Minimal lepton flavor violating realizations of minimal seesaw models

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Abstract

We study the implications of the global $U(1)_R$ symmetry present in minimal lepton flavor violating implementations of the seesaw mechanism for neutrino masses. In the context of minimal type I seesaw scenarios with a slightly broken $U(1)_R$, we show that, depending on the $R$-charge assignments, two classes of generic models can be identified. Models where the right-handed neutrino masses and the lepton number breaking scale are decoupled, and models where the parameters that slightly break the $U(1)_R$ induce a suppression in the light neutrino mass matrix. We show that within the first class of models, contributions of right-handed neutrinos to charged lepton flavor violating processes are severely suppressed. Within the second class of models we study the charged lepton flavor violating phenomenology in detail, focusing on $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$ and $\mu - e$ conversion in nuclei. We show that sizable contributions to these processes are naturally obtained for right-handed neutrino masses at the TeV scale. We then discuss the interplay with the effects of the right-handed neutrino interactions on primordial $B - L$ asymmetries, finding that sizable right-handed neutrino contributions to charged lepton flavor violating processes are incompatible with the requirement of generating (or even preserving preexisting) $B - L$ asymmetries consistent with the observed baryon asymmetry of the Universe.

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1 Introduction

The observation of neutrino flavor oscillations constitutes an experimental proof of lepton flavor violation [1]. In principle, other manifestations of such effects could be expected to show up in the charged lepton sector as well. However, the lack of a definitive model for neutrino mass generation implies that conclusive predictions for lepton flavor violating processes can not be made, and even assuming a concrete model realization for neutrino masses, predictions for such effects can only be done if the flavor structure of the corresponding realization is specified.

A problem that one faces when dealing with charged lepton flavor violating phenomenology is related with the arbitrariness of the free parameters that define these observables. In this regards the minimal lepton flavor violation (MLFV) hypothesis [5, 6, 7, 8] is a very useful guide for constructing predictive models in which lepton violating signals are entirely determined by low-energy neutrino data. The relationship between the free parameters and neutrino observables can arise via either a restrictive MLFV hypothesis or in some cases due to the intrinsic structure of the corresponding model (whenever the number of free parameters is comparable to the number of neutrino observables).

In the canonical seesaw (type-I) the kinetic sector of the model is invariant under a global large flavor symmetry group $G_F = SU(3)_e \times SU(3)_\ell \times SU(3)_N$, where $\ell$, $e$ and $N$ denote triplets in flavor space constructed from the electroweak lepton doublets and singlets and RH neutrinos. In addition there is an invariance under an extra global $U(1)_R$ [4, 8]. In principle the charges associated with this global transformation (hereafter denoted by $R$) are arbitrary, and thus different $R$-charge assignments define different models with their own consequences for lepton flavor violating phenomenology. In particular, models for which the $R$ charges allow for large Yukawa couplings and TeV RH neutrino masses should lead to sizable charged lepton flavor violating processes.

Consistent models of $\mathcal{O}$(TeV) RH states and large Yukawa couplings are achievable if cancellations among different pieces of the light neutrino mass matrix are allowed, and the RH neutrino mass spectrum is not strongly hierarchical [9]1. The class of TeV scale seesaw models arising from the presence of the $U(1)_R$ symmetry are expected to be in that sense different: no cancellations are needed because the suppression in the effective light neutrino mass matrix is no longer constrained to be related with the heaviness of the RH neutrinos.

In this paper we study the implications of a slightly broken $U(1)_R$ symmetry in minimal type-I seesaw models (with two RH neutrinos) 2 3. We show that the intrinsic structure of the relevant models leads to a flavor pattern completely determined

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1In the case of a large hierarchy among the different RH neutrino masses the one-loop finite corrections to the light neutrino mass matrix can exceed the corresponding tree-level contributions. Neglecting such corrections can in this case lead to a model inconsistent with neutrino data [10].
2Models with an arbitrary number of RH neutrinos were considered for the first time in [11].
3For related discussion within Type III and mixed Type I+III seesaw scenarios c.f. [12].
by low-energy neutrino observables, thus realizing in that way the MLFV hypoth-

thesis. This feature in addition to a slightly broken $U(1)_R$ leads to sizable $\mu \to e\gamma$, $\mu \to 3e$ and $\mu - e$ processes. This is in contrast to models where $U(1)_L$ is slightly broken so that lepton number and lepton flavor violation occur at the same scale.

In models with slightly broken lepton number it has been shown that leptogenesis is reconcilable with large charged lepton flavor violating rates, as the washouts induced by the RH neutrino states are controlled by the amount of lepton number violation [13]. We show however, that in the class of models considered here this statement does not hold, and that indeed leptogenesis and sizable charged lepton flavor violating effects are mutually exclusive. However, since both phenomena cover non-overlapped regions of parameter space their analyses are complementary. We therefore also explore the constraints on these models derived from the requirement of not erasing—via the RH neutrino dynamics—a preexisting $B - L$ asymmetry below the value consistent with the observed baryon asymmetry of the Universe.

The rest of this paper is organized as follows: in section 2 we discuss in detail the scenarios arising from the different $R$-charge assignments. In section 3 we discuss the phenomenology of charged lepton flavor violating decays in the representative models. In section 4 we analyze the implications of such constructions for scenarios of high scale baryogenesis by quantifying—via the Boltzmann equations—the effects of the RH neutrino dynamics on preexisting $B - L$ asymmetries. In section 5 we present our conclusions and final remarks. Explicit formulas used in the calculation of the different lepton flavor violating processes under study are given in appendix A.

## 2 The setups

The kinetic and gauge interaction Lagrangian of the standard model extended with two RH neutrinos exhibits a global $G = U(3)_e \times U(3)_\ell \times U(2)_N$ symmetry. Factorizing three $U(1)$ factors from $G$, the global symmetry can be rewritten as $U(1)_Y \times U(1)_L \times U(1)_R \times G_F$ where $U(1)_{Y,L}$ can be identified with global hypercharge and lepton number whereas the $U(1)_R$ is a “new” global symmetry [4, 8]. The remaining direct product group $G_F = SU(3)_e \times SU(3)_\ell \times SU(2)_N$ determines the flavor symmetry which is explicitly broken in the Yukawa sector.

In minimal lepton flavor violating seesaw models the Yukawa (mass) matrices are treated as spurion fields transforming under $G_F$ in such a way that the corresponding Yukawa (mass) terms in the leptonic Lagrangian remain invariant under the global flavor symmetry. The usual procedure is then based on an effective theory approach in which a set of non-renormalizable effective operators are constructed from the spurions.\(^4\) With the operators at hand and under certain restrictions on $G_F$ the lepton flavor violating effects can be estimated by means of low-energy neutrino data [5, 8]. Here, instead, we explicitly consider the seesaw Lagrangian with a slightly broken $U(1)_R$ and classify the possible realizations according to the $R$-charge

\(^4\)An exception are the explicit MLFV models discussed in Ref. [7].
assignments. Under this consideration, in the models featuring sizable lepton flavor violating effects, the flavor structure is determined by low-energy observables as well (up to a global normalization factor), not as a consequence of a restricted MLFV hypothesis but by the intrinsic structure of the resulting models.

Depending on the $R$-charge assignments two classes of generic models can be identified. Let us discuss this in more detail. Requiring $U(1)_R$ invariance of the charged lepton Yukawa terms determines $R(e)$ in terms of $R(\ell, H)$. After fixing $R(H) = 0$, to avoid charging the quark sector, the remaining charges can be fixed by starting with $R(N_{1,2})$. In order to have sizable lepton flavor violating effects both the $N_{1,2}$ Majorana mass terms should be suppressed by $R$-breaking parameters (generically denoted by $\epsilon$), so $R(N_{1,2}) \neq 0$. Thus one has only three possibilities: (A) $R(N_1) = R(N_2)$, (B) $R(N_1) = -R(N_2)$ and (C) $|R(N_1)| \neq |R(N_2)|$. The phenomenology of case (C), however, is expected to be similar to the one arising from models with $R$-charge assignments of type (A). The reason being that in that case the $N_1 - N_2$ mixing is always $U(1)_R$ suppressed, which in turn implies suppressed charged lepton flavor violating processes (equivalently, the effective neutrino mass matrix will not be $\epsilon$ suppressed, forcing tiny Yukawas or heavy $N_{1,2}$ states). This is to be compared with models based on $R$-charge assignments of type (B), where the $N_1 - N_2$ turns out to be maximal and a set of unsuppressed $\ell - N$ Yukawa couplings can be always obtained by properly chosing $R(\ell)$ (in these models the effective neutrino mass matrix involves extra $\epsilon$ suppression factors).

In summary depending on the $R$-charge assignments two classes of generic models can be identified: models in which the mechanism that suppresses the light neutrino masses propagates to lepton flavor violating observables, thus rendering their values far below planned experimental sensitivities; and models in which the mechanism decouples in such a way that lepton flavor violating effects become sizable. In what follows, in order to illustrate this is actually the case, we will discuss two examples of models of type A and B.

**Type A models** [8]: $R(N_a) = +1$ and $R(\ell, e, H) = 0$ ($H$ being the Higgs electroweak doublet)

In this case, in the basis in which the charged lepton Yukawa couplings and the Majorana RH neutrino mass matrix are diagonal, the Lagrangian reads

$$\mathcal{L} = -\bar{\ell} \tilde{Y}_e e H - \epsilon \bar{\ell} \lambda^* N \tilde{H} - \frac{1}{2} \epsilon^2 \mu N^T C \tilde{Y}_N N + \mathrm{h.c.}.$$  \hspace{1cm} (1)

Here $\tilde{H} = i\sigma_2 H^*$, $C$ is the charge conjugation operator and $\tilde{Y}_e, \lambda^*$ and $\tilde{Y}_N = \text{diag}(Y_{N_1}, Y_{N_2})$ are the Yukawa coupling matrices (we denote matrices in bold-face). The dimensionless parameter $\epsilon \ll 1$ slightly breaks $U(1)_R$ whereas, due to the assignment $L(N) = 1$, the lepton number $U(1)_L$ factor is broken by $\mu$. With this setup the $5 \times 5$ neutral fermion mass matrix can be written as

$$M_N = \begin{pmatrix} 0 & \epsilon v \lambda \\ \epsilon v^T \lambda & \epsilon^2 \mu \tilde{Y}_N \end{pmatrix},$$  \hspace{1cm} (2)
where \( \langle H \rangle = v \simeq 174 \) GeV. In the seesaw limit, which in this case reads \( v \lambda \ll \epsilon \mu \hat{Y}_N \)
the effective light neutrino mass matrix is given by

\[
m_{\nu}^{\text{eff}} = -\frac{v^2}{\mu} \sum_{a=1,2} \frac{\lambda_a \otimes \lambda_a}{Y_{Na}},
\]

(3)

with \( \lambda_a = (\lambda_{ea}, \lambda_{\mu a}, \lambda_{\tau a}) \). The corresponding light neutrino masses are obtained
from the leptonic mixing matrix \( U = V \hat{P} \) (with \( V \) having a CKM form and \( \hat{P} = \text{diag}(e^{i\phi}, e^{i\phi}, 1) \) containing the Majorana CP phase) after diagonalization:

\[
U^T m_{\nu}^{\text{eff}} U = \hat{m}_{\nu}^{\text{eff}}.
\]

(4)

In the 2 RH neutrino mass model the constrained parameter space enforces one of
the light neutrinos to be massless. Thus, in the normal hierarchical mass spectrum
case \( m_{\nu_1} = 0 \) and \( m_{\nu_2} < m_{\nu_3} \) whereas in the inverted case \( m_{\nu_3} = 0 \) and \( m_{\nu_1} < m_{\nu_2} \).

Since the dimension five effective operator is \( U(1)_R \) invariant the neutrino mass
matrix does not depend on \( \epsilon \); the suppression ensuring light neutrino masses is solely
provided by the lepton number breaking parameter \( \mu \). On the other hand, the RH
neutrino mass spectrum is determined by

\[
\hat{M}_N = \epsilon^2 \mu \hat{Y}_N.
\]

(5)

From this expression it can be seen that as long as the \( U(1)_R \) global symmetry is
an approximate symmetry of the Lagrangian (\( \epsilon \ll 1 \)) the RH neutrino mass scale is
decoupled from the lepton number violating scale. Thus, the RH neutrino masses
do not lie at the same scale at which lepton number breaking takes place.

Assuming \( \hat{Y}_N, \lambda \lesssim \mathcal{O}(1) \) an estimation of the lepton number breaking parameter
\( \mu \sim 10^{15} \) GeV can be obtained using \( \sqrt{\Delta m_{31}^2} \sim 0.05 \) eV [14] as a measure of the
largest light neutrino mass in these scenarios. From this estimation and eq. (5) it
can be seen that values of \( \epsilon \) of the order of \( \sim 10^{-6} \) allow to lower the lightest RH
neutrino mass below 1 TeV.

Formal invariance of the Lagrangian under \( G_F \) is guaranteed if the Yukawa matrices,
promoted to spurion fields, transform according to

\[
Y_e \sim (\bar{3}_e, 3_\ell, 1_N), \quad \lambda^* \sim (1_e, 3_\ell, \bar{2}_N), \quad Y_N \sim (1_e, 1_\ell, 3_N).
\]

(6)

The constraints imposed by \( G_F \) imply

\[
\lambda = \lambda_\ell \otimes \lambda_N,
\]

(7)

where \( \lambda_\ell \) is a \( SU(3)_\ell \) triplet and \( \lambda_N \) a \( SU(2)_N \) doublet in flavor space. Accordingly,
in these kind of models a unequivocal determination of the flavor structure via the
MLFV hypothesis is possible by means of a restrictive flavor symmetry \( G'_F \subset G_F \).
Though several possibilities may be envisaged we do not discuss further details since,
as we show below, in this type of models contributions to lepton flavor violating
processes of charged leptons are always negligible.
Type B models: $R(N_1, \ell_i, e_i) = +1$, $R(N_2) = -1$, $R(H) = 0$. Changing $R$ charges to $L$ charges, this case resembles models where lepton number is slightly broken (see for example [7, 15, 16, 17]). The Lagrangian is given by

$$\mathcal{L} = -\bar{\ell} Y_e e H - \bar{\ell} \lambda_1^* N_1 \tilde{H} - \epsilon_\lambda \bar{\ell} \lambda_2^* N_2 \tilde{H} - \frac{1}{2} N_1^T C M N_2 - \frac{1}{2} \epsilon_N N_a^T C M_{aa} N_a + \text{h.c.}. \quad (8)$$

The $\epsilon_{\lambda, N}$ parameters determine the amount of $U(1)_R$ breaking and are thus tiny.

The diagonalization of the Majorana RH neutrino mass matrix leads to two quasi-degenerate states with masses given by

$$M_{N_{1,2}} = M \mp \frac{M_{11} + M_{22}}{2} \epsilon_N. \quad (9)$$

In the basis in which the RH neutrino mass matrix is diagonal the Yukawa couplings become

$$\lambda_{ka} \rightarrow -\frac{(i)^a}{\sqrt{2}} \left[ \lambda_{k1} + (-1)^a \epsilon_\lambda \lambda_{k2} \right], \quad (k = e, \mu, \tau \text{ and } a = 1, 2), \quad (10)$$

and the $5 \times 5$ neutral fermion mass matrix is similar as in type A models. However, due to the structure of the Yukawa couplings the effective light neutrino matrix, up to $O(\epsilon_N \epsilon_\lambda^2)$, has the following form

$$m_{\nu}^{\text{eff}} = -\frac{v^2 \epsilon_\lambda}{M} |\lambda_1|||\Lambda| \left( \hat{\lambda}_1^* \otimes \hat{\Lambda}^* + \hat{\Lambda}^* \otimes \hat{\lambda}_1^* \right), \quad (11)$$

with

$$\hat{\Lambda}^* = \hat{\lambda}_2^* - \frac{M_{11} + M_{22}}{4M} \frac{\epsilon_\lambda}{\epsilon_N} \hat{\lambda}_1^*. \quad (12)$$

Here with the purpose of relating the flavor structure of these models with low energy observables, and following ref. [7], we expressed the parameter space vectors $\lambda_1$, $\Lambda$ in the light neutrino mass matrix in terms of their moduli $|\lambda_1|$, $|\Lambda|$ and unitary vectors $\hat{\lambda}_1$, $\hat{\Lambda}$.

Note that in these models lepton number is broken even when $U(1)_R$ is an exact symmetry of the Lagrangian. However due to the Yukawa structure and degeneracy of the RH neutrino mass spectrum at this stage $m_{\nu}^{\text{eff}} = 0$. Although a non-zero Majorana neutrino mass matrix arises only once the $R$ breaking terms are present this does not imply that in the absence of lepton number violating interactions a Majorana mass matrix can be built. In that case—as can seen from eq. (8)—only Dirac masses can be generated, as it must be.

Since $\epsilon_\lambda \ll 1$ small neutrino masses do not require heavy RH neutrinos or small Yukawa couplings, thus potentially implying large lepton flavor violating effects. In that sense, as already stressed, these models resemble those in which lepton number is slightly broken but with lepton number as well as lepton flavor violation taking place at the same scale $M$. 

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In contrast to models of type A, in this case due to the structure of the light Majorana neutrino mass matrix the vectors $\lambda_1$ and $\Lambda$ can be entirely determined by means of the solar and atmospheric mass scales and mixing angles, up to the factors $|\lambda_1|$ and $|\Lambda|$, without invoking a restrictive MLFV hypothesis. The relations are different for normal and inverted light neutrino mass spectra [7]:

- **Normal hierarchical mass spectrum**

  \[ \lambda_1 = |\lambda_1| \hat{\lambda}_1 = \frac{|\lambda_1|}{\sqrt{2}} \left( \sqrt{1 + \rho U_3^*} + \sqrt{1 - \rho U_2^*} \right), \quad (13) \]

  \[ \Lambda = |\Lambda| \hat{\Lambda} = \frac{|\Lambda|}{\sqrt{2}} \left( \sqrt{1 + \rho U_3^*} - \sqrt{1 - \rho U_2^*} \right), \quad (14) \]

  where $U_i$ denote the columns of the leptonic mixing matrix and

  \[ \rho = \frac{\sqrt{1 + r - \sqrt{r}}}{\sqrt{1 + r + \sqrt{r}}}, \quad r = \frac{m_{\nu_2}^2}{m_{\nu_3}^2 - m_{\nu_2}^2}. \quad (15) \]

- **Inverted hierarchical mass spectrum**

  \[ \lambda_1 = |\lambda_1| \hat{\lambda}_1 = \frac{|\lambda_1|}{\sqrt{2}} \left( \sqrt{1 + \rho U_2^*} + \sqrt{1 - \rho U_1^*} \right), \quad (16) \]

  \[ \Lambda = |\Lambda| \hat{\Lambda} = \frac{|\Lambda|}{\sqrt{2}} \left( \sqrt{1 + \rho U_2^*} - \sqrt{1 - \rho U_1^*} \right), \quad (17) \]

  with

  \[ \rho = \frac{\sqrt{1 + r - 1}}{\sqrt{1 + r + 1}}, \quad r = \frac{m_{\nu_2}^2 - m_{\nu_1}^2}{m_{\nu_1}^2}. \quad (18) \]

With these results at hand we are now in a position to calculate the most relevant lepton flavor violating processes, which we discuss in turn.

### 3 Lepton flavor violating processes

In type A models the RH neutrino masses can be readily at the TeV scale for $\epsilon \sim 10^{-6}$. Since the Yukawa couplings scale with $\epsilon$ as well, type A models are—in that sense—on the same footing as the canonical type-I seesaw model i.e. TeV RH neutrino masses imply tiny Yukawa couplings and thus negligible charged lepton flavor violating decay branching ratios. The main difference is that in the canonical case, sizable charged lepton flavor violation can still be induced via fine-tuned cancellations in the effective neutrino mass matrix, while no such effects are possible in the minimal type A models, since the relevant couplings are completely determined by light neutrino mass and mixing parameters.
Type B models, in contrast, may exhibit naturally large Yukawa couplings even for TeV scale RH neutrino masses (or even lighter masses, depending on the value of the $U(1)_R$ breaking parameter $\epsilon_\lambda$). Accordingly, several charged lepton flavor violating transition rates—induced by the RH neutrinos at the 1-loop level—can in principle reach observable levels. In what follows we study the allowed mass and Yukawa normalization factor ranges by considering the following lepton flavor violating processes: $l_i \rightarrow l_j \gamma$, $l_i \rightarrow 3l_j$ and $\mu - e$ conversion in nuclei.

3.1 $l_i \rightarrow l_j \gamma$ processes

Among these lepton flavor violating processes, presently the $\mu \rightarrow e\gamma$ transition is most severely constrained. The MEG collaboration recently established an upper bound of $2.4 \times 10^{-12}$ at the 90% C.L. [18]. For $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ on the other hand, the bounds are $3.3 \times 10^{-8}$ and $4.4 \times 10^{-8}$ at 90% C.L. [20], respectively. In the limit $m_{l_j} \ll m_{l_i}$ the partial decay width for $l_i \rightarrow l_j \gamma$ processes reads [21]

$$\Gamma(l_i \rightarrow l_j \gamma) = \frac{\alpha \alpha_W^2}{256 \pi^2} \frac{m_i^5}{M_W^4} \left| G_{ij}^{\ell\gamma} \right|^2 = \frac{\alpha}{1024 \pi^4} \frac{m_i^5}{M_W^4} \left| (\lambda Y_N \lambda^\dagger)^{ij}_l \right|^2, \quad (19)$$

where $M_W$ is the $W^\pm$ mass, $\alpha_W = g^2/4\pi$ and the elements of the diagonal matrix $G_\gamma$ are given in eq. (37) in the appendix. This function is such that in the limit $M_{N_a} \gg M_W$, $(G_\gamma)_{aa} \rightarrow M_W^2/2M_{N_a}^2$. The corresponding decay branching ratios are determined from the partial decay width after normalizing to $\Gamma_{\text{Tot}}^{l_i} = \hbar \tau_l$, with $\tau_l$ the $l_i$ charged lepton mean lifetime. In the limit $r_a \ll 1$, using eq. (5) and taking into account the Yukawa rescaling $\lambda \rightarrow \epsilon \lambda$ the decay branching ratio in type A models can be written as

$$\text{BR}(l_i \rightarrow l_j \gamma) \simeq \frac{\alpha}{4096 \pi^4} \frac{m_i^5}{\mu^4 \epsilon^4} \frac{1}{\Gamma_{\text{Tot}}^{l_i}} \left| (\lambda Y_N \lambda^\dagger)^{ij}_l \right|^2. \quad (20)$$

Thus, assuming $O(\lambda, Y_N) \sim 1$, for which $\mu \sim 10^{15}$ GeV and taking $\epsilon = 10^{-6}$, the value required for $O(\text{TeV})$ RH neutrino masses, we get $\text{BR}(\mu \rightarrow e\gamma) \simeq 10^{-30}$. This behavior, being extensible to other lepton flavor violating processes, shows that in type A models lepton flavor violating effects are negligibly small.

In type B models in contrast such lepton flavor violating effects may be sizable. Using expression (10) for the Yukawa couplings, neglecting the piece proportional to $\epsilon_\lambda$ and taking the limit $M_{N_a} \gg M_W$ the decay branching ratios can be expressed in terms of the parameters $\lambda_1$:

$$\text{BR}(l_i \rightarrow l_j \gamma) \simeq \frac{\alpha}{1024 \pi^4} \frac{m_i^5}{M_W^4} \frac{1}{\Gamma_{\text{Tot}}^{l_i}} \left| \lambda_{ii} \lambda_{jj}^* \right|^2. \quad (21)$$

Since the components of the unitary vector $\hat{\lambda}_1$ are entirely determined by low-energy observables (see eqs. (13) and (16)) the size of these branching ratios—and all the
Figure 1: Decay branching ratio $BR(\mu \rightarrow e\gamma)$ normalized to $|\lambda_1|^4$ for normal light neutrino mass spectrum as a function of the common RH neutrino mass (left hand side plot). The upper horizontal dashed line indicates the current limit on $BR(\mu \rightarrow e\gamma)$ from the MEG experiment [18], whereas the lower dotted one marks prospective future experimental sensitivities [19]. The corresponding bounds on $|\lambda_1|$ from the present [18] (upper gray band) and prospective future [19] (lower brown band) experimental searches are shown in the right hand side plot. The widths of the bands are due to the uncertainties in the neutrino mass matrix parameters. The results for the inverted neutrino spectrum are very similar and are thus not shown separately.

In order to quantify the size of these lepton flavor violating effects we randomly generate neutrino masses, mixing angles and Dirac and Majorana CP violating phases in their $2\sigma$ ranges for both normal and inverted hierarchical light neutrino mass spectra [1]. We also randomly generate the parameters $|\lambda_1|$ and $M$ in the ranges $[10^{-5}, 1]$ and $[10^2, 10^6]$ GeV allowing RH neutrino mass splittings in the range $[10^{-8}, 10^{-6}]$ GeV. With the numerical output we calculate the different lepton flavor violating decay branching ratios from eq. (19), using the full loop function given in the appendix, eq. (37).

We find that radiative $\tau$ decay rates are always below their current bounds and barely reach values of $10^{-9}$ for RH neutrino masses around 100-200 GeV (values exceeding the current bounds are not consistent with the seesaw condition, that for concreteness we take as $m_D M_N^{-1} < 10^{-1}$), we thus focus on the $\mu \rightarrow e\gamma$ process. The results for the normal mass spectrum case are displayed in fig. 1 as a function of the common RH neutrino mass, $M_N = M$. We observe that $BR(\mu \rightarrow e\gamma)$ can reach the current experimental limit reported by the MEG experiment [18] for RH neutrino masses $M_N < 0.1$ TeV, $1$ TeV, $10$ TeV provided $|\lambda_1| \gtrsim 2 \times 10^{-2}, 10^{-1}, 1$, respectively. The results for the inverted light neutrino mass hierarchy are very similar and consistent with these values. Finally we note that the widths of the
bands in fig. 1 (and similarly for all the other considered processes below) are solely due to the uncertainties in the light neutrino mass matrix parameters (mainly $\theta_{13}$ and the CP violating phases) and can thus be improved with more precise light neutrino data.

### 3.2 $l_i^- \rightarrow l_j^- l_j^− l_j^+$ processes

The decay branching ratios for these processes have been calculated in [21]. The most constrained process in this case is $\mu^- \rightarrow e^+e^-e^-$ for which the SINDRUM experiment has placed a bound on the decay branching ratio of $10^{-12}$ at 90% C.L. [22]. For $\tau^- \rightarrow e^+e^-e^-$ and $\tau^- \rightarrow \mu^+\mu^-\mu^-$ the current bounds are $2.7 \times 10^{-8}$ and $2.1 \times 10^{-8}$, respectively [23]. The decay branching ratios for these lepton flavor violating reactions are given by [21]

$$\text{BR}(l_i^- \rightarrow l_j^+ l_j^- l_j^+ ) = \frac{\alpha_W^4}{24576\pi^3} \frac{m_i^5}{M_W^2} \frac{1}{\Gamma_{\text{Total}}} \left\{ 2 \left| \frac{1}{2} F_{\text{Box}}^{d,l_j} + F_Z^{d,l_j} - 2s_W^2 \left( F_Z^{d,l_j} - F_{\gamma}^{d,l_j} \right) \right|^2 + 4s_W^4 \left| F_Z^{d,l_j} - F_{\gamma}^{d,l_j} \right|^2 + 16s_W^2 \text{Re} \left[ \left( F_Z^{d,l_j} + \frac{1}{2} F_{\text{Box}}^{d,l_j} \right) G_{\gamma}^{d,l_j} \right] \right\}$$

$$+ 48s_W^4 \text{Re} \left[ \left( F_Z^{d,l_j} - F_{\gamma}^{d,l_j} \right) G_{\gamma}^{d,l_j} \right] + 32s_W^4 \left| G_{\gamma}^{d,l_j} \right|^2 \left( \ln \frac{m_i^2}{m_j^2} - \frac{11}{4} \right).$$

(22)

Here $s_W = \sin \theta_W$, where $\theta_W$ is the weak mixing angle, and the functions $F_{\gamma}^{d,l_j}$, $F_Z^{d,l_j}$ and $F_{\text{Box}}^{d,l_j}$ are form factors that involve the Yukawa couplings and loop functions arising from the $\gamma$, $Z$ penguins and box diagrams that determine the full process (see appendix A for a compilation of these expressions and ref. [21] for their derivation).

Following the same numerical procedure as in the $l_i \rightarrow l_j \gamma$ case and using the form factors given in the appendix we evaluate the $\mu^+ \rightarrow e^+e^-e^-$, $\tau^+ \rightarrow e^+e^-e^-$ and $\tau^+ \rightarrow \mu^+\mu^-\mu^-$ decay branching ratios for both, the normal and inverted light neutrino mass spectra. We find that $\tau^+ \rightarrow e^+e^-e^-$ and $\tau^+ \rightarrow \mu^+\mu^-\mu^-$ processes are always below $\sim 10^{-9}$ (due to the constraint enforced by the seesaw condition when $|\lambda_1| > 10^{-1}$), so in fig. 2 we only display the results for $\mu^+ \rightarrow e^+e^-e^-$. We observe that the branching ratio can saturate the current experimental bound for RH neutrino masses $M_N < 0.1 \text{ TeV}, 1 \text{ TeV}, 10 \text{ TeV}$ provided $|\lambda_1| \gtrsim 2 \times 10^{-2}, 10^{-1}, 1$, respectively, very similar to the $\mu \rightarrow e\gamma$ case. The results for the inverted light neutrino mass hierarchy are again very similar and consistent with these values. As can be seen by comparing figs. 1 and 2, with the sensitivities of the planned future experiments this process has the potential to probe considerably larger values of

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[5] The proposed Mu3e experiment at PSI aims for a sensitivity of $10^{-15}$ in its first phase and
Figure 2: Decay branching ratio $BR(\mu^- \rightarrow e^- e^+ e^-)$ normalized to $|\lambda_1|^4$ for normal light neutrino mass spectrum as a function of common RH neutrino mass (left hand side plot). The upper horizontal dashed line indicates the current bound on the $\mu^- \rightarrow e^+ e^- e^-$ rate placed by the SINDRUM experiment [22], whereas the lower dotted one illustrates prospective future experimental sensitivities of the Mu3e experiment [24]. The corresponding bounds on $|\lambda_1|$ from the present [22] (upper gray band) and prospective future [24] (lower brown band) experimental searches are shown in the right hand side plot. The widths of the bands are due to the uncertainties in the neutrino mass matrix parameters. The results for the inverted neutrino spectrum are very similar and are thus not shown separately.

the RH neutrino masses (compared with $\mu \rightarrow e \gamma$), reaching RH neutrino mass scales in excess of $\mathcal{O}(10^5 \text{ GeV})$ for $|\lambda_1| \sim 1$. Finally we note that due to the strong $|\lambda_1|$ dependence, values of $|\lambda_1|$ below $10^{-3}$ are not expected to yield observable rates at near future experimental facilities even for RH neutrino masses of the order 100 GeV.

3.3 $\mu - e$ conversion in nuclei

Competitive lepton flavor violation constraints can also be obtained from searches for $\mu - e$ conversion in nuclei. Currently the strongest bounds on $BR_{\mu e} \equiv \Gamma_{\text{conversion}}/\Gamma_{\text{capture}}$ were set by the SINDRUM collaboration from experiments on titanium with $BR_{\mu e}^{(\text{Ti})} < 4.3 \times 10^{-12}$ [25] and gold target setting $BR_{\mu e}^{(\text{Au})} < 7 \times 10^{-13}$ [26], both at 90%CL. The $\mu - e$ conversion bounds are expected to be further improved in the future by several orders of magnitude. According to proposals [27] and [28, 29], one can expect a sensitivity of $10^{-16}$ or even $10^{-18}$ by the PRISM/PRIME experiment.

To get the constraint in the $\mu - e$ channel from these experiments, one needs to compute the relevant transition matrix elements in different nuclei. A detailed numerical calculation has been carried out by [30] and we use their formula in eq. (14) to calculate the desired conversion rates. They receive one-loop contributions from photonic penguins contributing to both effective dipole ($A_R$) and vector ($g_{LV}^{(u,d)}$)
Nucleus | $D[m^5/2]$ | $V^{(p)}[m^5/2]$ | $V^{(n)}[m^5/2]$ | $\Gamma_{\text{capture}}[10^6 \text{s}^{-1}]$
---|---|---|---|---
$^{48}\text{Ti}$ | 0.0864 | 0.0396 | 0.0468 | 2.59
$^{197}\text{Au}$ | 0.189 | 0.0974 | 0.146 | 13.07

Table 1: Data taken from Tables I and VIII of [30].

couplings, as well as $Z$ penguins and $W$ box diagrams (these only contribute to $g^{(u,d)}_{\text{LV}}$). Using the notation of [30] we thus have

$$\Gamma_{\text{conversion}} = 2G^2_F \left| A^*_R D + (2g^{(u)}_{\text{LV}} + g^{(d)}_{\text{LV}}) V^{(p)} + (g^{(u)}_{\text{LV}} + 2g^{(d)}_{\text{LV}}) V^{(n)} \right|^2 ,$$  \hspace{1cm} (23)

where $G_F$ is the standard model Fermi coupling constant and $A_R, g^{(u,d)}_{\text{LV}}$ are found to be $(Q_{u,d} = 2/3, -1/3)$:

$$A^*_R = \frac{\sqrt{2}}{8G_F M^2_W} \frac{\alpha_W}{8\pi} G^{\mu e}_{\gamma} ,$$ \hspace{1cm} (24)

$$g^{(u)}_{\text{LV}} = \frac{\sqrt{2} \alpha^2_W}{8G_F M^2_W} \left[ (F^\mu e_Z + 4F^{\mu 3e}_{\text{Box}}) - 4Q_u s^2_W (F^\mu e_{\gamma} - F^{\mu e}_{\gamma}) \right] ,$$ \hspace{1cm} (25)

$$g^{(d)}_{\text{LV}} = -\frac{\sqrt{2} \alpha^2_W}{8G_F M^2_W} \left[ (F^\mu e_Z + F^{\mu 3e}_{\text{Box}}) + 4Q_d s^2_W (F^\mu e_{\gamma} - F^{\mu e}_{\gamma}) \right] .$$ \hspace{1cm} (26)

Following the same numerical procedure as in sections 3.1 and 3.2 we evaluate the resulting $\mu - \epsilon$ conversion branching ratios. Since both Ti and Au processes feature the same flavor structure, the differences between them are entirely determined by the numerical factors quoted in table 1. The Ti parameters entering in the conversion rate are on average a factor $\sim 2.5$ smaller than the ones for Au, whereas the capture rates differ by a factor $\sim 5$. Accordingly the difference between these branching ratios is a factor of $\sim 2$. Due to its more stringent experimental upper bound we thus display only the results for Au in fig. 3 for the case of the normal light neutrino mass spectrum (the differences with the inverted mass spectrum case are again tiny). For $M_N \sim 1 \text{ TeV}$ the pieces proportional to $V^{(p),(n)}$ in eq. (23) cancel, so the $\mu - \epsilon$ conversion rate around this RH neutrino mass value is mainly controlled by the dipole contribution, $A^*_R D$, which is roughly two orders of magnitude smaller. The dip in fig. 3 is due to this cancellation.

From fig. 3 it can be seen that the current experimental bound on this process imposes a constraint on the RH neutrino mass (as a function of $|\lambda_1|$), which is roughly a factor of 2 stronger compared to bounds from $\mu \to e \gamma$ and $\mu \to 3e$ (except for the region around $M_N \sim 1 \text{ TeV}$, as explained above). Furthermore, given the expected future sensitivities, $\mu^- Au^{197}_{79} \to e^- Au^{197}_{79}$ (and $\mu^- Ti^{48}_{22} \to e^- Ti^{48}_{22}$) could probe RH neutrino masses up to $O(10^3 \text{ TeV})$, far above the values accessible in $\mu \to e \gamma$ and $\mu^- \to e^- \epsilon^+\epsilon^-$, and thus constitutes the primary search channel for such scenarios of heavy RH neutrinos.
Figure 3: Branching ratio \( BR(\mu^- Au^{197}_{79} \rightarrow e^- Au^{197}_{79}) \) normalized to \(|\lambda_1|^4\) as a function of the common RH neutrino mass for the light neutrino normal mass spectrum (left hand side plot). The upper horizontal dashed line indicates the current bound on this process settled by the SINDRUM experiment [26], whereas the lower one indicates future experimental sensitivities of the PRISM/PRIME experiment [27, 28, 29]. The corresponding bounds on \(|\lambda_1|\) from the present [26] (upper gray band) and prospective future [27, 28, 29] (lower brown band) experimental searches are shown in the right hand side plot. The widths of the bands are due to the uncertainties in the neutrino mass matrix parameters. The results for the inverted neutrino spectrum are very similar and are thus not shown separately.

4 Primordial lepton asymmetries

We now turn to the issue of primordial lepton asymmetries and the related dynamics of the RH neutrinos. As we have discussed, different \( R \)-charge assignments allow to define two types of models of which type B may yield sizable charged lepton flavor violating decays. For these effects to take place RH neutrino masses at or below the TeV scale as well as Yukawa couplings of order \( 10^{-2} \) or larger are needed. The washouts induced by such couplings and in this mass range are so large that any lepton asymmetry generated via the out-of-equilibrium decays of the RH neutrinos will always yield a baryon asymmetry much smaller than the observed one [31].

Either producing a baryon asymmetry consistent with the observed value or not erasing a preexisting one via the dynamics of the RH neutrino states (in case the RH neutrinos are still light and the resonant condition \( M_{N_2} - M_{N_1} \sim \Gamma_{N_1} \) is not satisfied) requires small Yukawa couplings, thus rendering charged lepton flavor violating decay branching ratios negligibly small. The phenomenological requirements of sizable charged lepton flavor violating effects and the generation of a \( B-L \) asymmetry (or of not erasing a preexisting one) are therefore mutually exclusive. Since these requirements cover non-overlapped regions in parameter space they are from that point of view complementary.

\[ \text{In models with a slightly broken lepton number the washout is tiny, as it is determined by the amount of lepton number violation \[13\]. In our case since lepton number is broken even in the } U(1)_R \text{ symmetric phase the washouts are dominated—as usual—by } N_{1,2} \text{ inverse decays.} \]
The generation of a $B - L$ asymmetry in the type B models discussed here follows quite closely the analysis done in ref. [32]. Thus, we do not discuss this issue here and instead study the constraints on parameter space derived from the condition of not erasing an assumed preexisting $B - L$ asymmetry. Note that in type-I seesaw models with flavor symmetries in the lepton sector, as for example in MLFV models, the CP violating asymmetry in RH neutrino decays vanishes in the limit of exact flavor symmetry [33]. However, since in type B models the MLFV hypothesis is a consequence of the intrinsic structure of the model this does not happen.

In order to quantify these effects from now on we focus on the normal hierarchical light neutrino spectrum. Results for the inverted hierarchical case resemble quite closely the ones reported here. We start by recalling that the washouts induced by both RH neutrino states (at $T \sim M$) on any primordial $B - L$ asymmetry are determined by the following set of kinetic equations:

$$\frac{dY_{\Delta_i}}{dz} = -\frac{\kappa_i}{4} \sum_{j=e,\mu,\tau} C_{ij}^{(\ell)} Y_{\Delta_j} K_1(z) z^3.$$  \hspace{1cm} (27)

Here $Y_X = (n_X - n_X)/s$ (where $n_X$ is the number density of particle $X$ and $s$ is the entropy density), $z = M/T$ and $\Delta_i = B/3 - L_i$ with $L_i = 2\ell_i + e_i$. The function $K_1$ is the modified Bessel function of the first type and the flavor coupling matrix $C^{(\ell)}$ is determined by the chemical equilibrium conditions imposed by the reactions that at the relevant temperature regime ($T \sim M$) are in thermal equilibrium [34]. The parameter $\kappa_i$, that determines the strength of the flavored washouts, is given by

$$\kappa_i = \frac{\tilde{m}_i}{m_*} \text{ where } \tilde{m}_i = \frac{v^2}{M} |\lambda_{i1}|^2.$$  \hspace{1cm} (28)

The factor $m_* \simeq 1.1 \times 10^{-12}$ GeV. Note that in the basis in which the RH Majorana neutrino mass matrix is diagonal $N_{1,2}$ couple to the lepton doublets with strength $\lambda_{i1}$, the factor 2 in $\tilde{m}_i$ is due to this fact.

According to the parametrization in eq. (13) the $\kappa_i$ parameters can be written as

$$\kappa_i = \frac{v^2}{m_*} |\lambda_{i1}|^2 \left| \tilde{\lambda}_{i1} \right|^2 = \frac{v^2}{m_*} |\lambda_{i1}|^2 \left| \sqrt{1 + \rho U_{i3}^*} + \sqrt{1 - \rho U_{i2}^*} \right|^2.$$  \hspace{1cm} (29)

Thus, after fixing low-energy observables the values of the parameters $\kappa_i$ depend only on $M$ and $|\lambda_{i1}|$. Fig. 4 (left hand side plot) shows an example for the values of $\kappa_{e,\mu}$ (the $\kappa_\tau$ is smaller than $\kappa_\mu$ by less than a factor 10) obtained by enforcing neutrino data to lie within their 2$\sigma$ experimental ranges [1] and fixing for concreteness $|\lambda_{i1}| = 10^{-5}$.

As can be seen, if the preexisting asymmetry is sufficiently large even in the case of light RH neutrinos a sizable asymmetry in the electron flavor could be stored.

An estimation of the $N_{1,2}$ washout effects can be easily done in the one-flavor approximation by taking $C^{(\ell)} = 1$ in eq. (27). The resulting equation can be analytically integrated yielding the following result for the final baryon asymmetry:

$$Y_{\Delta_B} = \frac{12}{37} Y_{\Delta_B}^{(in)} e^{-3\pi \kappa / 8}.$$  \hspace{1cm} (30)
Figure 4: Left hand side plot: washout factors for muon and electron lepton flavors as a function of the common RH neutrino mass in the case of a normal hierarchical spectrum. Right hand side plot: $|Y_{\Delta_B}|$ as a function of the common RH neutrino mass for several values of the assumed primordial $B - L$ asymmetry. The solid (black) horizontal line indicates the observed value of the baryon asymmetry. See the text for more details.

From this equation a parametric relation between the relevant parameters can be calculated, namely

$$\frac{|\lambda_1|^2}{M} = \frac{8 m_*}{3 \pi v^2} \log \left( \frac{12 Y_{\Delta_B}^{(in)}}{37 Y_{\Delta_B}} \right),$$

(31)

thus fixing $Y_{\Delta_B}$ to its central value ($Y_{\Delta_B} = 8.75 \times 10^{-11}$ [31]) and taking $Y_{\Delta_B}^{(in)} \subset [10^{-8}, 10^{-2}]$ it turns out that as long as $|\lambda_1|^2 / M \subset [1, 5] \times 10^{-16}$ GeV$^{-1}$ a primordial asymmetry may always survive the $N_{1,2}$ related washouts and yield a value consistent with the observed one.

A precise treatment, however, requires the inclusion of flavor. In the mass range we are interested in ($[10^3, 10^6]$ GeV) all the standard model Yukawa processes (quarks and leptons) are in thermodynamical equilibrium [34]. Neglecting order one spectator processes, the kinetic eqs. (27) consist of three coupled differential equations accounting for the evolution of the $\Delta_{r,\mu,e}$ asymmetries. Defining the asymmetry vector $\mathbf{Y}_{\Delta} = (Y_{\Delta_r}, Y_{\Delta_\mu}, Y_{\Delta_e})$ the system of coupled equations can be arranged in a single equation

$$\frac{d}{dz} \mathbf{Y}_{\Delta} = -\frac{v^2}{4 m_*} \frac{|\lambda_1|^2}{M} \bar{C}^{(\ell)} \mathbf{Y}_{\Delta} K_1(z) z^3,$$

(32)

where $\bar{C}^{(\ell)}_{ij} = |\hat{\lambda}_{i1}|^2 C^{(\ell)}_{ij}$ and the matrix $\mathbf{C}^{(\ell)}$, at this stage, is given by [34]

$$\mathbf{C}^{(\ell)} = \frac{1}{711} \begin{pmatrix} 221 & -16 & -16 \\ -16 & 221 & -16 \\ -16 & -16 & 221 \end{pmatrix}.$$
By rotating the *asymmetry vector* in the direction in which \( \tilde{C}^{(\ell)} \) becomes diagonal \( (Y_\Delta' = PY_\Delta) \) the system of equations can be decoupled and thus solved analytically for \( Y_\Delta' \) as in the unflavored regime:

\[
\frac{d}{dz} Y_\Delta' = -\frac{v^2}{4m_*} \frac{|\lambda_1|^2}{M} \tilde{C}_{\text{diag}}^{(\ell)} Y_\Delta' K_1(z) z^3 \quad \text{with} \quad P\tilde{C}^{(\ell)} P^{-1} = \tilde{C}_{\text{diag}}^{(\ell)}.
\]

(34)

The solution reads

\[
Y_\Delta' = Y_\Delta' \left(\text{in}\right) e^{-3\frac{\pi\kappa \tilde{c}_i}{8}},
\]

(35)

where the \( \tilde{c}_i \)'s \( (i = \tau, \mu, e) \) are the eigenvalues of the matrix \( \tilde{C}^{(\ell)} \). The final baryon asymmetry in this case is therefore given by

\[
Y_{\Delta B} = \frac{12}{37} \sum_{j=\tau,\mu,e} Y_{\Delta j} = \frac{12}{37} \sum_{j=\tau,\mu,e} \left(P^{-1}\right)_{ji} Y_{\Delta j} \left(\text{in}\right) e^{-3\frac{\pi\kappa \tilde{c}_i}{8}}.
\]

(36)

In order to illustrate the effects of the \( N_{1,2} \) related washouts on a preexisting \( B - L \) asymmetry we fix the light neutrino mixing angles and the atmospheric and solar scales to their best fit point values \([1], \delta = \pi/2, \phi = 0 \) and again \( |\lambda_1| = 10^{-5} \). Assuming the same primordial \( \Delta_i \) asymmetries in each flavor, varying them from \( 10^{-8} - 10^{-2} \), and using eq. (36) we calculate the resulting \( Y_{\Delta B} \) asymmetry. The results are displayed in fig. 4 (right hand side plot). It can be seen that for the set of parameters chosen a \( Y_{\Delta B} \) in the observed range can always be obtained.

## 5 Conclusions

Besides the global total lepton number \( U(1)_L \) the canonical seesaw mechanism also breaks a global \( U(1)_R \) symmetry respected by the kinetic and gauge terms in the SM Lagrangian. In the context of MLFV models, this \( U(1)_R \) can be identified with global phase rotations of the charged lepton electroweak singlets \( e \) or RH neutrinos \( N \). In this paper we have explored the implications of a slightly broken \( U(1)_R \) symmetry in the context of minimal seesaw setups (with two RH neutrinos). We have shown that depending on the \( R \)-charge assignments two classes of generic models can be identified: (type A) models where the small breaking of \( U(1)_R \) allows to decouple the lepton number breaking scale from the RH neutrino mass scale \([8]\); (type B) models where the parameters that slightly break the \( U(1)_R \) induce a suppression in the light neutrino mass matrix.

We have studied the implications of these models for charged lepton flavor violating decays. We found that in type A models the decoupling of the RH neutrino masses from the lepton number breaking scale implies also a suppression of the corresponding Yukawa couplings, thus leading to non-observable charged lepton flavor violating effects. Type B models realize the MLFV hypothesis in the sense that due to the structure of the light neutrino mass matrix their flavor patterns are—up to normalization factors—entirely determined by low-energy neutrino observables.
Moreover, the suppression induced by the slightly broken $U(1)_R$ on the neutrino mass matrix allows large Yukawa couplings and TeV RH neutrino masses, and thus potentially large flavor violating $\mu$ processes. We have studied the $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$ and $\mu - e$ conversion in nuclei for normal and inverted neutrino mass spectra, finding that the three processes have branching ratios accessible in present experiments as long as the relevant overall Yukawa normalization factor is larger than $\sim 10^{-2}, 10^{-1}, 1$ and the RH neutrino masses are below $\sim 0.1$ TeV, 1 TeV, 10 TeV, respectively. For heavier RH neutrinos $\mu \rightarrow e\gamma$ is below prospective future sensitivities while $\mu \rightarrow 3e$ and $\mu - e$ conversion in nuclei would remain observable, up to $M_N \sim 100$ TeV and $M_N \sim 10^3$ TeV respectively. On the other hand in both type A and B models, RH neutrino contributions to LFV tau lepton decays are restricted below the present and near future experimental sensitivities.

Sizable $\mu$ flavor violating decays require large Yukawa couplings and light RH neutrinos. These values imply large RH neutrino inverse decay effects, that render the dynamics of these states incompatible with either the generation of a $B - L$ asymmetry (consistent with the observed $B$ asymmetry) or with the preservation of a preexisting one. Accordingly, sizable lepton flavor processes and small RH neutrino inverse decay effects are phenomenological requirements that cover non-overlapping regions of parameter space, from that point of view the analysis of both of them turns out to be complementary. In the low mass range ($M \lesssim 10^6$ GeV), instead of studying the generation of a $B - L$ asymmetry via resonant leptogenesis, we have considered the influence of the RH neutrino dynamics on a primordial $B - L$ asymmetry. We have demonstrated that a preexisting asymmetry yielding the observed $B$ asymmetry can survive the RH neutrino related washouts provided the overall Yukawa coupling normalization is below $\sim 10^{-5}$.

Acknowledgments

We want to thank G. Isidori and E. Nardi and M. Hirsch for useful comments and remarks. DAS is supported by a belgian FNRS fellowship. The work of JFK was supported in part by the Slovenian Research Agency. DAS and AD want to thank the Josef Stefan Institute for the kind hospitality during the completion of this work.

A Formulas for $l_i \rightarrow l_j\gamma$ and $l_i^- \rightarrow l_j^- l_j^+ l_j^-$ processes

In this appendix we summarize the formulas we use for the calculation of the charged lepton flavor violating decays discussed in sections 3.1 and 3.2. The results presented here were extracted from ref. [21] and adapted to our notation. In what follows the parameters $r_a$’s are defined according to $r_a = M_W^2/M_{N_a}^2$.

The process $l_i^- \rightarrow l_j^- l_j^+ l_j^-$ is determined by $\gamma$, and $Z$ penguins and box diagrams (for the full set of Feynman diagrams see ref. [21]). The $\gamma$ penguin contribution can be split in two pieces corresponding to the photon being either on-shell or off-shell.
For the on-shell piece, the one that determines the $l_i \to l_j \gamma$ process, we have

$$G_{\gamma}^{l_i l_j} = \frac{2}{g^2} \left( \lambda \cdot G_{\gamma} \cdot \lambda^\dagger \right)_{ij} ,$$

$$G_{\gamma}(r_a) = \frac{r_a}{4(1 - r_a)^4} \left( 2 + 3r_a - 6r_a^2 + r_a^3 + 6r_a \log r_a \right) ,$$

whereas for the off-shell photon piece

$$F_{\gamma}^{l_i l_j} = \frac{2}{g^2} \left( \lambda \cdot F_{\gamma} \cdot \lambda^\dagger \right)_{ij} ,$$

$$F_{\gamma}(r_a) = -\frac{r_a}{12(1 - r_a)^4} \left[ 7 - 8r_a - 11r_a^2 + 12r_a^3 - (2 - 20r_a + 24r_a^2) \log r_a \right] .$$

The $Z$ penguin contribution can be split in two parts, namely

$$F_{Z}^{l_i l_j} = F_{Z}^{l_i l_j(1)} + F_{Z}^{l_i l_j(2)} ,$$

where the first piece can be written as

$$F_{Z}^{l_i l_j(1)} = \frac{2}{g^2} \left[ \lambda \cdot \left( \hat{F}_Z + \hat{G}_{Z}^{(1)} \right) \cdot \lambda^\dagger \right]_{ij} ,$$

$$F_{Z}(r_a) = \frac{5r_a}{2(1 - r_a)^2} \left( 1 - r_a + \log r_a \right) ,$$

$$G_{Z}^{(1)}(r_a) = -\frac{r_a}{1 - r_a} \log r_a ,$$

while the second contribution according to

$$F_{Z}^{l_i l_j(2)} = \frac{4}{g^4} \left[ \lambda \cdot \left( \hat{G}_{Z}^{(2)} + \hat{G}_{Z}^{(3)} + \hat{G}_{Z}^{(4)} + \hat{H}_Z \right) \cdot \lambda^\dagger \right]_{ij} ,$$

$$\tilde{G}_{Z}^{(A)}(r_a, r_b) = (\lambda^\dagger \cdot \lambda)_{ab} G_{Z}^{(A)}(r_a, r_b) \quad \text{with} \quad A = 2, 3, 4 ,$$

$$G_{Z}^{(2)}(r_a, r_b) = -\frac{r_a r_b}{2(r_a - r_b)} \left( \frac{1 - r_b}{1 - r_a} \log r_a - \frac{1 - r_a}{1 - r_b} \log r_b \right) ,$$

$$G_{Z}^{(3)}(r_a, r_b) = \frac{r_a r_b}{2(1 - r_a)} \log r_a ,$$

$$G_{Z}^{(4)}(r_a, r_b) = \frac{r_a r_b}{2(1 - r_b)} \log r_b ,$$

$$\hat{H}_Z(r_a, r_b) = (\lambda^T \cdot \lambda^\dagger)_{ab} H_Z(r_a, r_b) ,$$

$$H_Z(r_a, r_b) = -\frac{\sqrt{r_a r_b}}{4(r_a - r_b)} \left[ r_b(1 - 4r_a) \log r_a - r_a(1 - 4r_b) \log r_b \right] .$$

Note that due to the constraint implied by the $SU(3)_{f+\bar{N}}$ flavor symmetry the off-diagonal elements of the matrices $\tilde{G}_{Z}^{(A)}(r_a, r_b)$ and $\hat{H}_Z(r_a, r_b)$ vanish.
The box diagram contributions can be split in three parts as follows

\[
F_{\text{Box}}^{A,3l_j} = \sum_{A=1,2,3} F_{\text{Box}}^{A,3l_j(A)},
\]

(52)

For the first part we have

\[
F_{\text{Box}}^{A,3l_j(1)} = \frac{2}{g^2} \left[ \lambda \cdot \tilde{F}_{\text{Box}}^{(1)} \cdot \lambda \right]_{ij},
\]

(53)

\[
F_{\text{Box}}^{(1)}(r_a) = -\frac{2r_a}{(1-r_a)^2} (1 - r_a + r_a \log r_a).
\]

(54)

For the second is given by

\[
F_{\text{Box}}^{A,3l_j(2)}(j) = \frac{4}{g^4} \left[ \lambda \cdot \left( \tilde{F}_{\text{Box}}^{(2)}(j) + \tilde{F}_{\text{Box}}^{(3)}(j) \right) \cdot \lambda \right]_{ij},
\]

(55)

\[
\tilde{F}_{\text{Box}}^{(A)}(r_a, r_b)(j) = \lambda_{ja}^{*} F_{\text{Box}}^{(A)}(r_a, r_b) \lambda_{jb} \quad \text{with} \quad A = 2, 3,
\]

(56)

\[
F_{\text{Box}}^{(2)}(r_a, r_b) = \frac{r_a r_b}{4(r_a - r_b)} \left[ \frac{1 - 4r_a(2 - r_b)}{(1 - r_a)^2} \log r_a - \frac{1 - 4r_b(2 - r_a)}{(1 - r_b)^2} \log r_b 
- \frac{r_a - r_b}{(1 - r_a)(1 - r_b)} (7 - 4r_a r_b) \right],
\]

(57)

\[
F_{\text{Box}}^{(3)}(r_a, r_b) = 2r_a r_b \left[ \frac{r_b}{(1 - r_b)^2} (1 - r_b + \log r_b) + \frac{1}{(1 - r_a)^2} (1 - r_a + r_a \log r_a) \right],
\]

(58)

where in \( \tilde{F}_{\text{Box}}^{(A)}(r_a, r_b)(j) \) no summation over the indices \( a, b \) is performed. Finally, the third term in (52) can be written as

\[
F_{\text{Box}}^{A,3l_j(3)}(j) = \frac{4}{g^4} \left[ \lambda \cdot \tilde{G}_{\text{Box}}(j) \cdot \lambda \right]_{ij},
\]

(59)

\[
\tilde{G}_{\text{Box}}(r_a, r_b)(j) = \lambda_{ja} G_{\text{Box}}(r_a, r_b) \lambda_{jb}^{*},
\]

(60)

\[
G_{\text{Box}}(r_a, r_b) = -\frac{\sqrt{r_a r_b}}{r_a - r_b} \left[ \frac{r_a [1 - 2r_b(1 - 2r_a)]}{(1 - r_a)^2} \log r_a - \frac{r_b [1 - 2r_a(1 - 2r_b)]}{(1 - r_b)^2} \log r_b 
+ \frac{(r_a - r_b)}{(1 - r_a)(1 - r_b)} (1 + 2r_a r_b) \right],
\]

(61)

where, again, in \( \tilde{G}_{\text{Box}}^{(A)}(r_a, r_b)(j) \) no summation over the indices \( a \) and \( b \) is performed.

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