Dihadron fragmentation: in vacuum and in matter

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Abstract. Two particle correlations within a single jet produced in deeply inelastic scattering (DIS) off a large nucleus as well as in heavy-ion collisions are explored. This is performed within the framework of the medium modified dihadron fragmentation functions. The modification occurs due to gluon bremsstrahlung induced by multiple scattering. The modified fragmentation functions for dihadrons are found to follow closely that of single hadrons leading to a weak nuclear suppression of their ratios as measured by HERMES in DIS experiments. Meanwhile, a moderate medium enhancement of the near-side correlation of two high $p_T$ hadrons is found in central heavy-ion collisions, partially due to trigger bias caused by the competition between parton energy loss and the initial Cronin effect.

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1 Introduction

The modification of the properties of jets as they pass through dense matter has emerged as a new diagnostic tool in the study of the partonic structure of such an environment. Such modification goes beyond a mere suppression of the directed energy leading to a suppression of the multiplicity and could in principle be extended to include the modification of many particle observables. Theoretically, such observables may be computed through a study of $n$-hadron fragmentation functions. Such functions may be defined as the $n$-hadron expectation values of partonic operators. These may be factorized from the hard collision and their evolution with energy scale and modification in matter may be systematically studied in perturbative Quantum Chromodynamics (pQCD). This is quite similar to the usual single inclusive hadron fragmentation functions.

Experimentally, such objects may be estimated through the measurement of semi-inclusive distributions of hadrons emanating from such reactions. Along with the multiplicity distribution at a given momentum or momentum fraction, which corresponds to the single fragmentation function, a variety of triggered distributions may also be measured. In such experiments, the measurement of one or more associated hadrons is only undertaken in the event of the appearance of a trigger hadron with a certain characteristic momentum or momentum fraction. Such measurements have recently been performed in DIS off cold nuclear matter by the HERMES experiment at DESY. Measurements in hot and dense nuclear matter have been performed at the Relativistic Heavy-Ion Collider (RHIC) by the STAR and PHENIX detectors. Due to issues related to background subtraction and requisite statistics, such measurements were restricted to two hadron correlations. While the two-hadron correlation is found to be slightly suppressed in DIS off a nucleus versus a nucleon target, it is moderately enhanced in central $Au + Au$ collisions relative to that in $p + p$. This is in sharp contrast to the observed strong suppression of single inclusive spectra in both DIS and central $A + A$ collisions. The theoretical description of this behaviour forms the topic of this effort.

2 Definitions

Similar to single hadron fragmentation functions, dihadron fragmentation functions of a quark can be defined as the overlapping matrices of the quark fields and two-hadron final states,

$$D_{q,h_1 h_2}^{h_1 h_2}(z_1, z_2) = \frac{z_1^4 z_2^4}{4z_1 z_2} \int \frac{d^2 q_\perp}{4(2\pi)^2} \int \frac{d^4 p}{(2\pi)^4} \delta \left( z - \frac{p_\perp^2}{p^2} \right) \times \text{Tr} \left[ \frac{\gamma^+}{2p_h^+} \int d^4 x e^{i p \cdot x} \sum_{S=2} \langle 0 | \psi_q(x) | p_1, p_2, S - 2 \rangle (p_1, p_2, S - 2 | \bar{\psi}_q(0) | 0) \right], \quad (1)$$

and can be factorized from the hard processes, where $p_h^+ \equiv p_1^+ + p_2^+$ is the total momentum of the two hadrons with flavors $h_1$ and $h_2$, $z \equiv z_1 + z_2$ is the corresponding total momentum fraction and $q_\perp = p_{1\perp} - p_{2\perp}$ is the relative transverse momentum.

They also satisfy the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations as have been derived.
in Ref. [3]. One unique feature of the DGLAP equations for dihadron fragmentation functions is the contribution from independent fragmentation of two partons after the parton splitting, which involves the product of two single fragmentation functions. These DGLAP equations for dihadron fragmentation functions have been solved numerically [10] and the $Q^2$ evolution of the dihadron fragmentation functions agrees very well with results from JETSET Monte Carlo simulations of $e^+ e^- \rightarrow h_1 + h_2 + X$ processes. Although both the single and dihadron fragmentation functions evolve rapidly with $Q^2$, their ratio has a very weak $Q^2$ dependence.

As in the case for the single fragmentation functions, dihadron fragmentation functions may not be predicted entirely within QCD, but need to be measured for all momentum fractions at a given energy scale. Since there are no experimental data available for dihadron fragmentation functions, JETSET Monte Carlo results will be used as the initial condition for the vacuum dihadron fragmentation functions in this study. For single hadron fragmentation functions, the BKK parameterization [13] will be used; this also agrees well with JETSET results.

Besides predicting the factorization and evolution of appropriately defined parton matrix elements, pQCD also predicts a universality of these matrix elements. The fragmentation functions defined in $e^+ e^- \rightarrow$ collisions are identical to that in Deep-Inelastic Scattering (DIS) and in hadron-hadron or nucleus-nucleus collisions. The hard scattering cross section ($\sigma_0$) is however different in each case.

### 3 Modification in a cold medium

Applying factorization to dihadron production in single jet events in DIS off a nucleus, $e(L_1) + A(p) \rightarrow e(L_2) + h_1(p_1) + h_2(p_2) + X$, one can obtain the dihadron semi-inclusive cross section as,

$$E_{L_2} \frac{d\sigma_{D\text{DIS}}}{d^3L_2dz_1dz_2} = \frac{\alpha^2}{2\pi s} \frac{1}{Q^4} L_{\mu\nu} \frac{dW^{\mu\nu}}{dz_1dz_2}$$

in terms of the semi-inclusive tensor at leading twist,

$$\frac{dW^{\mu\nu}}{dz_1dz_2} = \sum_q \int dx f_q^A(x, Q^2) H^{\mu\nu}(x, p, q) \times D_h^h_1, h_2 (z_1, z_2, Q^2).$$

In the above, $D_h^h_1, h_2 (z_1, z_2)$ is the dihadron fragmentation function, $I_{\mu\nu} = (1/2) \text{Tr} (\not{L}_1 \Gamma_\mu \not{L}_2 \Gamma_\nu)$, the factor $H^{\mu\nu}$ represents the hard part of quark scattering with a virtual photon which carries a four-momentum $q = [-Q^2/2q^2, q^-\not{q}, 0_\perp]$ and $f_q^A(x, Q^2)$ is the quark distribution in the nucleus with a total momentum $A[p^\perp, 0_\perp]$. The hadron momentum fractions, $z_1 = p_1^-/q^-$ and $z_2 = p_2^-/q^-$, are defined with respect to the initial momentum $q^-$ of the fragmenting quark.

At next-to-leading twist, the dihadron semi-inclusive tensor receives contributions from multiple scattering of the struck quark off soft gluons inside the nucleus with induced gluon radiation. One can reorganize the total contribution (leading and next-to-leading twist) into a product of effective quark distribution in a nucleus, the hard part of photon-quark scattering $H^{\mu\nu}$ and a modified dihadron fragmentation function $\tilde{D}_h^h_1, h_2 (z_1, z_2)$. The calculation of the modified dihadron fragmentation function at the next-to-leading twist in a nucleus proceeds [12] similarly as that for the modified single hadron fragmentation functions [2] and yields,

$$\tilde{D}_h^h_1, h_2 (z_1, z_2) = D_h^h_1, h_2 (z_1, z_2) + \int_0^{Q^2} \frac{d\alpha_s}{\alpha_s^2} \int_{l_\perp^2} \frac{d^2z}{2\pi}$$

$$\times \left[ \int_{z_1}^{z_2} \frac{dy}{y^2} \left( \Delta P_{q\rightarrow qg}(y, x_B, x_L, l_\perp^2) D_h^h_1, h_2 \left( \frac{z_1}{y}, \frac{z_2}{y} \right) \right) + \Delta P_{g\rightarrow qg}(y, x_B, x_L, l_\perp^2) D_h^h_1, h_2 \left( \frac{z_1}{y}, \frac{z_2}{y} \right) \right]$$

$$\times D_h^h_1 \left( \frac{z_1}{y} \right) \left( \frac{z_2}{y} \right) + (h_1 \rightarrow h_2) \right]$$

in the above, $x_B = -Q^2/2p^+\not{q}$, $x_L = l_\perp^2/2p^+\not{q} - y(1-y)$, $l_\perp$ is the transverse momentum of the radiated gluon, $\Delta P_{q\rightarrow qg}$ and $\Delta P_{g\rightarrow qg}$ are the modified splitting functions whose forms are identical to that in the modified single hadron fragmentation functions [2]. The switch ($h_1 \rightarrow h_2$) is only meant for the last term, which represents independent fragmentation of the quark and gluon after the induced bremsstrahlung. The corresponding modified splitting function,

$$\tilde{\Delta P}_{q\rightarrow qg} = \frac{1 + y^2}{1 - y} \frac{C_A 2\pi\alpha_s T^{A,qg}_{qg}(x_B, x_L)}{1 - y (l_\perp^2 + \langle k_{1\perp}^2 \rangle) N_C T^{A}_{qg}(x_B, Q^2)},$$

is similar to $\Delta P_{q\rightarrow qg}$ but does not contain contributions from virtual corrections. In the above, $C_A = 3$, $N_C = 3$, and $\langle k_{1\perp}^2 \rangle$ is the average intrinsic parton transverse momentum inside the nucleus.

Note that both modified splitting functions depend on the quark-gluon correlation function in the nucleus $T^{A}_{qg}$, which also determines the modification of the single hadron fragmentation functions [14]. For a Gaussian nuclear distribution, it can be estimated as [2]

$$T^{A}_{qg}(x_B, x_L) = \tilde{C}(Q^2) m_N R_A f_q^A(x_B)/(1 - e^{-x_B^2/a_A^2}),$$

where $x_A = 1/m_N R_A$, $m_N$ is the nucleon mass and $R_A = 1.12 A^{1/3}$ is the nuclear radius. The overall constant $\tilde{C} \propto x_L G(x_L)$, where $G(x_L) = (k_{1\perp}^2)/2p^+\not{q} - y(1-y)$ is the only parameter in the modified dihadron fragmentation function which might depend on the kinematics of the DIS process but is identical to the parameter in the modified single fragmentation functions. In the phenomenological study of the single hadron fragmentation functions in DIS off nuclei, $C = 0.006 \text{GeV}^2$ is determined within the kinematics of the HERMES experiment [8].
of nuclear modification of the single hadron fragmentation function on the momentum fraction \( z \), initial quark energy \( \nu = q^- \) and the nuclear size \( R_A \) agrees very well with the HERMES experimental data \cite{12}.

With no additional parameters in Eq. (4), one can predict the nuclear modification of dihadron fragmentation functions within the same kinematics. Since dihadron fragmentation functions are connected to single hadron fragmentation functions via sum rules \cite{10}, it is more illustrative to study the modification of the conditional distribution for the second rank hadrons,

\[
R_2(z_1, z_2) \equiv D_{q_1, q_2}^{h_1, h_2}(z_1, z_2)/D_q^{h_1}(z_1),
\]

where \( z_1 \) and \( z_2 < z_1 \) are the momentum fractions of the triggered (leading) and associated (secondary) hadrons, respectively. Shown in Fig. 1 is the predicted ratio of the associated hadron distribution in DIS off a nitrogen (\( A = 14 \)) target to that off a proton (\( A = 1 \)), as compared to the HERMES experimental data \cite{5}.

The modification of the dihadron fragmentation function in a nucleus with atomic number \( A \) is estimated experimentally by the HERMES experiment \cite{5,8} by measuring the number of events with at least two hadrons with momentum fractions \( z_1, z_2 ) (N_A^2(z_1, z_2)) \) and the number of events with at least one hadron with momentum fraction \( z ) (N_A(z)) \). In an effort to eradicate systematic errors, a double ratio is presented (using similar measurements off deuterium, \( i.e., A = 2 \)),

\[
\mathcal{R}_2(z_2) = \frac{\sum_{z_1 > 0.5} N_A^2(z_1, z_2)}{\sum_{z_1 > 0.5} N_A(z)} \times \frac{\sum_{z_1 > 0.5} N_A^2(z_1, z_2)}{\sum_{z_1 > 0.5} N_A(z)}.
\]

This double ratio for different \( z_2 \) is plotted in Fig. 1 as the circular points.

In comparison with the experiment we present the associated hadron distribution: the ratio of the modified dihadron to the single hadron fragmentation function in a nucleus normalized by the same ratio in the vacuum. As in the experiment, we integrate over \( z_1 > 0.5 \) in both the single and dihadron fragmentation functions. The agreement between the prediction and the data is remarkable given that no free parameters are used. The suppression of the associated hadron distribution at large \( z_2 \) due to multiple scattering and induced gluon bremsstrahlung in a nucleus is quite small compared to the suppression of the single fragmentation functions \cite{8,12}. Since \( R_2 \) is the ratio of double and single hadron fragmentation functions, the effect of induced gluon radiation or quark energy loss is mainly borne by the single spectra of the leading hadrons. This is similar to the evolution of the dihadron fragmentation function with momentum scale in the vacuum \cite{3,10}. At small values of \( z_2 \), the modified dihadron fragmentation function rises above its vacuum counterpart more than the single fragmentation functions. This is due to the new contribution where each of the detected hadrons emanates from the independent fragmentation of the quark and the radiated gluon.

\[
\frac{N_{\text{trig}}(z_1, z_2, N)}{N_{\text{trig}}(z_1, N)}(\text{Rat. D})
\]

\[D_{q_1, q_2}(z_1, z_2)/D_{q_1}(z_1)(\text{Rat. D})
\]

Fig. 1. Results of the medium modification of the associated hadron distribution, in a cold nuclear medium versus its momentum fraction. The momentum fraction of the leading hadron \( z_1 \) is integrated over all allowed values above 0.5. The ratio of this quantity in Nitrogen (N) with that in deuterium (D) are the HERMES data points.

4 Modification in a Hot medium

In high-energy heavy-ion (or \( p + p \) and \( p + A \) collisions), jets are always produced in back-to-back pairs. Correlations of two high-\( p_T \) hadrons in azimuthal angle generally have two Gaussian peaks \cite{9,7}. Relative to the triggered hadron, away-side hadrons come from the fragmentation of the away-side jet and are related to single hadron fragmentation functions. On the other hand, near-side hadrons come from the fragmentation of the same jet as the triggered hadron and therefore are related to dihadron fragmentation functions.

To extend the study of intrajet dihadron correlations to heavy-ion collisions, one can simply assume \( \langle k_T^2 \rangle \approx \mu^2 \) (the Debye screening mass) and that \( C \) in Eq. (6) is proportional to the local gluon density of the produced dense matter. In addition, one also has to include the effect of thermal gluon absorption \cite{15} such that the effective energy dependence of the energy loss will be different. Such a procedure was applied to the study of the modification of the single hadron fragmentation functions and it successfully described the quenching of single inclusive hadron spectra, their azimuthal anisotropy and suppression of away-side high \( p_T \) hadron correlations \cite{16}. The change of the near-side correlation due to the modification of dihadron fragmentation functions in heavy-ion collisions can be similarly calculated.

The near-side correlation with background subtracted and integrated over the azimuthal angle \cite{17} can be related to the associated hadron distribution or the ratio of dihadron to the single hadron fragmentation functions as in Eq. (7). However, one has to average over the initial jet energy weighted with the corresponding production cross sections. For a given value of \( p_T^{\text{trig}} \) of the triggered
hadron, one can calculate the average initial jet energy \( \langle E_T \rangle \). Because of trigger bias and parton energy loss, \( \langle E_T \rangle \) in heavy-ion collisions is generally larger than that in \( p + p \) collisions for a fixed \( p_T^{\text{trig}} \). In these proceedings, we simply use the calculated \( \langle E_T \rangle \) and its centrality dependence for a given \( p_T^{\text{trig}} \) from Ref. [1] which in turn determines the average value of \( z_1 = \frac{p_T^{\text{trig}}}{\langle E_T \rangle} \) and \( z_2 = \frac{p_T^{\text{assoc}}}{\langle E_T \rangle} \). In the same calculation, one can also obtain the average parton energy loss as the difference of the average initial parton energy with and without parton energy loss, \( \langle \Delta E \rangle = \langle E_T \rangle^{\text{loss}} - \langle E_T \rangle^{\text{no-loss}} \), which is completely determined from the suppression of single inclusive hadron spectra and away-side correlation. Since the high \( p_T \) biases the jet production position towards the surface of the overlapped region, \( \langle \Delta E \rangle \) associated with a triggered hadron is always smaller than the average energy loss of both the away-side and jets in non-triggered events.

Using \( \langle \Delta z \rangle = \langle \Delta E \rangle / \langle E_T \rangle \), we first determine the parameter \( C \) and in turn calculate both the modified single and dihadron distributions. The ratio of such associated hadron distributions in \( Au + Au \) versus \( p + p \) collisions, referred to as \( I_{AA} \), is plotted as the solid line in Fig. 4 together with the STAR data, as a function of the number of participant nucleons. In central \( Au + Au \) collisions, triggering on a high \( p_T \) hadron biases toward a larger initial jet energy and therefore smaller \( z_1 \) and \( z_2 \). This leads to an enhancement in \( I_{AA} \) due to the shape of the dihadron fragmentation functions. This is in contrast to the slight nuclear suppression of the associated hadron distribution at large \( z_2 \) in DIS off a nucleus (Fig. 1). The enhancement increases with \( N_{\text{part}} \) because of increased initial gluon density and system size of the produced dense matter which leads to an increased total energy loss. In the most peripheral collisions, the effect of smaller energy loss is countered by the Cronin effect due to initial state multiple scattering that biases toward smaller \( \langle E_T \rangle \) relative to \( p + p \) collisions. As a result, the associated hadron distribution is slightly suppressed.

5 Conclusions

In summary, we have studied the medium modification of dihadron fragmentation functions in both hot and nuclear matter due to multiple parton scattering and induced gluon radiation. The modification is found to follow closely that of single hadron fragmentation functions so that the associated hadron distributions or the ratios of dihadron and single fragmentation functions are only slightly suppressed in DIS off nuclei but enhanced in central heavy-ion collisions due to trigger bias. With no extra parameters, our calculations agree very well with experimental data.

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