Microwave transmission through an artificial atomic chain coupled to a superconducting photonic crystal

Guo-Zhu Song\textsuperscript{1}, Leong-Chuan Kwek\textsuperscript{2,3,4,5}, Fu-Guo Deng\textsuperscript{6}, and Gui-Lu Long\textsuperscript{1,7,8}\textsuperscript{*}

\textsuperscript{1}State Key Laboratory of Low-Dimensional Quantum Physics and Department of Physics, Tsinghua University, Beijing 100084, China
\textsuperscript{2}Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, Singapore 117543
\textsuperscript{3}Institute of Advanced Studies, Nanyang Technological University, Singapore 639798
\textsuperscript{4}National Institute of Education, Nanyang Technological University, Singapore 639798
\textsuperscript{5}MajuLab, CNRS-UNS-NUS-NTU International Joint Research Unit, UMI 3654, Singapore
\textsuperscript{6}Department of Physics, Applied Optics Beijing Area Major Laboratory, Beijing Normal University, Beijing 100875, China
\textsuperscript{7}Tsinghua National Laboratory of Information Science and Technology, Beijing 100084, China
\textsuperscript{8}Collaborative Innovation Center of Quantum Matter, Beijing 100084, China

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Emitters strongly coupled to a photonic crystal provide a powerful platform for realizing novel quantum light-matter interactions. Here we study the optical properties of a three-level artificial atomic chain coupled to a one-dimensional superconducting microwave photonic crystal. A sharp minimum-energy dip appears in the transmission spectrum of a weak input field, which reveals rich behavior of the long-range interactions arising from localized bound states. We find that the dip frequency scales linearly with both the number of the artificial atoms and the characteristic strength of the long-range interactions when the localization length of the bound state is sufficiently large. Motivated by this observation, we present a simple model to calculate the dip frequency with system parameters, which agrees well with the results from exact numerics for large localization lengths. Furthermore, we find that the model remains valid even though the coupling strengths between the photonic crystal and artificial atoms are not exactly equal and the phases of external driving fields for the artificial atoms are different. Thus, we may infer valuable system parameters from the dip location in the transmission spectrum, which provides an important measuring tool for the superconducting microwave photonic crystal systems in experiment. With remarkable advances to couple artificial atoms with microwave photonic crystals, our proposal may be experimentally realized in the near future.

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I. INTRODUCTION

In recent years, one-dimensional (1D) waveguide quantum electrodynamics (QED) systems have emerged as an exciting frontier in quantum information science \cite{1, 2, 3}. The waveguide systems benefit from the confinement of continuous electromagnetic modes over a large bandwidth, which couple to nearby atoms or embedded artificial atoms. There are a wide variety of systems that can act as waveguide platforms such as plasmonic nanowires \cite{4, 5}, optical nanofibers \cite{6, 7}, diamond waveguides \cite{8, 9}, superconducting transmission lines \cite{10, 11}, and photonic crystal waveguides \cite{12}. Due to the intrinsically tailorable band structure, photonic crystal waveguides are of particular interest and enable tunable long-range interactions in many-body systems \cite{13, 14}.

Photonic crystals are highly dispersive periodic dielectric media in which the refractive index is modulated periodically due to photonic band gaps \cite{15, 16}. In this configuration, when a qubit trapped nearby the photonic crystal is excited at a frequency inside the band gap, it cannot radiate into the dielectric but gives rise to a qubit-photon bound state \cite{17, 18}. The photonic component of the bound state is an exponentially decaying envelope spatially centered at the qubit position, which facilitates coherent excitation exchange with proximal qubits \cite{19, 20}. Although significant progress has been made \cite{21, 22}, realizing efficient coupling in the optical regime faces the challenging task of interfacing emitters with photonic crystal waveguides. Recently, superconducting quantum circuits provide an alternative approach to study the physics of the bound state in the microwave regime \cite{23}. Using a stepped-impedance microwave photonic crystal and a superconducting transmon qubit, Liu et al. first experimentally observed the bound state in superconducting transmission lines \cite{24}. In their device, by adjusting the detuning between the qubit and the band edge, the localization length of the bound state is widely tunable. Later, Sundaresan et al. experimentally realize strong coupling between two transmon qubits and a superconducting microwave photonic crystal, which is a promising benchmark to create 1D chains of qubit-photon bound states with tunable long-range interactions \cite{25}.

Inspired by these remarkable advances, we here study the scattering properties of a weak incident field travel-
ling through an array of $\Delta$-type artificial atoms coupled to a superconducting microwave photonic crystal. In this work, one transition of the $\Delta$-type artificial atom is inside the band gap, which gives rise to the bound state. Besides, another transition is coupled to the electromagnetic modes of the superconducting microwave photonic crystal, and is utilized to explore the long-range interactions arising from the bound states.

With an effective non-Hermitian Hamiltonian, we calculate the transmission spectrum of a weak microwave input field and observe a sharp minimum-energy dip. We analyze the relation between the dip frequency and the system parameters, such as the number of the artificial atoms and the characteristic strength of the long-range interactions. The results reveal that the dip frequency scales linearly with both the number of the artificial atoms and the characteristic strength for large localization lengths of the bound states, which may provide an important measuring tool for the superconducting microwave photonic crystal systems. Motivated by this observation, we give a simple model to calculate the dip frequency with system parameters. We find that, when the localization length of the bound state is large enough, the results of our model agree well with exact numerics. Moreover, we study the effects of a Gaussian inhomogeneous broadening of the coupling strength and different phases of the external driving field for the artificial atoms on the dip frequency, respectively. The results show that, our model remains valid even though the coupling strengths between the photonic crystal and artificial atoms are not exactly equal and the phases of the driving fields for the artificial atoms are different. That is, in experiment, one may infer the system parameters by measuring the dip frequency in the transmission spectrum.

This article is organized as follows: In Sec. II, we present the physics of an artificial atomic chain coupled to a superconducting microwave photonic crystal, and an effective Hamiltonian is introduced for the system. In Sec. III, we study the transmission spectrum of the weak microwave coherent field, and give a simple model to estimate the dip frequency with system parameters. Moreover, we analyze the effects of the inhomogeneous broadening of the coupling strength and different phases of the driving field for the artificial atoms on the dip frequency, respectively. Finally, we summarize the results, and discuss the advantage and feasibility of the superconducting microwave photonic crystal systems in Sec. IV.

II. MODEL AND HAMILTONIAN

We model an array of $\Delta$-type artificial atoms coupled to an infinite superconducting microwave photonic crystal, as shown in Fig. 1(a). The superconducting microwave photonic crystal is implemented by periodically alternating sections of low and high impedance coplanar waveguides, which can be realized via changing the gap width and centre pin of the coplanar waveguide [78][79]. The dispersion relation of the guided modes in the superconducting microwave photonic crystal can be obtained by the transfer matrix method [78], and is given by

$$\cos(kd) = \cos\left(\frac{\omega_x d}{v_g} \right) - \frac{1}{2} \frac{Z_l}{Z_s} \sin\left(\frac{\omega_x d}{v_g}\right) \sin\left(\frac{\omega_y d}{v_g}\right) \sin\left(\frac{\omega_z d}{v_g}\right) \sin\left(\frac{\omega_x d}{v_g}\right),$$

where $k$ is Bloch wave vector, $d = d_l + d_s$ is the unit cell length, and $\omega_x$ is the guided mode frequency with velocity $v_g$. $Z_l$ ($Z_s$) and $d_l$ ($d_s$) represent the impedance and length of the high (low) impedance coplanar waveguide in the unit cell, respectively. Each artificial atom has three energy levels $|g\rangle$, $|e\rangle$ and $|s\rangle$. The $\Delta$-type artificial atom can be realized by a flux-based supercon-
ducting quantum circuit when the external magnetic flux through the loop $\Phi_e \neq \Phi_g/2$, where $\Phi_e$ is the flux quantum. Since the second band is smoother than the first one, each artificial atom is purposely placed in the center of the high-impedance section, which maximizes (minimizes) the coupling between the artificial atom and the second (first) band. Moreover, the width of the second band can be sufficiently large so that the influence of other band is ignored. In detail, the $\Delta$-type artificial atom is coupled to the high-impedance section of the unit cell through the loop-line mutual inductance $M$.

Here, we assume that the resonance frequency $\omega_{sg}$ of the transition $|g\rangle \leftrightarrow |s\rangle$ is inside the band gap, and close to second band edge frequency $\omega_e$ with detuning $\delta = \omega_e - \omega_{sg}$, as shown in Fig. 1(b). In this domain, due to the van Hove singularity in the density of states, the transition $|g\rangle \leftrightarrow |s\rangle$ of the artificial atom is predominantly coupled to the modes close to the second band edge. In this case, we can approximate the dispersion relation near the second band edge to be quadratic $\omega_s \approx \omega_e + \alpha(k - k_e)^2$, where $\alpha$ denotes the curvature of the band structure and $k_e = \pi/d$ is the band edge wave vector. Once such an artificial atom is excited to the state $|s\rangle$, a localized bound state appears. Specifically, as shown in Fig. 1(a), in real space, the photonic component of the bound state is exponentially localized around the artificial atom

$$\text{described by an effective non-Hermitian Hamiltonian} \quad H_{\text{non}} = -\sum_j \left[ (\Delta + i\Gamma_e/2)\sigma_{ze}^j + (\Delta - \Delta_c + i\Gamma_s/2)\sigma_{ss}^j \right],$$

$$+\Omega_c(\sigma_{sz}^j + \text{H.c.}) - i\frac{\Gamma_s}{2} \sum_{j,k} e^{ik_e z_j - z_k} |\sigma_{ej}\sigma_{ge}\rangle,$$

$$-V \sum_{j,k} (-1)^{j}|z_j - z_k|/\epsilon - |z_j - z_k|/L |\sigma_{sg}^j\sigma_{gs}^k\rangle. \quad (2)$$

Here $\Delta = \omega_{in} - \omega_{eg}$ is the detuning between the frequency $\omega_{in}$ of the input field with wave vector $k_{in}$ and the resonance frequency $\omega_{eg}$. $\Gamma_c (\Gamma_s)$ is the decay rate of the state $|e\rangle (|s\rangle)$ into free space, $n$ is the number of the artificial atoms, and $\Delta_c = \omega_e - \omega_{sg}$ denotes the detuning between the frequency $\omega_e$ of the external driving field and the frequency $\omega_{es}$ of the transition $|e\rangle \leftrightarrow |s\rangle$. $\Gamma_e = 4\pi g^2/\epsilon g$ is the single-atom spontaneous emission rate into the photonic crystal modes. $z_j$ represents the position of artificial atom $j$, and $V$ characterizes the strength of the long-range coherent interactions arising from the bound states. Note that, we include the case $j = k$ in the last term of Eq. (2), which corresponds to the Stark shift experienced by artificial atom $j$ due to its own bound photon.

Here, we mainly focus on the transport properties of a weak coherent probe field propagating through the artificial atomic chain. The corresponding driving part is given by $H_{\text{dr}} = \sqrt{\frac{\gamma_c}{2}}E \sum_j (\sigma_{ej}^j e^{ik_{in}z_j} + \text{H.c.})$, where $E$ is the amplitude of the weak probe field (Rabi frequency $\sqrt{\frac{\gamma_c}{2}}E$). As a consequence, the dynamics of the system is governed by the total Hamiltonian $\hat{H} = H_{\text{non}} + H_{\text{dr}}$, and the initial state is the ground state $|\psi_0\rangle = |g\rangle \otimes |n\rangle$ of the artificial atomic chain. When the probe field is sufficiently weak ($\sqrt{\frac{\gamma_c}{2}}E \ll \Gamma_e$), the occurrence of quantum jumps is infrequent and can be neglected. Using the input-output formalism, the transmitted field operator is

$$a_t(z) = E e^{ik_{in}z} + i \sqrt{\frac{\Gamma_o}{2\epsilon g}} \sum_{j=1}^{n} \sigma_{ge}^j e^{ik_e (z - z_j)}. \quad (3)$$

Consequently, the transmittance ($T$) of the weak probe field is given by

$$T = \frac{\langle \psi | a_t^\dagger a_t | \psi \rangle}{\epsilon^2}, \quad (4)$$

where $|\psi\rangle$ represents the steady state.
Γ′ = 3Γ (blue circles), L squares), characteristic strength V parameters: (a)-(d) are equally spaced along the 1D infinite superconducting microwave photonic crystal. To minimize the reflection of the input field from the artificial atomic chain, we choose the configuration $k_d d = \pi/2$. As shown in Figs. 2(a) and 2(b), we calculate the transmission spectra of the weak monochromatic coherent input field with localization length $L = 10^2 d$ for two choices of characteristic strength $V$. As illustrated in Fig. 2(a), for $V = 0$ (i.e., a conventional superconducting transmission line), we observe the electromagnetically induced transparency phenomenon in the transmission spectrum. While for $V \neq 0$, such as for the case $V = 3\Gamma_0$ shown in Fig. 2(b), some new sharp dips emerge in the transmission spectrum, which arise from the long-range coherent interactions between the artificial atoms.

In the following, we mainly focus on the minimum resonance frequency $\omega_{\text{min}}$, since it may reveal rich behavior of the long-range interactions. For simplicity, we adopt $|\omega_{\text{min}}|$ instead of $\omega_{\text{min}}$ in our discussions, which does not qualitatively influence the conclusions. As shown in Fig. 2(c), we plot $|\omega_{\text{min}}|$ as a function of the number of the artificial atoms coupled to the photonic crystal for different choices of the localization length $L$. The results show that in the case $L/d \gg 1$, $|\omega_{\text{min}}|$ scales linearly with the number of the artificial atoms. In particular, for infinite-range interaction such as $L/d = 10^4$, we conclude the relation $|\omega_{\text{min}}| \approx nV$. In fact, by diagonalizing the last term of Eq. (2) for $L/d \to \infty$, we obtain $(n - 1)$ degenerate resonance energies zero and one resonance energy $-nV$. While, for finite-range interaction such as $L/d = 10$, the frequency $|\omega_{\text{min}}|$ scales sub-linearly with the number of artificial atoms. Moreover, for short-range interaction such as $L = d$, the frequency $|\omega_{\text{min}}|$ almost remains constant despite we increase the number of the superconducting artificial atoms, as shown in Fig. 2(c). In addition, we also give the frequency $|\omega_{\text{min}}|$ versus the characteristic strength $V$ for different choices of the localization length $L$. Fig. 2(d) reveals that the frequency $|\omega_{\text{min}}|$ scales linearly with the characteristic strength $V$ in all cases.

The above observation motivates us to present a simple model for estimating the frequency $|\omega_{\text{min}}|$ with the parameters $V$, $L$, $n$. As shown in Figs. 2(a) and 2(b), the minimum resonance energy $\omega_{\text{min}}$ arises from the long-range interaction Hamiltonian

$$ H_b = -V \sum_{j,k} e^{-|z_j - z_k|/L} \sigma_j^z \sigma_k^z. $$

Here, for brevity, we have ignored the phase factor, which does not qualitatively change the conclusions. Intuitively, for any two atoms $j$ and $k$ chosen from $n$ artificial atoms, we may sum over all possible cases for separations, and get one energy $A$, i.e.,

$$ A = -V \sum_{l=0}^{n-1} e^{-ld/L}. $$

Note that, here we include the term $l = 0$, which corresponds to Stark shift experienced by an artificial atom. 

III. NUMERICAL RESULTS

A. The transmission properties of the input field

Here, we consider the case that $n = 10$ artificial atoms are equally spaced along the 1D infinite superconducting microwave photonic crystal.
due to its own bound photon. Summing all the terms, we easily obtain

\[ |A| = V e^{d/L} - e^{-(n-1)d/L} e^{d/L - 1}. \]  

(7)

The result of this model agrees well with the exact numerics shown in Fig. 2(d). That is, the frequency \( |\omega_{\min}| \) calculated from exact numerics scales linearly with the characteristic strength \( V \) for fixed number of the artificial atoms and localization length \( L \) of the bound states. In the limit \( L \gg nd \), using the approximation \( e^x \approx (1 + x) \) when \( x \to 0 \), the energy \( |A| \) in Eq. 7 can be written approximately as

\[ |A| \approx nV. \]  

(8)

It is consistent with the result \( |\omega_{\min}| \approx nV \) by diagonalizing the long-range interaction Hamiltonian for infinite localization length mentioned above. Consequently, with this model, in the limit \( L \gg d \), we may use the energy \( |A| \) to estimate the frequency \( |\omega_{\min}| \) with system parameters \( L, V, n \) and \( d \).

To demonstrate the validity of our model, we compare its results with exact numerics, as shown in Fig. 3(a). We observe that this model agrees well with the exact numerics in the limit \( L \gg d \). While for finite localization length \( L \), such as \( L=10d \), there is a difference between the two results, and the frequency \( |\omega_{\min}| \) scales sub-linearly with the number \( n \) of the artificial atoms in both cases. To obtain the validity of the localization length \( L \), we give the results from our model and exact numerics as a function of the localization length \( L \), respectively, as shown in Fig. 3(b). We observe that, when \( L \leq 10^2d \) in our system, this simple model agrees well with exact numerics. In other words, for a sufficiently large localization length \( L \), one may infer the characteristic strength \( V \) of the long-range interactions from Eq. 8 for fixed number of the artificial atoms, by measuring the dip frequency \( |\omega_{\min}| \) in the transmission spectrum.

As shown in Fig. 4 with \( n=10 \) and five choices of the localization length \( L \), we give the coupling strengths of the nearest and farthest two artificial atoms arising from the long-range coherent interactions in Eq. 5, respectively. For simplicity, here we omit the minus sign in Eq. 5, which does not influence the conclusions. In fact, the interaction strength between two atoms is determined by the overlap of their photonic envelopes with the atoms, as shown in Fig. 1(a). Here, we take the case \( j=1, k=2 \) as an example for the nearest two atoms, and \( j=1, k=10 \) for the farthest two atoms. The results reveal that, for infinite-range interaction such as \( L=10^4d \), the interaction strengths for the nearest and farthest two atoms are almost the same, as shown in the second row of Fig. 4. While, when the localization lengths of the bound states decrease gradually, the difference of the interaction strengths between the two cases becomes obvious. Specifically, for short-range interaction such as \( L=d \), the interaction strength between the nearest two atoms is approximately \( 0.37V \), while the interaction between the farthest two atoms becomes very weak and negligible, as

| L/d | j=1, k=2 | j=1, k=10 |
|-----|---------|---------|
| 10000 | 1.00 | 1.00 |
| 100   | 0.99  | 0.91  |
| 50    | 0.98  | 0.84  |
| 10    | 0.90  | 0.41  |
| 1     | 0.37  | 0.00  |

FIG. 4: The interaction strengths of the nearest two atoms \((j = 1, k = 2)\) and farthest two atoms \((j = 1, k = 10)\) arising from Eq. 5 with \( n = 10 \) for \( L = 10^4d, L = 10^2d, L = 50d, L = 10d \) and \( L = d \). The numbers (in units of \( V \)) in the second and third columns retain two digits after the decimal point. Here, we omit the minus sign in Eq. 5.
the dip frequency \( \omega_{\text{min}} \) is robust to the inhomogeneous broadening of the coupling strength. In fact, as shown in Figs. 2(a) and 2(b), the frequency \( \omega_{\text{min}} \) originates from the long-range interaction term in Eq. 2. We conclude that, although the coupling strengths between the photonic crystal and artificial atoms are not exactly equal in experiment, our model is valid.

As mentioned above, the transitions \(|e\rangle \leftrightarrow |s\rangle\) of the artificial atoms are driven by external microwave fields. In practical superconducting microwave photonic crystal systems, the phase of the external driving field for each artificial atom may be different, which have been regarded to be identical in above sections. Here, we assume that the external classical field driving the \( j \)th artificial atom has a Rabi frequency \( \Omega_j \) and a phase \( \phi_j \), where \( \Omega_c \) is regarded to be identical for all artificial atoms and \( \phi_j \) is randomly chosen from \([0, \eta]\) in our calculations. As shown in Fig. 2(b), we plot the dip frequency \( \omega_{\text{min}} \) calculated from exact numerics as a function of the parameter \( \eta \). Evidently, the dip frequency \( \omega_{\text{min}} \) is immune to the phase of the external driving field. In other words, our model remains valid even the phases of the driving field for the artificial atoms are different.

**IV. CONCLUSION**

In conclusion, we have studied the transport properties of a \( \Delta \)-type artificial atomic chain coupled to an infinite superconducting microwave photonic crystal. In this work, we take advantage of the superconducting quantum circuits, which provide widely tunable artificial atoms with long coherence times \([80, 87]\). Here, to reach the strong-coupling or ultrastrong-coupling regimes, the artificial atomic chain can be realized by superconducting transmon qubits \([78, 79]\) or flux qubits \([80, 81, 82, 83]\). Besides the method adopted in this paper, there are other approaches to realize superconducting microwave photonic crystals, such as lumped element circuits and Josephson junction arrays.

In our system, we place the resonance frequency \( \omega_{\text{sg}} \) of the transition \(|g\rangle \leftrightarrow |s\rangle\) inside the band gap, which gives rise to the long-range coherent dipole-dipole interactions between the artificial atoms. To probe the above mechanism in many-body regime, we utilize the coupling between the atomic transition \(|g\rangle \leftrightarrow |e\rangle\) and the second band modes of the superconducting microwave photonic crystal. According to the phenomena concluded from exact numerics, we present a simple model to estimate the dip frequency with known system parameters. The results reveal that our model agrees well with exact numerics when the localization length is sufficiently large. Moreover, we analyze the influence of a Gaussian inhomogeneous broadening of the coupling strength and different phases of the external driving field for the artificial atoms on the dip frequency, respectively. We show numerically that, the above conclusions still hold when the coupling strengths between the photonic crystal and

![Image](image_url)
artificial atoms are not exactly equal and the phases of the driving field for the artificial atoms are different in practical condition. That is, with this model, one may acquire valuable system parameters by measuring the dip frequency in the transmission spectrum, which opens up a new avenue to explore the superconducting microwave photonic crystal systems.

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