Mass and pressure constraints on galaxy clusters from interferometric
Sunyaev–Zel’dovich observations

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ABSTRACT
Following on from our previous study of an analytic parametric model to describe the baryonic
and dark matter distributions in clusters of galaxies with spherical symmetry, we perform a
Sunyaev–Zel’dovich (SZ) analysis of a set of simulated relaxed clusters and present their
mass and pressure profiles. The simulated clusters span a wide range in mass, $2 \times 10^{14} < M_{\text{tot}}(r_{200}) < 1.0 \times 10^{15} \, M_\odot$, and observations with the Arcminute Microkelvin Imager are
simulated through their SZ effect. We assume that the dark matter density follows a Navarro,
Frenk and White (NFW) profile and that the gas pressure is described by a generalized NFW
profile. By numerically exploring the probability distributions of the cluster parameters given
simulated interferometric SZ data in the context of Bayesian methods, we investigate the
capability of this model and analysis technique to return the simulated clusters input quantities
when using two parametrizations. In the first parametrization, we assume that the cluster
halo concentration parameter, $c_{200}$, is an independent free parameter where as in the second
parametrization we consider the dependency of $c_{200}$ on cluster mass and its redshift. The results
of the analysis using the first parametrization show that $c_{200}$ is completely unconstrained and
its mean value is driven by the prior range assumed on this parameter. We find that considering
the mass and redshift dependency of the cluster halo concentration parameter in the second
parametrization is crucial in obtaining an unbiased cluster mass estimate and hence deriving
the radial profiles of the enclosed total mass and the gas pressure out to $r_{200}$. We define the bias
in the cluster mass to be the ratio of the mean mass difference from the two parametrizations to
their average. We show that although the mean mass differences from the two parametrizations
are within $1\sigma$ given our prior range on $c_{200}$, the bias varies between 2–20 per cent as the mass
of each of the clusters increases in the sample.

Key words: methods: data analysis – cosmology: observations.

1 INTRODUCTION

Determining the properties of clusters of galaxies such as their total
and baryonic mass offers an independent and powerful cosmolog-
ical tool to constrain the parameters of the $\Lambda$ cold dark matter
($\Lambda$CDM) model. The mass distribution of clusters is usually mea-
sured using a variety of observational methods, including X-ray,
Sunyaev–Zeldovich (SZ; Sunyaev & Zeldovich 1970; Birkinshaw
1999; Carlstrom, Holder & Reese 2002) and gravitational lensing
analyses. These methods are often based on parametrized cluster
models for the distribution of the cluster dark matter and the ther-
modynamical properties of its intracluster medium (ICM). However,
these various approaches usually lead to different estimates of the
cluster mass. This is due to either extrapolating to halo masses and
redshifts that are not well sampled by the data or fitting for model
parameters to which the data are insensitive.

In this paper, we perform a detailed analysis of a sample of nine
simulated SZ observations of galaxy clusters in a mass range $2 \times
10^{14} \, M_\odot < M_{\text{tot}}(r_{200}) < 1.0 \times 10^{15} \, M_\odot$ at redshift $z = 0.3$ as if
observed with Arcminute Microkelvin Imager (AMI) (Zwart et al.
2008). To study the cluster total mass, we use the model described
in Olamaie, Hobson & Grainge (2012b), which has the following
main characteristic features: (1) host halo density profile follows a
Navarro, Frenk and White (NFW; Navarro, Frenk & White 1997)
profile and the gas pressure is described by a generalized NFW
(GNFW) profile (Nagai, Kravtsov & Vikhlinin 2007) with fixed
shape parameters, both in accordance with numerical simulations;
(2) the gas distribution is in hydrostatic equilibrium with the cluster
total gravitational potential dominated by dark matter and both
dark matter and gas are spherically symmetric; and (3) the local gas fraction is much less than unity throughout the cluster, i.e. \( \frac{n_{\text{gas}}(r)}{n_{\text{tot}}(r)} \ll 1 \) for all \( r \). This final assumption allows us to write \( n_{\text{gas}}(r) = n_{\text{gas}0}(r) + n_{\text{gas}1}(r) \approx n_{\text{gas}0}(r) \). We show that assuming the dark matter halo concentration parameter as an independent, free parameter in the analysis has the potential to introduce biases in the cluster mass estimate, and hence it is crucial to consider the mass and redshift dependency of this parameter in the analysis. Throughout, we assume a \( \Lambda \)CDM cosmology with \( \Omega_M = 0.3 \), \( \Omega_\Lambda = 0.7 \), \( \sigma_8 = 0.8 \), \( h = 0.7 \), \( w_0 = -1 \) and \( w_a = 0 \).

2 MODELLING AND ANALYSIS OF SIMULATED INTERFEROMETRIC SZ OBSERVATIONS

As the SZ surface brightness is proportional to the line-of-sight integral of the pressure of the hot plasma in the ICM, SZ analysis of galaxy clusters provides a direct measurement of the pressure distribution of the ICM.

The observed SZ surface brightness in the direction of electron reservoir may be described as

\[
\delta I_e = T_{\text{CMB}} y f(v) \frac{\partial B_e}{\partial m} \bigg|_{T=T_{\text{CMB}}} .
\]

Here, \( B_e \) is the blackbody spectrum, \( T_{\text{CMB}} = 2.73 \ \text{K} \) (Fixsen et al. 1996) is the temperature of the cosmic microwave background (CMB) radiation, \( f(v) = (\frac{\nu^3}{e^{\nu/T} - 1}) - (1 + \delta(x, T_e)) \) is the Planck function at the cluster redshift, \( \nu = \frac{h v}{k_{\text{B}} T_e} \) is the frequency and \( k_{\text{B}} \) is the Boltzmann constant, \( \delta(x, T) \) takes into account the relativistic corrections due to the relativistic thermal electrons in the ICM and is derived by solving the Kompaneets equation up to the higher orders (Rephaeli 1995; Challinor & Lasenby 1998; Itoh, Kohyama & Nozawa 1998; Nozawa, Itoh & Kohyama 1998; Pointecouteau, Giard & Barret 1998). It should be noted that at 15 GHz (AMI observing frequency) \( x = 0.3 \) and therefore the relativistic correction, as shown by Rephaeli (1995), is negligible for \( k_{\text{B}} T_e \leq 15 \ \text{keV} \) unless in the studies that involve stacking up clusters. The dimensionless parameter \( \gamma \), known as the Comptonization parameter, is the integral of the number of collisions multiplied by the mean fractional energy change of photons per collision, along the line of sight

\[
y = \int_{-\infty}^{+\infty} n_e(r) k_{\text{B}} T_e(r) \frac{\partial y}{\partial m} dr = \frac{\sigma_T}{m_e c^2} \int_{-\infty}^{+\infty} P_e(r) \frac{\partial y}{\partial m} dr ,
\]

where \( n_e(r) \), \( P_e(r) \) and \( T_e \) are the electron number density, pressure and temperature at radius \( r \), respectively, \( \sigma_T \) is Thomson scattering cross-section, \( m_e \) is the electron mass, \( c \) is the speed of light and \( d \) is the line element along the line of sight. It should be noted that in equation (2) we have used the ideal gas equation of state.

Moreover, the integral of the Comptonization \( y \) parameter over the solid angle \( \Omega \) subtended by the cluster \( (Y_{\text{ij}}) \) is proportional to the volume integral of the gas pressure. It is thus a good estimate for the total thermal energy content of the cluster and hence its mass (see e.g. Bartlett & Silk 1994). The \( Y_{\text{ij}} \) parameter in both cylindrical and spherical geometries may be described as

\[
Y_{\text{ij}}(R) = \frac{\sigma_T}{m_e c^2} \int_{-\infty}^{+\infty} \frac{d l}{d \Omega} \int_0^R P_e(r) 2\pi r ds ,
\]

\[
Y_{\text{ij}}(r) = \frac{\sigma_T}{m_e c^2} \int_0^r P_e(r') 4\pi r'^2 dr' ,
\]

where \( R \) is the projected radius of the cluster on the sky.

In this context, we use the model described in Olamaie et al. (2012b), with its corresponding assumptions on the dynamical state of the ICM, to model the SZ signal and determine the radial profiles of \( M_{\text{gas}} \) and \( P_e \) for nine simulated clusters. The model assumes that the dark matter density follows a NFW profile (Navarro et al. 1997) and the ICM plasma pressure is described by the GNFW profile (Nagai et al. 2007),

\[
\rho_{\text{DM}}(r) = \frac{\rho_i}{\left( \frac{r}{R_i} \right)^2 \left( 1 + \frac{r}{R_i} \right)^2} ,
\]

\[
P_e(r) = \frac{P_0}{\left( \frac{r}{r_i} \right)^3 \left( 1 + \frac{r}{r_i} \right)^3} ,
\]

where \( \rho_i \) is an overall normalization coefficient, \( R_i \) is the scale radius where the logarithmic slope of the profile \( d \ln \rho(r)/d \ln r \approx -2 \), \( P_0 \) is also an overall normalization coefficient of the pressure profile and \( r_i \) is the scale radius. It is common to define the latter in terms of \( \rho_{500} \), the radius at which the mean enclosed density is 500 times the critical density at the cluster redshift, and the gas concentration parameter, \( c_{500} = r_{500}/r_i \). The parameters \((a, b, c)\) describe the slopes of the pressure profile at \( r \approx r_i, r > r_i \) and \( r \ll r_i \), respectively. In the simplest case, we follow Arnaud et al. (2010) and fix the values of the gas concentration parameter and the slopes to be \( (c_{500}, a, b, c) = (1.156, 1.0620, 5.4807, 0.3292) \) (appendix B in Arnaud et al. 2010). It is also common practice to define the halo concentration parameter, \( c_{200} \), as an input free parameter and studied the cluster profiles for a distribution of concentrations at a given mass and redshift. However, the results of our analysis showed that \( c_{500} \) remains unconstrained. \( c_{200} \) is a physical parameter, which reflects the background density of the Universe, and so is not a parameter that just defines the shape or the slope of the profile that cannot be constrained by SZ only analysis of galaxy clusters. This therefore suggests that the concentration depends on the other physical sampling parameters for a given set of cosmological parameters, i.e. mass and redshift. Hence, in this paper, we consider such a dependency in our analysis and instead derive \( c_{200} \).

The results of the studies on the relation between the structural properties of the dark matter haloes such as concentration, spin and shape with mass and the redshift from both \( N \)-body simulations of cosmological structure formation in a CDM Universe and observations of clusters of galaxies show a clear dependence of the concentration parameter on the halo mass and its redshift. This is based on the fact, as first discussed by Navarro, Frenk & White (1996) and Navarro et al. (1997), that the concentration parameter reflects the background density of the Universe at its formation time and hence as small objects form first in a hierarchical universe, lower mass haloes will be more concentrated than the massive ones (Navarro et al. 1997; Bullock et al. 2001; Eke, Navarro & Steinmetz 2001; Pointecouteau, Arnaud & Pratt 2005; Vikhlinin et al.)

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1 It should be noted that we repeated the whole analysis with parameters given in equation (12) in Arnaud et al. (2010) [i.e. \( (c_{500}, a, b, c) = (1.177, 1.0510, 5.4905, 0.3081) \)] and there was no change in the results (Fig. 6).
c_{200} = \frac{5.26}{1+z} \left( \frac{M_{\text{tot}}(200)}{10^{14} h^{-1} M_\odot} \right)^{-0.1}. \tag{7}

The above correlation between $M_{\text{tot}}(200)$ and $c_{200}$ is for relaxed haloes. The choice of this relation was motivated by the facts that: first we simulated a sample of relaxed clusters and secondly; as has been mentioned in Muñoz-Cuartas et al. (2011), unrelaxed haloes often have poorly defined centres, which makes the determination of a radial density profile, and hence of the concentration parameter, an ill-defined problem. Moreover, unrelaxed haloes often have shapes that are not adequately described by either a sphere or an ellipsoid, making the shape parameters poorly defined as well. However, we will consider the mass–concentration relation given for the complete halo sample in equation 5 in Neto et al. (2007) in our future study when we may have unrelaxed clusters in our sample. We also studied the $c_{200}$–$M_{\text{tot}}$ relation given by Muñoz-Cuartas et al. (2011), where the normalization coefficient and the slope vary with cosmic time. However, the results were the same as using equation (7) within our cluster halo mass range.

We generate a sample of nine simulated relaxed SZ clusters equally spaced in the mass range $2 \times 10^{14} M_\odot < M_{\text{tot}}(200) < 1.0 \times 10^{15} M_\odot$ using the above mentioned model, equation (7) and the input parameters $M_{\text{tot}}(r_{200})$, $z$ and $f_{\text{gas}}(r_{200})$ listed in Table 1; this set of parameters fully describes the Comptonization parameter. Further details of generating simulated SZ skies and observing them with a model AMI small array (SA) are described in Hobson & Maisinger (2002), Grainge et al. (2002), Feroz, Hobson & Bridges (2009a) and Olamaie et al. (2012a).

Table 1. $M_{\text{tot}}(r_{200})$ used to generate simulated clusters and the thermal noise levels reached in the simulated observations of the cluster sample. Here, $\sigma_{\text{SA}}$ refers to the thermal noise levels reached in SA maps. All the clusters are generated at redshift $z = 0.3$ and fixed $f_{\text{gas}}(r_{200}) = 0.13$.

| Cluster | $M_{\text{tot}}(r_{200})$ ($10^{14} M_\odot$) | $\sigma_{\text{SA}}$ (mJy beam$^{-1}$) |
|---------|---------------------------------|-------------------|
| clsim1  | 2.0                             | 0.05              |
| clsim2  | 3.0                             | 0.06              |
| clsim3  | 4.0                             | 0.07              |
| clsim4  | 5.0                             | 0.06              |
| clsim5  | 6.0                             | 0.077             |
| clsim6  | 7.0                             | 0.08              |
| clsim7  | 8.0                             | 0.087             |
| clsim8  | 9.0                             | 0.07              |
| clsim9  | 10.0                            | 0.067             |

Table 2. Summary of the priors on the sampling parameters when we assume a delta function prior on redshift and uniform in log prior for the cluster mass. Note that $N(\mu, \sigma)$ represents a Gaussian probability distribution with mean $\mu$ and standard deviation of $\sigma$ and $U(a, b)$ represents a uniform distribution between $a$ and $b$.

| Parameter | Prior |
|-----------|-------|
| $x_c$, $y_c$ | $N(0, 60) \text{ arcsec}$ |
| $M_{\text{tot}}(r_{200})$ | Joint prior with $z$ between $1 \times 10^{14} M_\odot$ and $6 \times 10^{15} M_\odot$. |
| $z$ | Joint prior with $f_{\text{gas}}(r_{200})$ between 0.25 and 0.35. |
| $f_{\text{gas}}(r_{200})$ | $N(0.13, 0.02)$ |

position on the sky. We further assume that the priors on sampling parameters are separable (Feroz et al. 2009b) such that

$$\pi(\mathbf{\Theta}_c) = \pi(x_c) \pi(y_c) \pi(M_{\text{tot}}(r_{200})) \pi(f_{\text{gas}}(r_{200})) \pi(z). \tag{8}$$

We use Gaussian priors on cluster position parameters, centred on the pointing centre and with standard deviation of 1 arcmin and adopt a $\delta$ function prior on redshift $z$. The prior on $M_{\text{tot}}(r_{200})$ is taken to be uniform in log $M$ in the range of $M_{\text{min}} = 10^{14} M_\odot$ to $M_{\text{max}} = 6 \times 10^{15} M_\odot$ and the prior of $f_{\text{gas}}(r_{200})$ is set to be a Gaussian centred at $f_{\text{gas}} = 0.13$ with a width of 0.02 (Vikhlinin et al. 2005, 2006; Komatsu et al. 2011; Larson et al. 2011). It should be noted that for the two low-mass clusters we set the minimum mass to $M_{\text{min}} = 0.4 \times 10^{14} M_\odot$ in the prior range. A summary of the priors and their ranges are presented in Table 2.

Further, we assume a prior probability for the comoving number density of clusters as a function of total mass and redshift. This is given by previous theoretical and simulation work (see e.g. Press & Schechter 1974; Jenkins et al. 2001; Evrard et al. 2002; Tinker et al. 2008). We here use the prediction of Tinker et al. (2008) and repeat the analysis using the Tinker mass function prior for the mass and the redshift. A summary of the priors and their ranges for this case is presented in Table 3.

More details on our Bayesian methodology, modelling interferometric SZ data, primordial CMB anisotropies, and resolved and unresolved radio point source models are given in Hobson & Maisinger (2002), Feroz & Hobson (2008), Feroz et al. (2009b), Davies et al. (2011) and Olamaie et al. (2012a).

3 RESULTS AND DISCUSSION

Figs 1 and 2 show 1D marginalized posterior distributions of sampling parameters for clsim2 (one of the lowest mass clusters), clsim4 and clsim9 (the most massive cluster in our sample), respectively.
Figure 1. Left: 1D marginalized posterior distributions of clusters parameters for clsim2, clsim4 and clsim9 when $c_{200}$ is also assumed to be an input parameter. Right: 1D marginalized posterior distributions of sampling parameters of clsim2, clsim4 and clsim9 when $c_{200}$ is calculated using equation (7). In both panels, the priors used are the ones given in Table 2. The green vertical lines are the true cluster parameter values as given in Table 1 and the red vertical lines are the mean of the probability distributions of the parameters.
Figure 2. Left: 1D marginalized posterior distributions of clusters parameters for clsim2, clsim4 and clsim9 when $c_{200}$ is also assumed to be an input parameter. Right: 1D marginalized posterior distributions of sampling parameters of clsim2, clsim4 and clsim9 when $c_{200}$ is calculated using equation (7). In both panels, the priors used are the ones given in Table 3. The green vertical lines are the true cluster parameter values as given in Table 1 and the red vertical lines are the mean of the probability distributions of the parameters.
Figure 3. Integrated mass (left) and pressure (right) profiles as a function of $r$ for nine simulated SZ clusters. In each panel, the plots with * show the profiles when $c_{200}$ is assumed as an input (sampling) parameter and the plots with $\diamond$ show the profiles when $c_{200}$ is calculated using equation (7). This analysis is performed assuming the priors as given in Table 2. The parameters means and standard deviations ($\sigma$) at each point are calculated from the posterior samples provided from MultiNest.

The analysis in Fig. 1 is performed assuming the priors given in Table 2 and the analysis in Fig. 2 is carried out assuming the Tinker mass function for the priors on the cluster mass and redshift as given in Table 3. The left-hand panel in both figures show the results when $c_{200}$ is assumed to be a sampling parameter (Olamaie et al. 2012b) and the right-hand panel shows the results when $c_{200}$ – $M_{200}(r_{200})$ dependency is taken into account. The green vertical lines are the true values of the simulated clusters parameters while the red
vertical lines represent the mean values of the posterior probability distributions.

It should be pointed out that similar results are obtained for all clusters in our sample. The results presented in the left-hand panels of Figs 1 and 2 show that while assuming \( c_{200} \) as a sampling parameter can constrain cluster projected position on the sky and \( M_{\text{tot}}(r_{200}) \), the halo concentration parameter, \( c_{200} \), is unconstrained and its mean value is just the mean of the prior range which suggests that the mean value of the distribution is strongly driven by the prior. This result, as we have seen in our previous studies of clusters of galaxies (Olamaie et al. 2012a), also suggests a correlation between \( c_{200} \) and the other two physical sampling parameters, i.e. \( M_{\text{tot}}(r_{200}) \) and \( \varepsilon \). The existence of such a correlation means that the analysis may result in a biased estimate of cluster parameters if it is not considered in the analysis.

The results presented in the right-hand panels of Figs 1 and 2 show 1D marginalized posterior distributions of sampling parameters when we take into account the dependency of halo concentration on both the formation time and the dynamical state of the halo using equation (7). From the results, it is clear that this form of the parametrization can constrain \( c_{200} \) as well as other cluster parameters. Moreover, while NFW profiles are usually fitted using the two parameters \( R_{\text{s}} \) and \( c_{200} \), this parametrization makes the profile a one-parameter profile. We note that similar results were obtained when analysing all the clusters in our sample.

The left-hand panel in Fig. 3 presents the integrated mass profiles for our sample of nine SZ simulated galaxy clusters and the right-hand panel shows the pressure profiles of these clusters. The means and standard deviation (\( \sigma \)) of the parameters are calculated from the posterior samples provided from MultiNest (see equation 2.3 in Feroz et al. 2008). The results were obtained assuming the priors as given in Table 2. In each panel, we have plotted the profiles out to \( r_{200} \) using the two forms of parametrizations: (1) assuming \( c_{200} \) as an input parameter in the analysis (*) and (2) calculating \( c_{200} \) using equation (7), (\( \diamond \)).

From the plots, the difference in estimating the cluster mass and its gas pressure using two forms of parametrizations is clear and becomes more significant as \( M_{\text{tot}}(r_{200}) \) increases. The difference in the mass estimate within a specific radius varies by an order of magnitude from the lowest mass cluster to the most massive one. For example, the difference in mass estimate within \( r_{200} \) is \( 1 \times 10^{14} M_{\odot} \) for clsim1 and \( 5 \times 10^{13} M_{\odot} \) for clsim9.

We also investigated the possibility of a change in the results of our study when using different sets of sampling parameters describing the GNFW pressure profile; i.e. \( c_{500}, a, b, c \). The results are shown in Fig. 6. The left-hand panel shows the radial profiles of three cluster masses when \( c_{200} \) is assumed as an input (sampling) parameter and the right-hand panel shows the results when \( c_{200} \) is calculated using equation (7). In both panels, plots with the blue * show the profiles when \( c_{200} \) is assumed as an input (sampling) parameter and the plots with the green \( \diamond \) show the profiles when \( c_{200} \) is calculated using equation (7).

We also note that \( f_{\text{g}}(r_{200}) \) is hardly constrained in both parametrizations indicating that the gas fraction cannot be constrained using SZ only data. Cluster gas mass fraction can be measured using X-ray, or a combination of X-ray and SZ data. In our previous study (Olamaie et al. 2012b), we showed that \( f_{\text{g}}(r) \) is indeed a function of radius showing a pronounced dependency on the cluster mass which in turn reflects the dependency on the temperature (see e.g., Allen et al. 2004; Ettori et al. 2004; Sadat et al. 2005; Vikhlinin et al. 2005, 2006; LaRoque et al. 2006; Afshordi et al. 2007; McCarthy, Bower & Balogh 2007). As it was discussed in Section 2, the observed SZ surface brightness is proportional to Comptonization \( y \) parameter which is proportional to \( M_{\text{tot}} T_{e} \) or \( f_{\text{tot}} T_{e} \). The integral of the \( y \) parameter over the solid angle \( \Omega \) subtended by the cluster \( (Y_{\Omega}) \) on the other hand is proportional to \( M_{\text{tot}} \).
Figure 5. Left: radial profiles of three cluster masses when $c_{200}$ is assumed as an input (sampling) parameter. Right: radial profiles of three cluster masses when $c_{200}$ is calculated using equation (7). In both panels, plots with green ⋄ show the profiles when we assume the priors as given in Table 2 and the plots with blue * show the profiles when we assume priors as given in Table 3.

Figure 6. Left: radial profiles of three cluster masses when $c_{200}$ is assumed as an input (sampling) parameter. Right: radial profiles of three cluster masses when $c_{200}$ is calculated using equation (7). In both panels, plots with blue * show the profiles when $(c_{500}, a, b, c) = (1.156, 1.0620, 5.4807, 0.3292)$ and plots with green ⋄ show the profiles when $(c_{500}, a, b, c) = (1.177, 1.0510, 5.4905, 0.3081)$.

Thus, SZ data can constrain the cluster total mass and the fact that $f_g$ is not constrained using SZ only data results from a degeneracy between $T_g$ and $f_g$; see Fig. 7.

4 CONCLUSION

We have studied the recovery of $M_{\text{tot}}(r)$ and $P_e(r)$ from the SZ effect for a sample of nine simulated galaxy clusters ($2.0 \times 10^{14} M_{\odot} < M_{\text{tot}}(r_{200}) < 1.0 \times 10^{15} M_{\odot}$) using the model described in Olamaie et al. (2012b). This is motivated by the fact that SZ surface brightness is proportional to the line-of-sight integral of the ICM plasma so that SZ data can potentially constrain the cluster total mass.

To obtain an unbiased mass estimate we have carried out a detailed analysis of a series of simulated clusters using two different parametrizations within our model and its corresponding assumptions (Olamaie et al. 2012b). In the first parametrization, we assume that the halo concentration parameter $c_{200}$ is also a sampling parameter. However, the results of the analysis show that the simulated SZ data cannot constrain this parameter and therefore its mean value is driven by the prior range which may introduce biases in the ultimate cluster mass estimate. This result also suggests that $c_{200}$ depends on the other model parameters.

In the second parametrization, we consider the correlation of $c_{200}$ with $M_{\text{tot}}(r_{200})$ and $z$ within $\Lambda$CDM Universe (equation 7) as higher mass haloes that are forming today are less concentrated than haloes of lower mass that built up at an earlier epoch, where the mean density was higher. This parametrization clearly constrains $c_{200}$ as AMI SZ data can constrain $M_{\text{tot}}(r_{200})$. We hence conclude that in order to obtain a robust estimate on cluster physical parameters including its mass it is crucial to consider the mass and redshift dependency of $c_{200}$ as precisely as possible.

We calculate the difference in mass estimates and the corresponding bias when using the two parametrizations. We find that although the difference in mass estimates lie within 1σ error bars the fact that $c_{200}$ cannot be constrained using SZ only data indicates that the bias and difference in mass estimate may increase depending on the choice of prior on $c_{200}$.
Figure 7. 1D and 2D posterior probability distributions of four cluster physical parameters out to $r_{200}$ of clsim4 including $M_{\text{tot}}(r_{200}), f_g(r_{200}), M_g(r_{200})$ and $T_g(r_{200})$ showing the degeneracy between these parameters. The analysis is performed using priors given in Table 2. The left-hand panel shows the results when $c_{200}$ is assumed as an input (sampling) parameter and the right-hand panel shows the results of the analysis when $c_{200}$ is calculated using equation (7).

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