Diffraction properties study of reflection volume holographic grating in dispersive photorefractive material under ultra-short pulse readout

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Abstract. Based on the modified Kogelnik diffraction efficiency equation, we study the diffraction intensity spectrum and the total diffraction efficiency of reflection volume holographic gratings in photorefractive media. Taking photorefractive LiNbO₃ crystal as an example, the effect of the grating parameters and the pulse width on the diffraction properties is presented under the influence of crystal material dispersion. Under the combined effects, the diffraction pulse profiles and the total diffraction efficiency are compared with and without crystal material dispersion. The results show that the dispersion will decrease the diffraction intensity. Moreover, when pulse width is smaller or the grating spacing and the grating thickness are larger, the influence of dispersion on diffraction is large. The results of our paper can be used in pulse shaping applications.

1. Introduction

Volume holographic gratings (VHGs) have a diverse range of applications, such as add/drop multiplexers, fiber and laser and beam coherent and incoherent combination etc [1-5]. Recently, the application of VHGs for manipulating laser output pulses to achieve functional waveforms with large bandwidths that can be utilized for a variety of applications has attracted a significant amount of attention because of the flexibility and the possibility of using VHGs to implement dynamic processing [6-10]. A most widely used basis for description of volume gratings is the theory of coupled waves developed by Kogelnik in 1969 [11]. However, the Kogelnik theory is from the plane wave discussion. In deduction, the effect of the material dispersion on diffraction is neglected, because there has been only one frequency component involved for plane wave; but for pulsed beam, the
dispersion can not be neglected because of the large bandwidth of pulse beam, where each frequency component corresponds to a specific refractive index in the material, which will affect the diffraction greatly. Chen Yang and Xiaona Yan [11] investigated the diffraction properties of transmitting volume holographic grating (TVHG) in dispersion photorefractive material (InP:Fe crystal) for femtosecond pulse readout, but the work on reflection volume holographic grating (RVHG) has not been found yet.

In this paper, the diffraction properties of reflection volume holographic grating are investigated using the modified diffraction efficiency equation of Kogelnik. The analytical expressions of the normalized diffraction intensity spectrum and the total diffraction efficiency are derived by modified diffraction efficiency equation of Kogelnik in section 2. The numerical analysis of the normalized diffraction intensity spectrum and the total diffraction efficiency are presented in section 3. A summary of the results and conclusions are given in the final section.

2. Theory analysis

Our discussion is based on the diagram shown in Fig. 1. we consider an ultrashort pulsed beam incident on a lossless and unslanted RVHG, which is characterized by a refractive index in the modulated region \(0 \leq z \leq d\) of the form

\[
n = n_0 + n_1 \cos(K \cdot x) \quad n_1 << n_0
\]

where \(n_0\) is the refractive index of the grating region, \(n_1\) is the amplitude of index modulation and \(K\) is the grating vector oriented perpendicular to the fringe planes with a module \(|K_0| = 2\pi / \Lambda\), where \(\Lambda\) is the period of the grating, \(d\) is the thickness of grating. \(\theta_r\) is the incident angle (the angle inside crystal is \(\theta'_r\)).

The time variation of the electric field of the incident ultrashort pulse can be described as

\[
E_r(t) = \exp(-i\omega_0 t - \frac{t^2}{T^2})
\]
where \( \omega_0 = \frac{2\pi c}{\lambda_0} \) is the central angular frequency corresponding to the central wavelength \( \lambda_0 \), parameter \( T \) is denoted by \( T = \Delta \tau / (2\sqrt{\ln 2}) \), with full width at half maximum (FWHM) \( \Delta \tau \). The Fourier transform of \( E_r(t) \) is defined as

\[
E_r(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E_r(t) \exp(i\omega t) dt = \frac{T}{2\sqrt{\pi}} \exp \left[ -\frac{T^2(\omega - \omega_0)^2}{4} \right]
\]

(3)

If the readout beam is a monochromatic plane wave, the diffraction efficiency is [12]

\[
\eta = \left(1 + \frac{1 - \xi^2/\nu^2}{\sinh^2 \sqrt{\nu^2 - \xi^2}}\right)^{-1}
\]

(4)

where \( \nu = j\pi n_\omega d / \lambda(c_s c_s)^{1/2} \), \( \xi = -\partial d / 2c_s \), \( c_s \) and \( c_s \) are obliquity factors, \( \vartheta \) is the dephasing measure, and \( d \) is the grating thickness.

When the readout beam is a pulse beam, parameter \( \nu \) and \( \xi \) are frequency dependent, so \( \eta \) is frequency dependent. In our discussion, we assume all spectrum components incident at the same angle \( \theta_r \) to readout. Due to the material dispersion, the frequency components in the ultra-short pulse will be separated according to the Snell refraction law \( \sin \theta_r = n(\omega)\sin \theta_r(\omega) \). Accordingly, the obliquity factors are changed to frequency-dependent, for an unslanted RVHG \( c_r = -c_s = \cos \theta_r(\omega) \), so

\[
\nu(\omega) = \frac{\omega n_\omega d}{2c \cos \theta_r(\omega)}
\]

(5)

As the central wavelength component of the incident pulse satisfies the Bragg diffraction condition, we have \( \cos \theta_r = \lambda_0 / 2n(\omega_0) \), then other wavelength components will not satisfy the Bragg condition, the dephasing measure can be redefined as

\[
\vartheta(\omega) = K^2 c / 2n(\omega)(1/\omega_0 - 1/\omega)
\]

(6)

where \( c \) is the velocity of light in free space, \( n(\omega) \) is the refractive index of the crystal to frequency \( \omega \).

In LiNbO_3 crystal, the wavelength dependence of the refractive index is given by [13]

\[
n^2(\omega) = a_0 + \frac{a_1 \omega^2 \times 10^{-12}}{(2\pi \omega)^2 - a_2 \omega^2 \times 10^{-12}} - a_3(2\pi \omega)^2 \times 10^{12}
\]

(7)

(3)
where \( a_0 = 4.9048 \), \( a_1 = 0.117680 \), \( a_2 = 0.047500 \), \( a_3 = 0.027169 \).

By substituting Eqs. (5) and (6) into (4), the diffraction efficiency spectrum of the RVHG is obtained

\[
\eta(\omega) = \left(1 + \frac{1 - \xi(\omega)^2/v(\omega)^2}{\sinh^2 \sqrt{v(\omega)^2 - \xi(\omega)^2}}\right)^{-1}
\]

which is applicable to all readout components \( \omega \). In our discussion, the central frequency component satisfies the Bragg condition, so the diffraction efficiency comes to its maximum. However, the other readout components deviate from the central frequency, thus the diffraction efficiencies decrease; and the larger the deviation is, the smaller the diffraction efficiency becomes.

According to the definition of the diffraction efficiency, the diffraction intensity spectrum is obtained

\[
I_d(\omega) = I_r(\omega) \cdot \eta(\omega) = |E_r(\omega)|^2 \cdot \eta(\omega)
\]

where \( I_d(\omega) \) is the diffracted intensity corresponding to frequency \( \omega \).

Defining the total diffraction efficiency as the total intensity of diffracted beam expressed by Eq. (7) to that of incident intensity, and the representation is:

\[
\eta = \frac{\int_{-\infty}^{\infty} I_d(\omega) d\omega}{\int_{-\infty}^{\infty} |E_r(\omega)|^2 d\omega}
\]

From Eqs. (8) and (10), we can see that the diffraction intensity spectrum and total diffraction efficiency are affected by the grating parameters, readout pulse width and the material dispersion.

3. Numerical analysis and discussion of diffraction properties of RVHG illuminated by an ultrashort pulse

Assuming that the central wavelength of the incident pulse beam is 1.55 \( \mu \)m, and the corresponding central angular frequency is \( \omega_0 = 1.2161 \times 10^{15} \) Hz. Other parameters are \( n_1 = 1 \times 10^{-4} \), \( d = 1 \) mm and \( \Lambda = 1 \) \( \mu \)m. In order to compare diffracted pulses with input pulses, according to Eq. (2), the input pulses at different duration \( \Delta \tau \) versus angular frequency \( \omega \) are shown in Fig. 2.
3.1 Discussion of normalized diffraction intensity spectrum

First, we calculate the normalized diffraction intensity as a function of angular frequency. In order to facilitate compare, we only give a part frequency range of diffracted spectrum. The diagram is shown in fig.3. It’s shown that the dispersion of the material has effect on the diffraction. When the readout pulse is 50fs, the influence of dispersion becomes obvious and cannot be neglected, while to the 2ps readout pulse, the effect of the dispersion on the diffraction can be neglected. The result is accordance with the TVHG, the reason is that the wider the temporal width of the pulse beam, the narrower the spectral width of it is, the smaller the dispersion of the volume holographic grating.

Figure 2. Spectral distribution of input Gaussian pulse at different pulse duration

(a) Readout pulse 50fs
(b) Readout pulse 500fs
Figure 3. Normalized diffraction intensity spectrum against the angular frequency $\omega$ on conditions of different readout pulse width: (a) $\Delta\tau=50\text{fs}$; (b) $\Delta\tau=500\text{fs}$; (c) $\Delta\tau=2\text{ps}$.

Figure 4. Normalized diffraction intensity spectrum against the frequency $\omega$ on condition of different grating thickness $\Lambda=1\mu\text{m}$ and $d=2\mu\text{m}$, where $d=1\text{mm}$ and $\Delta\tau=500\text{fs}$.

Figure 5. Normalized diffraction intensity spectrum against the frequency $\omega$ on condition of different grating thickness $d=0.5\text{mm}$ and $d=1\text{mm}$, where $\Lambda=1\mu\text{m}$ and $\Delta\tau=500\text{fs}$.

Fig.4 shows the relation between the diffraction spectrum and the grating periods when pulse duration is $500\text{fs}$. We find that when the grating period increases, the decreasing speed of diffraction intensity becomes slower and the bandwidth of the diffracted pulse increases, the influence of the dispersion of diffraction becomes larger, which means that more frequency components have been diffracted out. With the increasing of the grating spacing, the diffraction angle of the readout frequency component becomes large, the angle deviates more from the Bragg angle, thus the more obviously of the dispersion effect on diffraction. This is the direct opposite to the TVHG.
Fig. 5 is the diagram of the normalized diffraction intensity spectrum against $\omega$ on condition of different grating thicknesses. In order to survey the effect of dispersion, the figure only gives part of frequency range. It shows that the thicker the grating thickness is, the smaller the bandwidth of the diffracted pulse becomes and large influence of dispersion on diffraction. This can be explained as the following: when the thickness is large, the process between the grating and beam will become longer and more frequency components are filtered out. The larger grating thickness, the distance of diffraction is farther, the deflection angle becomes larger, this lead to the angle deviates more from the Bragg angle, so the dispersion is more obvious. The result is consistent with the TVHG.

From above discussion, we can know the function of the recorded grating can be acted as a band-pass filter, and the bandwidth can be changed by choosing different grating spacing or thickness. The effect of the dispersion on diffraction in these two diagrams can be interpreted as, with the decreasing of the grating spacing, the diffraction angle of the readout frequency component becomes larger; while to the larger grating thickness, the deflection angle becomes larger. The results of these two conditions make the angle deviate more from the Bragg angle, thus the diffraction decrease.

3.2 Discussion of the total diffraction efficiency

Fig. 6 is the diagram of the total diffraction efficiency against the pulse width as the grating period is 0.5\(\mu\)m, 1\(\mu\)m and 2\(\mu\)m, the other parameter is the same as section 3.1, and Fig. 7 show the relationships of the total diffraction efficiency to the grating spacing as pulse duration is 20fs, 100fs and 500fs. These two diagrams can be explained from Figs.3-5. For fixed grating spacing, Bragg selectivity is specified. To the larger duration pulse, more frequency components will be diffracted out and the total diffraction efficiency will increase. While to the fixed $\Delta\tau$, the smaller the grating spacing is, the stricter the Bragg condition becomes, thus fewer components will be diffracted out, the diffraction efficiency is decreasing. Moreover, the dispersion will reduce the total diffraction efficiency, and its influence on the diffraction is consistent with those in Figs3-5.

Fig. 8 is the diagram of the total diffraction efficiency against the grating thickness as the pulse width is 100ps and grating spacings are 0.5, 1 and 2\(\mu\)m. From Fig. 8 we know the total diffraction efficiency increases when the grating thickness increases from 0.5mm to 2mm. It's also shown that the dispersion becomes more obvious with the increasing of grating thickness, which is consistent with the discussion in Fig.5.
Figure 6. Diagram of the total diffraction efficiency as $\Delta \tau$ for both considering and neglecting the dispersion effect: with grating spacing of 0.5$\mu$m, 1$\mu$m and 2$\mu$m.

Figure 7. Diagram of the total diffraction efficiency against the grating spacing from 0.2 to 2$\mu$m with readout pulse width of 20fs, 100fs, 500fs.

Figure 8. Diagram of the total diffraction efficiency against the grating thickness as the grating spacing is 0.5$\mu$m, 0.75$\mu$m and 1$\mu$m.

4. Conclusion
Based on modified the diffraction efficiency equation of Kogelnik, the total diffraction efficiency and diffraction intensity spectrum of the reflection holographic volume grating under ultrashort pulse illumination are investigated in the dispersion material. The results show that by controlling the grating spacing, thickness and readout pulse width, the diffraction will have much difference. Moreover, the dispersion effect decreases the total relative intensity and the total diffraction efficiency. Especially when the readout pulse is small and the grating thickness is relatively large, the influence of the dispersion on diffraction becomes obvious, this result is consistent with the TVHG, but when the grating spacing is larger, the influence of the dispersion on diffraction is obvious, which is contrary to the TVHG. It’s also show the effect of dispersion in LiNbO$_3$ crystal of RVHG is smaller than the TVHG in InP:Fe crystal. The results presented in this paper could be applied in pulse shaping and signal processing.

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