DEFORMATIONS OF THE SCHWARZSCHILD BLACK HOLE

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Due to its large number of symmetries the Schwarzschild Black Hole can be described by a specific two-dimensional dilaton gravity model. After reviewing classical, semi-classical and quantum properties and a brief discussion of virtual black holes deformations are studied: the first part is devoted to deformations of the Lorentz-symmetry, the second part to dynamical deformations and its role for the long time evaporation of the Schwarzschild Black Hole.

1. Introduction

Although the focus will be on the Schwarzschild Black Hole (SS BH) it is emphasized that most considerations below are valid for generic dilaton gravity models in two dimensions (2d). In this context the review[1] may be consulted. The main idea behind a study of the SS BH as a 2d effective theory is to gain insight into the otherwise more difficult accessible realm of quantum gravity by exploiting advantageous features of dilaton gravity in its first order formulation.

2. Classical SS BH

The SS BH may be studied at the level of 2d dilaton gravity by means of spherical reduction[2,3]. Especially the first order formulation[4]

\[ L = \text{const} \cdot \int_{\mathcal{M}_2} [X_a(D \wedge e)^a + X \wedge \omega + e\mathcal{V}(X, X^aX_a)] , \]  

(1)

turned out to be very useful already classically[5] because it allowed the construction of the corresponding Carter-Penrose (CP) diagram with particular ease. For the SS BH the potential reads \( \mathcal{V} = -X^+X^-/(2X) - \text{const} \).
3. Semi-classical SS BH
Particle creation due to the Hawking effect exists also in 2d models. For technical details cf. refs. and references therein.

4. Quantum SS BH
Kuchař has shown that the mass of the SS BH obeys a Schrödinger equation. At the level of dilaton gravity this can be argued as follows: since—in the absence of matter and test particles—the only physical observable is the so-called Casimir function (which for the SS BH is essentially given by the ADM mass) any Schrödinger functional can only depend on it (and its conjugate variable, the extrinsic time).

5. Virtual SS BH
Instead of fixing a geometric background and quantizing matter on it, it is possible to integrate out geometry first and to perform a perturbative expansion in the matter degrees of freedom afterward. If this is done to lowest order for scattering of s-waves of massless scalars virtual BHs appear at an intermediate stage. Due to our choice of “temporal gauge” (which was a technical pre-requisite for a successful path integration of geometry) the ensuing line-element has (outgoing) Sachs-Bondi form

\[(ds)^2 = 2drdu + \left(1 - \frac{2m(u,r)}{r} - a(u,r)r + d(u,r)\right)(du)^2,\]

with \(m\), \(a\) and \(d\) being localized on the cut \(u = u_0\) with compact support \(r < r_0\). These quantities depend on the second set of coordinates \(u_0\), \(r_0\). A corresponding CP diagram can be constructed as follows (cf. Figs.):

- Take Minkowski spacetime (or whatever corresponds to the geometry implied by the chosen boundary conditions on the auxiliary fields \(X, X^a\)) and draw \(N\) different points in its CP diagram; see left diagram of Fig.
- Draw \(N\) copies of this CP diagram and add one light light cut to each (always ending at a different point); remove the other \(N - 1\) points; the line element is given by; see middle diagram of Fig. (note: each of these CP diagrams is equivalent to the one depicted in Fig. with varying endpoint \(y\)).
- Glue together all CP diagrams at \(I^\pm\) and \(i^0\) (which is a common boundary to all these diagrams); see right diagram of Fig.
- Take the limit \(N \to \infty\).

The localization of “mass” and “Rindler acceleration” on a light-like cut is not an artifact of an accidental gauge choice, but has a physical interpretation in terms of the Ricci-scalar. Certain parallels to Hawking’s Euclidean VBHs can be observed, but also essential differences. The main one is our Minkowski signature which we deem to be a positive feature.
Figure 1. CP diagram of a single VBH; the point $y$ corresponds to $u = u_0$, $r = r_0$ in (2).

Figure 2. Constructing the CP diagram of all VBHs

One should not take the effective geometry at face value – this would be like over-interpreting the role of virtual particles in a loop diagram. It is a nonlocal entity and one still has to “sum” (read: integrate) over all possible geometries of this type in order to obtain the nonlocal vertices and the scattering amplitude. Nonetheless, the simplicity of this geometry and the fact that all possible configurations are summed over are nice features of this picture. Because all VBH geometries coincide asymptotically the boundaries of the diagram, $\mathcal{I}^\pm$ and $i^0$, behave in a classical way\(^c\) (thus enabling one to construct an ordinary Fock space like in fixed background QFT), but the more one zooms into the geometry the less classical it becomes.

\(^c\)Clearly the imposed boundary conditions play a crucial role in this context. They produce effectively a fixed background, but only at the boundary.
The coherent sum over all virtual BHs yields a finite S-matrix for ingoing modes with momenta $q, q'$ and outgoing ones $k, k'$,

$$T(q, q'; k, k') = -\frac{i\kappa \delta (k + k' - q - q')}{E^3 T},$$

(3)

with the total energy $E = q + q'$, $\kappa = 8\pi G_N$,

$$\tilde{T}(q, q'; k, k') := \frac{1}{E^3} \left[ \Pi \ln \frac{\Pi^2}{E^6} + \frac{1}{\Pi} \sum_{p \in \{k, k', q, q'\}} p^2 \ln \frac{p^2}{E^2} \left( 3kk'qq' - \frac{1}{2} \sum_{r \neq p, s \neq r, p} (r^2 s^2) \right) \right],$$

(4)

and the momentum transfer function $\Pi = (k + k')(k - q)(k' - q)$. The interesting part of the scattering amplitude is encoded in the scale independent factor $\tilde{T}$. The forward scattering poles occurring for $\Pi = 0$ should be noted.

Physically the s-waves of the massless Klein-Gordon field are scattered on their own gravitational self-energy. By rearrangement of the outer legs also a decay of an ingoing s-wave into three outgoing ones is possible.

6. Deformed SS BH

Inspired by recent work on deformed Lorentz symmetry\textsuperscript{11} (the different original proposal has been presented a decade before\textsuperscript{12}) Mignemi constructed deformed dilaton gravity models\textsuperscript{13}. However, it was shown\textsuperscript{14} that either the deformation is completely trivial (i.e. the metric is unchanged) or the resulting “metric” has pathological features of non-invariance. Thus, deformations implemented in this way do not seem to be very useful.

7. Dynamically deformed SS BH

A different implementation of deformations has been proposed recently\textsuperscript{15}: exploiting the result on the most general consistent deformation (in the sense of Barnich and Henneaux\textsuperscript{16}) of 2d BF-theory\textsuperscript{17} together with assumptions on the asymptotics, the global structure and boundedness of the asymptotic matter flux an evolution equation for the deformation parameter has been established. For the SS BH an attractor solution has been found corresponding to flat spacetime. The evolution could be described in this manner only down to a mass of $M_{\text{Planck}}/2$. This remnant mass supposedly is emitted in a final “thunderbolt”. The CP diagram (cf. Fig. 6) has similarities to the one of the Frolov-Vilkovisky model\textsuperscript{18} at $v = v_0$ a shock wave creates the BH (thus region I is Minkowski spacetime); at $u = 0$ the Hawking process commences (thus region II is Schwarzschild spacetime); at $u = u_f$ the thunderbolt is emitted (in region III dynamical deformation occurs); region IV is the remnant geometry (flat spacetime).
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