Weak Semileptonic Decays of Heavy Baryons

Containing Two Heavy Quarks

Xin-Heng Guo\textsuperscript{1,2}, Hong-Ying Jin\textsuperscript{2} and Xue-Qian Li\textsuperscript{3,4}

\textsuperscript{1}Department of Physics and Mathematical Physics, 
and Special Research Center for the Subatomic Structure of Matter, 
University of Adelaide, SA 5005, Australia 
\textsuperscript{2}Institute of High Energy Physics, Academia Sinica, Beijing 100039, P. R. China 
\textsuperscript{3}CCAST(World Laboratory) P.O.Box 8730, Beijing 100080, P. R. China 
\textsuperscript{4}Department of Physics, Nankai University, Tianjin 300071, P.R. China

Abstract

In the heavy quark limit a heavy baryon which contains two heavy quarks is believed to be composed of a heavy diquark and a light quark. Based on this picture, we evaluate the weak semileptonic decay rates of such baryons. The transition form factors between two heavy baryons are associated with those between two heavy mesons by applying the superflavor symmetry. The effective vertices of the W-boson and two heavy diquarks are obtained in terms of the Bethe-Salpeter equation. Numerical predictions on these semileptonic decay widths are presented and they will be tested in the future experiments.

PACS numbers: 12.39.Hg, 11.10.St, 13.30.-a, 12.39.-x
1 Introduction

The heavy flavor physics has been an interesting subject for many years. The meson case has been studied much more intensively both in experiments and in theory than the baryon case. The existence of three valence quarks in a baryon makes the theoretical study much more complicated. Recently more and more data for heavy baryons which contain one heavy quark have been accumulated [1] and in the near future we may expect even more data from LEP and other experimental groups. Although we do not have any data for the heavy baryons containing two heavy quarks (later we will call such baryons $B_{QQ'}$ where Q and Q’ could be b-quark or c-quark) at present, it would be interesting to make predictions on their properties which will be tested in the future experiments. In our previous paper [2] we have studied the production of a pair of $B_{QQ'}$ in electron-position collisions. It is the aim of the present work to study the weak semileptonic decays of $B_{QQ'}$.

The basic problem is how to deal with the transition form factors between $B_{QQ'}$ and $B_{QQ''}$ (or $B_{Q'Q''}$) where one flavor transits (explicitly $b \rightarrow c$) with another heavy quark and the light flavor remaining unchanged. Since it is determined by the non-perturbative QCD effects, the solution is by no means trivial. The heavy quark effective theory (HQET) provides a way to appropriately simplify the evaluation of the hadronic matrix elements [3] because by applying the HQET we are able to find relations among the form factors, and consequently reduce the independent number of these form factors. It is well known that in the heavy quark limit the extra symmetries $SU(2)_f \times SU(2)_s$ manifest and the non-perturbative effects are attributed to the well-defined Isgur-Wise function $\xi(v \cdot v')$, where $v$ and $v'$ are the four-velocities of the concerned heavy quarks.

It is pointed out in our previous paper [2] that in a heavy baryon which contains two heavy quarks, these two heavy quarks constitute a relatively stable heavy diquark (which will be called $\chi_{QQ'}$ later). This allegation has also been suggested
by other authors. The leftover light quark moves in the color field induced by
the heavy diquark. The size of the heavy diquark is much smaller compared with
the QCD scale $\Lambda_{QCD}$. In this scenario, the three-body problem is simplified into
a two-body problem. The ground state heavy diquark can be a spin-1 or spin-0
object. Due to the Pauli principle, when $Q=Q'$, the cc- or bb-diquark can only be in
the spin-1 state while for bc-diquark its spin may be either 0 or 1. Therefore, from
QQ-diquark we can construct a heavy baryon either with spin-$\frac{3}{2}$ ($B^*_{(QQ)_1}$) or with
spin-$\frac{1}{2}$ ($B_{(QQ)_1}$). On the other hand, from bc-diquark we may have spin-$\frac{1}{2}$ baryon
which is constructed from $\chi_{(bc)_0}$ ($B_{(bc)_0}$) or from $\chi_{(bc)_1}$ ($B_{(bc)_1}$), and also spin-$\frac{3}{2}$ baryon
from $\chi_{(bc)_1}$ ($B^*_{(bc)_1}$). In the present paper we will study the weak transition hadronic
matrix elements between these heavy baryons and then give the predictions for the
semileptonic decay widths of $B_{QQ'}$.

Due to the analogue of a heavy meson and a heavy baryon with a heavy diquark,
the superflavor symmetry is applicable to associate the transition matrix elements
between two heavy baryons $B_{QQ'}$ with those between two heavy mesons. The su-
perflavor symmetry was first established by Georgi and Wise for interchanging a
heavy quark and a heavy scalar object, later Carone generalized it to the sym-
metry of interchanging a heavy quark and a heavy axial vector object. Since the
heavy diquark is not really point-like with respect to weak transitions, we need to
derive the explicit expressions of the effective vertices $\chi\chi'W^\pm$ by taking into ac-
count the inner structure of heavy diquarks. Obviously these vertices are associated
with the bound state properties of heavy diquarks $\chi$ and $\chi'$. Therefore, some non-
perturbative model has to be adopted. As in our previous work we will apply
the Bethe-Salpeter (B-S) equation model to obtain such vertices.

The paper is organized as the following: In sect.2 we give a detailed derivation
of the transition form factors between two heavy diquarks with a virtual $W-$boson
being emitted. Consequently we obtain the effective currents for heavy diquark weak
transitions. Then in section 3. we apply superflavor symmetry to give the formulation for the weak matrix elements between heavy baryons and the semileptonic decay widths. The numerical results will be presented in section 4. Finally the last section is devoted to summary and discussions.

2 Derivation of the heavy diquark transition form factors

Since in the heavy quark limit the two heavy quarks in a baryon constitute a heavy diquark, in the decay process this diquark may be treated as a color-triple quasiparticle. It is noted that for applying the HQET to associate a baryon case to a meson case, the diquark should be of a point-like structure, the reason is that all non-perturbative effects are attributed into a well-defined Isgur-Wise function, therefore the necessary condition is that the diquark is seen by the light quark as a point-like color source. However, it by no means demands that in the weak transition the weak current see a point-like structureless object, by contraries, there is complicated structure due to the bound state effects of the diquark. The structure effects of the heavy diquark should be described by the bound state equation. Hence we have to adopt a plausible method to deal with the diquark structure effects which are governed by the non-perturbative QCD. In this section we solve the B-S equation [7] to obtain the bound state wave function of the heavy diquark and then give the transition form factors between such heavy diquarks in the weak decay processes.

Since the bound state B-S wave functions and the transition form factors between two heavy diquarks are obtained in the same framework, in our formulation one does not need to invoke some phenomenological inputs except the commonly accepted parameters such as $\alpha_s$ and $\kappa$ in the Cornell potential model.
The B-S equation for a heavy diquark can be written in the following form

\[
\chi_P(p) = S_1(\lambda_1 P + p) \int G(P, p, q) \frac{d^4q}{(2\pi)^4} S_2(\lambda_2 P - p), \tag{1}
\]

where \(S_j(j = 1, 2)\) are the propagators of heavy quark 1 and quark 2 in the diquark respectively and \(G(P, p, q)\) is the B-S equation kernel defined as the sum of all the irreducible diagrams concerning the interaction between the two quarks of the diquark, \(\lambda_1 = \frac{m_1}{m_1 + m_2}, \lambda_2 = \frac{m_2}{m_1 + m_2}\), and \(m_1, m_2\) are the quark masses. \(P\) is the total momentum of the diquark and can be expressed as \(P = M v\) where \(M\) is the mass of the diquark and \(v\) is its four-velocity.

Using the relation

\[
S_j(p) = i \left[ \frac{\Lambda_j^+(p_t)}{p_t - W_j + i\epsilon} + \frac{\Lambda_j^-(p_t)}{p_t + W_j - i\epsilon} \right] \phi, \quad (j = 1, 2) \tag{2}
\]

where \(p_t = p \cdot v, p_t = p - p_t v, W_j = \sqrt{|p_t|^2 + m_j^2}\) and \(\Lambda_j^\pm(p_t) = \frac{W_j \pm \sqrt{p_t^2 + m_j^2}}{2W_j}\), Eq. (1) can be expressed explicitly as

\[
\chi_P^{++}(p) = \frac{-\Lambda_1^+(p_t)\phi}{\lambda_1 M + p_t - W_1 + i\epsilon} \int G(P, p, q) [\chi^{++}(q) + \chi^{--}(q)] \frac{d^4q}{(2\pi)^4}, \tag{3}
\]

\[
\chi_P^{--}(p) = \frac{-\Lambda_1^-(p_t)\phi}{\lambda_1 M + p_t + W_1 - i\epsilon} \int G(P, p, q) [\chi^{++}(q) + \chi^{--}(q)] \frac{d^4q}{(2\pi)^4}, \tag{4}
\]

where \(\chi_P^{\pm\pm}(p) = \Lambda_1^\pm(p_t)\chi_P(p)\Lambda_2^\mp(-p_t)\).

In the heavy quark limit it can be shown that \(\Lambda_1^+(p_t) \approx \frac{1 + \frac{\phi}{2}}{2}, \Lambda_2^+(p_t) \approx \frac{1 + \frac{\phi}{2}}{2}\), and \(\chi_P^{--}\) is small and negligible. In the following we will only consider the large component \(\chi_P^{++}\).

So for a scalar or an axial vector diquark, the B-S wave function can be written in the forms

\[
\chi_P^S(p) = \frac{1 + \frac{\phi}{2}}{2} \sqrt{2M}\phi(p), \quad \chi_P^A(p) = \frac{1 + \frac{\phi}{2}}{2} \sqrt{2M}\gamma_5\phi(p).
\]
The superscript S and A denote the scalar and axial vector diquark respectively and \( \eta \) is the polarization vector of the vector diquark.

Now we assume the kernel \( G \) to have the form

\[
-iG = 1 \otimes 1V_1 + \not{\!p} \otimes \not{\!q}V_2,
\]

(5)

and

\[
V_1(p, q) = \frac{8\pi\beta_1\kappa}{[(pt - qt)^2 + \mu^2]^2} - (2\pi)^3\delta^3(p_t - q_t) \int \frac{8\pi\beta_1\kappa}{(k^2 + \mu^2)^2} \frac{d^3k}{(2\pi)^3},
\]

and

\[
V_2(p, q) = -\frac{16\pi\beta_2\alpha_s}{3(|p_t - q_t|^2 + \mu^2)},
\]

where \( V_1 \) and \( V_2 \) are the parts of the kernel associated with the scalar confinement and one-gluon-exchange diagram respectively\[3\]. The parameters \( \beta_1 \) and \( \beta_2 \) are different for various color states. For mesons, \( \beta_1 = 1, \beta_2 = 1 \), while for color-triplet diquarks, \( \beta_2 \) is directly associated to the color factor caused by the single-gluon exchange, so should be 0.5. In contrast, \( \beta_1 \) which is related to the linear confinement cannot be determined so far and we just take it as a free parameter within a range of 0 \( \sim \) 1 in numerical evaluations. As a matter of fact, later we pick up two typical values 0.5 and 1 for \( \beta_1 \) for demonstrating the influence of the color factor. In fact, the final results are not sensitive to its value, so that our predictions made with the value within a certain range can give rise to a reasonable order of magnitude, even not a precise number. The parameters \( \kappa \) and \( \alpha_s \) are well determined by fitting experimental data of heavy meson spectra. From the heavy meson experimental data, \( \kappa = 0.18, \alpha_s = 0.4 \)[4]. After substituting the form of the kernel Eq. (4) into Eq. (1) we have the following form of the B-S equation

\[
\tilde{\phi}(p_t) = \frac{-1}{M - W_1 - W_2} \int (V_1 - V_2)\tilde{\phi}(q_t) \frac{d^3q_t}{(2\pi)^3},
\]

(6)

where \( \tilde{\phi}(p_t) = \int \phi(p) \frac{dp}{2\pi} \). The above equation can be solved out numerically and by applying the relation between \( \phi(p_t, p_l) \) and \( \tilde{\phi}(p_t) \) we finally obtain the numerical
solution of the B-S equation. This solution will be applied to calculate the weak transition matrix elements of heavy diquarks.

The weak transition form factors of heavy diquarks are closely associated with their inner structure. Namely, to evaluate a transition \( b \to c \) which are constituent quarks of the initial and final diquarks, some \( Q^2 \)-dependent form factors would naturally emerge.

The form factors are process-dependent. For the semileptonic decay \( \chi_{bQ'}(v) \to \chi_{cQ'}(v') + l + \bar{v} \) with the light quark being a spectator, the fundamental vertex \( J_\mu \) corresponds to a radiation of a virtual \( W^- \) boson, so that

\[
J^\mu = \frac{g_w}{2\sqrt{2}} V^*_{cb} \bar{c}\gamma^{\mu}(1 - \gamma_5)b,
\]

where \( g_w \) is the weak coupling constant.

The effective currents \( L_\mu \) in the expressions of the heavy diquark transition matrix elements are calculated by means of the B-S equation for heavy diquarks.

For scalar or axial-vector diquark transitions, one has the following four types:

\[
\begin{align*}
\langle M'_S(v') | J_\mu | M_S(v) \rangle &= 2\sqrt{M'M}[f_1(v \cdot v')v'_\mu + f_2(v \cdot v')v_\mu], \\
\langle M'^A(v', \eta') | J_\mu | M^A(v, \eta) \rangle &= 2\sqrt{M'M}[f_3(v \cdot v')\eta' \cdot \eta'_\mu + f_4(v \cdot v')\eta' \cdot \eta v_\mu \\
&+ f_5(v \cdot v')\eta \cdot \eta'_\mu \cdot vv'_\mu + f_6(v \cdot v')\eta \cdot \eta' \cdot vv_\mu \\
&+ f_7(v \cdot v')\eta \cdot \eta'_\mu + f_8(v \cdot v')\eta' \cdot vv_\mu \\
&+ f_9(v \cdot v')i\epsilon_{\mu\nu\rho\sigma}\eta'^\nu v'^\rho v^\sigma \\
&+ f_{10}(v \cdot v')i\epsilon_{\mu\nu\rho\sigma}\eta^\nu v^\rho v'^\sigma],
\end{align*}
\]

\[
\begin{align*}
\langle M'^A(\eta', \nu) | J_\mu | M_S(v) \rangle &= 2\sqrt{M'M}[f_{11}\eta'_\mu + f_{12}\eta' \cdot vv'_\mu + f_{13}\eta' \cdot vv_\mu \\
&+ f_{14}i\epsilon_{\mu\nu\rho\sigma}\eta^\nu v'^\rho v^\sigma],
\end{align*}
\]

\[
\begin{align*}
\langle M'^S(v') | J_\mu | M^A(\eta, v) \rangle &= 2\sqrt{M'M}[f_{15}\eta_\mu + f_{16}\eta \cdot vv'_\mu + f_{17}\eta \cdot vv_\mu \\
&+ f_{18}i\epsilon_{\mu\nu\rho\sigma}\eta^\nu v^\rho v'^\sigma].
\end{align*}
\]

On the other hand, the effective matrix elements of heavy diquark transitions
can be expressed by the B-S wave functions in the following

\[ < M'(v')|J_\mu|M(v) > = \int Tr[\bar{\chi}_M'(p') \Gamma \chi_M(p) S^{-1}(p_2)(2\pi)^4 \delta^4(p_2 - p'_2) \frac{d^4p}{(2\pi)^4} \frac{d^4p'}{(2\pi)^4}] \]

(12)

where \( S(p_2) \) is the propagator of \( m_2 \) quark and \( \Gamma \) is the vertex of \( J_\mu \). \( p'_1 \) and \( p_i (i=1,2) \) are

\[ p'_1 = \lambda'_1 M'v' + p', \quad p'_2 = \lambda'_2 M'v' - p', \]
\[ p_1 = \lambda_1 Mv + p, \quad p_2 = -\lambda_2 Mv - p, \]

(13)

where \( M, M' \) are the masses of initial and final diquarks.

So the form factors \( f_i (i=1,...,18) \) in Eqs.(8) to (11) can be expressed as an integral of the two diquarks’ wave functions along with specific coefficients. The numerical values for the coefficients \( f_i (1,...,18) \) in Eqs.(8) through (11) are derived by the combination of Eq. (12) and Eqs.(8) to (11). The effective currents \( L_\mu \) inducing weak transitions between heavy diquarks can be expressed as

\[ L_\mu = \sum_{i=1}^{18} f_i J^{(i)}_\lambda \]

(14)

where the explicit expressions for \( J^{(i)}_\lambda \) are given in Eq. (40) in Appendix A. For instance, the terms in \( L_\mu \) which contribute to \( B_{(bb)} \to B_{(bc)} \) are \( \sum_{i=3}^{10} f_i J^{(i)}_\lambda \). In next section we will apply the effective currents to calculate the hadronic transition matrix elements with the aid of superflavor symmetry.

From the heavy meson experimental data, \( \kappa = 0.18 \) GeV\(^2\), \( \alpha_s = 0.4 \), \( m_b = 4.8 \) GeV, \( m_c = 1.45 \) GeV.

From the B-S equation, the numerical results of \( M \) (the heavy diquark mass) corresponding to the various quarks \( m_i (i = 1, 2) \) and \( \beta_1 \) are listed in Table 1.

**Table 1 Values of heavy diquark masses**

| \( \beta_1 \) | 0.5 | 1 | 0.5 | 1 | 0.5 | 1 |
|---------------|-----|---|-----|---|-----|---|
| \( m_1(\text{GeV}) \) | 4.8 | 4.8 | 4.8 | 4.8 | 1.45 | 1.45 |
| \( m_2(\text{GeV}) \) | 4.8 | 4.8 | 1.45 | 1.45 | 1.45 | 1.45 |
| \( M(\text{GeV}) \) | 9.68 | 9.74 | 6.46 | 6.58 | 3.27 | 3.33 |
3 Formulation for the transition matrix elements and decay widths

(i) The transition amplitudes

For semileptonic decays, the process can be described as a transition of a heavy baryon into another heavy baryon radiating a virtual W-boson which turns into a lepton pair $l\bar{\nu} (\bar{l}\nu)$. In the process, the factorization is perfect, so that the total transition amplitude can be written as

$$T \approx \langle B'|J_\alpha|B \rangle l^\alpha \left( \frac{i}{M_W^2} \right),$$

where the contribution from the leptonic current is

$$l^\alpha \equiv \frac{g_w}{2\sqrt{2}} \bar{u}(p_l)\gamma^\alpha(1-\gamma_5)v(\nu)(p_\nu), \quad \text{for } b \to c l\bar{\nu}_l.$$

At the concerned decay energy scale, the W-boson propagator $\frac{i}{q^2-M_W^2}(-g_{\mu\nu}+q_\mu q_\nu/M_W^2)$ can be approximated as $-ig_{\mu\nu}/M_W^2$. In our calculations, we neglect the lepton masses, because $\tau-$lepton production is hard to measure, we only discuss the cases of $e^-\bar{\nu}_e$ and $\mu^-\bar{\nu}_\mu$ radiation.

Thus we need to derive the forms of the hadronic matrix elements $\langle B'|J_\mu|B \rangle$. The effective currents $L_\mu$ are derived in Section 2, so we obtain the hadronic transition matrix elements $\langle B'|J_\mu|B \rangle$ by calculating $\langle B'|L_\mu|B \rangle$ in the diquark-quark picture. The scalar or axial-vector diquark is treated as a point-like object of color-$\bar{3}$ and spin-0 or -1 with definite form factors which are reflected in the coefficients $f_i$'s of the effective currents $L_\mu$, and combines with the light quark to constitute a baryon of spin-1/2 or -3/2. Thus we can use the superflavor symmetry to evaluate the transition matrix elements at the hadron level. In this scenario, there is only one uncertain function which is determined by non-perturbative QCD, i.e. the Isgur–Wise function $\xi(v \cdot v')$, unlike the case for transitions between light baryons where there are many form factors. Therefore, here we may expect to reduce the
uncertainty and improve the prediction power, and it exactly is the advantage of employing the superflavor symmetry.

In the scenario of superflavor symmetry [5], the wave function for a baryon consisting of a scalar diquark would be

\[ \tilde{\Psi}_X = \begin{pmatrix} 0 \\ u^T C / \sqrt{2M_X} \end{pmatrix}, \quad (16) \]

where \( M_X \) is the mass of the scalar diquark and \( C \) is the charge-conjugation operator satisfying \( C^{-1}\gamma^T \mu C = -\gamma^\mu \). For the spin-1 diquark case [6],

\[ \tilde{\Psi}_{1/2}(v) = \frac{1}{\sqrt{6M_A}} \begin{pmatrix} 0 \\ u^T C \sigma^{\mu\beta} v_{\beta\gamma} \end{pmatrix}, \quad (17) \]

and

\[ \tilde{\Psi}_{3/2} = \frac{1}{\sqrt{2M_A}} \begin{pmatrix} 0 \\ \psi^{\mu T} C \end{pmatrix}, \quad (18) \]

where \( M_A \) is the mass of the spin-1 diquark and the subscripts 1/2 and 3/2 denote the spins of baryons.

Thus the hadronic transition matrix element can be obtained as

\[ T_{\mu} \equiv \langle B'J'(v')|L_{\mu}|BJ(v) >= -\xi(v \cdot v') Tr[\tilde{\Psi}_{J'}(v') \sum_i f_i \Gamma_i \tilde{\Psi}_J(v)], \quad (19) \]

where \( \Gamma_i \)'s are the corresponding vertices in the effective current \( L_{\mu} \). In Ref.[5], the authors presented some transition matrix elements with certain effective currents, instead, here our effective currents correspond to the weak interaction.\(^1\) The explicit expressions for various hadronic transition matrix elements \( T_{\mu i}(i = 1, ...12) \) are listed in Eqs.(28) to (39) in Appendix A.

(ii) The amplitude square

To calculate the cross section, we need to take square of the amplitudes which are given in (i) and Appendix A. In the derivations we use the following relations

\[ \sum_s u(v, s)\bar{u}(v, s) = \frac{\theta + 1}{2}, \quad (20) \]

\(^1\)For evaluating a radiative decay, one can have similar effective currents with only small changes from that given for weak interactions.
\[
\sum_s \Psi_\lambda(v,s) \bar{\Psi}_\delta(v,s) = \frac{\not{p} + 1}{2} \left[-g_\lambda \not{s} + \frac{1}{3} \gamma_\lambda \gamma_\delta + \frac{1}{3} (\gamma_\lambda \gamma_\delta - \gamma_\delta \gamma_\lambda) + \frac{2}{3} v_\lambda v_\delta\right],
\]

for heavy baryons of spin 1/2 and 3/2 respectively and we neglect the lepton masses for simplicity.

\[
\sum_s u_{(l)}(p_3, s) \bar{u}_{(l)}(p_3, s) = (\not{p}_3 + m_l) \approx \not{p}_3, \quad (22)
\]

\[
\sum_s v_{(\nu)}(p_4, s) \bar{v}_{(\nu)}(p_4, s) = \not{p}_4. \quad (23)
\]

It is noted that here we adopt the above conventions for the heavy baryons and leptons, because at the limit \(m_l \sim 0\) this choice provides us with much convenience (also see below for the integration over the final state phase space).

Then the amplitude square \(|T(B_J(v) \rightarrow B'_J(v') + l\bar{\nu})|^2\) can be obtained. The results are given in Appendix B.

(iii) The integration over the final state phase space

To obtain the partial decay width, one needs to integrate out the phase space of the three-body final state. In the limit of \(m_l \sim 0\), the integration becomes much simplified.

It is easy to notice that the amplitude square can be written in a general form

\[
\sum_{\text{spins}} |T_{i\lambda}^{(\lambda)}|^2 \frac{1}{M_W^2} \equiv F_1(p_1 \cdot p_2)(p_3 \cdot p_4) + F_2(p_1 \cdot p_3)(p_2 \cdot p_4) + F_3(p_3 \cdot p_1)(p_4 \cdot p_1) + F_4(p_3 \cdot p_2)(p_4 \cdot p_2) + F_5(p_2 \cdot p_3)(p_1 \cdot p_4) + F_6(p_3 \cdot p_4), \quad (24)
\]

where \(p_3\) and \(p_4\) are the momenta of emitted lepton and neutrino, \(p_1 = mv\) is the decaying baryon momentum, so can be \(m(1, \vec{0})\) and \(p_2 = m'v'\) is the momentum of the decay product which should be integrated over, \(m\) and \(m'\) are the masses of initial and final baryons respectively, and \(T_{i\lambda} (i=1,...,18)\) are given in Appendix A.

Accordingly, the integration of the final state phase space should be properly written according to the above conventions. We write the expressions down explicitly,
\[ \Gamma = \frac{1}{(2s+1)} \int \frac{d^3p_2}{(2\pi)^3 E_2} \int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4} \sum_{spins} |T_{\lambda i\lambda}|^2 \frac{1}{M_W^4} \cdot (2\pi)^4 \delta^4(p_1 - p_2 - p_3 - p_4) \] (25)

where all \( p_i \)'s are defined above.

Thus after a simple manipulation, the final form of the decay width can be written (in the following expression the spin factor \( 2s + 1 = 2 \) for the spin-1/2 baryon decay while for spin-3/2 baryon decay \( 2s + 1 = 4 \)) as

\[ \Gamma = \frac{m'}{16(2s+1)\pi^3} \int_{0}^{(m-m')^2} ds_2 \left( \frac{F_1}{16m^2} s_2 [m^2 + m'^2 - s_2] \right. \\
\left. + \frac{F_2 + F_5}{96m^2} [(m^2 - m'^2 + s_2)(m^2 - m'^2 - s_2) + s_2(m^2 + m'^2 - s_2)] \right. \\
\left. + \frac{F_3}{48} \left( \frac{m^2 + s_2 - m'^2}{2m^2} \right)^2 + s_2 \right) + \frac{F_4}{48m^2} [(m^2 - m'^2 - s_2)^2 + m'^2 s_2] \\
\right. + \frac{F_6}{8m^2 s_2} \right) \cdot \lambda^{1/2}(m^2, m'^2, s_2) \] (26)

where

\[ \lambda(a, b, c) \equiv a^2 + b^2 + c^2 - 2ab - 2bc - 2ca, \]

and \( F_1 \) through \( F_6 \) are given in the expressions of \( \sum |T_{\lambda i\lambda}|^2 / M_W^4 \) by rearranging the corresponding terms so that they are expressed in terms of the form factors \( f_i(i = 1, \ldots, 18) \) which have been calculated in the B-S approach. The concrete forms are obtained by running the REDUCE computer programs and it is a very lengthy and tedious procedure. The relations between \( F_i(i = 1, \ldots, 6) \) and \( f_i(i = 1, \ldots, 18) \) are very complicated and we will not list them here.

### 4 Numerical results

In the relations between \( F_i(i = 1, \ldots, 6) \) and \( f_i(i = 1, \ldots, 18) \) there is an uncertain function, the Isgur-Wise function. Its behavior is controlled by non-perturbative
QCD effects which have to be dealt with in some phenomenological model. Because in the heavy quark limit, the spin of the heavy quark has no effects on the dynamics inside the hadron one expects that the Isgur-Wise function is totally determined by the light degrees of freedom. Therefore, the Isgur-Wise function between the transition of the heavy baryons consisting of two heavy quarks should be the same as that of the corresponding heavy mesons. Actually this is the plausibility of applying the superflavor symmetry. Hence we can simply use the form of the Isgur-Wise function for $B \to D$ in our numerical calculations for the decay width of heavy baryons which contain two heavy quarks. There are some model calculations for the Isgur-Wise function for $B \to D$ [10] [11] [12]. In our following numerical calculations we will use the following simple form given in [11]

$$\xi(\omega) = \frac{1}{1 - \omega^2/\omega_0^2},$$  \hspace{1cm} (27)

where the constant is taken to be $\omega_0 = 1.24$. It is noted that different forms of the Isgur-Wise function will give somehow different predictions. However our numerical computations show that with various Isgur-Wise function forms the order is not changed. The numerical results for the semileptonic decay widths for different processes are listed in Table 2. From the numerical results in Table 2 we can see that the decay widths are around the order $10^{-13} \sim 10^{-14}$s$^{-1}$. It can also be seen that the results are insensitive to the parameter $\beta_1$. 

5 Summary and discussions

In the present work we discuss the weak transitions between heavy baryons which consist of two heavy quarks in the heavy quark limit. The three-body system is simplified into a two-body system of a heavy diquark and a light quark. In the heavy quark limit, the heavy diquark is a point-like spin-0 or spin-1 object and the light quark is blind to the spin and flavor of the heavy diquark. With the help of the superflavor symmetry the matrix elements between heavy baryons and those between heavy mesons are related to each other and they can be described by the same Isgur-Wise function. To deal with the weak transitions between heavy diquarks we work in the B-S equation approach. We obtain the numerical solutions of the B-S equation by assuming the kernel which contains linear scalar confinement and one-gluon-exchange vector terms. The numerical solutions are used to obtain the effective currents between two heavy diquarks. These effective currents are expressed in terms of the coefficients $f_i (i = 1, \ldots, 18)$ which can be solved out numerically from the B-S equation for heavy diquarks. Then the weak transition matrix elements between

| $\beta_1$ | 1 | 0.5 |
|-----------|---|-----|
| $B_{(bb)} \to B_{(bc)}$ | $2.85 \times 10^{-13}$ | $2.78 \times 10^{-13}$ |
| $B_{(bb)} \to B_{(bc)^*}$ | $4.28 \times 10^{-14}$ | $4.81 \times 10^{-14}$ |
| $B_{(bc)} \to B_{(bc)}$ | $2.72 \times 10^{-13}$ | $2.69 \times 10^{-13}$ |
| $B_{(bc)} \to B_{(bc)^*}$ | $1.29 \times 10^{-13}$ | $1.41 \times 10^{-13}$ |
| $B_{(bc)^*} \to B_{(bc)}$ | $5.20 \times 10^{-13}$ | $5.15 \times 10^{-13}$ |
| $B_{(bc)^*} \to B_{(bc)^*}$ | $8.57 \times 10^{-14}$ | $9.61 \times 10^{-14}$ |
| $B_{(bc)} \to B_{(cc)}$ | $1.72 \times 10^{-13}$ | $1.68 \times 10^{-13}$ |
| $B_{(bc)^*} \to B_{(cc)}$ | $2.75 \times 10^{-14}$ | $2.81 \times 10^{-14}$ |
| $B_{(cc)} \to B_{(cc)}$ | $1.41 \times 10^{-13}$ | $1.43 \times 10^{-13}$ |
| $B_{(cc)^*} \to B_{(cc)^*}$ | $8.93 \times 10^{-14}$ | $9.32 \times 10^{-14}$ |
| $B_{(cc)} \to B_{(cc)}^*$ | $2.88 \times 10^{-13}$ | $2.91 \times 10^{-13}$ |
| $B_{(cc)^*} \to B_{(cc)^*}$ | $7.76 \times 10^{-14}$ | $7.82 \times 10^{-14}$ |
heavy baryons are expressed in terms of the Isgur-Wise function and $f_i(i = 1, \ldots, 18)$. Consequently we give the predictions for the semileptonic decay widths for all the possible twelve decay channels between two heavy baryons. The decay widths are around the order $10^{-13} \sim 10^{-14}\text{s}^{-1}$. These predictions will be tested in the future experiments.

There are some uncertainties in our work. First, as we have been working in the heavy quark limit, all $1/m_Q$ corrections are ignored. In the heavy quark limit physics is greatly simplified and we have only one unknown function, the Isgur-Wise function. Therefore, if one wishes to make a precise comparison of the theoretically calculated numbers with data, the $1/m_Q$ especially $1/m_c$ corrections must be taken into account. Besides this approximation when we calculate the effective currents between two heavy diquarks we work in the B-S equation approach in which the most uncertain point is the kernel which depends on non-perturbative QCD effects. Motivated by potential model we use the simple form which has linear scalar confinement and one-gluon-exchange terms. In the confinement part the parameter $\beta_1$ is not fixed and we pick up two typical values as 0.5 and 1. Fortunately, the decay widths are insensitive to this parameter. The Isgur-Wise function is another uncertain point since it is also controlled by non-perturbative QCD dynamics between heavy diquark and light quark and thus its evaluation is model dependent. To get the numerical results we use the simple form obtained in Ref. [10]. Different forms for the Isgur-Wise function may result in different decay widths. However, the order is not changed.

Acknowledgment:

This work was supported in part by the Australian Research Council and the National Science Foundation of China.
Appendix A. The hadronic transition matrix elements

(1) $B_{(bb)_1} \to B_{(bc)_1}$

$$
T_{1\lambda} = < B_{(bc)_1}^A \left( \frac{1}{2} \right) | L_{\lambda} | B_{(bb)_1}^A \left( \frac{1}{2} \right) >
$$

$$
= 2 \sqrt{M_1 M_2} < B_{(bc)_1}^A \left( \frac{1}{2} \right) | f_3 J_\lambda^{(3)} + f_4 J_\lambda^{(4)} + f_5 J_\lambda^{(5)} + f_6 J_\lambda^{(6)} + f_7 J_\lambda^{(7)} + f_8 J_\lambda^{(8)} + f_9 J_\lambda^{(9)} + f_{10} J_\lambda^{(10)} | B_{(bb)_1}^A \left( \frac{1}{2} \right) >
$$

$$
= \frac{1}{3} \xi (v \cdot v') \{ -f_3 v'_\lambda - f_4 v_\lambda (2 + v \cdot v') + (f_5 v'_\lambda + f_6 v_\lambda) (1 - (v \cdot v')^2) - (f_7 v'_\lambda + f_8 v_\lambda) (1 + v \cdot v') | \bar{u}' u + (f_7 + f_8) (1 + v \cdot v') | \bar{u}' \gamma_\lambda u \\
+ i (f_9 v^\rho \nu^\sigma - f_{10} v'^\rho \nu'^\sigma) \epsilon_{\lambda \delta \rho \sigma} \bar{u}' \gamma^\delta u \\
- i( f_9 v'^\rho + f_{10} v'^\sigma ) \epsilon_{\lambda \delta \rho \sigma} \bar{u}' \gamma^\delta \gamma^\rho u \}.
$$

(28)

(2) $B_{(bb)_1} \to B_{(bc)_0}$

$$
T_{2\lambda} = < B_{(bc)_0}^S \left( \frac{1}{2} \right) | L_{\lambda} | B_{(bb)_1}^A \left( \frac{1}{2} \right) >
$$

$$
= 2 \sqrt{M_1 M_2} < B_{(bc)_0}^S \left( \frac{1}{2} \right) | f_{15} J_\lambda^{(15)} + f_{16} J_\lambda^{(16)} + f_{17} J_\lambda^{(17)} + f_{18} J_\lambda^{(18)} | B_{(bb)_1}^A \left( \frac{1}{2} \right) >
$$

$$
= \frac{i}{\sqrt{3}} \xi (v \cdot v') \{ [(1 + v \cdot v') (f_{16} v'_\lambda + f_{17} v_\lambda) + f_{15} v_\lambda | \bar{u}' \gamma_5 u \\
+ f_{15} \bar{u}' \gamma_\lambda \gamma_5 u - i f_{18} \epsilon_{\lambda \delta \rho \sigma} \bar{u}' \gamma_5 \gamma^\delta \gamma^\rho u \}.
$$

(29)

(3) $B_{(bb)_1} \to B_{(bc)_1}^*$

$$
T_{3\lambda} = < B_{(bc)_1}^A \left( \frac{3}{2} \right) | L_{\lambda} | B_{(bb)_1}^A \left( \frac{1}{2} \right) >
$$

$$
= 2 \sqrt{M_1 M_2} < B_{(bc)_1}^A \left( \frac{3}{2} \right) | f_3 J_\lambda^{(3)} + f_4 J_\lambda^{(4)} + f_5 J_\lambda^{(5)} + f_6 J_\lambda^{(6)} + f_7 J_\lambda^{(7)} + f_8 J_\lambda^{(8)} + f_9 J_\lambda^{(9)} + f_{10} J_\lambda^{(10)} | B_{(bb)_1}^A \left( \frac{1}{2} \right) >
$$

$$
= \frac{i}{\sqrt{3}} \xi (v \cdot v') \{ [f_3 v'_\lambda v_\delta + f_4 v_\lambda v_\delta + (f_5 v'_\lambda v_\delta + f_6 v_\lambda v_\delta) (1 + v \cdot v') \\
+ f_8 v_\lambda v_\delta | \bar{\psi} \gamma^5 u \\
+ (f_3 v'_\lambda + f_4 v_\lambda) \bar{\psi} \gamma_5 \gamma_\delta \gamma_5 u + f_8 v_\delta \bar{\psi} \gamma_\delta \gamma_\lambda \gamma_5 u \\
+ i f_9 v^\rho v'^\sigma \epsilon_{\lambda \delta \rho \sigma} \bar{\psi} \gamma^5 u + i( f_9 v'^\rho + f_{10} v'^\sigma ) \epsilon_{\lambda \delta \rho \sigma} \bar{\psi} \gamma^\delta \gamma^\rho \gamma_5 u \}. 
$$

(30)
(4) \( B^*_{(bb)_1} \to B^*_{(bc)_1} \)

\[
T_{4\lambda} = <B_{(bc)}^A(\frac{3}{2})|L_\lambda|B_{(bb)}^A(\frac{3}{2}) >
= 2\sqrt{M_1 M_2} < B_{(bc)}^A(\frac{1}{2}) |f_3 J^{(3)}_\lambda + f_4 J^{(4)}_\lambda + f_5 J^{(5)}_\lambda + f_6 J^{(6)}_\lambda + f_7 J^{(7)}_\lambda + f_8 J^{(8)}_\lambda + f_9 J^{(9)}_\lambda + f_{10} J^{(10)}_\lambda | B_{(bb)}^A(\frac{1}{2}) >
\]

\[
= \xi (v \cdot v') \{ (f_3 v'_\lambda + f_4 v_\lambda) \bar{\Psi}^\alpha \Psi^\delta + (f_5 v'_\lambda v'_\nu \nu + f_6 v_\lambda v'_\nu \nu) \bar{\Psi}^{\mu} \Psi^\nu + (f_7 v'_\delta + f_8 v_\delta) \bar{\Psi}^\alpha \Psi + i(f_9 v^\sigma + f_{10} v^\sigma) \epsilon_{\lambda \delta \rho \sigma} \bar{\Psi}^\alpha \Psi^\rho \}. \quad (31)
\]

(5) \( B^*_{(bb)_1} \to B_{(bc)_1} \)

\[
T_{5\lambda} = <B_{(bc)}^A(\frac{1}{2})|L_\lambda|B_{(bb)}^A(\frac{3}{2}) >
= 2\sqrt{M_1 M_2} < B_{(bc)}^A(\frac{1}{2}) |f_3 J^{(3)}_\lambda + f_4 J^{(4)}_\lambda + f_5 J^{(5)}_\lambda + f_6 J^{(6)}_\lambda + f_7 J^{(7)}_\lambda + f_8 J^{(8)}_\lambda + f_9 J^{(9)}_\lambda + f_{10} J^{(10)}_\lambda | B_{(bb)}^A(\frac{1}{2}) >
\]

\[
= \frac{i}{\sqrt{3}} \xi (v \cdot v') \{ [(f_3 v'_\lambda v'_\alpha + f_4 v_\lambda v'_\alpha) + \nu + \nu') + f_7 v'_\lambda v'_\alpha \bar{u}' \gamma_5 \Psi^\alpha + f_8 (1 + v \cdot v') \bar{u}' \gamma_5 \Psi^\lambda - (f_3 v'_\lambda + f_4 v_\lambda) \bar{u}' \gamma_5 \Psi^\alpha - f_7 v'_\lambda \bar{u}' \gamma_5 \Psi^\lambda - f_8 (1 + v \cdot v') \bar{u}' \gamma_5 \Psi^\lambda \}
\]

\[
+ i f_{10} v^\delta v^\sigma \epsilon_{\lambda \delta \rho \sigma} \bar{u}' \gamma_5 \Psi^\rho - i(f_9 v^\sigma + f_{10} v^\sigma) \epsilon_{\lambda \delta \rho \sigma} \bar{u}' \gamma_5 \Psi^\rho \}. \quad (32)
\]

(6) \( B^*_{(bb)_1} \to B_{(bc)_0} \)

\[
T_{6\lambda} = <B_{(bc)}^S(\frac{1}{2})|L_\lambda|B_{(bb)}^A(\frac{3}{2}) >
= 2\sqrt{M_1 M_2} < B_{(bc)}^S(\frac{1}{2}) |f_{15} J^{(15)}_\lambda + f_{16} J^{(16)}_\lambda + f_{17} J^{(17)}_\lambda + f_{18} J^{(18)}_\lambda | B_{(bb)}^A(\frac{3}{2}) >
\]

\[
= \xi (v \cdot v') \{ f_{15} \bar{u}' \Psi^\lambda + (f_{16} v'_\delta v'_\lambda + f_{17} v'_\delta v_\lambda) \bar{u}' \Psi^\delta
+ i f_{18} \epsilon_{\lambda \delta \rho \sigma} v^\rho v^\sigma \bar{u}' \Psi^\delta \}. \quad (33)
\]

(7) \( B^*_{(bc)_1} \to B^*_{(cc)_1} \)

\[
T_{7\lambda} = <B_{(cc)}^A(\frac{3}{2})|L_\lambda|B_{(bc)}^A(\frac{3}{2}) >
= 2\sqrt{M_1 M_2} < B_{(bc)}^A(\frac{1}{2}) |f_3 J^{(3)}_\lambda + f_4 J^{(4)}_\lambda + f_5 J^{(5)}_\lambda + f_6 J^{(6)}_\lambda + f_7 J^{(7)}_\lambda + f_8 J^{(8)}_\lambda + f_9 J^{(9)}_\lambda + f_{10} J^{(10)}_\lambda | B_{(bb)}^A(\frac{1}{2}) >
\]

\[
= \xi (v \cdot v') \{ (f_3 v'_\lambda + f_4 v_\lambda) \bar{\Psi}^\delta + (f_5 v'_\lambda v'_\nu \nu + f_6 v_\lambda v'_\nu \nu) \bar{\Psi}^{\mu} \Psi^\nu + (f_7 v'_\delta + f_8 v_\delta) \bar{\Psi}^\alpha \Psi + i(f_9 v^\sigma + f_{10} v^\sigma) \epsilon_{\lambda \delta \rho \sigma} \bar{\Psi}^\alpha \Psi^\rho \}. \quad (34)
\]
\[ T_{8\lambda} = \langle B_{(bc)}(\frac{1}{2})|L_\lambda|B_{(bc)}(\frac{3}{2}) \rangle \]
\[ = 2\sqrt{M_1 M_2} < B_{(bc)}(\frac{1}{2})|f_3 J_\lambda^{(3)} + f_4 J_\lambda^{(4)} + f_5 J_\lambda^{(5)} + f_6 J_\lambda^{(6)} + f_7 J_\lambda^{(7)} + f_8 J_\lambda^{(8)} + f_9 J_\lambda^{(9)} + f_{10} J_\lambda^{(10)} |B_{(bb)}(\frac{1}{2})\rangle > \]
\[ = \frac{i}{\sqrt{3}} \xi(v \cdot v') \{(f_3 v_\lambda^a v_\delta^a + f_4 v_\lambda v_\delta + f_5 v_\lambda^a v_\delta + f_6 v_\lambda v_\delta)(1 + v \cdot v') \}
\[ + f_8 (1 + v \cdot v') \bar{u} \gamma_5 \Psi \lambda - (f_3 v_\lambda^a + f_4 v_\lambda) \bar{u} ' \gamma_5 \gamma_5 \Psi \alpha - f_7 v_\lambda^a \bar{u} ' \gamma_5 \gamma_5 \Psi \alpha \]
\[ + i f_9 v^a \frac{v^a}{v^p} \epsilon_{\lambda p s} \bar{u} ' \gamma_5 \gamma_5 \Psi \rho - i (f_9 v^a + f_10 v^a) \epsilon_{\lambda p s} \bar{u} ' \gamma_5 \gamma_5 \Psi \rho \}. \]

(9) \[ B_{(bc)} \rightarrow B_{(cc)} \]

\[ T_{9\lambda} = \langle B_{(bc)}(\frac{3}{2})|L_\lambda|B_{(bb)}(\frac{1}{2}) \rangle \]
\[ = 2\sqrt{M_1 M_2} < B_{(bc)}(\frac{3}{2})|f_3 J_\lambda^{(3)} + f_4 J_\lambda^{(4)} + f_5 J_\lambda^{(5)} + f_6 J_\lambda^{(6)} + f_7 J_\lambda^{(7)} + f_8 J_\lambda^{(8)} + f_9 J_\lambda^{(9)} + f_{10} J_\lambda^{(10)} |B_{(bb)}(\frac{1}{2})\rangle > \]
\[ = \frac{i}{\sqrt{3}} \xi(v \cdot v') \{(f_3 v_\lambda^a v_\delta^a + f_4 v_\lambda v_\delta + f_5 v_\lambda^a v_\delta + f_6 v_\lambda v_\delta)(1 + v \cdot v') \}
\[ + f_8 v_\lambda v_\delta \bar{v} ' \gamma_5 u 
\[ - (f_3 v_\lambda + f_4 v_\lambda) \bar{v} ' \gamma_5 \gamma_5 u - f_8 v_\delta \bar{v} ' \gamma_5 \lambda u \]
\[ + i f_9 v^p \frac{v^p}{v^q} \epsilon_{\lambda p s} \bar{v} ' \gamma_5 \gamma_5 u - i (f_9 v^p + f_10 v^p) \epsilon_{\lambda p s} \bar{v} ' \gamma_5 \gamma_5 \gamma^p u \}. \]

(10) \[ B_{(bc)} \rightarrow B_{(cc)} \]

\[ T_{10\lambda} = \langle B_{(bc)}(\frac{1}{2})|L_\lambda|B_{(bb)}(\frac{1}{2}) \rangle \]
\[ = 2\sqrt{M_1 M_2} < B_{(bc)}(\frac{1}{2})|f_3 J_\lambda^{(3)} + f_4 J_\lambda^{(4)} + f_5 J_\lambda^{(5)} + f_6 J_\lambda^{(6)} + f_7 J_\lambda^{(7)} + f_8 J_\lambda^{(8)} + f_9 J_\lambda^{(9)} + f_{10} J_\lambda^{(10)} |B_{(bb)}(\frac{1}{2})\rangle > \]
\[ = \frac{1}{3} \xi(v \cdot v') \{(-f_3 v_\lambda + f_4 v_\lambda)(2 + v \cdot v') + (f_5 v_\lambda + f_6 v_\lambda)(1 - (v \cdot v')^2) \}
\]
\[-(f_7 v'_\lambda + f_8 v_\lambda)(1 + v \cdot v') \bar{u}' u + (f_7 + f_8)(1 + v \cdot v') \bar{u}' \gamma_\lambda u \]
\[+ i(f_9 v^\rho v^\sigma - f_{10} v^\rho v^\sigma) \epsilon_{\lambda \delta \rho \sigma} \bar{u}' \gamma^\delta u \]
\[-i(f_9 v^\rho + f_{10} v^\rho) \epsilon_{\lambda \delta \rho \sigma} \bar{u}' \gamma^\delta \gamma^\rho u \}. \quad (37)\]

(11) $B_{(bc)\alpha} \rightarrow B_{(bc)1}^*$

$$T_{11\alpha} = \langle B_{(bc)1}^* | L_\lambda | B_{(bc)}(1/2) \rangle \rangle$$
\[= 2 \sqrt{M_1 M_2} < B_{(cc)}(1/2) | f_{11} J_\lambda^{(11)} + f_{12} J_\lambda^{(12)} + f_{13} J_\lambda^{(13)} + f_{14} J_\lambda^{(14)} | B_{(bc)}(1/2) \rangle > \]
\[= \xi(v \cdot v') \{ (f_{12} + f_{13} v_\lambda) 1/v \} \bar{u}' \gamma_{5} u \]
\[-i(f_{14} \epsilon_{\lambda \delta \rho \sigma} v^\rho \gamma_\delta \gamma_5 u \} \}. \quad (38)\]

(12) $B_{(bc)\alpha} \rightarrow B_{(cc)1}$

$$T_{12\alpha} = \langle B_{(cc)1}^* | L_\lambda | B_{(bc)}(1/2) \rangle \rangle$$
\[= 2 \sqrt{M_1 M_2} \langle B_{(cc)}(1/2) | f_{11} J_\lambda^{(11)} + f_{12} J_\lambda^{(12)} + f_{13} J_\lambda^{(13)} + f_{14} J_\lambda^{(14)} | B_{(bc)}(1/2) \rangle \rangle \]
\[= \frac{i}{\sqrt{3}} \xi(v \cdot v') \{ (f_{11} v'_\lambda + f_{12} v'_\lambda + f_{13} v_\lambda)(1 + v \cdot v') \} \bar{u}' \gamma_{5} u \]
\[-f_{11} \bar{u}' \gamma_{5} u - i f_{14} \epsilon_{\lambda \delta \rho \sigma} v^\rho \gamma_\delta \gamma_5 u \}. \quad (39)\]

In the Eqs. (38) to (39) we define

\begin{align*}
J_1^{(1)} &= \chi_f^{(1)}(v') v' \chi_i(v), & J_2^{(1)} &= \chi_f^{(1)}(v') v \chi_i(v), \\
J_3^{(1)} &= \chi_f^{(1)}(v') v' \chi_{\lambda \mu}(v), & J_4^{(1)} &= \chi_f^{(1)}(v') v \chi_{\lambda \mu}(v), \\
J_5^{(1)} &= (\chi_f^{(1)}(v') v) v' \chi_i(v), & J_6^{(1)} &= (\chi_f^{(1)}(v') v) v \chi_i(v), \\
J_7^{(1)} &= \chi_f^{(1)}(v') v' \chi_i(v), & J_8^{(1)} &= \chi_f^{(1)}(v') v \chi_i(v), \\
J_9^{(1)} &= i \epsilon_{\lambda \delta \rho \sigma} \chi_f^{(1)}(v') v^\rho v^\sigma v, & J_{10}^{(1)} &= i \epsilon_{\lambda \delta \rho \sigma} \chi_f^{(1)}(v') v^\rho v^\sigma v, \\
J_{11}^{(1)} &= \chi_f^{(1)}(v') v \chi_i(v), & J_{12}^{(1)} &= \chi_f^{(1)}(v') v \chi_i(v), \\
J_{13}^{(1)} &= (\chi_f^{(1)}(v') v) v \chi_i(v), & J_{14}^{(1)} &= (\chi_f^{(1)}(v') v) v \chi_i(v), \\
J_{15}^{(1)} &= \chi_f^{(1)}(v') v \chi_i(v), & J_{16}^{(1)} &= \chi_f^{(1)}(v') v \chi_i(v), \\
J_{17}^{(1)} &= \chi_f^{(1)}(v') v \chi_{\lambda \mu}(v), & J_{18}^{(1)} &= \chi_f^{(1)}(v') v \chi_{\lambda \mu}(v),
\end{align*}

where $\chi_i(v)$, $\chi_f(v')$, $\chi_i^\dagger(v)$ and $\chi_f^\dagger(v')$ stand for the initial scalar, final scalar, initial vector and final vector diquark fields in the baryons respectively.

It is noted that for the electromagnetic currents, the flavors do not change at the two sides of the vertex, so that there are some extra symmetries as used by Georgi,
Wise and Carone and the fact is expressed in the relations between the coefficients. For example, in the case of the $\gamma - M - M^*$ vertex, due to the current conservation (CVC), $f_1 = -f_2$ is required, and from our formulae, one can immediately prove that.

**Appendix B: The amplitude square**

(1) $B_{(bb)_1} \to B_{(bc)_1} + l + \bar{\nu}$

$$\Gamma_1 = \sum_{\text{spins}} |T_{1\lambda}^{l\lambda}|^2$$

$$= \frac{8}{9} \xi(v \cdot v')^2 Tr[A_\lambda A_\lambda^\dagger u^\dagger u u' + B^2 u^\dagger \gamma_\lambda u^\dagger \gamma_\lambda^\dagger u' + (A_\lambda B u^\dagger \gamma_\lambda u^\dagger u' + C.T)$$

$$\epsilon_{\lambda \delta \rho \sigma} \epsilon_{\lambda' \delta' \rho' \sigma'} \bar{u}^\dagger \gamma^\delta u^\dagger \gamma^\delta \gamma^\delta \gamma^\delta u' - (C'_{\rho \sigma} D^\sigma \epsilon_{\lambda \delta \rho \sigma} \epsilon_{\lambda' \delta' \rho' \sigma'}) \bar{u}^\dagger \gamma^\rho \gamma^\rho \gamma^\rho \gamma^\rho u'$$

$$\cdot (p_3^\lambda p_4^\lambda + p_3^\lambda p_4^\lambda - (p_3 \cdot p_4) g^{\lambda \lambda}) \tag{41}$$

where

$$A_\lambda = (-f_3 v_\lambda' - f_4 v_\lambda)(2 + v \cdot v') + (f_5 v_\lambda' + f_6 v_\lambda)(1 - (v \cdot v')^2) - (f_7 v_\lambda' + f_8 v_\lambda)(1 + v \cdot v'), \tag{42}$$

$$B = (f_7 + f_8)(1 + v \cdot v'), \tag{43}$$

$$C'_{\rho \sigma} = f_9 v^\rho v^\sigma - f_{10} v^\rho v^\sigma, \tag{44}$$

$$D^\sigma = f_9 v^\sigma + f_{10} v^\sigma. \tag{45}$$

It is noted that in $Tr[p_3^\lambda \gamma_\lambda(1 - \gamma_5) p_4^\lambda \gamma_\lambda(1 - \gamma_5)] = 2Tr[p_3^\lambda \gamma_\lambda p_4^\lambda \gamma_\lambda - p_3^\lambda \gamma_\lambda p_4^\lambda \gamma_\lambda \gamma_5]$ there is a term $\epsilon_{\alpha \lambda \alpha' \lambda'} p_3^\alpha p_4^\alpha$ which is antisymmetric to an exchange of $p_3$ and $p_4$, so should make null contribution after integration over the final state phase space as long as we omit the masses of leptons. The ”C.T” means the conjugate term, for example

$$(\bar{u}^\dagger \Gamma u^\dagger u' u')^* = \bar{u}^\dagger \gamma_0^\dagger \gamma_0 u' u^\dagger \gamma_0 \Gamma^\dagger \gamma_0 u, \tag{19}$$
and the other parts would be changed accordingly. Later the terms concerning the spinor-vector $\psi_{\mu}$ are also of their conjugate correspondence which can be obtained in a rule similar to the spinors.

(2) $B_{(bb)} \rightarrow B_{(bc)} + l + \bar{\nu}$

$$
\Gamma_2 = \sum_{\text{spins}} |T_{2\lambda}l^\lambda|^2 \\
= \frac{8}{3} |(v \cdot v')|^2 Tr[-A_\lambda A_\lambda' \bar{\psi}^\delta \gamma_5 u \bar{\psi} \gamma_5 + (A_\lambda B_{\lambda'} \bar{\psi}^\delta \gamma_5 u \bar{\psi} \gamma_5 + C.T) \\
+ B^2 \bar{u} \gamma_5 u \bar{u} \gamma_{5} u + C^\rho_{\sigma} C_{\rho'}_{\sigma'} \epsilon_{\lambda \delta \rho \sigma} \epsilon_{\lambda' \delta' \rho' \sigma'} \bar{u} \gamma_{5} u \bar{u} \gamma_{5'} u'] \\
\cdot [(p_3^3 \bar{p}_4^4 + p_3^\lambda p_4^\lambda - (p_3 \cdot p_4)g^{\lambda \lambda'}),

(46)

where

$$
A_\lambda = (f_{16} \bar{v}_\lambda + f_{17} v_{\lambda})(1 + v \cdot v') + f_{15} v_{\lambda},
$$

(47)

$$
B = f_{15},
$$

(48)

$$
C^\rho_{\sigma} = f_{18} v^\rho v^\sigma.
$$

(49)

(3) $B_{(bb)} \rightarrow B^*_{(bc)} + l + \bar{\nu}$

$$
\Gamma_3 = \sum_{\text{spins}} |T_{3\lambda}l^\lambda|^2 \\
= \frac{8}{3} |(v \cdot v')|^2 Tr[-A_\lambda A_\lambda' \bar{\psi}^\delta \gamma_5 u \bar{\psi} \gamma_5 \Psi^{\delta'} + (A_\lambda B_{\lambda'} \bar{\psi}^\delta \gamma_5 u \bar{\psi} \gamma_5 \Psi^{\delta'} + C.T) \\
+ (A_\lambda B_{\lambda'} \bar{\psi}^\delta \gamma_5 u \bar{\psi} \gamma_5 \Psi^{\delta'} + C.T) + B_\lambda B_{\lambda'} \bar{\psi}^{\delta'} \gamma_5 u \bar{\psi} \gamma_5 \Psi^{\delta'} \\
+ C_{\delta'} \bar{\psi}^{\delta'} \gamma_5 u \bar{\psi} \gamma_5 \Psi^{\delta'} + (B_\lambda C_{\delta'} \bar{\psi}^{\delta'} \gamma_5 u \bar{\psi} \gamma_5 \Psi^{\delta'} + C.T) \\
+ D_{\rho'} D_{\sigma'} \epsilon_{\lambda \delta \rho \sigma} \epsilon_{\lambda' \delta' \rho' \sigma'} \bar{\psi}^{\delta'} \gamma_5 u \bar{\psi} \gamma_5 \Psi^{\delta'} \\
+ (E_{\rho'} D_{\sigma'} \epsilon_{\lambda \delta \rho \sigma} \epsilon_{\lambda' \delta' \rho' \sigma'} \bar{\psi}^{\delta'} \gamma_5 u \bar{\psi} \gamma_5 \Psi^{\delta'} + C.T) \\
-E_{\rho'} E_{\sigma'} \epsilon_{\lambda \delta \rho \sigma} \epsilon_{\lambda' \delta' \rho' \sigma'} \bar{\psi}^{\delta'} \gamma_5 u \bar{\psi} \gamma_5 \Psi^{\delta'}] \cdot (p_3^\lambda p_4^\lambda + p_3^\lambda p_4^\lambda - (p_3 \cdot p_4)g^{\lambda \lambda'}), 

(50)

where

$$
A_{\lambda \delta} = f_3 v_\lambda^\lambda v_\delta + f_4 v_\lambda v_\delta + (f_5 v_\lambda^\lambda + f_6 v_\lambda)v_\delta (1 + v \cdot v') + f_8 v_\delta v_\lambda,
$$

(51)
\[B_\lambda = f_3 v_\lambda' + f_4 v_\lambda, \quad (52)\]
\[C_\delta = f_8 v_\delta, \quad (53)\]
\[D^\sigma = f_9 v^\sigma + f_{10} v^\sigma, \quad (54)\]
\[E^{\rho\sigma} = f_9 v^{\rho} v^\sigma. \quad (55)\]

(4) \(B^* \to B + l + \bar{\nu}\)

\[\Gamma_4 = \sum_{\text{spins}} |T_{4\lambda} l^\lambda|^2 \]
\[= 8|\xi(v \cdot v')|^2 Tr[A_\lambda A_\lambda' \bar{\bar{\Psi}}_{\delta'} \bar{\bar{\Psi}}_{\delta} + (A_\lambda B_{\nu\mu} A'_\lambda B_{\nu'\mu'} + C.T) \]
\[+ (A_\lambda C_\delta \bar{\bar{\Psi}}_{\delta} + C.T) + B_{\nu\mu} A'_{\lambda'\nu'} \bar{\bar{\Psi}}_{\delta'} + C.T) + B_{\nu\mu} A'_{\lambda'\nu'} \bar{\bar{\Psi}}_{\delta'} + C.T) \]
\[+ D^\sigma D^{\sigma'} \epsilon_{\lambda \delta \rho \sigma} (\bar{\bar{\Psi}}_{\delta} \bar{\bar{\Psi}}_{\delta'} + C.T)] \cdot \left[(p_3^A p_4 + p_3^A p_4 - (p_3 \cdot p_4) g^{\lambda \lambda'})\right], \quad (56)\]

where

\[A_\lambda = f_3 v_\lambda' + f_4 v_\lambda, \quad (57)\]
\[B_{\nu\mu} = f_5 v_\mu v_\nu + f_6 v_\mu v_\nu, \quad (58)\]
\[C_\delta = f_7 v_\delta + f_8 v_\delta, \quad (59)\]
\[D^\sigma = f_9 v^\sigma + f_{10} v^\sigma. \quad (60)\]

(5) \(B^* \to B + l + \bar{\nu}\)

\[\Gamma_5 = \sum_{\text{spins}} |T_{5\lambda} l^\lambda|^2 \]
\[= \frac{8}{3}|\xi(v \cdot v')|^2 Tr[-A_\lambda A_\lambda' \bar{\bar{\Psi}}_{\delta'} \bar{\bar{\Psi}}_{\delta} + (A_\lambda B_{\nu\mu} A'_\lambda B_{\nu'\mu'} + C.T) \]
\[+ (A_\lambda C_\delta \bar{\bar{\Psi}}_{\delta} + C.T) + B_{\nu\mu} A'_{\lambda'\nu'} \bar{\bar{\Psi}}_{\delta'} + C.T) + B_{\nu\mu} A'_{\lambda'\nu'} \bar{\bar{\Psi}}_{\delta'} + C.T) \]
\[+ D^\sigma D^{\sigma'} \epsilon_{\lambda \delta \rho \sigma} (\bar{\bar{\Psi}}_{\delta} \bar{\bar{\Psi}}_{\delta'} + C.T)] \cdot \left[(p_3^A p_4 + p_3^A p_4 - (p_3 \cdot p_4) g^{\lambda \lambda'})\right]. \]
\[-F^{\delta\sigma}F^{\delta'\sigma'}\epsilon_{\lambda\delta\rho\sigma}\epsilon_{\lambda'\delta'\rho'\sigma'}\bar{u}'\gamma_5\psi^{\rho'}\gamma_5u'\]

\[-(G^{\sigma}F^{\delta\sigma'}\epsilon_{\lambda\delta\rho\sigma}\epsilon_{\lambda'\delta'\rho'\sigma'}\bar{u}'\gamma_5\psi^{\rho'}\gamma_5u' + C.T)\]

\[+G^{\sigma}G^{\delta\sigma'}\epsilon_{\lambda\delta\rho\sigma}\epsilon_{\lambda'\delta'\rho'\sigma'}\bar{u}'\gamma_5\psi^{\rho'}\gamma_5u']\]

\[\cdot(p_3^\lambda p_4^\lambda + p_3^\lambda p_4^\lambda - (p_3 \cdot p_4)g^{\lambda\lambda'}),\]  \hspace{1cm} \text{(61)}

where

\[A_{\lambda\alpha} = f_3 v'_\lambda v'_\alpha + f_4 v_\lambda v'_\alpha + f_5(1 + v \cdot v')v'_\alpha v'_\lambda + f_6(1 + v \cdot v')v_\lambda v'_\alpha + f_7 v'_\lambda v'_\alpha,\]  \hspace{1cm} \text{(62)}

\[B = f_8(1 + v \cdot v'),\]  \hspace{1cm} \text{(63)}

\[C = -f_3 v'_\lambda - f_4 v_\lambda,\]  \hspace{1cm} \text{(64)}

\[D_{\alpha} = f_7 v'_\alpha,\]  \hspace{1cm} \text{(65)}

\[F^{\delta\sigma} = f_9 v^{\delta} v^{\sigma},\]  \hspace{1cm} \text{(66)}

\[G^{\sigma} = -f_9 v^{\sigma} - f_{10} v^{\sigma}.\]  \hspace{1cm} \text{(67)}

(6) $B_{(bb)}^* \rightarrow B_{(bc)}^* + l + \bar{\nu}$

\[\Gamma_6 = \sum_{\text{spins}} |T_{6\lambda}^\lambda|^2\]

\[= 8|\xi(\nu \cdot v')|^2Tr[A^2 u' \bar{\psi}_\lambda \bar{\psi}' \bar{u}' + (AB_{\delta\lambda} u' \bar{\psi}_\lambda \bar{\psi}' \bar{u}' + C.T)\]

\[+B_{\delta\lambda} B_{\delta'\lambda} u' \bar{\psi}_{\delta'} \bar{\psi}' \bar{u}' + C_{\rho\sigma} C_{\rho'\sigma'} \epsilon_{\lambda\rho\sigma} \epsilon_{\lambda'\rho'\sigma'} u' \bar{\psi}_\delta \bar{\psi}'_\delta u']\]

\[\cdot(p_3^\lambda p_4^\lambda + p_3^\lambda p_4^\lambda - (p_3 \cdot p_4)g^{\lambda\lambda'}),\]  \hspace{1cm} \text{(68)}

where

\[A = f_{15},\]  \hspace{1cm} \text{(69)}

\[B_{\delta\lambda} = f_{16} v'_\delta v'_\lambda + f_{17} v'_\delta v_\lambda,\]  \hspace{1cm} \text{(70)}

\[C_{\rho\sigma} = f_{18} v^{\rho} v^{\sigma}.\]  \hspace{1cm} \text{(71)}

(7) $B_{(bc)}^* \rightarrow B_{(cc)}^* + l + \bar{\nu}$

\[\Gamma_7 = \sum_{\text{spins}} |T_{7\lambda}^\lambda|^2\]
\[ (8) \quad B_{(bc)1} \rightarrow B_{(cc)1} + l + \nu \]

\[ \Gamma_8 = \sum_{\text{spins}} |T_{8\lambda}|^2 \]

\[ = \frac{8}{3} [\xi(v \cdot v')]^2 Tr \left[ \Lambda_A \Lambda_{\lambda'} \bar{\psi}^\lambda \psi_{\lambda'} \bar{\psi}^{\lambda'}_{\gamma_5} + (A_{\lambda} B_{\nu \lambda'} \Lambda_{\nu'} \bar{\psi}^\lambda \psi^{\lambda'} \bar{\psi}^{\lambda'}_{\gamma_5} + C.T) + (B_{\nu \lambda} A_{\nu'} \Lambda_{\lambda'} \bar{\psi}^\lambda \psi^{\lambda'} \bar{\psi}^{\lambda'}_{\gamma_5} + C.T) + (B_{\nu \lambda} C_{\delta} \Lambda_{\lambda'} \bar{\psi}^\lambda \psi^{\lambda'} \bar{\psi}^{\lambda'}_{\gamma_5} + C.T) + C_{\delta} A_{\nu} \Lambda_{\nu'} \bar{\psi}^\lambda \psi^{\lambda'} \bar{\psi}^{\lambda'}_{\gamma_5} \right] \]

\[ + D^\sigma D^{\sigma'} \epsilon_{\lambda'\delta\rho\sigma} \epsilon_{\lambda'\delta'\rho'\sigma'} \bar{\psi}^\lambda \psi^{\lambda'} \bar{\psi}^{\lambda'}_{\gamma_5} \]

\[ \cdot (p_3^{\lambda} p_4^{\lambda'} + p_3^{\lambda'} p_4^{\lambda} - (p_3 \cdot p_4) g^{\lambda\lambda'}) \], \quad (77) \]

where

\[ A_{\lambda} = f_3 v_\lambda^{\prime} + f_4 v_{\lambda} \], \quad (73) \]

\[ B_{\nu \lambda} = f_5 v^{\prime}_{\nu} v^{\prime}_{\lambda} + f_6 v_{\nu} v_{\lambda} \], \quad (74) \]

\[ C_{\delta} = f_7 v_{\delta} + f_8 v_{\delta} \], \quad (75) \]

\[ D^\sigma = f_9 v^{\prime} + f_{10} v^\sigma \]. \quad (76)
\[ B = f_8(1 + v \cdot v'), \quad (79) \]
\[ C = -f_3v' - f_4v, \quad (80) \]
\[ D_\alpha = f_7v'_\alpha, \quad (81) \]
\[ F^{\delta\sigma} = f_{10}v'^\delta v'^\sigma, \quad (82) \]
\[ G^\sigma = -f_9v'^\sigma - f_{10}v^\sigma. \quad (83) \]

(9) \( B_{(bc)_1} \rightarrow B^*_{(cc)_1} + l + \bar{\nu} \)

\[ \Gamma_9 = \sum_{spins} |T_{9\lambda\lambda'}|^2 \]
\[ = \frac{8}{3} [\xi (v \cdot v')]^2 Tr \left[ -A_{\lambda\delta}A_{\lambda'\delta'} \bar{\Psi}^{\delta\gamma_5} u \bar{u} \gamma_5 \Psi^{\delta'} + (A_{\lambda\delta} B_{\lambda'} \bar{\Psi}^{\delta\gamma_5} u \bar{u} \gamma_5 \gamma_5 \Psi^{\delta'}) + C.T \right] \]
\[ + (A_{\lambda\delta} C_{\delta'} \bar{\Psi}^{\delta\gamma_5} u \bar{u} \gamma_5 \gamma_5 \Psi^{\delta'}) + B_{\lambda'} \bar{\Psi}^{\delta\gamma_5} u \bar{u} \gamma_5 \gamma_5 \Psi^{\delta'} + C.T) + C_{\delta'} \bar{\Psi}^{\delta\gamma_5} u \bar{u} \gamma_5 \gamma_5 \Psi^{\delta'} + (B_{\lambda} C_{\delta'} \bar{\Psi}^{\delta\gamma_5} u \bar{u} \gamma_5 \gamma_5 \Psi^{\delta'}) + C.T) \]
\[ + D^\sigma D^{\sigma'} \epsilon_{\lambda\rho\sigma} \epsilon_{\lambda'\rho'\sigma'} \bar{\Psi}^{\rho\gamma_5} u \bar{u} \gamma_5 \gamma_5 \Psi^{\rho'} \]
\[ + (E^{\rho\sigma} D^{\rho'} \epsilon_{\lambda\rho\sigma} \epsilon_{\lambda'\rho'\sigma'} \bar{\Psi}^{\rho\gamma_5} u \bar{u} \gamma_5 \gamma_5 \Psi^{\rho'}) + C.T) \]
\[ - E^{\rho\sigma} E^{\rho'} \epsilon_{\lambda\rho\sigma} \epsilon_{\lambda'\rho'\sigma'} \bar{\Psi}^{\rho\gamma_5} u \bar{u} \gamma_5 \gamma_5 \Psi^{\rho'} \]
\[ \cdot (p_3^\lambda p_4^\lambda + p_3^\rho p_4^\rho - (p_3 \cdot p_4) g^{\lambda\rho}), \quad (84) \]

where

\[ A_{\lambda\delta} = f_3v'_\lambda v_\delta + f_4v_\lambda v_\delta + (f_5v'_\lambda + f_6v_\lambda) v_\delta (1 + v \cdot v') + f_8v_5v, \quad (85) \]
\[ B_{\lambda} = +f_3v'_\lambda + f_4v_\lambda, \quad (86) \]
\[ C_{\delta} = f_8v_\delta, \quad (87) \]
\[ D^\sigma = f_9v'^\sigma + f_{10}v^\sigma, \quad (88) \]
\[ E^{\rho\sigma} = f_9v^\rho v'^\sigma. \quad (89) \]

(10) \( B_{(bc)_1} \rightarrow B_{(cc)_1} + l + \bar{\nu} \)

\[ \Gamma_{10} = \sum_{spins} |T_{10\lambda\lambda'}|^2 \]

24
where

\[ A_\lambda = (-f_3 v'_\lambda - f_4 v_\lambda)(2 + v \cdot v') + (f_5 v'_\lambda + f_6 v_\lambda)(1 - (v \cdot v')^2) \]
\[ -(f_7 v'_\lambda + f_8 v_\lambda)(1 + v \cdot v'), \quad (91) \]
\[ B = (f_7 + f_8)(1 + v \cdot v'), \quad (92) \]
\[ C^{\rho\sigma} = f_9 v^\rho v^\sigma - f_{10} v^\rho v^\sigma, \quad (93) \]
\[ D^\sigma = f_9 v^\sigma + f_{10} v^\sigma. \quad (94) \]

(11) \[ B_{(bc)\lambda} \rightarrow B_{(cc)\lambda} + l + \bar{\nu} \]

\[ \Gamma_{11} = \sum_{\text{spins}} |T_{11\lambda} l\lambda|^2 \]
\[ = 8 |\xi(v \cdot v')|^2 Tr \left\{ A^2 \bar{\Psi}'_\lambda u \bar{\Psi}'_{\lambda'} + (A B_{g\lambda'} \bar{\Psi}'_\lambda u \bar{\Psi}'_{g\lambda'} + C.T) \right. \]
\[ + B_{8\lambda} B_{g\lambda'} \bar{\Psi}' g\bar{\Psi}' g_{\lambda'} + C^{\rho\sigma} C^{\rho'\sigma'} \epsilon_{\lambda\rho\sigma\rho'\sigma'} \bar{\Psi}' g\bar{\Psi}' g_{\lambda'} \left\} \right. \]
\[ \cdot (p_3^\lambda p_4^\lambda + p_3^\lambda p_4^\lambda - (p_3 \cdot p_4) g^{\lambda\lambda'}), \quad (95) \]

where

\[ A = f_{11}, \quad (96) \]
\[ B_{8\lambda} = f_{12} v_8 v_\lambda' + f_{13} v_8 v_\lambda, \quad (97) \]
\[ C^{\rho\sigma} = f_{14} v^\rho v^\sigma. \quad (98) \]

(12) \[ B_{(bc)\lambda} \rightarrow B_{(cc)\lambda} + l + \bar{\nu} \]

\[ \Gamma_{12} = \sum_{\text{spins}} |T_{12\lambda} l\lambda|^2 \]
\[ = \frac{8}{3} |\xi(v \cdot v')|^2 Tr \left\{ -A_\lambda A_{\lambda'} \bar{u}_5 u_{\gamma_5} u_{\gamma_5} u' + (A_\lambda B_{\bar{u}_5} u_{\gamma_5} u_{\gamma_5} u' + C.T) \right. \]
\[ + B^2 \bar{u}' \gamma_5 u \bar{u} \gamma_\lambda \gamma_5 u' + C^{\rho \sigma} C^{\nu \rho'} \epsilon_{\lambda \delta \rho \sigma} \epsilon_{\nu \delta' \rho' \sigma'} \bar{u}' \gamma_\delta \gamma_5 u \bar{u} \gamma^{\delta'} \gamma_5 u' \] 
\[ \cdot (p_3^\lambda p_4^{\lambda'} + p_3^{\lambda'} p_4^\lambda - (p_3 \cdot p_4) g^{\lambda \lambda'}) , \]

(99)

where

\[ A_\lambda = f_{11} v'_\lambda + f_{12} (1 + v \cdot v') v'_\lambda + f_{13} (1 + v \cdot v') v_\lambda , \]

(100)

\[ B = -f_{11} , \]

(101)

\[ C^{\rho \sigma} = f_{14} v'^\rho v^\sigma . \]

(102)
References

[1] UA1 Collaboration, C. Albarjar et al., Phys. Lett. B273 (1991) 540; CDF Collaboration, F. Abe et al., Phys.Rev. D47 (1993) 2639; R. Akers et al., OPAL collaboration, Z. Phys. C69 (1996) 195; Phys. Lett. B353 (1995) 402; S.E.Tzmarias, invited talk presented in the 27th International Conference on High Energy Physics, Glasgow, July 20-27, 1994; P. Abreu et al., Phys. Lett. B374 (1996) 351; CDF Collaboration, F. Abe et al., Phys. Rev. D55 (1997) 1142.

[2] X.-H. Guo, H.-Y. Jin and X.-Q. Li, Phys. Rev. D53 (1996) 1153.

[3] N. Isgur and M.B. Wise, Phys. Lett. B232 (1989) 113, B237 (1990) 527; H. Georgi, Phys. Lett. B264 (1991) 447; see also M. Neubert, Phys. Rep. 245 (1994) 259 for the review.

[4] M. Savage and M.Wise, Phys.Lett. B248 (1990) 177; A. Falk, M. Luke, M. Savage and M. Wise, Phys.Rev. D49 (1994) 555.

[5] H. Georgi and M. Wise, Phys. Lett. B243 (1990) 279.

[6] C. Carone, Phys. Lett. B253 (1991) 408.

[7] E.E. Salpeter and H.A. Bethe, Phys. Rev. 84 (1951) 1232.

[8] H.-Y. Jin, C.-S. Huang and Y.-B. Dai, Z. Phys. C56 (1992) 707.

[9] E. Eichten, K. Gottfried, T. Kinoshita, K.D. Lane and T.-M. Yan, Phys. Rev. D17 (1978) 3090.

[10] J.L. Rosner, Phys. Rev. D42 (1990) 3732.

[11] M. Neubert and V. Rieckert, Nucl. Phys. B382 (1992) 97; M. Neubert, Phys. Rev. D45 (1992) 2451.
[12] A. El-Hady, K. Gupta, A. Sommerer, J. Spence and J. Vary, Phys. Rev. D51 (1992) 5245.