Model selection based on the angular-diameter distance to the compact structure in radio quasars

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Introduction. – The luminosity distance is often used in cosmology for measurements involving standard candles, such as Type-Ia SNe [1] and gamma-ray bursts [2]. By comparison, the angular-diameter distance, \( d_A(z) \), uniquely reaches a maximum value at some finite redshift \( z_{max} \) and then decreases to zero towards the Big Bang. This effect has been difficult to observe due to a lack of reliable, standard rulers, though refinements to the identification of the compact structure in radio quasars may have overcome this deficiency. In this letter, we assemble a catalog of 140 such sources with \( 0 \leq z \leq 3 \) for model selection and the measurement of \( z_{max} \). In flat \( \Lambda \)CDM, we find that \( \Omega_m = 0.243^{+0.07}_{-0.09} \), fully consistent with the Planck optimized value, with \( z_{max} = 1.69 \). Both of these values are associated with a \( d_A(z) \) indistinguishable from that predicted by the zero active mass condition, \( \rho + 3p = 0 \), in terms of the total pressure \( p \) and total energy density \( \rho \) of the cosmic fluid. An expansion driven by this constraint, known as the \( R_h = ct \) universe, has \( z_{max} = 1.718 \), which differs from the \( \Lambda \)CDM optimized value by less than \( \sim 1.6\% \). Indeed, the Bayes Information Criterion favours \( R_h = ct \) over flat \( \Lambda \)CDM with a likelihood of \( \sim 81\% \) vs. 19\%, suggesting that the optimized parameters in Planck \( \Lambda \)CDM mimic the constraint \( p = -\rho/3 \).

Abstract. – Of all the distance and temporal measures in cosmology, the angular-diameter distance, \( d_A(z) \), uniquely reaches a maximum value at some finite redshift \( z_{max} \) and then decreases to zero towards the Big Bang. This effect has been difficult to observe due to a lack of reliable, standard rulers, though refinements to the identification of the compact structure in radio quasars may have overcome this deficiency. In this letter, we assemble a catalog of 140 such sources with \( 0 \leq z \leq 3 \) for model selection and the measurement of \( z_{max} \). In flat \( \Lambda \)CDM, we find that \( \Omega_m = 0.243^{+0.07}_{-0.09} \), fully consistent with the Planck optimized value, with \( z_{max} = 1.69 \). Both of these values are associated with a \( d_A(z) \) indistinguishable from that predicted by the zero active mass condition, \( \rho + 3p = 0 \), in terms of the total pressure \( p \) and total energy density \( \rho \) of the cosmic fluid. An expansion driven by this constraint, known as the \( R_h = ct \) universe, has \( z_{max} = 1.718 \), which differs from the \( \Lambda \)CDM optimized value by less than \( \sim 1.6\% \). Indeed, the Bayes Information Criterion favours \( R_h = ct \) over flat \( \Lambda \)CDM with a likelihood of \( \sim 81\% \) vs. 19\%, suggesting that the optimized parameters in Planck \( \Lambda \)CDM mimic the constraint \( p = -\rho/3 \).

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within the redshift range $1.0 \lesssim z \lesssim 2.7$, which showed no change in apparent angular size with angular-diameter distance, somewhat consistent with Friedmann-Robertson-Walker cosmology without any significant evolution (see fig. 1 below) [29].

It eventually became apparent that ultracompact radio sources are more likely to produce standard measuring rods than the large-scale jets in quasars and radio galaxies. The emission from these compact regions is dominated by self-absorbed synchrotron emission [16], forming at least partially opaque features with angular diameters in the milliarcsecond (mas) range, and linear sizes of the order of 10 parsecs [17,30]. Their significant advantage over larger structures, such as galaxies and kpc-scale jets, is that these central cores are much smaller than their parent active galactic nuclei (AGN), so their ambient physical environment should be similar from source to source and be reasonably stable, unlike the variations one expects in the intergalactic medium over large distances and times [31,32]. The compact structures in these sources therefore evolve principally under the influence of the central engine itself, which is typically characterized by only a few physical parameters, such as the mass of the black hole and its spin. Dynamical timescales in such environments are only tens of years, much shorter than the age of the Universe. The compact structures in radio quasars should therefore be free of long-term evolutionary effects [18].

In one of the more significant studies involving the compact structure of radio quasars, Gurvits, Kellermann and Frey [18] showed that a large sample of images taken with Very Large Baseline Interferometry (VLBI) may be used to establish some general constraints on cosmological parameters. This work has formed the basis of many subsequent investigations [33–35], leading to a second significant advancement with the use of these sources that we shall discuss shortly [20].

A persistent complication with compact radio jets has been that they are found in a mixed population of radio galaxies and AGNs —quasars, BL Lacs, OVVs, etc.— so systematic differences are not always easy to disentangle from true cosmological variations. Recent work by Cao et al. [19], however, has included the analysis of different AGN sub-samples based on different source optical counterparts and lying in different redshift ranges, leading to the conclusion that radio galaxies and quasars need to be handled with distinct strategies. This result is the basis for the current study of such sources, focusing only on compact structures in radio quasars [20,21]. The net outcome of this effort has been a significant reduction in scatter to produce a reliable sample of compact structures in radio quasars for use as standard rulers, allowing us to carry out the study reported in this letter, which compares models in ways not previously feasible with other measures of cosmological distance.

Data and analysis. – Following the suggestion by Gurvits et al. [18] and Vishwakarma [36] that the exclusion of sources with low luminosities $L$ and extreme spectral indices, $\alpha$, might curb the dependence of the core size on the source luminosity and redshift, several workers have refined the process of compiling from the many hundreds of available VLBI images a reduced sample with manageable scatter, free of evolutionary effects.

With very long baseline interferometry, the signal from a distant radio source is received at multiple radio telescopes across Earth’s surface, whose registration of intensities may then be correlated taking into account the slightly different arrival times at the various facilities. The net result is a combined observation made by a telescope with a baseline equal to the maximum separation of the radio antennae. For the compact structure in radio quasars, the characteristic angular size inferred from VLBI is defined as

$$\theta_{\text{core}} = \frac{2\sqrt{-\ln\Gamma \ln 2}}{\pi B},$$

where $B$ is the interferometer baseline and $\Gamma$ is the ratio of total flux density to the correlated flux density [37]. The linear size of the core may then be written as

$$\ell_{\text{core}} = \theta_{\text{core}}(z) \times d_A(z),$$

where $d_A(z)$ is the model-dependent angular-diameter distance.

It is now understood that the dispersion in linear size is greatly mitigated [18,20] by retaining only those sources with $-0.38 < \alpha < 0.18$. Additionally, Cao et al. [20] have recently pointed to a strong dependence of the core size $\ell_{\text{core}}$ on luminosity, not just at the low end, but at the high end as well. Using the parametrization $\ell_{\text{core}} = \ell_0 L^\beta(1+z)^n$, where $\ell_0$ is a scaling constant, they demonstrated that only a sub-sample of intermediate-luminosity radio quasars ($10^{27}$ W/Hz $< L < 10^{28}$ W/Hz) have a core size with negligible dependence on $L$ and $z$. For these objects, $\beta \approx 10^{-4}$ and $|n| \approx 10^{-3}$, yielding a compilation of compact structures in radio quasars with a rather robust standard linear size.

The data we use here were assembled by Jackson and Jannetta [38] using the 2.29 GHz VLBI survey of Preston et al. [37] and additions by Gurvits [39], resulting in a catalog of 613 sources. In order for us to extract the sub-sample with luminosities restricted to the range alluded to above, we use the Planck optimized parameters [40] to estimate the luminosity distance and thereby the value of $L$ from the measured total flux density at 2.29 GHz. These parameters are used merely to estimate $L$ for the purpose of identifying the intermediate-luminosity sources, and are not otherwise employed in the fitting procedure described below. We have carried out a simple test to ensure that the model selection is not biased with this approach by relaxing the constraint on $L$ by as much as 50%, which produced no discernable effect. In addition, note that using the Planck parameters would benefit ACDM, if at all, so an outcome favouring $R_0 = ct$ could not be viewed as having been facilitated by this approximation. The sample is further reduced by restricting $\alpha$ to the aforementioned
At a redshift $z$, the apparent angular size $\theta$ depends on how far back in time we look. Since all sources were closer to us as we see them, their angular size increases as $z$ increases initially, reaches a peak at $z \approx 1.37$, and then decreases at higher redshifts, reaching zero at $z \approx 2.6$. We will keep this analysis as simple and parameter-free as possible, concentrating solely on what is absolutely needed in order for us to extract information concerning the geometry of the Universe from the compact radio-jet data shown in fig. 1. For a given core size $\ell_{\text{core}}$, the predicted angular size of the compact structure in radio quasars is obtained from eq. (2), in terms of the angular-diameter distance, which, in flat $\Lambda$CDM, is given as

$$d_A(z) = \frac{c}{H_0} \frac{1}{1 + z} \int_0^z \frac{du}{[\Omega_m(1 + u)^3 + \Omega_k(1 + u)^2 + \Omega_\Lambda(1 + u)^{3+3\omega_\Lambda}]^{1/2}}.$$  

In this expression, $\Omega_k$ is the density of species “$i$” scaled to today’s critical density, $3c^2H_0^2/8\pi G$, and $w_\Lambda = -1$ is the equation-of-state parameter for a cosmological constant. Given that radiation energy density is relatively negligible up to $z \approx 3$, there are only two free parameters in the expression for $\theta_{\text{core}}$ once we marginalize over the unknowns $\ell_{\text{core}}$ and $H_0$. We do this by combining eqs. (2) and (3) and writing

$$\theta_{\text{core}}(z) = \eta \frac{1 + z}{I(z)},$$

where $\eta \equiv \ell_{\text{core}}H_0/c$ and

$$I(z) = \int_0^z \frac{du}{[\Omega_m(1 + u)^3 + \Omega_k(1 + u)^2 + \Omega_\Lambda(1 + u)^{3+3\omega_\Lambda}]^{1/2}}.$$  

\begin{table}[h]
\centering
\caption{Model selection using the compact structure of radio quasars.}
\begin{tabular}{lcccc}
\hline
Model & $\Omega_m$ & $\eta$ & $\chi^2_{\text{dof}}$ & BIC & Probability \\
\hline
$\Lambda$CDM & $0.24^{+0.1}_{-0.09}$ & $0.58 \pm 0.05$ & 0.31 & 11.6 & 19.8\% \\
$R_h = ct$ & $-0.05^{+0.03}_{-0.02}$ & $0.31$ & 8.8 & & 80.2\% \\
\hline
\end{tabular}
\end{table}

Using standard $\chi^2$ minimization, we optimize the values of $\eta$ and $\Omega_m$ that produce the best fit, shown as a solid, black curve in fig. 1. The results of this fitting are summarized in table 1, and the corresponding 1$\sigma$, 2$\sigma$ and 3$\sigma$ confidence regions are plotted on the $\eta$-$\Omega_m$ plane in fig. 2. To further emphasize the quality of the fit, we also show in fig. 1 the 1$\sigma$ confidence region estimated by keeping $\eta$ fixed and allowing $\Omega_m$ to vary. The curves bounding this confidence region correspond to $\Omega_m = 0.08$ and 0.39. We therefore find that the value of $\Omega_m$ optimized with fits to the compact radio-jet data is fully consistent with Planck $\Lambda$CDM at a level of confidence better than 1$\sigma$. 

Discussion. – Let us now turn our attention to the primary goal of using these sources for model selection purposes based solely on the inferred geometry of the Universe. The turning point in the fitted $\theta_{\text{core}}(z)$ function is a

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{(Colour online) Angular size of 140 individual compact structures in radio quasars binned into groups of 7, as a function of the redshift. The data points represent the median value in each bin. The thick solid curve is the optimized flat $\Lambda$CDM model, with angular size constant $\eta = 0.58 \pm 0.05$ and $\Omega_m = 0.24^{+0.1}_{-0.09}$ (see text). Also shown are the values of $\Omega_m$ (i.e., 0.08 and 0.39) setting the bounds of the 1$\sigma$ shaded region (when $\eta$ is held constant).}
\end{figure}
parameters $\Omega$ swing of about 46% from top to bottom. $\sigma$ between 1.42 and 2.20 across the 1

$z$ 

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new dataset has unfolded. What has emerged is an indica-

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$p = p_m + p_r + p_{de}$ and $\rho = \rho_m + \rho_r + \rho_{de}$, where $\rho_r$ and $\rho_i$ are 

the pressure and energy density, respectively, of species “i” 

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latter may or may not be a cosmological constant. The 

expansion dynamics is based on the assumption that $p_m = 0$, 

$\rho_r = \rho_i/3$ and (typically) $\rho_{de} = -\rho_{de}$, while the densities 

$\rho_m \sim (1+z)^{-3}$, $\rho_r \sim (1+z)^{-4}$ and $\rho_{de} \sim (1+z)^0$ each 

evolve with redshift independently of the others. But the 

data appear to be telling us something different, pointing to 

a coupling of the densities in order to preserve a con-

stant equation of state with $w = -1/3$, known as the “zero 

active mass” condition in general relativity. In test after 
test, the predictions of $\Lambda$CDM with this additional con-

straint, a model referred to as the $R_h = ct$ Universe [43–47] in the literature, have been a better match to the data than 
those of basic $\Lambda$CDM without it. These comparisons have 

been carried out using a broad range of observations, from 

byproduct of $d_A(z)$’s unique maximum value at a model-

dependent redshift $z_{\text{max}}$. The best-fit curve in fig. 1 has 

a turning point at $z_{\text{max}} = 1.69$, with a possible variation 

between 1.42 and 2.20 across the 1σ confidence region, a 

swing of about 46% from top to bottom.

We wish to understand what is so special about the values $\Omega_m = 0.24$ and $z_{\text{max}} = 1.69$ that the Universe 

would have “chosen” these to characterize its geometry and expansion. Such questions have been asked repeatedly 

over the past several decades as the analysis of each 

new dataset has unfolded. What has emerged is an indication 

that $\Lambda$CDM may be lacking an additional constraint on 

its constituents that may resolve the origin of these 

parameter values.

$\Lambda$CDM adopts the equation of state $p = \frac{\rho}{\omega}$, with 

$p = p_m + p_r + p_{de}$ and $\rho = \rho_m + \rho_r + \rho_{de}$, where $\rho_r$ and $\rho_i$ are 

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those of basic $\Lambda$CDM without it. These comparisons have 

been carried out using a broad range of observations, from 

the angular correlation function of the cosmic microwave 

background [48] and high-z quasars [40,50] in the early 

Universe, to gamma ray bursts [51] and cosmic chronometers [52] at intermediate redshifts, and to the relatively 

nearby Type-Ia SNe [53]. A recently compiled list of these 

comparative studies may be found in table 1 of [54].

The angular-size data presented in fig. 1 allow us to 

examine this growing body of evidence from an entirely 
different perspective, because we have never had an op-

portunity before the identification of this sample of 

compact radio structures by key workers in this field, including 

Gurvits [39], Jackson [32], Cao et al. [20,21] and others, 

of testing the prediction that $d_A(z)$ ought to have a max-

imum at a finite redshift $z_{\text{max}}$. When we impose the ad-

ditional constraint $w = -1/3$ on $\Lambda$CDM, eq. (5) is simply 

$$I(z) = \ln(1+z).$$

(6)
We are therefore seeing a strong confirmation of previous results based on other kinds of measurements. This outcome suggests that the optimized value of Ω_m in ΛCDM arises because the formulation of w in this model needs it to mimic the integral in eq. (6) associated with the zero active mass condition w = −1/3. But the most convincing evidence in support of this conclusion comes from an evaluation of zm in Rh = ct, a new probe of the Universe’s geometry — never seen before the recent work of Cao et al. [20,21] with any other kind of measurement. From eqs. (4) and (6) one finds that zm = 1.718 when the zero active mass condition is imposed on ΛCDM. This value lies within 1.6% of the turning point in θ_core found with the formulation of w in the standard model suggesting, once again, that the measured value of zm is not random at all, but is a direct consequence of the Universe’s expansion at a rate consistent with the zero active mass condition.

Finally, let us consider what the optimized value of η in table 1 implies for the Hubble constant H_0, should the core size ℓ_core be known from other data. A difficulty generally arises in the optimization of H_0 due to its model dependence. There is no universal way of measuring its value without assuming a particular model. The Hubble constant has been measured locally, e.g., using Cepheid variables, though it disagrees with the Planck value by more than 9%. It is not yet clear why this happens, but some have speculated that a local “Hubble bubble” [55–57] may be influencing the local dynamics within a distance ~300 Mpc (i.e., z ≤ 0.07). If true, such a fluctuation might lead to anomalous velocities within this region, causing the nearby expansion to deviate somewhat from a pure Hubble flow. For consistency, H_0 must therefore be measured on large, smooth scales.

With our value of η, H_0 may be inferred once ℓ_core is known. An estimate of its value was made recently by ref. [21], who used measurements of the expansion rate H(z) based on cosmic chronometers to break the degeneracy between ℓ_core and the Hubble constant. Their analysis estimated the core size to be ℓ_core = 11.03 ± 0.25 pc. Thus, for the optimized value η = 0.5 ± 0.03 in Rh = ct (see table 1), the implied Hubble constant is H_0 = 66.0±2.0 km s⁻¹ Mpc⁻¹. As of today, H_0 in Rh = ct has been measured 4 times: 63.2 ± 1.6 km s⁻¹ Mpc⁻¹ [52], 63.3 ± 0.7 km s⁻¹ Mpc⁻¹ [58], 62.3 ± 1.5 km s⁻¹ Mpc⁻¹ [59] and 63.0 ± 2.0 km s⁻¹ Mpc⁻¹ [60]. Cao et al.’s estimate of ℓ_core therefore yields a Hubble constant H_0 for Rh = ct consistent with these previous measurements.

The corresponding Hubble constant in ΛCDM is 76.6 ± 6.6 km s⁻¹ Mpc⁻¹. But while this value is still consistent with the Planck optimization, it is nonetheless somewhat on the high side, in slight tension with the estimates made in ref. [21]. It is very instructive to examine the cause of this non-trivial difference. There are actually two principal reasons why our inferred value of H_0 for ΛCDM does not agree completely with that obtained earlier by ref. [21].

First, while Cao et al. carried out a χ²-minimization using the individual quasars in our sample of 120, our approach calls for the binning of these sources into 20 redshift intervals before optimizing the model parameters. A quick inspection of their data and best fit curves shows that many of the individual sources lie several σ’s away from the theoretical curves. In other words, the reported errors are far too small to represent the actual scatter in the data. For this reason, we have chosen to bin the individual sources and use population variance based on assumed Gaussian variation within each bin to more reliably estimate the error associated with each data point. Not surprisingly, our errors are larger than those reported for each individual source because they better reflect the overall scatter in the data. A quick inspection of figs. 1 and 3 shows that all but one of the data in these plots lie within 1σ of the best-fit curves. We suggest that carrying out a model optimization with these binned data is therefore more reliable than simply trying to do this with data whose errors are unrealistically small.

Second, and more importantly, Cao et al. [21] optimized ℓ_core and H_0 separately, while (as noted above), we optimize the sole parameter η. Why is this important? Cao et al. did not base their parameter optimization solely on the quasar-core data. Since they needed additional information to separate these two unknowns, they combined their angular-size measurements with observations of H(z) based on cosmic chronometers, as noted earlier. As such, their optimized parameters reflect the joint analysis of several different data sets, as opposed to just the quasar-core observations — the principal focus in this paper. Each approach has its advantages, of course. Ours allows model selection to be carried out based solely on the quasar-core data. This is not a trivial step, because each kind of measurement should be studied on its own, not only in joint analyses with other observations that may introduce unknown biases.

Conclusion. — Needless to say, the identification of the compact structure in radio quasars as standard rulers has opened up an entirely new chapter in cosmology. With them, we may now map the geometry of the Universe well beyond the reach of Type-Ia SNe, sampling even the epoch during which the apparent size of sources increases with redshift, an effect not seen with any other kind of measurement probe. The results thus far point to the zero active mass condition in general relativity as the influence guiding the Universe’s expansion. Developing this notion further, and testing it with even higher-precision measurements, promises a very exciting future in observational cosmology.

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