Nuclear Effect in Higher-Dimensional Factorial Moment Analysis of the $^{16}$O-, $^{32}$S- and $^{197}$Au-Em Interaction Data at 200, 60 and 11 A GeV/c

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The anomalous behavior of 2-dimensional factorial moment in nucleus-nucleus collisions is studied in some detail using both mini-bias and central collision data of $^{16}$O-, $^{32}$S- and $^{197}$Au-Em interactions from EMU01 experiment. The correct value for the effective Hurst exponent in the analysis of higher-dimensional factorial moment is found to be greater than unity, showing clearly the existence of superposition effect in nucleus-nucleus collisions.
I Introduction

The scaling property of factorial moment (FM) defined as

\[ F_q(M) = \frac{1}{M} \sum_{m=1}^{M} \frac{\langle n_m(n_m - 1) \cdots (n_m - q + 1) \rangle}{\langle n_m \rangle^q} \]  

(1)

has been studied widely in various kinds of collisions, from hadron-hadron, \( e^+e^- \) to nucleus-nucleus at various energies\[^1\], and a rich pattern of phenomena has been obtained. In Eq.(1) \( M \) is the partition number of a phase space region \( \Delta \), \( n_m \) is the multiplicity in the \( m \)th subdivision cell.

The goal of this kind of study is primarily due to the hope of exploring the possible existence of dynamical fluctuation (or intermittency) in high energy collisions\[^2\]. Remarkable success has been obtained recently in this respect for hadron-hadron collisions — experimental evidences for anisotropical dynamical fluctuation (self-affine fractal) have been observed in \( \pi^+p \) and \( K^+p \) collisions at 250 GeV/c\[^3\]. It should be noticed, however, that the “intermittency” phenomena in different kind of collision processes may come from different physical origins, e.g. QCD parton shower, Bose-Einstein correlation, second order phase transition, etc. In some cases, especially in nuclear collision data from emulsion experiments the influence of \( \gamma \)-conversion should also be considered\[^4\]. In the present paper we will not deal with the physical origin of the phenomena and will concentrate on the comparison of the phenomena in A-A and h-h collisions in order to study the characteristic of nuclear effect and its possible application.

In comparing the scaling property of factorial moment in nucleus-nucleus and hadron-hadron collisions, noticeable universal phenomena can be observed:

1) In one-dimension, the raise of the logarithm of FM (lnFM) as the increasing of that of the phase space partition number \( M \) is much weaker for nucleus-nucleus than for hadron-hadron collisions, and the heavier the colliding nuclei are, the weaker is the rising of lnFM.

2) In higher-dimension, the lnFM for nucleus-nucleus collisions turns out to be bending upwards strongly, much stronger than for hadron-hadron collisions, and the heavier the colliding nuclei are, the stronger is the upward-bending of lnFM.

It has been shown in Ref.[5] that these two apparently contradictory facts are both due to the superposition effect of the contribution from the large number of elementary collisions in a nuclear collision process. This is a geometrical effect and is valid regardless of what is the physical origin of the phenomena. It may thus be useful in the search for new physics, such as quark-gluon plasma (QGP)\[^5\].

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In this paper these phenomena will be studied in more detail using the nucleus-nucleus collision data from the EMU01 experiment.

II  Method of analysis

Let us first recall briefly the superposition effect in the scaling behavior of factorial moment in nuclear collisions\textsuperscript{[5]}.

As is well known, in normal cases a nuclear collision process consists of a large number of elementary nucleon-nucleon collisions. Due to the complexity of the collision process, the rapidity centers of individual elementary collisions do not coincide but are scattered randomly within an interval on the rapidity axis. For example, in central nuclear collision each participating nucleon in an incident nucleus is facing a tube of nucleons in the other nucleus, so that each nucleon will collide with a number of nucleons successively. The difference in physical condition between these successive collisions makes the rapidity centers of individual elementary collisions scattered on the rapidity axis. When the (pseudo)rapidity region of each elementary collision is divided into \( M_{\parallel} \) pieces, their superposition makes the whole rapidity region be divided into a much larger number (\( M_{\parallel \perp} \)) of pieces, cf. Fig.1.

On the other hand, similar effect does not exist in the transverse direction (\( p_t, \varphi \)) where the phase space region is the same for all the elementary collisions, being \( (0, p_{t_{\max}}) \) for \( p_t \) and \( (0, 2\pi) \) for \( \varphi \), and their superposition makes no change in phase space partition.

A quantity \( H \) called Hurst exponent can be used to characterize the way of phase space partition. It is defined as

\[
H_{ab} = \frac{\ln M_a}{\ln M_b},
\]

where \( M_a \) and \( M_b \) are the partition numbers in the directions \( a \) and \( b \) respectively. The anomalous scaling of FM, if exists, is definitely connected with a certain value of \( H \). If the way of phase space partition is incorrect, that is to say, if the FM is not calculated with the right value of \( H \), the resulting \( \ln F_q \) vs. \( \ln M \) will be bending upwards\textsuperscript{[6]}.

In nucleus-nucleus collisions, due to the superposition of elementary collisions, the effective Hurst exponent

\[
H_{\parallel \perp}^{eff} = \frac{\ln M_{\parallel \perp}^{eff}}{\ln M_{\parallel \perp}} \gg \frac{\ln M_{\parallel}}{\ln M_{\perp}} = H_{\parallel \perp}.
\]

Therefore, when we calculate FM with \( H_{\parallel \perp}^{cal} \approx H_{\parallel \perp} \ll H_{\parallel \perp}^{eff} \), we will observe that \( \ln F_q \) vs. \( \ln M \) is bending strongly upwards. On the contrary, if we take a larger value for \( H_{\parallel \perp}^{cal} \), the phenomenon of
upward bending should be weakened and eventually tend to vanish as $H_{\parallel \perp}^{\text{cal}}$ get close to $H_{\parallel \perp}^{\text{eff}}$.

In order to characterize the degree of upward-bending we use a simple quadratic function $y = ax^2$ to fit the $\ln F_2$ vs. $\ln M$ data\footnote{In general, a linear term $bx$ should also be present in the function used for fitting. However, in the present case, similar to the reasoning leading to phenomenon 1 listed in Introduction, due to the large number of nucleons in the colliding nuclei the coefficient $b$ of the linear term, characterizing the strength of anomalous scaling, is very small and can therefore be omitted.} In the fitting the first few data points have been omitted to reduce the effect of momentum conservation\cite{7}. The origin of coordinates is then moved to the starting point $(M_0, F_2(M_0))$ of fitting and the variables are changed correspondingly to

$$x = \ln M - \ln M_0, \quad y = \ln F_2(M) - \ln F_2(M_0).$$

The fitting value of $a$, being proportional to the second order derivative $d^2y/dx^2$, is taken to be the characteristic parameter for the degree of upward-bending of $\ln F_2$ vs. $\ln M$.

III The experimental setup

The EMU01 collaboration has collected data from collisions between various projectiles and targets at different incident energies in the ultra-relativistic region\cite{8}. Two different techniques have been employed, both utilizing nuclear emulsion; ordinary emulsion stacks with exposures parallel to the emulsion plates and emulsion chambers in which the exposures are perpendicular to the plates. The second technique is best suited for an analysis in which the azimuthal emission angles are of interest since, in this case, the detector has azimuthal symmetry and high resolution. With this technique a resolution of $\Delta \eta \simeq 0.013$ rapidity units in the central region can be obtained\cite{9}. Pseudo-rapidity is given by $\eta = -\ln(\tan(\theta/2))$, where $\theta$ is the emission angle with respect to the beam direction. A major fraction of the chambers are equipped with thin target foils, providing possibilities to study interactions with various targets. In this paper, we have analysed data from interactions induced by the CERN/SPS 200 A GeV Oxygen and Sulphur beams, 60 A GeV Oxygen beams and the BNL/AGS 11A GeV Gold beams on targets of emulsion. The data samples used are as the following:
Experiments & Energy & Number of events &
(A GeV) & Mini-Bias Samples & Central Samples &

| Experiments | Energy (A GeV) | Number of events |
|-------------|--------------|------------------|
| 197 Au-Em   | 11           | 45               |
| 16 O-Em     | 60           | 903              |
| 16 O-Em     | 200          | 32               |
| 32 S-Em     | 200          | 889              |

Further details on the experiment, measurements and experimental criteria can be found elsewhere\cite{10}.

IV Results

We have analysed the second order FM for the experimental data with several values of Hurst exponent \((H = 1.0, 0.8, 2.0, 3.0)\) in the plane \((\eta, \varphi)\). Both central and minimum-bias events have been used.

The partition numbers in longitudinal and transverse directions are chosen respectively as

\[
M_\varphi = 1, 2, \ldots, 50, \quad M_\eta = M_\varphi^H, \text{ for } H = 1.0, \ 0.8 \ ; \\
M_\eta = 1, 2, \ldots, 50, \quad M_\varphi = M_\eta^{1/H}, \text{ for } H = 2.0, \ 3.0 .
\]

The method proposed in Ref.\cite{11} has been used for noninteger \(M\).

In Fig’s.2 and 3 the results for the minimum-bias data samples from \(^{32}\text{S}-\text{Em} (200\text{ A GeV})\) and \(^{16}\text{O}-\text{Em} (200\text{ and } 60\text{ A GeV})\) are shown respectively.

In Fig.4 the same analysis is repeated for central collision data samples from 11 A GeV \(^{197}\text{Au}-\text{Em}\) and 60 A GeV \(^{16}\text{O}-\text{Em}\).

The pseudo-rapidity region used is \([\eta_{\text{center}} - 2, \eta_{\text{center}} + 2]\), and the azimuthal region is \([0, 2\pi]\). In order to reduce the effect of non-flat average distribution the cumulative variables \(\chi_\eta\) and \(\chi_\varphi\) have been used instead of \(\eta\) and \(\varphi\)\cite{12}. The corresponding regions then become \([0, 1]\).

It can be seen from the figures that when \(H = 1\), the curves bend upwards strongly, and the smaller is the value of \(H\), the more strongly is the upward-bending. When \(H\) increases, the upward-bending is weakened, and at \(H = 2\) or 3 the curves are almost straight lines. This means that, in order to recover the anomalous scaling of FM, the phase space should be divided finer in the longitudinal direction than in the transverse direction. This is in contrary to the case of hadron-hadron collisions, where the anomalous scaling of FM is obtained with \(H < 1\)\cite{3}, i.e. when the phase space is divided finer in the transverse direction than in the longitudinal direction. Since each elementary collision process in the nuclear collision is expected to mimic hadron-hadron collision, the finer partition in longitudinal
direction for nuclear collisions could only be due to the superposition effect shown schematically in Fig.1.

Next, we try to get a quantitative description of the phenomena by fitting the upward-bending curves with the quadratic function \( y = ax^2 \) as described in section II. In order to show the quality of fitting we plot in Fig.5 the results of fitting for the \( H = 1 \) curve of \(^{197}\text{Au-Em} \) central collision (the first figure in Fig.4a). In Fig.5a all the points have been used and the origin is shifted to the first point. In Fig.5b, c, d the first 3, 4, 5 points are omitted and the origin is shifted to the 4-, 5-, 6th point respectively. It can be seen from the figures that the fit is good after omitting the first few points and the result is insensitive to the exact number of omitted points provided it is not too small, e.g. not smaller than 3.

The fit is unsatisfactory when all the points are included, cf. Fig.5a. This is due to momentum conservation. The later tends to spread the particles to opposite directions and thus reduce the value of FM. This effect becomes weaker when \( M \) increases. This explains why, when all the points are included, the data points for medium values of \( M \) lie below the fitting curve, cf. Fig.5a.

In order to study the dependence of the above-mentioned superposition effect on the mass of colliding nuclei, we have compared the results from central collision samples shown in Fig.4, where the impact parameter smearing is the minimum.

The fitting parameter \( a \) for all the curves in Fig.4, with the first three points omitted, are shown in Fig.6. From Fig.6 we see that \( a \) decreases with the increasing of \( H \), i.e. the \( \ln F_2 \) vs. \( \ln M \) curves bend upwards stronger for smaller \( H \), and tend to become straight lines as \( H \) increases.

From Fig.6 we also find that the values of \( a \) is always bigger for \(^{197}\text{Au-Em} \) than for \(^{16}\text{O-Em} \) collisions at the same value of \( H \). This means that the \( \ln F_2 \) vs. \( \ln M \) curves bend upwards stronger for heavier colliding nuclei than for lighter ones at the same \( H \). This is just what to be expected, because for heavier colliding nuclei the number of elementary collisions is larger, making the \( H^\text{eff}_{\|\perp} \) bigger. When we calculate with the same value of \( H^\text{cal}_{\|\perp} \) then this value will be further away from \( H^\text{eff}_{\|\perp} \) for heavier colliding nuclei than for lighter ones.

It can also be observed in Fig’s.2–4 that for those values of \( H > 1 \), which make the \( \ln F_2 \) vs. \( \ln M \) curves approximately straight, the slope of these straight lines are small. This can also be understood from the superposition effect in nucleus-nucleus collisions, which reduces the fluctuations characterized by the slope of \( \ln F_2 \) vs. \( \ln M \) (cf. the phenomenon 1 listed in Introduction).
It should be stressed that the 2-D results from EMU01 have main contribution from $\gamma$-conversion\cite{4} and so cannot be used to discuss the physical property of the final state hadronic system. However, the superposition effect discussed above is mainly a geometrical effect. It concerns only the different ways of phase space division and does not depend on the concrete property of the particles in consideration. Therefore, the above discussion is appropriate for the 2-D data from EMU01 as well.

IV Conclusion

We have investigated the 2-dimensional ($\eta, \varphi$) $\ln F_2$ vs. $\ln M$ with different values of $H$ from both the minimum-bias and the central collision data of EMU01 experiments. The value of $H_{\parallel, \perp}^{\text{eff}}$ in nucleus-nucleus collision is found to be larger than unity. This means that only when we use a right value of $H_{\parallel, \perp}^{\text{cal}}$, which is larger than unity, in the analysis, can the superposition of the contributions from elementary collisions in nucleus-nucleus collision be correctly accounted for, and thus a straight line in $\ln F_2$ vs. $\ln M$ be obtained.

A parameter $a$ is introduced to characterize the degree of upward-bending of $\ln F_2$ vs. $\ln M$. Using this parameter it is found that the heavier are the colliding nuclei, the stronger is the upward-bending of $\ln F_2$ vs. $\ln M$, in consistence with the fact that the number of elementary collisions is larger for heavier colliding nuclei.

Our results show clearly the existence of superposition effect in the behavior of higher-dimensional FM of nucleus-nucleus collisions and thus support the argument proposed in Ref.\cite{5} that the disappearance of strongly upward-bending of higher-dimensional factorial moments may be taken as a signal for the melting into a unique system of the produced particles from elementary collisions in relativistic heavy ion collision.

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Figure Captions

Fig. 1 Schematic plot of the superposition effect on longitudinal phase space division.

Fig. 2 Log-Log plot of 2-D $(\eta, \varphi)$ $F_2$ vs. the partition number $M(\eta)$ in longitudinal direction ($\eta$) from EMU01 200 A GeV $^{32}$S-Em data for different values of $H$.

Fig. 3 The same as Fig. 2 from 200 and 60 A GeV $^{16}$O-Em data.

Fig. 4 The same as Fig. 2 from 11 A GeV $^{197}$Au-Em and 60 A GeV $^{16}$O-Em central collision data.

Fig. 5 Fit the curve ($H = 1$) in Fig. 4 by square function $y = ax^2$ after shifting the origin to the first, fourth, five, and six point. Solid line stands for the fitting curve of $ax^2$.

Fig. 6 Comparison of the fitting parameter $a$ of the curves in Fig’s.4 with the origin of coordinate shifted to the fourth point.
Longitudinal phase space partition in an elementary collision process

Effective partition of phase space in the superposition of elementary collision processes

Fig 1

Fig 2

Fig 3a

Fig 3b
