Simple estimation of dipole source z-distance with compact magnetic gradiometer

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Abstract. A compact magnetometer/gradiometer with combined homogeneous and gradient outputs facilitates precise measurement of both \( H \) and \( G \) values with good spatial and temporal coherence. By evaluating combination of both signals, it is possible to estimate distance to a dipole source with relatively small error and largely independent from precise knowledge of source strength, orientation and lateral displacement. The performance is limited primarily by ambient noise. With an AC-driven source, tool navigation or distance sensing is also possible.

1. Introduction

We have previously presented a compact (10-cm head size, 3-cm base) axial magnetic gradiometer with two ring-core fluxgate sensors and combined homogeneous feedback and gradient feedback coils [1]. This arrangement provides simultaneously information about the homogeneous field strength \( B_z \) as well as the gradient \( dB_z/dz \) in one axis with minimal geometric error. Thanks to that we can utilize simple method of dipole source distance estimation [2, 3]. Such an information is needed e.g. for unexploded-ordnance-detection or mine-hunting [4]. In our method, we do not obtain the radial distance, but the \( z \)-, or the vertical distance to the detected object, which is one of the most important informations to distinguish between deep and strong sources and shallow and weak ones. Assuming constant background conditions (both homogeneous field \( H \) and gradient \( G \)), the vertical distance (\( z \)-component) to a source positioned along the gradiometer axis can be estimated with relatively low error simply from the ratio of \( H_z \) and \( G_{zz} \) signals, largely independent from radial distance to the dipole, and its strength. If we calculate \( 3 \times H_z/G_{zz} \), we get a figure characterizing the distance to source directly from one reading - not depending on its actual amplitude – this approach can be used also with AC-current driven coils for distance measurements or tool navigation tasks.

2. Experiment

The sensor was fixed in space after \( H \) and \( G \) readings in "infinity" (i.e. far from dipole source) were recorded as offsets for later subtraction from data. The dipole source in our case is small (\( 13\times4\times5\text{mm}^3 \)) NdFeB permanent magnet. In the first experiment, the sensor and dipole were aligned on the same axis (\( z \)), with the dipole axis oriented either coaxially or perpendicularly - see figure 1 (left). The vertical distance (\( z \)) was modified and the corresponding distance estimate from \( H \) and \( G \)

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data was calculated. In the following experiment, the vertical ($z$) distance was fixed at 15 cm and lateral displacement ($y$) was modified - again, see figure 1 (left) for the geometry of the setup.

**Figure 1.** Left: the geometrical situation of dipole and sensor. Right: the estimated distance vs. real (reference) distance to a dipole in coaxial or perpendicular orientation to sensor. For coaxial orientation, an offset must be subtracted from the estimate. Here, the lateral displacement $y$ is 0 cm.

3. Theory

As was shown in [2] and several related papers, measurement of full tensor gradient and field vector provides information for dipole source position estimate, irrespective of dipole orientation relative to sensor. In [2] the source is loop antenna transmitting AC signal. In our approach, we simplified the setup to just combined single-axis magnetometer and gradiometer. Also the dipole in our case was permanent magnet within Earth’s field. In spite of these limitations, we achieved reasonably good position estimates, albeit in limited range of distances.

If we assume $z$- orientation of the magnetic dipole (magnetic moment $m_z$), we can write for the $z$-component of the magnetic field $B_z$:

$$B_z = -\nabla \left( \frac{\mu_0 \cdot m_z \cdot z}{4\pi \cdot z^3} \right) = -\nabla \left( \frac{\mu_0 \cdot m_z}{4\pi \cdot z^2} \right) = \frac{2\mu_0 \cdot m_z}{4\pi \cdot z^3}$$

(1)

The $G_z$ gradient can be then written as:

$$G_z = \frac{\partial B_z}{\partial z} = -\nabla \left( \frac{\mu_0 \cdot m_z}{2\pi \cdot z^3} \right) = -\frac{3\mu_0 \cdot m_z}{2\pi \cdot z^4}$$

(2)

By dividing $B_z$ and $G_z$, we get finally: $z = -3 \frac{B_z}{G_z}$

(3)

This result is consistent with the tensor representation presented in [2].

4. Results and discussion

As shown in figure 1 (right), the estimated distance from $3\times H/G$ matches with reference distance quite well in 15-30 cm range. The estimate for coaxial case contained an unwelcome position offset (about 7cm), but after subtraction thereof, the error is 1cm or less. The data from perpendicular case are more influenced by noise due to lower dipole signal strength in 2nd Gauss position.
In the second measurement, the influence of lateral displacement (along y-axis) on distance (z) estimate was examined. The dependence of homogeneous field $H_z$ and $G_{zz}$ gradient on y-axis position of dipole is shown in figure 2, again for coaxial and perpendicular orientation of dipole. The position $y = 0$ corresponds to alignment of dipole with sensor from first experiment. From the same data, position estimates are again calculated. The quality of estimated z-distance quickly deteriorates by more than 100% - see figure 3 and figure 4, but for small lateral displacements (< 5 cm) the error is quite acceptably low. Dashed line represents true z-distance 15 cm. Note that in figure 3, the estimated distance z is not corrected for offset.

**Figure 2.** The measured homogeneous magnetic field and gradient in coaxial (left) and perpendicular (right) orientation of dipole to sensor vs. lateral displacement along y-axis.

**Figure 3.** Coaxial orientation. The estimate of distance z vs. lateral displacement along y-axis. In limited range of lateral displacements (< 5cm), the estimate error is small (< 5cm after offset removal). The inset shows overall response for large lateral displacements (detail in main graph).
5. Conclusions

We conducted several experiments with dipole distance estimation in 15 cm to 30 cm distance. After offset subtraction, the distance estimation error in our experiment with small NdFeB source was < 5 cm for coaxial case and < 3 cm for perpendicular case in 15 cm to 30 cm dipole-to-gradiometer distance. Moreover, the z-axis distance estimation was largely independent (within some limits) to axial misalignment (lateral shift) between the source and gradiometer axes. The performance was mainly limited by ambient noise, which was about 100× larger that our gradiometer noise (1 nT/m/√Hz). AC-driven coils experiment was also conducted showing feasibility of the proposed algorithm e.g. for tool navigation purposes or for distance sensors.

References

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