An exponential smoothing channel estimation method for MIMO-OFDM system

Guo Yan1*, Su Sheng2

1 Nanjing Research Institute of Electronics Technology, Nanjing 210016, China
2 College of Astronautics, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China
* E-mail: zhongwz@nuaa.edu.cn

Abstract. In order to solve the problem of inaccurate channel estimation of least square (LS) algorithm for multiple input multiple output orthogonal frequency division multiplexing (MIMO-OFDM) system under time-varying channel, a novel channel estimation method using exponential smoothing algorithm (ESA) is proposed in this paper. Based on LS estimation algorithm and ESA theory, the proposed method fully exploits the variation law of time-varying channel, adopts the predicted results of the previous time to predict the estimation value of the current time by iterative computations, and utilizes the low-pass character of the ESA equivalent filter to reduce the random disturbance. Therefore, the additive white Gaussian noise (AWGN) existing in LS algorithm is effectively suppressed and the estimation accuracy is further improved. Simulation results show that, compared with the traditional channel estimation approaches, the proposed method can provide high estimation accuracy with low computational complexity, especially under time-varying channel and low SNR conditions.

1. Introduction
MIMO-OFDM is an attractive air-interface solution to meet the demands of 4G/5G wireless mobile communications [1]. The combining of MIMO and OFDM technology makes it possible for high data rate transmission and large channel capacity without the impact of frequency-selective fading [2]. However, a dynamic estimation of channel is necessary for MIMO-OFDM system since the radio channel is frequency-selective and time-varying for wideband mobile communication systems. Therefore, the channel estimation method for MIMO-OFDM systems has received extensive attention [3].

Channel estimation method mainly includes three categories, the first category is blind channel estimation, the second category is semi-blind channel estimation, and the third is the training sequence based non-blind channel estimation. For the shortage of high computational complexity and low-rate convergence of blind and semi blind estimation, the researches with non-blind estimation gained extensive attention [4]. According to least squares criterion, a LS estimation algorithm for MIMO-OFDM system was proposed in paper [5]. Due to the ignoring of additional noise from wireless environment, LS estimation algorithm gets a poor performance in low SNR. In order to solve this problem, a minimum mean-square error (MMSE) estimation algorithm was introduced in paper [6]. With consideration of noise and the correlation of channel, MMSE estimation algorithm can suppress the noise effectively and get high precision. However, with the increasing of correlation matrix dimension, the computation complexity of MMSE increases dramatically. Meanwhile, due to the
difficulty of getting channel statistical characteristics, the algorithm is seldom used in practice. For the sake of high estimation precision and low computational complexity at the same time, many researches focus on improving the precision of LS estimation algorithm. Among the improved algorithms [7,8,9], DFT-based estimation algorithm presented in paper [9] gives us a choice. By setting the channel impulse response (CIR) over the maximum time delay to zero, DFT-based estimation algorithm can depress the AWGN effectively. Moreover, the complexity of the algorithm is very low, therefore, it can be used in practical applications. Nevertheless, maximum delay spread employed in DFT-based estimation algorithm is unknown without some extra estimation work. To solve the problems above, combining with ESA, a novel LS estimation algorithm applied to MIMO-OFDM system is proposed in this paper. For time-varying channel, the variation of CSI from adjacent time is regular. Combining with this character and based on LS estimation algorithm, the proposed estimation algorithm modifies the CSI of current time according to the predicted value of previous CSI and its predicted error. By iterative computations, the predicted CSI of all the time can be obtained. At the same time, due to the low-pass filtering effect of ESA, the random disturbance can be effectively suppressed. Therefore, relative to LS algorithm, the precision of estimation can be improved. Moreover, compared with DFT-based estimation algorithm, it is not necessary to know the maximum delay spread. Hence, the problem existing in DFT-based estimation algorithm is solved.

The rest of this paper is organized as follows. The system model of MIMO-OFDM is established in Section 2. Then, in Section 3, typical LS estimation algorithm and proposed ESA based improved method for MIMO-OFDM system are introduced. Meanwhile, the reason why the proposed algorithm has the ability of noise suppression is theoretically deduced in this part. Simulation results and conclusions are finally presented in Section 4 and 5, respectively.

2. System Model

2.1 Channel Model of MIMO

In this section, a centralized MIMO system is considered and the mathematical model for the MIMO channel is established. The model based on the tapped delay line can be described as follows [10]

\[ H(t, \tau) = \sum_{l=1}^{L} A_l(t) \delta(\tau - \tau_l(t)) \]  

(1)

where \( L \) and \( \tau_l(t) \) represent the total number of propagation paths and the \( l \)th path delay at time \( t \), respectively. \( \delta(.) \) denotes the impulse function and \( A_l(t) \) is channel coefficients matrix of the \( l \)th path at time \( t \), which can be expressed as follows [11]

\[
A_l(t) = \begin{bmatrix}
\alpha_{11}'(t) & \alpha_{12}'(t) & \cdots & \alpha_{1N_r}'(t) \\
\alpha_{21}'(t) & \alpha_{22}'(t) & \cdots & \alpha_{2N_r}'(t) \\
\vdots & \vdots & \ddots & \vdots \\
\alpha_{N_t1}'(t) & \alpha_{N_t2}'(t) & \cdots & \alpha_{N_tN_r}'(t)
\end{bmatrix}
\]  

(2)

where \( \alpha_{ji}'(t) \) indicates the channel coefficients of the \( l \)th path from the \( i \)th transmit antenna to the \( j \)th receive antenna, \( N_t \) and \( N_r \) denote the number of transmit antenna and receive antenna, respectively. Hence, the channel impulse response from the \( i \)th transmit antenna to the \( j \)th receive antenna at time \( t \) can be given as follows

\[ h_{ij}(t, \tau) = \sum_{l=1}^{L} \alpha_{ji}'(t) \delta(\tau - \tau_{ij}'(t)) \]  

(3)

where \( \tau_{ij}'(t) \) represents the \( l \)th path delay from the \( i \)th transmit antenna to the \( j \)th receive antenna. It assumes that the channel impulse response remains unchanged during one OFDM symbol but changes between symbols. Hence, formula (1) can be simplified as follows
\[ H(\tau) = \sum_{i=1}^{L} A(i) \delta(\tau - \tau_i) \]  

(4)

In this condition, \( h_j(\tau, \tau) \) in formula (3) can be described as

\[ h_j(\tau) = \sum_{i=1}^{\tau_j} \alpha^j_i \delta(\tau - \tau^j_i) \]  

(5)

For discrete-time system, \( h_j(\tau) \) can be approximated as a vector [12]

\[ h_j = [h_j(1), h_j(2), \ldots, h_j(L)]^T \]  

(6)

### 2.2 System Model of MIMO-OFDM

Fig. 1 displays the MIMO-OFDM baseband model used in this paper, where \( N_t \) and \( N_r \) represent the number of transmit and receive antennas, respectively. After space-time coding, pilot signals are inserted to different subcarriers. At the receiver side, channel estimation is implemented after FFT. In order to eliminate inter-symbol interference caused by multipath propagation, the length of CP \( L_g \) should be longer than \( L \) which is the maximum delay of multipath channel [13].

Taking the \( j \)th receive antenna as an example, the received signal in time-domain after CP removing can be expressed as

\[ y_j = \sum_{i=1}^{N_r} X_{ij} h_j + z_j \]  

(7)

where \( y_j = [y_j(1), y_j(2), \ldots, y_j(K)]^T \) denotes the received signal in time-domain at the \( j \)th receive antenna, \( h_j \) is given by formula (6), \( X_i \) represents a matrix consisting of transmit signals from the \( i \)th transmit antenna and is given as

\[ X_i = \begin{bmatrix} x_i(1) & x_i(2) & \cdots & x_i(K-L+2) \\ x_i(2) & x_i(1) & \cdots & x_i(K-L+3) \\ \vdots & \vdots & \ddots & \vdots \\ x_i(K) & x_i(K-1) & \cdots & x_i(K-L+1) \end{bmatrix} \]  

(8)

\( z_j = [z(1), z(2), \ldots, z(K)]^T \) denotes AWGN whose mean value and variance is zero and \( \sigma^2 \), respectively [14].

After \( K \)-point FFT transformation of received signals in time-domain, the received signals in frequency-domain can be expressed as follows.
\[ y_j^f = \sum_{i=1}^{N_T} X_j^f h_{ji}^f + z_j^f \]  
where \( y_j^f = [y_j^f(1), y_j^f(2), \ldots, y_j^f(K)]^T \) denotes the received signals in frequency-domain at the \( j \)th receive antenna. \( X_j^f = \text{diag}[x_j^f(1), x_j^f(2), \ldots, x_j^f(K)] \) is a matrix consisting of signals from the \( i \)th transmit antenna in frequency-domain. \( h_{ji}^f \) and \( z_j^f \) are the frequency-domain expression of \( h_{ji} \) and \( z_j \) respectively.

3. The Proposed Approach

In this section, the typical LS channel estimation algorithm is briefly introduced, and the proposed novel estimation method is described emphatically. In the two methods, it is assumed that channel impulse response between OFDM symbols varies slowly, and pilot arrangement is in comb type. At the same time, pilot signals from different antennas are located in crossed positions [15].

3.1 Typical LS Estimation Algorithm

According to least squares criterion, LS estimation algorithm estimates CSI according to the pilot signals and the corresponding received signals. Based on formula (9), received signals at pilot positions in frequency-domain can be given as

\[ y_j^p = \sum_{m=1}^{N_T} X_m^p h_{jm}^p + z_j^p \]  
where superscript \( P \) denotes the position of pilot signals. For the convenience of description, pilot signals from all transmit antennas in formula (10) are arranged as follows

\[ Q_m(n) = x_m^p(m + (n - 1)p) \]  
where \( Q_m(n) \) represents the \( n \)th pilot signals from the \( m \)th transmit antenna. By sorting out all pilot signals, matrix \( Q \) can be expressed as follows

\[ Q = \text{diag}(Q_1(1), Q_1(2), \ldots, Q_{N_T}(1), Q_2(1), Q_2(2), \ldots, Q_{N_T}(2), \ldots, Q_M(M), Q_M(M), \ldots, Q_{N_T}(M)) \]  
where \( M \) denotes the number of pilot signals from one OFDM symbol, \( K \) and \( p \) denote the length of FFT and the interval of pilot signals, respectively. The mathematic relation between the three parameters is \( M = \left\lceil \frac{K}{p} \right\rceil \). Corresponding to \( Q \), channel frequency response from all the transmit antennas to the \( j \)th receive antenna is given by

\[ h_{jm}^p = [h_{j1}^p(1), h_{j1}^p(2), \ldots, h_{j1}^p(N_t), h_{j2}^p(1), h_{j2}^p(2), \ldots, h_{j2}^p(N_t), \ldots, h_{jM}^p(1), h_{jM}^p(2), \ldots, h_{jM}^p(N_t)]^T \]  

Hence, formula (10) can be converted to the following form

\[ y_j^p = Qh_{jm}^p + z_j^p \]  

Based on formula (14), the channel state parameter \( \hat{h}_{jm}^p \) obtained by the LS channel estimation algorithm can be expressed as

\[ \hat{h}_{jm}^{\text{LS}} = Q^+ y_j^p + Q^+ z_j^p \]  
where

\[ \hat{h}_{jm}^{\text{LS}} = [\hat{h}_{j1}^{\text{LS}}(1), \hat{h}_{j1}^{\text{LS}}(2), \ldots, \hat{h}_{j1}^{\text{LS}}(N_t), \hat{h}_{j2}^{\text{LS}}(1), \hat{h}_{j2}^{\text{LS}}(2), \ldots, \hat{h}_{j2}^{\text{LS}}(N_t), \ldots, \hat{h}_{jM}^{\text{LS}}(1), \hat{h}_{jM}^{\text{LS}}(2), \ldots, \hat{h}_{jM}^{\text{LS}}(N_t)]^T \]  

After grouping channel frequency response from the same transmit antenna as \( \hat{h}_{j1}^{\text{LS}}(1), \hat{h}_{j2}^{\text{LS}}(p + 1), \ldots, \hat{h}_{jM}^{\text{LS}}([M - 1]p + 1) \), channel frequency response of all subcarriers can be obtained
by interpolation algorithm such as linear interpolation, second-order interpolation, spline interpolation and so on [16].

As seen in formula (15), \( Q^{-1} z_j^p \) is the estimation error generated by AWGN, and the greater the noise is, the worse performance is obtained. Therefore, the random noise is the main factor that affects the performance of the LS algorithm. To reduce the effect of noise, the improved channel estimation method based on ESA is proceeded below.

3.2 Improved Channel Estimation Method

3.2.1 Exponential Smoothing

Exponential smoothing [17] is a kind of method that analyses the variation tendency of time series with low complexity. It can effectively suppress disturbance caused by random factor. For sequences of time, ESA predicts the value of current time according to the variation tendency of all the time before. In details, in order to reduce the estimation error, an iterative method is needed to correct the predicted value of the current time by using the value of the previous time. Because the forecast process is regular, the disturbance caused by random factor can be weakened effectively.

The recursion formula of ESA is given as [18]

\[
\hat{H}_t = \alpha (\hat{H}_t - \hat{H}_{t-1}) + \hat{H}_{t-1}
\]

(17)

where \( \alpha \) denotes the smoothing coefficient varying from 0 to 1, the two sequences of time \( \hat{H}_t \) and \( \hat{H}_t \) are smoothed value and observed value at time \( t \), respectively. Formula (17) also can be described as

\[
\hat{H}_t = \sum_{i=0}^{t-1} \alpha (1-\alpha)^i \hat{H}_{t-i} + (1-\alpha)^t \hat{H}_0
\]

(18)

where \( \hat{H}_0 \) equaling to \( \hat{H}_0 \) is supposed in this paper. Hence, ESA can be regarded as a recursion algorithm.

Formula (17) can be transformed as follows

\[
\hat{H}(n) = \alpha \hat{H}(n) + (1-\alpha)\hat{H}(n-1)
\]

(19)

In fact, formula (19) is the expression of a digital filter and its system function can be denoted as

\[
H(z) = \frac{\hat{H}(z)}{\hat{H}(z)} = \frac{\alpha z}{z - (1-\alpha)}
\]

(20)

The frequency response of this filter is

\[
H(e^{j\omega}) = \frac{\alpha e^{j\omega}}{e^{j\omega} - (1-\alpha)}
\]

(21)

\[
= \frac{\alpha}{[1-(1-\alpha)\cos \omega] + j(1-\alpha)\sin \omega}
\]

Then the amplitude-frequency response can be given by

\[
|H(e^{j\omega})| = \frac{\alpha}{\sqrt{1 + (1-\alpha)^2 - 2(1-\alpha)\cos \omega}}
\]

(22)

For \( \alpha \) varying from 0 to 1, this filter is a digital low-pass filter essentially [19]. Due to the disturbance caused by random factor distributes in all frequency, those noises locating in medium and high frequency can be filtered by the ESA equivalent filter. The filtering effect of different \( \alpha \) is shown in Fig. 2. With the decreasing of smoothing coefficient \( \alpha \), better effect of low-pass filtering is obtained.
3.2.2 Improved LS Estimation Algorithm Based on ESA

In the following section, an ESA based estimation of the channel attenuations $\hat{h}_{ji}^{LS}$ from the estimation results of typical LS algorithm $\hat{h}_{ji}$ is presented. According to paper [19], channel impulse response during one OFDM symbol can be regarded as low-frequency signal under the assumption that the channel impulse response varies slowly between OFDM symbols. Meanwhile, the AWGN, which is the reason of inaccurate estimation results of LS algorithm, distributes in all frequency. Hence, the ESA equivalent low-pass filter can be used to suppress noise in typical LS estimation algorithm. The ESA based LS estimator consists of two separate steps. In the first step, IDFT is implemented to obtain the results of typical LS algorithm in time-domain, and the results are regarded as the observed value. In the second step, based on the observed value, ESA is employed to predict the CSI of every time and suppress noise. The procedure of the improved LS estimation algorithm is as follows:

**Step 1:**
$$\hat{h}_{ji}^{LS} = \text{IDFT}\{\hat{h}_{ji}^{LS}\}$$

**Step 2:**
$$\hat{h}_{ji}^{LS}(q) = \alpha(\hat{h}_{ji}^{LS}(q) - \hat{h}_{ji}^{LS}(q-1)) + \hat{h}_{ji}^{LS}(q-1)$$
where $\hat{h}_{ji}^{LS}(q)$ denotes smoothed channel impulse response of the $q$th OFDM symbol.

**Step 3:**
$$\hat{h}_{ji}^{LS} = \text{DFT}\{\hat{h}_{ji}^{LS}(q)\}$$
where the point of DFT is $K$.

The smoothing coefficient $\alpha$ varies from 0 to 1 ($0 < \alpha < 1$). At the same time, $\hat{h}_{ji}^{LS}$ is the smoothed CSI which is closer to the real CSI when compared with $\hat{h}_{ji}^{LS}$.

4. Simulation

In this section, based on the analysis above, the performance of proposed method is presented by comparing with the LS, DFT and MMSE estimation algorithm. Simulation parameters are displayed in Table 1, where pilot pattern is comb-type and pilot from different antennas are in crossed position. Furthermore, the channel impulse response varies between OFDM symbols.

| Parameters                     | Specifications |
|--------------------------------|---------------|
| Number of Transmit/Receive Antennas | 2             |
| FFT Size                       | 1024          |
| Modulation Mode                | QPSK          |
| Cyclic Prefix                   | 64            |
| Number of Multipath            | 6             |
| Pilot Interval                 | 8             |
As displayed in Fig. 3, with the decreasing of $\alpha$, better estimation results can be obtained in the condition of low SNR. However, in the condition of higher SNR, the performance becomes worse. Hence, three optimal smoothing coefficient $\alpha$ is set to 0.1, 0.3, 0.4 according to the best mean square error (MSE) when signal-to-noise ratio (SNR) is equal to 0dB, 14dB, 28dB.

The estimation error of the proposed algorithm is depicted in Fig. 4. Compared with LS, the ES based estimation method performs better than typical LS estimation algorithm when $\alpha$ is set to 0.4 and 0.3. This is due to ESA has low-pass characteristic, which can reduce the random noise existing in LS algorithm. Meanwhile, as mentioned before, the major problem of DFT-based estimation algorithm is that extra algorithm is needed to estimate the maximum delay spread. This problem is avoided in the proposed method and when $\alpha$ is set to 0.4, the performance of the proposed method is similar to that of DFT-based estimation algorithm. Moreover, in the condition of high SNR, smaller $\alpha$ brings worse performance. The reason for this is that useful signals play a leading role in the condition of high SNR, while smaller $\alpha$ brings stronger filtering effect as shown in Fig. 2. Hence, the performance becomes terrible as ESA filters noise and much of the useful signals at the same time. Therefore, for all SNRs, it is necessary to choose the best $\alpha$ to filter noise and protect useful signals simultaneously. In the simulation, to maintain a balance of the performance among the low, medium and high SNR, $\alpha$ equaling to 0.4 is the best choice. Then, even though MMSE has the best performance, its computational complexity is much more than ESA.

The bit error rate (BER) performance of LS, DFT, MMSE and ESA is displayed in Fig. 5, where coefficient $\alpha$ is set to the best choice 0.4. Consistent with the previous analysis, ESA improves the performance of LS estimation algorithm. At the same time, BER of ESA is similar to that of DFT-based estimation algorithm without the problem of some extra work to obtain maximum delay spread.

Therefore, the conclusion that ESA is an effective method to improve the performance of LS estimation algorithm is obtained. Compared to DFT-based estimation algorithm which is frequently-used for LS improving, ESA avoids the problem of extra estimation to obtain maximum delay spread. At the same time, the computational complexity of ESA is much lower than that of MMSE estimation algorithm.

Figure 3 MSE of ESA for different SNR and $\alpha$
5. Conclusion

1) The ESA is capable of revealing the variation regularity hidden in time series and suppressing random disturbance. Therefore, in the condition of time-varying channel, the inaccurate estimation results of typical LS algorithm caused by AWGN can be improved by the proposed ESA estimation method.

2) The smoothing effect of ESA varies with different smoothing coefficient $\alpha$. In the condition of low SNR, the smaller $\alpha$ is chosen, the better performance is obtained. On the contrary, in the condition of higher SNR, smaller $\alpha$ incurs worse performance. Hence, an optimal choice of $\alpha$ is necessary to fit different SNRs.

3) The benefits of ESA have been shown in terms of MSE and BER by considering a multipath channel model. Simulation results show that ESA improves the performance of typical LS estimation algorithm effectively. In addition, another remarkable enhancement is that the problem of extra estimation work to obtain maximum delay spread in DFT-based algorithm is avoided in ESA and the estimation precision of ESA is similar to that of DFT-based algorithm. Finally, compared with MMSE estimation algorithm, ESA has lower computational complexity.
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