FEM-Calculations on the Frequency Dependence of Hysteretic Losses in Coated Conductors

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Abstract. Calculations based on two different finite-element models have been carried out to investigate the flux flow behaviour of High Temperature Superconductors (HTS), in particular of Coated Conductors (CC) based on 123-HTS. The models allow the simulation of the response of the CC to various experimental operating conditions: e.g. a fast ramping of the transport current typically done in measurements of the critical current $I_c$ or sinusoidal changes of an external magnetic field typically used in AC loss measurements. The models also allow calculating the response to arbitrary combinations of current and field changes. The superconductor is modelled by using either a simple power-law $E(J)$ characteristic or one which also accounts for field and temperature dependences. The obtained results go beyond Bean’s approximation, which is mostly employed for interpreting such flux penetration effects. One consequence is that hysteretic losses, which in Bean’s model are frequency independent, show a dependence on the time scales of current or field changes. The field and frequency ranges where such deviations from Bean’s model should be taken into account are discussed.

1. Introduction
The critical state model (CSM) which is based on Bean’s approximation [1], implies that the penetration of the magnetic flux does not depend on its variation rate. In particular, the AC losses per cycle are independent of the frequency of the current and/or magnetic field source, as can also be seen in the analytical formulae derived by Norris [2], Brandt and Indenbom [3] and Zeldov et al. [4]. This is no longer true when a power-law $E(J)$ characteristic is used to describe the electrical behavior of superconductors, especially in the case of HTS, which exhibit a relatively low value of n (20-40), and as such are not correctly represented by the CSM.

The frequency dependence of AC losses has been investigated by several authors [5-11], but not yet for CC in a frequency range from 1mHz to 1kHz. In this paper we utilize two models to study the frequency dependence of the field profiles and the hysteretic AC losses in a thin superconducting strip subjected to a sinusoidal AC transport current and/or AC magnetic field. This is seen as a test case for more complex configurations and / or processes like those relevant to pulsed magnet applications like Superconducting Magnetic Energy Storage (SMES).
2. Models and Calculations

The utilized models follow two fundamentally different approaches.

The first model (IE) solves the integral equations for the current density distribution in thin conductors by using finite-elements. A general description of the model can be found in [12]. This model has also been applied to stacks and windings of thin superconductors [13].

The second finite element model (MS) was initially developed to successfully model Multi-Pulse Processes with Stepwise Cooling used to pulse magnetize YBCO bulk parts [14]. Here the versatile software which is based on an equivalent circuit approach [15] is applied to a CC tape.

Both models compute the sheet current density distribution $J$ in the tape for discrete times of the cycle, and the AC losses $Q$ are computed as follows:

$$Q = \frac{1}{T} \int_{-\alpha}^{\alpha} J \cdot E \, dx \, dt$$

where $T$ is the period, $\alpha$ is the tape’s half-width and $E$ is the electric field, respectively. In order to use a lighter notation, in the rest of the paper we use the general term “current density” for indicating the sheet current density.

Since we want to compare the results of our models with those obtained in the framework of the CSM, all presented figures for AC losses are normalized to the corresponding values obtained with the formulae of Norris (transport current, [1]) and Brandt (external perpendicular field, [2]). The data are listed in Table 1.

In particular, the transport losses per cycle (in J/m) are given by:

$$Q_t = \frac{\mu_0 I_c^2}{2\pi} \left[ (1-i)\ln(1-i) + (1+i)\ln(1+i) - i^2 \right]$$

where $i = I_{\text{max}}/I_c$ is the amplitude of the transport current normalized to the critical current $I_c$.

The magnetization losses are given by:

$$Q_m = 4\alpha^2 J_c B_{\text{max}} g\left(\frac{B_{\text{max}}}{B_c}\right)$$

$$g(x) = \frac{2}{x} \ln \cosh(x) - \tanh(x)$$

where $J_c$ is its sheet critical current density, $B_{\text{max}}$ the amplitude of the external field and $B_c = \mu_0 J_c/\pi$.

In a first set of calculations both models describe the superconductor by means of a power-law relation between the electric field and the current density: $E = E_c (J/J_c)^n$. In this paper the following parameters were used: $a = 6$ mm, $I_c = 270$ A, $E_c = 1\times10^{-4}$ V/m, $n = 28$, which are typical values for commercially available YBCO coated conductors at 77 K. In a second set of calculations the MS model also takes into account the dependence of the $E(J)$ characteristic on the magnetic field ([15] and $I_s(B_{\text{max}})$ in Table 1). It should be noted that in all calculation the temperature is kept constant.
Table 1. AC Losses of Norris & Brandt for $I_c = 270 \text{A}$ and Critical Currents $I_c(B_{\text{max}})$ used in MS-$E(J,B,77K)$

| $I_{\text{max}} / I_c$ | Norris’ AC Loss $Q_n$ [J/m] | $B_{\text{max}}$ [mT] | Brandt’s AC Loss $Q_m$ [J/m] | MS-$E(J,B,77K)$: $I_c(B_{\text{max}})$ [A] |
|-------------------------|----------------------------|----------------------|----------------------------|---------------------------------|
| 0.35                    | 7.676E-5                   | 10                   | 4.312E-3                   | 259.4                           |
| 0.40                    | 1.331E-4                   | 20                   | 0.027                      | 248.2                           |
| 0.50                    | 3.39E-4                    | 30                   | 0.057                      | 237.5                           |
| 0.60                    | 7.433E-4                   | 100                  | 0.284                      | 176.1                           |
| 0.70                    | 1.484E-3                   | 300                  | 0.932                      | 84.1                            |
| 0.80                    | 2.803E-3                   | 1,000                | 3.2                        | 34.6                            |

3. Results and Discussion

Figure 1 shows the frequency dependence of the magnetization (A) and transport (B) losses for various values of the applied field $B_{\text{max}}$ and transport current $I_{\text{max}}$, respectively. The data were calculated using the power law, and as stated above, the losses are normalized to Brandt’s and Norris’ values. The frequency ranges from 1 mHz to 1 kHz.

In the applied field case the losses are rather close to the Brandt’s value, the highest difference being in the order of 50%. It can be noticed that the frequency dependence changes as a function of the applied field. At low fields (10 mT), the losses decrease with increasing frequency. Then at 20 mT they become almost independent of the frequency. Finally, starting from 30 mT, they increase with frequency. As pointed out already by Wakuda et al. [7] for the different geometry of a hollow cylinder and a much more restricted frequency range, this behaviour has to be put in relation with the different degree of penetration of the magnetic flux in the superconductor. For sufficiently low fields the flux can penetrate only into the outer regions of the CC, the inner region is shielded. With increasing frequency the shielding currents increase, the field-free region grows, the magnetic flux that can contribute to dissipation is reduced together with the overall AC losses. On the contrary, for full flux penetration the higher induced currents lead to increased losses.

Figure 1. Frequency dependence of AC losses calculated with the power law: for different applied magnetic fields without transport current and normalized to Brandt’s formula (A) and for different transport currents without magnetic field and normalized to Norris’ formula (B).
As far as transport current losses are concerned, the magnetic flux caused by the current is gradually penetrating the superconductor. Similarly to the case with magnetic field below full penetration, the losses decrease as a function of frequency. The difference of the losses with respect to the Norris’ value reaches a factor 4 at low frequencies. This “driven mode” is different from the “induced mode” of $B_{\text{max}}$ where the overall external flux change as the driving force is definitely independent of the duration of the sine wave. For long periods (low frequencies) a transition occurs from AC losses to essentially DC losses (resistive transport current losses), particularly for transport currents approaching $I_c$. This effect cannot be accounted for by Bean’s or Norris’ models, where DC losses are zero by definition. In our models, on the contrary, the use of a smooth $E(J)$ characteristic makes it possible to have finite DC losses already below $J_c$.

Figure 2 shows the frequency dependence of the losses when a sinusoidal current and field are applied simultaneously. In the data presented, the peak transport current is kept constant to 0.5 $I_c$. The frequency ranges from 1 mHz to 1 kHz.

The curves of figure 2A were calculated with the power law. For low fields the transport current produces a field which is in the same order of magnitude as the external field. As a consequence the mutual influence of transport current and external field is rather strong, especially for the 10 mT case, and leads to AC losses that well exceed the sum of both separate effects. When going to higher fields the losses are widely dominated by the external field, and the frequency dependence essentially reflects the one of figure 1A. This is already expected from the data of Table 1 showing Brandt’s and Norris’ AC losses for $I_c = 270$A. Even for a transport current $I_{\text{max}}$ of 0.8 $I_c$ the AC loss is smaller than the one for a field $B_{\text{max}}$ of 10 mT and is three orders of magnitude smaller than for 1 T.

Figure 2B shows the corresponding curves when the field dependent characteristic MS-$E(J,B,77K)$ is applied. For the normalization here the field-dependent $I_c(B_{\text{max}})$ was used. The only significant difference occurs for larger fields at higher frequencies. The rather strong increase of the losses in this regime again reflects the frequency dependence of the losses for the field alone (not shown here). This effect can be partly understood when taking into account that during the cycle the $I_c$ strongly varies with the field, and that with increasing frequency also the field-induced currents strongly increase.

![Figure 2. Frequency dependence of AC losses for different applied magnetic fields with a transport current of 0.5 $I_c$ normalized to the sum of Norris’ and Brandt’s formula: calculated with the power law (A) and calculated with the field-dependent MS-$E(J,B,77K)$ characteristic taking into account $I_c(B_{\text{max}})$ (B)](image-url)
Figure 3 shows the current density (A & B) and field (C & D) distributions at different times during a half cycle (phase from 0 to Π) for 10 mT, 0.5 \( I_c \) and for 1 mHz (A & C) or 1 kHz (B & D) calculated with the two models for the power law. It is worth mentioning that the two models show excellent agreement even at the level of local quantities such as the current and the field. Transport current and external field are applied simultaneously, and the resulting asymmetric current and field distributions have quite complex structures over time leaving an inner part of the CC always field-free.

The most interesting feature is the difference of the internal profiles at the two different frequencies. For the low frequency the flux-free region (figure 3C) is strongly reduced with respect to the high-frequency case (figure 3D) where stronger shielding currents flow. The maximum current density level inside the tape reaches about 80% of \( J_c \) at low frequency (figure 3A), whereas at high frequency it is as high as 130% of \( J_c \) (figure 3B). In the framework of the CSM, the maximum current density is set to \( J_c \). As we have already noticed in the previous figures, these differences have important repercussions on the AC loss values.

![Figure 3](image)

**Figure 3.** Current density (A & B) and field (C & D) distribution over width for different times of a half cycle (phase from 0 to Π) calculated with the power law for 10 mT, 0.5 \( I_c \) and for 1 mHz (A & C) or 1 kHz (B & D)

Figure 4 shows the current density (A & B) and field (C & D) distributions at different times during a half cycle (phase from 0 to Π) for 100 mT, 0.5 \( I_c \) and for 1 mHz (A & C) or 1 kHz (B & D) calculated with the field-dependent \( MS-E(J,B,77K) \) characteristic. As in figure 3, transport current and external field are applied simultaneously.
Here the external magnetic field strongly penetrates. The maximum field generated by the current distribution is remarkably smaller than the maximum external field and leads to only small modulations at peak external field. The overall field distribution in the tape is much less asymmetric. This is also reflected in the current distribution which for the zeros of the cycle is nearly anti-symmetric relative to the tape centre.

Again the distributions differ significantly for the two frequencies. The current levels are substantially higher for the higher frequency. Local peaks well exceeding 500% of the $J_c(B_{\text{max}})$ appear where the local field changes its sign and consequently the local $J_c(B=0)$ is at maximum (figure 4B). For the lower frequency case the maximum current densities reach only about 150% of the $J_c(B_{\text{max}})$ (figure 4A). This results in correspondingly reduced local dips and peaks in the field distributions (figure 4 C) compared with the higher frequency (figure 4D).

Figure 4. Current density (A & B) and field (C & D) distribution over width for different times of a half cycle (phase from 0 to $\pi$) calculated with the field-dependent MS-$E(J,B,77K)$ characteristic for 100 mT, $0.5 I_c$ and for 1 mHz (A & C) or 1 kHz (B & D)
4. Conclusion
We have utilized two different models to study the influence of the frequency of transport currents and external magnetic fields on the current/field distributions and on the hysteretic AC losses of a superconducting thin tape. We found that, when a power-law is utilized to describe the $E(J)$ relation in the superconductor, the current/field distribution and, consequently, the losses, significantly depend on the variation rate of the applied current and/or field. In the considered case, hysteretic losses mostly change with frequency by up to a factor 2. We also found excellent agreement between the two models, which provides good confidence in their correctness, given the very different approaches they are based on. When a field dependent $E(J)$ relation is used the losses for higher fields substantially increase with frequency. This has to be taken into account when magnets are to be cycled on short time scales like in some SMES applications.

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