Gravastars and bifurcation in quasistationary accretion

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We investigate the newtonian stationary accretion of a polytropic perfect fluid onto a central body with a hard surface. The selfgravitation of the fluid and its interaction with luminosity is included in the model. We find that for a given luminosity, asymptotic mass and temperature of the fluid there exist two solutions with different cores.

Keywords: gravastars, nonuniqueness of solutions, stationary accretion

1. Introduction

The question we want to address in this paper is the following inverse problem: having a complete set of data describing a compact body immersed in a spherically symmetric accreting fluid, find the mass of the central body. We assume that we know the total mass, luminosity, asymptotic temperature, the equation of state of the accreting gas and the gravitational potential at the surface of the core.

The fundamental question is whether observers can distinguish between gravastars [1] versus black holes as engines of luminous accreting systems (see a controversy in [2, 3]). While we do not address this problem here, we show a related ambiguity in a simple newtonian model.

2. The Shakura model

The first investigation of stationary accretion of spherically symmetric fluids, including luminosity close to the Eddington limit, was provided by Shakura [4]. It was later extended to models including the gas pressure, its selfgravity and relativistic effects [5–8].

In the following we will denote the areal velocity by $U(r,t) = \partial_t R$ (where $t$ is comoving time and $R$ the areal radius), the local, Eddington and total luminosities by $L(R)$, $L_E$ and $L_0$, quasilocal mass by $m(R)$ and total by $M$, pressure by $p$, the baryonic mass density by $\rho$ (the polytropic equation of state will be $p = K\rho^\Gamma$, $1 < \Gamma \leq 5/3$) and the gravitational potential by $\phi(R)$. The radius of the central body is $R_0$ while its “modified radius” is defined by $\tilde{R}_0 = GM/|\phi(R_0)|$. Under the assumption that at the outer boundary of the fluid the following holds true:

$$U_\infty^2 \ll \frac{Gm(R_\infty)}{R_\infty} \ll a_\infty^2,$$  (1)

we have the following set of equations:

$$\dot{M} = -4\pi R^2 \rho U,$$  (2)

$$U \partial_R U = -\frac{Gm(R)}{R^2} - \frac{1}{\rho} \partial_R p + \alpha \frac{L(R)}{R^2},$$  (3)
\[ \partial_R \dot{M} = 0, \quad (4) \]
\[ L_0 - L(R) = \dot{M} \left( \frac{a^2}{\Gamma - 1} - \frac{a^2}{\Gamma - 1} - \frac{U^2}{2} - \phi(R) \right). \quad (5) \]

\( \alpha \) is a dimensional constant \( \alpha = \sigma_T/4\pi m_p c. \) The details of solving the system are provided elsewhere \([9, 10]\) and we will present only the main results here.

We assume that the accretion is critical, i.e., there exists a sonic point, where the speed of accreting gas \( U \) is equal to the speed of sound \( a. \) All values measured at that point will be denoted with an asterisk. We define:

\[ x = \frac{L_0}{L_E}, \quad y = \frac{M_\ast}{M}, \quad \gamma = \frac{R_0}{R_\ast}, \]

and obtain the total luminosity:

\[ L_0 = \phi_0 \chi_\infty G^2 \pi^2 \frac{M^3}{a^3_\infty} (1 - y) y^2 \left[ y - x \exp(-x\gamma) \right]^2 \left( \frac{2}{5 - 3\Gamma} \right)^{(5-3\Gamma)/2(\Gamma-1)}. \quad (6) \]

\( \chi_\infty \) is approximately the inverse of the volume of the gas located outside the sonic point. In sake of brevity we will use

\[ \beta = \alpha \phi_0 \chi_\infty G^2 \pi^2 \frac{M^2}{a^3_\infty} \left( \frac{2}{5 - 3\Gamma} \right)^{(5-3\Gamma)/2(\Gamma-1)} \]

to obtain Eq. (6) in a form using the relative luminosity:

\[ x = \beta (1 - y) [y - x \exp(-x\gamma)]^2. \quad (7) \]

3. Bifurcation

For the relative luminosity, fulfilling Eq. (7) we proved the following theorem:

(i) For the functional \( F(x, y) = x - \beta (1 - y) [y - x \exp(-x\gamma)]^2 \) there exists a critical point \( x = a, \ y = b \) such that \( F(a, b) = 0 \) and \( \partial_y F(a, b) = 0 \) with \( 0 < a < b < 1 \) and \( a = 4\beta (1 - b)^3, \ b = \left[ 2 + a \exp(-a\gamma) \right]/3. \)

(ii) For any \( 0 < x < a \) there exist two solutions \( y(x)^\pm \) bifurcating from \((a, b).\) They are locally approximated by:

\[ y^\pm = b \pm \left( \frac{(a - x)[b + a \exp(-a\gamma)(1 - 2a\gamma)]}{\beta[b - a \exp(-a\gamma)](1 - a \exp(-a\gamma))] \right)^{1/2}. \quad (8) \]

(iii) The relative luminosity \( x \) is extremized at the critical point \((a, b).\)

4. Discussion

In the paper we have assumed the existence of an accreting system which satisfies certain conditions. Under those assumptions the complicated set of integro-differential nonlinear equations (2-5) can be simplified to an algebraic one (7). We
checked numerically that the performed simplification causes errors of the order of $10^{-3}$ (see [9] for details).

The analysis of Eq. (7) shows that there exist two different solutions, having the same total luminosity and total mass, but different masses of the core objects. One can also conclude that for sufficiently large $\beta$ the maximal relative luminosity $a$ can get close to 1, i.e., the total luminosity approaches the Eddington limit.

As the two solution branches bifurcate from the point $(a, b)$, there is no much difference between the central masses of bright objects (see [9, 10] for plots). However, when luminosity is small ($L_0 \ll L_E$), this difference can become arbitrarily large. This can be understood intuitively, because the radiation is small for test fluids (since the layer of gas is thin), or when the central object is light (therefore weakly attracting surrounding gas).

The results obtained here are consistent with relativistic analysis neglecting interaction between the gas and the radiation [11, 12].

Acknowledgments

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