Hyperdeformed band in $^{36}$Ar populated in the $^{12}$C + $^{24}$Mg elastic scattering

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Abstract. The strongly oscillating angular distributions of the elastic scattering of $^{12}$C + $^{24}$Mg at energies around the Coulomb barrier ($E_{cm}=10.67 - 16.00$ MeV) [1] were reproduced adding three Breit-Wigner resonance terms with $J$ values of 6, 7 and 8 $\hbar$ respectively to the $l = 6, 7$ and 8 terms of the elastic S-matrix. The elastic scattering S-matrix was calculated using the double folding, deep, optical potential with non-local interaction, also called, São Paulo Potential [2]. All fifteen angular distributions could be well reproduced by the 3 resonances, located respectively at $E_{cm} = 14.15, 15.8$ and $16.9$ MeV in the entrance channel, which correspond to excitation energies of 30.45, 32.1 and 33.2 MeV in the $^{36}$Ar compound nucleus. The $J=6, 7$ and 8 $\hbar$ resonances fit well into a rotational molecular band, together with the $J=18, 20, 22$ and 24 $\hbar$ resonances observed in the $^{16}$O + $^{20}$Ne elastic scattering [3]. The band head is at 29.5 MeV excitation energy in the $^{36}$Ar compound nucleus and has a large moment of inertia indicating a large deformation. Calculations of Rae and Merchant [4] propose the existence of a hyperdeformed band in $^{36}$Ar with $^{12}$C + $^{24}$Mg and $^{16}$O + $^{20}$Ne cluster structure.

1. Introduction

The existence of nuclear molecular states has been recognized long time ago. Several review papers, conference proceedings, written in the last decade give details of the large amount of experimental evidence and theoretical explanations [5, 6, 7, 8, 9].

Nuclei with $N=Z$, with even and equal number of protons and neutrons, also called n-α or α-conjugate nuclei, present cluster structures composed of α particles [7], as observed in many experiments during the last 30 years. The α cluster states usually do not exist in the ground state of the composed nuclei, but at excitation energies close to the decay thresholds into clusters. This observation was first made by Ikeda [10].

The elastic scattering of α-conjugate nuclei has also shown evidence of clustering. The most striking example is the $^{12}$C + $^{12}$C system, where in the elastic scattering sharp structures were observed even below the Coulomb barrier [11]. These same structures were observed in many other reaction channels, with emission of p, α, n, or γ, or in the fusion and reaction cross-sections. The narrow (few hundreds of keV) resonances are strongly correlated in all channels and present a very large partial width for decay into $^{12}$C + $^{12}$C. In heavier systems, as $^{16}$O + $^{28}$Si, $^{12}$C + $^{28}$Si, $^{16}$O + $^{24}$Mg, $^{12}$C + $^{24}$Mg and others [12], the...
anomalous large angle scattering effect (ALAS) and correlated resonances were observed in the elastic, inelastic and α - transfer reactions, indicating the existence of two-body cluster states in the composed nucleus.

The complete set of 15 elastic scattering angular distributions of the $^{12}\text{C} + ^{24}\text{Mg}$ system [1], measured at energies very close to the Coulomb barrier, at $E_{cm} = 10.67 - 16.00$ MeV, has proven to be a difficult case to be interpreted solely as potential scattering. All angular distributions show strong oscillations and were first reproduced [1] by a shallow, energy dependent, phenomenological optical potential. The imaginary part of the optical potential has to be extremely, anomalously shallow, 1-2 % of the real depth, not only on the surface but also in the nuclear interior, where the wave-functions can penetrate without being absorbed.

More recently many attempts have been made and published to reproduce these data using optical potentials with deep real and very shallow imaginary parts [13, 14, 15, 16, 17]. In order to reproduce the strong oscillations present even at energies at the Coulomb barrier, they introduced deviations adding small additional derivatives to the deep real potential, or using modified Ginocchio type potentials. Thus conventional deep optical potentials were not successful to reproduce the low energy data even with very shallow imaginary parts.

In our analysis we have used the double-folding, deep, optical potential with non-local interaction, also called, São Paulo Potential [2]. In order to reproduce the oscillations we added 3 resonances to the optical model elastic scattering matrix.

2. Optical model analysis including resonances in the S-matrix

The fifteen angular distributions of the elastic scattering of $^{12}\text{C} + ^{24}\text{Mg}$ were measured at the University of São Paulo Pelletron tandem [1], at energies close to the Coulomb barrier, between $E_{cm} = 10.67$ and 16.00 MeV. An angular distribution measured at $E_{cm} = 16.53$ MeV by Mermaz [18] was also included in our analysis. Our angular distributions were measured up to $\theta_{cm}=164^\circ$ and they all show strong oscillations. We have used the elastic scattering excitation function measured by Mermaz and collaborators [18] at $\theta_{cm}=180^\circ$ to complete our angular distributions and due to possible differences in normalization we added a systematic error of 30 %.

The optical potential used in all these calculations is a double folding, deep, optical potential, where the Pauli non-locality, related with energy dependence, is taken into account. This potential is also called, São Paulo Potential (SPP) [2]. The imaginary part of the potential has the same form factor as the real one, and it has been used to reproduce quantitatively a large amount of elastic scattering data, going from energies close to the Coulomb barrier up to energies of several hundreds of MeV/nucleon. The normalizations of the real and imaginary parts are respectively 1.0 and 0.78, and no free parameters are in use.

The elastic scattering cross-section calculated using this potential reproduces the average behaviour of the angular distributions without the pronounced oscillations.

In order to reproduce the oscillations we have added a Regge-pole to the optical model S-matrix $S_l$, as indicated by the equation below:

$$S_l = S_l^0 (1 + \frac{iDe^{2i\Phi}}{(l - l_0) - \frac{i\Gamma}{2}})$$

All angular distributions could be reproduced by a pole with width $\Gamma = 0.05 \hbar$, which means that only the $l = l_0$ partial wave is affected by the pole. The other parameters $l_0$, D and $\Phi$ were determined by fit procedure to the angular distributions. The angular distributions measured at energies $E_{cm} = 11.33, 12.00, 12.33, 12.67, 13.00, 13.33, 13.67, 14.00, 14.33$ and 14.67 MeV could be well reproduced by a single pole with $l_0 = 6\hbar$, as we can see on Fig.1, but not the ones measured at higher energies (see Fig. 2). At $E_{cm} = 15.0, 15.33, 15.67$ and 16.0 MeV the best fit was obtained by using a pole with $l_0=7\hbar$, and at $E_{cm} = 16.53$ MeV, $l_0=8\hbar$. The next step was
Figure 1. Some angular distributions with calculations including an l=6 pole.

Figure 2. Some angular distributions with calculations including an l=6 or l=7 pole.

Table 1. The resonance parameters used in the calculation of the elastic scattering of $^{12}$C+$^{24}$Mg.

| $l_0$ | $E_0$ | $D$ | $\Gamma$ | $\Phi$  |
|------|-------|-----|---------|---------|
|      | [MeV] | [MeV] | degree  |
| 6    | 14.15 | 0.5  | 2.0     | $-30^\circ$ to $-90^\circ$ |
| 7    | 15.80 | 0.3  | 0.7     | $20^\circ$ to $65^\circ$ |
| 8    | 16.90 | 0.4  | 0.7     | $80^\circ$ to $125^\circ$ |

to include three resonances in the calculation, corresponding to the three spin values determined by the calculations with poles. We added three energy dependent Breit-Wigner terms to the optical model S-matrix, one with angular momentum $l = 6$, one for $l = 7$ and one for $l = 8\hbar$, as indicated in the equation below:

$$S_{l_0} = S_{l_0}^0 (1 - \frac{i D e^{2\phi}}{(E - E_0) + \frac{\Gamma}{2}})$$

(2)

The resonance parameters were determined by a fitting procedure to all 15 angular distributions. Their values are presented in Table 1. The phase $\Phi$ was varied as a function of the energy in order to improve the fit. The 15 angular distributions and the backward angle excitation function of Mermaz were quite well reproduced by the inclusion of these three resonances, as can be seen in figures 3, 4, 5 and 6.
Figure 3. Angular distributions with calculations including the 3 resonances.

Figure 4. Angular distributions with calculations including the 3 resonances.

Figure 5. Angular distributions with calculations including the 3 resonances.

Figure 6. The excitation function at $180^0$ [18] compared with calculations including the 3 resonances (dots) and without resonances (stars).
3. Highly deformed molecular band in $^{36}$Ar

We interpret the three resonances used to reproduce the 15 angular distributions as two-body cluster states in the compound nucleus $^{36}$Ar. This same compound nucleus can also be attained in the scattering of $^{16}$O + $^{20}$Ne. Shimizu [3] and Gai [19] have observed strong correlated structures at $E_{cm} = 24.5, 27.9, 31.7$ and $35.5$ MeV in the excitation functions of the elastic, inelastic scattering and $\alpha$-transfer reactions for the $^{16}$O + $^{20}$Ne system. Angular distributions of the $\alpha$-transfer, measured on top of the resonance structures, showed strong oscillations and the ALAS effect. They were fitted by Legendre polynomials in the angular region between 10 and 60 degrees and the $J$ values for the above cited energies were: 18, 20, 22 and 24 $h$.

The energies of the resonances observed in the $^{12}$C+$^{24}$Mg and $^{16}$O + $^{20}$Ne scattering were transformed into excitation energies in the $^{36}$Ar compound nucleus, adding the energy thresholds of 16.29 MeV and 18.45 MeV respectively, for the two systems. In Fig. 7 we plot the excitation energy in $^{36}$Ar against $J(J+1)$ and we see a clear linear dependence, indicating that these states belong to a rotational band.

The superdeformed (SD) band in $^{36}$Ar, indicated on Fig.7, was observed recently by Svensson et al [20]. It has a large quadrupole deformation of $\beta_2 = 0.46(3)$ at low spins. The slope parameter ($h^2/2I$) of the SD-band in the low-spin region is 0.08234 corresponding to a moment of inertia of $I = 2.36 \times 10^5$ MeVfm$^2$/c$^2$.

The band head of the rotational band built on the high lying resonances with two-body cluster structure of $^{12}$C+$^{24}$Mg and/or $^{16}$O + $^{20}$Ne is 29.5(5) MeV. It has a smaller slope parameter, thus a larger deformation, than the SD band. The slope parameter is 0.041(1) ($h^2/2I$, corresponding to a moment of inertia of $I = 4.1 \times 10^5$ MeVfm$^2$/c$^2$.

4. Theoretical predictions

Inakura [21] has predicted the presence of SD and hyperdeformed (HD) bands in $^{36}$Ar, $^{40}$Ca, $^{44}$Ti and $^{48}$Cr, performing cranked Skyrme-Hartree-Fock calculations in Z=N nuclei. Rae and Merchant [4] have used the cranked Bloch-Brink alpha-cluster model and found hyperdeformed bands in $^{36}$Ar and $^{48}$Cr. Their predicted structures are respectively chains of $^{16}$O + $^{16}$O + $\alpha$ and
16O + 16O + 16O with a ratio of major to minor axis of 3:1. They predict that members of the HD band in 36Ar should be seen as resonances in both 12C+24Mg and 16O + 20Ne scattering. They calculate the energy of the band head in 36Ar at 30.1 Mev in a subsequent work [22]. In a recent work Cseh [23] studied the allowed and forbidden binary cluster configurations of the ground, superdeformed (SD) and hyperdeformed (HD) states of 36Ar, based on the U(3) selection rule. The 12C+24Mg binary cluster configuration is allowed in the ground, the SD and the HD states, while the 16O + 20Ne binary cluster configuration is only allowed in the HD states. Thus, both Rae-Merchant and Cseh predict a hyperdeformed band in 36Ar with binary cluster configurations of 12C+24Mg and 16O + 20Ne, in agreement with our findings.

5. Conclusions
The complete set of 15 elastic scattering angular distributions of the 12C + 24Mg system [1] were measured at energies very close to the Coulomb barrier, at Ecm = 10.67 - 16.00 MeV, and all angular distributions show strong oscillations and conventional deep optical potentials need some modifications and very shallow imaginary parts to reproduce the data. In this work we have used a double folding, deep, optical potential with non-local interaction and added 3 resonances to the elastic scattering S-matrix, with spins of 6, 7 and 8h. The angular distributions and the excitation function [18] at 180 degrees are well reproduced by these calculations. The 3 resonances are located at 14.15, 15.8 and 16.9 MeV in the entrance channel. They fit well into a highly deformed molecular rotational band with Em = 29.5 MeV, together with resonances at higher energy, observed in the 16O + 20Ne scattering. Theoretical calculations based on cluster model predict the presence of a hyperdeformed band in 36Ar at the same band head energy and with binary cluster configurations of 12C+24Mg and 16O + 20Ne, in good agreement with our results.

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