CONTROLLABILITY AND STABILITY CAPACITY OF
A ROLL PAIR MOTION RESEARCH

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Abstract
The process of deriving the equation of the perturbed motion of a roll pair during the processing of leather material is discussed in the article, taking into account the influence of dynamic factors. It is shown that one of the reasons for the unstable stress state on the contact surfaces in the roller mechanism are dynamic factors arising from inaccuracies in the manufacture of its individual parts, assembly defects, and the occurrence of an oscillatory process in the roller mechanism, as well as due to the non-uniform thickness of the processed material at its gripping during starting and stopping the machine. Methods for determining optimal controls are shown; they provide asymptotic stability of the unperturbed motion of a roll pair and the torque applied to the upper roll as a function of generalized coordinates.

It is shown that the width of the contact strip of the clamp, which depends on the radii of the rolls and the hardness of the coatings, has a significant effect on the efficiency of the rolls. The larger the shaft radius, the lower the actual pressure per unit contact area.

It is shown that the squeezing efficiency increases with the improvement of the conditions for the removal of the squeezable liquid from the rolls (with their horizontal arrangement), with an increase in its temperature and a decrease in viscosity. Efficiency decreases with increasing material speed and thickness.

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Keywords: processing, spreading, pressure, shafts, clamping, skin, stability, humidity, deformation, technology.

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1. Introduction
Roller mechanisms are widely used due to the simplicity of design, the possibility of a continuous technological operation when processing the material and the combination of several functions. The variety of operations performed by roller mechanisms has not made it possible to create a unified system for their design, due to the difference in technological tasks and phenomena occurring in the contact zone of the rollers.

Most roller mechanisms perform two or more functions at the same time: active and auxiliary ones. The latter most often refers to the feeding the leather material; the squeezing mechanism can be wringing, pressing and deforming one.

Now consider the dynamic factors affecting the operation of the roll pair. One of the reasons for the unstable stress state on the contact surfaces in the roller mechanism is a dynamic factor arising from inaccuracies in the manufacture of its individual parts, assembly defects, and the occurrence of an oscillatory process in the roller mechanism, as well as due to the non-uniform thickness of the processed material, during its capture, machine start and stop operations.

The reason for the occurrence of an oscillatory process in the roller mechanism may be inaccuracies in the manufacture of machine parts. Let, for example, the geometric axis of the roller do not coincide with its axis of rotation. Then there is a kinematic excitation of harmonic nature, which leads to vibrations and resonance in the system. Vibrations in the roller mechanism are transmitted to the machine frame and require a definition of the inertial and elastic characteristics of its elements [1, 2].

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With the effective use of a biocatalytic modifier, the stiffness index of the semi-finished leather product is reduced by two times compared to the semi-finished leather product developed according to the existing technology, while maintaining its mechanical strength [3]. It should be noted that this paper does not consider the removal of excess fluid from the semi-finished leather product.

The articles [4–6] present the results of studies on the control of technological parameters of the operation to extract excess liquid from multilayer moisture-saturated fibrous materials using leather as an example. Mathematical dependences of the amount of moisture removed for each layer of wet leather on the feed rate between the rotating working rollers and their pressing force were obtained.

However, a critical analysis of the operation of the experimental roller stand used in these works shows that when processing a multilayer package of wet leathers, the control of their feed rate between the rotating working shafts is of great importance. Consequently, the greater the thickness of the leather package, the correspondingly lower the feed rate should be, but the authors of these works did not take this into account when conducting experiments. From this it should be noted that by controlling the feed rate of processed wet skins between the working shafts, it would be possible to achieve a significant increase in the quality of their processing.

There are publications devoted to the development and improvement of leather machines. In [7], the results of studies on determining the ratio of forces in the process of feeding a semi-finished leather product to the working area of a multi-operational machine by a conveying device are presented, and in [8] a solution to the problems of ensuring the stable motion of the feeding mechanism of multi-operation machines is proposed. These studies were carried out for manual work, where the issues of process control and the stability of the movement of the roll pair, depending on the uneven thickness of the processed material, were not considered.

The study in [9] is devoted to the solution of contact interaction in the roll pair. Mathematical models of the contact stresses distribution patterns are developed. Research is underway aimed at solving the problems of using tooth-lever differential gears in technological roller machines [10, 11]. However, these works do not take into account the dynamic factors of the process of contact interaction in roller pairs with a leather semi-finished product.

2. Materials and methods

The aim of the study is to determine the optimal controls that ensure the asymptotic stability of the unperturbed motion of the roll pair and the torque applied to the upper roll as a function of generalized coordinates.

Roll pairs are widely used in many industries. When studying the dynamics of a roll pair, it is necessary to proceed from the forces acting on the rolls during operation. The magnitude and direction of the acting forces during the capture of the processed material and in the steady state are different. They also depend on many parameters and factors, i.e. from the diameters of the rolls, which may be equal or different, from the kinematic connection between the rolls, which may be rigid or one of the rolls is free, which will rotate due to friction; from the installation of rolls that can be installed horizontally or obliquely, one above the other with the location of their axes of rotation on a vertical or inclined plane, also vertically; at the same time, to create a clamp between the rolls, the upper, lower or both rolls can be movable; swaths can be with hard or elastic coatings, which can be moisture-permeable or impervious, depending on the technology, combinations of them can be selected. In addition, one of the rolls or both rolls may be composite. In all cases, the action of force, movement must be continuous and stable. Therefore, it is important to ensure the stability of the process of pressing leather materials.

To determine the value of the basic parameters for controlling the motion of a roll pair, let’s first compose a differential equation of motion in the Lagrange form with holonomic servo constraints, obtained in [12]:

\[
\frac{d}{dt}\left(\frac{\partial \tau}{\partial \dot{q}_i}\right) - \frac{\partial \tau}{\partial q_i} = \dot{Q}_i + \sum_j \mu_j \alpha_j. \tag{1}
\]
Here

\[ \hat{T} = \hat{T}(q_j, \dot{q}_j, t). \]

is the kinetic energy of the system;

\[ \dot{q}_j = \sum_i Q_i \frac{\partial q_i}{\partial q_j}, \]

are the generalized forces referred to coordinates \( q_j \);

\[ a_{ij} = \sum_i A_i \frac{\partial q_i}{\partial q_j}, \]

are the known functions of time and \( q_j \).

In the center of the fixed roll, let's install a fixed system of coordinates \( Oxyz \), and the \( x \) axis is directed along the roll axis, and the \( y \) axis in the direction of leather motion and the \( z \) axis is directed vertically upwards. The origin of the moving coordinate system \( O_1x_1y_1z_1 \) is set to the center of mass of the moving roller. Let's direct the \( x_1 \) and \( y_1 \) axes parallel to the \( x \) and \( y \) axes, respectively, and let's direct the \( z \) axis along the \( z \) axis.

To determine the positions of the fixed roll, semi-finished leather product and the upper movable roll, five independent parameters must be set. Let's take the following values as generalized coordinates:

\[ \dot{q}_1 = \varphi - \text{angle of rotation of the fixed roll}; \]

\[ \dot{q}_2 = \varphi - \text{angle of rotation of the movable roll around the } x \text{ axis}; \]

\[ \dot{q}_3 = y - \text{ordinates of leather}; \]

\[ \dot{q}_4 = z \text{c} - \text{applicate of the center of mass of the movable roll}; \]

\[ \dot{q}_5 = \psi - \text{deviation of the movable roll from the } x \text{ axis}. \]

At the end of the movable roll, let's install special devices that at each time point provide for the following conditions:

\[ \vartheta = \vartheta_M = \vartheta_N, \]

where \( \vartheta_M \) and \( \vartheta_N \) are the linear velocities of the point lying on the rim of the movable and fixed rolls (Fig. 1).

In order for condition (2) to be fulfilled, the velocity of a point lying on a movable roll relative to the \( x_1 \) axis must be zero. In addition, to fulfill condition (2) at each time point (if the system moves from a state of rest), the following conditions should be fulfilled:

\[ \varphi_1 = \frac{\vartheta}{R_i}. \]
or

\[
\dot{\phi}_2 = \frac{\dot{\theta}}{R_2},
\]

\[
\dot{\phi}_1 = \frac{1}{R_1} y, \quad \dot{\phi}_2 = \frac{1}{R_2} y.
\] (3)

Since angular velocities of the movable and fixed rolls, and the leather feed rate are small, the fulfillment of condition (2) could be done using special equipment.

Let’s compose the equations of motion (1) taking into account relation (3) obtained in [13]:

\[
2a_1 \ddot{y} = a_5 + a_6 \mu_1,
\] (4)

\[
2a_2 \ddot{z}_c + a_3 \ddot{\psi} \cos \psi - a_3 \sin \psi \ddot{\psi} = a_7 - 2c \dot{z}_c + \mu_2,
\] (5)

\[
a_3 \cos \psi \cdot \ddot{z}_c + 2a_4 \cos^2 \psi \cdot \ddot{\psi} - 2a_5 \sin \psi \cdot \dot{z}_c \dot{\psi} - 3a_4 \sin 2\psi \cdot \ddot{\psi}^2 = a_8 \cos \psi - a_9 \sin 2\psi + \mu_3,
\] (6)

where

\[
a_1 = \frac{1}{2} \left( J_1 \frac{1}{R_1^2} + J_2 \frac{1}{R_2^2} + M_3 \right), \quad a_2 = \frac{1}{2} (M_2 + M_4 + M_5), \quad a_3 = \frac{1}{2} ( M_4 - M_5),
\]

\[
a_4 = \frac{I^2}{8} (M_4 + M_5), \quad a_5 = (M_1^0 - M_1^1) \frac{1}{R_1^1} \left( M_2 - M_2^1 \right) \frac{1}{R_2^1},
\]

\[
a_6 = \frac{R_2^1 + R_2^2 + R_2^3 R_2^1 R_2^2}{R_1^1 R_2^1}, \quad a_7 = g(M_5 - M_2 - M_4), \quad a_8 = -(m_4 + m_5) \cdot g \frac{1}{2}, \quad a_9 = \frac{c^2}{4}.
\]

The resulting equations (4)–(6), together with the constraint equations (2), completely describe the motion of the squeezing machine shown in Fig. 1.

3. Results and discussion

Next, let’s consider the problems of controllability of technological processes during the pressing of a semi-finished leather product.

When implementing technological processes, the parameters that characterize these processes must change according to certain laws (or be constant).

The purpose of the control is to ensure the necessary velocity between the various nodes of the system or to achieve the maximum power acting to a certain part of the squeezing machine. The processes that must change according to certain laws are velocity and power under the influence of external sources, realized by signals.

Now let’s compose the equation of the perturbed motion of a roll pair. For the unperturbed motion, let’s take the motion defined by equations (4)–(6) and assume that these equations have a particular solution [14, 15]:

\[
y = y_0, \quad \psi = 0, \quad z_c = \frac{a_7}{2c}, \quad \mu_1 = -\frac{a_5}{a_6}, \quad \mu_2 = 0, \quad \mu_3 = -a_8.
\]

Under perturbed motion there are:

\[
y = y_0 + x_1, \quad z_c = \frac{a_7}{2c} + x_2, \quad \psi = x_3, \quad \mu_1 = -\frac{a_5}{a_6} + u_1, \quad \mu_2 = u_2, \quad \mu_3 = -a_8 + u_3,
\]

where \(x_1, x_2\) and \(x_3\) are the deviations of the perturbed motion from the unperturbed motion.
Then the equations of perturbed motion have the following form:

\[ 2a_1 \ddot{x}_1 = a_6 u_1, \]  
\[ 2a_2 \ddot{x}_2 + a_3 \ddot{x}_3 \cos(x_3) - a_3 \sin(x_3) \cdot \dot{x}_3^2 = -2c \ddot{x}_2 + u_2, \]  
\[ a_3 \cos x_3 \ddot{x}_2 + 2a_4 \cos^2(x_3) \cdot \ddot{x}_3 - 2a_3 \sin x_3 \ddot{x}_2 \ddot{x}_3 - 3a_4 \sin x_3 \dot{x}_3^2 \frac{\dot{x}_3}{a_3} = a_6 (\cos x_3 - 1) - a_9 \sin 2x_3 + u_3. \]

It is easy to see that the right-hand sides of these equations vanish under conditions. It is easy to see that the right-hand sides of these equations vanish under conditions.

\[ x_1 = x_2 = x_3 = u_1 = u_2 = u_3 = 0. \]
Expanding the right-hand sides into series in powers of \( x_1, x_2, x_3 \) and restricting ourselves to terms of the first order of smallness, let’s obtain the equations of the first approximation:

\[ 2a_1 \ddot{x}_1 = a_6 u_1, \]
\[ 2a_2 \ddot{x}_2 + a_3 \ddot{x}_3 + 2c \ddot{x}_2 = u_2 + R_2, \]
\[ a_3 \ddot{x}_2 + 2a_4 \ddot{x}_3 + 2a_9 x_3 = u_3 + R_3, \]

where symbols \( R_2 \) and \( R_3 \) denote the terms the measurement of which in \( x_2, x_3, \dot{x}_3 \) is higher than the first making the necessary calculations, let’s write these equations in normal form. To do this, introducing notation, \( x_{2i-1} = y_i, x_{2i} = y_i (i = 1, 2, 3) \) and doing the necessary calculations, let’s obtain:

\[ \dot{y}_1 = y, \quad \dot{y}_2 = a_6 u_1, \quad \dot{y}_3 = a_6 u_1, \quad \dot{y}_4 = \frac{c(a_2^2 + a_4)}{a_2 a_1} y_3 + \frac{2a_2 a_9}{a_1} y_5 + \left( \frac{a_2^2 + a_4}{a_1} \right) u_2 - \frac{a_3}{a_1} u_3 + R_3, \quad \dot{y}_5 = y_6, \]
\[ \dot{y}_6 = \frac{2a_3 c}{a_1} y_1 - \frac{4a_2 a_9}{a_1} y_5 - \frac{2a_3}{a_1} u_2 + \frac{2a_2}{a_1} u_3 + R_4, \]

where

\[ a_1 = 4a_2 a_4 - a_2^2 = \frac{l^2}{4} (M_2 M_4 + M_3 M_5 + 4 M_4 M_5). \]

The system of equations (11) in a matrix formulation takes the following form or in a short form:

\[ \dot{y} = Ay + Bu + W, \]

where \( W \) denotes that the terms the dimension of which in \( y_1, y_2, y_6 \) and \( \dot{y}_1, \dot{y}_2, \ldots, \dot{y}_6 \) is higher than the first.

\[
\begin{bmatrix}
\dot{y}_1 \\
\dot{y}_2 \\
\dot{y}_3 \\
\dot{y}_4 \\
\dot{y}_5 \\
\dot{y}_6
\end{bmatrix}
=

\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & -c(a_2^2 + a_4) & 0 & 2a_2 a_3 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 2a_3 c & 0 & 4a_2 a_3 & 0
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4 \\
y_5 \\
y_6
\end{bmatrix}
+

\begin{bmatrix}
0 & 0 & 0 \\
a_6 & \frac{2a_1}{a_6} & 0 \\
0 & 0 & 0 \\
a_1 & \frac{a_2^2 + a_4}{a_1} & -\frac{a_3}{a_1} \\
0 & 0 & 0 \\
\frac{a_3}{a_1} & \frac{2a_2}{a_1} & \frac{a_2}{a_1}
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2 \\
u_3
\end{bmatrix}
+
W,
Now consider the matrix $[14–18]$:

$$K = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 & -a_3 & 0 & 0 & a_2 & 0 & 0 & 0 & a_4 & 0 & c_4 & 0 \\
1 & 0 & 0 & -a_3 & 0 & 0 & a_2 & 0 & 0 & 0 & -a_4 & 0 & 0 & 0 & c_4 & 0 \\
0 & 0 & 0 & 0 & 1 & M_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_5 & M_4 M_5 & 0 & 0 & e^2 M_4 M_5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_5 & M_4 M_5 & 0 & 0 & 0 & 0
\end{bmatrix}.$$  

Let’s take the first 6 columns of matrix $K$:

$$(B, AB) = \begin{bmatrix}
0 & 0 & 0 & 1 & a_6 & 0 \\
a_6 & 0 & 0 & 0 & 0 & 0 \\
\frac{a_6}{2a_1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a_2^2 + a_{10} & 0 & -a_3 \\
0 & \frac{a_2^2 + a_{10}}{2a_2 - a_{10}} & -a_3 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{2a_2}{a_{10}} & -\frac{2a_2}{a_{10}} & 0 \\
0 & -\frac{a_3}{a_{10}} & \frac{2a_2}{a_{10}} & 0 & 0 & 0
\end{bmatrix}.$$  

Given that, the determinant does not change if any linear combination of other rows is added to one of its rows, then:

$$(B, AB) = 4a_2 a_4 (2a_4 - a_3) - a_2^2 (2a_4 - a_3) = (4a_2 a_4 - a_2^2) (2a_4 - a_3).$$

where

$$2a_4 - a_3 = \frac{j^2}{4} (M_4 + M_5) - \frac{j^2}{2} (M_4 - M_5) \neq 0,$$

$$4a_2 a_4 - a_2^2 = 4 \left( \frac{1}{4} (M_2 + M_4 + M_5) - \frac{j^2}{8} (M_4 + M_5) - \frac{j^2}{4} (M_4^2 - 2M_4 M_5 + M_5^2) \right) =$$

$$\frac{j^2}{4} (M_2 + M_4 + M_5) (M_4 + M_5) - \frac{j^2}{4} (M_4^2 - 2M_4 M_5 + M_5^2) = \frac{j^2}{4} M_2 (M_4 + M_5) \neq 0.$$  

Thus, the rank of matrix $K$ is 6, and, consequently, the system (7)–(9) is completely controllable.
To solve the stabilization problem, let’s use the following theorem [19]: Controlled system (12) can always be stabilized with respect to the manifold defined by constraints (2).

Let’s choose values of $u_j$ for the nonlinear system (12) as control actions:

$$u_j^0 = \gamma_j \cdot y_j,$$

or in expanded form:

$$u_1^0 = \gamma_{11} y_1 + \gamma_{21} y_2 + \gamma_{31} y_3 + \gamma_{41} y_4 + \gamma_{51} y_5 + \gamma_{61} y_6,$$

$$u_2^0 = \gamma_{12} y_1 + \gamma_{22} y_2 + \gamma_{32} y_3 + \gamma_{42} y_4 + \gamma_{52} y_5 + \gamma_{62} y_6,$$

$$u_3^0 = \gamma_{13} y_1 + \gamma_{23} y_2 + \gamma_{33} y_3 + \gamma_{43} y_4 + \gamma_{53} y_5 + \gamma_{63} y_6,$$

(14)

where $\gamma_j$ are the constants, $(i = 1, \ldots, 6); (j = 1, 2, 3)$.

Substituting (14) into equations (12), let’s obtain the nonlinear equations of perturbed motion:

$$\dot{y}_1 = y_2, \quad \dot{y}_2 = c_{21} y_1 + c_{22} y_2 + c_{23} y_3 + c_{24} y_4 + c_{25} y_5 + c_{26} y_6,$$

$$\dot{y}_3 = y_4, \quad \dot{y}_4 = c_{41} y_1 + c_{42} y_2 + c_{43} y_3 + c_{44} y_4 + c_{45} y_5 + c_{46} y_6 + R_1^2,$$

$$\dot{y}_5 = y_6, \quad \dot{y}_6 = c_{61} y_1 + c_{62} y_2 + c_{63} y_3 + c_{64} y_4 + c_{65} y_5 + c_{66} y_6 + R_1^2,$$

(15)

where

$$c_{21} = \frac{a_6}{2a_6} \gamma_{11}, \quad c_{22} = \frac{a_6}{2a_6} \gamma_{21}, \quad c_{23} = \frac{a_6}{2a_6} \gamma_{31}, \quad c_{24} = \frac{a_6}{2a_6} \gamma_{41}, \quad c_{25} = \frac{a_6}{2a_6} \gamma_{51},$$

$$c_{26} = \frac{a_6}{2a_6} \gamma_{61}, \quad c_{41} = \frac{(a_2^2 + a_{10}) \cdot \gamma_{12} - 2a_2 a_3 \cdot \gamma_{13}}{2a_2 \cdot a_10}, \quad c_{42} = \frac{(a_2^2 + a_{10}) \cdot \gamma_{22} - 2a_2 a_3 \cdot \gamma_{23}}{2a_2 \cdot a_10},$$

$$c_{43} = \frac{(a_2^2 + a_{10}) \cdot \gamma_{32} - 2a_2 a_3 \cdot \gamma_{33}}{2a_2 \cdot a_10}, \quad c_{44} = \frac{(a_2^2 + a_{10}) \cdot \gamma_{42} - 2a_2 a_3 \cdot \gamma_{43}}{2a_2 \cdot a_10},$$

$$c_{45} = \frac{4a_2 a_3 a_0 + (a_2^2 + a_{10}) \cdot \gamma_{52} - 2a_2 a_3 \cdot \gamma_{53}}{2a_2 \cdot a_10}, \quad c_{46} = \frac{(a_2^2 + a_{10}) \cdot \gamma_{62} - 2a_2 a_3 \cdot \gamma_{63}}{2a_2 \cdot a_10},$$

$$c_{61} = \frac{2a_2 \cdot \gamma_{13} - a_3 \cdot \gamma_{12}}{a_10}, \quad c_{62} = \frac{2a_2 \cdot \gamma_{23} - a_3 \cdot \gamma_{22}}{a_10}, \quad c_{63} = \frac{2a_2 \cdot \gamma_{33} - a_3 \cdot \gamma_{32} + 2a_2 \cdot \gamma_{33}}{a_10},$$

$$c_{64} = \frac{2a_2 \cdot \gamma_{43} - a_3 \cdot \gamma_{42}}{a_10}, \quad c_{65} = \frac{2a_2 \cdot \gamma_{53} - a_3 \cdot \gamma_{52} - 4a_2 a_3}{a_10}, \quad c_{66} = \frac{2a_2 \cdot \gamma_{63} - a_3 \cdot \gamma_{62}}{a_10}. $$

Without violating the constancy of coefficients $\gamma_j$, let’s assume that:

$$\gamma_{13} = \frac{a_3}{2a_2} \gamma_{12}, \quad \gamma_{23} = \frac{a_3}{2a_2} \gamma_{22}, \quad \gamma_{32} = 2c, \quad \gamma_{43} = \frac{a_3}{2a_2} \gamma_{42},$$

$$\gamma_{52} = 0, \quad \gamma_{53} = 2a_9, \quad \gamma_{63} = \frac{a_3}{2a_2} \gamma_{62}.$$
Then equation (15) takes the following form:

\[
\begin{align*}
\dot{y}_1 &= y_2, \quad \dot{y}_2 = c_{21} y_1 + c_{22} y_2 + c_{23} y_3 + c_{24} y_4 + c_{25} y_5 + c_{26} y_6, \\
\dot{y}_3 &= y_4, \quad \dot{y}_4 = c_{41} y_1 + c_{42} y_2 + c_{44} y_3 + c_{46} y_6, \\
\dot{y}_5 &= y_6, \quad \dot{y}_6 = c_{63} y_3 + c_{65} y_5.
\end{align*}
\]

(16)

Let’s construct a characteristic determinant for system (15):

\[
\begin{vmatrix}
-\lambda & 1 & 0 & 0 & 0 & 0 \\
c_{21} & c_{22} - \lambda & c_{23} & c_{24} & c_{25} & c_{26} \\
0 & 0 & -\lambda & 1 & 0 & 0 \\
c_{41} & c_{42} & c_{43} & c_{44} - \lambda & 0 & c_{46} \\
0 & 0 & 0 & 0 & -\lambda & 1 \\
0 & 0 & c_{63} & 0 & c_{65} & -\lambda
\end{vmatrix} = 0.
\]

(17)

Expanding the determinant (17) and transforming, let’s obtain the characteristic equation in the following form:

\[
\begin{align*}
\lambda^6 + (-c_{22} - c_{44}) \cdot \lambda^5 + (c_{22} c_{44} - c_{24} c_{42} - c_{21}) \cdot \lambda^4 + & (c_{21} c_{44} - c_{24} - c_{23} c_{41}) \cdot \lambda^3 + \\
& (c_{43} c_{22} - c_{63} c_{46} - c_{42} c_{23} + c_{21} c_{46} c_{63} - c_{23} c_{41}) \cdot \lambda^2 + (c_{22} c_{63} c_{46} - c_{42} c_{63} c_{26} - c_{41} c_{63} c_{26}) \cdot \lambda - & c_{41} c_{63} c_{25} - c_{41} c_{25} c_{63} = 0,
\end{align*}
\]

or

\[
b_0 \lambda^6 + b_1 \lambda^5 + b_2 \lambda^4 + b_3 \lambda^3 + b_4 \lambda^2 + b_5 \lambda + b_6 = 0,
\]

(18)

where

\[
b_0 = 1, \quad b_1 = -c_{22} - c_{44}, \quad b_2 = c_{22} c_{44} - c_{24} c_{42} - c_{21}, \quad b_3 = c_{21} c_{44} - c_{24} - c_{23} c_{41}, \\
b_4 = c_{43} c_{22} - c_{63} c_{46} - c_{42} c_{23} + c_{21} c_{46} c_{63} - c_{23} c_{41}, \\
b_5 = c_{22} c_{63} c_{46} - c_{42} c_{63} c_{26} - c_{41} c_{63} c_{26}, \quad b_6 = -c_{42} c_{63} c_{25} - c_{41} c_{25} c_{63}.
\]

Let’s construct the following matrix, the so-called Hurwitz matrix [20] from coefficients \(b_0, b_1, ..., b_6\) of equation (18):

\[
\begin{bmatrix}
b_1 & b_3 & b_5 & 0 & 0 & 0 \\
b_0 & b_2 & b_4 & b_6 & 0 & 0 \\
0 & b_1 & b_3 & b_5 & 0 & 0 \\
0 & b_0 & b_2 & b_4 & b_6 & 0 \\
0 & 0 & b_1 & b_3 & b_5 & 0 \\
0 & 0 & b_0 & b_2 & b_4 & b_6
\end{bmatrix}
\]

(19)

It is known that if for \(b_0 > 0\) all principal diagonal Hurwitz minors \(\Delta_1, \Delta_2, ..., \Delta_6\) are positive, then the unperturbed motion is asymptotically stable, regardless of the terms higher than the first order of smallness. Therefore, let’s determine the coefficients of equation (14) in such a way that the Hurwitz conditions are satisfied [20]:

\[
\Delta_1 = b_0 = \frac{2}{3}, \quad \Delta_2 = b_1 b_2 - b_3 b_0 = \frac{2}{3}, \quad \Delta_3 = b_3 \cdot \Delta_2 - b_7 b_8 b_1 + b_9 b_5 = \frac{2}{3} \frac{a_2 a_3}{a_{10}} > 0,
\]

since \(a_2 > 0, a_3 > 0\) and \(a_{10} > 0\).
\[ \Delta_4 = b_1 \Delta_3 - b_2 b_5 \Delta_2 + b_1^2 b_2 b_6 + b_0 b_3 (b_5 - b_0^2 b_3 b_6) = \frac{2a_x a_y a_{10} - a_x^2 a_y^2}{9a_{10}^2} + 0.004 > 0, \]

since \(a_{10} > a_x a_y.\)

\[ \Delta_5 = b_5 \Delta_4 - b_0 \Delta_3 + b_2 b_5 \Delta_2 - b_0^2 b_3 b_6 = \frac{2a_x^2 a_y^2 a_{10} - a_x^3 a_y^3 + 0.009 a_x a_y a_{10}^2}{27a_{10}^2} - 0.000029 > 0, \]

\[ \Delta_6 = \Delta_2 \cdot b_6 = \Delta_2 \cdot 0.01 > 0, \]

where the values of the coefficients of the control action (14) are determined by the following formulas:

\[
\gamma_{11} = -\frac{16a_1}{7a_6}, \quad \gamma_{12} = 2a_3, \quad \gamma_{13} = \frac{a_1^2}{a_2}, \quad \gamma_{21} = \frac{a_1}{a_6}, \quad \gamma_{22} = -\frac{205}{42}, \quad \gamma_{23} = \frac{205a_3}{84a_2},
\]

\[
\gamma_{31} = \frac{84a_x a_y a_3}{(205 - 84a_2)a_6}, \quad \gamma_{32} = 2c, \quad \gamma_{33} = \frac{a_{10}}{c}, \quad \gamma_{41} = \frac{2a_x a_y}{a_6}, \quad \gamma_{42} = -\frac{7}{3} a_2,
\]

\[
\gamma_{43} = -\frac{7}{6} a_3, \quad \gamma_{51} = \frac{0.84 a_c}{a_6 (205 - 84a_2a_3)}, \quad \gamma_{52} = 0, \quad \gamma_{53} = 2a_y,
\]

\[
\gamma_{61} = \frac{28(3a_x a_3 + 7a_{10})}{3a_{10}(205a_6 - 168a_2a_6)}, \quad \gamma_{62} = -\frac{14}{9} c, \quad \gamma_{63} = -\frac{7a_3c}{9a_2}.
\]

(20)

Let’s substitute the values of the coefficients (20) into equation (14):

\[
u_y = \frac{16a_1}{7a_6} y_1 + \frac{a_1}{a_2} y_2 + \frac{84a_x a_y a_3}{(205 - 84a_2)a_6} y_5 + \frac{2a_x a_y}{a_6} y_4 + \frac{0.84 a_c}{a_6 (205 - 84a_2a_3)} y_3 + \frac{28(3a_x a_3 + 7a_{10})}{3a_{10}(205a_6 - 168a_2a_6)} y_6
\]

(21)

\[
u_y = 2a_x y_1 - \frac{205}{42} y_2 + 2cy_3 - \frac{7}{3} a_2 y_4 - \frac{14}{9} cy_6,
\]

\[
u_y = \frac{84a_x a_y a_3}{(205 - 84a_2)a_6} y_1 + \frac{205a_3}{84a_2} y_2 + \frac{a_{10}}{c} y_3 - \frac{7}{6} a_3 y_4 + 2a_y y_5 - \frac{7a_3 c}{9a_2}.
\]

If the masses of hydraulic servomotors 8 and 9 are considered equal, i.e. \(m_4 = m_2\) then the optimal control actions (14) take the form:

\[
u_y = -\frac{16a_1}{7a_6} y_1 + \frac{a_1}{a_2} y_2 + \frac{2a_x a_y}{a_6} y_4 + \frac{0.84 a_c}{205a_6} y_3 - \frac{14}{9} cy_6,
\]

\[
u_y = -\frac{205}{42} y_2 + 2cy_3 - \frac{7}{3} a_2 y_4 - \frac{14}{9} cy_6,
\]

\[
u_y = -\frac{a_{10}}{c} y_3 + 2a_y y_5.
\]

When designing a squeezing machine, all parameters of the machine must satisfy condition (20). Equations (21) are the sought-for laws of guidance of control parameters \(u_1, u_2, u_3\).

Let’s substitute relation (21) into equation (10) and obtain:

\[
2a_x \dot{x}_1 = a_0 \left( -\frac{16a_1}{7a_6} x_1 + \frac{a_1}{a_2} x_2 + \frac{2a_x a_y}{a_6} x_4 + \frac{0.84 a_c}{205a_6} x_3 - \frac{14}{9} cx_5 \right).
\]

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\[2a_2 \dot{x}_2 + 2c x_2 = \frac{205}{42} \dot{x}_1 + 2c \dot{x}_2 - \frac{7}{3} a_2 \ddot{x}_2 - \frac{14}{9} c \ddot{x}_3,\]
\[2a_4 \ddot{x}_3 + 2a_5 x_3 = \frac{a_{10}}{c} \ddot{x}_2 + 2a_6 x_3,\]
or
\[\dot{x}_1 = -\frac{8}{7} x_1 + \frac{a_6}{2a_2} \dot{x}_1 + 2a_2 \ddot{x}_2 + \frac{0.84a_6 c}{205} \ddot{x}_3 - \frac{14}{9} a_6 c \ddot{x}_3,\]
\[\dot{x}_2 = -\frac{205}{42} \frac{\ddot{x}_1}{2a_2} - \frac{7}{6} \ddot{x}_2 - \frac{14}{9} \frac{a_6 c}{2a_2} \ddot{x}_3, \quad \ddot{x}_3 = -\frac{a_{10}}{2c \cdot a_4} x_2. \quad (22)\]

From (Fig. 2) let’s determine:
\[\psi = \frac{z_2 - z_1}{l}, \quad z_c = -\frac{z_1 l_2}{l} + z_2 \frac{l_1}{l} \quad \text{or} \quad l \cdot \psi = \dot{z}_2 - \dot{z}_1, \quad \dot{z}_c = -\frac{1}{2} \dot{z}_1 + \frac{1}{2} \dot{z}_2.\]

From this equation, let’s obtain:
\[l \cdot \psi = 2 \dot{z}_c \quad \text{or} \quad \dot{x}_2 = \frac{1}{2} \dot{x}_3,\]
integrating, let’s obtain:
\[x_2 = \frac{1}{2} x_3 + c_1^*,\]
where \(c_1^*\) is the integration constant.

From equation (22), let’s obtain:
\[\ddot{x}_3 = -\frac{a_{10}}{2c \cdot a_4} \frac{1}{2} (x_3 + c_1^*) \quad \text{or} \quad \ddot{x}_3 + k^2 x_3 = -\frac{a_{10} c_1^*}{4c \cdot a_4}, \quad (23)\]
where
\[k^2 = \frac{a_{10}}{4c \cdot a_4}.\]

The general solution to equation (23) has the following form:
\[x_3 = c_2^* \cos kt + c_3^* \sin kt - \frac{a_{10} \cdot c_1^*}{4c \cdot a_4} \cdot \frac{a_{10}}{a_{10}} \cdot \frac{4c \cdot a_4}{a_{10}}\]
or
\[x_3 = c_2^* \cos kt + c_3^* \sin kt - c_1^*, \quad (24)\]
where \(c_1^*, c_2^*\) and \(c_3^*\) are the constants of integration, determined from the initial conditions: \(t = 0;\)
\[x_3 = \psi_0 = 0, \quad \dot{x}_3 = 0, \quad x_2 = \frac{a_7}{2c}, \quad \ddot{x}_2 = 0, \quad x_1 = y_0, \quad \dot{x}_1 = y_0. \quad (25)\]

Using equations (25), let’s determine:
\[c_1^* = \frac{a_7}{2c}, \quad c_2^* = 0, \quad c_3^* = \frac{a_7}{2c}.\]

The law of changing the angle of rotation of the roll from the x axis is:
\[x_3 = \frac{a_7}{2c} \sin kt - \frac{a_7}{2c}.\]
From the second equation of (22), let’s obtain:
\[ \dot{x}_2 = -\frac{205}{42a_2} y - \left( \frac{7}{6} + \frac{7}{9a_2} c \right) \dot{x}_2. \]

Considering that the squeezing process \( \dot{y} \) has constant values, let’s obtain:
\[ \frac{dx_2}{dt} = b_1 \dot{y} + b_2 \dot{x}_2 \]

or
\[ \frac{dx_2}{dt} = b_2 \left( \frac{b_1}{b_2} \dot{y} + \dot{x}_2 \right). \]

where
\[ b_1 = -\frac{205}{84a_2}, \quad b_2 = \frac{7}{6} + \frac{14}{9a_2} c. \]

Integrating the last equation, let’s obtain:
\[ \ell c_4 \left[ \frac{b_1}{b_2} \dot{y} + \dot{x}_2 \right] = b_2 t + \ell c_4^*, \]

where \( \ell c_4^* \) is the constant of integration.

From this equation, let’s determine \( \dot{x}_2 \):
\[ \dot{x}_2 = c_4^* e^{\ell \dot{y}} = \frac{b_1}{b_2} \dot{y}. \]

Integrating again, let’s determine:
\[ x_2 = c_4^* \frac{e^{\ell \dot{y}}}{b_2} \frac{b_1}{b_2} \dot{y} \cdot t + c_5^*. \]

With initial conditions (25), let’s determine \( c_4^* \) and \( c_5^* \):
\[ c_4^* = \frac{b_1}{b_2} \dot{y}, \quad c_5^* = \frac{a_2 b_2 l_2 - 2c b_1 \dot{y}}{2c b_2 l_2}. \]

Thus, the law of change of the center of mass of the roll has the following form:
\[ x_2 = \frac{b_1}{b_2 l_2} \dot{y} \cdot e^{\ell \dot{y}} - \frac{b_1}{b_2} \dot{y} \cdot t + \frac{b_2 a_2 l_2 - 2c b_1 \dot{y}}{2c b_2 l_2}. \]
or
\[
x_2 = \dot{y} \left( \frac{b_1}{b_{2t}} e^{\dot{t}} - \frac{b_1}{b_{2r}} t - \frac{b_2}{b_{2r}} \right) + \frac{a_2 b_2}{2 c l_2}.
\]

Further, setting \( \dot{y} = \text{const} \) and \( \ddot{y} = 0 \), from equation (4), let’s determine:
\[
a_5 = \frac{16 a_1}{7} x_1 - \frac{a_1}{a_2} x_1 - 2 a_2 x_2 = -\frac{0.8 a_1 c}{205} x_3 + \frac{14 a_6 c}{9} x_3,
\]

or given the values of \( a_5 \), let’s determine the torque \( M_1^a \):
\[
M_1^a = M_1^r + M_2^r \frac{R_1}{R_2} \frac{16 a_1 R_1}{7} x_1 - \frac{a_6 R_1}{a_2} x_1 - 2 a_2 R_2 x_2 = -\frac{0.8 a_1 c R_1}{205} x_3 + \frac{14 a_6 c R_1}{9} x_3,
\]

where
\[
a_1 = \frac{1}{2} \left( \frac{J_1}{R_1^2} + \frac{J_2}{R_2^2} + 2 m_3 \right), \quad a_2 = \frac{1}{2} (m_1 + 2 m_4), \quad a_6 = \frac{(R_1 + R_2)^2 - 2 R_2 R_1 \left(1 - \frac{1}{2} \frac{R_1 R_2}{R_1^2} \right)}{(R_1 R_2)^2}.
\]

When determining \( M_1^a \), it is assumed that one roll of the pair (a drive roll), the material being processed and the second roll (a non-drive roll) receive motion due to the force of friction on the contact surfaces.

If both rolls are driven, i.e. are connected with a rigid kinematic constraint and a drive, the material being processed is set in motion due to the friction force on the contact surfaces with the rolls, then it is recommended that the radii of both rolls and the monchons (fabric tires, that absorb and remove moisture well) should be the same. In this case, \( M_1^a \) and \( M_2^a \) will be equal.

Therefore, for the dynamic stability of the movement of a roll pair, the coefficients of equation (14) must satisfy equalities (20).

Without compiling the equation of perturbed motion and the requirement that the condition of asymptotic stability of the process under consideration be satisfied, this method loses its meaning. In order to apply the results obtained in practice, using the initial data (satisfying the conditions of the control function), let’s substitute equalities (20) into the equations of motion and, with the exact implementation of the servo constraint, determine the reaction forces of the constraints and the kinematic characteristics of motion.

To develop this method in the future, it is necessary to select servomotors, sensors and create programs that describe all technological processes of machining; to develop a wringing machine operating in automatic mode.

4. Conclusions

It is substantiated that the primary task that needs to be solved when designing automatic control systems is the construction of a kinematic diagram and mathematical model of the control object. It is shown that the basic aspect in the theory of automatic control is precisely the formulation of a mathematical description of the control object functioning, its properties and relationships, which makes it possible to evaluate (predict) information about the change in the state of the object when external impacts are applied to it.

It is shown that one of the reasons for the unstable stress state on the contact surfaces in the roller mechanism is dynamic factors arising from inaccuracies in the manufacture of its individual parts, assembly defects, the occurrence of an oscillatory process in the roller mechanism, and the non-uniform thickness of the processed material during its capture, starting and stopping the machine.

Optimal controls are determined that ensure the asymptotic stability of the unperturbed motion of the roll pair and the torque applied to the upper roll as a function of generalized coordinates.
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