Numeric Study on Skin Friction Reduction of Engine Oil-WS2 Nanofluid by MRT Lattice Boltzman Method

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Abstract: New lubricants use nanoparticles like WS2 to improve lubrication performance. The aim of the present study is to determine numerically the skin friction of nanofluid on moving surfaces in a lubricating system. The base fluid is 5W-30 engine oil and the nano-additive is WS2 nanoparticles. This numerical study is based on the multiple-relaxation-time Lattice Boltzmann method (MRT-LBM). The two-dimensional nine-velocity (D2Q9) model is adopted to simulate the nanofluid flow confined by two moving surfaces. The parameters considered are the nanoparticle concentration \( \varphi \) and the flow Reynolds number \( Re \). The results obtained show a reduction of skin friction factor when we increase the nanoparticle concentration.

Key words: Lubrication, friction factor, MRT-LBM, nanoparticle.

List of Symbols

| Symbol | Description                          |
|--------|--------------------------------------|
| \( Re \)  | Reynolds number                      |
| \( u, v \) | Components of velocity               |
| \( U, N \) | Dimensionless of velocity component  |
| \( x, y \) | Cartesian coordinates                |
| \( X, Y \) | Dimensionless of Cartesian coordinates|
| \( p \)  | Pressure                              |
| \( L \)  | Length of the cavity                 |
| \( H \)  | Cavity height                         |
| \( f_i \) | Density distribution function        |
| \( c_i \) | Discrete particle velocity           |
| \( U_p \) | Uniform velocity of moving plates    |
| \( \mathcal{C}_f \) | Local skin friction                  |
| \( \mathcal{C}_f \) | Average friction factor              |
| MRT     | Multiple relaxation time             |
| LBE     | Lattice Boltzmann equation           |

Greek Symbols

| Symbol | Description                          |
|--------|--------------------------------------|
| \( \mu_{nf} \) | Nanofluid dynamic viscosity          |
| \( \mu_{bf} \) | Dynamic viscosity of base fluid      |
| \( \rho_{nf} \) | Nanofluid effective density          |
| \( \rho_{bf} \) | Base fluid density                   |
| \( \rho_s \)  | Nanoparticles density                |
| \( \varphi \) | Volumetric concentration of nanofluid|

1. Introduction

Lubricating fluids play an important role in reducing the friction and wear of moving parts of moving mechanical systems. The improvement of lubricants rheological properties by using nanoparticle is the best solution for reducing friction. Recent research is increasingly interested in the study of nanofluids as a new alternative to improve lubrication effects. The studies conducted by Tao et al. [1] on the lubrication characteristics of diamond nanoparticles dispersed in oil, showed that this lubricant has a reducing effect in friction for the boundary type lubrication system. A study carried out by Gu et al. [2] on the properties of a nanofluid lubricant based on MoS2 nanoparticles showed that this lubricant had a Newtonian behavior and had friction reducing effects. Various experimental studies have been carried out to reveal the effects of nanofluids on the reduction of wear and friction, using metals [3-5], metal oxides [6, 7] and non-metal [8, 9] nanoparticles. Hwang et al. [10] studied the effect of shape and size of different nanoparticles in mineral oils. The nanoparticles studied were graphite, graphite nanofibers, carbon black and carbon nanotubes. The results showed that
the use of spherical shapes of particles has considerably reduced the friction factor. There are few numerical studies on improving lubrication using nanoparticles. This can be attributed to the fact that the lubrication mechanism in most cases leads to the deformation of the nanoparticles which makes modeling difficult. However, lubrication characteristics have been revealed by some studies using numerical methods such as the molecular dynamics method (MDM). Hu et al. [11] numerically studied the tribological properties of two hard nanoparticles species confined by two iron blocks using MDM. This study has shown that the nanoparticles separate the two blocks and behave like ball bearings when working at low load and at low speed. However at high speed and at high load, this bearing effect quickly disappears due to the deformation of the nanoparticles. Azaditalab et al. [12] carried out a numerical study on the reduction of the friction coefficient of a lubricant fluid in Taylor-Couette System by adding nanoparticles of MoS2, WS2 and diamond. This numeric study found that MoS2 and WS2 nanoparticles were more efficient in reducing friction. The present numerical study is motivated by the importance of recent developments of nanofluid simulation flow using the LBM method.

In this work, the Lattice Boltzmann method with multiple-relaxation-time (MRT-LBM) is used to simulate the nanofluid flow confined by two moving surfaces. The main purpose of the present study is to determine skin friction of WS2-Oil (5W30) nanofluid on moving surfaces in a lubricating system.

2. Problem Statement and Governing Equations

The geometry of the current problem is shown in Fig. 1. It consists of a two-dimensional enclosure of height $H$ and width $L$. The cavity is filled with nanofluid. The base fluid is 5W-30 engine oil and the nano-additive is WS2 nanoparticles. The two upper and lower plates move with the same speed in the opposite direction imposing shear stresses on the fluid. In the simulation we choose $H = L$.

Measurements of the rheological properties of nanofluids indicate that most nanofluids behave like Newtonian-fluid mixtures at low particle volume concentrations [13, 14]. The study will be limited to volumetric concentration $\varphi \leq 5\%$.

The effective density of nanofluid is calculated using the relation [15]:

$$\rho_{nf} = (1 - \varphi)\rho_b + \varphi\rho_s$$

$\rho_{nf}$ is the nanofluid effective density, $\rho_s$ is the nanoparticles density, $\rho_b$ is the base fluid density and $\varphi$ is the nanofluid volumetric concentration.

The effective dynamic viscosity of nanofluid with $(\varphi < 0.04)$ is given by Brinkman’s equation [16]:

$$\mu_{nf} = \frac{\mu_b}{(1 - \varphi)^{2.5}}$$

$\mu_{nf}$ and $\mu_b$ are dynamic viscosities of nanofluid and base fluid, respectively.

![Fig. 1 Physical problem scheme.](image-url)
For \( \varphi > 0.04 \) the effective dynamic viscosity is calculated from Batchelor equation [17] valid in the range \( (\varphi \leq 0.1) \) for spherical nanoparticles.

\[
\mu_{nf} = \mu_b f(1 + 2.5\varphi + 6.5\varphi^2)
\]

The density of 5W-30 engine oil at 100 °C is \( \rho_{bf} = 860 \) kg·m\(^{-3} \) and the density of WS\(_{2} \) at the same temperature is \( \rho_s = 7,500 \) kg·m\(^{-3} \).

The dynamic viscosity of 5W-30 engine oil at 100 °C is \( \mu_{bf} = 0.00946 \) kg·m\(^{-1}\)·s\(^{-1} \).

Fig. 2 shows that the increase in the nanoparticle volume concentration decreases the kinematic viscosity of nanofluid.

The fluid flow is supposed to be incompressible and viscous. The flow is 2D and the heat transfers are neglected. It is also supposed that the liquid and the solid are in thermal equilibrium at 100 °C and they flow at the same velocity. The governing dimensionless equations are as follows:

Continuity equation:

\[
\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0
\]

Momentum equations:

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -\frac{\partial P}{\partial x} + \gamma_d \left( \frac{\delta^2 U}{\delta x^2} + \frac{\delta^2 U}{\delta y^2} \right)
\]

The macroscopic boundary conditions are:

\[ u(x, 0) = -U_0, \; u(x, H) = U_0 \]

3. Lattice Boltzmann Simulation

The lattice Boltzmann method describes the fluid flow by using two processes: collision and stream processes of particle distribution function, \( f \), on each lattice node \( x \). There are two main collision models for the application of the lattice Boltzmann method. The single relaxation time (SRT), proposed by Bhatnagar, Gross and Krook (BGK) [18] in which all moments have the same relaxation rate. The simplicity of the lattice BGK model comes at the expense of numerical instability [18] and inaccuracy in implementing boundary conditions [19]. The second model called
Lattice Boltzmann Method with multiple relaxation time (MRT-LBM) introduced by d’Humières [20] is more stable than the BGK because the different relaxation times can be adjusted individually to obtain “optimal” stability [21].

### 3.1 D2Q9-MRT-LB Equation for the Flow Fields

In the MRT, the collision step happens on the moment space and the stream step is executed in the velocity space. According to Refs. [21, 22], the MRT-LBE equation is expressed as:

\[ f_i(x + c_i \Delta t, t + \Delta t) - f_i(x, t) = M^{-1} S (m_i(x, t) - m_{eq}) \]

where \( m \) and \( m_{eq} \) are vectors of moments, \( m = (m_1, m_2, ...)^T \), \( m = M f \) and \( m_{eq} = M f_{eq} \). \( S \) is the relaxation matrix. \( M \) is an orthogonal transformation matrix constricted from velocity. The D2Q9 model (Fig. 3) with standard two dimensional, nine velocities square lattice for flow is used in this work. The nine discrete velocities \( c_i \) are given by:

\[ c_x = c \begin{bmatrix} 0,1,0,-1,0,1-1,-1,1 \end{bmatrix} \]
\[ c_y = c \begin{bmatrix} 0,0,1,0,-1,1,1,1,-1 \end{bmatrix} \]

where \( c = \frac{\Delta x}{\Delta t} \) is the lattice speed, \( \Delta x \) and \( \Delta t \) are the lattice width and time step, respectively (these quantities are fixed at the unity (\( \Delta x = \Delta t = 1 \))).

The nine velocity moments are:

\( m = (\rho, e, j_x, q_x, j_y, q_y, P_{xx}, P_{xy}, P_{yx})^T \) where \( \rho \) is the density, \( j_x = \rho u_x \) and \( j_y = \rho u_y \) are \( x \) and \( y \) components of the flow momentum, respectively, \( e \) is related to energy , \( e \) is related to energy square, \( j_x = \rho u_x \) and \( j_y = \rho u_y \) are \( x \) and \( y \) components of the flow, respectively, \( q_x \) and \( q_y \) correspond to the \( x \) and \( y \) components of the energy flux, \( P_{xx} \) and \( P_{xy} \) correspond to the symmetric and traceless components of the strain-rate tensor. In D2Q9 lattice, the matrix \( M \) is defined as:

\[
M = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
-1 & -1 & -1 & -1 & 2 & 2 & 2 & 2 & 2 \\
4 & -2 & -2 & -2 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & -1 & 0 & -1 & -1 & -1 & -1 \\
0 & 0 & 1 & 0 & -1 & 1 & 1 & 1 & 1 \\
0 & 0 & -2 & 0 & 2 & 1 & 1 & 1 & 1 \\
0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & -1
\end{bmatrix}
\]

The equilibrium density distribution function which depends on the local velocity and density is given by:

\[ f_i^{eq}(x, t) = \rho \omega_i \left[ 1 + 3 \cdot (C_i \cdot U) + \frac{9}{2} (C_i \cdot U)^2 \right] - \frac{3}{2} U^2 \]

where \( \omega_i \) is the weighting factor defined as:

\[ \omega_i = \frac{4}{9}, i = 1, \omega_i = \frac{1}{9}, i = 2, 3, 4, \]
\[ 5 \omega_i = 1/36, i = 6, 7, 8, 9. \]

The diagonal relaxation matrix \( S \) is given by:

\[ S = \text{diag}(s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9) \] . For stability, relaxation rates must verify the double inequality \( 0 \leq S \leq 2 \). Using the multi-scale technique Chapman-Enskog expansion we can deduce the incompressible Navier Stokes equation and the viscosity under the condition \( s_7 = s_8 = 1/\tau \). The parameter \( \tau \) is the dimensionless relaxation time related to the viscosity in lattice by the following relation:

\[ \gamma_{ib} = C_s^2 \cdot \delta t (\tau - 0.5) \] where \( C_s = \frac{c}{\sqrt{3}} \) is the sound speed of the lattice. For mass and momentum conservation before and after collision, we assume in this simulation that \( s_1 = s_4 = s_6 = 1 \). We also consider \( s_2 = s_3 = 1.4 \); \( s_5 = s_7 = 1.2 \). The macroscopic variables such as density \( \rho \) and velocity \( \bar{u} \) are calculated as the moments of the
distribution function:
\[
\rho = \sum_{i=1}^{9} f_i
\]
\[
\rho \vec{u} = \sum_{i=1}^{9} \vec{e}_i \cdot f_i
\]

The main objective of this work is to determine the friction factor. The local skin friction coefficient is defined as
\[
C_f = \frac{F_d}{\frac{1}{2} \rho_n u_r^2}
\]
where \( F_d \) is the local skin friction drag at the bottom surface written as
\[
F_d(x) = -\mu_{nf} \frac{\partial u}{\partial y}(x, 0)
\]
and \( u_r \) is the reference velocity taken equal to the characteristic velocity \( U_0 \).
The local value of skin friction at the bottom surface is determined from the numerical solution as:
\[
C_f(X) = \frac{-2}{R_e} \frac{F(\varphi)}{(1 - \varphi + \varphi \frac{\varphi^2}{\rho_f})} \frac{\partial U}{\partial Y}(X, 0)
\]
where \( F(\varphi) \) function of \( \varphi \) is defined as:
\[
F(\varphi) = \begin{cases} 
\frac{1}{(1 - \varphi)^{2.5}} & \text{for } \varphi \leq 0.04 \\
1 + 2.5 \varphi + 6.5 \varphi^2 & \text{for } 0.04 < \varphi \leq 0.1 
\end{cases}
\]

The average friction factor at the bottom surface is calculated as
\[
C_{f} = \frac{1}{L_x} \int_{0}^{L_x} C_f(X) dX
\]

3.2 Boundary Conditions

Implementation of boundary conditions is very important for the simulation. The unknown distribution functions pointing to the fluid zone at the boundaries nodes must be specified. Concerning the no-slip boundary condition, a bounce-back boundary condition is used on the solid boundaries, which means that incoming boundary populations equal to out-going populations after the collision. For instance, for the left boundary, the following conditions are imposed: \( f_6 = f_8 \), \( f_2 = f_4 \) and \( f_9 = f_7 \). The Zou-He boundary conditions were applied on boundary moving with a given speed. For example, for the top boundary, unknown density distribution functions can be determined by the following conditions:
\[
f_5 = f_3, \quad f_6 = f_9 + \frac{1}{2} (f_2 - f_4) - \frac{1}{2} \rho U_0 \quad \text{and} \quad f_9 = f_7 + \frac{1}{2} (f_4 - f_2) + \frac{1}{2} \rho U_0
\]

4. Code Validation

A MATLAB code has been developed to run the simulation. A convergence criterion of \( 10^{-6} \) for velocity has been used to terminate the numerical simulation. The performance of the code is established by comparing its predictions with the relevant limiting cases available in the literature. We tested our LBM code in the problem of two-sided lid-driven cavity in case of anti-parallel wall motion. It is a square cavity where top and bottom walls are moving with uniform velocity in the opposite direction. We compare the developed velocity profiles with the results of Perumal [23]. As shown in Fig. 4 a good agreement between our LBM-MRT results and the compared results is observed. The agreement with these results gives credibility to our LBM code.

5. Results and Discussion

The governing equations were solved using the MRT lattice Boltzmann method and the results are shown in several plots for the effects of different parameters on the velocity and skin friction factor.

As seen in Fig. 5, the horizontal velocity is symmetrical with respect to the horizontal center line \( (Y = 0.5) \). Likewise, the local skin friction is symmetrical with respect to the vertical center line \( (X = 0.5) \). This observation is compatible with the studied physical problem.

As seen in Fig. 6, the increase in the volumetric concentration of nanoparticles in laminar regime slightly affects the velocity gradients. This effect is not observable when \( Re \) increases as shown in Fig. 7.
On the other hand, the local friction factor always decreases when the volumetric concentration is increased. This result is shown in Figs. 8 and 9.

When we increase in the volumetric concentration, the decreasing in average friction factor is observed for all the Reynolds numbers (see Figs. 10 and 11). As we observed from Fig. 2, by increasing volumetric concentration for nanoparticles, the kinematic viscosity of the nanofluid is greatly reduced leading to direct contact of the nanoparticles with the sliding surfaces. Their accumulation on the surface helps reduce the resulted friction factor and improve lubrication.
Fig. 6  Horizontal velocity along vertical centerline for $Re = 100$.

Fig. 7  Horizontal velocity along vertical centerline for $Re = 2,000$.

Fig. 8  Local skin friction factor at bottom surface for $Re = 100$.

Fig. 9  Local skin friction factor at bottom surface for $Re = 2,000$. 

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As we observe from Fig. 12, the friction factor of the WS2-Oil nanofluid 5% is the least for all Reynolds number. However, the reduction in the friction factor is the highest for $Re = 500$ and decreases for large values of $Re$. This downward trend in reduction of the friction coefficient when $Re$ increases can be explained by the existence of a main vortex at the center of the cavity. Higher value of Reynolds number leads to formation of bigger and stronger vortex which will entrain the nanoparticles towards the center of the cavity thus moving away from the surface. This effect explains their lower efficiency on the friction factor.

![Fig. 10](image1)

**Fig. 10** Average friction factor with volumetric concentration for $Re = 100$ and 1,000.

![Fig. 11](image2)

**Fig. 11** Average friction factor with volumetric concentration for $Re = 2,000$ and 4,000.
6. Conclusion

In the present work, an incompressible D2Q9 MRT model is employed to simulate the flow of WS2-5W30 nanofluid confined by two moving surfaces. The results obtained show that the addition of nanoparticles reduces the friction factor at bottom surface. This reduction increases with the volume concentration of nanoparticles and decreases for large values of Reynolds number. It is found that the best reduction is obtained for a volume concentration $\varphi = 5\%$ and for Reynolds number $Re = 500$.

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