Image Completion via Inference in Deep Generative Models

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Abstract
We consider image completion from the perspective of amortized inference in an image generative model. We leverage recent state of the art variational auto-encoder architectures that have been shown to produce photo-realistic natural images at non-trivial resolutions. Through amortized inference in such a model we can train neural artifacts that produce diverse, realistic image completions even when the vast majority of an image is missing. We demonstrate superior sample quality and diversity compared to prior art on the CIFAR-10 and FFHQ-256 datasets. We conclude by describing and demonstrating an application that requires an in-painting model with the capabilities ours exhibits: the use of Bayesian optimal experimental design to select the most informative sequence of small field of view x-rays for chest pathology detection.

1. Introduction
Given an image with missing pixel values, the image completion task is to infer, or inpaint, these missing values. Research on this subject has often been motivated by applications such as image restoration (where damaged parts of a photograph may be removed and inpainted) and object removal (where a user deletes and then inpaints parts of an image containing an unwanted object). Image completion without uncertainty is as an ill-posed problem, since it is impossible to exactly recover the missing pixel values. Exact recovery is often not even desired: an object removal tool would be of little use if it draws back objects deleted by the user. Instead, traditional image completion techniques (Bertalmio et al., 2001; Barnes et al., 2009) aim to inpaint missing regions to be similar to the rest of the image and sometimes explicitly penalise completed regions with features that “stand out” (Barnes et al., 2009). These techniques work well for many applications, but break down if more than a small portion of the image is missing, or if multiple diverse samples are required.

We view image completion from a Bayesian perspective. This requires us to first specify a prior distribution over images \( p(\mathcal{I}) \), which we model with a variational auto-encoder (VAE). We then consider how pixels can be removed from \( \mathcal{I} \) by modeling the known pixel values \( \hat{\mathcal{I}} \) as being drawn from the conditional distribution \( p(\hat{\mathcal{I}}|\mathcal{I}) \). This leads to a posterior distribution over image completions via Bayes’ rule:

\[
p(\mathcal{I}|\hat{\mathcal{I}}) = \frac{p(\mathcal{I})p(\hat{\mathcal{I}}|\mathcal{I})}{\int p(\mathcal{I})p(\hat{\mathcal{I}}|\mathcal{I})\,d\mathcal{I}}.
\]

The image completion task is then to draw approximate samples from this posterior. Basing our approach on this perspective allows us to inherit two major advantages of Bayesian inference. First, we can naturally incorporate uncertainty, meaning that our sampled image completions are diverse without the need for ad-hoc loss terms. Second, our approach can work well with few observations; i.e., provided we have a good generative model, we can produce plausible completions for images with arbitrarily many missing pixels.

Our approach to sampling from \( p(\hat{\mathcal{I}}|\mathcal{I}) \) draws heavily from the VAE framework (Kingma & Welling, 2013). VAEs can generate images by sampling latent variables \( z \sim p_{\text{model}}(\cdot) \) and then the image \( \mathcal{I} \sim p_{\text{model}}(\cdot|z) \). They can also reconstruct images if \( z \) is instead sampled from an encoder \( q(z|\hat{\mathcal{I}}) \). Image completion can be viewed as an interpolation between these two extremes; we wish to reconstruct observed pixels and sample the values of the remainder. This motivates us to tackle image completion via the learning of an additional component: a partial encoder \( \hat{q}(z|\hat{\mathcal{I}}) \) which maps from a corrupted image to a distribution over \( z \). In the extreme case where all pixels are missing from \( \hat{\mathcal{I}} \), the optimal \( \hat{q}(z|\hat{\mathcal{I}}) \) is the prior \( p_{\text{model}}(z) \). Similarly if no pixels are missing, \( \hat{q}(z|\hat{\mathcal{I}}) \) should match the encoder. We show that, in between these two extremes, \( 
\hat{q}(z|\hat{\mathcal{I}}) \) can learn distributions over \( z \) which lead to highly diverse and plausible image completions. An additional benefit to basing our architecture on a VAE is that, when a pretrained VAE is available, we can use it to speed up training.

In Section 5 we demonstrate an application of an uncertainty-preserving image completion system. In particular, we show...
how it can be used as part of a Bayesian optimal experimental design (BOED) (Chaloner & Verdinelli, 1995) pipeline to control a sensor. The sensor can observe only a small portion of a scene at a time. After observing each portion, we sample scene completions based on the observed part. Further processing of these completions can be used to determine where in the scene will be most informative to observe next. This is useful if there is a high cost to observing the scene. We consider a scenario where the BOED pipeline is used for x-ray imaging. In this case, it is desirable to use as few observations as possible so as to limit the patient’s exposure to radiation.

To summarise, we present an approach to convert a VAE into an image completion model. We then derive and compare two training objectives and show that one of them consistently outperforms all baselines. Finally, we show results indicating that this method could prove useful in the field of medical imaging.

2. Background

2.1. Image completion

Early diffusion-based approaches to image completion (Bertalmio et al., 2000; 2001; Ballester et al., 2001; Levin et al., 2003) could complete small gaps in images, such as those made by creases in an old photograph. A later strand of work filled larger gaps by copying and aligning patches from the rest of the image (Criminisi et al., 2003; Barnes et al., 2009). These methods produce convincing continuations of the background but no new structure.

Breakthroughs in deep learning enabled a plethora of advances in image completion. Köhler et al. (2014); Ren et al. (2015) trained convolutional neural networks (CNNs) to map directly from masked images to estimates of the missing pixel values using, e.g., a mean squared error (MSE) reconstruction loss. While this works well with small missing patches, the networks are deterministic and give blurry completions when the missing regions are large. Attempts to make more realistic completions have led to increasingly complex methods being used. Image completions are now often produced in several stages, with intermediate outputs such as segmentation masks (Song et al., 2018) or coarse-grained images (Yu et al., 2018; 2019) allowing additional losses to be imposed. In addition to reconstruction losses, it is also now common to use an adversarial loss (Pathak et al., 2016), or more often several adversarial losses from different discriminators (Iizuka et al., 2017). Domain-specific losses have also been considered: Li et al. (2017) crafted a loss term specifically for the completion of images of faces. Although adversarial losses were introduced as a tool to learn distributions (Goodfellow et al., 2014), most of these works are incapable of generating diverse completions from a single input. Creating diverse completions has therefore been the focus of several recent studies (Zheng et al., 2019; Zhao et al., 2020). However, these impose diversity using ad-hoc loss terms weighted by hyperparameters, and it is not clear how well they match the desired posterior $p(I|\hat{I})$. They also typically require many image pixels to be observed to generate compelling completions.

Related to this paper, conditional VAEs (CVAEs) have been used for image completion (Sohn et al., 2015; Zheng et al., 2019), but can lack sample diversity. The main difference is that the CVAE decoder receives a conditioning input as well as sampled latent variables. Our approach enforces all conditioning through distributions over the latent variables. Another related approach is that of Song et al. (2020), who present a stochastic differential equation-based image model. This can be conditioned on a subset of image pixels to perform image completion, but is slow to sample from.
We describe V AEs in terms of three components. A decoder maps from latent variables $z$ to data $I$ (e.g. an image). The prior over latent variables, $p_{\text{model}}(z)$, may have learnable parameters, which we consider to be part of $\theta$. Together, the prior and decoder define a joint distribution, $p_{\text{model}}(z, I)$. Finally, an encoder with parameters $\phi$ maps from data to an approximate posterior distribution over latent variables, $q(z|I) \approx p_{\text{model}}(z|I)$. Ideally, these networks would be trained to maximise the data likelihood $p_{\text{model}}(I) = \int p_{\text{model}}(z) p_{\text{model}}(I|z) dz$, averaged over some dataset. Since this is intractable, $\theta$ and $\phi$ are instead trained jointly to maximise an average of the evidence lower-bound (ELBO) over the data distribution $p_{\text{data}}(I)$:

$$E_{p_{\text{data}}(I)} \left[ \text{ELBO}(\theta, \phi, I) \right]$$

$$= E_{p_{\text{data}}(I)q(z|I)} \left[ \log \frac{p_{\text{model}}(z)p_{\text{model}}(I|z)}{q(z|I)} \right]$$

$$= \mathcal{H}[p_{\text{data}}(I)] - \text{KL}(r(z, I) \parallel p_{\text{model}}(z, I))$$

(2)

where we define $r(z, I) = p_{\text{data}}(I)q(z|I)$ to be the joint distribution defined by the data distribution and learned encoder. As Eq. (3) shows, maximising this objective will drive $p_{\text{model}}(z, I)$ towards $r(z, I)$, and so drive the marginal $p_{\text{model}}(I)$ towards $p_{\text{data}}(I)$.

A major weakness of V AEs in the past has been difficulty scaling to produce convincing high-resolution images. However, recent hierarchical architectures have made significant progress towards this end (Vahdat & Kautz, 2020; Child, 2020). Hierarchical V AEs differ from traditional V AEs by partitioning their latent variables $z$ into $L$ disjoint groups, $z_1, \ldots, z_L$ (Gregor et al., 2015; Kingma et al., 2016; Sønderby et al., 2016; Klushyn et al., 2019). We show a typical hierarchical V AE architecture in Fig. 2a. The decoder (black) maintains a hidden state $h_l$, which is initialized with a fixed $h_0$ and then incorporates each group of latent variables as they are sampled. By conditioning the prior for each $z_l$ on the previous hidden state, the prior for $z$ is factorised as

$$p_{\text{model}}(z) = \prod_{l=1}^{L} p_{\text{model}}(z_l|z_{<l})$$

(4)

where $z_{<l}$ is the null set for $l = 1$ and $\{z_1, \ldots, z_{l-1}\}$ otherwise. The distribution produced by the encoder (shown in orange) for each $z_l$ also depends on the previous hidden state $h_{l-1}$ and therefore factorises as $q(z|I) = \prod_{l=1}^{L} q(z_l|z_{<l}, I)$. This dependence on the decoder’s hidden state also means that $q(z|I)$ depends on $\theta$ as well as the encoder parameters $\phi$. Both $p_{\text{model}}(z_l|z_{<l})$ and $q(z_l|z_{<l}, I)$ usually assume independence between dimensions of $z_l$, and are often diagonal Gaussian distributions (Sønderby et al., 2016; Vahdat & Kautz, 2020; Child, 2020).

We will present a method to complete images through inference in the model defined by a V AE with a subset of image pixel values observed. For this inference to be well-posed, it is important for the observed pixels to have non-zero probability under the model; the denominator in Bayes’ rule (Eq. (1)) would otherwise be zero. Hierarchical V AEs are well-suited to satisfying this requirement for two reasons. First, the V AE objective optimises $\theta$ with a mass-covering KL divergence (Eq. (3)), which harshly penalises $\theta$ for as-
signing low probability density to any images in the data distribution. Second, recent hierarchical architectures use latent variables with extremely high dimensionality: the VDV-VAE (Child, 2020) uses 412 000 latent variables to model the CIFAR-10 dataset of 3072-dimensional images. This “over-parameterisation” ensures that the images produced are not constrained to a manifold. We confirm that these architectures can produce extremely mass-covering distributions; we find that a VD-VAE trained on images of faces can near-perfectly reconstruct wildly different data, including images of text and chest x-rays (see appendix). This property prevents pitfalls similar to those observed by Yeh et al. (2017) whose method, which completes images by estimating latent variables in a GAN, would often fail to reconstruct the known pixel values.

3. Method

To convert an unconditional VAE architecture to a conditional architecture, we introduce a partial encoder with parameters \( \hat{\phi} \). This is fed an observation \( \hat{I} \) (e.g. a corrupted image) and defines an approximate posterior over the latent variables, \( q(z|\hat{I}) \). We can then approximate the posterior over image completions,

\[
P_{\text{model}}(I|\hat{I}) = \int p_{\text{model}}(I|z)p_{\text{model}}(z|\hat{I})dz \quad \approx \int p_{\text{model}}(I|z)\hat{q}(z|\hat{I})dz. \tag{5}
\]

We can sample from this approximation by simply sampling \( z \sim \hat{q}(z|\hat{I}) \) and then \( I \sim p_{\text{model}}(I|z) \).

Our method requires that we have a model of how \( \hat{I} \) is generated from \( I \). More precisely, we need to define the conditional distribution \( p(I|\hat{I}) \). We do so by defining a distribution over image-sized binary masks. Then to sample from \( p(I|\hat{I}) \), we sample a mask \( m \) before returning \( \hat{I} = \text{concatenate}(I \odot m, m) \). Here, \( \odot \) is a pixel-wise multiplication operation which removes information from the missing pixels. The concatenation is performed along the channel dimension and makes it possible to distinguish between unobserved pixels and zero-valued pixels.

3.1. Training objectives

Recall that a VAE defines two joint distributions over \( z \) and \( I \): the distribution of samples from the generator, \( p_{\text{model}}(z, I) \); and the distribution resulting from sampling data \( I \) and encoding it, which we will now call \( r(z, I) = p_{\text{data}}(I|z)q(z|I) \). For notational convenience, we define the following joint distributions:

\[
p_{\text{model}}(z, I, \hat{I}) = p_{\text{model}}(z)p_{\text{model}}(I|z)p(\hat{I}|I) \tag{7}
\]
\[
p_{\text{data}}(I, \hat{I}) = p_{\text{data}}(I)p(\hat{I}|I) \tag{8}
\]
\[
r(z, I, \hat{I}) = p_{\text{data}}(I, \hat{I})q(z|I) \tag{9}
\]

We will use \( p_{\text{model}}, p_{\text{data}}, \) and \( r \) in this paper to refer to marginals and conditionals of these joint distributions, with the specific marginal or conditional made clear from context. With this notation, we now derive and compare two objectives for training the conditional generative model, with particular focus on the properties of the learned \( q(z|\hat{I}) \).

3.1.1. Forward KL

Consider maximising \( \log p_{\text{model}}(I|\hat{I}) \) on expectation over \( I \sim p_{\text{data}}(\cdot) \) and \( \hat{I} \sim p(\cdot|I) \). We can lower bound this as

\[
O_{\text{forward}}(\theta, \phi, \hat{\phi}) = \mathbb{E}_{q(z|I)p_{\text{data}}(I, \hat{I})} \left[ \log \frac{p_{\text{model}}(I|z)\hat{q}(z|\hat{I})}{q(z|\hat{I})} \right] \tag{10}
\]
\[
\leq \mathbb{E}_{p_{\text{data}}(I, \hat{I})} \left[ \log p(I|\hat{I}) \right] \tag{11}
\]

That is, we can lower-bound \( \log p_{\text{model}}(I|\hat{I}) \) very similarly to how the ELBO in an unconditional VAE lower-bounds \( \log p_{\text{model}}(I) \) (Eq. (2)). The only difference is that the prior, \( p_{\text{model}}(z) \), is replaced by \( \hat{q}(z|\hat{I}) \). This is reflected in Fig. 2b, where each \( z_i \) is now conditioned on \( \hat{I} \) via the partial encoder (blue).

We now consider the effect this objective has on the learned distribution \( \hat{q}(z|\hat{I}) \). We can decompose Eq. (10) as follows. Starting by multiplying both sides of the fraction by the intractable conditional distribution \( r(z|\hat{I}) \),

\[
O_{\text{forward}}(\theta, \phi, \hat{\phi})
\]

\[
= \mathbb{E}_{r(z, I, \hat{I})} \left[ \log \frac{p_{\text{model}}(I|z)\hat{q}(z|\hat{I})r(z|\hat{I})}{q(z|\hat{I})r(z|\hat{I})} \right] \tag{12}
\]
\[
= \mathbb{E}_{r(z, I, \hat{I})} \left[ \log \frac{p_{\text{model}}(I|z)r(z|\hat{I})}{q(z|\hat{I})} \right] - \mathbb{E}_{r(z, \hat{I})} \left[ \log \frac{r(z|\hat{I})}{\hat{q}(z|\hat{I})} \right] \tag{13}
\]
\[
= C - \mathbb{E}_{p_{\text{data}}(\hat{I})} \left[ \text{KL}(r(z|\hat{I}) \| \hat{q}(z|\hat{I})) \right] \tag{14}
\]

where \( C \) does not depend on the partial encoder’s parameters \( \hat{\phi} \). This shows that, when maximising \( O_{\text{forward}}(\theta, \phi, \hat{\phi}) \), the partial encoder will be trained to minimise the forward KL divergence to \( r(z|\hat{I}) \). Minimising the forward KL leads to mass-covering behaviour (Bishop, 2006) and so this objective could be expected to lead to diverse samples of the
V AE (Fig. 2a): the partial encoder with input $\hat{I}$. To reveal the properties of the learned partial encoder, we consider the two objectives we propose and the standard unconditional V AE objective to be consistent in the sense that they can all be simultaneously maximised. Specifically, all three objectives will be maximised if we have (1) a generator and full encoder network learned such that $\text{KL}(r(\hat{z}, \hat{I}) \parallel p_{\text{model}}(\hat{z}, \hat{I})) = 0$; and (2) a partial encoder such that $q(\hat{z}|\hat{I}) = r(\hat{z}|\hat{I})$. Training an unconditional V AE with the standard ELBO will lead to a generator and full encoder which satisfy the first condition (assuming convergence and sufficient expressivity). We can therefore speed up training of the image completion model by using a pretrained generator and full encoder, and only training the partial encoder. This leads to faster convergence, as well as faster training iterations since we only need to compute gradients for, and perform update steps on, the partial encoder’s parameters $\hat{\phi}$. Another possibility is to use the pretrained full encoder’s weights as an initialisation for the partial encoder’s weights. In our experiments we do this with AIPO-R, for which we found it made training more stable, but not with AIPO, for which it made training less stable. For experiments on CIFAR-10 and FFHQ-256, we made use of the pretrained models released by Child (2020). This allowed us to leverage models trained for about 2 GPU-weeks and 1 GPU-year, respectively. Training each complementary partial encoder in our experiments took no more than 1 GPU-week.

3.2. Faster training with a pretrained V AE

We consider the two objectives we propose and the standard unconditional V AE objective to be consistent in the sense that they can all be simultaneously maximised. Specifically, all three objectives will be maximised if we have (1) a generator and full encoder network learned such that $\text{KL}(r(\hat{z}, \hat{I}) \parallel p_{\text{model}}(\hat{z}, \hat{I})) = 0$; and (2) a partial encoder such that $q(\hat{z}|\hat{I}) = r(\hat{z}|\hat{I})$. Training an unconditional V AE with the standard ELBO will lead to a generator and full encoder which satisfy the first condition (assuming convergence and sufficient expressivity). We can therefore speed up training of the image completion model by using a pretrained generator and full encoder, and only training the partial encoder. This leads to faster convergence, as well as faster training iterations since we only need to compute gradients for, and perform update steps on, the partial encoder’s parameters $\hat{\phi}$. Another possibility is to use the pretrained full encoder’s weights as an initialisation for the partial encoder’s weights. In our experiments we do this with AIPO-R, for which we found it made training more stable, but not with AIPO, for which it made training less stable. For experiments on CIFAR-10 and FFHQ-256, we made use of the pretrained models released by Child (2020). This allowed us to leverage models trained for about 2 GPU-weeks and 1 GPU-year, respectively. Training each complementary partial encoder in our experiments took no more than 1 GPU-week.

4. Experiments

We create AIPO and AIPO-R image completion models based on the VD-VAE unconditional architecture (Child, 2020). We evaluate them on two datasets: CIFAR-10 (Krizhevsky et al., 2009) and FFHQ-256 (Karras et al., 2019) and compare against three baselines: Pluralistic Image Completion (PIC) (Zheng et al., 2019), Context Encoders (CE) (Pathak et al., 2016) and Attentive Neural Pro-
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Figure 3. **Top row:** FID scores (lower is better) for completions on CIFAR-10 and FFHQ-256. FID-n is the FID between the test set and a set consisting of one completion from n patches of each test image. FID-agg is the FID between the test set and the union of all completions generated for each FID-n. **Bottom row:** Inception scores for CIFAR-10 completions (left) and LPIPS scores for FFHQ-256 completions (right). When these are high, it indicates that the method can generate diverse completions for a single \( \hat{I} \). Each line shows an aggregation of results on 10 test images. AIPO achieves both the lowest FID scores and the highest diversity scores.

Our focus is on completing images when the majority of pixel values are unknown. Therefore, where not otherwise specified, all results we show are with networks trained and evaluated using the type of mask seen in the left column of Fig. 1. In these, all pixels are masked out except for those within one of \( n \) squares (where \( n \in \{0, 1, 2, 3, 4, 5\} \) is sampled uniformly for each image). The position of each square is randomly sampled and the side length is always 35% of that of the image. We now describe the metrics used to evaluate each method, for which results are summarised in Table 1.

4.1. Quantitative metrics

**FID for completed images** The FID score (Heusel et al., 2017) provides a measure of distance between image distributions. We use it to quantify the distance between the data distribution, \( p_{\text{data}}(\mathcal{I}) \), and the distribution of completed images. By this, we mean the distribution resulting from sampling a dataset image, masking out some pixels, and replacing them by performing inpainting. These distributions should only match if the distribution over completed images exactly matches \( p_{\text{data}}(\mathcal{I}|\hat{\mathcal{I}}) \), since \( \int p_{\text{data}}(\mathcal{I}|\hat{\mathcal{I}})p_{\text{data}}(\mathcal{I})d\hat{\mathcal{I}} = p_{\text{data}}(\mathcal{I}) \). Although this metric is only zero when an image completion model perfectly captures \( p_{\text{data}}(\mathcal{I}|\hat{\mathcal{I}}) \), it does not explicitly consider multiple completions of the same observations. We therefore view it as a measure of image completion quality more than diversity. Figure 3 shows FID scores for completions by each method with varying numbers of observed image patches. AIPO achieves significantly lower FID scores than the nearest method (which is AIPO-R on CIFAR-10 and PIC on FFHQ-256). The FID scores aggregated over each number of patches is reported in Table 1. This also reports the FID scores (described as “holes”) for a different mask distribution where all pixels are observed other than those covered by a patch; essentially the inverse of the masks from our standard distribution. This is closer to the standard inpainting setting where only small patches are missing. For this evaluation, we used the AIPO and AIPO-R models trained on the standard mask distribution. However, to ensure a fair comparison rather than a test of the baselines’ generalisation, we used baseline models trained on masks sampled from the “holes” distribution. Despite this, both AIPO and AIPO-R achieve significantly better FID scores than the baselines for this task.

**Inception score** The inception score (IS) (Salimans et al., 2016) is a widely-adopted heuristic for assessing the quality of image generative models. It takes into account both how diverse and how realistic the generated images are. We compute inception scores of the distribution over image completions for a fixed input \( \hat{\mathcal{I}} \). We then report the average of this inception score over multiple inputs. In Fig. 3 (bottom left), we show inception scores for CIFAR-10 with varying numbers of observed patches. As recommended...
by Barratt & Sharma (2018), these were computed with a classifier trained on CIFAR-10 (details in the appendix). With no, or few, observed patches, AIPO and AIPO-R both significantly outperform the baselines. Of these two, AIPO achieves the higher score; this may reflect greater diversity due to the mass-covering property of its objective. As more patches are observed, the inception scores for all methods decrease towards their minimum of 1. This is to be expected since there is little uncertainty in the completion when most of the image is observed. FFHQ-256 has no labels for training a classifier, so we report an LPIPS distance (described in the following paragraph) instead.

LPIPS Following Zhu et al. (2017); Zheng et al. (2019), we use a measure of image completion diversity based on the LPIPS metric (Zhang et al., 2018), a measure of distance between two images. we assess the diversity of an image completion distribution for observation \( \hat{I} \) by drawing 50k pairs of images and averaging the LPIPS for each pair. We average the result over 10 different observations. A higher LPIPS score means more dissimilar pairs of images are generated, implying a higher diversity among the drawn images. It is important to note that, in contrast to the inception score, LPIPS measures only diversity and does not take into account the quality or coherency of the images. Figure 3 (bottom right) plots the LPIPS on FFHQ-256 for image completions from various numbers of patches. We see that AIPO creates the most diverse completions, closely followed by PIC. AIPO-R creates the least diverse completions.

4.2. Qualitative results

Each row of Fig. 4 shows an example image completion task. The images are randomly sampled from the test set. The observation masks for the first two tasks are manually selected to cover most of the face parts. The last observation mask is sampled from the mask distribution \( p(\hat{I}|I) \) used for training the models. While both AIPO and AIPO-R produce more plausible images than the baselines, the samples from AIPO are more diverse. PIC achieves performance close to that of our models but struggles when most of the image is missing; for an example, see the hair in the second task. Samples from ANP and CE, although consistent with the general context of the patches, are blurry. See the appendix for more examples.

5. Application to medical imaging

In this section, we investigate a possible application for image completion in medical imaging. We do not claim that our system, as it is now, is suitable for use in a clinical setting but believe this is a worthwhile avenue to explore. In particular, we consider whether it is possible to automatically target a chest x-ray at areas most likely to reveal abnormalities. This could avoid the need to scan the entire chest and so bring benefits including reducing the patient’s radiation exposure. Specifically, we consider performing a series of x-ray scans, each targeted at only a small portion of the area of interest. We can select the coordinates

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**Figure 4.** Uncurated qualitative results of different models. Best viewed zoomed in. Each row shows an image completion task where the leftmost column is the input and the rest are outputs by different models. The lighter regions of (a) are the masked out pixels.

**Figure 5.** Accuracy against the number of scans, with scan positions chosen with BOED or randomly, compared against a CNN with access to the whole image. This is computed on a random subset of 1024 test images. Using BOED allows performance to quickly approach the level achieved using the whole image.
We experiment on the NIH Chest X-ray 14 dataset (Wang & Verdinelli, 1995) provides a framework to select a value $c_t$ of the scan at each step $t$, and this selection can be informed by what was observed in the previous scans. The task we consider is how to select $c_t$ to be most informative. In particular, assume we wish to infer a variable $v$ (representing, e.g., whether the patient has an illness). Bayesian optimal experimental design (BOED) (Chaloner & Verdinelli, 1995) provides a framework to select a value $c_t$ that is maximally informative about $v$. One measure of this is the expected information gain, or EIG. The EIG can be estimated as follows (Harvey et al., 2019). We define $\hat{x}_{c_1, \ldots, c_t-1}$ to be a sequence of observations from times $1, \ldots, t-1$; and $f_{c_1, \ldots, c_t}(I)$ to be a function which maps from a (latent) image $I$ to observations $\hat{x}_{c_1, \ldots, c_t}$. We also require a classifier $g$ which maps from the partially observed image to a distribution over the variable of interest, $v$. Then the expected information gain from directing the scanner at coordinates $c_t$ is

$$
\text{EIG}(c_t; \hat{x}_{c_1, \ldots, c_{t-1}}) = \mathcal{H}[g(v|\hat{I}_{1, \ldots, c_{t-1}})] - \frac{1}{N} \sum_{n=1}^{N} \mathcal{H}[g(v|f_{c_1, \ldots, c_t}(I^{(n)}))]
$$

where $I^{(1)}, \ldots, I^{(N)}$ are drawn from an approximation of $p(I|\hat{I}_{1, \ldots, c_{t-1}})$. In particular, we approximate samples from $p(I|\hat{I}_{1, \ldots, c_{t-1}})$ with the image completion method we presented in this paper. To select $c_t$, we simply evaluate $\text{EIG}(c_t; \hat{x}_{c_1, \ldots, c_{t-1}})$ for many different values of $c_t$ and select the value which minimises it.

We experiment on the NIH Chest X-ray 14 dataset (Wang et al., 2017) with images downsampled to $256 \times 256$. We simulate a scanner which returns a $64 \times 64$ image patch from this image. We train the classifier $g$ with the training procedure and Densenet-121 (Huang et al., 2017) architecture used by Harvey et al. (2019). The image completion network we use has the same architecture as that used for FFHQ-256. See the supplementary material for more detail.

Figure 5 shows the performance for the binary prediction of whether the patient has any of 14 conditions when $g$ is used to make a classification after varying numbers of scans. Using BOED allows more accurate classifications than random selection of $c_t$, and the performance gap increases as more glimpses are taken. This may be because, with BOED, the result of the first scan can be used to select informative, image-specific locations for the next scan. Using BOED also makes it possible to interpret why a particular coordinate was chosen with visualisations similar to Fig. 6. This shows several quantities from Eq. (17): the sampled images $I^{(n)}$ approximating $p(I|\hat{I}_{1, \ldots, c_{t-1}})$; the information gain given $I^{(n)}$ and each possible value of $c_t$; and finally the expected information gain for each value of $c_t$.

### 6. Discussion and conclusion

We have presented a method for generating diverse and high-quality image completions through amortized inference in the latent space of a VAE. We derived two training objectives, one of which significantly outperformed all baselines on every metric we computed. We demonstrated an application for such a model by using it to perform BOED. There are numerous directions for future research. One is to scale AIPO to larger images: Child (2020) scale the VD-VAE architecture to $1024 \times 1024$, albeit with increasing computational cost. Another is to further improve the image quality, for example by using a partial encoder with more expressive distributions. Preliminary experiments revealed...
that normalizing flows could help the partial encoder better match the posterior, but with little impact on FID scores.

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A. Reconstructions of o.o.d. images

Figure 7 shows images from various domains, all reconstructed by a VAE trained on a different domain (the FFHQ-256 dataset). The fact that the VD-VAE architecture can reconstruct most of these images alludes to its mass-covering property.

B. Experimental details

B.1. AIPO/AIPO-R

Architectures The AIPO encoder and decoder architectures we used for CIFAR-10 and FFHQ-256 were the same as those used by Child (2020) for the same datasets, with 45 and 62 groups of latent variables respectively. The encoder and decoder had 39 million parameters for CIFAR-10 and 62 groups of latent variables respectively. The encoder contained 18 million and 65 million parameters respectively. The encoder and decoder had 39 million parameters for CIFAR-10 and 62 groups of latent variables respectively. The encoder and decoder of a VD-VAE trained on images of faces. All but the uniformly sampled image are faithfully reconstructed.

Training The AIPO and AIPO-R networks on CIFAR-10 were each trained for 160,000 iterations with a batch size of 14. This took 17 hours for AIPO and 21 hours for AIPO-R. The learning rates used were selected with a search over three values and were $2 \times 10^{-4}$ for AIPO and $2 \times 10^{-5}$ for AIPO-R. On FFHQ-256, AIPO and AIPO-R were trained for 240,000 and 160,000 iterations respectively, taking 141 and 110 hours. These used learning rates of $1.5 \times 10^{-4}$ and $5 \times 10^{-5}$. As for CIFAR-10, these learning rates were selected with a search over three values. Other hyperparameters are the same as those used by Child (2020) for training VAEs on these datasets.

B.2. Baselines

For all the baselines, we based our implementations on publicly available (official or unofficial) implementations. A link to these repositories is provided for each baseline. All the training procedures were modified to use the same distribution of partially observed images as for training AIPO (see Section 4).

PIC We adapted the official implementation of Zheng et al. (2019)\textsuperscript{1} for the FFHQ-256 and CIFAR-10 datasets. We used the same architecture, hyperparameters and objective functions as their implementation. The networks for both CIFAR-10 and FFHQ-256 had 9,128,390 parameters. They were trained using Adam optimizer (Kingma & Ba, 2015) with a batch size of 20, a learning rate of $10^{-4}$ and $\beta_1, \beta_2$ parameters of 0.0 and 0.999 respectively. The model for FFHQ-256 was trained for 180 epochs and the model for CIFAR-10 was trained for 240 epochs. It took 118 hours to train each of these models on an RTX 2080 Ti GPU.

The PIC architecture is defined for $256 \times 256$ images. Therefore, in order to test it on the CIFAR-10 dataset, we resized the $32 \times 32$ CIFAR-10 images to $256 \times 256$ (via bilinear interpolation) before feeding them into the PIC model. We then down-sampled the inpainted images back to the original size of $32 \times 32$ for evaluation.

ANP Our ANP network architecture was based on that of Kim et al. (2019)\textsuperscript{2}, differing only in that we used hidden and latent dimensions of 512. Both the CIFAR-10 and FFHQ-256 models had 11,563,526 parameters and were trained for 220 epochs with a batch size of 16 using Adam optimizer. Learning rates of $5 \times 10^{-5}$ and $10^{-4}$ was used for the CIFAR-10 and FFHQ-256 models respectively. Training took 29 hours for CIFAR-10 and 43 hours for FFHQ-256 on a Tesla V100 GPU.

In the original image inpainting experiments of Kim et al. (2019)\textsuperscript{3} the images are $32 \times 32$. Since ANPs embed each pixel separately and their self-attention and cross-attention layers attend to every other pixel in the observed and target sets, it is computationally expensive to scale them to larger images. We therefore downsampled the FFHQ-256 images to $64 \times 64$, and upsampled the inpainted image back to $256 \times 256$ via bilinear interpolation. Additionally, at training time, we randomly dropped half of the observed pixels (a.k.a. context set in neural process literature) to reduce the computational cost. Finally, the target set at training time was half of the

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\textsuperscript{1}https://github.com/lyndonzheng/Pluralistic-Image-Inpainting

\textsuperscript{2}Our implementation of ANP is based on https://github.com/EmilienDupont/neural-processes
unobserved pixels. For CIFAR-10, on the other hand, no modification was done to the image resolution or observation masks i.e. images were fed in at $32 \times 32$ resolution, the context sets were the set of all observed pixels in partial images $\mathcal{I}$, and the target sets consisted of all unobserved pixels.

**CE** We used the same architecture as reported by Pathak et al. (2016)\(^3\). The models for CIFAR-10 and FFHQ-256 had 34,094,528 and 71,321,536 parameters respectively. Both models were trained for 500 epochs with a batch size of 64. We used Adam optimizer with a learning rate of $2 \times 10^{-4}$, $\beta_1 = 0.5$, and $\beta_2 = 0.999$. It took 5 hours to train the CIFAR-10 model and 48 hours to train the FFHQ-256 model on a RTX 2080 Ti GPU.

**B.3. Inception score**

**Definition** The inception score is defined based on an image classifier. A high score requires the individual samples to be classifiable with high confidence and, at the same time, the marginal distribution of the samples’ classes to be diverse. More formally, let $G$ be a generator producing samples $x \in \mathcal{X}$ and let the classifier output $p(y|x)$ where $y \in \mathcal{Y}$ denotes the classification labels specified by the classifier. Then the logarithm of the inception score is

$$ \log IS = \mathbb{E}_{x \sim G} \left[ \text{KL}(p(y|x) \parallel p(y)) \right] $$

$$ = -\mathbb{E}_{x \sim G} \left[ H(p(y|x)) - H(p(y|x), p(y)) \right]. \quad (18) $$

From an information-theoretic perspective, the inception score is the mutual information between class labels and generated images $I(x; y)$.

**Classifier network** Results in the GAN literature suggest that the inception score is unreliable when applied to image domains other than ImageNet. We therefore replaced the standard, ImageNet-trained, Inception network with a classifier trained on CIFAR-10. In particular, we used a ResNet-18 (He et al., 2016) architecture, pretrained on ImageNet\(^1\). We fine-tuned the model on CIFAR-10 for 200 epochs using stochastic gradient descent with an initial learning rate of 0.1 and a cosine annealing learning rate scheduler (Loshchilov & Hutter, 2017). We evaluated the classifier after each training epoch and picked the one with the highest test accuracy as the final classifier. The best accuracy of 95.15% correct classification rate was achieved after 198 epochs of training.

**B.4. BOED**

The procedure we use for BOED closely follows Harvey et al. (2019). Like them, we approximate the EIG as in Eq. (17) with two learned components. (1) The first is an “attentional variational posterior CNN” (AVP-CNN). This is denoted $g$ in Section 5 and uses exactly the same architecture and training procedure as Harvey et al. (2019), described in the paragraph below. (2) The second is a “stochastic image completion” mechanism. Harvey et al. (2019) generate image completions by retrieving images from a database which roughly match the observed pixel values. Although completions from this are diverse, they can match the observed values poorly. We therefore replace this stage with an image completion network trained using AIPO.

The AVP-CNN, $g$, is trained to map from masked images, $\mathcal{I}$, to distributions over the class labels. Each image in the Chest X-ray 14 dataset has labels indicating the presence or absence of each of 14 pathologies. We train $g$ to produce 15 outputs: an estimated probability of the presence or absence of each of the 14 conditions individually; and an additional estimate of the probability that any (one or more) of these conditions is present. We train $g$ to estimate these using a cross-entropy loss. Masked images are sampled using almost the same mask distribution as for training the image completion networks (described in Section 4); the only difference is that patches now have 25% rather than 35% of the image width, to match the observations we use in the experiments with BOED. We use an AVP-CNN pretrained on ImageNet and then trained on Chest X-ray 14 for 32,000 iterations with a batch size of 32 and learning rate $1 \times 10^{-5}$.

To use AIPO for the Chest X-ray dataset, we first trained an unconditional VAE. While AIPO allows us to train all components from scratch, training this unconditional model allowed us to speed up later experimentation with AIPO. The unconditional VAE was trained for 200,000 iterations (about 5 days) and the checkpoint with the best validation ELBO was used. The hyperparameters used were the same as those of Child (2020) on FFHQ-256, but with a larger skip threshold of 15,000\(^5\) and a smaller batch size (of 8 rather than 32). We trained AIPO for 50,000 iterations (less than 1 day on 4 GPUs) using this unconditional model with a batch size of 8 and the original skip threshold of 100 used by Child (2020).

Selecting each scan location was done by evaluating Eq. (17) at every point in an evenly-spaced $17 \times 17$ grid over the image, and choosing the maximum. We evaluate the EIG with $N = 10$ sampled image completions. This is repeated

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\(^1\)Our implementation of CE is based on [https://github.com/kuangliu/pytorch-cifar](https://github.com/kuangliu/pytorch-cifar), with some modifications according to [https://github.com/pathak22/context-encoder](https://github.com/pathak22/context-encoder) to support larger image sizes and non-centered observation masks.

\(^3\)Our implementation of the CIFAR-10 classifier is based on [https://github.com/kuangliu/pytorch-cifar](https://github.com/kuangliu/pytorch-cifar).

\(^5\)Child (2020) improve training stability by skipping updates with gradients larger than a set threshold. Using a larger threshold, and so skipping fewer updates, was found to be necessary to train their architecture on the Chest X-ray dataset.
to select the scan location for each \( t = 1, \ldots, 3 \) (with the sampled images conditioned on observations up to \( t - 1 \) at each stage)\(^6\).

**C. Additional BOED results**

Figure 9 shows additional visualisations of the Bayesian experimental design process for different test images, adding to the one shown in the main paper. For each, using experimental design with AIPO leads to an apparently reasonable sequence of scan locations. This helps to explain the quantitative improvement from BOED shown in Fig. 5.

It can be seen that the information gain is sometimes negative; this is expected, since a posterior after making an observation can sometimes have higher entropy than the prior. However, the expected information gain can never be negative. We see in the second column from the right of Fig. 9 that our estimate of the expected information gain is usually positive (blue) at every location in the grid. When it is slightly negative (red), this is a result of the approximations made to estimate it.

\(^6\)Line 429 of the manuscript mistakenly states we select a coordinate to *minimise* the EIG, when in fact we *maximise* it.

**D. Image samples**

**Chest X-ray 14**  
Figure 8 shows sampled completions of x-ray images from AIPO.

**FFHQ-256**  
Figure 10 on page 17 shows sampled image completions from each method (AIPO, AIPO-R, and all baselines) for an FFHQ-256 image. The long form of the appendix\(^7\) contains more completions for different FFHQ-256 images. For each image, completions from six different observation masks (with zero to five observed patches) are shown. For each mask, we plot five sampled completions from each stochastic method and the single deterministic completion from CE.

**CIFAR-10**  
Figure 11 (on page 18) shows sampled completions for various observations from a CIFAR-10 image, formatted in the same way as Figure 10. See the long form of the appendix for completions of different CIFAR-10 images.

As is apparent from the image samples, AIPO-R fails to generate meaningful completions when the number of observed pixels is small. It also has less sample diversity than AIPO, which can be explained by the mode-seeking behaviour of

\(^7\)https://www.cs.ubc.ca/˜wsgh/icidgm-appendix.pdf
Figure 9. Visualisations in the style of Fig. 6 for three more test images. Each shows the experimental design process to choose three scan locations. As in Fig. 6, the left column shows the observations made before selecting each scan location. The next five columns show five of the $N = 10$ sampled image completions conditioned on these observations. Each is overlaid with the information gain predicted after scanning any location. The second column from the left shows the expected information gain at each location, given by averaging the information gains arising from each sampled image completion. The final column shows the updated observation after scanning the location which maximises the expected information gain.
the reverse-KL and subsequent under-estimation of posterior uncertainty (Minka et al., 2005).

Finally, although AIPO generally produces more realistic images than the others, it is still far from perfect. In particular, a lack of bilateral symmetry is visible in the FFHQ-256 samples, especially with smaller numbers of observed pixels. This issue can be seen in unconditional samples from the underlying VAEs as well (see the samples in Child (2020)). Therefore, it is likely that any future advances in image modeling with VAEs could be integrated to improve this aspect of the results.

**Free-form observation masks** We also investigated how the models generalise to observation masks which are outside the training distribution (as defined by $p(\hat{I}|I)$). To do so, we created masks manually. The results can be seen in Figure 12 (on page 18). More available in the long form of the appendix. All methods generalise reasonably well to these masks, although better results may be obtained by changing the mask distribution use in training.

**E. Code**

We have attached code for training AIPO and AIPO-R. We will later release code for using this with Bayesian experimental design. This will all be made publicly available, along with weights for networks trained with AIPO. Additionally, we will make the weights for the baseline models and the classifier used for inception score computation publicly available.
Figure 10. Sampled completions from each method for an FFHQ-256 image. Panels (a) and (d) both show the true image, and the observation mask on which the samples in each row are conditioned. Panels (b) and (c) show completions from AIPO and AIPO-R. The remaining panels show completions from the baselines: (e) and (f) show five completions for the stochastic baselines PIC and ANP. Panel (g) shows the single deterministic completion produced by CE.
Figure 11. Sampled completions from each method for a CIFAR-10 image. Panels (a) and (d) both show the true image, and the observation mask on which the samples in each row are conditioned. Panels (b) and (c) show completions from AIPO and AIPO-R. The remaining panels show completions from the baselines: (e) and (f) show five completions for the stochastic baselines PIC and ANP. Panel (g) shows the single deterministic completion produced by CE.
Figure 12. Sampled completions from each method for an FFHQ-256 image with a free-form observation mask. Panels (a) and (d) both show the true image, and the observation mask on which the samples in each row are conditioned. Panels (b) and (c) show completions from AIPO and AIPO-R. The remaining panels show completions from the baselines: (e) and (f) show five completions for the stochastic baselines PIC and ANP. Panel (g) shows the single deterministic completion produced by CE.