Critical Stresses in Materials with Cracks

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The fracture mechanics concepts, as well as the concepts introduced on the basis of principle of critical energy, correlated with strength of materials with cracks is analysed. The equivalent stress method of fracture mechanics, the condition that the loading of a mechanical structure with cracks should be admissible is expressed in one of the following relationships:

\[ K_i \leq K_{ic}(t) \]

where \( K_i \) is the stress intensity factor; \( K_{ic}(t) \) is the critical value of \( K_i \).

In fracture mechanics, the condition that the loading of a mechanical structure with cracks should be admissible is expressed in one of the following relationships:

\[ J_i \leq J_{ic}(t) \]

where \( J_i \) is the integral and \( J_{ic}(t) \) is the critical value of \( J_i \).

Keywords: crack, critical stress, deterioration, principle of critical energy, equivalent stress

From the very manufacturing stage, mechanical structures often feature micro-cracks or small cracks whose dimension may become critical when the structure is under operating conditions.

This raises the problem of determining whether a crack at any given time is dangerous to the structure.

a. In the literature, one can find expressions for stress at the tip of the crack and for stress at a certain distance from the crack tip [1; 2]. On this basis, new concepts, correlated with the crack dimensions, which provide the ground for the chapter of fracture mechanics, have been defined, namely [3; 4]:

- **stress intensity factor**, for materials with linear-elastic behavior at the crack tip

\[ K_i = \sigma \cdot F \cdot \sqrt{\pi a} \cdot f_i(r; \theta); \]
\[ K_{ic} = \tau_{ic} \cdot F \cdot \sqrt{\pi a} \cdot f_{ic}(r; \theta); \]

b. V.V. Jinescu [5] proposed a new philosophy for the calculation of mechanical structures with cracks, different from the one in fracture mechanics.

- In the case of static loading, based on the principle of critical energy, one calculates the total participation of the specific energy \( P_{i}(t) \) determined by the local load, which is compared to the critical participation \( P_{ic}(t) \); the latter includes the influence of the crack [5-8]. Both variables are dimensionless and generally time-dependent (t).

The total participation of specific energies at a given moment t has the expression [5],

\[ P_{i}(t) = \sum \left( \frac{\delta_{ij}}{\delta_{i,j}} \right)^{\ast i} \cdot \delta_{ij} \]

where \( P_{i}(t) \) is the applied stress; \( \sigma_{ij} \) - the critical value of \( \sigma_{ij} \); \( \delta_{i,j} \) acts in the direction of the deformation process; \( \delta_{i,0} = 0 \) if \( \sigma_{ij} \) has no influence and \( \delta_{i,0} = -1 \) if \( \sigma_{ij} \) opposes the deformation process.

The exponent depends on the behavior of the material expressed by the power law,

\[ \sigma = \sigma_{0} \cdot e^{k \cdot \varepsilon} \]

where \( \sigma_{0} \) is the applied stress; \( \varepsilon \) - strain; \( M_{o}, k \) - constants.

The critical participation at time t when critical stresses are deterministic variables, has the expression,

\[ P_{i}(t) = 1 - D(t) \]
where $D(t)$ is the deterioration at time $t$.

If the crack increases over time, deterioration will also increase. Critical participation depends on the value of the deterioration which may be calculated with the existing relationships in the literature [9-11].

In work [25], the deterioration was correlated with the dimensions of the crack by applying fracture mechanics concepts, namely,

$$D(a,c)=\left(\frac{K_c}{K_c}\right)^2$$

or,

$$D(a,c)=\left(\frac{\sigma_{cr}}{\sigma_{cr}}\right)^{\frac{1}{k}}.$$

where exponent $k$ is derived from the law of behavior (8).

The influence of the crack is included in the strength calculation based on the principle of critical energy, without the need for the experimental determination of new material characteristics, as is the case with fracture mechanics.

This method of calculating the strength of cracked mechanical structures has been applied to many cases of mechanical stresses [5-8, 11-18]. The superposition of loads of the same nature or of a different nature proved to be possible only on the basis of the relations obtained through the use of the principle of critical energy [19-30].

C. In a second step, one moved from the calculation based on the participation of specific energy induced by the loads, to the classical calculation based on the concept of equivalent stress, $\sigma_{eq}$. In this case the influence of the crack was introduced into the expression of critical stress, $\sigma_{D}$ which depends on deterioration $D$.\[V.V.] Ionescu and V-I. Nicolof [13] proposed the following relationship for the local critical stress of the structure with cracks,

$$\sigma_{cr}(D)=\sigma_{cr}[1-D(a,c)]^{\frac{1}{k}}, \quad (10)$$

where $D(a,c)$ is the deterioration caused by a crack with depth $a$ and length $2c$, and $\sigma_{cr}$ is the critical stress of the material without crack.

Instead of the concepts of fracture mechanics, one uses the concept of local critical stress of the mechanical structure with cracks. With this concept it has become possible to use strength theories based on the equivalent stress concept. The condition for a cracked structure to undergo admissible loads is - structurally - identical with the condition for the material without crack.

In order to use relationship (11) for checking a cracked mechanical structure, it is necessary to know the interdependence between $\sigma_{cr}(D)$ and the deterioration $D$ produced by the crack.

In the papers [31-33], relationships have been proposed for yield loading in tubular cylindrical specimens with cracks, which may be written as in equation, obtained in [13],

$$Y_{y}(D)=Y_{y}[1-D(a,c)]^{\frac{1}{k}}, \quad (13)$$

where $Y(D)$ is the limit load of the cracked tubular specimen; $Y_{y}$ is the limit load of the crackless tubular specimen and $\alpha_{cr}=0$.

Local critical stresses and loads in mechanical structures with cracks

a. From previous research, one has found that the local critical stress or load decreases with an increase in the crack depth, $a$, length $2c$ or angular opening $2\theta$. For example, with hollow tubular specimens with semielliptical crack at the inner surface (fig 1, a, b) under axial force $F$, processing [14] the data from [34] the dependence of $\sigma_{cr}(a,\theta)$ or $\sigma_{cr}$ were obtained as a function of $a/s$ for three values of reported angle $\theta/\pi$ (fig. 2). The behavior of the pipe material has been considered as ideally-plastic, i.e. the maximum stress $\sigma_{cr}$ is the yield stress.

Higher $a/s$ and $\theta/\pi$, lower the critical stress $\sigma_{cr}(a,\theta)$.

b. The influence of the crack width, which has not been considered in research carried so far. We shall further present the influence of the crack on the fracture strength of some OL 304 (5NiCr180) steel specimens, used in the construction of equipment in the chemical, pharmaceutical, petroleum, nuclear, food industries.

Rectangular section specimens (fig. 3) have been created with cracks of $2c=5$ or 10 mm, unpenetrated ($a=1mm$) or penetrated $a=2 mm$, perpendicular to the direction of loading ($\beta=90^\circ$) or inclined ($\beta=45^\circ$).

Cracks with width $e$ were obtained by electro-erosion, which allowed for cracks with clean edges and crack depth control. The crack widths were $e=0.3; 0.4; 0.6; 0.8$ and 0.9.

The experiments were performed on a universal test machine A900 (TC100), LGB Testing Equipment, connected to a computer equipped with the LBG Easy Test software. Unpenetrated specimens of OL 304 were used to determine the behavior of the tested steel.
Based on the graphical representation of the interdependence between natural stress $\sigma_n$ and natural-strain, $\varepsilon_n$, the exponent $k$ from relation (8) was calculated. Figure 4 shows the dependence between natural concepts $\sigma_n(\varepsilon_n)$ and engineering concepts $\sigma(\varepsilon)$.

![Graph image](image)

**Fig. 4.** Dependencies $\sigma_n=\varepsilon_n$ and $\sigma-\varepsilon$ for an OL304 steel specimen without cracks under traction load

In this case, the result was exponent $k=0.205$.

For an OL304 specimen with unpenetrated crack, $(a=1\text{mm})$, featuring $2c=5\text{mm}$, and $\beta=90^\circ$ and $e=0.8\text{mm}$, under traction load, dependencies $\sigma_n-\varepsilon_n$ and $\sigma-\varepsilon$ have been shown in figure 5.

It has been found that the ultimate stress of the cracked specimen is much smaller than the one in the uncracked specimen.

By increasing the crack width, $e$, the ultimate stress decreases.

![Graph image](image)

**Fig. 5.** Dependencies $\sigma_n-\varepsilon_n$ and $\sigma-\varepsilon$ for OL 304 specimens with unpenetrated crack $(a=1\text{mm})$ featuring: a-$2c=5\text{mm}; e=0.8\text{mm}$ and $\beta=90^\circ$; b-$2c=10\text{mm}; e=0.6\text{mm}$ and $\beta=45^\circ$.

The ultimate stress $\sigma_{u,n}$ and $\sigma_{u}$, respectively, of the specimen with unpenetrated crack diminishes with the enlargement of crack width $e$ (fig. 6), both when $2c=5\text{mm}$ and $\beta=90^\circ$ as well as in the case when $2c=10\text{mm}$ and $\beta=45^\circ$.

![Graph image](image)

**Fig. 6.** The variation of ultimate stress $(\sigma_{u,n}$ and $\sigma_{u})$ with the width of the crack $e$, in the case $2c=5\text{mm}$ and $\beta=90^\circ$ (a), and if $2c=10\text{mm}$ and $\beta=45^\circ$ (b).

In the case of OL 304 specimens with the penetrating crack $(a=2\text{mm})$ under traction load there have been obtained the dependencies required for traction, the dependencies represented in figure 7 for cracks with $\beta=90^\circ$ and $2c=5\text{mm}, e=0.6\text{mm}$ (fig.7, a) and $2c=10\text{mm}, e=0.3\text{mm}$ (fig.7, b). It is noted that with an increasing crack length the fracture strength decreases.

The dependence of the fracture strength on the crack width is shown in figure 8, which also demonstrates the influence of the crack length. For $2c=5\text{mm}$ the values of the fracture strength (fig.8, a) are higher than when $2c=10\text{mm}$ (fig.8, b).

Since the crack width influences the fracture strength, in general, the expression of the critical stress (here the ultimate stress) as well as the critical load will be written as follows,

$$\sigma_{u}(a,c,e,\beta) = \sigma_{u} - \left[1-D(a,c,e,\beta)\right]^{1/2}$$

$$Y_{u}(a,c,e,\beta) = Y_{u} - \left[1-D(a,c,e,\beta)\right]^{1/2}$$

(14)
where one has emphasized the findings from the experiments performed and presented in the foregoing figures, that the critical parameters depend both on the configuration of the crack \((a, c, \varepsilon)\) and on its inclination.

**Conclusions**

The paper elaborates on the approach to the problem of material fracture by resorting to the concepts based on the principle of critical energy, namely:

- comparison of the total participation of the specific energies corresponding to the loadings, to the critical participation, dependent on crack induced deterioration;
- comparison of the equivalent stresses corresponding to the stresses loading the structure, to the local critical stress dependent on the crack configuration and inclination.

The experiments performed on OL 304 specimens have shown that the evaluation of the material fracture may be made more easily by resorting to the proposed concepts than with those currently used in fracture mechanics.

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