Computing the jump-term in space-time FEM for arbitrary temporal interpolation

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ABSTRACT

One approach with rising popularity in analyzing time-dependent problems in science and engineering is the so-called space-time finite-element method that utilized finite-elements in both space and time. A common ansatz in this context is to divide the mesh in temporal direction into so-called space-time slabs, which are subsequently weakly connected in time with a Discontinuous Galerkin approach. The corresponding jump-term, which is responsible for imposing the weak continuity across space-time slabs can be challenging to compute, in particular in the context of deforming domains. Ensuring a conforming discretization of the space-time slab at the top and bottom in time direction simplifies the handling of this term immensely. Otherwise, a computationally expensive and error prone projection of the solution from one time-level to another is necessary. However, when it comes to simulations with deformable domains, e.g. for free-surface flows, ensuring conforming meshes is quite laborious. A possible solution to this challenge is to extrude a spatial mesh in time at each time-step resulting in the so-called time-discontinuous prismatic space-time (D-PST) method[1]. However, this procedure is restricted to finite-elements of 1st order in time.

We present a novel algorithmic approach for arbitrarily discretized meshes by flipping the mesh in time-direction for each time-step. This ansatz allows for a simple evaluation of the jump-term as the mesh is always conforming. The cost of flipping the mesh around its symmetry plane in time scales with the number of nodes, which makes it computationally cheaper than an additional update of the mesh to enforce conformity or the evaluation of a projection. We validate the approach on various physical problems with and without deforming domains.

REFERENCES

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