Bayesian Modeling of Intersectional Fairness: The Variance of Bias

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Abstract

Intersectionality is a framework that analyzes how interlocking systems of power and oppression affect individuals along overlapping dimensions including race, gender, sexual orientation, class, and disability. Intersectionality theory therefore implies it is important that fairness in artificial intelligence systems be protected with regard to multi-dimensional protected attributes. However, the measurement of fairness becomes statistically challenging in the multi-dimensional setting due to data sparsity, which increases rapidly in the number of dimensions, and in the values per dimension. We present a Bayesian probabilistic modeling approach for the reliable, data-efficient estimation of fairness with multi-dimensional protected attributes, which we apply to novel intersectional fairness metrics. Experimental results on census data and the COMPAS criminal justice recidivism dataset demonstrate the utility of our methodology, and show that Bayesian methods are valuable for the modeling and measurement of fairness in an intersectional context.

1 Introduction

With the rising influence of machine learning algorithms on many important aspects of our daily lives, there are growing concerns that biases inherent in data can lead the behavior of these algorithms to discriminate against certain populations [2, 3, 4, 6, 12, 24, 25]. In recent years, substantial research effort has been devoted to the development and enforcement of mathematical definitions of bias and fairness in algorithms [12, 16, 22, 19].

In this work, our guiding principle regarding fairness is intersectionality, the core theoretical framework underlying the third-wave feminist movement [10, 8]. Intersectionality theory states that racism, sexism, and other social systems which harm marginalized groups have interlocking effects, such that the lived experience of, e.g., Black women, is very different than that of, e.g., white women. We therefore focus on the fairness scenario where there are multiple protected attributes to consider, such as gender, race, sexual orientation, and disability status. While it is conceptually straightforward to extend fairness methods to multiple protected attributes, as studied by [19, 17], data sparsity rapidly becomes an issue as the number of dimensions (and their number of values) increases, leading to uncertainty in the measurement of fairness. For example, Table 1 shows how the number of instances per value at the intersections of the protected attributes, and especially the minimum of these counts, de-

| Protected attributes | gender | gender, nationality | gender, nationality, race |
|----------------------|--------|---------------------|---------------------------|
| Median # instances   | 14,719 | 5,195               | 172                       |
| Minimum # instances  | 9,216  | 963                 | 5                         |

Table 1: Number of instances at each intersection of the protected attributes’ values, UCI Adult census training dataset. Nationality was binarized as USA vs other, while gender and race had 2 and 5 values, respectively.
creases as protected attributes are introduced, on the UCI Adult census dataset [21]. It may be difficult, for instance, to estimate the overall behavior of a classifier on those individuals who are LGBT women of Native American descent, due to a lack of recorded data on such individuals. To detect discrimination, we need to compare the system’s relative behavior between these intersecting groups, which is unreliable to estimate in the resulting “small N” regime [28].

We can, however, still model and predict the behavior of the system, leveraging data from similar groups. The goal of this work, therefore, is to address the challenge of reliably modeling and measuring fairness in an intersectional context, despite data sparsity. While small data uncertainty [28], intersectionality [7], and multiple attribute definitions [19] have been studied, we are the first to consider them concurrently. Our primary contributions are:

1. We introduce a novel fairness metric, differential fairness, that is philosophically concordant with intersectionality, by adapting ideas from differential privacy to fairness. Based on this metric, we develop a second, more politically conservative fairness metric for bias amplification.

2. We then propose a probabilistic modeling approach for reliably estimating fairness, e.g. under our metrics, in the data-sparse intersectional regime. Here, we also design a hierarchical extension of Bayesian logistic regression which is potentially an appropriate choice for this setting.

3. Finally, we study the performance of our intersectional fairness models on census and criminal justice data. Our results demonstrate the importance of Bayesian estimation techniques in this context. To illustrate the real-world applicability of our methods, we perform a case study on the differential fairness and bias amplification of the COMPAS system for predicting criminal recidivism.

The remainder of the paper is structured as follows. We begin by discussing intersectionality theory, which motivates our multi-dimensional approach to fairness. We then introduce two intersectional fairness metrics, differential fairness and its extension which measures bias amplification. Next, we propose probabilistic models for estimating fairness metrics (including but not limited to ours) in the multi-dimensional fairness regime. Finally, we empirically study the behavior of the models in estimating intersectional fairness metrics, and showcase their real-world application with a case study on the COMPAS recidivism dataset.

2 Motivation: Fairness and Intersectionality

It is important to connect fairness and bias in algorithms to the broader context of fairness and bias in society, which has long been the concern of civil rights and feminist scholars and activists [25]. In particular, the technical problems addressed by this work are motivated by the framework of intersectionality from the third-wave feminist movement [10]. Intersectional feminist scholars have noted that systems of oppression built into society lead to systematic disadvantages along intersecting dimensions, which include not only gender, but also race, nationality, sexual orientation, disability status, and socioeconomic class [9, 8, 10, 18, 23, 30]. These systems are interlocking in their effect on individuals at the intersection of the affected dimensions [9, 10]. Intersectionality was defined by Kimberlé Crenshaw in the 1980’s [10] and popularized in the 1990’s, e.g. by Patricia Hill Collins [8], although the ideas are much older [9, 30]. In the context of machine learning and fairness, intersectionality was recently considered by [7], who studied the impact of the intersection of gender and skin color on computer vision performance, and by [19, 17], who aimed to protect certain subgroups in order to prevent “fairness gerrymandering.” From a humanities perspective, [25] critiqued the behavior of the Google search engine with an intersectional lens, by examining the search results for terms relating to women and people of color, e.g. “Black girls.”
Intersectionality has further implications for algorithmic fairness, beyond the use of multiple protected attributes. Many fairness definitions aim to uphold the principle of infra-marginality, which states that differences in the distributions of the “merit” or “risk” (e.g. the probability of carrying contraband at a policy stop) of individuals from protected groups should be taken into account when determining bias [29]. Intersectionality theory provides a counterpoint: these differences, while acknowledged, are frequently due to systemic structural disadvantages such as racism, sexism, intergenerational poverty, the school-to-prison pipeline, mass incarceration, and the prison-industrial complex [9, 10, 11, 18, 32]. Systems of oppression can lead individuals to perform below their potential, for instance by reducing available cognitive bandwidth [31], or by increasing the probability of incarceration [11, 1]. Distributions of merit and risk are hence influenced by unfair societal processes.

As an example, consider the task of predicting prospective students’ academic performance for use in college admissions decisions. As discussed in detail by [31], and references therein, individuals belonging to marginalized and non-majority groups are disproportionately impacted by challenges of poverty and racism (in its structural, overt, and covert forms), including chronic stress, access to healthcare, under-treatment of mental illness, micro-aggressions, stereotype threat, disidentification with academics, and belongingness uncertainty. Similarly, LGBT and especially transgender, non-binary, and gender non-conforming students suffer bullying, discrimination, self-harm, and the burden of concealing their identities. These challenges are often further magnified at the intersection of affected groups. A survey of 6,450 transgender and gender non-conforming individuals found that the most serious discrimination was experienced by people of color, especially Black respondents [15]. Verschelden explains the impact of these challenges as a tax on the “cognitive bandwidth” of non-majority students, which in turn affects their academic performance. She states:

The evidence is clear that racism (and classism, homophobia, etc.) has made people physically, mentally, and spiritually ill and damped their chance at a fair shot at higher education (and at life and living).

A classifier trained to predict students’ academic performance from historical data hence aims to emulate outcomes that were substantially affected by unfair factors. Here, we must be careful to distinguish between the statistical problem of classification, and the economic problem of the assignment of outcomes to individuals based on classification. In this context, an accurate predictor for a student’s GPA may not correspond to a fair decision-making procedure for admissions. A reliable measurement tool for intersectional fairness issues would enable the University to detect unfairness in both the classifier and in the outcomes of its students, and make appropriate interventions to each of these.

Note also that other recent papers have proposed fairness definitions with regard to multiple protected attributes, and which aim to address the targeting of certain classes of subgroups of a protected group (subset targeting, or fairness gerrymandering) [12, 19] introduced the statistical parity subgroup fairness and false positive subgroup fairness definitions, which ensure that each of a specified collection of subgroups simultaneously satisfies the analogous univariate definition, unless that subgroup is a very small fraction of the population (a technical restriction required to prove generalization properties). While these definitions could be applied in our context, intersectionality theory suggests that we should aim to protect all intersections of the protected categories, not only those for which we have a lot of data. This is the challenge that this work aims to address. Contemporaneously publishing with [19] at ICML 2018, [17] introduced multicalibration, a similar fairness definition to subgroup fairness, but...

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1Reuters, https://www.reuters.com/article/us-amazon-com-jobs-automation-insight/amazon-scrap-secret-ai-recruiting-tool-that-showed-bias-against-women-1dUSKCN1MR05G has reported that Amazon abandoned a classifier for job candidate selection which was found to be gender biased. We speculate this was likely due to similar issues to the above scenario. (Retrieved 10.24.2018.)
for calibration of probabilities. While elegant, multi-calibration is a relatively weak definition in the context of intersectionality and civil rights, since it focuses on calibration instead of fairness of outcomes.

In summary, we argue that an intersectional definition of fairness for algorithms should have the following properties:

- Multiple protected attributes should be considered.
- All of the intersecting values of the protected attributes, e.g. Black women, should be protected under the definition. At the same time, we should ensure that the individual protected attributes, e.g. women, are protected overall.
- The definition should aim to ensure that systematic differences in the behavior of the algorithm, due to structural oppression, are rectified, rather than codified.

We propose such a definition in the next section, followed by a more politically conservative variant of our definition.

### 3 Differential Fairness (DF) Metric

Suppose we would like to build or critique a system for assigning outcomes to individuals, e.g. a classifier used to make lending decisions for loan applicants. The goal is to detect and prevent discriminatory (or other) bias with respect to a set of protected attributes, such as gender, race, and disability status. Specifically, according to our intersectional fairness criteria stated above, we aim to ensure that the individuals at each intersecting value of the protected attributes, e.g. white, male, and physically disabled, will on average be treated similarly by the algorithm. We further consider that the user of the classifications (the vendor), who may not be the data owner, may be untrusted, and should not access the input data.

We address this by defining a fairness criterion which ensures that the classifications are not informative of the intersection of the protected attributes. Suppose $M(x)$ is a (possibly randomized) mechanism which takes an instance $x$ and produces an outcome $y$ for the corresponding individual, $S_1, \ldots, S_p$ are discrete-valued protected attributes, $A = S_1 \times S_2 \times \ldots \times S_p$, and $\Theta$ is a set of distributions $\theta$ which could plausibly generate each instance $x$.

(Alternatively, $\Theta$ is the set of possible beliefs that the vendor or an adversary may have about the data.) For example, the mechanism $M(x)$ could be a deep learning model for a lending decision, $A$ could be the applicant’s possible gender and race, and $\Theta$ could be the set of Gaussian distributions over credit scores per value of the protected attributes, with mean and standard deviation within a certain range. The protected attributes are included in the attribute vector $x$, although $M(x)$ is free to disregard them (e.g. if this is disallowed).

Our first proposed criterion, **differential fairness (DF)**, is a measurement of the degree of (un)fairness of the mechanism. Differential fairness measures the **fairness cost** of $M(x)$ with a parameter $\epsilon$:

**Definition 3.1.** A mechanism $M(x)$ is $\epsilon$-differentially fair (DF) in a framework $(A, \Theta)$ if for all $\theta \in \Theta$ with $x \sim \theta$, and $y \in \text{Range}(M)$,

$$e^{-\epsilon} \leq \frac{P_{M,\theta}(M(x) = y|s_i, \theta)}{P_{M,\theta}(M(x) = y|s_j, \theta)} \leq e^\epsilon,$$

for all $(s_i, s_j) \in A \times A$ where $P(s_i|\theta) > 0$, $P(s_j|\theta) > 0$.

In Definition 3.1, $s_i, s_j \in A$ are tuples of all protected attribute values, e.g. gender, race, and nationality. This is an intuitive intersectional definition of fairness: regardless of the combination of protected attributes, the probabilities of the outcomes will be similar, as measured by the ratios versus other possible values of those variables, for small values of $\epsilon$. For example, the probability of being given a loan would be similar regardless of a protected group’s intersecting combination of gender, race, and nationality, marginalizing over the remaining attributes in $x$. If the probabilities are always equal, then $\epsilon = 0$.

Differential fairness is inspired by differential privacy, which also bounds ratios of probabilities.
by \( \exp(\epsilon) \), and it similarly provides privacy and economic guarantees whose strength depends on \( \epsilon \), by preventing inference on the protected attributes given outcomes, and bounding differences in utility between groups (see the Appendix for details). From another perspective, differential fairness is an extension of the 80% rule, used in legal settings, i.e. a fair system must bound a similar ratio between a disadvantaged and an advantaged group by at least 0.8 for a particular favorable outcome [14]. Differential fairness instead protects multi-dimensional categories, with respect to multiple output values, and measures fairness on a sliding scale that can be interpreted similarly to that of differential privacy.

We can also measure fairness in data, i.e. observed outcomes assigned by a black-box algorithm or social process.

**Definition 3.2.** A labeled dataset \( \{(x_1, y_1), \ldots, (x_N, y_N)\} \) is \( \epsilon \)-differentially fair (DF) in \( A \) with respect to model \( P_{\text{Model}}(x, y) \) if mechanism \( M(x) = y \sim P_{\text{Model}}(y|x) \) is \( \epsilon \)-differentially fair in framework \( (A, \{P_{\text{Model}}(x)\}) \), for \( P_{\text{Model}} \) trained on the dataset.

### 3.1 Differential Fairness and Intersectionality

In the intersectional setting, where \( A \) contains multiple protected attributes, differential fairness implies fairness with respect to each of the protected attributes individually, or indeed, any subset of the protected attributes.

**Theorem 3.1.** Let \( M \) be an \( \epsilon \)-differentially fair mechanism in \( (A, \Theta) \), \( A = S_1 \times S_2 \times \ldots \times S_p \), and let \( D = S_1 \times \ldots \times S_k \) be the Cartesian product of a nonempty proper subset of the protected attributes included in \( A \). Then \( M \) is \( 2\epsilon \)-differentially fair in \( (D, \Theta) \).

For example, if a loan approval mechanism \( M(x) \) is \( \epsilon \)-DF in \( A = \text{Gender} \times \text{Race} \times \text{Nationality} \), it is \( 2\epsilon \)-DF in, e.g., \( \text{Gender} \) by itself, or \( \text{Gender} \times \text{Nationality} \). In other words, by ensuring fairness at the intersection of gender, race, and nationality under our criterion, we also ensure a similar degree of fairness between genders overall, and between gender/nationality pairs overall, and so on. Note that in the above, \( 2\epsilon \) is a worst case, and DF may also hold for lower values of \( \epsilon \). We provide a proof in the Appendix. This property is philosophically concordant with intersectionality, which emphasizes empathy with all overlapping marginalized groups. However, its benefits are mainly practical: while one could manually enforce Theorem 3.1, e.g. by specifying binary indicator protected attributes \( S_i \) for all possible values of all \( 2^p \) subsets of the protected attributes, cf. [19], this quickly becomes computationally and statistically infeasible.

### 4 Bias Amplification Metric

Similarly to differential privacy, non-negative differences \( \epsilon_2 - \epsilon_1 \) between two mechanisms \( M_1(x) \) and \( M_2(x) \) are meaningful (for fixed \( A \) and \( \Theta \), and for tightly computed minimum values of \( \epsilon \)), and measure the additional “fairness cost” of using one mechanism instead of the other. From an economic perspective, \( M_2(x) \) admits at most an \( \exp(\epsilon_2 - \epsilon_1) \approx 1 + \epsilon_2 - \epsilon_1 \) (for small values of \( \epsilon_2 - \epsilon_1 \)) multiplicative increase in the disparity of expected utility between pairs of protected intersections of groups with \( s_i \in A, s_j \in A \), relative to \( M_1(x) \), for any utility function (see Equation 5 in the Appendix). When \( \epsilon_1 \) is the differential fairness of a labeled dataset and \( \epsilon_2 \) is the differential fairness of a classifier measured on the same dataset, \( \epsilon_2 - \epsilon_1 \) is a measure of the extent to which the classifier increases the unfairness over the original data, a phenomenon that [33] refer to as bias amplification. Politically, \( \epsilon \)-DF is a relatively progressive notion of fairness which we have motivated based on intersectionality (disparities in societal outcomes are largely due to systems of oppression), and which is reminiscent of demographic parity [12]. On the other hand, \((\epsilon_2 - \epsilon_1)\)-DF bias amplification is a more politically conservative fairness metric which does not seek to correct unfairness in the original dataset, and reflects the principle of inframarginality (a system is biased only if disparities in its behavior are worse than those in society) [29]. Informally, \( \epsilon \)-DF and \((\epsilon_2 - \epsilon_1)\)-DF bias amplification represent “upper and lower bounds” on the unfairness of the sys-
tem in the case where the relative effect of structural oppression on outcomes is unknown.

5 Method: Modeling Intersectional Fairness

The central challenge for measuring fairness in an intersectional context, either via our definitions or related notions such as $SP$-subgroup fairness [19], is to estimate $M(x)$’s marginal behavior $P_{M,\theta}(y|s, \theta)$ for each $(y, s)$ pair, with potentially little data for each of these. Naively, directly evaluating Equation 1 requires learning a class of plausible distributions $\Theta$ over the data $x$ (which may simply be a single point estimate $\Theta = \{\theta\}$), and applying Monte Carlo integration to estimate $P_{M,\theta}(M(x) = y|s, \theta) = \int_{x \in X} P_{M}(M(x) = y|x)P_{\theta}(x|s, \theta)$ for all $y, \theta \in \Theta$. Instead, our approach is to simplify the problem and model $P_{M,\theta}(y|s, \theta)$ directly. This is a “collapsed” model where we have marginalized over instances $x$ given $s$. We now need only learn parameters $\theta$ encoding $P_{M,\theta}(y|s, \theta)$, based on a dataset $D_s = \{(s_1, y_1), \ldots, (s_N, y_N)\}$, with labels $y'$ provided by $M(x)$. We consider several options.

5.1 Empirical Fairness Estimation

The simplest method to do this is to use the empirical data distribution, in which case we refer to the $DF$ criterion as empirical differential fairness (EDF).

Assuming discrete outcomes, $P_{Data}(y|s) = \frac{N_{y,s}}{N_s}$, where $N_{y,s}$ and $N_s$ are empirical counts of their subscripted values in the dataset. EDF corresponds to verifying that for any $y, s_i, s_j$, we have

$$e^{-\epsilon} \leq \frac{N_{y,s_i}N_{y,s_j}}{N_{s_i}N_{y,s_j}} \leq e^\epsilon,$$

where $\epsilon$ is the empirical differential fairness (EDF) criterion for any $y, s_i, s_j$.

Whenever $N_{s_i} > 0$ and $N_{s_j} > 0$. However, in the intersectional setting, the counts $N_{y,s}$ at the intersection of the values of the protected attributes become rapidly smaller as the dimensionality and cardinality of protected attributes increase, and will generally have high uncertainty (or variance, from a frequentist perspective) [28]. The counts may even be zero, which can make the estimate of $\epsilon$ in Equation infinite/undefined.

5.2 Smoothed Empirical Fairness Estimation

Alternatively, we propose to generalize beyond the training set by learning $P_{M,\theta}(y|s, \theta)$ via a probabilistic model. As a simple baseline model, we can use a Dirichlet prior on the probabilities in Equation 2.

Estimating $\epsilon$-DF via the posterior predictive distribution of the resulting Dirichlet-multinomial model, the criterion for any $y, s_i, s_j$ becomes

$$\epsilon^{-\epsilon} \leq \frac{N_{y,s_i}}{N_{s_i} + |Y|\alpha} \frac{N_{y,s_j}}{N_{y,s_j} + \alpha} \leq e^\epsilon,$$

where scalar $\alpha$ is each entry of the parameter of a symmetric Dirichlet prior with concentration parameter $|Y|\alpha$, $Y = \text{Range}(M)$. We refer to this as smoothed EDF.

5.3 Probabilistic Model-Based Fairness Estimation

More generally, we propose to estimate $P_{M,\theta}(y|s, \theta)$ via a probabilistic classifier that predicts the outcome $y$ given protected attribute values $s \in A$, trained on $D_s$. The complexity of the model determines the trade-off between (statistical) bias and variance in the estimation. For instance, ordered from high statistical bias to high variance, we could consider naive Bayes, logistic regression, or deep neural networks. As a compromise between statistical bias and variance in this setting, we suggest a hierarchical extension of logistic regression, where the “prior” on statistical bias is not to be confused with unfairness.
Input: Development set $\mathcal{D} = \{(x_i, y_i)\}$, mechanism $M(x)$, protected attributes $A$

Output: $\hat{\epsilon}_{\text{data}}, \hat{\epsilon}_{M(x)}$, boxplots of posterior uncertainty in $\epsilon_{\text{data}}, \epsilon_{M(x)}, \epsilon_{M(x)} - \epsilon_{\text{data}}$

Apply $M(x)$ to $x_i \in \mathcal{D}$, obtain mechanism labels $y'_i$;
Fit Bayesian classifier $p_1(y_i|s, \theta_1)$ on $\mathcal{D}_1 = \{(s_i, y'_i)\}$;
Fit Bayesian classifier $p_2(y'|s, \theta_2)$ on $\mathcal{D}'_2 = \{(s_i, y'_i)\}$;
Estimate $\hat{\epsilon}_{\text{data}}$ via Equation 1 posterior predictive distribution $p_1(y|s)$;
Estimate $\hat{\epsilon}_{M(x)}$ via Equation 1 posterior predictive distribution $p_2(y'|s)$;
Plot post. uncertainty in $\epsilon_{\text{data}}, \epsilon_{M(x)}, \epsilon_{M(x)} - \epsilon_{\text{data}}$;

Algorithm 1: Bayesian estimation of differential fairness and its uncertainty.

$\logit(P(y = 1|s))$ is a Gaussian around the prediction of a jointly trained logistic regression, allowing deviations justified by sufficient data. Let $\bar{s}_j$ be an encoding of protected attribute values $s_j$ with a binary indicator for each attribute’s value, with integer $j$ indexing each possible value of $s$, and $\beta_j$ be a regression coefficient for each entry of the $\bar{s}_j$’s. The model’s generative process is:

- $\sigma_2 \sim \text{Exponential}(\lambda)$
- $\beta_j \sim \text{Normal}(\mu, \sigma_1)$, $c \sim \text{Normal}(\mu, \sigma_1)$
- $\gamma_j \sim \text{Normal}(\beta_j \bar{s}_j + c, \sigma_2)$
- $P(y = 1|s_j) = \sigma(\gamma_j)$

For most typical models and datasets, to manage uncertainty in the data-sparse intersectional regime we recommend that the probabilistic classifier be trained via fully Bayesian inference. Fully accounting for parameter uncertainty, a single best estimate of the conditional distributions $\hat{\theta}$ to compute $\epsilon$ is the posterior predictive distribution, $\hat{\theta} = \hat{\theta} = P_{\text{Model}}(y|s, D) = \int \theta P_{\text{Model}}(y|s, \theta)P_{\text{Model}}(\theta|D)_s$, for model parameters $\theta$. This can be approximated by, e.g., averaging $P_{\text{Model}}(y|s, \hat{\theta})$ over MCMC samples of $\hat{\theta}$ or a variational posterior. We then report uncertainty in $\epsilon$ by plotting the posterior distribution over $\epsilon$ based on posterior samples of $\hat{\theta}$ (Algorithm 1). Alternatively, we can set $\Theta$ to be a set of high probability samples of $\hat{\theta}$, in which case $\epsilon$ is calculated as the worst-case over these via Equation 1. This will generally result in a higher estimated value of $\epsilon$, i.e. a more conservative estimate of the privacy and economic guarantees of $M(x)$ (see the Appendix), but a correspondingly less conservative estimate of its “unfairness.” Bootstrap estimators of $\epsilon$, bias amplification, and their variances, can also be used as a frequentist alternative to posterior samples.

6 Experimental Results

The goals of our experiments were to compare different models for intersectional fairness, to study the effect of uncertainty/variance in intersectional fairness estimation, and to illustrate the practical application of our methods. We performed all experiments on two datasets: the Adult 1994 U.S. census income data from the UCI repository [21], and the COMPAS dataset regarding a system that is used
Table 2: Comparison of predictive performance of intersectional fairness models with respect to average negative cross-entropy per intersection on the test set (higher is better), Adult dataset. Here, PE = point estimate, FB = fully Bayesian estimate.

| Models | Actual-labeled test set (full training set) | $M(x)$-relabeled test set (held-out training subset) | Actual-labeled test set (20% of training set) | $M(x)$-relabeled test set (20% of the training subset) |
|--------|---------------------------------------------|--------------------------------------------------|---------------------------------------------|--------------------------------------------------|
|        | PE  | FB  | PE  | FB  | PE  | FB  | PE  | FB  |
| EDF    | -0.437 | -0.436 | -0.358 | -0.358 | -0.435 | -0.441 | -0.436 | -0.368 |
| NB     | -0.433 | -0.433 | -0.365 | -0.351 | -0.433 | -0.431 | -0.362 | -0.351 |
| LR     | -0.442 | -0.43 | -0.382 | -0.35 | -0.443 | -0.43 | -0.377 | -0.349 |
| DNN    | -0.436 | -0.434 | -0.366 | -0.361 | -0.441 | -0.44 | -0.366 | -0.363 |
| HLR    | X    | -0.432 | X    | -0.353 | X    | -0.433 | X    | -0.35 |

Figure 2: Differential fairness estimates using variational posterior distribution to model uncertainty, COMPAS dataset: (a) $\epsilon_1$-DF estimates on true recidivism label of data, (b) $\epsilon_2$-DF estimates on COMPAS system $M(x)$-relabeled data, and (c) $(\epsilon_2 - \epsilon_1)$-bias amplification by the COMPAS system. The red “X” on top of the box-plots indicates DF and bias amplification estimates from the posterior predictive distribution.

to predict criminal recidivism [2]. We focus on the former for benchmarking, and the latter for a case study, and we report all remaining results in the Appendix.

Prediction performance: We first studied the predictive performance for models of $P_{M,\theta}(y|s, \theta)$: EDF, naive Bayes (NB), logistic regression (LR), deep neural networks (DNN), and our hierarchical logistic regression model (HLR). For each model, we compare point estimates (PE) (MAP, except for EDF), and fully Bayesian inference via the posterior predictive distribution (FB). Results on the Adult dataset are shown in Table 2. This dataset consists of 14 attributes regarding work, relationships, and demographics for individuals, who are labeled according to whether their income exceeds $50,000 per year, pre-split into a training set of 32,561 instances and a test set of 16,281 instances. We select race, gender, and nationality as the protected attributes. As most instances have U.S. nationality, we treat nationality as binary between U.S. and “other.” Gender is also coded as binary. The race attribute originally had 5 values. We merged the Native American category with “other,” as both contained very few instances. All models were trained using PyMC3, with ADVI used for Bayesian inference. We trained all models on the training set and reported negative cross-entropy from the test set’s empirical $P(y|s)$ and $P(y'|s)$, averaged over intersections $s$. Here, $y'$ is assigned by a logistic regression model $M(x)$ trained on half of the training set (which was held out from the $P_{M,\theta}(y|s, \theta)$ models). Furthermore, to study performance with sparse data, we repeated these experiments using only 20% of the training data (6,512 instances for actual labels, and 3,256 instances for $M(x)$ labels). Generally, we found that
the probabilistic models outperformed EDF, and that fully Bayesian inference outperformed point estimates. Our Bayesian HLR method performed the best (while the PE version performed too poorly to be shown). These differences were magnified when only 20% of the data was used.

**Variance in estimation:** We studied the stability of the estimation of differential fairness versus data sparsity, by estimating $\epsilon$ from bootstrap samples, varying the number of samples (Figure 1). We generated 10 bootstrap datasets for each number of instances, and reported the average $\epsilon$-DF, $(\epsilon_2 - \epsilon_1)$-DF bias amplification, and their variances. Bayesian models were found to converge more quickly to the full-data estimates. Although Bayesian DNN (DNN-FB) had low estimated variance, its estimates of $\epsilon$ deviated substantially from the other models. Our proposed HLR-FB model showed consistently stable behavior in all experiments for both a small and large number of instances. The variance in the estimates of fairness was substantial, but averaging over bootstrap samples mitigated this. Thus, if an analyst prefers not to use Bayesian analysis, we recommend the use of bootstrap estimation of $\epsilon$ and its variance.

**COMPAS case study:** Finally, we performed a case study on the fairness of the COMPAS criminal recidivism predictor, which has been criticized as potentially biased [2]. We used race and gender as protected attributes. Gender was coded as binary. Race originally had 6 values, but we merged “Asian” and “Native American” with “other,” as all three contained very few instances. We used “actual recidivism” (within a 2-year period), which is binary, as the true label of the data generating process and the COMPAS system’s prediction as the labels from $M(x)$. We merged the “medium” and “high” labels to make COMPAS scores binary, since the actual labels are binary. We estimated $\epsilon$-DF and DF bias amplification, and their uncertainty, using samples from the variational posteriors (Figure 2). To interpret $\epsilon$, note that the 80% rule finds $\epsilon \geq -\log 0.8 = 0.2231$, calculated on the favorable outcome only, is evidence of discrimination [14]. All three models put most of their posterior density on values higher than this for true recidivism, COMPAS, and its bias amplification. The most accurate model, HLR-FB, gave the highest predicted unfairness levels.

## 7 Conclusion

We have proposed probabilistic modeling approaches to estimate fairness in the intersectional regime in a data-efficient manner, using two novel intersectional fairness metrics that are related to differential privacy. Our empirical results show the benefits of the model-based approach in this setting, especially when using Bayesian inference. We plan to develop extensions to model continuous protected attributes, and learning algorithms that are regularized via our metrics.

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Appendix

8 Privacy Interpretation of Differential Fairness

The differential fairness definition, and the resulting level of fairness obtained at any particular measured fairness parameter $\epsilon$, can be interpreted by viewing the definition through the lens of privacy. Differential fairness ensures that given the outcome, an untrusted vendor/adversary can learn very little about the protected attributes of the individual, relative to their prior beliefs, assuming their prior beliefs are in $\Theta$:

$$e^{-\epsilon} \frac{P(s_i|\theta)}{P(s_j|\theta)} \leq \frac{P(s_i|M(x) = y, \theta)}{P(s_j|M(x) = y, \theta)} \leq e^{\epsilon} \frac{P(s_i|\theta)}{P(s_j|\theta)}.$$  (4)

E.g., if a loan is given to an individual, the vendor or adversary’s Bayesian posterior beliefs about their race and gender will not be substantially changed. Thus, an adversary will not be able to make an inference such as “this individual was given a loan, so they are probably white and male.” Our definition can thus provide fairness guarantees when the user of $M(x)$ is untrusted, cf. [12]. This can prevent subsequent discrimination, e.g. in retaliation to any fairness-preserving correction that was applied to the algorithm. Also note that although differential fairness is a population-level definition, it provides a privacy guarantee for individuals. The privacy guarantee only holds if $\theta \in \Theta$, which may not always be the case. Regardless, the value of $\epsilon$ may typically be interpreted as a privacy guarantee against a “reasonable adversary.”

The level of privacy provided by $\epsilon$-differential privacy (and hence, differential fairness) can be interpreted through a corresponding economic guarantee. In our case, an $\epsilon$-differentially fair mechanism admits a disparity in expected utility of as much as a factor of $\exp(\epsilon) \approx 1 + \epsilon$ (for small values of $\epsilon$) between pairs of protected groups with $s_i \in A$, $s_j \in A$, for any utility function that could be chosen. For example, consider a loan approval process, where the utility of being given a loan is 1, and being denied is 0. Suppose the approval process is $\ln(3)$-differentially fair. The approval process
could then be three times as likely to award a loan to white men as to white women, and thus award white men three times the expected utility as white women. The proof follows the case of differential privacy [13]. Let \( u(y) : \text{Range}(M(x)) \rightarrow \mathbb{R}_{\geq 0} \) be a utility function. Then:

\[
E_{P_M}[u(y)|s_i] = \int P_M(y|s_i)u(y)dy
\]

\[
\leq \int e^\epsilon P_M(y|s_i)u(y)dy
\]

\[
= e^\epsilon E_{P_M}[u(y)|s_i].
\]  

(5)

9 Proof of Theorem 3.1

Proof. Let \( M \) be an \( \epsilon \)-differentially fair mechanism in \((A, \Theta)\), \( A = S_1 \times S_2 \times \ldots \times S_p \), and let \( D = S_q \times \ldots \times S_k \) be the Cartesian product of a nonempty proper subset of the variables included in \( A \). We want to show that \( M \) is \( 2\epsilon \)-differentially fair in \((D, \Theta)\).

Define \( E = S_1 \times \ldots \times S_{a-1} \times S_{a+1} \ldots \times S_k \times \ldots \times S_p \) as the Cartesian product of the protected attributes included in \( A \) but not in \( D \). Let \( \theta \in \Theta \). Then for any outcome \( y \) and \((s_i, s_j) \in D \times D\) where \( P(s_i|\theta) > 0 \) and \( P(s_j|\theta) > 0 \), let \( q \) be any value of \( E \) where \( P(q, s_i|\theta) > 0 \). Such a \( q \) must exist, since \( P(s_i|\theta) = \sum_{e \in E} P(E = e, s_i|\theta) > 0 \). We have:

\[
P_{M,\theta}(M(x) = y|D = s_i, \theta) = \frac{\sum_{e \in E} P_{M,\theta}(M(x) = y|e, s_i, \theta) P_{M,\theta}(E = e|s_i, \theta)}{\sum_{e \in E} P_{M,\theta}(M(x) = y|e, s_i, \theta) P_{M,\theta}(E = e|s_j, \theta)}
\]

\[
= \frac{\sum_{e \in E} P_{M,\theta}(M(x) = y|e, s_i, \theta) P_{M,\theta}(E = e|s_i, \theta)}{\sum_{e \in E} P_{M,\theta}(M(x) = y|e, s_j, \theta) P_{M,\theta}(E = e|s_j, \theta)}
\]

\[
\leq \sum_{e \in E} e^\epsilon P_{M,\theta}(E = e|s_i, \theta) = e^{2\epsilon}.
\]  

Note that in showing the last inequality, we must be careful to consider the case where \( P_{M,\theta}(E = e, s_i|\theta) = 0 \), in which case the ratio in the numerator is not bounded by \( e^\epsilon \). However, the term drops out in this case as \( P_{M,\theta}(E = e|s_i, \theta) = 0 \) follows from the assumption that \( P(s_i|\theta) > 0 \). A similar argument applies for the case where \( P_{M,\theta}(E = e, s_j|\theta) = 0 \). Reversing \( s_i \) and \( s_j \) and taking the reciprocal shows the other inequality.

10 Additional Experimental Results

Table 3 shows the average negative cross-entropy of the \( P_{M,\theta}(y|s, \theta) \) models over the intersections of protected attributes in the COMPAS dataset. We split the COMPAS dataset into train and test sets with 5,410 and 1,804 data instances, respectively. We then trained the models on the training instances and computed negative cross-entropy on the empirical \( P(y|s) \) of the test set, with both the “actual recidivism” labels, and the labels provided by the blackbox COMPAS system \( M(x) \). As in our experiments on the Adult dataset, to test performance with data sparsity we repeated the experiments with 20% of train dataset (1,804 data instances). The results were similar to those on the Adult dataset. We found that probabilistic models improve over the raw empirical probabilities (EDF), Bayesian models are slightly but consistently better than the corresponding point estimates, and that these differences are increased when only a subset of the training data is used.

Figure 3 shows a comparison between intersectional fairness models when varying the number of data instances via bootstrap samples, on the COMPAS dataset. Our conclusions are the same as for the Adult dataset. Bayesian models are found to be more stable in DF estimates for all the experiments. Once again, Bayesian DNNs (DNN-FB) gave a poor performance in estimating \( \epsilon \) with a small number of instances (relative to the \( \epsilon \) estimated with more data), despite the fact that the estimated variance was relatively low (Figure 3(c) and (d)). Like on the Adult dataset, HLR-FB exhibited consistently stable estimates of \( \epsilon \) and bias amplification, with similar estimates whether it was given a small or a large number of instances, after averaging the estimates over bootstrap samples.
We also modeled uncertainty of DF and bias amplification measurements on the Adult dataset, using samples from the variational posteriors of the Bayesian models (Figure 4). We once again found that DNN-FB models produced substantially lower estimates of $\epsilon$-DF and bias amplification, compared to Bayesian logistic regression and its hierarchical extension. The latter two models both predicted that substantial bias amplification was likely, while DNN-FB seemed to predict that the algorithm was more likely to have exhibited a slight reduction in bias, which would be a surprising result. Given that the DNN-FB model performed poorly at predicting the $M(x)$ labels, while HLR-FB performed the best (Table 3), the HLR predictions seem more plausible.

### 11 Related Work

This section discusses relationships with other concepts in fairness, privacy, and in the treatment of subsets of protected groups.

#### 11.1 Fairness Definitions

An overview of fairness research can be found in [1]. We briefly describe several of the most influential mathematical definitions of fairness below, and discuss their relationships to our proposed differential fairness criterion.

**The 80% rule:** Our criterion is related to the 80% rule, a.k.a. the four-fifths rule, a guideline for identifying unintentional discrimination in a legal setting which identifies disparate impact in cases where $P(y = 1|s_1)/P(y = 1|s_2) \leq 0.8$, for a favourable outcome $y = 1$, disadvantaged group $s_1$, and best performing group $s_2$ [14]. This corresponds to testing that $\epsilon \geq - \log 0.8 = 0.2231$, in a version of Equation 1 where only the outcome $y = 1$ is considered.

**Demographic Parity:** [12] defined (and criticized) the fairness notion of demographic parity, a.k.a. statistical parity, which requires that $P(y|s_i) = P(y|s_j)$ for any outcome $y$ and pairs of protected attribute values $s_i, s_j$ (here assumed to be a single attribute). This can be relaxed, e.g. by requiring the total variation distance between the distributions to be less than $\epsilon$. Differential fairness is closely related as it also aims to match probabilities of outcomes, but measures differences using ratios, and allows for multiple protected attributes. The criticisms of [12] are mainly related to ways in which subgroups of the protected groups can be treated differently while maintaining demographic parity, which they call “subset targeting,” and which [19] term “fairness gerrymandering.” Differential fairness explicitly protects the intersection of multiple protected attributes, which can be used to mitigate some of these abuses.

**Equalized Odds:** To address some of the limitations with demographic parity, [16] propose to instead ensure that a classifier has equal error rates for each protected group. This fairness definition, called equalized odds, can loosely be understood as a notion of “demographic parity for error rates instead of outcomes.” Unlike demographic parity, equalized odds rewards accurate classification, and penalizes systems only performing well on the majority group.
Figure 3: Differential fairness estimates of COMPAS algorithm $M(x)$ using different $P_M(y|x, \theta)$ models on COMPAS dataset with respect to number of data instances: (a) DF measurement of COMPAS algorithm $M(x)$, (b) $(\epsilon_2 - \epsilon_1)$-DF bias amplification by $M(x)$, (c) variance in DF estimates of $M(x)$ with bootstrap data samples, and (d) variance in $(\epsilon_2 - \epsilon_1)$-DF bias amplification estimation with bootstrap data samples.

However, theoretical work has shown that equalized odds is typically incompatible with correctly calibrated probability estimates [27]. It is also a relatively weak notion of fairness from a civil rights perspective compared to demographic parity, as it does not ensure that outcomes are distributed equitably. Hardt et al. also propose a variant definition called equality of opportunity, which relaxes equalized odds to only apply to a deserving outcome. An advantage of this approach is that it preserves the privacy of the individuals, which can be important when the user of the classifications (the vendor), e.g. a banking corporation, cannot be trusted to act in a fair manner. However, this is difficult to implement in practice as one must define “similar” in a fair way. The individual fairness property also does not necessarily generalize beyond training set. In this work, we take inspiration from Dwork et al.’s untrusted vendor scenario, and the use of a privacy-preserving fairness definition to address it.

**Counterfactual Fairness:** [22] propose a causal definition of fairness. Under their counterfactual fairness definition, changing protected attributes $A$, while holding things which are not causally dependent on $A$ constant, will not change the predicted distribution of outcomes. While theoretically appealing, there are difficulties in implementing this in practice. First, it requires an accurate causal model at the fine-grained individual level, while even obtaining a correct population-level causal model is generally very difficult. To implement it, we must solve a challenging causal inference problem over unobserved variables, which generally requires approximate inference algorithms. (In the case of differential fairness, we advocate the use of Bayesian models which typically require approximate inference as well, although empirical distributions can be used if sufficient data is available.) Finally, to achieve counterfactual fairness, the predictions (usually) cannot make direct use of any descendant of $A$ in the causal model. This generally precludes using any of the observed features as inputs.

**Threshold Tests:** [29] address infra-marginality by modeling risk probabilities for different subsets (i.e. attribute values) within each protected category, and requiring algorithms to threshold these probabilities at the same points when determining outcomes. In contrast, based on intersectionality theory, our proposed differential fairness criterion specifies protected categories whose intersecting subsets should be treated equally, regardless of differences in risk across the subsets. Our definition is appropriate when the differences in risk are due to structural systems of oppression, i.e. the risk probabilities themselves are impacted by an unfair process. We also provide a bias amplification version of our metric, following [33], which is more in line with the infra-
marginality perspective.

### 11.2 Privacy Definitions

**Differential Privacy:** Our work on fairness is inspired by differential privacy, the gold-standard notion of privacy for data-driven algorithms [13]. Essentially, differential privacy is a promise: if an individual contributes their data to a dataset, their resulting utility, due to algorithms applied to that dataset, will not be substantially affected. The privacy guarantee is obtained via the use of randomized algorithms, typically by adding sufficient noise, e.g. from the Laplace distribution, in order to obfuscate the impact of any one data point on the algorithms’ outputs.

**Definition 11.1.** $\mathcal{M}(x)$ is $\epsilon$-differentially private if

$$
P(M(x) \in S) \leq e^\epsilon P(M(x') \in S)
$$

for all outcomes $S$, and pairs of databases $x, x'$ differing in a single element.

Similarly to differential privacy, our proposed differential fairness definition bounds ratios of probabilities of outcomes resulting from a mechanism. However, there are several important differences. When bounding these ratios, differential fairness considers different values of a set of protected attributes, rather than databases that differ in a single element. It posits a specified set of possible distributions which may generate the data, while differential privacy implicitly assumes that the data are independent [20]. Finally, since differential fairness considers randomness in data as well as in the mechanism, it can be satisfied with a deterministic mechanism, while differential privacy can only be satisfied with a randomized mechanism.

**Pufferfish:** [20] generalized differential privacy by using a variation of Equation 1 to hide the values of an arbitrary set of secrets.

**Definition 11.2.** A mechanism $\mathcal{M}(x)$ is $\epsilon$-pufferfish private in a framework $(S, Q, \Theta)$ if for all $\theta \in \Theta$ with $x \sim \theta$, for all secret pairs $(s_i, s_j) \in Q$ and $y \in \text{Range}(M)$,

$$
e^{-\epsilon} \leq \frac{P_{\mathcal{M},\theta}(M(x) = y | s_i, \theta)}{P_{\mathcal{M},\theta}(M(x) = y | s_j, \theta)} \leq e^{\epsilon}, \quad (7)
$$

when $s_i$ and $s_j$ are such that $P(s_i | \theta) > 0, P(s_j | \theta) > 0$.

The differential privacy criterion corresponds to a special case of pufferfish where the secrets are each individual’s data, the individuals’ data points are assumed to be independent, and any datasets differing by one individual must be indistinguishable. Differential fairness adapts pufferfish to the task of defining algorithmic fairness, by selecting a set of protected attributes as the secrets, and ensuring that the values of these attributes are indistinguishable.
Thus, differential fairness provides a closely related privacy guarantee to differential privacy.

11.3 Other Related Work

**Fairness and Intersectionality:** Of particular relevance to this work, fairness in an intersectional setting has been considered by [7] in a computer vision context, and by [19] and [17], who aim to protect certain subgroups by preventing “fairness gerrymandering.”

**Fairness and Uncertainty:** Bayesian modeling of fairness has been performed by [29] in the context of stop-and-frisk policing, and by [22], who use Bayesian inference on a causal model. As an alternative to the Bayesian methodology, adversarial methods are another strategy for managing uncertainty in a fairness context, e.g. [5] apply this approach to the setting of ensuring fairness given a limited number of observations in which demographic information is available. [28] study various hypothesis testing methods for the 80% rule in the small data regime.

**Relationship between Differential Fairness and Simpson’s Paradox:** Simpson’s reversal occurs when the sign of the association between two variables reverses when conditioning on a third, for all (or most) of the third variable’s values [26]. This can be counter-intuitive, leading to it being termed a “paradox.” For example, a university could admit men at a higher rate than women overall, yet admit women at a higher rate than men for each race of applicants. Thus, the direction of discriminatory bias in admissions “paradoxically” depends on the granularity of measurement.

On the other hand, Theorem 3.1 implies, roughly speaking, that if the university is the most inequitable in its admissions with respect to, e.g., Black men, versus other groups, it cannot be inequitable to a substantially higher degree against men overall, or against women overall. Ensuring that a satisfactory degree of differential fairness is obtained in the intersectional case (where we aim to protect gender alone, or race alone), even in the situation of a Simpson’s reversal.