Triply charmed and bottom baryons in a constituent quark model

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In this work, we study the mass spectrum of the \( \Omega_{cc} \) and \( \Omega_{bbb} \) baryons up to the \( N = 2 \) shell within a nonrelativistic constituent quark model (NRQCM). The model parameters are adopted from the determinations by fitting the charmonium and bottomonium spectra in our previous works. The masses of the \( \Omega_{cc} \) and \( \Omega_{bbb} \) baryon states predicted in present work reasonably agree with the results obtained with the Lattice QCD calculations. Furthermore, to provide more knowledge of the \( \Omega_{cc} \) and \( \Omega_{bbb} \) states, we evaluate their radiative decays with the available masses and wave functions from the potential model.

PACS numbers:
Keywords:

I. INTRODUCTION

In the discovery of the heavy baryons, the Large Hadron Collider (LHC) facility has shown its powerful abilities in recent years. For example, in 2017 the first doubly charmed baryon \( \Xi_{cc}^{+}(3621) \) was discovered in the \( \Lambda_{c}+K^{+}\pi^{-}\pi^{-} \) mass spectrum [1], and was confirmed in the \( \Xi_{c}^{\pm}K^{\mp} \) channel one year later [2] by the LHCb Collaboration. In 2017, five extremely narrow \( \Omega_{c}(X) \) states, \( \Omega_{c}(3000), \Omega_{c}(3050), \Omega_{c}(3066), \Omega_{c}(3090) \) and \( \Omega_{c}(3119) \), were observed in the \( \Xi_{c}^{+}K^{-} \) channel by the LHCb Collaboration [3]. Very recently four bottom baryon resonances \( \Xi_{b}(6227)^{-} \) [4], \( \Sigma_{b}(6097)^{\pm} \) [5], \( \Lambda_{b}(6146/6152)^{0} \) [6] were observed at LHCb as well. Except for the singly and doubly heavy baryons, the LHC facility may provide good opportunities for discovering the missing triply heavy baryons [7, 8].

The triply heavy baryons, as a system of fully heavy quarks, may provide a new window for understanding the structure of baryons. The complications of light-quark interaction are absent in the triply heavy baryons, thus, they provide an ideal place for our better understanding of the heavy quark dynamics. From the theoretical point of view, the potential models might be able to describe triply heavy baryons to a similar level of precision as their success in heavy charmonia. Just as the quark-antiquark interactions are examined in charmonia and bottomonia, the studies of the triply heavy baryon spectra will probe the quark-quark interactions in the heavy quark sector [9]. In the past years, many studies about the triply heavy baryons have been found in the literature. Most of them focused on the predictions of the masses [9–38] and the production [7, 8, 39–44]. However, only several works have paid attentions to the weak decays [45–47], magnetic moments [33, 48], \( M_{1} \) decays [48] of triply heavy baryons.

Stimulated by the large discovery potentials of the heavy baryons at the LHC facility, in this work we carry out a systematical study of the triply heavy baryon spectra of \( \Omega_{cc} \) and \( \Omega_{bbb} \) within a nonrelativistic potential model. Recently, with this model we have studied the spectra of the charmonium, bottomonium, \( \Lambda_{c} \) meson, \( \Omega \) baryon, and fully-heavy tetraquark states. For there are no measurements of the triply heavy baryons which can be used to constrain the parameters potential model, the model parameters of this work are adopted with the determinations by fitting the charmonium and bottomonium spectra in our previous works [49–51]. The masses of the \( \Omega_{cc} \) and \( \Omega_{bbb} \) baryon states predicted in present work reasonably agree with the results obtained with the Lattice QCD calculations [9, 15].

Furthermore, to provide more knowledge of the \( \Omega_{cc} \) and \( \Omega_{bbb} \) states, we evaluate their radiative decays with the available wave functions from the potential model. It should be emphasized that the \( OZI \) allowed two-body strong decay channels are absent for the low-lying \( 1P_{1}, 1D_{1}, \) and \( 2S_{1} \) wave \( \Omega_{cc} \) and \( \Omega_{bbb} \) states, thus, the radiative transitions become important in their decays. Consequently, the radiative decay processes of the excited \( \Omega_{cc} \) or \( \Omega_{bbb} \) states may be crucial for establishing them if they are produced in experiments. In this work, radiative decays of the \( \Omega_{cc} \) and \( \Omega_{bbb} \) states are calculated within a nonrelativistic constituent quark model developed in our previous study of the heavy quarkonia [49, 52]. This model was also successfully extended to deal with the radiative decays of the \( \Lambda_{c} \) meson states [53], \( \Omega \) baryon states [54], singly baryon states [55–57], and doubly heavy baryon states [58, 59].

This paper is organized as follows. In Sec. II, a brief review of the potential model is given and the mass spectra of the \( \Omega_{cc} \) and \( \Omega_{bbb} \) baryons are calculated. Then, in Sec. III, we give a review of the radiative decay model, and calculate radiative decays of the excited \( \Omega_{cc} \) and \( \Omega_{bbb} \) states by using the masses and wave functions obtained from the potential model. In Sec. IV, we give our discussions based on the obtained radiative decay properties and masses of the \( \Omega_{cc} \) and \( \Omega_{bbb} \) resonances. Finally, a summary is given in Sec. V.
II. MASS SPECTRUM

A. Hamiltonian

To calculate the spectrum of the \( \Omega_{cc} \) and \( \Omega_{bb} \) baryons, the following nonrelativistic Hamiltonian is adopted in this work

\[
H = \left( \sum_{i=1}^{3} m_i + T_i \right) - T_G + \sum_{i<j} V_{ij}(r_{ij}),
\]

where \( m_i \) and \( T_i \) stand for the constituent quark mass and kinetic energy of the \( i \)-th quark, respectively; \( T_G \) stands for the center-of-mass (c.m.) kinetic energy of the baryon system; \( r_{ij} \equiv |\mathbf{r}_i - \mathbf{r}_j| \) is the distance between the \( i \)-th quark and \( j \)-th quark; and \( V_{ij}(r_{ij}) \) stands for the effective potential between the \( i \)-th and \( j \)-th quark. In this work, we adopt a widely used potential form for \( V_{ij}(r_{ij}) \) [49, 52, 60–67], i.e.,

\[
V_{ij}(r_{ij}) = V^{conf}\(_{ij}\)(r_{ij}) + V^{sd\(_{ij}\)}(r_{ij}),
\]

where \( V^{conf}\(_{ij}\) \) stands for the potential for confinement, and is adopted the standard Cornell form:

\[
V^{conf}\(_{ij}\)(r_{ij}) = \frac{b}{2} r_{ij} - \frac{2}{3} \alpha_{ij},
\]

while \( V^{sd\(_{ij}\)}(r_{ij}) \) stands for the spin-dependent interaction, which is the sum of the spin–spin contact hyperfine potential \( V^{SS\(_{ij}\)} \), the tensor term \( V^{T\(_{ij}\)} \), and the spin-orbit term \( V^{LS\(_{ij}\)} \):

\[
V^{sd\(_{ij}\)} = V^{SS\(_{ij}\)} + V^{T\(_{ij}\)} + V^{LS\(_{ij}\)}.
\]

The spin-spin potential \( V^{SS\(_{ij}\)} \) and the tensor term \( V^{T\(_{ij}\)} \) are adopted the often used forms:

\[
V^{SS\(_{ij}\)} = -\frac{2\alpha_{ij}}{3} \left\{ -\frac{\pi}{2} \cdot \frac{\sigma_{ij}^3 e^{-\sigma_{ij} r_{ij}}}{\pi^{3/2}} \cdot \frac{16}{3m_i m_j} (\mathbf{S}_i \cdot \mathbf{S}_j) \right\},
\]

\[
V^{T\(_{ij}\)} = \frac{2\alpha_{ij}}{3} \cdot \frac{1}{m_i m_j} \left\{ 3(\mathbf{S}_i \cdot \mathbf{r}_{ij})(\mathbf{S}_j \cdot \mathbf{r}_{ij}) - (\mathbf{S}_i \cdot \mathbf{S}_j) \right\}.
\]

In this work, a simplified phenomenological spin-orbit potential is adopted as that suggested in the literature [54, 68, 69], i.e.,

\[
V^{LS\(_{ij}\)} = \frac{\alpha_{SO}}{\rho^2 + \lambda^2} \cdot \frac{\mathbf{L} \cdot \mathbf{S}}{3(m_1 + m_2 + m_3)^2}.
\]

In the above equations, the \( \mathbf{S}_i \), \( \mathbf{S} \) and \( \mathbf{L} \) are the spin operator of the \( i \)-th quark, the total spin of the baryon and the total orbital angular momentum of the baryon, respectively; the parameter \( b \), \( \alpha_{ij} \), and \( \alpha_{SO} \) denote the strength of confinement potential, strong coupling, and spin-orbit potential, respectively.

The seven parameters \( m_c, m_b, \alpha_{cc}, \alpha_{bb}, \sigma_{cc}, \sigma_{bb} \), and \( b \) have been determined by fitting the charmonium and bottomonium spectra in our previous works [49–51]. In this work, we use the same value of parameter \( \alpha_{SO} \) as in Ref. [54]. The quark model parameters adopted in present work are collected in Table I.

| Parameter | Value |
|-----------|-------|
| \( m_c \) (GeV) | 1.4830 |
| \( m_b \) (GeV) | 4.8520 |
| \( \alpha_{cc} \) | 0.5461 |
| \( \alpha_{bb} \) | 0.4311 |
| \( \sigma_{cc} \) (GeV) | 1.1384 |
| \( \sigma_{bb} \) (GeV) | 2.3200 |
| \( b \) (GeV\(^2\)) | 0.1425 |
| \( \alpha_{SO} \) (GeV) | 1.9000 |

B. States classified in the quark model

The \( \Omega_{cc} \) and \( \Omega_{bb} \) spectra should satisfy the requirements of the SU(6)\( \times \)O(3) symmetry. The states in the SU(6)\( \times \)O(3) representation up to the \( N = 2 \) shell are given in Table II. We denote the baryon states as \( |N_6, S^2 + 1, N, L, J^P\rangle \), where \( N_6 \) stands for the irreducible representation of spin-flavor SU(6) group, \( N \) stands for the irreducible representation of flavor SU(3) group, and \( N, S, L \), and \( J^P \) stand for the principal, spin, total orbital angular momentum, and spin-parity quantum numbers, respectively. The SU(6)\( \times \)O(3) wave functions, which correspond to the \( |N_6, S^2 + 1, N, L, J^P\rangle \) states, are also listed in Table II. The \( \psi_{NLM\(_{ij}\)}^{\sigma}(\rho, \lambda) \) and \( \chi_{M\(_{ij}\)}^{\sigma}(\rho, \lambda) \) are the spatial and spin wave functions, respectively, where \( \sigma = s, \rho, \lambda, \alpha \) denotes the representation of the SU(3) group. In the spatial wave functions, \( \rho \) and \( \lambda \) are the internal Jacobi coordinates. The explicit forms of the \( \psi_{NLM\(_{ij}\)}^{\sigma}(\rho, \lambda) \) and \( \chi_{M\(_{ij}\)}^{\sigma}(\rho, \lambda) \) have been given in the Ref. [54, 70].

C. Numerical calculation

The key problem of our numerical calculations is how to deal with the spatial wave functions. To work out the spatial wave functions, in this work we expand them in terms of Gaussian basis functions. The spatial wave function \( \psi_{NLM\(_{ij}\)}^{\sigma}(\rho, \lambda) \) may be expressed as [54]

\[
\psi_{NLM\(_{ij}\)}^{\sigma}(\rho, \lambda) = \sum_{N=2(n_1+n_2)}^{N_{max}} \sum_{\lambda_{ij}} \sum_{m_{ij}} \sum_{\rho_{ij}} C^{\sigma}_{n_1,n_2,m_{ij}}(\rho_{ij})\psi_{n_1,n_2,m_{ij}}(\lambda)_{NLM\(_{ij}\)}^{\sigma}.
\]

The coefficients \( C^{\sigma}_{n_1,n_2,m_{ij}} \) in the spatial wave function \( \psi_{NLM\(_{ij}\)}^{\sigma}(\rho, \lambda) \) up to the \( N = 2 \) shell have been given in our previous work [54]. In the above equation, \( \psi_{n_1,n_2,m_{ij}}(\rho_{ij}) \) and \( \psi_{n_1,n_2,m_{ij}}(\lambda) \) stand for the spatial wave functions of the \( \rho \)- and \( \lambda \)-mode excitations, respectively.

The radial wave functions of the \( \rho \)- and \( \lambda \)-mode excitations, \( R_{n_1}(\xi) \) (\( \xi = \rho, \lambda \)), are expanded by a series of Gaussian basis functions [54]:

\[
R_{n_1}(\xi) = \sum_{\ell=1}^{\infty} C_{\xi\ell} \phi_{n_1\ell}(d_{\xi\ell}, \xi).
\]
where

\[ \phi_{\alpha\ell}(d_{\alpha\ell}, \xi) = \left( \frac{d_{\alpha\ell}}{\xi} \right)^\ell \left[ \frac{(\xi^{2} - \frac{1}{4})^{m_{\alpha} + 1}}{\sqrt{\Gamma(m_{\alpha} + 1)}} \right]^{\frac{1}{2}} \left\{ \langle \xi \rangle \right\}^\ell \times e^{-\frac{1}{2} \left( \frac{d_{\alpha\ell}}{\xi} \right)^2} F \left( -n_{\xi}, \ell + \frac{1}{2}, \left( \frac{d_{\alpha\ell}}{\xi} \right)^2 \right). \]

(10)

The \( F \left( -n_{\xi}, \ell + \frac{1}{2}, \left( \frac{d_{\alpha\ell}}{\xi} \right)^2 \right) \) is the confluent hypergeometric function. The parameter \( d_{\alpha\ell} \) can be related to the harmonic oscillator frequency \( \omega_{\ell} \) with \( 1/d_{\ell}^2 = M_{\ell} \omega_{\ell}. \) The reduced masses \( M_{\rho,\lambda} \) are defined by \( M_{\rho} = \frac{2m_{\rho}m_{\lambda}}{m_{\rho} + m_{\lambda}}, \) \( M_{\lambda} = \frac{2m_{\rho}m_{\lambda}}{m_{\rho} + m_{\lambda}}. \) On the other hand, the harmonic oscillator frequency \( \omega_{\ell} \) can be related to the harmonic oscillator stiffness factor \( K_{\ell} \) with \( \omega_{\ell} = \sqrt{\frac{3K_{\ell}}{M_{\ell}}}. \) For the identical quark system, one has \( d_{\ell} = d_{\ell} = d_{\ell} = (3m_{Q}K_{\ell})^{-1/4}. \) With this relation, the spatial wave function \( \psi_{\ell M_{\rho},\lambda}(\rho, \lambda) \) can be simply expanded.
where $\Psi_{\Omega_{cbb}}(d, \rho, \lambda)$ stands for the trial harmonic oscillator functions,

$$\psi_{\Omega_{cbb}}(d, \rho, \lambda) = \sum_{n=2p_{n}+2m_{n}+l_{n}} C_{n}^{m_{n}, l_{n},} \phi_{n_{1}, l_{1}, m_{1}}(d_{l}, \rho) \phi_{n_{2}, l_{2}, m_{2}}(d_{l}, \rho) \chi_{n_{1} l_{1} m_{1}}^{\sigma_{1}} \chi_{n_{2} l_{2} m_{2}}^{\sigma_{2}}. \quad (12)$$

To solve the Schrödinger equation, the variation principle is adopted in this work. Following the method used in Refs. [50, 71], the oscillator length $d_{\ell}$ is set to be

$$d_{\ell} = d_{\ell} a^{-1} \quad (\ell = 1, ..., n), \quad (13)$$

where $n$ is the number of Gaussian functions, and $a$ is the ratio coefficient. There are three parameters $[d_{1}, d_{n}, n]$ to be determined through variation method. It is found that when we take parameters [0.068fm, 2.711fm, 15] and [0.050fm, 2.016fm, 15] for $\Omega_{cc}$ baryons and $\Omega_{cbb}$ baryons, respectively, we will obtain stable solutions for the $\Omega_{cc}$ and $\Omega_{cbb}$ baryons.

Finally, the problem of solving the Schrödinger equation become a problem of solving the generalized matrix eigenvalues of the following equation

$$\sum_{\ell=1}^{n} \sum_{\ell'=1}^{n} (H_{\ell \ell'} - E_{\ell} N_{\ell \ell'}) \zeta_{\ell}^{\ell'} = 0, \quad (14)$$

where $H_{\ell \ell'} \equiv \langle \Psi(d_{\ell'}) | H | \Psi(d_{\ell}) \rangle$ and $N_{\ell \ell'} \equiv \langle \Psi(d_{\ell'}) | \Psi(d_{\ell}) \rangle$. The function $\Psi(d_{\ell})$ is given by

$$\Psi(d_{\ell}) = \sum_{M_{L}+M_{S}=M} \langle LM_{L}, SM_{S} | J \rangle \psi_{\Omega_{cbb}}(d_{\ell}, \rho, \lambda) \chi_{M_{L}}^{\sigma_{1}}. \quad (15)$$

The calculations of matrix elements $H_{\ell \ell'}$ and $N_{\ell \ell'}$ have been detailed discussed in Ref. [54]. The physical state corresponds to the solution with a minimum energy $E_{m}$. By solving this generalized matrix eigenvalue problem, the masses and special wave functions of the $\Omega_{cc}$ and $\Omega_{cbb}$ baryons can be determined.

The predicted masses of the $\Omega_{cc}$ and $\Omega_{cbb}$ baryons up to $N = 2$ shell have been given in Table III and also shown in Fig 1.

### III. RADIATIVE DECAYS

In this work the radiative decays of the $\Omega_{cc}$ and $\Omega_{cbb}$ baryon states are evaluated within a nonrelativistic constituent quark model developed in our previous study of the heavy quarkonia [49, 52]. This model has been extended to deal with the radiative decays of the $B_{c}$ meson states [53], $\Omega$ baryon states [54], singly baryon states [55–57].

| $n^{2S+1}L_{J_{P}}$ | $|N_{c}, N_{s}, N_{L}, J'_{P} \rangle$ | $\Omega_{cc}$ | $\Omega_{cbb}$ |
|-----------------|---------------------------------|-----------|-----------|
| $1S_{\frac{1}{2}}$ | $[56, 4, 10, 0, 0, \frac{1}{2}]$ | 4828 | 14432 |
| $1P_{\frac{1}{2}}$ | $[70, 2, 10, 1, 1, \frac{1}{2}]$ | 5142 | 14773 |
| $1P_{\frac{3}{2}}$ | $[70, 2, 10, 1, 1, \frac{3}{2}]$ | 5162 | 14779 |
| $2S_{1}$ | $[70, 2, 10, 2, 0, \frac{1}{2}]$ | 5373 | 14959 |
| $2S_{\frac{3}{2}}$ | $[56, 4, 10, 2, 0, \frac{3}{2}]$ | 5285 | 14848 |
| $1D_{\frac{3}{2}}$ | $[70, 2, 10, 2, 2, \frac{3}{2}]$ | 5412 | 15016 |
| $1D_{\frac{5}{2}}$ | $[70, 2, 10, 2, 2, \frac{5}{2}]$ | 5433 | 15022 |
| $1F_{\frac{3}{2}}$ | $[56, 4, 10, 2, 2, \frac{3}{2}]$ | 5352 | 14971 |
| $1G_{\frac{5}{2}}$ | $[56, 4, 10, 2, 2, \frac{5}{2}]$ | 5368 | 14975 |
| $1F_{\frac{5}{2}}$ | $[56, 4, 10, 2, 2, \frac{5}{2}]$ | 5392 | 14981 |
| $1G_{\frac{7}{2}}$ | $[56, 4, 10, 2, 2, \frac{7}{2}]$ | 5418 | 14988 |

In this model, the quark-photon electromagnetic (EM) coupling at the tree level is adopted as

$$H_{e} = - \sum_{j} e_{j} \bar{\psi}_{j} \gamma_{\mu} A_{\mu}(k, r_{j}) \psi_{j}, \quad (16)$$

where $A_{\mu}$ is the photon field with three momentum $k$, while $r_{j}$ and $e_{j}$ stand for the coordinate and charge of the $j$th quark field $\psi_{j}$.

In order to match the nonrelativistic wave functions of the baryons, we should adopt the nonrelativistic form of Eq. (16) in the calculations. Including the effects of the binding potential between quarks [72], the nonrelativistic expansion of $H_{e}$ may be written as [73–75]

$$h_{e} \simeq \sum_{j} e \cdot r_{j} - \frac{e_{j}}{2m_{j}} \sigma_{j} \cdot \epsilon \cdot k = e^{-\kappa r_{j}}, \quad (17)$$

where $m_{j}$ and $\sigma_{j}$ stand for the constituent mass and Pauli spin vector for the $j$th quark. The vector $\epsilon$ is the polarization vector of the photon. This nonrelativistic EM transition operator has between widely applied to meson photoproduction reactions [73, 74, 76–85].

Then, the standard helicity transition amplitude $\mathcal{A}_{j}$ between the initial baryon state $|B\rangle$ and the final baryon state $|\bar{B}\rangle$ can be calculated by

$$\mathcal{A}_{j} = -i \sqrt{\frac{\omega_{e}}{2}} \langle \bar{B} | h_{e} | B \rangle. \quad (18)$$

where $\omega_{e}$ is the photon energy.

Finally, we can calculate the EM decay width by

$$\Gamma_{\gamma} = \frac{|k|^{2}}{2} \frac{2}{J_{f}+1} M_{f} \sum_{J_{f}, J_{e}} |\mathcal{A}_{j_{f}, J_{e}}|^{2}, \quad (19)$$

where $J_{f}$ is the total angular momentum of an initial meson, $J_{f}$ and $J_{e}$ are the components of the total angular momenta along the $z$ axis of initial and final mesons, respectively.
In our calculations, the masses and the wave functions of the \( \Omega_{cc} \) and \( \Omega_{bbb} \) baryon states are adopted by solving the Schrödinger equation in Sec.II. The radiative decay widths of \( \Omega_{cc} \) and \( \Omega_{bbb} \) baryons up to \( N = 2 \) are listed in Table IV. For simplicity one can fit the numerical wave functions with a single Gaussian (SG) form by reproducing the root-mean-square radius of the \( \rho \)-mode excitations. The determined harmonic oscillator strength parameters, \( \alpha \), for the \( \Omega_{cc} \) and \( \Omega_{bbb} \) baryon states are listed in Table V. With the the SG effective wave functions, we also calculated the radiative decay widths of the \( \Omega_{cc} \) and \( \Omega_{bbb} \) baryon states, these results are listed in Table IV for a comparison. From Table IV, it is find that the partial widths obtained with the SG effective wave functions show less differences with those obtained with the real numerical wave functions.

IV. DISCUSSIONS

A. Ground states

For the ground states \( \Omega_{cc} \) and \( \Omega_{bbb} \), our predicted masses are \( \sim 4828 \) MeV and \( \sim 14432 \) MeV, respectively. There are many predictions of the masses of \( \Omega_{cc} \) and \( \Omega_{bbb} \) in the literature \cite{9–37}. For a comparison, our results and those of other works are collected in Table VI and also shown in Fig. 2.

It is found that in most of the studies the masses of the ground states \( \Omega_{cc} \) and \( \Omega_{bbb} \) are predicted to be in the range of \( \sim 4800 \pm 50 \) MeV and \( \sim 14410 \pm 170 \) MeV, respectively. Our predicted masses are reasonably consistent with the previous studies, although our results are slightly larger most of the other predictions (see Fig. 2). Compared with the results of the lattice QCD, it is found that our predicted mass for \( \Omega_{cc} \) just lies the upper limit of the predictions in Refs. \cite{10, 12, 13}, while our predicted mass for \( \Omega_{bbb} \) is about 60 MeV above the predictions in Refs. \cite{10, 15}.

B. 1P-wave states

There are two 1P-wave \( \Omega_{QQQ} \) \((Q = c, b)\) states with \( J^P = 1/2^– \) and \( J^P = 3/2^– \) according to the quark model classification (see Table II). For a comparison, our predicted masses of the 1P-wave \( \Omega_{cc} \) and \( \Omega_{bbb} \) states together with those of other theoretical predictions have been listed in Table VII and shown in Figure 3. Our predictions of the radiative decay properties of the 1P-wave \( \Omega_{cc} \) and \( \Omega_{bbb} \) states are also given in Table IV.

1. \( \Omega_{cc}(1P) \) states

In our calculations, the masses of the 1P-wave states \( \Omega_{cc}(1^2P_{1/2}) \) and \( \Omega_{cc}(1^2P_{3/2}) \) are predicted to be \( \sim 5142 \) MeV and \( \sim 5162 \) MeV, respectively, which are close to the values \( \sim 5120(9) \) MeV \( \sim 5124(13) \) MeV from the Lattice QCD calculation \cite{9}. Our results are also compatible with the other model predictions in Refs. \cite{16, 17, 22, 30, 34}. The mass splitting between \( \Omega_{cc}(1^2P_{1/2}) \) and \( \Omega_{cc}(1^2P_{3/2}) \) might be small. With a simplified phenomenological spin-orbit potential as adopted in the study of the \( \Omega \) spectrum in our previous work \cite{54}, we predict that the mass splitting between these two 1P-wave states might be \( \sim 20 \) MeV, which is slightly larger than the value of several MeV predicted in the literature \cite{9, 16}.

The decays of the 1P-wave \( \Omega_{cc}(1^2P_{1/2}) \) and \( \Omega_{cc}(1^2P_{3/2}) \) states may be dominated by the radiative transitions into the ground \( 1S \)-wave state \( \Omega_{cc} \), for their OZI-allowed two body strong decay processes are absence. We further estimate the radiative decays of the \( \Omega_{cc}(1^2P_{1/2}) \) and \( \Omega_{cc}(1^2P_{3/2}) \) states by using the wave functions calculated from the potential model. It is found that both \( \Omega_{cc}(1^2P_{1/2}) \) and \( \Omega_{cc}(1^2P_{3/2}) \) have a comparable radiative decay width into the ground \( 1S \)-wave state \( \Omega_{cc} \), i.e.,

\[
\Gamma[\Omega_{cc}(1^2P_{1/2}) \rightarrow \Omega_{cc} \gamma] = 3.10 \text{ keV}, \tag{20}
\]

\[
\Gamma[\Omega_{cc}(1^2P_{3/2}) \rightarrow \Omega_{cc} \gamma] = 4.07 \text{ keV}. \tag{21}
\]

The radiative transitions \( \Omega_{cc}(1^2P_{1/2}) \rightarrow \Omega_{cc} \gamma \) may be crucial to established them in future experiments. It should be mentioned that few studies radiative decay properties of the excited \( \Omega_{cc} \) are found in the literature. More theoretical analysis is need to better understand these 1P-wave states.

2. \( \Omega_{bbb}(1P) \) states

The masses of the 1P-wave states \( \Omega_{bbb}(1^2P_{1/2}) \) and \( \Omega_{bbb}(1^2P_{3/2}) \) are predicted to be \( \sim 14773 \) MeV and \( \sim 14779 \) MeV, respectively. Our predictions are about \( \sim 40 \) – \( \sim 100 \) MeV larger than those predictions in Refs. \cite{15, 17, 30}, while about 200 MeV smaller than the those predictions in Refs. \cite{16, 32}. It should be mentioned that our predicted mass of \( \Omega_{bbb}(1^2P_{3/2}) \) is in good agreement with that predicted with Faddeev Equation \cite{24}. Furthermore, our predicted mass splitting between \( \Omega_{bbb}(1^2P_{1/2}) \) and \( \Omega_{bbb}(1^2P_{3/2}) \), \sim 6 MeV, is similar to the Lattice QCD prediction of \( \sim 8 \) MeV in Ref. \cite{15}.

In the following we give a brief discussion of the relations of the mass splitting between the 1P-wave states in the \( \Omega_{QQQ} \) \((Q = \{s, c, b\})\) baryon spectrum. If the mass splitting between two 1P-wave states is due to the spin-orbit interaction, from Eq. (7) one finds that the mass splitting \( \Delta m[\Omega_{QQQ}(1P)] \propto \frac{1}{m_Q} \frac{1}{J^P+\alpha} \), where \( m_Q \) is the mass of constituent quark \( Q \). With a simple harmonic oscillator wave function, one can relate the element matrix \( \frac{1}{J^P+\alpha} \) to the harmonic oscillator strength parameter \( \alpha \). One further finds that \( \frac{1}{J^P+\alpha} \propto \alpha^2 \). Then we obtain an useful relation for the mass splitting:

\[
\Delta m[\Omega_{QQQ}(1P)] \propto \left( \frac{\alpha}{m_Q} \right)^2. \tag{22}
\]

Taking the constituent quark masses and effective harmonic oscillator strength parameters \( \alpha \) for the 1P-wave \( \Omega \), \( \Omega_{cc} \), and \( \Omega_{bbb} \) states determined in present work and our previous
TABLE IV: Partial widths (keV) of radiative decays for the $\Omega_{ccc}$ and $\Omega_{bbb}$ baryons up to $N = 2$ shell. Case I and Case II stand the results obtained within the real numerical wave functions and the single Gaussian wave functions, respectively.

| Initial state | $\Gamma[\Omega_{ccc}(1^3S_{1/2}-)]$ Case I | $\Gamma[\Omega_{ccc}(1^3S_{1/2}-)]$ Case II | $\Gamma[\Omega_{bbb}(1^3S_{1/2}-)]$ Case I | $\Gamma[\Omega_{bbb}(1^3S_{1/2}-)]$ Case II |
|---------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\Omega_{ccc}(1^2P_{1/2}-)$ | 3.10 | 2.78 | 0.035 | 0.028 |
| $\Omega_{ccc}(1^2P_{3/2}-)$ | 4.07 | 3.64 | 0.038 | 0.031 |
| $\Omega_{ccc}(2^3S_{1/2}-)$ | 20.14 | 18.07 | 0.99 | 0.82 |
| $\Omega_{ccc}(2^3S_{3/2}-)$ | 0.002 | 0.001 | < 0.001 | < 0.001 |
| $\Omega_{ccc}(1^2D_{3/2})$ | 106.88 | 98.80 | < 0.0001 | < 0.0001 |
| $\Omega_{ccc}(1^2D_{5/2})$ | 0.25 | 0.22 | 0.99 | 0.82 |
| $\Omega_{ccc}(1^4D_{1/2})$ | < 0.001 | < 0.0001 | < 0.0001 | < 0.0001 |
| $\Omega_{ccc}(1^4D_{3/2})$ | 0.38 | 0.35 | 0.002 | 0.002 |
| $\Omega_{ccc}(1^4D_{5/2})$ | 0.22 | 0.21 | < 0.0001 | < 0.0001 |

![Diagram](image_url)

FIG. 2: A comparison of the masses of the ground states $\Omega_{ccc}$ and $\Omega_{bbb}$ from various model predictions.

work [54], we obtain the following ratios

$$\Delta m[\Omega(1P)] : \Delta m[\Omega_{ccc}(1P)] : \Delta m[\Omega_{bbb}(1P)] \approx 9 : 3 : 1.$$  (23)

Future experimental measurements of these ratios may provide a crucial test for the spin-orbit interactions adopted in present work.

The radiative decay properties of the $\Omega_{bbb}(1^2P_{1/2}-)$ and $\Omega_{bbb}(1^2P_{3/2}-)$ states are also estimated in present work by using the wave functions calculated from the potential model. The partial widths for the $\Omega_{bbb}(1^2P_{1/2}-)$ and $\Omega_{bbb}(1^2P_{3/2}-)$
Combing the partial widths of the $1P$-wave $\Omega_{cc}$ and $\Omega_{bb}$ baryons up to $N=2$ shell, we find
\[ R = \frac{\Gamma[\Omega_{bb}(1P) \rightarrow \Omega_{bb}(1S)\gamma]}{\Gamma[\Omega_{cc}(1P) \rightarrow \Omega_{cc}(1S)\gamma]} \approx \frac{1}{100} \] (26)
Since the partial widths for the $1P$-wave $\Omega_{cc}$ states are about two orders of magnitude smaller than those corresponding processes of the $1P$-wave $\Omega_{bb}$ states, the radiative decay process of $\Omega_{bb}(1P) \rightarrow \Omega_{bb}\gamma$ may be more difficultly observed than $\Omega_{cc}(1P) \rightarrow \Omega_{cc}\gamma$.

C. 1D-wave states

There are six 1D-wave states $|1^4D_{3/2}^+, 5/2^+, 3/2^+, 1/2^+\rangle$ and $|1^2D_{5/2}^-, 3/2^-, 1/2^-\rangle$ in $\Omega_{QQQ}$ spectrum according to the quark model classification (see Table II). For a comparison, the masses of the 1D-wave $\Omega_{cc}$ and $\Omega_{bb}$ states predicted in present work together with those from other works are listed in Table VIII. Our predictions of the radiative decay properties of the 1D-wave states are also given in Table IV. To our knowledge, no studies of the radiative decay properties of the 1D-wave triply heavy baryons can be available in the literature.

1. $\Omega_{cc}(1D)$ states

The masses for the spin quartets $\Omega_{cc}(1^4D_J)$ are predicted to be in the range of $\sim 5.35 - 5.42$ GeV in present work, which

\[ \Gamma[\Omega_{bb}(1^2P_{1/2}^-) \rightarrow \Omega_{bb}\gamma] = 0.0535 \text{ keV}, \] (24)
\[ \Gamma[\Omega_{bb}(1^2P_{3/2}^-) \rightarrow \Omega_{bb}\gamma] = 0.038 \text{ keV}. \] (25)
is compatible with the predictions from Lattice QCD [9] and NRQCM [17]. From Table VIII, it is found that the mass order for the spin quartets predicted in the literature is very different, in this work we predict a normal order, i.e.,

\[ M[\Omega_{ccc}(1^4D_{1/2})] < M[\Omega_{ccc}(1^4D_{3/2})] \]
\[ < M[\Omega_{ccc}(1^4D_{5/2})] < M[\Omega_{ccc}(1^4D_{7/2})]. \]  

(27)

The mass splitting between two adjacent states is about 20 MeV.

For the spin doublets \( \Omega_{ccc}(1^2D_{3/2}) \) and \( \Omega_{ccc}(1^2D_{5/2}) \), their masses are predicted to be \( \sim 5412 \) MeV and \( \sim 5433 \) MeV, respectively, which are compatible with those of Lattice QCD [9] and HCQM [32]. In this work we find the mass of \( \Omega_{ccc}(1^2D_{5/2}) \) is about 20 MeV above that of \( \Omega_{ccc}(1^2D_{3/2}) \). The mass order

\[ M[\Omega_{ccc}(1^2D_{3/2})] < M[\Omega_{ccc}(1^2D_{5/2})]. \]  

(28)

predicted by us is different from the predictions in Refs. [9, 32]. To clarify the mass order of these spin multiplets, the spin dependent interactions should be further studied in future works.

The radiative decay properties of the 1D-wave \( \Omega_{ccc} \) states are also studied, our results are listed in Table IV. The spin quartets \( \Omega_{ccc}(1^4D_J) \), their decay rates into the 1P-wave state \( \Omega_{ccc}(1P) \) are small. The maximum radiative decay width in the radiative transitions \( \Omega_{ccc}(1^4D_J) \rightarrow \Omega_{ccc}(1P) \) is no more than 1 KeV. However, the spin doublets \( \Omega_{ccc}(1^2D_{3/2}) \) and \( \Omega_{ccc}(1^2D_{5/2}) \) have large decay rates into the 1P-wave states via the E1 dominated processes \( \Omega_{ccc}(1^2D_{3/2}) \rightarrow \Omega_{ccc}(1P) \gamma \) and \( \Omega_{ccc}(1^2D_{5/2}) \rightarrow \Omega_{ccc}(1^2P_{3/2}) \gamma \). These radiative decay widths are predicted to be

\[ \Gamma[\Omega_{ccc}(1^2D_{3/2}) \rightarrow \Omega_{ccc}(1^2P_{1/2}) \gamma] \approx 107 \text{ keV}, \]  
\[ \Gamma[\Omega_{ccc}(1^2D_{3/2}) \rightarrow \Omega_{ccc}(1^2P_{3/2}) \gamma] \approx 34 \text{ keV}, \]  
\[ \Gamma[\Omega_{ccc}(1^2D_{5/2}) \rightarrow \Omega_{ccc}(1^2P_{3/2}) \gamma] \approx 122 \text{ keV}. \]  

(29, 30, 31)

If the spin doublets \( \Omega_{ccc}(1^2D_{3/2}) \) and \( \Omega_{ccc}(1^2D_{5/2}) \) can be produced in future experiments, these radiative processes may be useful for establishing them.

2. \( \Omega_{bbb}(1D) \) states

As shown in Table VIII, the masses of the 1D-wave states \( \Omega_{bbb}(1D) \) are predicted to be in the range of \( \sim 14.97 - 15.02 \) GeV in present work. The mass order for the six 1D-wave states is

\[ M[\Omega_{bbb}(1^4D_{1/2})] < M[\Omega_{bbb}(1^4D_{3/2})] \]  
\[ < M[\Omega_{bbb}(1^4D_{5/2})] < M[\Omega_{bbb}(1^4D_{7/2})] \]  
\[ < M[\Omega_{bbb}(1^4D_{9/2})] < M[\Omega_{bbb}(1^4D_{11/2})]. \]  

(32)

The mass splitting between two adjacent states is about several MeV. It is interesting to find that our prediction of the masses, mass order, and mass splitting for 1D-wave states \( \Omega_{bbb}(1D) \) are consistent with those of the Lattice QCD [15]. However, the masses of the 1D-wave states predicted in this work are about 100-300 MeV lower than those predicted in Refs. [16, 32], while about 100 MeV higher than those predicted in Ref. [17]. Our predicted mass order for the 1D-wave states is also different from those predictions in Refs. [16, 32]. As a whole there are large uncertainties in the predictions of the mass spectrum of the 1D-wave states \( \Omega_{bbb}(1D) \), more theoretical studies are needed.

In present work we also give our predictions of the radiative decays of 1D-wave states \( \Omega_{bbb}(1D) \). Our results are collected in Table IV. From the table it is found that for the spin quartets \( \Omega_{ccc}(1^4D_J) \), their partial widths of the radiative decays into the 1P-wave states \( \Omega_{ccc}(1P) \) are tiny (O(1) eV) for the absence of the E1 transitions. However, for spin doublets \( \Omega_{bbb}(1^2D_{3/2}) \) and \( \Omega_{bbb}(1^2D_{5/2}) \), there are fairly large decay rates into the 1P-wave states \( \Omega_{bbb}(1P) \) via the E1 dominant transitions. These radiative decay widths are predicted to be

\[ \Gamma[\Omega_{bbb}(1^2D_{3/2}) \rightarrow \Omega_{bbb}(1^2P_{1/2}) \gamma] \approx 7.8 \text{ keV}, \]  
\[ \Gamma[\Omega_{bbb}(1^2D_{3/2}) \rightarrow \Omega_{bbb}(1^2P_{3/2}) \gamma] \approx 2.8 \text{ keV}, \]  
\[ \Gamma[\Omega_{bbb}(1^2D_{5/2}) \rightarrow \Omega_{bbb}(1^2P_{3/2}) \gamma] \approx 8.4 \text{ keV}. \]  

(33, 34, 35)

These partial widths for the 1D-wave \( \Omega_{bbb} \) states are about one order of magnitude smaller than those corresponding processes of the 1D-wave \( \Omega_{ccc} \) states.
TABLE VII: Our predicted masses (MeV) of the 1P-wave $\Omega_{cc}$ and $\Omega_{bb}$ states compared with those of other works.

| Method                  | $\Omega_{cc}(1^2P_{1/2})$ | $\Omega_{cc}(1^2P_{3/2})$ | $\Omega_{bb}(1^2P_{1/2})$ | $\Omega_{bb}(1^2P_{3/2})$ |
|-------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| NRCQCM (Ours)          | 5142                        | 5162                        | 14773                       | 14779                       |
| Lattice QCD [9]        | 5120(9)                     | 5124(13)                    | ...                         | ...                         |
| Lattice QCD [15]      | ...                         | ...                         | 14706.3 ± 9.8 ± 18.4        | 14714 ± 9.5 ± 18.2          |
| NRCQCM [16]           | 5155                        | 5160                        | 14975                       | 14976                       |
| NRCQCM [17]           | 5129                        | 5129                        | 14688                       | 14688                       |
| QCD Sum Rule [21]     | ...                         | 4900 ± 100                  | ...                         | 14900 ± 200                 |
| QCD Sum Rule [22]     | ...                         | 5110 ± 150                  | ...                         | 14950 ± 110                 |
| Faddeev Equation [24] | ...                         | 5027                        | ...                         | 14771                       |
| Bag model [30]        | 5140                        | ...                         | 14660                       | ...                         |
| HCQM [32]             | 5002                        | 4982                        | 14941                       | 14935                       |
| Regge theory [34]     | ...                         | 5073 ± 107                  | ...                         | ...                         |
| Regge theory [35]     | ...                         | ...                         | 15055 ± 101                 | ...                         |
| Bathe-Salpeter Equation [37] | 5019               | 5014                        | ...                         | ...                         |

TABLE VIII: Our predicted masses (MeV) of the 2S- and 1D-wave $\Omega_{cc}$ and $\Omega_{bb}$ states compared with those of other works.

| Method                  | $\Omega_{cc}$ | $\Omega_{bb}$ |
|-------------------------|---------------|---------------|
| $\rho^{2S+1}L_J$        | NRCQCM Ours  | NRCQCM Ref. [16] | NRCQCM Ref. [17] | NRCQCM Ref. [32] | Lattice Ref. [9] | NRCQCM Ref. [16] | NRCQCM Ref. [17] | NRCQCM Ref. [32] |
| $2^2S_1$               | 5373          | 5405(14)      | 5332              | ...              | 5300              | 14959          | 14938 ± 18 ± 23 | 15097              | ...              |
| $2^2S_2$               | 5285          | 5317(31)      | 5313              | 5286             | 5300              | 14848          | 14840 ± 15 ± 20 | 15089              | 14805             | 15163           |
| $1^2D_2$               | 5412          | 5465(13)      | ...               | ...              | 5436              | 15016          | 15005 ± 18 ± 24 | ...                | ...              | 15298           |
| $1^2D_3$               | 5433          | 5464(15)      | 5343              | ...              | 5404              | 15022          | 15007 ± 18 ± 24 | 15109              | ...              | 15291           |
| $1^2D_4$               | 5352          | 5399(13)      | 5325              | 5376             | 5473              | 14971          | 14953 ± 17 ± 24 | 15102              | 14894             | 15306           |
| $1^2D_5$               | 5368          | 5430(13)      | 5313              | 5376             | 5448              | 14975          | 14958 ± 17 ± 23 | 15089              | 14894             | 15300           |
| $1^2D_6$               | 5392          | 5406(15)      | 5329              | 5376             | 5416              | 14981          | 14964 ± 17 ± 23 | 15109              | 14894             | 15293           |
| $1^2D_7$               | 5418          | 5397(49)      | 5331              | 5376             | 5375              | 14988          | 14969 ± 16 ± 23 | 15101              | 14894             | 15286           |

D. 2S states

There are two 2S-wave $\Omega_{cc}$/ $\Omega_{bb}$ states with $J^P = 1/2^+$ and $J^P = 3/2^+$ according to the quark model classification (see Table II). For a comparison, our predicted masses of the 2S-wave $\Omega_{cc}$ and $\Omega_{bb}$ states together with those of other theoretical predictions have been listed in Table VIII. Furthermore, our predictions of the radiative decay properties of the 2S-wave $\Omega_{cc}$ and $\Omega_{bb}$ states are given in Table IV. To our knowledge, no studies of the radiative decay properties of the 2S-wave triply heavy baryons can be available in the literature.

1. $\Omega_{cc}(2S)$ states

Our predicted masses for the 2S-wave $\Omega_{cc}$ states $\Omega_{cc}(2^2S_{1/2})$ and $\Omega_{cc}(2^2S_{3/2})$ are ~ 5373 MeV and ~ 5285 MeV, respectively, which are compatible with the Lattice QCD predictions in Ref. [9]. The mass splitting between these two 2S-wave state predicted in present work, ~ 90 MeV, is also in good agreement with that of the Lattice QCD [9]. It should be mention that there are only a few predictions of masses of the 2S-wave $\Omega_{cc}$ states. The mass range predicted in this work roughly agrees with the other quark model predictions [16, 17, 32], although the predicted mass splitting between the two 2S-wave $\Omega_{cc}$ states is different with each other.

The radiative decay rates of $\Omega_{cc}(2^2S_{1/2})$ into $\Omega_{cc}(1^2P_{1/2})\gamma$ and $\Omega_{cc}(1^2P_{3/2})\gamma$ final states are relatively large. The radiative partial widths are predicted to be

$$\Gamma[\Omega_{cc}(2^2S_{1/2}) \rightarrow \Omega_{cc}(1^2P_{1/2})\gamma] \approx 20 \text{ keV}, \quad (36)$$

$$\Gamma[\Omega_{cc}(2^2S_{1/2}) \rightarrow \Omega_{cc}(1^2P_{3/2})\gamma] \approx 27 \text{ keV}. \quad (37)$$

These radiative processes may be useful for establishing the 2S-wave $\Omega_{cc}$ states in future experiments. However, the radiative decay widths of $\Omega_{cc}(2^2S_{3/2})$ into the 1P-wave states $\Omega_{bb}(1P)$ are tiny ($O(1)$ eV) for the absence of the $E1$ transitions.

2. $\Omega_{bb}(2S)$ states

Our predicted masses for the 2S-wave $\Omega_{bb}$ states $\Omega_{bb}(2^2S_{1/2})$ and $\Omega_{bb}(2^2S_{3/2})$ are ~ 14959 MeV and ~ 14848 MeV, respectively, which are compatible with the Lattice QCD predictions in Ref. [15]. The mass splitting between these two 2S-wave state predicted in present work,
~ 110 MeV, is also in good agreement with that of the Lattice QCD [15]. Our predicted mass of $\Omega_{bb}(2^2S_{1/2})$ is also close to recent quark model prediction, 14805 MeV, in Ref. [17]. However, our predictions of the masses for these $2S$-wave $\Omega_{bb}$ states are about 200-300 MeV lower than the other quark model predictions in Refs. [16, 32].

The $E1$ dominant radiative transitions of $\Omega_{bb}(2^2S_{1/2}) \rightarrow \Omega_{bb}(1P)\gamma$ might play a crucial role in its decays. The radiative partial widths for these processes are predicted to be

$$\Gamma[\Omega_{bb}(2^2S_{1/2}) \rightarrow \Omega(1^2P_{1/2})\gamma] = 0.99 \text{ keV},$$

$$\Gamma[\Omega_{bb}(2^2S_{1/2}) \rightarrow \Omega(1^3P_{3/2})\gamma] = 1.46 \text{ keV}.$$  \hspace{1cm} (38)

(39)

Compared them with the partial widths of $\Omega_{cc}(2^3S_{1/2}) \rightarrow \Omega_{cc}(1^2P_{1/2})\gamma, \Omega_{cc}(1^3P_{3/2})\gamma$, it is found that

$$\mathcal{R} = \frac{\Gamma[\Omega_{bb}(2^2S_{1/2}) \rightarrow \Omega_{bb}(1P)\gamma]}{\Gamma[\Omega_{cc}(2^2S_{1/2}) \rightarrow \Omega_{cc}(1P)\gamma]} \approx \frac{1}{20}.$$  \hspace{1cm} (40)

It indicates that the $\Omega_{bb}(2^2S_{1/2})$ state may be more difficultly observed than $\Omega_{cc}(2^3S_{1/2})$ via the radiative decay processes. Finally, it should be mentioned that the radiative decay rates of $\Omega_{bb}(2^2S_{3/2})$ into the $1P$-wave states $\Omega_{bb}(1P)$ are tiny for the absences of the $E1$ transitions. The predicted partial widths are less than 1 eV. Thus, the radiative decay processes of $\Omega_{bb}(2^2S_{3/2})$ might be less helpful for establishing it in experiments.

V. SUMMARY

In this work, we calculate the $\Omega_{cc}$ and $\Omega_{bb}$ spectrum up to the $N = 2$ shell within a potential model. The potentials are determined by fitting the mass spectra of charmonium and bottomonium in our previous works. For the ground states $\Omega_{cc}$ and $\Omega_{bb}$, our predicted masses are ~ 4828 MeV and ~ 14432 MeV, respectively. Compared with the results of the lattice QCD, it is found our predicted mass for $\Omega_{cc}$ just lies the upper limit of the predictions in Refs. [10, 12, 13], while our predicted mass for $\Omega_{bb}$ is about 60 MeV above the predictions in Refs. [10, 15]. Furthermore, our predictions of the mass ranges for the $1P$, $1D$, and $2S$-wave excited $\Omega_{cc}$ and $\Omega_{bb}$ states are in good agreement with the Lattice QCD predictions [9, 15]. It should be pointed out that mass orders for the spin multiplets in the excited $\Omega_{cc}$ and $\Omega_{bb}$ states predicted in the literature is very different. To clarify the mass order of these spin multiplets, the spin dependent integrations should be further studied in future works.

Moreover, by using the predicted masses and wave functions from the potential model, the radiative transitions for the $1P \rightarrow 1S$, $1D \rightarrow 1P$, and $2S \rightarrow 1P$ are evaluated for the first time with a constituent quark model. For the $\Omega_{cc}$ sector, the transition rates for $1P \rightarrow 1S$ might be sizeable, the partial widths are about several keV; the transition rates for the $E1$ dominant decay processes $1^2D_{3/2} \rightarrow 1^2P_{1/2}, 1^2P_{3/2}, 1^2D_{5/2} \rightarrow 1^2P_{3/2}, 2^2S_{1/2} \rightarrow 1^2P_{1/2,3/2}$ are relatively large, their partial widths are predicted to be about 10s keV. For the $\Omega_{bb}$ sector, the partial widths of the corresponding transitions mentioned above are about one or two order of magnitude smaller than those for the $\Omega_{cc}$ sector. To better understand the radiative decay properties of the excited $\Omega_{cc}$ and $\Omega_{bb}$ baryon states, more studies are hoped to be carried out in theory.

Acknowledgement

This work is supported by the National Natural Science Foundation of China under Grants No. 11775078, No. U1832173, and No. 11705056.

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