3-D MHD Numerical Simulations of Cloud-Wind Interactions

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ABSTRACT

We present results from three-dimensional (3-D) numerical simulations investigating the magnetohydrodynamics of cloud-wind interactions. The initial cloud is spherical while the magnetic field is uniform and transverse to the cloud motion. A simplified analytical model that describes the magnetic energy evolution in front of the cloud is developed and compared with simulation results. In addition, it is found the interaction of the cloud with a magnetized interstellar medium (ISM) results in the formation of a highly structured magnetotail. The magnetic flux in the wake of the cloud organizes into flux ropes and a reconnection, current sheet is developed, as field lines of opposite polarity are brought close together near the symmetry axis. At the same time, magnetic pressure is strongly enhanced at the leading edge of the cloud from the stretching of the field lines that occurs there. This has an important dynamical effect on the subsequent evolution of the cloud, since some unstable modes tend to be strongly enhanced.

Subject headings: ISM: clouds – ISM: kinematics and dynamics – magnetic fields

1. Introduction

Magnetic fields are a pervasive element of the interstellar and intergalactic medium and they are often extremely relevant in characterizing local and global behaviors of these media. Magnetohydrodynamic (MHD) cloud-wind interactions are believed to be important in determining the observed filamentary and clumpy morphology associated with clouds moving through a magnetized interstellar medium. These processes may also result in a local enhancement of the background magnetic field, which, in turn, provides an important feedback on the subsequent evolution of the cloud. Jones et al. (1994, 1996) have shown that these regions of strong magnetic pressure (what we call “magnetic bumpers”) develop in front of the cloud as a result of the stretching of the field lines that anchors on the cloud surface. Such regions may also be adjacent to shocks, which can serve

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as acceleration sites of high energy particles \((e.g., \text{Jones} \& \text{Kang} 1993)\). Within some supernova remnants there is also clear evidence for supersonic clumps \((e.g., \text{Jones} \text{et al.} 1998)\). Consequently the strong field regions can show enhanced nonthermal radio emission \((\text{Jones} \text{et al.} 1994)\). \text{Miniati et al.} (1997, 1999a) investigated the role played by magnetic fields in cloud collisions by comparing two dimensional (2-D) hydrodynamic and magnetohydrodynamic simulations. They concluded that a magnetic field transverse to the cloud motion can dramatically alter the outcome of the collisions, preventing the disruption of the clouds otherwise occurring in almost all other scenarios. Since the development of the magnetic bumper was crucial for this result, \text{Miniati et al.} (1999b) have extended the \text{Jones et al.} (1994, 1996) work. In particular, they further investigated the formation of such a bumper under a broader range of initial conditions by studying the propagation of clouds through various oblique magnetic fields. In addition they also assessed the issue of the exchange of magnetic and kinetic energy during the evolution of diffuse clouds in the ISM. There, in fact, they offer a possible explanation for the comparable values of magnetic and kinetic energy densities observed in some \text{H I} complexes \((\text{Heiles} 1989; \text{Verschuur} 1989; \text{Myers} \& \text{Khersonsky} 1995; \text{Myers et al.} 1995)\). \text{Gregori et al.} (1999) have presented three dimensional (3-D) numerical simulations that show dramatic dynamical effects of the magnetic field in determining the cloud evolution during its propagation through a magnetized medium. In particular, those authors found that a strong field enhances the development of Rayleigh-Taylor unstable modes, thus hastening the cloud disruption. On one hand this demonstrates that magnetic fields in 3-D do not simply slip around the cloud surface, somewhat similar to the 2-D case. On the other hand, the 3-D influence of the field was disruptive, opposite to that seen in 2-D simulations.

In this paper we present 3-D numerical simulations of moderately supersonic cloud motion in a magnetized interstellar medium. The cloud is treated as non self-gravitating and adiabatic. First we present a detailed analytical model for the stretching mechanism of the field lines and the consequent magnetic field amplification at the cloud nose. We also show that the final outcome in a cloud-wind interaction is the development of complex features analogous to the ones observed in cometary plasma tails, resulting in the formation of a highly structured magnetotail. The magnetic field itself organizes in coherent tails, or flux ropes, in the wake of the cloud. These ropes are associated with the development of a Sweet-Parker reconnection sheet that alters the field topology there. Our aim is to give a picture of the basic processes that develop in cloud-wind interactions, in order to provide useful insight for observations in a large variety of astrophysical environments \((\text{see e.g.,} \text{Dgani} \& \text{Soker} 1998)\). The use of 3-D simulations is clearly a considerable advantage over previous work, since now the full spatial domain can be investigated without the geometrical limitations imposed by 1-D or 2-D calculations.

The paper is organized as follows. In §2 we describe the numerical setup, and the characteristic physical parameters of the problem. The computational results are introduced in §3. The development of the Rayleigh-Taylor instability and the cloud disruption are briefly discussed in §3.1. The analysis of the magnetic energy evolution is given in §3.2. In section §3.3 we investigate the formation of flux ropes in the wake of the cloud and in §3.4 the magnetic reconnection. Our
results are summarized in §4.

2. Numerical Setup and Definition of the Problem

The numerical computation is based on a total variation diminishing (TVD) scheme for ideal MHD (Ryu & Jones 1995). This is an explicit, conservative finite-difference method with second order accuracy in space and time. We have used the multidimensional, Cartesian version of the code (Ryu, Jones & Frank 1995) with a constrained transport scheme for preserving $\nabla \cdot B = 0$ (Ryu et al. 1998). Neglecting self-gravity and radiative energy losses, the complete set of the simulated equations for the velocity ($u$), magnetic field ($B$), density ($\rho$), pressure ($p$) and “color tracer” ($C$) can be conveniently written as

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0, \tag{2-1}
\]

\[
\frac{\partial u}{\partial t} + u \cdot \nabla u + \frac{1}{\rho} \nabla p - \frac{1}{\rho} (\nabla \times B) \times B = 0, \tag{2-2}
\]

\[
\frac{\partial p}{\partial t} + u \cdot \nabla p + \gamma p \nabla \cdot u = 0, \tag{2-3}
\]

\[
\frac{\partial B}{\partial t} - \nabla \times (u \times B) = 0. \tag{2-4}
\]

\[
\frac{\partial C}{\partial t} + u \cdot \nabla C = 0. \tag{2-5}
\]

The last equation has been added to the standard set of ideal MHD equations in order to be able to follow the motion of the cloud material itself. $C$ corresponds, in fact, to the mass fraction of the cloud gas inside the computational cell.

Initially, all the cloud material is labeled with $C = 1$, and the ambient medium with $C = 0$. At any time $t$, the density of cloud material in a fluid cell is given by $\rho_c = \rho C$, where $\rho$ is the total fluid density at that point. In the MHD equations, the magnetic field is normalized such that the factor $4\pi$ does not appear, giving an Alfvén speed $v_A = B/\sqrt{\rho}$. We assume an adiabatic index $\gamma = 5/3$, initial pressure equilibrium at $p_0 = 3/5$, and an initial density in the background medium $\rho = \rho_i = 1$. Thus, the velocity is expressed in units of the sound speed in the ambient medium: $c_s = (\gamma p_0/\rho_i)^{1/2} = 1$. The initial cloud density is $\rho_c = \chi \rho_i$, with $\chi = 100$. A thin transition layer $\sim 0.2 R_c$ ($R_c$ is the cloud radius) around the cloud, introduced to reduce the Richtmyer-Meshkov instability at startup, brings the density to the value of the intercloud medium. A finite transition layer is in general expected due to both thermal condution (Balbus 1986) and photoionzation (Tielens & Hollenbach 1985) at the cloud boundary, although at this point in our simulations it is considered only for reasons of numerical stability. The cloud is initially spherical in shape. The numerical value for its radius, $R_c$, is set to unity, and this is chosen as the unit of length. This also sets the unit time to the cloud sound crossing time, $\tau_{cs} = R_c/c_s$ (= 1 in numerical units). At time
$t = 0$, the cloud is set in motion with respect to the uniform background medium. Its velocity is $u_c = Mc_s$, with a starting value for the intercloud Mach number, $M = 1.5$. The magnetic field is conveniently expressed in terms of the familiar parameter

$$\beta = \frac{p}{p_B},$$

(2-6)

where $p_B = B^2/2$ is the magnetic pressure. In our numerical simulations we have considered both the cases of an initially strong field ($\beta = 4$) and a weak field ($\beta = 100$). To be able to compare with pure hydrodynamic effects, a case with $\beta = \infty$ (no magnetic field) has also been computed. A summary of all the simulations performed is given in Table 1. In addition to the $\beta$ parameter, another important dimensionless number often used to describe the action of the magnetic field on the fluid motion is the Alfvénic Mach number $M_A = u/v_{A_i}$, where $v_{A_i} = B/\sqrt{\rho_i}$ is Alfvén velocity in the intercloud medium. So, we initially have $M_A = 2.74$ for to $\beta = 4$, and $M_A = 13.7$ for $\beta = 100$.

The computational domain is outlined in Fig. 1. Symmetrical boundary conditions are employed on the $y = 0$ and $z = 0$ planes, inflow conditions are applied on the $x = 0$ plane, while open conditions are used on all other boundaries. This choice eliminates odd modes of instabilities, but in companion, fully 3-D simulations at somewhat lower resolution we saw no evidence that such modes play a deciding role in cloud evolution. We employed a uniform grid, $N_x \times N_y \times N_z = 416 \times 208 \times 416$, spanning $\frac{1}{4}$ the volume of interest. The volume computed was bounded along $x$-$y$, $x$-$z$ planes through the initial cloud center. This gives a resolution of 26 zones per cloud radius, less than that in our previous 2-D MHD simulations (Miniati et al. 1999b, Jones et al. 1996). So, small scale surface perturbations were relatively more damped. However, from our experience, the adopted resolution is sufficient to capture basic cloud evolution over the time interval considered. In addition, we have carried out several lower resolution 3D simulations spanning the full volume of interest (see Table 1). Some aspects of those companion simulations are reported as well in the following sections. The cloud shapes (sphere and cylinder) that we have considered are quite ideal and they need to be considered to give only a qualitative picture of the true interaction of a magnetized wind with an interstellar cloud which has no distinctive shape. In this respect, the simulations with spherical and cylindrical clouds show a quite similar behavior. Tests carried out with a different shape, an elliptical cloud, have also confirmed the same pattern. They all show qualitative agreement with the simulations reported here.

Since the cloud motion is supersonic, its motion leads to the formation of a forward, bow shock and a reverse, crushing shock propagating through the cloud. The approximate time for the latter to cross the cloud is referred to as the “crushing time$^4$ (e.g., Jones et al. 1994):

$$\tau_{cr} = \frac{2R_c\chi^{1/2}}{Mc_s}.$$  

(2-7)

$^4$This form of the crushing time is a factor of 2 larger than the one of Klein et al. (1994), since our definition is based on the cloud diameter instead of the cloud radius. We use this definition since it more closely measures the actual time before the crushing shock emerges.
Since the crushing time corresponds to the typical scale for the cloud evolution, in the following figures and discussions, time will be expressed in terms of $\tau_{cr}$.

Finally, it is necessary to add a few words about the significance of our choice in the initial direction of the field lines. In our simulations, the initial magnetic field has been set up transverse (perpendicular) to the cloud motion. As pointed out in previous 2-D simulations (Miniati et al. 1999b; Jones et al. 1996; and Mac Low et al. 1994), a magnetic field aligned with the direction of the cloud motion never becomes directly dynamically relevant in terms of body forces, even if it may have some stabilizing effects. For a general orientation, there will always be some component of the field transverse to the cloud velocity, which will be stretched around the cloud body. In this respect, our simulations can be viewed as an approximate solution of the more general problem of the oblique field orientation. As stressed by Jones et al. (1996) and Miniati et al. (1999b), for supersonic bullets most field directions will indeed produce effects similar to the transverse field case. To confirm that the field evolution in the experiments described here are not special cases geometrically, we have also carried out fully 3-D simulations at low resolution, including oblique initial field orientations, confirming the general behaviors for the more restricted symmetries imposed.

3. Results

3.1. Dynamical Evolution & Cloud Disruption

As discussed in a companion paper (Gregori et al. 1999), the magnetic field amplification that occurs in front of the cloud inhibits instabilities in the $x$-$y$ plane, but hastens those in the $x$-$z$ plane, thus accelerating the process of cloud disruption. An example of the effects of this magnetically enhanced Rayleigh-Taylor (R-T) instability can be seen in Fig. 2. This shows density slices in the $y = 0$ plane for the $\beta = \infty$ (top), $\beta = 100$ (middle) and $\beta = 4$ (bottom) simulations, at $t = 0.94\tau_{cr}$ (left) and $t = 2.25\tau_{cr}$ (right). Clearly, for the $\beta = 4$ case, the growth of a R-T instability is much more rapid and, unlike the other cases, very pronounced density fingers have developed at the simulation end. In addition, to illustrate how the cloud shape evolves, we define generalized cloud sizes in terms of the coordinate moments of inertia. Following Klein et al. (1994) and Xu & Stone (1995), the cloud extension in the $i$th direction, $R_i$, at time $t$ is given by

$$R_i^2(t) = \frac{\left(\int r_i^2 \rho C dV\right)_t}{\left(\int r_i^2 \rho C dV\right)_0} R_{c}^2,$$

(3-1)

where $r_i$ is the $i$th position coordinate with respect to the center of mass, and the integrals are intended over the entire computational domain. In Fig. 3 we have plotted $R_y$ and $R_z$ for both the MHD and the hydrodynamic simulations. Compared to the hydrodynamic case, the wrapping of the field lines around the cloud in the $x$-$y$ plane produces a strong radial magnetic pressure gradient that squeezes the cloud gas. Such pressure, consequently, forces an extrusion of the cloud along the $z$ direction. This explains the trends visible in Fig. 3. There we can read that
for the $\beta = 4$ case, after one crushing time, $R_y$ has already been reduced by 30-40% while $R_z$ has been increased by a similar amount. We may observe that at $t \gtrsim 1.5\tau_{cr}$, the expansion in the $z$ direction is sustained at a very large rate, while $R_y$ stays almost constant. Finally, in the $\beta = 100$ simulation the change in the cloud form follows the same qualitative pattern as for the $\beta = 4$ case, although the evolution is considerably less dramatic and rapid.

3.2. Magnetic Energy Evolution

3.2.1. Model

In the attempt to understand the process of the magnetic bumper formation and its interaction with the surrounding flow, Miniati et al. (1999b) and Jones et al. (1996) have proposed an approximate model based on the Faraday induction equation. Their analysis showed that the magnetic energy first increases with time exponentially (Jones et al. 1996) and then according to a power-law of index between one and two (Miniati et al. 1999b). In hopes that it may serve as a simple tool for understanding field growth in such problems, we examine again the magnetic energy evolution explicitly using the flux freezing condition while incorporating the important and more realistic assumption of a limited region of velocity shear around the cloud nose.

In fact, the time evolution of the magnetic field, in ideal MHD is given by the solution of the following equation in Lagrangian form

$$\frac{d}{dt}\left(\frac{B}{\rho}\right) = \frac{B}{\rho} \cdot \nabla u. \tag{3-2}$$

This equation can be formally integrated as (e.g., Batchelor 1967) to give

$$\left(\frac{B}{\rho}\right) = \left(\frac{B}{\rho}\right)_0 \cdot \frac{\partial}{\partial a} X(a,t). \tag{3-3}$$

Here, $(B/\rho)_0$ refers to the value at time $t = 0$, and $a$ is the initial position vector of the fluid element being followed. It is worth stressing that the previous equations are only valid far from shocks. This is indeed the case in our analysis since the magnetic energy tends to evolve mostly in front of the cloud, far from the bow shock and the crushing shock penetrating into the cloud. The mapping function of the flow field is $X(a,t)$, which locates the fluid element at subsequent times. Its derivative, $\partial X/\partial a$, parametrically measures the deformation of a fluid element, and, therefore determines the local growth of the magnetic field. From eq. (3-3) the strength of the magnetic field, $|B| = B$, then evolves according to:

$$\left(\frac{B}{\rho}\right) = \left(\frac{B}{\rho}\right)_0 \frac{\partial}{\partial a_i} X(a,t) \simeq \left(\frac{B}{\rho}\right)_0 \frac{\partial X}{\partial a} , \tag{3-4}$$

where $X = |X|$ and $a = |a|$. In order to derive the evolution of the term $\partial X/\partial a$ we assume steady
flow and write the equation for a fluid element along a flow line as (e.g., Aris 1962):

\[
d\left(\frac{dX}{dt}\right) = \frac{\partial u_i}{\partial x_j} \frac{dx_j}{dX} \frac{dx_i}{dX} dX \simeq \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) \cos \theta \sin \theta dX,
\]

(3-5)

where we neglect the terms involving \(u_z\) and identify \(dx_j/dX\) and \(dx_i/dX\) as the direction cosines of the flow line. To integrate equation (3-5) we make the important and realistic assumption that the velocity shear vanishes outside a finite region in front of the cloud. This assumption is confirmed by Fig. 10 where we can see that in most cases the velocity field is characterized by a strong shear only around the cloud front. We emphasize that if a fixed portion of a fluid line is subject to stretching, then its length, and, hence, its strength, will grow only linearly (as shown below by eq. 3-6), as opposed to an exponential increase occurring when the stretching is constant over the full extent of the fluid line. In addition, we assume for simplicity that such a shear pattern is approximately constant in space. Therefore, integrating eq. (3-5) in both space and time we obtain in that restricted region:

\[
X(a, t) \simeq \int_0^t dt \int_a^{a + \ell_a} \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) \cos \theta \sin \theta dX \simeq a + \frac{1}{2} \ell_a \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) t.
\]

(3-6)

where \(\ell_a\) represents the length of the flow line over which the velocity shear is non-null and the term \(\sin \theta \cos \theta\) has been taken to contribute a factor \(\frac{1}{2}\) coming from integrating along an arc size of the order of \(\frac{\pi}{2}\), as appropriate for the cloud nose. We are now able to derive the quantity of interest, namely \(\partial X/\partial a\). Assuming a laminar flow regime before the cloud and around its nose, it is possible to see that the change of the length (along which the strain occurs) \(\delta \ell_a\) between two adjacent flow lines is of the order of their separation \(\delta a\), or, equivalently \(\delta \ell_a/\delta a = 1\). We approximate also the velocity shear \(\frac{1}{2} \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) \sim \lambda(t) u_c/R_c\), since the flow speed increases from zero on the cloud nose to roughly the cloud speed, \(u_c\), along an arc of length \(\sim R_c\) at the start of the simulation. Here, \(\lambda(t)\) is a slowly varying quantity introduced to correct for all the details of the MHD flow dynamics that have not been included in this simplified analysis. From equations (3-4) and (3-6) we have then

\[
\frac{B}{\rho} \simeq \frac{B_0}{\rho_0} \left(1 + \lambda \frac{u_c}{R_c} t\right),
\]

(3-7)

or in terms of the magnetic energy density (magnetic pressure) \(p_B = B^2/2\),

\[
p_B \simeq p_{B0} \left(\frac{\rho}{\rho_0}\right)^2 \left(1 + \lambda \frac{u_c}{R_c} t\right)^2.
\]

(3-8)

The quantity \(\lambda(t)\) parameterizes the evolution in the flow. In the beginning this mostly represents an expansion of the flow field in response to increased magnetic pressure; that is, the flow lines become more spread out. Eventually, however, as the cloud begins to decelerate, it includes reductions in its asymptotic speed, as well. The details of the cloud dynamics embodied in \(\lambda(t)\) are difficult to estimate. Nevertheless, based on the results of our 3-D simulations and on the theoretical model of Miniati et al. (1999b), we believe that for \(t > \tau_s = R_c/u_c\) a functional form \(\lambda(t) \sim (t/\tau_s)^{-q}\) (with
\( q \sim \frac{1}{2} \) is adequate to capture the main features in the magnetic field evolution. Then, we can rewrite eq. (3-8) as becoming asymptotically

\[
p_B \simeq p_{B0} \left( \frac{\rho}{\rho_0} \right)^2 \left( \frac{t}{\tau_s} \right)^{2(1-q)}.
\] (3-9)

For \( q = \frac{1}{2} \) the magnetic pressure would continue to grow linearly with time. As discussed below, our simulations indeed suggest that the field pressure tends to increase with a power law exponent of order unity, at least until the crushing shock has completely crossed the cloud.

### 3.2.2. Quantitative Analysis and Comparison with Numerical Simulations

Neglecting the initial times \( (t \lesssim \tau_s) \), when compressibility is dominant, in the following we will assume \( \rho \approx \rho_0 \) and only consider the time regime in which equation (3-9) is appropriate in the flow around the cloud. Then, let us estimate the integral form of equation (3-9) over the entire computational volume \( V \). We indicate with \( V_\epsilon \) the region around the cloud where the velocity shear is large, and, therefore, field amplification takes place. Then \( \lambda u_c/R_c \) becomes the average rate of strain over such a volume, outside of which the magnetic field remains approximately equal to its initial value. Typically, \( V_\epsilon \) is of the order of \( V_c \), the cloud volume.

\[
E_B = \int_V p_B dV \simeq p_{B0} V_c \left( \frac{t}{\tau_s} \right)^{2(1-q)} + p_{B0}(V - V_c).
\] (3-10)

Then, we can rearrange (3-10) as

\[
\frac{E_B - E_{B0}}{E_{B0}^c} \equiv \frac{\Delta E_B}{E_{B0}^c} \simeq \left( \frac{t}{\tau_s} \right)^m,
\] (3-11)

where \( m = 2(1 - q) \), \( E_{B0} = p_{B0} V \) is the initial magnetic energy in the computational volume and \( E_{B0}^c = p_{B0} V_c \) is the initial cloud magnetic energy. In Fig. 4 (top panel) we plot the time dependence of \( \Delta E_B/E_{B0}^c \) obtained directly from the numerical simulation to compare with this relation. A lower resolution simulation result (16 zones per cloud radius) is also given for the \( \beta = 4 \) case. Both the high and the low resolution curves are sufficiently close to suggest a good convergence for the global magnetic energy evolution. It is clear that the total magnetic energy in the volume tends to increase monotonically over the simulated time.

Fitting the numerical results in the time interval \( 0.01 \lesssim t/\tau_{cr} \lesssim 1.0 \) to a \( t^m \) dependence, gives a power law index \( m \approx 1.1 \) for \( \beta = 4 \) and \( m \approx 1.4 \) for \( \beta = 100 \). These results suggest \( m \to 1 \), at least during this interval in the cloud evolution. Indeed, a linear increase in the magnetic energy is to be expected. In fact, up to \( t \simeq \tau_{cr} \) the cloud does not suffer any deceleration. Since the system is then supplied with a steady flux of kinetic energy, we may expect that a constant fraction of the latter is converted into magnetic energy at a constant rate. However, Fig. 4 shows that, in both cases, at \( t \gtrsim \tau_{cr} \) the rate of magnetic energy growth drops below linear. This is a signature of the
beginning of the cloud deceleration and the consequent reduction in energy supply, although some outflow of magnetic energy from the computational volume is also occurring (see below). It is of interest to compare the increment in magnetic energy to the evolution of the total kinetic energy inside the computational box. This is also shown in Fig. 4 (bottom panel). From it we learn that at about one crushing time more than 10% of the initial kinetic energy has been converted into magnetic energy for the $\beta = 4$ case. This fraction becomes larger that 15% at simulation end. On the other hand, for the $\beta = 100$ case, the conversion amounts to only a few percent, in accordance with a much larger conversion timescale as determined later (see §3.2.3). We point out that the increments in the magnetic energy correspond to and are responsible for the decrements over time of the kinetic energy (measured with respect to an observer who sees the intercloud medium at rest) for both values of $\beta$ as plotted in Fig. 5. In reality, a closer analysis reveals that at simulation end, the decrements in kinetic energy are somewhat larger than the correspondent magnetic energy increments, indicating that some magnetic energy flux has escaped the computational box. Thus, the final values for the $\Delta E_B$ in Fig. 4 are understood only as lower limits. Comparing results at different resolutions for the $\beta = 4$ case, again indicates a good convergence of the simulations in this property. It is worth mentioning that this strong increase in the magnetic pressure in front of the cloud turns out to be a result rather independent of the initial cloud geometry. In fact, Maxwell stresses will tend to squeezed the cloud into a cylinder-like structure with the axis aligned perpendicular to the plane defined by the initial field and the direction of motion. In this respect, low resolution simulations started with a cylinder as the initial cloud shape, revealed a qualitatively similar behavior in the magnetic energy evolution, although an even stronger increase in the magnetic pressure occured at the cloud nose. Overall these results compare well with the previous 2-D simulations of Miniati et al. (1999b). While the increase of magnetic energy in the 3-D simulation is slightly lower than for the 2-D case, the values are comparable in order of magnitude.

3.2.3. Timescales

In the evolution of the cloud several dynamical timescales can be identified. As the magnetic pressure in front of the cloud increases, the flow progressively tends to be dominated by Maxwell stresses and its global behavior radically changes from the pure hydrodynamical case. Typically, this occurs when the magnetic pressure becomes comparable to the ram pressure of the gas. Gregori et al. (1999) have shown that the onset of this transition is characterized by magnetically enhanced R-T modes that ultimatively disrupt the entire cloud. In Fig. 6 the ratio of the maximum magnetic pressure, $p_B$, along the symmetry axis ($y = z = 0$) with respect to the ram pressure of the intercloud fluid, $\rho_i u_c^2$, is plotted as a function of time for the two values of $\beta$. At the beginning of the simulation $p_{B0}/(\rho_i u_c^2) \approx 0.067$ for $\beta = 4$ and $p_{B0}/(\rho_i u_c^2) \approx 0.003$ for $\beta = 100$. Thus, the ram pressure is initially dominant. However, as shown in Fig. 6 for the $\beta = 4$ simulation, at about one crushing time the magnetic pressure maximum becomes comparable and even larger than the initial ram pressure. Its subsequent decrease is connected to the rapid development of the R-T instability; in fact the field lines move away from the symmetry axis, towards the newly developing indentations (see Fig. 2).
Our analysis of the field inside such structures reveals that the magnetic pressure keeps growing there and at \( t \sim 2\tau_{cr} \) reaches a ratio of 1.66 with respect to the ram pressure. For the \( \beta = 100 \) case we note that, despite a remarkable increase with respect to its initial value, the magnetic pressure maximum remains well below the ram pressure limit throughout the simulation time. Based on the model developed in the previous sections, after the initial transient, from equation (3-9) we estimate

\[
\frac{p_B}{\rho_i u_c^2} \simeq \frac{2}{M_A^2} \left( \frac{t}{\tau_s} \right)^m,
\]

where \( M_A \) is the initial Alfvénic Mach number. Using \( m \sim 1 \), we estimate that the magnetic pressure becomes comparable to the initial ram pressure for \( t \gtrsim (M_A^2/2)^{1/m} \tau_s \sim (M_A^2/2)\tau_s \), or \( t \gtrsim 0.28\tau_{cr} \) (\( \beta = 4 \)) and \( t \gtrsim 7.03\tau_{cr} \) (\( \beta = 100 \)). This is indeed confirmed from Fig. 6. For the \( \beta = 4 \) simulation at \( t > 0.3\tau_{cr} \) the magnetic pressure maximum is already comparable with the ram pressure. On the other hand, in the weak field case, the magnetic pressure maximum remains much below the initial ram pressure even at simulation end. These results roughly confirm our findings for the behavior of the magnetic field at the cloud nose.

Another significant timescale can be defined in terms of the energy exchange between kinetic and magnetic energy, since the stretching of the field lines acts as a conversion mechanism in which the cloud kinetic energy is transformed into magnetic energy. By analogy with Miniati et al. (1999b) we can estimate the time required for the magnetic energy to equal the initial kinetic energy of the cloud by solving the equation

\[
\frac{1}{2} \rho_c u_c^2 V_c = E_B - E_{B0} \simeq p_{B0} V_c \left( \frac{t}{\tau_s} \right)^m.
\]

We easily get the characteristic timescale

\[
\tau_{ma} = \left( \frac{\chi M_A}{4} \right)^{1/m} \tau_s \simeq \left( \frac{\chi M_A}{4} \right) \tau_s.
\]

where the last term has been obtained by setting \( m = 1 \), as previously observed. In this respect, \( \tau_{ma} \) must be intended as a characteristic braking time, since it sets an upper limit for the cloud motion across the ISM. We should note that in absence of magnetic field, the characteristic time required to stop the cloud is given by \( \tau_d = \chi \tau_s \) (e.g., Jones et al. 1994). It is then evident that the cloud motion is magnetically dominated only if \( \tau_{ma} \lesssim \tau_d \). By comparison with (3-14), we see that this is the case if \( M_A < 4 \). This indicates that for \( \beta = 4 \) \( (M_A = 2.74) \) the magnetic field plays a dynamically important role from the beginning. On the other hand, in the \( \beta = 100 \) case, \( M_A = 13.69 \) and the hydrodynamic drag is the relevant timescale in determining the cloud evolution; thus in this case magnetic effects are likely not to be important. Mouschovias & Paleologou (1979) and Elmegreen (1981) have calculated the characteristic time required by the magnetic field to stop the cloud using a different approach based on the radiated energy through Alfvén waves. Typically, for transverse field geometry and an infinitely long cylindrical cloud, they estimate that the timescale for magnetic braking is of the order of \( \frac{1}{2} \chi M_A \tau_s \) which, apart from a numerical factor, 2, is equivalent to our result.
The timescale (3-14) can be recast in a dimensional form:

\[
\tau_{ma} = 7.04 \times 10^{10} \chi \beta^{1/2} \left( \frac{R_c}{0.1 \text{ pc}} \right) \left( \frac{10^6 \text{ cm/s}}{c_s} \right) \text{s.} \tag{3-15}
\]

The characteristic time for radiative cooling is given by (Spitzer 1978)

\[
\tau_{cool} = \frac{3 k_B T}{2 n_i \Lambda} \simeq 1.6 \times 10^{14} \text{s,} \tag{3-16}
\]

where \(n_i\) is the number density in the intercloud medium and \(\Lambda\) is the interstellar cooling function. For temperatures of the interstellar gas in the order of \(10^4 \text{ K}\) and densities \(n_i \sim 0.1 \text{ cm}^{-3}\), we have \(n_i \Lambda \sim 1.3 \times 10^{-26} \text{ erg/s}\) (Ferrara & Field 1994) to give the numerical value in equation (3-16). We clearly see that for the values of the parameters \(\chi\) and \(\beta\) used in the simulations, the evolution of the system remains always strictly adiabatic (that is, \(\tau_{ma} < \tau_{cool}\)) only for clouds with radius \(R_c \lesssim 0.1 \text{ pc}\). In the case of larger clouds, radiative effects have still a small influence on the cloud dynamics if the cooling time is larger than the characteristic time for re-expansion of the postshock interstellar material as it flows around the cloud. Such time is in the order of \(R_c/c_s\) (Miniati et al. 1997). From the condition \(\tau_{ma} > R_c/c_s\) we then obtain that cooling remains negligible for clouds radii \(R_c \lesssim 10 \text{ pc}\).

### 3.3. Flux Ropes

When the magnetized fluid flows into the cloud wake, the field lines frozen in it produce elongated structures of strong field concentration surrounded by a thin vortex and current sheet. These filamentary structures are clearly visible in Figs. 7 and 8. Following Mac Low et al. (1994) we identify these regions as “flux-ropes”, where individual flux tubes are organized in a coherent pattern. In the simulations presented here, however, we see no signs of twist in them. The complete 3-D structure is illustrated in Fig. 9 for \(t = 0.94 \tau_{cr}\). In both the the strong and the weak field simulations, the flux ropes converge toward the symmetry axis where the thermal pressure is low (Mac Low et al. 1994). There are then strong similarities between our results and the 2-D simulations of Mac Low et al. (1994) and Jones et al. (1996), which show the same flux rope structures for the transverse field geometry. From Fig. 7 we can see that in the weak field (\(\beta = 100\)) simulation the flux rope remains very close to the symmetry axis for almost its entire length. Inside the flux rope the field strength at this time (\(t = 0.94 \tau_{cr}\)) is comparable to that at the leading edge of the cloud. In the cloud wake the dominant enhancement of the field strength is probably produced by the compression of the rope on the symmetry axis. Such compression, in turn, is due to the large gradient of thermal pressure produced there by the cloud motion. We should also note that some additional field amplification may occur as the fluid crosses the tail shocks. Conversely, in the strong field, \(\beta = 4\), simulation (Fig. 8), the flux rope is kept close to the symmetry axis only in a small region just at the cloud rear (a few cloud diameters in size). Farther away it opens up in “wings” that tend asymptotically to align with the initial unperturbed field direction. Finally, the
magnetic field value \((t = 0.94\tau_{cr}, \beta = 4)\) in these ropes is on average a factor ~ 2 larger than the background field.

As we can see in Fig. 10, an important feature of the flux ropes is that the plasma in them is dynamically tied to the cloud; that is, the plasma there moves with the cloud. This is a very remarkable characteristic because the flux ropes, formed by magnetic field lines stretched around the shape of the cloud, do not carry cloud material per se (demonstrated through the tracer variable \(C\)). The separation of the flux ropes from the external flow occurs through a tangential discontinuity, with the magnetic field parallel to the interface and no mass flow across it. Such a discontinuity is characterized by a constant total pressure (magnetic plus thermal) across the separation interface. However, the higher magnetic pressure inside the rope is balanced by a considerably lower density there, compared to the outside fluid material. As already noted, there is little or no twist generated by magnetic or current helicity in the flux ropes. They are sheet-like. Similar sheet-like morphologies were noted in strong magnetic structures formed in high resolution 3-D MHD Kelvin-Helmholtz instability simulations (Ryu et al. 2000). In that case it was clear that the sheet-like morphology was a consequence of the strong magnetic field and not a resolution effect. This may be true here as well, but our resolution is not sufficient to assess that.

### 3.4. Magnetic Reconnection

The ideal MHD approximation breaks down in regions where resistive effects or diffusion dominate the field evolution. Even if the scale length of these regions is very small, their development may change the global topology of the field lines (Song & Lysak 2000). The most important of such events is magnetic reconnection. This has been observed in the heliosphere and in various 2-D and 3-D numerical simulations (e.g., Biskamp 1994, Miniati et al. 1999b, Antiochos & De Vore 1999, Ryu et al. 2000). Miniati et al. (1999b), using the 2-D version of the MHD code applied here, showed clear 2-D reconnection in the wake of a cloud, apparently due to tearing mode instabilities. Our new numerical simulations confirmed the previous results for clouds and showed that, indeed, reconnection takes place in the wake of the cloud. In fact from Figs. 7 and 8, it is clear that on the symmetry plane, \(y = 0\), a pair of thin current sheets form and that field annihilation occurs there. Here, the reconnection process seems steady, however, that can be understood by examining the properties of the reconnection region. The two figures show that the ratio \(L/\delta\) between the length and the full thickness of the current sheet is ~ 15-25 for both the strong and weak field simulations. The current sheet structure, for the \(\beta = 4\) case, reveals strong similarities with the classic 2-D Sweet-Parker current sheet model (e.g., Biskamp 1993). In this respect, the recent 2-D resistive MHD simulations of Uzdensky & Kulsrud (1999) also seem to confirm that reconnection tends spontaneously to evolve toward a Sweet-Parker steady state. Additional analogies with other reconnection models, e.g., Syrovatskii's solutions (Biskamp 1993), can also be found in the branching of the “separatrix” on the right side of the sheet. In the presence of a reconnection layer the issue regarding its stability immediately is raised. Stability against tearing modes is expected if
\( S \sim (L/\delta)^2 \lesssim 10^4 \) (Biskamp 1993), where \( S \) is the Lundquist number of the current sheet. This is indeed in agreement with our present simulations, which show no development of such an instability. That contrasts as expected with our previous 2-D simulations where the tearing mode instability was, in fact, observed in the analogous regions Miniati et al. (1999b). In order to understand the difference between the two cases, we recall that the Lundquist number is actually defined as

\[
S = \frac{v_A L}{\eta},
\]

(3-17)

where \( v_A \) is the Alfvén velocity of the magnetic field on the top and bottom of the current sheet and \( \eta \) is the fluid resistivity. In the Sweet-Parker reconnection model \( S \) takes the form \( S = (L/\delta)^2 \). In simulations of ideal MHD \( \delta \) is limited by numerical discretization and its value is \( \sim 2-3\Delta x \), where \( \Delta x \) is the size of a numerical zone. The length of the reconnection region in the cloud wake will generally be several cloud radii, typically 5-10. This implies that the values for \( L/\delta \) are numerically limited: \( S \lesssim (2-3 \times N_c)^2 \), where \( N_c \) is the number of numerical zones per cloud radius. In our current 3-D simulations, \( N_c = 26 \), so \( S < 10^4 \), and the layer should be stable to tearing modes as, indeed, it is. On the other hand, our 2-D simulations were done at roughly twice the resolution across the clouds, with \( N_c = 50 \), so the current sheet in the tail was characterized by \( S > 10^4 \). It should have been, and was, tearing mode unstable.

4. Summary & Concluding Remarks

In this paper we have presented the results of a series of 3-D MHD numerical simulations of cloud-wind interactions. We have considered an initial spherical cloud that moves transverse to the magnetic field, with two different cases for its initial strength; namely \( \beta = p/p_B = 4 \) and \( \beta = 100 \). Both the weak (\( \beta = 100 \)) and strong field (\( \beta = 4 \)) simulations showed a qualitatively comparable behavior with a substantial enhancement of the magnetic pressure at the leading edge of the cloud. This confirms and extends previous 2-D results (Jones et al. 1996, and Miniati et al. 1999b). A new, detailed analysis of the field stretching that occurs in front of the cloud is presented here. Qualitative agreement with the numerical simulations is obtained. In particular, it is shown that magnetic effects tend to be important only if the initial Alfvénic Mach number is sufficiently small. In this case the cloud dynamical evolution (acceleration and disruption) occurs on shorter timescales and a substantial rapid conversion of kinetic into magnetic energy takes place. Thus, at \( t \lesssim \tau_{cr} \), the magnetic pressure becomes comparable with or greater than the ram pressure. On the other hand, if the initial Alfvénic Mach number is large, then the evolution of the cloud is still dominated by the hydrodynamic drag and conversion of kinetic into magnetic energy occurs at a much smaller rate.

For typical values of the magnetic field in the ISM, the propagation of a cloud through a magnetized medium should produce inhomogeneities on a scale comparable to the cloud size. As already pointed out in Miniati et al. (1999b), this could help explain observed fluctuations in the galactic magnetic field (Heiles 1989, Meyers et al. 1995). Observations of Gloeckler et al. (1997)
in the local interstellar cloud also seem to suggest the possibility of an inhomogeneous distribution of the field intensity. Moreover, we illustrate a complex series of topological modifications that take place in the cloud wake and characterize the dynamics of the flow there. In the tail of the cloud the field lines aggregate in a coherent pattern to form long, sheet-like flux ropes. The plasma entrained there moves approximately with the cloud. Additionally, along the symmetry axis, magnetic reconnection takes place. Evolutionary details such as the formation of vortical structures at the cloud interface, as well as the development of instabilities of the reconnection current sheet were prevented by the limited numerical resolution. Despite these limitations, the simulations reported here give a significant picture of the fundamental processes governing cloud-wind interaction in a magnetized ISM, setting important constraints for further observational and theoretical studies. In this respect, our results may give support to recent models of cloud-wind interactions such as those proposed to explain the formation of nonthermal filaments in the galactic center (Shore & LaRosa 1999). Investigations of planetary nebulae interacting with a magnetized fluid may also take advantage of the results presented here (see e.g., Soker & Dgani 1997, Dgani & Soker 1998).

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FIGURE CAPTIONS

Fig. 1.— Schematic of the numerical setup.

Fig. 2.— Slice of log(\(\rho\)) on the plane y = 0 at t = 0.94\(\tau_{cr}\) (left column) and t = 2.25\(\tau_{cr}\) (right column). Top: \(\beta = \infty\), center: \(\beta = 100\), bottom: \(\beta = 4\).

Fig. 3.— Relative cloud momenta in the y and z directions (see text).

Fig. 4.— Top: magnetic energy increment in the computational volume normalized with respect to the initial magnetic energy inside the cloud (\(\Delta E_B/E_{B0}^c\)). Bottom: magnetic energy increment in the computational volume normalized with respect to the initial kinetic energy of the cloud (\(\Delta E_B/E_{K0}^c\)). The high resolution (hr) simulation corresponds to case 3 in Table 1 and the low resolution (lr) simulation corresponds to case 4 in Table 1.

Fig. 5.— Total kinetic energy in the computational volume normalized with respect to its initial value (\(E_K/E_{K0}\)). The high resolution (hr) simulation corresponds to case 3 in Table 1 and the low resolution (lr) simulation corresponds to case 4 in Table 1.

Fig. 6.— Maximum value of the magnetic pressure on the symmetry axis (\(y = z = 0\)) with respect to its initial ram pressure of the fluid.

Fig. 7.— Slice in the plane \(z = 0\) for the \(\beta = 100\) simulation at t = 0.94\(\tau_{cr}\). log(\(\rho\)) top left, log(\(B^2/2\)) top right, log(\(\omega^2 + 1\)) bottom left, log(\(j^2 + 1\)) bottom right. Here, \(\omega = \nabla \times \mathbf{u}\) is the vorticity and \(\mathbf{j} = \nabla \times \mathbf{B}\) is the current density. The \(B_x\) and \(B_y\) components of the magnetic field are represented by the arrows.

Fig. 8.— Slice in the plane \(z = 0\) for the \(\beta = 4\) simulation at t = 0.94\(\tau_{cr}\). log(\(\rho\)) top left, log(\(B^2/2\)) top right, log(\(\omega^2 + 1\)) bottom left, log(\(j^2 + 1\)) bottom right. Here, \(\omega = \nabla \times \mathbf{u}\) is the vorticity and \(\mathbf{j} = \nabla \times \mathbf{B}\) is the current density. The \(B_x\) and \(B_y\) components of the magnetic field are represented by the arrows.

Fig. 9.— Left column: volume rendering of the magnetic pressure (log scale) for the \(\beta = 100\) simulation (top) and \(\beta = 4\) (bottom). Right column: volume rendering of the cloud density (log scale) for the \(\beta = 100\) simulation (top) and \(\beta = 4\) (bottom). Time is expressed in units of \(\tau_{cr}\).

Fig. 10.— Slice of log(\(\rho\)) on the plane \(z = 0\) at t = 0.94\(\tau_{cr}\) (left column) and t = 2.25\(\tau_{cr}\) (right column). Top: \(\beta = \infty\), center: \(\beta = 100\), bottom: \(\beta = 4\). The \(u_x\) and \(u_y\) components of the velocity field are represented by the arrows.
Table 1. Summary of 3-D MHD cloud simulations.

| Case | Symmetry plane(s)\(^a\) | Resolution\(^b\) | Initial field direction \((l_x, l_y, l_z)\)\(^c\) | Shape | \(M\) | \(\chi\) | \(\beta\) | \(M_A\) |
|------|-------------------------|-----------------|---------------------------------|-------|------|-------|-------|-------|
| 1    | \(x-y, y-z\)            | 26              | (0,1,0)                         | (1,1,1) | 1.5  | 100   | \(\infty\) | \(\infty\) |
| 2    | \(x-y, y-z\)            | 26              | (0,1,0)                         | (1,1,1) | 1.5  | 100   | 100   | 13.7  |
| 3    | \(x-y, y-z\)            | 26              | (0,1,0)                         | (1,1,1) | 1.5  | 100   | 4     | 2.74  |
| 4    | \(x-y, y-z\)            | 16              | (0,1,0)                         | (1,1,1) | 1.5  | 100   | 4     | 2.74  |
| 5    | none                    | 16              | \(\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)\) | (1,1,2) | 3.0  | 100   | 10    | 8.66  |
| 6    | none                    | 16              | \(\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)\) | (1,1,2) | 3.0  | 10    | 10    | 8.66  |
| 7    | none                    | 16              | \(\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)\) | (1,1,2) | 3.0  | 30    | 10    | 8.66  |
| 8    | none                    | 16              | \(\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)\) | (1,1,2) | 3.0  | 100   | 2     | 3.87  |
| 9    | none                    | 16              | \(\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)\) | (1,1,2) | 3.0  | 100   | 4     | 5.48  |

\(^a\)plane(s) across which symmetric boundary conditions are employed in the numerical computations.

\(^b\)in number of zones per cloud radius, \(R_c\).

\(^c\)initial cloud axis in units of \(R_c\). (1,1,1) corresponds to a sphere and (1,1,2) corresponds to a cylinder with the axis in the \(z\) direction.
\[ \frac{R}{R_e} \]

\[ \frac{t}{\tau_{ce}} \]

- \( R_\tau (\beta = 4) \)
- \( R_\gamma (\beta = 4) \)
- \( R_\gamma (\beta = 00) \)
- \( R_\gamma (\beta = 100) \)

\( R_\gamma = R_e (\beta = \infty) \)
