Analysis of $B \to \phi K$ Decays in QCD Factorization

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Abstract

We analyze the decay $B \to \phi K$ within the framework of QCD-improved factorization. We found that although the twist-3 kaon distribution amplitude dominates the spectator interactions, it will suppress the decay rates slightly. The weak annihilation diagrams induced by $(S - P)(S + P)$ penguin operators, which are formally power-suppressed by order $(\Lambda_{QCD}/m_b)^2$, are chirally and logarithmically enhanced. Therefore, these annihilation contributions are not subject to helicity suppression and can be sizable. The predicted branching ratio of $B^- \to \phi K^-$ is $(3.8 \pm 0.6) \times 10^{-6}$ in the absence of annihilation contributions and it becomes $(4.3^{+3.0}_{-1.4}) \times 10^{-6}$ when annihilation effects are taken into account. The prediction is consistent with CLEO and BaBar data but smaller than the BELLE result.
I. INTRODUCTION

Previously CLEO has put an upper limit on the decay mode $B \to \phi K$:

$$B(B^\pm \to \phi K^\pm) < 5.9 \times 10^{-6}. \quad (1.1)$$

However, both CLEO [2], BELLE [3] and BaBar [4] have recently reported the results:

$$B(B^\pm \to \phi K^\pm) = \begin{cases} 
(5.5^{+2.1}_{-1.8} \pm 0.6) \times 10^{-6} \quad \text{CLEO}, \\
(7.7^{+1.0}_{-1.0} \pm 0.8) \times 10^{-6} \quad \text{BaBar}, \\
(13.9^{+3.7+1.4}_{-3.3-2.4}) \times 10^{-6} \quad \text{BELLE}, 
\end{cases} \quad (1.2)$$

and

$$B(B^0 \to \phi K^0) = \begin{cases} 
(5.4^{+3.7}_{-2.7} \pm 0.7) \times 10^{-6} < 12.3 \times 10^{-6} \quad \text{CLEO}, \\
(8.1^{+3.1}_{-2.5} \pm 0.8) \times 10^{-6} \quad \text{BaBar}, \\
< 16.0 \times 10^{-6} \quad \text{BELLE}. 
\end{cases} \quad (1.3)$$

It is known that the neutral mode $B^0 \to \phi K^0$ is a pure penguin process, while the charged mode $\phi K^-$ receives an additional (though very small) contribution from the tree diagram. The predicted branching ratio is very sensitive to the nonfactorizable effects which are sometimes parameterized in terms of the effective number of colors $N_{\text{eff}}^c$; it falls into a broad range $(13 \sim 0.4) \times 10^{-6}$ for $N_{\text{eff}}^c = 2 \sim \infty$ [5]. Therefore, a theory calculation of the nonfactorizable corrections is urgently needed in order to have a reliable prediction which can be used to compare with experiment.

A calculation of $B \to \phi K$ within the framework of QCD-improved factorization has been carried out recently in [6]. However, the analysis of [6] is limited to the leading order in $1/m_b$ and hence the potentially important annihilation contributions which are power-suppressed in the heavy quark limit are not included.

In the present paper we will analyze the decay $B \to \phi K$ within the framework of QCD-improved factorization. We will study the important twist-3 effects on spectator interactions and also focus on the annihilation diagrams which are customarily assumed to be negligible based on the helicity suppression argument. However, weak annihilations induced by the $(S - P)(S + P)$ penguin operators are no longer subject to helicity suppression and hence can be sizable. This is indeed what we found in this work.

II. GENERALIZED FACTORIZATION

The effective Hamiltonian relevant for $B \to \phi K$ has the form

$$\mathcal{H}_{\text{eff}}(\Delta B = 1) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{us}^* \left[ c_1(\mu)O_1(\mu) + c_2(\mu)O_2(\mu) \right] \\
- V_{tb}V_{ts}^* \left( \sum_{i=3}^{10} c_i(\mu)O_i(\mu) + c_9(\mu)O_9(\mu) \right) \right\} + \text{h.c.}, \quad (2.1)$$
where

\[
O_1 = \langle \bar{u}b \rangle_{V-A}(\bar{s}u)_{V-A}, \quad O_2 = \langle \bar{u}b \beta \rangle_{V-A}(\bar{s}u\alpha)_{V-A}, \\
O_{3(5)} = \langle \bar{s}b \rangle_{V-A} \sum_{q'}(\bar{q}'q')_{V-A(V+A)}, \quad O_{4(6)} = \langle \bar{s}a \beta \rangle_{V-A} \sum_{q'}(\bar{q}'q')_{V-A(V+A)},
\]

\[
O_{7(9)} = \frac{3}{2}\langle \bar{s}b \rangle_{V-A} \sum_{q'}e_q'(\bar{q}'q')_{V-A(V-A)}, \quad O_{8(10)} = \frac{3}{2}\langle \bar{s}a \beta \rangle_{V-A} \sum_{q'}e_q'(\bar{q}'q')_{V-A(V-A)},
\]

\[
O_g = \frac{g_s}{8\pi^2} m_b \bar{s}\sigma^{\mu\nu} C_{\mu\nu}^a \frac{\chi^a}{2} (1 + \gamma_5)b,
\]

with \((\bar{q}_1 q_2)_{V\pm A} \equiv \bar{q}_1 \gamma_\mu (1 \pm \gamma_5) q_2\), \(O_3-O_6\) being the QCD penguin operators, \(O_7-O_{10}\) the electroweak penguin operators, and \(O_g\) the chromomagnetic dipole operator.

In the generalized factorization approach for hadronic weak decays, the decay amplitudes of \(B \to \phi K\) read (in units of \(G_F/\sqrt{2}\)) \[4, 8\]

\[
A(B^- \to K^- \phi) = -V_{tb}V_{ts}^* \left\{ a_3 + a_4 + a_5 - \frac{1}{2}(a_7 + a_9 + a_{10}) \right\} X^{(B^-K^-,\phi)}
\]

\[
+ \left[ a_4 + a_{10} - 2(a_6 + a_8) \frac{m_{B^-}}{(m_s + m_u)(m_b + m_u)} \right] X^{(B^-\phi K^-)}
\]

\[
+ V_{ub}V_{us}^* a_1 X^{(B^-\phi K^-)},
\]

\[
A(B^0 \to K^0 \phi) = -V_{tb}V_{ts}^* \left\{ a_3 + a_4 + a_5 - \frac{1}{2}(a_7 + a_9 + a_{10}) \right\} X^{(B^0K^0,\phi)}
\]

\[
+ \left[ a_4 - \frac{1}{2} a_{10} - 2(a_6 - a_8) \frac{m_{B^0}}{(m_s + m_d)(m_b + m_d)} \right] X^{(B^0\phi K^0)},
\]

where the factorized terms

\[
X^{(BK,\phi)} \equiv \langle \phi | \langle \bar{s}s \rangle_{V-A} | 0 \rangle \langle K | \langle \bar{q}b \rangle_{V-A} | B \rangle = 2f_{\phi} m_{\phi} F_{1BK}(m_{\phi}^2)(\varepsilon^* \cdot p_B),
\]

\[
X^{(B,K\phi)} \equiv \langle \phi K | \langle \bar{s}q \rangle_{V-A} | 0 \rangle \langle \bar{q}b \rangle_{V-A} | B \rangle = 2f_B m_{\phi} A_{0BK}^\phi (m_B^2)(\varepsilon^* \cdot p_B),
\]

(2.4)

can be expressed in terms of the form factors \(F_{1BK}, A_{0BK}^\phi\) (for the definition of form factors, see [4]) and the decay constants \(f_{\phi}\) and \(f_B\), and the nonfactorized contributions parametrized in terms of \(\chi_i\) are lumped into the effective number of colors \(N_{\text{eff}}\):

\[
\left( \frac{1}{N_{\text{eff}}} \right)_i \equiv \frac{1}{N_c} + \chi_i,
\]

(2.5)

so that the effective parameters \(a_i\) appearing in Eq. (2.3) read

\[
a_{2i} = c_{2i} + \frac{1}{(N_{\text{eff}})_i} c_{2i-1}, \quad a_{2i-1} = c_{2i-1} + \frac{1}{(N_{\text{eff}})_i} c_{2i}.
\]

(2.6)

It is known that the parameters \(a_3\) and \(a_5\) depend strongly on \(N_{\text{eff}}\), while \(a_4\) is \(N_{\text{eff}}\)-stable (see, for example, [4, 8]). Therefore, the prediction of \(B \to \phi K\) rates is sensitive to \(N_{\text{eff}}\) and
hence to the nonfactorizable terms $\chi_i$; it varies from $13 \times 10^{-6}$ to $0.4 \times 10^{-6}$ for $N_c^{\text{eff}}$ ranging from 2 to $\infty$ [3].

Owing to the unknown form factor $A_0^{\phi K}(m_B^2)$ at large $q^2$, it is conventional to neglect the annihilation contribution based on the argument of helicity suppression, which amounts to having a vanishing form factor $A_0^{\phi K}(m_B^2)$. However, this argument is valid only for $(V - A)(V - A)$ interactions but not for $(S - P)(S + P)$ ones. This explains the large enhancement factor of $m_B^2/(m_b m_s)$ for the penguin contributions [see Eq. (2.3)]. Therefore, it is conceivable that the annihilation contribution could be sizable and significant.

III. NONFACTORIZABLE EFFECTS IN PENGUIN AMPLITUDES

We next proceed to compute the nonfactorizable effects in the QCD-improved factorization approach. For simplicity we will neglect the light quark masses. In the chiral limit, kaon is massless, but the $\phi$ meson has a finite mass. We consider the vertex corrections and hard spectator interactions depicted in Fig. 1 as well as the annihilation diagrams shown in Fig. 2. Recently we have analyzed $B \to J/\psi K$ decays within the framework of QCD factorization [10]. The study of $B \to \phi K$ is quite similar to the $J/\psi K$ mode except for the absence of weak annihilations in the latter. The reader is referred to [10] for details. The resultant amplitudes are

$$A(B^- \to K^- \phi) = -V_{tb}V_{ts}^* \left\{ a_3 + a_4 + a_5 - \frac{1}{2} (a_7 + a_9 + a_{10}) \right\} X^{(B^- K^-, \phi)}$$

$$+ \left\{ (c_3 + c_9) A_{n_1} + (c_5 + c_7) A_{n_2}^2 + (c_6 + c_8 + \frac{1}{3} (c_5 + c_7)) A_f \right\} + V_{ub}V_{us}^* c_2 A_{n_f}^1;$$

$$A(B^0 \to K^0 \phi) = -V_{tb}V_{ts}^* \left\{ a_3 + a_4 + a_5 - \frac{1}{2} (a_7 + a_9 + a_{10}) \right\} X^{(B^0 K^0, \phi)}$$

$$+ \left\{ (c_3 - \frac{1}{2} c_9) A_{n_1}^1 + (c_5 - \frac{1}{2} c_7) A_{n_2}^2 + (c_6 + \frac{1}{3} c_5 - \frac{1}{2} c_8 - \frac{1}{6} c_7) A_f \right\},$$

where

$$a_3 = c_3 + \frac{c_4}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_4 \left[ - \left( \frac{18}{14} \right) - 12 \ln \frac{\mu}{m_b} + f_I + f_{II} \right];$$

$$a_4 = c_4 + \frac{c_3}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} \left\{ c_3 \left[ - \left( \frac{18}{14} \right) - 12 \ln \frac{\mu}{m_b} + f_I + f_{II} \right] + \left( c_3 - \frac{c_9}{2} \right) G(s_u) + G(s_d) \right\}$$

$$- c_1 \left( \frac{\lambda_u}{\lambda_t} G(s_u) + \frac{\lambda_c}{\lambda_t} G(s_c) \right) + \sum_{q=u,d,s,c,b} (c_4 + c_6 + \frac{3}{2} c_q (c_8 + c_{10})) G(s_q) + c_3 G_g \right\};$$

$$a_5 = c_5 + \frac{c_6}{N_c} - \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_6 \left[ - \left( \frac{6}{18} \right) - 12 \ln \frac{\mu}{m_b} + f_I + f_{II} \right];$$

$$a_7 = c_7 + \frac{c_8}{N_c} - \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_8 \left[ - \left( \frac{6}{18} \right) - 12 \ln \frac{\mu}{m_b} + f_I + f_{II} \right] - \frac{\alpha}{9\pi} N_c C_e,$$
FIG. 1. Vertex and spectator corrections to \( B \rightarrow \phi K \).

\[
\begin{align*}
\alpha_9 &= c_9 + \frac{c_{10}}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_{10} \left[ -\left( \frac{18}{14} \right) - 12 \ln \frac{\mu}{m_b} + f_I + f_{II} \right] - \frac{\alpha}{9\pi} N_c C_e, \\
\alpha_{10} &= c_{10} + \frac{c_9}{N_c} + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} c_9 \left[ -\left( \frac{18}{14} \right) - 12 \ln \frac{\mu}{m_b} + f_I + f_{II} \right] - \frac{\alpha}{9\pi} C_e.
\end{align*}
\]

In Eq. (3.2), the upper entry of the matrix is evaluated in the naive dimensional regularization (NDR) scheme for \( \gamma_5 \) and the lower entry in the 't Hooft-Veltman (HV) renormalization scheme, \( C_F = (N_c^2 - 1)/(2N_c) \), \( s_q = m_q^2/m_b^2 \), \( \lambda q' = V_{q'b}V'^*_{qs} \), and \( \alpha \) the electromagnetic fine-structure coupling constant. The other terms in (3.2) are

\[
\begin{align*}
G(s) &= \frac{2}{3} - \frac{4}{3} \ln \frac{\mu}{m_b} + 4 \int_0^1 d\xi \Phi^\phi(\xi) \int_0^1 du u(1-u) \ln[s-u(1-u)(1-\xi)]], \\
G_g &= - \int_0^1 d\xi \Phi^\phi(\xi) \frac{2}{1-\xi}, \\
C_e &= \left( \frac{\lambda_u}{\lambda_t} G(s_u) + \frac{\lambda_c}{\lambda_t} G(s_c) \right) (c_2 + \frac{c_1}{N_c}),
\end{align*}
\]

where \( \Phi^\phi(\xi) \) is the light-cone distribution amplitude (LCDA) of the \( \phi \) meson, which will be discussed shortly. \( B \rightarrow \phi K \) do not receive factorizable contributions from \( \alpha_6 \) and \( \alpha_8 \) except for the annihilation topologies. The nonfactorizable annihilation contributions \( A_{nf}^{1,2} \) and the factorizable annihilation amplitude \( A_f \) will also be elucidated on below.

Note that the effective parameters \( \alpha_i \) appearing in Eqs. (3.2) are renormalization scale and \( \gamma_5 \)-scheme independent. Since only one gluon exchange has been considered in the
annihilation diagrams (see Fig. 2), one may wonder the scale and scheme dependence of the annihilation amplitude given in Eq. (3.1). Fortunately, as we shall see below, the annihilation contribution is predominated by the penguin effects characterized by the parameters \( a_6 = c_6 + \frac{1}{3}c_5 \) and \( a_8 = c_8 + \frac{1}{3}c_7 \) multiplied by \( \mu_\chi \). It turns out that the scale dependence of \( a_6 \) and \( a_8 \) is canceled by the corresponding dependence in \( \mu_\chi \) owing to the running quark masses. Consequently, the annihilation amplitude is essentially scale independent.

The hard scattering kernel \( f_I \) appearing in Eq. (3.2) reads

\[
f_I = \int_0^1 d\xi \Phi^\phi(\xi) \left\{ \frac{3(1-2\xi)}{1-\xi} \ln \xi - 3i\pi + \frac{2z(1-\xi)}{1-z\xi} + 3 \ln(1-z) \\
+ \left( \frac{1-\xi}{(1-z\xi)^2} - \frac{\xi}{1-z(1-x)^2} \right) z^2 \xi \ln z \xi + \frac{z^2 \xi^2 [\ln(1-z) - i\pi]}{[1-z(1-x)]^2} \right\},
\]

where \( z \equiv m_\phi^2/m_B^2 \). For completeness, we have included the \( \phi \) mass corrections to \( f_I \) though such corrections are very small. In the \( m_\phi \to 0 \) limit, \( f_I \) has the same expression as that in \( B \to \pi \pi \) decay \([11]\), as it should be. The hard scattering kernel \( f_{III} \) arises from the hard spectator diagrams Figs. 1e and 1f and has the form \([10]\)

\[
f_{III} = \frac{\alpha_s(\mu_h) 4\pi^2}{\alpha_s(\mu) N_c} \frac{f_K f_B}{F_{1BK}(m_\phi^2) m_B^2} \frac{1}{1-z} \int_0^1 d\bar{\rho} \Phi^K_\rho(\bar{\rho}) \int_0^1 d\xi \frac{\Phi^\phi(\xi)}{\xi} \\
\times \int_0^1 d\bar{\eta} \left( \Phi^K_\eta(\bar{\eta}) + \frac{2\mu_\chi(\mu_h)}{m_B} \frac{1}{(1-z)^2} \frac{\Phi^K_\sigma(\bar{\eta})}{6\bar{\eta}} \right),
\]

where

\[
2\mu_\chi(\mu) = \frac{2m_\phi^2}{m_\phi(\mu) + m_u(\mu)} = \frac{-4\langle \bar{q}q \rangle}{f_K^2}
\]

(3.6)
is proportional to the quark condensate, the \( B \) meson wave function \( \Phi^K_\rho \) is defined by \([11]\)

\[
\langle 0| \bar{q}_\alpha(x) b_{\beta}(0) | B(p) \rangle \big|_{x_+ = x_-} = -\frac{i f_B}{4} [(\bar{p} + m_B) \gamma_5]_{\beta\gamma} \int_0^1 d\bar{\rho} e^{-i\bar{\rho}p_+ x^-} [\Phi^K_\rho(\bar{\rho}) + \Phi^K_\sigma(\bar{\rho})]_{\gamma\alpha}.
\]

(3.7)

with \( n_- = (1, 0, 0, -1) \), and \( \Phi^K_\sigma \) is a twist-3 kaon LCDA defined in the tensor matrix element \([13]\):

\[
\langle K^- (P) | \bar{s}(0) \gamma_\mu \gamma_5 u(x) | 0 \rangle = \frac{i f_K m_K^2}{6 m_u + m_s} \left[ 1 - \left( \frac{m_s + m_u}{m_K} \right)^2 \right] \\
\times (P_\mu x_\nu - P_\nu x_\mu) \int_0^1 d\bar{\eta} e^{i(\bar{P} \cdot x) \Phi^K_\sigma(\bar{\eta})}.
\]

(3.8)

\( ^* \)Eq. (3.4) can be obtained from Eq. (19) of \([12]\) or from Eqs. (2.22) and (2.23) of \([10]\) by neglecting the \( \xi^2 \) terms arising from the transverse wave function \( \Phi^\phi_\perp \) and applying the relation

\[
F_{0BK}(m_\phi^2)/F_{1BK}(m_\phi^2) = (m_B^2 - m_\phi^2)/m_B^2
\]

for form factors.
Since asymptotically $\Phi^K_\sigma(\eta) = 6\eta(1-\eta)$, the logarithmic divergence of the $\bar{\eta}$ integral in Eq. (3.5) implies that the spectator interaction is dominated by soft gluon exchanges between the spectator quark and the strange or anti-strange quark of $\phi$. Hence, QCD factorization breaks down at twist-3 order. Note that the hard gluon exchange in the spectator diagrams is not as hard as in the vertex diagrams. Since the virtual gluon’s momentum squared there is $k^2 = (-\bar{p}_B + \bar{\eta}p_K)^2 \approx -\bar{\eta}m^2_B \sim \mu_h m_b$, where $\mu_h$ is the hadronic scale $\sim 500 \text{ MeV}$, we will set $\alpha_s \approx \alpha_s(\sqrt{\mu_h m_b})$ in the spectator diagrams. For the second term in (3.5), due to the end point divergence, the scale may correspond to a softer scale $\mu_s$. However since $\alpha_s \mu$ is weakly scale-dependent, we can treat it at the $\sqrt{\mu_h m_b}$ scale. The corresponding Wilson coefficients in the spectator diagrams are also evaluated at the $\sqrt{\mu_h m_b}$ scale.

The infrared divergence is manifested in the integral $\int_0^1 d\bar{\eta}/\bar{\eta}$. However, it is known that the collinear expansion cannot be correct in the end point region owing to the transverse momentum $\langle k_T \rangle$ of the quark which is averagely about 300 MeV, the order of the meson’s size. Thus the lower limit of $\int_0^1 d\bar{\eta}/\bar{\eta}$ should be approximately proportional to $2\langle k_T \rangle/m_b$; or equivalently, $\int_0^1 d\bar{\eta}/\bar{\eta}$ can be approximately replaced by $\int_0^1 d\eta/(\eta + \langle 2k_T \rangle/m_b)$. A consistent treatment of $k_T$ in the calculation is still an issue since $k_T$ itself is a higher twist effect in the QCD factorization approach. Thus we will treat the divergent integral as an unknown “model” parameter and write

$$Y \equiv \int_0^1 \frac{d\bar{\eta}}{\bar{\eta}} = \ln \left( \frac{m_B}{\mu_h} \right) (1 + \rho_H),$$

with $\rho_H$ being a complex number whose phase may be caused by soft rescattering [14]. We see that although the scattering kernel induced by the twist-3 LCDA of the kaon is formally power suppressed in the heavy quark limit, it is chirally enhanced by a factor of $(2\mu/\Lambda_{QCD}) \sim \mathcal{O}(10)$, logarithmically enhanced by the infrared logarithms.

Finally, we wish to remark that the leading-twist LCDAs of the $\phi$ meson are given by [15]

$$\langle \phi(P, \lambda)|\bar{s}(x)\gamma_\mu s(0)|0\rangle = f_\phi m_\phi \varepsilon^\lambda*(x) \cdot P \cdot x P_\mu \int_0^1 d\xi e^{i\bar{x}_P \cdot x\Phi^\parallel(\xi)},$$

$$\langle \phi(P, \lambda)|\bar{s}(x)\sigma_{\mu\nu}s(0)|0\rangle = -if_\phi^T (\varepsilon^\lambda*(x) P_\nu - \varepsilon^\mu*(x) P_\mu) \int_0^1 d\xi e^{i\bar{x}_P \cdot x\Phi^\perp(\xi)},$$

(3.10)

where $\varepsilon^*$ is the polarization vector of $\phi$, $\xi$ is the light-cone momentum fraction of the strange quark in $\phi$, $f_\phi$ and $f_\phi^T$ are vector and tensor decay constants, respectively, but the latter is scale dependent. Although $\Phi^\parallel$ and $\Phi^\perp$ have the same asymptotic form, it is found that the transverse DA does not contribute to $f_I$ and $f_{II}$ if light quarks are massless. The contribution of $\Phi^\perp$ to vertex corrections is suppressed by a factor of $m_\phi/m_B$ and hence can be neglected.
IV. ANNIHILATION AMPLITUDES

As shown in [11], the annihilation amplitude is formally power suppressed by order $\Lambda_{\text{QCD}}/m_b$. Nevertheless, it has been stressed in the PQCD approach that annihilation contributions in hadronic charmless $B$ decays are not negligible [16]. There are four weak annihilation diagrams as depicted in Fig. 2. We first consider the annihilation amplitudes induced by $(V - A)(V - A)$ operators. The first two diagrams, Figs. 2a and 2b, are factorizable diagrams and their contributions are of order $m_{\phi}^2/m_B^2$ and hence can be neglected. Indeed, the factorizable annihilation amplitude should vanish in $m_{\phi} \to 0$ limit owing to current conservation. It is easily seen that only $O_{\text{odd}}$ operators contribute to the nonfactorizable annihilation diagrams Figs. 2c and 2d. It turns out that the nonfactorizable annihilations are dominated by Fig. 2d owing to an endpoint contribution. Explicit calculations yield [see Eq. (3.1)]:

$$A_{nf} = -2H \left\{ \int_0^1 d\bar{\rho} d\bar{\xi} d\eta \frac{\Phi^B(\bar{\rho})\Phi^\phi(\bar{\xi})\Phi^K(\eta)}{(\bar{\rho} - \xi)(1 - \eta)} + \int_0^1 d\rho d\bar{\xi} d\eta \frac{\Phi^B(\rho)\Phi^\phi(\bar{\xi})\Phi^K(\eta)}{(\rho - \xi)(\rho - \eta)} \right\},$$

$$\simeq 2H \left\{ 6(Y' - 1) \int_0^1 d\eta \frac{\Phi^K(\eta)}{\eta} - \int_0^1 d\xi d\eta \frac{\Phi^\phi(\bar{\xi})\Phi^K(\eta)}{\eta(\xi - \eta)} \right\},$$

(4.1)

where

$$H = \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} \frac{4\pi^2}{N_c} \frac{f_B f_K f_\phi m_\phi}{m_B^2} \langle \varepsilon^* \cdot p_B \rangle,$$

(4.2)

and we have applied the approximation $\rho \approx 1$ and $\bar{\rho} = 1 - \rho \approx 0$. In Eq. (4.1) the first term in brackets comes from Fig. 2d, while the second term from Fig. 2c. Since the soft phase of annihilation diagrams is not necessarily the same as that of the spectator diagram, we write

$$Y' \equiv \int_0^1 \frac{d\bar{\eta}}{\bar{\eta}} = \ln \left( \frac{m_B}{\mu_h} \right) (1 + \rho_A),$$

(4.3)

where the phase is characterized by the complex parameter $\rho_A$.

For $(V - A)(V + A)$ operators $O_5 - O_8$, the twist-2 kaon DA makes no contribution. Therefore, we need to consider the twist-3 kaon LCDAs $\Phi^K_\rho$ and $\Phi^K_\sigma$ with the former being defined in the pseudoscalar matrix element [13]:

$$\langle K^- (P) | \bar{s}(0) i \gamma_5 u(x) | 0 \rangle = \frac{f_K m_K^2}{m_s + m_u} \int_0^1 d\bar{\eta} e^{i P \cdot x} \Phi^K_\rho (\bar{\eta}).$$

(4.4)

The factorizable annihilation amplitude has the expression:

$$A_f = 4N_c H \left( \frac{2\mu_s}{m_B} \right) \int_0^1 d\xi \int_0^1 d\eta \Phi^\phi(\bar{\xi}) \left[ \Phi^K_{\rho}(\eta) \left( \frac{1}{\xi^2} - \frac{1}{2\xi} \right) \frac{1}{\eta} + \Phi^K_{\sigma}(\eta) \frac{1}{4\xi^2} \right],$$

$$= 24N_c H \left( \frac{2\mu_s}{m_B} \right) Y' (Y' - \frac{1}{2}),$$

(4.5)
where we have applied the LCDAs: \( \Phi^K(\bar{\xi}) = 6\bar{\xi}(1 - \bar{\xi}) \), \( \Phi^K_p(\eta) = 1 \) and \( \Phi^K_s(\eta) = 6\eta(1 - \eta) \). Likewise, the nonfactorizable annihilations induced by the penguin operators \( O_5 \) and \( O_7 \) read

\[
A_{nf}^2 = -H\left(\frac{2\mu_\chi}{m_B}\right) \left\{ 6Y'(Y' - 1) + \int_0^1 d\bar{\xi} \int_{\frac{\Lambda_{QCD}}{m_b}}^1 d\eta \Phi^K(\bar{\xi})\Phi^K_p(\eta) \frac{2 - \eta}{\xi\eta(\xi\eta - \xi - \eta)} \right\},
\]

where the dominated first term in brackets stems from Fig. 2d. Note that we have introduced a cutoff \( \Lambda_{QCD}/m_b \) to regulate the infrared divergence occurred in the second term.

Although the annihilation amplitudes \( A_f \) and \( A_{nf}^2 \) are formally of order \( (\Lambda_{QCD}/m_b)^2 \), they receive two large enhancements: one from the chiral enhancement \( (2\mu_\chi/\Lambda_{QCD}) \sim \mathcal{O}(10) \) and the other from the logarithmic endpoint divergence of the infrared divergent integral \( Y' \). Consequently, the annihilation effects can be sizable. Physically, this is because the penguin-induced annihilation contributions are not subject to helicity suppression.

V. RESULTS AND DISCUSSIONS

To proceed for numerical calculations, we employ the meson LCDAs as follows:

\[
\Phi^K(\bar{\eta}, \mu^2) = 6\bar{\eta}(1 - \bar{\eta}) \left( 1 + \sum_{n=1}^{\infty} a^K_{2n}(\mu^2) C^{3/2}_{2n}(2\bar{\eta} - 1) \right),
\]

\[
\Phi^K_p(\bar{\eta}) = N_B\bar{\rho}^2(1 - \bar{\rho})^2 \exp \left[ -\frac{1}{2} \left( \frac{\bar{\rho} m_B}{\omega_B} \right)^2 \right],
\]

\[
\Phi^K_s(\bar{\eta}) = \Phi^K(\bar{\xi}) = 6\bar{\xi}(1 - \bar{\xi}),
\]

where \( C^{3/2}_n \) are Gegenbauer polynomials and the values of the Gegenbauer moments \( a^K_n \) are available in [13], \( \omega_B = 0.25 \) GeV, and \( N_B \) is a normalization constant. We use the decay
constants $f_K = 0.16$ GeV, $f_B = 0.19$ GeV, $f_\phi = 0.237$ GeV, and the running quark masses: $m_b(m_b) = 4.40$ GeV, $m_s(m_b) = 90$ MeV $m_d(m_b) = 4.6$ MeV and $m_u(m_b) = 2.3$ MeV. The next-to-leading-order Wilson coefficients $c_i(\mu)$ in NDR and HQE $\gamma_5$-schemes are taken from Table XXII of [17]; they are evaluated at $\mu = \overline{m}_b(m_b) = 4.40$ GeV and $\Lambda_k^{(5)} = 225$ MeV. For form factors we use $F_1^{BK}(m_\phi^2) = 0.38$ as a benchmarked value. Note that $F_1^{BK}(m_\phi^2) = 0.407$ in the Bauer-Stech-Wirbel model [9], while it is 0.37 in a QCD sum rule calculation [18].

For the parameters $\rho_H$ in (3.9) and $\rho_A$ in (4.3), in principle they may be complex due to final-state soft rescattering. We find that the decay rate is much more sensitive to $\rho_A$ than to $\rho_H$. Presumably, some information on the parameter $\rho$ can be extracted from the study of $B \to K\pi$ modes. It has been shown recently in [14] that increasing the parameter $|\rho_A|$ from 1 to 2 would increase the corresponding error on the $K\pi$ branching ratios in which case it would require considerable fine-tuning of the strong interaction phase of $Y'$ in annihilation diagrams to reproduce the experimental value of the branching ratio. Hence, it is reasonable to assume that the model parameters are in the range $|\rho| \leq 1$. Writing $\rho_A = |\rho_A| \exp(i\delta)$, the branching ratio of $B \to \phi K$ vs. the phase $\delta$ is plotted in Fig. 3. We obtain

$$\mathcal{B}(B^- \to \phi K^-) = (4.3^{+3.0}_{-1.4}) \times 10^{-6}, \quad \mathcal{B}(B^0 \to \phi K^0) = (4.0^{+2.9}_{-1.4}) \times 10^{-6}, \quad (5.2)$$

where the central value corresponds to the default values $\rho_A = \rho_H = 0$ and the errors come from the variation of $|\rho_H|$ and $|\rho_A|$ from 0 to 1; that is, the theoretical uncertainties come from power corrections of twist-3 spectator interactions and annihilation contributions. Therefore, the predicted branching ratio is consistent with CLEO and BaBar numbers, but it is smaller than the BELLE result [see (1.2) and (1.3)]. The corresponding absolute ratio of the annihilation to penguin amplitudes depends on the annihilation phase and is at most of order 0.25. In the absence of annihilation effects, the branching ratios are given by

$$\mathcal{B}(B^\pm \to \phi K^\pm) = \left(\frac{F_1^{BK}(m_\phi^2)}{0.38}\right) (3.8 \pm 0.6) \times 10^{-6}, \quad (5.3)$$

where the error arises from the variation of $\rho_H$ from 0 to 1.

Needless to say, the major theoretical uncertainty stems from the unknown model parameters $\rho_H$ and $\rho_A$. It should be stressed that the infrared divergence here is always of the logarithmic type and other possible linear divergence occurred in annihilation diagrams with twist-3 wave functions are explicitly canceled out. As stressed in passing, the infrared divergence stems from the misuse of the collinear expansion in the end point region where the effect of the quark’s transverse momentum is important. Since $k_T$ is a higher-twist effect in QCD factorization, at present we treat the infrared divergent integral in a model manner.

Several remarks are in order. (i) The calculations are rather insensitive to the unitarity angle $\gamma$ as the CKM matrix element $V_{ub}V_{us}^*$ is considerably suppressed. (ii) The scattering
FIG. 3. Branching ratio of $B^- \rightarrow \phi K^-$ vs. the phase of the complex parameter $\rho_A$ [see Eq. (4.3)], where the dark (horizontal) and light bands correspond to $|\rho_A| = 0, 1$, respectively, with the variation of $|\rho_H|$ from 0 to 1.

kernel $f_{III}$ is dominated by the twist-3 effect. However, since the Wilson coefficients $c_{2i}$ and $c_{2i-1}$ have opposite signs, it turns out that the magnitudes of $a_{3-10}$ [see Eq. (3.2)] are slightly reduced by the twist-3 terms and therefore the branching ratio is suppressed by about 10% in the presence of twist-3 effects in spectator interactions. (iii) In the QCD factorization approach, the strong phase of the annihilation amplitude is of order $\alpha_s^2$ since it comes from the annihilation diagrams in which the gluon line is inset with an enclosed quark loop, resembling the vacuum-polarization bubble. Consequently, the phase of the annihilation contribution is likely dominated by the soft one induced by soft scattering as characterized by the parameter $\rho_A$. This is in contrast to the PQCD approach where the annihilation contributions have large strong phases [16].

VI. CONCLUSIONS

We have analyzed the decay $B \rightarrow \phi K$ within the framework of QCD-improved factorization and taken into account some of power-suppressed corrections. Our conclusions are:

1. Although the twist-3 kaon distribution amplitude dominates the spectator interactions, it will suppress the decay rates of $B \rightarrow \phi K$ slightly by about 10%. In the absence of annihilation contributions, the branching ratio is $(3.8 \pm 0.6) \times 10^{-6}$ for $\phi K^-$ and $(3.6 \pm 0.6) \times 10^{-6}$ for $\phi K^0$. 
2. The weak annihilation diagrams induced by \((S - P)(S + P)\) penguin operators, which are formally power-suppressed by order \(\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^2\), are chirally and logarithmically enhanced. Therefore, these annihilation contributions are not subject to helicity suppression and in principle can be sizable.

3. The branching ratio is predicted to be \((4.3^{+3.0}_{-2.1}) \times 10^{-6}\) for \(B^{-} \to \phi K^{-}\) and \((4.0^{+2.9}_{-1.4}) \times 10^{-6}\) for \(B^{0} \to \phi K^{0}\), where theoretical uncertainties come from power corrections of twist-3 spectator interactions and annihilation contributions. The corresponding absolute ratio of annihilation to penguin amplitudes depends on the annihilation phase and is at most 25%.

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