Sensorless control for the brushless DC motor: an unscented Kalman filter algorithm

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(Received 18 August 2014; accepted 28 October 2014)

In this paper, a new mathematical model is built according to the characteristics of the brushless DC (BLDC) motor and a new filtering algorithm is proposed for the sensorless BLDC motor based on the unscented Kalman filter (UKF). The proposed UKF algorithm is employed to estimate the speed and rotor position of the BLDC motor only using the measurements of terminal voltages and three-phase currents. In order to observe the drive performance, two simulation examples are given and the feasibility and effectiveness of the UKF algorithm are verified through the simulation results, and the accurate estimate performance is shown in simulation figures.

Keywords: BLDC; sensorless control; UKF algorithm; nonlinear wave function; trigonometric series

1. Introduction

With the rapid development of modern industries, the brushless DC (BLDC) motor is increasingly being used in computer peripherals, office automation and machine tool industries because of its high efficiency, easy control, compact form, etc. (Pillay and Krishnan, 1991), and the BLDC motor also has the characteristics of trapezoidal electromagnetic force (emf) and quasi-rectangular current waveforms. In order to obtain the appropriate commutation signals every 60 electrical degrees, rotor position sensors must be used, such as hall sensors and photoelectric encoders. However, speed or position sensors require additional mounting space, they increase the cost and the complexity of the system as well as reduce the reliability of the system. In recent years, the sensorless control technology has received wide attention. The sensorless technology improves the system reliability and it is of great significance to further expand the application fields of the BLDC motor.

At present, there are so many papers that have reported the sensorless control technology. Among them, the most popular and widely used method is the back-EMF method (Iizuka, Uzuhashi, Kano, Endo, and Mohri, 1985). However, this method generally has a drawback that the back-EMF cannot be detected exactly under low-speed conditions. In addition, some other methods have been discussed, such as the estimation flux method (Ji and Li, 2008) and the freewheeling diodes detection method. Nevertheless, these methods cannot provide continual rotor position information when a high accuracy of the rotor speed and position are required. The extended Kalman filter (EKF) is an optimal recursive estimation algorithm for nonlinear systems and has been applied for estimating the state variable of the BLDC motor (Lenine, Reddy, and Kumar, 2007; Terzic and Jadric, 2001). However, there are also some limitations for the EKF algorithm, such as the complex computation of the Jacobian matrices, only first-order accuracy, etc.

In this paper, a new method has been introduced to overcome the above drawbacks by means of the unscented Kalman filter (UKF) algorithm. The UKF algorithm uses a deterministic sampling approach to capture the mean and covariance estimates with a minimal set of sample points. The related applications of the UKF have been reported in many papers, such as permanent magnet synchronous motor drive (Chan and Borsje, 2009) and induction motor drive (Rigatos and Siano, 2012). The simulation results by MATLAB/Simulink indicate that the speed could be tracked and adjusted precisely, and the dynamical property of systems is evidently improved.

2. Mathematical model of the BLDC

In this paper, the three-phase BLDC motor with star connection is considered, and the voltage equation of the phase A of the stator is as follows (Pillay and Krishnan, 1989):

\[ u_a = R_i a + (L - M) \frac{d i_a}{dt} + e_a + u_n, \]  

(1)

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where \( u_a \) and \( u_b \) are the terminal voltage of phase A and neutral point voltage, respectively, \( R \), \( i_a \), \( L \) and \( e_a \) are the phase resistance, the phase current, the phase inductance and the phase back-emf of the phase A, respectively, and \( M \) is the mutual inductance. Especially, we have similar voltage equations for phases B and C.

According to the operation principle of the BLDC motor, it is known that only two phases conduct in three-phase stator winding at each time point. Thus, the neutral point voltage can be deduced as follows:

\[
    u_n = \frac{1}{2} [ (u_a + u_b + u_c) - (e_a + e_b + e_c) ],
\]

where \( u_b \) and \( e_b \) are the voltage and the back-emf of the phase B, respectively, \( u_c \) and \( e_c \) are the voltage and the back-emf of the phase C, respectively.

By substituting Equation (2) into the voltage equation of phase A, we can obtain the following equation:

\[
    \frac{di_a}{dt} = \frac{u_{ab} - u_{ca}}{3(L - M)} - \frac{R}{L - M} i_a + e_b + e_c - 2e_a,
\]

where \( u_{ab} = u_a - u_b \) and \( u_{ca} = u_c - u_a \).

According to the structure of the BLDC motor, the back-emf of phase A can be written as (Chen, Huang, Wang, and Wu, 2011)

\[
    e_a = \omega \varphi_m f_a(\theta),
\]

where \( \omega \) is the motor angular velocity, \( \varphi_m \) is the magnet flux linkage of the stator winding, \( f_a(\theta) \) changing along with the rotor position is the wave function of the back-emf of phase A and its maximum value and minimum value are 1 and \(-1\), respectively.

The nonlinear function \( f_a(\theta) \) can be described as follows:

\[
    f_a(\theta) = \begin{cases} 
      \frac{6}{\pi} \cdot \theta, & 2\pi \leq \theta \leq \frac{\pi}{6} + 2k\pi, \\
      1, & \frac{\pi}{6} + 2k\pi \leq \theta \leq \frac{5\pi}{6} + 2k\pi, \\
      -\frac{6}{\pi} \cdot (\theta - \pi), & \frac{5\pi}{6} + 2k\pi \leq \theta \leq \frac{7\pi}{6} + 2k\pi, \\
      -1, & \frac{7\pi}{6} + 2k\pi \leq \theta \leq \frac{11\pi}{6} + 2k\pi, \\
      \frac{6}{\pi} \cdot (\theta - 2\pi), & \frac{11\pi}{6} + 2k\pi \leq \theta \leq 2\pi + 2k\pi. 
    \end{cases}
\]

and represented by the trigonometric series as follows:

\[
    f_a(\theta) = 1.21 \sin \theta + 0.27 \sin 3\theta + 0.05 \sin 5\theta - 0.02 \sin 7\theta - 0.03 \sin 9\theta \ldots.
\]

For the symmetry structure of the BLDC motor, we have \( f_b(\theta) = f_a(\theta - 2\pi/3) \) and \( f_c(\theta) = f_a(\theta + 2\pi/3) \).

From Equations (3) and (4), it can be derived that

\[
    \frac{di_a}{dt} = \frac{u_{ab} - u_{ca}}{3(L - M)} - \frac{R}{L - M} i_a \\
    + \frac{\omega \varphi_m [f_a(\theta) + f_b(\theta) - 2f_c(\theta)]}{3(L - M)}. 
\]

In the motor control community, because of the extensive application of the digital control, the discrete-time system is widely used to describe the motor motions, and hence we consider the corresponding discrete-time equation of Equation (7) in this paper:

\[
    i_a(k+1) = \frac{T[u_{ab}(k) - u_{ca}(k)]}{3(L - M)} + \left( 1 - \frac{TR}{L - M} \right) i_a(k) \\
    + \frac{T\omega(\varphi_m [f_a(\theta_k) + f_b(\theta_k) - 2f_c(\theta_k)]}{3(L - M)},
\]

where \( T \) is the sampling time.

Similarly, we have

\[
    i_b(k+1) = \frac{T[u_{bc}(k) - u_{ab}(k)]}{3(L - M)} + \left( 1 - \frac{TR}{L - M} \right) i_b(k) \\
    + \frac{T\omega(\varphi_m [f_b(\theta_k) + f_c(\theta_k) - 2f_a(\theta_k)]}{3(L - M)},
\]

where \( i_a \) and \( i_c \) are the phase current of phases B and C, respectively.

According to Equation (4), we can obtain the following torque equation of the BLDC motor:

\[
    T_e = \frac{e_a i_a + e_b i_b + e_c i_c}{\Omega} \\
    = \frac{\omega \varphi_m [f_a(\theta) i_a + f_b(\theta) i_b + f_c(\theta) i_c]}{\omega / p} \\
    = p \varphi_m [f_a(\theta) i_a + f_b(\theta) i_b + f_c(\theta) i_c],
\]

where \( T_e \) is the electromagnetic torque, \( \Omega \) is the mechanical angular velocity and \( p \) is the number of pole pairs of the BLDC motor.

Consider the motion equation of the BLDC motor:

\[
    T_e - T_L = J \frac{d\omega}{dt} + B_c \omega,
\]

where \( T_L \) is the load torque of motor, \( J \) is the rotational inertia and \( B_c \) is the viscous friction coefficient.

From Equations (11) and (12) and by converting the mechanical angular velocity into the electrical angular
velocity, we have
\[
\omega(k+1) = \frac{p^2\varphi_m T}{J} [f_b(\theta) i_a + f_c(\theta) i_b + f_c(\theta) i_c] - \frac{p T}{J} T_L
+ \left(1 - \frac{B_c T}{J}\right) \omega(k).
\]
(13)

According to Newton’s law of motion, we can have
\[
\theta(k+1) = T\omega(k) + \theta(k).
\]
(14)

By combining the relevant equations (7), (9), (10), (13) and (14), the nonlinear state equations can be expressed in the following form:
\[
x_{k+1} = F_k(x_k)x_k + G_k u_k,
\]
\[
y_k = H x_k,
\]
(15)

where \( x_k = \begin{bmatrix} i_k & i_k & \omega & \theta \end{bmatrix}^T, y_k = \begin{bmatrix} i_a & i_b & i_k \end{bmatrix}^T; u_k = [u_{ab} - u_{ac} u_{bc} - u_{ab} u_{ca} - u_{bc} T L_k]^T \). 

3. UKF algorithm

For the Kalman filtering problem of a nonlinear system, although the EKF algorithm maintains the efficient recursive update form of the Kalman filter, it suffers a number of serious limitations. For instance, the calculation of the Jacobian matrices may be a difficult and error-prone process. For the purpose of overcoming the limitations of the EKF algorithm, the unscented transformation (UT) was proposed by Julie and Uhlman and a new Kalman filter algorithm (UKF) was presented in Julie and Uhlmann (2004). In this algorithm, a set of sample points are used to parameterize the mean and covariance of the probability distribution of the state variables.

In this section, the basic UT is reviewed first and the UKF algorithm is then introduced in detail.

3.1. Basic UT

Consider the following nonlinear function:
\[
y = h(x),
\]
(16)

where \( x \) is the \( n \)-dimensional variable with the mean \( \bar{x} \) and covariance matrix \( P_1 \); \( y \) is the \( m \)-dimensional variable with the mean \( \bar{y} \) and covariance matrix \( P_2 \).

For the variable \( x \) in Equation (16), a set of \( 2n \) sigma points are selected as follows (Simon, 2006):

\[
\begin{align*}
\bar{x}^{(i)} &= \bar{x} + \bar{\sigma}^{(i)}, & i = 1, \ldots, 2n, \\
\bar{\sigma}^{(i)} &= \sqrt{n P_1}^T, & i = 1, \ldots, n, \\
\bar{\sigma}^{(n+i)} &= -(\sqrt{n P_1}^T), & i = 1, \ldots, n,
\end{align*}
\]
(17)

where \( \sqrt{n P_1} \) is the matrix square root of \( n P_1 \) with \( \sqrt{n P_1}^T \cdot \sqrt{n P_1} = n P_1 \). Since \( n P_1 \) is a positive-definite symmetric matrix, the square root can be calculated using the Cholesky factorization which can simplify the calculation procedure.

Each sample point \( x^{(i)} \) is propagated through the nonlinear function to yield corresponding transformed sigma points \( y^{(i)} \), that is
\[
y^{(i)} = h(x^{(i)}) \quad i = 1, \ldots, 2n.
\]

The mean \( \bar{y} \) and covariance \( P_2 \) are approximated by the average mean and covariance of the transformed sigma points.

\[
\bar{y} = \frac{1}{2n} \sum_{i=1}^{2n} y^{(i)},
\]
\[
P_2 = \frac{1}{2n} \sum_{i=1}^{2n} (y^{(i)} - \bar{y})(y^{(i)} - \bar{y})^T,
\]
where \( 1/2n \) is the weight being used to calculate the mean and covariance.
3.2. UKF algorithm
Consider the following BLDC motor model:

\[ x_{k+1} = F_k(x_k)x_k + G_ku_k + w_k, \]
\[ y_k = Hx_k + v_k, \]  
\[ \text{(18)} \]

where \( w_k \) and \( v_k \) are, respectively, the process noise and the measurement noise, which is assumed as Gaussian white noise with covariance matrices \( Q_k \) and \( R_k \).

In the actual motor systems, the noises are unavoidable, such as the unmodeled noise, the detecting noise and so on. And these noises are likely to affect the normal operation of the motor. In the state-space model (18), all kinds of noises are described by the random variables \( w_k \) and \( v_k \).

Based on Equation (18), the UKF algorithm can be summed up as follows:

1. Compute the set of sigma points \( x_{k-1}^{(i)} \) based on the current optimal state estimation \( \hat{x}_{k-1} \) and the covariance estimation \( P_{k-1} \) according to Equation (17), that is,
\[ \hat{x}_{k-1}^{(i)} = \hat{x}_{k-1} + \tilde{\chi}^{(i)}, \quad i = 1, \ldots, 2n, \]
\[ \tilde{\chi}^{(i)} = (\sqrt{nP_{k-1}^{+}})^T, \quad i = 1, \ldots, n, \]
\[ \tilde{\chi}^{(n+i)} = - (\sqrt{nP_{k-1}^{+}})^T, \quad i = 1, \ldots, n. \]

2. Propagate the sigma points \( \hat{x}_{k-1}^{(i)} \) to \( \hat{x}_{k}^{(i)} \) through the nonlinear systems of the BLDC motor
\[ \hat{x}_{k}^{(i)} = F_{k-1}(\hat{x}_{k-1}^{(i)})\hat{x}_{k-1}^{(i)} + G_ku_k, \quad i = 1, \ldots, 2n. \]

3. Obtain the predicted mean \( \hat{x}_k^- \)
\[ \hat{x}_k^- = \frac{1}{2n} \sum_{i=1}^{2n} \hat{x}_{k}^{(i)} \]

4. Compute the predicted covariance \( P_k^- \)
\[ P_k^- = \frac{1}{2n} \sum_{i=1}^{2n} (\hat{x}_{k}^{(i)} - \hat{x}_k^-)(\hat{x}_{k}^{(i)} - \hat{x}_k^-)^T + Q_{k-1}. \]

5. Select the sigma points \( \hat{x}_k^{(i)} \) based on the predicted mean and covariance
\[ \hat{x}_k^{(i)} = \hat{x}_k^+ + \tilde{\chi}^{(i)}, \quad i = 1, \ldots, 2n, \]
\[ \tilde{\chi}^{(i)} = (\sqrt{nP_k^-})^T, \quad i = 1, \ldots, n, \]
\[ \tilde{\chi}^{(n+i)} = - (\sqrt{nP_k^-})^T, \quad i = 1, \ldots, n. \]

6. Transform the new sigma points through the measurement model
\[ \hat{y}_k^{(i)} = H\hat{x}_k^{(i)}, \quad i = 1, \ldots, 2n. \]

(7) Calculate the predicted mean of the observation \( \hat{y}_k \)
\[ \hat{y}_k = \frac{1}{2n} \sum_{i=1}^{2n} \hat{y}_k^{(i)}. \]

(8) Compute the predicted covariance matrices of the observation \( P_y \)
\[ P_y = \frac{1}{2n} \sum_{i=1}^{2n} (\hat{y}_k^{(i)} - \hat{y}_k)(\hat{y}_k^{(i)} - \hat{y}_k)^T + R_k. \]

(9) Compute the cross covariance matrices
\[ P_{xy} = \frac{1}{2n} \sum_{i=1}^{2n} (\hat{x}_k^{(i)} - \hat{x}_k^-)(\hat{y}_k^{(i)} - \hat{y}_k)^T. \]

(10) Update the estimation using the Kalman filter algorithm
\[ K_k = P_{xy} \cdot P_y^{-1}, \]
\[ \hat{x}_k = \hat{x}_k^- + K_k(y_k - \hat{y}_k), \]
\[ P_k = P_k^- - K_kP_y K_k^T. \]

4. Simulation results
In this section, under the environment of MATLAB/Simulink, the control system model of sensorless BLDC motor is built in Figure 1 and simulation examples are presented to illustrate the effectiveness of the UKF filter design method developed in this paper.

The parameters of the BLDC motor are given as follows: the stator resistance \( R = 0.62 \Omega \), the equivalent inductance of the stator \( L = M = 1.0 \times 10^{-3} \text{H} \), the maximum of each phase winding permanent magnet flux \( \psi_m = 0.066 \text{Wb} \), the inertia \( J = 0.362 \times 10^{-3} \text{kg m}^2 \), the viscous friction coefficient \( B_p = 9.444 \times 10^{-5} \text{N m s} \), poles of the permanent magnet \( p = 4 \) and simulation step length \( T = 5 \times 10^{-7} \text{s} \), \( x_0 = [0 0 0 0 0]^T \).

According to the UKF algorithm, the results will be more accurate by using the appropriate noise covariance matrices \( Q_k \) and \( R_k \). After repeated experiments, they are chosen as follows: \( Q_k = \text{diag}(0.01 \ 0.01 \ 0.01 \ 0.01 \ 0.01) \), \( R_k = \text{diag}(0.1 \ 0.1 \ 0.1) \).

In order to verify the estimate performance of our algorithm, two different experiments are given to check the speed tracking performance by the effect of the constant load and the changing load.

4.1. Constant load
In this example, the constant load torque is assumed to be \( 2 \text{N m} \), and the reference speed changes from 2000 to 2500 rpm at time \( t = 0.2 \text{s} \). Then, the experiment results (i.e. the performance curves) are obtained and presented in Figure 2.
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Figure 1. The system model.

Figure 2. Performance curves of speed change.

From Figure 2, we can see that the estimated speed can track the actual speed accurately when the reference speed changes. The error between the estimated speed and the actual speed is about 0.2 rpm. From the first performance curve, we can see that the estimated position and the actual position of the rotor are almost the same. And the error is about $1 \times 10^{-3}$ electrical angle, which confirms that the motor can operate normally with little torque ripple. The experiment results illustrate the effectiveness of our filtering algorithm.

4.2. Changing load

In this simulation experiment, the speed changes from 0 to reference speed 2000 rpm. Then, the load torque $T_L = 2$ N m is added to this motor at time $t = 0.1$ s and removed at $t = 0.3$ s. The performance curves are presented in Figure 3.

From Figure 3, we can see that the estimated and the actual speed are almost the same when the load torque suddenly changes. Moreover, the error curve changes slightly.
It can be concluded that the designed UKF algorithm is very effective when the torque suddenly changes.

5. Conclusion

In this paper, a mathematical model of a sensorless BLDC motor system has been built and a new filtering problem has been considered for this model based on the UKF algorithm. In order to evaluate the estimate performance, simulation experiments are presented in the paper. It is obvious to see that, from the simulation results, the accurate estimation performance can be obtained and the effectiveness of our designed algorithm can be demonstrated. Moreover, the sensorless BLDC motor can be controlled precisely according to the designed UKF algorithm.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

This work was supported in part by the National Natural Science Foundation of China [grant number 61374039]; the Program for Professor of Special Appointment (Eastern Scholar) at Shanghai Institutions of Higher Learning; the Program for New Century Excellent Talents in University [grant number NCET-11-1051] and Shanghai Pujiang Program under [grant number 13PJ1406300].

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