Influence of different wavelet filtering reconstruction techniques applied to bidimensional surface texture characterization

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Abstract: This article develops a preliminary study of the wavelet filtering application in surface finishing, extracting two-dimensional information from the surface microgeometry in a profile. First, the key elements to be considered in the analysis are described, starting with the surface finishing process of measurement. Then, the fundamentals of wavelet -based filtering and the traditional filtration procedure is shown. This procedure eliminates the values of the detail coefficients obtained in the filtering process before the reconstruction of the signal. Other alternatives of filtration, based on the shrinkage wavelet methodology, are also considered. Next, the features of the surface finish characteristic signals are highlighted, and the alternatives considered are applied on three different profiles generated from the result of a Brinell hardness test. Finally, the main conclusions drawn from the study are presented.

Keywords: Surface texture, Filtering, Wavelet, Surface finishing, Threshold, Wavelet shrinkage.

1. Introduction

Characterization of the surface finish or surface texture has a great relevance in manufacturing engineering processes. This is mainly due to the large number of functional behaviors of different components that are affected by the surface finishing condition [1]. Mechanical contact [2], corrosion [3], hydrodynamic lubrication [4], thermal conductivity [5], electrical resistance [6], and many others are some examples of these operational behaviors that prove the importance of surface finishing in the accomplishment of manufacturing procedures.

The surface finish measurement and characterization methodologies are schematised in figure 1. From the dimensional data collected by the measuring instrument, 2D (profile) or 3D (area) measurement results are obtained. This information is filtered to separate the microgeometric irregularities associated with different wavelengths. Once this division has been conducted, the surface finishing parameters will be evaluated, in particular Ra or Rq, which are the quantifying factors of the surface finish characterization.

2. Wavelet Filtering

As previously described, in the surface finish, the filtering process aim is to separate the microgeometric irregularities associated with different wavelengths in order to properly characterise the microgeometry of the component and its influence on its functional behaviour during service. In the two-dimensional...
analysis using profiles, the raw data is contained in the so-called effective profile or total profile (T). The first step of the filtering procedure is to eliminate really small wavelength irregularities (2.5 µm is taken as the standard value), this leads to the primary profile (P) which is subsequently filtered again, finally obtaining the roughness profile (R) and the wave profile (W).

Figure 1. Measurement and characterization processes to determine surface texture.

The Gaussian filter, defined in the ISO 16610-21 and ISO 16610-61 standards in their two- and three-dimensional versions, is the most common filtering technique used in the surface finish analysis. In general, it provides representative results although it presents some issues processing the edges of the signal [7] and the surroundings of outliers and steps, in which other filters such as Splines or Wavelet can offer significant advantages [8].

Figure 2. Direct and inverse multilevel Discrete Wavelet Transform DWT.

Wavelet-based filters are an interesting alternative in measuring surface finish that can reduce some of the Gaussian filtering drawbacks. However, they are not sufficiently advanced or widespread yet. The ISO 16610-29 and ISO 16610-69 standards contemplate this type of filtering, but their current versions are still immature in development which makes its application difficult. Wavelet filtering is based on the application of the Discrete Wavelet Transform (DWT) to a Z vector. In the case of surface finishing, Z has a size N, and contains the heights zᵢ {i = 1, .., N} of the N equally spaced points which define the
primary profile. The direct discrete transform carries out the decomposition of Z into two output vectors, designated as zA (vector of approximations) and zD (vector of details) whose size is half the initial vector, Z, size as shown in figure 2.

On the other hand, the inverse discrete transform combines two vectors of the same size, generating an output vector with twice the size of the input vectors. If the inverse transform is applied to the output vectors of the direct transform, the obtained result is the reconstruction of the initial vector. The terms "decomposition" and "reconstruction" are generally used in association with the direct and inverse wavelet transforms, respectively. In the implementation of the wavelet filtering, it is usual to perform a multiresolution analysis (MRA), which consists of the iteration of the filtering process in multiple stages, applying in each of them the direct discrete transform to the output signal of the preceding stage until the required decomposition level is reached, as shown in figure 2.

Since the number of data points of the output signal is reduced by half in each decomposition process, there is a limiting maximum level of decomposition L_{max} that depends on the number of points N of the initial vector, which is defined by L_{max}=\log_2(N). The MRA is carried out until completion by applying the inverse discrete transform exactly as many times as the direct transform was applied in the decomposition process, which allows the reconstruction of a vector with the same size as the initial one. If the corresponding approximation and detail vectors obtained in each stage are implemented in the decomposition-reconstruction procedure, and the same wavelet families are used, the final result will be identical to the starting vector. The current wavelet filtering technique is based on the reconstruction of the Z vector using modified versions of the detail and/or approximation vectors, zA or zD. The most commonly utilized transformation of these vectors, named as ZERO_T hereinafter, consists of removing the detail coefficients of each decomposition stage before applying the reconstruction. Nevertheless, due to the advent of the wavelet filtering usage and its potential features, researching other transformation alternatives of the approximation or detail vectors, zA or zD, is of significant relevance as well as analysing their impact on the final filtered profile obtained.

3. Wavelet shrinkage

Wavelet shrinkage is a noise elimination method in digital signal processing. Its principles are based on the application of a thresholding to the vectors resulting from the wavelet decomposition obtained by means of the previously mentioned direct discrete wavelet transform implementation. The wavelet shrinkage diagram is displayed in figure 3.

Figure 3. Wavelet shrinkage scheme.
First, the direct DWT is applied consecutively to the input vector Z (Primary profile), multiresolution analysis, until reaching a fixed decomposition level \( k \). Next, a threshold evaluation is performed. The computed threshold \( \lambda \) can be the same for every decomposition stage or different for each one, depending on the chosen thresholding technique. In the following step, by using the calculated threshold, a certain transformation is applied to the decomposition output vectors aiming to modify the subsets of the vector’s components following a specific reconstruction method. This transformation is selected on the basis of several parameters that are applicable. Finally, the iDWT performs the multiple reconstruction process with the modified vectors which leads to the shrinkage wavelet output, W (Waveness profile). Key pioneering achievements in the use of this methodology were possible because of Donoho et al work [9] which allowed the establishment of the wavelet shrinkage foundation.

In table 1 the different threshold evaluation techniques used in this paper alongside their respective equations are shown. It must be noted that, in the Universal, Simple and SURE Threshold methods, a single threshold value, \( \lambda \), is determined for every decomposition level, while in the Gradual and SURE Gradual Threshold methods, a different threshold value, \( \lambda_k \), is calculated for each level of decomposition.

| Method               | Acronym | Formulae                                                                 |
|----------------------|---------|---------------------------------------------------------------------------|
| Universal Threshold  | UN_T    | \( \lambda = \sigma \ast \sqrt{2 \ast \ln N} \)                           |
| Simple Threshold     | SI_T    | \( \lambda = \text{mean}(\overline{d}) + |y \ast \text{std}(\overline{d})| \) |
| Gradual Threshold   | GR_T    | \( \lambda_k = |\text{mean}(d_k)| + |y \ast \text{std}(d_k)| \) |
| SURE Threshold       | SU_T    | \( \lambda = t^*, \text{where } t^* \text{ minimizes } R(\overline{d}, t) \) |
| SURE Gradual Threshold | SG_T   | \( \lambda_k = t^*_k, \text{where } t^*_k \text{ minimizes } R_k(\overline{d_k}, t_k) \) |

\( a \) is the threshold value independently of the decomposition level, \( \lambda_k \) is the threshold computed at a specific decomposition level \( k \).

\( \overline{d} \) represents the vector resultant from merging all the detail vectors obtained for each level of decomposition and \( N \) is its size, \( \overline{d_k} \) is the detail vector at the decomposition level \( k \), its size is \( N_k \), and \( d_{k,i} \) is the value of the coefficient \( i \) of the detail vector at the decomposition level \( k \).

\( \text{mean} \) and \( \text{std} \) indicate the mean and standard deviation of a certain vector and \( y \) is a positive scalar.

Once the \( \lambda \) value, or \( \lambda_k \) values, of the threshold has been established according to the selected methodology, the next step is to apply a transformation to the detail vectors, result of the wavelet decomposition, by modifying certain coefficients which objective is to remove the roughness component from the final reconstructed profile. The complexity associated with the creation of each transformation techniques is based on the need to not lose relevant information contained in the frequencies linked with roughness. Therefore, the aim is to obtain new vectors of details that optimise the elimination of “noise”, which in this particular case is roughness.

In most of the transformation methods, the previously evaluated thresholds are implemented, which allows separating the interval of detail coefficients that define the roughness from the rest of the coefficients which provide notable data about the profile. The transformed detail vectors will be denoted by \( \overline{d'_k} \) while the detail vectors prior to the transformation will be \( \overline{d_k} \). In the table 2, it is shown the different threshold transformations considered in this work.
Table 2. Reconstruction techniques and their mathematical formulae.

| Acronym | Threshold Transformation | Formulae |
|---------|--------------------------|----------|
| ZERO_T  | Zero Transformation      | $d'_k = 0$, $|d_k| \leq \lambda$ |
| HARD_T  | Hard Transformation      | $d'_k = \begin{cases} \frac{2\lambda}{1 + e^{(\lambda - \text{sign}(d_k) \cdot d_k)}}, & |d_k| > \lambda \\ 0, & |d_k| \leq \lambda \end{cases}$ |
| SOFT_T  | Soft Transformation      | $d'_k = \begin{cases} \text{sign}(d_k) \cdot (|d_k| - T \cdot \lambda), & |d_k| > \lambda \\ 0, & |d_k| < \lambda \end{cases}$ |
| SSO_T   | Semisoft Transformation  | $d'_k = \begin{cases} \text{sign}(d_k) \cdot (|d_k| - T \cdot \lambda), & |d_k| > \lambda \\ 0, & |d_k| < \lambda \\ \frac{\lambda}{e^{(\frac{|d_k| - \lambda}{\lambda})}}, & |d_k| \geq \lambda \end{cases}$ |
| HUI_T   | Huimin Transformation    | $d'_k = \begin{cases} 0, & |d_k| < \lambda \\ \frac{2\lambda}{1 + e^{(\lambda - \text{sign}(d_k) \cdot d_k)}}, & |d_k| \geq \lambda \end{cases}$ |
| JIN_T   | Jing Transformation      | $d'_k = \begin{cases} \frac{\lambda}{e^{(\frac{|d_k| - \lambda}{\lambda})}}, & |d_k| \geq \lambda \end{cases}$ |

4. Some considerations and application example

In the previous sections, the key concepts and algorithms required to apply a filtering based on the wavelet shrinkage methodology to a vector have been described. When this vector contains the microgeometric information of a surface, it is convenient to carry out a series of preliminary considerations in order to take into account the peculiarities of the surface finishing theory in the results.

- **Objective sought with the shrinkage wavelet implementation in the surface finish.** Although the answer to this statement may seem obvious, it is important to mention some differences between the traditional purpose of denoising applications and the aim pursued in surface finish. Indeed, since its inception, wavelet shrinkage has been used as a noise removal technique in digital signals [9]. The signal is the relevant information, and the noise is the secondary element. In surface finishing, the roles are reversed, that is, the noise (the roughness) is important, while the signal (the waveness) generally occupies a secondary place. Although mathematically this fact does not suppose any algorithmic change, it must be taken into consideration as its interpretation conditions the type of signals to be analysed. Thus, for example, discontinuities (step functions based on the Dirac delta theory) are a constant critical issue in traditional wavelet shrinkage applications. By contrast in the study of surface finish those non-continuous values are not significant because they do not represent a common phenomenon in surface microgeometry. The surface finishing field seeks to quantify microgeometric irregularities through computed parameters with the filtered profile data (e.g. arithmetic mean roughness Ra or mean square roughness Rq), while in traditional applications the final result is the filtered signal itself.

- **Wavelet families.** As in other science and engineering sector where wavelets are applied, there is no one wavelet family that is "better" than another [12] for a given application. This fact implies the need to perform more exhaustive studies in the typology of wavelet families with the aim of finding the most suitable one for each type of microgeometry.

- **Decomposition level.** In traditional applications, the highest level of decomposition tends to be achieved in multiresolution filtering using wavelets. In surface finish, this condition is not applicable since, if the maximum level of decomposition is reached, the filtering process is not effective enough. A more detailed analysis on this aspect can be found in [13]. The number of data points of the treated signal also falls within this consideration. In traditional applications this number is usually a power of two (number of data points $= 2^n$; $n \in \mathbb{N}$) but in surface finish, the number of attainable data points depends on the characteristics of the measurement.
instrumentation and generally does not meet the principle previously mentioned.

- **Type of signal analysed.** Regardless of other options, the signals studied - in this particular case, the profiles - are usually divided into two types: experimental and synthetic. The first, the experimental ones, are those that come directly from the equipment or instrumentation signal measurement, therefore are conditioned to their operational characteristics. Secondly, the synthetic ones, are those that are generated using mathematical algorithms. In these latter profiles, there are numerous variants that depend on the key profile features to be represented.

- **Contrasting the results.** This aspect considers the procedure to be followed with the purpose of validating the adequacy of the analysis conducted. In traditional applications, and when working with synthetic signals, the difference between the original signal (noiseless) and the signal obtained after the denoising process is usually evaluated. This methodology may be valid in the first instance throughout the surface finishing study, however, taking also into account the considerations previously described, it is possible that this contrasting technique is insufficient, being necessary to contrast with other filtering processes. Likewise, the selection of the quantifying element used in the comparison should be also assessed, such as the MSE (Mean Square Error), the NSR (Noise Signal Ratio) or others.

Once the previous considerations contained in this section have been explained, the following proceeding is to evaluate the surface finish profiles represented in figure 4.

The aforementioned profiles are based on the hemispherical footprint with a diameter of 1.6 mm generated by carrying out a Brinell hardness test on an ASTM 329 steel. Three theoretical roughness profiles have been added to the already stated footprint, generated as follows: 1) Aerofoil Profile (AP), from an experimental roughness measurement of an aerofoil, 2) Mill Profile (MP), from a milled profile obtained from NIST [14] and 3) Normal Profile (NP), from a Normal distribution N (0.1). A symlet4 wavelet was utilised in the calculations and the level of decomposition was L=10, selected as a function of the number of data points contained in the profile [13]. As a comparative element of the results obtained against the initial theoretical profile, the parameter Root Mean Square Error (RMSE) will be determined, which is defined for a vector of size N as:

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{N}(z'_i - z_i)^2}{N}}
\]  

(1)

Where \(z_i\) and \(z'_i\) are the components of the original and filtered profiles respectively. The results
obtained corresponding to the profiles represented in figure 4 are summarized in tables 3, 4 and 5.

**Table 3.** Normal profile results for every threshold evaluation method and reconstruction technique.

|       | HARD_T | SOFT_T | HUI_T | SSO_T | JIN_T | ZERO_T |
|-------|--------|--------|-------|-------|-------|--------|
| SI_T  | 0.6109 | 0.4640 | 0.5115 | 0.5437 | 1.7337 |        |
| GR_T  | 1.2267 | 1.5309 | 2.3073 | 1.2866 | 1.3577 | 1.7337 |
| UN_T  | 0.5745 | 0.4682 | 0.6155 | 0.4859 | 0.5049 | 1.7337 |
| SU_T  | 0.7398 | 0.6508 | 0.5924 | 0.6922 | 0.7287 | 1.7337 |
| SG_T  | 0.7464 | 0.7464 | 0.7464 | 0.7464 | 1.7337 |        |

Table 4. Aerofoil profile results for every threshold evaluation method and reconstruction technique.

|       | HARD_T | SOFT_T | HUI_T | SSO_T | JIN_T | ZERO_T |
|-------|--------|--------|-------|-------|-------|--------|
| SI_T  | 0.5604 | 0.3761 | 0.4195 | 0.4472 | 0.5065 | 1.7386 |
| GR_T  | 1.2457 | 1.6715 | 2.5295 | 1.3686 | 1.5831 | 1.7386 |
| UN_T  | 0.5274 | 0.3656 | 0.5656 | 0.4019 | 0.4395 | 1.7386 |
| SU_T  | 0.6128 | 0.5561 | 0.5114 | 0.5833 | 0.6097 | 1.7386 |
| SG_T  | 0.6151 | 0.6151 | 0.6151 | 0.6151 | 1.7386 |        |

Table 5. Mill profile results for every threshold evaluation method and reconstruction technique.

|       | HARD_T | SOFT_T | HUI_T | SSO_T | JIN_T | ZERO_T |
|-------|--------|--------|-------|-------|-------|--------|
| SI_T  | 0.5904 | 0.3420 | 0.5007 | 0.4142 | 0.3517 | 1.7352 |
| GR_T  | 1.1656 | 1.5660 | 2.3595 | 1.2789 | 1.3941 | 1.7352 |
| UN_T  | 0.3905 | 0.3875 | 0.6290 | 0.3348 | 0.2714 | 1.7352 |
| SU_T  | 0.9619 | 0.7503 | 0.6066 | 0.8507 | 0.9343 | 1.7352 |
| SG_T  | 0.9850 | 0.9850 | 0.9850 | 0.9850 | 1.7352 |        |

Taking into consideration the RMSE parameter, the Zero Transformation technique, despite of being the traditional procedure, offers the least representative results. In order to satisfy the need for using wavelets in denoising applications, other threshold and reconstruction methodologies must be studied.

Regarding the threshold evaluation techniques, the gradual methods lead to the most inadequate reconstructed profile with respect to the original one. Furthermore, the SURE Threshold technique performance is lower than the Universal Threshold and Simple Threshold. These two latter procedures provide the most consistent establishment of the threshold for the subsequent reconstruction; therefore, the effectiveness of the final result will only depend on the reconstruction technique itself.

As for the reconstruction techniques, there is not a distinctive behavior pattern that clearly identifies the most determinant reconstruction method. Primarily, when implementing the Simple Threshold method, the Soft Transformation technique is the optimal one. Nevertheless, the best alternative will depend on the profile to be treated.

5. Conclusions

The results obtained in this preliminary study allow to reach various conclusions. Wavelet filtering is suitable for the study of surface finish and its application is algorithmic and computationally viable.
Nevertheless, there are numerous factors to be considered that require a more detailed study, such as the wavelet family or families to implement depending on the profile type, the optimal decomposition/reconstruction level based on the available signal information, among others. The application of the threshold in the filtering reconstruction process opens up new possibilities that also need to be evaluated in greater depth. In the present work, five threshold evaluation algorithms have been compiled, which have been combined with six reconstruction techniques. In wavelet filtering, the details removal procedure is traditionally used, although in view of the results obtained, there may be other reconstructions methods that lead to more precise values.

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