Adaptive region tracking control for underwater vehicles with large initial deviation and general uncertainty

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Abstract
A region tracking control system is developed for underwater vehicles with large initial deviation and general uncertainty. The developed system is a nonlinear cascaded system, consisting of two subsystems in series. The fixed-gain controller of the first subsystem is designed to compensate for the region tracking error caused by the large initial deviation. In the second subsystem, the radial basis function neural network is adopted to approximate the general uncertainty along with the external disturbance and modelling uncertainty. According to the Lyapunov theory, the control law of the two subsystems and adaptive law of the second subsystem are derived to ensure the region tracking errors asymptotically converge to zero. The validity of the control system is verified by a series of comparisons on simulation results.

Keywords
Underwater vehicle, region tracking control, neural network, initial deviation, general uncertainty

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Introduction
In the complex marine environment, underwater vehicles are widely used to accomplish the specified missions. To make the underwater vehicle, a highly refined machine, many scholars have done a lot of research on precise attitude and trajectory control. However, the underwater vehicle usually only needs to be within the range of the desired area, and there is no need to pursue excessive tracking control accuracy. To improve operation efficiency and reduce energy consumption, some scholars have proposed the concept of area tracking control. A variety of region tracking control methods have been proposed to deal with the adverse effects due to the external disturbance and modelling uncertainty for underwater vehicles in recent years. Ismail and Dunnigan proposed an adaptive region tracking control method based on sliding mode for modelling uncertainty of underwater vehicles. Zhang and Chu propose an adaptive area tracking control method combining radial basis function (RBF) neural network and boundary...
potential energy function. This method uses the RBF neural network to approximate the unknown term of dynamics model online and makes the tracking error reach the desired region through the boundary potential energy function. Chu and Zhu\cite{Chu03} utilized the RBF neural network to identify the uncertainty in the underwater vehicle system but only studied the three-DOF motion of the underwater vehicle, without considering the influence of the coupling of the six-DOF motion on the control effect. Ismail et al.\cite{Ismail04} have designed a region control scheme with robustness to external disturbance. Ismail et al.\cite{Ismail05} propose a region tracking control method based on the combination of boundary potential energy function and weighted pseudo-inverse and verify the control effect without current disturbance.

The above methods do not involve the control of the six-DOF motion of the underwater vehicle with the external disturbance and modelling uncertainty. In this article, the region tracking control experiment under the environment of severe current disturbance and modelling uncertainty is carried out, and it is found that the conventional region tracking control method\cite{Ismail04} cannot make the underwater vehicle reach the desired area.

All the above methods do not consider the problem of large initial deviation. In practice, the tracking control of underwater vehicles is usually long-time and long-range, so the deviation between the actual initial position of the underwater vehicles and the initial point of the desired area is inevitable. In this article, it is found that when the initial deviation is large, the conventional region tracking controllers\cite{Ismail04,Chu03} have the obviously worse control effects or even maybe completely invalid. In the end, the underwater vehicle will not be able to perform subsea detection operations or return safely.

Different from the previous region tracking control methods, this study focuses on solving the control error caused by large initial deviation while considering modelling uncertainty and external disturbance. The first subsystem of nonlinear cascaded system in this article is designed to compensate for tracking errors caused by large initial deviation through a fixed-gain controller. The second subsystem approximates current interference and modelling uncertainty online through the RBF neural network.

This article is organized as follows. The second section describes the six-DOF underwater vehicle kinematics and dynamics models under the environment of external disturbance and modelling uncertainty and the purpose of the region tracking control. In the third section, the design process of the proposed region tracking control system in detail and the stability analysis of the proposed control system based on the Lyapunov theory are performed. In the fourth section, the performance of the control system is verified through simulation studies, and the results are given. Furthermore, in the fifth section, a brief conclusion of this study is made.

**Problem formulation**

### Underwater vehicle modeling

Establish the kinematics and dynamics models of an underwater vehicle by taking a common standard method. The kinematic motion equations in the earth-fixed frame are defined as follows\cite{Ismail04}

\[
\begin{bmatrix}
\dot{\eta}_1 \\
\dot{\eta}_2
\end{bmatrix} =
\begin{bmatrix}
J_1(\eta_2) & 0_{3 \times 3} \\
0_{3 \times 3} & J_2(\eta_2)
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix}
\]

(1)

where \(M\) represents the inertial matrix; \(C(v)\) represents rigid-body Coriolis and Centripetal matrix; \(D(v)\) represents the drag coefficient matrix; \(g(\eta)\) represents the vector of combined gravitational and buoyancy forces and moments; \(\tau\) is the control forces and moments acting on the underwater vehicle center of mass; and \(\tau_d\) represents the external force.

### Problem formulation

To make the underwater vehicle reach the desired region, it is very important to design a suitable control law to make the maximum absolute error of the underwater vehicle less than the tracking error. Lyapunov function is constructed to prove the stability of the tracking errors under the environment of external disturbance and modelling uncertainty.

\[
\begin{bmatrix}
\dot{\eta}_1 \\
\dot{\eta}_2
\end{bmatrix} =
\begin{bmatrix}
\cos \theta \cos \psi & \sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi & \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi \\
\sin \theta \cos \psi & \sin \phi \sin \theta \sin \psi + \cos \phi \sin \psi & \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \\
-\sin \theta & \sin \phi \cos \theta & \cos \phi \cos \theta
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix}
\]

(2)

\[
M\ddot{v} + C(v)v + D(v)v + g(\eta) = \tau - \tau_d
\]

(4)
In the region tracking control for underwater vehicles, the desired trajectory \( \eta_d \) is the central trajectory of the desired region directly. The desired region \( \eta(i) \) is defined as follows

\[
y(\bar{\eta}) = [y_1(\bar{\eta}), \ldots, y_6(\bar{\eta})]^T \leq 0 \tag{5}
\]

where \( \bar{\eta} = \eta - \eta_d \) is the trajectory tracking error; \( \eta \) is the real trajectory; \( y_i(\bar{\eta}) = 1/2(\bar{\eta}^2 - e_i^2) \) \( i = 1, \ldots, 6 \); and \( e(t) = [e_1(t), \ldots, e_6(t)]^T \) represents the desired tracking error.

The desired function is defined as follows

\[
Y(\bar{\eta}) = \sum_{i=1}^{6} k_i \max(y_i(\bar{\eta})_i)^2 \tag{6}
\]

where \( k_i \) is a positive constant and \( \bar{\eta}_i \) is the trajectory tracking error on each DOF.

When the underwater vehicle is outside the desired region, the desired function \( Y(\bar{\eta}) > 0 \). To make the desired function converge to zero, we calculate the desired velocity by the gradient descent method. The desired velocity is expressed as follows

\[
v_d = J^{-1}(\eta) \left( \bar{\eta}_d - \sum_{i=1}^{6} k_i \frac{\partial Y(\bar{\eta})}{\partial \bar{\eta}_i} \right) \tag{7}
\]

**Region tracking control system design**

Based on the conventional RBF fault-tolerant control system, this article focuses on the failure of conventional control systems with the general uncertainty and large initial deviation and designs a nonlinear cascade system. The first subsystem of the control system uses a fixed-gain controller to compensate for the tracking error caused by the large initial deviation. And its second subsystem uses the RBF neural network to estimate the general uncertainty for underwater vehicles.

**Design idea**

Intelligent fault-tolerant control mainly performs reliable and precise control of complex nonlinear systems in real time.8 Artificial neural networks have achieved good results in intelligent fault-tolerant control applications. It is found that the fault-tolerant control effect of the conventional RBF fault-tolerant control systems9–21 is better when the initial point of the center trajectory of the desired region coincides with the actual initial point or the deviation is small. However, the controller parameters will be over adjusted, and the degree of change of the controller output will be very intense, when the initial deviation is large. And when the output of the controller exceeds the saturation constraint of the thruster change rate, the tracking error will continue to increase, which will lead to the final failure of the control.

In practice, the initial point of tracking target cannot be set artificially, and the complex ocean environment will cause the position fluctuation of the underwater vehicle. Therefore, the problem of large initial point deviation cannot be ignored. To address this problem, a cascade system for region tracking control of underwater vehicles is designed as follows.

In Figure 1, underwater vehicle model in the first subsystem is unaffected by the external disturbance and modelling uncertainty. The fixed-gain controller in the first subsystem compensates the tracking error caused by the large deviation between the virtual track \( \eta_d \) and the desired track \( \eta_d \), which can avoid the tracking error \( \eta_d \) caused by the controller with over-adjusted parameters. In the second subsystem, the tracking error \( \eta_d \) between the virtual track \( \eta_d \) and the actual track \( \eta_d \) is compensated by RBF neural network.

**Region tracking control system design**

At first, the control law of the controller with fixed gain of the first subsystem of the cascade system is designed, and the Lyapunov function is constructed to theoretically prove the tracking error asymptotically converge to zero, which compensates the region tracking errors caused by the initial deviation between the virtual track and the desired track. Then, the second subsystem of the cascade system is designed. The RBF neural network is adopted to online estimate the general uncertainty. And the Lyapunov function is made to theoretically prove the region tracking errors between the virtual trajectory and the actual trajectory. To sum up, the region control tracking errors of the control system can converge to zero when underwater vehicle has the general uncertainty and the large initial deviation.

**First subsystem design and stability analysis**

According to Figure 1, the region tracking error of the first subsystem as follows
where \( \ddot{v}_{rd} = \ddot{v}_r - \ddot{v}_d = J^{-1}(\eta)(\dot{\eta}_r - \dot{\eta}_d) \)

\[
= J^{-1}(\eta)(f_r(\eta_r) - A_d\eta_d + B_u - B_d r)
\]

where \( \ddot{v}_{rd} = v_r - v_d \) is the error between the virtual velocity and the desired velocity in the body-fixed frame; \( \eta_d \) is defined by \( \dot{\eta}_d = A_d\eta_d + B_d r; \) \( r \) is the desired trajectory; \( B_d = \begin{bmatrix} 0_{6 \times 6} & I_{6 \times 6} \end{bmatrix} ; \)

\( A_d = \begin{bmatrix} 0_{6 \times 6} & -\omega^2 \xi I_{6 \times 6} - 2\omega_n \xi I_{6 \times 6} \end{bmatrix} ; \) and \( \omega_n \) and \( \xi \) are the positive constants.

According to equations (7), (8), and the design method of state feedback controller, the control law of the first subsystem is as follows

\[
u_r = B^+(A_d\eta_d - f_r(\eta_r) + B_d r) - B^+J(\eta)(B_dK + BW)\ddot{v}_{rd} - B^+J(\eta)P^+J^T(\eta)K_p \sum_{i=1}^{6} k_i \frac{\partial Y(\dot{\eta}_{rd})}{\partial \dot{\eta}_{rd_i}}
\]

(9)

\[
A_d^TP + PA_d - PB_dA_dB_d^TP \leq 0
\]

(10)

where \( B^+ \) is the pseudo-inverse matrix of \( B; \) \( P^+ \) is the pseudo-inverse matrix of \( P \) which is a positive definite matrix; \( K_p = \text{diag}(K_{p1}; \cdots, K_{pn}); \)

\( K = \Lambda_kB_d^TP; \)

\( W = \Lambda_uB_d^TP; \) and \( \Lambda_u \) and \( \Lambda_k \) are positive constants.

Take the appropriate transformation to describe equation (10)

\[
\begin{bmatrix}
A_d^TP + PA_d & PB_d \\
B_d^TP & -\Lambda_kI_{6 \times 6}
\end{bmatrix} \leq 0
\]

(11)

where \( \Lambda_k \) and \( P \) can be obtained by linear matrix inequality Toolbox.22

To compensate the region tracking errors, \( \ddot{v}_{rd} \) caused by the initial deviation between the virtual track and the desired track, define a Lyapunov function as follows

\[
V_1 = \frac{1}{2}\ddot{v}_{rd}^TP\ddot{v}_{rd} + \sum_{i=1}^{6} K_{pi}k_i \left[ \max(0, y_i(\dot{\eta}_{rd})) \right]^2
\]

(12)

According to equations (8) and (9), the derivative of equation (12) is

\[
\dot{V}_1 = \ddot{v}_{rd}^TPJ^{-1}(\eta)(f_r(\eta_r) - A_d\eta_d + B_u - B_d r)
\]

\[
+ \sum_{i=1}^{6} K_{pi}k_i \max(0, y_i(\dot{\eta}_{rd}))\ddot{\eta}_{rd_i}^2
\]

\[
= \ddot{v}_{rd}^T(A_d^TP + PA_d + PB_d\Lambda_kB_d^TP)\ddot{v}_{rd} - \ddot{v}_{rd}^TPB_d\Lambda_uB_d^TP\ddot{v}_{rd}
\]

\[
- \ddot{v}_{rd}^TJ(\eta)K_p \sum_{i=1}^{6} k_i \max(0, y_i(\dot{\eta}_{rd}))\ddot{\eta}_{rd_i}
\]

\[
+ \sum_{i=1}^{6} K_{pi}k_i \max(0, y_i(\dot{\eta}_{rd}))\ddot{\eta}_{rd_i}^2
\]

(13)

According to the following equation, we can simplify equation (13)

\[
\ddot{v}_{rd}J^T(\eta)K_p \sum_{i=1}^{6} k_i \max(0, y_i(\dot{\eta}_{rd}))\ddot{\eta}_{rd_i}
\]

\[
= \sum_{i=1}^{6} K_{pi}k_i \max(0, y_i(\dot{\eta}_{rd}))\ddot{\eta}_{rd_i}
\]

\[
+ \left( \sum_{i=1}^{6} k_i \max(0, y_i(\dot{\eta}_{rd})) \right)\ddot{\eta}_{rd_i}^2 \right)
\]

(14)

When the parameters in equation (14) satisfy equation (11), we can get as follows

\[
\dot{V}_1 \leq -\dddot{v}^T \times \dddot{v} - \sum_{i=1}^{6} K_{pi}k_i^2 \left[ \max(0, y_i(\dot{\eta}_{rd})) \right]^2 \dddot{\eta}_{rd_i}^2
\]

\[
\leq -\rho_1 V_1
\]

where \( \rho_1 = \min(0, k_i^2 \dddot{\eta}_{rd_i}^2). \)

According to the range of values of \( k_i, \rho_i \) is equal to 0, equation (16) can be calculated as follows

\[
\dot{V}_1 \leq 0
\]

(15)

Therefore, according to equation (16) and the Lyapunov theory, it has that the control error is consistent and ultimately bounded.

**Second subsystem design and stability analysis**

According to Figure 1, the region tracking error of the second subsystem is expressed as follows

\[
\dot{v}_r = \dot{v}_r - \dot{v}_d = J^{-1}(\eta)(f_r(\eta_r) - f_r(\eta_r) + B(u - u_d))
\]

(17)

where \( \dot{v}_r = v - v_r \) is the error between the real velocity and the virtual velocity in the body-fixed frame.

Estimate \( f_r(\eta_r) - f(\eta) \) by the RBF neural network

\[
f_r(\eta_r) - f(\eta) = W\varphi(v, c, b) + \varepsilon \leq \varepsilon_0
\]

(18)

where \( W \) represents the hidden-to-output layer interconnection weighted matrix; \( \varphi(v, c, b) \) represents the output of the hidden layer; \( c \) is the center parameter; \( b \) is the wide parameter; \( \varepsilon \) is the neural network estimation error; and \( \varepsilon_0 \) is a positive constant.

Here, we adopt RBF neural network with three layers to online approximate the general uncertainty \( \dot{F} \)

\[
\dot{F} = \ddot{W}\varphi(v, c, b)
\]

(19)

where \( \ddot{W} \) is the estimation value of \( W. \)

Design the control law and adaptive adjustment law \( \dot{W} \) of are as follows
\[ u - u_d = B^+ \left[ \bar{\Phi} \varphi(v, c, b) - k_s \text{sgn} (\bar{v}_r) \right] \]
\[ - B^+ J(\eta) J^T(\eta) K_p \sum_{i=1}^{6} k_i \max(0, y_i(\tilde{\eta}_i)) \tilde{\eta}_i \]
\[ \dot{\tilde{W}} = -\frac{\lambda}{1 + \rho} \left( \bar{v}_r \varphi(v, c, b) \right)^T + m \tilde{W} \]  \hspace{1cm} (21)

where \( K_p \) and \( K_s \) are positive definite diagonal matrices; \( k_s \geq 0; \lambda \) is the adjust step of \( W; m \) is a positive constant; and \( \gamma = [\gamma_1, \cdots, \gamma_6] \) \( \gamma_i = \begin{cases} |\tau_i| - \tau_{Mi} & |\tau_i| > \tau_{Mi} \\ 0 & |\tau_i| \leq \tau_{Mi} \end{cases} \)

When the control law is saturated, the tracking error always exists, resulting in the real trajectory of the underwater vehicle cannot converge to the target region in a limited time. To solve the problem, in this article, the saturation factor \( 1/(1 + \gamma) \) is used. When the control law is saturated, the adaptive adjustment law decreases as the control law increases. So, even if the control law is saturated, the system is stable.

To compensate the region tracking errors between the real track and the virtual track, define a Lyapunov function as follows

\[ V_2 = \frac{1}{2} \bar{v}_r^T \bar{v}_r + \frac{1}{2} \text{tr} \left[ \bar{W}^T \lambda^{-1} \bar{W} \right] + \sum_{i=1}^{6} K_{pi} k_i \left[ \max(0, y_i(\tilde{\eta}_i)) \right]^2 \] \hspace{1cm} (22)

The derivative of equation (22) is as follows

\[ \dot{V}_2 = \bar{v}_r \dot{\bar{v}}_r + \text{tr} \left[ \bar{W}^T \lambda^{-1} \bar{W} \right] + \sum_{i=1}^{6} K_{pi} k_i \max(0, y_i(\tilde{\eta}_i)) \dot{\tilde{\eta}}_i \dot{\tilde{\eta}}_i \] \hspace{1cm} (23)

Substituting equation (17) into equation (23)

\[ \dot{V}_2 = \bar{v}_r^T J^{-1}(\eta) [f(\eta) - f_r(\eta)] + B(u - u_d)] + \text{tr} \left[ \bar{W}^T \lambda^{-1} \bar{W} \right] \]
\[ + \sum_{i=1}^{6} K_{pi} k_i \max(0, y_i(\tilde{\eta}_i)) \dot{\tilde{\eta}}_i \dot{\tilde{\eta}}_i \] \hspace{1cm} (24)

Substituting equation (18) and equation (20) into equation (24)

\[ \dot{V}_2 = \bar{v}_r^T J^{-1}(\eta) \left[ (\bar{W} - W) \varphi(v, c, b) - (k_s \text{sgn} (\bar{v}_r) + c) \right] \]
\[ - \bar{v}_r^T J^T(\eta) K_p \sum_{i=1}^{6} k_i \max(0, y_i(\tilde{\eta}_i)) \tilde{\eta}_i + \text{tr} \left[ \bar{W}^T \lambda^{-1} \bar{W} \right] \]
\[ + \sum_{i=1}^{6} K_{pi} k_i \max(0, y_i(\tilde{\eta}_i)) \dot{\tilde{\eta}}_i \dot{\tilde{\eta}}_i \] \hspace{1cm} (25)

where \( \bar{W} = \bar{W} - W \) is the error of \( W \).

According to the following equation and range of values of \( k_s \), we can simplify equation (25)

\[ \dot{V}_2 \leq \text{tr} \left[ \bar{W}^T \lambda^{-1} \bar{W} \right] - \text{tr} \left[ \bar{W}^T \lambda^{-1} \bar{W} \right] \]
\[ - \sum_{i=1}^{6} K_{pi} k_i^2 \max(0, y_i(\tilde{\eta}_i)) \dot{\tilde{\eta}}_i \dot{\tilde{\eta}}_i \] \hspace{1cm} (26)

According to following equations, we can simplify equation (26)

\[ \text{tr} \left[ \bar{W}^T \lambda^{-1} \bar{W} \right] \leq \text{tr} \left[ \bar{W}^T \lambda^{-1} \bar{W} \right] \]
\[ \leq ||\bar{W}||^2 ||\bar{W}|| \]
\[ - \text{tr} \left[ \bar{W}^T \lambda^{-1} \bar{W} \right] \leq -m \text{tr} \left[ \bar{W}^T (\bar{W} + W) \right] \]
\[ \leq -\frac{m}{2} ||\bar{W}||^2 + \frac{m}{2} ||W||^2 \]
\[ - \frac{m}{4} ||\bar{W}||^2 + ||\bar{W}|| ||\bar{W}|| ||\varphi(v, b, c)|| \]
\[ \leq -\frac{1}{4m} ||\bar{v}_r||^2 ||\varphi(v, b, c)||^2 \] \hspace{1cm} (29)

Equation (26) can be transformed as the following form

\[ \dot{V}_2 \leq -\frac{1}{4} ||\varphi(v, b, c)|| \dot{\bar{v}}_r - \frac{m}{4} ||\bar{W}||^2 \]
\[ - \sum_{i=1}^{6} K_{pi} k_i^2 \max(0, y_i(\tilde{\eta}_i)) \dot{\tilde{\eta}}_i \dot{\tilde{\eta}}_i + \frac{m}{2} ||W||^2 \leq -\rho_2 V_2 + \delta \] \hspace{1cm} (30)

We can get \( \rho_2 \) and \( \delta \) according to equation (30)

\[ \rho_2 = \min \left( \frac{1}{2} ||\varphi(v, b, c)||^2, \frac{m||\dot{\bar{v}}_r||}{2}, k_i^2 \right) \] \hspace{1cm} (31)
\[ \delta = \frac{m}{2} ||W||^2 \] \hspace{1cm} (32)

It can be proved by equation (30) that the real trajectory of the underwater vehicle can reach the desired area. According to equation (32), the degree of convergence between the real trajectory of the underwater vehicle and the desired region can be determined by \( m \).
Experimental results

To prove that the control system in this article has a good control effect on the six-DOF motion of underwater vehicle under general uncertainty, compared with the conventional controller based on the boundary potential energy function, the conventional controller based on RBF, and the control system in this article, carries on the simulation experiment under the system uncertainty.

To verify that the control system has good control effect under general uncertainty and large initial deviation, this article compares the conventional controller and the control system in this article, there is a system under the general uncertainty and large initial deviation, simulation experiments are performed.

The control effect of the underwater vehicle is judged by whether the region tracking error is stable within the desired area and whether the maximum absolute tracking error can reach the desired area.

Experimental conditions

The experimental conditions are as follows:

- Ocean current is shown in Figure 2.
- To embody the modeling uncertainty, 10% modeling error is assumed in the simulation.

Experiment with general uncertainty

In this section, when general uncertainty exists, tracking the desired region the center trajectory of which is the straight-line. The experimental results are shown in Figure 3.

The desired and real trajectories are shown in Figure 3(a). Figure 3(b) and (c) shows the tracking trajectory and error of six-DOF motion, respectively.

The lateral and pitch tracking errors of the conventional controller based on boundary potential energy obviously exceed the respective desired error, which can be seen from

\[ y(t) = \left[ 0.4m, 0.4m, 0.4m, 0.05\text{ rad}, 0.05\text{ rad, 0.05 rad} \right]^T \]

Ellipses: \( (t) = \left[ 0.4m, 0.4m, 0.4m, 0.2\text{ rad}, 0.2\text{ rad, 0.2 rad} \right]^T \)

The thruster thrust amplitude limit is \( \pm 100N \).

With the reference, the initial position of the AUV is appropriately set to \( \eta(0) = [-0.5, -0.5, -0.5, 0, 0, 0]^T \).

The first-order Gauss-Markov process is used to simulate the ocean current

\[ \dot{V}_c + \mu V_c = \omega \quad (33) \]

where \( V_c \) represents the magnitude of ocean current; \( \omega \) represents Gaussian white noise with mean \( 1 \) and variance \( 1 \); and \( \mu = 3 \). In addition, we assume that the direction of ocean current is constant. Therefore, the sideslip angle is equal to \( \pi/4 \) and angle of attack is equal to \( \pi/6 \).

In this article, the uncertainty of current disturbance \( \tau_f \) is as follows:

\[ \tau_f = \begin{cases} 
[0, 0, 0, 0, 0, 0]^T & t \leq 30s \\
0, \sin\left(\frac{\pi}{4}\cos\left(\frac{\pi}{6}\right)ight), 0, 0.5r + 0.725 \sin\left(\frac{\pi}{6}\right), 0.5r - 0.725 \cos\left(\frac{\pi}{6}\right) \cos\left(\frac{\pi}{4}\right), 0.5r & 30s < t < 50s \\
[0, 0, 0, 0, 0, 0]^T & t \geq 50s 
\end{cases} \]

The desired tracking error is as follows:

- Straight line: \( \dot{\eta}(t) = [0.02t, 0.02t, 0.001t, 0, 0, 0]^T \)
- Ellipse: \( \dot{\eta}(t) = \left[ 2, (1 - \cos\left(\frac{\pi}{60}\right)), \sin\left(\frac{\pi}{60}\right), 0, 0, 0 \right]^T \)
Figure 3(c). In Figure 3(a), the growth trend of lateral error of conventional controller is obvious. The real position and angle errors of the RBF controller and the control system in this article are less than the desired tracking error, respectively. And the convergence time is 20 s, which can be seen from Figure 3(c).

For a clear description, the maximum absolute errors are presented in Table 1.

In Table 1, the maximum absolute errors of the conventional controller based on boundary potential energy are [0.4732 m, 0.2222 m, 0.0964 m, 0.0226 rad, 0.0288 rad, and 0.1689 rad]. And its lateral and pitch tracking errors obviously exceed the respective desired error. That is to say, the controller based on boundary potential energy cannot reach the desired straight-line region with general uncertainty.

However, the maximum absolute tracking errors of the RBF controller and the control system in this article are [0.1933 m, 0.3511 m, 0.1062 m, 0.0100 rad, 0.0089 rad, and 0.0081 rad] and [0.1668 m, 0.3438 m, 0.0009 m, 0.0085 rad, 0.0023 rad, and 0.0078 rad], respectively. The real position and angle errors of the RBF controller and the control system in this article are less than 0.4 m, and the angle errors of the RBF controller and the control system in this article are less than 0.05 rad. That is to say, the RBF controller and the control system in this article can reach the desired straight-line region with general uncertainty. And in Figure 3(c), they can be stable maintained within the desired region. Because the maximum absolute errors of the control system are less than that of the RBF controller, the accuracy of the control system is higher.

Tracking the desired region the center trajectory of which is the ellipse. The experimental results are shown in Figure 4.

The desired and real trajectories are shown in Figure 4(a). Figure 4(b) and (c) shows the tracking trajectory and error of six-DOF motion, respectively.

The six-DOF tracking errors of the conventional controller based on boundary potential energy obviously exceed the respective desired error, which can be seen from Figure 4(c). The real position and angle errors of the RBF controller and the control system in this article are less than the desired tracking error, respectively. And the convergence time is very short, which can be seen from Figure 4(c).

For a clear description, in Table 2, the maximum absolute errors are presented.

In Table 2, the maximum absolute tracking errors of the conventional controller based on boundary potential energy are [21.45 m, 103.2 m, 49.50 m, 57.80 rad, 0.9246 rad, and 20.54 rad]. And its six-DOF tracking errors obviously exceed the respective desired error. That is to say, the controller based on boundary potential energy cannot reach the desired ellipse region with general uncertainty.

However, the maximum absolute tracking errors of the RBF controller and the control system in this article are [0.0089 m, 0.1051 m, 0.0056 m, 0.0051 rad, 0.0039 rad, and 0.0084 rad] and [0.0052 m, 0.1045 m, 0.0022 m, 1.9361 x 10^{-15} rad, 2.0941 x 10^{-18} rad, and 3.3509 x 10^{-11} rad], respectively. The real position errors of the
RBF controller and the control system in this article are less than 0.4 m, and the angle errors of the RBF controller and the control system in this article are less than 0.2 rad. That is to say, the RBF controller and the control system in this article can reach the desired ellipse region with general uncertainty. And in Figure 4(c), they can be stable maintained within the desired region. Because the maximum absolute errors of the control system are less than that of the RBF controller, the accuracy of the control system is higher.

To sum up, the experimental results show that when the external disturbance and modelling uncertainty exist, the controller based on boundary potential energy cannot make the underwater vehicle into the desired region, while the RBF controller and the control system in this article can make the underwater vehicle into the desired region and maintain the stability in the desired region. And the accuracy of the control system in this article is improved.

**Experiment with large initial deviation and general uncertainty**

When the large initial deviation and general uncertainty exist, tracking the desired region the center trajectory of which is the straight-line. The experimental results are shown in Figure 5.

The desired and real trajectories are shown in Figure 5(a). Figure 5(b) and (c) shows the tracking trajectory and error of six-DOF motion, respectively.

In Figure 5(c), the six-DOF tracking errors of the conventional controller based on boundary potential energy obviously exceed the respective desired error. And the lateral and longitudinal tracking errors of the RBF controller exceed the respective desired error. The real position and angle errors of the control system in this article are less than the desired tracking error. And the convergence time is about 23 s, which can be seen from Figure 5(c).

For a clear description, in Table 3, the maximum absolute errors are shown.

In Table 3, the maximum absolute tracking errors of the conventional controller based on boundary potential energy are [0.9526 m, 0.5753 m, 0.4568 m, 0.1346 rad, 0.9848 rad, and 0.4745 rad]. And its six-DOF tracking errors obviously exceed the respective desired error. That is to say, the controller based on boundary potential energy cannot reach the desired straight-line region with large initial deviation and general uncertainty.

The maximum absolute tracking errors of the RBF controller are [0.9630 m, 0.4322 m, 0.1847 m, 0.0232 rad, 0.0312 rad, and 0.0034 rad]. Its lateral and longitudinal tracking errors obviously exceed the desired error which

![Figure 4. (a to c) Experimental results with general uncertainty in ellipse tracking.](image_url)

| Maximum absolute error | X (m) | Y (m) | Z (m) | \( \Phi \) (rad) | \( \theta \) (rad) | \( \psi \) (rad) |
|------------------------|-------|-------|-------|-----------------|-----------------|-----------------|
| Desired                | 0.4   | 0.4   | 0.4   | 0.2             | 0.2             | 0.2             |
| Boundary potential energy | 21.45 | 103.2 | 49.50 | 57.80          | 0.9246          | 20.54           |
| RBF                    | 0.0089| 0.1051| 0.0056| 0.0051          | 0.0039          | 0.0084          |
| Control system         | 0.0052| 0.1045| 0.0022| 1.9361 \times 10^{-18} | 2.0941 \times 10^{-18} | 3.3509 \times 10^{-11} |

RBF: radial basis function.
is 0.4 m. That is to say, the RBF controller cannot reach the desired straight-line region with large initial deviation and general uncertainty.

However, the maximum absolute tracking errors of the proposed control system are [0.2311 m, 0.1953 m, 0.1023 m, 0.0264 rad, 0.0367 rad, and 0.0015 rad]. The real position errors of the control system in this article are less than 0.4 m, and the angle errors of the proposed control system are less than 0.05 rad. That is to say, the control system in this article can reach the desired straight-line region with large initial deviation and general uncertainty. And in Figure 5(c), it can be seen that they can be stable maintained within the desired region.

Tracking the desired region the center trajectory of which is the ellipse. The experimental results are shown in Figure 6.

### Table 3. Maximum absolute errors with large initial deviation and general uncertainty in straight-line tracking.

| Maximum absolute error | $X$ (m) | $Y$ (m) | $Z$ (m) | $\phi$ (rad) | $\theta$ (rad) | $\psi$ (rad) |
|------------------------|---------|---------|---------|--------------|---------------|--------------|
| Desired                | 0.4     | 0.4     | 0.4     | 0.05         | 0.05          | 0.05         |
| Boundary potential energy | 0.9526  | 0.5753  | 0.4568  | 0.1346       | 0.9848        | 0.4745       |
| RBF Control system     | 0.9630  | 0.4322  | 0.1847  | 0.0232       | 0.0312        | 0.0034       |

RBF: radial basis function.
The desired and real trajectories are shown in Figure 6(a). Figure 6(b) and (c) shows the tracking trajectory and error of six-DOF motion, respectively.

The six-DOF tracking errors of the conventional controller based on boundary potential energy obviously exceed the respective desired error, which can be seen from Figure 6(c). And the lateral, vertical, yaw, and pitch tracking errors of the RBF controller exceed the respective desired error. The real position and angle errors of the control system in this article are less than the desired tracking error. And the convergence time is very short, which can be seen from Figure 6(c).

For a clear description, in Table 4, the maximum absolute errors are shown.

In Table 4, the maximum absolute tracking errors of the conventional controller based on boundary potential energy are [212.3 m, 386.4 m, 457.9 m, 1753 rad, 1.225 rad, and 46.35 rad]. And its six-DOF tracking errors obviously exceed the respective desired error. That is to say, the controller based on boundary potential energy cannot reach the desired ellipse region with large initial deviation and general uncertainty.

The maximum absolute tracking errors of the RBF controller are [0.5739 m, 0.3826 m, 0.4397 m, 0.0367 rad, 1.742 rad, and 46.84 rad]. Its lateral, vertical, yaw, and pitch tracking errors obviously exceed the desired error which is 0.4 m. That is to say, the RBF controller cannot reach the desired ellipse region with large initial deviation and general uncertainty.

However, the maximum absolute tracking errors of the proposed control system are [0.0528 m, 0.0254 m, 0.0423 m, 0.0137 rad, 0.0203 rad, and 0.1339 rad]. The real position errors of the conventional control system are less than 0.4 m, and the angle errors of the control system in this article are less than 0.2 rad. That is to say, the control system in this article can reach the desired ellipse region with large initial deviation and general uncertainty. And in Figure 6(c), they can be stable maintained within the desired region.

To sum up, the experimental results show that when the large initial deviation and general uncertainty exist, neither the controller based on boundary potential energy nor the RBF controller can make underwater vehicles into the desired region, while the control system in this article can make underwater vehicles into the desired region and maintain the stability in the desired region.

Conclusions
In this study, the region tracking control system of underwater vehicles is designed in the presence of large initial deviation and general uncertainty. The design of the region tracking control system combines the design method of state feedback controller with the RBF neural network well. Some problems such as controller parameter overadjustment, general uncertainty, and large initial deviation are effectively avoided. And the tracking accuracy of the control system is slightly improved compared with that of the conventional RBF controller. Therefore, the control system has increased the practical feasibility and stability of region tracking control. The stability of the region tracking control system is proved through Lyapunov function. Finally, it is fully compared with the controllers based on boundary potential energy and RBF to further prove the superiority of the control system with large initial deviation and general uncertainty.

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