Numerical validation of a simplified inverse analysis method to characterize the tensile properties in strain-softening UHPFRC

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Abstract. In previous research a simplified closed-form non-linear hinge model based on the Third Point Bending Test (TPBT) developed by the authors to derive the tensile material properties of Ultra-High-Performance Fiber-Reinforced Concrete (UHPFRC) was numerically validated. This simplified non-linear hinge model was calibrated for UHPFRC that exhibits strain-hardening constitutive behavior. The aim of this work is the numerical validation of the simplified inverse analysis method to characterize the tensile properties of UHPFRC even with those UHPFRCs which exhibit strain-softening behavior. To get this objective a Finite Element Model (FEM) is carried out. The parameters to characterize the concrete properties from the simplified inverse analysis method by means of TPBT are used in the numerical modelling. The constitutive model for UHPFRC is modelled using a discrete crack approach for the macrocrack position. Different parameters are analysed to quantify the accuracy of the FEM. As a result, the model shows conservative load-deflection response when it is compared to the experimental curves.

1. Introduction
UHPFRC can be considered a special type of high-performance fiber reinforced cement composite (HPFRCC) if the definition in the Preface of the 6th and 7th Symposium on HPFRCC [1,2] and JSCE recommendations [3] is used. It states that HPFRCC can be considered all those concretes that exhibit a strain-hardening tensile stress-strain response accompanied by multiple cracking and a relatively large energy absorption capacity. Reaching strain-hardening behavior in a UHPC matrix depends on fiber type and amount, the matrix strength, bond between matrix and fibers, and also on specimen size and geometry, the pouring system, support conditions, structural redundancy, etc.

The characterization of UHPFRC tensile behavior is still a challenge. Four-point bending test (4PBT) arises as one of the best tests to get this result because of its simplicity. Nevertheless, it requires the use of an inverse analysis methodology to derive the tensile properties from the results obtained from it. Different inverse analysis methods have already been developed to obtain the parameters that define the UHPFRC behavior in tension from 4PBT [4–12].

To determine the validity and accuracy of inverse analysis methods, it is important to develop numerical models. The numerical model presented in [13,14] with the objective of validating the non-linear hinge model and its derived simplified five-point inverse analysis method presented in [15] to derive the tensile UHPFRC’s properties from load-deflection response obtained from Third Point
Bending experimental tests (TPBT) was developed for UHPFRCs which exhibits strain-hardening behavior. The aim of this work is the numerical validation of the simplified inverse analysis method to characterize the tensile properties of UHPFRC even with UHPFRCs exhibiting strain-softening behavior.

2. Simplified Four-Point Inverse Analysis Method (4P-IA)

The Simplified Four-Point Inverse Analysis Method (4P-IA) developed and explained in [16,17] is a simplification of the Simplified Five-Point Inverse Analysis Method (5P-IA) fully described in [15,17]. This new simplified methodology is based on the closed-form non-linear hinge model developed in [15,17]. It entails having to select four specific key points extracted from the experimental equivalent bending strength-displacement at mid-span curve (Figure 1). Using these points, parameters that define the assumed quadrilinear stress-strain law inside the hinge can be determined by a back-of-the-envelope calculation. This law is used to determine the constitutive tensile behavior of UHPFRC.

Constitutive model for UHPFRC proposed is depicted in Figure 1 which is defined as a function of six parameters: elastic modulus (E) cracking strength (f_t); ultimate cracking strength (f_u) and its associated strain (ε_u); crack opening at the intersection of the line that defines the initial slope to the w axis (w_0); and the characteristic crack opening (l_f/4) defined as one fourth the fiber length. Only the first five are determined using this procedure.

The proposed method requires six input parameters, which derive from only four key points extracted from the experimental flexural strength versus displacement at mid-span curve obtained from a TPBT. The method’s output generates the constitutive parameters that define the constitutive behavior of UHPFRC (Figure 1).

![Figure 1. Simplified 4P-IA [16,17].](image-url)
This 4P-IA (the same with the 5P-IA) is defined as follows: From the non-linear hinge model, analytical load-deflection curves from a TPBT with a specific slenderness ratio can be obtained from a hypothetical stress-strain response of the hinge. By varying the stress-strain parameters, different analytical strain-hardening behavior concretes can be modelled. For each set of tensile parameters, the analytical $\sigma_{fl}$-$\delta$ curve can be obtained according to the non-linear hinge model developed. From these curves, a set of points $(\delta_i, \sigma_{fl,i})$ can be extracted and statistically analyzed to establish relationships between these points and the tensile parameters.

Taking these into account, what happens if the UHPFRC does not exhibit strain-hardening behavior? Is the 4P-IA reliable enough to obtain the tensile parameters of UHFRC that exhibits strain-softening behavior in tension?

To distinguish between UHPFRCs that exhibit strain-hardening behavior and those that exhibit strain-softening behavior in tension, a hardening ratio ($\gamma$), has been defined (1). If $\gamma \geq 1$ the UHPFRC exhibits strain-hardening behavior. If $\gamma < 1$ the UHPFRC exhibits strain-softening behavior.

$$\gamma = \frac{f_{tu}}{f_t}$$

3. Numerical model

In this work, a numerical model is developed using the FE software DIANA [18]. To model the tensile UHPFRC constitutive behavior a discrete cracking approach is used. In this approach the constitutive model for UHPFRC is based on the discrete cracking model as an interface behavior. The constitutive law for discrete cracking in DIANA is based on a total deformation theory [18]. This behavior is only forced at the central section of the beam (Figure 2). To get this objective the tensile strength ($f_t$) and the ultimate tensile strength ($f_{tu}$) are reduced in a 2%. The rest of the beam is modelled using a smeared cracking approach based on a fixed total strain crack model expressed as function of crack opening curve (the “fibre reinforced concrete model, FRCCON” from fédération internationale du béton/International Federation for Structural Concrete (fib) working groups).

![Figure 2. Discrete cracking approach.](image)

The TPBT has been modelled with a 2D quadratic plane stress eight node quadrilateral element. Moreover, a quadratic 2D 3+3 nodes line interface element is situated in the central section of the beam. The load is applied on the steel load plates by a gradual increasing displacement.

A Nonlinear Analysis has been carried out using an incremental-iterative solution procedure. The theoretical formulation of the model, the adopted numerical procedures and solution strategy used are described in detail in DIANA user’s manual [18].

4. Experimental program

For the purpose of using the constitutive tensile parameters obtained as a result of the 4P-IA in the FEM developed, eighteen square cross-section specimens with 100mm depth and 500mm length have been tested in a FPBT (Figure 3). A 2% in volume of smooth-straight (13/0.20) steel fibers were used
in an Ultra-High-Performance cementitious matrix with an average concrete compressive strength of 145.46MPa, obtained from 32 100mm long cubes with a 1.63% coefficient of variation.

According to Figure 3, two displacement transducers were used to record the displacement at mid-span on the front side and in the back side each one.

![Figure 3. Third Point Bending Test (TPBT).](image)

5. Numerical model application

Table 1 shows the constitutive UHPFRC tensile parameters obtained when the 4P-IA is applied for each specimen and the hardening ratio defined in (1).

| Specimen id. | $f_t$ (MPa) | $f_{tu}$ (MPa) | $\varepsilon_{tu}$ (%) | $E$ (MPa) | $w_d$ (mm) | $w_o$ (mm) | $\gamma$ |
|--------------|-------------|---------------|-----------------|-------|---------|---------|-------|
| 4PBT-1       | 10.05       | 7.53          | 4.25            | 51300 | 2.53    | 3.80    | 0.75   |
| 4PBT-2       | 10.25       | 9.87          | 4.15            | 52400 | 2.09    | 3.14    | 0.96   |
| 4PBT-3       | 9.52        | 9.83          | 3.66            | 53600 | 1.98    | 2.97    | 1.03   |
| 4PBT-4       | 8.30        | 6.73          | 2.25            | 55000 | 2.49    | 3.74    | 0.81   |
| 4PBT-5       | 8.20        | 6.99          | 2.97            | 50400 | 1.24    | 1.86    | 0.85   |
| 4PBT-6       | 9.71        | 10.16         | 4.50            | 53300 | 1.29    | 1.94    | 1.05   |
| 4PBT-7       | 9.91        | 8.88          | 3.39            | 51400 | 1.49    | 2.24    | 0.90   |
| 4PBT-8       | 10.96       | 9.63          | 2.34            | 49800 | 1.22    | 1.83    | 0.88   |
| 4PBT-9       | 10.52       | 8.33          | 5.98            | 49800 | 2.19    | 3.28    | 0.79   |
| 4PBT-10      | 8.84        | 8.88          | 3.83            | 50000 | 2.14    | 3.21    | 1.00   |
| 4PBT-11      | 10.28       | 8.22          | 3.25            | 48000 | 1.77    | 2.66    | 0.80   |
| 4PBT-12      | 9.78        | 10.01         | 7.26            | 51600 | 2.13    | 3.19    | 1.02   |
| 4PBT-13      | 9.66        | 9.22          | 6.73            | 54100 | 2.01    | 3.02    | 0.95   |
| 4PBT-14      | 10.42       | 9.95          | 3.57            | 51600 | 1.75    | 2.62    | 0.95   |
| 4PBT-15      | 10.45       | 10.42         | 4.71            | 48100 | 2.04    | 3.06    | 1.00   |
| 4PBT-16      | 8.73        | 5.17          | 4.92            | 51259 | 2.11    | 3.17    | 0.59   |
| 4PBT-17      | 9.07        | 5.74          | 6.26            | 48500 | 1.94    | 2.91    | 0.63   |
| 4PBT-18      | 8.79        | 7.97          | 1.54            | 52900 | 1.24    | 1.85    | 0.91   |
The material properties obtained from the application of the simplified 4P-IA in Table 1 are implemented into the numerical model described in Section 3 and compared to the experimental program results (Figure 4).

In Figure 5, a comparison between the experimental and numerical tension-deflection ($\sigma$-$\delta$) curve for a specimen that exhibits strain-hardening (Figure 5a) and a specimen that exhibits strain-softening (Figure 5b) has been depicted.

Figure 4. FEM of the Third Point Bending Test specimen.

Figure 5. Specimen with strain-hardening (a), specimen with strain-softening (b).
As it can be seen in Figure 5, the $\sigma$-$\delta$ curve from the finite element model for specimens that exhibit strain-hardening tensile response ($\gamma \geq 1$) is more accurate than for specimens that exhibit strain-softening tensile behavior ($\gamma < 1$). Even though, despite this fact, the inaccuracy observed in the model’s curve for the specimens that show strain-softening behavior is on the safe side.

To quantify the accuracy of the finite element model in comparison to the experimental response, a coefficient of security (CS) has been defined as the ratio between the experimental stress and the stress obtained from the model (2). This coefficient of security has been obtained at four levels of the maximum experimental deflection ($\delta_{\text{maxexp}}$): $0.25 \cdot \delta_{\text{maxexp}}$, $0.50 \cdot \delta_{\text{maxexp}}$, $\delta_{\text{maxexp}}$ and $1.25 \cdot \delta_{\text{maxexp}}$ (see Figure 6). Moreover, the energy of the curve delimitated by the $\delta_{\text{maxexp}}$ level (A1) and by the $1.25 \cdot \delta_{\text{maxexp}}$ level (A2) for the experimental and model curves have been obtained. Table 2 shows the values of the parameters above defined.

$$CS = \frac{\sigma_{\text{exp}}}{\sigma_{\text{model}}}$$

(2)

**Figure 6.** Delimitation of the CS levels.

Figure 7 shows the relation between the hardening ratio ($\gamma$) and the Security Coefficient (CS) at the four levels of the maximum experimental deflection ($\delta_{\text{maxexp}}$): $0.25 \cdot \delta_{\text{maxexp}}$ (Figure 7a), $0.50 \cdot \delta_{\text{maxexp}}$ (Figure 7b), $\delta_{\text{maxexp}}$ (Figure 7c) and $1.25 \cdot \delta_{\text{maxexp}}$ (Figure 7d) for the 18 specimens analyzed in this work.
As it can be observed in Figure 7, the CS is near 1 in the early stages of the $\sigma - \delta$ curve (at 0.25·$\delta_{\text{max}}$ and 0.50·$\delta_{\text{max}}$) and clearly over 1 at the $\delta_{\text{max}}$ and 1.25·$\delta_{\text{max}}$ levels. It means that at the first stages of the $\sigma - \delta$ curve the model adjusts accurately the experimental curve. When the highest level of the curve is reached and the descending branch takes part the accuracy of the model is not as good as in the elastic branch but it seems to be on the safe side because for the same level of $\delta$, the $\sigma$ value of the model is lower than the experimental one. So the model is reliable enough. If more attention is paid in Figure 7, it can be observed that there is certain inaccuracy in the first two levels, 0.25·$\delta_{\text{max}}$ and 0.50·$\delta_{\text{max}}$ (elastic branch of the $\sigma - \delta$ curve), in the unsafety part with the specimens made of UHPFRC that exhibits strain-hardening ($\gamma \geq 1$). Nevertheless, this inaccuracy is negligible. If the other last levels are observed, although they are on the safe side, the stress experimental values are higher than the numerical ones especially in the strain-softening ($\gamma < 1$) case. It seems that the model is more accurate at these stages for specimens made of UHPFRC with strain-hardening behavior.

Figure 8 shows the relation between the energy calculated for the experimental ($A_{1\exp}$) and numerical ($A_{1\text{model}}$) curves for the $\delta_{\text{max}}$ level (Figure 8a) and the energy calculated for the experimental ($A_{2\exp}$) and numerical ($A_{2\text{model}}$) curves for the 1.25·$\delta_{\text{max}}$ level (Figure 8b).
As it can be seen in Figure 8 the energy obtained in the experimental test is slightly higher than that obtained in the model for the same specimen. The same trend can be observed for the rest of specimens either with respect to the energy limited by $\delta_{\text{maxexp}}$ (A1) and the energy limited by $1.25 \cdot \delta_{\text{maxexp}}$ (A2). So this demonstrates that the model is slightly conservative when it is compared to the experimental test.

Figure 9 shows the relation between the experimental and numerical $\sigma_{\text{max}}$.

As it is observed in Figure 9, if the maximum stress ($\sigma_{\text{max}}$) obtained in the experimental test and in the numerical model are compared for each specimen, it can be seen that the experimental value of $\sigma_{\text{max}}$ is higher than the numerical one for each specimen. So, as it happens with the energy, this value demonstrates that the model is conservative with respect to the experimental results. Moreover, as it can be observed in Figures 7, 8 and 9, the level of accuracy is high.

6. Concluding remarks
The numerical findings presented in the present paper allows to draw the main concluding remarks listed in the following:
1. The Simplified Four-Point Inverse Analysis Method (4P-IA) to obtain the tensile constitutive behavior of UHPFRC from the Third-Point Bending Test, has been validated resorting to a robust non-linear finite element modelling and a set of Third-Point Bending tests.

2. When using the tensile parameters resulting from the application of the 4P-IA, the numerical model shows a slightly conservative response with a considerable level of accuracy for modelling the tensile behavior of UHPFRC that exhibits strain-softening constitutive stress-strain response in a Third-Point Bending test.

3. As a result, inverse analysis methodologies based on the closed-form non-linear hinge model and the derived Simplified Four-Point Inverse Analysis Method proposed can be recommended to derive UHPFRC’s tensile properties in Third-Point Bending Tests in both cases: when the UHPFRC exhibits strain-hardening constitutive stress-strain behavior and when exhibits strain-softening behavior.

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