Nonperturbative dynamics in the colour–magnetic QCD vacuum

A. V. Nefediev and Yu. A. Simonov

Institute of Theoretical and Experimental Physics,
117218, B.Cheremushkinskaya 25, Moscow, Russia

Abstract

In the deconfinement phase of QCD quarks and gluons interact with the dense stochastic colour–magnetic vacuum. We consider the dynamics of quarks in this deconfinement phase using the Field Correlators Method and derive an effective nonperturbative interquark potential, in addition to the usual perturbative short–ranged interaction. We find the resulting angular–momentum–dependent interaction to be attractive enough to maintain bound states and, for light quarks (and gluons), to cause emission of quark and gluon pairs. Possible consequences for the strong interacting quark–gluon plasma are briefly discussed.

1 Introduction

The picture of the strong interacting quark–gluon plasma (SQGP) seems to be adequate for explaining the recent data on ion–ion collisions [1]. It was found on the lattice [2] that colour–magnetic vacuum fields do not change across the phase transition, thus supporting the conjecture made in Ref. [3] that, above the temperature of deconfinement $T_c$, the QCD vacuum loses its confining colour–electric part, while the colour–magnetic part remains intact. This idea can be economically expressed in the formalism of the Field Correlators Method (FCM) [4], where the Gaussian correlators of the colour–electric and colour–magnetic fields, $\langle\langle E_i(x)E_j(y)\rangle\rangle$ and $\langle\langle H_i(x)H_j(y)\rangle\rangle$ ($\langle\langle \ldots \rangle\rangle$ denotes irreducible correlators), are parametrised through the correlation functions $D^E$, $D^E_1$, $D^H$, and $D^H_1$, respectively (see Ref. [4] for the details of the formalism). Notice that among those only $D^E$ vanishes above the $T_c$ [2, 3]. The usual (time-like) string tension $\sigma_E$ and the so-called spatial string tension $\sigma_s$ are given by the double integrals from the corresponding correlators,

$$\sigma_E = \frac{1}{2} \int D^E(\xi) d^2 \xi, \quad \sigma_s \equiv \sigma_H = \frac{1}{2} \int D^H(\xi) d^2 \xi. \quad (1)$$

Both string tensions coincide below the $T_c$ but $\sigma_E$ is expected to vanish in the deconfinement phase, whereas $\sigma_H$ remains nearly constant in the vicinity of the critical temperature, both below and above the $T_c$ [4, 5]. Moreover, $\sigma_H$ grows at large $T$, as $\sqrt{\sigma_H} \propto T g^2(T)$ [6], signalising that $D^H$ grows as $\mathcal{O}(T^2 g^4(T))$. At the same time it was conjectured in Ref. [7], and confirmed later on the lattice [2, 8], that the non-confining correlator $D^E_1$ does not vanish either above the

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The correlators $D^E$ and $D^H$ appear at the structure $\delta_{ij}$ in these colour–electric and colour–magnetic correlators, respectively (see also Eq. (3)), whereas $D^E_1, H_1$ parametrise the terms with derivatives.
$T_c$ and leads to strong interaction between quarks and gluons. In Refs. [3, 4], bound states due
to $D^E_1$ in quark and gluonic systems where found in qualitative agreement with the lattice data,
Ref. [10]. Many efforts based on the colour–electric type forces have been applied to clarify
the dynamics of QCD above the deconfinement phase transition. Still, without the colour–
magnetic forces, the dynamical picture of the SQGP is not complete since colour–electric forces
cannot bind quarks and gluons for large angular momentum where colour–magnetic forces
are most important. Thus the purpose of the present Letter is to clarify the matter and to
study orbitally excited bound states of quarks above the $T_c$. For the sake of simplicity, we
consider only the confining correlators $D^{E,H}_1$ and neglect the other two, $D^{E,H}_2$. Furthermore,
we study a strongly interacting system at the fixed temperature $T$, which can be either below
or above the deconfinement temperature $T_c$, and develop the standard Hamiltonian approach
to it. We consider the situation when the colour–electric interaction is switched off above
the deconfinement temperature and the dynamics of quarks and gluons is governed by the
colour–magnetic forces only. Finally, we use the interaction under consideration to estimate
the strength of the interaction in the SQGP by comparing the mean potential energy of light
quarks (and gluons) in the plasma with their mean kinetic energy. We find the corresponding
parameter to be of order $\sigma_H/T^2$. Since this parameter is large for quarks and even $9/2$ times
larger for gluons we conclude that SQGP is very strongly coupled and it should be viewed as
a liquid, at least.

\section{QCD string and the spin–independent interaction in the quark–antiquark system}

In this section we derive the effective spin–independent quark–antiquark interaction arising due
the QCD string formation.

Following the method proposed in Refs. [11, 12] we consider the gauge–invariant Green’s
function in the vacuum at temperature $T$:

$$G(x_1,x_2|y_1,y_2) = \langle \Psi_{out}(x_1,x_2)\Psi^\dagger_{in}(y_1,y_2) \rangle,$$  \hfill (2)

where the wave functions of the initial and final colourless $q\bar{q}$ states are built with the help
of the parallel transporter $\Phi(x,y) = P \exp \left[ i \int_y^x dz_\mu A_\mu(z) \right]$. Using the standard path integral
approach and the Feynman–Schwinger representation for the single–quark propagator, one
arrives at the Green’s function \cite{2} in the form \cite{11}:

$$G(x_1,x_2|y_1,y_2) = \int_0^\infty ds \int (Dz)_{x_1y_1}^w e^{-K} \int_0^\infty d\bar{s} \int (D\bar{z})_{x_2y_2}^w e^{-\bar{K}} \langle TrW(C) \rangle,$$  \hfill (3)

with the kinetic energies $K = \frac{1}{2} \int_0^\infty d\tau z_\mu^2$ and $\bar{K} = \frac{1}{2} \int_0^\infty d\bar{\tau} \bar{z}_\mu^2$ and with the “winding path mea-
sure” $(Dz)_{xy}^w$ taking into account Matsubara periodic boundary conditions. All spin–dependent
terms are neglected in Eq. (3) — they will be restored below, in Section 3. The closed contour
$C$ runs over the quark trajectories and the dynamics of the system is defined by the averaged
Wilson loop $\langle TrW(C) \rangle$. In the Gaussian approximation, one finds that \cite{3}

$$\frac{1}{N_C} \langle TrW(C) \rangle = \exp \left( -\frac{1}{2} \int_S d\sigma_{\mu\nu}(x) \int_S d\sigma_{\lambda\rho}(x') \langle \langle TrF_{\mu\nu}(x)\Phi(x,x')F_{\lambda\rho}(x')\Phi(x',x) \rangle \rangle \right),$$  \hfill (4)

where $S$ is the minimal surface bounded by the contour $C$. Keeping only the string–generating
field strength correlators, we have

$$\langle \langle TrF_{\mu\nu}(x)\Phi(x,x')F_{\lambda\rho}(x')\Phi(x',x) \rangle \rangle = (\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\lambda}) D((x - x')^2).$$  \hfill (5)

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\end{tabular}
In what follows we distinguish between the electric and magnetic contributions in Eq. (5), so that the structure $d\sigma_{ab}d\sigma_{0i}$ enters Eq. (1) multiplied by $D^E$, whereas the spatial part $d\sigma_{jk}d\sigma_{jk}$ is accompanied by $D^H$. The electric and magnetic string tensions are defined then according to Eq. (1).

We synchronise the quarks in the laboratory frame, putting $x_{10} = x_{20} = t$, and adopt the straight–line ansatz for the minimal string writing

\[ d\sigma_{\mu\nu}(x) = \varepsilon^{ab}\partial_{\mu}w(t, \beta)\partial_{\nu}w(t, \beta)dtd\beta, \quad \{a, b\} = \{t, \beta\}, \]

where $0 \leq t \leq t_{\text{max}}$, $0 \leq \beta \leq 1$, and the profile function is defined by the trajectories of the quarks, $w(t, \beta) = \beta x_{1\mu} + (1 - \beta)x_{2\mu}$. For further convenience let us introduce two vectors:

\[ \vec{r} = \vec{x}_1 - \vec{x}_2, \quad \vec{\rho} = [(\vec{x}_1 - \vec{x}_2) \times (\beta \vec{x}_1 + (1 - \beta)\vec{x}_2)] \equiv r\vec{J}, \]

which allow one to write the differentials in a compact form,

\[ d\sigma_{0i}(x)d\sigma_{0i}(x') = \vec{r}(t)\vec{r}(t')dtdt'd\beta d\beta', \quad d\sigma_{jk}(x)d\sigma_{jk}(x') = 2\vec{\rho}(t)\vec{\rho}(t')dtdt'd\beta d\beta'. \]

Presenting the averaged Wilson loop as

\[ \frac{1}{N_C}\langle \text{Tr}W(C)\rangle = e^{-J}, \quad J = J^E + J^H, \]

one can write for the electric and magnetic contributions separately:

\[ J^E = \int_0^{t_{\text{max}}} dt\int_0^1 d\beta d\beta' \vec{r}(t)\vec{r}(t') D^E((x - x')^2), \]
\[ J^H = \int_0^{t_{\text{max}}} dt\int_0^1 d\beta d\beta' \vec{\rho}(t, \beta)\vec{\rho}(t', \beta') D^H((x - x')^2). \]

The correlation functions $D^E, D^H$ decrease in all directions of the Euclidean space–time with the correlation length $T_g$ which is measured on the lattice to be rather small, $T_g \approx 0.2 \div 0.3$ fm [13]. Therefore, only close points $x$ and $x'$ are correlated, so that one can neglect the difference between $\vec{r}(t)$ and $\vec{r}(t')$, $\vec{\rho}(t, \beta)$ and $\vec{\rho}(t', \beta')$ in Eqs. (9), (10) and also write:

\[ (x - x')^2 = (x(t, \beta) - x(t', \beta'))^2 = g^{ab}\xi_a\xi_b, \quad \xi_a = t - t', \quad \xi_b = \beta - \beta'. \]

The induced metric tensor is $g^{ab} = g^a\delta^{ab}$, $g^1 g^2 = \det g = r^2 + \rho^2 = r^2(1 + \omega^2)$. Now, after an appropriate change of variables and introducing the string tensions, according to Eq. (11), one readily finds:

\[ J^E = \sigma_E \int_0^{t_{\text{max}}} dt\int_0^1 d\beta \frac{r^2}{\sqrt{r^2 + \rho^2}} = \sigma_E \int_0^{t_{\text{max}}} dt\int_0^1 d\beta \frac{1}{\sqrt{1 + \omega^2}}, \]
\[ J^H = \sigma_H \int_0^{t_{\text{max}}} dt\int_0^1 d\beta \frac{\rho^2}{\sqrt{r^2 + \rho^2}} = \sigma_H \int_0^{t_{\text{max}}} dt\int_0^1 d\beta \frac{\omega^2}{\sqrt{1 + \omega^2}}. \]

For $\sigma_E = \sigma_H = \sigma$ the sum of $J^E$ and $J^H$ reproduces the well-known action of the Nambu–Goto string.

For further analysis we resort, as was done in Ref. [12], to the Hamiltonian description of the quark–antiquark system under consideration. We also turn over to Minkowski space–time.
Then the Lagrangian of the quark–antiquark system can be derived from the exponent in Eq. (3) in the form:

\[
L = -m_1\sqrt{1 - \dot{x}_1^2} - m_2\sqrt{1 - \dot{x}_2^2} - \sigma_E r \int_0^1 d\beta \frac{1}{\sqrt{1 - [\vec{n} \times (\beta \dot{x}_1 + (1 - \beta) \dot{x}_2)]^2}} + \sigma_H r \int_0^1 d\beta \frac{[\vec{n} \times (\beta \dot{x}_1 + (1 - \beta) \dot{x}_2)]^2}{\sqrt{1 - [\vec{n} \times (\beta \dot{x}_1 + (1 - \beta) \dot{x}_2)]^2}}, \quad \vec{n} = \frac{\vec{r}}{r}.
\]

(14)

Two particular cases of the Lagrangian (14) are of most interest. The first such case corresponds to equal masses, whereas in the other case one mass is assumed infinitely large. Then, using the standard technique, one can proceed to the Hamiltonian of the quark–antiquark system,

\[
H = \frac{1}{\xi} \left( \frac{p_r^2 + m^2}{\mu} + \mu \right) + \int_0^1 d\beta \left( \frac{\sigma_1^T r^2}{2\nu} + \frac{\nu}{2} + \sigma_2 r \right) + \frac{\vec{L}^2}{r^2[\xi\mu + 2\int_0^1 d\beta \nu(\beta - \xi/2)^2]},
\]

(15)

where \(\xi = 1\) for the case of equal masses \((m_1 = m_2 = m)\) and \(\xi = 2\) for the case of the heavy–light system \((m_1 \rightarrow \infty, m_2 = m)\). Here \(\sigma_1 = \sigma_H + \eta^2(\sigma_H - \sigma_E), \sigma_2 = 2\eta(\sigma_E - \sigma_H)\). The fields \(\mu, \nu(\beta),\) and \(\eta(\beta)\) are the auxiliary fields, also called in the literature the einbeins [14]. Generally speaking, the einbein fields appear in the Lagrangian and, even in absence of the corresponding velocities, they can be considered as extra degrees of freedom introduced to the system. The einbeins can be touched upon when proceeding from the Lagrangian of the system to its Hamiltonian and thus they mix with the ordinary particles coordinates and momenta. Besides, in order to preserve the number of physical degrees of freedom, constrains are to be imposed on the system and then the formalism of constrained systems quantisation [15] is operative (see, for example, Ref. [16] for the open straight–line QCD string quantisation using this formalism). A nontrivial algebra of constraints and the process of disentangling the physical degrees of freedom and non-physical ones make the problem very complicated. In the meantime, a simpler approach to einbeins exists which amounts to considering all (or some) of them as variational parameters and thus to taking extrema in the einbeins either in the Hamiltonian or in its spectrum [17]. Being an approximate approach this technique appears accurate enough (see, for example, Ref. [18]) providing a simple but powerful and intuitive method of investigation. In this Letter we follow the given technique, so that extrema in all three einbeins are understood in the Hamiltonian (15). Notice that, for \(\sigma_E = \sigma_H = \sigma\), the field \(\eta\) drops from the Hamiltonian and the standard expression for the string with quarks at the ends [12] readily comes out from Eq. (15).

Notice that the kinetic part of the Hamiltonian (15) has a very clear structure: the radial motion of the quarks happens with the effective mass \(\mu\), whereas for the orbital motion the mass is somewhat different, containing the contribution of the inertia of the string.

We take the extrema in \(\nu(\beta)\) and \(\eta(\beta)\) now, approximating \(\eta(\beta)\) by a uniform in \(\beta\) distribution. Then the Hamiltonian (15) takes the form

\[
H = \frac{1}{\xi} \left( \frac{p_r^2 + m^2}{\mu} + \mu \right) + V_{SI}(r),
\]

(16)

2The einbein \(\mu\) is introduced in the Lagrangian via the substitution \(2\sqrt{AB} \rightarrow A/\mu + B\mu\) and allows one to simplify the quark kinetic term. The continuous einbein \(\nu(\beta)\) enters through the same trick for the string term. Finally, the second continuous einbein \(\eta(\beta)\) appears due to the substitution \(A^2/B \rightarrow -B\eta^2 + 2\eta A\). As soon as extrema are taken in all einbeins, the initial form of the Lagrangian is restored.
and the spin–independent potential reads:

$$V_{SI}(r) = \eta_0 \sigma_E r + \left( \frac{1}{\eta_0} - \eta_0 \right) \sigma_H r + \frac{1}{\xi} \mu y^2, \quad \eta_0 = \frac{y}{\arcsin y},$$

(17)

with $y$ being the solution of the transcendental equation

$$\sqrt{l(l+1)} \frac{\sigma_H r^2}{\xi \mu r^2} = \frac{\xi}{4y} \left( 1 + \eta_0^2 \left( 1 - \frac{\sigma_E}{\sigma_H} \right) \right) \left( \frac{1}{\eta_0} - \sqrt{1 - y^2} \right) + \frac{\mu y}{\sigma_H r}. \quad (18)$$

The interested reader can find the details of a similar evaluation performed for the Hamiltonian \((15)\) with $\sigma_E = \sigma_H = \sigma$ in Ref. [19, 20].

The remaining einbein $\mu$ is to be considered as the variational parameter to minimise the spectrum of the Hamiltonian \((16)\). Obviously, the extremal value of $\mu$ depends on quantum numbers and acquires two contributions: one coming from the current quark mass $m$ and the other, purely dynamical, contribution coming from the mean value of the radial component of the momentum $p_r$. It is instructive to pinpoint the difference in the potential \((17)\) below and above the $T_c$.

At small $r$'s, the potential \((17)\) turns to the centrifugal barrier $l(l+1)/(\xi \mu r^2)$, whereas its large–r behaviour differs dramatically for the temperatures below and above the $T_c$. Indeed, the leading large-$r$ contribution to the inter-quark potential corresponds to $y \ll 1$ and, for $T < T_c$, reads:

$$V_{\text{conf}}(r) = \sigma_E r. \quad (19)$$

This is the linear confinement which is of a purely colour–electric nature and which admits angular–momentum–dependent corrections (see Refs. [12, 19]).

In the deconfinement phase, at $T > T_c$, the colour–electric part of the potential \((17)\) vanishes, the leading long–range term coming from the angular–momentum–dependent part of the interaction:

$$V_{SI}(r) = \frac{3l(l+1)}{\xi^2 \sigma_H r^3} + \ldots. \quad (20)$$

Interestingly, in the deconfinement phase in absence of the confining potential, the spin–independent interaction becomes short–ranged decreasing as $1/r^3$ at large inter-quark separations. This feature means the full compensation of the centrifugal barrier which would naively behave as $1/r^2$ instead. The reason is obvious: at large inter-quark separations, the effective quark mass $\mu$ is to be compared to the “mass” of the string $\sigma r$. The bound–state problem solved in the potential \((19)\) gives a large value $\langle p_r \rangle \propto \sigma_E \langle r \rangle$, so that, even for light (massless) quarks, their effective mass $\mu$ appears quite large ($\mu \gg m$). On the contrary, for light quarks and in absence of the strong confining interaction \((19)\), the values of $\mu$ are small ($\mu \approx m$) and can be neglected as compared to the string contribution $\sigma_H r$. This makes the spin–dependent terms in the effective inter-quark interaction important in this regime, as opposed to the confinement phase, where they give only small corrections to the bound states formed in the confining potential \((19)\).

In the next section we turn to the derivation of spin–dependent contributions to the inter-quark potential.

### 3 Nonperturbative spin–dependent interactions

In this section we return to the Green’s function of the quark–antiquark system \((3)\) and restore spin–dependent terms. To this end we notice that the interaction of the quark spins with the
background gluonic field is to be added at the exponent. It enters in the standard combination 
\[ \sigma_{\mu\nu} F_{\mu\nu}, \] 
with \[ \sigma_{\mu\nu} = \frac{1}{4i} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu), \]
and appears under the integral in the proper time \( \tau \). After averaging over the background field, the resulting expression for the Wilson loop can be again written in the form of Eq. (11),

\[ \frac{1}{N_C} \langle Tr W(C) \rangle = \exp \left( -\frac{1}{2} \int S d\pi_{\mu\nu}(x) \int S d\pi_{\lambda\rho}(x') \langle \langle Tr F_{\mu\nu}(x) \Phi(x, x') F_{\lambda\rho}(x') \Phi(x', x) \rangle \rangle \right), \]  
(21)

but with the differential \( d\pi_{\mu\nu} \) containing the spinor part,

\[ d\pi_{\mu\nu}(x) = ds_{\mu\nu}(x) - i\sigma_{\mu\nu} d\tau. \]  
(22)

The nonperturbative spin–dependent interaction appears from the combination of differentials involving \( \sigma_{\mu\nu} \). We skip the details of the derivation which can be found, for example, in Refs. [21, 22] and quote the result here. The leading spin–dependent term is the spin–orbit interaction (we omit contributions of \( D_{E,H} \) which bring about short–range terms \( O(r^{-3}) \)),

\[ V_{SO}(r) = \left( \frac{\vec{S}_1 \vec{L}_1 - \vec{S}_2 \vec{L}_2}{2\mu_1^2 - 2\mu_2^2} \right) \left( \frac{dV_0}{r \, dr} + \frac{2dV_1}{r \, dr} \right), \]  
(23)

where \( V_i(r) \) can be expressed through the colour–electric and colour–magnetic field correlators,

\[ \frac{1}{r} \frac{dV_0}{dr} = \frac{2}{r} \int_0^\infty d\tau \int_0^\tau d\lambda D^E(\tau, \lambda), \quad \frac{2}{r} \frac{dV_1}{dr} = -\frac{4}{r} \int_0^\infty d\tau \int_0^\tau d\lambda \left( 1 - \frac{\lambda}{r} \right) D^H(\tau, \lambda). \]  
(24)

Notice that this result [21, 22], is not due to the \( 1/m \) expansion, but is obtained with the only approximation made being the Gaussian approximation for field correlators. Accuracy of this approximation was checked both at \( T = 0 \) [23] and at \( T > T_c \) [21] to be of the order of one percent.

Only the \( V_1 \) potential survives above the \( T_c \). Besides the corresponding \( \mu \)–dependent denominator is to be corrected according to the discussion of the previous section — namely, one of \( \mu \)'s is to be augmented by the string rotation term yielding

\[ V_{SO}(r) = -\frac{2\xi \vec{S} \vec{L}}{\mu r (\xi \mu + 2\langle \nu(\beta - \xi/2)^2 \rangle)} \int_0^\infty d\tau \int_0^\tau d\lambda D^H(\tau, \lambda) \left( 1 - \frac{\lambda}{r} \right), \]  
(25)

where

\[ \langle \nu(\beta - \xi/2)^2 \rangle \equiv \int_0^1 d\beta \nu(\beta - \xi/2)^2 = \frac{\sigma_{Hr}}{2\xi^2 y^2} \left( 1 + \eta_0^2 \right) \left( \frac{1}{\eta_0} - \sqrt{1 - y^2} \right), \]  
(26)

and \( \vec{S} = \vec{S}_1 + \vec{S}_2 \), for the light–light system, and \( \vec{S} \) is the light–quark spin, for the heavy–light quarkonium.

## 4 Bound states of heavy quarks above the deconfinement temperature

In the previous sections we derived the effective nonperturbative inter–quark potential, including spin–independent terms and the spin–orbital interaction. It follows from Eq. (25) that above the deconfinement temperature, for the states with the total momentum \( J = l + S, \vec{S} \vec{L} > 0 \) and the
potential \( V_{SO}(r) \) becomes attractive, with a possibility to maintain bound states. Furthermore, its slow decrease as \( r \to \infty \) suggests that an infinite number of bound states exists, with the binding energies asymptotically approaching zero. Let us study these bound states in more detail. Hereafter \( \sigma_E = 0 \) and we use the notation \( \sigma \) for the magnetic tension \( \sigma_H \).

In view of an obvious similarity of the light–light and heavy–light cases (the difference manifesting itself only in numerical coefficients), we investigate numerically only the light–light system, as a paradigmatic example. Furthermore, for \( r \gg T_g \), the potential (25) does not depend on the form of the correlator \( D^H \) since

\[
2 \int_0^\infty d\tau \int_0^r d\lambda D^H(\tau, \lambda) \left( 1 - \frac{\lambda}{r} \right) \approx r \gg T_g \sigma. \tag{27}
\]

Finally, we neglect the perturbative part of the inter-quark interaction for it is screened to a large extend contributing to short–ranged forces only whereas the effect discussed in this work is essentially a long–ranged effect.

Therefore we study the spectrum of bound states in the potential

\[
V(r) = \left( \frac{\arcsin y}{y} - \frac{y}{\arcsin y} \right) \sigma r + \mu y^2 - \frac{\sigma l}{\mu r + 2\langle \nu(\beta - 1/2)^2 \rangle}, \tag{28}
\]

which is the sum of the spin–independent term (17) and the spin–orbital term (25); \( y \) is the solution of Eq. (18) with \( \sigma_E = 0 \). In Fig. 1 we plot the effective potential (28) for three values of the quark mass: \( m = 1\text{GeV}, 2\text{GeV}, \) and \( 3\text{GeV} \).

The resulting eigenenergy \( \varepsilon_{n,l}(\mu) \) is added then to the free part of the Hamiltonian (15),

\[
M_{n,l}(\mu) = \frac{m^2}{\mu} + \mu + \varepsilon_{n,l}(\mu), \tag{29}
\]

and this sum is minimised with respect to the einbein \( \mu \),

\[
\frac{\partial M_{n,l}(\mu)}{\partial \mu} \bigg|_{\mu=\mu_0} = 0, \quad M_{n,l} = M_{n,l}(\mu_0). \tag{30}
\]
| Parameter | $m_b$, GeV | $m_c$, GeV | $m_s$, GeV | $\sigma$, GeV$^2$ | $T_c$, fm |
|-----------|-----------|-----------|-----------|----------------|----------|
| Value     | 4.8       | 1.44      | 0.22      | 0.2            | 0.2      |

Table 1: The set of parameters used for the numerical evaluation.

| $n_r$ | $b\bar{b}$ | $c\bar{c}$ | $s\bar{s}$ |
|-------|------------|------------|------------|
| 0     | -0.007     | -0.19      | -45        |
| 1     | -5$\times$10$^{-4}$ | -0.015 | -2.7       |

Table 2: The binding energy $E_{n_r,l} \equiv M_{n_r,l} - 2m$ (in MeV) for the ground state and for the first radial excitation in the potential (28) with $l = 1$ for the $b\bar{b}$, $c\bar{c}$, and $s\bar{s}$ quarkonia.

In Table 1 we present the set of parameters used in our numerical calculations, whereas in Table 2 we give the results for the binding energy for the $b\bar{b}$, $c\bar{c}$, and $s\bar{s}$ quarkonia above the $T_c$ for $l = 1$ and $n_r = 0, 1$. We ensure therefore that for $l \neq 0$ the potential (28) does support bound states. The binding energy is small ($|E_{n_r,l}| \ll T$ for the $b$ and $c$ quarks and $|E_{n_r,l}| \lesssim T$ for $s$ quarks) so these bound states can dissociate easily.

5 Bound states of light quarks above the deconfinement temperature

In this section we turn to the problem of binding of light quarks. The effective potential (28) admits different forms at different inter-quark separations, depending on which contribution, of the quark mass term $\mu$ or of the “string mass” $2\langle \nu(\beta - 1/2) \rangle$, gives the dominating contribution, that is for $\mu \gg \sigma r$ and $\mu \ll \sigma r$. If large distances contribute most to the bound state formation (the latter case), then $V(r) = \mathcal{O}(l(l + 1)/(\sigma r^3)) + \mathcal{O}(l/(\mu r^2))$, where the first term comes from the spin–independent interaction (see Eq. (24)) and the other stems from the spin–orbit potential. The dependence of the binding energy on $\mu$ is expected then to be rather moderate, approximately as $1/\mu$.

On the contrary, in the former case with the string dynamics giving a correction to the quark mass term, the potential (28) can be approximated as

$$V(r) \approx \frac{l(l + 1)}{\mu r^2} - \frac{\sigma l}{\mu^2 r},$$

(31)

that is by the sum of the centrifugal barrier and the attractive Coulomb-like potential with the effective coupling

$$\alpha_{\text{eff}} = \frac{\sigma l}{\mu^2}.$$

(32)

The corresponding eigenenergy can be found in any textbook in Quantum Mechanics and gives a stronger dependence on $\mu$,

$$\varepsilon(\mu) \propto -\mu \alpha_{\text{eff}}^2 \propto -\frac{\sigma^2 l^2}{\mu^3}.$$

(33)

Let us consider the states with $l = 1$ and $n_r = 0$. We follow now the procedure described in detail in the previous section, solve the full problem numerically, and find the dependence...
of the eigenenergy $\varepsilon$ on the einbein $\mu$ to be

$$\varepsilon(\mu) \propto -\frac{1}{\mu^{2.79}}.$$  (34)

Comparing this to Eq. (33) we find a good agreement, with the small deviation in the power resulting from the proper string dynamics. We conclude therefore that the dynamics of the system develops at the inter-quark separations $T_g \ll r \lesssim m/\sigma$ (since, for the set of parameters given in Table 1, $m \gtrsim \sigma T_g$ then there is room for such separations).

Let us discuss now the procedure of minimisation of the spectrum (29) in $\mu$. First of all, let us notice that, in the einbein field formalism, the calculation of the spectrum naively looks like a nonrelativistic calculation due to the “nonrelativistic” form of the kinetic energy in the Hamiltonian with the einbein field $\mu$ introduced. In the meantime, the full relativistic form of the quark kinetic energy is readily restored as $\mu$ takes its extremal value and hence this is the procedure of taking extremum in $\mu$ in the masses (29) to sum up an infinite series of relativistic corrections and thus to restore the relativistic spectrum. For example, the relativistic ground state eigenenergy $E_0 = m\sqrt{1-(Z\alpha)^2}$ of the one–body Dirac equation with the Coulomb potential $-Z\alpha/r$ can be reproduced exactly with the help of the einbein technique. Finally, one can visualise the form of $\mu$ considering the effective Dirac equation for the light quark in the field of the static antiquark source. When written in the form of a second–order differential equation, it contains the spin–orbit term of the same form as given in Eq. (23) but with $\mu$ replaced by the combination $\epsilon + m + U - V$, where $U$ and $V$ are scalar and vector potentials, respectively. For light (massless) quarks this combination takes drastically different values below and above the deconfinement temperature. Indeed, in the confining phase of QCD, when spontaneous chiral symmetry breaking leads to a strong effective, dynamically generated scalar potential $U$, this effective “$\mu$” is large. On the contrary, above the $T_c$, when $U$ is small “$\mu$” is also small (it can be even negative since the eigenenergy $\epsilon$ may have any sign).

We find numerically that the extremum in $\mu$ for $M_{n,l}(\mu)$, Eq. (29), exists for $m$ exceeding the value of approximately 0.22GeV (for the given $\sigma = 0.2$GeV), and no extremum exists for smaller values of the quark mass (see Fig. 2 for the dependence of the binding energy $E_{n,l}$ on the quark mass). This property of the bound state spectrum can be easily understood using the analogy with the bound state problem for the Dirac equation with the potential in the form of a deep square well or Coulomb potential discussed above. For example, for the Coulomb potential, a

![Figure 2: The binding energy of the quark–antiquark system versus the mass of the quark for $l = 1$ and $n_r = 0$ (first plot) and $n_r = 1$ (second plot).](image-url)
problem appears as the coupling exceeds unity — the well–known problem of $Z > 137$. From Eq. (32) we easily find this critical phenomenon to happen at $m \approx \mu \lesssim \sqrt{\sigma} \approx 0.4 \text{GeV}$. This estimate is in good agreement with the result of our direct numerical calculations quoted above.

Physically this situation means that many quark–antiquark and/or gluon pairs are formed and finally stabilise the vacuum. Formally the problem is not anymore a two–body problem, but rather many–body, so that many–body techniques are to be applied. For example, in electrodynamics with $Z > 137$, one can derive the resulting self–consistent field of the Thomas–Fermi type [25]. A similar situation can be expected in the deconfinement phase of QCD. In absence of the linear potential, the einbein $\mu$ (playing the role of the effective quark mass) is not anymore bounded from below by the values of order $\sqrt{\sigma_E} \approx 0.4 \text{GeV}$ coming from the binding energy in the linearly rising potential. To see the onset of this phenomenon in the framework of our two–body (one–body for the heavy–light case) Hamiltonian, one should take into account the negative–energy part of the spectrum, when the full matrix form of the Hamiltonian is considered [26]. Indeed, the matrix structure of the Hamiltonian occurs in the path–integral formalism from the two–fold time–forward/backward motion described by the positive/negative values of $\mu$. Off–diagonal terms in the matrix Hamiltonian produce the turning points in the particle trajectory and result in $Z$–graphs.

Notice that the same is true for the glueballs and gluelumps since in this case equations are the same as for light–light and heavy–light quarkonia, respectively, but with the quark spin replaced by the gluon spin and $\sigma_H$ by $\frac{9}{4} \sigma_H$.

Concluding this section one can say that colour–magnetic (spin–dependent) interaction acting on light quark or gluonic systems enforces nonperturbative creation of light $q\bar{q}$ and $gg$ pairs.

6 Discussion

The results obtained in this Letter allow us to comment on the general situation with the existence of bound quark–antiquark states in the deconfinement phase of QCD. First of all, contrary to naive expectations, the colour–Coulomb potential is screened down to a short–ranged interaction and bound states appear due to nonperturbative colour–electric (see Refs. [8, 9]) and colour–magnetic interactions in the vacuum. Indeed, although quark–antiquark pairs in the relative S–wave cannot be bound by such interactions for $T \gtrsim 1.5 T_c$ (see, for example, Refs. [8, 9, 10]), pairs with a nonzero relative angular momentum can form bound states at all $T > T_c$ since $\sigma_H$ grows with $T$. Second, formation of such bound states above the $T_c$ is energetically favourable since it lowers the system energy as compared to the ensemble of free, unbound particles. For heavy quarks the binding is weak and the system dissociates easily. Finally, the dependence of the binding energy on the quark mass is strong — the corresponding eigenenergy for strange quarks is around $10 \div 100 \text{MeV}$ rather then below $1 \text{MeV}$ for the charmed and bottom quarks (see Table 2). The situation becomes even more dramatic for the lightest quarks. The effective inter-quark potential for light quark flavours (and gluons!) becomes extremely strong and may lead to pair creation — the effect similar to the critical phenomenon in QED for the centre charge $Z$ exceeding 137. Vacuum polarisation effects become important and they lead to a complete rebuilding of the vacuum structure of the theory. We anticipate a similar phenomenon to take place for light quarks (and gluons) in QCD in the deconfinement phase.

It is important to notice that it was a separated quark–antiquark pair which was considered in this paper. In reality such quark–antiquark pairs are to be considered in the medium formed
by other quarks and gluons, that is as a part of the SQGP. As a measure of the interaction in SQGP one can consider the ratio of the mean potential energy to the mean kinetic energy of the particles in the plasma, \( \Gamma = \langle V \rangle / \langle K \rangle \). It is easy to estimate that \( \langle K \rangle \approx T \) and \( \langle V \rangle \approx \sigma_H / T \). This gives \( \Gamma = \sigma_H / T^2 \) and so this parameter is large for quarks and it is several times larger for gluons. Therefore, SQGP is a strongly interacting medium which looks like a liquid, rather then as a gas. With the growth of the temperature the medium becomes more dense, and the mean distance between particles decreases. As this distance becomes comparable to the radius of the bound states discussed in this Letter, the latter will dissociate because of the screening effects. In other words, the hot medium plays the role of a natural cut-off for the effect of bound pair creation discussed above. Notice however that, for the quark masses around 0.2GeV, the radius of the bound state is of the order of one fm and it is expected to decrease further with the decrease of the quark mass, even if the pair creation process is properly taken into account. This means that indeed there is room for such bound states for the temperatures above the \( T_c \). Breakup, with the growth of the temperature, of such high-\( l \) states for quarks, and especially for gluons which possess more degrees of freedom than quarks, may affect such characteristics of the plasma as its free energy and it entropy (for a recent attempt of explaining the near-\( T_c \) behaviour of these characteristics see Ref. [27]). This work is in progress and will be reported elsewhere.

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