Do Märzke-Wheeler effects influence on measured data in nature?

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We wonder whether Märzke-Wheeler effects influence on measured data in nature. Through a formula developed in this letter for the calculation of the Märzke-Wheeler map of a general accelerated observer, we study the influence of the Märzke-Wheeler acceleration effect on the NASA’s Pioneer anomaly and found that it is about a fifth of the anomaly value. Due to statistical errors in the measured anomaly, it is not possible to neither confirm nor neglect the influence of the Märzke-Wheeler acceleration effect on the measured Pioneer data. We hope that the ideas presented here could encourage other research teams in the search for other observational objects that could finally answer the question posed in this letter.

I. INTRODUCTION

Nowadays, it is very clear how special relativity effects influence on measured data. The first celebrated example of this fact was the atmospheric muons decay explanation as a time dilation effect. This is the Rossi-Hall experiment. Considering the Märzke-Wheeler synchronization as the natural generalization to accelerated observers of Einstein synchronization in special relativity, we wonder whether Märzke-Wheeler effects influence on measured data in nature. This question is also motivated by the fact that recently the twin paradox was completely solved in (1+1)-spacetime (1) and it is natural to ask for empirical confirmation. Of course these effects comprehend the well known special relativistic ones for inertial observers as well as the new ones. These new effects can be seen as corrections of the special relativistic ones due to the acceleration of the involved observer.

A small deviation towards the sun from the predicted Pioneer acceleration: 8.09 ± 0.20 × 10^{-10} m/s^2 for Pioneer 10 and 8.56 ± 0.15 × 10^{-10} m/s^2 for Pioneer 11, was reported for the first time in (2). The analysis of the Pioneer data from 1987 to 1998 for Pioneer 10 and 1987 to 1990 for Pioneer 11 made in (3) improves the anomaly value and it was reported to be 8.74 ± 1.33 × 10^{-10} m/s^2. This is known as the Pioneer anomaly.

Considering that Märzke-Wheeler tiny effects are difficult to measure, we careful looked for some observational object for which the searched effect could be appreciable. This search led us to the Pioneer 10. In fact, through a simple analytic formula for the Märzke-Wheeler map exact calculation developed in this letter, computing the acceleration difference between the Märzke-Wheeler and Frenet-Serret coordinates for the earth’s translation around the sun, we see that this Märzke-Wheeler long range effect is between 0 and ≈ 17% of the Pioneer anomaly value. Unfortunately, due to statistical errors in the measured anomaly, it is not possible to confirm the influence of the Märzke-Wheeler acceleration effect on the measured Pioneer data. Moreover, a recently numerical thermal model based on a finite element method (4) has shown a discrepancy of 20% of the actual measured anomaly and due to the mentioned statistical errors, it was concluded there that the pioneer anomaly has been finally explained within experimental error of 26% of the anomaly value:

...To determine if the remaining 20% represents a statistically significant acceleration anomaly not accounted for by conventional forces, we analyzed the various error sources that contribute to the uncertainties in the acceleration estimates using radio-metric Doppler and thermal models... We therefore conclude that at the present level of our knowledge of the Pioneer 10 spacecraft and its trajectory, no statistically significant acceleration anomaly exists.

Although it is tempting to think that the 20% discrepancy found in (4) is due to a long range Märzke-Wheeler acceleration effect, it cannot be confirmed. We hope that the ideas presented here could encourage other research teams in the search for other observational objects that could finally answer the question posed in this letter.

II. MÄRZKE-WHEELER MAP

Consider the (1 + n)-spacetime \( \mathcal{M} \) spanned by the vectors \( \sigma_0, \sigma_1 \ldots \sigma_n \) with the Lorentz metric:

\[
\begin{align*}
    ds^2 &= (dx^0)^2 - (dx^1)^2 - \ldots (dx^n)^2
\end{align*}
\]
FIG. 1: M"arzke-Wheeler map

respect to the basis \{\sigma_0, \sigma_1 \ldots \sigma_n\}. An observer is a smooth curve \(\gamma : \mathbb{R} \rightarrow \mathcal{M}\) naturally parameterized with timelike derivative at every instant; i.e. \(|\dot{\gamma}(s)|^2 = 1\). We will say a vector is spatial if it is a linear combination of \{\sigma_1 \ldots \sigma_n\}. A spatial vector \(\vec{u}\) is unitary if \(|\vec{u}|^2 = -1\).

**Definition II.1** Consider a timelike vector \(a\) in \(\mathcal{M}\); i.e. \(|a|^2_L \geq 0\). We define the scaled Lorentz transformation \(\mathcal{L}(a)\):

\[
\mathcal{L}(a) = |a|_L \mathcal{L}_a
\]

where \(\mathcal{L}_a\) is the orthochronous Lorentz boost transformation sending \(\sigma_0\) to the unitary vector \(a/|a|_L\); i.e. the original and transformed coordinates are in standard configuration (\(x', y'\) and \(z'\) are colinear with \(x, y\) and \(z\) respectively where the prime denote the spatial transformed coordinates and the others denote the original spatial coordinates).

The scaled Lorentz transformation has the following properties:

\[
|\mathcal{L}(a)(b)|^2_L = |a|^2_L |b|^2_L
\]

\[
\mathcal{L}(a)(\sigma_0) = a
\]

**Definition II.2** A smooth map \(\Omega_\gamma : \mathcal{M} \rightarrow \mathcal{M}\) is a M"arzke-Wheeler map of the observer \(\gamma\) if it verifies:

\[
|\Omega_\gamma(s \sigma_0 + r\vec{u}) - \gamma(s \pm r)|^2_L = 0
\]

for every real \(s\), positive real \(r\) and unitary spatial vector \(\vec{u}\) (see Figure 1).

This map is clearly an extension of the Einstein synchronization convention for non accelerated observers; i.e. It is the natural generalization of a Lorentz transformation in the case of accelerated observers.

**Proposition II.1** Consider an observer \(\gamma : \mathbb{R} \rightarrow \mathcal{M}\). Then,

\[
\Omega_\gamma(s \sigma_0 + r\vec{u}) = \frac{\gamma(s + r) + \gamma(s - r)}{2} + \mathcal{L}\left(\frac{\gamma(s + r) - \gamma(s - r)}{2}\right)(\vec{u})
\]

is a M"arzke-Wheeler map of the observer \(\gamma\) such that \(\vec{u}\) is a unitary spatial vector.

**Proof:** Recall that for every \(a\) such that \(|a|^2_L \geq 0\) we have that \(\mathcal{L}(a)(\sigma_0) = a\). This way,

\[
|\Omega_\gamma(s \sigma_0 + r\vec{u}) - \gamma(s \pm r)|^2_L = |\pm \frac{\gamma(s + r) - \gamma(s - r)}{2} + \mathcal{L}\left(\frac{\gamma(s + r) - \gamma(s - r)}{2}\right)(\vec{u})|^2_L
\]

\[
= |\mathcal{L}\left(\frac{\gamma(s + r) - \gamma(s - r)}{2}\right)(\vec{u} \mp \sigma_0)|^2_L
\]

\[
= \frac{\gamma(s + r) - \gamma(s - r)}{2} |\vec{u} \mp \sigma_0|^2_L = 0
\]
because \(|(\vec{u} \mp \sigma_0)|^2_L = 0\). From the formula it is clear that \(\Omega_\gamma\) is smooth.

The last M"arzke-Wheeler map formula was written for the first time in\(^5\) for \((1+1)\)-spacetime where it was shown, in this particular case, that it is actually a conformal map. Moreover, the twin paradox is solved in \((1+1)\)-spacetime. In the general case treated here, the M"arzke-Wheeler map is no longer conformal.

As an example, consider the uniformly accelerated observer in \((1+3)\)-spacetime along the \(\sigma_1\) axis:

\[\gamma(s) = p + R \left( \sinh \left( \frac{s}{R} \right) \sigma_0 + \cosh \left( \frac{s}{R} \right) \sigma_1 \right)\]

where \(s\) is its natural parameter and \(R = c^2/a\) such that \(a\) is the observer acceleration. Its M"arzke-Wheeler map is:

\[
\Omega_\gamma(s, x, y, z) = p + R \sinh \left( \frac{s}{R} \right) \left[ \cosh \left( \frac{s}{R} \right) + \sinh \left( \frac{s}{R} \right) \frac{x}{r} \right] \sigma_0 \\
+ R \cosh \left( \frac{s}{R} \right) \left[ \cosh \left( \frac{s}{R} \right) + \sinh \left( \frac{s}{R} \right) \frac{x}{r} \right] \sigma_1 \\
+ \frac{R}{r} \sinh \left( \frac{r}{R} \right) [y \sigma_2 + z \sigma_3]
\]

where \(r^2 = x^2 + y^2 + z^2\). In this example, it is interesting that besides \(\Omega_\gamma\) restricted to the \((\sigma_0, \sigma_1)\) plane is a conformal map (as it was expected from\(^5\)), it is also also conformal restricted to the \((\sigma_2, \sigma_3)\) plane.

### III. PIONEER ANOMALY

The Pioneer 10/11 data is measured from Earth’s DSN antennas (Deep Space Network) and we wonder whether this data is affected by earth’s translation around the sun. We comment about earth’s rotation at the end of the section.

We model the Earth’s translation as the uniformly rotating observer

\[\gamma(s) = \frac{s}{k} \sigma_0 + R \left[ \cos \left( \frac{\omega cs}{ck} s \right) \sigma_1 + \sin \left( \frac{\omega cs}{ck} s \right) \sigma_2 \right]\]

where \(s\) is its natural parameter, \(k = \sqrt{1 - R^2\omega^2/c^2}\) is its Lorentz contraction factor and \(R|\omega| < c\). Its M"arzke-Wheeler map is:

\[
\Omega_\gamma(s, x, y, z) = \left[ \frac{s}{k} + \frac{R}{r} \sin \left( \frac{\omega cs}{ck} r \right) y \right] \sigma_0 \\
+ \left[ R \cos \left( \frac{\omega cs}{ck} r \right) + x \sqrt{\frac{1}{k^2} - \left( \frac{R}{r} \sin \left( \frac{\omega cs}{ck} r \right) \right)^2} \right] \vec{a}(s) \\
+ \frac{1}{k} y \vec{b}(s) \\
+ z \sqrt{\frac{1}{k^2} - \left( \frac{R}{r} \sin \left( \frac{\omega cs}{ck} r \right) \right)^2} \sigma_3
\]

where \(r^2 = x^2 + y^2 + z^2\). We have chosen the framing \(\{\vec{a}, \vec{b}, \sigma_3\}\) corresponding to the \(x, y, z\) coordinates such that \(\{\vec{b}, -\vec{a}, \sigma_3\}\) is the Frenet-Serret framing of the observer (see Figure 2):

\[a(s) = \cos \left( \frac{\omega cs}{ck} s \right) \sigma_1 + \sin \left( \frac{\omega cs}{ck} s \right) \sigma_2 \]

\[b(s) = -\sin \left( \frac{\omega cs}{ck} s \right) \sigma_1 + \cos \left( \frac{\omega cs}{ck} s \right) \sigma_2 \]

This expression was also obtained in\(^5\) in the particular case \(z = 0\). It is interesting to notice the oscillatory term of the above map. In order to compare the spatial M"arzke-Wheeler coordinates with the Frenet-Serret coordinates we
consider the difference $\Omega_{\gamma}(s, x, y, z) - \gamma(s)$. Because $\omega = 2\pi/\text{year}$ and $R = 1\text{AU}$, we have that $\omega R/c \approx 10^{-4}$ and restricted to the region $r < c/\omega \approx 10^4\text{AU}$ we have the approximation:

$$\Omega_{\gamma}(s, x, y, z) - \gamma(s) = \left[ R \left( \cos \left( \frac{\omega}{c} r \right) - 1 \right) + x \right] \bar{a}(s) + y \tilde{b}(s) + z \sigma_3$$

Because the $\sigma_0$ component is zero, we have the following transformation between the spatial Märzke-Wheeler coordinates and the Frenet-Serret coordinates:

$$x' = R \left( \cos \left( \frac{\omega}{c} r \right) - 1 \right) + x$$
$$y' = y$$
$$z' = z$$

where $r^2 = x^2 + y^2 + z^2$. Because the Pioneer’s velocity and acceleration are very small respect to the natural scale of the problem $c/\omega \approx 10^4\text{AU}$, differentiating the above expression we have:

$$a_{x}' = a_x - R \frac{\omega^2}{c^2} \cos \left( \frac{\omega}{c} r \right) \dot{r}^2 - R \frac{\omega^2}{c^2} \sin \left( \frac{\omega}{c} r \right) \ddot{r}$$
$$a_{y}' = a_y$$
$$a_{z}' = a_z$$

where $r$ is the distance from the sun and $a$ is the acceleration. Because the recorded Pioneer data (at least for Pioneer 10) corresponds to the region between $1\text{AU}$ and $\approx 80\text{AU}$, we can consider that $r_{\text{Pioneer}} < c/\omega \approx 10^4\text{AU}$ where $r_{\text{Pioneer}}$ is the Pioneer’s distance from the sun and we have the following approximation:

$$a_{x}' = a_x - R \frac{\omega^2}{c^2} v^2 \cos^2 \varphi$$
$$a_{y}' = a_y$$
$$a_{z}' = a_z$$

where $v$ is the Pioneer’s speed and $\varphi$ is the angle between its radius vector from the sun and its velocity vector. Computing the acceleration difference $\Delta a_x$ between the Märzke-Wheeler and Frenet-Serret coordinates at the Pioneer’s maximal speed $v_{\text{max}} = 48.000 \text{ m/s}$, we have the result:

$$0 \leq |\Delta a_x| \leq R \frac{\omega^2}{c^2} v^2 \approx 1.5 \times 10^{-10}\text{ m/s}^2$$
and we see that it is between 0 and \( \approx 17\% \) of the measured Pioneer anomaly \( a_p = 8.74 \pm 1.33 \times 10^{-10} m/s^2 \).

The calculated difference \( \Delta a_x \) points towards the \( z \) edge when \( x > 0 \) and in the opposite direction when \( x < 0 \). This would contradict the claim that the anomaly always points towards the sun made in the data analysis\(^1\) and\(^2\). However, in the data analysis made in\(^3\), it is claimed that it cannot be confirmed whether the anomaly is sunwards, contrary to the earlier claim.

Finally, we would like to comment about a possible numerical analysis on the influence of Earth’s rotation on the measured data. In order to do so, we define the following framing dependent non abelian product of observers:

\[
\gamma \cdot \xi = \Omega_\gamma \circ \xi
\]

This product is the generalization of the special relativistic velocities addition and has the following property:

\[
\Omega_{\gamma \cdot \xi} = \Omega_\gamma \circ \Omega_\xi
\]

This way, the observer \( \gamma = \gamma_{\text{Translation}} \cdot \gamma_{\text{Rotation}} \) is the one we should consider and its Märzke-Wheeler map is just the composition of the previously exactly calculated map of the uniformly rotating observer. Unfortunately, the map gets really involved and the analysis must be done numerically. An analysis of the parameters involved in the rotation analysis, shows that the magnitude order of the long range Märzke-Wheeler acceleration effect coincides with the one of the Pioneer anomaly and should also be considered.

**IV. CONCLUSION**

Although after strongly numerical evidence it is tempting to think that the 20\% discrepancy of the anomaly value found in\(^7\) is due to a long range Märzke-Wheeler acceleration effect described in this letter, due to statistical errors in the measured anomaly it cannot be neither confirmed nor neglected. We hope that the ideas presented here could encourage other research teams in the search for other observational objects that could finally answer whether Märzke-Wheeler effects influence on measured data in nature.

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