Abstract—This paper presents a novel strategy for synchronization of grid-connected Voltage Source Converters (VSCs) in power systems with low rotational inertia. The proposed model is based on emulating the physical properties of an induction machine and capitalizes on its inherent grid-friendly properties such as self-synchronization, oscillation damping and standalone capabilities. A detailed mathematical model of an induction machine is derived, which includes the possibility of obtaining the unknown grid frequency by processing the voltage and current measurements at the converter output. This eliminates the need for the phase-locked loop unit, traditionally employed in grid-following VSC control schemes, while simultaneously preserving the applied system-level and device-level control. Furthermore, the appropriate steps for obtaining an index-1 DAE representation of the induction-machine-based synchronization unit within the VSC control scheme are provided. The EMT simulations validate the mathematical principles of the proposed model, whereas a small-signal analysis provides guidelines for appropriate control tuning and reveals important properties pertaining to the nature of the underlying operation mode.

Index Terms—voltage source converter, induction machine, phase-locked loop, self-synchronization, virtual inertia emulation

I. INTRODUCTION

The primary frequency control of Synchronous Machines (SMs) is naturally based on the measure of the rotor angular speed of the machine itself. Since the dynamic response of synchronous machines imposes frequency variations, the rotor speeds are clearly the correct measurements to use for frequency control [1]. However, the situation changes substantially when it comes to defining the frequency signal for converter-interfaced generation. Unlike synchronous machines, and depending on the respective mode of operation, power electronics-based distributed generators do not necessarily impose the frequency at the point of connection. In particular, for the purpose of grid-following power converter control [2], a brittle local bus frequency signal has to be estimated by using available measurements, usually AC voltages and/or currents at the point of connection.

In order to regulate a grid-connected Voltage Source Converter (VSC) in the grid-following mode, a control sequence comprising system-level control with outer control loops and a synchronization unit, and device-level controls has become prevalent in industry for providing adequate VSC voltage, active and reactive power outputs [3], [4]. Additionally, the standard of having a Phase-Locked Loop (PLL) as a synchronization unit has been established [5], together with its numerous variants [6]–[9]. However, despite being widely utilized for frequency estimation, this additional, inherently nonlinear estimator introduces additional complexity to the system. As its input signal undergoes fast electromagnetic transients, the PLL can experience numerical issues and be affected by jumps and discontinuities following discrete events in the system such as faults or line outages [1]. Moreover, by the nature of its design, it introduces a non-negligible delay which can limit the performance of controls depending on frequency estimation, and in addition may be extremely difficult to tune [10], [11]. Several publications have recognized the impact of PLLs on the operation of non-synchronous generation [12], [13], but also highlighted the potential instabilities arising from high penetration of power converters employing such synchronization devices [4], [14], [15].

Recent studies have addressed some of the aforementioned issues by developing PLL-less converter regulation in the form of power-synchronization [16], virtual oscillator control [17], [18] and self-synchronizing synchronverters [11]. Although the proposed methods demonstrate numerous advantages, drawbacks have also been observed. Namely, the power-synchronization is mostly focused on VSC-HVDC applications and faces challenges with weak AC system connections. Virtual oscillator control faces obstacles in terms of reference tracking, whereas the synchronverter concept still requires a back-up PLL and improvements in operation under unbalanced and distorted grid voltages. Furthermore, all aforementioned controls apply to the grid-forming operation mode. While grid-forming and grid-supporting VSCs are an integral part of a future low-inertia power grid, the existing systems are primarily composed of renewable generation interfaced to the network via grid-following inverters.

Contrarily, a recently proposed VSC control method under the name of inducverter introduces the notion of a grid-connected converter operating under Induction Machine (IM) working principles and without a dedicated PLL unit [19]. Despite the concept still being at its early stages, it can potentially resolve the issues associated with the conventional PLL-based synchronization loop. However, [19] proposes a control design that integrates the system-level controller and synchronization unit into one compact structure. As a consequence, the frequency regulation and stabilization properties are attributed to the inducverter, whereas these functionalities are actually a sole consequence of the implemented PI-based droop power control. Additionally, the controller is implemented in a hybrid abc-dq frame, where the dq-axis current references are obtained according to the real and reactive power errors and translated to abc voltage references through an adaptive virtual impedance in the abc frame. This significantly complicates the analytical representation of the model and its analysis. As a consequence, for the controller tuning it is suggested to revert back to adopting the parameters of an existing induction machine. Finally, the fact that the cascaded inner control loop is replaced by a simple adaptive lead or lag compensator raises concerns in terms of fast voltage reference tracking and overcurrents during transients.

We improve the aforementioned design by incorporating the Virtual Induction Machine (VIM) framework in [20] as an independent synchronization unit into a detailed state-of-the-art converter model and analyze the time-domain performance of a grid-connected VSC through EMT simulations. More precisely, this paper reformulates the mathematical principles of an emulated induction machine from [19], [20] and extends them by deriving an appropriate index-1 DAE representation of such grid-connected VSC control scheme. This allows for a detailed small-signal analysis, which in turn helps to improve
the control tuning and reveals that replacing the PLL by a VIM results in a hybrid operation mode with both grid-forming and grid-following properties. It also enables us to draw accurate conclusions regarding the overall emulation properties and system response.

The remainder of the paper is structured as follows. In Section II, the mathematical model of an induction machine and the VIM control principles are presented. Section III describes the VIM control design and its implementation into a state-of-the-art VSC controller, as well as the model formulation as an index-1 DAE system. Section IV showcases the EMT simulation results and model validation, whereas Section V provides an insightful small-signal stability analysis. Finally, Section VI discusses the outlook of the study and concludes the paper.

II. VIRTUAL INDUCTION MACHINE STRATEGY

A. Induction vs. Synchronous Machine: Operating Principles

The physical mechanism behind the machine rotor movement and the subsequent synchronization to the grid are the most notable differences between the synchronous and induction machine. While the SM always operates at synchronous speed, the IM relies on a mismatch between the synchronous and induction speed. Nonetheless, such implementation should not be contrary to synchronous generators, induction machines do have an excitation system in the rotor. Thus, the ElectroMagnetic Force (EMF) induced in the rotor of an IM is a consequence of its rotation and the subsequent change of the magnetic flux linkage through the circuit. Given that the rotor is closed through an external resistance or a short-circuit ring, the induced EMF generates a current flow in the rotor conductor, which finally produces the synchronizing torque that drives the movement of the rotor. Hence, the IM can never reach the synchronous speed, since there would be no EMF in the rotor to continue its movement.

Considering the previously described properties, it can be observed that the IM with an arbitrary initial rotor speed close to the synchronous speed has a self-start capability, i.e., has the potential to synchronize with a grid of an unknown frequency and voltage magnitude. This implies that a VIM-based synchronization unit has the potential to replace the traditional PLL in the converter control design and eliminate its inherent drawbacks pertaining to time delay and stability margins. Nonetheless, such implementation should not be confused with system-level regulation, e.g., droop control, virtual synchronous machine or virtual oscillator control, as it does not inherently yield an emulation of inertia or frequency oscillation damping. Nevertheless, such services could easily be provided by an appropriate outer control loop, as will be shown later.

B. Induction Machine Emulation Strategy

For the purpose of emulating the operating principles of an IM through VSC control, let us observe a dynamical model of an IM [21] in a synchronously-rotating dq-frame and SI units:

\[
\begin{align*}
\dot{v}_d^r &= R_s i_d^r + \psi_d^r - \omega_s \psi_q^r, \\
\dot{v}_q^r &= R_s i_q^r + \psi_q^r + \omega_s \psi_d^r, \\
\dot{v}_d^s &= 0 = R_s i_d^s + \psi_d^s - \omega_s \psi_q^s, \\
\dot{v}_q^s &= 0 = R_s i_q^s + \psi_q^s + \omega_s \psi_d^s, \\
\dot{\psi}_d^r &= 0 = R_s i_d^r + \dot{\psi}_d^r + \omega_s^r \psi_q^r, \\
\dot{\psi}_q^r &= 0 = R_s i_q^r + \dot{\psi}_q^r + \omega_s^r \psi_d^r.
\end{align*}
\]

where \( v_d^r, v_q^r \in \mathbb{R}^2 \), \( v_d^s, v_q^s \in \mathbb{R}^2 \), \( \psi_s \in \mathbb{R}^2 \) and \( \psi_r \in \mathbb{R}^2 \) are the vectors of stator and rotor voltages and flux linkages, respectively, and \( R_s \in \mathbb{R}_{>0} \) and \( R_r \in \mathbb{R}_{>0} \) are the stator and rotor circuit resistances. The superscripts \( d \) and \( q \) refer to the vector component in the corresponding axis of the \( dq \)-reference frame, rotating at the synchronous speed \( \omega_s \). The first two expressions in (2) describe the stator voltage, whereas the latter two reflect the voltage circuit balance of a short-circuited rotor, hence \( v_r = 0 \). Note that the slip frequency \( \omega_s \) is involved in the last terms of (2c)-(2d). Moreover, the stator and rotor flux linkages can be described as

\[
\begin{align*}
\psi_s &= L_s i_s + L_m i_r, \\
\psi_r &= L_r i_r + L_m i_s,
\end{align*}
\]

with \( i_s = (i_d^s, i_q^s) \in \mathbb{R}^2 \) and \( i_r = (i_d^r, i_q^r) \in \mathbb{R}^2 \) denoting the vectors comprising stator and rotor current components in different axes, and \( L_s \in \mathbb{R}_{>0}, L_r \in \mathbb{R}_{>0}, L_m \in \mathbb{R}_{>0} \) being the stator, rotor and mutual inductance respectively.

The electric power \( P_e \in \mathbb{R} \) transferred between stator and rotor can now be expressed in terms of currents and flux linkages, either on the stator or on the rotor side as:

\[
P_e = \frac{3}{2} \left( \psi_d^s i_d^r - \psi_d^r i_d^s \right) = \frac{3}{2} \left( \psi_q^s i_q^r - \psi_q^r i_q^s \right),
\]

which yields the virtual electrical torque

\[
\tau_e = \frac{3}{2} \left( \psi_d^s i_d^r - \psi_d^r i_d^s \right) = \frac{3}{2} L_m \left( i_d^s \dot{i}_q^r - i_d^r \dot{i}_q^s \right).
\]

It is worth emphasizing that the expression for \( \tau_e \in \mathbb{R} \) is the same as for a synchronous machine [21].

Considering the fact that the converter model will not involve a PLL (and hence the synchronous speed and slip are unknown to the controller), the presence of \( \omega_s \) and \( \omega_r \), terms in (2a)-(2b) and (2c)-(2d), respectively, poses an obstacle for the control design. In other words, \( \omega_s \) and \( \omega_r \) are unknown variables and need to be computed internally based on available measurements. For that purpose, a field-oriented IM control first presented in [22] is employed. Considering that the direction of the \( dq \)-frame can arbitrarily be selected, we assume that in steady state the rotor flux is aligned with the \( d \)-axis, resulting in a simplified model with \( \psi_q = 0 \). The above assumption resembles the ones used in conventional PLLs, where the calculation of the voltage angle is based on aligning the voltage vector with the \( d \)-axis of a synchronously-rotating reference-frame [23].

According to the proposed approximation, (3b) is reformulated as

\[
\begin{align*}
\dot{i}_d^r &= \psi_q^r - L_m \dot{i}_q^d - L_r, \\
\dot{i}_q^r &= -L_m \dot{i}_d^q + L_r.
\end{align*}
\]

and the expressions for rotor voltage components in (2c) and (2d) can now be rewritten as

\[
\begin{align*}
0 &= R_s i_d^r + \dot{\psi}_d^r, \\
0 &= R_s i_q^r + \dot{\psi}_q^r + \omega_s \psi_d^r.
\end{align*}
\]

Substituting (6a) into (7a) yields

\[
\dot{i}_d^r = -R_s i_d^r = -R_s \left( \psi_q^r - L_m i_d^q \right),
\]
which in frequency domain can be expressed as
\[ \psi_{r}^{\prime d}(s) = \frac{R_r L_m}{R_r + s L_r} \psi_{r}^{d}(s) = K_{\psi}(s) \psi_{r}^{d}(s). \]  
(9)

Similarly, the slip frequency of the induction machine is computed by combining (6b) and (7b):
\[ \omega_{s} = -\frac{R_r}{\psi_{r}^{d}} \psi_{r}^{q} = \frac{R_r L_m}{L_r} \psi_{r}^{q}, \]
(10)
and subsequently substituting \( \psi_{r}^{d}(s) \) from (9), which gives
\[ \omega_{s}(s) = \left( \frac{R_r}{L_r} + s \right) \psi_{r}^{q}(s) = K_{\omega_{s}}(s) \psi_{r}^{q}(s). \]  
(11)

The expression (11) describes the dynamics of the slip frequency as a PD controller \( K_{\omega_{s}}(s) \) applied to the ratio of dq-components of the stator current. As such, this term is clearly sensitive to the variations in grid frequency and machine power output. Nevertheless, in order to complete the PLL-less control design, one needs to determine the synchronous speed. Having in mind that \( \omega_{s} = \omega_{r} + \Delta \omega_{r} \), an exact estimation of the rotor’s angular velocity is sufficient for achieving the targeted objective. Let us observe the power balance of an induction machine via the swing equation and the mechanical dynamics of the rotor, given by:
\[ J \dot{\omega}_{r} = \tau_{m} - \tau_{e} - \tau_{d}. \]  
(12)

Here, \( J \in \mathbb{R}_{>0} \) is the rotor’s moment of inertia, and \( \tau_{m} \in \mathbb{R}, \tau_{e} \in \mathbb{R} \) and \( \tau_{d} \in \mathbb{R} \) correspond to the mechanical, electrical and damping torque. By declaring \( \Delta \omega_{r} \in \mathbb{R} \) as a deviation of \( \omega_{r} \) from an initial (nominal) rotor speed\(^{1} \) \( \omega_{0} \in \mathbb{R}_{>0} \), the expression (12) becomes
\[ \Delta \dot{\omega}_{r} = \frac{1}{J} \left( \tau_{m} - \tau_{e} - \tau_{d} \right). \]  
(13)

By expressing all three torque components in (13) as functions of converter input signals, one could finalize the closed-form VIM formulation. We elaborate on mathematical details and derivations in the remainder of this section.

The electrical torque component is defined in (5), but can be further simplified by substituting the following expressions for stator flux linkage components:
\[ \psi_{s}^{d} = \left( L_s - \frac{L_m^2}{L_r} \right) i_{s}^{d} + \frac{L_m}{L_r} \psi_{r}^{d}, \]  
(14a)
\[ \psi_{s}^{q} = \left( L_s - \frac{L_m^2}{L_r} \right) i_{s}^{q}, \]  
(14b)
previously obtained from (3). The electrical torque in time-domain is therefore of the form
\[ \tau_{e} = \frac{3}{2} \frac{L_m}{L_r} \psi_{s}^{d} \psi_{s}^{q}, \]  
(15)
whereas in frequency domain it yields
\[ \tau_{e} = \frac{3}{2} \frac{L_m}{L_r} K_{\psi}(s) \psi_{s}^{d} \psi_{s}^{q} = K_{\psi}(s) \psi_{s}^{d} \psi_{s}^{q}. \]  
(16)

In (16), \( K_{\psi}(s) \) represents a first-order transfer function
\[ K_{\psi}(s) = \frac{3}{2} \frac{L_m}{L_r} K_{\psi}(s) = \frac{3}{2} \frac{R_r L_m}{2 R_r L_r + s L_r^2}, \]  
(17)
defined by the circuit parameters of the underlying induction machine, namely the rotor’s resistance and reactance as well as the mutual inductance.

On the other hand, the mechanical torque is determined by the machine’s mechanical power output and the angular speed of the rotor. Assuming a lossless converter, the mechanical input power of an IM can be approximated by the output power measured at the converter terminal (denoted by \( P_{c} \in \mathbb{R} \)), and given by
\[ \tau_{m} = \frac{p_{m} \omega_{r}}{\omega_{r}} \approx \frac{P_{c} \omega_{r}}{\omega_{r}}. \]  
(18)

Since the converter’s terminal current and voltage measurements, corresponding to stator current and voltage of a virtual induction machine\(^{2} \), are available and actively employed in device-level control (i.e., inner loop control presented in the following section), the output active power can be expressed as \( P_{c} := v_{f}^{T} i_{g} := v_{s}^{T} i_{s} \), therefore transforming (18) into
\[ \tau_{m} = -v_{s}^{d} i_{s}^{d} + v_{s}^{q} i_{s}^{q}, \]  
(19)

Finally, the damping torque \( \tau_{d} = D \Delta \omega_{r} \) is proportional to the rotor frequency deviation, which yields the following low-pass filter characteristic of the induction machine in frequency domain:
\[ \Delta \omega_{r} = \frac{1}{J + D} \left( \tau_{m} - \tau_{e} \right), \]  
(20)
where \( D = \mathbb{R}_{>0} \) denotes the damping constant. Substituting (16) and (19) into (20) results in a closed-form expression for \( \Delta \omega_{r} \) of the form:
\[ \Delta \omega_{r} = \frac{1}{J + D} \left( v_{s}^{d} i_{s}^{d} + v_{s}^{q} i_{s}^{q} \right) = 3 \frac{R_r L_m}{\omega_{0}^2 + D} \]  
(21)
which corresponds to \( \Delta \omega_{r} = F_{r}(u, p) \), with the measurement input vector \( u \in \mathbb{R}^4 \) and parameter vector \( p \in \mathbb{R}^6 > 0 \) defined as
\[ u = (v_{s}^{d}, v_{s}^{q}, i_{s}^{d}, i_{s}^{q}), \]  
(22a)
\[ p = (J, D, R_r, L_r, L_m, \omega_{0}^2). \]  
(22b)

Having obtained the desired analytical expressions for all frequency components, the synchronous speed can now be computed from the frequency slip \( \omega_{s} \) in (11) and \( \Delta \omega_{r} \) in (21), as follows:
\[ \omega_{s} = \omega_{r} + \Delta \omega_{r} = \omega_{r} + \omega_{s}^* + \frac{3 R_r L_m}{\omega_{0}^2 + D} \]  
(23)
\[ := F_{s}(u, p), \]
Similarly to any PLL, the angle reference is determined by integrating the frequency signal, i.e., \( \vartheta_{s} = \omega_{s} \). The resulting expression in (23) clearly shows that the closed-loop estimator \( F_{s}(u, p) \) emulates the synchronous speed and thus the synchronization properties of an IM based solely on the voltage and current measurements \( v_{f} \) and \( i_{g} \) at the converter terminal, therefore entirely replacing the conventional PLL-based synchronization. On the downside, the difference between the true synchronous speed and initial rotor speed setpoint can have an impact on synchronization accuracy. In particular, a proper selection of \( \omega_{0}^* \) prior to the grid connection of the VSC reduces \( \Delta \omega_{r} \) and the subsequent transients. Nevertheless, it is reasonable to assume that the VSC is connected to the
\[ ^{1} \text{We denote the initial (nominal) rotor speed by } (\dot{\theta})^* \text{ as it will later serve as an input setpoint to the VIM controller.} \]
\[ ^{2} \text{In other words, } v_{f} := v_{s} \text{ and } i_{g} := i_{s}, \text{ using the notation from Section III.} \]
grid during steady-state operation. Thus, a very basic PLL can be introduced only to estimate \( \omega_{p} \). However, even if this functionality is not available, any reasonable \( \omega_{p} \) guess will still allow the VIM to synchronize at the cost of some minor transients. A sensitivity analysis addressing the underlying phenomena is provided in Section IV-B. Another potential drawback of the frequency estimator \( (23) \) is the fact that the slip frequency \( \omega_{s} \) is computed using a PD controller imposed on the quotient of the current components in \( dq \)-frame. On the one hand, the derivative control is sensitive to fast signal changes. As the quotient of \( i_{q} / i_{d} \) can experience high oscillations during transients, the PD controller might be prone to overshoots and even instability. On the other hand, the given input-output structure of the PD controller might lead to an index-2 DAE form, which in turn increases the computational burden and imposes restrictions on the selection of the DAE solver. The aspects of DAE formulation will be discussed in detail in Section III-E.

III. VSC MODELING & CONTROL DESIGN

An overview of the prevalent control architecture for two-level power converters is shown in Fig. 1. In this configuration, an outer system-level control provides a reference for the converter’s terminal current that is subsequently tracked by a device-level controller. In the following, the model of a two-level voltage source converter is first presented and subsequently, the individual control blocks depicted in Fig. 1 are discussed.

A. Power Converter Model

The power converter model considered in this study is composed of a DC-link capacitor, a lossless switching block which modulates the DC-capacitor voltage \( v_{dc} \in \mathbb{R}_{>0} \) into an AC voltage \( v_{sw} \in \mathbb{R}^{2} \), and an output filter. Furthermore, we assume that the DC-source current \( i_{dc} \in \mathbb{R}_{>0} \) is supplied by a controllable source, in the form of energy storage or curtailed renewable generation, and can be used as a control input. Averaging the dynamics over a single switching period and expressing them in per-unit and \( dq \)-domain [4] yields:

\[
\frac{c_{dc}}{\omega_{b}} v_{dc} = -g_{dc} v_{dc} - i_{sw} + i_{dc},
\]

\[
\frac{\ell_{f}}{\omega_{b}} i_{f} = v_{sw} - v_{f} - (r_{f} + j \omega_{f} \ell_{f}) i_{f},
\]

\[
\frac{c_{f}}{\omega_{b}} \dot{i}_{f} = i_{f} - i_{g} - j \omega_{g} c_{f} v_{f},
\]

where \( c_{dc} \in \mathbb{R}_{>0} \) and \( g_{dc} \in \mathbb{R}_{>0} \) denote the DC capacitance and conductance, \( c_{f} \in \mathbb{R}_{>0} \), \( \ell_{f} \in \mathbb{R}_{>0} \), \( r_{f} \in \mathbb{R}_{>0} \) represent the AC filter capacitance, inductance and resistance, respectively. The DC-side current is represented by \( i_{sw} \in \mathbb{R} \), whereas \( i_{f} \in \mathbb{R}^{2} \), \( v_{f} \in \mathbb{R}^{2} \), and \( i_{g} \in \mathbb{R}^{2} \) denote the filter current, converter voltage and current injection into the system. System base frequency is represented by \( \omega_{s} \), and equals the nominal frequency, while \( \omega_{g} \in \mathbb{R}_{>0} \) is the normalized reference for the angular velocity of the \( dq \)-frame.

The converter is typically interfaced to the network through a transformer, with the transformer dynamics described by

\[
\dot{i}_{g} = \frac{\omega_{b}}{\ell_{t}} (v_{f} - v_{t}) - \left( \frac{r_{t}}{\ell_{t}} \omega_{b} + j \omega_{b} \omega_{g} \right) i_{g},
\]

where \( r_{t} \in \mathbb{R}_{>0} \) and \( \ell_{t} \in \mathbb{R}_{>0} \) denote the transformer’s resistance and inductance, and \( v_{t} \in \mathbb{R}^{2} \) is the voltage at the connection terminal.

B. System-Level Control

In the system-level control of grid-following VSCs, the measurements \( y_{k} = (v_{f}, p_{c}, q_{c}) \in \mathbb{R}^{4} \) are commonly the output voltage and the instantaneous active and reactive power given by

\[
p_{c} := v_{f}^{T} i_{g}, \quad q_{c} := v_{f}^{T} j v_{f} i_{g},
\]

where \( j \in \mathbb{R}^{2 \times 2} \) is the \( 90^\circ \) rotation matrix. Moreover, a synchronizing device - a PLL or a VIM - estimates the

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Fig. 1. General configuration of the implemented VSC control structure.
phase angle $\theta_s \in [-\pi, \pi)$ of the voltage $v_f$ as well as the synchronous (grid) frequency $\omega_s \in \mathbb{R}_{\geq 0}$ at the Point of Common Coupling (PCC), and provides them as reference $(\theta_s, \omega_s)$ to the device-level control. In addition, the outer control loop is used to calculate the current reference $i_f^* \in \mathbb{R}^2$ based on the mismatch between measured signals $u_d, u_q$ and prescribed setpoints $u_d = (\hat{p}_c, q_c, \omega_s, V_s^c) \in \mathbb{R}^4$.

The most common PLL implementation is a so-called type-2 SRF PLL [25], which achieves synchronization by diminishing the $q$-component of the measured voltage $v_f^q \in \mathbb{R}$ via PI control, thus aligning the $d$-axis of the Synchronously-rotating Reference Frame (SRF) with the output voltage vector $v_f$ [26]:

$$\dot{\varepsilon} = v_f^q, \quad \omega_s = \omega_s^* + K_P^q v_f^q + K_I^q \varepsilon, \quad \dot{\theta}_s = \omega_s - \omega_s^*, \quad (27a)$$

Here, $(K_P^q, K_I^q) \in \mathbb{R}_{\geq 0}^2$ are the proportional and integral control gains of the synchronization unit, $\omega_s^* = 1 \text{ p.u.}$ is the nominal angular frequency, and $\varepsilon \in \mathbb{R}$ is the integrator state.

Having determined the synchronous angle and frequency $(\theta_s, \omega_s)$, the outer control loop subsequently computes the current reference $i_f^* \in \mathbb{R}^2$, with $i_f = (p_f, q_f)$ being the current and voltage droop controller $(R_p^f, R_q^f) \in \mathbb{R}_{\geq 0}^2$ in combination with integral controllers $K_{P}^f, K_{I}^f \in \mathbb{R}_{\geq 0}$ and $K_{I}^f, K_{I}^f \in \mathbb{R}_{\geq 0}$. More precisely, the outer control loop, described by internal state variables $(\hat{p}_c, \hat{q}_c, x_c) \in \mathbb{R}^4$, regulates the power output $(p_c, q_c)$ to its respective setpoint $(p_d, q_d)$, as follows:

$$\dot{\hat{p}}_c = K_{P}^f (p^*_c - p_c - R_p^f (\omega_s - \omega_s^*)), \quad \dot{\hat{q}}_c = K_{I}^f (q^*_c - q_c - R_q^f (v_f^q - V_s^c)). \quad (28a)$$

Due to the $P - f$ and $Q - V$ droop characteristics, the active power reference is adjusted in response to a deviation of the measured frequency $\omega_s$ with respect to the frequency setpoint $\omega_s^*$, whereas the reactive power reference is modified according to the mismatch between the magnitude of the output voltage $||v_f||$ and the converter voltage setpoint $V^c$. Hence, the internal state vector of the system-level controller is $x_c = (\varepsilon, \omega_s, \hat{p}_c, \hat{q}_c)$.

The computed active and reactive power references are then transformed into the corresponding current reference signal $i_f^*$, with two commonly used implementations for balanced systems: a constant current and a constant power mode. The first approach directly feeds the power references to the device-level control $i_f^* = (\hat{p}_c, \hat{q}_c)$, while the second mode adjusts them based on output voltage measurement such that the converter’s power output is kept constant:

$$i_f^a = \frac{v_f^d \hat{p}_c + v_f^q \hat{q}_c}{||v_f||}, \quad i_f^q = \frac{v_f^d \hat{p}_c - v_f^q \hat{q}_c}{||v_f||}. \quad (29)$$

C. Device-Level Control

This control layer provides both AC and DC-side reference signals for the VSC device. The AC-side controller operates in a synchronously-rotating $dq$-frame, with the reference angle $\theta_s$ and velocity $\omega_s$ provided by the synchronization unit. In particular, given a current reference $i_f^* \in \mathbb{R}^2$ in $dq$-coordinates defined by $(\theta_s, \omega_s)$, the device-level control is described by a current controller representing the inner control loop and computing a switching voltage reference $v_{sw}^* \in \mathbb{R}^2$ [27], as follows:

$$\gamma^* = i_f^* - i_f, \quad (30a)$$

$$v_{sw}^* = K_P^d (i_f^* - i_f) + K_I^d \gamma + K_f^d v_f + j \omega_s f i_f, \quad (30b)$$

where $K_P^d, K_I^d \in \mathbb{R}_{>0}, K_f^d \in \mathbb{R}_{\geq 0}$ and $K_f^d \in \{0, 1\}$ are the respective proportional, integral, and feed-forward gains, $\gamma \in \mathbb{R}^2$ represents the integrator state, and superscript $d$ denotes the current controller. Note that the angular velocity $\omega_s$ of the SRF is reflected in the last term of (30b).

Finally, the DC voltage $v_{dc}$ is controlled through the DC current source and a PI controller, as follows:

$$\chi = v_{dc}^* - v_{dc}, \quad (31a)$$

$$i_{dc} = K_P^{dc} (v_{dc}^* - v_{dc}) + K_I^{dc} \chi + K^{dc} i_{dc}^*, \quad (31b)$$

with DC voltage setpoint $u_d = v_{dc}^* \in \mathbb{R}_{\geq 0}$ being an external control input, $\chi \in \mathbb{R}$ is the internal state variable, and proportional, integral, and feed-forward gains denoted by $K_P^{dc}, K_I^{dc} \in \mathbb{R}_{>0}, K_f^{dc} \in \mathbb{R}_{\geq 0}$, and $K_f^{dc} \in \{0, 1\}$, respectively. The DC current reference $i_{dc}^* \in \mathbb{R}_{\geq 0}$ is determined by the operating point $(V_s^c, p_c, q_c)$ and the converter losses, which indicates that for $v_{dc} = v_{dc}^*$, the DC-side current will be $i_{dc} = i_{dc}^*$.

Having computed the AC voltage $v_{sw}$ and DC current $i_{dc}^*$ reference signals, the device-level control output $r_d = (\theta_s, \omega_s, v_{sw}, i_{dc})$ is sent to the power converter, with $(\theta_s, \omega_s)$ being the angle and frequency references obtained by a synchronization unit within the system-level control. The state vector of the controller comprises AC and DC current computations in (30)-(31) and is described by $x_d = (\gamma, \chi) \in \mathbb{R}^4$.

Unlike the widely used virtual synchronous machine control [28], [29], and in contrast to the claims raised in [19], the inertia and damping in (21) will not be reflected in the converter’s frequency response and oscillation damping performance. They do, however, contribute to a more robust and resilient frequency estimation technique, as will be later shown in Section VI.

D. Grid Synchronization: PLL vs. VIM

The traditional PLL unit previously presented is now replaced by the proposed VIM synchronization technique, with the converter model, system-level and device-level control structure preserved. For comparison and clarity, the control-block implementation of both synchronization units (i.e., a type-2 SRF-PLL and a VIM) is depicted in Fig. 2.

The basic structure and operating principles of a type-2 SRF-PLL [25] have already been presented in Section III-B. This synchronization device acts as an observer and tracks the synchronous speed by measuring the stationary output voltage $v_f$, transforming it into an internal $dq$-SRF, and passing it through a PI-controller $(K_P^d, K_I^d)$ that acts on the phase angle difference:

$$\omega_s = \omega_0 + \left(\frac{K_P^d + K_I^d}{s}\right) v_f^q, \quad (32)$$

with $\omega_0 \in \mathbb{R}_{>0}$ denoting the nominal angular velocity. The synchronization is achieved by aligning the $d$-axis of the internal SRF with the stationary abc-frame and diminishing the $q$-component, as described in [2], [25] and illustrated in Fig. 2. It should be pointed out that the combined Clarke and Park transformation within the PLL is completely independent of the transformation defined by the angle and frequency of the active power controller used for the rest of the VSC control scheme and therefore introduces a second SRF into the system. Hence, the internally computed filter voltage $v_f$ must be aligned with the corresponding voltage vector in the main SRF.

The VIM control design illustrated in Fig. 2 is based on (23). Similarly to other synchronization methods including the PLL, the unknown grid frequency can be obtained by simply...
measuring the three-phase current and voltage at the filter output \( (i_{abc}^f, v_{abc}^f) \). Note that the VIM also operates in a separate SRF defined by the internally computed synchronization angle \( \theta_s \), and requires alignment of the transformed \( dq \)-quantities with the main SRF. As previously explained in Section II, another necessary input for the controller is the initial rotor frequency \( \omega_0^s \), which determines the VIM’s oscillation level at start-up. However, the requirement for the initial guess of \( \omega_0^s \) is not very strict, as it should only be “close enough” to the synchronous speed and subsequently let the emulated physical machine bring the VSC to synchronism. Moreover, in order to cope with potential frequency slip spikes during transients, induced by the PD control \( K_r(s) \) acting on the quotient \( i_g^d/i_g^q \) in (11), the frequency slip is constrained by the saturation limits \( \omega_s \in [\omega_{\bar{s}}, \omega_{\bar{s}}]\).

### E. DAE Formulation

The DAE formulation of a conventional converter control scheme has previously been presented in detail. As shown in Section III-B, the PLL controller (32) can be expressed as a second-order dynamic system with state vector \( x_{\text{PLL}} := (\epsilon, \theta_s) \in \mathbb{R}^2 \) and algebraic output vector \( y_{\text{PLL}} := \tau_s \).

In contrast, obtaining an appropriate (i.e., index-1) DAE representation of the VIM controller is not straightforward due to the aforementioned issues pertaining to the computation of the slip frequency in (11), as well as the fact that the electrical torque is described by a first-order transfer function of the slip frequency \( \omega_s \) in (11), as well as the fact that the computation of the slip frequency in (11) is constrained by the saturation limits \( \omega_s \in [\omega_{\bar{s}}, \omega_{\bar{s}}] \). Moreover, we define \( \varphi_d := \dot{\omega}_d \) and \( \varphi_q := \dot{\omega}_q \), and apply the quotient rule to (33), which yields

\[
\varphi = \frac{\dot{i}_g^q i_d^q - \dot{i}_g^d i_d^q}{i_g^q} = \frac{1}{i_g^q} \varphi_d - \frac{i_g^q}{i_g^q} \varphi_q.
\]

Note that the terms \((\varphi_d, \varphi_q) \in \mathbb{R}^2\) correspond to the differential equation (25), describing the dynamics of the current flowing through a transformer. Redefining (25) in SI yields

\[
\begin{aligned}
\dot{\varphi}_d &= \frac{\omega_0^s I_b}{\ell_t} (v_d^f - v_d^q) - I_b \left( \frac{r_t}{\ell_t} \omega_0^s i_d^q + \omega_0^s \omega_s i_g^q \right), \\
\dot{\varphi}_q &= \frac{\omega_0^s I_b}{\ell_t} (v_q^f - v_q^q) - I_b \left( \frac{r_t}{\ell_t} \omega_0^s i_q^q + \omega_0^s \omega_s i_g^q \right).
\end{aligned}
\]

Here, \( \omega_0 \in \mathbb{R}_{>0} \) and \( I_b \in \mathbb{R}_{>0} \) denote the base angular velocity and current used for conversion between the per unit and SI. The rest of the notation is adopted from (25). By rewriting (16) and (21) as

\[
\begin{aligned}
\dot{\tau}_e &= -\frac{R_r}{L_r} \tau_e + \frac{3R_r L_m^2}{2L_r} \dot{i}_g^q, \\
\Delta \omega_r &= \frac{1}{J} \left( \frac{v_d^f i_g^q}{\omega_0^s + \Delta \omega_r} - \tau_e \right) - \frac{D}{J} \Delta \omega_r,
\end{aligned}
\]

and transforming (11) and (23) respectively into

\[
\begin{aligned}
\dot{\omega}_s &= \frac{R_r}{L_r} \dot{\tau}_e + \varphi, \\
\omega_s &= \omega_{\bar{s}} + \Delta \omega_r + \bar{\omega}_s,
\end{aligned}
\]

we obtain an index-1 DAE form comprising (36) as differential equations and (34)-(35), and (37) as algebraic equations. The state vector representing the VIM dynamics is thus \( x_{\text{VIM}} := (\tau_e, \Delta \omega_r, \bar{\omega}_s) \in \mathbb{R}^3 \), and is of the same order as the PLL controller, whereas the vector of algebraic variables consists of \( y_{\text{VIM}} := (\varphi, \varphi_d, \varphi_q, \bar{\omega}_s, \bar{\omega}_r) \in \mathbb{R}^5 \). The newly defined variable \( \bar{\omega}_s \in \mathbb{R} \) represents the unsaturated frequency slip.

In order to include the frequency slip saturation limits into the DAE model, we employ the well-known expressions for the minimum and maximum of two variables:

\[
\begin{aligned}
\min\{a, b\} &= \frac{a + b - |b - a|}{2}, \\
\max\{a, b\} &= \frac{a + b + |b - a|}{2},
\end{aligned}
\]

and re-declare \( \omega_s \in [\bar{\omega}_s, \bar{\omega}_s] \) as the saturated slip counterpart of \( \omega_s \) determined by the following algebraic equation:

\[
\omega_s = \frac{1}{2} \left( \bar{\omega}_s + \omega_s + |\omega_s - \bar{\omega}_s| \right) + \omega_s - \frac{1}{2} (\omega_s + \omega_s + |\omega_s - \omega_s|).
\]

The derivation of (39) is given in Appendix A.

By including \( \bar{\omega}_s \) into \( \bar{y}_{\text{VIM}} \) we complete the DAE formulation of the VIM controller. Combining it with the internal dynamics of converter’s system- and device-level control, as well as with the network-side dynamics pertaining to the device model representation, results in a 16th-order converter model for both the PLL and VIM-based synchronization principle.

---

\(^*\)The state vectors \( v_f, i_f \) and \( v_t \) are included in the dynamical model of the converter control and hence do not contribute to the model order of the synchronization unit.
IV. Simulation Results

A. System Setup and VIM Tuning

In this section, the performance of the proposed control scheme is studied for various transient scenarios using the detailed EMT model of a grid-following VSC connected to a stiff grid, developed in MATLAB Simulink and described in Section III. We focus on real-time operation events such as start-up and synchronization, response to setpoint variation (i.e., voltage and power reference tracking) and islanding (i.e., converter disconnection from the main grid). Finally, the impact of the initial rotor speed estimate $\omega_r^\ast$ on the converter’s synchronization process with the grid is studied.

Understandably, the VIM’s response is highly dependent on the selection of tuning parameters, in particular the parameters of the equivalent physical induction machine. This mainly refers to the rotor resistance and inductance, but also to the mutual inductance included in the transfer function $K_v(s)$. Additionally, proper values for the moment of inertia and damping are crucial for the dynamics of the rotor frequency, which in turn affects the sinusoidal nature of the voltage and current at the converter terminal. The initial parameters used in this case study have been obtained from a physical induction generator of a 1.5 MW type-I wind turbine, with the most relevant parameters for VIM design listed in Table I. Note that the VIM input frequency is set to $f_0^\ast = 50$ Hz.

The dynamics of the frequency slip are described by a PD controller $K_v(s)$ in (11), with the proportional gain $K_v^P = \frac{R_v}{L_v}$ given by the IM parameters and a unity derivative gain $K_v^D = 1$. Nevertheless, such high value of the derivative gain can destabilize the converter control during transients. This problem is overcome by re-tuning the PD controller via the Ziegler-Nichols method [30], i.e., determining the optimal $K_v^D$ component while assuming the existing proportional gain $K_v^P$, which results in $K_v^D = 0.001$. The parameters of the system-level control (i.e., droop and integral gains included in the power control) and the device-level control (i.e., PI gains of the current controllers) have been adopted from [4] in order to test the plug-n-play properties of the VIM.

B. Case Studies

First, the connection and synchronization of a VIM-based converter to the grid is studied. The VSC is connected to the grid at $t = 0 \, \text{s}$, with the initial input frequency $f_0^\ast = 50$ Hz set equal to the grid frequency. The voltage reference is initialized at $V_\ast = 1$ p.u., whereas the active and reactive power setpoints are set to $p_\ast = 0.5$ p.u. and $q_\ast = 0$ p.u., respectively.

The transient response illustrated in Fig. 3 confirms the soft-start and self-synchronization capabilities of the VIM as well as an adequate oscillation damping characteristic. The setpoints are correctly tracked and the voltage and current overshoots during start-up are acceptable. Furthermore, the start-up overshoots can be avoided if the VSC is slowly ramped-up from the zero power setpoint. Note that the initial overcurrents are in accordance with the characteristic response of an induction generator, but can also be assigned to numerical initialization of the model. The initial transients are better understood by observing the estimated synchronous frequency $f_s$ and its time-variant components $f_e$ and $\Delta f_e$. The frequency slip is very volatile during the first 300 ms, unlike the rotor’s frequency dynamics term $\Delta f_r$, which can be associated with two aspects: (i) the frequency slip is proportional to the quotient $\frac{q_{sl}}{p_{sl}}$, which can reach very high values when $\frac{q_{sl}}{p_{sl}} \approx 0$; (ii) $K_v(s)$ behaves as a PD controller, with its derivative actions ($K_v^D$) being mostly utilized throughout the first 300 ms of the start-up. After 500 ms, both frequency components stabilize and the synchronous and converter output frequency reach a steady state value of $f_s \approx 50.008$ Hz.

| Parameter                  | Symbol | Value  |
|----------------------------|--------|--------|
| Nominal rated power        | $P_n$  | 1.5 MW |
| Total moment of inertiaa   | $J$    | 152.14 kg m² |
| Damping constant           | $D$    | 10 N m/s |
| Rotor resistance           | $R_r$  | 0.00005 p.u. |
| Rotor inductance           | $L_r$  | 0.05 p.u. |
| Mutual inductance          | $L_m$  | 0.6 p.u.  |
| Initial frequency setpointb| $f_0^\ast$ | 50 Hz |

| Parameter                  | Symbol | Value  |
|----------------------------|--------|--------|
| aCorresponding to the normalized inertia constant $H = 5 \, \text{s}$. |
| bCorresponding to $\omega_0^\ast = 314.16$ rad/s. |
Another important aspect of control performance is the reference tracking, i.e., converter’s ability to follow sudden changes in voltage and power setpoints. Both scenarios are simulated independently, with setpoint changes occurring at $t = 0.5\, \text{s}$ in each case. The voltage reference exhibits a step increase of 5\%, whereas the active power reference increases by 20\%. Both step changes last for 1.5\,s before setpoints returning to their original values.

The results depicted in Fig. 4 indicate that power and voltage reference tracking is achieved within reasonable time, as both the output voltage (i.e., the voltage $v_f$ after the filter) and active power follow closely the respective setpoints. This is an expected outcome, as the reference tracking capability comes from the proper design of system- and device-level controls, which remain intact compared to the model presented in Sec. III. Nevertheless, an inefficient synchronization device would have deteriorated the performance, which is clearly not the case for the VIM. A somewhat delayed response and excessive overshoot in the voltage output is solely an artifact of the employed PI tuning of the inner control loops, since the selected tuning favors the power tracking over the voltage tracking, and can easily be addressed with a different set of PI control gains.

Let us recall that one of the control inputs to the VIM is the so-called “estimated” initial rotor frequency $f_0^*$. Previous examples have shown that under the $f_s \approx f_0^*$ condition the system experiences a satisfactory performance with good synchronization and damping properties. However, having knowledge of the exact grid frequency prior to the connection of the VSC might not be achievable in real-world applications. Thus, the impact of an inaccurate $f_0^*$ guess on converter synchronization is investigated by studying the frequency and voltage response for $f_0^* = 49.9\, \text{Hz}$ and $f_0^* = 50.1\, \text{Hz}$, and comparing it against the results presented in Fig. 3 for the “ideal” case where $f_0^* = 50\, \text{Hz}$. The analysis is focused on the first second of the response after start-up, with the frequency and voltage response presented in Fig. 5. We can conclude that the synchronization is successfully achieved within 500\,ms for all three scenarios, with no distinctive differences between the three initialization points. Hence, as stated previously in Section II, the VIM will experience fast synchronization for any reasonable guess of the initial rotor frequency prior to the grid connection.

Finally, we investigate the behavior of the VIM-controlled converter after the disconnection from the network. One of the main benefits of employing a VIM synchronization scheme is its standalone capability, i.e., the ability to operate even after being disconnected from the main grid if desirable; a characteristic of a physical induction machine. Such property is not attainable by traditional grid-following VSCs employing a phase-locked loop. The islanding is simulated\(^3\) at $t = 0.5\, \text{s}$ and the inverter response is showcased in Fig. 6. The PLL-based unit loses synchronism immediately after the disconnection, with frequency plummeting below 49\,Hz within 3\,s of the disconnection. On the other hand, after some negligible initial transients, the VIM restores the converter to a new steady-state point and proceeds with normal operation.

**V. Stability Analysis**

Having validated the theoretical concept of VIM through EMT simulations, we dedicate this section to small-signal analysis of the DAE model presented in Section III. Moreover, we consider three different converter operation modes: (i) a grid-following VSC with a PLL; (ii) a grid-following VSC with a VIM; and (iii) a grid-forming VSC from [4]. Some very insightful observations can be made by studying the stability

\(^3\)We assume $t = 0$ to be the time instance at which the initialization transients have decayed and the system is in synchronism.

---

**Fig. 4.** VIM response to the variation of controller setpoints: (a) Variation of the voltage setpoint. (b) Variation of the active power setpoint.

**Fig. 5.** Impact of initial rotor frequency term on the synchronization process of a VIM during start-up: synchronous VIM frequency (top) and converter output voltage (bottom).

**Fig. 6.** Behavior of different synchronization units after grid disconnection.
maps in the $R_q^c - R_d^c$ plane of a single inverter connected to a stiff grid. One such map is illustrated in Fig. 7 for a wide range of active and reactive power droop gains. Clearly, the two grid-following controllers have significantly different stability regions. However, for a reasonable tuning range considered in practice (i.e., $R_q^c < 0.1$ p.u. and $R_d^c < 0.05$ p.u.), the two regions are identical. Nevertheless, a very interesting observation can be made by comparing the aforementioned stability maps to the corresponding map of a grid-forming unit. Indeed, the stable region of a grid-forming VSC closely resembles the one of a VIM-based inverter for the whole parameter range, suggesting that replacing a PLL with a VIM-based synchronization device might provide some “forming-like” properties to the grid-following VSCs. A notion that could be substantiated by the fact that the VIM has standalone capabilities.

We continue this line of investigation by comparing the stability margins of all three converter modes depending on the strength (i.e., the short-circuit ratio $\mu \in \mathbb{R}$) of the grid at the connection terminal. As can be seen from Fig. 8, the critical SCR for a PLL-based inverter is $\hat{\mu} = 1$, whereas the grid-forming VSC does not impose any requirements on the minimum grid strength. In that sense, the VIM-controlled inverter again resembles the grid-forming one, since it can withstand any SCR level. Moreover a similar analysis is done with respect to the maximum permissible penetration of inverter-interfaced generation. The movement of the most critical eigenvalue, i.e., the evolution of its real part with the increase in VSC installation level $\eta \in [0, 100]\%$, is depicted in Fig. 9 for the two-bus test case comprising one synchronous and one inverter-interfaced generator. While the maximum permissible installed capacity of PLL-based units is slightly below 70%, this level can be increased by $\approx 7\%$ by substituting the PLL with the VIM, therefore almost reaching the maximum permissible penetration of grid-forming VSCs of $\eta_{\text{max}} = 78.5\%$. Note that the eigenvalue movement for scenarios $SG - VSC_{\text{pll}}$ and $SG - VSC_{\text{vim}}$ is taken from [4]. This indicates that, despite not being able to provide black start and independently generate stable frequency reference, the VIM-based grid-following converters clearly share conceptual similarities with the grid-forming mode of operation. Furthermore, it should be pointed out that the instabilities arising at the maximum converter installation levels obtained from Fig. 9 are solely associated with device-level control, with the highest state participation given in Table II. Hence, the instability is related to the inner voltage and current control and is not affected by the selection of the synchronization unit.

Finally, we briefly address the topic of control tuning pertaining to the proposed VIM design. As previously pointed out in Section II, the tuning of a VIM can be based on the parameters of a physical induction machine. This particularly applies to the selection of resistance and inductance values involved in the PD controller $K_r(s)$ in (11) and the transfer function $K_\nu(s)$ in (17). However, emulating the existing machine has its drawbacks, as it does not guarantee an optimal control performance. This was already discussed in Section IV, as the derivative gain $K^D$ had to be re-tuned in order to achieve a satisfactory response during transients. Moreover, some induction generators might simply result in an unstable system. One such example is provided in Fig. 10 with the stability surface of a VIM-based converter illustrated in the $R_q - L_r - L_m$ space. It reflects the possible obstacles one could face when applying heuristic methods for VIM tuning, as the surface in Fig. 10 has distinctively nonlinear segments. Nevertheless, since VIM is an emulation of a physical machine, the tuning process does not have to incorporate exact physical parameters, but can rather use them as an initial starting point when designing the controller. Therefore, the multi-dimensional stability mapping, such as the one illustrated in Fig. 10, can be of great importance when optimizing the VIM performance.

**VI. Conclusion**

This paper proposes a novel control strategy for synchronization of grid-connected VSCs based on the emulation of induction generator principles. For that purpose, a detailed mathematical model of an induction machine was derived and implemented within a detailed converter control scheme. It therefore eliminates the need for a dedicated PLL unit, while also providing additional services such as standalone operation after an islanding event. Moreover, similar to the PLL, the

| TABLE II | STATE PARTICIPATION [%] IN THE UNSTABLE MODES. |
|----------|------------------------------------------------|
| State    | $\xi^d$ | $\xi^q$ | $\xi^q_f$ | $\xi^d_f$ | $\xi^g$ | $\xi^g_f$ | $\xi^0$ | $\xi^p$ | $\xi^p_f$ | other |
| PF       | 30.2    | 30.5    | 8.1       | 8.4       | 6.2     | 6.8       | 4.5     | 4.1     | 4.5       | 1.2   |

**Fig. 7.** Stability maps of different converter modes in the $R_q^c - R_d^c$ plane. Shaded areas correspond to stable regions of PLL and VIM-based converters, whereas the region left from the yellow line is stable for forming VSCs.

**Fig. 8.** SCR influence on system stability for different converter modes.

**Fig. 9.** Impact of penetration of inverter-based generation on system stability for different converter operation modes.
VIM also has plug-n-play properties and can be combined with any system-level and device-level control method. The EMT simulations showcase smooth start-up and synchronization to the grid and an accurate computation of grid frequency, independent of the initial rotor speed input. The proposed synchronization device does not hinder the performance of other converter controls and preserves accurate voltage and power setpoint tracking. Finally, it was shown that a VIM-based grid-following converter resembles a grid-forming unit in certain operational aspects, therefore allowing for a higher penetration of VIM-controlled DGs compared to the PLL-based ones. A path for future work should go in a similar direction and further explore the dynamical properties of a VIM, as well as its interactions with conventional SGs and different converter control schemes.

**APPENDIX A**

**DERIVATION OF SATURATED FREQUENCY SLIP**

Let us first recall the expressions for the minimum and maximum of two variables from (38). The goal is to impose the lower and upper saturation limits in an unsaturated frequency slip signal \( \hat{\omega}_r \), i.e., to obtain a new signal \( \omega_r \) such that \( \omega_r \in [\omega_{\hat{r}}, \omega_r^*] \). This is equivalent to finding the maximum of the underlying signal and its lower bound (let us denote it by \( \omega_{\hat{r}} \)), and subsequently finding the minimum of that signal and the upper bound. In other words,

\[
\hat{\omega}_r = \max \{ \omega_{\hat{r}}, \omega_r^* \},
\]

\[
\omega_r = \min \{ \omega_{\hat{r}}, \omega_r^* \},
\]

We can rewrite (40) as:

\[
\hat{\omega}_r = \frac{1}{2} \left( \omega_{\hat{r}} + \omega_r^* + |\omega_{\hat{r}} - \omega_r^*| \right),
\]

\[
\omega_r = \frac{1}{2} \left( \omega_{\hat{r}} + \omega_r^* - |\omega_{\hat{r}} - \omega_r^*| \right),
\]

and subsequently substitute (41b) into (41a), resulting in the expression given by (39).

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