$B \rightarrow D^{**}$ – puzzle 1/2 vs 3/2

Benoît Blossier

Laboratoire de Physique Théorique
CNRS/Université Paris-Sud, Bât 210, F-91405 Orsay Cedex, FRANCE

Understanding the composition of final states in $B \rightarrow X_c l \nu$ could help to get a feedback on the persisting disagreement between exclusive and inclusive determinations of $V_{cb}$. In particular the series of orbital excitations $D^{**}$ and radial excitations ($D', D^{*'}$) has received a lot of attention; a misinterpretation as a scalar state of the ($D' \rightarrow D \pi$) spectrum tail could have induced an experimental overestimate of the broad states contribution to the total $B \rightarrow X_c l \nu$ width with respect to theoretical expectations, all of them made however in the infinite mass limit: it is the so-called 1/2 vs 3/2 puzzle. We describe first attempts to measure on the lattice form factors of $B \rightarrow D^{**} l \nu$ at realistic quark masses. Cleaner processes, like hadronic decays $B \rightarrow D^{**} \pi$ and semileptonic decays $B_s \rightarrow D_s^{**} l \nu$ in the strange sector have recently been examined by phenomenologists, putting new interesting ideas on those issues with, again, the need of lattice inputs.

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Table 1: Low-lying spectrum in the $D$ sector; it is convenient to decompose the total orbital momentum as $J = \frac{1}{2} \oplus j_l$, where $j_l$ is the orbital momentum of the light degrees of freedom.

1 Introduction

Understanding the long-distance dynamics of QCD is crucial in the control of the theoretical systematics on low-energy processes that are investigated at LHCb and, in the next years, at Super Belle, to detect indirect effects of New Physics. It is particularly relevant for processes involving excited states, that occur often in experiments. With that respect beauty and charmed mesons represent a very rich sector. An intriguing question concerns the origin of the $\sim 3\sigma$ discrepancy between $|V_{cb}|^{\text{excl}}$ and $|V_{cb}|^{\text{incl}}$ \[1\]: expressed differently, it is welcome to know more about the composition of the final hadronic state $X_c$ in the semileptonic decay $B \to X_c l \nu$. We sketch in Table 1 the low-lying spectrum of $D$ mesons. The $D$ states of the $j_l^P = \frac{1}{2}^+$ doublet are broad while those of the $j_l^P = \frac{3}{2}^+$ doublet are narrow: indeed, the main decay channels are the non leptonic transitions $D^{**} \to D^{(*)} \pi$. Parity conservation implies that the pion has an even angular momentum $\ell$ with respect to $D^{(*)}$. Orbital momentum conservation implies that $\ell = 0$ or 2. That’s why $D_0^*$ and $D_1^*$ decay with a pion in the $S$ wave and $D_2^*$ decays with the pion in the $D$ wave. The decay $D_1 \to D^* \pi$ occurs with the pion in the $S$ or $D$ waves; however, thanks to Heavy Quark Symmetry, the latter is favored. Therefore, decays of the $j_l^P = \frac{3}{2}^+$ doublet are suppressed compared to decays of the $\frac{1}{2}^+$ doublet. But $X_c$ could be made of radial excitations as well: the Babar Collaboration claimed to have isolated a bench of $D^*$ states \[2\]. Among them, a structure in the $D^* \pi$ distribution is interpreted as $D(2550) \equiv D'$. After a fit, experimentalists obtain $m(D') = 2539(8)$ MeV and $\Gamma(D') = 130(18)$ MeV. A question raised about the correctness of this interpretation because, in theory, quark models predict approximately the same $D'$ mass (2.58 GeV) but a quite smaller width (70 MeV) \[3\]. However a well known caveat is that excited states properties are very sensitive to the position of the wave functions nodes, themselves depending strongly on the quark model. We collect in Table 1 the branching ratios of the $B \to X_c$ semileptonic decays. We are interested by $\sim 25\%$ of the total width $\Gamma(B \to X_c l \nu)$: 1/3 of it comes from the channel $B \to D_{\text{strong}}^{**}$. Studying the channel $B \to D^* l \nu$, assuming it is quite large \[4\] and using the fact that $\Gamma(D' \to D_{1/2}^*) \gg \Gamma(D' \to D_{3/2}^*)$, one concludes that an excess of $B \to (D_{1/2}^*) l \nu$ events could be observed with respect to their $B \to (D_{3/2}^*) l \nu$ counterparts. A question is then whether such a potentially large $B \to D^* l \nu$ width could explain the "1/2 vs. 3/2" puzzle: $\Gamma(B \to D_{1/2} l \nu) \propto \Gamma(B \to D_{3/2} l \nu)^{\text{exp}}$ while $\Gamma(B \to D_{1/2} l \nu) \ll \Gamma(B \to D_{3/2} l \nu)^{\text{theory}}$ \[5\]. A kinematical factor explains partly this suppression: $\frac{\Gamma(B \to D_{1/2})}{\Gamma(B \to D_{3/2})} = \frac{2}{(w+1)^2} \left(\frac{\tau_{1/2}(w)}{\tau_{3/2}(w)}\right)^2$. A detailed comparison between theory and experiment is made in the center panel of Table 1. The main tension
Table 2: Branching ratio of $B \rightarrow X_l \ell \nu$ (left panel); comparison between theory and experiment for the different $B \rightarrow D^{**} l \nu$ channels (center panel); comparison between theory and experiment for the different $B \rightarrow D^{*\pi} l \nu$ channels (right panel).

is for $B \rightarrow D_0^* l \nu$. On the experimental side, there are issues about identifying the $D_0^*$ state and the disagreement in $B(B \rightarrow D_1^* l \nu)$ between Belle (no events) and BaBar (claim of a signal). On the theory side, the limitation is that the predictions are made essentially in the infinite mass limit, including lattice QCD calculations of Isgur-Wise functions $\tau_{1/2}$ and $\tau_{3/2}$.

2 $B \rightarrow D^{**} l \nu$ and lattice QCD

2.1 Infinite mass limit

In the Heavy Quark Effective Theory framework, with the trace formalism, the transitions between two heavy-light mesons $H_\nu^{J,J'}$ and $H_\nu^{J',J''}$ are expressed in terms of universal form factors, the Isgur-Wise functions $\Xi(w \equiv v \cdot v')$, where $v$ is the velocity of the meson. Their number is limited thanks to Heavy Quark Symmetry: $\xi(w)$ parameterizes the elastic transition $H_\nu^\frac{J}{2} \rightarrow H_\nu^\frac{J'}{2}$ and is normalised at zero recoil: $\xi(1) = 1$. One has also $\langle H_\nu^\frac{J}{2} | \overline{H}_\nu^{J',\gamma''} h^{\gamma''}_w | H_\nu^\frac{J''}{2} \rangle = \tau_{J/2}(\mu, w) |(v - v')^\mu| \langle H_\nu^{J''} | \overline{H}_\nu^{J',\gamma'} h^{\gamma'}_w | H_\nu^{J'} \rangle = \sqrt{3} \tau_{J/2}(\mu, w)[(w + 1)\epsilon^\mu\nu \epsilon_{\alpha\beta} - \epsilon^*_{\alpha\beta} \epsilon^{\mu\nu} v \cdot v']$. $\tau_{1/2}$ and $\tau_{3/2}$ are not normalised at zero recoil; however, any scale dependence vanishes: $\tau_{\frac{1}{2},\frac{3}{2}}(\mu, 1) = \tau_{\frac{1}{2},\frac{3}{2}}(1)$. A quenched lattice study obtained $\tau_{\frac{1}{2}}(1) = \tau_{\frac{3}{2}}(1)$, even if the analysis was based on quite short plateaus of the $J^{P} = 2^+$ state effective mass and of $\tau_{\frac{1}{2},\frac{3}{2}}(1)$ data got from ratios of 3-pt and 2-pt correlation functions [6]. A similar computation was then led with $N_f = 2$ dynamical quarks, using a set of ETMC gauge ensembles, with acceptable signals for effective masses and $\tau_{1/2,3/2}(1)$. After a smooth extrapolation to the chiral limit, the authors found again that $\tau_{1/2}(1)$ seems significantly smaller than $\tau_{3/2}(1)$ [7]: lattice results point in the same direction as quark models [8, 9] and Operator Production Expansion based sum rules [10, 11].
2.2 Towards realistic \(b\) and \(c\) quark masses

More recently a direct computation in QCD has been tried [12]. The starting point is the definition of a set of form factors:

\[
\langle D_0^*|A^\mu|B \rangle = \tilde{u}^+(p_B + p_D)^\mu + \tilde{u}^-(p_B - p_D)^\mu,
\]

\[
\langle D_2^*(\epsilon^{(\lambda)})|V^\mu|B \rangle = i\tilde{h} \epsilon^{\mu\nu\rho\sigma} \epsilon^{(\lambda)*}_{\alpha\beta} p_B(\rho p_B + \sigma p_D)\rho(\rho p_B - \sigma p_D)\sigma,
\]

\[
\langle D_2^*(\epsilon^{(\lambda)})|A^\mu|B \rangle = \tilde{k} \epsilon^{(\lambda)\mu\nu\rho\sigma} \epsilon^{(\lambda)*}_{\alpha\beta} p_B^\nu + \epsilon^{(\lambda)\mu\nu\rho\sigma} \epsilon^{(\lambda)*}_{\alpha\beta} p_B^\rho + \epsilon^{(\lambda)\mu\nu\rho\sigma} \epsilon^{(\lambda)*}_{\alpha\beta} p_B^\sigma + \tilde{u}^- (p_B + p_D)^\mu + \tilde{u}^-(p_B - p_D)^\nu,
\]

with \(V_\mu = \bar{c} \gamma_\mu b\) and \(A_\mu = \bar{c} \gamma_\mu \gamma^5 b\). Choosing the kinematical configuration \(\vec{p}_D = \vec{0}\), \(\vec{p}_B = (\theta, \theta, \theta)\) and defining the tensors of polarisation accordingly, it has been shown that the leading form factors that contribute to the widths are

\[
\tilde{k} = -\sqrt{6} \mathcal{F}^{(0)1}_A = -\frac{\sqrt{6}}{\theta} \mathcal{F}^{(0)2}_A = \frac{\sqrt{6}}{2 \theta} \mathcal{F}^{(0)3}_A,
\]

\[
\tilde{k} = \frac{1}{\theta} \left[ \mathcal{F}^{(+2)1}_A + \mathcal{F}^{(-2)1}_A \right] = -\frac{1}{\theta} \left[ \mathcal{F}^{(+2)2}_A + \mathcal{F}^{(-2)2}_A \right],
\]

\[
\tilde{u}^+ = -\frac{1}{2 m_{D_0^*}} \left[ \frac{E_B - m_{D_0^*}}{3 \theta} (\mathcal{F}^1_A + \mathcal{F}^2_A + \mathcal{F}^3_A) - \mathcal{F}^0_A \right],
\]

where \(\mathcal{F}^{(\lambda)\mu}_A \equiv \langle D_2^*(\epsilon^{(\lambda)})|A^\mu|B \rangle\) and \(\mathcal{F}^\mu_A \equiv \langle D_0^*|A^\mu|B \rangle\). The preliminary study was performed using \(N_f = 2\) ETMC ensembles: the charm quark was tuned at the physical point, while several ”light” \(b\) quarks were simulated to extrapolate to \(m_b\); cut-off effects were investigated on 2 lattice spacings, a third one will finally be considered. Twisted boundary conditions are required to give a momentum to the \(B\) meson in 2-pt and 3-pt correlators. In the twisted-mass formalism it is difficult to isolate the signal for \(D_0^*\) because of the mixing with \(D\) state due to a breaking parity cut-off effect: solving a generalized eigenvalue problem is beneficial, as shown in the left panel of Figure 1. Isolating the signal for \(D_2^*\) is difficult because of the noise, despite averaging over different interpolating fields that belong to the same representation \((E\) or \(T_2)\) of the \(O_h\) cubic group. At zero recoil, it seems possible to isolate the signal for \(\mathcal{F}^0_A\) but it deteriorates if the \(b\) quark mass gets closer to \(m_b\), as shown in the right panel of Figure 1. Concerning the decay of \(D_2^*\), it is known that \(\mathcal{F}^{(\lambda)\mu}_A(1) = 0\): one needs to inject large momenta, where the data are also noisy.
3  \( B(s) \to D(s)\pi \): a more favorable situation?

A comparison between theory and experiment non leptonic \( B \to D \) decays is made in the right panel of Table 2. Though a (not so conclusive) experimental disagreement in \( B(B_d \to D_s^0\pi) \) between Belle and BaBar, and the fact that theoretical predictions are based on the factorisation approximation, that works well for the so called Class I decays, we globally observe a much better agreement between theory and experiment for \( B_d \to D_s^0\pi \) than for \( B_d \to D_s^0\nu \).

3.1 Largeness of \( B \to D'\ell\nu \) checked on \( B \to D' \)

It was proposed in [13] to check the hypothesis of a large branching ratio \( B(B \to D'\ell\nu) \) by studying non leptonic decays. By examining the Class I process \( \overline{B}^0 \to D'^+\pi^- \), one has in the factorisation approximation

\[
\frac{B(\overline{B}^0 \to D'^+\pi^-)}{B(\overline{B}^0 \to D^+\pi^-)} = \left( \frac{m_B^2 - m_{D'}^2}{m_B^2 - m_D^2} \right)^2 \left( \frac{\lambda(m_B, m_{D'}, m_{\pi})}{\lambda(m_B, m_D, m_{\pi})} \right)^{1/2} \left| \frac{f_{B \to D'}^{B}(0)}{f_{B \to D}^{B}(0)} \right|^2,
\]

where \( \lambda(x, y, z) = |x^2 - (y + z)^2||x^2 - (y - z)^2| \) and \( f_{B \to D'}^{B}(0) = f_{B \to D}^{B}(0) \). With \( V_{cb} f_{B \to D}(0) = 0.02642(8) \) from Babar [14] and \( |V_{cb}|_{\text{incl}} = 0.0411(16) \), we obtain \( f_{B \to D}(0) = 0.64(2) \). Next, with \( m_{D'} = 2.54 \) GeV, we get

\[
\frac{B(\overline{B}^0 \to D'^+\pi^-)}{B(\overline{B}^0 \to D^+\pi^-)} = \left( \frac{f_{B \to D'}^{B}(0)}{f_{B \to D}^{B}(0)} \right)^2 \times (4.7 \pm 0.4) \times 10^{-3}.
\]

Letting vary the \( f_{B \to D'}^{B}(0) \) form factor in the conservative range \([0.1, 0.4]\), according to the existing theoretical estimates [4], [15], we conclude that \( B(\overline{B}^0 \to D'^+\pi^-)^{\text{th}} \sim 10^{-4}: \) the measurement can be performed with the B factories samples and at LHCb. Having a look to the Class III process \( B^- \to D^{0,\pi^-} \), the factorised amplitude reads:

\[
A_{\text{fact}}^{III} = -i \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left[ a_1 f_{\pi} (m_B^2 - m_{D'}^2) f_{B \to D'}^{B}(m_{\pi}^2) + a_2 f_{D'}^* (m_B^2 - m_{D}^2) f_{B \to D}^{B}(m_{\pi}^2) \right].
\]

When the corresponding branching ratio is normalised by the Class I counterpart, we find

\[
\frac{B(B^- \to D^{0,\pi^-})}{B(\overline{B}^0 \to D^+\pi^-)} = \frac{\tau_{B^-}}{\tau_{\overline{B}^0}} \left[ 1 + \frac{a_2}{a_1} \times \frac{m_B^2 - m_{\pi}^2}{m_B^2 - m_{D'}^2} \times \frac{f_{B \to D}^{B}(m_{D}^2)}{f_{B \to D'}^{B}(0)} \times \frac{f_D}{f_{D'}}^2 \right] \frac{1}{f_D f_{\pi}}.
\]

The ratio of Wilson coefficients \( a_2/a_1 \) is extracted from \( \frac{B(B^- \to D^{0,\pi^-})}{B(\overline{B}^0 \to D^+\pi^-)} \), known experimentally [1], and it remains the computation on the lattice of the ratios of decay constants \( \frac{f_{D'}}{f_D} \) and \( \frac{f_{D'}}{f_{\pi}} \). Combining ETMC data at different \( a \) and \( m_{\text{sea}} \) in a common fit we get

\[
\frac{m_{D'}}{m_D} = 1.53(7), \quad \frac{f_{D'}}{f_D} = 0.59(11),
\]

\[
\frac{m_{D'}}{m_D} = 1.55(9), \quad \frac{f_{D'}}{f_{\pi}} = 0.57(16).
\]
Using the experimental value 

\[ B(D_{s1}^+ \rightarrow D_s^{\ast+}\pi^0) = (48 \pm 11)\% \]
\[ B(D_{s1}^+ \rightarrow D_s^\ast\gamma) = (18 \pm 4)\% \]
\[ B(D_{s1}^+ \rightarrow D_s^\ast\pi^+\pi^-) = (4.3 \pm 1.3)\% \]
\[ B(D_{s1}^+ \rightarrow D_{s0}^{\ast}\gamma) = (3.7^{+5.0}_{-2.4})\% \]

Table 3: Branching ratios of non leptonic \( D_{s1}^\ast \) decays.

The experimental result is \( (m_{D'}/m_D)^{\exp} = 1.36 \), 2\( \sigma \) smaller than our value. For the moment that discrepancy remains unexplained despite several checks described in [13]. With \( a_2/a_1 = 0.368 \), \( \tau_{D'}/\tau_B = 1.079(7) \), \( f_{B\rightarrow D'}(0) = 0.64(2) \) and \( f_{B\rightarrow \pi}(m_{D}^{2}) = 0.29(4) \) [16], we obtain

\[ \frac{B(B^- \rightarrow D^0\pi^-)}{B(B^- \rightarrow D^\ast+\pi^-)} = \left[ 1 + \frac{0.14(4)}{f_{B\rightarrow D'}(0)} \right]^2 \frac{B(B^- \rightarrow D^\ast+\pi^-)}{B(B^- \rightarrow D^\ast+\pi^-)} = (1.24 \pm 0.21) \times |f_{B\rightarrow D'}(0)|^2. \]

Using the experimental value \( m_{D'}/m_D = 1.36 \), we get \( \frac{B(B^- \rightarrow D^0\pi^-)}{B(B^- \rightarrow D^\ast+\pi^-)} = (1.65 \pm 0.13) \times |f_{B\rightarrow D'}(0)|^2 \): the dependence on \( m_{D'} \) of that ratio is actually small. Fixing \( f_{B\rightarrow D'}(0) = 0.4 \) and taking \( (m_{D'}/m_D)^{\exp} \) we have also

\[ \frac{B(B^- \rightarrow D^0\pi^-)}{B(B^- \rightarrow D^\ast+\pi^-)} = 1.6(3), \quad \frac{B(B^- \rightarrow D^0\pi^-)}{B(B^- \rightarrow D^\ast+\pi^-)} = 1.4(3). \]

It means that if \( f_{B\rightarrow D'} \) is large, as claimed by many authors, the measurement of \( B(B \rightarrow D'_\pi) \) should be as feasible as \( B(B \rightarrow D_s^\ast\pi) \).

### 3.2 \( B_s \rightarrow D_{s}^{\ast\ast}\pi \)

The situation of the \( D_s \) spectrum is peculiar: indeed, \( D_{s0}^*(2317) \) and \( D_{s1}^*(2460) \) are below the \( DK \) and \( D^\ast K \) thresholds. The main consequence is that they are narrow states. Thus it is very advantageous to examine them because there is no experimental issue from their broadness. It has been proposed to study hadronic decays \( B_s \rightarrow D_{s0}^*(2317)\pi^- \) and \( B_s \rightarrow D_{s1}^*(2460)\pi^- \) [17]. At the moment, only upper limits on \( B(D_{s0}^{\ast+} \rightarrow ...) \) are available: \( B(D_{s0}^{\ast+} \rightarrow D_s^+\gamma, D_{s0}^{\ast+} \rightarrow D_s^{\ast+}\gamma\gamma) < 0.2\% \). In phenomenological analyses, the range \( B(D_{s0}^{\ast+} \rightarrow D_s^{\ast+}\pi^0) = (97 \pm 3)\% \) is taken. There are more data concerning the decay of \( D_{s1}^* \), that we collect in Table 3. According to [17], at LHCb, one measures the cascade \( B_s \rightarrow D_{s0}^{\ast+}\pi^+, D_{s0}^{\ast-} \rightarrow D_{s}^{\ast-}\pi^0, D_{s}^{\ast-} \rightarrow K^+K^-\pi^-; \) the 4-momentum of the non detected \( \pi^0 \) is extracted from the \( B_s \) flight direction and the known \( m_{B_s} \) and \( m_{\pi^0} \). The narrow peak in the \( D_{s0}^{\ast\pi^0} \) mass distribution can be observed, depending on the accuracy of tracking capabilities. Neglecting SU(3) breaking effects, with \( B(B_s \rightarrow D_s^{\ast+}\pi^-) = (2.95 \pm 0.28) \times 10^{-3} \) and \( B(B_s \rightarrow D_{s0}^{\ast}\pi^+) = (1 \pm 0.5) \times 10^{-4} \), the number of expected events with 1 fb\(^{-1}\) of integrated luminosity is

\[ N(B_s \rightarrow D_{s0}^{\ast}\pi^+) = 600 \times (1 \pm 0.5) \times B(D_{s0}^{\ast}\rightarrow D_{s}^{\ast-}\pi^0) \times \epsilon_{\pi^0} : \sim 100. \]
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