Intersecting M-branes
as Four-Dimensional Black Holes

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Abstract

We present two 1/8 supersymmetric intersecting p-brane solutions of 11-dimensional supergravity which upon compactification to four dimensions reduce to extremal dyonic black holes with finite area of horizon. The first solution is a configuration of three intersecting 5-branes with an extra momentum flow along the common string. The second describes a system of two 2-branes and two 5-branes. Related (by compactification and T-duality) solution of type IIB theory corresponds to a completely symmetric configuration of four intersecting 3-branes. We suggest methods for counting the BPS degeneracy of three intersecting 5-branes which, in the macroscopic limit, reproduce the Bekenstein-Hawking entropy.

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1. Introduction

The existence of supersymmetric extremal dyonic black holes with finite area of the horizon provides a possibility of a statistical understanding [1] of the Bekenstein-Hawking entropy from the point of view of string theory [2,3,4]. Such black hole solutions are found in four [5,6,7] and five [4,8] dimensions but not in $D > 5$ [9,10]. While the D-brane BPS state counting derivation of the entropy is relatively straightforward for the $D = 5$ black holes [4,11], it is less transparent in the $D = 4$ case, a complication being the presence of a solitonic 5-brane or Kaluza-Klein monopole in addition to a D-brane configuration in the descriptions used in [12,13].

One may hope to find a different lifting of the dyonic $D = 4$ black hole to $D = 10$ string theory that may correspond to a purely D-brane configuration. A related question is about the embedding of the $D = 4$ dyonic black holes into $D = 11$ supergravity (M-theory) which would allow to reproduce their entropy by counting the corresponding BPS states using the M-brane approach similar to the one applied in the $D = 5$ black hole case in [14].

As was found in [15], the (three-charge, finite area) $D = 5$ extremal black hole can be represented in M-theory by a configuration of orthogonally intersecting 2-brane and 5-brane (i.e. $2\perp 5$) with a momentum flow along the common string, or by configuration of three 2-branes intersecting over a point ($2\perp 2\perp 2$). A particular embedding of (four-charge, finite area) $D = 4$ black hole into $D = 11$ theory given in [15] can be interpreted as a similar $2\perp 5$ configuration ‘superposed’ with a Kaluza-Klein monopole.

Below we shall demonstrate that it is possible to get rid of the complication associated with having the Kaluza-Klein monopole. There exists a simple 1/8 supersymmetric configuration of four intersecting M-branes ($2\perp 2\perp 5\perp 5$) with diagonal $D = 11$ metric. Upon compactification along six isometric directions it reduces to the dyonic $D = 4$ black hole with finite area and all scalars being regular at the horizon.

The corresponding $2\perp 2\perp 4\perp 4$ solution of type IIA $D = 10$ superstring theory (obtained by dimensional reduction along a direction common to the two 5-branes) is $T$-dual to a $D = 10$ solution of type IIB theory which describes a remarkably symmetric configuration of four intersecting 3-branes.

1 Similar D-brane configuration was discussed in [16,17]. Note that it is a combination of four and not three intersecting 3-branes that is related (for the special choice of equal charges) to the non-dilatonic ($a = 0$) RN $D = 4$ black hole. $T$-dual configuration of one 0-brane and three intersecting 4-branes of type IIA theory was considered in [18].
Our discussion will follow closely that of [15] where an approach to constructing intersecting supersymmetric p-brane solutions (generalising that of [19]) was presented. The supersymmetric configurations of two or three intersecting 2- and 5-branes of \(D = 11\) supergravity which preserve 1/4 or 1/8 of maximal supersymmetry are \(2 \perp 2, 5 \perp 5, 2 \perp 2 \perp 2, 5 \perp 5 \perp 5, 2 \perp 2 \perp 5\) and \(2 \perp 5 \perp 5\). Two 2-branes can intersect over a point, two 5-branes – over a 3-brane (which in turn can intersect over a string), 2-brane and 5-brane can intersect over a string [19]. There exists a simple ‘harmonic function’ rule which governs the construction of composite supersymmetric p-brane solutions in both \(D = 10\) and \(D = 11\): a separate harmonic function is assigned to each constituent 1/2 supersymmetric p-brane.

Most of the configurations with four intersecting M-branes, namely, \(2 \perp 2 \perp 2 \perp 2\), \(2 \perp 2 \perp 2 \perp 5\) and \(5 \perp 5 \perp 5 \perp 2\) are 1/16 supersymmetric and have transverse \(x\)-space dimension equal to two (\(5 \perp 5 \perp 5 \perp 5\) configuration with 5-branes intersecting over 3-branes to preserve supersymmetry does not fit into 11-dimensional space-time). Being described in terms of harmonic functions of \(x\) they are thus not asymptotically flat in transverse directions. There exists, however, a remarkable exception – the configuration \(2 \perp 2 \perp 5 \perp 5\) which (like \(5 \perp 2 \perp 2\), \(5 \perp 5 \perp 5\) and \(5 \perp 5 \perp 5\)) has transverse dimension equal to three and the fraction of unbroken supersymmetry equal to 1/8 (Section 3). Upon compactification to \(D = 4\) it reduces to the extremal dyonic black hole with four different charges and finite area of the horizon.

Similar \(D = 4\) black hole background can be obtained also from the the ‘boosted’ version of the \(D = 11\) \(5 \perp 5 \perp 5\) solution [15] (Section 2) as well from the \(3 \perp 3 \perp 3 \perp 3\) solution of \(D = 10\) type IIB theory (Section 4). The two \(D = 11\) configurations \(5 \perp 5 \perp 5 + \text{‘boost’}\) and \(2 \perp 2 \perp 5 \perp 5\) reduce in \(D = 10\) to \(0 \perp 1 \perp 4 \perp 4\) and \(2 \perp 2 \perp 4 \perp 4\) solutions of \(D = 10\) type IIA theory which are related by \(T\)-duality.

In Section 5 we shall suggest methods for counting the BPS entropy of three intersecting 5-branes which reproduce the Bekenstein-Hawking entropy of the \(D = 4\) black hole. This seems to explain the microscopic origin of the entropy directly in 11-dimensional terms.

\[\text{2} \text{ Intersecting p-brane solutions in [19,15] and below are isometric in all directions internal to all constituent p-branes (the background fields depend only on the remaining common transverse directions). They are different from possible virtual configurations where, e.g., a (p-2)-brane ends (in transverse radial direction) on a p-brane [20]. A configuration of p-brane and p’-brane intersecting in p+p’-space may be also considered as a special anisotropic p+p’-brane. There may exist more general solutions (with constituent p-branes effectively having different transverse spaces [19,21]) which may ‘interpolate’ between intersecting p-brane solutions and solutions with one p-brane ending on another in the transverse direction of the latter.}\]

\[\text{3} \text{ The ‘boost’ along the common string corresponds to a Kaluza-Klein electric charge part in the \(D = 11\) metric which is ‘dual’ to a Kaluza-Klein monopole part present in the \(D = 11\) embedding of dyonic black hole in [15].}\]
2. ‘Boosted’ $5\perp 5\perp 5$ solution of $D = 11$ theory

The $D = 11$ background corresponding to $5\perp 5\perp 5$ configuration [19] is [14]

$$ds^2_{11} = (F_1 F_2 F_3)^{-2/3} \left[ F_1 F_2 F_3 (- dt^2 + dy_1^2) + F_2 F_3 (dy_2^2 + dy_3^2) + F_1 F_3 (dy_4^2 + dy_5^2) + F_1 F_2 (dy_6^2 + dy_7^2) + dx_s dx_s \right],$$

$\mathcal{F}_4 = 3 (s dF_1^{-1} \wedge dy_2 \wedge dy_3 + s dF_2^{-1} \wedge dy_4 \wedge dy_5 + s dF_3^{-1} \wedge dy_6 \wedge dy_7).$ (2.2)

Here $\mathcal{F}_4$ is the 4-form field strength and $F_i$ are the inverse powers of harmonic functions of $x_s \ (s = 1, 2, 3)$. In the simplest 1-center case discussed below $F_i^{-1} = 1 + P_i/r \ (r^2 = x_s x_s)$. The metric (2.1) with $y_i \ (i = 2, 3, 4, 5)$ internal to the three 5-branes can be identified according to the $F_i$ factors inside the square brackets in the metric: $(y_1, y_4, y_5, y_6, y_7)$ belong to the first 5-brane, $(y_1, y_2, y_3, y_6, y_7)$ to the second and $(y_1, y_2, y_3, y_4, y_5)$ to the third. 5-branes intersect over three 3-branes which in turn intersect over a common string along $y_1$. If $F_2 = F_3 = 1$ the above background reduces to the single 5-brane solution [22] with the harmonic function $H = F_1^{-1}$ independent of the two of transverse coordinates (here $y_2, y_3$). The case of $F_3 = 1$ describes two 5-branes intersecting over a 3-brane. The special case of $F_1 = F_2 = F_3$ is the solution found in [19].

Compactifying $y_1, ..., y_7$ on circles we learn that the effective ‘radii’ (scalar moduli fields in $D = 4$) behave regularly both at $r = \infty$ and at $r = 0$ with the exception of the ‘radius’ of $y_1$. It is possible to stabilize the corresponding scalar by adding a ‘boost’ along the common string. The metric of the resulting more general solution [15] is (the expression for $\mathcal{F}_4$ remains the same)

$$ds^2_{11} = (F_1 F_2 F_3)^{-2/3} \left[ F_1 F_2 F_3 (dudv + K du^2) + F_2 F_3 (dy_2^2 + dy_3^2) + F_1 F_3 (dy_4^2 + dy_5^2) + F_1 F_2 (dy_6^2 + dy_7^2) + dx_s dx_s \right].$$

Here $u = y_1 - t, \ v = 2t$ and $K$ is a harmonic function of the three coordinates $x_s$. A non-trivial $K = 1 + Q/r$ describes a momentum flow along the string direction.\footnote{The corresponding 1/4 supersymmetric background also has 3-dimensional transverse space and reduces to a $D = 4$ black hole with two charges (it has $a = 1$ black hole metric when two charges are equal). The $5\perp 5\perp 5$ configuration compactified to $D = 10$ gives $4\perp 4$ solution of type IIA theory which is $T$-dual to $3\perp 3$ solution of type IIB theory.}

\footnote{The metric (2.3) with $F_i = 1$ (i.e. $ds^2 = -K^{-1} dt^2 + K (dy_1 + (K^{-1} - 1) dt)^2 + dy_s dy_s + dx_s dx_s$) reduces upon compactification along $y_1$ direction to the $D = 10$ type IIA R-R 0-brane background [23] with $Q$ playing the role of the KK electric charge.}

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$Q$ also has an interpretation of The $D = 11$ metric (2.3) is regular at the $r = 0$ horizon and has a non-zero 9-area of the horizon (we assume that all $y_n$ have period $L$)

$$A_9 = 4\pi L^7 [r^2 K^{-1/2} (F_1 F_2 F_3)^{-1/2}]_{r \to 0} = 4\pi L^7 \sqrt{QP_1 P_2 P_3}.$$  (2.4)

Compactification along $y_2, ... y_7$ leads to a solitonic $D = 5$ string. Remarkably, the corresponding 6-volume is constant so that one gets directly the Einstein-frame metric

$$ds^2_5 = H^{-1} (du dv + K du^2) + H^2 dx_s dx_s, \quad H \equiv (F_1 F_2 F_3)^{-1/3}.$$  (2.5)

Further compactification along $y_1$ or $u$ gives the $D = 4$ (Einstein-frame) metric which is isomorphic to the one of the dyonic black hole [6]

$$ds^2_4 = -\lambda(r) dt^2 + \lambda^{-1}(r) (dr^2 + r^2 d\Omega^2_5), \quad \lambda(r) = \sqrt{K^{-1} F_1 F_2 F_3} = \frac{r^2}{\sqrt{(r + Q)(r + P_1)(r + P_2)(r + P_3)}}.$$  (2.6)

Note, however, that in contrast to the dyonic black hole background of [6] which has two electric and two magnetic charges here there is one electric (Kaluza-Klein) and 3 magnetic charges. From the $D = 4$ point of view the two backgrounds are related by $U$-duality. The corresponding 2-area of the $r = 0$ horizon is of course $A_9/L^7$.

In the special case when all 4 harmonic functions are equal ($K = F_i = H^{-1}$) the metric (2.3) becomes

$$ds^2_{11} = H^{-1} du dv + du^2 + dy_2^2 + ... + dy_6^2 + H^2 dx_s dx_s \quad (2.8)$$

$$= -H^{-2} dt^2 + H^2 dx_s dx_s + [dy_1 + (H^{-1} - 1) dt]^2 + dy_2^2 + ... + dy_6^2,$$

and corresponds to a charged solitonic string in $D = 5$ or the Reissner-Nordström ($a = 0$) black hole in $D = 4$ (‘unboosted’) $5\perp 5\perp 5$ solution with $K = 1$ and equal $F_i$ reduces to $a = 1/\sqrt{3}$ dilatonic $D = 4$ black hole [19]).

A compactification of this $5\perp 5\perp 5$+‘boost’ configuration to $D = 10$ along $y_1$ gives a type IIA solution corresponding to three 4-branes intersecting over 2-branes plus additional Kaluza-Klein (Ramond-Ramond vector) electric charge background, or, equivalently, to the $0\perp 4\perp 4\perp 4$ configuration of three 4-branes intersecting over 2-branes which in turn intersect over a 0-brane. If instead we compactify along a direction common only to two of the three 5-branes we get $4\perp 4\perp 5$+‘boost’ type IIA solution. Other related solutions of type IIA and IIB theories can be obtained by applying T-duality and $SL(2, Z)$ duality.

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6 This may be compared to another type IIA configuration (consisting of solitonic 5-brane lying within a R-R 6-brane, both being intersected over a ‘boosted’ string by a R-R 2-brane) which also reduces [12,15] to the dyonic $D = 4$ black hole.
\section*{3. $2\perp2\perp5\perp5$ solution of $D = 11$ theory}

Solutions with four intersecting M-branes are constructed according to the rules discussed in \cite{15}. The $2\perp2\perp5\perp5$ configuration is described by the following background

\begin{equation}
\begin{aligned}
 ds^2_{11} &= (T_1T_2)^{-1/3}(F_1F_2)^{-2/3} \left[-T_1T_2F_1F_2 \, dt^2 \right. \\
 &\left. + T_1F_1dy_1^2 + T_1F_2dy_2^2 + T_2F_1dy_3^2 + T_2F_2dy_4^2 + F_1F_2(dy_5^2 + dy_6^2 + dy_7^2) + dx_s dx_s \right], \\
 F_4 &= -3dt \wedge (dT_1 \wedge dy_1 \wedge dy_2 + dT_2 \wedge dy_3 \wedge dy_4) \\
 &+ 3(*dF_1^{-1} \wedge dy_2 \wedge dy_4 + *dF_2^{-1} \wedge dy_1 \wedge dy_3). 
\end{aligned}
\tag{3.1}
\end{equation}

Here $T_i^{-1}$ are harmonic functions corresponding to the 2-branes and $F_i^{-1}$ are harmonic functions corresponding to the 5-branes, i.e.

\begin{equation}
T_i^{-1} = 1 + \frac{Q_i}{r}, \quad F_i^{-1} = 1 + \frac{P_i}{r}. \tag{3.3}
\end{equation}

$(y_1, y_2)$ belong to the first and $(y_3, y_4)$ to the second 2-brane. $(y_1, y_3, y_5, y_6, y_7)$ and $(y_2, y_4, y_5, y_6, y_7)$ are the coordinates of the two 5-branes. Each 2-brane intersects each 5-brane over a string. 2-branes intersect over a 0-brane $(x = 0)$ and 5-branes intersect over a 3-brane.

Various special cases include, in particular, the 2-brane solution \cite{24} $(T_2 = F_1 = F_2 = 1)$, as well as $5\perp5$ $(T_1 = T_2 = 1)$ \cite{19} and $2\perp5$ $(T_1 = F_2 = 1)$, $2\perp2\perp5$ $(F_2 = 1)$, $2\perp5\perp5$ $(T_2 = 1)$ \cite{13} configurations (more precisely, their limits when the harmonic functions do not depend on a number of transverse coordinates).

As in the case of the $5\perp5\perp5$+‘boost’ solution \cite{2.3}, \cite{2.2}, the metric \cite{3.1} is regular at the $r = 0$ horizon (in particular, all internal $y_n$-components smoothly interpolate between finite values at $r \to \infty$ and $r \to 0$) with the 9-area of the horizon being \cite{2.4}

\begin{equation}
A_9 = 4\pi L^7[r^2(T_1T_2F_1F_2)^{-1/2}]_{r \to 0} = 4\pi L^7 \sqrt{Q_1Q_2P_1P_2}. \tag{3.4}
\end{equation}

The compactification of $y_n$ on 7-torus leads to a $D = 4$ background with the metric which is again the dyonic black hole one \cite{2.6}, now with

\begin{equation}
\lambda(r) = \sqrt{T_1T_2F_1F_2} = \frac{r^2}{\sqrt{(r + Q_1)(r + Q_2)(r + P_1)(r + P_2)}}. \tag{3.5}
\end{equation}

In addition, there are two electric and two magnetic vector fields (as in \cite{14}) and also 7 scalar fields. The two electric and two magnetic charges are directly related to the 2-brane and 5-brane charges (cf. \cite{3.2}).

When all 4 harmonic functions are equal $(T_i^{-1} = F_i^{-1} = H)$ the metric \cite{3.1} becomes \cite{2.8}

\begin{equation}
\begin{aligned}
 ds^2_{11} &= -H^{-2} dt^2 + H^2 dx_s dx_s + dy_1^2 + ... + dy_7^2, 
\end{aligned}
\tag{3.6}
\end{equation}

i.e. describes a direct product of a $D = 4$ Reissner-Nordström black hole and a 7-torus.

Thus there exists an embedding of the dyonic $D = 4$ black hole into $D = 11$ theory which corresponds to a remarkable symmetric combination of M-branes only. In contrast to the embeddings with a Kaluza-Klein monopole \cite{15} or electric charge (‘boost’) \cite{2.3}, \cite{2.8} it has a diagonal $D = 11$ metric.
4. $3\perp3\perp3\perp3$ solution of type IIB theory

Dimensional reduction of the background (3.1), (3.2) to $D = 10$ along a direction common to the two 5-brane (e.g. $y_7$) gives a type IIA theory solution representing the R-R p-brane configuration $2\perp2\perp4\perp4$. This configuration is $T$-dual to $0\perp4\perp4\perp4$ one which is the dimensional reduction of the $5\perp5\perp5\perp5\perp5$‘boost’ solution. This suggests also a relation between the two $D = 11$ configurations discussed in Sections 2 and 3.

$T$-duality along one of the two directions common to 4-branes transforms $2\perp2\perp4\perp4$ into the $3\perp3\perp3\perp3$ solution of type IIB theory. The explicit form of the latter can be found also directly in $D = 10$ type IIB theory (i.e. independently of the above $D = 11$ construction) using the method of [15], where the 1/4 supersymmetric solution corresponding to two intersecting 3-branes was given. One finds the following $D = 10$ metric and self-dual 5-form (other $D = 10$ fields are trivial)

$$ds_{10}^2 = (T_1T_2T_3T_4)^{-1/2} \left[ -T_1T_2T_3T_4 \ dt^2 + T_1T_2dy_1^2 + T_1T_3dy_2^2 + T_1T_4dy_3^2 + T_2T_3dy_4^2 + T_2T_4dy_5^2 + T_3T_4dy_6^2 + dx_s dx_s \right],$$

$$F_5 = dt \wedge (dT_1 \wedge dy_1 \wedge dy_2 \wedge dy_3 + dT_2 \wedge dy_1 \wedge dy_4 \wedge dy_5 + dT_3 \wedge dy_2 \wedge dy_4 \wedge dy_6 + dT_4 \wedge dy_3 \wedge dy_5 \wedge dy_6)$$

$$+ *d T_1^{-1} \wedge dy_4 \wedge dy_5 \wedge dy_6 + *d T_2^{-1} \wedge dy_2 \wedge dy_3 \wedge dy_6 + *d T_3^{-1} \wedge dy_1 \wedge dy_3 \wedge dy_5 + *d T_4^{-1} \wedge dy_1 \wedge dy_2 \wedge dy_4.$$  

The coordinates of the four 3-branes are $(y_1, y_2, y_3)$, $(y_1, y_4, y_5)$, $(y_2, y_4, y_6)$ and $(y_3, y_5, y_6)$, i.e. each pair of 3-branes intersect over a string and all 6 strings intersect at one point. $T_i$ are the inverse harmonic functions corresponding to each 3-brane, $T_i^{-1} = 1 + Q_i/r$. Like the $2\perp2\perp5\perp5$ background of $D = 11$ theory this $D = 10$ solution is 1/8 supersymmetric, has 3-dimensional transverse space and diagonal $D = 10$ metric.

Its special cases include the single 3-brane [23, 25] with harmonic function independent of 3 of 6 transverse coordinates ($T_2 = T_3 = T_4 = 1$), $3\perp3\perp3$ solution found in [17] ($T_3 = T_4 = 1$) and also $3\perp3\perp3$ configuration ($T_4 = 1$). The 1/8 supersymmetric $3\perp3\perp3$ configuration also has 3-dimensional transverse space but the corresponding $D = 10$ metric

$$ds_{10}^2 = (T_1T_2T_3)^{-1/2} \left[ -T_1T_2T_3 \ dt^2 + T_1T_2dy_1^2 + T_1T_3dy_2^2 + T_1dy_3^2 + T_2T_3dy_4^2 + T_2dy_5^2 + T_3dy_6^2 + dx_s dx_s \right],$$

7 Similar configurations of three and four intersecting 3-branes, and, in particular, their invariance under the 1/8 fraction of maximal supersymmetry were discussed in $D$-brane representation in [17, 18].
is singular at \( r = 0 \) and has zero area of the \( r = 0 \) horizon.

As in the two \( D = 11 \) cases discussed in the previous sections, the metric of the \( 3 \perp 3 \perp 3 \perp 3 \) solution \((1.1)\) has \( r = 0 \) as a regular horizon with finite 8-area (cf.\((2.4),(3.4)\))
\[
A_8 = 4\pi L^6 [r^2 (T_1 T_2 T_3 T_4)^{-1/2}]_{r \to 0} = 4\pi L^6 \sqrt{Q_1 Q_2 Q_3 Q_4}.
\]
\((4.4)\)
\(A_8/L^6\) is the area of the horizon of the corresponding dyonic \( D = 4 \) black hole with the metric \((2.6)\) and
\[
\lambda(r) = \sqrt{T_1 T_2 T_3 T_4} = \frac{r^2}{\sqrt{(r + Q_1)(r + Q_2)(r + Q_3)(r + Q_4)}}. \quad (4.5)
\]
The gauge field configuration here involves 4 pairs of equal electric and magnetic charges. When all charges are equal, the \( 3 \perp 3 \perp 3 \perp 3 \) metric \((4.1)\) compactified to \( D = 4 \) reduces to the \( a = 0 \) black hole metric (while the \( 3 \perp 3 \perp 3 \perp 3 \) metric \((4.3)\) reduces to the \( a = 1/\sqrt{3} \) black hole metric \([26]\)).

5. Entropy of \( D = 4 \) Reissner-Nordström black hole

Above we have demonstrated the existence of supersymmetric extremal \( D = 11 \) and \( D = 10 \) configurations with finite entropy which are built solely out of the fundamental \( p \)-branes of the corresponding theories (the 2-branes and the 5-branes of the M-theory and the 3-branes of type IIB theory) and reduce upon compactification to \( D = 4 \) dyonic black hole backgrounds with regular horizon.

Namely, there exists an embedding of a four dimensional dyonic black hole (in particular, of the non-dilatonic Reissner-Nordström black hole) into \( D = 11 \) theory which corresponds to a combination of M-branes only. This may allow an application of the approach similar to the one of \([14]\) to the derivation of the entropy \((3.4)\) by counting the number of different BPS excitations of the \( 2 \perp 2 \perp 5 \perp 5 \) M-brane configuration.

The \( 3 \perp 3 \perp 3 \perp 3 \) configuration represents an embedding of the \( 1/8 \) supersymmetric dyonic \( D = 4 \) black hole into type IIB superstring theory which is remarkable in that all four charges enter symmetrically. It is natural to expect that there should exist a microscopic counting of the BPS states which reproduces the Bekenstein-Hawking entropy in a \((U\text{-}duality\text{\textdagger})\) way that treats all four charges on an equal footing.

Although we hope to eventually attain a general understanding of this problem, in what follows we shall discuss the counting of BPS states for one specific example discussed above: the M-theory configuration \((2.3),(2.2)\) of the three intersecting 5-branes with a common line. Even though the counting rules of M-theory are not entirely clear, we see an advantage to doing this from M-theory point of view as compared to previous discussions in the context of string theory \([12],[13]\): the 11-dimensional problem is more symmetric. Furthermore, apart from the entropy problem, we may learn something about the M-theory.

\[^8\] This is similar to what one finds for the ‘unboosted’ \( 5 \perp 5 \perp 5 \) configuration \((2.3),(2.2)\). As is well-known from 4-dimensional point of view, one does need \textit{four} charges to get a regular behaviour of scalars near the horizon and finite area.
5.1. Charge quantization in M-theory and the Bekenstein-Hawking entropy

Upon dimensional reduction to four dimensions, the boosted 5⊥5⊥5 solution \( (2.3), (2.2) \), reduces to the 4-dimensional black hole with three magnetic charges, \( P_1, P_2 \) and \( P_3 \), and an electric charge \( Q \). The electric charge is proportional to the momentum along the intersection string of length \( L \), \( \mathcal{P} = 2\pi N/L \). The general relation between the coefficient \( Q \) in the harmonic function \( K \) appearing in \( (2.3) \) and the momentum along the \( D = 5 \) string (cf.\( (2.3) \)) wound around a compact dimension of length \( L \) is (see e.g. \[27\])

\[
Q = \frac{2\kappa^2_{D-1}}{(D-4)\omega_{D-3}}, \quad \frac{2\pi N}{L} = \frac{\kappa^2_4 N}{L} = \frac{\kappa^2 N}{L^8},
\]

(5.1)

where \( \kappa^2_4/8\pi \) and \( \kappa^2/8\pi \) are the Newton’s constants in 4 and 11 dimensions. All toroidal directions are assumed to have length \( L \).

The three magnetic charges are proportional to the numbers \( n_1, n_2, n_3 \) of 5-branes in the (14567), (12367), and the (12345) planes, respectively (see \( (2.1),(2.3) \)). The complete symmetry between \( n_1, n_2 \) and \( n_3 \) is thus automatic in the 11-dimensional approach. The precise relation between \( P_i \) and \( n_i \) is found as follows. The charge \( q_5 \) of a \( D = 11 \) 5-brane which is spherically symmetric in transverse \( d + 2 \leq 5 \) dimensions is proportional to the coefficient \( P \) in the corresponding harmonic function. For \( d + 2 = 3 \) appropriate to the present case (two of five transverse directions are isotropic, or, equivalently, there is a periodic array of 5-branes in these compact directions) we get

\[
q_5 = \frac{\omega_{d+1} d}{\sqrt{2}\kappa} P \rightarrow \frac{\omega_2 L^2}{\sqrt{2}\kappa} P = \frac{4\pi L^2}{\sqrt{2}\kappa} P.
\]

(5.2)

At this point we need to know precisely how the 5-brane charge is quantized. This was discussed in \[4\], but we repeat the argument here for completeness. A different argument leading to equivalent results was presented earlier in \[28\]. Upon compactification on a circle of length \( L \), the M-theory reduces to type IIA string theory where all charge quantization rules are known. We use the fact that double dimensional reduction turns a 2-brane into a fundamental string, and a 5-brane into a Dirichlet 4-brane. Hence, we have

\[
T_2\kappa^2 = T_1\kappa_{10}^2 , \quad T_5\kappa^2 = T_4\kappa_{10}^2 ,
\]

(5.3)

where the 10-dimensional gravitational constant is expressed in terms of the 11-dimensional one by \( \kappa_{10}^2 = \kappa^2/L \). The charge densities are related to the tensions by

\[
q_2 = \sqrt{2}\kappa T_2 , \quad q_5 = \sqrt{2}\kappa T_5 ,
\]

(5.4)

and we assume that the minimal Dirac condition is satisfied, \( q_2q_5 = 2\pi \). These relations, together with the 10-dimensional expressions \[29\]

\[
\kappa_{10} = g(\alpha')^2 , \quad T_1 = \frac{1}{2\pi\alpha'} , \quad \kappa_{10} T_4 = \frac{1}{2\sqrt{\pi\alpha'}} ,
\]

(5.5)
fix all the M-theory quantities in terms of \( \alpha' \) and the string coupling constant, \( g \). In particular, we find
\[
\kappa^2 = \frac{g^3(\alpha')^{9/2}}{4\pi^{5/2}}, \quad L = \frac{g\sqrt{\alpha'}}{4\pi^{5/2}}.
\]
The tensions turn out to be
\[
T_2 = \frac{2\pi^{3/2}}{g(\alpha')^{3/2}}, \quad T_5 = \frac{2\pi^2}{g^2(\alpha')^3}.
\]
Note that \( T_2 \) is identical to the tension of the Dirichlet 2-brane of type IIA theory, while \( T_5 \) – to the tension of the solitonic 5-brane. This provides a nice check on our results, since single dimensional reduction indeed turns the M-theory 2-brane into the Dirichlet 2-brane, and the M-theory 5-brane into the solitonic 5-brane. Note that the M-brane tensions satisfy the relation \( 2\pi T_5 = T_2^2 \), which was first derived in [28] using toroidal compactification to type IIB theory in 9 dimensions. This serves as yet another consistency check.

It is convenient to express our results in pure M-theory terms. The charges are quantized according to
\[
q_2 = \sqrt{2}\kappa T_2 = n\sqrt{2}(2\kappa\pi^2)^{1/3}, \quad q_5 = \sqrt{2}\kappa T_5 = n\sqrt{2}(\frac{\pi}{2\kappa})^{1/3},
\]
i.e.
\[
P_i = \frac{n_i}{2\pi L^2}\left(\frac{\pi\kappa^2}{2}\right)^{1/3}.
\]
The resulting expression for the Bekenstein-Hawking entropy of the extremal Reissner-Nordström type black hole, (2.6), (2.7), which is proportional to the area (2.4), is
\[
S_{BH} = \frac{2\pi A_9}{\kappa^2} = \frac{8\pi^2 L^7}{\kappa^2}\sqrt{P_1P_2P_3Q} = 2\pi\sqrt{n_1n_2n_3N}.
\]
This agrees with the expression found directly in \( D = 4 \) [2, 12, 13].

In the case of the 2 \perp 2 \perp 5 \perp 5 configuration we find (for each pair of 2-brane and 5-brane charges) \( q_2 = \frac{4\pi L^2}{\sqrt{2}\kappa^2}Q, \quad q_5 = \frac{4\pi L^2}{\sqrt{2}\kappa^2}P \). The Dirac condition on unit charges translates into \( q_2q_5 = 2\pi n_1n_2 \), where \( n_1 \) and \( n_2 \) are the numbers of 2- and 5-branes. We conclude that \( Q_1P_1 = \frac{\kappa^2}{4\pi L^2}n_1n_2 \). Then from (3.4) we learn that
\[
S_{BH} = \frac{2\pi A_9}{\kappa^2} = \frac{8\pi^2 L^7}{\kappa^2}\sqrt{Q_1P_1Q_2P_2} = 2\pi\sqrt{n_1n_2n_3n_4}.
\]

\footnote{In [30] it was argued that the 2-brane tension, \( T_2 \), satisfies \( \kappa^2 T_2^3 = \pi^2/m_0 \), where \( m_0 \) is a rational number that was left undetermined. The argument of [28], as well as our procedure [9], unambiguously fix \( m_0 = 1/2 \).}
Remarkably, this result does not depend on the particular choice of M-brane quantization condition (choice of $m_0 = \pi^2 \kappa^{-2} T_2^{-3}$) or use of D-brane tension expression since the $2 \perp 2 \perp 5 \perp 5$ configuration contains equal number of 2-branes and 5-branes. This provides a consistency check. Note also that the $D = 4$ black holes obtained from the $2 \perp 2 \perp 5 \perp 5$ and from the $5 \perp 5 \perp 5$ M-theory configurations are not identical, but are related by U-duality. The equality of their entropies provides a check of the U-duality.

The same expression is obtained for the entropy of the $D = 10$ configuration $3 \perp 3 \perp 3 \perp 3$ (4.3) (or related $D = 4$ black hole). Each 3-brane charge $q_3$ is proportional to the corresponding coefficient $Q$ in the harmonic function (cf. (5.2))

$$q_3 = \frac{1}{\sqrt{2}} (\omega_{d+1} d) Q \rightarrow \frac{\omega_2 L^3 Q}{2\kappa_{10}} = \frac{2\pi L^3 Q}{\kappa_{10}} , \quad (5.13)$$

where $\kappa_{10}^2 / 8\pi$ is the 10-dimensional Newton’s constant and the overall factor $\frac{1}{\sqrt{2}}$ is due to the dyonic nature of the 3-brane. The charge quantization in the self-dual case implies (see 4) $q_3 = n\sqrt{\pi}$ (the absence of standard $\sqrt{2}$ factor here effectively compensates for the ‘dyonic’ $\frac{1}{\sqrt{2}}$ factor in the expression for the charge). Thus, $Q_i = \kappa_{10}^2 n_i$, and the area (4.4) leads to the following entropy,

$$S_{BH} = \frac{2\pi A_8}{\kappa_{10}^2} = \frac{8\pi^2 L^6}{\kappa^2} \sqrt{Q_1 Q_2 Q_3 Q_4} = 2\pi \sqrt{n_1 n_2 n_3 n_4} . \quad (5.14)$$

5.2. Counting of the microscopic states

The presence of the factor $\sqrt{N}$ in $S_{BH}$ (5.11) immediately suggests an interpretation in terms of the massless states on the string common to all three 5-branes. Indeed, it is well-known that, for a $1 + 1$ dimensional field theory with a central charge $c$, the entropy of left-moving states with momentum $2\pi N/L$ is, for sufficiently large $N$, given by

$$S_{stat} = 2\pi \sqrt{\frac{1}{6} c N} . \quad (5.15)$$

We should find, therefore, that the central charge on the intersection string is, in the limit of large charges, equal to

$$c = 6n_1 n_2 n_3 . \quad (5.16)$$

10 This agrees with the D3-brane tension, $\kappa_{10} T_3 = \sqrt{\pi}$, since in the self-dual case $q_p = \kappa_{10} T_p$.

11 As pointed out in [31], this expression is reliable only if $N \gg c$. Requiring $N$ to be much greater than $n_1 n_2 n_3$ is a highly asymmetric choice of charges. If, however, all charges are comparable and large, the entropy is dominated by the multiply wound 5-branes, which we discuss at the end of this section.
The fact that the central charge grows as $n_1n_2n_3$ suggests the following picture. 2-branes can end on 5-branes, so that the boundary looks like a closed string \[ [20,32,33] \]. It is tempting to associate the massless states with those of 2-branes attached to 5-branes near the intersection point. Geometrically, we may have a two-brane with three holes, each of the holes attached to different 5-dimensional hyperplanes in which the 5-branes lie. Thus, for any three 5-branes that intersect along a line, we have a collapsed 2-brane that gives massless states in the $1 + 1$ dimensional theory describing the intersection. What is the central charge of these massless states? From the point of view of one of the 5-branes, the intersection is a long string in $5 + 1$ dimensions. Such a string has 4 bosonic massless modes corresponding to the transverse oscillations, and 4 fermionic superpartners. Thus, we believe that the central charge arising from the collapsed 2-brane with three boundaries is $4(1 + \frac{1}{2}) = 6$.\[12\]

The upshot of this argument is that each triple intersection contributes 6 to the central charge. Since there are $n_1n_2n_3$ triple intersections, we find the total central charge $6n_1n_2n_3$. One may ask why there are no terms of order $n_1^3$, etc. This can be explained by the fact that all parallel 5-branes are displaced relative to each other, so that the 2-branes produce massless states only near the intersection points.

One notable feature of our argument is that the central charge grows as a product of three charges, while in all D-brane examples one found only a product of two charges. We believe that this is related to the peculiar $n^3$ growth of the near-extremal entropy of $n$ coincident 5-branes found in \[9\] (for coincident D-branes the near-extremal entropy grows only as $n^2$). This is because the intersecting D-brane entropy comes from strings which can only connect objects pairwise. The 2-branes, however, can connect three different 5-branes. Based on our observations about entropy, we conjecture that the geometries where a 2-brane connects four or more 5-branes are forbidden (otherwise, for instance, the near-extremal entropy of $n$ parallel 5-branes would grow faster than $n^3$). Perhaps such configurations are not supersymmetric and do not give rise to massless states.

The counting argument presented above applies to the configuration where there are $n_1$ parallel 5-branes in the $(14567)$ hyperplane, $n_2$ parallel 5-branes in the $(12367)$ hyperplane, and $n_3$ parallel 5-branes in the $(12345)$ hyperplane. As explained in \[31\], if $n_1 \sim n_2 \sim n_3 \sim N$ we need to examine a different configuration where one replaces a number of disconnected branes by a single multiply wound brane. Let us consider, therefore, a single 5-brane in the $(14567)$ hyperplane wound $n_1$ times around the $y_1$-circle, a single 5-brane in the $(12367)$ hyperplane wound $n_2$ times around the $y_1$-circle, and a single 5-brane in

\[12\] Upon compactification on $T^7$, these massless modes are simply the small fluctuations of the long string in $4 + 1$ dimensions which is described by the classical solution \[2.5\]. One should be able to confirm that the central charge on this string is equal to 6 by studying its low-energy modes.
the (12345) hyperplane wound $n_3$ times around the $y_1$-circle. Following the logic of \[31\], one can show that the intersection string effectively has winding number $n_1n_2n_3$: this is because the 2-brane which connects the three 5-branes needs to be transported $n_1n_2n_3$ times around the $y_1$-circle to come back to its original state.\[13\] Therefore, the massless fields produced by the 2-brane effectively live on a circle of length $n_1n_2n_3L$. This implies \[34\] that the energy levels of the 1 + 1 dimensional field theory are quantized in units of $2\pi/(n_1n_2n_3L)$. In this theory there is only one species of the 2-brane connecting the three 5-branes; therefore, the central charge on the string is $c = 6$. The calculation of BPS entropy for a state with momentum $2\pi N/L$, as in \[34,31\], once again reproduces (5.11).

While the end result has the form identical to that found for the disconnected 5-branes, the connected configuration is dominant when all four charges are of comparable magnitude \[31\]. Now the central charge is fixed, and the large entropy is due to the growing density of energy levels.

### 6. Black Hole Entropy in $D = 5$ and Discussion.

The counting arguments presented here are plausible, but clearly need to be put on a more solid footing. Indeed, it is not yet completely clear what rules apply to the 11-dimensional M-theory (although progress has been made in \[14\]). The rule associating massless states to collapsed 2-branes with three boundaries looks natural, and seems to reproduce the Bekenstein-Hawking entropy of extremal black holes in $D = 4$. Note also that a similar rule can be successfully applied to the case of the finite entropy $D = 5$ extremal dyonic black holes described in 11 dimensions by the ‘boosted’ $2 \perp 5$ configuration \[15\]. Another possible $D = 11$ embedding of the $D = 5$ black hole is provided by $2 \perp 2 \perp 2$ configuration \[15\]. The relevant $D = 10$ type IIB configuration is $3 \perp 3$ (cf. \[4,3\]) with momentum flow along common string. In the case of $2 \perp 5$ configuration the massless degrees of freedom on the intersection string may be attributed to a collapsed 2-brane with a hole attached to the 5-brane and one point attached to the 2-brane. If the 5-brane is wound $n_1$ times and the 2-brane – $n_2$ times, the intersection is described by a $c = 6$

---

\[13\] The role of $n_1n_2n_3$ as the effective winding number is suggested also by comparison of the $D = 5$ solitonic string metric, \[2,3\], with the fundamental string metric, $ds^2 = V^{-1}(dudv + Kdu^2) + dx_sdx_s$, where the coefficient in the harmonic function $V$ is proportional to the tension times the winding number of the source string (see e.g. \[27\]). After a conformal rescaling, \[2,3\] takes the fundamental string form with $V = H^3 = (F_1F_2F_3)^{-1}$ so that near $r = 0$ the $dudv$ part of it is multiplied by $P_1P_2P_3 \sim n_1n_2n_3$. Thus, the source string may be thought of as wound $n_1n_2n_3$ times around the circle.
theory on a circle of length $n_1 n_2 L$. Following the arguments of [31], we find that the entropy of a state with momentum $2 \pi N/L$ along the intersection string is

$$S_{stat} = 2 \pi \sqrt{n_1 n_2 N}. \quad (6.1)$$

This seems to supply a microscopic M-theory basis, somewhat different from that in [14], for the Bekenstein-Hawking entropy of $D = 5$ extremal dyonic black holes.

We would now like to show that (6.1) is indeed equal to the expression for the Bekenstein-Hawking entropy for the ‘boosted’ $2 \perp 5$ configuration [15] (cf. (5.11))

$$S_{BH} = \frac{2 \pi A_9}{\kappa^2} = \frac{4 \pi^3 L^6}{\kappa^2} \sqrt{QPQ'}. \quad (6.2)$$

$Q$ and $P$ are the parameters in the harmonic functions corresponding to the 2-brane and the 5-brane, and $Q'$ is the parameter in the ‘boost’ function, i.e. $T^{-1} = 1 + Q/r^2$, $F^{-1} = 1 + P/r^2$, $K = 1 + Q'/r^2$. Note that here (cf. (5.1))

$$Q' = \frac{\kappa^2 N}{\pi L^i}, \quad q_2 = \frac{4 \pi^2 L^4}{\sqrt{2 \kappa}} Q, \quad q_5 = \frac{4 \pi^2 L}{\sqrt{2 \kappa}} P. \quad (6.3)$$

As in the case of the $2 \perp 2 \perp 2 \perp 5$ configuration, we can use the Dirac quantization condition, $q_2 q_5 = 2 \pi n_1 n_2$, to conclude that $QP = \frac{\kappa^2 N}{4 \pi L^i} n_1 n_2$. This yields (6.1) when substituted into (6.2). A similar expression for the BPS entropy is found in the case of the completely symmetric $2 \perp 2 \perp 2$ configuration,

$$S_{BH} = \frac{4 \pi^3 L^6}{\kappa^2} \sqrt{Q_1 Q_2 Q_3} = 2 \pi \sqrt{n_1 n_2 n_3}, \quad (6.4)$$

where we have used the 2-brane charge quantization condition (5.8), which implies that $Q_1 = n_i L^{-4} (\frac{\kappa}{\sqrt{2 \pi}})^{4/3}$. Agreement of different expressions for the $D = 5$ black hole entropy provides another check on the consistency of (5.8),(5.9).

Our arguments for counting the microscopic states applies only to the configurations where M-branes intersect over a string. It would be very interesting to see how approach analogous to the above might work when this is not the case. Indeed, black holes with finite horizon area in $D = 4$ may also be obtained from the $2 \perp 2 \perp 5 \perp 5$ configuration in M-theory, and the $3 \perp 3 \perp 3 \perp 3$ one in type IIB, while in $D = 5$ – from the $2 \perp 2 \perp 2$ configuration. Although from the $D = 4, 5$ dimensional point of view these cases are related by U-duality to the ones we considered, the counting of their states seems to be harder at the present level of understanding. We hope that a more general approach to the entropy problem, which covers all the solutions we discussed, can be found.

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