Dynamics of spin-2 fields in Kerr background geometries

W. KRIVAN, P. LAGUNA, P. PAPADOPOULOS
Dept. of Astronomy & Astrophysics and
Center for Gravitational Physics & Geometry
Penn State University, University Park, PA 16802

We have developed a numerical method for evolving perturbations of rotating black holes. Solutions are obtained by integrating the Teukolsky equation written as a first-order in time, coupled system of equations, in a form that explicitly exhibits the radial characteristic directions. We follow the propagation of generic initial data through the burst, quasi-normal ringing and power-law tail phases. Future results may help to clarify the role of black hole angular momentum on signals produced during the final stages of black hole coalescence.

At first instance, a direct derivation of the equations governing the perturbations of Kerr spacetimes is to consider perturbations of the metric. This path, however, leads to gauge-dependent formulations. A theoretically attractive alternative is to examine curvature perturbations. Using the Newman-Penrose formalism, Teukolsky derived a master equation governing not only gravitational perturbations (spin weight $s = \pm 2$) but scalar, two-component neutrino and electromagnetic fields as well. For the case $s = 0$, it yields the equation for a scalar wave propagating in a Kerr background, a system which we had studied previously.

To our knowledge, most of the work on the dynamics of perturbations of Kerr spacetimes has been performed under the assumption of a harmonic time dependence. Here we are interested in the time integration of physical initial data, possibly from the inspiral collision of binary black holes. Fourier transformation of the data and subsequent evolution of such data in the frequency domain approach is, in principle, possible but numerically cumbersome. The main complication one faces by keeping the equation in the time domain is that one cannot longer benefit from the reduction of dimensionality implied by the separation of variables. We have chosen the option to evolve one single 2+1 PDE instead of the equivalent approach of solving the set of ODEs corresponding to the Fourier spectrum. The computational burden of both approaches is likely similar. The resulting evolution equation is a hyperbolic, linear equation which is quite amenable to numerical treatment, provided suitable coordinates, variables and boundary conditions are chosen.

The two key factors in successfully solving the Teukolsky equations were: first, to carefully select the evolution field and its asymptotic behavior, and second to rewrite the Teukolsky equation in a form that explicitly exhibits the
radial characteristic directions. On the analytical level, one obtains bounded solutions for any direction of propagation by choosing \( s = -2 \) and rescaling by an appropriate function of \( r \). A convenient choice is simply \( r^3 \), a factor that is regular at the horizon. Regarding the choice of spatial coordinates, we use the Kerr tortoise coordinate \( r^* \) and the Kerr \( \tilde{\phi} \) coordinate instead of the Boyer-Lindquist coordinate \( \phi \). Then the ansatz for the solution to the Teukolsky equation is: 

\[
\Psi(t, r^*, \theta, \tilde{\phi}) \equiv e^{im\tilde{\phi}} r^3 \Phi(t, r^*, \theta).
\]

After a series of unsuccessful numerical experiments with this second-order in time and space equation for \( \Phi \), we found that numerical instabilities due to the first order in time derivatives in the Teukolsky equation were suppressed by introducing an auxiliary field \( \Pi \equiv \partial_t \Phi + b \partial_{r^*} \Phi \), that converts the Teukolsky equation to a coupled set of first-order equations in space and time, where \( b \equiv (r^2 + a^2)/\Sigma \) and \( \Sigma^2 = (r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta \). The resulting first order system is hyperbolic in the radial direction. Stable evolutions were achieved using a modified Lax-Wendroff method, discretizing the equation on a uniform two dimensional polar grid. Typically we used a computational domain of size \(-100 M \leq r_i^* \leq 500 M\) and \(0 \leq \theta_j \leq \pi\) with \(0 \leq i \leq 8000\) and \(0 \leq j \leq 32\). Details of the numerical scheme are described in \(3\). The stability of the code was verified with long-time evolutions, of the order of \(1000 M\). Its accuracy in turn was tested using standard convergence tests.

The evolution of generic initial data consists of three stages, as seen from an observer located away from the hole. During the first stage, the observed signal depends on the structure of the initial pulse and its reflection from the angular momentum barrier (burst phase). This phase is followed by an exponentially decaying quasinormal ringing of the black hole (quasinormal phase). In the last stage, the wave slowly dies off as a power-law tail (tail phase). The numerical algorithm described in the previous section is used to obtain the time evolution of generic perturbations impinging on the rotating black hole. The broad features of the evolution are demonstrated in Fig. 1, where quasinormal ringing and tail behavior are clearly manifested. We then performed a series of simulations with \( m = 0 \) for different values of \( a \). Focusing on the later part of the ringing sequence, which clearly depicts the least damped mode of oscillation, we read off the oscillation frequencies. A comparison of our values with those given by Kokkotas\(6\) and Leaver\(7\) shows an agreement of better than 1%.

We have shown\(2\) that the late time evolution of scalar fields in the background of rotating black holes is qualitatively similar to the non-rotating case. We extend here this result to the physically more interesting spin-2 field evolution. Our calculations show that the exponents governing the behavior for \( a \neq 0 \) do not exhibit a significant change when compared to the Schwarzschild
case, if the initial data pulse is not given by the lowest allowed mode for a particular value of $m$. That is, the power-law tail behavior is basically determined by the dominant asymptotic form of the potential. When the initial data pulse is not given by the lowest allowed mode for a particular value of $m$, mixing of modes occurs\[2\].

We thank H.-P. Nollert, R. Price and J. Pullin for helpful discussions. This work was supported by the Binary Black Hole Grand Challenge Alliance, NSF PHY/ASC 9318152 (ARPA supplemented) and by NSF grants PHY 96-01413, 93-57219 (NYI) to PL. WK was supported by the Deutscher Akademischer Austauschdienst (DAAD).

References

1. S. A. Teukolsky, *Phys. Rev. Lett.* **29**, 1114 (1972).
2. W. Krivan, P. Laguna, and P. Papadopoulos, *Phys. Rev. D* **54**, 4728 (1996).
3. W. Krivan, P. Laguna, and P. Papadopoulos, *Dynamics of perturbations of rotating black holes*, to be submitted to *Phys. Rev. D* (1997).
4. R.H. Price, *Phys. Rev. D* **5**, 2419 (1972).
5. C. Gundlach, R.H. Price, and J. Pullin, *Phys. Rev. D* **49**, 883 (1994).
6. K.D. Kokkotas, *Class. Quantum Grav.* **8**, 2217 (1991).
7. E.W. Leaver, *Proc. R. Soc. Lond. A* **402**, 285 (1985)