**ZZ' Mixing and Radiative Corrections at LEP I**

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**Abstract**

We present a method for a common treatment of $Z'$ exchange, QED corrections, and weak loops in $e^+e^-$ annihilation. QED corrections are taken into account by convoluting a hard-scattering cross section containing $\gamma$, $Z$, and $Z'$ exchange. Weak corrections and $ZZ'$ mixing are treated simultaneously by a generalization of weak form factors. Using the properly extended Standard Model program for the $Z$ line shape, $zF_{Z\,T_E T_{ER}}$, we perform and compare two different analyses of the 1990 LEP I data in terms of theories based on the $E_6$-group and in terms of LR-symmetric models. From the LEP I data alone, the $ZZ'$ mixing angle may be limited to $|\theta_M| \leq 0.01$ and the $Z'$ mass to $M_2 > 118$–148 GeV, depending on the model (95% CL).


1 Introduction

The Standard Model [1] has been verified with a precision including one-loop corrections [2]. Nevertheless, there is a general consensus that we are far away from a final understanding of the elementary particle world. A unification of forces seems to happen at much higher mass scales than are accessible to present accelerators. Candidates for a truly unifying theory usually predict additional, heavy neutral gauge bosons $Z'$ (see e.g. [3]).

A search for a $Z'$ at LEP I energies or below relies on minor quantitative modifications of the neutral current cross sections, and one needs very precise predictions for cross sections and asymmetries. For a dedicated search, the fermion pair production reactions at LEP I are good candidates:

$$e^+e^- \rightarrow (\gamma, Z, Z') \rightarrow f^+f^- (\gamma).$$

(1)

A study of these reactions is the subject of the present article. In principle, the $Z'$ influences cross sections in three different ways:

- virtual $Z'$ exchange (also present without $ZZ'$ mixing);
- shift of the mass of the standard $Z$ boson seen at LEP I, due to $ZZ'$ mixing;
- modifications of the couplings of the standard $Z$ boson, due to $ZZ'$ mixing; this in fact concerns two different, although related observables – the $Z$ width [∼ peak height] and cross sections [∼ line shape]. For sufficiently large $Z'$ masses, the direct cross-section contributions originating from $Z'$ exchange may be neglected at LEP I energies. On the other hand, LEP I is the ideal place to search for the $ZZ'$ mixing phenomenon.

From existing measurements at LEP I [4, 5, 6], neutrino physics, and atomic parity violation [7, 8, 9], it is known that the mixing is very small if not vanishing. In such a situation, one has to disentangle with great care both the QED bremsstrahlung and weak standard-theory loop effects from the $Z'$ signals. Since QED corrections are model-independent (i.e. well-defined if vector- and axial-vector couplings, mass and width of the $Z'$ are fixed), the usual convolution formulae can be applied for the total cross section $\sigma_T$ and the forward–backward asymmetry $A_{FB}$ [10]:

$$\sigma_T(s) = \int dv \, \sigma^\text{Born}_T(s') R_T(v),$$

(2)

$$A_{FB}(s) = \frac{1}{\sigma_T} \int dv \, \sigma^\text{Born}_{FB}(s') R_{FB}(v),$$

(3)

with $v = 1 - s'/s$; the flux factors $R_{T,FB}$ are not influenced by the $Z'$.

There are two possible approaches to the $Z$ line shape:

- **Indirect data analysis.** Usually, one unfolds the cross sections and asymmetries with some model-independent ansatz in order to derive e.g. effective couplings or $Z$ partial widths. Afterwards, the $Z'$ analysis is performed. This seems to be a reliable procedure with the present data, but may prove to be insufficient in the future.

- **Direct data analysis.** Alternatively, one can confront (2) and (3) or, equivalently, $\sigma^\text{Born}_{T,FB}(s)$ directly with the data. The necessary modifications of these improved Born cross sections due to the $Z'$ will be described below. An advantage of the method is the possibility to study e.g. the top quark and $Z'$ influences on the cross sections simultaneously. Further, including the $Z'$ propagator opens a window to the $Z'$ mass $M_2$. 

In section 2 we introduce the gauge-boson mixing and define the notations, while in section 3 the modifications of the weak form factors due to a $ZZ'$ mixing are explained. Section 4 contains an application of both analysis methods to LEP I data, their comparison, and a discussion of the perspectives.

## 2  Gauge-Boson Mixing

The Lagrangian of the neutral gauge-boson interactions with fermions

$$\mathcal{L} = e A_\beta J^\beta_\gamma + g Z_{\beta\delta} J^\delta_Z + g' Z'_{\beta\delta} J'^\delta_{Z'},$$

contains currents of the form

$$J_n^\beta = \sum_f \tilde{f} \gamma^\beta \left[ v_f(n) + \gamma_5 a_f(n) \right] f, \quad n = \gamma, Z, Z'.$$

The $Z$-boson couplings are:

$$g = \sqrt{2} G_{\mu} M_Z, \quad a_f(Z) \equiv a_f = I_3^f(f), \quad v_f(Z) \equiv v_f = a_f(1 - 4|Q_f| \sin^2 \theta_W).$$

The photon couplings are defined such that $Q_e = -1$. The couplings $a_f(Z') \equiv a'_f$ and $v_f(Z') \equiv v'_f$ depend on the particular $Z'$ model. Some popular choices are the $E_6$ model and the left–right-symmetric model $^{3}$. In the following, we will assume that the mass eigenstates $Z_1$ and $Z_2$ result from a mixing of symmetry eigenstates $Z$ and $Z'$:

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_M & \sin \theta_M \\ -\sin \theta_M & \cos \theta_M \end{pmatrix} \begin{pmatrix} Z \\ Z' \end{pmatrix}.$$  \hspace{1cm} (7)

In the on-mass-shell renormalization scheme, the weak mixing angle $\theta_W$ and the gauge-boson mixing angle $\theta_M$ are related to the gauge-boson masses:

$$\cos \theta_W = \frac{M_W}{M_Z}, \quad \tan^2 \theta_M \equiv t^2_M = \frac{s_M^2}{c_M^2} = \frac{M_Z^2 - M_{Z'}^2}{M_{Z1}^2 - M_{Z2}^2}.$$  \hspace{1cm} (8)

Here, $M_W, M_1, M_2$ are particle masses and $M_Z$ has been introduced for convenience. Without mixing, $M_Z = M_1$. The resonance, which is being observed at LEP I, has mass $M_1$ and width $\Gamma_1$. From (8), we deduce the following couplings of $Z_1$ to fermions:

$$a_f(1) = c_M a_f + \frac{g'}{g} s_M a'_f \equiv (1 - y_f) a_f,$$  \hspace{1cm} (9)

$$v_f(1) = c_M v_f + \frac{g'}{g} s_M v'_f \equiv a_f(1) \left[ 1 + \left( \frac{v_f}{a_f} - 1 \right) (1 - x_f) \right]$$

$$= a_f(1) \left[ 1 + 4|Q_f| \sin^2 \theta_W (1 - x_f) \right].$$  \hspace{1cm} (10)

Here the $y_f$ are corrections of the axial couplings and the $x_f$ of the weak mixing angle in the vector couplings. They are approximately linear in the $ZZ'$ mixing angle:

$$y_f = -s_M \frac{g'd'_f}{ga_f} + (1 - c_M) \sim -s_M \frac{g'd'_f}{ga_f},$$

$$x_f = (1 - v_f/a_f)^{-1} \left( \frac{v_f + t_M v'_f g'/g}{a_f + t_M a'_f g'/g} \right) \sim s_M \frac{g'd'_f v'_f/a'_f - v_f/a_f}{g/a_f v_f/a_f}.$$  \hspace{1cm} (11)
3 Weak Form Factors

With a $ZZ'$ mixing, the matrix element for reaction (1) may be written in the form:

$$\tilde{M}_1 \sim \frac{1}{s - m_1^2} \frac{G_\mu M_1^2}{\sqrt{2}} a_e a_f \rho_{ef}^M \left[ L_\beta \otimes L_\beta - 4 |Q_e| \sin^2 \theta_W \kappa_{e}^M \gamma_\beta \otimes L_\beta - 4 |Q_f| \sin^2 \theta_W \kappa_{f}^M \gamma_\beta \otimes L_\beta + 16 |Q_e Q_f| \sin^4 \theta_W \kappa_{ef}^M \gamma_\beta \otimes \gamma_\beta \right].$$

(12)

The following short notations are used:

$$A_\beta \otimes B_\beta = \left[ \bar{u}_e A_\beta u_e \right] \cdot \left[ \bar{u}_f B_\beta u_f \right], \quad L_\beta = \gamma_\beta(1 + \gamma_5).$$

(13)

In the propagator, $m_2^2 = M_1^2 - i s \Gamma_1 / M_1$ denotes the complex mass parameter including finite-width effects. The form factors $\rho_{ef}^M, \kappa_{e}^M, \kappa_{f}^M$ and $\kappa_{ef}^M$ are composed of Standard Model weak corrections (contained in the weak form factors $[11, 12, 13] \rho_{ef}, \kappa_e, \kappa_f, \kappa_{ef}$) and additional factors due to gauge-boson mixing:

$$\rho_{ef}^M = \rho_{mix}(1 - y_e)(1 - y_f)\rho_{ef}, \quad \kappa_{f}^M = (1 - x_f)\kappa_f, \quad \kappa_{ef}^M = (1 - x_e)(1 - x_f)\kappa_{ef}.$$

(14)

In (12), the coupling constant $\alpha$ of the on-mass-shell scheme has been replaced by the muon decay constant:

$$\frac{\pi \alpha}{2 \sin^2 \theta_W \cos^2 \theta_W} = \rho_{mix} \frac{G_\mu}{\sqrt{2}} M_1^2 (1 - \Delta r).$$

(15)

The factor $(1 - \Delta r)$ is absorbed in the definition of $\rho_{ef}$. The $\rho_{mix}$ was introduced in (13), and consequently in (12), in order to eliminate $M_Z$ in favour of $M_1$:

$$\rho_{mix} \equiv \frac{M_2^2}{M_1^2} = \frac{1 + t_M^2}{1 + t_M^2} = 1 + s_\gamma \left( \frac{M_2^2}{M_1^2} - 1 \right) = \frac{M_W^2}{M_1^2 \cos^2 \theta_W}. \quad (16)$$

The last one in the above sequence of equations is valid only for restricted Higgs sectors. In the general case, $\rho_{mix}$ is an additional free parameter $[14, 15]$.

The four form factors $\rho^M, \kappa^M$ describe the weak radiative corrections completely in the case of massless fermions; the Born amplitude is obtained for $\rho = \kappa = 1$. The form factor $\rho_{ef}^M$ can be absorbed by the Fermi constant:

$$G_\mu \rightarrow \tilde{G}_\mu^M = \rho_{ef}^M(s, \cos \theta; m_t, M_H, M_1; M_2, \theta_M, \ldots) G_\mu.$$

(17)

Similarly, the form factors $\kappa^M$ can be interpreted as renormalizations of the weak mixing angle $\sin^2 \theta_W$:

$$\sin^2 \theta_W \rightarrow \begin{cases} \kappa_e^M \sin^2 \theta_W \\
\kappa_f^M \sin^2 \theta_W \\
\sqrt{\kappa_{ef}^M} \sin^2 \theta_W. \end{cases}$$

(18)

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$^1$ We do not discuss here problems connected with the definition of gauge-boson masses depending on the handling of the energy dependence of the width.
At LEP I energies, an effective weak mixing angle is often used,
\[
\sin^2 \theta^\text{eff}_W = \kappa \sin^2 \theta_W,
\]
where \(\kappa\) may be any (real part of) one of the form factors \(\kappa^M_f\), calculated at \(s = M_Z^2\). For further details see [13, 16, 17].

To complete the discussion of the \(Z\)-boson matrix element, we must define yet the decay width, which is the sum over all open fermion channels at the \(Z_1\) mass:
\[
\Gamma_1 = \sum_f \Gamma(1)_f = \sum_f c_f \frac{G_F M_Z^3}{\sqrt{2} \, 6\pi} \left[ \bar{v}^T_f(1)^2 + \bar{a}^T_f(1)^2 \right].
\]

For the partial widths, the effective couplings are:
\[
\begin{align*}
\bar{a}^T_f(1) &= \sqrt{\rho^{M,Z}_f} I^T_f(f), \\
\bar{v}^T_f(1) &= \bar{a}^T_f(1) \left[ 1 - 4|Q_f| \sin^2 \theta_W \kappa^{M,Z}_f \right],
\end{align*}
\]
where again weak corrections and the \(ZZ'\) mixing are properly combined:
\[
\begin{align*}
\rho^{M,Z}_f &= \rho_{\text{mix}} (1 - y_f)^2 \rho^Z_f, \\
\kappa^{M,Z}_f &= (1 - x_f) \kappa^Z_f.
\end{align*}
\]
The \(\rho^Z_f, \kappa^Z_f\) are the weak form factors of the Standard Model [13, 18]. As is well-known, at LEP I energies the couplings in the partial widths differ only slightly from those in the cross sections.

We shortly mention the matrix element \(\mathcal{M}_2\) with exchange of the heavy-mass eigenstate \(Z_2\):
\[
\mathcal{M}_2 \sim \frac{g^2}{s - m_2^2} \left\{ \gamma^\beta [a_\gamma(2) \gamma_5 + v_\gamma(2)] \otimes \gamma^\alpha [a_\gamma(2) \gamma_5 + v_\gamma(2)] \right\},
\]
where \(v_f(2), a_f(2)\) are vector- and axial-vector couplings of the \(Z'\). After adding up the photon-exchange diagram \(\mathcal{M}_\gamma\) with running QED coupling \(\alpha(s)\), the net matrix element is obtained,
\[
\mathcal{M} = \mathcal{M}_\gamma + \mathcal{M}_1 + \mathcal{M}_2,
\]
and the improved Born cross sections \(\sigma_{T,FB}^{\text{Born}}(s) \sim |\mathcal{M}|^2\) can be calculated and convoluted in (2) and (3).

At the end of this section, we should mention that the above derivations of matrix elements and form factors are equally valid for Bhabha and \(ep\) scattering. Another remark concerns some underlying assumptions, made in the numeric investigations of the next section, which are not inherent in the formalism. Additional degrees of freedom from exotic fermion mixing and Higgs structures are investigated in detail in [3, 19, 20] and will be neglected here. Further, it has been pointed out in [20] that including only the Standard Model radiative corrections (as is done here) is, in fact, a reasonable approximation to a complete treatment.
4 Applications and Discussion

Based on the above considerations, we created a FORTRAN program ZEFIT\cite{2}, which allows, together with the Standard Model program ZFITTER\cite{13}, to search for signals from both the $Z'$ propagator and a $ZZ'$ mixing in $e^+e^-$ annihilation.

In Fig. 1, the combined effect of $Z'$ mass and gauge-boson mixing at the $Z$ peak is shown for one of the $E_6$-based models, the $\chi$ model with $\theta_E = 0$ (which is, at the same time, one of the LR-models with $\alpha_{LR} = \sqrt{2/3}$).

![Figure 1](image)

**Figure 1:** The ratio $\sigma_{\gamma,Z,Z'}(\gamma,Z,Z')/\sigma_{\gamma,Z}(\gamma,Z)$ in the $E_6$-based $\chi$ model as a function of the $ZZ'$ mixing angle $\theta_M$ at $\sqrt{s} = M_1 = 91.180$ GeV, $m_t = 150$ GeV, $m_H = 300$ GeV. Parameter: the $Z'$ mass $M_2$. 
The ratio of muon-production cross sections $\sigma_T^\mu$ with and without $Z'$ is shown as a function of $\theta_M$ for different values of the $Z'$ mass. For $\theta_M \leq 0.05$, the ratio is linear in $\theta_M$ and independent of $Z_2$. This is a consequence of the vanishing $ZZ'$ interference and of the cancellation of $\rho_{\text{mix}}$ in the numerator and denominator of the cross-section formula at $\sqrt{s} = M_1$. A similar behaviour may be observed for the forward–backward asymmetry.

For the same model, Figs. 2a and b show this cross-section ratio as a function of the centre-of-mass energy for two different $Z'$ mass values. At the $Z$ peak, the predictions for different values of $M_2$ agree, while they show a different behaviour off the resonance position. At extreme LEP I energies, the differences reach the order of a percent even for not too large mixing angles. In view of plans for a high-luminosity version of LEP \cite{22}, it could be worthwhile to study possible prospects of this behaviour.

After these introductory remarks, we now outline the results from two different $Z'$ search strategies.

### 4.1 Indirect analysis using model-independent parameters

For our first series of fits we used the following input parameters, which we have taken from a model-independent analysis of 1990 data from all LEP I collaborations (Tables 1 and 2 of \cite{23}):

$$M_1, \Gamma_1, \sigma_{\text{had}}^{0,\text{peak}}, \gamma_T^2(1), a_l^2(1),$$

which are mass and width of the $Z$ boson, the improved hadronic Born cross section at the peak, and the squared effective leptonic couplings to the $Z$-mass eigenstate, respectively.
Figure 3: The 95% CL limits for the ZZ' mixing angle $\theta_M$ and Z' mass $M_2$, derived from a model-independent analysis of LEP I data for two classes of models: (a) $E_6$-based GUTs, (b) LR-symmetric theories. Parameters: $\alpha_s = 0.12$, $m_t = 150$ GeV, $M_H = 300$ GeV.

Error correlations as given in [23] are exactly taken into account. Allowed regions for the ZZ' mixing angle are shown in Figs. 3a and b for the $E_6$- and LR-models as functions of their parameters. The limits depend only weakly on the Z' mass and (not shown here) on the values of the top-quark mass $m_t$ and strong-interaction constant $\alpha_s$.

With Fig. 3, we obtain limits similar to those of other authors, e.g. our Fig. 3a is numerically comparable with Fig. 2 of [3] where, in a slightly different approach, 90% CL limits are derived from the 1990 LEP data; our Fig. 3b is in agreement with e.g. Fig. 3 of [24]. Both our figures contain slightly better limits than Figs. 3 and 4 of [25], which summarize an analysis of the preliminary 1991 LEP data (seemingly 90% CL).

4.2 Direct analysis of $\sigma_T(s)$ and $A_{FB}(s)$

Now we discuss direct fits to cross sections and asymmetries, taking into account their energy dependence. With the rising quality of the data, this approach will become more and more advantageous in comparison to the indirect fits. An important feature is the immediate use of line-shape formulae, including the virtual Z' exchange. The influence of the latter, and the resulting sensitivity of LEP I data to $M_2$ may be estimated as follows (similar estimates for the mixing angle $\theta_M$ are left to the reader): For sufficiently small ZZ' mixing, the dominant Z' term at LEP I is the ZZ' interference. In a self-explanatory notation, the line-shape is, without the Z':

$$
\sigma(s) \sim r_{\gamma} \frac{s}{s} + R \frac{s + R_f(s - M_0^2)}{(s - M_1^2)^2 + M_1^4 \Gamma_1^2} + \ldots,
$$

where $R_f = i/R$, and $i$ is the $\gamma Z$ interference. The ZZ' interference may be interpreted as a small correction to the $\gamma Z$ interference [13]:

$$
\Delta R_f(Z') \equiv R'_f = -2 \frac{g^2}{g^2} \frac{M_1^2}{M_2^2 - M_1^2} \frac{(v_e v'_e + a_e a'_e) \sum_q (v_q v'_q + a_q a'_q)}{(v_e^2 + a_e^2) \sum_q (v_q^2 + a_q^2)}.
$$

(26)
With \( \frac{2g^2}{g'2} = (10/3)\sin^2\theta_W \approx 0.77 \), and assuming, for instance, for a first estimate, formally \( v' = v, a' = a \), this is a rather simple expression, depending only on the two masses. Further, it is known how the peak position is shifted by such a \( \gamma Z \) interference:

\[
\Delta \sqrt{s_{\text{max}}} = \frac{1}{4M_1} R_f' \approx 17 \text{ MeV}.
\]  

(27)

A neglect of this peak shift leads to a systematic error of sign opposite to that of the \( Z \) mass \( M_1 \). Thus, (26) and (27) allow a rough estimate of the sensitivity of LEP I to a \( Z' \) propagator; for instance, with a \( \Delta M_1 = \pm 8 \text{ MeV} \), a \( Z' \) with a mass of 150 GeV and Standard-Model couplings cannot be excluded.

In practice, however, the sensitivity may deviate from this crude estimate. As an example, we use the hadronic line-shape data and the leptonic line-shape and asymmetry data of the 1990 LEP runs as quoted in [23], and references therein, for a search of the allowed region in the \( \theta_M - M_2 \) plane. The result is shown in Fig. 4 for three often analyzed \( E_6 \)-based models \((\theta_\chi = 0, \theta_\psi = \pi/2, \theta_\eta = -52.24^\circ = -0.9117) \). The top-quark mass dependence is indicated and, although present, not too large. For the \( Z' \) masses, the (95\% CL) exclusion limits are: \( M_\chi > 148 \text{ GeV}, M_\psi > 122 \text{ GeV}, M_\eta > 118 \text{ GeV} \). In obtaining these values, we have checked, that the lower \( Z' \) mass limits are stable against a variation of the \( Z \) mass within its experimental error. Our limits are to be compared with the ones derived in [27] from the CDF search for heavy bosons [28], \( M_2 > 148, 140, 165 \text{ GeV} \), respectively, and similar limits derived mainly from low-energy physics [8]. Although the present LEP I \( Z' \) mass limits cannot compete with the world’s best estimates, they indicate the potential of this device if used in the high-luminosity regime.

Basically, with the exclusion of the low-mass region of the \( \eta \) model, the limits to the \( ZZ' \) mixing are nearly independent of \( M_2 \). We should like to compare the allowed regions of the \( ZZ' \) mixing determined in the two approaches. The limits on the \( ZZ' \) mixing angle in Fig. 3a agree perfectly, for the available data, in their findings for the \( \chi \) and \( \psi \) models (Figs. 4a,b). For the \( \eta \) model, there are slight deviations. For instance, for \( M_2 = 200 \text{ GeV} \), one derives from Fig. 3a \( \theta_\eta = -0.06 - 0.01 \), while from Fig. 4c \( \theta_\eta = -0.04 - 0.015 \). We interpret this as an indication of the importance of the \( Z' \) propagator and of the correct energy dependences in general for the results in this parameter region.
Figure 4: Regions of $\theta_M$ and $M_2$ values in the $E_6$-based models $\chi, \psi, \eta$, which are compatible with the 1991 LEP I data (95% CL). Parameters are $\alpha_s = 0.12$, $M_H = 300$ GeV; $m_t = 100, 150, 200$ GeV (solid, dashed, dash-dotted curves).
To summarize, we developed two descriptions of fermion pair production at LEP I for $Z'$ models, one of them including the $Z'$ propagator and $Z\tilde{Z}$ mixing together with weak corrections and QED corrections. Some typical applications have been performed with data from the 1991 LEP I running periods. Both a fit to model-independent parameters and a direct line-shape analysis have been performed; they agree for most of the mixing-angle limits with each other and with earlier determinations. Additionally, from the direct fit one may determine $Z'$ mass limits. Future applications have been indicated.

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