Actuation curvature limits for a composite beam with embedded shape memory alloy wires

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Abstract

Shape memory alloy composites were manufactured using NiTi wires and woven glass fiber pre-impregnated fabrics. A closed form analytical model was developed to investigate the curvature achievable during actuation. The experimental results of actuation showed reasonable agreement with the model. Actuation temperatures were between \( \sim 55 \) and \( 110 \) °C, curvatures of \( 0.25 \)–\( 0.5 \) m\(^{-1}\) were obtained and the stresses in the wires were estimated to have reached 265 MPa during actuation. An actuation curvature map was produced, which shows the actuation limits and approximate temperature–curvature curves for the general case of a composite containing shape memory alloy wires.

Keywords: shape memory alloy composites, actuation, curvature limits

(Some figures may appear in colour only in the online journal)

1. Introduction

Temperature-dependent shape change in shape memory alloy composites (SMA composites) is accomplished by embedding shape memory alloys (SMAs) off-center in a composite host. The shape change property can be used to achieve useful mechanical actuation in aerospace [1, 2], automotive [3] and robotic application [4–6]. There are many material factors that limit the manufacture and performance of such SMA composites. The most obvious is the extent of shape change achievable, which is sensitive to the attainable length change of the embedded SMA and the stiffness of the composite host. Naturally, the temperatures of manufacture and actuation are also critical; one must apply temperature to achieve a cured composite host but this temperature must not destroy the ‘memory’ of the SMA. During service, the application of temperature must generate the desired shape change while cooling must reverse it. And one must be able to apply all these temperature changes without degrading the host. The stresses built up in the wires during actuation can also become limiting; they must not exceed yielding- or fatigue-based design limits. To facilitate the exploration of all these operational limits and to determine design possibilities, it is desirable to have a simple practical mathematical model for the shape change.

In the present study, we are interested in the limits and performance of a relatively stiff composite host containing thin SMA wires. This approaches the situation of full constrained actuation where almost zero strain is recovered [7, 8]. In such a case the stresses in the SMA can be quite high but good control can potentially be exerted over the process because actuation occurs more gradually over a wider temperature range. Only a small portion of the available actuation strain in the SMA is employed in the actuation.

Whilst a number of models [9–16] and experimental results [1, 17–22] can be found in the literature for similar materials, this specific case does not appear to have received direct attention. Certainly, a simple map for describing the actuation limits for wire-embedded composites does not appear to have been published. In the present work, SMA characterization is combined with a series of experimental SMA composite trials and a simple actuation model is developed to examine the achievable bending actuation limits of a glass fiber–SMA composite panel. A more general actuation limit map is then derived.
Figure 1. Profile view of the actuated beam showing strain distribution; h is the thickness, and \(x_w\) and \(x_n\) are the positions of the wire and neutral axis, respectively.

2. Background

A simple analytical approach based on a balance of bending moments and a convenient approximation for the Clausius–Clapeyron law is presented below after first considering the basic aspects of actuation in the present NiTi SMA.

Actuation depends on the influence of temperature and stress on phase transformation. In the current case, commercial NiTi is considered and thus the relevant phases are B2 austenite at elevated temperatures, the \(R\)-phase at intermediate temperatures (upon cooling) and the monoclinic B19’ martensite phase at low temperatures [23–25]. The as-received wires consist of ‘deformed’ martensite, which is crystallographically preferentially orientated and is thus often termed ‘POM’ [7, 22, 26–30]. As will be shown below, the as-received material in the present case contains \(\sim 5\%\) of recoverable pre-strain. Upon heating, in the absence of any constraint, the material transforms to austenite and the wires shrink back to their pre-deformed length. However, when the SMA wires are constrained from shrinking, the transformation is impeded and the stress in the wires rises [7, 22, 26–30]. In the present material, the wires are (near) fully constrained during curing but are only partially (but relatively heavily) constrained during actuation.

During the curing cycle (at 160 °C in the present case) complete or near complete transformation to the austenite phase is expected during heating. Due to the constraint, the wires are heavily stressed at this point [31]. Upon cooling after curing, the high stresses lead to the formation of crystallographically oriented martensite. Some \(R\)-phase formation is possible but the high stresses favor martensite [25]. Once cooled to room temperature the stresses in the wires are expected to be totally relaxed.

During service, the application of heat will lead to the formation of austenite from the preferentially orientated martensite formed after curing. This will lead to a contraction of the wires, and because the wires are embedded away from the composite beam mid-plane, the beam will bend. Upon cooling, the stresses caused by the constraining composite will lead to the re-formation of ‘deformed’ martensite and the \(R\)-phase. This will permit re-actuation in subsequent heating cycles.

The strains induced in the composite beam during actuation are shown in figure 1. The contraction of the wire, at \(x_w\), shortens the composite as well as bends it. This moves the neutral plane away from the mid-plane to \(x_n\), as shown.

Balancing the moment of a bent beam with the moment caused by the stress in the off-centered wires enables the radius of curvature to be written as:

\[
\rho = \frac{E_c I}{M} = \frac{E_c I}{\sigma_w A_w x_w} = \frac{1}{6} \frac{h E_c}{f x'_w \sigma_w} \tag{1}
\]

where \(M\) is the moment, \(E_c\) is the composite Young’s modulus, \(I\) is the second moment of the area, \(\rho\) is the radius of curvature, \(\sigma_w\) is the stress generated in the wire, \(A_w\) is the cross section area of the wires, \(x_w\) is the position of the wires in relation to the mid-plane, \(f\) is the fraction of SMA wires and \(x'_w\) is the relative position of the wire between the mid-plane and the surface \(x'_w = x_w/(h/2)\).

The stress in the SMA wire can be given using the relation between stress, ratio of transformation and temperature conceived by Liang and Rogers, [32], namely:

\[
\sigma_w = \frac{d\sigma}{dT}(T - A_s) - \left[\frac{(d\sigma /dT)(A_f - A_s)}{\pi} \arccos(2\xi - 1)\right]. \tag{2}
\]

where \(\xi\) is the martensite fraction, \(A_s\) and \(A_f\) are the austenite start and finish temperatures under zero stress and \(d\sigma /dT\) is the slope of Clausius–Clapeyron line for austenite transformation.

In the present case we are interested in the actuation of ‘stiff’ composites where the fraction of transformation is relatively small, i.e. \(\xi \sim 1\). This allows us to make the following rather convenient approximation, which forms an upper bound in relation to the stress carried by the wires:

\[
\sigma_w \sim \frac{d\sigma}{dT}(T - A_s). \tag{3}
\]

Substituting this into equation (1), results in a very simple relationship for the radius of curvature containing the key material, geometrical and actuating parameters:

\[
\rho = \frac{1}{6} \frac{h}{f x'_w (T - A_s)} \frac{E_c}{\sigma_w} \frac{dT}{d\sigma}. \tag{4}
\]

At first sight, it might seem strange that this expression does not contain the available SMA transformation strain. However,
in the limit considered, it is assumed that the actuation strains achieved are well below those ‘available’ in the wire, due to the low transformation fraction. Under such a case the curvature is dictated primarily by the stress, which is dominated by the slope of the stress–transformation temperature curve. The influence of transformation strain will be introduced in section 5 when we construct a map of the actuation limits.

For all else constant, equation (4) shows that smaller radii of curvature will be achieved during actuation with an increase in SMA wire fraction, high values of $x'_t$ and higher temperatures. Less actuation strain (higher radii of curvature) will arise with higher beam thickness and higher composite Young’s modulus, as would be expected.

The remainder of the article describes the experimental characterization of the SMA behavior and the manufacture of prototype glass fiber composites expected to fall in the range of applicability of equation (4). The study aims to reveal the extent of achievable actuation and the degree to which this can be understood and mapped out using equation (4). Finally a map of actuation is constructed.

3. Experimental procedure

NiTi shape memory alloy wires (Ni 49.3, Ti 50.4 at.%%) were obtained commercially (NDC, Fremont, CA) in the straight annealed condition. Diameters of 0.2 and 0.123 mm were used in the study. The host material was HexPly 913 prepreg, comprising an epoxy resin with woven 7781 glass fiber reinforcement with 293 gsm fiber areal weight and 37% resin content.

The phase transformations and base mechanical properties of the SMA wires were investigated using differential scanning calorimetry (DSC—TA instrument Q200) and dynamic mechanical analysis (DMA—TA instrument Q800). To reveal the stress dependency of the transformation temperatures, a given tensile stress below the yield stress was first applied, using the DMA unit, to the wire at a temperature above the $A_f$ temperature, and the temperature was cycled at a rate of $0.5 \degree C \text{ min}^{-1}$ while monitoring the strain [33, 34].

To investigate the effect of the curing cycle on subsequent thermo-mechanical behavior, NiTi wires that had been through the curing process were compared with uncured wires. Thermal cycles were conducted with a heating/cooling rate of $10 \degree C \text{ min}^{-1}$.

Composite stiffness characterization was conducted using DMA with a single cantilever set-up. The temperature was increased to $200 \degree C$ at a $2 \degree C \text{ min}^{-1}$ under a controlled force.

In order to create an SMA composite, a customized jig was designed and constructed along similar lines to that reported in [30] (figure 2). Pre-impregnated glass fiber fabric was cut into $10 \text{ mm} \times 200 \text{ mm}$ pieces and placed above and below the wires at $90^\circ$ orientation (which is identical to $0^\circ$ in woven prepreg materials). A surface finish cloth and a perforated release film were placed on top of the plies and a steel plate was added to the stack to provide a similar heating/cooling rate as at the bottom of the sample. Lastly, a layer of breather material was placed over the entire lay-up and sealed with bagging film and sealant tape. A schematic diagram of the lay-up is shown in figure 3. The samples then were placed under vacuum overnight to allow removal of any volatiles, this minimized the appearance of voids in the samples after curing. Curing employed three stages of consecutive heating at $140, 150$ and $160 \degree C$ for $40, 20$ and $10$ min. After curing, the samples were left in the closed oven for slow cooling to avoid production of any thermal residual stresses. Four SMA–glass fiber composite samples were made with different numbers of plies and SMA wire diameters (table 1). To investigate the mechanical properties of the composite host, a composite sample without NiTi wires was made under the same conditions used for each SMA composite sample.

Actuation testing was performed in a simple convection glass oven to $120 \pm 5 \degree C$ for a number of cycles. Two thermocouples were attached to the sample to monitor the temperature while the actuation was filmed. Only a few degrees of temperature difference was observed along the sample. The deflection angles were measured from snapshots of the actuation footage with an accuracy of $\pm 0.01^\circ$. The curvatures of deflection then were calculated and plotted as a function of temperature for each sample.

4. Results

4.1. SMA properties

Phase transformation temperatures for the 0.123 mm NiTi wire under stress-free conditions can be identified in the DSC
Figure 3. Schematic diagram of the lay-up system used for manufacturing SMA composites. One ply of glass fiber composite is put below and the rest of the plies are placed on top of the constrained SMA wires. The whole set-up is held under vacuum overnight before curing.

Figure 4. DSC curve for as-received 0.123 mm wire, transformation temperatures are measured using the tangent line technique [35]; only the second cycle is shown.

Phase transformation temperatures are measured to be $M_f = 3^\circ C$, $M_s = 15^\circ C$, $R_f = 46^\circ C$, $R_s = 52^\circ C$, $A_s = 56^\circ C$ and $A_f = 64^\circ C$; $M$, $R$ and $A$ stand for martensite, $R$-phase and austenite, respectively; subscripts $f$ and $s$ indicate finish and start temperatures. It is seen that on cooling, the $R$-phase forms before martensite and that the martensite transformation temperatures are below room temperature.

The tensile behavior of 0.123 mm TiNi wires under different temperatures is shown in figure 5. The NiTi wires were first cooled to $-20^\circ C$ to obtain a fully martensitic structure. The onset of yielding due to de-twinning occurs at 170 MPa at room temperature. For test temperatures above the austenite finish temperature ($64^\circ C$), the plateau arises from the stress-induced formation of martensite [23]. The rise in plateau stress with temperature thus reflects the rise in transformation stress with temperature. Failure is seen to occur at stresses between 1000 and 1150 MPa. The elastic modulus for the martensite phase is measured to be $E_M = 22 700$ MPa and for the austenite phase it is $E_A = 72 000$ MPa. These values agree with what is normally seen [25, 36].

Figure 5. Engineering stress strain curve for 0.123 mm NiTi wire from room temperature to $300^\circ C$, as with all samples used in the present study, the samples were first cooled to $-20^\circ C$ to obtain a fully martensitic structure.

The influence of stress on the transformation temperatures are seen in figure 6, which shows the changes in strain over a thermal cycle carried out under an applied stress of 20 MPa. The finish and start temperatures are illustrated by tangent lines drawn over the curve according to the method in [23]. Similar curves were obtained for stresses in the range 10–120 MPa. The critical temperatures thus identified are shown in figure 7. Figure 7 shows that all the transformation temperatures rise with increasing stress but that the $R$-phase transformation temperatures are less sensitive to stress, as is commonly seen [37]. The $A^{\sigma}_{s}$ and $A^{\sigma}_{f}$ lines are parallel with a slope of $d\sigma/dT = 5.8$ MPa $^\circ C^{-1}$. The gap between $A^{\sigma}_{s}$ and $A^{\sigma}_{f}$ is $\sim 5^\circ C$.

It should be noted that because of the different techniques used, the zero stress transformation temperatures determined here are slightly different to those determined using DSC. This has also been seen elsewhere [38]. Although the experiments were performed only up to the plateau stress; it has been shown that these lines can be extrapolated to higher stresses [24, 38].
Figure 6. A thermal cycle of the NiTi wire under 20 MPa of tensile stress. A strain change indicates a phase transformation. Transformation temperatures are determined by intersections of tangent lines. Wire diameter is 0.123 mm.

The results of stress-free thermal cycles of as-received and cured NiTi samples are shown in figure 8. All samples were first cooled to $-20^\circ C$ before the test and two cycles of heating and cooling were applied. It is seen that the as-received samples shrink by a strain of 4.6% and that the cured wires recover a strain of 5.4%, through the first heating. The fact that even more strain is recovered in the cured samples is probably due to some extra strain inadvertently introduced during the curing process [28, 39].

A comparison between the as-received wire cooled to below $M_f$ with the one cooled to room temperature shows that a small strain is recovered during consecutive heating in the former. This is due to a partial two-way memory that can occur in martensite transformation due to the dislocation density retained from manufacture [40, 41]. The magnitude of the recovered strain is greater in all cases than that recoverable from the $R$-phase transformation alone ($\sim 1\%$ [25]) so there is evidently a significant fraction of ‘deformed’ martensite in the wires after curing. Differences between the first and subsequent cycles are often reported; the following cycles are repeatable after the first heating [22, 42].

4.2. Composite properties

Figure 9 shows the storage modulus of a four-ply glass fiber composite measured using a DMA single cantilever test as a function of temperature. Storage modulus is a measurement of the recoverable strain energy and is equivalent to the material’s Young’s modulus at small deformations [43]. As shown in figure 9, it decreases exponentially with increasing temperature which is typical for many polymers [44]. The glass transition temperature ($T_g$) was measured to be 120 $^\circ C$ for this resin system. At the glass transition temperature, the polymer reversibly transforms from a glassy to a rubbery state; the stiffness drops significantly but can be recovered during cooling [45]. The glass transition temperature represents an upper limit to the actuation temperature so the influence of temperature on the composite stiffness in the present case is neglected. Tests were repeated on all the SMA–glass fiber composite samples to obtain values of the composite modulus (see table 2).

4.3. Actuation of SMA composites

SMA–glass fiber composites were made based on the specifications reported in table 1. After making the samples, a cross section was cut using a diamond saw to permit thickness and $x_w$ to be measured using optical microscopy. The dimensions are listed in table 2.

Figure 10 shows actuation over five thermal cycles for SMA–glass fiber composite no. 3. The curvature $(1/\rho)$ gradually rises during heating and drops during cooling. The minimum radius of curvature obtained was $\rho = 2$ m. At this condition the stress in the wires, given according to equation (1), is $\sim 265$ MPa.

There is clearly a hysteresis between cooling and heating in figure 10. This is due to the dissimilarity in the
Table 2. The specifications for the present SMA–glass fiber composites.

| Sample | $f$  | $h$ (mm) | $x_w$ (mm) | $x'_w = x_w/(h/2)$ | $E_c$ (MPa) |
|--------|------|----------|------------|---------------------|------------|
| SMA–glass fiber composite 1 | 0.013 | 0.8      | 0.18       | 0.45                | 22 070     |
| SMA–glass fiber composite 2 | 0.007 | 1.5      | 0.5        | 0.66                | 16 240     |
| SMA–glass fiber composite 3 | 0.02  | 1.4      | 0.3        | 0.43                | 19 430     |
| SMA–glass fiber composite 4 | 0.01  | 1.28     | 0.3        | 0.47                | 19 430     |

Figure 9. Storage modulus as a function of temperature for woven glass fiber composite with the sequence of four plies of 90°. The $T_g$ was assigned to be the onset of stiffness reduction.

Figure 10. Actuation cycles for SMA–glass fiber composite 3, a hysteresis is observed between heating and cooling. An error of ±5°C is considered for temperature measurement at all data points.

Figure 11. Curvature changes as a function of temperature for SMA–glass fiber composites and their models. Only first heating is shown. The error bar is shown to indicate the ±5°C uncertainty in the temperature.

5. Discussion

The present study shows that the simple closed form expression, given as equation (4), can provide reasonable estimates for the actuation curvatures of ‘stiff’ NiTi SMA-based glass fiber composites. In the following, this expression will be employed to map out key actuation limits. First, the assumption of a low transformation fraction in the present experiments will be verified.

If perfect strain transfer between the composite and the wire exists, one can write the following for the balance of strains for composite and the wire at $x_w$:

$$\varepsilon_{tr} + \varepsilon_{cl}^{w} + \varepsilon_{th}^{w} = \varepsilon_{cl}^{c} + \varepsilon_{th}^{c}. \quad (5)$$

Here we focus on the heating cycle.

The overlap between the subsequent cycles in figure 10 shows good repeatability though there is indication of some residual curvature. This is largely due to the existence of temperature variation; i.e. some undetected residual heat in the samples after cooling at the end of each cycle.

To further examine the influence of different SMA composite configurations, the curvature–temperature curves for the all SMA–glass fiber composites prepared in the present work are given in figure 11. The curvatures achieved in composites 2 and 4 are less than those attained in composites 1 and 3 due to lower wire fractions and greater thicknesses.

The predictions made using equation (4) are also shown in figures 10 and 11, and it is seen that the model reproduces the main trends in the data. The predicted actuation curves are seen to be displaced by $\sim 10°C$ to lower temperatures from the experimental values. Part of this difference may be due to temperature measurement differences but it also may reflect the increased transformation temperatures seen in figure 9. It is clear that accurate knowledge of the transformation temperatures in service is essential for accurate prediction of performance.
The limits set by the available (or desirable portion) of recoverable transformation strain. The maximum recoverable strain for equiatomic NiTi alloys is $−0.08$, this can be recovered if deformed martensite is made earlier in the NiTi during pre-straining. Note that only a strain of $−0.01$ can be recovered upon $R$-phase transformation. The transformation strain limit can be estimated by noting that when this limit becomes important the transformation strain is most likely to be considerably higher than both the elastic strain in the wire and thus the limit can be written, using equations (5) and (6) as:

$$
\left( \frac{h}{\rho} \right)_{\text{lim}} \sim - \frac{\varepsilon_{tr}}{(x''_w/2 + 1/6x''_w)}.
$$

(iv) The limiting stress that can be borne by the wires during reversible actuation is ultimately that which causes irreversible dislocation-mediated plasticity in the austenite. A more readily detected, but higher limit, is the ultimate tensile stress, which in the present case can be seen to be in the order of $1000–1100$ MPa. In fully constrained heating of NiTi wires it is seen that the maximum stress achieved is often approximately half of the ultimate yield strength [30]. Certainly in practice, to avoid problems with fatigue, stresses higher than this should be avoided. To define a limit in terms of stress, we re-write equation (3) to obtain a conservative limit for the temperature which should not be exceeded during actuation, to ensure the stress remains below its limit, $\sigma_{\text{lim}}$:

$$
T_{\text{lim}} \sim \sigma_{\text{lim}} \left( \frac{d\sigma}{dT} \right)^{-1} + A_s.
$$

(v) Finally, although equation (4) holds strictly for stiff composites and low transformation fractions, it still provides a useful approximation (within $A_f - A_s$ when plotted against temperature) for the other extreme of full transformation in a compliant composite. We therefore re-write the expression so that it gives a dimensionless curvature $h/\rho$ in terms of the composite elastic modulus over the SMA wire fraction (i.e. the ‘actuation stiffness’, $E_c/f$):

$$
\left( \frac{h}{\rho} \right) = 6 \left( \frac{d\sigma}{dT} \right) \left( \frac{E_c}{f} \right)^{-1} x''_w(T - A_s).
$$

Table 3. Elastic moduli and coefficients of thermal expansion for NiTi wire and the composite host used in the experiments.

| NiTi wire       | Martensite | Austenite | HexPly 913 fiberglass composite |
|----------------|------------|-----------|--------------------------------|
| Elastic modulus (MPa) | 22 700     | 72 000    | 30 000 [46]                    |
| Coefficient of thermal expansion ($10^{-6}$ °C$^{-1}$) | 6 [24]    | 11 [24]  | 6–8 [46]                       |

The superscripts w and c stand for wire and composite; where $\varepsilon_{tr}, \varepsilon_{el}$ and $\varepsilon_{w}$ are the transformation, elastic and thermal expansion strains; respectively. The elastic strain in the composite can be expressed in terms of the curvature ($\varepsilon_{w}^t = -\frac{x''_w}{x''_w}$), see figure 1, and the elastic strain in the wire is $\varepsilon_{el}^t = \frac{x''_w}{E_w}$. The thermal strains of the wire and the composite are almost equal (see table 3) and thus cancel each other in equation (5).

The position of the neutral plane can be obtained from a balance of forces and is found to be a function of thickness and $x''_w$:

$$
x_n = -\frac{h^2}{12x''_w}.
$$

So, substituting, one can write:

$$
\varepsilon_{tr} = -\frac{h}{\rho} \left( \frac{x''_w}{2} + \frac{1}{6x''_w} \right) \frac{\sigma_w}{E_w},
$$

where $x''_w = x_w/(h/2)$. Assuming a representative value for $E_w$ of $33000$ MPa, one finds that the transformation strain for the maximum radius of curvature of $2$ m in SMA composite $3$ ($h = 1.4$ mm, $\sigma_w = 265$ MPa, $x''_w = 3/7$) is $\sim-0.0085$. If one takes the total available recovery strain to be $-0.05$ (see figure 8), this gives the fraction of recovery strain used during actuation as $17\%$. Consequently, it is seen that the present material is being actuated via only modest amounts of transformation, despite being taken near to the temperature limit set by the composite host.

Turning now to the limits placed by the material on the achievable actuation curvatures, we first note that there are two obvious limits to the actuation temperature:

(i) SMA austenite start temperature. $A_s$ is the temperature that must be exceeded for transformation, and therefore bending, to begin. $A_s$ can be manipulated by changing the chemical composition or the thermo-mechanical processing of the SMA wire.

(ii) The composite glass transition temperature. Although this transition is reversible [45], the modulus drops dramatically through it. In practice, such a drop is likely to be localized to the material surrounding the wires, which are best heated by passing a current through them, and such localized variation of stiffness is likely to be detrimental.

There are also two other important limits that are set by the maximum stresses and strains that can be borne by the wires:

(iii) The limit set by the available (or desirable portion of) recoverable transformation strain. The maximum

$$
\sigma_{\text{lim}} = \frac{\rho}{h} \left( \frac{d\sigma}{dT} \right)^{-1} + A_s.
$$

The dimensionless curvature ($h/\rho$) is plotted as a function of temperature, for different values of actuation stiffness ($E_c/f$) in figure 12. The minimum and maximum allowable actuation temperatures $A_s = 54$ °C and $T_{\bar{g}} = 120$ °C are also plotted. For a lower actuation stiffness, a smaller radius of curvature can be obtained, as would be expected. To give an idea of the extent of the deflection achievable for a given radius

Table 3.

| NiTi wire       | Martensite | Austenite | HexPly 913 fiberglass composite |
|----------------|------------|-----------|--------------------------------|
| Elastic modulus (MPa) | 22 700     | 72 000    | 30 000 [46]                    |
| Coefficient of thermal expansion ($10^{-6}$ °C$^{-1}$) | 6 [24]    | 11 [24]  | 6–8 [46]                       |
of curvature, the deflection of the end of a beam of a specific length is also shown at the right-hand vertical axis.

The temperatures corresponding to stress limits of 200–500 MPa are shown in figure 12. This limit could become an important issue for host materials with a higher $T_g$ than for the present material. Limiting curvatures set by the magnitude of the available transformation strains are plotted horizontally. It is seen that these limits are only important for compliant composites with a low actuation stiffness. In such cases it is also seen that the actuation curvature increases rapidly with temperature, which may pose control challenges if intermediate curvatures are desired.

Finally a limit not considered here is the strength of the wire/matrix interface, which is set by the shear properties of the constituent materials [7] and the imperfections produced at the interface during manufacturing. This limit is likely to be important for high stresses in the SMA wires and for high numbers of cycles (see also [22, 47]).

It also should be noted that the stresses in the wires assumed in equation (4) are at the upper bound. A lower bound is readily obtained by replacing $A_s$ in this expression by $A_f$. This will shift the predictions in figures 10 and 11 to higher temperatures by the difference between $A_s$ and $A_f$, which is $\sim 5 ^\circ \text{C}$. The lower bound will thus provide a marginally better agreement but it does not have a good physical justification. However, the small difference between the upper and lower bound predictions means that the model assumption with regard to the stress in the wire is not overly restrictive.

6. Conclusions

In this study, a simple closed form analytical model was developed for stiff SMA composites based on a simplified Clausius–Clapeyron expression.

As-received NiTi SMA wires exhibited an $R$-phase transformation above room temperature and a martensite phase transformation temperature below it. The transformation temperatures were seen to vary with stress at the rate 5.8 $^\circ \text{C MPa}^{-1}$.

SMA–glass fiber composite beams were fabricated and their actuation curvatures monitored as a function of temperature. Observed curvature–temperature traces fell within $\sim 10 ^\circ \text{C}$ of the predicted values.

A map of the actuation limits was constructed for an SMA composite beam for a given offset of the wires from the beam mid-plane. The austenite start temperature was considered as the lower temperature limit, composite glass transition temperature and elastic stress were considered as the upper temperature limits for actuation. Maximum transformation strain defines the upper limit for curvature.

It was observed that the limiting stress in the SMA wires defines an upper actuation temperature, which falls in the range of composite glass transition temperatures. Which of these two factors become limiting will depend on the system.

The SMA actuation strain will only become limiting for the most compliant of composite hosts.

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