Constraining quark angular momentum through semi-inclusive measurements

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The determination of quark angular momentum requires the knowledge of the generalized parton distribution \( E \) in the forward limit. We assume a connection between this function and the Sivers transverse-momentum distribution, based on model calculations and theoretical considerations. Using this assumption, we show that it is possible to fit at the same time nucleon magnetic moments and semi-inclusive single-spin asymmetries. This imposes additional constraints on the Sivers function and opens a plausible way to quantifying quark angular momentum.

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At the starting scale $Q_0$ and following the notation of Ref. [19], we use the unpolarized distribution and fragmentation functions

$$f_1^a(x, k_T^2; Q_0^2) = \frac{f_T^a(x; Q_0^2)}{\pi(k_T^2)} e^{-k_T^2/(k_T^2)},$$

(5)

$$D_1^a(z, P_T^2; Q_0^2) = \frac{D_T^a(z; Q_0^2)}{\pi(P_T^2)} e^{-P_T^2/(P_T^2)},$$

(6)

where $z$ is the fraction of the energy of the fragmenting parton $a$ carried by the detected hadron. For $f_T^a(x)$ we use the MSTW08LO set [17], for $D_T^a(z)$ we use the DSS LO set [18]. We fix the width of the transverse-momentum distributions for the initial parton and final hadron, respectively, as

$$\langle k_T^2 \rangle = 0.14 \text{ GeV}^2, \quad \langle P_T^2 \rangle = 0.42 z^{0.54}(1 - z)^{0.37} \text{ GeV}^2.$$  

(7)

These parameters have been implemented in the HERMES ganc._trans Monte Carlo generator and are known to give a good description of the HERMES data [20]. In principle, these functions should be evolved according to TMD evolution [21]. However, we choose here to implement only the evolution of their collinear part.

Neglecting the contribution of heavier $c, b, t$ flavors, we parametrize the Sivers function in the following way (inspired by [15]):

$$f_{1T}^{l,a}(x, k_T^2; Q_0^2) = f_{1T}^{l,(0)a}(x; Q_0^2)$$

$$= \frac{M_1^2 + \langle k_T^2 \rangle}{\pi M_1^2 \langle k_T^2 \rangle} e^{-k_T^2/M_1^2} e^{-k_T^2/\langle k_T^2 \rangle}.$$  

(8)

where $M_1$ is a free parameter related to the width of the transverse-momentum distribution, and

$$f_{1T}^{l,(0)a}(x; Q_0^2) = C^{\alpha q} \sqrt{2e} \frac{M_1}{M_1^2 + \langle k_T^2 \rangle} \frac{1 - x/\alpha q}{[\alpha q - 1]} (1 - x)f_1^{q'}(x; Q_0^2),$$

$$f_{1T}^{\perp (0)a}(x; Q_0^2) = C^{\perp} \sqrt{2e} \frac{M_1}{M_1^2 + \langle k_T^2 \rangle} (1 - x) f_1^{q}(x; Q_0^2).$$  

(9)

Note that at $Q_0$ we establish a relation between the Sivers function for the combinations $q_q, q$, and the corresponding unpolarized PDF, at variance with what has been done in the literature [13][16]. This will turn out to be important when establishing a relation with the anomalous magnetic moment, since it guarantees that the valence Sivers function is integrable at any scale. We multiply the unpolarized PDF by $(1 - x)$ to respect the predicted high-$x$ behavior of the Sivers function [22]. We introduce the free parameter $\alpha q$ to allow for the presence of a node in the Sivers function at $x = \alpha q$, as suggested by diquark model calculations [9][10] and phenomenological studies [23] (see the discussion in Ref. [24]).

For the gluons, we consider all three projections but we multiply their errors in quadrature and neglect the experimental normalization uncertainty. Since the HERMES and COMPASS data sets (binned in three different ways: in $x$, $z$, $P_T$), we consider three projections but we multiply their errors in quadrature and neglect the experimental normalization uncertainty. Since the HERMES and COMPASS data sets (binned in three different ways: in $x$, $z$, $P_T$), we consider three projections but we multiply their errors in quadrature and neglect the experimental normalization uncertainty. Since the HERMES and COMPASS data sets (binned in three different ways: in $x$, $z$, $P_T$), we consider three projections but we multiply their errors in quadrature and neglect the experimental normalization uncertainty.
moments are known to a precision of $10^{-7}$ or higher. However, given the typical uncertainties on PDF extrac-
tions, our computation of $\kappa$ is affected by a theoretical
error of the order of $10^{-3}$. Therefore, for our present
purposes we take $\kappa^p = 1.793 \pm 0.001$, $\kappa^n = -1.913 \pm 0.001$.

We started from considering 15 free parameters. They are $C^q$, $C^{qv}$, $\alpha^{qv}$, with $q = u, d, s$, the gluon coefficient $C^g$, $M_1$, the lensing parameters $K$ and $\eta$, and the scales $Q_0$ and $Q_L$. However, after some explorations, we made a common set of assumptions in all attempted fits. In all cases, we fixed $\alpha^{qv} = 0$ (no nodes in the valence down and strange Sivers functions, as suggested in Refs. [9, 10, 23, 24]). We also set $C^g = 0$ (the influence of the gluon Sivers function through evolution is anyway limited). Finally, all fits indicated that $Q_0 = Q_L = 1 GeV$ was an acceptable choice. Therefore, the actual number of free parameters is at most 10. In this framework, we conclude that it is possible to give a simultaneous description of the SIDIS data and of the nucleon anomalous magnetic moments assuming the relation in Eq. [3].

We explored several scenarios characterized by different choices of the parameters related to the strange quark. We considered fits with fixed $C^g = 0$, or with fixed $C^{sv} = 0$, or with both parameters free (but constrained within positivity limits), or with both fixed $C^{sv} = C^g = 0$. In all cases, we obtained very good values of $\chi^2$ per degree of freedom ($\chi^2$/dof) between 1.323 and 1.347. All fits lead to a negative Sivers function for $u_v$ and large and positive for $d_v$, in agreement with previous studies [21, 16] and with some models [25, 35]. The data are compatible with vanishing sea-quark contributions (with large uncertainties). However, in the $x$ range where data exist, large Sivers functions for $\bar{u}$ and $\bar{d}$ are excluded, as well as large and negative for $s$. The Sivers function for $s_v$ is essentially unconstrained. The parameter $M_1$ is quite stable around 0.34 GeV, as well as the strength of the lensing function $K$ around 1.86 GeV. The parameter $\eta$ is typically around 0.4 but can vary between 0.03 and 2. The node $\alpha^{sv}$ appears only above $x \approx 0.78$.

We now discuss in detail the case with fixed $C^{sv} = C^g = 0$, because it gives the best $\chi^2$/dof (1.323) and suggests that it is possible to fit the present SIDIS data for Sivers asymmetries in kaon emission without the strange contribution to the Sivers function. The best-fit values of the parameters are listed in Tab. [4] together with their statistical errors corresponding to $\Delta \chi^2 = 1$.

| $C^{uv}$ | $C^{dv}$ | $C^{u}$ | $C^{d}$ |
|----------|----------|---------|---------|
| $-0.229 \pm 0.002$ | $1.591 \pm 0.009$ | $0.054 \pm 0.107$ | $-0.083 \pm 0.122$ |

$M_1$ [GeV] $K$ [GeV] $\eta$ $\alpha^{uv}$

| $0.346 \pm 0.015$ | $1.888 \pm 0.009$ | $0.392 \pm 0.040$ | $0.783 \pm 0.001$ |

TABLE I: Best-fit values of the 8 free parameters for the case $C^{sv} = C^g = 0$. The final $\chi^2$/dof is 1.323. The errors are statistical and correspond to $\Delta \chi^2 = 1$.

In Fig. [4] we show the corresponding outcome for $x f_1^{(1)a}(x; Q^2_0)$ with $a = u, d, \bar{u}, \bar{d}$. The Sivers functions for $s$, $\bar{s}$ vanish identically. The uncertainty bands are produced by propagation of the statistical errors of the fit parameters including their correlations, and correspond to $\Delta \chi^2 = 1$. Our results are comparable with other extractions of the Sivers function [13, 15, 16]. They are also qualitatively similar to the forward limit of the GPD $E$ extracted from experiments [31, 32, 33, 34]. We can now compute the contribution to the anomalous magnetic moment of each valence quark flavor $q_v$ using Eqs. [14]. We obtain

$\kappa^{uv} = 1.673 \pm 0.003^{+0.011}_{-0.000}$, $\kappa^{dv} = -2.033 \pm 0.002^{+0.011}_{-0.000}$, $\kappa^{sv} = 0^{+0.011}_{-0.000}$.

The first symmetric error is statistical and comes again from the errors of the fit parameters ($\Delta \chi^2 = 1$). The second asymmetric error is purely theoretical. It is computed by considering the other possible scenarios (corresponding to different choices for $C^{sv}$ and $C^g$) which give good $\chi^2$ fits as well. However, a precise estimate of this error can be obtained only by performing a neural network fit [31]. The strange contribution to the anomalous magnetic moment is negligible, because the positivity bounds severely limit the Sivers function for $s$ and, in
turn, also $E^{a*}$ and $\kappa^{a*}$. Our results are similar to other estimates of the strange Pauli form factor and lattice QCD calculations.

Using Eq. (1), we can compute the total longitudinal angular momentum carried by each flavor $q$ and $\bar{q}$ at our initial scale $Q^2 = 1 \text{ GeV}^2$. Using the standard evolution equations for the angular momentum (at leading order, with 3 flavors only, and $\Lambda_{\text{QCD}} = 257 \text{ MeV}$), we obtain the following results at $Q^2 = 4 \text{ GeV}^2$:

\[
J^u = 0.229 \pm 0.002^{+0.008}_{-0.012}, \quad J^d = 0.015 \pm 0.003^{+0.001}_{-0.000}, \quad J^s = 0.006^{+0.002}_{-0.006}, \quad J^\bar{s} = 0.006^{+0.000}_{-0.005}.
\]

As before, the first symmetric error is statistical and related to the errors on the fit parameters, while the second asymmetric error is theoretical and reflects the uncertainty introduced by the other possible scenarios. In the present approach, we cannot include the (probably large) systematic error due to the rigidity of the functional form in Eqs. (8)-(10). The bias induced by the choice of the functional form may affect in particular the determination of the sea quark angular momenta, since they are not directly constrained by the values of the nucleon anomalous magnetic moments. Our present estimates (at $Q^2 = 4 \text{ GeV}^2$) agree well with other analyses.

In summary, we have presented a determination of the quark angular momentum assuming a connection between the collinear limit of the generalized parton distribution $E$ and the Sivers transverse-momentum distribution. We have shown that it is possible to fit at the same time the nucleon anomalous magnetic moments and data for semi-inclusive single-spin asymmetries produced by the Sivers effect. Several different scenarios produce equally good $\chi^2$ fits. Our strategy opens a plausible way to quantifying the quark angular momentum, and imposes additional constraints on the Sivers function.

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