Mathematical analysis of spacecraft control cyclograms

N A Testoedov¹,², V E Kosenko¹, E N Golovenkin¹, A V Kuzovnikov¹ and I V Kovalev²³⁴⁵

¹ JSC Academician M.F. Reshetnev «ISS» Zheleznogorsk, Krasnoyarsk region, Russia
² Reshetnev Siberian University of Science and Technology, Krasnoyarsk, Russia
³ Siberian Federal University, Krasnoyarsk, Russia
⁴ Krasnoyarsk State Agrarian University, Krasnoyarsk, Russia
⁵ Krasnoyarsk Science and Technology City Hall, Krasnoyarsk, Russia

E-mail: gne@iss-reshetnev.ru

Abstract. In this paper, the mathematical analysis of spacecraft control cyclograms is considered. The proposed solution is applied to a ground-based control complex of spacecrafts, where a plenty of cyclograms is a part of the general technological control cycles of spacecrafts, which include the whole of spacecrafts system life cycle. The problem of ground control system software optimization is solved using static strategies and cost criteria. Spacecraft control processes cover all the system control cycle and regulate an execution of the information processing and control tasks in the ground-based and board control complexes. Analyzing characteristics of such the processes has ascertained a strong relation between spacecraft technological control cycles and their computer-based realization for the mentioned complexes.

1. Introduction

An intercommunication of the spacecraft control cyclograms (SCC) and their realization processes on the spacecraft control equipment shows that we must provide a computer-aided modeling of the spacecraft control cyclograms [1]. We consider a control complex which is multicomponent. This control complex has a plenty of processors.

A serial process of operations of the spacecraft control equipment components can be associated to every control cyclogram. The time of this serial process realization depends on the characteristics of hardware components and software reliability [2].

Above the solution was realized for a step of the cyclogram modeling under the hypotheses which reflect static strategy [3]. This strategy shows that it is possible to choose an optimal serial process in advance and the optimal conditions will be satisfied. Our solution is applied to a ground-based control complex of spacecrafts, where a plenty of cyclograms is a part of the general technological control cycles (TCC) of spacecrafts, which include the whole of spacecrafts system life cycle.

2. Problem statement

Let us formulate the problem, where a basis intercommunication of TCC and their realization processes at a computer network of the ground control complex is defined. It is necessary to map TCC onto a multicomponent computer network [1-3].
Let the TCC can be described by an oriented graph [4]. Spacecraft control problems and information processing problems can be associated to all the tops of the graph. Every problem has a specification. The other oriented graph corresponds to the computer network structure. Identified with components of the structure all the tops of the graph has specifications too.

We must define a possibility of the TCC realization for an initial structure of the computer network. If the TCC realization is possible then it is necessary to indicate an operation program [5] or a set of operation programs of the structural components providing the given TCC realization. A minimal time of the TCC realization must be satisfied subject to a reliability of the network hardware and software. A quality of the TCC realization depends on these characteristics [6-8].

A brief mention of a concept of the operation program should be made. It is apparent that the same hardware/software complex may realize both various TCC's and one and the same TCC with various time characteristics. Hence we shall be able to define the operation program under the conditions [3]:

- for all the structural component operations the components giving a data input and consuming a data output must be defined;
- the moments when the component switches (the initialization moments) are indicated, i.e. the problems of TCC are started to fulfill in these moments and it corresponds to the computer system job scheduling;
- a problem type and a problem duration are indicated for all the actuation moments when the component switches.

We shall note that the last condition is necessary if some component of the computer network may fulfill either various problem of TCC or one-type problems having various time characteristics.

As we could see from the foregoing, a solution of the formulated problem is in that we establish a one-to-one correspondence between the TCC graph tops and some sets of the operating sequences, which are realized by the computer network components. This correspondence identically determines the whole information exchange in the network [9, 10].

Having chosen the component initialization moments, we attain two variants. Either the network components come into action and realize TCC or they do not operate and come to a stop. In that case the TCC realization is not qualitative. Even if all the network components have come into action, the process of TCC realization may not satisfy for many reasons. For example, the time realization will be too large or the data channels will be low-loaded and so on.

In any case the following parameters must be defined [11]:

- the function establishing a one-to-one correspondence between the TCC-graph tops and every moment when the computer network components come into the action (the component initialization moments);
- the data streams between the components;
- the actuation times of the components.

The realization of the indicated conditions corresponds to the static strategy as noted above, for dynamic strategy is not the characteristic property of the ground-based control complex [86]. In this situation an initial distribution of the TCC problems with respect to the nodes of a network is up to the start time of the control process.

2.1 Analysis of spacecraft control cyclograms realization

As it was shown in [3], the problem of ground control system software optimization is solved using static strategies and cost criteria. Since a board control complex has several constraints for the TCC realization time, then the time characteristics for this control complex are more important than for a ground control complex. Considering the problem of a cyclogram realization time optimization we use the time criteria (in general case the probable time criteria). This cyclogram realization time depends on the choice of the program module (PM) type of a given control problem implementation.

Since the description of spacecraft control cyclograms does not depend on the type of control complex, then we have the evolution time vector (ETV) of TCC-graph for an initial date

\[ t = (t_1, t_2, \ldots, t_n), \]
where \(t_i\) is the moment when a component of the network structure comes into action to realize the problem of the \(i\)-th top of the TCC-graph. The evolution time vector determines all data intercommunications between the network components.

Let it be possible to describe the SCC graph realized on this structure by an incidence matrix

\[
A \equiv [a_{ij}].
\]

It is \(n \times m\) matrix and

\[
a_{ij} = \begin{cases} 
1 & \text{if the } j \text{-th arc starts from the top } i; \\
-1 & \text{if the } j \text{-th arc goes at the top } i; \\
0 & \text{otherwise.}
\end{cases}
\]

We use the realization vector as well

\[
h = (h_1, h_2, ..., h_m),
\]

where \(h_i\) is the fulfillment duration of the TCC problem, which is into the \(j\)-th arc origin.

It can be said with confidence that the realization vector is determined by the computer network structure. It is necessary to ascertain a possibility to realize this ETV onto the network having the given structure by the realization vector \(h\).

Using the computer-aided generating of the TCC, we realize the above cited algorithm of the formulated solution.

State the following proposition.

To realize TCC onto a given structure of the network for a given ETV \(t\) it is necessary and sufficient to meet the following conditions: if the \(j\)-th arc of the TCC-graph is directed from the \(i\)-th top to the \(\nu\)-th top then the difference \(t_\nu - t_i\) must be no less than the operation time of the component which is located at the \(i\)-th top.

\[
t_\nu - t_i \geq h_\nu a_{ij}
\]

The necessity of the conditions is evident since a work of all components giving the data for every component must be completed before an indicated component comes into action. As a demonstration of the sufficiency it would be appropriate to use the canonical parallel-maximal form of TCC by analogy with the same form of a computer algorithm graph realized on parallel computer systems [2]. A control loop of a TCC model corresponds to a layer of an algorithm graph. The given in the paper [3] TCC model description is more detailed.

According to the above stated, to solve the problem it is necessary and sufficient the following inequality to be fulfilled:

\[
-A' t > h.
\]

The TCC realization means the realization of an operation program on a network. But the rows of this vector-matrix inequality are none other than a formalized expression of the previous conditions of our statement, which defines a possibility of such the realization. Thanks to the matrix \(A\) and the vectors \(t\) and \(h\) it is possible to determine many characteristics of the TCC realization process. For example, the time of a TCC fulfillment will be equal to:

\[
T = \max_i (t_i + z_i) - \min_i t_i,
\]

where \(z_i\) - the realization time of the TCC problem started to be fulfilled at the moment \(t_i\).

The TCC problems fulfillment sequence corresponds to ordering the vector coordinates in order of increasing. If some of its coordinates are equal, then the corresponding problems of TCC are fulfilled by various components of the computer network in parallel. In succession all the groups of the ETV different coordinates are renumbered and a topological sorting of the TCC graph tops is conducted; at that the tops corresponding to the same \(t_i\) are marked with an identical index. This sorting will determine a parallel form of TCC realized on the computer network under the time condition, which are determined by ETV of TCC.
The software tools of computer-aided TCC generating provide primarily a generating stage for an inspection which is interactively conducted by a control technology engineer. The engineer formulates the specifications for the operation program of the spacecraft control complex hardware and software.

2.2 Correction algorithm of TCC

The algorithm discussed above is the second step, when (1) and (2) give exact result of the analysis. For the case when the inequality (1) is not fulfilled we change ETV (the component initialization moments) for a fixed realization vector correcting the TCC time parameters. This is a transfer to the third step discussed below. Corresponding to this ETV correction algorithm denotes the necessity of stating the model boundary conditions. Many different approaches to the analysis of the network problems are possible [1-3]. We are considering three groups of inequalities:

1. The condition of non-negativity

\[ t_i \geq 0, \quad (i \in I), \quad (3) \]

where \( I=\{i\} \) is a totality of the TCC generating problem;

2. The condition of termination of the TCC previous problems

\[ \sum_{t_j=t_i} z_i \leq t_i, \quad (i, j \in I); \quad (4) \]

3. The condition of a logical sequence

\[ t_i \geq \alpha_j t_j, \quad (i, j \in I), \quad (5) \]

where \( \alpha_{ij} \) is a coefficient of the TCC problems interconnection, i.e.

\[ a_{ij} = \begin{cases} 1, & \text{if a problem } j \text{ precedes a problem } i; \\ 0, & \text{in otherwise} \end{cases} \]

\( t_i \) and \( h_i \) are the components of ETV and the realization vector accordingly.

For each TCC problem the parameter \( t_i^f \) is a moment up to which the realization of a problem in the \( i \)-th top of TCC graph is maximal. However, the problem maximal value can be reached only in the case when the actual data are processed. Therefore we introduce the symbol \( t_i^s \) which is a minimal moment up to which a realization with the TCC problem in the \( i \)-th top is impossible

\[ t_i^s \leq t_i \leq t_i^f. \quad (6) \]

It is evident, that according to the regulations

\[ 0 \leq t_i^s < t_i^f < T, t_i^s - t_i^s \geq z_i, \]

where \( T \) is the duration of TCC realization.

If the conditions (3) are met then the conditions \( t_i^s < t_j^s, t_i^f < t_j^f \) for interconnected problems \( i \) and \( j \) (\( i \) precedes \( j \)) are correct too. On the other hand, if an admissible TCC exists then the following condition is sure to fulfil:

\[ i < j \Rightarrow t_j^f \geq t_i^s; \quad t_j^s \geq t_i^f + z_i; \]

where \( t_j^s \) and \( t_i^f \) are the moments of the \( j \)-th and \( i \)-th problem starting and finishing accordingly.

Therefore, a changing of values \( t_i^o \) by \( \max(t_i^s, t_j^s - z_j) \) and \( t_j^o \) by \( \max(t_j^f, t_i^f + z_i) \) during the correction of ETV does not affect on the existence of an admissible TCC (with regard to the intercommunication) for every connected pair \( i \) and \( j \).

Thus, solving the problem of analysis with regard to the problems interconnection when the inequality (1) is not satisfied is conducted in accordance with the suggested method of correction, i.e.
the components of ETV \( t_r = \{ t_r^i \} \) and \( t_j = \{ t_j^r \} \) are replaced with the vectors \( t_r^i \) and \( t_j^r \) satisfying the conditions \( t_j^r \) is a minimal and \( t_r^i \) is a maximal value in the coordinatewise vector comparison:

1) \( i < j \Rightarrow t_r^i \geq t_r^j, t_j^r \leq t_j^r \);
2) \( i < j \Rightarrow t_r^i \geq t_j^r + z_i ; \)
3) \( i < j \Rightarrow t_r^i \leq t_j^r - z_i \).

In a conclusion it may be noted that the mathematical modeling of TCC is the important part of designing a spacecraft control system and its software [4]. As it was indicated above, in this paper the case of a ground-based control complex has been considered, the generalized model of TCC for description of the cyclograms of a spacecraft board control complex was considered in the paper [11].

3. Conclusion
Speaking about the developed analytic and optimization procedure, a spacecraft TCC formalization allows to solve the tasks of an analysis of a TCC realizability, correction and optimization for both deterministic and stochastic types of network models.

Using the TCC stochastic description we can solve the problem where we determine the average time and the standard deviation of the TCC realization or the TCC step realization for conditions of uncertainty. In order this description to have meaning, it is necessary to obtain the variance estimates of each TCC problem. We must determine the stochastic characteristics of each TCC component (the spacecraft control problems and the information processing problems) for normal conditions of the control complex functioning. Many different variants are possible. This description of the TCC problems is more detailed and more exactly.

References
[1] Kovalev I V 2001 System of Multi-Version Development of Spacecrafts Control Software (Sinzheim: Pro Universitate Verlag)
[2] Antamoshkin A N et al 1993 System Analysis, Design and Optimization (Krasnoyarsk: Ofset Press)
[3] Antamoshkina O I and Kovalev I V 1996 Modeling, Optimization and Computer-Realization of Control Cyclograms (Krasnoyarsk: SAA Press)
[4] Kovalev I V 2014 Analysis of problems in the field of software reliability research: multi-stage and architectural aspect Vestnik of SibGAU 3(55) 78-92
[5] Kovalev I V et al 2017 Multiversion environment creation for control algorithm execution by autonomous unmanned objects IOP Conf. Series: Materials Science and Engineering 173 012025
[6] Kovalev I V and Younoussov R V 2002 Fault-tolerant software architecture creation model based on reliability evaluation Advanced in Modeling & Analysis 48(3-4) 31-43
[7] Levendel Y 1990 Reliability analysis of large software systems: Defect data modeling IEEE Trans. Software Engineering 16 141-52
[8] Lyu M R 1996 Handbook of Software Reliability Engineering (McGraw-Hill Book Company: IEEE Computer Society Press)
[9] Lyu M R 1996 Software Fault Tolerance (Published by John Wiley & Sons Ltd)
[10] Cherif A 1999 Improving the Efficiency of Replication for Highly Reliable Systems (FastAbstract ISSRE Copyright)
[11] Hecht H 1979 Fault tolerant software IEEE Trans. Reliability R-28 227-32
[12] Koltyshev A A et al 2018 The implementation of measures for reduction an inertia of the interaction components of the heterogeneous environment for the automated process control system IOP Conf. Series: Materials Science and Engineering 450 042015