Radius stabilization and brane running in RS1 model

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Abstract

We study the effective potential of a scalar field based on the 5D gauged supergravity for the RS1 brane model in terms of the brane running method. The scalar couples to the brane such that the BPS conditions are satisfied for the bulk configuration. The resulting effective potential implies that the interbrane distance is undetermined in this case, and we need a small BPS breaking term on the brane to stabilize the interbrane distance at a finite length. We also discuss the relationship to the Goldberger-Wise model.

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1 Introduction

It is quite probable that our 4d world is formed from a three brane embedded in a higher dimensional space, probably according to a ten-dimensional superstring theory. As such a model, we can consider a thin three-brane (Randall-Sundrum brane) embedded in AdS$_5$ space [1, 2]. In this case the coordinate transverse to the brane is considered as the energy scale of the field theory on the boundary, in the sense of the AdS/CFT correspondence [3, 4, 5, 6].

In the original Randall-Sundrum (RS) model, the brane action is expressed in terms of a tension parameter only. It has recently been pointed out that many higher derivative terms appear in the brane action when we make a change of the brane position without changing the solutions for the background and the Kaluza-Klein (KK) modes [7, 8, 9, 10, 11]. This procedure is called brane running, and is considered to be an approximate method of obtaining renormalization group flows for the field theory on the brane. This implies that the parameters of the brane-action on a different position are related through this flow. Inspired by this idea, we examine the running behavior of the brane action for a scalar field and derive the effective action to examine the stability of the two brane system via the brane running method. We should notice that the effective potential, given by Eq.(7), is here estimated from gravity side classical potential by pulling the hidden brane to the position of the visible one [7]. This point is different from many other approaches to explain the radius stabilization.

We consider a scalar field of a 5d gauged supergravity model, and restrict ourselves to the case of one scalar field coupled to the brane in a form consistent with the Bogomolnyi-Plasad-Sommerfield (BPS) conditions. The flow equations for the brane action of the scalar are solved in a nonlinear form with self-interactions, in order to find a non-trivial solution. It is shown for the RS1 (the two brane model) that the effective action implies the interbrane distance to be arbitrary for a BPS solution. When the BPS condition is slightly broken through the brane action, the distance between the two branes is stabilized in spite of the particular form of the potential used here. The model of Goldberger-Wise for the effective potential is also discussed in our context.

In Section 2, the model used here is set, and the flow equation for the brane is derived and solved in Section 3. In Section 4, a possible stability of the braneworld solutions is examined. Concluding remarks are given in the final section.
2 Setting of RS1 brane-world

As bulk action, consider a model of 5d gauged supergravity in the Einstein frame as

\[ S_g = \int d^4x d^l y \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R - \frac{1}{2} \sum_I (\partial_\phi I)^2 - V(\phi) \right\} + \frac{2}{\kappa^2} \int d^4x \sqrt{-g} K , \]  

(1)

where \( K \) is the extrinsic curvature on the boundary. The five-dimensional gravitational constant \( \kappa^2 \) is taken to be \( \kappa^2 = 2 \) for simplicity. The potential \( V \) is written in terms of a superpotential \( W(\phi_I) \) as

\[ V = \frac{v^2}{8} \sum_I \left( \frac{\partial W}{\partial \phi_I} \right)^2 - \frac{v^2}{3} W^2. \]  

(2)

The gauge coupling parameter \( v \) is fixed from the AdS_5 vacuum \[12\] in which \( \phi_I = 0 \), and is given as \( v = -2 \) by using the radius of AdS as a unit length. As for the brane action, it is given such that it satisfies the BPS conditions on the boundary \[14\],

\[ S_b = -v \int d^4x d^l y \sqrt{-g} W(\phi_I) (\delta(y - y_b) - \delta(y - y_v)). \]  

(3)

The background solutions are obtained under the following ansatz for the metric,

\[ ds^2 = A^2(y) \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \]  

(4)

where \( \eta_{\mu\nu} = \text{diag}(-1,1,1,1) \) and the coordinates parallel to the brane are denoted by \( x^\mu = (t, x^i) \), while \( y \) is the coordinate transverse to the brane.

For the bulk action, the BPS solution is given by solving the first order equations \[13, 15\],

\[ \phi_\prime_I = \frac{v}{2} \frac{\partial W}{\partial \phi_I}, \quad \frac{A'}{A} = -\frac{v}{3} W, \]  

(5)

where \( \prime = d/dy \). It is known that these equations are the necessary conditions for the supersymmetry of the solution. And the solutions of (5) satisfy the equations of motion for any \( W \).

Also at the position of the brane, the equations (5) are satisfied due to the special form of the brane action as given in [14], which is taken such that we can preserve the form of the supersymmetric or BPS bulk solutions [16, 17].

Our purpose is to find an effective potential \( V^{\text{eff}}(\phi) \) for scalars in a simple bulk background to obtain a solution, instead of solving the above equation (5). The metric is restricted to be of the AdS_5 form, which corresponds to \( A = e^{-|y|} \). It will be possible to estimate the effective brane action \( S_b^{\text{eff}} \) from the viewpoint of AdS/CFT.

\[ ^4 \text{Here we take the following definition, } R_{\mu\nu\lambda\sigma} = \partial_\lambda \Gamma_{\mu\nu} - \cdots, \quad R_{\nu\sigma} = R_{\mu\sigma}^\mu \text{ and } \eta_{AB} = \text{diag}(-1,1,1,1,1). \text{ Five dimensional suffices are denoted by capital Latin and four dimensional ones by Greek letters.} \]
correspondence as in the way [18, 19, 20, 21], but it would be very complicated due to the estimation of the bulk part. So, here, we consider a situation where it would be possible to estimate $S_b^{\text{eff}}$ from gravity side classical potential as follows. We pull the hidden brane from $y = y_h$ to the position of the visible brane, $y = y_v$, by the brane running method. As a result the bulk space vanishes, then we obtain

$$S_b^{\text{eff}} = \frac{1}{2} \tilde{S}_b,$$  \hspace{1cm} (6)

where $\tilde{S}_b$ denotes the sum of the actions of hidden and visible branes after the running, as mentioned above. Then we can estimate the effective potential to be

$$V_{\text{eff}} = -\frac{1}{2} \tilde{S}_b,$$  \hspace{1cm} (7)

and the solution of $\phi$ must be studied in order to estimate the above effective action.

3 Brane running and BPS

Consider the case of one scalar $\phi$. Then the equation for it is obtained as

$$\phi'' + \frac{4A'}{A} \phi' + q^2 \frac{\phi}{A^2} = \frac{\partial V(\phi)}{\partial \phi} - 2 \frac{\partial W(\phi)}{\partial \phi} \delta(y),$$  \hspace{1cm} (8)

where $q^2$ is the four dimensional momentum square of $\phi$ and we take $y_h = 0$ for simplicity. The boundary condition for $\phi$ is written as

$$\phi'(0) = -\left. \frac{\partial W(\phi)}{\partial \phi} \right|_{y=0}.$$  \hspace{1cm} (9)

We extend this equation to the position $y > 0$ where the running brane arrived, by introducing the action of this running brane as

$$S_b^{(R)} = -2 \int d^4x \sqrt{-g} \sum_{n=0}^{\infty} \frac{\phi^n}{n!} \tau^{(n)}(y),$$  \hspace{1cm} (10)

where $0 < y < y_v$. We should notice the following point for this brane action. In general, many kinds of derivative terms, kinetic and higher derivative terms, appear in the effective action after a running, but here only the non-derivative terms are retained in order to find a vacuum state which should be dominated by low frequency modes.

The coefficients for each power of the scalar field $\phi^n$ in (10) are defined as the running coupling constants, $\tau^{(n)}(y)$, since they in general vary with $y$. This variation is determined by the bulk configuration of the fields which are the solution of the classical equations including [S]. On the other hand, the running behavior of $\tau^{(n)}(y)$ is considered as a renormalization group equation obtained due to the interactions with
the dual field theory of the bulk background. In this sense, the approach considered here provides a quantum information for the brane action in a sense of gauge/gravity correspondense. Then it would be natural to consider the brane action obtained after a brane running as an effective action in which the quantum effects of the dual theory are reflected through $\tau^{(n)}(y)$. So we can see the dynamical properties of the dual gauge theory from the $y$-dependence of $\tau^{(n)}(y)$.

The interesting point of this brane running method is that they are determined by the boundary condition defined at the arrived new position, $y_h = y$, of the hidden brane. It is given by using (10) as

$$\phi'(y) = \sum_{n=1}^{\infty} \frac{\phi^{n-1}}{(n-1)!} \tau^{(n)}(y). \tag{11}$$

Since the boundary condition at $y = 0$ is written by the potential $W(\phi)$, which is used at the initial point $y = 0$, then the initial values, $\tau^{(n)}(0)$, are determined by $W$. Expanding $W$ as

$$W(\phi) = \sum_{n=0}^{\infty} \frac{\phi^n}{n!} W^{(n)}(\phi), \quad W^{(n)} = \frac{d^n W}{d\phi^n}, \tag{12}$$

and considering (11) at $y = 0$, we obtain the boundary condition for $\tau^{(n)}(y)$ as

$$\tau^{(n)}(0) = -W^{(n)}(0). \tag{13}$$

Further, for the region of $0 < y < y_v$, we obtain $\tau^{(n)}(y) = -W^{(n)}$ from (9) and (11), then

$$\tau^{(n)}(y) = \tau^{(n)}(0). \tag{14}$$

Therefor, for any BPS solution the coefficients $\tau^{(n)}(y)$ are at the fixed point. Then, the potential of hidden part coincides with the one of visible brane with opposite sign when the hidden brane arrived at the position of the visible brane. After all, we arrive at the following result

$$V^{\text{eff}} = 0. \tag{15}$$

Then, the potential is independent of the interbrane distance and it is arbitrary in this case. In other words, it can not be determined. In order to obtain some non-zero effective potential, we should consider a non-BPS background.

### 4 Non BPS case

Here we derive a non-BPS solution by solving directly the second order equation (8) instead of the first order equations (5). Then, the BPS condition is broken. Namely, we fix the bulk as AdS$_5$:

$$A'/A = -1, \quad \tau^{(0)}(y) = -W^{(0)}(0) = -\frac{3}{2}. \tag{16}$$
Further we assume
\[ \tau^{(1)}(y) = 0 \]  
(17)
to make it possible to solve the flow equation. Other \( \tau^{(n)}(y) \) are obtained from the brane running method.

In Ref. [7], tadpole and quadratic terms are considered in a truncated form by introducing a fixed point of the brane running. In this sense, we are considering a different model. The trivial BPS solution, \( \phi = 0 \), is consistent with these conditions, but we search for a non-trivial solution. The consistency of such a non-trivial solution and the background AdS(5) would be justified when the back-reaction is negligibly small, that is, for sufficiently small \( \phi \). Thus, this derivation is based on the perturbation.

The differential equations for \( \tau^{(n)} \) for \( n \geq 2 \) are obtained as follows. Differentiate (11) with respect to \( y \), and rewrite \( \phi'' \) and \( \phi' \) in terms of \( \phi \) by using (11) and (8). And, expand \( W(\phi) \) and \( V(\phi) \) in the series of \( \phi \)

\[ W(\phi) = \sum_{n=1}^{\infty} W(n) \phi^n, \quad V(\phi) = \sum_{n=1}^{\infty} V(n) \phi^n, \]

where \( V(n) \equiv \frac{\partial^n V}{\partial \phi^n} \). Then, by observing the coefficient of \( \phi^n \), we obtain the following results,

\[ (\tau^{(n)})' = -4 \frac{2}{3} W^{(0)}(0) \tau^{(n)} + V^{(n)} - \sum_{m=2}^{n} \frac{(n-1)!}{(m-2)!(n-m+1)!} \frac{\tau^{(m)} \tau^{(n+2-m)}}{\tau^{(n+2-m)}} + \frac{q^2}{A^2} \delta_{n,2}, \]

(18)

\[ V^{(n)} \equiv \frac{\partial^n V}{\partial \phi^n} = -4 \frac{2}{3} W^{(0)}(0) W^{(n)} + \sum_{m=2}^{n} n! \frac{W(m) W^{(n+2-m)}}{2(m-1)!(n-m+1)!} - \frac{4}{3} \sum_{m=2}^{n-2,n\geq4} n! W(m) W^{(n-m)} \]

\[ m!(n-m)! \]

(19)

for \( n \geq 2 \).

Here we consider the small \( q^2 \) region to see the effective potential, so we neglect the term \( \frac{q^2}{A^2} \) in (18). In this case, we find

\[ \tau^{(2)}(0)' = \tau^{(3)}(0)' = 0, \quad \tau^{(4)}(0)' = -8(\tau^{(2)}(0))^2. \]

(20)

These results imply that \( \tau^{(2)} \) and \( \tau^{(3)} \) are not running, but \( \tau^{(4)} \) is running and should decrease with \( y \) for any \( W \). This means that the effective potential behaves as \( V^{\text{eff}} \propto -\phi^4 \), so the braneworld is unstable for the present model. This instability is related to the breaking of the BPS conditions. In the next section, we shall see the details of this point in terms of a simple model of gauged supergravity.

### 4.1 Modified superpotential model

Here we consider a truncated superpotential [12, 13]

\[ W = -e^{-2\phi/\sqrt{6}} \left( 1 + \frac{1}{2} e^{6\phi/\sqrt{6}} \right). \]

(21)

For this potential, \( \tau^{(i)} \) for \( i \leq 3 \) are not running as shown above and we find

\[ \tau^{(2)} = 2, \quad \tau^{(3)} = \frac{4}{\sqrt{6}}. \]

(22)
For $i \geq 4$, $\tau^{(i)}$ are running and we obtain the next two terms as follows

\[ \tau^{(4)} = 8(e^{-4y} - 1/2), \quad \tau^{(5)} = \frac{40}{9\sqrt{6}} e^{-6y} \left( 32 - 36e^{2y} + 7e^{6y} \right). \]  

(23)

Up to this order $V^{\text{eff}}$ is written as

\[ V^{\text{eff}} = A^4 \left( \frac{\Delta \tau^{(4)}}{4!} \phi^4 + \frac{\Delta \tau^{(5)}}{5!} \phi^5 \right), \]  

(24)

where we set $y_v = y$ and $\Delta \tau^{(i)} \equiv \tau^{(i)}(y) - \tau^{(i)}(0)$. This potential has a minimum at

\[ \phi = -4\Delta \tau^{(4)}/\Delta \tau^{(5)} \equiv < \phi >, \]

which is negative, and we see that $| < \phi > | \geq 3\sqrt{6}/5 > 1$. Then this minimum point is outside the perturbation regime. In this sense, we can say only that the braneworld solution given here is unstable since the potential $V^{\text{eff}}$ indicates the existence of other stable points.

Another point to be noticed is the $y$-behavior of $V^{\text{eff}}(<\phi>)$. When it shows a non-trivial minimum at some finite $y$, it indicates the stabilization of the distance between the two branes. Although the value of $<\phi>$ given above is outside the approximation region considered here, we could study the $y$-dependence of $V^{\text{eff}}(<\phi>)$ for this case. As shown in Fig.1, it indicates no stable point. Then, we find again that this solution is unfavorable.

Fig. 1: The solid curve represents the potential, $V^{\text{eff}}(<\phi>)$, for the BPS brane solution up to $\phi^5$. The dashed curve represents $<\phi>$. 

is unfavorable.
Next, we consider a possible mechanism to stabilize the braneworld. It is realized by adding a small mass term, $-\epsilon\phi^2/2$, to the brane action:

$$ S_b = -v \int d^4x dy \sqrt{-g} \left\{ W(\phi) - \frac{1}{2} \epsilon \phi^2 \right\} (\delta(y - y_h) - \delta(y - y_v)). $$  \hspace{1cm} (25)

In this case, only the boundary condition for $\tau^{(2)}$ is modified as $\tau^{(2)}(0) = -(W^{(2)}(0) - \epsilon)$. Then $\tau^{(2)}$ is running and we obtain

$$ \tau^{(2)} = 2 + \frac{\epsilon}{1 + \epsilon y}, $$  \hspace{1cm} (26)

$$ \tau^{(3)} = \frac{1}{\sqrt{6(1 + \epsilon y)^3}} \left\{ 3\epsilon e^{-2y}(2 - 2\epsilon + \epsilon^2) + 4 + 6(-1 + 2y)\epsilon + 6(1 - 2y + 2y^2)\epsilon^2 \right. $$

$$ + \left. (-3 + 6y - 6y^2 + 4y^3)\epsilon^3 \right\}, $$  \hspace{1cm} (27)

up to order $\phi^3$. To this order, the effective potential is written as

$$ V^{\text{eff}} = A^4 \left( \frac{\Delta\tau^{(2)}}{2} \phi^2 + \frac{\Delta\tau^{(3)}}{3!} \phi^3 \right), $$  \hspace{1cm} (28)

and it has a minimum at $< \phi > = -2\Delta\tau^{(2)}/\Delta\tau^{(3)}$. For small $\epsilon$, we have

$$ < \phi > \sim \sqrt{\frac{2}{3}} \frac{y}{1 - e^{-2y}\epsilon}. $$  \hspace{1cm} (29)

Hence, a small value of $< \phi >$ can be realized, as long as $\epsilon$ is small enough and $y$ is not large.

The next problem is to see the $y$-dependence of $V^{\text{eff}}(< \phi >)$. The result is shown in Fig.2. As seen from this figure, $V^{\text{eff}}(< \phi >)$ has a minimum at $y \sim 0.4$ where $< \phi > \sim 0.06 << 1$ as expected. The figure is shown for $\epsilon = 0.1$, but this behavior changes only little with varying $\epsilon$ and its sign is not important. Then the small modification of the brane action given here is enough to stabilize the braneworld solution with an interbrane distance of $y \sim 0.4$. The back-reaction from this modification will be negligible since we can take very small $\epsilon$.

### 4.2 Goldberger-Wise model

Let us briefly consider the Goldberger-Wise model $[22]$ and its stabilization via the brane running. The brane running method has already been applied to this model with success in reproducing the stabilization $[7]$, indicating that the brane running method is a reasonable way of evaluating the effective action. In the previous analysis, only the brane running (renormalization group flow) near the fixed point is considered. Our reanalysis has no such restriction.
The brane action of the model is
\[ S_b = -v \int d^4xdy \sqrt{-g} (W_h \delta(y - y_h) - W_v \delta(y - y_v)) \ , \] (30)
where \( W_x(\phi) = (\phi^2 - \alpha_x)^2 + \beta_x \) and \( W_x(0) = \alpha_x^2 + \beta_x = -3/2 \) for \( x = h \) and \( v \), and the bulk potential \( V \) is defined as \( V = -6 + m^2 \phi^2 / 2 \). This model obviously breaks the bulk supersymmetry and the BPS conditions, since \( V \) and \( W_x \) defined above do not satisfy (2). It is found that \( W_h^{(2)} = -4 \alpha_h \), \( W_v^{(2)} = -4 \alpha_v \), \( W_h^{(4)} = W_v^{(4)} = 4! \) and \( V^{(2)} = m^2 \). So \( \tau^{(2)}(y) \) is obtained as
\[ \tau^{(2)}(y) = \frac{\lambda_1(\lambda_2 + W_h^{(2)})e^{\lambda_1 y} - \lambda_2(\lambda_1 + W_h^{(2)})e^{\lambda_2 y}}{(\lambda_2 + W_h^{(2)})e^{\lambda_1 y} - (\lambda_1 + W_h^{(2)})e^{\lambda_2 y}} \] (31)
where \( \lambda_1 = 2 + \sqrt{4 + V^{(2)}} \), \( \lambda_2 = 2 - \sqrt{4 + V^{(2)}} \). Furthermore, it is found from the initial conditions \( W_h^{(3)} = 0 \) and \( W_h^{(4)} = 4! \) that \( \tau^{(3)}(y_v) = 0 \) and \( \tau^{(4)}(y_v) \neq 0 \), that is,
\[ \tau^{(4)}(y_v) = -W_h^{(4)} \exp \left\{ \int_{y_h}^{y_v} 4(1 - \tau^{(2)})dy \right\} . \] (32)
The explicit form of \( \tau^{(4)}(y_v) \) is lengthy, so it is not shown here. Assuming that \( \phi \) is small, we then consider the brane solution up to \( \phi^4 \). Up to this order, the effective potential is
\[ V^{\text{eff}} = A^4 \left( \frac{\Delta \tau^{(2)}}{2!} \phi^2 + \frac{\Delta \tau^{(4)}}{4!} \phi^4 \right) , \] (33)
where \( \Delta \tau^{(i)} = \tau^{(i)}(y_v) + W_v^{(i)} \) for \( i = 2, 4 \), and it has a minimum at \( < \phi >^2 = -3! \Delta \tau^{(2)}/\Delta \tau^{(4)} \). When parameters are taken as \( \alpha_h = O(m^2) \) and \( \alpha_v = O(m^2) \), it...
is easily found that $\Delta \tau^{(2)} = O(m^2)$ and $\Delta \tau^{(4)} = O(1)$, leading to $\langle \phi \rangle^2 = O(m^2)$ and $V_{\text{eff}}(\langle \phi \rangle) = O(m^4)$. Hence, the back-reaction is negligible for small $m^2$. Figure 3

Fig. 3: The solid curve represents $V_{\text{eff}}(\langle \phi \rangle) \times 10^4$ for the brane solution up to $\phi^4$. The dashed curve represents $\langle \phi \rangle^2$. Parameters taken here are $\alpha_h = 5m^2/4$, $\alpha_v = m^2/4$ and $m^2 = 10^{-4}$. The effective potential has a minimum at $y \sim 1.0$ where $\langle \phi \rangle^2 \sim m^2 \ll 1$. The effective potential thus obtained would include some sort of quantum corrections in the sense that $\tau^{(2)}$ and $\tau^{(4)}$ are running in the renormalization group procedure. As an interesting point, the effective potential shows a concave shape which is also seen in the classical limit of $V_{\text{eff}}(\langle \phi \rangle)$ as reported in the original work [22] of this model.

Lastly, we make a comment on another possible origin to stabilize the interbrane distance [23, 24]. Consider the temperature on the brane and make use of the finite temperature field theory for this system. In this case, the Casimir energy between two branes for fields in the bulk contributes to the free energy, and its fermionic part could give a minimum of the free energy at a finite interbrane distance. Then, the effective potential could have a nontrivial minimum even for $\langle \phi \rangle = 0$ contrary to the case of the Goldberger-Wise stabilization method mentioned above. However, we notice that both stabilization mechanisms would be compatible.

5 Concluding remarks

We study the interbrane distance of the two brane system with a bulk scalar field. It is examined in terms of the effective potential obtained by the method of brane running. Here, the bulk scalar is considered in the gauged supergravity and it couples to the brane in a form consistent with the BPS conditions for the bulk solutions. In this case, for bulk BPS solutions, the parameters in the potential of the hidden brane are not
running even if the brane position is changed by the brane running. Then the effective potential, which is defined as the sum of two branes, becomes zero when the hidden branes arrived at the position of the visible one. As a result, the effective potential is independent of the value of the interbrane distance. Thus, the interbrane distance is arbitrary in this case. This property is true for any type of superpotential and BPS solutions. So it would be necessary to consider a non-BPS solution to obtain a finite and stable interbrane distance. We can show such two non-BPS examples.

We consider the case that one of the BPS conditions is broken by considering a nontrivial solution for the scalar $\phi$, while the bulk configuration keeps AdS$_5$ intact. This solution is justified when $\phi$ is small. In this case, we find that the effective potential behaves as $V^{\text{eff}} = -8(W^{(2)})^2 \phi^4 + O(\phi^5)$, and there is no stable point for the interbrane distance in this effective potential within the small $\phi$ approximation. We find that the braneworld of a non-trivial scalar can be stabilized for small $\phi$ when a small BPS breaking term is added to the brane action. We could show this by choosing the superpotential given in [12,13] as an example. The stable interbrane distance obtained in this way is not large. However, if necessary, it may be possible to get a larger stable distance by breaking the BPS conditions more severely. Actually, when we break the BPS conditions for both brane and bulk actions as in the Goldberger - Wise approach [22], we succeed as the second example in making a stable braneworld with a larger interbrane distance.

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