Background Dependent Lorentz Violation from String Theory

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Abstract

We revisit Lorentz violation in the Type IIB string theory with D3-branes and D7-branes. We study the relativistic particle velocities in details, and show that there exist both subluminal and superluminal particle propagations. In particular, the additional contributions to the particle velocity \(\delta v \equiv (v - c)/c\) from string theory is proportional to both the particle energy and the D3-brane number density, and is inversely proportional to the string scale. Thus, we can realize the background dependent Lorentz violation naturally by varying the D3-brane number density in space time. To explain the superluminal neutrino propagations in the OPERA and MINOS experiments, the string scale should be around \(10^5\) GeV. With very tiny D3-brane number density on the interstellar scale, we can also explain the time delays for the high energy photons compared to the low energy photons in the MAGIC, HESS, and FERMI experiments simultaneously. Interestingly, we can automatically satisfy all the stringent constraints from the synchrotron radiation of the Crab Nebula, the SN1987a observations on neutrinos, and the cosmic ray experiments on charged leptons. We also address the possible phenomenological challenges to our models from the relevant experiments done on the Earth.

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I. INTRODUCTION

The OPERA neutrino experiment at the underground Gran Sasso Laboratory (LNGS) has recently determined the muon neutrino ($\nu_\mu$) velocity with high accuracy through the measurement of the flight time and the distance (730 km) between the source of the CNGS neutrino beam at CERN (CERN Neutrino beam to Gran Sasso) and the OPERA detector at the LNGS [1]. The mean neutrino energy is 17 GeV. Very surprisingly, the OPERA experiment found that neutrinos arrived earlier than expected from luminal speed by a time interval

$$\delta t = (60.7 \pm 6.9_{\text{stat}} \pm 7.4_{\text{syst}}) \text{ns.}$$

(1)

This implies a superluminal propagation velocity for neutrinos by a relative amount

$$\delta v_\nu \equiv \frac{v_\nu - c}{c} = (2.48 \pm 0.28_{\text{stat}} \pm 0.30_{\text{syst}}) \times 10^{-5} \quad \text{(OPERA)},$$

(2)

where $c$ is the vacuum light speed. Also, there is no significant difference between the values of $\delta v_\nu$ measured for the lower- and higher- energy data with $\langle E \rangle \sim 13$ GeV and 43 GeV, respectively. Interestingly, this is compatible with the MINOS results [2]. Although not statistically significant, the MINOS Collaboration has found [2]

$$\delta v_\nu = (5.1 \pm 2.9) \times 10^{-5} \quad \text{(MINOS)},$$

(3)

for muon neutrino with a spectrum peaking at about 3 GeV, and a tail extending above 100 GeV. Moreover, the earlier short-baseline experiments have set the upper bounds on $|\delta v_\nu|$ around $4 \times 10^{-5}$ in the energy range from 30 GeV to 200 GeV [3]. Of course, the technical issues in the OPERA experiment such as pulse modelling, timing and distance measurement deserve further experimental scrutiny. Other experiments like MINOS and T2K are also needed to do independent measurements for further confirmation due to neutrino oscillations. From theoretical point of view, many groups have already studied the possible solutions to the OPERA anomaly [4–15]. For an early similar study, see Ref. [16]. Especially, the Background Dependent Lorentz Violation (BDLV) has been proposed by considering the spin-2, spin-1 and spin-0 particles, respectively in Refs. [7, 12, 13].

Moreover, the detection of neutrinos emitted from SN1987a gave us a lot of information not only on the process of supernova explosion, but also on neutrino properties. The Irvine-Michigan-Brookhaven (IMB) [17], Baksan [18], and Kamiokande II [19] experiments collected
8 + 5 + 11 neutrino events (presumably mainly $\bar{\nu}_e$) with energies between 7.55 MeV and 395 MeV within 12.4 seconds. In particular, the neutrinos arrived on the Earth about 4 hours before the corresponding light. Although this is compatible with the supernova explosion models, we can still obtain the upper limit on $\delta v_\nu$

$$|\delta v_\nu(15 \text{ MeV})| \leq 2 \times 10^{-9}.$$  \hfill (4)

This limit should be understood with an order-one uncertainty since the precise time delay between light and neutrinos is unknown.

In addition, the MAGIC \cite{20}, HESS \cite{21}, and FERMI \cite{22,23} Collaborations have reported time-lags in the arrival times of high-energy photons, as compared with photons of lower energies. The most conservative interpretations of such time-lags are that they are due to emission mechanisms at the sources, which are still largely unknown at present. However, such delays might also be the hints for the energy-dependent vacuum refractive index, as first proposed fourteen years ago in Ref. \cite{24}. Assuming that the refractive index $n$ depends linearly on the $\gamma$-ray energy $E_\gamma$, i.e., $n_\gamma \sim 1 + E_\gamma/M_{\text{QG}}$ where $M_{\text{QG}}$ is the effective quantum gravity scale, it was shown that the time delays observed by the MAGIC \cite{20}, HESS \cite{21}, and FERMI \cite{22,23} Collaborations are compatible with each other for $M_{\text{QG}}$ around $0.98 \times 10^{18}$ GeV \cite{25}. Also, there are the stringent constraints coming from the synchrotron radiation of the Crab Nebula \cite{26–28}. The D-particle models of space-time foam have been proposed to explain all these effects within the framework of string/brane theory, based on a stringy analogue of the interaction of a photon with internal degrees of freedom in a conventional medium \cite{27,29–32}. However, FERMI observation of GRB 090510 seems to allow only much smaller value for time delay and then requires $M_{\text{QG}} > 1.22 \times 10^{19}$ GeV \cite{33}. Because these data probe different redshift ranges, they may be compatible with each other by considering a redshift dependent D-particle density \cite{32}.

In this paper, we revisit the Lorentz violation in the Type IIB string theory with D3-branes and D7-branes \cite{31}. We study the relativisite particle velocities in details, and show that there exist both subluminal and superluminal particle propagations. In particular, the extra contributions to the particle velocity $\delta v$ from string theory is proportional to both the particle energy and the D3-brane number density, and is inversely proportional to the string scale. Thus, we can realize the background dependent Lorentz violation naturally by varying the D3-brane number density in space time. To explain the OPERA and MINOS
experiments, the string scale should be around $10^5$ GeV. With very tiny D3-brane number density on the interstellar scale, we can also explain the MAGIC, HESS, and FERMI experiments simultaneously. Interestingly, we can automatically satisfy all the stringent constraints from the synchrotron radiation of the Crab Nebula [28], the SN1987a observations on neutrinos [17–19], and the cosmic ray experiments on charged leptons [34–36]. The possible phenomenological challenges to our models from the relevant experiments done on the Earth will be briefly addressed as well.

II. TYPE IIB STRING MODELS

We consider the Type IIB string theory with D3-branes and D7-branes where the D3-branes are inside the D7-branes [31]. The D3-branes wrap a three-cycle, and the D7-branes wrap a four-cycle. Thus, the D3-branes can be considered as point particles in the Universe, i.e., the D-particles, while the SM particles are on the world-volume of the D7-branes. For the particles (called ND particles) arising from the open strings between the D7-branes and D3-branes which satisfy the Neumann (N) and Dirichlet (D) boundary conditions respectively on the D7-branes and D3-branes, their gauge couplings with the gauge fields on the D7-branes are

$$\frac{1}{g_{37}^2} = V \frac{g_7^2}{g_7^2},$$

where $g_7$ are the gauge couplings on the D7-branes, and $V$ denotes the volume of the extra four spatial dimensions of the D7 branes transverse to the D3-branes [37]. Because the Minkowski space dimensions are non-compact, $V$ is infinity and then $g_{37}$ is zero. Thus, the SM particles have no interactions with the ND particles on the D3-brane or D-particles.

To have the interactions between the particles on D7-branes and the ND particles, we consider the D3-brane foam, i.e., the D3-branes are distributed in the whole Universe. We assume that the $V_{A3}$ is the average three-dimensional volume which has a D3-brane locally in the Minkowski space dimensions, and $R'$ is the radius for the fourth space dimension transverse to the D3-branes in the D7-branes. Especially, $V_{A3}$ is the inverse of the D3-brane number density and can vary in the space time. In addition, the D-branes have widths along the transverse dimensions which are about $1.55\ell_s$ from the analysis of tachyon condensation [38]. Here, $\ell_s$ is string length, i.e., the square root of the Regge slope $\sqrt{\alpha'}$. Thus, our ansatz for the gauge couplings between the gauge fields on the D7-branes and the
ND particles is

\[
\frac{1}{g_{\text{37}}^2} = \frac{V_{A3} R' \ell_s^4}{(1.55 \ell_s)^4 g_T^2} = \frac{V_{A3} R'}{(1.55)^3 g_T^2}.
\]

\( \text{FIG. 1: (a): The splitting/capture/re-emission process of a generic matter string by a D-particle from a target-space point of view. (b): The same process for photons and in general particles in the Cartan subalgebra of the SM gauge group in the intersecting brane world scenario. (c): The four-point string scattering amplitude (corresponding to the parts inside the dashed box of (a)) between the constituent open ND strings of the splitting process. Latin indices at the end-points of the open string refer to the brane worlds these strings are attached to.}

We denote a generic SM particle as an open string \(a\bar{b}\) with both ends on the D7-branes. For \(a = b\), we obtain the gauge fields related to the Cartan subalgebras of the SM gauge groups, and their supersymmetric partners (gauginos), for example, the photon, \(Z^0\) gauge boson and the gluons associated with the \(\lambda^3\) and \(\lambda^8\) Gell-Mann matrices of the \(SU(3)_C\) group. For \(a \neq b\), we obtain the other particles, for example, the electron, neutrinos, and \(W^\pm\) boson, etc. As in Fig. 1(a), when the open string \(a\bar{b}\) passes through the D3-brane, it can be split and become two open strings (corresponding to the ND particles) \(a\bar{c}\) and \(c\bar{b}\) with one end on the D7-brane (\(a\) or \(\bar{b}\)) and one end on the D3-brane (\(c\) or \(\bar{c}\)). Then, we can have the two to two process and have two out-going particles arising from the open strings \(a\bar{c}\) and \(c\bar{b}\). Finally, we can have an out-going particle denoted as open string \(a\bar{b}\). In particular, for \(a = b\), we can have \(s\)-channel process at the leading order, and plot it in the Fig. 1(b). Interestingly, the time delays or advances at the leading order arise from the two to two process in the box of Fig. 1(a), and we plot the corresponding string diagram in Fig. 1(c).
Moreover, for \( a \neq b \), we have time delays or advances at the next to the leading order in Fig. 1(a).

To calculate the time delays or advances, we consider the four-fermion scattering amplitude and use the results in Ref. [39] for simplicity. We can discuss the other scattering amplitudes similarly, for example, the four-scalar scattering amplitude, and the results are the same. The total four-fermion scattering amplitude is obtained by summing up the various orderings [39]

\[
\mathcal{A}_{\text{total}} \equiv \mathcal{A}(1, 2, 3, 4) + \mathcal{A}(1, 3, 2, 4) + \mathcal{A}(1, 2, 4, 3) ,
\]

where

\[
\mathcal{A}(1, 2, 3, 4) \equiv A(1, 2, 3, 4) + A(4, 3, 2, 1) .
\]

\( A(1, 2, 3, 4) \) is the standard four-point ordered scattering amplitude

\[
(2\pi)^4 g^{(4)} \langle \sum_a k_a \rangle A(1, 2, 3, 4) = \frac{-i}{g s^2} \int_0^1 dx \langle \mathcal{V}(1)(0,k_1)\mathcal{V}(2)(x,k_2)\mathcal{V}(3)(1,k_3)\mathcal{V}(4)(\infty,k_4) \rangle ,
\]

where \( k_i \) are the space-time momenta, and we used the SL(2,R) symmetry to fix three out of the four \( x_i \) positions on the boundary of the upper half plane, representing the insertions of the open string vertex fermionic ND operators \( \mathcal{V}(i) \), \( i = 1, \ldots, 4 \), defined appropriately in Ref. [39], describing the emission of a massless fermion originating from a string stretched between the D7 brane and the D3 brane.

The amplitudes depend on kinematical invariants expressible in terms of the Mandelstam variables: \( s = -(k_1 + k_2)^2 \), \( t = -(k_2 + k_3)^2 \) and \( u = -(k_1 + k_3)^2 \), for which \( s + t + u = 0 \) for massless particles. The ordered four-point amplitude \( \mathcal{A}(1, 2, 3, 4) \) is given by [39]

\[
\mathcal{A}(1_{j_1}t_1, 2_{j_2}t_2, 3_{j_3}t_3, 4_{j_4}t_4) =
- g s^2 \int_0^1 dx \ (1 - x)^{-1-t^2} \frac{1}{[F(x)]^2} \times
\left[ \bar{u}^{(1)} \gamma_\mu u^{(2)} \bar{u}^{(3)} \gamma_\mu u^{(4)}(1 - x) + \bar{u}^{(1)} \gamma_\mu u^{(4)} \bar{u}^{(2)} \gamma_\mu u^{(3)} x \right]
\times \left\{ \eta_{\delta_{j_1}t_1, \delta_{j_2}t_2} \delta_{j_3}t_3 \delta_{j_4}t_4 \sum_{m \in \mathbb{Z}} e^{-\pi m^2 \ell_2^2/R^2} \right.
\left. + \delta_{j_1}t_1 \delta_{j_2}t_2 \delta_{j_3}t_3 \delta_{j_4}t_4 \sum_{n \in \mathbb{Z}} e^{-\pi n^2 R^2/\ell_2^2} \right\} ,
\]

(10)
where $g_s$ is the string coupling, $u$ is a fermion polarization spinor, $j_i$ and $I_i$ with $i = 1, 2, 3, 4$ are indices on the D7-branes and D3-branes, respectively, and the dependence of the appropriate Chan-Paton factors has been made explicit. Also, $F(x) \equiv F(1/2; 1/2; 1; x)$ is the hypergeometric function, $\tau(x) = F(1 - x)/F(x)$, and $\eta$ is

$$\eta = \frac{(1.55 \ell_s)^4}{V_{\text{AdS}} R}.$$  \hspace{1cm} (11)$$

Thus, taking $F(x) \simeq 1$ we obtain

$$A(1, 2, 3, 4) \propto g_s \ell_s^2 \left( t \ell_s^2 \pi \gamma_\mu u^{(1)} \bar{u}^{(2)} \bar{u}^{(4)} \gamma_\mu u^{(3)} \right)$$

$$+ s \ell_s^2 \pi \gamma_\mu u^{(4)} \bar{u}^{(3)} \bar{u}^{(2)} \gamma_\mu u^{(3)} \times \frac{\Gamma(-s \ell_s^2) \Gamma(-t \ell_s^2)}{\Gamma(1 + u \ell_s^2)} ,$$

$$A(1, 3, 2, 4) \propto g_s \ell_s^2 \left( u \ell_s^2 \pi \gamma_\mu u^{(1)} \bar{u}^{(3)} \bar{u}^{(4)} \gamma_\mu u^{(2)} \right)$$

$$+ u \ell_s^2 \pi \gamma_\mu u^{(4)} \bar{u}^{(3)} \bar{u}^{(2)} \gamma_\mu u^{(2)} \times \frac{\Gamma(-u \ell_s^2) \Gamma(-t \ell_s^2)}{\Gamma(1 + s \ell_s^2)} ,$$

$$A(1, 2, 4, 3) \propto g_s \ell_s^2 \left( u \ell_s^2 \pi \gamma_\mu u^{(1)} \bar{u}^{(3)} \bar{u}^{(2)} \gamma_\mu u^{(4)} \right)$$

$$+ s \ell_s^2 \pi \gamma_\mu u^{(3)} \bar{u}^{(2)} \bar{u}^{(4)} \gamma_\mu u^{(4)} \times \frac{\Gamma(-s \ell_s^2) \Gamma(-u \ell_s^2)}{\Gamma(1 + t \ell_s^2)} ,$$  \hspace{1cm} (12)$$

where the proportionality symbols incorporate Kaluza-Klein or winding mode contributions, which do not contribute to the time delays or advances.

Similarly to the discussions in Ref. [40], time delays or advances arise from the amplitude $A(1, 2, 3, 4)$ by considering backward scattering $u = 0$. Noting that $s + t + u = 0$ for massless particles, the first term in $A(1, 2, 3, 4)$ in Eq. (12) for $u = 0$ is proportional to

$$t \ell_s^2 \pi \gamma_\mu u^{(1)} \bar{u}^{(2)} \bar{u}^{(4)} \gamma_\mu u^{(3)} \times \frac{\Gamma(-s \ell_s^2) \Gamma(-t \ell_s^2)}{\Gamma(1 + u \ell_s^2)} = - s \ell_s^2 \pi \sin (\pi s \ell_s^2) \sin (\pi t \ell_s^2) \Gamma(s \ell_s^2) \Gamma(t \ell_s^2) \Gamma(u \ell_s^2).$$  \hspace{1cm} (13)$$

It has poles at $s = n/\ell_s^2$. The divergence of the amplitude at the poles is an essential physical feature of the amplitude, a resonance corresponding to the propagation of an intermediate string state over long space-time distances. To define the poles we use the correct $\epsilon$ prescription with $\epsilon > 0$. First, we replace $s \rightarrow s + i \epsilon$, which shift the poles off the real axis. Thus, we obtain

$$\frac{1}{\sin (\pi s \ell_s^2)} = - i \sum_{n \geq 0} e^{i(2n+1)\pi s \ell_s^2} + \mathcal{O} (\epsilon).$$  \hspace{1cm} (14)$$

So the particle velocity is subluminal. On noting that $s = E^2$, we obtain the time delay due to one D3-brane

$$\Delta t = (2n + 1) \pi E \ell_s^2 , \quad \text{where } n \geq 0 .$$  \hspace{1cm} (15)$$
Thus, the velocity of the particle is
\[
v = \frac{1}{1 + \frac{(2n + 1)\pi \xi M_{St}}{E c}}, \quad \delta v \simeq \frac{(2n + 1)\pi E}{\xi M_{St}},
\]
where
\[
M_{St} \equiv \frac{1}{\ell_s}, \quad \xi \equiv M_{St} \times V_{A3}^{1/3}.
\]
Here, \(M_{St}\) is the string scale. We emphasize that \(V_{A3}\) is the local value around the D3-brane.

Second, we consider \(s \to s - i\epsilon\), which also shift the poles off the real axis. Thus, we obtain
\[
\frac{1}{\sin(\pi s \ell_s^2 / s)} = i \sum_{n \geq 0} e^{-i(2n+1)\pi s \ell_s^2} + O(\epsilon).
\]
Interestingly, we obtain the superluminal particle propagation. The time advance due to one D3-brane is
\[
\Delta t = -(2n + 1)\pi E \ell_s^2, \quad \text{where } n \geq 0.
\]
Thus, the velocity of the particle is
\[
v = \frac{1}{1 - \frac{(2n + 1)\pi \xi M_{St}}{E c}}, \quad \delta v \simeq \frac{(2n + 1)\pi E}{\xi M_{St}}.
\]
We emphasize that these results on relativistic particle velocities are brand new.

Let us discuss the time delays or advances at the leading order for concrete particles. We will assume that \(\eta\) is a small number for a perturbative theory. Then, for the gauge fields and their corresponding gauginos which are related to the Cartan subalgebras of the SM gauge groups, all the amplitudes \(A(1, 2, 3, 4), A(1, 3, 2, 4), \) and \(A(1, 2, 4, 3)\) will give the dominant contributions to the total amplitude due to \(j_1 = \bar{j}_2\). Thus, they will have time delays or advances as given in Eq. (15) or (19), respectively. The resulting delay or advance for photon is independent of its polarization, and thus there is no birefringence, thereby leading to the evasion of the relevant stringent astrophysical constraints [41–43]. Especially, the photon refractive index is
\[
n_\gamma = 1 \pm \frac{(2n + 1)\pi E_\gamma}{\xi M_{St}} \equiv 1 \pm \frac{(2n + 1)E_\gamma}{M_{QG}},
\]
where the corresponding effective quantum gravity scale is

\[ M_{QG} = \frac{\xi M_{St}}{\pi}. \]  

(22)

However, for all the other SM particles, we have \( j_1 \neq \bar{j}_2 \), and then only the amplitude \( \mathcal{A}(1, 3, 2, 4) \) gives dominant contribution. Considering backward scattering \([40]\) \( u = 0 \) and \( s + t + u = 0 \), we obtain

\[
\mathcal{A}(1, 3, 2, 4) \propto g_s \ell_s^2 \left( \frac{1}{u \ell_s^2} \gamma^u_1 \gamma^u_2 \gamma^u_3 \gamma^u_4 \gamma^u_1 \right) - \frac{1}{s \ell_s^2} u \ell_s^2 \gamma^u_1 \gamma^u_2 \gamma^u_3 \gamma^u_4 \gamma^u_1. \]  

(23)

Because they are just the pole terms, we do not have time delays or advances for the other particles with \( j_1 \neq \bar{j}_2 \) at the leading order, for example, \( W^\pm \) boson, electron, and neutrinos, etc. At the order \( \eta (\mathcal{O}(\eta)) \), we have time delays or advances for these particles, which arise from the \( \eta \) term in Eq. \([10]\). Thus, we obtain \( \delta v \) for all the other SM particles as follows

\[
\delta v \simeq \pm \frac{(2n + 1) \eta E}{M_{QG}} = \pm \frac{(2n + 1) \eta E}{M_{QG}}. \]  

(24)

In short, the additional contributions to the particle velocity \( \delta v \) from string theory is proportional to both the particle energy and the D3-brane number density, and is inversely proportional to the string scale. Thus, we can realize the background dependent Lorentz violation naturally by varying the D3-brane number density in space time.

### III. PHENOMENOLOGICAL CONSEQUENCES

In this Section, we shall use the background dependent Lorentz violation to explain the OPERA, MINOS, MAGIC, HESS, and FERMI experiments simultaneously. In particular, we assume that \( V_{A3} \), which is the inverse of the D3-brane number density, is space-time dependent. For simplicity, we assume that \( R' = 2.5\ell_s \), and we consider the lowest order for time delays or advances, \( i.e. \), \( n = 1 \). To explain the MAGIC, HESS, and FERMI experiments, the photons must propagate subluminally. And to explain the OPERA and MINOS experiments, the muon neutrinos must propagate superluminally. Therefore, we assume that the muon neutrinos propagate superluminally, while the photons propagate subluminally. The fundamental explanation on this assumption deserves further study. The
possible reason is the following: photon is a vector boson while muon neutrino is a fermion, and the propagator of a fermion is similar to the square root of the propagator of a boson.

First, let us consider the OPERA and MINOS experiments. We denote the parameters $V_{A3}$, $\eta$, and $\xi$ on the Earth as $V_{A3}^E$, $\eta^E$, and $\xi^E$, respectively. For simplicity, we choose $(V_{A3}^E)^{1/3} = 2.5 \ell_s$, and then we have $\eta^E = 0.148$ and $\xi^E = 2.5$. Using the OPERA mean muon neutrino energy 17 GeV and the central value for $\delta \nu_\mu$, we obtain

$$M_{St} = 1.27 \times 10^5 \text{ GeV}.$$  \hspace{1cm} (25)

Thus, the physical volume of the three extra dimensions transverse to the D7-branes is very large. Because $\delta \nu_\mu$ is linearly proportional to the neutrino energy, we emphasize that our result is consistent with the $\delta \nu_\mu$ weak dependence on the neutrino energy from the OPERA experiment. Moreover, we understand that this may be a fruitless exercise, but it is certainly an imagination stretcher and see how far off we can find ourselves in order to explain the OPERA anomaly. Let us subscribe to the Eddington’s dictum: “Never believe an experiment until it has been confirmed by theory”.

Second, we consider the MAGIC, HESS, and FERMI experiments. We denote the parameters $V_{A3}$, $\eta$, and $\xi$ on the interstellar scale as $V_{A3}^{IS}$, $\eta^{IS}$, and $\xi^{IS}$, respectively. To have the effective quantum gravity scale $M_{QG} \simeq 0.98 \times 10^{18}$ GeV, we get

$$(V_{A3}^{IS})^{1/3} = 1.9 \times 10^8 \, [\text{GeV}]^{-1}, \quad \eta^{IS} = 1.6 \times 10^{-40}, \quad \xi^{IS} = 2.4 \times 10^{13}. \hspace{1cm} (26)$$

Moreover, the FERMI observation of GRB 090510 may be explained by choosing smaller $V_{A3}^{IS}$ in the corresponding space direction [33].

Because $\eta$ is equal to $1.6 \times 10^{-40}$, our models can automatically satisfy not only the constraints from the SN1987a observations on neutrinos, but also the following astrophysical constraints on charged leptons:

- The constraint from the Crab Nebula synchrotron radiation observations on the electron dispersion relation [28].
- The electron vacuum Cherenkov radiation via the decay process $e \rightarrow e\gamma$ becomes kinematically allowed for $E_e > m_e/\sqrt{\delta \nu_e}$. Note that the cosmic ray electrons have been detected up to 2 TeV, we have $\delta \nu_e < 10^{-13}$ from cosmic ray experiments [34].
• The high-energy photons are absorbed by CMB photons and annihilate into the electron-positron pairs. The process $\gamma \gamma \rightarrow e^+ e^-$ becomes kinematically possible for $E_{\text{CMB}} > m_e^2 / E_\gamma + \delta v_e E_\gamma / 2$. Because it has been observed to occur for photons with energy about $E_\gamma = 20$ TeV, we have $\delta v_e < 2m_e^2 / E_\gamma^2 \sim 10^{-15}$ from cosmic ray experiments [35].

• The process, where a photon decays into $e^+ e^-$, becomes kinematically allowed at energies $E_\gamma > m_e \sqrt{-2\delta v_e}$. As the photons have been observed up to 50 TeV, we have $-\delta v_e < 2 \times 10^{-16}$ from cosmic ray experiments [34]. The analogous bound for the muon is $-\delta v_\mu < 10^{-11}$ [36].

Furthermore, there are possible challenges to our models from the relevant experiments done on the Earth. Let us address them in the following:

• The agreement between the observation and the theoretical expectation of electron synchrotron radiation as measured at LEP [44] gives the bound $|\delta v_e| < 5 \times 10^{-15}$. The possible solution is that the vacuum in the LEP tunnel is similar to the vacuum on the interstellar scale.

• For the electron neutrinos at the KamLAND, the non-trivial energy dependence of the neutrino survival probability implies that the Lorentz violating off-diagonal elements of the $\delta v_\nu$ matrix in the flavor space are smaller than about $10^{-20}$ [45, 46]. The solution is that our D3-branes are flavour blind.

IV. CONCLUSIONS

We revisited the Lorentz violation in the Type IIB string theory with D3-branes and D7-branes. We studied the relativistic particle velocities in details, and showed that there exist both subluminal and superluminal particle propagations. In particular, the additional contributions to the particle velocity $\delta v$ from string theory is proportional to both the particle energy and the D3-brane number density, and is inversely proportional to the string scale. Thus, we can realize the background dependent Lorentz violation naturally by varying the D3-brane number density in the space time. To explain the OPERA and MINOS experiments, we obtained that the string scale is around $10^5$ GeV. With very tiny D3-brane number density on the interstellar scale, we can also explain the MAGIC, HESS, and
FERMI experiments simultaneously. Interestingly, we can automatically satisfy the stringent constraints from the synchrotron radiation of the Crab Nebula, the SN1987a observations on neutrinos, and the cosmic ray experiments on charged leptons. We also briefly discussed the solutions to the possible phenomenological challenges to our models from the relevant experiments done on the Earth.

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