Growth index of matter perturbations in the light of Dark Energy Survey

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We study how the cosmological constraints from growth data are improved by including the measurements of bias from Dark Energy Survey (DES). In particular, we utilize the biasing properties of the DES Luminous Red Galaxies (LRGs) and the growth data provided by the various galaxy surveys in order to constrain the growth index (γ) of the linear matter perturbations. Considering a constant growth index we can put tight constraints, up to ∼ 10% accuracy, on γ. Specifically, using the priors of the Dark Energy Survey and implementing a joint likelihood procedure between theoretical expectations and data we find that the best fit value is in between γ = 0.64 ± 0.075 and 0.65 ± 0.063. On the other hand utilizing the Planck priors we obtain γ = 0.680 ± 0.089 and 0.690 ± 0.071. This shows a small but non-zero deviation from General Relativity (γ_{GR} ≈ 6/11), nevertheless the confidence level is in the range ∼ 1.3 − 2σ. Moreover, we find that the estimated mass of the dark-matter halo in which LRGs survive lies in the interval ∼ 6.2 × 10^{12} h^{-1} M_{⊙} and 1.2 × 10^{13} h^{-1} M_{⊙}, for the different bias models. Finally, allowing γ to evolve with redshift [Taylor expansion: γ(z) = γ_0 + γ_1 z / (1+z)] we find that the (γ_0, γ_1) parameter solution space accommodates the GR prediction at ∼ 1.7 − 2.9σ levels.

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1. INTRODUCTION

The past and present analysis of various cosmological data (SNIa, Cosmic Microwave Background-CMB, Baryonic Acoustic Oscillations-BAOs, Hubble parameter measurements etc) converge to the following cosmological paradigm, the observed Universe is spatially flat and the cosmic fluid consists of ∼ 4% luminous (baryonic) matter, ∼ 26% dark matter and ∼ 70% some sort of dark energy (hereafter DE) which plays a key role in explaining the accelerated expansion of the universe (cf. [1, 7] and references therein). Despite the fact that there is an agreement among the majority of cosmologists concerning the ingredients of the cosmic fluid however, there are different explanations regarding the physical mechanism which causes the accelerated expansion of the universe. In brief, the general avenue that one can design in order to study cosmic acceleration is to treat DE either as a new field in nature or as a modification of General Relativity (see for review [8, 10])

It has been proposed [11–14] that in order to discriminate between scalar field DE and modified gravity one may utilize the evolution of the linear growth of matter fluctuations δ_m(z) = δ ρ_m / ρ_m. In particular, we introduce the growth rate of clustering, which is given by f(a) = D_m(a) / a = Ω_m(a), where D_m(a) = δ_m(a) / a = 1 is the linear growth factor (normalized to unity at the present epoch), a(z) = (1 + z)^{-1} is the scale factor of the universe, Ω_m(a) is the dimensionless matter density parameter and γ is the so called growth index [15, 16]. In fact the determination of the growth index is considered one of the main targets in these kind of studies because it can be used in order to test General Relativity (GR) on extragalactic scales, even in a model independent fashion [17]. Indeed, in the literature one may find a large family of studies in which the functional form of the growth index is given analytically for several cosmological models namely, scalar field DE [12, 14, 18, 20], DGP [12, 21, 23], f(R) [22, 24, 25], f(T) [26] Finsler-Randers [27], running vacuum models [28], clustered and Holographic dark energy [29].

From the view point of large scale structure, the study of the distribution of matter on extragalactic using different mass tracers (galaxies, AGNs, clusters of galaxies etc) provides important constraints on theories of structure formation. Specifically, owing to the fact that gravity reflects, via gravitational instability, on the physics of clustering [15] it is natural to utilize the clustering/biasing properties of the extragalactic mass tracers in constraining cosmological models (see [30–32]) as well as to test the validity of GR on cosmological scales (see [34, 35]). Following the above lines, in the current article we combine the linear bias data of Luminous Red Galaxies (hereafter LRGs; [36]), recently released by the DES group, with the growth rate data as provided by Sargedo et al. [39], in order to place constraints on (γ, M_h). Notice that M_h is the dark matter halo in which the LRG live.

The structure of the paper is as follows. In section II we present the DES bias data and the growth data. In section III we provide the family of basic bias models, while in section IV we discuss the evolution of linear
matter fluctuations. The outcome of our analysis is presented in section V, while our main conclusions can be found in section VI.

2. DESY1 RED GALAXIES BIAS DATA AND GROWTH DATA

In a sequence of previous theoretical articles we have proposed to use the biasing properties of extragalactic sources in order to constrain the growth index of matter fluctuations \[34\]. Therefore, in the light of recent Dark Energy Survey (DES) bias data, we attempt to compare the predictions of the most popular linear bias models (see below) with the data. Specifically, the DES bias data \[36\] were extracted in the context of the angular correlation function (ACF) using the 1-year DES sample of \(\sim 6.6 \times 10^5\) LRGs as tracers of the LSS. This population of galaxies can be observed in the redshift range \(0.15 < z < 0.9\). It is important to mention, that during the derivation of the bias data Elvin-Poole et. al. \[36\] imposed the assumption of linear bias. Indeed, following the notations of Krause et. al. \[37\] the scale of \(\sim 8h^{-1}\text{Mpc},\) used by the DES team, ensures that the impact of non-linear effects on biasing is practically negligible. In Table \[\text{I}\] we list the numerical values of the DES bias data with the corresponding errors. Regarding the cosmic expansion we restrict the present analysis to DES/Planck/JLA/BAO ΛCDM cosmology, namely \(\Omega_m = 1 - \Omega_{\Lambda} = 0.301,\ h = 0.682,\ \Omega_{\Lambda} = 0.048,\ n = 0.973\) and \(\sigma_8 = 0.801\) \[33\]. In this context, the normalized Hubble parameter of the ΛCDM model is written as

\[
E(z) = \left[\Omega_m (1+z)^3 + \Omega_{\Lambda} \right]^{1/2}. \tag{2.1}
\]

In addition to DES bias data, we use in our analysis the growth data and the corresponding covariances as collected by Sargedo et al. \[39\] (see their Table I and references therein). This sample contains 22 entries for which the product \(f(z)\sigma_8(z)\) is available as a function of redshift, where \(f(z)\) is the growth rate of clustering. It is well known that the product \(f\sigma_8\) is almost a model-independent parametrization of expressing the observed growth history of the universe \[40\].

### TABLE I: The measured bias data of the 1-year DES LRGs from Elvin-Poole et al. \[36\].

| Red. Range | Median Redshift | DESY1 bias |
|------------|----------------|------------|
| 0.15 < z < 0.3 | 0.225 ± 0.075 | 1.40 ± 0.077 |
| 0.3 < z < 0.45 | 0.375 ± 0.075 | 1.61 ± 0.051 |
| 0.45 < z < 0.6 | 0.525 ± 0.075 | 1.60 ± 0.040 |
| 0.6 < z < 0.75 | 0.675 ± 0.075 | 1.93 ± 0.045 |
| 0.75 < z < 0.9 | 0.825 ± 0.075 | 1.99 ± 0.066 |

3. BIAS MODELS

Let us first briefly present the main bias models. In particular, from the so called merging bias family we include here the models of Sheth, Mo & Tormen \[41\], Jing \[42\] and De Simeone et al. \[43\].

For these models the bias factor is written as a function of the peak-height parameter, \(\nu = \delta_c(z)/\sigma(M_h, z)\) where \(\delta_c\) is the linearly extrapolated density threshold above which structures collapse. Here we use the accurate fitting formula of Weinberg & Kamionkowski \[44\] to estimate \(\delta_c(z)\). Moreover, the mass variance is written as

\[
\sigma(M_h, z) = \left\{ \int_0^\infty k^2 P(k) W^2(k R) dk \right\}^{1/2} \tag{3.1}
\]

where \(W(k R) = 3[\sin(k R) - k R \cos(k R)]/(k R)^3\) is the top-hat smoothing kernel with \(R = [3M_h/(4\pi \rho_m)]^{1/3},\)

\(M_h\) is the halo mass and \(\rho_m\) is the present value of the mean matter density, namely \(\rho_m \approx 2.78 \times 10^{11}\text{M}_\odot\text{Mpc}^{-3}\). The quantity \(P(k, z)\) is the CDM linear power spectrum given by \[45\]:

\[
T(k) = \frac{L_0}{L_0 + C_0 q^2} \tag{3.2}
\]

with \(L_0 = \ln(2e + 1.8q),\ e = 2.718,\ C_0 = 14.2 + \frac{731}{1 + 62.5q}\) and \(q = k/\Gamma\) with \(\Gamma\) being the shape parameter given by \[46\]:

\[
\Gamma = \Omega_m h \exp(-\Omega_\Lambda - \sqrt{2h/\Omega_m}). \tag{3.3}
\]

Taking the aforementioned quantities into account and using Eq. \[3.1\], the normalization of the power spectrum becomes

\[
P_0 = 2\pi^2 \sigma_8^2 \left[ \int_0^\infty T^2(k) k^{n+2} W^2(k R_s) dk \right]^{-1} \tag{3.4}
\]

where \(\sigma_8 \equiv \sigma(R_s, 0)\).

Below we provide some details concerning the bias models.

A. SMT and JING

Sheth, Mo & Tormen \[41\], hereafter SMT based on the ellipsoidal collapse model they found the following bias formula

\[
b(\nu) = 1 + \frac{1}{\sqrt{\alpha}} \delta_c(z) \sqrt{\alpha (\alpha
\nu^2) + \sqrt{\alpha \nu^2}}^{1-c} - f(\nu) \tag{3.5}
\]
with
\[ f(\nu) = \frac{(\alpha \nu^2)^c}{(\alpha \nu^2)^c + b(1 - c)(1 - c/2)}. \] (3.6)

Using N-body simulations they evaluated the free parameters of the model, \( \alpha = 0.707 \), \( b = 0.5 \), \( c = 0.6 \)
Also, Jing [42] proposed the following bias form
\[ b(\nu) = \left( \frac{0.5}{\nu^4} + 1 \right)^{0.06 - 0.02\nu} \left( 1 + \frac{\nu^2 - 1}{\delta_c} \right). \] (3.7)

### B. DMR

De Simone et. al. [43] (hereafter DMR) generalized the original Press-Schether formalism incorporating a non-Markovian extension with a stochastic barrier. In this model, the critical value for spherical collapse was assumed to be a stochastic variable, whose scatter reflects a number of complicated aspects of the underlying dynamics. Therefore, the bias factor is
\[ b(\nu) = 1 + \sqrt{\frac{\alpha \nu^2}{\delta_c}} \left[ 1 + 0.4 \left( \frac{1}{\alpha \nu^2} \right)^{0.6} \right] \]
\[ - \frac{1}{\sqrt{\alpha \delta_c} \left[ 1 + 0.067 \left( \frac{1}{m a^2} \right)^{0.6} \right]}. \] (3.8)

### C. BPR

In addition to merging bias models we shall use the generalized model of Basilakos et al. [34] (hereafter BPR). This form of bias is valid for any dark energy model including those of modified gravity. In this case, using the hydrodynamic equations of motion, linear perturbation theory and the Friedmann-Lemaitre solutions a second differential equation of bias is derived [34]. The solution of the differential equation is given by:
\[ b(z) = 1 + \frac{b_0 - 1}{D(z)} + C_2 \frac{J(z)}{D(z)} \] (3.9)
with \( J(z) = \int_0^z \frac{1}{E(x)} dx \) where \( b_0 \) is the bias factor at the present time. The integration constants \( b_0 \) and \( C_2 \) can be found in [34], namely
\[ b_0 = 0.857 \left[ 1 + \left( \frac{\Omega_m}{0.27} \frac{M_h}{10^{14} h^{-1} M_\odot} \right)^{0.55} \right] \] (3.10)
and
\[ C_2 = 1.105 \left( \frac{\Omega_m}{0.27} \frac{M_h}{10^{14} h^{-1} M_\odot} \right)^{0.255}. \] (3.11)

### 4. EVOLUTION OF LINEAR GROWTH

In this section we discuss the main points of the linear growth of matter fluctuations via which the growth index, \( \gamma \), enters in the current analysis. Focusing on subhorizon scales the differential equation that governs the linear matter perturbations ([11, 12, 47–50] and references therein) is given by
\[ \ddot{\delta}_m + 2H \dot{\delta}_m = 4\pi G_{\text{eff}} \rho_m \delta_m, \] (4.1)
where \( \rho_m \propto a^{-3} \) is the matter density, \( G_{\text{eff}} = G_N Q(t) \) with \( G_N \) being the Newton’s gravitational constant, while the effects of modified gravity are encapsulated in the quantity \( Q(t) \). Of course for those DE models which adhere to General Relativity \( G_{\text{eff}} \) reduces to \( G_N \), hence \( Q(a) = 1 \).

The solution of the aforementioned equation (4.1) is \( \delta_m \propto D(a) \), where \( D(a) \) is the growth factor. For any type of gravity the growth rate of clustering is given by the following useful parametrization [12, 15, 16]
\[ f(a) = \frac{d\ln \delta_m}{d\ln a} \approx \Omega_m^\gamma(a) \] (4.2)
and thus we have
\[ D(a) = \exp \left[ \int_1^a \frac{\Omega_m(y)}{y} dy \right], \] (4.3)
where \( \Omega_m(a) = \Omega_{m0} a^{-3} / E^2(a) \) and \( \gamma \) is the growth index. Notice that the growth factor is normalized to unity at the present epoch.

Now, inserting the operator \( d/dt = H \, d/d\ln a \) and Eq. (4.2) into Eq. (4.1) we arrive at
\[ \frac{df}{d\ln a} + f^2 + \left( \frac{\dot{H}}{H^2} + 2 \right) f = \frac{3}{2} Q(a) \Omega_m(a). \] (4.4)

Considering the concordance \( \Lambda \)CDM model, namely \( Q(a) = 1 \) it is easy to show that
\[ \frac{\dot{H}}{H^2} + 2 = \frac{1}{2} - \frac{3}{2} w(a) [1 - \Omega_m(a)], \] (4.5)
where \( w(a) = -1 \). In this case the Hubble parameter \( H(a) = H_0 E(a) \), where \( E(a) \) is given by Eq. (2.1) and \( H_0 \) is the Hubble constant.

Generally speaking the growth index may not be a constant but rather evolve with redshift; \( \gamma \equiv \gamma(z) \). In this framework, substituting Eq. (4.2) into Eq. (4.1) we find
\[ -(1+z) \gamma \ln(\Omega_m) + \Omega_m^\gamma + 3w(1-\Omega_m) \left( \gamma - \frac{1}{2} \right) + \frac{3}{2} = \frac{3}{2} Q \Omega_m^{1-\gamma} \] (4.6)

\footnote{For the comoving distance and for the dark matter halo mass we use the traditional parametrization \( H_0 = 100kms^{-1}/Mpc \). Of course, when we treat the power spectrum shape parameter \( \Gamma \) we utilize \( h \equiv h = 0.68 \).}
where the prime denotes derivative with respect to redshift. In the present work we restrict our analysis to the following two cases [20, 51, 53]:

\[
\gamma(z) = \begin{cases} 
\frac{\gamma_0}{1 + z}, & \Gamma_1\text{-parametrization} \\
\gamma_0 + \gamma_1 z/(1 + z), & \Gamma_2\text{-parametrization}
\end{cases}
\]  

(4.7)

Using the latter \(\Gamma_2\) parametrization, which is nothing else but a Taylor expansion around \(a(z) = 1\), together with Eq. (4.9) evaluated at the present time \((z = 0)\), we can write the parameter \(\gamma_1\) in terms of \(\gamma_0\)

\[
\gamma_1 = \frac{\Omega_{m0}^{\gamma_0} + 3w_0(\gamma_0 - \frac{1}{2})(1 - \Omega_{m0}) - \frac{3}{2}Q_0\Omega_{m0}^{\gamma_0} + \frac{1}{2}}{\ln \Omega_{m0}}.
\]  

(4.8)

At large redshifts \((z \gg 1)\) \(\Omega_A(z) \approx 0\) the asymptotic value of the growth index becomes \(\gamma_{\infty} = \gamma_0 + \gamma_1\). In general, plugging \(\gamma_0 = \gamma_\infty - \gamma_1\) into Eq. (4.8) we can define the constants \(\gamma_i\) as a function of \(\Omega_{m0}, \gamma_\infty, w_0, Q_0\).

For example, in the case of \(\Omega_{m0} = 0.301, \gamma_\infty \approx 6/11, w_0 = -1\) and \(Q_0 = 1\), the above calculations give \(\gamma_0^{(th)} \approx 0.556, \gamma_1^{(th)} \approx -0.011\).

5. THE LIKELIHOOD ANALYSIS

In this section we provide the statistical method that we adopt in order to constrain the growth index, presented in the previous section. We implement a standard \(\chi^2\) minimization analysis in order to constrain either the \((\gamma, M_h)\) parameter space. Specifically, in our case the situation is as follows:

1. For the DES biasing cosmological probe we use

\[
\chi^2_{\text{DES}}(p_1) = \sum_{i=1}^{5} \frac{(b_{0,\text{obs}}(z_i) - b_{1h}(z_i, p_1))}{\sigma_{bi}}^2
\]  

(5.1)

where the various forms of the theoretical \(b_{1h}\) are given in section III. Notice that \(\sigma_{bi} = \sqrt{\sigma_1^2 + \sigma_2^2}\), where \(\sigma_1\) and \(\sigma_2 = 0.075\) are the uncertainties of the observed bias and redshift respectively (see Table I).

2. Regarding the analysis of the growth-rate data we use

\[
\chi^2_{\text{gr}}(p_2) = (MC_{\text{cov}}^{-1}M)^T
\]  

(5.2)

where \(M = \{f\sigma_{8,\text{obs}}(z_1) - f\sigma_{8}(z_1, p_2), \ldots, f\sigma_{8,\text{obs}}(z_5) - f\sigma_{8}(z_5, p_2)\}\) and \(C_{\text{cov}}^{-1}\) is the inverse covariance matrix [39]. The theoretical growth-rate is given by:

\[
f\sigma_8(z, p_2) = \sigma_8 D(z)\Omega_{m0}(z)^{\gamma(z)}.
\]  

(5.3)

The vectors \(p_1\) and \(p_2\) provide the free parameters that enter in deriving the theoretical expectations. The first vector includes the free parameters which are related to the expansion and the environment of the parent dark matter halo in which the LRGs DES galaxies live. Specifically, for constant \(\gamma\) we have \(p_1 = \{p_2, M_h\} = \{\Omega_{m0}, h, \sigma_8, \gamma, M_h\}\), while for the case of evolving \(\gamma\), the vector is defined as: \(p_1 = \{\Omega_{m0}, h, \sigma_8, \gamma_0, \gamma_1, M_h\}\).

We remind the reader that the cosmological parameters \(\{\Omega_{m0}, h, \sigma_8\} = \{0.301, 0.682, 0.801\}\) are given in section II [38].

Since we wish to perform a joint likelihood analysis of the two cosmological probes and owing to the fact that likelihoods are defined as \(L_i \propto \exp(-\chi^2_i/2)\), the overall likelihood function becomes

\[
L(p_1) = L_{\text{DES}} \times L_{\text{gr}},
\]  

(5.4)

which is equivalent to:

\[
\chi^2(p_1) = \chi^2_{\text{DES}} + \chi^2_{\text{gr}}.
\]  

(5.5)

Based on the above we will provide our results for each free parameter that enters in the \(p_1\) vector. The uncertainty of each fitted parameter will be estimated after marginalizing one parameter over the other, providing as its uncertainty the range for which \(\Delta \chi^2 \leq 1\).

As a further quality measure over the fits, we have used the AIC [54] criterion, in a modified form that is appropriate for small data sets, [53]. Considering Gaussian errors AIC is given by

\[
\text{AIC} = -2\ln L_{\text{max}} + 2k + \frac{2k(k + 1)}{N - k - 1},
\]  

(5.6)

(5.7)

where \(N\) is the total number of data and \(k\) is the number of fitted parameters (see also [53]). Of course, a smaller value of AIC implies a better model-data fit. In order to test the performance of the different bias models in fitting the data we need to utilize the model pair difference, namely \(\Delta \text{AIC} = \text{AIC}_{\text{model}} - \text{AIC}_{\text{min}}\). From one hand, the restriction \(\Delta \text{AIC} \leq 2\) indicates consistency between the two comparison models. On the other hand, the inequalities \(4 < \Delta \text{AIC} < 7\) indicate a positive evidence against the model with higher value of \(\text{AIC}_{\text{model}}\) [56, 57], while the condition \(\Delta \text{AIC} \geq 10\) suggests a strong skeptical evidence.

## A. Observational constraints

Below, we provide a qualitative discussion of our constraints, giving the reader the opportunity to appreciate the new results of the current study.

### 1. Constant growth index

Here we focus on the \(\Gamma_1\) parametrization, which means that the parameter space contains the following free parameters \((\gamma, M_h)\). The presentation of our constraints is provided in Table II for the case of DES/Planck/JLA/BAO reference cosmology (see section II). The Table includes the goodness of fit statistics (\(\chi^2_{\text{min}}, \text{AIC}\)), for the specific bias models. Also, in Figure
1 we present the $1\sigma$, $2\sigma$ and $3\sigma$ confidence contours in the $(\gamma, M_h)$ plane.

In particular, we find:

- For SMT model: $\chi^2_{\text{min}} = 15.042$ (AIC=19.542),
  $\gamma = 0.640 \pm 0.071$ and $\log(M_h/h^{-1}M_\odot) = 13.000 \pm 0.072$.

- For JING model: $\chi^2_{\text{min}} = 15.975$ (AIC=20.475),
  $\gamma = 0.650 \pm 0.063$ and $\log(M_h/h^{-1}M_\odot) = 12.910 \pm 0.062$.

- DMR model: $\chi^2_{\text{min}} = 15.098$ (AIC=19.598),
  $\gamma = 0.640 \pm 0.066$ and $\log(M_h/h^{-1}M_\odot) = 12.730 \pm 0.074$.

- For BPR model: $\chi^2_{\text{min}} = 17.048$ (AIC=21.548),
  $\gamma = 0.640 \pm 0.075$ and $\log(M_h/h^{-1}M_\odot) = 13.080 \pm 0.073$.

We observe that the aforementioned bias models provide very similar results (within $1\sigma$ errors) as far as the growth index is concerned. The corresponding best fit values show a small but non-zero deviation from the theoretically predicted value of GR $\gamma_{GR} \approx 6/11$ (see solid lines of Fig. 1), where the range of the confidence level is $\sim 1.3\sigma - 1.7\sigma$. Such a small discrepancy between the predicted and observationally fitted value of $\gamma$ has also been discussed by other authors. For example recently, [58] found $\gamma = 0.656_{-0.042}^{+0.048}$, while [59] obtained $\gamma = 0.628_{-0.039}^{+0.035}$. Also, similar results can be found in previous papers [60] in which the tension can reach to $\sim 2.5\sigma$.

Furthermore, we find that the best bias model is the SMT, however the inequality $\Delta\text{AIC} \leq 2$ indicates that the SMT bias model is statistically equivalent with rest of the models. The second result is that the differences of the bias models are absorbed in the fitted value of the DM halo mass in which LRGs live, and which ranges from $\sim 6.2 \times 10^{12}h^{-1}M_\odot - 1.2 \times 10^{13}h^{-1}M_\odot$, for the different bias models and in the case of DESY1COSMO bias. As it can also be seen from Table II, our derived mass of the host DM halo mass is consistent with that of Papageorgiou et al. [61], while Sawangwit et al. [62] and Pouri et al. [63] found $M_h \approx (1.9 - 2) \times 10^{13}h^{-1}M_\odot$.

In order to complete the present investigation we repeat the likelihood procedure in the case of Planck TT+TE+EE+low+lensing $\Lambda$CDM cosmology, hence $\Omega_{m0} = 1 - \Omega_{\Lambda0} = 0.3153$, $h = 0.6736$, $\Omega_{\Lambda0} = 0.0493$, $n = 0.9649$, and $\sigma_8 = 0.811$ [7]. Specifically, for the explored bias models we obtain (see also Table II):

- For SMT model: $\chi^2_{\text{min}} = 15.057$ (AIC=19.557),
  $\gamma = 0.680 \pm 0.076$ and $\log(M_h/h^{-1}M_\odot) = 13.070 \pm 0.064$.

- For JING model: $\chi^2_{\text{min}} = 15.952$ (AIC=20.452),
  $\gamma = 0.690 \pm 0.071$ and $\log(M_h/h^{-1}M_\odot) = 12.970 \pm 0.058$.

- DMR model: $\chi^2_{\text{min}} = 15.104$ (AIC=19.604),
  $\gamma = 0.680 \pm 0.075$ and $\log(M_h/h^{-1}M_\odot) = 12.800 \pm 0.073$.

- For BPR model: $\chi^2_{\text{min}} = 16.947$ (AIC=21.447),
  $\gamma = 0.680 \pm 0.089$ and $\log(M_h/h^{-1}M_\odot) = 13.060 \pm 0.080$.

![Fig. 1: The iso-likelihood contours for $1\sigma - 2\sigma - 3\sigma$ levels in the $(\gamma, M_h)$ parameter space for different bias models. Upper row: From left to right, JING and SMT models. Lower row: DMR and BPR models. For further details regarding the models, please see the relevant subsection. Notice that we use the DES/Planck/JLA/BAO $\Lambda$CDM cosmology [38]. The best fit values are given in Table 2. The vertical dashed line corresponds to $\gamma_{GR} \approx 6/11$.](image1)

![Fig. 2: The iso-likelihood contours for $1\sigma - 2\sigma - 3\sigma$ levels in the $(\gamma, M_h)$ parameter space for different bias models. Upper row: From left to right, JING and SMT models. Lower row: DMR and BPR models. For further details regarding the models, please see the relevant subsection. Notice that we use the Planck TT+TE+EE+low+lensing $\Lambda$CDM cosmology [38]. The best fit values are given in Table 2. The vertical dashed line corresponds to $\gamma_{GR} \approx 6/11$.](image2)
TABLE II: Observational constraints for the joint analysis of bias (see Table I) and growth rate data: The 1st column shows the expansion model (see section II [58]), the 2nd column indicates the bias models (see section III), the 3rd column corresponds to \( \gamma \) and the 4th column provides the fitted DM halo mass. The remaining columns present the goodness-of-fit statistics \( \chi^2_{\text{min}} \), AIC and \( \Delta \text{AIC} = \text{AIC}_{\gamma} - \text{AIC}_{\text{min}} \). The index \( i \) corresponds to the indicated bias model.

| ACeDM Expansion Model | Bias Model | \( \gamma \)         | \( \log(M/h^{-1} M_{\odot}) \) | \( \chi^2_{\text{min}} \) | AIC | \( \Delta \text{AIC} \) |
|-----------------------|------------|---------------------|-----------------------------|-----------------|-----|---------------------|
| DES/Planck/JLA/BAO (Abbott et al. [58]) | SMT | 0.640 ± 0.071 | 13.000 ± 0.072 | 15.042 | 19.542 | 0 |
|                     | JING | 0.650 ± 0.063 | 12.910 ± 0.062 | 15.975 | 20.475 | 0.933 |
|                     | DMR  | 0.640 ± 0.066 | 12.730 ± 0.074 | 15.096 | 19.598 | 0.056 |
|                     | BPR  | 0.640 ± 0.075 | 13.080 ± 0.073 | 17.048 | 21.548 | 2.006 |
| Planck TT+TE+EE+low+lensing (Aghanim et al. [7]) | SMT | 0.680 ± 0.076 | 13.07 ± 0.064 | 15.057 | 19.557 | 0 |
|                     | JING | 0.690 ± 0.071 | 12.97 ± 0.058  | 15.952 | 20.452 | 0.895 |
|                     | DMR  | 0.680 ± 0.075 | 12.80 ± 0.073  | 15.104 | 19.604 | 0.047 |
|                     | BPR  | 0.680 ± 0.089 | 13.06 ± 0.080  | 16.947 | 21.447 | 18.49 |

Obviously, our statistical results remain quite robust (within 1σ) against the choice of the underlying expansion [7, 58]. Moreover, as it can be seen from Fig.2 the growth index of the Planck TT+TE+EE+low+lensing ACeDM cosmology deviates with respect to that of GR \((\gamma_{\text{GR}} \approx 6/11)\) at \( \sim 1.5 - 2\sigma \) levels.

2. Constraints on \( \gamma(z) \)

In this section we implement the overall likelihood procedure in the \((\gamma_0, \gamma_1)\) parameter space. Based on the considerations discussed in the previous section the statistical vector takes the form \( p_1 = \{ \Omega_{m0}, h, \sigma_8, \gamma_0, \gamma_1, M_H \} \).

In Fig.3 we plot the results of our statistical analysis in the \((\gamma_0, \gamma_1)\) plane for the SMT bias model, since we have verified that using the other bias models we get similar contours. The predicted \((\gamma_0^{(th)}, \gamma_1^{(th)})\) ACeDM values are indicated by the solid point, while the star corresponds to our best fit values. In brief for the DES/Planck/JLA/BAO \( \Lambda \) cosmology we find \( \gamma_0 = 0.630 \pm 0.072, \gamma_1 = 0.040 \pm 0.403 \) with \( \chi^2_{\text{min}} = 15.033 \) (AIC=19.533), while in the case of Planck TT+TE+EE+low+lensing ACeDM cosmology we get \( \gamma_0 = 0.670 \pm 0.073, \gamma_1 = 0.100 \pm 0.422 \) with \( \chi^2_{\text{min}} = 14.988 \) (AIC=19.488). Notice that for the sake of simplicity we have marginalized the likelihood analysis over the LRG dark matter halo, namely \( \log(M/h^{-1} M_{\odot}) = 13.00 \) and 13.07 respectively (see SMT model in Table II).

We conclude that the joint statistical analysis put tight constraints \( \gamma_0 \), however for \( \gamma_1 \) the corresponding error bars remain quite large. Also the range of deviation from GR is \( 1.7 - 2.9\sigma \). We argue that with the next generation of data (mainly from Euclid) we will be able to test whether the growth index of matter fluctuations depends on time.

FIG. 3: Iso - likelihood contours for \( \Delta \chi^2 = -2\ln \mathcal{L}/\mathcal{L}_{\text{max}} \) equal to 2.30, 6.18 and 11.83, corresponding to 1σ, 2σ and 3σ confidence levels in the \((\gamma_0, \gamma_1)\) plane in the case of \( \Gamma_1 \) parametrization. The bias model is that of SMT, while the star corresponds to the best-fit point and the dot to the theoretical ACeDM point \((\gamma_0, \gamma_1) = (0.556, -0.011)\). In the right panel we present the contours that obtained using the Planck TT+TE+EE+low+lensing ACeDM cosmology [7], while in the left the contours obtained using DES/Planck/JLA/BAO ACeDM cosmology.

6. CONCLUSIONS

Testing the validity of general relativity (GR) on extragalactic scales is considered one of the most important tasks in cosmological studies, hence it is crucial to minimize the amount of priors needed to successfully complete such an effort. One such prior is the growth index \( \gamma \) of matter perturbations. It is well known that a necessary step toward testing GR is to measure \( \gamma \) at the \( \sim 1\% \) accuracy level. Obviously, in order to control the systematic effects that possibly affect individual methods and tracers of the growth of matter perturbations we need to have independent estimations of \( \gamma \).

In this article we used the biasing properties of the Luminous Red Galaxies, recently released by the group of Dark Energy Survey (DES), together with growth rate data in order to constrain the growth index of matter perturbations. Specifically, in the framework of concordance \( \Lambda \) cosmology, we study the ability of four bias models to fit the DES bias data. Then we combined bias in a joint...
analysis with the growth rate of matter fluctuations to place constraints on the parameters.

Considering a constant growth index we placed constraints, up to $\sim 10\%$ accuracy, on the growth index. Specifically, using the priors of the Dark Energy Survey we found that the constraints remain mostly unaffected by using different forms of bias. In particular, we obtained $0.640 \pm 0.071$, $0.650 \pm 0.063$, $0.640 \pm 0.066$ and $\gamma = 0.640 \pm 0.075$ for SMT [11], JING [12], DMR [13] and BPR [14] bias models. Also utilizing the Planck priors we got $\gamma = 0.680 \pm 0.076$, $\gamma = 0.690 \pm 0.071$, $\gamma = 0.680 \pm 0.075$ and $\gamma = 0.680 \pm 0.089$ for the aforementioned bias factors. Obviously, we found a small but non-zero deviation from GR ($\gamma_{GR} \approx 6/11$), where the confidence level lies in the interval $\sim 1.3\sigma - 2\sigma$. Such a small discrepancy between the predicted and observationally fitted value of $\gamma$ has also been reported in several studies [58, 59] and [60]. Moreover, the intrinsic differences of the bias models are absorbed in the fitted value of the dark-matter halo mass in which LRGs survive, and which belongs in the range $\sim 6.2 \times 10^{12} h^{-1} M_\odot - 1.2 \times 10^{13} h^{-1} M_\odot$.

Under the assumption that the growth index varies with time, namely $\gamma(z) = \gamma_0 + \gamma_1 z/(1+z)$, we showed that the ($\gamma_0, \gamma_1$) parameter solution space accommodates the GR ($\gamma_0, \gamma_1$) values at $\sim 1.7\sigma \sim 2.9\sigma$ level utilizing the DES/Planck/JLA/BAO (Planck) priors. Similar to previous studies, we placed tight constraints on $\gamma_0$, however the corresponding uncertainties of $\gamma_1$ remain large. The next generation (mainly from Euclid) of dynamical data are expected to improve the constraints on $\gamma_1$, hence the validity of general relativity on extragalactic scales will be effectively checked.

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