N = 2 Supersymmetric Quantum Black Holes
in Five Dimensional Heterotic String Vacua

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Abstract

Exact black hole solutions of the five dimensional heterotic $S$-$T$-$U$ model including all perturbative quantum corrections and preserving $1/2$ of $N = 2$ supersymmetry are studied. It is shown that the quantum corrections yield a bound on electric charges and harmonic functions of the solutions.
In [1] Strominger and Vafa considered five dimensional string theory with \( N = 4 \) supersymmetry to derive the Bekenstein-Hawking entropy [2] by counting black hole microstates. In this letter the low-energy effective action of the five dimensional S-T-U model in heterotic string vacua with \( N = 2 \) supersymmetry is studied. This model yields the Strominger-Vafa black hole including, in addition, perturbative quantum corrections.

The action of five dimensional \( N = 2 \) supergravity coupled to \( N = 2 \) vector multiplets has been constructed in [3] and the compactification of \( N = 1, D = 11 \) supergravity [M-theory] down to five dimensionens on Calabi-Yau 3-folds (\( CY_3 \)) with Hodge numbers \( (h_{1,1}, h_{2,1}) \) and topological intersection numbers \( C_{\Lambda \Sigma \Delta} \) has been given in [3,3]: The \( N_V \)-dimensional space \( \mathcal{M}(N_V = h_{1,1} - 1) \) of scalar components of \( N = 2 \) abelian vector multiplets coupled to supergravity can be regarded as a hypersurface of a \( h_{1,1} \)-dimensional manifold whose coordinates \( X(\phi) \) are in correspondence with the vector bosons (including the graviphoton). The defining equation of the hypersurface is \( \mathcal{V}(X) = 1 \) and the prepotential \( \mathcal{V} \) is a homogeneous cubic polynomial in the coordinates \( X(\phi) \):

\[
\mathcal{V}(X) = \frac{1}{6} C_{\Lambda \Sigma \Delta} X^\Lambda X^\Sigma X^\Delta, \quad \Lambda, \Sigma, \Delta = 1, \ldots h_{1,1} \tag{I.1}
\]

In five dimensions the \( N = 2 \) vector multiplet has a single scalar and \( \mathcal{M} \) is therefore real. Moreover, if the prepotential is factorizable, it is generically symmetric and of the form

\[
\mathcal{V}(X) = X^1 Q(X^{\Lambda+1}), \quad \Lambda = 1, \ldots N_V \tag{I.2}
\]

where \( Q \) denotes a quadratic form. It follows that the scalar fields parametrize the coset space

\[
\mathcal{M} = SO(1,1) \times \frac{SO(N_V - 1,1)}{SO(N_V - 1)} \tag{I.3}
\]

The bosonic action of \( N = 2 \) supergravity coupled to \( N_V \) vector multiplets is given by (omitting Lorentz indices)

\[
e^{-1} \mathcal{L} = -\frac{1}{2} R - \frac{1}{2} g_{ij} \partial \phi^i \partial \phi^j - \frac{1}{4} G_{\Lambda \Sigma} F^\Lambda F^\Sigma + \frac{e^{-1}}{48} C_{\Lambda \Sigma \Delta} \epsilon F^\Lambda F^\Sigma A^\Delta. \tag{I.4}
\]

The corresponding vector and scalar metrics are encoded in the function \( \mathcal{V} \) completely

\[
G_{\Lambda \Sigma} = -\frac{1}{2} \partial_\Lambda \partial_\Sigma \ln \mathcal{V}(X)_{|\nu=1}, \tag{I.5}
\]

\[
g_{ij} = G_{\Lambda \Sigma} \partial_i X^\Lambda(\phi) \partial_j X^\Sigma(\phi)_{|\nu=1}. \tag{I.6}
\]
Here the derivatives in the scalar metric are with respect to the $h_{1,1}$ coordinates $X^\Lambda(\phi)$ and the $h_{1,1} - 1$ scalar fields $\phi^i$, respectively. It is useful to introduce special coordinates $t^\Lambda$ and their duals $t_\Lambda$:

\[
t^\Lambda(\phi) = 6^{-1/3} X^\Lambda(\phi) = C^{\Lambda\Sigma}(\phi)t_\Sigma(\phi),
\]
\[
t_\Lambda(\phi) = C_{\Lambda\Sigma\Delta} t^\Sigma(\phi)t^\Delta(\phi) = C_{\Lambda\Sigma}(\phi)t^\Sigma(\phi)
\]

From these definitions follows $t^\Lambda t_\Lambda = 1$ and $C_{\Lambda\Sigma} C^{\Sigma\Delta} = \delta^\Delta_\Lambda$. In these special coordinates one finds for the gauge coupling matrix

\[
G_{\Lambda\Sigma} = -\frac{6^{1/3}}{2} (C_{\Lambda\Sigma} - \frac{3}{2} t^\Lambda t_\Sigma),
\]
\[
G^{\Lambda\Sigma} = -\frac{2}{6^{1/3}} (C^{\Lambda\Sigma} - 3 t^\Lambda t^\Sigma)
\]

with $G_{\Lambda\Sigma} G^{\Sigma\Delta} = \delta^\Delta_\Lambda$ and $g_{ij} = -3C_{\Lambda\Sigma}\partial_i t^\Lambda \partial_j t^\Sigma$.

It has been shown in [7,4] that the supersymmetry transformations of the gaugino and the gravitino vanish if the (electric) central charge $Z = t^\Lambda q_\Lambda$, appearing in the supersymmetry algebra, has been minimized in moduli space ($\partial_i Z = 0$). This minimization procedure yields the fixed values of the moduli on the black hole horizon [7,8]. Equivalently one may use the “stabilisation equations”

\[
q_\Lambda = t_\Lambda Z_{\text{fix}}, \quad Z_{\text{fix}}^2 = C^{\Lambda\Sigma}_{\text{fix}} q_\Lambda q_\Sigma.
\]

The geometry of the corresponding extreme $D = 5$ black holes is determined by the following metric

\[
ds^2 = -e^{-4V(r)} \, dt^2 + e^{2V(r)} (dr^2 + r^2 \, d\Omega_3^2)
\]

where the metric function $e^{2V(r)}$ is a function of harmonic functions. The moduli for so-called double-extreme black holes are constant and given by their fixed values throughout the entire space-time [7]. For these double-extreme black holes the gauge fields satisfy $2\sqrt{-g} G_{\Lambda\Sigma} F^{\Sigma} = q_\Lambda$. Moreover, the entropy [2] of extreme black holes in five dimensions is given by [7]

\[
S_{BH} = \frac{A}{4G_N} = \frac{\pi^2}{2G_N} \left( \frac{Z_{\text{fix}}}{3} \right)^{3/2}.
\]

In $D = 5$ point-like objects are dual to string-like objects. Thus, corresponding to the electric central charge $Z$ exist the dual magnetic central charge $Z_m = t_\Lambda p^\Lambda$ with magnetic
charges $p^A$. The electric and magnetic charges arise in M-theory from two- and five-brane solitons which wrap even cycles in the CY-space \([5,15]\).

$$q_\Lambda = \int_{C^4 \times S^5} G_7, \quad p^\Lambda = \int_{C^2 \times S^2} F_4. \tag{I.12}$$

Here, $F_4$ is the field-strength of the three-form in $D = 11$ supergravity while $G_7 = \frac{\delta L}{\delta F_4}$ is its dual; $C^4 [C^2]$ denotes a four- [two-] cycle in $CY_3$. From the point of view of the heterotic string $q_{2,3}$ correspond to perturbative electric charges of Kaluza-Klein excitations and winding modes, $p^1$ is the charge of the fundamental string and $p^{2,3}$ [$q_1$] arise from $D = 10$ solitonic five-branes wrapping around $K_3$ [$K_3 \times S_1$]. The magnetic central charge $Z_m$ determines the tension of magnetic string states as a function of the moduli. Thus, analogous to the fixed value of the electric central charge, there exist a fixed value for the string tension \([12,10]\).

$$p^\Lambda = t^\Lambda Z_{m,\text{fix}}, \quad Z_{m,\text{fix}}^3 = 27C_{\Lambda \Sigma \Delta} p^\Lambda p^\Sigma p^\Delta. \tag{I.13}$$

It follows that the $D = 5$ entropy-density of the magnetic string is given by \([12]\)

$$S_S \sim |Z_m|_{\text{fix}}^2 \sim \left(C_{\Lambda \Sigma \Delta} p^\Lambda p^\Sigma p^\Delta\right)^{2/3}. \tag{I.14}$$

Compactifying the $D = 10$ effective heterotic string on $K3 \times S_1$ one can construct the $D = 5$, $N = 2$ $S$-$T$-$U$ model \([14]\). This model contains 244 neutral hypermultiplets, which we will ignore in the following. Moreover it contains three vector moduli $S$, $T$ and $U$, where $S$ denotes the heterotic dilaton and $T, U$ are associated to the graviphoton and the additional $U(1)$ gauge boson of the $S_1$ compactification. The $D = 5$ heterotic $S$-$T$-$U$ model is dual to M-theory compactified on a Calabi-Yau threefold \([5]\). Further compactification on $S_1$ yields the rank 4 $S$-$T$-$U$ model in $D = 4$, which is dual to the $X_{24}(1,1,2,8,12)$ model of the type II string compactified on a Calabi-Yau \([14]\). In special coordinates the prepotential reads

$$\mathcal{V}(S, T, U) = STU + h(T, U) \tag{I.15}$$

The function $h(T, U)$ denotes perturbative quantum corrections, which have been determined in \([6]\).

$$h(T, U) = \frac{a}{3} U^3 \theta(T - U) + \frac{a}{3} T^3 \theta(U - T). \tag{I.16}$$

Here we have introduced the parameter $a = 1$ in order to discuss the classical limit $a \to 0$ in the following explicitly. In the classical limit the scalar fields parametrize the coset \([13]\).
with $N_V = 2$. Using very special geometry the dilaton field $S$ can be eliminated through the algebraic equation

$$ S = \frac{1 - h(T, U)}{TU}. \quad (I.17) $$

For convenience we define the functions

$$ f(x, y) = \frac{2a}{3} x^3 \theta(y - x) - \frac{a}{3} y^3 \theta(x - y), $$

$$ g(x, y) = \frac{a}{3} x^3 \delta(y - x) - \frac{a}{3} y^3 \delta(x - y). \quad (I.18) $$

It follows

$$ \partial_T S = -\frac{1 + f(T, U)}{T^2 U} - \frac{g(U, T)}{TU}, $$

$$ \partial_U S = -\frac{1 + f(U, T)}{U^2 T} - \frac{g(T, U)}{TU}. \quad (I.19) $$

If we take $t^{1,2,3} = (S, T, U)$, we find for the dual coordinates

$$ t_1 = \frac{1}{3} TU, $$

$$ t_2 = \frac{1}{3} SU + \frac{a}{3} T^2 \theta(U - T) $$

$$ t_3 = \frac{1}{3} ST + \frac{a}{3} U^2 \theta(T - U). \quad (I.20) $$

Thus, for the matrix $C$ (with components $C_{\Lambda \Sigma}$) we obtain

$$ C = \frac{1}{6} \begin{pmatrix} 0 & U & T \\ U & 2aT\theta(U - T) & \frac{1 - h(T, U)}{TU} \\ T & \frac{1 - h(T, U)}{TU} & 2aU\theta(T - U) \end{pmatrix}. \quad (I.21) $$

Hence, the gauge coupling matrix reads

$$ G = \frac{1}{2 \cdot 6^{2/3}} \begin{pmatrix} T^2 U^2 & U f(T, U) & T f(U, T) \\ U f(T, U) & \frac{1}{T^2} [1 - 2h(T, U) + f^2(T, U)] & 2h(T, U) \frac{1 - h(T, U)}{TU} \\ T f(U, T) & 2h(T, U) \frac{1 - h(T, U)}{U} & \frac{1}{U^2} [1 - 2h(T, U) + f^2(U, T)] \end{pmatrix}. \quad (I.22) $$

Moreover, it is straightforward to compute the metric $g_{ij}$ of the scalar fields

$$ g = \begin{pmatrix} \frac{1}{T^2} [1 - h(T, U) + T g(U, T)] & \frac{1}{TU} [1 + 2h(T, U) + T g(U, T) + U g(T, U)] \\ \frac{1}{T^2} [1 + 2h(T, U) + T g(U, T) + U g(T, U)] & \frac{1}{U^2} [1 - h(T, U) + U g(T, U)] \end{pmatrix}. \quad (I.23) $$
It follows in the weak coupling regime $S > T > U > 0$

\[
\det g = \frac{3}{4} \frac{1}{T^2 U^2} - \frac{a}{T^2} \quad \text{(I.24)}
\]

\[
\det G = \frac{1}{288} \left( 1 - \frac{a}{3} U^3 \right) \left( 1 - \frac{a}{3} U^3 - a^2 U^6 + \frac{a^3}{27} U^9 \right) \quad \text{(I.25)}
\]

Note that the gauge coupling matrix depends only on $U$. Thus, one obtains for the boundaries of the Weyl-chamber $S > T > U$

| boundary  | $\det g$   | critical points |
|-----------|------------|----------------|
| $U \to 0$ | diverges   | -              |
| $S \to T$ | regular    | $U_{\text{crit.}} = (\frac{3}{a}^{1/3}, (\frac{3}{4a})^{1/3})$ |
| $S \to T \to U$ | degenerates | -              |
| $T \to U$ | regular    | $U_{\text{crit.}} = (\frac{3}{4a})^{1/3}$ |

Here the boundaries are regular up to the critical points with $\det g_{\text{crit.}} = (0, \infty)$. The chamber $S > T > U > 0$ has three boundaries. The lines $S = T$ and $T = U$ are generically regular. These two lines intersect at one point in moduli space ($S = T = U$). Classically this intersection point is a “double self-dual point”, i.e. this point is self-dual with respect to T-duality ($R = 1$) and S-duality ($g_5 = 1$). Including quantum corrections one obtains

\[
U_0 = (1 + \frac{a}{3})^{-1/3} \equiv U_{\text{crit.}}(S \to T) \equiv U_{\text{crit.}}(T \to U) \quad \text{(I.26)}
\]

at this point. Thus, the scalar metric degenerates at this point and, therefore, the moduli space simply ends here [11].

For convenience we will restrict ourselves now to the fundamental Weyl chamber $T > U$. Moreover, we will consider first of all double-extreme black hole solutions before studying the bigger class of extreme solutions given in [16]. Starting with the prepotential (I.13) and the constraint $\mathcal{V}(X) = 1$ one obtains\(^1\) from the electric stabilisation equations

\[
3q_1 = ZTU, \quad 3q_2 = ZSU, \quad 3q_3 = ZST + aZU^2. \quad \text{(I.27)}
\]

It follows

\[
(2aU^3 + 3)Z - 9q_3 U = 0,
\]

\[
aZ^2 U^4 - 3q_3 ZU^2 + 9q_1 q_2 = 0. \quad \text{(I.28)}
\]

\(^1\)In this double-extreme context all the operators take their fixed values in moduli space.
In the classical limit \((a = 0)\) one obtains for the fixed values of the fields

\[
S = \left(\frac{q_2q_3}{q_1^2}\right)^{1/3}, \quad T = \left(\frac{q_1q_3}{q_2^2}\right)^{1/3}, \quad U = \left(\frac{q_1q_2}{q_3^2}\right)^{1/3}.
\] (I.29)

and the central charge \(Z = 3(q_1q_2q_3)^{1/3}\). Thus, we obtain the Strominger-Vafa black hole \([1]\) with entropy

\[
S_{BH} = \frac{\pi^2}{2G_N} \sqrt{q_1q_2q_3}.
\] (I.30)

Including the quantum corrections \((a = 1)\) one obtains a quadratic equation in \(U^3\) with solution

\[
U^3 = -\gamma(1 - \sqrt{1 - \delta/\gamma^2})
\]

\[
\gamma = \frac{3}{2a} \left(\frac{4aq_1q_2 - 3q_3^2}{4aq_1q_2 + 3q_3^2}\right)
\]

\[
\delta = \frac{9q_1q_2}{4a^2q_1q_2 + 3aq_3^2}
\] (I.31)

Since \(U\) is real we obtain a bound \(\gamma^2 - \delta \geq 0\), which becomes, in terms of the charges,

\[
q_3^2 \geq 4q_1q_2.
\] (I.32)

The appearance of this bound is a true quantum effect. The corresponding fixed values of the moduli \(S, T\) and the central charge follow from the solution straightforward. Note that the solution also has to satisfy the inequality \(S > T > U\) in terms of the charges. In the classical limit this condition is satisfied if \(q_3 > q_2 > q_1\). It follows \(q_3^2 > q_1q_2\) and, therefore, the quantum bound is stronger\([1]\). If we consider, for convenience, the case where (I.32) is saturated, we obtain for the fixed values of the fields

\[
S = \sqrt[3]{\frac{q_2}{q_1}} \left(\frac{3}{4}\right)^{1/3}, \quad T = \sqrt[3]{\frac{q_1}{q_2}} \left(\frac{3}{4}\right)^{1/3}, \quad U = \left(\frac{3}{4}\right)^{1/3}
\] (I.33)

It follows that the black hole entropy is given by

\[
S_{BH} = \frac{\pi^2}{6G_N} (q_3)^{3/2}.
\] (I.34)

\(^2\)I thank M. Green for a discussion on this point.
Clearly this result does not coincide with the classical entropy (I.30) in the limit \( q_3^2 = 4q_1q_2 \). Note that the metric function is always given by \( e^{2V} = 1 + \frac{Z}{r} \) in the double extreme limit \([16]\). Moreover, the entropy vanishes if one of the electric charges vanishes. The dual string solution as been extensively discussed in the literature \([13,12,10]\). The fixed values of the scalar fields are given by \( S, T, U = p^{1,2,3}/Z_m \) and the fixed value of the magnetic central charge reads

\[
Z_m = 3 \left( p^1 p^2 p^3 + \frac{a}{3}(p^3)^3 \right)^{1/3}.
\]

In the classical limit the electric and magnetic central charge are dual to each other, if one exchanges electric and magnetic charges. This property does not hold at the quantum level. It follows that some of the magnetic charges can vanish to give a non-trivial entropy-density of the dual magnetic string.

Now we will consider the more general class of black hole solutions of \([16]\). The static, spherically symmetric BPS black hole solution of \([16]\) has metric (I.10) and

\[
2G_{\Lambda\Sigma}F_{0m}^\Sigma = e^{-4V(r)} \partial_m H_\Lambda, \quad n, m = 1, 2, 3, 4
\]

\[
\eta^{nm} \partial_n \partial_m H_\Lambda(r) = 0 \quad \Rightarrow \quad H_\Lambda = h_\Lambda + \frac{q_\Lambda}{r^2}
\]

(I.36)

Here the five-dimensional harmonic functions \( H_\Lambda \) are characterized by the electric charge \( q_\Lambda \) of the three abelian gauge fields (including the graviphoton) and the arbitrary constants \( h_\Lambda \). For special values of \( h_\Lambda \) we obtain the double-extreme solution discussed above. Moreover, the solution satisfies

\[
\sqrt{-g} t_\Lambda = \frac{1}{3} H_\Lambda.
\]

(I.37)

From (I.37) follows

\[
e^{-2V} H_1 = TU, \quad e^{-2V} H_2 = SU, \quad e^{-2V} H_3 = ST + aU^2.
\]

(I.38)

Thus, analogous to the double-extreme black hole solution we obtain

\[
(2aU^3 + 3) e^{2V} - 3H_3 U = 0,
\]

\[
ae^{4V} U^4 - H_3 e^{2V} U^2 + H_1 H_2 = 0.
\]

(I.39)

In the classical limit (\( a = 0 \)) one finds \([16]\).
\[ S = \left( \frac{H_2 H_3}{H_1^2} \right)^{1/3}, \quad T = \left( \frac{H_1 H_3}{H_2^2} \right)^{1/3}, \quad U = \left( \frac{H_1 H_2}{H_3^2} \right)^{1/3}. \]  

(I.40)

Including the quantum corrections \((a = 1)\) one obtains again a quadratic equation in \(U^3\) with solution

\[
U^3 = -\gamma (1 - \sqrt{1 - \delta/\gamma^2})
\]

\[
\gamma = \frac{3}{2a} \left( \frac{4a H_1 H_2 - 3H_3^2}{4a H_1 H_2 + 3H_3^2} \right)
\]

\[
\delta = \frac{9H_1 H_2}{4a^2 H_1 H_2 + 3a H_3^2}
\]

(I.41)

Since \(U\) is real we obtain the bound \(\gamma^2 - \delta \geq 0\). If we take, for instance, \(4H_1 H_2 + 3H_3^2 > 0\) we obtain, in terms of the harmonic functions,

\[ H_3^2 \geq 4H_1 H_2. \]  

(I.42)

The corresponding values for the moduli \(S, T\) and the metric function \(e^{2V}\) in terms of harmonic functions follow straightforward. Note that this black hole configuration exhibits a \(Z_2\) symmetry: \(H_\Lambda \rightarrow e^{i\pi} H_\Lambda\) for integer \(n\). The corresponding black hole entropy of this extreme black hole solution is by definition the same as for the double-extreme solution. Although we can compute now the full quantum solution, i.e. the values of the moduli on the horizon, the entropy and the metric, these expressions are not very illuminating for the exact solution. Instead we give here the first order quantum corrections to various quantities to give a qualitative discussion, i.e. we omitt contribution of order \(O(a^2)\). The corresponding fixed values of the moduli on the horizon are

\[
S_{\text{fix}} = \left( \frac{q_2 q_3}{q_1^2} \right)^{1/3} (1 - \alpha), \quad T_{\text{fix}} = \left( \frac{q_1 q_3}{q_2^2} \right)^{1/3} (1 - \alpha), \quad U_{\text{fix}} = \left( \frac{q_1 q_2}{q_3^2} \right)^{1/3} (1 + 2\alpha)
\]

(I.43)

with \(\alpha = \frac{2a q_1 q_2}{q_3^2}\). It follows for the central charge \(Z_{\text{fix}} = 3(q_1 q_2 q_3)^{1/3} (1 - \alpha)\). The corresponding black hole entropy is

\[
S_{BH} = \frac{\pi^2}{2G_N} \sqrt{q_1 q_2 q_3} \left( 1 - \frac{2}{3} \alpha \right).
\]

(I.44)

Moreover, the leading order correction for the metric function \(e^{2V}\) is given by

\[ e^{2V} = (H_1 H_2 H_3)^{1/3} (1 - \Delta), \quad \Delta = \frac{2a H_1 H_2}{9 H_3^2}, \]

(I.45)

Near the horizon \((r = 0)\) the metric becomes approximately
\[ ds^2 = -\frac{r^4}{\lambda^2} \, dt^2 + \frac{\lambda^2}{r^2} \, dr^2 + \lambda^2 d\Omega_3, \quad \lambda^2 = (q_1 q_2 q_3)^{1/3} (1 - \alpha) \] (I.46)

It follows that the five-dimensional space-time manifold \( \mathcal{M}_5 \) is a product space near the horizon \( \mathcal{M}_5 = AdS_2 \times S^3 \) with symmetry group \( SO(2, 1) \times SO(3) \). It is straightforward to obtain the leading order quantum correction to the ADM-mass of this extreme black hole. Using diffeomorphism invariance the metric can always be brought into the following form:

\[ ds^2 = - \left( 1 - \frac{8G_N M_{\text{ADM}}}{3\pi} \frac{r^2}{r^2} + \cdots \right) dt^2 + \cdots \] (I.47)

Introducing “dressed charges” \( \hat{q}_\Lambda = q_\Lambda / h_\Lambda \) and expanding the metric function one obtains

\[ M_{\text{ADM}} = \frac{\pi}{4G_N} \left\{ \left( 1 + \frac{a}{3} \frac{h_1 h_2}{h_3^2} \right) \sum_{\Lambda=1,2,3} \frac{\hat{q}_{\lambda^*} - a}{h_1 h_2} h_3 \hat{q}_3 \right\}. \] (I.48)

In the classical limit we obtain the results of [16]. Moreover, we find that there are no leading order quantum corrections to the ADM-mass if

\[ \frac{\hat{q}_1 + \hat{q}_2}{\hat{q}_3} = 2. \] (I.49)

In addition, the extreme black hole solution has vanishing ADM-mass if

\[ \frac{h_3^2}{h_1 h_2} = \frac{a}{3} \frac{2\hat{q}_3 - \hat{q}_1 - \hat{q}_2}{\hat{q}_1 + \hat{q}_2 + \hat{q}_3}. \] (I.50)

Although this result only holds to the leading order one expects a similar condition for the massless black hole configuration including all quantum corrections.

To conclude, exact black hole solutions preserving 1/2 of \( N = 2 \) supersymmetry in the five dimensional \( S-T-U \) model including all perturbative quantum corrections have been studied. It has been shown that the quantum corrections yield a new bound on electric charges and harmonic functions of the solutions. The appearance of bounds of this kind in \( N = 2 \) supersymmetric models in five and four dimensions has been previously studied in [17,10]. It would be very interesting to find the corresponding statistical mechanical interpretation of the black hole entropy analogous to the analysis of Strominger and Vafa [1] including this quantum bound.

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