I. INTRODUCTION

Current candidates for the fundamental theory of particle physics such as string theory or M-theory are all defined as a theory in higher dimension. To obtain an appropriate 4-dimensional effective theory starting with such a theory, a certain dimensional reduction is necessary. One well-known scheme of dimensional reduction is the Kaluza-Klein compactification. In this scheme, the size of the extra dimension is supposed to be very small so as not to excite the modes which have momentum in the direction of the extra dimension. This scheme seems to work well as a mechanism to shield the effect of extra dimensions. This Kaluza-Klein scheme, however, is not a unique possible scheme for dimensional reduction. Recently, the braneworld scenario has been attracting a lot of attention as an alternative possibility.

The essential feature of the braneworld scenario distinct from the ordinary Kaluza-Klein compactification is that the matter fields of the standard model are supposed to be localized on the brane, while the graviton can propagate in a higher dimensional spacetime which we call “bulk”. Owing to the assumption that the ordinary matter fields are localized on the brane the braneworld models can be consistent with the particle physics experiments even if the length scale of the extra dimension is not extremely small. Then, the gravity is possibly altered at a rather macroscopic length scale, while the experimental constraint on the deviation of the gravitational force from the Newton’s law obtained so far is not severe below sub mm scale. Hence, a characteristic length scale can be as large as sub mm scale in the braneworld scenario. Therefore future experiments may detect an evidence of the existence of extra dimensions.

In the course of studies on braneworld, a new scenario was proposed by Randall and Sundrum (RS) In their models, the gravity is effectively localized due to the warped compactification even though the extension of the extra dimension is infinite. So far any results significantly distinguishable from the 4D general relativity have not been reported.

In this paper, we first review the current status of the studies on the gravity in this model. In §2 we explain the setup of the RS model with infinite extra dimension. In §3 we review the geometrical approach to the gravity in this model to find the limitation of the analysis without solving the 5D equations of motion. In §4 and §5 we summarize the results for linear perturbations and for nonlinear perturbations of this model, respectively. Even in the non-linear regime, 4D general relativity seems to be recovered. However, black hole solutions as a symbol of strong gravity have not been found yet. In §6 we discuss the possibility that there is no static black hole solution in this model, applying the argument of the AdS/CFT correspondence. In §7 we discuss the importance of the conjectured possibility. If the conjecture is true, the existence of long-lived solar mass black holes will give the strongest constraint on the bulk curvature radius. At the same time, we can propose a new method to simulate the evaporation of a 4D black hole due to the Hawking radiation as a 5D process.

II. WARPED EXTRA DIMENSION

In the model proposed by Randall and Sundrum, 5D Einstein gravity with negative cosmological constant $\Lambda$ is assumed. The ordinary matter fields are confined on a 4-dimensional object called “brane”. This brane has positive tension $\sigma$, and the spacetime has reflection symmetry ($Z_2$-symmetry) at the position of the brane, $y = y_b$. Here $y$ is the coordinate normal to the brane. 5D Einstein equations are

$$(5)G_{ab} = -\Lambda g_{ab} + 8\pi G_5 S_{ab}\delta(y - y_b), \quad (2.1)$$

with

$$S_{ab} = -\sigma \gamma_{ab} + T_{ab}, \quad (2.2)$$

where $T_{ab}$ is the energy momentum tensor of the matter field localized on the brane, $\gamma_{ab}$ is the 4D metric induced on the brane, and $G_5$ is the 5D Newton’s constant.

One solution of (2.1) is 5D anti-de Sitter (AdS) space

$$ds^2 = dy^2 + e^{-2|y|/\ell}(-dt^2 + dx^2), \quad (2.3)$$

with a single positive tension brane located at $y = 0$. Here, $\ell$ is the curvature radius of 5-dimensional AdS.
space. The five dimensional cosmological constant and the brane tension are set to $\Lambda = -6/\ell^2$ and $\sigma = 3/(4\pi G_5 \ell)$. It is convenient to introduce conformal coordinates defined by $z = \text{sign}(y) \ell(e^{|y|}/\ell - 1)$. In these coordinates the metric is expressed as

$$ds^2 = \frac{\ell}{(|y| + \ell)^2} (d\tau^2 + \eta_{\mu\nu} dx^\mu dx^\nu),$$

(2.4)

where $\eta_{\mu\nu}$ is the 4D Minkowski metric. The outstanding feature of this model is that the 4D general relativity is seemingly reproduced as an effective theory on the brane in spite of the fact that the extension in the direction of the extra dimension is infinite.

### III. GEOMETRICAL APPROACH

A quick way to see why the 4D general relativity is expected to be realized on the brane will be the geometrical approach introduced by Shiromizu, Maeda and Sasaki. We use the 4+1 decomposition of the 5D Einstein tensor. The components parallel to the brane are decomposed by the Gauss equation as

$$(4)G_{\mu\nu} = (5)G_{\mu\nu} + (5)R_{\mu
u\gamma\lambda} + KK_{\mu\nu} - K_{\mu}^\rho K_{\rho\nu} - \frac{1}{2} \gamma_{\mu\nu}(K^2 - K^{\alpha\beta}K_{\alpha\beta}) - (5)R_{\mu\nu\gamma\lambda} ,$$

(3.1)

where $K_{\mu\nu}$ is the extrinsic curvature tensor of the $y=$constant hypersurfaces. Taking account of $Z_2$ symmetry at $y=0$, the Israel’s junction condition gives

$$K_{\mu\nu}(y = \pm \ell) = -4\pi G_5 \left( S_{\mu\nu} - \frac{1}{3} \gamma_{\mu\nu} S \right).$$

(3.2)

Substituting the above two equations into Eq. (2.4), we obtain

$$(4)G_{\mu\nu} = 8\pi G_4 T_{\mu\nu} + (8\pi G_5)^2 \pi_{\mu\nu} - E_{\mu\nu},$$

(3.3)

where the 4D effective Newton’s constant is given by

$$G_4 = \frac{4\pi G_5^2 \sigma}{3} = \frac{G_5}{\ell},$$

(3.4)

and $E_{\mu\nu}$ is a projected Weyl tensor defined by

$$E_{\mu\nu} = (5)C_{\mu\nu\gamma\lambda} .$$

(3.5)

$\pi_{\mu\nu}$ is a tensor quadratic in $T_{\mu\nu}$ whose explicit form is given in Ref. [3]. By construction $E_{\mu\nu}$ is traceless. From the 5D conservation law for the localized matter fields, the 4D effective conservation law for the matter fields $^{(4)}D_\nu T_{\mu}^{\nu} = 0$ follows, which also implies

$$(4)D_\nu E_{\mu}^{\nu} = (8\pi G_5)^2 \pi_{\mu}^{\nu} ,$$

(3.6)

owing to the Bianchi identity.

If we can neglect the last two terms in Eq. (3.3), the dynamics of 4D general relativity is recovered. The order of magnitude of the $\pi_{\mu\nu}$ term is easily evaluated to be smaller by the factor of $T_{\mu\nu}/\sigma$ than the first term, $8\pi G_5 T_{\mu\nu}$. Therefore we can neglect the $\pi_{\mu\nu}$ term at a low energy. On the other hand, the equation (3.6) is not sufficient to determine the evolution of $E_{\mu\nu}$ in general. The equations that determine $E_{\mu\nu}$ are essentially 5 dimensional and are not obtained in the form of a 4D effective theory.

### IV. LINEAR PERTURBATION

As we have reviewed in the previous section, we need to solve a 5D equation in order to fully determine the evolution of the metric induced on the brane. Since solving 5D equation in general is not easy, we consider linear perturbations of the RS model. To discuss metric perturbations in the bulk, the RS gauge is convenient. In this gauge, $y$-components of metric perturbations are set to be zero; $h_{\mu\alpha} = 0$, and also $h_{\mu\nu}$ satisfies the transverse and traceless conditions; $h^{\mu\nu} = h^{\nu\mu} = 0$. The homogeneous equations for bulk metric perturbations become

$$\left[ -\partial_z^2 + V(z) \right] \psi_{\mu\nu} = \eta^{\rho\sigma} \partial_\rho \partial_\sigma \psi_{\mu\nu} ,$$

(4.1)

where $\psi_{\mu\nu} = \sqrt{|z| + \ell} h_{\mu\nu}$ and

$$V(z) = \frac{15}{4(|z| + \ell)^2} - 3\ell^{-1} \delta(z) .$$

(4.2)

The solution of Eq. (4.1) can be found in the form of $u_m(z) e^{ikz} x^\nu$. The separation constant $m^2 = -k^2 + \ell k^\mu k^\mu$ can be understood as the mass of the effective 4D field which corresponds to the mode $u_m(z)$. The equation that $u_m(z)$ satisfies is $\left[ -\partial_z^2 + V(z) \right] u_m(z) = m^2 u_m(z)$. The solution of this equation with the $Z_2$-symmetry is $u_m(z) = N_m \sqrt{|z| + \ell} J_1(m\ell) Y_2(m(|z| + \ell)) - Y_1(m\ell) J_2(m(|z| + \ell))$. The normalization $N_m$ determined so as to satisfy $\int_{|z|}^\infty u_m(z) u_m(z) dz = \delta(m - m')$ is given by $N_m = \sqrt{m/2} / \sqrt{J_1(m\ell)^2 + Y_1(m\ell)^2}$.

The basic feature of these wave functions can be understood without resorting to the explicit form of the solution. Note that the 5D metric given in Eq. (3.4) satisfies 5D Einstein equations even if we replace the Minkowski metric $\eta_{\mu\nu} dx^\mu dx^\nu$ in it with any vacuum solution of 4D Einstein equations. Corresponding to this type of solutions, there is a discrete mass spectrum at $m = 0$ with the wave function $h_{\mu\nu} \propto 1/z^2$. We call it zero mode. This zero mode wave function is nodeless. Hence, there is no bound state for $m^2 < 0$. On the other hand, the potential $V(z)$ goes to zero at $|z| \to \infty$. Hence, the mass spectrum is continuous for $m^2 > 0$. The potential $V(z)$ has barrier near the brane with the height of $O(\ell^{-2})$. For the modes with $0 < m \lesssim \ell^{-1}$, the wave function is suppressed near the brane due to this potential barrier. For
the modes with \( m \gtrsim \ell^{-1} \), their excitation is kinematically suppressed. Therefore the zero mode is the only active degrees of freedom, and it is a massless and spin-2 field in the language of the 4D effective theory. Hence, 4D general relativity is expected to be recovered at least at the linear level.

Linear metric perturbations induced on the brane were first explicitly evaluated in Ref. [6]. The result is summarized as

\[
\eta_{\mu\nu} = -16\pi G_5 \int d^4x' G(x, x') \left( T_{\mu\nu} - \frac{1}{3} \eta_{\mu\nu} T \right) + \frac{8\pi G_5 \ell^{-1}}{3} \gamma_{\mu\nu}^{(4)} \Box^{-1} T,
\]

(4.3)

where

\[
G(x, x') = -\int \frac{d^4k}{(2\pi)^4} e^{ik_\mu (x^n - x'^n)} \left[ \frac{z^{-2} z^{-2} \ell^{-1}}{k^2 - (\omega + i\epsilon)^2} + \int_0^\infty dm \frac{u_m(y) u_m(y')}{m^2 + k^2 - (\omega + i\epsilon)^2} \right],
\]

(4.4)

is the 5D scalar Green function. If we assume static and spherically symmetric configuration for the matter source localized on the brane, the gravitational field outside the matter distribution is evaluated as [7].

\[
h_{00} \approx \frac{2G_4 M}{r} \left( 1 + \frac{2\ell^2}{3r^2} \right),
\]

(5.1)

\[
h_{ij} \approx \frac{2G_4 M}{r} \left( 1 + \frac{\ell^2}{3r^2} \right).
\]

(5.2)

The correction to 4D general relativity is suppressed by the ratio between the 5D curvature scale \( \ell \) and the distance from the center of the star \( r \). The correction to the gravitational field inside the star also stays small by the factor of \( O(\ell^2/r_*^2) \), where \( r_* \) is the typical size of the star. If we neglect the contribution due to massive modes \( (m^2 > 0) \) in the Green function [14], Eq. (1.3) exactly reduces to the results for the linearized 4D general relativity.

V. NON-LINEAR PERTURBATION

At the linear level, perturbations of the RS model can be expressed as a 4D effective theory with an infinite tower of massive gravitons [13]. However, we will notice that the asymptotic behavior of the wave function at large \( z \) is not very regular. The zero mode wave function behaves as

\[
h_{\mu\nu}(\text{zero mode}) \approx \frac{1}{z^2}.
\]

An invariant obtained by contracting the Weyl tensor with itself \( C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \) behaves as \( \approx z^4 \). Thus, the infinity in the direction of the extra dimension is a curvature singularity. For the massive modes the situation is worse. The wave function behaves as

\[
h_{\mu\nu}(\text{massive mode}) \approx \frac{1}{\sqrt{z}}.
\]

Hence, the same invariant more severely diverges as \( C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \approx z^7 \).

However, such a divergence does not indicate the breakdown of the perturbation analysis. The perturbed metric induced by the matter fields on the brane consists of a superposition of various modes. For the static case, the 5D Green function is approximately evaluated in Ref. [8]. The result clearly showed that perturbations at large \( z \) are regular. Also in the dynamical cases, the asymptotic regularity of perturbations was shown in Ref. [8]. An interesting point which we wish to stress here is that perturbations become regular only after summation over all massless and massive modes.

Non-linear perturbations in this model are more complicated. If we adopt the picture of the 4D effective theory with a tower of massive gravitons, one may think that the higher order perturbations can be treated by taking into account the effective coupling between gravitons with various masses. However, this approach does not work. Let us consider the three point interaction vertex. The effective coupling constant between various massive gravitons will be obtained by expanding the action \( (\propto R) \) to the third order with respect to the metric perturbation \( h_{ab} \) and integrating out the dependence on the extra dimension. The quantity to be calculated will take the form

\[
\int d^4x \int_0^\infty dz \sqrt{-g} \sum g^{**} g^{**} g^{**} h_{***} h_{***} h_{***}
\]

(5.3)

The asymptotic behavior of respective component is given by \( \sqrt{-g} \sim z^{-5} \) and \( g^{**} \sim z^2 \). If we substitute the massive mode wave function, \( h_{**} \) and also \( h_{**} \) are \( \sim z^{-1/2} \). Hence, the integrand of (5.3) behaves as \( \propto z^{3/2} \), and the \( z \)-integration does not converge. Therefore we cannot define the effective coupling constant in this manner.

Nevertheless, this does not directly imply any catastrophe at least at the classical level. For the static and spherically symmetric configurations in the 4D sense, second order perturbations were calculated to show that the perturbations behave well, and the correction to 4D general relativity is suppressed by the factor of \( O(\ell^2/r_*^2) \) [13]. The approximate reproduction of the results for 4D general relativity is also confirmed numerically in Ref. [11], in which the strong gravity regime was also investigated. Hence, one may be able to conclude that the gravity in the RS infinite braneworld is well approximated by

\*
\*There is an alternative way of describing the model as a 4D higher derivative theory [13].
4D general relativity, although non-linear perturbations have not been computed in dynamical cases at all \[\text{[12]}\].

VI. BLACK HOLE AND ADS/CFT CORRESPONDENCE

In the preceding section we have observed that the induced metric on the RS brane mimics the results of 4D general relativity well. This seems to work even in the strong gravity regime. However, no black hole solution which is asymptotically AdS has been found so far.

In Ref. \[\text{[13]}\], a black string solution given by

\[
n s^2 = \frac{\ell^2}{(|z| + \ell)^2} \left[ dz^2 + q^{(4)}_{\mu\nu} dx^\mu dx^\nu \right],
\]

was discussed. Here \(q^{(4)}_{\mu\nu}\) is the usual 4D Schwarzschild metric. The induced geometry on the brane at \(z = 0\) is exactly 4D Schwarzschild spacetime. However, the asymptotic value of \(C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\) behaves as \(z^4 r^{-6}\), where \(r\) is the Schwarzschild radial coordinate. If we take the \(z \to \infty\) limit for a fixed \(r\), this curvature invariant diverges \[\text{[13]}\]. Also, this configuration is unstable \[\text{[14]}\]. Hence, the black string solution will not be an appropriate candidate for the final state of the gravitational collapse in the RS braneworld. A conjecture raised in Ref. \[\text{[13]}\] is that there will be a configuration called “black cigar” for which there is an event horizon localized near the brane.

However, as we have mentioned above, no black hole solution which is asymptotically AdS has not been found so far, although there were several works aiming at finding it. Here we suggest that the brane black hole may not exist, based on the argument of AdS/CFT correspondence. For an introduction to AdS/CFT correspondence, we follow Refs. \[\text{[17,18]}\]. Here we consider the RS model without matter fields on the brane. The AdS/CFT correspondence implies the relation

\[
S_{\text{RS}} = S_{\text{EH}}^{(4)} + 2W_{\text{CFT}},
\]

where \(W_{\text{CFT}}\) is the connected Green function with a high frequency cutoff for certain 4D CFT fields evaluated on the background metric induced on the brane, and

\[
S_{\text{EH}}^{(4)} = -\frac{\ell}{16\pi G_5} \int d^4 x \sqrt{-q^{(4)}} R,
\]

is the ordinary 4D Einstein-Hilbert action for the induced metric on the brane. This formula indicates that the RS infinite braneworld is equivalently described by 4D general relativity coupled to conformal fields. The number of degrees of freedom of the conformal fields is \(O(\ell^4 G_4)\), which is supposed to be large.

It is known that the quantum effect by means of 4D CFT corresponds to the classical effect due to the bulk graviton in 5D picture. This fact can be also understood in the following way. Let us consider the energy momentum tensor of the 4D CFT due to the vacuum polarization effect induced by the curved geometry. In this case, the contribution to the energy momentum tensor from each field will be \(O(1/L^4)\), where \(L\) is the characteristic length scale of the spacetime curvature. Thus, the vacuum polarization part of the energy momentum tensor \((4)^{4}T_{\mu\nu}^{(Q)}\) in total will become \(O(\ell^2/G_4 L^4)\), where we have taken into account the number of degrees of freedom. Hence, from 4D Einstein equations \((4)^{\Box} h_{\mu\nu} \approx G_4 (4)^{T_{\mu\nu}}\), the additional metric perturbation caused by this energy momentum tensor \((4)^{T_{\mu\nu}^{(Q)}}\) is estimated as \(h_{\mu\nu}^{(Q)} = O(\ell^2/L^2)\). On the other hand, the effective energy momentum tensor induced by the quantum effect in the 5D point of view is also given by the curvature scale of the spacetime. When we discuss in 5D picture, there are two characteristic length scales, \(\ell\) and \(L\). We denote both of them by \(\tilde{L}\) without distinguishing them. Then, we will have \((5)^{\Box} h_{\mu\nu} \approx \tilde{L}^{-5}\). Then, from 5D Einstein equations, \((5)^{\Box} h_{\mu\nu} \approx G_5 (5)^{T_{\mu\nu}}\), we will obtain \(h_{\mu\nu}^{(5)} = O(\tilde{L}^{-3} G_5) = O(\tilde{L}^{-2} G_4)\). Note that the number of degrees of freedom is a few in this case. Since the power of \(G_4\) does not coincide in the above two expressions for \(h_{\mu\nu}^{(Q)}\), it is almost impossible to expect that the contribution due to the quantum effect in 5D picture corresponds to that in 4D CFT picture. This mismatch comes from the large number of CFT fields of \(O(\ell^2/G_4)\).

The correspondence at the level of linear perturbation was made explicit in Ref. \[\text{[19]}\]. The leading correction to 4D general relativity starts with \(O(\ell^2/L^2)\) in \(h_{\mu\nu}\), and this leading term was shown to satisfy the correspondence relation. The possible error of the correspondence is higher order in \(\ell\).

It is very difficult to verify the correspondence beyond the linear perturbation. However, there is a supporting evidence related to the trace anomaly. Here we follow Ref. \[\text{[17]}\]. The trace of the effective Einstein equations \((3.3)\) becomes

\[
(4)^{G} = 8\pi G_4 T + \frac{(8\pi G_4 \ell^2)^2}{4} \left( T_{\mu\nu} T^{\mu\nu} - \frac{1}{3} T^2 \right),
\]

where the second term on the right hand side comes from the trace of \(\pi_{\mu\nu}\). On the other hand, the trace part of the energy momentum tensor of CFT is solely determined by the trace anomaly, and hence the trace of the Einstein equation with CFT becomes

\[
(4)^{G} = 8\pi G_4 T + \ell^2/4 \left( (4)^{G_{\mu\nu}} (4)^{G_{\mu\nu}} - \frac{4}{3} (4)^{G_{\mu\nu}} G_{\mu\nu} \right).
\]

In the situation that the correction to 4D general relativity is suppressed by the factor of \(O(\ell^2/L^2)\), we have \((4)^{G_{\mu\nu}} = 8\pi G_4 T_{\mu\nu} + O(\ell^2/L^2)\). Then the difference between \((6.4)\) and \((6.5)\) becomes \(O(\ell^2/L^2)\). Hence, the AdS/CFT correspondence for the trace part of the Einstein equation holds for the leading correction proportional to \(\ell^2\) even in the non-linear perturbation.
Now, let us apply the argument of the AdS/CFT correspondence to the formation of a black hole in the RS braneworld. In 4D CFT picture, a black hole is formed under the presence of a large number of conformal fields. Then, the back reaction due to the Hawking radiation will be much more efficient than in the ordinary 4D theory by the factor of $\ell^2/G_4$. If the AdS/CFT correspondence is valid in this situation, the quantum back reaction due to the Hawking radiation in 4D picture must be described as a classical dynamics in 5D picture. This means that the black hole evaporates as a classical process in 5D picture. This may implies that there is no stationary black hole solution in the 5D RS model. The result obtained in Ref. [12] that the exterior of the dynamically collapsing star cannot be static is also in line with the present conjecture. Of course, when the size of black hole becomes as small as the AdS curvature radius $\ell$, the correspondence may cease to hold.

There is a static black hole solution when a 2-brane in 4 dimensional bulk is considered. One may think this lower dimensional example as an evidence against the conjectured absence of black hole solution in braneworld. This solution was found by Emparan, Horowitz and Myers [13]. The 3-dimensional metric induced on the brane looks similar to the 4-dimensional Schwarzschild black hole:

$$ds^2 = -\left(1 - \frac{2\mu\ell}{r}\right)dt^2 + \left(1 - \frac{2\mu\ell}{r}\right)^{-1}dr^2 + r^2 d\phi^2.$$  \hfill (6.6)

The period of identification in $\phi$-direction is not $2\pi$ but

$$\Delta\varphi \approx \frac{4\pi}{3(2\mu)^{1/3}},$$

where we assumed that $\mu \gg 1$. For this black hole geometry, we have

$$E_{\mu\nu} = \frac{\mu\ell}{r^3} \text{diag}(1,1,-2).$$ \hfill (6.7)

If we apply the AdS/CFT correspondence, the energy momentum tensor of CFT is estimated by $T_{\mu\nu}^{CFT} \approx -(8\pi G_3)^{-1}E_{\mu\nu}$. To maintain a static configuration under the existence of the Hawking radiation, there should exist thermal bath which supplies the incoming energy flux to balance with the Hawking radiation. However, $T_{\mu\nu}^{CFT}$ estimated above decays too fast for large $r$ to explain this thermal bath.

However, we do not think that this is a counter example against the conjecture of the classical evaporation of black holes in the RS braneworld. An important difference of this lower dimensional example from the 4D Schwarzschild black hole is that the induced metric is not similar to a solution of 3D vacuum Einstein equation. It will be also worth pointing out that this effective energy momentum tensor [14] can be understood as the Casimir energy of CFT. This spacetime is very compact in the $\phi$-direction. The period in this direction is $r\Delta\varphi \approx r/\mu^{1/3}$, and is much shorter than the curvature scale $L \approx (\mu\ell/r^3)^{-1/2}$ even at the horizon. Hence, the approximation by a cylinder with a fixed radius $r_0 = r/\mu^{1/3}$ will work. For such a configuration, the energy momentum tensor for one conformal field is given by $\sim (\mu/r^3)\text{diag}(1,1,-2)$. The number of CFT species of $O(\ell/G_3)$ consistently explains the correspondence relation.

VII. SUMMARY AND DISCUSSION

In this paper, we reviewed the studies on the gravity in the Randall-Sundrum infinite braneworld. All the computations performed so far suggest that this model recovers the 4D general relativity as an effective theory induced on the brane. However, no black hole solution with the regular asymptotic behavior has been obtained. We discussed the possibility that there is no static black hole solution in this model, applying the argument of the AdS/CFT correspondence.

The first question is whether a brane black hole looks like an ordinary 4D BH or not. If possible black hole solutions are quite different from the ordinary black hole in 4D general relativity, the situation will be very interesting because we may have a chance to probe the extra-dimension by astronomical observations of black holes. If the solutions are similar to the ordinary ones, the next question arises whether the AdS/CFT correspondence applies for the black hole configuration. The case that the correspondence does not hold in such a strong gravity regime is still interesting because it may be possible to distinguish the model described by 4D CFT picture from the 5D RS model observationally in this case. The remaining possibility that the answer to the above question is "YES", i.e., that the correspondence applies even for a black hole configuration, is extremely interesting.

In this case, we can evaluate the mass loss rate due to the Hawking radiation as $M/M \sim N \times (1/G_3^2 M^3) \sim \ell^2/(G_3 M)^3$. Then the evaporation time scale becomes $\tau = (M/M_\odot)^3(1mm/\ell)^2 \times 1$ year. The existing black holes in X-ray binaries will give a stronger constraint on the value of $\ell$ than that from the laboratory experiment. The energy lost, say, for the last 10 second of the evaporation will be $E = (\ell/1mm)^2/3 \times 10^{52}$ erg. The amount of energy can be very large. However, this energy is radiated into CFT (= KK gravitons), which are almost decoupled from the ordinary matter fields. Hence, the direct observation of this energy seems to be difficult.

What does this evaporation look like in 5d picture? Although this is just a speculation, we would like to propose one possible explanation. We assume that blobs of the size $\ell^3$ are continuously created at the dynamical time scale of the BH, $G_4 M$, with the area of 5D horizon kept constant. The size of the blobs is suggested by the maximum length scale for the instability of black string [14]. Since the area of the main part of the horizon close to the brane will be $\approx M^2\ell$, we have
\( \frac{4}{\pi} M^2 \ell \approx \ell^3 / G_4 M \). From this estimate, we obtain the same order of magnitude for the mass loss rate as before. If the blobs escape to the 5th direction, this picture will consistently explain the propagation of CFT energy to the spatial infinity in 4D picture [21].

This correspondence, if it holds, can have an important meaning even in the case that our real universe is not described by the RS braneworld. Since 4D black hole evolution including back reaction due to the Hawking radiation can be described by a 5D classical problem in this case, it may become possible to solve the back reaction problem by using 5D numerical relativity. If we assume spherical symmetry in 4 dimensional sense, we have only to solve a problem with axial symmetry in 5D picture.

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