Impacts of government and market on firm’s efforts to reduce pollution

Bowon Kim1* and Jeong Eun Sim1

Abstract: We examine how the government and the market affect firm’s pollution abatement efforts, i.e. firm’s efforts to reduce its pollution emission. The way for the government to control firm’s pollution is to impose penalty, whereas the consumers (the market) make their purchasing decision by taking into account the pollution, i.e. the demand is affected by the stock of pollution. In effect, we consider two forces, government penalty and consumer’s sensitivity to pollution, as primary factors to control firm’s pollution and analyze their interaction in relation to the firm’s pollution reduction efforts. The analysis suggests as follows. The government penalty and the consumer’s awareness are substitutes either (1) when the market size is relatively large or (2) when the market is relatively small, but the government penalty is relatively heavy. On the contrary, the two factors are complements when the market size is relatively small and the government penalty is relatively light. We discuss managerial and economic implications of the analysis results.

Subjects: Environmental Economics; Environmental Management; Environmental Policy

Keywords: pollution reduction; government penalty; consumer awareness

1. Introduction

Environmental sustainability has become a critical issue both economically and managerially. That is, it is an important issue not only for the economy, but also for the firm, since a vast majority of...
pollution is emitted during the firm’s production process. There are stakeholders, who are concerned about pollution in the environment. For instance, to curb the economic disutility, the government might impose a penalty on the firm for its pollution emission. Pollution could also directly affect the utility of the consumer: it might reduce the consumer’s utility (Agrawal, Ferguson, Toktay, & Thomas, 2012). If the consumer dislikes and is sensitive to pollution, she would adjust her demand for the product from the firm, which emits pollution. That is, the way the consumer penalizes the firm’s pollution emission is to reduce its demand for the firm’s product. In this paper, we model the firm’s investment in reducing pollution, which is affected by two factors, the government penalty and the consumer’s sensitivity to the firm’s pollution. Based on the optimal control theory analysis, we also derive managerial implications from the numerical analysis. We structure the paper as follows. In the next section, we review the relevant literature, followed by model development and analysis. Then, we carry out numerical analysis, from which we derive managerial implications and conclusion.

2. Literature review
Researchers investigated how the government’s policy affects the firm’s pollution and environmental performance (Conrad, 1993; Lee, 1975). Milliman and Prince (1989) found that emission taxes effectively facilitate firm’s technological innovation in pollution abatement compared to other regulatory regimes such as direct controls, subsidies, or free marketable permits. In a similar vein, Jung, Krutilla, and Boyd (1996) examined diverse environmental policy instruments and found that emission taxes provide relatively high incentives for firms to invest in advanced pollution abatement technologies. Morley (2012) also empirically found that pollution decreases as environmental taxes increase. On the contrary, some suggested that the effect of government’s policy to reduce firm’s emission is rather complex (Pearce, 1991). Krass, Nedorezov, and Ovchinikov (2013) found that increasing environmental tax did not necessarily promote the firm’s choice of greener technology, arguing that increasing taxes has two effects, i.e. reducing the variable production cost of greener technology and raising total production cost of the firm.

In the literature, there is another important perspective focused on the consumer’s role in reducing pollution. Bansal and Gangopadhyay (2003) found that the subsidy to the firm adopting a cleaner process reduces total pollution, in the presence of environmentally conscious consumers. Kassinis and Soteriou (2003) found that the firm’s environmental practice increases the firm’s performance by improving customer satisfaction and loyalty, reflecting the rise of consumer awareness where consumers play an important role in heightening corporate environmental responsibility. Similarly, Liu, Anderson, and Cruz (2012) found that an increase in consumer’s environmental awareness always results in higher profits for environment-friendly supply chain players; however, it has a complex effect to supply chain players with inferior environmental performance, depending on the level of competition.

Despite the importance of government and customer’s role in inducing firm’s pollution abatement efforts, however, the interaction between these two mechanisms has rarely been investigated. Yalabik and Fairchild (2011) recognized that these two pressures from consumers and regulators, i.e. consumer’s demand sensitivity to emission and government penalty for emission, are important factors to increase firm’s investment in greener production. But they only examined the impact of the two factors on the environmental production independently, without explicitly investigating how the two factors interact in reducing firm’s emission.

In examining the dynamics of firm’s environmental efforts, differential games and optimal control theory models have been widely developed. Benchekroun and van Long (1998) found the optimal tax policy that induces the firm to produce at socially optimal output, based on a differential game model. Li (2013) investigated how the production and inventory strategy of the firm changes by the firm’s pollution abatement effort and emission permit banking, using the optimal control theory. El Ouardighi, Benchekroun, and Grass (2014) examined the optimal control problem of production and emission reductions, focusing on the absorption capacity of the environment. Boucekkine,
Krawczyk, and Vallée (2011) also studied the optimal control problem to investigate firms’ trade-offs between improving environmental quality and increasing economic performance. They found that, in equilibrium, firms often behave selfishly without taking into account other’s decisions.

### 3. Optimal control theory model and analysis outcomes

Our research context is described in Figure 1, where there are three economic entities, the government, consumer, and firm. The firm emits pollution while producing its product. In order to minimize pollution, the government imposes a penalty on the firm for its pollution emission. Since pollution reduces consumer utility, the consumer also wants to penalize the firm for its pollution by reducing her demand for the firm’s product. In developing the firm’s optimal control theory model, we define the variables and parameters as in Table 1.

![Figure 1. A general context of sustainable value chain.](image)

| Table 1. Definitions of variables and parameters |
|-----------------------------------------------|
| \( y(t) \) | Cumulative pollution at \( t \) |
| \( v(t) \) | Firm’s effort to reduce the emission of pollutants at \( t \) |
| \( f \) | Cost parameter associated with government’s penalty on the cumulative pollution |
| \( e \) | Cost parameter associated with firm’s pollution abatement effort |
| \( \theta \) | Firm’s plant capacity |
| \( l \) | Unit pollutant emission per manufacturing capacity |
| \( p(t) \) | Sales price at \( t \) |
| \( c \) | Unit production cost of the product |
| \( c_1 \) | Cost parameter associated with the deviation from the manufacturing capacity \( \hat{\theta} \) |
| \( D(t) \) | Demand at \( t; D = \alpha - \beta p - \gamma y \) |
| \( \alpha \) | Potential market size |
| \( \beta \) | Coefficient in the demand function associated with the sales price |
| \( \gamma \) | Coefficient in the demand function associated with the cumulative pollution |
| \( \delta \) | Decay rate of the cumulative pollution |
| \( r \) | Discount rate |
| \( y_{LR} \) | Long-run equilibrium of cumulative pollution |
| \( v_{LR} \) | Long-run equilibrium of firm’s pollution abatement effort |
| \( p_{LR} \) | Long-run equilibrium of the sales price |
The firm’s objective function is written as:

\[
\text{Maximize } J = \int_0^\infty e^{-rt} \left[ (p - c)D - c_1(D - \bar{U})^2 - ev^2 - fy^2 \right] dt
\]  

(1)

In Equation 1, \((p - c)D\), where \(p \geq 0\), is the total net profit of the firm, where \((p - c)\) is the unit profit, sales price minus unit production cost, and \(D = \alpha - \beta p - \gamma y\) is the demand function, i.e. the consumer’s demand for the product is a function of the sales price, \(p\) and the pollution stock, \(y\): the higher the sales price and the larger the pollution stock, the smaller the demand. In addition, cost incurs in a quadratic pattern as the total production amount deviates from the firm’s effective capacity, \(U\): the more the production amount deviates from the effective capacity, the larger the quadratic cost, i.e. \(c_1(D - \bar{U})^2\). While manufacturing the product, the firm emits pollutants harmful to the environment: \(y\) is the stock of pollution accumulated by \(t\). The government imposes a penalty on the pollution stock, i.e. \(fy^2\). This quadratic form of cost related to the pollution stock is often utilized in environmental studies (e.g. Bertinelli, Camacho, & Zou, 2014; Chung, Weaver, & Friesz, 2013; Li, 2014). In order to reduce government penalty, the firm makes an effort to cut its emission of pollutants. The effort level is denoted as \(v\) and an associated cost incurs in a quadratic way like \(ev^2\), implying the increasing marginal cost of the abatement activity as widely assumed in the literature (e.g. Chung et al., 2013; Liu et al., 2012; Ni, Li, & Tang, 2010). Finally, the firm’s profit is discounted with the rate of \(r\): note the discounting factor, \(e^{-rt}\).

The firm maximizes its objective function subject to the constraint:

\[ y = \bar{U}(l - v) - \delta y, \quad y(0) = y_0 > 0, \text{ where } 0 \leq v < l \]

If the firm doesn’t make any effort to reduce pollution, it emits pollution as much as \(\bar{U}l\) at \(t\), i.e. the amount of pollution emission is proportional to the firm’s capacity (Grant, Jones, & Bergesen, 2002; Laplante & Rilstone, 1996): one unit of capacity emits \(l\) units of pollution. If the firm’s effort level to reduce pollution is \(v\), one unit of capacity emits only \((l - v)\) units of pollution. The pollution stock \(y\) decays naturally by \(\delta y\) at \(t\). Now, we have the dynamic evolution of pollution stock as \(y = \bar{U}(l - v) - \delta y\). The resulting optimization problem of the firm writes as follows:

**Model:**

\[
\text{Maximize } J = \int_0^\infty e^{-rt} \left[ (p - c)(\alpha - \beta p - \gamma y) - c_1(\alpha - \beta p - \gamma y - \bar{U})^2 - ev^2 - fy^2 \right] dt
\]

subject to

\[ y = \bar{U}(l - v) - \delta y \]

\[ y(0) = y_0 > 0, \text{ where } 0 \leq v < l \text{ and } p \geq 0 \]

After solving the optimal control theory model, we summarize the analysis results for the long-term equilibrium in Table 2.

| Table 2. Summary of long-term equilibrium solutions |
|---------------------------------------------------|
| **Variable**                                      | **Long-run equilibrium** |
| \(y_L\)                                          | \(\frac{4\alpha\delta l(1+c_1)}{4\alpha+\delta+\frac{c_1}{2}(\alpha+\frac{l}{\delta}+\frac{\gamma}{\alpha})} \) |
| \(v_L\)                                          | \(l - \frac{c_1}{2\delta} y_L\) |
| \(p_L\)                                          | \(\frac{4(\alpha+\frac{l}{\delta}+\frac{\gamma}{\alpha})+\delta}{2\alpha+\delta} y_L\) |
4. Theorems
Since we are interested in the long-term dynamics of pollution reduction efforts at the firm level, we postulate theorems to characterize the long-term equilibrium relationship between government penalty and consumer awareness. In enhancing the firm’s activity for pollution abatement. The first theorem is concerned with the effect of government penalty (\( f \)) on the firm’s effort to mitigate pollution (\( y_{LR} \)) and pollution stock (\( v_{LR} \)): the government penalty imposed on the firm increases the firm’s long-term effort to reduce the pollution, which in turn decreases long-term pollution stock.

**Theorem 1** It holds that \( \frac{\partial y_{LR}}{\partial f} < 0 \), \( \frac{\partial y_{LR}}{\partial f} > 0 \), \( \frac{\partial v_{LR}}{\partial f} > 0 \), and \( \frac{\partial v_{LR}}{\partial f} < 0 \)

**Proof** See Appendix A.

Theorem 2 postulates the interaction between government penalty and consumer awareness, i.e. whether the two forces are complements or substitutes. The interaction relationship depends on the market size and the initial magnitude of the government penalty.

**Theorem 2** For any feasible \((r, f)\):

(i) if \( G \leq 0 \), \( \frac{\partial y_{LR}}{\partial r} \geq 0 \) and \( \frac{\partial y_{LR}}{\partial f} \leq 0 \) for all \( f \)

(ii) if \( G > 0 \), \( \frac{\partial y_{LR}}{\partial r} \geq 0 \) and \( \frac{\partial y_{LR}}{\partial f} \leq 0 \) for \( f \geq h(\gamma) \)

(iii) if \( G > 0 \), \( \frac{\partial y_{LR}}{\partial r} < 0 \) and \( \frac{\partial y_{LR}}{\partial f} > 0 \) for \( f < h(\gamma) \), where

\[
A = 3\bar{U}^4 (a - \beta c + 2\beta c_1 \bar{U}) > 0,
\]

\[
B = 16\beta \varepsilon \bar{U}^3 (1 + \beta c_1) (r + \delta) > 0,
\]

\[
C = 4\beta \bar{U}^4 (1 + \beta c_1) (a - \beta c + 2\beta c_1 \bar{U}) > 0,
\]

\[
D = 4\beta \varepsilon \delta \bar{U}^2 (1 + \beta c_1) (r + \delta) (a - \beta c + 2\beta c_1 \bar{U}) > 0,
\]

\[
G = B^2 - 4AD = 16\beta \varepsilon \bar{U}^6 (r + \delta) (1 + \beta c_1) \left[16\beta \varepsilon \bar{U}^2 (1 + \beta c_1) (r + \delta) - 3\delta (a - \beta c + 2\beta c_1 \bar{U})^2 \right]
\]

\[
G \leq 0, \text{if } \alpha \geq \bar{a} = \sqrt{\frac{16\beta \varepsilon \bar{U}^2 (1 + \beta c_1) (r + \delta)}{3\delta}} + \beta c - 2\beta c_1 \bar{U}; \text{G > 0, if } \alpha < \bar{a}
\]

\[
h(\gamma) = -\frac{A}{C} \gamma^2 + \frac{B}{C} \gamma - \frac{D}{C} = -\frac{A}{C} \left( \gamma - \frac{B}{2A} \right)^2 + \frac{B^2}{4AC} - \frac{D}{C}
\]

**Proof** See Appendix A.

We summarize the implications of Theorem 2 as in Figure 2. First, we need to consider two separate cases, one where the market is relatively large (i.e. \( \alpha \geq \bar{a} \)) and the other where the market is relatively small (i.e. \( \alpha < \bar{a} \)). The other criterion is the current level of government penalty in relation to consumer awareness, i.e. relatively heavy (if \( f \geq h(\gamma) \)) or relatively light (if \( f < h(\gamma) \)). Using the two criteria, we can recapitulate the analysis results. When the market size is relatively large, government penalty and consumer awareness are substitutes, implying that for a large market, it is better to utilize either government penalty or consumer awareness, but not both simultaneously. Such a
substitute relationship remains the same when the market is relatively small, but the current government penalty is heavy. Finally, government penalty and consumer awareness are complements when the market size is small and the government penalty is light. It implies that the two factors are complementing each other, e.g. the government penalty enhances the positive effect of consumer awareness on the firm’s effort to reduce pollution and vice versa.

To visualize the analysis outcomes, we conduct a numerical analysis with the parameter values in Table 3, based on a smartphone manufacturing industry in Korea. Table 4 shows the long-term equilibrium values of the numerical analysis.

In Figure 3, we show the two relationships between government penalty and consumer awareness. Since \( h(\gamma) \) is a concave function with a negative \( f \)-intercept \( \left( -\frac{\bar{D}}{\bar{C}} \right) \) and the two roots satisfying \( h(\gamma) = 0 \) are strictly positive when they are real, general patterns of Theorem 2 can be illustrated with Figure 3(a and b).

In Figure 3(a), the function \( h(\gamma) \) lies under the \( \gamma \)-axis. Therefore, \( G \leq 0 \), \( \frac{\partial^2 y_{LR}}{\partial f \partial \gamma} \geq 0 \) and \( \frac{\partial^2 v_{LR}}{\partial f \partial \gamma} \leq 0 \) hold for all \( f \). In Figure 3(b), however, a feasible \((\gamma, f)\) pair falls into one of two regions, i.e. \( f \geq h(\gamma) \) or \( f < h(\gamma) \): for \( f < h(\gamma) \), \( \frac{\partial^2 y_{LR}}{\partial f \partial \gamma} < 0 \) and \( \frac{\partial^2 v_{LR}}{\partial f \partial \gamma} > 0 \) hold, but for \( f \geq h(\gamma) \), \( \frac{\partial^2 y_{LR}}{\partial f \partial \gamma} \geq 0 \) and \( \frac{\partial^2 v_{LR}}{\partial f \partial \gamma} \leq 0 \).

### Table 3. Parameter values for numerical analysis

| \( r \) | \( c \) | \( c_1 \) | \( e \) | \( f \) | \( l \) | \( \delta \) | \( \alpha \) | \( \beta \) | \( \gamma \) | \( \bar{U} \) | \( y_0 \) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.004 | 20 | \( 10^{-5} \) | \( 10^9 \) | 0.01 | 0.01 | 0.1 | 200,000 | 2,000 | 0.1 | 100,000 | 0 |

### Table 4. Long-term equilibrium for base case

| Variable | Long-term equilibrium |
|---|---|
| Cumulative pollution (\( y \)) | 760.07 |
| Pollution abatement effort (\( v \)) | 0.00924 |
| Sales price (\( p \)) | 59.78 |
| Market demand per period | 80,355 |
| Firm’s profit for entire periods | \( 7.75595 \times 10^8 \) |
5. Discussion and conclusion

We discuss economic as well as managerial implications of the analysis results. In most cases, the government penalty and the consumer awareness are substitutes. As such, implementing either one of the measures is sufficient. For instance, if the consumers are already very sensitive to pollution, the government does not have to impose an extra penalty on the firm’s pollution emission. Similarly, if the government penalty on the firm’s pollution is already heavy, the consumer awareness is of little use. Another intriguing implication is that if it is difficult to educate consumers to become more sensitive to pollution, then it is better for the government to impose heavy penalty on the firm’s pollution emission rather than to spend resources on educating the consumers. On the contrary, if the consumers are very sophisticated and well educated so as to be aware of pollution problems, then the government is better not to intervene by imposing an extra penalty: such an excessive intervention will reduce the effectiveness of the already high consumer awareness.

Theorem 2 and the numerical example in Figure 3(b), however, indicate that it is possible for the two factors to be complements in a relatively small pocket of the feasible region, where the market size is relatively small and the government penalty is relatively light (in relation to the consumer awareness). That is, when the market potential is relatively small and the government penalty is relatively light (i.e. \( f < h(\gamma) \)), the government penalty enhances the effectiveness of the consumer awareness on pollution reduction and vice versa. In this situation, the two measures should be implemented together to maximize the firm’s effort to reduce pollution. Note that when the government penalty becomes larger than the threshold level (i.e. \( f \geq h(\gamma) \)), the relationship between government penalty and consumer awareness changes to substituting.

The economic implication is clear. In most cases, the policy-maker can have more latitude to choose an appropriate measure, either imposing penalty or enhancing consumer awareness in order to push the firm to make more efforts to reduce pollution. In this situation, she does not have to implement two measures simultaneously, since doing so reduces policy effectiveness. When the market is relatively small and the government penalty is not yet heavy, the situation can be delicate: the policy-maker should skillfully implement both measures simultaneously in order to maximize policy effectiveness.

In this paper, we have tried to examine the dynamic interaction between government penalty and consumer awareness in motivating the firm to make an effort to reduce its pollution emission. After developing an optimal control theory model using generalizable assumptions, we derived a few theorems and their economic implications. We put forth that the policy-maker should be able to understand conditions under which a certain measure is more effective than the other in order to maximize policy effectiveness. Since we are mainly interested in the impact of government and market forces on an individual firm’s incentive to carry out environmental activities, we have focused on pollution emission and reduction at a firm level. But, we believe there might be other external forces (other than the firm’s own efforts) that influence pollution accumulation and abatement.
studying such forces could be a promising future research. It also would be an important future study to look into a dynamic game context, where multiple firms compete for the market by dynamically adjusting their efforts to reduce pollution, which are in turn impacted by factors in addition to government and market.

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Author details
Bowon Kim1
E-mail: BowonKim@business.kaist.ac.kr
Jeong Eun Sim1
E-mail: JeongEunSim@business.kaist.ac.kr
1 Operations Strategy and Management Science, KAIST Business School, Seoul 130-722, Korea.

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Appendix A

The Hamiltonian function is given by:

\[ H = (p - c)(a - \beta p - \gamma y) - c_1(a - \beta p - \gamma y - \bar{U})^2 - ev^2 - fy^2 + \lambda (l - v) - \delta y \]

Assuming interior solutions, we obtain from the optimality conditions:

\[
p = \frac{(1 + 2\beta c_1)(a - \gamma y) - 2\beta c_1 \bar{U} + \beta c}{2\beta(1 + \beta c_1)} \tag{A1}
\]

\[
v = -\frac{\lambda}{2e} \tag{A2}
\]

The solutions that satisfy the necessary conditions are optimal. The objective function is concave in \((v, p)\). All constraints are linear in \((v, p)\).

Costate equation, using Equation A1, is:

\[
\dot{\lambda} = (r + \delta)\lambda + \left[ 2f + 2\gamma^2 c_1 - \frac{\gamma^2(1 + 2\beta c_1)^2}{2\beta(1 + \beta c_1)} \right] y
\]

\[
+ \left\{ \frac{\gamma(1 + 2\beta c_1)}{2\beta(1 + \beta c_1)}(a(1 + 2\beta c_1) - 2\beta c_1 \bar{U} + \beta c) - [\gamma c + 2\gamma c_1(a - \bar{U})] \right\} \tag{A3}
\]

From (A2) and the state equation, \(\lambda = \frac{\partial}{\partial p} (y + \delta y - \bar{U}) \tag{A4}\)

Substituting (A4) into (A3) and solving the second-order differential equation of \(y\) yield:

\[ y(t) = A_1 e^{m_1 t} + A_2 e^{m_2 t} + K_2 \tag{A5} \]

where

\[
m_1 = \frac{r + \sqrt{r^2 + 4\delta(r + \delta) + \frac{\gamma^2}{\beta(1 + \beta c_1)}}}{2} > r, \quad m_2 = \frac{r - \sqrt{r^2 + 4\delta(r + \delta) + \frac{\gamma^2}{\beta(1 + \beta c_1)}}}{2} < 0
\]

\[
K_1 = \frac{\delta(r + \delta) + \frac{\bar{U}^2}{2e} \left[ 2f + \frac{\gamma^2}{2\beta(1 + \beta c_1)} \right]}{4e\beta(1 + \beta c_1)(r + \delta) + 4fU^2 \beta(1 + \beta c_1) - \gamma^2 U^2}
\]

\[
K_2 = \frac{4e\beta(1 + \beta c_1)(r + \delta) - \gamma U^2(a - \beta c + 2\beta c_1 \bar{U})}{4\beta(1 + \beta c_1) \left[ U^2 + e\delta(r + \delta) - \gamma^2 \bar{U} \right]}
\]

Note that \(K_1 > 0\) would hold under the reasonable ranges of parameters, assuming a positive market demand.

From (A4) and (A5), \(A_1 = 0\) is obtained to guarantee that the limiting transversality condition \(\lim_{T \to \infty} \lambda(T) = 0\) holds under all parameters. Also, \(A_2 = y_0 - K_2\).

Considering \(m_2 < 0\), the long-run equilibrium solutions in Table 2 are readily determined from (A1), (A2), and (A5).
Proof of Theorem 1

\[
\frac{\partial V_{LR}}{\partial f} = -\frac{4\beta (1 + \beta c_1) \hat{U}^2 K_2}{\left\{ \frac{4\beta (1 + \beta c_1)}{f(\hat{U}^2 + \epsilon \delta (r + \delta))} \right\} - \gamma^2 \hat{U}^2}.
\]

Since \( K_1 \) is assumed to be positive and \( K_2 \) is nonnegative, \( \text{sgn} \left( \frac{\partial V_{LR}}{\partial f} \right) < 0 \)

Similarly, \( \frac{\partial^2 V_{LR}}{\partial f^2} = \frac{\partial V_{LR}}{\partial f} \cdot K_2 > 0 \)

Also, it holds that \( \frac{\partial^2 V_{LR}}{\partial f \partial \gamma} = \frac{32\beta^2 (1 + \beta c_1) \hat{U}^4}{\left\{ \frac{4\beta (1 + \beta c_1)}{f(\hat{U}^2 + \epsilon \delta (r + \delta))} \right\} - \gamma^2 \hat{U}^2} \cdot K_2 > 0 \) and \( \frac{\partial^2 V_{LR}}{\partial \gamma^2} = -\frac{\partial V_{LR}}{\partial \gamma} < 0 \)

Proof of Theorem 2

\[
\frac{\partial^2 V_{LR}}{\partial f \partial \gamma} = \frac{\partial \left( \frac{\partial V_{LR}}{\partial f} \right)}{\partial \gamma}
\]

\[
= \frac{4\beta (1 + \beta c_1) \hat{U}^2}{\left\{ \frac{4\beta (1 + \beta c_1)}{f(\hat{U}^2 + \epsilon \delta (r + \delta))} \right\} - \gamma^2 \hat{U}^2} \cdot \left[ 3\hat{U}^4 (\alpha - \beta c + 2\beta c_1 \hat{U}) \gamma^2 
- 16 \beta e \hat{U}^4 (1 + \beta c_1) (r + \delta) \gamma + 4 \beta \hat{U}^4 (1 + \beta c_1) (\alpha - \beta c + 2\beta c_1 \hat{U}) \gamma^2 
+ 4 \beta e \hat{U}^2 (r + \delta) (1 + \beta c_1) (\alpha - \beta c + 2\beta c_1 \hat{U}) \right]
\]

It is easily shown that \( \frac{\partial^2 V_{LR}}{\partial f \partial \gamma} = -\frac{\partial \left( \frac{\partial V_{LR}}{\partial f} \right)}{\partial \gamma} = \text{sgn}(g(\gamma, f)) \) hold as \( K_1 > 0 \)

Therefore,

If \( (\gamma, f) \in \{(\gamma, f) : g(\gamma, f) > 0\} \), \( \frac{\partial^2 V_{LR}}{\partial f \partial \gamma} < 0 \) and \( \frac{\partial^2 V_{LR}}{\partial \gamma^2} < 0 \)

If \( (\gamma, f) \in \{(\gamma, f) : g(\gamma, f) < 0\} \), \( \frac{\partial^2 V_{LR}}{\partial f \partial \gamma} > 0 \) and \( \frac{\partial^2 V_{LR}}{\partial \gamma^2} > 0 \)

Let \( h(\gamma) = -A \gamma^2 - B \gamma - \frac{C}{\beta} = -A \left( \gamma - \frac{B}{2A} \right)^2 + \frac{C}{4A} - \frac{B}{2A} \).

Let \( G = B^2 - 4 AD = 16 \beta e \hat{U}^2 (r + \delta) (1 + \beta c_1) \left[ 16 \beta e \hat{U}^2 (1 + \beta c_1) (r + \delta) - 3 \delta (\alpha - \beta c + 2\beta c_1 \hat{U})^2 \right] \)

\( G \) determines whether \( h(\gamma) = 0 \) has real root(s) and the sign of maximum value of \( h(\gamma) \).

Note that \( g(\gamma, f) \geq 0 \) is equivalent to \( f \geq h(\gamma) \) and \( h(\gamma) \) has a concave form.

To summarize,

If \( G \leq 0 \), \( g(\gamma, f) \geq 0 \) for all feasible \( f \).

If \( G > 0 \), \( g(\gamma, f) \geq 0 \) for \( f \geq h(\gamma) \) and \( g(\gamma, f) < 0 \) for \( f < h(\gamma) \)
Note that $\text{sgn}(G) = \text{sgn} \left[ 16 \beta_{el}^2 \left( 1 + \beta_c \right) (r + \delta) - 3 \delta \left( \alpha - \beta_c + 2 \beta_c \hat{U} \right)^2 \right]$

$$= -\text{sgn} \left[ \alpha - \left( \frac{16 \beta_{el}^2 \left( 1 + \beta_c \right) (r + \delta)}{3 \delta} + \beta_c - 2 \beta_c \hat{U} \right) \right]$$

Also, assuming $G > 0$, the real roots of $h(y) = 0$ are strictly positive as $A, B, D > 0$