The unity between quantum field computation, real computation, and quantum computation

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October 27, 2018

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Abstract

It is indicated that principal models of computation are indeed significantly related. The quantum field computation model contains the quantum computation model of Feynman. (The term "quantum field computer" was used by Freedman.) Quantum field computation (as enhanced by Wightman’s model of quantum field theory) involves computation over the continuum which is remarkably related to the real computation model of Smale. The latter model was established as a generalization of Turing computation. All this is not surprising since it is well known that the physics of quantum field theory (which includes Einstein’s special relativity) contains quantum mechanics which in turn contains classical mechanics. The unity of these computing models, which seem to have grown largely independently, could shed new light into questions of computational complexity, into the central P (Polynomial time) versus NP (Non-deterministic Polynomial time) problem of computer science, and also into the description of Nature by fundamental physics theories.

1 Introduction

The two great physical theories of the twentieth century were quantum theory and relativity, both of which generalize classical Newtonian mechanics. Quantum mechanics and classical mechanics are special limiting cases of quantum field theory. By taking the limit as the velocity of light $c \to \infty$ we expect to get non-relativistic quantum mechanics. The limit as Planck’s constant $\hbar \to 0$ gives classical mechanics $\square$. Neither quantum theory nor relativity can be ignored.

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Quantum field theory is a logical and natural result of combining quantum theory and relativity.

In the same twentieth century, the mathematical theory of computation was developed and the electronic computer was invented. Gate implementation for the standard classical computer or Turing machine can be based on classical mechanics; in the sense that the gates, ideally in the absence of perturbations (depicted by the electrical engineering term noise), could even consist of perfect billiard balls.

Meanwhile, we have become increasingly dependent on computing machines, and there are more models of computation. By Church’s thesis, to state it simply, all reasonable models of computation are equivalent. At the present time, there appear to be three principal models of computation, which have grown largely independently. The models are, quantum field computation, real computation and quantum computation. It is proposed and argued here that these three models of computation are indeed significantly related. Hopefully, from a unified point of view, there is much more to be learned about computation.

Unity of these computational models is not surprising because physicists generally believe that quantum field theory contains quantum mechanics, which in turn contains classical mechanics.

Correspondingly, we present the (quantum field computation) thesis:

A. Quantum field computation contains quantum computation as a proper subset.

B. Quantum field computation involves more powerful computational tools than quantum computation (e.g. infinite dimensional Hilbert spaces, some methods of real computation, and holomorphic functions).

On the other hand, quantum methods may not be suitable for certain real computing problems. A simple argument is that mathematics is a much wider subject than physics; not all mathematics is necessarily applicable to physics.

At present, quantum field theory is regarded only as an asymptotically valid theory [3]. We expect that well developed future generalizations of physics theories, which include general relativity (Einstein gravitation), and perhaps supersymmetry, will replace quantum field theory in the above thesis, with additional and more powerful computational tools brought into play.

1.1 Quantum Computation

Based on quantum theory, Feynman proposed quantum computation [3]. In quantum computation, as opposed to Turing computation, qubits (quantum bits) are used in place of classical bits. A bit could be in one of two discrete states, 0 or 1. A qubit, on the other hand, corresponds to a 2 level quantum system, like a spin 1/2 state of an electron. We can have a complex linear superposition of wave functions (eigenstates) of a 2 level quantum system so that two complex number amplitudes (in $\mathbb{C}^2$) are involved in each qubit. Feynman hoped to exploit the quantum system itself by making it do the computation, but practically, decoherence noise is a serious implementation problem even with very much less than 10 qubits.
As a result of Feynman’s proposal there has been an enormous amount of research, not only on quantum computation, but also on quantum cryptography. The efforts in this regard are to seek improved ways of performing computations, including a refinement of Church’s thesis by Deutsch [4] to tackle quantum computation, or building new types of computing machines. It is hoped that not only exponentially faster computation will be achieved [3], but that better understanding of computational complexity will come about [3, 5].

1.2 Real Computation

In another direction, the classical discrete digital Turing computer [8] has been generalized to include the possibility of computing over the continuum [9, 10]. This generalization is called real computation. The need for doing this is because computing over the continuum is more appropriate to the way we do analysis, physics, numerical analysis and engineering problems. Accordingly, the classic logical theory of computation was enhanced with analysis, topology and algebraic geometry.

Until recently, it was considered unthinkable to speak of computing over a continuum, for example, over the infinite number of points in the real interval [0, 1], without approximating at a finite number of points. But Tarski, in a little known paper [11], proved completeness over the reals for elementary algebra and geometry. The complexity was extremely high (exponential), but Smale et al [9, 10] have rectified that situation. Tarski’s result is in contrast to Gödel’s famous theorem [12] of incompleteness of arithmetic over the integers $\mathbb{Z}$, and to Turing’s theorem [8] of undecidability of the Halting problem for computation over the integers $\mathbb{Z}$.

A question was raised by Penrose [13] as to whether the Mandelbrot set was an (albeit beautiful, picturesque) example of an undecidable set (i.e. a recursively enumerable set that is not recursive). It was concluded that it was not possible to answer this question because there was no proper definition for computing over the continuum. One problem is: how does one feed a real number, consisting of an infinitely long sequence of bits, into a computing machine in finite time? A proper definition was indeed given in the work of Smale et al [9, 10] on real computation, and the question on the Mandelbrot set was answered in the affirmative. (The proof hinges on the fact that the Hausdorff-Besicovitch dimension of the boundary of the Mandelbrot set is indeed equal to 2.)

In fact, Tarski hoped to build a machine which would compute over the reals. But it is now possible to do some simple real computation even on a Turing machine. Our thesis on quantum field computation relies heavily, not only on the fact that quantum field theory generalizes quantum theory, but also on possibilities of computing over the continuum.

Real computation is a computing model that is based on classical mechanics and classical dynamical systems. But classical mechanics could also be extended to include relativity, resulting in relativistic mechanics [14].
1.3 Quantum Field Computation

In studying atomic phenomena, classical mechanics has been replaced by quantum mechanics. Correspondingly the classical computer could be improved with a quantum computer. But we could also think of more general models of computation based on adding relativity to quantum theory to get relativistic quantum field theory, and consider appropriate quantum field computation models.

In an approach to the central computer science problem of the P (Polynomial time) versus NP (Nondeterministic Polynomial time) [12] complexity classes, Freedman proposed a quantum field computer [15]. Under consideration were topological quantum field theories, and physical systems which contained non-Abelian gauge terms in the Lagrangian. The initial preparation of states was supposed to be consistent with knot types [1].

Of course, in a general situation, as in non-Abelian gauge theories, string theories (including general relativity), superstring theories, or topological field theories [16], quantum field computation would be an immensely difficult undertaking.

But due to the work of Wightman on relativistic quantum field theory (incorporating Einstein’s special relativity and employing analytic functions of several complex variables) many of the components for some quantum field computation are already available. There is extensive literature on Wightman’s model, for example, in [17, 1, 18] as well as the references cited there. Some additional computation methods are described in [19]. We concentrate therefore on explaining the unity among computation models.

In this article the approach is based on mathematical physics but the results also impact computer science.

2 Relationships Between Computation Models

There is a remarkable relationship between quantum field computation and real computation. Computation over the continuum appears in quantum field computation as well as in real computation. In the former, it is already possible to compute over cells which are actually certain chunks of the continuum space $\mathbb{C}^n$ of $n$ complex variables.

We might say this comes about because it is natural to consider a physical or quantum system in the continuum limit. In fact Isaac Newton, when studying gravitation, found it natural to consider a continuous distribution of matter to model the earth’s gravitational action at external points. From continuum quantum mechanics, by combining relativity, we have quantum field theory, a system with an infinite number of degrees of freedom. The development in real computation of the Newton endomorphism method in numerical analysis follows naturally from Newton’s continuum limit.

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1. The user-friendly, interactive and animated color graphics “SnapPea” program for creating knots and studying hyperbolic 3-manifolds is available at: http://thames.northnet.org/weeks/index/SnapPea.html.
Real computation could also be regarded as a stepwise form of analog computation working within a continuum.

Conversely, quantum (mechanics) computation would be suspected to be a discrete finite case of quantum field computation where the number of qubits is finite, and the corresponding Hilbert vector space is a finite dimensional vector space.

At the present time, quantum computation proceeds as a time evolution over a finite number of discrete time intervals, whereas time must be regarded as a continuous variable. But space and time are interwoven in relativity, depending on the frame of reference: thus the need to handle the problem in a covariant manner. Also, because a quantum field computation model does exist, it is important to say that quantum computation can therefore benefit by including considerations of relativity, methods of computing over the continuum, and an unbounded number of qubits (infinite dimensional Hilbert space).

The important concepts for quantum computation are unitary transformations, finite superposition of states, entanglement, and quantum cryptography. Superposition is standard also in quantum field theory. Entanglement is a rather interesting form of superposition, with applications to quantum teleportation considerations and quantum cryptography, and relies on a basis of EPR (named after Einstein, Podolsky and Rosen) or Bell states. The EPR gedanken (thought) experiment itself, from the point of view of quantum measurement theory, is not further discussed in quantum computation theory because the absence of hidden variables is now an accepted fact.

Just as the rotation group is of importance in non-relativistic quantum mechanics (with Euclidean geometry), the Lorentz group (with Minkowski geometry) is relevant to relativistic quantum mechanics. The Lorentz group contains the rotation group as a subgroup.

Thus a basic symmetry group in quantum mechanics is SU(2), the special (determinant = 1) unitary group of $2 \times 2$ complex matrices. This is also the universal covering group of the rotation group (real special orthogonal group) $\text{SO}(3)$ in 3-dimensional space.

$\text{SU}(2)$ is a proper subgroup of $\text{SL}(2, \mathbb{C})$, the universal covering group of the Lorentz group, which is the symmetry group for relativity in the usual 1-time and 3-space dimensions. Hence $\text{SL}(2, \mathbb{C})$ is the group appropriate for quantum field computation.

Consider, for example, the EPR states. One particular EPR state, based on electron spins, can be written as

$$\{|01\rangle - |10\rangle\}/\sqrt{2},$$

or equivalently as

$$\{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\}/\sqrt{2},$$

where, in the usual description, the first qubit refers to Alice and the second to Bob. They prepared the entangled state, perhaps on Earth when they were together, and now Bob could be in the Alpha Centauri system, at a space-like separation from Alice on Earth. This is simply the singlet state for adding two
spins of 1/2, where we have a simultaneous eigenstate of the total spin angular momentum \( S = 0 \), and the total \( z \)-component of spin \( S_z = 0 \). (It is possible also to have entangled states for photons, which can have horizontal or vertical polarization. Entanglement produces quantum interference between photons.)

Addition of angular momentum of spins 1/2 appears here as \( D^{1/2} \times D^{1/2} = D^{1} + D^{0} \), in terms of decomposition of representations of the rotation group in 3-dimensional space. In quantum field computation, this group is enlarged to the group \( SL(2, \mathbb{C}) \), which covers the Lorentz group. In general, one considers irreducible representations \( D^{(1/2, k/2)} \) of \( SL(2, \mathbb{C}) \).

The concept of electron spin 1/2 is added on to (non-relativistic) quantum computation in an ad hoc fashion. But Dirac showed that electron spin naturally follows from considerations of relativity, and the requirement of first order differential equations [20]. Entanglement of quantum states is also applicable to the relativistic theory, i.e. to quantum field theory. Indeed, experiments verify that quantum rather than classical field theory gives the correct results [21, 22].

3 Field Theory Enhancements to Quantum Computation

Quantum field theory not only includes all of quantum mechanics, and classical mechanics, but much more in the form of well-known results. Examples are, discrete anti-unitary symmetries, CPT invariance, and the spin statistics connection [17]. Our purpose here is to exploit results that enhance quantum computation, through working with a relativistic quantum field theory model.

Non-relativistic quantum mechanics is not complete because radiative corrections have to be made to it, using field theory. In dealing with a system corresponding to an infinite number of degrees of freedom, it is well known historically that formulations of quantum field theory like perturbation theory lead to infinities resulting in the need for renormalization. Nevertheless, quantum electrodynamics has turned out to be “the most accurate theory known to man” [2]. Dirac, Schwinger and Feynman are some of the principal contributors to quantum electrodynamics (the spectacular history of which is related in [23]) and hence to quantum field theory [2]. Relativistic covariance is of paramount importance in correctly performing the renormalization process.

If there is some way we can avoid approximations due to series expansions in perturbation theory, and also avoid renormalization problems, at least up to our point of departure of computational enhancements, we should do so. Fortunately we can achieve this by working within the Wightman formulation [17, 1, 18] of quantum field theory. Reasons for the fruitfulness and utility of this formulation, from a current perspective, are available in [18]. We are dealing with fields in the Heisenberg picture without using perturbation theory nor any particular time frame related Hamiltonians. The theory is in terms of analytic functions (Wightman functions) of several complex variables. These functions arise from

\[^2\text{This statement is attributed to Feynman.}\]
their boundary values which are vacuum expectation values (in Dirac’s bra-ket notation) of the form

$$\mathcal{W}_m(x_1, x_2, \ldots, x_m) = \langle \Omega | \phi_1(x_1) \phi_2(x_2) \cdots \phi_m(x_m) | \Omega \rangle$$

of products of $m$ quantum field operators in a separable Hilbert space. The field operators transform according to appropriate unitary spin representations of the Poincaré (inhomogeneous $SL(2, \mathbb{C})$) group, for 3+1 space-time dimensions and generally transform as spinors in $s$-dimensions. Spinor indices have been suppressed here, but are available in [17]. Wightman reconstructs quantum fields uniquely from these analytic functions. This is called the reconstruction theorem.

Let the $(m$-point) Wightman function be denoted by $W(n; z)$ where $z$ denotes the set of $n$ complex variables. Here, $n = sm$ where $s \geq 2$ is the space-time dimension; space-time will consist of 1-time and $(s - 1)$-space dimensions. $m$ is also called the function order, and $n$ will be called the function index. It was only recently known, how to physically understand concepts like closed time-like loops in more than one time dimension [24], where the second time dimension is in a tiny loop of a Kaluza-Klein type brane universe theory. However, we will restrict ourselves here to the conventional single dimension in time [25]. We use a general space-time dimension $s$ for the sake of considering uniformity of computation (to approach universality of computation), for what appear to be computational problems in their own right; whereas certain values of $s$, such as 2, 3, 4, 5, 10, 11 and 26 have turned out to be more appropriate for purely physics problems.

Because these analytic functions are fundamental to the theory, one is led to computation of holomorphy domains for these functions over the space of several complex variables, $\mathbb{C}^n$. The mass spectrum is assumed to be reasonable in the sense that momentum vectors $p^\mu$ lie in the closed forward light cone, with time component $p^0 > 0$ except for the unique vacuum state having $p = 0$.

Thus the many complex numbers (or amplitudes) that need to be handled in quantum field computation were indeed tamed as complex variables in analytic functions, i.e. the Wightman functions. (However, these are not the same entanglement type complex amplitudes used in quantum computation.)

Computation over $\mathbb{C}^n$ is common also in real computation. But, in the Wightman model, we seem to have stronger computation because of the use of holomorphic functions (over $\mathbb{C}^n$) of several complex variables. Not only the physics of quantum theory and special relativity, but also microcausality is utilized.

4 Analog Computation and Symbolic Computation

It is interesting that the strengths of analog and symbolic computation come into play as quantum field computation supplements and enhances quantum
computation. We think that, it is sometimes debatable as to what is symbolic or analog computation when it comes to computing over the continuum. In real computation there seems to be a subtle re-emergence of the old analog computer in a new and powerful form. This new form is effectively digitally clamped to avoid noise problems (such as voltage drifts in potentiometers) which plagued the old analog computer.

When \( s = 2 \), i.e. in 1-dimensional space and 1-dimensional time, deterministic exact analog computation \([13]\) (computation over \( cells \) in the continuum of \( \mathbb{C}^n \)) is used to obtain what are called primitive extended tube domains of holomorphy for \( W(n; z) \). The computation can be done with essentially reversible logic, as a Horn clause satisfiability problem (HORNSAT), and simulating on a Turing machine. But HORNSAT is in the complexity class \( P \) (polynomial time) \([12]\). This is now a deterministic problem of complexity \( P \), but also implies non-deterministic polynomial time computation, in the complexity class \( \text{NP} \), as discussed below.

We note a couple of points in this connection. First, we rely here on the soundness theorem and the converse theorem, namely, Gödel’s theorem of completeness of first order predicate calculus \([12]\). Secondly, reversibility of computation is an asset because information content is maximized, or equivalently, the entropy increase is minimized.

Just as the classical computer, Turing machine, computes over \( \mathbb{Z} \) or (up to polynomial time) equivalently over \( \mathbb{Z}_2 \) (the classical bit representation of numbers), we now have what can be called a complex Turing machine, in fact, a severally complex Turing machine.

The primitive extended tube domains are bounded by analytic hypersurfaces, namely several Riemann cuts, and other analytic hypersurfaces of types denoted by \( S \) and \( F \), which too play a role. These domains are in the form of semi-algebraic sets in the language used in real computation. Since the computation is symbolic, it is also exact, which is important in handling holomorphic functions.

Because of Lorentz invariance properties of the physics involved, the domains have a structure referred to as Lorentz complex projective spaces. (These Lorentz complex spaces are different, but physical, “non-Euclidean” views of complex projective spaces which are well known in mathematics.) Related to this invariance are certain continuum \( cells \) over which the computation occurs. Thus this computation is also like analog computation which would otherwise be regarded as impossible to do exactly.

In this simple case, it is possible to think that (suitably encoded) pieces or whole continuous group orbits are being fed into the Turing machine. Hopefully there will be more possibilities like this in the future.

### 4.1 Analytic Extensions

In relativistic quantum field theory it is possible to implement the physical requirement of microcausality. There exists quantum microcausality (field operators commute or anti-commute) at space-like separations.
Together with the consequence of permutation invariance of the domains, the edge-of-the-wedge theorem provides enlargements of the original primitive domains of analyticity into analyticity in unions of permuted primitive domains. Mapping these union domains creates some Boolean satisfiability problems. In fact, the novel methods of computation raise interesting issues of computability and complexity.

4.2 Non-deterministic Holomorphic Extensions

By the nature of analytic domains in more than one complex variable, it is in general possible to further extend these domains towards the maximal domains called envelopes of holomorphy. By considering boundary related semi-algebraic sets, there are non-deterministic computations of holomorphic extensions of domains. After the guessing step, the verification is by deterministic processes mentioned above. Historically, this method was used by Källén and Wightman in computation, for the first time, of the holomorphy envelope for $m = 3$.

Because HORNSAT is in P, [12], this results in an NP type problem, i.e. guessing the result and verifying in polynomial time. So this part of the problem is in the complexity class NP. (We note also that HORNSAT is P-complete.)

Built-in permutation invariance has considerable power just as $n!$ rapidly dominates over $2^n$ for large $n$. In applying local commutativity, it might appear that we have to generate permutations of $m$ objects; in fact, no algorithm is known to do this in polynomial time. But because of the power of non-deterministic computation [12], we are allowed to guess a candidate for a permutation; and then we can verify, in polynomial time, whether the guess is indeed a permutation, throwing out the candidate in case it is not a permutation.

5 Uniformity of Computation

Uniformity in the direction of universal computation has been discussed [10], in different contexts, including numerical analysis. We do have certain types of uniformity here.

First we note that the computation is independent of any particular form of Lagrangian or dynamics, and is uniform in $n$, qualifying for a universal quantum machine over $\mathbb{C}^\infty$. The latter space is defined in Appendix B.

5.1 Function Index Uniformity

When the logic program runs for $s = 2$, dynamic memory allocation is used through the operating system. Because $n$ can be input as a variable, only part of the whole memory management cost is outside the program. The program itself is independent of $n = sm$ and therefore is uniform in $n$, which is unbounded above. We can call this function index uniformity in $n^\infty$. 
5.2 Space-time Dimension Uniformity

In addition, there is uniformity in the dimension $s \geq 2$ of space-time, in the following manner. Given a dimension $s \geq 2$ of space-time, looking at the semi-algebraic sets defining the primitive extended tube domains of holomorphy (with hypersurface boundaries) and at function orders, there are three different classes of orders. These classes comprise, a) lower order W functions, b) intermediate order W functions, and c) high order W functions. Extended tube domains for all high order W functions have the same complicacy. For a) we have $m \leq s + 1$, and for c), $m > s(s - 1)/2 + 2$. The remaining cases lie in class b). For example, there is no class b) for $s = 2$ (i.e. class b) is empty), the most complicated primitive domain being for the 3-point function. If $s = 3$, then $m = 5$ is the only case in class b). When $s = 4$, we have in class b), the cases, $m = 6, 7$ and 8.

Since $s \geq 2$ is unbounded above, we can call this space-time dimension uniformity in $s^\infty$.

5.3 Uniformity of WHOLO

We recall that although deterministic complexity classes are closed under complements, the non-deterministic complexity class NP is not necessarily closed under complements. In fact, it is known [12] that the complexity class P is a subset of both complexity classes co-NP and NP. Also the problem PRIMES ("given an integer, is it a prime?"") belongs to both complexity classes co-NP and NP. But it is not known whether PRIMES belongs to the complexity class P i.e. no polynomial time algorithm is known for PRIMES. This lack of knowledge is the basis for the success of trapdoor cipher type encryption algorithms like RSA.

Let us denote the Wightman problem of computing holomorphy envelopes with the notation WHOLO. Thus we have seen above that the problem WHOLO has uniformity in $n^\infty$ and $s^\infty$.

The holomorphy envelopes for different orders $m$ of Wightman functions are related; the holomorphy envelope for order $m$ is contained in the intersection of holomorphy envelopes for lower order functions [13]. (This is further explained in Appendix A.)

For example, in $s = 2$, the 4-point function cannot be continued beyond the 2-point function Riemann cuts nor the (permuted) 3-point function Källén-Wightman domains of holomorphy.

This is a statement regarding analyticity that does not exist, and thus refers to the complements of domains of holomorphy; hence the use of the prefix co-. Because computations of analytic extensions of domains are non-deterministic (hence the notation $N$), we can say that we have co-$N$ uniformity over $s^\infty$, and in particular, co-NP complexity for $s = 2$.

In the case that the holomorphy domains are Schlicht, which is the only case known at present in this quantum field model, then the domains of holomorphy in Appendix A are closed under complements. This implies, in $s = 2$ for the
relevant part of the WHOLO problem, that the succinct certificates (or polynomial witnesses) of co-NP complexity for higher order functions are contained in those for lower order functions. This could have implications regarding problems which are in co-NP and not in NP.

6 Discussion

We have not used the non-linear positive definiteness conditions for W- functions in Hilbert space. These conditions are required for the reconstruction theorem. On the other hand, we want to exploit the complexity conditions for the linear-program problem as computational problems in their own right.

The original problem posed by Freedman [15] for a quantum field computer, was motivated by the existence of a great deal of mathematical physics relating to the case \( s = 3 \). In this 3-dimensional space-time, space itself is 2-dimensional, and there are a host of fruitful statistical mechanics and field theory problems in this case [1]. For example, instead of particles having to be Bosons or Fermions as in \( s = 4 \), we have Anyons corresponding to braid-group statistics. (The knot problem and 3-dimensional manifolds studied as knot complements, show up here.) There is also the fractional quantum Hall effect, which not only has produced some of the most accurate experimental results to date, but is the fertile testing ground for new physical theories as well. In particular, Chern-Simons type gauge interaction terms in the Lagrangian [26] give more insight into field theories, including gravitation. In the future, we should expect such theories to be part of quantum field computation.

At the time of Turing, a computer was a human being doing calculations. In the present era, computers are machines on which humans are extremely dependent, not only for calculations but also for modeling natural phenomena. Quantum computers indeed have the potential of greater power than classical computers. Exploiting real computation methods and quantum field computation enhancements by invoking special relativity, gives even stronger computational tools. In quantum cryptography, more powerful computation means stronger private code distribution and weaker public code methods. In the private code case, when Eve eavesdrops on the transmission of quantum information from Alice to Bob, the quantum data is disturbed so that Bob can decide it is so and discard those data items, requesting Alice to re-transmit. In the public code case, for example in the well-known RSA encryption and decoding algorithm, the code will be easier to break.

There is discrete translational invariance in quantum computation, compared to continuous translational invariance in quantum field computation. The discrete Fourier transform is of profound importance to the power of quantum computation. In the early days of quantum field theory, it was usual to quantize over a finite, rather than an infinite, box. The finite box incorporates discrete translational invariance and allows discrete Fourier transforms.

Since, in quantum field theory, particles with arbitrary spins can be annihilated and created, we can talk about qubits, qutrits, ququads, ..., and in general,
Relying on a fruitful set of models, we have related what appeared to be different models of quantum and classical computation based on relativistic and non-relativistic quantum mechanics and classical mechanics. Exact deterministic and non-deterministic computation over continuous domains appear naturally. Furthermore there is uniformity in computation over, unbounded above, or arbitrarily high index \( n \) of \( W(n; z) \) and arbitrarily high dimension \( s \) of space-time.

It is good to break up a complex problem into several parts and analyze the complexity of each part separately. Three parts of the problem WHOLO have been identified above. (There is a fourth part, namely, the representation of unions of domains, which has been possible to do only by human interaction.) In the case \( s = 2 \) the first part is in the complexity class \( P \) (and is \( P \)-complete), the second in \( NP \), and the third in \( co-NP \).

Identification, within quantum field computation, of these methods of computation raise interesting issues of computability and complexity, and possibly could shed more light, not only on computability, but also on the description of Nature by fundamental physics theories themselves.

7 Conclusion

By \textit{unity} between computation models we mean that the models are actually parts of a whole, higher (or broader) model of computation. Viewed from such a broader perspective it should be possible to better understand how the different parts, namely different computation models, fit together. The situation here is quite analogous to the situation in physics theories, where quantum field theory is the higher model (in this article), which contains quantum mechanics. Correspondingly we have quantum field computation as the higher level model which contains quantum computation.

Although some parts of the Wightman model of quantum field theory are exploited here, and in fact the only way employed up to the present of connecting up with the real computational model, these parts of the Wightman model should not be regarded as the only possible way of thinking in the future. The higher level model in physics is now quantum field theory, but this model might need to be expanded later (by including more symmetry groups, general relativity, topological fields, etc).

Each mathematical physics theory could possibly have some interesting, novel, computational and complexity ramifications. This idea was suggested by Freedman [5]. Accordingly, within quantum field theory we have identified \( P \) versus \( NP \) consequences and certain uniformities of computation. These uniformities are helpful in thinking of universality of computation, a hopeful problem for the future.

Through Einstein’s relativity, we have shown why there is unity between quantum field computation, real computation (computation over the continuum) and quantum computation. The Church Turing thesis for computation is
supposed to be presently enhanced with the quantum field computation thesis we have proposed above. Thus the ingenious methods in quantum computation, of dealing with discrete Fourier transforms, entangled states and fault-tolerant quantum error corrections could be profitably supplemented with concepts of infinite dimensional Hilbert spaces and (some) methods of computation over the continuum.

A Relations Between Holomorphy Envelopes

The holomorphy envelopes $H[D_m]$ for different orders $m$ of Wightman functions are related in the following way.

For $0 < r < m$, and relative to $H[D_m],$

$$H[D_m] \subset \bigcap_{\sigma \in P_m} \{H[\sigma D_{m-r}] \times (\sigma \mathbb{C}^s)\},$$

where $\sigma$ denotes permutations in $P_m$, the permutation group in the $m$ points of the $m$-point W function. This is a theorem which is referred to in [19]. In the case of Schlicht domains (analogous to single sheeted Riemann surfaces in $\mathbb{C}$), the $\subset$ sign means set theoretic inclusion (but more subtle otherwise).

B The complex space $\mathbb{C}^\infty$.

This space is defined to be the infinite disjoint union $\bigsqcup_{s=1}^{\infty} \mathbb{C}^{sm}$. We have followed the type of definition used in real computation [10]. There is a certain lattice pattern in the values of $n$ allowed in the subsequence $n = sm$. For example, since $s \geq 2, m \geq 2$, $n$ is never a prime number. Of course $\mathbb{C}^\infty$ involves holomorphy in the variables, which is a much stronger condition than differentiability, so that the space is different from $C^\infty$, the space of infinitely differentiable functions.

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