Prediction of hidden charm strange molecular baryon states with heavy quark spin symmetry

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Abstract

We have studied the meson-baryon $S$–wave interaction in the isoscalar hidden-charm strange sector with the coupled-channels, $\eta_c\Lambda$, $J/\psi\Lambda$, $D\Xi_c$, $D_s\Lambda_c$, $\bar{D}\Xi'_c$, $\bar{D}^*\Xi'_c$, $\bar{D}^*\Xi''_c$, $\bar{D}^*\Xi''''_c$ in $J^P = 1/2^-$, $J/\psi\Lambda$, $D^*\Xi_c$, $D^*_s\Lambda_c$, $D^*\Xi'_c$, $D^*\Xi''_c$, $D^*\Xi''''_c$, $D^*\Xi''''''_c$ in $3/2^-$ and $D^*\Xi'_c$ in $5/2^-$. We impose constraints of heavy quark spin symmetry in the interaction and obtain the non vanishing matrix elements from an extension of the local hidden gauge approach to the charm sector. The ultraviolet divergences are renormalized using the same meson-baryon- loops regulator previously employed in the non-strange hidden charm sector, where a good reproduction of the properties of the newly discovered pentaquark states is obtained. We obtain five states of $1/2^-$, four of $3/2^-$ and one of $5/2^-$, which could be compared in the near future with forthcoming LHCb experiments. The $5/2^-$, three of the $3/2^-$ and another three of the $1/2^-$ resonances are originated from isoscalar $\bar{D}(s)^\ast\Xi'_c$ and $\bar{D}(s)^\ast\Xi''_c$ interactions. They should be located just few MeV below the corresponding thresholds (4446, 4513, 4588 and 4655 MeV), and would be SU(3)-siblings of the isospin $1/2$ $\bar{D}(s)^\ast\Sigma_c$ quasi-bound states previously found, and that provided a robust theoretical description of the $P_c(4440)$, $P_c(4457)$ and $P_c(4312)$ LHCb exotic states. The another two $1/2^-$ and $3/2^-$ states obtained in this work are result of the $\bar{D}(s)^\ast\Xi_c - D^*_s\Lambda_c$ coupled-channels isoscalar interaction, are significantly broader than the others, with widths of the order of 15 MeV, being $\bar{D}(s)^\ast\Lambda_c$ the dominant decay channel.
I. INTRODUCTION

The discovery of two pentaquark states of hidden charm in Refs. [1, 2] by the LHCb collaboration gave a boost to hadron physics, providing one example of baryon states that challenge the standard wisdom of the three quark structure. The Run-2 experiment of the collaboration [3] has added more precise information where the old narrow state has split into two differentiated structures and a former small fluctuation has given rise to a new distinct peak.

Predictions for these hidden charm states in that energy region had been done before [4–12] and, after their discovery, a large number of works were devoted to explain their possible structure or new reactions where they could be alternatively obtained. Their detailed discussion and comparison with other work has been tackled in a series of review papers [13, 14]. The vast data [3] has stimulated again a large number of theoretical papers [14–35]. The vast majority of those works concludes that the new states are of molecular nature, mostly \( \bar{D}\Sigma_c \) \((J^P = 1/2^-, 3/2^-)\) with isospin \( I = 1/2 \). In some works predictions for more states are done, concretely in Refs. [16] and [36] where four more states of molecular nature involving \( \bar{D}^*(\Sigma_c^*) \) are reported.

The information provided by the new LHCb experiment [3] has been very useful to tune the theories for unknown information. In this sense the masses of the three states reported in Ref. [3] have been used in Ref. [36] to fix the only free parameter of the scheme of Ref. [10] (a subtraction constant in the regularized meson baryon loop functions) to agree with the average experimental mass. With this only experimental input three masses and three widths are obtained in agreement with experiment in Ref. [36].

Certainly a theoretical framework is more appreciated when predictions are made prior to experiment and the latter corroborates the predictions made. In this sense we intend with the present work to make predictions for hidden charm molecular states with strangeness, with the hope that experiments leading to the finding of these states are conducted in the near future.

Predictions for such states of molecular nature were done in Ref. [4], but lacking the present information to establish an origin for the energies, only approximate masses could be predicted. The success of Ref. [36] to describe the experimental data of Ref. [3] encourages us to use the same formalism, adapted to the analogous states of Ref. [36] with \( c\bar{c} \) and a strange quark, to make predictions for hidden-charm strange isoscalar molecules. For this purpose we use the same scheme as in Ref. [11], implementing heavy quark spin symmetry (HQSS) in the interaction of the coupled channels and use dimensional regularized loops with the same subtraction constant employed in Ref. [36].

The work of Ref. [4] has been complemented recently to evaluate decay modes and rates of the states found in Refs. [4, 37], estimating also the uncertainties [38].

II. FORMALISM

We study states that can couple to \( J/\psi\Lambda \), the channel where, by analogy to the \( J/\psi p \) of Ref. [3], the new states could be observed. Thus we study states with isospin \( I = 0 \). The spin, however, can be 1/2 or 3/2 with negative parity for the \( S \)–wave interaction that we shall consider.

The couple channels considered are:
i) $J = 1/2$, $I = 0$
\[ \eta_c \Lambda, J/\psi \Lambda, \bar{D}\Xi_c, \bar{D}^*\Xi_c, \bar{D}_s \Lambda_c, \bar{D}_s^* \Lambda_c, \bar{D}_s^* \Xi_c, \bar{D}_s^* \Xi_c. \]

ii) $J = 3/2$, $I = 0$
\[ J/\psi \Lambda, \bar{D}^*\Xi_c, \bar{D}_s \Lambda_c, \bar{D}_s^* \Xi_c, \bar{D}_s^* \Xi_c. \]

In addition, $\bar{D}_s^* \Xi_c$ could also couple to $J = 5/2$ in $S$–wave.

A. Lowest order HQSS constraints

We take into account the lowest order (LO) constraints of HQSS \[39, 41\], which states that interactions should be independent of the spin of the heavy quark ($Q$), up to corrections of the order of $\mathcal{O}(\Lambda_{QCD}/m_Q)$, with $m_Q$ the heavy quark mass. The spin-dependent interactions are proportional to the chromomagnetic moment of the heavy quark, and hence, they are of the order of $1/m_Q$. The total angular momentum $\vec{J}$ of the hadron is always a conserved quantity, but in this case the spin of the heavy quark $\vec{S}_Q$ is also conserved in the $m_Q \to \infty$ limit. Consequently, the spin of the light degrees of freedom $\vec{S}_l = \vec{J} - \vec{S}_Q$ is a conserved quantity in that limit.

Here we follow the same formalism as in Ref. \[10\] which we briefly describe below. As we have seen, in the isoscalar hidden-charm strange sector, there are 16 orthogonal states in the physical basis composed of meson-baryon $S$–wave pairs. Next we introduce a different basis, that we will call HQSS basis, for which it is straightforward to implement the LO HQSS constraints. In the HQSS basis we will classify the states in terms of the quantum numbers,

- $J = 1/2$, $3/2$ and $5/2$: total angular momentum of the meson-baryon system
- $L = 1/2$ and $3/2$, and $S_{cc} = 0$ and 1: total angular momentum of the light-quarks and the $c\bar{c}$ subsystems, respectively.
- $\ell_M = 0 \ [\eta_c, J/\psi]$ and $1/2 \ [D^{(*)}_{(c)}]$, and $\ell_B = 1/2 \ [\Lambda]$, $0 \ [\Xi_c]$ and $1 \ [\Xi_c^{(*)}]$: total angular momentum of the light quarks in the meson and in the baryon, respectively. Note that for the set of meson-baryon states considered here to construct the isoscalar hidden-charm strange sector, these quantum numbers also determine the total angular momentum of the heavy quarks in the meson and baryon $s_M$ and $s_B$, respectively.
- $S = 0 \ [D^{(*)}]$ and $-1 \ [D^{(*)}_s]$: strangeness of the meson, which also fixes that of the baryon since the total strangeness of the meson-baryon pair must be $-1$.

Note that we assume that all orbital angular momenta are zero, since we are dealing with ground state baryons, and that we have considered that the $\Xi_c$ and $\Xi_c'$ baryons are HQSS states with well defined $\ell_B = 0$ and 1, respectively. This latter approximation seems to be quite accurate \[42, 43\], and it is also implicitly assumed in the Review of Particle Physics \[44\].

The approximate HQSS of QCD leads (neglecting $\mathcal{O}(\Lambda_{QCD}/m_Q)$ corrections) to:

$$\langle \ell_M | H^{QCD} | S_{cc} L; J \rangle \langle \ell_M \ell_B S' | H^{QCD} | \ell_M \ell_B S \rangle = \delta_{J,J'} \delta_{S_{cc}} \delta_{L,L'} \langle \ell_M | H^{QCD} | \ell_M \ell_B S' \rangle = \langle \ell_M | H^{QCD} | \ell_M \ell_B S \rangle$$

(1)

\[1\] Throughout this work we use $\Xi_c^{(*)}$ to refer to $\Xi_c'$ or $\Xi_c$.  

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Hence, the reduced matrix elements neither depend on $S_{c\bar{c}}$ nor on $J$, because QCD dynamics is invariant under separate spin rotations of the charm quark and antiquark. Thus, one can transform a $c\bar{c}$ spin singlet state into a spin triplet state by means of a rotation that commutes with $H_{QCD}$, i.e., a zero cost of energy. Thus, we have a total of 11 unknown low energy terms (LET’s):

- One LET associated to $L = 3/2$

$$\hat{\lambda} = \langle \ell'_M = 1/2, \ell'_B = 1, S' = 0 | H_{QCD} | \ell_M = 1/2, \ell_B = 1, S = 0 \rangle_{L = 3/2}$$  \hspace{1cm} (2)

- Ten LET’s associated to $L = 1/2$ that form a $4 \times 4$ symmetric matrix, $\hat{\mu}_{ij}$, defined as

$$\hat{\mu}_{ij} = \langle j | H_{QCD} | i \rangle_{L = 1/2}$$

$$|i\rangle, |j\rangle = |\ell_M = 0, \ell_B = 1/2, S = 0 \rangle_{L = 1/2}, |\ell_M = 1/2, \ell_B = 0, S = 0 \rangle_{L = 1/2},$$

$$|\ell_M = 1/2, \ell_B = 0, S = 1 \rangle_{L = 1/2}, |\ell_M = 1/2, \ell_B = 1, S = 0 \rangle_{L = 1/2}$$  \hspace{1cm} (3)

In the HQSS basis, the $H_{QCD}$ is a block diagonal matrix, i.e., up to $O(\Lambda_{QCD}/m_Q)$ corrections, $H_{QCD} = \text{Diag}(\hat{\mu}, \hat{\mu}, \hat{\mu}, \hat{\lambda}, \hat{\lambda}, \hat{\lambda}, \hat{\lambda})$, where $\hat{\mu}$ and $\hat{\lambda}$ are symmetric matrices of dimension 4 and 1, respectively. This represents an enormous simplification, since a $16 \times 16$ symmetric matrix has, in principle, 136 independent elements.

To exploit Eq. (1), one should express the isoscalar strange hidden-charm uncoupled meson–baryon states in terms of the HQSS basis. The two basis are related by a Racah rotation, which is discussed in detail in Ref. [10],

$$|\ell_M s_M j_M S; \ell_B s_B j_B; J\rangle = \sum_{L S_{c\bar{c}}} \left[ (2S_{c\bar{c}} + 1)(2L + 1)(2j_M + 1)(2j_B + 1) \right]^{1/2}$$

$$\times \left\{ \begin{array}{c} \ell_M \ell_B L \\ s_M s_B S_{c\bar{c}} \\ j_M j_B J \end{array} \right\} |L S_{c\bar{c}}; J\rangle \langle \ell_M \ell_B S |$$  \hspace{1cm} (4)

where the angular momenta of the light and heavy degrees of freedom in the meson $\ell_M$ and $s_M$, and in the baryon $\ell_B$ and $s_B$, together with the meson ($j_M$) and baryon ($j_B$) spins are coupled, by means of the $9j$–symbol [14], to $L$, $S_{c\bar{c}}$ and $J$, respectively, in the HQSS basis. At the same time, looking at the columns of the $9j$, $\ell_M$ and $\ell_B$, $s_M$ and $s_B$, and $L$ and $S_{c\bar{c}}$ are coupled to $j_M$, $j_B$ and $J$, respectively.

From Eq. (1), we find that in the isoscalar strange hidden-charm sector the most general interactions compatible with LO HQSS read...
\begin{itemize}
  \item $J = 1/2, \ I = 0$

  \[
  \begin{pmatrix}
  \eta_{c}\Lambda & J/\psi\Lambda & \bar{D}\Xi_{c} & \bar{D}_{s}\Lambda_{c} & \bar{D}\Xi'_{c} & \bar{D}^*\Xi_{c} & \bar{D}^*\Xi'_{c} & \bar{D}^*\Xi^*_{c} \\
  \mu_{1} & 0 & -\frac{\mu_{12}}{2} & -\frac{\mu_{13}}{2} & -\frac{\mu_{14}}{2} & \frac{\sqrt{3}\mu_{12}}{2} & \frac{\sqrt{3}\mu_{13}}{2} & \frac{\sqrt{3}\mu_{14}}{2\sqrt{3}} \\
  0 & \mu_{1} & \frac{\sqrt{3}\mu_{12}}{2} & \frac{\sqrt{3}\mu_{13}}{2} & \frac{\sqrt{3}\mu_{14}}{2} & \frac{\mu_{12}}{2} & \frac{\mu_{13}}{2} & \frac{5\mu_{14}}{6} \\
  -\frac{\mu_{12}}{2} & \frac{\sqrt{3}\mu_{12}}{2} & \mu_{2} & \mu_{23} & 0 & 0 & 0 & -\frac{2\lambda - \mu_{4}}{3\sqrt{3}} \\
  -\frac{\mu_{13}}{2} & \frac{\sqrt{3}\mu_{13}}{2} & \mu_{23} & \mu_{3} & 0 & 0 & 0 & -\frac{2\lambda - \mu_{4}}{3\sqrt{3}} \\
  \mu_{12} & \frac{\mu_{12}}{2\sqrt{3}} & 0 & 0 & \frac{\mu_{24}}{\sqrt{3}} & \mu_{2} & \mu_{23} & \frac{2\mu_{24}}{3} \\
  -\frac{\mu_{14}}{2\sqrt{3}} & \frac{\mu_{14}}{2} & 0 & 0 & \frac{\mu_{24}}{\sqrt{3}} & \mu_{2} & \mu_{3} & \frac{2\mu_{24}}{3} \\
  -\frac{\mu_{14}}{6} & \frac{\mu_{14}}{2\sqrt{3}} & \frac{\mu_{24}}{\sqrt{3}} & \frac{\mu_{34}}{\sqrt{3}} & -\frac{2(\lambda - \mu_{4})}{3\sqrt{3}} & \frac{2\mu_{24}}{3} & \frac{2\mu_{34}}{3} & \frac{1}{9}(2\lambda + 7\mu_{4}) \\
  \sqrt{\frac{2}{3}}\mu_{14} - \frac{\sqrt{2}\mu_{14}}{3} - \sqrt{\frac{2}{3}}\mu_{24} - \sqrt{\frac{2}{3}}\mu_{34} & \frac{1}{3}\sqrt{2}(\mu_{4} - \lambda) & \frac{\sqrt{2}\mu_{24}}{3} & \frac{\sqrt{2}\mu_{34}}{3} & \frac{1}{3}(\lambda - \mu_{4}) & \frac{1}{3}(\lambda + 8\mu_{4}) \\
  \end{pmatrix}
  
  \end{equation}

  \item $J = 3/2, \ I = 0$

  \[
  \begin{pmatrix}
  J/\psi\Lambda & \bar{D}^*\Xi_{c} & \bar{D}^*\Xi'_{c} & \bar{D}\Xi^*_{c} & \bar{D}^*\Xi^*_{c} \\
  \mu_{1} & \mu_{12} & \mu_{13} & -\frac{\mu_{14}}{3} & \frac{\mu_{14}}{\sqrt{3}} & \frac{\sqrt{5}\mu_{14}}{3} \\
  \mu_{12} & \mu_{2} & \mu_{23} & -\frac{\mu_{24}}{3} & \frac{\mu_{24}}{\sqrt{3}} & \frac{\sqrt{5}\mu_{24}}{3} \\
  \mu_{13} & \mu_{23} & \mu_{3} & -\frac{\mu_{34}}{3} & \frac{\mu_{34}}{\sqrt{3}} & \frac{\sqrt{5}\mu_{34}}{3} \\
  -\frac{\mu_{14}}{3} & -\frac{\mu_{24}}{3} & -\frac{\mu_{34}}{3} & \frac{1}{3}(8\lambda + \mu_{4}) & \frac{\lambda - \mu_{4}}{3\sqrt{3}} & \frac{1}{9}\sqrt{5}(\lambda - \mu_{4}) \\
  \frac{\sqrt{5}\mu_{14}}{3} & \frac{\sqrt{5}\mu_{24}}{3} & \frac{\sqrt{5}\mu_{34}}{3} & \frac{\sqrt{5}\mu_{4}}{3} & \frac{1}{3}(2\lambda + \mu_{4}) & \frac{1}{3}\sqrt{5}(\mu_{4} - \lambda) \\
  \frac{\sqrt{5}\mu_{14}}{3} & \frac{\sqrt{5}\mu_{24}}{3} & \frac{\sqrt{5}\mu_{34}}{3} & \frac{1}{3}\sqrt{5}(\lambda - \mu_{4}) & \frac{1}{3}\sqrt{5}(\mu_{4} - \lambda) & \frac{1}{3}(4\lambda + 5\mu_{4}) \\
  \end{pmatrix}
  
  \end{equation}

  \item $J = 5/2, \ I = 0$

  \[
  \bar{D}^*\Xi^*_{c} : \hat{\lambda} 
  \end{equation}

\end{itemize}
B. Coupled-channels unitarity and the Bethe-Salpeter equation (BSE)

For each $J$, we use the BSE in coupled channels

$$T = [1 - V G]^{-1} V,$$  \hspace{1cm} (8)

where $V_{ij}$ is the two-particle irreducible amplitude (potential), and $G$ is a diagonal matrix constructed out of the loop functions for intermediate meson baryon states.

For any choice of $V$ and of the renormalization scheme adopted to evaluate Re$[G]$ on the real axis, we find that above the lowest of the meson-baryon thresholds, the discontinuity of $T^{-1}$ is equal to that of $-G$, fulfilling in this way exact unitary in coupled channels [46].

Here we implement LO HQSS relations in the kernel $V$, which is taken from Eqs. (5) and (6) for $J = 1/2$ and $J = 3/2$ respectively, while for $D^* \Xi^*_c \to D^* \Xi^*_c$, $J = 5/2$, the single-channel potential is $\hat{\lambda}$ (Eq. (7)). On the other hand, we calculate the loop functions in dimensional regularization using the formula of Refs. [47, 48], and take advantage of our previous work of Ref. [36] to fix $a(\mu = 1 \text{ GeV}) = -2.09$. This subtraction constant was determined in this latter reference to agree with the average experimental mass of the three non-strange pentaquarks reported by the LHCb Collaboration in [3].

C. Interactions from the local hidden gauge (LHG) approach

HQSS gives us the structure of $V$ in terms of the irreducible matrix elements of the interaction, but the strength is not given. Then, as in Ref. [10], we take these matrix elements using an extension of the LHG approach [49–52]. This picture is based on the exchange of vector mesons between the meson and the baryon. Actually, it has a direct connection with HQSS. Indeed, the dominant terms are those which exchange light vector mesons ($\rho$, $\omega$, $\phi$, $K^*$), since the exchange of heavy mesons is suppressed by large masses in the propagators involving them. The $c$ and $\bar{c}$ quarks in this case act as spectators in the interaction, which then does not depend upon them, and automatically the independence on the spin of the heavy quarks (or any other property) is fulfilled. This is not the case when heavy vectors are exchanged, and neither should it be, since these terms are of order $(1/m_Q^2)$ and then sub-leading in the $1/m_Q$ counting.

The evaluation of the matrix elements in Ref. [10] was done following the work of [4] where an extrapolation to SU(4) was done. In between it has become apparent that the use of SU(4) is unnecessary and that a perfect counting stems from the overlap of quarks of suitable wave functions that were used in Ref. [55]. We use the same procedure here and explain it below adapted to the present case.

We first write down the baryon wave functions that we use, where we single out the heavy quarks, and impose the spin-flavour symmetry in the light quarks:

1) Λ: \[ \frac{1}{\sqrt{2}}(\phi_{MS}\chi_{MS} + \phi_{MA}\chi_{MA}), \] where $\chi_{MS}$, $\chi_{MA}$ are the mixed symmetric and mixed antisymmetric representations of the spin $1/2$ of the baryons [53], $\phi_{MS}$, $\phi_{MA}$ are the flavour mixed symmetric and antisymmetric wave functions for the SU(3) baryons. Here, we must divert from the prescription of [53] to make the phase convention consistent with the use of chiral Lagrangians and we follow the convention of Refs.
Hence, with the symmetry in the last two particles, we have

\[ \phi_{MS} = -\frac{1}{\sqrt{2}} \left[ \frac{uds - dus}{\sqrt{2}} + \frac{usd - dsu}{\sqrt{2}} \right], \]
\[ \phi_{MA} = -\frac{1}{\sqrt{6}} \left[ \frac{uds - dus}{\sqrt{2}} + \frac{dus - uds}{\sqrt{2}} - 2 \frac{sd}{\sqrt{2}} \right]. \] (9) (10)

2) \( \Lambda_c^+ : c \frac{1}{\sqrt{2}} (ud - du) \chi_{MA}. \)

3) \( \Xi_c^+ : c \frac{1}{\sqrt{2}} (us - su) \chi_{MA} \) and \( \Xi_c^0 : c \frac{1}{\sqrt{2}} (ds - sd) \chi_{MA}. \)

4) \( \Xi_c'^+ : c \frac{1}{\sqrt{2}} (us + su) \chi_{MS} \) and \( \Xi_c'^0 : c \frac{1}{\sqrt{2}} (ds + sd) \chi_{MS}. \)

5) \( \Xi_c^* : c \frac{1}{\sqrt{2}} (us + su) \chi_S \) and \( \Xi_c^* : c \frac{1}{\sqrt{2}} (ds + sd) \chi_S. \)

with \( \chi_S \) the 3/2 spin symmetric wave function [53]. The light-quark parts of the above wave-functions are consistent with \( \ell_B = 0 \) for \( \Lambda_c \) and \( \Xi_c \), and \( \ell_B = 1 \) for \( \Xi_c' \) and \( \Xi_c'^* \). On the other hand, our conventions for the isospin doublets are \( (D^+, -D^0), (\bar{D}^0, D^-), (\Xi_c^+, \Xi_c^0). \)

Thus, for instance, we have that the isoscalar \( \bar{D} \Xi_c \) wave function is

\[ |\bar{D} \Xi_c, I = 0 \rangle = \frac{1}{\sqrt{2}} |\bar{D}^0 \Xi_c^0 - D^- \Xi_c^0 \rangle. \] (11)

![Diagram](image-url)

FIG. 1. Diagrams for \( \bar{D} \Xi_c \rightarrow \bar{D} \Xi_c \) transitions.

We must then evaluate the matrix elements of Fig. 1. Let us evaluate explicitly the first one of these diagrams. The lower vertex is easily evaluated using the Lagrangian

\[ \mathcal{L}_{VBB} \equiv g \left\{ \frac{1}{\sqrt{2}} (u\bar{u} - d\bar{d}), \rho^0 \right\}, \]
\[ \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}), \omega \} \], (12)

which implicitly assumes that the \( \gamma^\mu \) vertex for low energy baryons is converted into \( \gamma^0 \) which is unity in this case. Hence, we have

\[ \frac{1}{\sqrt{2}} \langle c(ds - sd) \chi_{MA} | g \left\{ \frac{1}{\sqrt{2}} (u\bar{u} - d\bar{d}) \right\} | c(ds - sd) \chi_{MA} \rangle = \frac{g}{\sqrt{2}} \left\{ -1, \rho^0 \right\}, \]
\[ 1, \omega \}, (13)\]
where $g$ is the coupling of the LHG, $g = M_V/2f_\pi$ ($M_V \approx 800$ MeV, $f_\pi = 93$ MeV). The upper vertex can be evaluated in a similar way using wave functions for the mesons as shown in Ref. [58] (see section IIA of that reference), but it is also shown there that for practical reasons it is easier to get the vertex from the Lagrangian

$$\mathcal{L}_{VPP} = -i g \langle [P, \partial_\mu P] V^\mu \rangle,$$

with $\langle \ldots \rangle$ the matrix trace, and

$$P = \begin{pmatrix} \pi^0/\sqrt{2} + \eta/\sqrt{6} & | & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & | & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 & D^- \\ K^- & | & \bar{K}^0 & -\eta/\sqrt{3} + \sqrt{2} \eta' D^- & \bar{D}^0 \\ D^0 & | & D^+ & -\eta/\sqrt{3} + \sqrt{2} \eta' D^+ & \eta_c \end{pmatrix},$$

$$V_\mu = \begin{pmatrix} \rho^0/\sqrt{2} + \omega/\sqrt{2} & | & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & | & -\rho^0/\sqrt{2} + \omega/\sqrt{2} & K^{*0} & D^{*-} \\ K^{*-} & | & \bar{K}^{*0} & \bar{D}^{*-} & \phi \\ D^{*0} & | & D^{*+} & D^{*++} & J/\psi \end{pmatrix}_\mu.$$

One finds readily that the vertex for $\bar{D}^0 D^0 \rho$ ($\omega$) is given by

$$t = -g(p^\mu + p'^\mu) \epsilon_\mu \left\{ \frac{1}{\sqrt{2}}, \frac{\rho^0}{\sqrt{2}} \right\}.$$ (17)

with $\epsilon_\mu$ the polarization vector of the exchanged virtual vector meson. As mentioned above, the baryon vertex selects the exchange of the temporal component. We thus see that in the first and last diagrams of Fig. 1 there is a cancellation of the $\rho$ and $\omega$ contributions, assuming equal $\rho$ and $\omega$ masses, and the contribution only comes from the two crossed terms, second and third diagrams of Fig. 1. Repeating the procedure for these terms we finally find

$$\hat{\mu}_2 = -g^2 (p^0 + p'^0) \frac{1}{m_V^2} = -\frac{1}{4} f_\pi^2 (p^0 + p'^0),$$ (18)

where $p^0$ and $p'^0$ are the energies of the external mesons.

It is easy to see that in $D_s \Lambda_c \rightarrow D_s \Lambda_c$ we cannot exchange $\rho^0$, $\omega$ since there are no $u$, $d$ quarks in $D_s$ and we cannot exchange $\phi$ since there are no strange quarks in $\Lambda_c$. Thus,

$$\hat{\mu}_3 = 0.$$ (19)

We can follow the same steps to find $\hat{\mu}_{23}$ and the other coefficients. It will be also needed to consider $D_{s(\bar{s})}^*$ vector-mesons as external legs. For the three vector vertex, we use

$$\mathcal{L}_{VV^2}^{(3V)} = ig \langle [V^\nu, \partial_\mu V^\nu] V^\mu \rangle,$$ (20)
and it is shown in Ref. [54] that in the limit of small external three momenta, which we assume here, it has the same structure as Eq. (14) for the equivalent pseudoscalar mesons, with $V^\mu$ playing the role of the exchanged vector, and the additional $\vec{c}\vec{c}'$ factor, with $\vec{c}$ and $\vec{c}'$ the polarization vectors of the external vector-mesons. Indeed, it is easy to see that $V^\mu$ cannot be an external meson. If this were the case, $\mu$ should be a space component since $\epsilon^0 = 0$ for vectors at rest, then $\partial_i V_i (i = 1, 2, 3)$ is proportional to the three momenta, which are neglected in this approach.

Finally we obtain

$$\hat{\mu}_1 = \hat{\mu}_3 = \hat{\mu}_{24} = \hat{\mu}_{34} = 0$$

$$\hat{\mu}_2 = \hat{\mu}_{23}/\sqrt{2} = \hat{\lambda} = -F, \quad F = \frac{1}{4f^2}(p^0 + p')$$

$$\hat{\mu}_{12} = -\hat{\mu}_{13}/\sqrt{2} = \hat{\mu}_{14}/\sqrt{3} = -\sqrt{\frac{2}{3}} \frac{m_{c'}}{m_{D^*}} F,$$

where we have explicitly implemented the reduction factor $m_{c'}/m_{D^*}$ in the matrix elements that involve change of $c$ quark content in the mesons, which proceed in our approach via the exchange of $D^*$. The null $\hat{\mu}_{34}$ and $\hat{\mu}_{24}$ values stem from our neglect of pion exchange, as done in Refs. [10, 36].

We should make here two important remarks:

- Up to some minus signs in off diagonal matrix elements, which can be re-absorbed by conveniently redefining the overall phase of the involved mesons and baryons, the $\bar{D}^{(*)}\Xi_c - \bar{D}^{(*)}\Xi_c'$ sub-matrices here are identical to those used in Ref. [36] for the $\bar{D}^{(*)}\Sigma_c - \bar{D}^{(*)}\Sigma_c'$ in the non-strange hidden charm sector with $I = 1/2$. This is the case for all $J = 1/2, 3/2$ and $5/2$ angular momenta and when the LET’s $\hat{\lambda}$ and $\hat{\mu}_4$ that appear here are identified to $\lambda_2$ and $\mu_3$ introduced in Ref. [36]. Moreover all these terms are predicted to be equal to $-F$ within the LHG approach, as deduced from SU(3)-light flavor symmetry. The correspondence between $(\bar{D}^{(*)}\Xi'_c)_{I=0}$ and $(\bar{D}^{(*)}\Sigma_c)_{I=3/2}$, and between $(\bar{D}^{(*)}\Xi'_c)_{I=0}$ and $(\bar{D}^{(*)}\Sigma'_c)_{I=1/2}$ is natural because in all these baryons the light quarks are coupled to $\ell_B = 1$, and in both cases the isospin meson-baryon coupling is symmetric.

A straightforward consequence is that the pattern of seven states obtained in Ref. [36], located few MeV below the $\bar{D}^{(*)}\Sigma_c$ thresholds, is expected to be also found here. We will see that seven quasi-bound $\bar{D}^{(*)}\Xi'_c$ isoscalar states, SU(3) partners of the former ones, are dynamically generated. Two $(J = 1/2, 3/2)$ HQSS doublets and a $(J = 1/2, 3/2, 5/2)$ HQSS triplet. The latter multiplet is originated from the $\bar{D}^*\Xi_c'$ interaction, the heaviest doublet from the $\bar{D}^*\Xi_c$, while the other one corresponds to $\bar{D}\Xi'_c (J = 1/2)$ and $\bar{D}\Xi'_c (J = 3/2)$. In the non-strange sector, the first doublet was identified in [36] with the $P_c(4440)$ and the $P_c(4457)$ LHCb states, while the quasi-bound $D\Sigma_c$ state had a mass and width compatible with those of the $P_c(4312)$.

- In the strict heavy quark limit, $\hat{\mu}_{12}$ and $\hat{\mu}_{13}$ also vanish within the HLG approach. Then, the $\bar{D}^{(*)}\Xi_c$ and $\bar{D}^{(*)}_s\Lambda_c$ decouple from the rest of channels in the $J = 1/2$ and $J = 3/2$ sectors. In both cases, the isoscalar interaction is determined from $\hat{\mu}_2$ and $\hat{\mu}_{23}$, and it reads

$$V_{\bar{D}^{(*)}\Xi_c, \bar{D}^{(*)}_s\Lambda_c} = F \left( \begin{array}{cc} -1 & -\sqrt{2} \\ -\sqrt{2} & 0 \end{array} \right)$$

(24)
Diagonalizing the above matrix, we find eigenvalues \( F \) and \(-2F\). The latter one is attractive and the corresponding eigenvector is dominated by the \( D^{(*)}\Xi_c \) component, i.e. \( (\sqrt{3} \hat{D}^{(*)}\Xi_c, \hat{D}^{(*)}\Lambda_c)/\sqrt{3} \). Hence, we should expect the existence of a \( J = 1/2 \) isoscalar \( D\Xi_c \) quasi-bound state around 4337 MeV and an isoscalar \( (J = 1/2, 3/2) \) HQSS doublet located near the \( D^{*}\Xi_c \)-threshold (4479 MeV).

On the other hand, in this \( m_Q \to \infty \) limit, the \( \hat{D}^{(*)}\Xi_c^* \) interactions become diagonal in the meson-baryon basis, with a common strength of \(-F\) since \( \hat{\mu}_4 = \hat{\lambda} \). Therefore, the \( \hat{D}^{(*)}\Xi_c \) states should be more bound than those generated from the \( \hat{D}^{(*)}\Xi_c^* \) interaction. Moreover in the latter case, all the binding energies should be similar, as it occurred in the non-strange case discussed in [30]. There, all states were found around 10 MeV below the corresponding \( \hat{D}^{(*)}\Sigma_c^{(*)} \)-thresholds.

Finally, let us note that in the non-strange hidden charm sector the channel equivalent to \( \hat{D}^{(*)}\Xi_c \) would be the \( \hat{D}^{(*)}\Lambda_c \), with \( \ell_B = 0 \) in both cases, but with a totally different meson-baryon isospin structure. Indeed, because of the isospin couplings, the \( \hat{D}^{(*)}\Lambda_c \) interaction turns out to be repulsive \((+F)\) [10], instead of attractive as we found here. As a consequence, we did not find \( \hat{D}^{(*)}\Lambda_c \) hadron molecules in our previous study of Ref. [36] in the non-strange sector.

\section{Results}

In Fig. 2 we show the results for \(|T|^2\) for the diagonal matrix elements of Eq. (8) and \( J = 1/2 \). The peaks indicate where states appear and one can also see qualitatively the width of these states. More details can be seen by looking explicitly for poles in the second Riemann sheet [10], and evaluating the couplings obtained from the amplitude close to the pole,

\[
T_{ij} \simeq \frac{g_i g_j}{\sqrt{s} - \sqrt{s_R}}, \quad \sqrt{s_R} = M + i \Gamma/2 \quad (25)
\]

We show this information in Table II. We find five states with \( J = 1/2 \), at \((M + i \Gamma/2) = (4276.59 + i 7.67)\) MeV, \((4429.89 + i 7.92)\) MeV, \((4436.70 + i 1.17)\) MeV, \((4580.96 + i 2.44)\) MeV, and \((4650.86 + i 2.59)\) MeV. We observe that the widths are small in all cases. In the table we stress with thick lettering the biggest couplings, which indicate the dominance of some channels. We see that the dominant channels for the states reported before are \( \hat{D}\Xi_c \), \( \hat{D}^*\Xi_c \), \( \hat{D}\Xi_c' \), \( \hat{D}^*\Xi_c' \) and \( \hat{D}^*\Xi_c^* \), respectively. The two lightest states have also some significant couplings to the open \( \hat{D}_s\Lambda_c \) and \( \hat{D}_s^*\Lambda_c \) channels in each case, as expected from the discussion at the end of Subsect. II C, and that gives rise to widths of around 15 MeV. The couplings of all states to \( J/\psi \Lambda \), the channel where these states are most likely to be observed, are relatively small, but sufficiently large to provide production rates in observable ranges, if one compares their strengths with the ones obtained for \( J/\psi N \) in the hidden charm sector without strangeness [10] where the new pentaquark peaks have been observed [3].

In the \( J = 3/2 \) sector we also find states, first depicted by means of \(|T|^2\) in Fig. 3. Once again, we summarize the information of the poles and couplings in Table III. We find four states at \((4429.52 + i 7.67)\) MeV, \((4506.99 + i 1.03)\) MeV, \((4580.96 + i 0.34)\) MeV and \((4650.58 + i 1.48)\) MeV. They are again narrow and couple mostly to the \( \hat{D}^*\Xi_c \), \( \hat{D}\Xi_c \), \( \hat{D}^*\Xi_c' \) and \( \hat{D}^*\Xi_c^* \) channels, respectively. The lightest state also couples to the open \( \hat{D}_s^*\Lambda_c \) channel, as expected, which leads to a sizable width of around 15 MeV. The couplings to \( J/\psi \Lambda \) are again sufficiently large compared to those of \( J/\psi N \) in the hidden charm non-strange sector.
FIG. 2. Results of the modulus squared of some diagonal elements of the amplitude matrix, as a function of $\sqrt{s}$, for the $J = 1/2$, $I = 0$ sector.

FIG. 3. Same as Fig. 2 for $J = 3/2$.

case, such that the observation of peaks in the $J/\psi \Lambda$ channel should not be a problem. We should note that the channels $\bar{D}^* \Xi_c$, $\bar{D}^* \Xi'_c$, $\bar{D}^* \Xi^*_c$ participate both in $J = 1/2$ and $J = 3/2$. The $J = 1/2$ and $J = 3/2$ states obtained which couple mostly to each of these channels are practically degenerate but have a different width. This is very similar to the situation found in Ref. [36] for the $P_c(4440)$ and $P_c(4457)$ states of Ref. [3], which in our approach appear near degenerate in $1/2^−$, $3/2^−$ but with a larger width for the $1/2^−$ state. Here, the situation is similar and, although all the states are quite narrow, the $1/2^−$ states have slightly larger widths than their corresponding $3/2^−$ ones.

In the $J = 5/2^−$ sector we have just the $\bar{D}^* \Xi^*_c$ state with a mass 4650.56 MeV with a zero width, which is also degenerate with the $1/2^−$ and $3/2^−$ states of the same structure.

One should remind the reader that there are suggestions of reactions to see these states, like the $\Lambda_b \rightarrow J/\psi K^0 \Lambda$ reaction [59], $\Lambda_b \rightarrow J/\psi \eta \Lambda$ [60], $\Xi_b^0 \rightarrow J/\psi K^- \Lambda$ [61]. It should be noted that the $\Xi_b^0 \rightarrow J/\psi K^- \Lambda$ reaction has already been observed by the LHCb collaboration [62] and results of the LHCb Run-2 experiment should be under present analysis, as anticipated in Ref. [62], searching for possible peaks in the $J/\psi \Lambda$ mass distribution.
| \( \eta_c \Lambda \) | \( J_\psi \Lambda \) | \( D \Xi_c \) | \( D_s \Lambda_c \) | \( D \Xi_c' \) | \( D* \Xi_c \) | \( D* \Lambda_c \) | \( D* \Xi_c' \) | \( D* \Xi_c^* \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| **4276.59 + i7.67** |
| \( g_i \) | 0.17 - i0.03 | 0.29 - i0.07 | **2.93 + i0.08** | 0.76 + i0.31 | 0.00 + i0.01 | 0.01 + i0.02 | 0.01 + i0.04 | 0.01 - i0.02 | 0.01 - i0.03 |
| \(|g_i|\) | 0.17 | 0.30 | **2.93** | 0.82 | 0.01 | 0.02 | 0.05 | 0.02 | 0.03 |
| **4429.84 + i7.92** |
| \( g_i \) | 0.29 - i0.11 | 0.17 - i0.07 | 0.00 - i0.00 | 0.00 - i0.00 | 0.15 - i0.26 | **2.78 + i0.01** | 0.66 + i0.32 | 0.01 + i0.05 | 0.01 + i0.03 |
| \(|g_i|\) | 0.31 | 0.18 | 0.00 | 0.00 | 0.30 | **2.78** | 0.73 | 0.05 | 0.04 |
| **4436.70 + i1.17** |
| \( g_i \) | 0.24 + i0.03 | 0.14 + i0.01 | 0.00 - i0.00 | 0.00 - i0.00 | **1.72 - i0.04** | 0.22 - i0.31 | 0.06 - i0.01 | 0.01 - i0.04 | 0.01 - i0.03 |
| \(|g_i|\) | 0.24 | 0.14 | 0.00 | 0.00 | **1.72** | 0.38 | 0.07 | 0.04 | 0.03 |
| **4580.96 + i2.44** |
| \( g_i \) | 0.12 - i0.00 | 0.37 - i0.04 | 0.02 - i0.01 | 0.02 - i0.01 | 0.03 - i0.00 | 0.02 - i0.02 | 0.03 - i0.02 | **1.57 - i0.17** | 0.00 + i0.02 |
| \(|g_i|\) | 0.12 | 0.37 | 0.02 | 0.02 | 0.03 | 0.03 | 0.03 | **1.58** | 0.02 |
| **4650.86 + i2.59** |
| \( g_i \) | 0.32 - i0.05 | 0.19 - i0.03 | 0.02 - i0.01 | 0.03 - i0.02 | 0.02 - i0.00 | 0.01 - i0.01 | 0.02 - i0.01 | 0.01 - i0.00 | **1.41 - i0.23** |
| \(|g_i|\) | 0.32 | 0.19 | 0.03 | 0.04 | 0.02 | 0.02 | 0.02 | 0.02 | **1.43** |
IV. CONCLUSIONS

We have studied the interaction of coupled channels of meson-baryon involving $c\bar{c}$ quarks, an $s$ quark and zero isospin. There are nine channels coupling to $1/2^-$ in $S$-wave, six channels coupling to $3/2^-$ and just one channel contributing to $5/2^-$. The interaction is constructed implementing the symmetries of heavy quark spin physics which provides a few independent matrix elements. The strength of these terms is taken from an extension of the LHG approach to the charm sector which turns out to be rather successful in the description of the $\Omega_c$ states of LHCb [63] in Refs. [55, 64], and the recent pentaquark states of Ref. [3] in Ref. [36]. This approach in the light sector leads to the LO chiral Lagrangians. In the leading terms, obtained from the exchange of light vectors, the $c$ quarks are mere spectators and hence the interaction is independent of them and automatically respects the rules of heavy quark spin symmetry. We obtain five $1/2^-$ states, four $3/2^-$ states and one $5/2^-$ state. We clearly identify three near degenerate HQSS multiplets corresponding to states that couple most strongly to $D^*_s\Xi_c$ ($1/2^-$, $3/2^-$), $\bar{D}^*\Xi_c$ ($1/2^-$, $3/2^-$) and $D^*\Xi_c^*$ ($1/2^-$, $3/2^-$, $5/2^-$). Other states appear with just one spin, which are the states coupling mostly to $\bar{D}\Xi_c$ ($1/2^-$), $D\Xi_c^*$ ($1/2^-$) and $D\Xi_c^*$ ($3/2^-$), though the mass difference between the two latter states is very similar to that of the $\Xi_c^*$ and $\Xi_c$ baryons. Indeed, these two dynamically generated states form a further HQSS doublet broken by the $\Xi_c^* - \Xi_c$ mass splitting.

The spectrum of states found in this work can be easily understood from the symmetry remarks made at the end of Subsec. II C. We have found two distinct sets of states. The first set (two $1/2^-$ resonances and a $3/2^-$ one) results from the $D^{(*)}\Xi_c - \bar{D}^{(*)}\Lambda_c$ coupled-channels isoscalar interaction. These resonances are significantly broader than the others, with widths of the order of 15 MeV, and they decay mostly to $D_s^{(*)}\Lambda_c$. Seen as $D^{(*)}\Xi_c$ quasi-bound states, they would be placed significantly below the corresponding thresholds, around 50-60 MeV.

The second set is composed of quasi-bound $\bar{D}^{(*)}\Xi_c$ isoscalar states, located around 5 to 10 MeV below their thresholds. They are very narrow and would be the SU(3)-partners

| $J/\psi\Lambda$ | $\bar{D}^*\Xi_c$ | $D^*_s\Lambda_c$ | $D^*\Xi'_c$ | $\bar{D}\Xi'_c$ | $D\Xi'^*_c$ | $\bar{D}\Xi'^*_c$ |
|----------------|-----------------|-----------------|-------------|-------------|-------------|-------------|
| 4429.52 + i7.67 | $g_1$ 0.31 – i0.10 | $D^{*}\Xi'_c$ 0.67 + i0.32 0.00 + i0.00.02 0.00 – i0.06 0.00 + i0.0004 | $|g_1|$ 0.32 | $2.77$ | $0.74$ | $0.02$ | $0.06$ | $0.04$ |
| 4506.99 + i1.03 | $g_1$ 0.27 – i0.02 0.02 – i0.03 0.02 – i0.02 0.00 – i0.03 | $1.56 - i0.07$ | $0.00 - i0.05$ | $|g_1|$ 0.27 | 0.03 | 0.03 | 0.03 | $1.56$ | 0.05 |
| 4580.96 + i0.34 | $g_1$ 0.14 – i0.01 0.01 – i0.01 0.01 – i0.01 | $1.54 - i0.02$ | $0.02 - i0.00$ | $0.00 - i0.04$ | $|g_1|$ 0.14 | 0.01 | 0.02 | $1.54$ | 0.02 | 0.04 |
| 4650.58 + i1.48 | $g_1$ 0.29 – i0.02 0.02 – i0.01 0.03 – i0.02 0.03 – i0.01 0.03 – i0.00 | $1.40 - i0.13$ | $|g_1|$ 0.29 | 0.03 | 0.03 | 0.03 | 0.03 | $1.41$ |
of the isospin 1/2 \( \bar{D}^{(*)}\Sigma^* \) states obtained in [36] using the same formalism, that naturally accommodated the \( P_c(4440) \), \( P_c(4457) \) and \( P_c(4312) \) LHCb exotic states.

The success in the description of the pentaquark states reported in Ref. [3] with the present approach, and the use here of the same regulator used there for the loop functions, makes the results obtained here rather credible, and it will be very interesting to compare them with results of future experiments, in particular from the study of the \( \Xi^- \rightarrow J/\psi K^-\Lambda \) reaction that could be the first one providing information of these states from the analysis of the Run-2 experiments.

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1. R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 115, 072001 (2015).
2. R. Aaij et al. [LHCb Collaboration], Chin. Phys. C 40, no. 1, 011001 (2016).
3. R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 122, no. 22, 222001 (2019) doi:10.1103/PhysRevLett.122.222001 [arXiv:1904.03947 [hep-ex]].
4. J. J. Wu, R. Molina, E. Oset and B. S. Zou, Phys. Rev. Lett. 105, 232001 (2010).
5. W. L. Wang, F. Huang, Z. Y. Zhang and B. S. Zou, Phys. Rev. C 84, 015203 (2011).
6. Z. C. Yang, Z. F. Sun, J. He, X. Liu and S. L. Zhu, Chin. Phys. C 36, 6 (2012).
7. S. G. Yuan, K. W. Wei, J. He, H. S. Xu and B. S. Zou, Eur. Phys. J. A 48, 61 (2012).
8. J. J. Wu, T.-S. H. Lee and B. S. Zou, Phys. Rev. C 85, 044002 (2012).
9. C. Garcia-Recio, J. Nieves, O. Romanets, L. L. Salcedo and L. Tolos, Phys. Rev. D 87, 074034 (2013) doi:10.1103/PhysRevD.87.074034 [arXiv:1302.6938 [hep-ph]].
10. C. W. Xiao, J. Nieves and E. Oset, Phys. Rev. D 88, 056012 (2013).
11. T. Uchino, W. H. Liang and E. Oset, Eur. Phys. J. A 52, no. 3, 43 (2016).
12. M. Karliner and J. L. Rosner, Phys. Rev. Lett. 115, no. 12, 122001 (2015).
13. H. X. Chen, W. Chen, X. Liu and S. L. Zhu, Phys. Rept. 639, 1 (2016).
14. Y. R. Liu, H. X. Chen, W. Chen, X. Liu and S. L. Zhu, Prog. Part. Nucl. Phys. 107, 237 (2019) doi:10.1016/j.ppnp.2019.04.003 [arXiv:1903.11976 [hep-ph]].
15. H. X. Chen, W. Chen and S. L. Zhu, [arXiv:1903.11001 [hep-ph]].
16. M. Z. Liu, Y. W. Pan, F. Z. Peng, M. Sanchez Sanchez, L. S. Geng, A. Hosaka and M. P. Valderrama, [arXiv:1903.11560 [hep-ph]].
17. J. He, Eur. Phys. J. C 79, no. 5, 393 (2019) [arXiv:1903.11872 [hep-ph]].
18. F. K. Guo, H. J. Jing, U. G. Meißner and S. Sakai, Phys. Rev. D 99, no. 9, 091501 (2019) [arXiv:1903.11503 [hep-ph]].
19. R. Chen, Z. F. Sun, X. Liu and S. L. Zhu, [arXiv:1903.11013 [hep-ph]].
20. H. Huang, J. He and J. Ping, [arXiv:1904.00221 [hep-ph]].
21. A. Ali and A. Y. Parkhomenko, Phys. Lett. B 793, 365 (2019) [arXiv:1904.00446 [hep-ph]].
22. Y. Shimizu, Y. Yamaguchi and M. Harada, [arXiv:1904.00587 [hep-ph]].
[23] C. J. Xiao, Y. Huang, Y. B. Dong, L. S. Geng and D. Y. Chen, arXiv:1904.00872 [hep-ph].
[24] Z. H. Guo and J. A. Oller, Phys. Lett. B 793, 144 (2019) arXiv:1904.00851 [hep-ph].
[25] X. Cao and J. p. Dai, arXiv:1904.06015 [hep-ph].
[26] H. Mutuk, arXiv:1904.09756 [hep-ph].
[27] X. Z. Weng, X. L. Chen, W. Z. Deng and S. L. Zhu, arXiv:1904.09891 [hep-ph].
[28] R. Zhu, X. Liu, H. Huang and C. F. Qiao, arXiv:1904.10285 [hep-ph].
[29] J. R. Zhang, arXiv:1904.10711 [hep-ph].
[30] X. Y. Wang, X. R. Chen and J. He, Phys. Rev. D 99, no. 11, 114007 (2019) doi:10.1103/PhysRevD.99.114007 arXiv:1904.11706 [hep-ph].
[31] M. I. Eides, V. Y. Petrov and M. V. Polyakov, arXiv:1904.11616 [hep-ph].
[32] Z. G. Wang, arXiv:1905.02802 [hep-ph].
[33] L. Meng, B. Wang, G. J. Wang and S. L. Zhu, arXiv:1905.04113 [hep-ph].
[34] T. Gutsche, M. A. Ivanov, J. G. Krner and V. E. Lyubovitskij, Particles 2, 339 (2019) doi:10.3390/particles2020021 arXiv:1905.06219 [hep-ph].
[35] J. B. Cheng and Y. R. Liu, arXiv:1905.08605 [hep-ph].
[36] C. W. Xiao, J. Nieves and E. Oset, arXiv:1904.01296 [hep-ph].
[37] J. J. Wu, R. Molina, E. Oset and B. S. Zou, Phys. Rev. C 84, 015202 (2011) arXiv:1011.2399 [nucl-th].
[38] C. W. Shen, J. J. Wu and B. S. Zou, arXiv:1906.03896 [hep-ph].
[39] N. Isgur and M. B. Wise, Phys. Lett. B 232, 113 (1989).
[40] M. Neubert, Phys. Rept. 245, 259 (1994) [hep-ph/9306320].
[41] A.V. Manohar and M.B. Wise, Heavy Quark Physics, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, vol. 10 (Cambridge University Press, Cambridge, England, 2000).
[42] K. C. Bowler et al. [UKQCD Collaboration], Phys. Rev. D 54, 3619 (1996) doi:10.1103/PhysRevD.54.3619 [hep-lat/9601022].
[43] C. Albertus, J. E. Amaro, E. Hernandez and J. Nieves, Nucl. Phys. A 740, 333 (2004) doi:10.1016/j.nuclphysa.2004.04.114 [nucl-th/0311100].
[44] M. Tanabashi et al. [Particle Data Group], Phys. Rev. D 98, no. 3, 030001 (2018). doi:10.1103/PhysRevD.98.030001
[45] M. E. Rose, Elementary Theory of Angular Momentum, John Wiley, 1957.
[46] J. Nieves and E. Ruiz Arriola, Nucl. Phys. A 679, 57 (2000) doi:10.1016/S0375-9474(00)00321-3 [hep-ph/9907469].
[47] J. A. Oller and U. G. Meissner, Phys. Lett. B 500, 263 (2001) [hep-ph/0011146].
[48] E. Oset, A. Ramos and C. Bennhold, Phys. Lett. B 527, 99 (2002) Erratum: [Phys. Lett. B 530, 260 (2002)] [nucl-th/0109006].
[49] M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, Phys. Rev. Lett. 54, 1215 (1985).
[50] M. Bando, T. Kugo and K. Yamawaki, Phys. Rept. 164, 217 (1988).
[51] U. G. Meissner, Phys. Rept. 161, 213 (1988).
[52] H. Nagahiro, L. Roca, A. Hosaka and E. Oset, Phys. Rev. D 79, 014015 (2009) arXiv:0809.0943 [hep-ph].
[53] F. F. Close, An introduction to Quarks and Partons (Academic Press, London 1979).
[54] E. Oset and A. Ramos, Eur. Phys. J. A 44, 445 (2010) doi:10.1140/epja/i2010-10957-3 arXiv:0905.0973 [hep-ph].
[55] V. R. Debastiani, J. M. Dias, W. H. Liang and E. Oset, Phys. Rev. D 97, no. 9, 094035 (2018).
[56] K. Miyahara, T. Hyodo, M. Oka, J. Nieves and E. Oset, Phys. Rev. C 95, no. 3, 035212 (2017) [arXiv:1609.00895 [nucl-th]].
[57] R. P. Pavao, W. H. Liang, J. Nieves and E. Oset, Eur. Phys. J. C 77, no. 4, 265 (2017) [arXiv:1701.06914 [hep-ph]].
[58] S. Sakai, L. Roca and E. Oset, Phys. Rev. D 96, no. 5, 054023 (2017).
[59] J. X. Lu, E. Wang, J. J. Xie, L. S. Geng and E. Oset, Phys. Rev. D 93, 094009 (2016) [arXiv:1601.00075 [hep-ph]].
[60] A. Feijoo, V. K. Magas, A. Ramos and E. Oset, Eur. Phys. J. C 76, no. 8, 446 (2016) [arXiv:1512.08152 [hep-ph]].
[61] H. X. Chen, L. S. Geng, W. H. Liang, E. Oset, E. Wang and J. J. Xie, Phys. Rev. C 93, no. 6, 065203 (2016) [arXiv:1510.01803 [hep-ph]].
[62] R. Aaij et al. [LHCb Collaboration], Phys. Lett. B 772, 265 (2017) doi:10.1016/j.physletb.2017.06.045 [arXiv:1701.05274 [hep-ex]].
[63] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 118, no. 18, 182001 (2017) [arXiv:1703.04639 [hep-ex]].
[64] G. Montaña, A. Feijoo and Á. Ramos, Eur. Phys. J. A 54, no. 4, 64 (2018) [arXiv:1709.08737 [hep-ph]].