LOW-$x$ STRUCTURE FUNCTIONS

Stefano Catani

I.N.F.N., Sezione di Firenze
and Dipartimento di Fisica, Università di Firenze
Largo E. Fermi 2, I-50125 Florence, Italy

Abstract

QCD predictions on the low-$x$ behaviour of the structure functions are reviewed and compared with the recent measurements of $F_2(x,Q^2)$ at HERA. The present theoretical accuracy of these predictions is discussed.
1 Introduction

At present and future hadron colliders, precise quantitative tests of QCD and searches for new physics within the standard model (top and Higgs production) and beyond are carried out in the small-$x$ kinematic region. By small $x$ we mean that the ratio $x = Q^2/S$, between the typical transferred momentum $<p_t> = Q$ in the process and the centre-of-mass energy $\sqrt{S}$ of the colliding hadrons, is much smaller than unity. For these small-$x$ processes reliable and accurate theoretical predictions are clearly necessary.

Despite the general relevance of this topic, the present contribution does not intend to be a comprehensive (even if concise) review on the theory of small-$x$ physics. I shall concentrate on the low-$x$ behaviour of the (proton) structure functions in deep inelastic electron-proton scattering. There are two very simple motivations for limiting ourselves to considering structure functions. First, the study is relevant by itself for investigating hadron physics at very high energy (since $S = Q^2/x$, the small-$x$ limit at fixed $Q^2$ is equivalent to $S \to \infty$). Second, according to the parton model the structure functions are proportional to the parton densities of the hadron and their knowledge is essential for accurate phenomenology at any hadron collider.

In the following, I shall review some of the main QCD predictions about the small-$x$ behaviour of the structure functions and, in particular, I shall try to discuss the present accuracy of these predictions. Some other aspects of small-$x$ physics are considered elsewhere at this meeting [1].

2 Structure functions at HERA

The natural starting point of our discussion on low-$x$ structure functions is represented by the recent HERA data on the proton structure function $F_2$ [2,3].

HERA is the first electron-proton collider ever built and is operating at present with the centre-of-mass energy $\sqrt{S} = 296$ GeV. Due to this large centre-of-mass energy, it is able to see into very small values of the Bjorken variable $x$, $x \sim 10^{-4}$, even in the deep-inelastic-scattering (DIS) region $10 \, \text{GeV}^2 \lesssim Q^2 \lesssim 10^5 \, \text{GeV}^2$ (I use standard notations for the DIS kinematic variables, namely $Q^2 = -q^2$, $x = Q^2/2p \cdot q$, $y = p \cdot q/p \cdot k$ with $S = Q^2/xy$; see Fig. 1) This is an entirely new DIS regime which extends for two orders of magnitude, both in $x$ and $Q^2$, outside that ($x \sim 10^{-2}$, $Q^2 \lesssim 10^3 \, \text{GeV}^2$) investigated by previous fixed-target experiments [5].

For values of $Q^2 \lesssim 10^5 \, \text{GeV}^2$, the electron-proton inclusive cross section is dominated by the exchange of a single off-shell photon and given by

$$\frac{d\sigma^{ep}}{dx \, dQ^2} = \frac{4\pi\alpha^2}{xQ^4} \left( 1 - y + \frac{1}{2} \frac{y^2}{1 + R(x, Q^2)} \right) F_2(x, Q^2). \quad (1)$$

Here $F_2$ is the customary proton structure function, which is proportional to the sum $\sigma_L + \sigma_T$, $\sigma_i$'s being the cross sections off a transversely ($i = T$) or longitudinally ($i = L$) polarized photon. The function $R$ denotes the ratio $R = \sigma_L/\sigma_T$: it is smaller than $10\%$ in the region which is being studied at HERA. It follows that the HERA data on the cross section [1] provides a direct measurement of $F_2(x, Q^2)$.

The data collected at HERA in 1992 [2,3] have already shown clearly that $F_2(x, Q^2)$ increases very steeply at small $x$. This behaviour has to be contrasted with that measured at higher $x$-values (Fig. 2). Previous data in the region of $x \sim 10^{-2}$ are indeed consistent with a constant (or almost constant) behaviour of $F_2$ for $x \to 0$. Moreover, HERA data on $F_2$ as to be regarded as
a qualitatively new (although, possibly, not unexpected) experimental result: a similar strong rise with the energy has not been observed so far in total hadronic cross sections.

Fig. 1: Kinematic variables for deep inelastic electron-proton scattering.

Fig. 2: H1 data on $F_2(x, Q^2)$. Data points of the NMC and BCDMS experiments are shown for comparison.
3 Total hadronic cross sections

A compilation of data on total hadronic cross sections is reported in Fig. 3. We can see that they increase slowly with increasing $\sqrt{S}$.

The curves in Fig. 3 are the results of a Regge-type fit performed by Donnachie and Landshoff. It provides a very simple and economical description of all data in the form:

$$\sigma_{TOT} = X \, S^{\varepsilon} + Y \, S^{-\eta}. \quad (2)$$

The parameters $X$ and $Y$ depend on the type of colliding hadrons whilst the powers $\varepsilon$ and $\eta$ are constrained to be ‘universal’ and found (by the fit) to be $\varepsilon = 0.08$ and $\eta = 0.45$. In this Regge-type phenomenological model $\alpha_P(0) = 1 + \varepsilon$ and $1 - \eta$ are thus respectively interpreted as the intercept of pomeron trajectory (it carries the vacuum quantum numbers) and of the $\rho$-meson trajectory ($\rho, \omega, f, a, \ldots$).
The increase of the cross section (2) with the energy is due to the pomeron exchange and is indeed very slowly because the pomeron trajectory is above unity by a small amount, i.e. \( \varepsilon = 0.08 \). Actually, since \( \varepsilon \ll 1 \), to a good approximation we have \( S^\varepsilon \simeq 1 + \varepsilon \ln S + \frac{1}{2} \varepsilon^2 \ln^2 S \). This is the reason why double-logarithmic expressions of the type \( \sigma_{TOT} = AS^{-n} + B + C \ln S + D \ln^2 S \) are equally successful in describing the data.

It is worth noting that this slow (approximately logarithmic) increase of the cross section holds true also for the photoproduction case. HERA data on the \( \gamma p \) cross section \( ^8 \) (Fig. 4) are in remarkable agreement with the value predicted by Donnachie and Landshoff almost ten years ago \( ^9 \).

**Fig. 4:** Fit of type (2) for the \( \gamma p \) cross section data below \( \sqrt{S} = 20 \) GeV. The two data points at \( \sqrt{S} = 200 \) GeV correspond to the H1 and ZEUS measurements at HERA.

On the basis of this successful phenomenological description of the energy dependence of total cross sections, one can assume a corresponding \( x \)-dependence for the structure function \( F_2 \) \( ^{10,11} \). In particular, one can extrapolate the Regge-type parametrization (2) from the photoproduction limit (\( Q^2 = 0 \)) to the low-\( Q^2 \) (\( Q^2 \lesssim 10 \) GeV\(^2 \)) regime by using the following expression \( ^{11} \)

\[
F_2(x, Q^2) = A x^{-0.08} \frac{Q^2}{Q^2 + a^2} + B x^{0.45} \frac{Q^2}{Q^2 + b^2}.
\]  

This extrapolation is obtained by a simple dipole model (\( a \simeq 750 \) MeV) for the \( Q^2 \)-dependence: \( F_2 \) is forced to vanish for \( Q^2 \to 0 \) (gauge invariance) and to be \( Q^2 \)-independent at high \( Q^2 \) (Bjorken scaling).

The comparison between the parametrization \( ^3 \) and HERA data (Fig. 5) shows that this type of models, based on the energy behaviour of low-\( p_t \) hadronic interaction (soft processes), fails in describing the structure function \( F_2(x, Q^2) \) as measured in the DIS regime. Hard physics is necessary for explaining the \( x \)-shape: a steeper \( F_2 \) is indeed expected from hard (perturbative) QCD.
Fig. 5: Comparison of the measured $F_2(x, Q^2)$ (H1 data) with the parametrization in Eq. (3) (DOLA) and with several QCD expectations (see Sect. 5). The overall normalization uncertainty of 8% is not included in the error bars.

4 Structure functions in perturbative QCD

4.1 The QCD improved parton model

According to the naïve parton model the hadron is built up by point-like partons (quarks and gluons) which are almost non-interacting and scattered incoherently when their transverse momentum $p_{\perp i}$ is larger than some value $Q_0$ of the order of the inverse hadronic radius $1/R_h \sim$
1 GeV. Therefore, in the large-$Q^2$ limit ($Q^2 \gg Q^2_0$) the structure function $F_2(x, Q^2)$ is given by

$$F_2(x, Q^2) = \sum_{i=1}^{N_f} e_i^2 \left[ f_{q_i}(x) + f_{\bar{q}_i}(x) \right] ,$$

where $f_{q_i}(x) \equiv x q_i(x)$ are the scale-invariant quark densities of the hadron and $e_i$ are the quark charges.

Perturbative QCD modifies this simple picture. The large $k_\perp$-gap ($Q^2_0 \ll k_\perp \ll Q^2$) between the incoming proton and the scattered quark can be filled up by QCD radiation of massless partons. They have a logarithmic spectrum $dk^2_\perp/k^2_\perp$ in transverse momentum thus leading to corrections of the type

$$\alpha_s \int_{Q^2_0}^{Q^2} \frac{dk^2_\perp}{k^2_\perp} = \alpha_s \ln \frac{Q^2}{Q^2_0}$$

that can be large ($\alpha_s \ln Q^2/Q^2_0 \sim 1$ when $Q^2 \gg Q^2_0$) even if the partons are weakly coupled ($\alpha_s(Q^2) \ll 1$). The large logarithmic contributions in Eq. (5) have to be computed to all orders in $\alpha_s$ and their resummation gives rise to parton densities evolving with $Q^2$ (scaling violations) in a predictable way.

The QCD improved parton model is summarized by the following factorization formula [12]

$$F_2(x, Q^2) = \sum_{Q=q,g} \int_x^1 \frac{dz}{z} F_{2a}(x/z, Q^2; Q^2_0) f_a(z, Q^2_0) + \mathcal{O}(1/Q^2) .$$

Here $f_a(x, Q^2_0)$ are the input (non-perturbative) parton densities ($f_g(x) \equiv x g(x)$ is the gluon density) at the scale $Q^2_0$, $F_{2a}$ accounts for the QCD evolution from $Q^2_0$ to $Q^2$ and is computable in perturbation theory and the $\mathcal{O}(1/Q^2)$ term stands for the so-called higher-twist contributions, i.e. contributions vanishing as inverse powers of $Q^2$ in the high-$Q^2$ regime.

Note that the $Q^2$-dependence is completely factorized in Eq. (6) and thus predicted by the perturbative QCD component $F_{2a}$. On the contrary, the $x$-dependence of $F_2(x, Q^2)$ enters Eq. (6) through the convolution of $F_{2a}$ and $f_a$ with respect to the longitudinal-momentum fraction $z$. As a result, the $x$-shape of $F_2$ is determined both by the input parton density and by its perturbative evolution. For instance, if we consider a model in which both the input and the evolution have a power dependence on $x$, i.e. $f_a(x) \sim x^{-\lambda}$ and $F_{2a}(x) \sim x^{-\alpha}$, from Eq. (6) we see that the steeper one wins as $x \rightarrow 0 : F_2(x) \sim x^{-\max\{\alpha, \lambda\}}$.

The perturbative QCD component $F_{2a}$ in Eq. (6) is given by

$$F_{2a}(x, Q^2; Q^2_0) = \sum_b \int_x^1 \frac{dz}{z} C_{2b}(x/z, \alpha_s(Q^2)) F_{ba}(z, Q^2; Q^2_0) ,$$

where $C_{2a}$ is the process-dependent coefficient function and $F_{ab}$, the universal (i.e. process independent) Green function for the evolution of the parton densities, fulfils the following generalized Altarelli-Parisi equation

$$F_{ab}(x, Q^2; Q^2_0) = \delta_{ab} \delta(1-x) + \sum_c \int_{Q^2_0}^{Q^2} \frac{dk^2_\perp}{k^2_\perp} \int_x^1 dz P_{ac}(z, \alpha_s(k^2_\perp)) F_{cb}(x/z, k^2_\perp; Q^2_0) .$$

The parton densities at the hard scale $Q^2$ are obtained from the corresponding inputs as follows

$$f_a(x, Q^2) = \sum_b \int_x^1 \frac{dz}{z} F_{ab}(x/z, Q^2; Q^2_0) f_b(z, Q^2_0) .$$
The coefficient function $C_2(z, \alpha_S)$ and the Altarelli-Parisi splitting function $P(z, \alpha_S)$ are both computable in QCD perturbation theory and their power series expansion has the form:

$$C_2(z, \alpha_S) = \delta(1 - z) + \alpha_S C_2^{(1)}(z) + \alpha_S^2 C_2^{(2)}(z) + \ldots ,$$

$$P(z, \alpha_S) = \alpha_S P^{(1)}(z) + \alpha_S^2 P^{(2)}(z) + \alpha_S^3 P^{(3)}(z) + \ldots ,$$

The coefficient function $C_2$ measures the effective coupling $\gamma^*\text{-parton}$ whereas Eq. (8) has a simple partonic interpretation. It describes a generalized parton evolution along a space-like cascade in which the successive $k_\perp$'s are strongly ordered (Fig. 6). The Altarelli-Parisi splitting function $P_{ab}(z, \alpha_S)$ can be consistently interpreted as the probability of emission of a bunch of partons (jets) whose transverse momenta are of the same order. In particular $P^{(1)}, P^{(2)}, \ldots, P^{(n)}$, in Eq. (11) respectively represent the emission of one, two, ... , $n$ partons with almost equal transverse momenta (Fig. 6b). We see that the radiation of an additional parton without $k_\perp$-ordering costs an extra power of $\alpha_S$ and, hence, is a subdominant effect with respect to the $Q^2$-evolution of $F_2(x, Q^2)$.

Fig. 6: Partonic picture of the perturbative QCD evolution: (a) strong ordering of transverse momenta ($Q^2 > k^2_{\perp n} > \ldots > k^2_{\perp 1} > Q^2_0$) along the space-like cascade; (b) splitting function for the emission of partons with comparable transverse momenta.

For the sake of simplicity, in the following I do not consider the higher-order contributions $C_2^{(n)}$ ($n \geq 1$) to the coefficient function. Note, however, that by no means this implies that their quantitative effect is negligible in the small-$x$ region [13,14,15].

In order to proceed in our discussion, it is also convenient to introduce $N$-moments as follows

$$F_N(Q^2; Q^2_0) \equiv \int_0^1 dx \; x^{N-1} \; F(x, Q^2; Q^2_0) \; .$$

The limit $x \to 0$ corresponds to the small-$N$ limit in the $N$-moment space.
The Mellin transformation in Eq. (12) diagonalizes the convolution over $z$ in Eq. (8). The solution of the latter is

$$F_N(Q^2; Q_0^2) = \exp \left\{ \int_{Q_0^2}^{Q^2} \frac{dk^2_{1\perp}}{k^2_{1\perp}} \gamma_N(\alpha_S(k^2_{1\perp})) \right\},$$

(13)

where the anomalous dimensions $\gamma_N$ are related to the $N$-moments of the splitting functions as follows

$$\gamma_N(\alpha_S) = \int_0^1 dz \; z^N \; P(z, \alpha_S) = \alpha_S \gamma_N^{(1)} + \alpha_S^2 \gamma_N^{(2)} + \ldots \, .$$

(14)

The expression (13) explicitly resums the large logarithmic contributions $\alpha^n_S(\ln Q^2/Q_0^2)^k$ we have mentioned before. Inserting the one-loop ($\gamma_N^{(1)}$), two-loop ($\gamma_N^{(2)}$), ... anomalous dimensions into Eq. (13), one can systematically resums leading ($k = n$), next-to-leading ($k = n - 1$), ... logarithmic terms.

### 4.2 Altarelli-Parisi evolution to leading order

The perturbative QCD evolution of the parton densities to leading order is particularly simple in the small-$x$ limit [12]. In this case the gluon channel dominates because the splitting function

$$P_{gg}(z, \alpha_S) \simeq \frac{C_A \alpha_S}{\pi} \frac{1}{z} + \mathcal{O}(\alpha^2_S), \quad (z \to 0),$$

(15)

has a $1/z$ singularity, related to the exchange of a spin-one particle, the gluon, in the $t$-channel. The corresponding anomalous dimension ($\bar{\alpha}_S \equiv C_A \alpha_S/\pi$)

$$\gamma_{gg,N}(\alpha_S) \simeq \frac{\bar{\alpha}_S}{N} + \mathcal{O}(\alpha^2_S), \quad (N \to 0),$$

(16)

is singular for $N \to 0$. Inserting Eq. (16) into Eq. (13) and transforming back to the $x$-space, one finds:

$$F_{gg}(x, Q^2; Q_0^2) \simeq \exp \left\{ 2 \sqrt{\bar{\alpha}_S \ln \frac{1}{x} \ln \frac{Q^2}{Q_0^2}} \right\}.$$

(17)

The result in Eq. (17) shows that, due to the perturbative QCD evolution, the parton densities raise rapidly for $x \to 0$. Their increase is faster than any power of $\ln x$, although slower than any power of $1/x$. Note also that, because of the factor $\ln Q^2/Q_0^2$, the effective slope in $x$ increases proportionally to the evolution range in $k^2_{1\perp}$.

Equation (17) resums the double logarithmic contributions $(\alpha_S \ln \frac{1}{x} \ln \frac{Q^2}{Q_0^2})^n$ to all orders in $\alpha_S$. It has been obtained by assuming a fixed coupling $\alpha_S$. The inclusion of the running coupling and of the next-to-leading term $\gamma_N^{(2)}$ for the anomalous dimensions does not change qualitatively the small-$x$ behaviour of $F(x, Q^2; Q_0^2)$ (see Sect. 5).

In order to gain physical insight into the result (17), let us consider a simplified derivation. The probability of emission of a gluon with a large rapidity $y = \ln 1/z$ and transverse momentum $k_{1\perp}$ is:

$$dw = \bar{\alpha}_S \; dy \; \frac{dk^2_{1\perp}}{k^2_{1\perp}}.$$

(18)

The Green function $F_{gg}$ is obtained by integrating the spectrum (18) over the rapidity range $L = \ln 1/x$ and summing over any number $n$ of final state gluons as follows

$$F_{gg}(x, Q^2; Q_0^2) \sim \sum_n \frac{1}{n!} \bar{\alpha}_S^n \int_0^L dy_1 \ldots \int_0^L dy_n \int \frac{dk^2_{1\perp}}{k^2_{1\perp}} \ldots \frac{dk^2_{n\perp}}{k^2_{n\perp}} \Theta(Q^2 > k^2_{1n} > \ldots > k^2_{n1} > Q_0^2)$$

$$= \sum_n \frac{1}{n!} (\bar{\alpha}_S L)^n \int \frac{dk^2_{1\perp}}{k^2_{1\perp}} \ldots \frac{dk^2_{n\perp}}{k^2_{n\perp}} \Theta(Q^2 > k^2_{1n} > \ldots > k^2_{n1} > Q_0^2).$$

(19)
The statistical factor $1/n!$ in Eq. (15) is due to the identity of the $n$ final-state gluons. The $\Theta$-function denotes the constraint of transverse-momentum ordering which is appropriate for the $Q^2$-evolution to leading accuracy. After $k_\perp$-integration, this constraint produces an additional factor of $1/n!$, thus leading to $(n!)^2 \simeq (2n)!/2^{2n}$ for $n \gg 1$

$$F_{gg}(x, Q^2; Q_0^2) \simeq \sum_n \left( \frac{1}{n!} \right)^2 \left( \sqrt{\alpha_S L \ln(Q^2/Q_0^2)} \right)^{2n} \simeq \exp \left\{ 2 \sqrt{\alpha_S L \ln(Q^2/Q_0^2)} \right\} . \quad (20)$$

This discussion shows that the rise of the parton densities at small $x$ is due to the perturbative emission of many gluons (each of them giving a constant amplitude) over a large rapidity gap. Actually, at very small values of $x$, the $L$ factors associated with the large rapidity gap may dominate the analogous factors $\ln Q^2/Q_0^2$ due to the $k_\perp$-evolution. In this case one has to go beyond the leading logarithmic approximation in Eqs. (15)-(17). The higher-order terms $P^{(n)}(z) (n > 1)$ in the splitting functions (11) can be large and have to be taken into account. In fact, the additional powers of $\alpha_s$ due to the emission of gluons with comparable $k_\perp$'s can be compensated by enhancing factors $\ln 1/z$. Equivalently, we can say that we must give up $k_\perp$-ordering and compute the large $\ln x$-factors associated with the large rapidity gap and any value of transverse momenta.

The possible impact of this improved treatment in the small-$x$ region can be argued from Eqs. (19) and (20). Releasing the $k_\perp$-ordering constraint in Eq. (19), we miss a $1/n!$ contribution to Eq. (20), thus obtaining

$$F_{gg}(x, Q^2; Q_0^2) \sim \sum_n \frac{1}{n!} (\text{const. } \alpha_s L)^n \sim \exp \{ \text{const. } \alpha_s L \} , \quad (21)$$

where the constant factor in the exponent is related to the transverse-momentum integrations. As I shall discuss in the next subsection, the behaviour in Eq. (21) is precisely the result of the resummation of the terms $(\alpha_s \ln x)^n$ in perturbation theory.

### 4.3 The BFKL equation

In order to perform this resummation it is convenient to introduce the unintegrated parton density $\mathcal{F}(x, k; Q_0^2)$ [13]. It gives the Green function for the QCD evolution with a fixed total transverse momentum $k$ in the final state. The customary Green function $F(x, Q^2; Q_0^2)$ in Eqs. (7) and (9) is then recovered by $k_\perp$-integration:

$$F(x, Q^2; Q_0^2) = \int_0^{Q^2} d^2k \ F(x, k; Q_0^2) . \quad (22)$$

In terms of this function the resummation of leading contributions $(\alpha_s \ln x)^n$ is achieved by an integral equation whose kernel describes parton radiation without $k_\perp$-ordering constraint:

$$\mathcal{F}(x, k; Q_0^2) = \pi \delta(1-x) \delta(k^2 - Q_0^2) + \alpha_s \int_x^1 \frac{dz}{z} \int d^2k' \ K(k; k') \ F(x/z, k; Q_0^2) . \quad (23)$$

The explicit form of the kernel is the following

$$\int d^2k' \ K(k; k') \ F(x, k) = \int \frac{d^2q}{\pi q^2} \left[ \mathcal{F}(x, k + q) - \Theta(k^2 - q^2) \mathcal{F}(x, k) \right]$$

and was derived by Balitskii, Fadin, Kuraev and Lipatov (BFKL) almost two decades ago [17].
The detailed discussion of Eq. (23) within the context of the present paper can be found elsewhere [18-20]. Here I limit myself to recall its main features.

Since the BFKL equation (23) does not enforce $k_\perp$-ordering, it is not an evolution equation in $k_\perp^2$. Its general solution is thus contaminated by higher-twist contributions. Nonetheless, in the large $k_\perp$-regime, one can disentangle the leading-twist behaviour in the form ($\mathcal{F}_N$ are the $N$-moments of $\mathcal{F}(x)$):

$$\mathcal{F}_N(k; Q_0^2) = \frac{\gamma_N(\alpha_s)}{\pi k_\perp^2} \left( \frac{k_\perp^2}{Q_0^2} \right)^{\gamma_N(\alpha_s)} \left[ 1 + \mathcal{O}(Q_0^2/k_\perp^2) \right].$$

This power behaviour in $k_\perp$ implies that the resummation of the $(\alpha_s \ln x)$-terms performed by the BFKL equation is consistent with an effective gluon anomalous dimension $\gamma_{gg,N}(\alpha_s) \simeq \gamma_N(\alpha_s)$ [19-21]. Its explicit expression is obtained by solving the following implicit equation

$$1 = \bar{\alpha}_S N \chi(\gamma_N(\alpha_s)),$$

where the function $\chi(\gamma)$ is related to the eigenvalues of the BKFL kernel and given by ($\psi(z)$ is the Euler $\psi$-function)

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma).$$

The power series solution of Eq. (26) is ($\zeta(n)$ is the Riemann zeta function):

$$\gamma_N(\alpha_s) = \sum_{n=1}^{\infty} c_n \left( \frac{\bar{\alpha}_S}{N} \right)^n = \frac{\bar{\alpha}_S}{N} + 2\zeta(3) \left( \frac{\bar{\alpha}_S}{N} \right)^4 + 2\zeta(5) \left( \frac{\bar{\alpha}_S}{N} \right)^6 + \mathcal{O} \left( \left( \frac{\bar{\alpha}_S}{N} \right)^7 \right).$$

The one-loop contribution on the r.h.s. of Eq. (28) reproduces the small-$N$ limit [14] of the Altarelli-Parisi splitting function $P^{(1)}$ in Eq. (11). The higher orders are the result of the resummation of the leading logarithmic contributions $(\alpha_s^n \ln^{n-1} z)/z$ in $P_{gg}(z, \alpha_s)$.

**Fig. 7:** The BFKL characteristic function $\chi(\gamma)$ for $0 < \gamma < 1$. $\gamma_N$ is the BFKL anomalous dimension.

The characteristic function $\chi(\gamma)$ in Eq. (27) has approximately a parabolic shape (Fig. 7). As $x \to 0$, $N$ decreases and reaches a minimum value $N_{\text{min}} = 4\bar{\alpha}_S \ln 2$ at which $\gamma_N$ has a
branch point singularity. Therefore the resummation of the singular terms \( (\alpha_S/N)^n \) builds up a stronger singularity at \( N = N_{\text{min}} \). This singularity, known as the perturbative QCD (or BFKL) pomeron, is responsible for a very steep behaviour of the QCD evolution at small \( x \) (and large \( k_{\perp}^2 \)):

\[
\mathcal{F}(x, k; Q_0^2) \sim \frac{1}{\pi k^2} \sqrt{\frac{k^2}{Q_0^2}} x^{-4\alpha_S \ln 2},
\]

(29)

\[
F_{gg}(x, Q^2; Q_0^2) = \int_0^{Q^2} d^2k \mathcal{F}(x, k; Q_0^2) \sim \sqrt{\frac{Q^2}{Q_0^2}} x^{-4\alpha_S \ln 2}.
\]

(30)

I have so far reviewed the main qualitative predictions of perturbative QCD about the small-\( x \) behaviour of the parton densities and, hence, of the structure function \( F_2(x, Q^2) \). The latter is expected to increase faster than any power of \( \ln 1/x \). To leading logarithmic order in \( \ln Q^2/Q_0^2 \), the parton densities are driven by the Green function in Eq. (17). Moreover, at very small values of \( x \), the complete (i.e. including subleading terms in \( \ln Q^2/Q_0^2 \)) resummation of the contributions \( (\alpha_S \ln x)^n \) gives rise to the power increase in Eq. (30). Note that the transition from the small-\( x \) behaviour in Eq. (17) to that in Eq. (30) is also accompanied by strong scaling violations (compare \( \exp \sqrt{\ln Q^2/Q_0^2} \) with \( \sqrt{Q^2/Q_0^2} \)).

### 4.4 Comment on unitarization

Before presenting a more detailed quantitative discussion, I should add some comments on unitarity.

Because of unitarity, the DIS cross section \( (1) \) cannot increase indefinitely. At asymptotic energies it must approach a constant (modulo \( \ln S \)-corrections) limit as given by the geometrical size of the hadron. This limit implies the following unitarity bound on the structure function \( (R_h is the hadron radius) \)

\[
\frac{1}{Q^2} F_2(x, Q^2) \lesssim \pi R_h^2.
\]

(31)

If we consider the leading-twist factorization formula \( (4) \) and the fact that gluon evolution dominates at small \( x \), from Eq. (33) we obtain a corresponding constraint on the gluon density, namely

\[
\alpha_S f_g(x, Q^2) \lesssim \pi R_h^2 Q^2.
\]

(32)

On the other hand, from Eqs. (17) and (31), we see that both the leading-order Altarelli-Parisi equation and the BFKL equation violate the constraint (32) at sufficiently small values of \( x \). This contradiction has a simple physical explanation. As soon as the gluon density becomes very large, the partons tend to overlap and are no longer scattered incoherently by the hard photon. In this regime we should take into account parton rescattering and parton recombination: they are responsible for fulfilling the unitarity bound in Eq. (31).

From a formal viewpoint, this means we have to include higher-twist contributions. Parton recombination is a process of order \( 1/(R_h^2 Q^2) \) and the probability of recombination of two gluons is approximately given by \( \alpha_S (f_g(x, Q^2))^2 \). Therefore this process produces an higher-twist correction of relative order \( \alpha_S f_g(x, Q^2)/(R_h^2 Q^2) \) on the r.h.s. of Eq. (31). As soon as this corrections approaches unity, higher-twist terms cannot be neglected in the factorization formula (3). We have thus recovered the constraint (32), which has to be correctly interpreted as a limit on the validity of the leading-twist approximation.
This argument shows that the perturbative QCD approach to small-$x$ physics is (at least) self-consistent as regards to unitarity. The factorization formula \( \square \) has to be used in a proper way by including higher-twist corrections whenever they are (estimated to be) large.

In spite of many efforts and partial results \([2]^\dagger\), a general theory of rescattering has not yet been set up. Reasonable physical models are however available \([18]\). Recent numerical estimates \([23]\) based on these models lead to the conclusion that rescattering effects give a small contribution (at most 10\%) to \( F_2 \) in the HERA kinematic range. On the other side, at present, no evident signal of unitarity corrections (i.e. flattening of \( F_2 \) at small \( x \)) emerges from the HERA data. This is not surprising since, even for values of \( Q^2 \) as small as 1 GeV\(^2\), the bound \([\square] \) gives \( F_2 \lesssim 25 \left( \pi R_h^2 \simeq 1 \text{ fm}^2 \right) \). Because of these reasons, in the following I shall not discuss any longer the issue of unitarity.

5 NLO parton densities

In order to perform a quantitative comparison between HERA data and perturbative QCD predictions, I shall consider some of next-to-leading order (NLO) parton densities that were available before HERA started operating. These parton densities are obtained by evolving (as in Eq. \( \square \)) some input distributions according to the generalized Altarelli-Parisi equation \([8]\) in NLO, i.e. including the full one- and two-loop \([24]\) splitting functions (anomalous dimensions) in Eq. \([\square] \) (Eq. \([\square\dagger]\)). The input parton densities at some scale \( Q_0^2 \) are fixed by fitting pre-HERA data on muon and neutrino DIS, prompt-photon production and Drell-Yan type processes in hadron-hadron collisions. The ensuing predictions have a high (better than 10\%) overall accuracy, including the theoretical uncertainties due to higher-twist effects and perturbative corrections beyond two-loop order. For this reason, they provide a good starting point for detailed quantitative investigations of the small-$x$ region.

Actually, since pre-HERA data concern the high-$x$ region, the input gluon and sea-quark distributions \( f_g \) and \( f_{\text{sea}} \) are essentially unconstrained for values of \( x \) smaller than 10\(^{-2}\). The representative sets of NLO parton densities listed below precisely differ on the assumptions about the small-$x$ behaviour of the input densities.

The GRV parton distributions \([25]\) are obtained by starting the perturbative QCD evolution at a very low input scale, \( Q_0^2 = 0.3 \text{ GeV}^2 \). At this scale the input distributions are chosen to vanish for \( x \to 0 \): \( f_g(x) \sim x^2 \), \( f_{\text{sea}}(x) \sim x^{0.7} \). Because of this ‘valence-like’ behaviour of the input, any steeper behaviour at high \( Q^2 \) is entirely generated by perturbative QCD evolution.

The input scale used in the MRS analysis \([26]\) is \( Q_0^2 = 4 \text{ GeV}^2 \). They provide two representative sets of parton densities. Set \( D'_0 \) is obtained using input densities which are flat at small \( x \): \( f_g(x) \sim f_{\text{sea}}(x) \sim \text{const.} \). Set \( D' \) instead uses very steep input distributions: \( f_g(x) \sim f_{\text{sea}}(x) \sim x^{-0.5} \). Roughly speaking, the flat input corresponds to the so-called ‘critical-pomeron’ (with intercept equal to unity) behaviour whereas the steep input is reminiscent of the BFKL pomeron.

As in the case of the MRS analysis, the CTEQ Collaboration \([27]\) starts the perturbative evolution at the input scale \( Q_0^2 = 4 \text{ GeV}^2 \). Among the different sets of parton distributions they consider, set CTEQ1MS is that which has the steepest input densities, namely \( f_g(x) \sim x^{-0.38} (\ln 1/x)^{0.09} \), \( f_{\text{sea}}(x) \sim x^{-0.27} \). This small-$x$ behaviour is just the result of a certain parametrization of the input and is not related to any ‘hybrid pomeron’.

Figures 5 and 8 show the comparison between the 1992 data\([4]\) of the H1 and ZEUS Collabora-

---

\(^\dagger\)The comparison with the 1993 preliminary data of the ZEUS Collaboration, presented at this meeting \([4]\), does not substantially modify the discussion that follows.
rations and the structure function $F_2$ as obtained from different sets of NLO parton densities. We see that the expectations from the GRV and MRSD’ sets seem favoured by the data.

**Fig. 8:** Comparison of the measured $F_2(x,Q^2)$ (ZEUS data) with the expectations from NLO parton densities. The overall normalization uncertainty of 7% is not included in the error bars.

This observation, rather than being the main conclusion of the present contribution, gives me the opportunity for addressing some points which require further investigations and that I consider relevant for the understanding of small-$x$ physics.

The GRV parton distributions on one side and the MRSD’$_0$, MRSD’$^-$, CTEQ1MS distributions on the other side represent, in a sense, two very different QCD expectations.

In the GRV framework, due to the valence-like behaviour of the input densities, the fast increase of $F_2$ at small-$x$ and high $Q^2$ is entirely generated by perturbative QCD evolution as in Eq. (17). More precisely, according to Eq. (17), a structure function as steep as that reported in
Figs. 5 and 8 can be obtained only by means of a very long $Q^2$-evolution. In the GRV case this is achieved by using a very low input scale. The weak (or, perhaps, amazing) point is that no higher-twist contribution is included (present) even at an input scale as low as $Q_0^2 = 0.3 \text{ GeV}^2$. Moreover, no small-$x$ resummation (either from the BFKL equation or from higher orders in the Altarelli-Parisi equation) is considered.

In the case of the MRS and CTEQ parton distributions the starting scale for the NLO QCD evolution is reasonably large, namely $Q_0^2 = 4 \text{ GeV}^2$. The different expectations for $F_2$ at small $x$ are entirely due to the different behaviour of the input densities. The MRSD’ set, which is favoured by the data, has the steepest input densities. According to a widespread wisdom, the MRSD’ framework is motivated by the BFKL equation. If one considers the BFKL pomeron intercept $N_{\text{min}} \approx 4 \bar{\alpha}_S \ln 2 \approx 2.65 \bar{\alpha}_S$ and a ‘reasonable’ value for $\alpha_S$, say $\alpha_S \approx 0.2$, one gets $N_{\text{min}} \approx 0.5: x^{-0.5}$ is indeed the small-$x$ behaviour of the input parton densities in the MRSD’ set. However, the perturbative (BFKL) pomeron is used here as non-perturbative input. On the contrary, the perturbative QCD evolution is performed with the NLO Altarelli-Parisi equation. In particular, since the input densities have an $x$-shape which is steeper than the one generated by the NLO QCD evolution, it follows (see the comment before Eq. (8)) that the small-$x$ behaviour of $F_2$ is dominated by that of the non-perturbative input. I find this situation a little uncomfortable, although consistent at present. As a matter of fact, there are theoretical and phenomenological problems which are still to be solved.

As regards to the theory, the BFKL equation resums the leading terms $(\alpha_S \ln x)^n$ in QCD perturbation theory but it is not an evolution equation in $k_\perp^2$. Its matching with the Altarelli-Parisi equation at larger values of $x$ (or $Q^2$) and its extension (if any) to subleading orders in $\ln x$ requires the inclusion of the running coupling $\alpha_S(k_\perp^2)$. Due to the lack of $k_\perp$-ordering, this leads to infrared instabilities coming from the Landau pole of $\alpha_S(k_\perp^2)$ in the low-$k_\perp$ (non-perturbative) region.

As regards to phenomenology, in any perturbative expansion the leading term predicts just the order of magnitude of a given quantity. The accuracy of the prediction is instead controlled by the size of the higher-order corrections.

The inclusion of subleading terms at small $x$ is therefore necessary for precise quantitative studies.

6 Beyond leading order at small $x$

A possible approach [15,28] for improving and controlling the accuracy of the perturbative QCD predictions at small $x$ (and large $Q^2$) consists in combining small-$x$ resummation with leading-twist factorization.

The generalized Altarelli-Parisi equation (8) for the evolution of the parton densities to leading-twist order systematically takes into account QCD corrections via the perturbative expansion (11) of the splitting functions $P_{ab}(z,\alpha_S)$. However, as discussed in Sects. 4.2 and 4.3, higher-loop contributions to $P_{ab}$ are logarithmically enhanced at small $x$. More precisely, in the small-$x$ limit, the $n$-loop order splitting function $P_{ab}^{(n)}$ behaves as

$$P_{ab}^{(n)}(x) \sim \frac{1}{x} \left[ \ln^{n-1} x + \mathcal{O}(\ln^{n-2} x) \right]. \quad (33)$$

Correspondingly, the anomalous dimensions $\gamma_N$ in Eq. (12) have singularities for $N \to 0$ in the form:

$$\gamma_{ab,N}(\alpha_S) = \sum_{n=1}^{\infty} \left( \frac{\alpha_S}{N} \right)^n A_{ab}^{(n)} + \alpha_S \left( \frac{\alpha_S}{N} \right)^n B_{ab}^{(n)} + \ldots, \quad (34)$$
These singularities may spoil the convergence of the perturbative expansion at small $x$. Nonetheless one can consider an improved perturbative expansion obtained by resumming the leading $(A_{ab}^{(n)})$, next-to-leading $(B_{ab}^{(n)})$, etc., coefficients in the small-$x$ regime. Once these coefficients are known, they can unambiguously be supplemented with non-logarithmic (finite-$x$) contributions exactly calculable to any fixed-order in perturbation theory. Moreover, the effects of the running coupling can be consistently included according to Eq. (13), thus avoiding the infrared instabilities encountered in phenomenological attempts [23,29] to extend the BFKL equation beyond leading order. Obviously, in this approach, the absolute behaviour of the structure function $F_2$ at small $x$ is still affected by that of the input parton densities. However, since logarithmic scaling violations are systematically under control, our confidence in making quantitative predictions at high $Q^2$ is enhanced.

The present status of small-$x$ resummation is the following. The gluon anomalous dimensions $\gamma_{gg,N}$ contain leading singularities of the type $(\alpha_s/N)^n$. These contributions are given by the BFKL anomalous dimensions in Eq. (28). The next-to-leading corrections $\alpha_s(\alpha_s/N)^n$ in the gluon sector are still unknown. Beyond leading accuracy, the quark sector has to be considered on an equal footing with the gluon sector. The next-to-leading corrections to the quark anomalous dimensions have been computed recently [28]. The first perturbative terms are explicitly given by ($T_R = 1/2$ in QCD)

$$\gamma_{gg,N}(\alpha_s) = \frac{\alpha_s}{2\pi} T_R \frac{2}{3} \left\{ 1 + \frac{5}{3} \frac{\alpha_s}{N} + \frac{14}{9} \left( \frac{\alpha_s}{N} \right)^2 + \mathcal{O}\left( \left( \frac{\alpha_s}{N} \right)^3 \right) \right\} , \quad (35)$$

and higher-loop contributions can be found in Ref. [15].

A numerical analysis of the effect of next-to-leading corrections at small $x$ has been presented recently by Ellis, Kunszt and Levin [30]. They consider both the flat ($D'_0$) and steep ($D'_-$) input distributions used by MRS and, besides carrying out a full two-loop evolution (as in the MRS analysis), they include also the $\mathcal{O}((\alpha_s/N)^4)$-term of the gluon anomalous dimensions [28] and the $\mathcal{O}((\alpha_s/N)^3)$-term of the quark anomalous dimensions (35). Their results on the structure function $F_2$ are reported in Fig. 9. We can see that, for $x \lesssim 10^{-2}$, the effect of the higher-order corrections strongly depends on the $x$-shape of the input distributions. The $Q^2$-evolution of the steep input is marginally affected by the inclusion of higher-loop contributions. On the contrary, the $Q^2$-evolution of the flat input is perturbatively unstable.

A first conclusion is that, if the input distribution is very steep, higher-order perturbative corrections are probably negligible, at least in the small-$x$ region which is being investigated at HERA. On the other side, this does not mean that a flat input has to be excluded due to its perturbative instability. In this case one should rather include higher-loop corrections at small $x$ and, possibly, recover reliable predictions after their all-order resummation. The difference between these two alternatives (steep input + fixed-order and flat input + resummation) is not just a matter of nominalism. Although the respective predictions for $F_2$ at a certain value of $Q^2$ can be very similar, the scaling violations (i.e. the $Q^2$-dependence) are stronger in the second case. Further studies are necessary for quantifying precisely the phenomenological consequences of this discussion for the HERA kinematic region, as well as, at future hadron colliders.

‡A similar perturbative stability is likely to occur in the GRV framework because of the very low input scale $Q_0$ and, hence, of the very long evolution in $Q^2$.

§Since in DIS the virtual photon couples directly to quarks (and not gluons), the quark anomalous dimensions (35) (despite being formally subleading with respect to the gluon anomalous dimensions in Eq. (28)) are largely responsible for the perturbative instability in the HERA region.
Fig. 9: Results of the QCD evolution of the steep (upper curves) and flat (lower curves) input distributions. The effect of including higher-order terms (NLL) is compared with the full one-loop (L) and two-loop (NL) evolution.

7 Conclusions

The structure function $F_2(x,Q^2)$ measured at HERA increases very steeply for $x \lesssim 10^{-2}$. A similar rise with the energy is not observed for total cross sections in soft hadronic processes. Therefore the structure function data at low-$x$ cannot be explained without hard QCD contributions.

Perturbative QCD qualitatively accounts for the steep increase of $F_2$. As regards to the sets of NLO parton densities available before HERA, the GRV and MRSD’ parton densities are favoured by the HERA data. Both sets of parton densities are obtained by perturbative QCD evolution in two-loop order. In the GRV framework ‘valence-like’ input densities at a very low input scale undergo a very long $Q^2$-evolution, thus acquiring the steep behaviour given by the Altarelli-Parisi equation \( (17) \). In the MRSD’ framework, input distributions as steep as $x^{-0.5}$ (inspired by the BFKL equation) are considered at a much higher input scale. The low-$x$ behaviour of $F_2$ at high $Q^2$ is thus dominated by that of the input densities\(^\dagger\).

Estimating the present accuracy of the perturbative QCD predictions requires the evaluation of higher-order contributions and more detailed quantitative studies. For instance, from the discussion in Sect. 6, one can argue that flat (or almost flat, as suggested by the soft-pomeron behaviour \( [6] \)) input densities at a scale of the order of 1 GeV$^2$, combined with a dynamically generated perturbative pomeron (namely, all-order resummation in the Altarelli-Parisi splitting functions), may lead to structure functions consistent with HERA data.

On the theoretical side, more calculations on next-to-leading terms at small $x$ are necessary. On the phenomenological side, I consider relevant the analysis of the $Q^2$-dependence and of the scaling violations. Studies on small-$x$ effects in the structure of the hadronic final states are

\( \dagger \)The small-$x$ behaviour of the input densities can be ‘tuned’ to describing better $F_2$. Recent fits to the 1992 HERA data, carried out by MRS and the CTEQ Collaboration, give the behaviour $x^{-0.3} \ [31]$. 

16
also important \[\text{20,32,33,34}\]. More (and more accurate) experimental information will certainly be available in the very near future.

**Acknowledgements** – I wish to thank Giorgio Bellettini and Mario Greco for the pleasant and stimulating atmosphere at this Conference.

**References**

1. B. Pope, these proceedings; J. Smith, these proceedings.
2. H1 Coll., I. Abt et al., Nucl. Phys. B407 (1993) 515.
3. ZEUS Coll., M. Derrick et al., Phys. Lett. 316B (1993) 412.
4. D. Newton, these proceedings.
5. M. Virchaux, in Proc. of the Aachen Conf. *QCD - 20 years later*, eds. P.M. Zerwas and H.A. Kastrup (World Scientific, Singapore, 1993), pag. 205 and references therein.
6. Particle Data Group, K. Hikasa et al., Review of particle properties, Phys. Rev. D45, No. 11 (1992).
7. A. Donnachie and P.V. Landshoff, Nucl. Phys. B231 (1983) 189, Phys. Lett. 296B (1992) 227.
8. ZEUS Coll., M. Derrick et al., Phys. Lett. 293B (1992) 465; H1 Coll., T. Ahmed et al., Phys. Lett. 299B (1993) 374.
9. A. Donnachie and P.V. Landshoff, Nucl. Phys. B244 (1984) 322.
10. H. Abramowicz, E.M. Levin, A. Levy and U. Maor, Phys. Lett. 269B (1991) 465.
11. A. Donnachie and P.V. Landshoff, Cambridge preprint DAMTP 93-23.
12. G. Altarelli, Phys. Rep. 81 (1982) 1 and references therein.
13. S. Catani, M. Ciafaloni and F. Hautmann, Phys. Lett. B242 (1990) 97, Nucl. Phys. B366 (1991) 135.
14. E.B. Zijlstra and W.L. van Neerven, Nucl. Phys. B383 (1992) 525.
15. S. Catani and F. Hautmann, Cambridge preprint Cavendish–HEP–94/01.
16. A. Bassetto, M. Ciafaloni and G. Marchesini, Phys. Rep. 100 (1983) 201.
17. L.N. Lipatov, Sov. J. Nucl. Phys. 23 (1976) 338; E.A. Kuraev, L.N. Lipatov and V.S. Fadin, Sov. Phys. JETP 45 (1977) 199; Ya. Balitskii and L.N. Lipatov, Sov. J. Nucl. Phys. 28 (1978) 822.
18. L.V. Gribov, E.M. Levin and M.G. Ryskin, Phys. Rep. 100 (1983) 1.
19. M. Ciafaloni, Nucl. Phys. B296 (1987) 249.
20. S. Catani, F. Fiorani and G. Marchesini, Phys. Lett. 234B (1990) 339, Nucl. Phys. B336 (1990) 18; S. Catani, F. Fiorani, G. Marchesini and G. Oriani, Nucl. Phys. B361 (1991) 645.

21. T. Jaroszewicz, Phys. Lett. B116 (1982) 291.

22. A.H. Mueller and J. Qiu, Nucl. Phys. B268 (1986) 427; A.H. Mueller, Nucl. Phys. B335 (1990) 115; J. Bartels, preprint DESY-91-074, Phys. Lett. 298B (1993) 204, Zeit. Phys. C60 (1993) 471; E.M. Levin, M.G. Ryskin and A.G. Shuvaev, Nucl. Phys. B387 (1992) 589; E. Laenen, E.M. Levin and A.G. Shuvaev, preprint Fermilab-PUB-93/243-T.

23. A. J. Askew, J. Kwiecinski, A. D. Martin and P. J. Sutton, Phys. Rev. D 47 (1993) 3775, Phys. Rev. D 49 (1994) 4402.

24. E.G. Floratos, D.A. Ross and C.T. Sachrajda, Nucl. Phys. B129 (1977) 66 (E Nucl. Phys. B139 (1978) 545), Nucl. Phys. B152 (1979) 493; A. Gonzalez-Arroyo, C. Lopez and F.J. Yndurain, Nucl. Phys. B153 (1979) 161; A. Gonzalez-Arroyo and C. Lopez, Nucl. Phys. B166 (1980) 429; G. Curci, W. Furmanski and R. Petronzio, Nucl. Phys. B175 (1980) 27; W. Furmanski and R. Petronzio, Phys. Lett. 97B (1980) 437; E.G. Floratos, P. Lacaze and C. Kounnas, Phys. Lett. 98B (1981) 89, 225.

25. M. Glück, E. Reya and A. Vogt, Zeit. Phys. C53 (1992) 127, Phys. Lett. 306B (1993) 391.

26. A.D. Martin, R.G. Roberts and W.J. Stirling, Phys. Lett. 306B (1993) 145.

27. CTEQ Coll., J. Botts et al., Phys. Lett. 304B (1993) 159.

28. S. Catani and F. Hautmann, Phys. Lett. 315B (1993) 157.

29. J. Collins and J. Kwiecinski, Nucl. Phys. B316 (1989) 307; J. Kwiecinski, A.D. Martin and P.J. Sutton, Phys. Rev. D 44 (1991) 2640.

30. R.K. Ellis, Z. Kunszt and E. M. Levin, preprint Fermilab-PUB-93/350-T.

31. A.D. Martin, R.G. Roberts and W.J. Stirling, Durham preprint DTP/93/86; CTEQ Coll., J. Botts et al., CTEQ2 sets of parton distributions.

32. L.V. Gribov, Yu.L. Dokshitzer, S.I. Troyan and V.A. Khoze, Sov. Phys. JETP 67 (1988) 1303.

33. G. Marchesini and B.R. Webber, Nucl. Phys. B349 (1991) 617, Nucl. Phys. B386 (1992) 215; B.R. Webber, in Proceedings of the HERA Workshop, eds. W. Buchmüller and G. Ingelman (DESY Hamburg, 1991), pag. 285.

34. J. Kwiecinski, A.D. Martin, P.J. Sutton and K. Golec-Biernat, Durham preprint DTP/94/08.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9406325v1
This figure "fig2-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9406325v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9406325v1
This figure "fig2-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9406325v1
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9406325v1
This figure "fig2-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9406325v1