Relativistic analysis of Michelson-Morley experiments and Miller’s cosmic solution for the Earth’s motion

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Abstract

A simple relativistic treatment of Michelson-Morley type of experiments shows the remarkable internal consistency of 1932 Miller’s cosmic solution $v_{\text{earth}} \sim 208$ km/s deduced from the experimental fringe shifts observed with his apparatus. The same analysis of present-day experiments is in good agreement with the existing data.
The aim of this Letter is to illustrate the remarkable internal consistency of 1932 Miller’s cosmic solution for the Earth’s motion. This was deduced from the data obtained with his Michelson-Morley interferometer [1] on the base of the theory exposed by Nassau and Morse [2]. In this way, from the variations of the magnitude and azimuth of the ether-drift effect with the sidereal time, one can determine the apex [3] of the motion of the solar system. By requiring consistency among the determinations obtained in four epochs of the year (see Fig. 23 of ref. [1]), Miller obtained values for the Earth’s velocity lying in the range 200-215 km/s (see page 233 of ref. [1]) with the conclusion that “…a velocity

\[ v_{\text{earth}} \sim 208 \text{ km/s} \] (1)

for the cosmic component, gives the closest grouping of the four independently determined locations of the cosmic apex”.

In this context, it is essential to stress that the fringe shifts observed in the classical experiment of Michelson-Morley [4] (and in the subsequent one of Morley and Miller [5]) although smaller than the expected magnitude corresponding to the orbital motion of the Earth, were not negligibly small. While this had already been pointed out by Hicks [6], Miller’s refined analysis of the half-period, second-harmonic effect observed in the experimental fringe shifts showed that they were consistent with an effective velocity lying in the range 7-10 km/s (see Fig. 4 of ref. [1]). For instance, the Michelson-Morley experiment gave a value \( \sim 8.8 \) km/s for the noon observations and a value \( \sim 8.0 \) km/s for the evening observations. By including Miller’s Mount Wilson results, whose typical effective velocities were also lying in the range 7-11 km/s (see Fig. 22 of ref. [1]), one concludes that all classical ether-drift experiments were consistent with an ‘observable’ velocity

\[ v_{\text{obs}} \sim 9 \pm 2 \text{ km/s}. \] (2)

The problem with Miller’s analysis was to understand the large discrepancy between Eq. (1), as needed to describe the variations of the ether-drift effect at different sidereal times, and Eq. (2), as determined by the magnitude of the fringe shifts themselves.

It has been recently pointed out by Cahill and Kitto [7] that an effective reduction of the Earth’s velocity from values \( v_{\text{earth}} = \mathcal{O}(10^2) \) km/s down to values \( v_{\text{obs}} = \mathcal{O}(1) \) km/s can be understood by taking into account the effects of the Lorentz contraction and of the refractive index \( N_{\text{medium}} \) of the dielectric medium used in the interferometer.

In this way, the observations become consistent [7] with values of the Earth’s velocity that are comparable to \( v_{\text{earth}} \sim 369 \) km/s as extracted by fitting the COBE data for the
cosmic background radiation [8]. The point is that the fringe shifts are proportional to 
\[ \frac{v_{\text{earth}}^2}{c^2} \left( 1 - \frac{1}{N_{\text{medium}}^2} \right) \] rather than to \( v_{\text{earth}}^2 \) itself. For the air, where \( N_{\text{air}} \sim 1.00029 \), assuming a value \( v_{\text{earth}} \sim 369 \) km/s, one expects fringe shifts governed by an effective velocity \( v_{\text{obs}} \sim 9 \) km/s consistently with Miller’s analysis Eq.(2) of the classical experiments.

This would also explain why the experiments of Illingworth [9] (performed in an apparatus filled with helium where \( N_{\text{helium}} \sim 1.000036 \)) and Joos [10] (performed in the vacuum where \( N_{\text{vacuum}} \sim 1.00000... \)) were showing smaller fringe shifts and, therefore, lower effective velocities.

In the following, I shall re-formulate the argument using Lorentz transformations. As a matter of fact, in this case there is a non-trivial difference of a factor \( \sqrt{3} \) that makes Miller’s solution Eq.(1) entirely consistent with Eq.(2).

2. As a first step, I’ll start from the idea that light propagates in a medium with refractive index \( N_{\text{medium}} > 1 \) and small Fresnel’s drag coefficient

\[ k_{\text{medium}} = 1 - \frac{1}{N_{\text{medium}}^2} \ll 1 \] (3)

(if the medium is the vacuum itself, the physical interpretation of \( N_{\text{vacuum}} \) represents a further step, see refs.[11, 12]). Let us also introduce an isotropical speed of light \( c = 2.9979 \times 10^{10} \) cm/s)

\[ u \equiv \frac{c}{N_{\text{medium}}} \] (4)

The basic question is to determine experimentally, and to a high degree of accuracy, whether light propagates isotropically with velocity Eq.(1) for an observer \( S' \) placed on the Earth. For instance for the air, where the relevant value is \( N_{\text{air}} = 1.00029... \), the isotropical value \( \frac{c}{N_{\text{air}}} \) is usually determined directly by measuring the two-way speed of light along various directions. In this way, isotropy can be established, at best, at the level \( 10^{-6} - 10^{-7} \). If we require, however, a higher level of accuracy, say \( 10^{-9} \), the only way to test isotropy is to perform a Michelson-Morley type of experiment and look for fringe shifts upon rotation of the interferometer.

Now, if one finds experimentally fringe shifts (and thus some non-zero anisotropy), one can explore the possibility that this effect is due to the Earth’s motion with respect to a preferred frame \( \Sigma \neq S' \). In this perspective, light would propagate isotropically with velocity as in Eq.(1) for \( \Sigma \) but not for \( S' \).

Assuming this scenario, the degree of anisotropy for \( S' \) can easily be determined by using Lorentz transformations. By defining \( \mathbf{v} \) the velocity of \( S' \) with respect to \( \Sigma \) one finds
\( (\gamma = 1/\sqrt{1 - \frac{v^2}{c^2}}) \)

\[
\mathbf{u}' = \mathbf{u} - \gamma \mathbf{v} + \mathbf{v}(\gamma - 1) \frac{\mathbf{v} \cdot \mathbf{u}}{\gamma (1 - \frac{v^2}{c^2})} \tag{5}
\]

where \( v = |\mathbf{v}| \). By keeping terms up to second order in \( v/u \), one obtains

\[
\frac{|\mathbf{u}'|}{u} = 1 - \alpha \frac{v}{u} - \beta \frac{v^2}{u^2} \tag{6}
\]

where \( (\theta \) denotes the angle between \( \mathbf{v} \) and \( \mathbf{u} \))

\[
\alpha = (1 - \frac{1}{\mathcal{N}_{\text{medium}}^2}) \cos \theta + \mathcal{O}((\mathcal{N}_{\text{medium}}^2 - 1)^2) \tag{7}
\]

\[
\beta = (1 - \frac{1}{\mathcal{N}_{\text{medium}}^2}) P_2(\cos \theta) + \mathcal{O}((\mathcal{N}_{\text{medium}}^2 - 1)^2) \tag{8}
\]

with \( P_2(\cos \theta) = \frac{1}{2}(3 \cos^2 \theta - 1) \).

Finally defining \( u'(\theta) = |\mathbf{u}'| \), the two-way speed of light is

\[
\bar{u}'(\theta) = \frac{1}{u} \frac{2u'(\theta)u'(\pi + \theta)}{u'(\theta) + u'(\pi + \theta)} = 1 - \frac{v^2}{c^2}(A + B \sin^2 \theta) \tag{9}
\]

where

\[
A = \mathcal{N}_{\text{medium}}^2 - 1 + \mathcal{O}((\mathcal{N}_{\text{medium}}^2 - 1)^2) \tag{10}
\]

and

\[
B = -\frac{3}{2}(\mathcal{N}_{\text{medium}}^2 - 1) + \mathcal{O}((\mathcal{N}_{\text{medium}}^2 - 1)^2) \tag{11}
\]

The above results will be useful in the following.

Let us now address the theory of the Michelson-Morley interferometer by considering two light beams, say 1 and 2, that for simplicity are chosen perpendicular in \( \Sigma \) where they propagate along the \( x \) and \( y \) axis with velocities \( u_x(1) = u_y(2) = u = \frac{c}{\mathcal{N}_{\text{medium}}} \). Let us also assume that the velocity \( v \) of \( S' \) is along the \( x \) axis. In this case, to evaluate the velocities of 1 and 2 for \( S' \), we can apply Lorentz transformations with the result

\[
u'_{x}(1) = \frac{u - v}{1 - \frac{v^2}{c^2}} \quad u'_y(1) = 0 \tag{12}
\]

and

\[
u'_{x}(2) = -v \quad u'_y(2) = u \sqrt{1 - \frac{v^2}{c^2}} \tag{13}
\]

Let us now define \( L'_P \) and \( L'_Q \) to be the lengths of two optical paths, say P and Q, as measured in the \( S' \) frame. For instance, they can represent the lengths of the arms of an interferometer.
which is at rest in the $S'$ frame. In the first experimental set-up, the arm of length $L'_P$ is taken along the direction of motion associated with the beam 1 while the arm of length $L'_Q$ lies along the direction of the beam 2. Notice that the two arms, in the $S'$ frame, form an angle that differs from $90^\circ$ by $O(v/c)$ terms.

Therefore, using the above results, the time for the beam 1 to go forth and back along $L'_P$ is

$$T'_P = L'_P \left( \frac{1 - uv/c^2}{u - v} + \frac{1 + uv/c^2}{u + v} \right) \sim \frac{2L'_P}{u} \left( 1 + k_{\text{medium}} \frac{v^2}{u^2} \right)$$  \hspace{1cm} (14)$$

To evaluate the time $T'_Q$, for the beam 2 to go forth and back along the arm of length $L'_Q$, one has first to compute the modulus of its velocity in the $S'$ frame

$$u'(2) \equiv \sqrt{(u'_x(2))^2 + (u'_y(2))^2} = u \sqrt{1 + k_{\text{medium}} \frac{v^2}{u^2}}$$  \hspace{1cm} (15)$$

and then use the relation $u'(2) T'_Q = 2L'_Q$ thus obtaining

$$T'_Q = \frac{2L'_Q}{u'(2)} \sim \frac{2L'_Q}{u} \left( 1 - k_{\text{medium}} \frac{v^2}{2u^2} \right)$$  \hspace{1cm} (16)$$

In this way, the interference pattern, between the light beam coming out of the optical path P and that coming out of the optical path Q, is determined by the delay time

$$\Delta T' = T'_P - T'_Q \sim \frac{2L'_P}{u} \left( 1 + k_{\text{medium}} \frac{v^2}{u^2} \right) - \frac{2L'_Q}{u} \left( 1 - k_{\text{medium}} \frac{v^2}{2u^2} \right)$$  \hspace{1cm} (17)$$

On the other hand, if the beam 2 were to propagate along the optical path P and the beam 1 along Q, one would obtain a different delay time, namely

$$(\Delta T')_{\text{rot}} = (T'_P - T'_Q)_{\text{rot}} \sim \frac{2L'_P}{u} \left( 1 - k_{\text{medium}} \frac{v^2}{2u^2} \right) - \frac{2L'_Q}{u} \left( 1 + k_{\text{medium}} \frac{v^2}{u^2} \right)$$  \hspace{1cm} (18)$$

so that, by rotating the apparatus, there will be a fringe shift proportional to

$$\Delta T' - (\Delta T')_{\text{rot}} \sim \frac{3(L'_P + L'_Q)}{u} k_{\text{medium}} \frac{v^2}{u^2}$$  \hspace{1cm} (19)$$

This coincides with the pre-relativistic expression provided one replaces $v$ with an effective observable velocity

$$v_{\text{obs}} = v \sqrt{k_{\text{medium}} \sqrt{3}}$$  \hspace{1cm} (20)$$

Eq. (19) can also be obtained by using the equivalent form of the Robertson-Mansouri-Sexl parametrization [13, 14] for the two-way speed of light defined above in Eq. (9). In fact, using the relations

$$\Delta T' = \frac{2L'_P}{u'(0)} - \frac{2L'_Q}{u'(\pi/2)}$$  \hspace{1cm} (21)$$
\[(\Delta T')_{\text{rot}} = \frac{2L'_p}{u'(\pi/2)} - \frac{2L'_Q}{u'(0)}\]  

(22)

and Eqs. (10) and (11), one obtains

\[\Delta T' - (\Delta T')_{\text{rot}} \sim (-2B) \frac{(L'_p + L'_Q)}{u} v^2 \frac{u}{u^2}\]  

(23)

that agrees with Eq. (19) up to \(O(k_{\text{medium}}^2)\) terms.

3. Now, upon operation of the interferometer, one is faced with several alternatives:

i) there are no fringe shifts at all. This corresponds to the usual point of view that light propagates isotropically on the Earth so that \(S' \equiv \Sigma\)

ii) there are fringe shifts but their magnitude turns out to be unrelated to any meaningful definition of the Earth’s velocity (think for instance of some anisotropy due to the Earth’s magnetic field)

iii) there are fringe shifts and their magnitude, observed with different dielectric media and within the experimental errors, points consistently to a unique value of the Earth’s velocity

Case iii) would represent experimental evidence for the existence of a preferred frame \(\Sigma \neq S'\). In practice, to \(O(v^2/c^2)\), this can be decided by re-analyzing [7] the experiments in terms of the effective parameter \(\epsilon = \frac{v^2}{u^2} k_{\text{medium}}\). The conclusion of Cahill and Kittto [7] is that the classical experiments are consistent with the value \(v_{\text{earth}} \sim 369\) km/s obtained from the COBE data.

However, in our expression Eq. (20) determining the fringe shifts there is a difference of a factor \(\sqrt{3}\) with respect to their result \(v_{\text{obs}} = v\sqrt{k_{\text{medium}}}\). Therefore, using Eqs. (2) and (20), for \(N_{\text{air}} \sim 1.00029\), the relevant value of the Earth’s velocity is not \(v_{\text{earth}} \sim 369\) km/s but rather

\[v_{\text{earth}} = 216 \pm 47\text{ km/s}\]  

(24)

This is completely consistent with the value Eq. (11) obtained by Miller from the variations of the magnitude and of the azimuth of the ether-drift effect with the sidereal time on the base of the theory of Nassau and Morse [2].

4. Still today, the original Michelson-Morley experiment of 1887 [4] is considered a proof that absolute motion cannot be detected. Actually, the results of that experiment were smaller than expected but non-zero. As pointed out by Cahill and Kittto [7], the key-ingredient to understand the reduction from values \(v_{\text{earth}} = O(10^2)\) km/s down to values \(v_{\text{obs}} = O(1)\)
km/s, consists in taking into account the Lorentz contraction and the refractive index of the
dielectric medium filling the arms of the interferometer. However, a full treatment on the
base of Lorentz transformations introduces a factor $\sqrt{3}$ with respect to their analysis. As a
consequence the relevant value of the Earth’s velocity is not $v_{\text{earth}} \sim 369$ km/s but rather
$v_{\text{earth}} \sim 216$ km/s. This value, which is completely consistent with Miller’s determination
Eq. (1), suggests that the magnitude of the fringe shifts is determined by the typical velocity
of the solar system within our galaxy and not, for instance, by the velocity of the solar system
relatively to the centroid of the Local Group. In the latter case, one would get higher values
as $v_{\text{earth}} = 300 \pm 25$ km/sec ref. [15], $v_{\text{earth}} = 315 \pm 15$ km/sec ref. [16], $v_{\text{earth}} = 308 \pm 23$ km/sec
ref. [17], $v_{\text{earth}} = 336 \pm 17$ km/sec ref. [18].

Notice that such ambiguities, say $v_{\text{earth}} \sim 200, 300, 369, ...$ km/s, on the actual value of
the Earth’s velocity determining the fringe shifts, can only be resolved experimentally in view
of the many theoretical uncertainties in the operative definition of the preferred frame where
light propagates isotropically. At this stage, I believe, one should just concentrate on the
internal consistency of the various frameworks. In this sense, the simple relativistic analysis
presented in this Letter shows that this is certainly true for Miller’s 1932 solution.

I am aware that my conclusion goes against the widely spread belief that Miller’s results
were only due to statistical fluctuations and/or local temperature conditions (see the Abstract
of ref. [19]). However, it is also true that the same authors of ref. [19] were admitting that
"...there can be little doubt that statistical fluctuations alone cannot account for the periodic
fringe shifts observed by Miller" (see page 171 of ref. [19]). Even more, although "...there is
obviously considerable scatter in the data at each azimuth position...the average values...show
a marked second harmonic effect" (see page 171 of ref. [19]). In any case, interpreting the
observed effects on the base of the local temperature conditions is certainly not the only
solution since "...we must admit that a direct and general quantitative correlation between
amplitude and phase of the observed second harmonic on the one hand and the thermal
conditions in the observation hut on the other hand could not be established" (see page 175
of ref. [19]). This unsatisfactory explanation should instead be compared with the excellent
agreement that was obtained by Miller once the final parameters for the Earth’s velocity
were plugged in the theoretical predictions (see Figs. 26 and 27 of ref. [1]). Finally, it seems
appropriate to remark that Miller’s experiments represented the most refined version of that
‘interferometric art’ initiated by Michelson and Morley. There is some inner contradiction
in concluding that Miller was simply wrong in 1932 but Michelson and Morley, nevertheless,
performed in 1887 an experiment that changed the history of physics.
I conclude with a brief comparison with present-day, ‘vacuum’ Michelson-Morley experiments of the type first performed by Brillet and Hall [20] and more recently by Müller et al. [21]. In this case, by definition $N_{\text{vacuum}} = 1$ so that $v_{\text{obs}} = 0$ and no anisotropy can be detected. However, as anticipated above, one can explore [11, 12] the possibility that, even in this case, a very small anisotropy might be due to a refractive index $N_{\text{vacuum}}$ that differs from unity by an infinitesimal amount. In this case, the natural candidate to explain a value $N_{\text{vacuum}} \neq 1$ is gravity. In fact, by using the Equivalence Principle, any freely falling frame $S'$ will locally measure the same speed of light as in an inertial frame in the absence of any gravitational effects. However, if $S'$ carries on board an heavy object this is no longer true. For an observer placed on the Earth, this amounts to insert the Earth’s gravitational potential in the weak-field isotropic approximation to the line element of General Relativity [22]

$$ds^2 = (1 + 2\varphi)dt^2 - (1 - 2\varphi)(dx^2 + dy^2 + dz^2)$$

(25)

so that one obtains a refractive index for light propagation

$$N_{\text{vacuum}} \sim 1 - 2\varphi$$

(26)

This represents the ‘vacuum’ analogue of $N_{\text{air}}, N_{\text{helium}},...$ so that from

$$\varphi = -\frac{G_N M_{\text{earth}}}{c^2 R_{\text{earth}}} \sim -0.7 \cdot 10^{-9}$$

(27)

and using Eq.(11) one predicts

$$B_{\text{vacuum}} \sim -4.2 \cdot 10^{-9}$$

(28)

For $v_{\text{earth}} \sim 208$ km/s, this implies an observable anisotropy of the two-way speed of light in the vacuum Eq.(9)

$$\frac{\Delta \tilde{c}_\theta}{c} \sim |B_{\text{vacuum}}| \frac{v_{\text{earth}}^2}{c^2} \sim 2 \cdot 10^{-15}$$

(29)

This prediction is in good agreement with the experimental value $\frac{\Delta \tilde{c}_\theta}{c} = (2.6 \pm 1.7) \cdot 10^{-15}$ determined by Miiller et al. [21]. Notice that the anisotropy experiment is sensitive to the product $B \frac{v_{\text{earth}}^2}{c^2}$ while the extraction of $B$ from the data was performed [21] assuming the fixed value $v_{\text{earth}} = 369$ km/s. Therefore, their determination $B^{\text{exp}} = (-2.2 \pm 1.5) \cdot 10^{-9}$, in addition to the purely experimental error, contains a theoretical uncertainty due to the rigid identification of the cosmic background radiation with the preferred frame where light propagates isotropically. This uncertainty represents a kind of systematic error whose magnitude can be estimated by comparing with alternative definitions of the Earth’s velocity. For
instance, using the alternative value $v_{\text{earth}} \sim 208$ km/s, the same experimental data would produce a value $B_{\text{exp}} = (-7.2 \pm 4.9) \cdot 10^{-9}$ that is certainly consistent with the prediction in Eq. (28).

Acknowledgements

I thank A. Pagano and V. Rychkov for useful discussions.

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