The Star Formation Rate in the Gravoturbulent Interstellar Medium

Blakesley Burkhart
Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA

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Abstract

Stars form in supersonic turbulent molecular clouds that are self-gravitating. We present an analytic determination of the star formation rate (SFR) in a gravoturbulent medium based on the density probability distribution function of molecular clouds having a piecewise lognormal (LN) and power-law (PL) form. This is in contrast to previous analytic SFR models that are governed primarily by interstellar turbulence, which sets purely LN density probability distribution functions (PDFs). In the gravoturbulent SFR model described herein, low-density gas resides in the LN portion of the PDF. Gas becomes gravitationally unstable past a critical density \( \rho_{\text{crit}} \), and the PDF begins to form a PL. As the collapse of the cloud proceeds, the transitional density \( \rho_t \) between the LN and PL portions of the PDF moves toward lower density while the slope of the PL \( (\alpha) \) becomes increasingly shallow. The SFR per free-fall time is calculated via an integral over the LN from \( \rho_{\text{crit}} \) to \( \rho_t \) and an integral over the PL from \( \rho_t \) to the maximum density. As \( \alpha \) becomes shallower, the SFR accelerates beyond the expected values calculated from an LN density PDF. We show that the star formation efficiency per free-fall time in observations of local molecular clouds increases with shallower PDF PL slopes, in agreement with our model. Our model can explain why star formation is spatially and temporally variable within a cloud and why the depletion times observed in local and extragalactic giant molecular clouds vary. Both star-bursting and quiescent star-forming systems can be explained without the need to invoke extreme variations of turbulence in the local interstellar environment.

Key words: galaxies: star formation – magnetohydrodynamics (MHD)

1. Introduction

Stars are the fundamental link between galaxy evolution, planet formation, and astrobiology (Chyba & Hand 2005; McKee & Ostriker 2007; Krumholz 2014). Stars inject energy and momentum into the surrounding interstellar and intergalactic medium, an effect commonly known as stellar feedback, via winds and supernova (Fall et al. 2010; Lopez et al. 2011; Rosen et al. 2014; Ageretz et al. 2015; Rosen et al. 2016; Hayward & Hopkins 2017). Despite the importance of stars for galaxy evolution, a full observational and theoretical understanding of their formation remains elusive. If gravity alone was relevant, galactic interstellar clouds with density \( \rho \) should collapse to form stars on roughly a free-fall time of

\[
\tau_{\text{ff}} = \frac{3\pi}{32G\rho}.
\]

However the dynamics of the star-forming molecular interstellar medium are complicated by energetics that can counteract gravity, including magnetic fields and supersonic turbulence (Mouschovias & Spitzer 1976; Larson 1981; Shu et al. 1987; McKee et al. 1993; Mac Low & Klessen 2004; Lazarian 2007; McKee & Ostriker 2007; Crutcher et al. 2009, 2010; Kowal et al. 2009; Crutcher 2012; Lazarian et al. 2012; Mocz et al. 2017), and the timescales for star formation are consequently much longer than the free-fall time. Additional factors that complicate our understanding of star formation include the many orders of magnitude in spatial and temporal scales and observational biases such as line-of-sight (LOS) effects, limited resolution, and incomplete density tracers (Goodman et al. 2009; Beaumont et al. 2015; Lombardi et al. 2015; Schneider et al. 2015a; Alves et al. 2017; Chen et al. 2018).

One of the main challenges for a theory of star formation is determining the star formation rate (SFR) and star formation efficiency (SFE) in both local molecular clouds and in galaxies (Krumholz 2014). The current paradigm for the initial conditions of star formation is that stars form in dense filamentary molecular clouds that result from gravito-magnetohydrodynamic (MHD) turbulent fragmentation (Mac Low & Klessen 2004; Krumholz & McKee 2005; Hennebelle et al. 2011; Padoan & Nordlund 2011; Burkhart & Lazarian 2012; Collins et al. 2012; Federrath & Klessen 2012; Federrath 2013a; Hopkins 2013; Lee et al. 2015; Semenov et al. 2016; Mocz et al. 2017; Semenov et al. 2017). The initial conditions imprinted on the diffuse and molecular gas on parsec scales (e.g., the level of turbulence, the cloud density, and the structure of the magnetic field) may determine the key properties of the initial mass function (IMF) and the SFRs in galaxies (Krumholz 2014).

In the absence of self-gravity, MHD turbulence sets a lognormal (LN) initial density distribution in star-forming clouds (Vazquez-Semadeni 1994; Padoan et al. 1997; Scalo et al. 1998),

\[
p_{\text{LN}}(s) = \frac{1}{\sqrt{2\pi}\sigma_s^2} \exp\left(-\frac{(s-s_0)^2}{2\sigma_s^2}\right),
\]

expressed in terms of the logarithmic density,

\[
s \equiv \ln(\rho/\rho_0),
\]

and where \( \sigma_s \) is the standard deviation of the LN. The quantities \( \rho_0 \) and \( s_0 \) denote the mean density and mean logarithmic density,\(^1\) the latter of which is related to \( \sigma_s \) by

\[
s_0 = -\frac{1}{2}\sigma_s^2.
\]

Analytic calculations for the SFR have used the LN probability distribution function (PDF) (Krumholz & McKee 2005;...
where \( \left( \text{equation of state is non-isothermal, the SFR calculation can vary as well as on the properties of the LN PDF, set by supersonic turbulence. In particular, the width of the LN PDF is controlled by the sonic Mach number} (M_s) \) and a dimensionless turbulent forcing parameter \( (b; \text{Federrath et al. 2008}) \),

\[
\sigma_t^2 = \ln[1 + b^2 M_s^2].
\]

(5)

Deviations from the LN form have been observed but do not strongly affect the SFR calculation. For example, if the gas equation of state is non-isothermal, the SFR calculation can decrease by roughly a factor of a few (Federrath & Banerjee 2015).

Simulations of self-gravitating MHD turbulence suggest that the gas density PDF stems from a combination of turbulence, which dominates the dynamics at low densities, and self-gravity, which dominates at high densities. Gravitationally unstable free-falling density structures are characterized by a power-law (PL) PDF (Ballesteros-Paredes et al. 2011; Collins et al. 2012; Girichidis et al. 2014; Myers 2015; Burkhardt et al. 2017; Mocz et al. 2017; Myers 2017; Padoan et al. 2017),

\[
p_{\text{PL}}(s) = C e^{-\alpha s}, \quad s > s_c,
\]

(6)

where \( s_c = \ln(\rho_c/\rho_0) \) is the logarithm of the normalized transitional density between the LN and PL forms of the density PDF. Observational studies of giant molecular clouds (GMCs) have confirmed that the highest column density regime (corresponding to visual extinction \( A_V > 1 \)) of the PDF often has a PL distribution, while the lower column density material in the PDF is well-described by an LN form (Kainulainen et al. 2009; Lombardi et al. 2010; Hennebelle et al. 2011; Federrath & Klessen 2012; Kainulainen et al. 2013; Burkhardt et al. 2015; Lombardi et al. 2015; Schneider et al. 2015a; Stutz & Kainulainen 2015; Imara & Burkhart 2016; Bialy et al. 2017b).

We explore an analytic determination of the SFR based on the density PDF having a piecewise LN and PL form. This paper is motivated by the observational and numerical evidence discussed above for the existence of a PL PDF at high densities as well as observational/numerical signatures of accelerating SFRs (Palla & Stahler 2000; Lee et al. 2015, 2016; Caldwell & Chang 2018). In particular, Palla & Stahler (2000) used pre-main-sequence evolutionary tracks to show that the SFR has accelerated rapidly in a number of nearby GMCs over a timescale of roughly \( 10^7 \) yr. Lee et al. (2016) showed that variations in the SFE observed in molecular clouds cannot be explained by the larger scale fluctuations in the cloud turbulent velocity dispersions, while Caldwell & Chang (2018) have shown that the SFE grows quadratically in time. These observations cannot be easily explained by the LN density star formation theories that predict constant SFRs for given turbulence parameters. Moreover, Leroy et al. (2017) found that in the M51 galaxy, as observed by the PdBI Arcsecond Whirlpool Survey (PAWS) survey, the SFE per free-fall time seems to be anticorrelated with the velocity dispersion, demonstrating further tension with turbulent star formation models. However, as we will show, the inclusion of a time varying PL density PDF can account for both accelerated and enhanced star formation without needing to invoke extreme variations in interstellar turbulence.

This paper is organized as follows. In Section 2 we review the LN density PDF calculations of the SFR. In Section 3 we review the analytic derivations of Collins et al. (2012) and Burkhardt et al. (2017) for the piecewise LN plus PL (LN + PL) PDF transition density and normalization. Then in Section 4 we calculate the SFR of the LN + PL PDF and compare this calculation to the LN PDF models described in Section 2. We compare the LN + PL model to observations in Section 5. Finally, we discuss our results in Section 6 followed by our conclusions in Section 7.

2. Review of the SFR based on LN Density PDFs

A number of past studies have developed an analytic derivation of the SFR based on the statistics of supersonic magnetized turbulence (Krumholz & McKee 2005; Hennebelle & Chabrier 2011; Padoan & Nordlund 2011; Federrath & Klessen 2012). All of these models take an integral over the LN density PDF (Equation (2)) in order to estimate the gas mass above a given density threshold that forms stars. Here we briefly review the basic points of these works, namely how the SFR is derived from the LN PDF and its dependencies on the properties of turbulence. We note that many other LN SFR models exist in the literature but we focus our comparison on a subset of them as the basic features are similar. The methodology outline in this section will be repeated for the LN + PL density PDF model derived in this work.

The analytic models of Krumholz & McKee (2005), Padoan & Nordlund (2011), and Hennebelle & Chabrier (2011) for the SFR per free-fall (SFRff) integrate the LN density PDF, Equation (2) multiplied by \( \rho/\rho_0 \), and a normalized free-fall factor. This then provides the SFR per free-fall time (SFRff), first described in Krumholz & McKee (2005), which is the mass fraction going into stars per free-fall time,

\[
\text{SFR}_{\text{ff}} = \epsilon_0 \int_{s_c}^{\infty} \frac{t_{\text{ff}}(\rho)}{t_{\text{ff}}(\rho_0)} \frac{\rho}{\rho_0} p_{\text{LN}}(s) \, ds,
\]

(7)

where \( \epsilon_0 \) is the local efficiency for converting gas into stars and depends on the properties of feedback, and \( s_c = \ln(\rho_c/\rho_0) \) is the critical density for collapse. Given the SFR per free-fall time one can also calculate the SFR as

\[
\text{SFR} = \frac{M_{\text{cloud}}}{t_{\text{ff}}(\rho_0)} \text{SFR}_{\text{ff}}.
\]

(8)

There are two main distinctions between different LN SFR models: the free-fall factor used, i.e., the treatment of the \( \frac{t_{\text{ff}}(\rho)}{t_{\text{ff}}(\rho_0)} \) term in Equation (7), and the definition of the critical density \( (s_{\text{crit}}) \). Krumholz & McKee (2005) estimate the SFRff with a factor of \( t_{\text{ff}}(\rho)/t_{\text{ff}}(\rho_0) = 1 \), while Padoan & Nordlund (2011) use a factor of \( t_{\text{ff}}(\rho)/t_{\text{ff}}(\rho_{\text{crit}}) \). Both of these studies use a free-fall factor that is independent of density, and can therefore be taken out of the integral. \( t_{\text{ff}}(\rho)/t_{\text{ff}}(\rho_{\text{crit}}) \) appears inside the integral in studies by Hennebelle & Chabrier (2011) and Federrath & Klessen (2012) because gas with different densities has different free-fall times. These studies are therefore termed “multifree-fall” SFR models. As for the differences in the critical density, we summarize the different critical densities and corresponding SFRff for the models that...
employ an LN density PDF (see also Table 1 of Federrath & Klessen 2012).

1. Krumholz & McKee (2005). The critical density for Equation (7) for the Krumholz & McKee (2005, KM) model is their Equation (27),

\[
\rho_{\text{crit,KM}} / \rho_0 = \frac{\pi^2}{15} \phi_t^{2} \alpha_{\text{vir}} M_\star^2,
\]

where \(\alpha_{\text{vir}}\) is the virial parameter, \(\phi_t\) is of order unity and accounts for the uncertainty in the timescale factor originally introduced in Krumholz & McKee (2005), and \(M_\star\) is the sonic Mach number. The corresponding SFR\(_{\text{ff}}\) with \(t_{\text{ff}}(\rho_0) / t_{\text{ff}}(\rho) = 1\) is

\[
\text{SFR}_{\text{ff,KM}} = \frac{\epsilon_0}{2} \left( 1 + \text{erf} \left[ \frac{\sigma_v^2 - 2 \sigma_{\text{crit}}}{2 \sqrt{2} \sigma_v} \right] \right),
\]

(10)

2. Padoan & Nordlund (2011). The critical density for Equation (7) for the Padoan & Nordlund (2011, PN) model is their Equation (18),

\[
\rho_{\text{crit,PN}} / \rho_0 = 0.067 \theta^{-2} \alpha_{\text{vir}} M_\star^2 f(\beta),
\]

where \(\theta \approx 0.35\) is the ratio of the cloud size over the turbulent integral scale. The PN critical density therefore has the same proportionality with the sonic Mach number and virial parameter as the KM model. The effect of the magnetic field in slowing down star formation is encompassed in the function with dependency on plasma Beta \((\beta, \text{the ratio of thermal to magnetic pressure})\),

\[
f(\beta) = \frac{(1 + 0.925 \beta^{-3/2})^{2/3}}{(1 + \beta^{-1/2})},
\]

(12)

and the corresponding SFR\(_{\text{ff}}\) with \(t_{\text{ff}}(\rho_0) / t_{\text{ff}}(\rho)\) pulled out of the integral in Equation (7) is

\[
\text{SFR}_{\text{ff,PN}} = \frac{\epsilon_0}{2} \left( 1 + \text{erf} \left[ \frac{\sigma_v^2 - 2 \sigma_{\text{crit}}}{2 \sqrt{2} \sigma_v} \right] \right) \exp(s_{\text{crit}}/2).
\]

(13)

The form of SFR\(_{\text{ff}}\) in Padoan & Nordlund (2011) therefore is nearly identical to the SFR in KM, but multiplied by an additional exponential term which arises from including \(t_{\text{ff}}(\rho_0) / t_{\text{ff}}(\rho) = t_{\text{ff}}(\rho_0) / t_{\text{ff}}(\rho_{\text{crit}})\). In both cases \(\rho_{\text{crit}} \propto \alpha_{\text{vir}} M_\star^2\).

3. Hennebelle & Chabrier (2011) and Federrath & Klessen (2012). The critical density for Equation (7) for the Hennebelle & Chabrier (2011, HC) is specified in follow-up papers by Federrath & Klessen (2012) and Hennebelle & Chabrier (2013). In the HC model, the critical density is defined by requiring that the turbulent Jeans length at the critical density is a fraction (which they denote as \(\gamma_{\text{cut}}\)) of the cloud size scale. This is in contrast to the KM and PN models in which the critical density is proportional to \(M_\star^2\), implying that only very dense structures will lead to star formation. In the HC definition, any structure can collapse if its gravitational energy dominates over all sources of support (thermal, turbulent, and magnetic), as long as the associated perturbation can grow and become unstable,

\[
s_{\text{crit,HC}} = \ln \left[ \rho_{\text{crit,th}} / \rho_0 + \rho_{\text{crit,turb}} / \rho_0 \right],
\]

(14)

where the (magneto)thermal contribution is

\[
\rho_{\text{crit,th}} / \rho_0 = \frac{\pi^2}{5} \gamma_{\text{cut}}^{-2} \alpha_{\text{vir}} M_\star^{-2}(1 + \beta^{-1}),
\]

(15)

and the turbulent contribution is

\[
\rho_{\text{crit,turb}} / \rho_0 = \frac{\pi^2}{15} \gamma_{\text{cut}}^{-4} \alpha_{\text{vir}}.
\]

(16)

The HC model keeps the factor of \(t_{\text{ff}}(\rho_0) / t_{\text{ff}}(\rho)\) inside the integral. The SFR\(_{\text{ff,HC}}\) as reviewed in Federrath & Klessen (2012) is their Equation (40),

\[
\text{SFR}_{\text{ff,HC}} = \frac{\epsilon_0}{2} \left( 1 + \text{erf} \left[ \frac{\sigma_v^2 - 2 \sigma_{\text{crit}}}{2 \sqrt{2} \sigma_v} \right] \right) \exp\left[ (3/8) \sigma_v^2 \right].
\]

(17)

We note that the multifree-fall SFR\(_{\text{ff}}\) extension discussed in Federrath & Klessen (2012) for the KM and PN models has the same form as Equation (17).

3. The Piecewise LN + PL Density PDF

The analytic models for the SFR rate of Krumholz & McKee (2005), Padoan & Nordlund (2011), and Hennebelle & Chabrier (2011), as well as follow-up modifications discussed by Federrath & Klessen (2012), Federrath & Banerjee (2015), and Gribel et al. (2017), are all based on integrals over the LN density PDF and thereby neglect contributions to the SFR that arise from the PDF PL tail. Numerous studies have now shown that the LN form of the PDF of column density or density describes the behavior of diffuse molecular and atomic gas (Berkhuijsen & Fletcher 2008; Hill et al. 2008; Burkhart et al. 2010; Imara & Burkhart 2016; Bialy et al. 2017). However, as reviewed in the Introduction, the dense star-forming gas PDF predominantly takes a piecewise LN + PL form in both 3D density (from simulations) and column density tracers (Kainulainen et al. 2009; Collins et al. 2012; Burkhart et al. 2015a, 2015b; Schneider et al. 2015a; Burkhart et al. 2015b; Imara & Burkhart 2016). In some cases the PDF of GMCs may be fully PL (Lombardi et al. 2015; Alves et al. 2017).

In light of these studies, we now consider the SFR calculation for a piecewise form of the density PDF in and around a star-forming molecular cloud that consists of an LN at low density, a PL at high density, and a transition point \((s_t = \ln(\rho / \rho_0))\) between the two. This form of the PDF was considered in recent works such as Collins et al. (2012) and Burkhart et al. (2017) as

\[
P_{\text{LN+PL}}(s) = \begin{cases} 
\frac{N}{\sqrt{2\pi} \sigma_v} e^{-(s-s_t)^2}, & s < s_t \\
N C e^{-\alpha s}, & s > s_t,
\end{cases}
\]

(18)

where again, \(s_0 = -\frac{1}{2} \sigma_v^2\).

Here the normalization, \(N\), is determined by the normalization requirement, \(\int_{-\infty}^{\infty} p_{\text{LN+PL}}(s) ds = 1\), and is given by

\[
N = \left( \frac{C e^{-\alpha s}}{\alpha} + \frac{1}{2} + \frac{1}{2} \text{erf} \left( \frac{2 s_t + \sigma_v^2}{2 \sqrt{2} \sigma_v} \right) \right)^{-1}.
\]

(19)

We note that in our definition of the PL slope \(\alpha\) is positive since the minus sign appears in the exponent separately. Assuming that \(P_{\text{LN+PL}}(s)\) is continuous and differentiable, we can formulate an analytic estimate for \(C\) and \(s_t\). These two
conditions yield

\[ s_t = (\alpha - 1/2) \alpha^2. \] (20)

And the amplitude of the PL is

\[ C = \frac{e^{1/2(\alpha-1)\alpha^2}}{\alpha \sqrt{2\pi}}. \] (21)

The transition density value between the LN and PL PDFs therefore depends on the slope of the PL, the width of the LN, and the mean density since \( s_t = \ln(\rho/\rho_0) \).

We pause here to discuss the physical meaning of the PL slope in the context of the piecewise PDF and the transition density. Numerical simulations of gravoturbulence suggest that the PDF of noncollapsing regions retracts the characteristics of the initial supersonic turbulence field (e.g., remain in the LN), while the density PDFs of collapsing regions show a clear PL at high density (Collins et al. 2012; Lee et al. 2015; Mocz et al. 2017). Once the critical density for gravitational collapse is reached the PL begins to form. The characteristic slope of that PL changes roughly from the cloud mean free-fall time from steep, \( \alpha \approx 3 \), to shallow, \( \alpha \approx 1.5 \) to 1 (Girichidis et al. 2014; Burkhart et al. 2017). The exact timescale for \( \alpha \) to saturate depends on the strength of the magnetic field (Burkhart et al. 2015a). The saturation value for the PL slope can be analytically determined from scale-free gravitational collapse (Girichidis et al. 2014; Guszejnov et al. 2017). As \( \alpha \) shallow, the transition density \( s_t \), between the PDF LN and PL moves toward lower density (Equation (20)). We illustrate the change of the PL slope and transition density in Figure 1.

By combining Equations (20) and (5), the transition point can be further related to the physics of the medium (Burkhart et al. 2017):

\[ s_t = \frac{1}{2}(2\alpha - 1)\ln[1 + b^2M^2_f]. \] (22)

The PDF transition density can then be expressed in terms of the isothermal post-shock density, \( \rho_{ps} = \rho_0M^2_f \), which is the density at which the turbulent energy density is equal to the thermal pressure,

\[ \frac{1}{2}\rho_{ps}c_s^2 = \frac{1}{2}\rho_0v^2. \] (23)

Manipulating this relation we find that \( M^2_f = \rho_{ps}/\rho_0 \), and Equation (22) becomes

\[ s_t = (\alpha - 1/2)\ln\left[1 + b^2\frac{\rho_{ps}}{\rho_0}\right]. \] (24)

In the limit of strong collapse, \( \alpha \) tends to 1.5 (Girichidis et al. 2014) and therefore the \( (\alpha - 1/2) \) term is of order unity. Therefore,

\[ \frac{\rho_f}{\rho_0} \approx \left(1 + b^2\frac{\rho_{ps}}{\rho_0}\right) \] (25)

For \( \alpha = -1.5 \) Equation (25) provides a direct physical interpretation for the PDF transition density in terms of the driving of the turbulence \( b \) and the post-shock density:

\[ s_t(\alpha = 1.5) = \sigma^2. \] (26)

The transition density is proportional to the critical density for collapse based on the post-shock density derived in Krumholz & McKee (2005) and Padoan & Nordlund (2011). We will explore the relationship between the transition density and the critical density for collapse in companion papers: Burkhart & Mocz (2018) and Mocz & Burkhart (2018).

4. The Dynamic SFR in the Gravoturbulent Media

Given the analytic expressions for the normalization constants and transition density point (Equations (19)–(22)), we can now integrate the piecewise density PDF and predict the SFR.

In what follows, we consider the original formulation for the critical density for collapse \( \rho_{crit} \) as defined in Padoan & Nordlund (2011) in Equation (11) with \( \alpha_{vir} = 1 \), a weak magnetic field \( (\beta = 20) \), and ratio of cloud size to turbulence integral scale of \( \theta = 0.35 \). In general, this is similar to the formulation of Krumholz & McKee (2005), \( \rho_{crit,KM}/\rho_0 = (\pi^2/15)\phi^2 \times \alpha_{vir}M^2_f \), where \( \alpha_{vir} = \phi^2 = 1 \). We also consider the multifree-fall piecewise density PDF SFR calculation derived in Federrath & Klessen (2012), again using the critical density of Padoan & Nordlund (2011) in Equation (11). We plan to explore the full parameter space of \( \alpha_{vir}, \beta \), and choice of critical density elsewhere.

The integral to calculate the SFR now splits into an integral over the LN from \( \rho_{crit} \) to \( \rho_t \) and an integral over the PL from \( \rho_t \) to infinity,

\[ SFR_{\text{LN+PL}} = \int_{\rho_{crit}}^{\rho_t} \frac{t_f(\rho)}{t_f(\rho_0)} \rho \, p(s)_{\text{LN}} \, ds + \int_{\rho_t}^{\infty} \frac{t_f(\rho)}{t_f(\rho_0)} \rho \, p(s)_{\text{PL}} \, ds. \] (26)

If we consider the free-fall factor weighting to be a constant \( t_f(\rho)/t_f(\rho_{crit}) \), and pull it out of the integral, then...
Equation (26) evaluates to

\[
\text{SFR}_{\text{ff}}^{\text{LN+PL}} = \exp(\alpha/2)N_{\text{f}} \left[ \frac{1}{2} \text{erf} \left( \frac{\sigma_{s}^{2} - 2s_{\text{crit}}}{\sqrt{8\sigma_{s}^{2}}} \right) \right]
\]

which is limited to \( \alpha > 1.0 \). If the free-fall factor is unity then the factor of \( \exp(\alpha/2) \) is removed.

When the free-fall factor is kept inside the integral, the LN + PL SFR per free-fall time becomes

\[
\text{SFR}_{\text{multif}}^{\text{LN+PL}} = \exp(3\sigma_{s}^{2}/8)N_{\text{f}} \left[ \frac{1}{2} \text{erf} \left( \frac{\sigma_{s}^{2} - s_{\text{crit}}}{\sqrt{2\sigma_{s}^{2}}} \right) \right]
\]

which is limited to \( \alpha > 1.5 \).

The SFR can be calculated according to Equation (8). As shown in Figure 1, when \( \alpha \) is steep, the density transition point is larger and hence the PL portion contributes very little to the SFR_{ff}. The PL tail appears first at high densities and steadily extends to lower densities as time proceeds, which has been shown analytically (Girichidis et al. 2014) and numerically (Collins et al. 2012; Burkhart et al. 2015a, 2017). The SFR_{ff} depends increasingly on the PL portion of the piecewise PDF as \( \alpha \) become smaller.

In Figure 2 we compare the SFR_{ff} versus the sonic Mach number for a pure LN SFR_{ff} (Equation (13)) to the LN + PL SFR_{ff} calculation. The left panel compares the original derivation from Padoan & Nordlund (2011) to Equation (27), while the right panel compares the multifree-fall expression derived in Federrath & Klessen (2012; Equation (17)) to the LN + PL multifree-fall expression in Equation (28).

We chose \( b = 0.4 \) (a mix of solenoidal and compressive forcing) and allow the sonic Mach number (x-axis) and \( \alpha \) to vary. Different colored lines in Figure 2 indicate different values of \( \alpha \). For steep values of the PL tail slope, the SFR_{ff}^{\text{LN+PL}} is very similar to the LN expression (SFR_{ff}^{\text{LN}}). However, as \( \alpha \) becomes increasingly shallow the SFR can grow to more than an order of magnitude above the pure LN calculation for both the multifree-fall expression as well as the expression with the free-fall factor pulled out of the integrals.

Figure 2 also predicts that when a time variable PL is included, the SFR is not constant. This is in contrast to the LN theories reviewed in Section 2 that are constant with the properties of MHD turbulence, which set the width of the LN, e.g., Equation (5).

Figure 3 shows the SFR per free-fall time (SFR_{ff}) versus the density PDF PL slope (\( \alpha \)) for both the LN + PL (Equation (27)) and LN models. In the top panel, we compare the SFR_{ff}^{\text{LN}} expression of Padoan & Nordlund (2011; Equation (13)), with the SFR_{ff}^{\text{LN+PL}} shown in Equation (27). The bottom panel shows the same comparison but using a free-fall factor of unity as prescribed in Krumholz & McKee (2005; Equation (10)). In both panels, the LN SFR calculation only depends on the properties of the turbulence (sonic Mach number and forcing parameter \( b \)) and has no dependency on \( \alpha \). When using the Padoan & Nordlund (2011) free-fall factor (top panel), the SFR_{ff} for both LN and LN + PL models increases with the increasing sonic Mach number (i.e., increasing PDF width).

The primary difference between the top and bottom panels stems from the factor of \( \exp(\alpha/2) \), which is present in the Padoan & Nordlund (2011) and absent in the Krumholz & McKee (2005) model due to the different treatment of the free-fall factor. As noted in Krumholz & McKee (2005), the combination of critical density increasing as the sonic Mach number squared and a free-fall factor of unity produces a SFR_{ff} that is slightly anticorrelated with the sonic Mach number, in contrast to the models of Padoan & Nordlund (2011), Hennebelle & Chabrier (2011), and Federrath & Klessen (2012). The Mach number dependency is also shown in Figure 1 of Federrath & Klessen (2012). This demonstrates that different treatments of the free-fall factor become important in both the LN and LN + PL models and can produce more than an order of magnitude difference in the value of SFR_{ff}.

As \( \alpha \) becomes increasingly shallow, the LN + PL models begin to diverge from the LN models. The width of the LN portion of the PDF sets the value of \( \alpha \) for which the two models diverge. Lower values of \( M_{s} \) show divergence at earlier stages of cloud evolution, i.e., steeper values of \( \alpha \), regardless of the free-fall factor criterion used. The models begin to diverge strongly around \( \alpha = 2 \).

5. Comparison with Observations

In what follows, we compare the SFR, using the LN + PL density PDF, to observations of local GMCs. We also investigate how our model can predict the slope and scatter of the relationship between the mass of molecular gas and the
5.1. Local GMCs and the SFE

A number of observational studies have been conducted to understand the relationship between the dense gas mass fraction, the gas density PDF, and the properties of star formation such as the SFE per free-fall time \( \dot{\epsilon}_{ff} \). We define \( \dot{\epsilon}_{ff} \) as in Krumholz & McKee (2005),

\[
\dot{\epsilon}_{ff} = \text{SFR} \frac{t_{ff}(\rho_0)}{M_{\text{cloud}}}.
\]  

Examples of such studies include Lada et al. (2010), Heiderman et al. (2010), and Kainulainen et al. (2014), who investigated the level of star formation activity and cloud structure in nearby GMCs. Krumholz et al. (2012) summarized the observed data from Lada et al. (2010) and Heiderman et al. (2010) and derived the SFE per free-fall time (see their Table 2). For many of the clouds studied in Lada et al. (2010), Lombardi et al. (2015) also calculated the slope of their column density PDF PL (\( \alpha_{CD} \)) using dust extinction/emission data from Planck, Herschel, and 2MASS. Burkhart et al. (2015a) also noted that there is a correlation in local GMCs with shallower column density PL tails and an increase in the number of young stellar objects (NYSO), which is proportional to the SFR. More recently, Kainulainen et al. (2014) derived 3D density GMC PDFs for 16 clouds using a wavelet-based reconstruction technique. The reported cloud masses, cloud densities, and NYSO can all be used to derive the SFRs and SFEs. Kainulainen et al. (2014) also measured the exponent of the radial density distribution (\( \kappa \)), which is related to the PDF density slope as \( \kappa = 3/\alpha \). We compile values of \( \alpha, \alpha_{CD} \), and the average value of \( \dot{\epsilon}_{ff} \) from the above mentioned studies and present them in Table 1.

We plot \( \dot{\epsilon}_{ff} \) versus PL slope in Figure 4. Error bars on the observations are computed from the standard deviation of the different reported values of \( \dot{\epsilon}_{ff} \) in the literature. The left panel shows \( \dot{\epsilon}_{ff} \) versus PL slope for all GMCs. The right panel presents \( \dot{\epsilon}_{ff} \) versus the column density PDF slopes published in Lombardi et al. (2015). For both density and column density PDF PL slopes we

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Figure 3. Analytic calculations for the SFR per free-fall time (SFR\( _{ff} \)) vs. the density PDF PL slope (\( \alpha \)). The top panel is using the free-fall factor of Padoan & Nordlund (2011). Solid lines are for the SFR calculated using the LN + PL model (Equation (27)) and the straight dashed–dotted lines are for the pure LN models (Equation (13)). The bottom panel shows the same but uses the free-fall factor of Krumholz & McKee (2005), e.g., the factor of exp(\( \kappa \)crit/2) is removed. For both the top and bottom panel, the critical density used is the same (Equation (11)). Different colored lines indicate changing of the sonic Mach number, with black representing the smallest value considered (\( M_s = 4 \)) and blue representing the largest value considered (\( M_s = 22 \)). The LN-only SFR calculation only depends on the properties of the turbulence (sonic Mach number and forcing parameter \( b \)) and hence has no dependency on \( \alpha \). Regardless of the free-fall factor used, as \( \alpha \) becomes increasingly shallow, the PL + LN model presented here begins to diverge from the LN model, with differences in the sonic Mach number becoming less significant.
note a clear trend of rising $\epsilon_{\text{ff}}$ with shallower PL tail slope in the observations. This trend is within the expectations of the LN + PL model (black lines) range for sonic Mach numbers of 5–25 and $b$ values of 1/3–0.7, although we emphasize that the amplitude depends primarily on the feedback parameter ($\epsilon_0$) and which version of the free-fall factor is employed. Regardless of these, the rise with shallower $\alpha$ is expected from the model and observed in the local GMCs.

We confirm that there are LOS effects, which may produce a difference between the actual density PDF $\alpha$ and the measured column density PDF $\alpha_{\text{CD}}$. These LOS effects may also apply to the reconstructed density PL slope measured in Kainulainen et al. (2014). Burkhart et al. (2015a, 2017) studied the column density PDF properties and compared them to the density PDF properties from Collins et al. (2012). They found that for the same time step and plasma $\beta$ (mean magnetic field strength), the PL slopes of column density PDF can change slightly depending on the LOS orientation. Girichidis et al. (2014) and Guszejnov et al. (2017) performed analytic calculations for the evolution of the PL PDF slopes for both density and column density of all of the materials that undergo complete fragmentation (e.g., go on to form stars). In particular, the results of Guszejnov et al. (2017) suggest that the density and column density PL slopes should converge to values of $-1$. However, they argue that the column density PDF is affected by LOS effects which steepens the slope, most likely producing final slopes roughly between $-1$ and $-2$. Indeed for all the clouds examined here $\alpha_{\text{CD}} > \alpha$. Additional studies comparing the analytic results of Girichidis et al. (2014) and Guszejnov et al. (2017) to numerical simulations will be useful to determine the correspondence between the column density PDF $\alpha_{\text{CD}}$ and the density PDF $\alpha$.

Despite the observational complications of measuring $\epsilon_{\text{ff}}$ and the PDF properties (Alves et al. 2017), a very clear trend of increasing $\epsilon_{\text{ff}}$ with shallower $\alpha$ is seen for the local GMCs. An important distinction between the analytic SFR model derived in this work and the works of Krumholz & McKee (2005), Padoan & Nordlund (2011), and Hennebelle & Chabrier (2011) is that they provide a constant SFR$_{\text{ff}}$ and/or $\epsilon_{\text{ff}}$ for given turbulence parameters, such as the sonic Mach number and the forcing, which set the LN width. However, it is not likely that differences in $\epsilon_{\text{ff}}$ observed in the data are due the properties of turbulence, as most of these clouds have similar sonic Mach numbers and similar line width size relations (Kainulainen & Federrath 2017). The LN + PL model SFR$_{\text{ff}}$ calculation presented here is inherently time varying as the slope of the PL varies with cloud evolution due to gravitational collapse and feedback. The level of turbulence is secondary to the SFR$_{\text{ff}}$ when the PL is taken into account (see Figure 3).

### 5.2. From Galactic to Extragalactic: Dense Gas Mass and the SFR

A number of insightful observational surveys have shed light on the relationship between ISM properties and extragalactic SFR scaling laws. Gao & Solomon (2004) found a linear relation between the SFR traced by total infrared luminosity and the luminosity in a dense gas tracer (HCN) averaged over entire galaxies (see also Wu et al. 2005). Similar relations between dense gas mass or gas surface density and SFRs have been observed in a number of recent extragalactic surveys (Bigiel et al. 2008, Hughes et al. 2013, Pety et al. 2013, Schinnerer et al. 2013, Colombo et al. 2014, Faesi et al. 2014, Usero et al. 2015, Bigiel et al. 2016, Federrath et al. 2017, Leroy et al. 2017), as well as in local well-resolved Milky Way GMCs (Heiderman et al. 2010, Lada et al. 2010).

The relationship between molecular gas mass and the SFR was further extended in Lada et al. (2012) and Faesi et al. (2014), who showed that the same linear correlation smoothly extends across more than five orders of magnitude when connected to local galaxy dense gas masses and SFRs and is consistent with the Gao & Solomon (2004) and Wu et al. (2005) results to within a factor of three. One interpretation of this linear slope is an approximately constant depletion time $t_{\text{depl}}$ (yr) $= M_{\text{mol}}(M_*) / \text{SFR}(M_\odot \text{ yr}^{-1})$, i.e., the timescale for the molecular gas to be converted into stars. Extragalactic studies consistently derive average depletion times of about 2 Gyr (Bigiel et al. 2008, Leroy et al. 2013a, 2013b), however deviations have been noted (Leroy et al. 2011; Saintonge et al. 2011a, 2011b, Burkert & Hartmann 2013, Sandstrom et al. 2013). These deviations could be due to systematics and/or or true environmental differences in galaxies. Furthermore, local MW GMCs and resolved GMCs in nearby galaxies have a median $t_{\text{depl}}$ of 180–230 Myr, an order of magnitude shorter than the typical values found in unresolved extragalactic studies (Lada et al. 2010, 2012, Faesi et al. 2014). Dense gas tracers have even shorter depletion times of around 20 Myr (Lada et al. 2012).

We explore these trends using the predictions of the LN + PL SFR model and compare with the LN-only model. Figure 5 shows the SFR versus the cloud mass for local GMCs and galaxies, with the solid black line showing the fit from Lada et al. (2012) and data from Gao & Solomon (2004). Lada et al. (2010, 2012) published values of local GMCs (diamonds), spiral galaxies (upward triangles), luminous infrared galaxies (circles), and ultra-luminous infrared galaxies (squares). GMCs from NGC 300 were published in Faesi et al. (2014; downward triangles). Opened symbols correspond to diffuse gas tracers (either dust extinction at 0.1 mag or CO), while closed symbols indicate dense gas tracers (either dust extinction at 0.8 mag or HCN). The rainbow colored solid lines are different LN + PL model predictions for the SFR with $M_\odot = 5$, $b = 1/3$ and different values of $\alpha$, which can greatly change the amplitude of the SFR and hence contribute to the scatter in the SFR.
despite the constant turbulence properties. For each of the LN + PL models we overplot the corresponding depletion times. Two LN-only models are shown in brown dotted ($M_* = 75$, $b = 1/3$) and dashed lines ($M_* = 5$, $b = 1/3$).

The LN and steep LN + PL models (red, orange, and yellow lines) show agreement with observations of diffuse molecular gas tracers (open symbols). These tracers are more likely to lie within the LN portion of the gas PDF since they are diffuse. However the high-density gas tracers (e.g., HCN, filled symbols) show better correspondence with shallower PL slope values of the SFR (green, blue, and purple lines). The LN SFR models could explain these points as well but only for extreme sonic Mach numbers in the range of $M_* = 75-100$ (Federrath 2013b; Usero et al. 2015; Federrath et al. 2017) or invoking large variations in the SFE. Typical sonic Mach numbers measured in local star-forming Milky Way clouds lie in the range of $M_{\text{M.K.W.}} = 4 - 20$ (Burkhart et al. 2010; Kainulainen & Tan 2013; Kainulainen & Federrath 2017).

As seen in the previous section in Figure 2, the LN + PL model SFR can change by more than an order of magnitude for different values of $\alpha$ and hence can explain the scatter in the SFR with the average observed Mach numbers in the ISM. A shallower value of $\alpha$ corresponds to increasing the fraction of dense gas ($f_{\text{dense}}$), which is representative of star formation law discussed in Lada et al. (2012): $\text{SFR} \propto f_{\text{dense}} M_{\text{cloud}}$.

For each of the three LN + PL models we overplot the corresponding depletion times, $t_{\text{depl}} = 7$, 20, 60, 400 Myr for $\alpha = 1.5$, 2.0, 2.5, 3.7, respectively. Clouds with shallow PL tails have the shortest depletion times, the most dense self-gravitating gas in the PL tail, and rapid star formation, while clouds with steep PL tails or LN distributions with moderate sonic Mach numbers have longer depletion times.

The SFR and depletion time are primarily controlled by the local dense gas fraction which, in the context of the density PDF SFR model presented here, are characterized by the amount of material in the PL portion of the PDF. Once the critical density is reached, the value of $\alpha$ rapidly shallows and the LN portion of the SFR calculation becomes insignificant. Therefore the value of $f_{\text{dense}}$, and hence the overall SFR, is determined by the competition of gravity versus feedback and secondarily on magnetic fields and large scale turbulence.

6. Discussion

The LN + PL model discussed here can be retermed as a graviturbulent star formation model since the development of the LN + PL density distribution is consistent with the picture of a turbulent molecular cloud undergoing gravitational collapse. Both numerical experiments and observations point to a picture of a dynamic SFR, which increases in time as clouds become gravitationally unstable (Hartmann et al. 2001; Vázquez-Semadeni et al. 2007; Heitsch & Hartmann 2008; Heitsch et al. 2008; Hennebelle & Teyssier 2008; Banerjee et al. 2009; Zamora-Avilés et al. 2012; Burkert & Hartmann 2013; Girichidis et al. 2014; Zamora-Avilés & Vázquez-Semadeni 2014) and later halts as feedback/supernova destroy the star-forming region. The model presented here, which includes both the LN and PL components, is inherently time varying as the PL slope is expected to change as the cloud contracts and later is destroyed by feedback. Previous studies have investigated the SFR or IMF from a PL-only distribution with single values of $\alpha$ (Collins et al. 2012; Girichidis et al. 2014; Lee & Hennebelle 2017). However, GMCs have different observed values of the PL slope and $\alpha$ is expected to become shallow as the cloud evolves from rapidly collapsing filaments (Collins et al. 2012; Federrath & Klessen 2012; Federrath 2013b; Burkhart et al. 2015b; Lee et al. 2015; Schneider et al. 2015b; Lee & Hennebelle 2017; Mocz et al. 2017; Caldwell & Chang 2018) and the SFR increases. $\alpha$ may steepen again as feedback from O and B stars destroys...
the GMC as may be indicated in GMCs that show multiple PL tail slopes (Schneider et al. 2015a).

As mentioned above, an important distinction between the analytic model presented here and the SFR models of Krumholz & McKee (2005), Padoan & Nordlund (2011), and Hennebelle & Chabrier (2011) is that they provide a constant SFR$_{ff}$ for a given sonic Mach number (i.e., LN width), $\beta$, $\alpha_{\text{vir}}$, and $\epsilon_0$. The model presented here provides the same value of the SFR when the PL tail is very steep and the PDF is dominated by the LN. However for clouds with prominent high-density PL PDFs the SFR is observed to increase and our model accounts for this effect without the need to invoke stronger turbulence or alternative environmental parameters.

The model presented here suggests that SFRs and efficiencies are very sensitive to the increasing influence of the gas self-gravity and feedback. The key observable is the manifestation of the PL in the density PDF. In that sense we can explain why observed SFRs do not show sensitivity or strong correlation with local measurements of velocity dispersion or second moment maps (Leroy et al. 2017) and why star formation correlates strongly with the dense gas fraction. We will explore this point further in a follow-up paper (Burkhart & Mocz 2018, submitted).

We emphasize that in the analytic formulation for the SFR and SFE outlined in this work there is no need to invoke the ionization state of the cloud or complex plasma processes such as the Hall effect, or ambipolar diffusion. Turbulence, turbulent magnetic fields, and gravity set the basic initial conditions for the SFRs and efficiencies. Turbulence sets the initial density distribution (i.e., with a LN PDF) and can also control the fluctuation, amplification, and diffusion of the magnetic field. A process known as reconnection diffusion (Lazarian & Vishniac 1999; Lazarian et al. 2012) is mediated by the properties of turbulence. Numerical simulations have shown that magnetic reconnection diffusion can be an important process for mediating collapse (Lazarian et al. 2012; Santos-Lima et al. 2012; Mocz et al. 2017), especially in clouds with sub-Alfvénic and magnetically subcritical initial conditions (Crutcher et al. 2009, 2010; Crutcher 2012). In this paper we only explored a weakly magnetized regime of parameter space with our choice of $\rho_{\text{crit}}$ but plan to explore the effects of strong magnetization, where reconnection might play a more important role, in a future work.

The gravoturbulent star formation model has implications for both the Kennicutt–Schmidt relationship as well as the NYSOs in local GMCs. The analytic model outlined here suggests that star formation scaling laws from galaxy scales to cloud scales are driven primarily by variation in the amount of dense gas (set by the PL slope $\alpha$) and secondarily on the properties of turbulence in the clouds. This is in agreement with recent observational work by Lada et al. (2017) on the California GMC as well as work by Elmegreen (2018) on explaining scatter in the Kennicutt–Schmidt relation. The gravoturbulent model can explain the scatter in the SFR versus dense gas mass
observations (Heiderman et al. 2010; Lada et al. 2010, 2012; Faesi et al. 2014; Leroy et al. 2017), without the need to invoke extremely large turbulent energies (e.g., large $M_\text{f}$). The gravoturbulent model SFR changes by more than an order of magnitude for different values of $\alpha$ and hence could be an explanation for the scatter in the SFR in both local GMCs as well as in extragalactic quiescent versus star-bursting systems. Clouds with shallow PL tails have SFRs dominated by self-gravity and we predict they also have higher dense gas fractions, shorter depletion times, and rapid star formation. This is in agreement with the observational studies of Usero et al. (2015) and Leroy et al. (2017), who found that gas with stronger self-gravity forms stars at a higher rate (lower depletion time). Moreover, these studies found that the SFE is anticorrelated with the velocity dispersion (Mach number), which is in tension with some LN turbulence theories of star formation.

The star formation theory derived in this paper is suitable for application to galaxy/cosmological simulation subgrid models (Hopkins et al. 2013). Our model predicts that star formation depends primarily on the amount of dense self-gravitating gas and secondarily on the properties of ISM turbulence, which set the initial density fluctuations. Gravitational collapse primarily controls star formation until it is shut off by mass loss and/or feedback processes. Galaxy formation simulations that seek to include ISM physics should adopt a SFR law that has a primary dependency on a dense gas probability and secondarily on the conditions of the ISM including, $\alpha$, $M_\text{f}$, $\alpha_{\text{crit}}$, and $\beta$.

4. Clouds with shallow PL tails have a higher dense gas fraction and therefore short depletion times and rapid star formation.

5. Our model can explain both star-bursting and quiescent star-forming systems without the need to invoke extreme variations in the local interstellar environment or large sonic Mach numbers.

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