M/D/1 Feedback Queueing Models with Retention of Reneged Customers

Dr. S. K. Tiwari\textsuperscript{1}, Dr. V. K. Gupta\textsuperscript{2}, Tabi Nandan Joshi\textsuperscript{3}

\textsuperscript{1}School of Studies in Mathematics, Vikram University, Ujjain, India
\textsuperscript{2}Govt. Madhav Science Colleges, Vikram University, Ujjain, India
\textsuperscript{3}School of Studies in Mathematics, Vikram University, Ujjain, India

Abstract: Every organization is facing the problem of customer impatience. Customer retention is the key issue in this context. Organizations are applying strategies to sustain their businesses. An impatient customer (due to reneging) may be convinced to stay in service system for his service by utilizing certain convincing mechanisms. Such customers are termed as retained customers. Queueing with feedback represents customer dissatisfaction because of unsuitable quality of service. In case of feedback, after getting partial or incomplete service, customer retries for service. Customer Retention is incorporated in a single-server Markovian feedback queueing model. The steady-state solution of the models is derived. Some useful measures of performance are derived. A particular case of the model is discussed.

Keywords: Customer retention, Feedback, Reneging; Steady-State Solution.

1. Introduction

A queue, or a waiting line, involves arriving items that wait to be served at the facility which provides the service they seek. Queueing theory is concerned with the statistical description of the behavior of the queues with result, e.g., the probability distribution of the number in the queue from which the mean and variance of queue length and the probability distribution of waiting time for a customer, or the distribution of a server’s busy period can be found. In this paper we have discussed about a steady state solution of the ordered queueing problem with reneging.

A customer may enter the queue, but after a time lose patience and decide to leave. In this case the customer is said to have reneged. A unit reneges (i.e., becomes impatient and leaves without having been served) after joining the queue if it is decided that the wait will be longer than can be tolerated. Here the waiting line is of Poisson balking probability which depend not only on the number of customers in the system, but also on the rate of service in the system. A queuing situation with the following characteristics has been considered. A customer receives the service immediately, when the system is empty. But a customer may rejoin the system as a feedback customer for receiving another regular service with probability \(q = 1 - p_2\). Recently, Kumar and Sharma [14] study the retention of reneged customers in an M/M/1/N queueing model and perform sensitivity analysis of the model. Kumar and Sharma [15] study M/M/1/N queueing system with retention of reneged customers and balking. They extend the work of Kumar and Sharma [14] by taking balking aspects in their model to study the effect of probability of retaining the reneged customers on expected system size. They perform the sensitivity analysis of the model. We assume that after the completion of service, each customer may rejoin the system as a feedback customer for receiving another regular service with probability \(p_1\) and may not join with complementary probability 1 - \(p_1\).

When a customer gets impatient, he may leave the queue with some probability, say \(p_2\) and may remain in the queue for service with some complementary probability \(q_2 = 1 - p_2\). Customers are the backbone of any business, because without customers there will be no reason for a business to operate. Customer impatience leads to loss of potential customers. It has become a highly challenging problem in the current era of cut-throat competition. Queueing with customer impatience has special significance for the business world as it has a very negative effect on the revenue generation of a firm. Therefore, the concept of customer retention assumes a tremendous importance for the business management. Customer retention is the key issue in the organizations facing the problem of customer impatience. Firms are employing a number of customer retention strategies to maintain their businesses. An impatient customer (due to reneging) may be convinced to stay in service system for his service by utilizing certain persuasive mechanisms. Such customers are termed as retained customers.
Sharma and Kumar [22] further study M/M/1/N feedback queuing model with balking and retention of reneged customers in the same year. They obtain steady-state solution of the model. They derive important performance measures of the model. Some queueing models are derived as special cases of the model. This paper discussed the queueing model for deterministic service time. Some useful measures of performances are derived. Some queueing models are obtained as particular cases of the model. Literature survey is also presented. Here, also present the steady-state analysis of a single-server Markovian feedback queuing system with retention of reneged customers. M/D/1 queueing model with retention of reneged customers considered.

2. Literature Survey

The earlier work on feedback is found in Takacs [23]. He studies queue with feedback to determine the stationary process for the queue size and the first two moments of the distribution function of the total time spent in the system by a customer. Davignon and Disney [6] study single server queues with state dependent feedback. Santhakumaran and Thangaraj [20] consider a single server feedback queue with impatient and feedback customers. They study M/M/1 queueing model for queue length at arrival and obtain result for stationary distribution, mean and variance of queue length. Thangaraj and Vanitha [24] obtain transient solution for stationary distribution, mean and variance of queue length. They study M/D/1 feedback queuing model with retention of reneged customers. M/D/1 queueing model with balking is also considered.

In this paper, infinite capacity, single-server deterministic service feedback queueing model with customer retention is studied. The steady-state analysis is performed. The steady state probabilities are obtained iteratively in case of reneging and retention of reneged customers. The performance measures are derived and some queuing models are discussed as particular cases of the model.

3. M/D/1 Feedback Queueing Model with Retention of Reneged Customers

In case of feedback, after the completion of service, each customer may rejoin the system as a feedback customer for receiving another regular service with probability p_1 and may not join with complementary probability 1- p_1. We consider an M/D/1 feedback queueing model with reneging. The reneging times are assumed to exponentially distributed with parameter \( \xi \). It is visualized that a reneged customer may be convinced by applying certain convincing method to stay in the system for his service. Thus, there is a probability say, \( q_2 \) that a reneged customer may be retained in the system and may not be retained with some complementary probability say, \( p_2 = 1 - q_2 \).

The differential-difference equations of the model are:

\[
\frac{d}{dt} P_n(t) = -\lambda P_n(t) + \mu q_1 P_1(t)
\]

Where \( P_n(t) \) is the probability of \( n \) in the system at time \( t \). [11]

Here, we use deterministic service time model. The approach we use is similar to to be found in saaty [19] and which is essentially originally due to Crommelin [5]. In the M/D/1 queueing model the arrival rate is \( \lambda \) and the constant service time (say \( b=1/\mu \)).

Now we rescale our parameter as \( \lambda = \lambda/b \) and \( \mu = 1/\mu \) so that the traffic intensity ‘\( \rho \)’ \( (= \lambda/b) \) remains unaffected.

The differential-difference equation of the model according to above condition becomes:

\[
\frac{d}{dt} P_n(t) = -\frac{\lambda}{b} P_n(t) + q_1 P_1(t) \quad \ldots \ldots \quad (1)
\]

\[
\frac{d}{dt} P_n(t) = \left[ \frac{\lambda}{b} + q_1(n-1)\xi p_2 \right] P_n(t) + \left[ q_1 + n\xi p_2 \right] P_{n+1}(t) + \frac{\lambda}{b} P_{n-1}(t) \quad n \geq 1 \ldots \ldots \quad (2)
\]

In steady state, \( \lim_{t \to \infty} P_n(t) = P_n \) and therefore \( \frac{d}{dt} P_n(t) = 0 \) as \( t \to \infty \) and hence the equation (1) and (2) gives the difference equations as

\[
0 = -\frac{\lambda}{b} P_n(t) + q_1 P_1(t) \quad \ldots \ldots \quad (3)
\]

\[
0 = \left[ \frac{\lambda}{b} + q_1(n-1)\xi p_2 \right] P_n(t) + \left[ q_1 + n\xi p_2 \right] P_{n+1}(t) + \frac{\lambda}{b} P_{n-1}(t) \quad n \geq 1 \ldots \ldots \quad (4)
\]
In this section, we derive some important Markovian queueing models from the \( M/M/1 \) feedback queueing model with retention of reneged customers.

In the absence of retention of reneged customer, When the probability of retention of Reneged customer is zero. i.e., \( q_r=0 \)

\[
P_n = \prod_{k=1}^{\infty} \left( \frac{\lambda}{q_1 + (k-1)\xi p_2} \right) P_0; \quad n \geq 1 \quad \ldots \quad (5)
\]

With \[
P_0 = \frac{1}{\sum_{n=1}^{\infty} \prod_{k=1}^{n} \left( \frac{\lambda}{q_1 + (k-1)\xi p_2} \right)} \quad \ldots \quad (6)
\]

The steady state probabilities exist if

\[
\left( 1 + \sum_{n=1}^{\infty} \prod_{k=1}^{n} \left( \frac{\lambda}{q_1 + (k-1)\xi p_2} \right) \right) < \infty
\]

### 4. Measures of Performance

In this section, some important measures of performance are derived. These can be used to study the performance of the queueing system under consideration.

#### Measures of Performance:

1. **The Expected System size, \( L_s \)**

\[
L_s = \sum_{n=0}^{\infty} nP_n
\]

2. **The Expected Queue Length \( L_q \)**

\[
L_q = \sum_{n=0}^{\infty} (nP_n - \frac{\lambda}{bq_1}) = \frac{\lambda}{bq_1} - \frac{\lambda}{bq_1} \prod_{n=1}^{\infty} \left( \frac{1}{q_1 + (k-1)\xi p_2} \right) P_0
\]

3. **The Expected waiting time \( W_s \)**

\[
W_s = \frac{\lambda}{b} \sum_{n=1}^{\infty} (nP_n - \frac{\lambda}{bq_1}) = \frac{\lambda}{bq_1} \prod_{n=1}^{\infty} \left( \frac{1}{q_1 + (k-1)\xi p_2} \right) P_0
\]

4. **The Expected waiting time in the queue \( W_q \)**

\[
W_q = \frac{b}{\lambda} \sum_{n=1}^{\infty} \left( \frac{\lambda}{bq_1} \prod_{k=1}^{n} \left( \frac{1}{q_1 + (k-1)\xi p_2} \right) P_0 - \frac{1}{q_1} \right)
\]

5. **The Expected number of customers served, \( E_{cs} \)**

\[
E_{cs} = q_1 \sum_{n=1}^{\infty} P_n
\]

6. **Rate of Abandonment, \( R_{aband} \)**

\[
R_{aband} = \frac{\lambda}{b} \sum_{n=0}^{\infty} P_n - E_{cs}
\]

#### 4.7 Expected number of waiting customers actually waits, \( E_{cw} \)

\[
E_{cw} = \frac{\sum_{n=2}^{\infty} (n-1)P_n}{\sum_{n=2}^{\infty} P_n}
\]

#### 4.8 Particular Cases of the Model

In this section, we derive some important Markovian queueing models from the \( M/M/1 \) feedback queueing model with retention of reneged customers.

In the absence of retention of reneged customer, When the probability of retention of Reneged customer is zero. i.e., \( q_r=0 \)

\[
P_n = \prod_{k=1}^{\infty} \left( \frac{\lambda}{q_1 + (k-1)\xi p_2} \right) P_0; \quad n \geq 1
\]

With \[
P_0 = \frac{1}{\sum_{n=1}^{\infty} \prod_{k=1}^{n} \left( \frac{\lambda}{q_1 + (k-1)\xi p_2} \right)}
\]

When the capacity of the system is finite

\[
P_n = \prod_{k=1}^{\infty} \left( \frac{\lambda}{q_1 + (k-1)\xi p_2} \right) P_0; \quad 1 \leq n \leq N
\]

With \[
P_0 = \frac{1}{\sum_{n=1}^{\infty} \prod_{k=1}^{n} \left( \frac{\lambda}{q_1 + (k-1)\xi p_2} \right)}
\]

When the Probability of Retention of Reneged Customer (i.e. for \( q_r=0 \)) In the absence of retention of reneged customer (i.e. for \( q_r=0 \)) Our model reduces to an \( M/D/1 \) feedback queueing model with reneging. When the Capacity of the System is Finite When the capacity of the system is taken as finite say, \( N \), the resulting model is an \( M/D/1/N \) feedback queueing model with retention of reneged customers.

### 5. Conclusions

In this paper, we have studied customer retention in an \( M/D/1 \) feedback queueing model with reneging and in an \( M/D/1 \) feedback queueing model with reneging and balk. The steady-state solution is obtained for the queueing model. Some important measures of performance are derived. Some important queueing models are obtained as particular cases of the model.

### References

[1] Ancker Jr., C. J. and Gafarian, A. V., Some Queuing Problems with Balancing and Reneging. I, Operations Research, vol. 11, no. 1, pp. 88-100, 1963.

[2] Ayyapan, G., Muthu Ganapathi Subramanian, A. and Sekar, G., M/M/1 Retrial Queueing System with Loss
and Feedback under Non-pre-emptive Priority Service
by Matrix Geometric Method, Applied Mathematical
Sciences, vol. 4, no. 48, pp. 2379-2389, 2010.
[3] Al-Seedy, R.O., El-sherbiny, A.A., El-Shehawy, S.A,
and Ammar, S.I., Transient Solution of the M/M/c
queue with balking and reneging, Computer and
Mathematics with Applications, vol. 57, no. 8, pp.
1280-1285, 2009.
[4] Choudhury, A., and Medhi, P., Balking and reneging in
multiserver Markovian queuing systems, International
Journal of Mathematics in Operational Research, vol. 3,
no. 4, 377-394, 2011.
[5] Crommelin, C. D. 1932. Delay Probability formulae
when the holding times are constant. P. O. Electrical
Engineering Journal 25, 41-50.
[6] D' Avignon, G.R. and Disney, R.L., Single Server
Queue with State Dependent Feedback, INFOR, vol. 14,
pp. 71-85, 1976.
[7] D.Y. Barrer, Queueing with impatient customers and
indifferent clerks, Operation Research, Vol.5, No.3, 1957.
[8] E. Koenigsberg, Queueing with special service,
Operation Research, Vol.4, 213-220, 1956.
[9] G.G.O. Brien, The solution of some queueing problems,
J. Soc. Ind. Appl. Math., Vol.2, 133-142, 1954.
[10] Gross, D. Shortle, J. Thompson, J. and Harris, C.,
Fundamentals of Queueing Theory, Forth edition,
Wiley, Poisson Input, Constant Service, pp. 294. 2013.
[11] Haight, F. A., Queueing with balking, I, Biometrika, vol.
44, pp. 360-369, 1957.
[12] Haight, F. A., Queueing with Reneging, Metrika vol. 2,
pp. 186-197, 1959.
[13] Kapodistria, S., The M/M/1 queue with synchronized
abandonments, Queuing Systems, vol. 68, pp. 79-109,
2011.
[14] Kumar, R. and Sharma, S.K., M/M/1/N queuing system
with retention of reneged customers, Pakistan Journal
of Statistics and Operation Research, vol. 8, no. 4, pp. 859-
866, 2012.
[15] Kumar, R. and Sharma, S.K., M/M/1 queueing model
with retention of reneged customers and balking,
American Journal of Operational Research, vol. 2,
3(2A) no. 1, pp. 1-6, 2013.
[16] Kumar, R. and Sharma, S.K., Managing congestion and
revenue generation in supply chains facing customer
impatience, Inventi Impact: Supply Chain & Logistics,
vol. 2012, pp. 13-17, 2012.
[17] Kumar, R. and Sharma, S.K., Formulation of Product
Replacement Policies for Perishable Inventory Systems
using Queuing Theoretical Approach, American Journal
of Operational Research, vol. 2, no. 4, pp. 27-30, 2012.
[18] R.G. Miller, A contribution to the theory of balk queues,
J. Roy Statist. Soc., Vol.21, No.2, 320-337, 1959.
[19] Saaty, T. L. 1961, Elements of Queueing Theory with
Applications McGraw Hill, New York.
[20] Santhakumaran, A. and Thangaraj, V., A Single Server
Queue with Impatient and Feedback Customers,
Information and Management Science, vol. 11, no. 3,
pp. 71-79, 2000.
[21] Sharma S.K. and Kumar R., A Markovian feedback
Queue with Retention of Reneged Customers, AMO-
Advanced Modelling and Optimization, vol. 14, no. 3,
pp. 673-679, 2012.
[22] Sharma S.K. and Kumar R., A Markovian feedback
Queue with Retention of Reneged Customers and
Balking, AMO-Advanced Modelling and Optimization,
vol. 14, no. 3, pp. 681-688, 2012.
[23] Takacs, L., A Single Server Queue with Feedback, The
Bell System Tech. Journal, vol. 42, pp. 134-149, 1963.
[24] Thangaraj, V. and Vanitha, S., On the Analysis of
M/M/1 Feedback Queue with Catastrophes using
Continued Fractions, International Journal of Pure and
Applied Mathematics, vol. 53, no. 1, pp. 131-151, 2009.