Improving students' mathematical creative reasoning on polyhedron through concept-based inquiry model

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Abstract. Mathematical reasoning is defined as an activity of mathematical thinking to produce statements or conclusions based on existing facts. Mathematical reasoning is divided into creative reasoning and imitative reasoning. According to constructivism theory, the idea in mathematics should be built by students, therefore mathematics become logic, meaningful, and real. One of the constructivist learning models that facilitate students to actively discover their own concepts is CBI model. The study aims to know the effectiveness of the CBI model on teaching mathematics toward improving MCR. The research is conducted in the year 2019/2020 at junior high school one in Citeureup-Bogor. Two classes, as the sample of the research, the first class as experiment class, and the second class as control class, both of them consists of 36 students. The instrument used to identify students' MCR abilities was an essay test on the polyhedron material. By using t-test analysis at a significant level of 5%, we obtained the Sig. (2-tailed) value obtained was 0.018. Therefore, Sig. (2-tailed) = 0.018 < α = 0.05. It means that students' MCR taught by CBI model is higher than students' MCR who taught with conventional learning models.

1. Introduction
Reasoning is one of the essential abilities in understanding and solving mathematical problems [13]. Without reason, learning mathematics becomes memorizing formulas and use the formula to solve routine problems. If we study mathematics, we integrated science with mathematics itself. When we learn about geometry, we could be related our insights to the context of everyday life, for example, tailors, how much material is needed, or home builders, how much paint is required to cover the surface, etc. Therefore, mathematics can be interpreted as the result of one's thoughts related to ideas, processes, and reasoning [7]. In line with Ruseffendi's opinion, students who often solve math problems using formulas directly without going through the reasoning process will have difficulty solving non-routine math problems [1].

Mathematical reasoning is defined as an activity of mathematical thinking to produce statements or conclusions based on existing facts or data so that mathematical problems are resolved [7,11,12,13]. Lithner divides mathematical reasoning into creative reasoning and imitative reasoning [2,3,4]. Creative reasoning is defined as a person's ability to solve non-routine mathematical problems in a way or mindset that is different from usual. Still, the reasoning process has a solid mathematical foundation. The results of these thoughts contain aspects of novelty, reason, and logic. [1,3,7]. Therefore, creative reasoning involves four criteria of thinking processes, namely fluency, flexibility, originality, and elaboration [4,8,9,14]. While imitative reasoning is a simple mathematical thinking
process where students can identify previous problems, then solve new mathematical problems used in previous problems. In imitative reasoning, thought processes tend to imitate the methods used.

Furthermore, Lithner mentioned three indicators of MCR abilities, namely creativity, plausibility, and anchoring. [2,3]. Creativity is defined as the ability of students to solve mathematical problems not only by remembering formulas or imitating the previous problem-solving algorithms but solving mathematical problems using unusual thinking methods or procedures so that their novelty can be seen. Plausibility is defined as the student's ability to provide logical, correct, and reasonable arguments for solutions to mathematical problems. Plausibility is not the ability to provide arguments by guessing because the answers to guesses will be difficult to justify. Plausibility will appear in the logical arguments presented by students when students provide solutions to the problems given. Anchoring is defined as the ability of students to solve mathematical problems that show the basic concepts of mathematics clearly. Students do not answer that contains feelings.

Mathematical reasoning abilities are also developed in geometry. Based on Indonesia's mathematics education curriculum, the polyhedron is one of the geometry subjects taught at the junior high school level. A polyhedron (plural polyhedra or polyhedrons) is a three-dimensional shape with flat polygonal faces, straight edges, and sharp corners or vertices. In this study, polyhedron consists of cubes, blocks, pyramids, and prisms.

By implementation of curriculum 2013 in Indonesia, teachers are required to develop the learning process from learning with lecture method to learning with a scientific approach in the class. Likewise, with learning geometry, various learning methods are used, both technology-based or using teaching aids. Even though the learning method has been changed, at the end of the lesson, students prefer to memorize formulas rather than develop reasoning skills. This is evidenced by the difficulties students have when given questions in non-routine math. In contrast to routine questions that only apply formulas, students can solve them easily. The following is an example of non-routine math problems on polyhedron material (TIMSS 2011, question number M052206):

![Figure 1. Example of a TIMSS 2011 problem (item number M052206).](image)

Figure 1 shows that the questions were not directed to ask the concept, volume, or surface area. This problem does not only require students to know the formula of volume or the surface area of a flat shape. To solve these problems, students must have good mathematical reasoning. Therefore students know which concepts can use to solve problems, and they are able to connect between book volume and box volume.

To solve math problems, as shown in Figure 1, learning with the transfer of knowledge and memorizing formulas will cause students difficulty solving it. The transfer of knowledge and memorization of formulas are causing students' reasoning abilities to not develop properly [12,13]; as
a result, students only have procedural skills and do not have a conceptual understanding. Meanwhile, according to constructivism theory, mathematical knowledge or concepts must be built by students under the guidance of the teacher to avoid misconceptions; in other words, the ideas built by students become logical, meaningful, and real [12,13].

One of the constructivist learning models that facilitate students to actively discover their own concepts is the Concept-Based Inquiry (CBI) model. The CBI model is a model that develops mathematical ideas that can be transferred from teachers to students through active question and answer activities. This model focuses on students so that they can build concepts in each step of their learning and apply the concepts they have to new situations [10,14].

Figure 2. The stage of concept-based Inquiry.

The main idea of the CBI model is "concept" as the basis for understanding mathematics. Students' mathematical reasoning abilities will develop well if a good knowledge of mathematical concepts supports them. The CBI model, as in Figure 2, according to Marschall, consists of seven stages, namely: (1) Engage. Engage is an introductory stage where students are involved emotionally and intellectually in learning. At this stage, the teacher activates the knowledge previously possessed by students by presenting videos, pictures, or questions that can trigger student curiosity. (2) Focus. At the focus stage, students' knowledge is directed at the concepts being studied by introducing factual examples that are relevant or not relevant to the concept of learning. This helps form schemas in students' thinking. (3) Investigate. At this stage, students explore various factual examples or case studies related to the concept of learning. The teacher provides examples or case studies that are more complex. These activities will train students to carry out investigations individually, in small groups, or throughout the class. When the investigation results are carried out, students can gather a lot of relevant information to build the concepts taught. (4) Set. After students have the data, students are given the opportunity to find patterns from the data collected so that they can find mathematical ideas to finally build concepts. At this stage, students are directed to process data using pictures, graphics, or other relevant tools. (5) Generalization. This stage demands the ability of students to communicate their findings. So that there will be interactions between students to criticize the findings of other groups. (6) Transfer. At this stage, the transfer does not mean the transfer of student knowledge, but the concepts obtained by students in the previous learning process will be applied to new situations, where the teacher prepares and presents questions in different situations. At this stage, the teacher can assess students' creative reasoning abilities based on understanding their concepts. (7) Meditate. This stage is a reflection activity guided by the teacher as an evaluation process to improve students' creative reasoning abilities during the learning process [6,14].
2. Experimental Method
The method of this research is experimental with a Randomize Control Group Post Test Only design. The study aims to know the effectiveness of the CBI model on teaching mathematics toward improving mathematical creative reasoning. The research is conducted in the year 2019/2020 at state junior high school one in Citeureup-Bogor. Two classes, as the sample of the research, the first class as experiment class, consists of 36 students. The students learn mathematics by Concept-Based Inquiry model, and the second class as control class consists of 36 students, where the students learn mathematics by conventional. The instrument used to identify students’ MCR abilities was an essay test on the polyhedron material consisting of five questions. The instrument is developed from three MCR indicators: creativity, plausibility, and anchoring. The instrument was validated by six experts and practitioners of mathematics education based on the suitability of the items with indicators, the suitability of the context with the curriculum, and the suitability of the items with the students’ understanding for junior high school level. Analysis of expert count by CVR formula: \( CVR = \frac{n_e - N}{\frac{N}{2}} \), the criteria is when the CVR > min score than the instrument can say valid [5]. The result of analysis expert shows at Table 1

| Respondent | Essential (e) | not-essential (ne) | not-relevant | CVR | Min Score | Conclusion |
|------------|---------------|--------------------|--------------|-----|-----------|------------|
| 1          | 10            | 0                  | 0            | 1   | 0.78      | valid      |
| 2          | 10            | 0                  | 0            | 1   | 0.78      | valid      |
| 3          | 10            | 0                  | 0            | 1   | 0.78      | valid      |
| 4          | 10            | 0                  | 0            | 1   | 0.78      | valid      |
| 5          | 10            | 0                  | 0            | 1   | 0.78      | valid      |
| 6          | 10            | 0                  | 0            | 1   | 0.78      | valid      |

The instrument was also tested in the field to find out the feasibility. Based on the validation test results, there are five questions declared valid with a reliability value of 0.78, and then the valid questions were used to collect research data about students’ MCR abilities. Furthermore, the instrument is used to collect the data from both groups. The data were analyzed by t-test. Its to know the difference in students’ MCR between experiment class and group class. The obtained student MCR data were then tested using the t-test at \( \alpha = 0.05 \) to determine whether there was a significant difference between the MCR of students taught through the CBI model and the conventional model.

3. Result and Discussion
This part firstly presents students’ MCR data in a table following by its interpretations.

| Table 2. Students’ mathematical creative reasoning ability data. |
|---------------------------------------------------------------|
| **Descriptive Statistic** | **Experiment Class** | **Control Class** |
|---------------------------|----------------------|-------------------|
| N                         | 36                   | 36                |
| \( \bar{x} \)             | 70.69                | 60.56             |
| Median                    | 75                   | 60                |
| Varians                   | 231.65               | 401.11            |
| Std. Deviation            | 15.22                | 20.03             |
| Minimum                   | 35                   | 25                |
| Maximum                   | 95                   | 95                |
| Range                     | 60                   | 70                |
| Skewness                  | -0.59                | -0.04             |
Table 2 shows that the average value of the experimental group was 70.67, with a standard deviation of 15.22, while the average value of the control group was 60.56 with a standard deviation of 20.03. In other words, the average of students who study by the CBI model was higher than students who learn by the Conventional model. The data distribution of the experimental group was more homogeneous than the control group data. Look at Figure 3.

![Figure 3. The distribution on students’ MCR for experiment class (blue line) and control class (red line).](image)

To determine whether the two groups have significant differences, it is necessary to analyze using the t-test with $\alpha = 0.05$. The results of the t-test calculations are presented in Table 3.

**Table 3. Hypothesis test results of mathematical creative reasoning ability.**

| t | Df | Sig. (2-tailed) |
|---|----|----------------|
| 2.418 | 70 | 0.018 |

Table 3 shows that at the significance level $\alpha = 0.05$, the Sig. (2-tailed) value obtained was 0.018. Therefore, Sig. (2-tailed) = 0.018 $< \alpha = 0.05$. This means that H0 is rejected. So, the findings in this section are that students' MCR abilities who study by the CBI model are higher than students' MCR abilities who study by the conventional model. Furthermore, the analysis of students' MCR abilities in the experimental group (students learning with the CBI model) based on three indicators (creativity, plausibility, and anchoring) is presented in Figure 2.

![Figure 4. Mathematical creative thinking at experiment class viewed from the indicator.](image)
Figure 4 shows the score of achievement for each indicator. The indicator of creativity is the highest among the other ones. It shows that learning mathematics by the CBI model can significantly increase students' MCR on the indicators of creativity.

In this study, indicators of creativity show the students' ability to answer questions in different ways. The following example is one of the questions that measure students' creative abilities: A cake maker will make several snack boxes from a sheet of thick paper with an area of 21,600 cm$^2$. Snack box in the shape of a cube with two types is of size options. Type I box measuring 15 cm and type II box measuring 20 cm. Determine the number of each type of box that must be made. Therefore the remaining paper is not more than 500 cm$^2$.

This problem requires students' reasoning abilities to provide different solutions regarding the number of each type of box that will be made by the cake maker, and the remaining paper students are not more than a certain size. The following is an example of a student's answer:

Figure 5. Example of students answer in creativity indicator.

Figure 5 shows the results of students' answers who are able to provide alternative answers in more than one way. Based on the specified area (the total paper area is 21,600 cm$^2$, and the remaining paper area should not be more than 500 cm$^2$), students can predict how many types I and type II boxes can be made. The results of the students' calculations in Figure 3, it turns out that students can find three combinations of box types that can be made. Type I (15 cm) and Type II (20 cm), the students' answers consist of:

- a. Five pieces type I and six types II boxes
- b. Seven pieces type I boxes and five pieces type 2 boxes
- c. only made 16 pieces of type I boxes.

Students' MCR abilities who are learning by the CBI model are continuously trained. Therefore students' MCR ability can develop properly. The process of finding mathematical concepts is always directed to the mathematical reasoning process. The stage of learning in CBI that support developing students' reasoning are investigation, organization, and generalization. At this stage, students are required to be able to solve the cases and/or factual examples. Students provided and be given the freedom to find mathematical concepts in their group discussions. This is in line with Ruseffendi's opinion that creativity will grow through exploration, inquiry, discovery, and problem solving [1]. Apart from that, the CBI model also includes a transfer knowledge stage that trains students in solving math problems well. In contrast, the teacher's role is more dominant in concept finding in groups of students who learn through conventional models. Therefore, students who learn through the CBI model have a higher score than students who learn through the conventional model.

The second indicator of students' creative reasoning abilities is plausibility. The plausibility indicator is shown by the student's ability to provide correct and reasonable arguments. The following is an example of a problem measuring the plausibility indicator. Fadhillah has a fishing pond with a length of 2 m, a width of 5 m, and a depth of 1 m. He wants to expand the size of the fishing pond, than
the volume becomes six times the volume before. Fadhillah has plans to dismantle and rebuild the pool by changing its size to 6 times its previous size. Is Fadhillah’s plan in accordance with what he expected? If not, what should Fadhillah do? Give your opinion.

This question expects students to give their opinion correctly about the plan made by Fadhillah. Students can provide precise and clear reasons based on the use of the right concept, as well. Examples of student answers to the plausibility indicator are presented in Figure 6.

Figure 6 shows one student’s opinion to comment on the plan to be carried out by Fadhilah as needed. Indicator plausibility, students can provide their arguments logically and show their reasons by using the correct concept. From question number 2, students have one key, its the volume pool of 60 m$^3$. The students already know the concept of block volume, which is length x width x height (lwh). From that formula, students try to find combination of 3 numbers, which when multiplied by the three results in 60. The combination of numbers obtained by students is as follows:

- Option 1: l = 2 m; w = 5 m; and h = 6 m
- Option 2: l = 6 m; w = 5 m; and h = 2 m
- Option 3: l = 4 m; w = 15 m; and h = 1 m
- Option 4: l = 4 m; w = 5 m; and h = 3 m
- Option 5: l = 6 m; w = 10 m; and h = 1 m

If we pay attention to the five combinations above, there are still other combinations of numbers showing a volume of 60 m$^3$; length = 6, widths = 1, and height = 10. However, the combination of these numbers is not used as an alternative pool size chosen by Fadhilah. Fish ponds are only 1 m in widths, while the depth is 10 m. So, it is clear that the five pool size combinations mentioned by the students are reasonable answers and are based on the concept of volume.

Students' CMR ability to argue is continuously trained through the learning process using the CBI model. The CBI learning stage that accommodates these abilities is the focus stage, the investigation stage, the organization, and generalization. At the focus stage, students are given cases to practice their reasoning skills. Therefore students can provide arguments and the reasons for their argument. At the stage of investigation, organization, and generalization, students are trained to express their opinions through discussion and solving the problem. And finally, students will be accustomed to expressing opinions logically and rationally.

The third indicator of CMR is anchoring, meaning that students can create mathematical solving strategies based on mathematical concepts. The following is an example of a math problem on the anchoring indicator.
The swimming pool is known as in the left Figure. The shape of the pool is a polyhedron with a depth of 1 m. If the base and the walls of the pool area to be covered by ceramic tile, how much ceramic tile does it take to cover the inside of the pool?

In this problem, students are expected to provide solutions based on relevant mathematical concepts. Simply ways cannot solve this question, students use the formula of the surface area of the polyhedron, but students are required to analyze first for the problem, starting from the shape of the pool, then which surface will be covered by ceramic tiles, and what it is size. Here are the examples of student answers.

Student answer:
Keramik untuk sisi kolam:
= 1 m x (10m +6m+8m+6m+8m+6m+4m)
= 56 m²

Keramik untuk alas kolam = (10x6) +(6x8) +(2x6) = 60+48+12
=120 m²
Jadi kebutuhan keramik untuk kolam renang tersebut = 56 m² + 120 m²
= 176 m²

Figure 7. Students answer for anchoring indicator.

The students' answers, shown in Figure 7, indicate that the students have carried out the reasoning process first, where students find the area for the wall and the area for the base of the swimming pool. The Student find the area of the wall for swimming pool is multiplication between the depth of pool and perimeter of pool = 1m x (10m + 6m + 8m + 6m + 8m + 6m + 4m). Meanwhile, for the pool base, the student makes a partition on the sketch, the pool becomes some of a rectangle. Then all area of a rectangle is added. Area I + Area II + Area III = (10x6) + (6x8) + (2x6) (look Figure 6). So, students have shown their ability to provide solutions based on relevant mathematical concepts. By calculating the surface area, students can conclude the need for ceramics tile to cover the swimming pool's inner surface.

Students' anchoring ability is also developed in mathematics learning using the CBI model; for example, at the engage stage, students are given problems that are solved by combining several concepts. At the focus stage, students are trained to be observant in selecting and using relevant concepts. For the stage of investigating, organizing, and generalize, students accustomed to finding concepts and reasoning independently.

Based on the research data obtained, the Concept-Based Inquiry model is a learning model that supports students to improve their mathematical reasoning and conceptual understanding [12]. Students can develop their creative reasoning abilities with mastery of mathematical reasoning and conceptual understanding [2]. Without good mathematical reasoning and conceptual understanding, then the students' imitative mathematical reasoning will develop.

They were teaching mathematics by CBI model impact on improving students' MCR. The following is the stage of the CBI model used in this research. In the stage engage, learning mathematics begins with emotional and intellectual. Students' involvement Emotional and intellectual in the learning process is carried out by playing videos as motivation and generating student knowledge, which is the basis of learning polyhedron material. The video aims to inspire students' enthusiasm for learning and activate students' initial knowledge. The teacher asks questions to find out students' knowledge about
the characteristics of spatial and the concept of surface area. At this stage, students are directed to have basic concepts of solving mathematical problems.

At the focus stage, students are directed to the concepts that are being studied. The teacher ensures that students have the same understanding of the concept. At first, the teacher provides some factual examples and not examples that are relevant to the form of space, and then students are directed to look for other examples by providing reasons based on the characteristics of these forms. To solve these problems, the teacher prepares worksheets. Therefore, the learning process is more focused. Students seem very active in group discussions to solve the questions given in the worksheet.

At the investigation stage, students were exposed to factual examples or cases related to geometric concepts. One of the cases used in this research is about the problem of wrapping paper that will be used to wrap some of the gifts on Indonesian Independence Day. At this stage, students are directed to understand and analyze information relevant to solving a given case. Students are given space to discuss together with a group of friends so they can express their opinions. After students understand the case, then students solve the problem at the organizing stage. Students are directed to organize the data that has been collected related to topics at the investigation stage. Students recognize patterns and find some similarities from the data obtained. Figure 8 is an example of a worksheet that has been completed by students. In this worksheet, students are directed to determine the surface area of each spatial shape. On the worksheet, students are given the freedom to make net build-up from the shapes. It appears that students can solve the given case, and students can provide their arguments that connect the area of the net with the amount of paper needed. At the end of the CBI, the stage is a generalization; students are directed to communicate their findings.

Figure 8. Worksheet at organize stage.

4. Conclusion
The results showed that the students’ MCR ability taught using by CBI model was higher than students’ MCR abilities of students taught using conventional learning. This is evidenced by the results of the t-test on the student’s MCR ability in both classes. It can be concluded that the CBI learning model has a positive impact on students’ MCR.
Based on these findings, we recommend that the CBI learning model be an alternative learning process for mathematics teachers to develop students’ MCRs. Besides, other researchers can conduct further research on the effectiveness of the CBI model on other mathematical abilities.

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