DUAL LATTICE SIMULATIONS OF FLUX TUBES

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ABSTRACT

Abelian gauge theories formulated on a space-time lattice can be used as a prototype for investigating the confinement mechanism. In $U(1)$ lattice gauge theory it is possible to perform a dual transformation of the path integral. Simulating the obtained dual theory (which corresponds to a certain limit of a dual Higgs model) including external sources, we perform a very accurate analysis of flux tubes with respect to the dual superconductor picture. Dual flux tube simulations are also performed in the full Abelian Higgs model, in order to obtain non-perturbative control over quantum and string fluctuations, and for a comparison to the results of dual QCD.

1. Introduction

The formation of colour-electric flux tubes provides an intuitive physical picture for permanent confinement of quarks within hadrons. Moreover, this picture seems to be realized within QCD, as has widely been observed in lattice simulations. The mechanism leading to flux tube formation, however, is still subject to analytical and numerical investigations.

The shortcoming of the corresponding lattice calculations, however, is the fact that regarding sufficiently long flux tubes, i.e. large lattice sizes, becomes a numerically very difficult task. A simpler gauge theory which also exhibits confinement is $U(1)$; it can be used as a prototype for investigating the formation of flux tubes.

Furthermore, it was realized many years ago\[1\] that one can perform a duality transformation of the path integral in compact Abelian gauge theories. In this way a new partition function is obtained which can be regarded as a limit of the dual non-compact Abelian Higgs model.\[2\] Besides this fact, the dually transformed theory can be used as a very efficient tool for the calculation of expectation values in the presence of external charges.\[3,4\] The numerical advantages of such dual flux tube simulations also hold for the more general case of dual Higgs models.

In this talk we review the results obtained in dually transformed $U(1)$ lattice gauge theory and present some first calculations in the dual Abelian Higgs model. We will focus on one of the key questions within the dual superconductor picture, namely whether the confinement vacuum corresponds to a type-I or a type-II superconductor, and we will investigate the role of quantum fluctuations of the dual degrees of freedom.
2. Dually transformed $U(1)$ lattice gauge theory

The expectation value of a physical observable like the electric field or the magnetic current in the presence of external sources (represented by Polyakov loops) can be rewritten as averaging over the dual degrees of freedom, as has been shown in previous work. The dual representation of the Polyakov loops can be interpreted as a dual Dirac sheet connecting the electric charges.

This dual formulation opens the possibility for very accurate numerical calculations for several reasons: Contrary to standard lattice simulations, the confinement phase is the weakly coupled one in the dual theory, therefore there are less quantum fluctuations. Even more important is the fact that it is not necessary to project the charge–anticharge state out of the vacuum: On a lattice of same size charge pairs with arbitrary distance can be simulated with equal accuracy, and increasing the time extent of the loops (i.e., of the lattice) does not influence the quality of the result, either.

Further advantages are the implementation of doubly charged flux tubes by double Dirac strings which also yields results of equal accuracy, and the extension to periodically closed flux tubes. These “torelons” are free of end effects and allow for a more reliable calculation of pure flux tube properties like the string tension.

An interesting physical task is the testing of the validity of a dual London equation for $U(1)$ flux tubes. In the dual formulation we were able to investigate rather long flux tube lengths up to 20 lattice spacings. Our results showed that the agreement between the $U(1)$ data and the predictions of a classical model of Maxwell and London equations are very good for small charge distances, but there is absolutely no agreement at large distances (> 10 lattice spacings). A possible interpretation within the dual superconductor picture is that the confining $U(1)$ behaves as an effective type-I superconductor, rather than an extreme type-II as an exact validity of the dual London equation would suggest. In fact we got support for this conjecture also from a completely independent investigation: simulating doubly charged flux tubes. We found that in four-dimensional $U(1)$ there is an attractive interaction between flux tubes for the coupling $\beta$ approaching the phase transition.

Thus one might look at $U(1)$ lattice gauge theory from two points of view: From a “microscopical” it can be regarded as a double limit of a dual Higgs model with both vanishing London penetration length and vanishing coherence length. From a “macroscopical” point of view it looks like a classical dual type-I superconductor. This peculiarity is related to the fluctuations in the dual theory: The observed attraction between flux tubes was shown to be a purely “quantum-mechanical” effect of the dually transformed $U(1)$ theory.

3. The dual Abelian Higgs model

The action of the full Abelian Higgs model including a dual Dirac string (denoted with $*n$) can be written in the form

$$S = \beta \sum_{\text{plaqs.}} (d^*\theta + 2\pi n)^2 - 2\gamma \sum_{\text{links}} |\Phi|^2 \cos (d^*\chi - *\theta) + \kappa \sum_{\text{sites}} V(*\Phi),$$

(1)

where $*\theta$ is the dual gauge field and $*\Phi$ the dual Higgs field, located on the sites of the dual lattice. $V(*\Phi)$ is the symmetry-breaking Higgs potential with its minimum...
at $\Phi = 1$. For the squared covariant derivative of the Higgs current the compact formulation has been chosen, because this allows for fluctuations of the fluxoid string. In the classical model $\beta/\gamma$ is the squared London penetration depth, and $\gamma/\kappa$ the squared coherence length. The double limit $\gamma \to \infty$, $\kappa \to \infty$ exactly corresponds to the dually transformed $U(1)$ theory discussed in the last section.

If we compare this model to the parameters of dual QCD after an Abelian ansatz, we find the correspondence

$$
\beta = \frac{4}{3} \frac{1}{g^2}, \quad \gamma = 8 B_0^2 a^2, \quad \kappa = \frac{100}{3} \lambda B_0^4 a^4,
$$

where $g$ is the (chromo-)magnetic coupling, $B_0$ the location of the minimum and $\lambda$ the strength of the corresponding continuum Higgs potential, and $a$ is the lattice spacing. These parameters can be chosen to reproduce very well the infrared behaviour of QCD. The classical flux tube solutions of the Higgs model on sufficiently fine lattices give very good agreement with corresponding continuum calculations. It is now an interesting question how quantum and string fluctuations (of the fluxoid string) do effect the physical quantities, e. g. the string tension.

For the set of parameters which yields the desired classical solution, the system is not in the Higgs phase any more on a reasonably fine lattice, if fluctuations are turned on. For this reason we introduce a “quantum scale” $n$, by substituting $\exp(-S)$ in the path integral by $\exp(-n S)$, which allows for a smooth interpolation between the classical limit ($n = \infty$) and an ordinary simulation ($n = 1$). The changing of the string tension as a function of $n$ is depicted in Fig. for two cases: Considering only quantum fluctuations but keeping the physical fluxoid string fixed, and also considering string fluctuations, which leads to an increase of the energy per length (until the system undergoes a phase transition to the Coulomb phase).

Physical arguments suggest that the dual superconductor should be approximately at the borderline between type-I and type-II. This is fulfilled for the chosen classical solution. Investigating the influence of fluctuations on an effective Landau-Ginzburg parameter (defined by the ratio of the individual contributions to the string tension) showed us that the system is pushed towards type-I behaviour by fluctuations.

### 4. Conclusions

By dual simulations of flux tubes one can address a lot of interesting questions concerning the confinement mechanism. Here we focussed on the discussion whether the confinement vacuum is a dual type-I or type-II superconductor, and on the meaning of fluctuations in dual theories. Similarly to the behaviour of $U(1)$ in the vicinity of the phase transition, these fluctuations can produce an effective type-I superconductor also in the full Abelian Higgs model.

Of course, it is another interesting question, which set of (bare) parameters has to be used to describe the same physics as with the classical solution of the Abelian Higgs model. For this task, however, it will be necessary to improve the control over string fluctuations in our simulations.
Fig. 1. The changing of the string tension $\sigma$ with respect to the classical value $\sigma_{\text{class}}$ under the influence of quantum fluctuations (dotted line), and of both quantum and string fluctuations (solid line). This calculation has been performed for a torolon on a $8^4$ lattice.

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References

1. T. Banks, R. Myerson, J. Kogut, *Nucl. Phys.* B 129 (1977) 493.
2. J. Fröhlich, P. A. Marchetti, *Europhys. Lett.* 2 (1986) 933.
3. M. Zach, M. Faber, P. Skala, *Phys. Rev.* D 57 (1998) 123.
4. M. Zach, M. Faber, P. Skala, *Nucl. Phys.* B (1998), in press, hep-lat/9709017.
5. V. Singh, R. W. Haymaker, D. A. Browne, *Phys. Rev.* D 47 (1993) 1715.
6. For a review see e.g. M. Baker, J. S. Ball, F. Zachariasen, *Phys. Rev.* D 41 (1990) 2612.
7. M. Baker, N. Brambilla, H. G. Dosch, A. Vairo, *Phys. Rev.* D 58 (1998) 034010.