Letter

X-ray absorption via electron–ion bremsstrahlung in Maxwellian plasma at the exact consideration of Coulomb potential

A G Ghazaryan

Centre of Strong Fields Physics, Yerevan State University, 1 A. Manukian, Yerevan 0025, Armenia

E-mail: amarkos@ysu.am

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Abstract

Based on the results of exact consideration of electron–ion Coulomb interaction, we study the absorption of hard x-ray quanta of arbitrary frequency by numerical simulations. The one-photon absorption coefficient is investigated including the practically more interesting case when photon energy is of the order of plasma temperature. We consider the case of high temperature plasma within the high nuclear charges as well (at which the Born approximation is not applicable). It is shown that one can achieve the efficient absorption coefficient in these cases.

Keywords: x-ray quanta absorption coefficient, bremsstrahlung, laser-plasma interaction

1. Introduction

With the advent of contemporary intense x-ray free electron lasers (FELs) [1–3], interest has grown in x-ray stimulated bremsstrahlung (SB) absorption, because for the short laser pulses it may become the dominant mechanism of the absorption of strong laser radiation in plasma [4]. Furthermore, in the field of intense x-ray laser an electron in the SB process may acquire the essential energy, which makes it an effective mechanism for laser-plasma heating, absorbing even one to two photons [5, 6]. Many papers have been dedicated to the theoretical investigation of the SB process in gas or plasma medium in the strong electromagnetic (EM) radiation field in various approximations over the scattering potential or EM wave field using nonrelativistic [4, 7–17] or relativistic descriptions [18–22]. What concerns the multiphoton absorption of plasma at high radiation intensities, the relativistic theory of the latter has been succeeded in the Born approximation by the scattering potential of a plasma ion, which demonstrates that for the relativistic laser intensities the SB rate exhibits a tenuous dependence on plasma temperature. So for middle wave intensities, the consideration of dependence on the plasma temperature is of interest.

Meanwhile, for the optical or infrared lasers the quantum effects are small because of the smallness of the photon energy. However for intense x-ray radiation the nonlinear over the field quantum effects will be considerable. What concerns the multiphoton absorption of plasma at high (and super-high) radiation intensities, the relativistic theory of the latter has been majorly succeeded in the regime of small angles
scattering for electrons SB [5, 22]. In spite of this, and to find out the SB absorption rate dependence versus the electron temperature, in the present paper the inverse-bremsstrahlung absorption of an x-ray laser field in the classical Maxwellian plasma is considered by the quantum mechanical theory taking into account a scattering potential field exactly, while the x-ray radiation is treated by perturbation theory.

The earlier investigations of the one-photon absorption of high-frequency electromagnetic radiation by plasma via the stimulated bremsstrahlung of electrons on the ion scattering potentials with the exact consideration of Coulomb potential have been succeeded analytically only in two boundary cases of photon frequency: for \( \hbar \omega \ll \kappa T \) and \( \hbar \omega \gg \kappa T \) (\( \kappa \) is the Boltzmann’s constant, \( \omega \) is the EM wave frequency) [17]. Taking into account the significance of the problem involving, specifically, the laser heating of plasma by current x-ray FELs and its diagnostics, in the present paper, based on mentioned results of exact consideration of electron–ion Coulomb interaction, by further numerical simulations for ultrarelativistic absorption coefficient, we study the absorption of hard x-ray quanta of arbitrary frequencies, including the practically more interesting case of frequencies \( \omega \sim \kappa T /h \), in high-temperature plasma within the high nuclear charges \( Z \) as well (at which the Born approximation is not applicable), constituting also the temperature dependence of a one-photon absorption coefficient.

The organization of this paper is as follows. In section 2 the quantum dynamics of x-ray absorption via electron–ion SB in Maxwellian plasma is presented with analytical results for the inverse-bremsstrahlung absorption coefficient at the exact consideration of Coulomb field. In section 3, the analytic formulas are considered numerically. Conclusions are given in section 4.

2. Basic theory of SB taking Coulomb potential exactly

The above-mentioned problem lead to the quantum-mechanical investigation of the dynamics of the SB process. Let us investigate the inverse-absorption of x-ray radiation considering the Coulomb potential exactly, while the EM wave field is treated by perturbation theory. The absorption coefficient \( \alpha \) of the weak wave in the one-photon approximation determined by the following formula:

\[
\alpha = \frac{\omega_0}{J} \int f(k)|w_a(k) - w_e(k)|^2 dk \left(\frac{2\pi}{Z}\right)^3 \quad (1)
\]

where \( E_0 \) is the amplitude of the electric field strength \( E(t) = \tilde{X} E_0 \sin \omega t \) of a linearly polarized EM wave in dipole approximation (\( \tilde{X} \) is a unit vector), \( J = \frac{eE_0^2}{8\pi} \) is the intensity of the EM wave in linear polarization case, \( w_a \) and \( w_e \) total probabilities of one-photon absorption and emission respectively, \( f(k) \) is the distribution function of the electrons over the wavevectors \( k = p/h \), which is normalized on the electron number density \( N_e \) as follows:

\[
\int f(k) dk^3 = (2\pi)^3 N_e.
\]

\( \varphi(r) = -Ze/r \) is the the Coulomb attractive potential, \( e \) is the electron charge, \( Ze \) is the nuclear charge. In [17] it was calculated the absorption coefficient of the weak EM wave in a plasma with the help of obtained wavefunction of the SB process taking the Coulomb potential exactly. We using the results obtained according to earlier analytic calculations [17] for the isotropic plasma with Maxwellian distribution function:

\[
f(k) = N_e \left(\frac{2\pi \hbar^2}{\mu \kappa T}\right)^{3/2} \exp \left(-\frac{\hbar^2 k^2}{2\mu \kappa T}\right), \quad (2)
\]

where \( \mu \) is the electron mass, \( T \) is the temperature of electrons in plasma. After integration over the phase angles the absorption coefficient \( \alpha \) (1) have the form [17]

\[
\begin{align*}
\alpha &= \alpha_B \frac{1}{2} \chi^{3/2} \\
&\times \int_0^\infty \exp \left(-\chi t^2\right) I_1 \left(k_0, \sqrt{1 + t^2}, k_0 t\right) \\
&\times \left[ \exp \left(\frac{2\pi a}{k_0 t}\right) - 1 \right]^{-1} \left[ 1 - \exp \left(-\frac{2\pi a}{\sqrt{1 + t^2} k_0}\right) \right]^{-1} \\
&\times \exp (-\chi) \left[ \exp \left(-\frac{2\pi a}{\sqrt{1 + t^2} k_0}\right) - 1 \right]^{-1} \left[ 1 - \exp \left(\frac{2\pi a}{k_0 t}\right) \right]^{-1} dt,
\end{align*}
\]

where the integral \( I_1 \) [28] is equal to

\[
I_1(k_1, k_2) = z \frac{d}{dz} \left[ F \left( \frac{a}{k_1}, i \frac{a}{k_2}, z \right) \right]^2,
\]

and \( z = -4k_1 k_2 / (k_1 - k_2)^2 \), \( F(\alpha, \beta, z) \) is the confluent hypergeometric function.

In the case of hard radiation

\[
a/k_\omega \ll 1,
\]

the absorption coefficient [17] in the lowest order of \( a/k_\omega \) can been reduced to the form:

\[
\alpha = \alpha_B \sqrt{\hbar^3} \chi^{3/2} \left(\frac{a}{k_\omega}\right) \quad (5)
\]

where \( \chi = \hbar \omega / (\kappa T) \) is the plasma-wave interaction parameter, \( a = Ze \) is the Bohr radius, \( k_\omega = (2\mu \omega/h)^{1/2} \), \( N_i \) is the ion number density. Here the both EM and Coulomb fields dependent function \( I \left( \chi, a/k_\omega \right) \) is

\[
I \left( \chi, a/k_\omega \right) = \int_0^\infty \exp \left(-\chi t^2\right) \ln \left| t + \sqrt{1 + t^2} \right| \\
\times \left[ 1 - \exp \left(-\chi t^2\right) \right] \left[ 1 - \exp \left(-\frac{2\pi a}{k_0 t}\right) \right] dt,
\]

where

\[
\alpha_B = \frac{32\pi^2 N_i N_e}{3 \mu e^2 \hbar^2} \quad (7)
\]

is the absorption coefficient in the Born approximation, which is the same in the cases for anisotropic electron distribution—monochromatic beam [17], as well for isotropic plasma with
A G Ghazaryan

arbitrary momentum distribution of electrons in the corresponding hard quantum case ($a/kω ≪ 1$ or $ℏω ≫ ℏ^2a^2/2μ$, $ℏ^2a^2/2μ$ is the binding energy in the ground state) [4].

Previous analytical studies were successful in two bounded cases of photon frequency. The first, for high-temperature plasma, $κT ≫ ℏω ≫ ℏ^2a^2/2μ$, by expanding exponent $\exp(-χ)$ in power series and performing integration in (6) one was obtained the formula:

$$\alpha = α_B 4\pi \left( \frac{a}{kω} \right) \frac{ℏω}{κT}$$

(8)

The second, for the case of $ℏω ≫ κT$, when the absorption coefficient $α$ (3) in Maxwellian plasma was expressed by the following formula:

$$\alpha = α_B 4π \left( \frac{a}{kω} \right) \left( \frac{ℏω}{κT} \right)^{1/2} \left| \int_0^{x_0} \left( 1 - \frac{i a}{kω}, x_0 \right) F \left( 1 - i \frac{a}{kω}, 1, x_0 \right) dx_0 \right|^2$$

$$\times (1 - \exp(-2πa/kω))^{-1} (\exp(-2πa/kT) - 1)^{-1}.$$  (9)

where $k_T = \sqrt{2μκT/ℏ}$, $x_0 = i a/kω$, $x_0 = i a/kω$. In the Born approximation by the scattering potential, when $a/kω ≪ 1$ and $a/κT ≪ 1$, the expression (9) coincides with the expression (7).

Further we continue our study by numerical calculations for one-photon inverse-bremsstrahlung absorption coefficient [17] of x-ray radiation of arbitrary frequency, including the more interesting case of $ω ≅ κT/ℏ$, in Maxwellian plasma of arbitrary temperature $T$.

3. Numerical results for one-photon SB absorption coefficient of hard x-ray

To investigate the SB absorption coefficient $α$ by numerical simulations we will utilize equation (3). Besides, as follows from (3) or (5) with (6), when Coulomb potential is taken into account exactly the absorption coefficient in the limit of low temperatures $a/κT ≫ 1$ decreases exponentially $α ∼ \exp(-2πa/kT)$ (according to the formula (9)). The magnitude of the electron momentum change during the scattering process of low-temperature electrons in the Coulomb potential

![Figure 1](image1.png)

**Figure 1.** The absorption coefficient $α$ scaled to $α_B$ of one-photon hard radiation (of linear polarization) inverse bremsstrahlung in Maxwellian plasma versus the dimensionless parameter ($ℏω = 5$ keV).

![Figure 2](image2.png)

**Figure 2.** The same as figure 1 but for photon energies $ℏω ≅ κT$.

![Figure 3](image3.png)

**Figure 3.** The absorption coefficient $α/α_B$ of the one-photon hard radiation inverse bremsstrahlung absorption coefficient scaled to $1/T_n$ versus the temperature $T_n$ for setup of figure 1.

![Figure 4](image4.png)

**Figure 4.** The same as figure 3 but not scaled to the temperature for $T_n ≅ 1$. 

![Figure 5](image5.png)

**Figure 5.** The absorption coefficient $α/α_B$ of the one-photon hard radiation inverse bremsstrahlung absorption coefficient scaled to $1/T_n$ versus the temperature $T_n$ for setup of figure 1.
is much less than is necessary for real absorption of quantum energy many times greater than the electron’s energy $kT$.

We practically study the more interesting case of absorption of hard x-ray quanta, making integration numerically. To show the dependence of the one-photon inverse-bremsstrahlung rates on the photon energy, in the figure 1 the absorption coefficient $\alpha$ (3) scaled to $\alpha_B$ (7) of linearly polarized wave in Maxwellian plasma versus $\chi$ for various photon energies is shown. As seen from this figure, in the case of hard radiation in high-temperature plasma with high nuclear charges $Z$ (beyond the Born approximation) the inverse-bremsstrahlung absorption coefficient dependence on $\chi$ has the characteristic maximum after which it decreases with increasing of the parameter $\chi$, and the SB coefficient is considerably suppressed with the increase of the wave quanta energy. In figure 2 the absorption coefficient $\alpha$ scaled to $\alpha_B$ dependence versus parameter $\chi$ is showed for the frequencies $\omega \sim kT/\hbar$ ($T_e \equiv T/\hbar \omega \sim 1$), which demonstrates that as the energy of the quantum of the wave increases, the probability of SB absorption reaches the maximal value when $\hbar \omega \sim kT$, and it demonstrates that one can achieve the efficient absorption coefficient in this case.

Further, our numerical simulations give the characteristic curves for the dependence of $\alpha/\alpha_B$ versus plasma temperature (for linearly polarized wave). To compare with the case when the electron–ion interaction is considered in the Born approximation [4], in figure 3 the dependence of the one-photon hard radiation inverse-bremsstrahlung absorption coefficient $\alpha/\alpha_B$ scaled to $1/T_e$ versus the plasma temperature is shown for various laser radiation frequencies. This figure 3 demonstrates that when the electron–ion interaction is taken into account exactly the absorption coefficient of high-temperature plasma at $kT \gg \hbar \omega \gg \hbar^2 a^2/2\mu$ ($\chi \ll 1$) dependence versus the plasma temperature $T_e$ is suppressed. Thus, the results obtained now revealed a different dependence of absorption coefficient on the plasma temperature $T$, in contrast to the known results obtained for the weak wave in isotropic plasma in the Born approximation in the soft photon limit ($\hbar \omega/\kappa T \ll 1$) [4]. If the Coulomb potential is taken into account exactly the absorption coefficient depends on plasma electron temperature as $\alpha \sim 1/T$, it decreases at high temperature plasma more slowly with plasma temperature, than in the Born approximation by the electron–ion interaction ($\alpha \sim T^{-3/2} \ln T$) [4]. In figure 4 the normed absorption coefficient $\alpha/\alpha_B$ dependence versus $T$ in the vicinity of quanta energy at $\hbar \omega \approx kT$ is shown. It demonstrates that, for hard radiation in the case of such frequencies in high temperature plasma with the high nuclear charges $Z$, one-photon absorption coefficient dependence versus $T$ has the maximum.

4. Conclusion

We have presented the numerical investigation of x-ray radiation one-photon SB absorption coefficient under the limits on the main characteristic coefficients of the process, such as photon energy $\hbar \omega$, plasma temperature $kT$ and ionization energy $\hbar^2 a^2/2\mu$, based on earlier results of the exact quantum mechanical consideration of the dynamic of electron–ion Coulomb interaction in the presence of a transverse EM wave, when Coulomb potential is taken exactly, while the EM wave is treated by perturbation theory. The coefficient of SB absorption has been calculated considering the classical Maxwellian distribution, the EM wave is linearly polarized. We study in details the more interesting case of absorption of hard x-rays ($a/k\tau \ll 1$, $\chi \sim 1$) in high temperature plasma with high nuclear charges as well, at which the Born approximation by scattering potential is broken. The obtained results demonstrate that one can achieve the efficient absorption coefficient in these cases, and the SB rate is suppressed with the increase of the wave quanta energy. For high-temperature plasma the absorption coefficient $\alpha$ decreases as $1/T$ in contrast to the case, when the scattering potential considering as perturbation where one has the dependence $\ln T/T^{3/2}$ [4]. In the limit of low plasma electron temperatures, $a/k\tau \gg 1$, one-photon SB absorption coefficient exponentially decreases because of the classical nature of the electron interaction with the Coulomb potential, whereas it interaction with EM wave has quantum nature (one-photon inverse bremsstrahlung). The problem is significantly connected with the laser heating of plasma by current x-ray FELs and its diagnostics.

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A G Ghazaryan

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