Numerical study of heat transfer and Hall current effects on the flow of Johnson-Segalman fluid between two eccentric rotating disks

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Abstract

Johnson-Segalman fluid flow is examined in this article between two eccentrically rotating disks. Analysis of velocity and temperature profile is carried out in the presence of Hall current and non-coaxial rotation. The governing non linear momentum and energy equations of Johnson-Segalman fluid constitute a complicated system of equations corresponding to an intricate regime which are solved by using a step wise numerical algorithm (VIM). The graphical interpretations for velocity and temperature profiles have been made for various embedded parameters of interest.

Nomenclature

| Symbol | Description |
|--------|-------------|
| µ      | dynamic viscosity |
| ξ      | kinematic viscosity |
| B(x)   | magnetic field |
| φ      | Hall parameter |
| Ω      | angular velocity |
| λ₁     | fluid relaxation time |
| J      | current density |
| E      | total electric field |
| σ      | electric conductivity |
| 2l     | distance between axes |
| C_p    | specific heat |
| Pr     | Prandtl number |
| E_c    | Eckert number |
| Br     | Brinkman number |
| ρ      | density |

1. Introduction

The study of non-Newtonian fluids between two parallel disks constitutes an important problem from physical and engineering points of views. Such flows have great applications in industry and modern technology e.g., turbines, turbomachines, spin-stabilized missiles etc. Orthogonal rheometer was introduced by Maxwell and Chartoff [1] to determine the viscosity of the viscoelastic fluids. The orthogonal rheometer had two parallel disks rotating with same angular velocity about two axes normal to the disk [1]. Mohanty [2] looked at flows between
eccentric rotating disks with same angular velocities for symmetric cases when induced magnetic field is small as compared to the applied magnetic field. Kasiviswanathan and Gandhi [3] extended Mohanty’s analysis [2] for the case of MHD micropolar fluid. Numerical analysis of three dimensional flow between parallel rotating plates about a common axis was performed by Lei and Rajagopal [4]. Impulsively started unsteady flow caused by eccentric rotations of a disk and fluid at infinity was contributed by Pop [5]. Erdogan [6] studied the time-dependant Navier–Stokes equations for a viscous fluid flow between eccentric rotating disks performing non-torsional oscillations. The hydrodynamic flow due to an oscillating disk and viscous fluid rotating non-coaxially has also been studied by Erdogan [7]. Hayat et al [8] extended the analysis done in [7] for the case of magnetohydrodynamic effect. Siddiqui et al [9] obtained an analytical solution of the unsteady second grade fluid’s flow induced by non-coaxial rotations of a porous disk and fluid at infinity in the presence of constant magnetic field applied in transverse direction. Bhatnagar and Zago [10] examined the second order fluid flow due to rotating disk about an axis. A fourth order fluid’s flow and a special subclass of models of KKB–Z type between eccentric rotating disks were analyzed by Siddiqui and Kaloni [11] and Dupont and Crochet [12]. The MHD effect on an Ekman layer over an infinite horizontal plate at rest relative to an electrically conducting and rotating liquid about a vertical axis was examined by Gupta [13]. Numerical simulation for MHD flow of a third grade fluid due to non-coaxial rotations of a porous disk and was contributed by Hayat et al [14].

Hall effect on the hydromagnetic flow of a viscous fluid between two parallel plates was dealt by Sutton and Sherman [15]. Krishna and Rao [16] provided the analysis of Hall current on unsteady MHD boundary layer flow. Nagy and Demendy [17] considered the general wall condition on Hartmann flow with Coriolis force and Hall current effects. Ray and Mazumdar [18] investigated the electrically conducting flow and falling liquid film over a smooth vertical surface with Hall current effect. Takher et al [19] discussed the MHD flow over a moving plate in a rotating fluid in the presence of Hall current. Abdoul–Hasan and Atia [20] discussed the Hall effect on the steady flow generated by a rotating disk. Hayat et al [21] examined the Hall effect on the hydromagnetic flow of a second grade fluid. Some recent contributions regarding the Hall effects on the flows of non-Newtonian fluids are due to Hayat et al [22], Ersoy [23], Fetecau et al [24] and Atia [25].

The consideration of heat transfer effect in the rotating disk problems of non-Newtonian fluids further complicates the analysis since the resulting energy equation also become non linear. Attia [26], Banerjee and Borkakti [27] and Ramzan et al [28–37] investigated heat transfer effect on different non-Newtonian fluid models subject to various boundary conditions.

Keeping in view the importance of the Hall current and heat transfer effects, in this communication we have modeled the equation of motion for Johnson–Segalman fluid flow generated due to eccentric rotating disks for the steady state velocity field and heat transfer analysis in the presence of Hall effect. The Johnson Segalman fluid model is one of the viscoelastic fluid models exhibiting the non-affine deformations [38]. Johnson–Segalman model has been utilized by many investigators to discuss the ‘spurt’ phenomenon. The spurt is usually referred to as a jump in flow rate. This phenomena is important in the flow of many nonlinear fluids, such as Lim and Schowalter [39] observed this phenomena while considering the wall slip of narrow molecular weight distribution of polybutadienes. Moreover Rammurthy [40], Massoudi and Tan [41] and Corr et al [42] discussed this phenomena while studying the granular materials, viscoelastic fluids and chemically reacting nonlinear fluids.

As far as we are aware, because of complexity of Johnson Segalman fluid model, literature is scarce on the discussion of Johnson–Segalman model in rotating disk problems in the presence of Hall and heat transfer effects. Using the low Reynolds number approximation, the numerical solution of the above stated problem is developed by using the variation iteration method (VIM) proposed by [43]. VIM provides a rapid convergence of the solution of related boundary value problem and has been successfully employed to solve fluid flow problems [44, 45].

In this paper, section 2 explains the governing equations and the boundary conditions of the problem under consideration with heat transfer analysis. Section 3 presents the application of Variational Iteration Method on the given problem. In section 4, we have presented some useful results illustrated with the help of graphs. In section 5, the concluding remarks and the future prospects in continuation of this research are discussed.

## 2. Problem formulation

Consider the Johnson-Segalman (J-S) fluid flow bounded between two eccentric rotating disks which are $2h$ distance apart. The two infinite rotating disks having angular velocity $\Omega$ about two non-coaxial axes are $2l$ distance apart. The z-axis is in the transverse direction to the disks. A constant magnetic field of high strength $B_0$ is applied in the transverse direction to the plates and induced magnetic field is negligible under the small magnetic Reynolds’ number approximation. The disks at positions $z = -h$ and $h$ are pulled with constant
velocities — $U$ and $U$ respectively. The schematic diagram showing the 2D view of the geometry with applied magnetic field $B_0$ is depicted in figure 1.

The boundary conditions at $z = -h$ and $h$ are

$$u = -\Omega(y + l) - U_1, \quad v = \Omega x - U_2, \quad w = 0$$

and

$$u = -\Omega(y - l) + U_3, \quad v = \Omega x + U_2, \quad w = 0.$$

(1)

The eccentric rotating disks suggest that the velocity field is the sum of translational and rotational velocities

$$u = -\Omega y + f(z), \quad v = \Omega x + g(z), \quad w = 0.$$

(2)

The governing equations for the MHD convective flow of an incompressible J-S fluid are

$$\nabla \cdot \mathbf{V} = 0,$$

(3)

$$\rho(\mathbf{V} \cdot \nabla)\mathbf{V} = -\nabla p + \nabla \cdot \mathbf{T} + \mathbf{J} \times \mathbf{B},$$

(4)

and

$$\rho C_p \frac{d\tau}{dt} = \kappa \nabla^2 \tau + \mathbf{T} \cdot \mathbf{L},$$

(5)

where $\mathbf{T}$ is the Cauchy stress tensor for a J-S fluid and is given as

$$\mathbf{T} = 2\mu \mathbf{D} + \mathbf{S},$$

(6)

where

$$\begin{align*}
\mathbf{S} + \lambda_1 & \left\{ \frac{dS}{dt} + \mathbf{S}(\mathbf{W} - a\mathbf{D}) + (\mathbf{W} - a\mathbf{D})^T \mathbf{S} \right\} = 2\mu_e \mathbf{D}, \\
\mathbf{D} & = \frac{1}{2}(\mathbf{L} + \mathbf{L}^T), \quad \mathbf{W} = \frac{1}{2}(\mathbf{L} - \mathbf{L}^T), \quad \mathbf{L} = \nabla \mathbf{V},
\end{align*}$$

(7)

with $d/dt$ is the material time derivative, $\mathbf{D}$ is the symmetric and $\mathbf{W}$ is the skew symmetric part of the velocity gradient, $\mu$ is the viscosity, $\lambda_1$ is the relaxation time and $a$ is the slip parameter. The J-S fluid model reduces to the upper convected Maxwell fluid model when $\mu = 0$ and $a = 1$, and reduces to the classical Naiver-Stokes fluid model when $\lambda_1 = 0$.

To pursue the foregoing analysis we further require the Maxwell’s equations which are given as

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{B} = \mu_m \mathbf{J}, \quad \nabla \times \mathbf{E} = 0,$$

(8)

where $\mathbf{J}$ is a current density vector, $\mu_m$ is the magnetic permeability, $\mathbf{E}$ is the total electric field and $\sigma$ is the electrical conductivity respectively. Furthermore, the generalized Ohm’s law in the presence of a Hall current is

$$\mathbf{J} + \frac{\omega_e \tau_e}{B_0} (\mathbf{J} \times \mathbf{B}) = \sigma \left[ \mathbf{E} + \frac{1}{\epsilon n_e} \nabla p_e \right],$$

(9)

where $e$ is the electron charge, $B_0$ is the applied magnetic field, $\omega_e$ is the cyclotron frequency of electrons, $\tau_e$ is the electron collision time, $n_e$ is the number density of the electron and $p_e$ is the electron pressure. In problem under consideration ion-slip and electric field effects are neglected since in the presence of Hall effect magnetic field is strong as compared to the electric field and thus we can neglect the ion slip and electric field effects. Also $\omega_i \tau_i \ll 1$ ($\omega_i$ and $\tau_i$ are the cyclotron frequency and collision time for ions respectively). Utilizing the equations (6) and (7) into momentum equation (4) and energy equation (5) and after simplification we
obtain three equations in terms of normal and shear stresses as
\[
\frac{\partial p}{\partial x} = \rho \Omega v + \frac{\partial S_{xz}}{\partial z} + \mu \frac{\partial^2 f}{\partial z^2} + \frac{\sigma B_0^2 (u - \phi v)}{1 + \phi^2}, \tag{10}
\]
\[
\frac{\partial p}{\partial y} = \rho \Omega u + \frac{\partial S_{yz}}{\partial z} + \mu \frac{\partial^2 g}{\partial z^2} + \frac{\sigma B_0^2 (v + \phi u)}{1 + \phi^2}, \tag{11}
\]
\[
\frac{\partial p}{\partial z} = \frac{\partial S_{zz}}{\partial z}, \tag{12}
\]
and
\[
\kappa \frac{d^2 \tau}{\rho C_p \frac{dz^2}{dx^2}} + \mu \left[ \frac{2f' S_{xz} + 2g' S_{yz}}{a_1 a_3 + \frac{a_2}{4} (1 + a_4)(f'^2 + g'^2)} \right] = 0, \tag{13}
\]
where
\[
S_{xz} = \frac{\partial S_{xz}}{\partial z}, \tag{14}
\]
\[
S_{yz} = \frac{\partial S_{yz}}{\partial z}, \tag{15}
\]
and
\[
D_1 = \mu \left( a_1 (f' - \Omega \lambda g') + \frac{a_2}{4} \lambda^2 \left[(f'^2 + g'^2) (f' + 2 \Omega \lambda g')\right]\right), \tag{16}
\]
\[
D_2 = \mu \left( a_3 (g' + \Omega \lambda f') + \frac{a_2}{4} \lambda^2 \left[(f'^2 + g'^2) (g' - 2 \Omega \lambda f')\right]\right), \tag{17}
\]
\[
D_3 = a_1 a_3 + \frac{a_2}{4} \lambda^2 (1 + a_4)(f'^2 + g'^2), \tag{18}
\]
such that
\[
a_1 = (1 + 4 \Omega^2 \lambda^2), \quad a_2 = (1 - a^2), \quad a_3 = (1 + \Omega^2 \lambda^2) \quad \text{and} \quad a_4 = (5 + 8 \Omega^2 \lambda^2). \tag{19}
\]
Integrating equations (10) and (11) we obtain
\[
\hat{p} = \left( p - \frac{\sigma B_0^2 \Omega \phi \rho}{1 + \phi^2} \right) (x^2 + y^2) + \lambda \{(1 - a) f' S_{xx} + (1 - a) g' S_{yy}\} \pm \frac{\sigma B_0^2 \Omega \phi}{1 + \phi^2} S_{yy}. \tag{20}
\]
Equations (10) and (11) now take the form
\[
\frac{\partial \hat{p}}{\partial x} = \frac{\partial S_{xz}}{\partial z} + \Omega \rho g (x) - \frac{\sigma B_0^2 \Omega (f - \phi g)}{1 + \phi^2} + \mu \frac{\partial^2 f}{\partial z^2}, \tag{21}
\]
\[
\frac{\partial \hat{p}}{\partial y} = \frac{\partial S_{yz}}{\partial z} - \Omega \rho f (x) - \frac{\sigma B_0^2 \Omega (g + \phi f)}{1 + \phi^2} + \mu \frac{\partial^2 g}{\partial z^2}, \tag{22}
\]
with the relevant boundary conditions
\[
\begin{cases}
\{ f(z) = \Omega l + U_1, \quad g(z) = U_2 \quad \text{at} \quad z = h, \\
\{ f(z) = -\Omega l - U_1, \quad g(z) = -U_2 \quad \text{at} \quad z = -h,
\end{cases} \tag{23}
\]
In the light of above information equations (12), (21) and (22) can be written as
\[
\mu \frac{d^2 f}{dz^2} + \frac{d S_{xz}}{dz} + \rho \Omega g - \frac{\sigma B_0^2 \Omega (f - \phi g)}{1 + \phi^2} = 0, \tag{24}
\]
\[
\mu \frac{d^2 g}{dz^2} + \frac{d S_{yz}}{dz} - \rho \Omega f - \frac{\sigma B_0^2 \Omega (g + \phi f)}{1 + \phi^2} = 0, \tag{25}
\]
and
\[
\kappa \frac{d^2 \tau}{\rho C_p \frac{dz^2}{dx^2}} + \mu \left[ \frac{(f')^2 + (g')^2}{a_1 a_3 + \frac{a_2}{4} (1 + a_4)(f'^2 + g'^2)} \right] = 0, \tag{26}
\]
with the boundary conditions

\[
\begin{align*}
    f(h) &= \Omega l + U_1, & g(h) &= U_2, \\
    f(-h) &= -\Omega l - U_1, & g(-h) &= -U_2,
\end{align*}
\]

and

\[
\begin{align*}
    \tau(-h) &= \tau_1, \\
    \tau(h) &= \tau_2.
\end{align*}
\]

To non-dimensionalize the problem we introduce the following quantities.

\[
\begin{align*}
    z^* &= \frac{z}{h}, & f^* &= \frac{f}{\Omega l}, & g^* &= \frac{g}{\Omega l}, & S^*_{xz} &= \frac{hS_{xz}}{\mu \Omega l}, & S^*_{rz} &= \frac{hS_{rz}}{\mu \Omega l}, \\
    Pr &= \frac{C_p \mu}{K_e}, & M^2 &= \frac{\sigma R^2 h^2}{\mu}, & R &= \Omega \rho \beta^2, & L &= \frac{\lambda^2 \Omega^2 l^2}{h^2}, \\
    V_1 &= \frac{U_1}{\Omega l}, & V_2 &= \frac{U_2}{\Omega l}, & E_l &= \left(\frac{\Omega l^2}{C_p(\tau_1 - \tau_2)}\right), & \theta &= \frac{\tau - \tau_2}{\tau_1 - \tau_2}.
\end{align*}
\]

In proceeding analysis we have suppressed the asteriks for simplicity.

### 3. Variational Iteration method

VIM has proven to be a powerful tool for the solution of integro-differential equations. Abbasbandy and Shivanian [46] proposed VIM to solve system of nonlinear integro-differential equations. The results revealed that this method is very effective and promising in comparison with other numerical techniques. In applying VIM [43, 47], we construct:

1. A correction functional which is an iterative expression that is based on linear and non-linear problems together with Lagrange multiplier which is evaluated according to the differential operator and variational theory,

2. After determining the Lagrange multiplier we then solve the integro-differential equation iteratively by choosing an initial guess which satisfy the initial or boundary conditions.

We will solve the system of equations (30) to (32) subject to (33) by using Variational Iteration Method (VIM). Now equations (30)–(32) takes the following form

\[
\begin{align*}
    f'' + \frac{d}{dz}\left[ a_1(f'' - \Omega \lambda g') + \frac{a_4}{4}(f'^2 + g'^2)(f' + 2\Omega \lambda g') \right] &- \frac{M^2}{1 + \phi^2}(f - \phi g) + Rg = 0, \\
    g'' + \frac{d}{dz}\left[ a_1(g'' + \Omega \lambda f') + \frac{a_4}{4}(f'^2 + g'^2)(g' - 2\Omega \lambda f') \right] &- \frac{M^2}{1 + \phi^2}(g + \phi f) - Rf = 0, \\
    \frac{d^2}{dz^2} + Br\left( \frac{2a_6 + a_4 f'' + g''}{a_1 a_3 + a_4 (1 + a_4)(f'^2 + g'^2)} \right) & = 0
\end{align*}
\]

with boundary conditions

\[
\begin{align*}
    f(-1) &= -(1 + V_1), & f(1) &= 1 + V_1, \\
    g(-1) &= -V_2, & g(1) &= V_2, \\
    \theta(-1) &= 1, & \theta(1) &= 0.
\end{align*}
\]

In proceeding analysis we have suppressed the asteriks for simplicity.
\[
g'' + \frac{d}{dz} \left[ \frac{a(t'(g' + \Omega \lambda f') + \frac{aL}{4} (f'' + g'') (g' - 2\Omega \lambda f'))}{a_1 a_3 + \frac{aL}{4} (1 + a_3) (f'' + g'')} \right] - \frac{M^2}{1 + \phi^2} (g + \phi f) - Rf = 0, \tag{35}\]
\[
\theta'' + Br \left[ \left( 2a_1 + \frac{aL}{4} (f'' + g'') \right) (f'' + g'') \right] = 0. \tag{36}\]

We will now implement the basic procedure of VIM by introducing the following iterative formulae for equations (34) to (36) known as correction functional as

\[
f_{n+1} = f_n + \int_0^2 V_f \left[ f''_n(s) + \frac{d}{ds} \left( \frac{a(t'_n(g'_n + \Omega \lambda g''_n) + \frac{aL}{4} (f''_n + g''_n) (g'_n + 2\Omega \lambda g''_n))}{a_1 a_3 + \frac{aL}{4} (1 + a_3) (f''_n + g''_n)} \right) ds, \tag{37}\]
\[
g_{n+1} = g_n + \int_0^2 V_g \left[ g''_n(s) + \frac{d}{ds} \left( \frac{a(t'_n(g'_n + \Omega \lambda g''_n) + \frac{aL}{4} (f''_n + g''_n) (g'_n + 2\Omega \lambda g''_n))}{a_1 a_3 + \frac{aL}{4} (1 + a_3) (f''_n + g''_n)} \right) ds, \tag{38}\]
and
\[
\theta_{n+1} = \theta_n + \int_0^2 V_\theta \left[ \left( \frac{d}{ds} \left( \frac{a(t'_n(g'_n + \Omega \lambda g''_n) + \frac{aL}{4} (f''_n + g''_n) (g'_n + 2\Omega \lambda g''_n))}{a_1 a_3 + \frac{aL}{4} (1 + a_3) (f''_n + g''_n)} \right) ds, \tag{39}\right]
\]

It is necessary to determine the Lagrange multipliers \( V_f, V_g \) and \( V_\theta \) through restricted variation by using method of integration by parts. After determining the Lagrange multipliers, an iterative formula, without restricted variation will be used to calculate the next approximations \( f_n, g_n \) and \( \theta_n \); \( n \geq 0 \) for the solutions \( f(z), g(z) \) and \( \theta (z) \). To calculate the Lagrange multipliers, we will take the variation of both sides of equations (37) to (39) which result in

\[
\delta f_{n+1} = \delta f_n + \int_0^2 V_f \left[ f''_n(s) + \frac{d}{ds} \left( \frac{a(t'_n(g'_n + \Omega \lambda g''_n) + \frac{aL}{4} (f''_n + g''_n) (g'_n + 2\Omega \lambda g''_n))}{a_1 a_3 + \frac{aL}{4} (1 + a_3) (f''_n + g''_n)} \right) ds, \tag{40}\]
\[
\delta g_{n+1} = \delta g_n + \int_0^2 V_g \left[ g''_n(s) + \frac{d}{ds} \left( \frac{a(t'_n(g'_n + \Omega \lambda g''_n) + \frac{aL}{4} (f''_n + g''_n) (g'_n + 2\Omega \lambda g''_n))}{a_1 a_3 + \frac{aL}{4} (1 + a_3) (f''_n + g''_n)} \right) ds, \tag{41}\]
and
\[
\delta \theta_{n+1} = \delta \theta_n + \int_0^2 V_\theta \left[ \left( \frac{d}{ds} \left( \frac{a(t'_n(g'_n + \Omega \lambda g''_n) + \frac{aL}{4} (f''_n + g''_n) (g'_n + 2\Omega \lambda g''_n))}{a_1 a_3 + \frac{aL}{4} (1 + a_3) (f''_n + g''_n)} \right) ds, \tag{42}\right]
\]
The variational condition of \( f_{n+1}, g_{n+1} \) and \( \theta_{n+1} \) requires that \( \delta f_{n+1} = 0, \delta g_{n+1} = 0 \) and \( \delta \theta_{n+1} = 0 \), which implies that the equations (40) to (42) are zero and this will provide us the stationary conditions. We therefore obtain the Lagrange multipliers by solving the corresponding stationary conditions for each of \( f_{n+1}, g_{n+1} \) and \( \theta_{n+1} \). Equations (40), (41) and (42) therefore take the form
\[ f_{n+1} = f_n + \int_0^z (s - z) V_0 \left[ \frac{d}{ds} \left( \frac{a_n f_n(s) + \Omega \lambda_s g_n(s)}{a_n + \frac{\alpha_s}{4}(1 + a_4)(f_n''(s) + g_n''(s))} \right) + \frac{\alpha_s}{4}(1 + a_4)(f_n''(s) + g_n''(s)) \right] ds, \]

\[ g_{n+1} = g_n + \int_0^z (s - z) V_0 \left[ \frac{d}{ds} \left( \frac{a_n g_n(s) + \Omega \lambda_s f_n(s)}{a_n + \frac{\alpha_s}{4}(1 + a_4)(f_n''(s) + g_n''(s))} \right) + \frac{\alpha_s}{4}(1 + a_4)(f_n''(s) + g_n''(s)) \right] ds, \]

which are the iterative formulas. Considering the given initial conditions from equations (32) and (33), we obtained the zeroth approximations as

\[ f_0(z) = \frac{U_1}{\Omega l} z, \quad g_0(z) = \frac{U_2}{\Omega l} z, \quad \theta_0(z) = \frac{1}{2} (1 - z). \]

At \( n = 0 \), from equation (43) we obtained \( f_1, f_2, f_3, \ldots, f_n \) up to maximum order of accuracy and similar is the case for \( g \). We list here the \( (2n + 1) \)th order approximations for the velocity as

\[ f_n(z) = d_1 z + \frac{d_2}{3!} z^3 + \frac{d_4}{5!} z^5 + \frac{d_6}{7!} z^7 + \frac{d_8}{9!} z^9 + \ldots + \frac{d_{2n+2}}{(2n + 1)!} z^{2n+1}, \]

\[ g_n(z) = d_1 z + \frac{d_3}{3!} z^3 + \frac{d_5}{5!} z^5 + \frac{d_7}{7!} z^7 + \frac{d_9}{9!} z^9 + \ldots + \frac{d_{2n+1}}{(2n + 1)!} z^{2n+1}, \]

and for the heat transfer analysis the approximate solution is

\[ \theta_n(z) = a_1 + \frac{c_2}{2!} z^2 + \ldots + \frac{c_n}{2n!} z^{2n}. \]

The coefficients are obtained from the iterative procedure.

4. Numerical results and discussion

Graphical illustrations corresponding to an approximate solution of the J-S fluid model in the prescribed domain are presented in this section. In non-dimensionalized model, the two disks are at \( z = -1 \) and \( z = 1 \) respectively. Later for the smooth analytic and numerical computation we have calculated the numerical results in the domain \( -1 \leq z \leq 1 \).

In figures 2(a)–(b) we are presenting the variation in the velocity components relative to variation in the Hall current. The combination \( \omega_r T \) is used to characterize an experimental situation that whether it is in the weak-field limit \( (\omega_r T \ll 1) \) or it is a strong-field limit \( (\omega_r T \gg 1) \). We will confine our attention to the weak-field limit i.e \( \phi < 1 \) and will consider the Hall parameter \( \phi < 1 \). Figure 2(a) shows the axi-symmetric behavior of the horizontal component of velocity and 2(b) depicts that the variation in the vertical component of velocity is negligible for a small change in Hall parameter.

The variation in the horizontal and vertical components of velocity relative to change in rotation parameter are presented in figures 3(a)–(b). The rotation parameter is considered to be \( R \gg 1 \), which implies that the rotation \( \Omega \) is dominating. We can see the axi-symmetric behavior of the velocity which shows that the magnitude of horizontal velocity decreases by increasing rotation whereas magnitude of vertical velocity increases by increasing rotation.

Figures 4(a)–(b) present the change in velocity in the entire domain corresponding to the variation of magnetic Reynolds number \( (Re_m) \). The magnetic Reynolds number (it is the ratio of advection to diffusion which gives the approximate relative effect of advection due to movement of conducting fluid to magnetic field)
predicts an estimate of the effect of magnetic advection relative to magnetic diffusion, for $Re_m < 1$. We can see that for increasing values of $Re_m$, the horizontal and vertical components of velocity increases because of thinning of fluid and reduction in the friction to fluid flow.
To note the effect of heat transfer from a moving wall, figures 5(a)–(b) are plotted. Figure 5(a) shows the change in temperature relative to change in the Brinkman number $Br$ which is product of $Pr$ and $Ec$. The impact of Hall current on the heat transfer is presented in figure 5(b) respectively. Brinkman number causes to increase the thermal permeability so the amount of temperature increases by the increase in Brinkman number. Hall parameter causes to reduce the thermal permeability so by increasing Hall parameter temperature profile decreases i.e to reduce the temperature one can incorporate the effect of Hall current.

5. Concluding remarks

We made an attempt to explore the rheological features of J-S fluid between two disks rotating about non-coaxial axes. Using a numerical tool VIM, we have presented an approximate solution to the problem under consideration by calculating the velocity and temperature profiles in series of flow parameters, noteworthy of which are the magnetic Reynolds number, the rotation parameter and the Hall effect parameter. The combined effects of magnetic field and Hall current on the flow characteristics are examined in a way that the adequate transition will take place in the rate of rotation, rate of heat transfer and velocity gradient of the main body of the fluid. In conclusion, the results showed that VIM is remarkably efficient and in addition this method has been adopted since it has been proved in literature ([45, 46] and the references therein) that it has more accuracy for the kind of integro-differential problems.

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