Evolution of Magnetic Fields in Freely Decaying Magnetohydrodynamic Turbulence

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We study the evolution of magnetic fields in freely decaying magnetohydrodynamic turbulence. By quasi-linearizing the Navier-Stokes equation, we solve analytically the induction equation in quasilinear approximation. We find that, if the magnetic field is not helical, the magnetic energy and correlation length evolve in time respectively as $E_B \propto t^{-2(1+p)/(3+p)}$ and $\xi_B \propto t^{2/(3+p)}$, where $p$ is the index of initial power-law spectrum. In the helical case, the magnetic helicity is an almost conserved quantity and forces the magnetic energy and correlation length to scale as $E_B \propto (\log t)^{3/4} t^{-2/3}$ and $\xi_B \propto (\log t)^{-1/3} t^{2/3}$.

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The origin of presently-observed large scale magnetic fields throughout the universe is still unclear. Essentially, there are two possible classes of mechanisms to produce cosmic fields depending on when they are generated: Astrophysical mechanisms acting during or after large-scale-structure formation, and mechanisms acting in the primordial universe. Magnetic fields created in the early universe (except those generated during inflation), usually suffer from a “small-scale problem”, that is their comoving correlation length is much smaller than the characteristic scale of the observed cosmic fields. However, if magnetohydrodynamic (MHD) turbulence operates during their evolution, an enhancement of correlation length can occur, especially if the magnetic field is helical. As pointed out by Banerjee and Jedamzik, the evolution of a magnetic field in the early universe goes through different phases depending on the particular conditions of the primordial plasma. In this paper, we are interested in the case of magnetic fields evolving in the turbulent primordial universe well before recombination epoch and when kinematic dissipative effects are due to diffusing particles. Therefore, we are concerned with the so-called phase of “turbulent MHD”. In other phases, such as “viscous MHD” and “MHD with ambipolar diffusion” described in Ref. [2], the dynamics of the magnetic field is very different from that studied here. The problem of determining the evolution properties of magnetic fields in MHD turbulence has been deeply and widely discussed in the literature using different methods and approximations. A direct integration of the full set of MHD equations would allow us to deeply understand the dynamics of freely decaying MHD turbulence. However, MHD equations are quite difficult to handle due to their high non-linearity and it has not been yet brought in a definitive verdict for the evolution laws of magnetic energy and correlation length (for recent numerical studies of freely decaying magnetohydrodynamic turbulence see, e.g., Ref. [2, 3, 4]).

The turbulent MHD equations for incompressible fluids, in the case of non-expanding universe, are:

\[
\begin{align*}
\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \nabla p - \nu \nabla^2 \mathbf{v} &= \mathbf{J} \times \mathbf{B}, \\
\partial_t \mathbf{B} &= \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B},
\end{align*}
\]

and $\nabla \cdot \mathbf{v} = \nabla \cdot \mathbf{B} = 0$. Here, $\mathbf{v}$ is the velocity of bulk fluid motion, $\mathbf{B}$ the magnetic field, $\mathbf{J} = \nabla \times \mathbf{B}$ the magnetic current, $\nu$ the kinematic viscosity, $\eta$ the resistivity. The thermal pressure of the fluid, $p$, is not an independent variable since, taking the divergence of Eq. (1), it can be expressed as a function of $\mathbf{B}$ and $\mathbf{v}$.

In the case of expanding universe in the radiation era, it has been shown that the MHD equations are the same as Eqs. (1)–(2) provided that time, coordinates, and dynamical variables are replaced by the following quantities (see, e.g., Ref. [2]): $t \to \tilde{t} = \int a^{-1} dt$, $\mathbf{x} \to \tilde{\mathbf{x}} = \mathbf{ax}$, $\mathbf{B} \to \tilde{\mathbf{B}} = a^2 \mathbf{B}$, $\nu \to \tilde{\nu} = a^{-1} \nu$, $\eta \to \tilde{\eta} = a^{-1} \eta$, where $a(t)$ is the expansion parameter. Because of the formal coincidence of the MHD equations in the expanding and non-expanding universe, we can study the evolution of magnetic fields in MHD turbulence in both cases in a similar way. For definiteness, in this paper we shall consider only the case of non-expanding universe.

It is useful to define the kinetic and magnetic Reynolds numbers, $\text{Re} = vl/\nu$ and $\text{Re}_B = vl/\eta$, where $v$ and $l$ are the typical velocity and length scale of the fluid motion. Magnetohydrodynamic turbulence occurs when $\text{Re} \gg 1$ and $\text{Re}_B \gg 1$. We are interested in the evolution of statistically homogeneous and isotropic magnetic fields. This means that the two-point correlation tensor $C_{ij}(\mathbf{x}, \mathbf{y}) = \langle B_i(\mathbf{x}) B_j(\mathbf{y}) \rangle$, where $\langle \ldots \rangle$ denotes ensemble average, is a function of $|\mathbf{x} - \mathbf{y}|$ only and transforms as an $SO(3)$ tensor. In terms of the Fourier amplitudes of the magnetic field, $B_i(\mathbf{k}, t) = \int d^3x e^{i\mathbf{k} \cdot \mathbf{x}} B_i(\mathbf{x}, t)$, these conditions translate into:

\[
\begin{align*}
\langle B_i(\mathbf{k}, t) B_j(\mathbf{p}, t) \rangle &= [(2\pi)^3/2] \delta(\mathbf{k} + \mathbf{p}) \\
&\times \left[ \delta_{ij} - \hat{k}_i \hat{k}_j \right] \mathcal{S}(k, t) + i \varepsilon_{ijk} \hat{k}_k \mathcal{A}(k, t),
\end{align*}
\]

where $\hat{k}_i = k_i/k$, $k = |\mathbf{k}|$, and $\varepsilon_{ijk}$ is the totally antisymmetric tensor. The functions $\mathcal{S}$ and $\mathcal{A}$ denote the symmetric and antisymmetric parts of the correlator. They
are related to the magnetic energy and helicity densities in the volume \( V \) through \( E_B(t) = (1/2V) \int_V d^3 x (B^2) = \int_0^\infty dk \epsilon_B(k, t) \) and \( H_B(t) = (1/V) \int_V d^3 x (A \cdot B) = \int_0^\infty dk H_B(k, t) \), where \( \epsilon_B = k^2 S/(2\pi^2) \) and \( H_B = kA/(2\pi^2) \) are the magnetic energy and magnetic helicity density spectra, and \( A \) is the vector potential. The kinetic energy, \( E_v(t) \), is defined as the magnetic one with \( B \) replaced by \( v \). We remember that for all magnetic field configurations, the magnetic helicity spectrum must satisfy the “realizability condition” \([3]\): \( |H_B| \leq 2k^{-1} \epsilon_B \).

The magnetic field is said to be “maximally helical” if, for all \( k \), \( H_B \) is of the same sign and saturates the above inequality. Moreover, the magnetic helicity is conserved when \( \eta = 0 \) since \( \partial_t H_B = -2(\eta/V) \int_V d^3 x (J \cdot B) = -2\eta \int_0^\infty dk k^2 H_B \). The relevant length scale in MHD theory, the so-called magnetic correlation length, is the characteristic length associated with the large magnetic energy eddies of turbulence and is defined by: \( \xi_B(t) = E_B^{-1} \int_0^\infty dk k \epsilon_B \). The integral form of the realizability condition takes the form: \( |H_B| \leq 2\xi_B E_B \).

Since we are interested in the case of large Reynolds numbers, we neglect the dissipation term in Eq. (1). Moreover, as in Ref. [7], we quasi-linearize the Navier-Stokes equation (11) neglecting the quadratic term \((v \cdot \nabla)v\). This corresponds to neglect small scale components of velocity field and to assume that the Lorentz force, \( F_L = J \times B \), acting on the charged particles of the fluid "drives" the development of turbulence on larger scales: \( \partial_t v \simeq F_L \). Although the validity of this approximation can be verified only by a numerical analysis, its use is justified \textit{a posteriori} since our results, as we will find, are in agreement with a numerical simulations of full MHD equations performed in Ref. [2]. Finally, we make the common approximation, \( v \simeq \tau_d F_L \), (4)

where the “draging time” \( \tau_d \) is the fluid-response time to the Lorentz force introduced by Sigl in Ref. [8]. We note that \( \tau_d \) and the characteristic time associated with kinetic turbulence, the so-called eddy turnover time \( \tau_{edd} = l/v \), are related by \( \tau_d \simeq \Gamma \tau_{edd} \), since \( 1 \simeq |v|/|\tau_d F_L| \simeq \Gamma \tau_{edd}/\tau_d \). Here, \( \Gamma \) is the ratio of the kinetic and magnetic energy. What is observed in numerical simulation of MHD equations [2] is that, in the non-helical case, turbulence proceed toward a state of equipartition between magnetic and kinetic energies (\( \Gamma \simeq 1 \)) while, in the helical case, though there is no evidence of equipartition, the ratio \( \Gamma \) approaches asymptotically to a constant value.

Inserting the above expression for \( v \) into the induction equation (2), we get in Fourier space:

\[
(\partial_t + \eta k^2) B_i(k) \tau_d \int d^3 p (2\pi)^3 \int d^3 q (2\pi)^3 \epsilon_{ijk} k_j q_i B_s(q) \times \left[ \epsilon_{krs} B_n(p-q) B_n(k-p) - \epsilon_{rst} B_s(k-p) B_m(p-q) \right],
\]

where summation over repeated indexes is understood. We will work in “quasi-normal approximation” and suppose that the four-point correlator can be decomposed, in terms of two-point correlator, as \([3]\):

\[
\langle B_i(k) B_j(p) B_k(q) B_l(r) \rangle = \langle B_i(k) B_j(p) \rangle \langle B_k(q) B_l(r) \rangle + \langle B_i(k) B_l(r) \rangle \langle B_j(p) B_k(q) \rangle.
\]

Multiplying Eq. (5) respectively by \( B_i(k) \) and \( A_i(k) \), and then averaging out we arrive at the following equations for the magnetic energy and helicity spectra:

\[
\partial_t \epsilon_B = -2\eta k^2 \epsilon_B + \alpha_B \epsilon_B^2 H_B,
\]

\[
\partial_t H_B = -2\eta k^2 \epsilon_B + 4\alpha_B \epsilon_B,
\]

where we have introduced \( \eta_{eff}(t) = \eta + 4E_B \tau_d/3 \) and \( \alpha_B(t) = -H_B \tau_d/(3\eta) \). For simplicity, we will restrict our analysis to magnetic fields with initial 'fractional helicity': \( H_B(k, 0) = h_B H_B^{max}(k, 0) \), where \( 0 \leq h_B \leq 1 \) is the fraction of the initial maximal helicity \( H_B^{max}(k, t) = 2k^{-1} \epsilon_B(k, t) \). In this case, the solution of Eqs. (7)-(8) is:

\[
\epsilon_B(k, t) = \epsilon_B(k, 0) \exp(-2k^2 \ell_{diss}^2) \times [\cosh(2k\ell_{alpha}) + h_B \sinh(2k\ell_{alpha})],
\]

\[
H_B(k, t) = H_B^{max}(k, 0) \exp(-2k^2 \ell_{diss}^2) \times [\sinh(2k\ell_{alpha}) + h_B \cosh(2k\ell_{alpha})],
\]

where we have defined the "dissipation" and "alpha" lengths, \( \ell_{diss}^2(t) = \int_0^t dt \eta_{eff} \) and \( \ell_{alpha}(t) = \int_0^t dt \alpha_B \). From Eqs. (9)-(10) we immediately get that magnetic fields with maximal initial helicity, \( h_B = 1 \), remain maximally helical for all times: \( H_B = 2k^{-1} \epsilon_B \). To proceed further, we assume that the initial magnetic energy spectrum can be represented by the following simple function: \( \epsilon_B(k, 0) = \lambda_B k^p \exp(-2k^2 \ell_B^2) \), where \( \lambda_B \) and \( \ell_B \) are constants. For \( k \ll \ell_B^{-1} \), the magnetic energy spectrum possesses a power law behavior, while for large \( k \) it is suppressed exponentially in order to have finite energy. The exponential cut-off, \( \ell_B \), is related to the initial correlation length by \( \ell_B = \xi_B(0)/\zeta_B \), where \( \zeta_B = \sqrt{2}\Gamma(p/2)/\Gamma(1+p)/2 \) and \( \Gamma(x) \) is the Euler gamma function. In Ref. [9], it was shown that analyticity of the correlator \( C_{ij}(x, y) \) defined on a compact support forces the spectral index \( p \) to be even and equal or larger than 4. Now, inserting Eqs. (9)-(10) into the expressions for the magnetic energy and helicity we find:

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1 If we decompose, in the spirit of mean-field-theory [8], the velocity field into an (almost uniform) average part and a weak, small-scale fluctuating part, \( v = \bar{v} + \delta v \) with \( |\delta v| \ll |\bar{v}| \), we have \( v \cdot \nabla v = \bar{v} \cdot \nabla \bar{v} \). Comparing this term with the Lorentz force, we get \( |v \cdot \nabla v|/|\bar{F}_L| \sim \Gamma |\delta v|/|\bar{v}| \), where \( \Gamma = \delta v/\bar{E}_B \). Hence, the quasi-linear approximation is valid as long as the condition \( \Gamma \gg |\delta v|/|\bar{v}| \) is satisfied.
\[ E_B(t) = \frac{(1 + \zeta_{\text{diss}}^2)^{-(1+p)/2}}{E_B(0)} \left[ 1 + \frac{1}{2} \frac{p}{2} \frac{\chi^2}{2} \right] + \frac{p\hbar B}{2} \zeta_B \frac{1}{\chi} F_1 \left( \frac{2 + p}{2}, \frac{3}{2}, \frac{\chi^2}{2} \right), \]  
\[ H_B(t) = \frac{(1 + \zeta_{\text{diss}}^2)^{-p/2}}{H_B(0)} \left[ 1 + \frac{1}{2} \frac{p}{2} \frac{\chi^2}{2} \right] + \frac{1}{\hbar B} \frac{2}{\zeta B} \zeta B \frac{1}{\chi} F_1 \left( \frac{1 + p}{2}, \frac{3}{2}, \frac{\chi^2}{2} \right), \]  

where \( F_1(a, b, z) \) is the Kummer confluent hypergeometric function, and we have defined \( \zeta_{\text{diss}} = \zeta_{\text{diss}}/\ell_B \), \( \zeta_\alpha = \zeta_\alpha/\ell_B \), and \( \chi = \zeta_\alpha/(1 + \zeta_{\text{diss}}^2)^{1/2} \). Equations (11)-(12) are integral equations for the magnetic energy and helicity. They can be solved once the explicit expression for the drag time is given. This can be done if we consider the scaling properties of the induction equation. It is well-known that the full MHD equations (neglecting dissipative terms) are invariant under scaling transformations \( x \rightarrow x, t \rightarrow \ell^{1-\gamma}, v \rightarrow \ell^\gamma v, B \rightarrow \ell B \), where \( \ell > 0 \) is the “scaling factor” and \( r \) is an arbitrary real parameter. Now, imposing that also the “reduced” MHD equations (2) and (4) are invariant under these scaling transformations, we get that the drag time is linear in time. Taking into account the relation between \( \tau_{\text{eddy}} \) and \( \tau_{\text{eddy}} \) previously discussed, we also have that the eddy turnover time is asymptotically linear in time. This allow us to write the drag time as \( \tau_\gamma(t) \simeq \Gamma(0) [\tau_{\text{eddy}}(0) + \gamma t] \), where \( \gamma = [\Gamma(\infty)/\Gamma(0)] \lim_{t \rightarrow \infty} \tau_{\text{eddy}}(t)/t \) is a constant, whose explicit value is essential for the following discussion.

It is useful to define accurately the magnetic Reynolds number and the eddy turnover time: \( \Re_B = v_{\text{rms}} \xi_B/\eta \), and \( \tau_{\text{eddy}} = \xi_B/v_{\text{rms}} \), where, as typical length scale and velocity, we used the magnetic correlation length and the root-mean-square value of the velocity field, \( v_{\text{rms}} = (1/V) f_0 d^2 x (v^2) = 2E_v \). With the aid of the above definitions and introducing the normalized time \( \tau = t/\tau_{\text{eddy}}(0) \), the integral equations (11) and (12) can be transformed into the differential equations

\[ \frac{d\zeta_{\text{diss}}}{d\tau} = \frac{\zeta_{\text{diss}}^2}{\Re_B(0)} + \frac{2}{3} \frac{\zeta_{\text{diss}}^2}{\Re_B(0)} (1 + \gamma \tau) \frac{E_B(\tau)}{E_B(0)}, \]  
\[ \frac{d\zeta_\alpha}{d\tau} = -\frac{1}{3} \zeta_\alpha \hbar B \Re_B(0)(1 + \gamma \tau) \frac{d}{d\tau} \frac{H_B(\tau)}{H_B(0)}, \]  

where \( E_B \) and \( H_B \), as a function of \( \zeta_{\text{diss}} \) and \( \zeta_\alpha \), are given by Eqs. (11) and (12). For large magnetic Reynolds numbers, the first term in the left-hand-side of Eq. (13) can be neglected with respect to the second one.

In the non-helical case, \( \hbar_B = 0 \), the solution of Eqs. (13)-(14) is \( \zeta_{\text{diss}} = 0 \), that is \( H_B(t) = 0 \) for all times, and \( \zeta_{\text{diss}} = (1 + \kappa_{\text{diss}})^2 (2 + \tau + \tau^2)^{2/(3+p)} - 1 \), where \( \kappa_{\text{diss}} = \gamma(3 + p)\zeta_{\text{diss}}^2/6 \). This, in turn, gives for \( \tau \gg 1 \):

\[ E_B(\tau) \simeq \kappa_E E_B(0) \tau^{-(3+p)/(3+p)}, \]  
\[ \xi_B(\tau) \simeq \kappa_\xi \xi B(0) \tau^{2/(3+p)}, \]  

where \( \kappa_E = \kappa_{\text{diss}}^{-(1+p)/(3+p)} \) and \( \kappa_\xi = \kappa_{\text{diss}}^{1/(1+p)} \).

It is interesting to observe that, starting from self-similarity of MHD equations, Olesen obtained the following expression for the magnetic energy spectrum [10]:

\[ E_B(k, t) = \lambda_B k^p \psi_B(k) t^{(2\gamma - 3)/2}, \]  

where \( \lambda_B \) is a constant, \( \psi_B \) is an unknown scaling-invariant function, and \( p \) is the power-law exponent of the initial magnetic energy spectrum. Our approach to MHD equations fixes the expression of the scaling-invariant function to \( \psi_B(x) = \exp[-2(x/x_s)^2] \), with \( x_s = \kappa_\xi^{-1/2}(\tau_{\text{eddy}}(0))^{2/(3+p)} \).

In Fig. 1, we plot the spectrum of the magnetic energy for the case \( p = 4 \) at different times. It is clear that, in the non-helical case, the decay of the magnetic field and the growth of the correlation length proceed through the so-called selective decay discussed by Son in Ref. [11]: there is no direct transfer of magnetic energy from small scales (large wavenumbers) to large scales (small wavenumbers) but, simply, modes with larger wavenumbers decay faster than those whose wavenumbers are small. Consequently, as the turbulence operates, the magnetic field survives only on larger and larger scales.

In the helical case, the evolution of the system goes through two different regimes depending on the value of \( \chi(t) \) which is an increasing function of time. Until when \( \chi < 1 \) the system behaves as if the magnetic helicity were zero: the system evolves by selective decay and, consequently, the asymptotic solutions are the same obtained previously. Afterwards, when \( \chi \gg 1 \), the system enters and persists in a phase characterized by a transfer of magnetic energy from small to large scales, a mechanism known as inverse cascade [5]. The asymptotic \( \tau \rightarrow \infty \) solutions in this latter phase are:

\[ \zeta_{\text{diss}}(\tau) \simeq \kappa_{\text{diss}} (\ln \tau)^{1/6} \tau^{2/3}, \]  
\[ \zeta_\alpha(\tau) \simeq \kappa_\alpha (\ln \tau)^{2/3} \tau^{2/3}, \]  

where \( \kappa_{\text{diss}} = (\gamma^2 p/12)^{1/6} \zeta B^{-1} \hbar_B^{-1/3} \), and \( \kappa_\alpha = (4p/3)^{1/2} \zeta_{\text{diss}} \).

Consequently, we have:

\[ E_B(\tau) \simeq c_E E_B(0) (\ln \tau)^{1/3} \tau^{-2/3}, \]  
\[ \xi_B(\tau) \simeq \kappa_\xi B(0) (\ln \tau)^{-1/3} \tau^{2/3}, \]  

with \( c_E = (2p/3)^{1/3} \zeta B^{-1} \), and \( \zeta = \hbar_B c_E^{-1} \). From the above equations, we directly obtain the relation

![FIG. 1: Magnetic energy spectrum in the non-helical case for \( p = 4 \), with \( \gamma = 1 \). The dotted line corresponds to the initial spectrum, while continuous lines correspond, from left to right, to \( t/\tau_{\text{eddy}} = 1, 10, 10^2, \ldots, 10^7 \).](image-url)
$E_B \xi_B \simeq H_B/2$. This means that a magnetic field with initial fractional helicity becomes maximally helical approximately after the system enters into the inverse-cascade regime. More accurately, we can find the time when this happens, $\tau_h$, matching the product of asymptotic solutions $E_0(t) = 10^{15}$, $p = 4$, $h = 10^{-3}$, with $\gamma = 1$. Upper panel: magnetic energy; middle panel: correlation length; Dotted lines correspond to analytical expansions. Lower panel: magnetic energy spectrum; the dotted line corresponds to the initial spectrum, while continuous lines correspond, from left to right, to $t/t_{\text{eddy}}(0) = 1, 10, 10^2, \ldots, 10^7$.

$E_B \xi_B \simeq H_B/2$. This means that a magnetic field with initial fractional helicity becomes maximally helical approximately after the system enters into the inverse-cascade regime. More accurately, we can find the time when this happens, $\tau_h$, matching the product of asymptotic solutions [13-14] for $E_B(0) = 10^{15}$, $p = 4$, and $h = 10^{-3}$. It is evident from the figure that the analytical expansions [non-helical solution for $\tau \lesssim \tau_h$ and Eqs. (17)-(18) for $\tau \gtrsim \tau_h$] fit very well the numerical solution. Because of quasi-conservation of magnetic helicity, small-scale modes are not dissipated during the decay but their energy is transferred to larger scales: this process of inverse cascade is manifest in the magnetic energy spectrum shown in Fig. 2.

It is worth noting that our final results, Eqs. (15)-(18), apart logarithmic factors, agree very well both with scaling arguments and results of a numerical integration of full MHD equations presented in Ref. [2].

In conclusion, we have studied the evolution of statistically homogeneous and isotropic magnetic fields in the context of freely decaying magnetohydrodynamic turbulence. By quasi-linearizing the Navier-Stokes equation, we have solved analytically the induction equation in quasi-normal approximation. We have found that, if the initial magnetic field is not helical, the evolution of the magnetic field proceeds through selective decay of magnetic modes: magnetic power on small scales is washed out by turbulence effects more effectively than on large scales. During this process, the correlation length grows as $\xi_B \propto t^{2/(3+p)}$, while the magnetic energy decays in time as $E_B \propto t^{-2(1/p)/(3+p)}$, where $p$ is the index of the initial power-law spectrum. In the helical case, the evolution of the system goes through two different phases: selective-decay phase in which the system evolves as if the magnetic helicity were zero and inverse-cascade phase. The first phase ends when quasi-conservation of magnetic helicity starts to trigger an inverse cascade of the magnetic field: small-scale modes are no more completely dissipated during turbulence but their energy is partially transferred to larger scales. This causes a faster growth of the correlation length and a slower dissipation of the magnetic energy with respect to the non-helical case. The time when the system enters into the inverse-cascade regime is proportional to $H_B^{-3/(3+p)/2p}$ times the initial eddy turnover time, where $H_B$ is fraction of the maximal initial magnetic helicity. Moreover, the process of inverse cascade erases any information about the initial structure of the magnetic field, so that the evolution laws of energy and correlation length are $E_B \propto (\log t)^{1/3} t^{-2/3}$ and $\xi_B \propto (\log t)^{-1/3} t^{2/3}$, whatever is the value of $p$.

In a cosmological context, these results are of interest when studying the evolution of primordial magnetic fields before neutrino decoupling. Indeed, during the period of neutrino (or photon) free-streaming, as well as after recombination, the equations governing the evolution of magnetic fields differ from those studied here [2] and their results do not apply. Nevertheless, our approach to MHD equations can be suitably extended to these last cases and an appropriate analysis is in progress.

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