Three-Dimensional TIN Algorithm for Digital Terrain Modeling

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Abstract  The problem of taking an unorganized point cloud in 3D space and fitting a polyhedral surface to those points is both important and difficult. Aiming at increasing applications of full three dimensional digital terrain surface modeling, a new algorithm for the automatic generation of three dimensional triangulated irregular network from a point cloud is proposed. Based on the local topological consistency test, a combined algorithm of constrained 3D Delaunay triangulation and region-growing is extended to ensure topologically correct reconstruction. This paper also introduced an efficient neighboring triangle location method by making full use of the surface normal information. Experimental results prove that this algorithm can efficiently obtain the most reasonable reconstructed mesh surface with arbitrary topology, wherein the automatically reconstructed surface has only small topological difference from the true surface. This algorithm has potential applications to virtual environments, computer vision, and so on.

Keywords  three dimensional triangulated irregular network; digital terrain surface modeling; Delaunay triangulation

Introduction

The latest technology developments clearly show an improvement (resolution and accuracy) of three-dimensional (3D) data collection techniques: aerial and close range photogrammetry, airborne or ground-based laser scanning, and mobile mapping and 3D seismic technology. However, we have yet to fully utilize the 3D geospatial datasets. A lot of researches have been conducted toward the automation of 3D object reconstruction; reliable and efficient methods of complicated surface modeling are increasingly required for broad virtual geographic environments (VGEs) applications. The critical issue of fast and accurate modeling of irregular complex surfaces from point clouds (derived from LIDAR, image matching, or other geotechnologies) has always been a concern of researchers from the fields of computer vision and computer graphics, as well as geosciences. The problem of taking an unorganized point cloud in 3D space and fitting a polyhedral surface to those points is both important and difficult.

In planimetric terrain modelling, triangulated irregular networks (TINs) have been widely used to tessellate the terrain and, for simplicity, it is usually assumed that terrain models are monotonic in $X$ and $Y$, i.e., there is only one possible $Z$ for each $XY$ location. This limitation results in the inability of 2D TIN to reconstruct cliffs, faults, caves, tunnels, bridges...
and overpasses, which are the basic features of actual complex terrain surfaces. A full 3D surface model is therefore needed for Architecture Engineering and Construction (AEC)/CAD, geological, atmospheric and oceanographic modeling, where attributes need to be assigned to arbitrary locations in 3D space. For most CAD applications, a simple modification to the basic 2D triangulation algorithm allows one to interactively modify the terrain model in order to add complex features that are otherwise unavailable. Because one is still forming a connected surface, a variety of topological operations, such as neighborhoods selection and flow modeling, may be performed.

Clearly, another, even higher, layer of operations would permit one to add pre-designed features such as buildings, dams, tunnels and others to our terrain model. However, this paper aims at the automatic modeling of complex 3D terrain surfaces based on point clouds.

The remainder of this paper is organized as follows. After reviewing the up-to-date 3D TIN algorithms in Section 2, Section 3 proposes a new triangulation algorithm which enables the extraction of an arbitrary surface topology under the guide of intrinsic property of the data source and constrained information. The experimental results are then presented and analyzed in Section 4. Finally, some concluding remarks are given in Section 5.

1 Brief review of 3D TIN algorithms

In recent years, the surface reconstruction problem in three dimensions found some attraction from researchers in computer graphics as well as from researchers in computational geometry. Several critical issues of full 3D TIN generation include the efficiency, reliability and correct topology closest to the original surface. All the 3D TIN algorithms can be classified into two types: direct and indirect ways. The typical indirect method is herein called the 2D mapping approach; however, the direct methods are usually employed for most 3D surface modeling.

The 3D Delaunay triangulation approach is one of the most popular direct methods. As the natural extension of 2D Delaunay TIN, the Delaunay triangulation of the point set is first constructed, followed by the extraction of triangles or triangular patches representing the object shape. For example, Boissonnat’s method applied the 3D Delaunay triangulation to obtain the convex hull of the point set. If not all the points in the point set are on the boundary of the convex hull, then some tetrahedron in the triangulation must be deleted in turns until all points lie on the boundary $P$ of the polyhedral shape so obtained, while ensuring that $P$ remains a polyhedron. Other algorithms in this category are alpha-shape algorithm, crust algorithm, and umbrella filter algorithm. A drawback of all Delaunay-based algorithms is that in practice, their output need not be a topologically correct surface. All these algorithms consider candidate triangles from the Delaunay complex by some criterion. In practice, these triangles are unlikely to form a topologically correct surface due to noise, undersampling or sharp surface features.

The distance function approach is a kind of simple direct method, for which Hoppe’s is a typical example. They presented an algorithm which reconstructs the surface as the zero set of a signed distance function. The algorithm makes certain sampling assumptions for the input data that may not hold in practical situations. A similar algorithm by Curless and Levoy is capable to do so; their volumetric representation consists of a cumulative weighted signed distance function, which relies on additional information rather than just the sample points.

Another important direct method is the region-growing approach, which constructs the mesh starting with a seed triangle, and progressively adding new triangles attached to the partially constructed mesh. The boundary edges are considered active edges, to which new triangles are added. The critical issue of this approach is how to select the proper point to form a new triangle with an active edge. In the ball pivoting algorithm, a ball with user-specified radius pivots around an active edge until it touches another point in the point cloud; the point being touched is then selected. Huang and Menq projected the $k$ nearest points of each end point of an active edge, respectively, onto the plane defined by the triangle adjacent to the active edge. A point is chosen among the $k$ points based on the minimal length criterion to
form a triangle with the active edge. Lin\textsuperscript{[12]} presented a region-growing mesh reconstruction algorithm, called the intrinsic property driven (IPD) algorithm. The IPD algorithm searches for a new point based on the sampling uniformity degree. Unlike algorithms requiring user-specified parameters, the mesh surface reconstructed by the IPD algorithm completely relies on this intrinsic property of the point cloud.

As an indirect method (2D mapping approaches), Wang et al\textsuperscript{[13]} and references\textsuperscript{[13,14]} presented a 2D mapping mesh reconstruction algorithm, while first mapping the local 3D point cloud onto a 2D plane, and then using the traditional planar Delaunay TIN to network these points. The algorithm takes the local geometric and topological property to generate a local triangular mesh based on the Delaunay triangulation technique in two dimensions. This algorithm is useful to smooth regions of the surface with high efficiency, but it is difficult to fit more complicated 3D surfaces.

A few works on 3D terrain modeling have been done so far. Triangulation in 3D is a part of it, but measurement errors and smoothness requirements make clear that a 3D terrain model is more than a triangulation of the original measurements. The data structures for a TIN of 2.5D and 3D data may be the same, but the derivation criterions are different. In photogrammetry, 3D digital terrain modeling based on tetrahedronization is proposed\textsuperscript{[15]} for 3D GIS data. Heitzinger\textsuperscript{[16]} uses a knowledge-based approach for constructing the TIN while Verbree and van Oosterom\textsuperscript{[17]} make use of line-of-sight knowledge obtained during data gathering. Pfeifer\textsuperscript{[18]} further presented a subdivision algorithm for 3D terrain modeling, which can incorporate additional geometric information and provide a smooth surface.

From the comparison analysis of a wide variety of 3D TIN generation algorithms (Fig.2 and Table 1), there is still no universal approach of 3D TIN generation. Even existing algorithms for 3D TIN generation work nicely on dense and smooth data sets, especially since most of the 3D surface modeling in CAD and computer graphics is based on the local flatness of a surface, which still face the difficulties of low reliability and low efficiency in full 3D terrain modeling when cliffs, faults, caves, tunnels, bridges and overpasses have to be considered. In most cases where the data contain unsampling sharp edges and areas with high curvature, these algorithms would produce holes or overlap triangles in the vicinity of the poor regions. As a hopeful direct method, the region-growing approach driven by the intrinsic property of point clouds is suitable for any kind of complicated surface. This paper, therefore, pays attention to its extension to the 3D terrain modeling.

| Table 1 Typical 3D TIN algorithms’ comparison |
| Sampling | Distance function | Delaunay | Voronoi | 2D mapping | Region growing |
| Sampling | Uniform | Uniform | Nonuniform | Nonuniform | Nonuniform |
| Efficiency | Low | Low | High | Very high | Low |
| Robustness | Good | Bad | Fair | Good | Bad |
| Automatization | Full | Semi- | Full | Full | Semi- |
| Complexity | Fair | Fair | Fair | Low | Fair |

2 A constrained region-growing algorithm

In order to extract arbitrary surface topology under the guide of intrinsic property of the data source and constrained information, this paper employs the 3D Delaunay triangulation criteria and umbrella condition to improve the local topologic property of the result. After defining the concept of the umbrella condition, a constrained region-growing algorithm for surface reconstruction from 3D irregular points is investigated and extended.

Definition (umbrella condition). The neighborhood of a vertex $v$ in a $d$-dimensional simplicial complex $k$ is the union of the interiors of all simplices in $k$ incident to $v$, together with the vertex $v$ itself. A complex is called surface complex if the neighborhood of every point $v$ is homeomorphic to the open ball $D^d$. Such a neighborhood is called an umbrella as shown
Because of the difficulties of automatic 3D surface reconstruction from the unorganized point cloud, it is proposed to generate the final TIN surface model through constrained region-growing and postprocessing, which coincides with the best-first principle. As shown in Fig.2, this algorithm consists of the following basic procedures.

1) The initial 3D constrained Delaunay triangulation (CDT) is generated based on the random insertion of points with line and/or polygon constraints. This CDT just provides a kind of neighbourhood relationship for the arbitrary 3D points set.

2) According to the local normal information of the surface, a quick filtering method is designed to extract a preliminary set of candidate surface triangles.

3) Based on the topological umbrella conditions between surfaces and points, various rules are introduced to check the local surface patches’ topological consistency. Unstructured and out-of-order points are therefore transformed into structured description, which facilitates the subsequent surface patch recognition and extraction. A seed triangle with the best property, thus, can be selected.

4) An efficient neighboring triangle location algorithm is proposed by taking the full advantage of surface normal and local umbrella conditions to extract a preliminary set of candidate triangles. Since the local geometric and topological property can be clearly described with its neighboring triangles for an arbitrary edge, the method can efficiently locate a valid triangle for the current edge.

5) A method of local geometric integrity test is introduced to ensure the correct topological reconstruction via postprocessing of the 3D TIN. This procedure can automatically detect and locate the boundary of holes and then repair the surface through filling up of the holes. The reconstructed surface then has the minimum topological difference from the surface of the original object.

There are three critical factors for the constrained region-growing procedure: the selection of seed triangle, the growing of triangles and the topological consistency check. The region growing always starts from a seed triangle; therefore, it is important to select a seed triangle with good properties, such as that which is closest to the original surface and without hole or overlap. In order to ensure the correct selection of the seed triangle, both the filtering of redundant triangles from the initial 3D CDT and local topological consistency check are effective operations. As shown in Fig.3, The triangle $\triangle V_0V_1V_2$ is selected as a good seed triangle for further region growing operation.

2.1 Filtering the redundant triangles based on the surface normal and shape

For an arbitrary unorganized point/line set, the initial 3D TIN can be generated based on CDT criteria. However, this TIN is usually not the ideal surface
mesh closest to the original object but just the vicinity relationship of the point set. While there are straightforward approaches to extract the surface mesh from the CDT, it is hard to deal with the objects that have complicated shape characteristics and topological relationships. Based on the close relation among the Voronoi pole, the surface normal and the triangle facet closest to the original object, a basic procedure is herein proposed to filter the redundant facets from the initial CDT. As shown in Fig.4, the normal of the surface at each point \( p \) is estimated using poles. The poles of a point \( p \) are the two farthest vertices of its Voronoi cell, one on each side of the surface. Since the algorithm does not know the surface, only the discrete points, it chooses the pole by first choosing the farthest Voronoi vertex, and then choosing the farthest in the opposite half-space. In the filtering algorithm, the preliminary triangle may be selected as follows: Let \( p \) be a point and let \( e \) be a Voronoi edge in the Voronoi cell of \( p \). The surface normal at \( p \) can be estimated by the vector from \( p \) to the pole if \( e \) has a point \( x \), where \( px \) makes an angle close to \( \pi/2 \) with the estimated normal at point \( p \). If this condition is satisfied for all three Voronoi cells adjacent to \( e \), its dual is included in the set of candidate triangles. After the filtering operation, all the restricted Delaunay triangles are extracted to be the candidate surface triangle set for further processing. During this filtering procedure, the normal vector plays a key role of ensuring that both the reserved triangle’s normal and the point’s normal are as parallel as possible. The whole shape feature of the object can, therefore, be maintained.

![Fig.4 Redundant triangle filtering based on the local surface normal information.](image)

### 2.2 Topological consistency check

It is important to compute a topologically correct and a geometrically close terrain surface under certain conditions on sampling density. Unfortunately, this sampling condition is not always met in practice due to cliff, fault, non-smoothness or simply due to inadequate sampling. Certain geoscience applications such as 3D visualization and digital terrain analysis require a water-tight surface; hence, a seamless and non-overlap 3D TIN surface model is required. However, the redundant triangles filter algorithm, as described in the previous section, results in a preliminary set of candidate triangles possibly with holes and other overlap triangles in the surface. Observe that in 3D, the local TIN has to satisfy the umbrella condition in order to form a correct surface. Namely, a set of surface triangles incident to these adjacent points form a topological disk. To ensure that the reconstructed surface has the correct topology of the original surface, according to the topological umbrella characteristics between surfaces and points, a local topological consistency check algorithm is designed for topologically correct surface reconstruction. In order to ensure that the local triangles set conforms to the umbrella condition, three rules are defined for the topological checking, i.e., the closest edges rule, the maximum-minimum angle rule, and the obtuse angle rule. Such consistency check walks through the Delaunay triangulation in depth first manner using the vertex and tetrahedral adjacencies. After local topological processing, holes and overlap triangles in the surface are eliminated, as shown in Fig.5. In fact, the importance of a local topological consistency check is not to carry out the final correct results, but to provide a good set of candidate triangles for the seed triangle selection for region growing.

![Fig.5 Overlapped triangles elimination after topological consistency check](image)

### 2.3 Constrained region-growing

The above-mentioned operations only deal with the local topological consistency of the TIN but the global topological consistency also has to be consid-
ered for the surface reconstruction. As the preliminary candidate triangles have no topological relationship to each other, to achieve the correct topological relationship, the region-growing approach is an appropriate choice. The region-growing method incrementally builds the mesh from the seed triangle. The seed triangle is selected from the set of candidate triangles. A seed triangle is a valid triangle to initiate the valid region-growing. The selected triangle should be small and its compactness should be large. The larger the compactness and the more regular the triangle, the closer the shape is to an equilateral triangle.

After determining a seed triangle, the most time-consuming process is to search the neighbor triangle, which is called the neighbor triangle location. As we know, it is difficult to find a proper vicinity point in 3D for the given edge (called active edge). Fortunately, the former operations provide us the local topological and geometric information about the active edge, which could enable the neighbor triangle location to be more efficient. Starting from the seed triangle, two constraints of weighted minimum length and triangle validity are introduced to the region-growing procedure. The weighted minimum length constraint takes both the side length and area of the triangle into account, which guarantees the minimum difference between the reconstructed 3D TIN surface and the original object surface.

As shown in Fig. 6, the given active edge \( PQ \) of the seed triangle \( \triangle RPQ \), to select a new vertex \( n_i \) from the vicinity triangle set of \( PQ \) to enable the new triangle \( \triangle PQn_i \) to satisfy \( W_i \) minimum:

\[
W_i = k_{P,Q} |P - P_Q|^2 + k_{P,Q} |P - P_i|^2 + k_{Q,i} |P - P_i|^2
\]

\[
k_{P,Q} = \frac{(L_{P,R}^2 + L_{Q,R}^2 - L_{P,Q}^2)}{A_{P,Q,R}}
\]

\[
+ \frac{(L_{P,i}^2 + L_{Q,i}^2 - L_{P,Q}^2)}{A_{P,Q,i}}
\]

(1)

(2)

where \( W \) is the weighted length; \( K \) is the ratio of side length and area; \( L \) is the side length; and, \( A \) is the area.

\[
k_{P,j} \text{ and } K_{Q,j} \text{ are computed as:}
\]

\[
k_{P,j} = K_{Q,j} = 2 \times (L_{P,j}^2 + L_{Q,j}^2 - L_{P,Q}^2) / A_{P,Q,j}
\]

(3)

As stated above, if the active edge \( PQ \) contains some adjacent candidate triangles, the region-growing algorithm first chooses the triangle that minimizes Eq.(1) from them, then performs the triangle validity test. It checks whether the intersection between the new triangle and those existing triangles is empty or an existing constrained edge. If so, the choice of triangle is confirmed. Otherwise, the algorithm chooses another triangle that minimizes Eq.(1) from the remaining adjacent candidate triangles and performs the same triangle validity test.

### 3 Experimental analysis

In order to investigate the correctness and reliability of the 3D TIN generation algorithm described above, two real terrain data sets are adopted in our experiment by a PC with a Pentium 1.8GHz CPU and 256Mb memory.

As shown in Fig. 7(a), the terrain surface with steep cliff is reconstructed from randomized grid points. Fig. 7(b) shows a terrain surface model with a geological fault from a dense point cloud. The time efficiency of experiments is listed in Table 2, which shows that our algorithm is almost a linear-time algorithm.

Experimental results prove that the reconstructed surfaces by our proposed algorithm make very small topological difference from the original object surface. The topological umbrella check and region-growing process ensure both local and global topological consistency, which is useful for the large scale complicated 3D terrain surface modeling.
4 Conclusions

More and more practical applications require the full 3D TIN representation for terrain modeling while at the same time, increasing 3D geospatial datasets become available. These range from a high-resolution 3D seismic volume, which provides an unprecedented level of information on subsurface geological architectures, to data from laser scanners, LIDAR and digital mapping/surveying techniques that capture macro- to mega- scale surface information rapidly and accurately. However, 3D TIN generation from unorganized points and constraints is quite more difficult than the traditional 2D TIN generation. Experimental results show that our algorithm can efficiently obtain the most reasonable triangle mesh with arbitrary topology. Once the 3D TIN model has been created, contours, profiles, volumes between surfaces and 3D displays become available. The improved growing algorithm has potential applications to virtual geographic environments, computer vision, and reverse engineering.

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