AN OPTIMUM VEHICULAR PATH ALGORITHM FOR TRAFFIC NETWORK BASED ON HIERARCHICAL SPATIAL REASONING

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KEY WORDS optimum path algorithm; traffic network; hierarchical spatial reasoning

ABSTRACT Human beings' intellection is the characteristic of a distinct hierarchy and can be taken to construct a heuristic in the shortest path algorithms. It is detailed in this paper how to utilize the hierarchical reasoning on the basis of greedy and directional strategy to establish a spatial heuristic, so as to improve running efficiency and suitability of shortest path algorithm for traffic network. The authors divide urban traffic network into three hierarchies and set forward a new node hierarchy division rule to avoid the unreliable solution of shortest path. It is argued that the shortest path, no matter distance shortest or time shortest, is usually not the favorite of drivers in practice. Some factors difficult to expect or quantify influence the drivers' choice greatly. It makes the drivers prefer choosing a less shortest, but more reliable or flexible path to travel on. The presented optimum path algorithm, in addition to the improvement of the running efficiency of shortest path algorithms up to several times, reduces the emergence of those factors, conforms to the intellection characteristic of human beings, and is more easily accepted by drivers. Moreover, it does not require the completeness of networks in the lowest hierarchy and the applicability and fault tolerance of the algorithm have improved. The experiment result shows the advantages of the presented algorithm. The authors argued that the algorithm has great potential application for navigation systems of large-scale traffic networks.

1 Introduction

Heuristic is an important method to solve the shortest path problems. It indicates literally learning by experience, or more generally in the artificial intelligence literature, a heuristic is a “rule of thumb” and as such is the approach used by almost any human in conducting a search. In the context of computer search algorithms, heuristic implies simple search through specific knowledge. Under a special function, at every step these algorithms search the node with highest score as the next node to be extended. The major effect of heuristic search is that in some way it constrains the search space. The shortest path algorithms based on heuristic includes costing algorithm, branch-and-bound algorithm, greedy algorithms, hill-climbing algorithms, and A* algorithm, etc.

Among the known shortest path algorithms, many of them use the greedy strategies as the search strategies and explore how to design delicate running data structures and searching algorithms, so as to improve the running efficiency of sequential shortest path algorithms under the uniform time complexity. For the two branches of lossless shortest path algorithms: label setting and label correct-
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Ining algorithms, experts have set forward various running data structures and related literatures emerge endlessly. Refs. [15 ~ 19] have made detailed analysis and comparison for the shortest path algorithms presented before. Because the real networks, such as transportation networks, telecommunication networks, facility networks and hydrographic networks do not concern negative weights, a famous label setting algorithm adopting greedy strategy, namely Dijkstra’s algorithm, has attracted wide attention and got wide dissemination and applications. Dijkstra’s algorithm is the most mature shortest path algorithm theoretically up to date[8,16,20] and has become the crucial algorithm for the network analysis modules of many large GIS platforms. Different implementation methods have formed a large family of Dijkstra’s algorithms. For example, the network module of Arc/Info adopts a Dijkstra’s algorithm implemented with binary heap priority queue[21], and GeoStar adopts a Dijkstra’s algorithm implemented with quick sorted FIFO queue[11]. Ref. [22] presented a Dijkstra’s algorithm implemented with quad heap priority queue and backward star data structure and applied it in the vehicle navigation system.

However, greedy strategies are not the surrogate of heuristics. There are many kinds of heuristics, such as directional strategies and hierarchical strategies besides greedy strategies. The integration of different heuristics can obviously improve the running efficiency of shortest path algorithms. Ref. [23] set forward a directional strategy with the spatial distribution of traffic networks to rationally constrain the searching space, so as to improve the running efficiency of shortest path algorithm.

Hierarchical strategy is another kind of heuristic strategies and plays an important role in human thinking. However, it has not got wide applications in the shortest path algorithms. On the basis of greedy and directional strategies, it is detailed in this paper how to adopt the hierarchical reasoning method to construct an optimum path algorithm for traffic networks.

2 Hierarchical spatial reasoning

Hierarchical spatial reasoning is a method usually adopted to solve the problems with characteristic of spatial distribution[24,25]. Human cognition to the world has a distinct property of spatial hierarchies, and every hierarchy contains some necessary information to solve specific problems. Understanding the sophisticated geospatial problems from different spatial scales makes us more easily hold the essence of problems step by step. For example, facing a traffic map, a person can quickly find an optimum path between two spots chosen arbitrarily on the map. Even if the path is not a shortest path, it is also a good alternate to be referenced. Here the human thinking is none other but a procedure of typical hierarchical spatial reasoning. It does not need to carry out complicated calculation in the completely unwrapped detail, but make judgement on a generalized hierarchy, and decide the optimum path in this way.

Hierarchical spatial reasoning is a spatial analysis method dividing the problem to be solved on space or tasks according to some special rules[25]. The partition of spatial objects with hierarchical structures is formed with levels or sub-zones. Every level or sub-zone has the same structure, the homogeneous objects and the same relation operations. The level $i + 1$ is a spatial subset of level $i$. Relations and operations between objects are only pertinent to the unique task divided into different levels. For example, the optimum path between two nodes in a traffic network is determined according to the road levels (freeway, arterial highway, secondary road, sidewalk and alley), but does nothing with other different tasks.

When data is incomplete or data volume is great, a shortest path algorithm based on graph searching may be inefficient, yet an algorithm based on hierarchical spatial reasoning can exhibit its advantages. Such an algorithm is still a spatial heuristic. Its attention to network details is gradually deepen with the levels, so as to filter the details no longer need to be paid attention to during hierarchy transition. Yet the graph searching algorithm with single hierarchy always exhibit any detail at the first sight. It is obvious that the great deal of detail impertinent to the problems to be solved will hamper the solution efficiency remarkably.
It is noteworthy that in this paper the optimum path is different from shortest path conceptually. A shortest path, whether distance shortest or time shortest, is not surely an optimum path for vehicular traffic. In addition to driving distance and time, to determine an optimum path for vehicular traffic needs considering many factors. For instance, even if a time shortest path is determined by considering road level, signal waiting and turn blocking, a driver still needs to consider such factors as road surface situation, convenience of contrarotation of side screws, driving field of vision, degree of conforming to the rules and probability of unexpected emergency. These factors form examination for vehicles, driving skills and attention of driver. It explains the reason why drivers would not like to choose the paths with shortest time, yet narrow alleys or streets with order easily disturbed, but would like to choose secondary shortest paths, and spend a little more time to travel on trunk road. So absolute distance or time shortest paths are usually not the vehicular paths the drivers would like to choose.

The optimum path in this paper means optimum vehicular path. The core of algorithm is still shortest path algorithm. The hierarchical spatial reasoning joined into the shortest path algorithms makes the resulted shortest path meet the requirement of optimum path for vehicular traffic much better.

3 Hierarchical structure of a traffic network

GIS data expression in traffic network has the following features\[^{[26]}\]: 1) human may divide a large road network hierarchically; 2) the amount of detail increases from the top to the lowest level; 3) hierarchical levels are formed according to road levels, traffic volume and expected travel speed; 4) optimum path can be searched in small sub-networks.

In accordance with the features mentioned above the hierarchical spatial reasoning can restrict the path choice within specific sub-networks. This method is like the local searching method usually adopted in the field of artificial intelligence. However, in the local searching method, the recursive condition is to judge whether the current solution superior to the acquired optimum feasible solution, whereas in the hierarchical spatial reasoning, the recursive condition is to judge whether the procedure has reached the lowest hierarchy.

In order to apply analytical tools, the network is abstracted to a regular square network, which is reasonably approximate to street networks with a distance between nodes of \(d\), as shown in Fig. 1. The roads in the network are divided into different classes according to their grades. Hierarchies are composed of classes. Every hierarchy contains the roads in higher hierarchies, as shown in Fig. 2. The network is divided into sub-regions with boundary formed by the road sections in lower hierarchy. A sub-region is a grid in sub-region with higher hierarchy. Adjacent sub-regions share the boundary. The adjacent hierarchies share some common nodes i.e., all the nodes in the higher hierarchy, these nodes are the connecter between this hierarchy and the lower hierarchy.

![Fig. 1 Regular mesh expression for traffic networks](image1.png)

![Fig. 2 Hierarchy division for traffic networks](image2.png)

The rule of hierarchical division was first set forward by Car & Frank (1994)\[^{[26]}\]. This rule has a distinct deficiency, i.e., for the two adjacent hierarchies, the nodes in the higher hierarchy network constitute a proper subset of the nodes in the lower hierarchy network, whereas the nodes in the lower hierarchy network, except for shared nodes, are not kept in the higher hierarchy. It is helpful to keep the semantic homogeneity for the nodes in different hierarchies. But because the algorithm needs backtracking from the lower to the higher hierarchies.
during the recursion procedure, the rules can sometimes result in unreliable solution as seeking far and neglecting what lies close at hand. In this paper, the authors present an improved rule for hierarchical division: for the adjacent hierarchies, the shared nodes in the lower hierarchy are kept in the nodes set of the higher hierarchy, as shown in Fig. 3.

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![Fig. 3 Improved hierarchy division rule for traffic networks](image)

**4 Application of hierarchical spatial reasoning in the optimum vehicular path algorithm**

According to the improved rule of hierarchy division, the applied method of hierarchical spatial reasoning in the optimum path algorithm can be described as follows:

Given the source and the destination nodes S and T. Suppose that i and j are the highest hierarchies that contain S and T. S and T are marked as $S_i$ and $T_j$, and the optimum path algorithm can run on the sub-network decided by min($i, j$). Fig. 4 gives an example, therein the highest hierarchy that contains S is 2, and the highest hierarchy that contains T is 3, then the algorithm can run on hierarchy 2 of the network. It is more efficient than running on hierarchy 1 completely. Although the lossy algorithm can not guarantee the resultant path is all to nothing the shortest path, it is a preferable path and an alternative to the shortest path. Furthermore, the seeming deficiency of hierarchical spatial reasoning is coincident with the drivers’ thinking mode. It has been discussed that when choosing travelling path, the drivers have to consider many factors that are difficult to expect or quantify, besides driving distance and driving time. The hierarchical spatial reasoning requires the paths selected on higher hierarchies as much as possible, so as to decrease the dramatical existence and occurrence of those factors. Therefore, at the cost of some mileage or time, the optimum vehicular path based on hierarchical spatial reasoning, as compared with the absolute shortest distance or time paths, has a better feasibility and is more easily accepted by the drivers.

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![Fig. 4 Route choice for multi-hierarchical networks](image)

In city’s traffic networks, hierarchies are divided by road classes. To ensure the even distribution of the sub-networks of different hierarchies on the whole network, the authors divided the networks into three hierarchies, namely, trunk roads (including superhighways), ordinary road and alley. Adjacent lists are established for the nodes in the three hierarchies respectively. Supposed $S_i$ and $T_j$ are the source and the destination, then the shortest path algorithm can be reconstructed as follows:

1) Searching the nearest node to $S_i$, $T_j$ to find out the highest hierarchies $i$ and $j$ that contain $S_i$ and $T_j$, so as to determine the beginning hierarchy. We have $k = \min(i, j | i \leq 3, j \leq 3)$;  

2) Confirming the hierarchical zones that $S_k$ and $T_k$ are located;  

3) If $S_k$ and $T_k$ are located in the same zone of hierarchy 2, then the shortest path is calculated directly on hierarchy 1;  

4) Otherwise, if $S_k$ and $T_k$ are located in the same zone of hierarchy 3 but the same zone of hierarchy 2, we can find out the nearest nodes to $S_k$ and $T_k$ on hierarchy 2, marked as $S_2$ and $T_2$. (If $i < j$, then $T_k$ has been in hierarchy 2, only $S_2$ needs to be found out). The two nodes are the entrance to hierarchy 2. The shortest paths $S_k \rightarrow S_2$ and $T_2 \rightarrow$
$T_k$ are calculated on hierarchy 1. They are a part of the optimum path $S_k \rightarrow T_k$. Then the shortest path $S_2 \rightarrow T_2$ is calculated on hierarchy 2. The resulted paths $S_k \rightarrow S_2, S_2 \rightarrow T_2, T_2 \rightarrow T_k$ are linked up;

5) Otherwise, if $S_k$ and $T_k$ are not located in the same zone of hierarchy 3, we can first find out the nearest nodes $S_2$ and $T_2$ to $S_k$ and $T_k$ on hierarchy 2 by the method in 4), then calculate the shortest paths $S_k \rightarrow S_2$ and $T_2 \rightarrow T_k$ on hierarchy 1. We can also find out the nearest nodes $S_3$ and $T_3$ to $S_2$ and $T_2$ on hierarchy 3, then to calculate the shortest paths $S_2 \rightarrow S_3$ and $T_3 \rightarrow T_2$ on hierarchy 2. Lastly, the shortest path $S_3 \rightarrow T_3$ is calculated directly on hierarchy 3, and all of the resulted paths, i.e., $S_k \rightarrow S_2, S_2 \rightarrow S_3, S_3 \rightarrow T_3, T_3 \rightarrow T_2$ and $T_2 \rightarrow T_k$ are linked to get the final result.

The shortest path searching adopts an improved Dijkstra's algorithm based on the quad heap priority queue and a restriction strategy based on rectangle searching area, which were presented by the authors in Refs. [22] and [23].

Fig. 5 gives an instance that the time shortest path algorithm is adopted only on a single hierarchy. We can see from it that half of the path is ordinary roads. Fig. 6 presents an instance of three-hierarchy division of a traffic network, where thick dark lines compose hierarchy 3, thin dark lines compose hierarchy 2, and the others compose hierarchy 1. Fig. 7 gives an example of the optimum path algorithm designed with the rule presented by Car & Frank. We can see that the resultant path is unreliable. Fig. 8 shows the same example of the optimum path algorithm based on hierarchical spatial reasoning, designed by the improved hierarchy division rule. In comparison with Fig. 5, although the resultant optimum path is not time shortest path theoretically, most of which is located on trunk roads. It decreases the factors difficult to expect or quantify and makes the path more robust. The result is conformable with human being thinking, and is more easily accepted by the drivers. In comparison with Fig. 7, the improved hierarchical division rule makes the algorithm find out more rational adjacent nodes on higher hierarchies during the backtracking from lower hierarchies to higher hierarchies and avoid the emergence of unreliable solution that seeks far and neglects what lies close at hand. The resultant path is more reliable.
5 Efficiency analysis

Suppose that a regular network contains \( n = v \times v \) nodes on the lowest hierarchy, according to the algorithm the author set forward in Ref. [22], finding out a shortest path containing \( n \) nodes costs \( O(n \log n) \). Given the proportion \( k \) of grid width between the adjacent hierarchies, then the cost of finding out the shortest path between the nodes on the current hierarchy and the connective nodes on the higher hierarchy is only \( O(k^2 \log k) \), which is a constant. Consequently the cost of calculating the shortest path between the source and the destination is only the function of the number of hierarchies or recursions. The number of hierarchies or recursions can be expressed as a logarithm of path length, i.e., \( \log_{lv} \). Therefore, with hierarchical spatial reasoning, calculating an optimum path containing \( n \) nodes in a regular network only costs \( O(\log_{lv}) = O(\log n) \). It is a great improvement in comparison with non-hierarchical algorithms. The advantages will get more distinct with the expansion of network scale.

Fig. 7 Unreliability of the hierarchy division rule by car & Frank

The algorithm presented in this paper has been implemented with MS Visual C++ environment. Taking Beijing traffic network as an example, the network contains 12800 nodes and 17800 road sections. On this network, for arbitrarily selected source and destination, using the algorithm the author set forward in Ref. [22], the longest time shortest path costs 0.15 s on a Pentium Pro 200 computer. With the joining of restriction search area method the author presented in Ref. [23], the efficiency can be improved up to 60% at least. With the joining of the optimum path algorithm presented in this paper, depending on the hierarchy and location difference of the source and the destination nodes the efficiency can be improved by several times. Nevertheless, it is noticeable that the algorithm set forward in this paper is a kind of lossy algorithm, and comparing its efficiency with that of the lossless algorithms this algorithm does not have strict theoretical significance.

Fig. 8 A case of optimum path algorithm with the improved hierarchy division rule

6 Conclusion

From the view of hierarchical spatial reasoning, the authors have discussed the characteristics of human beings' intellection on spatial analysis, analyzed the hierarchical spatial feature of traffic network, and divided the urban traffic network into three hierarchies and made the optimum path selection completed in higher hierarchies as much as possible. The lossy optimum path algorithm presented in this paper is more conformable to human being's intellection and the results are more easily accepted.

A new node hierarchy division rule is set forward, which avoids the unreliable solution of shortest path that possibly emerges under the previous node division rule that does not consider the semantic homogeneity. Then a lossy optimum path algorithm based on hierarchical spatial reasoning is pre-
presented, with an efficiency analysis of an instance. The result shows the optimum path algorithm based on hierarchical spatial reasoning has a better efficiency in comparison with non-hierarchical algorithms, and its advantages will be more distinct with the expansion of path length. Furthermore, the hierarchical division of traffic network makes the completeness of lower hierarchies unnecessary, provides fault tolerance to a certain extent for the input data, and improves the applicability of the algorithm. The algorithm presented in this paper has an excellent potential application in the navigation of large-scale traffic networks.

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