AC Resistance of Driven Vortices of Superconductors Measured by Microwave Technique

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Abstract. To investigate the dynamics of driven vortices in superconductors, microwave resonance characteristics were measured under dc driving force in Nb. DC driving current was supplied to the resonator, and the current dependence of the resonance spectra was investigated in the vortex state. When dc driving current was increased across the critical current, a parabolic dependence of the center frequency on dc current as well as an increase of the inverse of the quality factor, $Q^{-1}$, were observed. In addition, the ac resistance resulting from the motion of vortices showed an increase around the dc critical current, and the increase depended on frequency. This result suggests that the pinning force and the resultant depinning of the vortices is frequency dependent even near the crossover frequency from the low-frequency reactive response to the high-frequency dissipative response.

1. Introduction

Dynamic pinning force, $F_p$, acting on driven vortices of superconductors, namely, the dissipation due to the pinning in the moving state, has attracted a lot of attention not only as a source of the dissipation in the moving vortices, but also as a model system for kinetic friction at the solid-solid interface[1]. However, the microscopic process resulting in $F_p$ has not been understood yet. The understanding of $F_p$ is challenging, since it treats the nonequilibrium systems with nonlinearity and randomness. Thus, the elucidation of the origin of $F_p$ is important not only for vortex physics, but also for fundamental understandings of deformable driven systems with nonlinearity and randomness.

The concept of $F_p$ was first introduced by Kim in the equation of force balance for flux flow[2], which is expressed as

$$F_v + F_p = F_d,$$

where $F_v$ is the viscous drag force, and $F_d$ is the driving force. Several theoretical attempts were made to explain the origin of $F_p$. For instance, $F_p$ was calculated by treating pinning force as a perturbation to the equation of motion[3]. Later, applying field theoretical method[4], Grundberg and Rammer calculated $F_p$ both analytically and numerically[5]. However, in these treatments, $F_p$ was not derived from the microscopic motion of each vortices. On the other hand, Yamafuji and Irie attributed $F_p$ to the relaxation of the elastic deformation of the vortex lattice[6,7]. On the basis of that theory, Lowell tried to explain the velocity dependence of $F_p$, but the theory was limited to the large velocity limit, and could not explain experimental results in a unified manner[8]. Despite those theoretical challenges, to our knowledge, no experimental studies have examined the microscopic process of $F_p$.

Thus, we planned to reveal the microscopic dissipation mechanism resulting in $F_p$ experimentally.
To investigate the dynamic properties of the moving state, we measured the ac resistance of driven vortices in the Nb films around the crossover frequency, $f_{cr}$, from the reactive response to the dissipative response[9]. As a result, we observed the increase of the ac resistance when the vortices are dc driven, which was found to be frequency dependent.

2. Methods

Microwave transmission line resonance method was used to measure the driving force dependence of the ac resistance of the vortex lattice. The resonators were made of Nb films, which were deposited on 5 mm $\times$ 5 mm $\times$ 0.5 mm MgO substrates by rf sputtering. Deposition rate was monitored by a quartz oscillator. The typical film thickness was approximately 100 nm, and $T_c$ of the film was 8.7 K, and the resistivity at 10 K was 7 $\mu$Ω cm. The upper critical field, $B_{c2}$, at 3 K was 15000 G and the lower critical field, $B_{c1}$, was estimated to be much less than 10 G, considering the demagnetization factor of the thin strip[10–12]. The dc critical current, $I_c$, at 3 K, 30 G was around 72 mA, and $f_{cr}$ of the Nb films was estimated to be 10–20 GHz[13].

To measure the center frequency, $f_0$, and the quality factor, $Q$, under dc driving current, coplanar waveguide $\lambda/2$ resonators were patterned on the Nb films by the photolithography technique (Figure 1). The patterned films were etched by reactive ion etching with CF$_4$. The width of the center strip was 10 $\mu$m and the gap between the center strip and the ground was 5 $\mu$m. The resonator is capacitively coupled to the external circuit through a 4 $\mu$m gap. The characteristic impedance of the resonators was about 50 $\Omega$. DC bias current was applied to the resonator through leads forming $\lambda/2$ filters[14]. These filters suppress the loss due to the coupling to the external circuit. Two resonators were fabricated, $f_0$ of which are 3.7 GHz and 7.8 GHz, and the quality factors were 5000 and 9000 at 3 K, 0 G, respectively.

The block diagram of the measurements is shown in figure 2. Magnetic field was applied perpendicular to the Nb film. To avoid hysteresis, current was increased to 75 mA after the field cooling. In order to suppress Joule heating, rectangular pulsed current was supplied to the resonator to drive vortices. By synchronizing the network analyzer with the function generator, the transmission power, $S_{21}$, was measured at the flat part of the pulsed current around $f_0$. The pulse width and the duty cycle were 25 ms and 0.1, respectively, which was limited by the measurement time of the network analyzer ($\sim$15 ms). The amplitude of the pulsed current was estimated by measuring the voltage drop at a 10 $\Omega$ resistance in series with the sample. The maximum current on the resonance, $I_{max}$, were roughly estimated by $I_{max} \approx (P_{circ}/2\pi f_0 d)^{1/2}$, following Bothner et al.[15]. Here, $P_{circ} = 4P_{in}(1 - r)Q/\pi$ is the rf power circulating in the resonator, $P_{in}$ ($P_{out}$) denotes the input (output) power at the resonator, and $r = (P_{out}/P_{in})^{1/2}$. Assuming the loss at the cables to be 5–10 dB, $P_{in}$ and $P_{out}$ was estimated to be $1 \times 10^{-7}$ W,
$1 \times 10^{-9}$ W, respectively. With $P_{\text{circ}}$ of $1.1 \times 10^{-3}$ W and the total line inductance, $L = 3.8 \times 10^{-8}$ H, $I_{\text{max}}$ was calculated to be $7.7 \times 10^{-5}$ A, which was two orders of magnitude smaller than $I_c$.

The measured resonance spectra were analyzed as follows. In the mixed state, several factors contribute to $Q^{-1}$ as

$$
\frac{1}{Q} = \frac{1}{Q_{\text{super}}} + \frac{1}{Q_{\text{vor}}} + \frac{1}{Q_{\text{die}}} + \frac{1}{Q_{\text{rad}}},
$$

where $Q_{\text{super}}^{-1}$, $Q_{\text{vor}}^{-1}$, $Q_{\text{die}}^{-1}$ and $Q_{\text{rad}}^{-1}$ are the superconductor loss, the vortices loss, the dielectric loss and the radiation loss, respectively. In a superconducting state, $Q^{-1}$ is known to be less than $10^{-4}$ at $4.2$ K[15], and $Q_{\text{vor}}^{-1}$ is absent for $B = 0$. $Q_{\text{die}}^{-1}$ of MgO is considered to be less than $10^{-4}$[16]. Additionally $Q_{\text{rad}}^{-1}$ is typically much less than $10^{-2}$ in the coplanar waveguide resonator undercoupled to the external circuit[17]. However, in this experiment, the extra leads supplying the dc bias current were attached to the resonators, leading to a finite coupling to the external circuit. Hence, $Q_{\text{rad}}^{-1}$ was considered to become larger than $10^{-4}$, and it mainly contributed to $Q^{-1}$ without magnetic field, which was $1.2 \times 10^{-5}$ at 7.8 GHz. When magnetic field was applied, the changes of $Q_{\text{die}}^{-1}$ and $Q_{\text{rad}}^{-1}$ are assumed to be small compared with that of $Q_{\text{vor}}^{-1}$. Since applied magnetic field was much less than $B_{c2}$, the change in $Q_{\text{super}}^{-1}$ was also neglected. Then, the changes in the ac resistance due to the vortex motion can be calculated as

$$
\Delta R(B, I) = \omega L \left( \frac{1}{Q_{\text{vor}}} - \frac{1}{Q} \right) (0 \text{mA, 0G}).
$$

In addition to $\Delta Q^{-1}(B, I)$, the total inductance $L$ was necessary to obtain $\Delta R$ by equation (3). $L = L_{\text{m}} + L_{\text{k}}$ is calculated as

$$
L_{\text{m}} = \frac{\mu_0 K(k')}{4} K(k), \quad L_{\text{k}} = \frac{\lambda^2}{2 \kappa^2} g(s, w, d),
$$

$$
g(s, w, d) = \frac{1}{2k^2 K(k)^2} \left[ - \ln \left( \frac{d}{4w} \right) - \frac{w}{w+2s} \ln \left( \frac{d}{4(w+2s)} \right) + \frac{2(w+s)}{w+2s} \ln \left( \frac{s}{w+s} \right) \right],
$$

where $L_{\text{m}}$ is the magnetic inductance of a coplanar waveguide, $L_{\text{k}}$ is the kinetic inductance of a superconductor, $K(k)$ is the complete elliptic integral of the first kind with the modulus $k = w/(w+2s)$, $k' = (1 - k^2)^{1/2}$, $\mu_0$ is the vacuum permeability, $d$ is the film thickness, $w$ is the width of the center strip, and $s$ is the gap between the ground and the center strip[18]. Penetration depth was estimated to be 96 nm by the following equation[19],

$$
\lambda = 1.05 (\rho(T_c)/T_c)^{1/2} \times 10^{-3} \text{ m}.
$$

Consequently, $L$ was obtained, thus the ac resistance due to vortices was calculated by equation (3).

3. Results and Discussion

Figure 3 shows the dc $I$-$V$ characteristics of the 7.8 GHz resonator measured at 3 K, 30 G by the four-probe method. From this data, $I_c$ was estimated to be 72 mA ± 2 mA. The uncertainty of $I_c$ comes from the voltage fluctuation due to the strong pinning at the low magnetic fields compared with $B_{c2}$. Indeed, the $I$-$V$ characteristics at low magnetic fields is known to show hysteresis as well as the abrupt voltage jump because of the pinning[20]. Figure 4 shows the resonance spectra of the 7.8 GHz resonator with increasing dc bias current. The shape and the position of the resonance spectra changed as the current increased from 2 mA to 75 mA. As the current amplitude became larger, $Q^{-1}$ increased, while $f_0$ continued decreasing. First, we focus on the decrease of $f_0$. Parabolic current dependence in $f_0$ , as shown in the inset of figure 5, was already observed by Gittlemen et al.[21], which was explained considering that the order parameter of the superconductor is suppressed by current. The solution of Ginzburg-Landau equation for sufficiently thin films is

$$
|\phi|^2 = \left( \frac{\lambda_0}{\lambda} \right)^2 \left[ 1 - \frac{8\pi^2 \lambda_0 d}{c^2 H_0^2} f_0(t) f^2 \right] = \left( \frac{\lambda_0}{\lambda} \right)^2 \left[ 1 - k^2 f^2 \right],
$$
The vortices are still reactive to some extent at 80 GHz resonator.

The dc response and the ac response is understood by remembering that vortices oscillate even for the uniform current density.

Assuming the uniform current density and $\sigma_2 \gg \sigma_1$, the inductance change can be written as

$$\Delta L \propto \sigma_2(j) - \sigma_2(0) \approx j^2.$$

Thus, from equations (7) and (10), the parabolic dependence of $f_0$ on $j$ can be understood in terms of the pair breaking effect by the dc driving current.

Next, we discuss the change in $Q^{-1}$. Using equation (4), the change in the ac resistance per unit length was estimated (figure 6), and two remarkable features were observed. First, the ac resistance decreased slightly in the low current regime. Although we do not understand the origin of this decrease, this may be due to the anharmonicity in the pinning potential. As the current density increases, vortices move from initial positions to different equilibrium positions where the pinning force and the driving force balance with each other[22]. The oscillation of vortices in the pinning potential can be described by the equation of motion, $\eta \dot{v} + kx = f_{\text{dc}} \Phi_0$, where $\eta$ is the viscous drag coefficient, $\Phi_0$ is the flux quantum. The pinning constant, $k$, is $k = \partial^2 U_p/\partial r^2$, where $U_p$ is the pinning potential[23]. If $U_p$ was harmonic everywhere, $k$ would be a constant and the resistance would be the same even when vortices change their position. The second remarkable feature is that the ac resistance increases gradually in the high current regime, which is in sharp contrast to the dc behavior shown in figure 3. The difference between the dc response and the ac response is understood by remembering that vortices oscillate even for the smaller current than $I_c$, leading to the finite resistance. As for the increase of the ac resistance with increasing the dc driving force, a qualitative explanation is as follows. Vortices suffer from the pinning effect around $f_{\text{cr}}$, which means that, the ac responses of the vortices are still reactive to some extent at these frequencies[9]. On the other hand, moving vortices are known to feel weaker pinning force than pinned ones[24]. Hence, if the effect of the pinning is reduced by the translational motion of the vortices, the ac response is expected to be more resistive (less reactive), leading to the increase of the ac resistance.

Figure 3. The dc $I$-$V$ characteristic of the 7.8 GHz resonator measured at 3 K, 30 G. The dotted line is given as a guide to the eye.

Figure 4. The resonance spectra of the 7.8 GHz resonator measured at 3 K, 30 G.
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Figures 5. The current dependence of $f_0$. The inset shows $I^2$ vs $f_0$ plot.

Finally, we discuss the frequency dependence. Figure 6 shows $\Delta R$ measured at 3.7 GHz and 7.8 GHz. $\Delta R$ of 7.8 GHz per unit length increases from 0.043 $\Omega$ cm$^{-1}$ at 53 mA to 0.051 $\Omega$ cm$^{-1}$ at 75 mA, while that of 3.7 GHz increases 0.002 $\Omega$ cm$^{-1}$ at most. We estimated the ratio of the change in $\Delta R$, ($\Delta R_{\text{max}} - \Delta R_{\text{min}})/\Delta R(2 \text{ mA})$, where $\Delta R_{\text{max}}$ ($\Delta R_{\text{min}}$) is the maximum (minimum) value of $\Delta R$. It was 8 % for 3.7 GHz and 16 % for 7.8 GHz, respectively. These differences suggest that the depinning process depends on frequency[9]. To obtain the further detailed picture, experiments at other frequencies are indispensable, which are now in progress.

Shklovskij and Dobrovolskiy numerically solved the equation of motion for a single vortex in the tilted washboard potential[25]. They calculated the current density dependence of the resistivity, $\rho$, at the normalized frequency $\Omega = 0.1$, which is one-tenth of $f_c$. Their calculation showed a sharp rise of the real part of $\rho$ at dc critical current density which was very different from a continuous increase observed in our experiment. Although there are several differences in the situations between their calculation and our experiment, such as the frequency, the ac current amplitude and the definition of the critical current density, the most significant difference seems to be the presence/the absence of the randomness in the pinning potential. Indeed, the periodic sinusoidal potential was assumed in their calculation, whereas the realistic pinning potential is with an extensive amount of disorder. More elaborate treatment is needed urgently even theoretically.

4. Conclusion

In conclusion, we investigated the microwave responses of the driven vortices with superconducting spectra, first, the parabolic current dependence of the center frequency was understood considering the current dependence of the order parameter. Second, the ac resistance decreased in the low current regime, while it increased around the dc critical current. The increase was understood by considering that driven vortices feel weaker pinning force than the pinned vortices. Finally, from the frequency dependence, the depinning process was found to depend on frequency even near the crossover frequency. To understand the microscopic origin of the dynamic pinning force, further studies including measurements of the ac response at other frequencies, voltage noise measurements and numerical simulations are needed, which are now in progress.
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