The nonlinear computed torque control of a quadrotor

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ABSTRACT
This paper work focused on the study of the nonlinear computed torque control of a quadrotor helicopter. In order to model the dynamic of the vehicles, kinematics and dynamics modeling of the X4 is presented. Euler angles and parameters are used in the formulation of this model and the technique of Computed Torque control is introduced. In the second part of the paper, we develop a methodology of control that allows the quadrotor to accomplish a prospecting mission of an environment, as the follow-up of a trajectory by the simulation. Results show that Computed Torque control method is suitable for X4.

1. INTRODUCTION
More recently, humans have seen the capabilities of robotics expand to the flight control of both commercial and military vehicles. This shift has even begun to include highly agile and low-cost miniature Unmanned Aerial Vehicles. There is currently great interest in the area of quadrotor UAV research including search and rescue, surveillance, traffic monitoring, fire detection [1-3]. The main advantage of quadrotor vehicles is that they can perform any maneuver in many directions with equal performance. Therefore, quadrotors are well suited for mission requiring operation in tight locations such as flying indoors and close to objects in which turning around is not possible or very dangerous.

The control of a quadrotor is the principally problem because the nonlinear behavior of quadrotors requires an advanced stabilizing. The modeling of X4 to account for various parameters which affects the dynamic of the quadrotor. Unlike to the terrestrial mobile robots, when the model can be limited to kinematic part, in order to counter for gravity effects, frictions due to the aerodynamic torques, drag forces and gyroscopic effects (which can be derived from experiments), we need a dynamics model for simplifying the control startegies [4, 5].

There have been a number of papers dealing with various problems inherent to the exploitation of quadrotors. For example, dynamic modeling issues were addressed in references [6, 7] whereas, nonlinear control laws, such as feedback linearization control, visual control, backstepping control and sliding-mode control were studied in many papers such as references [8-19]. Self-Tuned Fuzzy Inference System (STFIS) controller for the XSF(X4 Stationnary Flyer) was the subject of the main idea developed in [20, 21]. Sliding mode control for altitude and attitude stabilization of quadrotor UAV with external disturbance presented by [22]. In [23], a combination of the computed-torque controller and adaptive fuzzy trajectory feedforward compensator is used for the trajectory tracking control of uncertain underwater vehicle is developed and Attitude and altitude control of quadrotor by discrete PID control and non-linear model predictive control is presented in [24].
In this paper, the complete model (kinematic and dynamic) of the quadrotor helicopter is presented, based on the Newton-Euler approach with analysis of gravity, aerodynamics, damping and thrust. This model is developing by [2, 4, 25]. The main objective of this article is the synthesis of stabilizing control laws in terms of translation and orientation for the X4. It’s a vehicle has the difficulty of the control because its complex, non-linear dynamics, multivariable, of coupled nature, particularly in its operation and sensitivity to the external disturbance. The PD-CTC controller is proposed to solve this problem (control). The control strategy is based on the decomposition of the original system into two subsystems: the first concerns the position control while the second is to control orientation and the velocity.

The quadrotor is composed of four rotors in a cross configuration, this cross structure is quite thin and light, which will guarantee robustness by ensuring the mechanical links for the motors, for reasons to simplifying the model; the axes of propellers are fixed in parallel and each propeller is combined with a motor, these configurations will help us to maneuver the quadrotor just by varying the speed of propellers.

The most important thing in this configuration that the pair front-rear propellers rotate counter clockwise while the left-right one rotates clockwise that removes the need for a tail rotor (needed in the classic helicopter), in the hovering condition, all the propellers must have the same speed. In Figure 1 a block diagram of the quadrotor structure is presented in black, the fixed-body B-frame is shown in green and in blue is represented the angular speed of the propellers in addition to the name of the velocity variable, for each propeller, two arrows are drawn: the curved one represents the direction of rotation and the second represents the velocity. In order to perform a stationary flight (hovering), the propellers must rotate at the same speed for countering the acceleration due to the gravity, as depicted in Figure 1.

The quadrotor is equipped just with four rotors so for controlling the engine, we have to choose a number of freedoms that correspond to the number of actuators so for our case we were choose 3 degrees of freedom which are roll angle, pitch angle, yaw angle. The quadrotor can perform four basic movements which allow this engine to reach a certain altitude (targets); the principal movements are described:

Throttle (U1): this command is provided by increasing (or decreasing) all the propeller speeds by the same amounts \( \Delta \dot{A} \), like in Figure 2, where \( \dot{A} \) is the speed of propellers. It leads to a vertical force which raises or lowers the quadrotor, the amount \( \Delta \dot{A} \) must be too small because the model would eventually influenced by strong non-linearities or saturations.

Roll (U2): this command is provided by increasing (or decreasing) the left propeller speed and inverse operation for the right one. It deals with the torque around the \( x_B \) axis. The amounts \( \Delta \dot{A} \) and \( \Delta \dot{B} \) in the Figure 3 must be equals for the same reasons mentioned above and also to maintain the vertical thrust unchanged.

Pitch (U3): this command is similar to the roll and is provided by increasing (or decreasing) the rear propeller speed and inverse operation for the front one. It leads to a torque around the \( y_B \) axis. The positive amounts \( \Delta \dot{A} \) and \( \Delta \dot{B} \) are chosen to maintain the vertical thrust unchanged and can’t be too large as in Figure 4.

Yaw (U4): this command is provided by increasing (or decreasing) the front-rear propellers speed and the inverse operation for the left-right pair. It leads to a torque with respect to the \( z_B \) axis, hence the front-
rear couple rotate counter clockwise and the left-right one rotate clockwise. The same technic for $\Delta_A$ and $\Delta_B$ that are chosen to maintain the vertical thrust unchanged and they can’t be too large as shown in the Figure 5.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure5.png}
\caption{Yaw movement}
\end{figure}

2. QUADROTOR MODELING

In this section, we will describe the reference frames and coordinate systems which will served to extract the positions and orientations of quadrotor. A transformation matrix between these coordinate systems is developed for the following reasons:
- Newton’s equations of motion are given in the body coordinate frame.
- Aerodynamics forces and torques are applied in the earth frame.

Unlike GPS which is used to measure position, ground speed and course angle with respect to the earth frame, the on-board sensors like accelerometers and gyroscopes are used for gathering information in the body frame. For trajectory planification missions, all the collected information have to be transformed in the inertial frame. To switch from one coordinate frame to another, two basic operations are performed: rotations and translations.

As described in [2-4], the details of dynamic model of the quadrotor will not be discussed in this paper again. For reasons cited previously, we will consider two frames: earth frame and a body frame whose origin is in the center of mass of our engine, see Figure 1. The attitude vector of the quadrotor is given by the roll, pitch and yaw angles, forming the vector $\mathbf{\Psi} = (\phi, \theta, \psi)$ however the position of the vehicle in the inertial frame is given by the position vector $\mathbf{r} = (x, y, z)$. Conservation of momentum and angular momentum laws are used to extract the dynamic model of the quadrotor, considering the applied forces and torques into account [4, 26]. The thrust force generated by rotor $i$, $i = 1, 2, 3, 4$ is $F_i = b \cdot w_i^2$ with the thrust factor $b$ and the rotor speed $w_i$, and the law of conservation of momentum yields:

$$\dot{\mathbf{r}} = g \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - R(\mathbf{\Psi}) \cdot \frac{b}{m} \sum_{i=1}^{4} w_i^2 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{1}$$

Herein $R(\mathbf{\Psi})$ is a suitable rotation matrix and $m$ is the mass of quadrotor. With the inertia matrix $I$ (a pure diagonal matrix with the inertias $I_x, I_y$, and $I_z$ on the main diagonal), the rotor inertia $I_R$, the vector $\mathbf{M}$ of the torque applied to the vehicle’s body and the vector $\mathbf{M}_G$ of the gyroscopic torques of the rotors, the law of conservation of angular momentum yields:

$$I \dot{\mathbf{\Omega}} = - (\mathbf{\Omega} \times I \mathbf{\Omega}) - \mathbf{M}_G + \mathbf{M} \tag{2}$$

The vector $\mathbf{M}$ is defined as:

$$\mathbf{M} = \begin{pmatrix} Lb(w_2^2 - w_3^2) \\ Lb(w_1^2 - w_3^2) \\ d(w_1^2 - w_2^2 + w_3^2 - w_4^2) \end{pmatrix} \tag{3}$$

With the drag coefficient $d$ and the length $L$ of the lever. The gyroscopic torques caused by rotations of the vehicle with rotating rotors are:

$$\mathbf{M}_G = I_R \left( \Omega \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right) \cdot (w_1 - w_2 + w_3 - w_4) \tag{4}$$

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The four rotational velocities $w_i$ of the rotors are the real input variables of the vehicle, but for a simplification of the model, the following substitute input variables are defined:

\[
\begin{align*}
    u_1 &= b \left( w_1^2 + w_2^2 + w_3^2 + w_4^2 \right) \\
    u_2 &= b \left( w_2^2 - w_4^2 \right) \\
    u_3 &= b \left( w_1^2 - w_3^2 \right) \\
    u_4 &= d \left( w_1^2 - w_2^2 + w_3^2 - w_4^2 \right)
\end{align*}
\]

(5)

Defining: $u^T = (u_1, u_2, u_3, u_4)$ and $(w_1 - w_2 + w_3 - w_4) = g(u)$ and introducing the vector of state variables: $x^T = (\dot{x}, \dot{y}, \dot{z}, \dot{\phi}, \dot{\theta}, \dot{\psi})$ evaluation of (1) until (5) yields the following state variable model:

\[
x^T = \begin{bmatrix}
    - (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \frac{u_1}{m} \\
    - (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \frac{u_1}{m} \\
    \psi - (\cos \phi \cos \theta) \frac{u_1}{m} \\
    x_1 \psi I_1 - \frac{I_y}{I_x} x_3 g(u) + \frac{L}{I_x} u_2 \\
    x_3 \phi I_2 + \frac{I_x}{I_y} x_1 g(u) + \frac{L}{I_y} u_3 \\
    x_3 \phi I_3 + \frac{1}{I_z} u_4
\end{bmatrix}
\]

(6)

Herein, we use the abbreviations $I_1 = \frac{(I_y - I_z)}{I_x}$, $I_2 = \frac{(I_x - I_z)}{I_y}$ and $I_3 = \frac{(I_x - I_y)}{I_z}$, in order to study the dynamic behavior of the quadrotor a 3DOF model is designed in Matlab and the open loop response is shown in the Figure 6.

![Figure 6. Open loop response of the quadrotor](image.png)

From the Figure 6 we see that the system is unstable and diverges to the saturation that is introduced to counterbalance the problem of gimbal lock (the angles orientation must be limited to 90 degrees in order to...
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These equations are used to well describing the aerodynamic vehicle study, because of the possibility of formulating them from trimmed aerodynamic data and simple autopilot designs. Nevertheless, they give a realistic picture of the translational and rotational dynamics unless large angles and cross-coupling effects dominate the simulations.

3. CONTROL INVESTIGATION

The controller of vehicle has the task of stabilization of desired orientation (attitude). In order to achieve and maintain a desired angle orientation. In this section we will discuss the ability of PD-CTC controller to stabilize the attitude of quadrotor, first we need to decompose the state space vector and take just the three angles of orientation ($\phi, \theta, \psi$). The PD-CTC controller is applied to a nonlinear systems so for this reason is not to linearize our system, we choose to take some assumptions that can help us to easily manipulate the orientation and applied the controller, first, the gyroscopic effect is neglected because the rotor inertia is very small with relative to the quadrotor inertia, the structure of quadrotor is fully symmetric and the inertia matrix is diagonal because the coupled inertia between axis is very low with relative to the inertia of each axis, as shown in Figure 7.

![Figure 7. PD-CTC control of the quadrotor](image)

The computed torque algorithm to develop the control allowing the system to follow the desired trajectory. We consider the tracking error:

$$e_z = z - z_r$$

$V_r$: is the desired velocity.

Its derivative is computed as:

$$\dot{e}_z = \dot{z} - \dot{z}_r$$

The nonlinear feedback control law that guarantees tracking of desired trajectory. Selecting Proportional plus derivative (PD) feedback for $U(t)$ results in the PD-computed torque controller [7], the final control is:

$$U_1 = \frac{(\ddot{e}_z - K_{z1}e_z - K_{z2}\dot{e}_z - g)m}{\cos\psi_4\cos\psi_5}$$

and the resulting linear error dynamics are:

$$\left(\ddot{z}_{ref} - K_{z1}e_z - K_{z2}\dot{e}_z\right) = 0$$

According to linear system theory, convergence of the tracking error to zero is guaranteed:

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\[\begin{align*}
    u_2 &= \frac{l_x}{L} \left( \ddot{x}_{\text{ref}} - K_{41} \dot{e}_y - K_{42} \dot{e}_x - x_y \dot{x}_y J_1 - \frac{I_y}{l_x} g(u) x_6 \right) \\
    u_3 &= \frac{l_y}{L} \left( \ddot{y}_{\text{ref}} - K_{51} \dot{e}_x - K_{52} \dot{e}_y - x_x \dot{x}_x J_2 - \frac{I_x}{l_y} g(u) x_7 \right) \\
    u_4 &= I_z \left( \ddot{z}_{\text{ref}} - K_{61} \dot{e}_x - K_{62} \dot{e}_y - x_z \dot{x}_z J_3 \right) \\
    x_5 &= \arcsin \left( \frac{-m_3 (e_x - e_y) + \sin \theta \cos \phi + \sin \phi \cos \rho}{\cos \phi \sin \xi} \right) \\
    x_4 &= \arccos \left( \frac{-m_3 (\dot{e}_x - \dot{e}_y) - \sin \theta \sin \phi + \sin \phi \cos \rho}{\cos \phi \sin \xi} \right)
\end{align*}\]

where \(K_{X1}, K_{X2}, K_{Y1}, K_{Y2}, K_{Z1}\) and \(K_{Z2}\) are the coefficients ensuring stability and positive constants. Where: \(l\) is the arm length of quadrotor, \((U_2, U_3, U_4)\) are controllers, \((I_{xx}, I_{yy}, I_{zz})\) are the moments of inertia of quadrotor, and \((\dot{\phi}, \dot{\theta}, \dot{\psi})\) are angular accelerations.

The four rotors are coupled two by two in order to counterbalance the gyroscopic effect so from the Figure 7 we can see that there are two pwm motors which began positive and attenuate until they vanish and the two others began negative and arise until they vanish this indicate that these two motors must rotate in opposite with the two others for the reason mentioned above.

4. SIMULATION RESULTS

To evaluate the designed control system, repetitive simulation tests were performed via numerical simulation. The control system was simulated using the variable step Runge-Kutta integrator in MATLAB. The airship is tested in simulation in order to validate motion planning algorithm considering the proposed Computed Torque controller.

Figures 8 and 10 show the tracking of desired trajectory by the simulated one and the evolution of the X4-flyer and its stabilisation in 3D displacement for the the straight circle and conical trajectories respectively. The simulation results of PD-CTC control with accurate parameters and without external disturbances and to confirm the robustness of proposed trajectory tracking control method. As illustrated in the above figures, the convergence to the trajectory is guaranteed after a transient behavior. It can be observed that the X4 platform tends to the target point precisely, which demonstrates that the proposed approach succeeds in station keeping control for the X4-flyer platform with minimum energy (see Figure 9).

Simulation results with the quadrotor helicopter starting with initial condition different from the reference trajectory (see Figure 11).

The results performance of this technique are discussed and compared to the results presented in [22, 23]. The simulation results showed that the quadrotor UAV can be stabilized at the desired with minimum energy in control effort. PD-CTC in this paper has shown better performance of X4.

![Figure 8. Result of position trajectories](image-url)
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Figure 9. Results of the controllers

Figure 10. Result of the conical trajectories

Figure 11. Result of the conical trajectories with initial conditions
5. CONCLUSION

The nonlinear computed torque control for a quadrotor is investigated in this paper. The simulation results demonstrate the X4 can stabilize at the desired with minimum energy in control effort. Further investigations are undergoing on robust controllers that would be able compensate noise effects and parameters mismatch, to be afterwards implemented on a real time experimental setup. Obstacles avoidance and flying multi-drone are also envisaged in a related research work.

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