Weak Golbach’s Conjecture from Isomorphic and Equivalent Odd Prime Number Functions

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Abstract: Mathematicians has been trying to prove the weak Golbach’s conjecture by adding prime numbers, as stated in the conjecture. However, we believe that the solution does not need to be analytically solved. Instead of trying to add prime numbers to prove the conjecture, we developed a prime number function $P_{odd}(x)_{p>2}$, including odd primes $p > 2$, isomorphic and equivalent to a function $N_{odd}(x)_{n>1}$, including odd natural numbers greater than one, $n_{odd} > 1$, in which, the sum of three of its elements result in odd numbers greater than 7, proving the conjecture.

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1. Introduction
The weak Goldbach’s conjecture is a well know problem, ”Every odd number $n$ greater than 7 can be expressed as the sum of three odd primes $p > 2$” (An odd prime $p > 2$ may be used more than once in the same sum).

\[(p_{p>2} + p_{p>2} + p_{p>2}) \in n_{odd} > 7\] (1)

Mathematicians has been attacking the problem as stated, trying to add prime numbers. Obtaining only partial results, proving the conjecture for sufficiently large odd numbers [1] [2], in the range of $10^{32}$ [3], if taking into account the Generalized Riemann Hypothesis (GRH). Likewise, without taking into account the GRH, partial results has been obtained [4] [5] [6], nevertheless, the sufficiently large odd numbers increase drastically, in the range of $10^{7194}$ [7]. Thus, still quantities not feasible to be checked by computers.

The Goldbach’s conjecture is known to be true up to $10^{14}$. Deshouillers, te Riele and Saouter [8] have checked it up to $10^{14}$ and Richstein [9] up to $4 \times 10^{14}$.

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In the same way, there has been parallel attempts to solve the conjecture, Tao [10] in 2012 improved the result of Ramaré [11], proving that every odd number $N$ greater than 1 can be expressed as the sum of at most five primes, instead of six primes.

However, we believe the solution can be obtained in a different way. By creating a mathematical function $P_{\text{odd}}(x)_{p>2}$ for all odd primes, $p > 2$, such that it is isomorphic (structurally identical) and equivalent to another mathematical function $N_{\text{odd}}(x)_{n>1}$ for all odd natural numbers greater than one, $n_{\text{odd}} > 1$, in which the sum of three of its elements will result in odd numbers greater than 7, proving the conjecture.

## 2. Prime Number Function $P(x)$

Prime numbers are the building blocks of the positive integers, this was shown by Euler in his proof of the Euler product formula for the Riemann zeta function, Euler came up with a version of the sieve of Eratosthenes, better in the sense that each number was eliminated exactly once [12] [13].

\[
\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p \text{ prime}} \frac{1}{1 - p^{-s}}
\]  

(2)

The sieve of Eratosthenes is a simple algorithm for finding all prime numbers up to any given limit. It iteratively mark as composite the multiples of each prime, starting from that prime, while leaving without mark prime numbers [14]. In order to mimic this simple sieve algorithm as a mathematical function, the prime number function $p(x)$ includes the $\sin\left(\frac{1}{p}\right)$ periodic function to indicate the periodicity of the prime multiples, $\sin\left(\frac{1}{p}\right)$, starting from primes $p$ [15].

For a single prime number, the sieve equation is

\[
p(x) = p + \sin\left(\frac{1}{p}\right)
\]

(3)

While, for the entire set of primes, the equation can be represented as the set of all single prime functions $p(x)$

\[
P(x) = \{p(x)\} = \left\{p + \sin\left(\frac{1}{p}\right)\right\}
\]

(4)

In order to visualize the prime number functions $P(x)$, a one-dimensional number line is used, starting from zero and ending at infinite. Due to the nature of periodic functions, a zero-cross from the function over the number line, indicates the marking of a composite number. Consequently, by the inclusion of a periodic function in $p(x)$, the sieve works instantaneously from its starting prime $p$ to $\infty$. Likewise, by following a linear progression, starting at $p = 2$ to $p \to \infty$, the multiples of primes are zero-cross, while prime number are decoded at each cycle of the function [16].

At each number $n > 1$, the function $P(x)$ evaluates if the number $n$ in consideration has been zero-cross or not. If a number $n$ has not been zero-cross, then the number is a prime $p$, this process we called prime decoding. While,
if a number $n$ has been zero-cross, then the number is marked as *composite*, meaning that a previous prime or primes $p$ has been decoded and this zero-cross is a multiple of such prime or primes $p(x)$, this process we called *composite encoding*. Once a prime number $p$ is decoded, the function $p(x)$ starts the expansion of the prime into its multiples, mimicking the sieve of Eratosthenes, zero-crossing composite numbers and decoding primes by leaving intact the number line at prime numbers $p$, as shown in figure 1.

Figure 1. Zero-cross numbers by $P(x)$. In red $p(x)$ for $p = 2$, in black $P(x)$ for $p > 2$ (odd primes); white squares zero-cross represent *composite* numbers; white squares zero-cross by only $p(x)$ for $p = 2$ (red color) represent numbers power of two $2^m$, for $m > 1$.

The process is as follows: Starting at number $p = 2$ the function $p(x) = 2 + \sin \left(\frac{1}{2}\right)$ zero-crosses all multiples of 2, while the maximum value of the function exist at all odd numbers, $n_{odd} > 1$, as shown in figure 2. At number $p = 3$ no zero-cross occur. Thus, the function $p(x) = 3 + \sin \left(\frac{1}{3}\right)$ starts zero-crossing odd and even multiples of 3, as shown in figure 3. At numbers $n = 4$ $n = 6$, $n = 8$, $n = 10$ and all even numbers $n_{even} > 2$ a zero-cross occurs from the function $p(x) = 2 + \sin \left(\frac{1}{2}\right)$ meaning *composite* numbers, fig. 2. At number $p = 5$ no zero-cross occur. Therefore, the function $p(x) = 5 + \sin \left(\frac{1}{5}\right)$ starts to zero-cross odd and even multiples of 5, as shown in figure 4. At number $p = 7$ no zero-cross occur. Thus, the function $p(x) = 7 + \sin \left(\frac{1}{7}\right)$ starts to zero-cross odd and even multiples of 7, as shown in figure 5. At number $n = 9$ a zero-cross occurs from the function $p(x) = 3 + \sin \left(\frac{1}{3}\right)$ meaning a *composite* number has been found, fig. 3. At number $p = 11$ no zero-cross occur. Thus the function $p(x) = 11 + \sin \left(\frac{1}{11}\right)$ starts to zero-cross odd and even multiples of 11, as shown in figure 6. And so on. Consequently, all $p(x)$ prime functions for $p > 2$, zero-cross odd and even multiples of $p > 2$.

### 2.1. Natural Number Function $N(x)_{n>1}$

If, instead of include only prime numbers $p$ in the prime number function $P(x)$, natural numbers greater than one are included, $n > 1$. The function changes to $N(x)_{n>1}$, starting from natural numbers greater than one and zero-crossing the same numbers as the prime number function $P(x)$, the set of *composite* numbers.

The sieve equation for a single natural number function is
Figure 2. \( P(x) = \{2 + \sin \left( \frac{x}{2} \right) \} \) starts at \( p = 2 \) and zero-cross multiples of 2. In red \( p = 2 \).

Figure 3. \( P(x) = \{2 + \sin \left( \frac{x}{2} \right) , 3 + \sin \left( \frac{x}{3} \right) \} \) zero-cross multiples of 2 and 3. In red \( p = 3 \).

Figure 4. \( P(x) = \{2 + \sin \left( \frac{x}{2} \right) , 3 + \sin \left( \frac{x}{3} \right) , 5 + \sin \left( \frac{x}{5} \right) \} \) zero-cross multiples of 2, 3 and 5. In red \( p = 5 \).
Figure 5. \( P(x) = \{2 + \sin \left( \frac{x}{2} \right), 3 + \sin \left( \frac{x}{4} \right), 5 + \sin \left( \frac{x}{5} \right), 7 + \sin \left( \frac{x}{7} \right)\} \) zero-cross multiples of 2, 3, 5 and 7. In red \( p = 7 \).

Figure 6. \( P(x) = \{2 + \sin \left( \frac{x}{2} \right), 3 + \sin \left( \frac{x}{4} \right), 5 + \sin \left( \frac{x}{5} \right), 7 + \sin \left( \frac{x}{7} \right), 11 + \sin \left( \frac{x}{11} \right)\} \) zero-cross multiples of 2, 3, 5, 7 and 11. In red \( p = 11 \).

\[
n(x)_{n>1} = n_{n>1} + \sin \left( \frac{1}{n_{n>1}} \right) \\
\]

While, for the entire set of natural number functions \( n(x)_{n>1} \), the equation can be represented as

\[
N(x)_{n>1} = \{n(x)_{n>1}\} = \left\{n_{n>1} + \sin \left( \frac{1}{n_{n>1}} \right) \right\} \\
\]

There are two possible cases when applying this function:

**Case I.** If the function \( N(x)_{n>1} \) follows the sieve rules, meaning that single functions \( n(x)_{n>1} \) only start from non-zero-cross numbers greater than one (primes), then both functions are equal

\[
N(x)_{n>1} = P(x) \\
\]
Case II. On the other hand, if we change the rule, meaning that there exist a function \( n(x)_{n>1} \) at every point greater than one over the number line, regardless if the point in consideration has been zero-cross or not, then the functions are equivalent and isomorphic, but not equal.

\[
N(x)_{n>1} = \{n(x)_{n>1}\} = \left\{ n_{n>1} + \sin \left( \frac{1}{n_{n>1}} \right) \right\} \iff P(x) = \{p(x)\} = \left\{ p + \sin \left( \frac{1}{p} \right) \right\} \tag{8}
\]

However, in both cases, the sieve functions, \( N(x)_{n>1} \) and \( P(x) \), will lead to the same result, the decoding of prime numbers while encoding composite numbers.

### 2.2. Odd Natural Number Function \( N_{odd}(x)_{n>1} \)

The weak Goldbach’s conjecture only takes into account odd numbers, therefore, all the involved function must have as input and output odd numbers greater than one. For this purpose the function \( N(x)_{n>1} \) is modified to \( N_{odd}(x)_{n>1} \), starting from odd natural numbers greater than one and zero-crossing only its odd multiples by doubling its period, from odd periods \( \sin \left( \frac{1}{n_{n>1}\_odd} \right) \) to even periods \( \sin \left( \frac{1}{2n_{n>1}\_odd} \right) \).

The sieve equation for a single odd natural number function is

\[
n_{odd}(x)_{n>1} = n_{odd_{n>1}} + \sin \left( \frac{1}{2n_{odd_{n>1}}} \right) \tag{9}
\]

While, for the entire set of odd natural number functions, the equation can be represented as

\[
N_{odd}(x)_{n>1} = \{n_{odd}(x)_{n>1}\} = \left\{ n_{odd_{n>1}} + \sin \left( \frac{1}{2n_{odd_{n>1}}} \right) \right\} \tag{10}
\]

Similar to the function \( N(x)_{n>1} \), there are two ways to apply this developed function.

**Case I.** If the function \( N_{odd}(x)_{n>1} \) follows the sieve rules, in which, single odd functions \( n_{odd}(x)_{n>1} \) only start from non-zero-cross odd numbers (odd primes).

**Case II.** If the function \( N_{odd}(x)_{n>1} \) does not follows the sieve rules, meaning that there exist a function \( n_{odd}(x)_{n>1} \) at every odd point greater than one over the number line, regardless, if the point in consideration has been zero-cross or not.

However, if the function \( N_{odd}(x)_{n>1} \) follows the sieve rules or not, the result will be the same, the decoding of odd prime numbers, \( p > 2 \), while zero-crossing odd composite numbers.

### 3. Weak Golbach’s Conjecture from Odd Prime Functions \( P_{odd}(x)_{p>2} \)

The weak Goldbach’s conjecture states: "Every odd number \( n \) greater than 7 can be expressed as the sum of three odd primes \( p > 2 \)." (An odd prime \( p > 2 \) may be used more than once in the same sum).

If instead of odd primes \( p > 2 \), the conjecture states odd numbers greater than one, \( n_{odd} > 1 \).
"Every odd number \( n \) greater than 7 can be expressed as the sum of three odd numbers greater than one, \( n_{odd} > 1 \). (An odd number greater than one, \( n_{odd} > 1 \) may be used more than once in the same sum). Then this modified conjecture is proven.

**Corollary 3.1.**
The sum of three odd numbers bigger than one, \( n_{odd} > 1 \), will result in odd numbers greater than 7, \( n_{odd} > 7 \)

\[
(n_{odd} > 1 + n_{odd} > 1 + n_{odd} > 1) \in n_{odd} > 7
\] (11)

**Proposition 3.1.**
The mathematical function \( P_{odd}(x)_{p>2} \) for all odd primes, \( p > 2 \), is isomorphic (structurally identical) and equivalent to the mathematical function \( N_{odd}(x)_{n>1} \) for all odd natural numbers greater than one, \( n_{odd} > 1 \).

\[
P_{odd}(x)_{p>2} \iff N_{odd}(x)_{n>1}
\] (12)

**Proof.** By definition, to obtain a prime number involves a process. "A prime number is a natural number greater than 1 that has no positive divisors other than 1 and itself".

Observe that the process to obtain primes \( p \) can be written as the function \( P(x) \), equation 4, similar to the sieve of Eratosthenes, which iteratively mark as composite the multiples of each prime, starting from that prime, while leaving without mark prime numbers \([14][15]\).

The prime number function \( P(x) \), takes into account all even and odd primes. If taking out of consideration \( p(x) \), when \( p = 2 \), then the function only includes odd primes \( P(x)_{p>2} \), and the zero-cross composite numbers change, excluding only powers of two \( \sum_{m=1}^{\infty} 2^m \), as shown in figure 1.

In order to create an odd prime function \( P_{odd}(x)_{p>2} \) isomorphic and equivalent to \( N_{odd}(x)_{n>1} \), equation 10. It is necessary to modify the function \( P(x)_{p>2} \), to zero-cross only odd composite numbers. This is done by doubling the period of the function \( P(x)_{p>2} \), so the function instead of odd periods \( \left(\frac{1}{p_{p>2}}\right) \), consist of even periods \( \left(\frac{1}{2p_{p>2}}\right) \).

Consequently, the function for a single odd prime number is modified to

\[
p_{odd}(x)_{p>2} = p_{p>2} + \sin \left(\frac{1}{2p_{p>2}}\right)
\] (13)

Starting at each odd prime number, \( p > 2 \), and zero-crossing only its odd multiples. While the maximum value of the function exists at even multiples of odd primes, as shown in figure 11. In the same way, the function \( P_{odd}(x)_{p>2} \) represents the set of all single odd prime functions \( p_{odd}(x)_{p>2} \)

\[
P_{odd}(x)_{p>2} = \left\{ p_{odd}(x)_{p>2} \right\} = \left\{ p_{p>2} + \sin \left(\frac{1}{2p_{p>2}}\right) \right\}
\] (14)

Starting at \( p = 3 \), the function \( p_{odd}(x) = \left[3 + \sin \left(\frac{1}{3}\right)\right] \) zero-crosses odd multiples of 3, while the maximum value of the function exist at even multiples of 3, as shown in figure 7. At number \( p = 5 \) no zero-cross occur. Thus, the function \( p_{odd}(x) = \left[5 + \sin \left(\frac{1}{5}\right)\right] \) starts to zero-cross odd multiples of 5, as shown in figure 8. At number \( p = 7 \) no zero-cross occur. Thus, the function \( p_{odd}(x) = \left[7 + \sin \left(\frac{1}{7}\right)\right] \) starts to zero-cross odd multiples of 7, as shown
in figure 9. At number \( n = 9 \) a zero-cross occurs from the function \( p_{\text{odd}}(x) \) for \( p = 3 \), meaning the number is composite, fig. 7. At number \( p = 11 \) no zero-cross occur. Therefore, the function \( p_{\text{odd}}(x) = [11 + \sin \left( \frac{1}{11} \right)] \) starts to zero-cross odd multiples of 11, as shown in figure 10. And so on.

Consequently, the function \( \text{P}_{\text{odd}}(x)_{p>2} \), zero-crosses the entire set of odd composite numbers, starting from odd primes, as shown in figure 11.

Thus, the function \( \text{P}_{\text{odd}}(x)_{p>2} \) is equal to \( \text{N}_{\text{odd}}(x)_{n>1} \) for case I, when the sieve rules are applied, where single natural number functions \( n_{\text{odd}}(x)_{n>1} \) only start from non-zero-cross odd numbers (odd primes)

\[
\text{P}_{\text{odd}}(x)_{p>2} = \text{N}_{\text{odd}}(x)_{n>1}
\]

While, for case II, there exist a function \( n_{\text{odd}}(x)_{n>1} \) at every odd point greater than one over the number line, regardless, if the point in consideration has been zero-cross or not. Then the functions are isomorphic and equivalent, equation 12.

---

**Proposition 3.2.**

The mathematical function \( \text{P}_{\text{odd}}(x)_{p>2} \) represents odd prime numbers \( p > 2 \) with the property that the sum of three odd primes result in every odd number \( n \) greater than 7, \( n > 7 \). (Odd primes \( p > 2 \) may be used more than once in the same sum).

**Proof.** For Case I. \( P(x) = N(x)_{n>1} \)

\[
\left\{ p + \sin \left( \frac{1}{p} \right) \right\} = \left\{ n_{\text{odd}}_{n>1} + \sin \left( \frac{1}{n_{n>1}} \right) \right\}
\]

if the set of all elements are equal, then single elements are equal
Figure 8. $P_{odd}(x)_{p>2} = \{3 + \sin \left( \frac{1}{3} \right), 5 + \sin \left( \frac{1}{5} \right)\}$ zero-cross odd multiples of $p = 3$ and $p = 5$. In black $p = 3$, in blue $p = 5$.

Figure 9. $P_{odd}(x)_{p>2} = \{3 + \sin \left( \frac{1}{3} \right), 5 + \sin \left( \frac{1}{5} \right), 7 + \sin \left( \frac{1}{7} \right)\}$ zero-cross odd multiples of $p = 3$, $p = 5$ and $p = 7$. In black $p = 3$ and $p = 5$, in blue $p = 7$.

Figure 10. $P_{odd}(x)_{p>2} = \{3 + \sin \left( \frac{1}{3} \right), 5 + \sin \left( \frac{1}{5} \right), 7 + \sin \left( \frac{1}{7} \right), 11 + \sin \left( \frac{1}{11} \right)\}$ zero-cross odd multiples of $p = 3$, $p = 5$, $p = 7$ and $p = 11$. In black $p = 3$, $p = 5$, $p = 7$, in blue $p = 11$. 
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Figure 11. \( P_{\text{odd}}(x)_{p \geq 2} \) zero-crosses odd multiples of \( p > 2 \), starting from odd primes \( p > 2 \). In red \( p = 2 \), in black \( p > 2 \).

\[
p + \sin \left( \frac{1}{p} \right) = n_{n>1} + \sin \left( \frac{1}{n_{n>1}} \right)
\]

and the equation can be reduced to

\[
p = n_{n>1}
\]

by taking out of consideration the periods continuation.

While for Case II. \( P(x) \iff N(x)_{n>1} \)

\[
\{ p + \sin \left( \frac{1}{p} \right) \} \iff \{ n_{n>1} + \sin \left( \frac{1}{n_{n>1}} \right) \}
\]

if the set of all elements are equivalent and isomorphic, then single elements are

\[
p + \sin \left( \frac{1}{p} \right) \iff n_{n>1} + \sin \left( \frac{1}{n_{n>1}} \right)
\]

and the equation can be reduced to

\[
p \iff n_{n>1}
\]

In the same way, for Case I. \( P_{\text{odd}}(x)_{p \geq 2} = N_{\text{odd}}(x)_{n>1} \)

\[
\{ p_{p \geq 2} + \sin \left( \frac{1}{2p_{p \geq 2}} \right) \} = \{ n_{\text{odd}n>1} + \sin \left( \frac{1}{2n_{\text{odd}n>1}} \right) \}
\]

if the set of all elements are equal, then single elements are equal

\[
p_{p \geq 2} + \sin \left( \frac{1}{2p_{p \geq 2}} \right) = n_{\text{odd}n>1} + \sin \left( \frac{1}{2n_{\text{odd}n>1}} \right)
\]
and the equation can be reduced to

\[ p_{p>2} = n_{odd_{n>1}} \]  \hspace{1cm} (24)

by taking out of consideration the periods continuation.

While for **Case II**, \( P_{odd}(x)_{p>2} \iff N_{odd}(x)_{n>1} \)

\[
\left\{ p_{p>2} + \sin \left( \frac{1}{2p_{p>2}} \right) \right\} \iff \left\{ n_{odd_{n>1}} + \sin \left( \frac{1}{2n_{odd_{n>1}}} \right) \right\}
\]

\hspace{1cm} (25)

if the set of all elements are equivalent and isomorphic, then single elements are

\[ p_{p>2} + \sin \left( \frac{1}{2p_{p>2}} \right) \iff n_{odd_{n>1}} + \sin \left( \frac{1}{2n_{odd_{n>1}}} \right) \]

\hspace{1cm} (26)

and the equation can be reduced to

\[ p_{p>2} \iff n_{odd_{n>1}} \]  \hspace{1cm} (27)

Likewise, the weak Goldbach’s conjecture is proven, by been able to zero-cross all odd natural numbers greater than seven, \( n_{odd} > 7 \), starting from odd primes \( p > 2 \), as with odd numbers greater than 1, \( n_{odd} > 1 \).

\[ \square \]

**References**

[1] Hardy G. H. and Littlewood J. E., Some problems of Partitio Numerorum III: On the expression of a number as a sum of primes, Acta. Math., 44 (1923). pp.1-70.

[2] Effinger G., Some numerical implication of the Hardy and Littlewood analysis of the 3- primes problem, submitted for publication.

[3] Lucke B., Zur Hardy-Littlewoodschen Behandlung des Goldbachschen Problems, Doctoral Dissertation, Göttingen, 1926.

[4] Zinoviev D., On Vinogradov’s constant in Goldbach’s ternary problem, Journal of Number Theory, 65 (1997), 334358.

[5] Borodzkin K. G., On I. M. Vinogradois constant, Proc. 3rd All-Union Math. Conf., vol. 1., Izdat. Akad. Nauk SSSR, Moscow, 1956. (Russian) MR 20:6973a.

[6] Chen J. R. and Wang T. Z., On the odd Goldbach problem, Acta Math. Sinica 32 (1989), 702-718.

[7] Chen J. R. and Wang T. Z., The Goldbach problem for odd numbers, Acta Math. Sinica 39 (1996), 169-174.

[8] Deshouillers J. M., te Riele H. J. and Saouter Y., New Experimental Results Concerning the Goldbach Conjecture, In Proc. 3rd Int. Symp. on Algorithm Number Theory, LNCS, 1423 (1998), 204-215.
[9] Richstein J., Verifying Goldbach's conjecture up to $4 \times 10^{14}$, Mathematics and Computation 70 (2001), 1745-1749.

[10] Tao T., Every Odd number Greater Than 1 is the Sum of at most five Primes, http://arXiv:1201.6656v3

[11] Ramaré O. and Saouter Y., Short effective intervals containing primes, J. Number Theory 98 (2003), no. 1, 10-33.

[12] Edwards H. M., The Euler Product Formula, New York: Dover 2001, pp. 6-7.

[13] Shimura G., Euler Products and Eisenstein Series, Providence, RI: Amer. Math. Soc., 1997.

[14] O'Neill M. E., The Genuine Sieve of Eratosthenes, Journal of Functional Programming, Cambridge University Press 2008, pp. 10-11, DOI:10.1017/S0956796808007004.

[15] Mateos L.A., Dynamical Sieve of Eratosthenes, http://arxiv.org/pdf/1206.2791

[16] Mateos L.A., Chaotic Nonlinear Prime Number Function, AIP Conf. Proc. CMLS 2011, 1371, pp. 161-170, DOI:10.1063/1.3596639.

[17] Sieve Of Eratosthenes. http://c2.com/cgi/wiki?SieveOfEratosthenes.
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