The Single Electron Transistor (SET) has recently been suggested as a read-out device for solid-state charge qubits \[1, 2, 3, 4, 5\]. Presenting new state of the art figures for sensitivity in charge measurements, Assime et al. \[7, 8\] made probable that the Radio-Frequency SET (RF-SET) can be used for single shot read-out of the Single Cooper-pair Box (SCB) qubit \[7, 8\]. This is possible if the measurement time \( t_{ms} \) needed to resolve the two states of the qubit is much shorter than the time \( t_{mix} \) required to destroy the initial qubit state because of back-action due to voltage fluctuations on the SET.

The mixing rate \( 1/t_{mix} \) is proportional to the noise at the frequency \( \omega \) corresponding to the energy splitting between the qubit states \( \Delta E \). Moreover, in order to have well defined charge states to achieve single shot read-out, \( \Delta E \) should be comparable to the qubit charging energy \( E_{qb} \). The ratio \( E_{qb}/E_C \) may be varied between approximately 1/10 and 10 in order to optimize the performance of the system.

The voltage fluctuations on the SET island have previously been studied in the experimentally accessible low frequency limit \((\hbar \omega \ll E_C)\). The noise is here fully understood both in the classical transport regime of sequential tunneling, described by a master equation with frequency independent tunneling rates \[9, 10, 11\], as well as in the Coulomb blockade cotunneling regime \[14\]. In Ref. \[14\] the back-action of the SET was estimated using simple interpolation between the low frequency shot noise limit and the high frequency limit of Nyquist noise from two independent tunnel junctions.

In this paper we evaluate the finite frequency noise on a fully quantum mechanical basis, taking into account the frequency dependence of the tunneling rates in the presence of fluctuations, separating processes that absorb or emit energy. We present a simple expression for the voltage noise of a SET biased in the transport mode, valid in the whole frequency range from low frequency classical shot noise to high frequency quantum noise. By comparing \( t_{mix} \) with \( t_{ms} \) for reading out a SCB qubit with the RF-SET of Ref. \[14\], we conclude that \( t_{ms} \ll t_{mix} \), i.e. single-shot read-out should indeed be possible.

We calculate the spectral density of voltage fluctuations in a Single Electron Transistor (SET), biased to operate in a transport mode where tunneling events are correlated due to Coulomb interaction. The whole spectrum from low frequency shot noise to quantum noise at frequencies comparable to the SET charging energy \((E_C/\hbar)\) is considered. We discuss the back-action during read-out of a charge qubit and conclude that single-shot read-out is possible using the Radio-Frequency SET.

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Consider a small metallic SET island coupled via low transparency tunnel barriers to two external leads, and coupled capacitively to the SCB, which is controlled by a gate voltage (see Fig.1). First we calculate the voltage noise on the SET island taking the voltage \( V_g \) on the SCB to be constant. Then the back-action of these fluctuations on the state of the SCB is estimated. This approach is appropriate in the considered limit of weak SET-qubit coupling \((\kappa C_C/C_{qb}, \kappa \ll 1; C_L, C_R \sim C_{qb})\).

We follow the outline of Ref. \[12\] and model the SET by the Hamiltonian

\[
H = H_L + H_R + H_I + V + H_T = H_0 + H_T, \tag{1}
\]

where

\[
H_r = \sum_{kn} \epsilon_{kn} a_{kn}^\dagger a_{kn}, \quad H_I = \sum_{ln} \epsilon_{ln} c_{ln}^\dagger c_{ln} \tag{2}
\]

describe noninteracting electrons in the left/right lead \((H_r, r \in \{L, R\})\) and on the island \((H_I)\). The quantum numbers \( n \) denote transverse channels including spin, and \( k,l \) denote momenta. The Coulomb interaction on the island is described by

\[
V(\hat{N}) = E_C (\hat{N} - n_x)^2, \tag{3}
\]

where \( \hat{N} \) denotes the excess number operator, \( E_C = e^2/2C \) the charging energy \((C = C_L + C_R + C_C)\), \( n_x \) the fractional number of electrons induced by the external voltages \((n_x)\) is the fractional part of \((C_L V_L + C_R V_R + C_C V_C)\)
where the operator \( e^{±i\phi} \) changes the excess particle number on the island by \( ±1 \) and \( T^{|n}\) are the tunneling matrix elements. \( \hat{\Phi} \) is the canonical conjugate to \( \hat{N} ((\Phi, N) = i) \). In this case of a metallic island, containing a large number of electrons, the charge degree of freedom \( \hat{N} = 0, ±1, ... \) is to a very good approximation independent of the electron degrees of freedom \( l, n \).

The Cooper-pair box is a small superconducting island coupled to a superconducting reservoir, via a tunnel junction with Josephson energy \( E_J \) (see Fig. 1). The box is also characterized by its charging energy \( E_{qb} = e^2/2C_{qb} \), where \( C_{qb} \) is the total capacitance of the box. For \( E_J \ll E_{qb} \) two nearby charge states, corresponding to zero and one excess Cooper pair on the island, can be used as the \( |0\) and \( |1\) states of the qubit. The energy splitting \( \Delta E \) between these states is controlled by the gate voltage \( V_g \).

For a given charge measurement sensitivity \( \delta q \) and measurement time \( t_{ms} \), the uncertainty in charge is given by \( \Delta q = \delta q/\sqrt{t_{ms}} \). In order to separate the two qubit states we need the two intervals \( 0 ± \Delta q \) and \( 2e ± \Delta q \) not to overlap, which defines the measuring time \( t_{ms} = \delta q^2/(ke)^2 \).

During the measurement, the voltage fluctuations on the SET island will induce transitions between the qubit states. The rate for excitation/relaxation of the qubit is proportional to the SET noise spectral density \( S_V(\omega) \) at the negative/positive frequency corresponding to the transition \( \Delta E/\hbar \). The information of the initial state is destroyed at the combined rate \( \Gamma \).

\[
\Gamma = \frac{1}{t_{mix}} = \frac{e^2}{h^2} \kappa^2 \frac{E_J^2}{\Delta E^2} \left[ S_V(\Delta E/\hbar) + S_V(-\Delta E/\hbar) \right],
\]

(5)

here given in the relevant limit \( \Delta E \gg E_J \) used below.

The spectral density of voltage fluctuations on the SET island is described by the Fourier transform of the voltage-voltage correlation function

\[
S_V(\omega) = \frac{e^2}{C^2} \int_{-\infty}^{\infty} dt e^{-i\omega t} Tr \{ \rho_{eq}(t_0) \hat{N}(\tau) \hat{N}(t_0) \}.
\]

(6)

Here \( \rho_{eq}(t_0) \) is the density matrix of the system in steady-state, which is assumed to have been reached at some time \( t_0 \) before the fluctuation occurs \( (t_0 < \min(0, \tau)) \). \( \rho_{eq} \) is the tensor product of the equilibrium (Fermi distributed) density matrix \( \rho_{eq}^l \) for the electron degrees of freedom in each reservoir \( \{ \hat{L}, \hat{R}, \hat{I} \} \) and a reduced density matrix \( \rho_{eq}^c \), describing the charge degrees of freedom, which we assume to be diagonal with elements \( P_N^c \) denoting the probability of being in charge state \( N \).

To evaluate \( S_V(\omega) \) we make a perturbation expansion of the forward and backward time evolution operators in terms of the tunneling term \( H_T \), as shown in Fig. 2a. The reservoir degrees of freedom are traced out using Wick’s theorem. In the diagrammatic language of reference [12] (Fig. 2) we have forward and backward propagators of the Keldysh contour (horizontal lines), with internal charge transfer vertices \( e^{±i\phi} \) (small dots) connected by reservoir lines. The \( \hat{N}(0) \) and \( \hat{N}(\tau) \) operators form external vertices (big dots). Figure 2b shows the diagrammatic expression for \( S_V(\omega) \) in our approximation, neglecting processes involving other charge states than \( N \in \{0, 1\} \). Here \( \Pi_{NN'}(\omega) \) is the frequency dependent transition rate between charge states \( N \) and \( N' \).

One may graphically write down a Dyson type of equation for \( \Pi_{NN'}(\omega) \) (Fig. 3b). In matrix notation this reads

\[
\tilde{\Pi}(\omega) = \frac{i}{\omega} \tilde{\Sigma}(\omega) + \tilde{\Pi}(\omega) \tilde{\Sigma}(\omega) \quad \Rightarrow \quad \tilde{\Pi}(\omega) = \frac{i}{\omega} \left( 1 - \frac{i}{\omega} \tilde{\Sigma}(\omega) \right)^{-1},
\]

(7)

where \( \Sigma_{NN'}(\omega) \) represents the sum of irreducible diagrams, i.e. containing no free propagators, for transitions between \( N \) and \( N' \).

Since the SET is biased in transport mode, we consider only the dominating lowest order diagrams in \( \Sigma_{NN'}(\omega) \), corresponding to single tunneling events. In this approximation we get [12]

\[
\Sigma_{NN, N \pm 1}(\omega) = \gamma_N^+(\omega) + \gamma_N^-(\omega),
\]

where \( \gamma_N^+ (\omega) \) denotes the rate, in second order perturbation theory, to go from charge state \( N \) to \( N \pm 1 \) while the SET is absorbing \( (\omega) \) or emitting \( (-\omega) \) a quantum of energy \( |\hbar \omega| \). The well-known expressions for these rates
\begin{align}
\gamma_N^+(\omega) &= \frac{\pi}{\hbar} \sum_r (\hbar \omega + \Delta_N^r) \alpha_0^r n(\hbar \omega + \Delta_N^r), \\
\gamma_N^-(\omega) &= \frac{\pi}{\hbar} \sum_r (\hbar \omega - \Delta_{N-1}^r) \alpha_0^r n(\hbar \omega - \Delta_{N-1}^r)
\end{align}

where the dimensionless conductivity

$$\alpha_0^r = \sum_n |T_{nr}^\alpha|^2 \rho_n^r \rho_l^\alpha = \frac{R_K}{4\pi^2 R_T^r}$$

is the ratio of the quantum resistance $R_K = \hbar/e^2$ and the resistance of barrier $r \in \{L, R\}$, and $\beta = 1/k_B T$. $n(\epsilon) = 1/\{1 - \exp[-\beta \epsilon]\}$ is the Bose-Einstein distribution of electron-hole excitations and comes from the convolution of the Fermi distributions for filled initial states and empty final states. $\Delta_N^r = V(N) + \mu_r - V(N+1) - \mu_l$ is the energy gained by an electron tunneling from the chemical potential of lead $r$ ($\mu_r$) to the chemical potential of the island ($\mu_l$), thus changing the charge state from $N$ to $N+1$ (see Figs. 3a-c). We also assume that the relaxation processes on the island are fast on the timescales we are looking at and neglect the energy dependence of both tunneling matrix elements $T_{kr}^\alpha \approx T_{rr}^\alpha$ and reservoir densities of states $\rho_n^r(\epsilon), \rho_l^\alpha(\epsilon)$. To avoid renormalization effects we choose not too small bias, i.e. $\max_r \alpha_0^r n(E_c/|\Delta_0^r|) \ll 1$.

We also choose the bias not too high, so that the higher charge states are inaccessible at low frequency, i.e. $\gamma_N^+(0) = \gamma_N^-(0) = 0$ giving $P_N^\alpha = 0$ for $N \notin \{0, 1\}$. At low frequencies we may thus restrict ourselves to the two lowest charge states $N \in \{0, 1\}$.

The matrix inverse in Eq. (7) may then easily be performed analytically. For high enough frequencies also the charge states $N \in \{-1, 2\}$ are accessible. In this regime we may expand Eq. (6) to first order in $\Sigma/\omega$. The results in both these regimes combine to the following expression for the noise valid, within our approximations, for all frequencies:

$$S_N(\omega) = \frac{2e^2}{C} P_{0^+}^\alpha \left[ \gamma_0^+(\omega) + \gamma_0^-(\omega) + P_{1^+}^\alpha \left[ \gamma_1^-(\omega) + \gamma_1^+(\omega) \right] \right]$$

where $P_{0^+}^\alpha = \gamma_0^+(0)/\gamma_0^+(0) + \gamma_0^-(0)$ and $P_{1^+}^\alpha = 1 - P_{0^+}^\alpha$.

Note the simple structure of this expression: a sum over the probabilities of being in the states $N = 0$ and $N = 1$ times the rates for possible transitions from these states, normalized by a denominator containing their finite lifetime due to the bias.

At zero frequency Eq. (12) coincides with the classical shot noise result \cite{14,13}, which is recovered by approximating all rates with their zero frequency value. In the high-frequency limit, where the biasing energy is negligible, one recovers the high-frequency Nyquist noise from two resistances connected in parallel to ground.

It is informative to analyze Eq. (12) at zero temperature where the tunneling rates are linear functions of the possible energy gain (since $n(\epsilon) = \Theta(\epsilon)$, where $\Theta(x)$ is the unit step function). To be specific we assume that $\mu_r > \mu_l > \mu_r$ and $\Delta_0 > 0$ so that electrons tunnel from the left lead to the right and $N = 0$ is the lowest charge state. We also assume that the left bias is larger than the right one, $|\Delta_L^0| > |\Delta_R^0|$ (see Fig. 3a).

For large negative frequencies $\hbar \omega < -|\Delta_0^L|$ the fluctuations try to extract more energy than the bias can provide, thus there is no contribution to the noise from this frequency range (since we neglect higher order cotunneling effects). Between $-|\Delta_0^L| < \hbar \omega < -|\Delta_0^R|$ the SET may emit energy $|\hbar \omega|$ while tunneling through the left barrier, while for $-|\Delta_0^R| < \hbar \omega < 0$ tunneling through both barriers contribute. For positive frequencies the SET may absorb energy $\hbar \omega$ while tunneling through either barrier. For frequencies $\hbar \omega > |\Delta_0^L/R|$ there is also a contribution from electrons tunneling backwards, against the bias, at the $(L/R)$ junction. In the region $|\Delta_0^L/R| \ll \hbar \omega < \{|\Delta_L^L/R|, |\Delta_L^R/R|\}$ the higher charge states $N \in \{-1, 2\}$ are still inaccessible and the noise approaches exactly one half of the Nyquist noise. This may be explained by the strong correlation of fluctuations at the two barriers induced by the Coulomb interaction. At high frequencies $\hbar \omega > \{|\Delta_L^L/R|, |\Delta_L^R/R|\}$ also processes at barrier $L/R$ including the charge states $N \in \{-1, 2\}$ contribute, giving the real high-frequency limit of Nyquist noise from two completely uncorrelated tunnel junctions. Figure 4 illustrates the above discussion of which processes contribute in which frequency region.

In the low-frequency regime $|\hbar \omega| < |\Delta_0^R|$, where no backward processes are allowed, the noise spectral den-
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FIG. 4: Schematic picture of the processes contributing to the finite frequency voltage fluctuations of a SET island, which indicates that single-shot measurement is possible. The sum of positive and negative frequency SET noise, proportional to the back-action on the qubit, is plotted as a function of level splitting in the left inset of Fig. 5 (dotted). The right inset shows the dependence of the SNR on the level splitting, which is symmetrical compared with the classical expression (dotted). The circles denote, in order from left to right, the finite frequency voltage fluctuations of a SET island biased in the transport mode. Using this information about back-action we can conclude that single-shot read-out of a niobium qubit, the SET emits/absorbs energy via a barrier, is possible.

In conclusion, we have obtained a simple expression for the sum of positive and negative frequency SET noise, proportional to the back-action on the qubit, which is plotted as a function of level splitting in the left inset of Fig. 5. The right inset shows the frequency regime separating positive and negative frequencies compared with the classical expression (dotted). The left inset shows the SNR of a single shot read-out as a function of level splitting in the qubit.

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FIG. 5: Summed voltage noise $S_V(\omega) + S_V(-\omega)$ for a symmetrically biased SET with DC-current $I = 6.7 nA$ and $R_J = R_F = 22 k\Omega$, $n_x = 0.49$, $T = 20 mK$, $E_C/k_B = 2.5 K$. Full expression (solid), Classical shot noise (dashed), Nyquist noise (dotted). The circles denote, in order from left to right, $\Delta \omega_0 = \{4, 2, 1\}$ and the right inset shows the low frequency regime separating positive and negative frequencies (solid) compared with the classical symmetrical expression (dotted). The left inset shows the SNR of a single shot read-out as a function of level splitting in the qubit.

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