Gauge Invariance of Green Functions: 
Background-Field Method versus Pinch Technique

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Abstract:
Application of the background-field method to QCD and the electroweak Standard Model yields gauge-invariant effective actions giving rise to simple Ward identities. Within this method, we calculate the quantities that have been treated in the literature using the pinch technique. Putting the quantum gauge parameter equal to one, we recover the pinch-technique results as a special case of the background-field method. The one-particle-irreducible Green functions of the background-field method fulfil for arbitrary gauge parameters the desirable theoretical properties that have been noticed within the pinch technique. Therefore the background-field formalism provides a general framework for the direct calculation of well-behaved Green functions. Within this formalism, the pinch technique appears as one of arbitrarily many equivalent possibilities.

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All known successful theories describing the interactions of elementary particles are
gauge theories. However, in order to evaluate quantized gauge theories within perturba-
tion theory, one has to break gauge invariance in intermediate steps by choosing a definite
gauge. As a consequence, although the physical observables, i.e. the S-matrix elements,
are gauge-independent, the Green functions, the building blocks of the S-matrix elements,
are gauge-dependent in the conventional formalism.

Before we proceed, we remind the reader of the notion of gauge invariance and gauge
independence: gauge invariance means invariance under gauge transformations. The
gauge invariance of the classical Lagrangian gives rise to Ward identities between the
Green functions of the quantized theory. Gauge independence becomes relevant when
quantization is done by fixing a gauge. It means independence of the method of gauge
fixing.

The gauge dependence of Green functions poses no problem as long as one calculates
physical observables in a fixed order of perturbation theory. However, as soon as one does
not take into account all contributions in a given order one will in general arrive at gauge-
dependent results. This happens usually if one tries to resum higher-order corrections via
Dyson summation of self-energies or if one is only interested in particular contributions
like definite formfactors, e.g. magnetic moments for off-shell particles, without taking into
account the full set of diagrams. This has been frequently done in the literature.

Motivated by these facts, various attempts have been made to define gauge-indepen-
dent building blocks. In order to construct gauge-independent running couplings, several
proposals for gauge-independent self-energies have been made [1,2]. These were essen-
tially obtained by considering four-fermion processes and shifting parts of the box and
vertex diagrams to the self-energies to cancel the gauge-parameter dependence of the
latter within the class of $R_\xi$ gauges. As one can shift arbitrary gauge-independent contribu-
tions between the different building blocks the resulting quantities are not unique. This
freedom has been used to require certain desirable properties from the self-energies, like
a decent asymptotic behaviour and the vanishing of the photon–Z-boson mixing at zero
momentum transfer. It nevertheless resulted in different definitions of gauge-independent
building blocks. All these ad-hoc treatments only refer to four-fermion processes and do
not give a general prescription which is applicable to other vertex functions.

Such a prescription is given by the so-called pinch technique (PT) [3,4,5,6]. The
PT is an algorithm for the construction of (within $R_\xi$ gauges) gauge-independent vertex
functions by reorganizing parts of the Feynman diagrams contributing to a manifestly
gauge-independent quantity, leaving only a trivial gauge dependence in the tree propaga-
tors. The results obtained via the PT directly fulfil the desirable properties that had to be
explicitly enforced in the ad-hoc treatments mentioned above. But even more important,
it turns out that the vertex functions constructed according to the PT fulfil the simple
Ward identities related to the classical Lagrangian.

However, the PT leaves many questions unanswered. So far, it has only been realized
for specific vertex functions at the one-loop level. Its application to other vertex functions
is not always clear and its generalization to higher orders is non-trivial and non-unique.
Although the PT vertex functions are claimed to be process-independent this has to
the best of our knowledge not been proven but only shown for specific examples. It
is very unsatisfactory that no explanation exists for the fact that the PT rules yield
vertex functions with the desirable properties and in particular that these vertex functions
fulfil simple Ward identities. Finally, although the application of the PT to four-fermion
processes is rather simple, it turns out that the explicit construction of general PT vertex
functions can be technically quite involved.

The purpose of this letter is to provide some insight into these problems. We will
show that the existing results obtained via the PT can be reproduced as a special case
within the background-field method (BFM). The BFM provides a natural explanation
for the desirable properties of the vertex functions noticed in the PT by relating them
to the Ward identities of the BFM which follow directly from gauge invariance. The
BFM is applicable to all orders of perturbation theory and for all vertex functions. It not
only generalizes the PT but in addition yields an infinite set of different vertex functions
fulfilling the same properties. The vertex functions of the BFM are directly derived from
Feynman rules. Since no complicated rearrangement between different Green functions
is needed, the BFM is technically much simpler than the PT and the vertex functions
obviously are not plagued with any process dependence.

The BFM [7] is a technique for quantizing gauge theories without losing explicit
gauge invariance. To this end one decomposes in the classical Lagrangian the usual gauge
field $V'$ into a background field $\hat{V}$ and a quantum field $V$ and adds a suitable non-linear
gauge-fixing term such that a gauge-invariant effective action $\Gamma[\hat{V}]$ can be constructed.
Its invariance with respect to background-field gauge transformations gives rise to simple
Ward identities between the corresponding vertex functions which follow from

$$\delta_{\text{gauge}} \Gamma[\hat{V}] = 0.$$  \hspace{1cm} (1)

The S-matrix is constructed in the usual way by forming trees with vertices from $\Gamma[\hat{V}]$
connected by lowest-order background-field propagators [8].

The background-field vertex functions can be calculated using Feynman rules that
distinguish between quantum fields and background fields. Whereas the quantum fields
appear only inside loops the background fields appear only in tree lines. The gauge fixing
of the background and quantum fields is completely unrelated resulting in independent
gauge-parameters $\hat{\xi}$ and $\xi_Q$, respectively. The gauge-invariant effective action depends
only on the quantum gauge parameter $\xi_Q$. The background gauge parameter $\hat{\xi}$ only
enters the S-matrix elements via the tree propagators. There are no background ghost
fields and there is in general no need to split the matter fields. Thus, fermion and scalar
fields can be treated as usual, they have the conventional Feynman rules, and there is no
distinction between background and quantum fields.

The relation between the BFM and the PT can be most easily seen in QCD. The
Feynman rules for QCD within the BFM have been given in Ref. [8]. They are particu-
larly simple in the Feynman gauge, $\xi_Q = 1$. For example, the coupling of one background
gluon with momentum $q^\mu$ to two quantum gluons reads

$$\Gamma_{\alpha\mu\lambda}^{abc}(k_1, q, k_2) = -gf^{abc} \Gamma_{\alpha\mu\lambda}^{abc}(k_1, q, k_2) = -gf^{abc} [g_{\alpha\lambda}(k_1 - k_2)_\mu - 2g_{\mu\lambda}q_\alpha + 2g_{\mu\alpha}q_\lambda],$$  \hspace{1cm} (2)
where \( f^{abc} \) are the structure constants of the gauge group \( SU(N) \) and all momenta are incoming. Straightforward application of those rules for \( \xi_Q = 1 \) yields for the one-loop gluon self-energy within dimensional regularization (we omit the fermion contribution)

\[
\Sigma_{ab}^{\mu\nu}(q) = -i\Gamma_{\mu\nu}^{(1)}(q) = -\delta^{ab} \frac{ig^2N}{(2\pi)^D}\mu^{4-D} \int \frac{d^Dk}{k^2(q+k)^2} \left[ \frac{1}{2} \Gamma_{\mu\lambda}(k,q,-q-k) \Gamma_{\alpha\nu}^{\lambda}(k,q,-q-k) \right. \\
- (2k+q)_\mu (2k+q)_\nu \right].
\]

(3)

where the first term originates from the gluon loop (Fig. 1a) and the second term from the ghost loop (Fig. 1b). The graphs involving quartic couplings vanish within dimensional regularization. The result (3) is precisely the same as the one derived within the PT as given in (3.9) of Ref. [4]. Calculating \( \Sigma_{ab}^{\mu\nu}(q) \) in the BFM for arbitrary \( \xi_Q \) yields

\[
\Sigma_{ab}^{\mu\nu}(q) = -\delta^{ab} (q^2g_{\mu\nu} - q_\mu q_\nu) \left[ bg^2 \log \frac{|q^2|}{\mu^2} + c(\xi_Q) \right],
\]

(4)

where

\[
b = \frac{11N}{48\pi^2},
\]

(5)

and \( c(\xi_Q) \) are terms independent of \( \mu \) and \( q^2 \). The coefficient \( b \) of the renormalization-scale-dependent logarithm does not depend on \( \xi_Q \) and coincides with the coefficient of \(-g^3\) in the \( \beta \) function \( \beta(g) \). This is a direct consequence of the gauge invariance in the BFM [4].

For the one-loop contribution to the three-gluon vertex one obtains in the Feynman gauge from the diagrams in Fig. 2 (\( k_1 = k + q_2, k_2 = k - q_1, k_3 = k \))

\[
\Gamma_{\mu\nu\rho}^{abc,(1)}(q_1, q_2, q_3) = -igf^{abc} \frac{g^2N}{2(2\pi)^D}\mu^{4-D} \int d^Dk \\
\left[ \frac{1}{k_1^2k_2^2k_3^2} \left[ -\Gamma_{\mu\beta}(k_2, q_1, -k_3) \Gamma_{\nu\lambda}(k_3, q_2, -k_1) \Gamma_{\rho\alpha}(k_1, q_3, -k_2) \\
+ 2(k_2 + k_3)\mu(k_1 + k_3)_{\nu}(k_1 + k_2)_{\rho} \right] \\
- \frac{1}{k_2^2k_3^2} 8(q_{1\rho}g_{\mu\nu} - q_{1\nu}g_{\mu\rho}) - \frac{1}{k_1^2k_3^2} 8(q_{2\nu}g_{\rho\mu} - q_{2\rho}g_{\mu\nu}) - \frac{1}{k_1^2k_2^2} 8(q_{3\mu}g_{\nu\rho} - q_{3\nu}g_{\mu\rho}) \right].
\]

(6)

Here again the first term results from the gauge-boson loop (Fig. 2a) and the second term from the two oppositely directed ghost loops (Fig. 2b). The two-point contributions
Figure 2: Feynman diagrams contributing to the three-gluon vertex at the one-loop level. Each graph in the second line represents three different permutations.

originate directly from the diagrams represented in Fig. 2c; the diagrams of Fig. 2d vanish.  
We have applied the PT and checked that it yields exactly the same result.

While in the PT the results (3) and (6) were obtained after an involved rearrangement of terms between different Green functions, in the BFM they are just the 2-point and 3-point vertex functions, respectively. They follow directly from the Feynman rules and are manifestly process-independent.

Moreover, the Ward identity which relates the self-energy and the vertex,

\[ q^\mu \Gamma^{abc,(1)}_{\mu\nu\rho}(q_1, q_2, q_3) = -g \left[ f^{a\rho b\nu d} \sum_{\nu\rho} (-q_3) - f^{a\rho b\nu d} \sum_{\nu\rho} (q_2) \right], \]

holds in the BFM not only for \( \xi_Q = 1 \) but for arbitrary \( \xi_Q \). The same is true for the Ward identity relating the three- and four-gluon vertices stated in Ref. [9]. While in the PT the validity of these Ward identities was only verified by explicit computation at one-loop order, in the BFM they are a direct consequence of the gauge-invariance of the effective action \( \Gamma[\hat{V}] \) and therefore valid to all orders.

The BFM can be directly applied to the Glashow–Salam–Weinberg model as well. However, in order to avoid tree-level mixing between the gauge bosons and the corresponding unphysical Higgs bosons one has to generalize the ’t Hooft gauge to the BFM. In this case, also the Higgs field has to be split into a background and a quantum part. While the background Higgs field \( \Phi \) has the usual non-vanishing vacuum expectation value, the one of the quantum Higgs field \( \Phi \) is zero. Denoting all background fields with a caret, the background-field ’t Hooft gauge-fixing term reads [10]

\[
\mathcal{L}_{GF} = -\frac{1}{2\xi_W^2} \left[ (\delta^{ac}\partial_\mu + g_2 e^{abc} \hat{W}^b_\mu) W^{c,\mu} - ig_2 \xi_Q^W W^W_\mu \frac{1}{2} (\hat{\Phi}_i^\dagger \sigma^a_j \hat{\Phi}_j - \hat{\Phi}_i^\dagger \sigma^a_j \hat{\Phi}_j) \right]^2 \\
- \frac{1}{2\xi_Q^B} \left[ \partial_\mu B^\mu + ig_1 e^B \xi_Q^B \frac{1}{2} (\hat{\Phi}_i^\dagger \Phi_i - \hat{\Phi}_i^\dagger \Phi_i) \right]^2, \tag{8}
\]

\(^1\)Note that the signs in Ref. [4] are inconsistent.
where we have used the conventions of Ref. [12], and \( \sigma^a, a = 1, 2, 3 \), denote the Pauli matrices. We note that this gauge-fixing term translates to the conventional one upon replacing the background Higgs field by its vacuum expectation value and omitting the background \( SU(2)_W \) triplet field \( \hat{W}^0_\mu \). Background-field gauge invariance restricts the number of quantum gauge parameters to two, one for \( SU(2)_W \) and one for \( U(1)_Y \).

We note in passing that the BFM has already been applied to the Glashow–Salam–Weinberg model in Ref. [12]. However, the gauge-fixing term used there breaks background-field gauge invariance as no background-Higgs field has been introduced. Since this only concerns contributions involving Higgs-particles outside loops, the results of Ref. [12] are nevertheless unaffected.

We have evaluated the complete Feynman rules within the background-field 't Hooft gauge (8) including the associated ghost terms for \( \xi_Q = \xi^W_Q = \xi^B_Q \). The vertices that do not contain background Higgs fields coincide for \( \xi_Q = 1 \) with those of Ref. [12]. Despite the fact that, owing to the doubling of the fields, the Feynman rules seem to become more complicated actual calculations become in fact simpler. This is in particular the case in the 't Hooft–Feynman gauge. The number of diagrams contributing to a certain vertex function is approximately the same as in the conventional formalism, but the diagrams themselves become easier to evaluate. Moreover, the number of diagrams contributing to the full (reducible) Green functions can be reduced by choosing an appropriate background gauge, e.g. the unitary gauge or a non-linear gauge.

Based on those Feynman rules, we have evaluated the quantities that have been treated in the literature using the PT, i.e. the gauge-boson self-energies, the fermion–gauge-boson vertices, and the triple-gauge-boson vertices. We find that all of them coincide for \( \xi_Q = 1 \) with those obtained in the PT [6, 13, 14]. Moreover, all desirable properties known for the PT vertex functions hold within the BFM for arbitrary \( \xi_Q \).

The validity of simple Ward identities follows in the BFM from the manifest gauge invariance. We list some of those Ward identities for illustration using the conventions of Ref. [12] (all fields and momenta are incoming and for the 2-point functions only the momentum of the first field is given):

\[
k_\mu \Gamma^{\tilde{A}\tilde{A}}(k) = 0, \quad \text{i.e.} \quad \Sigma^{\tilde{A}\tilde{A}}(k^2) = 0, \tag{9}
\]

\[
k_\mu \Gamma^{\tilde{A}\tilde{Z}}(k) = 0, \quad \text{i.e.} \quad \Sigma^{\tilde{A}\tilde{Z}}(k^2) = 0, \tag{10}
\]

\[
k_\mu \Gamma^{\tilde{A}\tilde{\chi}}(k) = 0, \quad \text{i.e.} \quad \Sigma^{\tilde{A}\tilde{\chi}}(k^2) = 0, \tag{11}
\]

\[
k_\mu \Gamma^{\tilde{Z}\tilde{Z}}(k) - iM_Z \Gamma^{\tilde{Z}\tilde{Z}}(k) = 0, \tag{12}
\]

\[
k_\mu \Gamma^{\tilde{Z}\tilde{\chi}}(k) - iM_Z \Gamma^{\tilde{Z}\tilde{\chi}}(k) + \frac{ie}{2s_Wc_W} \Gamma^\tilde{H}(0) = 0, \tag{13}
\]

\[
k_\mu \Gamma^{\tilde{W}^+\tilde{W}^-}(k) \mp M_W \Gamma^{\tilde{\phi}^+\tilde{\phi}^-}(k) = 0, \tag{14}
\]

\[
k_\mu \Gamma^{\tilde{W}^+\tilde{\phi}^+}(k) \mp M_W \Gamma^{\tilde{\phi}^+\tilde{\phi}^+}(k) \pm \frac{e}{2s_W} \Gamma^\tilde{H}(0) = 0, \tag{15}
\]

\[
k_\mu \Gamma^{\tilde{\phi}^+\tilde{\phi}^+}(k, \bar{p}, p) = -eQ_f[\Gamma^{ff}(\bar{p}) - \Gamma^{ff}(-p)], \tag{16}
\]

\[
k_\mu \Gamma^{\tilde{Z}\tilde{f}f}(k, \bar{p}, p) - iM_Z \Gamma^{\tilde{Z}\tilde{f}f}(k, \bar{p}, p) = e[\Gamma^{ff}(\bar{p})(v_f - a_f \gamma_5) - (v_f + a_f \gamma_5)\Gamma^{ff}(-p)], \tag{17}
\]

\[
k_\mu \Gamma^{\tilde{W}^+\tilde{f}f}(k, \bar{p}, p) \mp M_W \Gamma^{\tilde{\phi}^+\tilde{f}f}(k, \bar{p}, p) = \frac{e}{\sqrt{2}s_W}[\Gamma^{ff}(\bar{p})\frac{1 - \gamma_5}{2} - \frac{1 + \gamma_5}{2}\Gamma^{ff}(-p)], \tag{18}
\]
\( k^\mu \Gamma_{\mu \rho \sigma}^{\hat{A} \hat{W}+\hat{W}^{-}}(k, k_+, k_-) = e[\Gamma_{\rho \sigma}^{\hat{W}+\hat{W}^{-}}(k_+) - \Gamma_{\rho \sigma}^{\hat{W}+\hat{W}^{-}}(-k_-)] \),

\( k_+^\rho \Gamma_{\mu \rho \sigma}^{\hat{A} \hat{W}+\hat{W}^{-}}(k, k_+, k_-) - M_W \Gamma_{\mu \rho}^{\hat{A} \hat{A}+\hat{W}^{-}}(k, k_+, k_-) = e \left[ \Gamma_{\rho \sigma}^{\hat{W}+\hat{W}^{-}}(-k_-) - \Gamma_{\rho \sigma}^{\hat{A} \hat{A}}(k) + \frac{c_W}{s_W} \Gamma_{\mu \sigma}^{\hat{A} \hat{A}}(k) \right] \),

\( k_-^\sigma \Gamma_{\mu \rho \sigma}^{\hat{A} \hat{W}+\hat{W}^{-}}(k, k_+, k_-) + M_W \Gamma_{\mu \rho}^{\hat{A} \hat{A}+\hat{A}+\hat{Z}^{-}}(k, k_+, k_-) = -e \left[ \Gamma_{\mu \rho}^{\hat{W}+\hat{Z}^{-}}(-k_-) - \Gamma_{\mu \rho}^{\hat{A} \hat{A}}(k) + \frac{c_W}{s_W} \Gamma_{\mu \sigma}^{\hat{A} \hat{A}}(k) \right] \).

These identities hold for arbitrary \( \xi_Q \) in all orders of perturbation theory. Note that the vertex functions are one-particle irreducible, i.e. they contain no tadpole contributions; these appear explicitly as \( \Gamma^H(0) \). We have checked the validity of all these Ward identities at tree level and one-loop level by explicit computation. Within the PT they have been verified partially in Refs. [6, 13, 14].

Having derived the Ward identities in the BFM, we can also explain the other desirable properties of the vertex functions noticed within the PT. To this end, we use these Ward identities, simple power-counting arguments, and the fact that in contrast to the PT the building blocks in the BFM are genuine vertex functions.

We first discuss the list of properties stated by Degrassi and Sirlin [6]:

(i) The fact that the pole positions of the propagators are not modified within the BFM at the one-loop level is a direct consequence of the gauge invariance of those poles, which are directly related to physical observables.

(ii) The vanishing of the photon–Z-boson mixing at zero momentum, \( \Sigma_{\hat{A} \hat{Z}}(0) = 0 \), follows from the Ward identity (10) and the analyticity of \( \Sigma_{\hat{A} \hat{Z}}(k) \) at \( k^2 = 0 \), in analogy to \( \Sigma_{\hat{A} \hat{A}}(0) = 0 \).

(iii) The equality of the Z and W self-energy in the limit \( \sin \theta_W \to 0 \) and fixed \( g_2 \) results from the fact that the global \( SU(2)_W \) symmetry is not broken by the background-field gauge-fixing term.

(iv) The UV finiteness of the fermion–gauge-boson vertex functions including fermion wave-function renormalization can be derived from (16), (17) and (18) together with power-counting arguments like in QED.

(v) As stated in Ref. [6], the fact that the asymptotic behaviour for \(|q^2| \to \infty \) of the running couplings defined directly via Dyson summation of the self-energies is automatically governed by the renormalization group is a consequence of the UV finiteness of the fermion–gauge-boson vertices.

Moreover, the IR finiteness of the PT self-energies is trivial within the BFM.

We stress that all these arguments do not only hold for \( \xi_Q = 1 \) but for arbitrary finite values of \( \xi_Q \). In particular, the coefficients of the leading logarithms of the self-energies turn out to be independent of \( \xi_Q \) in the high-energy limit. We have verified all these properties by explicit calculation of the relevant one-loop vertex functions for arbitrary \( \xi_Q \). As a consequence, these features are not related to the \( \xi \) independence of the PT self-energies as could be supposed from the PT point of view.
From our results for the $A\bar{t}$ and $Z\bar{t}$ vertices within the BFM we have obtained the magnetic dipole moment form factor (MDM) of the top quark. It coincides with the one obtained within the conventional $R_\xi$-gauge formalism. In contrast to the statements in Ref. [13] but in agreement with Ref. [14], we find that the MDM vanishes in the limit $|q^2| \to \infty$ for all $\xi_Q$. This has been checked both numerically and analytically. Moreover, it can be inferred from a simple power-counting argument for all renormalizable gauges.

In Ref. [14] the three-gauge-boson vertices were derived within the PT for W bosons coupled to conserved currents and the Ward identity (19) was verified for this case. These results cannot be used as suitable building blocks in processes where this restriction does not apply as e.g. in $\gamma\gamma \to W^+W^-$. Projecting our general BFM results to the case of conserved currents, we find agreement with (3.16) of Ref. [14]. For our general off-shell result we have explicitly checked the Ward identity (13) and the other two Ward identities (20) and (21) which have not been mentioned in the PT context. In contrast to the claim made in Ref. [14], we found that the AWW vertex within the PT and the BFM is not IR-finite. However, the anomalous-magnetic-moment form factor and the electric-quadrupole moment-form factor are IR-finite and vanish in the limit $|q^2| \to \infty$. These last two facts can again be derived in the BFM from the Ward identities and power-counting arguments.

In the PT formalism the vertex functions are by construction independent of the gauge parameters $\xi$ within $R_\xi$ gauges. The gauge parameters only appear in the tree propagators joining the vertex functions. In the BFM formalism these gauge parameters correspond to the background gauge parameters $\hat{\xi}$. In Ref. [5] the PT has been applied to four-fermion processes in a gauge theory with spontaneous symmetry breaking. It turned out that the self-energy contributions to these processes could be formulated in different ways. Within the BFM these different possibilities correspond just to different background gauges. The representation (4.11) of Ref. [5] corresponds to the unitary gauge ($\hat{\xi} \to \infty$), whereas the representation (4.21) corresponds to the Landau gauge ($\hat{\xi} = 0$). Thus the results of Ref. [5] follow naturally within the BFM.

In conclusion, we have shown that the BFM provides a systematic way — via direct application of Feynman rules — to obtain Green functions that are derived from a gauge-invariant effective action, fulfil simple Ward identities and in comparison to their $R_\xi$-gauge counterparts possess very desirable properties such as improved IR und UV properties and a decent high-energy behaviour.

We applied the BFM to those cases for which the PT has been used in the literature. We found that the PT results coincide with the special case of the BFM results with quantum gauge parameter $\xi_Q = 1$. In contrast to the PT, the BFM can directly be applied to all orders of perturbation theory and all types of vertex functions. Whereas in the BFM the Ward identities are a strict consequence of gauge invariance, no general proof of their validity exists in the PT, and the Ward identities can only be verified in specific examples. Moreover, the calculation of the vertex functions is much simpler in the BFM than in the PT. While the rearrangement of different contributions in the PT is quite cumbersome and not clear for complicated vertex functions, the calculation within the BFM is comparable or even simpler than the evaluation of the vertex functions in the conventional formalism.

2There are some sign errors in Ref. [14].
In addition, we have found in all cases considered that the BFM yields well-behaved vertex functions for arbitrary values of $\xi_Q$. This means that the requirement of gauge-parameter independence used in the PT and former treatments is not the criterion leading to well-behaved vertex functions. We showed instead that the desirable properties of the background-field vertex functions are a direct consequence of the BFM Ward identities. They reflect the underlying gauge invariance and are manifestly valid for all values of the quantum gauge parameter $\xi_Q$.

Our results show in particular that the choice $\xi_Q = 1$ corresponding to the PT is not distinguished on physical grounds but it is only one of arbitrarily many equivalent possibilities. Of course, the background-field 't Hooft–Feynman gauge technically facilitates the calculations. The ambiguity of the vertex functions — quantified in the BFM by $\xi_Q$ — is also inherent in the PT and all other constructions, since the definite prescription to eliminate the gauge-parameter dependence appears to be just a matter of convention.

One benefit of the BFM is to make this ambiguity apparent. It shows that the PT vertex functions cannot carelessly be used to get physical predictions. Instead, we propose to use the Ward identities of the BFM to investigate for which cases physically meaningful results can be obtained.

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