Extracting the CKM phase angle $\gamma(\equiv \phi_3)$ from isospin analysis in $B \to K\pi$ decays

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(Dated: December 15, 2008)

Abstract

We propose a new method to extract the CKM phase angle $\gamma(\equiv \phi_3)$ from the isospin analysis in $B \to K\pi$ decays. Unlike previously proposed methods, we do not employ flavor SU(3) symmetry, so that this method is free from the hadronic uncertainty coming from the SU(3) breaking effect. Neither we adopt any Dalitz-plot analysis, which may involve multiple strong phases and large final state interactions. After including small CP violating terms in $B^+ \to K^0\pi^+$ and color-suppressed electroweak penguin contribution in $B^0 \to K^+\pi^-$, whose values are estimated from the QCD factorization, we obtain $\gamma = (70^{+5+1+2}_{-14-1-3})^\circ$ or $106^\circ < \gamma < 180^\circ$. The first error is due to the experimental errors mainly caused by mixing-induced CP asymmetry $S_{K_S\pi^0}$. The second and third errors come from the theoretical uncertainty for two above-mentioned small contributions, respectively. Since we utilize only the isospin relations in $B \to K\pi$ decays, this method will work well, regardless of possible new physics effects unless the isospin relations do not hold.

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I. INTRODUCTION

Investigating validity of the Cabibbo-Kobayashi-Maskawa (CKM) unitary triangle has been one of the most challenging subjects for testing the standard model (SM). Great efforts in both experiment at B factories and theoretical side have been devoted to extract the length of each side and each angle of the triangle. In this letter, we focus on the CKM angle $\gamma (\equiv \phi_3)$, providing a new method to extract $\gamma$ by using $B \to K\pi$ decays only.

It was first proposed by Gronau, London and Wyler (GLW) to extract $\gamma$ from $B \to DK$ decays [1]. The idea is on the basis of the well-known method using two triangles that are constructed from the decay amplitude $A(B^+ \to D^+_K K^+)$ and its charge conjugate $A(B^- \to D^-_K K^-)$, respectively, with one common amplitude $A(B^+ \to \bar{D}^0 K^+) = A(B^- \to D^0 K^-)$ in a complex plane. This method has its own benefit of theoretical cleanness, but it suffers from some practical difficulty because the triangles are a bit squashed. Atwood, Dunietz and Soni (ADS) [2] improved the GLW method, and later the Dalitz analysis was introduced [3]. These three methods have been used as the most favored methods for extracting $\gamma$ in experiment. The current fitting result from UTfit group combining with these three methods gives $\gamma = 83^\circ \pm 19^\circ$ [4].

Another promising approach for extracting $\gamma$ is the combined analysis of $B \to K\pi$ and $B \to \pi\pi$ decays. As Gronau, Hernandez, London and Rosner (GHLR) proposed [5], similarly to the case using $B \to DK$ decays, one can utilize the decay amplitudes of $B^\pm \to K^\pm\pi^0$, $B^\pm \to K^0\pi^\pm$ and $B^\pm \to \pi^\pm\pi^0$ with the help of flavor SU(3) symmetry. However, apart from the hadronic uncertainty coming from the SU(3) breaking effect, the GHLR method had been known to be spoiled considerably by electroweak (EW) penguin amplitudes [6]. Therefore, many authors have elaborated on dealing with the EW penguins and developed alternative ways to extract $\gamma$ from $B \to K\pi$ decays [7, 8, 9].

We would like to propose a new method for extracting $\gamma$ to a good accuracy by using the well-known isospin relations in $B \to K\pi$ decays:

$$A(B^0 \to K^+\pi^-) - A(B^+ \to K^0\pi^+) = \sqrt{2}A(B^+ \to K^+\pi^0) - \sqrt{2}A(B^0 \to K^0\pi^0),$$

$$A(B^0 \to K^-\pi^+) - A(B^- \to \bar{K}^0\pi^-) = \sqrt{2}A(B^- \to K^-\pi^0) - \sqrt{2}A(B^0 \to \bar{K}^0\pi^0).$$

We do not employ flavor SU(3) symmetry in order to avoid hadronic uncertainties stemming from the SU(3) breaking effect. Since we do not impose any theoretical inputs on the $K^+\pi^0$.
and $K^0\pi^0$ modes which are potentially sensitive to new physics effects, this method will work well, regardless of possible new physics effects unless they spoil the isospin relations. We note that significant new physics contribution from the electro-weak penguin diagram may be present in $B \rightarrow K\pi$ decays, as discussed in Ref. [11]. It has been shown that current experimental data imply the $r_{\text{EW}}$, the ratio of electro-weak penguin amplitude to the strong penguin amplitude, deviates from the SM expectation value: for example, $r_{\text{EW}} = 0.29 \pm 0.13$ whereas the SM expects $r_{\text{EW,SM}} = 0.14 \pm 0.04$. However, even under the presence of new physics, the $\gamma$ that we measure in the $B \rightarrow K\pi$ decays is the SM $\gamma$ through the re-parametrization invariance [12] unless new physics comes into the tree diagram.

The isospin relations imply two isospin quadrangles in a complex plane. In order to extract $\gamma$ from the two isospin quadrangles, it is crucial to fix the two isospin quadrangles in a common complex plane using current experimental data. Belle, BABAR and CLEO collaborations have measured branching ratios (BRs) and direct CP asymmetries for $B \rightarrow K\pi$ decays accurately as shown in Table I. Using these data, the length of each side of the two quadrangles could be determined within $(1-5)\%$ error through following definitions,

$$B_{ij} \propto \tau_{B^{+},0} \frac{|A_{ij}|^2 + |\bar{A}_{ij}|^2}{2}, \quad A_{CP}^{ij} \equiv -\frac{|A_{ij}|^2}{|A_{ij}|^2 + |\bar{A}_{ij}|^2},$$

where $A^{ij}$ and $\bar{A}^{ij}$ denote the decay amplitudes of $B \rightarrow K^i\pi^j$ and its charge conjugate mode, respectively.

### II. EXTRACTING $\gamma$ USING ISOSPIN RELATION.

For a moment, we digress to explain the triangle analysis [7,13] on determination of the CKM angle $\gamma$. Applying the quark diagram approach [5,14] to the $K^0\pi^+$ decay, the
decay amplitude is parameterized by 

\[ A(B^+ \rightarrow K^0\pi^+) = \mathcal{P}_{tc} + (\mathcal{P}_{uc} + \mathcal{A})e^{i\gamma}. \]

One can roughly estimate that the CP violating terms are very small since 

\[ |\mathcal{P}_{uc}/\mathcal{P}_{tc}| \sim |\mathcal{A}/\mathcal{P}_{tc}| \sim \mathcal{O}(\lambda^3) \]

For the illustration, at first we neglect these small CP violating terms, and we include them later on. Then approximately, 

\[ A(B^+ \rightarrow K^0\pi^+) \simeq A(B^- \rightarrow \bar{K}^0\pi^-) \simeq \mathcal{P}_{tc}. \]

Similarly, the decay amplitudes of \( B^0 \rightarrow K^+\pi^- \) and its CP conjugate mode can be parameterized as

\[ A(B^0 \rightarrow K^+\pi^-) = -\mathcal{P}_{tc} - T e^{i\gamma}, \quad A(\bar{B}^0 \rightarrow K^-\pi^+) = -\mathcal{P}_{tc} - T e^{-i\gamma}, \quad (4) \]

where the small color-suppressed EW penguin contributions are neglected, as it has been done in Ref. \[16\]. However, we will include the contributions of color-suppressed EW penguin as well as \((\mathcal{P}_{uc} + \mathcal{A})/\mathcal{P}_{tc}\) on extracting \(\gamma\) in the last part of the analysis. As will be seen, it turns out that these contributions are minor ones. Using Eq. \[11\], one can draw two triangles in a complex plane with the common side \(|\mathcal{P}_{tc}|\). Note that this can be done with a four-fold ambiguity as shown in Fig. \[13\]. Since we obtain \(|\mathcal{P}_{tc}|, |A(B^0 \rightarrow K^+\pi^-)|\) and \(|A(\bar{B}^0 \rightarrow K^-\pi^+)|\) from the current experimental data, once \(|T|\) is given, one can fix the two triangles with a four-fold ambiguity. Then the CKM angle \(\gamma\) can be extracted as illustrated in Fig. \[11\]. Fleischer suggested to make use of flavor SU(3) symmetry in order to estimate \(|T|\) from the branching ratio of \( B \rightarrow \pi^+\pi^0 \) decay \[7\]. Also, an alternative way to estimate \(|T|\) was proposed by using the factorization hypothesis \[8\]. However, it is obvious that both of them would not be used for a precise measurement of \(\gamma\), due to the unpredictable theoretical uncertainties related to the determination on the value of \(|T|\). Note that, even with such a theoretical deficiency, this triangle relations look quite useful and robust due to the very unlikely new physics contributions to \(T\) and \(\mathcal{P}_{tc}\). In our new proposal, we do not bring \(|T|\) into play and, instead, make use of the isospin relations \[11\] and \[2\] in order to fix the two triangles in Fig. \[11\].

It should be noted that in general four sides and one relative angle between two adjacent sides of a quadrangle determine the shape of the quadrangle with possible discrete ambiguities. Thus, in order to fix the two quadrangles in a common complex plane, besides the length of each side of the quadrangles, one needs at least two additional pieces of information: each on an angle of the individual isospin quadrangle. The mixing induced CP asymmetry in \( B \rightarrow K_S\pi^0 \) (denoted by \( S_{K_S\pi^0} \)) plays an crucial role in our analysis, because it provides a piece of information on the angle between two sides \( A(B^0 \rightarrow K^0\pi^0)\) and \( A(\bar{B}^0 \rightarrow \bar{K}^0\pi^0)\).
FIG. 1: A four-fold ambiguity arising in constructing two triangles from Eq. (4).

FIG. 2: Two isospin quadrangles of $B \to K\pi$ decays. Here two triangles $\triangle QGR$ and $\triangle QGR'$ in the case (A) of Fig. 1 are attached to these isospin quadrangles.

The two quadrangles can be put together with the common side $|A^{0+}|$ in a complex plane as shown in Fig. 2. Notice that two triangles $\triangle QGR$ and $\triangle QGR'$ in Fig. 1 are attached to two isospin quadrangles in Fig. 2, which shows the case (A) of Fig. 1 as an example. As the required two additional information, the angle $\theta$ and the requirement $GR = GR'$ will fix the two isospin quadrangles in the complex plane with a certain discrete ambiguity. Then, the CKM angle $\gamma$ can be extracted from these two quadrangles. In other word, one can find the value of $\gamma$ as a function of $\theta$ and extract $\gamma$ from the $\theta$ value given above.

We now relate the angle $\theta$ to the ratio of the decay amplitudes as

$$\theta \equiv (\text{sign}) \angle SOS' = \arg \left( \frac{A^{00}}{A^{0+}} \right),$$

where the sign is “plus” for the case of $\overline{OS'}$ upper than $\overline{OS}$ and “minus” for the opposite case. The crucial point is that $\theta$ can be obtained from the mixing induced CP asymmetry.
FIG. 3: A four-fold ambiguity arising in constructing two isospin quadrangles of $B \rightarrow K\pi$ decays, which corresponds to the case (A) of Fig. 1.

$S_{KS\pi^0}$ of $B^0 \rightarrow K_S\pi^0$ decay. It can be easily seen from the expression of $S_{KS\pi^0}$:

$$S_{KS\pi^0} = -\frac{2|A^{00}|}{|A^{00}|^2 + |A^{00}|^2} \text{Im}(e^{i\theta} e^{-2i\beta}).$$  \hfill (6)

We use $2\beta = 42.7^\circ \pm 2.0^\circ$ [10] that is averaged over the mixing induced CP asymmetries of $b \rightarrow c\bar{c}s$ processes. Thanks to the recent BABAR measurement, the error of $S_{KS\pi^0}$ is a bit reduced, resulting in $S_{KS\pi^0} = 0.38 \pm 0.19$ [10]. Using this experimental result, we determine the value of $\theta$ from Eq. (6) as $\theta = 20^\circ \pm 12^\circ$ or $-115^\circ \pm 12^\circ$.

Next, let us discuss the discrete ambiguity involved in extracting $\gamma$. We recall that a four-fold ambiguity is involved in constructing $\triangle QGR$ and $\triangle QGR'$ as shown in Fig. 1. For two triangles fixed as in the case (A) of Fig. 1, a two-fold ambiguity arises when one constructs each isospin quadrangle (attached to each triangle), depending on whether the position of the remaining apex $S(S')$ is upper than or lower than $OR\overline{OR'}$. Thus, another four-fold ambiguity arises when the two isospin quadrangles are constructed for the two fixed triangles. In Fig. 3 this four-fold ambiguity is depicted as (A1), · · ·, (A4) for the case (A). Therefore, there appears a sixteen-fold ambiguity in total, which is called (A1), (A2), · · ·, (D4). Accordingly, there are sixteen distinct $\gamma$’s as a function of $\theta$.

Each plot of $\gamma$ versus $\theta$ is shown in Fig. 4. As this figure displays, one can extract the possible values of $\gamma$ using the value of $\theta$. The result within $1 \sigma$ is

$$A1: \quad \gamma = (71^{+4}_{-4})^\circ, $$
where the first errors are due to the error of $S_{K_S\pi^0}$, while the second errors come from the combined error of all branching ratios and direct CP asymmetries in quadrature. Here we have discarded the region where $|T/P_{tc}| \geq 1$, which is much more conservative compared to the SM estimation of $0.18 \pm 0.11$ for QCDF [17] and $0.15 \pm 0.12$ for PQCD [18] within $2\sigma$ range. We note that the $\gamma$ solutions for the $B_2$ and $C_2$ case are very sensitive to every experimental error. Consequently, the estimation of $\gamma$ for the region greater than 90° is poor in our method. Combining above all the results,

$$\gamma = (71^{+5}_{-13} \pm 5)\degree,$$

$$B_2 \text{ and } C_2 : \gamma = (109^{+57}_{-0} \pm 6)\degree,$$

$$B_3 : \gamma = (119^{+2}_{-3} \pm 2)\degree,$$

(7)

where the values in brackets show $2\sigma$ result. It should be emphasized that, as can be seen from Fig. 4, more accurate measurement of $S_{K_S\pi^0}$ as well as branching ratios and direct CP asymmetries will lead to more precise estimation of $\gamma$ especially for the region less than 90°.

FIG. 4: Plot of $\gamma$ versus $\theta$ for sixteen distinct cases. Each curve is obtained from the central values of branching ratios and direct CP asymmetries. The extracted values of $\theta$ is represented by the shaded region. The dashed lines imply the region where $|T/P_{tc}| \geq 1$. 

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One can find the maximum value of $\gamma$ for the cases (A) and (D): $\gamma \leq 71^\circ$, and the minimum value of $\gamma$ for the cases (B) and (C): $\gamma \geq 109^\circ$, from Fig. 4. These bounds on $\gamma$ are sharply consistent with the Fleischer-Mannel bound $\gamma_0$, $0^\circ < \gamma < \gamma_0$ $\bigvee 180^\circ - \gamma_0 < \gamma < 180^\circ$, where $\gamma_0$ can be obtained to be $72^\circ$.

As we mentioned, the result of Eq.(8) is obtained with neglecting the small contributions, $(P_{uc} + A)/P_{tc} \equiv \varepsilon a e^{i\delta_a}$ and the ratio of the color-suppressed EW penguin to the strong penguin $P_{EW}^C/P_{tc} \equiv \varepsilon C e^{i\delta_C}$. Now we include these contribution, using the theoretical estimation in the framework of QCD Factorization. The authors of Ref. 17 calculated that $\varepsilon_a = 0.02 \pm 0.004$, $\phi_a = 13.6^\circ \pm 4.4^\circ$, $\varepsilon_C = 0.017 \pm 0.011$, $\phi_a = -67.7^\circ \pm 49.7^\circ$. We scan these four parameters within 1$\sigma$ variation of the estimation. And we followed the method that we explained above in order to get the solution of $\gamma$ under the consideration of each scanned parameter values. Then we find the variation of $\gamma$ values according to the variation of the four parameters. Since $A_{CP}(K^0\pi^+) \approx 2\varepsilon_a \sin \gamma \sin \phi_a$, the current experimental data for $A_{CP}(K^0\pi^+)$ strongly constrain the parameter region of $\varepsilon_a$ and $\phi_a$. We discard the parameter values that go beyond this constraint. Then, following result is obtained:

$$\gamma = (70^{+5+1+2}_{-14-1-3})^\circ \ [0^\circ < \gamma < 80^\circ] \ or \ 106^\circ < \gamma < 180^\circ \ [104^\circ < \gamma < 180^\circ],$$

(9)

where the values in brackets show 2$\sigma$ result. The first error is due to the experimental errors mainly caused by mixing-induced CP asymmetry $S_{K_S\pi^0}$. The second and third errors come from the theoretical uncertainty of $\varepsilon_a e^{i\delta_a}$ and $\varepsilon C e^{i\delta_C}$, respectively. Comparing the central value and the errors with Eq.(8), we can see that the small parameters $\varepsilon_a e^{i\delta_a}$ and $\varepsilon C e^{i\delta_C}$ are not significant for extracting $\gamma$ in this method.

III. CONCLUSION.

We have presented a new method for extracting the CKM phase $\gamma$ using the isospin analysis in $B \to K\pi$ decays. In this method, flavor SU(3) symmetry is not employed so that the hadronic uncertainty arising from the SU(3) breaking effect does not spoil the method. Neither we adopt any Dalitz-plot analysis, which may involve multiple strong phases and large final state interactions. Since we utilize only the isospin relations in $B \to K\pi$ decays, this method will work well, regardless of possible new physics effects unless the isospin relations do not hold. Using the current data for $B \to K\pi$ including $S_{K_S\pi^0}$, we have found that
\( \gamma = (70^{+5+1^{+2}}_{-14-1^{-3}})^{\circ} \text{ or } 106^\circ < \gamma < 180^\circ \text{ at } 1\sigma \text{ level, where the first error is due to the experimental errors, and the second and the third errors come from the theoretical uncertainty of the CP violating terms in } B^+ \rightarrow K^0\pi^+ \text{ and color-suppressed EW penguin term, respectively.} \)

**ACKNOWLEDGMENTS**

The work of C.S.K. was supported in part by CHEP-SRC and in part by the KRF Grant funded by the Korean Government (MOEHRD) No. KRF-2005-070-C00030. The work of S.O. was supported by the Second Stage of Brain Korea 21 Project. The work of Y.W.Y. was supported by the KRF Grant funded by the Korean Government (MOEHRD) No. KRF-2005-070-C00030.

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