Research Article

Design of Feedback Control for Networked Finite-Distributed Delays Systems with Quantization and Packet Dropout Compensation

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This paper investigates the feedback control for networked discrete-time finite-distributed delays with quantization and packet dropout, and systems induce the $H_{\infty}$ control problem. The compensation scheme occurs in a random way. The quantization of system state or output signal is in front of being communicated. It is shown that the design of both a state feedback controller and an observer-based output feedback controller can be achieved, which ensure the asymptotical stability as well as a prescribed $H_{\infty}$ performance of the resulting closed-loop systems satisfying dependence on the size of the discrete and distributed delays. Numerical examples are given to illustrate the effectiveness and applicability of the design method in this paper.

1. Introduction

Recurrent networked control systems (NCSs) have been extensively studied in recent years due to their great significance for both practical and theoretical purposes, such as mobile sensor networks, vehicles and crafts, communication networks, and Internet-based control. The stability and control problem of NCSs has aroused increasing interest and has been widely studied [1–4]. On the other hand, it is well known that the insertion of communication networks in control loops results in some inevitable phenomena including signal-transmission delay, packet dropout, and quantization errors, which can degrade the system performance and even destabilize the system. Therefore, the main goal is how to model them more accurately and quantitatively eliminate or compensate the effect caused by the communication delays and data package loss [5, 6]. Almost all time delays studied in the aforementioned literature fall into the discrete case. In fact, there is still another type of time delay, namely, distributed delay, which occurs in a lot of practical plants and has been drawing increasing attention [7]. Naturally, it turns out to be meaningful to investigate the issue of how distributed delays influence the dynamical behavior of the discrete-time NCSs. However, only few papers are focused on discrete-time finite-distributed delay systems.

Quantization always exists in digital/analog interface control systems and quantization errors have adverse effects on the NCSs’ performance. In the early 1990s, the quantized problem for time-invariant discrete-time linear system was proposed in [8], where quantized state feedback was
employed to stabilize an unstable linear system. Excitingly, there is a new evolution of research on the quantization effect on NCSs where a quantizer is thought of as an information coder. Therefore, it is necessary to guide an analysis on the quantizers and comprehend how much effect the quantization makes on the overall systems.

For NCSs, packet dropouts and data missing are unavoidable phenomena existing due to the limited transmission capacity of the networks. Recently, the predictive controller design of networked systems with communication delay and data loss has been dealt with in [9], in which a networked predictive control scheme is employed to compensate for communication delay and data loss actively rather than passively. When input and output signals quantization was considered, a sufficient condition for the existence of quantized static output feedback controller was proposed in [10]. In addition, in the presence of packet dropouts, the system-performance requirements such as robustness and disturbance rejection attenuation were gained; the state feedback quantized $H_{\infty}$ control problem was solved in [11] via a linear matrix inequality (LMI) approach. It is noted that all the above results are derived for packet-loss problem with discrete delays. Recently, although the networked-based feedback control problem for systems with discrete and infinite-distributed delays involving quantization and dropout was found in [12], another typical kind of packet dropouts, which include the finite-distributed delays including quantization and packet dropout, the following quantizers and comprehend how much effect the quantization makes on the overall systems.

This paper tackles the state feedback robust $H_{\infty}$ control problem for networked control systems with discrete and infinite-distributed delays including quantization and packet dropouts. In the NCSs, it is assumed that the measurement signals are quantized before being communicated. The compensation scheme is presented to deal with the effect of random packet dropout through communication network, which obeys a Bernoulli distributed white sequence taking on values of zero and one with certain probability [13]. Both a state feedback controller and an observer-based output feedback controller are designed such that the closed-loop NCS is stable, and the prescribed $H_{\infty}$ disturbance-rejection-attenuation performance is also achieved. Both stability-analysis and controller synthesis problems are thoroughly investigated. It is shown that the controller-design problem under consideration is solvable if certain linear matrix inequalities (LMIs) are feasible. Two simulation examples are exploited to demonstrate the effectiveness of the proposed LMI approach.

Throughout this paper, the notation $X \succeq Y$ ($X > Y$) for symmetric matrices $X$ and $Y$ indicates that the matrix $X - Y$ is positive and semidefinite (resp., positive definite), $Z^T$ represents the transpose of matrix $Z$, and the vector norm $\| \cdot \|$ indicates the Euclidean vector norm; that is, $\|W\| = \lambda_M^{1/2}(W^TW)$, where $\lambda_M(W)$ (resp., $\lambda_m(W)$) denotes the operation of taking the maximum (resp., minimum) eigenvalue of $W$.

## 2. Preliminaries

Consider the following discrete-time networked control system:

$$
\begin{align*}
\dot{x}(k+1) & = Ax(k) + Bx(k-\tau(k)) + C \sum_{i=1}^{d(k)} h(x(k-i)) + Du(k) \\
& + Ew(k) , \\
y(k) & = E_j x(k) , \\
z(k) & = C_z x(k) , \\
x(j) & = \phi(j), \quad -\infty < j \leq 0,
\end{align*}
$$

where $x(k) = (x_1(k), x_2(k), \ldots, x_n(k))^T$ is the state vector; $u(k) \in R^l$ is the control input; $y(k) \in R^m$ is the measured system output; $z(k) \in R^p$ is the signal to be estimated; $A, B, C, D, E, E_j, C_z$ are known real matrices with appropriate dimensions; $h(x(k)) = [h_1(x(k)), h_2(x(k)), \ldots, h_n(x(k))]^T$ is nonlinear functions; $w(k) \in R^r$ is the exogenous disturbance signal belonging to $L_2[0, \infty); \phi(j), \quad -\infty < j < 0$, are the initial conditions. $\tau(k)$ and $d(k)$ denote the discrete time-varying delay and distributed time-varying delays, respectively, satisfying

$$
\begin{align*}
\tau_1 \leq \tau(k) \leq \tau_2, \quad k \in N^+ , \\
d_1 \leq d(k) \leq d_2, \quad k \in N^+ ,
\end{align*}
$$

where $\tau_1, \tau_2, d_1,$ and $d_2$ are known positive integers.

Note that the measured signals will be quantized through the network before they are transmitted to the controller. The set of quantized levels is described as

$$
U = \{ \pm \lambda_i, \ i = \pm 1, \pm 2, \ldots \} \cup \{0\} .
$$

A quantizer is called logarithmic if the set of quantized levels is characterized by

$$
U = \{ \pm \lambda_i, \ \lambda_i = \rho \lambda_0, \ i = \pm 0, \pm 1, \pm 2, \ldots \} \cup \{0\} ,
$$

where the parameter $\rho$ is called the quantization density. For the logarithmic quantizer, the associated quantizer $f(\cdot)$ is defined as follows [14, 15]:

$$
f(\nu) = \begin{cases} 
\lambda_1, & \text{if } \nu < \nu \leq \lambda_1, \\
0, & \text{if } \nu = 0, \\
-f(-\nu), & \text{if } \nu < 0,
\end{cases}
$$

where $\lambda_1 = (1/(1+\delta))\lambda_1$, and $\lambda_1 = (1/(1-\delta))\lambda_1$, with $\delta = (1-\rho)/(1+\rho)$.

To realize the state feedback control of system (1)--(4) including quantization and packet dropout, the following
compensation scheme is presented at the side of the controller so as to deal with the effect of data loss:

\[
\bar{x}(k) = (1 - \theta_k) f(x(k)) + \theta_k \bar{x}(k-1),
\]

(9)

where \(\bar{x}(k) \in R^k\) is the state vector of compensation scheme; \(f(\cdot)\) is the logarithmic quantizer defined in (8); the stochastic variable \(\theta_k \in R\) is Bernoulli distributed white sequence with

\[
P_r(\theta_k = 1) = E[\theta_k] = \bar{\theta}.
\]

(10)

Then, a state feedback controller based on quantized state information and packet dropout compensation is designed as

\[
u(k) = K\bar{x}(k),
\]

(11)

where \(K\) is appropriately dimensioned controller gain matrix to be designed later. Combining compensation scheme (9) with system (1), the closed-loop system with quantization can be obtained:

\[
x(k+1) = Ax(k) + Bx(k-\tau(k)) \\
+ C \sum_{i=1}^{d(k)} h(x(k-i)) + (1-\theta_k)DK\bar{x}(k-1) + Ew(k).
\]

(12)

In terms of the method given in [15], the quantizing effects can be obtained:

\[
f(x(k)) - x(k) = \Delta_k x(k), \quad \|\Delta_k\| \leq \delta.
\]

(13)

For a given quantization density \(\rho\), it should be pointed out that, in the closed-loop system (12), there appears stochastic quantity \(\theta_k\). This differs from the traditional deterministic systems such as linear, discrete delay, and no-packet-loss systems, which are special case of our paper. The purpose of this paper is to design controllers for the system with quantization, such that, in the presence of data packet dropout, for any constant time delays \(\tau_1, \tau_2, d_1,\) and \(d_2\) satisfying (5), the closed-loop system is mean-square stable and the \(H_{\infty}\) performance

\[
\sum_{k=0}^{\infty} E\{\|z(k)\|^2\} \leq \gamma^2 \sum_{k=0}^{\infty} E\{\|w(k)\|^2\},
\]

(14)

under the zero-initial condition for any nonzero \(w(k) \in L_2[0, \infty)\), where \(\gamma > 0\) is a given scalar.

In order to get our main results, nonlinear function \(h(\cdot)\) is assumed to be bounded and satisfy the following assumption.

**Assumption 1.** For each \(i \in \{1, 2, 3, \ldots, n\}\), the function \(h_i : R \rightarrow R\) is Lipschitz continuous with a Lipschitz constant \(\sigma_i\); that is, there exists constant \(\sigma_i\) such that, for any \(x_1, x_2 \in R\), \(x_1 \neq x_2\), the functions satisfy

\[
0 \leq \frac{h_i(x_1) - h_i(x_2)}{x_1 - x_2} \leq \sigma_i, \quad i = 1, 2, \ldots, n,
\]

(15)

where \(\sigma_i > 0, i = 1, 2, \ldots, n\).

The following lemmas are essential for developing our main results.

**Lemma 2** (see [12]). Let \(D, S, \) and \(F\) be real matrices of appropriate dimensions with \(F\) satisfying \(F^T F \leq I\). Then, for any scalar \(\varepsilon > 0\), the following equation is satisfied:

\[
DFS + (DFS)^T \leq \varepsilon^{-1}DD^T + \varepsilon S^T S.
\]

(16)

**Lemma 3** (see [16]). For real matrices \(Z_1 > 0, Z_2 > 0, E, T, \) and \(S\) with appropriate dimensions. The following matrix inequalities hold:

\[
\begin{bmatrix}
EZ_m^{-1}E^T + TZ_m^{-1}T^T & S \\
S^T & Z_{m1} + Z_{m2}
\end{bmatrix} \geq 0,
\]

(17)

\[
\begin{bmatrix}
EZ_m^{-1}E^T & E \\
Z_{m1}^T & Z_{m2}
\end{bmatrix} \geq 0,
\]

(18)

where \(S = E - T\).

**Lemma 4** (see [12]). Let \(Z \in R^{n 	imes n}\) be a positive semidefinite matrix, \(x_i \in R^n\), and constant \(\beta_i > 0 (i = 1, 2, \ldots)\). If the series concerned is convergent, then the following inequality holds:

\[
\left( \sum_{i=1}^{\infty} \beta_i x_i \right)^T Z \left( \sum_{i=1}^{\infty} \beta_i x_i \right) \leq \left( \sum_{i=1}^{\infty} \beta_i \right) \sum_{i=1}^{\infty} \beta_i x_i Z x_i.
\]

3. Mathematical Formulation of the Proposed Approach

For convenience, without loss of generality, the state of system (1)–(3) is assumed to be measurable and will be quantized before it is transmitted to the controller through communication network while the data packet dropout happens. A sufficient condition is established for the asymptotically mean-square stability of the closed-loop system with quantization (12) for \(w(k) = 0\) and for the signal to be estimated \(z\) for satisfying the \(H_{\infty}\) disturbance attenuation in (14). Before proof of the main results, for simplicity, we denote the following null equations:

\[
-2\eta^T(k) G \left[ x(k+1) - Ax(k) - B \\
- C \sum_{i=1}^{d(x)} h(x(k-i)) + (1-\theta_k)kf(x(k))
\right]
\]

(19)
\[-\theta_k Dk\bar{x}(k-1) - Ew(k) = 0,\]
\[2\eta^T(k) \Gamma [\bar{x}(k) - (1 - \theta_k) q(x(k)) - \theta_k \bar{x}(k-1)] = 0,\]
\[-2\eta^T(k) U [f(x(k)) - x(k) - \Delta_k x(k)] = 0,\]

(19)

where

\[\eta(k) = \left[ x^T(k) \ x^T(k - \tau(k)) \ x^T(k - \tau_2) \ h^T(x(k)) \left( \sum_{i=1}^{d(k)} h(x(k - i)) \right) \ f^T(x(k)) \ \bar{x}^T(k-1) \ x^T(k+1) \ \bar{x}(k) \ \omega^T(k) \right]^T,\]

\[G = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \end{bmatrix}^T,\]
\[\Gamma = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_1^T & 0 & 0 \end{bmatrix}^T,\]
\[U = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & U_1^T & 0 & 0 \end{bmatrix}^T,\]

where \(I\) denotes the identity matrix of appropriate dimension and \(U_1^T\) and \(\Gamma_1^T\) are free-weighting matrices with appropriate dimension:

\[\Phi_1 = 2 \left[ x^T(k) H_1 + x^T(k - \tau(k)) H_2 + x^T(t - \tau_2) H_3 + x^T(t - \tau_1) H_4 \right] \begin{bmatrix} x(k) \\
-x(k - \tau(k)) - \sum_{j=k-\tau_1+1}^{k-\tau_2} (x(j) - x(j - 1)) \end{bmatrix} = 0,\]

(21)

\[\Phi_2 = 2 \left[ x^T(k) S_1 + x^T(k - \tau(k)) S_2 + x^T(k - \tau_2) S_3 + x^T(t - \tau_1) S_4 \right] \begin{bmatrix} x(k - \tau(k)) - x(k - \tau_2) \\
-x(k - \tau(k)) - \sum_{j=k-\tau_2+1}^{k-\tau_1} (x(j) - x(j - 1)) \end{bmatrix} = 0,\]

(22)

\[\Phi_3 = -2 \left[ x^T(k) T_1 + x^T(k - \tau(k)) T_2 + x^T(k - \tau_1) T_3 + x^T(k - \tau_1) T_4 \right] \begin{bmatrix} x(k - \tau_1) \end{bmatrix} = 0,\]

(23)

\[\Phi_4 = 2h^T(x(k)) Rh(x(k)) - 2h^T(x(k)) Rh(x(k)) = 0,\]

(24)

Now, the following theorem reveals that such conditions can be expressed in terms of LMIs.

**Theorem 5.** Under Assumption 1, given any delays \(\tau_1, \tau_2, d_1,\) and \(d_2\) satisfying (5) and supposing quantization density \(\rho > 0\) and packet dropout rate \(\tilde{\theta}\), the closed-loop system (9) and (12) with controller (11) is asymptotically stable in the mean square and (14) is satisfied under zero-initial conditions for any nonzero \(w(k) \in L_2[0, \infty)\) if there exist matrices \(P_i > 0 (i = 1, 2), Q_i > 0 (i = 1, 2, 3),\) and \(Z_i > 0 (i = 1, 2, 3)\) and diagonal matrices \(R > 0\) and matrices \(H_i, S_i,\) and \(T_i (i = 1, 2, 3, 4)\) of appropriate dimensions and positive scalar \(\epsilon\) and scalar \(\gamma \geq 0\) such that the following LMI holds:

\[
\begin{bmatrix}
\Omega & \tau_2 H & \tau_{21} T & \delta U \\
\tau_2 E^T & -\tau_2 Z_1 & 0 & 0 \\
\tau_{21} T^T & 0 & -\tau_{21} Z_2 & 0 \\
\delta U^T & 0 & 0 & -\epsilon I
\end{bmatrix} < 0,
\]

(25)
where

\[
\Omega = \begin{bmatrix}
\Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} & \Omega_{17} & \Omega_{18} & \Omega_{19} & \Omega_{21} & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} & \Omega_{26} & \Omega_{27} & \Omega_{28} & \Omega_{29} & \Omega_{31} & \Omega_{32} & \Omega_{33} & \Omega_{34} & \Omega_{35} & \Omega_{36} & \Omega_{37} & \Omega_{38} & \Omega_{39} & \Omega_{41} & \Omega_{42} & \Omega_{43} & \Omega_{44} & \Omega_{45} & \Omega_{46} & \Omega_{47} & \Omega_{48} & \Omega_{49} & \Omega_{51} & \Omega_{52} & \Omega_{53} & \Omega_{54} & \Omega_{55} & \Omega_{56} & \Omega_{57} & \Omega_{58} & \Omega_{59} & \Omega_{61} & \Omega_{62} & \Omega_{63} & \Omega_{64} & \Omega_{65} & \Omega_{66} & \Omega_{67} & \Omega_{68} & \Omega_{69} & \Omega_{71} & \Omega_{72} & \Omega_{73} & \Omega_{74} & \Omega_{75} & \Omega_{76} & \Omega_{77} & \Omega_{78} & \Omega_{79} & \Omega_{81} & \Omega_{82} & \Omega_{83} & \Omega_{84} & \Omega_{85} & \Omega_{86} & \Omega_{87} & \Omega_{88} & \Omega_{89} & \Omega_{91} & \Omega_{92} & \Omega_{93} & \Omega_{94} & \Omega_{95} & \Omega_{96} & \Omega_{97} & \Omega_{98} & \Omega_{99} & \Omega_{101} & \Omega_{102} & \Omega_{103} & \Omega_{104} & \Omega_{105} & \Omega_{106} & \Omega_{107} & \Omega_{108} & \Omega_{109} & \Omega_{111}
\end{bmatrix},
\]

and where

\[
\begin{align*}
\Omega_{11} &= -P_1 + (1 + \tau_{21}) Q_1 + Q_2 + Q_3 + H_1 + H_1^T \\
&\quad + \tau_2 Z_1 + \tau_{21} Z_2 + C_T^T C_2 + \epsilon I, \\
\Omega_{12} &= -H_1 + H_1^T + T_1 + S_1, \\
\Omega_{13} &= -S_1 + H_3^T, \\
\Omega_{14} &= -T_1 + H_4^T, \\
\Omega_{15} &= \Sigma^T R, \\
\Omega_{16} &= 0, \\
\Omega_{17} &= U_1^T, \\
\Omega_{18} &= 0, \\
\Omega_{19} &= -\tau_2 Z_1 - \tau_{21} Z_2 + A^T, \\
\Omega_{110} &= 0, \\
\Omega_{111} &= 0, \\
\Omega_{22} &= -Q_1 - H_2 - H_1^T + S_2 + S_2^T + T_2 + T_2^T, \\
\Omega_{23} &= -S_2 - H_3^T + T_3 + S_3, \\
\Omega_{24} &= -H_4^T - T_4 + T_4^T + S_4^T, \\
\end{align*}
\]
\(\Omega_{47} = 0,\)
\(\Omega_{48} = 0,\)
\(\Omega_{49} = 0,\)
\(\Omega_{410} = 0,\)
\(\Omega_{411} = 0,\)
\(\Omega_{55} = -R^T - R\)
\[= - R - R^T + \left[ (2d_2 - d_1) + \frac{1}{2} (d_2 - d_1) (d_2 + d_1 - 1) \right] Z_3, \]
\(\Omega_{56} = 0,\)
\(\Omega_{57} = 0,\)
\(\Omega_{58} = 0,\)
\(\Omega_{59} = 0,\)
\(\Omega_{510} = 0,\)
\(\Omega_{511} = 0,\)
\(\Omega_{66} = - \frac{1}{d_2} Z_3,\)
\(\Omega_{67} = 0,\)
\(\Omega_{68} = 0,\)
\(\Omega_{69} = C^T,\)
\(\Omega_{610} = 0,\)
\(\Omega_{611} = 0,\)
\(\Omega_{77} = -U_1 - U_1^T,\)
\(\Omega_{78} = 0,\)
\(\Omega_{79} = ( (1 - \overline{d}) DK)^T,\)
\(\Omega_{710} = (1 - \overline{d})^T \Gamma_1,\)
\(\Omega_{711} = 0,\)
\(\Omega_{88} = - P_2,\)
\(\Omega_{89} = (\overline{d} DK)^T,\)
\(\Omega_{810} = \overline{g} \Gamma_1,\)
\(\Omega_{811} = 0,\)
\(\Omega_{99} = - 2I + P_1 + r_2 Z_1 + r_3 Z_2,\)
\(\Omega_{910} = 0,\)
\(\Omega_{911} = 0,\)
\(\Omega_{1010} = - \Gamma_1 - \Gamma_1^T + P_2,\)
\(\Omega_{1011} = E,\)
\(\Omega_{1111} = - \gamma^2 I,\)
Define the difference of (29) along the solution of (9) and (12) and (13) with \( \omega(k) = 0 \) and take the mathematical expectation
\[
E\{\Delta V(k)\} = E\{V_j(k+1)\} - E\{V_i(k)\}; \text{ one has}
\]
\[
E\{\Delta V_i\} = x^T(k+1) P_1 x(k) - x^T(k) P_2 x(k) + \bar{x}^T(k) P_2 \bar{x}(k) - \bar{x}^T(k-1) P_2 \bar{x}(k-1),
\]
\[
E\{\Delta V_2\} = x^T(k) \left[ \left( (\tau_2 - \tau_1) + 1 \right) Q_1 x(k) + x^T(k) \cdot Q_2 x(k) - x^T(k) \cdot \tau_2 \right] \theta x(k) - \tau_2 + x^T(k) Q_3 x(k) - x^T(k - \tau_1) \cdot Q_3 x(k - \tau_1),
\]
\[
E\{\Delta V_3\} = \tau_2 (x(k+1) - x(k))^T Z_1 (x(k+1) - x(k)) - x(k) - \tau_2 \sum_{j=k-\tau_2+1}^{k-\tau(k)} (x(j) - x(j-1))^T Z_1 (x(j) - x(j-1)),
\]
\[
E\{\Delta V_4\} = (\tau_2 - \tau_1) (x(k+1) - x(k))^T Z_2 (x(k+1) - x(k)) - x(k) - \tau_2 \sum_{j=k-\tau(k)+1}^{k-\tau(k)} (x(j) - x(j-1))^T Z_2 (x(j) - x(j-1)),
\]
\[
E\{\Delta V_5\} \leq \left[ 2d_2 - d_1 \right] + \frac{1}{2} \left( d_2 - d_1 \right) \left( d_2 + d_1 - 1 \right)
\cdot h^T(x(k)) Z_3 h(x(k)) - \frac{1}{d_2} \left( \sum_{i=1}^{d(k)} h(x(k-i)) \right)^T
\cdot Z_3 \left( \sum_{i=1}^{d(k)} h(x(k-i)) \right).
\]

It follows from Lemma 3 that (36) is achieved:
\[
\eta^T(k) \left[ \tau_2 H Z_1^{-1} H^T + \tau_2 T Z_2^{-1} T^T \right] \eta(k) - \sum_{j=k-\tau(k)+1}^{k-\tau(k)} \eta^T(k) \left[ H Z_1^{-1} H^T + T Z_2^{-1} T^T \right] \eta(k) = 0.
\]

On the other hand, based on Lemma 3, the following inequalities are also true:
\[
\sum_{j=k-\tau(k)+1}^{k-\tau(k)} \left\{ \eta^T(k) \left( H Z_1^{-1} H^T + T Z_2^{-1} T^T \right) \eta(k) + 2\eta^T(k) \right\} > 0,
\]
\[
\sum_{j=k-\tau(k)+1}^{k-\tau(k)} \left\{ \eta^T(k) T Z_2^{-1} T^T \eta(k) - 2\eta^T(k) \right\} > 0.
\]

Using Assumption 1 and noting that \( R > 0 \) is diagonal matrix, one has
\[
2h^T(x(k)) R h(x(k)) \leq 2h^T(x(k)) R \Sigma x(k),
\]
where \( \Sigma = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_n) \), in which \( \sigma_i > 0 \) (\( i = 1, 2, \ldots, n \)) are given in Assumption 1.
Combining (31)–(35), (19)–(24), and (36)–(38) and using Lemma 2 lead to
\[
E\{\Delta V\} \leq \eta^T(k) \left[ \prod_{j=k-\tau(k)+1}^{k-\tau(k)} \left( \tau_2 H Z_1^{-1} H^T + \tau_2 T Z_2^{-1} T^T + \epsilon^{-1} \delta^2 U U^T \right) \right] \cdot \eta(k),
\]
where \( \Pi = \Pi(i, j), i = 1, 2, \ldots, 11, j = 1, 2, \ldots, 11 \), with

\[
\Pi(1, 1) = -P_1 + (1 + \tau_{21}) Q_1 + Q_2 + Q_3 + H_1 + H_T^T
+ \tau_2 Z_1 + \tau_2 Z_2 + \varepsilon I,
\]

\[
\Pi(1, 2) = -H_1 + H_T^T + T_1 + S_1,
\]

\[
\Pi(1, 3) = H_T^T - S_1,
\]

\[
\Pi(1, 4) = H_T^T - T_1,
\]

\[
\Pi(1, 5) = \left(R \sum_{i=1}^{10}\right)^T,
\]

\[
\Pi(1, 7) = U_1^T,
\]

\[
\Pi(1, 9) = A^T - \tau_2 Z_1 - \tau_{21} Z_2,
\]

\[
\Pi(2, 2) = -Q_1 - H_2 - H_T^T + T_2 + T_2^T + S_2 + S_T^T,
\]

\[
\Pi(2, 3) = -H_T^T - S_2 + S_T^T + T_3,
\]

\[
\Pi(2, 4) = -H_T^T - T_4 + T_4^T + S_T^T,
\]

\[
\Pi(2, 9) = B^T,
\]

\[
\Pi(3, 3) = -S_3 - S_T^T - Q_2,
\]

\[
\Pi(3, 4) = -T_3 - S_T^T,
\]

\[
\Pi(4, 4) = -Q_3 - T_4 - T_4^T,
\]

\[
\Pi(5, 5) = -R - R^T
+ \left[2 d_2 - d_1\right] + \frac{1}{2} \left(d_2 - d_1\right) \left(d_2 + d_1 - 1\right) Z_3,
\]

\[
\Pi(6, 6) = -\frac{1}{d_2} Z_3,
\]

\[
\Pi(6, 9) = C^T,
\]

\[
\Pi(7, 7) = -U_1 - U_1^T,
\]

\[
\Pi(7, 9) = \left((1 - \theta) D K\right)^T,
\]

\[
\Pi(7, 10) = (1 - \theta)^T \Gamma_1,
\]

\[
\Pi(8, 8) = -P_2,
\]

\[
\Pi(8, 9) = (\theta D K)^T,
\]

\[
\Pi(8, 10) = \theta \Gamma_1,
\]

\[
\Pi(9, 9) = -2I + P_1 + \tau_2 Z_1 + \tau_{21} Z_2,
\]

\[
\Pi(10, 10) = -\Gamma_1 - \Gamma_1^T + P_2,
\]

\[
\Pi(10, 11) = E,
\]

in which the other elements vanish. Therefore, from \( w(k) = 0 \), we have \( E[\Delta V] < 0 \) for all nonzero \( \eta(k) \). This means that system (9) and (12) is asymptotically stable in the mean square for any delays \( \tau_1, \tau_2, d_1, \) and \( d_2 \) satisfying (5).

Next, we establish \( H_\infty \) performance of the controller process under zero-initial condition. We introduce the following performance index:

\[
J_N = \sum_{k=0}^{N} E \left[Z^T(k) Z(k) - r^2 w^T(k) w(k)\right]
= \sum_{k=0}^{N} E \left[Z^T(k) Z(k) - r^2 w^T(k) w(k)\right]
+ [V(k + 1) - V(k)] - E [V(N + 1)]
\leq \sum_{k=0}^{N} E \left[Z^T(k) Z(k) - r^2 w^T(k) w(k) + \Delta V(k)\right].
\]

Construct the same Lyapunov-Krasovskii functional as in Theorem 5. A similar manipulation as in the proof of Theorem 5 yields

\[
J_N \leq \sum_{k=0}^{N} \eta^T(k) \Psi \eta(k) ,
\]

where \( \Psi = \Omega + \tau_2 H Z_{21}^{-1} H_T^T + \tau_3 T Z_{21}^{-1} T_T^T + \varepsilon^{-1} \delta^2 U U^T \).

On the other hand, by the Schur complement, it follows from (25) which implies \( I_N \leq 0 \) for any nonzero \( w(k) \in L_2(0, \infty) \). Thus \( N \to \infty \); for any delays \( \tau_1, \tau_2, d_1, \) and \( d_2 \) satisfying (22) and (23), the inequality in (31) holds. This completes the proof.

Now, we are in position to present study to both a state feedback controller and an observer-based output feedback controller, which are designed such that the closed-loop NCS is stable, and the prescribed \( H_\infty \) disturbance-rejection-attenuation performance is also achieved. First, the following compensator is constructed to deal with the packet dropout

\[
y_c(k) = (1 - \theta_k) f(y(k)) + \theta_k y_c(k - 1),
\]

where \( y_c(k) \in R^k \) is the state vector of compensation scheme; \( f(\cdot) \) is the logarithmic quantizer defined in (8); the stochastic variable \( \theta_k \in R \) is Bernoulli distributed white sequence with (10). Then, output feedback controller based on
quantized state information and packet dropout compensation is designed as

\[ x(k + 1) = Ax(k) + Bx(k - \tau(k)) + Du(k) \\
+ L \left( y(k) - E_i x(k) \right) \]

\[ + C \sum_{i=1}^{d(k)} h(x(k - i)) \]

\[ u(k) = K x(k), \]

(44)

where \( K \) and \( L \) are appropriately dimensioned state feedback controller and observer-based output feedback controller gain matrices to be designed later, respectively. Similarly, the quantization error can be obtained by

\[ e(k + 1) = (A - LE_i) e(k) + Be(K - \tau(k)) \]

\[ + C \sum_{i=1}^{d(k)} h(e(k - j)) - (1 - \theta_k) Lf(y) \]

\[ + Ly(k) - \theta_k Ly_c(k - 1) + Ew(k), \]

(45)

Substituting (45) into (1), one has

\[ x(k + 1) = (A + DK)x(k) - DKe(k) \]

\[ + Bx(k - \tau(k)) + C \sum_{i=1}^{d(k)} h(x(k - j)) \]

\[ + Ew(k). \]

Next, we are able to focus on the analysis of \( H_{\infty} \) performance of the observer-based output feedback control design problem for (42) and (47) and (48), which depends on the size of the delays.

**Theorem 6.** Under Assumption 1, given any delays \( \tau_1, \tau_2, d_1, \) and \( d_2 \) satisfying (5) and supposing quantization density \( \rho > 0 \) and packet dropout rate \( \hat{\theta} \), the closed-loop system (42) and (47) and (48) with controllers (44) and (45) is asymptotically stable in the mean square and (14) is satisfied under zero-initial conditions for any nonzero \( w(k) \in L_2[0, \infty) \) if there exist matrices \( P_x > 0, P_c > 0, \) and \( Q_{ix} > 0 \) \((i = 1, 2, 3)\) and \( Q_{ic} > 0 \) \((i = 1, 2, 3)\) and \( Z_{ix} > 0 \) \((i = 1, 2, 3)\) and diagonal matrices \( R_k > 0 \) and \( R_c > 0 \) and matrices \( H_i, S_i, \) and \( T_i \) \((i = 1, 2, 3, 4)\) of appropriate dimensions and positive scalar \( \varepsilon \) and scalar \( \gamma \geq 0 \) such that the following LMI holds:

\[
\begin{bmatrix}
\Pi & \tau_2 M & \tau_{21} N & \tau_3 H & \tau_{31} T & \delta U \\
\tau_2 M^T & -\tau_2 Z_{1x} & 0 & 0 & 0 & 0 \\
\tau_{21} N^T & 0 & -\tau_{21} Z_{2x} & 0 & 0 & 0 \\
\tau_3 H^T & 0 & 0 & -\tau_2 Z_{4x} & 0 & 0 \\
\tau_{31} T^T & 0 & 0 & 0 & -\tau_{31} Z_{2x} & 0 \\
\delta U & 0 & 0 & 0 & 0 & -\varepsilon I
\end{bmatrix} < 0,
\]

where \( \Pi(i, j), i = 1, 2, \ldots, 19, j = 1, 2, \ldots, 19, \)

\[ \Pi(1, 1) = -P_x + [1 + (\tau_2 - \tau_1)] Q_{ix} + Q_{ic} + M_1 + M_{11} + \tau_1 Z_{1x} \]

\[ + (\tau_2 - \tau_1) Z_{3x} + C_i^T C_i, \]

\[ \Pi(1, 2) = -M_1 + M_{11}^T + N_1 + W_1, \]

\[ \Pi(1, 3) = -W_1 + M_{11}, \]

\[ \Pi(1, 4) = -N_1 + M_{11}^T, \]

\[ \Pi(1, 5) = (R_c S_0)^T, \]

\[ \Pi(1, 7) = (A + Db) F_i^T - \tau_2 Z_1 - (\tau_2 - \tau_1) Z_2, \]

\[ \Pi(2, 2) = -Q_{ix} + M_2 - M_{11} + N_2 + N_{11}^T + W_2 + W_{11}^T, \]

\[ \Pi(2, 3) = -M_{11}^T - W_1 + W_3 + N_{11}^T, \]

\[ \Pi(2, 4) = -M_{11}^T - N_2 + N_{11}^T + W_{11}^T, \]

\[ \Pi(2, 7) = B^T, \]

\[ \Pi(3, 3) = -W_3 - W_{11}^T - Q_{3x}, \]

\[ \Pi(3, 4) = -N_3 - W_{11}^T, \]

\[ \Pi(4, 4) = -Q_{3x} - N_3 - N_{11}^T, \]

\[ \Pi(5, 5) = -R_c - R_{11}^T + \left[ (2d_2 - d_1) + \frac{1}{2}(d_2 + d_1 - 1) \right] Z_{1x}, \]

\[ \Pi(6, 6) = -\frac{1}{d_2} Z_{3x}, \]

\[ \Pi(6, 7) = C_i^T, \]

\[ \Pi(7, 7) = -F_1 - F_{11}^T + P_x + \tau_2 Z_{1x} + (\tau_2 - \tau_1) Z_{2x}, \]

\[ \Pi(7, 8) = -DK, \]
\[
\Pi(8, 8) = [1 + \tau_1] Q_1 e + Q_2 e + Q_3 e - P e + H_1 + H_T 1 + \tau_2 Z_1 e + \tau_2 Z_2 e,
\]
\[
\Pi(8, 9) = - H_1 + H_T 2 ... 1)
\]
\[
\cdot P_c y c(k - 1),
\]
\[
V_2 = \sum_{j=k-\tau(k)}^{k-1} x^T(j) Q_{1e} x(j)
\]
\[
+ \sum_{i=\tau_1}^{\tau_1 - 1} \sum_{j=k+i+1}^{k-1} x^T(j) Q_{1e} x(j)
\]
\]
in which the other elements vanish, with \( \Sigma_{x_1} = \text{diag}(\sigma_{1x_1}, \sigma_{2x_1}, \ldots, \sigma_{nx_1}) \) and \( \Sigma_x = \text{diag}(\sigma_{1x}, \sigma_{2x}, \ldots, \sigma_{nx}) \), where \( \sigma_{1x} > 0 \) and \( \sigma_{ix} > 0 \) (i = 1, 2, \ldots, n) are given in Assumption 1, \( \tau_2 = \tau_1 - 1 \).

Proof. Pick the Lyapunov-Krasovskii functional candidate for system (42) and (47) and (48) as

\[
V_1 = x^T(k) P_x x(k) + e^T(k) P_e e(k) + y_c^T(k - 1)
\]
\[
\cdot P_y y_c(k - 1),
\]
\[
V_2 = \sum_{j=k-\tau(k)}^{k-1} x^T(j) Q_{1e} x(j)
\]
\[
+ \sum_{i=\tau_1}^{\tau_1 - 1} \sum_{j=k+i+1}^{k-1} x^T(j) Q_{1e} x(j)
\]
\[ + \sum_{j=k-\tau_2}^{k-1} x^T(j) Q_{2x} x(j) + \sum_{j=k-\tau_1}^{k-1} x^T(j) Q_{3x} x(j) \]
\[ \cdot \sum_{j=k-\tau} e^T(j) Q_{\omega} e(j) \]
\[ + \sum_{i=t_2}^{1} \sum_{j=k+i}^{k-1} e^T(j) Q_{e} e(j) + \sum_{j=k-\tau_2}^{k-1} e^T(j) Q_{2e} e(j) \]
\[ + \sum_{j=k-\tau_1}^{k-1} e^T(j) Q_{\omega} e(j) , \]
\[ V_3 = \sum_{i=t_2}^{1} \sum_{j=k+i}^{k-1} (x(j) - x(j-1))^T Z_{1x} x(j) \]
\[ - x(j-1) + \sum_{i=t_2}^{1} \sum_{j=k+i}^{k-1} (e(j) - e(j-1))^T \]
\[ \cdot Z_{1e} (e(j) - e(j-1)) , \]
\[ V_4 = \sum_{i=t_2}^{1} \sum_{j=k+i}^{k-1} (x(j) - x(j-1))^T Z_{2x} x(j) \]
\[ - x(j-1) + \sum_{i=t_2}^{1} \sum_{j=k+i}^{k-1} (e(j) - e(j-1))^T \]
\[ \cdot Z_{2e} (e(j) - e(j-1)) , \]
\[ V_5 = \sum_{i=1}^{d(k)} \sum_{j=k-i}^{k-1} h^T(x(j)) Z_{3x} h(x(j)) \]
\[ + \sum_{i=d+1}^{d(i+1)} \sum_{j=1}^{k} \sum_{l=k-j}^{k-1} h^T(x(j)) Z_{3x} h(x(j)) \]
\[ + \sum_{i=1}^{d(k)} \sum_{j=k-i}^{k-1} h^T(e(j)) Z_{3e} h(e(j)) \]
\[ + \sum_{i=d}^{d(i+1)} \sum_{j=1}^{k} \sum_{l=k-j}^{k-1} h^T(e(j)) Z_{3e} h(e(j)) . \]

Using the Schur complement to (52) and in view of LMI (48), it follows that \( \Theta < 0 \). Thus \( N \to \infty \) for any delays \( \tau_1, \tau_2, d_1, \) and \( d_2 \) satisfying (42) and (47) and (48), the inequality in (14) holds. This completes the proof. \( \square \)

**Remark 7.** Based on Lemma 3, (21)–(23) are integrated with (37) and (24) is combined with (38) by using Assumption 1 to Lyapunov-Krasovskii functional stability inequalities. Theorem 6 proposes a delay-dependent criterion such that the state feedback robust \( H_{\infty} \) control problem for the networked control systems with discrete and finite-distributed delays including quantization and packet dropouts can be achieved by solving an LMI. Free-weighting matrices \( G, \Gamma, U, S, H, \) and \( T \) are introduced into the LMI condition (49). It should be noted that these free-weighting matrices are not required to be symmetric. The purpose of introduction of these free-weighting matrices is to reduce conservatism for systems.

**Remark 8.** In deriving the delay-dependent observer-based controller in Theorem 6, no model transform technique incorporating Moon's bounding inequality \cite{17} to estimate the inner product of the involved crossing terms has been performed. This feature has the potential to enable us to obtain less-conservative results by means of Lemma 3.

**4. Examples**

In this section, numerical examples are now presented to illustrate the usefulness of the proposed approach on the state feedback \( H_{\infty} \) control problem for networked control systems with discrete and finite-distributed delays including quantization and packet dropouts.

**Example 1.** Consider system (1)–(4) with the following parameters:

\[ A = \begin{bmatrix} 0.5 & 0.1 & 0 \\ 0 & 0.3 & 0.1 \\ 0.1 & 0 & -0.2 \end{bmatrix} , \]
\[ B = \begin{bmatrix} 0.2 & -0.2 & 0 \\ 0.1 & -0.2 & 0 \\ 0 & -0.1 & -0.1 \end{bmatrix} , \]
\[ C = \begin{bmatrix} -0.2 & 0 & 0.1 \\ 0 & 0.2 & 0.1 \end{bmatrix} , \]
\[ D = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} , \]
\[ E = \begin{bmatrix} 0.1 \\ 0 \\ 0.2 \end{bmatrix} . \]

Similarly, the rest of the proof follows from Theorem 5. Therefore, in accordance with Theorem 5, the observer-based output feedback control system synthesis (42) and (47) and (48) satisfies (14). It follows from synthesis (42) and (47) and (48) that the following performance index

\[ J_N \leq \sum_{K=0}^{N} \eta^T(k) \Theta \eta(k) , \]

where \( \Theta = \Pi + \tau_2 M Z_{1x}^{-1} M^T + \tau_2 N Z_{2x}^{-1} N^T + \tau_2 H Z_{1e}^{-1} H^T + \tau_2 T Z_{2e}^{-1} T^T + \varepsilon^{-1} S^2 U U^T . \)
Choosing the constants $\beta_i = 2^{-5-i}$, we easily find that $\overline{\beta} = \sum_{i=1}^{\infty} \beta_i = 2^{-5} < \sum_{i=1}^{\infty} i \beta_i = 1/16 < +\infty$, which satisfies the convergence condition (18).

The nonlinear function in this example is assumed to satisfy Assumption 1 with $\alpha_1 = 0.102, \alpha_2 = 0.329$, and $\alpha_3 = 0.068$. Assume that the random variable $\theta_k$ satisfies $P_r(\theta_k = 1) = E[\theta_k] = \overline{\theta} = 0.8$. Then, it can be verified that the delay-dependent conditions of Theorem 1 in [11] cannot be satisfied for any interval time-varying $r(k)$. Thus, [11] cannot provide any results on the upper and lower maximum bounds allowed delay. By Theorem 6, an observer-based controller is designed such that the closed-loop NCS is asymptotically stable in the mean square with a guaranteed $H_{\infty}$ performance $\gamma$ for all discrete time-varying delays $r(k) = 3 + \sin(k\pi/2)$ and distributed time-varying delays $d(k) = 3 + (1 + (-1)^k)/2$; that is, $2 \leq r(k) \leq 4$ and $3 \leq d(k) \leq 4$, when $\gamma = 3.0613$ and quantization density $\rho = 0.6$. By solving (48), we can obtain the desired $H_{\infty}$ controller parameters as follows:

\begin{align*}
K &= [0.0208 \ 0.0467 \ 0.0326], \\
L &= [0.3643 \ 0.5310 \ 0.6124]^T, \\
P_x &= \begin{bmatrix}
1.4086 & -0.0087 & -0.0298 \\
-0.0087 & 1.4081 & -0.0214 \\
-0.0298 & -0.0214 & 1.3827 \\
\end{bmatrix}, \\
P_c &= \begin{bmatrix}
2.5919 & -0.0045 & -0.0181 \\
0.0045 & 2.6028 & -0.0103 \\
-0.0181 & 0.0103 & 2.5849 \\
\end{bmatrix}, \\
P_y &= \begin{bmatrix}
2.0113 & -0.0053 & -0.0166 \\
0.0053 & 2.0328 & -0.0113 \\
-0.0166 & 0.0113 & 2.2849 \\
\end{bmatrix}, \\
Q_{1x} &= \begin{bmatrix}
0.7739 & -0.0004 & -0.0016 \\
-0.0004 & 0.7741 & -0.0008 \\
-0.0016 & -0.0008 & 0.7725 \\
\end{bmatrix}, \\
Q_{2x} &= \begin{bmatrix}
0.4135 & -0.0003 & -0.0010 \\
-0.0003 & 0.4120 & -0.0008 \\
-0.0010 & -0.0008 & 0.4117 \\
\end{bmatrix}, \\
Q_{3x} &= \begin{bmatrix}
0.4586 & -0.0003 & -0.0008 \\
-0.0003 & 0.4571 & -0.0007 \\
-0.0008 & -0.0007 & 0.4570 \\
\end{bmatrix}, \\
R_x &= \begin{bmatrix}
0.6525 & 0.0003 & 0.0002 \\
0.0003 & 0.6507 & 0.0005 \\
0.0002 & 0.0005 & 0.6524 \\
\end{bmatrix}, \\
R_c &= \begin{bmatrix}
0.5817 & 0.0002 & 0.0005 \\
0.0002 & 0.5805 & 0.0003 \\
0.0005 & 0.0003 & 0.5824 \\
\end{bmatrix}, \\
H_1 &= \begin{bmatrix}
-0.1021 & 0.0149 & 0.0188 \\
0.0171 & -0.0804 & 0.0111 \\
0.0205 & 0.0138 & -0.1036 \\
\end{bmatrix},
\end{align*}
\[ H_2 = \begin{bmatrix} -0.1391 & 0.0150 & 0.0173 \\ 0.0097 & -0.0951 & 0.0121 \\ 0.1005 & 0.0147 & -0.1029 \end{bmatrix}, \]

\[ H_3 = \begin{bmatrix} 0.0598 & -0.0060 & -0.0133 \\ -0.0117 & 0.0691 & -0.0081 \\ -0.0162 & -0.0124 & 0.0071 \end{bmatrix}, \]

\[ S_1 = \begin{bmatrix} -1.0903 & -0.0061 & -0.0068 \\ -0.0061 & -1.1485 & -0.0222 \\ -0.0068 & -0.0222 & -1.1476 \end{bmatrix}, \]

\[ S_2 = \begin{bmatrix} -1.0722 & -0.0058 & -0.0103 \\ -0.0058 & -1.1501 & -0.0305 \\ -0.0103 & -0.0305 & -1.0988 \end{bmatrix}, \]

\[ S_3 = \begin{bmatrix} 0.9027 & -0.0019 & -0.0025 \\ -0.0019 & 0.9976 & 0.0031 \\ -0.0025 & 0.0031 & 1.0013 \end{bmatrix}, \]

\[ S_4 = \begin{bmatrix} 0.8167 & -0.0022 & -0.0041 \\ -0.0022 & 0.8816 & 0.0038 \\ -0.0041 & 0.0038 & 0.9055 \end{bmatrix}, \]

\[ T_1 = \begin{bmatrix} -0.0765 & 0.0103 & 0.0127 \\ 0.0105 & -0.0628 & 0.0052 \\ 0.0130 & 0.0054 & -0.0718 \end{bmatrix}, \]

\[ T_2 = \begin{bmatrix} -0.0815 & 0.0113 & 0.0131 \\ 0.0114 & -0.0758 & 0.0058 \\ 0.0134 & 0.0060 & -0.0803 \end{bmatrix}, \]

\[ T_3 = \begin{bmatrix} 0.1175 & -0.0170 & -0.0211 \\ -0.0173 & 0.0938 & -0.0079 \\ -0.0216 & -0.0081 & 0.1189 \end{bmatrix}, \]

\[ T_4 = \begin{bmatrix} 0.1204 & -0.0094 & -0.0200 \\ -0.0097 & 0.1018 & -0.0127 \\ -0.0201 & -0.0128 & 0.1322 \end{bmatrix}, \]

\[ \varepsilon = 0.9946. \]

\[ \gamma = 0.4803. \]

\[ H_\infty \text{ performance } \gamma = 3.0613 \text{ for satisfying all discrete time-varying delays. Figure 1 shows that the system state } x(k) \text{ converges to zero, where the initial condition is set to be } x(0) = [-1, 0.4, 1]. \]

\[ \text{Example 2. An Internet-based test rig is used to confirm the effectiveness of the observer-based controller-design approach. This test rig consists of a plant (DC servo system) and a remote controller. The plant and the controller are connected via the Internet [4]. According to [4], here we only consider the linear case with finite-distributed delays to show the effectiveness and application feasibility in networked control systems. Based on the results in [4], the DC servo system is recognized to be a third-order system and in state-space depiction has the following system matrices:} \]

\[ A = \begin{bmatrix} 1.12 & 0.213 & -0.333 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \]

\[ C = \begin{bmatrix} -0.2193 & 0.0219 & 0.0844 \\ 0.2177 & -0.0032 & -0.0662 \\ 0.1298 & -0.0087 & -0.0381 \end{bmatrix}. \]
parameters and by using the LMI toolbox in Matlab, the desired controller can be found as
\[
K = \begin{bmatrix} 0.1083 & 0.0419 & 0.3715 \end{bmatrix},
\]
\[
L = \begin{bmatrix} 0.0577 & 0.2209 & 0.0841 \end{bmatrix}^T,
\]
\[
P_x = \begin{bmatrix} 3.4427 & -0.0680 & -0.0093 \\ -0.0680 & 3.7182 & 0.1275 \\ -0.0093 & 0.1275 & 4.0112 \end{bmatrix},
\]
\[
P_e = \begin{bmatrix} 3.6439 & -0.0710 & -0.0087 \\ -0.0710 & 3.8223 & 0.1337 \\ -0.0087 & 0.1337 & 4.0855 \end{bmatrix},
\]
\[
P_y = \begin{bmatrix} 3.2215 & -0.0651 & -0.0778 \\ -0.0651 & 3.3019 & 0.1229 \\ -0.0778 & 0.1229 & 4.0112 \end{bmatrix},
\]
\[
Q_{1x} = \begin{bmatrix} 0.6692 & -0.0010 & -0.0013 \\ -0.0010 & 0.6703 & -0.0011 \\ -0.0013 & -0.0011 & 0.6714 \end{bmatrix},
\]
\[
Q_{2x} = \begin{bmatrix} 0.4245 & -0.0005 & -0.0010 \\ -0.0005 & 0.4126 & -0.0008 \\ -0.0010 & -0.0008 & 0.4117 \end{bmatrix},
\]
\[
Q_{3x} = \begin{bmatrix} 0.4566 & -0.0003 & -0.0008 \\ -0.0003 & 0.4583 & -0.0007 \\ -0.0008 & -0.0007 & 0.4570 \end{bmatrix},
\]
\[
Q_{1e} = \begin{bmatrix} 0.5836 & 0.0001 & 0.0007 \\ 0.0001 & 0.5904 & 0.0003 \\ 0.0007 & 0.0003 & 0.5909 \end{bmatrix},
\]
\[
Q_{2e} = \begin{bmatrix} 0.6296 & 0.0001 & 0.0007 \\ 0.0001 & 0.6362 & 0.0002 \\ 0.0007 & 0.0002 & 0.6293 \end{bmatrix},
\]
\[
Q_{3e} = \begin{bmatrix} 0.6308 & 0.0003 & 0.0006 \\ 0.0003 & 0.6307 & 0.0003 \\ 0.0006 & 0.0003 & 0.6289 \end{bmatrix},
\]
\[
Z_{1x} = \begin{bmatrix} 1.0833 & -0.0047 & -0.0171 \\ -0.0047 & 1.0905 & -0.0129 \\ -0.0171 & -0.0129 & 1.0751 \end{bmatrix},
\]
\[
Z_{2x} = \begin{bmatrix} 1.0228 & -0.0034 & -0.0113 \\ -0.0034 & 1.0251 & -0.0089 \\ -0.0113 & -0.0089 & 1.0125 \end{bmatrix}.
\]
\[ T_1 = \begin{bmatrix} -0.0735 & 0.0113 & 0.0125 \\ 0.0115 & -0.0638 & 0.0042 \\ 0.0132 & 0.0044 & -0.0698 \end{bmatrix}, \]

\[ T_2 = \begin{bmatrix} 0.0114 & -0.0748 & 0.0048 \\ 0.0157 & 0.0028 & -0.0813 \end{bmatrix}, \]

\[ T_3 = \begin{bmatrix} 0.1165 & -0.0160 & -0.0191 \\ -0.0163 & 0.0938 & -0.0079 \\ -0.0196 & -0.0081 & 0.1189 \end{bmatrix}, \]

\[ T_4 = \begin{bmatrix} 0.1214 & -0.0074 & -0.0210 \\ -0.0077 & 0.1009 & -0.0127 \\ -0.0211 & -0.0128 & 0.1212 \end{bmatrix} \]

\[ \varepsilon = 0.9767. \]

Therefore, in the presence of quantization and packet dropout, controllers (43) and (44) are designed to guarantee asymptotical stability in the mean square for the closed-loop DC servo system (47) and (48) and the prescribed \( H_\infty \) performance level in (14) is satisfied under zero-initial conditions, which verifies that packet-based control method is valid.

5. Conclusions

In this paper, the problem of \( H_\infty \) control problem for a class of networked systems with discrete and finite-distributed delays subject to quantization and packet dropout has been considered. The state of system (1)–(3) is assumed to be measurable and will be quantized before it is transmitted to the controller through communication network while the data packet dropout happens. An observer-based feedback controller has been designed to stabilize the networked system in the sense of mean square and also achieve the prescribed \( H_\infty \) performance bound for any delays \( \tau_1, \tau_2, d_1, \) and \( d_2 \) satisfying (5). Through numerical examples, the effectiveness of the proposed criterion design approach has been shown.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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