MULTIPLE-STAGE MULTIPLE-MACHINE CAPACITATED LOT-SIZING AND SCHEDULING WITH SEQUENCE-DEPENDENT SETUP: A CASE STUDY IN THE WHEEL INDUSTRY

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(Communicated by Changzhi Wu)

Abstract. This paper studies a real-world problem of simultaneous lot-sizing and scheduling in a capacitated flow shop. The problem combines two significant characteristics in production which are multiple-stage production with heterogeneous multiple machines and sequence-dependent setup time. Setup time does not hold the triangle inequality, thus there may be a setup for a product without actual production. Consequently, a novel mixed integer programming (MIP) formulation is proposed and tested on real data sets of wheel production. Exact approaches cannot find a feasible solution for the model in a reasonable time, so MIP-based heuristics are developed to solve the model more quickly. Test results show that the formulation is able to contain the problem requirements and the heuristics are computationally effective. Moreover, the obtained solution can improve on a real practice at the plant.

1. Introduction. The automotive industry is one of the most economically important industries. According to OICA (International Organization of Motor Vehicle Manufacturers) [11], vehicle production increased every year in average 3.2% since 2010. The automotive industry is also categorized by a high degree of technological sophistication. Manufacturers consist of many shops and stations. Each station has a number of operations. Therefore, companies in this industry have focused on their core competencies to preserve high efficiency. The industry offers broad and diversified product portfolios. Many companies encounter extensive setup times and costs within a growing range of products, consequently the production capacity may loss and lead to excessive inventories due to inefficient production planning. This has made the industry to seek for more efficient and effective production planning and control methods. A few mixed integer production planning models of automotive have been proposed, for instance Gnoni et al. [4]. They proposed a mixed-integer linear programming model dealing with a lot sizing and scheduling problem. The proposed model is applied to a multi-site manufacturing system of braking equipments for the automotive industry.

The production lot-sizing and scheduling problem in a wheel industry is difficult to determine because it has to consider several aspects simultaneously. Firstly,
there are various products to be produced in multiple-stage production. Secondly, several machines operate with different specificities and capacities. Most of all, setup (changeover) from one product to another requires sequence-dependent setup time which depends not only on that job but also the previous job. And the setup time does not always obey the triangular inequality which means it might be faster to changeover from one product to another by means of a third product. Most companies determine the production planning manually which can take days until a reasonable plan is completed.

Lot-sizing specifies the quantity of a product produced on a machine continuously without interruption. While scheduling determines the sequence of the production on a machine. If sequence-dependent setup times occur during production, a simultaneous lot-sizing and scheduling is essential because the sequence of the lots has an effect on the available capacity, and the capacity impacts the sizes of the lots [15].

This paper investigates the multiple-stage capacitated simultaneous lot-sizing and scheduling problem with sequence-dependent setups for flow shop production. Our problem does not hold the triangle inequality for setup time. Mixed integer programming models are applied and designed to map an industrial optimization problem in wheel production planning. Then, this research examines the significance of multiple-stage production and sequence-dependent setups in solving the MIP. The computational tests for large industrial problems indicate that the model is inefficiently solved by standard approaches. Therefore, we develop MIP-based heuristics that aim at producing good solutions in short calculation time.

The remainder of this section describes the wheel production process and planning following by literature review. Section 2 introduces the formulation for the multi-stage lot-sizing and scheduling problem with sequence-dependent setups. The MIP-based heuristics in solving the problem are presented in Section 3. Section 4 provides numerical results of our method. The paper ends with some conclusions from this work and directions for future research.

1.1. Wheel production process and planning. The production process of wheels is multiple-stage flow shop production with several production lines or machines operated on each stage. There are total of three stages. All types of wheels basically go through these three production stages. However, they are distinguished by production tools and machines at each stage. All products consume identical raw materials which are unlimited supplied. The three-stage production process can be seen in Figure 1. A machine (M1.1) in the first stage of production produces first-stage products from raw materials. Then, the first-stage products are stored in storage 1 as inventories. The second stage consisting of six machines (M2.1-M2.6) produces second-stage products by inputting the first-stage products from storage 1. Each machine with different capacities and specificities can process only one product at a time. One unit of the second-stage product is obtained by consuming one unit of the first-stage product. The second-stage products are kept in storage 2 as inventories and transferred to the final stage consisting of four machines (M3.1-M3.4) with the same principle as in the second stage.

Product structure in the wheel production is a divergent type. One type of predecessor may be used to produce multiple type of successors, but one type of successor can be produced from just one type of predecessor. The example of a bill of materials from one type of first-stage product is illustrated by Figure 2. The
company produces about 100 types of finish wheels with approximately 60 types of the first-stage and the second-stage products.

The production planning is done manually by spreadsheet in two sections, lot-sizing (approximate planning) and scheduling (detailed planning). The company has a just-in-time strategy that aims to meet a deterministic demand of the finish products which is called the external demand without backlogging under available machine capacity. Overtime is allowed for urgent needs. Starting with approximate planning, the company determines lot-size for the finish products to satisfy the demand under machine capacity. Then, the quantity of demands for the second-stage products are known and their due date is set to one period before production of finish products. After that, the planning proceed to the second and the first stage with the same procedure. For the first-stage and the second-stage products, there
are also minimum level of inventories to insure that the company has adequate supply for production while at the same time minimizing inventory lead time. Finally, the scheduling for each stage production is determined manually by avoiding high setup time which may result from changeovers between products of two different families. Product changeovers are very frequent, normally about 300 times per week in total. Therefore setups need to be well managed and controlled in order to make effective use of the production capacity.

1.2. Literature review. A considerable amount of literature has been published on the capacitated lot-sizing and scheduling problems. Reviews of the problem can be found in, e.g., Drexl and Kimms [3] and Jans and Degraeve [7]. Zhu and Wilhelm [18] gave a review of the literature on lot-sizing and scheduling with sequence-dependent setup. In their review, even though numerous researchers have studied sequence-dependent setup, a number of real-world instances have not received adequate attention. Furthermore, in the capacitated lot-sizing problem with extensions review by Quadt and Kuhn [12], there is no extension of sequencing, multiple-machine and multiple-stage together in one problem. Although extensive researches have been carried out on the capacitated lot-sizing and scheduling problems, modelling the multi-stage parallel-machine problem with sequence-dependent setup has not received much attention due to computational difficulty and model complexity.

Recently, Seeanner and Meyr [14] and Seeanner et al. [13] studied the multi-stage lot-sizing and scheduling problem for flow line production that includes sequence-dependent setup. They proposed new improvement of the general lot-sizing and scheduling problem for multiple production stage. Their model is a small time bucket (STB) model with micro periods where at most one or two products may be produced per period. Consequently, the number of periods included into the model is relative large. For the same planning horizon, the number of periods in STB models is larger than those in big time bucket (BTB) models, which resulted in higher computational effort.

There are also some BTB models in the lot-sizing and scheduling problem with sequence-dependent setup. For BTB models, setup of several products is allowed within one period. As a result, the sequence-dependent setup plays an important role in scheduling products in each period. Haase [5] developed a single-level BTB model in the capacitated lot-sizing problem with sequence dependent setup (CLSD). He determined setup sequences in which any tour or subtour is not allowed by using TSP-constraints. Almada-lobo et al. [2] presented two novel linear mixed integer programming formulations for a single machine multi-product capacitated
lot-sizing problem with sequence-dependent setups (CLSD-SM). In their model, the TSP-constraints was applied to eliminate only disconnected subtour (Figure 3) in the setup sequences. James and Almada-Lobo [6] proposed a new iterative MIP-based neighborhood search heuristic to solve parallel machine capacitated lot-sizing and scheduling problems with sequence-dependent time and costs (CLSD–PM). The model is a generalization of the single machine model introduced in Almada-lobo et al. [2]. All of mentioned literature considered the problem that setup times and costs hold the triangle inequality, whilst it is not mandatory in our problem.

The multi-stage model presented in this paper is the extension of CLSD–PM by James and Almada-Lobo [6] and Almada-lobo et al. [2]. Our problem does not hold the triangle inequality for setup time, thus the model has new improvement of no-subtour and some variations. Furthermore, our test instances is much larger than others.

2. Model development. The single-stage model by James and Almada-Lobo [6] which is foundation of our model will be briefly explained in subsection 2.1. Then, in subsection 2.2 we will develop a new model for multiple-stage multiple-machine lot-sizing and scheduling with sequence-dependent setup.

2.1. Single-stage model. The model suggested by James and Almada-Lobo [6] is single-stage parallel machine capacitated lot-sizing and scheduling with sequence-dependent setups. In the model, N products are produced on M machines in a single-stage system over T periods to satisfy given deterministic demands without backlogging. We assume that there are unlimited materials. Let \( N \) be a set of products, \( M \) be a set of machines, and \( T \) be a set of periods. Denote the demand of each product \( i \) for each period \( t \) by \( d_{ti} \). The problem is to find a cost optimal lot-sizing plan for all machines and the production scheduling over \( T \) periods.

At any time, a machine can process at most one product. Producing a lot of products consumes a production time and requires a setup time. Let \( p_{mi} \) denote the production time of one unit of each product \( i \in [N] \) on each machine \( m \in [M] \). A setup from each product \( i \) to \( j \in [N] \) on each machine \( m \) incurs the setup time \( s_{mj} \) and the relevance setup cost \( c_{mj} \). The inventory \( I_{t}^{i} \) is counted for each product \( i \) at the end of each period \( t \in [T] \). We assume that each initial inventory level \( I_{0}^{i} \) is zero. \( h_{i} \) denotes the cost of carrying one unit of inventory of each product \( i \) for one period. The objective is to minimize the sum of inventory holding costs and setup costs. Each machine \( m \) has the specific time capacity \( C_{m}^{i} \) and an upper bound on the production quantity \( b_{mi}^{t} \) for each product \( i \) in each period \( t \). Each machine has a given set of products that are able to be produce. Let \( A_{mi} \) be a binary data which is 1 if a machine \( m \) is able to produce a product \( i \) and 0 otherwise.

Further assumptions for the problem are as follows: Setups of a machine must be completed in a period. The setup state is preserved over time, which means that a product can be set up at the end of a time period and the actual production starts in the next time period. This is the same property as a setup carryover. The setup carryover occurs when a product is the last one to be produced in a period, thus no setup is required in the next period. In addition, we assume that the triangle inequality holds for setups, that is, for any three products, the cost and time required to directly set up the machine from one product to another is always less than or equals to the sum of those required when setting up via an intermediate product.
The following decision variables are used:

- $X_{mi}^t$ refers to the quantity of each product $i$ produced in each period $t$ on each machine $m$.
- $V_{mi}^t$ is an auxiliary variable that assigns each product $i$ on each machine $m$ in each period $t$. The larger $V_{mi}^t$ is the later of production.
- $Z_{mi}^t$ is defined as a binary setup variable which is 1 if a changeover occurs from each product $i$ to $j$ on each machine $m$ in each period $t$ and 0 otherwise.
- $Y_{mi}^t$ is a binary setup variable which is 1 if each machine $m$ is setup for each product $i$ at the beginning of each period $t$ and 0 otherwise.

Then the CLSD–PM model by James and Almada-Lobo [6] is stated as follows:

$$
\text{min} \sum_{m=1}^{M} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{t=1}^{T} c_{mi}^t Z_{mi}^t + \sum_{i=1}^{N} \sum_{t=1}^{T} h_i I_i^t
$$

s.t. $I_i^t = I_i^{t-1} - d_i^t + \sum_{m=1}^{M} X_{mi}^t, \quad i \in [N], t \in [T], (2)$

$I_i^0 = 0, \quad i \in [N], (3)$

$$
\sum_{i=1}^{N} p_{mi} X_{mi}^t + \sum_{i=1}^{N} \sum_{j=1}^{N} s_{mj} Z_{mj}^t \leq C_m^t, \quad m \in [M], t \in [T], (4)
$$

$$
X_{mi}^t \leq b_{mi}^t (\sum_{j=1}^{N} Z_{mj}^t + Y_{mi}^t), \quad m \in [M], i \in [N], t \in [T], (5)
$$

$$
Y_{mi}^{t+1} + \sum_{j=1}^{N} Z_{mj}^t = Y_{mi}^t + \sum_{j=1}^{N} Z_{mj}^t, \quad m \in [M], i \in [N], t \in [T], (6)
$$

$$
\sum_{i=1}^{N} Y_{mi}^t = 1, \quad m \in [M], t \in [T], (7)
$$

$$
V_{mi}^t + N Z_{mi}^t - (N-1) - N Y_{mi}^t \leq V_{mj}^t, \quad m \in [M], i \in [N], j \in [N] \setminus \{i\}, t \in [T], (8)
$$

$$
\sum_{t=1}^{T} X_{mi}^t \leq b_{mi}^t A_{mi}, \quad m \in [M], i \in [N], (9)
$$

$(X_{mi}^t, I_i^t) \geq 0, \quad (Z_{mi}^t, Y_{mi}^t) \in \{0, 1\}, \quad X_{mi}^t \in \mathbb{Z}, \quad V_{mi}^t \in \mathbb{R}. (10)$

The objective function (1), which is minimized, is the sum of setup and inventory holding costs. The constraints (2) are generalized inventory balance constraints. The constraints (3) set the initial inventory levels. The constraints (4) ensure that the sum of production and setup time does not exceed the available capacity. Due to the setup forcing constraints (5), production can only take place if the machine is set up accordingly. The constraints (6) keep track of the setup carryover information, whilst the constraints (7) guarantee that each machine is set up for one product at the beginning of each period. Requirements (8) link two consecutive periods. Disconnected subtours are eliminated by constraints (9). The constraints (10) ensure that production of a product can only occur on machines that are able to produce that product.
2.2. Multiple-stage model with no subtour. We will show the multiple-stage lot-sizing and scheduling problem with sequence-dependent setups with no subtour by extending the single-stage CLSD–PM model. Our multiple stage problem is a serial system where each single product must go through $S$ stages, beginning with stage 1 and ending with stage $S$. Each stage has heterogeneous parallel machines. Basic assumptions are the same as the single-stage model. However, the triangle inequality of setup costs and times may not hold. In addition, each product is set up at most once in each single period.

We consider $N$ products where the first $F$ products are produced in a stage from 1 to $S - 1$ and the rest $N - F$ products are produced in the finish stage $S$. These products are scheduled on $M$ machines, each of which belongs to a specific stage, over $T$ periods to satisfy given external demands without backlogging. Let $d_i^t$ denote the demand of each finish product $i \in \{F, \ldots, N\}$ in each period $t \in [T]$. Overtime is allowed and denoted by $O_m^t$ for each machine $m \in [M]$ at each period $t$.

The company seeks to minimize the summation of setup times, lead times, and overtimes with weights. A setup time from each product $i$ to $j$ is measured by a sequence-dependent setup time $s_{mij}$ for each machine $m$. The most obvious factor related to the lead time is the inventory $I_{ti}$ for each product $i$ at the end of each period $t$. Let $w_i$ be the inventory lead time weight for each product $i$ and $q$ be the overtime weight. Then, the objective of the model is formulated as

$$\sum_{m=1}^{M} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{t=1}^{T} s_{mij} Z_{mij}^t + \sum_{i=1}^{N} \sum_{t=1}^{T} w_i I_i^t + \sum_{m=1}^{M} \sum_{t=1}^{T} qO_m^t,$$

where $Z_{mij}^t$ is a binary setup variable as defined in the single-stage model.

In the multiple-stage production in the wheel industry, unlimited materials are input at stage 1 and are operated through the next stages until the finish stage $S$ to satisfy the external demands. A material in each stage from 2 to $S$ is supplied from the adjacent predecessor stage. A bill of the materials is indicated by $a_{ij}$ which is 1 if one unit of the product $i$ is required to produce one unit of the product $j$ and 0 otherwise. We assume that a lead time between production stages is one period, because one unit of the successor requires one unit of the predecessor prior one period. Demand of the intermediate product is defined as an internal demand $D_i^t$ for each product $i$ in each period $t$, which can be formulated as an internal demand constraints

$$D_i^{t-1} = \sum_{m=1}^{M} \sum_{j=1}^{N} a_{ij} X_{mj}^t, \quad i \in [N], t \in [T].$$

Next, we have the inventory balance constraints

$$I_i^t = I_i^{t-1} - D_i^t - d_i^t + \sum_{m=1}^{M} X_{mi}^t, \quad i \in [N], t \in [T]$$

over consecutive periods $t - 1$ and $t$. Each inventory of product $i$ must be not less than the minimum inventory level $e_i$. This is imposed by the minimum inventory constraints

$$I_i^t \geq e_i, \quad i \in [N], t \in [T].$$
The capacity constraints
\[
\sum_{i=1}^{N} p_m X_{mi}^t + \sum_{i=1}^{N} \sum_{j=1}^{N} s_{mij} Z_{mij}^t \leq C_m^t + O_m^t, \quad m \in [M], t \in [T] \tag{15}
\]
imply that the total production and setup time do not exceed the sum of the available capacity and the overtime of each machine in each period. Breakdowns and preventive maintenance are allowed which will affect the capacity $C_m^t$.

The assignment of machines to production stages is indirectly indicated by $b_{mi}^t$ which denotes an upper bound on the quantity of each product $i$ in each period $t$ to each machine $m$. It determines whether a machine can produce a product i.e. $b_{mi}^t = 0$ if machine $m$ cannot produce product $i$ in period $t$. The next constraints
\[
X_{mi}^t \leq b_{mi}^t (\sum_{j=1}^{N} Z_{mji}^t + Y_{mi}^t), \quad m \in [M], i \in [N], t \in [T] \tag{16}
\]
guarantee that a product is produced only if the machine has been set up for it and ensure production of a product can only occur on machines that are able to produce that product.

The setup carryover constraints (17), the setup beginning constraints (18), and disconnected subtour elimination constraints (19) are derived from the model CLSD–PM as follows:
\[
Y_{mi}^{t+1} + \sum_{j=1}^{N} Z_{mij}^t = Y_{mi}^t + \sum_{j=1}^{N} Z_{mji}^t, \quad m \in [M], i \in [N], t \in [T], \tag{17}
\]
\[
\sum_{i=1}^{N} Y_{mi}^t = 1, \quad m \in [M], t \in [T], \tag{18}
\]
\[
V_{mi}^t + N Z_{mij}^t - (N - 1) - N Y_{mi}^t \leq V_{mj}^t, \quad m \in [M], i \in [N], j \in [N] \setminus \{i\}, t \in [T]. \tag{19}
\]

However, Menezes et al. [9] point out and proof that constraints (19) still allow the connected subtour at the beginning of period as shown in Figure 4. However, if the triangle inequalities
\[
s_{mij} \leq s_{mik} + s_{mkj}, \quad i, j, k \in [N], i \neq j, j \neq k, k \neq i
\]
hold, then there exists an optimal solution of the CLSD–PM model where each product is produced at most once per period. In this case, the connected subtour

**Figure 4.** A subtour connected to a main sequence at the beginning of period.
will not occur. In our problem, the inequalities above may not hold, so there might be a setup via one product without production in order to changeover to another product. Therefore, we include the constraints (20) where each product is produced at most once per period and minimum lot-size constraints (21) to avoid setup state changes without actual product changeovers:

\[
\sum_{j=1}^{N} Z_{mij}^t \leq 1, \quad m \in [M], \quad i \in [N], \quad t \in [T],
\]

(20)

\[
X_{mi}^t \geq l_i \sum_{j=1}^{N} Z_{mij}^t, \quad m \in [M], \quad i \in [N], \quad t \in [T],
\]

(21)

where \( l_i \) is the minimum lot-size for product \( i \). By adding these constraints, subtours are not generated.

Finally, the binary, non-negativity and integrality constraints \((Z_{mij}^t, Y_{mi}^t) \in \{0, 1\}, \quad (X_{mi}^t, I_{mi}^t) \geq 0, \quad X_{mi}^t \in \mathbb{Z}, \quad V_{mi}^t \in \mathbb{R}\)

are added.

Now we can summarize the proposed model “CLSD–PM–MS with no subtour” as following:

\[
\begin{align*}
\min & \quad \sum_{m=1}^{M} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{t=1}^{T} s_{mij} Z_{mij}^t + \sum_{i=1}^{N} \sum_{t=1}^{T} w_i I_{mi}^t + \sum_{m=1}^{M} \sum_{t=1}^{T} qO_{mi}^t \\
\text{s.t.} & \quad (12)-(22)
\end{align*}
\]

3. MIP-based heuristics. A survey of scheduling problems with setup time or cost by Allahverdi et al. [1] stated that even a single-machine problem is NP-hard. The real size problems cannot be solved in reasonable time using a commercial mathematical programming solver. A variety of solution methods is used to solve capacitated lot-sizing and scheduling problems. Relax-and-Fix heuristics are one of the most practical ways for a large industrial problem due to significantly lower computational effort. Moreover, James and Almada-Lobo [6], Lang and Shen [8], Stadtler and Sahling [16] and Xiao et al. [17] obtained good results by using Relax-and-Fix and Fix-and-Optimize heuristics. These facts encourage us to develop these approximate approaches to deliver superior solutions in an acceptable computation time.

Both Relax-and-Fix and Fix-and-Optimize heuristics solve a sequence of subproblems and each subproblem includes a number of integer variables that are less enough to be quickly solved by a standard solver. In the Relax-and-Fix heuristic, as the sequences progress, fixed integer variables are increased and the relaxed variables are decreased in number. By contrast, there is no relaxation in Fix-and-Optimize heuristics. The detail of both methods will be explained in the following subsections. In both approaches, each subproblem can be solved to optimality in reasonable time by using any standard solver for MIPs, e.g. Gurobi.

3.1. Relax-and-fix heuristics. The Relax-and-Fix framework decomposes the large-scale MIP proposed in Section 2 into a number of smaller partially relaxed MIP subproblems which are solved in series. Let \([NT] = \{1, \ldots, N\} \times \{1, \ldots, T\}\) be a set of all product-period combinations \((i, t)\). In each subproblem, we decompose the set \([NT]\) in three disjunctive subsets \([NT]^{fix} \subseteq [NT], [NT]^{bin} \subseteq [NT], \) and \([NT]^{rel} \subseteq [NT]\). Then the subproblem is stated as
Figure 5. Relax and fix heuristic on multi-stage and over the periods.

\[
\begin{align*}
\min & \quad \sum_{m=1}^{M} \sum_{i=1}^{N} \sum_{t=1}^{T} s_{mij} Z_{mij}^t + \sum_{i=1}^{N} \sum_{t=1}^{T} w_i I_i^t + \sum_{m=1}^{M} \sum_{t=1}^{T} qO_m^t \\
\text{s.t.} & \quad \text{(12)-(21)}, \\
& \quad (X_{mi}^t, D_{mi}^t) \geq 0, \quad X_{mi}^t \in \mathbb{Z}, \quad V_{mi}^t \in \mathbb{R}, \\
& \quad Z_{mij}^t = Z_{mij}^0, \quad m \in [M], (i, t) \in [NT]^{fix}, j \in [N], \\
& \quad Y_{mi}^t = Y_{mi}^0, \quad m \in [M], (i, t) \in [NT]^{fix}, \\
& \quad Z_{mij}^t \in [0, 1], \quad m \in [M], (i, t) \in [NT]^{rel}, j \in [N], \\
& \quad Y_{mi}^t \in [0, 1], \quad m \in [M], (i, t) \in [NT]^{rel}, \\
& \quad Z_{mij}^t \in \{0, 1\}, \quad m \in [M], (i, t) \in [NT]^{bin}, j \in [N], \\
& \quad Y_{mi}^t \in \{0, 1\}, \quad m \in [M], (i, t) \in [NT]^{bin},
\end{align*}
\]

where \(Y_{mi}^t\) and \(Z_{mij}^t\) are fixed values of \(Y_{mi}^t\) and \(Z_{mij}^t\) in the current subproblem.

The solution quality and the computational time of the Relax-and-Fix heuristic is influenced by how the original MIP model is partitioned. Each subproblem should involve a small but sufficient number of integer variables to be quickly and optimally solved by using the exact methods \[6\]. Due to the structure of the original MIP model, we propose two-level partitions which are a stage partition and a period partition.

With the two-level partitions (as seen in Figure 5), each subproblem relates to a time-window of \(\lambda\) consecutive periods in one stage. Let \(t^f\) and \(t^e\) denote the
first and end period of the current time-window. Starting from the products in the finish stage \( S \) \((i, j = F + 1, \ldots, N)\), the variables \( Z_{mij}^t \) and \( Y_{mi}^t \) in the interval \([t^l, t^e]\), where \( t^l = 1 \) and \( t^e = t^l + \lambda - 1 \), are defined binary. Furthermore, the variables \( Z_{mij}^t \) and \( Y_{mi}^t \) with \( t < t^l \) are fixed due to previous iteration. All remaining binary variables are relaxed in the current subproblem. The current subproblem is solved to optimality. Then, the time-window is shifted forward \( \theta \) periods ahead \((\theta \leq \lambda)\), i.e. \( t^* = t^* + \theta \) and \( t^e = \min\{t^* + \theta, T\} \) until the end of the planning horizon is reached \((t^e \geq T)\). After all binary variables in the finish stage are fixed, the same decomposition method is applied to the binary variables in the previous stage. These steps are repeated until the first stage production, so that all binary variables are solved. The final solution computed from Relax-and-Fix heuristics will be improved in Fix-and-Optimize heuristics.

3.2. Fix-and-optimize heuristics. We use the solution from the Relax-and-Fix heuristic as an initial solution in the Fix-and-Optimize heuristic. Since the Relax-and-Fix heuristic decomposes the model on stages and periods, we use machine-decomposition in the Fix-and-Optimize heuristic. Each machine has difference in the working day, thereby it is very important to optimize all the periods at once.

Similar to the Relax-and-Fix heuristic, the Fix-and-Optimize heuristic solve a series of subproblems. Each subproblem can be stated with subsets \([M]^f_{ix}\) and \([M]^b_{in}\) of the machine-set \([M]\) as follows

\[
\min \sum_{m=1}^{M} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{t=1}^{T} s_{mij} Z_{mij}^t + \sum_{i=1}^{N} \sum_{t=1}^{T} w_i I_i^t + \sum_{m=1}^{M} \sum_{t=1}^{T} q O_m^t \quad (31)
\]

s.t. \([12-21], (X_{mi}^t, D_i^t) \geq 0, X_{mi}^t \in \mathbb{Z}, V_{mi}^t \in \mathbb{R}, \quad (32)\)

\[
Z_{mij}^t = \overline{Z}_{mij}^t, \quad m \in [M]^f_{ix}, i, j \in [N], t \in [T], \quad (33)\)

\[
Y_{mi}^t = \overline{Y}_{mi}^t, \quad m \in [M]^b_{in}, i \in [N], t \in [T], \quad (34)\)

\[
Z_{mij}^t \in \{0, 1\}, \quad m \in [M]^f_{ix}, i, j \in [N], t \in [T], \quad (35)\)

\[
Y_{mi}^t \in \{0, 1\}, \quad m \in [M]^b_{in}, i, j \in [N], t \in [T], \quad (36)\)

where \( Y_{mi}^t \) and \( Z_{mij}^t \) are fixed values of \( Y_{mi}^t \) and \( Z_{mij}^t \) in the current subproblem.

Starting with \([M]^b_{in} = \{m\}\) and \([M]^f_{ix} = [M] - \{m\}\) for a machine \( m \) in the finish stage \( S \), the subproblem is solved. This step is repeated until all the machines are visited at least once.

4. Computational test and results. Real operational data from the wheel production industry were collected to investigate the problem, to test the quality of methods, and to check whether the model is applicable. There are three stages in production, where the first, second, and finish stage consist of 1, 8 and 4 machines \((M = 13)\), respectively. Total of 196 product types \((N = 196)\) are to be scheduled for 7 day periods \((T = 7)\). Due to non-disclosure agreement with the company, we cannot explicitly reveal the data.

The model formulations and the heuristics were coded using Python. Gurobi 5 mixed integer programming solver then solved the subproblem to optimality and...
4.1. Parameters setting. In this section, parameters in our MIP-based heuristics and the proposed model will be tested in 8 real practice weekly data. In the Relax-and-Fix heuristic, there are two parameters which are time-window periods $\lambda$ and shifted periods $\theta$. Moreover, to solve MIP subproblem by Gurobi solver, the terminate parameters of MIP gap and limit time are needed to be determined. In the objective function (11) of our proposed model, there are two parameters, the inventory lead time weight $w_i$ and the overtime weight $q$.

In the Relax-and-Fix heuristic, different values of $\lambda$ and $\theta$ are tested in order to finding a good compromise between solution quality and computational time. When the time-window periods $\lambda$ are enlarged, better solution is obtained with cost of computation time. We found that $\lambda = 4$ periods and $\theta = 1$ period are the longest periods that produce good solutions at acceptable time. For MIP parameters of each subproblem in the Relax-and-Fix heuristic, the solver’s branch and cut algorithm is terminated when the MIP gap = 5% or time limit = 5000 sec is satisfied. MIP gap is reduced to 1% in the Fix-and-Optimize heuristic.

For our model, the parameters of the objective function will be assessed to make the solutions meet company target. There are three key concerns of company’s goal. The first important one is zero overtime since the overtime has high cost and will be allowed only in urgent need. Secondly, there are no inventory for the finish product in accordance with a just-in-time strategy. Therefore $w_i$ of the finish products is larger than $w_i$ for the intermediate products. Finally, the setup time is as minimum as possible. We test different values of $q$ and $w_i$ with four weeks historical data in order to meet the company’s objective. Average objective values are shown separately in setup time, inventory level, and overtime in Table 1, where $W$ is the value of $w_i$ for the intermediate products and $w_i$ of the finish product is three times of $W$. When $W$ is less than 100, it is apparent that the overtimes are minimum and quite steady from overtime column in Table 1. Then, we compare the inventory levels when $W$ is less than 100, we can see that all of the inventory levels at $W = 100$ are lower than that at $W = 10$. Finally the setup times are examined at $W = 100$, the objective value of the setup time at $q = 300$ is smallest. These results suggest to use $w_i = 100$ for the intermediate products, $w_i = 300$ for the finish products, and $q = 300$.

4.2. Solutions quality of the heuristic. To investigate the quality of our solution approach, we evaluate and compare the performance of proposed heuristics with MIP by Gurobi. Gurobi could not find a feasible solution from all real data sets within 120 hours. In order to compare the solution quality and computation time, we generate small problems. There are 3 different size of problems, where $N \times M \times T$ are less than 1000, 1000–4000, and 4000–6000. For each problem size,
Table 2. Numerical results of small problems.

| Problem size \((N \times M \times T)\) | \(< 1000\) | \(1000-4000\) | \(4000-6000\) |
|-------------------------------------|-----------|-------------|-------------|
| Avg. Time (sec)                    | 8716      | 958         | 35226       |
| Avg. Gap (%)                       | 3.94      | 5.67        | 5.44        |
| StDev. Gap                         | 1.81      | 4.06        | 1.88        |

Table 3. Numerical results of real problems by our heuristics.

|                      | Avg. Time (sec) | Avg. LBDev (%) |
|----------------------|-----------------|----------------|
| High variant of products family | 8330            | 18.54          |
| Low variant of products family | 2756            | 1.47           |

Table 4. Total objective value between the company solutions and our model solutions.

| Week | Company | Model |
|------|---------|-------|
| 1    | 1,473,400 | 1,209,100 |
| 2    | 1,973,405 | 1,294,400 |
| 3    | 2,008,300 | 1,885,500 |
| 4    | 15,855,500 | 11,345,500 |

10 instances are generated. Table 2 indicates the average of solution times and the average and standard deviation of MIP gaps of each different size of instances between MIP and proposed heuristics (indicated by Heu.).

To examine whether our proposed heuristics influenced the gaps, paired t-tests were studied to compare the average gaps of each problem of difference size between the MIP and the proposed heuristics. No significant difference in average gap was founded (paired t-test, \(P > 0.1\), \(n = 10\)) between the MIP and the proposed heuristics for the problem size \((N \times M \times T) < 1000\) and \(1000-4000\). Therefore, the average gap from the proposed heuristics that is higher is not significantly different from the MIP. The average gap in the problem size \((N \times M \times T) 4000-6000\) was significantly increased (paired t-test, \(P < 0.05\), \(n = 10\)). However, comparing the columns of MIP and heuristics in Table 2 our heuristics determines high quality solutions, as the average gap is less than 10 % with average computation time not more than 30 minutes.

Furthermore, we compared the heuristics solution (\(HeuSol\)) of 12 real problems data \((N \times M \times T > 6000)\) to the best lower bound (LB) computed by Gurobi MIP solver after 12 hours. Lower bound deviation (\(LBDev\)) indicates the solution quality for a single test instance and is defined as

\[
LBDev = \frac{(HeuSol - LB)}{HeuSol} \cdot 100\%.
\]

From the results, there are two different groups of data that has high \(LBDev\) (larger than 5 %) and low \(LBDev\) which can be seen in Table 3. In column “Avg. LBDev”, the average deviation from the lower bound is indicated. We observed the characteristics of the problems and found that the problem that has high-variant of products will result to high \(LBDev\). When the products to be scheduled have high-variant of product family, the sequence dependent setup time also varies. These results show that sequence dependent setup times play an important role in computation of the problem.
4.3. **Comparison with company own planning.** The current planning procedure of the company is done manually using a spreadsheet. It takes a few days and three workers to build all production planning. To compare with the company own planning, real historical 4 weeks data sets are used for backtesting. This investigation will verify whether the proposed model and heuristics are applicable.
Table 4 compares the total objective value between the company’s and the model’s solution. It is obvious that the solutions from our model are better than those from the company. Then, we examine each component of the objective value illustrated in Figure 6–Figure 8. Figure 6 provides the differences of total setup time for each week between the company planning and our mathematical model. It is clear that the total setup time by the model is less than those by the company. In Figure 7, the inventory levels by the company drop because they have strategy to reduce inventories at the end of the month at the expense of the overtime as seen in Figure 8. From Figure 8 the overtime occurs only in the last week and the overtime by our model is less. Overall, these results indicate that our proposed model is practical and provide a better solution.

5. Conclusion. The main goal of the current research is to address the capacitated lot-sizing and scheduling problem for multi-stage parallel-machine flow shop with sequence-dependent setup to minimize setup times, lead times, and overtimes. Within this case study, the industry plans and schedules without computerized tools leading to inefficient production planning. In order to cope with the problem, practical mathematical programming model is proposed. The model covers multiple-stage production and sequence-dependent setups. For our problem, setup times also do not hold the triangle inequality leading to a solution with subtours. Thus, we introduced a novel no subtour formulation.

Standard MIP solver cannot find a feasible solution for the large industrial problems. Therefore, MIP-based heuristics are developed to produce good solutions in an acceptable computation time. Better solutions is obtained compare to those current industrial practice and meet the company strategy. The results of this research also indicate that these two characteristics, multiple-stage production and sequence-dependent setup, play an important role on computation of the problem. A possible area of future research would be to find more efficient algorithms or heuristics to provide solutions in shorter time.

Acknowledgments. This research is supported in part by Grant-in-Aid for Science Research (A) 26242027 of Japan Society for the Promotion of Science.

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Received May 2015; 1st revision October 2015; 2nd revision November 2015.

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