Modeling and Simulation of Vibrating Online Viscometer System

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Abstract. Aiming at the present situation of the difficulties in the analysis and research of the vibration type on-line viscometer system, the structural characteristics and mathematical model of the vibrating online viscometer are analyzed and studied. Based on this, the closed-loop control principle is further elaborated. At the same time, the simulation model of the vibrating online viscometer system was built by closed-loop control technology. The MATLAB simulation results verify the accuracy of the closed-loop control and the effectiveness of the simulation model, which provides a theoretical basis and guiding ideas for the system design and debugging of the vibrating online viscometer.

1. Introduction
The vibrating viscometer studied is based on the viscous damping effect. When the vibrating structure is subjected to torsional vibration in the fluid to be tested, the amplitude will be attenuated. The greater the viscous damping, the greater the degree of attenuation. In order to maintain the amplitude, it is necessary to provide the energy consumed by the viscous damping from the outside, and the viscosity of the fluid at this time has the following correspondence with the energy provided by the outside[1][2]:

\[ E \propto \sqrt{\eta \rho} \]  

(1)

Where \( \eta \) is the viscosity of the fluid and \( \rho \) is the density of the fluid. The amplitude is kept constant through closed-loop control so that the fluid viscosity can be converted to the drive current. This closed-loop instrument improves response speed and reduces nonlinearity.

2. System Model
The simplified model of the vibrating online viscometer is shown in Figure 1. During the viscosity measurement process, the sensitive component is subjected to the viscous damping of the fluid, and the change in the magnitude of the damping reflects the change in viscosity, and the vibration amplitude of the torsion bar is changed by the spindle. The amplitude measuring circuit measures the change value of the amplitude, the single chip receives the amplitude change signal and changes the magnitude of the driving current, and controls the driving mechanism to maintain the amplitude of the torsion bar before the viscosity changes, and finds the relationship between the driving current and the viscosity of the fluid, and can calculate the viscosity of the fluid medium[3].
Figure 1. Vibration viscometer simplified model.

Separating the spindle, sensor and drive mechanism parts from the sensor. The equivalent model of the system is shown in Figure 2. It consists of a damper, an inertial disc and an elastic rod.

Figure 2. System equivalent model.

In order to analyze the motion characteristics of the vibration system, the system needs to be simplified into a dynamic model to determine the motion mode and degree of freedom of the system. The system is a torsional vibration system. The torsion angle can be used as the independent coordinate of the vibration system to determine the motion state of the system, and the torsional vibration system model with feedback control can simplify the system into a single-degree-of-freedom second-order closed-loop control system\cite{4}\cite{5}. The dynamic equation is:

\[ M_m \theta + C_m \frac{d\theta}{dt} + J_m \frac{d^2\theta}{dt^2} = 0 \]  

In the formula, \( M_m \) is the driving torque, \( K_m \) is the equivalent stiffness, \( C_m \) is the equivalent damping, \( J_m \) is the equivalent moment of inertia, \( \theta \) is the torsion angle. Among them, the natural frequency of the system is \( \omega_0 = \sqrt{K_m/J_m} \).

3. Closed Loop Control Method

According to the fluid mechanics, when the sensitive component of the viscometer is subjected to torsional vibration in the viscous medium, a viscous moment is generated under the action of shear stress, and the size is (3), which hinders the relative movement of the sensitive component in the medium. Further, the amplitude of the drive crossbar is reduced.

\[ M = \mu \int_{\Omega} K \cdot \Omega \cdot ds \]  

Where \( \Omega \) is the contact area of the sensitive element with the medium, and \( K \) is the proportionality factor.

When the viscosity of the medium is constant, a constant driving current is maintained, so that the sensitive component is torsionally vibrated at a resonant frequency in the viscous medium under the driving of the driving crossbar, and maintain a certain vibration amplitude, and measuring amplitude by amplitude detection circuit. when changing the medium, the viscosity of the medium becomes larger or smaller, which will affect the amplitude of the torsional vibration to become smaller or larger.
If the magnitude of the drive current is controlled by closed-loop feedback so that the torsional vibration amplitude remains the same, the relationship between the drive current and the viscosity of the fluid can be found. The closed loop control process is shown in Figure 3.

![Figure 3. Closed loop control flow char.](image)

4. Simulation analysis

4.1. Model construction

The system dynamics model can be seen from the previous analysis:

\[ J_m \frac{d^2 \theta}{dt^2} + C_m \frac{d\theta}{dt} + K_m \cdot \theta = M_m \]  \hspace{1cm} (4)

If the input excitation current is \( I = A \sin \omega t \), the system dynamics model can be written as:

\[ J_m \cdot \theta''(t) + C_m \cdot \theta'(t) + K_m \cdot \theta(t) = H \cdot I(t) \]  \hspace{1cm} (5)

If the input current is used as the excitation and the torsion angle is the response, by Laplace transformation of the above formula, we can get:

\[ J_m \cdot s^2 \cdot \theta(s) + C_m \cdot s \cdot \theta(s) + K_m \cdot \theta(s) = H \cdot I(s) \]  \hspace{1cm} (6)

So the transfer function of the system is:

\[ G(s) = \frac{\theta(s)}{I(s)} = \frac{H}{K_m \cdot s^2 + C_m \cdot s + K_m} = \frac{H \cdot K_m}{s^2 + C_m/s^2 + s + K_m} \]  \hspace{1cm} (7)

The general expression of the second-order oscillation link transfer function is:

\[ G(s) = \frac{K}{T^2 s^2 + 2 \xi Ts + 1} = \frac{K \cdot \omega_n^2}{s^2 + 2 \xi \omega_n s + \omega_n^2} \]  \hspace{1cm} (8)

So the system scale factor is \( K = \frac{H}{K_m} = \frac{n \pi BDL}{2K_m} \), natural angular frequency is \( \omega_n = \sqrt{K_m/J_m} \), time constant is \( T = 1/\omega_n = \sqrt{J_m/K_m} \), damping ratio is \( \xi = C_m/2\sqrt{K_m J_m} \).

The system is accurately calculated, system stiffness is \( K_m = 337 N/m \), system moment of inertia is \( J_m = 56 \times 10^{-6} kg \cdot m^2 \), system damping is \( C_m = 1.91 \times 10^{-5} \mu \), system drive torque is \( M_m = 7.85 I \), the time constant obtained by further calculation is \( T = 1.3 \times 10^3 \), damping ratio is \( \xi = 6.95 \times 10^{-5} \mu \), proportional coefficient is \( K = 0.023 \), so the system transfer function is as follows, the simulation diagram shown in Figure 4.

\[ G(s) = \frac{23000}{1.66s^2 + 5.56 \mu s + 1000000} \]  \hspace{1cm} (9)
4.2. Simulation analysis results
When the input is $I = \sin t$ and the medium is water, the dynamic viscosity of water is known to be 1 and the output amplitude is 0.04.

![Figure 5. Media water current amplitude 1 output.](image)

If the fixed medium is for honey $\mu = 3 \text{Pa} \cdot \text{s}$, change the input current to a magnitude of 1.5 and the output amplitude is 0.035.

![Figure 6. Media honey current amplitude 1.5 output results.](image)

According to this idea, when an output amplitude is fixed, the input current amplitude can be changed to reach the output amplitude for different viscosity media. The above simulation reflects the relationship between viscosity and excitation current amplitude. To find out the relationship between viscosity and current, the amplitude should be converted into the actual current size. The amplitude $A$ in the excitation current reflects the magnitude of the sinusoidal variation. Because the value of periodic current is different at every moment, the effective value is often used to measure the role of periodic current in the circuit. The effective value of the periodic current is:

$$ W = \int_0^T R i^2(t) dt $$

$$ I = \sqrt{\frac{1}{T} \int_0^T I_m^2 \sin^2 (\omega t + \phi) dt} = \frac{L_m}{\sqrt{2}} = 0.707 I_m $$
If the amplitude of the output is fixed at 0.04, the relationship between dynamic viscosity and current can be obtained by simulation in MATLAB using the viscosity of the known substance, as shown in Table 1.

Table 1. MATLAB simulation results for different media.

| Viscous substance     | Dynamic viscosity $\mu$ | Amplitude $A$   | Electric current $I$ |
|-----------------------|-------------------------|-----------------|----------------------|
| Head oil              | $5 \, (Pa \cdot s)$     | 1.73(Scalar)    | 1.22(A)              |
| Pear pulp             | $4 \, (Pa \cdot s)$     | 1.72(Scalar)    | 1.21(A)              |
| Paper glue            | $3 \, (Pa \cdot s)$     | 1.70(Scalar)    | 1.20(A)              |
| Semi-inverted syrup   | $2.4 \, (Pa \cdot s)$   | 1.69(Scalar)    | 1.19(A)              |
| Glycerol              | $1.5 \, (Pa \cdot s)$   | 1.66(Scalar)    | 1.17(A)              |
| Ketchup               | $1 \, (Pa \cdot s)$     | 1.65(Scalar)    | 1.15(A)              |
| Concentrated orange juice | $0.63 \, (Pa \cdot s)$ | 1.62(Scalar)    | 1.14(A)              |
| Chocolate milk        | $0.28 \, (Pa \cdot s)$  | 1.59(Scalar)    | 1.12(A)              |
| Emulsion              | $0.2 \, (Pa \cdot s)$   | 1.57(Scalar)    | 1.11(A)              |
| Egg                   | $0.15 \, (Pa \cdot s)$  | 1.55(Scalar)    | 1.09(A)              |

Taking the current $I$ as the abscissa and the dynamic viscosity $\mu$ as the ordinate, put the experimental data into the MATLAB simulation, and the curve is fitted by three mathematical models: exponential function, power function and Gaussian function. The fitting result is shown in Figure 7. The exponential function $f(x) = 1.12*\exp(1.128*x)$, the power function $f(x) = 0.01347*x^{29.78}$, Gaussian function $f(x) = 528.2*\exp(-((x-10.26)/4.134)^2)$. By comparison, it is known that the Gaussian function fits the curve best and can be used as an important theoretical basis for viscosity measurement.
5. Conclusion
In this paper, the structural characteristics and mathematical model of vibrating online viscometer are analyzed and researched, and the simulation model of vibrating online viscometer system is built by closed-loop control technology. The accuracy of closed-loop control and the effectiveness of the simulation model are verified by MATLAB simulation experiments. The relationship between viscosity and compensation current is established by Gaussian function model, which provides an important theoretical basis and guidance for system design and debugging of vibrating online viscometer.

References
[1] Chen Gang, Zhu Zhengang, Shui Jiapeng. Principle of measuring liquid viscosity coefficient by forced vibration torsion method[J]. Acta Physica Sinica, 1999(03): 40-44.
[2] Margaret Stautberg Greenwood, Judith Ann Bamberger. Measurement of viscosity and shear wave velocity of a liquid or slurry for on-line process control[J]. Ultrasonics, 2002, 39(9).
[3] He Zheng, Li Changxi. Research on vibration viscometer [J]. Chemical automation and instrumentation, 2010, 37 (12): 73-75.
[4] Hu Yanru, Chi Guoxuan. Calculation of Natural Frequency of Single Degree of Freedom Vibration[J]. Journal of Jiaomusi Institute of Technology, 1991(04): 230-236.
[5] Paul Van den Hof. Closed-loop issues in system identification[J]. Annual Reviews in Control, 1998, 22.