Determining parameters of the CSUH constitutive model by genetic algorithm

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ABSTRACT

In this paper, the genetic algorithm (GA) is applied to determine the parameters of unified model for clay and sand, referred to as CSUH constitutive model. When determining the parameters of CSUH model by conventional methods, many conventional tests are required to be done, including low and high confining pressure triaxial tests and isotropic compression tests with specific initial void ratio. That will take more efforts and long time, and a basic understanding of CSUH model should be requested at the same time. However, using GA to automatically determine those parameters only needs a small amount of conventional triaxial tests. Finally, taking the parameters determination of Toyoura sand in CSUH model as an example, the GA is used to determine the parameters according to the partial experimental data and then to predict all the experimental data, the prediction result is acceptable.

Keywords: CSUH constitutive model; genetic algorithm; parameters determination; conventional test

1 INTRODUCTION

Constitutive modelling is the cornerstone of soil mechanics. Since the classical modified Cam-clay model (Roscoe and Burland 1968) was proposed, the elastoplastic constitutive models of soil have been developed greatly. Many advanced models were proposed based on the modified Cam-clay model, such as the subloading surface model (Schofield and Wroth, 1968), the bounding surface model (Dafalias and Herrmann, 1986; Hashiguchi, 1989) and Pender’s model (Pender, 1978). By introducing a unified hardening parameter into the modified Cam-clay model, Yao et al. proposed a unified hardening (UH) model for overconsolidated clay. (Yao et al., 2008a; Yao et al., 2008b; Yao et al., 2008c; Yao et al., 2009) The UH model is able to describe the isotropic compression, shear yielding and dilatancy of overconsolidated clay. Yao et al. also proposed a model that takes into account temperature (Yao et al., 2013). The premise of constitutive model application is to determine the constitutive model parameters of soil. The methods to determine the model parameters roughly are divided into two categories. One is the conventional methods, such as definition analysis, which is to determine the parameter values according to the physical meaning of parameters in corresponding tests. Other methods are intelligent algorithms, such as genetic algorithms (Levasseur et al., 2008), neural networks (Ghaboussi and Sidarta, 1998; Obrzud et al., 2009), particle swarm optimization algorithm (Knabe et al., 2013), etc., among which the genetic algorithm (GA) has a strong robustness. The definition analysis method is only applicable to parameters with clear physical meaning, and the corresponding experiments used to acquire these parameters may need heavy workload, and also requires engineers to have sufficient understanding of constitutive models. Some complex but excellent constitutive models, such as anisotropy model (Guo et al., 2013), unsaturated soil model (Sun et al., 2007a; Sun et al., 2007b; Sun et al., 2007c), are not easy to understand. However, GA performs the least squares operation between the constitutive model calculation results and the experiment results, and optimize the ideal constitutive model parameters by minimizing the least squares automatically, only needing the conventional triaxial test results. Compared
with the conventional methods, the GA has the advantages of simplicity, automation and convenience.

Genetic algorithm is an evolutionary algorithm proposed by Holland in 1965 based on Darwin’s theory of natural selection (Holland, 1992). In recent years, various improved genetic algorithms have been proposed and applied to solve geotechnical engineering problems (Jin et al., 2016; Papon et al., 2012; Rokonuzzaman and Sakai, 2010; Wöhling et al., 2008). The coding methods of GA include binary coding methods suitable for small and medium scale problems with low precision, and real code methods suitable for high precision and large-scale problems (Goldberg, 1991). CSUH constitutive model has 8 parameters in which some parameters are highly sensitive, so real coding genetic algorithm (RCGA) is used to determine the parameters.

The purpose of this paper is to determine the CSUH model parameters of Toyoura sand based on conventional triaxial test using GA. In order to simplify the reading of experiment data under different stress paths, the uniform input format was proposed. In addition, Cubic spline interpolation of experiment data and normalization of error function were applied in GA. Finally, taking 7 group of Toyoura sand tests as an example, CSUH model parameters of Toyoura sand were determined using GA according to 4 group of data. And then the determined parameters are used to predict all of the test data, and the predicted results are acceptable. Through triaxial test combination calculation, the principle of optimizing test combination for genetic algorithm is given.

2 DETERMINING PARAMETERS BY CONVENTIONAL METHOD

2.1 CSUH model parameters

The CSUH model has 8 parameters, in which 5 parameters are same with the classical modified Cam-clay model including $M$, $v$, $\kappa$, $\lambda$ and $N$. The other 3 parameters are $Z_c$, $\chi$ and $m$, representing void ratio on the NCL at $p = 1kPa$, critical state parameter and dilatancy parameter respectively (Yao et al., 2019). If conventional methods were used to determine these parameters, conventional undrained triaxial compression test (CU test), conventional drained triaxial compression test (CD test) and isotropic compression test should be performed.

2.2 Steps to determine parameters

The steps to determine the parameters of CSUH constitutive model by conventional methods are as follows:

1. determining the critical state stress ratio $M$ and poisson's ratio $v$. The critical state stress ratio $M$ acquire critical stress ratio in CU test or CD test. Poisson's ratio $v$ can be determined empirically, usually within 0.1–0.3.

2. determining the parameter $Z$. The critical state line(CSL) can be obtained by connecting the critical state points that acquired in CD or CU test with small confining pressure. The intersection point of axis $e$ (when $p=0kPa$) and CSL is the maximum critical state void ratio $e_{c0}$, which can be approximately considered as parameter $Z$.

3. determining the normal compression line parameters $\lambda$, $N$ and $\kappa$. After the parameter $Z$ is determined, the normal compression line NCL can be obtained by performing the isotropic compression test with the initial void ratio of $Z$. After the isotropic compression is completed, unload it. The corresponding stress $p$ of point with maximum curvature on NCL was crushing stress $p_s$. In $e-ln(p+p_s)$ space, the NCL is a straight line with the slope $\lambda$, intercept $N$, and the unloading line also is a straight line with slope $\kappa$, as shown in Figure 1. The parameter $\lambda$ and $p_s$ can also be obtained by fitting the normal compression test loading part of the data directly using the normal compression line formula (1).

\[
\epsilon = Z - \lambda \ln \left( \frac{p + p_s}{1 + p_s} \right)
\]  

(1) 

4. determining the critical state parameters $\chi$. The critical state parameter $\chi$ can be obtained by fitting the critical state data (Yao et al., 2019).

5. determining the dilatancy parameter $m$. The dilatancy parameter $m$ can be determined by the characteristic state point of CU test (Yao et al., 2019).

The above steps are used to determine the
parameters of the CSUH model using conventional methods. It can be seen that conventional methods are complex approaches, which need not only CD and CU tests, but also isotropic compression test of specific initial void ratio.

3 DETERMINING PARAMETERS BY GA

3.1 Error function

In the GA, a population is assembled by multiple individuals that are assembled by multiple genes. An individual $X_i$ is assemble by all 8 parameters of CSUH:

$$X_i = [M, \nu, \kappa, \lambda, N, Z, \chi, m], i=1\ldots n$$  \hspace{1cm} (2)

Where $n$ is the population size, and also is the number of individuals in the population.

Conventional tests include CU test, conventional triaxial test with constant p (constant p test), CD test, lateral limit compression test ($K_0$ test) and isotropic compression test (ISO test). Numbers 1~5 can be used respectively to represent these test types. The test data of these test types can be arranged into a unified five-column input format:

$$\varepsilon_1, q(kPa), p'(kPa), e, u(kPa)$$  \hspace{1cm} (3)

The variable $\varepsilon_1$ represents axial strain, variable $q$ represents shear stress, variable $p'$ represents average effective principal stress, variable $e$ represents current void ratio and variable $u$ represents current pore pressure.

The benefit of this format is that it implies the initial conditions of test. The first line data of $p'$ and $e$, represent initial confining pressure and initial void ratio respectively in shear stage. With an additional test path information (represented by test type number 1~5) and constitutive model parameters, the theoretical data in formula (3) can be acquired. With theoretical and experiment data of each test type, perform least square calculation can acquire error as fellows:

$$Error(X_i) = \sqrt{\frac{1}{n_r} \sum_{i=1}^{n_r} \left( \frac{R_{\text{exp}_i} - R_{\text{th}_i}}{R_{\text{exp}_i}} \right)^2}$$  \hspace{1cm} (4)

Where $X_i$ represent a set of CSUH model parameters proposed in formula (2), $n_r$ represent the total number of rows of experimental data used for calculation, $R_{\text{exp}_i}$ and $R_{\text{th}_i}$ represent experimental and theoretical variables respectively in formula (3) except $\varepsilon_1$. Since it is not easy to judge the fitting effect of the least square results easily, normalization is adopted like formula (4), so that the error value is within $[0,1]$. And smaller the error value is, the fitting effect is better. Of course, the error function has other forms, and different kind of error functions may acquire different optimal parameters.

Not all variables are used in error calculation, and the effective variables used in error calculation in different test types are different, As shown in Table 1:

| Test type | $q$(kPa) | $p'$(kPa) | $e$ | $u$(kPa) |
|-----------|----------|-----------|-----|----------|
| CU        | ✓        | ×         | ×   | ✓        |
| Constant p| ✓        | ×         | ✓   | ×        |
| CD        | ✓        | ×         | ×   | ✓        |
| K0        | ×        | ✓         | ×   | ×        |
| ISO       | ×        | ✓         | ×   | ×        |

For example, when calculate the CD test error, $R_{\text{exp}}$ only take all shear stresses $q$ and void ratio $e$ in the test data as effective variables, and that is represented using symbol "$\times" in table 1. In order to make the theoretical data fit the test data better, cubic spline interpolation was applied to the test data in advance to reduce the discretization of the test data.

The flowchart of GA is shown in Figure 2.

3.2 Three major operators

Genetic algorithm is composed of three operators: selection operator, crossover operator and mutation operator. Each generation performs these three operator in turn.

(1) Selection operator

The effect of the selection operator is to eliminate the individuals with large error value. The selection operator is generally executed $n$ times in each generation, where $n$ is the population size. Binary tournament selection algorithm is adopted here (Arumugam et al., 2005; Mokhade and Kakde, 2014). In this way, two individuals in the population were
randomly selected each time, and only the individuals with small error value have the opportunity to carry out the crossover operator of the subsequent process of this generation.

2) Crossover operator

The effect of the crossover operator is to search possible solutions. Generating a random number in range [0, 1] and if it is less than the crossover probability \( P_c \) (generally in range [0.75, 0.9], where 0.75 is taken here), crossover operator is performed. Two old individuals can generate two new individuals through crossover operator, which increases the diversity of the population. The simulated binary operation, and

\[
B = \{B_i\}, \quad i=1...8
\]  

Where, \((1+B)\) means that each element of vector \( B \) is added by 1, and the dot symbol "." means that the corresponding elements of the vectors are multiplied respectively. \( X_i \) and \( X'_i \) represent lower and upper bound of parameter \( s \), respectively.

\[
X'_i = \frac{1}{2}(1-B) \cdot X_i + (1-B) \cdot X'_i
\]  

Where, \((1+B)\) means that each element of vector \( B \) is added by 1, and the dot symbol "·" means that the corresponding elements of the vectors are multiplied respectively. \( X_i \) and \( X'_i \) represent lower and upper bound of parameter \( s \), respectively.

\[
X'_i = X_i + s \cdot \Phi_0 \cdot (X'_i - X_i)
\]  

Where \( X_i \) is the currently selected mutant individual, \( s_m \) is the variation step distance. With the increase of generation number \( k \), step distance \( s_m \) gradually decreases, making the search of genetic algorithm more accurate and stable. The elements of vector \( \Phi_0 \) are within the scope of [-1, 1]. The vector length is 8 for CSUH constitutive model.

\[
s_m = \frac{1}{3} \arctan \left( a \left( \frac{k}{k_{\text{max}}} \right)^b \right) + \frac{1}{2}
\]  

Where \( k \) and \( k_{\text{max}} \) denote the current and the maximum generation number, respectively. Variable \( a \) equals 10 and variable \( b \) equals 1 here.

4 PARAMETERS DETERMINATION OF TOYOURA SAND IN CSUH MODEL BY GA

The test data (Verdugo and Ishihara, 1996) have 3 CD and 4 CU test. The initial conditions of the test were shown in Table 3.

Table 3. Initial conditions of triaxial tests.

| NO | test type | initial confining pressure (kPa) | initial void ratio |
|----|-----------|---------------------------------|-------------------|
| ① | CD        | 500                             | 0.960             |
| ② | CD        | 500                             | 0.886             |
| ③ | CD        | 500                             | 0.810             |
| ④ | CU        | 100                             | 0.833             |
| ⑤ | CU        | 1000                            | 0.833             |
| ⑥ | CU        | 2000                            | 0.833             |
| ⑦ | CU        | 3000                            | 0.833             |

The CSUH constitutive model parameters of Toyoura sand experiment data were determined by genetic algorithm. The specific test combination and corresponding error are shown in Table 4.

Table 4. Calculated error of different combinations.

| group | use test | self-error(%) | total error(①~⑦) (%) |
|-------|----------|---------------|----------------------|
| A     | ①~⑦     | 8.51          | 8.51                 |
| B     | ①~⑦     | 24.42         | 24.42                |
| C     | ①~⑦     | 26.93         | 26.93                |
| D     | ①~⑦     | 15.44         | 15.44                |
| E     | ①~⑦     | 18.74         | 18.74                |
| F     | ①~⑦     | 13.06         | 13.06                |
| G     | ①~⑦     | 11.84         | 11.84                |
| H     | ①~⑦     | 10.24         | 10.24                |
| I     | ①~⑦     | 10.38         | 10.38                |
| J     | ①~⑦     | 20.30         | 20.30                |
| K     | ①~⑦     | 10.37         | 10.37                |
| L     | ①~⑦     | 8.82          | 8.82                 |
| M     | ①~⑦     | 9.06          | 9.06                 |
| N     | ①~⑦     | 8.92          | 8.92                 |

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Where self-error is the error calculate between the constitutive model prediction and test data used to determine parameters. Total error is the error calculate between the constitutive model prediction in which parameters are determined by partial test data and all test data. As shown in Table 4, the prediction effect with all test data is best. When partial data are used, the total error is larger when the number of test data groups is less than 3. The total error is not necessarily small when self-error is small. The comparison between J and K groups shows that the prediction results of only two undrained tests are not necessarily ideal. As test data group L shows, its self-error may be greater than the total error, which is caused by the dispersion of test data.

Both CD and CU test data are recommended in determining parameters, and so are the test data groups with high initial confining pressure. At least three test data with different initial conditions are needed to determine parameters. In accordance with the above principles, determining CSUH parameters using group N is more effective than others. The determined parameters of Toyoura sand are shown in Table 5.

Table 5. CSUH model parameters for Toyoura sand.

|   | M   | v  | k   | λ   | N  | Z   | m   |
|---|-----|----|-----|-----|----|-----|-----|
| A | 1.19| 0.21| 0.089 | 0.163 | 2.230 | 0.948 | 0.59 | 0.15 |
| N | 1.18| 0.21| 0.070 | 0.155 | 2.151 | 0.986 | 0.59 | 0.25 |

Using test combinations in group N, the parameters of Toyoura sand determined by GA are used to predict all test data as shown in Fig. 3 and Fig. 4.

As shown in figure 3, the fitting effect is acceptable.

Fig. 3 Comparisons between CD test results and CSUH model predicted results using parameters in group N.

5 CONCLUSIONS

The procedures of determining CSUH constitutive model parameters by conventional method and GA are discussed. Several kinds of tests are used to determine parameters by conventional methods. The critical state line must be determined according to the CU and CD tests under small confining pressure first, and then the isotropic compression tests of specific initial void ratio should be conducted. However, the parameters determined by conventional methods are generally not satisfied under the practical conditions.

The CSUH constitutive model parameters are determined by the improved genetic algorithm. Taking Toyoura sand as an example, partial tests data are used to predict all the tests data. And the predict result is suitable. The improved genetic algorithm can be acceptable to determine the parameters of CSUH model.

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