Update of the Three-fluid Hydrodynamics-based Event Simulator: light-nuclei production in heavy-ion collisions

M. Kozhevnikova,1 Yu. B. Ivanov,2,3,4 Iu. Karpenko,5 D. Blaschke,6,2,3 and O. Rogachevsky1

1 Veksler and Baldin Laboratory of High Energy Physics, JINR Dubna, 141980 Dubna, Russia
2 Bogoliubov Laboratory of Theoretical Physics, JINR Dubna, 141980 Dubna, Russia
3 National Research Nuclear University "MEPhI", 115409 Moscow, Russia
4 National Research Centre "Kurchatov Institute", 123182 Moscow, Russia
5 Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague, 11519 Prague 1, Czech Republic
6 Institute of Theoretical Physics, University of Wroclaw, 50-204 Wroclaw, Poland

We present an update of the event generator based on the three-fluid dynamics (3FD), complemented by Ultra-relativistic Quantum Molecular Dynamics (UrQMD) for the late stage of the nuclear collision – the Three-fluid Hydrodynamics-based Event Simulator Extended by UrQMD (THESEUS). Two modifications are introduced. The THESEUS table of hadronic resonances is made consistent with that of the underlying 3FD model. The main modification is that the generator is extended to simulate the light-nuclei production in relativistic heavy-ion collisions, on the equal basis with hadrons. These modifications are illustrated by applications to the description of available experimental data. The first run of the updated generator revealed a surprisingly good reproduction of the NA49 data on the light nuclei. The reproduction is achieved without any extra parameters, while the coalescence approach in 3FD requires special tuning of the coalescence coefficients for each light nucleus separately.

PACS numbers: 25.75.-q, 25.75.Nq, 24.10.Nz
Keywords: relativistic heavy-ion collisions, hydrodynamics

I. INTRODUCTION

In Ref. [1] the THESEUS event generator was presented and its possible applications to the description of heavy-ion collisions were demonstrated. The THESEUS is based on the 3FD model [2, 3] extended by UrQMD [4, 5] for the afterburner stage. The 3FD was designed to simulate heavy-ion collisions at moderately relativistic energies of the Beam Energy Scan program (BES) at the Relativistic Heavy-Ion Collider (RHIC) at the Brookhaven National Laboratory (BNL) [6], CERN Super-Proton-Synchrotron (SPS) [7], the Facility for Antiproton and Ion Research (FAIR) in Darmstadt [8] and the Nuclotron-based Ion Collider Facility (NICA) in Dubna [9]. Precisely in this energy range the onset of deconfinement is expected [3].

The 3FD approximation is a minimal way to simulate the early, nonequilibrium stage of the produced strongly-interacting matter. It takes into account counterstreaming of the leading baryon-rich matter at the early stage of nuclear collisions [2]. This nonequilibrium stage is modeled by the means of two counterstreaming baryon-rich fluids, which are initially associated with the constituent nucleons of the projectile (p) and the target (t) nuclei. Later, these fluids may consist of any type of constituents rather than only nucleons. Newly produced particles, which dominantly populate the midrapidity region, are associated with a fireball (f) fluid. Each of these fluids is governed by conventional hydrodynamic equations coupled by friction terms in the right-hand sides of the Euler equations. These friction terms describe the energy–momentum exchange between the fluids.

Different equations of state (EoS) can be applied within the 3FD model. The recent series of 3FD simulations were based on three different types of the EoS: a purely hadronic EoS [10] (hadr. EoS) and two versions of the EoS with deconfinement [11], i.e. an EoS with a first-order phase transition (1PT EoS) and one with a smooth crossover transition (crossover EoS). Analysis of available experimental data indicated a strong preference of the deconfinement scenarios with the deconfinement already at moderately relativistic energies \( \sqrt{s_{NN}} \geq 5 \text{ GeV} \) [12].

When the system becomes dilute, the 3FD evolution is stopped and the system is frozen out. The local freeze-out criterion used in the 3FD model is \( \varepsilon < \varepsilon_{\text{frz}} \), where \( \varepsilon \) is the local energy density of all three fluids in their common rest frame and \( \varepsilon_{\text{frz}} \) is the freeze-out energy density serving as a freeze-out criterion. More details can be found in Refs. [13, 14]. The output of the model is recorded in terms of Lagrangian test particles (i.e. fluid droplets) for each fluid \( \alpha = [p, t \text{ or } f] \), which are characterized by a local flow velocity and thermodynamic quantities.

The original 3FD model still needs a certain refinement. An afterburner stage that can play an important role for some observables is absent in the model. From the practical point of view, the model is not well suited...
for data simulations in terms of experimental events, because the model output consists of fluid characteristics rather than of a set of observable particles since the 3FD observables are computed directly via the integrals over the freeze-out hypersurface.

The event generator THESEUS developed in Ref. [1], the corresponding TABLE I. List of hadrons incorporated in the THESEUS of Ref. [1], i.e. THESEUS-v1. The resonances used in 3FD simulations [2] are marked by bold font.

| light mesons | flavored mesons | N and Δ baryons | flavored baryons |
|--------------|-----------------|-----------------|-----------------|
| π            | K               | N               | Δ               |
| η            | K* (892)        | N* (1520)       | Δ* (1520)       |
| η′ (958)     | K* (1410)       | N* (1675)       | Δ* (1675)       |
| ρ(770)       | K1 (1270)       | N1 (1535)       | Δ1 (1535)       |
| ω(782)       | K1 (1400)       | N1 (1650)       | Δ1 (1650)       |
| φ (1020)     | K (1460)        | N (1710)        | Δ (1710)        |
| χ (1170)     | K2 (1580)       | N (1720)        | Δ (1720)        |
| χ (1170)     | K2 (1580)       | N (1720)        | Δ (1720)        |
| h (1123)     | K1 (1650)       | N (1920)        | Δ (1920)        |
| a (1260)     | K* (1680)       | Δ (1680)        | Σ               |
| f (1270)     | K2 (1770)       | Δ (1700)        | Σ (1700)        |
| χ (1285)     | K2 (1820)       | Δ (1700)        | Σ (1835)        |
| f (1420)     | K3 (2320)       | Δ (1950)        | Σ (1940)        |
| ...          | ...             | ...             | ...             |
| f (1430)     | D               | Ξ               |                 |
| ...          | ...             | ...             | ...             |
| η (1475)     | D* (2400)       | Ω               |                 |
| ...          | ...             | ...             | ...             |
| f (2340)     | Υ (11020)       | Ξb              |                 |

TABLE I. List of hadrons incorporated in the THESEUS of Ref. [1], i.e. THESEUS-v1. The resonances used in 3FD simulations [2] are marked by bold font.

longer than that incorporated in the 3FD. The hadronic states in the 3FD are marked with bold font in Tab. [1]. All the 3FD hadrons have well-known decay modes [15]. Only hadrons are displayed in Tab. [1] the corresponding anti-hadrons (where applicable) are implied. Different isotopic states of the listed hadrons are also implied but not explicitly displayed.

The 3FD list is used for the hadron-gas EoS in terms of which all the freeze-out densities (baryon, strange and energy ones) are transformed into the corresponding baryon, strange chemical potentials and temperature. In the 3FD model, all the available energy and baryon number is distributed between the hadrons from the 3FD hadron list, except for the Ξ and Ω hyperons. The latter two are just calculated with the deduced chemical potentials and temperature, and do not participate in the balance of conserved quantities because of their negligible multiplicity as compared with that of other strange hadrons included in the 3FD table. Any additional (to the 3FD table) hadronic resonance, sampled in THESEUS, brings an excess energy and baryon charge with respect to the 3FD baseline. This excess violates the energy, baryon number and strangeness conservation. If we use the original THESEUS-v1 table of hadronic resonances, it overestimates the total energy and baryon charge and produces non-zero net strangeness with respect to the pure 3FD calculation. Therefore, in the updated version, i.e. THESEUS-v2, we reduce the list of hadronic resonances precisely to the 3FD one in order for the average energy, strangeness and baryon charge of the produced events to be consistent with the total energy, strangeness and baryon charge of the underlying fluids in 3FD.

In principle, an alternative way to correct the conservation issue is possible. The local temperature, baryon and strange chemical potentials, provided by the 3FD input, could be recalculated proceeding from the respective conservation laws in the extended table of hadronic resonances. This alternative way also has certain shortcomings. We would have to deal with a large number of resonances with not well-known decay modes. This would result in some uncertainties in the numbers and spectra of produced stable particles. Though, we expect that the final yields and spectra of produced stable particles would only insignificantly change as compared to the 3FD case because approximately the same number stable particles (because of baryon-number, strangeness and energy conservation) will originate from a larger variety of primordial resonances. Of course, the above-mentioned uncertainties in decay modes of heavy resonances would also contribute to the change of these numbers and spectra.

Fig. [1] demonstrates the effect of the excess baryon charge, pronounced in the proton rapidity distribution. The weak-decay protons are included, consistently with the STAR data point [10], which is overlaid on the plot. One can see that the original extended THESEUS table of hadronic resonances (THESEUS-v1 without UrQMD) overshoots the original 3FD distribution by approximately 10%. The 3FD table of hadronic resonances (THESEUS-v2 without UrQMD) leads to a good reproduction of the 3FD result. The effect of the UrQMD...
FIG. 1. Rapidity distribution of protons in central \( (b = 2 \text{ fm}) \) \( \text{Au+Au} \) collisions at a collision energy of \( \sqrt{s_{NN}} = 9.2 \) GeV calculated with the crossover EoS \( [11] \): the 3FD result (short-dashed line), the result of \( \text{THESEUS-v1} \) without UrQMD (dotted line), the result of \( \text{THESEUS-v2} \) without UrQMD (long-dashed line) and the result of \( \text{THESEUS-v2} \) with UrQMD (solid line). Experimental data are from the STAR collaboration \( [16] \).

FIG. 2. The same as in Fig. 1 but for positive pions (excluding the weak-decay feed-down).

afterburner for protons turns out to be small, which one can see from \( \text{THESEUS-v2} \) curve in Fig. 1.

Similar results for pions are demonstrated in Fig. 2. The weak-decay contributions to the pion spectra are excluded, to be again consistent with the STAR data \( [10] \). As seen from Fig. 2, even the \( \text{THESEUS-v2} \) result differs from the 3FD one in spite of the identical lists of resonances. This is a consequence of different decay branching fractions of hadronic resonances in the \( \text{THESEUS-v2} \) and 3FD codes. The data on branching fractions collected by the Particle Data Group (PDG) for their Review of Particle Physics, are corrected in each edition thereof. Moreover, the branching ratios are experimentally known with an accuracy of 20–40%, as a rule, even for well-established decay modes. This concerns also the probabilities of decays with one and two final pions, which are of prime importance for the rapidity distributions. Therefore, implementations of the branching data differ among different models. In practice, the decay channels and their branching fractions for \( \text{THESEUS} \) are taken from the EPOS3 code \( [17] \). Therefore, somewhat different decay channels the \( \text{THESEUS-v2} \) and 3FD codes result in the difference displayed in Fig. 2. We have checked that the thermal pion distributions are identical in the \( \text{THESEUS-v2} \) and 3FD calculations. Moreover, the UrQMD afterburner only slightly changes the rapidity distribution of pions, similarly to that of protons.

The results for positive and negative kaons are presented in Figs. 3 and 4, respectively. The effects of the resonance table on \( K^+ \) and \( K^- \) are different in relative values. However, they are equal in absolute values because of strangeness conservation. \( \text{THESEUS-v2} \) without UrQMD perfectly reproduces the 3FD results. The UrQMD afterburner stronger affects kaons than protons and pions. In particular, it strongly reduces the \( K^- \) midrapidity density because of strong absorption of \( K^- \).
in reactions of the type $K^- + n \rightarrow \Lambda + \pi^-$. The effect of the resonance table is stronger at high collision energies because a larger number of hadronic resonances beyond the 3FD list gets noticeably excited. At $\sqrt{s_{NN}} < 5$ GeV the effect of the resonance table is negligible, it amounts to less than 1% in terms of the midrapidity proton density.

III. LIGHT NUCLEI

An important modification consists in inclusion of light nuclei in the list of particles: deuterons ($d$), tritons ($t$), helium isotopes $^3$He and $^4$He, and low-lying resonances of the $^4$He system, the decays of which contribute to the yields of stable species \cite{18}, see Tab. II. The momenta distributions of these nuclei are sampled similarly to those of other hadrons, i.e. according to their phase-space distribution functions at given flow velocity, chemical potentials and temperature. However, the sampling cannot be done straightforward, proceeding from the 3FD input.

The 3FD and, thereby, the THESEUS-v1 calculate spectra of the so-called primordial nucleons, i.e. both observable nucleons and those bound in the light nuclei. Therefore, the nucleons bound in the light nuclei should be subtracted from the primordial ones in order to obtain the observable nucleons which can be compared with data. Note that this subtraction is done in the 3FD, where the spectra of the light nuclei are calculated within the coalescence approach \cite{2, 19} rather than the statistical one. Therefore, in order to implement the thermodynamic approach of nuclei formation, we first should re-calculate the baryon chemical potential, proceeding from the local baryon number conservation

$$n_{\text{primordial}}(x; \mu_B, T) + \sum_{\text{hadrons}} n_i(x; \mu_B, \mu_S, T)$$
$$= n_{\text{observable}}(x; \mu'_B, T) + \sum_{\text{hadrons}} n_i(x; \mu'_B, \mu_S, T) + \sum_{\text{nuclei}} n_c(x; \mu'_B, \mu_S, T). \quad (1)$$

The sum over “hadrons” runs over the list of hadrons incorporated in the 3FD (bold entries in Tab. I) excluding the nucleon ($N$), the density of which is presented explicitly. The sum over “nuclei” runs over the list of light nuclei in Tab. II $n_i$ and $n_c$ are the local baryon densities of $i$-th species of hadron and $c$-th nucleus respectively, which depend on the local baryon ($\mu_B, \mu'_B$) and strange ($\mu_S$) chemical potentials and the temperature ($T$) at the freeze-out hypersurface. $\mu_B$ is the baryon chemical potential in terms of the primordial nucleons provided by the 3FD input, $\mu'_B$ is that in terms of the observable nucleons.

In fact, the recalculation of the baryon chemical potential also affects the energy and strangeness conservations. However, we do not additionally tune $\mu_S$ and $T$, because these conservations turn out to be quite good already: the total energy is conserved with the accuracy of 3% and on average a few units of net strangeness are gained in the sampling for the central Au+Au collision at $\sqrt{s_{NN}} = 2.7$ GeV. The total strangeness is not well conserved. However, it should be kept in mind that the strangeness production in the 3FD is poorly defined at this low energy. The strangeness produced at this collision energy should be reduced by a factor of $\gamma_S \approx 0.2$ \cite{12}, which accounts for additional strangeness suppression due to constraints of the canonical ensemble \cite{20}. This is the lowest explored collision energy, at which there is the largest fraction of nucleons bound in light nuclei. With the increase of collision energy the accuracy of the energy and strangeness conservation becomes better. For central Pb+Pb collisions at $E_{\text{lab}} = 20$ A GeV, the energy is conserved within 1%. Note that the coalescence implemented in the 3FD also reduces the total energy of the system because of a reduction of the number of degrees of freedom.

Results of such simulations within the THESEUS-v2 are demonstrated in Fig. 5 for central ($b = 2$ fm) Au+Au collisions at a collision energy of $\sqrt{s_{NN}} = 2.7$ GeV ($E_{\text{lab}} = 2$ A GeV in notation of E895 \cite{21}) in the crossover EoS scenario, which corresponds to the curve labeled ‘THESEUS-v2’. In order to illustrate the effect of the

| Nucleus($E$[MeV]) | $J$ | decay modes, in % |
|------------------|----|------------------|
| $d$              | 1  | Stable           |
| $t$              | 1/2| Stable           |
| $^3$He           | 1/2| Stable           |
| $^4$He           | 0  | Stable           |
| $^4$He(20.21)    | 0  | $p = 100$        |
| $^4$He(21.01)    | 0  | $n = 24, p = 76$ |
| $^4$He(21.84)    | 2  | $n = 37, p = 63$ |
| $^4$He(23.33)    | 2  | $n = 47, p = 53$ |
| $^4$He(23.64)    | 1  | $n = 45, p = 55$ |
| $^4$He(24.25)    | 1  | $n = 47, p = 50, d = 3$ |
| $^4$He(25.28)    | 0  | $n = 48, p = 52$ |
| $^4$He(25.95)    | 1  | $n = 48, p = 52$ |
| $^4$He(27.42)    | 2  | $n = 3, p = 3, d = 94$ |
| $^4$He(28.31)    | 1  | $n = 47, p = 48, d = 5$ |
| $^4$He(28.37)    | 1  | $n = 2, p = 2, d = 96$ |
| $^4$He(28.39)    | 2  | $n = 0.2, p = 0.2, d = 99.6$ |
| $^4$He(28.64)    | 0  | $d = 100$        |
| $^4$He(28.67)    | 2  | $d = 100$        |
| $^4$He(29.89)    | 2  | $n = 0.4, p = 0.4, d = 99.2$ |

TABLE II. Stable light nuclei and low-lying resonances of the $^4$He system (from BNL properties of nuclides \cite{5}). $J$ denotes the total angular momentum. The last column represents branching ratios of the decay channels, in per cents. The $p, n, d$ correspond to the emission of proton, neutron, or deuteron, respectively.

\footnotetext[4]{https://www.nndc.bnl.gov/nudat2/getdataset.jsp?nucleus=$4$He&unc=nds}
FIG. 5. Rapidity distributions of deuterons ($d$), tritons ($t$), $^3$He, $^4$He and protons in central ($b = 2$ fm) Au+Au collisions at collision energy of $\sqrt{s_{NN}} = 2.7$ GeV ($E_{\text{lab}} = 2A$ GeV) within the crossover scenario [11] calculated by means of: the 3FD with coalescence (short-dashed line), the THESEUS-v1 (dotted line), i.e. without recalculation of the baryon chemical potential but with the 3FD table of resonances, THESEUS-v2 without excited states of $^4$He (long-dashed line), and THESEUS-v2 (solid line). The proton data are from Ref. [21].

FIG. 6. The same as in Fig. 5 but for central ($b = 3$ fm) Pb+Pb collisions at a collision energy of $E_{\text{lab}} = 20A$ GeV. Experimental data are from the NA49 collaboration [23].

performed modifications, we also show the results without recalculation of the baryon chemical potential and those with recalculation but without contributions from the excited states of $^4$He. As one can see, the recalculation of the baryon chemical potential leads to a considerable reduction of the light-nuclei yields. The inclusion of the excited states of $^4$He also noticeably affects the light-nuclei yields.

The results of the 3FD calculation within the coalescence approach are also presented in Fig. 5. As the light-nuclei data for this reaction are absent, the coalescence coefficients in the 3FD were chosen solely on the condition that the proton result agrees with the E895 data [21] after subtraction of the light-nuclei contribution from the yield of the primordial protons. In practice, the coalescence coefficients are taken from Ref. [22] and are scaled to reproduce the observed proton rapidity distribution, with their ratios fixed. Therefore, a detailed comparison of THESEUS-v2 results with those of the 3FD is meaningful only for protons. Although the experimental data on the light nuclei are unavailable, we can conclude that the light-nuclei yield predicted by THESEUS-v2 is reasonable because the resulting proton rapidity distribution agrees well with the E895 data [21]. The agreement is achieved without any fitting (or extra) parameters, contrary to the coalescence approach in the 3FD.

Figure 6 shows similar results at a higher collision energy of $\sqrt{s_{NN}} = 6.4$ GeV ($E_{\text{lab}} = 20A$ GeV in notation of NA49 [23]) for central ($b = 3$ fm) Pb+Pb collisions. The shown protons do not include contributions from the weak decays. These results are confronted to the experimental data on the light-nuclei production by the NA49 collaboration [23]. The impact parameter of the simulation corresponds to the centrality selection of the data. The reproduction of the data turns out to be surprisingly good. Here the multiplicities of produced light nuclei are considerably lower than those at the low energy considered above. The readjustment of the baryon
chemical potential strongly affects this result as well. At the same time, the effect of the excited states of $^4$He becomes weak. The results of the 3FD coalescence calculation from Ref. [19] are shown as well. It is worthwhile to mention that the THESEUS-v2 simulation reproduces the data [23] well without any fit parameters, whereas the 3FD coalescence requires a special tuning of the coalescence coefficients for each light nucleus separately.

**IV. ISOTOPIC CONTENT OF PRODUCED HADRONS**

In the 3FD model, particles are not isotopically distinguished, i.e. the model deals with nucleons, pions, etc. rather than with protons, neutrons, $\pi^+$, $\pi^0$, $\pi^-$, etc. Therefore, all quantities proportional to the number of protons are calculated for nucleons (after decays of all resonances) and then are scaled with the factor $(Z_p + Z_t)/(A_p + A_t)$, where $Z_\alpha$ is the charge of the colliding nucleus ($\alpha = $ projectile or target) and $A_\alpha$ is its mass number. This is just an approximate recipe to estimate the difference between proton and neutron yields. Other species (pions, kaons, etc.) are equally distributed between isotopic states, e.g., numbers of pions and kaons are $N_{\pi^+} = N_{\pi^-} = N_{\pi^0} = N_\pi/3$ and $N_{K^+} = N_{K^-} = N_K/2$, respectively.

THESEUS distinguishes different isospin states in the multiplets. However, by default it simulates isospin-symmetric matter because no information on its isotopic content is available from the 3FD. Nevertheless, even in isospin-symmetric matter the multiplicities of $\pi^\pm$ and $\pi^0$ and those of $K^\pm$ and $K^0$ differ because of the small differences in their masses [15] which are taken into account in THESEUS. If the weak decays are allowed, which is of interest in THESEUS-v2, the difference between $\pi^\pm$ and $\pi^0$ becomes larger. The figures 2 and 3 are produced precisely this way. The above points, of course, do not exhaust the full list of reasons resulting in the isotopic difference. This prescription is a good approximation at high collision energies, while at low energies it ignores the difference in yields of different isotopic species.

For the protons, THESEUS-v2 uses a slightly improved 3FD recipe. The fraction of protons is calculated as

$$R_{\text{proton}} = \frac{Z_{\text{participants}} - N_t - N_i - 2N_{^3\text{He}} - 2N_{^4\text{He}}}{B_{\text{participants}} - 2N_t - 3N_i - 3N_{^3\text{He}} - 4N_{^4\text{He}}}$$

where $B_{\text{participants}}$ and $Z_{\text{participants}} = B_{\text{participants}}(Z_1 + Z_2)/(A_1 + A_2)$ are the total baryon number and electrical charge of participants, $N_{\text{nucleus}}$ is the multiplicity of the produced light nucleus. $B_{\text{participants}}$ is calculated within THESEUS-v2. Eq. (2) takes into account that the fraction of observed protons is changed if some protons are bound in light nuclei. The figures 1, 5 and 6 are produced precisely this way.

The numbers of originally generated tritons and $^3$He isotopes, $N_i$ and $N_{^3\text{He}}$, are equal because of the above mentioned isotopic symmetry of the 3FD input. Therefore, these numbers, $N_i$ and $2N_{^3\text{He}}$, are also scaled with the factors $(N_p + N_t)/(A_p + A_t)$ and $(Z_p + Z_t)/(A_p + A_t)$, respectively, in order to take into account the initial isotopic imbalance of colliding nuclei.

**V. SUMMARY**

An update of the event generator THESEUS – THESEUS-v2 is presented. We benchmark the update in Au-Au collisions at $\sqrt{s_{NN}} = 9.2$ and 2.7 GeV and Pb-Pb collisions at $\sqrt{s_{NN}} = 6.4$ GeV.

The first modification concerns the THESEUS table of hadronic resonances, which is made consistent with that of the underlying 3FD model. It is done in order to exactly conserve the energy, strangeness and baryon number. The effect of the resonance table is stronger at higher collision energies because a larger number of hadronic resonances beyond the 3FD list are noticeably excited. At $\sqrt{s_{NN}} < 5$ GeV the effect of the resonance table is negligible.

The production of light nuclei, i.e. deuterons ($d$), tritons ($t$), helium isotopes $^3$He and $^4$He, and low-lying resonances of the $^4$He, is implemented in the generator. The low-lying $^4$He resonances decay into nucleons and lighter nuclei, thereby contributing to their total yields. The yields and momenta of the light nuclei are sampled in the thermodynamic approach, similarly to those of hadrons, i.e. according to the local equilibrium distribution functions with a given flow velocity, temperature and chemical potentials. Note that within the 3FD the spectra of the light nuclei are calculated within the coalescence approach rather than the thermodynamic one. Therefore, a recalibration of the baryonic chemical potentials, provided by the 3FD output, was performed. It was demonstrated that the recalibration is very important for the light-nuclei production in the thermodynamic approach. The effect of the decays of low-lying resonances of the $^4$He is large at low collision energies and gradually dies out with increasing energy.

The first application of THESEUS-v2 revealed a surprisingly good reproduction of the NA49 data [23] on light nuclei. This reproduction was achieved without any extra (fit) parameters, while the 3FD coalescence [19] required a tuning of the coalescence coefficients for each light nucleus separately.

At present, there are several 3D dynamical models which include the coalescence mechanism of the light-nuclei production [2, 19, 24, 28], see also a recent review [29]. The recently developed transport models SMASH (Simulating Many Accelerated Strongly-interacting Hadrons) [30, 31] and PHQMD (Parton-Hadron-Quantum-Molecular-Dynamics) [32], treat light nuclei microscopically (so far, only deuterons in the SMASH [31]) on an equal footing with other hadrons. THESEUS-v2 incorporates the thermodynamic approach in the 3FD model, which is an alternative to the above.
approaches. We expect that all these models will serve as complementary approaches to study the light-nuclei production at collision energies of the BES-RHIC, SPS, NICA and FAIR.

ACKNOWLEDGMENTS

This work was partially supported by the Russian Foundation for Basic Research (RFBR) under Grant Nos. 18-02-40084, 18-02-40085 (Y.B.I.) and 18-02-40137 (D.B.). D.B., Y.B.I., M.K. and O.R. acknowledge support through the Russian Science Foundation under project No. 17-12-01427. D.B. and Y.B.I. received support from the Ministry of Education and Science of the Russian Federation within the Academic Excellence Project of the NRNU MEPhI under contract No. 02.A03.21.0005. I.K. acknowledges support by the project Centre of Advanced Applied Sciences with number CZ.02.1.01/0.0/0.0/16-019/0000778, which is co-financed by the European Union.

[1] P. Batyuk et al., Phys. Rev. C 94, 044917 (2016) [arXiv:1608.00965 [nucl-th]].
[2] Y. B. Ivanov, V. N. Russkikh and V. D. Toneev, Phys. Rev. C 73, 044904 (2006) [nucl-th/0503088].
[3] Y. B. Ivanov, Phys. Rev. C 87, no.6, 064904 (2013) [arXiv:1302.5766 [nucl-th]].
[4] S. A. Bass, R. Mattiello, H. Stöcker, W. Greiner and C. Hartnack, Phys. Lett. B 302, 381 (1993).
[5] S. A. Bass et al., Prog. Part. Nucl. Phys. 41, 255 (1998) [nucl-th/9803035].
[6] G. S. F. Stephans, J. Phys. G 32, S447 (2006) [nucl-ex/0607030].
[7] M. Gazdzicki [NA49 and NA61/SHINE Collaborations], J. Phys. G 38, 124024 (2011) [arXiv:1107.2345 [nucl-ex]].
[8] T. Abyzayimov et al. [CBM], Eur. Phys. J. A 53, no.3, 60 (2017) [arXiv:1607.01487 [nucl-ex]].
[9] V. D. Kekelidze, V. A. Matveev, I. N. Meshkov, A. S. Sorin and G. V. Trubnikov, Phys. Part. Nucl. 48, no.5, 727 (2017).
[10] V. M. Galitsky and I. N. Mishustin, Yad. Fiz. 29, 363 (1979).
[11] A. S. Khvorostukin, V. V. Skokov, V. D. Toneev and K. Redlich, Eur. Phys. J. C 48, 531 (2006) [nucl-th/0605069].
[12] Y. B. Ivanov, Phys. Rev. C 87, no.6, 064905 (2013) [arXiv:1304.1638 [nucl-th]].
[13] V. N. Russkikh, Yu. B. Ivanov, Phys. Rev. C 76, 054907 (2007) [nucl-th/0611094].
[14] Yu. B. Ivanov, V. N. Russkikh, Yad. Fiz. 72, 1288-1294 (2009) [arXiv:0810.2262 [nucl-th]].
[15] P.A. Zyla et al. [Particle Data Group], PTEP 2020, no.8, 083C01 (2020).
[16] B. I. Abelev et al. [STAR], Phys. Rev. C 81, 024911 (2010) [arXiv:0909.4131 [nucl-ex]].
[17] K. Werner, B. Guiot, I. Karpenko and T. Pierog, Phys. Rev. C 89, no.6, 064903 (2014) [arXiv:1312.1233 [nucl-th]].
[18] E. Shuryak and J. M. Torres-Rincon, Phys. Rev. C 101, no.3, 034914 (2020) [arXiv:1910.08119 [nucl-th]].
[19] Y. B. Ivanov and A. A. Soldatov, Eur. Phys. J. A 53, no.11, 218 (2017) [arXiv:1703.05040 [nucl-th]].
[20] P. Koch, B. Müller and J. Rafelski, Phys. Rept. 142, 167-262 (1986).
[21] J. L. Klay et al. [E-0895], Phys. Rev. C 68, 054905 (2003) [arXiv:nucl-ex/0306033 [nucl-ex]].
[22] V. N. Russkikh, Y. B. Ivanov, Y. E. Pokrovsky and P. A. Hemming, Nucl. Phys. A 572, 749-790 (1994).
[23] T. Anticic et al. [NA49 Collaboration], Phys. Rev. C 94, no.4, 044906 (2016) [arXiv:1606.04234 [nucl-ex]].
[24] H. Liu, D. Zhang, S. He, K. j. Sun, N. Yu and X. Luo, Phys. Lett. B 805, 135-152 (2020) [arXiv:1909.09304 [nucl-th]].
[25] L. Zhu, C. M. Ko and X. Yin, Phys. Rev. C 92, no.6, 064911 (2015) [arXiv:1510.05368 [nucl-th]].
[26] Z. J. Dong, G. Chen, Z. L. She, Y. L. Yan, F. X. Liu, D. M. Zhou and B. H. Sa, Eur. Phys. J. A 54, no.9, 144 (2018) [arXiv:1803.01547 [nucl-th]].
[27] S. Sombun, K. Tomuang, A. Limphirat, P. Hillmann, C. Herold, J. Steinheimer, Y. Yan and M. Bleicher, Phys. Rev. C 99, no.1, 014901 (2019) [arXiv:1805.11509 [nucl-th]].
[28] W. Zhao, C. Shen, C. M. Ko, Q. Liu and H. Song, Phys. Rev. C 102, no.4, 044912 (2020) [arXiv:2009.06959 [nucl-th]].
[29] D. Olinichenko, [arXiv:2003.05476 [hep-ph]].
[30] J. Weil, V. Steinberg, J. Staudenmaier, L. G. Pang, D. Olinichenko, J. Mohs, M. Kretz, T. Kehrenberg, A. Goldschmidt and B. Bächle, et al. Phys. Rev. C 94, no.5, 054905 (2016) [arXiv:1606.06642 [nucl-th]].
[31] D. Olinichenko, L. G. Pang, H. Ellner and V. Koch, Phys. Rev. C 99, no.4, 044907 (2019) [arXiv:1809.03071 [hep-ph]].
[32] J. Aichelin, E. Bratkovskaya, A. Le Fèvre, V. Kireyev, V. Kolesnikov, Y. Leifels, V. Voronyuk and G. Coci, Phys. Rev. C 101, no.4, 044905 (2020) [arXiv:1907.03860 [nucl-th]].