Bayes-ToMoP: A Fast Detection and Best Response Algorithm Towards Sophisticated Opponents

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Abstract

Multiagent algorithms often aim to accurately predict the behaviors of other agents and find a best response during interactions accordingly. Previous works usually assume an opponent uses a stationary strategy or randomly switches among several stationary ones. However, in practice, an opponent may exhibit more sophisticated behaviors by adopting more advanced strategies, e.g., using a bayesian reasoning strategy. This paper presents a novel algorithm called Bayes-ToMoP which can efficiently detect and handle opponents using either stationary or higher-level reasoning strategies. Bayes-ToMoP also supports the detection of previous unseen policies and learning a best response policy accordingly. Deep Bayes-ToMoP is proposed by extending Bayes-ToMoP with DRL techniques. Experimental results show both Bayes-ToMoP and deep Bayes-ToMoP outperform the state-of-the-art approaches when faced with different types of opponents in two-agent competitive games.

Introduction

In multiagent systems, the ideal behavior of an agent is contingent on the behaviors of coexisting agents. However, agents may exhibit different behaviors adaptively depending on the contexts they encounter. Hence, it is critical for an agent to quickly predict or recognize the behaviors of other agents, and make a best response accordingly. Bayesian Policy Reuse (BPR) (Rosman, Hawasly, and Ramamoorthy 2016), which was originally proposed for multi-task learning problems to determine the best policy when faced with different tasks. Hernandez-Leal et al. (Hernandez-Leal et al. 2016) proposed BPR+ by extending BPR to multiagent learning settings to detect the dynamic changes of an opponent’s strategies. BPR+ also extends BPR with the ability to generate new policies online against an opponent using previously unseen policies. However, BPR+ is designed for single-state repeated games only. Later, Bayes-Pepper is proposed for stochastic games by combining BPR and Pepper framework (Crandall 2012). However, Bayes-Pepper cannot handle an opponent which uses a previously unknown strategy.

The above problem can be partially addressed by introducing the concept of Theory of Mind (ToM) (Goldman 2009). ToM is a kind of recursive reasoning technique (Hernandez-Leal et al. 2017a) describing a higher cognitive mechanism of explicitly attributing unobservable mental contents such as beliefs, desires, and intentions to other players. Previous methods use explicit representations of nested beliefs and “simulate” the reasoning processes of other agents to predict their actions (Gmytrasiewicz and Durfee 2000; Gmytrasiewicz and Doshi 2005; Wunder et al. 2011; Wunder et al. 2012). However, these approaches show no adaptation to non-stationary opponents (Hernandez-Leal et al. 2017a). One exception is a reasoning framework proposed by (De Weerd, Verbrugge, and Verheij 2013), which enables an agent to predict the opponent’s actions explicitly by building an abstract model of the opponent’s behaviors using recursive nested beliefs. Additionally, they use a confidence value to help an agent to adapt to different opponents.

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However, the main drawbacks of this model are: 1) it works only if an agent holds exactly one more layer of belief than its opponent; 2) it is designed for predicting the opponent’s primitive actions instead of high-level strategies, resulting in slow adaptation to non-stationary opponents; 3) it shows poor performance against an opponent using previously unseen strategies.

To address the above challenges, we propose a novel algorithm, named Bayesian Theory of Mind on Policy (Bayes-ToMoP), which leverages the predictive power of BPR and ToM, to compete against sophisticated opponents. Bayes-ToMoP can quickly and accurately predict an opponent’s behaviors and compute a best response accordingly. Theoretical guarantees are provided for the optimal detection of the opponent’s strategies. Besides, Bayes-ToMoP also supports detecting whether an opponent is using a previously unseen policy and learning an optimal response against it. Furthermore, Bayes-ToMoP can be straightforwardly extended to DRL environment with a neural network as the value function approximator, termed as deep Bayes-ToMoP. Experimental results show that both Bayes-ToMoP and deep Bayes-ToMoP outperform the state-of-the-art approaches when faced with different types of opponents in two-agent competitive games.

Background

Bayesian Policy Reuse BPR was originally proposed in (Rosman, Hawasly, and Ramamoorthy 2016) as a framework for an agent to quickly determine the best policy to execute when faced with an unknown task. Given a set of previously-solved tasks \( \mathcal{T} \) and an unknown task \( \tau^* \), the agent is required to select the best policy \( \pi^* \) from the policy library \( \Pi \) within as small numbers of trials as possible. BPR uses the concept of belief \( \beta \), which is a probability distribution over the set of tasks \( \mathcal{T} \), to measure the degree to which \( \pi^* \) matches the known tasks based on the signal \( \sigma \). The belief is initialized with the prior probability and updated using Bayes’ rule at each timestep \( t \):

\[
\beta_k(\tau) = \frac{P(\sigma_k|\tau, \pi)\beta_{k-1}(\tau)}{\sum_{\tau' \in \mathcal{T}} P(\sigma_k|\tau', \pi)\beta_{k-1}(\tau')} \tag{1}
\]

A signal \( \sigma \) can be any information that is correlated with the performance of a policy (e.g., immediate rewards, episodic returns). BPR involves performance models of policies on previously-solved tasks, which describes the distribution of returns from each policy \( \pi \) on previously-solved tasks. A performance model \( P(U|\tau, \pi) \) is a probability distribution over the utility of a policy \( \pi \) on a task \( \tau \).

In order to select the best policy, a number of BPR variants with exploration heuristics are available, e.g., probability of improvement (BPR-PI) heuristic and expected improvement (BPR-EI) heuristic. BPR-PI heuristic utilizes the probability with which a specific policy can achieve a hypothesized increase in performance over the current best estimate: \( U = \max_{\pi \in \Pi} \sum_{\tau \in \mathcal{T}} \beta(\tau)E[U|\tau, \pi] \). Formally, it chooses the policy most likely to achieve the utility \( U^+ \):

\[
\pi^* = \argmax_{\pi \in \Pi} \sum_{\tau \in \mathcal{T}} \beta(\tau)P(U^+|\tau, \pi) \quad \text{where} \quad U^+ > U
\]

However, it is not straightforward to determine the appropriate value of \( U^+ \), thus another way of avoiding this issue is BPR-EI heuristic, which selects the policy most likely to achieve any possible utilities of improvement \( U < U^+ < U_{\max} \):

\[
\pi^* = \argmax_{\pi \in \Pi} \int_{U}^{U_{\max}} \sum_{\tau \in \mathcal{T}} \beta(\tau)P(U^+|\tau, \pi)dU^+ \tag{2}
\]

Rosman et al. (Rosman, Hawasly, and Ramamoorthy 2016) showed BPR-EI heuristic performs best among all BPR variants. Therefore, we choose BPR-EI heuristic for playing against different opponents.

Theory of Mind ToM model (De Weerd, Verbrugge, and Verheij 2013) is used to predict an opponent’s action explicitly by building an abstract model of the opponent’s actions using recursive nested beliefs. ToM model is described in the context of a two-player competitive game where an agent and its opponent differ in their abilities to make use of ToM. The notion of ToM_k indicates an agent that has the ability to use ToM up to the k-th order. De Weerd et al. (De Weerd, Verbrugge, and Verheij 2013) showed that the reasoning levels deeper than 2 do not provide significant benefits, so we briefly introduce the first two orders of ToM models.

A zero-order ToM (ToM_0) agent holds its zero-order belief in the form of a probability distribution on the action set of its opponent. The ToM_0 agent then chooses the action that maximizes its expected payoff. A first-order ToM agent (ToM_1) keeps both zero-order belief \( \beta^{(0)} \) and first-order belief \( \beta^{(1)} \). The first-order belief \( \beta^{(1)} \) is a probability distribution that describes what the ToM_1 agent believes its opponent believes about itself. The ToM_1 agent first predicts its opponent’s action under its first-order belief. Then, the ToM_1 agent integrates its first-order prediction with the zero-order belief and uses this integrated belief in the final decision. The degree to which the prediction influences the agents actions is determined by its first-order confidence \( 0 \leq c_1 \leq 1 \), which is increased if the prediction is right while decreased otherwise.

Bayes-ToMoP

Motivation

Previous works (Hernandez-Leal et al. 2016, Hernandez-Leal and Kaisers 2017, Hernandez-Leal et al. 2017b, Yan et al. 2018) assume that an opponent randomly switches its policies among a number of stationary policies. However, a more sophisticated agent may choose when to change its policy in a more principled way. For instance, it first predicts the policy of its opponent and then best responds towards the estimated policy accordingly. If the opponent’s policy is estimated by simply counting the action frequencies, it is then reduced to the well-known fictitious play algorithm (Shoham and Leyton-Brown 2009). However, in general, an opponent’s action information may not be observable during interactions. One way of addressing this problem is using BPR (Rosman, Hawasly, and Ramamoorthy 2016), which uses Bayes’ rule to predict the policy of the opponent according to the received signals (e.g., rewards), and can be regarded as the generalization of fictitious play.

Therefore, a question naturally arises: how an agent can effectively play against both simple opponents with stationary strategies and more advanced ones (e.g., using BPR)?
To address this question, we propose a new algorithm called Bayes-ToMoP, which leverages the predictive power of BPR and recursive reasoning ability of ToM to predict the strategies of such opponents and behave optimally. We also extend Bayes-ToMoP to DRL scenarios with a neural network as the value function approximator, termed as deep Bayes-ToMoP. In the following descriptions, we do not distinguish whether a policy is represented in a tabular form or a neural network unless necessary.

We use the notion of Bayes-ToMoP₀ to indicate an agent with the ability of using theory of mind up to the k-th order. Intuitively, Bayes-ToMoP₀ with a higher-order ToM could take advantage of any Bayes-ToMoP₀ with a lower-order ToM (i > j). Without loss of generality, we focus on Bayes-ToMoP₀ and Bayes-ToMoP₁. Bayes-ToMoPₖ (k > 1) can be naturally constructed by incorporating a higher-order ToM idea into our framework. Another reason that we focus on zero and first-order ToMoP is that it has been found that the benefit of considering deeper recursion (higher-level ToM, k > 1) is marginal [De Weerd, Verbrugge, and Verheij 2013]. Bayes-ToMoP₁ is capable of learning a new policy against its opponent if it detects the opponent is using a previously unknown strategy. In contrast, the above two features are not satisfied by the previous ToM model which works only if an agent holds exactly one more layer of belief than its opponent.

Bayes-ToMoP₀ Algorithm

We start with the simplest case of Bayes-ToMoP₀. Bayes-ToMoP₀ extends ToMo₀ by incorporating Bayesian reasoning techniques to predict the strategy of an opponent. Bayes-ToMoP₀ holds a zero-order belief β⁽⁰⁾ about its opponent’s strategies {j|j ∈ J}, each of which β⁽⁰⁾(j) is a probability that its opponent may adopt each strategy j: β⁽⁰⁾(j) ≥ 0, ∀j ∈ J. Summing j ∈ J β⁽⁰⁾(j) = 1. Given a utility U, a performance model Pself(U|j, π) describes the possibility of an agent using a policy π ∈ Π against an opponent’s strategy j ∈ J.

Algorithm 1 Bayes-ToMoP₀ algorithm

Initialize: Policy library II and J, zero-order belief β⁽⁰⁾, performance models Pself(U|J, II, Π)
1: for each episode do
2: Select the optimal policy: π* = argmaxπ∈Π I⁺β⁽⁰⁾(π)Pself(U+|j, π)dU+
3: Play and get the episodic reward ⟨rself, roppo⟩
4: for each opponent strategy j ∈ J do
5: Update zero-order belief β⁽⁰⁾:
β⁽⁰⁾(j) = Pself(rself|j, π)β⁽⁰⁾(j)
6: end for
7: New opponent detection and learning (see Algorithm 4)
8: end for

For Bayes-ToMoP₀ agent (Algorithm 1), it starts with its policy library II and its opponent’s strategy library J, its performance models Pself(U|J, II) and the zero-order belief β⁽⁰⁾. Then, in each episode, given the current belief β⁽⁰⁾, Bayes-ToMoP₀ agent evaluates the expected improvement utility defined following BPR-EI heuristic for all policies and then selects the optimal one (Line 2). Next, Bayes-ToMoP₀ agent updates its zero-order belief using Bayes’ rule [Rosman, Hawasly, and Ramamoorthy 2016] (Line 4-6). At last, Bayes-ToMoP₀ detects whether its opponent is using a previously unseen policy. If yes, it learns a new policy against its opponent (Line 7). The new strategy detection and learning algorithm will be described in Section 4.

Finally, note that without the new strategy detection and learning phase (Line 7), Bayes-ToMoP₀ agent is essentially equivalent with BPR [Rosman, Hawasly, and Ramamoorthy 2016] since they both first predict the opponent’s strategy (or type) and then select the optimal policy, each strategy of the opponent here can be regarded as a task in the original BPR. Besides, the full Bayes-ToMoP₁ is essentially equivalent with BPR+ [Hernandez-Leal et al. 2016] since both can handle previously unseen strategies.

Bayes-ToMoP₁ Algorithm

Algorithm 2 Bayes-ToMoP₁ Algorithm

Initialize: Policy library II and J, zero-order belief β⁽⁰⁾, performance models Pself(U|J, II) and Poppo(U|J, Π), zero-order belief β⁽⁰⁾, first-order belief β⁽¹⁾
1: for each episode do
2: Compute the first-order prediction: ̂j = argmaxj∈J I⁺β⁽¹⁾(β⁽¹⁾)Poppo(U+|j, β⁽¹⁾)dU+
3: Integrate ̂j with β⁽⁰⁾: I(β⁽⁰⁾, ̂j, c₁) (see Equation (3))
4: Select the optimal policy: π* = argmaxπ∈Π I⁺β⁽¹⁾(β⁽¹⁾, j, c₁)(j)Pself(U+|j, π)dU+
5: Play and get the episodic reward ⟨rself, roppo⟩
6: for each policy π ∈ Π do
7: Update first-order belief β⁽¹⁾:
β⁽¹⁾(π) = Poppo(roppo|π, β⁽¹⁾)β⁽¹⁾(π)∑π′∈Π Poppo(roppo|π′, β⁽¹⁾)β⁽¹⁾(π′)
8: end for
9: for each opponent strategy j ∈ J do
10: Update zero-order belief β⁽⁰⁾:
β⁽⁰⁾(j) = Poppo(roppo|j, π)β⁽⁰⁾(j)
11: end for
12: Update c₁ (following Equation (4))
13: New opponent detection and learning (see Algorithm 4)
14: end for

Next, we move to Bayes-ToMoP₁ algorithm. Apart from its zero-order belief, Bayes-ToMoP₁ also maintains a first-order belief, which is a probability distribution that describes the probability that an agent believes his opponent believes it will choose a policy π ∈ Π. Bayes-ToMoP₁ makes a prediction of its opponent’s policy based on its first-order belief. However, this prediction may conflict with its zero-order be-
lief. To address this conflict, Bayes-ToMoP₁ holds a first-order confidence \( c₁(0 ≤ c₁ ≤ 1) \) serving as the weighting factor to balance the influence degree between its first-order prediction and zero-order belief.

The overall strategy of Bayes-ToMoP₁ is shown in Algorithm 2. Given the policy library II and \( \mathcal{J} \), performance models \( P_{self}(U|J, II) \) and \( P_{oppo}(U|II, \mathcal{J}) \), zero-order belief \( \beta^{(0)} \) and first-order belief \( \beta^{(1)} \), Bayes-ToMoP₁ agent first predicts the policy \( j \) of its opponent assuming the opponent maximizes the utility based on BPR-EI heuristic under its first-order belief (Line 2). Then, an integration function \( I \) is introduced to compute the final prediction results which is defined as the linear combination of the first-order prediction \( \hat{j} \) and zero-order belief \( \beta^{(0)} \) weighted by the confidence degree \( c₁ \) following Equation 3 (De Weerd, Verbrugge, and Verheij 2013) (Line 3). Next, Bayes-ToMoP₁ agent computes the expected utility improvement based on the performance models and the integrated belief to derive an optimal policy (Line 4). At last, Bayes-ToMoP₁ agent updates its first-order belief and zero-order belief using Bayes’ rule (Rosman, Hawasly, and Ramamoorthy 2016) (Line 5-11).

\[
I(\beta^{(0)}, \hat{j}, c₁)(j) = \begin{cases} 
(1 - c₁)\beta^{(0)}(j) + c₁ & \text{if } j = \hat{j} \\
(1 - c₁)\beta^{(0)}(j) & \text{otherwise}
\end{cases}
\]

(3)

The next issue is how to adaptively update the first-order confidence degree \( c₁ \). Initially, we are not sure whether an opponent is making decisions using ToM reasoning or simply switching among stationary strategies. The value of \( c₁ \) can be understood as the exploration rate of using first-order belief to predict the opponent’s strategies. In previous ToMoP model (De Weerd, Verbrugge, and Verheij 2013), the value of \( c₁ \) is increased if the prediction is right while decreased otherwise based on the assumption that an agent can observe the actions of its opponent. However, in our settings, the prediction works on a higher level of behaviors (the policies), which usually are not available (agents are not willing to reveal their policies to others to avoid being exploited in competitive environments). Therefore, we propose using game outcomes as the signal to indicate whether our previous predictions are correct and adjust the first-order confidence degree accordingly. In a competitive environment, we can distinguish game outcomes into three cases: win, lose or draw by comparing two agents’ rewards. Thus, the value of \( c₁ \) is increased when the agent’s reward \( r_{self} \) is larger than its opponent’s reward \( r_{oppo} \) and decreased otherwise by an adjustment rate of \( \lambda \):

\[
c₁ = \begin{cases} 
(1 - \lambda)c₁ + \lambda & \text{if } r_{self} > r_{oppo} \\
(1 - \lambda)c₁ & \text{otherwise}
\end{cases}
\]

(4)

Following this heuristic, Bayes-ToMoP₁ can easily take advantage of Bayes-ToMoP₀ since it can well predict the policy of Bayes-ToMoP₀ in advance. However, this does not work when it plays against less sophisticated agents, e.g., an agent switching among several stationary policies without the ability of using ToM. This is due to the fact that the curve of \( c₁ \) becomes oscillating when it is faced with an agent who is unable to make use of ToM, thus fails to predict the opponent’s behaviors accurately. To this end, we propose an adaptive and generalized mechanism to adjust the value of \( c₁ \) and further detect the switches of the opponent’s strategies.

We first introduce the concept of winning rate \( v_i = \frac{\sum_{j=1}^{l} r_{self}}{l} \) during a fixed length \( l \) episodes to adaptively adjust the confidence degree \( c₁ \). Since Bayes-ToMoP₁ agent assumes its opponent is Bayes-ToMoP₀ at first, the value of \( l \) controls the number of episodes before considering its opponent may switch to a less sophisticated type. If the current episode’s performance is better than the previous episode’s performance \( v_i ≥ v_{i-1} \), we increase the weight of using first-order prediction, i.e., increasing the value of \( c₁ \) with an adjustment rate \( \lambda \); if \( v_i \) is smaller than \( v_{i-1} \) but still higher than a threshold \( \delta \), it indicates the performance of the first-order prediction diminishes. Then Bayes-ToMoP₁ decreases the value of \( c₁ \) quickly with a decreasing factor \( \Delta v_i = \frac{l_{Th}}{v_i - v_{Th}} \); if \( v_i ≤ \delta \), the rate of exploring first-order belief is set to 0 and only zero-order belief is used for prediction. Formally we have:

\[
c₁ = \begin{cases} 
((1 - \lambda)c₁ + \lambda)\psi(v_i) & \text{if } v_i ≥ v_{i-1} \\
\psi(\frac{1}{v_i - v_{Th}})c₁\psi(v_i) & \text{if } \delta < v_i < v_{i-1} \\
\psi(\frac{1}{v_i - \delta}) & \text{if } v_i ≤ \delta
\end{cases}
\]

(5)

where \( \delta \) is the threshold of the winning rate \( v_i \), which reflects the lower bound of the difference between its prediction and its opponent’s actual behaviors. \( \psi(v_i) \) is an indicator function to control the direction of adjusting the value of \( c₁ \). Intuitively, Bayes-ToMoP₁ detects the switching of its opponent’s strategies at each episode \( i \) and reverses the value of \( \psi(v_i) \) whenever its winning rate \( v_i \) is no larger than \( \delta \) (Equation 6). Finally, at the end of each episode, Bayes-ToMoP₁ learns a new optimal policy following Algorithm 3 if it detects a new opponent strategy (detailed in next section).

\[
\psi(v_i) := \begin{cases} 
1 & \text{if } (v_i ≤ \delta \wedge \psi(v_i) = 0) \\
0 & \text{if } (v_i ≤ \delta \wedge \psi(v_i) = 1)
\end{cases}
\]

(6)

New Opponent Detection and Learning

Algorithm 3 Detecting and learning opponent new strategy

**Input:** Policy library II, \( \mathcal{J} \), belief \( \beta^{(k)} \), performance models \( P_{self}(U|J, II) \)

1: if New opponent strategy \( j_{new} \) is detected
2: Learn an optimal policy \( \pi_{new} \) against \( j_{new} \)
3: Compute \( P_{self}(U|j_{new}, \pi_{new}) \)\( P_{oppo}(U|\pi_{new}, j_{new}) \)
4: Compute \( P_{self}(U|\pi_{new}, j_{new}) \)\( P_{oppo}(U|\pi_{new}, J) \wedge \pi_{new} \in \mathcal{J} \)
5: Compute \( P_{self}(U|j, \pi_{new}) \)\( P_{oppo}(U|\pi_{new}, j) \wedge j \in \mathcal{J} \)
6: Update each order of belief \( \beta^{(k)} \)
7: Update II, \( \mathcal{J} \)
8: end if

The new opponent detection and learning component is the same for all Bayes-ToMoPₖ agents (\( k ≥ 0 \)) (Algorithm 3). Bayes-ToMoPₖ first detects whether its opponent is using a new strategy (Line 1). This is achieved by recording a
length $h$ of game outcomes and checking whether its opponent is using one of the known strategies under the performance models at each episode. In details, Bayes-ToMoP keeps a length $h$ of memory recording the game outcomes (win, lose or draw) at each episode $i$, and uses the winning rate $\theta_i = \frac{\sum_{i-h+1}^{i} r}{h}$ over the most recent $h$ episodes as the signal indicating the performance under the current policy library. If the winning rate $\theta_i$ is lower than a given threshold $\delta$ ($\theta_i < \delta$), it indicates that all existing policies show poor performance against the current opponent strategy even with the confidence degree adjustment mechanism, in this way Bayes-ToMoP agent infers that the opponent is using a previously unseen policy outside the current policy library.

Since we can easily obtain the average winning rate of each policy $\pi$ against each known opponent strategy $j$, the lowest winning rate among the best-response policies \(\min_{\pi \in \Pi} \max_{j \in J} \theta(\pi, j)\) can be seen as an upper bound of the value of $\delta$. The value of $h$ controls the number of episodes before considering whether the opponent is using a previously unseen strategy. Note that the accuracy of detection is increased with the increase of the memory length $h$, however, a larger value of $h$ would necessarily increase the detection delay. The optimal memory length is determined empirically through extensive simulations.

After detecting the opponent is using a new strategy, the agent begins to learn the best-response policy against it (Line 2). Following previous work (Hernandez-Leal et al. 2016), we adopt the same assumption that the opponent will not change its strategy during the learning phase (a number of rounds). Otherwise, the learning process may not converge. For tabular Bayes-ToMoP, we adopt the traditional model-based RL: R-max (Brafman and Tennenholtz 2002) to compute the optimal policy. Specifically, once a new strategy is detected, R-max estimates the state transition function $T$ and reward function $R$ with $T$ and $R$. R-max computes $Q(s, a) = R(s, a) + \gamma \sum_{s'} T(s, a, s') \max_{a'} Q(s', a')$ for all state-action pairs and selects the action that maximizes $Q(s, a)$ according to $\epsilon$-greedy mechanism. For deep Bayes-ToMoP, it is hard to model the environment so that we apply DQN (Mnih et al. 2015) to do off-policy learning using the obtained interaction experience. DQN is a deep Q-learning method with experience replay, consisting of a neural network approximating $Q(s, a; \theta)$ and a target network approximating $Q(s, a; \theta^-)$, where $\theta$ and $\theta^-$ are the network parameters. DQN draws samples (or minibatches) of experience $(s, a, s', r) \sim U(D)$ uniformly from a replay memory $D$, and updates using the following loss function: $L(\theta) = E_{(s, a, s', r) \sim U(D)}[(r + \gamma \max_{a'}Q(s', a'; \theta^-) - Q(s, a; \theta))^2]$. To generate new performance models, we use a neural network to estimate the policy of the opponent based on the observed state-action history of the opponent using supervised learning techniques.

After the above learning phase, new performance models are generated using rewards obtained from a number of simulations of the agents policy against the opponents estimated strategy. These values are modeled as a Gaussian distribution to obtain the performance models (Line 3-5). Next, each order of belief is updated (Line 6). Finally, it adds the new policy $\pi_{\text{new}}$ and the estimated opponent policy to its policy library II and its opponent’s policy library $J$ respectively (Line 7).

### Theoretical Analysis

In this section, we provide a theoretical analysis that Bayes-ToMoP can accurately detect the opponent’s strategy from a known policy library and derives an optimal response policy accordingly.

**Theorem 1 (Optimality on Strategy Detection)** If the opponent plays a strategy from the known policy library, Bayes-ToMoP can detect the strategy w.p.1 and selects an optimal response policy accordingly.

**Proof 1** Suppose the opponent strategy is $j \in J$, Bayes-ToMoP receives a signal $s_t$ at step $t$, the belief $\beta_t(j)$ is updated as follows:

$$\beta_{t+1}(j) = \frac{P(s_t|j, \pi)\beta_t(j)}{\sum_{j' \in J} P(s_t|j', \pi)\beta_t(j')}$$

Then there exists a policy $\pi \in \Pi$, that makes the inequality $P_{\text{det}}(s_t|j, \pi) \geq P(s_t|j, \pi)$ establish for all $\forall j' \in J$. Besides, since $\beta_t(j)$ is bounded ($0 \leq \beta_t(j) \leq 1$) and monotonically increasing ($\beta_{t+1}(j) \geq \beta_t(j) > 0$), based on the monotone convergence theorem, we can easily know the limit of sequence $\beta_t(j)$ exists. Thus, if we limit the two sides of the above Equation, the following equation establishes,

$$P(s_t|j, \pi) = \sum_{j' \in J} P(s_t|j', \pi)\beta_t(j')$$

iff $\beta_t(j) = 1, \beta_t(j') = 0, \forall j' \neq j$. So that Bayes-ToMoP can detect the strategy w.p.1 and selects an optimal response policy accordingly.

As guaranteed by Theorem 1, Bayes-ToMoP behaves optimally when the opponent uses a strategy from the known policy library. If the opponent is using a previously unseen strategy, following the new opponent strategy detection heuristic, this phenomenon can be exactly detected when the winning rate of Bayes-ToMoP during a fixed length of episodes is lower than the accepted threshold. Then Bayes-ToMoP begins to learn an optimal response policy. The performance of Bayes-ToMoP against opponents using either known or previously unseen strategies will be extensively evaluated through empirical simulations in the next section.

### Simulations

In this section, we present experimental results of Bayes-ToMoP compared with state-of-the-art tabular approaches (BPR+ (Hernandez-Leal et al. 2016) and Bayes-Pepper (Hernandez-Leal et al. 2017b)). For deep environments, we compare deep Bayes-ToMoP with the following four baseline strategies: 1) BPR+, 2) Bayes-Pepper (BPR+ and Bayes-Pepper use a neural network as the value function approximator), 3) DRON (He and Boyd-Graber 2016) and deep BPR+ (Yan et al. 2018). We first evaluate the performance of Bayes-ToMoP by comparing it with state-of-the-art approaches in both tabular and deep settings. Following that is the comparison of Bayes-ToMoP and deep Bayes-ToMoP with previous works against an opponent using previously unseen strategies. Finally, we explore the influence
of key parameters of Bayes-ToMoP. Unless otherwise mentioned, all experiments use the same parameter settings: \( c_1 = 0.3, \lambda = 0.7, \delta = 0.7 \) (experimentally selected). DQN has two fully-connected hidden layers both with 50 hidden units, the output layer is a fully-connected linear layer with a single output for each valid action. All results are averaged over 1000 runs.

**Game Settings**

We evaluate the performance of Bayes-ToMoP on the following testbeds: rock-paper-scissors (RPS) (v. Neumann 1928; Shoham and Leyton-Brown 2009), soccer (Littman 1994; He and Boyd-Graber 2016) and thieves and hunters (Goodrich, Crandall, and Stimpson 2003; Crandall 2012). We consider two versions of soccer and thieves and hunters with different state representations. For the first version, we discretize the state space into a few numbers of discrete states, in which Q-values can be represented in a tabular form. We also consider the full state space which consists of different dimensions of information: for example, states in soccer includes coordinates of two agents and the ball possession. In this case, we evaluate the performance of deep Bayes-ToMoP.

RPS is a two-player stateless game in which two players simultaneously choose one of the three possible actions ‘rock’ (R), ‘paper’ (P), or ‘scissors’ (S). If both choose the same action, the game ends in a tie. Otherwise, the player who chooses R wins against the one that chooses S, S wins against P, and P wins against R.

Soccer (Figure 1) is a stochastic game on a 6 x 7 grid. Two players, A and B, start at one of starting points in the left and right respectively and can choose one of the following 5 actions: go left, right, up, down and stay. Any action that goes to black-slash grids or beyond the border is invalid. Player A scores one point if it takes the ball to its opponent’s goals. If neither player gets a score within 50 steps, the game ends with a tie. We construct 24 deterministic policies for the opponent and the corresponding pre-trained best-response policies make up the policy library. For deep Bayes-ToMoP, DQN is used to train optimal policies.

**Performance against Different Opponents**

Three kinds of opponents are considered: (1) a Bayes-ToMoP\(_0\) opponent (O\(_{ToMoP_0}\)); (2) an opponent that randomly switches its policy among stationary strategies and lasts for an unknown number of episodes (O\(_{as}\)) and (3) an opponent switching its strategy between stationary strategies and Bayes-ToMoP\(_0\) (O\(_{ToMoP_0+}\)). In this section, we assume an opponent only selects a policy from the known policy library. Thus Bayes-Pepper is functionally equivalent with BPR\(_s\) in our setting and we use BPR\(_s\) to denote both strategies.

Figure 3 depicts the average rewards of different approaches against O\(_{ToMoP_0}\) on various games. We can see from Figure 3(a-c) that only Bayes-ToMoP\(_1\) can accurately and quickly detect the opponent’s strategies and achieve the highest average rewards. In contrast, BPR\(_s\) fails against O\(_{ToMoP_0}\). Similar comparison results can be found for deep Bayes-ToMoP\(_1\) (Figure 3(d)). This is because Bayes-ToMoP\(_1\) explicitly considers two-orders of belief to do recursive reasoning first and then derives an optimal policy against its opponent. However, BPR\(_s\) makes decisions based on BPR heuristic which is essentially equivalent with Bayes-ToMoP\(_0\). Therefore, neither BPR\(_s\) nor Bayes-ToMoP\(_0\) could take advantage of each other and the winning percentages are expected to be approximately 50\%. Average winning rates shown in Table 1 also confirm our hypothesis. Deep BPR\(_+\) incorporates previous opponent’s behaviors into BPR, however, their model is still constructed by explicitly considered which kind of stationary strategy the opponent is using, thus is not enough to detect the policy of opponent O\(_{ToMoP_0}\). We can see from Figure 3(d) that DRON performs better than Deep BPR\(_+\) and BPR\(_s\). This is because it automatically adjusts the weights of K expert networks, thus leading to relatively good performance against overall kinds of opponents. However, it fails to achieve the highest average rewards because the dynamic adjustment of DRON cannot guarantee that each response policy is optimal against a particular type of opponent. Deep versions of two games are constructed in a similar way and the results in two games are also similar. Due to the space limitation, we only give experiments on the deep soccer game.

Next, we consider the case of playing against O\(_{as}\) which is the baseline of previous works. Figure 2(a-c) shows the comparison of Bayes-ToMoP\(_1\) with BPR\(_s\) on three games, where Bayes-ToMoP\(_1\) can quickly and accurately detect which stationary strategy the opponent is using and derive the optimal policy against it. We observe that Bayes-ToMoP\(_1\) requires a longer time before achieving the highest reward than BPR\(_s\). This happens because Bayes-ToMoP\(_1\) needs additional time to determine that the opponent is not
using a BPR-based strategy. This phenomenon is consistent with the slightly lower winning rate of Bayes-ToMoP than BPRs (see Table 1). Figure 5(d) depicts that our deep Bayes-ToMoP performs closer to BPR+ and deep BPR+. However, DRON only achieves the average rewards of 0.7. This is because DRON uses an end-to-end trained response sub-network, which cannot guarantee that each response policy is good enough against a particular type of opponent.

Finally, we consider the case of playing against $O_{ToMoP0}$ to show the robustness of Bayes-ToMoP. Figure 5(a-c) shows that only Bayes-ToMoP1 can quickly and accurately detect the strategies of both opponent $O_{ToMoP0}$ and non-stationary opponent. In contrast, BPRs fails when its opponent’s strategy switches to Bayes-ToMoP0. A similar comparison can be found in deep soccer shown in Figure 5(d) in which both BPRs and deep BPR+ fail to detect and respond to $O_{ToMoP0}$ opponent. Figure 5(d) also shows DRON performs poorly against both two kinds of opponents due to similar reason described above. Average winning rates are summarized in Table 1 which is consistent with the results in Figure 5(b).

### Table 1: Average winning rates with std.dev.(±) in tabular soccer.

| Approaches / Opponents | BPRs | Bayes-ToMoP1 | BPRs |
|------------------------|------|--------------|------|
| $O_{ToMoP0}$           | 49.78%±1.71% | 99.72%±0.16% |      |
| $O_{ns}$               | 99.37%±0.72% | 99.72%±0.37% |      |
| $O_{ToMoP0}$-s         | 33.4%±0.57%  | 98.48%±0.54% |      |

### New Opponent Detection and Learning

In this section, we evaluate Bayes-ToMoP against an opponent who may use previously unknown strategies. We manually design a new strategy for the opponent in soccer game. The opponent starts with one of known strategies and switches to the new strategy at the 200th episode. Figure 6 and 7 show the dynamics of average rewards of different approaches on two versions of soccer game respectively.

We can see from Figure 6 that when the opponent switches to an unknown strategy, both Bayes-ToMoP1 and BPR+ can quickly detect this change, and finally learn an optimal policy. However, Bayes-Pepper fails against an unknown opponent strategy. This is because Bayes-Pepper explicitly predicts the strategy of its opponent from a known policy library, which makes it fail to well respond to a previously unseen strategy. The learning curve of Bayes-ToMoP1 is closer to BPR+ since both methods learn from scratch. Similar results can be found for deep Bayes-ToMoP and BPR+ in Figure 7. DRON fails to respond to the unknown opponent strategy due to the fact that the number of expert networks is fixed and thus unable to handle such case. Deep BPR+ performs better than the other three methods. This is because Deep BPR+ uses policy distillation to transfer knowledge from similar previous policies to accelerate the online learning. This technique can be readily applied into our Bayes-ToMoP learning framework to accelerate online learning as future work.

### The Influence of Key Parameters

Figure 8 depicts the impact of the memory length $l$ on the average adjustment time of Bayes-ToMoP before taking advantage of $O_{ToMoP0}$. We observe a diminishing return phenomenon: the average adjustment time decreases quickly as the initial increase in the memory length, but quickly render
additional performance gains become marginal. The average adjustment time stabilizes around 2.8 when $l > 35$. We hypothesize that it is because the dynamic changes of the winning rate over a relatively small length of memory may be caused by noise thus resulting in inaccurate opponent type detection. As the increase of the memory length, the judgment about its opponent’s types is more precise. However, as the memory length exceeds a certain threshold, the winning rate estimation is already accurate enough and thus the advantage of further increasing the memory length diminishes.

Finally, the influence of threshold $\delta$ against opponent $O_{\text{ToMoP}}$ is shown in Figure 9. We note that the average adjustment time decreases as $\delta$ increases, but the decrease degree gradually stabilizes when $\delta$ is larger than 0.7. With the increase of the value of $\delta$, the winning rate decreases to $\delta$ more quickly when the opponent switches its policy. Thus, Bayes-ToMoP$_{\delta}$ detects the switching of its opponent’s strategies more quickly. Similar with the results in Figure 8 as the value of $\delta$ exceeds a certain threshold, the advantage based on this heuristic diminishes.

### Conclusion and Future Work

This paper presents a novel algorithm called Bayes-ToMoP, which leverages the predictive power of BPR and recursive reasoning ability of ToM to quickly and accurately predict the behaviors of not only switching, non-stationary opponents and also more sophisticated ones (e.g., BPR-based) and behave optimally. Our Bayes-ToMoP also enables an agent to learn a new optimal policy when encountering a previously unseen strategy. As future work, it is worth investigating how to accelerate the online new policy learning phase and how to extend Bayes-ToMoP to multi-opponent scenarios.

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