Electromagnetic Energy Penetration in the Self-Induced Transparency Regime of Relativistic Laser-Plasma Interactions

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Two scenarios for the penetration of relativistically intense laser radiation into an overdense plasma, accessible by self-induced transparency, are presented.

For supercritical densities less than 1.5 times the critical one, penetration of laser energy occurs by soliton-like structures moving into the plasma. At higher background densities laser light penetrates over a finite length only, that increases with the incident intensity. In this regime plasma-field structures represent alternating electron layers separated by about half a wavelength by depleted regions.

Recent developments of laser technology have opened possibilities to explore laser-matter interactions in regimes previously not achievable, [1]. This has meant a strong impulse to the theoretical investigation of phenomena occurring in such extreme conditions, when electrons quiver with relativistic velocities and new regimes may appear. In particular, penetration of ultra-intense laser radiation into sharp boundary, overdense plasmas is playing a fundamental role in the development of the fast ignitor fusion concept as well as of x-ray lasers, [2,3]. In this regime the optical properties of the plasma are substantially modified by the relativistic increase of the inertial electron mass and the consequent lowering of the natural plasma frequency.

In the Seventies it was shown that this relativistic effect enables super-intense electromagnetic radiation to propagate through classically overdense plasmas, the so called induced transparency effect, [4–7]. Recent numerical simulations based on relativistic PIC codes [3,8,9], multifluid plasma codes [10] and Vlasov simulations [11], as well as recent experiments [12,13], have revealed a number of new features of the interaction dynamics, such as laser hole boring, enhanced
incident energy absorption, multi-MeV electron beam, as well as ion beam production and generation of strong magnetic field.

An exact analytical study of the stationary stage of the penetration of relativistically strong radiation into a sharp boundary, semi-infinite, overdense plasma, taking into account both the relativistic and striction nonlinearity, has recently led to the determination of an effective threshold intensity for penetration [4]. It is known that, for incident intensities lower than the penetration threshold, an overdense plasma totally reflects the radiation with the formation of a nonlinear skin-layer structure close to the plasma-vacuum boundary, [6]. For higher intensities the radiation was found to propagate in the form of nonlinear traveling plane waves [6,7], or solitary waves [13]. Further analysis has shown that other scenarios are possible for incident intensities exceeding the threshold, depending on the supercritical plasma parameter, [16]. Namely, if \( n_o > 1.5 \) (\( n_o \) is the supercritical parameter defined as \( n_o = \omega^2_p/\omega^2 \), where \( \omega \) is the carrier frequency of the laser, \( \omega_p = (4\pi e^2 N_o/m)^{1/2} \) is the plasma frequency of the unperturbed plasma), a quasi-stationary state can be realized and, even if still in a regime of full reflection, the laser energy penetrates into the overdense plasma over a finite length which depends on the incident intensity. The subsequent plasma-field structure consists of alternating electron layers, separated by depleted regions with an extension of about half a wavelength which acts as a distributed Bragg reflector. How do these structures emerge as a consequence of relativistic laser-overdense plasma interactions? What kind of scenarios are realized? These are the questions we will try to answer in this Letter.

Our model is based on relativistic fluid equations for the electrons, in order to avoid plasma kinetic effects which may shade or complicate the problem (see, for example, [10,17]). Ions are considered as a fixed neutralizing background due to the very short time scales involved, and the slowly varying envelope approximation in time is assumed to be valid. The governing set of self-consistent equations for the 1D case of interest in the Coulomb gauge reads

\[
\frac{\partial p_\parallel}{\partial t} = \frac{\partial \phi}{\partial x} - \frac{\partial \gamma}{\partial x},
\]

\[
\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(n p_\parallel) = 0,
\]

(1)

(2)
\[ \frac{\partial^2 \phi}{\partial x^2} = n_o(n - 1), \quad (3) \]
\[ 2i \frac{\partial a}{\partial t} + \frac{\partial^2 a}{\partial x^2} + \left(1 - \frac{n_o}{\gamma}na\right) = 0. \quad (4) \]

Variables are normalized as: \( \omega t \rightarrow t, \omega x/c \rightarrow x \), the longitudinal momentum of the electrons \( p_\parallel/mc \rightarrow p_\parallel \), the scalar potential \( e\varphi/mc^2 \rightarrow \varphi \), electron density \( N/N_o = n, \gamma = (1 + p_\parallel^2 + a^2)^{1/2} \) is the Lorentz factor, \( m \) and \( e \) are the electron rest mass and charge, \( c \) is the speed of light in vacuum and we consider circularly polarized laser radiation with the amplitude of the vector potential normalized as \( eA/mc^2 = (a(x,t)/\sqrt{2}) \text{Re}(y + iz) \exp(i\omega t) \).

Eqs. (1)-(4) have been numerically integrated for the problem of normally incident laser radiation from vacuum \( (x < 0) \) onto a semi-infinite overdense plasma \( (x \geq 0) \), the numerical interval consisting of two parts: a short vacuum region to the left of the plasma boundary and a semi-infinite plasma region to the right.

As for the boundary conditions, at infinity in the plasma region the field must vanish, electrons are immobile and the electron density unperturbed, conditions that are valid until this right boundary is reached by field perturbations. At the vacuum-plasma boundary the radiation boundary condition reads

\[ a - i \frac{\partial a}{\partial x} = 2a_i(t), \quad (5) \]

where \( a_i(t) \) is the incident laser wave, which means that in the vacuum region the total field is the sum of the incident and reflected wave. At the initial time electrons are in equilibrium with ions, i.e., \( p_\parallel = 0, n = 1, \varphi = 0 \). Two different cases have been considered for the incident laser pulse: a semi-infinite envelope turning on as \( a_i(t) = a_o(\tanh t + 1) \) and a Gaussian envelope.

Finally, the analysis has been performed for overdense plasmas \( (n_o > 1) \) and for a quite wide range of incident intensities both higher and lower than the penetration threshold.

For maximum incident intensities lower than the threshold, after a transient stage, a stationary regime with the formation of nonlinear skin-layers is reached, which is in perfect agreement with previous analytical solutions \[6,14\]. Furthermore, good agreement is found with the calculated threshold for laser penetration, \[14\], for intensities above which the nonlinear skin-layer regime is broken and the interaction leads to the penetration of laser energy into the overdense...
plasma. Above this threshold, interactions drastically come into play and the analysis of this dynamical process, object of a second set of numerical studies, has revealed two qualitatively different scenarios of laser penetration into overdense plasmas, depending on the supercritical parameter \( n_o \). Ultimately, the qualitative behavior of the system occurs over a wide range of incident intensities and thus it does not sensitively depend on the specific values. If \( n_o \leq 1.5 \) we have only a dynamical regime where laser radiation slowly penetrates into the overdense plasma by moving soliton-like structures. In Fig. (1), the temporal evolution of the semi-infinite tanh-shaped laser radiation interacting with a plasma with \( n_o = 1.3 \) is depicted. Solitary waves are generated near the left boundary and then slowly propagate as quasi-stationary plasma-field structures with a velocity much lower than the speed of light. The contribution to the nonlinear dielectric permittivity due to electron density perturbations is weaker than the one due to the relativistic nonlinearity, therefore we may consider these solutions as the extension of pure low-relativistic soliton solutions, [15], to a regime of slightly higher amplitudes. Furthermore, the excitation dynamics of such structures is similar to that of structures described by the nonlinear Schrödinger equation with a cubic nonlinearity for a slightly overdense plasma in the low relativistic limit, (see, i.g., [18] and references therein). The generation of similar structures can be inferred from the results of PIC simulations, such as those presented in [19]. Thus, if this were the case, i.e., if \( n_o - 1 \ll 1 \), solitary structures excited by incident intensities slightly above the threshold may be considered as exact solutions.

When the incident pulse has a Gaussian shape, penetration is seen to occur by a finite number of soliton-like structures. As shown in Fig. 2(a) a Gaussian pulse with amplitude \( a_o = 0.74 \) and pulse duration \( \tau = 200 \), for the same plasma parameters as in Fig. 1, generates two propagating solitary structures instead of a continuous train. The corresponding spectral analysis, see Fig. 2(b), shows that the spectrum of the transmitted radiation is on average redshifted, while that of the reflected radiation presents an unshifted and a blueshifted part which can be accounted for in terms of Doppler shift due to the moving real vacuum-plasma boundary.

It should be underlined that in the limit of strongly relativistic intensities, when localized solutions have the form of few-cycle pulses as in [15], our model cannot be applied since the
slowly varying envelope approximation will break down, and the question of what happens at intensities largely exceeding the threshold is still open.

At higher background densities, $n_o > 1.5$, the dynamic regime of interaction is completely different, as shown in Fig. 3, where a $tanh$-like pulse with $a_o = 1.3$ that is an intensity of $3.6 \times 10^{18} W/cm^2$ for a wavelength of $1 \mu m$ interacts with a plasma with $N_o = 1.6N_{cr}$ ($N_{cr} = m\omega^2/4\pi e^2$ is the critical density).

The earliest stage of the spatial evolution presents the characteristic distribution of a nonlinear skin-layer, but the ponderomotive force acting at the vacuum-plasma boundary is pushing electrons into the plasma, thus shifting the real boundary to a new position. When the field amplitude on the real boundary exceeds the threshold calculated in [14], the interaction leads to the creation of a deep electron density cavity whose size is about half a wavelength and which acts as a resonator. The whole plasma-field structure then starts to slowly penetrate into the plasma and the same process is repeated at the boundary, where now the perturbed plasma has different parameters.

What is interesting is that, after a transient stage during which deep intensity cavities are produced, the plasma settles down into a quasi-stationary plasma-field distribution, allowing for penetration of the laser energy over a finite length only, which increases with increasing incident intensities. The electron density distribution becomes structured as a sequence of electron layers over the ion background, separated by about half a wavelength wide depleted regions. The peak electron density increases from layer to layer reaching an absolute maximum in the closest layer to the vacuum boundary. At the same time the width of the layers becomes more and more narrow. Such nonlinear plasma structures can act as a distributed Bragg reflector and they are very close to those described analytically in [16].

If the incident laser pulse has a finite duration, the electromagnetic energy penetrates into the plasma over a fixed finite length but, after the laser drive has vanished, the energy localized inside the plasma is reflected back towards the vacuum space, as in some sort of "boomerang" effect. The transient regime is obviously more complicated as the depleted regions surrounded by electron layers act like resonators, with the electromagnetic energy being excited by the incident pulse. Fig. 4 shows how these structures excited by a pulse $400 fs$ long ($\lambda = 1 \mu m$)
bounce back. Clearly, these excited localized plasma-field structures may live much longer than
the duration time of the drive pulse. However, it should be underlined that, on a longer time
scale, the dynamics can be rather unpredictable. For instance, in a run with a laser drive 200fs
long, the interaction between two structures has resulted in one long-lived cavity, whereas a
second run evolved into a moving localized structure similar to those presented in Fig. (1).

It is obvious that, when dealing with long-time dynamics, absorption processes acting on the
electromagnetic energy in the cavities should be taken into account.

In conclusion, we have shown that there are two qualitatively different scenarios of laser
energy penetration into overdense plasmas in the regime of relativistic self-induced trans-
parency, depending on the background supercritical density. For slightly supercritical densities
$N_o < 1.5N_{cr}$, the penetration of laser energy occurs in the form of long-lived soliton-like struc-
tures which are generated at the vacuum-plasma boundary plasma and then propagate into
the plasma with low velocity. At higher plasma densities $N_o > 1.5N_{cr}$, the interaction results
in the generation of plasma-field structures consisting of alternating electron and electron dis-
placement regions, with the electromagnetic energy penetrating into the overdense plasma over
a finite length only, as determined by the incident intensity.

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FIG. 1. Electron density (solid lines) and field amplitude (dashed lines) distributions at various moments, for $n_o = 1.3$ and semi-infinite pulse with the maximum incident intensity $a_o = 0.74$. The ion density distribution is in dotted line. All the quantities are dimensionless.

FIG. 2. Temporal distribution (a) of the field structures generated in a plasma with $n_o = 1.3$ and $a_{th} = 0.62$ by a Gaussian incident pulse with amplitude $a_o = 0.74$ and width $\tau = 200$ and relative spectra (b). All the quantities are dimensionless.

FIG. 3. Snapshots of the time evolution of the electron density (continuous line) and the solitary structures (dashed line) generated by a semi-infinite pulse $a_o(\tanh t + 1)$ with $a_o = 1.3$, propagating into a plasma with $n_o = 1.6$ and $a_{th} = 0.99$. All the quantities are dimensionless.

FIG. 4. Temporal distribution of the field structures generated in a plasma with $n_o = 1.6$ and $a_{th} = 0.99$ by a Gaussian incident pulse with amplitude $a_o = 1.5$ and width $\tau = 800$. All the quantities are dimensionless.
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