Existence and Stability of Spinning Embedded Vortices

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Leandros Perivolaropoulos

Division of Theoretical Astrophysics
Harvard-Smithsonian Center for Astrophysics
60 Garden St.
Cambridge, Mass. 02138, USA.

Abstract

I show the existence of a new type of vortex solution which is non-static but stationary and carries angular momentum. This spinning vortex can be embedded in models with trivial vacuum topology like a model with \( SU(2)_{\text{global}} \times U(1)_{\text{local}} \rightarrow U(1)_{\text{global}} \) symmetry breaking. The stability properties of the embedded spinning vortex are also studied in detail and it is shown that stability improves drastically as angular momentum increases. The implications of this result for vortices embedded in the electroweak model are under study.

\(^1\)E-mail address: leandros@cfata3.harvard.edu
\(^2\)Also, Visiting Scientist, Department of Physics, Brown University, Providence, R.I. 02912.
This work focuses on a new class of embedded vortices which are non-static but stationary and carry non-zero angular momentum. They may therefore be called spinning embedded vortices. It will be shown that these vortices have improved stability properties compared to their non-spinning counterparts (electroweak vortices with in a hypothetical model with Weinberg angle $\theta_w = \pi/2$\cite{1, 2, 3}) whose properties are discussed in the contributions of Klinkhamer and Vachaspati in this volume.

Let’s first clarify the motivation for generalizing the concept of the non-spinning electroweak (hereafter EW) vortex. As is well known the vacuum manifold of the standard EW model is a three sphere $S^3$. This implies that the model does not support any stable topological defects. However, almost a couple of years ago, Vachaspati\cite{4} pointed out that there is a vortex-like coherent state in the EW model which appears as an exact solution to the field equations. In a later paper\cite{5} we showed that this coherent state, called the embedded electroweak vortex\cite{6}, is in fact stable for a finite sector in parameter space. We also showed that the stability sector does not include the physically realized values of parameters. This was unfortunate but raised an exciting new question: Are there different types of embedded vortices with improved stability properties?

Until recently there had been two main classes of attempts to address this question. The first class originated from a work of Vachaspati and Watkins\cite{5} who showed that bound states on the embedded vortex may indeed improve its stability. Due to the complexity of the problem however, it was not clear if this improvement of stability was enough to stabilize the electroweak vortex for the physical values of parameters. The second class focused on embeddings in extensions of the standard EW model. There have been several interesting works on this subject\cite{6, 7, 8, 9} but of particular interest is the work of Dvali and Senjanovic\cite{8} who discovered topologically stable vortices in the two Higgs doublet EW model.

Here I focus on a new stabilization mechanism which is applicable to vortices of the standard, minimal EW model and introduces spin to improve the stability of the embedded vortices. An efficient way to introduce angular momentum to the vortex configuration is to embed it in a background of charge density which may be provided for example, by a charged external field with coherence length much larger than the width of the vortex.

Consider a toy model with symmetry breaking $SU(2)_{\text{global}} \times U(1)_{\text{local}} \rightarrow U(1)_{\text{global}}$. For our purpose, this model is identical to the bosonic sector of the
standard EW model with \( \theta_w = \pi/2 \). The vacuum manifold in this model is \( S^3 \). It has been shown\([1, 2]\) that the Nielsen-Olesen vortex\([10]\) is supported as an exact solution and that it is stable for a limited parameter range. Consider now an embedding ansatz\([11]\) which is a stationary generalization\([13]\) of the Nielsen-Olesen ansatz

\[
\Phi = \begin{pmatrix} 0 \\ f(r) e^{im\theta} e^{i\omega_0 t} \end{pmatrix}
\] (1)

\[
A_\theta = \frac{v(r)}{r}, \quad A_0 = \alpha(r)
\] (2)

The differences between the ansatz (1), (2) and the embedded Nielsen-Olesen vortex\([2]\) (EW string with \( \theta_w = \pi/2 \)) is the linear time dependence of the Goldstone field and the allowed possibility of having \( A_0 \neq 0 \). The Lagrangian density which supports this ansatz as a solution is a generalized Abelian Higgs Lagrangian where the scalar field \( \Phi \) has been promoted to an \( SU(2) \) doublet and also a background charge density has been introduced and coupled to the gauge field \( A_\mu \). It is of the form

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} |D_\mu \Phi|^2 - \frac{\lambda}{4} (|\Phi|^2 - \eta^2)^2 - A_\mu J^\mu
\] (3)

where \( J^\mu = (\rho(r), 0, 0, 0) \) is a background charge density. Let \( \rho(r) \to \rho_0 \) asymptotically, where \( \rho_0 \) is a constant. The two crucial parameters that determine the stability of the embedded vortex (1), (2) will be shown to be \( \beta \) and \( \rho_0 \). The angular velocity \( \omega_0 \) is fixed in terms of those parameters. The total charge of the vortex configuration plus the background is

\[
Q = \int_V d^2 x \left[ f^2 (\omega_0 - \alpha) + \rho(r) \right]
\] (4)

where the first term in the integral is due to the vortex field and the second is due to the background. This charge is in general non-zero, finite and conserved. The angular momentum \( \hat{M}_v \) of the vortex configuration may be shown to be proportional to the charge \( Q_v \) and the winding number \( m \) of the vortex field

\[
\hat{M}_v = \int_V d^2 x \hat{r} \times (\hat{E}_v \times \hat{B}) = -2 \int_V d^2 x f(r)^2 (\omega_0 - \alpha) v(r) \hat{e}_z \simeq -2 m Q_v \hat{e}_z
\] (5)
The crucial question that needs to be addressed is, *what are the stability properties of the vortex and how do they compare with the corresponding properties of its non-spinning counterpart (embedded EW vortex with $\theta_w = \pi/2$)?*

Consider a perturbation to the ansatz (1), (2) of the form

$$
\delta \Phi = \begin{pmatrix} g(r)e^{i n \theta} \\ \delta f(r, \theta) \end{pmatrix}, \quad \delta A_\mu(r, \theta)
$$

(6)

The energy of the perturbed configuration decouples as follows

$$
E = E_0(f, v, \alpha) + \delta E(\delta f, \delta A_\mu) + E_1(g)
$$

(7)

where the energy $E_0$ of the unperturbed configuration is identical in form to the energy of the topologically stable configuration obtained when the complex doublet in the ansatz is replaced by a complex singlet. The term $\delta E$ is identical in form to the perturbation of the topologically stable spinning vortex and must be positive definite since the topological vortex is stable. Thus the term whose sign determines the stability is the term $E_1(g)$. For those parameter values $(\beta, \rho_0)$ for which $E_1(g)$ can be negative, the embedded vortex (1), (2) is unstable. But $E_1$ may be shown\cite{11} to be positive definite iff the eigenvalue equation

$$
-g'' - \frac{g'}{r} + \frac{(v - n)^2}{r^2}g + \beta(f^2 - 1)g + \alpha^2 g = \omega^2 g
$$

(8)

has no negative eigenvalues. For the range of parameters $(\beta, \rho_0)$ for which this happens, the embedded spinning vortex is stable.

The unperturbed fields, $f, v, \alpha$ and therefore the sign of $E_1$, also depend on the detailed form of $\rho(r)$ close to the core of the vortex. I will therefore consider two opposite extreme forms of $\rho(r)$ and show that stability is dramatically improved in both cases. For the first type of background, the *soft* background, $\rho(r)$ adjusts in a way to completely neutralize the charge induced by the spin of the vortex *i.e.*

$$
\rho(r) = -\omega_0 f^2(r), \quad \alpha = 0
$$

(9)

For such a background, the field equations allow $\alpha = 0$ everywhere. This implies that the Schroedinger-type potential of the eigenvalue equation (8),
looses a positive definite contribution: the term $\alpha^2$. This makes the equation more receptive to negative eigenvalues. We are therefore led to the conjecture that the soft background is the background of minimum stability.

The other opposite extreme form is that of a hard background where $\rho(r)$ does not feel the presence of the vortex and is constant everywhere

$$\rho(r) = \rho_0, \quad \alpha \neq 0$$

In that case, $A_0$ (or $\alpha$) can not vanish\[11\] and we expect better stability properties for this background type.

**Figure 1:** The stability map is parameter space for the embedded spinning vortex. Sector III. (II. and III.) is the stability sector for the case of a soft (hard) background.

It is straightforward to solve the eigenvalue problem numerically using relaxation methods\[14\], for the two types of backgrounds and map the stability sectors in parameter space $(\beta, \rho_0)$ for each case. The stability sector for the soft (hard) background is on the right of the continuous (dotted) line in Fig. 1.

The case of the non-spinning EW vortex is obtained for $\rho_0 = 0$ and has been studied previously \[12, 3\] (stability only for $\beta < 1$). Clearly the range of $\beta$ corresponding to stability increases dramatically when spin is introduced ($\rho_0 \neq 0$). In fact it may be shown analytically\[11\] that for $\rho_0 \to \infty$ the embedded spinning vortex is stable for any $\beta$.

In conclusion, I have shown that angular momentum induced by a charged background can indeed stabilize the embedded EW vortex. This result raises
some interesting new questions. First, can spin stabilize the EW vortex for physical values of parameters? Second, can global embedded vortices (which are normally always unstable) be stabilized by the introduction of spin? These issues are currently under investigation.

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