Model Parameterization for Coherent Manipulation in Spin Current through FM-QD1-QD2-FM

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Abstract. In this work, model parameterization for coherent manipulation, for serially coupled double quantum dots embedded between ferromagnetic leads (FM-QD1-QD2-FM), is presented. Theoretical model based to the non-equilibrium Green's function approach is considered. Since the spin current and its spin channels are formulated incorporating the spin polarization and the type of spin configuration on the leads. Our model incorporates the inter-dot hopping, the intra-dot Coulomb correlation, the spin exchange energy and the coupling interactions between the quantum dots and leads. The results concerned to the parallel configuration at strong inter-dot coupling regime shows the preferable coherent state for spin down electrons only at relatively high polarization. While, for weak coupling regime and antiparallel configuration, the decoherent state is dominating. Interesting results have been concluded by calculating the molecular energy levels of the quantum dots. In all our calculations the same spin polarization on the leads is considered. The case of different spin polarization on the lead is also investigated but no unprecedented features are concluded as active region is symmetric.

1. Introduction
Nanotechnology has stimulated much interest in the study of quantum transport in mesoscopic structure. Devices based on quantum dot (QD) have particular attention, in semiconductor quantum dot a two-dimensional electron gas is wasted by an arrangement of gate barriers such that a zero-dimensional region is produced. Transport through this quantum dot takes place through spin-conserving tunnel events.

The advantage of these systems is that their parameters can be continuously changed by modifying the applied external potentials so that different coupling regimes can be studied. For such devices to be useful, an understanding of the processes which couple the microscopic device to the macroscopic environment is paramount, that is to say, the decoherent processes of such systems must
be well understood. In the presence of an interdot coupling, coherent electron states can extend over the whole DQD system, like the formation of chemical bonds in molecules. Depending on the strength of the interdot coupling, the two dots could be ionic like (weak tunnel coupling) or covalent like bonds (strong tunnel coupling). DQD systems exhibit a wide range of interesting and fundamental physical phenomena, such as Coulomb blockade oscillations of conductance [1] and the Kondo effect [2,3]. In the last years attention has been focused on properties of the coupled quantum dots system since they are expected to be basic building blocks for a quantum computer [4-7].

Double quantum dots have already paved their way to become underlying devices of spintronics not only because of beautiful physics emerging in those systems, but, more importantly, due to possible future applications and due to the possibility of manipulation of a single spin [8-12]. For studying a system consisting of a quantum dot coupled to leads one could use the Anderson impurity model [13,14], which is most relevant when there are many single particle levels within the energy window of interest.

In this work, which is a part of our extended work in studying the Thermospin effects of quantum dots structures, a mathematical model is constructed to study the spin current through a system consisting of two serially coupled double quantum dots embedded between two ferromagnetic leads. Since the spin transport occurs through the spin dependent states of the active region. The efficiency of spin transport in this system depends on the spin properties of the system such as, the chemical potential of the left and right lead \( \langle \mu_{\sigma}^L \rangle \), the energy levels of the two dots \( \langle E_{\sigma}^n \rangle \), as well as the energy band characteristics (such as band width and electron density of states) for the right and left leads.

2. The Mathematical Model

The system considered is DQD attached to ferromagnetic source and drain as illustrated in figure 1, we consider that there is only two spin-split levels per dot and that electrons experience an on-site interaction energy \( U_i \). The system is described by Anderson Hamiltonian, which take all the coupling and exchange interactions into consideration as follows [15-17]:

\[
H = H_{\text{DQD}} + H_{\text{Leads}} + H_{\text{QD-QD}} + H_{\text{DQD-Leads}}
\]

The first term in equation (1) represents the Hamiltonian of quantum dots molecule and is written as [18]:

\[
H_{\text{DQD}} = \sum_{\sigma} \sum_{i=1}^{2} E_{\sigma i}^{\alpha} n_{\alpha i}^{\sigma} + \sum_{i=1}^{2} U_i n_{\alpha i}^{\sigma} n_{\alpha i}^{\sigma}
\]

\( E_{\sigma i}^{\alpha} \) are the energy level for the dot \( i \) \( (i=1,2) \) with spin \( \sigma \) (\( \uparrow, \downarrow \)), while \( n_{\alpha i}^{\sigma} \) is the corresponding occupation numbers,

\[
n_{\alpha i}^{\sigma} = C_{\alpha i}^{\sigma} C_{\alpha i}^{\sigma}
\]

with,

\[
E_{\sigma 1}^{\alpha} = E_1 + U_1 n_{\sigma 1}^{\sigma} - J n_{\sigma 2}^{\alpha}
\]

\[
E_{\sigma 2}^{\alpha} = E_2 + U_2 n_{\sigma 1}^{\sigma} - J n_{\sigma 1}^{\alpha}
\]

\( E_1, E_2 \) are the effective energy levels of the two dots, \( U_i \) represent the intradot Coulomb repulsive energy of QD1 and QD2 sites respectively. \( J \) represents the spin exchange energy. The second term in equation (1) represents electrons of the two leads [19]:

\[
H_{\text{Leads}} = \sum_{\alpha} \sum_{i} \sum_{k_{a}} E_{\alpha k_{a}}^{\alpha} n_{\alpha k_{a}}^{\alpha}
\]
Figure 1. Symmetric double quantum dots embedded between two ferromagnetic leads in the case of (a) Parallel configuration and (b) Antiparallel configuration.

Where $E_{d_{\alpha}}^{\sigma}$ is the single electron energy level in the lead ($\alpha = L, R$) with momentum $k_{\alpha}$ and spin $\sigma$. $n_{d_{\alpha}}^{\sigma}$ is the corresponding occupation number. The third term in equation (1) depicts the Hamiltonian due to coupling between the two dots,

$$H_{QD-QD} = \sum_{\alpha} (V_{ij} C_{i_{\alpha}}^{\sigma} C_{j_{\alpha}}^{\sigma} + V_{ij} C_{i_{\alpha}}^{\sigma} C_{j_{\alpha}}^{\sigma})$$

$$+ \frac{1}{2} J (C_{i_{\alpha}}^{\sigma} C_{j_{\alpha}}^{\sigma} + h c.)$$

$V_{ij}$ is the inter-dot tunneling coupling between the two dots. Finally, the tunneling between the DQD molecule and the two leads is described by the last term of equation (1),

$$H_{QD-Leads} = \sum_{\alpha} \sum_{i} \sum_{i_{\alpha}} \sum_{i_{\alpha}} (V_{ai_{\alpha}} C_{i_{\alpha}}^{\sigma} C_{i_{\alpha}}^{\sigma} + h c.)$$

$C_{i_{\alpha}}^{\sigma}$ is the creation (annihilation) operator of the electronic state $k_{\alpha}$ of the leads. $V_{ai_{\alpha}}$ give the tunneling amplitude between the dot $i$ and the ferromagnetic leads. The occupation numbers of the dots energy levels $E_{d_{i\alpha}}^{\sigma}$ are defined as [20, 21];

$$n_{d_{\alpha}}^{\sigma} = \int_{-\infty}^{\infty} \rho_{ai}^{\sigma}(E) f_{ai}^{\sigma}(E, T_{\alpha}) \, dE$$

$\rho_{ai}^{\sigma}(E)$ is the electronic density of states on dot $i$ that connected to the lead $\alpha$ and $f_{ai}^{\sigma}(E, T_{\alpha})$ is the Fermi distribution function. Using the definition of $\rho_{ai}^{\sigma}(E)$ in terms of Green function such as [22,23];

$$\rho_{ai}^{\sigma}(E) = -\frac{1}{\pi} \text{Im} G_{ai}^{\sigma}(E)$$
Im$G_{i\sigma}^\omega(E)$ is the imaginary part of Green functions on the dot $i$ with spin $\sigma$ due to the coupling with the lead [23]:

$$
G_{i\sigma}^\omega(E) = \frac{1}{2} \left[ \frac{(1+W_i^\sigma/\mathcal{V}_i^\sigma)}{(E-E_{i\sigma}^\omega) + i\Gamma_{i\omega}} + \frac{(1-W_i^\sigma/\mathcal{V}_i^\sigma)}{(E-E_{i\sigma}^\omega) + i\Gamma_{i\omega}} \right]
$$

(10b)

$$
G_{i\sigma}^{\omega R}(E) = \frac{1}{2} \left[ \frac{(1-W_2^\sigma/\mathcal{V}_2^\sigma)}{(E-E_{2\sigma}^\omega) + i\Gamma_{2\omega}} + \frac{(1+W_2^\sigma/\mathcal{V}_2^\sigma)}{(E-E_{2\sigma}^\omega) + i\Gamma_{2\omega}} \right]
$$

(10a)

$\Gamma_{i\omega}$ is the broadening in the dot $i$ energy levels and $E_{i\sigma}^\omega$ are energies corresponding to the bonding and antibonding wave functions [24], defined by the following relations [25],

$$
E_{i+}^\omega = E_i + U_i n^{-\omega} - J n^\omega \pm \mathcal{V}_i
$$

(11)

where,

$$
\mathcal{V}_i^\sigma = (V_{12}^2 + (W_i^\sigma)^2)^{1/2}; \quad W_i^\sigma = U_i M^{-\sigma} + JM^\sigma
$$

(12)

$$
n^\sigma = \frac{n_{i1}^\sigma + n_{i2}^\sigma}{2}; \quad M^\sigma = \frac{n_{i1}^\sigma - n_{i2}^\sigma}{2}
$$

(13)

We treat the electrons of opposite spin $\sigma$ as static entities whose main influence is to change the effective site energy of the electrons of spin $\sigma$ by $\mathcal{V}_i$. Each Green function is simply a sum over effective non-interacting Green functions each weighted by the probability of a particular realization of the occupation numbers for the opposite spin states. Of course the single particle energy levels have to be adjusted to the correct occupations of the opposite spin states.

Substituting equation (10) in equation (9), the localized density of states on the dots will be:

$$
\rho^\sigma(E) = \sum_j \sum_{\omega} K_{j\omega}^\sigma \rho_{\omega j}^\sigma(E)
$$

(14)

Where $(j = +, -)$, if $i = 1$ then $\alpha = L$ and if $i = 2$ then $\alpha = R$. For simplicity, reduced function $K_{ij}^\sigma$ is used:

$$
K_{ii}^\sigma = \frac{1}{2\pi}(1 + j \frac{W_i^\sigma}{V_i^\sigma}); \quad K_{ij}^\sigma = \frac{1}{2\pi}(1 - j \frac{W_i^\sigma}{V_i^\sigma})
$$

(15)

One of the most important properties is the spin transport which must be determined by calculating the spin current. The spin current exists due to spin bias $(\mu \Rightarrow \mu + e\mathcal{V}_{ab})$ on the leads $(e\mathcal{V}_{ab}$ is the spin bias). The spin current $I^\sigma$ for spin certain channel $\sigma$ through double quantum dots is given by [26,27]:

$$
I^\sigma = \frac{e}{h} \int dE \left[ f_{L^\sigma}(E) - f_{R^\sigma}(E) \right] \tau^\sigma(E)
$$

(16)

$\tau^\sigma(E)$ is the transmission function for electron with spin $\sigma$ and is given by [28]:

$$
\tau^\sigma(E) = 2\pi \frac{\Gamma_{ii}^\sigma \Gamma_{ij}^\sigma}{\Gamma_{ii}^\sigma + \Gamma_{ij}^\sigma} \rho^\sigma(E)
$$

(17)

Where $\rho^\sigma(E)$ is defined by equation (9). Then the current $I^\sigma$ expression will be;

$$
I^\sigma = I_{ii}^\sigma + I_{ij}^\sigma
$$

(18)

since:

$$
I_{ii,\omega}^\sigma = \int dE \rho_{ii,\omega}^\sigma(E) f_{\omega}^\sigma(E)
$$

(19a)

$$
I_{ij,\omega}^\sigma = \int dE \rho_{ij,\omega}^\sigma(E) f_{\omega}^\sigma(E)
$$

(19b)
with,

\[ \rho_{i\sigma}^\sigma(E) = \frac{\Gamma_i^\sigma}{(E - E_{i\sigma}^\sigma)^2 + (\Gamma_i^\sigma)^2} \]  

(20)

By solving the integral in equations (19) analytically using Sommerfeld expansion at low temperature, we get the following forms:

\[
\begin{align*}
I_{1L,\sigma}^\sigma &= \sum_j (K_{ij}^\sigma \left( \tan^{-1}(\frac{\mu_{i\sigma}^\sigma - E_{1j}^\sigma}{\Gamma_{1L}^\sigma}) - \tan^{-1}(\frac{-2\beta - E_{1j}^\sigma}{\Gamma_{1L}^\sigma}) \right) \\
&\quad - \frac{\pi^2}{3} (k_B T_{\sigma})^2 \frac{(\mu_{i\sigma}^\sigma - E_{1j}^\sigma)}{(\mu_{i\sigma}^\sigma - E_{1j}^\sigma)^2 + (\Gamma_{1L}^\sigma)^2} \rho_{1ij}^\sigma (\mu_{i\sigma}^\sigma)) \right)  \\
I_{2R,\sigma}^\sigma &= \sum_j (K_{2j}^\sigma \left( \tan^{-1}(\frac{\mu_{i\sigma}^\sigma - E_{2j}^\sigma}{\Gamma_{2R}^\sigma}) - \tan^{-1}(\frac{-2\beta - E_{2j}^\sigma}{\Gamma_{2R}^\sigma}) \right) \\
&\quad - \frac{\pi^2}{3} (k_B T_{\sigma})^2 \frac{(\mu_{i\sigma}^\sigma - E_{2j}^\sigma)}{(\mu_{i\sigma}^\sigma - E_{2j}^\sigma)^2 + (\Gamma_{2R}^\sigma)^2} \rho_{2Rj}^\sigma (\mu_{i\sigma}^\sigma)) \right) 
\end{align*}
\]  

(21a)

and:

\[
\begin{align*}
I_{1L}^\sigma &= 2\Delta^\sigma (I_{1L,\sigma}^\sigma - I_{1R,\sigma}^\sigma)  \\
I_{2R}^\sigma &= 2\Delta^\sigma (I_{2L,\sigma}^\sigma - I_{2R,\sigma}^\sigma)  
\end{align*}
\]  

(22-23)

where,

\[ \Gamma^\sigma = \frac{\Gamma_{1L}^\sigma \Gamma_{2R}^\sigma}{\Gamma_{1L}^\sigma + \Gamma_{2R}^\sigma} \]  

(24)

Finally, the spin current for spin channel (\(\sigma\)) is calculated using the relations ((18)-(23)), and the total spin current will be:

\[ I_{\sigma}^\sigma = \frac{1}{h} \sum_{\sigma} \sigma I_{\sigma}^\sigma \]  

(25)

Where arbitrary units are adopted in all currents calculations.

3. Model Parameterization for Coherent Manipulation

Actually, electrons in quantum dots interact with many other particles: charge, impurity, atoms, electrons in surrounding gates, phonons in the substrate and photons from environmental. The interactions between the electrons on the dot with these abovementioned particles destroy the coherent behavior of the electrons on the dot. Due to these processes the phase of the electron wave acquires some dephasing and its phase-coherence time becomes finite. In general, only scattering processes during which an excitation of the environment is created or destroyed leads to a loss of phase-coherence. Since, the phase-coherence time is the scale on which the electrons preserve their quantum mechanical phase which is needed for the appearance of the quantum interference effects. The coherence of electronic states has arisen as physical bases for information processing projects that use two-state quantum system to give efficient computation and secure communications.

In this paper, we focus on the spin polarized transport through two serially coupled quantum dots coupled to two ferromagnetic leads, since the parallel (P) and antiparallel (AP) configurations are considered (see figure 1). In the parallel configuration, we have \(\Gamma_{1L}^\sigma = \Gamma_{2R}^\sigma = \Gamma_0(1+P)\) and...
\( \Gamma_{12}^{\sigma} = \Gamma_{2R}^{\sigma} = \Gamma_0(1 - P) \), where \( P \) is the leads degree of polarization \( 0 < P < 1 \). While in the antiparallel configuration, we have, \( \Gamma_{1R}^{\sigma} = \Gamma_{2L}^{\sigma} = \Gamma_0(1 + P) \) and \( \Gamma_{1L}^{\sigma} = \Gamma_{2R}^{\sigma} = \Gamma_0(1 - P) \) [29,30]. Since \( \Gamma_0 \) is the hybridization function in the absence of ferromagnetism. For the case of parallel alignment, the spin electrochemical potential of the leads are fixed at \( \mu_0^\sigma = -\mu_0^{-\sigma} = -\mu_0^{\alpha} = 0.05 \, \text{eV} \), while for antiparallel alignment, \( \mu_0^\sigma = \mu_0^{-\sigma} = -\mu_0^{\alpha} = 0.05 \, \text{eV} \).

The interactions included in our model calculation must be highlighted,

1- The spin independent interaction between the two dots \( (V_{12}) \).
2- The spin dependent broadening in the energy levels of the dots due to the interaction with lead \( \Gamma_{12}^{\sigma} \).
3- The interdot Coulomb correlation on each dot, where the dots are symmetric \( U_1 = U_2 \).
4- The effective exchange energy \( (J) \).

Equations (8) and (11) are solved self-consistently to get the occupation numbers of quantum dots energy levels and the “molecular orbitals energies” of the quantum dots molecule. In order to investigate the coherence state in our system, the calculations are accomplished in two cases parallel and antiparallel configurations. Firstly, for the case of parallel configuration, \( I^{\sigma} \) and \( I_{sp} \) are calculated using equations (18) and (25) for two different regimes, the strong interdot coupling regime \( (V_{12} > \Gamma_{12}^{\sigma}) \) and the weak interdot coupling regime \( (V_{12} < \Gamma_{12}^{\sigma}) \). The values of \( \Gamma_{12}^{\sigma} \) are tuned by the spin polarization on the leads, so two values are considered \( P = 0.3 \) (low spin polarization) and \( P = 0.7 \) (relatively high polarization). The spin exchange parameter may be antiferromagnetic \( (J < 0) \) or ferromagnetic \( (J > 0) \) nature which leads to a different spin transport characteristics for the device. So, in order to study the transport properties for our system a spin bias will applied to the leads and while other parameters are fixed at \( E_L = E_R = 0.1 \, \text{eV} \), \( U_1 = U_2 = 0.05 \, \text{eV} \), \( 0.2 \, \text{eV} \) and \( J = 0.3 \, \text{eV} \), \( -0.3 \, \text{eV} \).

Figures ((2)-(3)) represent \( I^{\sigma} \) and \( I_{sp} \) as a function of spin bias for different values of \( P = 0.3 \) and \( J = 0.3 \, \text{eV} \) at weak and strong coupling regimes except for \( I^{\sigma} \) at strong coupling with \( U_{sp} = 0.05 \, \text{eV} \) which can be considered as a dephasing case. In these figures, the spin current and its spin contributions shows a decoherence state. The spin current also shows a negative differential resistance for both values of \( P \).

In figures ((4)-(5)), \( I^{\sigma} \) and \( I_{sp} \) are calculated for relatively higher spin polarization \( P = 0.7 \), with \( \Gamma_{12}^{\sigma} = 0.102 \, \text{eV} \) and \( \Gamma_{12}^{\sigma} = 0.018 \, \text{eV} \). The same physical behavior can be reported for these figures, except for the spin down \( (I^{-\sigma}) \) at \( U_{sp} = 0.05 \, \text{eV} \) which shows a step stairs shape in the strong interdot coupling, but the decoherent state is dominated in the spin current where \( I^{-\sigma} < I^{\sigma} \). The antiferromagnetic nature of the spin exchange interaction is also highlighted with \( J = -0.3 \, \text{eV} \) (see figures ((6)-(9))). The spin down electrons shows dephasing behavior while the spin up current shows obvious decoherent state. It is concluded that the spin down electrons show the preferable coherent state at relatively high spin polarization for both values of \( E_{sp} \) and \( U_{sp} \) as the nature of the spin exchange interaction is ferromagnetic one with \( V_{12} > \Gamma_{12}^{\sigma} > \Gamma_{12}^{\sigma} \).
Figure 2. Spin-polarized currents (a) $I^\sigma (I^{-\sigma})$ (b) $I_{sp}$ as a function of spin bias for different values of correlation energy with $P = 0.3$, $V_{12} = 0.2$ eV, $J = 0.3$ eV, $E_1 = E_2 = 0.1$ eV, $T_L = T_R = 100$ K, $\Gamma_{0L}^\sigma = \Gamma_{0R}^\sigma = 0.078$ eV, $\Gamma_{0L}^{-\sigma} = \Gamma_{0R}^{-\sigma} = 0.042$ eV.

Figure 3. Spin-polarized currents (a) $I^\sigma (I^{-\sigma})$ (b) $I_{sp}$ as a function of spin bias for different values of correlation energy with $P = 0.3$, $V_{12} = 0.01$ eV, $J = 0.3$ eV, $E_1 = E_2 = 0.1$ eV, $T_L = T_R = 100$ K, $\Gamma_{0L}^\sigma = \Gamma_{0R}^\sigma = 0.078$ eV, $\Gamma_{0L}^{-\sigma} = \Gamma_{0R}^{-\sigma} = 0.042$ eV.
Figure 4. Spin-polarized currents (a) $I^\sigma (I^{-\sigma})$ (b) $I^\uparrow$ as a function of spin bias for different values of correlation energy with $P = 0.7$, $V_{12} = 0.2$ eV, $J = 0.3$ eV, $E_1 = E_2 = 0.1$ eV, $T_x = T_y = 100$ K, $\Gamma^\sigma_{1 \uparrow} = \Gamma^\sigma_{2 \uparrow} = 0.102$ eV, $\Gamma^\sigma_{1 \downarrow} = \Gamma^\sigma_{2 \downarrow} = 0.018$ eV.

Figure 5. Spin-polarized currents (a) $I^\sigma (I^{-\sigma})$ (b) $I^\uparrow$ as a function of spin bias for different values of correlation energy with $P = 0.7$, $V_{12} = 0.01$ eV, $J = 0.3$ eV, $E_1 = E_2 = 0.1$ eV, $T_x = T_y = 100$ K, $\Gamma^\sigma_{1 \uparrow} = \Gamma^\sigma_{2 \uparrow} = 0.102$ eV, $\Gamma^\sigma_{1 \downarrow} = \Gamma^\sigma_{2 \downarrow} = 0.018$ eV.
Figure 6. Spin-polarized currents (a) $I^{\sigma}(I^{-\sigma})$ (b) $I_{sp}$ as a function of spin bias for different values of correlation energy with $P = 0.3$, $V_{12} = 0.2$ eV, $J = -0.3$ eV, $E_1 = E_2 = 0.1$ eV, $T_L = T_R = 100$ K, $\Gamma_{1L}^{\sigma} = \Gamma_{2R}^{-\sigma} = 0.078$ eV, $\Gamma_{1L}^{-\sigma} = \Gamma_{2R}^{\sigma} = 0.042$ eV.

Figure 7. Spin-polarized currents (a) $I^{\sigma}(I^{-\sigma})$ (b) $I_{sp}$ as a function of spin bias for different values of correlation energy with $P = 0.3$, $V_{12} = 0.01$ eV, $J = -0.3$ eV, $E_1 = E_2 = 0.1$ eV, $T_L = T_R = 100$ K, $\Gamma_{1L}^{\sigma} = \Gamma_{2R}^{-\sigma} = 0.078$ eV, $\Gamma_{1L}^{-\sigma} = \Gamma_{2R}^{\sigma} = 0.042$ eV.
Figure 8. Spin-polarized currents (a) $I^\sigma (I^{-\sigma})$ (b) $I^\sigma_w$ as a function of spin bias for different values of correlation energy with $P = 0.7$, $V_{12} = 0.2$ eV, $J = -0.3$ eV, $E_1 = E_2 = 0.1$ eV, $T_x = T_y = 100$ K, $\Gamma_{x1}^\sigma = \Gamma_{x2}^\sigma = 0.102$ eV, $\Gamma_{x1}^{-\sigma} = \Gamma_{x2}^{-\sigma} = 0.018$ eV.

Figure 9. Spin-polarized currents (a) $I^\sigma (I^{-\sigma})$ (b) $I^\sigma_w$ as a function of spin bias for different values of correlation energy with $P = 0.7$, $V_{12} = 0.01$ eV, $J = -0.3$ eV, $E_1 = E_2 = 0.1$ eV, $T_x = T_y = 100$ K, $\Gamma_{x1}^\sigma = \Gamma_{x2}^\sigma = 0.102$ eV, $\Gamma_{x1}^{-\sigma} = \Gamma_{x2}^{-\sigma} = 0.018$ eV.
The spin current \( I_\sigma \) and its contributions are also calculated in the case of antiparallel configuration for strong and weak interdot couplings with \( E_i = 0.1 \) eV, \( U_i = 0.05 \) eV (see figures ((10)-(17))). In these figures \( I^\sigma \) and \( I^{-\sigma} \) are equal but they flow through the device in adverse directions. The results concerned to the weak coupling regime show decoherent state and the intradot Coulomb correlation has no role when \( J = 0.3 \) eV in the case of low and relatively high spin polarization (see figures ((10)-(13))). When the spin exchange interaction is tuned to \( -0.3 \) eV i.e. the antiferromagnetic nature (see figures ((14)-(17))), the results that calculated for weak and strong interdot coupling are nearly coincide for but they have a higher values in comparison with the parallel configuration.

The case of coherence can be checked throughout the molecular energy levels calculation which is the most important step of the self-consistently solution where \( E_{ij}^\sigma = E_{ij}^{-\sigma} \) for \( j = +, - \). The molecular energy levels are presented in figures (18) and (19) as a function of the spin bias in the case of parallel configuration for strong and weak interaction coupling, respectively. It is well known that the quantum tunneling process through the active region is determined by the position of active region energy levels with respect to the chemical potentials of the ferromagnetic leads for both spin, which means that two energy windows must be considered. Figure (18a) shows that the molecular energy levels \( E_{ij}^{\sigma} \) are lying below the energy reference \( E = 0 \) while \( E_{ij}^{-\sigma} \) are lying upper to it, with no crossing between them for \(-0.5 \) eV < \( eV_{ab} < 0.5 \) eV. This feature enhances the quantum tunneling keeping in mind that this calculation is accomplished for \( U_i = 0.05 \) eV. For high \( U_i (= 0.2 \) eV), the molecular energy levels are also separated except for \( eV_{ab} \) values about \( eV_{ab} = 0 \), which may enhance the dephasing case (see figure (18b)). In figure (19), the molecular energy levels are crossing which means that these is no energy window for spin up or spin down for both values of \( U_i \). This feature destroy the quantum interference effects and enhances the decoherence case.
Figure 10. Spin-polarized currents (a) $I^\sigma(I^{-\sigma})$ (b) $I_{sp}$ as a function of spin bias for different values of correlation energy with $P = 0.3$, $V_{12} = 0.2$ eV, $J = 0.3$ eV, $E_1 = E_2 = 0.1$ eV, $T_f = T_R = 100$ K, $\Gamma_{1\Gamma}^\sigma = \Gamma_{2\Gamma}^\sigma = 0.042$ eV, $\Gamma_{1\Gamma}^{-\sigma} = \Gamma_{2\Gamma}^{-\sigma} = 0.042$ eV.

Figure 11. Spin-polarized currents (a) $I^\sigma(I^{-\sigma})$ (b) $I_{sp}$ as a function of spin bias for different values of correlation energy with $P = 0.3$, $V_{12} = 0.01$ eV, $J = 0.3$ eV, $E_1 = E_2 = 0.1$ eV, $T_f = T_R = 100$ K, $\Gamma_{1\Gamma}^\sigma = \Gamma_{2\Gamma}^\sigma = 0.078$ eV, $\Gamma_{1\Gamma}^{-\sigma} = \Gamma_{2\Gamma}^{-\sigma} = 0.042$ eV.
Figure 12. Spin-polarized currents (a) $I^\sigma(I^{-\sigma})$ (b) $I_\psi$ as a function of spin bias for different values of correlation energy with $P = 0.7$, $V_{12} = 0.2$ eV, $J = 0.3$ eV, $E_1 = E_2 = 0.1$ eV, $T_L = T_R = 100$ K, $\Gamma_{ul} = \Gamma_{2R} = 0.102$ eV, $\Gamma_{ul} = \Gamma_{2R} = 0.018$ eV.

Figure 13. Spin-polarized currents (a) $I^\sigma(I^{-\sigma})$ (b) $I_\psi$ as a function of spin bias for different values of correlation energy with $P = 0.7$, $V_{12} = 0.01$ eV, $J = 0.3$ eV, $E_1 = E_2 = 0.1$ eV, $T_L = T_R = 100$ K, $\Gamma_{ul} = \Gamma_{2R} = 0.102$ eV, $\Gamma_{ul} = \Gamma_{2R} = 0.018$ eV.
Figure 14. Spin-polarized currents (a) $I^\sigma (I^{-\sigma})$ (b) $I_{sp}$ as a function of spin bias for different values of correlation energy with $P = 0.3$, $V_{12} = 0.2$ eV, $J = -0.3$ eV, $E_1 = E_2 = 0.1$ eV, $T_f = T_R = 100$ K, $\Gamma^{-\sigma}_{1t} = \Gamma^{-\sigma}_{1\sigma} = 0.042$ eV, $\Gamma_{1t} = \Gamma_{1\sigma} = 0.042$ eV.

Figure 15. Spin-polarized currents (a) $I^\sigma (I^{-\sigma})$ (b) $I_{sp}$ as a function of spin bias for different values of correlation energy with $P = 0.3$, $V_{12} = 0.01$ eV, $J = -0.3$ eV, $E_1 = E_2 = 0.1$ eV, $T_f = T_R = 100$ K, $\Gamma^{-\sigma}_{1t} = \Gamma^{-\sigma}_{1\sigma} = 0.042$ eV, $\Gamma_{1t} = \Gamma_{1\sigma} = 0.042$ eV.
Figure 16. Spin-polarized currents (a) $I^\sigma (I^{-\sigma})$ (b) $I_{sp}$ as a function of spin bias for different values of correlation energy with $P = 0.7$, $V_{12} = 0.2$ eV, $J = -0.3$ eV, $E_1 = E_2 = 0.1$ eV, $T_L = T_R = 100$ K, $\Gamma_1^{\sigma} = \Gamma_2^{\sigma} = 0.102$ eV, $\Gamma_{1L}^{\sigma} = \Gamma_{2R}^{\sigma} = 0.018$ eV.

Figure 17. Spin-polarized currents (a) $I^\sigma (I^{-\sigma})$ (b) $I_{sp}$ as a function of spin bias for different values of correlation energy with $P = 0.7$, $V_{12} = 0.01$ eV, $J = -0.3$ eV, $E_1 = E_2 = 0.1$ eV, $T_L = T_R = 100$ K, $\Gamma_1^{\sigma} = \Gamma_2^{\sigma} = 0.102$ eV, $\Gamma_{1L}^{\sigma} = \Gamma_{2R}^{\sigma} = 0.018$ eV.
Figure 18. The molecular energy levels for QD1 and QD2 as a function of spin bias for different values of correlation energy with $P = 0.7$, $V_{12} = 0.2$ eV, $J = 0.3$ eV, $E_1 = E_2 = 0.1$ eV, $T_L = T_R = 100$ K, $\Gamma_{1L}^{\sigma} = \Gamma_{2R}^{\sigma} = 0.102$ eV, $\Gamma_{1L}^{\sigma} = \Gamma_{2R}^{\sigma} = 0.018$ eV.

Figure 19. The molecular energy levels for QD1 and QD2 as a function of spin bias for different values of correlation energy with $P = 0.7$, $V_{12} = 0.01$ eV, $J = 0.3$ eV, $E_1 = E_2 = 0.1$ eV, $T_L = T_R = 100$ K, $\Gamma_{1L}^{\sigma} = \Gamma_{2R}^{\sigma} = 0.102$ eV, $\Gamma_{1L}^{\sigma} = \Gamma_{2R}^{\sigma} = 0.018$ eV.
4. Conclusions

In summary, we investigate theoretically the coherent tunneling of the spin current through a serially symmetric coupled double quantum dots attached to two ferromagnetic leads, using the nonequilibrium Green's function approach in the linear response regime. This paper is a part of our extended work in studying the spin dependency of thermal and electrical transport currents in the FM-QD1-QD2-FM system. Our model incorporates the inter-dot hopping, the intra-dot Coulomb repulsion and the exchange coupling in the spin current calculations.

The results that related to the case of parallel configuration declare that the spin down electrons show preferable coherent state at low and relatively high spin polarization. Also we noticed that the results concerned to the weak coupling regime show decoherent state and the intradot Coulomb correlation has no role when $J = 0.3 \text{ eV}$ in the case of low and relatively high spin polarization. When the spin exchange interaction is tuned to $-0.3 \text{ eV}$ i.e. the antiferromagnetic nature, the results that have been calculated for weak and strong interdot coupling are coincide at specific but they have a higher values in comparison with the parallel configuration. The spin down electrons shows dephasing behavior while the spin current electrons show obvious decoherent state. All our calculations for the antiparallel case show decoherence state and intradot Coulomb correlation has no role. The case of coherence has been checked throughout the molecular energy levels which are investigated for all case.

Finally, in all these calculations the case of $P = P_L = P_R$ is considered. The case of $P_L \neq P_R$ is also investigated, but no different spin features are concluded as the active region considered in our study is symmetric.

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