A FOREGROUND-CLEANED COSMIC MICROWAVE BACKGROUND MAP FROM NON-GAUSSIANITY MEASUREMENT

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ABSTRACT

In this Letter, we present a new method to estimate a foreground-cleaned cosmic microwave background (CMB) map at a resolution of 1° by minimizing the non-Gaussian properties of the cleaned map which arise dominantly due to diffuse foreground emission components from the Milky Way. We employ simple kurtosis statistic as the measure of non-Gaussian properties and perform a linear combination of five frequency maps provided by the Wilkinson Microwave Anisotropy Probe (WMAP) in its seven-year data release in such a way that the cleaned map has a minimum kurtosis which leads to a non-Gaussianity-minimized, foreground-cleaned CMB map. We validate the method by performing Monte Carlo simulations. To minimize any residual foreground contamination from the cleaned map we flag out the region near the galactic plane based upon results from simulations. Outside the masked region our new estimate of the CMB map matches well with the WMAP’s Internal Linear Combination (ILC) map. A simple pseudo-Cℓ-based CMB TT power spectrum derived from the non-Gaussianity minimized map reproduces the earlier results of WMAP’s power spectrum. An important advantage of the method is that it does not introduce any negative bias in angular power spectrum in the low multipole regime, unlike usual ILC method. Comparing our results with the previously published results we argue that CMB results are robust with respect to specific foreground removal algorithms employed.

Key words: cosmic background radiation – cosmology: observations – diffuse radiation

Online-only material: color figures

1. INTRODUCTION

Cosmic microwave background (CMB) has conceivably become the finest tool so far for probing the physics of the early universe. The problem of isolating a clean CMB signal from contaminations originating from Milky Way is of primary importance to a cosmologist. According to the slowly rolling single scalar field inflationary scenario the primordial perturbations follow Gaussianity to a very good approximation, any non-Gaussian properties predicted to be a small effect (Allen 2003). The importance to a cosmologist. According to the slowly rolling single scalar field inflationary scenario the primordial perturbations follow Gaussianity to a very good approximation, any non-Gaussian properties predicted to be a small effect (Allen et al. 1987; Falk et al. 1992; Gangui et al. 1994; Acquaviva et al. 2003; Maldacena 2003). Since CMB anisotropies are directly related to these perturbations via the spacetime metric one expects CMB to follow a Gaussian distribution. It was shown by Munshi et al. (1995) that the non-Gaussian effect introduced in CMB after decoupling is also small. It has been shown by Komatsu et al. (2003) that CMB data observed by the Wilkinson Microwave Anisotropy Probe (WMAP) satellite (Bennett et al. 1997) are consistent with primordial Gaussian fluctuation. Contrary to these, diffuse foreground components originating from our own galaxy exhibit highly non-Gaussian properties due to nonlinearities involved in the physical processes during their origin. Based upon simple assumption of pure Gaussian properties of CMB and the non-Gaussian nature of diffuse foregrounds, in this Letter we propose a new foreground removal method from CMB maps. One of the most important aspects of the method is that it continues to preserve a CMB signal even if it possesses a non-Gaussian nature which may result due to any possible primordial and/or secondary effects, e.g., CMB weak lensing. A detailed account on several other foreground removal methods may be found in existing literature (Brandt et al. 1994; Tegmark & Efstathiou 1996; Bouchet & Gispert 1999; Bennett et al. 2003; Tegmark et al. 2003; Saha et al. 2006; Hansen et al. 2006; Eriksen et al. 2006; Hinshaw et al. 2007; Saha et al. 2008; Eriksen et al. 2008b, 2008a; Leach et al. 2008; Delabrouille et al. 2009; Samal et al. 2010; Pietrobon et al. 2010). For discussions about searching for non-Gaussianity in foreground-minimized CMB maps we refer to Räth et al. (2009), Barreiro et al. (2000), Banday et al. (2000), Raeth et al. (2011), Hou et al. (2010), and Bernui & Rebouças (2010).

2. METHODOLOGY

2.1. Estimator for Non-Gaussianity

A measure of non-Gaussian properties inherent in a collection of N random samples is given by so-called kurtosis statistic, (K). For a set, S, of N random samples, \( S = \{x_i | i = 1, 2, 3, ..., N\} \), the K statistic is defined as

\[
K = \frac{1}{N} \sum_{i=1}^{N} \frac{(x_i - x_0)^4}{\sigma^4} - 3, \tag{1}
\]

where \( x_0 \) denotes the sample mean and \( \sigma \) is the standard deviation of samples. The sample kurtosis is zero in the mean for samples drawn from the Gaussian distribution, if \( \sigma \) is taken as the theoretical standard deviation of the underlying Gaussian distribution, whereas as discussed in Section 3.1 foreground distributions possess large positive kurtosis.

2.2. Foreground-cleaned Map

Let \( X_f \) denote a foreground-contaminated CMB map in thermodynamic temperature unit at 1° resolution at a frequency index \( f \) and \( n_f \) represent the total number of available frequency bands. \( X_f \) is a \( 1 \times n \) vector, where \( n \) is the number of surviving pixels at a frequency band after masking the known point-source positions. We form a foreground-cleaned map as follows:

\[
X^c = \sum_{f=1}^{n_f} w_f X_f, \tag{2}
\]
where \( w_f \) denotes the weight factor for a linear combination of frequency index \( f \). We choose these factors such that the \( \mathcal{K} \) value for \( X \) is minimized with an imposed constraint on them that \( \sum_{f=1}^{n_f} w_f = 1 \). This condition preserves the actual value of CMB temperature at each pixel in the foreground-cleaned map.

Using Equations (1) and (2) one can show \( \mathcal{K}(W) \) of the linearly combined map as follows:

\[
\mathcal{K}(W) = \left[ \frac{n}{(WTW^T)^2} \sum_{j=1}^{n} (WT_j W^T j)^2 \right] - 3 ,
\]

where \( W \) denotes a \( 1 \times n_b \) vector whose \( j \)th entry is given by \( w_f \) and \( T = \sum_{j=1}^{n} T_j \). Here, \( T_j \) is an \( n_b \times n_b \) symmetric matrix for pixel \( j \), \( T_{j(f)} = \Delta T_{j(f)} \), where \( \Delta T_{j(f)} \) denotes the temperature at pixel \( j \) of frequency index \( f \) after the mean temperature corresponding to this frequency has been subtracted from actual pixel temperature.

In principle, the solution for \( W \) for minimum \( \mathcal{K}(W) \) satisfying our conditions can be obtained by employing a Lagrange’s undetermined multiplier approach. However, because of the nontrivial nature of dependency of \( \mathcal{K} \) on \( W \) we find that such an approach is not feasible for our problem. Instead, we find the minimum of \( \mathcal{K}(W) \) by invoking a nonlinear search algorithm due to Powell (Press et al. 1992).

3. RESULTS

3.1. Kurtosis

The distributions of pixel temperatures due to diffuse galactic foregrounds are shown in the top panel of Figure 1. Each of these foreground distributions is strongly asymmetric and exhibits a long tail toward the positive temperature direction indicating a variation slower than \( \sim e^{-T^2} \), which is the case for a Gaussian distribution. The peak for the synchrotron distribution shows that at 23 GHz most likely contamination due to synchrotron occurs at a temperature \( \sim 75 \mu K \), although contaminations at high \( (>400 \mu K) \) and low \( (e.g., \text{as low as} <25 \mu K) \) pixel temperature are also likely.

How does \( \mathcal{K} \) vary with foreground contamination in CMB maps at different WMAP frequencies? To answer this question we generate 150 random full sky CMB maps using WMAP’s LCDM power spectrum. With each of these random realization of CMB maps we add WMAP’s maximum entropy method (MEM) foreground templates (Gold et al. 2011) at various strengths. The resulting behavior of the \( \mathcal{K} \) with foreground strength is shown in Figure 1 for all WMAP frequency bands. For each band \( \mathcal{K} \) increases as the amount of foreground contamination increases. For \( K \) band the variation takes a shape of a plateau, characterized by a slow increase of \( \mathcal{K} \), as the foreground level reaches \( \sim 20\% \) of its full level. For all bands \( \mathcal{K} \) becomes more than 150 when all foregrounds are operative at their 100\% level. Interestingly, \( Q \) band (not the \( K \) band which has the highest level of synchrotron and free–free contamination) shows the largest \( \mathcal{K} \) value among all the five bands at the maximum foreground strength. This can be explained by noting that kurtosis depends upon the nature of non-Gaussian foreground distributions, and not only on the strength of foregrounds. Although, none of the individual MEM foreground components shows a maximum kurtosis at \( Q \) band, we find that kurtosis for synchrotron plus thermal dust maps has a peak at \( Q \) band—implying the peak at \( Q \) band is caused due to these components.

3.2. Validation of the Method

We validate the methodology by performing Monte Carlo simulations. We use MEM foreground maps for synchrotron, free–free and thermal dust available from LAMBDA Web site. These maps are provided in a common resolution of \( 1^\circ \) and at pixel resolution parameter nsidestart sign \( =128 \) in antenna millikelvin temperature unit. We upgrade the pixel resolution of each map to nsidestart end sign = 512 and convert them to thermodynamic microkelvin unit. We add the composite foreground map of each WMAP frequency with a random realization of CMB compatible to LCDM model to make foreground-contaminated CMB maps at each of the WMAP frequencies. Finally, we mask the position of the known point sources using the WMAP’s seven-year point-source mask. We note that the detector noise can be ignored for \( 1^\circ \) resolution of this work.

We develop a C code (hereafter GaussMap) to implement constrained Powell’s conjugate gradient method. Using 200 Monte Carlo simulations of foreground removal procedure we find that foreground removal is effective over almost all parts of the sky. However, we find visible signature of some residual foreground emission in the inner plane of the galaxy. To find out the sky regions where the residual foreground could be significant compared to the expected CMB signal we subtract average of input CMB maps from the average of foreground-cleaned maps. Then we form an initial mask by assigning zero values to all pixels with absolute temperature values more than \( 20 \mu K \) of this map and unity at all other pixels. This mask contains a set of scattered pixels in the inner galactic plane. To remove these pixels we first smooth the initial mask by a Gaussian window of \( 1^\circ \). We transform this smoothed mask to a new mask by assigning all pixels of smoothed mask with values greater than or equal to 0.9 to a new value of unity and the all other pixels to zero. To exclude position of the known point sources from the analysis we make the final mask multiplying this mask by the WMAP’s point-source mask. We call the resulting mask as G20 mask. This retains \( \sim 87\% \) of the entire sky area.

After excluding positions of known point sources we estimate \( \mathcal{K} \) for each of the cleaned maps obtained from Monte Carlo simulations of foreground removal procedure. The mean kurtosis obtained from all the cleaned maps is given by \( \langle \mathcal{K} \rangle = 0.86 \pm 0.21 \). This corresponds to \( \sim 4\sigma \) detection of non-Gaussianity from the cleaned maps. We interpret the non-vanishing \( \langle \mathcal{K} \rangle \) in terms of residual foreground contamination originating from the galactic plane. After flagging of the pixels determined by the G20 mask we obtain, \( \langle \mathcal{K} \rangle = -0.02 \pm 0.06 \), which is consistent with zero. Henceforth, we use the G20 mask as the basic mask to remove pixels contaminated by the residual foreground while analyzing cleaned maps from Monte Carlo simulations as well as WMAP data. The mean weights for five frequency bands satisfy \( \mathcal{W} \) = \( (0.049 \pm 0.021, -0.419 \pm 0.063, -0.213 \pm 0.027, 1.643 \pm 0.063, -0.059 \pm 0.029) \).

3.2.1. Temperature Distribution

Using the Monte Carlo simulations we verify that the pixel temperature of cleaned maps outside the G20 mask follows a Gaussian distribution. For this we apply the G20 mask both to a randomly chosen CMB realization and its foreground-cleaned counterpart. Histograms of these maps agree well with each other. To verify the Gaussian nature of these distributions we fit the histogram of the input CMB map (after the G20 mask is applied) by a normalized Gaussian probability distribution,
Figure 1. Top panel: non-Gaussian distribution of pixel temperature of MEM foreground maps. The frequency bands are chosen such that respective foreground component is most dominant over other frequencies inside the WMAP window. Bottom panel: variation of kurtosis, $K$, as a function of foreground strength at WMAP frequencies. The solid lines show mean curves obtained from 150 random simulations of CMB and foreground with foreground strength as indicated by the ordinate of each points. The dashed lines show the variation of kurtosis for a randomly chosen realization. The error bars indicate cosmic variance. The mean kurtosis estimated from pure CMB maps is given by, $\langle K_{\text{CMB}} \rangle = -0.023 \pm 0.060$.

(A color version of this figure is available in the online journal.)

$$g(T) = \exp\left(-\frac{(T-a)^2}{2s^2}\right) / \sqrt{2\pi s^2}, \text{ where } a \text{ and } s \text{ denote, respectively, mean and standard deviation of the distribution. From the fit we find that } s = 69.95 \pm 0.09 \mu K \text{ and } a = 2.12 \pm 0.11 \mu K.$$ 

### 3.2.2. Power Spectrum

We apply the G20 mask on each of the 200 foreground-cleaned CMB maps and estimate the full sky estimate of the CMB power spectrum using the MASTER method (Hivon et al. 2002). The average of 200 power spectra obtained from the foreground-removed maps matches excellently with the average of the input CMB power spectra. We show both spectra in Figure 2 along with the cosmic variance. This verifies that no significant foreground contamination exists outside the G20 mask and the method outlined in this Letter can be used to estimate the CMB power spectrum to extract cosmological information.

In earlier publications (Saha et al. 2006; Souradeep et al. 2006; Saha et al. 2008; Samal et al. 2010) the authors reported that the multipole-based Internal Linear Combination (ILC) method gives rise to a negative bias in its power spectrum at the low multipoles. The bias appears due to a mere chance correlation between CMB and foregrounds at large angles due to availability of only a small number of modes in the large scales on the sky. In principle, the negative bias in the power spectrum can be present in both pixel-based and multipole-based ILC algorithms which rely upon minimization of any net non-CMB variance from the cleaned map. The empirical variance inevitably contains the chance correlation term which has a frequency dependence. The variance minimization algorithm...
Figure 2. Comparison of the average power spectrum obtained from foreground-cleaned CMB maps outside the G20 mask (blue line) with the average input CMB power spectrum (red line with error bars).

(A color version of this figure is available in the online journal.)

treats this frequency-dependent term as caused by a non-CMB component, and while minimizing this correlation, it creates a negative bias in the power spectrum (Saha et al. 2008). However, the foreground removal method described in this Letter relies upon the Gaussian nature of the final distribution, without explicitly minimizing the variance of the data. This leads to the advantage that the power spectrum obtained from the cleaned map does not have any negative bias.

3.3. Application on WMAP Data

3.3.1. Non-Gaussianity Minimized Map

We use the GaussMap code to estimate the best-fit weight factors corresponding to point source masked WMAP frequency maps. The code gives best choice of weights as \( W = (0.035, -0.378, -0.217, 1.632, -0.073) \) for \( K \) to \( W \) bands corresponding to \( k_{\text{max}}(W) = 1.17 \). The complete procedure to estimate the weights takes only about a couple of minutes on an Intel 2.26 GHz processor. As one might expect, \( V \) band gets the maximum positive weight since it is supposed to be the least foreground-contaminated frequency band in the WMAP observation window. The kurtosis for cleaned map outside the G20 mask becomes 0.093. Weights obtained from WMAP data have similar values to the mean weights obtained from Monte Carlo simulations in Section 3.2.

Using the weights described above we obtain the foreground-cleaned CMB map (top panel of Figure 3, hereafter GMAP). Although there are some visible signature of the presence of residual foreground contamination in this map on the galactic plane (e.g., Cygnus region, Cassiopeia A, Carina nebula) all

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Figure 3. Top panel: the foreground-removed GMAP at a beam resolution of 1° and a pixel resolution, \( n_{\text{side}} = 512 \). Bottom panel: the difference between GMAP and the WMAP’s seven-year’s ILC map outside the G20 mask. The dark regions near both poles result from converging longitudes in these regions.

(A color version of this figure is available in the online journal.)
contaminations are confined only near the galactic plane. We show the difference between GMAP and the WMAP’s seven-year ILC map outside the G20 mask in the bottom panel of Figure 3. In the unmasked difference map larger pixel amplitudes are confined near the galactic plane, making a narrow strip-like structure, with absolute pixel temperature exceeding 50 µK. The bottom panel of Figure 3 shows that applying the G20 mask significantly reduces the pixel amplitude of the difference map.

3.3.2. Temperature Distribution

We test for the distribution of pixel temperature of GMAP outside the G20 mask. The average pixel temperature of this map outside the G20 mask is only 0.85 µK. Hence, we fit for a Gaussian probability function with mean 0 and variance $s^2$ to the pixel temperature distribution of the masked GMAP. From the fit we obtain $s = 67.49 \pm 0.12 \mu K$. We show the distribution outside the G20 mask obtained from GMAP and the WMAP’s ILC map in Figure 4. We also show pixel temperature distributions of individual frequency bands. The long tails for $K$ and Ka bands are due to strong synchrotron contamination. We note that all the distributions corresponding to five frequency bands are asymmetric representing their non-Gaussian nature. Pixel temperature of simulated frequency maps have distributions similar to these histograms.

3.3.3. Power Spectrum

To estimate the power spectrum from GMAP we first apply the G20 mask and estimate the partial sky CMB power spectrum. We convert the partial sky power spectrum to the full sky estimate by inverting the mode–mode coupling matrix. Finally, we remove both beam and pixel effect from this power spectrum. We show the resulting power spectrum in Figure 5. As shown in this figure, this power spectrum matches well with the ILC power spectrum estimated from the same sky region until $l \sim 100$. Beyond this multipole range GMAP power spectrum has less power than the ILC power spectrum. From the GMAP we find $C_2 = 246.4 \mu K^2$ and $C_3 = 402.3 \mu K^2$ consistent with ILC estimates ($C_2 = 248.8 \mu K^2$ and $C_3 = 404.0 \mu K^2$).

4. DISCUSSIONS AND CONCLUSION

We developed, validated, and applied on seven-year WMAP data a global foreground minimization method from CMB sky based upon the measure of non-Gaussian nature of the diffuse galactic foregrounds. The GMAP obtained by this method matches well with the WMAP’s ILC map outside the G20 mask. The power spectrum from GMAP also matches excellently with the power spectrum of the ILC map until $l \sim 100$. At even higher $l$ we find less power than WMAP’s ILC map. The kurtosis value of GMAP outside the G20 mask is $0.093 \pm 0.06$, the error is estimated from Monte Carlo simulations of the foreground removal method. This is only a 1.5σ effect implying that the CMB sky outside the G20 region is sufficiently clean so that the signals from this region may be interpreted to have cosmological origin consistent with standard cosmological scenario. The empirical kurtosis of ILC map outside the G20 mask is 0.086, close to the corresponding value of GMAP.

The usual ILC method which relies upon the minimization of foreground variance produces inverse noise variance weights in the limit when foregrounds are negligible compared to the detector noise. Such property of weights are important if the method needs to be used at higher angular resolution. However, since we minimize the dimensionless quantity, $K^T(W)$, which represents non-Gaussian property due to foregrounds, we do not expect weights in our method would satisfy such property. However, this does not necessarily indicate a limitation of our method, since it may also imply that the method performs better in terms of non-Gaussian foreground removal when the data become noise dominated.

A crucial advantage of the method over the usual variance-based ILC method is that the former does not possess any negative bias at low multipoles. The quadrupole moment estimated by us matches excellently with the WMAP’s ILC estimate. Our result shows that the problem of low quadrupole moment persists in WMAP data. The quadrupole and octopole maps also show similar nature as the ones estimated from the WMAP’s ILC map. The problem of quadrupole and octopole alignment using GMAP would be investigated in a future paper—however, given the similarity between GMAP and ILC map (at the least, outside the G20 mask) the alignment is likely to remain. Another direction for future research would be to implement the method in the harmonic domain of the maps. The method can be generalized to estimate the CMB angular power spectrum down
to low angular scale following approaches described in Saha et al. (2006) and Saha et al. (2008). Since polarized foreground models are poorly known, an excellent research direction would be to apply our method on polarization sensitive CMB data released by WMAP and Planck satellite mission (Ade et al. 2011).

Our method shows that it is possible to remove foregrounds from the CMB sky purely based upon the non-Gaussian nature of non-cosmological signals. Moreover, since the weights are constrained to satisfy $\sum_{b=1}^{N_b} w_f = 1$ CMB remains preserved even if it possesses any non-Gaussian property. We verify this by performing Monte Carlo simulations of the foreground removal procedure with non-Gaussian CMB and MEM foregrounds. This is a very important advantage of the method since by analyzing GMAP it may be possible to directly quantify any primordial and/or secondary non-Gaussian signal. In this context, it would be interesting to investigate, if slightly excess $K$ for GMAP ($0.093$) over ILC map ($0.086$) outside the G20 region results from cosmological signal or any foreground residual. Estimating GMAP using data from current and future generation sensitive CMB experiments such as PLANCK and CMBPol would play an important role to unfold this scientific information.

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3 The HEALPix distribution is publicly available from the Web site http://healpix.jpl.nasa.gov.