Study of the reactions
$B \to D^*\pi\pi$ and $B \to D^*\rho\pi$

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ABSTRACT

We evaluate the non leptonic B meson decays $B \to D^*\pi\pi$ and $B \to D^*\rho\pi$, in the factorization approximation and in the limit of infinitely heavy quarks, assuming the dominance of intermediate positive parity charmed resonances. We find that the branching ratios are of the order $10^{-3}$. 
Positive parity charmed mesons have been recently investigated by a number of authors \cite{1,2,3,4} in the framework of the Heavy Quark Effective Theory (HQET). In particular in \cite{1} it has been shown that the semileptonic decays:

\[ B \rightarrow D^{**} \ell \nu \quad (1) \]

can be described in terms of two universal form factors, \( \tau_{1/2}(\omega) \) and \( \tau_{3/2}(\omega) \), where \( \omega = v \cdot v' \) and \( v^\mu, v'^\mu \) are the B and \( D^{**} \) meson four-velocities respectively. In eq.(1) \( D^{**} \) is one of the four positive parity charmed mesons: \( 2^+_{3/2}, 1^+_{3/2}, 1^+_{1/2}, 0^+_{1/2} \), that one expects in the infinite heavy quark mass limit; here we employ the notation \( J_{s_\ell}^P \), where \( s_\ell = 1/2 \) or \( 3/2 \) is the total angular momentum of the light degrees of freedom. The form factor \( \tau_{3/2}(\omega) \) describes the decays into the \( 2^+_{3/2} \) and \( 1^+_{3/2} \) states, whereas \( \tau_{1/2}(\omega) \) is related to the \( 1^+_{1/2} \) and \( 0^+_{1/2} \) states. Both form factors have been estimated in \cite{2} by using the QCD sum rules approach.

The \( 2^+_{3/2} \) state has been observed experimentally \cite{2} with a mass of 2460 \( MeV \): it is denoted by \( D_1^+(2460) \). The \( 1^+_{3/2} \) meson has to be basically identified with \( D_1(2420) \). As shown in refs.\cite{3,4} they are both narrow (\( \Gamma \leq 20 \; MeV \)) since their strong decays occur in \( D^- \)-wave, in contrast with the states \( D_0 \) (the \( 0^+_{1/2} \) state) and \( D_1^0 \) (the \( 1^+_{1/2} \) state) that can also decay by \( S^- \)-wave. On the experimental side, some evidence has been gathered on the semileptonic decays (1) \cite{3,4}, but it is not yet conclusive. A different way to study the transition \( B \rightarrow D^{**} \) is by the non leptonic reactions \cite{5}:

\[ B \rightarrow D^* \pi \pi \quad (2) \]
\[ B \rightarrow D^* \rho \pi \quad (3) \]

which can occur by the intermediate production of positive parity resonances with subsequent decay into \( D^* \pi \). Examples of these processes are as follows:

\[ B^- \rightarrow D^{**0} \pi^- \rightarrow D^{*+} \pi^- \pi^- \quad (4) \]
\[ B^- \rightarrow D^{**0} \rho^- \rightarrow D^{*+} \pi^- \rho^- \quad (5) \]
\[ \bar{B}^0 \rightarrow D^{**0} \pi^+ \pi^- \quad (6) \]

where in the last case we could have either \( D^{*0} \rho^0 \) or \( D^{**+} \pi^- \) as intermediate resonant states.

In this letter we wish to study the processes (4-6) in the framework of the factorization approximation. This approach has first been proposed by Feynman \cite{5}, and then has been extensively applied by Bauer, Stech and Wirbel (BWS) \cite{10} and by a number of other authors \cite{6}. As well known, in this approximation one considers matrix elements of the weak non leptonic effective hamiltonian

\[ H_{NL} = \frac{G}{\sqrt{2}} V_{cb} V_{ud} : a_1 \bar{c} \gamma_\mu (1 - \gamma_5) b \bar{d} \gamma^\mu (1 - \gamma_5) u + a_2 \bar{d} \gamma_\mu (1 - \gamma_5) b \bar{c} \gamma^\mu (1 - \gamma_5) u : \quad (7) \]

\(^1\)It is likely however that \( D_1(2420) \) also contains an admixture of \( 1^+_{1/2} \); for discussions see \cite{3} and \cite{4}.

\(^2\)We do not include final state strong interaction effects due to the lack of data on the phase shifts. For an analysis of such effects in \( B \rightarrow D^* \pi \) and \( B \rightarrow DD \) decays see ref. \cite{6}. For other studies of non leptonic \( B \) meson decays see refs. \cite{6,12} and references therein.
and evaluates them by inserting the hadronic vacuum state between the $V - A$ quark currents appearing in (7); the constants $a_1$ and $a_2$ are given by:

$$a_1 = c_1 + \frac{c_2}{N_c}$$

(8)

$$a_2 = c_2 + \frac{c_1}{N_c}$$

(9)

where $c_1 = 1.1$ and $c_2 = -0.24$ are Wilson coefficients evaluated at the $b-$quark mass scale. As for $N_c$, experimental data for $B \to D\pi$ and $B \to D\rho$ seem to favour the value of the BWS model $[10] N = \infty$ (see e.g. $[11]$) instead of the value $N_c = 3$; we shall comment on this point later on.

A test of factorization has been worked out in ref. $[13]$, where the experimental ratio between the $B^0 \to D^{*+}\pi^-$ non leptonic width and semileptonic spectrum $\frac{d\Gamma(B^0 \to D^{*+}\ell^-\nu)}{dq^2}$, evaluated at $q^2 = m^2_{\pi}$, has been used. If factorization works well, this quantity is a model independent prediction; the result shows that, within the accuracy of the present experimental data, factorization is correct at the 25% level.

In order to compute the hadronic matrix elements of $H_{NL}$ we need the weak current matrix elements between hadron states. In the infinite heavy quark mass limit they can be written as follows $[1, 14, 15]$: 

$$<D(v')|V_{\mu}(0)|B(v)> = \sqrt{m_Bm_D} \xi(v \cdot v') (v + v')_\mu$$

$$<D^*(v',\epsilon)|(V - A)_\mu(0)|B(v)> = \sqrt{m_Bm_D} \xi(v \cdot v') \left[ i\epsilon_{\mu\alpha\beta}\epsilon^{*\nu}v^\alpha v^\beta + \epsilon^*_\mu (1 + v \cdot v') + (\epsilon^* \cdot v) v'_\mu \right]$$

(11)

$$<D_0(v')|A_\mu(0)|B(v)> = -2\sqrt{m_Bm_D} \tau_{1/2}(v \cdot v') (v - v')_\mu$$

(12)

$$<D_1(v',\epsilon)|(V - A)_\mu(0)|B(v)> = \sqrt{m_Bm_D} \tau_{3/2}(v \cdot v') \left[ 2 (v \cdot v' - 1) \epsilon^*_\mu + 2 (\epsilon^* \cdot v) v'_\mu - i\epsilon_{\mu\alpha\beta}\epsilon^{*\nu}(v + v')^\alpha (v - v')^\beta \right]$$

(13)

$$<D_2(v',\epsilon)|(V - A)_\mu(0)|B(v)> = \sqrt{m_Bm_D} \tau_{3/2}(v \cdot v') \left[ i \frac{\sqrt{3}}{2} \epsilon_{\mu\alpha\beta}\epsilon^{*\nu}v_\nu(v + v')^\beta (v - v')^\gamma + \frac{1}{2\sqrt{2}} \epsilon_{\mu\alpha\beta}\epsilon^{*\nu}(v + v')^\alpha (v - v')^\beta \right]$$

(14)

$$<D_2^*(v',\epsilon)|(V - A)_\mu(0)|B(v)> = \sqrt{m_Bm_D} \tau_{3/2}(v \cdot v') \left[ i \frac{\sqrt{3}}{2} \epsilon_{\mu\alpha\beta}\epsilon^{*\nu}v_\nu(v + v')^\beta (v - v')^\gamma - \frac{1}{2\sqrt{2}} \epsilon_{\mu\alpha\beta}\epsilon^{*\nu}(v + v')^\alpha (v - v')^\beta \right]$$

(15)

where $V^\mu = \bar{c}_v\gamma^\mu b_v$, $A^\mu = \bar{c}_v\gamma^\mu\gamma_5 b_v$, $c_v$ and $b_v$ are quark operators in HQET, $m_{D^*}$ is the mass of the positive parity charmed resonances, and $\xi(v \cdot v')$, $\tau_{1/2}(v \cdot v')$ and $\tau_{3/2}(v \cdot v')$ are the Isgur-Wise universal form factors $[1]$. For the form factors $\tau_{1/2}(v \cdot v')$ and $\tau_{3/2}(v \cdot v')$ we take the results of Ref. $[2]$, where these form factors have been obtained by 3-point function QCD sum rules in the $m_Q \to \infty$ limit. The results of this approach are in reasonable agreement with an estimate based on the non relativistic potential model $[4]$; in particular at $v \cdot v' = 1.3$ which is the relevant value for $v \cdot v' = \frac{m_2^2 + m_3^2 - q^2}{2m_Bm_{D^*}} |q^2 - m_2^2 m_3^2|$ in our case, we use $\tau_{1/2}(v \cdot v') = 0.20$ and $\tau_{3/2}(v \cdot v') = 0.19$. 


We have also to consider the matrix elements of the currents between the vacuum and the charmed resonances in the $m_Q \to \infty$ limit:

$$< 0|V^\mu(0)|D_0(p) > = i f^{(+)} / \sqrt{m_c p^\mu}$$
$$< 0|A^\mu(0)|D_1'(\epsilon, p) > = f^{(+)} / \sqrt{m_c} \epsilon^\mu$$

where $f^{(+)}$ depends only logarithmically on the heavy quark mass and has been determined in ref. [2] by QCD sum rules $\langle m_c = 1.35 \text{ GeV} \rangle$:

$$f^{(+)} \approx 0.46 \text{ GeV}^{3/2}.$$  
(18)

In contrast, the matrix elements between the vacuum and the $s_\ell = 3/2$ resonances vanish in the $m_Q \to \infty$ limit [3].

Finally, we have to consider the weak current matrix elements between $B$ and a light meson $\pi, \rho$; they can be written as follows:

$$< \pi(p')|V^\mu|B(p) > = (p^\mu + p'^\mu - \frac{(m_B^2 - m_\pi^2)}{q^2} q^\mu) F_1(q^2)$$
$$+ \frac{(m_B^2 - m_\pi^2)}{q^2} q^\mu F_0(q^2)$$

$$< \rho(p', \ell^*)|V^\mu - A^\mu|B(p) > = \epsilon^{\mu\nu\rho\sigma} \epsilon^\nu_{\mu\rho} p'_{\rho} \frac{2V(q^2)}{(m_B + m_\rho)}$$
$$+ i \left\{ \epsilon^\nu (m_B + m_\rho) A_1(q^2) + (\ell^* \cdot p) \frac{A_2(q^2)}{(m_B + m_\rho)} (p + p')^\nu ight\}$$

$$+ (\ell^* \cdot p) \frac{q^\mu}{q^2} [(m_B + m_\rho) A_1(q^2)$$
$$- (m_B - m_\rho) A_2(q^2) - 2m_\rho A_0(q^2)] \right\}$$

where $F_0(0) = F_1(0)$ and $2m_\rho A_0(0) = (m_B + m_\rho) A_1(0) - (m_B - m_\rho) A_2(0)$.

For the $q^2$ dependence of the various form factors in (18) and (19) we have assumed a simple pole formula, i.e. $F(q^2) = F(0)/(1 - q^2/m^2)$ with $m = 5.32 \text{ GeV}$ for $F_1$ and $V$, $m = 5.99 \text{ GeV}$ for $F_0$, $m = 5.73 \text{ GeV}$ for $A_1$ and $A_2$. As for their values at $q^2 = 0$ we take the results of an analysis of the semileptonic $B$ decays into light mesons, obtained assuming both chiral symmetry and HQET [17]: $F_1(0) = 0.53$, $A_1(0) = 0.21$, $A_2(0) = 0.20$, $V(0) = 0.62$. These results are obtained by an effective lagrangian which incorporates the symmetries for light and heavy quarks, generalizing the results of ref. [16] to all the semileptonic B decays into light particles. The quoted values are obtained without $1/m_Q$ corrections, to be consistent with the choices (15)-(17); the role of possible $1/m_Q$ terms will be discussed later on.

The computed widths for the various $B^- \to D^{*0}\pi^-$, $\rho^-$ decay channels are collected in Table I and Table II. It is worth observing that the $a_2$ contributions, when present, are quite sizeable, mainly due to the small numerical value of the universal form factor $\tau_{1/2}$. On the other hand, the $a_2$ contributions are absent in the case of $B \to D^{**}(J_{3/2}^\pm)\pi, \rho$ since, as observed above, the weak current matrix elements between the vacuum and the $3/2$ doublet vanish in the limit of infinitely heavy quarks. This allows us to give the following predictions for the ratios

$$\frac{\Gamma(B^- \to D^{**}(2_{3/2}^+)^0 \pi^-)}{\Gamma(B^- \to D^{**}(3_{3/2}^0)^0 \pi^-)}$$
$$\frac{\Gamma(B^- \to D^{**}(2_{3/2}^+)^0 \rho^-)}{\Gamma(B^- \to D^{**}(1_{3/2}^0)^0 \rho^-)}$$

$$\Gamma(B^- \to D^{**}(2_{3/2}^+)^0 \pi^-) = \frac{4z (1 + \sqrt{z})^2}{(1 - \sqrt{z})^2[(1 + \sqrt{z})^2 - y]^2}$$

(21)
and

\[
\frac{\Gamma(B^- \rightarrow D^{*0}(2_{3/2}^+)\rho^-)}{\Gamma(B^- \rightarrow D^{*0}(1_{3/2}^+)\rho^-)} = \frac{4[y(3\omega + 2) + (\omega - 1)(1 + \sqrt{z})^2]}{[y(3\omega - 7) + 5\omega - 2\sqrt{z}\omega^2 + 5\omega - 6\sqrt{z}\omega + 3z]}
\]  (22)

\(z = (\frac{m_{D^*}}{m_B})^2, y = (\frac{m_D m_B}{m_B})^2\) and \(\omega = \frac{1+z-y}{2z}\) which are independent of \(\tau_{3/2}\); however, the experimental test of these relations is a difficult task.

In order to evaluate the transitions (4-6), we use the narrow width approximation for the positive parity charmed resonances, which is a good assumption for \(D'_2(2460)\) and \(D_1(2420)\). As for \(D'_1\) (\(D_0\) does not decay into \(D^*\pi\)), an estimate based on a constituent quark model \[15\] and HQET in the chiral limit \[3\] gives \(\Gamma(D'_1) \simeq 80\ MeV\), a value compatible with the narrow width approximation. We also observe that, in the limit \(m_{D_1} = m_{D'_1}\) and \(\Gamma_{D_1} = \Gamma_{D'_1} \simeq 0\) the mixing angle between \(D_1\) and \(D'_1\) does not affect the final results.

The branching ratios for the \(D^{**}\) decays into \(D^*\pi\) have been evaluated in ref. \[3\] in the framework of the Heavy Quark Effective Chiral Perturbation Theory, with the results:

\[
BR(D'_1^0 \rightarrow D^{*+}\pi^-) = 2/3
\]  (23)
\[
BR(D'_0^0(2420) \rightarrow D^{*+}\pi^-) = 2/3
\]  (24)
\[
BR(D'_2^0(2460) \rightarrow D^{*+}\pi^-) = 0.20 .
\]  (25)

Using these branching ratios and the results in Table I and Table II (with \(a_1 = 1.1\) and \(a_2 = -0.24\)) we estimate for the transitions (4-6) the following BR’s (\(\tau_B = 1.2\ ps\)):

\[
BR(B^- \rightarrow D^{*+}\pi^-\pi^-) = 1 \times 10^{-3}
\]  (26)
\[
BR(B^0 \rightarrow D^{*0}\pi^+\pi^-) = 6 \times 10^{-4}
\]  (27)
\[
BR(B^- \rightarrow D^{*+}\pi^+\rho^-) = 2 \times 10^{-3} .
\]  (28)

In computing Eq.(27) the channel \(B^0 \rightarrow D^{*0}\rho^0\), which is a pure \(a_2\) process, has been included incoherently.

Let us conclude with some comments on the accuracy of the results Eqs.(26-28). First, it should be observed that, in considering the processes (4-6), we have not taken into account non resonant pion production, since we presume dominance of resonant behaviour. Second, we have obtained the results (26-28) in the limit where both the beauty and the charmed quark masses are taken to infinity. The accuracy of the predictions obtained in this limit cannot be assessed by general arguments, since we know that in some cases the \(1/m_Q\) corrections may be relevant (for instance in the calculation of the B leptonic decay constant \(f_B\) \[20\]) and in other cases they are small \[21\]. Moreover we cannot present a complete analysis of such corrections for the \(B \rightarrow D^{**}\) semileptonic transitions, because a study of these processes at all orders in \(1/m_Q\) has been performed only for the states \(D_0\) and \(D'_1\) \[22\]. We can nevertheless estimate the size of the possible errors by looking at one particular channel, i.e. \(B^- \rightarrow D^{*0}(0_{1/2}^+)\pi^-\). In this case we can compare the entry in Table I with the result obtained including \(1/m_Q\) corrections. The last one follows from the matrix elements: \(<D^{*0}|A_\mu|B^->\) and \(<0|V_\mu|D^{**0}>\) computed at all the orders in \(1/m_Q\) by QCD sum rules \[22\], together with the matrix element: \(<\pi|V_\mu|B>\) computed in ref. \[17\] by including \(1/m_Q\) corrections (in this case one should use in (19) \(F_1(0) = 0.89\) instead of 0.53 ). We obtain for \(B.R.(B^- \rightarrow D^{*0}(0_{1/2}^+)\pi^-)\) a result which differs by less
than 10% from the asymptotic \((m_Q \to \infty)\) value reported in Table I, which provides a perhaps optimistic estimate of the theoretical errors introduced by the heavy quark mass limit. If we observe that uncertainties in the phenomenological value of \(N_c\) (see eqs.(8,9)) would strongly affect only the \(B^-\) decays into the \(0_{1/2}^+\) state, which however does not decay into \(D^*\pi\) in the infinite heavy quark limit, we can conclude that our predictions (26)-(28) should provide reasonable estimates of the non leptonic \(B^-\) decays into charmed multipion final states.

Acknowledgments

We thank S.Stone for having suggested this calculation, and N.Paver for useful discussions.
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Table Captions

**Tab. I** Widths and branching ratios (B.R.) of $B^- \to D^{**0}(J_{su}^P)\pi^-$. The $B^-$ lifetime is $\tau_B = 1.2 \times 10^{-12}$ sec. The branching ratios are obtained for $a_1 = 1.1$, $a_2 = -0.24$ and $V_{cb} = 0.045$.

**Tab. II** Widths and branching ratios (B.R.) of $B^- \to D^{**0}(J_{su}^P)\rho^-$. The $B^-$ lifetime is $\tau_B = 1.2 \times 10^{-12}$ sec. The branching ratios are obtained for $a_1 = 1.1$, $a_2 = -0.24$ and $V_{cb} = 0.045$. 
### Table I

| $J^{P}_{s_{I}}$ | Width (GeV) | B.R.   |
|-----------------|-------------|--------|
| $2^{+}_{3/2}$   | $2.64 a_{1}^{2} 10^{-16} \left( \frac{V_{cb}}{0.045} \right)^{2}$ | $6 \times 10^{-4}$ |
| $1^{+}_{3/2}$   | $2.02 a_{1}^{2} 10^{-16} \left( \frac{V_{cb}}{0.045} \right)^{2}$ | $4 \times 10^{-4}$ |
| $1^{+}_{1/2}$   | $1.01 (a_{1}^{2} + 48.1 a_{2}^{2}) 10^{-16} \left( \frac{V_{cb}}{0.045} \right)^{2}$ | $6 \times 10^{-4}$ |
| $0^{+}_{1/2}$   | $0.87 (a_{1} - 14.7 a_{2})^{2} 10^{-16} \left( \frac{V_{cb}}{0.045} \right)^{2}$ | $3 \times 10^{-3}$ |

### Table II

| $J^{P}_{s_{I}}$ | Width (GeV) | B.R.   |
|-----------------|-------------|--------|
| $2^{+}_{3/2}$   | $5.08 a_{1}^{2} 10^{-16} \left( \frac{V_{cb}}{0.045} \right)^{2}$ | $1 \times 10^{-3}$ |
| $1^{+}_{3/2}$   | $4.8 a_{1}^{2} 10^{-16} \left( \frac{V_{cb}}{0.045} \right)^{2}$ | $1 \times 10^{-3}$ |
| $1^{+}_{1/2}$   | $2.5 (a_{1}^{2} + 17.9 a_{2}^{2}) 10^{-16} \left( \frac{V_{cb}}{0.045} \right)^{2}$ | $1 \times 10^{-3}$ |
| $0^{+}_{1/2}$   | $1.96 (a_{1} + 3.2 a_{2})^{2} 10^{-16} \left( \frac{V_{cb}}{0.045} \right)^{2}$ | $4 \times 10^{-5}$ |