Screening in Strongly Coupled Plasmas: Universal Properties from Strings in Curved Space

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Abstract

We use the gauge/gravity correspondence to study the screening of a heavy quark-antiquark pair in various strongly coupled plasmas. Besides $\mathcal{N} = 4$ super Yang-Mills theory and the corresponding $AdS_5$ space we also study theories obtained as deformations of $AdS_5$, among them in particular a class of deformations solving supergravity equations of motion. We consider the dependence of the screening distance on the velocity and the orientation of the pair in the plasma. The value of the screening distance in $\mathcal{N} = 4$ SYM is found to be a minimum in the class of theories under consideration for all kinematic parameters.

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1 Introduction

The finding that the quark-gluon plasma created in heavy-ion collisions at RHIC (and soon to be created at the LHC) appears to be strongly coupled calls for a better theoretical understanding of strongly coupled gauge theories at finite temperature. Until recently, lattice QCD constituted the only viable approach to this problem, albeit only for static observables. With the discovery of the gauge/gravity (or AdS/CFT) correspondence [1, 2, 3] a new path has opened to attack the problem of strongly coupled gauge theories. It even allows one to study dynamical processes in the plasma. In our study we make use of the correspondence to study the screening of a heavy quark-antiquark pair in a strongly coupled plasma. In particular, we will consider the maximal distance for which the quark and antiquark form a bound state, also called the screening distance, and its dependence on the velocity and the orientation of the pair with respect to the plasma. A good understanding of this observable might be helpful for diagnosing the properties of the quark-gluon plasma with the help of heavy probes like charmonium or bottomonium.

In its original form the gauge/gravity correspondence is a holographic duality between supergravity on a five-dimensional $AdS_5$ space and four-dimensional $\mathcal{N} = 4$ super Yang-Mills (SYM) theory with gauge group $SU(N_c)$ in the large-$N_c$ limit. The most interesting property of the duality is that the weak-coupling (small curvature) limit on the gravity side corresponds to the strong-coupling limit on the gauge theory side. This makes it possible to solve hard problems in a gauge theory by doing simple calculations on the gravity side. Finite temperature can be accommodated by introducing a black hole in the bulk of the $AdS$ space.

Although the AdS/CFT duality was a major leap from a theoretical point of view its use for the phenomenology of QCD is far from obvious. Clearly, $\mathcal{N} = 4$ SYM is very different from QCD: it is maximally supersymmetric, features only particles in the adjoint representation of the gauge group, does not have a running coupling (i.e. is conformal), and exhibits neither confinement nor chiral symmetry breaking. It hence requires some optimism to apply the duality to QCD. The situation appears somewhat more promising when one considers finite temperature. Here, QCD is no longer confining and above $2T_c$ even appears close to conformal. At the same time, finite temperature breaks the exact conformal invariance of $\mathcal{N} = 4$ SYM. In addition, it is feasible that some properties of a gauge theory plasma are to some extent independent of the microscopic degrees of freedom. This is supported by the apparent validity of the hydrodynamical description of various observables in the quark-gluon plasma. Still, it remains unknown whether a gravitational theory dual to QCD exists.

In an attempt to come closer to a potential dual of QCD one can introduce deformations of the $AdS_5$ space which break the conformal invariance of the dual theory. Obviously one can think of a vast variety of deformations of this kind, and it will be very difficult to find a particular deformation that reproduces all properties of QCD. Therefore it seems more interesting to ask for universal properties of large classes of such deformations. In particular, a given observable
can be robust under deformations and change only very little. In this case one might hope that the value of an observable in \( \mathcal{N} = 4 \) SYM might already be a good approximation to the value in actual QCD. An observable can also be universal in the sense that it does not change at all under deformations, or changes consistently in one direction. A famous example for a universal observable is the ratio of viscosity to entropy density \( \eta/s \) which acquires the value \( 1/(4\pi) \) in all theories with gravity duals \[4\]. This value has even been conjectured to be a lower bound for all possible theories \[5\].

In the following we will show that the screening distance of a heavy quark-antiquark pair which moves in a hot plasma is in fact a universal observable in a large class of theories. We will restrict ourselves to the main results, a more detailed account of our study will be published elsewhere \[6\].

2 AdS/CFT at Finite Temperature and its Deformations

The gravity dual of \( \mathcal{N} = 4 \) SYM at finite temperature is an \( \text{AdS}_5 \times S^5 \) space with a Schwarzschild black hole. The \( S^5 \) factor will not be relevant for our considerations and will be suppressed from the beginning. The 5-dimensional AdS black hole metric is given by

\[
ds^2 = G_{\mu\nu} dx^\mu dx^\nu = -f(r) dt^2 + \frac{r^2}{R^2} (dx_1^2 + dx_2^2 + dx_3^2) + \frac{1}{f(r)} dr^2
\]

with the curvature radius \( R \) of \( \text{AdS}_5 \), and with

\[
f(r) = \frac{r^2}{R^2} \left( 1 - \frac{r_0^4}{r^4} \right).
\]

For each fixed value of the fifth coordinate \( r \) the metric describes a Minkowski space in the remaining four coordinates. The coordinate \( r \) has an interpretation as an energy variable in the boundary theory. The holographically dual \( \mathcal{N} = 4 \) SYM ‘lives’ at \( r = \infty \), that is at the boundary of \( \text{AdS}_5 \). The location \( r_0 \) of the black hole horizon is related to the temperature \( T \) of the boundary theory via \( T = r_0/(\pi R^2) \). The latter coincides with the Hawking temperature of the black hole. A large curvature radius \( R \) and thus the applicability of classical gravity on the AdS side requires large \('t\) Hooft coupling \( \lambda = g_{\text{YM}}^2 N_c \) on the gauge theory side.

A simple class of deformations of \( \text{AdS}_5 \) which break conformal invariance in the dual theory is the KTY model \[7\] in which the original metric is multiplied by an exponential factor depending on \( r \) such that

\[
ds^2 = \exp \left( \frac{29}{20} c \frac{R^4}{r^2} \right) \left[ -f(r) dt^2 + \frac{r^2}{R^2} (dx_1^2 + dx_2^2 + dx_3^2) + \frac{1}{f(r)} dr^2 \right].
\]

The function \( f \) is as in \[2\], and also here the temperature of the dual field theory is given by \( T = r_0/(\pi R^2) \). Several properties of QCD thermodynamics are well reproduced with \( T_c \simeq 170 \text{ MeV} \) when the dimensionful parameter
c is chosen as \( c \simeq 0.127 \text{ GeV}^2 \). In order to see how an observable depends on the deformation it is interesting to study how it changes with the actual conformality-breaking parameter \( c/T^2 \). (A ‘realistic’ range for that parameter is \( 0 \leq c/T^2 \leq 4 \).) However, the KTY model suffers from the problem that it does not solve supergravity equations of motion. Thus, its thermodynamic consistency is questionable.

More recently, also the construction of thermodynamically consistent deformations has been explored \[8\]. It is found that the dilaton potential \( V(\Phi) \) in the 5-dimensional gravitational action

\[
S_5 = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left( R - \frac{1}{2} (\partial_{\mu} \Phi)^2 - V(\Phi) \right)
\]

(4)
can be chosen such that one obtains a 2-parameter model \[9\] of the form

\[
ds^2 = e^{2A(r)} (-h(r)dt^2 + d\vec{x}^2) + \frac{e^{2B(r)}}{h(r)} dr^2
\]

(5)
with two parameters \( c/T^2 \) and \( \alpha = c/\phi \), where \( \Phi = \sqrt{\frac{3}{2}} \phi \frac{R^2}{r^2} \). One can use a residual gauge freedom to identify \( r = \Phi \). The temperature of the dual theory is given by

\[
T = \frac{e^{A(\Phi_h) - B(\Phi_h)} |h'(\Phi_h)|}{4\pi}
\]

(6)
where \( \Phi_h \) is the location of the horizon defined by the zero of \( h(r) \). Defining

\[
A(\Phi) = \frac{1}{2} \ln \left( \sqrt{\frac{3}{2}} \frac{R^2}{\alpha} \right) - \frac{1}{2} \ln \Phi - \frac{\alpha}{\sqrt{6}} \Phi
\]

\[
B(\Phi) = \ln \left( \frac{R}{2} \right) + \frac{1 + 2\alpha^2}{2\alpha^2} \ln \left( 1 + \alpha \sqrt{\frac{2}{3}} \Phi \right) - \ln \Phi - \frac{1}{\alpha \sqrt{6}} \Phi
\]

(7)
one can calculate \( h \) from supergravity equations of motion. This model becomes similar to the KTY model for the choice \( \alpha = \alpha_{\text{KTY}} = 20/49 \), when the exponential factor of (3) is reproduced in the first term of (5).

From the two-parameter model one can obtain a further model if one treats \( \Phi \) not as the dilaton but as an additional scalar field, assuming a trivial dilaton instead. In that case the Einstein frame and the string frame for the calculation of a string on this background coincide. In our figures below we denote this construction as ‘Einstein frame’ model, while the model described above (with \( \Phi \) being the dilaton) is denoted as ‘string frame’ model.

Clearly, the deformed AdS metrics described here define dual four-dimensional theories at strong coupling. However, the Lagrangians of these holographically dual theories are not known, and it is not even clear that they correspond to gauge theories. Nevertheless, they are perfectly fine if one is interested in the effects of conformality-breaking on various observables.
3 Screening Distance and Free Energy of a $Q\bar{Q}$ Pair

The free energy $E(L)$ of a heavy quark-antiquark pair separated by a distance $L$ in a gauge theory plasma is obtained from the temporal Wegner-Wilson loop

$$W(C) = \text{Tr} \mathcal{P} \exp \left[ i \oint_C dx^\mu A^\mu(x) \right]$$

via its expectation value $\langle W(C) \rangle = \exp \left[ -i \mathcal{T} E(L) \right]$, where $\mathcal{T}$ is the (large) temporal extension of the closed curve $C$. On the gravity side, one has

$$\langle W(C) \rangle \propto \exp \left[ -i (S - S_0) \right],$$

where $S$ is the Nambu-Goto action for an open string hanging down into the bulk of the AdS-type space. Its ends are attached to the quark and antiquark at the boundary $r = \infty$. For a plasma moving in $x_3$-direction this situation is illustrated in Fig. 1. $S_0$ is twice the Nambu-Goto action for an open string hanging down from a single quark.

![Figure 1: String configuration for a $Q\bar{Q}$ pair in a moving plasma. The heavy quarks are located at $r = \Lambda$, and the limit $\Lambda \to \infty$ is implied.](image)

The calculation of $E(L)$ and of related observables had been performed for $N = 4$ SYM in [10] and for the KTY model in [11]. In our work we have considered the full dependence on the velocity and on the orientation of the pair in the plasma and have extended it to the two-parameter models given above. In the following we outline the calculation for the simple case of $N = 4$ SYM. The calculation for the other models is done along the same lines but leads to formulae less suited for a short exposition.

The moving plasma is accommodated by boosting the metric with velocity $v = \tanh \eta$ in $x_3$-direction. The $Q\bar{Q}$ pair can be rotated w.r.t. the $x_3$-direction.
by an angle $\theta$. We parametrize the string world sheet as indicated in Fig. 1 and extremize the resulting Nambu-Goto action

$$S = \frac{T}{2\pi\alpha'} \int_{-\frac{T}{2}}^{\frac{T}{2}} d\sigma \sqrt{A \left( \frac{(\partial_\sigma r)^2}{f} + \frac{r'^2}{R^2} \right)}$$  \hspace{1cm} (10)$$

where

$$A = \frac{r^2}{R^2} \left[ 1 - \frac{r_0^4 \cosh^2 \eta}{r^4} \right].$$  \hspace{1cm} (11)$$

The solutions can be parametrized by the conserved Hamiltonian

$$\mathcal{H} \equiv \mathcal{L} - y' \frac{\partial \mathcal{L}}{\partial y'} = \frac{y^4 - \cosh^2 \eta}{\mathcal{L}} = q,$$  \hspace{1cm} (12)$$

and one can solve for the coordinate function $r = r_0 y$ of the string,

$$y' = \frac{1}{q} \sqrt{(y^4 - 1)(y^4 - y_c^4)} \quad \text{with} \quad y_c^4 \equiv \cosh^2 \eta + q^2.$$  \hspace{1cm} (13)$$

Using the boundary conditions one finally obtains the quark-antiquark distance as a function of $q$,

$$\frac{L \pi T}{2} = \int_{0}^{L_{\perp}} d\sigma = q \int_{y_c}^{y} dy \frac{1}{\sqrt{(y^4 - 1)(y^4 - y_c^4)}}.$$  \hspace{1cm} (14)$$

In AdS$_5$ and the deformations discussed above one finds that $L(q)$ has a maximum for all values of the rapidity $\eta$ and for all orientation angles. For all $L$ up to this $L_{\text{max}}$ there are two solutions with different $q$. For $L > L_{\text{max}}$, on the other hand, no string configuration connecting the quark and the antiquark exists. We call this maximally possible distance between the quark and the antiquark the screening distance. It depends on the rapidity $\eta$ and the orientation angle $\theta$ with respect to the moving plasma, and on the parameters of the deformation of the AdS metric.

For the case of $\mathcal{N} = 4$ SYM the behavior of the string configurations up to $L_{\text{max}}$ is shown in Fig. 2. For each distance $L$ up to $L_{\text{max}}$ there are two solutions. The configuration that stays higher up in the bulk has a smaller energy $E(L)$ than the configuration coming closer to the horizon. The latter can therefore be identified as the unstable solution. With increasing distance $L$ the two configurations move towards each other until for the screening distance $L_{\text{max}}$ there is only one solution. Note that none of the solutions touches the horizon.

The computation of the free energy $E(L)$ of the quark-antiquark pair from (9) requires the knowledge of the action $S_0$ of a single string hanging down from a moving quark into the bulk of AdS$_5$. This can be obtained from an AdS/CFT calculation of the drag force acting on the quark [12, 13]. That calculation has been extended to the KTY model in [14]. We have computed the drag force also for the two-parameter models solving supergravity equations of motion.

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[1] This configuration might also be metastable. A more precise statement would require an understanding of the dynamical mechanism of the transition between the two solutions.
Figure 2: String configurations for different quark-antiquark distances. The distance is given as the dimensionless \( l = L\pi T \) and is parametrized by \( \sigma \). The strings end on the quark and antiquark located at \( y \to \infty \).

Knowing \( S_0 \) we obtain the free energy \( E(L) \) also for the latter models and find its behavior to be qualitatively the same as in \( \mathcal{N} = 4 \) SYM and in the KTY model.

The screening distance should not be confused with the inverse Debye mass which is often called screening length. The latter describes the exponential fall-off of the free energy at distances larger than our screening distance \( L_{\text{max}} \). The Debye mass cannot be obtained from the calculations outlined above. It has been argued in [15] that it can be related to the exchange of the lowest supergravity modes between two open strings hanging into the bulk at a separation \( L > L_{\text{max}} \).

We have computed the screening distance for all deformations of the AdS space presented in section 2. We find that in all cases the screening distance has a very weak dependence of up to about 10% on the orientation angle of the \( Q\bar{Q} \) pair with respect to the plasma wind. The dependence on the velocity is dominated by a factor \((\sqrt{\cosh \eta})^{-1}\). The screening distance approaches this dominant behavior at large velocities in all models.

Finally, we have studied how the screening distance changes when deformations of the AdS space are introduced. This is illustrated in Fig. 3 where we show the dimensionless quantity \( \pi TL_{\text{max}}\sqrt{\cosh \eta} \) for the case \( \theta = 0 \) (\( Q\bar{Q} \) oriented parallel to the plasma wind). The last factor in this product compensates the dominant behavior of \( L_{\text{max}} \) with \( \eta \) just discussed so that the differences between the models become visible more clearly. In the KTY model we have chosen \( c/T^2 = 1 \) for this figure, while in the two-parameter deformations
Figure 3: Dimensionless screening distance $\pi T L_{\text{max}} \sqrt{\cosh \eta}$ of the $Q\bar{Q}$ pair in the plasma as a function of rapidity $\eta$.

$c/T^2 = 1$ and $\alpha = \alpha_{\text{KTY}}$.

We first find that the screening distance is a robust observable. Under ‘realistic’ deformations of the AdS space, i.e. deformations with thermodynamic observables not drastically different from those of QCD, its value changes only by up to 30%.

The most remarkable observation is that the screening distance has a universal behavior under the class of deformations studied here. For any given velocity and orientation angle of the $Q\bar{Q}$ pair with respect to the plasma and for all deformations considered here the value of $L_{\text{max}}$ is larger than in $\mathcal{N} = 4$ SYM. In other words: the screening distance in $\mathcal{N} = 4$ SYM is minimal in the class of theories under consideration for all kinematic parameters. It suggests itself to speculate that this might also apply to (all?) other theories obtained holographically as deformations of $AdS_5$.

4 Screening of Heavy Baryons

In the framework of the gauge/gravity correspondence heavy baryons can be constructed out of $N_c$ heavy quarks situated at the boundary of $AdS_5$. An open string is attached to each of the quarks and ends at a D5-brane that fills the 5 dimensions of $S^5$ and is located at a point $r_e$ in the bulk of $AdS_5$. For such a baryon configuration a similar analysis of screening in a moving plasma can be performed [16]. We have extended this study, previously done for $\mathcal{N} = 4$ SYM, to the deformation models described in section 2. In analogy to the screening distance of a $Q\bar{Q}$ pair one can define the maximally possible radius

\[ L_{\text{max}} \]
of the baryon configuration as a screening distance of the baryon. Also here we find that the screening distance of the baryon in $\mathcal{N} = 4$ SYM is minimal in the class of theories under consideration.

5 Summary

We have calculated the screening distance of a heavy quark-antiquark pair moving in different strongly coupled plasmas which are obtained holographically as duals of deformations of $AdS_5$. Our study includes deformations solving supergravity equations of motion. We observe that the screening distance is a robust observable and changes only little when deformations are introduced. We find the screening distance in $\mathcal{N} = 4$ SYM to be a minimum among the theories under consideration for all velocities and orientation angles of the pair in the plasma. A similar behavior is found for heavy baryons moving in strongly coupled plasmas. We conjecture that the screening distance found in $\mathcal{N} = 4$ SYM constitutes a lower bound for an even wider range of theories. It would obviously be interesting to show this analytically although that appears to be a challenging problem.

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