Analysis of Halbach Permanent Magnet Array with Convex Type Section

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Abstract. This paper proposes Halbach permanent magnet arrays with convex type section. The magnetic field is modeled according to Ampere’s hypothesis of molecular current. Based on the model, the fundamental component of the magnetic flux density and the sinusoidal distortion rate are selected as evaluation indexes. Compared with these of Halbach permanent magnet array with normal square section, the Halbach permanent magnet arrays with specific configuration own better performance, which provide direction for further optimization.

1. Introduction
Klaus Halbach developed a single-sided rare-earth magnet array for use in undulator and particle accelerators [1]. Fig. 1 shows such a square Halbach magnet array. This type of magnet array differs from conventional arrays in that each adjacent magnet segment is rotated around an axis perpendicular to the direction in which the array extends by a predetermined angle. Such a linear Halbach array has \(\sqrt{2}\) times stronger field than that of a conventional ironless magnet array with the same volume. And the magnetic field distribution is nearly sinusoidal, as shown in Fig. 2.

Figure 1. Linear Halbach magnet array with square section.
Figure 2. Magnetic flux density of Halbach magnet array with square section.

The blue line in Fig. 2 represents the magnetic field distribution, and the red line represents the standard sine wave. There exist differences between these two lines. Magnetic field distribution in standard sine wave is of vital importance in ultra-precision servo systems, but it can only be obtained in ideal Halbach array whose magnetization direction varies continuously sinusoidal along the array. Obviously ideal Halbach array can’t be manufactured in reality, so researches on Halbach arrays with different configurations are carried on. In this paper, linear permanent magnet Halbach arrays with convex type section in different size are proposed. The magnetic flux density of the Halbach arrays are modeled according to Ampere’s hypothesis of molecular current [2-3], and are analyzed by evaluating the fundamental component of the magnetic flux density and the sinusoidal distortion rate [4-6].
2. Halbach Arrays with Convex Type Section

Fig. 3 shows one permanent magnet segment with convex type section. Its size are labeled as $a$–$e$, which satisfy

$$a + 2c = e$$  \hspace{1cm} (1) \hspace{1cm}

Figure 3. Permanent magnet segment with convex type section.

When $e$ is fixed, $a$ and $c$ can be chosen differently, and the cross section shape change accordingly. In this paper, five permanent magnet segments compose one magnetic period, and the magnetic period length $l$ is chosen as 24mm [7]. To find out the tendency, three different size of the magnet segment are selected, as shown in Tab. 1. And these are called $a3c3$, $a5c1$ and $a1c5$, in which $b=d=3\text{mm}$.

| Dimension(mm) Type | a3c3 | a5c1 | a1c5 |
|--------------------|------|------|------|
| $a$                | 3    | 5    | 1    |
| $c$                | 3    | 1    | 5    |

Fig. 4 shows the schematic diagram of three types of magnet segments and the Halbach magnet arrays each type composed. Their total length is unequal since the different $e$. Type $a5c1$ owns the shortest total length, while $a1c5$ the longest.

Figure 4. Schematic diagram of magnet segment and magnet array.
3. Modeling of Halbach Array

According to Ampere’s hypothesis of molecular current, the magnetic field created by permanent magnet segment is equivalent to that excited by surface current. Based on the above theory, analytic coordinate system can be constructed, as shown in Fig. 5, the origin of coordinate is located in the center of the permanent magnet.

![The equivalent circuit model.](image)

The permanent magnet segment with positive y direction magnetized can be equivalent to surface current $I_1$ and $I_\pi$, as shown in Fig. 5. The magnetic field produced by surface current $I_1$ and $I_\pi$ at point $P(x, y)$ can be expressed as (2) and (3).

$$B_{I_1}(x, y, k_v) = \frac{\mu_0 k_v}{4\pi} \ln \frac{(y + d)^2 + \left(\frac{x - e}{2}\right)^2}{y^2 + \left(\frac{x - e}{2}\right)^2} + \ln \frac{y^2 + \left(\frac{x - e}{6}\right)^2}{(y - d)^2 + \left(\frac{x - e}{6}\right)^2}$$

$$B_{I_\pi}(x, y, k_v) = \frac{\mu_0 k_v}{2\pi} \left[ \arctan \left(\frac{y}{x - \frac{e}{2}}\right) + \arctan \left(\frac{y - d}{x - \frac{e}{6}}\right) - \arctan \left(\frac{y + d}{x - \frac{e}{6}}\right) - \arctan \left(\frac{y}{x - \frac{e}{6}}\right) \right]$$

$$B_{I_\pi}(x, y, k_v) = \frac{\mu_0 k_v}{4\pi} \ln \frac{(y + d)^2 + \left(\frac{x + e}{2}\right)^2}{y^2 + \left(\frac{x + e}{2}\right)^2} + \ln \frac{y^2 + \left(\frac{x + e}{6}\right)^2}{(y - d)^2 + \left(\frac{x + e}{6}\right)^2}$$

$$B_{I_\pi}(x, y, k_v) = \frac{\mu_0 k_v}{2\pi} \left[ \arctan \left(\frac{y + d}{x + \frac{e}{2}}\right) + \arctan \left(\frac{y}{x + \frac{e}{6}}\right) - \arctan \left(\frac{y}{x + \frac{e}{2}}\right) - \arctan \left(\frac{y - d}{x + \frac{e}{6}}\right) \right]$$

Where $k_v$ is the linear current density of the surface current. Then the magnetic field produced by the permanent magnet segment with positive y direction magnetized can be described as (4).
\[
B_x(x, y, k_x) = B_{l_x}(x, y, k_x) + B_{l_{nx}}(x, y, k_x)
\]
\[
B_y(x, y, k_y) = B_{l_y}(x, y, k_y) + B_{l_{ny}}(x, y, k_y)
\]

The derivation is similar to permanent magnet segment with different magnetization angle and shape dimension. Magnetic field produced by one Halbach magnetic array at point \(P(x, y)\) can be expressed as (5)

\[
\begin{align*}
B_x(x, y) &= \sum_{n=0}^{N} \sum_{i=1}^{4} B_{lx} \left( x - \left( n + \frac{e}{2} \right), y, k \right) \\
B_y(x, y) &= \sum_{n=0}^{N} \sum_{i=1}^{4} B_{ly} \left( x - \left( n + \frac{e}{2} \right), y, k \right)
\end{align*}
\]

Based on (5), evaluation indexes \(B_h\) and \(K_{BD}\) can be get, where \(B_h\) the fundamental component of the magnetic flux density is and \(K_{BD}\) is the sinusoidal distortion rate. \(K_{BD}\) Can be expressed as

\[
K_{BD} = \sqrt{\sum_{i \neq 1} \left( \frac{B_{li}}{B_{1i}} \right)^2}
\]

Where \(B_{li}\) represents the \(i\)th harmonic component of the magnetic flux density.

4. Result and Discussion
Fig. 6 shows the comparison of magnetic field of Halbach magnet arrays with different convex type section. And evaluation indexes are listed in Tab. 2.
Evaluation indexes on $x$ axis and $y$ axis are listed respectively since suspending and driving capability are required in different occasions [8-13]. The fundamental component of the magnetic field of Habach magnet arrays with convex type section is bigger than normal ones in common. The maximum of magnetic field’s fundamental component on $x$ axis is $a1c5$ type. Type $a5c1$ owns the maximum in total and on $y$ axis. Also, this type has the minimum sinusoidal distortion rate on $y$ axis, which is suitable for occasion that emphasizes suspending capability. Despite these advantages, sinusoidal distortion rate of the Habach magnet arrays with convex type section are too big.
Table 2. Comparison of the Evaluation Indexes of Halbach Magnet Arrays

| Index Type | normal | a3c3 | a1c5 | a5c1 |
|------------|--------|------|------|------|
| $B_{x_1}$ (T) | 0.1507 | 0.2089 | 0.2168 | 0.1886 |
| $B_{y_1}$ (T) | 0.1701 | 0.1757 | 0.1419 | 0.2095 |
| $B_{z_1}$ (T) | 0.2273 | 0.2729 | 0.2591 | 0.2819 |
| $K_{RDX}$ | 0.1629 | 0.4761 | 0.4316 | 0.4678 |
| $K_{RDY}$ | 0.1256 | 0.2193 | 0.2897 | 0.0958 |
| $K_{RD}$ | 0.2057 | 0.5242 | 0.5198 | 0.4775 |

Fig. 7 shows one permanent magnet segment of type $a3c3$ and the Halbach array it composed. Other types are difficult to manufacture because of the special mold they need and the high cost.

![Halbach magnet array with convex section.](image)

(a) One permanent magnet segment  
(b) Halbach array

**Figure 7.** Halbach magnet array with convex section.

5. Conclusion

To enhance the performance of Habach magnet array, permanent magnet segment with different configurations are studied. The magnetic field is modeled according to Ampere’s hypothesis of molecular current. The fundamental component of the magnetic flux density and the sinusoidal distortion rate obtained from the magnetic field model are compared, which lay a good foundation for subsequent optimization.

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