Hysteresis in consumer markets with focus on the mobile communications market

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Abstract. Our aim here is to try to identify hysteresis in the switching patterns of consumers in the Irish mobile phone industry. It was not until the introduction by the Communication Regulator of full-number portability that consumers began to take advantage of the savings that switching mobile phone operator could produce. As with most relatively new industries, the awareness of savings is clouded by a lack of understanding of what on offer and an underlying fear of change from something they have only just started to comprehend. With people changing company loyalties more frequently than ever at the prospect of better, more cost-efficient services, it is now the million euro question for the phone companies on how close they should match each others’ offers to maximize their profits, and what their best pricing strategy should be to obtain an even larger share of the market. Through the use of experimental economics and by modelling switching behaviour using the Preisach model, along with observed and market data, we hope to both pose this problem and start the journey to answering this question.

1. Simple switching model in payment methods

Consider the problem of a mobile phone customer making a choice between two different payment methods, namely bill-pay (B) and pay-as-you-go (P). If there is nothing to lose from switching from one service to another than the decision is quite easy. You pick the service that derives the highest utility i.e. the one that suits the individual customer best. However, switching from P to B is not that simple. It involves a non-monetary transaction cost in the form of a time commitment in investigating and physically switching to the new service, a financial commitment to the mobile phone operator (usually 12 months) and the prospect of self-discipline in controlling the size of each bill. Equally, switching from B to P involves effort, sacrificing a handset subsidy and accepting more expensive service costs. Obviously, one must weigh up the utility derived from each payment method and a switch will not occur until the utility difference becomes greater than the transaction costs.

1.1. Theoretical framework

Using general notation, let $u_b(t)$ be the utility of having the bill-pay method at time $t$, $u_p(t)$ be the utility of having the pay-as-you-go method at time $t$, and $a_{q,w}$ be the transaction cost of...
moving payment method from $Q$ to $W$. Customers who are currently in state $P$ should stick with that payment method until

$$u_b(t) - u_p(t) \geq a_{p,b}.$$  

Here, $a_{p,b}$ is the switching threshold for switching from $p$ to $b$.

Similarly customers who are currently in state $B$ should stick with that payment method until

$$u_p(t) - u_b(t) \geq a_{b,p}.$$  

Let us now define an operator $\gamma$ that indicates the payment state at a given time. The state of $\gamma$ will not only depend on the difference of utility

$$x(t) = u_b(t) - u_p(t)$$

but also on the state originally occupied

$$z(0) = z_0.$$  

These equations combined will be the input of $\gamma$, namely $u(t)$. The output, $f(t)$, will be:

- $z_0$ if $-a_{b,p} < x(t_k) < a_{b,p}$ for all $t_k \in [0,t]$
- $P$ if $\exists \tau \in [12 \text{ months}, t]$ such that $x(\tau) \leq -a_{b,p}$ and for all $t_k \in [\tau, t]$, $x(t_k) < a_{b,p}$
- $B$ if $\exists \tau \in [0, t]$ such that $x(\tau) \leq a_{p,b}$ and for all $t_k \in [\tau+12 \text{ months}, t]$, $x(t_k) < -a_{p,b}$

At all times $a_{b,p} + a_{p,b} \geq 0$ to prevent gains being made by constantly switching. $\gamma$ is a non-ideal relay. However, when the customer either begins in state $B$ or switches to state $B$ there is a 12 month contract signed to the company, and so this prevents any change in state for that amount of time. The customer therefore is unable to switch state for this 12 month period regardless of the magnitude in the difference of utility.

![Figure 1](image)

**Figure 1.** Thresholds $a_{b,p}$ and $a_{p,b}$ for hysteresis system $\gamma$ describing switching in payment methods.

If we consider a set of hysteresis operators with different threshold values, by use of weights we can set these up in parallel to produce what is known as the Preisach model [1]. The weight
function is known as the Preisach function and we can now describe the Preisach model by the following relation:

\[ f(t) = \int \int_{\alpha<\beta} \mu(\alpha,\beta) \gamma_{\alpha,\beta} u(t) \, d\alpha \, d\beta. \]

where \( u(t) \) is our input, \( f(t) \) is our overall output and distribution density function \( \mu(\alpha,\beta) \), known as the Preisach function. This will be used in our calculations when fitting the data.

1.2. Fitting the data

We propose that the main contributing factor to switching state, and so our measurement of derived utility, is the expected savings that will be made by customers. For each customer, it will take a certain level of expected savings to induce a switch from one payment method to another. This figure will vary from customer to customer but should have an identifiable distribution overall. Using both observed values from individuals and overall market data, we attempt to compare and interpret the resulting distributions from each set of data. The individual observed values were collected using market specific surveys and the results of 84 of these surveys contributed to this data set. The market data was made available through the Irish Commission for Communications Regulation [3]. The input is standardized as follows:

\[ X_{ij}^{P}(t_k) = \frac{\text{Expected savings per month at time of switch for the } p^{th} \text{ customer}}{\text{Average monthly spending for the } p^{th} \text{ customer}}, \]

for some time \( t_k \) when a switch occurs from state \( i \) to state \( j, i \neq j \). For values of \( X_{ij}^{PB}(t_k) \), which are independently distributed, we observed, without the use of statistical analysis, that the individual data followed that of an approximate normal distribution with mean 0.717308 and variance 0.197397. For values of \( X_{ij}^{BP}(t_k) \), which are also independently distributed, we observed that the data again followed approximately that of a normal distribution with mean 0.586591 and variance 0.362957. This result means that, within the confines of this model, the average pay as you go customer needs to make a saving of 58% per month to induce a switch to the bill pay service. The assumption of both of these data sets having normal distributions may appear crude but we will see that in this model it is important for simplifying calculations later on.

![Figure 2. Results of observed data fitted to the switching model.](image)

To estimate the Preisach function, \( \mu(\alpha,\beta) \), we will assume for simplicity that both thresholds above are also independently distributed. Denote \( X_{ij}^{PB}(t_k) = \tilde{\alpha} \) and \( X_{ij}^{BP}(t_k) = \tilde{\beta} \). Now \( \tilde{\mu}(\alpha,\beta) = P(\tilde{\alpha}) \times P(\tilde{\beta}) \) where
\[ P(\tilde{\alpha}) = \frac{1}{\sigma_1 \sqrt{2\pi}} e^{-\frac{(\tilde{\alpha} - m_1)^2}{2\sigma_1^2}} , \quad P(\tilde{\beta}) = \frac{1}{\sigma_2 \sqrt{2\pi}} e^{-\frac{(\tilde{\beta} - m_2)^2}{2\sigma_2^2}} . \]

Where \( m_1 \) and \( \sigma_1 \) are the mean and variance of \( \tilde{\alpha} \). Similarly \( m_2 \) and \( \sigma_2 \) are the mean and variance of \( \tilde{\beta} \). Thus the Preisach function is

\[ \tilde{\mu}(\alpha, \beta) = \frac{1}{2\pi \sigma_1 \sigma_2} e^{-\frac{(\alpha - m_1)^2 - (\beta - m_2)^2}{4\sigma_1^2 \sigma_2^2}} , \quad (1) \]

To simplify the calculations and the interpretation of the results, we adopt the relation

\[ \int_{-2}^{2} \left( \int_{-2}^{2} \tilde{\mu} \, dA \right) \, dB \simeq 0, \quad \int_{-2}^{2} \left( \int_{-2}^{B} \tilde{\mu} \, dA \right) \, dB \simeq 1. \quad (2) \]

This means that if a customer expects his/her service cost to double then the probability of them switching from their current service is zero. On the opposite boundary, if a customer stands to save twice as much money as they currently spend (highly unlikely as this would imply that the phone company would be paying you!), then customers would switch service with probability one.

The next step is to apply our observed Preisach function to the data made available by the Irish Commission for Communications Regulation. Market data tells us that 75% of customers are currently in state \( P \) and 25% are \( B \). Using the assumption from market data that the proportion of bill paying customers has been increasing we can assume a horizontal line on the alpha beta diagram below. Using the bill pay proportion of 0.25 we can calculate the current markets attitude to switching from \( P \) to \( B \). So to calculate what the current saving level is in the market for consumers who switch we simply double integrate the distribution function \( \tilde{\mu} \) to find out what proportion of savings, denoted \( Z \), results in a 25% market share.

\[ \int_{-2}^{Z} \left( \int_{-2}^{B} \tilde{\mu} \, dA \right) \, dB \simeq 0.25 \]

Solving this equation we get \( Z = -0.143 \).

Thus, the result we observe from the market is a saving of \(-0.143\). This means that if an average customer switches to the bill pay service, he/she will actually spend, based on the above model, an extra $14 per month per $100 monthly spending. This appears to be an illogical result based on our assumption of monthly savings being the solitary driving factor in inducing a switch.
1.3. Conclusion

Currently people are willing to switch to bill-pay while, knowingly or otherwise, facing the prospect of a more expensive service. If the above framework is correct we would expect both results to be the same or close, however this is not the case. According to the constructed framework above, the significant contrast in results between observed data and market data suggests that the average customer is misguided and misinformed in his/her decision to switch to bill pay. More realistically though, one can attribute this unexpected result to the enhanced services and mobile phone subsidies that a customer receives as a bill-pay customer. If we wanted to analyze this problem further and more rigorously it would be necessary to put a quantifiable price on these non-monetary perks.

2. Effect of a special offer

By 2003 the Irish mobile phone industry had almost reached total saturation. With each company’s stake in the market reaching equilibrium point, it was obvious that a shift of business strategy was needed if Meteor were to compete against a market dominated by O2 and Vodafone. When Meteor first announced its free Meteor-to-Meteor text offer it was seen as the first predatory type offer from a company in this market where the emphasis had now switched from attracting new customers to the market to hunting other companies’ customers. After an initial time lag, this offer caused a large flow of consumers to switch to Meteor from its two competitors. This sudden growth in market share grew as a result of targeting customers whose most influential factor was lowering monthly spending. Meteor’s free text offer meant that the more people you knew on the Meteor network the higher your expected savings per month would be by switching to Meteor.

2.1. Theoretical framework

Consider the problem of a mobile phone customer making a choice between either staying with a well-established company with traditional marketing strategy V O (Vodafone or O2) or switching to a new company with a predatory marketing strategy M (Meteor). The framework here is similar to that of the previous section but with subtle differences. As before we have access to both individual observed data on switching behavior from survey results and overall market data from the Irish Commission for Communications Regulation that we will use in our model. Again, using similar notation as earlier, let $u_{VO}(t)$ be the utility of being a customer of VO, $u_M(t)$ be the utility of being M and $a_{Q,W}$ be the transaction cost of moving from $Q$ to $W$. There is no monetary transaction cost in switching company, only a non-monetary cost in the form of time and effort, this transaction cost is the same for both types of company:

$$a_{VO,M} = a_{M,VO}.$$  

Customers who are currently in state V O should remain with that company until

$$u_M(t) - u_{VO}(t) \geq a_{M,VO}.$$  

Similarly customers who are currently in state M should remain with company until

$$u_{VO}(t) - u_M(t) \geq a_{VO,M}.$$
Let us now define an operator $\psi$ that indicates the state occupied at a given time. The state of $\psi$ will depend on the difference of utility

$$x(t) = u_M(t) - u_{VO}(t),$$

and also on the state originally occupied

$$z(0) = z_0.$$

These combined will be the input of $\psi$. The output will be:

- $z_0$ if $-a_{M,VO} < x(t_k) < a_{VO,M}$ for all $t_k \in [0,t]$
- $VO$ if $\exists \tau \in [0, t]$ such that $x(\tau) \leq -a_{M,VO}$ and for all $t_k \in [\tau, t], x(t_k) < a_{M,VO}$
- $M$ if $\exists \tau \in [0, t]$ such that $x(\tau) \leq a_{VO,M}$ and for all $t_k \in [\tau, t], x(t_k) < -a_{VO,M}$

At all times $a_{M,VO} + a_{VO,M} \geq 0$ to prevent gains being made by constantly switching company. This is a classical non–ideal relay operator. It is similar to the operator defined in section one, but note that there are now no time restraints on the customer.

![Figure 3. Thresholds $a_{M,VO}$ and $a_{VO,M}$ for switching from traditional strategy company to predatory strategy company.](image)

### 2.2. Fitting the data

When fitting the data to the above framework we use the assumption that a consumer is equally likely to switch from one state to another depending only on expected saving. Our observed values of $X_{p,MO}^{V,M}(t_k)$, (the observed threshold for each customer $p$ switching from $V$ to $MO$), which are independently distributed, appear visually to follow that of a symmetrical bell–shape. For simplicity we will assume that they are normally distributed. We calculate the mean to be 0.677247 and the variance to be 0.139892. Thus we can also say that $X_{p,MO}^{M,V}(t_k)$ is normally distributed with the same mean and variance, i.e. an expected monthly saving of 67.7% is the average savings requirement to induce a customer to switch from one company type to another.
Figure 4. Results of observed data fitted with symmetric thresholds for customers switching from traditional company to predatory company.

Using the assumption that both thresholds, $a_{M,VO}$ and $a_{VO,M}$, are normally and independently distributed, we again obtain (1) as an estimate of the distribution function of the Preisach model. We assume the same approximate boundaries (2).

Market data tells us that 79.7% are $VO$ (traditional company customers) and 20.3% are $M$ (predatory company customers). Using the assumption from market data that the proportion of predatory customers has been monotonically increasing over time, we can use the bill-pay proportion of 0.203, and calculate what level of savings has resulted in a switch of 20.3% of the market.

$$\int_Z^2 \left( \int_A^2 \mu \, dB \right) \, dA \approx 0.203$$

Solving this equation we get a value of $Z = -0.37$.

This result can be interpreted as meaning that the current level of saving for the average customer who has actually switched from the traditional type company to the predatory is negative, resulting in a loss of 37% per month as a consequence of switching.
2.3. Conclusion

Again, within the confines of the above framework, this result would appear irrational on the part of the customer. Unlike the problem in section 1 however, we cannot justify the result here by any simple logic or intuitive means. We propose that the discrepancy in our findings is as a result of a delay of information in the market. Potential savings may be available to consumers at any given instance but it takes time for these savings to be identified by the consumers and realized as switches in the market. Hence this delay exists in our constructed model also. With this idea in mind we hypothesize that the above result is not the current market level of savings but rather a level of savings from sometime in the past. This leads us to a series of discussions and a string of open questions which we will propose in our final discussion.

3. Discussion

In section one, we dealt with a dynamic (but not particularly volatile) switching model with a constant larger (if only marginally) flow of customers switching from the pay–as–you–go payment method to the bill–pay method. Both payment methods have been in existence for quite some time and in effect the market has leveled off to an extent. This has helped us in our calculations and in the interpretation of the results we acquired. We concluded that the average customer, although anticipating making a monthly saving, was in fact facing a more expensive service.

In section two, in contrast to section one, we have analyzed a far more volatile market. The introduction of a predatory type company to the market has served to both vastly increase competition in the market and highlight potential savings to the customers. In recent years the latter has lead to a more informed and also far less company-loyal brand of consumer. This, along with the introduction of full number portability [2], has resulted in the predatory company continuing to advance its share of the market. Due to the highly competitive nature of the market, we continuously see changes in the potential savings available to consumers by switching company. This is complicated further by the spectrum of packages available to customers to cater for their individual needs, each package offering specific savings unique to each customer’s particular mobile phone requirements.

So with savings so readily available, why do people not change company every day, week, month or year? There are several valid explanations such as a lack of information, laziness, a fear of the unknown or simply just personal preference. However, an influential factor which is of growing importance at the moment for every rational customer is simply the amount of people that a customer knows on a particular network. This is due to the substantial lower costs of intra–company services. When calculating the most cost effective company to be with, the customer must assign a weight to each person they will potentially contact according to how often they communicate with that contact and decide which company will offer him/her the lowest weighted total expected cost.

The main problems facing one looking to identify hysteresis in this particular market are the lack of available data due to confidentiality and sensitivity issues and also the lack of accurate data. For example, when a customer changes company, neither company knows how much the customer expects to save or ends up actually saving per month. The companies do however have access to a vast amount of data relating to each of their customers usage and behaviour. This leads to some open questions to carry on from the problems posed in this paper.

In conclusion we have verified that Preisach modeling is potentially a very useful and adequate tool for the sort of problems posed in the above and hopefully this will lead to further investigations of this kind.
4. Open questions

The mobile communications market is an extraordinarily intriguing area to examine and from our observations there are many problems that are yet to be researched that would make for fascinating investigations.

Following on from the above discussions one could look at both problems in terms of flows of customers. What we have observed in the second case, interestingly enough, is something similar to the dynamics of biological systems in the way that customers savings are directly related to the amount of customers of each company that he/she is in contact with. This would suggest that we could perhaps assemble a pair of first order, non-linear, differential equations of the Lotka-Volterra type whereby whereby the predator would be the predatory companies’ customers and the prey would be the traditional companies’ customers.

An area that would also be of relevant and practical importance to these companies would be to accurately quantify the non–monetary cost of a customer switching service. We have roughly attempted this in our above workings but if we can effectively and accurately put a price on the subjective factors that we struggle to enumerate, we would have a far superior analysis into the switching behaviour of consumers. This could be achieved, we speculate, if we had better resources for data such as more comprehensive individual data and access to more up–to–date and detailed market data.

A more advanced and complex aspect of the problems posed would be to consider a 3-state relay problem, and apply it to the problem posed in question two as a three–company problem. This would be an arduous but in no means impossible undertaking that if successfully worked would be highly beneficial and potentially profitable for any of the three companies in the market.

On a technical analytical level, statistical analyses would be required on the models proposed in the earlier frameworks to make more concrete inferences about the market. We have taken a parsimonious approach to the above problems for the sake of calculations. We might consider using lognormal distributions that are not independent of each other for the distributions of our thresholds. All in all, a more rigorous investigation into the distributions used and perhaps a goodness of fit test would help strengthen our acquired results.

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