Multiple quantum collapse of the inflaton field and its implications on the birth of cosmic structure

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Abstract
The standard inflationary account for the origin of cosmic structure is, without a doubt, extremely successful. However, it is not fully satisfactory as has been argued in Perez \textit{et al} (2006 \textit{Class. Quantum Grav.} \textbf{23} 2317). The central point is that, in the standard accounts, the inhomogeneity and anisotropy of our universe seem to emerge, unexplained, from an exactly homogeneous and isotropic initial state through processes that do not break those symmetries. The proposal made there to address this shortcoming calls for a dynamical and self-induced quantum collapse of the original homogeneous and isotropic state of the inflaton. In this paper, we consider the possibility of a multiplicity of collapses in each one of the modes of the quantum field. As we will see, the results are sensitive to a more detailed characterization of the collapse than those studied in the previous works, and in this regard two simple options will be studied. We find important constraints on the model, most remarkably on the number of possible collapses for each mode.

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1. Introduction
Modern cosmology has become a very successful field of research in recent years. One of the major ideas, incorporated in the cosmological model, is the existence of a period of accelerating...
IR expansion early in the Universe’s history, called inflation. One of the major successes of inflationary cosmology is its ability to ‘account for’ the spectrum of the temperature anisotropies in the cosmic microwave background (CMB), which is understood as the earliest observational data about the primordial density fluctuations that seed the growth of structure in our Universe.

However, when considering this account in more detail, one immediately notes that there is something odd about it. Namely that out of an initial situation, which is taken to be perfectly isotropic and homogeneous, and based on a dynamics that supposedly preserves those symmetries, one ends with a non-homogeneous and non-isotropic situation.

The problem described above has been acknowledged by some cosmologists\(^5\)\[^1\] and even by some authors in recent textbooks \[^2–4\]. Nevertheless, several researchers in the field continue to hold the belief that the issues have been successfully resolved \[^5–8\]. For an extensive discussion about why the standard explanations do not solve this problem, we invite the reader to consult reference \[^9\].

In a recent series of works \[^9–16\], the problem has been analyzed leading to the conclusion that we need some new physics to be able to fully address the problem. The essential idea (as exposed in \[^9–16\]) is to introduce a new ingredient to the inflationary paradigm—the self-induced collapse hypothesis—a phenomenological model incorporating the description of the effects of a dynamical collapse of the wavefunction of the inflaton on the subsequent cosmological evolution. The idea is inspired by Diósi \[^17–19\] and Penrose’s arguments \[^20–23\] in the sense that the unification of quantum theory and the theory of gravitation would likely involve modifications in both theories, rather than only the latter as is more frequently assumed. Moreover, Penrose’s idea is that the resulting modifications of the former should involve something akin to a self-induced collapse of the wavefunction occurring when the matter fields are in a quantum superposition corresponding to spacetime geometries which are ‘too different from each other’. This sort of self-induced collapse would, in fact, be occurring in rather common situations, and would ultimately resolve the long-standing ‘measurement problem’ in quantum mechanics.

The collapse hypothesis in this context was originally inspired by Penrose’s ideas; however, it might be compatible with other collapse mechanisms which attempt to give a reasonable solution to the measurement problem. In essence, the collapse hypothesis simply sustains that something intrinsic to the system, i.e. independent of observers, induces the collapse or reduction of the quantum mechanical state of the system. Various proposals of that sort have been considered \[^24–28\], and might well be compatible with the self-induced collapse of the inflaton’s wavefunction that we are considering. However, we are not following any previous proposed scheme as the intention at this point is to learn what characteristics are needed for it to work in the present context. The point is that, in the case at hand, the collapse hypothesis can be tested and exposed through strictly empirical analyses.

The proposal is, at this stage of the analysis, a purely phenomenological scheme. It does not attempt to explain the process in terms of some specific new physical theory, but merely gives a rather general parametrization of the quantum transition involved. We will refer to this phenomenological model as the collapse scheme. We will not further recapitulate the motivations and discussion of the original proposal and instead refer the reader to the above-mentioned works.

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\(^5\) Sometimes this problem is formulated as the quantum-to-classical transition.

\(^2\)
Previous works along these lines have focused on the times of collapse and the natural basis for the collapse [14], and the issue of fine tuning of the inflaton potential in the collapse schemes [16]. However, so far the analysis has been based on the consideration of a single collapse of the inflaton’s wavefunction for each mode. That limitation of scope has allowed the investigation to proceed without the post-collapse state being characterized beyond the specification of the expectation values of the field and the conjugate momentum in the corresponding modes. The motivation of this paper is to extract more information about the collapse by considering the possibility that \textit{multiple collapses} occur in each mode, a consideration that requires a further specification of the post-collapse states; in particular, we are going to focus in models where the post-collapse states can be regarded as \textit{coherent} or \textit{squeezed} states.

The paper is organized as follows. In section 2, we briefly review the quantum mechanical treatment of the field’s fluctuations introducing the collapse hypothesis; we emphasize how the self-induced collapse proposal is contrasted with the observations and, additionally, we describe the three \textit{collapse schemes} that have been studied so far, namely independent, Newtonian and Wigner schemes. In section 3, we generalize the collapse hypothesis of section 2 to the case of multiple collapses. In section 4, we characterize the multiple post-collapse states and obtain new information about the parameters describing the post-collapse state. In section 5, we end with a discussion of the results obtained in the previous sections.

Regarding notation we will use signature \((- + + +\)) for the metric and Wald’s convention for the Riemann tensor. We will use units where \(c = 1\) but will keep the gravitational constant \(G\) and \(\hbar\) explicit throughout the paper.

2. The collapse model for the quantum fluctuations in the inflationary scenario

In this section, we review the formalism used in analyzing the collapse process. The full formalism and motivation are presented in [10–13]. We will use a semi-classical description of gravitation in interaction with quantum fields as reflected in the semi-classical Einstein’s equation

\[
G_{ab} = 8\pi G \langle \mathcal{T}_{ab} \rangle,
\]

whereas the other fields are treated in the standard quantum field theory (in curved spacetime) fashion. This is supposed to hold at all times except when a quantum gravity-induced collapse of the wavefunction occurs. At that point, one would have to assume that the excitation of the fundamental quantum gravitational degrees of freedom must be taken into account, with the corresponding breakdown of the semiclassical approximation (the possible breakdown of the semi-classical approximation is formally represented by the presence of a term \(Q_{ab}\) on the left-hand side of the semi-classical Einstein’s equation which is supposed to become nonzero only during the collapse of the quantum mechanical wavefunction of the matter fields, see [10] for the detailed discussion).

The starting point is the action of a scalar field minimally coupled to gravity

\[
S[\phi] = \int \! d^4 x \sqrt{-g} \left[ \frac{1}{16\pi G} R[g_{ab}] - \frac{1}{2} g^{ab} \nabla_a \phi \nabla_b \phi - V[\phi] \right].
\]  

(1)

One then splits the corresponding fields into their homogeneous part and the perturbations. Thus, the metric and the scalar fields are written as \(g = g_0 + \delta g\) and \(\phi = \phi_0 + \delta \phi\), respectively.
With the appropriate choice of gauge⁶ (we will work with the longitudinal gauge also referred to as the Newtonian gauge) and ignoring the vector and tensor part of the metric perturbations, the spacetime metric can then be described by the line element

\[ ds^2 = a(\eta)^2 \left[ -(1 + 2\Psi(\eta, x)) d\eta^2 + (1 - 2\Psi(\eta, x)) \delta_{ij} dx^i dx^j \right]. \tag{2} \]

where \( \Psi(\eta, x) \) is referred to as the Newtonian potential.

The inflationary regime is characterized by a scale factor \( a(\eta) \approx -1/[H(1 - \epsilon)\eta] \), with \( H^2 \approx 8\pi G V / 3 \) (which is Friedmann’s equation) and \( \epsilon \approx \frac{1}{2} (M_\text{P}^2 / \hbar) (\partial_\phi V / V)^2 \) the slow-roll parameter (which during inflation \( \epsilon \ll 1 \)). \( M_\text{P} \) is the reduced Planck mass \( M_\text{P}^2 \approx \hbar / (8\pi G) \).

The normalization of the scale factor will be set so \( a = 1 \) at the ‘present cosmological time’. The inflationary regime would end at \( \eta = \eta_\text{r} \), a value which is negative and very small in absolute terms \( (\eta_\text{r} \approx -10^{-22} \text{ Mpc}) \). That is, the conformal time \( \eta \) during the inflationary era is in the range \( -\infty < \eta < \eta_\text{r} \); thus, \( \eta = 0 \) is a particular value of the conformal time that does not correspond to the inflationary period; in fact, it belongs to the radiation dominated epoch.

The background scalar field \( \phi_0 \) will be considered in the slow-roll regime, i.e. \( \phi_0'' = -(a^3 / 3a') \partial_\phi V \), where the primes denotes \( \Gamma' \equiv d/d\eta \).

Combining the background equations with Einstein’s equations to first order in the perturbations, we obtain

\[ \nabla^2 \psi + \mu \psi = 4\pi G (a \phi_0 + \phi_0' \phi_0'), \tag{3} \]

where \( \mu \equiv \mathcal{H}^2 - \mathcal{H}' \), \( u \equiv 3\mathcal{H} \phi_0'' + a^2 \partial_\phi V [\phi] \) and \( \mathcal{H} \equiv a'(\eta) / a(\eta) \). If one uses the expressions for the scale factor during a de Sitter phase, then \( \mu = 0 \), while the slow-rolling approximation \( \phi_0'' = -a^2 \partial_\phi V / 3(\mathcal{H}) \) corresponds to the condition \( u = 0 \). Under those simplifying conditions, the last equation becomes a Poisson-like equation

\[ \nabla^2 \psi = 4\pi G \phi_0' \phi_0' \equiv s \phi_0', \tag{4} \]

with \( s \equiv 4\pi G \phi_0' \), which can be rewritten, by using the slow-roll parameter, the background equation for \( \phi_0' \) in the slow-roll regime and Friedmann’s equation, as \( s \equiv a h \sqrt{\mathcal{V}'} / (\sqrt{6} M_\text{P}^2) \).

The next step involves the quantization of the field fluctuation. We emphasize that the background field \( \phi_0 \) is described in a classical⁷ fashion and it is only the fluctuation \( \delta \phi \) which is subjected to a quantum treatment.

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⁶ Although the equations in this gauge are formally identical to the gauge-independent equations [29], the analysis done here requires the choosing of a specific gauge. One cannot work with the so-called gauge invariant combinations, because in the approach followed here, the metric and field fluctuations are treated on a different footing. The metric is considered a classical variable (taken to be describing, in an effective manner, the deeper fundamental degrees of freedom of the quantum gravity theory that one envisions, lies underneath), while the matter fields, specifically the inflation field perturbations, are given a standard quantum field (in curved spacetime) treatment, with the two connected through the semiclassical Einstein’s equations. The choice of gauge implies that the time coordinate is attached to some specific slicing of the perturbed spacetime, and thus, our identification of the corresponding hypersurfaces (those of constant time) as the ones associated with the occurrence of collapses—something deemed as an actual physical change—turns what is normally a simple choice of gauge into a choice of the distinguished hypersurfaces, tied to the putative physical process behind the collapse. This naturally leads to tensions with the expected general covariance of a fundamental theory, a problem that affects all known collapse models, and which in the non-gravitational settings becomes the issue of compatibility with Lorentz or Poincaré invariance of the proposals. We must acknowledge that this generic problem of collapse models is an open issue for the present approach. One would expect that its resolution would be tied to the uncovering of the actual physics behind what we treat here as the collapse of the wavefunction (which we view as a merely an effective description). As has been argued in related works, and in following ideas originally exposed by Penrose [20–23], we hold that the physics that lies behind all this ties the quantum treatment of gravitation with the foundational issues afflicting quantum theory in general, and in particular those with connection to the ‘measurement problem’.

⁷ By classical, in this context, we mean that the homogeneous background field \( \phi_0(\eta) \) is taken as an approximated description of the quantum quantity \( \langle \psi| \hat{\phi}(x, \eta) |\psi \rangle \), where the state \( |\psi \rangle \) is the vacuum state of \( \delta \phi(x, \eta) \).
Actually, it is convenient to work with the auxiliary field $y = a \delta \varphi$. The equation of motion for this field is

$$y'' - \left( \nabla^2 + \frac{a''}{a} \right) y = 0.$$  \tag{5}$$

The conjugated canonical momentum of $y$ is $\pi = y' - ya'/a$. In order to avoid infrared problems, we will consider a restriction of the system to a box of side $L$, with periodic boundary conditions. The field and its momentum can be decomposed in Fourier’s modes as

$$\hat{y}(\eta, x) = \frac{1}{L^3} \sum_k e^{ikx} \hat{y}_k(\eta), \quad \hat{\pi}(\eta, x) = \frac{1}{L^3} \sum_k e^{ikx} \hat{\pi}_k(\eta),$$  \tag{6}$$

with the wave vectors satisfying $k_i L = 2\pi n_i$ for $i = 1, 2, 3$. The field operator coefficients are further written as $\hat{y}_k(\eta) = y_k(\eta) \hat{\alpha}_k + \overline{y}_k(\eta) \hat{\alpha}_k^\dagger$ and $\hat{\pi}_k(\eta) = g_k(\eta) \hat{\alpha}_k + \overline{g}_k(\eta) \hat{\alpha}_k^\dagger$. The functions $y_k(\eta)$ and $g_k(\eta)$ reflect the election of the vacuum state. In our case, as is customarily done in the literature, we choose the so-called Bunch–Davies vacuum [30], resulting from this choice,

$$y_k(\eta) = \frac{1}{\sqrt{2k}} \left( 1 - \frac{i}{\eta k} \right) \exp(-i k \eta), \quad g_k(\eta) = -i \frac{\sqrt{L}}{2} \exp(-i k \eta).$$  \tag{7}$$

The vacuum state is defined by the condition $\hat{\alpha}_k |0\rangle = 0$ for all $k$, and can be easily seen to be homogeneous and isotropic at all scales. The self-collapse is assumed to occur, independently, for each mode of the field. That is, one assumes that at a certain time $\eta c$ (from now on we will refer to this particular time as the time of collapse) the state of each mode $k$ of the field, which was initially the vacuum, changes spontaneously into another state. This self-collapse of the wavefunction is inspired by Penrose’s ideas [20–23], in which gravity plays a fundamental role on the collapse of the wavefunction and it does not require outside observers who perform a measurement in order to induce the collapse. The collapse scheme as employed here, however, does not propose at this point a concrete physical mechanism behind it, although one envisions that a more profound theory, presumably derived from quantum gravity, will eventually account for it. These ideas and motivations are discussed in detail in [10–13]. In order to study the possibility of multiple collapses, we will see that more detailed specifications of the states after the collapse are needed in contrast with the works [10–13].

Following [10], it is convenient to decompose the field $\hat{y}_k$ and its conjugated momentum $\hat{\pi}_k$ in their real and imaginary parts which are completely Hermitian $\hat{y}_k(\eta) = \hat{y}^R_k(\eta) + i \hat{y}^I_k(\eta)$ and $\hat{\pi}_k(\eta) = \hat{\pi}^R_k(\eta) + i \hat{\pi}^I_k(\eta)$ where

$$\hat{y}^R_k(\eta) = \frac{1}{\sqrt{2}} \left( y_k(\eta) \hat{\alpha}_k + \overline{y}_k(\eta) \hat{\alpha}_k^\dagger \right),$$  \tag{8}$$

$$\hat{\pi}^R_k(\eta) = \frac{1}{\sqrt{2}} \left( g_k(\eta) \hat{\alpha}_k + \overline{g}_k(\eta) \hat{\alpha}_k^\dagger \right),$$  \tag{9}$$

here

$$\hat{\alpha}_k^R = \frac{1}{\sqrt{2}} (\hat{\alpha}_k + \hat{\alpha}_{-k}), \quad \hat{\alpha}_k^I = -\frac{i}{\sqrt{2}} (\hat{\alpha}_k - \hat{\alpha}_{-k}).$$  \tag{10}$$

The commutators of the real and imaginary annihilation and creation operators are

$$\left[ \hat{\alpha}_k^R, \hat{\alpha}_k^R \right] = \hbar L^3 (\delta_{kk'} + \delta_{kk' - k}), \quad \left[ \hat{\alpha}_k^I, \hat{\alpha}_k^I \right] = \hbar L^3 (\delta_{kk'} - \delta_{kk' - k}).$$  \tag{11}$$

A full characterization of the state of each mode of the field would require the specification all statistical moments. In previous works [10, 14, 16], the collapse has been characterized only...
in terms of the expectation values of field and of the momentum conjugate for the new quantum state. However, in this work, as we are assuming the possibility of multiple collapses, we will need to focus on the first two statistical moments: the expectation value and the uncertainties (see section 3).

For any state $|\Xi\rangle$ of the field $\hat{y}$, we introduce the following quantities:
\[
d_k^{(R, I)} = \langle \hat{d}_k^{(R, I)} \rangle_{\Xi}, \quad c_k^{(R, I)} = \langle (\hat{d}_k^{(R, I)})^2 \rangle_{\Xi}, \quad e_k^{(R, I)} = \langle \hat{d}_k^{(R, I)} \hat{d}^*_k^{(R, I)} \rangle_{\Xi}.
\] (12)

The expectation values of the field modes can be written as
\[
\langle \hat{y}_k^{(R, I)}(\eta) \rangle_{\Xi} = \sqrt{2 \hbar} (y_k(\eta) a_k^{(R, I)}), \quad \langle \hat{y}^*_k^{(R, I)}(\eta) \rangle_{\Xi} = \sqrt{2 \hbar} (g_k(\eta) c_k^{(R, I)}),
\] (13)

while their uncertainties are
\[
\langle \Delta \hat{y}_k^{(R, I)}(\eta) \rangle_{\Xi}^2 = 3 \hbar (y_k^2(\eta) c_k^{(R, I)}) + \frac{1}{2} |y_k(\eta)|^2 (\hbar L^3 + 2 e_k^{(R, I)}) - 2 \left[ \hbar (y_k(\eta) a_k^{(R, I)}) \right]^2,
\] (14)

\[
\langle \Delta \hat{y}^*_k^{(R, I)}(\eta) \rangle_{\Xi}^2 = 3 \hbar (g_k^2(\eta) c_k^{(R, I)}) + \frac{1}{2} |g_k(\eta)|^2 (\hbar L^3 + 2 e_k^{(R, I)}) - 2 \left[ \hbar (g_k(\eta) d_k^{(R, I)}) \right]^2;
\] (15)

specifically for the vacuum state $|0\rangle$, one has, as expected, $d_k^{(R, I)} = c_k^{(R, I)} = 0$, and thus $\langle \hat{y}_k^{(R, I)}(\eta) \rangle_{|0\rangle} = 0$, $\langle \hat{y}^*_k^{(R, I)}(\eta) \rangle_{|0\rangle} = 0$, and their corresponding uncertainties
\[
\langle \Delta \hat{y}_k^{(R, I)}(\eta) \rangle_{|0\rangle}^2 = \frac{1}{2} |y_k(\eta)|^2 \hbar L^3, \quad \langle \Delta \hat{y}^*_k^{(R, I)}(\eta) \rangle_{|0\rangle}^2 = \frac{1}{2} |g_k(\eta)|^2 \hbar L^3.
\] (16)

Once we specify the expectation value of the field’s modes $\hat{y}_k^{(R, I)}$ and $\hat{y}^*_k^{(R, I)}$ in the post-collapse state $|\Xi\rangle$ at the time of collapse $\eta^c_k (|0\rangle \rightarrow |\Xi\rangle)$,
\[
\langle \hat{y}_k^{(R, I)}(\eta) \rangle_{\Xi} = (\Xi |\hat{y}_k^{(R, I)}(\eta) \rangle_{\Xi}), \quad \langle \hat{y}^*_k^{(R, I)}(\eta) \rangle_{\Xi} = (\Xi |\hat{y}^*_k^{(R, I)}(\eta) \rangle_{\Xi}),
\] (17)

we can obtain the expectation values evolved at any time after the collapse, provided that there is no additional collapse. In fact, by comparing (17) with (13), we obtain
\[
\langle \hat{y}_k^{(R, I)}(\eta) \rangle_{\Xi} = A(\eta, \eta^c_k) \langle \hat{y}_k^{(R, I)}(\eta) \rangle_{\Xi} + B(\eta, \eta^c_k) \langle \hat{y}^*_k^{(R, I)}(\eta) \rangle_{\Xi},
\] (18a)

\[
\langle \hat{y}^*_k^{(R, I)}(\eta) \rangle_{\Xi} = C(\eta, \eta^c_k) \langle \hat{y}_k^{(R, I)}(\eta) \rangle_{\Xi} + D(\eta, \eta^c_k) \langle \hat{y}^*_k^{(R, I)}(\eta) \rangle_{\Xi},
\] (18b)

where $A$, $B$, $C$, and $D$ are the time-dependent functions which describe the temporal evolution of the quantum system between $\eta^c_k$ and $\eta$. In particular, in the inflationary stage, these functions are

\[
A(\eta, \eta^c_k) = \cos (k \eta - k \eta^c_k) + \frac{\sin (k \eta - k \eta^c_k)}{k \eta^c_k},
\] (19a)

\[
B(\eta, \eta^c_k) = -k \sin (k \eta - k \eta^c_k),
\] (19b)

\[
C(\eta, \eta^c_k) = \frac{\cos (k \eta - k \eta^c_k)}{k} \left( \frac{1}{k \eta^c_k} - \frac{1}{k \eta} \right) + \frac{\sin (k \eta - k \eta^c_k)}{k} \left( \frac{1}{k^2 \eta^c_k} + 1 \right),
\] (19c)

\[
D(\eta, \eta^c_k) = \cos (k \eta - k \eta^c_k) - \frac{\sin (k \eta - k \eta^c_k)}{k \eta^c_k}.
\] (19d)

Equations (18a) and (18b) can be rewritten in the matrix form
\[
\Upsilon(\eta, \Xi) = U(\eta, \eta^c_k) \Upsilon(\eta^c_k, \Xi),
\] (20)

where
\[
\Upsilon(\eta, \Xi) = \begin{pmatrix} \langle \hat{y}_k^{(R, I)}(\eta) \rangle_{\Xi} \\ \langle \hat{y}^*_k^{(R, I)}(\eta) \rangle_{\Xi} \end{pmatrix},
\] (21)
\[
U(\eta, \eta_c^k) = \begin{pmatrix} A(\eta, \eta_c^k) & B(\eta, \eta_c^k) \\ C(\eta, \eta_c^k) & D(\eta, \eta_c^k) \end{pmatrix},
\]

(22)

\[
\Upsilon(\eta_c^k, \Xi) \equiv \left\langle \frac{\hat{\pi}(R, I, \eta_c^k)}{\Xi_1} \right\rangle /\left\langle \frac{\hat{y}(R, I, \eta_c^k)}{\Xi_1} \right\rangle.
\]

(23)

In this notation, it is clear that the matrix \(U(\eta, \eta_c^k)\) represents the standard unitary evolution (this refers to the standard quantum mechanical evolution of states or operators as it might be the case, and should not be taken to mean that the matrix \(U(\eta, \eta_c^k)\) is unitary. It is not, and there is no reason for it to be so), for the expectation value of the fields, from the time \(\eta_c^k\) to the arbitrary time \(\eta\).

The evolution of the uncertainties \(\left(\Delta \hat{\pi}_k^{(R,I)}(\eta)\right)_Z^2\) and \(\left(\Delta \hat{\pi}_k^{(R,I)}(\eta)\right)_Z^2\) depends on the specific post-collapse state. In particular, the quantities \(c_k^{(R,I)}\) and \(e_k^{(R,I)}\) depend on the state after the collapse. That is, once we specify the post-collapse state (and thus the quantities \(c_k^{(R,I)}\) and \(e_k^{(R,I)}\) are fixed), we can use (14) and (15) to obtain the evolution of the uncertainties.

2.1. Connection to observations

In order to connect the predicted quantities with the observed ones, we start from (4),

\[\nabla^2 \Psi(\eta, x) = s \delta \phi'(\eta, x).\]

For the mode \(\Psi_k\), after a Fourier’s decomposition, we obtain

\[\Psi_k(\eta) = -\frac{s}{k^2} \delta \phi'_k(\eta).\]

(24)

After describing the parametrization of the collapse in the previous section, we proceed to evaluate the perturbed metric using the semi-classical Einstein’s field equations: \(G_{ab} = 8\pi G \langle \hat{T}_{ab} \rangle\) which we described at the beginning of this section. To the lowest order, this set of equations reduces to

\[\Psi_k(\eta) = -\frac{s}{a(\eta)} \langle \hat{\pi}_k(\eta) \rangle,\]

(25)

where we used that \(\langle \delta \phi'_k \rangle_Z\) is connected to the expectation value of the momentum field by \(\langle \delta \phi'_k \rangle_Z = \langle \hat{\pi}_k \rangle_Z / a(\eta)\) on the state \(|\Xi\rangle\).

Recalling that \(s \equiv a\hbar \sqrt{V\epsilon} / \sqrt{6M_p^2}\), the expression for the Newtonian potential is

\[\Psi_k(\eta) = -\frac{\hbar}{k^2M_p^2} \sqrt{\frac{V\epsilon}{6}} \langle \hat{\pi}_k(\eta) \rangle.\]

(26)

We note that before the collapse occurs, the state of the field is the Bunch–Davies vacuum for which \(\langle \hat{\pi}_k(\eta) \rangle = 0\), consequently \(\Psi_k(\eta) = 0\), and the spacetime is homogeneous and isotropic (at that scale). However, after the collapse takes place, the new state will generically have \(\langle \hat{\pi}_k(\eta) \rangle \neq 0\) and the gravitational perturbations appear. That is, the onset of the inhomogeneity and anisotropy at each scale is associated with the first collapse of the corresponding mode.

In order to obtain a theoretical prediction and contrast it with the observations, we strictly cannot use the expression of \(\Psi_k(\eta)\) as given in (26) because it was obtained using the slow-roll approximation which is only valid in the inflationary epoch, while the observations made today by our satellites depend on the Newtonian potential at the last scattering surface. That is, the observations rely on \(\Psi(\eta_D, x_D)\), with \(\eta_D\) the time of decoupling and
where $R_D$ is the radius of the last scattering surface, and $\theta, \phi$ are the standard spherical coordinates in the sky.

The conformal time of decoupling lies in the matter-dominated epoch. Nevertheless, we will work with the expression for $\Psi_k(\eta)$ in the radiation-dominated era, extending if one wants its range of validity which is from $\eta$ to $\eta_{eq}$ (where $\eta_{eq}$ is the conformal time of the radiation-matter equality epoch). The changes during the brief period from the start of ‘matter domination’ to ‘decoupling’ (where the scale factor changes only by a factor of 3, i.e. $a(\eta_D)/a(\eta_{eq}) \approx 3$) are naturally considered to be irrelevant for the issues concerning us here, and thus the approximated value for the quantities of observational interest obtained using $\Psi(\eta)$ in the radiation-dominated regime should be a very good approximation for the exact value of these quantities. Therefore, our goal here is to obtain an estimate for $\Psi_k(\eta)$ during the radiation epoch.

The analysis can be simplified by working with a quantity whose evolution is rather simple, the so-called intrinsic curvature perturbation $\zeta$ [31–33], which is defined as

$$\zeta \equiv \frac{2}{3(w + 1)} (H^{-1} \Psi' + \Psi) + \Psi,$$ (27)

where $w \equiv P/\rho$. During the inflationary regime, $P = -\frac{1}{2} g^{ab} \partial_a \phi \partial_b \phi - V$ represents the ‘pressure’ of the scalar field and $\rho = -\frac{1}{2} g^{ab} \partial_a \phi \partial_b \phi + V$ the energy density.

It is a known result [2, 34] that $\zeta$ is, for modes larger than the Hubble radius (commonly referred to as modes ‘larger than the horizon’, i.e. modes with $k \ll \mathcal{H}$) and for ‘adiabatic perturbations’, roughly a ‘constant quantity’, irrespective of the cosmological regime and the nature of the dominant kind of matter. The constancy of this quantity is used to obtain a relation between the values of the Newtonian potential during the two relevant regimes $\Psi_{k}^{\text{inf}}(\eta)$ and $\Psi_{k}^{\text{rad}}(\eta)$,

$$\zeta^{\text{inf}} = \zeta^{\text{rad}} \Rightarrow \Psi_k^{\text{inf}} \left[ \frac{2}{3} \left( \frac{1}{w_{\text{inf}} + 1} \right) + 1 \right] = \frac{3}{2} \Psi_k^{\text{rad}},$$ (28)

where, in obtaining the right-hand side of (28), the use of the equation of state $P = \rho/3$ was made, and the left-hand side was obtained using the equation of state $P = w_{\text{inf}} \rho$ where $w_{\text{inf}} + 1 = \phi_0^2/a^2 \rho$. Finally, by relying on the assumption of validity of the slow-roll approximation during inflation, $\phi_0^2/a^2 = \frac{3}{2} V \epsilon$, (28) becomes

$$\Psi_k^{\text{rad}} = \frac{2}{3} \Psi_k^{\text{inf}} \epsilon.$$ (29)

Thus, substituting (26) into (29), the expression for the Newtonian potential, in the radiation-dominated epoch, becomes

$$\Psi_k^{\text{rad}}(\eta) = \frac{-\hbar}{2M_p^2} \sqrt{\frac{8V}{27\epsilon}} \frac{\langle \delta_k(\eta) \rangle}{k^2}.$$ (30)

The expression above is valid for modes with $k \ll \mathcal{H}$, which are actually the modes of interest from the observational point of view. That is, we need to consider that $k/\mathcal{H} \ll 1$ in $\langle \delta_k(\eta) \rangle$. Furthermore, result (30) shows that for a generic collapse scheme, there is an amplification $1/\epsilon$ in the Newtonian potential, in accordance with the generic findings of the detailed study for the collapse scheme presented in [16].

In order to connect with the observations we note that the quantity that is observed is $\frac{\delta_k}{T}(\theta, \phi)$, which is expressed in terms of its spherical harmonic decomposition $\sum \alpha_{lm} Y_{lm}(\theta, \phi)$. The theoretical calculations make a prediction for the most likely value of
the coefficients $\alpha_{lm}$ which are expressed in terms of the Newtonian potential on the 2-sphere corresponding to the intersection of our past light cone with the surface of last scattering

$$\alpha_{lm} = \int d^2 \Omega \Psi(\eta_D, x_D) Y_{lm}(\theta, \phi).$$

(31)

After a Fourier decomposition of the Newtonian potential

$$\Psi_1(\eta_D, x_D) = \sum_k (\Psi_k(\eta_D)/L^3) e^{ikx_D},$$

and using (30), we obtain

$$\alpha_{lm} = \int d^2 \Omega \sum_k -\hbar \frac{8V}{27\epsilon} \langle \hat{\pi}_k(\eta_D) \rangle Y_{lm}^*(\theta, \phi) e^{ikx_D}.$$  

(32)

Using the standard spherical harmonic relations

$$e^{ikx_D} = 4\pi \sum_{iljl}(kRD) Y_{lm}(\theta, \phi) Y_{lm}^*(\hat{k}),$$

where $jl$ are spherical Bessel functions, we obtain

$$\alpha_{lm} = 4\pi \sum_k -\hbar \frac{8V}{27\epsilon} \langle \hat{\pi}_k(\eta_D) \rangle j_l(kRD) Y_{lm}^*(\hat{k}).$$

(33)

The quantity $\alpha_{lm}$ is the sum of contributions from the collection of modes, each contribution being a complex number, leading to what is in effect a sort of 'two-dimensional random walk' whose total displacement corresponds to the observational quantity (this will be seen more clearly in the next section when we specify $\langle \hat{\pi}_k(\eta_c) \rangle$). It is clear that, as in the case of any random walk, such a quantity cannot be evaluated and the only thing that can be done is to evaluate the most likely value for such total displacement, with the expectation that the observed quantity will be close to that value. As is now standard in our treatments, we do this with the help of the imaginary ensemble of universes and the identification of the most likely value with the ensemble’s mean value

$$|\alpha_{lm}|^2_{M.L.} = \left(\frac{4\pi}{L^d}\right)^2 \sum_{kk'} \frac{2\hbar^2 V}{27\epsilon M_p^d k^2 k'^2} \langle \hat{\pi}_k(\eta_D) \rangle \langle \hat{\pi}_{k'}(\eta_D) \rangle j_l(kRD) j_l(k'R_D) Y_{lm}^*(\hat{k}) Y_{lm}^*(\hat{k'}).$$

(34)

The rest of the present work focuses on obtaining the quantity $\langle \hat{\pi}_k(\eta) \rangle \langle \hat{\pi}_k^*(\eta) \rangle$ under specific conditions on the post-collapses states.

An important observation follows directly from the point of view adopted to relate the metric effective description of gravity to the quantum aspect of the matter fields: the source of the fluctuations that lead to anisotropies and inhomogeneities lies in the quantum uncertainties for the scalar field, which collapses, due to some unknown quantum gravitational effect. Once collapsed, these density inhomogeneities and anisotropies feed into the gravitational degrees of freedom leading to nontrivial perturbations in the metric functions, in particular, the Newtonian potential. However, the metric itself is not a source of the quantum gravitational-induced collapse. Therefore, as the scalar field does not act as a source for the gravitational tensor modes—at least not at the lowest order considered here—the tensor modes cannot be excited. Thus, as already discussed in [10, 11], the scheme naturally leads to the prediction of a zero—or at least a strongly suppressed—amplitude of gravitational waves to the CMB.

8 However, it is worthwhile pointing out that such a conclusion is directly tied to our underlying approach that favors the semi-classical Einstein’s equations augmented with a collapse proposal as a way to deal with the gravity quantum interface faced in the current problem. It is of course conceivable, although seems harder to understand in a wider context (see the discussion in section 8 of [9]), that a collapse might be incorporated into a setting where both the gravitation and scalar filed perturbations are simultaneously treated at the quantum level. If the latter happened to be the correct approach, something that would be possible to ascertain when we have a fully satisfactory theory of quantum gravity, our conclusion about the tensor modes would be modified.
2.2. Quantum collapse schemes

In order to proceed, we must specify the quantum collapse scheme which drives the inflaton field out of homogeneity and isotropy. In past works [10, 14], three different schemes were considered. Two of them, called independent collapse and Newtonian collapse, were presented in [10] and the last one, denominated Wigner’s collapse, was presented in [14]. In [14], these schemes are further studied but limiting the consideration to a single collapse. In the following, we describe them briefly.

2.2.1. Independent collapse scheme. In this scheme, one assumes that the expectation values of the field’s mode $\hat{y}(R,I)^{(R,I)}$, and their conjugate momentum $\hat{\pi}(R,I)^{(R,I)}$, acquire ‘random’ independent values. The expectation value was considered as randomly selected

$$\langle \hat{y}(R,I)^{(R,I)}(\eta_c^k) \rangle_{\xi_1} = x(R,I)^{k,II} \sqrt{\Delta_{\hat{y}}(R,I)^{(R,I)}(\eta_c^k)}_0,$$

$$\langle \hat{\pi}(R,I)^{(R,I)}(\eta_c^k) \rangle_{\xi_1} = x(R,I)^{k,II} \sqrt{\Delta_{\hat{\pi}}(R,I)^{(R,I)}(\eta_c^k)}_0.$$

(35)

In this scheme, the expectation value jumps to a random value $x(R,I)^{(R,I)}$ multiplied by the uncertainty of the vacuum state of the field. The random variables $x(R,I)^{k,II}$ are selected from a Gaussian distribution centered at zero, of spread one (normalized), and are statistically uncorrelated, that is the rationale of the name. This means that we are ignoring the natural correlation that exists in the conjugate fields in the pre-collapse state.

2.2.2. Newtonian collapse scheme. This scheme is motivated by the observation that in the Poisson-like equation (26), only the expectation value of $\hat{\pi}(R,I)^{(R,I)}$ appears. Thus, following Penrose’s ideas regarding the quantum uncertainties that the gravitational potential would be inheriting from the matter fields’ quantum uncertainties, as fundamental factors triggering the collapse, one is led to consider a scheme where ‘only $\hat{\pi}(R,I)^{(R,I)}$ collapses’, leaving the expectation value of $\hat{y}(R,I)^{(R,I)}$ unchanged,

$$\langle \hat{y}(R,I)^{(R,I)}(\eta_c^k) \rangle_{\xi_1} = 0, \quad \langle \hat{\pi}(R,I)^{(R,I)}(\eta_c^k) \rangle_{\xi_1} = x(R,I)^{k,II} \sqrt{\Delta_{\hat{\pi}}(R,I)^{(R,I)}(\eta_c^k)}_0.$$

(36)

As before, $x(R,I)^{k,II}$ represents a random Gaussian variable normalized and centered at zero.

2.2.3. Wigner’s collapse scheme. The last collapse scheme considered in [14, 15] attempts to take into account the correlation between $\hat{y}(R,I)^{(R,I)}$ and $\hat{\pi}(R,I)^{(R,I)}$ existing in the pre-collapse state, and to characterize it in terms of the Wigner function. The Wigner function of the vacuum state of the inflaton is a bi-dimensional Gaussian function. This fact will be used to model the resulting collapse of the quantum field state. The assumption will be that, at a certain (conformal) time $\eta_c^k$, the part of the state characterizing the mode $k$ will collapse, leading to a new state in which the fields will have the expectation values given by

$$\langle \hat{y}(R,I)^{(R,I)}(\eta_c^k) \rangle_{\xi_1} = x(R,I)^{k,II} \Lambda_k \cos \Theta_k, \quad \langle \hat{\pi}(R,I)^{(R,I)}(\eta_c^k) \rangle_{\xi_1} = x(R,I)^{k,II} \Lambda_k \sin \Theta_k,$$

(37)

where $x(R,I)^{k,II}$ is a random variable, characterized by a Gaussian distribution centered at zero with a spread one. $\Lambda_k$ is given by the major semi-axis of the ellipse characterizing the bi-dimensional Gaussian function (the ellipse corresponds to the boundary of the region in ‘phase space’ where the Wigner function has a magnitude larger than 1/2 its maximum value), and $\Theta_k$ is the angle between that axis and the $\hat{y}(R,I)^{(R,I)}$ axis. The quantities $\Lambda_k$ and $\Theta_k$ can be expressed in terms of $\eta_c^k$ [14] as

$$\Lambda_k = \frac{4\eta_c^k \sqrt{hL^3}}{\sqrt{1 + 5(k\eta_c^k)^2} - \sqrt{1 + 10(k\eta_c^k)^2} + 9(k\eta_c^k)^4}}.$$  

(38)
\[ 2\theta_k = \arctan \left( \frac{4k\eta_k^c}{1 - 3(k\eta_k^c)^2} \right). \]  

### 3. Multiple quantum collapses

Once one hypothesizes that there is a new kind of physical process which affects the system under investigation, it seems logical to consider the possibility that it occurs more than once and in circumstances different from those for which it was first proposed. The extensive study of that issue is well beyond the present paper and would require its merger with other studies of collapse models for more general circumstances. However, the cosmological situation is one where further analysis can be done with relative ease and where it is natural to assume that the effect must manifest in a rather unmodified version in all its occurrences. This work can be considered as the first exploration (see also [15]) of the effects of multiple collapses in the situation which lead to the first proposals suggesting that they play a fundamental role in cosmology.

#### 3.1. Collapse scheme relations for multiple collapses

We analyze the provided schemes in the case of multiple quantum collapses by focusing on the expectation values of the relevant quantities at any time after exactly \( n \) quantum collapses.

First, we generalize the notation in order to handle in a unified fashion the three different quantum collapse schemes (developed in [10, 14]). Let us assume that at time \( \eta_{c1}^k \), a single collapse has occurred taking the state \( |c_0 \rangle \) to the state \( |c_1 \rangle \).

Then, a natural generalization of the expectation value of the field (and its conjugated momentum) in the post-collapse state \( |c_1 \rangle \) is assumed to be given by

\[ \langle \hat{y}(R,I)_{k \eta_{c1}^k} \rangle_{c_1} = x_{(1)(R,I)}^{k,I} \sigma_{(1)}^{y}(\eta_{c1}^k, c_0) + \langle \hat{y}(R,I)_{k \eta_{c1}^k} \rangle_{c_0}, \]  

\[ \langle \hat{\pi}(R,I)_{k \eta_{c1}^k} \rangle_{c_1} = x_{(1)(R,I)}^{k,II} \sigma_{(1)}^{\pi}(\eta_{c1}^k, c_0) + \langle \hat{\pi}(R,I)_{k \eta_{c1}^k} \rangle_{c_0}, \]  

where \( x_{(1)(R,I)}^{k,J} \) and \( x_{(1)(R,I)}^{k,II} \) stand for a random value characterizing the change in the expectation value of \( \hat{y}^{(R,I)}_{k \eta_{c1}^k} \) and \( \hat{\pi}^{(R,I)}_{k \eta_{c1}^k} \), respectively. The superscript (1) indicates that the random variables are associated with the first collapse, while the quantities in the last term of the right-hand side of (40) and (41) represent the value of the corresponding operators if there had been no collapse. The correlation between these random variables depends on the particular collapse scheme. The functions \( \sigma_{(1)}^{y}(\eta_{c1}^k, c_0) \) and \( \sigma_{(1)}^{\pi}(\eta_{c1}^k, c_0) \) denote the uncertainties of the expectation values of the fields for the particular collapse scheme considered. The notation employed reminds us the principal quantities that characterize the expectation values. That is, it depends on the previous collapse state \( |c_0 \rangle \) (which in the case of a single collapse is the vacuum state), the time of collapse \( \eta_{c1}^k \), and the random variables \( x_{(1)(R,I)}^{k,I} \) and \( x_{(1)(R,I)}^{k,II} \).

Note that the left-hand side (lhs) of (40) and (41) is in the post-collapse state \( |c_1 \rangle \), while the right-hand side (rhs) is in the pre-collapse state \( |c_0 \rangle \), i.e. the new state depends on the old state. Let us also note that the whole expressions (40) and (41) are evaluated in \( \eta_{c1}^k \), the time at which the first collapse occurs. The second term on the rhs is the expectation value of the mode in the state \( |c_0 \rangle \) evolved up to \( \eta_{c1}^k \). This dynamical evolution is dictated by (20).

9 In this section, we have changed the notation slightly, the post-collapse state will be denoted by \( |c_n \rangle \) instead of \( |\Xi_n \rangle \) as in the previous section. That is, the state \( |c_n \rangle \) represents the vacuum state; \( |c_1 \rangle \) will denote the first collapse state and so on.
The generalization of these ideas allows us to write the collapse scheme for the $n$th collapse $|c_{n-1}\rangle \rightarrow |c_{n}\rangle$

\[
\begin{align*}
\langle \hat{y}_{k}^{(R,I)} (\eta_{k}^{c_{n}}) \rangle_{c_{n}} &= x_{k,i}^{(\eta_{k}^{c_{n}})} \sigma_{y}^{(R,I)} (\eta_{k}^{c_{n}}, c_{n-1}) + \langle \hat{y}_{k}^{(R,I)} (\eta_{k}^{c_{n}}) \rangle_{c_{n-1}}, \\
\langle \hat{r}_{k}^{(R,I)} (\eta_{k}^{c_{n}}) \rangle_{c_{n}} &= x_{k,i}^{(\eta_{k}^{c_{n}})} \sigma_{\pi}^{(R,I)} (\eta_{k}^{c_{n}}, c_{n-1}) + \langle \hat{r}_{k}^{(R,I)} (\eta_{k}^{c_{n}}) \rangle_{c_{n-1}}.
\end{align*}
\]  

(42)

(43)

The second term on the rhs of (42) and (43) is the expectation value of the $(n - 1)$st collapse evaluated at the $n$th collapse time $\eta_{k}^{c_{n}}$. If we employ the matrix notation introduced in the previous section, we can rewrite (42) and (43) as

\[
\Upsilon(\eta_{k}^{c_{n}}, c_{n}) = \Delta(x_{k,i}^{(\eta_{k}^{c_{n}})}), \eta_{k}^{c_{n}}, c_{n-1}) + \Upsilon(\eta_{k}^{c_{n}}, c_{n-1}),
\]

(44)

where we introduced a new object

\[
\Delta(x_{k,i}^{(\eta_{k}^{c_{n}})}), \eta_{k}^{c_{n}}, c_{n-1}) = \left(\begin{array}{c}
\sigma_{y}^{(R,I)} (\eta_{k}^{c_{n}}, c_{n-1}) \\
\sigma_{\pi}^{(R,I)} (\eta_{k}^{c_{n}}, c_{n-1})
\end{array}\right),
\]

with $i = I, II$.

3.2. Evolution between collapses

Equation (20) characterizes the evolution of the state between two successive collapses, e.g., $n$ and $n - 1$. In other words, this means that (20) is valid from $\eta_{k}^{c_{n}}$ (their initial condition) to $\eta_{k}^{c_{n-1}}$. We can rewrite (20), with the notation adopted in this section, in order to see this more clearly,

\[
\Upsilon(\eta, c_{n}) = U(\eta, \eta_{k}^{c_{n}}) \Upsilon(\eta_{k}^{c_{n}}, c_{n}).
\]

(45)

Thus, the evolution from $\eta_{k}^{c_{n}}$ to $\eta_{k}^{c_{n-1}}$ of the expectation values of the state $|c_{n-1}\rangle$, is determined by (45) but using, as an initial condition, the expectation value given by the collapse $n - 2$, which evolved in a similar manner. This will lead us to a recursive relation for the dynamical equation of the field’s expectation value after $n$ collapses. Note that this description is just the orthodox quantum evolution following the standard rules of quantum mechanics: between ‘measurements’ the wavefunction continues to evolve according to Schrödinger’s equation but now with the initial condition of the ‘post-measurement’ quantum state, etc.

On account of the discussion above, we note that (45) depends on the $(n - 1)$st, $(n - 2)$nd, . . . , 1st collapse states. Therefore, we will obtain a new expression for (45) which will show this dependence explicitly.

We start by substituting (44) into (45) obtaining

\[
\Upsilon(\eta, c_{n}) = U(\eta, \eta_{k}^{c_{n}}) \Delta(x_{k,i}^{(\eta_{k}^{c_{n}})}) + U(\eta, \eta_{k}^{c_{n}}) \Upsilon(\eta_{k}^{c_{n}}, c_{n-1}).
\]

(46)

The quantity $\Upsilon(\eta_{k}^{c_{n}}, c_{n-1})$ contains information of the expectation value of the fields in the state $|c_{n-1}\rangle$ at the time $\eta_{k}^{c_{n}}$, but (45) gives us the value of $\Upsilon$ for any time $\eta$ and any state $|c_{n}\rangle$. In other words, we can use (45) and the collapse ‘recipe’ (44) to obtain $\Upsilon(\eta_{k}^{c_{n}}, c_{n-1})$ to calculate $\Upsilon(\eta_{k}^{c_{n}}, c_{n-1})$. This calculation will result in a term $\Upsilon(\eta_{k}^{c_{n}}, c_{n-1})$ which, again, can be computed from (45) and (44); therefore, (46) is a recursive relation which depends explicitly from the very first to the $(n - 1)$st post-collapse state. For example, if a single collapse occurs, we have

\[
\Upsilon(\eta, c_{1}) = U(\eta, \eta_{k}^{c_{1}}) \Delta(x_{k,i}^{(1)(R,I)}) + U(\eta, \eta_{k}^{c_{1}}) \Upsilon(\eta_{k}^{c_{1}}, c_{0}).
\]

(47)
because \(|c_0\rangle\) is taken to be the vacuum and \(\Upsilon(\eta^c_1, c_0) = 0\). For two collapses, one obtains
\[
\Upsilon(\eta, c_2) = U(\eta, \eta^c_2)U(\eta^c_2, c_1) + U(\eta, \eta^c_2)U(\eta^c_2, \eta^c_1)U(\eta^c_1, c_0).
\]
Thus, the general expression for \(\Upsilon(\eta, c_n)\) after \(n\) collapses is
\[
\Upsilon(\eta, \eta^c_n) = U(\eta, \eta^c_n)U(\eta^c_n, \eta^c_{n-1})U(\eta^c_{n-1}, \eta^c_{n-2}) \cdots U(\eta, \eta^c_n)U(\eta^c_n, \eta^c_1)U(\eta^c_1, c_0).
\]

From (45) it is evident that the matrix \(U(\eta^c_n, \eta^c_{n-1})\) represents the unitary evolution for the expectation value of the fields in the state \(|\eta^c_{n-1}\rangle\) from \(\eta^c_n\) to \(\eta^c_{n-1}\). Because of the unitary evolution, we have \(U(\eta, \eta^c_n)U(\eta^c_n, \eta^c_{n-1}) \cdots U(\eta^c_2, \eta^c_1) = U(\eta, \eta^c_1)\). Using this property in (49), we finally obtain
\[
\Upsilon(\eta, \eta^c_n) = \sum_{m=1}^{n} U(\eta, \eta^c_m)U(\eta^c_m, \eta^c_{m-1}).
\]

Equation (50) allows us to extract the evolution for the expectation value of \(\hat{\pi}(R,I)\) after \(n\) collapses,
\[
\{\hat{\pi}(R,I)\}_{\eta^c_n} = \sum_{m=1}^{n} \left[ -k \sin (k\eta - k\eta^c_m)\chi^{(m)(R,I)}(\eta^c_m, \eta, c_m) + \sqrt{k\eta^c_m} \frac{\sin(k\eta - k\eta^c_m)}{k\eta^c_m} \chi^{(m)(R,I)}(\eta^c_m, \eta, c_m) \right].
\]

Result (51) is the generalization of (18a) for multiple collapses during the inflationary epoch. We observe that the evolution of \(\hat{\pi}(R,I)\) resembles a superposition of many independent one-collapse evolutions of the expectation values of \(\hat{\pi}(R,I)\), any of which suffered a collapse at different times.

As mentioned at the end of section 2.1, to connect the theoretical predictions with the observational quantities, we need to compute \(|\alpha_{mL}|^2\), as given in (34). That is, we need to obtain \(\{\hat{\pi}(R,I)\}_{\eta^c_n} \langle \hat{\pi}(R,I) \rangle_{\eta^c_n}\), which will be our next task.

First, we note that, in the notation introduced in (40) and (41), the expectation value of each collapse scheme was decomposed generically as \(x^{(1)(R,I)}_k(\eta^c, c)\) and \(x^{(1)(R,I)}_k(\eta^c, c)\), where the random variables are dimensionless, and \(\chi^{(R,I)}\) are the part of the collapse scheme carrying the units (e.g., in the independent collapse scheme \(\chi^{(R,I)}(\eta^c, c) = \sqrt{h/2}\chi_\eta(\eta)\) and \(\chi^{(R,I)}(\eta^c, c) = \sqrt{h/2}\chi_\eta(\eta)\)). The mean value of the product of \(x^{(n)(R,I)}_{k,i}\) depends on the particular scheme considered. In the independent scheme, the product mean value is given by
\[
\chi^{(n)(R,I)}_{k,i}\chi^{(m)(R,I)}_{k,j} = (\delta_{k,k'} + \delta_{k,-k'})\delta_{m,n}, \quad \chi^{(m)(R,I)}_{k,i}\chi^{(n)(R,I)}_{k,j} = (\delta_{k,k'} - \delta_{k,-k'})\delta_{m,n},
\]
where \(i = I, II,\) and with all the other possible combinations equal zero. Meanwhile, in the Newtonian scheme it is
\[
\chi^{(n)(R,I)}_{k,i}\chi^{(m)(R,I)}_{k,j} = (\delta_{k,k'} + \delta_{k,-k'})\delta_{m,n}, \quad \chi^{(m)(R,I)}_{k,i}\chi^{(n)(R,I)}_{k,j} = (\delta_{k,k'} - \delta_{k,-k'})\delta_{m,n},
\]
and in the Wigner’s scheme, we have
\[
\chi^{(n)(R,I)}_{k,i}\chi^{(m)(R,I)}_{k,j} = (\delta_{k,k'} + \delta_{k,-k'})\delta_{m,n}, \quad \chi^{(m)(R,I)}_{k,i}\chi^{(n)(R,I)}_{k,j} = (\delta_{k,k'} - \delta_{k,-k'})\delta_{m,n}.
\]
\[ C(n) \]

This expression is breakdown the variables associated with different collapses, respectively (e.g., the random variable \( x(14) \), after Class. Quantum Grav. 28 (2011) 155010 G León et al.

After choosing a particular collapse scheme (with the corresponding characterization for the mean value of the random variables (52), (53), (54)), and recalling that \( \langle \hat{\eta}_k (\eta) \rangle_{\text{c}_{s}} = \langle \hat{\eta}_k (\eta) \rangle_{\text{c}_{s}} + i \langle \hat{\tau}_k (\eta) \rangle_{\text{c}_{s}} \) (using (51)), one obtains the quantity \( \langle \hat{\eta}_k (\eta) \rangle_{\text{c}_{s}} \) for each collapse scheme. The calculation will be simplified due to the fact that, as usual, the average over the random variables (in the three collapse schemes) will lead to a cancellation of the cross terms. Thus, after going to the continuum limit \((L \to \infty)\), the expression for \( |a_{lm}|_{M.L.}^2 \)

\[ \frac{4Vh^3}{27\pi M_p^2} \int \frac{dk}{k^2} |j_i(kR_D)|^2 \sum_{n=1}^{N} C_{l}^{(a)}(k, \eta_D) \] (55)

where \( C_{l}^{(a)}(k, \eta_D) \) depends on the collapse scheme considered. In the independent scheme, this expression is

\[ C_{l}^{(a)}(k, \eta_D) = \left( \cos \left( k \eta_D - k \eta_n^\alpha \right) + \sin \left( k \eta_D - k \eta_n^\alpha \right) \right)^2 \left( \Pi_k^+ + (-1)^i \Pi_k^- \right); \] (56)

meanwhile, in the case of the Newtonian scheme

\[ C_{l}^{(a)}(k, \eta_D) = \left( \cos \left( k \eta_D - k \eta_n^\alpha \right) + \sin \left( k \eta_D - k \eta_n^\alpha \right) \right)^2 \left( \Pi_k^+ + (-1)^i \Pi_k^- \right). \] (57)

the quantities \( Y_k^\pm \) and \( \Pi_k^\pm \) are defined as

\[ Y_k^\pm = \left( \Delta \hat{\eta}_k^R (\eta_n^\pm) \right)_{c_{s-1}} \pm \left( \Delta \hat{\eta}_k^R (\eta_n^\pm) \right)_{c_{s+1}} \] \( \Pi_k^\pm = \left( \Delta \hat{\eta}_k^R (\eta_n^\pm) \right)_{c_{s+1}} \pm \left( \Delta \hat{\eta}_k^R (\eta_n^\pm) \right)_{c_{s-1}} \) (58)

Finally, in the Wigner scheme, \( C_{l}^{(a)}(k, \eta) \) is given by

\[ C_{l}^{(a)}(k, \eta_D) = 2k^2 \Lambda_{k,n} \sin \left( k \eta_D - k \eta_n^\alpha \right) \sin \Theta_{k,n} + \left( \cos \left( k \eta_D - k \eta_n^\alpha \right) + \sin \left( k \eta_D - k \eta_n^\alpha \right) \right)^2 \cos \Theta_{k,n} \] (59)

with \( \Lambda_{k,n} \) and \( \Theta_{k,n} \) defined as

\[ \Lambda_{k,n} = \frac{4 \eta_n^\alpha \sqrt{\hbar k}}{\sqrt{1 + 5(k \eta_n^\alpha)^2 - \sqrt{1 + 10(k \eta_n^\alpha)^2 + 9(k \eta_n^\alpha)^4}}}, \] (60a)

\[ 2 \Theta_{k,n} = \arctan \left( \frac{4k \eta_n^\alpha}{1 - 3(k \eta_n^\alpha)^2} \right). \] (60b)

It is worthwhile to comment that uncertainties in (55) are always evaluated at the \((n-1)\)st collapse state.

Before discussing the physical implications of the general result (55), let us start by analyzing the assumption of a single collapse in the independent scheme; in this case, (55)
reduces to
\[
|\alpha_{lm}|^2_{M.L.} = \frac{4Vh^3}{54\pi\epsilon M_p^2} \int \frac{dk}{k} |j_l(kR_D)|^2 \left( 1 + \frac{\sin^2(k\eta_D - k\eta_k^c)}{(k\eta_k^c)^2} + \frac{\sin 2(k\eta_D - k\eta_k^c)}{k\eta_k^c} \right).
\]  

(61)

Considering again a single collapse and working within the Newtonian scheme, (55) leads to
\[
|\alpha_{lm}|^2_{M.L.} = \frac{4Vh^3}{54\pi\epsilon M_p^2} \int \frac{dk}{k} |j_l(kR_D)|^2
\]
\[
\times \left[ 1 + \sin^2(k\eta_D - k\eta_k^c) \left( \frac{1}{(k\eta_k^c)^2} - 1 \right) + \frac{\sin 2(k\eta_D - k\eta_k^c)}{k\eta_k^c} \right].
\]  

(62)

Results (61) and (62) are consistent with the findings presented in [10] and [14]. The result obtained from (55), for a single collapse in the Wigner scheme, also corresponds with the one presented in [14].

We observe that, for the three schemes considered, in the case of a single collapse \(N = 1\), only the uncertainties of the vacuum state contribute to the integral in (55). The point is that for a single collapse, (55) does not contain any information characterizing the post-collapse state (the information that defines a particular post-collapse state is contained in the uncertainties evaluated in that precise state). That is, we do not need to specify the post-collapse state. However, if we assume multiple collapses, then the uncertainties of the post-collapse states will contribute to the integral in (55), and since the uncertainties will depend on the pre-collapsed states, which are now different from the vacuum, we will need to specify every pre-collapse state (which will be the subject of the next section).

An important feature arises in the independent and Newtonian schemes, since for these cases, (55) exhibits an explicit dependence of \(l\) (there is also another dependence on \(l\) in the term \(|j_l(kR_D)|^2\); however, this dependence will not affect the compatibility of the theoretical predictions obtained in our approach with the ones from the standard treatment, since the latter, also involves this dependence on \(l\) in the spherical Bessel function \(j_l(kR_D))\) in the terms \(k^l + (-1)^l Y_k^l\) and \(\Pi_k^l + (-1)^l \Pi_k^l\). If \(l\) is even, \(Y_k^l + (-1)^l Y_k^l = 2(\Delta\hat{\nu}_k^R(\eta_k^c))_{c_{-1}}^2\) and \(\Pi_k^l + (-1)^l \Pi_k^l = 2(\Delta\hat{\nu}_k^R(\eta_k^c))_{c_{-1}}^2\); if \(l\) is odd, \(Y_k^l + (-1)^l Y_k^l = 2(\Delta\hat{\nu}_k^I(\eta_k^c))_{c_{-1}}^2\) and \(\Pi_k^l + (-1)^l \Pi_k^l = 2(\Delta\hat{\nu}_k^I(\eta_k^c))_{c_{-1}}^2\). Thus, depending on the parity of \(l\), the predicted quantity \(|\alpha_{lm}|^2_{M.L.}\) will involve the uncertainty of the real or imaginary parts of \(\hat{\nu}_k(\eta_k^c)\) and \(\hat{\nu}_k(\eta_k^c)\) which is not entirely compatible with the standard prediction, namely a flat spectrum. In order to recover the standard theoretical prediction, the dependence of \(l\) should be avoided, and the most natural option is that the uncertainties satisfy
\[
(\Delta\hat{\nu}_k^R(\eta_k^c))_{c_{-1}}^2 = (\Delta\hat{\nu}_k^I(\eta_k^c))_{c_{-1}}^2 = (\Delta\hat{\nu}_k^I(\eta_k^c))_{c_{-1}}^2.
\]  

(63)

It is clear, of course, that this is not the most generic case\(^{10}\), and need not be taken as a necessary condition for the compatibility of the theoretical predictions in our approach with the observations from the CMB, since we still need to consider the physics of the cosmological epochs after the end of the inflationary regime that leads to the so-called acoustic oscillations. It is also interesting to note that the condition on the uncertainties of the real and imaginary parts of \(\hat{\nu}_k(\eta_k^c)\) and \(\hat{\nu}_k(\eta_k^c)\) only applies to the independent and Newtonian schemes. In the Wigner scheme, we do not find a similar condition for the parameters \(\Lambda_{k,n}\) and \(\Theta_{k,n}\) that characterize the uncertainties in that case.

\(^{10}\) However, as we will show in the next section, if we assume that the post-collapse states are coherent states, this condition is fulfilled automatically.
Finally, we note that all the quantities involved in (55) are positive. In other words, we have a sum of positive definite terms. Therefore, if we set $N \to \infty$, the sum will generically diverge, which implies that we cannot set an infinite number of collapses because the predicted value for $|\alpha_{\text{ML}}|^2$ will tend to infinity. Thus, we must restrict consideration to the case with a finite number of collapses. It is important to note that the calculations that lead to result (55) have not considered any particular post-collapse state. Of course (and we will do it in the next section), we can consider a particular post-collapse state and that information will enter in the uncertainties. However, the conclusion obtained from (55) related to the finiteness of the collapses is valid for a generic post-collapse state in the three schemes considered.

4. Characterization of the post-collapse states

The information that characterizes a particular post-collapse state will enter in the uncertainties of the field (and its momentum) through the parameters that characterize the post-collapse state. Thus, our first task will be to focus on obtaining the uncertainties for the coherent and squeezed states and afterward we are going to use the results of the previous sections to obtain predicted values for the observational quantities.

4.1. Coherent states as post-collapse states

A simple election for a post-collapse state is a coherent state. A coherent state is a specific state of the harmonic oscillator and its dynamic is very similar to the one of the classic harmonic oscillator. The coherent states $|\xi\rangle$ are defined as the eigenstates of the annihilation operator $\hat{a}$, 

$$\hat{a}|\xi\rangle = \xi|\xi\rangle,$$  

(64)

since $\hat{a}$ is not a Hermitian operator, $\xi$ is a complex number and can be represented in the complex polar form $\xi = |\xi|e^{i\chi}$, where $|\xi|$ is the amplitude and $\chi$ is the phase.

Equation (64) physically implies that the coherent state $|\xi\rangle$ is not affected by the detection and annihilation of one particle. In a coherent state, the quantum uncertainties of $\hat{p}$ and $\hat{q}$ (the momentum and position of the quantum oscillator, respectively) take the minimum value, i.e. $\Delta\hat{p}\Delta\hat{q} = \frac{\hbar}{2}$.

With the exception of the vacuum state $|0\rangle$ (which is also a coherent state), every coherent state can be produced by the application of the Displacement operator $\hat{D}(\xi) = \exp(\xi \hat{a}^\dagger - \xi^* \hat{a})$ to the vacuum state $|\xi\rangle = \hat{D}(\xi)|0\rangle$.

Using the simple properties of the coherent states, we can calculate the quantities $d_k^{(R,I)}$, $e_k^{(R,I)}$ and $c_k^{(R,I)}$ (12) when the post-collapse state of each mode of the field is a coherent state $|\xi_k\rangle$,

$$d_k^{(R,I)} = \xi_k^{(R,I)}, \quad c_k^{(R,I)} = (\xi_k^{(R,I)})^2, \quad e_k^{(R,I)} = |\xi_k^{(R,I)}|^2,$$  

(65)

Expressions (65), (14) and (15) allow us to obtain the evolution of the uncertainties of the field and its conjugate momentum for any coherent state. Making use of the same arguments that led (20) to generalization (45) in the case of multiple collapses, (14) and (15) can also be considered in conjunction with the assumption that every post-collapse state is a coherent state ($|\xi_{kn}^{(n)}\rangle = |c_{kn}\rangle$),

$$\left(\Delta \hat{y}_k^{(R,I)}(\eta)\right)^2_{\text{MC}} = \Re\left[y_k^2(\eta)\left(\xi_k^{(n)(R,I)}\right)^2 + \frac{1}{2}|y_k(\eta)|^2\left(hL^3 + 2|\xi_k^{(n)(R,I)}|^2\right) - 2\Re\left[y_k(\eta)\xi_k^{(n)(R,I)}\right]^2\right] = \frac{1}{2}|y_k(\eta)|^2hL^3 = \frac{\hbar L^3}{4k}\left(1 + \frac{1}{(k\eta)^2}\right),$$  

(66)
\[
\left( \Delta \hat{S}_{k}(R,I)(\eta) \right)^2_{\xi_k} = \text{Re} \left[ g_{k}(\eta) \left( \hat{S}_{k}(R,I)(\eta) \right)^2 \right] + \frac{1}{4} \left| g_{k}(\eta) \right|^2 \left( \hbar L^3 + 2 |\xi_k|^2 \right) - 2 \hbar \left[ g_{k}(\eta) \xi_k \right]^2 \\
= \frac{1}{2} |g_{k}(\eta)|^2 \hbar L^3 = \frac{\hbar L^3}{4}.
\]

This last result shows that the uncertainties of the \( n \)th coherent post-collapse state have the same form as those of the vacuum state. We also note that the uncertainty of the conjugate momentum is constant in the inflationary era.

### 4.2. Squeezed states as post-collapse states

Squeezed states can be considered as a more general case of the coherent states. Qualitatively, a squeezed state is a state that has the minimal uncertainty, not in the standard position and momentum variables, but in a new pair of ‘rotated’ canonical variables (commonly referred to as quadrature variables [35]). Let us call them \( \hat{Q} \) and \( \hat{P} \). For a squeezed state, one can have ‘more (or less)’ uncertainty in either \( \hat{Q} \) or \( \hat{P} \), as long as their product is equal to the minimum value allowed by Heisenberg’s principle. The parameters of the squeezed state control the angle of ‘rotation’ and the ‘squeezing’ of the uncertainties.

The work with squeezed states is simplified by the introduction of the following operator:

\[
\hat{S}(\omega) = \exp \left( \frac{i}{2} \omega \hat{a}^\dagger \hat{a} - \frac{1}{2} \omega \hat{a}^\dagger \hat{a}^\dagger^2 \right),
\]

where the parameter \( \omega \) is a complex number. In particular, \( \omega \) can be written as \( \omega = r e^{i\theta} \).

The operator \( \hat{S}(\omega) \) is known as the squeeze operator. Applying the squeeze and displacement operators to the vacuum state, we obtain a squeezed state

\[
|\xi \omega \rangle \\equiv \hat{D}(\xi) \hat{S}(\omega) |0 \rangle.
\]

We note that the squeezed state \( |\xi \omega \rangle \) is completely defined by four parameters: \( |\xi|, \chi, r \) and \( \theta \).

Some well-known properties of the operators \( \hat{D}(\xi) \) and \( \hat{S}(\omega) \) are as follows.

(i) \( \hat{D}(\xi) \hat{D}(\xi^*) = \hat{D}(\xi + \xi^*) \)

(ii) \( \hat{D}(\xi) \hat{D}(\xi^*) = \hat{D}(\xi + \xi^*) \)

(iii) \( \hat{S}(\omega) \hat{S}(\omega^*) = \hat{S}(\omega \omega^*) \)

(iv) \( \hat{S}(\omega) \hat{S}(\omega^*) = \hat{S}(\omega \omega^*) \)

(v) Both \( \hat{D} \) and \( \hat{S} \) are unitary operators.

By regarding the post-collapse state of each mode as a squeezed state and using the properties (i)–(v), one can obtain \( d_k^{(R,I)} \), \( c_k^{(R,I)} \) and \( e_k^{(R,I)} \) from (12),

\[
d_k^{(R,I)} = \xi_k^{(R,I)},
\]

\[
c_k^{(R,I)} = -\hbar L^3 \cosh r_k^{(R,I)} \sinh \theta_k^{(R,I)} e^{-i\omega_k^{(R,I)}} + \left( \xi_k^{(R,I)} \right)^2,
\]

\[
e_k^{(R,I)} = \hbar L^3 \sinh^2 r_k^{(R,I)} + |\xi_k^{(R,I)}|^2.
\]

Equations (14) and (15) give us the evolution of the uncertainties, in terms of the quantities \( d_k^{(R,I)} \), \( c_k^{(R,I)} \) and \( e_k^{(R,I)} \). As in the coherent state, (14) and (15) can be generalized straightforward to the case of multiple collapse. Thus, by considering the post-collapse states of each mode as squeezed states, we can substitute (70), (71) and (72) into (14) and (15), which for multiple collapses yields

\[
\left( \Delta \hat{S}_{k}(R,I)(\eta) \right)^2_{\xi_k} = \frac{\hbar L^3}{4k} \left[ 1 + \frac{1}{(k\eta)^2} \right] - \sinh (2r_k^{(R,I)}) \\
\times \cos \left[ \theta_k^{(R,I)} + 2 \arctan \left( \frac{1}{k\eta} \right) + 2k\eta \right] + \cosh (2r_k^{(R,I)}),
\]

17
\[(\Delta \hat{\gamma}_k^{(R,I)}(\eta_i))^2 = \frac{\hbar L}{4} \left[ \sinh (2\gamma_k^{(R,I)}) \cos (\eta_k^{(R,I)} + 2k\eta) + \cosh (2\gamma_k^{(R,I)}) \right]. \] (74)

The squeezing parameters \(\gamma_k^{(R,I)}\) and \(\eta_k^{(R,I)}\) refer to the squeeze parameters of the \(n\)th post-collapse squeezed state of each mode. The situation at hand is totally different from the coherent case, in which the uncertainties are completely characterized by the vacuum state despite \(n\) collapses have occurred. In the squeeze state case, it is evident from (73) and (74) that the dispersions \((\Delta \hat{\gamma}_k^{(R,I)}(\eta_i))^2\) and \((\Delta \hat{\theta}_k^{(R,I)}(\eta_i))^2\) are determined by the squeeze parameters \(\gamma_k^{(R,I)}\) and \(\eta_k^{(R,I)}\). This is a crucial difference from the coherent case in which the uncertainties are independent of the parameters characterizing the coherent state.

4.3. Connections with the observational quantities

The uncertainties of the field are characterized by both the particular post-collapse state and the collapse scheme. In the rest of this section, we will focus on the independent collapse scheme; however, similar conclusions as those obtained from these results can be derived when considering the other two collapse schemes that have been proposed so far.

4.3.1. Squeezed states as post-collapse states

The connection with the observations will be made under the following assumptions. (I) The wavefunction of the field has collapsed \(N\) times and the \(N\) post-collapse states are squeezed states. (II) The uncertainties \((\Delta \hat{\gamma}_k^{(R)}(\eta_i^c))_c\) and \((\Delta \hat{\gamma}_k^{(I)}(\eta_i^c))_c\) are equal (as well as the uncertainties \((\Delta \hat{\gamma}_k^{(R)}(\eta_i^c))_c\) and \((\Delta \hat{\gamma}_k^{(I)}(\eta_i^c))_c\)), which is motivated by the discussion at the end of section 3.

Under assumption (II), (55) (recall that we are working under the independent scheme) takes the simplified form

\[
|\alpha_{lm}|_{M,L}^2 = \frac{8V\hbar^3}{27\pi^2\epsilon M_p^2} \int \frac{dk}{k} \left| f_i(kR_D) \right|^2 \sum_{n=1}^{N} \left[ \left( k \sin (k\eta_D - k\eta_k^c) \right)^2 \left( \Delta \hat{\gamma}_k(\eta_k^c) \right)^2 \right. \\
+ \left( \cos (k\eta_D - k\eta_k^c) + \frac{\sin (k\eta_D - k\eta_k^c)}{k\eta_k^c} \right)^2 \left( \Delta \hat{\theta}_k(\eta_k^c) \right)^2 \left. \right] . \] (75)

After a little algebra, the expression for \(|\alpha_{lm}|_{M,L}^2\) obtained by substituting (73) and (74) into (75) becomes

\[
|\alpha_{lm}|_{M,L}^2 = \frac{2V\hbar^3}{27\pi^2\epsilon M_p^2} \int \frac{dk}{k} \left| f_i(kR_D) \right|^2 \sum_{n=1}^{N} \left[ \left( 1 + \frac{\sin (2(k\eta_D - k\eta_k^c))}{k\eta_k^c} \right) \right. \\
\times \left( \cosh 2\gamma_k^{c-1} \sinh 2\gamma_k^{c-1} \cos (\theta_k^{c-1} + 2k\eta_k^c) \right) + \frac{2 \sin^2 (k\eta_D - k\eta_k^c)}{(k\eta_k^c)^2} \right. \\
\left. \times \left( \cosh 2\gamma_k^{c-1} \sinh 2\gamma_k^{c-1} \sin (\theta_k^{c-1} + 2k\eta_k^c) \right) \right] . \] (76)

The above result leads us to conclude that, in order to obtain a reasonable power spectrum, that is, a nearly flat Harrison–Zel’dovich spectrum, there seems to be one simple case characterized by two particular conditions.

First, for each one of the \(n\) post-collapse states, \(k\eta_k^c\) should be independent of \(k\) but dependent on \(n\), i.e. the time of collapse for the \(n\) post-collapse states of the different modes should depend on the mode’s frequency according to \(\eta_k^c = fn/k\) (where \(fn\) is a real number.
that changes for each collapse). This condition is the generalization for \( n \) collapses of the result presented in [10] where a single collapse was considered (a possible deviation of such ‘recipe’ for the time of collapse was studied in [14]). In other words, result (76) generalizes the condition \( \eta_k^c \propto 1/k \) in the case of multiple collapses.

Second, a nearly flat spectrum is recovered if the parameters characterizing the \( n \) squeezed states are also independent of the mode’s frequency, that is, if \( r_k^c = r^c \) and \( \theta_k^c = \theta^c \) are independent of \( k \) but dependent on \( n \). This does not mean that the uncertainties for each mode are all the same, because the uncertainties are also characterized by the time of collapse of each mode and its frequency, as can be seen in (73) and (74).

### 4.3.2. An upper bound for the number of collapse using coherent states as post-collapse states.

As already noted generically, the number of collapses in each mode must be finite, and we expect to provide a simple estimate in this subsection. We will continue the consideration of the independent collapse scheme, but we will assume that all the \( N \) post-collapse states are coherent states (which, after all, are just a particular class of squeezed states with \( r_k = 0 \)). That is, we can use the uncertainties (66) and (67) to obtain a predicted value for \( |\alpha_{lm}|^2_{M,L} \).

Substituting (66) and (67) into (55) yields

\[
|\alpha_{lm}|^2_{M,L} = \frac{2}{27\pi} \frac{V h^3}{\epsilon M_p^2} \int \frac{dk}{k} |j_l(k R_D)|^2 \sum_{n=1}^N \left( 1 + \frac{\sin 2(k \eta_D - k \eta_n^c)}{k \eta_n^c} + \frac{2 \sin^2 \left( k \eta_D - k \eta_n^c \right)}{(k \eta_n^c)^2} \right).
\]

From this last expression, it is a relatively simple task to obtain information regarding the maximum number of collapses allowed by observations. If we assume that \( |k \eta_n^c| \gg k \eta_D \), that is, the time for the 1st, 2nd, \ldots, \( N \)th collapse occurs at a very early stage of the inflationary regime, (77) is approximated by

\[
|\alpha_{lm}|^2_{M,L} \approx \frac{2}{27\pi} \frac{V h^3}{\epsilon M_p^2} \int \frac{dk}{k} |j_l(k R_D)|^2 N; \quad (78)
\]

using \( \int x^{-1} j_l^2(x) \, dx = \pi / (l + 1) \), the expression above reduces to

\[
|\alpha_{lm}|^2_{M,L} \approx \frac{2}{27} \frac{V h^3}{\epsilon M_p^2} \frac{N}{l(l + 1)}. \quad (79)
\]

In general, \( |\alpha_{lm}|^2_{M,L} \) is independent of \( m \) and the quantity that is presented as the result of observations is \( \mathcal{O}_B l = l(l + 1) C_l \), where \( C_l = (2l + 1)^{-1} \sum_m |\alpha_{lm}|^2 \). If we ignore the physics of the plasma that follows after the reheating era, \( \mathcal{O}_B l \) is essentially independent of \( l \) corresponding to the amplitude of the metric perturbations (which is roughly \( 10^{-10} \)). Thus, setting \( \mathcal{O}_B l \equiv A \), the maximum number of collapses \( N_{\text{max}} \) allowed by the observations is

\[
N_{\text{max}} \approx \frac{27\epsilon M_p^2 A}{2V h^3}. \quad (80)
\]

We believe that this constraint might be of great help in studying the viability of the actual proposals for the detailed physical mechanism that lies behind the collapse we have been considering.
5. Discussion

As first reviewed in [10], the inflationary account of the origin of cosmic structure possesses a serious shortcoming, namely the emergence of structure from an initial state that was homogeneous and isotropic. The proposal to address this existing issue was through the introduction of a modification of standard quantum theory corresponding to a dynamical reduction of the wavefunction. The present study represents a continuation of the investigation of such a proposal.

In this paper, we have examined the possibility that multiple collapses take place in each of the modes of the quantum field. This study required a much more detailed characterization of the post-collapse states. This, in turn, required the introduction of extra assumptions. We focused here in the possibility that the states are coherent or squeezed and under these assumptions we were able to further constrain, beyond the results of previous analyses, the features of the collapse hypothesis required for agreement with observations. These we will discuss in the following.

The first result obtained in this paper is that in order to recover a flat spectrum, and assuming that multiple collapses occur, the uncertainties of the real and imaginary parts of the fluctuation of the inflaton field, i.e. \( \delta_k \) and its conjugated momentum \( \pi_k \), must be equal. We can interpret this result as the most natural option for selecting simple candidates for post-collapse states since the uncertainty of each mode of the field and its conjugated momentum is characterized by specifying the post-collapse state. Therefore, given a particular state \( |\Xi\rangle \) for each mode \( k \), one can calculate the uncertainties of the field and its conjugated momentum for that state. If the uncertainties for each mode satisfy the relation \( (\Delta \hat{y}^R_k)^2 = (\Delta \hat{y}^I_k)^2 \) and \( (\Delta \hat{\pi}^R_k)^2 = (\Delta \hat{\pi}^I_k)^2 \) (as well as \( (\Delta \hat{\theta}^R_k)^2 = (\Delta \hat{\theta}^I_k)^2 \)), then \( |\Xi\rangle \) can be regarded as a reasonable choice for a post-collapse state.

In fact, in section 4.1, we found that, for coherent states, the relation between the uncertainties of the real and imaginary parts of \( y_k \) and \( \pi_k \) is satisfied automatically. Consequently, a coherent state is a natural candidate for a post-collapse state. The fact that a coherent state acts as a good candidate for a post-collapse state is consistent with the notion that a coherent state of the field is the closest quantum mechanical state to a classical description of the field, i.e. a state for which the semiclassical approximation of gravity given by \( G_{ab} = 8\pi G (\hat{T}_{ab}) \) is valid in the sense of Ehrenfest’s theorem and thus qualifies for a reasonable candidate for a post-collapse state.

Nevertheless, for a generic squeezed state \( |\Sigma\rangle \), \( (\Delta \hat{y}^R_k)^2 = (\Delta \hat{y}^I_k)^2 \) and \( (\Delta \hat{\pi}^R_k)^2 = (\Delta \hat{\pi}^I_k)^2 \), but this does not mean that post-collapse squeezed states are forbidden. That is, one can select a set of squeezed states, characterized by the squeezing parameters \( r_k^{(R,I)} \) and \( \theta_k^{(R,I)} \), such that \( r_k^R = r_k^I \) and \( \theta_k^R = \theta_k^I \) for which the relation in the uncertainties holds. Furthermore, in section 4.3.1, we argued that, given a collection of multiple post-collapse squeezed states characterized by \( r_k^R \) and \( \theta_k^R \), the simplest choice that allows the recovering of the standard flat spectrum is that the squeezing parameters are independent of \( k \). The point is that we again used the observations as a guide to uncover the particular characteristics of a squeezed state that could be regarded as a reasonable post-collapse state. We should note that, as discussed in [14], we cannot expect such a strict pattern to be followed in an exact manner in a theory involving a collapse controlled by some fundamentally random events, and as such one can in principle investigate the effects of the expected deviations on the observational data. The investigation of the detailed signature of those deviations, as well as the observational bounds on them (i.e. analogs of those considered in [14]), is part of our ongoing research program.

Another important result from this work is that the number of collapses must be finite under generic conditions. However, we could, in principle, select a set of post-collapse states and adjust the uncertainties of the field (and its conjugated momentum) and the times of
collapse in a way that the predicted observational quantity (the sum in (55)) would remain finite, even for an infinite number of collapses. Evidently, this would amount to a fine tune of the scheme which we do not see as an attractive choice. On the other hand, we should say that if the collapse of the state, which gives birth to the inhomogeneities observed in the CMB, is a process that keeps occurring indefinitely even after inflation ends, the Newtonian potentials would also be changing, thus affecting in a rather random way the propagation of photons from the last scattering surface to our satellites. These ideas might be considered as related, at least at the phenomenological level, to those explored in [36]. We did not investigate these issues here. In the present work, we rather concentrated on the generic sort of conditions for the collapse during inflation and not only found that the number of collapses should be finite, but also obtained,—under the extra hypothesis on the form of post-collapse states—a rough estimate on the number of collapses in terms of the parameters of the inflaton potential.

All of the previous discussion shows that, even though in principle we do not know precisely what is the nature of the physics behind what we call the collapse, we can, in fact, obtain some insights on the ‘rules’ that govern it, i.e. those determining the nature of post-collapse states and the number of collapses of each mode, by comparing the observations with our theoretical predictions.

We are beginning to investigate the possible connection of our proposal with other more developed collapse mechanisms involving similar non-unitary modifications of quantum theory. Henceforth, the path to follow in our future research is to explore the connections of our proposal with other collapse mechanisms compatible with the conclusions obtained in this and previous works.

We believe that, in the case of the inflationary paradigm, we cannot content ourselves with the fact that calculations lead to results that match the observations but which cannot be fully justified within the context of the interpretations provided by our current physical theories. We readily acknowledge that, although our proposal seems to offer a clearer picture of the emergence of the seeds of cosmic structure, it might be ultimately an incorrect proposal which might need to be replaced by something even more complex and distant from the established physical paradigms. What seems clear is that the standard account of the genesis of the cosmic structure, something intimately tied with the rise in the conditions that are a prerequisite for our own existence, is not fully satisfactory and that on the other hand, our present and future access to detailed empirical data makes the issue not only susceptible to scientific inquire, but from our point of view, one of the most promising fertile grounds where some fundamental questions can be explored.

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