The Parameter-free Finger-of-God Model and Its Application to 21 cm Intensity Mapping

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Abstract

Using the galaxy catalog built from ELUCID N-body simulation and the semianalytical galaxy formation model, we have built a mock H I intensity mapping map. We have implemented the Finger-of-God (FoG) effect in the map by considering the galaxy H I gas velocity dispersion. By comparing the H I power spectrum in redshift space with a measurement from the IllustrisTNG simulation, we have found that the FoG effect can explain the discrepancy between current mock maps built from the N-body simulation and the IllustrisTNG simulation. Then we built a parameter-free FoG model and a shot-noise model to calculate the H I power spectrum. We found that our model can accurately fit both the monopole and quadrupole moments of the H I matter power spectrum. Our approach to building the mock H I intensity map and the parameter-free FoG model will be useful for upcoming 21 cm intensity mapping experiments, such as CHIME, Tianlai, BINGO, FAST, and SKA. It is also vital for studying nonlinear effects in 21 cm intensity mapping.

Unified Astronomy Thesaurus concepts: Line intensities (2084); Large-scale structure of the universe (902)

1. Introduction

The 21 cm emission line comes from the spin-flip of the electrons in neutral hydrogen. Therefore, the intensity distribution of 21 cm in the universe represents the distribution of neutral hydrogen. The neutral hydrogen traces the underlying matter field, which is also known as the large-scale structure of the universe. Just like galaxy surveys, 21 cm intensity mapping can also be used to measure the tomographic baryon acoustic oscillations, but much cheaper and faster (Wyithe et al. 2008). With upcoming experiments such as CHIME (Bandura et al. 2014), Tianlai (Chen 2012), BINGO (Wuensche & BINGO Collaboration 2019), FAST (Bigot-Sazy et al. 2016; Smoot & Debono 2017; Hu et al. 2020), SKA (Santos et al. 2015) and so on, 21 cm intensity mapping may be able reveal the properties of dark matter and dark energy (Kovetz et al. 2017).

As we can only observe the redshifted emission lines of neutral hydrogen rather than their true distance from us, the distribution of neutral hydrogen that we can map from the 21 cm intensity is distorted by their peculiar velocity. This is well known as redshift-space distortion (RSD). Since the peculiar velocity of galaxies is dominated by gravity and the clustering of matter, it contains information on the growth of large-scale structure. The RSD effect in galaxy spectroscopy surveys is used to measure the growth factor of the matter power spectrum (Icaza-Lizaola et al. 2020), and to test general relativity (Anagnostopoulos et al. 2019; Julio et al. 2019) and other cosmological models (Costa et al. 2017; An et al. 2019; Cheng et al. 2019; Yang et al. 2019).

The RSD effect can be understood as a combination of two effects, the Kaiser effect (Kaiser 1987) and the Finger-of-God (FoG) effect (Jackson 1972). The Kaiser effect is dominant at large scales, and squeezes the distribution of galaxies in the line-of-sight direction. The FoG effect is dominant on small scales, and elongates the distribution of galaxies, making them look like fingers pointing at the observer. The RSD effect in 21 cm intensity mapping was studied in detail in Sarkar & Bharadwaj (2019). They proposed a model for calculating the 2D redshift space H I power spectrum, which contains a free parameter σ_p for the FoG effect at every redshift. This free parameter limits our ability to constrain cosmological parameters more precisely. It would be better if we could find a parameter-free model for the FoG effect.

In order to study the RSD effect in 21 cm intensity mapping and build a better RSD model, especially at small scales, we need to construct a H I distribution map from high-resolution simulations. We choose to use the ELUCID simulation (Wang et al. 2016), together with a semianalytical model (Luo et al. 2016), to build a galaxy catalog. Due to the reionization that happened at z ~ 10, the universe at z < 5 is mostly ionized (Becker et al. 2001; Fan et al. 2006a, 2006b). The only region in which neutral hydrogen can “survive” is high-density regions (Prochaska et al. 2005; Wolfe et al. 2005; Zafar et al. 2013), where the column depth can be sufficient for self-shielding (Pritchard & Loeb 2012). Therefore, at low redshift (z < 5), most H I gas is inside galaxies, in the form of individual H I clouds. This is also confirmed in the state-of-the-art hydrodynamic simulation IllustrisTNG (Villaescusa-Navarro et al. 2018).

The collective redshifted 21 cm emission from these H I clouds appears as a smooth background signal in the radio surveys. Comparing to the resolution of 21 cm intensity mapping surveys, which is square degree level, the size of dark matter halos is negligible. In other words, the galaxies, the dark matter halos, and the H I gas inside them can be safely considered as point mass. By smoothing the collective H I mass in redshift space, we can get a mock H I map. In Hall et al. (2013), it was pointed out that the leading contributions to the perturbation of the 21 cm intensity (represented in the brightness temperature T_b) are the H I density perturbation and RSDs.
Thus, the brightness temperature \( T_b \) fluctuation map of 21 cm emission is almost equivalent to the H I density fluctuation map.

We first introduce the method with which we generate the H I density map from simulations in Section 2. We introduce our RSD model, particularly the parameter-free FoG model, in Section 3. We introduce the model for the shot-noise power spectrum in Section 4. We present a summary and discussions in Section 5.

2. Mock H I Map Building Methodology

2.1. Galaxy Catalog

The galaxy catalog was generated using the ELUCID N-body simulation (Wang et al. 2016), with the semianalytical galaxy formation model described in Luo et al. (2016). The ELUCID simulation is run with 3072\(^3\) particles in a periodic cubic box of 500 h\(^{-1}\) Mpc on a side. WMAP5 cosmology (Dunkley et al. 2009) was assumed in the ELUCID simulation. In the galaxy catalog, we use the position, velocity, H I mass, and dark matter halo mass of the galaxies; the lower limit of the dark matter halo mass is about \(1.85 \times 10^{10} M_\odot / h\). We assume that all the H I mass in the universe is inside galaxies and their hosting halos, concentrated in the center of the dark matter halo. As described in Villaescusa-Navarro et al. (2018; hereafter F18) from IllustrisTNG simulation (Naiman et al. 2018; Nelson et al. 2018, 2019; Pillepich et al. 2018; Springel et al. 2018) results, at \(z < 3.0\), more than 90% of the H I gas is inside the galaxies and more than 95% of the H I mass is inside the halos. Therefore, our assumption is reasonable.

The H I gas velocity dispersion inside each galaxy, especially inside high-mass galaxies, is also important for incorporating the FoG effect into the mock 21 cm map (Sarkar & Bharadwaj 2019). We adopted the empirical fitting function provided in F18,

\[
\sigma_v(M) = \sigma_{10} \left( \frac{M}{10^{10} h^{-1} M_\odot} \right)^{\alpha},
\]

where \(\sigma_{10}\) is in units of km s\(^{-1}\), \(M\) is the dark matter halo mass, and \((\sigma_{10} = 4.8857 z + 29.95238'\) and \(\alpha = 0.00914286 z + 0.35714286; z\) means redshift\) are fitted from the data provided in F18.

We can construct the H I distribution in both real and redshift spaces from the galaxy catalog, with the H I mass distribution in real space, the bulk velocity of H I mass, and the internal velocity dispersion of H I gas.

2.2. Redshift Space Distortion

The mapping between the real space position of the H I gas and the redshift space can be separated into two parts, the bulk motion contribution and the internal H I velocity dispersion contribution. The bulk motion contribution can be calculated with the plane-parallel approximation as,

\[
s = x + \frac{1 + z}{H(z)} v_p(r),
\]

where \(v_p(r)\) is the peculiar velocity of the galaxy (and its H I gas) along the line of sight. In the mock galaxy catalog, all the galaxies and their H I gas inside are simplified as point mass. After moving the positions of the galaxies in real space to redshift space according to their peculiar velocity, they are still present as point mass. However, the internal galaxy H I gas velocity dispersion will stretch the galaxies into needles along the line of sight. The amount of stretch can be described by the velocity dispersion

\[
\delta_s(M) = \frac{1 + z}{H(z)} \frac{\sigma_v(M)}{\sqrt{3}},
\]

where \(M\) is the galaxy dark matter halo mass. Therefore, every galaxy will be stretched differently, according to its halo mass. Since we have assumed that the internal H I velocity distribution follows the Gaussian distribution and is described by \(\sigma_v(M)\), the mass profile in the redshift space also follows a Gaussian distribution.

2.3. Making a 3D Map

We cannot identify a single galaxy in 21 cm intensity mapping; we can only observe the collective intensity of the 21 cm emission. Thus, we should create a 21 cm intensity map with grids. For simplicity, we generated a 3D brightness–temperature fluctuation map from the mock galaxy distribution box. To make a 3D map in real space and redshift space without internal velocity dispersion being considered, we first smooth the H I mass into a density distribution with a nearest-grid-point (NGP) algorithm. For a better illustration, we further smooth the H I density field with a Gaussian kernel with \(\sigma = 1 h^{-1}\) Mpc. However, we did not perform such smoothing when measuring the power spectrum from the mock map. The H I density is related to the brightness temperature by

\[
T_b(x) = 189 h \left( \frac{H_0(1 + z)^2}{H(z)} \right) \frac{\rho_{HI}(x)}{\rho_{crit}} \text{mK}. \tag{4}
\]

It should be noticed that Equation (4) is only applied for the average brightness temperature. The fluctuation of the brightness temperature contains additional contributions, which is well introduced in Hall et al. (2013). However, since the leading contributions are the H I density distribution and RSDs, we assume that the residual contributions can be neglected. This assumption is also adopted by F18 to generate the mock map. Under this assumption, the brightness–temperature fluctuation can be easily calculated from

\[
\delta_{T_s(x)} = \frac{T_b(x) - \bar{T}_b}{T_b} = \delta \rho_{HI}(x) = \frac{\rho_{HI}(x) - \rho_{H1}}{\rho_{H1}}. \tag{5}
\]

These are the steps to create a 3D brightness–temperature fluctuation map with internal H I velocity dispersion, which is more realistic.

1. Slice the 500 h\(^{-1}\) Mpc box into 500\(^3\) grids; each grid is 1 h\(^{-1}\) Mpc in a side.
2. Label the slice in the line-of-sight direction.
3. For the \(n\)th slice, pick out the galaxies in the mock catalog within 3\(\delta_s(M)\) away from the slice. The periodic boundary condition should be taken into account properly.
4. Calculate the H I mass-weighted contribution from every selected galaxy. Since we assume the velocity distribution of H I in each galaxy follows a Gaussian distribution, the H I mass distribution in redshift space along the line of sight also follows a Gaussian distribution. Thus, the H I mass weight from each galaxy to the \(n\)th slice can be calculated by
the error function \( w = \text{erf} \left( \frac{z - z(n + 1)}{\delta_M} \right) - \text{erf} \left( \frac{z - z(n)}{\delta_M} \right) \),

where \( z \) is the line-of-sight redshift-space position of the galaxy, \( z(n) \) and \( z(n + 1) \) are the edges of the slice. The mass contribution of the galaxy is thus given by \( M \times w \).

5. Smooth the mass in the \( n \)th slice into grids with the NGP algorithm.

6. Loop through all 500 slices, which gives us the final 3D \( \text{H} \text{I} \) density map, and translate into a brightness temperature fluctuation map.

In Figure 1, we show the mock brightness–temperature fluctuation map in real space and redshift space, in both \( x-y \) (perpendicular to the line of sight) and \( x-z \) directions (parallel to the line of sight). The FoG effect contributed from the internal \( \text{H} \text{I} \) velocity dispersion is hard to determine by comparing the lower middle panel and the lower right panel. The Kaiser effect can be easily seen by comparing the lower left panel and the lower middle panel, especially at high-density regions such as the location around \((x = 150, z = 20)\) and \((x = 450, z = 20)\).

The power spectrum of our mock map was shown in Figure 2. In F18, the authors used the \( N \)-body simulation together with the \( \text{H} \text{I} \)-halo mass relation to assign \( \text{H} \text{I} \) mass into dark matter halos as point mass, which is similar to what we have done here. Without taking the internal \( \text{H} \text{I} \) velocity dispersion into account, we found that the power spectrum in redshift space measured from hydrodynamic simulations is significantly lower than that of their mock map using \( N \)-body simulations. This can be seen in Figure 2 by comparing the black dashed line and the black solid line. They claimed that such suppression at small scales is due to the FoG effect caused by internal velocity dispersion. In Figure 2, comparing the red

![Figure 1](image1.png)

**Figure 1.** Here is the brightness–temperature fluctuation map at \( z = 0 \). The axes are in units of \( h^{-1} \) Mpc. The upper panels are face-on maps with the same slice position and the lower panels are line-of-sight maps with the same slice position. The maps are smoothed by a Gaussian kernel with \( \sigma = 1 h^{-1} \) Mpc. From left to right, the panels are in real space, redshift space without velocity dispersion considered, and redshift space with velocity dispersion considered. Comparing the upper panels shows that for the same slice, that is, the same observational frequency band, the structures we can observe are different in real and redshift spaces. By comparing the lower panels, we can see the squeezing effect, which is the Kaiser effect in high-density regions, and the FoG effect is too tiny to be identified by eye.

![Figure 2](image2.png)

**Figure 2.** The power spectra measured from our mock map are shown as the blue solid line (mock in real space), the red dashed line (mock in redshift space without FoG), and the red solid line (mock in redshift space with FoG). The results from F18 are shown as the black dashed line (similar method as our red dashed line) and the black solid line (measured from hydrodynamic simulation). The lower panel show the ratio between the solid line and the dashed line in the upper panel. In the lower panel, the difference between the red line and black line is very small at large scales and slightly larger but still within 10% at small scales. This proves that our method for generating mock 21 cm intensity maps, considering internal \( \text{H} \text{I} \) velocity dispersion, is quite successful.
dashed line and the blue line, we see that the Kaiser effect can increase the power at large scales, thus tilting the curve more. The ratio between the black solid line and the black dashed line, and the ratio between the red solid line and the red dashed line, in the upper panel of Figure 2, are shown in the lower panel. The only difference between the red solid line and the red dashed line is whether or not the internal H1 velocity dispersion is taken into account. If the claim raised in F18 is correct, that the internal H1 velocity dispersion is the reason for such suppression on small scales, the red line representing the ratio will be very close to the black line. We can clearly see that such suppression is really similar, as shown in the lower panel of Figure 2.

The red dashed line is similar to the black dashed line at small scales, where they all become flat. The same flattened or even tilted up curve is also seen in the blue and red solid lines. This is likely due to the shot noise, since we have assumed that H1 mass is all concentrated in the center of the galaxy. This phenomenon is not shown in the F18 hydro curve since the HI mass distribution in the Illustris simulation is better represented with much less shot noise. The shot noise is also part of the reason why the suppression ratio shown as a red and black line is still different by about 10%. We will discuss the shot noise effect in more detail in Section 4.

We contend that the claim made in F18, that the additional FoG effect caused by the internal H1 velocity dispersion can explain the discrepancy between the black solid and dashed lines, is correct. After considering the additional FoG effect caused by the H1 velocity dispersion inside halos, the discrepancy between the mock and the hydrodynamic simulation results can be resolved to about 10%, which is likely due to shot noise. Our method for creating a mock 21 cm intensity map is better than the traditional methods, which ignore the internal H1 velocity dispersion. This is helpful for further studying nonlinear effects in 21 cm intensity mapping.

### 3. Redshift Space Distortion Model

In this section, we introduce the model for calculating the redshift-space power spectrum and perform a comparison with our mock map. Previous studies such as Villaescusa-Navarro et al. (2018) and Sarkar & Bharadwaj (2019) proposed RSD models to calculate the H1 power spectrum in redshift space. Sarkar & Bharadwaj (2019) built several mock maps and tried to determine the model by fitting to the power spectrum measured from the mock maps. We improved this model with less parameters and good precision. We follow the model used in Sarkar & Bharadwaj (2019):

\[
P^{2}_{H1}(k, \mu) = (1 + \beta \mu^2)^2 P^{2}_{H1}(k)D_{FoG}(k, \mu, \sigma_p),
\]

(6)

where \(1 + \beta \mu^2\)^2 is the term representing the Kaiser effect and \(\beta\) is set to be a free parameter when fitting the model to the mock map. We choose the best-fit functional form for the FoG term \(D_{FoG}(k, \mu, \sigma_p) = \left(1 + \frac{1}{2}k^2\mu^2\sigma_p^2\right)^{-2}\), where \(\sigma_p\) is also used in Sarkar & Bharadwaj (2019) as a free parameter to fit the mock map.

However, from the steps that we use to generate the mock 3D map, we note that the FoG effect is closely related to the internal H1 velocity dispersion \(\sigma(M)\). Therefore, it is natural to investigate the relation between \(\sigma_p\) and \(\sigma(M)\). We found that \(\sigma_p\) is a weighted average of \(\sigma(M)\):

\[
\sigma_p = \int_{0}^{\infty} \sigma(M)\Theta\left(\frac{1}{H(z)} \sigma(M) - 1\right) \frac{dn}{dm} dm \frac{1 + z}{H(z)},
\]

(7)

where \(\Theta\) function represents the Heaviside step function, and \(\frac{dn}{dm}\) is the halo mass function, which can be calculated following Tinker et al. (2008). We use the yt python package to calculate the halo mass function (Turk et al. 2011).

The physical meaning of Equation (7) is the weighted average of velocity dispersion of each halo. The weight is given by the H1 velocity dispersion of the halo, or in other words, the halo mass. Since the minimum grid size in our mock map is \(1 h^{-1}\) Mpc, any FoG effect that “stretches” the galaxy less than the grid size cannot be captured by the mock map. It is also similar in real observations. The frequency resolution is limited to about 1 MHz (Wuensch & BINGO Collaboration 2019), which is about 2 Mpc/h. We may also have to artificially set bins in redshift to collect enough signals in the bins. This bin size can be about 10 MHz, which is about 20 Mpc/h. In our model, we have taken this bin size effect into account. Any “stretch” caused by the H1 velocity dispersion being less than half of the grid size is not taken into account in the model, since it also cannot be captured because of the limited size of the bin. Therefore, our FoG model has no free parameter if the \(\sigma(M)\) function can be determined either by simulations or other independent observations. On the other hand, with our FoG model, if set to be free for fitting, we can also find the \(\sigma(M)\) function by fitting to the real 21 cm intensity mapping observations.

In Figure 3, we present a comparison between the measured power spectrum in redshift space from our mock map and the calculation from the RSD model. We separate the power spectrum using Legendre polynomials. The monopole is \(P^{0}_{H1}(k) = \frac{5}{2} \int_{-1}^{1} P_{H1}^{0}(k, \mu) d\mu\) and the quadrupole is \(P^{2}_{H1}(k) = \frac{5}{2} \int_{-1}^{1} \frac{3\mu^2 - 1}{2} P_{H1}^{2}(k, \mu) d\mu\). For the monopole, our model prediction reached 10% accuracy for all redshifts down to \(k = 0.5 h\) Mpc\(^{-1}\), and reached \(\sim 20\%\) accuracy down to \(k = 1.0 h\) Mpc\(^{-1}\). The tilt-up tails in the power spectrum measured from mock maps are due to the shot noise (Villaescusa-Navarro et al. 2018). A detailed discussion of shot noise is given in Section 4. The quadrupole measurement is very noisy due to a lack of multiple realizations and large enough boxes. Given the noisy quadrupole measurement from mock maps, the model prediction is reasonably good.

We also calculate the H1 power spectrum without considering the FoG effect; the result is shown in Figure 4. It is quite clear that without modeling the FoG effect, the calculated monopole moment of the H1 power spectrum is too high at small scales, compared to the mock result. The quadrupole moment of the H1 power spectrum is very far from the mock result at small scales. Even at large scales \(k > 0.1 h\) Mpc\(^{-1}\), the monopole moment of the H1 power spectrum calculated from a model without the FoG effect is wrong by more than 10% at \(z = 0.0, 0.66\). This error grows larger with increasing \(k\). Thus, for a highly accurate observational project aiming to measure the H1 power spectrum in the future, we need a mock that properly accounts for the FoG effect due to galaxy internal H1 velocity dispersion, and a model that accounts for the FoG effect as well. Our mock building methodology and FoG model
provide an important step forward for future 21 cm intensity mapping surveys.

4. Shot Noise

Since we have placed all the HI mass in the center of the halos to generate the mock map, a question naturally arises: how will such a point mass assumption affect the power spectrum? This question is discussed in detail in Castorina & Villaescusa-Navarro (2017). The shot noise power spectrum for HI density distribution can be calculated by

$$P_{\text{HI}}^{\text{SN}}(z) = \int_0^\infty n(M, z)M_{\text{HI}}^2(M, z)dM \left( \int_0^\infty n(M, z)M_{\text{HI}}(M, z)dM \right)^2$$

$$= \frac{V}{N} \left( \langle M_{\text{HI}}^2 \rangle \right)^2,$$

where $V$ is the volume of the simulation box and $N$ is the total number of galaxies in the SAM galaxy catalog that have neutral hydrogen. We have transformed Equation (8) into its final form on the right side in order to more easily calculate it with the catalog, from which we already have the HI mass inside each galaxy. We list the values in Table 1.

However, if we would like to calculate the shot noise power spectrum in redshift space, we need to consider the internal HI velocity dispersion. The HI gas in a galaxy is stretched into a line in the line-of-sight direction in redshift space and keeps its point mass in the direction perpendicular to the line of sight. Therefore, Equation (8) is no longer correct. The difference is clear: HI mass is not considered the point mass, but rather the one-dimensional Gaussian density distribution along the line-of-sight direction. So the modified shot noise power spectrum should be

$$P_{\text{HI}}^{\text{SN}}(k) = \frac{1}{2} \int_0^\infty n(M, z)M_{\text{HI}}^2(M, z)dM \left( \int_0^\infty n(M, z)M_{\text{HI}}(M, z)dM \right)^2,$$

where

$$u(k) = \int_0^\infty \delta(x) \delta(y) e^{-ik_x x} e^{-ik_y y} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2\sigma^2}} e^{-ik_z z} dx dy dz$$

$$= e^{-\frac{1}{2} \sigma^2 k^2},$$

and for simplicity,

$$P_{\text{HI}}^{\text{SN}}(k, \mu) \approx P_{\text{HI}}^{\text{SN}} u(k) \mu$$

$$= \frac{V}{N} \left( \langle M_{\text{HI}}^2 \rangle \right)^2 e^{-\frac{1}{2} \sigma^2 \mu^2 k^2}.$$

We combine the model for the redshift-space power spectrum shown in Equation (7) and shot noise contribution in Equation (11). The total power spectrum is the sum of Equations (7) and (11). The monopole moment of the shot noise power spectrum can be simply calculated by

$$P_{\text{HI}}^{\text{SN}} = \int_{-1}^{+1} \frac{1}{2} P_{\text{HI}}^{\text{SN}}(k, \mu) d\mu,$$
and the quadrupole moment is
\[ P_{\text{HI}}^{\text{SN}} = \int \frac{5}{4}(\mu^2 - 1)P_{\text{HI}}^{\text{SN}}(k, \mu) d\mu. \] (13)

Since the stretch in redshift space caused by the H\textsc{i} velocity dispersion is quite small, the anisotropy of the shot noise is also very small, and the quadrupole moment of the shot noise power spectrum is negligible. For the monopole moment, compared to the pure point mass shot noise, \( P_{\text{HI}}^{\text{SN}} \), \( P_{\text{HI}0}^{\text{SN}} \) has a smaller value at large \( k \). The shot noise contributions are shown as dotted lines in Figure 3. The dotted lines have not been artificially shifted, while the solid and dashed lines are shifted, for better illustration. The dotted lines are, in fact, always lower than the solid lines and dashed lines. Note that in Equation (8), the shot noise power spectrum is independent of \( k \), as expected. This is because of the point mass assumption we made in calculating the shot noise. However, in Equation (11), the shot noise power spectrum in redshift space is dependent on \( k \) because after considering the FoG effect, the point mass assumption is no longer valid. The point mass is replaced by line mass in redshift space. The size of the line is decided by the galaxy H\textsc{i} gas velocity dispersion; therefore the shot noise power spectrum becomes \( k \)-dependent.

In order to justify our scale-dependent shot noise calculation, we have performed a test on random points. We have generated 10,000 random points in a 100 Mpc/\( h \) box. Therefore, the number density \( n = 0.01h^3 \text{Mpc}^{-3} \). Then we stretched the random points into lines in the \( z \) direction according to the process for generating the mock catalog, with \( \delta_c = 5 \text{ Mpc}/h \). The result is shown in Figure 5. We can see that, below the Nyquist limit, \( 1/n \) shot noise can describe the random point power spectra. However, after stretching, the

![Figure 4](image_url)

**Figure 4.** Same as Figure 3: on the left (right) panel, we show the power spectrum monopole \( P_{\text{HI}}^0 \) (quadrupole \( P_{\text{HI}}^2 \)). In the upper panels, the measurement from the mock map is shown as solid lines and the calculation from our RSD model is shown as dashed lines. Different colors represent different redshifts. The lines in the upper panels are artificially shifted by factors of 4 (2, 1, 0.5, 0.25, and 0.125) for \( z = 3.09 \) (2.52, 1.94, 1.25, 0.66, and 0), for better illustration. (The shot noise contribution is shown as dotted lines; note that they have not been artificially shifted.) In the lower panels, the relative difference \( \left(P_{\text{model}}/P_{\text{mock}} - 1\right) \) between the model calculation and measurement is shown. It is clear that without properly modeling the FoG effect, the calculated H\textsc{i} power spectrum is far from the measurement of the mock.

![Figure 5](image_url)

**Figure 5.** The power spectra of random points are shown in blue. The \( 1/n \) shot noise is shown in black. The power spectra of stretched random points are shown in orange. The calculated power spectra with Equation (11) are in green. Our model fits the stretched random sample much better than the traditional \( 1/n \) shot noise.

| Redshift | 0 | 0.66 | 1.25 | 1.94 | 2.52 | 3.09 |
|----------|---|------|------|------|------|------|
| \( \Sigma \sqrt{(h^{-1} \text{Mpc})^3} \) | 1.82 | 1.80 | 1.82 | 1.94 | 2.13 | 2.44 |
| \( \frac{\langle M_{\text{HI}} \rangle}{(M_{\text{HI}})^2} \) | 26.5 | 14.7 | 9.46 | 6.03 | 4.35 | 3.31 |

Table 1: Terms of the Shot Noise Power Spectrum Calculated from the ELUCID SAM Galaxy Catalog, Necessary for Calculating the Shot Noise Power Spectrum with Equations (8) and (11)
power spectra drops on smaller scales. Our scale-dependent shot noise model can reasonably describe such power spectra. The random point test shows that it is necessary to take the modification of the shot noise power spectrum model into account. Note that $\delta_c = 5 \text{ Mpc}/h$ is not a realistic number, but is much larger than the real case. We use such a large number to enlarge the effect for illustration.)

5. Discussion and Summary

In this paper, we have provided a novel method to generate a mock 21 cm intensity map from simulations. This method can take the galaxy internal H I gas velocity dispersion into account correctly. Therefore, it can generate a mock map with fruitful details about the FoG effect at small scales. By comparing the power spectrum of our mock map and hydrodynamic simulation shown in F18, we concluded that our method can accurately predict the power spectrum measured from the mock map and the RSD model, we found that our model can accurately generate a mock map that includes the FoG effect. We have also concluded that the claim raised in F18, that the FoG effect can cause additional suppression in the power spectrum, is correct.

We have also proposed a novel RSD model that includes a parameter-free FoG model. By comparing the power spectrum measured from the mock map and the RSD model, we found that our model can accurately predict the power spectrum measured from the mock maps in redshift space. The parameter-free FoG model is quite useful for future constraints using 21 cm intensity mapping. If the H I velocity dispersion versus galaxy virial mass relation can be calibrated by simulations or independently measured from other observations, we need one less parameter than the traditional RSD model to fit the power spectrum in redshift space, which will possibly provide better constraints for the cosmological parameters that we are interested in.

However, our mock map still overestimated the power spectrum by about 10%, compared to the result from F18. It is likely due to the low resolution of our mock map ($1h^{-1}\text{Mpc}$ grid size) and the related shot noise. We also provided a novel calculation for the shot noise power spectrum applicable for our mock.

In fact, since most of the near future observations for 21 cm intensity mapping are very poor in resolution, the large beam size can effectively suppress the power spectrum at $k > 0.2h \text{Mpc}^{-1}$. Therefore, linear perturbation theory might still be sufficient in the near future. However, if we would like to gain more information and constrain power from the near future surveys, such as CHIME, Tianlai, BINGO, FAST, and SKA, a good understanding of nonlinear effects on small scales is necessary. We have pointed out that the FoG effect due to galaxy internal H I velocity dispersion can lead to an incorrect calculation of the H I power spectrum even at $k \sim 0.2h \text{Mpc}^{-1}$. Moreover, if we can achieve higher-resolution 21 cm intensity mapping surveys in the future, nonlinear effects will be more important.

Throughout this paper, we have heavily used numpy, scipy, and matplotlib for data analysis and generating plots (Hunter 2007; van der Walt et al. 2011; Virtanen et al. 2020). We have also used Pylians\(^7\) python package to calculate the power spectrum and density field. This work was supported by IBS under the project code, IBS-R018-D1. Y. Luo acknowledges the support from NSFC (No. 11730391). This work was also partially supported by NNSFC key project No. 11835009.

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\(^7\)https://github.com/franciscovillaescusa/Pylians