Minimal Low Scale Orientifold Models

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Abstract. We present the minimal low energy effective action of a large class of models which extend the Standard Model by a certain number of extra $U(1)$ gauge symmetries, whose cubic trace may be non-vanishing. Such models are encountered both in string compactifications (low scale orientifold vacua for instance) and in the bottom-up approach. We show how, by means of the four-dimensional version of the stringy Green-Schwarz mechanism these models can be rendered consistent with gauge invariance.

1. The effective action of the mLSOM

We will consider models which can originate from a non-supersymmetric string compactification where the Standard Model is localized on D-branes and/or intersections of D-branes in the presence of orientifold planes. The low energy limit of such models, assuming that they contain the Standard Model spectrum, is marked by the presence of extra $U(1)$ gauge bosons and of a certain number of scalar fields with axion and St"uckelberg couplings. Consistency of these models and more specifically the cancellation of anomalies requires also certain Chern-Simons type of couplings.

Therefore apart from the Standard model fields, we have three more $U(1)$’s, three scalars (axions) that mix with the $U(1)$’s, and two Higgs doublets.

The minimal Lagrangian consistent with these features is [1]

\[
\mathcal{L} = - \frac{1}{2} \text{tr} \, G_{\mu\nu}G^{\mu\nu} - \frac{1}{2} \text{tr} \, W_{\mu\nu}W^{\mu\nu} - \frac{1}{4} F_{\mu\nu}^{\text{I}}F^{\mu\nu,\text{I}} \\
- |(\partial_{\mu} + ig_{2}^{a}W_{\mu}^{a} + i q_{l}^{(H_{u})} g_{l} A_{\mu}^{l})H_{u}|^{2} - |(\partial_{\mu} + ig_{2}^{a}W_{\mu}^{a} + i q_{l}^{(H_{d})} g_{l} A_{\mu}^{l})H_{d}|^{2} \\
+ Q_{Li}\sigma^{\mu\nu}D_{\mu}Q_{Li} + u_{Ri}\sigma^{\mu\nu}D_{\mu}u_{Ri} + d_{Ri}\sigma^{\mu\nu}D_{\mu}d_{Ri} \\
+ L_{Li}\sigma^{\mu\nu}D_{\mu}L_{Li} + e_{Ri}\sigma^{\mu\nu}D_{\mu}e_{Ri} + \nu_{Ri}\sigma^{\mu\nu}D_{\mu}\nu_{Ri} \\
+ \gamma_{ij}^{u}H_{u}^{T} \tau^{2} (Q_{Li}\sigma^{2}u_{Rj}) + \gamma_{ij}^{d}H_{d}^{T} \tau^{2} (Q_{Li}\sigma^{2}d_{Rj}) + \text{c.c.} \\
+ \gamma_{ij}^{e}H_{u}^{T} (L_{Li}\sigma^{2}e_{Rj}) + \gamma_{ij}^{\nu}H_{d}^{T} \tau^{2} (L_{Li}\sigma^{2}\nu_{Rj}) + \text{c.c.}
\]

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where we have introduced two dimensional notations for the fermion interactions, as specified below. The gauge symmetry under which this Lagrangian is invariant is

\[ SU(3)_c \times SU(2)_W \times U(1)_G, \quad G_1 = \prod_{i=1}^{4} U(1)_i. \]  

(1.2)

The \( U(1) \) factors are all anomalous in general. In the above the indices \( l, m, n = 1, \ldots, 4 \) count the \( U(1) \)'s in the D-brane basis. There is also a sum over \( SU(2) \) indices \( a = 1, 2, 3 \) and a sum over flavor indices \( i = 1, 2, 3 \). \( G_{\mu \nu} \) is the field strength for the gluons and the \( W_{\mu \nu} \) is the field strength of the weak gauge bosons \( W_\mu \). The fermions in eq. (1.1) are either left handed Weyl spinors \( f_L \), or right handed Weyl spinors \( f_R \) and they fall in the usual \( SU(3) \) and \( SU(2) \) representations of the Standard Model. The covariant derivatives act on the fermions \( f_L, f_R \) as

\[ D_\mu f_L = \left( \partial_\mu + i A_\mu + iq_{f_L} g_{lA} \right) f_L \]

\[ D_\mu f_R = \left( \partial_\mu + i A_\mu - iq_{f_R} g_{lA} \right) f_R \]

(1.3)

where \( A_\mu \) is a non abelian Lie algebra element. The matrices \( \sigma^a = (\sigma^0, \sigma^a) \) where \( \sigma^0 = \text{diag}(1,1) \), \( \sigma^a \) are the Pauli matrices and \( \overline{\sigma}^a = (\sigma^0, -\sigma^a) \). We have also introduced two Higgs \( SU(2) \) doublets \( H_u \) and \( H_d \). The matrices \( \tau^3 \) are Pauli matrices acting on \( SU(2) \) indices.

In the Yukawa sector, the Pauli matrix \( \tau^2 \) acts on the \( SU(2) \) indices while the Pauli matrix \( \sigma^2 \) acts on the spinor indices. The symbol \( T \) (t) suggests transposition with respect to \( SU(2) \) (spinor) indices. To lighten the notation we do not show explicitly the \( SU(3) \) contraction. It should be understood however that the quarks are, on the top of all contractions explicitly shown, contracted as \( q_L \dagger q_R \) in the \( SU(3) \) sense. The \( \gamma^u_{ij} \) etc. are complex three by three matrices. The standard procedure is to bring them in a form as close as possible to diagonal. The result of this is

\[ \mathcal{L}_{\text{Yuk.}} = \sum_{i,j} H_d^T \tau^2 (Q^i_{Li} \sigma^2 U_{ji} \Gamma^{u}_{jj} u_{Rj}) + \sum_i H_d^T (Q^i_{Li} \sigma^2 \Gamma^{d}_{ii} d_{Ri}) + \text{c.c.} \]

\[ + \sum_{i,j} H_u^T (L^i_{Li} \sigma^2 U_{ji} \Gamma^{e}_{jj} e_{Rj}) + \sum_i H_u^T \tau^2 (L^i_{Li} \sigma^2 \Gamma^{\nu}_{ii} \nu_{Ri}) + \text{c.c.} \]

(1.4)

where the \( \Gamma^{u,d,e,\nu} \) are diagonal matrices and \( U^u \) is the CKM matrix which appears in the Yukawa sector of this model in a similar way as in the Standard Model. The \( U^\nu \) matrix is the MNS neutrino mixing matrix. In the electroweak vacuum the Higgs couples universally to the Yukawa sector and the Yukawa couplings turn into mass terms for the fermions. The CKM and MNS matrices disappear from the Yukawa couplings but they appear explicitly in the gauge boson-fermion-fermion interactions, as we will see later.
The couplings $\mathcal{M}_I^l$, $F_I$, $D_I$, $C_{Imn}$ and $E_{lmn}$ are known once a specific string vacuum has been chosen. One feature of the action, as we are going to describe below, is the presence of both dimension-4 and dimension-5 operators, which render it an effective non-renormalizable extension of the Standard Model. The mechanism of cancellation of the anomalies which is enforced on the model is different from the Standard Model one and for this reason all the couplings the $E$, $D$, and $C$ carry an intrinsic power of $h$, the Planck constant, in their definition. The index $I = 1, \cdots, N_a$ runs over the scalars with axion couplings whose number is in general different (and usually much larger) than the number of $U(1)$ fields. In the mLSOM the number of relevant axions will be taken to be always one less than the number of $U(1)$s (i.e. the number of D-brane stacks), in our case $N_a = 3$.

Finally, the Higgs potential is one that is consistent with the symmetries of the theory and breaks the electroweak symmetry spontaneously down to electromagnetism as in the SM. In general it can depend on all scalar fields present in the spectrum, namely both on the Higgs fields and on the axions, provided it is compatible with the gauge invariances. We will split the Higgs potential in two parts. The one in eq. (2.1) which does not depend on the axions, and the one in eq. (2.48) which mixes the Higgs doublets with the pseudo-scalars.

Models with similar structure have been considered before both from the top-bottom and the bottom-up point of views [2, 4, 3, 5, 6, 7, 10, 9, 11, 8].

1.1. Anomalous couplings

The net effect of the Green-Schwarz mechanism on the four-dimensional effective action is a number of scalar fields with St"uckelberg and axion-like couplings and certain Chern-Simons couplings. It is also interesting to point out that these unusual couplings are remnants of the interplay between closed and open strings from the string theory point of view or the gravitational and gauge sectors in the language of the low energy effective action. The pseudoscalar axions originate from (closed string sector) RR fields coupled to the (open string sector) gauge fields of the D-brane world volume through the Wess-Zumino effective action. Besides their theoretical interest, the presence of these terms may provide us with a unique opportunity to test string theory experimentally.

The D-brane basis St"uckelberg couplings in eq. (1.1) can be then written in matrix form as

$$\mathcal{L}^{\text{Stack}} = \frac{1}{2} \sum_I (\partial_\mu a^I + \mathcal{M}_I^l A_\mu^l)(\partial_\mu a^I + \mathcal{M}_I^l A_\mu^l)$$  \hspace{1cm} (1.5)

and as we have seen in detail, ensure that some of the $U(1)$s pick up masses of the order of the string scale.

The other Green-Schwarz couplings in eq. (1.1) consist of the axion-like terms

$$\mathcal{L}^{\text{axion}} = D_I a^I \text{tr} \{G \wedge G\} + F_I a^I \text{tr} \{W \wedge W\} + C_{Imn} a^I F^m \wedge F^n$$  \hspace{1cm} (1.6)

where we have introduced the dimensionfull couplings $D_I$, $F_I$ and $C_{Imn}$ and the Chern-Simons terms,

$$\mathcal{L}^{\text{C-S}} = E_{lmn} \epsilon^{\mu\nu\rho\sigma} A_\mu^l A_\nu^m F^\rho_n \wedge F^\sigma_n.$$  \hspace{1cm} (1.7)
In the above the sum over \( l, m, n \) is implied. Under the \( U(1) \) gauge transformation

\[
A^I_\mu \longrightarrow A^I_\mu + \partial_\mu \epsilon^I
\]

with \( \epsilon \) the gauge transformation parameters, the anomalous variation of the Lagrangian is \(^1\)

\[
\mathcal{L}^{1\text{-loop}} = \epsilon \left[ g g_3^2 \Lambda_3 t^{(3)}_l G \wedge \Lambda + g g_2^2 \Lambda_2 t^{(2)}_l W \wedge W + \Lambda_1 g g_m g_n t^{(1)}_{lnm} F^m \wedge F^n \right]
\]

where

\[
t^{(1)}_{lnm} = \text{tr}(q_l q_m q_n), \quad t^{(1)}_{llm} = \frac{1}{2!} \text{tr}(q^2_l q_m), \quad t^{(1)}_{lll} = \frac{1}{3!} \text{tr}(q^3_l)
\]

and

\[
t^{(3)}_l = \text{tr}(q_l T^A T^A), \quad t^{(2)}_l = \text{tr}(q_l T^j T^j).
\]

Here the index \( A (a) \) labels the generators of \( SU(3) \) (\( SU(2) \)). The nature and meaning of the quantities \( \Lambda_1, \Lambda_2 \) and \( \Lambda_3 \) is clear once the anomaly diagrams are explicitly computed in momentum space. They can be seen to be the shift necessary to be performed in the momentum integration of the triangle anomaly diagram so that the Green-Schwarz anomaly cancellation mechanism is reflected by the Ward identities.

The axions transform under the \( U(1) \) transformations as

\[
a^I \longrightarrow a^I - \mathcal{M}^I_l \epsilon^l.
\]

The St"uckelberg and the axion-gauge-gauge couplings are gauge invariant separately but the Chern-Simons term is not. The gauge variation of the latter is cancelled by the anomaly. By comparing the different gauge variations, we can easily read off the four dimensional version of the Green-Schwarz anomaly cancellation conditions

\[
D^I \mathcal{M}^I_l = \Lambda_3 g g_3^2 t^{(3)}_l \quad (1.13)
\]

\[
F^I \mathcal{M}^I_l = \Lambda_2 g g_2^2 t^{(2)}_l \quad (1.14)
\]

\[
C_{lnm} \mathcal{M}^I_l + (E_{lnm} - E_{mln}) = \Lambda_1 g g_m g_n t^{(1)}_{lnm}. \quad (1.15)
\]

The first two of the above, eqs. (1.13) and (1.14) represent the cancellation of the anomalous triangle graph with a \( U(1) \) gauge boson and two gluons and \( SU(2) \) gauge bosons for external legs respectively. The third, eq. (1.15) represents the mixed \( U(1) \) anomaly cancellation.

We can put some restrictions on the couplings \( E_{lnm} \). Define

\[
S^{lnm} = \int \epsilon^{\mu \nu \rho \sigma} F^m A^l_\mu A^n_\nu F^l_{\rho \sigma} \quad (1.16)
\]

which transforms as

\[
\delta S^{lnm} = \int (-\epsilon^l F^m \wedge F^n + \epsilon^m F^l \wedge F^n). \quad (1.17)
\]

\(^1\) We use a symmetric regularization scheme.
It is easy to see that $S_{lmn}$ satisfy
\begin{equation}
S_{lmn} = -S_{mln}
\tag{1.18}
\end{equation}
and that the transformation property of the Chern-Simons couplings is
\begin{equation}
\delta(E_{lmn}S_{lmn}) = \int (E_{mln} - E_{lmn})\epsilon^lF^m \wedge F^n,
\tag{1.19}
\end{equation}
which was used to derive eq. (1.15). An immediate consequence of eq. (1.18) is that $E_{lmn}S_{lmn}$ vanishes identically unless $E_{lmn}$ is antisymmetric in the first two indices. Now, if $E_{lmn}$ is totally antisymmetric, then $E_{lmn}S_{lmn}$ can be seen to be again identically zero by using the identity
\begin{equation}
S_{lmn} + S_{nlm} + S_{mnl} = 0
\tag{1.20}
\end{equation}
which can be derived by integrating by parts. Therefore the only choice left is the one where $E_{lmn}$ is antisymmetric in $lm$. Then, eq. (1.15) reduces to
\begin{equation}
C_{lmn}M_{1l} + 2E_{lmn} = \Lambda_1g g_m g_n i_{lmn}^{(1)}
\tag{1.21}
\end{equation}
and the gauge transformation to
\begin{equation}
\delta(E_{lmn}S_{lmn}) = -2 \int E_{lmn}\epsilon^l F^m \wedge F^n.
\tag{1.22}
\end{equation}

2. Electroweak symmetry breaking

The electroweak symmetry breaking in mLSOM is a very interesting effect, because the Higgses are charged under the anomalous U(1) gauge symmetries.

In order to discuss EW symmetry breaking we have to be more specific about the Higgs potential. Before EW breaking the Abelian gauge symmetry in the $D$-brane basis is $G_1$ and the Higgs potential $V_{PQ}$ is the most general $SU(2)_L \times G_1$ invariant constructed from the two Higgs $SU(2)$ doublets $H_u$ and $H_d$:
\begin{equation}
V_{PQ}(H_u, H_d) = \sum_{a=u,d} \left( \mu_a^2 H_a^\dagger H_a + \lambda_{aa}(H_a^\dagger H_a)^2 \right) - 2\lambda_{ud}(H_u^\dagger H_d)(H_d^\dagger H_d) + 2\lambda'_{ud}|H_u^\dagger H_d^\tau_2 H_d|^2.
\tag{2.1}
\end{equation}

We can parameterize the Higgs fields in terms of 8 real degrees of freedom as
\begin{equation}
H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^+ \\ H_d^0 \end{pmatrix}
\tag{2.2}
\end{equation}
where $H_u^+, H_d^+$ and $H_u^0, H_d^0$ are complex fields. Specifically
\begin{equation}
H_u^+ = \frac{H_u^+ + iH_u^0}{\sqrt{2}}, \quad H_d^+ = \frac{H_d^+ + iH_d^0}{\sqrt{2}}, \quad H_u^- = H_u^+, \quad H_d^- = H_d^+.
\tag{2.3}
\end{equation}
Expanding around the vacuum we get for the uncharged components
\begin{equation}
H_u^0 = v_u + \frac{H_u^0 + iH_u^0}{\sqrt{2}}, \quad H_d^0 = v_d + \frac{H_d^0 + iH_d^0}{\sqrt{2}}.
\tag{2.4}
\end{equation}
The Weinberg angle is defined via \( \cos \theta_W = g_2/g, \sin \theta_W = g_Y/g \), with \( g^2 = g_Y^2 + g_2^2 \). We also define \( \cos \beta = v_d/v, \sin \beta = v_u/v \) and \( v^2 = v_u^2 + v_d^2 \).

As in the MSSM one can set \( H_u^+ = 0 \) at the minimum by an \( SU(2) \) transformation. Then a minimum with \( \partial V/\partial H_u^+ = 0 \) must also have \( H_d^+ = 0 \). A necessary condition for the potential \( V_{PQ} \) to be bounded from below can be obtained by requiring that the potential is non-negative definite around the electroweak breaking vacuum:

\[
\mu_u^2 v_u^2 + \mu_d^2 v_d^2 + \lambda_{uu} v_u^4 + \lambda_{dd} v_d^4 - 2 \lambda_{ud} v_u^2 v_d^2 \geq 0. \tag{2.5}
\]

The above constraint should be satisfied simultaneously with the constraint coming from the requirement that the vacuum \( <H_u^0> = 0, <H_d^0> = 0 \) (which does not trigger electroweak symmetry breaking) is an unstable minimum of the potential. This is the case when

\[
\mu_u^2 \mu_d^2 \leq 0. \tag{2.6}
\]

Minimizing the potential with respect to \( H_u^0 \) and \( H_d^0 \) one can see that the Higgs vevs

\[
H_u = v_u \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad H_d = v_d \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.7}
\]

do not break electric charge and minimize \( V_{PQ} \) (at tree level) if

\[
\begin{pmatrix} \mu_u^2 \\ \mu_d^2 \end{pmatrix} = 4 \begin{pmatrix} -\lambda_{uu} & \lambda_{ud} \\ \lambda_{ud} & -\lambda_{dd} \end{pmatrix} \begin{pmatrix} v_u^2 \\ v_d^2 \end{pmatrix} \tag{2.8}
\]

with \( \lambda_{uu}, \lambda_{dd} \) and \( \lambda_{ud} \) all real. Using the above conditions, the constraint eq. (2.5) becomes

\[
\mu_u^2 v_u^2 + \mu_d^2 v_d^2 \geq 0. \tag{2.9}
\]

Furthermore, the couplings \( \lambda_{uu}, \lambda_{dd} \) and \( \lambda_{ud} \) should be such that eq. (2.6) and eq. (2.9) are also consistent.

### 2.1. The gauge boson masses

The vevs eq. (2.7), in addition to breaking \( SU(2)_L \times G'_1 \) down to \( U(1)_{\gamma} \), should not be in contradiction with the low energy EW data. The previous discussion for the gauge boson masses still applies with appropriate adjustments that take into account the effects of EW breaking. Technically speaking, the neutral \( U(1) \) mass matrix \( M^{EW} \) should have precisely one zero eigenvalue consistent with an unbroken \( U(1)_{\gamma} \).

\( M^{EW} \) is the 5 by 5 matrix that can be read off the quadratic form

\[
|D_\mu H_u|^2 + |D_\mu H_d|^2 + \frac{1}{2} \sum_I (\partial a'_I + M_I A')^2 \tag{2.10}
\]

and whose eigenvalues and eigenvectors we will now compute. Notice that as before, we have absorbed in \( M_I \) a factor of \( g_I \) in the Stückelberg part of the above formula. One can easily put
we obtain explicitly when we discuss NG bosons. The covariant derivatives are
\[ D_\mu H_u = \left( \partial_\mu + \frac{i}{\sqrt{2}} g_2 \left( T^+ W^+ + T^- W^- \right) + \frac{i}{2} g_2 \tau_3 W_{3\mu} + \frac{i}{2} g^Y A_\mu^Y + \frac{i}{2} \sum_I q^I_u A^I_\mu \right) H_u \]
\[ D_\mu H_d = \left( \partial_\mu + \frac{i}{\sqrt{2}} g_2 \left( T^+ W^+ + T^- W^- \right) + \frac{i}{2} g_2 \tau_3 W_{3\mu} + \frac{i}{2} g^Y A_\mu^Y + \frac{i}{2} \sum_I q^I_d A^I_\mu \right) H_d \]
where \( q^I_u, q^I_d \) are the \( U(1) \) charges of the two Higgses in the rotated basis. We also have
\[ q^Y_u = q^Y_d = \frac{1}{2}. \tag{2.11} \]

The \( SU(2) \) generators and gauge bosons are defined as
\[ T^j = \frac{\tau^j}{2}, \quad T^\pm = T^1 \pm i T^2, \quad W^\pm = \frac{1}{\sqrt{2}} \left( W^1 \mp i W^2 \right), \tag{2.12} \]
we obtain explicitly
\[ D_\mu H_u = \left( \partial_\mu + \frac{i}{\sqrt{2}} g_2 W_{3\mu} + \frac{i}{2} g^Y A^Y_\mu + \frac{i}{2} \sum_I q^I_u A^I_\mu \right) H_u \]
\[ \partial_\mu - \frac{i}{\sqrt{2}} g_2 W_{3\mu} + \frac{i}{2} g^Y A^Y_\mu + \frac{i}{2} \sum_I q^I_u A^I_\mu \right) H_u \]
and a similar expression for the covariant derivative of \( H_d \). The mass matrix in the mixing of the neutral gauge bosons can be then computed from
\[ \frac{1}{2} \sum_I M^2_I (A^I_\mu)^2 + \frac{1}{4} \left( -g_2 W_{3\mu} + g^Y A^Y_\mu + \sum_I q^I_u A^I_\mu \right)^2 v^2_u \]
\[ + \frac{1}{4} \left( -g_2 W_{3\mu} + g^Y A^Y_\mu + \sum_I q^I_d A^I_\mu \right)^2 v^2_d, \tag{2.14} \]
and it reads
\[ \mathbf{M}^2 = \frac{1}{4} \begin{pmatrix} g_2^2 v^2 & -g_2 g^Y v^2 & -g_2 x_I \\ -g_2 g^Y v^2 & g^2 v^2 & g^Y x_I \\ -g_2 x_I & g^Y x_I & 2M^2_I \delta_{IJ} + N_{IJ} \end{pmatrix} \tag{2.15} \]
where
\[ v^2 = v^2_u + v^2_d, \quad N_{IJ} = q^I_u q^I_d v^2_u + q^I_d q^I_u v^2_d, \quad x_I = q^I_u v^2_u + q^I_d v^2_d. \tag{2.16} \]
The zero eigenvalue corresponds to the photon
\[ A_\gamma = \frac{g^Y}{g} W_3 + \frac{g_2}{g} A^Y, \quad g = \sqrt{g^2_Y + g^2_2}. \tag{2.17} \]
We will now assume that the UV masses \( M_I \) are much larger than other mass scales as expected in realistic orientifold vacua. Then we can treat all other parameters of the mass matrix as of order one.

We use normalized kinetic terms for all gauge bosons.
There is an extra non-zero eigenvalue which is of order one, corresponding to the $Z$ gauge boson, with mass

$$m_Z^2 = m_{Z^0}^2 - \frac{g^2}{8} \sum_I \epsilon_I x_I + \mathcal{O}(M_I^{-4})$$

with $m_{Z^0}$ the SM value of the neutral gauge boson mass and

$$\epsilon_I = \frac{x_I}{M_I^2}$$

small parameters.

The other eigenvalues are of order $M_I$

$$(m_{Z'}^2)^2 = \frac{1}{2} M_I^2 + \mathcal{O}(\epsilon_I)$$

The decoupling limit can be studied in terms of the parameters $\epsilon_I$. In order to identify the modifications introduced by the new model on the masses of the $W$ and $Z$ bosons and to the Standard Model $\rho$ parameter we recall that in any 2-Higgs doublet extensions of the Standard Model the kinetic terms for the $W^\pm$ and $Z$ gauge bosons are given by

$$\mathcal{L}_{\text{kin}} = \frac{g^2}{4 \cos^2 \theta_W} v^2 Z_{\mu}^0 Z_{\mu}^0 + \frac{g_2^2}{4} v^2 W^{+\mu} W^{-\mu} + \frac{g_2^2}{4} v^2 W^{-\mu} W^\mu + \mathcal{O}(M_I^{-4})$$

which bring in the identifications

$$m_W^2 = \frac{g_2^2}{4} v^2$$
$$m_{Z^0}^2 = \frac{g_2^2}{4 \cos^2 \theta_W} v^2.$$  

We can now compute the tree-level corrections to the $\rho$ parameter, which are given by

$$\rho = \frac{m_W^2 g_2^2}{m_{Z^0}^2 g_2^2} = 1 + \frac{1}{2} \sum_I \epsilon_I \frac{x_I}{v^2} + \mathcal{O}(M_I^{-4}).$$

Using the experimental fact that the deviation of the $\rho$ parameter from unity should be $\lesssim 2 \times 10^{-4}$, we can obtain constraints on the UV parameters of the theory which should be understood as an approximate lower bound on the $Z_I'$ gauge bosons mass and consequently on the string scale $M_{\text{str}}$.

The small $\epsilon_I$ limit can be also studied directly in the mixing matrix which however yields typically similar, but weaker constraints than the ones derived from the $\rho$ parameter.

### 2.2. Higgs-axion mixing and NG-bosons

From the third and fourth lines of eq. (1.1) and more specifically from the parts linear in the partial derivatives, we can extract the linear combinations of fields that are physical and the linear combinations that are NG-bosons. A new feature with respect to conventional extensions...
of the SM is the mixing of the axions with the fields appearing in the Higgs sector. It is important therefore to describe the unitary gauge of this model in detail. The results from this analysis will be useful also in the gauge fixing process.

The vacuum should break $3+3=6$ generators, corresponding to the symmetry breaking pattern

$$SU(2)_L \times G_1' \rightarrow U(1)_\gamma.$$  

(2.24)

Thus, we expect 5 real physical fields to appear and corresponding to the 6 broken generators we expect to find in total 6 NG-bosons. Even though we did our counting for the case of 3 extra $U(1)$s, it would be straightforward to generalize it for arbitrary $N_s$ (as for any of the other formulas shown here for $N_s = 4$).

We now apply these general arguments to our model. Defining

$$C_{a\mu}^u = -H_a^{+\mu}(\partial_\mu H_a^+ + H_a^+(\partial_\mu H_a^+)) + H_a^{0\mu}(\partial_\mu H_a^0) - H_a^{0}(\partial_\mu H_a^0), \quad a = u, d$$  

(2.25)

$$C_{2\mu}^a = -H_a^{+\mu}(\partial_\mu H_a^+ + H_a^+(\partial_\mu H_a^+)) - H_a^{0\mu}(\partial_\mu H_a^0) + H_a^{0}(\partial_\mu H_a^0), \quad a = u, d$$  

(2.26)

$$C_{a\mu}^- = H_a^{0\mu}(\partial_\mu H_a^0) - H_a^{+\mu}(\partial_\mu H_a^0), \quad C_0^a = C_{a\mu}^-, \quad a = u, d$$  

(2.27)

the terms contained in the Higgs kinetic terms linear in the derivatives can be written as

$$- \frac{i}{2} \left( g_2 W^+ \cdot (C_u^u + C_d^d) - g_2 W^- \cdot (C_u^u + C_d^d) + g_2 W^3 \cdot (C_1^u + C_1^d) + g_Y A_Y \cdot (C_2^u + C_2^d) \right)$$

$$+ \sum_l \left[ g_l A_l^I (q_1^l C_2^l + q_2^l C_2^l) - 2i g_1 M_I A_l^I (\partial \mu a^l_I) \right],$$

(2.28)

where in the terms proportional to $M_I$ we have put back the factor of $g_I$.

Let us first look at the charged terms. They can be written as

$$- \frac{i}{2} (g_2 W^{+\mu} v \partial_\mu G^+ - g_2 W^{-\mu} v \partial_\mu G^+)$$

(2.29)

where

$$G^- = \sin \beta H_a^{+\mu} + \cos \beta H_a^{+\mu}, \quad G^+ = (G^-)^*. $$

(2.30)

Two NG bosons gave been accounted for ($G^+$ and $G^-$) and therefore we expect to find the other 4 NG bosons in the neutral sector.

We can bring eq. (2.28) into the form

$$- \frac{i}{2} A_y \cdot \left\{ g_2 O_{a W_3}^A (C_1^a + C_1^d) + g_Y O_{a Y}^A (C_2^a + C_2^d) \right\}$$

$$- \frac{i}{2} Z \cdot \left\{ g_2 O_{a Z_3}^A (C_1^u + C_1^d) + g_Y O_{a Y}^A (C_2^u + C_2^d) \right\}$$

$$+ \sum_l \left[ g_l O_{a Z_I}^A (q_1^l C_2^l + q_2^l C_2^l) - 2i g_1 M_I O_{a Z_I}^A (\partial_\mu a^l_I) \right]$$

$$- \frac{i}{2} \sum_j Z_{jI} \cdot \left\{ g_2 O_{a Z_{j3}}^A (C_1^u + C_1^d) + g_Y O_{a Y}^A (C_2^u + C_2^d) \right\}$$

$$+ \sum_l \left[ g_l O_{a Z_{jI}}^A (q_1^l C_2^l + q_2^l C_2^l) - 2i g_1 M_I O_{a Z_{jI}}^A (\partial_\mu a^l_I) \right].$$

(2.31)
Notice now that when we expand the Higgs away from its vacuum value and keep terms linear in the fluctuations we find that the coefficient of $A_\gamma$ vanishes identically and

$$C^a_{2\mu} = -C^a_{\mu} = 2i \Im [v_a (\partial_\mu H_\gamma^a)] \equiv 2i \partial_\mu C^a, \quad C^a = v_a \Im H_\gamma^0, \quad a = u, d. \quad (2.32)$$

We can then rewrite eq. (2.31) as

$$Z^\mu \partial_\mu \left\{ f_u C^a + f_d C^d + \sum_l g_l M_l O^A_{Z_l} a^l \right\} + \sum_j Z^\prime_j \partial_\mu \left\{ f_{u,j} C^u + f_{d,j} C^d + \sum_l g_l M_l O^A_{Z_l} j^l \right\}, \quad (2.33)$$

where

$$f_u = g_2 O^A_{ZW_3} - g_y O^A_{Z Y} - \sum_l q^l g_l O^A_{Z_l}, \quad f_d = g_2 O^A_{ZW_3} - g_y O^A_{Z Y} - \sum_l q^l g_l O^A_{Z_l}$$

$$f_{u,j} = g_2 O^A_{ZJW_3} - g_y O^A_{Z J Y} - \sum_l q^l g_l O^A_{Z_l}, \quad f_{d,j} = g_2 O^A_{ZJW_3} - g_y O^A_{Z J Y} - \sum_l q^l g_l O^A_{Z_l}. \quad (2.34)$$

By means of the orthogonal rotation

$$\begin{pmatrix} \Im H^0_u \\ \Im H^0_d \\ a^l' \end{pmatrix} = O^X \begin{pmatrix} \chi \\ G^0_1 \\ G^0_2 \end{pmatrix} \quad (2.35)$$

with $O^X$ an 5 dimensional orthogonal matrix, we can transform to the mass eigenstate basis. We have denoted the physical field by $\chi$ and the 4 NG-bosons by $G^{0}_{1,\ldots,4}$. Then, eq. (2.33) becomes

$$Z^\mu \partial_\mu \left\{ \chi \left[ f_u v_u O^{\chi}_{11} + f_d v_d O^{\chi}_{21} + \sum_l g_l M_l O^A_{Z_l} O^{\chi}_{l+2,1} \right] + m_{Z^0} G^{Z} \right\}$$

$$\sum_j \sum^l Z^{\prime,\mu}_j \partial_\mu \left\{ \chi \left[ f_{u,j} v_u O^{\chi}_{1j} + f_{d,j} v_d O^{\chi}_{2j} + \sum_l g_l M_l O^A_{Z_l j} O^{\chi}_{l+2,1} \right] + m_{Z^\prime_j} G^{Z^\prime}_j \right\}, \quad (2.36)$$

where

$$G^{Z} = G^{0}_1 \left[ f_u \frac{v_u}{m_{Z^0}} O^{\chi}_{12} + f_d \frac{v_d}{m_{Z^0}} O^{\chi}_{22} + \sum_l g_l \frac{M_l}{m_{Z^0}} O^A_{Z_l} O^{\chi}_{l+2,2} \right] +$$

$$\cdot G^{0}_2 \left[ f_u \frac{v_u}{m_{Z^0}} O^{\chi}_{1,5} + f_d \frac{v_d}{m_{Z^0}} O^{\chi}_{2,5} + \sum_l g_l \frac{M_l}{m_{Z^0}} O^A_{Z_l} O^{\chi}_{l+2,5} \right], \quad (2.37)$$

$$G^{Z^\prime}_j = G^{0}_1 \left[ f_{u,j} \frac{v_u}{m_{Z^\prime_j}} O^{\chi}_{1j} + f_{d,j} \frac{v_d}{m_{Z^\prime_j}} O^{\chi}_{2j} + \sum_l g_l \frac{M_l}{m_{Z^\prime_j}} O^A_{Z_l j} O^{\chi}_{l+2,2} \right] +$$

$$\cdot G^{0}_2 \left[ f_{u,j} \frac{v_u}{m_{Z^\prime_j}} O^{\chi}_{1j,1} + f_{d,j} \frac{v_d}{m_{Z^\prime_j}} O^{\chi}_{2j,5} + \sum_l g_l \frac{M_l}{m_{Z^\prime_j}} O^A_{Z_l j} O^{\chi}_{l+2,5} \right]. \quad (2.38)$$
Let us try to elucidate a bit these apparently complicated expressions. The simplest example is the case of the potential $V_{PQ}$ of eq. (2.1) where the axions do not couple to the Higgs fields, which translates into applying eq. (2.35) with all but the upper left two by two sub-matrix of $O^\chi$ set to zero:

$$
\begin{pmatrix}
\text{Im} H_0^u \\
\text{Im} H_0^d
\end{pmatrix} = O^\chi_2
\begin{pmatrix}
A^0 \\
G^0
\end{pmatrix} .
$$
(2.39)

In the above we have added a subscript to the rotation matrix in order to emphasize its dimension and called the physical mass eigenstate $A^0$ (as in the MSSM) instead of $\chi$, a term reserved for fields with axion-like couplings. From eq. (2.39) it is clear that since in $V_{PQ}$ the Higgs fields do not mix with axions the physical state in the CP-odd sector does not acquire an axion coupling. Furthermore, since the mass matrix in the CP-odd sector $M_3$ is identically zero not only the NG-boson $G^0$ but also $A^0$ remains massless. On the other hand, according to our general discussion, from the axion sector all $a'_I = G^0_I$ are (3) massless Goldstone modes. The total number of fields is then 5 physical Higgs fields (four massive and one massless) and 6 NG-bosons taking into account also the 2 NG modes from the charged sector.

For the potential $V_{PQ} + V_{PQ}'$ in eq. (2.48) of the next section the situation is quite different. The fields $G^0_1, \ldots, G^0_4$ will turn out to be massless accounting for that many NG bosons and the field $\chi$ will turn out to be a massive physical field with an axion coupling because of the mixing of the D-brane basis axions with the CP-odd Higgs sector. Again, the counting is 5 physical fields, four Higgs and one axion (all five massive) and 6 NG-bosons.

The expression eq. (2.36) contains unphysical couplings. Requiring that the gauge fields mix only with NG-bosons, introduces the constraints

$$
\begin{align*}
  f_u v_u O^\chi_{11} + f_d v_d O^\chi_{21} + \sum_I g_I M_I O^A_{ZI} O^\chi_{I+2,1} &= 0 \\
  f_u v_u O^\chi_{11} + f_d v_d O^\chi_{21} + \sum_I g_I M_I O^A_{ZI} O^\chi_{I+2,1} &= 0.
\end{align*}
$$
(2.40)

These have the simple solution

$$
\begin{align*}
  O^\chi_{11} &= -N \cos \beta, \\
  O^\chi_{21} &= N \sin \beta \\
  O^\chi_{I+2,1} &= -\frac{q^u_I - q^d_I}{2} \frac{u}{M_I} N \sin 2\beta \equiv \Theta_I
\end{align*}
$$
(2.41)

normalized as

$$
N = \frac{1}{\sqrt{1 + \frac{\sin^2 2\theta}{4} \sum_I \left( \frac{q^u_I - q^d_I}{M_I} \right)^2}}.
$$
(2.43)

Eqs. (2.41) and (2.42) represent the first column of $O^\chi$ which is an $SO(5)$ matrix with 10 independent rotation angles. Having fixed its first column (which is essentially a consequence of the fact that there is one physical linear combination) leaves a freedom of $SO(4)$ rotations on the vacuum manifold. The number of NG bosons is therefore

$$
dim \frac{SO(5)}{SO(4)} = 4.
$$
(2.44)
corresponding to the $4$ rotation angles parameterizing the vacuum manifold $S^4$, as expected. We will present the rotation matrix in its full form when we encounter it again while we are discussing the Higgs and axion masses in the next section where we will extract the rotation matrix $O^\chi$ from the full Higgs potential. Of course the two methods give the same result which means in particular that if we had computed the rotation matrix first and then the mixings between $\chi$ and the $Z$-bosons, we would have found them to be all identically zero.

Evidently, the charged and CP-odd parts of the original Higgs kinetic terms together with the gauge boson mass terms contained in eq. (2.10) can be written in this new basis as

$$
(\partial G^+ - im_W W^+)(\partial G^- + im_W W^-) + (\partial \chi)^2 + (\partial G^Z + m_{Z^0} Z)^2 + \sum_I (\partial G^Z_I + m_{Z'^I} Z'^I)^2,
$$

(2.45)
a form that clearly suggests that indeed the $G^\pm$, $G^Z$ and $G^Z_I$ are the $6$ NG bosons we are after.

2.3. Higgs-axion mixing in the potential

The potential $V_{PQ}$ does not give a mass to one of the scalars. In order to avoid this, one must take into account new types of contributions to the scalar potential, where not only the Higgs fields enter but also the the axion fields $a_I$ which transform under $U(1)$ transformations as (see eq. (??))

$$
a_I' \rightarrow a_I' - M_I \epsilon_I.
$$

(2.46)
The gauge invariant Higgs potential is then

$$
V_{PQ} = \sum_{a=u,d} \left( \mu_a^2 H_a^\dagger H_a + \lambda_{aa}(H_a^\dagger H_a)^2 \right) - 2\lambda_{ud}(H_u^\dagger H_u)(H_d^\dagger H_d) + 2\lambda'_{ud}|H_u^T \tau_2 H_d|^2
$$

(2.47)
as before, plus the new terms

$$
V_{PQ} = b (H_u^\dagger H_u e^{-i\sum_i (q_u^I - q_u^I') \frac{\phi_i^J}{\pi f_I}}) + \lambda_1 (H_u^\dagger H_u e^{-i\sum_i (q_u^I - q_u^I') \frac{\phi_i^J}{\pi f_I}})^2 + \lambda_2 (H_u^\dagger H_u)(H_d^\dagger H_d e^{-i\sum_i (q_u^I - q_u^I') \frac{\phi_i^J}{\pi f_I}}) + \lambda_3 (H_d^\dagger H_d)(H_u^\dagger H_u e^{-i\sum_i (q_u^I - q_u^I') \frac{\phi_i^J}{\pi f_I}}) + c.c.
$$

(2.48)

In the above, $b$ has dimension of mass squared and $\lambda_{1,2,3}$ are dimensionless. As before, we can set $H_u^+ = 0$ at the minimum by an $SU(2)$ rotation and then consistency requires that also $H_d^+ = 0$ at the minimum. To avoid a stable vacuum with unbroken electroweak symmetry, the (MSSM-like) condition

$$
\mu_u^2 \mu_d^2 \leq b^2
$$

(2.49)

must hold. Contribution of terms proportional to $\lambda_{1,2,3}$ do not appear in this condition since they all correspond to terms that mix neutral and charged components of the Higgs fields. The potential, around the correct vacuum, is non-negative definite when

$$
\mu_u^2 v_u^2 + \mu_d^2 v_d^2 + \lambda_{uu} v_u^4 + \lambda_{dd} v_d^4 - 2\lambda_{ud} v_u^2 v_d^2 + 2b v_u v_d + 2v_d^2(\lambda_1 + \lambda_2 \tan \beta + \lambda_3 \cot \beta) \geq 0.
$$

(2.50)
This is a necessary condition so that the potential is bounded from below. The vevs eq. (2.7) still minimize $V_{PQ} + V_{PQ//}$ if
\[
\mu_u^2 = -b \frac{v_d}{v_u} - 2\lambda_{uu} v_u^2 + 2\lambda_{ud} v_d^2 - 2\lambda_1 v_u^2 - 3\lambda_2 v_u v_d - \lambda_3 \frac{v_d^3}{v_u}
\]
\[
\mu_d^2 = -b \frac{v_u}{v_d} - 2\lambda_{dd} v_d^2 + 2\lambda_{ud} v_u^2 - 2\lambda_1 v_u^2 - 3\lambda_2 v_u v_d
\] (2.51)
and consistency of the minimum of the potential requires that the couplings $b\mu, \lambda_{1,2,3}$ are all real. Furthermore, eq. (2.50) and eq. (2.51) are compatible when
\[
\mu_u^2 v_u^2 + \mu_d^2 v_d^2 \geq -2b v_u v_d.
\] (2.52)
Finally, the parameter range of the couplings should be such that eq. (2.52) is consistent also with eq. (2.49).

One should not forget that these statements about the minimum of the potential are tree level statements. At 1-loop the potential will change and in general one has to do the minimization from the beginning and make sure that the chosen Higgs vacuum expectation values still correspond to a stable minimum that can break the electroweak symmetry in the desired way. It is possible that there exists an energy regime where the 1-loop correction to the effective potential is negligible (as it is the case in the MSSM) and then the tree level results can be still trusted. In this paper however we restrict ourselves to the tree level analysis.

In order to find the masses of the physical Higgs fields we have to expand $V_{PQ} + V_{PQ//}$ away from eq. (2.7) and collect the terms quadratic in the fields. Our discussion here is similar to that of section 4 with the obvious modifications.

The quadratic sector is given by
\[
V_q(H) + V_q'(H, a_I') = \begin{pmatrix} H_u^+, H_d^- \end{pmatrix} N_1 \begin{pmatrix} H_u^+ \\ H_d^+ \end{pmatrix} + \begin{pmatrix} \text{Re} H_u^0, \text{Re} H_d^0 \end{pmatrix} N_2 \begin{pmatrix} \text{Re} H_u^0 \\ \text{Re} H_d^0 \end{pmatrix} + \begin{pmatrix} \text{Im} H_u^0, \text{Im} H_d^0, a_I' \end{pmatrix} N_3 \begin{pmatrix} \text{Im} H_u^0 \\ \text{Im} H_d^0 \\ a_I' \end{pmatrix}.
\] (2.53)

In the charged sector, the mass matrix elements are
\[
N_1(1,1) = -2 \cot \beta \left( \lambda_3 \cos^2 \beta + (\lambda_1 - \lambda_{ud}') \sin 2\beta + \lambda_2 \sin^2 \beta \right) v^2
\]
\[-2b \cot \beta
\]
\[
N_1(1,2) = 2 \left( \lambda_3 \cos^2 \beta + (\lambda_1 - \lambda_{ud}') \sin 2\beta + \lambda_2 \sin^2 \beta \right) v^2 + 2b
\]
\[
N_1(2,2) = -2 \left( \lambda_3 \cos^2 \beta + (\lambda_1 - \lambda_{ud}') \sin 2\beta + \lambda_2 \sin^2 \beta \right) v^2 \tan \beta
\]
\[-2b \tan \beta
\] (2.54)
and we find a zero eigenvalue, corresponding to the goldstone mode $G^+$ and the nonzero eigenvalue
\[
m_{H^+}^2 = 4\lambda_{ud}' v^2 - 2 \left( \frac{2b}{v^2 \sin 2\beta} + 2\lambda_1 + \tan \beta \lambda_2 + \cot \beta \lambda_3 \right) v^2
\] (2.55)
corresponding to the charged Higgs mass.

In the neutral sector both a CP-even and a CP-odd sector are present. The CP-even sector is described by \( \mathcal{N}_2 \). The mass matrix in the CP-even sector is given by

\[
\mathcal{N}_2(1,1) = -2(-4v^2\lambda_{uu}\sin^2\beta + v^2\lambda_3 \cos^2\beta \cot\beta - \frac{3}{2}v^2\lambda_2 \sin 2\beta + b \cot\beta)
\]

\[
\mathcal{N}_2(1,2) = 2(3v^2\lambda_3 \cos^2\beta + 3v^2\lambda_2 \sin^2\beta + 2v^2\lambda_1 \sin 2\beta - 2v^2\lambda_{ud} \sin 2\beta + b)
\]

\[
\mathcal{N}_2(2,2) = -2 \sec\beta (-4\lambda_{dd}v^2 \cos^2\beta - 3\lambda_3 v^2 \sin\beta \cos^2\beta + \lambda_2 v^2 \sin^3\beta + b \sin\beta).
\]

(2.56)

The eigenvalues corresponding to the physical neutral Higgs fields are given by

\[
m^2_{h^0} = \frac{1}{2} \left( \mathcal{N}_2(1,1) + \mathcal{N}_2(2,2) - \sqrt{\Delta} \right)
\]

\[
m^2_{H^0} = \frac{1}{2} \left( \mathcal{N}_2(1,1) + \mathcal{N}_2(2,2) + \sqrt{\Delta} \right).
\]

(2.57)

The lighter of the two, \( h^0 \), is the state which is expected to be the one that corresponds to the Standard Model Higgs field.

Finally, the symmetric matrix describing the mixing of the CP-odd Higgs sector with the axion fields \( a'_I \) reads

\[
\mathcal{N}_3 = -\frac{1}{2}v_u v_d c_{\chi'} \begin{pmatrix}
\cot\beta & -1 & v_d \frac{q_u - q_d}{M_1} & v_d \frac{q_u - q_d}{M_2} & v_d \frac{q_u - q_d}{M_3} \\
-1 & \tan\beta & -v_u \frac{q_u - q_d}{M_1} & -v_u \frac{q_u - q_d}{M_2} & -v_u \frac{q_u - q_d}{M_3} \\
v_d \frac{q_u - q_d}{M_1} & -v_u \frac{q_u - q_d}{M_1} & -v_d \frac{q_u - q_d}{M_2} \\
v_d \frac{q_u - q_d}{M_2} & -v_u \frac{q_u - q_d}{M_2} & v_u v_d \frac{(q_u' - q_d')(q_u' - q_d')}{M_f M_f} \\
v_d \frac{q_u - q_d}{M_3} & -v_u \frac{q_u - q_d}{M_3} & v_u v_d \frac{(q_u' - q_d')(q_u' - q_d')}{M_f M_f}
\end{pmatrix}
\]

(2.58)

with

\[
c_{\chi'} = \frac{4b}{v^2 \sin 2\beta} + 4\lambda_1 + \lambda_2 \tan\beta + \lambda_3 \cot\beta.
\]

(2.59)

The mass eigenvalue in this sector is

\[
m^2_{\chi} = -\frac{1}{2} \left[ 1 + \sum_I \left( \frac{q_u^I - q_d^I}{2} \frac{v}{M_f} \sin 2\beta \right)^2 \left[ \frac{4b}{v^2 \sin 2\beta} + 4\lambda_1 + \lambda_2 \tan\beta + \lambda_3 \cot\beta \right] v^2. \right.
\]

(2.60)

2.4. Properties of the axi-Higgs

Let us now discuss the properties of the physical field that appears in the CP-odd scalar sector. We call this field \( \chi \) the axi-Higgs since it is a gauge invariant mixture of the bulk axions and the Higgs phase. In a unitary gauge it is proportional to \( a'_I \) which have axionic couplings. As its D-brane basis cousins \( a'_I \), it appears in the Lagrangian through a dimension five operator. We have already computed its coupling in the broken phase to the gauge bosons. We can also compute the coupling of \( \chi \) to the fermions.
The Yukawa couplings provide mass terms for the fermions as well as cubic Higgs-fermion interactions and axion-fermion-fermion interactions. All these can be extracted from

\[ \mathcal{L}_{\text{Yuk}} = -H_u^0(u_{Li})^\dagger \partial^2 \Gamma u_i u_{Ri} + H_u^+(d_{Li})^\dagger \partial^2 \Gamma u_i u_{Ri} + H_d^{+\nu}(u_{Li})^\dagger \partial^2 \Gamma d_i d_{Ri} + H_d^{0\nu}(d_{Li})^\dagger \partial^2 \Gamma d_i d_{Ri} + H_u^+ \nu_{Li} \sigma^2 \Gamma u_i e_{Ri} + H_d^{0\nu} \nu_{Li} \sigma^2 \Gamma d_i \nu_{Ri} + H_u^+ e_{Li} \sigma^2 \Gamma u_i \nu_{Ri} + \text{c.c.} \]

(2.61)

with

\[
\begin{align*}
H_u^0 &= v_u + \frac{1}{\sqrt{2}}(h^0 \sin \theta - H^0 \cos \theta) + i O_\text{Y}_{11} \\
H_u^+ &= -\frac{1}{\sqrt{2}} H^+ \cos \beta \\
H_d^0 &= v_d + \frac{1}{\sqrt{2}}(h^0 \cos \theta + H^0 \sin \theta) + i O_{21} \chi \\
H_d^+ &= \frac{1}{\sqrt{2}} H^+ \sin \beta
\end{align*}
\]

(2.62)

the unitary gauge expression for the Higgs fields. In eq. (2.61) the bold notation and the dagger on the quarks reflects their non-trivial \( SU(3) \) transformation property. The Higgs field expanded around its vacuum expectation value is to lowest order

\[
\begin{align*}
H_u^0 &= (v_u + \cdots) e^{\frac{N \cos \beta}{v_u + \cdots}} \chi \simeq v_u + i N \cos \beta \chi \\
H_d^0 &= (v_d + \cdots) e^{\frac{N \sin \beta}{v_d + \cdots}} \chi \simeq v_d + i N \sin \beta \chi
\end{align*}
\]

(2.63)

where the dots stand for the contribution of the (small) fluctuations of \( h^0 \) and \( H^0 \) and which are negligible for this discussion. Defining

\[
\begin{align*}
m_{ui} &= -v_u \Gamma^u_{ii}, & m_{ei} &= -v_u \Gamma^e_{ii} \\
m_{di} &= -v_d \Gamma^d_{ii}, & m_{\nu_i} &= -v_d \Gamma^\nu_{ii}
\end{align*}
\]

(2.64)

we can write the parts of the effective action that the axion appears (suppressing the spinor contraction) as

\[
\begin{align*}
m_{ui} u_{Li}^\dagger u_{Ri} e^{\frac{N \cos \beta}{v_u + \cdots}} \chi + m_{di} d_{Li}^\dagger d_{Ri} e^{\frac{N \sin \beta}{v_d + \cdots}} \chi + m_{ei} e_{Li} e_{Ri} e^{\frac{N \cos \beta}{v_u + \cdots}} \chi + m_{\nu_i} \nu_{Li} \nu_{Ri} e^{\frac{N \sin \beta}{v_d + \cdots}} \chi + \text{c.c.}
\end{align*}
\]

(2.65)

From the above equations one can see that the couplings of the Higgs fields to the fermions in the Yukawa sector will induce an axion-fermion-fermion coupling

\[
- i \Gamma^{u,e}_{ii} O^\chi_{11} = i \Gamma^{u,e}_{ii} N \cos \beta
\]

(2.66)

to the up quark and electron sector respectively and

\[
i \Gamma^{d,\nu}_{ii} O^\chi_{21} = i \Gamma^{d,\nu}_{ii} N \sin \beta
\]

(2.67)

to the down quark and right handed neutrino sector respectively. As expected, by doing a chiral rotation on the quarks one can make the \( \theta \)-parameter of QCD vanish.
The decay rate of $\chi$ into two gauge bosons $A_1$ and $A_2$ of mass $m_1$ and $m_2$ is given by

$$\Gamma(\chi \rightarrow A_1A_2) = \frac{1}{16\pi m_\chi} \Phi^{1/2} \left[ \left| A_1^2 \right| \right] < |A|^2 >,$$

where

$$\Phi(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$$

and the part involving the amplitude can be computed to be

$$|A|^2 = -\left( g^{\chi A_1 A_2} \right)^2 \left[ m_1^2 m_2^2 - \frac{1}{2} \left( m_\chi^2 - m_1^2 - m_2^2 \right) \right].$$

In our case, the gauge bosons $A_{1,2}$ can be two gluons, a $W^\pm$ pair or any of a photon, a $Z$ and a $Z'$. Clearly, in the electroweak channel, the decay that dominates is the one into two photons. Including a factor of 1/2 when averaging over the final state, we obtain

$$\Gamma(\chi \rightarrow \gamma\gamma) = \frac{(g^{\chi \gamma \gamma})^2 m_\chi^3}{64\pi}.$$ 

This decay rate is to be compared with the rate of the 1 loop decay $h^0 \rightarrow \gamma\gamma$ (the only channel for a scalar decaying into two photons available in the SM) which is

$$\Gamma(h^0 \rightarrow \gamma\gamma) \sim \frac{e^4 \sin^2 \alpha m_{h^0}^3}{1024 \pi^5 m_W^2}.$$ 

In low scale models these two rates could be comparable in magnitude or even the axi-Higgs decay could be dominating. When $\chi$ is off shell, the axi-Higgs-photon-photon vertex gives a tree level contribution to the $p\bar{p} \rightarrow \gamma\gamma$ cross section.

The state $\chi$ is peculiar since it is neither a typical PQ-type of axion nor a typical Higgs field. It is something in between. It inherits properties from both precisely because it is a linear combination of the original Higgs and axion fields. We can summarize then by saying that the mLSOM axion is massive with mass $m_\chi$ generated by the $V_{PQ/}$ part of the scalar potential. Strictly speaking to this one should add the usual mass that is generated non-perturbatively.

This is a small contribution to the mass, proportional to the coupling of the axion to the gauge bosons. For the PQ axion this is the only mass generating source which implies that if its coupling to the gauge bosons is suppressed then its mass is automatically tiny. This (strong) correlation between the (small) mass and the coupling of the PQ axion results in computable cosmological and astrophysical effects that put severe bounds on models with such axions. Here, as can be seen from eq. (2.60) the mLSOM axion acquires from spontaneous symmetry breaking a rather large mass which is generically expected to be of the order of $O(100 \text{ GeV})$ since it is proportional to $v$. This property is one inherited by its Higgs nature. On the other hand, its coupling to the gauge bosons is suppressed by the explicit factor of $1/M_{str}$ contained in $D$, $F$ and $C$ and further suppressed by the factor $v/M_f$ contained in $\Theta_f$ defined in eq. (2.42). This property is a remnant of its axion nature. Evidently the mass is essentially not related to the gauge boson couplings, i.e. a suppressed coupling does not imply a tiny mass as in typical axion extensions of the SM.
In the fermion sector the situation is slightly different. The PQ axion has a coupling to the fermions proportional to its coupling to the gauge bosons and therefore it is equally suppressed. The mLSOM axion on the other hand from eqs. (2.66) and (2.67) is seen to have an $O(1)$ coupling to the fermions. Some relative suppression in the latter due to $\beta$ (by one or two orders of magnitude) is perhaps still allowed.

3. Conclusions

The Minimal Low Energy Orientifold models constitute an interesting class of models extending the Standard Model. They reflect the stringy Green-Schwarz mechanism. Their spectrum involves extra neutral gauge bosons of the order of a few TeV and a peculiar state which is between an axion and a Higgs field. For these reasons these models have possibly interesting experimental signals.

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