Simulation of the process $e^+ e^- \rightarrow e^+ e^- \gamma$
within electroweak theory
with longitudinally polarized initial electrons.

M. Caffo $^{ab}$, H. Czyż $^c$ *, E. Remiddi $^{ba}$

$a$ INFN, Sezione di Bologna, I-40126 Bologna, Italy
$b$ Dipartimento di Fisica, Università di Bologna, I-40126 Bologna, Italy
$c$ Institute of Physics, University of Silesia, PL-40007 Katowice, Poland

E-mail:
caffo@bologna.infn.it
czyz@usctouxlim.cto.us.edu.pl
remiddi@bologna.infn.it

Abstract

We present simple analytic expressions for the distributions of the Bhabha scattering process with emission of one hard photon, including weak boson exchanges, and with longitudinal polarization of the initial electron. The results from the Monte Carlo generator BHAGEN-1PH, based on these expressions, are presented and compared, for the unpolarized case, with those existing in literature.

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1. Introduction.

The cross section for $e^+ e^- \rightarrow e^+ e^- \gamma$ in QED has been calculated for different purposes and various distributions have been produced [1-16]. The availability of high energies (TRISTAN, LEP, SLC) and longitudinally polarized electron beam (SLC) makes it necessary to include weak-boson and polarization effects. For the unpolarized case a relatively simple expression for the square of the matrix element was obtained [2,3], for the polarized case it was advocated [4] that it is easier to use just a matrix element and not its square. This is true for the general case with multiphoton emission, however for the experimentally interesting case of longitudinally polarized electron beam, by careful choice of the variables, in [5] was obtained a reasonably compact expression for the square of the matrix element. In this work that expression is improved with some other terms relevant on the $Z$ boson peak and with all the relevant mass-corrections for the configurations in which the final fermions have angles with the initial direction larger than, say, 1 mrad. For extremely forward final fermions the mass-corrections reported here are not sufficient as discussed in [1,5,6,7,8]. We present the complete analytical form of the differential cross-section, omitting the details of the calculation; the expression is simple enough and can be easily cross-checked. Because of the experimental interest we also implemented it within a Monte Carlo program, allowing for a fast event generation of the process. Furthermore the procedure do not require the introduction of new peaks as frequently occurring in spinor techniques. Some results and the numerical tests are given in Section 3, while the details of the structure of the program are presented elsewhere [9].

2. The distributions.

We consider the process

$$(2.1) \quad e^+(p_+) + e^-(p_-) \rightarrow e^+(q_+) + e^-(q_-) + \gamma(k).$$

We define

$$(2.2) \quad s = (p_+ + p_-)^2, \quad t = (p_+ - q_+)^2, \quad u = (p_+ - q_-)^2, \quad s_1 = (q_+ + q_-)^2,$$
$$t_1 = (p_- - q_-)^2, \quad u_1 = (p_- - q_+)^2, \quad k_\pm = p_\pm.k, \quad h_\pm = q_\pm.k,$$

and

$$(2.3) \quad \mathcal{P} = \epsilon_{\mu\nu\rho\sigma} p_+^\mu p_-^\nu q_+^\rho q_-^\sigma, \quad \epsilon_{0123} = 1.$$  

In the following $\alpha$ is the fine structure constant, $P_L$ is the degree of longitudinal polarization of the initial electron ($P_L = 1$ for pure right-handed states) and $C_V, C_A$ the $Z$ couplings to the electron. In the standard electroweak theory one has in particular

$$(2.4) \quad C_V = -\frac{1 - 4 \sin^2 \theta_W}{4 \sin \theta_W \cos \theta_W}, \quad C_A = -\frac{1}{4 \sin \theta_W \cos \theta_W},$$

where $\theta_W$ is the weak mixing angle. We define, for later convenience, also the combinations (the factor 2 in $C_{ZZ}^0$ was absent in [5], due to a misprint)

$$(2.5) \quad C_{\gamma Z}^\pm = C_V^2 \pm C_A^2, \quad C_{\gamma Z}^\pm = (C_V^2 + C_A^2)^2 \pm 4C_V^2C_A^2,$$
$$C_{\gamma Z}^0 = C_V C_A, \quad C_{ZZ}^0 = 2C_V C_A (C_V^2 + C_A^2).$$
With $M_Z$ and $\Gamma_Z$ the mass and width of the $Z$, $\Theta$ the usual step function and
\begin{equation}
D(s) = (s - M_Z)^2 + M_Z^2 \Gamma_Z^2 \Theta(s),
\end{equation}
we define the following functions (slightly different from [5], to keep the expressions as compact as possible)
\begin{align}
F^+(s, t, a) &= 1 + \left[ \frac{s(s - M_Z^2)}{D(s)} + \frac{t(t - M_Z^2)}{D(t)} \right] \left( C_Z^+ - 2P_L C^{0}_{\gamma Z} (1 + a) \right) \\
&+ \frac{st \left[ (s - M_Z^2)(t - M_Z^2) + M_Z^2 \Gamma_Z^2 \Theta(s)\Theta(t) \right]}{D(s)D(t)} \left( C_{ZZ}^+ - 2P_L C_{ZZ}^{0} (1 + a) \right),
\end{align}
\begin{align}
F^-(s, t) &= 1 + \left[ \frac{s(s - M_Z^2)}{D(s)} + \frac{t(t - M_Z^2)}{D(t)} \right] C_Z^- \\
&+ \frac{st \left[ (s - M_Z^2)(t - M_Z^2) + M_Z^2 \Gamma_Z^2 \Theta(s)\Theta(t) \right]}{D(s)D(t)} C_{ZZ}^-,
\end{align}
\begin{align}
E^+(s, t) &= M_Z \Gamma_Z \left[ \left( \frac{s\Theta(s)}{D(s)} - \frac{t\Theta(t)}{D(t)} \right) \left( C_Z^{0} - \frac{1}{2} P_L C_{\gamma Z}^+ \right) \right. \\
&+ \left. \frac{st \left( \Theta(s)(t - M_Z^2) - \Theta(t)(s - M_Z^2) \right)}{D(s)D(t)} \left( C_{ZZ}^{0} - \frac{1}{2} P_L C_{ZZ}^+ \right) \right],
\end{align}
\begin{align}
E^-(s, t) &= -\frac{P_L}{2} M_Z \Gamma_Z \left[ \left( \frac{s\Theta(s)}{D(s)} - \frac{t\Theta(t)}{D(t)} \right) C_Z^- \right. \\
&+ \left. \frac{st \left( \Theta(s)(t - M_Z^2) - \Theta(t)(s - M_Z^2) \right)}{D(s)D(t)} C_{ZZ}^- \right].
\end{align}

The differential cross section for the process (2.1) can be written as
\begin{equation}
d\sigma = \frac{\alpha^3}{2\pi^2 s} \left( X + Y + Z \right) \frac{d^3 q_+ \, d^3 q_- \, d^3 k}{E_+ \, E_- \, E_\gamma} \delta^4(p_+ + p_- - q_+ - q_- - k),
\end{equation}
where $E_+, E_-, E_\gamma$ are the energies of the final positron, electron and photon respectively. The quantities $X, Y, Z$ refer to the annihilation, the coulomb and the interference part of the square amplitude respectively and, omitting the terms which are numerically irrelevant at the interested energies and for the kinematical configurations considered (final fermions...
not too forward), can be expressed in the form

\[
(2.12) \quad X = \left[ F^-(s_1, s_1)(t^2 + t_1^2) + F^+(s, s_1, 0)(u^2 + u_1^2) \right] \frac{1}{4s s_1} \left[ \frac{u}{k_+ h_-} + \frac{u_1}{k_- h_+} - \frac{t}{k_+ h_-} - \frac{t_1}{k_- h_+} \right] \\
+ \left[ F^-(s_1, s_1)(t^2 + t_1^2) + F^+(s, s_1, 0)(u^2 + u_1^2) \right] \frac{1}{4s_1 k_+ k_-} \\
+ \left[ F^-(s, s)(t^2 + t_1^2) + F^+(s, s, 0)(u^2 + u_1^2) \right] \frac{1}{4s h_+} \\
+ \mathcal{P} \left[ E^-(s_1, s_1)(t^2 - t_1^2) + E^+(s, s_1)(u^2 - u_1^2) \right] \frac{s - s_1}{2s s_1 k_+ k_- h_+ h_-} \\
- \frac{m_e^2}{2s^2} \left[ F^-(s, s) \left( \frac{t_1^2}{(h_+)^2} + \frac{t^2}{(h_-)^2} \right) + F^+(s, s, 0) \left( \frac{u^2}{(h_+)^2} + \frac{u_1^2}{(h_-)^2} \right) \right] \\
- \frac{m_e^2}{2s_1^2} \left[ F^-(s_1, s_1) \left( \frac{t_1^2}{(k_+)^2} + \frac{t^2}{(k_-)^2} \right) + F^+(s_1, s_1, 0) \frac{u_1^2}{(k_+)^2} + F^+(s_1, s_1, a) \frac{u^2}{(k_-)^2} \right],
\]

\[
(2.13) \quad Y = \left[ F^-(t, t_1)(s^2 + s_1^2) + F^+(t, t_1, 0)(u^2 + u_1^2) \right] \frac{1}{4t t_1} \left[ \frac{u}{k_+ h_-} + \frac{u_1}{k_- h_+} + \frac{s}{k_+ k_-} + \frac{s_1}{h_+ h_-} \right] \\
- \left[ F^-(t, t)(s^2 + s_1^2) + F^+(t, t, 0)(u^2 + u_1^2) \right] \frac{1}{4t k_- h_-} \\
- \left[ F^-(t_1, t_1)(s^2 + s_1^2) + F^+(t_1, t_1, 0)(u^2 + u_1^2) \right] \frac{1}{4t_1 k_+ h_+} \\
- \frac{m_e^2}{2t_1^2} \left[ F^-(t, t_1) \left( \frac{s^2}{(h_+)^2} + \frac{s_1^2}{(k_+)^2} \right) + F^+(t, t_1, 0) \left( \frac{u^2}{(h_+)^2} + \frac{u_1^2}{(k_+)^2} \right) \right] \\
- \frac{m_e^2}{2t^2} \left[ F^-(t, t) \left( \frac{s^2}{(h_-)^2} + \frac{s_1^2}{(k_-)^2} \right) + F^+(t, t, 0) \frac{u_1^2}{(h_-)^2} + F^+(t, t, a) \frac{u^2}{(k_-)^2} \right],
\]

\[
(2.14) \quad Z = \frac{u^2 + u_1^2}{4} \\
\left[ \frac{F^+(s, t, 0)}{s t} \left( \frac{u}{k_- h_+} + \frac{s}{h_- h_-} - \frac{t}{k_- h_-} \right) + \frac{F^+(s, t_1, 0)}{s_1 t} \left( \frac{u_1}{k_- h_+} + \frac{s_1}{k_- k_-} - \frac{t_1}{k_- k_-} \right) \right] \\
+ \left[ \frac{F^+(s_1, t_1, 0)}{s_1 t_1} \left( \frac{u_1}{k_- h_+} + \frac{s_1}{k_- k_-} - \frac{t_1}{k_- k_-} \right) + \frac{F^+(s, t_1, 0)}{s t_1} \left( \frac{u}{k_- h_+} + \frac{s}{k_- k_-} - \frac{t_1}{k_- k_-} \right) \right] \\
+ \mathcal{P} \left[ \frac{E^+(s, t) k_+}{s t} + \frac{E^+(s, t_1) k_-}{s_1 t_1} - \frac{E^+(s_1, t) h_+}{s t_1} - \frac{E^+(s_1, t_1) h_-}{s_1 t} \right] \frac{u^2 - u_1^2}{k_+ k_- h_+ h_-} \\
- \frac{m_e^2}{s t_1} \left[ \frac{F^+(s, t_1, 0)}{h_+} \frac{u_1^2}{(k_+)^2} - \frac{m_e^2}{s_1 t_1} \frac{F^+(s_1, t_1, 0)}{h_-} \frac{u^2}{(k_-)^2} \right] \\
- \frac{m_e^2}{s t} \frac{F^+(s, t, 0)}{h_-} \frac{u_1^2}{(k_-)^2} - \frac{m_e^2}{s_1 t} \frac{F^+(s_1, t, a)}{h_+} \frac{u^2}{(k_+)^2}.
\]
where \( m_e \) is the electron mass and \( a = \frac{(s-s_1)^2}{s s_1} \). The mass-terms (i.e. the terms proportional to \( m_e^2 \)) included here are the relevant ones when the final fermions have angles with the initial direction larger than, say, 1 mrad. For even smaller angles a few more terms have to be included as discussed in [1,5,6,7,8]. The contributions to Eq.s (2.12-14) proportional to the pseudoscalar of Eq. (2.3), not reported in [5], are relevant at the \( Z \) boson peak. The unpolarized cross section can be easily recovered by putting \( P_L = 0 \).

We have verified that the unpolarized annihilation part agrees with the result of Ref.[17] for \( e^+e^- \rightarrow \mu^+\mu^-\gamma \), in the limit of massless fermions. The coulomb part can be verified by noting that it must be reproduced by crossing from the annihilation, i.e. by the exchange of \( p_+ \) with \( -q_- \). Finally, after some algebra, one can recover from our expressions the QED result of Ref.[2] in the appropriate limit; as a consequence of the presence of the \( Z \)-exchange terms, our formulae do not possess the factorization observed there. Such a factorization is recovered only in the unrealistic \( M_Z = 0 \) limit, suggesting that the connection between hard and soft photon emission observed there is related to the absence of a mass scale. Our results reproduce also the results of Ref. [3] for the unpolarized case (in the t-channel \( Z \) boson contribution we put to zero the absorptive part, i.e. the part proportional to the \( Z \) width).

3. Numerical results and comparisons.

On the basis of the above formulae we have constructed a Monte Carlo event generator, BHAGEN-1PH. The algorithms used in the generation and a long write-up can be found in Ref. [9]. However an indispensable part of the construction of the Monte Carlo event generator is its test, which assure that the algorithms used for the generation are well constructed and applied. Numerical tests check also a number of bugs in the program. The tests through comparisons with existing independent programs where possible only for the situation where the beams are unpolarized as no program which can provide cross section for polarized beam(s) is available to our knowledge. However as the algorithm used in BHAGEN-1PH for generation in the situation with the polarized electron beam is identical to the one used for unpolarized beams, no further tests of the algorithm are necessary. On the occasion of the estimation of the theoretical error of the luminosity measurement at LEP [10,11], extensive tests of the QED only \( t \)-channel part of the program were done, as BHAGEN-1PH was included as a part of BHAGEN95, a Monte Carlo program to calculate the cross-section for Bhabha scattering, completed with radiative corrections. There the accuracy of \( 3 \times 10^{-4} \) was reached in the \( \mathcal{O}(\alpha) \) comparisons; no deviation up to this accuracy was observed [10,11] from OLDBIS [12], SABSPV [13], NNLBHA [14], BHLUMI [15]. To complete this tests we present here a comparison with OLDBIS, where only contributions to the cross section of the hard photon spectrum are separately compared in Table 1. This specific tests were not done in [10,11] as only the total error on the complete Bhabha scattering was relevant there. The event selection used here is the BARE1 type [11] and the other cuts and parameters used are described in the caption of Table 1. The results of the various programs agree again within their statistical errors, although the accuracy reached in this comparisons is much lower especially at the hard end of the spectrum, due to the long time necessary to run OLDBIS to obtain the required accuracy.

The program BHAGEN-1PH can be used of course also at lower energies, for example
in the DAΦNE energy range, for which one can simulate the so called large-large angle electron-positron double tagging [6]. For the other extremely forward configurations described in [6] the formulae used in BHAGEN-1PH and presented in this paper are not adequate. In the Table 2 we show a comparisons of the results of BHAGEN-1PH with the results of PHIPHI, which were presented in [6] and [7]. The cuts used are described in the caption of Table 2, where the results are seen to agree well, although the large numerical errors. Again the accuracy of the comparisons is limited by the accuracy of the PHIPHI program results. As the energy is low these comparisons do not test the Z contributions to the cross-section and again t-channel contributions are mostly concerned, due to the angular cuts used. The tests are completed by the comparison with the BABAMC program [16] for large angles around the Z boson resonance, which allows for the tests of s-channel Z boson exchange contributions. As one can see from the Table 3 the agreement with BABAMC is good except for the hard part (0.5 $E_b < E_\gamma < 0.9 E_b$) of the photon spectrum at beam energy $E_b = 47.5$ GeV, where we have found the difference of 0.60(17)%. We guess that this difference is due to the approximation used in BABAMC to simulate this part of the spectrum, although the accuracy seems to be sufficient for every expected experimental precision in this part of the allowed phase space. Finally in Table 4 we present some cross-section results obtained with the longitudinally polarized initial electron, with beam degree of polarization $P_L = \pm 1$, in the experimentally accessible region $0.1 < E_\gamma/E_b < 0.5$, cuts and parameters are specified in the table caption.

4. Conclusions.

We have obtained simple analytic formulae for the cross-section of the $e^+e^- \to e^+e^-\gamma$ process, with Z exchange included and longitudinally polarized initial electron. A fast Monte Carlo event generator simulating the process, BHAGEN-1PH, has been implemented to cover the experimental event selection, although in its present version the program is not suitable for configurations where final fermions are extremely forward ($\theta_\pm < 1\text{mrad}$).

Apart from the intrinsic interest for a direct independent test of the electroweak couplings, the bremsstrahlung process constitutes an important background to other processes. Together with the existing soft and collinear corrections the program is used [10,11] to calculate radiative corrections to the Bhabha scattering, including Z exchange, for different detector arrangements.

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Table 1. Total cross-section (in nanobarns) are compared between OLDBIS [12] and BHAGEN-1PH for $t$-channel contribution to the $e^+e^- \rightarrow e^+e^-\gamma$ process, for different ranges of the photon emitted energy ($\Delta = \frac{E_{\gamma}}{E_b}$), for the electron and positron angular range $3^\circ \leq \theta_\pm \leq 8^\circ$ and for beam energy $E_b = 47.585$ GeV. No cuts are applied on photon angles. The statistical variance (one standard deviation) is given in brackets. In the third column is the ratio of BHAGEN-1PH cross-section to OLDBIS cross-section.

| $\Delta$ | OLDBIS | BHAGEN-1PH | BG/OB |
|-----------|--------|------------|-------|
| 0.01 - 0.1 | 12.235(4) | 12.2374(5) | 1.0002(4) |
| 0.1 - 0.3 | 4.861(4) | 4.8557(3) | 0.9989(9) |
| 0.3 - 0.5 | 1.631(4) | 1.63236(15) | 1.001(3) |
| 0.5 - 0.7 | 0.630(4) | 0.63183(9) | 1.003(7) |
| 0.7 - 0.9 | 0.303(4) | 0.31061(5) | 1.025(14) |

Table 2. Total cross-section (in microbarns) are compared between PHIPHI (values in [6] and correction factor in [7]) and BHAGEN-1PH for the $e^+e^- \rightarrow e^+e^-\gamma$ process, for different ranges of the photon emitted energy ($E_{\gamma}^{min} < E_{\gamma} < E_b - m_e^2/E_b$), for the electron and positron angular range $8.5^\circ \leq \theta_\pm \leq 171.5^\circ$ and for beam energy $E_b = 0.51$ GeV. The minimal allowed angle between photon and final lepton is $0.5^\circ$. The statistical variance (one standard deviation) is given in brackets. In the third column is the ratio of BHAGEN-1PH cross-section to PHIPHI cross-section.

| $E_{\gamma}^{min}$ (MeV) | PHIPHI | BHAGEN-1PH | BG/PH |
|------------------------|--------|------------|-------|
| 2 | 15.9(6) | 16.18(7) | 1.02(4) |
| 5 | 13.0(5) | 13.09(6) | 1.01(5) |
| 10 | 10.7(4) | 10.78(5) | 1.01(4) |
| 50 | 5.66(21) | 5.68(2) | 1.00(4) |
| 100 | 3.6(1) | 3.689(14) | 1.02(3) |
Table 3. Total cross-section (in picobarns) are compared between BABAMC [16] and BHAGEN-1PH for the $e^+e^- \rightarrow e^+e^-\gamma$ process, for different ranges of the photon emitted energy ($\Delta = \frac{E_\gamma}{E_b}$) and beam energy $E_b$, for the electron and positron angular range $40^\circ \leq \theta_{\pm} \leq 140^\circ$. No cuts are applied on photon angles. The statistical variance (one standard deviation) is given in brackets. In the third column is the ratio of BHAGEN-1PH cross-section to BABAMC cross-section. The following parameters are used: $Z$ boson mass $M_Z = 91.175$ GeV, $Z$ boson width $\Gamma_Z = 2.3355$ GeV and weak mixing angle $\sin \theta_W = 0.2247$.

| $E_b$ (GeV) | $\Delta$ | BABAMC     | BHAGEN-1PH | BG/BC   |
|-------------|-----------|-------------|------------|--------|
| 43.5        | 0.1 - 0.5 | 74.143(15)  | 74.150(5)  | 1.0001(3) |
| 43.5        | 0.5 - 0.9 | 15.970(13)  | 15.9824(14)| 1.0008(9) |
| 45.5        | 0.1 - 0.5 | 241.24(8)   | 241.29(3)  | 1.0002(5) |
| 45.5        | 0.5 - 0.9 | 55.34(7)    | 55.43(1)   | 1.0016(14)|
| 47.5        | 0.1 - 0.5 | 63.780(11)  | 63.758(5)  | 0.99966(25)|
| 47.5        | 0.5 - 0.9 | 8.325(13)   | 8.3750(8)  | 1.0060(17)|

Table 4. Total cross-section (in picobarns) from BHAGEN-1PH for the process $e^+e^- \rightarrow e^+e^-\gamma$, for longitudinally polarized electron beam with degree of polarization $P_L$, for $0.1 \leq \Delta \leq 0.5$ ($\Delta = \frac{E_\gamma}{E_b}$) and for different beam energies $E_b$, for the electron and positron angular range $40^\circ \leq \theta_{\pm} \leq 140^\circ$. No cuts are applied on photon angles. The statistical variance (one standard deviation) is given in brackets. The parameters are as in Table 3.

| $E_b$ (GeV) | $\Delta$ | $P_L = +1$ | $P_L = 0$ | $P_L = -1$ |
|-------------|-----------|------------|-----------|------------|
| 43.5        | 0.1 - 0.5 | 68.095(3)  | 74.150(4) | 80.198(5) |
| 45.5        | 0.1 - 0.5 | 200.90(1)  | 241.29(2) | 281.69(2) |
| 47.5        | 0.1 - 0.5 | 57.469(3)  | 63.760(4) | 70.062(5)|

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