Charm quarks in medium and their contribution to di-electron spectra in relativistic heavy ion collisions

Hao-jie Xu, Xin Dong, Li-juan Ruan, Qun Wang, Zhang-bu Xu, and Yi-fei Zhang

1Department of Modern Physics, University of Science and Technology of China, Anhui 230026, People’s Republic of China
2Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA
3Physics Department, Brookhaven National Laboratory, Upton, New York 11973-5000, USA

We study the dynamics of charm quarks in the partonic medium and its implication to the di-electron spectra in high energy heavy ion collisions. The charm quarks traversing the thermal medium is simulated by the relativistic Langevin equation for elastic scatterings of charm quarks by thermal partons in a hydrodynamically expanding fireball. The transport coefficients of charm quarks are calculated by the in-medium T-matrix method, where a static heavy quark potential is used with parameters by fitting the lattice QCD results. The di-electron invariant mass spectra are computed in the most central collisions and are compared to the STAR preliminary data. The agreement is as good as the previous results. The angular correlations of di-electrons are almost the same in p+p and Au+Au collisions in the mass range $1 < M < 2.5$ GeV with the back-to-back feature, which means that the correlation is intact even with medium interaction. Such a feature can be used to identify the di-electron background of the open charm origin from thermal sources.

I. INTRODUCTION

The goal of heavy ion collision experiments at Relativistic Heavy Ion Collider (RHIC) [1, 2] and Large Hadron Collider (LHC) [3] is to search for and then study properties of the new state of matter, the quark-gluon plasma (QGP). The jet quenching and strong elliptic flow observed at RHIC [4, 5] and LHC [6] indicate that the hot and dense medium interacts strongly and behaves as a nearly prefect fluid [7, 8], so it is called the strongly coupled QGP or sQGP. Among all observables to pin down the sQGP, electromagnetic probes such as photons and dileptons are expected to provide clean signatures due to their weak couplings to the hot and dense matter [9, 10].

The dilepton invariant mass spectrum is usually divided into the low, intermediate and high mass regions (LMR, IMR and HMR), based on the notion that each region is dominated by different sources. In the LMR, $M \lesssim 1$ GeV, the medium modification of the $\rho$ meson spectral function is the key to describe the di-muon enhancement in the NA60 experiment at the Super Proton Synchrotron (SPS) [11–13], as well as the di-electron enhancement in the STAR experiment at RHIC [14, 15]. In the IMR, $1 \lesssim M \lesssim 3$ GeV, the thermal quark-antiquark annihilation in the QGP phase was proposed to provide a measurable signal for the de-confinement phase transition at the RHIC energy [16]. However, in this mass region, the dilepton yields from semi-leptonic decays of open charm hadrons increase rapidly with collisional energies. In Ref. [14] by some of us, a naive model was used to estimate electrons from open charm decays. It was found that electrons from open charm hadrons out-populate the thermal ones at the RHIC energy in the IMR.

The charm and bottom quarks are expected not to fully equilibrate in the hot and dense medium, so they are regarded as hard probes to the partonic medium due to their large mass scales compared to the temperature. Perturbative Quantum Chromodynamics (PQCD) calculations predicted a less energy loss for heavy quarks than for light quarks [17] due to the “dead-cone” effect [18]. But the measurements of non-photonic electrons from semi-leptonic decays of heavy flavor hadrons at RHIC [19] give strongly suppressed nuclear modification factor $R_{AA}$ and large elliptic flow $v_2$. This implies a substantial modification of heavy quark spectra when they traverse the hot and dense medium.

There are a variety of models on the market for heavy quarks in partonic medium, such as the heavy quark diffusion model using the Fokker-Planck-Langevin equation [20–22] and the model based on the Boltzmann transport simulation [24, 25], etc.. These models show that in order to obtain the same suppression of $R_{AA}$ for heavy quarks as for light quarks, much larger transport coefficients of heavy quarks than standard PQCD prediction have to be used in the diffusion process. This implies non-perturbative effects. The in-medium T-matrix method is one of non-perturbative models for heavy-light and heavy-gluon interactions [22, 26, 28] with a static heavy quark potential extracted from the lattice QCD (LQCD) with relativistic corrections. Such a static reduces the thermalization time of heavy quarks 3-4 times shorter than the PQCD prediction.

In high energy heavy ion collisions, charm quark pairs are produced back to back in their center of mass frame in the gluon fusion process, $gg \rightarrow c\bar{c}$. The angular correlation of charm quark pairs are expected to be modified by the interaction of charm quarks with the surrounding partonic medium [29, 30]. In the previous work by some of us [14], the medium modification of the angular correlation was neglected. In this paper, we will adopt a more realistic description for the dynamics of charm quarks in partonic medium and then provide a better model for the background di-electrons from open charm hadrons.

This paper is organized as follows. In Sec. I we summarize the relativistic Fokker-Planck-Langevin equation for charm quark diffusion in partonic medium. The space-time evolution is described by a $(2+1)$-dimension hydrodynamic model. In Sec. II we calculate observables related to charm hadrons, such as transverse momentum spectra of $D_0$ mesons and the nuclear modification factor of electrons from semi-leptonic decays of open charm hadrons. We calculate the invariant mass spectra of di-electrons...
including the contribution from open charm hadrons and compare to the STAR preliminary data. We finally give a summary of our results in Sec. [IV].

II. CHARM QUARKS IN PARTONIC MEDIUM

In this section we will describe our model for charm quarks traversing the partonic medium based on the Fokker-Planck-Langevin equation for heavy quark diffusion and (2+1)-dimension hydrodynamics for the fireball expansion.

A. Fokker-Planck-Langevin equation

The charm quarks traversing in a partonic medium will change their momentum distribution by interaction with thermal partons. Due to its heavy mass, the movement of a charm quark in the partonic medium can be treated as Brownian motion governed by the Langevin equation \[31, 32\],

\[
\frac{dx^i}{dt} = \frac{p^i}{E}, \quad \frac{dp^i}{dt} = -a_j^i p^j dt + c_j^i \circ dB^i(t),
\]

where \( x^i \) and \( p^i \) denote the \( i \)-th spatial components of the position and momentum vectors \( x \) and \( p \) of the charm quark, \( a_j^i \) and \( c_j^i \) are coefficients which are related to drag force and diffusion, \( dB^i(t) \) are noise variables which are specified by their correlations

\[
\langle dB^i(t) \rangle = 0, \quad \langle dB^i(t) dB^j(t') \rangle = \begin{cases} \delta_{ij} \sqrt{dt}, & t = t' \\ 0, & t \neq t', \end{cases}
\]

We can choose the standard Gaussian noise satisfying the above correlation,

\[
\mathcal{P}(dB(t) \in [y, y + dy]) = \left( \frac{1}{2\pi dt} \right)^{3/2} \exp \left( -\frac{y^2}{2dt} \right),
\]

where \( dB(t) = \sqrt{\sum (dB^i)^2} \). The symbol \( \circ \) in Eq. (2) denotes the discretization rule.

We use post-point discretization, then the corresponding Fokker-Planck equation is \(32\)

\[
\frac{\partial}{\partial t} f = \frac{\partial}{\partial p^i} \left( a_j^i p^j - c_j^i \frac{\partial}{\partial p^j} c_j^i \right) f + \frac{1}{2} \frac{\partial}{\partial p^i} \left( c_j^i c_j^i f \right)
\]

where \( f \) is the phase-space distribution of heavy quarks. If the heat bath is stationary, isotropic and homogeneous, the coefficients take the simple diagonal form \(32\)

\[
a_j^i = \Gamma(E) \delta_j^i, \quad c_j^i = \sqrt{2D(E)} \delta_j^i,
\]

where \( \Gamma(E) \) and \( D(E) \) are the drag and diffusion coefficient respectively. The generalized fluctuation-dissipation relation (the equilibrium condition) gives

\[
D(E) = E \Gamma(E),
\]

where \( E = \sqrt{p^2 + m_Q^2} \) is the charm quark energy. We will use the form \(4\) for \( a_j^i \) and \( c_j^i \) in our calculation.

The Fokker-Planck equation can be derived from the Boltzmann equation in the Landau approximation \(31\),

\[
\frac{\partial f}{\partial t} = \frac{\partial}{\partial p^i} \left( A(E)p^i f + \frac{\partial}{\partial p^i} \langle B^i(p) f \rangle \right),
\]

where the relaxation rate is given by

\[
A(E) = \frac{1}{2E_p} \int \frac{d^3 q}{(2\pi)^3 E_q} f_i(x, q) \times \int \frac{d^3 p'}{(2\pi)^3 E_{p'}} \int \frac{d^3 q'}{(2\pi)^3 E_{q'}} \left( 1 - \frac{p \cdot p'}{p^2} \right) \times \mathcal{M}(s)^2 (2\pi)^4 \delta^4(p + q - p' - q'),
\]
where $f_i(x,q)$ is the phase-space distribution for light quarks or gluons, and $q$ and $q'$ denote their 4-momenta. The heavy quark 4-momenta are denoted by $p, p'$ whose spatial components are $p, p'$ respectively. The function $B^{ij}(p)$ is a momentum integral similar to $A(E)$. Comparing with Eqs. (5,8), we have

$$\Gamma(E) = A(E) + \frac{1}{E} \frac{\partial}{\partial E} D(E),$$

where we used $\frac{\partial}{\partial p^k} = (p^k/E) \frac{\partial}{\partial E}$. In Eq. (9) $\mathcal{M}(s)$ is the scattering amplitude of a heavy quark by a light quark or a gluon. Note that $A(E)$ and $\Gamma(E)$ depend on temperature via $f_i(x,q)$ and $\mathcal{M}(s)$.

### B. Charm quark diffusion in expanding partonic medium

As we have mentioned in the introduction, the PQCD calculation of heavy quark transport coefficients cannot reproduce the data for the heavy quark $R_{AA}$ and the elliptic flow, so non-perturbative effects have to be included. To this end, we adopt the in-medium T-matrix method based on the static heavy quark potential [22, 26] to calculate $\mathcal{M}(s)$. The potential was given by a microscopic model with a Coulomb and a confinement component, where the free parameters were fixed by comparison with the color-averaged free energy of charm quarks from LQCD [27].

We choose “potential-1” in Ref. [27] which corresponds to the (2+1)-flavors LQCD result. The internal energy was used as the interaction kernel in the T-matrix equation. The 4-dimensional Bethe-Salpeter type T-matrix equation can be reduced to 1-dimensional equation by the Thompson method and partial-wave expansion. The charm quark scatterings with light quarks [27] and gluons [28] are all considered. In Fig. 1(a) we show the relaxation rate of the charm quark as functions of 3-momenta.
at different temperatures. The contributions from $u/d$ and $s$ quarks and from gluons at $T = 294$ MeV are shown in Fig. 1b). The relaxation rate given by the in-medium $T$-matrix is much larger than the PQCD result [28]. This is because the charm-light quark scattering amplitude shows a Feshbach resonance structure in the $T$-matrix calculation [27]. The resonance feature greatly enhances the cross section of heavy-light quark scattering and shortens the charm quark equilibration time.

The space-time evolution of the partonic medium is given by a (2+1)-dimensions ideal hydrodynamic model [14, 16]. The distribution of the initial energy density is determined by the Glauber model. We use the equation of state (EOS) from LQCD, namely S95P-PCE, which has a wide range of phase transition or cross-over temperatures from 184 to 220 MeV. We consider the most central collisions at the RHIC energy $\sqrt{s_{NN}} = 200$ GeV and fix the impact parameter at $b = 2.4$ fm. The initial energy density at the center of the colliding nuclei is set to $\varepsilon_0 = 55$ GeV/fm$^3$ ($b = 0$ fm) and equilibration time is set to $\tau_0 = 0.4$ fm, which are determined by fitting transverse momentum spectra of long-life hadrons such as pions, kaons and protons [14].

We choose the freezeout temperature to be the transition temperature $T_c = 184$ MeV. Actually there is no rigorous definition of the transition temperature for a crossover manifested by the lattice equation of state. By the transition temperature here we mean the hadronic/partonic phase below/above it. Note that the freezeout temperature of light hadrons is much lower than the transition temperature. We assume that charm quarks hadronize at the freezeout temperature to form charm hadrons (mostly open charm mesons) and decouple from the medium immediately. It was shown in Ref. [33] that if open charm hadrons undergo further interaction with the hadronic medium, their transverse momentum spectra are suppressed by about 20 – 25\% for $p_T = 3 – 10$ GeV at RHIC energy. We will discuss below that such an effect can be partially achieved by decreasing the freezeout temperature.

In high energy nuclear collisions, charm quark pairs are produced back to back in their rest frame in the leading order gluon fusion process: $gg \rightarrow c\bar{c}$. We use the event generator PYTHIA [35] (MSEL=4, charm production with massive matrix elements) to simulate the initial momentum distribution of charm quark pairs with angular correlation. After performing Brownian motion in the partonic medium, the charm quarks are put back to PYTHIA to undergo hadronization and resonance decays. In p+p collisions, the di-electron yield in the mass range [1.1, 2.5] GeV is dominated by semi-leptonic decays of open charm hadrons. In the PHENIX acceptance the integrated yield of di-electrons per event from heavy-flavor decays is $(4.21 \pm 0.28 \pm 1.02) \times 10^{-8}$ [34]. With the branching ratio of charm quarks to electrons [36] and the correction for the geometrical acceptance, the rapidity density of $c\bar{c}$ pairs can be estimated [34]. We use PYTHIA with the PHENIX detector acceptance to reproduce the di-electron invariant mass spectra from open charm hadrons in p+p collisions, see Fig. 2. The initial charm quark pairs in heavy ion collisions are determined by multiplying the number of binary collisions [23]. Before the equilibration time $\tau_0$, we assume charm quarks move as free streaming.

To solve the Fokker-Planck-Langevin equation in a expanding fluid, at each space-time point, we boost the charm quark momentum to the local rest frame of the fluid cell at the position of the charm quark, then let the charm quark undergo Brownian motion and change its position and momentum, and finally boost it back to the lab frame [23]. So the phase space state of the charm quark is traced in the lab frame. The time interval for the position and momentum update in the Langevin equation are kept equal to that for the temperature and fluid velocity update in hydrodynamical evolution. We set it to $d\tau = 0.01$ fm in the lab frame. The time interval in one fluid cell is given by [37]

$$\Delta \tau = \sqrt[\gamma d \tau (E_Q - p_1 u_1 - p_2 u_2)} / E_Q$$  (11)
Figure 3: (a) The $p_T$ spectra of charm quarks in the initial state without medium modification and those after Hydro-Langevin evolution in the final state. The nuclear modification factor is shown in the inset. (b) The angular correlation of charm quark pairs in the initial and final states. The different $p_T$ cutoffs are chosen. The freezeout temperature is set to $T_c = 184$ MeV.

where $E_Q, p_1, p_2$ are the energy and the spatial components of a charm quark momentum, $u_1$ and $u_2$ are the spatial components of the fluid velocity at the charm quark position, and $\gamma = 1/\sqrt{1 - u_1^2 - u_2^2}$ is the Lorentz contraction factor. All above quantities are defined in the lab frame. We then put charm quarks to perform the Hydro-Langevin simulation step by step. In each step, the momentum diffusion of the charm quark is controlled by the drag and diffusion coefficients as functions of the charm quark’s 3-momentum and the temperature of the fluid cell at the position of the charm quark.

As long as the local temperature of the fluid cell at the position of the charm quark is below $T_c$, we stop the Langevin evolution of the charm quark and record its final position and momentum. After all charm quarks complete their evolution, we obtain the $p_T$ spectra as well as the angular correlation of charm quark pairs. In Fig. 3 we show the final $p_T$ distribution of charm quarks and the angular correlation of charm quark pairs in Hydro-Langevin simulation compared with that of initial charm quarks given by PYTHIA. The nuclear modification factor $R_{AA}$, which measures the charm quark energy loss in medium, is shown in the inset of Fig. 3(a).

It is shown in Fig. 3(b) that the medium modification of the angular correlation of charm quark pairs is small in both low and high $p_T$ range. At high $p_T$, the angular correlation is almost kept same after charm quark pairs pass through the medium, this is because the relaxation rate at high $p_T$ is small, see Fig. 1, so the angular deflection of the charm quark is small in each scattering by thermal partons. At low $p_T$, though a large angular deflection may occur in each scattering which would reduce the angular correlation, since the original angular correlation is already very weak, such a reduction can hardly be observed. The reason for such results is that a large default value $\langle k_T \rangle = 2$ GeV for the Gaussian $k_T$ width of promordial partons in colliding protons is used in PYTHIA 6.4 which dismisses the angular correlation of low $p_T$ charm quark pairs. This is different from the result of Ref. [30] which used momentum-independent transport coefficients and a lower $\langle k_T \rangle$ value. It was shown that if the thermal medium has large collective flow, such as the partonic medium created at the LHC energy, the near side instead of the
back-to-back correlation of charm quark pairs would appear. At such a high collisional energy, the next-to-leading order PQCD processes for charm quark pairs become more important, and regeneration of charm quarks in the partonic medium should also be taken into account [24], which will further modify the angular correlation of charm quark pairs at low $p_T$.

To look at the dependence of the angular correlation on the value of $\langle k_T \rangle$, we set $\langle k_T \rangle = 1 \text{ GeV}/c$, i.e. the default value of PYTHIA 6.3. This corresponds to a softer $p_T$ distribution of charm quarks in the initial state. Then the change of the angular correlation at low $p_T$ is more obvious, but the correlation at high $p_T$ is insensitive to the value of $\langle k_T \rangle$. The nuclear modification factors for two values of $\langle k_T \rangle$ in PYTHIA are shown in Fig. 4(b).

The freezeout temperature $T_c$ influences the lifetime of the partonic medium and then the degree of equilibrium of charm quarks. To look at the $T_c$ dependence, we tune it to a lower value, $T_c = 159 \text{ MeV}$, which corresponds to energy density $\epsilon_c = 0.445 \text{ GeV/fm}^3$. The lower freezeout temperature gives a longer evolution time of charm quarks in the partonic medium and more interaction with thermal partons, which give additional contribution to their energy loss, as shown in Fig. 5. The diffusion effect of open charm hadrons in hadronic medium can partially be accounted by lower freezeout temperature, in this sense our result is similar to a Langevin simulation of charm hadrons in hadronic medium [33].

### III. SEMI-LEPTONIC DECAYS OF OPEN CHARM HADRONS AND DI-ELECTRON BACKGROUND

After Hydro-Langevin evolution in the partonic medium, charm quarks are put back to the same event in PYTHIA to undergo hadronization and resonance decays. In Fig. 6(a), we show the $p_T$ spectra and the nuclear modification factor of $D_0$ mesons. The result of $R_{AA}$ for single electrons from semi-leptonic decays of charm hadrons is shown in Fig. 6(b), in comparison with the PHENIX data in central Au+Au collisions [38]. Our result agrees with the data at intermediate $p_T$.  

![Figure 4](image-url)  

Figure 4: (a) Same as Fig.3(b) but with $\langle k_T \rangle = 1 \text{ GeV}/c$. (b) The nuclear modification factors with Hydro-Langevin evolution for charm quarks in partonic medium for two values of $\langle k_T \rangle$ in PYTHIA.
Figure 5: Comparison the energy loss of charm quarks with different decoupling temperature. The medium effect of charm quarks is given by the Hydro-Langevin evolution.

Figure 6: (a) The $p_T$ spectra and the nuclear modification factor of $D_0$ mesons. (b) The nuclear modification factor of electrons from semi-leptonic decays of charm hadrons. The data are taken from PHENIX [38].
Now we try to reproduce the STAR data for invariant mass spectra of di-electrons in the LMR and IMR. We implement the STAR detector acceptance (transverse momentum $p_T > 0.2$ GeV and pseudo-rapidity $|\eta_e| < 1$ for an individual electron, rapidity $|y_{ee}| < 1$ for a pair of electrons) and $p_T$ resolution as in the previous work by some of us [14]. The di-electrons from open charm hadrons are simulated by the Hydro-Langevin evolution. We compare in Fig. 7(a) our result in most central collisions with the STAR preliminary data of 0–10% centrality [39] and with the cocktail (no medium modification). The contribution from open charm hadrons is slightly less suppressed than the previous result in Ref. [14].

In the dilepton invariant mass range $1.1 < M < 2.5$ GeV, we look at the $p_T$ spectra and angular correlation of di-electrons...
with the STAR detector acceptance in Fig. 7(b,c). The black-dashed line includes the open charm contribution without medium modification whose spectra are the same as in p+p collisions up to normalization of the binary collision number. The blue-solid line includes the open charm contribution with medium modification from the Hydro-Langevin evolution. One can see the suppression of the \( p_T \) spectra from the medium effect, however the angular correlation is almost intact. As a comparison, we also show the angular correlation from QGP in Fig. 7(c), which is very different from the Langevin result dominated by open charm contribution. So the away side correlation of dilepton is the feature of the charm background which we can identify when we extract thermal sources of dileptons from the QGP phase. On the other hand, our current results indicate that the previous naive model [14] for the medium modification of charm quarks, albeit simple, works well in the IMR.

For the charm quark hadronization, in this paper, we use the Lund string fragmentation model encoded in PYTHIA in both p+p and Au+Au collisions. Actually the hadronization mechanism in p+p and Au+Au collisions can be different in some \( p_T \) range. It is known that in the low \( p_T \) range, the coalescence rather than fragmentation is more important [40–42]. Here we focus on the IMR dileptons, most of which are from open charm hadron decays with intermediate and high transverse momenta. This can be seen by the obvious back-to-back angular correlations of di-electrons in Fig. 7(c). Given the uncertainties of coalescence probability in the IMR, we expect that the modification from the coalescence/recombination would be small. We will study the charm quark hadronization in a separate paper in details.

### IV. SUMMARY AND DISCUSSIONS

We investigate the medium modification of charm quarks when they traverse the partonic medium using the relativistic Fokker-Planck-Langevin equation for elastic scatterings of charm quarks by thermal partons in a hydrodynamically expanding fireball. The transport coefficients of charm quarks are calculated by the in-medium T-matrix approximation with the static heavy quark potential, where the free parameters are fitted by the color-average free energy of charm quarks from LQCD. The space-time history of the fireball is provided by the (2+1)-dimensions ideal hydrodynamic model. We find that the suppression of charm quark transverse momentum spectra depends on the transport coefficients in the Langevin equation, the life-time of the partonic medium, and the input transverse momentum spectra of charm quarks in the initial state. The medium modification of the angular correlation of charm quark pairs turns out to be small in both the low and high \( p_T \) range, but it becomes obvious in the low \( p_T \) range when a smaller \( \langle k_T \rangle \) of primordial partons in colliding protons is used in PYTHIA.

With hadronization of charm quarks, we calculate the \( p_T \) spectra and nuclear modification factor of \( D_0 \) mesons as well as the nuclear modification factor of single electrons from charm hadron decays. Our results are in good agreement with the PHENIX data in the intermediate \( p_T \) range. With the STAR detector acceptance, we compute the di-electron invariant mass spectra in most central collisions and compare them with the STAR preliminary data of 0–10\% centrality. Our current result is consistent to the previous one by some of us. We find that the angular correlations of di-electrons are almost identical in p+p and Au+Au collisions in the mass range \( 1.1 < M < 2.5 \) GeV. This implies that such a feature of the back-to-back angular correlation for di-electrons from open charm decays can be used to identify in the IMR the open charm background in extracting thermal dileptons from the QGP phase.

We try to give a quantitative study of the open charm contribution to dileptons in the IMR. There are still some uncertainties which could be improved in a future study. Firstly the initial charm quark distribution in p+p and Au+Au collisions has to be fixed. There is a recent measurement of the charm meson production cross section and transverse momentum spectra [43] which may shed light on this, but the statistics is not high enough and needs to be improved. Secondly the charm hadron diffusion in the hadronic medium might be relevant. Though this effect can be partially achieved by tuning the freezeout temperature to lower values, a rigorous study is necessary. Thirdly the charm quark hadronization has to be treated in a better model. Though we argue that the hadronization process may not modify our current IMR results very much, it is expected to have impact on the low and intermediate \( p_T \) region. Finally the charm quark regeneration in the partonic medium may also be important at the LHC energy.

### Acknowledgments

HX thanks Y.-K. Song, B.-C. Huang and J. Song for helpful discussions. QW is supported by National Natural Science Foundation of China under the grant no. 11215254. This work was supported in part by the Offices of NP and HEP within the U.S. DOE Office of Science under the contracts of DE-FG02-88ER40412 and DE-AC02-98CH10886.USTC.

[1] J. Adams et al. (STAR Collaboration), Nucl.Phys. A757, 102 (2005), nucl-ex/0501009.
[2] K. Adcox et al. (PHENIX Collaboration), Nucl.Phys. A757, 184 (2005), nucl-ex/0410003.
