Research on the efficiency of equipment repair works based on the random risk process

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Abstract. In order to increase the efficiency of equipment repair work it is proposed to introduce an insurance fund that performs two functions: accumulation of payments with different frequency and cost for performing various types of repair work; payment of these works if it is necessary. The random risk process is used for the mathematical description of the insurance fund. This approach allowed us to introduce the indicators of efficiency of repair works as resource-cost risk, numerical average operating time and numerical gamma-percent resource which takes into account the fact of unrealized repair works due to lack of financial resources. A modeling program based on the event approach was created to study these indicators. Experiments based on modeling program allowed us to conclude that both risk values and reliability indicators should be preferred the variant when the frequency of payments to the insurance fund depends on the type of repair work.

1. Introduction
Modern automated equipment is complex and multicomponent. It operates in conditions of uncertainty, which complicates its monitoring, diagnostics, maintenance and repair. Monitoring and diagnostics of equipment allow us to predict possible failures, organize the maintenance, and repair work. [1, 2].

Now the education of specialists by programs related to automation the technological processes, production and transport means provides the study directed to calculating the reliability values of repairable and non-repairable objects, methods of estimating the reliability values non-redundant and redundant systems, principles of equipment construct that provides reliable systems [3, 4]. Less attention is aimed to servicing complex robotic equipment under uncertainty and limited financial resources. Although, it is also an important area related to using efficiency [5, 6].

It is assumed that there is an insurance fund to increase the efficiency of repair works. The insurance fund performs two functions [5, 7]: accumulation of payments with different frequency and cost for performing various types of repair work, for example: a) current; b) emergency; c) major. For each type of work are established the frequency of replenishment of the insurance fund (day) and their cost (million rubles); payment of these works if it is necessary. Moreover, the frequency of using the insurance fund (day) and their cost (million rubles) are established for each type of work. Those variables are random with known functions of distribution up to the values of their parameters.

It is proposed to use a random risk process for the mathematical description of the insurance fund in the efficiency study of repair works of complex equipment [8].
2. Mathematical description of efficiency indicators based on random risk process

First of all, the value of payments for various types of automated equipment repair works is determined ($X$, million rubles). Further, the value of payments is distributed by types of work.

$$X^{(1)} = c_1 \cdot X, \quad X^{(2)} = c_2 \cdot X, \quad X^{(3)} = c_3 \cdot X; \quad c_1 + c_2 + c_3 = 1. \quad (1)$$

Here $c_j$ are the coefficients that take into account a part of the payments for current, emergency and major repairs, $X^{(j)}$ are annual payments to the insurance fund for these works, $j = 1, 2, 3$. The cost of one payment to the insurance fund for the $j$-th type repair work based on (1)

$$Y_j = h_j \cdot X^{(j)} / T_g = c_j \cdot h_j \cdot X / T_g, \quad j = 1, 2, 3, \quad (2)$$

where $T_g$ is the number of days in a year; $h_j$ is the frequency of payments to the insurance fund (day) for the $j$-th type of work. These time intervals are determinate variables in that research.

The total accumulations of payments to the insurance fund for $j$-th type repair works both based on (1) and (2) and assumptions of replenishment frequency are equal

$$Y_j(t) = Y_j \cdot N_j(t) = (c_j \cdot h_j \cdot X / T_g) \cdot N_j(t), \quad j = 1, 2, 3, \quad (3)$$

where $N_j(t)$ is the number of payments to the insurance fund over time $t$ for the $j$-th type of work.

The time in case of emergency situations are described by the process

$$T_j = T_{j,i} + t, \quad i = 1, 2, ..., N_a(t), \quad T_0 = 0, \quad (4)$$

where $t_i$ is time intervals between emergency situations; $N_a(t)$ is the number of emergency situations during time $t$. Time $T_i$ (4) corresponds to the cost of eliminating the $i$-th emergency situation $- Z_i$. It is a random variable with a known distribution function. So the total cost of repair work in emergency situations for time $t$ are equal.

$$YA(t) = \sum_{i=1}^{N_a(t)} z_i. \quad (5)$$

Similarly to (4), we can propose processes that describe situations related to current and capital work. Then the total costs for these works are equal to:

$$YT(t) = \sum_{i=1}^{N_c(t)} u_i, \quad (6)$$

where $u_i$ is the value of the cost of the $i$-th current work; $N_c(t)$ is the number of current work during time $t$;

$$YK(t) = \sum_{i=1}^{N_m(t)} \omega_i, \quad (7)$$

where $\omega_i$ is the value of the cost of the $i$-th major work; $N_m(t)$ is the number of major works over time $t$. The time intervals and the cost in that research are random variables with known distribution for example: gamma, Weibull, etc.

The state of the insurance fund in proposed notations is described by a random risk process

$$R(t) = X_0 + Y1(t) + Y2(t) + Y3(t) - YA(t) - YT(t) - YK(t), \quad (8)$$

where $X_0$ is the initial funds of the insurance fund for the year; $Yj(t)$ is the total accumulation of payments by type of work (3) at time $t$, ($j = 1, 2, 3$); $YA(t)$ is the total cost for emergency situations (5); $YT(t)$ is the total cost for current work (6); $YK(t)$ is the total cost for major work (7).

The point of time $\tau$ for the random risk process (8) is determined if $R(t) < 0$ (there are no financial resources for repair work).

For the random risk process (8), the time instant $\tau$ is determined when the condition $R(t) < 0$ is fulfilled for the first time (there are no financial resources to carry out repair work),

$$\tau = \min \{ t : R(t) < 0 \}. \quad (9)$$
The point of time (9) depends on the organization's efficiency of repair work in turn of the payments distribution by types (3).

It is proposed to use the method of simulation modeling and created a simulation program based on the event approach to research the random risk process (8). This program reproduces three processes for received payments by types of work and three processes for the cost of these works.

The possible process implementations of the process (8) are shown in Figure 1. It was received by the simulation modeling: the situation when the process does not cross the time axis during the simulation time (sample value (9) is not created) is shown on Figure 1,a; the situation when a sample value (τ) is created is shown on Figure 1,b.

**Figure 1.** Graphical representation of a random risk process (8):

a – no sample value; b – sample value (τ) is created

As a result of multiple process modeling (8), for the variable (9) a sample of volume m is created

\[ T = (\tau_1, \ldots, \tau_i, \ldots, \tau_m) \]  

(10)

It is proposed to consider sample values (10) either as a resource-cost risk, taking into account uncompleted repair works in case of limited financial resources, or as an operating time characterizing a "denial of service" of equipment for the same reason.

It is proposed to estimate resource-cost risk as an indicator of efficiency, as the probability of a special event.

\[ r_i = P(\tau < T_i) \]

(11)

where \( T_i \) is set time (day).

The indicator (11) in simulation modeling is estimated by point (\( R_i \)) and interval (\( \tau_i, \tau_2 \)) estimates

\[ R_i = k_i / n, \]

(12)

where \( k_i \) is the number of process implementations (8) for which the condition is fulfilled, \( n \) is the total number of created implementations by simulation method;

\[ \tau_i = k_i / [k_i + (n-k_i+1) \cdot F_i(v_1,v_2)], \]

(13)

where \( F_i(v_1,v_2) \) is the critical value for the \( F \)-distribution with \( v_1 \) and \( v_2 \) degrees of freedom and confidence probability \( \gamma = 0.95; v_1 = 2 \cdot (n-k_i+1), v_2 = 2 \cdot k_i; \)

\[ \tau_2 = \frac{(k_i+1) \cdot F_2(v_3,v_4)}{[n-k_i+(k_i+1) \cdot F_2(v_3,v_4)]}, \]

(14)

where \( F_2(v_3,v_4) \) is the critical value for the \( F \)-distribution with \( v_3 \) and \( v_4 \) degrees of freedom and confidence probability \( \gamma = 0.95; v_3 = 2 \cdot (k_i+1), v_4 = 2 \cdot (n-k_i). \)

In the case when the sample values are considered as operating time, the efficiency indicators are:

a) numerical average operating time
\[ t_j = \frac{b}{J} \left( 0.5 + \sum_{j=1}^{k_j} 1 \right) \]  
(15)

b) numerical gamma percentage resource

\[
y_j = t_j - \frac{(\gamma - k_{j-1})}{k_j - k_{j-1}} \cdot \frac{b}{J}
\]
(16)

In formulas (15) and (16): \( b \) is the maximum sample value; \( J \) is the number of intervals in the grouped sample; \( k_j \) are inverse relative to unit accumulated relative frequencies, \( k_j = 1 - d_j; j = 1,...,J; k_0 = 1; \) \( d_j = \sum_{i=1}^{j} m_i / m \), \( d_j = 1; \) \( t_j \) are nodes of the grouped sample; \( \gamma \) is the probability for the gamma percentage resource.

3. The research of efficiency indicators using the simulation modelling method

The calculation of indicators (12)–(16) is done by the created modelling program. The obtained sample (10) is used in calculating these indicators. The number of simulated process implementations \( n = 20000 \).

The distribution laws and their numerical characteristics for the time intervals between types of work and cost of these works are shown in Table 1: \( m_o \) is the mathematical expectation, \( k_v \) is the coefficient of variation. The help of experts is proposed to select the distribution laws and their numerical characteristics.

| Work  | Intervals of time, day | Costs, million rubles |
|-------|------------------------|-----------------------|
| Current | Birnbaum-Saunders       | Lognormal             |
|       | \( m_o = 15.0 \)       | \( m_o = 3.5 \)       |
|       | \( k_v = 0.20 \)       | \( k_v = 0.20 \)       |
| Emergency | Weibull                | Pareto                |
|       | \( m_o = 45.0 \)       | \( m_o = 10.0 \)       |
|       | \( k_v = 0.30 \)       | \( k_v = 1.50 \)       |
| Major  | Gamma                  | Normal                |
|       | \( m_o = 60.0 \)       | \( m_o = 20.0 \)       |
|       | \( k_v = 0.25 \)       | \( k_v = 0.15 \)       |

The variables of income and expenses in the insurance fund for repair works are on average equal to each other. Time (9) is called the ruin moment in insurance mathematics. The total income should be more than total expenses so that the ruin probability be less than 1. It is achieved in the model (8) due to existing the initial funds of the insurance fund for the year.

The receipt of payments to the insurance fund for repair work (current, emergency, major) can be organized by the different ways. This research includes four options (O):

1) \( c_1 = 0.296; \ c_2 = 0.282; \ c_3 = 0.422; \ h_1 = 10, \) day; \( h_2 = 30, \) day; \( h_3 = 90, \) day;
2) \( c_1 = 0.333; \ c_2 = 0.333; \ c_3 = 0.334; \ h_1 = 10, \) day; \( h_2 = 30, \) day; \( h_3 = 90, \) day;
3) \( c_1 = 0.333; \ c_2 = 0.333; \ c_3 = 0.334; \ h_1 = 10, \) day; \( h_2 = 10, \) day; \( h_3 = 10, \) day;
4) \( c_1 = 0.333; \ c_2 = 0.333; \ c_3 = 0.334; \ h_1 = 15, \) day; \( h_2 = 15, \) day; \( h_3 = 15, \) day.

Table 2 shows the simulation results for these options for replenishing the insurance fund. The values in the ranges from 0 to 90 days and from 0 to 180 days were extracted from sample (10).
Table 2. Simulation results

| O | $R_t(90)$ | $R_t(180)$ | $\bar{t}_f(90)$ | $\bar{t}_f(180)$ | $y_r(90)$ | $y_r(180)$ |
|---|-----------|------------|-----------------|-----------------|----------|----------|
| 1 | 0.071     | 0.150      | 66.74           | 113.02          | 28.36    | 56.61    |
| 2 | 0.072     | 0.159      | 63.80           | 109.37          | 29.34    | 39.88    |
| 3 | 0.102     | 0.219      | 56.50           | 97.94           | 24.99    | 32.45    |
| 4 | 0.096     | 0.207      | 57.02           | 98.25           | 26.08    | 32.07    |

Based on the analysis results of Table 1 can conclude that in terms of risk and reliability indicators, preference should be given to the first option, when the frequency and amount of payments to the insurance fund depend on the type of repair work.

The histograms of the frequencies of the sample used for option 1 are shown in Figure 2: in figure 2a, it is shown for 90 days, and in figure 2b – for 180 days. The histograms of frequencies look like “saws” with a period of 90 days. Moreover, the left sides of the “saw teeth” are changed non-monotonically in ascending order. This is due to the fact that in this model payments are made: at the beginning of each decade for current work (2.334 million rubles), at the beginning of the month for emergency work (6.667 million rubles), at the beginning of the quarter (through 90 days) for major work, which are the largest in money (30 million rubles). But the most important fact is that all three payments are made at the beginning of each quarter.

4. Conclusion

In this article research on the efficiency of equipment repair works is conducted and proposed to use an insurance fund. The state of insurance fund is described by a random process of risk. A program based on the event approach was created to simulate this process. The modeling program creates a sample point of times when there are no financial resources repairs. Sample values are processed using the proposed algorithms for estimating resource-cost risk, numerical average operating time, and numerical gamma-percent resource. These indicators allow estimating efficiency of equipment repairs works and get practically important results. For example, based on the results of computational experiments, it is shown that according to the criteria of minimum risk assessments and maximum reliability ratings, preferentially should be given to the option when the frequency of payments to the insurance fund depends on the type of repair work.
5. References

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