Research on pasted FBG-based accelerometer’s sensitization process method and its characteristics

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Abstract: A pasted FBG based cantilever beam accelerometer’s sensitization process method has been proposed in this paper. We have built the pasted FBG-based cantilever beam accelerometer’s sensitization process model. The sensor’s sensitivity was improved by adjusting glue thickness between FBG and cantilever beam. Theoretical model has been proved by ANSYS simulation and experimental analysis: Simulation analysis shows that when distance between fiber core and cantilever beam is greater than 0.4 mm, it can be used to enhance sensitivity. Experimental analysis shows that when distance between fiber core and cantilever beam is 1 mm, sensitivity of sensor is increased by 2.9 times compared with the conventional process, but it has no effect on the sensor dynamic characteristics. Compared with conventional sensitization method, it’s more general, and can be applied to all pasted FBG accelerometer for enhancing sensitivity.

Keywords: pasted FBG, accelerometer, sensitization process

Classification: Fiber optics, Microwave photonics, Optical interconnection, Photonic signal processing, Photonic integration and systems

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1 Introduction

Optical fiber Bragg grating (FBG) has been invented more than 30 years ago, they possess several advantages over the conventional electrical sensors, such as resistance to electromagnetic interference, high reliability, the advantages of low cost, small volume, multiplexing capabilities, especially suitable for radioactive, corrosive or dangerous environment. So FBG is widely used in many kinds of fields, it’s one of the most promising optical fiber sensor at present [1, 2].

At present, there are many reports about FBG based vibration sensor all over the world. Some scholars designed vibration sensor based on principle of FBG suspension, but a large number of scholars usually directly pasted FBG on the surface of the elastomer to design vibration sensor [3, 4, 5]. For example, Wenjun Zhou et al made FBG glue in a slanted direction onto the lateral side of a right-angled triangle cantilever beam. When vertical vibration applied to the cantilever beam, the FBG could be chirped and its reflection bandwidth and optical power change with the deflection of the beam, So that vibration was measured by optical power measurement [6]; Wang Tao et al proposed a novel temperature self-compensation FBG vibration sensor, FBGs were fixed on symmetrical position of a cantilever girder’s surface, its sensitivity was about 28 pm/g [7]; Yinian Zhu et al pasted FBG on the cantilever beam, and measured the reflected optical power of a strain induced chirped FBG. The sensor’s maximum linear acceleration range was approximately 8 g with a dynamic range of 26 dB [8]; literature [9] proposed a FBG-based accelerometer designed from a dual flexural beam approach that was tunable in resonant frequency and sensitivity by the addition of a concentrated mass at the center of the diaphragm. Its sensitivity reached 2–12.5 pm/g; literature [10] presented a novel pasted FBG accelerometer based on a diaphragm with a sensitivity of 36.6 pm/g. As we all know, sensitivity is an important index of vibration sensors. From the all of above FBG vibration sensors, we can find that they were all directly pasted on the cantilever beam. So their sensitivity is limited by the cantilever structure. Though some designed the new elastomer structure to improve vibration sensor’s sensitivity, it’s very complex operations and maybe make the dynamic properties of sensor change [11, 12].

In this paper has proposed a pasted FBG-based cantilever beam accelerometer’s sensitization process method without change the cantilever beam structure. We have built the pasted FBG-based cantilever beam accelerometer’s sensitization process model by analysis of theoretical and simulation, through increase the
separation between surface of the cantilever and the FBG to enhance sensitivity. Compared with others sensitization method, it’s more simple and general; also it can be applied in all pasted FBG accelerometer sensitization. This paper mainly falls into the following aspects: the basic principle of the sensor and sensitization process model, sensitization process simulation analysis, dynamic characteristics simulation analysis of sensor, experimental analysis of sensing properties of sensors and discussions.

2 Principle and sensitization process model of the sensor

2.1 Principle of pasted FBG cantilever beam accelerometer

Schematic diagraph of FBG-based cantilever beam accelerometer is shown in Fig. 1. From the figure, we can find that FBG was pasted on the surface of the cantilever beam, and the lumped mass was installed at the end of the cantilever beam. When the sensor vibrates along with measured body, the lumped mass imposes certain inertial load $F = ma$ to the cantilever beam. Then strain of the cantilever beam surface corresponding change, eventually it makes the pasted FBG wavelength drift. Thus, measured body vibration acceleration would be obtained by FBG center wavelength shift.

Supposed the lumped mass is $m$; Young’s modulus of cantilever beam is $E$; $L$, $b$ and $h$ are the cantilever beam length, width and thickness, respectively. And deflection of the end of cantilever beam is $Y$. According to mechanics of materials, the cantilever beam stiffness is $k = Ebh^3/6L^3$. When alternating load imposes to the end of the cantilever beam, we can get kinetic equation without the structural damping, which can be described by Eq. (1):

$$m\ddot{Y} + kY = F \sin(wt)$$  \hspace{1cm} (1)

From the above differential equation, the cantilever beam deflection can be expressed by:

$$Y = \frac{F}{m(w_0^2 - w^2)} = \frac{a}{(w_0^2 - w^2)}$$  \hspace{1cm} (2)

Where the $w_0$ is the inherent frequency, it can be expressed to $w_0^2 = k/m$.

Also we can get the strain of cantilever beam each layer $\varepsilon$ and deflection at the end of cantilever beam $y$ through the mechanics of materials, which are expressed as follow:
\[
\varepsilon = \frac{12FLz}{Ebh^3} \\
y = \frac{6FL^3}{Ebh^3}
\] (3)

\(z\) represents the distance between any longitudinal section and neutral layer of cantilever beam.

Based on Eqs. (2), (3) and (4), the relationship between strain \(\varepsilon\) and acceleration \(a\) can be deduced as Eq. (5):

\[
\varepsilon = \frac{2yz}{L^2} = \frac{2z}{(w_0^2 - w^2)L^2}a
\] (5)

When the \(w \ll w_0\), Eq. (5) can be simplified to Eq. (6):

\[
\varepsilon = \frac{2z}{w_0^2L^2}a
\] (6)

According to the principle of FBG sensing, the shift of a FBG wavelength due to strain and temperature can be expressed as Eq. (7):

\[
\frac{\Delta \lambda_B}{\lambda_B} = (1 - \rho_e)\Delta \varepsilon + (a_n + a_A)\Delta T
\] (7)

Where \(\rho_e\) is the strain-optic coefficient of an optical fiber, \(a_n\) is the coefficient of thermal expansion, \(a_A\) is the thermo-optic coefficient, and \(\Delta T\) is the value of temperature change.

When cantilever beam strain transfers to FBG, it could loss same. So the strain of the FBG can be described by Eq. (8):

\[
\Delta \varepsilon = \alpha \varepsilon
\] (8)

Where \(\alpha (0 < \alpha < 1)\) is the glue transfer factor. Using above Eqs. (6), (7) and (8) with the assumption of no temperature change, we can measure the acceleration from the wavelength shift as:

\[
\Delta \lambda_B = (1 - \rho_e) \frac{2az\lambda_B}{w_0^2L^2}a
\] (9)

We can get the sensor sensitivity from the Eq. (9):

\[
S_1 = (1 - \rho_e) \frac{2az\lambda_B}{w_0^2L^2}
\] (10)

### 2.2 Sensitization process model

According to Eqs. (5) and (6), we can find that when \(w \ll w_0\) and the \(z\) is a constant, the strain \(\varepsilon\) of cantilever beam surface is proportional to the acceleration \(a\). But there exists glue line between fiber and cantilever beam in actual situation (Fig. 2), and the strain of FBG isn’t equal to cantilever beam surface. So we could reset the relationship between FBG strain and the acceleration \(a\). In order to ensure the equation, we need to make the following assumptions: 1) Each part always stay in touch in the bending process; 2) Fiber core and optical fiber coating have the same physical parameters.

According to the static equilibrium theorem, in the \(x\) direction of the model we can get the equation:
\[ F_1 = \int \sigma_1 dA + \int \sigma_2 dA + \int \sigma_3 dA = 0 \quad (11) \]

\[ \sigma_1, \sigma_2 \text{ and } \sigma_3 \text{ are stress of cantilever beam, glue and FBG, respectively; } A_1, A_2 \text{ and } A_3 \text{ are cross-sectional area of cantilever beam, glue and FBG, respectively.} \]

According to material deformation geometrical with physical relationship, the Eq. (11) can be simplified to Eq. (12):

\[ E_1 \int \frac{y}{\rho} dA + E_2 \int \frac{y}{\rho} dA + E_3 \int \frac{y}{\rho} dA = E_1 \cdot S_1 + E_2 \cdot S_2 + E_3 \cdot S_3 = 0 \quad (12) \]

Eq. (12) can be reduced to Eq. (13):

\[ E_1 \cdot S_1 + E_2 \cdot S_2 + E_3 \cdot S_3 = E_1 A_1 (y_1 - y) + E_2 A_2 (y_2 - y) + E_3 A_3 (y_3 - y) = 0 \quad (13) \]

Where the \( E_1, E_2 \text{ and } E_3 \) are stress of cantilever beam, glue and FBG, respectively; \( y_1, y_2 \text{ and } y_3 \) are distances between centroid positions of cantilever beam, glue and FBG and lower surface of cantilever beam (Fig. 3). From the Eq. (13), we can obtain equation of the \( y \) which is the distance between neutral layer and lower surface of cantilever beam.

\[ y = \frac{E_1 A_1 y_1 + E_2 A_2 y_2 + E_3 A_3 y_3}{E_1 A_1 + E_2 A_2 + E_3 A_3} = \frac{\sum_{i=1}^{3} E_i A_i y_i}{\sum_{i=1}^{3} E_i A_i} \quad (14) \]

Due to the acceleration is made of \#45 steel cantilever beams, and the AB glue, it’s obvious to know that \( E_1, E_2 \ll E_3, \) so we can approximate get: \( y \approx y_1. \)

Based on Eq. (6) and (14), we can obtain the relationship between FBG strain and acceleration, it’s expressed by Eq. (15):
\[ \epsilon_{\text{op}} = \frac{2(h/2 + d + r)}{w_0^2L^2} a \]  

(15)

Where \( d \) is the glue thickness between FBG and cantilever; \( r \) is radius of the FBG coating.

After Sensitization processing, based on Eq. (7), (8) and (15), the wavelength variation response to acceleration could be given by:

\[ \Delta \lambda_B = (1 - \rho_e) \frac{2a(h/2 + d + r)\lambda_B}{w_0^2L^2} a \]  

(16)

We can get the sensor sensitivity with sensitization process from the Eq. (10):

\[ S_2 = (1 - \rho_e) \frac{2a(h/2 + d + r)\lambda_B}{w_0^2L^2} \]  

(17)

According to Eqs. (10) and (17), we can find that the sensitivity of sensor is increased by \( \eta \) compared with the FBG directly pasted on the cantilever beam:

\[ \eta = \frac{S_2}{S_1} = \frac{h/2 + d + r}{h/2 + r} \]  

(18)

3 Simulated analysis

In this paper, we made sensitization process model analysis, modal analysis and harmonic response analysis on the partial assembly of the sensor using the finite element analysis method with Ansys software. The physical parameters of the sensor’s main structure are listed in Table I. The sensor’s cantilever beam size is: length \( L = 60 \text{ mm} \), the maximum width \( b = 12 \text{ mm} \), thickness \( h = 1 \text{ mm} \); lumped mass: external diameter = 4 mm, height = 4 mm; glue line length = 16 mm, thickness = 2 mm, width = 3 mm; the effective length of the fiber is 16 mm, and all of the fiber is embedded in the glue line.

| Material          | #45 Steel | Glue | Coating | Fiber | Lumped mass |
|-------------------|-----------|------|---------|-------|-------------|
| \( E/10^9 \text{ pa} \) | 210       | 3.3  | 0.1     | 72    | 115         |
| Poisson ratio     | 0.3       | 0.35 | 0.35    | 0.17  | 0.31        |
| Density/Kg/m\(^3\) | 7850      | ---  | ---     | ---   | 8960        |

3.1 Sensitization process model simulation analysis

In order to quickly achieve many times sensitization process simulation analysis by ANSYS, so we didn’t consider structure of lumped mass in this simplified simulation model. We set a node in the center hole of the simulation model. A MPC186 unit was used to make the rigid node, and we created coupling with nodes in the circle. Then we apply a loaded concentrate with 1N in the center hole of the node. And we continuously adjusted the \( D = d + r \) (0.2 mm, 0.4 mm, 0.6 mm, 0.8 mm, 1.0 mm, 1.2 mm, 1.4 mm, 1.6 mm and 1.8 mm) which is the distance between fiber core centre and cantilever surface to repeat simulation analysis.
FBG’s versus the different distances D is shown in Fig. 4, we have selected the middle of FBG as zero point on the x axis. From the figure, we can find that FBG’s strain increases with the increase of the distances D. When the D is (i) <0.4 mm, the FBG’s strain is less than the cantilever beam surface strain 142\(\mu e\), because there is almost no change for D, so the \(\alpha\) became the basic factor in this ranges; (ii) >0.4 mm, the FBG’s strain is more than the cantilever beam surface strain 142\(\mu e\), and now the variational D became the leading role. So we can select appropriate D in the ranges of D > 0.4 mm to enhance sensitivity of the sensor.

From the above simulated analysis, we chose the D = 1 mm to do the further research. The strain transfer cloud map at the distance D = 1 mm is shown in Fig. 5. From the figure, we can find that the maximum of strain locate in the middle of glue line, so we could make the FBG in the middle of the glue line during the experiment; the FBG’s strain is increased by 1.7 times compared with the surface strain of cantilever beam.
3.2 Dynamic characteristics simulation analysis

In this dynamic simulation model, we did not consider the effect of glue line and fiber, as it is too small compared with #45 steel young’s modulus and has little impact on the overall analysis. We chose the solid element to mesh the structure of FBG accelerometer. The first-order natural frequency obtained from the modal analysis is 192.65 Hz (Fig. 6). The harmonic load amplitude from the harmonic response analysis is about 318310 Pa (4 N), and the frequency ranges within 0–250 Hz. Harmonic response of FBG acceleration sensor without glue line is shown in Fig. 7, which shows that the frequency range of 0–120 Hz is the dynamic measurement range of FBG acceleration sensor, and the resonant frequency is 192 Hz.

Fig. 6. First-order mode shape of FBG acceleration sensor without glue line

Fig. 7. Harmonic response of FBG acceleration sensor without glue line

4 Experiments of sensing characteristics

Based on the above theory and simulation analysis, we selected two pasted FBG based cantilever beam accelerometer to do research on their static and dynamic characteristic analysis. The FBG is directly pasted on the surface of cantilever beam which is labeled as #1 FBG accelerometer; the other is labeled as #2FBG
accelerometer which exists 1 mm glue line between FBG and the cantilever beam. Physical map of FBG acceleration sensors is shown in Fig. 8. According to the above Eq. (18), we detected #2FBG accelerometer sensitivity is

$$\eta = \frac{h/2 + d + r}{h/2 + r} = \frac{0.5 + 1 + 0.125}{0.5 + 0.125} = 2.6\text{ times as great as } #1\text{ FBG accelerometer.}$$

In order to realize the experimental study of FBG accelerometers’ dynamic and static characteristics, we built experimental system which is shown in Fig. 9. The experimental system was mainly composed of B&K vibration test system, B&K-4508b piezoelectric acceleration sensor, signal generator, power amplifier, and FBG interrogator (sampling rate: 4 k, resolution: 0.1 pm) etc. We used 3M glue to fix sensor on the exciter (Fig. 9), and adjusted the output of amplitude and frequency of signal generator, combined with power amplifier and piezoelectric acceleration vibration sensor to make vibration exciter work in predetermined value. Finally we can obtain the response signal of the FBG accelerometer by FBG interrogator under the different excitation.

### 4.1 Static characteristics

We set vibration exciter’s frequency was 30 Hz, and adjusted the acceleration value of vibrator within the ranges of 10–90 m/s². The experiment was repeated three times. Fig. 10 is shown the responses of two FBG accelerometers in the 100 Hz with amplitude of 2 g excitation. Average of wavelength drift differences obtained in the repeated experiments. Then we used the least square method to make linear fitting, the test data and the liner fitting are shown in Fig. 11. Through analyzing the experimental data and the fitting line, two sensors’ static characteristics are shown in Table II. The Fig. 11 and Table II show that the sensitivity $S_2$ of #2FBG accelerometer is 9.1 pm/g, and sensitivity $S_1$ of #1 FBG accelerometer is 3.1 pm/g; $S_2$ is about 2.94 times as great as $S_1$. From the Table II, we can find that all of the

![Fig. 8. Physical map of FBG acceleration sensors](image)

![Fig. 9. (a) The schematic diagram of the static experimental system; (b) Physical map of experiment system](image)

![Fig. 10. Responses of two FBG accelerometers in the 100 Hz with amplitude of 2 g excitation](image)

![Fig. 11. Average of wavelength drift differences obtained in the repeated experiments](image)

![Fig. 12. The least square method to make linear fitting](image)

![Table II. Sensitivity of FBG accelerometers](image)
#2FBG accelerometer’s performances which is used to sensitization process are better than #1 FBG accelerometer.

![Graph showing responses of two FBG accelerometers](image)

**Fig. 10.** Responses of two FBG accelerometers in the 30 Hz with amplitude of 6 g excitation

| Performance | Hysteresis error/% | Linearity/% | Sensitivity/µm/g | Fitted equation |
|-------------|--------------------|-------------|-----------------|----------------|
| #1 FBG accelerometer | 6.05 | 1.96 | 3.1 | $\Delta \lambda = 0.31 \cdot a + 3.44$ |
| #2 FBG accelerometer | 2.95 | 1.41 | 9.1 | $\Delta \lambda = 0.91 \cdot a + 3.15$ |

**Table II.** Static performance parameters of acceleration sensors

![Graph showing static characteristic curves](image)

**Fig. 11.** Static characteristic curve FBG acceleration sensor

### 4.2 Dynamic characteristics

In this Experiment, we fixed acceleration amplitude of vibration exciter was 2 g, and adjusted the frequency within the scope of 10–250 Hz. The experiment was repeated 4 times. Fig. 12 shows responses of two FBG accelerometers in the 100 Hz with amplitude of 2 g excitation. Dynamic characteristic curve of FBG accelerometers’ are shown in Fig. 13. It shows that the working band and the resonant frequency of the #1FBG accelerometer are about 0–150 Hz and 195 Hz, respectively; #2FBG accelerometer’s dynamic characteristics is similar to #1FBG accelerometer, its resonant frequency is 200 Hz, and the working band from 0 to 150 Hz.
5 Discussions and analyses

The results of $S_2/S_1$ were calculated by three kinds of analysis methods which are shown in Table III. Through contrastive analysis of Table III, we can obtain the below conclusions: I) $S_2/S_1$ of the theory and experiment are 2.6 and 2.9 times, respectively. The two ratios are almost the same; it directly proved the correctness of the theoretical model. The results of $S_2/S_1$ are still exist the certain error, there are some reasons could account for it. i). The thickness of glue line has certain error in experiment; ii). We ignored the influence young’s modulus of optical fiber and glue line in the simplified model. II) $S_2/S_1$ value of simulation and experiment is large difference. The main reason is that we set the material properties of glue line and fiber in the simulation model isn’t completely consistent with the real model. But the strain distribution trend of simulation is consistent with the experiment; and we can use it to do some qualitative analysis.

Table III. Results of $S_2/S_1$ were calculated by three kinds of analysis methods

|       | $S_2/S_1$ |
|-------|-----------|
| Theory| 2.6       |
| Simulation | 1.7   |
| Experiment | 2.9    |
The dynamic characteristics were obtained by three kinds of analysis methods which are shown in Table IV. Through contrastive analysis of Table IV, we can get that: I) The resonance frequency and working band of simulation are consistent with experiment, it’s verified the validity of the simulation model; II) There exists certain difference between simulation and experiment. The main reason is that glue line and fiber was neglected in the dynamic simulation analysis. III) Dynamic properties values of the #1 FBG accelerometer is difference from the #2FBG accelerometer, but the dynamic properties difference between two sensor is very small. It’s indirectly shown that sensitization process method have little effect on the dynamic properties of FBG accelerometer.

| #1 FBG accelerometer | #2 FBG accelerometer |
|----------------------|----------------------|
| Working band/Hz      | Resonant frequency/Hz |
| Simulation           | 0–120                 | 192                   |
| Experiment           | 0–150                 | 195                   |
|                     | 0–120                 | 192                   |
|                     | 0–150                 | 200                   |

6 Conclusions

A pasted FBG based cantilever beam accelerometer’s sensitization process method has been proposed in this paper. We have built the pasted FBG-based cantilever beam accelerometer’s sensitization process model. The sensor’s sensitivity was improved by adjusting glue thickness between FBG and cantilever beam. We used the ANSYS simulation and experimental to analysis theoretical model. Simulation analysis shows that when distance between fiber core and cantilever beam is greater than 0.4 mm, the FBG’s strain is more than the cantilever beam surface strain, so it can be used to enhance sensitivity. Also experimental analysis shows that i). All of the #2FBG accelerometer’s performances which is used to sensitization process are better than #1 FBG accelerometer; ii). When distance between fiber core and cantilever beam is 1 mm, sensitivity of sensor is increased by 2.9 times compared with the conventional process, which is consistent with the theoretical analysis 2.6; iii). Dynamic properties values of the #1FBG accelerometer is little difference from the #2FBG accelerometer, it is show that there has no effect on the sensor dynamic characteristics. Compared with conventional sensitization method, it’s more general, and can be applied to all pasted FBG accelerometer for enhancing sensitivity.

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