New method of the estimation of the bending strength of ultrafine-grained structural ceramics for application in the conditions of multiaxial stress-strain state

A A Popov, N N Berendeev, M S Boldin, A V Nokhrin and V N Chuvil’deev

Lobachevsky State University of Nizhny Novgorod, Nizhny Novgorod, 603950 Russia

E-mail: popov@nifti.unn.ru

Abstract. A new method of mechanical testing of high-strength ultrafine-grained ceramic materials by the method of spark plasma sintering (SPS) is proposed. A promising scheme for testing cylindrical SPS-samples is the “Ball on three balls test” (B3B) scheme. The B3B test method has advantages over the standard four-point bending test method, since it is not necessary to cut standard samples from disks, cylindrical samples are aligned on a special support. In this paper, we present a methodology for processing experimental B3B test data developed using the finite element method. The finite element method for estimating the parameters allows us to consider a larger number of parameters that affect the absolute value of the maximum principal stress. The experimental data processing technique takes into account the influence of the sample size, equipment geometry, physical and mechanical properties of the material, and friction coefficient.

1. Introduction

Spark Plasma Sintering (SPS) is a new technology for the sintered ceramics and ceramic composites. A distinctive feature of SPS is the possibility of sintering micro- and nanosized ceramic materials with high mechanical properties due to ultrahigh heating rates (up to 2500 °C/min) in vacuum or inert medium with simultaneous application of pressure [1-7]. Samples of ceramics sintered by the SPS method have the shape of a cylinder, in which the ratio of height (t) to radius (R): t/R <0.5. The study of the strength characteristics of ceramics obtained by the SPS method is an urgent task of assessing and predicting the behavior of materials for the development of new products, structures and machine components.

The standard method for testing ceramics for bending strength under four-point bending conditions is described in standards [8, 9]. The surface fibers of the material undergo uniaxial tension in standard tests. However, for standard tests, it is required to prepare a sample of standard shapes and sizes, which requires machining of the material. The mechanical treatment of the ceramic surface leads to the accumulation of defects in the surface layer, which will introduce a significant error in the assessment of the bending strength of the material [10]. The process of preparing standard rectangular samples from cylindrical ceramic samples sintered by the SPS method introduces defects into the structure and increases the cost of research. Thus, to test cylindrical ceramic samples sintered by the SPS method, a test method and a method for calculating experimental data without significant mechanical preparation of the samples under multiaxial stress state (MSS) are required.
Studies [11, 12] devoted to loading schemes with a complex stress-strain state (SSS) showed the effectiveness of using the loading scheme “Ball on three balls test” (or B3B in the terminology of the National Association of Corrosion Engineers standards) [12]. “Ball on three balls test” (B3B test) is a method for measuring the bending strength of ceramic cylindrical shape samples with a height to radius ratio $\nu R <0.35$ [13].

The scheme of sample loading during tests according to scheme B3B is shown in Figure 1. With such a scheme of loading a third-order axis of symmetry appears in the system. Contact zones also arise between the ball and the sample. This makes stress analysis rather complicated and requires the use of modern computer simulation methods. In [14–16], the maximum principal stress was adopted as an estimate of the bending strength of ceramics $\sigma_{B3B}$. Publications [13, 14, 17, 18] show the applicability of this method for studying the bending strength of brittle cylindrical samples and is the completion of the preparatory phase of the adoption of this method as a standard.

The proposed loading scheme simplifies sample preparation in comparison with the preparation of standard samples. Cylindrical ceramic samples can be tested after sintering. In the experiment, repeatability of SSS for a series of samples is observed. The requirements for geometric faceting of the sample before testing are reduced. An analysis of the works [11, 17, 18] shows the absence of studies of the influence of the physical and mechanical constants of the material, the geometry of the loading scheme B3B on the calculation of maximum tensile stresses. The formulas proposed in the works for calculating the maximum principal stresses are linear approximations or are the result of calculations by numerical simulation. This makes it difficult for the experimenter to use this method for evaluating the strength of ceramics.

In [13], an analytical formula was proposed for calculating the maximum principal stress when all the balls have the same size $\sigma_{B3B} = f(\alpha, \beta, \mu) F/t^2$, where $\sigma_{B3B}$ is the maximum principal stress; $F$ is the breaking load, $t$ is the thickness of the sample, $\alpha = t/R$, $\beta = t/R_o$, $R$ - is the radius of the sample, $R_o$ - is the radius of the circle on which the three balls rest, $\mu$ is the Poisson's ratio. The above expression does not allow to take into account the material parameters and equipment features (when varying the size of the balls). Obtaining an analytical expression for calculating the maximum principal stresses is a very difficult task. This involves the introduction of a contact model between the loading ball and the sample. On the other hand, an expression for calculating the maximum principal tensile stresses can be obtained by applying the numerical experiment by the finite element method [17]. In the present paper, it is precisely the approximation approach to obtaining expressions for calculating the maximum principal stresses that is used.

This study focuses on the development of a methodology for processing experimental data of the B3B test when testing brittle materials (ceramics) under multiaxial stress state according to the “Ball on three balls test” loading scheme and formulating the limits of applicability of this technique using the finite element method.

In this paper, to calculate the bending strength, we propose a method for processing experimental results using the B3B loading scheme for ceramic cylindrical samples. The influence of various parameters on the stress-strain state was studied by numerical simulation in the ANSYS WORKBENCH 17.2 software package. The effects of various parameters (Young's modulus, friction
coefficient, radius of support balls, etc.) were considered. This is done by sequentially modeling the stress-strain state of the 1/6 model of a cylindrical specimen. The obtained dependencies of the maximum principal stresses on the parameters for calculating the maximum principal stresses. The obtained expressions allow the experimenter to process the experimental data without using the finite element modeling method.

2. Experimental

2.1 Test procedure «Ball on three balls test»

The “Ball on three balls test method” (B3B test) is a method for measuring the bending strength of cylindrical ceramic materials [13] under biaxial stress state (BSS). The dimensions of the sample may vary in diameter from 10 to 40 mm. A cylindrical sample (of radius \( R \), height \( t \)) is mounted on three balls (radius \( R_b \)) lying in the corners of an equilateral triangle inscribed in the support circle (radius of the support circle of the three balls \( R_a \)) and is symmetrically loaded with the fourth ball on the opposite side (Figure 1a). An experiment to determine bending strength is carried out in a specially designed tooling. The lower punch and pusher are made of hardened stainless steel with a yield strength above 1000 MPa. Tests are performed on a R-50 universal tensile testing machine using reverse tooling, which sets the specimen loading. The sample is placed in a centered liner. With this design, the sample is always coaxial to all the snap. The traverse speed is 70-100 \( \mu \text{m/min} \).

2.2 Finite Element Modelling

In the present work, the finite element method was used to study the SSS depending on various parameters. It is based on two main ideas: discretization of the studied object and approximation of the studied functions [19]. A numerical experiment was performed in the ANSYS WORKBENCH 17.2 environment.

The mathematical model of the material is linear elastic. On the surface G4 (Figure 2d), the displacement along the Z axis (Figure 2d) is specified, on the surface G1 (Figure 2a) the direction of the force along the Z axis for the loading ball is specified. On the surface G2 and G3 (Figures 2b, c) forbidden to move along the normals to these surfaces on the basis of symmetry conditions. On the surface G5 (Figure 2e) it is forbidden to move along the Z axis for supporting balls. To solve the problem using the finite element method, a discretization of the area of the object was carried out (Figure 2e). When sampling, a twenty-node three-dimensional finite element “Solid 186” with a quadratic shape function was used.

3. Results and discussion

3.1 Finite element modelling

3.1.1 Radial, tangential and axial normalized stress components. The plane-parallel cylinder (\( R = 6.3, 10.3, \) and 15.3 mm) are the object of study. The dimensions of the supporting circles are set by the separator and are \( R_a = 5.5, 9.25 \) and 14.25 mm. In a numerical experiment, the radii of all the balls and the height of the sample were varied.

The finite element method was used to calculate the radial, tangential, and axial normalized stress components. Stress curves were recorded along the directions \( K_1 \rightarrow K_2, K_1 \rightarrow K_3, K_1 \rightarrow K_4 \). At point \( K_1 \) (Figure 1b), the values of three principal stresses were controlled. A generalization of the results shows that the distribution of stresses is not affected by the radius of the support balls and the radius of the support circle. Figure 3 show the generalized results of the curves for samples of different thicknesses and with different sizes of balls (the abscissa axis is the coordinate normalized to \( R_a \), the ordinate axis is the stress dependence on the coordinate \( \sigma(x) \) normalized to the maximum tensile stress \( \sigma_{\text{max}} \).
The boundary conditions for the balls: the application of force (a) and displacement along the Z axis (d); prohibition of movement based on symmetry (b, c); prohibition of movement along the X axis of the ball (e); discrete model (f).

Figure 2. The simulation of the stress state of a ceramic sample at various radii of the supporting circle and the radius of the supporting balls led to the following results. Depending on the size of the support balls, the stresses along the line K1 → K2 in the region of contact of the ball with the sample have different values from each other. The greatest "curvature" has a dependence for the smallest diameter of the ball. An increase in diameter leads to a smoothing curve. However, the stress curves along the line K1 → K2 differ only in the area of contact with the ball. On the rest of the line, the stress values coincide.

The stress state on the opposite surface from the loading ball does not change. The stress values along the directions K1 → K4 and K1 → K3 inside the support circle coincide with each other. The influence of the radius of the balls and the geometric dimensions of the sample does not affect the stress curve. An increase in the diameter of the support balls from 5 to 9 mm leads to an increase in the friction force between the balls and the sample. This does not affect the result of calculating the first principal stress.
Figure 3. Radial, tangential and axial normalized stress components for sample Ø30.6x0.5. (a) Along a path in the 0°-direction (K1–K2), (b) along the 60°-direction of the tensile plane of the disc (K1–K3) and (c) along the axis of the disc (K1–K4)

3.1.2 The dependence of stresses on the geometric parameters of the loading circuit. Of particular interest is the dependence of the principal stresses at point K1 on the height of the test sample. To do this, a numerical simulation of the stress state was carried out and the dependence of the main stresses at the point of measurement of the maximum tensile stress on the height of the test sample was determined (Figure 4).

From the obtained results it follows that the dependence of the main stresses on the value \( t/R_a \) is of the same nature when scaling the dimensions of the sample. An analysis of the influence of the diameter (R) of the test sample on the change in the stress-strain state showed that the radius of the test sample does not change the stress state in the region of maximum tensile material. The stresses at the extreme points were considered at \( t/R_a = 0.04 \) and \( t/R_a = 0.38 \), the stress distribution along three directions: K1-> K4, K1-> K2 and K1-> K3.

Figure 4. Dependence \( \sigma_1(t/R_a) \) (a) and \( \sigma_3(t/R_a) \) (b) for the support circle \( R_a = 5.5 \) mm
3.1.3 The dependence of stresses on the maximum load force. In Section 3.1.1, we studied the dependence of the principal stresses on the parameter \( t/R_a \) for one force \( (F = \text{const}) \) of the numerical experiment and established the dependences for expressions 1 and 3 for \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) respectively. Power coefficients were set for all dependencies:

\[
\sigma_1 \approx A \cdot (t/R_a)^{-2.2},
\]

\[
\sigma_3 = B \cdot (t/R_a)^{-3.7}.
\]

Let us analyze by methods of numerical simulation the dependence of the coefficients \( A \) and \( B \) on the loading force. To do this, we carry out numerical experiments for the selected values of \( t/R_a \) at different loading forces for the three radii of the supporting circle \( (R_a = 5.5, 9.25, 14.25 \text{ mm}) \). The families of curves \( \sigma_1 \approx \sigma_2 = f(t/R_a, F) \) and \( \sigma_3 = f(t/R_a, F) \) are shown in Figures 5.

Consider the dependence of the coefficients \( A \) and \( B \) (formulas 1 and 2) on the applied force for the three radii of the supporting circle \( (R_a = 5.5, 9.25, 14.25 \text{ mm}) \). Figure 5 shows the dependencies for \( R_a = 5.5 \text{ mm} \). The dependence of the coefficients \( A \) and \( B \) on the force are shown in Figure 6.

According to the results of the study, dependencies are obtained for calculating the principal stresses without using numerical simulation:

\[
\sigma_1 \approx \sigma_2 = (x_1 \cdot R_a + x_2) \cdot F - (x_3 \cdot R_a + x_4) \cdot (t/R_a)^{-2.2},
\]

\[
\sigma_3 = (y_1 \cdot R_a + y_2) \cdot F - (y_3 \cdot R_a + y_4) \cdot (t/R_a)^{-3.7},
\]

where \( x_i \) and \( y_i \) are numerical coefficients.

3.1.4 The dependence of stresses on the maximum load force. A variety of brittle materials makes it necessary to check the dependences of the maximum principal stresses on the Young's modulus (150-570 GPa) and Poisson's ratio (0.09-0.32).
To assess the influence of the elastic modulus on the calculation of the maximum principal stresses, we will carry out numerical modeling for 9 values of the elastic modulus (Figure 7). It is important to note that in the existing numerical model, the coefficient of friction between the sample, supporting and loading balls contributes to the nonlinearity of the solution. Based on the existing reference data, we estimate the effect of the coefficient of friction in the range of 0.08-0.32. The experimental results are shown in Figure 8.

![Figure 7](image1.png)

**Figure 7.** The dependence of the principal stresses \(\sigma_1\) (a) and \(\sigma_3\) (b) for various values of elastic modulus (\(\mu = 0.08\))

![Figure 8](image2.png)

**Figure 8.** Dependence of the pre-determined coefficient on the elastic modulus for \(\sigma_1\) (a) and \(\sigma_3\) (b)

The following conclusions follow from the analysis of the pre-existing coefficients. First, the maximum principal stresses do not depend on the elastic modulus of the test sample in the range \(E = 150-1000\) GPa. With a decrease in Young's modulus below 200 GPa, the maximum principal stresses begin to depend on Young's modulus according to a power law:

\[
\sigma_1 \simeq \sigma_2 = A' \cdot (E)^{-2.5} \cdot \left(\frac{t}{R_{a}}\right)^{-2.2},
\]

\[
\sigma_3 = B' \cdot (E)^{-3.8} \cdot \left(\frac{t}{R_{a}}\right)^{-3.7},
\]

where \(A'\) and \(B'\) are the redefined coefficients.

The observed result occurs when the stiffness of the contact between the ball and the samples under friction conditions becomes comparable with the stiffness of the test sample. Thus, for most structural high-strength ceramics (\(E > 200\) GPa), this effect will not be observed. Secondly, the maximum principal stresses depend on the coefficient of friction between the sample and the ball. Dependence on the coefficient of friction:

\(E < 200\) GPa

\[
\sigma_1 \simeq \sigma_2 = A''' \cdot (a_1 \mu^2 + a_2 \mu + a_3) \cdot (E)^{-2.5} \cdot \left(\frac{t}{R_{a}}\right)^{-2.2},
\]

\(E > 200\) GPa

\[
\sigma_3 = B''' \cdot \left(\frac{t}{R_{a}}\right)^{-3.7},
\]
8

E > 200 GPa

3.2 Processing technique

3.2.1 The limits of applicability of method B3B. Based on the results of numerical modeling and experimental results, the limits of applicability of a new methodology for studying the bending strength of ceramics in conditions of complex stress-strain state were formulated. The limits of the methodology (based on the criteria) for this snap-in are given in table 1.

### Table 1. The limits of applicability of the method B3B

| Parameter | Value | Criteria |
|-----------|-------|----------|
| Roughness class | > 7 | ASTM C1161 [9] |
| \( R_a \), mm | 5.5; 9.25; 14.25 | Design constraints |
| \( R_{tp} \), mm | 2.5 | Steel balls |
| \( R \), mm | 12.0÷38 | Range SPS-simple |
| Young's modulus, GPa | \( < 200 \), affects | From research |
| | \( > 200 \), does not affect | |
| Poisson's ratio | does not affect | Fitting parameter |
| Parameter \( \frac{t}{R_a} \) | 0.06÷0.38 | Design constraints |

3.2.2 Functional dependence of the maximum principal stresses. In chapter 3, empirical dependences are obtained for the maximum principal stresses on the geometrical dimensions of the tooling (\( R_a \)) and the specimen (\( t \)) on the loading force (\( F \)), the dependences on the friction coefficient (\( \mu \)) between the balls and the specimen, and also studied the influence of Young’s modulus (\( E \)).

Generalization, the results of numerical modeling by the finite element method of the flat cylindrical specimen under complex SSS conditions with various system parameters made it possible to formulate expressions for calculating the maximum principal tensile stresses at \( E < 200 \) GPa:

\[
\sigma_1 = \frac{1}{3} \left[ \sigma_1 \mu^2 + \sigma_2 \mu + \sigma_3 \right] \cdot \left[ \sigma_4 \cdot R_a + \sigma_5 \right] \cdot \left[ \frac{F}{(\sigma_6 \cdot R_a + \sigma_7)} \right] \cdot \left[ ((E)^{2.5}) \cdot \left[ (t/R_a)^{-2.2} \right] \right]
\]

At a stress of \( E > 200 \) GPa:

\[
\sigma_1 = \frac{1}{3} \left[ \sigma_1 \mu^2 + \sigma_2 \mu + \sigma_3 \right] \cdot \left[ \sigma_4 \cdot R_a + \sigma_5 \right] \cdot \left[ \frac{F}{(\sigma_6 \cdot R_a + \sigma_7)} \right] \cdot \left[ ((E)^{3.9}) \cdot \left[ (t/R_a)^{-3.7} \right] \right]
\]

where \( \sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7 \) – coefficients.

4. Conclusions

In this work, we propose a method for processing experimental data for cylindrical ceramic samples tested according to the B3B test loading scheme using the finite element method. The result of the research was the functional dependence of the maximum principal tensile stresses on the geometric dimensions of the sample, balls, friction coefficients, Young's modulus, etc.

It is shown that the main stresses in the zone of maximum tensile stresses do not depend on the geometric parameters of the loading scheme and the dimensions of the sample. Studies of the effect of friction between the ball and the sample show the functional dependence of the stress values. It is established that when testing materials with an elastic modulus not exceeding the stiffness of contact
with friction between the sample and the ball, nonlinearity arises in the numerical solution. This result is extremely important for the processing of experimental results of testing samples with a small Young's modulus sintered SPS.

The obtained dependences of the principal stresses on the geometric and physical parameters of the system during the B3B test are the desired result of this work. The studied methodology for processing experimental data of the B3B test allows the experimenter to calculate the maximum principal stresses without using the finite element method.

Acknowledgements
This study was performed with the support of the Russian Science Foundation (Grant No. 18-73-10177).

References
[1] Olevsky E and Froyen L 2009 J. Am. Ceram. Soc. 92 [S1] 122
[2] Holland B, Dat V, Tran B and Mukherjee K 2012 Journal of the European Ceramic Society 32 3667
[3] Dudina D V, Bokhonov B B and Olevsky E A 2019 MDPI AG 12 541
[4] Chuvil'deev VN, Blagoveshchenskiy Yu V, Nokhrin A V, Boldin M S, Sakharov N V, Isaeva N V, Shotin S V, Belkin O A, Popov A A, Smirnova E S and E.A. Lantsev 2017 Journal of Alloys and Compounds 708 547
[5] Shafeiey A, Enayati M and Alhaji A 2018 Ceramics International 44 3536
[6] Romaric C and Sophie L 2017 Journal of Alloys and Compounds 692 1
[7] Wolfrum A, Quitzke C, Matthey B, Herrmann M and Michaelis A 2018 Wear 172 369
[8] GOST 24409-80 (2003). Ceramic electrotechnical materials. Test Methods Interstate Standard (in Russian)
[9] ASTM C1161 - 13. Standard Test Method for Flexural Strength of Advanced Ceramics at Ambient Temperature. – P. 19.
[10] Evans A and Langdon T 1980 Structural ceramics (Moscow: Metallurgy) P. 256 (in Russian)
[11] Fett T, Rizzi G, Ernst E, Muller R and Oberacker R 2007 Journal of the European Ceramic Society 27 1
[12] Raschea S, Stroblb S, Kunac M, Bermejoa R and Lubea T 2014 Procedia Materials Science 3 961
[13] Strobl S and Raschea S 2014 Journal of the European Ceramic Society 1 34
[14] Danzer R, Harrer W, Supancic P, Lubea T, Wang Z and Borger A 2007 Journal of the European Ceramic Society 27 1481
[15] Morrell R and McCormick N 1999 British Ceramic Transactions 5 234
[16] Serkan N 2011 Ceramics International 38 2411
[17] Borger A and Supancic P 2002 Journal of the European Ceramic Society 22 1425
[18] Borger A, Supancica P and Danzera R 2014 Journal of the European Ceramic Society 24 2917
[19] Zenkevich O 1975 The finite element method in technology (Moscow: Mir) P.543 (in Russian)