Research Article

Analysis of CP Violation in $D^0 \to K^+ K^- \pi^0$

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We study the CP violation induced by the interference between two intermediate resonances $K^+ (892)^*$ and $K^+ (892)^-$ in the phase space of singly-Cabibbo-suppressed decay $D^0 \to K^+ K^- \pi^0$. We adopt the factorization-assisted topological approach in dealing with the decay amplitudes of $D^0 \to K^+ K^- (892)^*$. The CP asymmetries of two-body decays are predicted to be very tiny, which are $(-1.27 \pm 0.25) \times 10^{-5}$ and $(3.86 \pm 0.26) \times 10^{-5}$, respectively, for $D^0 \to K^+ K^- (892)$ and $D^0 \to K^+ K^- (892)^*$, while the differential CP asymmetry of $D^0 \to K^+ K^- \pi^0$ is enhanced because of the interference between the two intermediate resonances, which can reach as large as $3 \times 10^{-4}$. For some NPs which have considerable impacts on the chromomagnetic dipole operator $O_{8j}$, the global CP asymmetries of $D^0 \to K^+ K^- (892)^*$ and $D^0 \to K^+ K^- (892)^-$ can be then increased to $(0.56 \pm 0.08) \times 10^{-3}$ and $(-0.50 \pm 0.04) \times 10^{-3}$, respectively. The regional CP asymmetry in the overlapped region of the phase space can be as large as $(1.3 \pm 0.3) \times 10^{-3}$.

1. Introduction

Charge-Parity (CP) violation, which was first discovered in $K$ meson system in 1964 [1], is one of the most important phenomena in particle physics. In the Standard Model (SM), CP violation originates from the weak phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2, 3] and the unitary phases which usually arise from strong interactions. One reason for the smallness of CP violation is that the unitary phase is usually small. Nevertheless, CP violation can be enhanced in three-body decays of heavy hadrons, when the corresponding decay amplitudes are dominated by overlapped intermediate resonances in certain regions of phase space. Owing to the overlapping, a regional CP asymmetry can be generated by a relative strong phase between amplitudes corresponding to different resonances. This relative strong phase has nonperturbative origin. As a result, the regional CP asymmetry can be larger than the global one. In fact, such kind of enhanced CP violation has been observed in several three-body decay channels of $B$ meson [4–7], which was followed by a number of theoretical works [8–19].

The study of CP violation in singly-Cabibbo-suppressed (SCS) $D$ meson decays provides an ideal test of the SM and exploration of New Physics (NP) [20–23]. In the SM, CP violation is predicted to be very small in charm system. Experimental researches have shown that there is no significant CP violation so far in charmed hadron decays [24–33]. CP asymmetry in SCS $D$ meson decay can be as small as

$$ A_{\text{CP}} \sim \left| \frac{V_{ub}^* V_{ub}}{V_{ub}^* V_{ub}} \right| \frac{\alpha_s}{\alpha_s} \pi \sim 10^{-4}, $$

or even less, due to the suppression of the penguin diagrams by the CKM matrix as well as the smallness of Wilson coefficients in penguin amplitudes. The SCS decays are sensitive to new contributions to the $\Delta C = 1$ QCD penguin and chromomagnetic dipole operators, while such contributions can affect neither the Cabibbo-favored (CF) ($c \to s \bar{d} u$) nor the doubly-Cabibbo-suppressed (DCS) ($c \to d \bar{s} u$) decays [34]. Besides, the decays of charmed mesons offer a unique opportunity to probe CP violation in the up-type quark sector.

Several factorization approaches have been wildly used in nonleptonic $B$ decays. In the naive factorization approach [35, 36], the hadronic matrix elements were expressed as a product of a heavy to light transition form factor and a decay constant. Based on Heavy Quark Effect Theory, it is shown
in the QCD factorization approach that the corrections to the hadronic matrix elements can be expressed in terms of short-distance coefficients and meson light-cone distribution amplitudes [37, 38]. Alternative factorization approach based on QCD factorization is often applied in study of quasi-two-body hadronic $B$ decays [19, 39, 40], where they introduced unitary meson-meson form factors, from the perspective of unitarity, for the final state interactions. Other QCD-inspired approaches, such as the perturbative approach (pQCD) [41] and the soft-collinear effective theory (SCET) [42], are also widely used in $B$ meson decays.

However, for $D$ meson decays, such QCD-inspired factorization approaches may not be reliable since the charm quark mass, which is just above 1 GeV, is not heavy enough for the heavy quark expansion [43, 44]. For this reason, several model-independent approaches for the charm meson decay amplitudes have been proposed, such as the flavor topological diagram approach based on the flavor $SU(3)$ symmetry [44–47] and the factorization-assisted topological-amplitude (FAT) approach with the inclusion of flavor $SU(3)$ breaking effect [48, 49]. One motivation of these aforementioned approaches is to identify as complete as possible the dominant sources of nonperturbative dynamics in the hadronic matrix elements.

In this paper, we study the $CP$ violation of SCS $D$ meson decay $D^0 \rightarrow K^+ K^- \pi^0$ in the FAT approach. Our attention will be mainly focused on the region of the phase space where two intermediate resonances, $K^*(892)^+$ and $K^*(892)^-$, are overlapped. Before proceeding, it will be helpful to point out that direct CP asymmetry is hard to be isolated for decay process with $CP$-eigen-final-state. When the final state of the decay process is $CP$ eigenstate, the time integrated $CP$ violation for $D^0 \rightarrow f$, which is defined as

$$a_f \equiv \left[ \frac{1}{\Gamma(D^0 \rightarrow f)} \int_0^\infty \Gamma(D^0 \rightarrow f) dt \right] - \left[ \frac{1}{\Gamma(D^0 \rightarrow f)} \int_0^\infty \Gamma(D^0 \rightarrow f) dt \right] \text{d}t,$$

(2)

can be expressed as [34]

$$a_f = a_f^d + a_f^m + a_f^i,$$

(3)

where $a_f^d$, $a_f^m$, and $a_f^i$ are the $CP$ asymmetries in decay, in mixing, and in the interference of decay and mixing, respectively. As is shown in [34, 50, 51], the indirect $CP$ violation $a_{ind}^d \equiv a_{ind}^m + d$ is universal and channel-independent for two-body $CP$-eigenstate. This conclusion is easy to be generalized to decay processes with three-body $CP$-eigenstate in the final state, such as $D^0 \rightarrow K^+ K^- \pi^0$. In view of the universality of the indirect $CP$ asymmetry, we will only consider the direct $CP$ violations of the decay $D^0 \rightarrow K^+ K^- \pi^0$ throughout this paper.

The remainder of this paper is organized as follows. In Section 2, we present the decay amplitudes for various decay channels, where the decay amplitudes of $D^0 \rightarrow K^+ K^*(892)^+$ are formulated via the FAT approaches. In Section 3, we study the $CP$ asymmetries of $D^0 \rightarrow K^+ K^*(892)^+$ and the $CP$ asymmetry of $D^0 \rightarrow K^+ K^- \pi^0$ induced by the interference between different resonances in the phase space. Discussions and conclusions are given in Section 4. We list some useful formulas and input parameters in the Appendix.

2. Decay Amplitude for $D^0 \rightarrow K^+ K^- \pi^0$

In the overlapped region of the intermediate resonances $K^*(892)^+$ and $K^*(892)^-$ in the phase space, the decay process $D^0 \rightarrow K^+ K^- \pi^0$ is dominated by two cascade decays, $D^0 \rightarrow K^+ K^*(892)^- \rightarrow K^+ K^- \pi^0$ and $D^0 \rightarrow K^+ K^*(892)^+ \rightarrow K^+ K^- \pi^0$, respectively. Consequently, the decay amplitude of $D^0 \rightarrow K^+ K^- \pi^0$ can be expressed as

$$\mathcal{M}_{D^0 \rightarrow K^+ K^- \pi^0} = \mathcal{M}_{D^0 \rightarrow K^+ K^- \pi^0} + \delta \mathcal{M}_{D^0 \rightarrow K^+ K^- \pi^0},$$

(4)

in the overlapped region, where $\mathcal{M}_{D^0 \rightarrow K^+ K^- \pi^0}$ and $\mathcal{M}_{D^0 \rightarrow K^+ K^- \pi^0}$ are the amplitudes for the two cascade decays and $\delta$ is the relative strong phase. Note that nonresonance contributions have been neglected in (4).

The decay amplitude for the cascade decay $D^0 \rightarrow K^+ K^*(892)^+ \rightarrow K^+ K^- \pi^0$ can be expressed as

$$\mathcal{M}_{K^+ \rightarrow K^+ K^- \pi^0} = \frac{\sum l \mathcal{M}_{K^+ \rightarrow K^+ K^- \pi^0} \cdot \mathcal{M}_{D^0 \rightarrow K^+ K^- \pi^0}}{s_{pK^+} \cdot [m_{K^+}^2 - m_{\pi^0}^2 + i m_{K^+} \cdot \Gamma_{K^+}]}.$$

(5)

where $\mathcal{M}_{K^+ \rightarrow K^+ K^- \pi^0}$ and $\mathcal{M}_{D^0 \rightarrow K^+ K^- \pi^0}$ represent the amplitudes corresponding to the strong decay $K^+ \rightarrow K^- \pi^0$ and weak decay $D^0 \rightarrow K^+ K^-$, respectively, $\lambda$ is the helicity index of $K^+$, $s_{pK^+}$ is the invariant mass square of $\pi^0 K^-$ system, and $m_{K^+}$ and $\Gamma_{K^+}$ are the mass and width of $K^*(892)^+$, respectively. The decay amplitude for the cascade decay, $D^0 \rightarrow K^+ K^*(892)^- \rightarrow K^- K^+ \pi^0$, is the same as (5) except replacing the subscripts $K^+$ and $K^-$ with $K^+$ and $K^-$, respectively.

For the strong decays $K^*(892)^+ \rightarrow \pi^0 K^+$, one can express the decay amplitudes as

$$\mathcal{M}_{K^+ \rightarrow K^+ K^- \pi^0} = g_{K^+ \rightarrow K^+ K^- \pi^0} \cdot \epsilon_{K^+ \rightarrow \pi^0 K^+} (\rho, \lambda),$$

(6)

where $\rho_{K^+}$ and $\rho_{\pi^0 K^+}$ represent the momentum for $\pi^0$ and $K^+$ mesons, respectively, and $g_{K^+ \rightarrow K^+ K^- \pi^0}$ is the effective coupling constant for the strong interaction, which can be extracted from the experimental data via

$$g_{K^+ \rightarrow K^+ K^- \pi^0}^2 = \frac{6 m_{K^+}^2 \Gamma_{K^+ \rightarrow \pi^0 K^+}}{\lambda_{K^+ \rightarrow \pi^0 K^+}},$$

(7)

with

$$\lambda_{K^+ \rightarrow \pi^0 K^+} = \frac{1}{2 m_{K^+}} \cdot \left[ \sqrt{m_{K^+}^2 - (m_{\pi^0} + m_{K^+})^2} \right] \cdot \left[ m_{K^+}^2 - (m_{\pi^0} - m_{K^+})^2 \right],$$

(8)

and $\Gamma_{K^+ \rightarrow \pi^0 K^+} = \text{Br}(K^+ \rightarrow \pi^0 K^+) \cdot \Gamma_{K^+ \rightarrow \pi^0 K^+}$. The isospin symmetry of the strong interaction implies that $\Gamma_{K^+ \rightarrow \pi^0 K^+} \approx (1/3)\Gamma_{K^- \rightarrow \pi^0 K^-}$. The decay amplitudes for the weak decays, $D^0 \rightarrow K^+ K^*(892)^+$ and $D^0 \rightarrow K^+ K^*(892)^-$, will be handled with
The two tree diagrams in first line of Figure 1 represent the color-favored tree diagram for $D \rightarrow PV$ transition and the $W$-exchange diagram with the pseudoscalar (vector) meson containing the antiquark from the weak vertex, respectively. The amplitudes of these two diagrams will be, respectively, denoted as $T_{P(\nu)}$ and $E_{P(\nu)}$.

According to these topological structures, the amplitudes of the color-favored tree diagrams $T_{P(\nu)}$, which are dominated by the factorizable contributions, can be parameterized as

$$T_p = \frac{G_F}{\sqrt{2}} \lambda_e a_2(\mu) f_p m_V A_0^{D \rightarrow V} \left( m^2_\nu \right) 2 \left( e^* \cdot p_D \right), \quad (9)$$

and

$$T_V = \frac{G_F}{\sqrt{2}} \lambda_e a_2(\mu) f_p m_V A_0^{D \rightarrow V} \left( m^2_\nu \right) 2 \left( e^* \cdot p_D \right), \quad (10)$$

respectively, where $G_F$ is the Fermi constant, $\lambda_e = V_{ts} V_{cs}^*$, with $V_{ts}$ and $V_{cs}$ being the CKM matrix elements, $a_2(\mu) = c_2(\mu) + c_1(\mu)/N_c$, with $c_1(\mu)$ and $c_2(\mu)$ being the scale-dependent Wilson coefficients, and the number of color $N_c = 3$, $f_{\nu(p)}$ and $m_{\nu(p)}$ are the decay constant and mass of the vector (pseudoscalar) meson, respectively, $f_{\nu(p)}^{D \rightarrow P}$ and $A_0^{D \rightarrow V}$ are the form factors for the transitions $D \rightarrow P$ and $D \rightarrow V$, respectively, $e$ is the polarization vector of the vector meson, and $p_D$ is the momentum of $D$ meson. The scale $\mu$ of Wilson coefficients is set to energy release in individual decay channels [52, 53], which depends on masses of initial and final states and is defined as [48, 49]

$$\mu = \sqrt{\Lambda m_D (1 - r^2_D) (1 - r^2_\nu)}.$$  

with the mass ratios $r_{\nu(P)} = m_{\nu(p)}/m_D$, where $\Lambda$ represents the soft degrees of freedom in the $D$ meson, which is a free parameter.

For the $W$-exchange amplitudes, since the factorizable contributions to these amplitudes are helicity-suppressed, only the nonfactorizable contributions need to be considered. Therefore, the $W$-exchange amplitudes are parameterized as

$$E_{P(\nu)}^q = \frac{G_F}{\sqrt{2}} \lambda_e a_2(\mu) f_p m_V A_0^{D \rightarrow V} \left( m^2_\nu \right) 2 \left( e^* \cdot p_D \right), \quad (12)$$

where $m_D$ is the mass of $D$ meson, $f_D$, $f_\pi$, and $f_\rho$ are the decay constants of the $D$, $\pi$, and $\rho$ mesons, respectively, and $\lambda_q^E$ and $\phi_q^E$ characterize the strengths and the strong phases of the corresponding amplitudes, with $q = u, d, s$ representing the strongly produced $q$ quark pair. The ratio of $f_p/f_V$ over $f_\pi/f_\rho$ indicates that the flavor SU(3) breaking effects have been taken into account from the decay constants.

The penguin diagrams shown in the second line of Figure 1 represent the color-favored, the gluon-annihilation, and the gluon-exchange penguin diagrams, respectively, whose amplitudes will be denoted as $T_{P(\nu)}$, $P_E(\nu)$, and $P_A(\nu)$, respectively.

Since a vector meson cannot be generated from the scalar or pseudoscalar operator, the amplitude $T_p$ does not include contributions from the penguin operator $O_5$ or $O_6$. Consequently, the color-favored penguin amplitudes $P T_p$ and $P T_V$ can be expressed as

$$P T_p = - \frac{G_F}{\sqrt{2}} \lambda_e a_4(\mu) f_p m_V A_0^{D \rightarrow P} \left( m^2_\nu \right) 2 \left( e^* \cdot p_D \right), \quad (13)$$

and

$$P T_V = \frac{G_F}{\sqrt{2}} \lambda_e \left[ a_4(\mu) - \frac{1}{\Lambda} a_6(\mu) \right] f_p m_V A_0^{D \rightarrow V} \left( m^2_\nu \right) \cdot 2 \left( e^* \cdot p_D \right), \quad (14)$$

with $a_4(\mu)$ and $a_6(\mu)$ being the penguin operators.

![Figure 1: The relevant topological diagrams for $D \rightarrow PV$ with (a) the color-favored tree amplitude $T_{P(\nu)}$, (b) the $W$-exchange amplitude $E_{P(\nu)}$, (c) the color-favored penguin amplitude $P T_{P(\nu)}$, (d) the gluon-annihilation penguin amplitude $P E_{P(\nu)}$, and (e) the gluon-exchange penguin amplitude $P A_{P(\nu)}$.](image)
respectively, where \( \lambda_b = V_{ub} V_{ub}^* \) with \( V_{ub} \) and \( V_{ub}^* \) being the CKM matrix elements, \( a_{45}(\mu) = c_{45}(\mu) + c_{45}(\mu)/N_c \), with \( c_{4,5,6} \) being the Wilson coefficients, and \( r_x \) is a chiral factor, which takes the form

\[
r_x = \frac{2m_p^2}{(m_u + m_q)(m_u + m_c)},
\]

with \( m_{u(q)} \) being the masses of \( u(c, q) \) quark. Note that the quark-loop corrections and the chromomagnetic-penguin contribution are also absorbed into \( c_{4,5,6} \) as shown in [49].

Similar to the amplitudes \( E_{PV} \), the amplitudes \( PE_{PV} \) only include the nonfactorizable contributions as well. Therefore, the amplitudes \( PE_{PV} \), which are dominated by \( O_4 \) and \( O_5 \) [48], can be parameterized as

\[
PE_{PV}^3 = -\frac{G_F}{\sqrt{2}} \lambda_b \left[ (2a_6(\mu) / 2g_5) \right] \chi_q e^{i\phi_q} f_D m_D
\]

\[
\cdot \frac{f_{P,E}}{f_{\pi F_P}} (e^* \cdot P_D).
\]

For the amplitudes \( PA_p \) and \( PA_V \), the helicity suppression does not apply to the matrix elements of \( O_5, \) so the factorizable contributions exist. In the pole resonance model [54], after applying the Fierz transformation and the factorization hypothesis, the amplitudes \( PA_p \) and \( PA_V \) can be expressed as

\[
PA_p^3 = -\frac{G_F}{\sqrt{2}} \lambda_b \left[ (2a_6(\mu) / 2g_5) \right] \chi_q e^{i\phi_q} f_D m_D
\]

\[
\cdot \frac{f_{P,E}}{f_{\pi F_P}} (e^* \cdot P_D).
\]

\[
PA_V^3 = -\frac{G_F}{\sqrt{2}} \lambda_b \left[ (2a_6(\mu) / 2g_5) \right] \chi_q e^{i\phi_q} f_D m_D
\]

\[
\cdot \frac{f_{P,E}}{f_{\pi F_P}} (e^* \cdot P_D).
\]

respectively, where \( g_5 \) is an effective strong coupling constant obtained from strong decays, e.g., \( \rho \rightarrow \pi \pi \), \( K^* \rightarrow K \pi \), and \( \phi \rightarrow KK \), and is set as \( g_5 = 4.5 \) [54] in this work, \( m_p \) and \( f_p \) are the mass and decay constant of the pole resonant pseudoscalar meson \( P^* \), respectively, and \( \chi_q^A \) and \( \phi_q^A \) are the strengths and the strong phases of the corresponding amplitudes.

From Figure 1, the decay amplitudes of \( D^0 \rightarrow K^+K^- (892)^- \) and \( D^0 \rightarrow K^+K^- (892)^+ \) in the FAT approach can be easily written down

\[ \mathcal{M}_{D^0 \rightarrow K^+K^-} = T_K^- + E_{K^-} + PT_{K^-} + PE_{K^-} + PE_{K^-}, \]

and

\[ \mathcal{M}_{D^0 \rightarrow K^+K^+} = T_K^+ + E_{K^+} + PT_{K^+} + PE_{K^+} + PE_{K^+}, \]

respectively, where \( \lambda \) is the helicity of the polarization vector \( \epsilon(\rho, \lambda) \). In the FAT approach, the fitted nonperturbative parameters, \( \chi_q^A, \phi_q^A, \lambda_q^A, \lambda_q^A, \phi_q^A \), are assumed to be universal and can be determined by the data [49].

In Table 1, we list the magnitude of each topological amplitude for \( D^0 \rightarrow K^+K^0(892)^- \) and \( D^0 \rightarrow K^-K^0(892)^+ \) by using the global fitted parameters for \( D \rightarrow PV \) in [49]. One can see from Table 1 that the penguin contributions are greatly suppressed. \( PT \) is dominant in the penguin contributions of \( D^0 \rightarrow K^+K^0(892)^- \), while \( PT \) is small in \( D^0 \rightarrow K^-K^0(892)^+ \), which is even smaller than the amplitude \( PA \). This difference is because of the chirally enhanced factor contained in (14) while not in (13). The very small \( PE \) do not receive the contributions from the quark-loop and chromomagnetic penguins, since these two contributions to \( c_4 \) and \( c_5 \) are canceled with each other in (16). Besides, the relations \( PE_{PV} = PE_{PV}^+, PE_{PV} = PE_{PV}^+ \) and \( PE_{PV} = PE_{PV}^+ \) can be read from Table 1; this is because that the isospin symmetry and the flavor \( SU(3) \) breaking effect have been considered.

Since the form factors are inevitably model-dependent, we list in Table 2 the branching ratios of \( D^0 \rightarrow K^+K^0(892)^- \) and \( D^0 \rightarrow K^-K^0(892)^+ \) predicted by the FAT approach, by various form factor models. The pole, dipole, and covariant light-front (CLF) models are adopted. The uncertainties in Table 2 mainly come from decay constants. The CLF model agrees well with the data for both decay channels, and other models are also consistent with the data. However, the model-dependence of form factor leads to large uncertainty of the branching fraction, as large as 20%. Because of the smallness of the Wilson coefficients and the CKM-suppression of the penguin amplitudes, the branching ratios are dominated by the tree amplitudes. Therefore, there is no much difference for the branching ratios whether we consider the penguin amplitudes or not.

### 3. CP Asymmetries for \( D^0 \rightarrow K^+K^- (892)^- \) and \( D^0 \rightarrow K^+K^- (892)^+ \)

The direct CP asymmetry for the two-body decay \( D \rightarrow PV \) is defined as

\[
A_{CP}^{D \rightarrow PV} = \frac{M_{D \rightarrow PV} - M_{D \rightarrow PV}^\ast}{M_{D \rightarrow PV} + M_{D \rightarrow PV}^\ast},
\]

where \( M_{D \rightarrow PV} \) represents the decay amplitude of the CP conjugate process \( D \rightarrow \bar{PV} \), such as \( D^0 \rightarrow K^+K^- (892)^- \) or \( D^0 \rightarrow K^+K^- (892)^+ \). In the framework of FAT approach,
we predict very small direct \(CP\) asymmetries of \(D^0 \rightarrow K^+K^*(892)^-\) and \(D^0 \rightarrow K^{-}K^*(892)^-\) presented in Table 3. The uncertainties induced by the model-dependence of form factor to the \(CP\) asymmetries of \(D^0 \rightarrow K^+K^*(892)^-\) and \(D^0 \rightarrow K^{-}K^*(892)^-\) are about 30% and 10%, respectively.

The differential \(CP\) asymmetry of the three-body decay \(D^0 \rightarrow K^+K^-\pi^0\), which is a function of the invariant mass of \(s_{K^+K^-}\) and \(s_{\pi K}\), is defined as

\[
A_{CP}^{D^0 \rightarrow K^+K^-\pi^0} (s_{K^+K^-}, s_{\pi K}) = \frac{|\mathcal{M}_{D^0 \rightarrow K^+K^-\pi^0}| - |\mathcal{M}_{D^0 \rightarrow K^-K^+\pi^0}|}{|\mathcal{M}_{D^0 \rightarrow K^+K^-\pi^0}| + |\mathcal{M}_{D^0 \rightarrow K^-K^+\pi^0}|},
\]

where the invariant mass \(s_{K^+K^-} = (p_{K^+} + p_{K^-})^2\). As can be seen from (4), the differential \(CP\) asymmetry \(A_{CP}^{D^0 \rightarrow K^+K^-\pi^0}\) depends on the relative strong phase \(\delta\), which is impossible to be calculated theoretically because of its nonperturbative origin. Despite this, we can still acquire some information of this relative strong phase \(\delta\) from data. By using a Dalitz plot technique [55, 58, 59], the phase difference \(\delta_{\text{exp}}\) between \(D^0\) decays to \(K^+K^*(892)^-\) and \(K^-K^*(892)^-\) can be extracted from data. One should notice that \(\delta_{\text{exp}}\) is not the same as the strong phase \(\delta\) defined in (4). The strong phase \(\delta\) is the relative phase between the decay amplitudes of \(D^0 \rightarrow K^+K^*(892)^-\) and \(D^0 \rightarrow K^-K^*(892)^-\). On the other hand, the phase \(\delta_{\text{exp}}\) is defined through

\[
\mathcal{M}_{D^0 \rightarrow K^+K^-\pi^0} = (|\mathcal{M}_{K^+K^-}| + e^{i\delta_{\text{exp}}} |\mathcal{M}_{K^-K^+}|) e^{i\delta_{K^+K^-}}.
\]

in the overlapped region of the phase space, where \(\delta_{K^+K^-}\) is the phase of the amplitude \(\mathcal{M}_{K^+K^-}\):

\[
\mathcal{M}_{K^+K^-} = |\mathcal{M}_{K^+K^-}| e^{i\delta_{K^+K^-}}.
\]

Therefore, neglecting the CKM suppressed penguin amplitudes, \(\delta_{\text{exp}}\) and \(\delta\) can be related by

\[
\delta_{\text{exp}} - \delta = \delta_{K^+K^-} - \delta_{K^-K^+},
\]

where \(\delta_{K^+K^-}\) = arg \((T_{K^+K^-} + P_{K^+K^-})\) are the phases in tree-level amplitudes of \(D^0 \rightarrow K^+K^*(892)^-\) and are equivalent to \(\delta_{K^-K^+}\) if the penguin amplitudes are neglected. With the relation of (25), and \(\delta_{\text{exp}} = -35.5^\circ \pm 4.1^\circ\) measured by the BABAR Collaboration [56], we have \(\delta = -51.85^\circ \pm 4.1^\circ\).

In Figure 2, we present the differential \(CP\) asymmetry of \(D^0 \rightarrow K^+K^-\pi^0\) in the overlapped region of \(K^+(892)^-\) and \(K^-(892)^-\) in the phase space, with \(\delta = -51.85^\circ\). Namely, we will focus on the region \(m_{K^-} - 2\Gamma_{K^-} < \sqrt{s_{\pi K}} < \sqrt{s_{\pi K}} < m_{K^+} + 2\Gamma_{K^+}\) of the phase space. One can see from Figure 2 that the differential \(CP\) asymmetry of \(D^0 \rightarrow K^+K^-\pi^0\) can reach \(3.0 \times 10^{-4}\) in the overlapped region, which is about 10 times larger than the \(CP\) asymmetries of the corresponding two-body decay channels shown in Table 3.

The behavior of the differential \(CP\) asymmetry of \(D^0 \rightarrow K^+K^-\pi^0\) in Figure 2 motivates us to separate this region into four areas, area A \((m_{K^-} - 2\Gamma_{K^-} < \sqrt{s_{\pi K}} < m_{K^-} + 2\Gamma_{K^-}, m_{K^-} < \sqrt{s_{\pi K}} < m_{K^+})\), area B \((m_{K^-} < \sqrt{s_{\pi K}} < m_{K^-} + 2\Gamma_{K^-}, \sqrt{s_{\pi K}} < m_{K^+} + 2\Gamma_{K^+})\), area C \((m_{K^-} < \sqrt{s_{\pi K}} < m_{K^-} + 2\Gamma_{K^-}, m_{K^-} < \sqrt{s_{\pi K}} < m_{K^+} + 2\Gamma_{K^+})\), and area D \((m_{K^-} < \sqrt{s_{\pi K}} < m_{K^-} + 2\Gamma_{K^-}, \sqrt{s_{\pi K}} < m_{K^-} < m_{K^+})\). We further consider the observable of regional \(CP\) asymmetry in areas A, B, C, and D displayed in Table 4, which is defined by

\[
A_{CP}^{\Omega} = \frac{\int_{\Omega} \left( |\mathcal{M}_{\text{tot}}|^2 - |\mathcal{M}_{s_{K}K^+s_{K^+K^-}}|^2 \right) ds_{K^+K^-}}{\int_{\Omega} \left( |\mathcal{M}_{\text{tot}}|^2 + |\mathcal{M}_{s_{K}K^+s_{K^+K^-}}|^2 \right) ds_{K^+K^-}},
\]

where \(\Omega\) represents a certain region of the phase space.

Comparing with the \(CP\) asymmetries of two-body decays, the regional \(CP\) asymmetries, from Table 4, are less sensitive to the models we have used. We would like to use only the CLF model for the following discussion. The uncertainties in Table 4 come from decay constants as well as the relative phase \(\delta_{\text{exp}}\). In addition, if we focus on the right part of area A, that is, \(m_{K^+} < \sqrt{s_{\pi K}} < m_{K^-} + 2\Gamma_{K^-}, m_{K^-} - 2\Gamma_{K^-} < \sqrt{s_{\pi K}} < m_{K^+} + 2\Gamma_{K^+}\), the regional \(CP\) violation will be \((1.09 \pm 0.16) \times 10^{-4}\).

The energy dependence of the propagator of the intermediate resonances can lead to a small correction to \(CP\) asymmetry. For example, if we replace the Breit-Wigner
arise significant contributions to the up squark-gluino loops in supersymmetry (SUSY) can be realized if some NPs effects are pulled in. For example, tree-level operators, however, such large contribution can in [48, 64] lying within the range $K_c$.

Moreover, if we follow [49] taking $\Delta A_{CP}$ measured by LHCb [69], is a $CP$ asymmetry of $D^0 \rightarrow K^+ K^- \pi^0$, respectively. Our discussion shows that the $CP$ violation can be enhanced by the interference effect in three-body decay $D^0 \rightarrow K^+ K^- \pi^0$. The differential $CP$ asymmetry can reach $3.0 \times 10^{-4}$ when the interference effect is taken into account, while the regional one can be as large as $(1.09 \pm 0.16) \times 10^{-4}$.

Besides, since the chromomagnetic dipole operator $O_{sg}$ is sensitive to some NPs, the inclusion of this kind of NPs will lead to a much larger global $CP$ asymmetries of $D^0 \rightarrow K^+ K^- \pi^0$ and $D^0 \rightarrow K^- K^+ \pi^0$, which are $(0.56 \pm 0.08) \times 10^{-3}$ and $(0.50 \pm 0.04) \times 10^{-3}$, respectively, while the regional $CP$ asymmetry of $D^0 \rightarrow K^+ K^- \pi^0$ can be also increased to $(1.3 \pm 0.3) \times 10^{-3}$ when considering the interference effect in the phase space. Since the $O(10^{-3})$ of

**Table 3: CP asymmetries (in unit of $10^{-4}$) of $D^0 \rightarrow K^+ K^- (892)^-$ and $D^0 \rightarrow K^- K^+ (892)^+$ predicted by the FAT approach with pole, dipole, and CLF models adopted. The uncertainties in this table are mainly from decay constants.**

| Form factors | $A_{CP}(D^0 \rightarrow K^+ K^- (892)^-)$ | $A_{CP}(D^0 \rightarrow K^- K^+ (892)^+)$ |
|--------------|----------------------------------------|----------------------------------------|
| Pole         | $-1.45 \pm 0.25$                      | $3.60 \pm 0.23$                       |
| Dipole       | $-1.63 \pm 0.26$                      | $3.70 \pm 0.24$                       |
| CLF          | $-1.27 \pm 0.25$                      | $3.86 \pm 0.26$                       |

**Figure 2:** The differential $CP$ asymmetry distribution of $D^0 \rightarrow K^+ K^- \pi^0$ in the overlapped region of $K^+ (892)^-$ and $K^- (892)^+$ in the phase space.

The propagator by the Flatté Parametrization [60], the correction to the regional $CP$ asymmetry will be about $1\%$.

Since the $CP$ asymmetry of $D^0 \rightarrow K^+ K^- \pi^0$ is extremely suppressed, it should be more sensitive to the NP. For example, some NPs have considerable impacts on the chromomagnetic dipole operator $O_{sg}$ [34, 61–66]. Consequently, the $CP$ violation in SCS decays may be further enhanced. In practice, the NP contributions can be absorbed into the corresponding effective Wilson coefficient $c_{sg}^{eff}$ [67, 68]. For comparison, we first consider a relative small value of $c_{sg}^{eff}$ (as in [48, 64]) lying within the range $(0, 1)$ and the global $CP$ asymmetry of $D^0 \rightarrow K^+ (892)^+ K^-$ are no larger than $5 \times 10^{-5}$. Moreover, if we follow [49] taking $c_{sg}^{eff} = 10$ (while $c_{sg}^{eff} = 10$, which is extracted from $\Delta A_{CP}$ measured by LHCb [69], is a quite large quantity even for the coefficients corresponding tree-level operators, however, such large contribution can be realized if some NPs effects are pulled in. For example, the up squark-gluino loops in supersymmetry (SUSY) can arise significant contributions to $c_{sg}^{eff}$. More details about the squark-gluino loops and other models in SUSY can be found in [34, 62, 70–72]), the global $CP$ asymmetries of $D^0 \rightarrow K^+ K^- (892)^-$ and $D^0 \rightarrow K^- K^+ (892)^+$ are then $(0.56 \pm 0.08) \times 10^{-3}$ and $(-0.50 \pm 0.04) \times 10^{-3}$, respectively.

We further display the $CP$ asymmetry of $D^0 \rightarrow K^+ K^- \pi^0$ in the overlapped region of $K^+ (892)^-$ and $K^- (892)^+$ in Figures 3(a) and 3(b) for $c_{sg}^{eff} = 1$ and $c_{sg}^{eff} = 10$, respectively. After taking the interference effect into account, the differential $CP$ asymmetry of $D^0 \rightarrow K^+ K^- \pi^0$ can be increased as large as $5.5 \times 10^{-4}$ and $2.8 \times 10^{-3}$ for $c_{sg}^{eff} = 1$ and $c_{sg}^{eff} = 10$, respectively. The regional ones (in phase space of $\sqrt{0.74}$ GeV $< \sqrt{s_{K^+ K^-}} < \sqrt{0.81}$ GeV, $\sqrt{0.57} < \sqrt{s_{K^+ K^-}} < m_{K^+} + 2 I_{K^+}$) can reach $(2.7 \pm 0.5) \times 10^{-4}$ and $(1.3 \pm 0.3) \times 10^{-3}$ for $c_{sg}^{eff} = 1$ and $c_{sg}^{eff} = 10$, respectively.

**4. Discussion and Conclusion**

In this work, we studied $CP$ violations in $D^0 \rightarrow K^+ (892)^+ K^-$, $K^+ K^- \pi^0$ via the FAT approach. The $CP$ violations in two-body decay processes $D^0 \rightarrow K^+ K^- (892)^-$ and $D^0 \rightarrow K^- K^+ (892)^+$ are very small, which are $(1.27 \pm 0.25) \times 10^{-5}$ and $(3.86 \pm 0.26) \times 10^{-5}$, respectively. Our discussion shows that the $CP$ violation can be enhanced by the interference effect in three-body decay $D^0 \rightarrow K^+ K^- \pi^0$. The differential $CP$ asymmetry can reach $3.0 \times 10^{-4}$ when the interference effect is taken into account, while the regional one can be as large as $(1.09 \pm 0.16) \times 10^{-4}$.
Table 4: Three from factor models: the pole, dipole, and CLF models are used for the regional CP asymmetries (in unit of $10^{-4}$) in the four areas, A, B, C, and D, of the phase space.

| Form factors | $A^\text{P}_{\text{CP}}$ | $A^\text{D}_{\text{CP}}$ | $A^\text{C}_{\text{CP}}$ | $A^\text{B}_{\text{CP}}$ | $A^\text{All}_{\text{CP}}$ |
|--------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Pole         | $0.87\pm0.11$   | $0.42\pm0.08$   | $0.39\pm0.07$   | $-0.30\pm0.08$  | $0.33\pm0.05$   |
| Dipole       | $0.87\pm0.11$   | $0.41\pm0.08$   | $0.38\pm0.07$   | $-0.30\pm0.08$  | $0.32\pm0.05$   |
| CLF          | $0.84\pm0.10$   | $0.45\pm0.08$   | $0.42\pm0.07$   | $-0.25\pm0.08$  | $0.36\pm0.06$   |

Appendix

Some Useful Formulas and Input Parameters

(1) Effective Hamiltonian and Wilson Coefficients. The weak effective Hamiltonian for SCS D meson decays, based on the Operator Product Expansion (OPE) and Heavy Quark Effective Theory (HQET), can be expressed as [78]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ \sum_{q=d,s} \lambda_q (c_1 O_1^q + c_2 O_2^q) + \lambda_b \left( \sum_{i=3}^6 c_i O_i + c_{bg} O_{bg} \right) \right] + h.c.,$$

where $G_F$ is the Fermi constant, $\lambda_q = V_{iq} V_{cq}^*$, $c_i (i = 1, \ldots, 6)$ is the Wilson coefficient, and $O_1^q, O_2^q, O_3, O_4, O_5, O_6, O_{bg}$ are four-fermion operators which are constructed from different combinations of quark fields. The four-fermion operators take the following form:

$$\begin{align*}
O_1^q &= \overline{u}_a Y_q (1 - y_3) \gamma_\mu (1 - y_5) \gamma_\nu c_a, \\
O_2^q &= \overline{u}_a Y_q (1 - y_3) \gamma_\mu (1 - y_5) c, \\
O_3 &= \overline{u}_a Y_\mu (1 - y_3) \sum_{q'} \bar{q}' \gamma^\mu (1 - y_5) q', \\
O_4 &= \overline{u}_a Y_\mu (1 - y_3) \bar{c} \sum_{q'} \bar{q}' \gamma^\mu (1 - y_5) q', \\
O_5 &= \overline{u}_a Y_\mu (1 - y_3) \sum_{q'} \bar{q}' \gamma^\mu (1 + y_5) q', \\
O_6 &= \overline{u}_a Y_\mu (1 - y_3) \bar{c} \sum_{q'} \bar{q}' \gamma^\mu (1 + y_5) q', \\
O_{bg} &= \frac{-g_8}{8\pi^2 M_{\text{q}_{bg}}} \overline{u}_a \gamma_\mu (1 + y_5) \gamma^{\nu} c_b,
\end{align*}$$

where $a$ and $b$ are color indices and $q' = u, d, s$. Among all these operators, $O_1^q$ and $O_2^q$ are tree operators, $O_3 - O_5$ are QCD penguin operators, and $O_{bg}$ is chromomagnetic dipole.
operator. The electroweak penguin operators are neglected in practice. One should notice that SCS decays receive contributions from all aforementioned operators while only tree operators can contribute to CF decays and DCS decays.

The Wilson coefficients used in this paper are evaluated at $\mu = 1 \text{ GeV}$, which can be found in [48].

(2) CKM Matrix. We use the Wolfenstein parameterization for the CKM matrix elements, which up to order $\sigma(\lambda^3)$ read [79, 80]

$$V_{ud} = \lambda - \frac{1}{2} A^2 \lambda^2 (\rho^2 + \eta^2),$$
$$V_{us} = 1 - \frac{1}{2} \lambda^2 - \frac{1}{8} \lambda^4 (1 + 4 A^2),$$
$$V_{ub} = \frac{\lambda}{16} \lambda^6 (1 - 4 A^2 + 16 A^2 (\rho + i \eta)),$$
$$V_{cb} = A \lambda^3 (\rho - i \eta),$$
$$V_{tb} = A \lambda^3 - \frac{1}{2} A^3 \lambda^5 (\rho^2 + \eta^2),$$

where $A, \rho, \eta,$ and $\lambda$ are the Wolfenstein parameters, which satisfy following relation:

$$\rho + i \eta = \frac{\sqrt{1 - A^2 \lambda^4}}{\sqrt{1 - \lambda^2}} \frac{(\mu + i \eta)}{(1 - A^2 \lambda^4 (\mu + i \eta))}.$$  \hspace{1cm} \text{(A.4)}

Numerical values of Wolfenstein parameters which have been used in this work are as follows:

$$\lambda = 0.22548^{+0.00068}_{-0.00054},$$
$$A = 0.810^{+0.018}_{-0.024},$$
$$\mu = 0.145^{+0.013}_{-0.007},$$
$$\eta = 0.343^{+0.011}_{-0.012}.$$ \hspace{1cm} \text{(A.5)}

(3) Decay Constants and Form Factors. In (17) and (18), the pole resonance model was employed for the matrix element $\langle PV | \gamma_1 q_2 | 0 \rangle$ in the annihilation diagrams. By considering angular momentum at weak vertex and all conservation laws are preserved at strong vertex, the matrix element $\langle PV | \gamma_1 q_2 | 0 \rangle$ is therefore dominated by a pseudoscalar resonance [54],

$$\langle PV | \gamma_1 q_2 | 0 \rangle = \langle PV | P^* \rangle \langle P^* | \gamma_1 q_2 | 0 \rangle = g_{PV} \epsilon_\mu \epsilon_P \frac{m_P}{m_P^2 - m_P^2} f_P,$$ \hspace{1cm} \text{(A.6)}

where $g_{PV}$ is a strong coupling constant and $m_P$ and $f_P$ are the mass and decay constant of the pseudoscalar resonance $P^*$. Therefore, $\eta$ and $\eta'$ are the dominant resonances for the final states of $K^{*\pm} K^\mp$, which can be expressed as flavor mixing of $\eta_3$ and $\eta_2$,

$$\left( \frac{\eta_3}{\eta_2} \right) = \left( \begin{array}{cc} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{array} \right) \left( \frac{\eta_3}{\eta_2} \right)$$ \hspace{1cm} \text{(A.7)}

where $\phi$ is the mixing angle and $\eta_3$ and $\eta_2$ are defined by

$$\eta_3 = \frac{1}{\sqrt{2}} (u \bar{u} + d \bar{d}),$$ \hspace{1cm} \text{(A.8)}

The decay constants of $\eta$ and $\eta'$ are defined by

$$\langle 0 | \gamma_\mu \gamma_5 u | \eta (p) \rangle = i f_\eta \mu \omega,$$
$$\langle 0 | \gamma_\mu \gamma_5 u | \eta' (p) \rangle = i f_\eta' \mu \omega,$$
$$\langle 0 | \gamma_\mu \gamma_5 d | \eta (p) \rangle = i f_\eta \mu \omega,$$
$$\langle 0 | \gamma_\mu \gamma_5 d | \eta' (p) \rangle = i f_\eta' \mu \omega,$$
$$\langle 0 | \gamma_\mu \gamma_5 s | \eta (p) \rangle = i f_\eta \mu \omega,$$
$$\langle 0 | \gamma_\mu \gamma_5 s | \eta' (p) \rangle = i f_\eta' \mu \omega,$$

where

$$f_\eta = f_{\eta'} = \frac{1}{\sqrt{2}} f_\eta,$$ \hspace{1cm} \text{(A.10)}$$
$$f_\eta = f_{\eta'} = \frac{1}{\sqrt{2}} f_\eta.$$ \hspace{1cm} \text{(A.11)}

According to [81, 82], the decay constants of $\eta$ and $\eta'$ can be expressed as

$$f_\eta = f_\eta \cos \phi,$$
$$f_\eta' = f_\eta \sin \phi,$$
$$f_\eta = -f_\eta \sin \phi,$$
$$f_\eta' = f_\eta \cos \phi,$$

where $f_\eta = (1.07 \pm 0.02) f_\eta$ and $f_\eta = (1.34 \pm 0.02) f_\eta$, and the mixing angle $\phi = (40.4 \pm 0.6)^\circ$ [83]. Other decay constants used in this paper are listed in Table 5.

The transition form factors $A_{0}^{D^{*} \rightarrow K^{*} \pi^{0}}$ and $A_{1}^{D^{*} \rightarrow K^{*} \pi^{0}}$, based on the relativistic covariant light-front quark model [85], are expressed as a momentum-dependent, 3-parameter form (the parameters can be found in Table 6):

$$A \left( q^2 \right) = \frac{A(0)}{1 - a (q^2/m_P^2) + b (q^2/m_P^2)^2}.$$ \hspace{1cm} \text{(A.12)}

(4) Decay Rate. The decay width takes the form

$$\Gamma_{D \rightarrow K \pi} = \frac{|P_{D} |^3}{8 \pi m_D^2} \left| M_{D \rightarrow K \pi} \right|^2.$$ \hspace{1cm} \text{(A.13)}
where $p_1$ represents the center of mass (c.m.) 3-momentum of each meson in the final state and is given by

$$|p_1| = \sqrt{\left(m_3^2 - (m_{K^*} + m_K)^2\right)\left(m_2^2 - (m_{K^*} - m_K)^2\right)} / 2m_2$$

(A.14)

$\mathcal{M}$ is the corresponding decay amplitude.

**Data Availability**

No data were used to support this study.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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**References**

[1] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, "Evidence for the 2π Decay of the $K_{S}^{0}$ Meson," Physical Review Letters, vol. 13, no. 4, pp. 138–140, 1964.

[2] M. Kobayashi and T. Maskawa, "CP-violation in the renormalizable theory of weak interaction," Progress of Theoretical and Experimental Physics, vol. 49, pp. 652–657, 1973.

[3] C. Cabibbo, "Unitary symmetry and leptonic decays," Physical Review Letters, vol. 10, no. 12, pp. 531–533, 1963.

[4] R. Aaij, LHCB Collaboration et al., "Measurement of CP Violation in the Phase Space of $B^0 \rightarrow K^0 \pi^+ \pi^-$ and $B^0 \rightarrow K^0 \bar{K}^0$ Decays," Physical Review Letters, vol. 111, no. 2, Article ID 101801, 2017.

[5] R. Aaij, LHCB Collaboration et al., "Measurement of CP Violation in the Phase Space of $B^\pm \rightarrow K^\mp K^\pm \pi^\pm$ Decays," Physical Review Letters, vol. 112, no. 2, Article ID 011801, 2014.

[6] R. Aaij, LHCb Collaboration et al., "Measurements of CP violation in the three-body phase space of charmless $B^\pm$ decays," Physical Review D, vol. 90, Article ID 112004, 2014.

[7] J. H. A. Nogueira, S. Amato, A. Austregesilo et al., "Summary of the 2015 LHCb workshop on multi-body decays of $D$ and $B$ mesons," https://arxiv.org/abs/1605.03889.

[8] Z.-H. Zhang, X.-H. Guo, and Y.-D. Yang, "CP violation in $B^\pm \rightarrow \pi^\pm \pi^\mp \pi^\pm$ in the region with low invariant mass of one $\pi^-\pi^+$ pair," Physical Review D, vol. 87, Article ID 076007, 2013.

[9] I. Bediaga, T. Frederico, and O. Lourenço, "CP violation and CPT invariance in $B^\pm$ decays with final state interactions," Physical Review D, vol. 89, Article ID 094013, 2014.

[10] H.-Y. Cheng and C.-K. Chua, "Branching fractions and direct CP violation in charmless three-body decays of $B$ mesons," Physical Review D, vol. 88, Article ID 114014, 2013.

[11] Z.-H. Zhang, X.-H. Guo, and Y.-D. Yang, "CP violation induced by the interference of scalar and vector resonances in three-body decays of bottom mesons," https://arxiv.org/abs/1308.3242.

[12] B. Bhattacharya, M. Gronau, and J. L. Rosner, "CP asymmetries in three-body $B^\pm$ decays to charged pions and kaons," Physics Letters B, vol. 726, no. 1–3, pp. 337–343, 2013.

[13] D. Xu, G.-N. Li, and X.-G. He, "Large SU(3) Breaking Effects and CP Violation in $B^{\pm}$ Decays Into Three Charged Octet Pseudoscalar Mesons," International Journal of Modern Physics A, vol. 29, Article ID 1450011, 2014.

[14] W.-E. Wang, H.-C. Hu, H.-N. Li, and C.-D. Liu, "Direct CP asymmetries of three-body $B$ decays in perturbative QCD," Physical Review D, vol. 89, Article ID 074031, 2014.

[15] Z.-H. Zhang, C. Wang, and X.-H. Guo, "Possible large CP violation in three-body decays of heavy baryon," Physics Letters B, vol. 751, pp. 430–433, 2015.

[16] C. Wang, Z.-H. Zhang, Z.-Y. Wang, and X.-H. Guo, "Localized direct CP violation in $B^{\pm} \rightarrow \pi^\pm (\omega)\pi^\mp \rightarrow \pi^\pm \pi^\mp \pi^\mp$; The European Physical Journal C, vol. 75, p. 356, 2015.

[17] J. H. A. Nogueira, I. Bediaga, A. B. R. Cavalcante, T. Frederico, and O. Lourenço, "CP violation: Dalitz interference, CPT, and final state interactions," Physical Review D, vol. 92, Article ID 054010, 2015.

[18] J. Dedonder, A. Furman, R. Kamiński, L. Leśniak, and B. Loiseau, "$S$-, $P$- and $D$-wave $\pi\pi\pi$ final state interactions and CP violation in $B^{\pm} \rightarrow \pi^\pm \pi^\mp \pi^\mp$ decays," Acta Physica Polonica B, vol. 42, no. 9, Article ID 2013, 2011.

[19] B. El-Bennich, A. Furman, R. Kamiński, L. Leśniak, and B. Loiseau, "Interference between $f_1(980)$ and $\rho(770)$ resonances in $B \rightarrow \pi^- \pi^- K$ decays," Physical Review D, vol. 74, Article ID 114009, 2006.

[20] I. Bigi and A. Sanda, "On $D^*D^*$ mixing and CP violation," Physics Letters B, vol. 171, no. 2–3, pp. 320–324, 1986.

[21] G. Blaylock, A. Seiden, and Y. Nir, "The role of CP violation in $D^*D^*$ mixing," Physics Letters B, vol. 355, no. 3, pp. 555–560, 1995.
[22] S. Bergmann, Y. Grossman, Z. Ligeti, Y. Nir, and A. A. Petrov, "Lessons from CLEO and FOCUS measurements of $D^{0}\overline{D}^{0}$ mixing parameters," *Physics Letters B*, vol. 486, no. 3-4, pp. 418–425, 2000.

[23] U. Nierste and S. Schacht, "Neutral $D\rightarrow K\overline{K}$ decays as discovery channels for charm CP violation," *Physical Review Letters*, vol. 119, Article ID 251801, 2017.

[24] G. Bonvicini, CLEO Collaboration et al., "Search for CP violation in $D^{0} \rightarrow K_{s}^{0}\overline{K}_{s}^{0}$, $D^{0} \rightarrow \pi^{+}\pi^{-}$ and $D^{0} \rightarrow K_{s}^{0}\overline{K}_{s}^{0}$ decays," *Physical Review D*, vol. 63, Article ID 071101, 2001.

[25] J. M. Link, "Search for CP Violation in the decays $D^{+} \rightarrow K\pi^{+}$ and $D^{+} \rightarrow K\pi^{+}$," *Physical Review Letters*, vol. 88, Article ID 041602, 2002.

[26] T. Aaltonen, CDF Collaboration et al., "Measurement of CP-violating asymmetries in $D^{0} \rightarrow \pi^{+}\pi^{-}$ and $D^{0} \rightarrow K\overline{K}$ decays at CDF," *Physical Review D*, vol. 85, Article ID 012009, 2012.

[27] R. Cenci, "Mixing and CP Violation in Charm Decays at BABAR," in *Proceedings of the 7th International Workshop on the CKM Unitarity Triangle (CKM 2012)*, Cincinnati, Ohio, USA, 2012.

[28] J. P. Lees, BABAR Collaboration et al., "Limits on CP violation in $D^{0} \rightarrow K\overline{K}$, $\pi^{+}\pi^{-}$, and $\pi^{+}\pi^{-}$ decays with the full Belle data set," *Physics Letters B*, vol. 753, pp. 412–418, 2016.

[29] R. Aaij, R. Aaij, B. Adeva et al., "Measurement of CP asymmetries in $D^{0} \rightarrow \eta\pi^{+}$ and $D^{0} \rightarrow \eta\pi^{+}$ decays," *Physics Letters B*, vol. 771, pp. 20–39, 2017.

[30] R. Aaij, LHCB Collaboration et al., "Measurements of charm mixing and CP violation using $D^{0} \rightarrow K^{+}\pi^{-}$ decays," *Physical Review D*, vol. 95, Article ID 052004, 2017.

[31] LHCB Collaboration, "Search for CP violation in the phase space of $D^{0} \rightarrow \pi^{+}\pi^{-}\pi^{+}\pi^{-}$ decays," *Physics Letters B*, vol. 769, pp. 345–356, 2017.

[32] Y. Bhardwaj, "Latest Charm Mixing and CP results from B-factories," in *Proceedings of the 9th International Workshop on the CKM Unitarity Triangle (CKM2006)*, vol. 139, Mumbai, India, 2017.

[33] Y. Grossman, A. L. Kagan, Y. Nir et al., "New physics and CP violation in singly Cabibbo suppressed D decays," *Physical Review D*, vol. 75, Article ID 036008, 2007.

[34] J. D. Bjorken, "Topics in B-physics," *Nuclear Physics B (Proceedings Supplements)*, vol. II, no. C, pp. 325–341, 1989.

[35] M. J. Dugan and B. Grinstein, "QCD basis for factorization in decays of heavy mesons," *Physics Letters B*, vol. 255, no. 4, pp. 583–588, 1991.

[36] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, "QCD factorization for $B \rightarrow \pi\pi$ decays: strong phases and CP violation in the heavy quark limit," *Physical Review Letters*, vol. 83, no. 10, pp. 1914–1917, 1999.

[37] M. Beneke and M. Neubert, "QCD factorization for $B \rightarrow PP$ and $B \rightarrow PV$ decays," *Nuclear Physics B*, vol. 675, no. 1-2, pp. 335–415, 2003.

[38] D. R. Boito, J. Dedonder, B. El-Bennich, O. Leitner, and B. Loiseau, "Scalar resonances in a unitary," *Physical Review D*, vol. 96, Article ID 113003, 2017.

[39] A. Furman, R. Kaminska, L. Lesniak, and B. Loiseaub, "Long-distance effects and final state interactions in $B \rightarrow \pi\pi K$ and $B \rightarrow K\overline{K}$ decays," *Physics Letters B*, vol. 622, pp. 207–217, 2005.
