Experimental and computational dynamic structural analysis of free-flight rockets

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Abstract. Aerospace vehicles’ designers often strive to approach the lightweight structures in order to improve flight performance without compromising vehicles durability. However, this goal may be achieved on the expense of structural rigidity of the vehicle. A missile with high slenderness ratio is a typical vehicle that should be considered as flexible body if precise dynamic behaviour analysis is sought. This analysis is crucial to identify the flight performance of such missile with high accuracy. The present paper discusses the results of a side-by-side experimental and numerical analysis of the vibration characteristics of a full-scale free-flight missile having 70mm caliber. The experimental modal analysis is conducted on the missile with empty motor to represent its unpowered flight regime. Experimental modal analysis setup involved accelerometer sensors while vibration excitation is achieved using an impact hammer. Modal analysis is also conducted numerically using a well-used high-fidelity commercial tool. Results of experimental modal analysis have shown that the first four modes as bending modes with frequencies of 134.4, 400.6, 819.4 and 1173.6 Hz, respectively. The results also demonstrate the close agreement between numerical and experimental approaches.

1. Introduction
Dispersion is one of the key issues related to unguided missiles that can degrade their accuracy and lethality and increase their undesired lateral impact. Enhancing dispersion is thus an essential practice for missile flight experts. One main source of dispersion is aeroelasticity and structural dynamic behavior of missiles during their flight which is more pronounced for those with high fineness (slenderness) ratios; understanding and predicting such missile features is one of the designers’ goals. Aeroelasticity is concerned with the coupled dependency between loads and deformation and involves three disciplines namely, elasticity, dynamics, and aerodynamics. Structural dynamic characteristics are fundamental aspects of aeroelasticity. This interaction of elastic and inertial forces that effects on the structure body, leads to mechanical vibrations. All structural bodies that have mass and elasticity are capable of free vibration in the absence of an external excitation. To better investigate structural vibration problems, the natural frequencies and mode shapes need to be studied. Moreover, exploring structural dynamics helps finding new useful ways to solve aeroelasticity and, hence, dispersion problems. Unguided air-to-surface missiles that are fired against stationary ground targets are vulnerable to dispersion. In addition, they are commonly of a high fineness ratio and unpowered flight dominates their trajectory. As a result of increasing slenderness ratio, the structure flexibility of a missile is increased and the vibration longitudinal and transverse natural frequencies are decreased. Accordingly, it is of a high importance to study the dynamic behavior of such “flexible” missiles to ensure the safety of flight and accurate arrival at the target. Modal analysis is a more and more paramount engineering tool that is used for representing structural dynamic behavior.
Modal analysis can be performed in many approaches namely, theoretically, experimentally, or numerically. The numerical (finite element) method is widely used in industry for producing a true engineering structure with an excellent representation. However, results of finite element method need to be validated against experimental counterparts. The coupling of both finite element method and experimental modal testing is widely used for determining dynamic characteristics of complicated engineering structure with maximum confidence.

In the literature, a considerable body of researches investigated the structural characteristics of missiles using the different approaches. For instance, Vernon et al. [1] studied the natural frequencies and mode shapes of the multistage launch vehicle. The first three natural frequencies and mode shapes of a full-scale four-stage launch vehicle were measured experimentally and calculated numerically. The results of the experimental and numerical analysis of the vibration characteristics of the launch vehicle were compared. Larry et al. [2] verified experimentally the numerically calculated vibration characteristics and natural and bending frequencies of complex launch vehicle structure. Similarly, Basant et al. [3] studied the vibration characteristics of a two-stage launch vehicle experimentally and numerically. The first three natural frequencies and mode shapes were calculated and compared with the experimental results. Robert and Theodore [4] studied experimentally the vibration characteristics of a launch vehicle. The natural frequencies, mode shapes and damping characteristics of this launch vehicle were measured in different conditions of flight. Platus [5] investigated numerically the effect of structural damping on dynamic behavior of hypersonic launch vehicle. The natural frequencies and mode shapes were calculated for the launch vehicle. A similar missile category was examined by Yun et al. [6], who investigated numerically dynamic behavior, natural frequencies, and mode shapes of hypersonic launch vehicle. Abbas et al. [7] calculated theoretically the natural vibration characteristics of a two-stage launch vehicle under variable axial load. The natural frequencies and mode shapes were calculated and compared with flight test results and numerical simulations [1]. Launch vehicles were also examined numerically in [8, 9]. The first five natural frequencies and bending mode shapes were deduced numerically by Li et al. [8] whereas Chen et al. [9] computed the first six natural frequencies and mode shapes theoretically and numerically.

Numerical finite element approach was adopted in a number of studies. Natural frequencies and mode shapes of a flexible launch vehicle models were examined by Rajan and Narasimhan [10] and Başkut and Akgül [11] whereas Tsushima et al. [12] studied the dynamic bending vibrations of such vehicles; the results were compared with experimental results in [4]. Recently, Li and Ye [13] investigated numerically the dynamic behavior of a flexible spinning missile.

In the present paper, modal analysis has been implemented on 70 mm missile. An experiment was carried out on the missile's structure without presence of propellant and explosive for safety to confirm the numerical calculation that were performed using ANSYS Workbench.

2. Case-study
The case study in concern is an unguided, fin-stabilized, free-flight air-to-surface missile with caliber 70 mm as shown in figure 1. The dummy empty missile has a total mass of 4.208 kg and 1.445 m total length (fineness ratio is 20.6). As illustrated in table 1, the missile is represented as a non-uniform beam consisting of five parts. Material properties for each part are listed in table 2.

| Table 1. Dimensions, masses, and materials of missile body sections. |
|-------------------------|-----------------|----------------|
| Part                    | Length [cm]     | Mass [kg]      |
| Nose tip                | 8.5             | 0.189          |
| Ogival warhead fairing  | 8.9             | 0.886          |
| Dummy warhead           | 27.7            | 0.717          |
| Motor case              | 86.8            | 1.102          |
| Nozzle                  | 12.6            | 1.314          |
Table 2. Density and mechanical properties of missile body material.

| Material     | Density [kg/m³] | Young’s modulus [GPa] | Poisson’s ratio |
|--------------|-----------------|-----------------------|-----------------|
| AL-1050      | 2710            | 71                    | 0.33            |
| Steel Alloy  | 7850            | 210                   | 0.30            |

Figure 1. Configuration of the case study model.

Figure 2. Locations of accelerometer sensors along the missile.

3. Methodology

3.1. Experiment setup and instruments
For experimental analysis, the missile body is horizontally hanged by using polyester ropes that allow one degree of freedom around its hang point. Experimental modal analysis of the missile is observed using accelerometers, impact hammer slightly excites the body vibration. Accelerometer sensors measure the local accelerations due to vibration and convert this mechanical motion into electrical signal. Vibration is excited by applying an impact for infinitely short duration to generate constant amplitude in the frequency domain yielding all modes of vibration. Five accelerometer sensors are fitted on the missile airframe at the positions as shown in figure 2.

Three accelerometer sensors (type PCB352C33) with voltage sensitivity (±10%): 100 mV/g [14] and two sensors (type PCB352C03) with voltage sensitivity (±10%): 10mV/g [15] are used. Accelerometers of the first type are fitted at positions 1, 3 and 5 as shown in figure 2. Frequency range for both types is (±10%): 0.3 to 15000 Hz. The used impact hammer is (type PCB086C03) with voltage sensitivity (±15%): 2.25 mV/N, constant current excitation: 2 to 20 mA, and Quartz sensing
element with Epoxy sealing [16]. Data acquisition is performed using NI (National Instruments) PXIe-4499 hardware along with NI Signal Express control software [17].

Firstly, one accelerometer sensor was used in the experiment. It was located in the middle of the missile. The NI Signal Express program was established for measuring ten times in one experiment. Then, the impact hammer was used to excite the missile for ten times at the same point in the nose part. After every strike, the experiment was held until the NI Signal Express reading data. Then, NI Signal Express was stopped automatically after the last one. Then, the output data were exported in an excel sheet from NI Signal Express. This experiment with the same settings was repeated five times. For every time data were exported to ensure the validity of the results of the experiment. The experimental work of the modal shape analysis and sample data is shown in figure 3.

Finally, the experiment was done but by using five accelerometer sensors. These five accelerometer sensors were placed at certain positions on the missile as shown in figure 2. This experiment had been done similarly. However, in this experiment, the output data were exported in Excel sheet for each accelerometer.

### 3.2. Numerical modal analysis

Finite element simulation approach is applied to the missile using a commercial software (ANSYS). An unstructured mesh is constructed on the structure with high smoothing. A grid sensitivity check is performed for five grid resolutions to enhance solution accuracy. In the sensitivity check, a free-free boundary condition is represented and with option twenty max modes to find because the first six calculated modes in ANSYS were taken as a rigid body. The results of the grid sensitivity check are shown in table 3. As a result, grid 4 is adopted in the present study.

**Table 3. Results of the grid sensitivity check.**

| Grid trail | No. of mesh elements | 1\(^{\text{st}}\) bending mode [Hz] | Change [%] |
|------------|----------------------|-----------------------------------|------------|
|            |                      | Y-direction                        | Z-direction |            |
| 1          | 30687                | 147.16                            | 148.26      | -----      |
| 2          | 71791                | 144.57                            | 145.37      | 1.76       |
| 3          | 89555                | 142.53                            | 142.53      | 1.41       |
| 4          | 280906               | 142.50                            | 142.50      | .02        |
| 5          | 1667890              | 142.47                            | 142.47      | .02        |

**Figure 3.** Missile modal analysis experiment setup.
4. Results
The first four natural frequencies and mode shapes were obtained for the missile with 70mm caliber numerically by using finite element model (ANSYS Workbench). Also, the natural frequencies of the same missile were obtained experimentally. The modal analysis was performed as a free-free body.

4.1 Experiment results
Through experiment procedure illustrated in subsection 3.1, the first four natural frequencies of bending mode are obtained. The experiment has been repeated five times, where corresponding results are listed in table 4.

4.2 Numerical results
The obtained first four natural frequencies of bending mode for the missile using ANSYS Workbench is shown in Table 5. The obtained mode shapes for a free-free boundary conditions of the same missile are shown in Figures 4, 5, 6 and 7. A comparison has been implemented between bending natural frequencies obtained with the experimental results as illustrated in table 5.

| Mode | EXPERIMENT [Hz] | NUMERICAL [Hz] | ERROR |
|------|-----------------|----------------|-------|
| 1    | 133             | 134            | 6%    |
| 2    | 397             | 400.6          | 9%    |
| 3    | 815             | 819.4          | 11%   |
| 4    | 1174            | 1173.6         | 22%   |

Table 4. The first four bending natural frequencies resultant from the five experiments.

Table 5. Comparison between numerical and experimental results for bending modes.

Figure 4. The 1$^{\text{st}}$ bending mode shape.
5. Conclusion

In present work, numerical and experimental modal analysis of the missile with 70mm caliber were performed in free-free boundary conditions. In the experiment modal analysis, it was not possible to determine the higher modes in a free-free boundary conditions of the missile using the impact hammer. The numerical and experimental results for the missile were found to have a highly good correlation as showed in table 5. Some errors were introduced between the results of the numerical and experimental modal analysis of the missile. After analyzing the results of overall modal analysis, it can
be concluded that the results are all acceptable within the affordable error margin. The presented techniques of modal analysis in this paper are extremely helpful in dynamic analysis, optimizing and developing of designing complex engineering structures and components.

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