Reply to Comment on
’A new method to calculate the spin-glass order parameter of the two-dimensional $\pm J$ Ising model’

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Abstract. In response to the comment made by Dr. Shirakura et al (cond-mat/0011235), we explain that their scaling forms of the order parameter distribution are inadequate. We then present an appropriate scaling form of the order parameter distribution, which gives a good scaling plot of the order parameter distribution itself and also gives consistent results with our previous results[1].

In a recent paper[1], we discussed the spin-glass phase transition in the two-dimensional $\pm J$ Ising model. We analysed the Binder parameter, $g_L$, and the spin-glass susceptibility, $\chi_{SG}$, by finite-size scaling including the corrections to scaling. We concluded that the estimation of $T_c$ is strongly affected by the corrections to scaling, so that there still remains the possibility that $T_c = 0$. In the comment by Shirakura et al[2], they have performed the scaling analysis of the spin-glass order parameter distribution, $P_L(q)$. They insisted that two scaling forms of $P_L(q)$ reproduce the scaling forms of $g_L$ and $\chi_{SG}$ in [1], but do not give good scaling plots of the order parameter distribution itself. Thus they concluded that there is no possibility that $T_c = 0$.

In this reply, we first explain that the scaling forms of $P_L(q)$ proposed in [2] are inadequate. Then, we present an appropriate scaling form of $P_L(q)$, which gives a good scaling plot of $P_L(q)$ itself, and gives consistent results with our previous results[1].

First, the integral of rhs of equation (1)(equation (2)) in [2] gives $1/(1 + c/L^\alpha) ((1 + d/L^\omega)^{1/2})$. Namely, both the scaling forms of $P_L(q)$ proposed in [2] are not normalized correctly. This should not be ignored, since the difference from 1 is $O(1/L^\omega)$, which is the same order with the difference of the ratios of peak heights for various $L$ in the scaling plots of $P_L(q)$. Furthermore, when the scaling forms of $P_L(q)$ (equations (1) and (2)) in [2] are correctly normalized, we obtain that $a = 0$ and $b = 2c = -d$, where $a$ and $b$ are the coefficients of the correction terms in the scaling forms of $g_L$ and $\chi_{SG}$ (equations (16) and (17)) in [1]. Namely, the scaling forms of $P_L(q)$ proposed in [2] give no correction term of $O(1/L^\omega)$ in the scaling form of $g_L$ for any values of $c$ and $d$, when they are appropriately normalized. Thus, we conclude that the scaling forms proposed in [2] are inadequate.

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We did not consider the scaling form of $P_L(q)$ in [1], since our method only gives $[<q^n>_{L,T}]_p$. In principle, however, the correction term of $[<q^n>_{L,T}]_p$ for any $n$ comes from the correction term of $P_L(q)$. Thus, from now on, we discuss the scaling form of $P_L(q)$ including the corrections to scaling at $T = T_c$.

We think that an appropriate scaling form of $P_L(q)$ at $T = T_c$ can be written as follows:

$$P_L(q) = \frac{L^{n/2} \bar{P}(qL^{n/2})(1 + \frac{\bar{f}(qL^{n/2}) - k}{L^\omega})}{},$$  \hspace{1cm} (1)

where $\bar{P}(x)$ and $\bar{f}(x)$ are some scaling functions. $P_L(q)$ is normalized when we take $k$ as

$$k = \int \bar{P}(x)\bar{f}(x)dx.$$  \hspace{1cm} (2)

Now, we show that a simple form of $\bar{f}(x)$ gives a good scaling plot of $P_L(q)$ itself, and gives consistent results with our previous results[1]. We take $\bar{f}(x)$ as follows:

$$\bar{f}(x) = \begin{cases} 1 + 0.1 & \text{for } |x| \leq 1.1 \\ 0 & \text{for } 1.1 < |x| . \end{cases}$$  \hspace{1cm} (3)

First, we show the scaling plot of $P_L(q)$ with $\omega = 0.5$, $k = 1.252$ and $\eta = 0.2$[4] in figure 1, where we can see that the data with different $L$ fit very well on one scaling function. (Here, we have used the data of $P_L(q)$ used in [2] and [3].) This $P(x)$ gives $\bar{g}(T = 0) = 0.938$ and $a = -0.319$, which are consistent with our previous results (figure 7 in [1]). However, it gives $\bar{\chi}_{SG}(T = 0) = 1.07$ and $b = -0.729$, which are different from our previous results (figure 8 in [1])[5]. Here, we show a scaling plot of $\chi_{SG}$ with $\omega' = 0.5$, $\nu = 2.6$, $\eta = 0.2$ and $T_c = 0$ in figure 2. We find that the data fit rather well on one scaling function, though the data with small linear sizes deviate from the scaling function. Further, $\bar{\chi}_{SG}(T = 0) = 1.07$ is also consistent with that in figure 2. Thus, we conclude that the value of $b$ in equation (17) in [1] should be modified. It is noted that this modification does not change the main result in [1] that the estimation of $T_c$ is strongly affected by the corrections to scaling, so that there still remains the possibility that $T_c = 0$.

In conclusion, we have shown that the scaling forms of the order parameter distribution, $P_L(q)$, proposed in [2] are inadequate. Further, we propose an appropriate scaling form of $P_L(q)$. A simple example of $\bar{f}(x)$ gives a nice scaling plot of $P_L(q)$, and also gives consistent results with those in [1], though we have had to modify the value of the coefficient, $b$, in the scaling form of the spin-glass susceptibility, $\chi_{SG}$.

Acknowledgments

The authors would thank Dr. Shirakura for sending the detailed data in [2] and [3].

References

[1] Kitatani H and Sinada A 2000 J. Phys. A 33 3545
Comment

[2] Shirakura T Matsubara F and Shiomi M (preceding comment) cond-mat/0011235
[3] Shiomi M Matsubara F and Shirakura T 2000 J. Phys. Soc. Jpn. 69 2798
[4] Bhatt R N and Young A P 1988 Phys. Rev. B 37 5606
[5] It is noted that slightly different boundary conditions have been used in [1] and [3]. Therefore, $P_L(q)$ may be different in both studies, which may lead to different values of $a$, $b$ and $\omega$. However, the values of $\bar{g}(T=0)$ and $\bar{\chi}_{SG}(T=0)$ should be free from the boundary condition.

![Figure 1](image-url)  
**Figure 1.** A scaling plot of the order parameter distribution, $P_L(q)$, with $\omega = 0.5$, $k = 1.252$ and $\eta = 0.2$, using the scaling form of equation (1).
Figure 2. A scaling plot of $\chi_{SG}$ with $\omega' = 0.5$, $b = -0.729$, $\nu = 2.6$, $\eta = 0.2$ and $T_C = 0$, using the scaling form of equation (17) in [1].