Anomalous fluctuations in observations of Q0957+561 A,B: smoking gun of a cosmic string?

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Abstract. We report the detection of anomalous brightness fluctuations in the multiple image Q0957 + 561 A,B gravitational lens system, and consider whether such anomalies have a plausible interpretation within the framework of cosmic string theory. We study a simple model of gravitational lensing by an asymmetric rotating string. An explicit form of the lens equation is obtained and approximate relations for magnification are derived. We show that such a model with typical parameters of the GUT string can quantitatively reproduce the observed pattern of brightness fluctuations. On the other hand explanation involving a binary star system as an alternative cause requires an unacceptably large massive object at a small distance. We also discuss possible observational manifestations of cosmic strings within our lens model.

Key words. cosmology: miscellaneous – gravitational lensing – quasars: individual: Q0957+561 – dark matter – elementary particles

1. Introduction

Recent observations of the Q0957+561 A,B gravitational lens system show unexpected synchronous (without the expected time delay) fluctuations of brightness of the two quasar images. An ordinary binary star system, which might theoretically explain such fluctuations, would be too massive and close to us, i.e., would be clearly visible, which is not the case. Therefore, we attempt to explain these data by lensing on a cosmic string, particularly on a loop of string. The existence of cosmic strings is predicted by particle physics (Vilenkin & Shellard 1994) and gravitational lensing effects are a promising signature of these astrophysical objects. Sazhin et al. (2003) claimed the detection of the first case of cosmic string lensing. We demonstrate here another possible signature of a string: microlensing by oscillating loops of cosmic strings, which results in quasiperiodic fluctuations of the observed brightness of the source. In section 2 we discuss the observational data, and in section 3 a quantitative model of string lensing is elaborated. In section 4 an explanation of the observational data is presented, with discussion and conclusions in section 5.

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2. Observations of an anomaly in the Q0957+561 A,B gravitational lens system

The Q0957 system was the first discovered multiple image gravitational lens system, and already at the time of its discovery in 1979 (Walsh et al. 1979) it was understood that it was extremely important to astrophysics. Measurement of the time delay between fluctuations in the two known images would allow determination of the Hubble constant from simple theory, independent of uncertainty in local distance estimates for the supernovae and Cepheid variable stars (Refsdal 1964). Thus from the time of discovery, monitoring of the brightness of the two images, separated by 6 arcsec, was undertaken so that the quasar’s intrinsic brightness fluctuations could be recognized in the two images separately, and a time delay measured.

With the Schild and Cholfin (1986) measurement of time delay (including numerous subsequent refinements; see Colley et al. 2003 for a summary) it was soon recognized that there were differences between the time delay corrected brightness curves, although the basic pattern could be easily recognized. The differences were attributed to microlensing by individual massive objects, presumably stars, in the lens galaxy (Schild & Smith 1991). The prospect that such microlensing might allow detection of any baryonic missing mass objects justified intensive mon-
Fig. 1. Brightness of the two quasar images displayed with no correction for gravitational lens time delay. The brightness of quasar image A (upper record with square data markers), has been fitted with a sine curve having 0.04 magnitude amplitude. The lower record, with triangular markers, appears to have the same sinusoidal brightness curve with zero lag, even though at most epochs data for the gravitationally lensed images show a lag of 417 days.

Monitoring campaigns, and in the 24 years since discovery the source has been consistently observed on more than 1500 nights.

Such monitoring reveals two principal components in the quasar’s brightness fluctuations: a component due to intrinsic quasar brightness fluctuations, first seen in image A and then seen 417.1 days later in image B, and a microlensing component arising in only one image component due to individual stars along the A or B image line of sight.

We now illustrate what appears to be a third component of quasar brightness fluctuations, seen in the combination of Figs. 1,2, and 3. Here we plot the measured brightnesses of the two quasar images as measured during the 1994-1995 and 1995-1996 seasons. In Fig. 1 no correction has been made for time delay; the plotted brightness measurements in magnitudes are shown for the Julian dates of observation. The plotted symbols are the size of the typical 1 – σ error bars previously established for this data set (.006 and .007 mag for images A and B). In Fig. 2 a sine curve has been fitted to the A image data but not B image to allow the eye to judge whether there appears to be a repeating pattern of fluctuations for 0 lag.

Fig. 2. Quasar brightness displayed for measured time delay. The upper record shows the same data and fit for image A as displayed in Fig. 1. The lower data markers (triangles) are the brightness measurements for image B measured 417 days (the gravitational lens time delay) later, but with 417 subtracted from the Julian dates for plotting. If the image A brightness fluctuations are intrinsic to the quasar, they should be seen also in image B 417 days later, but the two are seen not to match as well as the 0 lag comparison in Fig. 1.
If the observed fluctuations are caused by random microlensing events, it would be unexpected for them to be apparently in phase. We have not yet devised a statistical test defining the significance level or error limits on simultaneity because the basic statistical process is non-Gaussian and has not yet been simulated. Moreover, any statistical analysis cannot be perfect in the presence of the usual stochastic microlensing variation taking into account the limited time interval where the anomalous effect has been observed. If the fluctuations are intrinsic to the quasar, and seen simultaneously by some highly improbable coincidence, they must be seen in the observations of the preceding and following years, as illustrated in Figs. 2 and 3. Thus we show in Fig. 2 that if the B data of Fig. 1 are compared to A image data measured 417 days previously, the fluctuations are probably not seen. Similarly we illustrate in Fig. 3 that if the A image pattern from Fig. 1 is compared to B image 417 days later, the pattern is again not seen. If the brightness fluctuations are intrinsic to the quasar and seen simultaneously as in Fig. 1 by chance, they must also be seen in the lagged data for the opposite quasar image; thus they would be seen in both Fig. 2 and Fig. 3.

The importance of these observations relates to the fact that there should be no causal connection between brightness fluctuations seen simultaneously. If the fluctuations were due to the quasar’s intrinsic brightness changes, then they should be seen at the measured time delay, which is 417.09 ± 0.07 days (Colley et al. 2003). If they were produced in proximity to the lens galaxy at redshift 0.37 (the quasar redshift is 1.41) they should similarly be seen with a large time delay. Only fluctuations produced locally (i.e., between the lens galaxy and the observer but close to the observer) can be observed to be simultaneous.

This problem is even more serious because of the relatively large separation of the two quasar images, 6.2 arcsec on the sky. Supposition that the above oscillations are induced by orbiting of binary stars leads to anomalously large masses of the components, as shown in subsection 4.1. Therefore we consider below the possibility that the oscillations are due to time variations of the gravitational field of a cosmic string.

3. Gravitational lensing by a cosmic string

3.1. Cosmic string characteristics

Cosmic strings are linear defects that could be formed at a symmetry breaking phase transition in the early Universe (Vilenkin & Shellard 1994). A horizon-sized volume at any cosmological time \( t \) should contain a few long strings stretching across the volume as well as a large number of small closed loops. At the moment of creation \( t \), typical loop length is \( l \sim \alpha ct \), i.e., about \( \alpha \) of horizon size \( ct \). During the string evolution, loops constitute some fixed part of total string network; this scaling results in the following loop number density

\[
n_l(t) \sim \alpha^{-1}(ct)^{-3}. \tag{1}
\]

Typically \( \alpha \) is determined by the gravitational back-reaction, so that \( \alpha \sim k_s G \mu / c^3 = k_s \epsilon \), where \( k_s \) is a numerical coefficient, \( G \) is gravitational constant, \( c \) is the speed of light, \( \mu \sim \eta^2 / \hbar c^3 \) is the mass per unit length
of string, \( h \) is the Planck constant, and \( \eta \) is the symmetry breaking scale of strings. For a Grand Unified Theory (GUT) string \( \eta_{\text{GUT}} \sim 10^{25} \text{ eV} \) and \( \mu_{\text{GUT}} \sim 10^{22} \text{ g/cm} \), \( \epsilon_{\text{GUT}} \sim 10^{-6} \) and \( \alpha_{\text{GUT}} \sim 10^{-4} \).

The loops oscillate and lose their energy, mostly by gravitational radiation. For a loop of length \( l \), the oscillation period is \( T_l = l/2c \) and the lifetime determined by gravitational radiation losses is \( \tau_l \sim l/c(k_g \mu/c^2) \).

Gravitational lensing by cosmic strings has been considered by many authors (see references in Vilenkin & Shellard 1994 and de Laix & Vachaspati 1996). Straight cosmic strings have a distinctive feature: they produce two identical images. However they cannot explain the oscillatory character of our data. Therefore we consider gravitational action of cosmic string loops, which cause effects similar to ordinary oscillating systems (binary stars and others), but are more massive and move with relativistic speeds.

### 3.2. Lensing by oscillating loops

De Laix & Vachaspati (1996) considered in detail, lensing by cosmic string loops. Here we use their approach for the interpretation of the observed oscillations. In the simplest idealized case of a circular loop, with oscillations reduced to variations of loop radius, they find that the image brightness of a point source will not oscillate if the loop does not overlap the source. Consequently we should take an asymmetric loop to explain the observed oscillations. We consider a maximally asymmetric string configuration in the form of a rotating double line segment of length \( 2R \) lying transverse to the line of sight and having coordinates:

\[
\begin{align*}
  x_1^{\text{str}} &= R \cos \left( \frac{ct}{R} \right) \sin \sigma, \\
  x_2^{\text{str}} &= R \sin \left( \frac{ct}{R} \right) \sin \sigma, \\
  x_3^{\text{str}} &= 0.
\end{align*}
\]

This is a particular solution from the family of known solutions to string equations (Turok 1984). Vilenkin & Shellard 1994). Here axis \( x_3 \) is directed to observer, \( x_1^{\text{str}} \), \( x_2^{\text{str}} \) are coordinates of the loop in the lens plane, \( \sigma \) indicates position on the string and varies from 0 to \( 2\pi \) and \( t \) is the time. This configuration is a limiting case of asymmetric loop; in a more realistic case a loop will have an ellipsoidal form with large eccentricity.

The lens equations can be obtained from general result of de Laix & Vachaspati (1996). After some calculations taking into account the particular solution (2) we have:

\[
\frac{D_l}{D_s} \bar{y}_1 = \bar{x}_1 - q_s R^2 \sgn(\bar{x}_1) \times \sqrt{\frac{(R^2 + \bar{x}_1^2 + \bar{x}_2^2)^2 - 4R^2 \bar{x}_1^2 - R^2 - \bar{x}_2^2 + \bar{x}_1^2}{2((R^2 + \bar{x}_1^2 + \bar{x}_2^2)^2 - 4R^2 \bar{x}_1^2)}},
\]

\[
\frac{D_l}{D_s} \bar{y}_2 = \bar{x}_2 - q_s R^2 \sgn(\bar{x}_2) \times \sqrt{\frac{(R^2 + \bar{x}_1^2 + \bar{x}_2^2)^2 - 4R^2 \bar{x}_1^2 - R^2 - \bar{x}_2^2 + \bar{x}_1^2}{2((R^2 + \bar{x}_1^2 + \bar{x}_2^2)^2 - 4R^2 \bar{x}_1^2)}},
\]

where \( q_s \sim 8\pi \frac{G \rho}{c^2} \frac{D_s}{D_l} \) (\( D_s \) and \( D_l \) are distances from us to source plane, to lens plane and from source to lens plane respectively), \( \bar{x}_1 = x_1 \cos \left( \frac{ct}{R} \right) + x_2 \sin \left( \frac{ct}{R} \right) \), \( \bar{x}_2 = x_2 \cos \left( \frac{ct}{R} \right) - x_1 \sin \left( \frac{ct}{R} \right) \) (where \( x_1 \), \( x_2 \) are coordinates in the lens plane) and \( \bar{y}_1 = y_1 \cos \left( \frac{ct}{R} \right) + y_2 \sin \left( \frac{ct}{R} \right) \), \( \bar{y}_2 = y_2 \cos \left( \frac{ct}{R} \right) - y_1 \sin \left( \frac{ct}{R} \right) \) (where \( y_1 \), \( y_2 \) are coordinates in the source plane). It may be shown that all the relations under the root signs are non-negative.

Magnification of a point-like source by such a string is

\[
m = \left| 1 - q_s^2 R^4 \left( x_1^2 + x_2^2 \right)^2 - 4R^2 \left( x_1 \cos \frac{ct}{R} + x_2 \sin \frac{ct}{R} \right)^2 \right|^{3/2} \left( \left( \frac{R^2 + x_1^2 + x_2^2}{R^2 + x_1^2 + x_2^2} \right) + \frac{3q_s^2 R^6}{\rho^2} \left( \frac{y_1^2 - y_2^2}{y_1^2 - y_2^2} \right) \cos \frac{2ct}{R} + 2y_1 y_2 \sin \frac{2ct}{R}. \right.
\]

This yields amplitude of source brightness fluctuations as follows

\[
\Delta m \approx \left| 6q_s^2 R^6 \rho^{-2} \right| \equiv \frac{32\pi^2 G^2 \mu^2 q_s^4}{c^4 \theta_t^2},
\]

where \( \theta_t = \rho/D_s \) is the angular impact distance of the line-of-sight with respect to the center of the loop, \( \theta_R = R/D_l \) is half of the visible angular size of the loop.

### 3.3. Lensing by a binary system

Now we consider for comparison the gravitational lensing by a binary system of two equal point masses \( M \) orbiting their center of mass with the period \( T \). Further, \( r \) is half of the distance between the masses and \( \omega = \pi/T \). The magnification of a point-like source by such lens system is (Schneider, Ehlers & Falco 1992):

\[
m = \left| 1 - q_0^2 r^4 \left( x_1^2 + x_2^2 + r^2 \right)^2 - 4r^2 \left( x_1 \sin \omega t - x_2 \cos \omega t \right)^2 \right|^{3/2} \left( \left( x_1^2 + x_2^2 + r^2 \right)^2 - 4r^2 \left( x_1 \cos \omega t + x_2 \sin \omega t \right)^2 \right)^{-1},
\]

where \( q_0 = \frac{8\pi GM}{c^2} \frac{D_s}{D_l} \) and the other values are defined as in the previous section.
Analogously to the previous case we obtain approximate formulae for magnification
\[ m = 1 + \frac{g_0^2 r_u^4}{\rho^4} - 4 \frac{g_1 r_u^6}{\rho^6} + \frac{6 g_2 r_u^6}{\rho^8} \left((y_1^2 - y_2^2) \cos 2\omega t + 2y_1 y_2 \sin 2\omega t\right) \]  
(9)

(where \( r_u = rD_s/D_1 \)) and for the amplitude of fluctuations in case of circular motion of the binary system:
\[ \Delta m \approx \frac{12 g_0^2 r_u^6}{\rho^6} \approx \frac{1.2 \times 10^6 }{c^4 T^4} D_s^4 D_1^2, \]  
(10)

where \( \theta_l = \rho / D_s \) and \( \theta_r = r / D_1 \) (angles in radians). Here we have used the relation \( GM = 4\pi^2 r^3 / T^2 \) for the circular motion.

4. Explanation of the experimental data

Finally we apply the above calculations to explain the observed brightness oscillations. These oscillations are nearly sinusoidal, their period is approximately 100 days and their amplitude is about 4% of the quasar image brightness. At least three oscillations were observed during the period of the observations. We consider the possibility that this phenomenon is caused by the cosmic string loop passing through the neighborhood of images A and B at a small distance from the observer. Obviously to fit the observational data described in the section 2 we are forced to restrict the parameters of our model. Also, because the synchronous oscillations have been observed within a limited time interval, we include into the consideration the motion of the loop. At that the number of observed oscillations (3-4) restricts a transverse velocity of the loop to the values \( v_1, v_2 \leq 0.1c \), but leaves a considerable freedom for the velocity component \( v_3 \) along the line of sight. In this case the only correction for the lens equations, as can be shown, is to change the parameter \( q_3 \) by \( q_s = q_3 (1 + v_3 / c)^{-1} \).

4.1. String-induced and binary star-induced oscillations of quasar brightness

As we mentioned above, the period of observed oscillations \( T \approx 100 \) days is related to the string length \( l = 2\pi R \) as \( R = c T_l / \pi \). Taking into account relativistic motion of the string along the line of sight we have \( T_l = T(1 - v_3^2 / c^2)^{1/2} \).  

Equation (4) can be rewritten as:
\[ \Delta m \approx 5.6 \left( \frac{\theta_l}{3''} \right)^{-6} \left( \frac{\theta_R}{1.5''} \right)^4 \times \frac{\mu}{10^{22} \text{g/cm}^2} \left(1 + \frac{v_3}{c}\right)^{-2}. \]  
(11)

To provide almost equal amplitude of the brightness variations in both images the loop should fly close to the mid-point. In this case \( \theta_l \approx 3'' \), i.e. about half of the distance between the images A and B. For numerical estimates we put, e. g., \( v \approx 0.7c \). In order to have 3 oscillations with possible phase shift (see Fig. 5) we need \( v_1 \approx 0.03 \) and \( v_2 \approx 0.11 \) (Fig. 4).

In order to have quasi-sinusoidal variations, \( \theta_R \) must be considerably smaller than the angular distance between images A,B of the Q0957+561 (otherwise there will be sharp spikes and/or discontinuities in the dependence of brightness upon time) and \( \theta_R \) cannot be too small leading to large loop mass in virtue of equation (11) (to avoid a large monopole input into the effective lens potential leading to additional amplification and corresponding unobserved slow increase and decrease of image brightness superimposed on smaller oscillations due to loop rotation). Therefore we should take \( \theta_R \approx 1.5'' \) and consequently \( D_l \approx 3 \) kpc. From equation (11) we can find that \( \mu \approx 4 \times 10^{20} \text{g/cm} \) for observed amplitude \( \Delta m \approx 0.04 \).
More accurate values of loop parameters follow from numerical solution of equations (3) - (5) without assumption $\sqrt{y_1^2 + y_2^2} \gg R_y$. For explanation of observations we need $q_s = 1.3$ and consequently $\mu \approx 8 \times 10^{21} \text{g/cm}$ (see Fig. 5). Remarkably, this value is close to one predicted by the GUT, $\mu \sim \mu_{\text{GUT}}$.

In the alternative case of lensing by a binary system we obtain from equation (10):

$$
\Delta m \approx 0.04 \left( \frac{T}{100 \text{ days}} \right)^{-4} \left( \frac{\theta_r}{1.5''} \right)^8 \\
\times \left( \frac{\theta_I}{3''} \right)^{-6} \left( \frac{D_l}{1.2 \text{ pc}} \right)^4,
$$

(12)

In order to explain the observed fluctuations the requirement on the trajectory should be the same as in the loop case: $\theta_I \approx 3''$, i.e. about half of the distance between images A and B. The minimum distance from us to this system must be 1.2 pc, the orbital radius should be $\theta_I D_l \approx 1.8 \text{ a.u.}$ and the masses of the components should equal $78 M_\odot$ to supply 4% amplitude fluctuations of period 100 days. At such distance the stars would be observable. For larger distances to the binary star the masses of the components need to be larger as well. Therefore we consider such a binary star model to be unacceptable.

4.2. Influence of the loop on the brightness of the lensing galaxy

Let us now consider the influence of a loop on the brightness of the lens galaxy which is visible between the images of the quasar and makes a [3%, 15%] contribution to the observed brightness in the [A, B] apertures, respectively (Colley & Schild 1999, Fig. 3). For a galaxy at distance approximately $1''$ from image B with Gaussian brightness distribution ($\sigma = 2''$), we obtain from numerical calculation that relative oscillations of galaxy brightness in the measured apertures are equal to 9% and 6%. This corresponds to 1.6% and 1.1% of total signal in measured apertures A and B, which include quasar images. Oscillations will be superimposed on a background of monopole component with amplitudes of 12% and 14% of galaxy brightness. This corresponds to 2.2% and 2.5% of total brightness variations. A view of the lens galaxy as modified by the loop at $t = 0$ is shown in Fig. 5a.

The loop overlapping a galaxy can result in significant microlensing effect. Caustics and critical curves are shown...
in Fig. 4 (b and c). When a star of radius $R_\star$ is located on the caustic near its edge, the star’s brightness is increased by a factor of $3 \times 10^3 (R_\star/R_C)^{-1/2}$ according to equation for magnification near a straight caustic (see Schneider, Ehlers, & Falco [1992]). The corresponding increase of the galaxy brightness will be about $3 \times 10^{-7}$ i.e., unobserved in our case.

The largest magnification is expected for stars crossing the cusp of the caustic. Our calculations show that magnification of a star with $R_\star = R_C$, is equal to $m \approx 1.1 \times 10^9$ (0.1% relative to the galaxy brightness). The visible speed of the cusp motion through the galaxy plane is $v_{\text{cusp}} \approx 2 \times 10^{-3}$ pc/s. In one pixel of a telescope image plane, light is collected from about $(D_g \theta_{\text{pixel}})^2 n_{\text{star}} \approx 5 \times 10^8 (\theta_{\text{pixel}}/0.1'')^2$ stars ($n_{\text{star}} \approx 1000$ pc$^{-2}$ is the surface density of stars, $D_g \approx 1.4$ Gpc is the distance to the galaxy). Therefore during about $10^{-8} (\theta_{\text{pixel}}/0.1'')^{-3} \approx 4 \times 10^{-11}$ s the brightness of the star is larger than the brightness of the pixel without lensing.

For telescope integration time $T_{\text{int}}$, the average magnification of the star in the cusp is: $< m > = 1.2 \times 10^4 (T_{\text{int}}/1s)^{-2/3} \approx 400$. The typical distance between projections of stars in the galaxy plane equals $l_{\text{ss}} \approx 0.03$ pc. Therefore such flashes will be repeated approximately every $l_{\text{ss}}/v_{\text{cusp}} \sim 10$ seconds.

In our case the galaxy brightness is about 20% of the quasar B image brightness and therefore the maximum of these flashes will be approximately $2 \times 10^{-4}$ of the image brightness, which is below observational precision. So lensing of the galaxy is observable only from distortion of the total galaxy image, as shown in Fig. 4a.

5. Conclusions

Our main motivation to apply the cosmic string model for explanation of brightness oscillations in question is their specific characteristics. It is difficult to propose a less exotic model, such as a double star model to explain the observed oscillations. We have shown (see subsection 4.1) that in case of a double star the masses of the components must be of order of $78 M_\odot$ at 1.2 pc distance in order to meet all observational requirements. For larger distances the masses of the components should be larger as well. This seems to be unacceptable.

On the other hand the property of fast oscillations is typical for cosmic strings. To show viability of the string interpretation of observed oscillations we have chosen a particular “degenerate” (highly asymmetrical) analytical solution of string equations. This solution is a limiting case of strongly elongated rotating loop configuration with sufficient quadrupole moment. Of course, more realistic case should include additional modes of loop oscillations in order to avoid self-intersections and annihilation. Nevertheless, even in more general cases solutions with sufficient quadrupole moment can provide the same level of brightness variations.

The results presented here show that loops of cosmic strings supply quantitative explanations of synchronous variations in the two images of the gravitationally lensed quasar Q0957+561 A,B. The derived value of the string parameter $\mu \sim 10^{22} g/cm$ lies just in the theoretically preferable range. Atypical (with small probability of realization, but not impossible) in our model are the distances to the loops and their sizes - both are about $10^{-3} - 10^{-4}$ of statistically expected values. The reason for this is the relatively short observed period, only of order 100 days, which limits the length of the loop. Moreover, the observed angular separation of the images predicts the distance to the loop, while we fixed its size according to the period of flux fluctuation. Consequently in another hypothetical object with different observational parameters, more typical loops will probably work. Of course, a single event need not follow statistical rules, especially since our observational sampling rate might strongly favor this particular specimen. Moreover, some physical mechanism might cause the concentration of loops in galactic halos. Therefore, further observational efforts towards uncovering more objects with similar properties are extremely important.

Searches for brightness oscillations, similar to those described above, can be a promising way of discovering the gravitational signatures of cosmic strings.

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