Study on complex dynamics for the waste electrical and electronic equipment recycling activities oligarchs closed-loop supply chain

Meihong Zhu1,2 · Xiao Li1,2 · Junhai Ma1 · Tiantong Xu1 · Liqing Zhu1

Received: 17 March 2021 / Accepted: 11 August 2021 / Published online: 19 August 2021
© The Author(s), under exclusive licence to Springer-Verlag GmbH Germany, part of Springer Nature 2021

Abstract
Nowadays, with more and more WEEE (waste electrical and electronic equipment) being abandoned, WEEE recycling activities are increasingly popular. In this paper, we build a closed-loop supply chain model and focus on the recycling behaviors of the members in this supply chain, which contains two manufacturers, the retailer, and the consumer. In the reverse chain, we set up dual channels and design two recycling methods: sell-back method and trade-in method. We use classical backward induction to run the model. And then we analyze the stability of the system and the impacts of some essential parameters by numerical simulation. The speed of the manufacture’s decision adjustment has a significant effect on the stability of the model. Manufacturers should adopt a stable rather than radical price adjustment strategy in order to achieve stable profit growth in the whole supply chain and avoid a chaotic price war. Manufacturers should increase innovation investment in recycling technology to reduce the cost of remanufacturing products and improve the initiative in market competition. In the end, we adopt the parameter control method and the decision-making method to control chaos, and they both have a good control effect.

Keyword Reverse chain · WEEE recycling · Game theory · Oligarchs · Complex system · Closed-loop supply chain

Introduction
An increasing number of states pay more attention to environmental issues including climate change, air pollution, and breakage of land. With the continuous improvement of global informatization, e-waste occupies a large part of the environmental damage. The generally accepted concept of “e-waste” is discarded electrical or electronic devices, such as common household appliances and the recovery of mobile phones, washing machines, televisions, and air conditioners computers (Guo et al. 2018). Compared with the recycling of traditional materials, WEEE products are composed of more complicated material and components (Gu et al. 2017).

With the increasingly frequent update and upgrade of electrical and electronic equipment, the trade of related products rises and a great deal of electrical and electronic equipment is abandoned every day. In the EU, the amount of WEEE has been increasing by 3–5% per year. In 2012, the electrical waste sums up to 3.6 billion kg, of which over 70% was recycled (Zlamparet et al. 2017). In China, according to the calculated theoretical scrap amount of waste electrical and electronic products, the theoretical scrap amount of 14 products in “The Disposal Catalogue of Waste Electrical and Electronic Products” in 2017 is shown in Table 1 (White paper WEEE recycling industry in China 2017). Among them, the theoretical scrap quantity of the first batch of products is about 100 million, including 3.216 million TV sets, 2.439 million refrigerators, 16.2 million washing machines, and 27.23 million room air conditioners. Compared with 2016, the theoretical scrap volume of the first batch of products presents an overall upward trend. For this reason, WEEE recycling activities have become an extensively researched issue in recent years.

Affected by the growth of e-waste, recovery of WEEE (waste electrical and electronic equipment) products has
become a hot issue in the economic field. Many corporations need a more stable and sustainable business model. It is the requirement for environmental consequences, social factors, and economic constraints. Proper disposal of WEEE not only reduces environmental pollution but also enhances resource utilization efficiency which benefits manufacturers as well as the environment. For this reason, product recycling activities have been paid attention to around the world (Lu et al. 2018; Polat et al. 2018). We take China as an example since the Chinese government puts great emphasis on WEEE recycling activities. A model of green design, green production, green sales, green recycling, and disposal led by production enterprises is emerging now. Under great pressure from the government, other firms and consumers are responsible for environmental and social as well, and manufacturers would choose to recycle used products. For example, HUAWEI is collecting used products through VWALL; Changhong and Haier are also actively building green recycling systems for used products. Meanwhile, retailers such as SuNing and JingDong also participate in collecting used products activities. Based on the background, this paper tries to discuss the following research questions:

(1) How is the complex chaotic dynamical process in the reverse channel structure?
(2) How to control and coordinate the WEEE recycling activities under the closed-loop supply chain structures?

The rest of this paper is organized as follows. In the “Literature review” section, we reviewed the relevant literatures. In the “Methodology” section, we describe the problem and show parameters and assumptions and a closed-loop supply chain network which is composed of two manufacturers and one retailer. We solve the model, analyze the local stability, and further analyze the influence of parameters on the model’s stability in the “Results and discussion” section. Based on what we have stated, we introduce the method to control the chaos system in the “Chaos control” section. Finally, we make conclusions in the “Conclusion and policy implications” section.

### Literature review

The recycling process leads to better economic viability and environmental sustainability which makes the closed-loop supply chain more attractive (Hong et al. 2017). The closed-loop supply chain contains the forward logistics and reverse logistics. In forward logistics, the supply chain process includes purchasing, manufacturing, and selling; in reverse logistics, the supply chain differs from traditional supply chain, realizes recycle and remanufacture, and is designed to increase the utilization ratio of raw materials and reduce pollution emissions to protect the environment and lower costs at the same time. Therefore, constructing a closed-loop supply chain can promote sustainable development. The recycling activity has become a new trend in logistics and supply chain management. Based on related research in the closed-loop supply chain and reverse supply chain, the collector recycling used products in the closed-loop supply chain can be classified into three categories: manufacturer, retailer, and the third party (Savaskan et al. 2004).

A great number of researches about WEEE recycling have been done in recent years. Xu et al. (2017) investigated the design of a global reverse supply chain. The results show the effect of uncertainties and carbon constraints on decisions to reduce costs and emissions. Bal and Satoglu (2018) investigated a multi-facility, multi-product, and multi-period model which collects WEEE products and recovers the waste materials. Bing et al. (2015) redesigned a reverse supply chain to provide useful insights for managing waste as a resource in the view of a global perspective. Liu et al. (2017) studied a manufacturer who owes a WEEE recycling qualification under fund policy and analyzed the effect of fund policy on the manufacturer. Liu et al. (2016) built a dual-channel WEEE recycling model that contains a formal section and informal section. They considered price competition based on quality and government subsidy and obtained the equilibrium prices as well as the influence of government subsidy under four competitive scenarios. Ma et al. (2020a) constructed a closed-loop supply chain and studied how different channel power affects the dynamic characteristic of the system. In terms of competition in the reverse supply chain, most papers

| Electric & electronic products | Scrap amount (million) | Electric & electronic products | Scrap amount (million) |
|-------------------------------|------------------------|-------------------------------|------------------------|
| TV                            | 32.16                  | Gas water heater              | 9.33                   |
| Refrigerator                  | 24.39                  | Printer                       | 30.99                  |
| washing machine               | 16.20                  | Copier                        | 6.69                   |
| Air conditioner               | 27.23                  | Fax machine                   | 5.06                   |
| Micro-computer                | 25.24                  | Fixed telephone               | 23.80                  |
| Oil absorption                | 39.87                  | Mobile phone                  | 232.75                 |
| Electric water heater         | 25.85                  | Monitors                      | 0.48                   |

Table 1. The theoretical scrap amount of China electric and electronic products in 2017.
focus on competitions between two retailers, between two third parties, or between one retailer and one third party. For example, Savaskan and Van Wassenhove (2006) studied two competitive retailers in 2006. Therefore, we consider the fact that some manufacturers recycle used products by themselves and build a dual-channel supply chain in which one manufacturer collects used products from consumers directly and the other manufacturer collects used products through retailers. Yin et al. (2010) discussed the effect of the retail used goods market and the electronic peer-to-peer (P2P) market on the primary market. Inspired by Yin et al. (2010), we set a trade-in recycling method where the retailer buys back a used product and sells a new one to consumers. Zhou et al. (2017) explored a two-period model in which a manufacturer sells products and two competitive recyclers recycle used products under extended producer responsibility (EPR) and further discussed the decision about setting subsidy from the government. Considering the situations that the buyers promise to buy back the unsold products from the seller, Xie et al. (2021) studied the effect of income uncertainty and relative bargaining power on the performance of repurchase contracts in supply chain. Based on current research, we further consider a three-period model while putting more emphasis on the reverse supply chain.

Analyzing product recycling activities, most literature focus on the closed-loop supply chain. Based on different strategies of collecting used products, Savaskan et al. (2004) analyzed three different reverse loops of the supply chain and analyzed the results. Östlin et al. (2008) focused on remanufacturing. They expanded seven different types of closed-loop supply chains and analyzed the advantages and disadvantages of each model. Qiang et al. (2013) built a closed-loop supply chain which contains raw material suppliers, retail outlets, and manufacturers and gave the optimality conditions of the various decision-makers. Huang et al. (2013) explored the optimal strategies in the closed-loop supply chain with a dual recycling channel in which retailers and the third party collect used products. De Giovanni and Zaccour (2014) introduced a two-period closed-loop supply chain and analyzed whether to collect end-of-life products by remanufacturer exclusively or introduce a retailer or a third-service provider. Wei et al. (2015) analyzed a closed-loop supply chain under symmetric and asymmetric information. Gao et al. (2016) focused on exploring power structures’ effect on optimal strategies in a closed-loop supply chain. In a word, for the reverse channel in the closed-loop supply chain, most literature focus on the method of collection, power structure, and so on. At present, competition between two different recycling methods in the waste recycling market is less mentioned. Consequently, based on Guo and Ma (2013) and Ma et al. (2013) and associated with reality, we enter into discussion in the two different recycling methods in a closed-loop supply chain and expect to make some useful supplements on this.

Besides, most literature are based on two hypotheses: complete information and “completely rational person” hypothesis. Actually, in the long run, players would adjust their decisions according to the information they received. Despite that, there is a lot of literature with WEEE recycling activities, and the study of complexity theory is not wide in the current literature. Thus, we introduce dynamic game when analyzing the closed-loop supply chain in the light of Ma and Guo (2014) in which they introduced dynamic system concepts to simulate the evolution of the strategic decisions over multiple periods. Tian et al. (2020) analyze the complexity of a multi-channel supply chain and revealed that a larger adjustment speed of the sale effort causes the unstable state of the system. Based on bounded rationality, Ma et al. (2020b) explored the impact of decision sequence on the profits of the enterprise and how the adjustment speed affects the stability of the system. Considering a supply chain with two different manufacturers who produce fuel vehicle and battery vehicle, respectively, Bao et al. (2020) derived the optimal decisions and make comparative analysis between three different games, finding that the adjustment speed of the ecofriendly level has the greatest impact on the market.

To investigate how WEEE recycling activities affect the oligarch’s closed-loop supply chain, a method that has dual channels sales and two recycling is designed. Based on observations from current literature and actual situation, our model’s innovation can be summarized as follows:

1. We build a closed-loop supply chain including two manufacturers and one retailer. Two different recovery modes are taken into account. It can be expanded by applying other models from literature in the future.
2. We consider competition between two different recycling methods in the waste recycling market. Besides, based on Complex System Theory, we adopt a dynamic decision-making method instead of a single period of static decision-making.
3. Chaos is controlled by adjusting the means of relevant parameters and decision-making methods.

The contribution of this paper is to enrich the analytical analysis on CLSC, investigate the complexity of the decision-making process, and inspire the operations method in recycling activities.

**Methodology**

During the last 4 decades, nonlinear dynamics have made great progress and have attracted increasing attention from different fields including supply chain management (Xie et al. 2017, Ma et al. 2021). Ma and Wu (2014) explored a discrete triopoly dynamical model and analyzed the
complexity of this system. Elsadany (2010) proposed and analyzed a duopoly delayed bounded rationality game, and the analysis proved that delayed bounded rationality contributed to the higher possibility of Nash equilibrium point. Du et al. (2013) analyzed a dynamic duopoly game with heterogeneous players. They found that the chaos was harmful to at least one player of the two, and an appropriate limiter would be conducive to the players’ performance. As an essential tool, the dynamic game also has been applied to the reverse supply chain. Xie and Ma (2016) researched a duopoly market of color TV recycling, analyzed the influence of coefficients changing on the system’s stability, and introduced the method of chaos control. Ma and Wang (2014) first built a closed-loop supply chain in which the retailer sells and collects products and the manufacturer produces and recovers products and then presents the dynamic phenomena.

Model framework

In this paper, we consider a two-period model, but we focus on the recycling behavior of the members in a typical supply chain including two manufacturers, one retailer, and consumers. In the forward chain, two manufacturers sell their products to the retailer and the retailer sells products to consumers. In the reverse chain, we set up dual channels and design two recycling methods: sell-back method and trade-in method. Consumers sell the used product back to the retailer utilizing the sell-back recycling or trade the used product in for a new one through trade-in recycling. When using the method of sell-back, consumers can get money by selling waste products. When using the method of trade-in, consumers can sell back waste products to the retailer in exchange for lower prices of new products. And then the retailer sells the used products back to manufacturer 1. Meanwhile, consumers are allowed to sell used products to manufacturer 2 directly. In the next stage, manufacturer 1 and manufacturer 2 remanufacture used products and sell new products to consumers. Our model in this paper is shown in Fig. 1.

In Fig.1, M1 represents manufacturer 1, and in the same way, M2 shows manufacturer 2. R represents the retailer. And C is the consumer.

Symbols and assumptions

The relevant parameters are shown in Table 2.

To simplify the analysis, we made those assumptions as follows.

a. The manufacturer and retailer are bounded rational.
b. Remanufactured products and new products have the same level of quality. The gap between the two products is negligible.
c. The transportation cost and other expenses are not taken into account.
d. The demand is linearly dependent on price.
e. The recycling quantity is linearly dependent on the recycling price.

Model construction

In our model, manufacturer 1 produces product 1, and manufacturer 2 produces product 2. Those two products are made of the same raw materials and possess the same function, quality, price, et cetera (Lou and Ma 2018; Ma and Sun 2017). We assumed that the demand is linearly dependent on price. Meanwhile, the rival product’s retail price also affects demand. Thus, the demand for new product 1 and new product 2 can be given as follows.

\[
\begin{align*}
Q_1 &= a_1 - b_1 P_1 + d_1 P_2 = a_1 - b_1 (1 + \lambda_1) W_1 + d_1 (1 + \lambda_1) W_2 \\
Q_2 &= a_2 - b_2 P_2 + d_2 P_1 = a_2 - b_2 (1 + \lambda_2) W_1 + d_2 (1 + \lambda_2) W_2
\end{align*}
\]  

(1)

Comparably, we set the recycling quantity linearly depending on the recycling price. So the sell-back quantity of used products for the retailer and the sell-back quantity of used products for manufacturer 2 can be given as follows.

\[
\begin{align*}
Q_{r1} &= k_1 + g_1 R_1 - h_1 R_2 - t_1 N = k_1 + g_1 F_1 (1 - \lambda_3) - h_1 R_2 - t_1 N \\
Q_{r2} &= k_2 + g_2 R_2 - h_2 R_1 - t_1 N = k_2 + g_2 R_2 - h_2 F_1 (1 - \lambda_3) - t_1 N
\end{align*}
\]

(2)

Similarly, the trade-in quantity can be given as follows.

\[
Q_c = \beta + \gamma N - t_2 (R_1 + R_2) = \beta + \gamma N - t_2 F_1 (1 - \lambda_3) - t_2 R_2
\]

(3)
Table 2. The parameters and their meanings

| Parameters | Meanings |
|------------|----------|
| $Q_i$      | Demand for the product $i$ |
| $a_i$      | Basic market size of product $i$ |
| $b_i$      | Sensitivity of consumers to the retail price of product $i$ |
| $d_i$      | Sensitivity of consumers to the rival products’ retail price of product $i$ |
| $p_i$      | Unit retail price of product $i$ |
| $W_i$      | Unit wholesale price of product $i$ |
| $Q_i^R$    | Recycling quantity of product $i$ |
| $N$        | Difference between the sell-back price and the trade-in price |
| $g_i$      | Sensitivity of consumers to the sell-back price of product $i$ |
| $h_i$      | Sensitivity of consumers to the rival products’ sell-back price of product $i$ |
| $t_1$      | Sensitivity of the difference between the sell-back price and the trade-in price |
| $F_1$      | Sell-back price of manufacturer 1 |
| $R_1$      | Sell-back price of the retailer |
| $R_2$      | Sell-back price of manufacturer 2 |
| $C_i$      | Cost of product $i$ |
| $Q_i$      | Trade-in quantity |
| $C_{ni}$   | Cost of product $i$ made from the used product |
| $\Pi_1$    | The profit of manufacturer $i$ |
| $\Pi_2$    | Retailer’s profit |
| $\lambda_3$| Potential maximum waste quantity of product $i$ for recycling in the market |
| $k_i$      | Price coefficient of retailer |
| $\nu_i (i=1,2,3)$ | Indicate the adjusting speed of $F_1$, $R_2$, and $N$, respectively |
| $\beta$    | Sensitivity of the difference between the sell-back price and the trade-in price |
| $t_2$      | Sensitivity of the whole sell-back prices |

In the set recycling supply chain, the profits of manufacturers include two parts: the profits of new products and the profits of used products. And the profits of the retailer come from selling and recycling products 1 and 2. The profit model can be given as follows.

\[
\begin{align*}
\Pi_1 & = (W_1-C_1)(Q_1-Q_{r1}) + (W_1-C_1-F_1)(Q_e + Q_{r1}) \\
\Pi_2 & = (W_2-C_2)(Q_2-Q_{r2}) + (W_2-C_2-R_2)Q_{r2} \\
\Pi_i & = (P_1-W_1)Q_1 + (F_1-R_1)Q_{r1} + (F_1-R_1-N)Q_e + (P_2-W_2)Q_2
\end{align*}
\]

To maximize the profit, the three parties (manufacturer 1, manufacturer 2, and the retailer) have to make appropriate decisions. According to the profit model (4), the decision variable of manufacturer 1 is the sell-back price of manufacturer 1, $F_1$; the decision variable of manufacturer 2 is the sell-back price of manufacturer 1, $R_2$; and the decision variable of the retailer is the difference between the sell-back price and the trade-in price, $N$.

As to manufacturer 1, its profit can be written in the form

\[
\Pi_1 = (W_1-C_1)(Q_1-Q_{r1}) + (W_1-C_1-F_1)(Q_e + Q_{r1})
\]  

(5)

where

\[
\frac{\partial(W_1-C_1)Q_1}{\partial F_1} = \frac{\partial(W_1-C_1)Q_1}{\partial R_2} = \frac{\partial(W_1-C_1)Q_1}{\partial N} = 0
\]  

(6)

Thus, $(W_1-C_1)Q_1$ is independent of the three decision variables. To simplify the analysis, model (5) can be simplified as

\[
\Pi_1' = (C_1-C_r-F_1)(Q_e + Q_{r1})
\]  

(7)

As to manufacturer 2, its profit can be written in the form

\[
\Pi_2 = (W_2-C_2)(Q_2-Q_{r2}) + (W_2-C_2-R_2)Q_{r2}
\]  

(8)

where

\[
\frac{\partial(W_2-C_2)Q_2}{\partial F_1} = \frac{\partial(W_2-C_2)Q_2}{\partial R_2} = \frac{\partial(W_2-C_2)Q_2}{\partial N} = 0
\]  

(9)

Comparably, $(W_2-C_2)Q_2$ is independent of the three decision variables. To simplify the analysis, the model (8) can be simplified as

\[
\Pi_2' = (C_2-C_r-R_2)Q_{r2}
\]  

(10)

As to the retailer, its profit is given as

\[
\Pi_r = (P_1-W_1)Q_1 + (F_1-R_1)Q_{r1} + (F_1-R_1-N)Q_e + (P_2-W_2)Q_2
\]  

(11)
Similarly, \((P_1 - W_1)Q_1\) and \((P_2 - W_2)Q_2\) are independent of the three decision variables. To simplify the analysis, the model (11) can be simplified as

\[\Pi' = (F_1 - R_1)Q_{r1} + (F_1 - R_1 - N)Q_e\]  
\[\text{(12)}\]

So the simplified profit model can be given as

\[\begin{align*}
\Pi_1' &= (C_1 - C_{r1} - F_1)(Q_e + Q_{r1}) \\
\Pi_2' &= (C_2 - C_{r2} - R_2)Q_{r2} \\
\Pi_e' &= (F_1 - R_1)Q_{r1} + (F_1 - R_1 - N)Q_e
\end{align*}\]  
\[\text{(13)}\]

For further simplification, if we set

\[\begin{align*}
C_1 - C_{r1} &= dC_1 \\
C_2 - C_{r2} &= dC_2
\end{align*}\]  
\[\text{(14)}\]

then the model (13) can be written in the form

\[\begin{align*}
\Pi_1' &= (dC_1 - F_1)(Q_e + Q_{r1}) \\
\Pi_2' &= (dC_2 - R_2)Q_{r2} \\
\Pi_e' &= (F_1 - R_1)Q_{r1} + (F_1 - R_1 - N)Q_e
\end{align*}\]  
\[\text{(15)}\]

The derivatives of the manufacturers’ profit models are given as

\[\begin{align*}
\frac{\partial \Pi_1'}{\partial F_1} &= h_1 R_2 - k_1 - (dC_1 - F_1) \left( \frac{\lambda t_1}{2} + \frac{\lambda t_2}{2} + \frac{R t_2}{2} + \frac{R_2 t_2}{2} + \frac{g_1 (\lambda - 1)}{2} \right) \\
&- t_2 (\lambda - 1) - t_1 (t_2 + t_1 + t_3) - t_1 (t_2 - t_1) + g_1 (\lambda - 1)
\end{align*}\]  
\[\text{(19)}\]

\[\frac{\partial \Pi_2'}{\partial R_2} = (dC_2 - R_2) \left( g_2 - \frac{t_1 t_2}{2} - g_2 R_2 - k_2 + \frac{t_1 (t_2 - t_1 - t_3) + F_1 \lambda t_1 + F_1 \lambda t_2}{2} + F_1 g_1 (\lambda - 1) \right)
\]

### Results and discussion

#### Model analysis

Our model mainly involves three parties, and they all have their own decision variable. To maximize the profit, they will adjust the value of decision variables in the game. We first consider the retailer’s decision. The derivative of the retailer’s profit model with respect to \(N\) is given as

\[\frac{\partial \Pi_e'}{\partial N} = \frac{r_2 t_2 - t_1 (F_1 + F_1 (\lambda - 1)) - \gamma \beta}{2} + \gamma (F_1 - n + F_1 (\lambda - 1)) - F_1 t_2 (\lambda - 1)\]  
\[\text{(16)}\]

We set equation (16) equal to zero. Then we can get the optimal decision of retailer

\[N^* = \frac{-\beta F_1 t_2 - R_2 t_2 - F_1 (\gamma - 1) + F_1 \lambda t_1 + F_1 \lambda t_2}{2 \gamma}\]  
\[\text{(17)}\]

Substituting the optimal decision of retailer \(N^*\) into manufacturers profit models, we get the manufacturers profit models to the retailer’s decision.
In our model, manufacturers are of bounded rationality and they play the leading roles. And the retailer makes decisions following the manufacturers. So the evolution game model for the three parties is given as

\[
\begin{align*}
F_1(t+1) &= F_1(t) + v_1 F_1(t) \frac{\partial \Pi_1}{\partial F_1} \\
R_2(t+1) &= R_2(t) + v_2 R_2(t) \frac{\partial \Pi_2}{\partial R_2} \\
N(t+1) &= N(t) + v_3 \left(N(t)^* - N(t)\right)
\end{align*}
\]  
(20)

**Proposition 1**

The Nash equilibrium of the game \((F_1^*, R_2^*, N^*)\) could be derived by solving the equations of \(\frac{\partial N^*}{\partial F_1} = 0, \frac{\partial N^*}{\partial R_2} = 0\).

**Proposition 2**

For the optimal trade-in strategy, we can observe that:

1. The trade-in price will ramp up with the increasing of sell-back price of manufacturer 1 when \(t_2 + \lambda_3(\gamma - t_1 - t_2) > 0\).
2. The trade-in price will ramp up with the increasing of sell-back price of manufacturer 1 when \(t_2 + \lambda_3(\gamma - t_1 - t_2) < 0\).
3. The trade-in price will definitely ramp up with the increasing of sell-back price of manufacturer 2.

**Proof:** It can be obtained that \(\frac{\partial N^*}{\partial F_1} = \frac{t_2 + \lambda_3(\gamma - t_1 - t_2)}{2\gamma}\). Hence, the first and the second lemma are easily proved by figuring out the in equation of \(\frac{\partial N^*}{\partial F_1} > 0\) and \(\frac{\partial N^*}{\partial R_2} < 0\). Additionally, we can obtain that \(\frac{\partial N^*}{\partial F_1} = \frac{t_2}{2\gamma}\), which can generate the third lemma of Proposition 2.

From Proposition 1, we can know that if manufacturer 2 sets a higher sell-back price, the trade-in price will be increased. This finding reveals the competition of recycling between the retailer and the manufacturer 2—a player will increase the recycling price if his competitor sets a higher recycling price.

**Inherent complexity analysis**

In order to better show the complex dynamic properties of the game process, we pre-analyze the model by numerical simulation on the premise of meeting the conditions listed above. We make the following model parameter assumptions with reference (Xie et al. 2021; Gao and Ma 2009.)

\[k_1 = 0.5, k_2 = 0.5, g_1 = 1, g_2 = 1, h_1 = 0.3, h_2 = 0.3, \beta = 0.3, \gamma = 0.3, t_2 = 1, dC_1 = 2.0, dC_2 = 2.5, \lambda_3 = 0.2\]

Based on the assumption, we assume that the manufacturer’s potential quantity of recycled products from the market is consistent, and the manufacturer’s cost reduction by using recycled products for reproduction is consistent. The dynamic changes of decision variables \(F_1\) and \(R_2\) from cycle 100 to cycle 200 are shown in Fig. 2.

As shown in Fig. 2, under the set parameters, the changes of decision variables are cyclical. The whole game is in an unstable state. Thus, we take a further analysis of the stability of the model.

The Jacobi matrix of the system can be given as

\[
\text{Jac} = \begin{bmatrix}
J_{11} & v_1 F_1 \left( h_1 + \frac{t_2}{2} + \frac{t_1 t_2}{2\gamma} \right) & 0 \\
-v_2 R_2 \left( h_2 (\lambda_3 - 1) + \frac{t_1 (\lambda_3 t_1 - \gamma \lambda_3 + t_2 (\lambda_3 - 1))}{2\gamma} \right) & J_{22} & 0 \\
-(v_3 (\lambda_3 t_1 - \gamma \lambda_3 + t_2 (\lambda_3 - 1)) \right) & \frac{v_3 t_2}{2\gamma} & 1-v_3
\end{bmatrix}
\]  
(21)

\[\mathfrak{S} \text{ Springer}\]
According to the Jury criterion, the stability of the discrete dynamic system relates to whether the eigenvalues are in the unit circle. The eigenvalues of the model can be written as

\[
|\text{Jac} - \lambda E| = \begin{vmatrix}
J_{31} - \lambda_1 \\
-\nu_2 R_2 \left( h_2 (\lambda_3 - 1) + \frac{(t_1 (\lambda_3 t_1 - \gamma \lambda_3 + t_2 (\lambda_3 - 1)))}{2 \gamma} \right)
\end{vmatrix}
\Rightarrow \lambda_3 = 1 - \nu_3
\]

**Proposition 3**

The dynamic system will be stable if the adjustment speed of the retailer is in the range of \( \nu_3 \in [0, 2] \).

**Proof**: According to equation (22), if \( \nu_3 \in [0, 2] \), then \( \lambda_3 \in [-1, 1] \). In such a condition, the third eigenvalue is in the unit circle.

Proposition 3 indicates that if the retailer adjusts his trade-in speed in a not so high speed, the dynamic system will not trapped into unstable state accordingly. Hence, the stability of the model is related to manufacturers’ decisions.

Then, we further analyze the influence of the decision adjustment speed of manufacturer 1 and manufacturer 2 on the stability of the game. We set the decision adjustment speed of manufacturer 1 \( \nu_1 \) as a variable and the decision adjustment speed of manufacturer 2 as \( \nu_2 = 0.1 \) and \( \nu_2 = 0.3 \). The results of numerical simulation are shown in Fig. 3 and Fig. 4.

The results show that with the increase of the decision adjustment speed of manufacturer 1, the decision variables of manufacturer 1 and manufacturer 2 go from a stable state into chaos. Besides, the decision adjustment speed of manufacturer 2 has a significant impact on the decision variables.

Next, we set the decision adjustment speed of manufacturer 2 \( \nu_2 \) as a variable and the decision adjustment speed of manufacturer 1 as \( \nu_1 = 0.1 \) and \( \nu_1 = 0.5 \). The results of numerical simulation are shown as Fig. 5 and Fig. 6.

According to the results, the trends of the change of the decision variables are similar to those of the previous numerical simulation, which also go from a stable state into chaos. Hence, we can get the conclusion that the stability of the game is related to both \( \nu_1 \) and \( \nu_2 \). To make a better illustration, we draw the stability domain diagram concerning \( \nu_1 \) and \( \nu_2 \).

In Fig. 7, different color represents a different stability state. The dark blue represents the model in a stable state, the light blue represents the model in the bifurcation phase, and the color of scattering represents the model in a chaotic
state. According to this figure, the stability of the model decreases with the increase of adjustment speed $v_1$ and $v_2$. In other words, the swifter response indicates weaker market stability.

Next, we will further analyze the impact of the instability on the profits of manufacturers and retailers. The three-dimensional figures from Fig. 8, Fig. 9, and Fig. 10 reveal the trend of the profits of the manufacturers and the retailer.

As expected, with the increase of the decision adjustment speed $v_1$ and $v_2$, the decision variables go unstable, resulting in the instability of the profits of manufacturers and retailers. And on average, both manufacturers and retailers have a profit loss in multiple cycles. Thus, the chaos is unfavorable for the
entire market, which seriously affects the normal operation of the parties in the game.

**Impact from parameters**

In addition to analyzing the speed of adjustment $v_1$ and $v_2$, how to impact the market of manufacturers and retailers. We also analyze the impact of several key parameters in the game on the evolution process from the perspective of profit, which is the concern of all parties.

**Cost reduction of manufacturer 1**

According to the above results, the stability of the system is related to the manufacturer’s adjustment speed $v_1$ and $v_2$. With the increase of adjustment speed $v_1$ and $v_2$, the system goes from a stable state into an unpredictable instability. Here, we analyze the impact of cost reduction of manufacturer 1 $dC_1$ and the profits of the three parties in the game under the stable state and the unstable state.
Based on the above analysis, we set $v_1 = 0.2$ and $v_2 = 0.2$ for the stable state while $v_1 = 0.3$ and $v_2 = 0.5$ for the unstable state.

As shown in Fig. 11, with the increase in cost reduction of manufacturer 1, the profit of manufacturer 1 and the retailer has a similar trend in the stable state. The change is indistinctive at the beginning and then goes significant. The profit trend of manufacturer 2 stays at first, different from those of manufacturer 1 and the retailer, which decreases with a cost reduction of manufacturer 1 increasing. The profit with respect to the diagram in the unstable state can be seen in Fig. 12.

However, in the unstable state, the profit of manufacturer 1 and that of the retailer has a similar trend. They both have little change with the cost reduction $dC_1$ and then go up and fall off a cliff finally. After that, the profit of manufacturer 1 keeps increasing and becomes fluctuant; meanwhile, the profit of the retailer keeps decreasing and becomes fluctuant. The explanation is that in the recycling stage, the retailer is responsible for recycling used products and sells them back to manufacturer 1, and a cooperative relationship forms between manufacturer 1 and the retailer. The similar trend of the profit of manufacturer 1 and that of the retailer is shown in Fig. 12.
Manufacturer 2 also collects used products from the market, so he competes with manufacturer 1 and the retailer. In Fig. 12, with the cost reduction $dC_1$ increasing, the profit of manufacturer 2 remains stable first and then goes decreasing which is different from the trend of manufacturer 1 and the retailer. In the end, the profit of manufacturer 2 also becomes fluctuant.

For further analysis, we make a numerical simulation of the profit of manufacturer 1 with respect to the adjustment speed $v_1$ and cost reduction $dC_1$. Fig. 13 shows the result. From Fig. 13, we can get a similar conclusion that with the increase of cost reduction, the profit of manufacturer 1 is increasing. However, when the adjustment speed $v_1$ is far too high, the system will step into chaos and the manufacture will suffer profit losses.

**Cost reduction of manufacturer 2**

Similarly, we analyze the impact of cost reduction of manufacturer 2 $dC_2$ on the profits of the three parties in the game in the stable state and the unstable state. And again we set $v_1 = \ldots$
0.2 and $v_2 = 0.2$ for the stable state while $v_1 = 0.3$ and $v_2 = 0.5$ for the unstable state.

The profit with respect to $dC_2$ when $v_1 = 0.2$, $v_2 = 0.2$ a diagram can be seen in Fig. 14.

It is easy to find out that in the stable state, with the increase of its cost reduction $dC_2$, the profit of manufacturer 2 rises first and then decreases suddenly; after that, the profit of manufacturer 2 stays approximate to zero. While the profit of manufacturer 1 stays the same first and then drops, the profit of the retailer has no change at the same time. Then the profit of manufacturer 1 and that of the retailer increase simultaneously. There is a minimum value for the total profit of the three parties in the game. In general, the profit of each party is stable.

The profit with respect to $dC_2$ when $v_1 = 0.3$, $v_2 = 0.5$, the diagram can be seen in Fig. 15.

In the unstable state, it goes similarly to Fig. 14 when the cost reduction $dC_2$ is small. Then the profit of manufacturer 1

---

**Fig. 11.** Profit with respect to $dC_1$
when $v_1 = 0.2$, $v_2 = 0.2$

---

**Fig. 12.** Profit with respect to $dC_1$
when $v_1 = 0.3$, $v_2 = 0.5$
and that of the retailer fall off a cliff. The profit of manufacturer 1 keeps decreasing; at the same time, the profit of manufacturer 2 decreases continuously. After that, with the increase in its cost reduction, the profit of the three parities fluctuates and becomes unstable, which is bad for all parties in the game.

As before, we analyze the profit of manufacturer 2 with respect to adjustment speed $v_2$ and cost reduction $dC_2$, and Fig. 16 presents the result. From Fig. 16, we can get a similar conclusion that with the increase of cost reduction $dC_2$, the profit of the manufacturer reaches the maximum and then drops off sharply. However, when the adjustment speed $v_2$ is too high, the system will step into chaos and it will take a toll on the profit.

The price coefficient of the retailer

Next, we move on to the price coefficient of the retailer $\lambda_3$ and explore its impact on the profit of the parties in the game. And we also set $v_1 = 0.2$ and $v_2 = 0.2$ for the stable state and $v_1 = 0.3$ $v_2 = 0.5$ for the unstable state.
From the comparison of Fig. 17 and Fig. 18, it can be concluded that as for manufacturer 1 and the retailer, the smaller price coefficient of the retailer can bring higher profits but also aggravates the chaotic condition in which the stability of the whole system will decrease. Thus, there is a contradiction between the profitability and the stability of the whole system. With the increasing price coefficient of the retailer, the profit of manufacturer 2 also increases in general, which is different from the trend of manufacturer 1 and the retailer.

From the above three-dimensional Fig. 19 and Fig. 20, we can also see that with the increase of the price coefficient of the retailer, the profit trend of manufacturer 1 and that of manufacturer 2 is opposite. As for manufacturer 1, the profit goes decreasing with the price coefficient of the retailer increasing; as for manufacturer 2, the profit goes increasing with the price coefficient of the retailer increasing.

**Chaos control**

As we stated, when the state of the system becomes chaotic, that is, the market is in a chaotic state, the profit of

---

**Fig. 15.** Profit with respect to $dC_2$

When $v_1 = 0.3$, $v_2 = 0.5$

**Fig. 16.** Profit of manufacturer 2 with respect to $v_2$ and $dC_2$
the system will decrease and the decision-making process of supply chain’s members will become more complicated than before, which is harmful to economic development. Therefore, we need to control the chaotic state to benefit the whole system. In this section, we control chaos from two aspects: adjusting the related parameters and the decision method.

**Parameter control method**

As for manufacturer 1 and manufacturer 2, according to the above discussion and conclusion, \( dC_1 \) and \( dC_2 \) are two important parameters that can be improved. Then, we set parameters \( v_1 = 0.4, v_2 = 0.4 \) and study the change of the system stability with the change of different manufacturer cost variables. The

---

**Fig. 17.** Profit with respect to \( \lambda_3 \) when \( v_1 = 0.2, v_2 = 0.2 \)

**Fig. 18.** Profit with respect to \( \lambda_3 \) when \( v_1 = 0.3, v_2 = 0.5 \)
results of numerical simulation are shown in Fig. 21 and Fig. 22.

In Fig. 21 and Fig. 22, we draw the trend of the change of decision variables with the change of $dC_1$ and $dC_2$. As can be seen in Fig. 21 and Fig. 22, with the increase of $dC_1$ and $dC_2$, the stability of the system declines, and bifurcation and chaos form. As for manufacturer 1 and manufacturer 2, $dC_1$ and $dC_2$ represent profit margin between remanufacturing and manufacturing, and according to the conclusion we got, the profit of manufacturer 1 will increase with $dC_1$ increasing and the profit of manufacturer 2 will increase with $dC_2$ increasing.

So profit and system stability need to be balanced; that is, manufacturers need to further optimize the manufacturing process of new products, and higher stability can be obtained through reducing $C_1$ and $C_2$.

As for the retailer, according to the above analysis, the retailer price coefficient $\lambda_3$, which represents the increase in retailer price, can be improved. The trend of the change of decision variables with the change $\lambda_3$ is shown in Fig. 23.

In Fig. 23, we can see that with the decrease of $\lambda_3$, the stability of the system declines, and bifurcation and chaos form, which means the higher $\lambda_3$ is, the higher retailer price
is, and the stronger system’s stability is. Therefore, retailers can enhance their earnings stability by increasing the price of corresponding products.

**Decision-making method**

Based on the characteristics of the whole decision-making process, we adopt an adaptive method to control the chaotic state. That is, when making decisions next period, we take not only decisions in this period into consideration, but also earlier decisions. Thus, the system can be summarized as follows.

\[
\begin{align*}
F_1(t+1) &= F_1(t) + v_1 F_1(t) \frac{\partial \Pi_1}{\partial F_1} + \nu_1(t-F_1(t)-F_1(t-1)) \\
R_2(t+1) &= R_2(t) + v_2 R_2(t) \frac{\partial \Pi_2}{\partial R_2} + \nu_2(t-R_2(t)-R_2(t-1))
\end{align*}
\]

(23)

We simplify Equation (23) based on topological equivalence and then adopt \( v_1 = 0.5, v_2 = 0.35 \) and make Fig. 24.
as follows to show the effect of the control parameter’s control of chaos.

It can be seen from Fig. 24 that with the increasing control parameter $v'$, the state of the system goes into period-doubling bifurcation from chaos and then becomes stable. When the system is in a stable state, players who make decisions in this game will obtain higher profits and make their own decisions more controllable.

In summary, we propose two methods of chaos control in the game process of two manufacturers and the retailer. In the parameter control method, we keep the enterprise decision-making adjustment speed and control system from the perspective of the actual production situation. As a result, we obtain final equilibrium. In the decision-making method, we maintain the original game situation and change the decision plan of the enterprises, which makes the response of the
enterprise to the market delay. Manufacturers and the retailer can consider two methods based on the actual situation in the actual decision-making process.

**Conclusion and policy implications**

Studying the WEEE recycling activities has important ramifications for corporations, the whole supply chain system, and the market. In an electronics CLSC, more and more large retailers (e.g., Mediaworld, Fnac, and Darty) offer recycling services to increase their attractiveness. Recycling services are extremely relevant for the key manufacturer, retailer, and customers in the supply chain. Especially, some kinds of electronic products are very difficult to transport and handle, which is beneficial to customers (De Giovanni 2018).

In this paper, a closed-loop supply chain game model is constructed based on two recycling methods for product 1. The influence of multiple decision variables on the stability of the model is analyzed, and the stability control of the unstable system by two control methods is explored. The main findings of the paper get the following results.

1. The results show that the stability of the model is related to the decision adjustment speed of the manufacturers. In the game process, manufacturers and retailers cannot use radical price adjustment strategy; otherwise, it will be difficult for the game participants to control the price changes in the game process. If the game turns into a chaotic price war, the profits of the participants themselves and the whole supply chain will decline.

2. At this time, the profits of the manufacturer and the retailer will become unstable, seriously affecting the normal operation of both sides of the game. If the adjustment speed of the manufacturers keeps in a rational value, players would achieve great benefits in the real market. Benign and sustainable price adjustment strategy will make both sides benefit from the market.

3. The manufacturer’s cost reduction is beneficial to itself as well as the retailer, but not to its competitor; manufacturers should strive to improve the technology of recycled products, so as to extract higher use value from recycled products. The benefits will benefit both manufacturers and retailers in the chain and can suppress the development of competitors through technological innovation. As the sales role of the whole system, the retailer’s price strategy will have the most direct impact on the system. The higher the retailer’s price coefficient, the higher the profit is, and the more unstable the system will be.

4. There are two kinds of methods analyzed in this paper. They can be used to control chaotic. The chaotic control of the unstable system is carried out by adjusting the relevant parameters and adjusting the decision method. Both methods have a good control effect.

Besides, given the model studied in this paper, further research may be done from following two aspects: one is to consider the environmental benefits brought by waste recycling process; the other is to consider government intervention in this process, such as government subsidies and preferential tax on remanufactured products. Our model can be extended to account for these aspects.

**Acknowledgements** We thank the reviewers and associate editor for their careful reading and helpful comments on the revision of paper.

**Author contribution** Meihong Zhu, Lixiao: Thesis architecture design and writing the original draft

Junhai Ma: Supervision and project administration

Tiantong Xu, Liqing Zhu: Reviewing, preparing final draft, and formatting

Junhai Ma Liqing Zhu: Investigation, validation, programming and calculation, and editing

Tiantong Xu: Methodology and experimental calculations

**Funding** The research was supported by the Innovation Fund of Tianjin University.

**Data availability** Supplementary data to this article will be provided upon request.

**Declarations**

**Ethics approval** Not applicable

**Consent to participate** Not applicable

**Conflict for publication** Not applicable

**Conflict of interest** The authors declare no competing interests.

**References**

Bal A, Satoglu SI (2018) A goal programming model for sustainable reverse logistics operations planning and an application. J Clean Prod 201:1081–1091

Ma J, Goh M (2020) Short-and long-term repeated game behaviours of two parallel supply chains based on government subsidy in the vehicle market. Int J Prod Res 58(24):7507–7530

Bing X, Bloemhof-Ruwaard J, Chaabane A, van der Vorst J (2015) Global reverse supply chain redesign for household plastic waste under the emission trading scheme. J Clean Prod 103:28–39

De Giovanni P (2018) A joint maximization incentive in closed-loop supply chains with competing retailers: the case of spent-battery recycling. Eur J Oper Res 268(1):128–147

De Giovanni P, Zaccour G (2014) A two-period game of a closed-loop supply chain. Eur J Oper Res 232(1):22–40
Du J-g et al (2013) Dynamics analysis and chaos control of a duopoly game with heterogeneous players and output limiter. Econ Model 33:507–516

Elsadany AA (2010) Dynamics of a delayed duopoly game with bounded rationality. Math Comput Model 52(9-10):1479–1489

Gao Q, Ma J (2009) Chaos and hopf bifurcation of a finance system. Nonlinear Dyn 58(1-2):209

Gao J, Han H, Hou L, Wang H (2016) Pricing and effort decisions in a closed-loop supply chain under different channel power structures. J Clean Prod 112:2043–2057

Gu Y, Wu Y, Xu M, Wang H, Zuo T (2017) To realize better extended producer responsibility: redesign of WEEE fund mode in China. J Clean Prod 164:347–356

Guo Y, Ma J (2013) Research on game model and complexity of retailer collecting and selling in closed-loop supply chain. Appl Math Model 37(7):5047–5058

Guo Q, Wang E, Nie Y, Shen J (2018) Profit or environment? A system dynamic model analysis of waste electrical and electronic equipment management system in China. J Clean Prod 194:34–42

Hong X, Govindan K, Xu L, du P (2017) Quantity and collection decisions in a closed-loop supply chain with technology licensing. Eur J Oper Res 256(3):820–829

Huang M, Song M, Lee LH, Ching WK (2013) Analysis for strategy of closed-loop supply chain with dual recycling channel. Int J Prod Econ 144(2):510–520

Hu L, Lei M, Deng H, Keong Leong G, Huang T (2016) A dual channel, quality-based price competition model for the WEEE recycling market with government subsidy. Omega 59:290–302

Liu Z, Tang J, Li B-y, Wang Z (2017) Trade-off between remanufacturing and recycling of WEEE and the environmental implication under the Chinese Fund Policy. J Clean Prod 167:97–109

Lou W, Ma J (2018) Complexity of sales effort and carbon emission reduction effort in a two-parallel household appliance supply chain model. Appl Math Model 64:398–425

Lu B et al (2018) Perspectives on reuse of WEEE in China: lessons from the EU. Resour Conserv Recyl 135:83–92

Ma J, Guo Z (2014) The parameter basin and complex of dynamic game with estimation and two-stage consideration. Appl Math Comput 248:131–142

Ma J, Sun L (2017) Complexity analysis about nonlinear mixed oligoplies game based on production cooperation. IEEE Trans Control Syst Technol 26(4):1532–1539

Ma J, Wang H (2014) Complexity analysis of dynamic noncooperative game models for closed-loop supply chain with product recovery. Appl Math Model 38(23):5562–5572

Ma J, Wu F (2014) The application and complexity analysis about a high-dimension discrete dynamical system based on heterogeneous triopoly game with multi-product. Nonlinear Dyn 77(3):781–792

Ma W-m, Zhao Z, Ke H (2013) Dual-channel closed-loop supply chain with government consumption-subsidy. Eur J Oper Res 226(2):221–227

Ma J, Zhang F, Jiang H (2020a) Dynamic pricing game under different channel power structures in a closed-loop supply chain. Int J Bifurcation Chaos 30(04):2050052

Ma J, Hou Y et al (2020b) A time-based pricing game in a competitive vehicle market regarding the intervention of carbon emission reduction. Energy Policy 142:111440

Ma J, Hou Y, Wang Z, Yang W (2021) Pricing strategy and coordination of automobile manufacturers based on government intervention and carbon emission reduction. Energy Policy 148

Ostlin J, Sundin E, Björkman M (2008) Importance of closed-loop supply chain relationships for product remanufacturing. Int J Prod Econ 115(2):336–348

Polat O, Capraz O, Gungor A (2018) Modelling of WEEE recycling operation planning under uncertainty. J Clean Prod 180:769–779

Qiang Q, Ke K, Anderson T, Dong J (2013) The closed-loop supply chain network with competition, distribution channel investment, and uncertainties. Omega 41(2):186–194

Savaskan RC, Van Wassenhove LN (2006) Reverse channel design: the case of competing retailers. Manag Sci 52(1):1–14

Savaskan R, Canan SB, Wassenhove LNV (2004) Closed-loop supply chain models with product remanufacturing. Manag Sci 50(2):239–252

Tian Y, Ma J et al (2020) Coordination and control of multi-channel supply chain driven by consumers’ channel preference and sales effort. Chaos, Solitons Fractals 132:109576

Wei J, Govindan K, Li Y, Zhao J (2015) Pricing and collecting decisions in a closed-loop supply chain with symmetric and asymmetric information. Comput Oper Res 54:257–265

White paper WEEE recycling industry in China (2017), http://img.thinker.com/D5C994E3-F0CF-4E81-676E-B20D75A511B0_thinky_2018-05_5adbe128602f.pdf. Accessed May 2018

Xie L., Ma J (2016) Study the complexity and control of the recycling-supply chain of China’s color TVs market based on the government subsidy. Commun Nonlinear Sci Numer Simul 38:102–116

Xie L., Ma J, Hongshuai H (2017) Implications of stochastic demand and manufacturers’ operational mode on retailer’s mixed bundling strategy and its complexity analysis. Appl Math Model 55:484–501

Xie L., Ma J, Goh M (2021) Supply chain coordination in the presence of uncertain yield and demand. Int J Prod Res 59(14):4342–4358

Xu Z, Elomri A, Pokharel S, Zhang Q, Xi G, Liu W (2017) Global reverse supply chain design for solid waste recycling under uncertainties and carbon emission constraint. Waste Manag 64:358–370

Yin S, Ray S, Gurnani H, Animesh A (2010) Durable products with multiple used goods markets: product upgrade and retail pricing implications. Mark Sci 29(3):540–560

Zhou W, Zheng Y, Huang W (2017) Competitive advantage of qualified WEEE recyclers through EPR legislation. EUR J Oper Res 245(2):641–655

Zlamparet GI, Ijomah W, Miao Y, Awasthi AK, Zeng X, Li J (2017) Remanufacturing strategies: a solution for WEEE problem. J Clean Prod 149:126–136

Publisher’s note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.