New, high statistics measurement of the $K^+ \rightarrow \pi^0 e^+ \nu$ ($K_{e3}^+$) branching ratio

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The experimentally determined Cabibbo-Kobayashi-Maskawa (CKM) matrix describes quark mixing in the Standard Model framework. Any deviation from the matrix’s unitarity would undermine the validity of the Standard Model. One unitarity condition involves the first row elements:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \delta$$  \hspace{1cm} (1)

where a non-zero value of $\delta$ indicates a deviation from unitarity. The $V_{ud}$ element is obtained from nuclear and neutron decays. $V_{ub}$, from the semileptonic decays of B mesons, is too small to affect Eq. 1. The $V_{us}$ element can be determined either from hyperon, $K \rightarrow \pi \mu \nu(K_{\mu3})$ or from $K \rightarrow \pi e \nu(K_{e3})$ decays. However, $K_{e3}$ decays provide a smaller theoretical uncertainty$^{1,2,3}$. The most precise value of $V_{ud}$ obtained from the nuclear superallowed Fermi beta decays leads to $\delta = (3.2 \pm 1.4) \cdot 10^{-3}$, a 2.3$\sigma$ deviation from unitarity.

Both experimental and theoretical efforts to improve the determination of $V_{ud}$ continue. Theoretical contributions to $V_{us}$ were reevaluated recently$^{2,6,7}$, but there has been little new experimental input on the $K_{e3}$ branching ratio. Since the $V_{ud}^2$ and $V_{us}^2$ uncertainties are comparable, a high statistics measurement of the $K_{e3}^+$ branching ratio (B.R.) with good control of systematic errors is useful.

The bare (without QED corrections) $K_{e3}^+$ decay rate$^{2,6,7}$ is:

$$d\Gamma(K_{e3}^+) = C(t)|V_{us}|^2|f_+(0)|^2\left[1 + \frac{t}{M_\pi^2}\right]^2dt$$  \hspace{1cm} (2)

where $t = (P_K - P_e)^2$, C(t) is a known kinematic function, and $f_+(0)$ is the vector form factor value at $t = 0$, determined theoretically$^2,5$. Two recent experiments$^{10,11}$ give $\lambda_+$ (the form factor slope) measurements consistent with each other and with previous measurements. An omitted negligible term contributing to Eq. 2 contains the form factor $f_-$, and is proportional to $M_\pi^2/M_K^2$.

![FIG. 1: Plan view of the E865 detector with a simulated $K^+ \rightarrow \pi^0 e^+ \nu$ decay followed by $\pi^0 \rightarrow e^+ e^- \gamma$.](image-url)

E865$^{12}$ searched for the lepton flavor violating decay $K^+ \rightarrow \pi^+ \mu^+ e^-$. The detector (Figure 1) resided in a 6 GeV/c positive beam$^{12}$. For the $K_{e3}^+$ running, the intensity was reduced by a factor of 10, to $10^7$ kaons, 2 x $10^8$ protons, and 2 x $10^8\pi$ per 2.8 second spill. The beam was intentionally debunched at extraction to remove rf structure at the experiment. The first dipole magnet separated particles by charge, while the second magnet together with four multiwire proportional cham-
bers (MWPCs: P1-P4) formed the spectrometer. The particle identification used the threshold multichannel Čerenkov detectors (C1 and C2, each separated into left and right volumes, for four independent counters) filled with gaseous methane (Čerenkov threshold $\gamma_l \approx 30$ and electron detection efficiency $\epsilon_e \approx 0.98$), an electromagnetic calorimeter [12], and a muon detector (not used for the $K_{e3}^+$ measurement). The D and A scintillator hodoscopes gave left/right and cruder vertical position.

The $\pi^0$ from the kaon decays was detected through the $e^+e^-$ from the $\pi^0 \rightarrow e^+e^-\gamma$ decay, with the $\gamma$ detected in some cases. To eliminate the uncertainty (2.7%) of the $\pi^0 \rightarrow e^+e^-\gamma$ B.R., and to reduce systematic uncertainty we used the other three major decay modes with a $\pi^0$ in the final state ($K^+ \rightarrow \pi^+\pi^0(K_{e2}^+), K_{\mu3}^+, K^+ \rightarrow \pi^+\pi^0\pi^0(K_{\pi3}^+)$) for the normalization sample ("Kdal").

The $K_{e3}^+$ data was collected in a one-week dedicated run in 1998, with special on-line trigger logic.

The Kdal and $K_{e3}^+$ data were collected by the "$e^+e^-$" trigger, which was designed to detect $e^+e^-$ pairs and required at least one D-counter scintillator slat on each (left and right) side of the detector and signals from each of the four Čerenkov counters. The Čerenkov efficiency trigger required only 3 out of 4 Čerenkov counters (no D-counter requirement). The "TAU" trigger, requiring only two D-counter scintillator hits (one left, and one right), collected events for the $K^+ \rightarrow \pi^+\pi^-\pi^+(K_\tau)$ sample, to study the detector unbiased by Čerenkov requirements. About 50 million triggers were accumulated, about 37 million in the "$e^+e^-$" trigger. About 75% of "$e^+e^-$" triggers included accidental tracks, often a $\mu$ from high momentum $K \rightarrow \mu\nu$ or $\pi \rightarrow \mu\nu$ decays partially satisfying the Čerenkov requirements.

Off-line reconstruction used the spectrometer only. The Čerenkov and D counter efficiencies were obtained from the Čerenkov efficiency triggers. The redundancy of the MWPCs (4 planes/chamber) and track reconstruction was used to extract MWPC efficiencies. The absence of the electromagnetic calorimeter from the trigger allowed its efficiency determination. Each efficiency was measured over its relevant phase space.

Relevant kaon decay chains [13] were simulated with GEANT [14] (including decays of secondary pions and muons). For $K_{e3}^+$, $\lambda^+ = 0.0278 \pm 0.0019$ [1] was used. The radiative corrections to the $K_{e3}^+$ decay phase-space density [5] were used. The $K_{e3\gamma}$ (inner bremsstrahlung) decays outside the $K_{e3}^+$ Dalitz plot boundary were explicitly simulated [5]. For $\pi^0 \rightarrow e^+e^-\gamma$ decay, radiative corrections were taken into account according to Ref. [15]. Measured efficiencies were applied [13], and accidental detector hits (from reconstructed $K_\tau$ events) were added. About 10% of both the $K_{e3}^+$ and Kdal samples had extra reconstructed tracks.

Selection criteria, common to $K_{e3}^+$ and Kdal, included requirements for a good quality three track vertex in the decay volume (no requirement for exactly three reconstructed tracks was applied), for the three tracks to cross the active parts of the detector, for the low ($M_{ee} < 0.05$ GeV) mass $e^+e^-$ pair to be identified in the Čerenkov counters, and for the second positive track to have less than 3.4 GeV/c momentum. The momentum cut rejects events where $\mu^+ + \pi^+ + K^+$ decays is above Čerenkov threshold and can be identified as $e^+$. A geometric Čerenkov ambiguity cut rejected events (27%, 15%, 25%, and 35% for $K_{e3}^+, K_{e2}^+, K_{\mu3}^+$, and $K_{\pi3}^+$ respectively) where the Čerenkov counter response could not be unambiguously assigned to separate tracks [12].

The $K_{e3}^+$ sample was then selected by requiring the second positive track to be identified as $e^+$ in 2 of the 3 electron detectors: C1, C2, or the calorimeter, each with $\epsilon_e \approx 98\%$. Events entering the Kdal sample had no response in at least one of the two Čerenkov counters. These criteria minimized systematic uncertainties [13], but resulted in a small overlap, $\approx 3\%$ of the $K_{e3}^+$ sample and $\approx 0.3\%$ of the Kdal which was accounted for in the B.R. calculation. The $K_{e3}^+$ acceptance is $\approx 1.2\%$. The $K_{e3}^+$ acceptance $\approx 0.7\%$ [13], somewhat lower because of the lower average $e^+$ momentum in the $K_{e3}^+$ decay.

The overall acceptance level of 1% can be approximately understood by assuming a factor of three loss for each charged particle, 30% for the Čerenkov ambiguity, and approximately a factor of 2 for other cuts. Final acceptances for the three modes in the Kdal sample differed by $\leq 4\%$ taking into account that either of the $\pi^0$s from $K_{\pi3}^+$ can decay into $e^+e^-\gamma$. The final $K_{e3}^+$ and Kdal samples were 71,204 and 558,186, respectively. Figure 2 shows some relevant spatial distributions.

**FIG. 2:** Distributions of X and Y positions of the first positive track (not $e^+$ from the $\pi^0$ decay) for the selected $K_{e3}^+$ and Kdal samples. X and Y positions are measured at the end of the second pair of the Čerenkov counters (C2). Histograms represent Monte Carlo; points with errors represent data.

Contamination of the $K_{e3}^+$ sample by other $K^+$ decays occurred when $\pi^+ + \mu^+$ from Kdal decays were misidentified as $e^+$, or as a result of $\pi^0 \rightarrow e^+e^-e^+e^-$. Contamination due to secondary particle decays was es-
estimated to be at the level of 0.1%. About 8% of final state pions decayed into muons inside the spectrometer. The careful MWPC simulation gave good agreement of reconstructed track χ^2 and vertex distributions between data and Monte Carlo. No tight track χ^2 cuts were applied, and the systematic uncertainties estimated by variation of the vertex cuts were included in the final result. The check of B.R. (K+/Kdal), described below, also tests the final state π and μ decays.

Total contamination of the Ke3 sample was estimated to be (2.49 ± 0.05 stat ± 0.32 sys)% , with the systematic uncertainty caused by the simulation accuracy of the C1 and C2 response to π+ and μ+. Contamination due to overlapping events was (0.25 ± 0.07)% and (0.12 ± 0.05)% of the Kdal and K+e3 respectively. Figure 4 shows the energy distribution in the calorimeter from the e+ in the K+e3 sample. The contamination is manifest in the minimum ionization spike at 250 MeV. The small excess of data in the spike agrees with our contamination uncertainty estimate.

The final K+e3 sample included ≈30% of events with a fully reconstructed π’s. We used the π⁰ information as a consistency check. Not requiring π⁰’s in our main analysis minimized the uncertainty arising from photon detection and reconstruction in the calorimeter, but increased vulnerability to contamination from upstream decays and photon conversion. Upstream decays whose photon produced pairs before the decay volume (evacuated to about 10⁻⁸ nuclear interaction length) were suppressed by requiring the three track vertex to be more than two meters downstream of the decay volume entrance. In addition, the results obtained from the two independent samples, one with and one without the π⁰ reconstructed, did not show a statistically significant discrepancy.

The K+e3 statistical precision is 0.4%. The systematic error estimate, summarized in Table I, was determined from the B.R. stability under variation of reconstruction procedure, selection criteria, assumed detector efficiencies, and subdivision of both K+e3 and Kdal samples. No significant correlations between any of the different systematic uncertainties were observed.

The two largest contributions to the systematic error come from the discrepancies between data and Monte Carlo in the momentum (Figure 4) and spatial distributions. These errors were determined by dividing the K+e3 and Kdal events into two roughly equal subsamples, using the relevant parameters, and observing the variation of the result. The errors were found to be uncorrelated. The sensitivity of the vertical spatial discrepancy to the MWPC alignment and of the momentum discrepancy to the spectrometer parameters is indicative of their possible origins. The Z-vertex position is also sensitive to the magnetic field, but has a smaller systematic error contribution as determined from both upstream and downstream cuts in Z.

As an additional consistency check, we estimated the K+/Kdal B.R. The result was (1.01 ± 0.02) × the PDG ratio, (the theoretical prediction was used for the π⁰ → e⁺e⁻γ decay rate). The 2% error was dominated by the uncertainty in the prescale factor of the TAU trigger. A second consistency check compared the K+e3 B.R. from 1998 and 1997 data. The 1997 K+e3 data used a trigger that required calorimeter hits, and A and D-counters. That trigger neither allowed measurement of these detector efficiencies nor of the trigger efficiency. While we did not use the 1997 data for our final result, the 1997 K+e3 branching ratio was statistically consistent (within one sigma) with that from 1998. This agreement is important.
since the momentum spectrum discrepancy between data and Monte Carlo in the 1997 data is qualitatively different from 1998. A preliminary reconstruction version was used for the 1997 data, without the final magnetic field and detector alignment. This bolsters our intuition that the discrepancies in decay product momenta and spatial distributions, which dominate the systematic uncertainties, reflect our imperfect knowledge of the magnetic field and detector positions but do not bias our result beyond our estimated systematic errors.

We estimated the form factor slope $\lambda_+$ from both 1998 and 1997 $K_{e3}^+$ data. We obtained: $\lambda_+ = 0.0324 \pm 0.0044_{\text{stat}}$ for 1998, and $\lambda_+ = 0.0290 \pm 0.0044_{\text{stat}}$ for the 1997 data, both consistent with the current PDG fit.

After contamination subtraction, our result is $BR(K_{e3}^+)/(BR(K_{e3}^+) + BR(K_{\mu3}^+) + BR(K_{\pi3}^+)) = 0.1962 \pm 0.0008_{\text{stat}} \pm 0.0035_{\text{sys}}$, where $K_{e3}^+$ includes all QED contributions (loops and inner bremsstrahlung). As noted above, the $\pi^0$ was detected using the $e^+e^-\gamma$ pair from the selected $K_{e3}^+$, and no photons were required.

Using current Kdal B.R.’s we infer $BR(K_{e3}^+)$ = (5.13$\pm$0.02$\text{stat}$+0.09$\text{sys}$+0.04$\text{norn})%$, where the normalization error was determined by the PDG estimate of the Kdal B.R. uncertainties. This result does not include the correction due to the correlation of the PDG kaon decay ratios, since it was estimated to be small compared to the systematic error. The PDG fit to the previous $K^+$ decay experiments yields $BR(K^+ \to \pi^0 e^+\nu) = (4.87 \pm 0.06)$% [8], $\approx 2.3\sigma$ lower than our result.

Radiative corrections for decays inside the $K_{e3}^+$ Dalitz plot boundary were estimated to be $-1.3\%$ using the procedure of Ref. [17]. $K_{e3}^+$ decays outside the Dalitz plot boundary gave $+0.5\%$. Thus the total radiative correction was $-0.8\%$ resulting in the bare $BR(K_{e3}^+) = (5.17 \pm 0.02_{\text{stat}} \pm 0.09_{\text{sys}} \pm 0.04_{\text{norn}})%$.

Using the PDG value for $G_F$, the short-distance enhancement factor $S_{EW}(M_\mu, M_Z) = 1.0232_2^{17}$, and our result for the bare $K_{e3}^+$ rate we obtain $|V_{us}f_+(0)| = 0.2243 \pm 0.0022_{\text{rate}} \pm 0.0007_{\text{sys}}$, which gives $|V_{us}| = 0.2272 \pm 0.0023_{\text{sys}} \pm 0.0007_{\text{sys}} \pm 0.0018_{f+(0)}$ if $f_+(0) = 0.9874 \pm 0.0084_{[2]}^{[17]}$. With this value of $V_{us}$ and $V_{ud}$ from superallowed nuclear Fermi beta decays, $\delta = 0.0003 \pm 0.0016$.

This result is consistent with CKM unitarity, but increases the discrepancy with the $V_{us}$ from $K^0_e$ decay if extracted under conventional theoretical assumptions about symmetry breaking. $K_{e3}^+$ measurements in progress (CMD2, NA48, KLOE) should help to clarify the experimental situation.

We thank V. Cirigliano for the $K_{e3}^+$ radiative corrections code. We gratefully acknowledge the contributions by the staffs of the AGS, and participating institutions. This work was supported in part by the U.S. Department of Energy under contract DE-AC02-98CH10886, the National Science Foundations of the USA, Russia and Switzerland, and the Research Corporation.

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