Natural units, numbers and numerical clusters

F. Pisano and N. O. Reis
Department of Physics, Federal University of Paraná, Curitiba, PR 81531-990, Brazil

Defined by Lord Kelvin as the science of measurement it is described a fundamental fact of physics. The so called ‘natural’ units represent the unique system of units conveniently used in the realm of High Energy Physics. The system of natural units is defined by the consistent providing of the adimensional unit value to the velocity \( c \) of the electromagnetic radiation in the vacuum, related to the Maxwell classical electromagnetic theory and to the special relativity theory, to the reduced Planck constant \( \hbar \equiv \frac{\hbar}{2\pi} \) having the dimension of action, which is the fundamental constant of the quantum mechanics as well as to the Boltzmann constant \( k_B \) which has the dimension of heat capacity, being the fundamental constant associated to the statistical mechanics, \( c = \hbar = k_B = 1 \).

Such prescription is mandatory as we investigate the ultimate constituents of matter and their dynamics, since the atoms, leptons and quarks up to the grand unification, strings, superstrings, and the supergravity theories, as well as the total unification of the interactions in the M-theory. The one-dimensional character of this phenomenology is totally featured by the use of the natural units, reflecting the fact that in order to be possible the investigation of the fundamental constituents of matter it is being necessary the resolution of lengths of the order of \( 10^{-16} \) cm or to reach energies of the order of at least 1 TeV. The application of this system of natural units results exceptionally adequate in the description of the Universe since the Planck scale, up to the Hubble large scale expanding Universe, resulting in the curious formation of the regular numerical clusters.

06.20.Fn: Units and standards
06.30.-k: Measurements common to several branches of physics and astronomy
I. INTRODUCTION

The consolidation of specific systems of units of measurement was determined by factors related to the environment, as well as by historical and physiological factors. Such fact is possible to be verified for instance through the case of the units inch, yard, metrical foot, palm and so on. Considering such natural arbitrariness in the definition of the patterns of measurement it was always necessary to try to establish systems of simple and convenient units such as, for instance, the case of the m.k.s. system, the base of the international system of units, or even the case of the c.g.s. system. Naturally, the choice of such standards is arbitrary. Nevertheless, the m.k.s. or c.g.s. systems has been becoming commonly used in the realm of physics and engineering. In the High Energy Physics the system of natural units is the exceptionally consistent system not requiring the definitions of the usual patterns of measurement, generically used, being possible the definition of new fundamental patterns of measurement. The strongly singular but universal postulate used in this case consists in the fact that the unique things that can retain their identities in any place of the Universe are the elementary particles, leptons and quarks, as well as their interactions described by the corresponding dynamics given by the general relativity and by the standard model of the non-gravitational interactions, which consist in the weak and strong nuclear interactions of short range, $10^{-16}$ cm for the weak interaction and $10^{-13}$ cm for the strong interaction, and the long range electromagnetic interaction. In such physics where we are searching the fundamental laws of the Universe as a whole and the bits of matter, measurement is essential again, but the quantum and relativistic regimes imply, aside from any multicultural diversity, the simplest system of units, the ‘natural’ units.

II. THE PHYSICAL CONSTANTS AND THE NATURAL UNITS

As the description of the dynamics of the fundamental interactions was being elaborated it became a consolidated fact that the universal physical constants usually play the role of constants of proportionality between magnitudes in the equations of physics. The velocity of the electromagnetic radiation in the vacuum,

$$c = 299 792 458 \text{ m s}^{-1}$$

which is an exact value since the meter turn to be defined as the space covered by the light in the vacuum during the time of $1/299 792 458$ of 1 second, the reduced Planck constant

$$\hbar = \frac{\hbar}{2\pi} = \frac{6.626 068 72}{2\pi} \times 10^{-34} \text{ J s}$$

$$= 1.054 571 596 \times 10^{-34} \text{ J s}$$

and the Boltzmann constant,

$$k_B = 1.380 650 3 \times 10^{-23} \text{ J K}^{-1}$$

$$= 8.617 342 \times 10^{-5} \text{ eV K}^{-1},$$

define the system of natural units when are satisfied the dimensional relations

$$[c] = [\hbar] = [k_B] = 1$$

as well as, numerically, also

$$c = \hbar = k_B = 1.$$

These three universal physical constants are associated with the electromagnetic Maxwell theory, as well as with the special relativity, $c$, with the quantum mechanics, $\hbar$, and with the statistical mechanics, $k_B$, already appearing in the Boltzmann equation of calorimetry

$$\Delta Q = \Delta Q(\Theta) = k_B \Delta \Theta$$

which establishes the relation between the perceivable heat quantity $\Delta Q$ and the finite variation $\Delta \Theta$ of the absolute temperature $\Theta$. The Newton-Cavendish constant of universal gravitation, by its turn,

$$G = 6.672 599 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$$

is the constant appearing both in the Newtonian gravitation and in the general relativity.

Regarding the c.g.s. or m.k.s. systems the fundamental dimensions are length $[L]$, mass $[M]$, and time $[T]$ but taking the mass dimension $[M]$, action $[S]$, and velocity $[V]$ as fundamental dimensions, length and time dimensions are

$$[L] = \frac{[S]}{[M][V]}$$

$$[T] = \frac{[S]}{[M][V]^2}$$

and being $p$, $q$, $r$ real numbers the general dimensional relation

$$[M]^p [L]^q [T]^r = [M]^{p-q-r} [S]^{q+r} [V]^{-q-2r}$$

in the c.g.s. or m.k.s. units has the natural unit mass dimension $[M]^n$ with $n = p - q - r$. For action $p = 1$,

$^1$Notice the coincidence with the dimensional value $c = 1 \text{ ly} \times \text{yr}^{-1}$. 


mass natural unit as and we can write the dimension of the quantity with the conversion factor resulting in

\[ [A] = [M]^p [L]^q [T]^r \] (10)

and if we choose as fundamental unit the mass \([M]\) the natural units (NU) and the International System of units, SI, from the French terminology Système International d’Unités, of the \(A\) quantity are related according to

\[ [M]^\alpha = [A]_{\text{NU}} = [A]_{\text{SI}} [h]^{\frac{p}{2}} [c]^q \] (11)

where \([h] = [M] [L]^2 [T]^{-1}\) and \([c] = [M]^0 [L] [T]^{-1}\). Taking now \([A]\) written as in Eq. (10) with the \([h]\) and \([c]\) dimensional relations the Eq. (11) becomes

\[ [M]^\alpha = [M]^{p+x} [L]^{q+2x+y} [T]^{-x-y}. \] (12)

The values of \(\alpha, x,\) and \(y\) are obtained by realizing that two quantities are equal just when the potencies of the three fundamental units are the same. Therefore,

\[ [M]^\alpha = [M]^{p+x}, \]
\[ [L]^0 = [L]^{q+2x+y}, \]
\[ [T]^0 = [T]^{-x-y}, \] (13)

and to know \(\alpha, x\) and \(y\) in terms of the \(p, q,\) and \(r\) dimensional exponents it is just necessary to solve the system of algebraic equations,

\[ p + x = \alpha, \]
\[ q + 2x + y = 0, \]
\[ r - x - y = 0, \] (14)

resulting

\[ \alpha = p - q - r, \]
\[ x = - q - r, \]
\[ y = q + 2r, \] (15)

and we can write the dimension of the quantity \(A\) in a mass natural unit as

\[ [A]_{\text{NU}} = [M]^{p+q-r} \] (16a)

with the conversion factor

\[ [A]_{\text{NU}} = [A]_{\text{SI}} (h^{-p} c^{p-r}) \] (16b)

given in terms of the \(h\) and \(c\) universal constants.

The general procedure gives the following results,

\[ [A]_{\text{NU}} = [L]^{-p+q+r}, \] (17a)

when the fundamental unit is a length, and

\[ [A]_{\text{NU}} = [T]^{-p+q+r}, \] (18a)

\[ [A]_{\text{NU}} = [h^{-p} c^{2p-q}] \] (18b)

for the time as natural unit, which comply the linear algebra of fundamental units [6]. The conversion equations allows us to convert any physical quantity to NU, or back to the SI. The conversion is realized by setting explicitly the \([A]_{\text{SI}}\) as every SI unit which it contains. The dimensional pattern of some fundamental physical magnitudes in natural units is

\[ [E] = [M] = [\Theta] = [L]^{-1} = [T]^{-1} \] (19)

which can be verified in the following sequence,

(i) \([E] = [M] :\) it follows from the mass-energy relation \(E = mc^2;\)

(ii) \([E] = [\Theta] :\) take the energy-temperature Boltzmann equation, \(E(\Theta) = Q(\Theta) = k_B \Theta\) where \(\Theta\) is the absolute temperature;

(iii) \([E] = [L]^{-1} :\) from the energy-momentum relativistic equation for a massless particle, \(E = pc,\) where \(p = |p|\) is the scalar momentum, and the de Broglie relation \(\lambda = h/p,\) associating the wave character (\(\lambda\)) of the matter (\(p\)), it follows that \(\lambda = hc/E\) or \(E = hc/\lambda;\)

(iv) \([E] = [T]^{-1} :\) being \([c] = [L][T]^{-1} = 1\) in natural units, then from \([E] = [L]^{-1}\) we see that \([E] = [T]^{-1}.\)

All the diversity of the world [8] is found at different sizes, energy, temperature or any other magnitude. We find the nucleons and atoms from the Fermi length, 1 fermi \(\equiv 10^{-15}\) m, to the Angström scale, 1 Å \(\equiv 10^{-10}\) m, virus between \((20\ and\ 300) \times 10^{-9}\) m, bacteria and cellule at \(1\ \mu m.\) Atoms form molecules and there are molecules which contain hundreds of atoms such as proteins, the DNA and RNA extended up to several twelves of angströms. Today are synthetized molecules containing until \(10^4\) atoms. The human being is the most evolved structure, being perhaps the conscience of Universe [9]. It is composed by \(10^{28}\ \sim 10^{30}\) mol of atoms, being hydrogen, carbon, nitrogen, and oxygen the majority of them. Even if more than one hundred of chemical elements have been cataloged in the Mendeleev periodic classification just four of them are the essential ones to build up the basic molecular structures that allows life and intelligent life.

**III. THE NUMERICAL CLUSTERS**

With the fundamental constants \(c, G, h, k_B,\) and the electron mass

where 1 eV $\equiv 1.60217733 \times 10^{-19}$ joule, which is the lightest massive elementary fermion among all the elementary particles of the electroweak standard model, the mass of the proton

$$M_p^+ \simeq 938.27199 \text{ MeV } c^{-2}$$  \hspace{1cm} (21)

and of the neutron,

$$M_n^0 \simeq 939.56533 \text{ MeV } c^{-2}$$  \hspace{1cm} (22)

and the electron charge

$$e = -1.60217735 \times 10^{-19} \text{ coulomb}$$  \hspace{1cm} (23)

we can obtain adimensional numbers such as

$$\frac{M_n^0}{m_e^-} \simeq 1839, \quad \frac{M_p^+}{m_e^-} \simeq 1836$$  \hspace{1cm} (24)

or even the Sommerfeld fine structure constant

$$\alpha_{em} \equiv \frac{e^2}{\hbar c} \simeq \frac{1}{137}$$  \hspace{1cm} (25)

as well as the dimensional quantities such as the radius of the Bohr level of the Hydrogen atom

$$a_0 \equiv \frac{\hbar}{m_e^- c} \simeq 0.5 \times 10^{-8} \text{ cm} \equiv 0.5 \text{ Å}.$$  \hspace{1cm} (26)

Realize the absence of the universal constant of gravitation $G$ in these three last relations which cannot be combined with $h$ and $c$ to form a gravitational fine structure constant analogous to $\alpha_{em}$. The characteristic ondulatory length of the electron at velocities near to that of the light $c$, by its turn, is given by the reduced Compton wavelength

$$\lambda_e^- \equiv \frac{\lambda_c^-}{2\pi} = \frac{\hbar}{m_e^- c} = \alpha_{em} a_0 \simeq \frac{1}{137} a_0$$  \hspace{1cm} (27)

and when all energy of an electron, $m_e^- c^2$, is electrostatic potential energy kind, $e^2/a$, then

$$m_e^- c^2 = \frac{e^2}{a},$$  \hspace{1cm} (28)

resulting the electron classical radius,

$$a = \frac{e^2}{m_e^- c^2} \equiv 3 \times 10^{-13} \text{ cm} \simeq 3 \text{ fermi}$$  \hspace{1cm} (29)

When we build up numbers involving the constant $G$ it is possible to obtain numbers that are centered in a cluster of the order of $10^{40}$ as the ratio between the forces of the electrostatic and gravitational interactions among a proton and an electron,

$$\frac{F_{\text{electr}}}{F_{\text{grav}}} = \frac{e^2}{G M_{p^+} m_e^-} \simeq 10^{40}$$  \hspace{1cm} (30)

which is much larger than the Avogadro constant, $N_A = 6.02214199(47) \times 10^{23}$ mol$^{-1}$. In the atomic scale, electrostatic forces are dominant but in large scale such forces anulate each other and the gravitation becomes dominant. If a nucleon with the mass $M_n$ of almost 1 GeV is a black hole in rotation, of Schwarzschild radius, its radius is obtained by equalling the kinetic energy $\frac{1}{2} m v^2$, to the gravitational potential energy $G M_n/r$. Considering the limit $v = c$, the Schwarzschild radius is

$$r_S = 2 G M_n/c^2,$$  \hspace{1cm} (31)

corresponding to the gravitational lenght of a nucleon

$$a_g = \frac{G M_n}{c^2} \simeq 10^{-52} \text{ cm}$$  \hspace{1cm} (32)

in which we do not take into account the factor 2. If a nucleon of classical radius $a \equiv e^2/(m_e^- c^2)$ reaches the gravitational lenght $a_g$, the ratio between these lengths will be

$$\frac{a}{a_g} = \frac{e^2}{G M_n m_e^-} \simeq 0.2 \times 10^{40}$$  \hspace{1cm} (33)

that shows that a nucleon is much larger when compared to what it would be if it were collapsed in a Schwarzschild black hole. We classify two adimensional numerical groups. The first one near to the unit, such as $m_e^- /M_n \simeq 1/1836$ or $e^2/\hbar c \simeq 1/137$ and their inverses. Another cluster of adimensional numbers involve the constant of gravitation $G$, centered around $10^{40}$ as

$$\frac{e^2}{G M_n^2} = \frac{1}{1836} \times (0.2 \times 10^{40}),$$  \hspace{1cm} (34a)

$$\frac{\hbar c}{G M_n} = \frac{137}{1836} \times (0.2 \times 10^{40}),$$  \hspace{1cm} (34b)

$$\frac{e^2}{G M_n m_e^-} = 1 \times (0.2 \times 10^{40}),$$  \hspace{1cm} (34c)

$$\frac{\hbar c}{G M_n m_e^-} = 137 \times (0.2 \times 10^{40}),$$  \hspace{1cm} (34d)

$$\frac{e^2}{G m_e^-} = 1836 \times (0.2 \times 10^{40})$$  \hspace{1cm} (34e)

always proportional to $G^{-1}$. Let us denotate this cluster of numbers by $N_1$.

**IV. THE CHANGE OF UNIVERSAL PHYSICAL CONSTANTS**

With the scrutiny of the numerical clusters we have, this way, a cluster centered in $N_0 \sim 1$ being this last one centered in $N_1 \sim 10^{40}$. Considering the time variation $\frac{d}{dt}$ of the universal physical constants, the time functional behavior of the gravitation universal constant $G = G(t) \sim t^{-1}$, suggested for the first time by Dirac, \[10\][11].
it is possible to verify, sistematically, in this case that the unit group \( N_0 \sim 1 \) is not affected. However, we verify that all the elements of the cluster \( N_1 \) are shifted beneath \( N_0 \sim 10^{40} \). It is also considered the variation of the elementary electric charge\(^2\)\( e \), in such a manner that it diminishes with the expansion of the Universe. It is interesting to observe that the elements of the unit group \( N_0 \) as well as the ones of the cluster \( N_1 \) will spread-out.

V. THE HIGH ENERGY PHYSICS AND THE COSMOLOGY

On dimensional grounds it can be established the Planck scale, inaccessible to any conceivable experiment, 
Energy:
\[
E_{\text{Planck}} = \left( \frac{\hbar c^5}{G} \right) \frac{1}{2} \simeq 1.2 \times 10^{19} \text{ GeV}, \tag{35a}
\]
Mass:
\[
M_{\text{Planck}} = \left( \frac{\hbar c}{G} \right) \frac{1}{2} \simeq 2.1 \times 10^{-8} \text{ kg}, \tag{35b}
\]
Time:
\[
t_{\text{Planck}} = \left( \frac{\hbar G}{c^5} \right) \frac{1}{2} \simeq 5.4 \times 10^{-44} \text{ sec}, \tag{35c}
\]
Length:
\[
l_{\text{Planck}} = \left( \frac{\hbar G}{c^3} \right) \frac{1}{2} \simeq 1.6 \times 10^{-35} \text{ m}, \tag{35d}
\]
Density:
\[
\rho_{\text{Planck}} = \frac{c^5}{\hbar G^2} \simeq 5.1 \times 10^{96} \text{ kg m}^{-3} \tag{35e}
\]

Gravitation is closely connected to the Planck scale since \( E_{\text{Planck}} \) is the characteristic scale of gravity quantization \([12]\). However, the Planck scale is invisible to our highly selective detectors which are looking up to the kTeV or PeV scale, \([13]\) with the prefix ‘peta,’ \( P \equiv 10^{15} \), and very beyond in the Cosmic Ray Physics \([14,15]\) on the way of a systematic phenomenology to find new physics. The horizon of the observable Universe is determined by the Hubble length,

\[
L_{\text{Hubble}} = \frac{c}{H_0} \tag{37}
\]

where the universal constant \( H_0 = 100 h_0 \text{ km s}^{-1} \text{ Mpc}^{-1} \) is the Hubble present expansion rate of the Universe with the adimensional ignorance parameter \( \hbar_0 = (0.71 \pm 0.07) \times 1.15 \) and 1 pc (parsec) \( \simeq 3.262 \text{ ly} \simeq 3.086 \times 10^{16} \text{ m.} \)

In a energy natural unit
\[
H_0 = 2.13 h_0 \times 10^{-33} \text{ eV}. \tag{38}
\]

From the Planck time scale \( t_{\text{Planck}} \simeq 10^{-44} \text{ sec up to the Hubble time } \tag{16} \]
\[
t_{\text{Hubble}} = H_0^{-1} \simeq 10^{10} \text{ yr} \simeq 10^{17} \text{ sec} \tag{39}
\]

there are 61 orders of magnitude, the same as the spread of time in seconds to be scrutinized since the Big Bang to the age of the Universe. Taking the value \( L_{\text{Hubble}} \simeq 15 \times 10^9 \text{ ly,} \) in terms of the electron classical radius, \( a \), there is a third cluster,

\[
N_2 \equiv \frac{L_{\text{Hubble}}}{a} \simeq 5 \times 10^{40} \tag{40}
\]

coincident with the \( N_1 \) cluster. Such coincidence, \( N_1 = N_2 \), seems to indicate an unknown deep connection between the universal physical constants and the structure of the Universe. Perhaps such large numbers realize the \( N_1 = N_2 \) coincidence not so occasionally. The establishment of the correlation

\[
\text{numerical cluster } N_1 \longleftrightarrow \text{ elementary particles}
\]

and

\[
\text{numerical cluster } N_2 \longleftrightarrow \text{ large scale Universe},
\]

accordingly to the Dirac large numbers hypothesis, the \( N_1 \) and \( N_2 \) clusters ought to be connected by a simple mathematical relation. Particularly,

\[
N_1 = N_2 \equiv N. \tag{41}
\]

Thus, it is possible to verify the existence of a numerical series,

\[
N^0, N^{\pm 1}, N^{\pm 2}, \ldots, N^{\pm n/2}, \ldots, \tag{42}
\]

i.e., \( N^{\pm n/2} \) being \( n = 0, 1, 2, 3, 4, \ldots \), only, all along for the Universe, with \( N \) contained in the interval \( 10^{38} \leq N \leq 10^{42} \). Some numbers of such a series can be easily interpreted such, for instance,

\[
\text{density of a nucleon} : \text{density of the Universe} = \frac{N_2^2}{N_1^2} = N \tag{43}
\]

to the cluster \( N \),

\[
\text{Compton length of a nucleon} : \text{Planck length} = N_1^2, \tag{44}
\]

\^2As a matter of fact, the quantum of electric charge is the charge of the down-like quark flavors with \(-\frac{1}{3}|e| \) for a particle state.
or

\[
\frac{\text{Planck mass}}{\text{nucleon mass}} = \mathcal{N}_1^{\frac{1}{2}},
\]

(45)

for \(\mathcal{N}_1^{\frac{1}{2}}\), and

\[
\frac{\text{mass of the Universe}}{\text{Planck mass}} = \mathcal{N}_1^{\frac{3}{2}} \times \mathcal{N}_2 = \mathcal{N}_{\frac{3}{2}},
\]

(46)

or still,

\[
\frac{\text{extension of the Universe}}{\text{Planck length}} = \mathcal{N}_1^{\frac{1}{2}} \times \mathcal{N}_2 = \mathcal{N}_{\frac{1}{2}}
\]

(47)

for the cluster \(\mathcal{N}_{\frac{1}{2}}\), and finally,

\[
\text{total number of nucleons in the Universe} = \mathcal{N}_1 \times \mathcal{N}_2 = \mathcal{N}^2 \approx 10^{80}
\]

(48)

which is the Eddington number, the largest number of the whole physics together with the total number of photons, \(10^{89}\).

VI. CONCLUSIONS

The simple but meaningful algebraic combinations of the universal physical constants are grouped in regular numerical clusters building up a finite geometrical series with a step of \(10^{20}\) and opposite extremes \(10^{-80}\). Introducing the system of natural units in which \(c = \hbar = k_B = 1\), the High Energy Physics allows a one-dimensional system of units. Such inevitable one-dimensionality is the natural direct result of the Universe ultimate constituents search, according to Georgi “our pride and our curse” \[7\]. Admitting the inverse time variation of the gravitation universal constant, the whole cluster of numbers centered around the unit is not affected, even though the cluster centered in \(10^{40}\) is deviated below this scale. All clusters spread-out with the elementary electric charge reduction.

All surprises and the new physics islands must be in some specific places on a natural unit one-dimensional scale. Such islands contain an apparently inexhaustible source of new physics \[13\] such as the continuous constatations of the deterministic chaos \[14\] due to a hypersensible initial conditions dependence of the classical dynamical system, as well as the cellular automata possibility \[15\] and the uncertainty principle \[16\] even for classical systems \[17\] due to an irreducible limitation on the knowledge of the properties of the physical system.

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