AN INSTANTON-INDUCED CONTRIBUTION TO THE DECAY OF THE \( \eta_c \) INTO \( p\bar{p} \)

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ABSTRACT
We compute the decay rate for the process \( \eta_c \rightarrow p\bar{p} \) using an effective helicity-flipping proton-antiproton-gluon vertex which incorporates nonperturbative chiral symmetry breaking effects induced by instantons. We fix the strength of the vertex by requiring it to account for the screening of the proton’s axial charge observed in deep-inelastic scattering, and we estimate the size of the instanton effects by assuming them to depend linearly on the instanton density. We find that, despite a large suppression, the instanton-induced process occurs with a sizable rate comparable to the observed one, whereas the process is forbidden in perturbative QCD and not understood using standard methods.

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The current theoretical understanding of perturbative QCD has improved at the point that it is now possible to single out a few instances where the data seem to disagree with perturbative computations. Polarization effects, in particular, seem to display several cases where conventional perturbative techniques appear to be inadequate. At high energy, where perturbative methods should apply, polarized phenomena are controlled by the helicity selection rules which follow from the chiral symmetry of the QCD Lagrangian. However, as is well known [1], this symmetry is partly broken at the quantum level: flavor singlet chiral symmetry, which is an exact symmetry of the QCD Lagrangian, is broken by the axial anomaly. Besides its standard perturbative consequences (such as the perturbative mixing of the fermion singlet axial current with gluonic operators [2]), this allows nonperturbative symmetry-breaking effects, such as those required to solve the U(1) problem of QCD [1,3]. This suggests that a related symmetry-breaking mechanism may be at the origin of the observed discrepancies with perturbative expectations. Explanation of these effects may thus lead to a better quantitative understanding of chiral symmetry breaking, a nonperturbative phenomenon in QCD.

In previous work [4–6] we have suggested a specific scenario where these effects can be understood: we have shown that instanton-induced interactions [7] lead to chirality breaking processes which may be described [6] in terms of a family of effective pseudoscalar nucleon–nucleon–$n$-gluon vertices which flip the nucleon chirality in a way which depends on the momentum transfer at the nucleon vertex, but not on the energy of the process. Even though the strength of the effective coupling cannot be computed, general arguments allow to compute different processes in terms of a single coupling. In particular, assuming [4] this coupling to be responsible for the observed discrepancy [8] between the measured axial charge of the nucleon and its quark model value allows to compute the single-spin polarization in elastic proton-proton scattering at high energy and small momentum transfer, and predict that it satisfies a scaling law [6].

Here, we will apply the same physical mechanism to the decay of the $\eta_c$. The decay of this particle into proton-antiproton poses an outstanding puzzle to perturbative QCD [9–12]. This particle is the lightest $c\bar{c}$ meson; its mass ($m_{\eta_c} = 2980$ MeV) excludes decays into particles with open charm and all of its decays must proceed through gluon emission and are Zweig suppressed. The energy scale for these processes is large enough that they ought to be mediated by hard gluons, as required to explain their (observed) Zweig suppression, and in agreement with the successful description of analogous charmonium decays.
Nevertheless, a perturbative computation of the decay rate for $\eta_c \to p\bar{p}$ leads to a dramatic disagreement with the data: spin and parity considerations imply that the final $p\bar{p}$ state must be in a total spin $S = 0$ state, whereas, in the limit of exact chiral symmetry, helicity conservation forces the final $p\bar{p}$ state to have $S = 1$, hence, the process is strictly forbidden. Chiral symmetry breaking effects due to the quark masses can lead to a tiny decay rate, of the order of few eV \cite{12}. However, the decay is experimentally observed to take place with a width $\Gamma = (12.1 \pm 6.1)$ keV \cite{13}. Even introducing effective diquark constituents in the nucleon wave function does not help: although these would allow the decay without need for chiral symmetry violation (because of the presence of spin-1 constituents) the computed rates are typically too small by four orders of magnitude \cite{10}. An explanation of the large observed $\eta_c \to p\bar{p}$ decay rate seems to require more fundamental modifications of the usual perturbative QCD: for example, a nonperturbative gluonic coupling of the $\eta_c$ to the nucleon could provide such new mechanism \cite{14,15}.

The effective instanton-induced nucleon–nucleon–$n$-gluon interaction discussed in Ref. \cite{6} incorporates nonperturbative chiral symmetry breaking and could thus provide a dynamical model for this kind of process. However, the effect discussed in \cite{6} persists at arbitrarily high energies provided the limit of vanishing momentum transfer at the nucleon vertex is taken; here, instead, the momentum transfer is equal to the mass of the $\eta_c$. Furthermore, the effective action of Ref. \cite{6} has been derived in the semiclassical approximation, which is justified if the strong coupling is large, but it certainly fails in the asymptotic $\alpha_s \to 0$ limit; also, in this limit any instanton effect should be exponentially suppressed. Here we shall discuss these problems, and estimate the modifications to the quantitative consequences of the results of Ref. \cite{6} due to these effects. We will see that the $\eta_c$ mass falls within an intermediate energy range where the strong coupling $\alpha_s$ is small enough that a perturbative treatment is allowed, but (essentially because of its slow logarithmic fall-off) nonperturbative effects are still sizeable and observable in channels where conservation laws forbid the bulk of the usual perturbative process. Even though large theoretical uncertainties are involved in an estimate of instanton effects, we will be able to show that their size is significantly larger than any previously computed perturbative contribution and comparable to the experimentally observed rate.

Let us first review the derivation of the effective nucleon–nucleon–$n$-gluon Lagrangian which we shall use \cite{6}. This is based on the observation \cite{4} that instantons may contribute to the axial form factor $G_A(q^2)$ of the nucleon, defined by the decomposition

$$\langle p', \lambda' | j^A_5 | p, \lambda \rangle = G_A(q^2)\bar{u}_{\lambda'}(p')\gamma^\mu\gamma_5 u_{\lambda}(p) + G_P(q^2)q^\mu\bar{u}_{\lambda'}(p')\gamma_5 u_{\lambda}(p), \quad (1)$$
where $G_A$ and $G_P$ are respectively the axial and pseudoscalar form factors, $u_\lambda(p)$ is a nucleon spinor with momentum $p$, mass $m_N$ and helicity $\lambda$, and $q = p' - p$ for space-like processes and $p' + p$ for time-like ones. That the presence of a classical instanton background field may contribute directly to $G_A(q^2)$ can be understood by considering the time component of Eq. (1) in the forward direction, i.e.,

$$\langle p, \lambda|Q_5|p, \lambda \rangle = 2\lambda G_A(0). \quad (2)$$

Eq.(2) shows that $G_A(0)$ is the coefficient of proportionality between the nucleon’s total axial charge and its helicity: in the presence of instantons, the axial charge receives a contribution which is not conserved and corresponds to anomalous violation of chirality. An explicit mechanism through which such a contribution may arise has been presented in Ref. [5], in a simplified model (QCD with a single massless quark flavor and gauge group SU(2), and with the nucleon matrix elements in Eq.(2) replaced by a quark matrix element).

Whereas this kind of contribution can be computed exactly only in simplified models, such as that of Ref. [5], its size may be estimated by assuming that it is mainly responsible for the fact that the experimentally measured value of $G_A(0)$ is very small, $G_A(0) \sim 0$, whereas the quark model would lead to expect $G_A(0) \sim 0.6$. Such an assumption is supported by the model computation of Ref. [5]. This determines the value of the instanton contribution to $G_A(0)$. The latter, however, may be viewed as the strength of an effective instanton-nucleon-nucleon coupling. Indeed, because of the anomaly equation [1], Eq. (1) implies

$$\lim_{q \to 0} iG_A(q^2)q_\mu \bar{u}_{\lambda'}(p')\gamma^\mu\gamma_5 u_\lambda(p) =$$

$$= \langle p, \lambda'| (-2Q + \sum_{\text{flavors}} 2im_i \bar{\psi}_i \gamma_5 \psi_i) |p, \lambda \rangle, \quad (3)$$

where $Q$ equals the number density of instantons minus anti-instantons:

$$Q = -\frac{N_f}{32\pi^2}g^2tr\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}. \quad (4)$$

Hence, the instanton contribution to the first term on the r.h.s. of Eq.(3) provides (in the semiclassical approximation) an effective instanton-nucleon-nucleon coupling, which we may rewrite in the form

$$\langle p', \lambda'|Q_{\text{Inst}}|p, \lambda \rangle = -im_N G_A^{\text{Inst}}(q^2)\bar{u}_{\lambda'}(p')\gamma_5 u_\lambda(p), \quad (5)$$
where $G_A^{\text{Inst}}(q^2)$ depends only on $q^2$ (so that $G_A^{\text{Inst}}(0)$ is a universal coupling) \[6\]. Furthermore, in the semiclassical limit, the instanton can fragment into $n$ gluons (the semiclassical approximation being allowed if $n \gtrsim \frac{1}{\alpha_s}$), thereby leading to an effective nucleon–nucleon coupling with the production of $n$ gluons, described by the matrix element \[6,6\]

$$
\langle p', \lambda' | \prod_{i=1}^{n} [A_{\nu_i}(k_i)] | p, \lambda \rangle = (2\lambda)im_N G_A^{\text{Inst}}(q^2) \bar{u}_{\lambda'}(p') \gamma_5 u_\lambda(p) 
\times \prod_{i=1}^{n} \left( \frac{16\pi^2}{g_s} \right) \left( \frac{\bar{\eta}_{\mu_i,\nu_i}^{a_i} k_i^{\mu_i}}{k_i^4} \left( 1 - \frac{1}{2} K_2(\rho|k_i|) \rho^2 k_i^2 \right) \right),
$$

(6)

where $K_2$ denotes the modified Bessel function, $g_s$ is the strong coupling (i.e., $\alpha_s = \frac{g_s^2}{4\pi}$), $\rho$ is the instanton radius, which must eventually be integrated over, $a_i$, $k_i^{\mu_i}$, and $\nu_i$ are respectively the color*, four-momenta, and Lorentz indices of the $n$ gluons (over which the product runs), and $\bar{\eta}_{\mu_i,\nu_i}^{a_i}$ denotes the 't Hooft symbols \[7\] $\eta_{\mu_i,\nu_i}^{a_i}$ or $\bar{\eta}_{\mu_i,\nu_i}^{a_i}$ of the instanton or anti-instanton (which must also be summed over), respectively.

In Ref. \[6\] Eq.(6) was used to derive an effective coupling in the limit of small $k_i$ by the LSZ procedure; with $|G_A^{\text{Inst}}(q^2)| \sim |G_A^{\text{Inst}}(0)| \sim 1$ from the requirement that the instanton contributions to $G_A(0)$ cancels the quark model one in order to lead to the near-vanishing experimental value, and $\rho$ fixed as the average instanton radius. In such case the strong coupling is of order $\alpha_s \sim 1$ and the process is semiclassical with a small number of gluons. Here we would like to use the coupling Eq. (6) to describe the production of a gluon state which couples to the $\eta_c$, so that its decay may proceed through the diagram of Fig.1. To this purpose, we must make sure on the following points: first, we should establish that the process of Fig. 1 exists, in the sense that semiclassical gluons may couple perturbatively to the $c$ quark line; second, we should determine how the size of the instanton effects is affected by the the extrapolation from $q = 0$ to $q \sim m_{\eta_c}$.

The existence of the process of Fig. 1 relies on the slow fall-off of $\alpha_s$. At $m_c$ the value of the strong coupling is known with good accuracy from spectroscopy \[17\], and it is given by $\alpha_s(m_c) = 0.28$. This value is small enough that the coupling of the gluons to the charm quark may be treated perturbatively, as is borne out by ample spectroscopic evidence. The coupling of gluons emitted from the charmonium decay to the nucleon, however, will in general be nonperturbative. Because $\frac{1}{\alpha_s} \sim 3$, instanton-induced processes

* The indices $a_i$ in Eq.(6) run over an SU(2) subgroup of the color gauge group, which must be embedded into SU(3).
with few gluons are semiclassical at this scale. Hence gluons radiated from the \( \eta_c \) decay may couple in principle to the semiclassical vertex of Eq. (1). Because the \( \eta_c \) is \( C \)-even the leading contribution will be given by the two-gluon process of Fig. 1. Whether this is quantitatively significant is the question we address next.

Quite in general, one would expect that instanton effects are weighted by the Euclidean instanton action \( e^{-2\pi \alpha_s} \); thus even the moderate decrease in coupling from \( \alpha_s \sim 1 \) to \( \alpha_s \sim \frac{1}{3} \) yields a substantial suppression. In order to estimate this suppression quantitatively, recall that the instanton contribution to \( G_A(q) \) [5] is due to the fact that instantons behave in a dielectric way with respect to the axial charge, \( i.e. \), in presence of a source carrying axial charge they give rise to anomalous creation of axial charge anticorrelated to that of the source; thus, the effect will be weighted by the probability of interaction with an instanton. We will crudely estimate this probability by assuming it to be linear in the instanton density.

Now, because the effective coupling Eq. (5) is due to anomalous axial charge creation in the instanton background, it will only receive contributions from instantons of size \( \rho \lesssim \frac{1}{q} \), because the anomalous particle creation induced by the instanton occurs at a finite distance of order* \( \rho \). It follows that no particle creation is seen if an instanton of radius \( \rho \) is probed at a scale \( q > \frac{1}{\rho} \); consequently instantons with \( \rho > \rho_c \), with \( \rho_c \sim \frac{1}{q} \), do not contribute to the effective interaction Eq. (5) and its cognates Eq. (6). Accordingly, the instanton density, on which we assumed \( G_{A}^{\text{Inst}} \) to depend linearly, will decrease from its vacuum value \( n_0 \) (which includes instantons of all sizes) to its value \( n(\rho_c) \), computed including only small instantons with \( \rho < \rho_c \). Thus we estimate

\[
N(q) \equiv \frac{G_A^{\text{Inst}}(q)}{G_A^{\text{Inst}}(0)} \sim \frac{n(1/q)}{n_0}.
\]

The suppression due to the decrease in instanton density can now be estimated using Eq. (7) and the explicit expression of the differential instanton density* [7,19,20]

\[
\frac{dn(\rho)}{d\rho} = \frac{C}{\rho^5} [\alpha_s(\rho^{-1})]^{-6} \exp \left[ -\frac{2\pi}{\alpha_s(\rho^{-1})} \right]
\]

* For example, it can be proven rigorously [18] that particle creation in an instanton–anti-instanton valley background disappears if the separation of the instanton–anti-instanton pair is smaller than \( \frac{4}{3} \rho \).

* We use here the vacuum instanton density. In general, the quark-quark chirality flipping interaction [8] will carry extra powers of \( \rho \); these, however, are cancelled by corresponding powers of \( \rho \) in the quark propagators in the instanton background when computing a matrix element, such as that of Eq. (5), as it is clear on dimensional grounds.
where all the \( \rho \)-independent quantities have been lumped in the constant \( C \). The differential density Eq. (8) should then be integrated over all instanton radii. The density of instantons with radii up to \( \rho_c \) will be given integrating in the range \( 0 \leq \rho \leq \rho_c \); because in the case of interest \( \rho_c \) is in the perturbative region the integration can be performed by using the perturbative expression for \( \alpha_s \). The vacuum instanton density in the denominator of Eq. (7), instead, of course diverges for large instanton radii; this problem is cured phenomenologically [19] by assuming the growth of the instanton radius to be cut off at some scale \( \rho_0 \sim 1 \) Fm by a (poorly known) instanton repulsion mechanism. The corresponding value of the average instanton radius (obtained averaging with the measure Eq. (8) up to \( \rho_0 \)) is [20] of order of \( \langle \rho \rangle \approx \frac{1}{3} \) Fm.

In order to minimize the model dependence of the computation of \( N(q) \), Eq. (7), rather than using a phenomenological value for \( n_0 \) we compute the denominator of Eq. (7) by integrating Eq. (8) up to a cutoff value \( \rho_0 \sim 1 \) Fm. This indeed minimizes the model-dependence of the result because then the value of \( N(q) \) is controlled by the exponential dependence of \( n(\rho) \) on \( \rho \) around \( \rho \sim m_{\eta_c}^{-1} \), while it is essentially independent of pre-exponential factors. The former in turn is controlled by the value of \( \alpha_s(m_{\eta_c}) \), which is known rather accurately. On the contrary, as we will shortly see, the result is rather insensitive to both the upper limit of integration \( \rho_0 \), and the precise value of \( \alpha_s(\rho_0^{-1}) \), which is large, \( \alpha_s(\rho_0^{-1}) \gtrsim 1 \).

Due to sensitivity of the results to the perturbative running coupling, we use the next-to-leading form

\[
\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln t} \left[ 1 - \frac{\beta_1 \ln \ln t}{\beta_0^2 \ln t} \right],
\]

where \( \beta_0 = 11 - 2n_f/3, \beta_1 = 102 - 38n_f/3 \) and in the perturbative regime \( t = \ln(Q^2/\Lambda^2) \). In order to compute the suppression Eq. (9), we must introduce an infrared interpolation for the strong coupling \( \alpha_s \), so that at large \( Q^2 \) the perturbative behavior is reproduced, while at small \( Q^2 \) the strong coupling \( \alpha_s \) saturates to a constant value. To this purpose we set \( t = \delta + \ln(Q^2/\Lambda^2) \). For the perturbative QCD scale we take the value (with \( n_f = 4 \)) \( \Lambda = 263 \) [21], which yields \( \alpha_s(m_c) = 0.31 \), in good agreement with spectroscopy, whereas \( \delta \) is fixed by the value of \( \alpha_s \) in the infrared. For example, imposing \( \alpha_s(1 \text{ Fm}^{-1}) = 1 \) we get \( \delta = 3 \), while larger infrared values of \( \alpha_s \) can be obtained by reducing the value of \( \delta \) (which must anyway satisfy \( \delta > 1 \) to insure saturation).
With this form of the strong coupling*, \( \delta = 3 \), and taking the cutoff radius of the vacuum instanton density to be \( \rho_0 = 1 \text{ Fm} \), we get \( N^2(m_{\eta_c}) \simeq 3.9 \times 10^{-5} \). Varying the infrared value of \( \alpha_s \) by an order of magnitude as well as the value of \( \rho_0 \) from 1 Fm to 10 Fm this determination varies at most by a factor two, thus displaying the infrared stability of our estimate. Even though there is (as expected) a substantial suppression of the coupling Eq. (6) due to the decrease in instanton density, the strength of the coupling is still large enough to lead to a sizable decay rate, as we show next.

We proceed therefore to the computation of the diagram of Fig. 1. The coupling of gluons to the charm quark line is assumed to be given by perturbative QCD; the charm quark-antiquark pair then hadronizes with a wave function that we assume to have the static (nonrelativistic) form, [23,10]

\[
\psi_{\eta_c}(\vec{k}) = \frac{1}{4\sqrt{3}} R(0) \frac{\delta(|\vec{k}|)}{(k)^2} \delta_{ij} (c^i_+ \bar{c}^j_+ - c^i_- \bar{c}^j_-),
\]

where \( i, j \) are colour indices and \( \pm \) denote the quark helicities. The value of the radial charmonium wave function in the origin, \( R(0) \), can be fixed computing [23] the width of the electromagnetic process \( \eta_c \rightarrow \gamma\gamma \) and comparing to the experimental value [13]:

\[
\Gamma(\eta_c \rightarrow \gamma\gamma) = \frac{64}{27} \frac{\alpha^2}{m_{\eta_c}^2} |R(0)|^2 \left( 1 - 3.4 \frac{\alpha_s}{\pi} \right) = (6.6^{+2.4}_{-2.1}) \text{ keV},
\]

which yields \( R^2(0) = (0.67^{+0.24}_{-0.21}) \text{ (GeV)}^3 \).

The amplitude for the decay process is then given by

\[
A_{\lambda_p, \lambda_{\bar{p}}} = \pi R(0) \left[ M_{\lambda_p, \lambda_{\bar{p}}; + + (\vec{k} = 0)} - M_{\lambda_p, \lambda_{\bar{p}}; -- (\vec{k} = 0)} \right],
\]

where \( M \) are the elementary helicity amplitudes for the process \( c\bar{c} \rightarrow p\bar{p} \):

\[
M_{\lambda_p, \lambda_{\bar{p}}; \lambda_c, \lambda_{\bar{c}}} = \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4}
\times R^{\mu\nu}_{\lambda_c, \lambda_{\bar{c}}}(c, k) T^{\Lambda_{\lambda_p, \lambda_{\bar{p}}}(k_1, k_2, p)}_{\mu\nu}(2\pi)^4 \delta^{(4)}(k_1 + k_2 - 2c),
\]

* The computation of the integral over instanton radii has been performed by taking the appropriate number of flavors at any scale, i.e., \( n_f = 3 \) for \( \frac{1}{\rho} < m_c \), \( n_f = 4 \) for \( m_c \leq \frac{1}{\rho} < m_b \) and so forth, and suitably updating the value of \( \Lambda [22] \). The result turns out to be essentially the same as that obtained setting \( n_f = 4 \), due to the rapid falloff of the instanton density at large \( \rho \), as well as the insensitivity of the value of \( N(q) \) Eq. (7) to the precise form of the coupling in the infrared.
and we have set \( c = (m_c, \vec{0}) \), \( m_c = m_{\eta_c}/2 \) and \( k = (k_1 - k_2)/2 \), while \( R^{\mu\nu} \) and \( T^{\mu\nu} \) give respectively the perturbative part of the amplitude, and the nonperturbative coupling to the proton. Explicitly, the perturbative contribution is given by

\[
R^{\mu\nu}_{\lambda_c, \lambda_c}(c, k) = \bar{v}_{\lambda_c}(c) \gamma^\mu \frac{k + m}{k^2 + m_c^2} \gamma^\nu u_{\lambda_c}(c),
\]

where we have Wick-rotated to Euclidean space where the instanton coupling is defined. The nonperturbative coupling is constructed by taking \( n = 2 \) in Eq. (6) and calculating the color trace, which, due to the form of the wave function Eq. (10) amounts basically to projecting out the color singlet component, with the result

\[
T^{\mu\nu}_{\lambda_p, \lambda_p}(k_1, k_2, p) = \frac{(2\pi)^4}{3g_s^2} G_A(m_{\eta_c}) m_N 2 \lambda_p \bar{u}_{\lambda_p}(p) i\gamma_5 v_{\lambda_p}(p') \times \left[ k_1 \cdot k_2 g^{\mu\nu} - \frac{1}{2}(k_1^\mu k_2^\nu + k_2^\mu k_1^\nu) + (2\lambda_p) \epsilon^{\alpha\mu\beta\nu} k_1^\alpha k_2^\beta \right] \frac{\Phi(k_1)\Phi(k_2)}{k_1^4 k_2^4}. \tag{15}
\]

In Eq. (15) we have set \( p' = k_1 + k_2 - p \), and

\[
\Phi(k) = 4 \left(1 - \frac{1}{2}K_2(\rho|k|)\rho^2 k^2 \right). \tag{16}
\]

Using Eqs. (14) and (15) in Eq. (13) we get, through a somewhat tedious but straightforward computation

\[
M_{\lambda_p, \lambda_p, \lambda_c, \lambda_c} = \frac{(2\pi)^4}{3g_s^2} G_A(m_{\eta_c}) 4m_c^2 (4\lambda_c \lambda_p) \delta_{\lambda_c, \lambda_c} \delta_{\lambda_p, \lambda_p} \times \int \frac{d^4k}{(2\pi)^4} k^4(2c - k)^2 [(c - k)^2 + m_c^2] \Phi(k)\Phi(2c - k). \tag{17}
\]

The loop integration in Eq. (17) converges, as one would expect due to the effective nature of the interaction (3), because the function \( \Phi(k) \), Eq. (13), behaves as

\[
\phi(k) \underset{k \to 0}{\sim} \rho^2 k^2
\]

\[
\phi(k) \underset{k \to \infty}{\sim} 4 \left(1 - \frac{1}{2} \rho^2 k^2 \sqrt{\frac{\pi}{2\rho|k|}} e^{-\rho|k|} \right)
\]

thereby cutting off the ultraviolet tail of the integration while being finite in the infrared. Using the result Eq. (17) in the expression (12) of the decay amplitude and supplementing the required kinematical factors the width for the process is found to be

\[
\Gamma(\eta_c \to \bar{p}p) = \frac{m_N(m_c^2 - m_{\eta_c}^2)^{1/2}}{16\pi^2} \sum_{\lambda_p, \lambda_{\bar{p}}} |A_{\lambda_p, \lambda_{\bar{p}}}|^2. \tag{19}
\]
In order to determine the value of $\Gamma$, Eq. (19), we must still fix the values of the instanton radius and of the coupling constant in the nonperturbative portion of the amplitude, Eq. (15). In principle, all $\rho$-dependent quantities ought to be included in the averaging over instanton radii which also provides the decrease of $G_{\text{Inst}}^A$ according to Eq. (7); this, however, would unnecessarily (to the level of accuracy of this estimate) complicate the evaluation of the loop integral in Eq. (17). Hence, we evaluated instead the integral by fixing $\rho = \frac{1}{m_{\eta c}}$. Using the value quoted above of $N(m_{\eta c})$ we get thus finally

$$\Gamma \sim 0.8 \text{ keV}$$ (20)

We can get a more direct handle on the ingredients which enter the estimate Eq. (20) by noting that, because of the behavior Eq. (18) of the function $\Phi(k)$, the loop integration is effectively cut off in the ultraviolet at momenta of order $|k| \sim \frac{1}{\rho}$. If accordingly we neglect the contribution to the integral from momenta $|k| > \frac{1}{\rho}$, using the low-momentum form of $\Phi$, Eq. (18), the integration may be performed analytically, with the result

$$\Gamma = K \times |I|^2 \times |N|^2 \times \frac{g_4^4(m_c)}{g_4^4(\rho^{-1})},$$ (21)

where all kinematical factors have been lumped in

$$K = \frac{m_N m_{\eta c}^4 \sqrt{m_{\eta c}^2 - 4m_N^2}}{3\pi^2} |R(0)|^2,$$ (22)

while $I$ is the result of the loop integral,

$$I = \int d^4k \frac{1}{k^4(2c - k)^2 \left[(c - k)^2 + m_c^2\right]} \Phi(k)\Phi(2c - k)$$

$$\approx \rho^4 \int d^4k \frac{1}{(2c - k)^2 \left[(c - k)^2 + m_c^2\right]}$$

$$= \rho^4 \pi^2 \ln \frac{m_c^2 + \frac{1}{\rho^2}}{m_c^2}$$ (23)

and $N$ provides the overall suppression due to the instanton density, Eq.(4). Numerically, $K \simeq 0.3 \text{ (GeV)}^9$, while (with $\rho^{-1} = m_{\eta c}$) $I \simeq 0.2 \text{ (GeV)}^{-4}$ and $N^2 \simeq 3.9 \times 10^{-5}$, which indeed reproduces the result Eq. (20).

It thus appears that the decay rate induced by the chirality-flipping interaction is kinematically of order of several hundred MeV, but it is then reduced due to the large
suppression of instanton effects to the order of the keV, in good agreement with the experimental value. Clearly, this is a crude estimate of the order of magnitude of $\Gamma(\eta_c \to p\bar{p})$ induced by instantons. The main uncertainties involved in this estimate enter in the determination of the suppression factor $N(q)$ Eq. (7), mostly in the exact dependence on the cutoff instanton radius and in the precise value of the strong coupling. For example, if the value $\rho_c = \frac{3}{2}q$ were used (appropriate for the instanton–anti-instanton valley of Ref. [18]), rather than $\rho_c = \frac{1}{q}$, then the value of $N(q)$ would increase by an order of magnitude. Analogously, varying $\Lambda$ between 200 and 300 MeV the value of $N(q)$ fluctuates by about one order of magnitude. These uncertainties also affect the effective value of $\rho$ which should be used in the computation of the loop integral Eq. (17), on which the loop integral depends as $I \propto \rho^4$, according to Eq. (23). As discussed above, a smaller uncertainty comes from the infrared behavior of the instanton density. Finally, a minor uncertainty enters in the experimental determination of $R(0)$. In short, most of the uncertainty comes from the perturbative behavior close to the mass of the $\eta_c$, which is under theoretical control and may be reduced by a more accurate treatment and a better determination of $\Lambda$. The result turns out to be in remarkable qualitative agreement with experiment.

The instanton-induced coupling discussed here leads also to decay of the first radial excitation of the $\eta_c$ according to the same mechanism, i.e., to the process $\eta'_c \to p\bar{p}$. This has not been observed yet, even though data are expected to become available soon. If these processes proceed mainly through the instanton-induced mechanism discussed here, we expect the value of $\Gamma(\eta'_c \to p\bar{p})$ to be smaller than that for the $\eta_c$. Indeed, even though the larger value of the $\eta'_c$ mass ($m_{\eta'_c} \simeq 3.6$ MeV) leads to an increase of the phase space (compare Eq. (24)), this is largely compensated by the very rapid decrease of the instanton suppression factor Eq. (7) (and to a lesser extent, the loop integral Eq. (23)) as the characteristic scale of the process increases. Specifically, we get

$$ S \equiv \frac{\Gamma(\eta'_c \to p\bar{p})}{\Gamma(\eta_c \to p\bar{p})} \simeq 0.2 \times \frac{|R(0)(\eta'_c)|^2}{|R(0)(\eta_c)|^2} \quad (24) $$

The precise value of $S$ depends on the ratio of the radial wave functions in the origin. Assuming for instance this ratio to behave as that of successive $s$-wave radial excitations in a hydrogen-like potential, then $R_{20}(0)/R_{10}(0) = 1/\sqrt{8}$, leading to $S \simeq 3 \times 10^{-2}$.

In sum, we have estimated the magnitude for the width of the decay $\eta_c \to p\bar{p}$ using an instanton-induced effective Lagrangian to model the chiral symmetry breaking which is necessary in order for the decay to take place. The decay proceeds through the emission by
this effective Lagrangian of a gluon pair which is soft enough to be treated semiclassically at the proton vertex, but hard enough to couple perturbatively to a charmed quark line. The result turns out to be remarkably of the correct order of magnitude, whereas all previously known mechanism to describe exclusive processes within QCD had failed (by several orders of magnitude) to give the correct result. The main uncertainties involved in this estimate come from the determination of the strength of the instanton effects at the scale considered here, where, even though nonperturbative effects cannot be neglected, one is already well within the perturbative region. Since these effects can be brought under theoretical control, this provides a strong motivation for a systematic investigation of instanton-induced helicity effects at medium-high energy of which Ref. [6] (for forward elastic scattering) and the present work are the first steps.

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FIGURE CAPTION

The diagram which leads to the decay $\eta_c \rightarrow p\bar{p}$. The blob indicates the effective instanton-induced interaction of Eq. (3).