Power-law Magnetic Field Decay and Constant Core Temperatures of Magnetars, Normal and Millisecond Pulsars

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Abstract. The observed correlations, between the characteristic ages and dipole surface magnetic field strengths of all pulsars, can be well explained by magnetic field decay with core temperatures of $2 \times 10^8$ K, $2 \times 10^7$ K, and $10^5$ K, for magnetars, normal radio pulsars, and millisecond pulsars, respectively; assuming that their characteristic ages are about two orders of magnitude larger than their true ages, the required core temperatures may be reduced by about a factor of 10. The magnetic decay follows a power-law and is dominated by the solenoidal component of the ambipolar diffusion mode. In this model, all NSs are assumed to have the same initial magnetic field strength, but different core temperature which do not change as the magnetic field decays. This suggests that the key distinguishing property between magnetars and normal pulsars is that magnetars were born much hotter than normal pulsars, and thus have much longer magnetic field decay time scales, resulting in higher surface magnetic field strength even with the same ages of normal pulsars. The above conclusion agrees well with the observed correlations between the surface temperatures of magnetars and other young NSs, which do not agree with the cooling dominated evolution of neutron stars. This suggests a possible scenario that heating, perhaps due to magnetic field decay, balances neutron star cooling for observed pulsars.

1. Introduction

The strength of the surface dipole magnetic field of a neutron star may be estimated directly from the period and period derivative, i.e. $B_d \approx 3.3 \times 10^{10}(P\dot{P})^{1/2}$ G, which assumes that its rotational energy is lost entirely via magnetic dipole radiation. Then the spin-down age (characteristic age) can also be estimated as $\tau_c = P/2\dot{P}$. Then all pulsars can also be displayed in the $\tau_c - B$ diagram (Figure 1), instead of the $P - \dot{P}$ diagram. Clearly all pulsars can be divided into three clusters in Figure 1; from top to bottom, the three cluster are magnetars, normal pulsars, and millisecond pulsars, respectively.

Magnetars, which are normally referred to as neutron stars with surface magnetic fields larger than the quantum critical value, $B_{cr} = m^2c^3/eh \approx 4.4 \times 10^{13}$ G, have been proposed as the most promising energy source for Anomalous X-ray Pulsars (AXPs) and the Soft Gamma-ray Repeaters (SGRs). In the $\tau_c - B$ diagram, magnetars are composed of a distinctive group with higher magnetic field ($\sim 5 \times 10^{14} - 2 \times 10^{15}$ G) and younger characteristic ages ($\sim 10^3 - 10^5$ yrs). The two classes of X-ray Pulsars, AXPs and SGRs, differ from the common accretion-powered pulsars in massive X-
Figure 1. The $\tau_c - B$ diagram. Blue triangles: SGRs and AXPs. Magenta circles: other pulsars, including normal radio pulsars, radio-quiet pulsars, high energy pulsars, millisecond pulsars, binary pulsars, etc. Thin lines: the best fitted line for magnetars (dotted line), ordinary pulsars with dipole field strength $4 \times 10^{10} - 2 \times 10^{13} \text{G}$ (dashed line), and millisecond pulsars with dipole field strength $< 10^9 \text{G}$ (dot-dashed line). Thick solid lines: ‘death line’ (black), magnetic field decay for $T_8 = 2$ (blue), $T_8 = 0.2$ (red), $T_8 = 0.001$ (purple). For magnetars, their characteristic ages ($\tau_c$) and surface magnetic fields $B_{\text{surf}}$ ($B_d$) are from McGill SGR/AXP Online Catalog (http://www.physics.mcgill.ca/~pulsar/magnetar/main.html); for other pulsars, $\tau_c$ and $B_{\text{surf}}$ are from the ATNF Pulsar Catalogue (http://www.atnf.csiro.au/people/pulsar/psrcat/).

Ray binaries mainly in the following observational properties (see Mereghetti 2008, for a review): they probably have no binary companions, but their persistent X-ray luminosity can be larger than their spin-down power; they have periods of activities during which they emit numerous short bursts in hard X-ray/soft gamma-ray band, and the bursts have peak luminosity up to $\sim 10^{42} \text{ erg s}^{-1}$ and last for $\sim 0.01 - 1 \text{ s}$; they have a short hard spike, with luminosity larger than $\sim 5 \times 10^{44} \text{ erg s}^{-1}$, followed by a long pulsating tail, the so-called giant flares, a very striking phenomenon observed only from SGRs.

It is believed that the energy reservoir fueling the SGR/AXP activity is their extreme magnetic fields (Thompson & Duncan 1995, 1996, hereafter TD96). For instance, their magnetic fields have enough free energy to power the giant flares; meanwhile the short duration of their initial spikes is consistent with the propagation with Alfvén speed of the magnetic instability over a whole neutron star surface (Thompson & Duncan 1995), and the magnetic confinement of the hot plasma is responsible for the pulsating tails (Mereghetti 2008). However, the recent discovery of a SGR with a very low surface dipolar magnetic field, SGR 0418+5729, whose dipolar magnetic field cannot be
greater than $7.5 \times 10^{12}$ G, implying that a high surface dipolar magnetic field is not necessarily required for magnetar-like activities (Rea et al. 2010). And the outer gap model of magnetars may also be challenged by the null detection in a Fermi/LAT observation of AXP 4U 0142+61 (Tong et al. 2010). Thus, it is likely that the magnetar activity is driven by the magnetic energy stored in the internal toroidal field (Thompson & Duncan 1995, 2001) or the surface multipolar field. After the discovery of SGR 0418+5729, the magnetar population includes objects with a wider range of $B$-field strengths and ages; however their evolutionary stages still remain far from clear.

In this report, by adopting the magnetic field decay model to the magnetars, normal pulsars, and millisecond pulsars, we show that the inner core temperatures of the three types of pulsars are systematically different, consistent with the observed three clusters shown in Figure 1 and the observed surface temperature of magnetars and other young normal pulsars. It is found that the surface and core temperatures of magnetars are the highest (include the low magnetic field magnetar SGR 0418+5729) and remain constant for at least 24 Myrs.

2. Magnetic Field Decay of All Pulsars

Goldreich & Reisenegger (1992, hereafter GR92) studied several avenues for magnetic field decay in isolated neutron stars: ohmic decay, ambipolar diffusion, and Hall drift. Depending on the strength of the magnetic field, each of these processes may dominate the evolution. Heyl & Kulkarni (1998, hereafter HK98) applied the model to magnetars, and found that for sufficiently strong nascent fields, field decay alters significantly the cooling evolution relative to similarly magnetized neutron stars with constant fields. Following GR92 and HK98, a simple differential equation can be used to describe the dipole magnetic field decay (HK98)

$$\frac{dB_p}{dt} = -B_p\left(\frac{1}{t_{\text{ohmic}}} + \frac{1}{t_{\text{ambip}}} + \frac{1}{t_{\text{hall}}}\right),$$

where $B_p$ is the strength of the magnetic field at the pole at the surface of a neutron star, and $t_{\text{ohmic}}$, $t_{\text{ambip}}$, and $t_{\text{hall}}$ are the decay time scales due to ohmic decay, ambipolar diffusion, and Hall drift, respectively. Taking $\sigma_0 = 4.2 \times 10^{28} T_8^{-2} (\rho/\rho_{\text{nuc}})^3$ s$^{-1}$, where $T_8$ denotes the temperature in units of $10^8$ K, $\rho_{\text{nuc}} \equiv 2.8 \times 10^{14}$ g cm$^{-3}$, we have $t_{\text{ohmic}} \sim 2 \times 10^{11} \frac{L_5^2 \rho^3}{T_8^2 \rho_{\text{nuc}}}$ yr, where $L_5$ is a characteristic length scale of the flux loops through the outer core in units of $10^5$ cm. The time scales for Ambipolar diffusion are given by, $t_{\text{ambip}}^s \sim 3 \times 10^9 \frac{L_5^2 T_8^3}{B_{12}^2}$ yr, and $t_{\text{ambip}}^r \sim \frac{5 \times 10^{15} T_8^2}{B_{12}^2} (1 + 5 \times 10^{-7} L_5^2 T_8^8)$ yr, where $B_{12}$ is the field strength in units $10^{12}$ G, the $t_{\text{ambip}}^s$ represent for the time scale of solenoidal component and $t_{\text{ambip}}^r$ for irrotational component; note that the latter is only for the case of modified URCA reactions. Finally the time scale of the Hall cascade is $t_{\text{hall}} \sim 5 \times 10^8 \frac{L_5^2 T_8^2}{B_{12}^2} (\rho_{\text{nuc}}/\rho)^2$ yr.

To compare eq. (1) with data shown in Figure 1, we take $L_5 = 1$, $\rho/\rho_{\text{nuc}} = 1$, initial field strengths $B_{p,0} = 10^{16}$ G, and begin the integrations at $t = 0.1$ yr, and assume constant core temperatures for the three classes of pulsars. In Figure 1, we plot the calculated magnetic field decay histories with eq. (1) for three typical core temperatures; for comparison the pulsar ‘death line’ is also plotted (Chen & Ruderman 1993). Several
Figure 2. Observations of the surface temperatures of magnetars and young NSs. Left panel: surface temperature vs. characteristic age. The red solid line is a basic theoretical cooling curve of a nonsuperfluid neutron star with $M = 1.3 \, M_\odot$ (Yakovlev & Pethick 2004). Right panel: surface temperature vs. magnetic field. The green dotted line and dashed line are heating balance lines (HBL) for different parameters, and the band between them is heating balance area. The red solid line is a theoretical curve for magnetothermal evolution of magnetized neutron stars, including crustal heating by magnetic field decay (APM08). Each number marked in the plot is the characteristic age of the pulsar in unit of $10^3$ yrs.

Observations can be obtained from Figure 1: (i) Magnetars also follow the ‘death line’ (albeit with low statistics), calculated assuming the radio pulsar mechanism. (ii) The inner core temperature of magnetars is systematically higher than other types of pulsars. The core temperatures of magnetars has a very narrow range and is about $2 \times 10^8$ K. The core temperatures of ordinary pulsars distribute in a relatively wide range, from $2 \times 10^6$ K to $2 \times 10^8$ K; the center is about $2 \times 10^7$ K. The core temperature of millisecond pulsars is around $10^5$ K. (iii) Several radio-quiet pulsars and normal radio pulsars fall right on the curve for magnetars, and thus may have the same core temperature.

All observed NSs appear in the power-law parts of the model curves, i.e., $B_{\text{surf}} \propto \tau_c^{-0.5}$. As we have shown in a companion paper in the same proceedings (Zhang & Xie 2011), this corresponds to the solenoidal component of the ambipolar diffusion dominated decay mode with a constant core temperature. Interestingly, exactly the same relation is obtained in the magnetic dipole spin-down model with a constant $P$, i.e., $\dot{P} \ll 1$. This immediately suggests that the observed extremely small $\dot{P}$ of NSs is due to magnetic field decay with constant core temperature.

3. Core Temperatures of Magnetars and Young Pulsars

Currently the core temperature of a NS can only be inferred from its surface temperature $T_s$. The spectra of some NSs are well-described by a simple blackbody (BB) radiation which is believed to be its thermal surface emission. For NSs with spectra that include two BB components, we adopt the temperature of the component which dominates the spectrum. In Figure 2 we show the observed $T_s - \tau_c$ and $T_s - B$ relations for magnetars and young NSs. In the left panel of Figure 2, it is shown that the surface temperature distribution can be divided into two separate groups. The normal radio pulsars and
several radio-quiet pulsars may follow the cooling curve with large scatters, though a constant temperature may also describe the data almost equally well. However, the surface temperatures of magnetars (\(\gtrsim 4 \times 10^6\) K) are much higher than that of normal young NSs and also almost constant for at least twenty million years, consistent with the model parameters used in Figure 1.

Pons & Geppert (2007) reported a correlation between the surface temperature and the magnetic field and investigated similar diagram as the right panel. They approximated the correlation as \(T_{s,6}^4 \approx CB_{d,14}^2\), the so-called heating balance line (HBL), (see also APM08), where \(C\) is a constant that depends on the thickness of the crust with a typical value of \(C \approx 10\). They argued that the thermal evolution of NSs with \(B \gtrsim 10^{15}\) G is predominantly determined by the strength of the magnetic field, and NSs with field of \(\sim 10^{12}\) G should cool much more rapidly than those with field of \(\sim 10^{13}\) G and higher. They further suggested that “A NS will begin its life high on the figure with some \(B_d\). As it cools it moves vertically downward, until decay of its field causes the trajectory to bend to the left. The star eventually reaches its HBL, and then continues moving down it.”

However, the above scenario is not supported by data; the data are more consistent with a scenario that both magnetars and normal pulsars move horizontally to the left, but with initially very different temperatures, i.e., their magnetic fields decay but core temperatures remain the same, again consistent with the results in Figure 1. Rea et al. (2010) suggested that the magnetic energy stored in the internal toroidal field that power the violent activities for magnetars, rather than surface dipole field. Thompson & Duncan (1996) investigated the process that magnetic field decay can provide a significant source of internal heating, and the possibility of obtaining an equilibrium between neutrino cooling and heating through magnetic field decay for \(B \sim 10^{15}\) G and \(T \sim 10^8\) K. The internal toroidal field probably plays this role and therefore keeps the core temperature as well as surface temperature of magnetars nearly constant.

4. Summary and Discussion

In this report, we found that the observed \(\tau_c - B\) correlations of all pulsars can be well explained by magnetic field decay with core temperatures of \(2 \times 10^8\) K, \(\sim 2 \times 10^7\) K, and \(\sim 10^5\) K, for magnetars, normal radio pulsars, and millisecond pulsars, respectively; the decay is dominated by the solenoidal component of the ambipolar diffusion mode with a time scale of \(r_{ambip}^6 \sim 3 \times 10^9 \frac{T_8^2}{B_{12}^2}\) yr. With just one free parameter (the core temperature), all data of pulsars can be reasonably well described. We emphasize that in Figure 1, all NSs are assumed to have the same initial magnetic field strength, but different core temperature which do not change as the magnetic field decays. This suggests immediately that the key distinguishing property between magnetars and normal pulsars is that magnetars were born much hotter than normal pulsars, and thus have much longer magnetic field decay time scales, resulting in higher surface magnetic field strength even with the same ages of normal pulsars. The above conclusion agrees well with the observed correlations between the surface temperatures of magnetars and other young NSs, which do not agree with the cooling dominated evolution of neutron stars.

In the context of temperature evolution and magnetic field decay of magnetized NSs, several sophisticated models have developed in recent years (e.g. Geppert et al. 2006; Pons & Geppert 2007; Aguilera et al. 2008a). In these studies an isothermal core
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and a magnetize envelope were generally assumed. They showed that energy released by magnetic field decay and Joule heating in the crust is important for the thermal evolution of a NS with field strength \( \gtrsim 10^{13} \) G. Though the surface temperature of radio-quiet pulsars and normal radio pulsars may be broadly covered by the cooling curves of different strengths of initial magnetic field, the surface temperature of magnetars are still too high to reach even the initial field strength is as strong as \( \sim 10^{16} \) G (Aguilera et al. 2008b, hereafter APM08).

As pointed in our companion paper (Zhang & Xie 2011), the characteristic ages of NSs are typically much longer than the ages of their associated supernova remnants; the magnetic field decay is found to be responsible for this apparent discrepancy. Using the true ages of NSs, instead of the characteristic ages, will result in reduced core temperatures for them, by roughly one order of magnitudes; however all other conclusions reached above remain unchanged.

The locations of all magnetars on the left side of the ‘death line’ in Figure 1, in the same way as all other normal pulsars, would suggest that their activities are at least related to the mechanism responsible the radiation of normal pulsars. This is not expected in the currently accepted model for magnetars, since when their surface magnetic fields \( B \) exceeds \( 0.1B_{cr} \), their pulsar winds may be dominated by bound pairs rather than by freely streaming electrons and positrons (Usov & Melrose 1996).

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