Probing new physics through $B \rightarrow Kl^+l^-$ decays in R-parity violating minimal supersymmetric standard model

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Abstract

We study the decay rate of process $B \rightarrow K l^+l^-(l=e,\mu)$ and some of its other related observables, like forward backward asymmetry ($A_{FB}$), polarization asymmetry ($P_{A}$) and CP-asymmetry ($A_{CP}$) in R-parity violating (RPV) Minimal Supersymmetric Standard Model (MSSM). The analysis shows that RPV Yukawa coupling products contribute significantly to the branching fraction of $B \rightarrow K l^+l^-$ within $1\sigma$ and $2\sigma$. Study shows that $P_{A}$ and $A_{FB}$ are sensitive enough to RPV Yukawa coupling products and turn out to be good predictions for measurement in future experiments. The CP-asymmetry calculated in this framework agrees well with the recently reported value (i.e. 7%).

1 Introduction

$B$ physics has played a central role in clarifying issues related to the standard model (SM) like measurements of Cabibbo-Kobayashi-Maskawa (CKM) unitary angles, probing CP-violation and $B$ decays sensitive to new physics[1]. Belle and BaBar experiments have contributed significantly to this important area of particle physics which holds promise and potential to discover physics beyond the standard model or new physics (NP) and offers a motivation for construction of super $B$-factories [2]. Various authors have studied the contribution of NP, like 2 Higgs doublet Model(2HDM), MSSM and extra dimensions, etc, to $B$ decays[3].

There are some important processes in $B$-physics interesting to probe for NP involving $B\bar{B}$ mixing and leptonic $B$-decays [4]. Leptonic $B$ decays are simple to analyze and may provide some clear signals for exploring physics beyond SM. Among such processes are the FCNC (flavour changing neutral current) processes. FCNC proceed through higher order box and penguin diagrams in SM, whose contribution may compete with that of leading contributions of R-parity violating MSSM. While the branching fraction in such processes may be an obvious choice for testing bounds on parameters related to physics beyond the SM.
or NP, different observables like $A_{FB}$, $PA$ and $CP$ asymmetry may also show sensitivity to NP parameters. Several authors have studied NP contribution to these observables in different models like 2HDM and extra dimensions and confirmed that these observables are indeed sensitive to parameters of NP [5, 6].

Supersymmetry (SUSY) was proposed to extend the Standard Model and to define a more complete theoretical framework which will unify all interactions including gravity. SUSY among other issues solves the gauge hierarchy problem related to Higgs boson and leads to the hint for unification of forces [7]. As an extended symmetry of space-time, it gives rise to spin doubling of the constituent and force fields and defines a generalized superpotential which contains a part conserving R-parity and another allowing R-parity violation [9]. SUSY with finite neutrino Majorana mass leads to lepton flavor mixing and oscillations, an important area of recent research. Supersymmetric partners (sparticles) have, however, not been found in nature yet, though the search for SUSY is now focussed on LHC with expectations. SUSY, however, remains as a promising candidate theory and has made its way into many other NP scenarios like extra dimensions and strings [10].

One of the drawbacks of SUSY is that matter is no longer predicted to be stable as proton can decay through sparticles. In order to avoid this catastrophic scenario, R-parity was introduced [9]. R-parity is a discrete symmetry and is defined as $R = (-1)^{3B+L+2S}$, where $B$, $L$ and $S$ are baryon, lepton number, and spin of a particle respectively. This implies that sparticles cannot mediate an interaction and the sparticles are always produced in pair. The second consequence implies the existence of a lightest supersymmetric sparticle, which is a very good candidate for dark matter. R-parity, though, ensures the stability of matter, is an ad hoc assumption. R-parity violation, however, cannot be ruled out theoretically. R-parity violation ($R_p^p$), is introduced by relaxation of R-parity conservation with some constraints on couplings responsible for proton decay.

The $R_p$ superpotential approach results in a framework which is phenomenologically rich and has been actively pursued. Lepton flavor and number violating decays are mediated by sparticles. In addition, neutrino also acquires mass [11]. FCNC constitutes as one important part of $R_p$ SUSY phenomenology. The aim of this paper is to discuss how well the experimentally observed value of branching fraction of FCNC ($b \rightarrow s l^+ l^-$) fits with the theoretical predictions on the basis of $R_p$. Further, using SUSY, to examine, parameters which fit the decay rate in the process to explain forward backward, polarization, and CP asymmetries in this decay for comparison with future expected experiments.

In Sec. 2, we review the effective Hamiltonian and also $R_p$ SUSY potential approach for use in our calculation. In Sec. 3 we review and analyze some of the results on form factors from existing literature to be used in our study based on $R_p$ violating MSSM. Further, in this section we give the calculation, in
this framework, for the decay rate of the decay processes $B \to Kl^-l^+, l = e, \mu$, and the relevant various asymmetries to this process. In Sec. 4, we give results and implications of our results from the point of view of possible comparison of measured values in future experiments. Last section Sec. 5, gives conclusion and summarizes our results.

2 $B \to Kl^-l^+$ decay in SM and $R_p$ SUSY

The effective Hamiltonian for the given decay process is given by [12, 5]

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} \{ C_i(\mu) O_i(\mu) + C_Q(\mu) O_Q(\mu) \}.$$  

(1)

The first set of operators in eq. (1) describe the contribution from SM, while the second term describes the contribution from physics beyond SM. $\mu$ defines the energy scale of interactions. The corresponding Wilson coefficients for SM can be found in literature [5, 12]. Form factors are needed to calculate the decay rate of exclusive $B \to Kl^-l^+; l = e, \mu$ channel. We use the form factors analytically derived in literature [12].

Since we are interested in comparing the SUSY contribution along with that of SM in the observed value of branching fraction, we proceed to consider MSSM ($N = 1$) super potential [8, 9]

$$W_{R_p} = \frac{1}{2} \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \frac{1}{2} \lambda''_{ijk} U^c_i D^c_j D^c k + \mu_i H_u L_i,$$  

(2)

where $i, j, k$ are generation indices, $L_i$ and $Q_i$ are the lepton and quark left-handed $SU(2)_L$ doublets and $E^c, D^c$ are the charge conjugates of the right-handed leptons and quark singlets, respectively. $\lambda_{ijk}, \lambda'_{ijk}$ and $\lambda''_{ijk}$ are Yukawa couplings. Note that the term proportional to $\lambda_{ijk}$ is antisymmetric in the first two indices $[i, j]$ and $\lambda''_{ijk}$ is antisymmetric in the last two indices $[j, k]$, implying $9(\lambda_{ij}) + 27(\lambda'_{ij}) + 9 \left( \lambda''_{ij} \right) = 45$ independent coupling constants among which 36 are related to the lepton flavor violation (9 from $LLE^c$ and 27 from $LQD^c$). The last term can be rotated away by a unitary transformation. However this may induce additional terms involving Yukawa couplings, not relevant here [13]. In $R_p$ MSSM the relevant effective Lagrangian is given by [9]

$$L_{\text{eff}}^{R_p} (b \to s l_\beta + \bar{l}_\beta) = \frac{G_F}{2\sqrt{2\pi}} \left[ A_{\beta\beta} \left( \bar{l}_\beta \gamma^\mu P_L l_\beta \right) \left( \bar{s} \gamma_\mu P_R b \right) - P_R b \left( \bar{l}_\beta \gamma^\mu P_L l_\beta \right) \left( \bar{s} \gamma_\mu P_R b \right) - P_L b \left( \bar{l}_\beta \gamma^\mu P_L l_\beta \right) \left( \bar{s} \gamma_\mu P_R b \right) \right], \beta = e, \mu$$  

(3)

where $P_R = \frac{1+\gamma_5}{2}; \ P_L = \frac{1-\gamma_5}{2}$. The first term in eq.(3) comes from the up squark exchange and the remaining two terms come from sneutrino exchange. Here $s$ and $b$ denote strange and beauty down type quarks. The
dimensionless coupling constants $A_{\beta\beta}^{bs}$, $B_{\beta\beta}^{bs}$, and $C_{\beta\beta}^{bs}$ depend on the species of charged leptons and are given by [14]

$$A_{\beta\beta}^{bs} = \frac{2\sqrt{2}\pi}{G_F} \sum_{m,n,i=1}^{3} \frac{V_{ni}^\dagger V_{im}}{2m_{\nu_i}} \lambda_{\beta nk}^* \lambda_{\beta mp},$$

$$B_{\beta\beta}^{bs} = \frac{2\sqrt{2}\pi}{G_F} \sum_{i=1}^{3} \frac{2}{m_{\nu_i}^2} \lambda_{i\beta\beta} \lambda_{ikp}^*,$$

$$C_{\beta\beta}^{bs} = \frac{2\sqrt{2}\pi}{G_F} \sum_{i=1}^{3} \frac{2}{m_{\nu_i}^2} \lambda_{i\beta\beta} \lambda_{ikp}^*.$$  

In the Section 3 we study the asymmetries relevant to this process using the above described R-parity violating MSSM framework which essentially involves, in this problem, simple computation of Feynman diagrams with exchange of SUSY particles (Fig. 1).

3 Calculations and Analysis

The matrix element of the decay rate ($B \to Kl^+l^-$) is given by [12]

$$M = F_S i\bar{l}l + F_V p_{\mu} \tilde{\gamma}^\mu i\bar{l}l + F_A (p_B)_{\mu} i\tilde{\gamma}^\mu i\bar{l}l + F_P i\gamma^5 i\bar{l}l,$$  

where $(p_B)_\mu$ is the initial momentum of $B$ meson and $F_{S,V,A,P}$ are functions of lorentz invariant quantities like dilepton centre of mass energy squared ($s$). These functions involve the Wilson coefficients ($C_i(\mu)$ and $C_Q(\mu)$) as given below in eq.(11) are reproduced from [12].

The Forward-Backward Asymmetry ($A_{FB}$) is related to asymmetric angular distribution of dilepton pair with respect to the initial meson direction of momentum in the dilepton rest frame. The $A_{FB}$ is defined as:

$$A_{FB}(s) = \frac{f_0^1 d\cos\theta d^2\chi}{d\cos\theta d^2\chi} - \frac{f_{-1}^0 d\cos\theta d^2\chi}{d\cos\theta d^2\chi} \frac{f_0^1 d\cos\theta d^2\chi}{d\cos\theta d^2\chi} + \frac{f_{-1}^0 d\cos\theta d^2\chi}{d\cos\theta d^2\chi}$$

In the dilepton rest frame [15] [16]

$$A_{FB}(s) = \frac{1}{128\pi^3 m_B^3 \left( \frac{d\Gamma}{ds} \right)} m_B\lambda(\lambda) \Re(F_S F_V^*),$$

where

$$\frac{d\Gamma}{ds} = \frac{1}{256\pi^3 m_B^3} \beta(\lambda) \lambda^\dagger(s) R(s)$$  

(10)
\[
\beta(m_l, s) = \left(1 - \frac{4m_l^2}{s}\right)^{\frac{1}{2}},
\]
\[
\lambda(s) = m_B^4 + m_K^4 + s^2 - 2sm_B^2 - 2sm_K^2 - 2m_B^2m_K^2,
\]

where

\[
R(s) = |F_S|^2 2\beta(m_l, s)^2 + |F_P|^2 2s + |F_V|^2 \left[ \frac{1}{3} \lambda(s)(1 + \frac{2m_l^2}{s}) + |F_A|^2 \left[ \frac{1}{3} \lambda(s)(1 + \frac{2m_l^2}{s}) + 8m_B^2m_l^2 \right] + \text{Re}(F_P F_A^*) 4m_l(m_B^2 - m_K^2 + s) \right],
\]

and \( s \) (invariant mass squared of the dilepton system in its rest frame) is bounded as:

\[
4(m_l)^2 \leq s \leq (m_B - m_K)^2
\]

The coefficients \( F_V, F_A, F_P \) are determined by using eqs.(1, 7 & A-1 to A-4)

\[
F_V = \frac{G_F \alpha V_{tb} V_{ts}^*}{2 \sqrt{2\pi}} \left( 2 C^\text{eff}_{q} f^+ (s) - C_7 \frac{4m_b}{m_B + m_K} f^T (s) \right) + \frac{1}{4} f^+(s) \sum_{i,m,n=1}^3 \lambda i_{mn} \lambda^\ast i_{n,m} m_i^2 m_i^2, \tag{11}
\]

\[
F_A = \frac{G_F \alpha V_{tb} V_{ts}^*}{2 \sqrt{2\pi}} \left( 2 C_{10} f^+ (s) \right) - \frac{1}{4} f^+(s) \sum_{i,m,n=1}^3 \lambda i_{mn} \lambda^\ast i_{n,m} m_i^2 m_i^2, \tag{11}
\]

\[
F_S = \frac{1}{4} \frac{(m_B^2 - m_K^2)}{m_B - m_K} f o(s) \sum_{i=1}^3 \frac{(\lambda i_{33} \lambda_{23} - \lambda i_{13} \lambda_{32})}{m_{\beta_i}^2}, \tag{11}
\]

\[
F_P = \frac{G_F \alpha V_{tb} V_{ts}^*}{2 \sqrt{2\pi}} 2m_l C_{10} (f^+ (s) + f^- (s)) + \frac{1}{4} \frac{(m_B^2 - m_K^2)}{m_B - m_K} f o(s) \sum_{i=1}^3 \frac{(\lambda i_{33} \lambda_{23} + \lambda i_{13} \lambda_{32})}{m_{\beta_i}^2}, \tag{11}
\]

Notice that in eq. (11), \( F_V, F_A, F_P \), under \( H_{eff} \) (see eq.(1)) are split-up into two parts each carrying SM&MSSM contributions, while \( F_S \) contains only the contribution from MSSM.

Also here, \( f^\pm (s), f^T (s), fo(s) \) are form factors\[12\] related to decay \( B \rightarrow Kl^\pm l^- \). We use values of Wilson coefficients \( C_i(m_b) \[6 \quad 12 \quad 13\] to leading order evaluated at the rest mass \( m_b \) of b quarks (see eqs. A-5 to A-7 of Appendix A). Inegrated branching fracion for this process used in our calculation for comparison with the data on branching ratio is given

\[
BR(B^\pm \rightarrow Kl^\pm l^-) = \int_{4(m_l)^2}^{(m_B - m_K)^2} \frac{dT}{ds} \, ds \, \tau_o \tag{12}
\]

where \( \tau_o \) is the life time of \( B \) meson.

The polarization asymmetries are defined, as usual, as \[6 \quad 12\]:

5
\[
(PA)_i = \frac{d\Gamma(\hat{n} = -\hat{e}_i)}{d\hat{n}} - \frac{d\Gamma(\hat{n} = \hat{e}_i)}{d\hat{n}} + \frac{d\Gamma(\hat{n} = -\hat{e}_i)}{d\hat{n}} + \frac{d\Gamma(\hat{n} = \hat{e}_i)}{d\hat{n}}, \tag{13}
\]

where \((i = L, N, T)\) and \(\hat{n}\) is the spin direction of lepton \(l\). Following three polarization unit vectors are defined in the centre of mass of the \(l^+l^-\) system and are given in literature[6, 12]:

\[
\begin{align*}
\hat{e}_L &= \frac{\vec{p}_\ell}{|\vec{p}_\ell|}; \\
\hat{e}_N &= \frac{\vec{p}_K \times \vec{p}_\ell}{|\vec{p}_K \times \vec{p}_\ell|}; \\
\hat{e}_T &= \hat{e}_N \times \hat{e}_L;
\end{align*}
\]

where \(\vec{p}_\ell\) and \(\vec{p}_K\) are the three momenta of the \(l^-\) lepton and the \(K\) meson respectively. \(\hat{e}_L\) is the unit vector of lepton in the dilepton rest frame of reference. Unit vector \(\hat{e}_N\) is normal to the plane of \(K\) meson and \(\hat{e}_L\) and \(\hat{e}_T\) lie in the plane containing lepton and \(K\) meson momentum vector.

We plot \((PA)_L\) and \((PA)_T\) versus dilepton centre of mass energy and Yukawa couplings to gain an insight into the behavior of these observables in physics beyond SM(based on MSSM). Normal polarization asymmetry \((\hat{e}_N)\) is too small to be given any importance[6]. The relevant expressions for polarization asymmetries as taken from [5, 6, 12]

\[
(PA)_L = \frac{\beta(m_l, s) 2}{R(s)} \left[ \frac{\lambda}{3} \text{Re}(F_V^* F_A) - 4s \text{Re}(F_S^* F_P) - 4m_l((m_B)^2 - (m_K)^2 - s) \text{Re}(A_F^* F_S) \right], \tag{14}
\]

\[
(PA)_T = \frac{\pi \sqrt{\lambda(s)}}{R(s) \sqrt{s}} [m_l((m_B)^2 - (m_K)^2 - s) \text{Re}(F_V^* F_A) + s(\beta(m_l, s))^2 \text{Re}(A_F^* F_S) + s \text{Re}(F_V^* F_P)], \tag{15}
\]

where \(m_l\) is the mass of lepton.

We have also computed average polarization asymmetries, which are defined as

\[
< (PA)_i > = \frac{\int_{4(m_B - m_K)^2}^{4m_B^2} (PA)_i \frac{d\Gamma}{ds} ds}{\int_{4(m_B - m_K)^2}^{4m_B^2} \frac{d\Gamma}{ds} ds}, \tag{16}
\]

Number of events required to calculate \(PA\) at \(n\sigma\) level are given by \(N = n^2/(BR)(< (PA)_i >)^2\), where \(n\) is the desired \((1\sigma \text{ or } 2\sigma)\) level. \(BR\) represents the branching fraction of \(B \to K\ell\bar{\ell}\) and \(< (PA)_i >\) is the polarization asymmetry. CP asymmetries are defined as[17]

\[
A_{CP} = \frac{d\Gamma(B \to K^+\ell^-)}{d\ell} - \frac{d\Gamma(B \to K^-\ell^+)}{d\ell} + \frac{d\Gamma(B \to K^+\ell^+)}{d\ell} - \frac{d\Gamma(B \to K^-\ell^-)}{d\ell} \tag{17}
\]
In SM, $C_9^{eff}$ becomes complex due to non-negligible terms induced in $A$ & $B^{[13]}$. Its complex nature is responsible for CP-asymmetry. $C_7$ & $C_{10}$ being real do not contribute to CP-asymmetry. $\mathcal{R}_p$ Yukawa couplings can be imaginary so we have (see Appendix-B)

$$
\frac{d\Gamma(\bar{B} \rightarrow \bar{K}l^+l^-)}{ds} = \frac{1}{256\pi^3m_B^2}\beta(m_i,s)\lambda^2(s)\{|F_S|^22s\beta(m_i,s)^2 + |F_P|^22s + |F_V|^2\frac{1}{3}\lambda(s)(1 + \frac{2m_l^2}{s})^2 + |F_A|^2\big[\frac{1}{3}\lambda(s)(1 + \frac{2m_l^2}{s}) + 8m_B^2m_l^2\big] + \text{Re}(\bar{F}_P^*F_A)4m_i(m_B^2 - m_K^2 + s)\}\text{,} \tag{18}
$$

$$
A_{CP} = \lambda(s)(1 + \frac{2m_l^2}{s})(|F_V|^2 - |\bar{F}_V|^2)(D(s))^{-1} \tag{19}
$$

where

$$
D(s) = 6(|F_S|^22s\beta(m_i,s)^2 + |F_P|^22s + |F_A|^2\big[\frac{1}{3}\lambda(s)(1 + \frac{2m_l^2}{s}) + 8m_B^2m_l^2\big] + \text{Re}(F_P^*F_A^*4m_i(m_B^2 - m_K^2 + s) + \frac{\lambda(s)}{6}(1 + \frac{2m_l^2}{s})(|F_V|^2 + |\bar{F}_V|^2)) \tag{20}
$$

One can easily see from the expression of $F_V$ & $\bar{F}_V$ from appendix B that if SUSY terms containing $\lambda'_{j\alpha_3}\lambda'_{3\alpha_2}$ vanish in eq.(20), then one recovers SM result for $A_{CP}$.

Also it is clear from the above expression that $A_{CP}$ directly depends upon the squark exchange $\mathcal{R}_p$ coupling (see Appendix-B). We have calculated average CP-asymmetries to determine its dependence on $\mathcal{R}_p$ Yukawa coupling products.

In the next Section, we give some results on asymmetries in this process based on computations of the eqs (8-19) and discuss possibility of their measurement in future experiments.

### 4 Results and Discussion

Fig. 1 gives the tree s-particle exchange Feynman diagrams evaluated as leading SUSY contributions to this process. We have used data from pdg[20] to plot Figs (2-10). The total branching ratio of decay processes ($B^\pm \rightarrow K^\pm l^+l^-$) are numerically calculated to be $5.17 \times 10^{-7}$ for $l = e$ and $5.13 \times 10^{-7}$ for $l = \mu$(see eq.(12)). The small difference between the two modes is due to difference in phase space integration.

We have used bounds on $\mathcal{R}_p$ couplings from the literature[14]. In Figs. (2-3), integrated branching fraction has been plotted against $\mathcal{R}_p$ Yukawa couplings within $1\sigma$ (dashed) and $2\sigma$(solid) levels in observed value of branching fraction in the process $B^\pm \rightarrow K^\pm l^+l^-$. Single coupling dominance has been assumed in these
graphs to observe how significant is the contribution of individual couplings to the branching fraction. This is a clear demonstration of the fact that tree level graphs in $R_p$ have a significant contribution to FCNC. An interesting conclusion that can be drawn from these graphs Figs(2-3) is that sneutrino $R_p$ couplings interfere only constructively with the SM box and penguin graphs i.e. the branching fraction in Fig (2) is enhanced. While the squark $R_p$ couplings interfere destructively i.e. the branching fraction in Fig (3) can be smaller than the SM prediction. This is because of a positive contribution in $F_V$ and a negative contribution in $F_A$ (see eq.11). Figs (2-3) also constrain the bounds on squark and sneutrino $R_p$ couplings. Fig. (4) represents combined effect of both squark and sneutrino $R_p$ couplings. We have ignored the effects of positive contributions in $F_V$ and negative contributions in $F_A$ (see eq.11). Figs (2-3) also constrain the bounds on squark and sneutrino $R_p$ couplings. Fig. (4) represents combined effect of both squark and sneutrino $R_p$ couplings. We have ignored the effects of $\lambda'_{323}\lambda'_{311}$ as it only changes the given graphs slightly. We also ignore the phase of $\lambda'_{323}\lambda'_{33}\beta\beta$ ($\beta = 1, 2$) in these graphs for a simple analysis. The significant contribution made by these $R_p$ Yukawa couplings indicate that one must look for more events of ($B^\pm \rightarrow K^\pm \mu^+\mu^-$) in LHCb and Super B factory.

The decay process $B^\pm \rightarrow K^\pm \mu^+\mu^-$ is ideal to measure new physics observables like $A_{FB}$, $(PA)_L\&(PA)_T$. Firstly this decay has been observed experimentally[19] and secondly these observables are no longer suppressed in the decay process for muons as they are in the decay process involving electrons and positrons due to relatively large mass of muons. In Figs.(5-7), $A_{FB}$, $(PA)_L\&(PA)_T$ have been plotted against dilepton centre of mass energy squared and different set of $R_p$ Yukawa couplings. We have ignored the effects of phase of $\lambda'_{323}\lambda'_{33}\beta\beta$ ($\beta = 1, 2$) in these graphs for a simple analysis. $A_{FB}$ is found to lie between (-5 and +5)%. It may be measured in experiments expected to take place at Super B factory [2]. $(PA)_L$ in SM is negative at high dilepton centre of mass energy values but in the presence of $R_p$ Yukawa couplings it also changes sign and can become as large as 50% as shown in Fig. (6). Thus a change of sign of $(PA)_L$ will be a strong indication of new physics beyond SM. The magnitude of $(PA)_T$ is (10-60)% in SM, but its magnitude may rise upto 90% in the presence of $R_p$ Yukawa couplings as shown in Fig. (7). Thus PAs are indeed sensitive to $R_p$ Yukawa couplings and are a better probe to NP than $A_{FB}$.

We next calculate average $PA$. For $B^\pm \rightarrow K^\pm \mu^+\mu^-$, we find that $< (PA)_T >_{SM} = -0.11$ and $< (PA)_L >_{SM} = -0.49$. Fig(8) shows that average PAs change significantly in the presence of $R_p$ Yukawa couplings. Calculations further show an estimate of $6\times10^6n^2 BB$ events are required for the measurement of $(PA)_T$, while $1\times10^7n^2$ events are required for the measurement of $(PA)_L$, where $n$ is 1σ or 2σ level. We have also calculated average $A_{FB}$ in Fig(9). It is found to be $(\pm 4)\%$ i.e. it requires an order of $(10^9 - 10^{11})n^2 BB$ events. In Super B factories there is an estimate of $10^9 BB$ available events. Thus the transverse polarization asymmetry in this process appears to be a dominant observable for looking beyond SM in a framework based on MSSM. We, thus look forward for measuring PAs and $A_{FB}$ in Super B factories for comparison with asymmetry predictions made in this work. In the following we argue that CP-asymmetry is another
significant and important observable for looking for physics beyond standard model in such processes.

In Fig (10a), we show the behavior of CP-asymmetry in SM. Its average value is $-1 \times 10^{-3}$, whose magnitude is quite small as compared to the experimentally measured value i.e. $0.07^{[20]}$. Fig (10(b)) shows the behavior of CP-asymmetry in $\mathcal{R}_p$ MSSM. Since the contribution by the squark exchange term is proportional to $G_F\alpha V_{tb}^* V_{ts}$, whereas the pure SM contribution to $A_{CP}$ is proportional to $(G_F\alpha V_{tb}^* V_{ts})^2$ (see appendix B). It shows that $CP$ asymmetry is influenced by the imaginary value of squark exchange term significantly as compared to the SM contribution. We study CP-asymmetry in the absence of sneutrino exchange term because it is not much influenced by them. The maximum contribution by imaginary part of squark exchange term is fairly close to the experimental value, i.e $7\%$, although the experimental errors are rather large (see Fig. 10b). Also $CP$ asymmetry becomes vanishingly small when the squark exchange term is real (i.e. imaginary part is zero in eq.(19)). We believe that better statistics will be available in Super B factories such that the CP asymmetries in such processes will be measured with better precision to match these predictions.

5 Conclusion & Summary

We have carried out an analysis of the decay processes $(B^\pm \rightarrow K^\pm l^+l^-)$ in $\mathcal{R}_p$ SUSY. The analysis shows that the contribution of $\mathcal{R}_p$ Yukawa couplings to branching fraction of this decay along with that of SM is significant. This makes $\mathcal{R}_p$ SUSY a good framework to study FCNC semileptonic decays. The new physics observables including asymmetries($A_{FB}$,$PA$s and $A_{CP}$) have also been calculated. Sneutrino $\mathcal{R}_p$ Yukawa couplings play an important role in these observables by providing different kinds of enhancements. Thus a non zero $< A_{FB} >$ and a relatively high magnitude $< (PA)_L >$ and $< (PA)_T >$ will indicate direct signs of new physics. Transverse polarization asymmetry is one of the most favorable physics observable in these searches as it is negligible in SM but rises to a significant magnitude in the presence of $\mathcal{R}_p$ Yukawa couplings. In addition, $\mathcal{R}_p$ Yukawa squark couplings also have sizeable contribution to CP-asymmetry. These observables may be measured at future experiments at Super B factory$^{[2]}$ and may provide a good probe for physics beyond Standard Model.

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Appendix-A

Wison Coefficients

We reproduce the matrix element of the decay rate \((B \rightarrow Kl^+l^-)\) from\(^{[12]}\)

\[
M = F_S \bar{l}l + F_V p_\mu \bar{l} \gamma_\mu l + F_A (p_B)_\mu \bar{l} \gamma_\mu \gamma^5 l + F_P \bar{l} \gamma^5 l
\]

The hadronic matrix elements contained in \(F_{V,A,S,P}\) are given in terms of form factors \(f^\pm(s), f^T(s)^{[6,12,15]}\)

\[
< K(p_K) | \bar{s} \gamma_\mu (1-\gamma_5) b | B(p_B) > = (p_B + p_K)_\mu f^+(s) + p_\mu f^-(s)
\]

\(\text{(A-1)}\)

\[
< K(p_K) | \bar{s} \sigma_{\mu \nu} \gamma^\nu (1+\gamma_5) b | B(p_B) > = (p_B + p_K)_\mu s - p_\mu (m_B^2 - m_K^2) \frac{f^T(s)}{m_B + m_K}
\]

\(\text{(A-2)}\)

\[
< K(p_K) | \bar{s} b | B(p_B) > = \frac{(m_B^2 - m_K^2)}{m_b - m_s} f_o(s)
\]

\(\text{(A-3)}\)

where \(p_\mu = (p_B - p_K)_\mu\) is the momentum transfer to dilepton pair and \(s = p_\mu p^\mu, m_B, m_K, m_b, m_s\) are masses of B and K mesons, b quark and s quark respectively. We have computed form factors \(f^\pm(s), f^T(s)\) from the analytic form given by\(^{[12]}\), which are used in our plots. \(f_o(s)\) is defined as

\[
f_o(s) = f^+(s) + \frac{s}{m_B^2 - m_K^2} f^-(s)
\]

\(\text{(A-4)}\)

The effective Hamiltonian for the given decay process is given by\(^{[12]}\)

\[
H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} \{ C_i(\mu) O_i(\mu) + C_{Qi}(\mu) O_{Qi}(\mu) \}
\]

\(\text{(A-5)}\)

The Wilson coefficients relevant to our calculation are evaluated at \(\mu = m_b\):
The expression for $C_{9}^{eff}$ in the next to leading order approximation used here is given by \[6, 12, 15\]

$$C_{9}^{eff}(\hat{m}_b, \hat{s}) = A(\hat{s}) + B(\hat{s})$$  \hspace{1cm} (A-7a)

where

$$A(\hat{s}) = 4.227 + 0.124w(\hat{s}) + \sum_{i=1}^{6} \alpha_i(\hat{s}) C_i$$ \hspace{1cm} (A-7b)

$$\alpha_1(\hat{s}) = 3g(\hat{m}_c, \hat{s}); \quad \alpha_2(\hat{s}) = g(\hat{m}_c, \hat{s}); \quad \alpha_3(\hat{s}) = 3g(\hat{m}_c, \hat{s}) - \frac{1}{2}g(\hat{m}_s, \hat{s}) - 2g(\hat{m}_b, \hat{s}) + \frac{2}{3}; \quad \alpha_4(\hat{s}) = \frac{2}{3} + g(\hat{m}_c, \hat{s}) - \frac{1}{2}g(\hat{m}_b, \hat{s});$$

$$\alpha_5(\hat{s}) = \frac{2}{3} + 3g(\hat{m}_c, \hat{s}) - \frac{2}{3}g(\hat{m}_b, \hat{s}); \quad \alpha_6(\hat{s}) = \frac{2}{3} + g(\hat{m}_c, \hat{s}) - \frac{1}{2}g(\hat{m}_b, \hat{s})$$

$$B(\hat{s}) = \lambda_t(3C_1 + C_2)(g(\hat{m}_c, \hat{s}) - g(\hat{m}_u, \hat{s}))$$ \hspace{1cm} (A-7c)

where coefficients $C_1 - C_6$ (evaluated at $m_b$), $g(\hat{m}_q, \hat{s})$ and $w(\hat{s})$ \[6, 12, 15\] used in our computation are listed as:

| $C_1$  | $C_2$  | $C_3$  | $C_4$  | $C_5$  | $C_6$  |
|-------|-------|-------|-------|-------|-------|
| -0.249| 1.107 | 0.011 | -0.025| 0.007 | -0.031|

$g(\hat{m}_q, \hat{s}) = \frac{-8}{9} \log[\hat{m}_q] + \frac{4}{9}y_q - \frac{2}{3}(1+y_q)\sqrt{1-y_q} + \{\theta(1-y_q)\log(1+\frac{1}{\sqrt{1-y_q}} - i\pi) + \theta(y_q - 1)\tan^{-1}(\frac{1}{\sqrt{y_q - 1}})}\}$ \hspace{1cm} (A-7d)

$w(s) = \frac{2}{9}\pi^2 - \frac{4}{3}Li_2(s) - \frac{2}{3}\ln[\hat{s}]\ln(1-\hat{s}) - \frac{5+4\hat{s}}{3}(1+2\hat{s})\ln(1-\hat{s}) - \frac{2s(1+\hat{s})(1-2\hat{s})}{3(1-\hat{s})^2(1+2\hat{s})}\ln[\hat{s}] + \frac{5+9\hat{s} - 6(\hat{s})^2}{6(1-\hat{s})(1+2\hat{s})}$ \hspace{1cm} (A-7e)

where
\[
\lambda_t = \frac{V^*_t V_{ts}}{V^*_c V_{cs}}, \quad \hat{m}_q = \frac{m_q}{m_b}, \quad \hat{s} = \frac{s}{(m_b)^2}, \quad y_q = \frac{(2\hat{m}_q)^2}{s}
\]

\(C_7\) belongs to photon Penguin and \(C_9^{eff}\) and \(C_{10}\) belong to \(W\) box and \(Z\) Penguin Feynman diagrams contributing to the process under discussion here. \(B\) is continuum part of \(u\bar{u}\) and \(c\bar{c}\) loops proportional to \(V^*_u V_{uq}\) and \(V^*_c V_{cq}\) respectively. It is solely responsible for the Wilson contribution to \(CP-\)asymmetry.

Appendix-B

**CP-Asymmetry**

We calculate the \(A_{CP}\) as

\[
A_{CP} = \frac{d\Gamma(B \to Kl^- l^+)}{ds} - \frac{d\Gamma(\bar{B} \to \bar{K}l^+ l^-)}{ds} + \frac{d\Gamma(B \to Kl^- l^-)}{ds} - \frac{d\Gamma(\bar{B} \to \bar{K}l^+ l^+)}{ds}
\]

\[
\frac{d\Gamma(B \to Kl^- l^+)}{ds} = \frac{1}{256\pi^3 m_B^2} \beta(m_t, s) \lambda^\frac{1}{2}(s) \left( |F_S|^2 2s |\beta|^2 + |F_P|^2 2s + |F_V|^2 \frac{1}{3} \lambda(s)(1 + \frac{2m_t^2}{s}) \right) \\
+ |F_A|^2 \left[ \frac{1}{3} \lambda(s)(1 + \frac{2m_t^2}{s}) + 8m_B^2 m_t^2 \right] + \text{Re}(F_P^* F_A^* 4m_t (m_B^2 - m_K^2 + s)^2), \quad (B-1a)
\]

\[
\frac{d\Gamma(B \to Kl^- l^-)}{ds} = \frac{1}{256\pi^3 m_B^2} \beta(m_t, s) \lambda^\frac{1}{2}(s) \left( |F_S|^2 2s |\beta|^2 + |F_P|^2 2s + |F_V|^2 \frac{1}{3} \lambda(s)(1 + \frac{2m_t^2}{s}) \right) \\
+ |F_A|^2 \left[ \frac{1}{3} \lambda(s)(1 + \frac{2m_t^2}{s}) + 8m_B^2 m_t^2 \right] + \text{Re}(F_P^* F_A^* 4m_t (m_B^2 - m_K^2 + s)^2), \quad (B-1b)
\]

\[
A_{CP} = \lambda(s)(1 + \frac{2m_t^2}{s})(|F_V|^2 - |\bar{F}_V|^2)(D(s))^{-1}
\]

where

\[
D(s) = 6(|F_S|^2 2s \beta(m_t, s)^2 + |F_P|^2 2s + |F_A|^2 \left[ \frac{1}{3} \lambda(s)(1 + \frac{2m_t^2}{s}) + 8m_B^2 m_t \right] + \\
\text{Re}(F_P F_A^* 4m_t (m_B^2 - m_K^2 + s) + \lambda(s) \frac{2m_B^2}{6}(1 + \frac{2m_t^2}{s})(|F_V|^2 + |\bar{F}_V|^2)) \quad \text{ (B-2)}
\]

where
\[ F_V = \frac{G_F \alpha V_b V_s^*}{2\sqrt{2\pi}} \left( 2C_9 \epsilon f f (\bar{m}_b, \bar{s}) f^+(s) - C_7 \frac{4m_b}{m_B + m_K} f^T(s) \right) + \frac{1}{4} f^+(s) \sum_{i,m,n=1}^{3} V_{ni}^\dagger V_{im} \frac{\lambda_{\beta m 2}^{*}}{m_{u_i}^2} \] (B-3)

\[ F_A = \frac{G_F \alpha V_b V_s^*}{2\sqrt{2\pi}} (2C_{10} f^+(s)) - \frac{1}{4} f^+(s) \sum_{i,m,n=1}^{3} V_{ni}^\dagger V_{im} \frac{\lambda_{\beta 3 n m 2}^{*}}{m_{u_i}^2}, \]

\[ F_S = \frac{1}{4} \frac{(m_B^2 - m_K^2)}{m_b - m_s} f_0(s) \sum_{i=1}^{3} \left( \chi_{\beta \beta 3}^{*} \chi_{23}^{*} - \lambda_{\beta \beta 3}^{*} \chi_{32}^{*} \right) \frac{1}{m_{\nu Li}^2}, \]

\[ F_P = \frac{G_F \alpha V_b V_s^*}{2\sqrt{2\pi}} 2m_tC_{10} (f^+(s) + f^-(s)) + \frac{1}{4} \frac{(m_B^2 - m_K^2)}{m_b - m_s} f_0(s) \sum_{i=1}^{3} \left( \chi_{\beta \beta 3}^{*} \chi_{23}^{*} + \lambda_{\beta \beta 3}^{*} \chi_{32}^{*} \right) \frac{1}{m_{\nu Li}^2}, \]

\[ F_V = \frac{G_F \alpha V_b V_s^*}{2\sqrt{2\pi}} \left( 2C_9 \epsilon f f f (\bar{m}_b, \bar{s}) \right) f^+(s) - C_7 \frac{4m_b}{m_B + m_K} f^T(s) + \frac{1}{4} f^+(s) \left( \sum_{m,n=1,i=1}^{3} V_{ni}^\dagger V_{im} \frac{\lambda_{\beta m 2}^{*}}{m_{u_i}^2} \right)^*, \] (B-4)

\[ F_A = F_A^*, \quad F_S = F_S^*, \quad F_P = F_P^*, \quad C_9 \epsilon f f (\bar{m}_b, \bar{s}) = C_9 \epsilon f f (\bar{m}_b, \bar{s}, \lambda_0), \]

\[ \chi_{\beta n 3 \lambda_{\beta m 2}^{*}} = |\chi_{\beta n 3 \lambda_{\beta m 2}^{*}}| e^{i\theta}, \beta = e, \mu \] (B-5)

Eq. (18) for \( A_{CP} \) contains the following factor

\[ |F_V|^2 - |F_V|^2 = 4 \left( \frac{G_F \alpha V_b V_s^*}{2\sqrt{2\pi}} \right)^2 f^+(s) \text{Im}(A'(s)B'(s)^*) \text{Im}(\lambda_0) + \]

\[ \frac{G_F \alpha V_b V_s^*}{2\sqrt{2\pi}} f^+(s) \text{Im}(A'(s)) \text{Im}(\sum_{m,n=1,i=1}^{3} V_{ni}^\dagger V_{im} \frac{\lambda_{\beta m 2}^{*}}{m_{u_i}^2}) + \]

\[ \frac{G_F \alpha V_b V_s^*}{2\sqrt{2\pi}} (f^+(s))^2 \text{Im}(B'(s)) \text{Im}(\lambda_0^* \sum_{m,n=1,i=1}^{3} V_{ni}^\dagger V_{im} \frac{\lambda_{\beta m 2}^{*}}{m_{u_i}^2}), \] (B-6)

where

\[ A'(s) = 2Af^+(s) - C_7 \frac{4m_b}{m_B + m_K} f^T(s) \]

\[ B'(s) = 2(3C_1 + C_2)(g(\bar{m}_c, \bar{s}) - g(\bar{m}_u, \bar{s})), \]
$$|F_V|^2 + |F_V|^2 = G(s) + H(s),$$

$$G(s) = 2(f^+(s))^2 \left( \sum_{m,n,i=1}^{3} V_{ni}^* V_{im} \frac{\lambda^{\prime}_{3n3} \lambda^{\prime}_{3m2}}{m_{u_i}^2} \right)^2 + 4 \left( \frac{G_F a V_{tb} V_{ts}^*}{2\sqrt{2}\pi} \right) f^+(s) \operatorname{Re}(A') \right.$$ 

\[\operatorname{Re} \left( \sum_{m,n,i=1}^{3} V_{ni}^* V_{im} \frac{\lambda^{\prime}_{3n3} \lambda^{\prime}_{3m2}}{m_{u_i}^2} \right) + 4 \left( \frac{G_F a V_{tb} V_{ts}^*}{2\sqrt{2}\pi} \right) f^+(s) \operatorname{Re}(A') \right.$$ 

\[\operatorname{Re} \left( \sum_{m,n,i=1}^{3} V_{ni}^* V_{im} \frac{\lambda^{\prime}_{3n3} \lambda^{\prime}_{3m2}}{m_{u_i}^2} \right) \operatorname{Re}(B'), \right.$$ 

$$H(s) = \left( \frac{G_F a V_{tb} V_{ts}^*}{2\sqrt{2}\pi} \right)^2 \left( 2C_f f f (m_b, s) f^+(s) - C_T \frac{4m_b}{m_B + m_K} f^T(s) \right)^2 + \left( 2C_f f f (m_b, s) f^+(s) - C_T \frac{4m_b}{m_B + m_K} f^T(s) \right)^2$$

The factors contributing to the CP- asymmetry at various energies are:

$$\operatorname{Im}(A'(s)) = 2f^+(s) \operatorname{Im}(A(s)) = 2f^+(s) \left. \frac{1}{2\pi} (C_3 + 3C_4) \right.$$ 

$$\operatorname{Im}(B'(s)) = 2(3C_1 + C_2) \operatorname{Im}(g(m_c, s) - g(m_u, s)), \left. \frac{1}{2\pi} (2 + y_u) \sqrt{1 - y_u} \right.$$ 

$$\left( \frac{1}{2\pi} (2 + y_u) \sqrt{1 - y_u} - (2 + y_c) \sqrt{1 - y_c} \right) \left\{ \begin{array}{l} m_c^2 > s > m_u^2 \\ s > m_c^2 \end{array} \right.$$ 

$$\lambda_t = -C \lambda^2 e^{i\delta}$$

Since for simplicity, we have considered only the contribution from $i = 3$ in $\mathbb{R}_p$. Yukawa couplings, $V_{tb}$ contributes in $F_{A,V}$ only. Further, since $(V_{td}, V_{ts}) \times 10^{-3}$, we neglect these numbers and take $V_{tb} \sim 1/20$. Since $17, 20$

$$0.23 < C < 0.59; \ 0.216 < \lambda_t < 0.223; \ \delta = 1.3439$$

we use the central value of $\lambda_t$ and $C$, i.e. 0.41 and 0.22 respectively. This parameterises the CP-violating phase used in Fig 10.
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Fig. 1  Tree-Level diagrams contributing to charged meson decays

\[ B^+ \rightarrow K^+ l_\beta^+ l_\beta^- (b \rightarrow s l_\beta^+ l_\beta^-). \]
Fig 2. Dependence of total branching fraction with real R-parity Yukawa couplings within \(1\sigma\) (dashed) and \(2\sigma\) (solid) level in observed value of branching fraction. SM prediction is given by Dotted dash line. Single coupling dominance has been assumed. \(\lambda'_{332} \lambda_{3\alpha\alpha} \) (\(\alpha=1, 2\)) is expressed in the units of \(\left(\frac{m_{\nu_e}}{100\, GeV}\right)^2\).
Fig 3. Dependence of total branching fraction with R-parity Yukawa coupling and its complex phase within 1σ (a, c) and 2 σ (b, d) level in observed value of branching fraction. The dashed plane represents the SM prediction of branching fraction. Single coupling dominance has been assumed. $\lambda'_{\alpha 33} \lambda'_{\alpha 32}$ ($\alpha=1, 2$) is expressed in the units of $\frac{m_{\nu_L}}{100 GeV}$. 
Fig 4. Dependence of total branching fraction with R-parity Yukawa coupling within 1σ (a, c) and 2σ level (b-d) in observed value of branching fraction. The dashed plane represents the SM prediction of branching fraction. Single coupling dominance has been assumed. R-parity Yukawa couplings have been assumed real. \( \lambda'_{333} \lambda'_{332} (\alpha=1, 2) \) is expressed in the units of \( \left( \frac{m_{\nu}}{100 \text{GeV}} \right)^2 \). \( \lambda'_{332} \lambda'_{33\alpha} (\alpha=1, 2) \) is expressed in the units of \( \left( \frac{m_{\nu}}{100 \text{GeV}} \right)^2 \).
Figure 5: Variation of FBA w. r. t. Yukawa coupling $\lambda'_{332} \lambda_{322}$ and $p^2$ at various values of $\lambda'_{323} \lambda_{322}$ and $\lambda'_{233} \lambda_{232}$ given by Ref [16]. $\lambda_{332} \lambda_{322}$ is expressed in the units of $(\frac{m_{\tau}}{100 GeV})^2$. 
Figure 6: Variation of Longitudinal Polarization Asymmetry w.r.t. $p^2$ (a) in SM (b) and Yukawa $\lambda'_{332} \lambda_{322}$ at various values of $\lambda'_{323} \lambda_{322}$ and $\lambda'_{233} \lambda_{232}$ given by Ref [16]. $\lambda'_{332} \lambda_{322}$ is expressed in the units of $(\frac{m_{\nu_1}}{100 GeV})^2$. 

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Figure 7: Variation of transverse polarization asymmetry w.r.t. $p^2$ (a) in SM (b) and $\lambda'_{332} \lambda_{322}$ at various values of $\lambda'_{323} \lambda_{322}$ and $\lambda'_{233} \lambda'_{232}$ given by Ref [16]. $\lambda'_{332} \lambda_{322}$ is expressed in the units of $(\frac{m_{\phi}}{100 GeV})^2$. 
Figure 8: Variation of (a) $<PA_L>$ (b) $<PA_T>$ w. r. t. $\lambda'_{312} \lambda_{322}$, $\lambda'_{233} \lambda'_{323}$ at various values of $\lambda'_{332} \lambda_{322}$, which are given by Ref [16]. $\lambda'_{332} \lambda_{322}$ is expressed in the units of $\left(\frac{m_{\nu_i}}{100 GeV}\right)^2$. $\lambda'_{233} \lambda'_{323}$ is expressed in the units of $\left(\frac{m_{\nu_i}}{100 GeV}\right)^2$. 

$\langle PA_L \rangle_{SM} = -0.49$

$\langle PA_T \rangle_{SM} = -0.11$
Figure 9: Variation of $<A_{FB}>$ w. r. t. $\lambda'_{332}$ $\lambda'_{322}$, $\lambda''_{233}$ $\lambda'_{232}$ at various values of $\lambda'_{332}$ $\lambda'_{322}$, which are given by Ref [16]. $\lambda'_{332}$ $\lambda'_{322}$ is expressed in the units of $(\frac{m_{\nu_L}}{100 GeV})^2$. $\lambda''_{233}$ $\lambda'_{232}$ is expressed in the units of $(\frac{m_{\nu_L}}{100 GeV})^2$. 
Figure 10: (a) Plot showing variation of $A_{\text{cp}}$ in SM. (b) Variation of $<A_{\text{cp}}>$ w. r. t R-parity violating Yukawa coupling products. The shaded plane represents the SM prediction for $A_{\text{cp}}$. $\lambda'_{232}\lambda'_{333}$ is expressed in the units of $(\frac{m_{\mu}}{100\text{GeV}})^2$. 