Classical sum rules and spin correlations in photoabsorption and photoproduction processes

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Abstract

In this paper we study the possibility of generalizing the classical photoabsorption ($\gamma a \rightarrow bc$) sum rules, to processes $bc \rightarrow \gamma a$ and crossed helicity amplitudes. In the first case, using detailed balance, the sum rule is written as

$$\int_{\nu_{th}}^{\infty} K \Delta \sigma_{\text{Born}}(\nu) = 0$$

where $K$ is a kinematical constant which depends only on the mass of the particles and the center of mass energy. For other crossed helicity amplitudes, we show that there is a range of values of $s$ and $t$ for which the differential cross section for the process $\gamma b \rightarrow ac$ or $ac \rightarrow \gamma b$ in which the helicities of the photon and particle $a$ have specific values, is equal to the differential cross section for the process in which one of these two helicities is reversed (parallel-antiparallel spin correlation).

I. INTRODUCTION

Deep inelastic and photon scattering sum rules have been historically an important tool for obtaining information about hadron structure \[1\]. Some of them, such as the Bjorken polarized deep inelastic sum rule, are exact QCD results, and therefore constitute stringent tests of this theory. Others are based on various assumptions which in some cases have been proved to be incorrect, but even here these sum rules have been useful. For example,
the failure of the Gottfried sum rule gave essential information about the proton sea. More recently, the failure of the Ellis-Jaffe sum rule indicated the presence of polarized strange quarks inside the nucleon.

In 1965 Drell and Hearn [2], and independently Gerasimov [3], found an amazing equation that relates the square of the anomalous magnetic moment of a particle to the logarithmic integral of the difference of the cross section for the absorption of a photon with spin parallel or anti-parallel to the target spin:

\[ \mu_a^2 = \frac{4\pi\alpha S}{M^2}(g - 2)^2 = \frac{S}{\pi} \int_{\nu_{th}}^\infty \frac{d\nu}{\nu} \Delta \sigma(\nu), \] (1)

for a target of spin \( S \), whether elementary or composite.

This sum rule has interesting theoretical implications. For example, in pure QED \( \Delta \sigma \) is of order \( \alpha^2 \) (elastic Compton scattering), which means that \( g - 2 = 0 + \mathcal{O}(\alpha) \), and therefore \( g = 2 \) at tree level for a particle of any spin. But then:

\[ \int_{\nu_{th}}^\infty \frac{d\nu}{\nu} \Delta \sigma_{\text{QED}}(\nu) = \mathcal{O}(\alpha^3), \] (2)

This remarkable result was derived by Altarelli, Cabibbo and Maiani [4], and later generalized using loop counting arguments [5] to any process of the form \( \gamma a \to bc \):

\[ \int_{\nu_{th}}^\infty \frac{d\nu}{\nu} \Delta \sigma_{\text{tree}}(\nu) = \int_{\nu_{th}}^\infty \frac{d\nu}{\nu} (\sigma_{P}^{\gamma a\to bc}(\nu) - \sigma_{A}^{\gamma a\to bc}(\nu)) = 0, \] (3)

where the initial photon and particle \( a \) are polarized (parallel (P) or antiparallel (A) to each other). This result provides non-trivial checks on calculations, and is a potential diagnostic tool for new physics. A further consequence is the fact that the vanishing of this logarithmic integral also implies that there must be a center of mass energy where \( \Delta \sigma(s) \) vanishes, a result which can be used in phenomenological studies. In fact, it has been applied to the process \( \gamma e \to W\nu \) to probe anomalous trilinear gauge couplings \( \gamma WW \) [6], in heavy quark electro and photoproduction reactions [7,8], in studies of the spin-dependent photon structure function [9], and even in quantum gravity [10].
In this paper, we study the possibility of generalizing the later results for processes of the form \( bc \rightarrow \gamma a \), \( ac \rightarrow \gamma b \) or \( \gamma c \rightarrow ab \). In all cases we consider the correlation between the helicities of the photon and particle “a”.

The paper is organized as follow. In section II we obtain a DHG-like sum rule for the process \( bc \rightarrow \gamma a \). Section III is devoted to the study of some properties of crossed helicity amplitude in order to determine whether it is possible to build a DHG-like sum rule for processes of the form \( ac \rightarrow \gamma b \) or \( \gamma c \rightarrow ab \). We finish summarizing our main conclusions.

II. A CLASSICAL PHOTOPRODUCTION SUM RULE

A. Deduction of the Sum Rule

The processes \( \gamma a \rightarrow bc \) and \( bc \rightarrow \gamma a \) are described by the same amplitude and their center of mass systems are the same, so in general we can write, using detailed balance:

\[
\sigma(\gamma a \rightarrow bc) = K \sigma(bc \rightarrow \gamma a)
\]

where \( K \) is a constant that depends only on the masses and the center of mass energy and which takes into account the different phase spaces of the processes. This constant can be easily calculated in the center of mass system and we obtain:

\[
K = \frac{1}{(1 - \frac{m^2}{s})^2} \left(1 - 2\left(\frac{m_b^2 + m_c^2}{s}\right) + \frac{(m_b^2 - m_c^2)^2}{s^2}\right).
\]

Therefore equation \( (3) \) becomes:

\[
\int_{\nu_{th}}^{\infty} \frac{d\nu}{\nu} K(\sigma_P^{bc\rightarrow\gamma a}(\nu) - \sigma_A^{bc\rightarrow\gamma a}(\nu)) = 0.
\]

In this case \( \sigma_P \) and \( \sigma_A \) refer to the cross sections for parallel and anti-parallel spins for the final state photon and particle \( a \), and \( \nu \) is the energy of the photon.

Because the kinematical constant \( K \) is defined as a ratio of cross sections (see eq. \( (2) \)), it cannot be zero while the two channels are open. In fact \( K \) is always positive. This
observation and equation (6) imply that the quantity \((σ_P^{bc→γa}(ν) − σ_A^{bc→γa}(ν))\) must vanish at a certain energy \(ν = ν_c\).

A further consequence is that in the photoabsorption process \(γa → bc\) there is a range of values of the Mandelstam variables \(s = s_0\) and \(t = t_0\) for which the differential cross sections for the processes where the initial photon and particle \(a\) are polarized parallel and antiparallel to each other are equal:

\[
\left(\frac{dσ}{dt}\right)_P(s_0, t_0) = \left(\frac{dσ}{dt}\right)_A(s_0, t_0),
\]

and the same holds for the photoproduction process \(bc → γa\). A specific example of these results will be shown in the next section.

Thus we have obtained a new sum rule for final state polarized photons, which implies a zero in the corresponding difference of total and differential cross sections.

B. An Illustrative Example

In order to illustrate the previous results, we consider the electron-positron annihilation to a pair of photons. In this case the difference between the cross sections for parallel and anti-parallel spins in the final states in the center of mass system can be written as:

\[
σ_P − σ_A = \frac{e^4}{32πm^2} \frac{(1 − y^2)}{y^2} \left[3y − \log \left(\frac{1 + y}{1 − y}\right)\right]
\]

where \(m\) is the mass of the electron and \(y\) is defined as:

\[
y = \sqrt{1 − \frac{4m^2}{s}}
\]

In this example the kinematical constant \(K\) and the photon energy \(ν\) take the simple form:

\[
K = y^2
\]

\[
ν = \frac{m}{\sqrt{1 − y^2}}
\]
With these ingredients we can write:

\[
\int_0^\infty \frac{du}{u} K (\sigma_P - \sigma_A) = \frac{e^4}{32 \pi m^2} \int_0^1 dy y \log \left( \frac{1 + y}{1 - y} \right) = 0 \quad (12)
\]

From eq. (8) we can see that the quantity \( \sigma_P - \sigma_A \) vanishes for \( y \approx 0.859 \) or, using (9), \( \sqrt{s} \approx 7.63 m \).

III. HELICITY CORRELATION BETWEEN AN INITIAL (FINAL) POLARIZED PHOTON AND A FINAL (INITIAL) PARTICLE

Let us consider now processes of the form \( bc \to \gamma a \) and \( \bar{a}c \to \gamma \bar{b} \) (crossed), or equivalently using detailed balance \( \gamma a \to bc \) and \( \gamma \bar{b} \to \bar{a}c \). It is evident that equation (6) implies the existence of some \( s = s_0 \) and \( t = t_0 \) which satisfy

\[
\left| M_{bc \to \gamma a} \right|^2 (s_0, t_0) - \left| M_{\bar{b}c \to \gamma \bar{a}} \right|^2 (s_0, t_0) = 0 \quad (13)
\]

where \( M_{bc \to \gamma a} \) designates the amplitude of the process \( bc \to \gamma a \). As noted before, this means that the corresponding differential cross sections are also equal at the same values \( s_0 \) and \( t_0 \). In order to study the crossed processes it is necessary to know how the left part of the previous equation transforms under crossing.

It is a known result that helicity amplitudes transform under crossing as [11]:

\[
M_{bc \to \gamma a}(s, t) = \sum_{a',b',c',\gamma'} d_{\gamma' \gamma}^{d_{a' a}(d_{b' b}(d_{c' c}^{\gamma}(\chi_{c} c)(\chi_{b} b)(\chi_{a} a)))} M_{\bar{c}a \to \gamma b}(s, t) \quad (14)
\]

where \( \gamma, a, b, c \) are helicity indexes of the particles represented by the same letter, \( s_i \) represents the spin of the particle “i”, \( d(\chi) \) are the Wigner matrices and \( M (\tilde{M}) \) represents the amplitude of the original (crossed) process.

From the previous equation it is easy to prove that the left part of equation (13) transforms as:

\[
\left| M \right|^2_p - \left| M \right|^2_A = \kappa \left( \left| \tilde{M} \right|^2_p - \left| \tilde{M} \right|^2_A \right) \quad (15)
\]

where
\[ \kappa = \begin{cases} 
-1 & \text{if } a \text{ is a fermion} \\
1 & \text{if } a \text{ is a massless vector} \\
\cos(\chi_a) & \text{if } a \text{ is a massive vector} 
\end{cases} \]

with, in our case,

\[ \cos(\chi_a) = \frac{(s + m_a^2)(t + m_a^2 - m_b^2) - 2m_a^2(m_a^2 - m_b^2 + m_c^2)}{(s - m_a^2)\sqrt{[t - (m_a + m_b)^2][t - (m_a - m_b)^2]}} \quad (16) \]

As an example of the meaning of equation (15), consider the processes \( e^+e^- \to \gamma\gamma \) and \( e^-\gamma \to e^-\gamma \), where we will study the helicity correlation between the photons. It can be shown that:

\[ |M|_P^2 - |M|_A^2 = \frac{2e^4}{(t - m^2)^2(u - m^2)^2} \left\{ (tu - m^4)(6m^4 + t^2 + u^2 - 4m^2t + 4m^2u) - m^2s^2(s - 2m^2) \right\} \quad (17) \]

\[ |\tilde{M}|_P^2 - |\tilde{M}|_A^2 = \frac{2e^4}{(\tilde{s} - m^2)^2(\tilde{u} - m^2)^2} \left\{ (\tilde{s}\tilde{u} - m^4)(6m^4 + \tilde{s}^2 + \tilde{u}^2 - 4m^2\tilde{s} + 4m^2\tilde{u}) - m^2\tilde{t}^2(\tilde{t} - 2m^2) \right\} \quad (18) \]

From equations (17) and (18), it is evident that if \( s = s_0 \) and \( t = t_0 \) are such that they satisfy equation (13) then \( \tilde{s} = t_0 \) and \( \tilde{t} = s_0 \) satisfy

\[ |\tilde{M}|_P^2 (\tilde{s} = t_0, \tilde{t} = s_0) - |\tilde{M}|_A^2 (\tilde{s} = t_0, \tilde{t} = s_0) = 0 \quad (19) \]

Figures 1 and 2 show the solution of equation (13) for the systems \( e^+e^- \to \gamma\gamma \) and \( W^+W^- \to \gamma\gamma \) respectively, in a range of \( t \) and \( s \) values. In both figures the continuous line shows the zeros of equation (13) that are physical for the direct process but unphysical for crossed system (\( \tilde{s} = t \) would be negative). On the other hand, the pointed curve shows the zeros of equation (13) which are unphysical for the direct process but are perfectly meaningful for the crossed one.
IV. CONCLUSIONS

In this paper we have shown that the classical polarized photoabsorption sum rules for the process $\gamma a \rightarrow bc$, given in equation \ref{eq:3}, can be easily generalize using detailed balance to polarized photoabsorption processes, $bc \rightarrow \gamma a$. This means that there are values of the Mandelstam variable $s$ for which the total cross section for the processes where the initial photon and particle $a$ are polarized parallel or antiparallel to each other are equal, and furthermore that there is a range of values of the Mandelstam variables $s = s_0$ and $t = t_0$ for which the differential cross sections for these processes are also equal. Moreover, using crossing symmetry, we also showed that there is a range of values of $s$ and $t$ for which the differential cross section for the process $\gamma b \rightarrow ac$ (or $ac \rightarrow \gamma b$) in which the helicities of the photon and particle $a$ have specific values, is equal to the differential cross section for the process in which one of these two helicities is reversed.

These results could have interesting phenomenological consequences. Perhaps a practical application is $q\bar{q} \rightarrow \gamma^* g$ in high transverse Drell-Yan processes. Here the photon polarization can be in principle measured via the lepton pair angular distribution. The parallel-antiparallel spin correlation could be that of the photon with the target proton polarization. Another possibility is in strange quark production, where its polarization is reflected in a leading $\Lambda$. One example is semiinclusive DIS, where the incident photon polarization can be controlled by the electron polarization. Of course in these two applications we are really considering virtual photons, and the idea is to consider the behavior of the spin correlation as a function of photon virtuality.

Acknowledgements

We thank Stanley Brodsky for helpful comments. This work was supported by Fondecyt (Chile) grants No. 8000017 and 3020002 (A.Z.).
FIG. 1. Values of $s$ and $t$ for which the differential cross section for the process in which the helicities of the photon and particle $a$ have specific values is equal to the differential cross section for the process in which one of the two helicities is reversed. In the particular case shown here $a$ is also a photon. So the continuous line shows the zero for $e^+e^- \rightarrow \gamma\gamma$, and the dotted line for the crossed process $\gamma e^- \rightarrow \gamma e^- \ (m$ is the electron mass).

FIG. 2. Same as figure 1, but for the process $W^+W^- \rightarrow \gamma\gamma \ (M$ is the $W$ mass).
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