Research on dynamic characteristics of motor vibration isolation system through mechanical impedance method

Xingqian Zhao¹,²,*, Wei Xu¹,², Changgeng Shuai¹,² and Zechao Hu¹,²

¹Institute of Noise & Vibration, Naval University of Engineering, Wuhan, P.R. China
²National Key Laboratory of Ship Vibration and Noise, Wuhan, P.R. China

*Corresponding author e-mail: xqzhao0613@163.com

Abstract. A mechanical impedance model of a coupled motor–shaft–bearing system has been developed to predict the dynamic characteristics and partially validated by comparing the computing results with finite element method (FEM), including the comparison of displacement amplitude in x and z directions at the two ends of the flexible coupling, the comparison of normalized vertical reaction force in z direction at bearing pedestals. The results demonstrate that the developed model can precisely predict the dynamic characteristics and the main advantage of such a method is that it can clearly illustrate the vibration property of the motor subsystem, which plays an important role in the isolation system design.

1. Introduction

The revolution of the proposition motor in a ship can create severe vibration which not only lowers the life cycle of equipment, but also causes radiating noise and further exposes the ship location. Isolation system is a good way to reduce the vibration transferred to the ship hull. In the design process of an isolation system, important parameters like the displacement difference between the two ends of flexible coupling, the forces transferred to the foundation by bearing pedestals are of great interest. In this paper, mechanical impedance method (MIM) is introduced to derive the interested parameters. The analyzed isolation system is shown in Figure 1, which is composed of a motor, vibration isolators, flexible coupling, transmission shaft and bearings. The transmission shaft is sheathed by the rotor of motor and connected with the rotor by a flexible coupling.

Figure 1. Schematic diagram of motor vibration isolation system
2. Kinematic Analysis of the motor vibration isolation system

The motor vibration isolation system is split into two: the motor subsystem including the stator, the rotor and isolators; the shaft subsystem including the transmission shaft and bearings. The two subsystems are connected with each other by the flexible coupling, as shown in Figure 2. In each subsystem, the y-axis is along the shaft, the x-y plane is horizontal and the y-z plane is vertical. The corner mark m indicates the motor subsystem and s indicates the shaft subsystem.

![Figure 2](image_url) (a) Schematic diagram of motor subsystem. (b) Schematic diagram of shaft subsystem.

2.1. Kinematic Analysis of the motor subsystem

The motor is isolated by low-frequency air springs, whose flexibility is much larger than the motor’s, so the motor can be regarded as rigid body. In that case, the motor subsystem presents 6-degree rigid modals. At the same time, the isolators can be treated as linear springs, which is the precondition of the subsequent kinematic analysis [1]. The motor weighs 50t, supported by 12 isolators which are π/6 obliquely placed, as shown in Figure 3.

![Figure 3](image_url) the layout of isolators in motor subsystem

The coordinate system is established as shown in Figure 2(a), where the origin is located at the centroid of the motor. The motion equation of motor subsystem is given by

\[ M_m \ddot{X}_m + K_m X_m = F'_m \]  \hspace{1cm} (1)

where \( M_m = \text{diag}(m, m, I_{xx}, I_{yy}, I_{zz}) \) is the mass matrix of the motor. \( m \) is the mass of the motor. When \( i=j \) (i, j=x, y, z), \( I_{ij} \) is moment of inertia; when \( i \neq j \), \( I_{ij} \) is cross inertia of the motor, which can be regarded as zero because of the symmetry of the motor. \( X_m = [x'_m, y'_m, z'_m, \alpha'_m, \beta'_m, \gamma'_m] \) is the displacement vector at motor centroid. \( F'_m \) is the load vector at the motor centroid. \( K_m \) is the global stiffness matrix of isolators, which is assembled by the stiffness of each isolators through coordinate transformation. \( K_m \) is given by

\[ K_m = \sum_{i=1}^{n} G_i^T K_{m_i} G_i \]  \hspace{1cm} (2)
where $G_i$ is the position transformation matrix, $K_m$ is the stiffness matrix of each isolators in global coordinate in motor subsystem. $G_i$ is given by

$$
G_i = \begin{bmatrix}
0 & a'_i & -a'_i \\
E_{i,j} -a'_i & 0 & a'_i \\
a'_i & a'_i & 0
\end{bmatrix}
$$

where $a'_i$ ($i=1,2...n$, $j=x,y,z$) is the distances between positions of each isolators and $x, y, z$ axis. $K_m$ is given by

$$
K_m = T^T k_i T
$$

where $k_i = \text{diag}(k_p, k_q, k_r)$ is the stiffness matrix of each isolators, $p, q, r$ are the stiffness spindles and $k_p, k_q, k_r$ is the linear stiffness of the corresponding isolators in local coordinate system. $T = \cos \theta$ is the direction transformation matrix which converts the stiffness matrix from local coordinate system to global coordinate system, $\theta$ is the angle matrix between local coordinate axes and corresponding global coordinate axes, which is given by

$$
\theta_i = \begin{bmatrix}
\phi_{px} & \phi_{py} & \phi_{pz} \\
\phi_{qx} & \phi_{qy} & \phi_{qz} \\
\phi_{rx} & \phi_{ry} & \phi_{rz}
\end{bmatrix}
$$

where $\phi, \varphi, \psi$ are the angles between stiffness spindles $p, q, r$ and $x, y, z$.

Under the application of stable harmonic excitation $F_m e^{j\omega t}$, the equation (1) can be given by

$$
Z_m' V_m' = F_m'
$$

where $Z_m' = (K_m - j\omega^2 M_m) / j\omega$ is the point impedance matrix at the centroid in the motor subsystem. The motor subsystem is connected with shaft subsystem by the flange, so the displacement and force at the centroid can be converted to the flange. The relationship is given by

$$
\begin{cases}
V_m = G_j X_m' \\
F_m' = G_j F_m
\end{cases}
$$

where $G_j = \begin{bmatrix} E & A \\ 0 & E \end{bmatrix}$. The $E$ is a diagonal matrix and $A = \begin{bmatrix} 0 & a_z & -a_y \\ -a_z & 0 & a_x \\ a_y & -a_x & 0 \end{bmatrix}$, where $a_i$ is the coordinate of flange in motor subsystem, so the motion equation of flange can be given by

$$
Z_m V_m = F_m
$$

where $Z_m = [G_j^T Z_m' G_j]$ is the point impedance matrix at the flange (station 6) in motor subsystem.

2.2. Kinematic Analysis of the shaft subsystem

The shaft subsystem has 6 stations, as shown in Figure 2(b) and a shaft element is shown in Figure 4. There are two types of methods to model the shaft element. An Euler-Bernoulli beam model is
appropriate for low-speed flexible shafts and a Timoshenko beam model is appropriate for high-speed, larger diameter shafts[2]. Allowing for that the rotary speed is less than 200r/min and the ratio between shaft length and diameter is larger than 25, the Euler-Bernoulli beam model is adopted to analyse the shaft subsystem.

![Figure 4. Shaft segment n in yz plane](image)

The relationship between the force and velocity vectors of shaft segment $n$ is given by

$$Z_n^* V^* = F_n^*$$

where $F_n^* = [F_{ix}^*, F_{iz}^*, M_{ix}^*, M_{iz}^*, F_{i\alpha}^*, M_{i\alpha}^*]^T$, $V_n^* = [V_{ix}^*, V_{iz}^*, \omega_{ix}^*, \omega_{iz}^*, V_{i\alpha}^*, \omega_{i\alpha}^*]^T$. $Z_n^*$ is given by[3]

$$Z_n^* = \begin{bmatrix} Z_{11}^n & Z_{12}^n \\ Z_{13}^n & Z_{14}^n \end{bmatrix}$$

Where

$$Z_{11}^n = \begin{bmatrix} a & 0 & b & 0 \\ 0 & a & 0 & b \\ b & 0 & e & 0 \\ 0 & b & 0 & e \end{bmatrix}, Z_{12}^n = Z_{13}^n = Z_{14}^n = \begin{bmatrix} c & 0 & d & 0 \\ 0 & c & 0 & d \\ -d & 0 & f & 0 \\ 0 & -d & 0 & f \end{bmatrix}, Z_{14}^n = \begin{bmatrix} a & 0 & -b & 0 \\ 0 & a & 0 & -b \\ -b & 0 & 0 & e \\ 0 & -b & 0 & e \end{bmatrix}$$. The matrix elements $a-f$ are functions of shaft segment, as follows:

$$a = \frac{\omega J k (\sin kL \cosh kL + \cos kL \sinh kL)}{\Delta}, b = \frac{\omega J \sin kL \sinh kL}{\Delta}, c = -\frac{\omega J k (\sinh kL + \sin kL)}{\Delta}, d = -\frac{\omega J (\cos kL - \cosh kL)}{\Delta}, e = -\frac{\omega J (\cos kL \sinh kL - \sin kL \cosh kL)}{\Delta}, f = -\frac{\omega J (\sin kL - \sinh kL)}{k \Delta}$$

where $\Delta = 1 - \cosh kL \cos kL$, $J = \rho A c K$, $c = \sqrt{E/\rho}$, $K = \sqrt{I/A}$, $k = \sqrt{\rho A \omega^2 / E I}$. $A$ is the cross sectional area of shaft segment; $L$ is the length of the shaft segment; $E$ is Young’s modulus; $I$ is the second moment of area of the shaft segment; $\omega$ is the circular frequency of vibration.

### 2.3. Kinematic Analysis of the fluid film journal bearings

In fluid film journal bearing, because the dynamic characteristics in orthogonal directions are cross-coupled, the relationship between the fluid film force and the displacement and velocity of journal is very complicated. Generally, under the assumption of linearization, the dynamic characteristics can be given by [4]

$$\begin{bmatrix} k_{bxx} & k_{bxc} \\ k_{bxc} & k_{bcc} \end{bmatrix} \begin{bmatrix} x_{hi} \\ z_{hi} \end{bmatrix} + \begin{bmatrix} c_{bxx} & c_{bxc} \\ c_{bxc} & c_{bcc} \end{bmatrix} \begin{bmatrix} \dot{x}_{hi} \\ \dot{z}_{hi} \end{bmatrix} = \begin{bmatrix} F_{bxx} \\ F_{bxc} \end{bmatrix}$$

where $k_{bij}$ ($i, j = x, z$) is the stiffness coefficient and $c_{bij}$ is the damping coefficient which is mainly attributed to the fluid film. Under the condition of stable vibration, equation (11) becomes
\[ Z_{ix} V_{ix} = F_{ni} \]  

\[ \text{where } Z_n = K_n / j\omega + C_n \text{ is the impedance matrix of bearing.} \]

2.4. Assembly of shaft subsystem impedance matrix

As shown in Figure 2, there are 5 stations on the shaft. Station 1, 2, 4, 5 are bearings. Station 3 is connected with station 6, which is on the motor, through the flexible coupling. The impedance matrices of shaft segments are assembled the way same as the manner used in finite element \([5]\) through applying the continuity of linear and angular velocities at each shaft station.

The assembled shaft subsystem impedance matrix \( Z' \) is of order 20\(\times\)20 (4 dofs\(\times\)5 stations). In equation (12), \( Z_{ii} \) is of order 2\(\times\)2, which is added to the \( \{1+4(i-1), 2+4(i-1)\} \)th rows, \( \{1+4(i-1), 2+4(i-1)\} \)th columns of \( Z' \), where \( i=1, 2, 4, 5 \). It is obtained the complete assembled shaft subsystem impedance matrix \( Z_s \).

2.5. The coupled system impedance matrix

The coupled impedance matrix is \( Z' = \text{diag}(Z_s, Z_m) \). Then it is needed to consider the influence of the flexible coupling. The motion relationship between two ends of the flexible coupling can be described as

\[
\begin{bmatrix}
F_{3x} \\
F_{3z}
\end{bmatrix} = \begin{bmatrix}
F_{6x} \\
F_{6z}
\end{bmatrix} = -K_c \begin{bmatrix}
x_3 - x_6 \\
z_3 - z_6
\end{bmatrix} = -Z_c \begin{bmatrix}
V_{3x} - V_{6x} \\
V_{3z} - V_{6z}
\end{bmatrix} \tag{13}
\]

\[ \text{where } K_c \text{ is the stiffness matrix of the flexible coupling. } Z_c = K_c / j\omega, \text{ the impedance matrix of flexible coupling, is of order } 2\times2, \text{ which is added to } \{9, 10\} \text{th rows, } \{9, 10\} \text{th columns and } \{21, 23\} \text{th rows, } \{21, 23\} \text{th columns of } Z'. \text{ At the same time, } -Z_c \text{ is added to } \{9, 10\} \text{th rows, } \{9, 10\} \text{th columns and } \{21, 23\} \text{th rows, } \{21, 23\} \text{th columns of } Z'. \text{ It is obtained the complete coupled impedance matrix } Z. \]

The motion equation of whole system is given by

\[ ZV = F \]  

\[ \text{where } F = [F^T_s, F^T_m]^T, V = [V^T_s, V^T_m]^T, \text{ in which } F_m \text{ is the exciting forces from the motor. It is assumed that the foundation is rigid, so } F_i = 0 \text{ and all external exciting comes from the motor.} \]

3. Model validation

A moment of 1000N/m is applied to the flexible coupling (station 6) to simulate the output vibration of the motor. The MIM and finite element method (FEM) are used to obtain some important parameters in the isolation system design. The Comparison of displacement amplitude in x and z directions at station 6 and 3 is presented in Figure 6. It is revealed that the results through two methods have good agreement under 150Hz. When above 150Hz, MIM tends to shift the peak of the curve to right, which can be attributed to the limitation of Euler-Bernoulli beam model discussed in section 2.2.
Figure 5. Comparison of displacement amplitude obtained by FEM and MIM in x and z directions at station 6 and 3. (a) Displacement amplitude in x direction at station 6, (b) Displacement amplitude in z direction at station 6, (c) Displacement amplitude in x direction at station 3, (d) Displacement amplitude in z direction at station 6.

In the spectrum of above 30Hz, there present several resonance peaks, which is the model frequency of the shaft subsystem. In the low frequency spectrum, the results of FEM tend to present more resonance peaks than MIM. This is because in MIM, the stiffness of isolators is concentrated at the center of the motor, so it only presents two or three rigid model. The FEM takes the different stiffness of isolators into consideration, so it presents violent fluctuation. The MIM can clearly illustrate the important information, so it has advantage in the isolation system design relative to FEM.

The comparison of normalized vertical reaction force obtained by FEM and MIM in z direction at station 1, 2, 4 and 5 is demonstrated in Figure 6. In the design process, we are interested in how much the force is transferred to the base by the bearing pedestals, especially where the resonance peaks happen and the MIM can offer sufficiently precise prediction.

Figure 6. Comparison of normalized vertical reaction force obtained by FEM and MIM in z direction at station 1, 2, 4 and 5. (a) Normalized vertical reaction force at station 1, (b) Normalized vertical reaction force at station 2, (c) Normalized vertical reaction force at station 4, (d) Normalized vertical reaction force at station 5.
4. Conclusion and further development

In this paper, a mechanical impedance model is developed to predict the dynamic characteristics of the isolation system in the design process for a coupled motor–shaft–bearing system and partially validated by comparing the computing results with finite element method. The main advantage of such a method is that it can clearly illustrate the vibration property of the motor subsystem, which plays an important role in the isolation system design. At the same time, the MIM can precisely predict the shaft subsystem model frequencies.

The foundation in this paper is assumed to be rigid. In the later research, the motor-shaft-bearing will be put on a huge raft which is supported by a number of isolators and forms an integrated isolation system. Further development of MIM on the base of the derivation in this paper and some experimental validation will be reported in the near future.

Acknowledgments

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