Formation of Spiral Structures in Galactic Discs from Reaction-Diffusion Networks

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Abstract

In this paper, we propose an extension of the Smolin-model of a galactic disc in isolated, flocculent Sc-type galaxies. This model supplies the necessary mechanism to suppress spatial inhomogeneities on short distance scales not present in the model proposed by Smolin.

I. INTRODUCTION

In astronomy, the problems of pattern formation appear on a broad variety of scales; from the formation of galaxy clusters on the scale of 50 to several hundred mpc down to the formation of stars on a scale of a few dozen pc. The most intensely studied of these is the problem of the formation of spiral structures in galactic discs, which has brought forth numerous theoretical and numerical approaches [1–8], which produce impressive results, but also have their own shortcomings. Models based on propagating starformation [2] do indeed reproduce the appearance of a range of galaxies [4,5], however, they either involve strong simplifications of the astronomy involved or need the fine tuning of rates in order to reproduce the observed structures. In the case of density wave models [3] where density waves facilitate the star formation process, a continuous recurrance of density waves passing through the galaxy is required in order to keep the star formation process continuous over the galaxies lifetime, hence there is the need for sufficient sources of density waves requiring large massive objects, like other galaxies, not being too distant from the galaxy in question. However, if one is concerned with isolated galaxies where there cannot be a continuous recurrance of density waves, there has to be another process or processes to produce the observed spiral structure other than density waves.

As a consequence of the above, in the following, we will be concerned with flocculent, spiral structure in so called Sc-type galaxies, which are sufficiently far from other galaxies, such that the spiral structure has to be endogenous, i.e. understood as a product of processes occurring within the disc. As was pointed out by the authors of Ref. [4,6] these galaxies show

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spiral structure in the blue light but not in the red light; hence the structure is mainly not due to a density wave but is rather a trace of a star formation process [1].

It is important to point out that star formation apparently occurs at a constant rate in these galaxies when averaged over the whole disc. This is an important clue, as pointed out by Smolin [1], since the implication of this constant rate is the presence of feedback mechanisms in the processes governing the rate of star formation, so as to keep the rate steady and slow [1,11]. Since the rate is constant over the history of a galaxy ($10^{10}$ years) as compared to the time scales of the dynamical process ($10^7$ years), there is no other known way to explain that galaxies with slow and steady rates of star formation are so common. Hence, one can put forth the hypothesis [1] that the star formation process can be explained to be a network of self-regulated and autocatalyzed processes resulting from the self-organization of the material in the disc. Thus non-equilibrium statistics govern the evolution of the disc system in which the network processes, involving matter- and energy-flows amongst its components, are governed by feedback-loops. In other words, the way spatial inhomogeneities are created and stabilized from non-equilibrium systems might explain the patterns produced by the star formation process.

There are numerous theoretical and experimental articles (the interested reader should consult the references contained in [1]) describing how spatial and temporal patterns are produced in non-equilibrium systems. However, we would like to point out one important aspect. Recent studies of non-equilibrium systems have produced impressive successes by which patterns, that can be reproduced in actual laboratory situations, are explained by simple models involving both partial differential equations and discrete elements like cellular automata. Hence, structure formation seems to occur in a broad variety of far-from-equilibrium steady-state systems. To see whether this is true in the above mentioned case of flocculent galaxies let us give a ‘check-list’ of the main elements that characterize systems to which non-equilibrium models of pattern formation are applicable. These include [13–21]:

- The system is in a steady-state with a slow and steady flow of energy (and matter) through it.
- The steady-state is far from thermodynamic equilibrium i.e a coexistence of several phases of matter exchanging matter and energy through closed cycles.
- The flow-rate of material around these cycles is governed by feedback-loops that have arisen during the organization of the system to the steady state.
- The reaction networks are autocatalytic i.e. substances serving as catalysts/inhibitors of the reaction network are produced by reactions within the network.
- The possibility of spatial segregation of different phases/materials in the network if catalysts/inhibitors, or more precisely their effects, propagate over different scales.

Given these characteristics, there are a variety of models, particularly of the reaction-diffusion type [13–15] which describe how spatial structure is formed and stabilized. It was shown in [1] that the above mentioned criteria are met by the Sc-type galaxies and hence, the galactic disc of these galaxies can be described as an autocatalytic network of reactions or put differently, the criteria, allowing the system to be described by a reaction-diffusion model, have been met.
In Sec. II the main processes are reviewed drawing heavily on the excellent exposition in [1], a homogenous “one zone model” very close to the Smolin-model [1] is set-up, and it is shown that the system reaches a steady state. In Sec. III the model is expanded to allow for spatial variations. In Sec. IV a linearized analysis is carried out and a closer look is taken at the region in parameter space for which spatial inhomogeneities occur in the model. It will be shown that the model proposed in this paper remedies one major drawback of the Smolin-model, namely the non-suppression of instabilities at small scales. Furthermore there are at least two distinct regions in the parameter space which allow for qualitatively different results in the sensitivity of the model to changes in the main parameters. Conclusions and future work will be discussed in Sec. V.

II. THE HOMOGENOUS “ONE ZONE MODEL”

Before describing the “one zone model”, let us briefly review the major processes involved in the model.

• Condensation of giant molecular clouds (GMC’s)

GMC’s are cold clouds condensing out of the interstellar medium (ISM), apparently forming scale invariant or fractal distributions of cold molecular gas and dust.

  – Catalysts: Dust, carbon and oxygen.

  – Inhibitors: The two main inhibitors are ultraviolet radiation from massive stars which heats the ambient ISM making condensation less likely and shockwaves from supernovae dispersing the GMC’s. UV radiation is inhibitory on a very long distance scale ($L_{long} \simeq$ size of galaxy) with a very short propagation time of about $10^5$ years, whereas the inhibitory effect of the shockwaves is on a short distance scale ($L_{short} \simeq 20$ pc) due to the fact that the ISM’s viscosity, with respect to shockwaves, slows these waves down, enough to be ineffective as an inhibitor on $L_{int} \simeq 100$ pc.

• Collapse of GMC’s

Star formation occurs when the cores of GMC’s collapse. There is a small possibility for a spontaneous rate of core collapse, however the collapses of cores massive enough to lead to formation of massive stars is usually catalyzed.

1 The motivation for using diffusion was taken from [1].

2 Note that these substances spread through the ISM over an intermediate distance scale $L_{int} \simeq 100$ pc, corresponding to the scale over which the products of massive stars and supernovae are spread due to the supernovae’s shockwaves.

3 This useful picture was introduced by Pavarrano et al. [1].
– Catalysts: The main catalyst are shockwaves from supernovae or HII-regions. These processes have a typical scale of $L_{\text{int}}$.4

– Inhibitors: The main inhibitors are stellar winds from young, massive objects disrupting GMC’s in which they are formed as well as UV radiation from young, massive stars, evaporating GMC’s. The last process is not directly related to the heating of the ISM. The typical distance scales are $L_{\text{short}} \approx$ size of one cloud complex.

The effect of collapsing GMC’s is the introduction of a latency time $\tau_L$ during which the star forming process will not recur. $\tau_L$ is of the order of the time it takes for a GMC to form out of gas, after another GMC in the same region has been evaporated as a result of radiation from newly formed stars.

• Star Formation

The final stages of star formation involve the formation of a protostar and an accretion disc. The process is self-limiting (see [12]) i.e. matter accretes onto the protostar until it is stopped by the outflow of matter due to stellar winds created in the accretion disc after the start of nuclear reactions.

Let us move on and start discussing the homogeneous “one zone model”. Neglecting spatial variations for the moment, the reaction network may be described by a “one zone model” analogous to a system of equations describing homogeneous, chemical reaction networks. Labeling the densities of components as:

\[ c = \text{cold gas in GMC’s} \]
\[ w = \text{warm, ambient gas} \]
\[ s = \text{massive stars} \]
\[ d = \text{light stars} \]
\[ r = \text{density of UV radiation} \]
\[ h = \text{density of shockwaves from supernovae} \]

we get, following the spirit of the chemical reaction models:

\[
\begin{align*}
\dot{c} &= \frac{\alpha'w^2}{r} - (\beta + \mu)ch - \gamma cs - \gamma' cr \\
\dot{s} &= \beta ch - \frac{s}{\tau} \\
\dot{w} &= -\frac{\alpha'w^2}{r} + \frac{s}{\tau} + \delta + \gamma' cr + \gamma cs \\
\dot{r} &= -\phi(c + w)r + \eta s \\
\dot{h} &= -\phi'(c + w)h + \eta' s \\
\dot{d} &= \mu ch. 
\end{align*}
\]

\(1\)

\[4\] Density waves can also cause core collapses. These are probably dominant in Grand Design Spirals but are of lesser importance for Sc-type galaxies
The parameters describe the rates of different, astrophysical processes:

\[ \tau = \text{life time of a typical massive star (10}^7\text{ years).} \]

\[ \alpha' \sim \text{characteristic rate for GMC's to condense from warm gas.} \]

\[ \beta, \mu, \gamma \text{ and } \gamma' \text{ govern the rates per unit mass density at which material flows from GMC's to other states of the ISM due to processes catalyzed by the action of shockwaves, UV radiation and massive stars.} \]

\[ \beta = \text{formation rate of massive stars per unit density of c.} \]

\[ \mu = \text{formation rate of light stars per unit density of c.} \]

\[ \gamma = \text{rate per unit density that cold gas is heated by massive stars.} \]

\[ \gamma' = \text{rate per unit density that cold gas is heated by UV radiation.} \]

\[ \delta = \text{rate at which warm gas accretes onto the galactic disc.} \]

\[ \phi, \phi' = \text{rate at which radiation and shockwaves are damped by the ISM.} \]

\[ \eta', \eta = \text{rate at which shockwaves and radiation are produced by massive stars.} \]

The simplifications in the physics are the same as those made by Smolin [1] in his model. Let us quickly recount: there are only two types of stars, light stars can evaporate spontaneously from GMC’s, the role of dust and carbon is not explicitly included, the Pavarrano process is given by a simple negative feedback and \( \delta \) is assumed to be constant. Notwithstanding these grains of salt, the model is an acceptable first model of how the ISM arrives at a steady-state. Solving Eq. [1] for the steady state (all time derivatives vanish except \( \dot{d} \)) one finds a unique solution:

\[ s_0 = \frac{\beta \tau \delta}{\mu} \]

\[ h_0 = \frac{\delta}{\mu c_0} \]

\[ r_0 = \frac{\eta' h_0}{\eta \phi} \]

\[ w_0 = \left[ \frac{\beta \tau \eta'}{\phi'} - 1 \right] c_0 \]

\[ 0 = \frac{\mu^2 \eta' \alpha'}{\delta^2 \eta} \left[ 1 + \beta \tau \eta' \right] c_0^3 - \gamma \beta c_0 - \left[ \beta + \frac{\gamma' \eta}{\eta'} \right], \quad (2) \]

\( \tau \) will also govern, as it turns out, the characteristic time scales of the instabilities. It is important for this scale not to be too long or too short such that the instabilities do neither grow too fast which would be in contradiction to the observed steady star formation rate nor too slow which too would not be in accord with the observation of a steady star formation rate. This means that a scale of the order of the dynamical time scale of the system will be appropriate.

The efficiency for the production of massive stars is small, hence one must choose parameters such that \( \frac{\alpha' w}{\tau} > \tau^{-1} \) in the steady state.

This is a a short scale/local effect.
also
\[ \dot{d} = \delta, \]  
so that the steady-state forms light stars at the rate \( \delta \). This, of course, has to be true since matter is conserved in the system!

### III. THE REACTION-DIFFUSION SYSTEM

The formalism of temporal and spatial inhomogeneities can be naturally understood in the realm of biology and chemistry, if autocatalytic reaction networks have catalysts/inhibitors spread through the system over different scales in space and time i.e. one has a hierarchy of scales over which catalytic/inhibitory reactions are alternatively more important (see Ref. [13–20,22] for details). The main claim of Ref. [1] and of this paper is that this may give a natural understanding about the appearance of structures in galactic discs. As was pointed out in Sec. [II] the above mentioned conditions are met (see [1]). The processes over \( L_{\text{int}} \) are all catalytic and over \( L_{\text{short}} \) and \( L_{\text{long}} \) inhibitory.

Now that one has met the conditions for pattern formation in the model, one has to incorporate spatial inhomogeneities. This is most commonly done through diffusion. Diffusion definitely occurs in biological systems, shockwaves and radiation however are probably best described by propagation or some Navier-Stokes type of flow-equation. Nevertheless the assumption of diffusion is a rather good first approximation, taking into account the rather impressive results of the Gerola-Seiden-Schulman model [4] based on cellular automata and the Elmegreen-Thomasson model [5] where shockwaves responsible for propagating star formation are indirectly modeled as either a direct effect on the catalyzation of star formation in neighbouring regions or a process by which “young stars” diffuse from their parent cloud and then give energy to a nearby GMC. In order to include diffusion in our equations we add a diffusion term \( D \nabla^2 \). We also simplify by normalizing all densities according to:
\[ \bar{c}(x, t) = \frac{c(x, t)}{c_0} \text{ etc.} \]

We then obtain the following system of equations:
\[
\begin{align*}
\dot{c} &= \alpha \frac{\bar{w}^2}{\bar{r}} + \left[ \nu + \frac{\rho}{\epsilon T} + \frac{1}{\epsilon T} - \alpha \right] \bar{c}s - \nu \bar{c} \bar{r} - \left[ \frac{\rho}{\epsilon T} + \frac{1}{\epsilon T} \right] \bar{c} \bar{h} \\
\dot{s} &= D_s \nabla^2 \bar{s} + \frac{1}{\tau} \left[ \bar{c} \bar{h} - \bar{s} \right] \\
\dot{w} &= \alpha \epsilon \left[ \bar{c} \bar{s} - \frac{\bar{w}^2}{\bar{r}} \right] + \epsilon \nu \left[ \bar{c} \bar{r} - \bar{c} \bar{s} \right] + \frac{\rho}{\tau} \left[ \bar{s} - \bar{c} \bar{s} \right] + \frac{1}{T} \left[ 1 - \bar{c} \bar{s} \right] \\
\dot{r} &= D_r \nabla^2 \bar{r} - \frac{\sigma \epsilon}{1 + \epsilon} \bar{c} \bar{r} - \frac{\sigma}{1 + \epsilon} \bar{w} \bar{r} + \sigma \bar{c} \\
\dot{h} &= D_h \nabla^2 \bar{h} - \frac{\sigma' \epsilon}{1 + \epsilon} \bar{c} \bar{h} - \frac{\sigma'}{1 + \epsilon} \bar{w} \bar{h} + \sigma' \bar{s} \\
\dot{d} &= \delta \bar{c} \bar{h}.
\end{align*}
\]

with the following parameters:
\[
\begin{align*}
\alpha &= \frac{\alpha' w_2}{c_0 r_0}, \quad \frac{1}{T} = \frac{\delta}{w_0} \\
\rho &= \frac{s_0}{w_0} \quad \nu = r_0 \gamma' \\
\epsilon &= \frac{c_0}{w_0} \quad \sigma = \eta \frac{s_0}{r_0} \\
\sigma' &= \eta' \frac{s_0}{r_0}.
\end{align*}
\] (6)

where \(\alpha\) gives the time of condensation of cold clouds, which can be taken to be about \(10 - 100\) (\(\tau = 10^7\) years, the typical time scale of our problem, is set to be 1), \(\rho\) is the ratio of mass in massive stars to mass in warm gas and can be set to 0.1 indicating that the efficiency of the star formation process is low and the massive-star production is suppressed by the power law in the initial mass function. \(\epsilon\) is the ratio of cold to warm gas and should be around unity since about 50% of the gas in the ISM is observed to be in GMC’s. \(T^{-1}\) is the time scale for accretion of warm gas onto the disc and should be about \(100 - 1000\) times longer than \(\tau\) since one needs a continuous supply of matter over the galaxies lifetime. \(\sigma\) and \(\sigma'\) are the parameters governing the “viscosity” of the ISM with respect to UV radiation and shockwaves and can be taken to be about 10 and 0.1 respectively, again with \(\tau = 1\), since \(\sigma^{(i)} \sim \frac{1}{T'}\) where \(T'\) (time of travel to the typical scale of the process) can be reasonably taken to be \(10^6\) and \(10^8\) years respectively. \(\nu\) is the parameter governing the propagation of the core collapsing effect of UV radiation and massive stars, taken to be \(10^2\) to \(10^3\) in units of \(\tau\), since the effect takes about \(10^5\) years to propagate through the disc. The diffusion constants \(D_s, D_r\) and \(D_h\) can, reasonably, be taken to be \(D_s = L_{int}^2/\tau = 10^{-3}\), \(10^5 \leq D_r \leq 10^9\) and \(D_h \simeq 10^{10}\) in units of \(\tau\). The greatest uncertainties lie in \(\nu, \sigma, \sigma'\) and the diffusion constants since there is no data available on these parameters, i.e one has to check the sensitivity of the model to changes in these parameters.

As a next step, we can assume the problem to be essentially two dimensional due to the fact that the disc is very “thin” as compared to its radius. One has thus arrived at a reaction-diffusion model governing the dynamics of the ISM. It has to be pointed out here, that the differential rotation of the disc is not included at this stage. It will be included in the full numerical analysis which will be the next stage of the project.

**IV. LINEARIZED ANALYSIS AND THE REGION IN PARAMETER SPACE**

In the following, the linearized analysis of the model proposed in Sec. III will be carried out to see if there are indeed unstable modes which may develop into spatial structure. To do this, one expands Eq. 5 to linear order from the steady-state:

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8Here the main effect comes from UV-radiation which travels with the speed of light.

9 This is about the size of the radius of the galactic disc times the speed of light, since UV radiation propagates through the whole disc.

10 This is about \(L_{int} \times \) average speed of shockwave \(O(10^5 \frac{m}{s})\).
\[ \dot{c} = 1 + C \text{ etc.} \] (7)

and arrives at the following system of equations:

\[
\begin{align*}
\dot{C} &= \alpha [2W - C - S] - \nu [R - S] - \left[ \frac{\rho}{\epsilon \tau} + \frac{1}{\epsilon T} \right] [H - S] \\
\dot{S} &= D_s \nabla^2 S + \frac{1}{\tau} [C + H - S] \\
\dot{W} &= \alpha \epsilon [-2W + C + S] + \epsilon \nu [R - S] - \left[ \frac{\rho}{\tau} + \frac{1}{T} \right] C - \frac{1}{T} S \\
\dot{R} &= D_r \nabla^2 R - \frac{\sigma}{1 + \epsilon} [\epsilon C + W] - \sigma [R - S] \\
\dot{H} &= D_h \nabla^2 H - \frac{\sigma'}{1 + \epsilon} [\epsilon C + W] - \sigma' [H - S] \\
\dot{d} &= \delta [1 + C + H]
\end{align*}
\] (8)

As an ansatz, one can take the following solutions to Eq. 8:

\[
\begin{align*}
C &= \tilde{C} e^{\lambda t} \cos(k \cdot x) \\
S &= \tilde{S} e^{\lambda t} \cos(k \cdot x) \\
W &= \tilde{W} e^{\lambda t} \cos(k \cdot x) \\
R &= \tilde{R} e^{\lambda t} \cos(k \cdot x) \\
H &= \tilde{H} e^{\lambda t} \cos(k \cdot x)
\end{align*}
\] (9)

which describe an instability with wavevector \( k \) growing exponentially with a time scale \( \lambda^{-1} \).

To discover instabilities, one has to find solutions for reasonable values of the parameters and wavelengths for which \( \lambda \) is real and positive. Plugging Eq. 9 into Eq. 8, one arrives at an eigenvalue problem

\[
M^b a^b = \lambda v^a 
\]

with \( \nu^a = (C, S, W, R, H) \) and the matrix:

\[
\begin{pmatrix}
-\alpha & -\alpha + \nu + \frac{\rho}{\epsilon \tau} + \frac{1}{\epsilon T} & 2\alpha & 0 & -\nu \\
-\frac{\alpha \epsilon}{\tau} & -\tau^{-1} - D_s k^2 & 0 & 0 & -\frac{\rho}{\tau} - \frac{1}{\epsilon T} \\
-\sigma & \sigma \epsilon - \epsilon \nu - \frac{1}{\tau} & -2\alpha \epsilon & \epsilon \nu & 0 \\
-\sigma' \epsilon & \sigma' & -\sigma - D_r k^2 & 0 \\
0 & \sigma' & -\sigma' & 0 & -\sigma' - D_h k^2
\end{pmatrix}
\] (10)

It is easy to study the behaviour of the eigenvalues of this matrix as a function of the parameters. For a broad range of parameters one finds three negative eigenvalues, one of which is very negative as found in the Smolin- model \[\] stemming from the very large \( D_r \).

There are also two positive eigenvalues, the largest of these governs the evolution of the dominant instability of the disc\[\]. In Fig.(1-2) the largest positive eigenvalue is plotted as a function of \( y \), where \( y = \log(k) \), in units of \( L_{int} \) i.e. \( y = 0 \) corresponds to for example \( L_{int} \), hence the largest scale are to the left towards the vertical axis and the smallest scales are to the right. One observes by changing the parameters of the model that the most unstable

\[\text{11 For every set of parameters there are values of } k \text{ which give complex eigenvalues for } \lambda \text{ simult-}\]
FIG. 1. \( \nu/\alpha = 3 \), \( D_r/\sigma = 10^5 \), \( D_h/\sigma' = 10 \) and \( D_s = 0 \).

FIG. 2. \( \nu/\alpha = 30 \), \( D_r/\sigma = 10^4 \), such that the unstable mode does not run out of the graph (clearly unphysical situation) and \( D_h/\sigma' \) as in Fig.1 with \( D_s = 10^{-3} \).
modes at large scales are very stable towards parameter changes, however this is not true for small scales.

The time scale of the instability is between $10^6$ and $10^8$ years as can be easily deduced from the figures. Even though at this stage the scales at which the instabilities occur seem to be right for the initiation of spiral structure formation as in Sc-type galaxies, the full non-linear analysis with rotation has to be carried out to be sure whether this model indeed works. As was observed above, the short and long distance suppression of instabilities is sensitive to the choice of $\sigma, \sigma', \nu, D_r, D_s, D_h$ and, amazingly enough, also to $\alpha$. In the following, the parameters, not yet too constrained by observations, are varied independently from one another up and down by 1 order of magnitude and the following conclusions can be drawn about the size of the parameter space allowing instabilities to form and the importance of the different parameters (see Fig.(3)):

neously above and below the real axis. It seems as if the system undergoes a phase transition but this has not been investigated. In the figures one can see the points beyond which the eigenvalues become complex by looking for the kinks in the graphs above the $y$-axes. One notices that the real positive eigenvalues do not quite reach 0 but this is of no consequence to the analysis since one can easily extrapolate from the point of the kink down to the $y$-axes due to the very regular behaviour of the largest positive eigenvalue with changing $k$. The reason why the figures were continued beyond the real positive eigenvalues is a purely technical one and has to do with the difficulty of manipulating large tables of numbers under Mathematica.

FIG. 3. a) $D_r/\sigma = 10^5$ b) $D_r/\sigma = 10^4$ c) $D_h/\sigma' = 1$ d) $D_h/\sigma' = 10$. Note $D_s = 0$ in Fig.3(a-d)
• The formation of instabilities and the length scales at which they occur, do not depend on the parameters separately but rather on the ratios \( \nu/\alpha, D_r/\sigma \) and \( D_h/\sigma' \). The time scale depends strongly on \( \nu/\alpha \) and much less strongly on the other ratios.

• Since \( D_s \) is at most \( 1^{12} \), one does not find any significant dependence of the model for \( 0 \leq D_s \leq 1 \) for \( \nu/\alpha = 3 \), hence one can set it to 0. For \( \nu/\alpha = 30 \), there is a strong dependence of the short-distance scale instabilities on \( D_s \). One recovers the same type of morphology as in Fig.2 for \( D_s = 1 \) but has no or only weak suppression of short-distance scale instabilities for \( D_s = 0 \).

• The ratio \( \nu/\alpha \) strongly determines the onset of instabilities as one obtains two different morphologies for e.g., \( \nu/\alpha = 3 \) or \( \nu/\alpha = 30 \) (see Fig.(1-2)) at two different time scales. This fact is not too surprising since \( \alpha \) is the time of core collapse and \( \nu \) determines the influence of UV radiation on cloud condensation. Let us point out once more that for \( \nu/\alpha = 3 \), at a time scale of about \( 10^8 \) years, there is only one type of morphology \( 1^{13} \), however for \( \nu/\alpha = 30 \), at a time scale of \( 10^6 \) years, one finds both morphologies from Fig.1,2 if one varies for example \( D_s \).

• The ratio \( D_h/\sigma' \) influences the scale of the onset of the instabilities more weakly (see Fig.(3c,d)). That the ratio influences the lower scale is not surprising since shockwaves act as an inhibitor on short scales and as a catalyst on intermediate scales.

• The ratio \( D_r/\sigma \) influences the maximum range of the instabilities, but not too strongly (see Fig 3a,b). This again is not surprising since UV radiation is supposed to inhibit star formation on large scales. However, it is interesting to note that for a particular ratio one obtains the typical size of a galaxy independent of what \( D_r \) is\( 1^{14} \) as long as the ratio stays constant. Also note that \( \sigma \) does not depend in any way on the size of a typical galaxy.

• From the above and the parameter space search exemplified by the figures, one can draw the conclusion that the region in parameter space allowing for instabilities to occur on the right scales, is fairly large, however not too large since varying the ratios by 2 orders of magnitude destroys the formation of unstable modes in the model.

V. CONCLUSIONS

The conclusion is that, albeit only a linearized analysis has been carried out, the model provides instabilities in time and space on the right scales and is not very arbitrary (parameter space region seems to be not too large). This gives rise to the hope that a full

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\(^{12}\)Massive stars diffuse very slowly

\(^{13}\)Unless one basically neglects the diffusion of shockwaves.

\(^{14}\)\( D_r \) is the only place where the size of a typical galaxy enters.
non-linear analysis which is currently underway, will be successful and the model indeed contains flocculent structures. Even if this is so, it is by no means correct to say that this is indeed the way structure formation occurs in Sc-type galaxies. For this statement to be made a renormalization group analysis, stage three of this project, will have to be carried out in order to make predictions for observations like the correlation function of for example cold clouds to cold clouds in Sc-type galaxies and these will then have to be confirmed by experiment before one can be confident that reaction-diffusion models work for flocculent galaxies.

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