Abstract

We revisit and extend previous work on neutrino mediated long range forces in a background at finite temperature. For Dirac neutrinos, we correct existing results. We also give new results concerning spin-independent as well as spin-dependent long range forces associated to Majorana neutrinos. An interesting outcome of the investigation is that, for both types of neutrinos whether massless or not, the effect of the relic neutrino heat bath is to convert those forces into attractive ones in the supra-millimeter scale while they stay repulsive within the sub-millimeter scale.
Neutrinos mediate long-range forces between macroscopic bodies [1], [2], [3], [4], [5]. Indeed double neutrino exchange among matter fermions generates spin-independent forces that extend coherently over macroscopic distances. The effect, however, is extremely weak, much too tiny to be experimentally detected with present day technology. Compared to their gravitational pull, the force between two nucleons 1 cm apart is about $10^{-28}$ times weaker. Not only their coupling strength is very small but also their decay with distance is fast. Indeed the potential drops as $r^{-5}$ so that the effects die off correspondingly. Phenomenological surveys on forces with this particular distance behaviour have been conducted in the literature (see e.g.[6]) over the whole span of distances from astronomical down to the micron scale. If at all, this forces will induce physical effects in the sub-millimeter (but macroscopic) end of the distance scale. Perhaps an exception to this is the case of a system with high density of matter such as the core of a neutron star where collective effects may show up [5], [7].

In a neutrino populated medium, such as the cosmic neutrino background or the hot core of a supernova, the helicity flip produced by single neutrino exchange can be balanced by the neutrinos in the medium and, as a consequence, a spin-independent interaction takes place that leads to a coherent effect over many particles in bulk matter.

The neutrino long-range forces in the presence of a neutrino thermal bath have been explored in reference [8] in the Dirac neutrino case. The long range forces mediated by Majorana neutrinos, on the other hand, have been studied only in the zero temperature case [4]. Here we wish to extend the nonzero temperature results to the Majorana case. Because the distinction between Dirac and Majorana neutrinos is superfluous for massless neutrinos, we shall consider the general $m \neq 0$ case.

We shall adopt the notation in [8] and write,

$$V(r) = -\int \frac{d^3Q}{(2\pi)^3} \exp(iQ \cdot r)T(Q)$$

where $T(Q)$ is the nucleon-nucleon elastic scattering amplitude (figure 1) in the static limit, i.e. momentum transfer $Q \simeq (0, Q)$, where matter is supposed to be at rest in the microwave background radiation (MWBR) frame. It can be cast in the form

$$T(Q) = -2iG_F^2 (g_V, -2g_AS)^\mu (g'_V, -2g'_AS')^\nu I_{\mu\nu}$$

with

$$I_{\mu\nu} = \int \frac{d^4k}{(2\pi)^4} Tr[\gamma_\mu O_iS_T(k)\gamma_\nu O_iS_T(k-Q)].$$
The operator $O$ is the left-handed projector $\frac{1}{2}(1-\gamma_5)$ for Dirac neutrinos and $\frac{\sqrt{2}}{2}\gamma_5$ for Majorana neutrinos. The temperature dependent propagator $S_T$ has the explicit form

$$S_T(k) = (k + m) \left[ (k^2 - m^2 + i\epsilon)^{-1} + 2\pi i \delta(k^2 - m^2)(\theta(k^0)n_+ + \theta(-k^0)n_-) \right]$$

where $n_+$ and $n_-$ are Fermi-Dirac distribution functions for particle and antiparticle, respectively. As discussed in [8], figure 1 evaluated with this propagator taken together with the usual Feynman rules is sufficient to calculate the potential. In equation (2), $g_{V,A}$ are composition-dependent weak vector and axial-vector couplings. We focus first on the spin-independent potential, that is the $g_Vg'_V$ component of equation (2).

Use of the first piece in equation (4) gives the zero temperature vacuum results [4], [5],

$$V_{\text{Dirac}}(r) = \frac{G_F^2 m^3 g_V g'_V}{4\pi^3 r^2} K_3(2mr)$$

and

$$V_{\text{Majorana}}(r) = \frac{G_F^2 m^2 g_V g'_V}{2\pi^3 r^3} K_2(2mr)$$

in terms of the modified Bessel functions $K_{2,3}$.

At very large distances ($mr \gg 1$), i.e. much larger than the Compton wavelength of the neutrino, these potentials exhibit the asymptotic behaviour

$$V_{\text{Dirac}}(r) \sim \frac{G_F^2 g_V g'_V}{8} \left( \frac{m}{\pi r} \right)^{5/2} e^{-2mr}$$

and

$$V_{\text{Majorana}}(r) \sim \frac{G_F^2 g_V g'_V}{4} \left( \frac{m^3}{\pi^5 r^7} \right)^{1/2} e^{-2mr}.$$ 

Of course, both potentials (equations (5) and (6)) coincide when $m = 0$. They give the well
known Feinberg and Sucher result:

\[ V(r) = \frac{G_F^2 g_V g'_V}{4\pi^3 r^5}. \] (9)

In a neutrino background, a contribution to the long range force can arise because a neutrino in the thermal bath may be excited and de-excited back to its original state in the course of the double scattering process. This effect is described by the crossed terms contained in \( I_{\mu\nu} \) that involve the thermal piece of one neutrino propagator along with the vacuum piece of the other neutrino propagator. This thermal component of the tensor \( I_{\mu\nu} \) can be written as

\[
I_{\mu\nu}^{T,D} = -\pi i \int \frac{d^4k}{(2\pi)^4} \delta(k^2 - m^2) \left[ \theta(k^0) n_+ + \theta(-k^0) n_- \right] \\
\times \left[ Tr\left[ \gamma^\mu (\frac{k + Q}{k}) \gamma^\nu (\frac{k}{k}) \right] \frac{1}{(k + Q)^2 - m^2 + i\epsilon} + Tr\left[ \gamma^\mu (\frac{k}{k} - \frac{Q}{k}) \gamma^\nu (\frac{k - Q}{k}) \right] \frac{1}{(k - Q)^2 - m^2 + i\epsilon} \right] \] (10)

in the Dirac case, and

\[
I_{\mu\nu}^{T,M} = -\pi i \int \frac{d^4k}{(2\pi)^4} \delta(k^2 - m^2) n \left[ Tr\left[ \gamma^\mu (\frac{k + Q + m}{k + Q}) \gamma^\nu (\frac{k^0}{k}) \right] \frac{1}{(k + Q)^2 - m^2 + i\epsilon} + Tr\left[ \gamma^\mu (\frac{k + m}{k} - \frac{Q}{k}) \gamma^\nu (\frac{k - Q - m}{k}) \right] \frac{1}{(k - Q)^2 - m^2 + i\epsilon} \right] \] (11)

for Majorana neutrinos, where in this latter case we put \( n_+ = n_- = n \) since the chemical potential vanishes. Note that in (10) there is no component proportional to \( \epsilon^{\mu\nu\alpha\beta} \) since after the integration over \( k \) the only four-vector available is \( Q^\alpha \). As a result there will be no parity violating potentials.

Far from degeneracy (i.e. for chemical potential \( \mu \ll T \)), as is probably the case for cosmological neutrinos, we can consider the neutrinos to be Boltzmann distributed, that is we take

\[ n_\pm = \exp[(\pm\mu - |k^0|)/T]. \] (12)

With this approximation, the integrations involved in the calculation of potentials can be easily done by conveniently choosing the order in which they are performed. The results can be expressed again in terms of Bessel functions and are as follows:

\[
V^{Dirac}_T(r) = -\frac{G_F^2 m^4 g_V g'_V}{\pi^3 r} \cosh (\mu/T) \left[ \frac{K_1(\rho)}{\rho} + \frac{4K_2(\rho)}{\rho^2} \right] \] (13)

and

\[
V^{Majorana}_T(r) = -\frac{4G_F^2 m^4 g_V g'_V}{\pi^3 r} \frac{K_2(\rho)}{\rho^2} \] (14)
where we have defined

\[ \rho \equiv \frac{m}{T} \sqrt{1 + (2rT)^2}. \]  

(15)

For massless neutrinos (and \( \mu = 0 \)) both potentials collapse to

\[ V_T(r) = -\frac{8G_F^2m^3g_Vg'_V}{\pi^3r^4} \frac{1}{\rho^4} \]  

(16)

which is the result given in reference [8]. Because the neutrino background temperature is \( T \sim (1.2\text{mm})^{-1} \), we see that for distances much larger than 1 mm (i.e. \( rT \gg 1 \)) the potential in equation (16) reads

\[ V_T(r) \approx -\frac{G_F^2g_Vg'_V}{2\pi^3r^5}. \]  

(17)

When added to the vacuum result (9), the total potential is

\[ V_{\text{tot}}(r) \approx -\frac{G_F^2g_Vg'_V T^4}{\pi^3r^5} \]  

(18)

that is, in the presence of the cosmic neutrino background the original Feinberg-Sucher force switches sign, i.e. a repulsive force turns into an attractive one. On the other hand, well within the sub-millimeter domain (\( rT \ll 1 \)), the temperature dependent potential (16) behaves as follows

\[ V_T(r) \approx -\frac{8G_F^2g_Vg'_V T^4}{\pi^3r^5} \]  

(19)

which is negligible compared to the vacuum contribution in equation (9).

In the general \( m \neq 0 \) case, we shall study the Dirac and Majorana potentials, equations (13) and (14) respectively, in various physically interesting limits. Consider first the cases where \( r \ll m^{-1} \ll T^{-1} \) or \( m^{-1} \ll r \ll T^{-1} \). Performing the relevant expansions of the Bessel functions in (13) and (14) leads to

\[ V_T^{\text{Dirac}}(r) \approx -\frac{G_F^2m^{5/2}g_Vg'_V}{2^{1/2}\pi^{5/2}r} T^{3/2} \cosh (\mu/T) e^{-m/T} \]  

(20)

and

\[ V_T^{\text{Majorana}}(r) \approx -\frac{2^{3/2}G_F^2m^{3/2}g_Vg'_V}{\pi^{5/2}r} T^{5/2} e^{-m/T}. \]  

(21)

Hence, thermal effects are exponentially damped in both distance domains.

A different behaviour is obtained for distances much larger than any inverse energy scale in the problem, i.e. for \( r \gg T^{-1} \gg m^{-1} \) or \( r \gg m^{-1} \gg T^{-1} \). Indeed, now we have

\[ V_T^{\text{Dirac}}(r) \approx -\frac{G_F^2g_Vg'_V}{4} \left( \frac{m}{\pi r} \right)^{5/2} \cosh (\mu/T) e^{-2mr} \]  

(22)

and

\[ V_T^{\text{Majorana}}(r) \approx -\frac{G_F^2g_Vg'_V}{2} \left( \frac{m^3}{\pi^5 r^7} \right)^{1/2} e^{-2mr}. \]  

(23)
Both expressions exhibit the characteristic Yukawa exponential damping associated to two-particle exchange. These results when added to their vacuum counterparts, equations (7) and (8), produce the inversion phenomenon already noticed in the massless case. At asymptotically large distances the resulting potential is equal in strength as it would be in vacuum but, contrary to what happens in vacuum, it is attractive instead.

There is no exponential suppression only when $r \ll T^{-1} \ll m^{-1}$ or $T^{-1} \ll r \ll m^{-1}$, where one essentially recovers the massless cases, equations (19) or (17), respectively. Indeed, for Majorana neutrinos one gets these equations as they stand, and for Dirac neutrinos both equations should be multiplied by the factor $\cosh(\mu/T)$ for non-zero chemical potential.

Let us note that the results given in equations (20) and (22) for the $m \neq 0$ Dirac case disagree with the corresponding results given in reference [8]. Indeed, their formulae do not show the Boltzmann or Yukawa suppression factors that enter the asymptotic expansions of the Bessel functions and which are bound to be there on physical grounds. For the sake of an explicit comparison we provide the reader, in the appendix at the end of the paper, with some details of the calculation.

Up to this point all calculations refer to spin-independent potentials, those that can coherently add over macroscopic samples of unpolarized matter. Let us, for the sake of completeness, consider briefly the question of potentials that depend on spin. Now we should focus on the spatial indices of the tensor $I_{\mu\nu}$ appearing in the scattering amplitude in equation (2). The Fourier transformation (1) is in this case somewhat more involved than before because the amplitude will depend on the components of the 3-momentum transfer. Nevertheless they can be easily performed and we get:

$$V_T^{\text{spin}}(r) = -\frac{4G_F^2m^4g_Ag'_A}{\pi^3r} \left[ (\mathbf{S} \cdot \mathbf{S'}) F(r) + 2\frac{(\mathbf{S} \cdot \mathbf{r})(\mathbf{S'} \cdot \mathbf{r})}{r^2} G(r) \right] \cosh(\mu/T) \tag{24}$$

where

$$F(r) \equiv a\frac{K_1(\rho)}{\rho} + 2\frac{K_2(\rho)}{\rho^2} - 8m^2r^2\frac{K_3(\rho)}{\rho^3} \tag{25}$$

and

$$G(r) \equiv 7\frac{K_2(\rho)}{\rho^2} - 4m^2r^2\frac{K_3(\rho)}{\rho^3} \tag{26}$$

with $a = 1$ for Dirac neutrinos and $a = 2$ for Majorana neutrinos and $S^2 = 3/4$. Of course, in the Majorana case we must put $\mu = 0$.

Both cases above, i.e. Dirac and Majorana, lead to the potential

$$V_T^{\text{spin}}(r) = -\frac{16G_F^2g_Ag'_A T^4}{\pi^3(1+4r^2T^2)^3} \left[ (\mathbf{S} \cdot \mathbf{S'}) (1 - 12r^2T^2) + \frac{(\mathbf{S} \cdot \mathbf{r})(\mathbf{S'} \cdot \mathbf{r})}{r^2} (7 + 12r^2T^2) \right] \tag{27}$$
for $m = 0$ and $\mu = 0$. This result, eq. (27), should be then added to the vacuum result [1]

$$V^{\text{spin}}(r) = \frac{G_{F}^{2} g_{V} g_{V}'}{2\pi^{3} r^{5}} \left[ \frac{(S \cdot r)(S' \cdot r)}{r^{2}} - 3(S \cdot S') \right].$$  (28)

The various regimes explored before can be studied also for the spin-dependent forces. The discussion involves the various asymptotic forms of the same modified Bessel functions and will lead to the same exponential damping whenever the temperature or the mass is the relevant energy parameter. Since these forces will be even more difficult to detect than the spin-independent ones, for they do not add up coherently in bulk matter, we do not bother here to display the explicit form for the different limits.

We end this paper with a short summary. Double neutrino exchange mediates (extremely feeble) long range forces. In vacuum these forces have been known (at least for Dirac neutrinos) for quite some time. Recently, it has been realised that a neutrino background will also induce long range interactions among bulk matter. The results were given for Dirac neutrinos. We have extended the work of Horowitz and Pantaleone [3] to include the case of Majorana neutrinos and, furthermore, we have derived the exact form of the potentials in either case, i.e. Dirac and Majorana, and explored physically relevant distance and energy scales. In so doing we have found important discrepancies with previous work ($m \neq 0$, Dirac case). Since matter is embedded in the cosmic neutrino background, a consequence of our analysis is that the forces are repulsive in the sub-millimeter scale and attractive for distances well beyond $1 \text{mm}$ for any kind of neutrino (massless or not). In fact, on the small scale the vacuum result (Feinberg and Sucher) dominates whereas on the larger scale the relic neutrino background is responsible for the dominant effect. This means that by experimentally detecting (admittedly a highly improbable event for laboratory experiments) such forces in both different regimes one would, not only establish these neutrino interactions, but one would in addition detect the relic neutrino background. Actually it is the neutrino background temperature ($T^{-1} \sim 1.2 \text{mm}$) which sets this $1 \text{mm}$ distance scale. Incidentally, the sub-millimeter scale has been subject recently of renewed theoretical as well as experimental interest [4]. For an experimental point of view of the actual possible detection of very weak long range forces we refer the reader to references [5].

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Appendix

We give here some details of the calculation for the massive Dirac case. All integrals used can be found in [11]. Starting with eq. (1) we have (in the formulae below $Q \equiv |Q|$):

$$V_{\text{Dirac}}^T(r) = \frac{iG^2}{\pi^2 r} \int_0^\infty dQ Q I_0(Q) \sin(Qr)$$

$$= \frac{G^2}{2\pi^4 r} \cosh(\mu/T) \int_0^\infty \frac{dk^2}{\sqrt{k^2 + m^2}} \exp\left(-\sqrt{k^2 + m^2}/T\right)$$

$$\times \int_{-1}^1 dz \left[(2kz)^2 - 2m^2 - 4k^2\right] \int_0^\infty dQ \frac{Q \sin(Qr)}{Q^2 - (2kz)^2}$$

(A.1)

Performing first the integration over $Q$, then over $z$, we obtain:

$$V_{\text{Dirac}}^T(r) = \frac{G^2 g_v g'_v}{2\pi^3 r^4} \cosh(\mu/T) \left[-(1 + (mr)^2) I(m, r, T) + r \frac{dI(m, r, T)}{dr}\right]$$

(A.2)

where

$$I(m, r, T) = \int_0^\infty \frac{dk}{\sqrt{k^2 + m^2}} \exp\left(-\sqrt{k^2 + m^2}/T\right) \sin(2kr)$$

$$= \frac{2rTm}{\sqrt{1 + (2rt)^2}} K_1\left(\frac{mT}{2} \sqrt{1 + (2rt)^2}\right)$$

(A.3)

Inserting (A.3) in (A.2) gives eq.(13).

For the spin-dependent part we decompose

$$T(Q) = (S \cdot Q)(S' \cdot Q) t_1(Q) + (S \cdot S') t_2(Q)$$

(A.4)

where the functions $t_1(Q)$ and $t_2(Q)$ depend only on $|Q|$ and we perform first the angular $\hat{Q}$ integration in eq.(1). The rest of the calculation goes along similar lines as in the spin-independent part above.

References

[1] G. Feinberg and J. Sucher, Phys. Rev. A166 (1968) 1638; G. Feinberg, J. Sucher and C.-K. Au, Phys. Rep. 180 (1989) 83

[2] S. D. H. Hsu and P. Sikivie, Phys. Rev. D49 (1994) 4951
[3] R. P. Feynman, F. B. Morinigo and W. G. Wagner, *Feynman Lectures on Gravitation*, Addison-Wesley, Readings, MA 1995

[4] J. A. Grifols, E. Masso and R., Toldra, Phys. Lett. B389 (1996) 363

[5] E. Fischbach, Ann. Phys. (N.Y.) 247 (1996) 213

[6] J. A. Grifols and S. Tortosa, Phys. Lett. B328 (1994) 98; see also F. Ferrer and J. A. Grifols, ‘Long Range Forces from Pseudoscalar Exchange’, UAB-FT-447, hep-ph/9805477

[7] A. Abada, M. B. Gavela and O. Pene, Phys. Lett. B387 (1996) 315; A. Y. Smirnov and F. Vissani, hep-ph/9604443

[8] C. J. Horowitz and J. Pantaleone, Phys. Lett. B319 (1993) 186

[9] S. Dimopoulos, M. Dine, S. Raby and S. Thomas, Phys. Rev. Lett. 76 (1998) 70; I. Antoniadis, S. Dimopouls and G. Dvali, Nucl. Phys. B516 (1998) 70

[10] E. G. Adelberger, B. R. Heckel, C. W. Stubbs and W. F. Rogers, Annu. Rev. Nucl. Part. Sci. 41 (1991) 269; C. I. Sukenik, M. G. Boshier, D. Cho, V. Sandoghdar and E. A. Hinds, Phys. Rev. Lett. 70 (1993) 560; J. C. Price in International Symposium on Experimental Gravitational Physics, ed. P. F. Michelson, H. Enke and G. Pizzella, World Scientific, 1987

[11] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, corrected and enlarged edition, Academic Press, INC 1980