The system with exponentially degenerate vacuum state

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Abstract

I suggest and examine artificial material which has exponentially degenerate vacuum state. The corresponding Hamiltonian contains only exotic four-spin interaction term. Each vacuum state is realized as a particular spin configuration separated from others by potential barriers. The benefit of such system in practical applications is that it can be used as high density magnetic recording system which can reduce storage of one bit information to nm scale. The information is stored as a particular vacuum state of the system. The process of recording can be visualized as a process in which the system moves from one vacuum state to another. Storing information in the form of different vacuum states separated by potential barriers will allow to protect it from fluctuations and for a longer time. These materials can be realized as lattices of nuclear spins with specially adjusted interactions. The planes of flipped spins can in principle be of atomic scale.

Dedicated to the memory of Professor Judah Eisenberg
1 Introduction

Lattice spin systems and different ferromagnetic materials with competing interaction have been catching attention in the last decades [1]. Spin glasses, alloys and amorphous systems which have randomly distributed competing interaction, are also of similar nature [2, 3, 4].

In the recent articles [5, 6, 7, 8] the authors formulated a new class of statistical systems in which the energy functional is proportional to the total length of edges of the interface [9]. These lattice spin systems have specially adjusted interaction between spins in order to simulate a given energy functional. The specific property of these systems is that they have very high – exponential degeneracy of the ground state. This happens simple because surface tension forces are tuned to vanish [9]. This peculiar property of the system could make them useful for practical applications. In this article I suggest and examine application of this system in memory devices and possibly to store bits in future quantum computers.

In three dimensions the corresponding Hamiltonian is equal to [6]

\[ H_{gonihedric}^{3d} = -2k \sum_{\vec{r}, \vec{\alpha}} \sigma_{\vec{r}} \sigma_{\vec{r}+\vec{\alpha}} + \frac{k}{2} \sum_{\vec{r}, \vec{\alpha}, \vec{\beta}} \sigma_{\vec{r}} \sigma_{\vec{r}+\vec{\alpha}+\vec{\beta}} - \frac{1}{2} \frac{k}{2} \sum_{\vec{r}, \vec{\alpha}, \vec{\beta}} \sigma_{\vec{r}} \sigma_{\vec{r}+\vec{\alpha}} \sigma_{\vec{r}+\vec{\alpha}+\vec{\beta}} \sigma_{\vec{r}+\vec{\beta}}, \tag{1} \]

where \( \vec{r} \) is a three-dimensional vector on the lattice \( \mathbb{Z}^3 \), the components of which are integer and \( \vec{\alpha}, \vec{\beta} \) are unit vectors parallel to the axes. This lattice system crucially depends on the coupling constant \( k \) called self-intersection coupling constant. The form of the Hamiltonian \( H^k \) and the symmetry of the system essentially depends on \( k \): when \( k \neq 0 \) one can flip spins on arbitrary parallel layers and thus the degeneracy of the ground state is equal to \( 3 \cdot 2^N \), where \( N^3 \) is the size of the lattice (see Figures 1). When \( k = 0 \) the system has even higher symmetry, all states, including the ground state are exponentially degenerate [7]. This degeneracy is equal to \( 2^{3N} \). This is because now one can flip spins on arbitrary layers, even on intersecting ones (see Figure 2). The corresponding Hamiltonian contains only exotic four-spin interaction term.

This simply means that the "crystal" of the size \( N^3 \) has \( 2^{3N} \) different ground states. This exponential degeneracy is much bigger than the degeneracy of the vacuum state of the Ising ferromagnet, which is simply equal to two, and in this respect has very close nature with spin glasses [11] and may describe liquid-glassy phase transition [12, 13].

In the usual Ising ferromagnet we have two different vacuum states, so in order to store more than one bit of information one should allow excited states as it is shown on Figure 1 A. With decreasing geometries of recording and reading heads and increasing magnetic media storage densities, these excitations become metastable thanks to the fluctuations on nm scale and thus can not be protected from damages. In about 10 years, storage of one bit of information is expected to cover an area of 100x100 nm², and metastability of excitations on nm scale becomes an increasingly limiting factor of performance [15].

Opposite to that situation the "crystal" of size \( N^3 \) which has special interaction between spins can "memorize" \( 2^{3N} \) different states, which are well separated by potential barriers (see Figure 1 and 2). Therefore we suggest that natural or artificial materials with the corresponding structure of interactions can be used as magnetic recording systems. This article considers the question of possible construction of artificial material with the

\[ ^{1} \text{This is very important property of the system which allows also to construct the dual system [7].} \]
Figure 1: Magnetic strip of the width $h$ with islands $C_i$ of flipped spins. For the 3D-Ising ferromagnet the energy of this configuration is equal to the length of the boundary $C_i$ times the strip width $h$ and is proportional to the area $S = h \sum_i \text{length}(C_i)$ of the interface. It is nonzero for both configurations A and B. For the gonihedric system (1) the energy is proportional to the curvature, of the boundary and of the intersections, multiplied by the strip width $h$. This is the size of the interface $A = h \sum_i (\text{Right Angles})_i + 4 \kappa \sum_i (\text{Intersections})_i$. It is nonzero for the configuration A, but is equal to zero for the configuration B, the configuration B is one of the ground states.

Figure 2: Magnetic strip of the width $h$ with islands $C_i$ of flipped spins. For the gonihedric system with $k = 0$ the energy is proportional to the curvature of the boundary times $h$, $A = h \sum_i (\text{Right Angles})_i$, but now without contributions coming from self-intersections, therefore any "chess-board" configuration is a ground state.
above property, its possible realization and application in memory devices and possibly to store bits in future quantum computers.

2 System with \(3 \times 2^N\) ground state degeneracy

The benefit of having system with exponentially degenerate vacuum state in practical applications is that it can be used as high density magnetic recording system. Each vacuum state is realized as a particular spin configuration separated from others by potential barriers of height \(U\) which is proportional to the width of the magnetic strip \(h\) (see formulas (3) and Figure 1). The information can be stored as a particular vacuum state of the system. The process of recording can be visualized as a process in which the system moves form one vacuum state to another. Storing information in the form of different vacuum states separated by potential barriers will allow to keep it safely away from fluctuations and for a longer time. We shall demonstrate that this material can be realized as a lattice of nuclear spins with specially adjusted interactions.

First let us consider the system which has exponentially degenerate vacuum state, but only at zero temperature. This "crystal" which has \(3 \times 2^N\) ground state degeneracy has been constructed in [5] and was studied in a number of articles analytically [6, 7, 8, 10] and numerically [11, 12] (see Figure 1). It corresponds to the case \(k = 1\) in the equation (1). The Hamiltonian of the system has the form

\[
H^{3d}_{gonihedric} = -J \sum_{\vec{r}, \vec{a}} \sigma_{\vec{r}} \sigma_{\vec{r}+\vec{a}} + \frac{J}{4} \sum_{\vec{r}, \vec{a}, \vec{\beta}} \sigma_{\vec{r}} \sigma_{\vec{r}+\vec{a}+\vec{\beta}}.
\]

The energy of a configuration is equal to the curvature of the boundary plus the energy at the intersections

\[
E = h \left[ \sum_i (Right\ Angles)_i + 4\kappa \sum_j (Intersections)_j \right].
\]

In this case the Hamiltonian includes only competing ferromagnetic and antiferromagnetic interactions. The ferromagnetic coupling constant is \(J_{ferromagnetic} = J\) and the antiferromagnetic coupling constant should be four times smaller \(J_{antiferromagnetic} = J/4\), thus the ratio is equal to four

\[
J_{ferromagnetic}/J_{antiferromagnetic} = 4.
\]

The critical temperature \(\beta_c = J/KT \approx 0.44\) has been predicted in [7] (see also [10]) and confirmed by Monte-Carlo simulations [11, 12] and the low temperature expansion [8], thus \(T_c \approx J/0.44K\). In order to have the phase transition point at high temperatures the coupling constant \(J\) should be large enough.

It is an interesting question if there exist a material with the above interactions. The crystalline \(EuS\) and \(Eu_xSr_{1-x}S\) [3], which is a ferromagnetic insulator, has exchange energy coupling constants equal to

\[
J_{ferromagnetic} = (0.221 \pm 0.003)K,
\]

\[
J_{antiferromagnetic} = (0.1 \pm 0.004)K,
\]

therefore the ratio is equal to two and one should look for other materials with appropriate coupling constants. If the material with these interactions will be found or constructed
it will be not so useful for direct applications because the exponential degeneracy of the vacuum state is lifted at nonzero temperatures. This is because nonzero surface tension is generated by quantum-thermal fluctuations [9, 12, 8], the area term in the energy functional. This effect will suppress the interface walls and thus the degeneracy is lifted. From other side, crystal of this type can be helpful for experimental verification of the string tension generation phenomena in string theory suggested in [9], sort of experimental laboratory for QCD string.

3 System with $2^{3N}$ ground state degeneracy

The system which has even higher degeneracy of the ground state than the one which we described in the previous section has been constructed in [7]. The advantage of this system is that the degeneracy of the vacuum state remain untouched even at nonzero temperatures.

In terms of Ising spin variables $\sigma_{\vec{r}}$ the Hamiltonian of the system with $2^{3N}$ ground state degeneracy can be written in the form [7]

$$H_{Gonihedric}^{3D} = -J_4 \sum_{\vec{r},\vec{\alpha},\vec{\beta}} \sigma_{\vec{r}} \sigma_{\vec{r}+\vec{\alpha}} \sigma_{\vec{r}+\vec{\alpha}+\vec{\beta}} \sigma_{\vec{r}+\vec{\beta}}$$

(5)

where $\vec{r}$ is a three-dimensional vector on the lattice $Z^3$, the components of which are integer and $\vec{\alpha}, \vec{\beta}$ are unit vectors parallel to the axes. This Hamiltonian corresponds to the case $k = 0$ in equation (4). We should stress that the Ising spins in (5) are on the vertices of the lattice $Z^3$ and are not on the links and the coupling constant $J_4$ should be positive. The Hamiltonian contains only exotic four-spin interaction term $\sigma\sigma\sigma\sigma$.

It is hard to see how this four-spin interaction term can be simulated even by artificial materials. In this section we propose to introduce additional spin which is located in the center of plaquette and then adjust it interaction so that effective interaction between four-spins located at the vertices of the plaquette will be of the form $\sigma\sigma\sigma\sigma$. One can consider this transformation as a modification of the decoration transformation and it is analogous to the star-triangle transformation [14].

Thus to generate four-spin interaction term we shall introduce central spin $\sigma$ which interacts with its four neighbors (see Figure 3). We should prove that integrating out the interaction with the central spin will generate the necessary four-spin interaction term. Therefore we have to prove the existence of the following relation

$$\frac{1}{2} \sum_{\sigma=\pm 1} e^{J_4 \sigma_{\sigma_1+\sigma_2+\sigma_3+\sigma_4}} = A e^{J_1(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_4+\sigma_4\sigma_1)} e^{J_2(\sigma_1\sigma_3+\sigma_2\sigma_4)} e^{J_4(\sigma_1\sigma_2\sigma_3\sigma_4)}$$

(6)

with nonzero coupling constant $J_4$. Here $\sigma$ is the central spin and $\sigma_i, i = 1, 2, 3, 4$ are spins on the vertices. If this relation holds then it means that the interaction of the central spin with its neighbors can be effectively replaced by direct $J_1$, diagonal $J_2$ and four-spin interaction $J_4$ (see Figure 3). To express coupling constants $J_1, J_2, J_4$ and $A$ through the coupling constant $J$ we have to solve the system of $2^4$ equations which appear when we substitute the values of the spins $\sigma_i, i = 1, 2, 3, 4$ into the equation (6). Only four equations are independent because of the global $Z_2$ invariance:

$$ch(4J) = A exp(4J_1 + 2J_2 + J_4),$$
Figure 3: Integration over the central spin $\sigma$ produces an effective interaction between four spins at the vertices of the plaquette.

\[ 1 = A \exp(-2J_2 + J_4), \]
\[ \text{ch}(2J) = A \exp(-J_4), \]
\[ 1 = A \exp(-4J_1 + 2J_2J_4), \]  
(7)

From the first, second and the fourth equations it follows that

\[ J_1 = J_2 \]

and our equations reduce to:

\[ \text{ch}(4J) = A \exp(6J_2 + J_4), \]
\[ 1 = A \exp(-2J_2 + J_4), \]
\[ \text{ch}(2J) = A \exp(-J_4). \]  
(8)

From these equations it follows that:

\[ \text{ch}(4J) = A^8/(\text{ch}2J)^4 \]

and thus

\[ A = (\text{ch}^4(2J) * \text{ch}(4J))^{1/8}. \]  
(9)

Using again equations (8) we can find coupling constants $J_2$ and $J_4$

\[ J_2 = \frac{1}{8} \ln(\text{ch}(4J)), \]
\[ J_4 = \frac{1}{8} \ln(\text{ch}(4J)/\text{ch}^4(2J)). \]  
(10)

The formulas (9) and (10) express the solution through the coupling $J$. It is easy to see that

\[ A \geq 1, \quad J_1 = J_2 \geq 0, \quad J_4 \leq 0. \]  
(11)

Let us rewrite our basic relation in the form

\[ e^{-J_2(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_4 + \sigma_4\sigma_1)}e^{-J_2(\sigma_1\sigma_3 + \sigma_2\sigma_4)} \frac{1}{2} \sum_{\sigma=\pm 1} e^{J\sigma(\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4)} = Ae^{J_4(\sigma_1\sigma_2\sigma_3\sigma_4)} \]  
(12)
Figure 4: Direct interactions AB and BC are ferromagnetic, BB and CA are antiferromagnetic. Diagonal interaction AB is ferromagnetic and BB is antiferromagnetic. Their intensities are given by the formula (10).

where as we have seen $J_4 \leq 0$. The physical interpretation of the last formula is as follows: the initial direct and diagonal antiferromagnetic interactions $J_1(J) = J_2(J)$ between spins $\sigma_i, i = 1, 2, 3, 4$ together with the interaction $J$ with the central spin $\sigma$ generate effective four spin interaction

$$e^{J_4(\sigma_1\sigma_2\sigma_3\sigma_4)}, \quad J_4 \leq 0.$$  \hspace{1cm} (13)

Unpleasant feature of this result is that the coupling constant $J_4$ is negative while we need it to be positive. To generate four spin interaction with positive, ferromagnetic coupling constant $J_4$ we have to change the interaction of one of the spins, let us say $\sigma_1 \rightarrow -\sigma_1$ in the formula (12)

$$e^{J_2(\sigma_1\sigma_2\sigma_3\sigma_4)}e^{J_2(\sigma_1\sigma_3-\sigma_2\sigma_4)}\frac{1}{2} \sum_{\sigma=\pm 1} e^{J\sigma(-\sigma_1+\sigma_2+\sigma_3+\sigma_4)} = Ae^{-J_4(\sigma_1\sigma_2\sigma_3\sigma_4)}$$  \hspace{1cm} (14)

where now the four-spin interaction comes with the right positive sign $-J_4 \geq 0$.

The interpretation of the last formula is as follows: one should introduce three types of spin-atoms $A, B, C$ as it is shown on Figure 4 with corresponding interactions between them and then after integration over central spin one can see that effective interaction is of the type (5). This structure can be periodically extended to the whole three-dimensional lattice. For that one should also use the structure similar to the one shown on Figure 4, in which $A$ and $B$ spin-atoms have been interchanged.

4 Discussion

As we already discussed in the introduction the high density magnetic recording systems require a storage of information in nm scale, but fluctuations on nm scale will produce damages which are difficult to prevent. We will face this fundamental problem in the near future. We have seen that at least theoretically one can construct lattice crystal with specially tuned interactions which has exponentially degenerate vacuum state and it is
suggested that it can store information in a form of different vacuum states. The planes of flipped spins representing different vacua can in principle be of atomic scale.

The Ising type spin systems can only mimic real magnetic materials and one should think about similar construction involving interaction between magnetic domains, but from the other side one can also think about materials in which electron or nuclear spin interaction is organized in a proper way.

We are facing close phenomena with computer circuits as well. As the components of the computer circuits become very small, in the extreme limit they can approach the atomic scale. Their description in the limit of atomic scale should be quantum-mechanical and in recent years there have been intensive studies in the physical limitations of the computational process which we shall review in Appendix.

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5 Appendix

Quantum computers were suggested and analyzed by Benioff and Feynman. Typically they consist of $N$ interacting Ising type two-state spin systems. The initial input state of quantum computer is a quantum binary string. Computation is accomplished through the unitary evolution. In the course of computation the intermediate states are in general superpositions of binary strings. The theoretical importance of quantum computers comes from the realization of the fact that quantum computation can be exponentially faster than the best known classical algorithms. The most important examples are quantum algorithms for integer factorization, the discrete logarithm and searching in unsorted database.

Although the theory is fairly well understood, the actual building of quantum computer is extremely difficult. The measurement of the state of the quantum computer lead to obstacles in making computation reversible. The other problem is that unknown quantum state cannot be perfectly duplicated. Nevertheless it was demonstrated that quantum error correction is possible in order to protect quantum information against corruption. Quantum teleportation and superdense coding was developed. The problem of maintaining the coherence in the process of quantum computation was discussed.

There is an increasing interest in practical realization of quantum computers. One of the ideas is to exploit quantum effects to build molecular-level computers, that is to induce parallel logic in arrays of quantum dots and in molecules. Real physical implementation comes with ions traps: laser cooling and thermal isolation of the gaseous Bose-Einstein condensate. The ion can be used to operate quantum logic gate that couples the hyperfine splitting of a single trapped ion $Be^+$ to its oscillation modes in the ion trap. Optical cavities have been used in the other setup: quantum phase gate, was demonstrated for photon pair coupled by a single atom in a quantum electrodynamics cavity. The control and target bits of quantum phase gate are two photons of different optical frequency, passing together through a low-loss QED cavity few microns long.
But most promising is probably the bulk nuclear magnetic resonance technique: nuclear spins act as quantum bits, and are particularly suited to this role because of their natural isolation from environment [29].

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