Collisionless Absorption of Intense Laser Beams by Anharmonic Resonance

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Collisionless absorption of intense sub-ps laser pulses in solid targets up to 70% and beyond is possible [1]. The results found their confirmation in numerous particle-in-cell (PIC) [2, 3] and Vlasov simulations [4, 5]. Since the beginning intensive search for physical mechanism supporting such high absorption values in the absence of collisions started and a whole variety of effects was made responsible for the observations. Pioneering work in this context has been done by several researchers. W. Krueer and K. Eastabrook introduced the concept of \( j \times B \)-heating in 1985 [6]. They realized for the first time the relevance of electron motion in laser beam direction for heating. A further, very significant step forward towards understanding strong collisionless absorption was made in 1987 due to F. Brunel by recognizing the role of the collective self-generated electric field in the plasma under oblique incidence or the action of the \( j \times B \)-force, respectively [7]. Finally, P. Gibbon discovered that a certain fraction of electrons pushed out into the vacuum do not return to the target with the rhythm of the laser oscillations. For this phenomenon, because of indicating irreversibility aspects, he coined the term “vacuum heating” in 1994 which subsequently has become very popular [8]. In our opinion it was unfortunate that this phenomenon did not find the systematic attention of researchers it deserved. So far also quantitative analytic absorption models have been elaborated, like anomalous skin layer absorption [9], stochastic heating [10], or Landau damping [11], which however yield absorption values approximately an order of magnitude lower than the values above we are interested in in this context here. For a more detailed analysis see [12].

According to some researchers’ view one of the phenomena makes the contribution to collisionless absorption, according to others altogether contribute. In summary, collisionless absorption in overdense matter is well confirmed by experiments and simulations but not understood physically to the desired extent.

We introduce the new concept of anharmonic resonance and show that it is the leading physical mechanism responsible for collisionless absorption. In a first stage we give a description of the idea for the hurried reader; without loss of physical insight he may skip all formulas. Then we show in detail how anharmonic resonance works with a single plasma layer. Subsequently we study numerically the interaction of multiple layers coupled together and, with the aid of test particle simulations, we present the proof of our assertion for realistic interaction, with the limitation to plane geometry for being basic in many respects.

Formally, the search for collisionless absorption reduces to the question where the phase shift \( \phi \) between the monochromatic electric field of the laser \( E \sim \cos \omega t \) of frequency \( \omega \) and the electron current density \( j \sim \sin(\omega t + \phi) \) in Poynting’s theorem comes from in the absence of collisions since, with \( \phi = 0 \), \( j \cdot E \sim \sin \omega t \cos \omega t = 0 \). The most efficient collisionless mechanism producing \( \phi \neq 0 \) is resonance. However, resonance absorption at \( \omega_p \gg \omega \) in the plasma has been categorically excluded so far by the scientific community. The insight that this statement is incorrect, because limited to the harmonic oscillator only, is the key to the solution. In fact, when the electrons of a plane plasma layer of thickness \( d \) are displaced by an amount \( \xi = d \) or larger their oscillation eigen-period \( T_0 \) lengthens from \( 2\pi/\omega_p \) to larger values and at a sufficiently big excursion \( \xi \) becomes equal to the laser period \( T = 2\pi/\omega \), i.e., it enters into resonance. This happens already at low laser intensities. The transition from non-resonant to resonant state is irreversible and is accompanied by the desired well-known phase shift of \( |\phi| = \pi \). This behavior persists also in the real plasma with crossing of multiple oscillators (breaking) and mutually influencing each other thereby lowering the single layer resonance threshold. Owing to the non-integrable (chaotic) nature of the governing equations already in the simplest case this final part of the assertion can...
be proved only with the help of numerical methods.

Now we proceed to the quantitative proof. A plane, fully ionized target is assumed to fill the half space $x > 0$. A plane wave $E(x, t) = E_0 \exp[i(k \cdot x - \omega t)]$ in $p$-polarization, wave vector $k$ and frequency $\omega$, is incident from $-\infty$ under an angle $\alpha$ onto the plasma surface (Fig. 1b). After applying a Lorentz boost $v_0 = c \sin \alpha$ the wave impinges normally. The target is thought to be cut into a sufficient number of thin parallel layers, each of which is exposed to a driving force $F$ acting along $x$ of magnitude $ev_0B$ and frequency $\omega$, and an additional component originating from the Lorentz force of the oscillatory motion along $y$ of frequency $2\omega$ ($e$ electron charge, $B$ magnetic field of the laser).

With the electron displacement $\xi$ in $x$-direction and immobile ions of density $n_0$ (corresponding to $\omega_{pe}$) the motion of a single layer (Fig. 1b) in the non-relativistic limit is well determined by the potential $V = m_e(\omega_{pe}d/2)^2(1 + \xi^2)^{1/2}$, $\xi = 2\xi/d$, $m_e$ electron mass (derivation is elementary). Hence, the plasma oscillator is governed by the dimensionless equation

$$\frac{d^2\xi}{d\tau^2} + \frac{\partial(1 + \xi^2)^{1/2}}{\partial\xi} = D(\tau)$$

(1)

where $\tau = \omega_{pe}t$ and $D = 2F/(\omega_{pe}^2d)$. The eigenperiod

$$T_0 = \int 4[2(E - V(\xi))/m_e]^{-1/2} d\xi$$

(2)

for a fixed energy $E$, at high excitation and $\omega_{pe} \gg \omega$ is given by $T_0 = 8(\omega_{pe}^2d)^{-1/2}\xi_0^{-1/2}$. The eigenfrequency $\omega_0 = 2\pi/T_0 = (\pi/4)(\omega_{pe}^2d)^{1/2}\xi_0^{-1/2}$ decreases with increasing oscillation amplitude $\xi_0$, in contrast to the linear harmonic oscillator. To make the essence clear we concentrate on large angles $\alpha$ and set $D = a_0(t) \cos \omega t$, $a_0(t)$ amplitude. At $\omega_0 = \omega$ the single plasma layer is resonantly excited to an amplitude $\xi_0 = \xi_r = (\pi \omega_{pe}/4\omega)^2d$ and $t = t_r$. Under the assumption that the oscillator is driven into resonance during half a laser cycle, i.e., when the driver amplitude $E_0 = m_e\omega_{pe}^2d/(4\epsilon)$, with $d = 0.1-0.2$ nm and $\omega_{pe} \gg 2 \times 10^{10}$ s$^{-1}$ this happens at the laser intensity $I = 10^{15}$ W cm$^{-2}$.

Under a weak driver $D(\tau)$ in (1) the eigenfrequency $\omega_0$ is much higher than $\omega$ and the excursion $\xi(\tau) \sim \cos(\omega t + \phi)$ follows adiabatically $\xi(\tau)$, i.e., $\dot{\phi} = 0$. In other words, $\xi(\tau)$ moves in phase with $D(\tau)$. At large displacements the restoring force weakens and $\omega_0 \ll \omega$, thus imposing $|\phi| = \pi$, like a free electron oscillating in the laser field. At an intermediate $\xi$-value the phase shift must have taken place with the consequence that the product $\xi D$ transmits from $\sin \tau \times \cos \tau$ to $\cos^2 \tau$. It is the resonance region. Only there, in the so-called neighborhood of stationary phase (13) resonance occurs and irreversible energy absorption by the oscillator is possible. A necessary condition for stationary phase is the existence of a point with $|\phi| = \pi/2$. Slow transitions through resonance can be visualized geometrically with the help of the Cornu spiral as explicitly shown for $\omega_0(t)$ in another context in (14). In order to get rid of such WKB-like excitation Eq. (1) is solved numerically with a smooth $N$-cycle sinus$^2$-shaped driver.

$$D = a_0(t) \cos \omega t = D_0 \sin^2[\omega t/(2N\omega_{pe})] \cos \omega t/\omega_{pe},$$

$$\omega/\omega_{pe} = 0.1, N = 20, (a) below resonance ($D_0 = 0.921$), (b) above resonance ($D_0 = 0.923$) with an 43 times higher final energy gain $\varepsilon_f$ than in (a) although the driver strength is changed only by 0.2%. The potential and the resonant energy level are indicated dashed and dotted, respectively. Note the different scales in a, b.

In Fig. 2b, with a driver $D_0 = 0.921$ the layer remains below resonance, the energy gain $\varepsilon_f = \varepsilon(2N\pi\omega_{pe}/\omega)$ after the pulse is negligible. By an increase of the driver of only $\Delta D_0 = 0.002$ resonance takes place. Due to dephasing above resonance the energy content shows the typical periodic variations attenuating in time towards a finite value when the pulse is over. Much energy is stored now in the oscillator (see the horizontal orbits in Fig. 2b). Compared to case (a) the energy gain is increased by a factor of 43.

Figure 3 is of particular relevance. It shows the driving field $D(\tau)$, the displacement $\xi(\tau)$, and the energy gained $\varepsilon(\tau)$. At position 1 the oscillator is entering resonance ($\varepsilon$ starts increasing, $\omega_0 > \omega$), $D$ and $\xi$ are in phase; at 2 it is leaving resonance ($\varepsilon$ starts decreasing, $\omega_0 < \omega$), $D$ and $\xi$ are dephased by $\pi$. Positions I and II (points of stationary phase) indicate maximum energy gain and maximum energy loss, $D$ and $\xi$ are dephased by $\pm \pi/2$. Thus, the resonance signature is preserved in a rapid transition. The phenomenon repeats in the second maximum.
of $\varepsilon$, etc. The definition of resonance as the points of stationary phases is the natural extension of the concept of resonance to anharmonic and nonlinear oscillators. An equivalent criterion to be used later is this: (i) the half widths of the local maxima of $\varepsilon(\tau)$ is not much shorter than half a driver cycle $T = 2\pi/\omega$ (in Fig. 2 it is $1.5T$), in contrast to a collisional event which is almost instantaneous, and (ii) $\varepsilon(\tau \rightarrow \infty) > 0$.

The motion of $N$ layers of free electrons and ions is described for arbitrarily large oscillations by the non-separable (non-integrable, chaotic), yet elementary Hamiltonian

$$H = \sum_{k=1}^{N} \left( \frac{p_k^2}{2} + \frac{1}{2} \sum_{k' \neq k}^{N} V_{kk'} + \sum_{l=1}^{N} V_{kl} - D(\tau)\zeta_k \right)$$

with $p_k = d\zeta_k / dt$, $V_{kk'} = [1 + (\zeta_k - \zeta_{k'})^{1/2}]$, $V_{kl} = -[1 + (\zeta_k - \zeta_{kl})^{1/2}]$, $\zeta_k = 2x_k/d$, $\zeta_{kl} = 2a_l/d$. When one of the layers is driven into resonance it starts moving opposite to the coherently moving non-resonant layers thereby crossing one or several adjacent oscillators. This is a new scenario of very effective wave breaking not described in the literature so far. We give it the name of resonant (wave) breaking. It means loss of coherence and leads to flattening of the collective potential and, in concomitance, to a reduction of the resonance threshold. As a representative case we study the dynamics of a 100 times overdense target, subdivided into 120 layers of $d = 0.125$ nm each, on which $I = 3.5 \times 10^{18}$ Wcm$^{-2}$ at $\lambda = 500$ nm is impinging. The typical scenario is as follows: After being pulled out into the vacuum and oscillating there for some time the layers are pushed back in a disruption-like manner into the target (formation of jets). When the layers leave from the back of the target they are replaced by new layers with zero momentum (cold return current). First indication of resonance: After 100 laser cycles more than half of the now more than 2700 layers have gained energies exceeding their quiver energy $U_p$. The power spectrum shows a plateau between $1U_p$ and the cut off at $6U_p$. To show the occurrence of resonance explicitly the phase of each layer with respect to the driving laser field was investigated. A typical example is shown in Fig. 4 with layer #32, LHS trajectory and driver field (a), middle absorbed energy (b), RHS phase $\varphi$ of velocity $v \sim \sin(\omega t + \varphi)$ with respect to the driver $E \sim \cos \omega t$ (c). Over $T/2$ there is a continuous and smooth transition of $\varphi$ through $-\pi/2$ at $t/T = 2.8$ (I) with a simultaneous strong increase in the absorbed energy (b) and the excursion $x$, with following disruption of the layer at $t/T = 3.6$. The change of $\varphi$ is clearly seen also in (a). Another resonance of the same kind is found at $t/T = 8.8$ (II). Other two passages of $\varphi$ through $-\pi/2$ at $t/T = 5.7$ (1) and $8.1$ (2) show rapid fluctuations and hence do show almost no energy gain and no disruption [see (b), (a)]. Transitions of this latter kind are morphologically clearly distinguishable from the former case, and for none of the 120 layers they are able to accelerate them across the target. This proofs in an impressive way that our definition of resonance as points of stationary phase and the resonance criterion (i)+(ii) are correct and meaningful. We conclude that in the cold plasma model all absorption is by anharmonic resonance. The layers disrupt in the chaotic order 6, 5, 4, 3, 2, 15, 17, 16, 9, 11, 10, 8, 12, 24, 31, 34, 28, 32, 14, 30, 1, 33, 45, 26, 36, 27, etc.; layer 113 disrupts before front layer 0. This is one of the fundamental differences in the dynamics in comparison to [16]. More essential is that the Hamiltonian Eq. (3) acts on the single layer like a half open potential, as sketched by the dashed line in Fig. 4b. We tested this explicitly and found that the degree of absorption does not depend much on the height of the potential barrier, even when
this latter lies considerably below the resonance level of the closed potential. Only when it is set equal to zero as in [7] the absorption almost vanishes because of the absence of resonance. The half open potential with finite threshold towards the target interior is clearly seen in our PIC simulations with the PSC code.

The final step of the proof, i.e., high degree of absorption as mentioned in the introduction [1-4] and absorption itself being accomplished by anharmonic resonance, at present can only be based on computer simulations. For this purpose a $10^6$ particles PIC simulation with the PSC code [15] in its collisionless mode under $45^\circ$ irradiation is performed. In the boosted frame a Gaussian Nd laser beam of $I = 10^{17}$ Wcm$^{-2}$ and halfwidth of 26 fs acts on a plane 80 times overcritical $7.3\mu$m thick target. The orbits of 200 test electrons equally distributed over the target are followed during 15 laser cycles of $T_{Nd} = 5$ fs. Again, the outcome of the statistics is overwhelming: all test electrons interacting with the laser field are resonantly accelerated and disrupt nearly immediately. In Fig. 5 the time history of test electron is depicted. The electron enters the laser field (shadowed interference pattern) and interacts resonantly (see momentum) and escapes into the target an instant later with 5 times $U_p$. All resonant orbits look nearly identical to each other. The resonance character is ensured by the high energy gain and the phase shift in comparison to the total laser field (white line superposed). The shadowed fine structure right of the laser field is due to plasmons. The simulation tells also important details on the heating mechanism. The primary effect is the generation of the fast spectrum by resonance. They generate "solitary", i.e. non-Bohm-Gross plasmons in the dense interior which, in turn, heat cold electrons by a mechanism resembling Landau damping. This completes our last part of the proof. It becomes clear now why perturbative theories when starting in zero approximation from straight orbits, e.g., anomalous skin effect, fail to explain strong absorption because anharmonic resonance is outside the validity of a linearized treatment of standard type. We want to point out that this mechanism is active also in long fs or ps pulses when profile steepening is so strong that no linear resonance can take place.

In summary we have addressed the leading physical mechanism of collisionless absorption and, in particular, we have discovered the phase shift between driver field and electron current, indispensable for absorption, as a resonance effect. The discovery may be viewed as the extension of the well-known fact that a single point charge cannot absorb a photon unless it resonates in an outer potential. As a byproduct we have found a new, very efficient scenario leading to (wave) breaking. Our results will have a major impact on the further progress in the theory of laser-dense matter interaction. So, for example, on the basis of anharmonic resonance the appearance of a hot temperature lying higher, often considerably higher, than the mean electron quiver energy and the fact that the Maxwellian-like tail of their energy spectrum is filled up nearly instantaneously (fs time scale) finds a very natural explanation. The latter phenomenon, never recently discussed in the literature, is particularly surprising as the fast electrons do almost not interact together. Finally, as the overwhelming majority of oscillatory motions in nature are anharmonic, the harmonic oscillator being the great exception, the model developed here will find its application in various other fields of fundamental and applied science, e.g., formation of cracks by fatigue under oscillatory stress, and in catastrophe theory. In the specific field of high power - overdense matter interaction the main relevance of our finding we see in the possibility to tailor the electron spectrum for various applications (electron and ion acceleration, fast ignition, etc.) by designing targets properly, for instance by choosing carefully their thickness. The energy spectrum of thin targets is more energetic owing to multiple resonances than the thick target spectrum. Finally, the simulations have also revealed that in the latter case the spectrum is subject to continuous metamorphosis in time, an aspect which may play an important role in fast ignition.

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