quark (gluon). This, as expected, is a sign of being in a confined phase.

3.2 Jet quenching parameter in noncommutative high temperature

The noncommutative background (3) in the light-cone coordinate is given by
\[
ds^2 = u^{3/2} \left( -\frac{(1+f)h}{2} dx^+ dx^- + dx_2^2 + dx_3^2 + dx_4^2 \right) - \frac{(1-f)h}{4} \left( (dx^+)^2 + (dx^-)^2 \right) + u^{-3/2} \left( \frac{du^2}{f} + u^2 d\Omega_4^2 \right). \] (11)

By choosing \( u(\sigma) \) and constant values for other coordinates the final action, in static gauge, is found by substituting (11) and (8) in (9) and then
\[
S = \frac{u_h^{3/2}}{4\pi\alpha'} \int_0^{L^-} d\tau \int d\sigma \sqrt{h(1 + \frac{u'^2}{f u^3})}, \] (12)
where \( u' = \frac{du}{d\sigma} \). Note that the second term in (9) does not contribute in the action as in the previous case. Light-cone time does not explicitly appear in the action and we will therefore have a conserved charge, energy of the system \( E \). We have then
\[
E = \mathcal{L} - u' \frac{\partial \mathcal{L}}{\partial u'}, \] (13)
3 Another configuration has been considered in the appendix.
and by using (12), the above equation leads to
\[ E^2 u'^2 = u^3 (h - E^2) f. \] (14)

The above equation can be solved to find \( L \) by using appropriate boundary condition. The boundary condition introduced in [9] is to consider curve \( C \) as a boundary of the string worldsheet by imposing \( u'(\pm L) = \infty \) which preserve the symmetry \( u(\sigma) = u(-\sigma) \). Moreover, as it was discussed in [12], the turning point of the string where \( u' = 0 \) is located at \( \sigma = 0 \) (where \( u(\sigma = 0) = u_h \)) as implied by the symmetry. In other words, string stretches from infinity to horizon and then returns to infinity.

As it was discussed in [16] in the AdS/CFT correspondence, IR limit of the thermal noncommutative gauge theory can be described by geometry (3). On the gravity side it means that \( u \) can not go to infinity and the value of \( \theta^3 u^3 \) must always be small for a fixed value of \( \theta \). Therefore we set \( u(\pm L) = f \) where \( f \) is the upper limit of the \( u \) coordinate. Hence (14) leads to
\[
\frac{L}{2} = \int_0^{L/2} d\sigma = E \int_{u_h}^{u_f} \frac{du}{\sqrt{(h - E^2)(u^3 - u_h^3)}}. \] (15)

By expanding the right hand side of (15) up to \( O(\theta^6) \) terms, we have [4]
\[
\frac{L}{2E} = \frac{1}{\sqrt{1 - E^2}} \left( \int_{u_h}^{u_f} \frac{du}{\sqrt{u^3 - u_h^3}} - \frac{\theta^3}{2(1 - E^2)} \int_{u_h}^{u_f} \frac{u^3 du}{\sqrt{u^3 - u_h^3}} \right). \] (16)

As it was mentioned earlier, since gravity theory with B-field describes IR region of the gauge theory [16], \( u \) can not go to infinity and for this reason we can consider \( u_f = bu_h \) and hence (16) becomes
\[
\frac{L}{2E} = \frac{1}{\sqrt{1 - E^2}} \left( \frac{2\Gamma\left(\frac{7}{6}\right)\sqrt{b\pi} - \Gamma\left(\frac{3}{2}\right)F\left(\frac{1}{2}, \frac{1}{2}, \frac{7}{6}, \frac{1}{b^2}\right)}{\Gamma\left(\frac{3}{2}\right)\sqrt{b\pi}} ight) \\
\quad - \frac{\theta^3}{2(1 - E^2)} \frac{1}{15} \int_{u_h}^{b u_h} \left[ 5\Gamma\left(-\frac{5}{6}\right)\sqrt{\pi} + 6b^{5/2} F\left(-\frac{5}{6}, \frac{1}{2}, \frac{1}{6}, b^{3/2}\right) \right], \] (17)

where \( F \) and \( b \) are Gauss hypergeometric function and a finite constant number. Setting \( b = 50 \), for simplicity, the above equation is simplified as
\[
L = \frac{4.3a}{\sqrt{u_h}} \left( 1 + (1 + a^2) \theta^3 u_h^3 \right), \] (18)

[4]Our expansion is reliable when \( \theta \int d\sigma u \ll 1 \).
where \(a^2 = \frac{E^2}{1-E^2}\) and \(\hat{\theta} = 1644.63\). Note that the different value of \(b\) changes coefficients of the above equation. (18) is a third order equation in \(a\) and has therefore three roots where its real one is given by

\[
a^2 = \frac{L^2 u_h}{(4.3)^2} \left( 1 - 2 \left[ 1 + \frac{L^2 u_h}{(4.3)^2} \hat{\theta}^3 u_h^3 \right] \right).
\]  

This gives us a relation between \(a\) and \(L\) which can be used to simplify the string action.

The string action at high temperature background is given by

\[
S = \frac{u_h^{3/2} L^-}{4\pi \alpha' E} \int d\sigma \, h \\
= \frac{u_h^{3/2} L^-}{2\pi \alpha'} \left[ \frac{h u}{\sqrt{(h - E^2)(u^3 - u_h^3)}} \right].
\]  

Similar to what is done in (16), one easily finds

\[
S = \frac{u_h^{3/2} L^-}{2\pi \alpha' \sqrt{1 - E^2}} \left( \int_{u_h}^{u_f} \frac{du}{\sqrt{u^3 - u_h^3}} - \frac{\theta^3(1 - 2E^2)}{2(1-E^2)} \int_{u_h}^{u_f} \frac{u^3 du}{\sqrt{u^3 - u_h^3}} \right) + O(\theta^6),
\]

and then

\[
S = \frac{2.15 u_h L^-}{2\pi \alpha' \sqrt{(1 - E^2)}} \left( 1 - \frac{1 - 2E^2}{1 - E^2} \hat{\theta}^3 u_h^3 \right).
\]  

Regarding to relation between \(a\) and \(E\), i.e. \(a^2 = \frac{E^2}{1-E^2}\), \(E\) can be found from (19) and by substituting in (22), we have

\[
S = \frac{2.15 L^- u_h}{2\pi \alpha'} \left( 1 - \hat{\theta}^3 u_h^3 \right) \sqrt{1 + 0.05 L^2 u_h} \\
\approx \frac{2.15 L^- u_h}{2\pi \alpha'} \left( 1 - \hat{\theta}^3 u_h^3 \right) \left( 1 + \frac{0.05}{2} L^2 u_h \right),
\]

where in the last line square root was expanded for small transverse distance, \(LT \ll 1\).

As was discussed in [9], the above calculation includes the self-energy of quark-antiquark system. In order to subtract the self-energy, we consider a trivial configuration given by two disconnected open strings, each of them...
descend from $u_f$ to $u_i$ at constant $x_2$, one at $+\frac{L}{2}$ and the other at $-\frac{L}{2}$. The string action then becomes

$$ S_s = \frac{L^-}{\pi\alpha'} \int_{u_h}^{u_f} du \sqrt{g_{uu}} $$

$$ = \frac{L^-}{2\pi\alpha'} \int_{u_h}^{u_f} du \sqrt{h(1-f)\left(1 - \hat{\theta}^3 u_h^3\right)} + O(\theta^6). \quad (24) $$

By substituting form (3) in (24), we finally have

$$ S_s = \frac{2.15 u_h L^-}{2\pi\alpha'} \left(1 - \hat{\theta}^3 u_h^3\right) + O(\theta^6). \quad (25) $$

The total action is given by

$$ S_t \equiv S - S_s = \frac{0.03}{\pi\alpha'} \left(1 - \hat{\theta}^3 u_h^3\right) L^2 L^- u_h^2. \quad (26) $$

Footnote 1 implies that in the large $N_c$ limit

$$ \langle W^A(C_{\text{Light-Like}}) \rangle = 2 \langle W^F(C_{\text{Light-Like}}) \rangle, \quad (27) $$

and the adjoint Wilson loop (5) is then given by $\exp(-2S_t)$. Jet quenching
parameter finally is
\[
\hat{q} = \frac{0.06}{\pi \alpha'} (1 - \hat{\theta}^3 u_h^3) u_h^2
\]
\[
= 7.35 \left(1 - \left(\frac{4\pi}{3}\right)^6 \hat{\theta}^3 T^6\right) \lambda T^4.
\]  
(28)

where \( \lambda = 4\pi g_s N_c \alpha'^{1/2} \). Setting \( \theta = 0 \), i.e. commutative case, jet quenching parameter is proportional to \( \lambda \) and \( T^4 \), as reported in [23]. Moreover in the noncommutative background we see a decrease in value of jet quenching parameter. However this reduction seems to be more important at high temperature, we should notice that the above result has been obtained assuming \( \hat{\theta} u_h \ll 1 \). Because this term is the first term of an expansion in terms of \( \theta \).

### 4 Summary

Jet quenching is one of the most important characteristic features of nuclear matter in the quark-gluon plasma phase, which has been observed at RHIC or heavy ion collisions at LHC. In this paper the effect of noncommutativity on the jet quenching parameter was studied at low and high temperatures of noncommutative Sakai-Sugimoto background.

At low temperature case where the system is in the confined phase, we expect the jet quenching phenomenon to be absent as the medium is transparent to the gluon or quark jets. In other words there is no interaction between test quark or gluon and the medium.

At high temperature case the effect of noncommutativity reduces the magnitude of the jet quenching parameter as compared to its value in the commutative case.

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### A A more general string configuration and jet quenching parameter

Here we consider a more general configuration but in commutative high temperature background. In static gauge by choosing \( u(\sigma), x_4(\sigma) \) and constant
values for other coordinates the string action becomes

\[ S = \frac{u_h^{3/2}}{4\pi\alpha'} \int_0^{L^-} d\tau \int d\sigma \sqrt{1 + u'^2 + \frac{u'^2}{f u^3}}, \tag{29} \]

where \( u' = \frac{du}{d\sigma} \) and \( x_4' = \frac{dx_4}{d\sigma} \). Note that in commutative case the second term in (9) is zero. As before light-cone time does not explicitly appear in the action and energy of the system \( \hat{E} \) is a constant which is given by

\[ \hat{E} = \mathcal{L} - u' \frac{\partial \mathcal{L}}{\partial u'} - x_4' \frac{\partial \mathcal{L}}{\partial x_4'}, \tag{30} \]

and then

\[ \hat{E} = \frac{h}{\sqrt{h(1 + x_4'^2 + \frac{u'^2}{f u^3})}}. \tag{31} \]

Equation of motion for \( x_4 \) is

\[ x_4' = \frac{C}{E}, \tag{32} \]

where \( C \) is a constant and as a result \( x_4' \) is also a constant. From (30) it is easy to find a relation among \( \hat{E} \), \( C \) and \( L \) defined in (15) which is

\[ L = \frac{4.84\hat{a}}{\sqrt{u_h}}, \tag{33} \]

where \( \hat{a}^2 = \frac{E^2}{1 + (\frac{C}{E})^2} \) and \( t = 1 + (\frac{C}{E})^2 \). The string action then becomes

\[ S = \frac{u_h^{3/2}}{4\pi\alpha'} \int_0^{L^-} d\tau \int d\sigma \sqrt{1 + \hat{x}_4'^2 + \frac{\hat{x}_4'^2}{f u^3}}, \]

where \( \hat{x}_4' = \frac{\hat{x}_4}{d\sigma} \) and \( \hat{x}_4 = \frac{dx_4}{d\sigma} \).

\[ \hat{E} = \mathcal{L} - u' \frac{\partial \mathcal{L}}{\partial u'} - x_4' \frac{\partial \mathcal{L}}{\partial x_4'}, \tag{30} \]

and then

\[ \hat{E} = \frac{h}{\sqrt{h(1 + \hat{x}_4'^2 + \frac{\hat{x}_4'^2}{f u^3})}}. \tag{31} \]

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\[ L = \frac{4.84\hat{a}}{\sqrt{u_h}}, \tag{33} \]

where \( \hat{a}^2 = \frac{E^2}{1 + (\frac{C}{E})^2} \) and \( t = 1 + (\frac{C}{E})^2 \). The string action then becomes
\[ S \simeq \frac{2.42L^{-u_h}}{2\pi\alpha'} (1 + 0.02L^2tu_h), \quad (34) \]

\( L^2t \) is a new parameter appearing in the jet quenching parameter. As it is shown in Fig. 3, this quantity is physical distance which must appear in physical quantity. In fact the equation of motion for \( x_4 \) states that \( \dot{\hat{q}} \) can be zero or constant. If \( x_4 = 0 \), \( L \) is physical distance and otherwise our results are written in terms of physical distance \( L^2t \).

Obviously, after subtracting the self-energy of the system, jet quenching parameter becomes

\[ \hat{q} = \frac{0.04}{\pi\alpha'}u_h^2. \quad (35) \]

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