THE POWER-LAW SPECTRA OF ENERGETIC PARTICLES DURING MULTI-ISLAND MAGNETIC RECONNECTION

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ABSTRACT

Power-law distributions are a near-universal feature of energetic particle spectra in the heliosphere. Anomalous cosmic rays (ACRs), super-Alfvénic ions in the solar wind, and the hardest energetic electron spectra in flares all have energy fluxes with power laws that depend on energy \( E \) approximately as \( E^{-1.5} \). We present a new model of particle acceleration in systems with a bath of merging magnetic islands that self-consistently describes the development of velocity-space anisotropy parallel and perpendicular to the local magnetic field and includes the self-consistent feedback of pressure anisotropy on the merging dynamics. By including pitch-angle scattering we obtain an equation for the omnidirectional particle distribution \( f(v, t) \) that is solved in closed form to reveal \( v^{-5} \) (corresponding to an energy flux varying as \( E^{-1.5} \)) as a near-universal solution as long as the characteristic acceleration time is short compared with the characteristic loss time. In such a state, the total energy in the energetic particles reaches parity with the remaining magnetic free energy. More generally, the resulting transport equation can serve as the basis for calculating the distribution of energetic particles resulting from reconnection in large-scale inhomogeneous systems.

Key words: acceleration of particles – magnetic reconnection – solar wind – Sun: corona – Sun: flares – Sun: heliosphere

Online-only material: color figure

1. INTRODUCTION

Accelerated particles with power-law spectra are a nearly universal feature of heliospheric plasmas and also characterize the cosmic-ray spectrum. Anomalous cosmic rays (ACRs; Stone et al. 2008; Decker et al. 2010), super-Alfvénic ions in the solar wind (Fisk & Gloeckler 2006), and the hardest energetic electron spectra in flares (Holman et al. 2003) all have energy fluxes with power laws that depend on energy \( E \) approximately as \( E^{-1.5} \). An important question is whether there is a common acceleration mechanism in these very disparate environments.

A range of acceleration mechanisms have been proposed to explain the spectra of energetic electrons (up to several MeV) and ions (up to several GeV) in impulsive flares, including the reconnection process itself and reconnection-driven turbulence (Miller et al. 1997; Dmitruk et al. 2004; Liu et al. 2006; Zharkova et al. 2011). The single x-line model of reconnection in flares, in which electrons are accelerated by parallel electric fields, cannot explain the large number of accelerated electrons (Miller et al. 1997). On the other hand, both observations (Sheeley et al. 2004; Savage et al. 2012) and modeling (Kliem 1994; Shibata & Tanuma 2001; Drake et al. 2006a, 2006b; Onofri et al. 2006; Oka et al. 2010; Huang et al. 2011; Daughton et al. 2011; Fermo et al. 2012) suggest that reconnection in flares involves the dynamics of large numbers of x-lines and magnetic islands or flux ropes.

The seed population of ACRs is interstellar pickup particles (Cummings & Stone 1996, 2007). However, the conventional idea that they are accelerated at the termination shock (TS; Pessas et al. 1981) was called into question when the Voyagers crossed the TS and found that the intensity of the ACR spectrum did not peak there (Stone et al. 2005, 2008). A possible alternate source is magnetic reconnection of the sector heliosheath (Lazarian & Öpher 2009; Drake et al. 2010). Simulations of reconnection in the sectored field region revealed that the dominant heating mechanism was Fermi reflection in contracting and merging islands (Drake et al. 2010; Kowal et al. 2011; Schoeffler et al. 2011). Because contraction increases the energy parallel to the local magnetic field and reduces the perpendicular energy, the heating mechanism drives the system to the firehose stability boundary \( \alpha = 1 - (\beta_1 - \beta_L)/2 = 0 \) where reconnection is throttled because the magnetic tension drive is absent (Drake et al. 2006b, 2010; Öpher et al. 2011; Schoeffler et al. 2011).

A rigorous model for particle acceleration in a multi-island, reconnecting system has not yet been developed. The Parker equation does not describe particle acceleration in nearly incompressible systems (Parker 1965) and extensions do not account for the geometry of reconnection and island merging (Earl et al. 1988). Here we explore particle acceleration in a bath of merging magnetic islands with a particle distribution function \( f(v_1, v_\perp) \) that accounts for the velocity space anisotropy along \( (v_1) \) and across \( (v_\perp) \) the local magnetic field. Thus, the pressure anisotropy can be directly evaluated and the feedback on island merging calculated.

2. PARTICLE DYNAMICS DURING ISLAND MERGER

We develop a probabilistic model of particle acceleration in a bath of merging two-dimensional (2D) magnetic islands with a distribution of magnetic flux \( \psi \) and area \( A \) given by \( g(\psi, A) \) (Fermo et al. 2010). The development of structure in 3D may ultimately be important and should be addressed, but observations (Phan et al. 2006) and simulations (Hesse et al. 2001) suggest that at the largest scales reconnection is nearly 2D. We first calculate the particle energy gain during the merging of two circular islands of radii \( r_1 \) and \( r_2 \) with \( r_j = \sqrt{A_j/\pi} \) as shown in Figure 1. Merging leads to a single island of area \( A_f = A_1 + A_2 \) and with magnetic flux \( \psi_f \) given by the larger
of $\psi_1$ and $\psi_2$ (Fermo et al. 2010). The reduction of energy by the factor $(\psi_1^2 + \psi_2^2)/\psi_2^2$ results from the shortening of the field lines as merging proceeds. Thus, energy release takes place not at the merging site, but as reconnected field lines contract after merger. As long as the kinetic-scale, boundary layer where reconnection occurs is small compared with the island radii, the dominant energy exchange with particles takes place on the closed, reconnected field lines that release magnetic energy as they contract.

We take advantage of two adiabatic invariants, the magnetic moment $\mu = mv^2/B$ and the parallel action $\int v_1d\ell$, which are constants if the gyration time of particles around the local magnetic field and their circulation time around islands are short compared with the merging time. The former describes the reduction in $v_1$ as $B$ decreases and the latter the increase in $v_1$ as $\ell$ decreases. The parallel action invariant is valid for velocities that exceed the local Alfvén speed, which implies that a seed heating mechanism is needed for low $\beta$ systems such as the solar corona. Fortunately, ions gain sufficient energy as they cross from upstream into reconnection exhausts (Drake et al. 2009; Knizhnik et al. 2011). To calculate $\ell$, we first calculate the merging velocity $\dot{r}_{\text{sep}}$ of two islands with differing radii and magnetic fields, $\dot{r}_{\text{sep}} = r_1 + r_2 = -\psi (B_1 + B_2)/(B_1 B_2)$, since merging magnetic islands reconnect their magnetic flux at the same rate. The reconnection rate is given by Cassak & Shay (2007), $\dot{\psi} = 2V_{12}B_1 B_2/(B_1 + B_2)$, with $V_{12} = \epsilon_r \sqrt{\alpha_{12} B_1 B_2/4\pi \rho}$, where $\epsilon_r \sim 0.1$ is the normalized rate of reconnection (Shay et al. 2007) and $\alpha_{12} = 1 - 4\pi (p_1 - p_\perp)/(B_1 B_2)$ is the firehose stability parameter. Thus, $\dot{r}_{\text{sep}} = -2V_{12}$ and $V_{12}$ is the island merging velocity. The rate of line shortening can now be calculated from the total merging time $(r_1 + r_2)/(2V_{12})$ and the difference between the initial field line length as merging starts and the final length using area conservation, $\ell = -2\pi h_{12} V_{12}$ with $h_{12} = 2(r_1 + r_2 - \sqrt{r_1^2 + r_2^2})/(r_1 + r_2)$. Parallel action conservation then yields an equation for $v_1$,

$$\dot{v}_1 = \frac{dv_1}{dt} = \frac{h_{12}V_{12}}{r_1 + r_2}.$$  \hfill (1)

To obtain the corresponding equation for $v_\perp$, we use the conservation of magnetic flux and area as a flux tube contracts so that $B/\ell$ is constant. Therefore, from $\mu$ conservation $v_\perp^2/\ell$ is also constant and

$$\dot{v}_\perp^2 = \frac{dv_\perp^2}{dt} = -v_\perp^2 \frac{h_{12}V_{12}}{r_1 + r_2}.$$  \hfill (2)

Thus, the perpendicular energy goes down during island merger as the parallel energy increases.

### 3. A KINETIC EQUATION FOR PARTICLE ACCELERATION DURING ISLAND MERGER

From the energy gain of particles in merging islands we can formulate a model of particle acceleration in a macroscopic current layer of length $L$ (Longcope & Cowley 1996). Particles are injected into the bath of interacting islands in the current layer from upstream as each individual island grows due to reconnection of the upstream field. They then undergo acceleration in the merging islands until they are convectively lost. The rate of injection of particles is given by the upstream particle distribution function $f_{\text{up}}(\psi)$ times the integrated rate of area increase of all of the magnetic islands $A_T$ (Fermo et al. 2010),

$$\dot{A}_T = 2\pi \epsilon_r c_{\text{Aup}} \int_0^\infty \int_0^\infty dA d\psi r g(\psi, A),$$  \hfill (3)

with the island radius given by $r = \sqrt{A/\pi}$. In the bath of islands, the average of $v_1$ and $v_\perp^2$ is calculated from the merging probability of two islands of radii $r_1$ and $r_2$, given by their overlap probability $4\pi r_1 r_2/L^2$, and then averaged over the distributions $g_1 = g(\psi_1, A_1)$ and $g_2 = g(\psi_2, A_2)$. We find $\langle v_1 \rangle = R v_1$ and $\langle v_\perp^2 \rangle = -R v_\perp^2$ with

$$R = \int \int dA d\psi g_1 g_2 \frac{4\pi r_1 r_2 h_{12} V_{12}}{L^2 (r_1 + r_2)},$$  \hfill (4)
with $di = dA_i d\psi_i$. The phase-space volume element for $f(v_i, v_\perp)$ is $\pi dv_i dv_\perp^2$ so $(\nabla v_i)$ and $(\nabla v_\perp^2)$ describe the convection of $f$ in the phase space of $(v_i, v_\perp^2)$. Setting the change in the number of particles in a closed volume equal to the flux through the surface and using the divergence theorem yields an evolution equation for $f(v_i, v_\perp, t)$,

$$\frac{\partial}{\partial t} f + \frac{\partial}{\partial v_i} (f v_i) + \frac{\partial}{\partial v_\perp} (v_\perp^2 f) = 0$$

$$= \frac{\partial}{\partial t} f + R \left( \frac{\partial}{\partial v_i} v_i - \frac{\partial}{\partial v_\perp} v_\perp^2 \right) f
= v \frac{\partial}{\partial \xi} (1 - \xi^2) \frac{\partial}{\partial \xi} f - \frac{c_{\text{Aug}}}{L} f + \dot{A}_T f_{\text{up}}, \quad \text{(5)}$$

where we have included the particle source from upstream of the layer and convective loss with $c_{\text{Aug}}$ the Alfvén speed based on the upstream magnetic field. In earlier simulations of multi-island reconnection, strong pressure anisotropy with $p_\parallel > p_\perp$ within the core of merging islands was limited by the firehose (Drake et al. 2010) and Weibel (Schoeffler et al. 2011) instabilities so we have included a phenomenological pitch-angle scattering operator of strength $v$ that acts on the angle $\xi = v_i/v$ to reduce anisotropy. Importantly, drive $R$ is independent of the particle velocity. It depends on the pressure anisotropy through the merging velocity $V_{12} \propto \sqrt{\delta_{12}}$ so that $V_{12} \rightarrow 0$ as the firehose condition is approached.

If $f$ were isotropic and therefore only a function of $v_i$, the energy drive operator in Equation (5) would vanish when averaged over the angle $\xi$. In this limit there is zero net energy gain, consistent with Parker’s equation in the incompressible limit (Parker 1965). Equation (5) is an equidimensional equation and therefore has no characteristic velocity scale. Solutions therefore take the form of power laws. An important property of such an equation is that the fluid moments of a given order completely decouple from those of a differencing order and their solutions can therefore be readily obtained from Equation (5) in closed form. Specifically, an equation for $p_\parallel$ and $p_\perp$ can be obtained so that $a_{12}$ in the energy drive $R$ can be evaluated explicitly. Thus, the feedback of energetic particles on the dynamics of reconnection can be computed. In the case of no source, sink or scattering, for example, Equation (5) yields $\partial p_\parallel/\partial t = 2R p_\parallel$ and $\partial p_\perp/\partial t = -R p_\perp$ so that $p_\parallel$ and $p_\perp$ increase and decrease in time, respectively, but the total energetic particle pressure $p = (p_\parallel + 2p_\perp)/3$ increases, $\delta p/\delta t = (2R/3)(p_\parallel - p_\perp)$.

Instead of directly evaluating the full moments of Equation (5), we simplify the equation by ordering the magnitudes of the rates $R$, $v_i$, and $c_{\text{Aug}}/L$. To estimate the scaling of $R$, we note that $N_f = \int dA_{\text{el}}$ is the total number of islands in the layer, so for densely packed islands we can define a characteristic island radius $r_N = L/2N_f$. Thus, $R \sim \epsilon_i^{1/2} \epsilon_{\text{CA}}^{1/2} N_f^2 r_N^2/L^2 \sim \epsilon_i^{1/2} \epsilon_{\text{CA}}^{1/2} r_N$. Since the scattering represented by $v$ arises from the pressure anisotropy driven by convection, we argue that $v$ scales like the contraction rate $\epsilon_i \epsilon_{\text{CA}}/r_N$. Thus, since $r_N \ll L$, the relative order is $v \sim R \gg c_{\text{Aug}}/L$. However, while this ordering is reasonable for calculating how particles gain energy, we will show that for $R \gg c_{\text{Aug}}/L$ there is no steady-state solution in which the pressure is bounded unless $v \gg R$. The resolution of this dilemma is that the system must bump against the firehose stability boundary so $R \propto \sqrt{\alpha}$ is reduced while $v$ increases due to increased scattering. We therefore take $v \gg R \gg c_{\text{Aug}}/L$. The large $v$

assumption allows us to solve Equation (5) by expanding $f$ in a series of Legendre polynomials $f = \sum_i P_i(\xi) f_i(\nu)$, where $P_i$ is the $i$th-order Legendre polynomial. By the symmetry in $v_i$, $f_1$ is zero. The equation for $f_2$ follows from balancing the reconnection drive acting on $f_0$ with the scattering operator acting on $f_2 P_2(\xi)$, $f_2(\nu) = -(Rv/6v_0)\partial f_0(\nu)/\partial \nu$. By averaging Equation (5) over $\xi$ we obtain an equation for $f_0(\nu)$. The scattering term vanishes and the energy drive term acting on $f_2 P_2(\xi)$ is evaluated by converting $(v_i, v_\perp)$ to $(\nu, \xi)$ before averaging over $\xi$,

$$\frac{\partial f_0}{\partial t} - \frac{R^2}{30v} \frac{\partial}{\partial \nu} \frac{v_\perp^4}{v^4} \frac{\partial}{\partial \nu} \frac{c_{\text{Aug}}}{L} f_0 + \dot{A}_T f_{\text{up}} = 0. \quad \text{(6)}$$

This equation is again of equidimensional form and has power-law solutions whose individual moments can be calculated. Evaluating the density in steady state, for example, by integrating over velocity, the drive term vanishes and the total number of particles undergoing acceleration $n_T$ is given by $n_T = \dot{A}_T r_{\text{up}}$, where $A_T = A_T L/c_{\text{Aug}}$ is the integrated area of all of the islands in the layer. The firehose parameter needs to be self-consistently evaluated and for this we need

$$p_\parallel - p_\perp = \frac{1}{A_T} \int_{\delta -1}^{\delta +1} d\xi \int_0^\infty dv \pi v^2 m \left( v_\perp^2 - \frac{1}{2} v_\parallel^2 \right) f_2(\nu) P_2(\xi), \quad \text{(7)}$$

Using the expression for $f_2$ and noting that $v_\perp^2 - v_\parallel^2/2 = v^2 P_2(\xi)$, we obtain

$$p_\parallel - p_\perp = \frac{R}{2v_0} p_0, \quad \text{(8)}$$

where $p_0$ is the isotropic pressure calculated by taking the pressure moment of Equation (6) for $f_0$,

$$p_0 = \frac{1}{3} \int_0^\infty dv 4\pi m v^4 f_0(v) = \frac{p_{\text{up}}}{1 - R^2L/3c_{\text{Aug}}v_0^2}. \quad \text{(9)}$$

The firehose parameter becomes

$$\alpha \approx 1 - \frac{4\pi p_{\text{up}}}{B^2} \frac{R/2v_0}{1 - R^2L/3c_{\text{Aug}}v_0^2}. \quad \text{(10)}$$

where $\bar{B}$ is the average island magnetic field strength based on the sum in Equation (4). A key feature of Equation (10) is its singular behavior when $\delta = R^2L/3c_{\text{Aug}}v_0^2 = 1$. This singularity can be understood from the power-law solutions for $f_0$, which describe its behavior at energies greater than that of the source $f_0$. Taking $f_0 \propto v^{-\gamma}$, from Equation (6) we obtain $\gamma(\gamma - 3) = 10/\delta$ so that when $\delta = 1$, $\gamma = 5$. The second solution, $\gamma = -2$, corresponds to divergent behavior and must be rejected. The singularity in Equation (10) therefore arises when $f_0 \propto v^{-5}$ and corresponds to a divergence of the pressure integral. Thus, it is clear that the requirement that the pressure be bounded requires that $\gamma > 5$ or $\delta < 1$. Based on our ordering for $v$ we find $\delta \sim \epsilon_i L/r_N$. We have assumed that the islands are much smaller than the system size so that $\epsilon_i L/r_N \gg 1$. We can therefore only satisfy $\delta < 1$ if $\alpha \ll 1$ so that the islands bump against the firehose stability boundary. Unless $p_{\text{up}}$ is very large, the only way that the firehose condition in Equation (10) can be reached is if $\delta \approx 1$ or $v \approx 2$ and $f_0 \propto v^{-5}$.

The total energy content $W_0 = 3p_0/2$ of this high-energy tail can be directly calculated from the pressure in Equation (9) using $\alpha \approx 0$ and $\delta \approx 1$,

$$W_0 = \frac{\bar{B}^2}{4\pi} \sqrt{3vL/c_{\text{Aug}}}. \quad \text{(11)}$$
We demonstrated that the particle acceleration in a bath of merging magnetic islands. This is the correct limit when the characteristic magnetic island radius is much smaller than the system scale size $L$. We argue therefore that the widely observed $E^{-1.5}$ spectrum in the heliosphere is a natural consequence of multi-island reconnection. The total energy content of this $E^{-1.5}$ spectrum reaches parity with the remaining magnetic field energy in the system.

Equation (5) can be readily generalized to a 2D system by replacing the factors $2r_{iL}$ by $4r_{iL}^2$ in the drive term $\mathcal{R}$. The estimate for the scaling of $\mathcal{R}$ is unchanged. The model loss term $c_{A\parallel} f / L$ should also be replaced by the convective loss rate $\mathbf{u} \cdot \nabla f$ with $\mathbf{u}$ the convective velocity of the system. The arguments leading to the $f \propto v^{-5}$ also apply to the 2D equations. In a system in which the driver $R$ is spatially non-uniform, the 2D version of Equation (5) could then be numerically solved for the spatial distribution of energetic particles from reconnection. The impact of the finite structure of magnetic islands that might develop in the third direction remains an important open issue (Onofri et al. 2006; Schreier et al. 2010; Daughton et al. 2011).

There have now been several published simulations of particle acceleration and associated spectra in 2D multi-current layer systems (Drake et al. 2010; Drake & Swisdak 2012). We can compare the spectra predicted from our equation with the results of those simulations. Since the simulations were doubly periodic, there was no convective loss. Further, the pressure anisotropy was strong so we consider the non-scattering limit of Equation (5) in which the source and loss terms are discarded. The exact solution for $f$ is given by

$$f(v_{\parallel}^2,v_{\perp}^2,t) = f(v_{\parallel}^2 e^{-2G(t)}, v_{\perp}^2 e^{G(t)}, 0),$$

where $G(t) = \int_0^t d\tau R(\tau)$. This is consistent with exponential growth of the effective parallel temperature and an exponential decrease in the perpendicular temperature. The omnidirectional distribution function can be computed numerically for any specified initial distribution function for comparison with simulation data. The comparison is made with a system with 16 initial current layers in a $409.6d_i \times 204.8d_i$ domain, where $d_i = c/\omega_{pi}$ is the ion inertial length (Drake et al. 2010). In Figure 2, we show the magnetic field strength at late time ($t = 100\Omega_{ci}^{-1}$) in the simulation after islands on adjacent current layers have overlapped. The typical island radius $r_{ci}$ at this time is around $15d_i$. The characteristic acceleration rate $R \sim c_{A\parallel}/r_{Xe} \sim 0.007\Omega_{ci}^{-1}$, where $\Omega_{ci}$ is the ion cyclotron frequency. Reconnection remains strong for a total time of around $100\Omega_{ci}^{-1}$ when the pressure anisotropy shuts off reconnection. Thus, the integrated acceleration rate is $G \sim 0.7$. The comparison between the model and the simulation data is shown in Figure 3. The particle energy spectrum from the simulation is shown in the initial state and at $t = 200\Omega_{ci}^{-1}$ in the solid lines. Note that the initial state is not a simple Maxwellian because of the shift in the ion velocity distribution that is required in the current layers. The fit of the initial spectrum with a single Maxwellian, shown in the dot-dashed line in Figure 3, therefore matches the low-energy portion of the spectrum very well but underestimates the number of particles at high energy. The late-time energy spectrum from the solution given in Equation (12) after integration over the angle $\zeta$ is given by the dashed line in Figure 3. The best fit corresponds to $G = 0.82$ rather than the estimate of 0.7. The model reproduces the overall late-time energy spectrum very well but modestly overestimates the number of particles in the high-energy tail.
Observations in the quiet solar wind have revealed that the super-Alfvénic ions display an $f(v) \propto v^{-5}$ distribution (Fisk & Gloeckler 2006). It has been suggested that solar wind turbulence would be dissipated in reconnection current layers (Servidio et al. 2009) and therefore that reconnection is an important dissipation mechanism in the turbulent solar wind. Solar wind observations also reveal that the pressure anisotropy bumps against the firehose threshold in some regions and that there are enhanced magnetic fluctuations at these locations (Bale et al. 2009). There are therefore mechanisms in solar wind turbulence driving anisotropy and the anisotropy is limited by enhanced scattering. Finally, the direct observations of reconnection events in the solar wind reveal heating but no localized regions of energetic particles (Gosling et al. 2005).

This is consistent with our picture that the energetic particle spectrum is not produced at a single x-line but requires that the ions interact with many reconnection sites.

The spectrum of energetic electrons in impulsive flares is not measured in situ and must be inferred from chromospheric X-ray emission. Nevertheless, the energetic particle fluxes do occasionally reveal spectra as hard as $E^{-1.5}$, which corresponds to $f \propto v^{-5}$ (Holman et al. 2003). In recent over-the-limb observations of flares in which the reconnection region high in the corona can be directly diagnosed, it was found that all of the electrons in the acceleration region became part of the energetic component, indicating that all electrons in the region of energy release underwent acceleration (Krucker et al. 2010) and that the $\beta$ of these electrons was of the order of unity, consistent with our model. Bursts of emission occur with durations of around a minute. Estimating the characteristic acceleration time from our model is complicated by the uncertainty in the typical island size $r_N$. From the estimated local parameters ($B \sim 50$ G, $n \sim 10^9$ cm$^{-3}$, $L \sim 10^5$ km), we take a range of $r_N \sim 10^2$-$10^3$ km to obtain $R \sim 0.4$-$4$ s$^{-1}$ and a time for accelerating electrons from 100 eV to 1 MeV of 1-$10$ s, which is within measured burst durations.

Whether the sectored heliosheath magnetic field has reconnected remains an open issue because the Voyager magnetometers are at the limits of their resolutions at the magnetic field strengths in the heliosheath (Burlaga et al. 2006). Large drops in the energetic electron and ACR population as Voyager 2 exited from the sectored zone are consistent with reconnection as the ACR driver (Opher et al. 2011). The spectral index of the ACR particle flux measured at Voyager 1 is slightly above 1.5 (Stone et al. 2008; Decker et al. 2010). Further, the integrated energy density of the measured ACR spectrum between 1 and 100 MeV is comparable to that of the magnetic field, which has a magnitude of around 0.1 nT, consistent with the predictions of our model.

The equations presented here were derived in the non-relativistic limit. However, the ideas can be easily extended to the case where the particles are relativistic but where reconnection itself is non-relativistic. We express the distribution of particles in terms of the particle momentum $p$. For the pressure integral to remain bounded, $\gamma > 4$ for power-law distributions with $f_0(p) \propto p^{-\gamma}$. The resulting particle flux per unit energy interval $\Gamma$ is given by $\Gamma \propto p^2 f(p) \propto p^{2-\gamma}$. Thus, the spectrum of the flux in the strongly relativistic limit should scale as $p^{-2}$.

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