Quantum mysteries for anybody: Solved

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Abstract

In 1981, Mermin published a now famous paper titled, “Bringing home the atomic world: Quantum mysteries for anybody.” Therein, he presented the ‘Mermin device’ that illustrates the conundrum of entanglement per the spin singlet state for the “general reader.” He then challenged the “physicist reader” to explain the way the device works “in terms meaningful to a general reader struggling with the dilemma raised by the device.” Herein, I show how conservation per no preferred reference frame (NPRF) answers that challenge, but still leaves a mystery for those who seek dynamical explanation via hidden variables or ‘causal influences’. Since NPRF also underwrites the postulates of special relativity, we see a common theme between relativistic and non-relativistic modern physics.
I. INTRODUCTION

Nearly four decades ago, Mermin revealed the conundrum of quantum entanglement for a general audience\[1\] using his “simple device,” which I will refer to as the “Mermin device” (Figure 1). To understand the conundrum of the device required no knowledge of physics, just some simple probability theory, which made the presentation all the more remarkable. In subsequent publications, he “revisited”\[2\] and “refined”\[3\] the mystery of quantum entanglement with similarly simple devices. In this paper, I will focus on the original Mermin device as it relates to the mystery of entanglement via the spin singlet state.

So as not to lose the “general reader” at this point, let me introduce “entanglement” and “the spin singlet state” in general terms now. This is by no means a replacement for an introductory course in quantum mechanics, but as Mermin showed, one needs only simple probability theory to understand the conundrum of quantum entanglement. My goal is to provide just enough background to make the conundrum and its proposed solution accessible to the “general reader.”

The “source” in the Mermin device (middle box in Figure 1) creates a pair of particles measured by Alice and Bob in settings 1, 2, or 3 of the boxes on the left and right, respectively, in Figure 1. There are two possible outcomes of a measurement and they are denoted “R” and “G” (indicated by the light bulbs in Figure 1). The two possible Mermin device outcomes R and G represent two possible spin measurement outcomes “up” and “down,” respectively, (Figure 2) and the three possible Mermin device settings represent three different orientations of the Stern-Gerlach magnets (Figures 3 and 4). Whenever Alice and Bob choose to measure their particles using the same setting, they obtain opposite results\[21\] (entanglement), 50% RG (Alice’s outcome is R and Bob’s outcome is G) and 50% GR. This 50% RG and 50% GR result for the same settings is the spin singlet state.

That is all you need to know about the terms “entanglement” and “the spin singlet state” to understand the conundrum. As I will explain, Mermin’s device shows that explaining the opposite outcomes for the same settings (per entanglement via the spin singlet state) cannot be accomplished without violating another fact about entanglement via the spin singlet state. That is, when the settings are different Alice and Bob only measure opposite results 25% of the time. That in a nutshell, is the conundrum of entanglement via the spin singlet state as shown by the Mermin device (details are in Section \[II\]).
FIG. 1. The Mermin Device. Alice has her measuring device on the left set to 2 and Bob has his measuring device on the right set to 1. The particles have been emitted by the source in the middle and are in route to the measuring devices.

FIG. 2. A Stern-Gerlach (SG) spin measurement showing the two possible outcomes, up and down, represented numerically by +1 and −1, respectively. Figure 42-16 on page 1315 of Physics for Scientists and Engineers with Modern Physics, 9th ed, by Raymond A. Serway and John W. Jewett, Jr. Reproduced by permission of Brooks/Cole.

Concerning his device Mermin wrote, “Although this device has not been built, there is no reason in principle why it could not be, and probably no insurmountable practical difficulties.” Sure enough, the experimental confirmation of the violation of Bell’s inequality per quantum entanglement can now be carried out in the undergraduate physics laboratory. Thus, there is no disputing that the conundrum of the Mermin device has been experimen-
FIG. 3. Alice and Bob making spin measurements on a pair of spin-entangled particles with their Stern-Gerlach (SG) magnets and detectors.

FIG. 4. Three possible orientations of Alice and Bob’s SG magnets.

tally well verified, vindicating its prediction by quantum mechanics.

While the conundrum of the Mermin device is now a well-established fact, Mermin’s challenge to explain the device “in terms meaningful to a general reader struggling with the dilemma raised by the device” arguably remains unanswered. To answer this challenge, it is generally acknowledged that one needs a compelling model of physical reality in which the conundrum of the Mermin device is resolved. Such a model needs to do more than the “Copenhagen interpretation,” which Mermin characterized as “shut up and calculate.” In other words, while the formalism of quantum mechanics accurately predicts the conundrum, quantum mechanics does not provide a model of physical reality or underlying physical principle to resolve the conundrum. While there are many interpretations of quantum mechanics, even one published by Mermin, there is no consensus among physicists on any given interpretation.
Rather than offer yet another uncompelling interpretation of quantum mechanics, I will share an explanation of the quantum correlations responsible for the conundrum of the Mermin device. While this explanation, conservation per no preferred reference frame (NPRF), may not be “in terms meaningful to a general reader,” it is pretty close. That is, all one needs to appreciate the explanation is a course in introductory physics, which probably represents the “general reader” interested in this topic. If one is also familiar with the spin singlet state of quantum mechanics, they will fully understand the nuances of the explanation.

Of course, entangled states like the spin singlet state result from conservation principles and quantum mechanics produces classical results on average, so this result is perhaps not surprising in a general sense. However, as we will see, the manner by which this ‘average’ conservation obtains does not require any ‘causal influences’ or hidden variables. In other words, there is no ‘deeper mechanism’ to explain how this average conservation principle over the ensemble is obeyed on a trial-by-trial basis. So, how does the Mermin device “know” what outcomes to produce in any given trial of the experiment? Therefore, while conservation per NPRF will likely strike the typical physicist as a sound principle, it only resolves the conundrum of the Mermin device if one is willing to accept the conservation principle as a constraint over the ensemble of measurement results without a corresponding ‘deeper mechanism’ at work to govern outcomes on a trial-by-trial basis.

Thus, the solution to the mystery of the Mermin device per the average conservation principle requires we move beyond the “ant’s-eye view” of physical reality to a “4D view,” like that of Minkowski spacetime, per Wilczek’s challenge (p. 37):

A recurring theme in natural philosophy is the tension between the God’s-eye [4D] view of reality comprehended as a whole and the ant’s-eye view of human consciousness, which senses a succession of events in time. Since the days of Isaac Newton, the ant’s-eye view has dominated fundamental physics. We divide our description of the world into dynamical laws that, paradoxically, exist outside of time according to some, and initial conditions on which those laws act. ... The God’s-eye [4D] view seems, in the light of relativity theory, to be far more natural. Relativity teaches us to consider spacetime as an organic whole whose different aspects are related by symmetries that are awkward to express if we insist on carving experience into time slices. ... To me, ascending from the ant’s-eye view to the God’s-eye [4D] view of physical reality is the most profound challenge for
fundamental physics in the next 100 years.

This is admittedly a non-trivial change in what constitutes explanation. An easy way to see the difficulty in moving from the ant’s-eye view to the 4D view is to consider the example of a light ray (path orthogonal to the light’s wavefront) emitted in air and arriving at a location in water (Figure 5).

The 4D view says that the path taken by the light ray is the path of least time (Fermat’s principle\[11\] (p. 26-3)). The path of shortest distance (straight line connecting “Light Ray Emitted” and “Light Ray Received” in Figure 5) takes longer, so it is not the path taken. The ant’s-eye view says that the light ray emitted towards the water proceeds without deviation, since nothing is interacting with it, until it hits the water. The water then refracts the light ray in accord with Snell’s Law, i.e., the water slows the speed of the light wave causing it to bend, towards the point where it is ultimately received.

While both views yield the same result, we tend to favor the ant’s-eye view since the 4D view sounds a bit like the light intended to go to its ultimate destination and calculated its path based on the presence of the water. According to the ant’s-eye view, the light that was received was refracted at the air-water interface, otherwise it would have continued along the dashed path shown (“Path without water present”). That is, the light did not adjust its emission direction with the intention of reaching its destination and it certainly does not “know” it will encounter water when it is emitted. But, the 4D view does not attribute any intentionality or prescience to the light ray, rather it simply relegates the ant’s-eye view to secondary status a priori.

This language about the light not “knowing” it will encounter water at its emission should remind the reader of Mermin’s language about the detectors in his device. There he writes, “Why do the detectors always flash [different] colors when the switches are in the same positions? Since the two detectors are unconnected there is no way for one to ‘know’ that the switch on the other is set in the same position as its own.” This leads him to introduce “instruction sets” to account for the behavior of the device when the detectors have the same settings. He writes, “It cannot be proved that there is no other way, but I challenge the reader to suggest any.” As I will show in Section II, the instruction sets that are required to account for the behaviour of the device when the settings are the same fail to account for the behavior of the device when the settings are different, thus the conundrum.

I will not elaborate further on this point, instead I refer the reader to our previous work\[12\]...
FIG. 5. A light ray emitted in air and received in water takes the path of least time rather than the shortest spatial path. This is called Fermat’s principle or the principle of least time.

I just want to be clear about the implications of accepting conservation per NPRF as a reasonable constraint resolving the mystery of the Mermin device, since there is no ‘deeper mechanism’ offered to explain said conversation on a trial-by-trial basis. Accepting this explanation of the Mermin device is akin to accepting Fermat’s principle without requiring Snell’s Law to explain the path of the light ray in Figure 5. This constitutes a significant change in what it means to explain something via physics, so it needs to be acknowledged up front.

II. THE MERMIN DEVICE AND ITS CONUNDRUM

Let me remind the reader how the Mermin device works and how it relates to the spin measurements carried out with Stern-Gerlach (SG) magnets and detectors (Figures 2 and 3). The Mermin device contains a source (middle box in Figure 1) that emits a pair of entangled particles towards two detectors (boxes on the left and right in Figure 1) in each trial of the experiment. The settings (1, 2, or 3) on the left and right detectors are controlled by Alice and Bob, respectively, and each measurement at each detector produces either a result of R or G. When Alice and Bob’s settings are the same in a given trial (Mermin’s
“case (a)”), their outcomes are always different, half the time RG (Alice’s outcome is R and Bob’s outcome is G) and half the time GR. On average, in all trials when their settings are different (Mermin’s “case (b)”), the outcomes are only different \( \frac{1}{4} \) of the time, \( \frac{1}{8} \) RG and \( \frac{1}{8} \) GR.

Since the particles do not “know” at time of emission how they will be measured in each trial (they do not “know” which setting they will encounter), in order to guarantee case (a), Mermin introduces “instruction sets.” Again, he writes, “It cannot be proved that there is no other way, but I challenge the reader to suggest any.” Now look at all trials when Alice’s particle has instruction set RRG and Bob’s has instruction set GGR, for example.

That means Alice’s outcome in setting 1 will be R, in setting 2 it will be R, and in setting 3 it will be G. Likewise, Bob’s outcome in setting 1 will be G, in setting 2 it will be G, and in setting 3 it will be R. Therefore, the particles will produce an RG result when Alice and Bob both choose setting 1 (referred to as “11”), an RG result when both choose setting 2 (referred to as “22”), and a GR result when both choose setting 3 (referred to as “33”). Thus, for case (b) (different settings) Alice and Bob will obtain different outcomes when Alice chooses setting 1 and Bob chooses setting 2 (referred to as “12”), which gives an RG outcome. And, they will obtain different outcomes when Alice chooses setting 2 and Bob chooses setting 1 (referred to as “21”), which also gives an RG outcome. That means we have different outcomes for different settings in \( \frac{1}{3} \) of the six possible case (b) situations (the others are 13, 31, 23, 32). This \( \frac{1}{3} \) ratio holds for any instruction sets with two R(G) and one G(R).

The only other possible instruction set pairs are RRR/GGG or GGG/RRR and all six case (b) settings for those instruction sets always produce different outcomes. Thus, the “Bell inequality” for the Mermin device says that instruction sets must produce different outcomes for case (b) in more than \( \frac{1}{3} \) of the trials. But, quantum mechanics says you only get different outcomes in \( \frac{1}{4} \) of the case (b) trials, thereby violating the Bell inequality. Thus, the conundrum of Mermin’s device is that the instruction sets needed for case (a) fail to yield the proper outcomes for case (b). The fact that quantum mechanics accurately predicts the observed phenomenon without spelling out any means a la “instruction sets” for how it works made Lee Smolin say recently, “[quantum mechanics] doesn’t make any sense because it’s wrong,” meaning it is incomplete. What is the nature of reality (how does the device work) such that both case (a) and case (b) obtain according to quantum mechanics?
Again, as Mermin points out, his device can be instantiated via SG measurements of the spin singlet state where the R and G outcomes of the Mermin device represent spin up and spin down, respectively. The three Mermin device settings then correspond to three SG magnet orientations shown in Figure 4.

To remind the reader, spin is a binary property associated with the angular momentum of electrons (and other particles) as shown in Figure 2. To create a pair of spin-entangled particles giving rise to cases (a) and (b), one could use processes such as the dissociation of a spin-zero diatomic molecule \(^{14}\) or the decay of a neutral pi meson into an electron-positron pair \(^{15}\) processes that conserve total spin angular momentum of zero. It just needs to be the case that the spin angular momentum of the two ejected particles adds to zero (angular momentum vectors cancel out) to conserve the source’s total zero value of angular momentum. Thus, whenever Alice and Bob measure the spins of their respective particles using the same SG magnet orientation, they get opposite results representing a total angular momentum of zero. Again, this defines the spin singlet state (Figure 6).

![Figure 6](image)

**FIG. 6.** Outcomes (yellow dots) in the same reference frame, i.e., outcomes for the same measurement (blue arrows represent SG magnet orientations), for the spin singlet state explicitly conserve angular momentum.

Quantum mechanics says the probability of getting different results (RG or GR) when Alice sets her SG magnet at angle \(\alpha\) and Bob sets his at angle \(\beta\) is \(P_{RG} = P_{GR} = \frac{1}{2} \cos^2 \left(\frac{\alpha - \beta}{2}\right)\) (Figure 3). Again, you do not need to know how the formalism of quantum mechanics renders its probabilities, you only need to know that quantum mechanics does predict that distribution. However, these probabilities follow from our conservation principle and NPRF, as we will see below. The case (a) orientations have \(\alpha = \beta\) so the probability of different results is 1. The case (b) orientations have \(\alpha - \beta = \frac{2\pi}{3}\), so the probability of different results
is $P_{GR} + P_{RG} = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$. The difference in the statistics between instruction sets and quantum mechanics is typically analyzed using the correlation function.

The correlation function between two outcomes over many trials is the average of the two values multiplied together. In this case, there are only two possible outcomes for any setting, $+1$ (up or R) or $-1$ (down or G), so the smallest average possible is $-1$ (total anti-correlation, RG or GR, as when the settings are the same) and the largest average is $+1$ (total correlation, RR or GG, as when $\alpha - \beta = \pi$). One way to write the equation for the correlation function is

$$\langle \alpha, \beta \rangle = \sum (i \cdot j) \cdot p(i, j \mid \alpha, \beta)$$

where $p(i, j \mid \alpha, \beta)$ is the probability that Alice measures $i$ and Bob measures $j$ when Alice’s SG magnet is at angle $\alpha$ and Bob’s SG magnet is at angle $\beta$. Examples below will clarify Eq. (1), so do not despair if it is unfamiliar now. Let us look at the correlation function for the Mermin device outcomes in more detail.

We have two sets of data, Alice’s set and Bob’s set. They were collected in $N$ pairs with Bob’s(Alice’s) SG magnet at $\alpha - \beta = \theta$ relative to Alice’s(Bob’s). Both sets are half $+1$ and half $-1$ results (NPRF). By definition, the correlation function for these $N$ pairs of results is

$$\langle \alpha, \beta \rangle = \frac{(+1)_A(-1)_B + (+1)_A(+1)_B + (-1)_A(-1)_B + \ldots}{N}$$

Now organize the numerator into two equal subsets, the first is that of all Alice’s $+1$ results $(+1)_A$ and the second is that of all Alice’s $-1$ results $(-1)_A$

$$\langle \alpha, \beta \rangle = \frac{(+1)_A(\sum BA+) + (-1)_A(\sum BA-)}{N}$$

where $\sum BA+$ is the sum of all of Bob’s results corresponding to Alice’s result $(+1)_A$ and $\sum BA-$ is the sum of all of Bob’s results corresponding to Alice’s result $(-1)_A$. We could just as well have used Bob’s results $(+1)_B$ and $(-1)_B$ and obtained averages over Alice’s results instead, since the situation is symmetric (NPRF). Now, rewrite Eq. (3) as

$$\langle \alpha, \beta \rangle = \frac{(+1)_A(\sum BA+)}{N} + \frac{(-1)_A(\sum BA-)}{N} = \frac{(+1)_A(\sum BA+)}{2 \frac{N}{2}} + \frac{(-1)_A(\sum BA-)}{2 \frac{N}{2}}$$

which is
\[ \langle \alpha, \beta \rangle = \frac{1}{2} (+1)_{A}BA+ + \frac{1}{2} (-1)_{A}BA- \] (5)

with the overline denoting average. Notice this correlation function is independent of the formalism of quantum mechanics, all we have assumed so far is that Alice and Bob measure +1 or −1 with equal frequency at any setting \( \alpha \) or \( \beta \), respectively, per NPRF.

To get a feel for what the correlation function is telling us, suppose Bob’s results are equally mixed between +1 and −1 in each of \( \sum BA+ \) and \( \sum BA- \). This is possible since Bob measures +1 and −1 with equal frequency overall. In that case, \( \langle \alpha, \beta \rangle = 0 \) and the \( N \) pairs of outcomes is said to be uncorrelated. In other words, there is no statistical evidence of any relationship between Alice’s outcomes and Bob’s outcomes. Now imagine trading some of the (−1)\( B \) results from \( \sum BA- \) for (+1)\( B \) results in \( \sum BA+ \). In that case, \( \sum BA+ \) would change from zero to a negative number and \( \sum BA- \) would change from zero to a positive number making \( \langle \alpha, \beta \rangle \) less than zero or slightly anti-correlated. Trading all the (−1)\( B \) results from \( \sum BA- \) for all the (+1)\( B \) results in \( \sum BA+ \) yields \( \langle \alpha, \beta \rangle = -1 \), i.e., the results are totally anti-correlated, as happens when the settings are the same. Likewise, trading some of the (+1)\( B \) results from \( \sum BA- \) for (−1)\( B \) results in \( \sum BA+ \) would change \( \langle \alpha, \beta \rangle \) from zero to something positive. And, trading all the (+1)\( B \) results from \( \sum BA- \) for all the (−1)\( B \) results in \( \sum BA+ \) yields \( \langle \alpha, \beta \rangle = +1 \), i.e., the results are totally correlated, as happens when \( \alpha - \beta = \pi \).

Again, the case (a) results require \( \langle \alpha, \beta \rangle = -1 \) and the instruction sets give us that, since RG or GR outcomes correspond to (+1)(−1) or (−1)(+1) in the SG counterpart and every case (a) setting pair (11, 22, or 33) gives RG or GR. Using Eq. (5), the case (b) correlation function for two R(G) and one G(R) instruction sets gives

\[ \langle \alpha, \beta \rangle = (+1)(-1)\left(\frac{1}{6}\right) + (-1)(+1)\left(\frac{1}{6}\right) + (+1)(+1)\left(\frac{2}{6}\right) + (-1)(-1)\left(\frac{2}{6}\right) = \frac{1}{3} \] (6)

since the probability of an RG(GR) outcome is \( \frac{1}{6} \) and the probability of an RR(GG) outcome is \( \frac{2}{6} \). Thus, \( \langle \alpha, \beta \rangle \) is skewed away from zero towards +1 meaning the results are more correlated, (+1)(+1) or (−1)(−1), than anti-correlated, (+1)(−1) or (−1)(+1). That makes sense since we saw in the Mermin device that the probability of obtaining RG or GR outcomes (anti-correlated) for case (b) is only \( \frac{1}{3} \) for two R(G) and one G(R) instruction sets. Thus, the correlation function is quantifying the fact that it is more likely we will get correlated outcomes (RR or GG), i.e., (+1)(+1) or (−1)(−1), than anti-correlated outcomes (RG
or GR), i.e., (+1)(–1) or (–1)(+1) for case (b) ($\frac{2}{3}$ correlated versus $\frac{1}{3}$ anti-correlated when the instruction sets contain two R(G) and one G(R)). Recall, the only other instruction sets RRR/GGG and GGG/RRR produce only anti-correlated results for case (b), so instruction sets give us $\langle \alpha, \beta \rangle < \frac{1}{3}$.

Now look at the case (b) correlation function using quantum probabilities instead of probabilities from instruction sets. As I stated above, the quantum probability of an RG(GR) outcome for case (b) is only $\frac{1}{8}$($\frac{1}{8}$). It is also the case that the quantum probability for an RR(GG) outcome for case (b) is $\frac{3}{8}$($\frac{3}{8}$), so Eq. (1) then gives

$$\langle \alpha, \beta \rangle = (+1)(-1)\left(\frac{1}{8}\right) + (-1)(+1)\left(\frac{1}{8}\right) + (+1)(+1)\left(\frac{3}{8}\right) + (-1)(-1)\left(\frac{3}{8}\right) = \frac{1}{2} \quad (7)$$

This means $\langle \alpha, \beta \rangle$ is skewed even more strongly towards +1 than its instruction sets counterpart ($\frac{1}{2}$ compared to less than $\frac{1}{3}$). Again, that makes sense since the quantum probability for RG or GR outcomes (skewing $\langle \alpha, \beta \rangle$ towards = –1 and away from +1) in case (b) is less than that for instruction sets ($\frac{1}{4}$ compared to at least $\frac{1}{3}$).

Thus, quantum mechanics predicts and we observe more strongly correlated outcomes for case (b) than its classical counterpart using instruction sets. Mermin’s challenge then amounts to explaining why that is true. Again, quantum mechanics predicts what is observed, no one disputes that, we just do not have any agreement (consensus) as to why it is true. In other words, how does the Mermin device work? As Bell stated (p. 140), “The scientific attitude is that correlations cry out for explanation.” While the case (a) result is prima facie easy to understand from conservation of angular momentum (explicit cancellation of +1 with –1 in every trial with the same settings), the case (b) result is not so clear. Quantum mechanics does not provide any model of physical reality or physical principle that explains why the case (b) correlation holds. That is the “quantum mystery for anybody,” at least for those who are not content to “shut up and calculate.”

III. THE CONSERVATION PRINCIPLE

To answer Mermin’s challenge, we will derive the quantum correlation function ($\langle \alpha, \beta \rangle$ as obtained from quantum probabilities) and quantum state (probability for each possibility) without using the formalism of quantum mechanics. Instead, we will use a physical principle accessible to the “general reader” interested in this mystery. This is akin to deriving time
dilation and length contraction from the light postulate of special relativity. The physical principle I will use is conservation per NPRF. This conservation principle requires only an understanding of introductory physics, so this derivation should be accessible to the “general reader” interested in this mystery.

Let us start with our target, the quantum correlation function. To compute $\langle \alpha, \beta \rangle$ using Eq. (1) for the spin singlet state, we need the quantum probabilities (given above) for different outcomes (ud or du) in the spin singlet state

$$P_{ud} = P_{du} = \frac{1}{2} \cos^2 \left( \frac{\alpha - \beta}{2} \right) \quad (8)$$

And, we need the quantum probabilities for the same outcomes (uu or dd) in the spin singlet state

$$P_{uu} = P_{dd} = \frac{1}{2} \sin^2 \left( \frac{\alpha - \beta}{2} \right) \quad (9)$$

Eq. (1) then gives the quantum correlation function for the spin singlet state when Alice’s SG magnet is at $\alpha$ and Bob’s SG magnet is at $\beta$, and Alice’s +1/–1 outcomes are labeled $i$ and Bob’s +1/–1 outcomes are labeled $j$ as

$$\langle \alpha, \beta \rangle = (+1)(-1)\frac{1}{2} \cos^2 \left( \frac{\alpha - \beta}{2} \right) + (-1)(+1)\frac{1}{2} \cos^2 \left( \frac{\alpha - \beta}{2} \right) +$$

$$(+1)(+1)\frac{1}{2} \sin^2 \left( \frac{\alpha - \beta}{2} \right) + (-1)(-1)\frac{1}{2} \sin^2 \left( \frac{\alpha - \beta}{2} \right) \quad (10)$$

$$= -\cos (\alpha - \beta) = -\cos (\theta)$$

where again I have used the fact that +1 corresponds numerically to up ($u$), –1 corresponds numerically to down ($d$), and $\alpha - \beta = \theta$ is the angle between Alice and Bob’s SG magnet orientations. I will now show that this quantum correlation function and the corresponding quantum state (Eqs. 8 & 9) can be derived using NPRF and the conservation of angular momentum on average. This makes the derivation well suited for the “general reader,” at least those familiar with introductory physics.

It is easy to see how this follows by starting with total angular momentum of zero for binary (quantum) outcomes $+1$ and $-1$. Alice and Bob both measure $+1$ and $-1$ results with equal frequency for any SG magnet angle (NPRF) and when their angles are equal they obtain different outcomes giving total angular momentum of zero (a defining aspect, Figure 6). The case (a) result is not difficult to understand via conservation of angular momentum, because Alice and Bob’s measured values of spin angular momentum cancel
directly when $\alpha = \beta$, that defines the spin singlet state. But, when Bob’s SG magnet is rotated by $\alpha - \beta = \theta$ relative to Alice’s, the situation is not as clear.

As I did above, divide Alice’s results into two subsets of $(+1)_A$ and $(-1)_A$, each occurring $\frac{1}{2}$ the time (again, the argument is symmetric with respect to Bob per NPRF). In introductory physics, one would say the projection of the angular momentum vector of Alice’s particle $\vec{S}_A = +1\hat{\alpha}$ along $\hat{\beta}$ is $\vec{S}_A \cdot \hat{\beta} = + \cos \theta$ where again $\theta$ is the angle between the unit vectors $\hat{\alpha}$ and $\hat{\beta}$. From Alice’s perspective, had Bob measured at the same angle, i.e., $\beta = \alpha$, he would have found the angular momentum vector of his particle was $\vec{S}_B = -1\hat{\alpha}$, so that $\vec{S}_A + \vec{S}_B = \vec{S}_{Total} = 0$. Since he did not measure the angular momentum of his particle at the same angle, he should have obtained a fraction of the length of $\vec{S}_B$, i.e., $\vec{S}_B \cdot \hat{\beta} = -1\hat{\alpha} \cdot \hat{\beta} = - \cos \theta$ (this also follows from counterfactual spin measurements on the single-particle state\textsuperscript{15}). Of course, Bob only ever obtains $+1$ or $-1$ (NPRF), so let us posit that Bob will average the required $- \cos \theta$ (Figures 7 & 8), which means

$$BA_+ = - \cos \theta$$

Likewise, for Alice’s $(-1)_A$ results we have

$$BA_- = \cos \theta$$

Putting these into Eq. (5) we obtain

$$\langle \alpha, \beta \rangle = \frac{1}{2}(+1)_A(- \cos \theta) + \frac{1}{2}(-1)_A(\cos \theta) = - \cos \theta$$

which is precisely the correlation function Eq. (10) given by quantum mechanics. As required by Mermin’s challenge, this derivation of the quantum correlation function is independent of the formalism of quantum mechanics, instead it follows from a compelling and simple physical principle, the conservation of angular momentum per NPRF.

We now use this conservation principle and NPRF to derive the quantum state, $P_{uu}$, $P_{dd}$, $P_{ud}$, and $P_{du}$. We need four independent conditions. Normalization gives

$$P_{uu} + P_{ud} + P_{du} + P_{dd} = 1$$

and our correlation function

$$\langle \alpha, \beta \rangle = (+1)_A(+1)_B P_{uu} + (+1)_A(-1)_B P_{ud} +$$

$$(-1)_A(+1)_B P_{du} + (-1)_A(-1)_B P_{dd}$$

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FIG. 7. A spatiotemporal (4D) ensemble of 16 experimental trials for the spin singlet state. Angular momentum is not conserved in any given trial, because there are two different measurements being made, i.e., outcomes are in two different reference frames, but it is conserved on average for all 16 trials. It is impossible for angular momentum to be conserved explicitly in each trial since the measurement outcomes are binary (quantum) with values of +1 (up) or –1 (down) per no-preferred reference frame. The conservation principle at work here assumes Alice and Bob’s measured values of angular momentum are not mere components of some hidden angular momentum with variable magnitude. That is, the measured values of angular momentum are the angular momenta contributing to this conservation.

along with our conservation principle represented by Eqs. (11), (12) & (13) give

\[ P_{uu} - P_{ud} = -\frac{1}{2} \cos \theta \] (16)

and

\[ P_{du} - P_{dd} = \frac{1}{2} \cos \theta \] (17)

Finally, NPRF gives \( P_{ud} = P_{du} \), since \( P_{ud} \) is Alice’s up results paired with Bob’s down results and \( P_{du} \) is Bob’s up results paired with Alice’s down results. Solving these four equations for \( P_{uu}, P_{dd}, P_{ud}, \) and \( P_{du} \) gives precisely Eqs. (8) & (9).

Since the angle between SG magnets \( \alpha - \beta \) is twice the angle between Hilbert space measurement bases, this result easily generalizes to conservation per NPRF of whatever the measurement outcomes represent when unlike outcomes entail conservation.

IV. DISCUSSION

Thus, if one subscribes to conservation per NPRF, the case (a) and case (b) results of the Mermin device follow (and violate the Bell inequality). We see explicitly in this result how quantum mechanics conforms statistically to a conservation principle without
FIG. 8. Reading from left to right, as Bob rotates his SG magnets relative to Alice’s SG magnets for her +1 outcome, the average value of his outcome varies from –1 (totally down, arrow bottom) to 0 to +1 (totally up, arrow tip). This obtains per conservation of angular momentum on average in accord with no preferred reference frame. Bob can say exactly the same about Alice’s outcomes as she rotates her SG magnets relative to his SG magnets for his +1 outcome. That is, their outcomes can only satisfy conservation of angular momentum on average, because they only measure +1/–1, never a fractional result. Thus, just as with the light postulate of special relativity, we see that no preferred reference frame requires quantum (+1/–1) outcomes for all measurements and that leads to a 4D view of physical reality.

need of a ‘causal influence’ or hidden variables acting on a trial-by-trial basis to account for that conservation. There are many attempts to add such classical mechanisms, but they are superfluous as far as the physics is concerned. So, contrary to Smolin’s claim that “[quantum mechanics] doesn’t make any sense because it’s wrong,” quantum mechanics is actually quite correct (complete) and consistent with relativity in this respect (makes perfect sense, Figure 9).

In special relativity, Alice is moving at velocity $V_a$ relative to a light source and measures the speed of light from that source to be $c$. Bob is moving at velocity $V_b$ relative to that same light source and measures the speed of light from that source to be $c$. Here “reference frame” refers to the relative motion of the observer and source which then defines a specific measurement of a specific quantity in the context of all its alternatives. NPRF in this context thus means all measurements produce the same outcome $c$. As a consequence of this constraint and NPRF proper (giving the relativity postulate of special relativity), one obtains the relativity of simultaneity and Wilczek’s 4D view of physical reality.

In quantum mechanics, Alice orients her SG magnet at $\alpha$ relative to a source of spin
FIG. 9. Comparing special relativity with quantum mechanics according to no preferred reference frame. Because Alice and Bob both measure the same speed of light $c$ regardless of their relative motion, Alice(Bob) may claim that Bob’s(Alice’s) length and time measurements are erroneous and need to be corrected (length contraction and time dilation). Likewise, because Alice and Bob both measure the same values for angular momentum $+1/–1\left(\frac{\hbar}{2}\right)$ regardless of their relative SG magnet orientation, Alice(Bob) may claim that Bob’s(Alice’s) individual $+1/–1$ values are erroneous and need to be corrected (averaged). It is possible that Alice and Bob’s outcomes are equally valid, i.e., neither need to be corrected, per no preferred reference frame. In special relativity, the apparently inconsistent results can be reconciled via relativity of simultaneity. In quantum mechanics, the apparently inconsistent results can be reconciled via ‘average-only’ conservation.

| Special Relativity                                      | Quantum Mechanics                                      |
|--------------------------------------------------------|--------------------------------------------------------|
| Empirical Fact: Alice and Bob both measure $c$, regardless of their relative motion | Empirical Fact: Alice and Bob both measure $+1/–1\left(\frac{\hbar}{2}\right)$, regardless of their relative SG orientation |
| Alice(Bob) says of Bob(Alice): Time dilation and length contraction | Alice(Bob) says of Bob(Alice): Must average results |
| NPRF: Relativity of simultaneity                        | NPRF: ‘Average-only’ conservation                      |

entangled particles and measures $+1$ or $–1$. Bob orients his SG magnet at $\beta$ relative to that same source of spin entangled particles and measures $+1$ or $–1$. Here “reference frame” refers to the relative orientation of the SG magnets and source which then defines a specific measurement of a specific quantity in the context of all its alternatives. NPRF in this context means all measurements produce the same outcome $+1$ or $–1$. As a consequence of this constraint, we can only conserve angular momentum on average in different reference frames, i.e., it cannot be conserved on a trial-by-trial basis unless the SG magnets or polarizers are co-aligned (Figure 6). As we saw, this constraint plus NPRF proper (giving $P_{+-} = P_{-+}$) then produce the quantum state, i.e., probability for each possible measurement outcome. This is quite unlike classical physics (Figure 7), in fact it is what uniquely distinguishes the quantum joint distribution from its classical counterpart. Thus, NPRF here leads to quantum outcomes ($+1/–1$ only) and ‘average-only’ conservation, a constraint in spacetime (Figure 8).
This is quite different from classical physics where Alice’s (Bob’s) particle would have a definite angular momentum $\vec{S}_A(\vec{S}_B)$ and it would be very unlikely that her (his) random choice of $\hat{\alpha}(\hat{\beta})$ would happen to align with $\vec{S}_A(\vec{S}_B)$. In that case, both Alice and Bob should measure various values of angular momentum, not +1 or –1 exclusively (recall the result “predicted by a classical analysis” in Figure 2). In the quantum case, Alice and Bob’s outcomes do not suggest any hidden or underlying $\vec{S}_A$ or $\vec{S}_B$, either individually or as correlated per the conservation of angular momentum. This conservation principle applies only to what is actually measured by Alice and Bob, regardless of their choices of SG magnet orientations.

Finally, while this conservation principle is compelling (who would argue with conservation per NPRF?), there are no hidden variables or ‘causal influences’ offered to underwrite it on a trial-by-trial basis. Indeed, since the conservation principle yields the quantum correlation function and the quantum correlation function is at odds with the results predicted by Mermin’s instruction sets, conservation per NPRF is not compatible with local hidden variables. The conservation principle in and of itself is invoked as a constraint to dispel the mystery of the Mermin device. In that sense, it is akin to Einstein’s light postulate of special relativity as invoked to explain time dilation or length contraction (Figure 9). In the case of the light postulate, Albert Michelson said:\textsuperscript{20} “It must be admitted, these experiments are not sufficient to justify the hypothesis of an ether. But then, how can the negative result be explained?” In other words, even Michelson of the Michelson-Morley experiment\textsuperscript{27} required some ‘deeper mechanism’ to explain why “the speed of light $c$ is the same in all reference frames.” Likewise here, if one requires some ‘deeper mechanism’ to explain conservation per NPRF, then this constraint is simply one mystery replacing another. Perhaps, as with special relativity, Nature is again hinting that physical reality is easier to model using the 4D view rather than the ant’s-eye view, precisely as Wilcek challenged. That is for the reader to decide.

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the A. Garg and N.D. Mermin paper cited herein.

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In Mermin’s paper, the results are the same when the settings are the same. At the end of the paper he then explains that Bob’s R(G) results actually correspond to Alice’s G(R) results per the spin singlet state. For this presentation, it is less confusing if R and G represent the spin outcomes +1 and −1, respectively, for both Alice and Bob.

The term “reference frame” has many meanings in physics related to microscopic and macroscopic phenomena, Galilean versus Lorentz transformations, relatively moving observers, etc. Here, a measurement configuration constitutes a reference frame, as with the light postulate of special relativity, as we will see below.

The light postulate of special relativity says “the speed of light c is the same in all reference frames.” This means if I am traveling towards you at half the speed of light 0.5c and shine a flashlight towards you, we will both measure the speed of the flashlight beam to be c. You would (incorrectly) expect that if the light moves away from me at c and I am moving towards you at 0.5c, then the light is moving towards you at 1.5c. As a consequence of this counterintuitive light postulate, we do not measure the same time or distance between events. This is called time dilation and length contraction, respectively.

I am not following Unnikrishnan’s argument, but instead I am using an argument based on NPRF.

I am suppressing the factor of $\hbar/2$.

What one observer claims are simultaneous events in spacetime will differ from what another observer in a different reference frame claims are simultaneous events in spacetime. This amounts
to saying different observers in different reference frames will slice spacetime differently into space plus time. Brian Greene has an excellent explanation of relativity of simultaneity (what he calls different ‘now’ slices) in “The Illusion of Time (Fabric of the Cosmos)” NOVA HD.

Michelson and Morley created a very precise measurement device for measuring the speed of light. At the time, it was believed that light was a wave in “the ether,” just as sound is a wave in the air. They used their device to measure differences in the speed of light due to Earth’s motion through the ether, but failed to detect any difference in the speed of light in agreement with the light postulate. That is what Michelson is calling “the negative result” in his quote.