Renormalization of Higher Derivative Operators in the Matrix Model

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Abstract

$M$-theory is believed to be described in various dimensions by large $N$ field theories. It has been further conjectured that at finite $N$, these theories describe the discrete light cone quantization (DLCQ) of $M$ theory. Even at low energies, this is not necessarily the same thing as the DLCQ of supergravity. It is believed that this is only the case for quantities which are protected by non-renormalization theorems. In $0 + 1$ and $1 + 1$ dimensions, we provide further evidence of a non-renormalization theorem for the $v^4$ terms, but also give evidence that there are not such theorems at order $v^8$ and higher. These results are compatible with known facts about the matrix model.
1 Introduction

The matrix model conjecture has two parts. In its original form, it was suggested that in the large $N$ limit, the matrix model should describe $M$ theory in the infinite momentum frame\cite{1,2}. Susskind proposed that for finite $N$, the matrix model describes the discrete light cone quantization of $M$ theory\cite{7}. Naively, one might take this to mean that for large distance processes, the predictions of the matrix model should agree with the DLCQ of 11-dimensional supergravity. Most of the initial matrix model tests seemed to support this hypothesis. For example, graviton-graviton scattering agrees at finite $N$. We will refer to this expectation as the “naive DLCQ.”

Seiberg and Sen have subsequently given a derivation of the finite $N$ version of the matrix model\cite{8,9}. Their argument begins with the observation that the DLCQ, in general, can be thought of as a large boost of a theory compactified on a small, space-like circle. However, their argument does not necessarily establish that amplitudes in the matrix model must agree with the naive DLCQ. As pointed out by Polchinski and Hellerman, this derivation implies that there is a small space-like distance in the problem, and so the DLCQ does not describe some simple, low energy limit of string theory\cite{10}. Thorn\cite{11} has stressed that even ignoring the zero modes, in field theory the DLCQ only “works” for generic momenta; for special kinematics (e.g. zero longitudinal momentum exchange, the kinematical situation most easily studied in the matrix model) there are subtle issues. Indeed, the matrix model does not reproduce the naive DLCQ for for scattering in non-trivial backgrounds\cite{12}.

The situation has been well-summarized in \cite{3}. It raises the question: why does the finite $N$ model agree with the naive DLCQ sometimes? Consider the case of graviton-graviton scattering. In the matrix model, the relevant terms are the $v^4$ terms in the effective action, i.e. the terms with four derivatives. It is believed that there is a non-renormalization theorem for these, so the lowest order term must reproduce the exact answer, explaining the agreement. The evidence for this theorem is limited. In four dimensions, it has been proven at infinite volume\cite{13}. In 0 + 1 dimensions, it has been shown by explicit computation that there is no renormalization of the $v^4$ terms at two loops in the case of $SU(2)$\cite{14}.

If $n$ graviton scattering is to work similarly, there should be a non-renormalization theorem for the $v^{2n}$ terms in the action. In \cite{15}, certain terms at order $v^6$, which are subleading in $N$, were shown to agree with the DLCQ of Einstein’s gravity. This suggests that some quantities at order $v^6$ are not renormalized. On the other hand, it would be suprising if there were an
infinite series of such non-renormalization theorems. The work of \[12\] suggests that there might be a problem at the level of four graviton scattering\[16\]. Investigations of this question will appear elsewhere.

In these notes, we will first give some further evidence supporting the non-renormalization theorem for $v^4$ in low dimensions, and direct evidence that there are not general non-renormalization theorems for $v^8$ and higher powers of the velocity (we will not be able to make a definite statement about $v^6$). In doing so, the first question we have to ask is: “non-renormalization of what?” In four dimensions we are used to the idea that non-renormalization theorems are statements about a Wilsonian effective action. For example, the non-renormalization theorem discussed in \[13\] is derived by considering the $N = 4$ theory on its Coulomb branch, and studying the effective action obtained by integrating out massive and high frequency modes. In $0 + 1$ and $1 + 1$ dimensions (or in finite volume), however, there is not a notion of a moduli space in the same sense. Instead, one must adopt a Born-Oppeheimer treatment of the problem, thinking of holding the slow modes fixed and solving for the dynamics of the fast modes.

The approach of most authors has been to compute the one particle irreducible effective action, using conventional field theory rules. Consider the case of $SU(2)$. At a given order in $v$, the loop expansion is formally an expansion in powers of $1/r^3$ ($1/r^2$), in $0 + 1$ ($1 + 1$) dimensions\[15\]. Here, $r$ is the expectation value of the adjoint fields (transverse separation of the gravitons, in the matrix model interpretation). The spectrum includes states with mass (frequency) of order $r$ and massless states. In the two-loop computation of \[14\], individual diagrams contributing to the effective action containing massless states are infrared divergent. The authors of this reference dealt with this by using dimensional regularization, defining

\[
\int \frac{d^d p}{p^2} = 0.
\]

(1)

With this regulator, these authors find that there is no renormalization of the $v^4$ term. The result involves not only fermi-bose cancellations, but also cancellations between diagrams containing only massive states and diagrams containing massless states.

This result is encouraging, but since infrared divergences usually signal real physics, one might worry about the regularization procedure. However, there are many infrared divergent diagrams, and, as we will see in section 2, in the case of $v^4$, the infrared divergences cancel and there is no sensitivity to the regularization procedure. We will also see that this cancellation is quite special to $v^4$, and there is no reason to expect it to occur for higher orders in velocity.

While it is true that we do not have a good definition of a Wilsonian effective action, for
the success of the naive DLCQ, what really interests us is the scattering amplitude. For the success of the naive DLCQ, at $\mathcal{O}(v^4)$, we actually require that there should be no corrections to this amplitude. This is, as we will explain in the next section, equivalent to the requirement that there should be no corrections to the 1PI effective action. We will see that, for two particle scattering, at order $v^8$ and higher, there are infrared divergences in the perturbation theory in powers of $1/r^3$\footnote{In the eikonal approximation appropriate to this problem, terms in the effective action of order $v^n$ translate into terms in the scattering amplitude of order $v^{n-1}$. We will count powers of $v$ and $r$ in this paper as appropriate for the effective action.} As a result, we believe that the expansion in powers of velocity breaks down.

![Figure 1: Infrared divergent contributions to the effective action.](image)

The origin of the infrared problem is easily understood. In $\ell$ loop order, $\ell \geq 2$, consider the diagram shown in fig. 1. Here the central loop contains a massive field, and the $\ell - 1$ smaller loops contain massless fields. In momentum space, this graph is proportional to

$$\frac{1}{r^{6+2(\ell-1)}(\int \frac{d^d p}{p^2})^{\ell-1}}.$$  \hspace{1cm} (2)

Alternatively, if the amplitudes are written in coordinate space, the propagator is ambiguous; individual diagrams are proportional to this ambiguity. In the infrared limit, one can think of the integral over the massive states as generating a local operator, and the massless integrals as giving the “vacuum matrix element” of this operator. This same type of analysis can be performed for all of the infrared divergent graphs. For the $v^4$ terms, we will see in the next section that this matrix element vanishes. However, this cancellation depends crucially on the fact that $1/r^7$ is the Green’s function for the nine-dimensional Laplace operator, and does not hold for higher powers of $v$. $v^6$ turns out to be special as well, because the one loop contribution
vanishes\textsuperscript{[15]}, and we cannot make a definite statement. However, starting at order $v^8$, it is easy to see that there are divergences, they do not cancel, and they become more severe in each order of perturbation theory.

These divergences presumably signify that there is not an expansion of the effective action (in the matrix model) in powers of $v$ and $1/r^3$. The problem seems analogous to difficulties encountered in finite temperature field theories, where the divergence is cut off by some dynamical mass scale (e.g. the Debye mass or the magnetic mass in the case of finite temperature QCD). In the present case, there is an effective mass proportional to $v^4/r^9$ ($v^4/r^8$) in $0 + 1$ ($1 + 1$) dimensions, which can cut off the integrals. If this is the relevant scale, the $0 + 1$ expansion breaks down at order $v^6$; in $1 + 1$, the problem occurs at the level of $v^8$.

In the case of $SU(3)$ (and higher rank groups), one can make the problem somewhat sharper by exhibiting certain finite renormalizations. In this case there are two (or more) scales, $R$ and $r$. As in \textsuperscript{[17]}, one can consider a hierarchy of scales (impact parameters, in the matrix model interpretation), $R \gg r$. Again, the diagrams contributing to the effective action contain infrared divergent terms. But there are also finite terms which behave as $(1/R^2r)^{\ell-1}$. It is easy to isolate these terms. Diagrams such as those of fig. 1, where now the small loops contain fields of mass $r$ and the big loop masses of order $R$, are of the form

$$\frac{v^4}{R^{6+2(\ell)-d}} \int \frac{d^d p}{p^2 + r^2} (\sim) \frac{v^4}{R^{6+2\ell-d} r^{\ell-1-d}}$$

(in $1 + 1$, the $r$ dependence is logarithmic). In the third section, we will see that there is a cancellation of the most singular term at order $v^4$ for $\ell = 2$. Based on the results for $SU(2)$, it seems quite plausible that this cancellation persists to all orders. From the perspective of the matrix model, this is reassuring, since there would be no sensible spacetime interpretation for such terms. As for $SU(2)$, it is not easy to decide what happens at order $v^6$, but at order $v^8$ and beyond, it is a simple matter to show that there are renormalizations.

However, to determine the full implications of these results requires settling some subtle issues. In particular, for these low dimension theories, the significance of the effective action is not completely clear, obscured, as we have noted, by infrared and (related\textsuperscript{[8]} ) operator ordering questions. We will offer some remarks on these issues, but will not completely resolve them. Whether there are discrepancies between the matrix model and four graviton scattering, as suggested by these remarks about $v^8$, will be discussed elsewhere.

\textsuperscript{2}We thank Nathan Seiberg for stressing this connection to us.
Consider, first, the matrix model with $N = 2$ in $d = 0 + 1$ and $1 + 1$. We will write the bosonic part in terms of a set of "fields," $x^i, i = 1, \ldots, 9$, and a "gauge boson," $A$. All of these fields are $SU(2)$ matrices. There are flat directions with $\vec{x}$ a diagonal matrix,

$$\vec{x} = \vec{r}_3/2.$$  

(4)

Correspondingly, there are a set of massive modes (i.e. modes with frequencies proportional to $r$) and massless modes. At one loop, integrating out the massive modes in this model is well-known to generate an effective action, whose leading bosonic term behaves as

$$L_1 = \frac{15}{16}r^4/r^7.$$  

(5)

When considering the scattering amplitude, in a path integral approach, one considers

$$\langle \vec{x}_f(t_f) | \vec{x}_i(t_i) \rangle$$  

(6)

where $\vec{x}_f$ and $\vec{x}_i$ are the eigenvalues of $\vec{x}_3$, the diagonal component of the matrix. Expanding $\vec{x}$ about the classical solution

$$\vec{x}(t) = \vec{x}_{cl} + \delta \vec{x}$$  

(7)

$$\vec{x}_i + \frac{\vec{x}_f - \vec{x}_i}{t_f - t_i} (t - t_i) + \delta \vec{x}(t)$$

$$= \vec{b} + \vec{v}t + \delta \vec{x}$$

one studies the region of large $|\vec{b}|$, small $|\vec{v}|$. In this regime, the amplitude can be expanded in powers of $\vec{v}$\[18, 20\]. At higher orders, as we have noted, there is a serious potential for infrared divergences. In $1+1$ dimension, the problem is familiar from string theory. Written in a fourier decomposition, the two dimensional massless propagator is:

$$\langle x(\sigma)x(\sigma') \rangle = \int d^2k \frac{e^{ik(\sigma-\sigma')}}{k^2},$$  

(8)

which is ill defined. Correspondingly, the coordinate space expression is

$$\langle x(\sigma)x(\sigma') \rangle = \ln(\sigma - \sigma')^2 + \text{constant}.$$  

(9)

In string theory, one only considers Green’s functions of translationally invariant combinations of operators, and these are infrared finite; equivalently, they are independent of the arbitrary constant.
In 0 + 1 dimensions, the divergences are even more severe. If we try to write a momentum (frequency) space propagator we have

\[ \langle \delta x_i(t) \delta x_j(t') \rangle = \delta_{ij} \int d\omega \frac{e^{-i\omega(t-t')}}{\omega^2} \]  

which is linearly divergent. Correspondingly, the coordinate space Green’s function is ambiguous (dropping the vector symbol):

\[ \langle \delta x(t) \delta x(0) \rangle = a t \theta(t) - b t \theta(-t) + ct + d, \]  

with \( a + b = 1 \).

When we say below that infrared divergences do (or do not) cancel in 1+1 or 0+1 dimension, we will mean that they cancel at the level of momentum space expansions, or alternatively that the quantities in question are not sensitive to the ambiguities in the propagators.

Figure 2: Some two loop corrections to the effective action.

Now consider two loop corrections to the effective action. Some sample diagrams are shown in fig. 2. Consider, in particular, diagrams with one massive state and one massless state running in the loop. Individual diagrams with massless states in the loop are infrared divergent, behaving as

\[ \int \frac{d\omega}{\omega^2} \]  

for small frequencies. Note that the external \( x \)'s must always attach to massive lines. Because of this fact, and because the leading infrared divergence always comes from such a small frequency region of integration, the leading divergent piece of each diagram always factorizes into a product of two one loop terms. One is a massive loop, with four external “scalars” \( (x \)'s), on which the time derivatives act, and two more without derivatives. The two without derivatives are then contracted with each other, forming the massless loop. In other words, the infrared divergent terms can all be organized in terms of operators generated at one loop of the form

\[ \mathcal{O} = v^4 \delta x^2 / v^9. \]  

7
The infrared divergence then arises from simply contracting the two factors of $x$ in this expression, i.e. taking the “vacuum matrix element.”

However, we do not need to compute all of the diagrams to determine the coefficient of this term in the effective action! In eqn. 5, we can interpret $r^2$ as $(\vec{x}_{cl} + \delta \vec{x})^2$, and expand in powers of $\delta \vec{x}$. This gives

$$O_1 = \frac{v^4}{x_{cl}^7} \left( 1 - \frac{7}{2x_{cl}^2} (2\vec{x}_{cl} \cdot \delta \vec{x} + \delta x^2) + \frac{7}{8x_{cl}^4} (2\vec{x}_{cl} \cdot \delta \vec{x} + \delta x^2)^2 \right).$$

(14)

Taking the expectation value, the last two (infrared divergent) terms in this expression cancel because there are nine $x$’s. A similar cancellation occurs in $1+1$ dimension.

It should be noted that there are no potential infrared divergences from other diagrams. Diagrams involving gauge fields (which exist in gauges other than $A^0 = 0$) are not divergent. The one loop effective action must be gauge invariant, and this means that it must be independent of the gauge field in $0+1$ dimensions, and involve at least two time derivatives in $1+1$ dimension. Diagrams involving fermions are not as divergent due to the structure of the fermion propagator.

It is easy to extend this argument for the cancellation of the most infrared singular terms to higher orders. At each order, the most singular contribution comes from diagrams where several massless scalars attach to a single loop of massive fields. These diagrams correspond to expanding the $1/r^7$ term to higher orders in $x$, and contracting the $x$’s. But $1/r^7$ is special, as it is the Green’s function for the nine-dimensional laplacian. This means that, for $r \neq 0$, $r \gg x$,

$$\nabla^2 \frac{1}{|\vec{r} + \delta \vec{x}|^7} = 0$$

(15)

where the derivatives act with respect to $\vec{r}$. Expanding in powers of $\delta \vec{x}$, this must be true for every term in the sum. It must also, then, be true when we average over $x$. But averaged over $\delta x$, each term is proportional to $\frac{1}{r^6}$ (times an infrared divergent integral). So, except for the leading term, the coefficient of every other term in the expansion must vanish, upon averaging. The skeptical reader is invited to check the next order explicitly.

Note that in the path integral framework, the non-renormalization of the $v^4$ terms (and the cancellation of ir divergences) in the effective action immediately implies the same for the scattering amplitude. Contributions which are not one particle irreducible are easily shown to be of higher order in velocity. The same holds true for the $v^8$ terms which we discuss shortly.

Now consider higher orders in velocity. At one loop, there is no $v^6$ term in the effective
action. There is a \( v^8 \) term,

\[
\frac{v^8}{r^{15}}.
\]  

Expanding the denominator as before, one now finds that there is an infrared divergence at two loops. Again, this cannot be cancelled by graphs with fermions or gauge bosons. While the expressions are ill defined, this presumably represents a breakdown of the non-renormalization theorems. The same statements also hold in 1 + 1 dimensions.

![Figure 3: Three loop correction to the effective action.](image)

Returning to the \( v^6 \) terms, as noted above, a \( v^6 \) term is not generated at one loop. Such a term is generated at two loop\[15\]. But we cannot simply apply our reasoning to the two loop case. The calculation of \[15\] includes graphs with both massive and massless states. At three loops, there are diagrams with zero, one or two massless particles in the loop. Expanding the two loop action in powers of \( \delta\vec{x} \), and contracting \( <\delta\vec{x}\delta\vec{x}> \) correctly reproduces the infrared parts of diagrams with one massless field, but double counts the diagrams with two (see fig. 3). So we cannot establish by this means whether there is an infrared divergence (and a breakdown of the non-renormalization theorem) for \( SU(2) \) at \( v^6 \). This is just as well. The fact that the calculation of \[15\] successfully reproduces the naive DLCQ strongly suggests that there is a non-renormalization theorem for this case.

Finally, we should note that the authors of \[14\] have computed, using their regulator, the coefficient of the \( v^8 \) term at two loops\[21\]. However, they are not able to perform a direct comparison with supergravity.

### 3 Finite Renormalizations in \( SU(3) \)

Consider, now, an \( SU(3) \) gauge group. In this case, taking \( x \) to be a \( U(3) \) field, we will consider “expectation values” of the fields \( x \) of the form:

\[
x^9 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & R \end{pmatrix}.
\]  

(17)
This is not the most general expectation value, but it is sufficient for our purposes. In the language of \( M \) theory or \( D0 \) branes, this corresponds to three gravitons (branes) at locations 0, \( r \) and \( R \) respectively.

Suppose that \( r \ll R \). Then there is an approximate \( SU(2) \) (\( U(2) \)) symmetry. We can then imagine first integrating out states with mass of order \( R \), and then those with mass of order \( r \), to obtain an effective action for the massless fields. At the first step, we expect to generate an operator of the form

\[
O_3 = \frac{v_3^4(\langle a\vec{x}^a \cdot \vec{x}^a \rangle + b \vec{R} \cdot \vec{x}^a \vec{R} \cdot \vec{x}^a)}{R^9},
\]

(18)

where \( \vec{x}_a \) are the \( SU(2) \) triplet fields. Then, replacing \( x^a x^a \) by

\[
\langle x^a_i x^a_j \rangle = \delta_{ij} \left( \frac{1}{r} + \int \frac{d\omega}{(2\pi)\omega^2} \right)
\]

in this expression, we obtain a result proportional to

\[
O_4 = \frac{v_3^4}{R^9 r},
\]

(20)
as well as a potentially infrared divergent term.

As in the case of \( SU(2) \), it is not difficult to verify that the coefficient of \( O_4 \), as well as the infrared divergence, vanishes to two loops. In the \( SU(3) \) case, the one-loop result is:

\[
\mathcal{L}_{x^4} \propto \left( \frac{v_4}{x_{12}^7} + \frac{v_4}{x_{13}^7} + \frac{v_4}{x_{23}^7} \right).
\]

(21)

Write

\[
\vec{x}_1 = \vec{x}_1 \quad \vec{x}_2 = \vec{r} + \vec{x}_2 \quad \vec{x}_3 = \vec{R} + \vec{x}_3,
\]

(22)

and expand the last two terms in powers of the fluctuations, \( \vec{x} \), keeping only the part proportional to \( v_3^4 \). The first order terms are \( SU(2) \) singlets (they are proportional to \( \vec{x}_1 + \vec{x}_2 \)). The quadratic terms contain the \( SU(2) \) non-singlet fields, \( \vec{x}_1 - \vec{x}_2 \). These couplings can be generalized to the \( SU(2) \) invariant coupling,

\[
\delta\mathcal{L} = \frac{v^4 x^a x^a}{R^9}.
\]

(23)

Taking the expectation value, we see that as in the case of the \( SU(2) \) infrared divergences, the leading \( 1/r \) and infrared divergent pieces cancel. It is not so easy to check higher orders, in this case, since one can’t generalize, e.g., the \( x^n \) terms unambiguously to \( SU(2) \)-invariant
expressions. However, we have checked explicitly the cancellation to next order, and expect the same will occur for higher orders.

Equally important, repeating this analysis for $v_8^3$ and higher, we see that there will be $1/r$ corrections at two loops for $v^8$. Again, because of the vanishing of the $v^6$ term at one loop, we cannot establish by this sort of reasoning whether or not there are corrections to the various $v^6$ operators at three loops.

One must ask if there are other operators which can contribute to $1/r$ effects. But it is again easy to see, on gauge invariance grounds, that diagrams containing gauge bosons cannot contribute. Diagrams involving fermions have the wrong dependence on $R$ and $r$.

This argument can be extended to $1 + 1$ dimensions. There is again no infrared problem at $O(v^4)$, and no terms which depend on $\ln(r/R)$ (the analog of the $1/r$ terms in the $0 + 1$ dimensional case). Infrared divergences and finite renormalizations do again appear at $O(v^8)$.

4 Conclusions: Implications for the Matrix Model

At order $v^4$, we have provided additional evidence that there are non-renormalization theorems. At order $v^6$, our method does permit one to say whether or not there are renormalizations at three loops. This is consistent with the result of [15] that the naive DLCQ works for the $v^6$ terms in two graviton scattering. For $v^8$ and beyond, we have found that there are both finite and infinite renormalizations. This suggests that in four graviton scattering and beyond, there may be differences between the matrix model and tree level supergravity predictions. This is under investigation, and will be reported elsewhere.

It is perhaps worth describing the three graviton calculation of [17] in the language we have used in this paper. On the gravity side, it was shown that a term in the scattering amplitude behaves as $\frac{v_3^2v_3^2v_3^2}{r^R}$, where $r$ and $R$ are the graviton separations and $R \gg r$. The claim of [17] is that there can be no terms on the matrix model side of the form $\frac{v_3^4v_3^2}{R^9}$. To see this, one can consider the problem, as in the previous section, by first integrating out states of mass $R$. At one loop, diagram by diagram, there are terms proportional to $1/R$, $v^2/R^3$, and $v^4/R^7$, where the $v$'s indicate various tensor structures. Because of the non-renormalization theorems for the terms with zero and two derivatives, the various contributions to the first two sets of operators cancel. Expanding the $v^4/R^7$, the terms which can yield $v_3^4$ are of the form $v_3^4x^a x^a /R^9$ and $v_3^2x^3 x^3 v^a v^a /R^9$, as in eqn. 23. Now allowing some momenta (frequency) to flow in the various
external lines, and contracting the two $x^a$ or $v^a$ factors, all terms with four $v^3$ factors come with $1/R^9$. It is worth stressing that this is not a diagram by diagram statement. Individual one loop diagrams, for example, will generate operators like $v^3 x^2 x^a x^a / R^7$. Contracting the $x^a$'s will then lead to terms containing $v^4 x_{12} / R^7 r^7$. But these terms must cancel. Indeed, in this paper, in considering the infrared divergences, we have implicitly assumed cancellations of this kind. For example, in $SU(2)$ at one loop, individual diagrams contribute to operators like $1/r \sim \delta x^2 / r^7$, and these would give rise to additional infrared divergences. But supersymmetry insures cancellation of such operators.

The type of analysis we have performed here, which relies heavily on the singular infrared behavior of low dimensional field theories, cannot be readily extended to higher dimensions. In $2 + 1$ dimensions and beyond, rather than yielding an infrared divergence, taking then vacuum matrix elements of operators like $O_1, O_3$ in the low energy theory yields ultraviolet divergences.

As we have stressed, the interpretation of the effective action we have considered here is not completely clear, and it is best to think in terms of calculating the path integral for the scattering amplitude. In the case of $SU(2)$, we have seen that while the formal expansion of the path integral amplitude in powers of velocity is potentially quite infrared divergent, the most severe divergences cancel through order $v^4$. We have argued that this will not persist beyond order $v^8$, and that this signals a breakdown of the velocity expansion. We have also seen that in the case of $SU(3)$, large finite corrections also cancel at order $v^4$, but this is not the case at $O(v^8)$ and higher.

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Note added: Since this paper appeared on hep-th, there have been a number of developments. First, it has been shown that supersymmetry uniquely determines the $v^4$ term in the quantum mechanics[23], which establishes the non-renormalization theorem. Still, one might worry that in the presence of infrared divergences, there are subtle issues in defining the effective action. The cancellation we have exhibited provides some insight into this question. A similar non-renormalization theorem has also been proven for the $v^6$ terms in $SU(2)$[24]. Second, in the original version of this paper, it was assumed that there were already discrepancies between the Matrix model and naive DLCQ at the level of three graviton scattering. The calculation of[25] shows that this is not the case. (Earlier criticisms of this work appeared in [?].) Ooguri[16] has
pointed out that the analysis of [12] implies that discrepancies are likely to arise at the level of four graviton scattering. The present authors have isolated the error in [17]. Those authors correctly concluded that there are no $1/R^7 r^7$ terms in the matrix model effective action. This is in agreement with the calculations of [25], and also with an argument of Taylor and Van Raamsdonk [22]. They went on to argue that there are no such terms in the matrix model S-matrix. This is not the case, as will be explained in [27]. The methods of ref. [17] do permit very simple calculations of terms in the matrix model effective action. The present authors have verified the calculation of [25] using these methods and have also exhibited agreement of certain terms in the four (and $n$) graviton scattering amplitudes. Other terms are currently being checked. The results of this investigation should appear shortly [27].

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