Improving the precision of \(\gamma/\phi_3\) via CLEO-c Measurements

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Quantum correlations in \(\psi(3770) \rightarrow D^0 \bar{D}^0\) provide unique access to information about strong phase differences. Precision determination of the CKM phase \(\gamma/\phi_3\) via \(B \rightarrow D K\) decays depends upon constraints on charm mixing amplitudes, measurements of doubly-Cabibbo suppressed amplitudes and relative phases, and studies of correlated charmed meson decays tagged by flavor or CP eigenstates. CP-tagged \(D^0 \rightarrow K^- \pi^+ \pi^-\) decays and CP-tagged \(D^0 \rightarrow K^0_S \pi^+ \pi^-\) Dalitz plots are only available at CLEO-c. Using the 818 pb\(^{-1}\) CLEO-c data sample produced by the Cornell Electron Storage Ring (CESR) at \(\sqrt{s} = 3.77\) GeV, we perform analyses of these decays. We describe the techniques used to measure the \(D\)-decay parameters, and the CLEO-c impact on measurements of \(\gamma/\phi_3\).

1. INTRODUCTION

1.1. Measuring the CKM Phase \(\gamma\)

Precision measurements of the weak phases that compose the unitarity triangle, \(\alpha\), \(\beta\) and \(\gamma\), allow us to test the internal consistency of the Cabbibo-Kaboyashi-Maskawa (CKM) model and search for signatures of New Physics. The CKM phase \(\gamma\) is only constrained by direct measurements to \((67^{+32}_{-25})^\circ\) [3]. The most promising methods of determining the CKM phase \(\gamma\) exploit the interference within \(B^\pm \rightarrow DK^\pm\) decays, where the neutral \(D\) meson is a \(D^0\) or \(\bar{D}^0\). The most straightforward of these strategies considers two-body final states of the \(D\) meson, but additional information can be gained from strategies that consider multi-body final states. The parameters associated with the specific final states needed for these analyses can be extracted from correlations within CLEO-c [2] \(\psi(3770)\) data.

1.2. Determination of the CKM phase \(\gamma\) from \(B^\pm \rightarrow DK^\pm\)

The interference between decays of the type \(B^\pm \rightarrow DK^\pm\) provide a theoretically clean method for extracting the CKM phase \(\gamma\) when the \(D^0\) and \(\bar{D}^0\) mesons decay to a common final state, \(f_D\). For example, we may write the ratio of the amplitudes between the suppressed amplitude and the dominant amplitude as:

\[
\frac{A(B^- \rightarrow \bar{D}^0 K^-)}{A(B^- \rightarrow D^0 K^-)} = r_B e^{i(\delta_B - \gamma)},
\]

and we may write a similar ratio for \(B^+ \rightarrow DK^+\). The ratio of these amplitudes is a function of the ratio of the amplitudes’ absolute magnitudes \((r_B)\), a CP invariant strong phase difference \((\delta_B)\), and the CKM weak phase \(\gamma\). Due to color and CKM suppression, \(r_B \sim 0.1\) [3]; therefore, the interference is generally small. A variety of strategies exist, however, that attempt to resolve this and maximize the achievable sensitivity to \(\gamma\).

2. The ADS Formalism and \(D \rightarrow K^- \pi^+\)

Atwood, Dunietz and Soni (ADS) [3] have suggested considering \(D\) decays to non-CP eigenstates as a way of maximizing sensitivity to \(\gamma\). Final states such as \(K^- \pi^+\), which may arise from either a Cabibbo favored \(D^0\) decay or a doubly Cabibbo suppressed \(\bar{D}^0\) decay, can lead to large interference effects and hence provide particular sensitivity to \(\gamma\). This can be observed by considering the rates for the two possible \(B^-\) processes:

\[
\Gamma(B^- \rightarrow (K^- \pi^+)DK^-) \propto 1 + (r_B r_{DK}^{K^+})^2 + 2r_B r_{DK}^{K^+} \cos(\delta_B - \delta_{DK}^{K^+} - \gamma),
\]

\[
\Gamma(B^- \rightarrow (K^+ \pi^-)DK^-) \propto r_B^2 + (r_{DK}^{K^+})^2 + 2r_B r_{DK}^{K^+} \cos(\delta_B + \delta_{DK}^{K^+} - \gamma),
\]
where \( r_D^{K\pi} = \sqrt{(0.3342 \pm 0.0084)\%} \) parameterizes the relative suppression between \( A_{D^0} \) and \( A_{\bar{D}^0} \), and \( \delta_D^{K\pi} \), the relative strong phase difference. By considering the other two rates associated with the \( B^+ \) decay, and combining this with information from decays to the CP-eigenstates \( K^+K^- \) and \( \pi^+\pi^- \), an unambiguous determination of \( \gamma \) can be made. CLEO-c has recently measured \( \delta_D^{K\pi} \) to be \((22^{+14}_{-15})^\circ\) through a quantum correlated analysis of completely reconstructed \( \psi(3770) \to D\bar{D} \) decays \([3]\).

3. \( D \to K^-\pi^+\pi^+\pi^- \)

3.1. Multi-body Extension to the ADS Method

The ADS formalism can be extended by considering multi-body decays of the \( D \) meson. However, a multi-body \( D \)-decay amplitude is potentially different at any point within the decay phase space, because of the contribution of intermediate resonances. It is shown in Ref. \([6]\) how the rate equations for the two-body ADS method should be modified for use with multi-body final states. In the case of the \( B^- \) rates, for some inclusive final state \( f \), Eq. \((3)\) becomes:

\[
\Gamma(B^- \to (\bar{f}D)K^-) \propto A_f^2 \bar{A}_f^2 + r_B^2 A_f^2 + 2r_B R_f A_f \bar{A}_f \cos(\delta_B + \delta_D^f - \gamma),
\]

where \( R_f \), the coherence factor, and \( \delta_D^f \), the average strong phase difference, are defined as:

\[
A_f^2 = \int |A_{D^0}(x)|^2 \, dx, \quad \bar{A}_f^2 = \int |A_{\bar{D}^0}(x)|^2 \, dx,
\]

\[
R_f e^{i\delta_D^f} = \frac{\int |A_{D^0}(x)||A_{\bar{D}^0}(x)|e^{i\zeta(x)} \, dx}{A_f \bar{A}_f} \quad \{R_f \in \mathbb{R} \mid 0 \leq R_f \leq 1\},
\]

where \( x \) represents a point in multi-body phase space and \( \zeta(x) \) is the corresponding strong phase difference.

3.2. Determining \( R_f \) and \( \delta_D^f \) at CLEO-c

It has been shown in Ref. \([3]\) that, double-tagged \( D^0\bar{D}^0 \) rates measured at \( \psi(3770) \) threshold provide sensitivity to both the coherence factor, \( R_f \), and the average strong phase difference, \( \delta_D^f \). Starting with the anti-symmetric wavefunction \( \Psi \) of the \( \psi(3770) \) and then calculating the matrix element for the general case of two inclusive final states, \( F \) and \( G \), the double-tagged rate is found to be proportional to:

\[
\Gamma(F|G) \propto A_F \bar{A}_G^2 + \bar{A}_F^2 A_G^2 - 2R_F R_G A_F \bar{A}_F A_G \bar{A}_G \cos(\delta_D^F - \delta_D^G).
\]

From this, one finds three separate cases of interest for accessing both the coherence factor and the average strong phase difference. These results are summarized in Ref. \([3]\), where CLEO-c has provided a preliminary determination of \( R_{K\pi\pi} \) and \( \delta_D^{K\pi\pi} \) for the instance of \( F = K\pi\pi\pi \) using 818 \( pb^{-1} \) of data taken at the \( \psi(3770) \) resonance. The resulting constraints on the parameters \( R_{K\pi\pi} \) and \( \delta_D^{K\pi\pi} \) from these preliminary measurements are shown in Fig. \([3]\). It is apparent, from Fig. \([3]\) that the coherence across all phase space is low, reflecting the fact that many out of phase resonances contribute to the \( K\pi\pi\pi \) final state. An inclusive analysis of this final state with the ADS analysis will therefore have low sensitivity to the phase \( \gamma \), although the structure of Eq. \((4)\) makes it clear that such an analysis will allow for a determination of the amplitude ratio \( r_B \), which is a very important auxiliary parameter in the \( \gamma \) measurement.

Shown in Figure \([2]\) are projections of the overall systematic uncertainty on \( \gamma \) at LHCb \([9]\). The figure demonstrates how the overall systematic uncertainty on \( \gamma \) improves as additional information from CLEO-c is used in concert with expected LHCb data samples documented in Reference \([10]\).
Figure 1: (Preliminary) resulting limits on $R_{K^{3}\pi}$ and $\delta_{D}^{K^{3}\pi}$ at 1$\sigma$, 2$\sigma$ and 3$\sigma$ levels.

Figure 2: Projections of the overall systematic uncertainty on $\gamma$ at LHCb, estimated for various values of of $\delta_{D}^{K^{3}\pi}$.

4. $D \to K_{S}^{0}\pi^{+}\pi^{-}$

Dalitz plot analyses of the three-body decay $D \to K_{S}^{0}\pi^{+}\pi^{-}$ together with studies of $B^{\pm} \rightarrow DK^{\pm}$ processes currently provide the best measurements of the CKM weak phase $\gamma$ 11, 12. However, $D \rightarrow K_{S}^{0}\pi^{+}\pi^{-}$ Dalitz analyses are sensitive to the choice of the model used to describe the three-body decay, which currently introduces a model systematic uncertainty on the determination of $\gamma$ which is greater than 5° 11. For LHCb and future Super-B factories, this uncertainty will become a major limitation. A model independent approach to understanding the $D$ decay has been proposed by Giri and further investigated by Bondar 13, which takes advantage of the quantum correlated $D^{0}/\bar{D}^{0}$ CLEO-c data produced at the $\psi(3770)$ resonance.

Consider a Dalitz plot in which we define $x = m_{K_{S}\pi^{-}}^{2}$ and $y = m_{K_{S}\pi^{+}}^{2}$. Both $D^{0} \rightarrow K_{S}^{0}\pi^{+}\pi^{-}$ and $\bar{D}^{0} \rightarrow K_{S}^{0}\pi^{+}\pi^{-}$


decays appear on this plot. We then divide the Dalitz plot into regions which are expected to have about the same relative strong phase difference between the $D^0$ and $\bar{D}^0$ decays, based on the $D^0 \rightarrow K_S^0\pi^+\pi^-$ decay model from BaBar [14], as shown in Figure 3. Assuming the amplitude for the $D^0 \rightarrow K_S^0\pi^+\pi^-$ process is $f_D(x,y)$, we can define the bin-averaged cosine, $c_i$, and bin-averaged sine, $s_i$, for each bin $i$ as follows:

\[ f_D(x,y) = |f_D(x,y)|e^{i\delta_D(x,y)} \]  
\[ c_i = \frac{1}{\sqrt{F_iF_\bar{i}}} \int_{D_i} |f_D(x,y)||f_D(y,x)|\cos(\delta_{x,y} - \delta_{y,x})dxdy \]  
\[ s_i = \frac{1}{\sqrt{F_iF_\bar{i}}} \int_{D_i} |f_D(x,y)||f_D(y,x)|\sin(\delta_{x,y} - \delta_{y,x})dxdy \]

Using the 818 pb$^{-1}$ $\psi(3770) \rightarrow D^0\bar{D}^0$ data sample collected by CLEO-c, we can measure the strong phase parameters, $c_i$ and $s_i$, using fully reconstructed $D^0\bar{D}^0$ pairs with $K_S^0\pi^+\pi^-$ vs. flavor states, CP eigenstates, and double $K_S^0\pi^+\pi^-$ samples.

We may create a CP-tagged sample $K_S^0\pi^+\pi^-$ events by requiring the neutral $D$ which does not decay to $K_S^0\pi^+\pi^-$ to decay to states of definite CP ($\pi^+\pi^-$, $K^+K^-$, $K_S^0\pi^0\pi^0$, $K_L^0\pi^0$, $K_S^0\pi^0$, $K_S^0\eta$, and $K_S^0\omega$). The CP-tagged $K_S^0\pi^+\pi^-$ sample only allows us to measure $c_i$, and not $s_i$, in each bin. It can be shown that the bin averaged cosine in each
of these bins is:

\[ c_i = \frac{(M_i^+/S_+ - M_i^-/S_-)}{(M_i^+/S_+ + M_i^-/S_-)} \frac{(K_j + K_i)}{2\sqrt{K_i K_j}} \]  

(11)

where \( M_i^+ (M_i^-) \) is the number of CP even(odd)-tagged \( K_0^{3/2} \pi^+ \pi^- \) events in each bin and \( K_i (K_j) \) is the number of flavor tagged \( K_0^{3/2} \pi^+ \pi^- \) events in each bin. In our analysis, we use hadronic flavor-tags (\( K^- \pi^+ \), \( K^- \pi^0 \), and \( K^- \pi^+ \pi^- \)), for which doubly-Cabbibo suppressed decays are considered in evaluation of the systematic error. There are \( \sim 800 \) CP-tagged events in the sample we use to determine \( c_i \) in each bin, which are shown in Figure 4.

Using the \( K_0^{3/2} \pi^+ \pi^- \) vs. \( K_0^{3/2} \pi^+ \pi^- \) sample, one can extract \( c_i \) and \( s_i \) simultaneously. The number of double-tagged events \( M_{i,j} \) can be related to the number of flavor-tags for each \( D \) decay:

\[ M_{i,j} = \frac{1}{2N_{D,D} B_f^2} (K_i K_j + K_j K_i - 2\sqrt{K_i K_j} K_i K_j (c_i c_j + s_i s_j)) \]  

(12)

where \( B_f \) is the branching ratio of \( K_0^{3/2} \pi^+ \pi^- \), and \( N_{D,D} \) is the number of \( \psi(3770) \) decays, assuming 100\% efficiency. There are \( \sim 450 \) \( K_0^{3/2} \pi^+ \pi^- \) vs. \( K_0^{3/2} \pi^+ \pi^- \) events in the sample of \( K_0^{3/2} \pi^+ \pi^- \) vs. \( K_0^{3/2} \pi^+ \pi^- \) events.

The latest preliminary CLEO results for \( c_i \) and \( s_i \) from both \( K_0^{3/2} \pi^+ \pi^- \) CP-Tags and \( K_0^{3/2} \pi^+ \pi^- \) vs. \( K_0^{3/2} \pi^+ \pi^- \) events are shown in Table 4.

With the measurements presented here, the systematic uncertainty resulting from our understanding of the \( D \) decays is lowered to \( \sim 2\% \), which is calculated using the methods reported in Reference [15].
Table I: Preliminary CLEO results for $c_i$ and $s_i$ with respect to a particular type of tag.

|   | $c_i$ ($K^0_S\pi^+\pi^-$ vs. CP-Tags) | $c_i$ ($K^0_S\pi^+\pi^-$ vs. $K^0_S\pi^+\pi^-$) | $s_i$ ($K^0_S\pi^+\pi^-$ vs. $K^0_S\pi^+\pi^-$) |
|---|---------------------------------|---------------------------------|---------------------------------|
| 1 | $0.706 \pm 0.069 \pm 0.028$    | $0.779 \pm 0.087 \pm 0.062$    | $0.380 \pm 0.179 \pm 0.085$    |
| 2 | $0.586 \pm 0.126 \pm 0.037$    | $0.874 \pm 0.120 \pm 0.113$    | $0.137 \pm 0.260 \pm 0.084$    |
| 3 | $0.041 \pm 0.120 \pm 0.043$    | $0.003 \pm 0.166 \pm 0.152$    | $0.749 \pm 0.145 \pm 0.053$    |
| 4 | $-0.510 \pm 0.178 \pm 0.074$  | $-0.165 \pm 0.323 \pm 0.152$  | $0.490 \pm 0.400 \pm 0.093$    |
| 5 | $-0.949 \pm 0.063 \pm 0.029$  | $-0.929 \pm 0.058 \pm 0.044$  | $0.141 \pm 0.268 \pm 0.085$    |
| 6 | $-0.807 \pm 0.108 \pm 0.039$  | $-0.472 \pm 0.196 \pm 0.099$  | $-0.679 \pm 0.203 \pm 0.059$   |
| 7 | $0.085 \pm 0.154 \pm 0.046$   | $0.459 \pm 0.204 \pm 0.170$   | $-0.558 \pm 0.367 \pm 0.106$   |
| 8 | $0.339 \pm 0.082 \pm 0.024$   | $0.526 \pm 0.109 \pm 0.114$   | $-0.376 \pm 0.169 \pm 0.060$   |

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