Symmetry Energy and Its Components in Finite Nuclei

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Abstract. We derive the volume and surface components of the nuclear symmetry energy (NSE) and their ratio within the coherent density fluctuation model. The estimations use the results of the model for the NSE in finite nuclei based on the Brueckner and Skyrme energy-density functionals for nuclear matter. The obtained values of the volume and surface contributions to the NSE and their ratio for the Ni, Sn, and Pb isotopic chains are compared with estimations of other approaches which have used available experimental data on binding energies, neutron-skin thicknesses, and excitation energies to isobaric analog states (IAS). Apart from the density dependence investigated in our previous works, we study also the temperature dependence of the symmetry energy in finite nuclei in the framework of the local density approximation combining it with the self-consistent Skyrme-HFB method using the cylindrical transformed deformed harmonic-oscillator basis. The results for the thermal evolution of the NSE in the interval $T=0$–$4$ MeV show that its values decrease with temperature. The investigations of the $T$-dependence of the neutron and proton root-mean-square radii and the corresponding neutron skin thickness point out that the effect of temperature leads mainly to a substantial increase of the neutron radii and skins, especially in nuclei which are more rich of neutrons.

1. Introduction

The nuclear symmetry energy, as a fundamental quantity in nuclear physics and astrophysics, represents a measure of the energy gain in converting isospin asymmetric nuclear matter (ANM) to a symmetric system. Its value depends on the density $\rho$ and temperature $T$. A natural and important way to learn more about the NSE is the transition from ANM to finite nuclei. Experimentally, the NSE is not a directly measurable quantity and is extracted indirectly from observables that are related to it (see, e.g., the review [1]). The need of information for the symmetry energy in finite nuclei (including the one theoretically obtained) is a major issue because it allows one to constrain the bulk and surface properties of the nuclear energy-density functionals (EDFs) quite effectively. More information on the nuclear symmetry energy is still required for understanding the structures of nuclei far away from the $\beta$-stability line, heavy-ion collisions, supernova explosions, and neutron star properties.
In Ref. [2] it has been demonstrated by Danielewicz that the ratio of the volume to surface symmetry energy is closely related to the neutron-skin thickness (see also Refs. [3, 4]). Discussions on the correlation between the bulk and surface symmetry energy are given also e.g., in Refs. [5, 6, 7, 8]. It has been shown in [9] by Lee and Mekjian by calculations of the thermal nuclear properties that the surface symmetry-energy term is more sensitive to the temperature than the volume energy term. The thermal behavior of the symmetry energy has a role in changing the location of the nuclear drip lines as nuclei warm up. Also, since the density derivative of the symmetry coefficient reflects the pressure difference on the neutrons and protons and is thus one of the determinants in fixing the neutron skin of nuclei, the nature and stability of phases within a warm neutron star, its crustal composition or its thickness [10] would be strongly influenced by the temperature dependence of the symmetry energy.

In the present work (see also Ref. [11]) we investigate the volume and surface contributions to the NSE within the coherent density fluctuation model (CDFM) [12, 13]. We use our results for NSE obtained using Brueckner EDF [14, 15] in Refs. [16, 17, 18], as well as the considerations of this subject mentioned above (e.g., [2, 3, 4, 6, 7, 8]). The present calculations are performed using both Brueckner and, in addition, Skyrme energy-density functionals with different Skyrme forces (e.g., Ref. [19]). The results are compared with those of other theoretical methods and with corresponding experimental data obtained from analyses of different nuclear quantities, such as binding energies, neutron-skin thicknesses, excitation energies to IAS and others. In this work (see also Ref. [20]) we focus on the determination of the symmetry energy coefficient depending on $\rho$ and $T$, for which we have explored the local density approximation [21, 22, 23, 24] with some modifications. The temperature-dependent densities of these nuclei are calculated within a self-consistent Skyrme-HFB method using the cylindrical transformed deformed harmonic-oscillator basis (HFBTHO densities) [25, 26]. The kinetic energy density is calculated by the HFBTHO code, as well as, for a comparison, by the Thomas-Fermi (TF) expression up to $T^2$ term [9]. In addition, in Ref. [20] some results are presented for the $^{208}$Pb nucleus with densities obtained within the extended TF method [27, 28] and the rigorous density functional approach [29]. The effect of temperature on the rms radii of protons and neutrons and the formation of neutron skin in hot nuclei is also analyzed and discussed.

2. Theoretical framework

An important result that expresses the ratio of the volume ($a^V_A$) to the surface ($a^S_A$) energy coefficients by means of the shape of the symmetry energy dependence on density $s(\rho)$ is given (in the local density approximation to the symmetry energy), e.g. in Refs. [2, 4, 7]:

$$\frac{a^V_A}{a^S_A} = \frac{3}{r_0} \int \rho(r) \left( \frac{s(\rho_0)}{s(\rho)} - 1 \right) \, dr.$$  \hspace{1cm} (1)

In Eq. (1) $\rho(r)$ is the half-infinite nuclear matter density, $\rho_0$ is the nuclear matter equilibrium density, and $r_0$ is the radius of the nuclear volume per nucleon that can be obtained from

$$\frac{4\pi r_0^3}{3} = \frac{1}{\rho_0}.  \hspace{1cm} (2)$$

For density-independent symmetry energy $s(\rho) = s(\rho_0) = a^V_A$ and, then it follows from Eq. (1) that the ratio $a^V_A/a^S_A = 0$ [4]. The density $\rho(r)$ in Eq. (1) is uniform in two Cartesian directions and generally nonuniform in the third, usually chosen to be $z$ [6]. The integral in Eq. (1) is across the nuclear surface involving the shape of the density dependence [3]. In the Danielewicz’s approximation only the symmetry energy of a finite nucleus has a mass dependence, while $a^V_A$, $a^S_A$, and their ratio $a^V_A/a^S_A$ are $A$-independent quantities. The values of $a^V_A$ and $a^S_A$ differ for various Skyrme interactions in wide intervals (see Table I of Ref. [6]). At the same time, as
shown in [2], a combination of empirical data on skin sizes and masses of nuclei constrains the
volume symmetry parameter to $27 \leq a_V^A / a_A^V \leq 31$ MeV and the ratio $a_V^A / a_A^V$ to $2.0 \leq a_V^A / a_A^V \leq 2.8$.

In the present paper and in [11] we develop, using as a base the Danielewicz’s model [Eq. (1)],
another approach to calculate the ratio $a_V^A / a_A^S$, as well as $a_V^A$ and $a_A^S$ within the CDFM. Our
motivation is that numerous analyses of the volume and surface components of the NSE using
a wide range of data on the binding energies, neutron-skin thicknesses and excitation energies
to IAS give estimations of these quantities as functions of the mass number $A$ that change in
some intervals for different regions of nuclei. For instance, the reported values of $a_V^A$ and $a_A^S$ are consistent with each other in a wide mass region ($30 \leq A \leq 240$). In the CDFM we take nuclear
matter values of the parameters to deduce their values in finite nuclei (using the self-consistently
calculated nuclear density) which become dependent on the considered nucleus. For this purpose,
we start from Eq. (1) but in it we replace the density $\rho(r)$ for the half-infinite nuclear matter
in the integrand by the density distribution of finite nucleus (Eq. (18) of Ref. [11]). Then, we
obtain approximately an expression that allows us to calculate the ratio $a_V^A / a_A^S$. It has the form:

$$
\kappa \equiv \frac{a_V^A}{a_A^S} = \frac{3}{r_0 \rho_0} \int_0^\infty dx |\mathcal{F}(x)|^2 x \rho_0(x) \left\{ \frac{s(\rho_0)}{s[\rho_0(\mathcal{F}(x))]} - 1 \right\}.
$$

In Eq. (3) $s(\rho_0) = s^{ANM}(\rho_0)$ and the quantity $s[\rho_0(\mathcal{F}(x))] = s^{ANM}[\rho_0(\mathcal{F}(x))]$ is the NSE within
the chosen approach for the EDF. The weight function $|\mathcal{F}(x)|^2$ in Eq. (3) can be expressed
by the density distribution $\rho(r)$ and in the case of the monotonically decreasing local density
($d\rho/dr \leq 0$) can be obtained using a known density (from experiments or from theoretical
models) for a given nucleus:

$$
|\mathcal{F}(x)|^2 = -\frac{1}{\rho_0(x)} \frac{d\rho(r)}{dr} \bigg|_{r=x}
$$

with the normalization $\int_0^\infty dx |\mathcal{F}(x)|^2 = 1$. Having calculated within the CDFM the values of $s$
and $\kappa$, the expressions from which we can estimate the values of $a_V^A$ and $a_A^S$ separately can be
obtained (see Ref. [11]):

$$
a_V^A = s(1 + A^{-1/3} \kappa),
$$

$$
a_A^S = \frac{s}{\kappa}(1 + A^{-1/3} \kappa).
$$

For finite systems, different definitions of the symmetry energy coefficient and its temperature
dependence are considered in the literature. In the present paper we develop an approach to
calculate the symmetry energy coefficient for a specific nucleus starting with the LDA expression
given in [21, 22]:

$$
e_{sym}(A, T) = \frac{1}{I^2 A} \int \rho(r) e_{sym}[\rho(r), T] \delta^2(r) d^3r.
$$

In Eq. (7) $I = (N - Z)/A$, $e_{sym}[\rho(r), T]$ is the symmetry energy coefficient at temperature
$T$ of infinite nuclear matter at the value of the total local density $\rho(r) = \rho_n(r) + \rho_p(r)$,
$\delta(r) = [\rho_n(r) - \rho_p(r)] / \rho(r)$ is the ratio between the isovector and the isoscalar parts of $\rho(r)$, with
$\rho_n(r)$ and $\rho_p(r)$ being the neutron and proton local densities. The symmetry energy coefficient
$e_{sym}(\rho, T)$ can be evaluated in different ways. Following Refs. [21, 24], we adopt in this work
the definition

$$
e_{sym}(\rho, T) = \frac{e(\rho, \delta, T) - e(\rho, \delta = 0, T)}{\delta^2},
$$

where $e(\rho, \delta, T)$ is the energy per nucleon in an asymmetric infinite matter, while $e(\rho, \delta = 0, T)$
is that one of symmetric nuclear matter. These quantities are expressed by $e = E(r, T)/\rho$, where $E(r, T)$ is the total energy density of the system (for the Skyrme EDF in our case). The
dependence on temperature of $\mathcal{E}(r, T)$ and nucleon effective mass comes from the $T$-dependence of the densities and kinetic energy densities (see Ref. [20]).

In our study aiming to investigate the temperature dependence of $e_{sym}$ within a given isotopic chain, we introduce other definitions of $e_{sym}(A, T)$ in LDA that, in our opinion, would be more appropriate in this case. They concern the problem of calculating the term $e(\rho, \delta = 0, T)$ of Eq. (8) for symmetric nuclear matter. In our LDA approach the latter is simulated by considering the $N = Z = A/2$ nucleus, but we analyze two possibilities. First, on the basis of Eqs. (7) and (8) with $e = \mathcal{E}(r)/\rho$, we present the integrand of the right-hand side of the following expression for $I^2e_{sym}(A, T)$ as a difference of two terms with transparent physical meaning:

$$I^2e_{sym}(A, T) = \int d\mathbf{r} \left[ \frac{\mathcal{E}(\rho_A(r), \delta, T)}{A} - \frac{\mathcal{E}(\rho_{A1}(r), \delta = 0, T)}{A1} \right],$$

in which the first one corresponds to the energy per volume and particle of nuclear matter $\mathcal{E}(\rho_A(r), \delta, T)/A$ with a density $\rho_A(r)$ equal to that of the considered nucleus with $A$ nucleons, $Z$ protons and $N$ neutrons from the given isotopic chain. The second term $\mathcal{E}(\rho_{A1}(r), \delta = 0, T)/A1$ is the analogous for the isotope with $A1 = 2Z$ ($N1 = Z = A1/2$). For example, for the Ni isotopic chain the nucleus $A1$ is the double-closed shell nucleus $^{56}\text{Ni}$ ($Z = N1 = 28$), while for the Sn isotopic chain the nucleus $A1$ is the double-closed shell nucleus $^{100}\text{Sn}$ ($Z = N1 = 50$) and both $^{56}\text{Ni}$ and $^{100}\text{Sn}$ isotopes play a role of reference nuclei. Our second new definition of $e_{sym}(A, T)$ is based on an expression given in Ref. [24] for finite nuclei. The latter allows us, using the LDA, to present $e_{sym}(A, T)$ in the form:

$$I^2e_{sym}(A, T) = \int d\mathbf{r} \left[ \mathcal{E}(\rho_A(r), \delta, T) - \mathcal{E}(\rho_{\bar{A}}(r), N = \bar{A}/2, Z = \bar{A}/2, \delta = 0, T) \right],$$

in which the mass number $\bar{A} = A$ is the same, but with different nucleon content, $A(Z, N)$ and $\bar{A}(Z = \bar{A}/2, N = \bar{A}/2)$. This consideration requires the even-even nucleus with $N = Z = \bar{A}/2$ to be bound.

3. Results and discussion

In the case of the Brueckner EDF the results of the calculations using Eq. (3) of the ratio $\kappa$ as a function of the mass number $A$ for the isotopic chain of Sn with different forces are given in Fig. 1. By means of Eqs. (5) and (6) and the values of the NSE obtained in our works [16, 17], we calculated the coefficients $a_A^V$ and $a_A^S$. Their values as functions of $A$ for the same isotopic chain are presented in Fig. 2. It can be seen from Fig. 1 (also for Ni and Pb isotopic chains [20]) that our results for the values of the ratio $\kappa$ are within the range $2.10 \leq \kappa \leq 2.90$. This range of values is similar to the estimations of the ratio $\kappa$ extracted from nuclear properties and presented in Ref. [7]:

$$2.6 \leq \kappa \leq 3.0$$

from IAS and skins [3],

$$2.0 \leq \kappa \leq 2.8$$

from masses and skins [2], and

$$1.6 \leq \kappa \leq 2.0$$

from the analyses in Ref. [7] of masses and skins. As can be seen the ranges (11) and (12) are in a good agreement with our results. In addition, there exists a ”kink” in the curve of $\kappa = a_A^V/a_A^S$ as a function of $A$ for the double-magic $^{132}\text{Sn}$ nucleus that can be seen in Fig. 1. It is seen from Fig. 2 (also for Ni and Pb isotopic chains [20]) that our CDFM results obtained with Brueckner EDF for $a_A^V$ are between 41.5 and 43 MeV, while for $a_A^S$ they are between 14 and 20 MeV. We
would like to note that "kinks" appear also in the cases of $a^V_A$ and $a^S_A$ as functions of the mass number $A$. In Fig. 2(a) a "kink" appears for $a^V_A(A)$ not only for the double-magic $^{132}$Sn, but also for the semi-magic $^{140}$Sn nucleus. The latter is related to the closed $2f_{7/2}$ subshell for neutrons. Reiss et al. discussed in Ref. [30] that the region around $N = 90$ for neutron-rich tin isotopes is an interesting one because the shell structure is somewhat fluctuating. In addition, in Ref. [31] $N = 90$ was predicted to be submagic with Gogny D1S and D1M interactions at $^{140}$Sn because the $2f_{7/2}$ orbit is fully occupied, but not with M3Y-P6 and P7 semi-realistic NN interactions.

The mass dependence of $e_{sym}(A)$ is displayed in Fig. 3 for Ni, Sn, and Pb isotopic chains for SLy4 and SkM* interactions at three temperatures, $T=0$, 2, and 4 MeV. The results are obtained by using HFBTHO densities and the TF method up to $T^2$ term for the kinetic energy density (more details can be found in [20]). From one hand, one can see that the values of $e_{sym}$ calculated with SLy4 overestimate those obtained with SkM* force. From another side, the difference between both sets of values decreases going to higher temperatures, in a way that it is small at the transition from $T=0$ to $T=2$ MeV and a "gap" appears between the results corresponding to $T=2$ and $T=4$ MeV. For Pb isotopic chain even a "crossover" of curves that correspond to temperatures $T=0$ and $T=2$ MeV and both parametrizations is observed in Fig. 3(c). We also would like to note the existence of a kink in the values of $e_{sym}(A)$ at zero temperature at the double-magic $^{78}$Ni and $^{132}$Sn nuclei (see Figs. 3(a) and 3(b)) as well as the lack of kinks in the Pb isotopic chain [Fig. 3(c)]. These results confirm our previous observations when studying the density dependence of the symmetry energy for Ni, Sn, and Pb isotopes [16, 17]. We also note that in the cases of $e_{sym}(A)$ for Ni and Sn isotopic chains the kinks exist for $T = 0$ MeV, but not for $T = 2$ and $T = 4$ MeV. The reason is the well-known fact that the shell effects can be expected up to $T \leq 2$ MeV.

4. Conclusions
The results of the present work can be summarized as follows:

i) We develop, using as a base the Danielewicz’s model [Eq. (1)], another approach within the CDFM to calculate the ratio $a^V_A/a^S_A$ between the volume and surface components of the symmetry energy $s$, as well as $a^V_A$ and $a^S_A$ separately, for finite nuclei. We obtain within the CDFM the expression for $\kappa \equiv a^V_A/a^S_A$ [Eq. (3)] that allows us to calculate this ratio using the ingredients of the model, the weight function $|F(x)|^2$ and the nuclear matter symmetry energy $s^{ANM}[\rho_0(x)]$ from two energy-density functionals, Brueckner and Skyrme ones. The first one of
them was used to calculate the NSE in our previous works [16, 17, 18]. In the CDFM we take nuclear matter values of the components of NSE to deduce their values in finite nuclei. Thus, our approach is different from the Danielewicz’s formalism. Being motivated by the available empirical data that show $A$-dependence of $a_V^A$, $a_S^A$, and their ratio, we obtained in our approach a possibility to find a (weak) $A$-dependence of the theoretical results for these quantities within the CDFM.

ii) Studying firstly the isotopic sensitivity of $a_V^A$, $a_S^A$, and their ratio in the case of using the Brueckner EDF we observe peculiarities (“kinks”) of these quantities as functions of the mass number $A$ in the cases of the double-magic $^{78}$Ni and $^{132}$Sn isotopes for $\kappa \equiv a_V^A / a_S^A$, $a_V^A$, and $a_S^A$, as well as a ”kink” of $a_V^A$ for $^{140}$Sn. The latter is related to the closed $2f_{7/2}$ subshell for neutrons. The origin of the “kinks” is in the different behavior of the density distributions $\rho(r)$ for the isotopes, because the derivative of $\rho(r)$ determines the weight function $|F(x)|^2$ [Eq. (4)] that takes part in the expression for the ratio $\kappa \equiv a_V^A / a_S^A$ [Eq. (3)]. Similarly to the case when Brueckner EDF is used, in the case of the Skyrme EDF one can see also a ”kink” in the behavior of $\kappa$ for the chain of Ni at $A = 78$ and for the Sn chain at $A = 132$, as well as a lack of ”kinks” for the Pb case. A ”kink” in the Ni chain at $A = 78$ can be seen also in the $A$-dependence of $a_S^A$, as well as of $a_V^A$ at $A = 132$ in the case of the Sn chain.

iii) A theoretical approach to the nuclear many-body problem has been used to study the temperature dependence of the symmetry energy coefficient in finite nuclei and other properties, such as the $T$-dependent nucleon densities and related rms radii, as well as the possibility of formation of neutron skins. The properties of hot nuclei were modelled in a temperature range $T = 0$–4 MeV. In general, we have found that the density distributions decrease with the temperature in the center of the nucleus. Following the trend of the corresponding proton and neutron rms radii, the neutron-skin thickness grows significantly with the increase of $T$ within a given isotopic chain.

iv) Our investigations of the $T$-dependent symmetry coefficients $e_{sym}(A, T)$ for finite nuclei (in particular, cases of Ni, Sn, and Pb isotopic chains) within the LDA with some modifications face the problem for the choice of density distributions and the kinetic energy densities. In our work both quantities are calculated through the HFBTHO code that solves the nuclear Skyrme-HFB problem by using the cylindrical transformed deformed harmonic-oscillator basis [26]. We have explored the LDA expression [Eq. (7)] for the symmetry energy $e_{sym}(A, T)$. Aiming to study the $T$-dependence of $e_{sym}$ within a given isotopic chain, we introduced two new definitions of $e_{sym}(A, T)$ [Eqs. (9) and (10)] within the LDA, as an attempt to analyze in a more appropriate...
way the symmetry energy coefficient of finite nuclei within a given chain. In general, the results of $\varepsilon_{sym}$ calculated for various isotopes in the present work are in good agreement with theoretical predictions for some specific nuclei reported by other authors.

Having in mind the dependence of $\varepsilon_{sym}(A,T)$ on its various definitions we note that more refined future investigations, for instance, of the temperature dependence of both volume and surface components of the symmetry energy coefficient [24], would provide better description of hot nuclei and could minimize the ambiguities due to the use of different definitions for the symmetry energy coefficient of finite nuclei.

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