Periodic giant-persistent current in sharp pulses on a ring

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We show here a mesoscopic device based on a narrow ring containing an electron. In the device, an amount of energy is stored in advance. Similar to the pendulum, an exact periodic motion of the electron is thereby initiated afterward. The motion appears as a series of sharp pulses, and is in nature different from the well known Aharonov-Bohm (A-B) oscillation. In particular, the pulses of current can be tuned to be very strong (say, more than two orders stronger than the usual A-B current). Related theory and numerical results are presented.

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I. INTRODUCTION

It is well known that mesoscopic devices are not only interesting in academic aspect but also promising in technical aspect. Among these devices, the quantum ring is distinguished due to its special geometry. Accordingly, special physical phenomena emerge, namely, the normal and fractional Aharonov-Bohm (A-B) oscillations of the ground state energy and persistent current[1–8]. The A-B oscillation is caused by the variation of an external magnetic flux threading the ring. At any instant, the current is uniform (i.e., it is the same everywhere on the ring). This kind of oscillation has already been extensively studied. In this paper, we report a new kind of persistent current which is a series of pulses and is different from the A-B current in nature. They move on the ring strictly periodically. Both the intensity and the period can be tuned. In particular, giant current (i.e., two or more orders stronger than the A-B current) can be obtained.

It is recalled that the oldest clock is based on the periodic motion of a classical pendulum, where the motion is initiated by an amount of potential energy stored in advance. Similarly, in a mesoscopic device, if an amount of energy has been stored in advance and can be transformed to kinetic energy afterward, and if dissipation can be neglected, a persistent periodic motion might exist based on the principle of the pendulum. In this paper, an idea is proposed so that an amount of energy can be stored in a system with an electron moving on a super-conducting ring, and an exact periodic motion of the electron can be thereby induced. Related theory and numerical results are as follows.

II. HAMILTONIAN AND RELATED THEORY

We consider a very narrow ring which is a quasi-one-dimensional system containing a free electron. A constant magnetic field is preset lying on a segment of the ring (say, caused by electrodes) is applied to localize the electron. The Hamiltonian reads

$$H = G (-i \frac{\partial}{\partial \theta} + \Phi)^2 + U(\theta)$$

(1)

where $\theta$ is the azimuthal angle of the electron, $G = \hbar^2/(2m^*R^2)$, $m^*$ is the effective mass, and $R$ is the radius of the ring. $\Phi$ is the magnetic flux threading the ring in the unit $\Phi_0 = hc/e$. $U(\theta) = 0$ if $\pi - d \leq \theta \leq \pi + d$, or $= U_o$ otherwise. $U_o$ is a positive number and $2d$ is the width of the square-well. The potential is deep enough so that at least a localized state is contained. The electron is assumed to be in the ground state $\Psi_g$ with energy $E_g > 0$ initially. Thus it is localized in the beginning.

Suddenly, the well is removed ($U(\theta)$ becomes zero everywhere) and at the same time the flux $\Phi$ is changed to $\Phi'$. Accordingly, the Hamiltonian is suddenly changed from $H$ to

$$H_{evol} = G (-i \frac{\partial}{\partial \theta} + \Phi')^2$$

(2)

Obviously, $\Psi_g$ is no more the eigen-state of the new Hamiltonian. Thus it begins to evolve.

Starting from $\Psi_g$, the formal time-dependent solution of the Schrödinger equation reads

$$\Psi(t) = \exp(-iH_{evol}t/\hbar)\Psi_g$$

(3)

which can be written in an applicable form when we know all the eigen-states of $H_{evol}$. This is easy for one-dimensional rings. They read simply $|k\rangle = \exp(ik\theta)/\sqrt{2\pi}$, where $k$ include all integers from $-\infty$ to $+\infty$. With them, eq. (3) can be rewritten as

$$\Psi(t) = \sum_k \exp(-iE_k t/\hbar)|k\rangle\langle k|\Psi_g$$

(4)
where $E_k = G(k + \Phi')^2$.

In order to obtain $\Psi_g$, $H^\Phi$ is diagonalized by using the set $|k\rangle$ as basis functions. For numerical calculation the summation of $k$ in eq. (4) can be confined within a range, say, from $-I_\Phi - 30$ to $-I_\Phi + 30$, where $I_\Phi$ is the integer closest to $\Phi$. This could provide the accuracy with at least four effective figures in numerical results.

The matrix element reads

$$
\langle k'|H^\Phi|k\rangle = \begin{cases} 
G(k + \Phi)^2 + U_o(1 - d/\pi), & \text{if } k = k' \\
-\frac{U_o}{\pi(k-k')\pi} \cos[(k-k')\pi] \sin[(k-k')d], & \text{else (5)}
\end{cases}
$$

After the diagonalization the ground state energy $E_g$ and the ground state $\Psi_g = \sum_k C_k^g |k\rangle$ can be obtained. In particular, $\langle k|\Psi_g\rangle = C_k^g$ can be known. There are two noticeable features:

(i) From eq. (5), we have $\langle k'|H^\Phi|k\rangle = \langle k' - I|H^{\Phi+I}|k - I\rangle$ where $I$ is an arbitrary integer. Therefore the Hamiltonian $H^{\Phi+I}$ and $H^\Phi$ have exactly the same ground state energy $E_g$. And the ground state of $H^{\Phi+I}$ reads $\Psi_g^{\Phi+I} = \sum_k C_k^{\Phi+I}|k\rangle$. It implies that, once the cases with $0 \leq \Phi \leq 1$ are clear, the cases with $\Phi < 0$ and $\Phi > 1$ are also clear.

(ii) When $\Phi$ is an integer or a half-integer, we have $\langle k'|H^\Phi|k\rangle = (-k' - 2\Phi|H^\Phi| - k - 2\Phi)$. This equality would impose a kind of symmetry between the coefficients $C_k^\Phi$ and $C_{-k-2\Phi}^\Phi$. Since the ground state should not have a node at $\theta = \pi$, it can be deduced that $C_k^\Phi = C_{-k-2\Phi}^\Phi$ (if $\Phi$ is an integer) or $C_k^\Phi = -C_{-k-2\Phi}^\Phi$ (if $\Phi$ is a half integer).

Let $t_o = \hbar/G$ be the unit of time, and $\tau = t/t_o$. From eq. (4), we obtain the time-dependent density

$$
\rho(\theta, \tau) = |\Psi(t)|^2 = \frac{1}{2\pi} \sum_{k,k'} \sum_{k,k'} C_k^\Phi C_{k'}^\Phi \cos[(k-k')\theta - (k+k' + 2\Phi')\tau] \quad (6)
$$

and, from the conservation of mass, the current

$$
J(\theta, \tau) = 2J_{A-B} \sum_{k,k'} C_k^\Phi C_{k'}^\Phi (k + \Phi') \cos[(k-k')\theta - (k+k' + 2\Phi')\tau] \quad (7)
$$

where $J_{A-B} = \hbar/4\pi m^* R^2$ is the maximal current of the A-B oscillation of the one-electron ground state. $J_{A-B}$ is served as a unit of current in the follows.

It was found from eqs. (6) and (7) that the motion of the electron would be periodic if $\Phi'$ satisfies the following rule:

Let $I$, $I_o$ and $I_e$ denote positive integers, where $I_o$ is an odd integer and $I_e$ even integer. (a) If $(I_e)_{\text{min}}$ is the smallest even integer so that $|\Psi'\rangle = I/(I_e)_{\text{min}}$, then the evolution is strictly periodic with a period $\tau_p = (I_e)_{\text{min}} \pi$. (b) However, for the special cases $|\Psi'| = I_o/2 I_o$, the period $\tau_p = I_o \pi$. (c) Otherwise, the evolution is not strictly periodic.

III. NUMERICAL RESULTS

How the density is distributed on the ring in the early stage of evolution is shown in Fig. 1. When $\tau = 0$, the electron is localized around $\theta = \pi$ as shown in (a) as mentioned, where the distribution is not sensitive to $\Phi$. When the evolution starts, the peak of $\rho$ will shift left (clockwise) as shown in (b) with $\tau = \pi/12$. However, it

![FIG. 1: (Color online). $\rho(\theta, \tau)$ plotted against $\theta$ when $\tau$ is given. $m^* = 0.063m_e$ is assumed and the ring has $R = 100nm$. The preset potential has $U_o = 2G$ and $d = \pi/4$. $\Phi'$ is given zero. The above parameters are also the same in following figures. The curves from "1" to "4" have $\Phi = 0$, 1.25, 2.5, and 3.75, respectively. $\tau = 0$ (a) and $\pi/12$ (b).](image-url)
would shift right if $\Phi < 0$. The shift would be larger if $\Phi$ is larger. The shift is caused by an initial current $J(\theta, 0)$ shown in Fig. 2. This current is created simultaneously with the sudden creation of the new Hamiltonian. When $\Phi = 0$, $J(\theta, 0) = 0$ (curve "1" of 2). When $\Phi \neq 0$, a larger $|\Phi|$ leads to a stronger $J(\theta, 0)$ as shown by "2" to "4" of 2, they are the motivity of the evolution. In 2a, the current of "1" is either negative (if $\theta < \pi$) or positive (if $\theta > \pi$), it implies that the current flows to both sides if $\Phi = 0$. However, when $\Phi > 0$, "2" to "4" of 2 are purely negative implying going left. Obviously, a more negative current leads to a larger shift of $\rho$ as previously shown in [4].

To better understand the origin of $J(\theta, 0)$, we study the structure of the initial state $\Psi_g$. It was found that $\Psi_g$ is less sensitive to the parameters of the potential, but very sensitive to $\Phi$. This fact is shown in Fig. 3. Although $|\Psi_g|^2 \equiv \rho(\theta, 0)$ plotted in 3a and 3b (in solid lines) are nearly the same disregarding the difference in $\Phi$, their wave functions are greatly different. In 3, there are many nodes in both the real and imaginary parts of $\Psi_g$. It is well known that the number of nodes would be a measure of kinetic energy if the magnetic field does not exist. Therefore, an amount of energy $E_{init}$ has been stored in $\Psi_g$ initially via the preset magnetic field. Once the field is canceled, the energy can be released and motivates the evolution.

The evolution of $\rho$ is shown in Fig. 4, where $\Phi = 1.25$ and $\tau$ is from 0 to $\pi$. In the duration $[0, \pi/4]$, the peak of $\rho$ starting from the initial end ($\theta \approx \pi$) keeps going left (clockwise) and becomes broader and broader as shown by the curves "1" to "4" of 4. In this duration, the distance that the peak has shifted increases remarkably with $\Phi$. In 4, the peak of "4" is apart from the initial end by $\sim 5\pi/8$. However, if $\Phi = 6.25$ for an example, it would become $13\pi/8$. In the next duration $[\pi/4, 3\pi/4]$, $\rho$ becomes diffused and is widely distributed on the ring with peaks and dips as shown by "5" to "7" of 4a and "1" to "3" of 4b. Meanwhile, the classical picture of motion is not clear. It can be called a duration of cruise. However, when $\tau = \pi/2$, $\rho$ is nearly half-to-half concentrated in both ends as shown by "7" of 4b (which is identical to
"1" of [4]). When $\tau = 3\pi/4$, the diffused density begins to be concentrated again and forms a broad main peak as shown by "4" of [4]. The broad peak is roughly apart from the opposite end ($\theta \approx 0$) by $5\pi/8$ (the same distance mentioned above). Afterward, in $[3\pi/4, \pi]$, the broad peak keeps going left, becomes narrower, and arrives at the opposite end exactly at $\tau = \pi$. This is shown by "4" to "7" of [4]. Thus the evolution of $\rho$ in the upper half period can be divided into three stages, namely, shift-cruise-shift. This qualitative feature is not affected by $\Phi$. In particular, $\rho$ is peaked at the initial end when $\tau = 0$, peaked at the opposite end when $\tau = \pi$, and nearly half-to-half concentrated in both ends when $\tau = \pi/2$ disregarding how $\Phi$ is. Nonetheless, how far the peak of $\rho$ would shift within the durations $[0, \pi/4]$ or $[3\pi/4, \pi]$ depends seriously on $\Phi$. Besides, during the cruise, a larger $\Phi$ leads to a stronger current in general.

When $\Phi' = 0$ as we have chosen, it is straightforward from eqs. (6) and (7) to obtain $\rho(-\theta, \pi + \tau) = \rho(\theta, \pi - \tau)$ and $J(-\theta, \pi + \tau) = J(\theta, \pi - \tau)$. Therefore, the evolution in $[\pi, 2\pi]$ is just a time-inverse of the evolution in $[0, \pi]$ together with a spatial reflection against $\theta = 0$. It implies that, when $\tau > \pi$, the peak would shift left from $\theta = 0$ to $-5\pi/8$ (if $\Phi = 1.25$), then a cruise, then a shift again from $\pi + 5\pi/8$ toward $\pi$. When $\tau = 2\pi$, the peak arrives exactly at the initial end $\theta = \pi$ and the period is ended, and a new period will begin, and so on.

From Fig. 4 we know that the density will highly concentrated in the initial (opposite) end once and once when $\tau = I_\pi (I_o \pi)$. Accordingly, when one observe the time-dependence of current at the initial (opposite) end, a series of pulses will appear one-after-one as shown in Fig. 5. For an example, there are five peaks in $[\Phi]$ at $\tau = 0.28\pi$, $0.49\pi$, $\pi$, $1.51\pi$, and $1.72\pi$, respectively. At each of these instants, a pulse of current will arrive at the opposite end. The pulse appearing at $\tau = \pi$ is much stronger (associated with the curve "7" of [4]). There are also five peaks in $[\Phi]$, the highest peak is at $\tau = 0$ (associated with the curve "1" of [4]). Once the strongest pulse has arrived at an end, the current at the other end is zero simultaneously. The maximal current of the highest peak (the strongest pulse) is found to be nearly linearly proportional to $|\Phi|$. With the above parameters, we found an approximate formula $J(0, \pi) = -6.9\Phi J_{A-B}$ (Say, when $\Phi = 1.25$, $J(0, \pi) = -8.6J_{A-B}$ as plotted in $[\Phi]$). When $\Phi = 30$, it is more than 200 times stronger than the usual A-B current). Thus, giant, punctual, and persistent sharp pulses of current can be obtained. This is the most distinguished feature of the device.

The assumption that $\Phi' = 0$ leads to a strict period $\tau_p = 2\pi$. When $\Phi'$ is nonzero, the periodic behavior depends on $\Phi'$ extremely sensitively. For an example, based on the rule, when $\Phi' = \frac{499}{1000}$, we have $\tau_p = 1000\pi$. However, when $\Phi' = \frac{500}{1000}$, we have $\tau_p = \pi$. That is, a very slight change in $\Phi'$ might lead to a great change in $\tau_p$. This high sensitivity is also a noticeable point.

### IV. CONCLUSIONS

In summary, a device with periodic persistent current in sharp pulses is proposed based on a narrow ring. A crucial point is how to store an amount of energy in advance. For this aim a potential and a magnetic field is preset. Then, the potential is removed suddenly and the flux jumps from $\Phi$ to $\Phi'$ simultaneously. In this way, the amount of energy can be released and motivate an exact periodic motion of the electron. The period is controllable (say, by altering the materials, the size of the ring, and/or $\Phi'$). On the contrary with the A-B current, the present current is far from uniform but in sharp pulses, and can be tuned to be very strong by increasing the difference $|\Phi' - \Phi|$. The appearance of sharp pulses is a distinguished feature of the device.

When $\Phi' = 0$, the evolution in a period has been studied in detail. Three stages (shift-cruise-shift) have been found. The distance that the peak of $\rho$ has shifted during the stage of "shift" depends seriously on $\Phi$, and a larger $|\Phi|$ will lead to a longer distance. At the two ends of the ring sharp pulses of current are found passing by one-after-one. The strongest pulses will emerge at the initial (opposite) end each time when $\tau = I_\pi (I_o \pi)$. When $\Phi > 15$, the strongest current $J(0, I_\pi)$ is two or more orders stronger than the A-B current.

When $\Phi' \neq 0$, the above periodic behavior will be changed. When $\Phi' = I_o/2$, the period would be the shortest $\pi I_o$. It is noted that the current given by eq. (4) contains the product $C_k^\rho (k, \Phi')$. Due to the feature of the ground state, $|C_k^\rho|$ would be larger only if $k$ is close to $-\Phi$. Therefore, the contribution of this product would be larger if $|\Phi' - \Phi|$ is larger. This explains why a big jump.
in the flux is necessary to produce the giant current.

Finally, it is noted that time-counting is important to developing micro-techniques. The central element for time-counting would be a device containing exact periodic motion. It is believed that, comparing with smooth periodic motion, a device with sharp pulses (refer to Fig. 5) might be a better candidate for this purpose. Furthermore, due to the sharp pulses, the period $2\pi t_o$ might be accurately measured. This might lead to a better understanding to the parameters of the system ($m^*$ and $R$).

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