Simplified model of Covid-19 epidemic prognosis under quarantine and estimation of quarantine effectiveness

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ABSTRACT
A simplified model of Covid19 epidemic dynamics under quarantine conditions and criteria to estimate quarantine effectiveness is developed. The model is based on the growth rate of new infection cases when total number of infected cases is significantly smaller than population size of infected country or region. The model is developed on collected epidemiological data of Covid19 pandemic, which shows that the growth rate of new infection cases has tendency of linearly decrease until its constant value during imposed quarantine according to effectiveness of quarantine actions. The growth rate of new infection cases can be used as criteria to estimate quarantine effectiveness.

Keywords: Novel Coronavirus, COVID-19, Pandemic, Modelling, Forecast

Introduction

The 2019–2020 coronavirus outbreak has started since December 29th, 2019 in Wuhan, Hubei province, People’s Republic of China, and has progressively expanded through almost all countries. This ongoing pandemic of coronavirus disease 2019 (COVID-19) caused by severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2).

In order to prevent the spread of the disease, many of the countries affected by the disease has been put under quarantine, which has led to 58\% of whole world population (4,5\cdot10^9 people) to be quarantined. However, the remaining problem of the current COVID-19 disease is that second wave of the epidemic is expected after imposed quarantine is discontinued in each country. It can be estimated that for most countries only 1\% of the population of the infected country/region will acquire the immunity after first wave of epidemic controlled by the effective quarantine. That would require next 50-60 similarly controlled waves to achieve herd (population) immunity of 50-70 \% in order to stop further spread of disease. Such scenario is improbable as it would require a significant amount of time, during which the individual immunity could be lost because even today, when more than 3 million people have been infected (28 April 2020), it is not clear whether the recovered patients have the immunity. However, hope for SARS-CoV-2 antiviral vaccine or sufficiently effective antiviral drugs, or SARS-CoV-2 virus mutation to less aggressive strain [1] gives us chance to survive through the course of the controlled epidemic.
Therefore, forecasting the spread of the pandemic on basis of mathematical models are extremely important for decisions how to prepare countries in order to avoid overloading of health system and manage other related problems. Valuable information that could be obtained from modelling is forecast of the expected time and number of most active infected cases and the effectiveness of applied infection control measures. It is current global trend, that the experience and available data from already affected countries are used to model the pandemic dynamics in other countries before the epidemic has reached the peak or to estimate effectiveness of various scenario of the next wave management [2].

Most popular epidemic dynamics models of Covid-19 are based on transmission model for a directly transmitted infectious disease, such as standard compartment models of disease SIR [3,4], or more advances derivates, such as SEIR and similar models [5–8]. Many of the models, which are used to forecast the COVID-19 epidemic, do not accurately capture the transient dynamics of epidemics; therefore, they give poor predictions of both the epidemic’s peak and its duration [9], because calibration of parameters are based on dynamics of such non-reliable epidemiological data as number of active infectious cases.

We propose to build epidemic analysis and model on the dynamics of rate of new infection cases as more reliable epidemiological data together with an assumption of effectiveness to isolate registered infectious during imposed quarantine. The proposed approach is based on SIR model.

**SIR model**

The simplest SIR model consists of three compartments: $S$ for the number of susceptible, $I$ for the number of infectious, and $R$ for the number of removed (recovered, deceased or immune) individuals. These variables ($S$, $I$, and $R$) represent the number of people in each compartment at a particular time. We denote the total population size by $N$. The dynamics of the simplest SIR system (excluding birth and death) can be described by the following set of ordinary differential equations [7]:

\[
\begin{align*}
\frac{dS}{dt} &= -\frac{\beta SI}{N} \\
\frac{dI}{dt} &= \beta SI - \gamma I \\
\frac{dR}{dt} &= \gamma I \\
N &= S + I + R = \text{const}
\end{align*}
\]

where $\gamma$ is the rate of recovery or mortality, $\beta$ is the infectious rate controlling the rate of spread that represents the probability of transmitting disease from infectious individual to susceptible individual. The disease transition rate $\beta/NI$ is defined as a product of $\beta$ and a probability of disease transmission during a contact between an infectious individual and a susceptible individual. In that case, $\beta$ is the average number of contacts per person per time unit and can be defined by the typical time between contacts $T_c = 1/\beta$. The transition rate between $I$ and $R$ defined by $\gamma$ and is estimated from typical time until recovery $T_r = 1/\gamma$. In case of isolation or self-isolation, $\gamma$ can be defined by the average number of days $T_{ri}$ that a person is infectious (before they are isolated or self-isolate), $\gamma = 1/T_{ri}$. 

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The dynamics of the infectious class depends on the following ratio:

\[ R_0 = \frac{\beta}{\gamma} \]  

(2)

the so-called basic reproduction number (or basic reproduction ratio) of an infection and represents the average number of infections generated by one individual over the course of the infectious period.

The estimation model parameters \( \beta \) and \( \gamma \), as well as \( R_0 \), may vary depending on country due to methodological issues, including different assumptions and choice of parameters, utilized models, used datasets and estimation period. In addition, during the spread of the SARS-CoV-2 virus infection, it was found that the model parameters are varying according to the dynamics of transmission of the novel coronavirus outbreak as well as the case reporting rate, which requires to build up more sophisticated and complex models [10]. The model parameters are constantly calibrated according to the last epidemiological data because forecast is constantly refreshed.

**Simplified model of epidemic dynamics under quarantine**

The wide and rapid spread of the disease forced many countries to impose quarantines, entry bans and other restrictions during the pandemic to reduce the movement of population and recent travellers in most affected regions [11]. Global restrictions that apply to all foreign countries and regions have also been imposed in other countries preventing their own citizens to travel overseas. These measures, especially quarantine, helped to suppress the spread of the disease within the population of various countries with different effectiveness.

Other important property of the pandemic is that total number of infected cases \( T \) is much smaller than population size of infected country or region due to quarantine:

\[ T = I + R \ll S = N \]  

(3)

This assumption allows simplify epidemiological models used to simulate Codiv-19 disease spread under quarantine.

Consequently, on basis of eq. (3), the parameter \( \beta \) of SIR model can be estimated as the number of new registered cases of infection to number of active cases ratio:

\[ \beta(t) = \frac{-dS}{dt} \frac{1}{S(t)} \approx -\frac{dS}{dt} = \frac{N_I}{I} \]  

(4)

where \( N_I \) is the rate of new infected cases

\[ N_I(t) = -\frac{dS}{dt} \]  

(5)

which can be estimated by counting new cases of infection, and usually is measured by number of registered new cases per time period \( \tau \)

\[ n_I(t, \tau) = \int_t^{t+\tau} N_I(t) dt = -\int_t^{t+\tau} \frac{dS}{dt} dt = -(S(t + \tau) - S(t)) \]  

(6)
The number of daily new infected cases is defined as

\[ n_{td}(t) = n_l(t, \tau = 1 \text{ day}) \]

(7)

So, expected number of new cases in next day could be predicted by the today number of active infected individuals multiplied by the infectious rate

\[ n_{td} = \beta I \]

(8)

In general, the infectious rate \( \beta \) is time dependent and usually can be described by a complex function with additional parameters that must be daily calibrated according to last epidemiological data. Behaviour of the infectious rate \( \beta \) during quarantine may differ in different countries, which reduces possibilities to build correct model of the infectious rate on basis of epidemiological data from other countries. Furthermore, there is no clear criteria to relate influence of quarantine actions with the infectious rate \( \beta \).

The dynamics of the parameter \( \beta \) estimated from active infected individuals and new cases by eq. (4) is demonstrated in Figure 1 for 38 countries with imposed quarantine. Time is aligned to the date of imposed quarantine in each country \( t' = t - t_q \), where \( t_q \) is the initialisation date of the quarantine [12]. Daily numbers of active infected individuals and new infection cases were collected from various open sources [13,14]. In order to damp daily and weekly fluctuations, \( I, n_{td} \) and estimated the parameter \( \beta \) were smoothed by moving average, where each average is calculated over a sliding window of length \( N_s = 7 \) days:

\[ \langle f(t) \rangle = \frac{1}{N_s} \sum_{i=-N_s}^{N_s} f(t + i), \; N_s = 1 + 2n_s \]

(9)

where \( f \) is any parameter to be smoothed.

As can be seen, the dynamics of the parameter \( \beta \) demonstrates similar non-linearly decreasing behaviour after the start of the quarantine for almost all countries.

![Figure 1. The infectious rate $\beta$ versus time for 38 countries. Vertical black line points the day of imposed quarantine.](https://doi.org/10.1101/2020.04.28.20083428)
In addition, an estimation of the number of active infected individuals \( I \) depends on the number of recovered active cases of infections \( R \), which in turn depends on testing protocols, tests number, delays in testing and other circumstances, which are different in each country. Consequently, this makes \( I = T - R \) unreliable parameter for model calibration. For example, in Lithuania the first pool consisting of 54 recovered cases of 999 total cases was reported only after 30 days from the first infection registration [15], which can be explained only by troubles in recording of recovered cases and can be also expected in other countries. In contrary, the rate of new infected cases \( n_{td} \), despite of countries specificity, is still the most reliable parameter allowing to estimate disease spreading.

In order to build model of Covid-19 disease spread during quarantine we make simplifying assumption, that registered infected individuals do not spread virus to health individuals, because of an effective isolation of the registered infected individuals. New cases of infectious individuals are generated by previously infectious individuals until they are registered; therefore, the number of new cases of the day is dependent on the number of new cases generated during previous day and the effectiveness of imposed quarantine actions: 1) restriction of social contacts and mobility, 2) identification of infectious individual and his contact tracing as soon as possible, 3) isolation of infectious individuals.

Let us analyse hypothetical simplified case. The time period, during which infected individual is registered and isolated after infection, is denoted \( T_i \). During this time period, the infected individual is infectious and infects \( \alpha \) individuals. In the end of this period, the infected individual is isolated and does not take more participation in the process of the epidemic spread. During time period \([t - T_i, t] \) \( n_i(t - T_i, T_i) \) of new infected cases are generated. These new infected cases will then generate more cases and the number of new infection cases for next time period \([t, t + T_i]\):\

\[
n_i(t, T_i) = \alpha \cdot n_i(t - T_i, T_i) \tag{10}
\]

where \( \alpha \) is the growth rate of new cases and is time-dependent. \( \alpha > 1 \) means that number of new cases is increasing, while \( \alpha < 1 \) - decreasing, and epidemic is finished when \( \alpha = 0 \).

According to eq. (6), the parameter \( \alpha \) can expressed as follows:

\[
\alpha = \frac{n_i(t + \tau, \tau)}{n_i(t, \tau)} = \frac{n_i(t + \tau, \tau) - n_i(t, \tau)}{n_i(t, \tau)} + 1 = \frac{n_i(t + \tau, \tau) - n_i(t, \tau)}{-S(t + \tau) + S(t)} + 1 \tag{11}
\]

It is evident that in limits of \( \tau \to 0 \), we have

\[
\alpha = 1 - \frac{dN_i}{dt} \frac{dS}{dt} \tag{12}
\]

which, together with eq. (5), gives differential equation for disease dynamics during quarantine:

\[
\frac{d^2S}{dt^2} = (\alpha - 1) \frac{dS}{dt} \tag{13}
\]
Keeping in mind eqs. (4) and (5), the parameter $\alpha$ can be related to the infectious rate $\beta$ as follows

$$\alpha = 1 + \frac{1}{\beta l} \frac{d\beta l}{dt} \quad (14)$$

Let us define $\alpha_0$ as the growth rate of new infection cases when epidemic starts and spreads uncontrolled. Because of $\alpha_0 > 1$, the number of new cases is exponentially increasing. It is expected that the population in any country would start to behave more safely even though no official quarantine actions were taken; consequently, the growth rate of new cases is expected to slowly decrease before strict quarantine rules are imposed. The growth rate of new infections when quarantine starts ($t = t_q$) can be defined as $\alpha_q(t = t_q) > 1$. Because of $\alpha_q > 1$, the number of new cases is still increasing. Let us assume that the imposed quarantine is ideally effective, which means that all infected individuals are isolated until the end of the time span and do not have contact with other individuals during time span $[t_q, t_q + T_q]$. Furthermore, no new cases will be generated for the next time period $[t_q + T_q, t_q + 2T_q]$. In such case, the growth rate of new cases becomes constant and equal to 0 after the quarantine starts $\alpha(t > t_q) = \alpha_q = 0$ (Figure 2 (a)), which means that the supposed quarantine effectiveness is equal to 1 and epidemic is stopped immediately.

If the quarantine is less effective, then the growth rate of new cases is non zero constant $\alpha(t > t_q) = \alpha_q > 0$, which leads to slower spread of the disease in case of $\alpha_q > \alpha > 1$ and suppression of the epidemic in case of $1 > \alpha_q > 0$ (Figure 2 (b)).

Consequently, quarantine effectiveness can be measured as

$$e_q = \frac{\alpha_0 - \alpha_q}{\alpha_0} = 1 - \frac{\alpha_q}{\alpha_0} \quad (15)$$

The zero effectiveness $e_q = 0$ means that the growth rate of new cases during quarantine remains the same as before: $\alpha_q = \alpha_q$. Therefore, in order to suppress the disease, the growth rate of new cases must be below 1, which means that quarantine effectiveness $e_q$ must be greater than $1 - 1/\alpha_0$.

![Figure 2. Idealized scenario of the growth rate of new cases $\alpha$ dynamics with quarantine effectiveness $e_q$: a) $e_q = 1$, b) $e_q < 1$.](image-url)
In realistic scenario, the growth rate of new cases $\alpha$ does not change sharply at time $t_q$ because of time lag due to incubation period, infections generated by non-registered infected individuals and so on. In addition, it takes time for people to adjust to the quarantine requirements after the beginning of the quarantine; therefore, there is a time lag before people start to strictly follow the rules. Consequently, the growth rate of new cases $\alpha$ decreases from initial value $\alpha_q0$ until reaches value $\alpha_q$ satisfying the effectiveness of applied quarantine at the time $t_{qc}$ (Figure 3). $T_{q,\text{peak}} = t_{q,\text{peak}} - t_q$ is the period during which number of new cases $n_t$ reaches maximum value after the quarantine start $t_q$. During the stage of constant $\alpha = \alpha_q < 1$, the epidemic is suppressing until the end of the epidemic at the time $t_{q,\text{end}}$, after which only small number new infection case are registered.

![Figure 3. Realistic scenario of the growth rate of new cases $\alpha$ dynamics.](image)

Duration of $\alpha$ decreasing stage depends on $\alpha_q0 - \alpha_q$ and properties of Covid-19 disease, such as incubation period and time span of individual being infectious. As a result, the angle of $\varphi$ depends on quarantine effectiveness, because $\varphi = 0$ condition satisfies zero quarantine effectiveness $\epsilon_q = 0$, while case of $\varphi$ approaching to $90^\circ$ corresponds to quarantine effectiveness approaching to 1.

The proposed parameters $\epsilon_q$ and $\varphi$ together with analysis of the population mobility and social contacts [16] can be used to estimate effectiveness of country or region lockdown measures.

In order to predict Covid-19 disease spread in infected country or region with imposed quarantine, a model of the growth rate of new cases $\alpha$ needs to be developed. It is possible to build up such model speculatively in general; however, it is reasonable to analyse dynamics of $\alpha$ in various countries. We analysed Covid-19 pandemic data from various countries [13,14]. The growth rate of new infection cases $\alpha$ was estimated on basis of the registered daily new infection cases $n_{t,d}(t)$ (defined by eq. (7)) and smoothed by moving average according to eq. (9) with sliding window of length $N_s = 7$ days in order to damp daily and weekly fluctuations:

$$\alpha(t) = \frac{\langle n_{t,d}(t + \tau) \rangle}{\langle n_{t,d}(t) \rangle}; \quad \tau = 1 \text{ day} \quad (16)$$

The dynamics of the obtained parameter $\langle \alpha \rangle$ is demonstrated in Figure 4 for 38 countries with imposed quarantine. Time is aligned to the date of imposed quarantine in each country $t' = t - t_q$, 

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where \( t_q \) is the time of start of the imposed quarantine [12]. The values of new infection cases were collected from various open sources.

![Figure 4. \( \alpha \) versus time for 38 countries. Vertical black line points the day of imposed quarantine.](image)

The typical example of above described scenario was realised in Australia, in which the epidemic started on 2020 January 26, the quarantine was imposed on 23 March 2020 [12] and epidemic almost finished on April 25, 2020 with total number 6703 of confirmed infected cases. The dynamics of the daily new infection cases \( n_{td} \) [13] is presented in Figure 5 (a) by solid blue line and the moving average (with sliding window of length \( N_s = 7 \) days) of daily new infection cases \( \langle n_{td} \rangle \) by solid green line. The growth rate of new cases \( \alpha \) estimated by eq. (16) is presented in Figure 5 (b) by solid blue line.

The growth rate of new cases \( \alpha \) was almost constant or slowly decreasing before quarantine, started to decrease sufficiently after quarantine had been imposed and had somewhat linear dependency on time during quarantine time, which allowed to approximate \( \alpha \) by descending straight line (Figure 5 (b)) during the time period \( t_{qc} - t_q = 11 \) days, where \( t_{qc} \) is the date of \( \alpha \) becoming equal to \( \alpha_q = 0.86 \). \( \alpha \) was then extended as constant until the epidemic end

\[
\alpha(t \geq t_{qc}) = \alpha_q \tag{17}
\]

Consequently, the model of the growth rate of new cases can be described as follows:

\[
\alpha_m(t) = \begin{cases} 
\alpha_{m1} = \alpha_{q0} + b_q(t - t_q), & t_q < t \leq t_{qc} \\
\alpha_q, & t > t_{qc}
\end{cases} \tag{18}
\]

where \( \alpha_{q0} = 1.075 \) is \( \alpha \) value in the beginning of the quarantine

\[
\alpha_{q0} = \alpha(t_q) \tag{19}
\]

the parameter \( b_q = -0.0207 \) characterizes decreasing of \( \alpha_m \) during the quarantine

\[
b_q = \frac{\alpha_q - \alpha_{q0}}{t_{qc} - t_q} \tag{20}
\]
The modelled daily new infection cases $n_{tdm}$ was calculated by the simple algorithm

$$n_{td.m}(t + \tau) = \alpha_m(t + \tau) \cdot n_{td.m}(t); \quad \tau = 1 \text{ day}$$

starting from the day of imposed quarantine $t = t_q$: $n_{td,m}(t_q) = \langle n_{td}(t_q) \rangle$. The dynamics of $n_{td,m1}$ estimated by $\langle \alpha_m t_q \rangle$ for the time period $t_q < t \leq t_{qc}$ is shown by red solid line and $n_{td,mq}$ estimated by $\alpha_q$ for time $t > t_{qc}$ by dashed red line.

The initial value of the growth rate of new cases can be estimated approximately as $\alpha_0 \approx 1.22$. Consequently according to eq. (15), the estimated quarantine effectiveness is equal to $e_q = 1 - \alpha_q/\alpha_0 \approx 0.30$.

We will shortly overview some other countries demonstrating applicability and possible shortcomings of the proposed approach.

The next example of the similar scenario is Switzerland. The government of Switzerland announced that no lockdown would be implemented; however, some restrictions were implemented [17]. On 13 March 2020, the Federal Council decided to cancel classes in all educational establishments until 4 April 2020 and banned all events (public or private) involving more than 100 people. Furthermore, the borders were closed, and border control was enacted. On 16 March 2020, the Federal Council announced further measures and a revised ordinance. Measures included the closure of bars, shops and other gathering places until 19 April 2020, but lefted open certain essentials, such as grocery shops, pharmacies, (a reduced) public transport and the postal service. Since 20 March 2020, all events or meetings over 5 people were prohibited, and economic activities would continue including construction.

The growth rate of new cases $\alpha$ began decreased before first official restrictions of social contacts (Figure 6 (b)) while daily numbers of new cases were relatively small. During three following
weeks after the restrictions initiation, $\alpha$ continued to decrease from 1.207 on 13 March 2020 until 0.918 and it is expected for $\alpha$ to remain constant until the end of the epidemic (Figure 6).

![Figure 6. Dynamics of the daily new cases $n_{td}$ (a) and the growth rate of new cases $\alpha$ (b) for Switzerland. Vertical black line points the day of first quarantine action on 2020 March 13. Data collected from [13].](image)

The similar scenario to Australia and Switzerland cases developed in Austria (Figure 7), where the epidemic started on 2020 February 25, 2020 and the quarantine was imposed on 2020 March 6 [18].

![Figure 7. Dynamics of the daily new cases $n_{td}$ (a) and the growth rate of new cases $\alpha$ (b) for Austria. Vertical black line points the day of first quarantine action on 2020 March 16. Data collected from [13].](image)

New Zealand is an example of slightly different scenario (Figure 8). On 21 March, 2020 at midday, New Zealand Prime Minister announced the introduction of a country-wide alert level system, similar to the existing fire warning systems [19]. There are four levels, with 1 being the least risk of infection and 4 the highest. At the time of Prime Minister's announcement she stated that New Zealand was at level 2. Each level brings added restrictions on activities or movements. Prime
Minister announced on 23 March at around 2 pm that, effective immediately, New Zealand would be at alert level 3, moving to level 4 at 11:59 pm on 25 March.

The dynamics of the daily new infection cases $n_{td}$ [13] and the calculated growth rate of new cases $\alpha$ are presented in Figure 8. The modelled $n_{td}$ was overestimated (Figure 8 (b)), because $\alpha$ was approximated by one straight in the time interval from $t_q$ (corresponding to 21 March 2020) to $t_{qc} = t_q + 13$ (Figure 8 (a)). To improve the model of $\alpha$, the change of alert level from 2 to 4 on fifth day after the beginning of the quarantine must be considered. Accordingly, $\alpha$ must be approximated by 2 straights in the time interval $[t_q, t_{qc}]$.

The similar scenario is demonstrated in Iceland, where universities and secondary schools were closed on 16 March 2020. Furthermore, public gatherings of over 100 were banned on the same day [20]. The same quarantine conditions were applied for the whole quarantine period. Nevertheless, there were three stages during the time interval $[t_q, t_{qc}]$, $t_{qc} = t_q + 25$ (Figure 9 (b)). The first stage of decreasing $\alpha$ lasted for 6 days since the beginning of the quarantine. Then the next stage followed up for 7 days, in which the $\alpha$ remained almost constant $\alpha \approx 1$. The third and the final stage of decreasing $\alpha$ took place for 12 days, during which $\alpha$ reached value of $\alpha_q = 0.807$. Therefore, $\alpha$ model for the whole period of 25 days, which describes the behaviour of $\alpha$ by one decreasing line $\alpha_{m1} = \alpha_{q0} + b_q(t - t_q), t_q < t < t_{qc}$, overestimates the rate of new infection cases $n_{td}$ in the second stage (Figure 9 (a)).
The example of Spain illustrates the scenario when the proposed method to predict epidemic dynamics under quarantine does not work straightforward. On 13 March, Prime Minister of Spain announced a declaration of a nationwide State of Alarm for 15 days [21]. During three following weeks after the beginning of the quarantine, $\alpha$ decreased from 1.226 to 0.945 and then stayed at almost constant level (Figure 10). In contrary to the country, after three weeks of $\alpha$ decrease during lockdown in Madrid, the growth rate of new cases started to increase demonstrating reduced quarantine effectiveness (Figure 11), what can be related to Easter celebration started from Good Friday, 10 April 2020.

Figure 9. Dynamics of the daily new cases $n_j$ (a) and the growth rate of new cases $\alpha$ (b) for Iceland. Vertical black line points the day of imposed quarantine on 2020 March 16. Data collected from [13].

Figure 10. Dynamics of the daily new cases $n_j$ (a) and the growth rate of new cases $\alpha$ (b) for Spain. Vertical black line points the day of imposed quarantine on 2020 March 14. Data collected from [13].
Figure 11. Dynamics of the daily new cases $n_{td}$ (a) and the growth rate of new cases $\alpha$ (b) for Madrid city. Vertical black line points the day of imposed quarantine on 2020 March 14. Data collected from [14].

Italy has been under quarantine since March 9, 2020. It seems that Italians are too tired of the quarantine, and therefore have intentions to finish the lock-down as soon as possible. Therefore, Italy celebrated Easter more quietly without visible breaks of quarantine restrictions (Figure 12). However, a high value of the growth rate of new infection cases during the quarantine $\alpha_q = 0.973$ revealed/suggested that quarantine actions are not sufficient and, there is little hope to reach the end of epidemic as fast as in Australia with $\alpha_q = 0.86$.

The evolution of $\alpha$ shows that the Easter celebration had no influence on quarantine effectiveness in Germany (Figure 13), which is under strict national quarantine since 23 March 2020 [22].
Figure 13. Dynamics of the daily new cases $n_{\text{d}}$ (a) and the growth rate of new cases $\alpha$ (b) for Germany. Vertical black line points the day of imposed quarantine on 2020 March 23. Data collected from [13].

Denmark is an example in which the proposed model cannot be applied to predict the epidemic dynamics, and clarification of the reasons for such discrepancy requires more detailed analysis of epidemic situation in the country (Figure 14).

Figure 14. Dynamics of the daily new cases $n_{\text{d}}$ (a) and the growth rate of new cases $\alpha$ (b) for Denmark. Vertical black line points the day of imposed quarantine on 2020 March 11. Data collected from [13].

Situation in Russia serves as an example of ineffective quarantine conditions during first 2 weeks of the country lockdown. This can be explained by huge size of the country and heterogeneous distribution of the population across the country, which is reason for sequence of arising infection clusters in different place/locations/regions at different times, despite that national quarantine was imposed on 30 March, 2020. Consequently, the growth rate of new infection cases $\alpha$ remained
above 1 (Figure 15), which demonstrates that effectiveness of the quarantine actions was insufficient.

Like Russia, United States of America is another huge country by the population number and size. The dynamics of the new cases $n_{td}$ and the growth rate of new infection cases $\alpha$ in USA show that quarantine is not effective enough (Figure 16) and suggested emergence of new clusters of infection. It should be taken into account, that differences of COVID-19 statistics across various states are huge: cases per 100 000 people are varying more than 10 times [24]. Therefore, overall US data can suffer from too high level of generalisation. Analysis and forecasting of epidemic situation should be done at state level to generate results with practical value.
Scenario realised in Sweden, where the quarantine started on 10 March, 2020 [23], is specific. Due to soft conditions of the quarantine (technically most of EU countries would not attribute Swedish regime a quarantine, just a gradual restriction of some social activities), it seems that the peak of the new cases $n_{td}$ is achieved only in 40 days after quarantine begin and further dynamics is unclear because no country has experience of such situation (Figure 17). However, the experience gained in Sweden is very important and will be used by other countries for the second wave management in the future.

![Figure 17. Dynamics of the daily new cases $n_{td}$ (a) and the growth rate of new cases $a$ (b) for Sweden. Vertical black line points the day of imposed initial quarantine actions on 2020 March 10. Data collected from [13].](image)

**Conclusions**

The experience gained during the first wave of the Covid-19 pandemic in winter and spring of 2020 could help countries to better prepare for the next wave, which is expected to take place in autumn of the same year. We proposed the simplified approach, which allows to estimate effectiveness of the imposed quarantine conditions/restrictions, to forecast the epidemic spread and to take appropriate decisions. The observed dynamics of pandemic in various countries shows that the growth rate of new infection cases linearly decreases when the quarantine is imposed in a country (or a region) until it reaches constant value, which corresponds to the effectiveness of quarantine measures taken in the country. The proposed parameters $e_q$ and $\varphi$ together with analysis of the population mobility and social contacts can be used to estimate effectiveness of country or region lockdown measures; and on the basis of these parameters, the countries experiencing ongoing epidemic can use the proposed approach to study effectiveness of taken measures in other countries yet affected by the Covid-19 disease.

The proposed approach has limitation because it cannot be applied directly for countries with large population size, which might have several epidemic clusters due heterogeneous population distribution. In this case, each cluster must be analysed separately.

On the basis of the proposed approach, more complex models can be developed.
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REFERENCES

[1] Tang X, Wu C, Li X, Song Y, Yao X, Wu X, et al. On the origin and continuing evolution of SARS-CoV-2. Natl Sci Rev 2020. doi:10.1093/nsr/nwaa036.
[2] Dandekar R, Barbastathis G. Quantifying the effect of quarantine control in Covid-19 infectious spread using machine learning. MedRxiv n.d. doi:10.1101/2020.04.03.20052084.
[3] Weiss Sir Ronald Ross H. The SIR model and the Foundations of Public Health. Mater MATemàtics 2013.
[4] Giordano G, Blanchini F, Bruno R, Colaneri P, Di Filippo A, Di Matteo A, et al. Modelling the COVID-19 epidemic and implementation of population-wide interventions in Italy. Nat Med 2020. doi:10.1038/s41591-020-0883-7.
[5] Berger DW, Kyle F. Herkenhoff, Mongey S. An SEIR Infectious Disease Model with Testing and Conditional Quarantine. NBER Work Pap Ser 2020. doi:10.3386/w26901.
[6] Wu JT, Leung K, Leung GM. Nowcasting and forecasting the potential domestic and international spread of the 2019-nCoV outbreak originating in Wuhan, China: a modelling study. Lancet 2020;395:689–97. doi:10.1016/S0140-6736(20)30260-9.
[7] Hethcote HW. The Mathematics of Infectious Diseases. SIAM Rev 2000;42:599–653.
[8] Nakata Y, Enatsu Y, Inaba H, Kuniya T, Muroya Y, Takeuchi Y. Stability of epidemic models with waning immunity. SUT J Math 2014;50:205–45.
[9] Grant A. Dynamics of COVID-19 epidemics: SEIR models underestimate peak infection rates and overestimate epidemic duration. MedRxiv 2020:2020.04.02.20050674. doi:10.1101/2020.04.02.20050674.
[10] Tang B, Bragazzi NL, Li Q, Tang S, Xiao Y, Wu J. An updated estimation of the risk of transmission of the novel coronavirus (2019-nCov). Infect Dis Model 2020;5:248–55. doi:10.1016/j.idm.2020.02.001.
[11] Leyen U von der, Michel C. Joint European Roadmap towards lifting COVID-19 containment measures 2020. https://ec.europa.eu/info/sites/info/files/communication_-_a_european_roadmap_to_lifting_coronavirus_containment_measures_0.pdf.
[12] Template:2020 coronavirus pandemic lockdowns n.d. https://en.wikipedia.org/wiki/Template:2020_coronavirus_quarantines_outside_Hubei.
[13] European Centre for Disease Prevention and Control n.d. https://www.ecdc.europa.eu/en/publications-data/download-todays-data-geographic-distribution-covid-19-cases-worldwide.
[14] Novel Coronavirus (COVID-19) Cases Data n.d. https://data.humdata.org/dataset/novel-coronavirus-2019-ncov-cases.
[15] Svarbiausia informacija apie koronavirusą (COVID-19) n.d. https://koronastop.lrv.lt/.
[16] See how your community is moving around differently due to COVID-19 n.d. https://www.google.com/covid19/mobility/.
[17] 2020 coronavirus pandemic in Switzerland n.d. https://en.wikipedia.org/wiki/2020_coronavirus_pandemic_in_Switzerland#Government_response.
[18] 2020 coronavirus pandemic in Austria n.d.
Declaration of interests
We declare no competing interests.

Data Availability
The data used to support the findings of this study are available from the corresponding author upon request. Data for the infected cases count from various countries is obtained from the data collected in [13,14]. Dates of the imposed quarantine were collected from [12].