SPATIAL CORRELATION FUNCTION OF QUASARS AND
POWER SPECTRUM OF COSMOLOGICAL MATTER DENSITY
PERTURBATIONS

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ABSTRACT

We examine the dependence of the spatial two-point correlation function of quasars \( \xi_{qq}(r, z) \) at different redshifts on the initial power spectrum in flat cosmological models. Quasars and other elements of the large-scale structure of the universe are supposed to form in the peaks of the scalar Gaussian field of density fluctuations of appropriate scales. Quasars are considered as a manifestation of short-term active processes at the centers of these fluctuations; such processes set in when dark matter counterflows and a shock wave appear in the gas. We propose a method for calculating the correlation function \( \xi_{qq}(r, z) \) and show its amplitude and slope to depend on the shape of the initial power spectrum and the scale \( R \) of the fluctuations in which quasars are formed. We demonstrate that in the CDM models with the initial power spectrum slope \( n = 0.7 \pm 1 \) it is possible to explain, by choosing appropriate values of \( R \), how the amplitudes and correlation radii of \( \xi_{qq}(r, z) \) may either increase or decrease with increasing redshift \( z \). In particular, the correlation radii of \( \xi_{qq}(r, z) \) grow from \( 6 - 10 h^{-1} \) Mpc when \( R \) grows from \( 0.45 h^{-1} \) to \( 1.3 h^{-1} \) Mpc. The H+CDM model at realistic values of \( R \) fails to account for the observational data according to which the \( \xi_{qq}(r, z) \) amplitude decreases with increasing \( z \).

Subject headings: cosmology: initial power spectrum - dark matter - quasars
1. Introduction

The most elaborated scenario of the origin of the large-scale structure in the universe is pictured by the model in which such a structure results from the evolution of the uniform isotropic Gaussian scalar field of cosmological density fluctuations under the effect of gravitational instability. The major unresolved problems in this case are the choice of an inflationary model for an early universe, determination of the nature of the "dark" non-baryon matter (DM) and its fraction in the total matter mass. Assumptions as to the inflationary model and DM nature define the shape of the initial (post-recombination) power spectrum of cosmological density fluctuations, \( P(k) \), and thus the principal parameters of large-scale structure can be theoretically calculated. Therefore it is of importance to test cosmological models with given power spectra \( P(k) \). This testing can be done by calculating spatial two-point correlation functions for large-scale structure elements on various scales and comparing them with observational data. The testing is based on the relationship between the characteristics of structure elements and their correlation functions, on the one hand, and the amplitude and slope of the initial spectrum \( P(k) \) at different \( k \), on the other hand. Information on the power spectrum on small and intermediate scales \( r \leq 100h^{-1}\) Mpc, \( h = H_0/(100\text{km} \cdot \text{s}^{-1}\text{Mpc}^{-1}, H_0 \) being the Hubble constant at the present epoch) resides in the correlation functions of bright massive galaxies, rich clusters of galaxies, and quasars. The "observed" correlation functions of all these three types of objects are described by the approximate expression \( \xi(r) = (r/r_0)^{-1.8} \), where \( r_0 \) is the correlation radius equal to 5.4\( h^{-1}\) Mpc, 18\( h^{-1}\) Mpc for galaxies, (6 \( \div \) 10)\( h^{-1}\) Mpc for the three types of objects, respectively. At the moment investigations of the "observed" correlation function of quasars in different redshift ranges give ambiguous results. For example, the correlation function amplitude found in [4, 5, 8] decreases with increasing \( z \), it remains unchanged in [4], and grows at \( z \leq 1.7 \) and diminishes at \( z > 1.7 \) in [4]. The common result of these studies is that the amplitude for quasars is larger than for galaxies but is smaller than for rich clusters of galaxies. This result was obtained by different authors from two quasar samples: a combined sample of all quasars observed in the \( z \) range from 0.1 to 4.5 and the "nearby" quasars at \( z \leq 1.5 \).

Here we look into the possibility of using the above results as a test for cosmological models with given power spectra \( P(k) \); to this end, the theoretical correlation function of quasars has to be calculated.

Theoretical methods for calculating correlation functions of galaxies and their clusters are based on the theory of Gaussian random fields [4, 6], they have been devised in detail. The results obtained within the scope of cosmological models with given initial power spectra were analyzed in [2, 4], for example. Calculation of the correlation function of quasars \( \xi_{qq} \) is complicated by a number of problems which have yet to be resolved. What scale is typical of the regions where quasars formed at different \( z \)? What are typical physical parameters of quasars: mass, duration of formation, lifetime, etc.? What is the relation between \( \xi_{qq} \) and these parameters? Answers to these questions essentially depend on the physical model chosen for the quasar phenomenon. In particular, the disk accretion of gas on a massive black hole at the center of a galaxy may be a model mechanism. Therefore, for the mass of the fluctuations in which quasars are formed we can take the mass of "parent" galaxy \( M_{g/q} \) which is able to ensure a high luminosity of the nucleus (observed as a quasar) during the quasar lifetime \( \tau_q \). Based on the results of [4, 8], we may take \( 10^{10} - 10^{12}\) M\( _\odot \) for \( M_{g/q} \) (the black hole mass \( \sim 10^8 - 10^9 M_\odot \), \( \sim 10^7 - 10^8 \) yr for \( \tau_q \), and \( \Delta t_q \leq 10^8 \) yr for the duration of quasar formation). Whether these parameters are the same for quasars at different \( z \) is still an open question.

2. Principal assumptions and formulation of the problem

In the cosmological scenario used by us, galaxies, rich clusters of galaxies, and quasars appear in the peaks of the scalar Gaussian field of matter density fluctuations on corresponding scales, the relative amplitude of fluctuations being \( \delta \equiv \delta \rho/\rho = \sigma \cdot \nu \) (\( \sigma \) is the rms amplitude, \( \nu \) is the peak height). It is assumed that galaxies and their clusters come into being when counterflows arise in the DM and a shock wave arises in the gas. The amplitude \( \delta \) at this moment \( t \) is determined from Tolmen’s model in terms of redshift: \( \delta(z) = 1.69 \cdot (z + 1) \). The amplitude corresponding to the objects that appeared earlier is \( \delta > \delta(z) \), it has a normal distribution: \( p(\delta) = (2\pi\sigma^2)^{-1/2} \cdot \exp(-\delta^2/2\sigma^2) \). The probability that a galaxy or a rich cluster of
galaxies occurs at a fixed \( z \) is
\[
P_1(z) = \int_0^\infty p(\delta) \, d\delta. \tag{1}
\]
The probability that two galaxies exist simultaneously at a fixed \( z \) at two different points \( \vec{x}_1 \) and \( \vec{x}_2 \) \((r = |\vec{x}_1 - \vec{x}_2|)\) is
\[
P_2(z) = \int_0^\infty \int_0^\infty p(\delta_1, \delta_2) \, d\delta_1 d\delta_2, \tag{2}
\]
where \( p(\delta_1, \delta_2) \) is the two-dimensional normal distribution of random amplitudes \( \delta_1 \) and \( \delta_2 \): \[13\]
\[
p(\delta_1, \delta_2) = (2 \cdot \pi)^{-1} \cdot \left( \frac{\xi_2(0, z) - \xi_2(r, z)}{\delta_1 \cdot \delta_2} \right) - 1 \\
\times \exp \left( -\frac{\xi(0, z) \cdot \delta_1^2 + \xi(0, z) \cdot \delta_2^2 - 2 \cdot \xi(r, z) \cdot \delta_1 \cdot \delta_2}{2 \cdot (\xi_2(0, z) - \xi_2(r, z))} \right),
\]
where \( r > 0 \) and \( \xi(r, z) \) is the correlation function of the density fluctuations in which the objects are formed. The function is calculated from the given initial power spectrum \( P(k, R_f) \) smoothed on the scale \( R_f \) which corresponds to the scale of the objects:
\[
\xi(r, z) = \frac{1}{2\pi^2} \cdot \int_0^\infty \frac{k^2 \cdot P(k, R_f)}{(1 + z)^2} \cdot \frac{\sin(kr)}{kr} \, dk, \tag{4}
\]
where
\[
P(k, R_f) = P(k) \cdot W^2(k R_f)
\]
and
\[
W(k R_f) = \exp \left( -\frac{1}{2} \cdot k^2 \cdot R_f^2 \right)
\]
is the smoothing function. The statistical correlation function of the fluctuation peaks in which cosmological objects are formed is, by definition,
\[
\xi_{oo}^{st}(r, z) \equiv \frac{P_2(z)}{P_1^2(z)} - 1. \tag{5}
\]
This function for rich clusters of galaxies or for galaxies at \( z = 0 \) is \[11\]
\[
\xi_{oo}^{st}(r) \equiv \xi_{oo}(r, z = 0) = \sqrt{\frac{2}{\pi}} \cdot \left( \text{erfc} \left( \frac{\nu}{\sqrt{2}} \right) \right)^{-2} \\
\times \int_{\nu}^\infty e^{-y^2} \cdot \text{erfc} \left( \frac{\nu - y \cdot \xi(r)/\xi(0)}{\sqrt{2} \cdot (1 - \xi_2(r)/\xi_2(0))} \right) \, dy - 1. \tag{6}
\]
Expression (6) is simplified at \( \xi(r) \ll 1 \) and \( \nu \gg 1 \)
\[
\xi_{oo}^{st}(r) \approx \left( \frac{\nu}{\sigma} \right)^2 \cdot \xi(r). \tag{7}
\]
In this case \( \xi_{oo}^{st}(r) \) for objects is related to \( \xi(r) \) for the fluctuations on the corresponding scale in which the objects are formed. The factor \( \nu/\sigma \) is called the statistical biasing of these objects, and the procedures for its determination for galaxies and rich clusters of galaxies are described in \[10-12, 20\]. In deriving the correlation function of quasars we take advantage of the approach proposed in \[11\] with allowance made for the specific nature of quasars.

3. Correlation function of quasars

To calculate correlation functions of galaxies and rich clusters of galaxies, we have to specify the scale of the corresponding fluctuations in which they are formed. For quasars, there are two more important quantities along with this scale: the duration of quasar formation \( \Delta t_q \) and quasar lifetime \( \tau_q \). The smallest mass of the galaxies which may experience the quasar phase in the course of their evolution is \( M_{g/q} \approx 2 \cdot 10^{11} M_\odot \) \[13-15\]. The duration \( \Delta t_q \leq 10^8 \) yr is much shorter than the cosmological evolution time of fluctuations in which such galaxies are formed at \( z \leq 5 \), and so we may ignore this quantity in first approximation. The same studies \[13-15\] reveal that the quasar phase is short-lived \( \tau_q \sim 10^7 - 10^8 \) yr. The quantities \( P_1(z) \) and \( P_2(z) \) in (1), (2) are the probabilities that the random amplitudes of corresponding fluctuations are \( \delta > \delta(z) \), i. e., they determine the probability of coming into being or the probability of the existence of objects on corresponding scales with redshifts larger than a given \( z \). For quasars, the interval \( \Delta(z, \tau_q) \) between the limits of integration in (1) and (2) is determined as the lifetime \( \tau_q \) of the quasars which came into being at the cosmological moment \( t(z) \) corresponding to a given \( z \) \[21-23\]:
\[
\Delta(z, \tau_q) = 1.13 \cdot (z + 1) \cdot \frac{\tau_q}{t(z)}. \tag{8}
\]
When \( \tau_q \sim t(z) \), the quantities \( P_1(z) \) and \( P_2(z) \) are in form (1), (2) with the upper limit of integration \( \delta(z) + \Delta(z, \tau_q) \). When \( \tau_q \ll t(z) \), expressions (1), (2) take the form
\[
P_1^q(z) = P(\delta(z)) \cdot \Delta(z, \tau_q), \quad P_2^q(z) = P(\delta_1, \delta_2) \cdot \Delta^2(z, \tau_q). \tag{9}
\]
in view of smallness of $\Delta(z, \tau_0)$. Then, on the basis of (5) and in view of (9), the correlation function of quasars is

$$\xi_{qq}(r, z) = \left(\frac{\sqrt{1 - \left(\frac{\xi(r)}{\xi(0)}\right)^2}}{\xi(0)}\right)^{-1} \times \exp\left(\frac{2.86 \cdot (z + 1)^2}{\xi(0)} \cdot \left(1 + \frac{\xi(0)}{\xi(r)}\right)^{-1} - 1\right),$$

As it follows from (4), the ratio $\xi(r, z)/\xi(0, z)$ is independent of $z$, and $\xi(r, z) = 0$.

The above expression for $\xi_{qq}(r, z)$ represents the statistical component in the correlation function of the fluctuation peaks where quasars are formed, but it takes no account of the dynamics of background large-scale inhomogeneities (the dynamical component). According to (11), the complete correlation function of objects is

$$\xi_{oo}(r, z) = \left(\frac{\sqrt{\xi_{qq}(r, z)^2 + \xi_{oo}(r, z)^2}}{\xi_{oo}(r, z)}\right)^2,$$

where the statistical component for quasars $\xi_{qq}(r, z)$ is represented by (10) and the dynamical component $\xi_{oo}(r, z) = \xi(r, z)$ is correlation function (4) of density fluctuations.

Let us examine the approximation $(\xi(r)/\xi(0)) \ll 1$. In this approximation $\xi_{qq}(r, z)$ (10) takes the form

$$\xi_{qq}(r, z) \approx \left(\frac{1.69 \cdot (1 + z)}{\sigma^2}\right)^2 \cdot \xi(r) = \left(\frac{\nu}{\sigma}\right)^2 \cdot \xi(r),$$

after its expansion into the Taylor series. It coincides with expression (7) for the correlation function of the galaxies and clusters which formed in high peaks $\nu \gg 1$ (that is, at high redshifts), but in the case of quasars there is no such a restriction to high peaks only.

According to (4), the dynamical component $\xi(r, z)$ is expressed in terms of $\xi(r)$

$$\xi(r, z) = \xi(r) \cdot (1 + z)^{-2}.$$

Then, in view of (11)-(13), we can write

$$\xi_{qq}(r, z) = \left(\frac{1.69 \cdot (1 + z)}{\sigma^2} + \frac{1}{1 + z}\right)^2 \cdot \xi(r).$$

As seen from (12) and (13), the statistical component in the correlation function of quasars increases with $z$, while the dynamical component diminishes. Therefore, when $z$ is variable and $r$ is fixed, the correlation function amplitude may have a minimum if a decrease in the dynamical component is not compensated by an increase in the statistical component at small $z$. The minimum point can be found from the condition $\partial \xi_{qq}(r, z)/\partial z = 0$ with expression (14):

$$z_* = \frac{\sigma}{1.3} - 1.$$

Thus, at $\sigma < 1.3$ we have $\partial \xi_{qq}(r, z)/\partial z > 0$ within the interval $z > 0$ the correlation function amplitude does nothing but grows. When $\sigma > 1.3$, the amplitude is minimum at $z = z_*$. Over the entire range $z \leq 5$, in which quasars are observed, the amplitude of the correlation function of quasars diminishes at $\sigma \geq 7.8$, over the range $z \leq 4$ it diminishes at $\sigma \geq 6.5$, and over the range $z \leq 3$ at $\sigma \geq 5.2$. These results were derived in the approximation $\left(\xi(r)/\xi(0)\right) \ll 1$, which is valid at $r \geq 10h^{-1}$ Mpc. On these scales, the large-scale inhomogeneities have the amplitude $<\delta \rho/\rho > \ll 1$ even at $z \sim 0$, so that the correction of the spectrum $P(k)$ for the nonlinear evolution does not affect the results.

4. Calculation results

We calculated the correlation function of quasars $\xi_{qq}(r, z)$ within the framework of models with various initial power spectra of fluctuations $P(k)$. Among models with various $\Omega_b$ (fraction of baryon density in the density of the whole matter in terms of the critical density $\Omega_b = \rho_b/\rho_0$), $\Omega_{CDM}$, and $\Omega_{HDM}$ (CDM stands for the cold dark matter of axion type and HDM for the hot dark matter of massive neutrino type), the CDM models with $\Omega_b \leq 0.1$ and $\Omega_{CDM} = 1 - \Omega_b$ are believed to be the most promising at the moment; they include the "standard" CDM model with the spectrum slope $n = 1$ ($P(k) = A \cdot k^n \cdot T^2(k)$), where $A$ is a spectrum normalization constant, $k$ is the wave number, and $T(k)$ is a transfer function depending on DM nature), "inclined" CDM models with $n = 0.8; 0.7$, and the "hybrid" H+CDM model with $\Omega_b = 0.1$, $\Omega_{CDM} = 0.6$, $\Omega_{HDM} = 0.3$, $n = 1$. Model parameters and methods of spectrum normalization on the COBE results [24, 25] were described in [13, 23, 24]. To calculate $\xi_{qq}(r, z)$, one has to specify the characteristic scale or mass of fluctuations in which quasars are formed and smooth the power spectrum $P(k)$ by a Gaussian filter with the radius $R_f$ corresponding to this scale. It follows from the expression
$M = 4.37 \cdot 10^{12} R_f^3 h^{-1} M_\odot$ \cite{10} that the radius corresponding to the mass $M_{\text{min}}^{g/q} \simeq 2 \cdot 10^{11} M_\odot$ \cite{15-18} is $R_f \approx 0.35 h^{-1}$ Mpc. So, we start from the assumption that quasars are an early short phase in the evolution of galaxies with this mass and calculate the correlation function of quasars at this value of $R_f$.

Figure 1 shows approximations of the observed correlation functions of galaxies \cite{2, 3} and rich clusters of galaxies \cite{1, 3} and the correlation function of quasars calculated by expression (11) for $z = 0.5, 1, 2, 3, 4$ in various cosmological models.

It is apparent that at large redshifts, where the correlation function is mainly specified by its statistical component and the dynamical component is of minor importance, the amplitudes of correlation functions of quasars diminish with increasing $n$ in various CDM models. This is a manifestation of the well-known tendency of Gaussian field peaks to decrease the degree of their clustering with decreasing height $\nu$. Calculations of these heights for a fixed $z$ by the expression $\nu = \delta(z)/\sigma = 1.69 \cdot (1 + z)/\sigma$ where $\sigma = \sigma(R_f) = \sqrt{\xi(0)}$ \cite{14}, reveal that they are the largest in the CDM model with $n = 0.7$ ($\sigma = 1.67$), somewhat smaller in the CDM model with $n = 0.8$ ($\sigma = 2.37$), and the smallest in the CDM model with $n = 1$ ($\sigma = 4.78$). The above rms amplitudes $\sigma$ allow one to follow the variations of correlation function amplitudes for quasars in these models at different $z$ (see expression (15)). Thus, the amplitude $\xi_{qq}(r, z)$ in the CDM model with $n = 0.7$ diminishes at small redshifts, reaches its minimum at $z \approx 0.3$, and then it grows. In the CDM model with $n = 0.8$, a similar minimum occurs at $z \approx 0.8$, in the CDM model $n = 1$ at $z \approx 2.7$, and in the H+CDM ($\sigma = 1.39$) at $z \approx 0.07$. These results are in good agreement with numerical calculations (Fig. 1).

When the plots in Fig. 1 are matched to observational data \cite{4, 5}, one can see that only the CDM model with $n = 1$ and results of \cite{5} are in accord as to the correlation radius $r_0(z = 0.5) \sim (6 \div 10) h^{-1}$ Mpc and the trend of the $\xi_{qq}(r, z)$ amplitude variations. As regards other observational data, the cosmological models discussed here under the assumption that quasars are formed in the fluctuations on the scale $R_f = 0.35 h^{-1}$ Mpc fail to explain the amplitudes of the observed correlation functions of quasars at various $z$ and their correlation radii. According to \cite{2, 4}, the radii $r_0(z) \sim (6 \div 10) h^{-1}$ Mpc at different redshifts. These correlation function parameters in the CDM models are smaller than in the observed func-

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Correlation functions of quasars $\xi_{qq}(r, z)$ calculated in different models at $R_f = 0.35 h^{-1}$ Mpc and correlation functions of galaxies (circles) and rich clusters of galaxies (squares) approximated by the expression $\xi(r) = (r/r_0)^{-1.8}$ at $r_0 = 5.4 h^{-1}$ Mpc and $r_0 = 18 h^{-1}$ Mpc.}
\end{figure}
tion, while in the H+CDM model they are too large. This may mean that the fluctuation scale chosen by us, \( R_f = 0.35 h^{-1} \) Mpc, is too small in the CDM models and too large in the H+CDM model. The correlation functions \( \xi_{qq}(r, z) \) plotted in Fig. 2 were calculated with the CDM models with such fluctuation scales \( R_f \) for each model that the functions do not contradict the data of \([4, 6, 8]\) on the amplitude (or correlation radius) on the scales \( r \leq 12 h^{-1} \) Mpc. At \( 12 < r \leq 40 h^{-1} \) Mpc the amplitudes of these correlation functions of quasars are no smaller than the amplitude of the observed correlation function of galaxies, with due regard for a large scatter of that function on these scales. Besides, to make more correct the comparison between the theoretical and observational data, we took into consideration that the observed correlation functions of quasars were determined for wide redshift intervals \( \Delta z \equiv [z_{\text{min}}, z_{\text{max}}] \) \([4, 6, 8]\), while the theoretical functions were calculated for fixed \( z \). Therefore, the observed correlation functions at different \( \Delta z \) were fitted by the theoretical functions calculated for \( \bar{z} \), the weighted mean in these intervals found from the observed distribution of quasars \( n_q(z) \) \([27]\) in the comoving reference frame:

\[
\bar{z} = \frac{\int z \cdot n(z) \cdot r^2(z) \, dz}{\int n(z) \cdot r^2(z) \, dz},
\]

where \( r(z) = 2c/H_0 \cdot (1 - (z + 1)^{-0.5}) \) is the distance in the comoving reference frame, \( dr(z) = c/H_0 \cdot (z + 1)^{-1.5} \, dz \), and \( c \) is the velocity of light. The weighting made to fit \( \xi_{qq}(r, z) \) to the results of \([4, 6, 8]\) gave \( \bar{z} = 1.3 \) for the interval \( \Delta z \equiv [0.1, 1.5] \) and \( \bar{z} = 2.6 \) for \([1.5, 4.5]\). For the intervals \( \Delta z \) used in studies \([4, 6, 8]\) we found \( \bar{z} = 1.05 \) for \([0.1, 1.1] \), \( \bar{z} = 1.49 \) for \([1.1, 1.7] \), and \( \bar{z} = 2.38 \) for \([1.7, 3.1] \).

Based on the correlation functions of quasars which correspond to the data of \([4, 6, 8]\) as to correlation radius \( r^2_{\text{q}}(z) \sim 6 \, 10 h^{-1} \) Mpc, we determined the scales \( R_f \) of fluctuations in which quasars are formed and their rms amplitudes \( \sigma(R_f) \) in the CDM models: \( R_f \approx 1.3 h^{-1} \) Mpc, \( \sigma(R_f) = 2.24 \) (in the model with \( n = 1 \)); \( R_f \approx 0.7 h^{-1} \) Mpc, \( \sigma(R_f) = 1.73 \) (in the model with \( n = 0.8 \)); \( R_f \approx 0.45 h^{-1} \) Mpc, \( \sigma(R_f) = 1.52 \) (in the model with \( n = 0.7 \)).

Judging from the above values of \( \sigma(R_f) \) and expression (15), the amplitude and the correlation radius of \( \xi_{qq}(r, z) \) in these models grow with \( \bar{z} \).

Fig. 2.— Correlation functions of quasars \( \xi_{qq}(r, z) \) calculated in CDM models for \( \bar{z} = 1.3, 2.6, 1.05, 1.49, 2.38 \) weighted means in the \( \Delta z \) intervals \( 0.1 - 1.5, 1.5 - 4.5, 0.1 - 1.1, 1.1 - 1.7, 1.7 - 3.1 \). The scale \( R_f \) is chosen such that the correlation radii may be within the range \( 6 - 10 h^{-1} \) Mpc. Other designations are the same as in Fig. 1.
5. Conclusion

We have elaborated a method for the theoretical calculation of the correlation function of quasars, it is based on the Gaussian statistics of the initial field of cosmological matter density fluctuations. The correlation functions of quasars were calculated for different redshifts in cosmological models with given initial power spectra $P(k)$ on the assumption that quasars are formed in the peaks of the fluctuations and exist over a time much shorter than the cosmological time. The amplitudes and slopes of the correlation functions are shown to depend on fluctuation scale and power $P(k)$ on corresponding scales. The amplitudes and correlation radii of the observed and theoretical correlation functions of quasars are consistent in the CDM models with the spectrum slope $n$ ranging from 0.7 to 1 when the scale $R_f$ of the corresponding fluctuations in which quasars are formed ranges from $0.45h^{-1}$ Mpc to $1.3h^{-1}$ Mpc. In this case, however, the function amplitudes increase with $z$. Although there is no evidence of any monotonic growth of the correlation function amplitude in studies [4, 5, 6, 7, 8], the estimates are so contradictory that we cannot use them to test the cosmological models. The H+CDM model fails to explain the correlation radius derived from observational data ($6h^{-1} \div 10h^{-1}$ Mpc) if the scale of fluctuations in which quasars are formed exceeds $0.35h^{-1}$ Mpc. Our results are in accord with the physical models which regard the quasar phenomenon as an early short phase in the evolution of massive galaxies or as a merger of galaxies situated in groups.

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