Geant4 Developments for the Radon Electric Dipole Moment Search at TRIUMF

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Abstract. An experiment is being developed at TRIUMF to search for a time-reversal violating electric dipole moment (EDM) in odd-A isotopes of Rn. Extensive simulations of the experiment are being performed with Geant4 to study the backgrounds and sensitivity of the proposed measurement technique involving the detection of γ rays emitted following the β decay of polarized Rn nuclei. Geant4 developments for the RnEDM experiment include both realistic modelling of the detector geometry and full tracking of the radioactive β, γ, internal conversion, and x-ray processes, including the γ-ray angular distributions essential for measuring an atomic EDM.

1. Introduction
The interest in particle and atomic electric dipole moments (EDMs) derives from the desire to understand the fundamental symmetries in the laws of physics and the most basic origins of matter in the universe. To date, all experimental evidence supports that CPT is a true symmetry, known as the CPT Theorem [1]. The CPT transformation is the combination of the charge conjugation transformation (C), parity transformation (P) and time-reversal transformation (T). A particle EDM has the effective operator $q_i \vec{r}_i \cdot \vec{J}/(J+1)$ where $\vec{r}_i$ is the position of any charge element $q_i$ and $\vec{J}$ is the total angular momentum. This operator is odd under both the P and T transformations. Therefore, a permanent non-zero EDM for an elementary particle or atom can only occur from parity and time-reversal violating fundamental interactions. The direct violation of time-reversal symmetry is equivalent to a violation in the charge conjugation and parity symmetry (CP) while conserving CPT as a true symmetry. CP violation is essential for baryogenesis, as first pointed out by Sakharov in 1967 [2]. In the Standard Model, CP violation enters via weak interaction flavor mixing represented by the complex phase $\delta_{\text{CKM}}$ of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and via $\theta_{\text{QCD}}$, the vacuum expectation value of the QCD gluon field. These sources of CP violation are not sufficient to account for the observed asymmetry between matter and anti-matter in our universe. Thus additional sources of CP violation are required and provide a strong motivation to search for new physics beyond the Standard Model.

Despite over 50 years of searching with increasing experimental sensitivity, no non-zero particle EDM has yet been detected. However, current theories beyond the Standard Model, such as multiple-Higgs theories, left-right symmetry, and supersymmetry (SUSY), generally predict

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EDMs within current experimental reach [3]. Present limits on the EDMs of the neutron, electron and $^{199}$Hg atom (Table 1) have, in fact, already significantly reduced the parameter spaces of these models.

| Species      | EDM Upper Limit | C.L. |
|--------------|-----------------|------|
| Neutron      | $< 2.9 \times 10^{-26}$ e·cm [4] | 90%  |
| Electron     | $< 1.6 \times 10^{-27}$ e·cm [5] | 90%  |
| $^{199}$Hg Atom | $< 3.1 \times 10^{-29}$ e·cm [6] | 95%  |

The search for an atomic EDM with odd-A radon isotopes is motivated by the predictions of large enhancements in the observable atomic EDM. Recent theoretical calculations predict an enhancement factor of $\sim 600$ for $^{223}$Rn relative to $^{199}$Hg [7], which is the most sensitive EDM measurement to date. This enhancement is derived from three sources: octupole deformation of the nucleus, close-lying parity doublet states in the nucleus that accompany octupole deformation, and the large $Z$ of the isotope.

Neutron-rich radon isotopes are predicted to have octupole deformed nuclei [8]. This implies large intrinsic Schiff moments ($S_{\text{int}}$), the lowest order time-reversal odd moment of a nucleus that is measurable in a neutral atom [9]. According to the Schiff theorem, the nuclear EDM is “screened” by the orbiting electrons. The observed EDM of the atom is rather induced by the Schiff moment, which is a measure of the difference between the charge and dipole distributions in the nucleus. The intrinsic Schiff moment is proportional to $\beta_3$, where $\beta_3$ measures the presence of the octupole ($L = 3$) spherical harmonic in the nuclear shape. Therefore, a large octupole deformation gives a large intrinsic Schiff moment.

The second enhancement factor in radon is due to the existence of close-lying parity doublet states that are associated with the nuclear octupole deformation [10]. The laboratory Schiff moment ($S_{\text{lab}}$) is enhanced by the mixing of these nearby opposite parity states by a $P$ and $T$ odd interaction ($V^{PT}$),

$$S_{\text{lab}} \sim \frac{\langle \psi | V^{PT} | \psi \rangle}{\Delta E} S_{\text{int}}.$$  \hspace{1cm} (1)

The final part of the enhancement factor derives from the high $Z$ of radon. The observable atomic EDM induced via the laboratory Schiff moment of the nucleus is approximately proportional to $Z^3$ [10]. These three factors, a collective octupole deformation of the nucleus, close-lying parity doublet states and the high $Z$ of radon, give the odd-A radon isotopes a large enhancement of the observable atomic EDM induced by an underlying CP-odd interaction, and thus make radon an excellent candidate for an EDM search.

2. The Rn EDM Search at TRIUMF

The neutron-rich odd-A radon isotopes which are of particular interest ($^{219,221,223,225}$Rn) will be produced by the ISAC (Isotope Separator and ACcellerator) facility at the TRIUMF laboratory in Vancouver, British Columbia, Canada. The laboratory is built around a 500 MeV proton (H$^-$) cyclotron which provides a beam of up to 100 $\mu$A that bombards a primary production target at the ISAC facility. Spallation reactions produce a variety of exotic nuclides which are then ionized and sent to the magnetic separator to select nuclei of a specific charge-to-mass ratio. Intense beams of neutron-rich Rn isotopes will become available with the implementation of the ISAC actinide production target.
The produced radon isotopes will be sent to the RnEDM apparatus and collected in a thin foil. The Rn atoms will be then transferred on-line to an EDM cell [11, 12] through cryogenic and gas transfer techniques [13]. The nuclei will be polarized via spin-exchange collisions with polarized rubidium vapour [13, 14]. The method of polarizing radioactive noble-gas nuclei via spin-exchange collisions with alkali atoms has been demonstrated at the ISODE isotope separator at CERN using Rn isotopes and has been successful in polarizing lighter noble-gas nuclei (specifically xenon) with polarizations > 70% [15]. Similar polarizations studies with Rn isotopes will be performed at the ISAC facility at TRIUMF following the implementation of the ISAC actinide production target.

Through the application of an RF pulse, the polarized radon atoms will begin to precess at a frequency

\[ \hbar \omega_{\pm} = 2 \mu B \pm 2dE, \]

where \( \mu \) is the magnetic moment, \( B \) is the magnetic field, \( d \) is the electric dipole moment and \( E \) is the electric field. The + (−) corresponds to the electric field oriented parallel (anti-parallel) to the magnetic field. The presence of a non-zero EDM is detected through the small change in the precession frequencies between the two configurations:

\[ d = \frac{\hbar \Delta \omega}{4E}. \]

To measure the precession frequencies eight TIGRESS/GRIFFIN detectors [16] will be used in a ring configuration around the EDM cell and oven, as shown in Figure 1. The spherical cell and cylindrical oven are both constructed of glass. The cell is located inside the oven and is approximately 1 mm thick with an inner radius of 1 cm. The oven is approximately 5 mm thick with an inner of radius 10 cm, allowing the TIGRESS/GRIFFIN detectors to be close-packed in their highest efficiency mode with an inner radius of 11 cm. The TIGRESS/GRIFFIN detectors, Figure 2, contain four high-purity germanium (HPGe) crystals surrounded by full bismuth germanate (BGO) Compton-suppression shields. Each of the four crystals has an efficiency of 40% at 662 keV relative to a 3” × 3” NaI(Tl) detector. The HPGe detectors will detect the emitted \( \gamma \)-rays following the \( \beta \) decay of the polarized radon nuclei. The angular distribution of the \( \gamma \) radiation is dependent on the polarization direction of the precessing ensemble of Rn nuclei.

3. **Geant4 Developments**

Geant4 is a C++ toolkit used to simulate the passage of particles through matter. The goal of the Geant4 developments for the RnEDM experiment at TRIUMF is to provide an

![Figure 1. Geant4 rendering of the experimental setup for the RnEDM experiment at TRIUMF](image1)

![Figure 2. Geant4 rendering of one TIGRESS/GRIFFIN detector in the forward position.](image2)
accurate simulation of the signal to background in the γ-ray spectra that, together with counting statistics, will determine the overall sensitivity of the RnEDM experiment. To achieve this, the TIGRESS/GRIFFIN detectors and the entire radioactive decay process has been accurately modelled in Geant4.

The TIGRESS/GRIFFIN detector simulations have been extensively studied and validated with measurements made from real TIGRESS detectors [17]. Methods to optimize the performance of these detectors have also been explored in Geant4 [18, 19]. These methods include the positioning of the Compton-suppression shields, vetoing events selectively and reconstructing γ-ray energy deposits in neighboring crystals and/or clovers. The absolute efficiency of a ring of eight TIGRESS/GRIFFIN detectors in the highest efficiency mode is 9.7% at 1MeV, as illustrated in Figure 3.

The developed radioactive decay package is capable of simulating any β-decaying nucleus given the half-life, the β-decay branching ratio to the ground state, Q-value, level spins and parities, γ-ray energies, intensities, mixing ratios, internal conversion coefficients, and the resulting x-ray energies and intensities. Along with reproducing the entire decay scheme and realistic β, γ, electron and x-ray energies and probabilities, the package also simulates the angular distribution of the emitted γ rays. In general, the distribution of the emitted γ rays depends on the multipolarity and the spin of the initial and final states in the daughter nucleus.

In the RnEDM experiment, the polarized radon nuclei align with the applied magnetic field. The polarization vector about this axis of orientation (η) precesses about the magnetic field with the application of an RF pulse. The strength of the magnetic field determines the rate of the precession frequency, with a 1 mG field producing a precession frequency on the order of a few Hz [20]. The initial polarization of the radon nuclei will begin to depolarize through the β and γ decays and through the spin-relaxation (T1) and the spin-decoherence (T2) times. The direction of the polarization vector is monitored as a function of time throughout the simulation, and the depolarization tracked through evolving m-state populations as a function of time.

The angular distribution about η is written as a sum of Legendre polynomials [21, 22],

$$W(\theta) = 1 + A_2 P_2(\cos \theta) + A_4 P_4(\cos \theta),$$

where $A_2$ and $A_4$ are calculated for each particular transition based on sampling of the simultaneous m-state population by Monte Carlo techniques. Figure 4 provides an example of such an angular distribution.

To begin a simulation, an initial number of radioactive nuclei are selected. The β branching ratios are calculated from the input γ-ray intensities. The time intervals between successive decays are selected by Monte Carlo from a determined by the half-life and the remaining number
of nuclei. This establishes the time of each decay and thus the spin orientation and parameters of the polarization of the precessing ensemble of Rn nuclei. Through a similar Monte Carlo process, the $\beta$ particle branch and energy are selected from calculated distributions and emitted into the GEANT4 geometry. Following the initial $\beta$ decay, the daughter nucleus may now exist in an excited state. The probabilities of $\gamma$ decay versus internal conversion are calculated and again through a Monte Carlo process, the particles are determined and are emitted in the GEANT4 geometry until the daughter reaches its ground state. The emitted $\gamma$ rays exhibit characteristic angular distributions which precess in time about the magnetic/electric field direction. The simulation of this precession for each $\gamma$-ray provides information on the sensitivity of the RnEDM experiment.

4. Results and Conclusions

To date, the GEANT4 simulations for the RnEDM experiment have been focused on the decay of $^{223}$Rn into $^{223}$Fr [23, 24]. The GEANT4 output data is analyzed to generate a precession frequency for each $\gamma$-ray. A resolution function is applied to the $\gamma$-ray energies in order to simulate the true energy resolution of the TIGRESS/GRIFFIN detectors. Backgrounds in the simulation include bremsstrahlung radiation, pair production and $\gamma$-ray scattering inside the detectors, cell and oven.

The simulated $\gamma$-ray spectrum for the decay of $10^9$ $^{223}$Rn nuclei detected by eight TIGRESS/GRIFFIN detectors around the EDM cell is shown in Figure 5. The inclusion of the internal conversion process is shown to be important as the resulting x-rays dominate the spectrum at low energies. Energy gates can be set on any of the photopeaks and projected on the time axis to generate a time projection. The time projection of the 592 keV photopeak for two detectors separated by 180° is shown in Figure 6. The counts in the two facing detectors are summed together as the $\gamma$ distribution is symmetric for these detectors. The 592 keV transition was simulated to be a mixed M1+E2 transition with a $\delta = 0.60$; matching the experimentally measured mixing ratio of 0.60(15) [23]. Figure 6 also clearly illustrates the depolarization of the Rn nuclei. The T1 used in the simulation was 30 seconds, consistent with the value attained for polarized $^{209}$Rn from work at Stony Brook [20].

Taking advantage of the predicted EDM enhancement factor of $\sim$600 for $^{223}$Rn relative to $^{199}$Hg, the RnEDM experiment would need a sensitivity of order $1 \times 10^{-26}$ e-cm [13], to be competitive with the current $^{199}$Hg measurement of $|d(^{199}\text{Hg})| < 3.1 \times 10^{-29}$ e-cm [6]. 100 days of running is expected to give approximately $10^{12}$ photopeaks in the HPGe detectors [13, 20], combined with fitting precession frequencies for multiple $\gamma$-ray peaks, we expect an EDM sensitivity below the $1 \times 10^{-26}$ e-cm level and to achieve an improved sensitivity to new fundamental $CP$-violating physics beyond the Standard Model.
Figure 5. The γ-ray singles spectrum from the β-decay of $1 \times 10^9 \text{^{223}Rn}$ nuclei detected by eight TIGRESS/GRIFiNN detectors in the highest efficiency configuration around the EDM cell.

At present, high-statistic simulations are being performed under a number of experimental conditions to study the backgrounds in the γ-ray spectra and thus the effect on the overall experimental sensitivity.

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