Beyond leading-order corrections to $\bar{B} \to X_s \gamma$ at large $\tan \beta$

The charged-Higgs-boson contribution

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Among the $O(\alpha_s \tan \beta)$ contributions to the Wilson coefficients $C_7$ and $C_8$, relevant for the decay $B \to X_s \gamma$, those induced by two-loop diagrams with charged-Higgs-boson exchange and squark-gluino corrections are calculated in supersymmetric models at large $\tan \beta$. The calculation of the corresponding Feynman integrals is exact, unlike in previous studies that are valid when the typical supersymmetric scale $M_{\text{SUSY}}$ is sufficiently larger than the electroweak scale $m_{\text{weak}}(\sim m_W, m_t)$ and the mass of the charged Higgs boson $m_H\geq m_{\text{weak}}$. Therefore, the results presented here can be used for any value of the various supersymmetric masses. These results are compared with those of an approximate calculation, already existing in the literature, that is at the zeroth order in the expansion parameter $(m_{\text{weak}}^2, m_H^2)/M_{\text{SUSY}}^2$, and with the results of two new approximate calculations in which the first and second order in the same expansion parameter are retained, respectively. This comparison allows us to assess whether the results of these three approximate calculations can be extended beyond the range of validity for which they were derived, i.e., whether they can be used for $m_H \geq M_{\text{SUSY}}$ and/or $M_{\text{SUSY}} \sim m_{\text{weak}}$. It is found that the zeroth-order approximation works well even for $m_H \geq M_{\text{SUSY}}$, provided $M_{\text{SUSY}}^2 \gg m_{\text{weak}}^2$. The inclusion of the higher-order terms improves the zeroth-order approximation for $m_H^2 \ll M_{\text{SUSY}}^2$, but it worsens it for $m_H \geq M_{\text{SUSY}}$.

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I. INTRODUCTION

While the rate for the inclusive decay $\bar{B} \to X_s \gamma$, $\text{BR}(\bar{B} \to X_s \gamma)$, has been calculated up to the next-to-leading order (NLO) in QCD within the standard model (SM)\textsuperscript{[1]}, similarly precise calculations exist for only some extensions of the SM. Achieving a NLO precision for this decay rate in such extensions, and in particular in supersymmetric models, is important. It seems unlikely that an increased experimental precision in the three apparatuses where this decay is measured, i.e. BABAR, BELLE and CLEO\textsuperscript{[2]}, combined with the increased theoretical precision of a possible estimate of the next-to-NLO corrections in the SM\textsuperscript{[3]}, will bring unequivocal signals of new physics. Nevertheless, calculations of $\text{BR}(\bar{B} \to X_s \gamma)$ with NLO accuracy for the models that are considered the most likely candidates to extend the SM could help in understanding where the effective scale of these models sets in, and the extent of the spreading of masses of additional particles around this scale.

NLO calculations exist for two-Higgs-doublet models of type I and type II\textsuperscript{[4],[5],[6]}, as well as models in which the couplings of the charged Higgs boson to fermions are, in absolute values, those of type II models, but are, in general, complex\textsuperscript{[6]}. In the case of supersymmetric models, the situation is as follows. For generic models, the QCD corrections to the electroweak rate, calculated first in\textsuperscript{[7]}, have been included only at leading-order (LO) precision\textsuperscript{[8]}. Higher-order QCD corrections have been evaluated for specific scenarios.

In one class of such scenarios\textsuperscript{[9]}, the two charginos, one $\tilde{t}$ squark, which is predominantly right handed, and the charged Higgs boson are assumed to be relatively light, while all the other squarks and the gluino are heavy. Moreover, no additional sources of flavor violation are present at the electroweak scale, other than the Cabibbo-Kobayashi-Maskawa mixing elements.

Other studies\textsuperscript{[10],[11]} considered a slightly different class of scenarios, in which $\tan \beta$ is large, and the supersymmetric spectrum is like the spectrum for the scenarios of Ref.\textsuperscript{[9]}, but without the assumption of a light $\tilde{t}$ squark. The same minimality in flavor violation is also assumed\textsuperscript{[12]}. The importance of supersymmetric models at large $\tan \beta$ hardly needs to be highlighted here, as $\tan \beta$ tends to be large whenever an attempt to unify Yukawa couplings is made, as required by a SO(10) grand unification\textsuperscript{[13]}. Indeed, many phenomenological calculations exists, in which the constraints imposed by the measured rate of $\bar{B} \to X_s \gamma$ to models at large $\tan \beta$ have been analyzed at LO in QCD\textsuperscript{[14]}. The papers in Refs.\textsuperscript{[10],[11]} allow one to refine these analyses in models which, as well as having large $\tan \beta$, predict the same type of supersymmetric spectrum that they assume.

The restriction to specific ranges of masses of the supersymmetric partners of the SM particles, made in Refs.\textsuperscript{[9],[10],[11]}, has made possible the use of an effective Lagrangian formalism, and has led to rather compact and simple
formulas. These very same restrictions, however, limit the usefulness of such calculations, unless it is proven that the formulas obtained in these papers can be safely used beyond the range of validity for which they have been derived. Nevertheless, they have often been employed also when the charged Higgs boson is as heavy as the squarks and the gluino. It is interesting to understand, in such cases, how far the resulting analyses are from ideal ones, in which exact formulas are used.

In this paper we address such an issue, focusing on the scenarios of Refs. [10, 11]. We consider the gluino-induced supersymmetric corrections of $O(\alpha_s \tan \beta)$ that are the largest beyond-leading-order corrections in scenarios with large $\tan \beta$. Corrections of this type are obtained by (i) including the finite corrections to the $b$-quark mass in the fermion–fermion–Higgs-boson and in the fermion–fermion–Higgsino couplings [15, 17], and (ii) “dressing” with squark-gluino subloops the one-loop diagrams mediated by the charged Higgs and the charged Goldstone bosons that contribute to $B \to X_s \gamma$ at the partonic level. A graphical representation of this “dressing” is explicitly shown in Fig. 1 for the diagrams contributing to $b \to s \gamma$ or to $b \to s g$ and mediated by the charged Higgs boson. (In this figure, the photon or the gluon is assumed to be attached in all possible ways.) Indeed, the substitution of the vertex $\bar{s}_L t_R H^- \to t_R H^-$ of the one-loop diagrams with $\bar{s}_L t_R H^-$, possible in the two-loop diagrams, allows the gain of a $\tan \beta$ factor, yielding the required $\alpha_s \tan \beta$.

As already pointed out, in the calculations of Refs. [10, 11], all scalar superpartners of the SM fermions, at the scale $M_{\text{SUSY}}$, are assumed to be heavier than the $t$ quark and the $W$ boson, whereas the charged Higgs boson $H^\pm$ is assumed to be around the electroweak scale $m_{\text{weak}}$, with $m_{\text{weak}} \sim M_W, m_t$. In the limit of large $M_{\text{SUSY}}$, the two-loop diagrams in Fig. 1 in which the photon is emitted only by the $t$ quark and the charged Higgs boson, and the gluon only by the $t$ quark, all with chirality flip on the $t$-quark line, are of nondecoupling nature. In the same limit, all other diagrams in Fig. 1 decouple. It is conceivable, therefore, that for $m_{H^\pm}^2 \sim m_t^2 \ll M_{\text{SUSY}}^2$, the former two-loop diagrams are the only ones that give a sizable contribution to the $O(\alpha_s \tan \beta)$ corrections to the decay amplitudes of $b \to s \gamma$ and $b \to s g$, and that the expression for their squark-gluino subloops is well approximated by the expression for the same subdiagrams with vanishing external momenta. Thus, in this approximation, these two-loop diagrams are factorized into two one-loop diagrams. Such a factorization is supported by the use of an effective two-Higgs-doublet Lagrangian, in which all heavy degrees of freedom are integrated out. (A similar formalism was used in Ref. [14], in which corrections of $O(\alpha \tan \beta)$ to the decay amplitude of $b \to s \gamma$ were calculated.)

A potential problem with this approach may arise when the supersymmetric spectrum is not much heavier than $m_{\text{weak}}$, and/or when the mass of the charged Higgs boson tends to be closer to $M_{\text{SUSY}}$ than to $m_{\text{weak}}$. (Papers with further improvements of the original calculations, such as those in Ref. [13], do not address this issue.) One plausible solution to this problem is to include in the original calculation higher-order terms, up to $O((m_{\text{weak}}^2, m_{H^\pm}^2/M_{\text{SUSY}}^2)^n)$, with a suitable $n$. That is equivalent to saying that the effective Lagrangian is extended to include higher-order operators. (A discussion on this point can be found in Ref. [12].) An efficient way to carry out this extension, consistently including all the operators that yield terms of the same order $n$, is to make use of the heavy mass expansion (HME) [20]. Starting, for example, from the regime in which $m_t^2, M^2_W \sim m_{H^\pm}^2 \ll M_{\text{SUSY}}^2$, this technique allows one to add all operators needed when $m_{H^\pm}^2$ tends to $M_{\text{SUSY}}^2$ from below. It remains, however, to be established up to what value of $n$ it is necessary to extend this expansion, in order to obtain an estimate of $\text{BR}(B \to X_s \gamma)$ adequate for all possible values of $M_{\text{SUSY}}$ and of $m_{H^\pm}^2$ predicted by different supersymmetric models at large $\tan \beta$. Clearly, an accurate answer to this question can be given only by a comparison with the exact calculation of all the two-loop diagrams that give rise to $O(\alpha_s \tan \beta)$ corrections, in which no assumption is made on the relative size of $m_{H^\pm}, M_{\text{SUSY}},$ and $m_{\text{weak}}$.

In this paper we perform such an exact calculation of these two-loop diagrams contributing to the two Wilson coefficients $C_7$ and $C_8$ related to the partonic processes $b \to s \gamma$ and $b \to s g$. Among all the contributions to these processes [21], we restrict ourselves to those mediated by the charged Higgs boson, addressing first the problem of obtaining results valid throughout all values of $m_{H^\pm}$. We postpone the presentation of the complete $\text{BR}(B \to X_s \gamma)$ to future work [22].
We also calculate the same diagrams using the HME technique, up to order \(O((m_{\text{weak}}^2, m_{H^\pm}^2/M_{\text{SUSY}}^2)^2)\). The hope is that, in the case in which the series expansion appears to be quickly converging, approximate and possibly still compact formulas can be provided, which nevertheless allow one to extend the results existing in the literature to all possible values of \(m_{H^\pm}\) and \(M_{\text{SUSY}}\). It turns out that the series expansion in \((m_{\text{weak}}^2, m_{H^\pm}^2/M_{\text{SUSY}}^2)\) is far from being as well behaved as hoped and that, in general, the use of the exact results cannot be avoided. However, whereas the approximate calculations including terms up to \(O((m_{\text{weak}}^2, m_{H^\pm}^2/M_{\text{SUSY}}^2)^2)\) give results in disagreement with the exact one for values of \(m_{H^\pm}\) in the \(M_{\text{SUSY}}\), the lowest-order term of the series expansion, i.e., the nondecoupling approximation of Refs. [10, 11] seems to be a good approximation of the exact result throughout all ranges of \(m_{H^\pm}\), provided \(M_{\text{SUSY}}\) is sufficiently larger than \(m_{\text{weak}}\). Deviations appear for supersymmetric particles not much above the electroweak scale.

The paper is organized as follows. In Sec. II we list all the diagrams needed to evaluate the charged-Higgs-boson mediated \(\alpha_s\tan \beta\) to the partonic decays \(b \to s\gamma\) and \(b \to sg\), are obtained from the right diagram of Fig. 1 after allowing a photon or/and a gluon to be emitted in all possible ways. They are shown explicitly in Fig. 2 where the photon may be replaced by a gluon, and vice versa, whenever possible.

It should be observed that, while the one-loop diagram on the left side of Fig. 1 has a chirality flip in the internal \(t\)-quark line (we neglect in this discussion the diagram with chirality flip on the external \(b\)-quark line, which is \(\tan \beta\) suppressed), the diagrams in Fig. 2 can have such a chirality flip on the \(t\)-squark line also. The number of contributions to be evaluated, therefore, amounts to 8 for the calculation of the Wilson coefficient \(C_7(\mu_W)\), and 8 for the coefficient \(C_8(\mu_W)\). The diagrams used for the calculations of \(\text{BR}(B \to X_s \gamma)\) in Refs. [10, 11] are the two at the top of Fig. 2 for the calculation of \(C_7(\mu_W)\), i.e., (a) and (b), and the diagram (a), with the photon replaced by a gluon, for the calculation of \(C_8(\mu_W)\), all with chirality flip on the \(t\)-quark line only.

We have calculated all the 16 contributing terms, by making use of results and techniques presented in Ref. [23]. Our normalization of the Wilson coefficients \(C_7(\mu_W)\) and \(C_8(\mu_W)\) is the conventional one, as follows from the definition of the effective Hamiltonian,

\[
H_{\text{eff}} \supset -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left[ C_7(\mu) O_7(\mu) + C_8(\mu) O_8(\mu) \right],
\]

and of the operators \(O_7\) and \(O_8\),

\[
O_7(\mu) = \frac{e}{16\pi^2} m_b(\mu) \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, \quad O_8(\mu) = \frac{g_s}{16\pi^2} m_b(\mu) \bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu},
\]

where \(F_{\mu\nu}\) and \(G^a_{\mu\nu}\) are the field strengths of the photon and the gluon, respectively. We denote by \(C_{7,H}(\mu_W)\) and \(C_{8,H}(\mu_W)\) the \(\tan \beta\)-unsuppressed charged-Higgs-boson contribution to \(C_7(\mu_W)\) and \(C_8(\mu_W)\), and we decompose them as

\[
C_{i,H}(\mu_W) = \frac{1}{1 + \Delta_{b_R,\beta} \tan \beta} \left[ C^0_{i,H}(\mu_W) + \Delta C^1_{i,H}(\mu_W) \right],
\]

where \(C^0_{i,H}(\mu_W)\) and \(\Delta C^1_{i,H}(\mu_W)\) are induced by the one-loop diagram in Fig. 1 and the two-loop diagrams in Fig. 2 respectively. The overall factor \(1/(1 + \Delta_{b_R,\beta} \tan \beta)\) (see notation of Ref. [10]) stems from expressing the \(H^+ t_L b_R\) Yukawa coupling in terms of \(m_b\), corrected up to \(O(\alpha_s \tan \beta)\) [13, 14]. \(\Delta_{b_R,\beta}\) is given by

\[
\Delta_{b_R,\beta} = \frac{C_F \alpha_s}{2\pi} \mu m_b^2 I(m_{b_1}^2, m_{b_2}^2, m_{b_3}^2),
\]

where the function \(I\) is
FIG. 2: Charged-Higgs-boson mediated diagrams contributing at order $O(\alpha_s \tan \beta)$ to the partonic decays $b \to s\gamma$ and $b \to sg$. The photon must be replaced by a gluon, and vice versa, whenever possible.

\[
I(m_1^2, m_2^2, m_3^2) = -\frac{m_1^2 m_2^2 \ln(m_1^2/m_2^2) + m_2^2 m_3^2 \ln(m_2^2/m_3^2) + m_3^2 m_1^2 \ln(m_3^2/m_1^2)}{(m_1^2 - m_2^2)(m_2^2 - m_3^2)(m_3^2 - m_1^2)}.
\]

(5)

As the photon can be emitted by the $t$ quark and the charged Higgs boson, we have two contributions to $C^0_{7H}(\mu_W)$:

\[
C^0_{7H}(\mu_W) = -\frac{1}{2} Q_t \frac{m_t^2}{m_{H^\pm}^2} F_3 \left( \frac{m_{H^\pm}^2}{m_t^2} \right)
\]

\[
C^0_{7H}(\mu_W) = \frac{1}{2} Q_H \frac{m_t^2}{m_{H^\pm}^2} F_4 \left( \frac{m_{H^\pm}^2}{m_t^2} \right)
\]

(6)

while only one to $C^0_b(\mu_W)$:

\[
C^0_{b}(\mu_W) = -\frac{1}{2} Q_t \frac{m_t^2}{m_{H^\pm}^2} F_3 \left( \frac{m_{H^\pm}^2}{m_t^2} \right)
\]

(7)

due to the emission of the gluon from the $t$ quark. The superscript indices $a$ and $b$ in the above expressions denote emission of the photon/gluon from the $t$ quark and the charged Higgs boson, respectively. The functions $F_3$ and $F_4$ are listed in Appendix A. $Q_t$ and $Q_H$ indicate the electric charges of the $t$ quark and the charged Higgs boson ($Q_H = -1$).

The contributions to $\Delta C^0_{7H}(\mu_W)$, due to the different two-loop diagrams in Fig. 2 are

\[
\Delta C^0_{7H}(\mu_W) = \frac{1}{2} Q_t C_F (\alpha_s \tan \beta) m_t \mu (4\pi)^3 \left[ (U_{i2})_{21}(U_{i2}^T)_{12} m_t m_{\tilde{g}} I_{i2} - (U_{i2})_{21}(U_{i2}^T)_{11} I_{i11} \right],
\]

\[
\Delta C^0_{7H}(\mu_W) = \frac{1}{2} Q_H C_F (\alpha_s \tan \beta) m_t \mu (4\pi)^3 \left[ (U_{i2})_{21}(U_{i2}^T)_{12} m_t m_{\tilde{g}} I_{i2} - (U_{i2})_{21}(U_{i2}^T)_{11} I_{i11} \right],
\]

\[
\Delta C^0_{7H}(\mu_W) = \frac{1}{2} Q_t C_F (\alpha_s \tan \beta) m_t \mu (4\pi)^3 \left[ (U_{i2})_{21}(U_{i2}^T)_{12} m_t m_{\tilde{g}} I_{i2} - (U_{i2})_{21}(U_{i2}^T)_{11} I_{i11} \right],
\]

\[
\Delta C^0_{7H}(\mu_W) = \frac{1}{2} Q_s C_F (\alpha_s \tan \beta) m_t \mu (4\pi)^3 \left[ (U_{i2})_{21}(U_{i2}^T)_{12} m_t m_{\tilde{g}} I_{i2} - (U_{i2})_{21}(U_{i2}^T)_{11} I_{i11} \right].
\]

(8)
In the above expressions, the definitions
\[
\tan \beta = \frac{v_U}{v_D}, \quad \bar{v}^2 = \frac{v_U^2 + v_D^2}{2}
\]
are adopted, with \(v_U\) and \(v_D\) the vacuum expectation values of the neutral components of the Higgs doublet with hypercharge \(+1/2\) and \(-1/2\). All phases for the supersymmetric parameters are assumed to be vanishing. Thus, the matrix \(U_t\) is the \(2 \times 2\) orthogonal diagonalization matrix of the matrix for the \(t\)-squark mass squared, and a summation over the index \(i\), identifying the two \(t\) eigenvalues, \(t_1\) and \(t_2\), is understood. Our conventions for the squark sectors are listed in Appendix C. We have also assumed that the left-right mixing in the matrix for the \(\tilde{s}\)-squark mass squared is nearly vanishing and therefore only one \(\tilde{s}\)-squark eigenstate, the left-handed one, denoted simply by \(\tilde{s}\), is needed for this calculation. \(Q_3\) and \(Q_1\) indicate the electric charges of the \(t\) and the \(\tilde{s}\) squarks. \(C_F = 4/3\) is a color factor. Finally, the scalar integrals \(I_{ti1}, I_{ti2}, I_{Hi1}, I_{Hi2}, I_{tii1}, I_{tii2}, I_{sii1},\) and \(I_{sii2}\) are listed in integral form in Appendix C. The explicit expression of the integrals introduced in this section can be obtained upon request as a FORTRAN code.

Similarly, the contributions to \(\Delta C_{8,H}^1(\mu_W)\) are
\[
\begin{align*}
\Delta C_{8,H}^{1,a}(\mu_W) &= \frac{1}{2} C_F \left( \alpha_s \tan \beta \right) m_t \mu \left[ (U_t)_2 (U_t^T)_1 m_{t_1} m_{t_2} (U_t^T)_{i1} I_{tii1} - (U_t)_{2i} (U_t^T)_{i1} I_{tii1} \right], \\
\Delta C_{8,H}^{1,b}(\mu_W) &= \frac{1}{2} C_F \left( 1 - \frac{C_V}{2 C_F} \right) \left( \alpha_s \tan \beta \right) m_t \mu \left[ (U_t)_2 (U_t^T)_1 m_{t_2} m_{t_1} I_{tii2} - (U_t)_{2i} (U_t^T)_{i1} I_{tii1} \right], \\
\Delta C_{8,H}^{1,c}(\mu_W) &= \frac{1}{2} C_F \left( 1 - \frac{C_V}{2 C_F} \right) \left( \alpha_s \tan \beta \right) m_t \mu \left[ (U_t)_2 (U_t^T)_1 m_{t_1} m_{t_2} I_{sii2} - (U_t)_{2i} (U_t^T)_{i1} I_{sii1} \right], \\
\Delta C_{8,H}^{1,d}(\mu_W) &= \frac{1}{2} C_F \left( 1 - \frac{C_V}{2 C_F} \right) \left( \alpha_s \tan \beta \right) m_t \mu \left[ (U_t)_2 (U_t^T)_1 m_{t_2} m_{t_1} I_{sii1} - (U_t)_{2i} (U_t^T)_{i1} I_{sii1} \right],
\end{align*}
\]
where, obviously, \(\Delta C_{8,H}^{1,b}(\mu_W) = 0\). Here, \(C_V = 3\) is another SU(3) group factor and the integrals \(I_{tii1}, I_{tii2}\) are also defined in Appendix C. The explicit expression of the integrals introduced in this section can be obtained upon request as a FORTRAN code.

The results shown in Refs. [10, 11] for the BR(\(\bar{B} \to X_s \gamma\)) are obtained using only the first term in the square brackets of \(\Delta C_{8,H}^1(\mu_W), \Delta C_{7,H}^1(\mu_W),\) and \(\Delta C_{8,H}^1(\mu_W),\) with the approximations for \(I_{tii2}\) and \(I_{Hi2}\) listed in Appendix C. As anticipated in the Introduction, we call the approximation of these references the nondecoupling approximation. It is indeed of nondecoupling type in the limit of heavy supersymmetric particles and collects all terms of \(O((m_{\tilde{s}}^2; m_{\tilde{t}}^2; M_{\tilde{B}}^2; M_{\tilde{T}}^2; M_{\tilde{B}}^2)^n)\) with \(n > 0\), i.e., terms that decouple in the limit \(M_{\tilde{B}} \to \infty\), were neglected except for those trivially accounted for by the couplings of squarks and their masses \(18\).

### III. EFFECTIVE LAGRANGIAN AND HEAVY MASS EXPANSION

The calculation of the two-loop diagrams discussed in the previous section was performed in Refs. [10, 11] under the assumption of heavy squarks and gluino, at the scale \(M_{\text{SUSY}}\), and a light charged Higgs boson, of \(O(m_{\text{weak}})\) \(m_{\text{weak}} \sim M_W, m_A\), with \(m_{\text{weak}} \ll M_{\text{SUSY}}^2\). In these calculations, terms of \(O((m_{\text{weak}}; M_{\text{SUSY}}^2)^n)\) with \(n > 0\), i.e., terms that decouple in the limit \(M_{\text{SUSY}} \to \infty\), were neglected except for those trivially accounted for by the couplings of squarks and their masses \(18\).

The amplitude of the diagrams in Fig. 2 can be expanded in \(O((m_{\text{weak}}; M_{\text{SUSY}}^2)^n)\). It seems plausible that by retaining in such an expansion terms up to \(O((m_{\text{weak}}; M_{\text{SUSY}}^2)^n)\) with \(n \geq 1\), may allow one to extend the results obtained to values of \(M_{\text{SUSY}}\) not too dissimilar from \(M_{\text{SUSY}}\), as well as to a not too large value for \(M_{\text{SUSY}}\). In this spirit, it seems worth performing these approximate calculations, in the hope that the results turn out to be still relatively compact.

A systematic way to include all the needed terms at \(O((m_{\text{weak}}; M_{\text{SUSY}}^2)^n)\) is provided by the heavy mass expansion technique \(20\). Before proceeding further, we recall here some basic properties of this expansion.

Let us assume that all masses of a given Feynman diagram \(\Gamma\) can be divided into a set of large \(M = \{M_1, M_2, \ldots\}\) and small \(m = \{m_1, m_2, \ldots\}\) masses. If all external momenta \(q_i = \{q_1, q_2, \ldots\}\) are small compared to the scale of the large masses \(M\), then the dimensionally regularized (unrenormalized) Feynman integral \(F_{\Gamma}\) associated with the Feynman diagram \(\Gamma\) can be decomposed as
\[
F_{\Gamma} \overset{\text{\(M \to \infty\)}}{\sim} \sum_{\gamma} F_{\Gamma / \gamma} \circ T_{\gamma} \circ m \cdot F_{\gamma} (q^\gamma, m^\gamma, M),
\]
(11)
where the sum is performed over all subdiagrams $\gamma$ of $\Gamma$ which (i) contain all lines with heavy masses ($M$), and (ii) are one-particle irreducible with respect to lines with small masses ($m$). The case $\gamma = \Gamma$ is always included in the sum $\sum_{\gamma}$. For each $\gamma$, $m_{\gamma}$ denotes the set of light masses, $q_{\gamma}$ the set of all external momenta with respect to the subdiagram $\gamma$, which can be internal momenta with respect to the full diagram $\Gamma$. The operator $\mathcal{T}$ performs a Taylor expansion in the variables $q_{\gamma}$ and $m_{\gamma}$ and it is understood to act directly on the integrand of the subdiagram $\gamma$. The diagram $\Gamma/\gamma$ is obtained by reducing $\gamma$ to a vertex in $\Gamma$. Thus, by factorizing the product of scalar propagators of the original, say, $l$-loop diagram $\Gamma$ as $\Pi_{\Gamma} = \Pi_{\Gamma/\gamma} \Pi_{\gamma}$, the decomposition of the original Feynman integral $F_{\Gamma}$ is simply

$$F_{\Gamma/\gamma} \circ T_{q_{\gamma}, m_{\gamma}} F_{\gamma} = \int dk_1 \cdots dk_l \Pi_{\Gamma/\gamma} T_{q_{\gamma}, m_{\gamma}} \Pi_{\gamma}.$$ 

(12)

Note that the Taylor operator $\mathcal{T}$ introduces additional spurious IR or UV divergences in the various terms of the sum $\sum_{\gamma}$, which cancel in the sum.

For the calculation of the diagrams in Fig. 2, we start assuming that

$$m_{\text{weak}}^2 \sim m_{\text{weak}}^2 \ll M_{\text{SUSY}}^2,$$

(13)

where $m_{\text{weak}}$ is, in turn, $\gg m_b$. In this limit, the diagram (b) in Fig. 2 with chirality flip on the $t$-quark line has a Feynman integral that can be decomposed into two terms contributing to the sum $\sum_{\gamma}$ of Eq. (11). In the first term, $\gamma_1$ is the upper right diagram of Fig. 3 and the corresponding Feynman integral must be expanded in the momenta of the external particles, of the charged Higgs boson $H^\pm$, and of the $t$ and $s$ quarks; $\Gamma/\gamma_1$ is the diagram on the upper left of the same figure. Although the calculation is actually done in the mass eigenbasis for squarks and Higgs bosons, the diagrams in this figure (as well as those in Figs. 4 and 5) are shown in the gauge eigenbasis. In an effective two-Higgs-doublet Lagrangian, obtained after integrating out all heavy degrees of freedom, i.e., squarks and gluino, the zeroth-order expansion of $\gamma_1$ is described by the term in the Lagrangian

$$\mathcal{L}_{\text{eff}}^{\gamma_1} = -V_{ts} \frac{m_t (\text{SM})}{v \sin \beta} \Delta_{tR,s} H^0_{D_{L}} H^0_{R} \sin \beta + (\text{H.c.}),$$

(14)

where, again, we follow the notation of Ref. [7]. and the coefficient $\Delta_{tR,s}$ is

$$\Delta_{tR,s} = \frac{C_F \alpha_s}{2\pi} \mu m_{\gamma}^3 (U_{t1})_2 (U_{t1}^T)_3 I (m_{\gamma}^2, m_{\gamma}^2, m_{\gamma}^2).$$

(15)

The function $I$ is the same as in Eq. (6). This zeroth order corresponds to the nondecoupling approximation of Refs. [10, 11] and gives rise to the following contribution to the Wilson coefficient $C_{7,H}(\mu W)$:

$$\Delta C_{7,H}(\mu W)_{\text{nondec}} = -\frac{1}{2} Q_H C_F (\alpha_s \tan \beta) \mu m_{\gamma} \frac{1}{2\pi} (U_{t1})_2 (U_{t1}^T)_3 I (m_{\gamma}^2, m_{\gamma}^2, m_{\gamma}^2) \frac{m_{\gamma}^2}{M_{H^\pm}} F_4 \left( \frac{m_{\gamma}^2}{M_{H^\pm}} \right)$$

$$= -\frac{1}{2} Q_H \tan \beta \Delta_{tR,s} \frac{m_{\gamma}^2}{M_{H^\pm}} F_4 \left( \frac{m_{\gamma}^2}{M_{H^\pm}} \right).$$

(16)
The second term contributing to the sum $\sum_{\gamma_1, \gamma_2}$ is the full diagram, and its Feynman integral is to be expanded in $m_t, m_{H^\pm}, m_b$, and the external momenta; $\Gamma/\gamma_2$ is trivially, the identity. Again, in an effective two-Higgs-doublet Lagrangian, the operator describing $\gamma_2$ is the very operator $O_7$ in Eq. (2). This gives rise to a vanishing zeroth-order term in the expansion of light masses, i.e., a vanishing nondecoupling contribution, and was, therefore, not included in the analyses of Refs. [10, 11].

The case in which the chirality flip is on the $\tilde{t}$-squark line instead of on the $t$-quark line can be treated in a similar way. In this case, the term $\gamma_1$, shown by the lower right diagram of Fig. 3, is already of decoupling type and was also not considered in Refs. [10, 11].

The procedure to be followed is similar in the case in which the photon is attached to the $t$-quark line. When the chirality flip is on the $t$-quark line, the diagram $\gamma_1$ is again the upper right diagram of Fig. 3 which at the zeroth-order expansion gives rise to the same term of the effective two-Higgs-doublet Lagrangian given in Eq. (14). The diagram $\Gamma/\gamma_1$ is analogous to the upper left diagram of Fig. 3. The corresponding contributions to the Wilson coefficient $C_7(\mu_W)$ and to $C_8(\mu_W)$, when the photon is substituted by a gluon, are

$$\Delta C_{7, H}(\mu_W)|_{\text{nondec}} = \frac{1}{2} Q_t \tan \beta \Delta t_{R,s} \frac{m_t^2}{M_{H^\pm}^2} F_3 \left( \frac{m_t^2}{M_{H^\pm}^2} \right),$$

$$\Delta C_{8, H}(\mu_W)|_{\text{nondec}} = \frac{1}{2} \tan \beta \Delta t_{R,s} \frac{m_t^2}{M_{H^\pm}^2} F_3 \left( \frac{m_t^2}{M_{H^\pm}^2} \right).$$

Again, the contributions from the second diagram $\gamma_2$ and the diagrams obtained when the chirality flip is on the $\tilde{t}$-squark line are of decoupling type. By comparing Eqs. (16), (17), and (18) with the $\tan \beta$-unsuppressed one-loop
contributions $C^0_{i,H}(\mu_W)$ in Eqs. 6 and 7, the relation

$$\Delta C^1_{i,H}(\mu_W)|_{\text{nondec}} = -\tan \beta \Delta t_{i,s} C^0_{i,H}(\mu_W)$$

follows, which shows clearly that $\Delta C^1_{i,H}(\mu_W)$ has the same $m_t$ and $m_{H^\pm}$ dependence as that of $C^0_{i,H}(\mu_W)$.

Slightly different is the case in which the photon is emitted by one of the two squarks $\tilde{t}, \tilde{s}$. In the case of emission from the $\tilde{s}$ squark, for example, the decomposition of the Feynman integral has, again, a term corresponding to the full diagram $F_{\gamma s}$, to be expanded in $m_t, m_{H^\pm}, m_b$, and the external momenta. The other term is given by the convolution of the Feynman integral $F_{\gamma t}$ corresponding to the upper or lower box diagrams $\gamma_1$ shown on the right of Fig. 4 (depending on where the chirality is flipped, i.e., on the $t$ quark or $\tilde{t}$ squark), expanded in the momenta of $H^\pm$, of the $t$ and the $s$ quarks and the momentum of the photon, and the Feynman integral $F_{\gamma/\gamma_1}$ of the upper or lower diagrams $\Gamma/\gamma_1$ on the left side of the same figure.

When it is a gluon to be radiated off during the interactions that lead the $b$ quark into an $s$ quark, there is the additional possibility of having the gluon emitted from the $\tilde{g}$ line. The decompositions in $\gamma_1$ and $\Gamma/\gamma_1$ of the corresponding diagrams are explicitly shown in Fig. 5.

The results of the calculation in the regime of Eq. 13 turn out to be rather involved, already at $O(m^2_{\text{weak}}, m^2_{H^\pm}/M^2_{\text{SUSY}})$ and $O((m^2_{\text{weak}}, m^2_{H^\pm}/M^2_{\text{SUSY}})^2)$. We report them only numerically in the next section, together with the numerical results of the exact calculation of Sec. III.

IV. $H^*$ CONTRIBUTION TO $C_{7,8}(\mu_W)$ UP TO $O(\alpha_s \tan \beta)$

We present here numerical results for the charged-Higgs-boson contributions to the Wilson coefficients $C_7$ and $C_8$ at the scale $\mu_W = M_W$, i.e., $C_{7,8}(\mu_W)$, including the $O(\alpha_s \tan \beta)$ corrections discussed in the previous sections. We make a comparison of the exact results for $C_{7,8}(\mu_W)$, which we denote by $C_{7,8}(\mu_W)|_{\text{exact}}$, and $C_{7,8}(\mu_W)|_{\text{approx}}$, obtained from Eqs. 8 and 10, with several approximate forms, $C_{7,8}(\mu_W)|_{\text{approx}}$, the nondecoupling approximation, and the two approximations in which the first- and second-order terms in the expansion of the exact coefficients in $(m^2_{\text{weak}}, m^2_{H^\pm}/M^2_{\text{SUSY}})$ are retained.

In Figs. 9 and 10 we plot the ratios

$$r_i(\mu_W) = \frac{C_{i,H}(\mu_W)|_{\text{approx}} - C_{i,H}(\mu_W)|_{\text{exact}}}{C_{i,H}(\mu_W)|_{\text{exact}}} \quad (i = 7, 8),$$

in which the $O(\alpha_s \tan \beta)$ corrections to the $b$-quark mass cancel out, showing the goodness of each approximation as a function of $m_{H^\pm}$. We denote these ratios for the nondecoupling approximation and those with an expansion in the first and second order in $(m^2_{\text{weak}}, m^2_{H^\pm}/M^2_{\text{SUSY}})$ by dotted, dashed, and dot-dashed lines, respectively.

Two sets of parameters for supersymmetric particles are used. For Fig. 6 we have chosen a superpartner spectrum, called here spectrum I, with $(m_{\tilde{e}_L}, m_{\tilde{Q}}, m_{\tilde{e}_R}, m_{\tilde{\tau}_R}) = (700, 450, 435, 470)$ GeV, $A_t = 150$ GeV, $m_{\tilde{g}} = 600$ GeV, and $\mu = 550$ GeV. For Fig. 7 a lighter spectrum is considered: $(m_{\tilde{e}_L}, m_{\tilde{Q}}, m_{\tilde{e}_R}, m_{\tilde{\tau}_R}) = (250, 230, 210, 260)$ GeV, $A_t = 70$ GeV, $m_{\tilde{g}} = 200$ GeV, and $\mu = 250$ GeV. This is denoted as spectrum II. As for other input parameters, we have used $\tan \beta = 30$, $m_t(\mu_W) = 176.5$ GeV, which corresponds to a pole mass $M_t = 175$ GeV, $m_b(\mu_W) = 3$ GeV, $A_b = 0$, $\alpha_s(\mu_W) = 0.12$, $M_Z = 91.2$ GeV, and $\sin^2 \theta_W = 0.23$.

For spectrum I, the difference between the exact calculation and the nondecoupling approximation is very small in the whole range of $m_{H^\pm}$, even for $m_{H^\pm} \gtrsim M_{\text{SUSY}}$. This is an unexpected result since, as discussed in Sec. III, the nondecoupling approximation is, in principle, theoretically justified only for $m^2_{H^\pm} \ll M^2_{\text{SUSY}}$. In the case of spectrum II, $r_{7,8}$ become larger for the nondecoupling approximation. The corrections beyond this approximation are of the same order as the SU(2)$\times$U(1) breaking effects in the supersymmetric particle subloops IIR and are no longer negligible. Nevertheless, $r_7$ and $r_8$ for the nondecoupling approximation remain of the same order of magnitude for increasing $m_{H^\pm}$, up to $m_{H^\pm} \gg M_{\text{SUSY}}$.

In both cases, the bulk of the difference between the results of the exact and nondecoupling calculations come from $I_{i,1}$ in $\Delta C^1_{7,H}$ and $\Delta C^1_{8,H}$, and also from $I_{i,2}$ in $\Delta C^1_{8,H}$, of Eqs. 8 and 10. The inclusion of the $(m^2_{\text{weak}}, m^2_{H^\pm}/M^2_{\text{SUSY}})$ terms by the HME, as described in Sec. III, improves the goodness of $r_i$ of the nondecoupling approximation when $m_{H^\pm} < M_{\text{SUSY}}$, but worsens it when $m_{H^\pm} \gtrsim M_{\text{SUSY}}$. This is clearly seen in Fig. 6.

In Fig. 8 we show $C_{7,H}(\mu_W)$ and $C_{8,H}(\mu_W)$ for the nondecoupling approximation and exact two-loop results, as well as the one-loop results $C^0_{7,H}(\mu_W)$, for spectrum II. The $O(\alpha_s \tan \beta)$ corrections are comparable to the one-loop
FIG. 6: Ratios $r_{7,8}(\mu_W)$ for various approximations of the $O(\alpha_s \tan \beta)$ corrections, as defined in the text, for spectrum I. The dotted, dashed, and dot-dashed lines show the nondecoupling approximation, first-order expansion, and second-order expansion in $(m_t^2, m_{H^\pm}^2)/M_{\text{SUSY}}^2$, respectively.

FIG. 7: $r_{7,8}(\mu_W)$ for spectrum II. Notation is the same as in Fig. 6.

contributions. The deviation of the exact calculation from the nondecoupling approximation is small, but nonnegligible for $C_{8,H}$. It is also shown that all three results have a similar dependence on $m_{H^\pm}$.

To understand the results for $m_{H^\pm} \gtrsim M_{\text{SUSY}}$ qualitatively, we focus on the diagram in Fig. 2(a), with chirality flip on the $t$-quark line. When $m_{H^\pm}$ is sufficiently larger than $m_t$, the contribution of this diagram to $\Delta C_{7,H}^1(\mu_W)$ and $\Delta C_{8,H}^1(\mu_W)$ is among the largest. Analytically, it is proportional to $\mu m_{\tilde{s}} I_{t_2}$, with the integral $I_{t_2}$ listed in Eq. (C3) of Appendix C. For the following discussion it is convenient to write $\mu m_{\tilde{s}} I_{t_2}$ in the form

$$\mu m_{\tilde{s}} I_{t_2}(m_t, m_{H^\pm}, m_{\tilde{t}}, m_{\tilde{s}}, m_{\tilde{g}}) = \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{[k^2 - m_{\tilde{t}}^2]^3} Y_{t_2} \left( k^2; m_{\tilde{t}}, m_{\tilde{s}}, m_{\tilde{g}} \right), \quad (21)$$

where $Y_{t_2}(k^2; m_{\tilde{t}}, m_{\tilde{s}}, m_{\tilde{g}})$ represents the subdiagram contribution to the vertex $H^- \tilde{s}_{LR}$ and is given by

$$Y_{t_2}(k^2; m_{\tilde{t}}, m_{\tilde{s}}, m_{\tilde{g}}) = \mu m_{\tilde{g}} \left[ -2F + (k^2 - m_{\tilde{t}}^2)G \right] \left( k^2; m_{\tilde{t}}^2, m_{\tilde{s}}^2, m_{\tilde{g}}^2 \right), \quad (22)$$
For \( y_k \) which is a constant with respect to with approximation, and exact two-loop result, respectively.

\[
F(k^2; m_{t_i}^2, m_{s}^2, m_{g}^2) = \int \frac{d^4l}{(2\pi)^4} \frac{1}{[l^2 - m_{t_i}^2] [I^2 - m_{g}^2]},
\]

\[
k^\mu G(k^2; m_{t_i}^2, m_{s}^2, m_{g}^2) = \int \frac{d^4l}{(2\pi)^4} \frac{l^\mu}{[l^2 - m_{t_i}^2] [I^2 - m_{g}^2]^2}.
\]

The nondecoupling approximation \( \mu m_{s} I_{t2}|_{\text{nondec}} \) of Eq. (21), where the integral \( I_{t2}|_{\text{nondec}} \) is given in Eq. (23), is obtained by approximating \( Y_{t2}(k^2; m_{t_i}, m_{s}, m_{g}) \) by

\[
Y_{t2}|_{\text{nondec}} = -2\mu m_{s} F(0; m_{t_i}^2, m_{s}^2, m_{g}^2),
\]

which is a constant with respect to \( k^2 \). Note the relation \( F(0; m_{t_i}^2, m_{s}^2, m_{g}^2) = (-i/16\pi^2)I(m_{t_i}^2, m_{s}^2, m_{g}^2) \), where \( I \) is the function defined in Eq. (24). To simplify our discussion of the behavior of these functions, we hereafter set \( m_{t_i}, \ m_{s}, \ m_{g}, \) and \( \mu \) equal to \( M_{\text{SUSY}} \).

For \( |k^2| \ll M_{\text{SUSY}}^2 \), \( F(k^2; M_{\text{SUSY}}^2) \) and \( G(k^2; M_{\text{SUSY}}^2) \) behave as

\[
F(k^2; M_{\text{SUSY}}^2) = O(\frac{1}{M_{\text{SUSY}}^2}) + O(\frac{k^2}{M_{\text{SUSY}}^2}),
\]

\[
G(k^2; M_{\text{SUSY}}^2) = O(\frac{1}{M_{\text{SUSY}}^2}).
\]

For \( |k^2| \gg M_{\text{SUSY}}^2 \), the behavior is

\[
F(k^2; M_{\text{SUSY}}^2) \to O(\frac{1}{k^2 \ln k^2 / M_{\text{SUSY}}^2}),
\]

\[
G(k^2; M_{\text{SUSY}}^2) \to O(\frac{1}{k^4 \ln k^2 / M_{\text{SUSY}}^2}).
\]

The behavior of \( Y_{t2}(k^2; M_{\text{SUSY}}^2) \) is therefore

\[
Y_{t2}(k^2; M_{\text{SUSY}}^2) \to \begin{cases} 
Y_{t2}|_{\text{nondec}} + O(\frac{k^2}{M_{\text{SUSY}}^2}, \frac{m_{t_i}^2}{M_{\text{SUSY}}^2}) & (|k^2| \ll M_{\text{SUSY}}^2), \\
O(\frac{M_{\text{SUSY}}^2}{k^2} \ln \frac{k^2}{M_{\text{SUSY}}^2}) & (|k^2| \gg M_{\text{SUSY}}^2),
\end{cases}
\]

FIG. 8: \( C_{7,H}(\mu_W) \) and \( C_{8,H}(\mu_W) \) for spectrum II. The dotted, dashed, and solid lines show the one-loop result, nondecoupling approximation, and exact two-loop result, respectively.
which supports the naive expectation that a substantial deviation of $I_{122}(m_t, m_{H^\pm}, M_{\text{SUSY}}^2)$ from $I_{122}(m_t, m_{H^\pm}, M_{\text{SUSY}}^2)_{\text{nondec}}$ may arise from the region $|k^2| > M_{\text{SUSY}}^2$.

The factor multiplying $Y_{122}(k^2; M_{\text{SUSY}}^2)$ in Eqs. (24), however, plays a rather important role, leading to the fact that this expectation does not hold in the case in which $M_{\text{SUSY}}$ is not rather light. Since for $|k^2| > m_{H^\pm}^2$ this factor drops as $d^4k/k^3$, the integral (24) gets its largest contribution from the region $|k^2| \approx m_{H^\pm}^2$. A closer inspection actually shows that it is the region of small $|k^2|$, up to $|k^2| = O(m_t^2)$, which determines the bulk of the value of this integral. If $m_t$ is sufficiently smaller than $M_{\text{SUSY}}$, $Y_{122}(k^2; M_{\text{SUSY}}^2)$ does not deviate substantially from $Y_{122(0)}_{\text{nondec}}$. The region in $k^2$ where the deviation $Y_{122}(k^2; M_{\text{SUSY}}^2) - Y_{122(0)}_{\text{nondec}}$ is largest, i.e., $|k^2| \sim M_{\text{SUSY}}^2$, is weighted by a rather efficient suppression factor in Eq. (24) if $m_t^2 \ll m_{H^\pm}^2 \ll M_{\text{SUSY}}$.

It is clear that an expansion of the integral $I_{122}(m_t, m_{H^\pm}, M_{\text{SUSY}}^2)$ in $m_t^2/M_{\text{SUSY}}^2$ and $m_{H^\pm}^2/M_{\text{SUSY}}^2$, as obtained from the HME, generates terms $O(m_t^2)$ at the first order, and terms $O(m_{H^\pm}^2)$ at the second order. These terms contribute to give a better approximation of the exact results for $C_{1,2}(\mu_W)$, when $m_{H^\pm}^2 < M_{\text{SUSY}}^2$, but have a dependence on $m_{H^\pm}$ that is rather different from that of the exact results, when $m_{H^\pm}^2 \gg M_{\text{SUSY}}^2$. As already observed, the $m_{H^\pm}$ dependence of the exact results for $C_{1,2}(\mu_W)$ at order $O(\alpha_s \tan \beta)$ is similar to that of their nondecoupling approximation, which is, in turn, identical to that of the one-loop results $C_{1,2}^0(\mu_W)$ (i.e., the results at LO in QCD) for the same Wilson coefficients. As for the series expansion obtained through the HME, the qualitative discussion sketched above and the limited number of terms we have calculated do not allow us to conclude whether it is convergent or not. Even if convergent, however, the numerical results we have plotted in the previous figures show clearly that it is not converging fast enough to be of any practical use in the region $m_{H^\pm}^2 \gg M_{\text{SUSY}}^2$, as was originally hoped.

V. CONCLUSION

In this paper we have evaluated the $O(\alpha_s \tan \beta)$ corrections to the charged-Higgs-boson mediated contributions to the Wilson coefficients relevant for the decay $B \to X_s \gamma$, in supersymmetric models with large $\tan \beta$. These corrections are generated by the shift of the $b$-quark mass in the Higgs-boson–quark couplings and by the dressing of the one-loop charged-Higgs-boson diagrams with squark-gluino subloops (see Fig. 2). The former corrections are very well known. In previous studies (10, 11), the contributions from these two-loop diagrams were calculated using an effective two-Higgs-doublet Lagrangian formalism where the squarks and gluino are integrated out. This method is theoretically justified in the limit in which the charged Higgs boson is light, i.e., at the electroweak scale $\sim m_{\text{weak}}$, whereas the squark and gluino masses, $\sim M_{\text{SUSY}}$, are rather large. The resulting approximation, in which only all the nondecoupling terms in the large $M_{\text{SUSY}}$ limit are retained, has clearly the nontrivial advantage of providing a rather compact result for these corrections. However, it is in general expected that its deviation from the exact two-loop result, of $O(m_{\text{weak}}^2, m_{H^\pm}^2/M_{\text{SUSY}}^2)$, becomes significant for $m_{\text{weak}} \sim M_{\text{SUSY}}$ and/or $m_{H^\pm} \gg M_{\text{SUSY}}$.

We have calculated the contributions of the two-loop diagrams in Fig. 2 exactly, without assuming any patterns for the masses of the particle involved. By making use of the heavy mass expansion technique, we have also evaluated two additional approximate forms for the same diagrams, including all terms up to $O(m_{\text{weak}}^2, m_{H^\pm}^2/M_{\text{SUSY}}^2)$ in one case, and all terms up to $O((m_{\text{weak}}^2, m_{H^\pm}^2/M_{\text{SUSY}}^2)^2)$ in the other. The exact calculation, not confined to specific values of the masses involved, has allowed us to establish the goodness of all three approximations.

Our findings can be summarized as follows. Surprisingly, the results of Refs. (10, 11) approximate the exact two-loop result quite adequately, irrespective of the value of $m_{H^\pm}$ relative to $M_{\text{SUSY}}$, provided $M_{\text{SUSY}}$ is large enough. The unexpected absence of a large deviation for the case of $m_{H^\pm} \gg M_{\text{SUSY}}$ with $m_{\text{weak}} \ll M_{\text{SUSY}}$ can be understood from the structure of the two-loop integrals. This points to the fact that the only relevant hierarchy in this problem is that between $m_{\text{weak}}$ and $M_{\text{SUSY}}$, whereas, for $m_{H^\pm} > m_{\text{weak}}$, the value of $m_{H^\pm}$ with respect to that of $M_{\text{SUSY}}$ plays a rather marginal role. Therefore, deviations between the exact result and that of the nondecoupling approximation can be found only for $M_{\text{SUSY}}$ not much larger than $m_{\text{weak}}$. The inclusion of the higher-order terms $O(m_{\text{weak}}^2, m_{H^\pm}^2/M_{\text{SUSY}}^2)$ and $O((m_{\text{weak}}^2, m_{H^\pm}^2/M_{\text{SUSY}}^2)^2)$ improves the approximation of Refs. (10, 11) for $m_{H^\pm}^2 \ll M_{\text{SUSY}}^2$, as expected, but makes it worse for $m_{H^\pm}^2 \gg M_{\text{SUSY}}^2$. This behavior is attributed to the fact that these higher-order terms in the $(m_{\text{weak}}^2, m_{H^\pm}^2/M_{\text{SUSY}}^2)$ expansion have a dependence on $m_{H^\pm}$ rather different from that of the lowest-order terms in this expansion (the terms of the nondecoupling approximation) and that of the exact two-loop result.

We have illustrated our findings by showing the values of the Wilson coefficients $C_7$ and $C_8$ at the electroweak matching scale $\mu_W$, for different gluino, squark, and charged Higgs boson spectra. We postpone a presentation of the
same coefficients $C_7$ and $C_8$ at a low scale $\sim m_b$ and of the actual branching ratio $\text{BR}(\bar{B} \to X_s \gamma)$ to future work \cite{22}.

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**APPENDIX A**

We report here the functions $F_3(x)$ and $F_4(x)$ introduced in the second paper of Ref. \cite{21}:

\[
F_3(x) = \frac{1}{2(x-1)^2} (x^2 - 4x + 3 + 2 \log x),
\]
\[
F_4(x) = \frac{1}{2(x-1)^2} (x^2 - 1 - 2x \log x).
\]

**APPENDIX B**

We have adopted the following conventions for squark mass eigenstates $\tilde{q}_1, \tilde{q}_2$ and mass eigenvalues $m_{\tilde{q}_1}^2, m_{\tilde{q}_2}^2$:

\[
\begin{pmatrix}
\hat{q}_L \\
\hat{q}_R
\end{pmatrix} = U_q \begin{pmatrix}
\tilde{q}_1 \\
\tilde{q}_2
\end{pmatrix},
\]

where the diagonalization matrix $U_q$ is such that

\[
U_q^T M_q^2 U_q \equiv \hat{M}_q^2 = \begin{pmatrix}
m_{\tilde{q}_1}^2 & 0 \\
0 & m_{\tilde{q}_2}^2
\end{pmatrix}.
\]

$M_q^2$ is the mass squared matrix for the squark $\tilde{q}$ and $\hat{M}_q^2$ is the diagonalized squark mass squared. For $\tilde{q} = \tilde{b}$ and $\tilde{t}$, the matrix $M_q^2$ is, respectively,

\[
M_{\tilde{b}}^2 = \begin{pmatrix}
m_{\tilde{Q}}^2 + m_{\tilde{b}}^2 + D_L^b & m_b(A_b - \mu \tan \beta) \\
m_b(A_b - \mu \tan \beta) & m_{\tilde{b}}^2 + D_R^b
\end{pmatrix},
\]

and

\[
M_{\tilde{t}}^2 = \begin{pmatrix}
m_{\tilde{Q}}^2 + m_{\tilde{t}}^2 + D_L^t & m_t(A_t - \mu \cot \beta) \\
m_t(A_t - \mu \cot \beta) & m_{\tilde{t}}^2 + D_R^t
\end{pmatrix},
\]

with $D_{L,R}^b$ and $D_{L,R}^t$ given by

\[
D_L^b = \cos 2\beta M_Z^2 \left( + \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right), \quad D_R^b = \cos 2\beta M_Z^2 \left( + \frac{2}{3} \sin^2 \theta_W \right),
\]
\[
D_L^t = \cos 2\beta M_Z^2 \left( - \frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \right), \quad D_R^b = \cos 2\beta M_Z^2 \left( - \frac{1}{3} \sin^2 \theta_W \right).
\]

**APPENDIX C**

After defining the following product of propagators:

\[
D = \frac{1}{[k^2 - m^2_{\ell}][k^2 - m^2_{H^\pm}]} \left[ (l + k)^2 - m^2_{\ell} \right] \left[ l^2 - m^2_{\tilde{q}} \right] \left[ l^2 - m^2_{\bar{q}} \right],
\]

\[
(C1)
\]
the integrals in Eqs. (8) and (10) can be cast in the form

\[
I_{i1} = \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \frac{D}{[k^2 - m_i^2]} \left\{ \frac{2m_i^2(l \cdot k)}{k^2 - m_i^2} - \frac{14(l \cdot k)^2 - l^2k^2}{3} \right\},
\]

\[
I_{i2} = \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \frac{D}{[k^2 - m_i^2]} \left\{ \frac{l \cdot k}{l^2 - m_i^2} - \frac{2k^2}{k^2 - m_i^2} \right\},
\]

for the case in which a photon/gluon is emitted by the \( t \) quark;

\[
I_{Hi1} = \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \frac{D}{[k^2 - m_{H_i}^2]} \left\{ \frac{4(l \cdot k)^2 - l^2k^2 + 3k^2(l \cdot k)}{k^2 - m_i^2} - \frac{4(l \cdot k)^2 - l^2k^2 + 3l^2(l \cdot k)}{l^2 - m_g^2} \right\},
\]

\[
I_{Hi2} = \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \frac{D}{[k^2 - m_{H_i}^2]} \left\{ \frac{l \cdot (l + k)}{l^2 - m_i^2} - \frac{k \cdot (l + k)}{k^2 - m_i^2} \right\}
\]

for the case of a photon emitted by the charged Higgs boson;

\[
I_{i1} = \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \frac{1}{[(l + k)^2 - m_{H_i}^2]} \left\{ \frac{4(l \cdot k)^2 - l^2k^2 + 3k^2(l \cdot k)}{k^2 - m_i^2} - \frac{4(l \cdot k)^2 - l^2k^2 + 3l^2(l \cdot k)}{l^2 - m_g^2} \right\},
\]

\[
I_{i2} = \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \frac{D}{[(l + k)^2 - m_{H_i}^2]} \left\{ \frac{l \cdot (l + k)}{l^2 - m_i^2} - \frac{k \cdot (l + k)}{k^2 - m_i^2} \right\}
\]

for the case of a photon/gluon emitted by the \( \tilde{t} \) squark;

\[
I_{s11} = \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \frac{D}{[l^2 - m_s^2]} \left\{ \frac{4(l \cdot k)^2 - l^2k^2 + 3k^2(l \cdot k)}{k^2 - m_i^2} - \frac{4(l \cdot k)^2 - l^2k^2 + 3l^2(l \cdot k)}{l^2 - m_g^2} \right\},
\]

\[
I_{s12} = \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \frac{D}{[l^2 - m_s^2]} \left\{ \frac{l^2}{l^2 - m_i^2} - \frac{l \cdot k}{k^2 - m_i^2} \right\}
\]

for the case of a photon/gluon emitted by the \( \tilde{s} \) squark;

\[
I_{g1} = \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \frac{D}{[l^2 - m_g^2]} \left\{ \frac{2m_g^2(l \cdot k)}{l^2 - m_g^2} - \frac{14(l \cdot k)^2 - l^2k^2}{3} \right\},
\]

\[
I_{g2} = \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \frac{D}{[l^2 - m_g^2]} \left\{ \frac{l \cdot k}{k^2 - m_i^2} - \frac{2l^2}{l^2 - m_g^2} \right\}
\]

for the case of a gluon emitted by the gluino \( \tilde{g} \).

These integrals were obtained from those corresponding to the diagrams (a)–(e) in Fig. 2 after an expansion on the external momenta and a reduction of all tensorial structures to scalar ones. They can be further reduced to linear combinations of the two-loop vacuum integrals

\[
G(m_1, p_1; m_2, p_2; m_3, p_3) = \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \frac{1}{[(l + k)^2 - m_1^2]^{p_1} [l^2 - m_2^2]^{p_2} [k^2 - m_3^2]^{p_3}},
\]

whose general solution is explicitly reported in Ref. 23.

In the nondecoupling approximation of Refs. 10, 11, only the integrals \( I_{i12} \) and \( I_{Hi2} \) are needed. They are evaluated after substituting \((l + k)^2\) with \(l^2\) in the expression for \(D\). The terms proportional to \(l \cdot k\) in the curly brackets of
Eqs. (C3) and (C5) are then dropped and the integrals are factorized into two one-loop integrals as

\[ I_{t\ell_2|\text{nondec}} = \int \frac{d^4 k}{(2\pi)^4} \left[ \frac{-2k^2}{[k^2 - m_{t_1}^2][k^2 - m_{H^\pm}^2]} \right] \int \frac{d^4 l}{(2\pi)^4} \left[ \frac{1}{[l^2 - m_{t_1}^2][l^2 - m_{H^\mp}^2][l^2 - m_{\tilde{g}}^2]} \right], \]  

(C13)

\[ I_{Ht\ell_2|\text{nondec}} = \int \frac{d^4 k}{(2\pi)^4} \left[ \frac{k^2}{[k^2 - m_{t_1}^2][k^2 - m_{H^\pm}^2]^2} \right] \int \frac{d^4 l}{(2\pi)^4} \left[ \frac{1}{[l^2 - m_{t_1}^2][l^2 - m_{H^\mp}^2][l^2 - m_{\tilde{g}}^2]} \right], \]  

(C14)

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