Search for New Physics in $CP$-violating $B$ Decays

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Abstract

We consider three possible scenarios of new physics in $B^0_d - \bar{B}^0_d$ mixing and propose a simple framework for analyzing their effects. This framework allows us to study the $CP$ asymmetry in semileptonic $B_d$ decays ($A_{SL}$) and those in nonleptonic transitions such as $B_d \rightarrow J/\psi K_S$ and $B_d \rightarrow \pi^+\pi^-$. Numerically we find that new physics may enhance the magnitude of $A_{SL}$ up to the percent level within the appropriate parameter space. So measurements of $A_{SL}$ and its correlation with other $CP$ asymmetries will serve as a sensitive probe for new physics in $B^0_d - \bar{B}^0_d$ mixing.

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1 Introduction

The $B$-meson factories under construction at KEK and SLAC will provide a unique opportunity to study $CP$ violation in weak $B$ decays. One type of $CP$-violating signals, arising from $B^0_d$-$\bar{B}^0_d$ mixing, is expected to manifest itself in the decay rate asymmetry between two semileptonic channels $B^0_d \to l^+\nu_l X^-$ and $\bar{B}^0_d \to l^-\bar{\nu}_l X^+$. This $CP$ asymmetry, denoted as $A_{SL}$ below, has been estimated to be of the order $10^{-3}$ within the standard model (SM) \cite{1,2}. Obviously the magnitude of $A_{SL}^{SM}$ is too small to be measured in the first-round experiments of any $B$ factory. The current experimental constraint on $A_{SL}$ is very rough: $|A_{SL}| < 0.18$ at the 90% confidence level \cite{3,4}. Nevertheless, one expects that this limit will be greatly improved once $B$-meson factories start collecting data.

Importance of the $CP$ asymmetry $A_{SL}$ has been repeatedly emphasized for the purpose of searching for new physics (NP) in the $B$-meson system (see, e.g., Refs. \cite{5,6}). The presence of NP in $B^0_d$-$\bar{B}^0_d$ mixing might enhance the magnitude of $A_{SL}$ to an observable level. For instance, $|A_{SL}| \sim 10^{-2}$ can be achieved from a specific superweak model proposed in Ref. \cite{6}, where the Kobayashi-Maskawa (KM) matrix is assumed to be real and the $CP$-violating phase comes solely from the NP. It is therefore worthwhile to search for $CP$ violation in semileptonic $B_d$ decays at $B$-meson factories, in order to determine or constrain both the magnitude and the phase information of possible NP in $B^0_d$-$\bar{B}^0_d$ mixing. The $CP$-violating phase of NP may also be isolated from measuring $CP$ asymmetries in some $B_d$ decays into hadronic $CP$ eigenstates, such as $B_d \to J/\psi K_S$ and $B_d \to \pi^+\pi^-$ modes \cite{7,8}.

In this paper we investigate the $CP$ asymmetry $A_{SL}$ and its correlation with the $CP$ asymmetries in $B_d \to J/\psi K_S$ and $B_d \to \pi^+\pi^-$, based on three NP scenarios for $B^0_d$-$\bar{B}^0_d$ mixing. Scenario (A) allows $B^0_d$-$\bar{B}^0_d$ mixing to contain a real SM-like term and a complex superweak contribution \cite{9}; scenario (B) requires $B^0_d$-$\bar{B}^0_d$ mixing to include the normal SM effect with an additional real superweak contribution \cite{10}; and scenario (C) is a general case in which both the SM and NP contributions to $B^0_d$-$\bar{B}^0_d$ mixing are complex ($CP$-violating).

For each scenario, we first propose a simple parametrization of the NP effect, and then calculate $CP$-violating asymmetries in the above mentioned $B$ decays. Some numerical estimates for these $CP$ asymmetries in scenarios (A) and (B) are also made. We find that for all three scenarios the magnitude of $A_{SL}$ can reach the percent level within the suitable parameter space. Thus an experimental study of the correlation between $CP$ asymmetries in the semileptonic and nonleptonic $B_d$ decays should impose useful constraints on possible NP in $B^0_d$-$\bar{B}^0_d$ mixing.

The remainder of this paper is organized as follows. In section 2, some necessary preliminaries for $B^0_d$-$\bar{B}^0_d$ mixing, $CP$ violation and the KM unitarity triangle are briefly reviewed. The SM prediction for $CP$ violation in semileptonic $B_d$ decays, i.e., $A_{SL}^{SM}$, is updated in section 3. Section 4 is devoted to parametrizing NP effects in $B^0_d$-$\bar{B}^0_d$ mixing and calculating $A_{SL}$ as well as its correlation with the $CP$ asymmetries in $B_d \to J/\psi K_S$ and $B_d \to \pi^+\pi^-$. 
on the basis of three different NP scenarios. We numerically illustrate the allowed parameter space and $CP$ asymmetries for scenarios (A) and (B) in section 5. Finally some concluding remarks are given in section 6.

2 Preliminaries

The mass eigenstates of $B^0_d$ and $\bar{B}^0_d$ mesons can be written, in the assumption of $CPT$ invariance, as

$$|B_L\rangle = p|B^0_d\rangle + q|\bar{B}^0_d\rangle,$$

$$|B_H\rangle = p|B^0_d\rangle - q|\bar{B}^0_d\rangle,$$

(2.1)

where $p$ and $q$ are complex mixing parameters. In terms of the off-diagonal elements of the $2 \times 2 B^0_d-\bar{B}^0_d$ mixing Hamiltonian $M - i\Gamma/2$, we express the ratio $q/p$ as

$$\frac{q}{p} = \sqrt{\frac{M^*_{12} - i\Gamma^*_{12}/2}{M_{12} - i\Gamma_{12}/2}}.$$  

(2.2)

To a good approximation (i.e., $|M_{12}| \gg |\Gamma_{12}|$), the mass difference of $B_H$ and $B_L$ (denoted by $\Delta M$) is related to $|M_{12}|$ through $\Delta M = 2|M_{12}|$, and $q/p = \sqrt{M^*_{12}/M_{12}}$ holds.

The $CP$-violating asymmetry $A_{SL}$, for either incoherent or coherent decays of $B^0_d$ and $\bar{B}^0_d$ mesons, is given by [11]

$$A_{SL} = \frac{1}{|p|^4 + |q|^4} \text{Im} \left( \frac{\Gamma_{12}}{M_{12}} \right),$$

(2.3)

At a $B$-meson factory, this signal can be extracted from the same-sign dilepton asymmetry on the $\Upsilon(4S)$ resonance.

Another observable, which is of particular interest for testing the KM mechanism of $CP$ violation, is the $CP$ asymmetry in $B^0_d$ vs $\bar{B}^0_d \to J/\psi K_S$ modes [12]:

$$A_{\psi K} = -\text{Im} \left( \frac{q}{p} \frac{V_{ub}V_{cs}^*}{V_{cb}V_{cs}^*} \frac{q_K^*}{p_K^*} \right),$$

(2.4)

where the minus sign comes from the $CP$-odd eigenstate $J/\psi K_S$, and $q_K/p_K$ describes the $K^0-\bar{K}^0$ mixing phase in the final state. Neglecting the tiny $CP$-violating effect measured from $K^0-\bar{K}^0$ mixing, one can find that $q_K/p_K$ is essentially unity in an appropriate phase convention, no matter whether NP exists or not. Thus the above asymmetry turns out approximately to be $A_{\psi K} = -\text{Im}(q/p)$, if one adopts the Wolfenstein phase convention for the KM matrix.

In the neglect of penguin effects, measuring the $CP$ asymmetry in $B^0_d$ vs $\bar{B}^0_d \to \pi^+\pi^-$ modes is also promising to probe the $CP$-violating weak phase [12]:

$$A_{\pi\pi} = \text{Im} \left( \frac{q}{p} \frac{V_{ub}V_{ud}^*}{V_{ub}V_{ud}} \right).$$

(2.5)
The correlation between $A_{\pi\pi}$ and $A_{\psi K}$ is sensitive to a variety of NP scenarios, such as superweak models (see, e.g., Refs. [10, 14]).

Within the SM, $A_{\psi K}$, $A_{\psi K}$ and $A_{\pi\pi}$ are related to the inner angles of the KM unitarity triangle formed by three vectors $\xi_i \equiv V_{id}^* V_{id} \ (i = u, c, t)$ in the complex plane (see Fig. 1 for illustration). The sides $|\xi_u|$ and $|\xi_c|$ have been model-independently measured, while the existence of NP may in general affect determination of the side $|\xi_t|$ from the rate of $B_d^0-\bar{B}_d^0$ mixing. In terms of the Wolfenstein parameters, one has

$$
\begin{align*}
\xi_u &\approx A\lambda^3 (\rho + i\eta), \\
\xi_c &\approx -A\lambda^3, \\
\xi_t &\approx A\lambda^3 (1 - \rho - i\eta).
\end{align*}
$$

For convenience in subsequent discussions, we define

$$
\chi \equiv \left| \frac{\xi_u}{\xi_c} \right| \approx \sqrt{\rho^2 + \eta^2},
$$

which is a NP-independent quantity.

Note that in this work we only assume the kind of NP which does not involve extra quark(s). This requirement implies that quark mixing remains to be described by the $3 \times 3$ KM matrix even though NP is present in $B_d^0-\bar{B}_d^0$ mixing. For some interesting extensions of the SM, e.g., the supersymmetric models, the above assumption is of course satisfied. We further assume that the penguin and tree-level contributions to $B_d \rightarrow \pi^+\pi^-$ may be separated from each other using some well-known techniques [15, 16], thus $A_{\pi\pi}$ is useful for probing the $CP$-violating weak phase(s).

### 3 $CP$ asymmetries in the SM

Within the SM, both $M_{12}$ and $\Gamma_{12}$ can be reliably calculated in the box-diagram approximation. Due to the dominance of the top-quark contribution, $M_{12}^{SM}$ reads [17, 18]

$$
M_{12}^{SM} = \frac{G_F^2 B_B f_B^2 M_B m_t^2}{12\pi^2} \eta_B F(z) (\xi_{t}^*)^2,
$$

(3.1)
where \( B_B \) is the “bag” parameter describing the uncertainty in evaluation of the hadronic matrix element \( \langle B_d^0 | B_{\gamma}^0 (1 - \gamma_5) d | B_d^0 \rangle \), \( M_B \) and \( f_B \) are the \( B_d \)-meson mass and decay constant respectively, \( m_t \) is the top-quark mass, \( \eta_B \) denotes the QCD correction factor, \( F(z) \) stands for a slowly decreasing monotonic function of \( z \equiv m_t^2 / M_W^2 \) [19]:

\[
F(z) = \frac{1}{4} + \frac{9}{4} \frac{1}{1 - z} - \frac{3}{2} \frac{1}{(1 - z)^2} - \frac{3}{2} \frac{z^2 \ln z}{(1 - z)^3} .
\]

In particular, \( F(0) = 1 \), \( F(1) = 3/4 \), and \( F(\infty) = 1/4 \).

Next, \( \Gamma_{12}^{\text{SM}} \) is given as follows [11, 13]:

\[
\Gamma_{12}^{\text{SM}} = - \frac{G_F^2 B_B f_B^2 M_B m_b^2}{8 \pi} \left[ (\xi_u^*)^2 T_u(\tilde{z}) + (\xi_c^*)^2 T_c(\tilde{z}) + (\xi_t^*)^2 T_t(\tilde{z}) \right] ,
\]

where \( m_b \) is the bottom-quark mass, and \( T_i(\tilde{z}) \) is a function of \( \tilde{z} \equiv m_c^2 / m_b^2 \). Explicitly, \( T_u(\tilde{z}) \), \( T_c(\tilde{z}) \) and \( T_t(\tilde{z}) \) read

\[
T_u(\tilde{z}) = \eta_4 \tilde{z}^2 \left( 3 - 2 \tilde{z} \right) + \frac{4}{3} \eta_5 \tilde{z} \left( 1 - \tilde{z} \right)^2 , \\
T_c(\tilde{z}) = \eta_4 \left( 1 + 2 \tilde{z} \right) \left[ \sqrt{1 - 4 \tilde{z}} - (1 - \tilde{z})^2 \right] - \frac{4}{3} \eta_5 \tilde{z} \left[ 2 \sqrt{1 - 4 \tilde{z}} - (1 - \tilde{z})^2 \right] , \\
T_t(\tilde{z}) = \eta_4 \left( 1 + 2 \tilde{z} \right) (1 - \tilde{z})^2 - \frac{4}{3} \eta_5 \tilde{z} \left( 1 - \tilde{z} \right)^2 ,
\]

in which \( \eta_4 \) and \( \eta_5 \) are two QCD correction factors. A numerical calculation shows that \( T_t(\tilde{z}) \) is dominant over \( T_u(\tilde{z}) \) and \( T_c(\tilde{z}) \) in magnitude (see below).

The \( CP \) asymmetry in semileptonic \( B_d \) decays (\( A_{SL}^{\text{SM}} \)) turns out to be

\[
A_{SL}^{\text{SM}} = C_m \left[ \text{Im} \left( \frac{\xi_u}{\xi_t} \right)^2 T_u(\tilde{z}) + \text{Im} \left( \frac{\xi_c}{\xi_t} \right)^2 T_c(\tilde{z}) \right] ,
\]

where \( C_m = 3 \pi m_b^2 / [2m_t^2 \eta_B F(z)] \), and

\[
\text{Im} \left( \frac{\xi_u}{\xi_t} \right)^2 \approx \frac{2 \eta \left[ \rho \left( 1 - \rho \right) - \eta^2 \right]}{[\left( 1 - \rho \right)^2 + \eta^2]^2} , \\
\text{Im} \left( \frac{\xi_c}{\xi_t} \right)^2 \approx \frac{2 \eta \left( 1 - \rho \right)}{[\left( 1 - \rho \right)^2 + \eta^2]^2} .
\]

The \( T_t(\tilde{z}) \) term, which dominates \( \Gamma_{12}^{\text{SM}} \), has no contribution to the \( CP \) asymmetry \( A_{SL}^{\text{SM}} \).

Indeed the \( CP \)-violating phases \( \phi_1 \) and \( \phi_2 \) can be determined from the \( CP \) asymmetries in \( B_d \to J/\psi K_S \) and \( B_d \to \pi^+ \pi^- \), respectively. Within the SM, \( q/p = \xi_t / \xi_t^* \) results from the box-diagram calculation. From Eqs. (2.4) and (2.5), it is easy to obtain

\[
A_{\psi K}^{\text{SM}} = \sin(2\phi_1) \approx \frac{2 \eta \left( 1 - \rho \right)}{\left( 1 - \rho \right)^2 + \eta^2} , \\
A_{\pi \pi}^{\text{SM}} = \sin(2\phi_2) \approx \frac{2 \eta \left[ \rho \eta^2 - \rho \left( 1 - \rho \right) \right]}{\left( \rho^2 + \eta^2 \right) \left[ \left( 1 - \rho \right)^2 + \eta^2 \right]} .
\]
A test of the correlation between \( A_{SL}^{SM} \) and \( A_{\psi K}^{SM} \) or that between \( A_{\psi K}^{SM} \) and \( A_{\pi\pi}^{SM} \) at \( B \) factories is necessary, in order to find possible NP which may affect these observables in different ways.

Let us illustrate the magnitudes of \( A_{SL}^{SM} \), \( A_{\psi K}^{SM} \) and \( A_{\pi\pi}^{SM} \) explicitly. The current quark masses are typically taken as \( m_c = 1.4 \) GeV, \( m_b = 4.8 \) GeV and \( m_t = 167 \) GeV; and the QCD correction factors are chosen to be \( \eta_B = 0.55 \), \( \eta_4 = 1.15 \) and \( \eta_5 = 0.88 \). Then one gets \( F(z) \approx 0.55 \), \( T_u(\tilde{z}) \approx 0.11 \), \( T_c(\tilde{z}) \approx -0.11 \), \( T_t(\tilde{z}) \approx 1.04 \), and \( C_m \approx 1.3 \times 10^{-2} \). An analysis of current data on quark mixing and \( CP \) violation yields \( \rho \approx 0.05 \) and \( \eta \approx 0.36 \) as favored values [20]. With these inputs we arrive at

\[
A_{\psi K}^{SM} \approx 0.66, \quad A_{\pi\pi}^{SM} \approx 0.43, \quad A_{SL}^{SM} \approx -9.8 \times 10^{-4}. \tag{3.8}
\]

If the large errors of relevant inputs are taken into account, we find that the magnitude of \( A_{SL}^{SM} \) may change a little bit around \( 10^{-3} \), but its sign remains negative. Clearly it is very difficult to measure such a small \( CP \) asymmetry.

4 Effects of NP on \( CP \) asymmetries

In most extensions of the SM, NP can significantly contribute to \( M_{12} \). However, NP is not expected to significantly affect the direct \( B \)-meson decays via the tree-level \( W \)-mediated channels. Thus \( \Gamma_{12} = \Gamma_{12}^{SM} \) holds as a good approximation. In the presence of NP, \( M_{12} \) can be written as

\[
M_{12} = M_{12}^{SM} + M_{12}^{NP}. \tag{4.1}
\]

The relative magnitude and the phase difference between \( M_{12}^{NP} \) and \( M_{12}^{SM} \) are unknown, while \(|M_{12}| = \Delta M/2\) holds by definition.

It is convenient to parametrize the magnitude of \( M_{12}^{SM} \) as

\[
|M_{12}^{SM}| = R_{SM} \frac{\Delta M}{2} \tag{4.2}
\]

in subsequent discussions. The allowed range of \( R_{SM} \) can be estimated by use of Eq. (3.1) and current data; i.e.,

\[
R_{SM} = \frac{G_F^2 B_B f_B^2 M_B m_t^2}{6\pi^2 \Delta M} \eta_B F(z) |\xi_t|^2. \tag{4.3}
\]

Using \( f_B \sqrt{B_B} = (200 \pm 40) \) MeV, \( m_t = (167 \pm 6) \) GeV, \( \Delta M = (0.464 \pm 0.018) \) ps\(^{-1}\), and \( \eta_B = 0.55 \pm 0.01 \) (see Ref. [21]), we get \( R_{SM} \approx (1.34 \pm 0.71) \times 10^4 |\xi_t|^2 \). The large error comes primarily from the input value of \( f_B \sqrt{B_B} \), which will be improved in more delicate lattice-QCD calculations. Since \(|\xi_u|\) and \(|\xi_c|\) have been measured, the most generous constraint on \(|\xi_t|\) (in the presence of NP) should be

\[
|\xi_c| - |\xi_u| \leq |\xi_t| \leq |\xi_c| + |\xi_u|, \tag{4.4}
\]
as one can see from Fig. 1. A measurement of the rare decay $K^+ \to \pi^+\nu\bar{\nu}$ will provide an independent determination of (or constraint on) $|\xi_t|$. By use of $|\xi_u| = 0.003 \pm 0.001$ and $|\xi_c| = 0.0087 \pm 0.0007$ \cite{4,20}, we get $0.1 \leq R_{SM} \leq 3.7$ as a conservative result. If only the central values of input parameters are taken into account, then a narrower range can be obtained: $0.43 \leq R_{SM} \leq 1.8$.

To illustrate the effect of NP on $CP$ asymmetries $A_{SL}$, $A_{\psi K}$ and $A_{\pi\pi}$, we subsequently consider three possible NP scenarios for $M_{12}$.

### 4.1 Scenario (A): $\text{Im}(M_{12}^{SM}) = 0$ and $\text{Im}(M_{12}^{NP}) \neq 0$

In this scenario, the KM matrix is assumed to be real and $CP$ violation arises solely from NP. Then $M_{12}^{SM}$, $M_{12}^{NP}$ and $M_{12}$ in Eq. (4.1) can be instructively parametrized as

$$
\{M_{12}^{SM}, M_{12}^{NP}, M_{12}\} = \{R_{SM}, R_{NP}e^{i2\theta}, e^{i2\phi}\} \frac{\Delta M}{2},
$$

where $R_{NP}$ is a real (positive or vanishing) parameter, $\theta$ stands for the phase of NP, and $\phi$ is an effective phase of $B_d^0$-$\bar{B}_d^0$ mixing. In the complex plane, $M_{12}^{SM}$, $M_{12}^{NP}$ and $M_{12}$ (or equivalently, $R_{SM}$, $R_{NP}e^{i2\theta}$ and $e^{i2\phi}$) form a triangle \cite{20}, as illustrated by Fig. 2. By use of the triangular relation, $R_{NP}$ can be expressed as

$$
R_{NP} = -R_{SM} \cos(2\theta) \pm \sqrt{1 - R_{SM}^2 \sin^2(2\theta)}.
$$

We find that two solutions exist for $R_{NP}$, corresponding to ($\pm$) signs on the right-hand side of Eq. (4.6). Since the magnitude of $R_{SM}$ has been constrained to some extent, we are able to obtain the allowed ($\theta, R_{NP}$) parameter space numerically.

The $CP$-violating phase $\phi$ can be measured from the $CP$ asymmetry in $B_d \to J/\psi K_S$. With the help of Eq. (2.4), we obtain

$$
A_{\psi K} = \sin(2\phi) = R_{NP} \sin(2\theta).
$$

Of course, $|A_{\psi K}| \leq 1$ holds for $R_{NP}$ and $\theta$ to take values allowed by Eq. (4.6).

The $CP$ asymmetry in $B_d \to \pi^+\pi^-$ is simply related to $A_{\psi K}$. By use of Eq. (2.5), we find

$$
A_{\pi\pi} = -\sin(2\phi) = -A_{\psi K}.
$$
Such a linear correlation between $A_{\psi K}$ and $A_{\pi\pi}$ is a straightforward consequence of the superweak scenario of NP considered here.

The $CP$ asymmetry in semileptonic $B_d$ decays, defined in Eq. (2.3), is given by

$$A_{SL} = C_m \, R_{SM} \, R_{NP} \, \sin(2\theta) \left[ \left( \frac{\tilde{\zeta}_u}{\tilde{\xi}_t} \right)^2 T_u(\bar{z}) + \left( \frac{\tilde{\zeta}_c}{\tilde{\xi}_t} \right)^2 T_c(\bar{z}) + T_t(\bar{z}) \right], \quad (4.9)$$

where $\tilde{\zeta}_u$, $\tilde{\zeta}_c$, and $\tilde{\xi}_t$ denote the real KM factors in scenario (A). Different from the SM result in Eq. (3.5), whose magnitude is associated only with $T_u(\bar{z}) \sim O(0.1)$ and $T_c(\bar{z}) \sim O(0.1)$, here the magnitude of $A_{SL}$ is dominated by $T_t(\bar{z}) \sim O(1)$. Hence $|A_{SL}/A_{SM}^SL| \geq 10$ is possible within an appropriate $(\theta, R_{NP})$ parameter space. We shall present numerical estimates for $A_{SL}$ and $A_{\psi K}$ in the next section.

### 4.2 Scenario (B): $\text{Im}(M_{12}^{NP}) = 0$ and $\text{Im}(M_{12}^{SM}) \neq 0$

In this scenario the NP contribution is of the so-called “real superweak” type [10, 24]. The phase of $M_{12}^{SM}$ comes from the KM matrix and amounts to $2\phi_1$, as one can see from Eq. (3.1) and Fig. 1. For simplicity, we parametrize $M_{12}^{SM}$, $M_{12}^{NP}$ and $M_{12}$ as follows:

$$\{M_{12}^{SM}, M_{12}^{NP}, M_{12}\} = \{R_{SM} \, e^{i2\phi_1}, R_{NP}, e^{i2\phi_1}\} \, \frac{\Delta M}{2}, \quad (4.10)$$

where $R_{NP}$ is a real parameter, and $\phi$ denotes the effective phase of $B_d^0\bar{B}_d^0$ mixing. Clearly $M_{12}^{SM}$, $M_{12}^{NP}$ and $M_{12}$ (or equivalently, $R_{SM} e^{i2\phi_1}$, $R_{NP}$ and $e^{i2\phi_1}$) form a triangle in the complex plane, as illustrated by Fig. 3. In terms of $R_{SM}$ and $\phi_1$, $R_{NP}$ can be written as [10]

$$R_{NP} = -R_{SM} \cos(2\phi_1) \pm \sqrt{1 - R_{SM}^2 \sin^2(2\phi_1)}. \quad (4.11)$$

We see that there are two solutions for $R_{NP}$, corresponding to $(\pm)$ signs on the right-hand side of Eq. (4.11).

The $CP$-violating phase $\phi$ can be determined from the $CP$ asymmetry in $B_d \to J/\psi K_S$:

$$A_{\psi K} = \sin(2\phi) = R_{SM} \sin(2\phi_1). \quad (4.12)$$

Of course, $|A_{\psi K}| \leq 1$ holds if $R_{SM}$ and $\phi_1$ vary in their allowed regions.
Figure 4: Triangular relation of $M_{12}^{SM}$, $M_{12}^{NP}$ and $M_{12}$ (rescaled by $\Delta M/2$) in scenario (C).

The CP asymmetry in $B_d \to \pi^+\pi^-$ reads:

$$A_{\pi\pi} = -\sin 2(\phi + \phi_3) = R_{SM} \sin(2\phi_2) - R_{NP} \sin(2\phi_3),$$

(4.13)

where $\phi_2$ and $\phi_3$ are the second and third angles of the unitarity triangle (see Fig. 1). The correlation between $A_{\psi K}$ and $A_{\pi\pi}$ does exist, because of $\phi_1 + \phi_2 + \phi_3 = \pi$. In the absence of NP (i.e., $R_{NP} = 0$ and $R_{SM} = 1$), Eqs. (4.12) and (4.13) will be simplified to the SM results as given by Eq. (3.7).

The CP asymmetry $A_{SL}$ in scenario (B) turns out to be

$$A_{SL} = C_m R_{SM} R_{NP} \left[ \text{Im} \left( \frac{\xi_u}{|\xi_t|} \right)^2 T_u(\tilde{z}) + \text{Im} \left( \frac{\xi_c}{|\xi_t|} \right)^2 T_c(\tilde{z}) + \text{Im} \left( \frac{\xi_t}{|\xi_t|} \right)^2 T_t(\tilde{z}) \right] + C_m R_{SM}^2 \left[ \text{Im} \left( \frac{\xi_u}{\xi_t} \right)^2 T_u(\tilde{z}) + \text{Im} \left( \frac{\xi_c}{\xi_t} \right)^2 T_c(\tilde{z}) \right].$$

(4.14)

One can see that $A_{SL}$ consists of two terms: the first comes from the interference between $M_{12}^{SM}$ and $M_{12}^{NP}$ in $M_{12}$, while the second is purely a SM-like contribution from $M_{12}^{SM}$ itself (see Eq. (3.5) for comparison). Within a suitable parameter space, the magnitude of $A_{SL}$ should be dominated by the term associated with $T_t(\tilde{z})$; thus $|A_{SL}/A_{SM}^{SL}| \geq 10$ is possible in this NP scenario.

### 4.3 Scenario (C): $\text{Im}(M_{12}^{NP}) \neq 0$ and $\text{Im}(M_{12}^{SM}) \neq 0$

This is a quite general NP scenario which can accommodate both scenarios (A) and (B). As done before, we parametrize $M_{12}^{SM}$, $M_{12}^{NP}$ and $M_{12}$ in the following way:

$$\left\{ M_{12}^{SM}, M_{12}^{NP}, M_{12} \right\} = \left\{ R_{SM} e^{i\phi_1}, R_{NP} e^{i\theta}, e^{i2\phi} \right\} \frac{\Delta M}{2},$$

(4.15)

where $R_{NP}$ is a real (positive or vanishing) parameter, $\theta$ represents the NP phase, and $\phi$ denotes the effective phase of $B^0_d - \bar{B}^0_d$ mixing. In this case, $M_{12}^{SM}$, $M_{12}^{NP}$ and $M_{12}$ (or equivalently, $R_{SM} e^{i\phi_1}$, $R_{NP} e^{i\theta}$ and $e^{i2\phi}$) form a triangle in the complex plane, as illustrated by Fig. 4. In terms of $R_{SM}$, $\phi_1$ and $\theta$, $R_{NP}$ can be expressed as

$$R_{NP} = -R_{SM} \cos 2(\theta - \phi_1) \pm \sqrt{1 - R_{SM}^2 \sin^2 2(\theta - \phi_1)}.$$

(4.16)
We observe that $R_{NP}$ depends on the phase difference between $\theta$ and $\phi_1$. Also there exist two solutions for $R_{NP}$, corresponding to $(\pm)$ signs on the right-hand side of Eq. (4.16). In the special case $\theta = \phi_1$, one arrives at $R_{NP} = -R_{SM} \pm 1$.

It is straightforward to derive the $CP$ asymmetry $A_{\psi K}$ in $B^0_d \to \bar{B}^0_d \to J/\psi K_S$ modes:

$$A_{\psi K} = \sin(2\phi) = R_{SM} \sin(2\phi_1) + R_{NP} \sin(2\theta). \quad (4.17)$$

Since $R_{NP}$, $R_{SM}$ and $\phi_1$, $\theta$ are dependent on one another through Eq. (4.16), $|A_{\psi K}| \leq 1$ is always guaranteed within the allowed parameter space.

In scenario (C), the $CP$ asymmetry in $B_d \to \pi^+\pi^-$ is given as

$$A_{\pi\pi} = -\sin 2(\phi + \phi_3) = R_{SM} \sin(2\phi_2) - R_{NP} \sin 2(\theta + \phi_3), \quad (4.18)$$

where $\phi_2$ and $\phi_3$ are two inner angles of the unitarity triangle in Fig. 1. In comparison with scenarios (A) and (B), here the correlation between $A_{\psi K}$ and $A_{\pi\pi}$ becomes more complicated. We see that the results in Eqs. (4.12) and (4.13) can be respectively reproduced from Eqs. (4.17) and (4.18) if $\theta = 0$ is taken.

The $CP$ asymmetry $A_{SL}$ in scenario (C) reads

$$A_{SL} = C_m R_{SM} R_{NP} \left[ \text{Im} \left( \frac{\xi_u}{|\xi_t|} \right)^2 T_u(\bar{z}) + \text{Im} \left( \frac{\xi_c}{|\xi_t|} \right)^2 T_c(\bar{z}) + \text{Im} \left( \frac{\xi_t}{|\xi_t|} \right)^2 T_t(\bar{z}) \right] \cos(2\theta)$$

$$+ C_m R_{SM}^2 \left[ \text{Im} \left( \frac{\xi_u}{\xi_t} \right)^2 T_u(\bar{z}) + \text{Im} \left( \frac{\xi_c}{\xi_t} \right)^2 T_c(\bar{z}) \right]$$

$$+ C_m R_{SM} R_{NP} \left[ \text{Re} \left( \frac{\xi_u}{|\xi_t|} \right)^2 T_u(\bar{z}) + \text{Re} \left( \frac{\xi_c}{|\xi_t|} \right)^2 T_c(\bar{z}) + \text{Re} \left( \frac{\xi_t}{|\xi_t|} \right)^2 T_t(\bar{z}) \right] \sin(2\theta). \quad (4.19)$$

Clearly the second term of $A_{SL}$ comes purely from $M_{12}^{SM}$ itself and its magnitude is expected to be of $O(10^{-3})$. The first and third terms of $A_{SL}$ arise from the interference between $M_{12}^{SM}$ and $M_{12}^{NP}$; but they depend on nonvanishing $\text{Im}(M_{12}^{SM})$ and $\text{Im}(M_{12}^{NP})$, respectively. Thus $A_{SL}(\theta = 0)$ is just the result given by Eq. (4.14) for scenario (B). For appropriate values of $\theta$ and $\phi_1$, magnitudes of both the first and third terms of $A_{SL}$ may be at the percent level. To obtain $|A_{SL}| \sim O(10^{-2})$, however, there should not be large cancellation between two dominant terms in Eq. (4.19).

5 Numerical estimates of $CP$ asymmetries

For the purpose of illustration, let us estimate the magnitudes of $CP$ asymmetries obtained from the above NP scenarios. Since scenario (C) involves several unknown parameters (though they are related to one another through Eq. (4.16)), a numerical analysis of its allowed parameter space would be complicated and less instructive [27]. Hence we shall only concentrate on scenarios (A) and (B) in the following.
5.1 Results for scenario (A)

Following the spirit of the Wolfenstein parametrization [13], the real KM matrix $\tilde{V}$ in scenario (A) can be parametrized in terms of three independent parameters $\tilde{\lambda}$, $\tilde{A}$ and $\tilde{\rho}$. Taking $\tilde{V}_{us} = \tilde{\lambda}$, $\tilde{V}_{cb} = \tilde{A}\tilde{\lambda}^2$, $\tilde{V}_{ub} = \tilde{A}\tilde{\lambda}^3\tilde{\rho}$ and using the orthogonality conditions of $\tilde{V}$, one can derive the other six matrix elements. In particular, we get $\tilde{V}_{td} \approx \tilde{A}\tilde{\lambda}^3(1 - \tilde{\rho})$. In view of current data on $|\tilde{V}_{us}|$, $|\tilde{V}_{cb}|$ and $|\tilde{V}_{ub}/\tilde{V}_{cb}|$ (see Ref. [4]), we find $\tilde{\lambda} \approx 0.22$, $\tilde{A} \approx 0.8$ and $\tilde{\rho} \approx \pm 0.36$ typically. The sign ambiguity of $\tilde{\rho}$ may affect the allowed parameter space of NP as well as the $CP$ asymmetries of $B$-meson decays, as one can see later on.

The size of $R_{SM}$ depends on the real KM factor $\tilde{\xi}_t$. With the help of Eq. (4.3), we get $R_{SM} \approx 1.8$ for $\tilde{\rho} \approx -0.36$ and $R_{SM} \approx 0.43$ for $\tilde{\rho} \approx +0.36$. The corresponding results of $R_{NP}$, changing with $\theta$, can be obtained from Eq. (4.6). We find that for $\tilde{\rho} \approx +0.36$ the ($-$) solution of $R_{NP}$ is not allowed. The allowed $(\theta, R_{NP})$ parameter space is shown in Fig. 5.

The $CP$ asymmetries $A_{SL}$ and $A_{\psi K}$ (or $A_{\pi\pi}$) in this scenario can then be calculated by use of the above obtained parameter space. For simplicity, we express the KM factors $\tilde{\xi}_u$, $\tilde{\xi}_c$ and $\tilde{\xi}_t$ in Eq. (4.9) in terms of $\tilde{\lambda}$, $\tilde{A}$ and $\tilde{\rho}$. Then the correlation between $A_{SL}$ and $A_{\psi K}$ reads

$$A_{SL} \approx C_m R_{SM} \left[ \frac{\tilde{\rho}^2}{(1 - \tilde{\rho})^2} T_u(z) + \frac{1}{(1 - \tilde{\rho})^2} T_c(z) + T_t(z) \right] A_{\psi K}$$

(5.1)

with $\tilde{\rho} \approx \pm 0.36$. The results of $A_{SL}$ and $A_{\psi K}$, changing with the $CP$-violating phase $\theta$, are depicted in Fig. 6. Two remarks are in order.

a) With the typical inputs mentioned above, we have found $A_{SL}/A_{\psi K} \approx 0.023$ for $\tilde{\rho} \approx -0.36$ and $A_{SL}/A_{\psi K} \approx 0.0042$ for $\tilde{\rho} \approx +0.36$. Because of $|A_{\psi K}| \leq 1$, only the former case
Figure 6: Illustrative plot for the CP asymmetries $A_{\text{SL}}$ and $A_{\psi K}$ in scenario (A).
likely to lead the magnitude of $\mathcal{A}_{\text{SL}}$ to the percent level.

b) In both cases, however, the $CP$ asymmetry $\mathcal{A}_{\psi K}$ may take promising values (e.g., $|\mathcal{A}_{\psi K}| \geq 0.5$). The correlation between $\mathcal{A}_{\psi K}$ and $\mathcal{A}_{\pi\pi}$, i.e., $\mathcal{A}_{\pi\pi} = -\mathcal{A}_{\psi K}$, is particularly interesting in this superweak scenario of NP. If this correlation and the one between $\mathcal{A}_{\text{SL}}$ and $\mathcal{A}_{\psi K}$ can be measured at a $B$-meson factory, they will provide a strong constraint on the underlying NP in $B^0_d - \bar{B}^0_d$ mixing.

5.2 Results for scenario (B)

To calculate $R_{\text{NP}}$ with the help of Eq. (4.11), we should first estimate $R_{\text{SM}}$ by use of Eq. (4.3). The magnitude of $R_{\text{SM}}$ depends on $|\xi_t|$:

$$|\xi_t| \approx A \lambda^3 \sqrt{(1 - \rho)^2 + \eta^2} \approx A \lambda^3 \sqrt{1 - 2\rho + \chi^2},$$

(5.2)

where $\chi$ has been defined in Eq. (2.7). From current data on $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}/V_{cb}|$, we get $\lambda \approx 0.22$, $A \approx 0.8$ and $\chi \approx 0.36$. The most generous range of $\rho$, due to the presence of NP, should be $-\chi \leq \rho \leq +\chi$. We find that the resultant region of $|\xi_t|$ is just the one given in Eq. (4.4). Note that $\phi_1$ is also a function of $\rho$ and $\chi$, i.e.,

$$\tan \phi_1 \approx \pm \frac{\sqrt{\chi^2 - \rho^2}}{1 - \rho},$$

(5.3)

thus it cannot take arbitrary values from 0 to $\pi$. For this reason, it is more convenient to calculate the $(\rho, R_{\text{NP}})$ parameter space of scenario (B) by taking $\rho \in [-0.36, +0.36]$. We get $0^\circ \leq \phi_1 \leq 20.1^\circ$ or $158.9^\circ \leq \phi_1 \leq 180^\circ$, corresponding to the (+) or (−) sign of $\tan \phi_1$. The allowed region of $R_{\text{NP}}$ changing with $\rho$ is shown in Fig. 7.

Now we calculate the $CP$ asymmetries $\mathcal{A}_{\psi K}$, $\mathcal{A}_{\pi\pi}$ and $\mathcal{A}_{\text{SL}}$ by use of Eqs. (4.12), (4.13) and (4.14), respectively. In terms of the parameters $\rho$ and $\chi$, the correlation between $\mathcal{A}_{\pi\pi}$ and $\mathcal{A}_{\psi K}$ can be given as

$$\mathcal{A}_{\pi\pi} \approx -\left[ \frac{\rho - \chi^2}{\chi^2(1 - \rho)} + \frac{\rho (1 - 2\rho + \chi^2)}{\chi^2(1 - \rho)} \frac{R_{\text{NP}}}{R_{\text{SM}}} \right] \mathcal{A}_{\psi K}.$$  

(5.4)

Similarly we obtain the correlation between $\mathcal{A}_{\text{SL}}$ and $\mathcal{A}_{\psi K}$ as follows:

$$\mathcal{A}_{\text{SL}} \approx C_m f(\rho, \chi) \mathcal{A}_{\psi K},$$  

(5.5)

where

$$f(\rho, \chi) \equiv R_{\text{SM}} \left[ \frac{\rho - \chi^2}{(1 - \rho)(1 - 2\rho + \chi^2)} T_u(\tilde{z}) + \frac{1}{1 - 2\rho + \chi^2} T_c(\tilde{z}) \right] + R_{\text{NP}} \left[ \frac{\rho}{1 - \rho} T_u(\tilde{z}) - T_l(\tilde{z}) \right].$$  

(5.6)

Note that the term associated with the KM factor $\text{Im}(\xi_c/|\xi_t|)^2$ in Eq. (4.14) does not appear in $f(\rho, \chi)$, because this factor approximately vanishes in the Wolfenstein parametrization.
Figure 7: Illustrative plot for the $(\rho, R_{NP})$ parameter space in scenario (B).

Figure 8: Illustrative plot for the correlation between $A_{\psi K}$ and $A_{\pi \pi}$ in scenario (B).
Figure 9: Illustrative plot for the $CP$ asymmetries $A_{SL}$ and $A_{\psi K}$ in scenario (B).
The numerical results for three CP asymmetries are shown in Figs. 8 and 9, where relevant inputs have been used to get the \((\rho, R_{NP})\) parameter space in Fig. 7. Three remarks are in order.

a) The two solutions of \(R_{NP}\) lead to identical results for the CP asymmetry \(A_{\psi K}\). The reason is simply that \(A_{\psi K}\) depends only on \(R_{SM}\) and \(\phi_1\), as given in Eq. (4.12).

b) The correlation between \(A_{\pi \pi}\) and \(A_{\psi K}\) is complicated here, compared with that in scenario (A) where the superweak relation \(A_{\pi \pi} = -A_{\psi K}\) holds. We observe that both CP asymmetries can take the same sign and promising magnitudes in scenario (B) [23].

c) Due to their correlation, behaviors of \(A_{SL}\) and \(A_{\psi K}\) changing with \(\rho\) are similar. The magnitude of \(A_{SL}\) can reach the percent level for appropriate values of \(\rho\), if \(R_{NP}\) takes its \((-)\) solution.

6 Concluding remarks

Importance of measuring the CP asymmetry \(A_{SL}\) in semileptonic \(B_d\) decays, which can serve as a sensitive probe of NP in \(B_d^0\bar{B}_d^0\) mixing, has been highly stressed. We have taken three NP scenarios into account, and proposed a simple framework for analyzing their effects on \(A_{SL}\) and other CP asymmetries. Some numerical estimates have also been made to illustrate possible enhancement of CP asymmetries in the presence of NP. We find that the magnitude of \(A_{SL}\) at the percent level cannot be excluded within the appropriate parameter space. The correlation of \(A_{SL}\) with the CP asymmetries in \(B_d \to J/\psi K_S\) and \(B_d \to \pi^+ \pi^-\) (i.e., \(A_{\psi K}\) and \(A_{\pi \pi}\)) have been discussed. Measuring the correlation between \(A_{\psi K}\) and \(A_{\pi \pi}\) may partly testify or abandon the superweak model of CP violation which was proposed long time ago [23], at least within the \(B_d^0\bar{B}_d^0\) system [27].

If the CP asymmetry \(A_{SL}\) is really of the order \(10^{-2}\), it should be detected at the forthcoming \(B\)-meson factories, where as many as \(10^8 B_d^0\bar{B}_d^0\) events will be produced at the \(\Upsilon(4S)\) resonance in about one year. Indeed the experimental sensitivities to \(A_{SL}\) are expected to be well within few percent for either single lepton or dilepton asymmetry measurements [28], if the number of \(B_d^0\bar{B}_d^0\) events is larger than \(10^7\) or so.

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