Negative Energies and the Limit of Classical Space–Time

Adam D. Helfer
Department of Mathematics, University of Missouri, Columbia, Missouri 65211, U.S.A.

Summary

Relativistic quantum field theories predict negative energy densities, contravening a basic tenet of classical physics and a fundamental hypothesis of the deepest results in classical general relativity. These densities may be sources for exotic general–relativistic effects, and may also lead to pathologies.

Combining Ford’s “quantum inequality” with quantum restrictions on measuring devices, I present an argument that these densities nevertheless satisfy a sort of “operational” positivity: the energy in a region, plus the energy of an isolated device designed to detect or trap the exotic energy, must be non–negative. This will suppress at least some pathological effects.

If we suppose also that Einstein’s field equation holds, then no local observer can measure the geometry of a negative energy density regime accurately enough to infer a negative energy density from the curvature. This means that the physics of a negative energy regime cannot be adequately modeled by a classical space–time.

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0. Preface

What follows is the essay I wrote for the Gravity Research Foundation’s 1998 prize competition. The editor and referee have suggested that I add a few remarks to amplify some issues and to place the work in context. I am grateful for the opportunity to do so, as constraints of space made this impossible in the original paper. Indeed, there is a temptation to write a Shavian preface twice as long as the piece. However, I can resist anything — except temptation.

This paper has very little speculation in it. The overall approach is to take conventional quantum field theory in curved space–time, and conventional quantum mechanics, and apply them at conventional scales. Thus I do not attempt to learn how physics might be modified at, for example, the Planck scale, or what exotic effects might be produced by string theory. I do not suppose my present approach will apply directly to such extreme regimes, which have been investigated by other authors [0]. I do not use any version of “quantum gravity” in the sense this term is usually understood. Still, I am led to infer a quantum character for space–time in certain regimes — by showing that a classical model is not tenable.

The regimes in question are the negative energy–density configurations arising in relativistic quantum field theories. These regimes are predicted to occur generically. Thus a central question is, Why are negative energy density effects not pervasive? Part of the answer, I suggest, is a restriction on quantum measurement, deducible from known physics but not previously considered. This is the “operational weak energy condition,” which requires that the energy in a regime, plus the energy of an isolated device in that regime constructed to measure or trap that energy, be non–negative.

It is worth emphasizing that this condition is a restriction on the ability of a measuring device in a specific space–time region to achieve certain results. This sort of restriction does not seem to have received much attention (beyond the causality requirements embodied in the space–like commutation of field operators).

I have avoided talking about “interpretations” of quantum mechanics, as those who feel strongly about the subject will certainly draw their own conclusions of my work’s significance. However, whatever view one has, I should like to reiterate the point of the previous paragraph: the present analysis can only be accommodated by considering what devices might measure the energy density, and where they are located. It could not be accommodated within any set of assumptions which presume the existence of ideal measuring devices (devices measuring arbitrary self–adjoint operators without otherwise interfering with the system), or without considering where the devices are situated.
The restriction on measurability comes from the restrictions on the mass of a clock with given resolving time (which may be attributed to Bohr, Einstein, Schrödinger, Salecker and Wigner); the operational positivity of energy density comes from combining this with “quantum inequalities” of Ford and Roman, which limit the times for which negative energy densities may persist. There has been previous work on limitations of quantum measurements of position based on related considerations [0]. On the other hand, the present work contrasts with restrictions on measurements which have been proposed to arise from deformations of the canonical commutation relations.

Assuming Einstein’s field equation, the geometry is related to the energy density, and so the restrictions uncovered forbid direct measurements of the curvature of space–time in regimes where the energy density is negative (at least, of those predicted by conventional quantum field theory). What this means is that one cannot verify by direct local means that the space–time geometry of a negative energy regime “is really there.” (There could be indirect evidence for it, however.) The geometry can only be measured locally by introducing devices so massive that they swamp the negative–energy effects.

The details of these arguments are set out more fully in [7]. Roughly speaking, at the simplest level, the usual “test particle” thought–experiments to measure the geometry of space–time cannot succeed in negative energy–density regimes, because particles with Compton wavelengths short enough to accurately probe the curvatures driven by negative energies turn out to be massive enough to destroy that negativity.

This undoubtedly has a queer sound to it, and it takes some work to understand what it means. In this essay I have given a simple example, relevant to Cosmic Censorship, in which it is easy to see the physical consequences of the restrictions. However, a more detailed analysis addresses the issue of to what extent we may say that there is a classical space–time in a negative energy–density regime and to what extent we are forced to impute a quantum character to the geometry. Such an analysis will be found in [7], where it is shown that it is very hard to ascribe any meaningful classical geometry to these regimes. Even if we abandon the test–particle picture, and attempt to take into account the interaction of the measuring device and the field, there are considerable obstacles to giving a meaningful classical character to the negative energy–density regime. The concern that quantum measurement processes might be incompatible with general relativity has been raised earlier [0’].

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0 It is here that the link between space–time geometry and quantum measurement theory is made. In this sense I am making a “quantum gravitational” assumption.
1. Introduction

All known classical forms of matter have positive energy density. Indeed, that the energy of a system be positive (or, in the non–relativistic context, bounded below) controls some of the most basic aspects of its behavior. It is what makes thermodynamic equilibrium and the laws of thermodynamics possible; and it is required for dynamic stability. In General Relativity, positivity of energy density (and similar “energy conditions”) are at the heart of the deepest results in the field: the singularity theorems, the area theorem for black holes, and the positivity–of–total–energy theorems.

Yet it is well–known that in some senses quantum fields may have negative energy densities. This has motivated the search for exotic effects driven by such densities. Serious workers have investigated possible thermodynamic paradoxes [1], as well as “traversable worm holes,” “warp drives” and “time machines” [2]. Of course, it is understood that such work is very speculative.

Curiously, many of the initial investigations have been followed by others suggesting that these exotic effects are difficult or impossible to attain [3]. Sometimes one needs devices at something like the Planck scale to create the negative densities demanded by particular hypothesized applications. In other cases there are problems in usefully controlling the effects. One is led to wonder if these results are manifestations of some deeper principle suppressing exotic negative energy effects.

A recent result suggests that such a principle exists. It has been shown that, in generic space–times, the energy density and even total energy operators are always unbounded below, and the set of states on which their expectations are $-\infty$ is dense in the Hilbert space $[4]$. In such a setting, it is hard to see how exotic negative energy effects could be avoided, unless there is a general principle which tends to suppress them. What, for example, prevents an ordinary particle from absorbing negative energy and becoming a tachyon? If there were any cross–section for such a process, it is hard to see how, given the pervasiveness of very negative energy states, we would have failed to see it.

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1 Precisely, if $\tilde{T}_{ab}$ is the renormalized stress–energy, then the Hamiltonian $\tilde{H}(\xi) = \int \tilde{T}_{ab} \xi^a d\Sigma^b$ associated to a timelike vector field $\xi^a$ is unbounded below, and the states on which its expectation is $-\infty$ are dense, unless $\xi^a$ can be chosen to be a Killing field at $\Sigma$. 
2. An “Operational” Weak Energy Condition

Important restrictions on negative energy densities were discovered by Ford [5]. The quantum inequalities limit the time a negative energy density may persist. For example, for the Klein–Gordon field on Minkowski space,

\[
\langle \Phi | \int_{-\infty}^{\infty} \hat{T}_{00}(t, 0, 0, 0) b(t) dt | \Phi \rangle \geq -(3/32\pi^2)\hbar c/(ct_0)^4
\]

for any normalized |\Phi\rangle , \hspace{1cm} (1)

where the sampling function \( b(t) = (t_0/\pi)/(t^2 + t_0^2) \) has area unity and characteristic width \( \sim t_0 \). While no similar results have been rigorously proved for generic curved four-dimensional space–times, there are good reasons for thinking that they will hold. I shall assume, as do most workers, that they do.

A crucial issue is pointed up by the quantum inequalities. When negative energy density effects are important, one cannot simply speak of the “energy in a region;” one must include a notion of the time scale over which the energy is averaged. The same state could have very negative energies when measured over a short time, and less negative energies when averaged over a longer time. We shall speak of the energy content of a regime, understanding that this refers not just to a region of space, but also to a temporal averaging.

We now combine the quantum inequalities with quantum restrictions on a device which might measure or trap energy. Consider an isolated device designed to measure or trap a negative energy density. Since the negative energy density can persist for only a limited time, the device must have a clock which turns it on and off, say on a time \( \sim t_0 \). A clock which resolves times of order \( t_0 \) must have rest–energy \( \gtrsim \hbar/t_0 \) [6]. On the other hand, the total negative energy detected or absorbed is restricted by the quantum inequalities and causality:

\[
|E_{neg}| \leq (4/3)\pi(ct_0)^3 \cdot (3/32\pi^2)\hbar c/(ct_0)^4 = (8\pi)^{-1}\hbar/t_0 .
\]

Thus the energy of the measuring device must be greater than the negative energy detected.

This may be called an operational weak energy condition for the Klein–Gordon field in Minkowski space: the energy in a regime, plus the energy of an isolated device constructed to measure or trap that energy, must be non–negative.

I suggest that the operational weak energy condition is valid generally, for all quantum fields. I should emphasize that the result has not been proved with mathematical rigor even for the Klein–Gordon field in Minkowski space. \(^2\) Nevertheless, the result is so suggestive, and the factor \( 8\pi \) so in excess of unity, that it seems at least worth considering.

\(^2\) The argument as given depends on a special choice of sampling function, and the inequality \( E_{\text{clock}} \gtrsim \hbar/t_0 \)
A thought experiment to measure energy density in a region gravitationally has been investigated in some detail, and the operational weak energy condition appears explicitly [7]. Indeed, at least for the device considered, timing errors prevent one from coming close to saturating the condition unless the Planck scale is approached.

It is not clear that this operational weak energy condition will rule out all pathological effects, let alone exotica. Each must be examined carefully. Certainly the condition would prevent an ordinary particle from absorbing negative energy and becoming a tachyon.

We shall see however that the consequences of the condition are of interest whether or not it resolves all the pathologies.

3. Limitations of a Classical Model for Space–Time

The operational weak energy condition would imply that negative energy densities have, in some sense, a will–o’–the–wisp character. While they might be definitely predicted by theory (as, for example, between the plates of a Casimir apparatus), they cannot be confirmed by a direct local experimental measurement — for this would always require a device massive enough to swamp the negative energy density.

To begin to understand the physical significance of this, let us consider a situation relevant to the Cosmic Censorship Conjecture. Suppose a singularity is present in a negative energy density regime. Could this be visible from infinity?

If the operational weak energy condition holds, then, while there might be mathematical null geodesics escaping from the singular region to infinity, these geodesics could not carry physical photons of short enough wavelength to give a detailed image of the singular region. This is because any measurement of the singular region accurate enough to measure the curvature would imply a measurement of the energy density, by Einstein’s equation. And if the energy density is negative, this is forbidden by the operational weak energy condition.

While this argument has a little interest as circumstantial support for the Cosmic Censorship Conjecture, it is much more important in that it shows that there may be a clear distinction between the mathematical model of a classical space–time and the physical possibilities for measurement of geometry and transmission of information by signals. This is in fact a general feature of negative energy density regimes.

is only known to hold in an order–of–magnitude sense.
We may say that a space–time region is modeled classically (to a desired accuracy) if it is possible in principle to introduce test particles (i.e., particles not interfering with the measurement) whose trajectories can be measured to determine the geometry of the region (to the desired accuracy).\(^3\)

Classical existence in this sense is forbidden by the operational weak energy condition in negative energy density regimes, whenever the accuracy is enough to infer the energy density from Einstein’s equation. Whether one is bold enough to say that such a regime is quantum space–time is a matter of temperament. However, the inadequacy of the classical model is fairly clear. Picturesquely:

*A space–time regime can have a negative energy density only if no one is there to measure it!*

4. Conclusion

General Relativity is ineffably beautiful, and only with the greatest caution should we seek to move beyond it. Yet the principle on which Einstein founded this theory, and which was essential to the development of quantum theory, was operationalism: that a theory should be formulated in terms of physical observables. Einstein himself was sensible that the geometry of space–time in the quantum regime would ultimately have to be justified operationally:

It is true that this proposed physical interpretation of geometry breaks down when applied immediately to spaces of sub–molecular order of magnitude.... Success alone can decide as to the justification of [attempts to do so].... It might possibly turn out that this extrapolation has no better warrant than the extrapolation of the idea of temperature to parts of a body of molecular order of magnitude. [8]

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\(^3\) The term “particle” does not imply a point mass, but only an object with some degree of localizability.
References

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