Noise and Bistabilities in Quantum Shuttles

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Abstract. We present a study of current fluctuations in two models proposed as quantum shuttles. Based on a numerical evaluation of the first three cumulants of the full counting statistics we have recently shown that a giant enhancement of the zero-frequency current noise in a single-dot quantum shuttle can be explained in terms of a bistable switching between two current channels. By applying the same method to a quantum shuttle consisting of a vibrating quantum dot array, we show that the same mechanism is responsible for a giant enhancement of the noise in this model, although arising from very different physics. The interpretation is further supported by a numerical evaluation of the finite-frequency noise. For both models we give numerical results for the effective switching rates.

Keywords: Current noise and fluctuations, bistabilities, quantum shuttles.

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Introduction – In 1998 Gorelik et al. proposed a nano-electromechanical system (NEMS), the charge shuttle, consisting of a movable nanoscopic grain coupled via tunnel barriers to source and drain electrodes [1]. Originally the motion of the grain was modelled using a classical harmonic oscillator. Here we present a study of current fluctuations in two models of (quantum) shuttles, where the oscillator is quantized.

Models – Two models have been proposed as quantum shuttles (the 1-dot shuttle [2] and the 3-dot shuttle [3]). The 1-dot shuttle consists of a single mechanically oscillating quantum dot situated between two leads. In the 3-dot shuttle the mechanically oscillating quantum dot is flanked by two static dots, thus making up an array of dots. Both devices are operated in the strong Coulomb blockade regime, and consequently only one excess electron at a time is allowed in the device. In the 1-dot (3-dot) model the coupling to the leads (the interdot coupling) depends exponentially on the position of the vibrating dot. For detailed descriptions of the models we refer to Refs. [2, 3, 4].

Both models are described using the language of quantum dissipative systems [5]. As the “system” we take in the 1-dot model (3-dot model) the single (three) electronic state(s) of the occupied dot (array) and the unoccupied state plus the quantum harmonic oscillator with natural frequency \( \omega_0 \). In the limit of a high bias between the leads [6], and assuming that the oscillator is damped due to a weak coupling to a heat bath, the time evolution of the reduced density matrix of the system \( \hat{\rho}(t) \) is governed by a Markovian generalized master equation (GME) of the form [2, 3, 4]

\[
\dot{\hat{\rho}}(t) = \mathcal{L} \hat{\rho}(t) = (\mathcal{L}_{\text{coh}} + \mathcal{L}_{\text{damp}} + \mathcal{L}_{\text{driv}}) \hat{\rho}(t).
\]  

(1)

Here \( \mathcal{L}_{\text{coh}} \) describes the internal coherent dynamics of the system, while \( \mathcal{L}_{\text{damp}} \) and
\( \mathcal{L}_{\text{drive}} \) give the damping and the coupling to the leads, respectively. In the following we consider the stationary state defined by \( \dot{\rho}^{\text{stat}}(t) = \mathcal{L} \dot{\rho}^{\text{stat}}(t) = 0 \). The GME is only valid in the high-bias limit, and hence we cannot use the applied bias as a control parameter. Instead, we vary in the 1-dot model the strength of the damping, denoted \( \gamma \), and in the 3-dot model the difference between the energy levels corresponding to the outer dots, referred to as the device bias and denoted \( \epsilon_b \).

**Theory** – We have recently developed a systematic theory for the calculation of the \( n \)'th cumulant of the current \( \langle \langle I^n \rangle \rangle \) for NEMS described by a Markovian GME of the form given in Eq. (1) [7]. In Ref. [7] a numerical evaluation of the first three cumulants showed that the 1-dot model in a certain parameter regime behaves as a bistable system switching slowly between two current channels. The first three cumulants of a bistable system switching slowly (compared to the electron transfer rates) with rates \( \Gamma_1 \leftarrow 2 \) and \( \Gamma_2 \leftarrow 1 \) between two current channels 1 and 2 with corresponding currents \( I_1 \) and \( I_2 \), respectively, are [8]

\[
\langle \langle I \rangle \rangle = \frac{I_1 \Gamma_1 \leftarrow 2 + I_2 \Gamma_2 \leftarrow 1}{\Gamma_2 \leftarrow 1 + \Gamma_1 \leftarrow 2},
\]

\[
\langle \langle I^2 \rangle \rangle = 2(I_1 - I_2)^2 \frac{\Gamma_1 \leftarrow 2 \Gamma_2 \leftarrow 1}{(\Gamma_1 \leftarrow 2 + \Gamma_2 \leftarrow 1)^3},
\]

\[
\langle \langle I^3 \rangle \rangle = 6(I_1 - I_2)^3 \frac{\Gamma_1 \leftarrow 2 \Gamma_2 \leftarrow 1(\Gamma_2 \leftarrow 1 - \Gamma_1 \leftarrow 2)}{(\Gamma_1 \leftarrow 2 + \Gamma_2 \leftarrow 1)^5}.
\]

As pointed out by Jordan and Sukhorukov [8, 9] these expressions are very general, i.e. they do not depend on the microscopic origin of the rates or the current channels. For the 1-dot model the two current channels were identified from phase space plots of the oscillating dot as a shuttling and a tunneling channel, respectively, with known analytic expressions for the corresponding two currents [7, 11]. By comparing the numerical results for the first two cumulants with the corresponding analytic expressions given above, the two rates \( \Gamma_1 \leftarrow 2 \) and \( \Gamma_2 \leftarrow 1 \) could be extracted, and finally a comparison of the numerical results for the third cumulant and the analytic expression given above (with the extracted rates\(^1 \) \( \Gamma_1 \leftarrow 2 \) and \( \Gamma_2 \leftarrow 1 \)) confirmed the conjecture about the bistable behavior (see Fig. 1). This in turn explained a giant enhancement of the zero-frequency current noise (the second cumulant) found in Ref. [11].

A similar enhancement of the zero-frequency current noise was found in a study of the 3-dot model [4]. Also in this case, the enhancement was tentatively attributed to a switching behavior, however, neither the number nor the nature of the individual current channels were clarified, and no quantitative explanation could be given. Phase space plots of the oscillating dot seem to indicate the existence of two current channels [4]: One channel, where electrons tunnel sequentially through the array of dots, and one channel, where electrons co-tunnel between the static dots. The current corresponding to each of the two channels can be read off from the numerical results obtained in Ref. [4]. By proceeding along the lines outlined above, the conjecture that the enhanced noise is due to a slow switching between the sequential and co-tunneling channel can be scrutinized.

\(^1 \) In a certain limit the rates may even be found analytically, see Ref. [10].
Results – In Figs. 1, 2 we show numerical results for the first three cumulants for the two models together with the analytic expression for the third cumulant of a bistable system with rates extracted from the first two cumulants. We take the agreement between the numerical and (semi-) analytic results as evidence that both models exhibit a bistable behavior. In Ref. [12] this interpretation was further supported by numerical studies of the finite-frequency current noise in the 1-dot model. Correspondingly, we show in Fig. 3 the agreement between the numerical results for the finite-frequency noise in the 3-dot model and semi-analytic results for a slow bistable switching process [12]. In Fig. 4 we show the extracted rates for both models. Most noteworthy is the crossing of the two rates in the 1-dot case, which results in the change of sign of the third cumulant seen in Fig. 1. On each side of the crossing one of the current channels dominates. In the 3-dot case, the two rates close in, however, without crossing each other. Consequently one of the current channels, the sequential tunneling channel, never dominates. It should also be noted that in both models one of the currents is comparable to one of the rates, which implies that some corrections to Eq. 2 are expected [9]. However, we have found that these corrections do not contribute significantly.

Conclusion – We have presented a study of noise in two models of quantum shuttles. By evaluating numerically the first three cumulants of the full counting statistics, we
FIGURE 3. Finite-frequency current noise $S(\omega)$ (normalized with respect to the current $I$) for the 3-dot model. Circles indicate numerical results, while full lines are the corresponding (semi-)analytic results for a slow bistable switching process [12]. The results correspond to Fig. 2 with $\epsilon_b = 2.6\hbar\omega_0$ (lower curve), $2.70\hbar\omega_0$, $2.79\hbar\omega_0$ (upper curve).

FIGURE 4. Left panel: The two switching rates for the 1-dot model as a function of the damping strength $\gamma$. Here the two current channels are the shuttling channel (1) and the tunneling channel (2). The rates correspond to the results shown in Fig. 1. Right panel: The two switching rates for the 3-dot model as a function of the device bias $\epsilon_b$. Here the two current channels are the sequential tunneling channel (1) and the co-tunneling channel (2). The rates correspond to the results shown in Fig. 2.

have shown that a giant enhancement of the zero-frequency current noise in both models can be explained in terms of a slow bistable switching behavior. For both models, this interpretation is supported further by a numerical evaluation of the finite-frequency current noise. We underline that although the two models behave very differently, it is the same mechanism that is responsible for the giant enhancement of the noise.

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