Application of the hierarchy analysis method for the choice of the computer mathematics system for the IT-sphere specialists preparation

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Abstract. Peculiarities of the usage of the hierarchy analysis method for the making decision on the choice of the most efficient computer mathematics system (CMS) used for the preparation of the IT-sphere specialists are in the focus of the research. Eight alternatives were selected for the hierarchy analysis method and seven criteria were considered. The alternative is chosen by means of figuring of the priority vector corresponding to each alternative. The alternative with the highest value is considered the right decision. The scale of impact was used that is the scale of evaluations for the paired comparisons of the advantage of the first object over other with meaning from 1 to 9. The basic criteria for which the alternatives are calculated were described in the course research; their characteristics were chosen, in particular functionalities, studying materials, on-line mode, mobile application, license, language support and an open code. The matrix of the paired comparisons for each criterion was constructed and numerical characteristics of these matrices were calculated – the highest own value, the index of compatibility and the index of the consistency of formulas. Every matrix is composed of the expert evaluations for the alternatives, which are the systems of computer mathematics used in the courses of mathematics. Based on the results of the calculations carried out the impact of the alternative was shown and the system having the highest impact was chosen.

1. Introduction

Nowadays there is a big number of information technologies including computer mathematics systems (CMS) used for teaching mathematics [2]. All of them have a wide-range functional set that is constantly developing and updating [4]. However, every CMS has its drawbacks. Thus, there is a need to choose the right CMS for teaching mathematics to the future IT-specialists [8].

There is no algorithm which can definitely state that every particular task has the most effective CMS [6]. That is why CMS is chosen for every specific context. Accounting for the big number of various systems of computer mathematics it is reasonable to try to automatize the process of choice of such learning tools [7]. To support the choice of the most suitable CMS we propose to use hierarchy analysis method.
The aim of the research is the usage of the hierarchy analysis method for the choice of the most effective CMS, which is used for teaching mathematics in the process of IT-specialists training.

The method is based on the knowledge of experts. And in our case, a specific group of experts preferred CMS for IT-specialists, because the basis of programming is the ability to build a mathematical model, knowledge of efficient algorithms, the process of creating algorithms to solve the problem.

2. Results
The hierarchy analysis method involves decomposition of the problem into the simpler constituents and following pair-wise comparison of the constituents at every next level of the hierarchy. As the result a relatively stable degree of cooperation of the constituents in the hierarchy can be calculated. These statements are expressed numerically. The method includes procedures of synthesis of multiple statements, obtaining of the priority criteria and establishment of the evaluations of the alternative decisions. The hierarchy is being plotted from the top (aim), through the intermediate levels (criteria determining the following levels) and to the lowest level which is a list of the alternatives [5]. The elements of the task in the hierarchy analysis method are compared in pairs with an account of their influence on their common characteristics.

It is showed later in this article the hierarchy analysis method for the choice of the most effective system of computer mathematics consisting of eight alternatives and including seven criteria.

The aim of the research is to choose one of the alternatives on the basis of multiple criteria. The choice of the alternative is carried out by means of the calculating of the elements of the priority vector corresponding to every alternative. The alternative with the highest value of this element is considered a decision made [1].

The hierarchy analysis method uses the scale of the expert evaluations (degree of impact) which are the evaluations for the paired comparisons for the determining of the advantage of the first object over other with meaning from 1 to 9. The general content of these evaluations is set in table 1 [3].

| Degree of impact | Definition | Commentary |
|------------------|------------|------------|
| 1                | Equal impact factor | Both object make equal contributions into the achieving of the aim |
| 3                | Weak impact factor | Experience and speculations make the first object a little bit advantageous over the other object |
| 5                | Significant or strong impact factor | Experience and speculations make the first object much more advantageous over the other object |
| 7                | Very strong and obvious impact factor | The advantage of the first object over the other object is very strong. Its advantage is practically obvious. |
| 9                | Absolute impact factor | Evidence is in favour of the highly convincing advantage of the first object over the other object. |
| 2, 4, 6, 8       | Reciprocal values of the given valuations | The situation requiring compromise decisions |
|                  | Intermediate values between neighbouring values of the scale | |

Based on the results of the analysis of the functionalities of the chosen CMS a multiplicity of the criteria was formed. These criteria should serve as a basis for the choice of the system grounding on the expert evaluations received with the help of the hierarchy analysis method.
The basic criteria for which the alternatives were calculated and their characteristics were chosen received such names:

1. functionalities;
2. studying materials;
3. on-line mode;
4. mobile application;
5. license;
6. language support;
7. open code.

The structure of the task for the decision making with the help of the hierarchy analysis method for the choice of CMS for teaching mathematics to future IT-sphere specialists is shown in figure 1.

![Diagram of CMS decision making](image)

**Figure 1.** The structure of the task for making of the decision with the help of the hierarchy analysis method for the choice of CMS for teaching mathematics.

To solve the task of the choice of CMS we need to use the hierarchy analysis method. To do this we need to plot matrixes for the paired comparisons for each criterion and calculate new numerical characteristics for theses matrixes, namely the highest own value, an index of compatibility and index of the sequence of the correlations. Every matrix consists of the expert evaluations concerning the pairs of the alternatives that are the systems of computer mathematics used for teaching of the mathematics courses.

In case of the application of the hierarchy analysis method as related to the plotting of the matrix of the paired comparisons for every criterion we will provide additional speculations concerning peculiarities of the usage of the system of computer mathematics used for teaching of the IT-specialists.

The criterion “functionality”. This criterion defines an opportunity of solving using a system with a particular range of tasks as the systems significantly differ from each other in their functionality (universal systems and system of the narrow focus). The matrix of the paired comparisons for the choice of the alternative system by the criterion “functionality” is showed in table 2.

Using the matrix (table 2) we can calculate the “impact factor” (priority vector). Speaking the language of mathematics we can interpret it as calculation of the main own vector which upon the normalization becomes a priority vector. To get an analytical evaluation of this vector we can use the algorithm:

1. We calculate the sum of the elements of each column:
   \[ S_i = a_{1i} + a_{2i} + \ldots + a_{ni}. \]

2. We divide all the elements of each column of the matrix to the sum of the elements of the corresponding column: \[ A_{ij} = \frac{a_{ij}}{S_i}. \]
Let us calculate such parameters for the matrix of the alternative system by the criterion “functionality”.

**Table 2.** The matrix of the paired comparisons for the choice of the alternative system by the criterion “functionality”.

| Alternatives | Maple  | Mathematica | Mathcad | Maxima | MATLAB | SMath Studio | SageMath | GeoGebra |
|--------------|--------|-------------|---------|--------|--------|--------------|----------|----------|
| Maple        | 1.00   | 1.00        | 6.00    | 2.00   | 2.00   | 6.00         | 3.00     | 1.00     |
| Mathematica  | 1.00   | 1.00        | 6.00    | 2.00   | 2.00   | 6.00         | 3.00     | 1.00     |
| Mathcad      | 0.17   | 0.17        | 1.00    | 0.25   | 0.25   | 1.00         | 0.33     | 0.17     |
| Maxima       | 0.50   | 0.50        | 4.00    | 1.00   | 1.00   | 4.00         | 2.00     | 0.50     |
| MATLAB       | 0.50   | 0.50        | 4.00    | 1.00   | 1.00   | 4.00         | 2.00     | 0.50     |
| SMath Studio | 0.17   | 0.17        | 1.00    | 0.25   | 0.25   | 1.00         | 0.33     | 0.17     |
| SageMath     | 0.33   | 0.33        | 3.00    | 0.50   | 0.50   | 3.00         | 1.00     | 0.33     |
| GeoGebra     | 1.00   | 1.00        | 6.00    | 2.00   | 2.00   | 6.00         | 3.00     | 1.00     |
| **Sum**      | 4.67   | 4.67        | 31.00   | 9.00   | 9.00   | 31.00        | 14.66    | 4.66     |

These two operations are called matrix normalizing.

3. Then we add up the elements of each line.

\[ S_l = A_{l1} + A_{l2} + \ldots + A_{ln}. \]

4. And find out the arithmetic mean of each line.

\[ w_l = \frac{S_l}{n} \]

The results are shown in table 3. The last column is a priority vector.

The priority vector is calculated as the evaluation of the main own vector of the matrix of the paired comparisons. The elements of this vector are the impact factors of the alternatives, which were calculated as an algebraic expression of the elements of the corresponding column of the table 3 divided by the general number of the alternatives – number of the elements of the line in table 2.

**Table 3.** The impact factor of the alternatives by the criterion “functionality”.

| Alternatives | Maple  | Mathematica | Mathcad | Maxima | MATLAB | SMath Studio | SageMath | GeoGebra | Sum | Impact factor of all alternatives |
|--------------|--------|-------------|---------|--------|--------|--------------|----------|----------|-----|---------------------------------|
| Maple        | 0.214  | 0.214       | 0.194   | 0.222  | 0.222  | 0.194        | 0.205    | 0.214    | 1.679| 0.210                           |
| Mathematica  | 0.214  | 0.214       | 0.194   | 0.222  | 0.222  | 0.194        | 0.205    | 0.214    | 1.679| 0.210                           |
| Mathcad      | 0.036  | 0.036       | 0.032   | 0.028  | 0.028  | 0.032        | 0.023    | 0.036    | 0.250| 0.031                           |
| Maxima       | 0.107  | 0.107       | 0.129   | 0.111  | 0.111  | 0.129        | 0.136    | 0.107    | 0.938| 0.117                           |
| MATLAB       | 0.107  | 0.107       | 0.129   | 0.111  | 0.111  | 0.129        | 0.136    | 0.107    | 0.938| 0.117                           |
| SMath Studio | 0.036  | 0.036       | 0.032   | 0.028  | 0.028  | 0.032        | 0.023    | 0.036    | 0.251| 0.031                           |
| SageMath     | 0.071  | 0.071       | 0.097   | 0.056  | 0.056  | 0.097        | 0.068    | 0.071    | 0.585| 0.073                           |
| GeoGebra     | 0.214  | 0.214       | 0.194   | 0.222  | 0.222  | 0.194        | 0.205    | 0.214    | 1.679| 0.210                           |
| **Sum**      | 1.000  | 1.000       | 1.000   | 1.000  | 1.000  | 1.000        | 1.000    | 1.000    | 8.000| 1.000                           |

Thus, by the criterion “functionality” the best alternatives are the three systems Maple, Mathematica and GeoGebra as they have the highest value of the impact factor – 0.210.

Let us calculate such parameters for the matrix of the paired comparisons plotted by the criterion “functionality”:

– the evaluation of the own value calculated: \( \lambda_{max} = \sum_{i=1}^{n} w_i S_i \)
where \( w_i \) – the impact factor of the alternatives with a number \( i \), \( S_i \) – the sum of the elements of the column with a number \( i \) of the matrix of the paired comparisons, \( n \) – a quantity of alternatives;

- the consistency index: \( CI = \frac{\lambda_{\max}}{n-1} \);
- the index of ratio sequence: \( CR = CI/RI \).  

\[ RI = 1.41 \] – random index for \( n = 8 \), the values of which are equal for the rest of the following calculations of the impact factors of the alternatives.

Upon the calculation of the matrix of the paired comparisons plotted by the criterion “functionality”, the stated parameters acquire the following values:

\[ \lambda_{\max} = 0.21 \cdot 4.67 + 0.21 \cdot 4.67 + 0.031 \cdot 31 + 0.117 \cdot 9 + 0.117 \cdot 9 + 0.031 \cdot 31 + +0.073 \cdot 14.66 + 0.21 \cdot 4.67 = 8.067 \]

- the consistency index: \( CI = \frac{\lambda_{\max} - n}{n-1} = \frac{8.067-8}{8-1} = 0.0097 \);
- the index of ratio sequence: \( CR = CI/RI = 0.01/1.41 = 0.007 \).  

As \( CR = 0.7\% < 10\% \), then the matrix of the paired comparisons by the criterion “functionality” is considered consistent.

By the criterion “teaching materials”. In this context we consider the availability of the teaching materials for easy mastering of the work in the system of computer mathematics. Special attention is paid to the instructional guidelines for the work in the system, video lessons, developed applets and algorithms of the tasks solution.

The results of the calculation of the impact factor of the alternatives by the criterion “teaching materials” are showed in table 5. The elements of the columns are were obtained as a result of the normalizing of the corresponding elements of table 4. Similar to the criterion “functionality” the priority vector is calculated as an evaluation of the own main vector of the matrix of the paired comparisons where the elements are the impact factors of the alternatives. The last were calculated as an algebraic expression of the elements of the corresponding line divided by the general number of the alternatives.

**Table 4. The matrix of the paired comparisons of the alternatives by the criterion “teaching materials”.**

| Alternatives | Maple | Mathematica | Mathcad | Maxima | MATLAB | SMath Studio | SageMath | GeoGebra |
|--------------|-------|-------------|---------|--------|--------|-------------|----------|----------|
| Maple        | 1.00  | 0.33        | 3.00    | 3.00   | 2.00   | 0.50        | 0.50     | 0.25     |
| Mathematica | 3.00  | 1.00        | 5.00    | 5.00   | 4.00   | 2.00        | 2.00     | 0.50     |
| Mathcad     | 0.33  | 0.20        | 1.00    | 1.00   | 0.50   | 0.25        | 0.25     | 0.14     |
| Maxima       | 0.33  | 0.20        | 1.00    | 1.00   | 0.50   | 0.25        | 0.25     | 0.14     |
| MATLAB     | 0.50  | 0.25        | 2.00    | 2.00   | 1.00   | 0.33        | 0.33     | 0.17     |
| SMath Studio | 2.00  | 0.50        | 4.00    | 4.00   | 3.00   | 1.00        | 1.00     | 0.33     |
| SageMath   | 2.00  | 0.50        | 4.00    | 4.00   | 3.00   | 1.00        | 1.00     | 0.33     |
| GeoGebra    | 4.00  | 2.00        | 7.00    | 7.00   | 6.00   | 3.00        | 3.00     | 1.00     |
| **Sum** | **13.16** | **4.98** | **27.00** | **27.00** | **20.00** | **8.33** | **8.33** | **2.86** |

Thus, by the criterion “teaching materials” the best alternative is the system GeoGebra as it has the highest impact factor – 0.324.

For the matrix of the paired comparisons plotted by the criterion “teaching materials” the following parameters were calculated:

- evaluation of the highest own value: \( \lambda_{\max} = 8.165 \);
- the consistency index: \( CI = 0.0236 \);
- the index of ratio sequence: \( CR = 0.017 \).
Table 5. The impact factors of the alternatives by the criterion “teaching materials”.

| Alternatives | Maple | Mathematica | Mathcad | Maxima | MATLAB | SMath Studio | SageMath | GeoGebra | Sum | Impact factor of the alternatives |
|--------------|-------|-------------|---------|--------|--------|--------------|----------|----------|-----|----------------------------------|
| Maple        | 0.076 | 0.066       | 0.111  | 0.111 | 0.100  | 0.060        | 0.060    | 0.087    | 0.672| 0.084                            |
| Mathematica  | 0.228 | 0.201       | 0.185  | 0.185 | 0.200  | 0.240        | 0.240    | 0.175    | 1.654| 0.207                            |
| Mathcad      | 0.025 | 0.040       | 0.037  | 0.037 | 0.025  | 0.030        | 0.030    | 0.050    | 0.274| 0.034                            |
| Maxima       | 0.025 | 0.040       | 0.037  | 0.037 | 0.025  | 0.030        | 0.030    | 0.049    | 0.273| 0.034                            |
| MATLAB       | 0.038 | 0.050       | 0.074  | 0.074 | 0.050  | 0.040        | 0.040    | 0.058    | 0.424| 0.053                            |
| SMath Studio | 0.152 | 0.100       | 0.148  | 0.148 | 0.150  | 0.120        | 0.120    | 0.115    | 1.054| 0.132                            |
| SageMath     | 0.152 | 0.100       | 0.148  | 0.148 | 0.150  | 0.120        | 0.120    | 0.115    | 1.054| 0.132                            |
| GeoGebra     | 0.304 | 0.402       | 0.259  | 0.259 | 0.300  | 0.360        | 0.360    | 0.350    | 2.594| 0.324                            |
| Sum          | 1.000 | 1.000       | 1.000  | 1.000 | 1.000  | 1.000        | 1.000    | 1.000    | 8.000| 1.000                            |

As $CR = 1.7% < 10\%$, then the matrix of the paired comparisons by the criterion “teaching materials” is considered consistent.

The criterion “on-line mode”. An opportunity to work with CMS online without loading and installation on the computer is considered. Convenience of work in the system is also taken into account. The results of the calculation of the impact factor by the criterion “on-line mode” is showed in table 7. The elements of the columns were obtained through normalizing of the corresponding elements of table 6.

Table 6. The matrix of the paired comparisons of the alternatives by the criterion “on-line mode”.

| Alternatives | Maple | Mathematica | Mathcad | Maxima | MATLAB  | SMath Studio | SageMath | GeoGebra  |
|--------------|-------|-------------|---------|--------|---------|--------------|----------|-----------|
| Maple        | 1.00  | 0.25        | 7.00    | 7.00   | 1.00    | 3.00         | 3.00     | 0.25      |
| Mathematica  | 4.00  | 1.00        | 9.00    | 9.00   | 4.00    | 5.00         | 5.00     | 1.00      |
| Mathcad      | 0.14  | 0.11        | 1.00    | 1.00   | 0.14    | 0.20         | 0.20     | 0.11      |
| Maxima       | 0.14  | 0.11        | 1.00    | 1.00   | 0.14    | 0.20         | 0.20     | 0.11      |
| MATLAB       | 1.00  | 0.25        | 7.00    | 7.00   | 1.00    | 3.00         | 3.00     | 0.20      |
| SMath Studio | 0.33  | 0.20        | 5.00    | 5.00   | 0.33    | 1.00         | 1.00     | 0.17      |
| SageMath     | 0.33  | 0.20        | 5.00    | 5.00   | 0.33    | 1.00         | 1.00     | 0.17      |
| GeoGebra     | 4.00  | 1.00        | 9.00    | 9.00   | 5.00    | 6.00         | 6.00     | 1.00      |
| Sum          | 10.94 | 3.12        | 44.00   | 44.00  | 11.94   | 19.40        | 19.40    | 3.01      |

Thus, the system GeoGebra is the best alternative by the criterion “on-line mode” as it has the highest impact factor – 0.308.

For the matrix of the paired comparisons plotted by the criterion “on-line mode” the following criteria were calculated:

- evaluation of the highest own value: $\lambda_{max} = 8.811$;
- the consistency index: $CI = 0.1159$;
- the index of ratio sequence: $CR = 0.082$.

As $CR = 8.2% < 10\%$, then the matrix of the paired comparisons by the criterion “on-line mode” is considered consistent.
The criterion “mobile application”. An opportunity of loading of CMS into the mobile device as well as opportunities to work in this mobile application are analyzed.

The results of the calculations of the impact factor of the alternatives by the criterion “mobile application” are showed in table 9. The elements of the columns were obtained through normalizing of the corresponding elements of table 8.

Table 8. The matrix of the paired comparisons of the alternatives by the criterion “mobile application”.

| Alternatives   | Maple | Mathematica | Mathcad | Maxima | MATLAB | SMath Studio | SageMath | GeoGebra |
|----------------|-------|-------------|---------|--------|--------|-------------|----------|----------|
| Maple          | 1.00  | 0.13        | 1.00    | 0.33   | 0.17   | 0.17        | 1.00     | 0.11     |
| Mathematica    | 8.00  | 1.00        | 8.00    | 5.00   | 2.00   | 2.00        | 8.00     | 0.50     |
| Mathcad        | 1.00  | 0.13        | 1.00    | 0.33   | 0.17   | 0.17        | 1.00     | 0.11     |
| Maxima         | 3.00  | 0.20        | 3.00    | 1.00   | 0.33   | 0.33        | 3.00     | 0.17     |
| MATLAB         | 6.00  | 0.50        | 6.00    | 3.00   | 1.00   | 1.00        | 6.00     | 0.25     |
| SMath Studio   | 6.00  | 0.50        | 6.00    | 3.00   | 1.00   | 1.00        | 6.00     | 0.25     |
| SageMath       | 1.00  | 0.13        | 1.00    | 0.33   | 0.17   | 0.17        | 1.00     | 0.11     |
| GeoGebra       | 9.00  | 2.00        | 9.00    | 6.00   | 4.00   | 4.00        | 9.00     | 1.00     |
| Sum            | 35.00 | 4.58        | 35.00   | 18.99  | 8.83   | 8.84        | 35.00    | 2.50     |

Thus, the system GeoGebra is the best alternative by the criterion “mobile application” as it has the highest impact factor – 0.354.

For the matrix of the paired comparisons plotted by the criterion “mobile application” the following criteria were calculated:

- evaluation of the highest own value: \( \lambda_{\text{max}} = 8.347; \)
- the consistency index: \( CI = 0.0496; \)
- the index of ratio sequence: \( CR = 0.035. \)

As \( CR = 3.5\% < 10\%, \) then the matrix of the paired comparisons by the criterion “mobile application” is considered consistent.
Table 9. The impact factor of the alternatives by the criterion “mobile application”.

| Alternatives | Maple | Mathematica | Mathcad | Maxima | MATLAB | SMath Studio | SageMath | GeoGebra | Sum | Impact factor of the alternatives |
|--------------|-------|-------------|---------|--------|--------|--------------|----------|----------|-----|----------------------------------|
| Maple        | 0.029 | 0.027       | 0.029   | 0.017  | 0.019  | 0.029        | 0.044    | 0.213    |     | 0.027                           |
| Mathematica  | 0.229 | 0.218       | 0.229   | 0.263  | 0.226  | 0.229        | 0.200    | 1.820    |     | 0.228                           |
| Mathcad      | 0.029 | 0.028       | 0.029   | 0.017  | 0.019  | 0.029        | 0.044    | 0.214    |     | 0.027                           |
| Maxima       | 0.086 | 0.044       | 0.086   | 0.053  | 0.037  | 0.086        | 0.067    | 0.495    |     | 0.062                           |
| MATLAB       | 0.171 | 0.109       | 0.171   | 0.158  | 0.113  | 0.171        | 0.100    | 1.108    |     | 0.138                           |
| SMath Studio | 0.171 | 0.109       | 0.171   | 0.158  | 0.113  | 0.171        | 0.100    | 1.108    |     | 0.138                           |
| SageMath     | 0.029 | 0.027       | 0.029   | 0.017  | 0.019  | 0.029        | 0.044    | 0.213    |     | 0.027                           |
| GeoGebra     | 0.257 | 0.437       | 0.257   | 0.316  | 0.453  | 0.452        | 0.257    | 0.400    | 8.00 | 0.354                           |
| Sum          | 1.000 | 1.000       | 1.000   | 1.000  | 1.000  | 1.000        | 1.000    | 8.000    |     | 1.000                           |

The criterion “license”. This criterion allows analyzing of the opportunities of application of CMS as the main drawback of the systems is that they are commercial that is one needs to get a license to use them. This fact significantly complicates the usage of such systems in the teaching process. The situation is quite different when the usual systems of computer mathematics are used.

The results of the calculations of the impact factor of the alternatives by the criterion “license” are showed in table 11. The elements of the columns were obtained through normalizing of the corresponding elements of table 10.

Table 10. The matrix of the paired comparisons of the alternatives by the criterion “license”.

| Alternatives | Maple | Mathematica | Mathcad | Maxima | MATLAB | SMath Studio | SageMath | GeoGebra |
|--------------|-------|-------------|---------|--------|--------|--------------|----------|----------|
| Maple        | 1.00  | 0.33        | 0.20    | 0.14   | 0.50   | 0.14         | 0.14     | 0.14     |
| Mathematica  | 3.00  | 1.00        | 0.25    | 0.14   | 0.00   | 0.14         | 0.14     | 0.14     |
| Mathcad      | 5.00  | 4.00        | 1.00    | 0.14   | 5.00   | 0.14         | 0.14     | 0.14     |
| Maxima       | 7.00  | 7.00        | 7.00    | 1.00   | 7.00   | 1.00         | 1.00     | 1.00     |
| MATLAB       | 2.00  | 0.33        | 0.20    | 0.14   | 1.00   | 0.14         | 0.14     | 0.14     |
| SMath Studio | 7.00  | 7.00        | 7.00    | 1.00   | 7.00   | 1.00         | 1.00     | 1.00     |
| SageMath     | 7.00  | 7.00        | 7.00    | 1.00   | 7.00   | 1.00         | 1.00     | 1.00     |
| GeoGebra     | 7.00  | 7.00        | 7.00    | 1.00   | 7.00   | 1.00         | 1.00     | 1.00     |
| Sum          | 39.00 | 33.66       | 29.65   | 4.56   | 34.50  | 4.56         | 4.56     | 4.56     |

Thus, the three systems Maxima, SageMath, SMath Studio, GeoGebra are the best by the criterion “license” as they have the same highest value of the impact factor – 0.213.

For the matrix of the paired comparisons plotted by the criterion “license”, the following criteria were calculated:

- evaluation of the highest own value: $\lambda_{max} = 8.743$;
- the consistency index: $CI = 0.1061$;
- the index of ratio sequence: $CR = 0.075$.

As $CR = 7.5\% < 10\%$, then the matrix of the paired comparisons by the criterion “license” is considered consistent.
The criterion “language support”. This criterion allows analyzing in which languages a particular system can function; an issue of Ukrainian-language and Russian-language interface of the system is in the focus of attention.

The results of the calculations of the impact factor of the alternatives by the criterion “language support” are showed in table 13. The elements of the columns were obtained through normalizing of the corresponding elements of table 12.

Table 12. The matrix of the paired comparisons of the alternatives by the criterion “language support”.

| Alternatives | Maple | Mathematica | Mathcad | Maxima | MATLAB | SMath Studio | SageMath | GeoGebra |
|--------------|-------|-------------|---------|--------|--------|--------------|----------|----------|
| Maple        | 1.00  | 0.33        | 0.25    | 0.33   | 1.00   | 0.14         | 1.00     | 0.11     |
| Mathematica  | 3.00  | 1.00        | 0.33    | 2.00   | 3.00   | 0.20         | 3.00     | 0.14     |
| Mathcad      | 4.00  | 3.00        | 1.00    | 4.00   | 4.00   | 0.33         | 4.00     | 0.33     |
| Maxima       | 3.00  | 0.50        | 0.25    | 1.00   | 3.00   | 0.25         | 3.00     | 0.14     |
| MATLAB       | 1.00  | 0.33        | 0.25    | 0.33   | 1.00   | 0.14         | 1.00     | 0.11     |
| SMath Studio | 7.00  | 5.00        | 3.00    | 4.00   | 7.00   | 1.00         | 7.00     | 0.50     |
| SageMath     | 1.00  | 0.33        | 0.25    | 0.33   | 1.00   | 0.14         | 1.00     | 0.11     |
| GeoGebra     | 9.00  | 7.00        | 3.00    | 7.00   | 9.00   | 2.00         | 9.00     | 1.00     |
| Sum          | 29.00 | 17.49       | 8.33    | 18.99  | 29.00  | 4.21         | 29.00    | 2.44     |

Thus, the system GeoGebra is the best alternative by the criterion “language support” as it has the highest impact factor – 0.368.

For the matrix of the paired comparisons plotted by the criterion “language support” the following criteria were calculated:
- evaluation of the highest own value: \( \lambda_{\text{max}} = 8.477 \);
- the consistency index: \( CI = 0.0681 \);
- the index of ratio sequence: \( CR = 0.048 \).

As \( CR = 4.8\% < 10\% \), then the matrix of the paired comparisons by the criterion “language support” is considered consistent.
The criterion “open code”. This criterion allows analyzing an opportunity of improvement of CMS by means of additional functionality. An availability of open code for the developers is checked.

The results of the calculations of the impact factor of the alternatives by the criterion “open code” are showed in table 15. The elements of the columns were obtained through normalizing of the corresponding elements of table 14.

Table 13. The impact factor of the alternatives by the criterion “language support”.

| Alternatives | Maple | Mathematica | Matlab | Maxima | MATLAB | SMath Studio | SageMath | GeoGebra | Sum | Impact factor of the alternatives |
|--------------|-------|-------------|--------|--------|--------|--------------|----------|----------|-----|---------------------------------|
| Maple        | 0.034 | 0.019       | 0.030  | 0.017  | 0.034  | 0.034        | 0.034    | 0.045    | 0.249| 0.031                           |
| Mathematica  | 0.103 | 0.057       | 0.040  | 0.105  | 0.103  | 0.048        | 0.103    | 0.057    | 0.618| 0.077                           |
| Mathecad     | 0.138 | 0.172       | 0.120  | 0.211  | 0.138  | 0.078        | 0.138    | 0.135    | 1.129| 0.141                           |
| Maxima       | 0.103 | 0.029       | 0.030  | 0.053  | 0.103  | 0.059        | 0.103    | 0.057    | 0.538| 0.067                           |
| MATLAB       | 0.034 | 0.019       | 0.030  | 0.017  | 0.034  | 0.034        | 0.034    | 0.045    | 0.249| 0.031                           |
| SMath Studio | 0.241 | 0.286       | 0.360  | 0.211  | 0.241  | 0.238        | 0.241    | 0.205    | 2.023| 0.253                           |
| SageMath     | 0.034 | 0.019       | 0.030  | 0.017  | 0.034  | 0.034        | 0.034    | 0.045    | 0.249| 0.031                           |
| GeoGebra     | 0.310 | 0.400       | 0.360  | 0.369  | 0.310  | 0.475        | 0.310    | 0.409    | 2.944| 0.368                           |
| Sum          | 1.000 | 1.000       | 1.000  | 1.000  | 1.000  | 1.000        | 1.000    | 1.000    | 8.000| 1.000                           |

Thus, the systems Maxima and GeoGebra are the best two alternatives by the criterion “open code” as they have the highest impact factor – 0.329.

For the matrix of the paired comparisons plotted by the criterion “open code” the following criteria were calculated:

- evaluation of the highest own value: \( \lambda_{max} = 8.159 \);
- the consistency index: \( CI = 0.0227 \);
- the index of ratio sequence: \( CR = 0.016 \).

As \( CR = 1.6% < 10% \), then the matrix of the paired comparisons by the criterion “open code” is considered consistent.

All the criteria have indexes of compatibility significantly lower than 10%. It means that the matrixes of the paired comparisons are compatible.

Table 14. The matrix of the paired comparisons of the alternatives by the criterion “open code”.

| Alternatives | Maple | Mathematica | Matlab | Maxima | MATLAB | SMath Studio | SageMath | GeoGebra |
|--------------|-------|-------------|--------|--------|--------|--------------|----------|----------|
| Maple        | 1.00  | 1.00        | 1.00   | 0.11   | 1.00   | 1.00         | 0.17     | 0.11     |
| Mathematica  | 1.00  | 1.00        | 1.00   | 0.11   | 1.00   | 1.00         | 0.17     | 0.11     |
| Mathecad     | 1.00  | 1.00        | 1.00   | 0.11   | 1.00   | 1.00         | 0.17     | 0.11     |
| Maxima       | 9.00  | 9.00        | 9.00   | 1.00   | 9.00   | 9.00         | 3.00     | 1.00     |
| MATLAB       | 1.00  | 1.00        | 1.00   | 0.11   | 1.00   | 1.00         | 0.17     | 0.11     |
| SMath Studio | 1.00  | 1.00        | 1.00   | 0.11   | 1.00   | 1.00         | 0.17     | 0.11     |
| SageMath     | 6.00  | 6.00        | 6.00   | 0.33   | 6.00   | 6.00         | 1.00     | 0.33     |
| GeoGebra     | 9.00  | 9.00        | 9.00   | 1.00   | 9.00   | 9.00         | 3.00     | 1.00     |
| Sum          | 29.00 | 29.00       | 29.00  | 2.88   | 29.00  | 29.00        | 7.85     | 2.88     |
Table 15. The impact factor of the alternatives by the criterion “open code”.

| Alternatives   | Maple | Mathematica | Mathcad | Maxima | MATLAB | SMath Studio | SageMath | GeoGebra | Sum | The impact factor of the alternatives |
|----------------|-------|-------------|---------|--------|--------|-------------|----------|----------|-----|--------------------------------------|
| Maple          | 0.034 | 0.034       | 0.034   | 0.039  | 0.034  | 0.034       | 0.021    | 0.038    | 0.270| 0.034                               |
| Mathematica    | 0.034 | 0.034       | 0.034   | 0.038  | 0.034  | 0.034       | 0.022    | 0.038    | 0.270| 0.034                               |
| Mathcad        | 0.034 | 0.034       | 0.034   | 0.038  | 0.034  | 0.034       | 0.022    | 0.038    | 0.270| 0.034                               |
| Maxima         | 0.310 | 0.310       | 0.347   | 0.310  | 0.310  | 0.382       | 0.347    | 2.628    |     | 0.329                               |
| MATLAB         | 0.034 | 0.034       | 0.034   | 0.038  | 0.034  | 0.034       | 0.022    | 0.038    | 0.270| 0.034                               |
| SMath Studio   | 0.034 | 0.034       | 0.034   | 0.038  | 0.034  | 0.034       | 0.022    | 0.038    | 0.270| 0.034                               |
| SageMath       | 0.207 | 0.207       | 0.207   | 0.115  | 0.207  | 0.207       | 0.127    | 0.115    | 1.391| 0.174                               |
| GeoGebra       | 0.310 | 0.310       | 0.310   | 0.347  | 0.310  | 0.382       | 0.347    | 2.628    |     | 0.329                               |
| Sum            | 1.000 | 1.000       | 1.000   | 1.000  | 1.000  | 1.000       | 1.000    | 8.000    |     | 1.000                               |

To determine the impact factor we need to evaluate the degrees of importance of the criteria using the method of paired comparisons. To simplify any further speculations we will consider all the criteria equally important; in this case all the boxes of the matrix with have the same values equal to one.

In this case, the column “total impact factor” in table 16 the alternative is calculated as an arithmetical average of all the impact factors by all the criteria. Grounding on the results of the presented calculations the impact factors of the alternatives are provided and on the basis the system with the highest impact factor is chosen. The analysis of the results of table 16 demonstrated that the system of the dynamic mathematics GeoGebra has the highest total impact factor.

Table 16. The results of evaluation of the impact factors of the alternatives.

| Function | Maple | Mathematica | Mathcad | Maxima | MATLAB | SMath Studio | SageMath | GeoGebra | Sum | Total value |
|----------|-------|-------------|---------|--------|--------|-------------|----------|----------|-----|-------------|
| Functionality | 0.2099 | 0.0840 | 0.1207 | 0.0266 | 0.0225 | 0.0311 | 0.0338 | 0.0755 |
| Teaching materials | 0.2099 | 0.2068 | 0.2847 | 0.2275 | 0.0297 | 0.0772 | 0.0338 | 0.1528 |
| Online mode | 0.0314 | 0.0343 | 0.0204 | 0.0267 | 0.0686 | 0.1412 | 0.0338 | 0.0509 |
| Mobile application | 0.1173 | 0.0342 | 0.0204 | 0.0619 | 0.2129 | 0.0673 | 0.3285 | 0.1203 |
| Language support | 0.1173 | 0.0530 | 0.1186 | 0.1385 | 0.0275 | 0.0311 | 0.0338 | 0.0743 |
| Open code | 0.0314 | 0.1318 | 0.0635 | 0.1385 | 0.2129 | 0.2529 | 0.0338 | 0.1235 |
| Total | 0.0731 | 0.1318 | 0.0636 | 0.1385 | 0.2129 | 0.0311 | 0.1739 | 0.3008 |
| Total compatibility | 0.0299 | 0.3243 | 0.3081 | 0.3537 | 0.2129 | 0.3680 | 0.3285 | 0.565 |

Thus, according to the expert evaluations obtained with the help of the hierarchy analysis method of CMS, GeoGebra is defined as the most effective to use in the process of IT-sphere specialists preparation.

It is advisable to evaluate an arithmetic average on hierarchies’ compatibility by means of summing up all the levels by the valued index of the sequence of the correlations:

$$ RC = \frac{0.0565}{1.41} = 0.0401 < 0.1. $$
In the course of the calculations, it was evaluated that the index of the sequence of correlations equals to zero if all the elements of the matrix of the paired comparisons are equal to zero. The obtained result testifies to the compatibility of all the hierarchies. The stated conclusions were drawn on the basis of the calculations and provided in figure 2 and table 17.

**Table 17.** The impact factors of the alternatives of the systems of computer mathematics.

| Alternative   | Priority |
|---------------|----------|
| Maple         | 0.0755   |
| Mathematica   | 0.1528   |
| Mathcad       | 0.0509   |
| Maxima        | 0.1203   |
| MATLAB        | 0.0743   |
| SMath Studio  | 0.1235   |
| SageMath      | 0.1019   |
| GeoGebra      | 0.3008   |

**Figure 2.** The diagram of distribution of the impact factor indexes of the alternative CMS for teaching mathematics.

3. **Conclusions**

In conclusion, in the course of the research, we analyzed eight alternatives and accounted for seven criteria for taking decision on the choice of the most efficient CMS used for teaching mathematics to the future IT-sphere specialists. The choice of the alternative is made by means of calculating of the elements of the priority vector corresponding to each alternative. The basic criteria for which the alternatives were calculated were their characteristics, namely the following: functionality; teaching materials; an online mode; a mobile application; a license; language support; and an open code. The matrix of the paired comparisons by each criterion was plotted and numerical characteristics of these matrices, namely the highest own value, an index of compatibility and an index of the sequence of correlations, were calculated. Every matrix consists of the expert evaluations of the pairs of alternatives, which are the systems of computer mathematics used for teaching of the courses on mathematics. Grounding on the results of the provided calculations impact factors of the alternatives were showed and on the basis a system of dynamic mathematics GeoGebra with the highest impact factor was chosen. Thus, according the expert evaluations obtained using hierarchy analysis method it is the CMS GeoGebra that is best for usage for teaching mathematics in the process of preparation of future IT-sphere specialists.
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