Spatial extreme modeling using student t copula approach in Ngawi Regency

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Abstract. Extreme rainfall is an unpredictable phenomenon which causes suffering effect such as flooding. Located on the Equator area, Indonesia results in a high intensity of extreme rainfall. Initial information regarding the patterns, characteristics, and rainfall prediction is needed in order to minimize the negative effect of such phenomenon. A method that can be used to predict extreme rainfall is the spatial extreme value using the copula approach. The copula approach used in this study is student-t copula. The Generalized Extreme Value (GEV) distribution used for the student-t copula with parameter estimation is Pseudo-Maximum Likelihood Estimation (PMLE). The proposed method was applied to model the extreme rainfall at Ngawi Regency. An extreme spatial dependency on location is shown by extreme coefficient graphic. The best model that is obtained is based on Akaike Information Criterion’s (AIC) lowest value. The best model then continues to be used to predict the rainfall intensity return level. The prediction result of the rainfall intensity return level value shows that the maximum value of rainfall intensity increases from year to year in each station.

1. Introduction

Indonesia is an archipelago with many straits and bays which make Indonesian regions to be vulnerable to climate change [2] moreover, the location of the Indonesia being on the equator causes Indonesia to have high rainfall intensity throughout the region. However, in some particular regions of Indonesia, extreme rainfall intensity can cause several negative impacts such as floods and landslides. One effort to minimize the negative impact of extreme rainfall is using the Extreme Value Theory.

The Extreme Value Theory is a statistical method that study patterns and characteristics for extreme events such as rainfall that occurs in several locations. Rainfall occurs in several locations that are close to making rainfall based on multivariate data. Therefore, the development of rainfall with a multivariate database is called the development of Spatial Extreme Value. There are several methods to analyze using Spatial Extreme Value: Max-Stable Process and Copula. The copula is a method for exploring the structure of dependencies between random variables through marginal distribution functions [8]. Copula will handle multivariate data that has abnormal marginal distributions that generally occur in extreme data. Several studies in Indonesia regarding extreme rainfall are by [7] extreme rainfall in Ngawi Regency discussion using Max-Stable Process with the Schlater model and [1] Smith model. Forecasting extreme rainfall spatially with copula approach at 19 rain stations in Banten Province [4]. Copula student t uses the multivariate distribution of t distribution.

This paper discusses the extreme rainfall model using Spatial Extreme Value modeling with the copula t student approach which is applied to daily rainfall data in Ngawi Regency. After obtaining the
best combination of models, the predicted return level is then calculated. This is done to anticipate and minimize the impact of losses that usually occur due to extreme rainfall. Therefore, the return level is the initial information in suppressing the loss rate.

2. Material and Methodology

Extreme Value Theory (EVT) is a statistical method developed to identify extreme events by looking at patterns and characteristics of extreme events. In Extreme Value Theory, there are some approaches: Block Maxima (BM) and Peaks Over Threshold (POT). But in this research uses the BM approach that will be explained in detail as follows.

2.1 Block Maxima (BM)

One of methods used for identifying extreme values based on the highest value of observational data grouped in a certain period. Observation data is divided into blocks over a certain period, for example monthly, quarterly, semester, and yearly, then each block is determined the highest value called the extreme value for each block. This value is included as a sample obtained from the BM method can be seen in Figure 1.

According to [10], the BM method applies the Fisher-Tippet Gnedenko (1928) theorem that the extreme value sample data taken from the BM method will follow the Generalized Extreme Value (GEV) distribution. The cumulative distribution function (cdf) form of the GEV distribution is as follows:

$$F(y; \mu, \sigma, \xi) = \begin{cases} \exp \left\{ - \left[ 1 + \xi \left( \frac{y - \mu}{\sigma} \right) \right]^{-\frac{1}{\xi}} \right\}, & \text{for } \xi \neq 0, \\ \exp \left\{ -\exp \left( - \frac{y - \mu}{\sigma} \right) \right\}, & \text{for } \xi = 0 \end{cases}$$

When viewed from the shape parameter ($\xi$) values, GEV is divided into three types of distributions namely, Gumbel distribution if value $\xi = 0$, Frechet distribution if value $\xi > 0$, and Weibull distribution if value $\xi < 0$. The greater the value ($\xi$), then it produces a tail that is getting heavier (heavy tail) so that there is a chance that the greater extreme value will occur. Therefore, based on the three types of GEV distribution, the distribution with heavy tail is the Frechet distribution [6].
2.2 Parameter Estimation of GEV Univariate Distribution

Parameter estimation in the GEV univariate distribution uses the Maximum Likelihood Estimation (MLE) method. The probability distribution function (PDF) GEV distribution form is as presented in equation (2):

\[
f(x; \mu, \sigma, \xi) = \begin{cases} 
\frac{1}{\sigma} \left[1 + \xi \left(\frac{x - \mu}{\sigma}\right)\right]^{\frac{-1}{\xi}} \exp \left(-\left[1 + \xi \left(\frac{x - \mu}{\sigma}\right)\right]^{\frac{-1}{\xi}}\right), & -\infty < x < \infty, \quad \xi \neq 0 \\
\frac{1}{\sigma} \exp\left(-\exp\left[-\frac{(x - \mu)}{\sigma}\right]\right), & -\infty < x < \infty, \quad \xi = 0 
\end{cases}
\]

If the completion of the parameter estimation results in the form of an equation that is not closed form, then proceed using the numerical approach namely Broyden Fletcher-Goldfarb-Shanno (BFGS) Quasi Newton. The general formula is based on the equation below:

\[\theta^{(k+1)} = \theta^{(k)} - H\left(\theta^{(k)}\right)^{-1} g\left(\theta^{(k)}\right)\]  
(3)

Furthermore, the distribution suitability test is conducted to show the suitability of the theoretical distribution. Distribution checks using the Anderson Darling test and probability plot.

Hypothesis Tests

\[H_0 : F(x) = F^*(x) \quad \text{(The data follows theoretical distribution } F^*(x))\]
\[H_1 : F(x) \neq F^*(x) \quad \text{(The data does not follows theoretical distribution } F^*(x))\]

The Test Statistics

\[AD = -n - \frac{1}{n} \sum_{i=1}^{n} (2i - 1) \left(\ln\left(F^*(x_i)\right) + \ln\left(1 - \left(F^*(x_{n+1-i})\right)\right)\right)\]  
(4)

Reject \(H_0\) if \(AD\) value > specified critical value or \(p\)-value < \(\alpha\) (significance level specified).

2.3 Spatial Extreme Modeling

The main thing that is considered in the spatial field is the existence of dependencies between locations, which means that if an event occurs in a neighboring location, it tends to have similar events, rather than events that are located farther away. Spatial extremes modeling begins with extreme value modeling with a multivariate extreme value distribution. Because multivariate extreme value is based on distribution with low or limited dimensions (finite dimensional). Therefore, an approach is needed for multivariate data, there are the Max-Stable Process approach and Copula approach.

The difference between the two methods of this approach is when modeling and the process of transformation. For the transformation process, both approaches use the same process, namely max-stable. This process brings data to the frechet distribution, but in the copula transformation process uses the first nature transformation, while the max-stable process uses the second nature transformation. The transformation properties are as follows:

1. The one-dimensional marginal distribution follows the GEV distribution
   \[X \perp GEV(\mu, \sigma, \xi)\] use the distribution function as follows:
   \[F(\mu, \sigma, \xi) = \exp\left[-\left(1 + \frac{\xi(x - \mu)}{\sigma}\right)^{\frac{1}{\xi}}\right], \quad -\infty < \mu, \quad \xi < \infty, \quad \sigma > 0\]  
(5)

where, \(\mu = \text{location parameter}, \ \sigma = \text{scale parameter}, \ \xi = \text{shape parameter}.\]
2. The marginal k-dimensional distribution follows the multivariate extreme value distribution. 
\( \{ Z(j) \} \) is a max-stable process that has a Frechet margin unit with a distribution function
\( F(Z_j) = \exp(-1/z), \ z > 0 \) so this process[9] can be obtained by transforming like the following equation:
\[
Z_j = \left[ 1 + \frac{\xi(x - \mu)}{\sigma} \right]^{\frac{1}{\xi}}
\]
(6)

2.4 Copula Approach
According to [8] copula is a method that can explore and characterize the structure of dependencies or dependencies between random variables through marginal distribution functions. By [11] t-copula shows flexibility in covariant structure and tail dependencies. Tail dependencies can be considered as conditional opportunities on extreme observations in one component under extreme circumstances. The Sklar theorem is a function that links the multivariate distribution function with its marginal distribution. Some copula families include the Archimedes copula, the Elliptic copula, the Bivariate Extreme Value copula, and the Marshal-Olkin copula. But the two families that are quite popular are elliptic copula (copula gaussian and student-t copula). Based on [5] the CDF form of gaussian copula and student-t copula can be found in equation (7).
\[
C(u_1, u_2, ..., u_m) = T_v \left\{ T_v^{-1}(u_1), T_v^{-1}(u_2), ..., T_v^{-1}(u_m) \right\}
\]
(7)
where,
\[ T \quad : \text{multivariate cumulative distribution function t} \]
\[ T_v^{-1} \quad : \text{inverse cdf multivariate distribution t with degree of freedom v} \]

Based on [3], the form of the student-t copula distribution function can be written as:
\[
c(u) = \frac{\Gamma\left(\frac{v + d}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\sqrt{(v\pi)^d |\rho(h)|^\frac{d}{2}} \left\{ 1 + \frac{u^T \rho^{-1} u}{v} \right\}^{-\frac{v+d}{2}}}
\]
(8)

Based on equation (8) pdf copula student-t is obtained as follows:
\[
c(u_1, ..., u_m) = \frac{\Gamma\left(\frac{v + d}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\sqrt{(v\pi)^d |\rho(h)|^\frac{d}{2}} \left\{ 1 + \frac{\text{a}^T \rho^{-1} \text{a}}{v} \right\}^{-\frac{v+d}{2}}}
\]
(9)
\[
c(u_1, ..., u_m) = \frac{\Gamma\left(\frac{v + d}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\sqrt{(v\pi)^d |\rho(h)|^\frac{d}{2}} \left\{ 1 + \text{a}^T \rho^{-1} \text{a} \right\}^{-\frac{v+d}{2}}}
\]
(10)
where \( \text{a} = \left( T_v^{-1}(u_1), ..., T_v^{-1}(u_m) \right) \). According to the Sklar theorem, each probability with copula can be written by multiplying between the marginal pdf distribution and the copula cdf function [10], so that it can be written:
\[
f(x_1, ..., x_m) = f_{x_1}(x_1) \cdot \cdots \cdot f_{x_m}(x_m) \cdot c(u_1, ..., u_m)
\]
(11)
2.5 Extremal Coefficient
In modeling extreme values using copula, measuring data dependencies between regions is the main thing. The extremal coefficient represents the extreme value relation of each location using the equation below:

$$\theta(s_1 - s_2) = -z \log \{Z(s_1) \leq z, Z(s_2) \leq z\}$$

(12)

The extremal coefficient values range from $1 \leq \theta(s_1 - s_2) \leq 2$. If a value is nearing 1, then it becomes complete dependence, whereas if the value is nearing 2, it becomes independence.

2.6 Selecting the Best Model
In this paper, the Akaike Information Criterion (AIC) will be employed to choose the best trend surface model formulated as:

$$\hat{\mu}(s) = \hat{\beta}_{\mu,0} + \hat{\beta}_{\mu,1}(lon) + \hat{\beta}_{\mu,2}(lat)$$

$$\hat{\sigma}(s) = \hat{\beta}_{\sigma,0} + \hat{\beta}_{\sigma,1}(lon) + \hat{\beta}_{\sigma,2}(lat)$$

$$\hat{\xi}(s) = \hat{\beta}_{\xi,0}$$

(13)

AIC is defined by:

$$AIC = -2\ell_p(\hat{\beta}) + 2g$$

(14)

Where, $\ell_p(\hat{\beta}) = \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{m} \ln \{f(u_{ij}, u_{ik}; \hat{\beta})\}$ is estimator obtained dari pseudo maximum likelihood and $i=1,2,...,n, j=1,2,...,m-1, k=2,3,...,m$, $g$ is the number of parameters estimated. The smallest AIC value indicates the best model combination. After getting the best combination of models, the last step is predicting the value of the return level. Return level prediction is used to find out how much return level will occur in the following year. To calculate the predicted return level based on the equation below (15):

$$z_p(s) = \hat{\mu}(s) - \hat{\sigma}(s) \left( 1 - \left[ - \ln \left( 1 - \frac{1}{T} \right) \right]^{-\hat{\xi}(s)} \right)$$

(15)

where $z_p(s) = \text{nilai return level}$, $T = \text{reset period}$.

2.7 Extreme Rainfall
Rainfall is the height of rainwater collected in a flat place, does not evaporate, does not sink in and does not flow. It is said that the region experiences extreme rainfall if rainfall is more than 100 millimeters per day [2]. Based on the distribution of average monthly rainfall data, rainfall in Indonesia is divided into 3 types:

a. Equatorial Type, Sumatra and Kalimantan regions.
b. Monsoon type, the southern tip of Sumatra, Java, Bali, Nusa Tenggara and South Maluku.
c. Local Type.

2.8 Procedure
2.8.1 Estimating univariate GEV parameters with the following steps:

1. Establishing the function of likelihood from the GEV distribution pdf
2. Establishing the function of ln likelihood/GEV distribution
3. Maximizing the function of $\ln L(\mu, \sigma, \xi)$
4. If the first derivative in step (c) results are not closed form, then a numerical optimization is performed using the BFGS optimization method.

2.8.2 Analysis of spatial extreme value modeling data with the copula studentt approach steps are as follows:

1. Identifying the heavytail data and extreme values using histograms
2. Derive extreme values sample from the original data using the BM approach
3. Obtaining the results of the estimated parameter values $\hat{\mu}, \hat{\sigma}, \hat{\xi}$ for each location univariately using MLE estimation, but because it was not close form, it is continued with numerical iteration using BFGS.
4. Transform the extreme sample data obtained from BM to the Frechet distribution, and then transform it again into the copula margin.
5. Obtain extreme coefficient values ranging from 1 to 2 as in sub (2.5).
6. Make a trend surface model combination and determine the best trend surface model based on the AIC value obtained using equation (13) on each model combination.
7. Calculate the value $\hat{\mu}(s), \hat{\sigma}(s), \hat{\xi}(s)$ for each item based on the best trend surface model and get the estimated results of the student copula model.
8. The result of step (g), the value is used to predict the return level for each station in Ngawi Regency.

3. Main Result
In this part, we discussed the univariate GEV distribution parameter estimation using Maximum Likelihood Estimation (MLE). Furthermore, the student t copula approach is applied to the daily rainfall data in Ngawi Regency. The first thing to do in the analysis is the description of the data to find out the characteristics of rainfall data in Ngawi Regency in six rain stations from 1989 until 2010.

3.1 Parameters Estimation of Univariate GEV Distribution
The estimation of univariate GEV parameters uses MLE by the steps outlined in section 2.8 by forming the likelihood function based on equation (2). The result parameters estimation of univariate GEV distribution based on below:

$$n - n \frac{n n}{\xi} + \frac{n}{\sigma} + \xi = \frac{1}{\sigma} \sum_{i=0}^{n} \left[ 1 + \xi \left( \frac{x_i - \mu}{\sigma} \right) \right]^{\frac{1}{\xi}}$$

$$\hat{\mu} = \frac{1}{-n \xi} \left( n - n \frac{n n}{\xi} + \frac{n}{\sigma} + \xi \right)$$

$$\hat{\sigma} = \sqrt{\left( 1 + \frac{1}{\xi} \sum_{i=0}^{n} \left[ 1 + \xi \left( \frac{x_i - \mu}{\sigma} \right) \right] \right) - \left( \frac{\xi}{\sigma^2} \sum_{i=0}^{n} \left[ 1 + \xi \left( \frac{x_i - \mu}{\sigma} \right) \right] \right) - \left( \frac{\xi}{\sigma^2} \sum_{i=0}^{n} \left[ 1 + \xi \left( \frac{x_i - \mu}{\sigma} \right) \right] \right)^{\frac{1}{\xi}}}$$

(16)
Based on the result from equations (16), (17), and (18), it is known that the results of the estimated parameters in the form of an equation are not close form. Then it must be continued using numerical iteration is Quasi Newton’s BFGS.

3.2 Spatial Extreme Value Modeling Data Analysis with Student t Copula Approach

This research monitors the daily rainfall data in six rain stations at Ngawi Regency from 1989 to 2010. The first thing to do is descriptive statistics, it is crucial to know the characteristics of rainfall pattern in six rain stations. The information of daily rainfall data in six rain stations can be seen in Table 1. The average daily rainfall at rain stations in Ngawi Regency ranges from 4,625 mm/day to 5,158 mm/day with the highest average rainfall of 5,158 mm/day at the Bekoh station. Overall, the highest minimum rainfall value is the Ngrambe station with 137 mm/day.

**Table 1.** Description of Daily Rainfall Data for Six Rain Stations in Ngawi Regency

| Station     | Min (mm/day) | Max (mm/day) | Mean (mm/day) |
|-------------|--------------|--------------|---------------|
| Kedungprahu | 0            | 151          | 4,625         |
| Ngrambe     | 0            | 137          | 4,860         |
| Tretes      | 0            | 185          | 4,730         |
| Bekoh       | 0            | 195          | 5,158         |
| Gemarang    | 0            | 160          | 4,940         |

In Figure 2, it can be seen the heavytail patterns in all rain stations at Ngawi Regency that the data distribution curve is tilted to the right and high frequency of data stands out around zero, which means that extreme rainfall data in six rain stations contains extreme values.
Next, to get samples of extreme values in rainfall data, the BM approach is used. At this stage, the rainfall data is divided into training data and testing data. The BM approach divides data into three-month blocks in the form of four season periods per year: December-January-February (DJF), March-April-May (MAM), June-July-August (JJA), September-October-November (SON). The divided blocks is based on the BMKG reference which classifies the monsoon rain pattern in most areas in Java. The Figure 3 it can be seen the probability plot that the data follows GEV distribution for six rain stations throughout the entire Ngawi Regency.

![Probability Plot](image)

**Figure 3.** Probability Plot

Then in the Table 2 it can be seen that the parameter estimation $\hat{\mu}, \hat{\sigma}, \hat{\xi}$ results for each location are univariate using Maximum Likelihood Estimation and continued with numerical optimization is BFGS Quasi Newton.

| No | Station   | $\hat{\mu}$ | $\hat{\sigma}$ | $\hat{\xi}$ |
|----|------------|------------|------------|------------|
| 1  | Kedungprahu| 45,873     | 31,837     | -0.182     |
| 2  | Ngrambe    | 40,318     | 30,079     | -0.162     |
| 3  | Tretes     | 48,201     | 36,849     | -0.109     |
| 4  | Bekoh      | 57,122     | 40,015     | -0.193     |
| 5  | Gemarang   | 56,054     | 39,444     | -0.309     |
| 6  | Kendal     | 62,364     | 40,002     | -0.307     |

After obtaining univariate parameter estimation, the next step is proceeded with testing extreme spatial dependencies between locations. From the Figure 4, it can be seen extremal coefficient that the points spread around 1.35 to 1.55 which means there are extreme spatial dependencies between locations, the picture is shown as follows.
Figure 4. Extremal Coefficient

Then, find the best trend surface model based on a combination trend surface model from equation (13). The best trend surface model is chosen based on the smallest AIC value. The smallest AIC value is 133,4807 from a combination of trend surface models.

\[
\hat{\mu}(s) = -0.61647 + 0.00938(\text{long})
\]

\[
\hat{\sigma}(s) = 0.56424 + 0.03422(\text{lat})
\]

\[
\hat{\xi}(s) = -0.4702
\]

Based on the trend surface model, the estimated value of the spatial GEV parameter of the student t copula model in each rain station is multivariate as it can be seen in Table 3.

| No | Station     | \(\hat{\mu}(s)\) | \(\hat{\sigma}(s)\) | \(\hat{\xi}(s)\) |
|----|-------------|-------------------|---------------------|-----------------|
| 1  | Kedungprahu | 0.4267            | 0.3094              | -0.4702         |
| 2  | Ngrambe     | 0.4265            | 0.3071              | -0.4702         |
| 3  | Tretes      | 0.4263            | 0.3092              | -0.4702         |
| 4  | Bekoh       | 0.4275            | 0.3076              | -0.4702         |
| 5  | Gemarang    | 0.4281            | 0.3111              | -0.4702         |
| 6  | Kendal      | 0.4274            | 0.3055              | -0.4702         |

Furthermore, the parameter estimation \(\hat{\mu}(s)\), \(\hat{\sigma}(s)\), and \(\hat{\xi}(s)\) is used to calculate the return level for prediction of the value rainfall in each rain station at Ngawi Regency. The return level calculation formula uses equation (15), where \(T = 2 \text{ years} \times 4 \text{ (number of blocks)} = 8\). The prediction of return level for 5 years is presented by Table 4 prediction return level, with the results of the calculation of the return level as follows:

| No | Station     | 2011 (mm) | 2012 (mm) |
|----|-------------|-----------|-----------|
| 1  | Kedungprahu | 77,928    | 91,88     |
| 2  | Ngrambe     | 70,540    | 83,998    |
| 3  | Tretes      | 86,530    | 104,515   |
| 4  | Bekoh       | 96,865    | 114,128   |
| 5  | Gemarang    | 93,375    | 108,199   |
| 6  | Kendal      | 99,539    | 114,110   |

Based on Table 4 it can be explained that the maximum predicted return level value in 2012 at Bekoh station with a predicted value of 114.128 mm and the minimum return level value occurs at the Ngrambe station with a predicted value of 83.998 mm. The predicted value of return levels in all rain stations are increased from year to year.
4. Conclusion
The result of this study is the estimation results of the best trend surface model is AIC value of 133,4807. Then the results of the predicted return level values show that the value of rainfall is increasing from year to year in all stations at Ngawi Regency. For further research with copula approach can using gaussian copula to compare the result between student’s t copula and gaussian copula.

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