3D Nasal Magnetic Resonance Image Registration by Maximization of Mutual Information

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Abstract. The magnetic resonance image (MRI) is widely used in medical treatment, which is harmless for human body. However, more noise than the CT image sometimes can be a problem when doctor focus on details of some narrow structure of human body such as nasal. It is necessary to process the MRI like image fusion before using it. Subjects have to move head during a few MR scans. Image registration can help all corresponding points find their own correct position, so that it can be processed and give us more information from not only one time scan. Mutual information is a good mathematics tool in statistics and information theory. It can describe the relationship between input signal and output signal. In this paper, they are reference image data set and float image data set, and the process between them are some little bit head move and some other influence during scanning. The rigid body transformation is used to reduce the impact of head move because the nasal consisting of bones almost don’t changes shape during scan. Registered images from another scan and reference image can get together to be used in image fusion and reconstruction.

1. Introduction
Medical image shows the internal structure of human’s body by different media such as ultrasound, X-ray and magnetic field. Researchers and doctors always try to find more information through medical image, no matter for research and medical treatment. For medical image better using, registration is a difficult and important point.

Besl provided a method for registration of 3D shape on model shape level based on the iterative closest point algorithm [1]. Huttenlocher used the Hausdorff distance but it is found that the curve of distance is not smooth [2]. It always makes trouble so that finding the best parameter is difficult. Whether the answer of the algorithm based on feature points is accurate and robust depends too much on parameter choice.

The method on the pixel level can be suitable for most cases. Woods used mutual information to rule the pair of images [3]. The mutual information of corresponding images should be maximal if they are geometrically aligned. Hill provided a method for calculating mutual information through the 2D joint histogram [4]. Maes firstly provided a method system including interpolation and quick search algorithm [5]. The method works on pixel level is most flexible and unconditional according to the information theory. Vajda’s work associated image registration with mutual information [6].

Previous work paid more attention on X-ray and CT image. It is probably because the great effect of these images after registration could be shown. The study on MRI may be more valuable with the growth
of technique on magnetic resonance imaging. Piuze developed the MRI registration through maximization of mutual information [7].

The purpose of this paper is to find out a universal method for MRI registration for following processing step. Previous work is introduced in section I. Section II presents some related theory and algorithm. Section III and section IV show process and result of experiments and discuss implications.

2. RELATED THEORY

2.1. Transformation

In theory, the result of some scans on the same body part should be same. But as all know, it is impossible for subjects to keep absolutely same in every MR scan if more than one scan is needed. Body move can be mathematically modelled by geometric transformation, such as rigid body transformation, affine transformation, projective transformation and nonlinear transformation.

Rigid body transformation is the most uncomplicated one. The distance between every two points in the body don’t change. That means internal structure of the body keep static. The data in this paper is from three consecutive scans on nasal and paranasal sinuses that is made up of bone and cartilage, which is suitable for rigid body transform. In a way, nasal is a good place to evaluate the efficiency of the method of registration.

Rigid body transformation is the combination of rotation and translation. Every situation in 3D space could be described as rotation and translation around X axis, Y axis and Z axis. So there are six parameters including three rotation angles ($\alpha_x$, $\alpha_y$, $\alpha_z$) and three translation distance ($t_x$, $t_y$, $t_z$).

For every point $R_n$, the relationship between $R_n$ and base point $R_0$ ($X_0$, $Y_0$, $Z_0$) could be described as a vector ($X$, $Y$, $Z$), which $R_0$ can be the middle point in the 3D space.

$$X = X_n - X_0$$

$$Y = Y_n - Y_0$$

$$Z = Z_n - Z_0$$

(1)

3D vector after $\alpha$ degree rotation around z axis only changes X and Y. The situation around x axis and y axis can be omitted.

$$X' = X \cdot \cos \alpha - Y \cdot \sin \alpha$$

$$Y' = X \cdot \sin \alpha + Y \cdot \cos \alpha$$

(2)

3D vector after ($X_t$, $Y_t$, $Z_t$) translation can be easily calculated. The $F_0$ ($X_0$, $Y_0$, $Z_0$) is a base point in floating image. The final coordinate of floating image is

$$X_F = X' + X_t + X_0$$

$$Y_F = Y' + Y_t + Y_0$$

$$Z_F = Z' + Z_t + Z_0$$

(3)

The new coordinate $F_n$ ($X_F$, $Y_F$, $Z_F$) in floating image is mapped by a certain rigid body transformation on $R_n$. As shown in fig. 1, X-X’ mapping the relationship of rigid body transformation on reference image and float image.

Fig.1 is just a 2D figure, and it is easy to associate it with 3D space. The E point in float image can’t map the E point in reference image, but $E'$ in float image can. Calculating the gray value of $E'$ in float
image could map the E point in reference. Image registration is to find the best E’ points and the best float image to map the reference image.

2.2. Interpolation

Fn (X_f, Y_f, Z_f) will not be integer in most cases. It is necessary to reach the gray value for Fn, which coordinate is made up of float number.

![Fig. 1](image) a 2D figure of rigid body transformation

There isn’t an existing point E’ and the directly available gray value in float image. The only way is to calculate the gray value by some points around E’.

Common method includes nearest neighbor interpolation and weighted mean interpolation like trilinear interpolation, TRI and partial volume distribution, PV. The former indicate that the gray value of nearest neighbor of four in 2D space takes place of the gray value of E’. The latter is more complicated. The weight depends on the distance between E’ and its four neighbors. Using weighed mean of gray value of all neighbors as the gray value of E’.

It must be noted that some of the points have to be out of the past scale in float image such as A’. So it is necessary to expand scale of float image set to make sure that A’ can be found.

2.3. Mutual Information

Some statistics describe the dependence and independence between two random variables A and B. As all know that p_{AB}(a,b) = p_A(a) * p_B(b) if A and B are statistically independent. The mutual information as a statistic, describes how similar they are by measuring the distance between joint distribution p_{AB}(a,b) and the distribution when completely independent p_A(a) * p_B(b). The formula [6] of calculating MI is

$$I(A, B) = \sum p(a, b) \log \frac{p(a,b)}{p(a)p(b)}$$

The float image set after rigid body transform and reference image set are random variables A and B. Mutual information measures the information redundancy through the image intensity histogram of two image sets and the 2D histogram on pairs of corresponding voxels’ gray value, which is from 0 to about 1000. The gray value is out of 256 because they are 16-bit images.
2.4. Search Algorithm

Image registration is a complex problem. There are many possible situations depended on six parameters in rigid body transformation. Searching for the best situation becomes a simpler question as optimizing the function of the maximization of mutual information.

There are global searching algorithm and local searching algorithm. Powell algorithm belong to the latter. It can search for the answer quickly because it not only searches for better value but also updates the direction vector immediately. Powell algorithm can be thought as climbing. Go along the direction to the highest position in the view at every moment. However, it sometimes just find the local best value just like climbing a mountain which is not the highest.

3. Experiments

3.1. Data Collection and Introduction

The image is from a 28-year-old male subject, lying on his back in MR machine. All three times scan are T1 scans, which has all three types of image sets, coronal sagittal and transverse. Data introduction is shown in table I.

| Process | Set | Role  | Type    | Size in voxel |
|---------|-----|-------|---------|---------------|
| original | (a) | Reference | Coronal | 784*960*224   |
|         | (b) | Float   | Transverse | 512*512*104 |
|         | (c) | Float   | Sagittal  | 512*512*128  |
| handled | (a) | Reference | Coronal | 160*160*200  |
|         | (b) | Float   | Coronal   | 200*200*240  |
|         | (c) | Float   | Coronal   | 200*200*240  |

The reference image set are 784*960*224 voxels and each voxel size is 0.25mm*0.25mm*0.5mm. Set one has 128 images and 512*512*128 voxels. Set two has 512*512*104 voxels. Both set one and set two contain voxels of 0.5mm*0.5mm*1mm.

3.2. Data Preprocessing

It is necessary to preprocess the data for registration and other operation. Voxel size unitization could help a lot. Make all voxels to be the 0.5mm*0.5mm*0.5mm through inter-slice interpolation and voxel merging interpolation. Using some simple transformation make three different views to be the coronal view. Cut these image set to make sure they are easy to register. It is important that the scale of flat image set should be wider than reference image set. This is why set (c) and set (c) have 200*200*240 voxels rather than 160*160*200 voxels.

It is obvious that there are huge difference between other image sets. Some difference of set (a) and set (b) are shown below. The image (a) is the first slice of set (a) and the image (b) is the first slice of set (b).

![Fig. 2](image_url) the first slices of reference image set (a) and float image set (b)
3.3. Image Registration
Using the Powell algorithm through the six-dimensional space to search for the best transformation parameter. There are six corresponding parameters consisting of three rotation angle $a$, $b$, $c$, and three translation distance $x$, $y$, $z$. The parameter $a$, $b$, $c$ are measured in degree and $x$, $y$, $z$ are measured in voxel, which is 0.5mm$^3$.

4. Results and discussion
After the experiments, maximization of mutual information will be the best situation for registration.

4.1. Result
Register set (b) referenced to set (a). The maximization of mutual information is 2.0424 while the mutual information is 0.5207 when all six transformation parameters are zero. The effect of registration is obvious.

![Fig. 3 first slices in reference image set (a) and registered image set (b)](image)

As shown in fig.3, the image (b) is the first slice of registered set (b). The landmark below nasal disappear totally. It can prove the effect of the registration visually.

All six parameters are shown in TABLE II. The x, y, z are measured in voxels, which is 0.5mm$^3$.

|    | $a$  | $b$  | $c$  | $x$  | $y$  | $z$  | MI referenced to (a) |
|----|------|------|------|------|------|------|----------------------|
|    | 0    | 0    | 0    | 0    | 0    | 0    | 0.5207               |
|    | 7.2° | -0.6°| 4.7° | 4.6  | 1.1  | -1.6 | 2.0424               |

The MI changes monotonically with one parameter changing from the top point. The trend between MI and $\alpha_x$ is shown in Fig.4.

![Fig. 4 trend between the MI and $\alpha_x$](image)
4.2. Evaluation
Mathematics tool is a necessary method for evaluation. The second mathematics tool is supposed after the MI. The distribution and standard deviation of difference value of corresponding points could make it through some experience and observation. Difference value of gray value of corresponding points between reference image and float image can evaluate the initial situation. So as the difference value between reference image and registered image. It evaluates the best situation. Fig.5 shows corresponding slices of three images. The registered image (c) is transformed from the float image (b), but changes a lot referenced to the reference image (a).

![Fig. 5](image)

Fig. 5 the 68th slice of reference image and float image & registered image

![Fig. 6](image)

Fig. 6 distribution of gray value of | reference – float | and | reference – float |

Fig.6 shows the distribution of difference value. The standard deviation of | (a) – (b) | is 42.8198. The standard deviation of | (a) – (c) | is 11.1844 and its distribution of difference value is more concentrated.
4.3. Discussion
There are still some problems to be solved. Using difference value need to be proved theoretically, not just caused by experience and observation. Some global searching algorithm should be used to search the scale of the best situation before the Powell algorithm for shorter time and better effect. Finally, more MRI data on nasal should be used and tested.

In short, MI is a good mathematics tool to judge the effect of 3D nasal MRI registration. It is an accurate and robust voxel-level medical image registration method. The distribution of difference value is supposed to evaluate the registration and it still need to be proved theoretically. The effect of 3D nasal MRI has been improved. Further research needs to use wiser search algorithm to avoid local best value. More nasal MRI of different resolution and different scan type could push the research of the nasal tract.

5. Acknowledgments
This work is supported by the National Nature Science Foundation of China (No.61175016, No.61876131).

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