Electrodisintegration of Deuteron into Dark Matter and Proton Close to Threshold

A. N. Ivanov,1 R. Höllwieser,1,2 N. I. Troitskaya,1 M. Wellenzohn,1,3 and Ya. A. Berdnikov4

1Atominstitut, Technische Universität Wien, Stadionallee 2, A-1020 Wien, Austria
2Department of Physics, Bergische Universität Wuppertal, Gaussstr. 20, D-42119 Wuppertal, Germany
3FH Campus Wien, University of Applied Sciences, Favoritenstraße 226, 1100 Wien, Austria
4Peter the Great St. Petersburg Polytechnic University, Polytechnicheskaya 29, 195251, Russian Federation

(Dated: November 10, 2021)

We discuss an investigation of the dark matter decay modes of the neutron, proposed by Fornal and Grinstein (Phys. Rev. Lett. 120, 191801 (2018)) and Ivanov et al. (arXiv:1806.10107 [hep-ph]) for solution of the neutron lifetime anomaly problem, through the analysis of the electrodisintegration of the deuteron d into dark matter fermions χ and protons p close to threshold. We calculate the triple–differential cross section for the reaction e− + d → χ + p + e− and propose to search for such a dark matter channel in coincidence experiments on the electrodisintegration of the deuteron e− + d → n + p + e− into neutrons n and protons close to threshold with outgoing electrons, protons and neutrons in coincidence. A missing of neutron signals should testify a detection of dark matter fermions.

PACS numbers: 11.10.Ef, 13.30a, 95.35.+d, 25.40.Fq

I. INTRODUCTION

Recently Fornal and Grinstein [1,5] have proposed a solution to the neutron lifetime anomaly (NLA) problem, related to a discrepancy between experimental values of the neutron lifetime measured in bottle and beam experiments, through a contribution of the dark matter decay mode n → χ + e− + e+, where χ is a dark matter fermion and (e−e+) is the electron–positron pair. However, according to experimental data [6–8], the decay mode n → χ + e− + e+ is suppressed. So at first glimpse it seems that the decay mode n → χ + e− + e+ cannot explain the NLA problem. In order to overcome such a problem we have assumed [9] that an unobservability of the decay mode n → χ + e− + e+ may only mean that the production of the electron–positron pair in such a decay is below the reaction threshold, i.e. a mass mχ of dark matter fermions obeys the constraint mχ > mν + mν, where mν and mν are masses of the neutron and electron (positron), respectively. Then, we have proposed that the NLA problem can be explained by the decay mode n → χ → νeνe, where (νeνe) is a neutrino–antineutrino pair [9]. Since neutrino νe and electron e− belong to the same doublet in the Standard Electroweak Model (SEM) [15, 16] (see also [17, 18]), neutrino–antineutrino (νeνe) pairs couple to the neutron–dark matter current with the same strength as electron–positron (−e−e+) pairs [9]. We have extended this effective quantum field theory by a gauge invariant quantum field theory model of the neutron–dark matter interactions invariant under the UY(1) × UY′(1) gauge symmetry. In the physical phase the dark matter sectors with UY′(1) and UY(1) symmetries are responsible for the effective interaction (νeνe) [9] and interference of the dark matter into dynamics of neutron stars [19–22], respectively. The dark matter sector with UY′(1) symmetry we have constructed in analogue with scenario proposed by Cline and Cornell [22]. This means that dark matter fermions with mass mχ < mν couple to a very light dark matter spin–1 boson Z′ providing a necessary repulsion between dark matter fermions in order to give a possibility for neutron stars to reach masses of about 2M⊙ [23], where M⊙ is the mass of the Sun [15]. The corrected versions of the dark matter sectors, invariant under the UY′(1) × UY(1) gauge symmetry is expounded in the Appendix.

In connection with different approaches to the explanation of the NLA, we have to mention another mechanism proposed by Berezhiani [10, 11], which is not related to the decay of the neutron into a dark matter but based on the n → n′ transitions, where n′ is a mirror neutron [12, 13]. Since, such a mechanism is far from being applicable to the analysis of the electrodisintegration of the deuteron, we will not discuss it in this paper. Another mechanism...
for the explanation of the NLA proposed by Berezhiani [14], assuming the existence of the neutron decays into dark matter particles, is similar to that by Fornal and Grinstein [15]. So our approach to the NLA seems also to be a modification of Berezhiani’s mechanism as well as a modifications of the mechanism by Fornal and Grinstein.

We would like to notice that the existence of the reaction \( n \rightarrow \chi + \nu_e + \bar{\nu}_e \) for the explanation of the NLA problem entails the existence of the reactions \( n + n \rightarrow \chi + \chi \), \( n + n \rightarrow \chi + \nu_e + \bar{\nu}_e \) and \( \chi + \chi \rightarrow n + n \), which together with the reaction \( n \rightarrow \chi + \nu_e + \bar{\nu}_e \) can serve as URCA processes for the neutron star cooling [30,35].

We would like to emphasize that a possibility to explain the NLA problem by the neutron dark matter decay mode \( n \rightarrow \chi + \nu_e + \bar{\nu}_e \) is not innocent and demands to pay the following price. As has been pointed out in [9], the explanation of the neutron lifetime \( \tau_n = 888.0(2.0) \text{ s} \) [1] within the Standard Model with the axial coupling constant \( g_A = 1.2764 \pm 0.003 \) [24, 25], reproducing the neutron lifetime \( \tau_n = 879.6(4) \text{ s} \) [26], is not possible and demands the account for the contributions of interactions beyond the Standard Model such as the Fierz interference term \( b = -1.44 \times 10^{-2} \) [35]. However, as has been shown in [35], the Fierz interference term \( b = -1.44 \times 10^{-2} \) does not contradict the existing experimental data on the correlation coefficients and asymmetries of the neutron beta decay. It is obvious that the decay channel \( n \rightarrow \chi + \nu_e + \bar{\nu}_e \), since all decay particles are neutral. Hence, such mechanism of the NLA can be confirmed experimentally by experimental investigations of reactions, where an emission of a dark matter fermion \( \chi \) is accompanying with emission of charged standard model particles.

Having assumed that the results of the experimental data [6,8] can be also interpreted as a production of electron–positron pairs below reaction threshold, we may assume that the neutron dark matter decay \( n \rightarrow \chi + \nu_e + \bar{\nu}_e \) can be confirmed, for example, in the process of the electrodisintegration of the deuteron into dark matter fermions and protons \( e^- + d \rightarrow \chi + p + e^- \) close to threshold, induced by the \((n\chi-e^-)\) interaction [9].

The paper is organized as follows. In section II we calculate the triple–differential cross section for the electrodisintegration of the deuteron \( e^- + d \rightarrow \chi + p + e^- \) into dark matter fermions \( \chi \) and protons \( p \). In section II we discuss the obtained results, make an estimate of the triple–differential cross section, calculated in section II, and propose an experimental observation of dark matter fermions in coincidence experiments on the electrodisintegration of the deuteron \( e^- + d \rightarrow n + p + e^- \) close to threshold by detecting outgoing electrons, protons and neutrons in coincidence. A missing of neutron signals should testify an observation of dark matter fermions. In the Appendix we extend the gauge invariant and renormalizable effective quantum field theory of strong and electroweak low-energy interactions, proposed in [17,18], by the dark matter sector invariant under \( U'_L \times U'_R \) gauge symmetry.

II. TRIPLE–DIFFERENTIAL CROSS SECTION FOR ELECTRODISINTEGRATION OF DEUTERON INTO DARK MATTER FERMIONS AND PROTONS \( e^- + d \rightarrow \chi + p + e^- \)

For the solution of the neutron lifetime anomaly problem we have proposed to use the following effective interaction [9]

\[
\mathcal{L}_{DMBL}(x) = -\frac{G_F}{\sqrt{2}} V_{ud} \bar{\psi}_e(x) \gamma_\mu(h_V + \bar{h}_A \gamma^5) \psi_n(x) [\psi_e(x) \gamma^\mu(1 - \gamma^5) \Psi_e(x)],
\]

where \( G_F = 1.1664 \times 10^{-11} \text{ MeV}^{-2} \) is the Fermi weak coupling constant, \( V_{ud} = 0.97370(14) \) is the Cabibbo-Kobayashi–Maskawa (CKM) matrix element [15], extracted from the \( 0^+ \rightarrow 0^+ \) transitions [19]. The phenomenological coupling constants \( h_V \) and \( h_A \) define the strength of the neutron-dark matter \( n \rightarrow \chi \) transitions. Then, \( \psi_e(x) \) and \( \bar{\psi}_e(x) \) are the field operators of the dark matter fermion and neutron, respectively. According to the SEM [17] (see also [18]), the field operator \( \Psi_e(x) \) is the doublet with components \( (\psi_{e\alpha}(x), \bar{\psi}_e(x)) \), where \( \psi_{e\alpha}(x) \) and \( \bar{\psi}_e(x) \) are the field operators of the electron–neutrino (electron–antineutrino) and electron (positron), respectively. The leptonic current \( \Psi_e(x) \gamma^\mu(1 - \gamma^5) \Psi_e(x) \) has the \( V-A \) structure, since electron–neutrinos are practically left–handed. The amplitude of the reaction \( e^- + d \rightarrow \chi + p + e^- \) is defined by

\[
M(e^- + d \rightarrow \chi + p + e^-) = \frac{G_F}{\sqrt{2}} V_{ud} \langle \bar{\psi}_e(k_e', \sigma_e') | [\bar{\psi}_e(k_e, \sigma_e) \gamma_\mu (h_V + \bar{h}_A \gamma^5) \psi_n(0)] | d(k_d, \lambda_d) \rangle \times \langle \bar{u}_e(k_e', \sigma_e') | \gamma^\mu(1 - \gamma^5) u_e(k_e, \sigma_e) \rangle,
\]

where \( \lambda_d = 0, \pm 1 \) define the polarization states of the deuteron, \( \bar{u}_e(k_e', \sigma_e') \) and \( u_e(k_e, \sigma_e) \) are Dirac wave functions of free electrons in the final and initial states of the reaction. In the matrix element of the \( d \rightarrow \chi + p \) transition \( \langle p(k_p', \sigma_p) | \chi(k_e', \sigma_e) \rangle \) and \( | d(k_d, \lambda_d) \rangle \) are the wave functions of the dark matter fermion and proton in the final state and the deuteron in the initial one. They are defined by [39]

\[
\langle p(k_p', \sigma_p) | \chi(k_e', \sigma_e) \rangle = \langle 0 | a_p(k_e', \sigma_e) a_p(k_p', \sigma_p) \rangle.
\]
and

\[ |d(\vec{k}_d, \lambda_d = \pm 1)) = \frac{1}{(2\pi)^3} \int \frac{d^3q_p}{\sqrt{2E_p(q_p)}} \frac{d^3q_n}{\sqrt{2E_n(q_n)}} \sqrt{2E_d(q_p + q_n)} \delta^{(3)}(\vec{k}_d - \vec{q}_p - \vec{q}_n) \times \Phi_d\left(\frac{\vec{q}_p - \vec{q}_n}{2}\right) a^\dagger_p(q_p, \pm 1/2) a^\dagger_n(q_n, \pm 1/2)|0\rangle, \]

\[ |d(\vec{k}_d, \lambda_d = 0)) = \frac{1}{(2\pi)^3} \int \frac{d^3q_p}{\sqrt{2E_p(q_p)}} \frac{d^3q_n}{\sqrt{2E_n(q_n)}} \sqrt{2E_d(q_p + q_n)} \delta^{(3)}(\vec{k}_d - \vec{q}_p - \vec{q}_n) \times \Phi_d\left(\frac{\vec{q}_p - \vec{q}_n}{2}\right) \frac{1}{\sqrt{2}} [a^\dagger_p(q_p, +1/2) a^\dagger_n(q_n, -1/2) + a^\dagger_p(q_p, -1/2) a^\dagger_n(q_n, +1/2)]|0\rangle, \] (4)

where \(|0\rangle\) is the vacuum wave function, \(a^\dagger_j(\vec{p}_j, \sigma_j)\) and \(a_j(\vec{p}_j, \sigma_j)\) are operators of creation and annihilation of a fermion \(j = n, p, \chi\) with a 3–momentum \(\vec{p}_j\) and polarization \(\sigma_j = \pm 1/2\) obeying standard relativistic covariant anti–commutation relations [39]. Then, \(\Phi_d(\vec{k})\) is the component of the wave function of the bound np–pair in the \(^3S_1\) state defined in the momentum representation. It is normalized to unity [39] (see also [40]):

\[ \int |\Phi_d(\vec{k})|^2 d^3k/(2\pi)^3 = 1. \] (5)

We neglect the contribution of the component of the wave function of the bound np–pair in the \(^3D_1\)–state [41][43] (see also [40][44]), which is not important for the analysis of the electrodisintegration of the deuteron into dark matter fermions and protons. The wave function of the deuteron Eq. (1) is normalized by [39]:

\[ \langle d(\vec{k}_d, \lambda'_d)|d(\vec{k}_d, \lambda_d)\rangle = (2\pi)^3 2E_d(\vec{k}_d) \delta^{(3)}(\vec{k}_d - \vec{k}_d) \delta_{\lambda'_d \lambda_d}. \] (6)

In the non–relativistic approximation for heavy fermions and in the laboratory frame, where the deuteron is at rest, the amplitudes of the electrodisintegration of the deuteron into dark matter fermions and protons are determined by

\[ M(e^- d \rightarrow \chi p e^-)^{\sigma'_\chi, \sigma_p, \sigma_\chi}_{\sigma_e, \lambda_d = \pm 1} = -\sqrt{4m_m m_p} G_F V_{ud} \Phi_d(\vec{k}_p) \delta_{\sigma_e, \pm 1/2} \left\{ h_V [\varphi^{\dagger}_{\chi}(\sigma_\chi) \varphi_n(\pm 1/2)] \times [\varphi^{\dagger}_{\chi}(\vec{k}_e, \sigma_\chi) \gamma^0 (1 - \gamma^5) u_e(\vec{k}_e, \sigma_e) - h_A [\varphi^{\dagger}_{\chi}(\sigma_\chi) \vec{\sigma} \varphi_n(\pm 1/2)] \times [\varphi^{\dagger}_{\chi}(\vec{k}_e, \sigma_\chi) \vec{\gamma}(1 - \gamma^5) u_e(\vec{k}_e, \sigma_e)] \right\}, \]

\[ \times \delta_{\sigma_p, +1/2} \left\{ h_V [\varphi^{\dagger}_{\chi}(\sigma_\chi) \varphi_n(-1/2)] \times [\varphi^{\dagger}_{\chi}(\vec{k}_e, \sigma_\chi) \gamma^0 (1 - \gamma^5) u_e(\vec{k}_e, \sigma_e) - h_A [\varphi^{\dagger}_{\chi}(\sigma_\chi) \vec{\sigma} \varphi_n(-1/2)] \times [\varphi^{\dagger}_{\chi}(\vec{k}_e, \sigma_\chi) \vec{\gamma}(1 - \gamma^5) u_e(\vec{k}_e, \sigma_e)] \right\} + \delta_{\sigma_p, -1/2} \left\{ h_V [\varphi^{\dagger}_{\chi}(\sigma_\chi) \varphi_n(+1/2)] \times [\varphi^{\dagger}_{\chi}(\vec{k}_e, \sigma_\chi) \gamma^0 (1 - \gamma^5) u_e(\vec{k}_e, \sigma_e) - h_A [\varphi^{\dagger}_{\chi}(\sigma_\chi) \vec{\sigma} \varphi_n(+1/2)] \times [\varphi^{\dagger}_{\chi}(\vec{k}_e, \sigma_\chi) \vec{\gamma}(1 - \gamma^5) u_e(\vec{k}_e, \sigma_e)] \right\}, \] (7)

where \(\varphi_\chi(\sigma_\chi)\) and \(\varphi_n(\sigma_n)\) are the Pauli wave functions of the dark matter fermion and neutron, respectively, \(\vec{\sigma}\) are \(2 \times 2\) Pauli matrices, \(m_m = m_n + m_p + \varepsilon_d\) is the deuteron mass, \(m_n\) and \(m_p\) are masses of the neutron and proton, and \(\varepsilon_d = -2.224575(9)\) MeV is the deuteron binding energy [45], and \(m_\chi\) is the dark matter fermion mass. The differential cross section for the reaction \(e^- d \rightarrow \chi p + e^- + p\), averaged over polarizations of the incoming electron and deuteron and summed over polarizations of the fermions in the final state, is equal to

\[ d^3\sigma(E'_e, E_c, \vec{k}_e', \vec{k}_e, \vec{k}_\chi, \vec{k}_p) = \left(1 + 3g_A^2\right) \frac{G_F^2 |V_{ud}|^2 \zeta^{(\text{dm})}}{8\pi^5} \frac{\zeta^{(\text{dm})}}{\beta_e} \left(1 + a^{(\text{dm})} \frac{\vec{k}_e' \cdot \vec{k}_e}{E'_e E_c}\right) |\Phi_d(\vec{k}_p)|^2 \delta(E'_e + E_p + E_c - m_d - E_e) \times \delta^{(3)}(\vec{k}_e + \vec{k}_p + \vec{k}_\chi' - \vec{k}_e') d^3k_\chi d^3k_p d^3k_e'. \] (8)

where \(\beta_e = k_e/E_c\) is the incoming electron velocity and \(g_A = 1.27641(56)\) is the axial coupling constant [24][25] introduced in [9] for convenience. The correlation coefficients \(\zeta^{(\text{dm})}\) and \(a^{(\text{dm})}\) are defined by

\[ \zeta^{(\text{dm})} = \frac{1}{1 + 3g_A^2} \left(\frac{|h_V|^2 + 3|h_A|^2}{|h_V|^2 + 3|h_A|^2}\right) \quad \text{and} \quad a^{(\text{dm})} = \frac{|h_V|^2 - |h_A|^2}{|h_V|^2 + 3|h_A|^2}. \] (9)
where \(m_n - m_\chi\) is measured in MeV. In the non–relativistic approximation for the dark matter fermion and proton and in the center–of–mass frame of the \(\chi p\)–pair we transcribe Eq.\((8)\) into the form

\[
d^3\sigma(E'_e, E_e, k'_e, k_e, \tilde{p}, \tilde{k}) = (1 + 3\lambda^2) \frac{G_F^2 |V_{ud}|^2}{8 \pi^3} \frac{\zeta^{(dm)}(\lambda, \tilde{p})}{\beta_e} \left(1 + a^{(dm)}_e \frac{k'_e \cdot \tilde{k}_e}{E'_e E_e}\right) \Phi_d(\tilde{k} + \frac{m_p}{m_p + m_\chi} \tilde{p})^2 \times \delta \left(\frac{\tilde{k}^2}{2M} + \frac{k^2}{2\mu} - (E_0 + E_e - E'_e)\right) \delta(\tilde{p} + \tilde{k}_e - k_e) d^3 p d^3 k d^3 k'_e,
\]

where \(E_0 = m_n - m_\chi + \varepsilon_d, \tilde{p}\) and \(\tilde{k}\) are the total and relative 3–momenta of the \(\chi p\)–pair, related to the 3–momenta of the dark matter fermion \(k'_e\) and the proton \(k_\tilde{e}\) as follows

\[
k'_e = -\tilde{k} + \frac{m_\chi}{m_p + m_\chi} \tilde{p}, \quad k_\tilde{e} = \tilde{k} + \frac{m_p}{m_p + m_\chi} \tilde{p}.
\]

Then, \(M = m_p + m_\chi\) and \(\mu = m_p m_\chi / (m_p + m_\chi)\) are the total and reduced masses of the \(\chi p\)–pair. Having integrated over \(\tilde{p}\) we arrive at the expression

\[
d^3\sigma(E'_e, E_e, k'_e, k_e, \tilde{n}) = \frac{\mu^2}{4 \pi^3} \frac{\zeta^{(dm)}(\lambda, \tilde{n})}{\tau_n f_n \beta_e} \left(1 + a^{(dm)}_e \frac{k'_e \cdot \tilde{k}_e}{E'_e E_e}\right) \Theta(\mu (E_0 + E_e - E'_e) - \frac{1}{4} q^2) \times \sqrt{2 \mu (E_0 + E_e - E'_e)} d^3 k d^3 k'_e,
\]

where \(\Theta(z)\) is the Heaviside function, \(\tilde{n} = \tilde{k}/k, d\Omega_e = 2 \pi \sin \theta_e d\theta_e\) and \(d\Omega_\tilde{n} = \sin \theta d\theta d\phi\) with the standard definition of the kinematics of the electrodisintegration of the deuteron [17][19] (see Fig. 1 of Refs. [17][19]), where \(\tilde{k}_e \cdot k_e = k'_e k_e \cos \theta_e\) and \(\tilde{n} \cdot \tilde{q} = q \cos \theta.\) Then, it is convenient to transcribe Eq.\((13)\) into the form

\[
\frac{d^3\sigma(E'_e, E_e, k'_e, k_e, \tilde{n})}{dE'_e d\Omega_e d\tilde{n}} = -\frac{\mu}{8 \pi^3} \frac{\zeta^{(dm)}(\lambda, \tilde{n})}{\tau_n f_n \beta_e} \left(1 + a^{(dm)}_e \frac{k'_e \cdot \tilde{k}_e}{E'_e E_e}\right) \Theta(\mu (E_0 + E_e - E'_e) - \frac{1}{4} q^2) \times \sqrt{\frac{\mu}{M} (E_0 + E_e - E'_e)} - \frac{1}{4} q^2 d^3 \tilde{n} d^3 k d^3 k'_e.
\]

For the wave function of the deuteron \(\Phi_d(\vec{\ell})\) we may use the expression

\[
\Phi_d(\vec{\ell}) = \sqrt{\frac{8\pi}{\sum_{i,j=1}^{n} C_i C_j \sum_{i=1}^{n} m_i m_j (m_i + m_j)^3}} \int d^3 \ell (2\pi)^3 |\Phi_d(\vec{\ell})|^2 = 1
\]

with parameters \(C_i\) and \(m_i\) taken from the paper by Machleidt et al. [11]. Also we may follow Gilman and Gross [44] and describe the wave function \(\Phi_d(\vec{\ell})\) by the expression

\[
\Phi_d(\vec{\ell}) = -\sqrt{\frac{8 \pi \sqrt{-m_{\varepsilon_d}}}{\varepsilon_d + \ell^2}} \frac{1 - \frac{m_{\varepsilon_d}}{P_0^2}}{p_0^2} \left(1 + \frac{\ell^2}{p_0^2}\right)^{3/2} \int d^3 \ell (2\pi)^3 |\Phi_d(\vec{\ell})|^2 = 1,
\]

where \(-m_{\varepsilon_d} = 0.940 \times 10^{-3}\) GeV\(^2 = 2.09 \times 10^{-3}\) GeV\(^2\) and \(p_0^2 = 0.15\) GeV\(^2\). The squared 3–momenta \(\ell^2\) and \(q^2\) are defined by

\[
\ell^2 = m_p (E_0 + E_e - E'_e) + q \sqrt{m_p (E_0 + E_e - E'_e)} - \frac{1}{4} q^2 \cos \theta, \\
q^2 = (k_e - k'_e)^2 = (k_e - k'_e)^2 + 4k'_e k_e \sin^2 \frac{\theta_e}{2}.
\]
Using the definition of the correlation function $\zeta^{(\text{dm})}$ (see Eq. (30)) we may rewrite the triple–differential cross section Eq. (16) as follows

$$\frac{d^3\sigma(E'_{e+}, E_{e+}, E_{e-}, \vec{n})}{dE_{e-}d\Omega_{e-}d\Omega_{e+}} = \sigma_0 \frac{k_{n}E'_{e+}}{16\pi^2\beta_e} \left(1 + a^{(\text{dm})} \frac{k_{e-} \cdot k_{e+}}{E'_{e+}E_{e+}} \right) \Theta \left(m_p(E_0 + E_{e+} - E_{e-}) - 4q^2\right)$$

$$\times \left|m_p(E_0 + E_{e+} - E_{e-}) - 4q^2\right|^2 \frac{1}{4} q^2 \left|P_{d}(\sqrt{m_p(E_0 + E_{e+} - E_{e-}) + q\sqrt{m_p(E_0 + E_{e+} - E_{e-}) - 4q^2 \cos \theta}})\right|^2,$$

where all momenta and energies and the dimension of the wave function of the deuteron are measured in MeV. Then, the scale parameter $\sigma_0$ is equal to

$$\sigma_0 = 2m_p \frac{\zeta^{(\text{dm})}}{\tau_{n\overline{f}_n}} = \frac{0.12}{m_n - m_{\chi}} \times \frac{\text{fb}}{\text{MeV}}.$$

The triple–differential cross section for the reaction $e^- + d \rightarrow \chi + p + e^-$, given by Eq. (18), can be used for the analysis of the experimental data on searches for dark matter fermions in coincidence experiments [16, 48, 50]. For $(m_n - m_{\chi}) \simeq 0.023 \text{ MeV}$ (see the Appendix and a discussion below Eq. (A–23)), the scale parameter $\sigma_0$ increases by four orders of magnitude $\sigma_0 \simeq 24.7 \text{ pb/MeV}$.

III. DISCUSSION

We have analyzed the electrodisintegration of the deuteron into dark matter fermions and protons $e^- + d \rightarrow \chi + p + e^-$ close to threshold. Such a disintegration is induced by the electron–neutron inelastic scattering $e^- + n \rightarrow \chi + e^-$ with energies of incoming electrons larger than the deuteron binding energy $|\varepsilon_d| = 2.224575(9) \text{ MeV}$. The strength of the reaction $e^- + n \rightarrow \chi + e^-$ is caused by the strength of the neutron dark matter decay mode $n \rightarrow \chi + \nu_e + \bar{\nu}_e$, which has been proposed in [9] for an explanation of the neutron lifetime anomaly in case of an unobservability (see [6, 7]) of the dark matter decay mode $n \rightarrow \chi + e^- + e^+$ [11], where the production of the electron–positron pair can be below the reaction threshold [9]. Following such an assumption that the production of the electron–positron pair can be below the reaction threshold for a confirmation of an existence of the dark matter decay mode $n \rightarrow \chi + e^- + e^+$ we have proposed in [9] to analyze the low–energy electron–neutron inelastic scattering $e^- + n \rightarrow \chi + e^-$, which can be in principle distinguished above the background of the low–energy electron–neutron elastic scattering $e^- + n \rightarrow n + e^-$. The effective interaction Eq. (1) is supported by the effective quantum field theory model with gauge $SU_L(2) \times U_Y(1) \times U^I_R(1) \times U^K_L(1)$ symmetry, where the SM and dark matter sectors are described by the effective low-energy Lagrangian $\mathcal{L}_{\text{fact-LMSET-DM}}$ (see the Appendix). The SM part of this effective field theory, determined by the effective low-energy Lagrangian $\mathcal{L}_{\text{fact-LMSET}}$ invariant under $SU(2)_L \times U_Y(1)$ gauge symmetry, is gauge invariant and renormalizable [17, 18]. This has been demonstrated in [17] by examples of the calculation of the radiative corrections of order $O(\alpha E_e/m_N)$ to the neutron lifetime and correlation coefficients of the neutron beta decay [53]. As has been shown in [9] (see also the Appendix) such a quantum field theory model allows i) to derive the effective interaction Eq. (1) in the tree–approximation for the dark matter spin–1 boson $Z'$ exchanges with $h_Y = h_4$ (see Eq. (A–23) in the Appendix), and ii) following the scenario by Cline and Cornell [22] to show that dynamics of dark matter fermions with mass $m_\chi < m_n \sim 1 \text{ GeV}$ and a light dark matter spin–1 boson $Z''$, responsible for a repulsion between dark matter fermions, does not prevent neutron stars to reach masses of about $2M_\odot$. It has been also noticed [9] that the processes $n \rightarrow \chi + \nu_e + \bar{\nu}_e$, $n + n \rightarrow \chi + \chi$, $n + n \rightarrow \chi + \chi + \nu_e + \bar{\nu}_e$ and $\chi + \chi \rightarrow n + n$, allowed in such a quantum field theory model, can serve as URCA processes for the neutron star cooling [36–38]. The effective quantum field theory, described by the Lagrangian $\mathcal{L}_{\text{fact-LMSET-DM}}$ (see the Appendix) is fully low-energy one. The application of this effective theory to the analysis of the searches of dark matter in the LHC experiments is not a straightforward and demands a special consideration, which we are planning to carry out in our forthcoming publication.

However, in order to have more processes with particles of the Standard Model in the initial and final states allowing to search dark matter in terrestrial laboratories we have turned to the analysis of the dark matter decay mode $n \rightarrow \chi + e^- + e^+$ through the electrodisintegration of the deuteron $e^- + d \rightarrow \chi + p + e^-$ induced by the interaction $(4\chi e^- e^+)$ [9]. We have calculated the triple–differential cross section for the reaction $e^- + d \rightarrow \chi + p + e^-$ (see Eq. (18)), which can be used for the analyze of traces of dark matter in coincidence experiments on the electrodisintegration of the deuteron $e^- + d \rightarrow n + p + e^-$ close to threshold [16, 49, 51]. An important property of this cross section is its independence of the azimuthal angle $\phi$ between the scattering and reaction planes (see Fig. 1 of Ref. [48]). Using the experimental conditions of Ref. [48]: $E_e = 50 \text{ MeV}$, $E_x = E_e = E'_{e+} = 8 \text{ MeV}$, $\theta_{\text{c.m.}} = 40^\circ$, $\theta = 0$ and the wave function of
Indeed, in [49, 50] the electrodisintegration of the deuteron into dark matter fermions and protons close to threshold can be performed in coincidence experiments on the electrodisintegration of the deuteron.

Nevertheless, we believe that an observation of the electrodisintegration of the deuteron into dark matter fermions and protons close to threshold detecting outgoing electrons and protons from the deuteron can be performed in coincidence experiments on the electrodisintegration of the deuteron. Above the background of neutrons, which are caused by the same interaction Eq. (1), on a formation of dark matter in the Universe during the evolution of the Universe [51–53].

Finally we would like notice that it would be very interesting to understand an influence of the reactions $n \rightarrow \chi + \nu_e + \bar{\nu}_e$, $e^- + n \rightarrow \chi + e^-$, $e^- + d \rightarrow \chi + p + e^-$, $\nu_e(\bar{\nu}_e) + n \rightarrow \chi + \nu_e(\bar{\nu}_e)$ and $\nu_e(\bar{\nu}_e) + d \rightarrow \chi + p + \nu_e(\bar{\nu}_e)$, which are caused by the same interaction Eq. (1), on a formation of dark matter in the Universe [51, 53].

IV. ACKNOWLEDGEMENTS

We are grateful to Hartmut Abele for fruitful discussions stimulating the work under this paper. The work of A. N. Ivanov was supported by the Austrian “Fonds zur Förderung der Wissenschaftlichen Forschung” (FWF) under contracts P31702-N27 and P26636-N20 and “Deutsche Forschungsgemeinschaft” (DFG) AB 128/5-2. The work of R. Höllwieser was supported by the Deutsche Forschungsgemeinschaft in the SFB/TR 55. The work of M. Wellenzohn was supported by the MA 23.
Appendix A: Effective quantum field theory of low-energy strong, electroweak and dark matter interactions for the description of neutron-dark-matter decays and related processes

Renormalizable and gauge invariant effective quantum field theory of strong and electroweak low-energy interactions of pions, nucleons, electrons and neutrinos

The problem of the neutron decays into dark matter is entirely the low-energy one. In [9] we have proposed a quantum field theoretic model of low-energy electroweak and dark matter interactions for the proton and neutron coupled to the electron and neutrino and dark matter particles. Unfortunately, the part of this quantum field theoretic model for the nucleon and leptons cannot be treated as a hadronized version of the Standard Model (SM) [15, 16]. This problem has been overcome in [17, 18], where we have proposed the effective quantum field theory of strong and electroweak low-energy interactions for pions, proton and neutron coupled to the electron and neutrino that we have called as the \( \mathcal{L}_{\sigma} \) model in SET. In this theory strong low-energy pion-nucleon interactions are described by the linear \( \alpha \)-model \( (L_{\alpha}) \) [69]. Such an equivalence of the low-energy Lagrangian of the infinite mass L and electroweak low-energy interactions for pions, proton and neutron coupled to the electron and neutrino, that we have have called as the \( \mathcal{L}_{\sigma} \) model (L\( \sigma \)) reproduces all results of the current algebra in the form of effective chiral perturbations of pion-nucleon interactions with non-linear realization of chiral \( SU(2) \times SU(2) \) symmetry and different parametrizations of the pion-field [61, 63]. For the exponential parametrization of the pion-field the Lagrangian \( L_{\sigma} \) of the \( \mathcal{L}_{\sigma} \) model, taken as \( m_{\sigma} \rightarrow \infty \), reduces to the Lagrangian \( L_{\text{HBPT}} \) of the chiral quantum field theory with the structure of low-energy interactions agreeing well with Gasser-Leutwyler’s chiral perturbation theory (ChPT) or the heavy baryon chiral perturbation theory (HBChPT) [61, 63] with chiral \( SU(2) \times SU(2) \) symmetry (see, for example, Ecker [69]). Such an equivalence of the \( \mathcal{L}_{\sigma} \) model with the Gasser-Leutwyler’s ChPT has been also proved in [62] in the leading logarithmic approximation (see also [18]).

As has been shown in [17, 18] the effective quantum field theory \( \mathcal{L}_{\sigma} \) is gauge invariant and renormalizable theory, allowing, for example, a quantitative analysis of next-to-leading order corrections in the large nucleon mass expansion \( O(\alpha E_{N}/m_{N}) \) to radiative corrections of order \( O(\alpha/\pi) \), calculated to leading order in the large nucleon mass expansion [Sirlin [63], Shamm [55] and Ivanov et al. [58, 59, 60]]. In this Appendix we extend the effective quantum field theory \( \mathcal{L}_{\sigma} \) by the dark matter sector as it has been done in [9]. As a result we obtain a renormalizable gauge invariant quantum field theoretic model allowing to take into account the contributions of neutron- and lepton-dark matter interactions at low energies.

In the symmetric phase the Lagrangian of the effective quantum field theory \( \mathcal{L}_{\sigma} \) takes the form [17]:

\[
\mathcal{L}_{\text{Lag}} = \Psi_{NL} i \gamma^\mu \left( \partial_\mu + i g \frac{1}{2} \sigma L \cdot W_\mu \right) \bar{\Psi}_{NL} + \bar{\psi}_{R} i \gamma^\mu \left( \partial_\mu + i g' B_\mu \right) \psi_{R} + \bar{\psi}_{N} i \gamma^\mu \partial_\mu \psi_{N}
\]

\[
- \sqrt{2} g_{N} \left( \Psi_{NL} \Phi \bar{\psi}_{R} + \bar{\psi}_{R} \Phi^\dagger \Psi_{NL} \right) - \sqrt{2} g_{N} \left( \Psi_{NL} \Phi^d \psi_{N} + \bar{\psi}_{N} \Phi^d \Psi_{NL} \right)
\]

\[
+ \left( \bar{\psi}_{N} \Phi^t - i g \frac{1}{2} \Phi^t \sigma L \cdot W_\mu + i g \frac{1}{2} \Phi B_\mu \right) \left( \partial^\mu \Phi^t + i g \frac{1}{2} \sigma L \cdot W_\mu - i g' \frac{1}{2} B_\mu \Phi^t \right)
\]

\[
+ \mu^2 \Phi^t \Phi - \frac{1}{4} \gamma \left( \Phi^t \Phi \right)^2 - \frac{1}{4} \bar{W}_{\mu \nu} \cdot W_{\mu \nu} - \frac{1}{4} B_{\mu \nu} B_{\mu \nu} + \Psi_{NL} i \gamma^\mu \left( \partial_\mu + i g \frac{1}{2} \sigma L \cdot W_\mu - i g' \frac{1}{2} B_\mu \right) \Psi_{\ell L}
\]

\[
+ \bar{\psi}_{N} i \gamma^\mu \left( \partial_\mu - i g' B_\mu \right) \psi_{N} - \sqrt{2} g_{N} \left( \Psi_{NL} e_{R} \bar{\psi}_{R} + \bar{\psi}_{R} \Phi^t \Psi_{NL} \right) + \left( \partial_\mu \phi - i g \frac{1}{2} \phi^t \cdot \bar{W}_\mu - i g' \frac{1}{2} \sigma L \phi^t \cdot W_\mu \right)
\]

\[
\times \left( \partial^\mu \phi + i g \frac{1}{2} \sigma \cdot W_\mu \phi + i g' \frac{1}{2} B_{\mu} \phi \right) + \mu^2 \phi \phi - \lambda \phi^4 \phi^2,
\]

where the hadron, lepton and Higgs-boson field operators are defined by [17]:

\[
\Psi_{NL} = P_{L} \Psi_{N} = P_{L} \left( \begin{array}{c} \psi_{p} \\ \psi_{n} \end{array} \right), \; \psi_{R} = P_{R} \psi_{p}, \; \psi_{N} = P_{R} \psi_{n},
\]

\[
\Phi = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \sigma + i \pi^3 \\ \pi^2 + i \pi^3 \end{array} \right), \; \Phi^d = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \sigma + i \pi^0 \\ \pi^0 + i \pi^3 \end{array} \right),
\]

\[
\Phi^t = -i \tau_{2 L} \Phi^d = \frac{1}{\sqrt{2}} \left( \begin{array}{c} i (\pi^1 - i \pi^2) \\ \pi^1 - i \pi^2 \end{array} \right), \; \Phi = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \sigma + i \pi^0 \\ \pi^0 + i \pi^3 \end{array} \right),
\]

\[
\Psi_{\ell L} = P_{L} \left( \begin{array}{c} \psi_{\nu} \\ \psi_{\ell} \end{array} \right), \; \psi_{\ell R} = P_{R} \psi_{\ell}, \; \phi = \left( \begin{array}{c} \phi^t \\ \phi^0 \end{array} \right)
\]
with \( P_{L,R} = (1 + \gamma^5)/2 \) are the left-right projection operators with the properties \( P_{L,R}^2 = P_{L,R} \) and \( P_L P_R = P_R P_L = 0 \).

In Eq. (A-2) \( \psi_p, \bar{\psi}_n, \sigma, \pi^\pm, \pi^0 \) are the field operators of the proton (\( p \)), neutron (\( n \)), \( \sigma \)-meson (\( \sigma \)) and pions (\( \pi^\pm, \pi^0 \)), respectively. Then, \( \bar{\psi}_n, \psi_p, \phi^+, \phi^0 \) are the field operators of the electron neutrino (\( \nu \)), electron (\( e^- \)) and Higgs-bosons (\( \phi^+, \phi^0 \)), respectively. For the electroweak gauge boson operators we use the standard notations \( \tilde{W}_\mu \) and \( B_\mu \) \ref{15} \ref{16}. The operators of the field strength tensors \( \tilde{W}_\mu \) and \( B_\mu \) of the electroweak gauge boson fields are given by

\[
\tilde{W}_\mu = \partial_\mu \bar{W}_\nu - \partial_\nu \bar{W}_\mu - g \bar{W}_\mu \times \bar{W}_\nu,
\]

\[
B_\mu = \partial_\mu \bar{B}_\nu - \partial_\nu \bar{B}_\mu \tag{A-3}
\]

The field operators Eq. (A-1) have the following properties under the \( SU(2)_L \times U(1)_Y \) infinitesimal transformations

\[
\Psi_{NL} \tilde{\alpha}_L \gamma^\mu \Psi_{NL}^* = \left(1 + i \frac{1}{2} \bar{\tau}_L \cdot \tilde{\alpha}_L + i \frac{1}{2} Y \gamma_Y \right) \Psi_{NL},
\]

\[
\psi_{pR} \tilde{\alpha}_L \gamma^\mu \psi_{pR}^* = \left(1 + i \frac{1}{2} Y \gamma_Y \right) \psi_{pR},
\]

\[
\Phi \tilde{\alpha}_L \gamma^\mu \Phi^* = \left(1 + i \frac{1}{2} \bar{\tau}_L \cdot \tilde{\alpha}_L + i \frac{1}{2} Y \gamma_Y \right) \Phi,
\]

\[
\psi_{eR} \tilde{\alpha}_L \gamma^\mu \psi_{eR}^* = \left(1 + i \frac{1}{2} Y \gamma_Y \right) \psi_{eR},
\]

\[
\phi \tilde{\alpha}_L \gamma^\mu \phi^* = \left(1 + i \frac{1}{2} \bar{\tau}_L \cdot \tilde{\alpha}_L + i \frac{1}{2} Y \gamma_Y \right) \phi,
\]

\[
\tilde{W}_\mu \tilde{\alpha}_L \gamma^\mu \tilde{W}_\mu^* = \tilde{W}_\mu + \tilde{W}_\mu \times \tilde{\alpha}_L - \frac{1}{g} \partial_\mu \tilde{\alpha}_L,
\]

\[
B_\mu \tilde{\alpha}_L \gamma^\mu B_\mu^* = B_\mu - \frac{1}{g} \partial_\mu \alpha_Y,
\]

\[
\tilde{W}_\mu \tilde{\alpha}_L \gamma^\mu \tilde{W}_\mu = \tilde{W}_\mu + \tilde{W}_\mu \times \tilde{\alpha}_L.
\]

\[
B_\mu \tilde{\alpha}_L \gamma^\mu B_\mu^* = B_\mu \tag{A-4}
\]

where \( \frac{1}{2} \bar{\tau}_L \) (or \( \bar{I}_L = \frac{1}{2} \bar{\tau}_L \)) and \( Y \) are the operators of the \textit{weak isospin} and \textit{weak hypercharge}, respectively, \( \tilde{\alpha}_L \) and \( \alpha_Y \) are infinitesimal parameters of the \( SU(2)_L \) and \( U(1)_Y \) gauge group transformations, respectively. The operators of the third component \( I_{3L} \) of the \textit{weak isospin} \( I_L \) and the \textit{weak hypercharge} \( Y \) are related by \( Q = I_{3L} + Y/2 \) \ref{59} \ref{60} (see also \ref{15} \ref{16}), where \( Q \) is the operator of electric charge, measured in the proton electric charge \( e \).

The eigenvalues of the third component of the \textit{weak isospin} and \textit{weak hypercharge} are \( (I_{3L})_{pL}, (I_{3L})_{nL} = (+1/2, +1), \) \( (I_{3L})_{pR}, (I_{3L})_{nR} = (0, +2), \) \( (I_{3L})_{nR}, (I_{3L})_{nR} = (0, 0), \) \( (I_{3L})_{\sigma, \pi, \mu, \tau, \phi} = (+1/2, -1), \) \( (I_{3L})_{\sigma, \pi, \mu, \tau, \phi} = (-1/2, +1), \) and \( (I_{3L})_{\sigma, \pi, \mu, \tau, \phi} = (-1/2, -1), \) respectively.

At \( g = g' = 0 \) the effective low-energy Lagrangian \( \mathcal{L}_{\text{Lag}} \) reduces to the Lagrangian \( \mathcal{L}_{\text{Lag}} \) of the linear \( \sigma \)-model \ref{17}

\[
\mathcal{L}_{\text{Lag}} = \bar{\psi}_{NL} i \gamma^\mu \partial_\mu \psi_{NL} + \bar{\psi}_{pR} i \gamma^\mu \partial_\mu \psi_{pR} + \bar{\psi}_{nR} i \gamma^\mu \partial_\mu \psi_{nR} - \sqrt{2} g_{\pi N} \left( \bar{\psi}_{NL} \Phi \psi_{pR} + \bar{\psi}_{pR} \Phi^\dagger \psi_{NL} \right)
\]

\[
- \sqrt{2} g_{\pi N} \left( \bar{\psi}_{NL} \Psi_{c} \psi_{nR} + \bar{\psi}_{nR} \Phi^\dagger \Psi_{c} \right) + \partial_\mu \Phi^\dagger \partial^\mu \Phi + \mu^2 \Phi^\dagger \Phi - \frac{1}{2} \gamma \left( \Phi^\dagger \Phi \right)^2 \tag{A-5}
\]

invariant under chiral \( SU(2) \times SU(2) \) transformations \ref{17}. It can be rewritten in the standard form \ref{55} \ref{57} (see \ref{17}):
Dark matter sector of the effective quantum field theory of strong, electroweak and dark matter low-energy interactions for the description of neutron-dark-matter decays and related processes

Now we are able to add the dark matter sector. According to [9], such a sector contains a dark matter fermion $\chi$, a dark matter spin–1 gauge boson $C_\mu$, and a complex scalar boson $\varphi$. The Lagrangian of the dark matter sector we define as follows [9]

$$\mathcal{L}_{\text{DM}} = \bar{\psi}_R i \gamma^\mu (\partial_\mu + i e_\chi C_\mu) \psi_R - \frac{1}{4} C_\mu C^{\mu \nu} + (\partial_\mu - i e_\chi C_\mu) \varphi (\partial_\mu + i e_\chi C_\mu) \varphi + \kappa^2 |\varphi|^2 - \gamma |\varphi|^4$$

$$+ \bar{\psi}_L i \gamma^\mu \partial_\mu \psi_L - \sqrt{2} f_\chi (\bar{\psi}_R \psi_L \varphi + \bar{\psi}_L \psi_R \varphi^* ) + \Psi_{LL} i \gamma^\mu (\partial_\mu + i g \frac{1}{2} \tau \cdot \tilde{W}_\mu - i g \frac{1}{2} B_\mu + i e_\chi C_\mu) \Psi_{LL}$$

$$- 2\zeta_\chi (\Psi_{LL} \psi_L e^\tau \varphi + \varphi^* \phi^\dagger \bar{\psi}_R \Psi_{LL} ) + 2\sqrt{2} \xi_\chi (\varphi^* \bar{\psi}_R \gamma^\mu (\partial_\mu + i e_\chi C_\mu) \psi_R - i (\partial_\mu - i e_\chi C_\mu) \bar{\psi}_R \gamma^\mu \psi_R \varphi ) \right) \right), \quad (A-7)$$

where $C_\mu = \partial_\mu C_\tau - \partial_\tau C_\mu$ is the field strength tensor operator of the dark matter spin–1 gauge boson field $\psi_R = P_L \psi_{\chi R}$ and $\psi_R = P_R \psi_{\chi R}$ are the field operators of the left- and right-handed dark matter fermions $\chi$, $e_\chi$ is a gauge coupling constant or the dark matter charge of the right-handed dark matter fermion $\chi$ and the left-handed SM electron and neutrino. The dark matter hypercharge $Y'$ is equal to $Y' = +1$ for the right-handed dark matter fermion $\psi_{\chi R}$, for the left-handed leptons $\Psi_{LL}$ and the complex scalar field $\varphi$, and $Y' = 0$ for the left-handed dark matter fermion $\psi_{\chi L}$, respectively.

The parameters $\kappa^2$ and $\gamma$ define a non–vanishing vacuum expectation value of the dark matter scalar field $\varphi$, that leads to a non–vanishing mass $m_\alpha$ of the dark matter fermion $\chi$, which should be proportional to the coupling constant $f_\chi$. Then, the coupling constant $\xi_\chi$ defines a mixing of the right-handed neutron with the right-handed dark matter fermion. The term, proportional to the coupling constant $\xi_\chi$, redefines the electron mass in $\mathcal{L}_{\text{LeMSET}}$ as follows

$$\sqrt{2} g_e (\bar{\Psi}_{LL} \psi_R \varphi + \varphi^* \phi^\dagger \bar{\psi}_R \Psi_{LL} ) \rightarrow 2\zeta_\chi (\bar{\Psi}_{LL} \psi_R \varphi + \varphi^* \phi^\dagger \bar{\psi}_R \Psi_{LL} ) \right), \quad (A-8)$$

The Lagrangian Eq. (A-7) is invariant under $U'_y(1)$ dark matter gauge transformations

$$\psi_R \rightarrow \psi'_R = e^{i a_\chi} \psi_R , \quad \Psi_{LL} \rightarrow \Psi'_L = e^{i a_\chi} \Psi_{LL} , \quad \varphi \rightarrow \varphi' = e^{i a_\chi} \varphi , \quad C_\mu \rightarrow C'_\mu = C_\mu - \frac{1}{e_\chi} \partial_\mu a_\chi,$$

$$\psi_L \rightarrow \psi'_L = \psi_L , \quad \psi_R \rightarrow \psi'_R = \psi_R , \quad \tilde{W}_\mu \rightarrow \tilde{W}'_\mu = \tilde{W}_\mu , \quad B_\mu \rightarrow B'_\mu = B_\mu. \quad (A-9)$$

Thus, the total Lagrangian of the effective quantum field theory of strong, electroweak and dark matter interactions we take in the following form

$$\mathcal{L}_{\text{LeMSET}} = \Psi_{NL} i \gamma^\mu \left( \partial_\mu + i g \frac{1}{2} \tau \cdot \tilde{W}_\mu + i g \frac{1}{2} B_\mu \right) \Psi_{NL} + \bar{\psi}_{R} i \gamma^\mu (\partial_\mu + i g \frac{1}{2} B_\mu) \psi_R$$

$$+ \bar{\psi}_{L} i \gamma^\mu \partial_\mu \psi_R - \sqrt{2} g_{2N} \left( \bar{\Psi}_{NL} \Phi \psi_R + \bar{\psi}_{R} \Phi^\dagger \Psi_{NL} \right) - \sqrt{2} g_{2N} \left( \bar{\Psi}_{NL} \Phi^\dagger \psi_R + \bar{\psi}_{R} \Phi^\dagger \Psi_{NL} \right)$$

$$+ \left( \partial_\mu \Phi^\dagger - i g \frac{1}{2} \Phi^\dagger \tau \cdot \tilde{W} + i g \frac{1}{2} \Phi^\dagger B \right) \left( \partial_\mu + i g \frac{1}{2} \tau \cdot \tilde{W} + i g \frac{1}{2} B \right) + \gamma (\Phi^\dagger \Phi)^2$$

$$- \frac{1}{4} \tilde{W}_\mu \cdot \tilde{W}^\mu - \frac{1}{4} B_\mu B^\mu + \bar{\psi}_{L} i \gamma^\mu (\partial_\mu - i g \frac{1}{2} B_\mu) \psi_R + \left( \partial_\mu \phi^\dagger + i g \frac{1}{2} \phi^\dagger \tau \cdot \tilde{W} - i g \frac{1}{2} \phi^\dagger B \right)$$

$$\times \left( \partial_\mu ^{\phi} + i g \frac{1}{2} \tau \cdot \tilde{W} \phi ^{\phi} + i g \frac{1}{2} B \phi ^{\phi} \right) + \mu^2 \phi^\dagger \phi^\dagger - \lambda (\phi^\dagger \phi)^2 + \bar{\psi}_{L} i \gamma^\mu \partial_\mu \psi_R - \sqrt{2} f_\chi (\bar{\psi}_{R} \psi_L \varphi + \bar{\psi}_{L} \psi_R \varphi^* )$$

$$+ \bar{\Psi}_{LL} i \gamma^\mu \left( \partial_\mu + i g \frac{1}{2} \tau \cdot \tilde{W} - i g \frac{1}{2} B + i e_\chi C_\mu \right) \Psi_{LL} - 2\zeta_\chi (\bar{\Psi}_{LL} \psi_L e^\tau \varphi + \varphi^* \phi^\dagger \bar{\psi}_R \Psi_{LL} )$$

$$+ 2\sqrt{2} \xi_\chi (\varphi^* \bar{\psi}_R \gamma^\mu (\partial_\mu + i e_\chi C_\mu) \psi_R - i (\partial_\mu - i e_\chi C_\mu) \bar{\psi}_R \gamma^\mu \psi_R \varphi ) \right) \right), \quad (A-10)$$

This Lagrangian is invariant under transformations of the gauge $SU(2) \times U(1) \times U'_y(1)$ group [9]. It is well-known [59, 60] that in the spontaneously broken or physical phase the Lagrangians like $\mathcal{L}_{\text{LeMSET}}$ should contain only physical states. It is exactly shown in [9, 17, 15]. The spontaneously broken phase for the LeMSET part of the Lagrangian $\mathcal{L}_{\text{LeMSET}}$ has been investigated in details in [17, 15]. In turn, for the $\mathcal{L}_{\text{DM}}$ the spontaneously broken phase has been analyzed in [9]. Here we reproduce the results obtained in [9] (see Eqs.(34) - (38) in Ref. [9]).

In order to define the dark matter sector in the spontaneously broken or physical phase we take the complex dark matter scalar field $\varphi$ in the following form $\varphi = e^{i a_\chi} \rho / \sqrt{2}$ [88, 89], where $\rho$ is a dark matter scalar field, and make a
gauge transformation $\psi_R \rightarrow e^{i\alpha} \psi_R$ and $\Psi_{\ell L} \rightarrow e^{i\alpha} \Psi_{\ell L}$. As a result we arrive at the Lagrangian

$$\mathcal{L}_{\text{DM}} = \bar{\psi}_R i \gamma^\mu (\partial_\mu + i \partial_\mu \alpha + i e \chi C_\mu) \psi_R + \bar{\psi}_L i \gamma^\mu (\partial_\mu - i \partial_\mu \alpha) \psi_L - f_\chi \bar{\psi}_L \psi_R - \frac{1}{4} C_{\mu \nu} C^{\mu \nu} + \frac{1}{2} \partial_\mu \partial^\mu \rho$$

+ $\frac{1}{2} (\partial_\mu \alpha + e \chi C_\mu (\partial^\mu \alpha + e \chi C^{\mu}) \rho^2 + \frac{1}{2} \gamma^2 \rho^2 - \frac{1}{2} V_{\psi L} i \gamma^\mu (1 + \gamma^5) \psi_R - \bar{\psi}_L \psi_R + \bar{\psi}_L \chi \gamma^\mu \gamma^5 \psi_R$$

- $\sqrt{V} C_\mu (\Psi_{\ell L} \psi_D \phi + \psi_D \Psi_{\ell L} \phi)$,

(A-11)

where the ellipses denotes the contribution of the electroweak gauge bosons. The field $C_\mu + \partial_\mu \alpha / e \chi$ can be treated as a new dark matter spin–1 field $Z'$. In the form of the dark matter scalar $\rho$–field $V(\rho) = -\frac{1}{2} \kappa^2 \rho^2 + \frac{1}{2} \gamma \rho^4$ possesses a minimum at $\langle \rho \rangle = v_\chi = \sqrt{\kappa^2/\gamma}$. Introducing a new scalar field $\rho = v_\chi + S$ and taking the SM part of the effective low-energy system, described by the Lagrangian $\mathcal{L}_{\text{SM}}$, in the spontaneously broken or physical phase, we transcribe the Lagrangian $\mathcal{L}_{\text{SM}}$ into the form

$$\mathcal{L}_{\text{SM}} = \bar{\psi}_R i \gamma^\mu (\partial_\mu - m_\chi) \psi_R + \bar{\psi}_L i \gamma^\mu (\partial_\mu - m_\chi) \psi_L + \bar{\psi}_L i \gamma^\mu \partial_\mu \psi_L = \frac{1}{4} Z_{\mu \nu} Z^{\mu \nu}$$

+ $\frac{1}{2} M_\chi^2 Z_{\mu \nu}^\dagger Z^{\mu \nu} + g_\chi (\bar{\psi}_L i \gamma^\mu (\partial_\mu + e \chi Z_\mu) (1 + \gamma^5) \psi_L - \bar{\psi}_L \gamma^\mu \gamma^5 \psi_L - \bar{\psi}_L \mu_\chi \gamma^\mu \gamma^5 \psi_L$$

- $\bar{\psi}_L \chi \gamma^\mu \gamma^5 \psi_L Z_\mu - 1/2 \nabla \phi \partial^\mu \phi - 1/2 m_S^2 S^2 - 1/4 \gamma S^4 + \ldots$,

(A-13)

where the ellipses denotes the contributions, which are not important for the analysis of the neutrino- and lepton-dark matter interactions. Then, $g_\chi = \xi_\chi v_\chi$ and $m_\chi$, $M_{Z'}$ and $m_S$ are masses of the dark matter fermion $\chi$, dark matter spin–1 $Z'$ and dark matter scalar $S$ fields, respectively, equal to

$$m_\chi = f_\chi v_\chi$$

$$M_{Z'} = e_\chi v_\chi$$

$$m_S = \sqrt{2} e_\chi v_\chi.$$  

(A-14)

In principle, a mass of the dark matter scalar $S$–field is arbitrary. In order to allow the $n \rightarrow \chi$ transitions only by virtue of the dark matter spin–1 boson $Z'$ we may delete the $S$–field from its interactions taking the limit $m_S \rightarrow \infty$. This agrees well with the Appelquist–Carazzone decoupling theorem [90]. Indeed, keeping the ratio $v_\chi = \sqrt{\kappa^2/\gamma}$ fixed one may set $\gamma \rightarrow \infty$. This is similar to the removal of the scalar $\sigma$–meson from its interactions in the linear $\sigma$–model of strong low-energy interactions [61] (see also [11] [38] and [69]). As a result, the Lagrangian Eq. (A-13) becomes equal to

$$\mathcal{L}_{\text{DM}'} = \bar{\psi}_R i \gamma^\mu (\partial_\mu - m_\chi) \psi_R + \bar{\psi}_L i \gamma^\mu (\partial_\mu - m_\chi) \psi_L + \bar{\psi}_L i \gamma^\mu \partial_\mu \psi_L$$

- $\frac{1}{4} Z_{\mu \nu} Z^{\mu \nu} + \frac{1}{2} M_\chi^2 Z_{\mu \nu}^\dagger Z^{\mu \nu} + \mathcal{L}_{\text{DM}'} + \ldots$,

(A-15)

where $\mathcal{L}_{\text{DM}'}$ is the Lagrangian of the effective low-energy neutron-lepton-dark matter interactions given by

$$\mathcal{L}_{\text{DM}'} = g_\chi (\bar{\psi}_L i \gamma^\mu (1 + \gamma^5) \partial_\mu \psi_L - \partial_\mu \bar{\psi}_L i \gamma^\mu (1 + \gamma^5) \psi_L - m_\chi) \psi_L + \bar{\psi}_L \chi \gamma^\mu \gamma^5 \psi_L$$

- $\frac{1}{2} e_\chi \bar{\psi}_L i \gamma^\mu (1 + \gamma^5) \psi_L Z_\mu - 1/2 e_\chi \bar{\psi}_L \gamma^\mu (1 - \gamma^5) \psi_L Z_\mu - 1/2 e_\chi \bar{\psi}_L \gamma^\mu (1 - \gamma^5) \psi_L Z_\mu$.

(A-16)

In [9] we have estimated the coupling constant $g_\chi$ by using the finite contribution to the neutron mass of the first term in the effective low-energy neutron- and lepton-dark matter interactions described by the Lagrangian Eq. (A-16). The
Summing up the contributions of the Feynman diagrams in Fig. 3 we obtain the amplitude of the neutron dark matter

g_n \rightarrow \bar{\ell} + \ell' + \bar{\ell}' with \ell = e^-, \nu_e and 
\ell' = e^+, \bar{\nu}_e.

Feynman diagram of such a contribution is shown in Fig. 2. Having assumed that such a contribution should be smaller
than the experimental error \pm 6 \times 10^{-9} MeV of the neutron mass \[15\], we have got \(|g_n| < 2.45 \times 10^{-3} \sqrt{m_n - m_X/m_n}.

Recall that the coupling constant \(g_X\) is dimensionless.

The amplitude of the neutron dark matter decays \(n \rightarrow \chi + \ell + \bar{\ell}\), where \(\ell = e^-, \nu_e\) and \(\bar{\ell} = e^+, \bar{\nu}_e\), is defined by the Feynman diagrams in Fig. 3. The analytical expressions for the Feynman diagrams in Fig. 3 are given by

\[
M(n \rightarrow \chi + \ell + \bar{\ell})_{\text{Fig. 3}} = \frac{g_X e^2}{2M_Z'} \frac{m_n^2}{m_n^2 - m_X^2} \left[ u_\chi(k_n, \sigma_n) \gamma^\mu(1 + \gamma^5)u_n(k_n, \sigma_n) \right] \frac{1}{M_Z' - q^2 - i0} \left( -\eta_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{M_Z'} \right) \times \left[ \bar{u}_\ell(k_{\ell}) \gamma^\nu(1 - \gamma^5)v_\ell(k_{\ell}, \sigma_\ell) \right]
\]  

(A-17)

and

\[
M(n \rightarrow \chi + \ell + \bar{\ell})_{\text{Fig. 3}} = -\frac{g_X e^2}{2M_Z'} \frac{m_n^2}{m_n^2 - m_X^2} \left[ u_\chi(k_n, \sigma_n) \gamma^\mu(1 + \gamma^5)u_n(k_n, \sigma_n) \right] \frac{1}{M_Z' - q^2 - i0} \left( -\eta_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{M_Z'} \right) \times \left[ \bar{u}_\ell(k_{\ell}) \gamma^\nu(1 - \gamma^5)v_\ell(k_{\ell}, \sigma_\ell) \right]
\]  

(A-18)

Summing up the contributions of the Feynman diagrams in Fig. 3 we obtain the amplitude of the neutron dark matter
decays \(n \rightarrow \chi + \ell + \bar{\ell}\)

\[
M(n \rightarrow \chi + \ell + \bar{\ell}) = -\frac{g_X e^2}{2M_Z'} \frac{m_n^2}{m_n^2 - m_X^2} \left[ u_\chi(k_n, \sigma_n) \gamma^\mu(1 + \gamma^5)u_n(k_n, \sigma_n) \right] \frac{M_Z'^2}{M_Z' - q^2 - i0} \left( -\eta_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{M_Z'} \right) \times \left[ \bar{u}_\ell(k_{\ell}) \gamma^\nu(1 - \gamma^5)v_\ell(k_{\ell}, \sigma_\ell) \right]
\]  

(A-19)

Assuming that \(M_Z' \gg q^2\) we arrive at the amplitude

\[
M(n \rightarrow \chi + \ell + \bar{\ell}) = \frac{g_X e^2}{2M_Z'} \frac{m_n^2}{m_n^2 - m_X^2} \left[ u_\chi(k_n, \sigma_n) \gamma^\mu(1 + \gamma^5)u_n(k_n, \sigma_n) \right] \left[ \bar{u}_\ell(k_{\ell}) \gamma^\nu(1 - \gamma^5)v_\ell(k_{\ell}, \sigma_\ell) \right],
\]  

(A-20)

which can be obtained from the effective local Lagrangian

\[
\mathcal{L}_{\text{ndm}(e+\nu)}(x) = -\frac{G_F}{\sqrt{2}} V_{ud} \left[ \bar{\psi}_e(x) \gamma^\mu(hV + h_A\gamma^5)\psi_e(x) \right] \left[ \bar{\psi}_\nu(x)\gamma^\nu(1 - \gamma^5)\psi_e(x) \right] \left[ \bar{\psi}_e(x)\gamma^\nu(1 - \gamma^5)\psi_\nu(x) \right],
\]  

(A-21)

where we have denoted

\[
-G_F \frac{V_{ud} h V}{\sqrt{2}} = \frac{g_X e^2}{2M_Z'} \frac{m_n^2}{m_n^2 - m_X^2}, \quad -G_F \frac{V_{ud} h A}{\sqrt{2}} = \frac{g_X e^2}{2M_Z'} \frac{m_n^2}{m_n^2 + m_X^2}.
\]  

(A-22)

In terms of the vacuum expectation value of the Higgs-field \(v = 1/\sqrt{2G_F} = 246\text{ GeV}\) the coupling constants \(hV\) and \(hA\) are defined by

\[
h V = h A = -\frac{g_X}{2V_{ud}} \frac{v^2}{v^2} \frac{m_n}{m_n - m_X},
\]  

(A-23)
O

annihilation. Some examples of Feynman diagrams of order of perturbation theory can appear, for example, in the processes of fermion–fermion scattering or fermion–antifermion

Lagrangian Eq.(A-11) is similar to renormalizability of the heavy baryon chiral perturbation theory (HBχ)

the contributions of the terms

In order to restore renormalizability to order of perturbation theory [91]–[93] by the Adler–Bell–Jackiw anomaly [94, 95]. A lowest order of perturbation theory, to

the Lagrangian Eq.(A-11) the term

Usector with

the amplitudes of fermion–antifermion annihilation by virtue of the Adler–Bell–Jackiw anomaly in the dark matter

The latter also deals with dimensional coupling constants.

We would like to notice that practical applications of the dark matter sector with Usector with

is invariant under Usector with

(1) gauge symmetry

ψXR → ψ′XR = eαXψXR , \( \varphi \rightarrow \varphi' = e^{i\alpha_X}\varphi \), ψXL → ψ′XL = ψXL , \( C_\mu \rightarrow C'_\mu = C_\mu - \frac{1}{e_X}\partial_\mu \alpha_X \), (A-25)

where αX is a gauge parameter. In the spontaneously broken or physical phase the Lagrangian Eq.(A-25) takes the form

where we have set \( m_\chi = m_n \) everywhere, except in the mass difference in the denominator. Taking into account the expression for \( \zeta^{(dm)} \) in Eq.(4), \( |V_{ud}| = 0.97370 \) [15], \( g_A = 1.2764 \) [24] and \( |g_\chi| < 2.45 \times 10^{-3}\sqrt{m_n - m_\chi}/m_n \) we may estimate the vacuum expectation value \( v_\chi \). We get \( v_\chi \sim 0.09 v (m_n - m_\chi) \sim 22 (m_n - m_\chi) \) GeV, where \( m_n - m_\chi \) is measured in MeV. For such a vacuum expectation value the mass of the dark matter spin-1 Z′-boson is defined by \( M_{Z'} \sim 22 v_\chi (m_n - m_\chi) \) GeV. Setting, for example, \( e_X = 2 \) and assuming that \( M_{Z'} \sim 1 \) GeV, that is enough to neglect the contributions of the terms \( g_\mu q_\mu/M_{Z'}^2 \) in Eqs.(A-17) and (A-18) we get \( (m_n - m_\chi) \sim 0.023 \) MeV. Apparently, this is the minimal value of the mass difference \( (m_n - m_\chi) \) in our approach.

**Adler–Bell–Jackiw anomalies and violation of renormalizability of renormalizable gauge theories**

FIG. 4: Examples of Feynman diagrams violating renormalizability of the dark matter sector with \( U_\nu^\prime(1) \) gauge symmetry to order \( O(e_\chi^6/\Lambda^2) \) (a) and \( O(e_\chi^8/\Lambda^2) \) (b) by the Adler–Bell–Jackiw anomaly in processes of fermion–antifermion annihilation (s-channel) or fermion–fermion scattering (t-channel).

FIG. 5: Example of Feynman diagrams violating renormalizability of the dark matter sector with \( U_\nu^\prime(1) \) gauge symmetry to order \( O(g_\sigma^2 e_\chi^4/\Lambda^2) \) by the Adler–Bell–Jackiw anomaly, caused by the \( n_\chi Z' \) interaction.

where \( Z' \) couples to fermions through the vertex \( \chi_R Z' \) described by one-fermion loops with virtual dark matter fermions, electrons and neutrinos. In order to restore renormalizability to order \( O(e_\chi^n/\Lambda^2) \), where \( n \geq 6 \), we propose to add to the Lagrangian Eq.(A-11) the term

\[
\delta \mathcal{L}_{DM'} = \bar{\psi}_X i\gamma^\mu (\partial_\mu + ie_\chi C_\mu) \psi_X - \sqrt{2} f_X (\bar{\psi}_X \psi_X \varphi + \bar{\psi}_X \psi_X \varphi^*) + \bar{\psi}_X i\gamma^\mu \partial_\mu \psi_X,
\]

(\ref{eq:FeynmanDiagrams})

where \( \delta \mathcal{L} \) is a gauge transformation.

\[
\psi_{XR} \rightarrow \psi'_{XR} = e^{i\alpha_X} \psi_{XR} , \quad \varphi \rightarrow \varphi' = e^{i\alpha_X}\varphi , \quad \psi_{XL} \rightarrow \psi'_{XL} = \psi_{XL} , \quad C_\mu \rightarrow C'_\mu = C_\mu - \frac{1}{e_X} \partial_\mu \alpha_X ,
\]

(\ref{eq:GaugeTransformation})

\[
\delta \mathcal{L}_{DM'} = \bar{\psi}_X (i\gamma^\mu \partial_\mu - m_X) \psi_X - \frac{1}{2} e_\chi \bar{\psi}_X \gamma^\mu (1 + \gamma^5) \psi_X Z'_\mu + \ldots ,
\]

(\ref{eq:GaugeTransformation2})
where \( m_X = f_X v_X \) is a mass of the dark matter fermion \( X \) such as \( m_X = f_X v_X \gg m_\chi \) and even \( m_X \gg M_{Z'} \). For the dark matter fermion \( X \) the dark matter hypercharge \( Y' \) is equal to \( Y' = +1 \). The anomalous diagrams are one-loop fermion \( Z'Z'Z' \)-diagrams of order \( e^4 \sqrt{2} / m_n \). The contributions of dark matter fermions \( \chi \) and \( X \) give the Adler–Bell–Jackiw terms with a sign \((-1)\), whereas the electron and neutrino contributions appear with the sign \((+1)\). Since the Adler–Bell–Jackiw anomaly does not depend on the mass of virtual fermions \([11, \text{ or } 35]\), the sum of the diagrams with dark matter fermion \( \chi \) and \( X \), electron and neutrino loops is free from the Adler–Bell–Jackiw anomaly.

An additional violation of renormalizability by virtue of the Adler–Bell–Jackiw anomaly can appear also because of the \( n\chi Z' \) interaction. For example, in the processes of fermion–fermion scattering and fermion–antifermion annihilation the contribution of the \( n\chi Z' \) interaction, violating renormalizability by virtue of the Adler–Bell–Jackiw anomaly, is of order \( O(e^4 g^2_e / 24) \) (see some examples of Feynman diagrams in Fig. 5). Unfortunately, such a violation of renormalizability cannot be repaired. It is important to emphasize that a contribution of the Feynman diagrams, violating renormalizability by virtue the Adler–Bell–Jackiw anomaly caused by the \( n\chi Z' \) interaction, relative to the main order contribution \( O(e^4 / 2) \) is of order \( O(4 e^4 / c^2) < 2 \times 10^{-13} \) at \( e_\chi = 2 \) and \( g_\chi < 2.45 \times 10^{-3} \sqrt{m_n - m_\chi} / m_n \sim 4 \times 10^{-7} \) for \( m_n - m_\chi \approx 0.023 \) MeV. One may argue that this possibility for renormalizability to such an order of perturbation theory with contributions of a relative order \( 2 \times 10^{-13} \) or even smaller cannot discredit any quantum field theory model moreover when practical applications of such a model to the analysis of observable phenomena can be restricted by the tree- and one-loop approximation only.

According to \([22]\), violation of renormalizability with a relative order smaller than \( 2 \times 10^{-13} \) to order \( O(g^2_e e^4 / 24) \) and higher orders of perturbation theory, caused by the \( n\chi Z' \) interaction, may lead to violation of gauge invariance only to the same order of magnitude, i.e. to a relative order smaller than \( 2 \times 10^{-13} \), and in the same orders of perturbation theory. So we may argue that up to fourth order of perturbation theory \( O(g^2_e e^4 / 24) < 2 \times 10^{-13} \) at \( e_\chi = 2 \) (see, for example, the Feynman diagram in Fig. 5 without fermion line hooked by two dark matter spin–1 boson \( Z' \), describing fermion–dark matter spin–1 boson \( Z' \) scattering \((f + Z' \to f + Z') \) in the t-channel or fermion–antifermion annihilation into \( Z'Z' \)-pair \((f + f \to Z' + Z') \) in the s-channel and other similar processes, renormalizability and gauge invariance of the dark matter sector with \( U_{\nu}^n(1) \) gauge symmetry are not violated by the \( n\chi Z' \) interaction.

Dark matter dynamics in neutron stars

The influence of the dark matter fermion \( \chi \), which can appear in the final state of the neutron dark matter decays, on dynamics of neutron stars has been investigated in \([10, \text{ or } 22]\). The main result is that dark matter fermions in the equilibrium state with the SM matter of neutron stars do not destroy the possibility for neutron stars to reach the maximum mass of about \( 2 M_\odot \) \([23]\), where \( M_\odot \) is the mass of the Sun \([13]\), only for \( m_\chi > 1.2 \) GeV. In other words dark matter fermions with masses \( m_\chi < m_n \) may appear in case of existence of a repulsive interaction between dark matter fermions mediated by a sufficiently light dark matter spin–1 bosons, the Compton wavelength of which is larger than inter-particle distances in neutron stars \([19]\). Such a possibility for dark matter fermions from the neutron decays has been realized in scenario by Cline and Cornell \([22]\) within \( U'(1) \) gauge quantum field theory model with dark matter fermions \( \chi \) coupled to a dark matter photon \( A' \), which mass is constrained by \( m_A' / g' \leq (45 - 60) \) MeV, where \( g' \) is a gauge coupling constant or a dark matter \textit{charge} of dark matter fermions. According to \([22]\), the ratio \( m_A' / g' \leq (45 - 60) \) MeV depends on the nuclear equation of state and has been derived from the requirement for neutron stars to have masses compatible with \( 2 M_\odot \) \([23]\).

Since in our approach to the neutron lifetime anomaly the mass of the dark matter fermion is smaller than the neutron mass \( m_\chi < m_n \), we have to accept the mechanism of the influence of dark matter fermions on dynamics of neutron stars, allowing to have masses of about \( 2 M_\odot \), developed by Cline and Cornell \([22]\). For this aim we extend the symmetry of our model from \( SU_L(2) \times U_Y(1) \times U_{\nu}^n(1) \) to \( SU_L(2) \times U_Y(1) \times U_{\nu}^n(1) \times U_{\nu}^n(1) \), where \( U_{\nu}^n(1) \) is a new dark matter gauge group. In other words we add to the effective low-energy field theory, described by the Lagrangian \( \mathcal{L}_{\text{SMSETANDM}} \), the Lagrangian

\[
\mathcal{L}_{\text{DM}''} = \bar{\psi}_X L i\gamma^\mu (\partial_\mu + i e_\chi \tilde{C}_\mu) \psi_{XL} - \frac{1}{4} \tilde{C}_{\mu
u} \tilde{C}^{\mu\nu} + (\partial_\mu - i e_\chi \tilde{C}_\mu) \tilde{\varphi} (\partial_\mu + i e_\chi \tilde{C}_\mu) \tilde{\varphi} + \bar{\psi}_X (\gamma^\mu \partial_\mu + \gamma^\nu \partial_\nu) \varphi + \ldots
\]  

(A.27)

invariant under \( U_{\nu}^n(1) \) gauge transformations, where \( \tilde{C}_{\mu
u} = \partial_\mu \tilde{C}_\nu - \partial_\nu \tilde{C}_\mu \) is the field strength tensor operator of the dark matter spin–1 field \( \tilde{C}_\mu \). \( e_\chi \) is the dark matter \textit{charge} of the left–handed dark matter fermions and the dark matter complex scalar field \( \tilde{\varphi} \). The ellipses denotes the contribution of right-handed dark matter fermions. The Lagrangian Eq. \((A.27)\) is invariant under gauge \( U_{\nu}^n(1) \) transformations. The dark matter \textit{hypercharge} \( Y'' \) is equal to
$Y'' = +1$ for the left-handed dark matter fermion field $\psi_L$, and the complex dark matter scalar boson field $\tilde{\varphi}$, and $Y'' = 0$ for the right-handed dark matter fermion field $\psi_R$, respectively. The first five terms in Eq. (A-27) define the extension of the term $\bar{\psi}_L i\gamma^\mu \partial_\mu \psi_L$ in Eq. (A-7). In turn, the last term in Eq. (A-27) is obtained from the term $f_X(\bar{\psi}_L \Sigma \phi + \bar{\psi}_L \Sigma \phi^*)\Sigma$ in Eq. (A-7) by the replacement

$$\sqrt{2} f_X(\bar{\psi}_L \Sigma \phi + \bar{\psi}_L \Sigma \phi^*) \rightarrow 2f_X(\bar{\psi}_L \Sigma \phi + \bar{\psi}_L \Sigma \phi^*).$$

(A-28)

This implies that the mass of the dark matter fermion $\chi$ appears in the phase of spontaneously broken $U'_Y(1)$ gauge symmetry. The Lagrangian Eq. (A-27) describes interactions of dark matter particles only. We would like to notice that the SM particles and the dark matter particles transforming under $SU_L(2) \times U_R(1)$ gauge transformations are invariant under gauge transformations of the $U'_Y(1)$ group.

Following Kibble [88, 89] and repeating the procedure expounded above, namely, assuming i) to replace $\tilde{\varphi}$ by $\varphi = e^{i\alpha}(\tilde{\varphi} + \tilde{S})/\sqrt{2}$, ii) to make a gauge transformation $\psi_L \rightarrow e^{i\beta} \psi_L$, and iii) to introduce a new spin–1 boson field $Z^\mu \equiv C^\mu + \partial_\mu \tilde{\alpha} = \tilde{\tilde{\alpha}}$, where $\tilde{\alpha} = \sqrt{2} \tilde{\alpha}/\tilde{\alpha}$ is the vacuum expectation value of the dark matter scalar field $\Phi$, we arrive at the Lagrangian

$$L_{DM'} = \bar{\psi}_L i\gamma^\mu(\partial_\mu + i\tilde{\tilde{\alpha}} Z^\mu) \psi_L - m_\chi \bar{\psi}_L \psi_L + \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{1}{2} M_{Z'}^2 Z'_{\mu\nu} Z'^{\mu\nu} + \tilde{\alpha}^2 \tilde{\tilde{\alpha}} Z_{\mu\nu} Z'^{\mu\nu} \tilde{S} + \frac{1}{2} Z_{\mu}^2 Z'^{\mu} \tilde{S}^2 + \frac{1}{2} \tilde{\tilde{\alpha}} \tilde{\tilde{S}} \tilde{S} - \frac{1}{2} m_\chi^2 \bar{\psi}_L \psi_L + \bar{\psi}_L \tilde{\tilde{\alpha}} \tilde{\tilde{S}} \tilde{S} \tilde{S} + \ldots,$$

where $m_\chi$, $M_{Z'}$, and $m_\tilde{S}$ are masses of the dark matter fermion $\chi$, dark matter spin–1 $Z''$ and dark matter scalar $S$ fields

$$m_\chi = \tilde{f}_X v \tilde{\alpha} \tilde{\alpha}, \quad M_{Z'} = \tilde{\tilde{\alpha}} \tilde{\tilde{\alpha}}, \quad m_\tilde{S} = \sqrt{2} \tilde{\alpha} \tilde{\alpha}.$$

(A-30)

Without loss of generality we may again set the mass of the dark matter scalar boson $S$ arbitrary heavy [17, 18, 61]. This leads to the decoupling of the dark matter scalar boson $S$ from the dark matter fermion $\chi$ and the dark matter spin–1 boson $Z'$ in agreement with the Appelquist–Carazzone decoupling theorem [30].

Since the dark matter spin–1 boson $Z'$ is too heavy to provide a repulsion at large inter–particle distances in neutron stars, so the contribution of its repulsion should be taken into account as some corrections to the repulsion produced by the dark matter spin–1 boson $Z''$. Indeed, following McKeen et al. [19] (see also [22]), the pressure and energy density of neutron stars (or the equation of state of neutron stars) should acquire the corrections (see Eq. (11) of Ref. [19] and Eq. (4) of Ref. [22])

$$\Delta P_\chi = \Delta E_\chi = \frac{1}{2} \left( \frac{c_2^2}{4 M_{Z''}^2} + \frac{c_2^2}{4 M_{Z'}^2} \right) n_\chi^2 = \frac{c_2^2}{8 M_{Z''}^2} \left( 1 + \frac{\bar{c}_2^2}{c_2^2} \right) n_\chi^2 = \frac{c_2^2}{8 M_{Z''}^2} \left( 1 + R_\chi \right) n_\chi^2$$

(A-31)

caused by the contributions of the dark matter spin–1 bosons $Z''$ and $Z'$, respectively, where we have used $M_{Z''} = \bar{c}_2 \bar{c}_2$ and $M_{Z'} = c_2 c_2$. Perturbative contributions of the dark matter spin–1 boson $Z'$ imply that the ratio $R_\chi = c_2 \bar{c}_2 / c_2 \bar{c}_2$ obeys the constraint $R_\chi \ll 1$. Having neglected the contribution of the dark matter spin–1 boson $Z'$ to the equation of state we may deal with the dark matter spin–1 boson $Z''$ only. For the confirmation of such an approximation we make an estimate of $R_\chi$ below Eq. (A-34).

Thus, the part of the total Lagrangian $L_{\Sigma M \& SET DM' \& DM''}$, which should be responsible for dark matter dynamics in neutron stars, can be written in the following form

$$L_{\Sigma M \& SET DM' \& DM''} = \bar{\psi}_L i\gamma_\mu \left( Z_{\mu\nu} - m_\chi \bar{\psi}_L \psi_L - \frac{1}{4} Z_{\mu\nu} Z'^{\mu\nu} + \frac{1}{2} M_{Z''}^2 Z''_{\mu\nu} Z'^{\mu\nu} - \frac{1}{2} \bar{c}_2 \bar{c}_2 \bar{c}_2 \bar{c}_2 \psi_\gamma_\gamma \psi_\gamma \psi_\gamma \right) \psi_L + \ldots,$$

(A-32)

where the ellipsis denotes the contributions of other kinetic and interaction terms of the SM and dark matter particles, which are not important for the analysis of the influence of the dark matter fermions with mass $m_\chi < m_\chi$ on the dynamics of neutron stars. In the non–relativistic approximation the potential of the dark matter spin–1 boson $Z'$ between two dark matter fermions $\chi$ is equal to

$$V_{Z''}(r) = \frac{c_2^2}{16\pi} \frac{e^{-M_{Z''} r}}{r}.$$

(A-33)

Since it coincides with the potential of the vector field with mass $M_{Z''}$, describing a repulsive interaction between two fermions with “charges” $\bar{c}_2 2$ separated by a distance $r$, we may apply it for the analysis of dark matter dynamics in neutron stars in the scenario by Cline and Cornell [22]. For a short confirmation of a validity of our model for the
analysis of dynamics of neutron stars we may use the estimate by Cline and Cornell [22]. Indeed, according to Cline and Cornell [22], a possibility for neutron stars with dark matter fermions lighter than neutron and light dark matter spin–1 bosons in the equilibrium with the SM particles to reach maximum masses compatible with $2\, M_\odot$ places the constraint (see Eq.(12) of Ref.[22]). Since the correction to the equation of state (see Eq.(A-31)), caused by repulsion between dark matter fermions with mass $m_\chi < m_n$, is fully defined by the dark matter spin–1 boson $Z''$, the inequality $m_{A'}/g' \leq (45 - 60)\, \text{MeV}$ (see Eq.(3.4) of Ref.[22]) should be saturated only by the dark matter spin–1 boson $Z''$. In our notations such a constraint reads

$$\frac{2M_{Z''}}{\tilde{e}_\chi} \lesssim (45 - 60)\, \text{MeV}. \quad (A-34)$$

This allows to estimate the vacuum expectation value $\tilde{v}_\chi$. Substituting $M_{Z''} = \tilde{e}_\chi \tilde{v}_\chi$ into Eq.(A-33) we get $\tilde{v}_\chi \lesssim (23 - 30)\, \text{MeV}$. Using $\tilde{v}_\chi \lesssim (23 - 30)\, \text{MeV}$ and $v_\chi \simeq 22 (m_n - m_\chi)$ GeV for the ratio $R_\chi = \tilde{v}^2_\chi/v^2_\chi$ we get the value $R_\chi \sim 1.5 \times 10^{-6}/(m_n - m_\chi)^2$. For $(m_n - m_\chi) \sim 0.023\, \text{MeV}$ we get $R_\chi \sim 3 \times 10^{-3}$. Thus, the contribution of the dark matter spin–1 boson $Z'$ to the equation of state of neutron stars makes up of about 0.03% with respect to the contribution of the dark matter spin–1 boson $Z''$. This, confirms our assertion that the contributions of the dark matter spin–1 boson $Z'$ can be taken into account perturbatively when it is required.

A specific value of the $Z''$–boson mass depends on the value of the gauge coupling constant $\tilde{e}_\chi$, which can be obtained from a detailed analysis of the interference of dark matter into dynamics of neutron stars. Of course, such an analysis, namely i) using the dark matter fermion mass obeying the constraint $m_\chi - m_n \simeq 0.023\, \text{MeV}$, ii) taking into account the neutron dark matter decay mode $n \to \chi + \nu_e + \bar{\nu}_e$, where the neutrino–antineutrino pair possesses a zero net chemical potential [20], and equations of state [21, 96], goes beyond the scope of this paper. We are planning to carry out this analysis in our forthcoming publications. Here we would like only to notice that there is practically nothing that can prevent for the dark matter spin–1 boson $Z''$ to have a mass as light as the dark matter spin–1 boson $A'$, introduced by Cline and Cornell [22].
Werder, Measurement of the weak axial-vector coupling constant in the decay of free neutrons using a pulsed cold neutron beam, Phys. Rev. Lett. **122**, 242501 (2019); DOI: https://doi.org/10.1103/PhysRevLett.122.242501; arXiv: 1812.04606 [nucl-ex].

[25] A. Czarnecki, W. J. Marciano, and A. Sirlin, The neutron lifetime and axial coupling constant connection, Phys. Rev. Lett. **120**, 202002 (2018); DOI: https://doi.org/10.1103/PhysRevLett.120.202002; arXiv:1802.01804 [hep-ph].

[26] A. N. Ivanov, M. Pitschmann, and N. I. Troitskaya, Neutron beta decay as a laboratory for testing the standard model, Phys. Rev. D **88**, 073002 (2013); DOI: https://doi.org/10.1103/PhysRevD.88.073002; arXiv:1212.0332 [hep-ph].

[27] A. N. Ivanov, M. Cargnelli, M. Faber, H. Fuhrmann, V. A. Ivanova, J. Marton, N.I. Troitskaya, and J. Zmeskal, Neutrino emissivities of neutron stars, Phys. Rev. C **101**, 035503 (2020); DOI: https://doi.org/10.1103/PhysRevC.101.035503.

[28] G. Gamow and M. Schönberg, Neutron dark matter decays and on the Fierz interference term b from a measurement of the beta asymmetry in neutron decay, Phys. Rev. Lett. **125**, 112501 (2020); DOI: https://doi.org/10.1103/PhysRevLett.125.112501.

[29] A. N. Ivanov, R. Höllwieser, N. I. Troitskaya, M. Wellenzohn, and Ya. A. Berdnikov, Precision analysis of pseudoscalar interactions in neutron beta decays, Nucl. Phys. B **951**, 114891 (2020); DOI: https://doi.org/10.1016/j.nuclphysb.2019.114891; arXiv:1905.04147 [hep-ph].

[30] A. N. Ivanov, R. Höllwieser, N. I. Troitskaya, M. Wellenzohn, and Ya. A. Berdnikov, Neutron dark matter decays and correlation coefficients of neutron beta decays, Nucl. Phys. B **938**, 114 (2019); DOI: https://doi.org/10.1016/j.nuclphysb.2018.11.005.

[31] G. Gamow and M. Schönberg, Neutrino theory of stellar collapse, Phys. Rev. **59**, 539 (1941); DOI: https://doi.org/10.1103/PhysRev.59.539.

[32] B. L. Friman and O. V. Maxwell, Neutrino emissivities of neutron stars, ApJ **232**, 541 (1979); DOI: 10.1086/157313.

[33] P. Hänsel, Urca processes in dense matter and neutron star cooling", Space Science Reviews. **74**, 427 (1995); DOI: https://doi.org/10.1007/s11214-017-0159-x.

[34] A. N. Ivanov, R. Höllwieser, H. Fuhrmann, V. A. Ivanova, J. Marton, N.I. Troitskaya, and J. Zmeskal, On kaonic deuterium: Quantum field theoretic and relativistic covariant approach, Eur. Phys. J. A **23**, 79 (2005); DOI: https://doi.org/10.1007/s10050-004-0155-3; arXiv: nucl-th/0406053.

[35] S. Christlmeier and H. W. Grießhammer, Pion-less effective field theory on low-energy deuteron electrodisintegration, Phys. Rev. C **77**, 064001 (2008); DOI: https://doi.org/10.1103/PhysRevC.77.064001.

[36] R. Machleidt, K. Holinde, and C. Elster, The Bonn meson exchange model for the nucleon nucleon interaction, Phys. Rept. **149**, 1 (1987); DOI: 10.1016/0370-1573(87)80002-9.

[37] M. Garçon and J. W. Van Orden, The deuteron: structure and form-factors, Adv. Nucl. Phys. **26**, 293 (2001); DOI: https://doi.org/10.1007/0-306-47915-X4.

[38] A. N. Ivanov, V. A. Ivanova, H. Oberhummer, N. I. Troitskaya, and M. Faber, On the D wave state component of the deuteron in the Nambu-Jona-Lasinio model of light nuclei, Eur. Phys. J. A **12**, 87 (2001); DOI: https://doi.org/10.1007/s100500170041.

[39] R. A. Gilman and F. Gross, Electromagnetic structure of the deuteron, J. Phys. G **28**, R37 (2002); DOI: https://doi.org/10.1088/0954-3899/28/4/201.

[40] C. Van der Leun and C. Alderliesten, The deuteron binding energy, Nucl. Phys. A **380**, 261 (1982); DOI: https://doi.org/10.1016/0375-9474(82)90105-1.

[41] W. Fabian and H. Arenhövel, Electrodisintegration of deuteron including nucleon detection in coincidence, Nucl. Phys. A **314**, 253 (1979); DOI: https://doi.org/10.1016/0375-9474(79)90599-2.

[42] D. Werder, Measurement of the weak axial-vector coupling constant in the decay of free neutrons using a pulsed cold neutron beam, Phys. Rev. Lett. **122**, 242501 (2019); DOI: https://doi.org/10.1103/PhysRevLett.122.242501; arXiv: 1812.04606 [nucl-ex].
DOI: https://doi.org/10.1016/0377-9983(82)90336-0.

[48] H. Arenhövel, W. Leidemann, and E. L. Tomusiak, *General multipole expansion of polarization observables in deuteron electrodissintegration*, Eur. Phys. J. A **14**, 491 (2002); DOI: 10.1140/epja/i2001-10207-7.

[49] T. Tamae, H. Kawahara, A. Tanaka, M. Nonoura, K. Namai, and M. Sugawara, *Out-of-plane measurement of the D(e, e′p) coincidence cross section*, Phys. Rev. Lett. **59**, 2919 (1987); DOI: https://doi.org/10.1103/PhysRevLett.59.2919.

[50] P. von Neumann-Cosel, A. Richter, G. Schrieder, A. Shevchenko, A. Stiller, and H. Arenhövel, *Deuteron breakup in the $^2$He(e, e′p) reaction at low momentum transfer and close to threshold*, Phys. Rev. Lett. **88**, 202304 (2002).

[51] C. Hernández-Monteagudo, Yin-Zhe Ma, F. S. Kitaura, W. Wang, R. Génova-Santos, J. Macías-Pérez, and D. Herranz, *Evidence of the missing baryons from the kinematic Sunyaev–Zel’dovich effect in Planck data*, Phys. Rev. Lett. **115**, 191301 (2015); DOI: https://doi.org/10.1103/PhysRevLett.115.191301.

[52] H. Ejiri and J. D. Vergados, *Neutron disappearance inside the nucleus*, J. Phys. G: Nucl. Part. Phys., **46**, 025104 (2019); DOI: https://doi.org/10.1088/1361-6471/aad55b [arXiv:1805.04477 [hep-ph]].

[53] G. K. Karananas and A. Kassiteridis, *Small-scale structure from neutron dark decay*, Phys. Lett. B **783**, 242 (2018); DOI: 10.1016/j.physletb.2018.04.012 [arXiv:1805.03656 [hep-ph]].

[54] M. Nowak, M. Rho, and I. Zahed, in *Chiral dynamics in nucleons and nuclei*, V. Bernard, N. Kaiser, and Ulf-G. Meißen, eds. (World Scientific, Singapore 1996). DOI: https://doi.org/10.1016/0378-4371(96)50003-7.

[55] J. Bijnens, *Chiral Lagrangians and Nambu-Jona-Lasinio - like models*, Phys. Rep. **265**, 369 (1996); DOI: 10.1016/0370-1573(95)00517-8.

[56] H. B. Lee, in *Chiral dynamics*, Gordon and Breach, New York, 1972.

[57] M. Nowak, M. Rho, and I. Zahed, in *Chiral nuclear dynamics*, World Scientific, Singapore – New Jersey – London – Hong Kong, 1996.

[58] A. N. Ivanov, R. Höllwieser, N. I. Troitskaya, and V. Bernard, *Theoretical description of the neutron beta decay in the standard model at the level of 10^−5*, Phys. Rev. D **104**, 033006 (2021); DOI: https://doi.org/10.1103/PhysRevD.104.033006; arXiv: 2104.11080 [hep-ph].

[59] S. Weinberg, *Chiral perturbation theory*, Physica **A 96** (1979); https://doi.org/10.1016/0378-4371(79)90223-1.

[60] S. Weinberg, *Chiral perturbation theory and effective Lagrangians*, Phys. Lett. B **193**, 126 (1987); DOI: https://doi.org/10.1016/0370-2693(87)90907-5.

[61] S. Weinberg, *Phenomenological Lagrangians*, Physica **A 96**, 327 (1979); DOI: 10.1016/0378-4371(79)90223-1.

[62] S. Weinberg, *Dynamical approach to current algebra*, Phys. Rev. Lett. **18**, 188 (1967); DOI: https://doi.org/10.1103/PhysRevLett.18.188.

[63] S. Weinberg, *Nonlinear realization of chiral symmetry*, Phys. Rev. **166**, 1568 (1968); DOI: https://doi.org/10.1103/PhysRev.166.1568.

[64] S. Weinberg, *Nucleons in chiral loops*, Nucl. Phys. B **779**, 65 (1996); DOI: https://doi.org/10.1016/0550-3213(96)00051-8.

[65] J. Gasser and H. Leutwyler, *Chiral perturbation theory*, Annals of Physics **158**, 142 (1984); DOI: https://doi.org/10.1016/0003-4916(84)90242-2.

[66] J. Gasser, *Chiral perturbation theory and effective Lagrangians*, Nucl. Phys. B **279**, 65 (1987); DOI: https://doi.org/10.1016/0550-3213(87)90307-5.

[67] J. Gasser, M. E. Sainio, and A. Švarc, *Nucleons in chiral loops*, Nucl. Phys. B **307**, 779 (1988); DOI: 10.1016/0550-3213(88)90108-3.

[68] V. Bernard, N. Kaiser, and Ulf-G. Meißen, *Chiral structure of the nucleon*, Nucl. Phys. B **388**, 315 (1992); DOI: https://doi.org/10.1016/0550-3213(92)90615-1.

[69] V. Bernard, N. Kaiser, and Ulf-G. Meißen, *Chiral dynamics in nucleons and nuclei*, Int. J. Mod. Phys. E **4**, 193 (1995); DOI: https://doi.org/10.1142/S021830139500092.

[70] G. Ecker, *Chiral perturbation theory*, Prog. Part. Nucl. Phys. **35**, 1 (1995); DOI: https://doi.org/10.1016/0146-6410(95)00041-G.

[71] G. Ecker, *Low-energy QCD*, Prog. Part. Nucl. Phys. **36**, 71 (1996); DOI: https://doi.org/10.1016/0146-6410(95)00011-7.

[72] J. Bijnens, *Chiral Lagrangians and Nambu-Jona-Lasinio - like models*, Phys. Rep. **265**, 369 (1996); DOI: 10.1016/0370-1573(95)00515-8.

[73] V. Bernard, N. Kaiser, and Ulf-G. Meißen, *Aspects of chiral pion - nucleon physics*, Nucl. Phys. A **615**, 483 (1997); DOI: https://doi.org/10.1016/S0375-9474(97)00021-3.

[74] N. Fettes, Ulf-G. Meißen, and S. Steininger, *Pion-nucleon scattering in chiral perturbation theory (I): Isospin-symmetric case*, Nucl. Phys. A **640**, 199 (1998); DOI: https://doi.org/10.1016/S0375-9474(98)00452-7.

[75] J. Gasser, *Chiral perturbation theory*, Nucl. Phys. B (Proc. Suppl.) **86**, 257 (2000); DOI: https://doi.org/10.1016/S0920-5632(00)00573-9.

[76] S. Scherer, *Introduction to chiral perturbation theory*, Adv. Nucl. Phys. **27**, 277 (2003); hep-ph/0210398.

[77] T. Fuchs, J. Gegelia, G. Japaridze, and S. Scherer, *Renormalization of relativistic baryon chiral perturbation theory and
power counting, Phys. Rev. D 68, 056005 (2003);
DOI: https://doi.org/10.1103/PhysRevD.68.056005.

[77] W. P. Alvarez, K. Kubodera, and F. Myhrer, Comparison of the extended linear σ model and chiral perturbation theory, Phys. Rev. C 72, 038201 (2005);
DOI: https://doi.org/10.1103/PhysRevC.72.038201.

[78] V. Bernard and Ulf-G. Meiβner, Chiral perturbation theory, Annu. Rev. Nucl. Part. Sci. 57, 33 (2007);
DOI: 10.1146/annurev.nucl.56.080805.140449.

[79] V. Bernard, Chiral perturbation theory and baryon properties, Prog. Part. Nucl. Phys. 60, 82 (2008);
DOI: https://doi.org/10.1016/j.ppnp.2007.07.001.

[80] S. Scherer, Chiral perturbation theory: introduction and recent results in one-nucleon sector, Prog. Part. Nucl. Phys. 61, 1 (2010);
DOI: https://doi.org/10.1016/j.ppnp.2009.08.002.

[81] M. R. Schindler and S. Scherer, Chiral effective field theories of the strong interactions, Eur. Phys. J. Special Topics 198, 95 (2011);
DOI: https://doi.org/10.1140/epjst/e2011-01485-0.

[82] M. Bisseger and A. Fuhrer, A renormalization group analysis of leading logarithms in ChPT, Eur. Phys. J. C 51, 75 (2007);
DOI: https://doi.org/10.1140/epjc/s10052-007-0292-9.

[83] A. Sirlin, General properties of the electromagnetic corrections to the beta decay of a physical nucleon, Phys. Rev. 164, 1767 (1967);
DOI:https://doi.org/10.1103/PhysRev.164.1767.

[84] A. Sirlin, Current algebra formulation of radiative corrections in gauge theories and the universality of the weak interactions, Rev. Mod. Phys. 50, 573 (1978);
DOI: https://doi.org/10.1103/RevModPhys.50.573.

[85] R. T. Shann, Electromagnetic effects in the decay of polarized neutrons, Nuovo Cimento A 5, 591 (1971);
DOI: https://doi.org/10.1007/BF02734506.

[86] A. N. Ivanov, R. Höllwieser, N. I. Troitskaya, M. Wellenzohn, and Ya. A. Berdnikov, Precision analysis of electron energy spectrum and angular distribution of neutron beta decay with polarized neutron and electron, Phys. Rev. C 95, 055502 (2017);
DOI: 10.1103/PhysRevC.95.055502; arXiv:1705.07330 [hep-ph].

[87] A. N. Ivanov, R. Höllwieser, N. I. Troitskaya, M. Wellenzohn, and Ya. A. Berdnikov, Test of the Standard Model in neutron beta decay with polarized electrons and unpolarized neutrons and protons, Phys. Rev. D 99, 053004 (2019);
DOI: 10.1103/PhysRevD.99.053004; arXiv:1811.04853 [hep-ph].

[88] T. W. B. Kibble, Spontaneously broken gauge theories of weak interactions and heavy leptons, Phys. Rev. D 7, 887 (1973);
DOI: https://doi.org/10.1103/PhysRevD.7.887.

[89] T. W. B. Kibble, History of electroweak symmetry breaking, Journal of Physics: Conf. Series 626, 012001 (2015);
DOI: https://doi.org/10.1088/1742-6596/626/1/012001.

[90] Th. Appelquist and J. Carazzone, Infrared singularities and massive fields, Phys. Rev. D 11, 2856 (1975);
DOI: https://doi.org/10.1103/PhysRevD.11.2856.

[91] C. Bouchiat, J. Iliopoulos, and P. Meyer, An anomaly free version of Weinberg’s model, Phys. Lett. B 38, 519 (1972);
DOI: https://doi.org/10.1016/0370-2693(72)90532-1.

[92] D. J. Gross and R. Jackiw, Effect of anomalies on quasi-renormalizable theories, Phys. Rev. D 6, 477 (1972);
DOI: https://doi.org/10.1103/PhysRevD.6.477.

[93] J. D. Bjorken and C. H. Llewellyn Smith, Spontaneously broken gauge theories of weak interactions and heavy leptons, Phys. Rev. D 11, 2856 (1975);
DOI: https://doi.org/10.1103/PhysRevD.11.2856.

[94] S. L. Adler, Axial vector vertex in spinor electrodynamics, Phys. Rev. 177, 2426 (1969);
DOI: https://doi.org/10.1103/PhysRev.177.2426.

[95] J. S. Bell and R. Jackiw, A PCAC puzzle: π0 → γγ in the σ-model, Nuovo Cim. A 51, 47 (1969);
DOI: https://doi.org/10.1007/BF02823296.

[96] S. Gandolfi, J. Carlson, and S. Reddy, Maximum mass and radius of neutron stars, and the nuclear symmetry energy, Phys. Rev. C 85, 032801 (2012);
DOI: https://doi.org/10.1103/PhysRevC.85.032801.