NEUTRINO’S NON-STANDARD INTERACTIONS; ANOTHER EEL UNDER A WILLOW?

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Abstract

I report some progress that occurred since NO-VE 08 in the field of non-standard interactions (NSI) of neutrinos. After briefly reviewing theoretical developments, I give a summary of the two works in which I was involved. Firstly, we have formulated a perturbative framework to illuminate the global features of neutrino oscillations with NSI, aiming at exploring method for determination of the standard mixing and the NSI parameters. We have recognized that the parameter degeneracy prevails with an extended form which involves the NSI elements. Furthermore, a completely new type of degeneracy is shown to exist. The nature of the former degeneracy is analyzed in detail in the second work. The work is primarily devoted to analyze the problem of discriminating the two CP violation, one due to the lepton Kobayashi-Maskawa phase and the other by phase $\phi$ of the NSI elements. We have shown that the near (3000 km)—far (7000 km) two detector setting in neutrino factory does have the discrimination capability and is sensitivities to CP violation due to NSI to $|\epsilon_{e\mu}|$ to $\simeq$ several $\times 10^{-4}$ in most of the region of $\delta$ and $\phi_{e\mu}$.

1. Introduction

The question I would like to address in my talk is: Are there something terribly new in neutrino properties after the discovery of neutrino masses and lepton mixing? The Japanese saying in the subtitle is meant to be that. Clearly this is an extremely interesting question. But, I must start with a cautionary remark.

What is the natural time scale for discovery of something extremely new in neutrino properties? Let us look back the history to obtain a hint for answering the question. It took more than 60 years from the Meitner-Hahn measurement of electron energy spectrum in nuclear beta decay in 1911 to the discovery of NC reaction in 1973. From the discovery of neutrino itself in 1953 by Reines and Cowan to the discovery of neutrino mass and lepton flavor mixing by Super-Kamiokande in 1998 needed 45 years. Thus, the right time scale, as history tells us, is $\sim$ 50 years. It implies a warning; What people think about the possible candidates for “terribly

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a Written version of a talk presented at XIII International Workshop on Neutrino Telescopes, Istituto Veneto di Scienze, Lettere ed Arti, Venice, Italy, March 10-13, 2009.

b Though the original saying in Japan is “Another loach under a willow?” I decided to keep the modified version because eel is much more familiar to us, and in particular thanks to a great support to the eel version by the spokesman of IceCube collaboration.
new in the neutrino properties” at the right time can be very different from those we consider today. Though I have to speak within the scope that I can think of today but this point has to be kept in mind as a word of caution.

It also has to be remarked that the scope of my presentation is very limited; Though I restrict myself into the so called non-standard interactions (NSI) of neutrinos in my talk, “new in the neutrino properties” may include various often more radical possibilities such as: departure from three flavor mixing, sterile neutrinos, violation of fundamental symmetries like CPT. See e.g., [9] for a status summary for these more exotic possibilities.

2. Non-Standard Interactions of Neutrinos

It has been proposed that neutrinos might possess yet unknown new neutrino interactions. Today we have Standard Model of particle physics, one of the most successful theories in physics. Therefore, when we discuss NSI it is natural (and to a large extent mandatory) to talk about it in a language of higher dimensional operators.

Suppose that there exist a new physics at energy scale $M_{NP}$, which I assume to be greater than $\sim 1$ TeV, but not too much larger than this value. I assume the type of higher-dimentional operators for effective interactions of neutrinos with matter

$$L_{\text{eff}}^{\text{NSI}} = -2\sqrt{2} \varepsilon_{\alpha\beta} f_\mu P \nu_\alpha \nu_\beta \left(\bar{f} \gamma^\mu P f\right),$$

where $G_F$ is the Fermi constant, and $f$ stands for the index running over fermion species in the earth, $f = e, u, d$, in which we follow the conventional notation. $P$ stands for a projection operator and is either $P_L \equiv \frac{1}{2}(1 - \gamma_5)$ or $P_R \equiv \frac{1}{2}(1 + \gamma_5)$. Given the dimension six operator in (1) and because we normalize the operator with Fermi constant $G_F$, $\varepsilon_{\alpha\beta}$ must be of the order of $(M_W/M_{NP})^2 \sim 0.01 (10^{-4})$ if $M_{NP} = 1(10)$ TeV. If we have to go to dimension eight operators their effective strength would be at most $(M_W/M_{NP})^4 \sim 10^{-4}$ even for $M_{NP} = 1$ TeV. The off-diagonal elements may have further suppression.

Since I gave a talk on NSI last year in Venice [12], I will restrict myself into developments that occurred after NO-VE 08 to show that the field is moving. The rest of my report has three parts: In section 3 I review the recent development in the theory of NSI. From section 4 I change gear to NSI effect in propagation in matter. In section 5 I discuss perturbative treatment of the system with NSI. Sections 6 and 7 are devoted to further clarifying the properties of the system and to discuss the question of discriminating two kind of CP violation, one from the lepton Kobayashi-Maskawa phase [13] and the other from phases of the NSI elements, the problem discussed in [14]. My presentation in the last three sections will be based on the two recent papers
The latter work is a natural continuation of our previous work \cite{17}.

3. Recent Development in the Theory of NSI

Let me start by reviewing the development in the theory of NSI that occurred very recently. Since long time ago, it has been noticed \cite{11} that phenomenological study with NSI of the type (1) has a potential caveat. To get to the point, let us agree on the following understanding: At a high-energy scale where NSI originates the $SU(2) \times U(1)$ gauge invariance holds. Then, the left-handed neutrino field in the operator (1) must be elevated into the lepton doublet of $SU(3)$. When we require this an obvious problem occurs; The resultant four charged lepton operators have to obtain severe constraints from experiments. The most stringent is the one imposed by the branching ratio of $\mu \rightarrow eee$, $BR(\mu \rightarrow eee) \leq 10^{-12}$ \cite{18}, which would yield the constraint $|\epsilon_{e\mu}| \leq 10^{-6}$. Of course, nothing is wrong with it. But, we would like to avoid this because we are interested in observable effects in near (or even remote) future neutrino experiments. Therefore, people looked for the possible higher dimensional operators which are free from the charged lepton constraints.

Some candidates which were discussed by people (see, for example, \cite{19,20,21}) are:

\[
\mathcal{O}_6^a = (\bar{L}_\gamma \gamma_2 L^\alpha)(\bar{L}_\delta \gamma_2 L_\beta) \quad (2)
\]

for dimension six operator where $L^c = C \bar{L}^T$ and $C$ is the charge conjugation operator. For dimension eight operators they are of the type

\[
\mathcal{O}_8^a = (\bar{L}_\beta \gamma_\mu L_\alpha)(\bar{L}_\delta \tilde{H} \gamma_\rho (\tilde{H}^T L_\gamma). \quad (3)
\]

See, for example, \cite{11,22,21,23,24} for relevant references. Intuitive understanding of (3) is that the Higgs field v.e.v. projects out only the neutrino component of left-handed doublet. The meaning of (2) becomes clear by writing it in a form with obvious antisymmetry in flavor space \cite{24},

\[
2\mathcal{O}_6^a = (\bar{\ell}_\alpha \gamma_\mu \ell_\beta)(\bar{\nu}_\gamma \gamma_\mu \nu_\delta)+ (\bar{L}_\gamma \gamma_\mu \ell_\delta)(\bar{\nu}_\alpha \gamma_\mu \nu_\beta)- (\bar{L}_\alpha \gamma_\mu \ell_\delta)(\bar{\nu}_\gamma \gamma_\mu \nu_\beta)-(\bar{\ell}_\gamma \gamma_\mu \ell_\beta)(\bar{\nu}_\alpha \gamma_\mu \nu_\delta) \quad (4)
\]

which implies

\[
\varepsilon_\alpha^\beta = -\varepsilon_\gamma^\beta = -\varepsilon_\delta^\beta = \varepsilon_\alpha^\gamma = \varepsilon_\beta^\gamma = \varepsilon_\delta^\gamma. \quad (5)
\]

The antisymmetric nature prohibits, for example, $\varepsilon_{e\mu}^{ee}$ which would produce NSI effects in neutrino propagation in matter.

Recently, this problem of searching for higher dimensional operators without charged lepton constraints has come to conclusion; It has been proved that the above two possibilities are unique in dimension six and eight operators, respectively, if one wants to avoid the charged lepton constraints at the tree level \cite{23}.

Now, the question is: Is avoiding the tree level constraint sufficient to be free from the charged lepton constraints? It is known that the answer is NO \cite{22}. Namely, the
dressing by $W$ and $Z$ bosons can produce four charged lepton processes which leads to highly restrictive bound in some channels, in particular on $|\varepsilon_{e\mu}|$. See [22] (new version) for details of the type of bound, and for a summary of the other constraints on NSI.

It turned out, however, that it was NOT the end of the story. Biggio, Blennow and Fernandez-Martinez [24] have recently pointed out that the bounds on NSI have to be relaxed to a large extent, a factor of $\sim 10^4$. Because of the antisymmetric nature of the dimension six operator (2), the contributions of diagrams with different flavor indices tend to cancel and add up to zero in the limit of neglecting the lepton masses. Turning on lepton masses leaves the contribution of the order of $(m_\ell/M_W)^2$.

Notice that it is the unique dimension six operator which is free from the tree-level four charged lepton counterpart so that we have to live with it. Another significant feature is that the bound on $\varepsilon_{ee}$ goes away, or in other word, it must vanish by construction of the operator (2).

I would like to note here that an important feature is hidden behind my shallow description of their results; SU(2) gauge invariance. That is, imposing the gauge invariance is essential to obtain gauge invariant results of logarithmically divergent terms of the one-loop diagrams. Go to the original reference [24] for more complete understanding, in particular on the meaning of quadratically divergent terms.

4. NSI in Neutrino Propagation in Matter

In the rest of my presentation I discuss NSI effects in neutrino propagation in matter. It should be remarked, however, that NSI effects are present also in production and detection processes of neutrinos, so that my discussion is obviously incomplete.

To summarize its effects on neutrino propagation it is customary to introduce the $\varepsilon$ parameters, which are defined as $\varepsilon_{\alpha\beta} \equiv \sum_{f,P} \frac{n_f}{n_c} \varepsilon_{fP}^{\alpha\beta}$, where $n_f$ is the number density of the fermion species $f$ in matter. Notice that only the vector combination of the NSI can be probed when we discuss neutrino propagation in matter. Approximately, the relation $\varepsilon_{\alpha\beta} \simeq \sum_{P} \left( \varepsilon_{eP}^{eP} + 3 \varepsilon_{uP}^{uP} + 3 \varepsilon_{dP}^{dP} \right)$ holds because of a factor of $\simeq 3$ larger number of $u$ and $d$ quarks than electrons in isosinglet matter. Notice that with the dimension six operator (2) part of the first term, the ones from $\varepsilon_{e\mu}^{ee}$ and $\varepsilon_{e\tau}^{ee}$ is absent. I, however, choose to proceed with a generic framework.

Using the $\varepsilon$ parameters the neutrino evolution equation which governs the neutrino propagation in matter is given as

$$
\frac{id}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \frac{1}{2E} \left[ U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U^\dagger + a \begin{pmatrix} 1 + \varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu}^* & \varepsilon_{e\mu} & \varepsilon_{e\mu} \\ \varepsilon_{e\tau}^* & \varepsilon_{e\mu}^* & \varepsilon_{e\tau} \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}
$$

Here, I restrict my discussion into the dimension six operators. But, it is fair to note that this discussion is much more relevant for the dimension eight operators. See [24].
where $U$ is the MNS matrix, and $a \equiv 2\sqrt{2}G_F n_e E$ where $E$ is the neutrino energy and $n_e$ denotes the electron number density along the neutrino trajectory in the earth. $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$ with neutrino mass $m_i$ ($i = 1 - 3$). Notice that the phase of $\varepsilon$ parameters may provide new source of CP violation. Another important point is that complexity of the system in (6) would lead to confusion in determination of the mixing and the NSI parameters.

5. Perturbation Theory of Neutrino Oscillation with NSI

Obviously, I am a newcomer to the field of NSI. When I started to work on this topic I tried to understand the features of neutrino oscillations with NSI. Alas, I found that not so many things are known. The questions I would like to know the answer were:

- From the experience in neutrino oscillation with standard interaction (SI) I would expect that the appearance channel $\nu_e \rightarrow \nu_\mu$ (or, $\nu_e \rightarrow \nu_\tau$) has great sensitivities to tiny effects of NSI. Then, the natural question is: Which NSI elements of $\varepsilon_{\alpha\beta}$ in (6) give the dominant contribution to $P(\nu_e \rightarrow \nu_\mu)$? Or, more concretely, how large is the contribution of e.g., $\varepsilon_{e\mu}$, $\varepsilon_{e\tau}$, and $\varepsilon_{\mu\tau}$ to $P(\nu_e \rightarrow \nu_\mu)$?

- What about the disappearance channels though they may be less attractive? Namely, what is the size of contributions of $\varepsilon_{ee}$ in $P(\nu_e \rightarrow \nu_e)$? What is the relative importance of $\varepsilon_{e\mu}$, $\varepsilon_{e\tau}$, $\varepsilon_{\mu\mu}$, and $\varepsilon_{\mu\tau}$ in $P(\nu_\mu \rightarrow \nu_\mu)$?

I was amazed by the fact that apparently nobody knew the answers to these questions. I believe that the questions are not only due to academic interests. It is because I think that treating the full system is really necessary. Though people (including myself) do make approximations of ignoring some elements keeping only a few of them, but they do so without good reasons. It is even more so now because most of the stringent bounds on $\varepsilon_{\alpha\beta}$ based on lepton processes, the model-independent ones, are gone. Only when we recognize the correct theory at high energy scale we can be sure that the approximation he/she is making is the correct one.

5.1. Perturbation theory

To answer these questions and to have a global bird-eye view of neutrino oscillation with NSI we have formulated a perturbative framework. Unfortunately, there is no unique framework because we still do not know the value of $\theta_{13}$, though the bound on

\[ \text{It might be a too strong statement, given the fact that so many people are working in this field. Any comments are welcome. In fact, it appears that answer to these questions were known at least partly by Jacobo Lopez-Pavon in UAM, Madrid, though the result was unpublished.} \]
it exists. The only parameter which we know to be small is the ratio \( \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \approx 0.03 \). Therefore, we take an ansatz
\[
\epsilon \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \sim s_{13} \sim \varepsilon_{\alpha\beta} \sim 10^{-2} \quad (\alpha, \beta = e, \mu, \tau) \tag{7}
\]
to formulate our perturbation theory, which we called the “\( \epsilon \) perturbation theory” in \( \textit{15} \). In doing so I assume \( \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \) is of order unity, anticipating very long-baseline neutrino experiments such as neutrino factory \( \textit{28} \), or the beta beam \( \textit{29} \). We do not take \( \left( \frac{1}{\sqrt{2}} - s_{23} \right) \) as an expansion parameter because a rather large range is currently allowed and the situation will not be changed even with the next generation experiments \( \textit{30} \).

The \( \epsilon \) perturbation theory based on (7) is a natural generalization of the framework taken by Cervera et al. \( \textit{31} \) for systems with SI into the one with NSI. Note that the Cervera et al. formula (which we call the SI second-order formula) is the most widely used perturbative formula to discuss various aspects of neutrino oscillations. We derive the NSI second-order formula which generalizes the SI second-order formula to the case with NSI to have an overview of the neutrino oscillation phenomena in systems with NSI. A different but related perturbative approach to neutrino oscillation with NSI has been studied in references \( \textit{32,33,34,35,17,36,37} \).

In passing I have a few remarks on \( \theta_{13} \). It is a big question whether \( \theta_{13} \) falls into the range which can be explored by the next generation accelerator \( \textit{38,39,40} \) and the reactor \( \textit{41} \) experiments. In fact, I argue sometimes rather strongly that \( \theta_{13} \) must be large, for example in \( \textit{42} \). The belief is one of the motivations for my works which proposed reactor measurement of \( \theta_{13} \) \( \textit{43} \) and superbeam measurement of lepton CP violation \( \textit{44} \). Nevertheless, I am a pessimist here with the ansatz (7). Well, the reason why I take the ansatz of small \( \theta_{13} \) is that it is the only natural perturbative framework of neutrino oscillation. For instance, the appearance oscillation probability \( P(\nu_e \rightarrow \nu_\mu) \) consists only of order \( \epsilon^2 \) terms. If I take a different ansatz \( s_{13} \sim \sqrt{\epsilon} = \sqrt{\frac{\Delta m^2_{21}}{\Delta m^2_{31}}} \) (which roughly correspond to the Chooz limit \( \textit{27} \)), the terms in \( P(\nu_e \rightarrow \nu_\mu) \) do not scale uniformly and we would have to keep terms of order \( \epsilon^3 \) to include effects of CP violation. It necessitates to keep order \( s^3_{13} \) terms.

5.2. NSI second-order formula; \( \nu_e \)-related sector

How can one go to the NSI second-order formula from the SI second-order formula? Though the task might look formidable, it is in fact trivial in \( \nu_e \)-related channels! What is necessary is to make replacements in the atmospheric and the solar variables

\[ \varepsilon \] The basic reasoning for my belief is simple: The MNS matrix is the product of the two unitary matrices which diagonalize the neutrino and the charged lepton mass matrices. The two angles in the MNS matrix are known to be large. Then, why should the third one extremely small?
in the SI second-order formula and that’s it:

\[
\begin{align*}
\frac{s_{13}}{a} & \rightarrow s_{13} \frac{\Delta m^2_{31}}{a} + (s_{23} \varepsilon_{e\mu} + c_{23} \varepsilon_{e\tau}) e^{i\delta}, \\
\frac{c_{12}s_{12}}{a} & \rightarrow c_{12}s_{12} \frac{\Delta m^2_{21}}{a} + c_{23} \varepsilon_{e\mu} - s_{23} \varepsilon_{e\tau}.
\end{align*}
\]

It is very easy to understand why the particular combinations of \( \varepsilon \) parameters come into the atmospheric and the solar variables, respectively. It is well known \(^{16} \) that in doing perturbation theory the convenient basis is the tilde basis \( \tilde{H} = U^\dagger_{23} H U_{23} \).

The combinations of the \( \varepsilon \) parameters are the ones that appear in the NSI part of \( \tilde{H}_{e3} \) and \( \tilde{H}_{e2} \), analogues of “\( \sin \theta_{13} \)” and “\( \sin \theta_{12} \)” (See equation (15) in \(^{15} \)).

The resultant NSI second-order formula in \( \nu_e \rightarrow \nu_\mu \) channel reads \(^{15} \)

\[
P(\nu_e \rightarrow \nu_\mu) = 4 \left| c_{23} \left( c_{12}s_{12} \frac{\Delta m^2_{21}}{a} + c_{23} \varepsilon_{e\mu} - s_{23} \varepsilon_{e\tau} \right) \sin \left( \frac{a L}{4E} \right) \exp \left( -i \frac{\Delta m^2_{31} L}{4E} \right) \\
+ s_{23} \left( s_{13} e^{-i\delta} \frac{\Delta m^2_{31}}{a} + s_{23} \varepsilon_{e\mu} + s_{23} \varepsilon_{e\tau} \right) \left( \frac{a}{\Delta m^2_{31} - a} \right) \sin \left( \frac{\Delta m^2_{31} - a L}{4E} \right) \right|^2.
\]

I hope the readers are convinced of my claim that the formula is surprisingly simple in its form. \( P(\nu_e \rightarrow \nu_\tau) \) can be obtained by doing the transformation \( c_{23} \rightarrow -s_{23} \) and \( s_{23} \rightarrow c_{23} \) in \( P(\nu_e \rightarrow \nu_\mu) \), but undoing any transformation in the generalized atmospheric and the solar variables defined in \(^{8} \).

A notable feature in \(^{8} \) is that only the elements \( \varepsilon_{e\mu} \) and \( \varepsilon_{e\tau} \) appears in the NSI second order probability formula. Because of the decoupling of the other \( \varepsilon \)'s it is in principle possible to determine \( \varepsilon_{e\mu} \) and \( \varepsilon_{e\tau} \) together with the SI parameters \( \theta_{13} \) and \( \delta \), 6 real parameters including phases. If one carries out this task by rate only analysis we need measurement of the oscillation probabilities in the following three channels \( \nu_e \rightarrow \nu_\mu, \nu_e \rightarrow \nu_\tau, \) and \( \nu_\mu \rightarrow \nu_e \) and their antineutrino counterpart.

5.3. NSI second-order formula; \( \nu_\mu - \nu_\tau \) sector

In \( \nu_\mu - \nu_\tau \) sector the situation is different. The NSI dependent piece in the oscillation probabilities \( P(\nu_\mu \rightarrow \nu_\tau), P(\nu_\mu \rightarrow \nu_\mu) \), and \( P(\nu_\tau \rightarrow \nu_\tau) \) is universal. See \(^{15} \) for explicit expressions. Because of this feature one cannot determine all the relevant NSI elements \( \varepsilon_{\mu\tau} \) and \( \varepsilon_{\mu\mu} - \varepsilon_{\tau\tau} \) (only the difference can be measured), 3 unknowns, by the rate only analysis.

The reasons for such curious feature of the universal NSI dependent term is simple to understand. By unitarity it follows that

\[
\begin{align*}
P(\nu_\mu \rightarrow \nu_\mu) + P(\nu_\mu \rightarrow \nu_\tau) &= 1 - P(\nu_\mu \rightarrow \nu_e), \\
P(\nu_\tau \rightarrow \nu_\tau) + P(\nu_\tau \rightarrow \nu_\mu) &= 1 - P(\nu_\tau \rightarrow \nu_e).
\end{align*}
\]

\(^{10} \)
We note that $P(\nu_\mu \to \nu_\epsilon)$ and $P(\nu_\tau \to \nu_\epsilon)$ do not contain $\varepsilon_{\mu\mu}$, $\varepsilon_{\tau\tau}$, and $\varepsilon_{\mu\tau}$ to second order in $\epsilon$. Then, it follows from the first equation in (10) that $P(\nu_\mu \to \nu_\tau; \varepsilon_{\mu\mu}, \varepsilon_{\mu\tau}, \varepsilon_{\tau\tau}) = -P(\nu_\mu \to \nu_\mu; \varepsilon_{\mu\mu}, \varepsilon_{\mu\tau}, \varepsilon_{\tau\tau})$. Noticing that the terms related to $\varepsilon$’s in the $\nu_\mu - \nu_\tau$ sector are $T$-invariant, the relations $P(\nu_\tau \to \nu_\tau; \varepsilon_{\mu\mu}, \varepsilon_{\mu\tau}, \varepsilon_{\tau\tau}) = -P(\nu_\mu \to \nu_\tau; \varepsilon_{\mu\mu}, \varepsilon_{\mu\tau}, \varepsilon_{\tau\tau})$ must also hold. Therefore, the $\varepsilon_{\alpha\beta} \ (\alpha, \beta = \mu, \tau)$ dependent term in the three channels are all equal up to the over-all sign. The necessity of spectrum analysis is obvious to determine all the NSI and the SI parameters.

5.4. Summary table

To answer to the questions raised above I present below the summary table. One of the features in Table 1 which requires comment is that in the last column. In standard neutrino oscillation only with SI the matter effect comes in into the oscillation probability only at the second order in $\epsilon$, the property dubbed “matter hesitation” in 15. It is the reason why it is so difficult to detect the matter effects in many accelerator neutrino experiments including NO$\nu$A 39. The matter hesitation is a highly nontrivial property because we treat the coefficient $a$ (I mean, $a/\Delta m_{31}^2$) as of order unity. For example, there exists first order $a$ dependent term in the $S$ matrix, but it does not survive in $P(\nu_\epsilon \to \nu_\epsilon)$ because it enters as a phase factor. Notice, however, that its validity relies on the particular framework of perturbation theory. For a (simple!) proof of this property see 15.

Table 1: Presented are the order in $\epsilon$ ($\sim 10^{-2}$) at which each type of $\varepsilon_{\alpha\beta}$ $(\alpha, \beta = \epsilon, \mu, \tau)$ and $a$ dependence ($a$ is Wolfenstein’s matter effect coefficient) starts to come in into the expression of the oscillation probability in $\epsilon$ perturbation theory. The last column is for the $a$ dependence in the standard oscillation without NSI. The order of $\epsilon$ indicated in parentheses implies the one for the maximal $\theta_{23}$ in which cancellation takes place in the leading order.

| Channel | $\varepsilon_{\epsilon\epsilon}$ | $\varepsilon_{\epsilon\mu}$ | $\varepsilon_{\epsilon\tau}$ | $\varepsilon_{\mu\mu}$ | $\varepsilon_{\mu\tau}$ | $\varepsilon_{\tau\tau}$ | a dep.(NSI) | a dep.(SI) |
|---------|-------------------------------|-----------------------------|-----------------------------|--------------------------|--------------------------|--------------------------|-------------|------------|
| $P(\nu_\epsilon \to \nu_\alpha)$: | $\epsilon^3$ | $\epsilon^2$ | $\epsilon^2$ | $\epsilon^3$ | $\epsilon^3$ | $\epsilon^2$ | $\epsilon^2$ | $\epsilon^2$ |
| $\alpha = \epsilon, \mu, \tau$ | | | | | | | | |
| $P(\nu_\alpha \to \nu_\beta)$: | $\epsilon^3$ | $\epsilon^2$ | $\epsilon^1$ | $\epsilon^1(\epsilon^2)$ | $\epsilon^1(\epsilon^2)$ | $\epsilon^1$ | $\epsilon^2$ |
| $\alpha, \beta = \mu, \tau$ | | | | | | | | |

One of the implication of the matter hesitation is that $\varepsilon_{\epsilon\epsilon}$ comes into the oscillation probability at order $\epsilon^3$ in all channels, as indicated in Table 1. It is because $\varepsilon_{\epsilon\epsilon}$ is nothing but a small shift of the matter effect coefficient $a$. Because of this property it is very difficult to measure $\varepsilon_{\epsilon\epsilon}$ in long-baseline experiments. It should be remarked, however, that assuming that the other NSI elements are vanishingly small it can be measured in a great precision of a few % at a neutrino factory 47,48. But, I must

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1 We emphasize that this feature itself is highly nontrivial, and can be realized only by an explicit computation.
note that it is only true under the assumption that the earth matter density along
the neutrino trajectory is accurately known.

Another notable feature in Table. 1 is that there exists first order term of NSI
element $\varepsilon_{\mu\tau}$ (and $\varepsilon_{\mu\mu} - \varepsilon_{\tau\tau}$ if $\theta_{23} \neq \frac{\pi}{4}$) in the $\nu_\mu - \nu_\tau$ sector. Clearly, they are due to
direct transition caused by these NSI elements. In fact, rather high sensitivities for
determining $\varepsilon_{\mu\tau}$ and $\varepsilon_{\mu\mu} - \varepsilon_{\tau\tau}$ observed in atmospheric\footnote{49} and future accelerator\footnote{50} neutrino analyses are benefited by this feature.

5.5. SI–NSI confusion

The structure of the oscillation probability (9) in which $\varepsilon_{e\alpha}$ enters into the ex-
pression only through the generalized solar and the atmospheric variables (8) is one
of the most significant features of the oscillation probability with NSI. It also implies
that, when the parameter determination is attempted, there exists severe confusion
between the SI and the NSI parameters. Hence, our result may be regarded as an
analytic proof of the general “confusion theorem” in $\varepsilon$ perturbation theory.

The uncovered structure may be helpful to formulate a strategy of resolving the
confusion, because (8) clearly dictates which SI parameters will be confused by which
NSI variables by which way. Notice that our confusion theorem is quite different in
nature from the one proved in\footnote{26} in which $\theta_{13}$ is confused with the NSI elements in
production and detection processes.

5.6. Parameter degeneracy in neutrino oscillation with NSI

It is now well understood that phenomenon of parameter degeneracy, existence of
the multiple solutions, occurs in neutrino oscillation measurement of SI parameters
\footnote{51,52,53}. Because of the large number of unknown (i.e., to be determined) param-
eters (2 standard and 8 NSI ones) the parameter degeneracy in the full system is a
formidable problem to work out, even under the approximation of ignoring NSI effects
in production and detection of neutrinos.

Yet, it was possible to recognize a completely new type of degeneracy\footnote{15}. Let us
denote the generalized atmospheric and the solar variables in (8) in section 5.2 as $\Theta_\pm$
and $\Xi e^{-i\delta}$, respectively. Then, if there is a solution $|\Theta_\pm^{(1)}|$ and $|\Xi^{(1)}|$, then the second
solution $|\Theta_\pm^{(2)}| = \sqrt{\frac{X_\pm}{Z_\pm}}|\Xi^{(1)}|$ and $|\Xi^{(2)}| = \sqrt{\frac{X_\pm}{Z_\pm}}|\Theta_\pm^{(1)}|$ exists. (See\footnote{15} for definitions
of $X_\pm$ etc.) It can be called the “atmospheric–solar variables exchange” degeneracy,
which arises because of large number of unknown parameters in the solar and the
atmospheric variables. Of course, it does not survive when NSI is turned off because
there is no solar degrees of freedom (as to be determined parameters) as can be seen
in (8).

What is the right way in this difficult problem of degeneracy in systems with
NSI? As a first step, we have worked out the problem in a region where the matter
effect can be treated as a perturbation. For early references of matter perturbation theory, see e.g., \[54, 55\]. It is known that analysis of the parameter degeneracy becomes particularly transparent in this setting \[52, 56, 57, 58\].

Our analysis is most transparent in the “discrete” degeneracy, the sign-$\Delta m_{31}^2$ and the octant ones. Let me describe first the sign-$\Delta m_{31}^2$ degeneracy. One can show that the NSI dependent terms in the oscillation probability $P(\nu_e \rightarrow \nu_{\alpha})$ ($\alpha = \mu, \tau$) to first order in $a$ is invariant under the transformation

$$
\begin{align*}
\Delta m_{31}^2 & \rightarrow -\Delta m_{31}^2, \\
\delta & \rightarrow \pi - \delta, \\
\phi_{e\alpha} & \rightarrow 2\pi - \phi_{e\alpha},
\end{align*}
$$

(11)

while keeping $\theta_{13}$ and $|\varepsilon_{e\alpha}|$ fixed. It nicely complements the discussion in \[52\] and it indicates that there exists a new (approximate) solution with differing sign of $\Delta m_{31}^2$. It is worth to note that the invariance is true only if the CP phase of NSI element is involved in the transformation \[11\].

Similarly, one can show that there is another invariance under the transformation (assuming $\theta_{23} \neq \frac{\pi}{4}$)

$$
\begin{align*}
c_{23} & \rightarrow s_{23}, \\
s_{23} & \rightarrow c_{23}, \\
(\varepsilon_{\mu\mu} - \varepsilon_{\tau\tau}) & \rightarrow -(\varepsilon_{\mu\mu} - \varepsilon_{\tau\tau}).
\end{align*}
$$

(12)

It means that the $\theta_{23}$ octant degeneracy prevails in the presence of NSI, and actually in an extended form which involves NSI parameter $\varepsilon_{\mu\mu} - \varepsilon_{\tau\tau}$. Since this NSI parameter decouples from $P(\nu_\mu \rightarrow \nu_e)$ to second-order in $\epsilon$, the presence of the $\theta_{23}$ octant degeneracy remains intact when the NSI is included though values of the degenerate solutions themselves are affected by the presence of $\varepsilon_{e\alpha}$. Thus, we have shown that the parameter degeneracy survives the presence of NSI provided that NSI elements are “actively involved” in the degenerate solutions.

The other salient feature of degeneracy in the mass perturbative regime is the property called the “decoupling between degeneracies” \[58\]. We have revisited this issue in the systems with and without NSI. The conclusion obtained in \[15\] is that the decoupling between the sign-$\Delta m_{31}^2$ and the $\theta_{23}$ octant degeneracies holds with and without NSI. On the other hand, decoupling between them and the intrinsic degeneracy does not holds with and without NSI, partly correcting the conclusion in \[58\]. The only exception is the special setting at the oscillation maximum, or more precisely, the shrunk ellipse limit \[59\], for which the decoupling holds.

5.7. Parameter degeneracy; An example
An example of the parameter degeneracy is presented in Fig. 1. This is one of the examples found in doing the work\textsuperscript{16} but is not presented in the reference. It clearly demonstrates the existence of the $\Delta m^2_{31}$-sign flipped and the intrinsic degeneracies, the natural extension of the one\textsuperscript{51, 52} to the systems with NSI. As pointed out in section 5.6 the phase $\phi_{e\mu}$ of the NSI element is indeed heavily involved\textsuperscript{15} though the relation (11) which is valid in the mass perturbative regime does not quite hold. In this example (as well as in the one which is presented in\textsuperscript{16}), the far detector measurement successfully lift the $\Delta m^2_{31}$-sign flipped degeneracy, but not and the intrinsic one.

How robust is the degeneracy in system with NSI? It is a difficult question to answer in general. But, it is worth to remark that sometimes the degeneracy is extremely hard to solve because the energy spectra corresponding the degenerate solutions are so similar\textsuperscript{8}.

\textsuperscript{8} It is often the case that in systems only with SI that the intrinsic degeneracy can be “easily”
values of the SI and NSI parameters. See Fig. 13 in16 for the corresponding figures for the $\varepsilon_{e\mu}$ system with exactly the same feature.

6. Discriminating CP violation due to SI and NSI phases

I emphasize that one of the most important features of the system with NSI is the coexistence of two kind of CP violation14, the one due to $\delta$ in the MNS matrix1, the leptonic version of the celebrated Kobayashi-Maskawa (KM) phase13 in the CKM matrix13,60 for quarks, and the other which come from the phases of NSI elements. Knowing the nature of CP violation seen in any kind of experiments61 is of decisive importance because of many reasons, in particular for possible connection to leptogenesis scenario62, currently the most promising one for baryon number generation in the universe.

6.1. Two-detector setting

I have discussed in the last year in Venice the possibility of resolving the $\theta_{13}$—NSI confusion25,26 by the near (3000 km)—far (7000 km) two-detector setting in neutrino factory based on our work17. I use the same setting to examine the question of whether the two-phase confusion can be resolved16. For a related work on the same subject see63. The similar question of distinguishing two kind of CP violation in resolved by the spectrum analysis. See for example, in the case of T2K or T2KK settings57,68.
the context of “unitarity violation” approach [64] has also been investigated in [65,66].

Setting of the second detector at around the magic baseline $\approx 7000$ km was motivated by high sensitivity to the matter effect [47]. It is in concordant with the similar two detector setting in a neutrino factory as a degeneracy solver [51,67,68]. Two-detector setting has been proposed in neutrino experiments in a variety of contexts [51,67,68]. The basic idea in the present context is to seek complementary role played by the far detector.

6.2. Use of the bi-probability plot

Though I told you that derivation of the NSI second-order formula of the oscillation probability is simple, it means neither that the dynamics of the system is simple, nor it is easy to understand. What makes the system so complicated is the very existence of two CP violating phases, $\delta$ and the phase $\phi_{\epsilon\alpha}$ of the NSI element $\epsilon_{\epsilon\alpha} = |\epsilon_{\epsilon\alpha}| e^{i\phi_{\epsilon\alpha}} (\alpha = \mu, \tau)$. In my talk, therefore, I report the work done with only a single type of NSI, either $\epsilon_{e\mu}$ or $\epsilon_{e\tau}$, as a first step of understanding the features of neutrino oscillation with NSI.

How complicated is the system with NSI? Seeing is believing. Presented in Fig. 3 is the bi-probability plot in $P(\nu_e \to \nu_\mu) - P(\bar{\nu}_e \to \bar{\nu}_\mu)$ space but by varying the two phases, $\delta$ and $\phi_{\epsilon\mu}$. As you see the ellipses move around in the plane such that the whole triangular region is (almost) swept over. So “anything can happen” with the two phases. We have characterized the behavior of ellipses as rotating ellipses in [16]; An ellipse drawn by varying the phase A rotates when the other phase B is varied. Because of the behavior of the probabilities rich phenomena such as confusion between SI and NSI parameters and the parameter degeneracy are expected.

In fact, various viewpoints have to be involved to really understand features of neutrino oscillations with NSI and the sensitivities to the NSI elements, $|\epsilon_{\epsilon\alpha}|$ and $\phi_{\epsilon\alpha}$ ($\alpha = \mu, \tau$), and the SI parameters, $\delta$ and $\sin^2 2\theta_{13}$, to be achieved by the detectors at $L = 3000$ km and $L = 7000$ km separately and in combination. They include:

- How prominent is the synergy between the near and the far detectors for determination of SI and NSI parameters? How it differs between the systems with

---

b Here is a brief history of two-detector settings in contemporary neutrino experiments: It was proposed as an appropriate setting for measuring CP violation [49] in the context of low energy superbeam experiment [69]. In a quite different context of reactor measurement of $\theta_{13}$ [44] the two-detector setting is the standard one to guarantee the near-far cancellation of systematic errors. It has triggered interests in the world wide scale [70], and led to the several international collaboration experiments [11]. The idea has also been applied to the Kamioka-Korea two detector complex with an upgraded neutrino beam from J-PARC to determine the mass hierarchy as well as discovering CP violation [57,68]. It “unifies” the two aspects of near-far cancellation and synergy between the two detectors, and can serve for a possible upgrade option of the T2K experiment [35]. For an overview of T2KK, see e.g., [71], and for a review of the two-detector setup [72].
Figure 3: Bi-probability plots in $P(\nu_e \rightarrow \nu_\mu) - P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$ space at $L = 3000$ km, for $E = 20$ GeV, with $\sin^2 2\theta_{13} = 10^{-3}$ and $\varepsilon_{e\mu} = 5 \times 10^{-3}$ computed numerically using the constant matter density $\rho = 3.6$ g/cm$^3$ assuming the electron number density per nucleon of 0.5. The both axes is labeled in units of $10^{-4}$. The values of the parameters taken are $\varepsilon_{e\mu} \sin^2 2\theta_{13}$

$\varepsilon_{e\mu}$ and $\varepsilon_{e\tau}$? What about dependence on values of the parameters, in particular on the size of NSI elements?

- How can the two-phase confusion be resolved? Does the answer depend on which NSI element is turned on?

- What is the nature of the parameter degeneracy in system with NSI, and whether it can be resolved by the two-detector setting? If so how can it be realized?

These points are fully discussed in our paper [16].

Here, we make comments only on a puzzle. The difference in sensitivity to NSI has non-trivial features. At relatively large values of NSI, $\varepsilon_{e\mu} = 10^{-3}$ and $\varepsilon_{e\tau} = 10^{-2}$, the size of the ellipses are similar in size. But, the sensitivity is in fact very different between the two systems; The one in the $\varepsilon_{e\mu}$ system is much higher than that in the $\varepsilon_{e\tau}$ system. On the other hand, the parameter degeneracy is much severer at the near detector in systems with $\varepsilon_{e\mu}$ compared to the ones with $\varepsilon_{e\tau}$ as is shown in the tables.

A unified understanding of the puzzling features becomes possible once one draws the bi-probability plots by varying $\delta$ for several different values of $\phi$ and $\theta_{13}$, as done in Fig. 4. The degeneracy is severer in the $\varepsilon_{e\mu}$ system because of the more dynamic behavior of the bi-probability ellipses as shown in the left panel of Fig. 4. Because the ellipses can locate themselves essentially everywhere in the bi-probability space there are chances that fake solutions can be produced at points far apart from the true solution. What about the difference in the sensitivities? We observe in the right
Figure 4: Bi-probability plots drawn by continuously varying $\delta$ for four different values of $\phi$. 

panel of Fig. 4 that the ellipses in the $\epsilon_{ee\tau}$ system remain in much more compact region when $\delta$ and $\theta_{13}$ are varied. Because of the finite resolution of the experimental data it appears that the dense concentration of the ellipses with different parameters leads to merging of many degenerate solutions. It probably explains lack of the sensitivities and at the same time much less frequent degenerate solutions in the $\epsilon_{ee\tau}$ system.

7. Discovery Potentials

This is the appropriate point to discuss the discovery potential of various quantities, $|\epsilon_{ee\mu}|$, $|\epsilon_{ee\tau}|$, $\phi_{ee\mu}$, $\phi_{ee\tau}$, as well as the standard parameters $\delta$ (and $\theta_{13}$ in principle) and the neutrino mass hierarchy. In this report we focus on CP violation caused by the lepton KM phase $\delta$, and the phase $\phi$ of NSI. The discovery potential for the rest of the quantities are discussed in [16].

By the way, I remind you that all the figures presented in this manuscript are new, i.e., no single figure which is identical to the one in [16]. To keep this tradition I will always give in this manuscript the sensitivity regions calculated with the inverted mass hierarchy as input. Notice that all the sensitivity plots given in [16] are calculated by taking the normal hierarchy as input. Great thanks to Hiroshi Nunokawa for his efforts to prepare them for this manuscript.

While I do not give any details of the quantitative analysis in this manuscript (for which see [16]), it should be remarked that all the systematic errors as well as backgrounds are ignored in our analysis. Here, I explain the reasons for this choice in my own language. The dimension six operator that can give neutrino’s NSI without producing unwanted four charged lepton NSI is unique [23], the anti-symmetrized one given in Eq. (2) in section 3. Then, unless someone is able to show that only the
dimension six operator naturally arises in a certain class of models of new physics at TeV scale\(^1\), we must prepare for search for the dimension eight (or higher) operators, with the size \(\varepsilon \sim 10^{-4}\) assuming no extra suppression.\(^2\) If it turned out to be the case one must think of the experimental technology which can accommodate this request. Since its realization is not known, we invented a model experiment using the neutrino factory setting with an ideal detector for which the systematic errors and background are ignored.

7.1. CP violation due to NSI

Let us start with the sensitivity to CP violation caused by the phase \(\phi\) of NSI. In Fig. 5 and Fig. 6 the regions sensitive to non-standard CP violation due to NSI are presented in \(\phi - |\varepsilon|\) space. In these regions one can detect non-standard CP violation by NSI \((\phi \neq 0 \text{ and } \phi \neq \pi)\) at 2\(\sigma\) (red thin lines) and 3\(\sigma\) (blue thick lines) CL. Fig. 5 and Fig. 6 are for the \(\varepsilon_{e\mu}\) and the \(\varepsilon_{e\tau}\) systems, respectively.

![Diagram of CP violation due to NSI](image)

Figure 5: Regions where the non-standard CP violation caused by \(\phi_{e\mu} \neq 0\) or \(\phi_{e\mu} \neq \pi\) can be established for the case \(\sin^2 2\theta_{13} = 10^{-3}\), \(\delta = \pi\) (left panel) and \(\delta = 3\pi/2\) (right panel). The inverted mass hierarchy is assumed as the input.

By comparing the Fig. 5 and Fig. 6 to Figs. 18 and 20 in \(^{10}\), respectively, one notices several notable differences between the normal and the inverted mass hierarchies. In the \(\varepsilon_{e\mu}\) system the sensitivities to non-standard CP violation at \(\delta = \pi\) are significantly worse both at the near detector (3000 km) and the near-far (7000 km) combined in comparison to those obtained with the input normal hierarchy. The results at \(\delta = 3\pi/2\), however, are very similar to the case of normal mass hierarchy.

\(^1\) From the reasoning below, I think it important to pursue this possibility.

\(^2\) I have assumed throughout this report that discussions on NSI in the lepton sector applies to the operators which involve neutrinos and quarks. Though I think it reasonable it can be subject to criticism.
Figure 6: The same as in Fig. 5 but for the non-standard CP violation caused by $\phi_{e\tau} \neq 0$ or $\phi_{e\tau} \neq \pi$.

A somewhat curious behavior seen in the upper-right panel of Fig. 5, no sensitivity to non-standard CP violation at the maximal CP violating inputs, $\phi = \pi/2$ and $\phi = 3\pi/2$ is explained as a consequence of the parameter degeneracy, the one called the $\phi$ degeneracy in [16].

In the $\varepsilon_{e\tau}$ system the sensitivities to non-standard CP violation are similar to the normal hierarchy case. The most notable difference is in the $\delta = \pi$ case; At the near detector the sensitivity to non-standard CP violation is a bit worse than that of the normal hierarchy case, but curiously enough it is a little better when the far detector is combined. It can well be the case because the features of synergy between the two detectors are highly nontrivial [17,16].

7.2. Standard CP violation

Let us go back to the sensitivity to CP violation due to the KM phase $\delta$. In Fig. 7 the regions sensitive to the standard CP violation due to $\delta$ are presented in the system without NSI. They are significantly worse than the normal hierarchy case given in Fig. 22 in [16]. Most probably, it is due to relatively smaller number of events in the antineutrino channel. Notably a peninsula like region without sensitivity develops from $\delta \simeq 0.8\pi$ to $\delta \simeq 0.5\pi$.

The gross features of the sensitivity regions remain unchanged even when the NSI degrees of freedom is turned on, as can be seen in Fig. 8. The sensitivities to the standard CP violation are slightly worse compared to the normal hierarchy case given in Fig. 23 in [16].

I must warn the readers that the sensitivity contours are unstable to inclusion of the systematic errors and backgrounds. Yet, I suspect that these sensitivity regions are similar to the ones obtained with the apparatus which can explore the NSI in the
whole region down to $|\varepsilon| \sim 10^{-4}$, the sensitivity which I argued to be necessary in the future NSI search.

Figure 7: Sensitivity to discovery of standard CP violation. Here no effect of NSI is assumed in the input data or in the fit. The upper panel shows the case where only the detector at 3000 km is considered, whereas the lower panel is the case corresponding to the combination of detectors at two different baselines. The inverted mass hierarchy is assumed as input.

Figure 8: The similar plots as in Fig. 7 but with non-zero NSI allowed in the fit; The input data was generated assuming the inverted mass hierarchy without NSI but non-zero values of $\varepsilon_{\mu\tau}$ (left panel) and $\varepsilon_{e\tau}$ (right panel) were allowed in the fit.

8. Conclusion

After reviewing the theoretical progresses on NSI recently made in section 3, I tried to explain the works done by our group on theoretical and phenomenological aspects of hunting the NSI in sections 5-7. I guess the former contributed to illuminate the global features of neutrino oscillation with NSI including method for parameter determination and recognition of the parameter degeneracy with NSI. While in the
latter we have investigated the problem of discriminating the two kind of CP violation, one due to the standard KM phase and the other by phases of NSI elements.

It appears that the near (3000 km) − far (7000 km) two detector setting in neutrino factory does have a rather high sensitivity to explore $|\varepsilon_{e\mu}|$ to $\simeq 10^{-4}$ in a lucky region of $\phi$ and to $\simeq$ several $\times 10^{-4}$ in most of the region of $\phi$. The sensitivity to $|\varepsilon_{e\tau}|$ is lower but still it can be explored to $\simeq 10^{-3}$. See Figs. 14-17 in [16]. The sensitivity to CP violation is also very good; The one due to NSI phase can be probed to $|\varepsilon_{e\mu}|$ to $\simeq$ several $\times 10^{-4}$ in most of the region of $\delta$ and $\phi_{e\mu}$. They are close but not quite the lower end of the required full region for exploration of NSI due to TeV scale new physics. Moreover, our estimation ignores the systematic errors and backgrounds, and hence is overly optimistic one.

How can this situation be overcome? Honestly, I don’t know the answer. However, a few comments may be made:

- Effects of NSI in production and detection of neutrinos, which are completely ignored in our works, can of great help.
- One can formulate the good enough reasoning to convince people that search for neutrino’s NSI to $|\varepsilon| \sim 10^{-2}$ is sufficiently informative to signal new physics at TeV scale.

However, I guess the former possibility is not easy to be realized. Nonetheless, I would like to recall that no discovery done in the past was an easy one. Also, upon identification or grasp of the new physics we should obtain the clearer view. A second eel can be a big one!

9. Bibliographical Note

Given the large number of references devoted to the subject of NSI it is not easy to find the appropriate one, in particular for a newcomer as I was sometime ago. Therefore, I tried some efforts to collect them here with classification by subjects. Yet, it is extremely difficult to find all of them, and therefore, the list should be considered as an incomplete one. I would like to apologize to those who will find their references missed. The categories I use are:

- Accelerator neutrinos; neutrino factory [25, 26, 32, 17, 16, 73].
- Accelerator neutrinos; excluding neutrino factory [35, 50, 74].
- Atmospheric neutrinos [14, 49, 75].
- Reactor or spallation source neutrinos or low-energy scattering [76].
- Solar neutrinos [77].
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