Abstract

The observed branching ratios for $B \to K\eta'$ decays are much larger than factorization predictions in the Standard Model (SM). Many proposals have been made to reconcile the data and theoretical predictions. In this paper we study these decays within the SM using flavor U(3) symmetry. If small annihilation amplitudes are neglected, one needs 11 hadronic parameters to describe $B \to PP$ decays where $P$ can be one of the $\pi$, $K$, $\eta$ and $\eta'$ nonet mesons. We find that existing data are consistent with SM with flavor U(3) symmetry. We also predict several measurable branching ratios and CP asymmetries for $B \to K(\pi)\eta(\eta')$, $\eta(\eta')\eta(\eta')$ decays. Near future experiments can provide important tests for the Standard Model with flavor U(3) symmetry.
Experimental data from CLEO, BaBar and Belle [1–4] have measured branching ratios of \( B \rightarrow K\eta' \) around \( 6 \times 10^{-5} \) which are substantially larger than theoretical calculations based on naive factorization approximation in the Standard Model (SM) [5]. Although there are some improvements in calculating the branching ratios in the last few years by using QCD improved factorization method [6], there are still large uncertainties in calculating the branching ratios for \( B \rightarrow K\eta' \) because of issues related to \( \eta_1 - \eta_8 \) mixing and QCD anomaly associated with \( \eta_1 \). There are also many speculations about possible new physics beyond the SM in these decays [7]. Before any claim can be made about new physics, one must study the SM contributions in all possible ways to see if it is really inconsistent with experimental data.

In this paper we carry out a systematic study of \( B \rightarrow K\eta' \), and more generally processes of \( B \rightarrow PP \) decays with at least one of the \( P \) to be \( \eta(\eta') \) in the final states by using flavor symmetry in the SM. This way one can relate different decays to predict unmeasured branching ratios and CP asymmetries. Drastic deviations between the predicted relations and experimental data can provide information about the SM and models beyond. Similar considerations based on \( SU(3) \) have been applied to \( B \rightarrow PP \) decays [8], and shown to be consistent with data [9]. If one sticks to flavor \( SU(3) \) symmetry, one needs to introduce the singlet \( \eta_1 \) in the theory and to add additional amplitudes to describe new decay modes [10]. One may also consider to promote the flavor \( SU(3) \) symmetry to flavor \( U(3) \) symmetry such that \( \eta_1 \) is automatically included in the theory.

Flavor \( U(3) \) symmetry has been studied in Kaon decays. There there are non-negligible symmetry breaking effects. For \( B \) decays one may also expect symmetry breaking effects to exist. There are also some studies of \( U(3) \) symmetry for \( B \) decays [11]. Present data, however, are not able to make clear statement about whether this symmetry is badly broken. In this paper we will take the flavor \( U(3) \) symmetry as working hypothesis and to study whether experimental data can be explained by carrying out a systematic analysis. We find that the SM with flavor \( U(3) \) symmetry can explain all existing data, in particular can obtain large branching ratios for \( B \rightarrow K\eta' \) decays. We also predict some unmeasured branching
ratios and CP asymmetries which can be used to further test the theory.

The quark level effective Hamiltonian can be written as [12]

$$H_{\text{eff}}^q = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{uq}^\ast (c_1 O_1 + c_2 O_2) - \sum_{i=3}^{11} (V_{ub} V_{uq}^* c_i^{uc} + V_{tb} V_{tq}^* c_i^{tc}) O_i \right].$$

(1)

Here $V_{ij}$ are KM matrix elements. The coefficients $c_{1,2}$ and $c_i^{jk}$ are the Wilson Coefficients which have been evaluated by several groups [12] with $|c_{1,2}| >> |c_i^{jk}|$. $O_i$ are operators consisting of quarks and gluons.

The $B \to PP$ decay amplitudes can be parameterized as

$$A(B \to PP) = < PP | H_{\text{eff}}^q | B > = \frac{G_F}{\sqrt{2}} | V_{ub} V_{uq}^* T + V_{tb} V_{tq}^* P |,$$

(2)

where $B = (B_u, B_d, B_s) = (B^-, \bar{B}^0, \bar{B}^0)$ which form a fundamental representation of $SU(3)$ (or $(U(3))$. The amplitudes $T$ and $P$ are related to the hadronic matrix elements $< PP | O_i | B >$ which are very difficult to calculate. For our purpose, however, we only need to know the fact that under $SU(3)$ (or $U(3))$ $O_{1,2}, O_{3-6,11},$ and $O_{7-10}$ transform as $\bar{3} + \bar{3'} + 6 + \bar{15}, 3$, and $\bar{3} + \bar{3'} + 6 + \bar{15}$, respectively [8], and to parameterize the amplitudes according to $SU(3)$ (or $U(3)$) invariant amplitudes to be discussed below.

As mentioned earlier that there are two approaches to the problem from flavor symmetry point of view. We first work with the $U(3)$ symmetry approach. In this case, the $\pi, K, \eta_8$ and $\eta_1$ form a nonet representation of $U(3), M_j^i,$ with

$$(M_j^i) = \left( \begin{array}{ccc} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -2 \frac{\eta_8}{\sqrt{6}} \end{array} \right) + \left( \begin{array}{ccc} 0 & \frac{1}{\sqrt{3}} \eta_1 & 0 \\ \frac{1}{\sqrt{3}} \eta_1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{3}} \eta_1 \end{array} \right).$$

One can write the $T$ amplitude for $B \to PP$ in terms of the flavor $U(3)$ invariant amplitudes as

$$T = A_{13}^T B_i H(\bar{3})^i (M_j^i M_k^j) + C_{13}^T B_i M_j^i M_k^j H(\bar{3})^i$$

$$+ \bar{A}_{6}^T B_i H(6)^{ij} M_j^i M_k^j + \bar{C}_{6}^T B_i M_j^i H(6)^{ij} M_k^i$$

$$+ A_{15}^T B_i H(\bar{15})^i M_j^i M_k^j + C_{15}^T B_i M_j^i H(\bar{15})^i M_k^j$$

$$+ A_{15}^T B_i H(\bar{15})^i M_j^i M_k^j + C_{15}^T B_i M_j^i H(\bar{15})^i M_k^j.$$
\( + B^T_3 B_i H(3)^i M_j^i M_k^k + B^T_6 B_i H(6)^i_k M_j^j M_l^l \\
+ B^T_{15} B_i H(15)^i_j M_j^j M_l^l + D^T_3 B_i M_j^j H(3)^j M_l^l ). \)  

(3)

In Table I we list all decay amplitudes involving \( \eta_{1,8} \). The amplitudes containing only \( K \) and \( \pi \) in the final states can be found in Ref. [9]. There are a few new features for the U(3) amplitudes in eq. (3) compared with the SU(3) amplitudes for \( B \to PP \). The last four terms are new. In SU(3) case because the traceless condition of \( M_j^j \), these terms are automatically zero. With SU(3) symmetry, the amplitudes \( \tilde{A}_6 \) and \( \tilde{C}_6 \) always appear in the combination of \( \tilde{C}_6 - \tilde{A}_6 \) [8]. This degeneracy is, naively lifted in processes with \( \eta_1 \) in the final states. It seems that there is the need of having both \( \tilde{C}_6 \) and \( \tilde{A}_6 \) to describe the decays increasing the total number of hadronic parameters by one. However, this is not true since that the \( \tilde{A}_6^T \) and \( \tilde{C}_6^T \) terms in decay modes with \( \eta_1 \) in the final state can be written as \( C_6^T = \tilde{C}_6^T - \tilde{A}_6^T \), and the additional \( \tilde{A}_6^T \) be absorbed by redefining the amplitude \( B_6^T = \tilde{B}_6^T + \tilde{A}_6^T \). In Table I we therefore have listed the decay amplitudes in terms of the independent U(3) invariant amplitudes, \( C_{3,6,15}^T \), \( A_{3,15}^T \), \( B_{3,6,15}^T \) and \( D_3^T \).

We now describe the other approach to include \( \eta_1 \) in \( B \to PP \) decays. Here one treats \( \eta_1 \) as a singlet of \( SU(3) \) and parameterizes the decay amplitudes according to SU(3) symmetry. In this case there are also additional four new terms,

\[
T_{\text{new}} = a^T B_i H(3)^i \eta_1 \eta_1 + b^T B_i M_j^j(8) H(3)^j \eta_1 \\
+ c^T B_i H(6)^i_k M_k^l(8) \eta_1 + d^T B_i H(15)^i_l M_l^l(8) \eta_1 .
\]

(4)

In the U(3) symmetry limit, we have

\[
a^T = A_3^T + 3B_3^T + \frac{1}{3} C_3^T + D_3^T , \quad b^T = \frac{2}{\sqrt{3}} C_3^T + \sqrt{3} D_3^T , \\
c^T = \frac{2}{\sqrt{3}} \tilde{A}_6^T + \sqrt{3} \tilde{B}_6^T + \frac{1}{\sqrt{3}} \tilde{C}_6^T , \quad d^T = \frac{2}{\sqrt{3}} A_{15}^T + \sqrt{3} B_{15}^T + \frac{1}{\sqrt{3}} C_{15}^T .
\]

(5)

We also see from the above that one can use \( C_6^T \) and \( B_6^T \) to absorb \( \tilde{A}_6^T \) by writing, \( c^T = \sqrt{3} B_6^T + C_6^T \). It is interesting to note that both approaches discussed above introduce the same number of new parameters, four of them, into the theory. In our analysis we will work with the flavor U(3) symmetry.
To obtain the amplitudes for $B$ decays with at least one $\eta(\eta')$ in the final states, one also needs to consider $\eta - \eta'$ mixing,

$$\begin{pmatrix} \eta' \\ \eta \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}. \quad (6)$$

The average value of the mixing angle $\theta$ is $-15.5^\circ \pm 1.3^\circ$ [13]. We will use $\theta = -15.5^\circ$ in our fit.

There are similar $U(3)$ invariant amplitudes for the penguin contributions. We indicate them as $C_{3,6,15}^P$, $A_{3,15}^P$, $B_{3,6,15}^P$ and $D_{3,6,15}^P$. The amplitudes $A_i$ and $B_i$ are referred as annihilation amplitudes because the $B$ mesons are first annihilated by the interaction Hamiltonian and two light mesons are then created. In total there are 22 complex hadronic parameters (44 real parameters with one of them to be an overall unphysical phase). However simplification can be made because the following relations in the SM,

$$C_6^P(B_6^P) = -\frac{3}{2} \frac{c_9^{lc} - c_10^{lc}}{c_1 - c_2 - 3(c_9^{ac} - c_10^{ac})/2} C_6^T(B_6^T) \approx -0.013 C_6^T(B_6^T),$$

$$C_{15}^P(A_{15}^P, B_{15}^P) = -\frac{3}{2} \frac{c_9^{lc} + c_10^{lc}}{c_1 - c_2 - 3(c_9^{ac} + c_10^{ac})/2} C_{15}^T(A_{15}^T, B_{15}^T) \approx +0.015 C_{15}^T(A_{15}^T, B_{15}^T). \quad (7)$$

Here we have used the Wilson Coefficients obtained in Ref. [12].

We comment that in finite order perturbative calculations the above relations are renormalization scheme and scale dependent. One should use a renormalization scheme consistently. We have checked with different renormalization schemes and find that numerically the changes are less than 15% for different schemes. In obtaining the above relations, we have also neglected small contributions from $c_{7,8}$ which cause less than 1% deviations.

Using the above relations the number of independent hadronic parameters are reduced which we choose to be, $C_3^{T,P}(A_3^{T,P}), C_6^T, C_{15}^{T}(A_{15}^{T}), B_3^{T,P}, B_6^T, B_{15}^T, D_3^{T,P}$. An overall phase can be removed without loss of generality, we will set $C_3^P$ to be real. There are in fact only 25 real independent parameters for $B \rightarrow PP$ in the SM with flavor $U(3)$ symmetry,

$$C_3^P, C_3^T e^{i\delta_3}, C_6^T e^{i\delta_6}, C_{15}^T e^{i\delta_{15}}, A_3^T e^{i\delta_A^T}, A_3^C e^{i\delta_A^C}, A_{15}^T e^{i\delta_A^{15}},$$

$$B_3^P e^{i\delta_B^P}, B_3^T e^{i\delta_B^T}, B_6^P e^{i\delta_B^6}, B_6^T e^{i\delta_B^6}, B_{15}^P e^{i\delta_B^{15}}, B_{15}^T e^{i\delta_B^{15}}, D_3^P e^{i\delta_D^P}, D_3^T e^{i\delta_D^T}.$$
Further the amplitudes $A_i$ and $B_i$ correspond to annihilation contributions and are expected to be small which is also supported by data [9]. If the annihilation amplitudes are neglected, there are only 11 independent hadronic parameters

$$C_3^P, \ C_6^P e^{i\delta_3}, \ C_6^T e^{i\delta_6}, \ C_{15}^T e^{i\delta_{15}}, \ D_3^T e^{i\phi_3^T}, \ D_3^P e^{i\phi_3^P}. \tag{8}$$

The phases in the above can be defined in such a way that all $C_i^{T,P}$ and $D_i^{T,P}$ are real positive numbers.

At present many $B \to PP$ decay modes have been measured at B-factories [2–4]. It is tempting to use experimental data to fix all the hadronic parameters described earlier. It has been shown that if processes involving $\eta(\eta')$ are not included, it is indeed possible to determine all the SU(3) invariant amplitudes, $A_i$ and $C_i$ [8]. When processes involving $\eta(\eta')$ are also included, a meaningful determination of all hadronic parameters (25 of them)
is, however, not possible at present because of too many parameters. We therefore in the following neglect the annihilation amplitudes, which are anticipated to be small, to see if all data can be reasonably explained, in particular to see if large $B \to K\eta'$ branching ratios can be obtained, with only 11 parameters given in eq. (8). This is a nontrivial task. Remarkably we find that all data can, indeed, be well explained.

We use the averaged CLEO, BaBar and Belle data [2–4] shown in Table III and IV to fix the unknown 11 hadronic parameters by carrying out a global $\chi^2$ analysis. The results are shown in Table II. In our analysis due to the lack of knowledge of the error correlations from experiments, in obtaining the averaged error bars, we have, for simplicity, taken them to be uncorrelated and assumed to obey Gaussian distribution taking the larger one between $\sigma_+$ and $\sigma_-$ to be on the conservative side. Experimental data on $\epsilon_K$, $B - \bar{B}$ mixing, $|V_{cb}|$, $|V_{ub}/V_{cb}|$, and $\sin 2\beta$ provide very stringent constraints on the KM matrix elements involved in our analysis [9,14,15]. We have treated them in our analysis as known parameters with the values $\lambda = 0.2196$, $A = 0.854$, $\rho = 0.25$ and $\eta = 0.34$ ($\gamma = 53.4^\circ$) determined from the most recent data.

Using the above determined hadronic parameters, we study several other unmeasured branching ratios and CP violating rate asymmetries $A_{CP}$ for $B \to PP$ defined by,

$$A_{CP} = \frac{\Gamma(B_i \to PP) - \Gamma(\bar{B}_i \to \bar{PP})}{\Gamma(B_i \to PP) + \Gamma(\bar{B}_i \to \bar{PP})}.$$

The results are shown in Tables III and IV.

We now discuss some implications of the results obtained and draw conclusions. We see from Tables III and IV that the best fit values for the known branching ratios are in good agreements with data, in particular large $B \to K\eta'$ can be obtained. The minimal $\chi^2$ in our fit is 16 with total of 23 data points from $B \to PP$ decays and the 11 hadronic parameters in eq.(8) as fitting parameters. The value of 1.33 for the $\chi^2$ per degree of freedom represents a reasonable fit. In our fit the $\eta - \eta'$ mixing parameter $\theta$ is fixed at the average value determined from other data [13]. We checked the sensitivity of the final results on $\theta$ within the allowed region and find the changes are small. We also find that if one reduces the U(3)
TABLE II. The best fit values and their 68% C.L. ranges for the hadronic parameters.

| Parameter | Central Value | Error Range |
|-----------|---------------|-------------|
| $C_3^P$   | $0.136$       | $0.003$     |
| $C_3^T$   | $0.174$       | $0.090$     |
| $C_6^T$   | $0.244$       | $0.077$     |
| $C_{15}^T$| $0.147$       | $0.011$     |
| $\delta_3$| $85.6^\circ$ | $29.8^\circ$|
| $\delta_6$| $79.0^\circ$ | $17.4^\circ$|
| $\delta_{15}$| $8.9^\circ$ | $15.3^\circ$|
| $D_3^P$   | $0.122$       | $0.011$     |
| $D_3^T$   | $0.940$       | $0.340$     |
| $\delta_{D_3^P}$ | $-85.0^\circ$ | $6.0^\circ$ |
| $\delta_{D_3^T}$ | $-83.7^\circ$ | $16.5^\circ$ |

Symmetry to the SU(3) case, just fitting data on $B \to \pi\pi, \pi K, KK$, the values obtained for $C_i$ are not very much different than what obtained here. This indicates that the parameters $C_i$ are stable when promoting $SU(3)$ to $U(3)$. The large branching ratios for $B \to K\eta'$ are due to the new parameters $D_3$. The $U(3)$ assumption can provide a good approximation for $B \to PP$ decays. The Standard Model with flavor $U(3)$ symmetry is not in conflict with existing data.

In our fit, we did not include the branching ratios which only have information on their upper bounds, such as $Br(B_d \to K^-K^+)$, $Br(B_d \to \bar{K}^0 K^0)$ and $Br(B_u \to \pi^- \eta')$. Since we neglected annihilation contributions, the mode $B_d \to K^-K^+$ has vanishing branching ratio which is consistent with data. The stringent upper limit [2–4] of $10^{-6}$ on $B_d \to K^-K^+$ branching ratio supports the expectation that annihilation contributions are small. The predicted branching ratio of $0.7 \times 10^{-6}$ for $B_d \to \bar{K}^0 K^0$ is safely below the experimental bound. The predicted branching ratio for $Br(B_u \to \pi^- \eta')$ is $16.8^{+16.0}_{-9.7} \times 10^{-6}$. The central
TABLE III. The central values and 68% C.L. allowed ranges for branching ratios (in units of $10^{-6}$) and CP asymmetries for processes with no $\eta$ or $\eta'$ in the final states.

| Process          | Branching Ratios |          |                      | CP Asymmetries |
|------------------|------------------|----------|----------------------|----------------|
|                  | Experiment       | Fit      | Experiment           | Fit            |
| $B_u \rightarrow \pi^- \pi^0$ | $5.6 \pm 0.9$   | $5.5_{-0.9}^{+0.9}$ | $0.06 \pm 0.16$     | $0.00$         |
| $B_u \rightarrow K^- K^0$    | $-0.6 \pm 0.8$  | $0.8_{-0.2}^{+0.4}$  |                      | $-0.49_{-0.46}^{+0.84}$ |
| $B_d \rightarrow \pi^+ \pi^-$ | $4.8 \pm 0.5$   | $4.7_{-0.5}^{+0.5}$  | $0.51 \pm 0.19$     | $0.45_{-0.12}^{+0.11}$ |
| $B_d \rightarrow \pi^0 \pi^0$ | $2.0 \pm 0.8$   | $1.9_{-0.7}^{+0.8}$  |                      | $0.27_{-0.33}^{+0.18}$ |
| $B_d \rightarrow \bar{K}^0 K^0$ | $0.7_{-0.2}^{+0.4}$ |                           | $-0.49_{-0.46}^{+0.84}$ |
| $B_u \rightarrow \pi^- \bar{K}^0$ | $18.2 \pm 1.7$  | $20.1_{-1.1}^{+1.1}$  | $0.04 \pm 0.08$     | $0.02_{-0.04}^{+0.03}$ |
| $B_u \rightarrow \pi^0 K^-$    | $12.9 \pm 1.2$  | $10.8_{-1.2}^{+0.6}$  | $-0.10 \pm 0.08$    | $-0.01_{-0.07}^{+0.06}$ |
| $B_d \rightarrow \pi^+ K^-$    | $18.5 \pm 1.0$  | $18.9_{-0.4}^{+0.9}$  | $-0.09 \pm 0.04$    | $-0.11_{-0.03}^{+0.03}$ |
| $B_d \rightarrow \pi^0 \bar{K}^0$ | $10.3 \pm 1.5$  | $8.9_{-0.5}^{+0.4}$   | $0.03 \pm 0.37$     | $-0.06_{-0.06}^{+0.06}$ |
| $B_s \rightarrow K^+ \pi^-$   | $4.4_{-0.5}^{+0.5}$ |                           | $0.45_{-0.12}^{+0.11}$ |
| $B_s \rightarrow K^0 \pi^0$   | $1.8_{-0.7}^{+0.7}$ |                           | $0.27_{-0.33}^{+0.18}$ |
| $B_s \rightarrow K^+ K^-$    | $17.8_{-0.4}^{+0.8}$ |                           | $-0.11_{-0.03}^{+0.03}$ |
| $B_s \rightarrow K^0 \bar{K}^0$ | $17.7_{-1.0}^{+0.5}$ |                           | $0.02_{-0.04}^{+0.03}$ |

value is slightly larger than the 90% C.L. allowed upper bound $12 \times 10^{-6}$. But the 68% C.L. range is consistent with data. At present it is too early to claim conflict of theory with data. But this mode can be used to test the theory. Should a branching ratio much smaller than the central value predicted here be measured in the future, it is an indication that the assumptions made need to be modified.

Using existing experimental data, we have determined 11 hadronic parameters needed to describe $B \rightarrow PP$ decays with flavor U(3) symmetry. We have compared with QCD improved factorization calculations and found that the magnitudes of the parameters are of the same order of magnitude. In our fit, we determined two U(3) invariant amplitudes,
TABLE IV. The central values and their 68% C.L. allowed ranges for branching ratios (in units of $10^{-6}$) and CP asymmetries with at least one of the final mesons to be a $\eta$ or $\eta'$.  

|                  | Branching Ratios | CP Asymmetries |
|------------------|------------------|-----------------|
|                  | Experiment       | Fit             | Experiment       | Fit             |
| $B_u \to \pi^- \eta$ | $4.1 \pm 0.9$   | $4.1^{+0.9}_{-0.9}$ | $-0.51 \pm 0.20$ | $-0.23^{+0.14}_{-0.14}$ |
| $B_u \to K^- \eta$ | $3.1 \pm 0.7$   | $3.3^{+0.6}_{-0.4}$ | $-0.32 \pm 0.22$ | $-0.37^{+0.08}_{-0.09}$ |
| $B_u \to K^- \eta'$ | $77.6 \pm 4.8$  | $72.8^{+3.9}_{-3.8}$ | $0.04 \pm 0.04$ | $0.07^{+0.04}_{-0.04}$ |
| $B_d \to K^0 \eta$ | $2.6 \pm 0.9$   | $2.4^{+0.5}_{-0.6}$ | $-0.21^{+0.07}_{-0.09}$ |
| $B_d \to K^0 \eta'$ | $58.3 \pm 6.0$  | $66.5^{+3.7}_{-3.6}$ | $0.08 \pm 0.16$ | $0.12^{+0.04}_{-0.04}$ |
| $B_u \to \pi^- \eta'$ | $16.8^{+16.0}_{-9.7}$ |                  | $-0.18^{+0.15}_{-0.09}$ |
| $B_d \to \pi^0 \eta$ | $1.2^{+0.6}_{-0.4}$ |                  | $-0.94^{+0.15}_{-0.03}$ |
| $B_d \to \pi^0 \eta'$ | $7.8^{+3.8}_{-4.3}$ |                  | $-0.38^{+0.19}_{-0.35}$ |
| $B_d \to \eta \eta$ | $3.1^{+1.3}_{-1.1}$ |                  | $-0.33^{+0.10}_{-0.13}$ |
| $B_d \to \eta \eta'$ | $7.6^{+5.3}_{-3.4}$ |                  | $-0.20^{+0.12}_{-0.20}$ |
| $B_d \to \eta' \eta'$ | $5.4^{+4.5}_{-3.1}$ |                  | $-0.11^{+0.14}_{-0.28}$ |
| $B_s \to K \eta$ | $2.8^{+1.5}_{-1.2}$ |                  | $0.17^{+0.07}_{-0.07}$ |
| $B_s \to K \eta'$ | $19.1^{+7.0}_{-9.0}$ |                  | $-0.39^{+0.17}_{-0.26}$ |
| $B_s \to \pi^0 \eta$ | $0.05^{+0.10}_{-0.10}$ |                  | $0.98^{+0.03}_{-0.14}$ |
| $B_s \to \pi^0 \eta'$ | $0.07^{+0.10}_{-0.10}$ |                  | $0.91^{+0.09}_{-0.10}$ |
| $B_s \to \eta \eta$ | $7.0^{+1.5}_{-1.5}$ |                  | $-0.23^{+0.09}_{-0.09}$ |
| $B_s \to \eta \eta'$ | $24.1^{+1.4}_{-1.5}$ |                  | $0.08^{+0.04}_{-0.04}$ |
| $B_s \to \eta' \eta'$ | $68.3^{+4.4}_{-4.5}$ |                  | $0.09^{+0.05}_{-0.05}$ |
$D_{3}^{T,P}$. These are particularly difficult to estimate from theoretical calculations because these amplitudes may be related to QCD anomalies. Also in factorization approach, it is not possible to reliably calculate the phases in the hadronic parameters. In our fit, we find that these phases can be sizeable. Further improved theoretical method is needed to have a better understanding of $B \rightarrow PP$ decays.

Using the hadronic parameters determined from existing data, we have predicted several unmeasured branching ratios. These predictions can be used to test the theory. There are six modes involving at least one $\eta$ (or a $\eta'$) in the final states for $B_{d}$ decays. Among them $B_{d} \rightarrow \pi^{-}\eta'$ has the largest branching ratio, it is a clear test for the theory. The other modes are also in the reach of near future data from B-factories. There are also seven $B_{s}$ decay modes with at least one $\eta$ or $\eta'$ in the final states. Several of the branching ratios are predicted to be large, in particular the predicted branching ratio for $B_{s} \rightarrow \eta'\eta'$ is about $7 \times 10^{-5}$ which can be measured at future hadron colliders and can provide another crucial test for theory.

We have also obtained interesting predictions for CP asymmetries in $B \rightarrow PP$ decay modes. Many of the predicted central values for the CP asymmetries are larger than 10% which can be measured in the near future. These modes can provide important information about CP violation in the Standard Model.

In conclusion, we have carried out an systematic analysis for $B \rightarrow PP$ decays in the SM with flavor U(3) symmetry. This approach allows one to study $B$ decays involving at least one $\eta$ or $\eta'$ in the final states. We find that all existing data can be explained, in particular large branching ratios for $B \rightarrow K\eta'$ are possible. There is no conflict between the Standard Model and present experimental data. We have also predicted several unmeasured branching ratios and CP asymmetries within the reach of near future B-factories. Future experimental data will provide crucial information on flavor symmetries and also the Standard Model.
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