A Model of Choice with Minimal Compromise

Mario Vázquez Corte†

This version: July 2016

Abstract

I formulate and characterize the following two-stage choice behavior. The decision maker is endowed with two preferences. She shortlists all maximal alternatives according to the first preference. If the first preference is decisive, in the sense that it shortlists a unique alternative, then that alternative is the choice. If multiple alternatives are shortlisted, then, in a second stage, the second preference vetoes its minimal alternative in the shortlist, and the remaining members of the shortlist form the choice set. Only the final choice set is observable. I assume that the first preference is a weak order and the second is a linear order. Hence the shortlist is fully rationalizable but one of its members can drop out in the second stage, leading to bounded rational behavior. Given the asymmetric roles played by the underlying binary relations, the consequent behavior exhibits a minimal compromise between two preferences. To our knowledge it is the first Choice function that satisfies Sen’s $\beta$ axiom of choice, but not $\alpha$.

J.E.L. codes: D0.

Keywords: Bounded Rationality, Multiple Preferences, Two-Stage Choice, Shortlisting, Altruism.

---

*This work draws from my work under the supervision of Levent Ülkü, who erroneously appeared as a coauthor in a previous draft.
†Department of Economics, ITAM
1 Introduction

The standard model of rational choice centers around a decision maker (DM) who maximizes a given preference in every menu. Experimental evidence and field data contain robust deviations from this model. The accumulation of such evidence has created an interest in developing new models of bounded rationality which rely on a richer set of psychological variables.

A particularly prominent idea which has been explored in this literature is that the DM might use multiple preferences in making choices. In the presence of multiple preferences, any conflict which may arise between preferences need to be resolved before making choices.\(^1\) Recent work has studied various ways of resolving such conflicts, mainly by explicitly attributing different roles to different preferences. In Manzini and Mariotti (2007) and Bajraj and Ülkü (2015), for example, the DM uses one preference to identify a shortlist of viable alternatives and a second preference to choose from the shortlist.\(^2\)

In this work I will study a model which, similarly, features a compromise between two preferences. In my model, the DM will choose to maximize a preference, with the proviso that, in case multiple alternatives are maximal, a second preference will be able to veto an alternative. To be precise, the model works as follows. The DM is endowed with a weak order (a utility) and a linear order (a utility where no distinct alternatives are indifferent.) The DM first shortlists all best alternatives in the weak order. If a unique alternative is shortlisted, it is chosen. Otherwise, in a second stage, she eliminates from the shortlist the worst alternative according to the linear order. The remaining alternatives form the choice set.

In view of the very asymmetric roles played by the two underlying preferences, this model features an idea of a minimal compromise. Note that the linear order plays no role if the weak order is decisive in the first round. Only if the weak order shortlists exactly two candidates, does the linear order make the choice in the second round. If more than two are shortlisted, the linear order can only veto one of them. Hence the departure from the maximization of the first stage preference as a result of a conflict with the second order is minimal.

\(^1\)Of course, conflicts between criteria need not be resolved and the DM may choose any alternative which is the best according to some preference. The resulting behavior is characterized by the famous Path Independence axiom. See, for instance, Moulin (1985).

\(^2\)This is further discussed in Horan (2016), and García-Sanz and Alcatud (2015). Both analyze the two-stage procedure inspired by Manzini and Mariotti (2007).
preference is, in some sense, minimal.

In terms of behavior, this model can explain violations of Sen’s $\alpha$ axiom, however it necessarily satisfies various other rationality axioms such as $\gamma$, $\beta$ and No Binary Cycles (NBC). My main result is a characterization of this model using five novel conditions.

My model is largely inspired by Manzini and Mariotti (2007). They study a two-stage choice procedure which depends on two asymmetric binary relations. In the first stage, the DM forms a shortlist consisting of all maximal alternatives according to the first binary relation. In the second stage she chooses from the shortlist using the second binary relation. They show that this two-stage procedure explains cyclical behavior whereby $x$ is chosen over $y$, $y$ over $z$ and $z$ over $x$. The main difference between the present work and Manzini and Mariotti (2007) is that I study a choice correspondence, while they characterize a choice function, which is a restrictive form of a choice correspondence which selects a unique alternative in every menu. Furthermore, in my model the role of the second binary relation is different. Instead of choosing its best-preferred alternative from the shortlist, it vetoes the choice of its least preferred shortlisted alternative. I should note that, as does the related two-stage model of Bajraj and Ülkü (2015), my model fails to account for cyclical behavior. Instead, my model can explain violations of $\alpha$, whereby an alternative drops out of the choice set in a smaller menu which contains it.

2 Interpretation

The Choice with Minimal Compromise can be interpreted as a two different agents choosing over a menu. Imagine you are in a restaurant with your significant other and decide to order pizza. You read the menu and enumerate your favorite options: Hawaiian, supreme, veggie and 4 cheeses. Your significant other then says he doesn’t want to eat supreme. So you compromise and decide to choose a pizza from the remaining three alternatives.

Formally, the first agent is represented by the preference relation $R$ shortlist her best alternatives. Then asks the second agent, to take out his least favorite option from the shortlist encoded by the preference relation $L$. then, she proceeds to choose any item from the remaining shortlist.
3 Model

I consider a standard choice environment. Let \( X \) be a finite set of alternatives. A binary relation \( R \) on \( X \) is (1) complete if for all \( x, y \in X \), either \( xRy \) or \( yRx \), (2) transitive if for all \( x, y, z \in X \), if \( xRy \) and \( yRz \), then \( xRz \), and (3) antisymmetric if for all \( x, y \in X \), if \( xRy \) and \( yRx \), then \( x = y \). A weak order is a complete and transitive binary relation. A linear order is an antisymmetric weak order. We will typically refer to weak orders by \( R \) and linear orders by \( L \). For any weak order \( R \), I will denote by \( I \) and \( P \) the symmetric and asymmetric parts of \( R \), respectively: \( xIy \Leftrightarrow xRy \) and \( yRx \); and \( xPy \Leftrightarrow xRy \) and \( \neg(yRx) \).

A menu is any nonempty subset of \( X \). \( 2^X = \{ A \subseteq X : A \neq \emptyset \} \) denotes the set of menus. If \( R \) is a weak order, let \( \max(A, R) \) denote the set of maximal alternatives in \( A \) according to \( R \), in other words, \( \max(A, R) = \{ x \in A : xRy \text{ for all } y \in A \} \). If \( L \) is a linear order, then \( \max(A, L) \) is a singleton, as is \( \min(A, L) = \{ x \in A : yLx \text{ for all } y \in A \} \). In this case, I will refer to the unique alternative in \( \max(A, L) \) (resp. \( \min(A, L) \)) by \( \max(A, L) \) (resp. \( \min(A, L) \)) as well.

A choice correspondence is a map \( c: 2^X \to 2^X \) satisfying \( c(A) \subseteq A \) for every menu \( A \). If \( c(A) = \{ x \} \) for some menu \( A \) and some alternative \( x \in A \), I will say that \( c \) is decisive at \( A \). A choice correspondence \( c \) is rational if there exists a weak order \( R \) such that \( c(A) = \max(A, R) \) for every menu \( A \). A choice correspondence \( c \) satisfies the weak axiom of revealed preference (WARP) if for every \( x, y, A \) and \( B \), if \( x, y \in A \cap B, x \in c(A) \) and \( y \in c(B) \), then \( x \in c(B) \) as well. It is well known that WARP is a necessary and sufficient condition for the rationality of choice correspondences. (See for instance Moulin, 1985.)

I can now define the class of choice correspondences of interest in this work.

**Definition 1** A choice correspondence \( c \) admits a minimal compromise representation if there exist a weak order \( R \) and a linear order \( L \) such that for every menu \( A \)

\[
c(A) = \begin{cases} 
\max(A, R) & \text{if } \max(A, R) \text{ is a singleton,} \\
\max(A, R) \setminus \min(\max(A, R), L) & \text{otherwise.}
\end{cases}
\]

If this is the case, I will call \( c \) an MC choice correspondence.

Hence a choice correspondence \( c \) with a minimal compromise representation operates
in two stages. In the first stage maximal alternatives according to $R$ are shortlisted. In the second stage the choice is made from the shortlist using $L$ as follows. If the shortlist contains only one alternative, then it is the choice, and $L$ has no role to play. If the shortlist contains multiple alternatives, however, the alternative which is the worst according to $L$ is eliminated. All remaining alternatives form the choice set. Hence the compromise between $R$ and $L$ is minimal, in the sense that $L$ can veto only one alternative from the shortlist, if the shortlist contains two or more alternatives.

MC choice correspondences can usefully explain failures of WARP. Consider the following two axioms:

$\alpha$: If $x \in c(A)$ and $x \in B \subset A$, then $x \in c(B)$.

$\beta$: If $x, y \in A \subset B$, $x, y \in c(A)$, and $y \in c(B)$, then $x \in c(B)$ as well.

**Fact 1:** (Moulin, 1985) A choice correspondence satisfies WARP if and only if it satisfies $\alpha$ and $\beta$.

My first result indicates that MC choice correspondences can fail $\alpha$, but they have to satisfy $\beta$.

**Lemma 1** Let $c$ admit a minimal compromise representation. Then $c$ satisfies $\beta$. However $c$ may fail $\alpha$.

**Proof.** Let $c$ admit a minimal compromise representation. Fix $x, y \in A, B \subseteq X$. Suppose $x, y \in c(A)$, $A \subset B$ and $y \in c(B)$. I need to show that $x \in c(B)$. If $x = y$ the condition is automatically satisfied since $y \in c(B)$. Suppose $x \neq y$. Since $y \in c(B)$, $y \in \max(B, R)$ and therefore $yRb$ for all $b \in B$. The fact that $x \in c(A)$ implies that $x \in \max(A, R)$, so $xRy$ since $y$ belongs to the set $A$. Now transitivity of $R$ gives $xRb$ for all $b \in B$, and therefore $x \in \max(B, R)$. Notice that $\max(B, R)$ can not be a singleton since $x$ and $y$ are different, therefore $c(B) = \max(B, R) \setminus \min(\max(B, R), L)$. I will now show that $x \neq \min(\max(B, R), L)$. By hypothesis $x, y \in c(A)$ so $\max(A, R)$ is not a singleton, recall $x \neq y$, which implies $c(A) = \max(A, R) \setminus \min(\max(A, R), L)$. Hence there exists some $z$ distinct from $x$ and $y$, such that $z = \min(\max(A, R), L)$. Hence $xLz$. Since $z \in \max(A, R)$, $zRy$. Since $y$ belongs to the set $A$, then, by transitivity of $R$, $zRb$ for all $b \in B$ which
shows that \( z \in \max(B, R) \). Hence \( x \) can not be the \( L \)-worst alternative in the shortlist \( \max(B, R) \).

The following example shows that \( c \) may fail \( \alpha \). Let \( c \) admit a minimal compromise representation. Additionally, let \( X = \{x, y, z\} \), with \( R \) ranking all three alternatives indifferent and \( xLyLz \). The consequent MC choices are given by the following table:

\[
\begin{array}{c|cccc}
A & \{x, y\} & \{x, z\} & \{y, z\} & \{x, y, z\} \\
c(A) & \{x\} & \{x\} & \{y\} & \{x, y\}
\end{array}
\]

Note \( y \) is chosen from the menu \( \{x, y, z\} \), but not from the menu \( \{x, y\} \) where it belongs, leading to a failure of \( \alpha \). This happens because the vetoed alternative is different between the two menus: \( y \) is the \( L \)-worst alternative in the shortlist \( \{x, y\} = \max(\{x, y\}, R) \), but \( z \) is the \( L \)-worst alternative in the shortlist \( \{x, y, z\} = \max(\{x, y, z\}, R) \).

There is another sense in which the deviation of MC choice correspondences from full rationality comes in the form of \( \alpha \) failures. Consider the following two axioms:

\( \gamma \): If \( x \in c(A) \cap c(B) \), then \( x \in c(A \cup B) \).

No binary cycles (NBC): If \( x \in c(\{x, y\}) \) and \( y \in c(\{y, z\}) \), then \( x \in c(\{x, z\}) \).

**Fact 2**: (Moulin, 1985) A choice correspondence satisfies WARP if and only if it satisfies \( \alpha, \gamma \) and NBC.

My next result indicates that MC choice correspondences satisfy \( \gamma \) and NBC. Hence in view of the particular characterization given in Fact 2, MC choice correspondences fail to be rational only because they may fail \( \alpha \).

**Lemma 2** Let \( c \) admit a minimal compromise representation. Then \( c \) satisfies \( \gamma \) and NBC.

**Proof.** Suppose \( c \) admits a minimal compromise representation. I will first show that \( c \) satisfies \( \gamma \). Let \( x \in c(A) \cap c(B) \). Then it must be the case that \( x \in \max(A, R) \) and \( x \in \max(B, R) \), which means \( x \in \max(A \cup B, R) \). There are two cases: \( x \) is the only \( R \)-maximal element in both menus or there exists an other \( R \)-maximal element, distinct from \( x \), in at least one menu. In the first case, \( \{x\} = \max(A \cup B, R) \) and \( c(A \cup B) = \{x\} \).

\[
\begin{array}{c|cccc}
A & \{x, y\} & \{x, z\} & \{y, z\} & \{x, y, z\} \\
c(A) & \{x\} & \{x\} & \{y\} & \{x, y\}
\end{array}
\]
max(A ∪ B, R) = \{x\} and x is chosen in A ∪ B as desired. In the second case, there exists y ≠ x in, say, menu A such that y ∈ max(A, R). Since yIx, y ∈ max(A ∪ B, R) as well. Since x, y ∈ max(A, R) it must be the case that c(A) = max(A, R) ∖ min(max(A, R), L). Notice that there exists z ∈ max(A, R) such that xLz, since x ∈ c(A) and therefore x ≠ min(max(A ∪ B, R), L). This last statement means zRx hence z ∈ max(A ∪ B, R) as well.

Since x, y ∈ max(A, R) it must be the case that c(A) = max(A, R) \ min(max(A, R), L).

Notice that there exists z ∈ max(A, R) such that xLz, since x ∈ c(A) and therefore x ≠ min(max(A ∪ B, R), L). This last statement means zRx hence z ∈ max(A ∪ B, R) as well.

To show that c satisfies NBC, let x ∈ c(\{x, y\}) and y ∈ c(\{y, z\}). I have to show that x ∈ c(\{x, z\}). Since c has a minimal compromise representation it must be the case that x ∈ max(\{x, y\}, R) and y ∈ max(\{y, z\}, R), so xRy and yRz. Then, by transitivity of R, xRz which means x ∈ max(\{x, z\}, R). If ¬(zRx) then c(\{x, z\}) = max(\{x, z\}, R) = \{x\}. If zRx then zRy and yRx since R is transitive. Consequently max(\{x, y\}, R) = \{x, y\}, max(\{y, z\}, R) = \{y, z\} and max(\{x, z\}, R) = \{x, z\}, hence L vetoes an alternative in all three doubleton menus. Recall x ∈ c(\{x, y\}) and y ∈ c(\{y, z\}), then xLy and yLz, which implies xLz by transitivity of L. Hence \{x\} = c(\{x, z\}) once again. This finishes the proof. ■

4 Revelation of Preferences

Suppose that c admits a minimal compromise representation with the underlying preferences R and L. How does the resulting behavior reveal R and L? The following example shows that there may be more than one way to rationalize observed choices by this model.

Example 1 Consider the following choice correspondence:

\[
\begin{array}{cccc}
A & \{x, y\} & \{x, z\} & \{y, z\} & \{x, y, z\} \\
c(A) & \{y\} & \{x\} & \{y\} & \{y\}
\end{array}
\]

Note that this choice correspondence is decisive in every menu. Furthermore it is rationalizable in the standard sense. One way to see that this behavior admits a minimal compromise representation is to let L be any linear order and take R as follows: yP xP z, where P is the strict part of R. Alternatively, if I take xIyP z for R, and zLyLx, the resulting choices are identical. ▲

Yet, through violations of α, a MC choice correspondence can quite tractably reveal at least parts of the underlying weak order R and the linear order L. First, note that any
chosen alternative must be at least as good as any other feasible alternative in \( R \). Hence \( xRy \) if \( x \in c(A) \) and \( y \in A \) in some menu \( A \). This mirrors the revelation of the underlying preference in a standard rationality framework. However in my model, if \( x \in c(A) \) and \( y \in A \setminus c(A) \), this does not mean that \( x \) is strictly preferred to \( y \). This is because \( x \) and \( y \) are perhaps indifferent, but \( y \) has been vetoed by the linear order \( L \). However, any such \( y \) can easily be detected as their removal would impact behavior. Indeed, if \( y \) has been vetoed, then its removal from \( A \) may lead to the veto of the next alternative in \( L \), meaning \( c(A \setminus y) \neq c(A) \) even though \( y \not\in c(A) \). If this happens, then I can also conclude that \( yRa \) for all \( a \in A \). Furthermore I can also conclude that \( xLy \) for all \( x \in c(A) \). However this revelation of \( L \) is not complete. Suppose that \( R \) shortlists only two alternatives \( x \) and \( y \) but \( y \) is vetoed by \( L \). In this case the removal of \( y \) has no impact on behavior as \( c(A) = \{x\} \) since \( y \) is vetoed, but \( c(A \setminus y) = \{x\} \) as well, as only \( x \) has been shortlisted, stripping \( L \) of its veto power.

The characterization exercise of the next section contains my main result, Theorem 1, where these revelations play the key role. In it, I define the binary relation \( R \) as follows: \( xRy \) iff \( y \) belongs to a menu where the removal of \( x \) affects behavior. I also define a binary relation \( L \) as follows: \( xLy \) iff \( c \) is decisive in the menu \( \{x, y\} \) in favor of \( x \). I show that, under certain conditions which I will specify, \( R \) is complete and transitive, i.e., a weak order. I also show that, again under conditions, \( L \) can be completed to a linear order. Furthermore, any \( c \) satisfying the conditions I will identify behaves identically to a MC choice correspondence defined by \( R \) and \( L \).

5 Characterization

In this section I will characterize the class of MC choice correspondences. Let me begin with some notation which will help in the statement of two of my conditions. For any choice correspondence \( c \) and any menu \( A \), let

\[
r^c(A) = \{x \in A : c(A \setminus x) \neq c(A)\}.
\]

In words, \( r^c(A) \) collects all members of \( A \) whose removal impacts behavior. Note \( r^c \) is a choice correspondence itself: \( r^c(A) \subseteq A \) and, since \( c(A) \subseteq r^c(A) \), \( r^c(A) \) is nonempty. Clearly, the removal of any chosen element will impact behavior. However the removal of
alternatives which are not chosen may also impact behavior. The following observation indicates that this never happens if \( c \) is rational.

**Lemma 3** If a choice correspondence \( c \) is rational, then \( r^c(A) = c(A) \) for every menu \( A \).

**Proof.** Take a choice correspondence \( c \). Suppose \( c \) is rational and let \( R \) be the weak order which \( c \) maximizes, i.e., \( c(A) = \max(A, R) \) for all \( A \). I will show that \( r^c = c \). By definition, \( c(A) \subseteq r^c(A) \). Take any \( x \in A \setminus c(A) \). If \( a \in \max(A, R) \), then \( a \neq x \), \( a \in A \setminus x \) and \( a \in \max(A \setminus x, R) \) as well. Hence \( c(A) \subseteq c(A \setminus x) \). If \( a \not\in \max(A, R) \) and \( a \neq x \), on the other hand, then there exists \( a' \neq x \) such that \( a'Pa \) and \( a \not\in \max(A \setminus x, R) \), giving \( c(A \setminus x) \subseteq c(A) \). I conclude that \( a \not\in r^c(A) \) and \( c(A) = r^c(A) \). ■

The next example show that the statement in the preceding Lemma can not be reversed, i.e., \( r^c = c \) is not sufficient for the rationality of \( c \).

**Example 2** Consider the following choice correspondence on \( X = \{x, y, z\} \).

\[
\begin{array}{cccc}
A & \{x, y\} & \{x, z\} & \{y, z\} & \{x, y, z\} \\
c(A) & \{x, y\} & \{x\} & \{y\} & \{x, y\}
\end{array}
\]

Note \( r^c = c \) but \( c \) is not rational as it fails \( \alpha \): \( x \in c(\{x, y, z\}) \) but \( x \not\in c(\{x, z\}) \). ▲

I am now ready to state the characterizing conditions for MC choice correspondences.

**Condition 1.** If \( x, y \in A \subseteq B \), \( x \in c(A) \), and \( y \in c(B) \), then \( x \in c(B) \).

This condition says that if an alternative \( y \) is chosen in a menu, and \( x \) is chosen in a smaller menu where \( y \) is present, then \( x \) must also be chosen in the larger menu. This condition strengthens \( \beta \) by weakening its ”if-part” as it does not insist that \( y \) should be chosen in the smaller menu \( A \) for the conclusion to follow. The following example shows that the strengthening is strict, as there are choice correspondences which satisfy \( \beta \) but fail Condition 1.
Example 3  The following choice correspondence satisfies $\beta$ but fails Condition 1.

\[
\begin{array}{cccc}
A & \{x, y\} & \{x, z\} & \{y, z\} & \{x, y, z\} \\
c(A) & \{x\} & \{x\} & \{y\} & \{y\}
\end{array}
\]

Note $\beta$ holds vacuously as $c$ is decisive in every menu. However Condition 1 fails: $x, y \in \{x, y\} \subset \{x, y, z\}$, $x \in c(\{x, y\})$, and $y \in c(\{x, y, z\})$, but $x \not\in c(\{x, y, z\})$. ▲

The next condition says that Condition 1 should hold for the map $r^c$ associated with the choice correspondence $c$.

**Condition 2.** For any $c$, $r^c$ satisfies Condition 1: If $x, y \in A \subset B$, $x \in r^c(A)$, and $y \in r^c(B)$, then $x \in r^c(B)$.

Hence if the removal of $y$ changes behavior in menu $B$, and the removal of $x$ changes behavior in a smaller menu $A$ where $y$ belongs, then the removal of $x$ should change behavior in menu $B$ as well.

The next condition says that $c$ should not choose every feasible alternative, except of course in singletons.

**Condition 3.** For any nonsingleton menu $A$, there exists $x \in A$ such that $x \not\in c(A)$.

Note that Condition 3 implies, in particular, that $c$ should be decisive in doubleton menus. Furthermore, I have the following result.

**Lemma 4** If $c$ satisfies Conditions 1 and 3, then it must satisfy NBC as well.

**Proof.** Suppose $c$ satisfies Conditions 1 and 3, $x \in c(\{x, y\})$ and $y \in c(\{y, z\})$ but $\{z\} = c(\{x, z\})$. Note, by Condition 3, this means $\{x\} = c(\{x, y\})$ and $\{y\} = c(\{x, y\})$. Consider the menu $\{x, y, z\}$. Since all alternatives in $\{x, y, z\}$ have been chosen in a smaller menu, Condition 1 implies that if one of them belongs to $c(\{x, y, z\})$, then all do so. This violates Condition 3. ■

Next consider the following condition.
Condition 4. If \( x \in A \setminus c(A) \) and \( x \in c(A \cup \{y\}) \), then \( y \notin c(A \cup \{y\}) \).

Imagine adding a new alternative \( y \) to a menu \( A \). Condition 4 says that this can not lead to the inclusion of both \( y \) and a previously unchosen alternative \( x \) in the choice set. If \( x \) jumps in the choice set as a result of the inclusion of \( y \), then \( y \) should not belong to the choice set.

Finally, my last condition is as follows.

Condition 5. For all \( A \), for all nonsingleton \( B \subseteq r^c(A) \) there exists some \( x \notin c(B) \) such that \( B = c(B) \cup \{x\} \).

Imagine choice in a menu of alternatives all of which impact choice in a larger menu. Condition 5 says that all but one of these alternatives must belong to the choice set.

I am now ready to state the main result.

**Theorem 1** A choice correspondence admits a minimal compromise representation if and only if it satisfies Conditions 1-5.

**Proof.** To begin, suppose that \( c \) admits a minimal compromise representation and let \( R \) and \( L \) be the underlying weak and linear orders.

To see that \( c \) satisfies Condition 1, fix \( x, y \in A \subset B \) such that \( x \in c(A) \) and \( y \in c(B) \). I need to show that \( x \in c(B) \). There is nothing to show if \( x = y \), so suppose \( x \neq y \). By definition \( x \in \max(A, R) \) and \( y \in \max(B, R) \), hence \( x \in \max(B, R) \) and \( y \in \max(A, R) \) as well. Since \( \max(A, R) \) is not a singleton, there exists some \( z \in \max(A, R) \setminus c(A) \) such that \( xLz \). Note \( z \in \max(B, R) \) as well, so \( x \neq \min(\{\max(B, R)\}, L) \), giving \( x \in c(B) \).

Next take \( x, y \in A \subset B \) such that \( x \in r^c(A) \) and \( y \in r^c(B) \). To establish Condition 2, I need to show that \( x \in r^c(B) \). There is nothing to show if \( x = y \) or if \( x \in c(B) \). Suppose \( x \neq y \) and \( x \notin c(B) \). Note that minimal compromise representation implies \( r^c(S) \subseteq c(S) \) for any menu \( S \). Hence \( x \in \max(A, R) \), \( y \in \max(B, R) \) and consequently \( \max(A, R) \subseteq \max(B, R) \) and \( x \in \max(B, R) \) as well. This means \( x \) is vetoed by \( L \) in \( B \). I need to show that \( c(B) \) contains at least two alternatives so that \( x \in r^c(B) \). Suppose \( x \in c(A) \). Note \( y \in \max(A, R) \) as well, so \( \max(A, R) \) contains at least two alternatives. Hence an alternative must be vetoed in \( A \), say some \( a \neq x \). Then \( a \in \max(A, R) \subseteq \max(B, R) \) and \( xLa \), hence \( x \) cannot vetoed in \( B \), a contradiction. If \( x \notin c(A) \), on the other hand, then \( x \) is vetoed in \( A \). However \( x \in r^c(A) \), meaning \( c(A) \) contains at least two distinct
alternatives $a_1, a_2 \in \max(A, R) \subseteq \max(B, R)$ such that $a_i L x$. Then $a_1, a_2 \in c(B)$, and the removal of $x$ from $B$ will result in a change in behavior, as I needed to show. I conclude that $x \in r^c(B)$.

To show that $c$ satisfies Condition 3, take a menu $A$ which is not a singleton and suppose $c(A) = A$. Since $c(A) \subseteq \max(A, R)$ whenever $c(A)$ contains multiple alternatives, $A = c(A) \subseteq \max(A, R)$, and impossibility.

Next, to show $c$ satisfies Condition 4, take $x, y$ and $A$ such that $x \in A \setminus c(A)$ and $x \in c(A \cup \{y\})$. I have to show that $y \notin c(A \cup \{y\})$. By definition of $r^c$, $x, y \in r^c(A \cup \{y\})$, which means $x, y \in \max(A \cup \{y\}, R)$ as well. Hence $x \in \max(A, R)$ but for all $a \in c(A)$, $a L x$. Now take any $a \in \max(A \cup \{y\}, R) \setminus \{y\}$. Note such $a \in \max(A, R)$ as well. If $y L a$, then $y L x$ also and consequently $x$ is the $L$-worst alternative in $\max(A \cup \{y\}, R)$, meaning $x \notin c(A \cup \{y\})$, a contradiction. Hence $a L y$ for all $a \in \max(A \cup \{y\}, R) \setminus \{y\}$ and $y \notin c(A \cup \{y\})$.

Finally, to see that $c$ satisfies Condition 5 take nonsingleton menus $A$ and $B$ such that $c(A) \neq c(A \setminus x)$ for all $x \in B$. Then $B \subseteq \max(A, R)$. Consequently $B = \max(B, R)$. Let $x = \min(B, L)$ so that $c(B) = B \setminus \{x\}$, and Condition 5 follows.

In the reverse direction, suppose $c$ satisfies Conditions 1-5. Define $x R y$ iff there exists some menu $A$ such that $x \in r^c(A)$ and $y \in A$. Also define $x L y$ iff $\{x\} = c(\{x, y\})$. I will now show that $R$ is a weak order and $L$ is a linear order.

Completeness of $R$ follows from the definition of a choice correspondence. Take any $x, y \in X$. If $x = y$, then $x R x$ since $\{x\} = c(\{x\})$. Otherwise since $c(\{x, y\}) \neq \emptyset$, $x R y$ or $y R x$ (or both). To see that $R$ is transitive, suppose $x R y$ and $y R z$. Then there exist menus $A$ and $B$ such that $x \in r^c(A)$ and $y \in r^c(B)$, $y \in A$ and $z \in B$. Take any $w \in r^c(A \cup B)$. If $w \in A$, then $x \in r^c(A \cup B)$ by Condition 2. If $w \in B$, then $y \in r^c(A \cup B)$ and therefore $x \in r^c(A \cup B)$, again, by Condition 2. Since $z \in A \cup B$, $x R z$ as desired. This proves that $R$ is a weak order.

To see that $L$ is complete, take any $x, y \in X$. If $x = y$, then $x L x$ as $\{x\} = c(\{x\})$. Otherwise Condition 3 implies $\{x\} = c(\{x, y\})$ or $\{y\} = c(\{x, y\})$. Hence $x L y$ or $y L x$ and $L$ is complete. Furthermore I can not have $x L y$ and $y L x$ for distinct $x$ and $y$, hence $L$ is asymmetric. The transitivity of $L$ follows from Lemma 4 and Condition 3 as follows. If $x L y$, $y L z$, then Lemma 4 implies $x \in c(\{x, z\})$ and Condition 3 implies $\{x\} = c(\{x, z\})$. Hence $x L z$ and $L$ is transitive. This proves that $L$ is a linear order.

Now let $c_{R,L}$ be the MC choice correspondence defined by $R$ and $L$. I will show that $c =
First suppose that $x \in c(A)$. I need to show that $x \in c_{R,L}(A)$. Since $c(A) \subseteq r^c(A)$, $x \in r^c(A)$. By definition of $R$, then, $x \in \max(A, R)$. There are two cases to consider. If $\max(A, R) = \{x\}$, then $\max(A, R) = \{x\} = c_{R,L}(A)$ by definition and $x \in c_{R,L}(A)$ as desired. Suppose now that $\max(A, R)$ contains multiple alternatives. I need to show that $x$ is not the $L$-worst alternative in $\max(A, R)$. In other words, I need to find an alternative $a \in \max(A, R) \setminus \{x\}$ such that $c(\{a, x\}) = \{x\}$. Let $y \in \max(A, R) \setminus x$. By Condition 3, $c(\{x, y\})$ is a singleton. If $c(\{x, y\}) = \{x\}$, then $xLy$ and I am done. Suppose that $c(\{x, y\}) = \{y\}$. Since $x \in c(A)$ and $\{x, y\} \subset A$, as I keep adding alternatives to menu $\{x, y\}$ to reach menu $A$, $x$ must jump in the choice set at some point. In other words, there must exist a menu $D$ and an alternative $z \in A \setminus D$ such that $\{x, y\} \subset D \subseteq A$, $x \notin c(D)$ and $x \in c(D \cup \{z\})$. (If no such $D$ and $z$ exist, then $x \notin c(A)$.) Note that $z \in r^c(D \cup \{z\})$ and therefore $zRx$. Hence $z \in \max(A, R)$ as well. However $z \notin c(D \cup \{z\})$ by Condition 4. Now consider the menu $\{x, z\}$. If $\{z\} = c(\{x, z\})$, then Condition 1 dictates that $z \in c(D \cup \{z\})$, a contradiction. Then, by Condition 3, $c(\{x, z\}) = \{x\}$ and $xLz$. This proves that $x$ is not the $L$-worst in $\max(A, R)$. I conclude that $x \in c_{R,L}(A)$.

To finish, take $x \in c_{R,L}(A)$. I need to show that $x \in c(A)$. By definition $x \in \max(A, R)$. If $\max(A, R) = \{x\}$, there exists no $y \in A \setminus \{x\}$ such that $yRx$. This implies that $c(A)$ can only contain $x$. Since $c(A)$ is nonempty, it has to contain $x$, as desired. Now suppose $\max(A, R) \neq \{x\}$, i.e., that there exists some $z \neq x$ such that $\{x, z\} \subseteq \max(A, R)$. Since $x \in c_{R,L}(A)$ and $\max(A, R)$ contains multiple alternatives, $x$ is not the $L$-worst alternative in $\max(A, R)$. Hence I can take $z$ such that $xLz$, i.e., $c(\{x, z\}) = \{x\}$.

Towards a contradiction suppose $x \notin c(A)$ and pick $y \in c(A)$. If $x \in c(\{x, y\})$, then by Condition 1, $x \in c(A)$ as well, a contradiction. By Condition 3, then $\{y\} = c(\{x, y\})$.

Since $zRx$ and $zRy$, there exist menus $B_x$ and $B_y$ such that $x \in B_x$, $y \in B_y$ and $z \in r^c(B_x) \cap r^c(B_y)$. By Condition 2, then $z \in r^c(B_x \cup B_y)$. Since $c(\{x, z\}) = \{x\}$, $x \in r^c(\{x, z\})$ and Condition 2 implies $x \in r^c(B_x \cup B_y)$. Similarly, since $c(\{x, y\}) = \{y\}$, $y \in r^c(B_x \cup B_y)$.

Now I will use Condition 5. Consider the menu $\{x, y, z\} \subseteq r^c(B_x \cup B_y)$. Conditions 3 and 5 imply that $c(\{x, y, z\})$ contains exactly two alternatives. If $c(\{x, y, z\}) = \{y, z\}$, then Condition 1 fails since $c(\{x, z\}) = \{x\}$. Similarly if $c(\{x, y, z\}) = \{x, z\}$, as $\{y\} = c(\{x, y\})$. Hence $c(\{x, y, z\}) = \{x, y\}$. Now Condition 1 implies $x \in c(A)$, as $y \in c(A)$. This contradiction finishes the proof. $\blacksquare$
6 Conclusion

In this work I study a two-stage choice correspondence defined by a weak order $R$ and a linear order $L$. In any menu $A$, first $R$ shortlists its maximal alternatives and next $L$ vetoes its worst alternative in the shortlist, provided that the shortlist contains multiple alternatives. Hence the behavior features a compromise from the maximization of $R$, but this compromise is minimal, as $L$ can only veto its least preferred alternative. Only if $R$ shortlists two candidates does $L$ make the choice. If three or more candidates are shortlisted, then $L$ can only veto a single one of them. Moreover if only a single alternative is shortlisted, $L$ has no effect on behavior.

I show that this model satisfies various rationality conditions, most notably $\beta$, $\gamma$ and no-binary-cycles. However it may fail the famous $\alpha$ axiom. I provide five novel conditions which together characterize this model. I leave for future work the generalization to the scenario where the second preference is also a weak order, and could veto multiple alternatives.

References

[1] Bajraj, G. and Ülkü, L., (2015). Choosing two Finalists and the Winner. Social Choice and Welfare, December 2015, Volume 45, Issue 4, pages 729-744

[2] Manzini, P. and Mariotti, M. (2007). Sequentially Rationalizable Choice. American Economic Review, 97(5), 1824-1839

[3] Moulin, H. (1985). Choice functions over a finite set: A summary. Social Choice and Welfare, 2(2), 147-160

[4] Horan, S. (2016). A Simple Model of Two-Stage Choice. Journal of Economic Theory, Issue C, 372-406

[5] García-Sanz, María D. & Alcantud, José Carlos R (2015). Sequential rationalization of multivalued choice, Mathematical Social Sciences, Elsevier, vol. 74(C), pages 29-33