INTRODUCTION

The recent experimental data of the hadronic transitions of heavy quarkonia, such as $\psi(nS)$ or $Y(nS)$ to lower states with emission of two pions, show a puzzling behavior. The experimental data show that the $X(4260)$, $X(4360)$ and $X(4660)$ states apparently decay only through particular channels. The $X(4260)$ resonance has been seen only in $J/\psi\pi^+\pi^-$, whereas the $X(4360)$ and $X(4660)$ appear only in the $\psi(2S)\pi^+\pi^-$ channel. Furthermore, these two resonances have an anomalously large width [1, 2, 3]. Moreover, such decays have not been observed for the $\psi(4415)$. In the bottom sector, compared to the ordinary $Y(nS) \rightarrow Y(mS)$ ($m < n$) transitions, the partial width of the $Y(10860)$ is out of line by two orders of magnitude [4].

The typical momentum involved in the transition is too low to use perturbative QCD and therefore such methods can not be applied. Then, non perturbative methods, like the QCD multipole expansion approach (QCDME), should be used to describe at least the transition of the lower lying states. In the single channel picture of QCDME the heavy quarkonium system serves as a compact color source which emits two soft gluons which hadronize into two pions. After the emission of the first gluon and before the emission of the second one there exists an intermediate state where the $Q\bar{Q}$ pair together with the gluon forms an hybrid state. The width of the transition critically depends on the particular spectrum of the hybrid states, therefore it is important to describe the $Q\bar{Q}$ states on the hybrid consistently using as few parameters as possible.

Apart from lattice calculations [8, 9], hybrid meson properties have been calculated in different models: the flux-tube model [10, 11], constituent gluons [12], Coulomb gauge QCD [13] and quark confining string model (QCS) [14, 15, 16] or QCD string model [17]. Among them we adopt the QCS model since it was used in the early works of QCDME and it incorporates finite quark mass corrections.

Above the open flavor threshold, the QCDME scenario may change due to the possible contribution of molecular components [5], which can modified the hadronic decay width.

In this work we will address the description of the new data of the hadronic transitions in heavy quarkonium using the QCDME in the framework of a constituent quark model (see references [6] and [7] for reviews) which has been successful in describing the hadron phenomenology and the hadronic reactions. Hybrid states are consistently generated in the original quark model using the QCS scheme. In this way we minimize the number of free parameters. Above the open flavor threshold molecular meson-meson channels coupled to the $c\bar{c}$ states are included in the calculation.

THEORETICAL FRAMEWORK

The constituent quark model we use is based on the assumption that the spontaneous chiral symmetry breaking generates the constituent quark mass. To compensate this mass term in the Hamiltonian the Lagrangian must include Goldstone boson fields which mediates the interaction between quarks. The minimal realization of this mechanism include a pseudoscalar boson and an scalar one. This fact does not affect the heavy quark sector but is of paramount importance in the molecular picture be-
cause the only remaining interaction between the two molecular component, due to its color singlet nature, is the one driven by the Goldstone boson exchanges between the light quarks. Below the chiral symmetry scale quarks still interact through one gluon exchange and confinement potential. Explicit expression for this interactions are given in Ref. [19].

In this model mesons are described as clusters of one pair of quark and antiquark. To found the quark-antiquark bound states we solve the Schrödinger equation using the Gaussian Expansion Method [20]. In the case of the molecular structures the four body problem is also solved using the gaussian expansion and the two body wave functions obtained from the solution of the Schrödinger equation. To derive the meson-meson interaction using the Gaussian Expansion Method [20]. In the confinement potential. Explicit expression for this interaction we use the Resonating Group Method (RGM) [21]. The coupling between the $q\bar{q}$ and the four quarks configuration are performed using the $3P_0$ model [22]. A more detailed discussion of the model and its application to the states mentioned above can be found in Ref. [5].

To calculate the hadronic transition we use the QCDME method. The multipole expansion has been widely used for studying radiation processes in which the electromagnetic field is radiated from local sources. In our case the energy difference between the initial and the final states is usually small and therefore the gluon wavelength is large compared to the typical size of the $Q\bar{Q}$ states and can be treated in a multipole expansion. The gauge-invariant formulation of multipole expansion within QCD was given by T.-M. Yan in Ref. [23].

The two pion hadronic transitions are dominated by double electric-dipole transitions (E1E1). The transition amplitude split into two factors. The first one concerns to the wave functions and energies of the initial and final quarkonium state as well as those of the intermediate states. All these quantities can be calculated using suitable quark models. The second one describes the conversion of the emitted gluons into light hadrons. It scale is the light hadron mass, which is very low and therefore cannot be calculated in a perturbative way. Usually one uses a phenomenological approach based on PCAC [24] which involves two parameters ($C_1$ and $C_2$). The $C_1$ term is isotropic (S-wave) while the $C_2$ term is angular dependent (D-wave). These two parameters are fitted to the well established $\psi(2S) \rightarrow J/\psi\pi^+\pi^- \text{ and } \psi(3770) \rightarrow J/\psi\pi^+\pi^- \text{ transitions.}$

As explained above, these transitions involve the emission of two gluons that hadronized into two pions. The intermediate states after the emission of the first gluon and before the emission of the second gluon are states with a gluon and a $Q\bar{Q}$ pair which are the so-called hybrid states. We describe these states in the QCS model.

The QCS model is defined by a relativistic-, gauge- and reparametrization-invariant action describing quarks interacting with color $SU(3)$ gauge fields in a two dimensional world sheet. The model has no gluonic degrees of freedom, but has instead string degrees of freedom.

The string can carry energy-momentum only in the region between the quark and the antiquark, thus the quarks appear to be at the ends of the string. The equation that describes the dynamics of the quark-antiquark pair linked by the string is the usual Schrödinger equation with a confinement potential. Gluon excitation effects are described by the vibration of the string. These vibrational modes provide new states beyond the naive meson picture. A complete description of the QCS model can be found in Ref. [15]. Its generalization to our constituent quark model is described in Ref. [25].

**RESULTS**

All the parameters of the quark model are taken from Ref. [26]. The quantum numbers of hybrid states which participate in the two pion transition are $J^{PC}=1^{-+}$. The mass of the two low lying states in the $c\bar{c}g$ sector are 4.35 and 4.64 GeV. The first one agrees with the results of the flux-tube model [11] (4.1 − 4.2), Coulomb gauge QCD [13] (4.47), QCD string model [17] (4.40), potential models [18] (4.23) and lattice calculation [9] (4.40). In the bottom sector the ground state mass is 10.785 GeV near to the mass of the potential model (10.79) [18]

Table 1 shows the calculated $\mathcal{B}_{\pi^+\pi^-\psi(nS)} \times \Gamma_{e^+e^-}$ for the $J^{P^C}=1^{--}$ charmonium states. As the decays $\psi(2S) \rightarrow J/\psi\pi^+\pi^-$ and $\psi(3770) \rightarrow J/\psi\pi^+\pi^-$ have been used to fit the $C_1$ and $C_2$ parameters, they are not included in the table. One can see that in the case of the decay channel $\psi(2S)\pi^+\pi^-$ the only significant values correspond to the decays of the $X(4360)$ and $X(4660)$ which are also in agreement with the recent experimental data.

In the decay channel $J/\psi\pi^+\pi^-$ a high value of the $\mathcal{B}_{\pi^+\pi^-\psi(nS)} \times \Gamma_{e^+e^-}$ is obtained for the $X(4360)$ resonance. This result apparently contradicts the experimental data because this decay has not been reported in the reaction $e^+e^- \rightarrow J/\psi\pi^+\pi^-$. The cross section of this reaction shows a resonance in the 4.2 − 4.4 GeV, energy region which has been attributed to the $X(4260)$. This resonance does not appear in our calculation as a $c\bar{c}$ meson and its nature is still under discussion. An interference between the $X(4260)$ and $X(4360)$ resonances would be possible and should be explored.

If one looks to the values of the cross section at peak (Table 2) one sees that in the decay channel $J/\psi\pi^+\pi^-$ the values of the rest of peak cross sections, besides the resonance at 4.36 GeV, are of the order of the experimental background and it is difficult to decide whether a
The decays of the molecular structures through the channel calculation including the probability. The results for the bottomonium sector are shown in Table 3. One can see that the theoretical values agree reasonably well with the experimental ones except in the resonance is present or not. These results justify why no signal of the \( \psi(4040) \), \( \psi(4160) \) and \( \psi(4415) \) has been reported.

The results for the bottomonium sector are shown in Table 3. One can see that the theoretical values agree reasonably well with the experimental ones except in the case of the \( \Upsilon(10860) \). In this case we do not find any hybrid state near its energy and the mechanism which explains the large width in the charm sector cannot be applied to this case.

The \( \Upsilon(10860) \) is above the open-bottom threshold and then contributions of molecular components of the wave function like \( BB, BB^* \) or \( B^*B^* \) may appear. The importance of these components lies in the fact that the decays of the molecular structures through the \( \Upsilon \pi \pi \) channel is OZI allowed whereas the decay of a \( b\bar{b} \) component is OZI forbidden. Therefore the presence of molecular components in the wave function enhances the transition probability.

Following Ref. [5], we have performed a coupled channel calculation including the \( b\bar{b} 5^2S_1 \) and \( 4^3P_1 \) together with the \( BB, BB^* \) and \( B^*B^* \) molecules. The result of this calculation is shown in Table 4. We obtain two resonances in the 10860 MeV mass region. The first one is too narrow and the second is wide. We identify the wide resonance with the \( \Upsilon(10860) \). The process \( \Upsilon(10860) \rightarrow \Upsilon(nS) \pi \pi \) involves the creation of a \( q\bar{q} \) pair and a quark rearrangement process. This can result into a nonresonant final state \( \Upsilon(\Upsilon(980)) \pi \pi \), or an intermediate state in which the two pion forms a resonance (like \( f_0(980) \)) following by a subsequent decay into the final channel. In the calculation we include the \( f_0(500) \) and the \( f_0(980) \) resonances together with the nonresonant contribution. Results are shown in Tables 5 and 6. The contribution of the molecular components is able to explain the branching ratio of the \( \Upsilon(10860) \rightarrow \Upsilon(1S) \pi \pi \) but fails in the \( \Upsilon(2S) \pi \pi \) and \( \Upsilon(3S) \pi \pi \) cases. In all cases the main contribution corresponds to the nonresonant process but this contribution decreases as the phase space decreases. The contribution of more resonances could help to improve the results.

**SUMMARY**

Using the constituent quark model quoted above we calculate the \( J/\psi \pi \pi \) and \( \psi(2S) \pi \pi \) decays of the \( J^{PC} = 1^{--} \)
We are able to explain the anomalously large decay width of the 1^−− ϒ(10860) due to the presence of hybrid states located near the masses of both resonances. However this mechanism does not work in the case of the 1^−− ϒ(10860). The contribution of molecular components improves the description of the 1^−− ϒ(10860) → 1^−− ϒ(1S)ππ branching ratio but fails to explain the branching ratio in the 1^−− ϒ(2S)ππ and 1^−− ϒ(3S)ππ cases. Our model also predicts a large value for the decay 1^−− ϒ(4360) → J/ψππ which has been not seen in the experiments. Therefore the puzzling situation of the hadronic decays still deserves more experimental and theoretical efforts.

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