Seesaw Neutrino Masses with Large Mixings from Dimensional Deconstruction

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Abstract

We demonstrate a dynamical origin for the dimension-five seesaw operator in dimensional deconstruction models. Light neutrino masses arise from the seesaw scale which corresponds to the inverse lattice spacing. It is shown that the deconstructing limit naturally prefers maximal leptonic mixing. Higher-order corrections which are allowed by gauge invariance can transform the bi-maximal into a bi-large mixing. These terms may appear to be non-renormalizable at scales smaller than the deconstruction scale.

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1 Introduction

In contrast to the quark sector, the present state-of-the-art neutrino experiments [1] demand large or even maximal mixings. An attempt to unification requires new physics beyond the standard model (SM). For instance, in SO(10) models, one can relate the spectra of both the charged and neutral fermions in agreement with known phenomenology [2]. Broadly, most efforts to explain flavour physics are either (i) strongly model dependent (see e.g. [3]) or (ii) require initial assumptions on the neutrino spectra (see e.g. [4]). Another option is to explore the framework of dimensional deconstruction [5], where the effects of higher dimensions originate as a pure dynamical effect in the infrared limit. In this context, it is interesting to study the impact on the Yukawa sector of a model [6]. Here, we recover the salient aspects of neutrino phenomenology, namely large mixings and light masses, from a completely massless four dimensional theory at some large scale. The lightness of the neutrino masses are an outcome of deconstruction which projects out the dimension-five seesaw operator [7]. This scenario contains massless Nambu-Goldstone modes corresponding to a symmetry which can be associated with large mixings. All of these basic features can be easily understood by considering a simple two-site lattice model.

2 The two-site model

Consider a $G = G_{SM} \times SU(m_1) \times SU(m_2)$ gauge theory for deconstructed extra dimensions, where $G_{SM}$ denotes the SM gauge group. The left-handed lepton doublets are denoted by $\ell_\alpha = (\nu_{\alpha L}, e_{\alpha L})^T$ and the corresponding right-handed charged leptons by $E_\alpha$, where the Greek indices denote the usual flavors ($e$, $\mu$ and $\tau$). We will assume that $\ell_\alpha$ and $E_\alpha$ transform as $\overline{m}_1$ under $SU(m_1)$ and $\ell_\beta$ and $E_\beta$ transform as $m_2$ under $SU(m_2)$. We introduce the right-handed neutrinos $N_\alpha$ and $N_\beta$, where $N_\alpha$ transforms as $\overline{m}_1$ under $SU(m_1)$ while $N_\beta$ transforms as $m_2$ under $SU(m_2)$. The scalar link field $\Phi$ connects as the bi-fundamental representation ($m_1, \overline{m}_2$) the neighboring $SU(m_1)$ groups. This field theory is summarized by the “moose” or “quiver” [8] diagram in Fig. 1. The most general renormalizable Yukawa interactions for the neutrinos are then given by

$$\mathcal{L}_Y = Y_\alpha \overline{\ell}_\alpha \tilde{H} N_\alpha + Y_\beta \overline{\ell}_\beta \tilde{H} N_\beta + f N_\alpha \Phi N_\beta + \text{h.c.} \ .$$ (1)
The kinetic term for the link field is $\sim (D_{\mu} \Phi)^{\dagger} D^\mu \Phi$; $D_{\mu} \Phi = (\partial_{\mu} - ig_1 A_{1\mu} T_a + ig_2 A_{2\mu} T_a) \Phi$, where $A_{i\mu}^a$ $(i = 1, 2)$ are the gauge fields and $T_a$ represent the group generators along with the dimensionless gauge couplings, $g_1$ and $g_2$. In (1), $\bar{H} = i\sigma^2 H^*$ is the charge conjugated Higgs doublet and $Y_\alpha, Y_\beta, f$ are complex Yukawa couplings of $O(1)$. Note that in (1) the bare Dirac and Majorana mass terms of the types $\sim N_\alpha^c N_\beta$ and $\sim \bar{N}_\alpha^c N_\alpha$ or $\sim \bar{N}_\beta^c N_\beta$ are forbidden by invariance under the group $G$. For a suitable scalar potential the field $\Phi$ can acquire a VEV such that $\langle \Phi \rangle = M_x$, thereby generating a mass for the scalar field. $M_x$ is identified with the deconstruction scale at which the $SU(m) \times SU(m)_{2}$ symmetry is broken down to the diagonal $SU(m)$, thereby eating one adjoint Nambu-Goldstone multiplet in the process. The corresponding lattice spacing is $a \sim 1/M_x$. After spontaneous symmetry breaking, the light effective Majorana mass matrix takes the form

$$\mathcal{M}_\nu = Y_\alpha Y_\beta \frac{\epsilon^2}{f M_x} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (2)$$

Here, $\epsilon \equiv \langle H \rangle \simeq 10^2$ GeV is the electroweak scale and $M_x \simeq 10^{15}$ GeV is the seesaw scale. This simple analysis leads to our main observations. It is not difficult to identify the mass matrix in (2) as the one which arises from the usual dimension-five operator of the type $\sim \nu \nu \bar{H} H$. In other words, for length scales $r \gg a \sim 1/M_x$, the renormalizable and gauge invariant Yukawa interaction in (1) reproduces the effects of the fifth dimension. Whereas, for $r \ll a$, we retain a completely renormalizable four-dimensional interaction as defined in (1). In addition, contrary to the conventional seesaw operator, dimensional deconstruction can naturally lead to maximal mixings between the two active neutrino flavors, $\nu_\alpha$ and $\nu_\beta$. This is realized due to the $\Phi$ field which mediates a symmetry between each of the fermions $(N_{\alpha,\beta})$. This symmetry can be interpreted as an interaction which conserves a charge $L_\alpha - L_\beta$ which is reflected in the resulting mass matrix for $N_{\alpha,\beta}$. This is retained after symmetry breaking as there exists the diagonal subgroup $SU(m)$ which respects the symmetry such that the $\Phi$ field would transform as $(m, \bar{m})$. In the gauge sector, this unbroken symmetry corresponds to the presence of a zero mode and is $A_{\mu}^{a(0)} \sim (g_2 A_{1\mu}^a + g_1 A_{2\mu}^a)$. The Dirac sector of the model remains diagonal due to the nature of this construction while maximal mixings are introduced from the heavy Majorana sector of the resulting seesaw operator. Equivalently, the qualitative features of this system are not altered even if one allows for both the fermions and scalars to be link variables. We use this freedom when we discuss the phenomenology for this mechanism in section 3.

Next, we would like to understand how generic is the interaction described in (2). To answer this question, we consider three possible modifications to Fig. 1 which are summarised in Fig. 2 as cases (i)-(iii). Clearly, in case (i), only higher-order terms of the form $\sim \bar{H} \Phi N_\beta$ are possible, but a priori, there is no information on mixings or masses. In case (ii), we have an interaction $\sim Y_\alpha \bar{N}_\alpha H N_\alpha + Y_\beta \bar{N}_\beta H N_\alpha + f \bar{N}_\alpha^c \Phi N_\beta$ which leads to $\mathcal{M}_\nu = 0$. Interestingly, for case (iii), depending on the representation of the fermionic fields, we can envisage two distinct interactions. The first one is of the type $\sim Y_\alpha \bar{N}_\alpha H N_\alpha + Y_\beta \bar{N}_\beta H N_\beta$
which gives Dirac masses with arbitrary masses and mixings. The second possibility is of the type \( \sim Y_\alpha \ell_\beta \bar{H} N_\alpha + f M_\nu \bar{N}_\alpha N_\beta \) which again results in \( \mathcal{M}_\nu = 0 \). Note that in the latter case, gauge invariance allows for a bare mass term which is in contrast to \( \Pi \). From the different cases (i)-(iii) we observe a restrictive pattern for the allowed fermion masses; this is unlike \( \Pi \) which ensures a renormalizable mass term for all of the resulting Dirac and Majorana fermions. This maximises the allowed Yukawa interactions and leads also to maximal mixings. In a realistic framework, the basic structure of \( \Pi \) is always expected to be borne out as we shall demonstrate.

3 A realistic model

We examine a generalization to the case of a moose mesh \( \Pi \). Consider a \( \Pi_{i=1}^4 SU(m) \) gauge theory containing five scalar link variables \( \Phi_i \ (i = 1, \ldots, 5) \) and the fermion fields \( \Psi_\alpha \) is the set \( \{ \ell_\alpha, E_\alpha, N_\alpha \} \). This model is depicted in Fig. 4 and up to mass dimension six, we have

\[
\phi_1 \Phi_2 \Phi_3 \Phi_4 \Phi_5 \Psi_r \Psi_e \Psi_\mu
\]
\[
\mathcal{L}_Y = \sum_{\alpha} Y_{\alpha} \bar{\nu}_{\alpha} \mathcal{N} \nu_{\alpha} + f_1 \bar{\nu}_e \Phi_2 N_{\mu} + f_2 \bar{\nu}_e \Phi_1 N_{\tau} + \frac{f_3}{\Lambda} N_{\mu} (\Phi_3)^2 N_{\mu} \\
+ \frac{f_4}{\Lambda} N_{\tau} (\Phi_4)^2 N_{\tau} + \frac{f_5}{\Lambda} \bar{\nu}_e \Phi_3 \Phi_4 N_{\tau} + \frac{f_6}{\Lambda} \bar{\nu}_e (\Phi_5)^2 N_e + \ldots + \text{h.c.,}
\]

where the dots represent non-renormalizable interactions of the leptons with effective scalar operators involving only the fields \(H\) and/or \(\Phi_i\). In (3), \(Y_{\alpha}\) and \(f_i\) \((i = 1, \ldots, 6)\) are complex couplings and \(\Lambda(\gg \langle \Phi_i \rangle)\) denotes the scale such that, for lattice spacing \(a \ll 1/\Lambda\), the theory is fully renormalizable. After symmetry breaking and giving universal VEVs \(\langle \Phi_i \rangle \equiv M_x\), the Dirac and Majorana mass matrices take the form

\[
\mathcal{M}_D = \epsilon \begin{pmatrix}
Y_e & \lambda^2 & \lambda^2 \\
\lambda^2 & Y_\mu & \lambda^2 \\
\lambda^2 & \lambda^2 & Y_\tau
\end{pmatrix}, \quad \mathcal{M}_R = M_x \begin{pmatrix}
\lambda f_6 & f_1 & f_2 \\
f_1 & \lambda f_3 & \lambda f_5 \\
f_2 & \lambda f_5 & \lambda f_4
\end{pmatrix}, \quad \lambda = \frac{M_x}{\Lambda} < 1,
\]

where only the order of magnitude of the terms with mass dimension \(\geq 6\) has been indicated. We note that in (4), as a consequence of the lattice geometry \(\mathcal{M}_D\) is nearly diagonal while the Majorana sector carries the \(\mathcal{M}_R = L_{\nu} - L_{\mu} - L_{\tau}\) symmetry which is softly broken by a nonzero \(\lambda\). In the limit \(\lambda \to 0\), both (1) and (3) reproduce similar features. Neglecting the small mixing in the Dirac sector\(^1\) and setting \(Y_\mu \approx Y_e\) along with real couplings, \(f_1 = -f_2 = f_3 = f_4 = f_5 \equiv f\), the effective light neutrino mass matrix comes to a familiar pattern \([10]\) with

\[
\mathcal{M}_\nu \simeq \frac{Y_\mu e^2}{4 f^3 M_x} \begin{pmatrix}
0 & 2 \lambda Y_e f^2 & -2 \lambda Y_e f^2 \\
2 \lambda Y_e f^2 & Y_\mu f (\lambda^2 f^2 - f) & -Y_\mu f (\lambda^2 f^2 + f) \\
-2 \lambda Y_e f^2 & -Y_\mu f (\lambda^2 f^2 + f) & Y_\mu f (\lambda^2 f^2 - f)
\end{pmatrix} + \mathcal{O}(\lambda^3).
\]

The relations between the solar and atmospheric mass-squared differences, \(\Delta m^2_{\odot}\) and \(\Delta m^2_{\text{atm}}\) respectively, and the solar mixing angle \(\theta_{12}\) are

\[
\frac{\Delta m^2_{\odot}}{\Delta m^2_{\text{atm}}} \simeq 2 \sqrt{2} \lambda^3 \left( \frac{Y_e f_6}{Y_\mu f} \right) + \mathcal{O}(\lambda^4),
\]

\[
\tan \theta_{12} \simeq 1 - \frac{\lambda}{2 \sqrt{2}} \left( \frac{Y_\mu f_6}{Y_e f} \right) + \frac{\lambda^2}{16} \left( \frac{Y_\mu f_6}{Y_e f} \right)^2 + \mathcal{O}(\lambda^4).
\]

For illustration, we choose \(\lambda = 0.22\), \(Y_e = f\), \(Y_\mu = f_6\), \(Y_\mu/Y_e = 2.5\) and we obtain an atmospheric mixing angle \(\theta_{23} \simeq \pi/4\) and a reactor mixing angle close to zero, \(i.e., U_{e3} \simeq 0\). Such an allowed choice minimally alters the basic features of a renormalizable Lagrangian, leading to a soft breaking of the \(\mathcal{L}\) symmetry. Furthermore, taking \(\Delta m^2_{\text{atm}} = 2.5 \times 10^{-3}\text{eV}^2\), we obtain a normal neutrino mass hierarchy with \(\Delta m^2_{\odot} \simeq 7.5 \times 10^{-5}\text{eV}^2\) and \(\theta_{12} \simeq 32^\circ\), which is in agreement with the MSW LMA-I solution \([11]\). For the above values, the system predicts an effective neutrinoless double beta decay mass, \(m_{ee} \simeq 10^{-3}\text{eV}\).
We briefly outline two different variations to (3). Let us first consider a $\Pi_{i=1}^3 SU(3)_i$ product gauge group with a representation content as specified in Fig. 5 where the arrows define as before the field transformations. We use the Froggatt-Nielsen mechanism [12] to break the $\mathcal{L}$ symmetry by putting on each of the sites $SU(3)_1$ and $SU(3)_2$ two extra SM singlet fields; these are two scalars ($\phi_e$, $\phi_\mu$) and two heavy Dirac fermion fields ($F_e$, $F_\mu$). To retain soft-breaking of the $\mathcal{L}$ symmetry, we need to impose a $Z_4$ symmetry $\Psi_\alpha \rightarrow -\Psi_\alpha$, $\phi_\alpha \rightarrow -\phi_\alpha$, $F_{\alpha L} \rightarrow iF_{\alpha L}$, $\Phi_3 \rightarrow -\Phi_3$, where $\alpha = e, \mu$. We assume that the $SU(3)_1$ and $SU(3)_2$ symmetries are broken by bare Majorana mass terms $\sim F_{\alpha R}^2 M_\alpha F_{\alpha R}$ at some scale $M_\alpha \gg M_x$. When the fields $\phi_\alpha$ acquire the VEVs $\langle \phi_\alpha \rangle = M_x$ the heavy right-handed fermions $F_{\alpha R}$ are integrated out leading to the dimension-five terms $\sim \lambda f_3 M_x N_\mu N_\mu$ and $\sim \lambda f_6 M_x N_e N_e$, where $\lambda \simeq M_x/M_\alpha$. The right-handed neutrino mass matrix is given by $M_{\nu}$ in (4) with $f_4, f_5 \simeq 0$ and hence one obtains the relations as in (6). Alternatively, if we perform the identification, $\Phi_2 \rightarrow 0$ and $SU(3)_i \rightarrow U(1)_i$ and the fields ($\ell_\alpha, N_\alpha$) ($\alpha = e, \mu, \tau$) are assigned appropriate $U(1)$ charges, it is not difficult to derive a model leading to (4).

In conclusion, we argue that upon deconstruction (i) a light neutrino mass is a general result and (ii) maximal mixing is inevitable due to the specific Yukawa interactions in (1). In the limit of a large lattice site model (of size $N \gg 1$), one can draw comparisons to the genuine extra-dimensional scenarios (of radius $R$) with the identifications to the five-dimensional gauge couplings, $g_5(y_i) \rightarrow \sqrt{R/N} g_i$, where $y_i$ denotes the fifth coordinate. In this analysis, we have limited ourselves to describing the physics of a periodic lattice where it is sufficient to examine the periodic interval of any one Brillouin zone. In general, we predict a small $U_{e3}$ which depends on the pattern of the underlying $\mathcal{L}$ symmetry breaking.

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1The contributions from the charged lepton sector are identical to $M_D$ and can be neglected.
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