Understanding the superfluid phase diagram in trapped Fermi gases

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(Dated: May 28, 2018)

Trapped ultracold Fermi gases provide a system that can be tuned between the BCS and BEC regimes by means of a magnetic-field Feshbach resonance. Condensation of fermionic atom pairs in a 40K gas was demonstrated experimentally by a sweep technique that pairwise projects fermionic atoms onto molecules. In this paper we examine previous data obtained with this technique that probed the phase boundary in the temperature-magnetic field plane. Comparison of the 40K data to a theoretically computed phase diagram demonstrates good agreement between the two.

cond-mat/0512596

The field of ultracold Fermi gases has seen enormous progress in the years since the first observation of Fermi degeneracy [1]. Upon variation of a magnetic field B, Feshbach resonances provide a means of controlling the strength of interactions between fermionic atoms, which is characterized by the s-wave scattering length a. The nature of the resultant superfluidity is expected to vary continuously [2] from BCS (a > 0) to BCS (a < 0) as a function of magnetic-field detuning from the resonance. In the BEC regime, evidence for condensation was obtained [3, 4, 5] by observing a bimodal distribution of the momentum profile, a standard technique originally developed for bosonic gases.

On the BCS side of resonance the situation is experimentally more complicated. To demonstrate condensation a momentum projection technique based on fast sweeps into the BEC regime was introduced [6]. Detailed time-dependent studies suggest that the sweeps used are sufficiently rapid that a condensate cannot be created during this sweep process. The presence of a condensate after a sweep then provides strong support for the existence of a condensate before the sweep on the BCS side of resonance. In this way the first Fermi gas normal-superfluid (NS) phase diagram was obtained experimentally for 40K [6] and later for 6Li [7]. Additional experiments in 6Li have since added to the evidence for fermionic superfluidity, including, collective mode observations [8, 9], thermodynamic measurements near unitarity [10] and, most conclusively, the demonstration of quantized vortices [11]. These data in conjunction with the sweep experiments serve to further constrain the NS boundary.

The purpose of this paper is to present a comparison of this important phase boundary measured in fast sweep experiments involving 40K to a theoretical computation of the NS boundary. Recent theoretical work examining the entropy of the trapped gas in the BCS-BEC crossover now makes it possible to make this comparison [12]. Thus we present previous data [6] in a new way and show that the experimentally obtained condensate fraction in 40K provides a good measure of this phase boundary. While the emphasis of Ref. [6] was on providing evidence for the condensation of atom pairs in an ultracold Fermi gas, here we show that these data moreover provide a universal NS phase diagram, as a function of temperature and interaction strength, that can be quantitatively compared with theory.

In making this quantitative comparison several important issues need to be considered. First we need to consider the fast sweep technique and what information it can provide. The essential and unique feature of this technique is that it can provide direct information about the condensate fraction by means of a measurement of a bimodal distribution in the particle density or momentum profile. For the fermionic regime this information is obtained [6, 7] by rapidly sweeping the magnetic field beginning on the BCS side of resonance to the BEC, or a > 0, side of resonance where time-of-flight imaging can be used to measure the momentum distribution of the weakly bound molecules. The projecting magnetic field sweep is completed on a time scale that allows molecule formation but is still too brief to allow additional pairs to condense.

In the 40K experiment it was observed that the fast sweep resulted in significant number loss [6, 7], presumably because of the relatively short lifetime of the molecules away from resonance [13]. The measured condensate fraction, which is defined as the number of condensed molecules divided by the total number of molecules observed after the fast sweep, could be affected by this loss. However, the loss process is almost certainly density dependent and thus one expects only suppression (and never enhancement) of the condensate fraction. Furthermore one expects a large loss only for large condensate fractions. Therefore, the NS phase line (separation between zero condensate fraction and a finite value) obtained in the experiment should be relatively unaffected. When plotted as a function of the temperature, it corresponds to the threshold curve below which a finite fraction of the molecules is observed to have near zero momentum.

Second, we note that a general difficulty in these Fermi gas experiments is the lack of model-independent thermometry in the strongly interacting regime. Therefore experiments typically rely on the combination of temperature measurements made away from resonance and slow adiabatic sweeps to the strongly interacting regime. In this paper we use a superscript “0” to denote quantities measured away from the Feshbach resonance in the weakly interacting Fermi gas regime. The temperature relative to the Fermi temperature (T/TF)0 is determined from surface fits to absorption images of the gas.
taken after expansion from the trap. We have checked that this yields accurate temperature measurements down to \( (T/T_F)^0 \approx 0.1. \) Below this temperature measuring \( (T/T_F)^0 \) this way becomes more difficult due to the fact that the momentum distribution of the Fermi gas approaches the \( T = 0 \) limit. The NS phase diagram also depends on the adiabaticity of the magnetic-field sweep toward resonance. Studies of the condensate fraction as a function of sweep rate suggest that the sweep toward resonance is sufficiently slow. More recent studies involving double ramps to the resonance and then back away suggest that extra heating during the ramp is not significant on the BCS side of resonance.

Third, for comparison with theory the magnetic-field values should be converted to the dimensionless parameter \( 1/k_F a \), which reflects the strength of the pairing interaction in BCS-BEC crossover theories. Here \( k_F \) is the Fermi wavevector at the trap center, and \( a \) is the two-body \( s \)-wave scattering length between fermionic atoms. For \( a \) we use a previous measurement of the scattering length as a function of magnetic field [6], and for the Fermi wavevector we use \( k_F \) measured in the weakly interacting regime, \( k_F^0 \). Here \( E_F^0 \equiv k_B T_F^0 = \hbar^2 (k_F^0)^2/2m \), where \( E_F^0 \) is the noninteracting Fermi energy. As defined here, \( k_F^0 = (2m\omega_0^2/\hbar)^{1/2}(3N_a)^{1/6} \), where \( N_a \) is the total number of atoms and \( \omega_0 = (\omega_x\omega_y\omega_z)^{1/3} \) is the geometric mean angular trap frequency. Results for the NS phase diagram are then plotted in terms of \( (T/T_F)^0 \) and the dimensionless parameter \( 1/k_F^0 a \). The phase diagram plotted this way is nearly universal and should be applicable to \(^6\)Li as well. However, because of the large resonance width \(^6\)Li experiments may have some difficulty in reaching the noninteracting Fermi gas regime, which sets the temperature scale we use here.

Our theoretical calculations are based on the finite temperature formalism described in Ref. [12]. Comparisons with the experimental data require not only an understanding of the BCS-BEC crossover system at finite \( T \), including the behavior of \( T_c \) in a trap and the temperature-dependent superfluid density \( N_s \), but also an understanding of the entropy \( S \). Note that \( (T/T_F)^0 \) is essentially a measure of the entropy of the gas. We take as a ground state the usual BCS-Leggett wave function and include excitations of finite momentum pairs. At unitarity, the resulting thermodynamics has been shown to compare favorably to experiment in Ref. [10]. More generally, the excitations that contribute to the temperature dependence of the energy can be attributed to both “bosonic” and fermionic degrees of freedom, which are strongly inter-dependent. The latter are associated with a finite excitation gap, which in turn is a measure of the presence of fermion pairs or “bosons”. Similarly, the superfluid density reflects both the bosonic and fermionic contributions found in the thermodynamics. The entropy is dominated by fermionic excitations in the BCS regime and bosonic excitations in the BEC regime. In an adiabatic sweep from BCS to BEC, the temperature increases. A detailed theory of the thermometry for adiabatic sweeps is presented in Ref. [12].

This approach leads to a self-consistent set of equations for the fermionic atoms and Cooper pairs throughout the crossover. The transition temperature \( T_c \) is associated with a vanishing of the chemical potential of the pairs. The magnitudes of this transition temperature (in a trap) are similar to those found elsewhere using a different ground state, where less is currently known about the superfluid density and thermodynamics.

We now address the implications for the phase diagram. In the experiment the temperature of the gas is determined in the noninteracting Fermi gas limit at high field; we denote this temperature by \( T^0 \). The destination field is accessed adiabatically by a slow magnetic-field sweep. Using the theory in Ref. [12], we calculate the entropy at different magnetic fields and temperatures. In this way, we can associate the physical temperature \( T \) with the effective temperature \( T^0 \). We then calculate the condensate fraction \( N_0/N \), here identified with the superfluid density \( N_s/N \), as a function of temperature \( T \) or \( T^0 \) and of magnetic field \( B \). The latter parameter is appropriately characterized by the dimensionless variable \( 1/k_F^0 a \), which provides a measure of the strength of pairing interaction. At a given field this parameter varies with the Fermi temperature.

In Fig. [1(a)], we show the results for the superfluid transition temperature \( T_c \) (black dashed line) and its corresponding value \( T_c^0 \) (black solid line) for an isentropic sweep into the Fermi gas regime as functions of \( 1/k_F^0 a \). In a similar fashion,
we plot the physical (red dashed line) and effective temperatures (red solid line) corresponding to $N_s/N = 0.01$. In Fig. 1(b), we plot the fermionic chemical potential $\mu(T_c)$ and the excitation gap $\Delta(T_c)$ at the trap center as a function of $1/k_F^0 a$. When the chemical potential is negative the system can be viewed as “bosonic”, whereas when $\mu$ is positive it is “fermionic.”

Because the entropy for a noninteracting gas at low $T/T_F^0$ is nearly linearly dependent on the temperature, one can conclude that $T_c^0$ is essentially proportional to the entropy at the transition $S(T_c)$. The latter is labeled on the right hand axis of Fig. 1(a). It follows that as a natural consequence of an isentropic sweep, $T_c^0$ is reduced substantially from the physical $T_c$ except in the BCS regime. As can be seen from the figure, this reduction is dramatic in the BEC regime ($1/k_F^0 a > 0.7$) and persists essentially to unitarity ($1/k_F^0 a = 0$). One can understand this reduction as reflecting the presence of bosonic degrees of freedom at $T_c$. Once noncondensed bosons or preformed pairs are present at the temperature of their condensation, the entropy curve for $S(T)$ for $T \leq T_c$ drops substantially below its counterpart for a noninteracting Fermi gas at the same temperature. One can alternatively say that when $T_c$ and $T_c^0$ are significantly different a normal state excitation gap or “pseudogap” is present at $T_c$. In the fermionic regime ($\mu > 0$) and at the transition temperature, this pseudogap is parametrized by $\Delta(T_c)$, which is also shown in Fig. 1(b) and should be viewed as an alternative measure of bosonic degrees of freedom. One can see that the difference between $T_c$ and $T_c^0$ reflects rather nicely the behavior of $\Delta(T_c)$ as a function of $1/k_F^0 a$. Beginning at unitarity and moving towards the BEC regime, $\Delta(T_c)$ increases rapidly, reflecting the rapid increase in the bosonic degrees of freedom. This leads then to a strong reduction from $T_c$ to $T_c^0$ and explains the existence of the maximum seen in $T_c^0$.

To illustrate these effects, in Fig. 2 the temperature dependence of the entropy $S(T)$ is shown for selected values of $1/k_F^0 a$ representing the Fermi gas, unitary, and BEC cases. Here we see that the noninteracting gas result at low $T/T_F^0$ is close to a straight line, and that as the system becomes more strongly interacting, $S(T)$ deviates from this line with decreasing temperature. This deviation sets in once the temperature goes below the pair formation temperature, $T^*$.

We are now in a position to compare our calculated phase diagram with experimental measurements. In Fig. 3 we replot the measured phase diagram from Ref. 6 as a function of $1/k_F^0 a$ and overlay our theoretical curves. The top (black) curve corresponds to the theoretical calculation for $(T_c/T_F)^0$, whereas the remaining (red) line is the effective temperature $(T/T_F)^0$ corresponding to the superfluid fraction $N_s/N = 0.01$. We present both theoretical curves because (as can be seen from the disproportionate breadth of the contour swath for $0 < N_s/N < 0.01$ in Fig. 4, see also Ref. 14), the superfluid density has a flat tail close to the transition temperature due to trap inhomogeneity effects. Consequently experimental noise may add a large uncertainty to the temperature for which $N_s/N = 0$; the $1\%$ contour should be an experimentally more robust boundary.

In general, the phase boundary for $N_s/N = 0.01$ is in good agreement with the experimentally measured phase boundary for $N_0/N = 0.01$. However, in the near-BEC regime there are a few experimental data points that show a finite condensate fraction above the theoretical transition line. The two well-known weaknesses of the mean-field approach may be partly responsible for this discrepancy: Both the overestimate of the interboson scattering length in the BEC regime and underestimate of the value of the “beta” factor at unitarity lead to a
slightly underestimated peak density of the trap profile, which in turn leads to an underestimate of $T_c$. Also, in the experiment, the slow sweeps that extend to the BEC side of the resonance are not perfectly adiabatic. Moreover, the experiment finds less than 100% conversion of atoms to molecules or pairs in this regime [24].

In Fig. 4 we plot the theoretical phase diagram and contour plot for $N_s/N$, which should be appropriate to a variety of different experiments. We can compare this calculation with its experimental counterpart in Fig. [1]. When comparing the theoretical values of $N_s/N$ at the lowest temperature $(T/T_F)^0 \approx 0.07$ accessed experimentally, we find that, at unitarity, the theoretical value is about two times as large as in the $^{40}$K experiment. This difference may be attributed to a number of factors. First, the condensate fraction may be partly destroyed during the projection sweep process. A possible sweep-induced reduction of the condensate fraction has been addressed elsewhere [21,22]. In Refs. [21,22], however, no attempt was made to distinguish between the actual physical temperature and that measured in the weakly interacting regime. Second, sweeps may not be 100% adiabatic; minor heating is known to be present for sweeps from the noninteracting Fermi gas to the BEC regime. Finally, in the fermionic and unitary regimes the condensate fraction is not a unique parameter and varies with the particular observation under consideration. In the present measurements, it is not known precisely how well the superfluid density represents this fraction. This uncertainty does not affect the NS phase boundary. The superfluid density, however, should be regarded as an upper bound to the condensate fraction. As an example, a recent Monte Carlo calculation defined a $T = 0$ condensate fraction that is nearly a factor of two lower than $N_s/N$ at unitarity [23]. Given these factors, the agreement between theory and experiment is reasonably good.

In summary, we have shown that previous measurements of Ref. [6] of the normal state-superfluid phase boundary in $^{40}$K are in good agreement with theoretical calculations. In addition to reinforcing the theoretical approach taken here, this work adds support to previous claims that fast sweep experiments do, indeed, provide a reliable indication of the phase boundary. A feature of the predicted phase boundary is that, when it is plotted in terms of the temperature $T_c^0$ in the noninteracting regime, there is a maximum near unitarity as a function of $1/k_F^0 a$. Indications for this maximum have also been observed [7] in $^6$Li. Here one finds that the biggest condensate fraction occurs near unitarity, as is predicted in Fig. [4].

We thank Eric Cornell for providing the impetus for this paper and Cheng Chin for very helpful comments. This work was supported by NSF, NASA, and NSF-MRSEC Grant No. DMR-0213745. C.A.R. acknowledges support from the Hertz foundation.

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