Abstract

We investigated the possibility of testing factorization hypothesis in non-leptonic exclusive decays of $B$-meson. In particular, we considered the non-factorizable $B^0 \to D^{(*)+}D^{(*)-}$ modes and $B^0 \to D^{(*)+}(\pi^-, \rho^-)$ known as well-factorizable modes. By taking the ratios $B(B^0 \to D^{(*)+}D^{(*)-})/B(B^0 \to D^{(*)+}(\pi^-, \rho^-))$, we found that under the present theoretical and experimental uncertainties there's no evidence for the breakdown of factorization description to heavy-heavy decays of the $B$ meson.
I. INTRODUCTION

Non-leptonic decays of heavy mesons are very important weak processes for the determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements [1] and the understanding of the CP violation mechanism. The analysis of decay processes of $B$-mesons, now produced in large numbers in the $B$-factories worldwide, will contribute to the stringent test of the Standard Model (SM) and search for new physics. Non-leptonic decays of $B$-mesons, however, are complicated processes due to inherent hadronic nature and final state interactions.

A simple formulation of the decay amplitude, so called the naive factorization scheme [2,3], has been widely used without full theoretical justification. And its phenomenological extension, the generalized factorization scheme, with process-dependent quantities from penguin effects and non-factorizable contributions has been also widely used in the literature [4,5]. In this latter scheme, the non-factorizable effects are contained in the effective color number, $N_{\text{eff}}^{c}$, which is a free parameter [4,5] of the scheme; the value of $N_{\text{eff}}^{c}$ was adjusted to $\infty$ for $D$ decays, and to 2 or 5 depending on the chiral structure of $B$ decays. However, the $N_{\text{eff}}^{c}$-dependence of the Wilson coefficient in the effective Hamiltonian is different for each individual coefficient. For example, the coefficients related to color-favored processes, $a_{1}$, $a_{4}$, $a_{6}$, $a_{9}$ are stable against the variation of $N_{\text{eff}}^{c}$, while those related to color-suppressed processes strongly depend on $N_{\text{eff}}^{c}$ [4–6]. This observation indicates that the non-factorizable contributions in the color-favored decays are negligible compared with the factorizable contributions.

Recently much progress has been achieved [7,8] towards understanding non-leptonic decay processes by separating out short distance physics from long distance effects in the well-defined manner; Beneke et al. [7] proved the validity of factorization for the $B$-meson decay amplitude in the context of perturbative QCD formalism. They showed that when a $B$-meson decays weakly to a heavy meson and emits a light meson, the decay amplitude factorizes in the same form as the naive factorization formula, but with calculable coefficients in the heavy quark limit. In the case of $B$-meson decaying to light meson rather than heavy
one, according to their formalizm, a contribution by hard spectator quark is added to the amplitude, therefore the total amplitude is still factorizable. However, for $B$-decays to a heavy or light meson with emitting a heavy meson, their amplitudes are not written as factorized forms, since the color transparency arguments cannot be applied for such decays.

Though the factorization of the decay amplitude for a $B$-meson decaying to heavy-heavy mesons has not been justified, there have been many calculations using the factorized formula in the literature \[5,9,10\]. Within the naive factorization scheme, Luo and Rosner \[10\] calculated the branching ratios of the $B$-meson decays, $\bar{B}^0 \to D^{(*)}+D_s^{(*)}$, after extracting the values of $|V_{cb}|$ and the slope of the universal Isgur-Wise form factor $\rho^2$, by comparing the decay rates of $\bar{B}^0 \to D^{(*)}+(\pi^-, \rho^-)$ with a differential distribution of $\bar{B}^0 \to D^{(*)}+l^-\bar{\nu}_l$ measured by the CLEO Collaboration \[11\]. They found theoretical predictions of the naive factorization approach are acceptable within present experimental errors.

Here we test the generalized factorization scheme for the color-favored $B$-meson decay to heavy-heavy mesons by comparing with the $B$-decay to heavy-light mesons. The selected decay processes are $\bar{B}^0 \to D^{(*)}+D_s^{(*)}$ for heavy-heavy and $\bar{B}^0 \to D^{(*)}+(\pi^-, \rho^-)$ for heavy-light, whose experimental branching ratios are well known. Compared to the work of Luo and Rosner, in which the authors used the naive factorization scheme neglecting penguin effects, we here include penguin effects and take ratios of the decay rates to reduce the form factor dependence and cancel the CKM matrix elements. In the next section, we will give theoretical descriptions of $\bar{B}^0 \to D^{(*)}+(\pi^-, \rho^-)$ and $\bar{B}^0 \to D^{(*)}+D_s^{(*)}$ within generalized factorization scheme, and present numerical analyses. Section III discusses experimental feasibility of our analyses and closes with a brief summary.

**II. THEORETICAL DESCRIPTION OF $\bar{B}^0 \to D^{(*)}+(\pi^-, \rho^-)$ AND $\bar{B}^0 \to D^{(*)}+D_s^{(*)}$ WITHIN GENERALIZED FACTORIZATION APPROACH**

As previously mentioned, there has been general consensus on the applicability of factorization approach to the color-favored heavy-light decays $B \to D^{(*)}(\pi, \rho)$, and this has
been recently justified within perturbative QCD formalism \[\text{[7]}\]. However, questions still
remain about the applicability of factorization to heavy-heavy decay of $B$-meson, such as
$B \rightarrow D^{(*)} D_s^{(*)}$. We here investigate the validity of factorization hypothesis by taking ratios of
branching fractions of presumably non-factorizable $B^0 \rightarrow D^{(*)} D_s^{(*)}$ modes to factorizable
$B^0 \rightarrow D^{(*)} (\pi^-, \rho^-)$ modes.

Based on the generalized factorization formalism, the decay amplitudes of our interest
are expressed as

$$A(B^0 \rightarrow D^{(*)} + M^-) = \frac{G_F}{\sqrt{2}} V_{cb} V^*_{ts} \tilde{a}(D^{(*)} M) \langle M^- | \bar{q} \gamma^\mu (1 - \gamma_5) q | 0 \rangle \langle D^{(*)} | \bar{c} \gamma_\mu (1 - \gamma_5) b | B^0 \rangle, \quad (1)$$

where $q(q') = u(d)$ for $M = \pi, \rho$ and $q(q') = c(s)$ for $D^{(*)}$. The coefficient $\tilde{a}$ includes penguin
effects and possible non-factorizable contributions in the generalized factorization scheme.

They are given, neglecting W-exchange diagram and using $V_{tb} V^*_{ts} \approx -V_{cb} V^*_{cs}$, as

$$\tilde{a}(D^{(*)}(\pi, \rho)) = a_1,$$
$$\tilde{a}(D D_s) = a_1 \left(1 + \frac{a_4 + a_{10}}{a_1} + 2 \frac{a_6 + a_8}{m^2_D} \right);$$
$$\tilde{a}(D^* D_s) = a_1 \left(1 + \frac{a_4 + a_{10}}{a_1} - 2 \frac{a_6 + a_8}{m^2_D} \right);$$
$$\tilde{a}(D^{(*)} D^*_s) = a_1 \left(1 + \frac{a_4 + a_{10}}{a_1} \right). \quad (2)$$

where $a_j$'s represent conventional effective parameters defined as $a_{2i} = c_{2i}^{\text{eff}} + c_{2i-1}^{\text{eff}} / N_c^{\text{eff}}$ and
$a_{2i-1} = c_{2i-1}^{\text{eff}} + c_{2i}^{\text{eff}} / N_c^{\text{eff}}$. Using the numerical values of $a_j$'s of Ref. \[\text{[9]}\], the effective parameters
$\tilde{a}$ defined above are related to $a_1$ by

$$|\tilde{a}(B \rightarrow D D_s)| = 0.847 a_1,$$
$$|\tilde{a}(B \rightarrow D^* D_s)| = 1.037 a_1,$$
$$|\tilde{a}(B \rightarrow D^{(*)} D^*_s)| = 0.962 a_1, \quad (3)$$

where the values are obtained by choosing $N_c^{\text{eff}} = 2$ for $(V - A)(V - A)$ interactions (i.e. for
operators $O_{1,2,3,4,9,10}$) and $N_c^{\text{eff}} = 5$ for $(V - A)(V + A)$ interactions (i.e. for operators $O_{5,6,7,8}$)
$\text{[9]}$. We note that the ratios, $|\tilde{a}/a_1|$, are numerically very stable over different $N_c^{\text{eff}}$ values;
for example, the numerical deviations are less than a few % for $N_c^{\text{eff}} = 2, 3, 5$ and $\infty$. From
the relations in Eq. (3) one can see that, at the amplitude level, the penguin contributions to \( \overline{B}^0 \to D^{*+}D^-_s \) decay (\( \sim 3.7\% \)) are much smaller than those for \( \overline{B}^0 \to D^+D^-_s \) mode (\( \sim 15.3\% \)). In fact the penguin effects on \( \overline{B}^0 \to D^+D^-_s \) decay are not small enough to be simply neglected. As previously mentioned, the penguin effects are neglected in the analyses of Ref. [4]. We will show that the inclusion of the penguin effect in \( \overline{B}^0 \to D^+D^-_s \) mode improves substantially the theoretical prediction to the experimental value. For \( \overline{B}^0 \to D^{*+}D^-_s \) decay mode, the penguin contribution can be neglected. This difference of penguin contributions between the similar decay modes \( \overline{B}^0 \to D^+D^-_s \) and \( \overline{B}^0 \to D^{*+}D^-_s \) is due to the different chiral structure of the final states: \( B \to D^* \) transitions occur through axial vector currents, while \( B \to D \) through vector currents.

The ratios

\[
\mathcal{R}_{D_s^*(\pi,\rho)} = \frac{\mathcal{B}(\overline{B}^0 \to D^+D_s^*(\pi,\rho))}{\mathcal{B}(\overline{B}^0 \to D^+D^-_s)}
\]

and

\[
\mathcal{R}_{D_s^*(\pi,\rho)} = \frac{\mathcal{B}(\overline{B}^0 \to D^{*+}D^*_- s)}{\mathcal{B}(\overline{B}^0 \to D^{*+}D^-_s)}
\]

are given as

\[
\mathcal{R}_{D_s/\pi} = \frac{\bar{a}(DD_s)}{\bar{a}(D\pi)} \left( \frac{f_{D_s}}{f_\pi} \right)^2 \left( \frac{p_{c}^{DD_s}}{p_{c}^{D\pi}} \right)^2 \left( \frac{F_0^{BD}(m_{D_s}^2)}{F_0^{BD}(m_s^2)} \right)^2,
\]

\[
\mathcal{R}_{D_s^*/\rho} = \frac{\bar{a}(DD_s^*)}{\bar{a}(D\rho)} \left( \frac{f_{D_s^*}}{f_\rho} \right)^2 \left( \frac{p_{c}^{DD_s^*}}{p_{c}^{D\rho}} \right)^3 \left( \frac{F_1^{BD}(m_{D_s^*}^2)}{F_1^{BD}(m_{\rho}^2)} \right)^2,
\]

\[
\mathcal{R}_{D_s/\pi} = \frac{\bar{a}(DD_s)}{\bar{a}(D\pi)} \left( \frac{f_{D_s}}{f_\pi} \right)^2 \left( \frac{p_{c}^{DD_s}}{p_{c}^{D\pi}} \right)^2 \left( \frac{A_0^{BD}(m_{D_s}^2)}{A_0^{BD}(m_{\pi}^2)} \right)^2,
\]

\[
\mathcal{R}_{D_s^*/\rho} = \frac{\bar{a}(DD_s^*)}{\bar{a}(D\rho)} \left( \frac{f_{D_s^*}}{f_\rho} \right)^2 \left( \frac{p_{c}^{DD_s^*}}{p_{c}^{D\rho}} \right)^3 \left( \frac{A_1^{BD}(m_{D_s^*}^2)}{A_1^{BD}(m_{\rho}^2)} \right)^2 \left( \frac{H(m_{D_s^*}^2)}{H(m_{\rho}^2)} \right),
\]

where \( p_{c}^{XY} \) is the c.m. momentum of the decay particles and we used \(|V_{cs}/V_{ud}| = 1\). Here the form factors have the following parameterization [3]:

\[
\langle P'(p')|V_\mu|P(p) \rangle = \left( p_\mu + p'_\mu - \frac{m_{\mu}^2 - m_{\mu'}^2}{q^2} q_\mu \right) F_1(q^2) + \frac{m_{\mu}^2 - m_{\mu'}^2}{q^2} q_\mu F_0(q^2),
\]
\[
\langle V(p', \epsilon) | V_{\mu} | P(p) \rangle = \frac{2}{m_P + m_V} \epsilon_{\mu\alpha\beta\gamma} \epsilon^{\nu\rho} p^\alpha p'^\beta V(q^2),
\]
\[
\langle V(p', \epsilon) | A_{\mu} | P(p) \rangle = i \left[ (m_P + m_V) \epsilon_{\mu} A_1(q^2) - \frac{\epsilon \cdot p}{m_P + m_V} (p + p')_{\mu} A_2(q^2) - 2m_V \frac{\epsilon \cdot p}{q^2} q_{\mu} [A_3(q^2) - A_0(q^2)] \right],
\]
(10)
where \( q = p - p' \), \( F_1(0) = F_0(0), A_3(0) = A_0(0) \),
\[
A_3(q^2) = \frac{m_P + m_V}{2m_V} A_1(q^2) - \frac{m_P - m_V}{2m_V} A_2(q^2),
\]
and \( P, V \) denote the pseudoscalar and vector mesons, respectively. For \( B \to V_1 V_2 \) decay (see Eq. (9)), three form factors \( A_1(q^2), A_2(q^2), \) and \( V(q^2) \) contribute. Here we factored out the dominant one \( A_1(q^2) \) and the other two are put in the function \( H(q^2) \) defined as
\[
H(q^2) = (a - bx)^2 + 2(1 + c^2y^2),
\]
(11)
with
\[
a = \frac{m_B^2 - m_1^2 - m_2^2}{2m_1m_2}, \quad b = \frac{2m_Bp_c^2}{m_1m_2(m_B + m_1)^2}, \quad c = \frac{2m_Bp_c}{(m_B + m_1)^2},
\]
(12)
\[
x = \frac{A_2^{BV_1}(q^2)}{A_1^{BV_1}(q^2)}, \quad y = \frac{V^{BV_1}(q^2)}{A_1^{BV_1}(q^2)},
\]
(13)
where \( m_1 (m_2) \) is the mass of the vector meson \( V_1 (V_2) \). Using the above ratios Eqs.(9)-(10), one can, in principle, test the validity of factorization without having dependence on CKM matrix elements. However, the analysis depends strongly on nonperturbative hadronic factors such as decay constants and form factors. \( B \to D^{(*)} \) transition form factors are rather well-constrained and the uncertainty in their ratios would be rather moderate. In the following numerical analysis, we consider three models for the form factors of \( B \to D^{(*)} \) transitions: the Bauer-Stech-Wirbel (BSW) model [3], Melikhov/Stech [12], and relativistic light-front (LF) quark model [13].

Another uncertainty comes from decay constants, especially \( f_{D_s^{(*)}} \), which has presently large uncertainty. Particle Data Group report [14] gives two distinct values depending on its decay modes:
\[ f_{D_s^+} = 194 \pm 35 \pm 20 \pm 14 \text{ MeV} \quad \text{from } D_s \to \mu \nu_\mu, \]  
(14)

\[ f_{D_s^+} = 309 \pm 58 \pm 33 \pm 38 \text{ MeV} \quad \text{from } D_s \to \tau \nu_\tau. \]  
(15)

Recently, a rather interesting value appeared in Ref. [15]:

\[ f_{D_s^+} = 323 \pm 44 \pm 12 \pm 34 \text{ MeV} \quad \text{from } D_s \to \mu \nu_\mu, \]  
(16)

which is obtained by measuring the branching fraction of \( D_s \to \mu \nu_\mu \) relative to the branching fraction \( D_s \to \varphi \pi \to K^+K^-\pi \). We use the statistical average of the above three,

\[ f_{D_s} = 252 \pm 31 \text{ MeV}. \]  
(17)

Then we get the theoretical predictions

\[ R_{D_s/\pi} = [3.38 \pm 0.841] \left( \frac{\tilde{a}(DD_s)}{\tilde{a}(D\pi)} \right)^2 \left( \frac{f_{D_s}}{0.252} \right)^2 \left( \frac{F_0^{BP}(m^2_{D_s})}{0.74} \right)^2 \left( \frac{A_0^{BP}(m^2_\pi)}{0.686} \right)^2, \]

\[ R_{D_*/\rho} = [0.92 \pm 0.236] \left( \frac{\tilde{a}(DD_{s*})}{\tilde{a}(D\rho)} \right)^2 \left( \frac{f_{D_*}}{0.252} \right)^2 \left( \frac{F_1^{BP}(m^2_{D_*})}{0.817} \right)^2 \left( \frac{A_1^{BP}(m^2_\rho)}{0.701} \right)^2, \]

\[ \tilde{R}_{D_s/\pi} = [2.17 \pm 0.654] \left( \frac{\tilde{a}(DD_{s*})}{\tilde{a}(D^*\pi)} \right)^2 \left( \frac{f_{D_s}}{0.252} \right)^2 \left( \frac{A_0^{BP}(m^2_{D_s})}{0.793} \right)^2 \left( \frac{A_0^{BP}(m^2_\pi)}{0.699} \right)^2, \]

\[ \tilde{R}_{D_*/\rho} = [2.15 \pm 0.545] \left( \frac{\tilde{a}(DD_{s*})}{\tilde{a}(D^*\rho)} \right)^2 \left( \frac{f_{D_*}}{0.252} \right)^2 \left( \frac{A_1^{BP}(m^2_{D_*})}{0.730} \right)^2 \left( \frac{A_1^{BP}(m^2_\rho)}{0.673} \right)^2, \]  
(18)

where the quoted errors are based on our estimates of uncertainties in the form-factor model-dependence and in the decay constants \( f_{D_{s(*)}} \). Here we assumed \( f_{D_*} = f_{D_s} \) for simplicity and used \( f_\pi = 131 \text{ MeV} \) and \( f_\rho = 209 \text{ MeV} \). As the ratios of \( \tilde{a} \)'s are factored out, the numerical predictions of Eqs. (18) correspond to those in the naive factorization approximation. As is shown, the main uncertainty comes from our ignorance on the decay constant \( f_{D_{s(*)}} \). Within the generalized factorization scheme and by including penguin effects, the central values of the ratios are shifted to

\[ R_{D_s/\pi}^{GF} = 2.43 \pm 0.61, \]

\[ R_{D_*/\rho}^{GF} = 0.85 \pm 0.22, \]

\[ \tilde{R}_{D_s/\pi}^{GF} = 2.33 \pm 0.70, \]

\[ \tilde{R}_{D_*/\rho}^{GF} = 2.00 \pm 0.50, \]  
(19)
where we used the explicit numerical values for $\tilde{a}$ of Eq. (3). Considering the current experimental branching ratios for each decay mode [10,14], one gets the following ratios:

\[
\begin{align*}
\mathcal{R}^{\text{exp}}_{D_s/\pi} &= 2.67 \pm 1.061, \\
\mathcal{R}^{\text{exp}}_{D^*_s/\rho} &= 1.27 \pm 0.671, \\
\tilde{\mathcal{R}}^{\text{exp}}_{D_s/\pi} &= 3.58 \pm 1.138, \\
\tilde{\mathcal{R}}^{\text{exp}}_{D^*_s/\rho} &= 2.16 \pm 0.817.
\end{align*}
\]

(20)

Comparing the ratios (18), (19) and (20), all the theoretical predictions are well within the present experimental constraints. We note that the inclusion of the penquin effects for $B_0 \to D_s^- D^+$, which add a sizable contribution, improves the central value so that it is much closer to the experimental value. Although presently the experimental errors are too large to say anything definite, our analysis indicates that factorization hypothesis is still a good method for describing the $B$-meson decaying to heavy-heavy mesons. Furthermore, one could even consider a possibility that the factorization may not be a consequence of only perturbative QCD, in contrast to the arguments of Ref. [7]. Similar arguments are given in Ref. [16], in which the authors considered $B \to D^{(*)}X$ decays and expected non-factorization effects would grow with the invariant mass $m_X^2$ of the multi hadronic state $X$ if the factorization is a consequence of perturbative QCD, but they found no such dependence on $m_X^2$.

### III. DISCUSSIONS ON EXPERIMENTAL FEASIBILITY AND SUMMARY

The comparison of (19) and (20) will give a test of generalized factorization model that we considered in this paper. As it stands now, the two sets of values are consistent well within uncertainties. On the theoretical side, the biggest uncertainty is in the determination of meson decay constants $f_{D_s^{(*)}}$, while on the experimental side the statistical errors of $\mathcal{B}(B \to D^{(*)}D_s^{(*)})$ give the largest uncertainty. Therefore, we need to improve the precision of such experimental measurements, for the method described in this paper to have any significance.
As of this writing, the combined data sample of BABAR and Belle experiments is more than 30 fb$^{-1}$ [7]. The peak instantaneous luminosity of each experiment is over $3 \times 10^{33}$ cm$^{-2}$s$^{-1}$, which corresponds to more than 1 fb$^{-1}$/week for each. At this rate, we expect to have more than 100 fb$^{-1}$ of data accumulated within a year. We will take this as our basis for considering experimental feasibility.

Currently, the most precise measurement of $f_{D_s}$ is obtained by the CLEO collaboration [18] in $D_s \rightarrow \mu \nu$ decays. Adding the errors in quadrature, they obtained $f_{D_s} = 280 \pm 48; 17\%$ total uncertainty (7\% statistical) in $f_{D_s}$, corresponding to 34\% error in our calculation of the ratios. With 100 fb$^{-1}$ data sample from BABAR and Belle, which is more than 20 times that of the existing result [18], the statistical error will be reduced to $1/\sqrt{20}$ of Ref. [18]. The systematic errors may not go down as fast, but better understanding of every other aspect of the analysis will help reduce the systematic uncertainties. Assuming that the systematic error can be reduced to $1/3$ of Ref. [18], the $f_{D_s}$ value will be determined to 5\% accuracy, hence resulting in 10\% error in our ratio. As for the form factor errors in the theoretical calculations, we hope that in the near future precision measurements in heavy-flavor physics processes from B or charm factories should help test and confirm the reliability of lattice QCD technique greatly. Then we may have much improved form factor errors.

In the experimental measurements of branching ratios, $B \rightarrow D^{(*)}\pi$ modes are measured with much better precision than $B \rightarrow D^{(*)}\rho$ modes. Similarly, $D^{(*)}D_s$ modes are determined with significantly higher precision than $D^{(*)}D_s^*$ modes. Therefore, we expect that $R_{D_s/\pi}$ and $\bar{R}_{D_s/\pi}$ will be determined with higher precision than other ratios. The most recent and precise measurement of $\mathcal{B}(B \rightarrow D^{*+}\pi^-) = (2.81 \pm 0.24 \pm 0.05) \times 10^{-3}$ is obtained by the CLEO collaboration with 3.1 fb$^{-1}$ of data sample. With a data sample of 100 fb$^{-1}$, we can measure this branching ratio with a precision at a few\% level. On the other hand, the most precise measurement of $\mathcal{B}(B \rightarrow D^{*+}D_s^-)$ also made by CLEO with 2.0 fb$^{-1}$ of data, is $(0.90 \pm 0.27 \pm 0.22) \times 10^{-2}$. Again, with 100 fb$^{-1}$ of data, the statistical error will be reduced to $\sim 1/7$ of its current value. If experimental systematic error can be reduced to $\sim 40\%$
of its current value, $\mathcal{B}(B \to D^{+} D_{s}^{-})$ will be determined to 10 % level. Therefore, it is likely that the experimental value of $\tilde{R}_{D_{s}/\pi}$ can be determined at the level of 10 % precision. Similar case can be made with $R_{D_{s}/\pi}$.

Comparing (19) and (20), we note that the experimental and theoretical values of $\tilde{R}_{D_{s}/\pi}$ show the biggest difference if we accept their central values. We also note that if both values can be determined within 10 % accuracy and if we assume that their central values stand as they are, then we will be able to see $3\sigma$ difference in $\tilde{R}_{D_{s}/\pi}$. In conclusion, we will have a good opportunity to test the generalized factorization scheme as discussed in this paper once we have $\sim 100$ fb$^{-1}$ of data from the $B$-factory experiments.

To summarize, we have investigated the possibility of testing factorization hypothesis from non-leptonic exclusive decays of $B$ meson into two meson final states. In particular, we considered the presumably non-factorizable $\mathcal{B}^{0} \to D^{(*)+} D_{s}^{(*)-}$ modes and $\mathcal{B}^{0} \to D^{(*)+}(\pi^{-}, \rho^{-})$ known as well-factorizable modes. By taking the ratios $\mathcal{B}(\mathcal{B}^{0} \to D^{(*)+} D_{s}^{(*)-})/\mathcal{B}(\mathcal{B}^{0} \to D^{(*)+}(\pi^{-}, \rho^{-}))$, the dependence on CKM matrix elements vanishes and some model-dependence on hadronic form-factors is reduced. We found that under the present theoretical and experimental uncertainties there’s no evidence for breakdown of factorization description to heavy-heavy decays of the $B$-meson.

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