Light Quark Spin-Flavor Symmetry for Baryons Containing a Heavy Quark in Large N QCD

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Abstract

The couplings and interactions of baryons containing a heavy quark are related by light quark spin-flavor symmetry in the large \( N \) limit. The single pion coupling constant which determines all heavy quark baryon-pion couplings is equal to the pion coupling constant for light quark baryons. Light quark symmetry relations amongst the baryon couplings are violated at order \( 1/N^2 \). Heavy quark spin-flavor symmetry is used in conjunction with large \( N \) light quark spin-flavor symmetry to determine the couplings of the degenerate doublets of heavy quark baryons.
The large $N$ limit of QCD \cite{1} originally led to a qualitative understanding of baryon-pion interactions \cite{2}. Recent developments \cite{3} imply rigorous quantitative results. In large $N$, it is possible to show that baryon-pion couplings obey light quark spin-flavor symmetry relations. Furthermore, violation of these symmetry relations occurs only at $\mathcal{O}(1/N^2)$, since the $1/N$ correction has the same group theoretic structure as the leading term \cite{4}. Large $N$ relations for other physical quantities can also be obtained \cite{3} \cite{5}. These results are identical to the predictions of the Skyrme model in large $N$ \cite{6}. Since group theoretic predictions of the Skyrme model are identical to the predictions of the quark model in large $N$ \cite{4} \cite{8}, these relations are also the same as those of the non-relativistic quark model. It is worth emphasizing, however, that these results are model-independent and are rigorous consequences of QCD in the large $N$ limit.

Another area of recent interest has been the study of baryons and mesons which contain a single heavy quark. In the heavy quark limit $m_Q \to \infty$, the heavy quark effective lagrangian possesses a heavy quark spin-flavor symmetry for a single heavy quark with a given velocity \cite{9}. This heavy quark symmetry can be used to derive relations between decay amplitudes and other physical quantities for baryons or mesons containing a single heavy quark. For the study of baryons containing a single heavy quark, it is possible to make use of both heavy quark symmetry and large $N$. Recent work has studied the properties of baryons containing a single heavy quark in the Skyrme model \cite{10}. The heavy quark baryons arise in the Skyrme picture as heavy meson-soliton bound states, where the solitons are the ordinary baryons containing no heavy quark of the Skyrme model. Quantum numbers and other properties of these heavy meson-soliton bound states are in good agreement with experiment \cite{11} \cite{12} \cite{13}.

The main emphasis of this work is the study of pion-heavy quark baryon interactions. In large $N$ QCD. It is shown that a single pion coupling constant parametrizes all heavy quark baryon-pion interactions. More importantly, this single pion coupling constant is identical to the single pion coupling constant describing baryon-pion interactions for baryons containing no heavy quarks. This argument can be generalized to baryons containing any finite number of heavy quarks. Thus, all pion-baryon interactions are determined by a single pion coupling constant in large $N$ QCD. As is the case for ordinary baryon-pion interactions, the large $N$ predictions for the heavy baryon-pion couplings respect light quark spin-flavor symmetry. Again, these symmetry relations are valid to $\mathcal{O}(1/N^2)$.
The large $N$ derivation of baryon-pion couplings depends only on the isospin and angular momentum assignments of the light degrees of freedom of the heavy quark baryons, since the spin and flavor of the heavy quark are conserved by low-energy QCD interactions. The decoupling of the heavy quark in the heavy quark $m_Q \to \infty$ limit makes the analysis of pion couplings for heavy quark baryons identical to the analysis for ordinary baryons. Heavy quark spin-flavor symmetry is used in conjunction with the large $N$ light quark spin-flavor symmetry to determine the couplings of the degenerate doublets of heavy quark baryons.

Although this work concentrates on baryon-pion couplings, the large $N$ arguments presented here have implications for other physical parameters of the chiral lagrangian. The general result of this work is that physical parameters for heavy quark baryons are described by an approximate light quark spin-flavor symmetry which becomes exact in the large $N$ limit and an approximate heavy quark spin-flavor symmetry which becomes exact in the heavy quark limit. Together, these two approximate symmetries greatly constrain chiral lagrangian parameters. In most cases, only a unique coupling remains for each type of process.

The derivation of the above results begins with a discussion of the spectrum of baryon states in the large $N$ limit. For the case of baryons containing no heavy quarks, the baryon spectrum for an odd number of colors and $N_f = 2$ light flavors consists of a degenerate tower of isospin and angular momentum multiplets $(I, J)$ equal to $(1/2, 1/2), (3/2, 3/2), \ldots, (N/2, N/2)$. It is possible to show that this tower constitutes the minimal set of states which can be present in large $N$ \footnote{In principle, there may be additional towers of states degenerate with the minimal set. We will simply assume the minimal choice.}.

For the case of baryons containing a single heavy quark, the minimal set of states consists of a degenerate tower of $(I, J)$ states equal to $(0, 0), (1, 1), (2, 2), \ldots, ((N - 1)/2, (N - 1)/2)$, where $J$ represents the angular momentum of the light degrees of freedom of the heavy quark baryon. (Note that each $(I, J)$ state in this tower corresponds to a degenerate doublet of heavy baryon multiplets with isospin $I$ and total spin equal to $I \pm \frac{1}{2}$, since the angular momentum of the light degrees of freedom $J = I$ and the spin of the heavy quark $S_Q = \frac{1}{2}$.) Each of these two towers of states has the correct quantum numbers to be generated from a single $SU(2N_f)$ multiplet.

For the first tower, the $SU(4)$ representation is the totally symmetric tensor product of $N$ fundamental representations of $SU(4)$. The Young tableau for this representation is
a row of \( N \) boxes. Under the breaking of the spin-flavor group to its isospin and spin subgroups, \( SU(4) \to SU(2) \times SU(2) \), this representation produces the states of the first tower. The tower of states for the light degrees of freedom of heavy quark baryons has the correct quantum numbers for it to arise from the \( SU(4) \) representation of \((N - 1)\) completely symmetrized boxes. Representative Young tableaux for the breaking of these representations under the isospin and spin subgroups are given in fig. 1.

It is possible to prove that the pion couplings of the multiplets in each degenerate tower of states are determined by a single coupling constant. The group theoretic structure of the couplings for each tower are the couplings which result if the tower forms an \( SU(4) \) representation. We first review the derivation of this result for ordinary baryon-pion interactions \[3\]; then the argument is extended to heavy quark baryons.

General large \( N \) considerations imply that pion-baryon scattering should be \( O(1) \) in the large \( N \) expansion \[2\]. Ref. \[3\] shows that this behavior only occurs if the there is an exact cancellation amongst graphs at leading order in \( N \), since for arbitrary couplings pion-baryon scattering will grow with \( N \). Thus, unitarity of pion-baryon scattering in the large \( N \) limit constrains pion-baryon couplings. As Ref. \[3\] shows, this large \( N \) constraint uniquely determines all pion-baryon couplings in terms of a single coupling constant.

Consider a general baryon-pion vertex

\[
\overline{B}_2 G^{ai} B_1 \frac{\partial^a \pi^i}{f_\pi}, \tag{1}
\]

where \( a = 1, 2, 3 \) labels the angular momentum channel of the \( p \)-wave pion, \( i = 1, 2, 3 \) labels the isospin of the pion, and \( G^{ai} \) is an operator with unit spin and isospin. Eq. (1) is written in terms of static baryon fields of the baryon-pion chiral lagrangian \[14\] in the baryon rest frame. All pion couplings amongst baryon multiplets in the degenerate tower which are allowed by isospin and angular momentum conservation are present and all couplings are unrelated. Because the emission of a pion can only change the isospin or spin of the initial baryon by one unit, only diagonal couplings and nearest neighbor off-diagonal couplings between multiplets in the tower are allowed. The couplings allowed by isospin and angular momentum conservation can be parametrized by the reduced matrix elements of \( G \),

\[
\langle I_2 I_{2z}, J_2 J_{2z} | G^{ai} | I_1 I_{1z}, J_1 J_{1z} \rangle
= N g(J_1, J_2) \sqrt{\frac{2J_1 + 1}{2J_2 + 1}} \left( \begin{array}{cc|c}
I_1 & 1 & I_2 \\
I_{1z} & i & I_{2z}
\end{array} \right) \left( \begin{array}{cc|c}
J_1 & 1 & J_2 \\
J_{1z} & a & J_{2z}
\end{array} \right), \tag{2}
\]
where $I_1 = J_1$ and $I_2 = J_2$ for the assumed tower of states and $g(J_1, J_2)$ are arbitrary couplings of $\mathcal{O}(1)$. An explicit factor of $N$ has been factored out of the reduced matrix elements in Eq. (2) to keep all $N$ dependence manifest. The explanation of this factor of $N$ is given by the same power counting argument that implies that the axial vector coupling constant of the nucleon $g_A \sim N$. Large $N$ considerations relate the couplings $g(J_1, J_2)$. First consider $\pi$ nucleon scattering, $\pi N \rightarrow \pi N$, in large $N$. The scattering occurs through direct and crossed diagrams, each with $N$ and $\Delta$ intermediate states. Thus, the scattering depends on the couplings $g_{NN}$ and $g_{\Delta N}$. Large $N$ counting of the diagrams is as follows. Each diagram receives a factor of $1/N$ from explicit factors of $f_\pi$ since each diagram contains two pion vertices and $f_\pi \sim \sqrt{N}$. The matrix elements at the vertices give $\langle G^{ai} \rangle \langle G^{bj} \rangle \sim \mathcal{O}(N^2)$ so that the scattering amplitude is of order $N$, unless the leading terms cancel. This cancellation requires $g_{\Delta N} = g_{NN} \equiv g$. All other couplings are determined recursively to equal $g$. For example, $\pi \Delta \rightarrow \pi N$ scattering involves the one additional coupling $g_{\Delta \Delta}$. The constraint that the matrix element cancels at leading order in $N$ implies $g_{\Delta \Delta} = g$. Scattering of $\pi \Delta$ to the $(5/2, 5/2)$ baryon involves one additional coupling, $g_{\Delta \Delta}$. Again, this coupling must equal $g$ to produce a scattering amplitude which is $\mathcal{O}(1)$. Thus, all the pion couplings $g(J_1, J_2)$ are determined recursively to be equal to $g$, 

$$
\langle I_2 I_{2z}, J_2 J_{2z} | G^{ai} | I_1 I_{1z}, J_1 J_{1z} \rangle = N g \sqrt{\frac{2J_1 + 1}{2J_2 + 1}} \left( \begin{array}{c|c} I_1 & 1 \\ \hline I_{1z} & i \end{array} \right) \left( \begin{array}{c|c} I_2 & 1 \\ \hline I_{2z} & a \end{array} \right) \left( \begin{array}{c|c} J_1 & 1 \\ \hline J_{1z} & a \end{array} \right) \left( \begin{array}{c|c} J_2 & 1 \\ \hline J_{2z} & a \end{array} \right). \tag{3}
$$

The coupling relations (3) are the relations obtained by grouping the degenerate tower of baryon states into the totally symmetric $SU(4)$ representation.

A similar analysis can be performed for pion couplings to the tower of states for the light degrees of freedom of the heavy quark baryons. In the heavy quark limit, the angular momentum of light degrees of freedom is separately conserved in low-energy QCD interactions, so the analysis can proceed without considering the heavy quark spin. For the tower of integral isospin and spin, a diagonal pion coupling to the $(0, 0)$ state is not allowed. The first coupling in the series connects the $(0, 0)$ and the $(1, 1)$ states. Exact cancellation of the $\mathcal{O}(N)$ contribution to $\pi + (0, 0) \rightarrow \pi + (1, 1)$ scattering relates this coupling to the diagonal $(1, 1)$ pion coupling. Again, all couplings can be determined recursively in terms of this single coupling. The pion coupling relations for this tower are 

$$
\langle I_2 I_{2z}, J_2 J_{2z} | G^{ai} | I_1 I_{1z}, J_1 J_{1z} \rangle = N g' \sqrt{\frac{2J_1 + 1}{2J_2 + 1}} \left( \begin{array}{c|c} I_1 & 1 \\ \hline I_{1z} & i \end{array} \right) \left( \begin{array}{c|c} I_2 & 1 \\ \hline I_{2z} & a \end{array} \right) \left( \begin{array}{c|c} J_1 & 1 \\ \hline J_{1z} & a \end{array} \right) \left( \begin{array}{c|c} J_2 & 1 \\ \hline J_{2z} & a \end{array} \right) \tag{4}
$$

\vspace{5mm}
where $g'$ denotes the coupling constant, $G^{ai}$ is an operator with unit spin and isospin, and the tower of states requires integral $I = J$. As before, these are the relations which follow if one forms the totally symmetric $SU(4)$ representation of $(N - 1)$ boxes from the tower of states for the light degrees of freedom of the heavy quark baryons.

Note that in considering pion-heavy quark baryon scattering, we have ignored the heavy quark entirely. The couplings obtained above are the couplings to the light degrees of freedom of the heavy quark baryons. In order to obtain the couplings of the heavy quark baryons, one must construct tensor products of the light quark angular momentum and the heavy quark spin. For instance, the $(0, 0)$ state corresponds to the spin-$\frac{1}{2}$ $Λ_Q$. All other states correspond to degenerate doublets of heavy quark baryons. For example, the $(1, 1)$ state corresponds to the spin-$\frac{1}{2}$ $Σ_Q$ and the spin-$\frac{3}{2} Σ_Q^*$. The pion couplings to the heavy baryon states can be easily derived using Eq. (4) for the matrix elements of the light degrees of freedom and the spin decomposition of the heavy baryon states

$$|JJ_z⟩ = \sum_{J_ℓ, S_Q z} |J_ℓ, J_ℓ z⟩ |S_Q S_Q z⟩ \left( \begin{array}{c|c} J_ℓ & S_Q \\ \hline J_ℓ z & S_Q z \end{array} \right) |J J_z⟩,$$

where $J_ℓ$ and $S_Q = \frac{1}{2}$ are the angular momentum of the light degrees of freedom and the spin of the heavy quark, respectively. The result of this computation is

$$\langle I^I, J^J |G^{ai}| II^I_z, JJ_z⟩ = N g' (-1)^{1+I+S_Q+J'} (-1)^{I'-I-1} (-1)^{J'-J-1} \sqrt{(2I+1)(2J+1)} \left\{ \begin{array}{ccc} 1 & I & I' \\ S_Q & J' & J \end{array} \right\} \left( \begin{array}{c|c} I & 1 \\ \hline I_z & a \end{array} \right) \left( \begin{array}{c|c} J & 1 \\ \hline J_z & a \end{array} \right),$$

where substitutions $J_ℓ = I$ and $J'_ℓ = I'$ have been made and the quantity in curly braces is the 6$j$ symbol. For a single heavy quark, $S_Q = \frac{1}{2}$. The spectrum of heavy baryon states thus consists of a degenerate tower of doublets with integer isospin $I$ and half-integral total spin $J = I \pm 1/2$.

To prove that the pion coupling constants for baryon-pion and heavy quark baryon-pion interactions are equal to each other, one needs to consider the coupling of heavy quark baryons to ordinary baryons and heavy quark mesons. The lowest-lying multiplet of mesons containing a single heavy quark is a degenerate doublet which consists of a pseudoscalar meson $P_Q$ and a vector meson $P_Q^*$. For $Q = b$, these are the $\overline{B}$ and $\overline{B}^*$, whereas for $Q = c$, they are the $D$ and $D^*$. The light degrees of freedom of this multiplet carry isospin and angular momentum quantum numbers ($\frac{1}{2}, \frac{1}{2}$). All the baryon → heavy
baryon couplings to a heavy meson can be parametrized in terms of the reduced matrix elements of an operator $H$,

$$
\langle I_2 I_{2z}, J_2 J_{2z} | H^{a_i} | I_1 I_{1z}, J_1 J_{1z} \rangle = h(J_1, J_2) \sqrt{\frac{2J_1 + 1}{2J_2 + 1}} \left( \begin{array}{c}
I_1 \\
I_{1z}
\end{array} \right) \left( \begin{array}{c}
\frac{1}{2} \\
i
\end{array} \right) \left( \begin{array}{c}
J_1 \\
J_{1z}
\end{array} \right) \left( \begin{array}{c}
\frac{1}{2} \\
J_2
\end{array} \right),
$$

where $H^{a_i}$ is an operator with isospin and spin $\frac{1}{2}$. The ordinary baryon has half-integral $I_1 = J_1$, whereas the light degrees of freedom of the heavy quark baryon has integral $I_2 = J_2$. This formula can be generalized by tensoring in the spin of the heavy quark, but the analysis is more transparent if only the angular momentum of the light degrees of freedom is used. The matrix elements $\langle H^{a_i} \rangle$ are $O(1)$. A simple way to see that this is the correct power of $N$ is to consider the scattering baryon + heavy meson $\rightarrow$ baryon + heavy meson through an intermediate heavy quark baryon. This scattering is a low momentum transfer process; the large heavy quark mass flows through the diagram since the intermediate baryon is a heavy quark baryon. There is no possible crossed diagram for this process since the heavy quark mass cannot be properly routed through the crossed diagram. Thus, no cancellation is possible, and the tree graph must be $O(1)$, which implies that each matrix element is $O(1)$. Alternatively, one can think of the heavy meson + baryon $\rightarrow$ heavy baryon coupling as replacing a quark in the baryon with a heavy quark of the same color. If the absorbed and emitted heavy mesons attach to the same quark line in the baryon, there are no factors of $N$ involved. If one of the $O(N)$ other lines is involved in the heavy meson emission, then there must be an exchange of a gluon between the different quark lines by momentum conservation. Gluon exchange introduces a compensating factor of $1/N$, so the scattering is $O(1)$.

The pion couplings of the two baryon towers can be related by studying the amplitudes for $\pi +$ heavy baryon $\rightarrow$ heavy meson + baryon. This process involves $g$, $g'$ and $h(J_1, J_2)$. The two tree graphs which contribute are given explicitly in fig. 2 for the special case $\pi + \Lambda_Q \rightarrow P_Q^{(*)} + N$. The graph in which the pion couples to the heavy meson is order $1/N$ with respect to these two diagrams and can be ignored. No large momentum transfer is involved in this process since the heavy quark mass flows through each diagram. The

† Note that an inverse factor of the $P^{(Q)}$ meson decay constant $f_P$ is implicit in the definition of the matrix elements Eq. 7. If this factor is not included, the matrix elements $\langle H^{a_i} \rangle$ must be redefined with a factor of $\sqrt{N}$. The heavy meson vertex is then given by $\langle H^{a_i} \rangle/f_P$, which is still $O(1)$.
typical momentum transfer is $m(\Lambda_Q) - m(P_Q^{(*)}) - m(N) \sim O(\Lambda_{QCD})$, which is $O(1)$ in both the $1/N$ and $1/m_Q$ expansions. The direct channel graph involves an intermediate heavy baryon state, whereas the crossed diagram involves an intermediate baryon. Since the pion-baryon couplings grow like $\sqrt{N}$, these diagrams will result in a scattering amplitude of $O(\sqrt{N})$ unless the diagrams cancel exactly at leading order. The condition for cancellation is

$$\langle I' I_z' J_z' | H^{bj} G^{ai} | II_z J J_z \rangle - \langle I' I_z' J_z' | G^{ai} H^{bj} | II_z J J_z \rangle = 0,$$

where summation over a complete set of intermediate states is implied. For the first term, the sum is over all states allowed by isospin and angular momentum conservation with integral $I = J$; for the second term, the sum is over all allowed states with half-integral $I = J$. Using Eqs. (3), (6), and (7), this constraint can be rewritten in terms of the couplings $g$, $g'$ and $h(J_1, J_2)$. For a given initial heavy baryon with spin of the light degrees of freedom $J$, the final baryon can have spin $J \pm \frac{1}{2}$ or $J \pm \frac{3}{2}$. The recursion relation for each of these channels are:

$$g'(J, J + 1)h(J + 1, J + \frac{3}{2}) - h(J, J + \frac{1}{2})g(J + \frac{1}{2}, J + \frac{3}{2}) = 0$$

(9)

for $J' = J + \frac{3}{2}$,

$$g'(J, J)h(J, J + \frac{1}{2}) - h(J, J + \frac{1}{2})g(J + \frac{1}{2}, J + \frac{3}{2}) + g'(J, J + 1)h(J + 1, J + \frac{1}{2}) - h(J, J - \frac{1}{2})g(J - \frac{1}{2}, J + \frac{1}{2}) = 0$$

(10)

for $J' = J + \frac{1}{2}$,

$$g'(J, J)h(J, J - \frac{1}{2}) - h(J, J + \frac{1}{2})g(J + \frac{1}{2}, J - \frac{1}{2}) + g'(J, J - 1)h(J - 1, J - \frac{1}{2}) - h(J, J - \frac{1}{2})g(J - \frac{1}{2}, J - \frac{1}{2}) = 0$$

(11)

for $J' = J - \frac{1}{2}$, and

$$g'(J, J - 1)h(J - 1, J - \frac{3}{2}) - h(J, J - \frac{1}{2})g(J - \frac{1}{2}, J - \frac{3}{2}) = 0$$

(12)

for $J' = J - \frac{3}{2}$, where $J$ is an integer. The initial conditions are that $h(0, -\frac{1}{2}) = h(0, -\frac{3}{2}) = h(1, -\frac{1}{2}) = 0$. In addition, all allowed pion couplings can be set equal to $g$ or $g'$. Eq. (4) implies

$$h(J, J + \frac{1}{2}) = \left(\frac{g}{g'}\right)^J h(0, \frac{1}{2}),$$

(13)

for $J = 0, 1, 2, \ldots$.
whereas Eq. (12) implies
\[
h(J, J - \frac{1}{2}) = \left( \frac{g'}{g} \right)^{J-1} h(1, \frac{1}{2}),
\]
for all integer \( J \). Eq. (10) for \( J = 0 \) (recall that \( g'(0, 0) = 0 \)) yields
\[
h(1, \frac{1}{2}) = \left( \frac{g}{g'} \right) h(0, \frac{1}{2}).
\]
Thus, if there is a consistent solution of the recursion relations, all of the \( h(J, J \pm \frac{1}{2}) \) couplings are given in terms of \( h(0, \frac{1}{2}) \). Finally, substituting Eqs. (13), (14), and (15) into Eq. (10), one obtains the constraint
\[
(g' - g) \left( \frac{g}{g'} \right)^J + g' \left( \frac{g'}{g} \right)^{J-1} - g \left( \frac{g'}{g} \right)^{J-2} = 0,
\]
which can only be satisfied for arbitrary \( J \) if \( g = g' \). Thus, the coupling constants for the baryon-pion and heavy baryon-pion interactions are equal and the heavy meson couplings of these baryons are parametrized by a single coupling constant \( h \),
\[
\langle I_2 J_{2z}, J_2 J_{2z} | H^{ai} | I_1 I_{1z}, J_1 J_{1z} \rangle = h \sqrt{\frac{2J_1 + 1}{2J_2 + 1}} \left( \begin{array}{c|c} I_1 & \frac{1}{2} \\ \frac{1}{2} & I_2 \end{array} \right) \left( \begin{array}{c|c} J_1 & \frac{1}{2} \\ \frac{1}{2} & J_2 \end{array} \right).
\]
Note that there is no constraint on the value of \( h \). The physical origin of this coupling is quite different than that for \( g \), so no relation is to be expected.

The corrections to the spin-flavor relations of the pion couplings for each tower of baryon states are order \( 1/N^2 \) \(^4\). The relation \( g = g' \) receives a correction at order \( 1/N \), however, since the \( 1/N \) symmetry-preserving corrections to the two pion couplings need not be equal. This same power counting was obtained in the soliton-heavy meson bound state model \(^{11}\). The large \( N \) approximation justifies these predictions.

The large \( N \) arguments given in this work can be generalized to baryons containing a finite number of heavy quarks. In this case, one treats the heavy quarks as a subsystem with total angular momentum \( J_Q \). The heavy quark subsystem must be in a state which is totally symmetric under the heavy quark spin-flavor symmetry. Thus, the heavy quark multiplets of baryons with a given number of heavy quarks form a single representation of heavy quark spin-flavor symmetry, namely the Young tableau with a row of \( N_h \) boxes, where \( N_h \) is the number of heavy quarks in the baryon. Low-energy QCD interactions do not couple the heavy quark subsystem to the light degrees of freedom of baryon. Thus,
analysis of pion couplings proceeds in terms of the flavor and angular momentum quantum numbers of the light degrees of freedom of the baryons as before. The tower of states for the light degrees of freedom of the baryons will be the half-integer (integer) \( I = J \) towers for baryons with an even (odd) number of heavy quarks. Unitarity of pion-baryon scattering amplitudes in large \( N \) implies that the pion couplings amongst baryons containing a given number of heavy quarks are described by a single coupling constant. Thus, in the large \( N \) limit, the tower of states for the light degrees of freedom of the baryon forms the representation of light quark spin-flavor symmetry given by the Young tableau with a row of \((N - N_h)\) boxes. Pion-baryon scattering \( \to \) heavy meson + baryon identifies the pion coupling constants of baryons containing \( N_h \) heavy quarks and baryons containing \((N_h - 1)\) heavy quarks. (Note that this scattering must obey angular momentum conservation for both the heavy degrees of freedom and the light degrees of freedom involved in the process.) Thus, large \( N \) \( SU(2N_f) \) light quark symmetry in conjunction with \( SU(2N_h) \) heavy quark symmetry implies that all baryon-pion couplings are given in terms of a single coupling constant up to corrections of order \( 1/N \) and \( 1/m_Q \).
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**Figure Captions**

Fig. 1. The towers of states $(I, J)$, $I = J$ equals half-integer or $I = J$ equals integer arise from the totally symmetric $SU(4)$ representation $(a)$ of $N$ or $(N - 1)$ boxes. Under the breaking $SU(4) \rightarrow SU(2) \times SU(2)$, the $SU(4)$ representation breaks into isospin and spin $SU(2)$ representations of all Young tableaux with two rows formed from $N$ or $(N - 1)$ boxes. The two highest dimensional representations are given by $(a)$ and $(b)$. A general representation has the form $(c)$. The lowest representation $(d)$ is the $(\frac{1}{2}, \frac{1}{2})$ or $(0, 0)$ state.

Fig. 2. Feynman diagrams for the scattering $\pi + \Lambda_Q \rightarrow P_Q^{(*)} + N$. In leading order in $N$, only the direct and crossed diagrams contribute. The $(I, J)$ quantum numbers of the light degrees of freedom of each particle are given. Note that the scattering is a low-momentum process which does not depend on the heavy quark mass or spin; the large heavy quark mass $m_Q$ and the heavy quark spin $S_Q$ are conserved in each diagram.
Figure 1
Figure 2