Understanding saturation and AA collisions with an eA collider

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Abstract

The initial conditions in high energy nucleus-nucleus collisions are determined by the small momentum fraction part of the nuclear wavefunction. This is the regime of gluon saturation and the most direct way to experimentally study it would be deep inelastic scattering at a high energy electron ion collider (EIC). This talk discusses some of the connections between physics at the EIC and the initial stage of relativistic heavy ion collisions. We argue that measurements at an EIC will provide detailed high-precision information about the parameters for the initial conditions, transverse geometry and longitudinal correlations that will be crucial in understanding the initial stage of a heavy ion collision.

1. Introduction

Hadrons and nuclei consist of partons, and the relevant degrees of freedom for describing their collisions at high energy are the quark and gluon fields. The most convenient kinematical variables to describe these degrees of freedom are the momentum transfer $Q^2$ (interpreted as a resolution scale in the transverse direction) and $x$, the fraction of longitudinal momentum carried by the parton in a frame where the hadron momentum is large. In a collision of two nuclei or protons it is usually not possible to experimentally determine the precise values of $x$ and $Q^2$ that were involved in the production of a final state particle; one measures only convolutions of the properties of the two wavefunctions. In deep inelastic scattering (DIS), by contrast, the outgoing electron is measured and it is therefore possible to know exactly the values of $x$ and $Q^2$ probed from the measured momenta. This is the feature that makes DIS the ideal way to obtain precision information about the QCD wavefunction.

With the planning process underway to build an Electron-Ion-Collider (EIC) [1] or, on a somewhat longer timescale, a Large Hadron-electron Collider (LHeC) [2] the subject is a topical one. This talk is, however, not intended to be a review of the experimental program at the EIC or the LHeC. We will instead concentrate on a few particular aspects in which nuclear DIS (nDIS) experiments can and already have been useful in understanding the initial stages of a heavy ion collision and therefore crucial for experimentally studying the properties of the Quark-Gluon Plasma.

2. The nonlinear high energy regime of QCD

In collisions of protons and nuclei the typical values of $x$ that are probed in the wavefunction are $x \sim p_{\perp} / \sqrt{s}$. Let us consider, in the center-of-mass frame of the collision, the nucleus moving...
in the +z direction. In light cone variables $p^\pm = (p^0 \pm p^3)/\sqrt{2}$ and $x^\pm = (t \pm z)/\sqrt{2}$ we consider $p^+$ as the longitudinal momentum, $p^-$ as the light cone energy, $x^+$ the light cone time and $x^-$ the longitudinal coordinate. Note that the variables $x^\pm$ are conjugate to $p^\pm$. The parton with momentum fraction $x$ will have longitudinal momentum $xp^+ \sim x \sqrt{\Sigma}/A$ and will thus probe the other, leftmoving, nucleus at a length scale $\Delta x^- \sim A/(x \sqrt{\Sigma})$. The longitudinal size of the leftmover is Lorentz-contracted from $R_x \sim A^{-1/3} R_p$ to $A^{-1/3} R_p (Am_N/\sqrt{\Sigma})$. We see that if $x \ll A^{-1/3} R_p m_N$, the partons in the rightmoving nucleus will not be able to resolve the individual nucleons of the leftmoving one. The whole nucleus must therefore be treated as one coherent target, not as a collection of independent nucleons. The observation that the large $x$ localized, valence-like, degrees of freedom are not resolved in the collision, but only the smaller $x$ partons that they radiate, naturally leads to the idea of treating the two separately in an effective field theory approach. This effective field theory is known as the Color Glass Condensate (CGC). We refer the reader to the reviews [3, 4] for further details, but it suffices here to emphasize the following. The CGC describes a high energy hadron in terms of a classical strong color field (the small $x$ gluons) radiated by an effective colour current (the large $x$ degrees of freedom). The classical color charges are stochastic random variables with a probability distribution $W_x[p]$. Both the initial field configuration in a heavy ion collision and observables in DIS at small $x$ can be computed in terms of these same classical gluon fields. The color charge distribution depends on nonperturbative input and cannot completely be computed from first principles. Its dependence on the energy scale (rapidity) that separates the large and small $x$ degrees of freedom can, however, be computed and expressed in terms of a renormalization group equation. The distribution of color charges is a universal object, it can be measured in one process (ideally DIS) and then used as an independent input to make prediction for another one (say, the initial field configurations in a heavy ion collision). In this sense the situation is analogous to collinear factorized perturbation; there is a universal, nonperturbative distribution (color charge distribution or parton distribution function), a separation scale (rapidity or virtuality) and a renormalization group equation derived from first principles that describes the dependence on this separation scale.

Gluon saturation appears as a very different phenomenon in different Lorentz frames. In the infinite momentum frame, where the parton model is defined, saturation arises from nonlinear interactions between gluons. A more convenient description of DIS at small $x$ is obtained in the dipole frame (roughly the target rest frame). In this frame the process can be viewed as a virtual photon with four-momentum $q$ and virtuality $q^2 = -Q^2$ splitting into a quark-antiquark dipole that then interacts with the target (see fig. [1]). At high energy (or small $x$) this fluctuation...
has a lifetime $\sim 1/(x m_N)$ which, for the values of $x$ that we are considering, is much larger than the size of the nucleus. The dipole therefore does not resolve individual nucleons, but interacts coherently with the nucleus as a whole.

The transverse size of the dipole $r$ is related, by the wavefunction of the virtual photon splitting into a fermion pair; to the momentum transfer, $r \sim 1/Q$. The interaction between the dipole and the target is described by a scattering amplitude $N(x, r, b_{\perp})$ or, integrated over the impact parameter $b_{\perp}$, the dipole cross section $\sigma(x, r) = 2 \int d^2 b_{\perp} N(x, r, b_{\perp})$. In the limit of small dipole sizes the scattering amplitude should vanish, because a dipole of size $r = 0$ is a colorless object. For small $r$ the dipole scattering amplitude behaves as $\sim r^2 x G(x, Q^2 \sim 1/r^2)$, where $x G(x, Q^2 \sim 1/r^2)$ is the conventional integrated gluon distribution. For large $Q^2$ this behavior dominates, and one recovers back the DGLAP description applicable in the dilute regime. The scattering amplitude is, however, bound by unitarity: $|N(x, r, b_{\perp})| \leq 1$. The growth as a function of $r$ cannot, therefore, continue indefinitely. It must be modified for large $r$, i.e. for $Q^2$ smaller than some characteristic scale $Q^2_s \sim x G(x, \bar{Q}^2 \sim Q_s^2)$; the saturation scale. In the infinite momentum frame this characteristic scale corresponds to the typical transverse momentum of the gluons in the wavefunction. Furthermore, it is observed experimentally and understood theoretically in terms of an exponentially growing cascade of bremsstrahlung gluons that the gluon distribution rises strongly at small $x$ as $x G(x, Q^2) \sim x^{-\lambda}$. This leads to the conclusion that the saturation scale must rise as a function of energy, typically as $Q^2_s \sim x^{-\lambda}$. If the collision energy is too small, the saturation scale is $\sim \Lambda_{QCD}$ and weak coupling methods can only be applied to rare high $Q^2$ phenomena, not the bulk dominated by $Q_s$. For large enough energies, however, $Q_s \gg \Lambda_{QCD}$ and weak coupling methods can be used.

In the infinite momentum frame the same phenomenon of gluon saturation looks completely different. There it is easy to understand the growth of the gluon distribution with energy; with more phase space available for radiation the gluons tend to split and their number grows exponentially with rapidity, i.e. as a negative power of $x$. When the phase space density of the gluons becomes of order $1/\alpha_s$, the nonlinear interactions among them become important and they start to recombine, which slows the growth of the gluon distribution. The curious thing about these two views of the same phenomenon is that in what in the infinite momentum frame looks like the result of nonlinear interactions and is more complicated to quantify is in the dipole frame a simple and precise statement based on the unitarity of the $S$-matrix. In many treatments of high energy evolution and saturation in the dipole frame the effect of increasing the energy is treated as a boost given the dipole (Lorentz-invariance guarantees that one can choose to boost either the dipole or the target as convenience dictates, the physical result must be the same), which makes the dynamics appear as splitting and merging of dipoles in the probe, instead of gluons in the target as in the infinite momentum frame. From the discussion in the dipole frame it should be clear that the relevant question is not whether parton saturation exists; the saturation in the gluon distribution at some energy-dependent transverse scale is required by unitarity. At asymptotically high energies this scale is bound to be large enough compared to $\Lambda_{QCD}$ for a weak coupling description of the process to be applicable. The relevant question is instead what the value of $Q_s(x)$ is, and whether the momenta and energies of the process one is studying are close enough to $Q_s$ that saturation has to be taken into account. As we shall argue in the following, there is strong evidence that this is the case for bulk particle production and forward jet production at RHIC, and for most of the properties of the initial state in heavy ion collisions at the LHC.
3. Parameters of the initial condition for AA

An important measure of the properties of the quark gluon plasma are the properties of hard particles or jets propagating though the medium. In order to isolate the effects of the medium one needs to be able to calculate the production rates of hard partons via pQCD methods, which requires knowing the nuclear parton distribution functions. Because there is so little data on nDIS, these, especially the gluon distribution are poorly constrained at small $x$; this is demonstrated in fig. 2. At RHIC the $x$ values relevant for jet production are, apart from forward rapidities, still quite large, but this will change significantly at LHC energies, where partons with $x \ll 10^{-2}$ can produce jets with $p_T > 10$ GeV. This means that the jet production is very sensitive to a region in $x$ where the nuclear gluon distribution is virtually unknown.

Bulk particle production in heavy ion collisions is dominated by the saturation scale $Q_s$. For a genuinely independent understanding of the initial conditions of a heavy ion collision we should be able to determine its value independently, without relying on modeling of the later stages of the evolution. The evolution of the saturation scale with the collision energy can be computed perturbatively, but its actual value depends on the initial condition, which is a nonperturbative input that has to be obtained from experimental data. This can be done using DIS experiments. Dipole model fits to HERA data constrain the saturation scale in a proton, which can then be combined with basic nuclear geometry to calculate $Q_s$ in a large nucleus [6] (see fig. 2). The existing nDIS data is from a too small energy range to provide a very stringent constraint on the nuclear $Q_s$, although it has been used in some attempts to parametrize the A dependence of $Q_s$ [7, 8]. A comparison of two parametrizations fitted to HERA data and extended to nuclei [6, 9] is shown in fig. 3. For the central rapidity region at RHIC this estimate yields a value $Q_s \approx 1.2$ GeV. This value can then be used to compute the initial gluon multiplicity using either a numerical solution of the classical Yang-Mills equations [10, 11, 12, 13] (see [14] for a discussion on relating the numerical value of $Q_s$ between CYM calculations and DIS observables) or in a $k_T$-factorized perturbative approximation [15, 16], with the result of somewhere around 1000 gluons per unit rapidity in the initial state of a heavy ion collision. The remarkable fact about
Figure 3: Left: Comparison of two impact parameter dependent dipole model parametrizations fitted to HERA data \[17\] to nDIS data. Right: Calculations of the charged multiplicity in in heavy ion collisions from RHIC to LHC energies from \[18\]. Shown are calculations using the same dipole cross section parametrizations, extrapolations using a constant \(\lambda\) in \(Q_s^2 \sim x^{-\lambda}\) and a logarithmic fit to RHIC data.

this value is how well it fits in with a picture of fast thermalization and subsequent ideal, i.e. entropy-conserving, hydrodynamical evolution of the system. One could say that this agreement is even too good for a leading order calculation since it leaves so little room for higher order contributions and an increase in the entropy from the thermalization phase.

When trying to extrapolate these ideas to the LHC, the situation is less settled, however. Initial stage gluon production is, to a very good accuracy, a one scale problem. The number of gluons in the initial stage of heavy ion collisions must then be \(\approx cQ_s^2(x)\pi R_A^2/\alpha_s\), where \(c\) is a nonperturbatively determined constant. The dependence of the multiplicity and initial energy density follows from the \(x\)-dependence of the saturation scale. Estimates from fits to HERA inclusive data (e.g. \([19, 20, 21, 17]\)) vary in the range \(\lambda = 0.2 \ldots 0.3\). Running coupling BK leads to a \(\lambda\) varying with \(x\), but within the same range for the energies between RHIC and LHC \[22\]. Extrapolated over the wide range of energy separating LHC from RHIC this turns into a major uncertainty on the prediction for the LHC initial gluon multiplicity, see fig. 3. Turning this argument around means that the measurement of the charged hadron multiplicity in AA-collisions at the LHC will provide a relatively simple and clear constraint on the interpretation of the HERA data. Disentangling the many effects that influence the energy dependence of \(Q_s\) would greatly benefit from precision measurements at the EIC.

4. Diffraction and transverse geometry

A major discovery at HERA was that a large fraction (~ 15%) of high energy DIS events are **diffractive**. In this context a diffractive event is defined as one where the virtual photon exchanges momentum with the target and dissociates into a hadronic system of invariant mass \(M_V^2\) (e.g. a vector meson) with the target staying intact. The experimental signature of these events is that the diffractive system is separated from the target by a large rapidity gap with no produced particles, indicating that no quantum numbers (in particular no color charge) has been exchanged. Diffractive DIS has a natural interpretation in the dipole picture, where it corresponds to elastic scattering between the dipole and the target. In the unitarity limit (and when the scattering amplitude is purely imaginary, as it is to a good approximation in high energy hadronic collisions) the elastic dipole-target cross section and therefore the diffractive DIS cross section is half of
the total. In nuclei the interaction of the dipole is closer to the unitarity limit than in protons at the same energy, and the fraction of diffractive events is even larger. According to one recent estimate [23] 20-25% of the events at an EIC could be diffractive. This is due to the combination of shadowing of the total cross section and enhancement of the diffractive one (see fig. 4). There are several fascinating features in diffractive DIS. One is that such a large fraction of the interactions in chromodynamics, even at high energy and $Q^2$ where asymptotic freedom should apply, happens without any net color being exchanged. Another is the large probability of the target staying intact. In the target rest frame one is (at HERA energies) hitting a proton with a TeV scale electron without breaking it. When the same electron beam is scattered on a nucleus the probability of the nucleus staying intact is even larger, in spite of the huge amount of energy deposited compared to the typical nuclear binding energies in the 10 MeV range.

The distribution in $t$ (the momentum kick given to the target proton or nucleus) in diffractive DIS is directly a Fourier transform of the impact parameter distribution of the gluons. In the proton this has led to the observation that the gluons are localized in a smaller radius around the center of the proton than the valence quarks. In the context of heavy ion collisions Monte Carlo Glauber modeling has been remarkably successful in describing most of the transverse geometrical features of the collision system at RHIC. There are some signals, such as $v_2$ fluctuations [24, 25], that for accurate enough measurements one will need to understand the geometry of the actual small $x$ gluonic degrees of freedom better than by Glauber modeling which is essentially extrapolating from the valence region, since it is based on density profiles measured from electric charge densities, and based on an assumption of independent nucleon scatterings. Measuring diffractive observables at an EIC will be challenging because of the smallness of the $t$ that dominate for large nuclei, but could significantly improve our understanding of the transverse geometry of the small $x$ glue in nuclei.

5. Longitudinal direction

The “Ridge” structure of two particle correlations in central heavy ion collisions [27, 28, 29, 30] and observations of long range rapidity correlations in particle multiplicities [31, 32] are striking signals of new dynamical effects at RHIC. Because particles that are produced far

![Figure 4: Left: Diffractive structure functions in nuclei compared to the proton, scaled by $A$ [23]. The additional kinematic variable introduced here is $\beta = Q^2/(Q^2 - t + M^2)$. Right: Nuclear modification to $\pi^+$ production due to parton energy loss in cold nuclear matter [26].]
away in rapidity can, by causality, only be correlated at early times; these correlations must have originated in the initial stages of the collision. They should therefore be present already in the wavefunctions of the initial nuclei, and correspond to a clearly and exactly measurable correlation observable in nuclear DIS. Formulating concisely and precisely what this observable would be is still to be done.

Another intriguing experimental observation, although still preliminary, is event-by-event CP violation in heavy ion collisions (also discussed at this conference). In the CGC framework this is naturally understood in terms of the same parametrically strong longitudinal glasma fields as the long range rapidity correlations in the ridge. The existence of the phenomenon was predicted before its observation, and a more detailed description of the mechanism is provided by the “Chiral Magnetic Effect”.

### 6. Energy loss in cold nuclear matter

In contrast to the situation at small $x$ outlined in sec. and discussed in most of this talk we should also mention the opposite limit, which is relevant for heavy ion collisions in another way. When the momentum fraction $x$ is large, the wavelength of the virtual photon in the target rest frame is very short. It will not have time to fluctuate, but will instead interact locally with a quark (typically valence, since we are at large $x$). In a large nucleus, this quark jet will then have to travel a long way in cold nuclear matter, where it will interact and lose energy; see fig. for one calculation compared to HERMES data at a much lower energy than would be reached at the EIC. Because of the well constrained kinematics in DIS, the initial momentum of the jet is very precisely. Measuring the hadronization products in the process will yield detailed information on the process of energy loss and hadronization in cold nuclear matter, and serve as a clear baseline and comparison for understanding energy loss in the quark-gluon plasma. We refer to the recent review for further details.

### 7. Conclusions

In this talk, we have reviewed a series of topics common to the physics that could be studied in future DIS experiments on nuclei and the initial stage of ultrarelativistic heavy ion collisions. We argued that in both cases, at high energy, the bulk of the physics is dominated by a single transverse momentum scale, the saturation scale $Q_s$. The saturation scale can be understood in different ways depending on the Lorentz frame in which one views the process. On one hand it is a transverse length scale at which QCD cross sections deviate from their perturbative rise to comply with unitarity. On the other hand it is the transverse momentum scale at which the occupation numbers of gluonic states grows so large that their nonlinear interactions start to limit further radiation. We then discussed some aspects of the high energy wave function that could be understood at an electron ion collider with a higher precision than can be achieved by looking at AA collisions alone. These include the numerical values for parameters, such as $Q_s$, that characterize the wavefunction (sec. 3), the transverse geometry of the small $x$ glue in the nucleus (sec. 4), the longitudinal structure and correlations in rapidity present in the wavefunction and the correspondint structure of the longitudinal glasma fields in AA collisions (sec. 5) and,
finally, the local properties of the nuclear medium as probed by a high energy jet propagating through it (sec. 6). This has hopefully conveyed the picture that DIS and AA experiments should not be seen as separate domains, but complementary methods of addressing common questions about the nature of QCD.

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References

[1] A. Deshpande, R. Milner, R. Venugopalan and W. Vogelsang, Ann. Rev. Nucl. Part. Sci. 55, 165 (2005) [arXiv:hep-ph/0506148].
[2] J. B. Dainton, M. Klein, P. Newman, E. Perez and F. Willeke, JINST 1, P10001 (2006) [arXiv:hep-ex/0603016].
[3] E. Iancu and R. Venugopalan, The color glass condensate and high energy scattering in QCD, in Quark gluon plasma, edited by R. Hwa and X. N. Wang, World Scientific, 2003, arXiv:hep-ph/0303204.
[4] H. Weigert, Prog. Part. Nucl. Phys. 55, 461 (2005) [arXiv:hep-ph/0501087].
[5] K. J. Eskola, H. Paukkunen and C. A. Salgado, JHEP 04, 065 (2009) [arXiv:0902.4154 [hep-ph]].
[6] H. Kowalski and D. Teaney, Phys. Rev. D68, 114005 (2003) [arXiv:hep-ph/0304189].
[7] A. Freund, K. Rummukainen, H. Weigert and A. Schafer, Phys. Rev. Lett. 90, 222002 (2003) arXiv:hep-ph/0210139.
[8] N. Armesto, C. A. Salgado and U. A. Wiedenmann, Phys. Rev. Lett. 94, 022002 (2005) [arXiv:hep-ph/0407018].
[9] H. Kowalski, T. Lappi and R. Venugopalan, Phys. Rev. Lett. 100, 022303 (2008) [arXiv:0705.3047 [hep-ph]].
[10] A. Krasnitz and R. Venugopalan, Nucl. Phys. B557, 237 (1999) [arXiv:hep-ph/9905433].
[11] A. Krasnitz, Y. Nara and R. Venugopalan, Phys. Rev. Lett. 87, 192302 (2001) [arXiv:hep-ph/0108092].
[12] T. Lappi, Phys. Rev. C67, 054903 (2003) [arXiv:hep-ph/0303076].
[13] A. Krasnitz, Y. Nara and R. Venugopalan, Nucl. Phys. A727, 427 (2003) [arXiv:hep-ph/0305112].
[14] T. Lappi, Eur. Phys. J. C55, 285 (2008) [arXiv:0711.3039 [hep-ph]].
[15] D. Kharzeev and M. Nardi, Phys. Lett. B507, 121 (2001) [arXiv:nucl-th/0012025].
[16] D. Kharzeev and E. Levin, Phys. Lett. B523, 79 (2001) [arXiv:nucl-th/0012025].
[17] H. Kowalski, L. Motyka and G. Watt, Phys. Rev. D74, 074016 (2006) [arXiv:hep-ph/0606272].
[18] T. Lappi, J. Phys. G35, 104052 (2008) [arXiv:0804.2338 [hep-ph]].
[19] K. J. Golec-Biernat and M. Wusthoff, Phys. Rev. D59, 014017 (1999) [arXiv:hep-ph/9807513].
[20] J. Bartels, K. J. Golec-Biernat and H. Kowalski, Phys. Rev. D66, 014001 (2002) [arXiv:hep-ph/0203258].
[21] I. Iancu, K. Itakura and S. Munier, Phys. Lett. B590, 199 (2004) [arXiv:hep-ph/0310338].
[22] J. L. Albacete, Phys. Rev. Lett. 99, 262301 (2007) [arXiv:0707.2545 [hep-ph]].
[23] H. Kowalski, T. Lappi, C. Marquet and R. Venugopalan, Phys. Rev. C78, 045201 (2008) [arXiv:0805.4071 [hep-ph]].
[24] STAR, P. Sorensen, J. Phys. G35, 104102 (2008) [arXiv:0808.0356 [nucl-ex]].
[25] PHOBOS, B. Alver et al., J. Phys. G35, 104101 (2008) [arXiv:0804.4297 [nucl-ex]].
[26] A. Accardi, Acta Phys. Hung. A27, 189 (2006) [arXiv:nucl-th/0510090].
[27] J. Putschke, J. Phys. G34, S679 (2007) [arXiv:nucl-ex/0701074].
[28] STAR, M. Daugherty, J. Phys. G35, 104090 (2008) [arXiv:0806.2121 [nucl-ex]].
[29] PHOBOS, B. Alver et al., arXiv:0812.1172 [nucl-ex].
[30] J. L. Nagle, arXiv:0907.2707 [nucl-ex].
[31] STAR, B. I. Abelev et al., arXiv:0905.0237 [nucl-ex].
[32] T. J. Tarnowsky, arXiv:0807.1941 [nucl-ex].
[33] F. Gelis, T. Lappi and R. Venugopalan, Phys. Rev. D79, 094017 (2009) [arXiv:0810.4829 [hep-ph]].
[34] STAR, S. A. Voloshin, arXiv:0907.2213 [nucl-ex].
[35] D. Kharzeev, A. Krasnitz and R. Venugopalan, Phys. Lett. B545, 298 (2002) [arXiv:hep-ph/0109253].
[36] T. Lappi and L. McLerran, Nucl. Phys. A772, 207 (2006) [arXiv:hep-ph/0602189].
[37] D. Kharzeev, Phys. Lett. B633, 260 (2006) [arXiv:hep-ph/0606125].
[38] D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A803, 227 (2008) [arXiv:0711.0950 [hep-ph]].
[39] A. Accardi, F. Arleo, W. K. Brooks, D. D’Enterria and V. Muccifora, arXiv:0907.3534 [nucl-th].