Role of five-quark components in radiative and strong decays of
the $\Lambda(1405)$ resonance

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(Dated: April 5, 2010)

Abstract

Within an extended chiral constituent quark model, three- and five-quark structure of the $S_{01}$
resonance $\Lambda(1405)$ is investigated. Helicity amplitudes for the electromagnetic decays ($\Lambda(1405) \rightarrow 
\Lambda(1116)\gamma$, $\Sigma(1194)\gamma$), and transition amplitudes for strong decays ($\Lambda(1405) \rightarrow \Sigma(1194)\pi$, $K^-p$)
are derived, as well as the relevant decay widths. The experimental value for the strong decay
width, $\Gamma_{\Lambda(1405)\rightarrow(\Sigma\pi)^0} = 50 \pm 2$ MeV, is well reproduced with about 50% of five-quark admixture in
the $\Lambda(1405)$. Important effects due to the configuration mixings among $\Lambda^2_P A$, $\Lambda^2_P M$ and $\Lambda^4_P M$
are found. In addition, transitions between the three- and five-quark components in the baryons
turn out to be significant in both radiative and strong decays of the $\Lambda(1405)$ resonance.

PACS numbers: 12.39.-x, 14.20.Jn, 13.30.Eg, 13.40.Hq
I. INTRODUCTION

The structure and properties of the $S_{01}$ resonance $\Lambda(1405)$, discovered in 1960’s, is still one of the puzzling issues in hadron physics. In the literature the $\Lambda(1405)$ is considered as an $s$-channel resonance [1] or as a quasi-bound ($\bar{K}N, \Sigma\pi$) state [2–13]. In quark-model approaches, this hyperon is treated as a pure $|qqq\rangle$-state [14–20], or still as an admixture of $|q^3 + q^4\rangle$ configuration [21, 22]. Other approaches take this hyperon as an “elementary” field [23] or as a quasi-bound state [24] using chiral perturbation theory, or consider it as composed of an SU(2) soliton and a kaon bound in an S-wave [25].

In recent years, possible unconventional or exotic structure for that resonance has received significant attention, suggesting the presence of states other than pure three-quark configuration.

QCD-sum Rules framework has been applied to investigate [26–30] the nature of the $\Lambda(1405)$. Using the $\Sigma^0\pi^0$ multiquark interpolation field the mass of that resonance is overestimated by about 100 MeV [27]. Introducing coupling between positive- and negative-parity baryons within the flavor-octet hyperons leads to the conclusion that the $\Lambda(1405)$ is not the parity partner of the $\Lambda$ and may be a flavor-singlet or exotic state. Mixing of three- and five-quark Fock components attributes [29] to this latter 90% of occupations, employing a non-unique flavor-singlet operator for it, composed of two flavor diquarks and one antiquark. Moreover, a recent work [30] predicts that resonance as an exotic $[udsg]$ strange hybrid and the mass of the lowest strange hybrid with $IJ^P = 0(1/2)^-$ turns out to be 1407 MeV.

Various lattice QCD calculations [31–36] have been devoted to predict the mass of $\Lambda(1405)$ and come up with masses higher than the observed one by 300-400 MeV. An interesting outcome of those works is nevertheless the need for five-quark components in $\Lambda(1405)$.

Investigations of the radiative and strong decay processes of baryons offer an appropriate case study in getting reliable insights to their internal structure. Several authors have studied the decay properties of the $\Lambda(1405)$ within constituent quark models [15–20]. However, the calculated strong decay width of the $\Lambda(1405)$ in the traditional constituent quark model turns out to be much smaller than the value $\Gamma = 50 \pm 2$ MeV reported by the Particle Data Group (PDG) [37].

Recently, extended constituent quark models, which include higher Fock components,
have been developed to describe the properties of baryon resonances \cite{38-43}. Those approaches strongly support the existence of significant genuine non-perturbative five-quark components in baryons (for a recent concise review see Ref. \cite{44}) and provide much better descriptions for the electromagnetic and strong decays of $\Delta(1232)$ \cite{38,39}, $N(1440)$ \cite{40,41} and $N(1535)$ \cite{42,43}.

Here we investigate the relevance of five-quark components in $\Lambda(1405)$, within an extended chiral constituent quark approach. The orbital-flavor-spin configuration for the four-quark subsystem of the five-quark components in $\Lambda(1405)$, with lowest energy being \cite{31}$_{\chi FS}$[4]$_{\chi}$[211]$_{\pi}$[22]$_{S}$ \cite{45,46}, allows for $u\bar{u}, d\bar{d}$ and $s\bar{s}$ components in this resonance, while the lowest energy five-quark component in the $S_{11}$ nucleon resonance $N(1535)$ can only be the $s\bar{s}$ component \cite{42,45}. Those features shed a light on the observed mass ordering of $\Lambda(1405)$ and $N(1535)$, which cannot be described within conventional constituent quark models.

In this work we focus on the radiative and strong decays widths of the $\Lambda(1405)$ in a truncated Fock space, which includes three- and five-quark components, as well as configuration mixings among them, namely, $qqq \leftrightarrow qqq\bar{q}$ transitions (here, we have omitted the $\gamma^{*}$ or the meson, $\pi$ and $K$, which intervene in those transitions). We find that the mixing mechanism contributes significantly to both strong and radiative decays.

The manuscript is organized in the following way. The wave functions for the three- and five-quark components in $\Lambda(1405)$ and that in the $SU(3)$ octet baryons are given in Section \textbf{II}. In Section \textbf{III} we give a brief account of to the formalism for the radiative and strong decays in the extended chiral constituent quark model. The numerical results are presented and discussed in Section \textbf{IV}. Finally, Section \textbf{V} contains our conclusions.

\section{Wave Function Model}

In our extended chiral constituent quark model, we assume that the wave function for a baryon can be expressed as

$$|B\rangle = A_{(B)3q}|qqq\rangle + A_{(B)5q}\sum_{i} A_{i}|qqqq\bar{q}_{i}\rangle + \cdots.$$  \hspace{1cm} (1)

Here $A_{(B)3q}$ and $A_{(B)5q}$ are the amplitudes for the 3-quark and 5-quark components, respectively, in the corresponding baryon. If we neglect higher Fock components, then
\[ A_{(B)3q}^2 + A_{(B)5q}^2 = 1. \] The sum over \( i \) runs over all the possible \( qqq, q_i \) components \((i = u, d, s)\), and the factors \( A_i \) denote the coefficient for the corresponding \( qqq, q_i \) component, implying \( \sum_i A_i^2 = 1. \)

In this paper, we consider the \( S_01 \) resonance \( \Lambda(1405) \) to be an admixture of the configurations \( \Lambda_0^2 P_A, \Lambda_0^2 P_M \) and \( \Lambda_8^2 P_M \). We also assume the \( SU(3) \) octet baryons to be an admixture of \( B_8^2 S_S, B_8^2 S'_S \), and \( B_8^2 S_M \) configurations. Concerning the mixing probability amplitudes for these latter configurations, we employ, for simplicity, the ones proposed in Refs. [15, 47]

\[
|\Lambda(1405)\rangle = 0.90|\Lambda_1^2 P_A\rangle - 0.43|\Lambda_8^2 P_M\rangle + 0.06|\Lambda_8^4 P_M\rangle, \tag{2}
\]

\[
|\Lambda(1116)\rangle = 0.93|\Lambda_8^2 S_S\rangle + 0.30|\Lambda_8^4 S'_S\rangle - 0.20|\Lambda_8^2 S_M\rangle, \tag{3}
\]

\[
|\Sigma(1193)\rangle = 0.95|\Sigma_8^2 S_S\rangle + 0.18|\Sigma_8^4 S'_S\rangle - 0.16|\Sigma_8^2 S_M\rangle, \tag{4}
\]

\[
|N(939)\rangle = 0.90|N_8^2 S_S\rangle + 0.34|N_8^2 S'_S\rangle - 0.27|N_8^2 S_M\rangle. \tag{5}
\]

Note that the signs of the second and third coefficients in the above equations are different from those in Ref. [13], due to our definitions for the spin states \(|\frac{1}{2}, \pm \frac{1}{2}\rangle_{(\rho, \lambda)}\), the orbital state \( \Phi^s_{\theta\rho} \) and the configuration \(|\Lambda_1^2 P_A\rangle\). We give the explicit wave functions for the components in \( \Lambda(1405) \) in the following two subsections.

### A. Wave functions for the three-quark components

Here we take the flavor-spin-orbital wave functions for the three-quark components in the considered configurations of \( \Lambda(1405) \) (\( \equiv \Lambda^* \)) to be of the following forms:

\[
|\Lambda(1405)\rangle_1^2 P_A, \frac{1}{2}^{-} = \frac{1}{\sqrt{6}} |\Lambda\rangle a X_a \Phi_{\Lambda^*}(q_{\Lambda}, q_{\rho}), \tag{6}
\]

\[
|\Lambda(1405)\rangle_8^2 P_M, \frac{1}{2}^{-} = -\frac{1}{2\sqrt{3}} (|\Lambda\rangle_\lambda X_\lambda + |\Lambda\rangle_\rho X_\rho) \Phi_{\Lambda^*}(q_{\Lambda}, q_{\rho}), \tag{7}
\]

\[
|\Lambda(1405)\rangle_8^4 P_M, \frac{1}{2}^{-} = \frac{1}{2\sqrt{3}} (|\Lambda\rangle_\lambda X'_\lambda + |\Lambda\rangle_\rho X'_\rho) \Phi_{\Lambda^*}(q_{\Lambda}, q_{\rho}), \tag{8}
\]

where \(|\Lambda\rangle_a\) and \(|\Lambda\rangle_\rho(\lambda)\) are the totally anti-symmetric (the flavor singlet) and mixed symmetric (the flavor octet) flavor wave functions; \( X_a, X_\rho(\lambda) \), and \( X'_\rho(\lambda) \) denote the completely anti-symmetric and mixed symmetric spin-orbital coupled wave functions, respectively; \( \Phi_{\Lambda^*}(q_{\Lambda}, q_{\rho}) \) the symmetric orbital wave function, and the Jacobi momenta are related to those of the quarks by

\[
q_{\rho} = \frac{1}{\sqrt{2}} (q_1 - q_2), \quad q_{\Lambda} = \frac{1}{\sqrt{6}} (q_1 + q_2 - 2q_3). \tag{9}
\]
For the considered configurations of the octet baryons, we employ the following flavor-spin-orbital wave functions:

\[
|B_{S_s}^2 S^+_s, \frac{1}{2} \rangle_{s_s} = \frac{1}{\sqrt{2}} \left( |B\rangle_{\lambda} \frac{1}{2}, s_{z}\rangle_{\lambda} + |B\rangle_{\rho} \frac{1}{2}, s_{z}\rangle_{\rho}\right) \Phi_{000}(\vec{q}_{\lambda}, \vec{q}_{\rho}),
\]

(10)

\[
|B_{S_s}^2 S^+_s, \frac{1}{2} \rangle_{s_s} = \frac{1}{\sqrt{2}} \left( |B\rangle_{\lambda} \frac{1}{2}, s_{z}\rangle_{\lambda} + |B\rangle_{\rho} \frac{1}{2}, s_{z}\rangle_{\rho}\right) \Phi_{200}^s(\vec{q}_{\lambda}, \vec{q}_{\rho}),
\]

(11)

\[
|B_{S_s}^2 S^+_s, \frac{1}{2} \rangle_{s_s} = \frac{1}{2} \left( \left(|B\rangle_{\lambda} \frac{1}{2}, s_{z}\rangle_{\rho} + |B\rangle_{\rho} \frac{1}{2}, s_{z}\rangle_{\lambda}\right) \Phi_{200}^\rho(\vec{q}_{\lambda}, \vec{q}_{\rho}) - \left(|B\rangle_{\lambda} \frac{1}{2}, s_{z}\rangle_{\lambda} + |B\rangle_{\rho} \frac{1}{2}, s_{z}\rangle_{\rho}\right) \Phi_{200}^\lambda(\vec{q}_{\lambda}, \vec{q}_{\rho}) \right).
\]

(12)

Here \(|B\rangle_{\rho(\lambda)}\) denotes the mixed symmetric flavor wave function for the corresponding baryon, and \(|\frac{1}{2}, s_{z}\rangle_{\rho(\lambda)}\) the mixed symmetric spin wave function. \(\Phi_{000}(\vec{q}_{\lambda}, \vec{q}_{\rho}), \Phi_{200}^s(\vec{q}_{\lambda}, \vec{q}_{\rho}), \Phi_{200}^\rho(\vec{q}_{\lambda}, \vec{q}_{\rho})\), and \(\Phi_{200}^\lambda(\vec{q}_{\lambda}, \vec{q}_{\rho})\) are the harmonic orbital wave functions with the subscripts being the corresponding nlm quantum numbers. The explicit forms for all of the flavor, spin, and orbital wave functions are given in Appendix A.

B. Wave functions for the five-quark components

Flavor-spin-orbital configurations of the four-quark subsystems in the five-quark components, with lowest energy for the \(J^p = \frac{1}{2}^-\) resonances, are \([31]_{FSX}[4]_{X}[31]_{FS}[211]_F[22]_s\), with the hyperfine interaction between the quarks (anti-quark) assumed to depend either on flavor and spin \([48]\) or on color and spin \([49]\). Accordingly, the octet baryon is \([31]_{FSX}[31]_{X}[4]_{FS}[22]_F[22]_s\).

Wave functions for the five-quark components in the \(\Lambda(1405)\) resonance, and for the octet baryons can be written, respectively, in the following general forms:

\[
|\Lambda(1405), s_{z}\rangle_{5q} = \sum_{abc} C_{[31]_{a}[211]_{a}}^{[14]} C_{[211]_{b}[22]_{c}}^{[31]_{a}} [4]_{X}[211]_F(b)[22]_s(c)[211]_C(a)\bar{\chi}_{s}\Psi(\vec{\kappa}_{i}),
\]

(13)

\[
|B_{octet}, s_{z}\rangle_{5q} = \sum_{a,b,c,m,s} C_{[31]_{a}[211]_{a}}^{[14]} C_{[211]_{b}[22]_{c}}^{[31]_{a}} [22]_C(a)[31]_{X,m}(a)[22]_F(b)[22]_s(c)\bar{\chi}_{s}\times\psi(\vec{\kappa}_{i}).
\]

(14)

Here the color, space, and flavor-spin wave functions of 4-quark subsystem are denoted in their Young patterns. The sum over \(a\) runs over the 3 configurations of the \([211]_C\) and \([31]_{XFS}\), those over \(b\) and \(c\) run over all the configurations of the \([22]\) and \([211]\) representations

5
of $S_4$, respectively. $C_{[14]}^{[31]a[211]}$ and $C_{[211]b[22]}^{[31]}$ are the Clebsch-Gordan coefficients of the $S_4$ permutation group, the values of which are $C_{[31]1[211]}^{[14]} = -C_{[211]2[21]}^{[14]} = C_{[211]3[21]}^{[14]} = \frac{1}{\sqrt{3}}$, $C_{[22]}^{[4]} = \frac{1}{\sqrt{2}}\delta_{bc}$ and the coefficients $C_{[211]b[22]}^{[31]}$ are shown in the decompositions of the $|31\rangle_{FS}$ configurations in Appendix B. The orbital, flavor, spin, and color wave functions are denoted by the Weyl tableau, and we give the explicit forms for those wave functions in Appendix B. $\Psi(\vec{\kappa}_i)$ and $\psi(\vec{\kappa}_i)$ in Eqs. (13) and (14) are the orbital symmetric wave functions for the five-quark components in $\Lambda(1405)$ and the octet baryons, respectively, with the Jacobi momenta

$$\vec{\kappa}_1 = \sqrt{\frac{1}{2}}(\vec{q}_1 - \vec{q}_2), \quad \vec{\kappa}_2 = \sqrt{\frac{1}{6}}(\vec{q}_1 + \vec{q}_2 - 2\vec{q}_3),$$

$$\vec{\kappa}_3 = \sqrt{\frac{1}{12}}(\vec{q}_1 + \vec{q}_2 + \vec{q}_3 - 3\vec{q}_4), \quad \vec{\kappa}_4 = \sqrt{\frac{1}{20}}(\vec{q}_1 + \vec{q}_2 + \vec{q}_3 + \vec{q}_4 - 4\vec{q}_5).$$

The orbital configuration $[4]_X$ for $\Lambda(1405)$ is completely symmetric, which means all of the quarks and anti-quark should be in their orbital ground states, and the explicit form of the mixed symmetric orbital configuration $[31]_X$ for the octet baryons is

$$|31\rangle_{X1} = \sqrt{\frac{1}{12}}\{3|0001\rangle - |0010\rangle - |0100\rangle - |1000\rangle\},$$

$$|31\rangle_{X2} = \sqrt{\frac{1}{6}}\{2|0010\rangle - |0100\rangle - |1000\rangle\},$$

$$|31\rangle_{X3} = \sqrt{\frac{1}{2}}\{|0100\rangle - |1000\rangle\},$$

where 0 and 1 correspond to the quark in its ground or first orbitally excited state, respectively. The explicit orbital wave function is the combination of the orbital configuration, Eqs. (17)-(19), and the symmetric wave function $\psi(\vec{\kappa}_i)$. Explicit color-orbital coupled wave function are reported in Appendix B.

In Table I we give for decomposition of baryon states the relevant flavor-spin configurations, as well as the Coefficients $A_i$, Eq. I. The corresponding five-quark components in $\Lambda(1405)^2P_M$ and $B^2S_S$ are taken from Ref. [50], and those for the other configurations are obtained by employing the weight diagram method [51]. In this latter case, one can also apply the $SU(3)$ uppering and lowering operators in the flavor space.
TABLE I: Five-quark components in \( \Lambda(1405) \), \( \Lambda \), \( \Sigma^0 \), the proton and the corresponding coefficients.

| Baryon        | Flavor-spin configuration | \( A_u \) | \( A_d \) | \( A_s \) |
|---------------|--------------------------|---------|---------|---------|
| \( \Lambda(1405) \) \( ^2 \) \( P_A \) | [211] \(_F\)             | \( \sqrt{\frac{2}{3}} \) | \( \sqrt{\frac{1}{3}} \) | \( \sqrt{\frac{1}{3}} \) |
| \( \Lambda(1405) \) \( ^2 \) \( P_M \) | [211] \(_F\)             | \( -\sqrt{\frac{1}{6}} \) | \( -\sqrt{\frac{1}{6}} \) | \( \sqrt{\frac{1}{3}} \) |
| \( \Lambda(1405) \) \( ^4 \) \( P_M \) | [211] \(_F\)             | \( -\sqrt{\frac{1}{6}} \) | \( -\sqrt{\frac{1}{6}} \) | \( \sqrt{\frac{1}{3}} \) |
| \( \Lambda(1116) \) \( ^2 \) \( S_S \) | [22] \(_F\)              | \( -\sqrt{\frac{1}{2}} \) | \( -\sqrt{\frac{1}{2}} \) | 0       |
| \( \Lambda(1116) \) \( ^2 \) \( S'_S \) | [22] \(_F\)              | \( -\sqrt{\frac{1}{2}} \) | \( -\sqrt{\frac{1}{2}} \) | 0       |
| \( \Lambda(1116) \) \( ^2 \) \( S_M \) | [22] \(_F\)              | \( -\sqrt{\frac{1}{2}} \) | \( -\sqrt{\frac{1}{2}} \) | 0       |
| \( \Sigma(1194) \) \( ^2 \) \( S_S \) | [22] \(_F\)              | \( -\sqrt{\frac{1}{6}} \) | \( \sqrt{\frac{1}{12}} \) | \( \sqrt{\frac{1}{2}} \) |
| \( \Sigma(1194) \) \( ^2 \) \( S'_S \) | [22] \(_F\)              | \( -\sqrt{\frac{1}{6}} \) | \( \sqrt{\frac{1}{12}} \) | \( \sqrt{\frac{1}{2}} \) |
| \( \Sigma(1194) \) \( ^2 \) \( S_M \) | [22] \(_F\)              | \( -\sqrt{\frac{1}{6}} \) | \( \sqrt{\frac{1}{12}} \) | \( \sqrt{\frac{1}{2}} \) |
| \( N(939) \) \( ^2 \) \( S_S \) | [22] \(_F\)              | 0       | \( \sqrt{\frac{7}{12}} \) | \( \sqrt{\frac{1}{2}} \) |
| \( N(939) \) \( ^2 \) \( S'_S \) | [22] \(_F\)              | 0       | \( \sqrt{\frac{7}{12}} \) | \( \sqrt{\frac{1}{2}} \) |
| \( N(939) \) \( ^2 \) \( S_M \) | [22] \(_F\)              | 0       | \( \sqrt{\frac{7}{12}} \) | \( \sqrt{\frac{1}{2}} \) |

III. FORMALISM FOR THE RADIATIVE AND STRONG DECAYS

Taking into account the five-quark components, the decays of a baryon embodies three types of possible transitions: i) between the three-quark, ii) between the five-quark, iii) between three- and five-quark. The first two processes are the so-called diagonal, and the last one nondiagonal transitions.

In the next two subsections, we describe briefly the formalism for radiative and strong decays of the baryons in a non-relativistic quark model.

A. Formalism for radiative decay

It is established that the radiative decay of baryons can be described by the helicity amplitudes for the electromagnetic transitions. For \( \gamma^* Y \rightarrow \Lambda(1405) \), with \( Y \equiv \Lambda(1116), \Sigma(1193) \), they are defined as follows:

\[
A_{1/2} = \sqrt{\frac{2\pi\alpha}{K}} \frac{1}{e} \langle \Lambda(1405), S_z^* \rangle = \frac{1}{2} |\epsilon^+_\mu J^\mu| B, S_z = -\frac{1}{2} \rangle.
\]
Here $\epsilon_\mu^+$ is the polarization vector for the right-handed photon, $J^\mu$ denotes the electromagnetic current, and $K$ the real photon three-momentum magnitude in the centre-of-mass frame of the $\Lambda(1405)$ resonance. For the $\Lambda(1405) \to \Lambda(1116)\gamma$, $\Sigma(1194)\gamma$ radiative decays, the values for $K$ are about 259 MeV/c and 195 MeV/c, respectively.

The diagonal electromagnetic transition operator in the non-relativistic constituent quark model takes \[20, 52\] the following form:

$$
\hat{T}_d = \sum_i \sqrt{2} \mu_i \phi_i^{ij} \left( \begin{array}{cc} \sqrt{2} q_i^+ & \frac{k}{\sqrt{2} q_i^+} \\ 0 & \sqrt{2} q_i^+ \end{array} \right) \phi_i^{ij}.
$$

Here the sum over $i$ runs over the quark contents of the corresponding components, i.e. $nq = 3$ for the three-quark and $nq = 5$ for the five-quark components. $\mu_i = \frac{e_i}{2m_i}$ denotes the magnetic moment operator of the $i^{th}$ quark, $\phi_i^{ij}$ and $\phi_i^{ij}$ are the $i^{th}$ quark spin operators for the initial and final states, respectively, and $q_i^+ = \frac{1}{\sqrt{2}}(q_{ix} + iq_{iy})$ with $\vec{q}_i$ being the momentum of the $i^{th}$ quark. Finally, $k$ is the z-component of the photon momentum. Note that we have taken the photon momentum to be $\vec{k} = (0, 0, k)$, and it is related to the square of the four-momentum transfer $Q^2$

$$
k^2 = Q^2 + \left( \frac{M^2_{\Lambda(1405)} - m^2_Y - Q^2}{4M^2} \right)^2,
$$

where $Y \equiv \Lambda(1160)$, $\Sigma(1193)$.

For the nondiagonal transitions, taking the $q\bar{q} - \gamma$ vertices to have the elementary forms

$$
\bar{u}(q_i)\gamma^\mu v(q) \ (3q \to 5q) \text{ and } \bar{v}(q)\gamma^\mu u(q_i) \ (5q \to 3q),
$$

then the transition operators in the non-relativistic constituent quark model can be derived

$$
\hat{T}_{35} = \sum_i 4 \sqrt{2} e_i \phi_i^{ij} \left( \begin{array}{cc} 0 & 1 \\ 0 & 0 \end{array} \right) \phi_i^{ij},
$$

$$
\hat{T}_{53} = \sum_i 4 \sqrt{2} e_i \phi_i^{ij} \left( \begin{array}{cc} 0 & 1 \\ 0 & 0 \end{array} \right) \phi_i^{ij}.
$$

Here $\hat{T}_{35}$ and $\hat{T}_{53}$ are the operators for the $\gamma^*qqq \to qqq\bar{q}$ and $\gamma^*qqqq \to qqq$ transitions, respectively.

Thus, the helicity amplitude $A_{1/2}$ for the electromagnetic transition $\gamma^*Y \to \Lambda(1405)$ can be written in the following form:

$$
A_{1/2} = \sqrt{\frac{2\pi\alpha}{K}} e^{i\arg(\Lambda(1405), \frac{1}{2})(\hat{T}_d + \hat{T}_a)}|Y, -\frac{1}{2}>,
$$
TABLE II: Helicity amplitude $A_{1/2}$ for electromagnetic transition $\gamma^*\Lambda \to \Lambda(1405)$. Note that the full amplitudes in columns 2 to 4 are obtained by multiplying each term by the following expressions: $\sqrt{\frac{2\pi\alpha}{\hbar}} A_{3g} A_{3q}^* \exp\{\frac{-k^2}{6\omega_3^2}\}$ for $3q \to 3q$, $\sqrt{\frac{2\pi\alpha}{\hbar}} A_{5q} A_{5q}^* \frac{1}{24} (\frac{1}{m} + \frac{2}{m_s}) \omega_5 \exp\{\frac{-k^2}{6\omega_3^2}\}$ for $5q \to 5q$, and $\sqrt{\frac{2\pi\alpha}{\hbar}} A_{3g} A_{3q}^* C_{35} \exp\{\frac{-3k^2}{20\omega_3^2}\}$ for $N - D$.

| Transition | $3q \to 3q$ | $5q \to 5q$ | $N - D$ |
|------------|-------------|-------------|---------|
| $A_S^2 S_S \to A_1^2 P_A$ | $\frac{1}{18} (\frac{1}{m} + \frac{2}{m_s}) \omega_3 (1 + \frac{k^2}{2\omega_3^2})$ | $1/\sqrt{3}$ | $\frac{1}{6}$ |
| $A_S^2 S_S \to A_2^2 P_M$ | $\frac{1}{36} (\frac{1}{m} - \frac{2}{m_s}) \frac{k^2}{\omega_3^2} - (\frac{1}{m} + \frac{2}{m_s}) 2\omega_3$ | $-1/\sqrt{6}$ | $\frac{\sqrt{2}}{12}$ |
| $A_S^2 S_S \to A_4^2 P_M$ | $\frac{1}{36m^3} \frac{k^2}{\omega_3^2}$ | $-1/\sqrt{6}$ | $\frac{\sqrt{2}}{12}$ |
| $A_S^2 S_S' \to A_1^2 P_A$ | $-\frac{1}{18\sqrt{3}} (\frac{1}{m} + \frac{2}{m_s}) \omega_3 [(1 + \frac{k^2}{6\omega_3^2}) - (1 - \frac{k^2}{6\omega_3^2}) \frac{k^2}{2\omega_3^2}]$ | $1/\sqrt{3}$ | $0$ |
| $A_S^2 S_S' \to A_2^2 P_M$ | $\frac{1}{54\sqrt{3}} [(\frac{1}{m} - \frac{2}{m_s}) \frac{k^2}{\omega_3^2} (1 - \frac{k^2}{6\omega_3^2}) + (\frac{1}{m} + \frac{2}{m_s}) 2\omega_3 (1 + \frac{k^2}{6\omega_3^2})]$ | $-1/\sqrt{6}$ | $0$ |
| $A_S^2 S_S' \to A_4^2 P_M$ | $\frac{1}{36\sqrt{3}m^3} (1 - \frac{k^2}{6\omega_3^2}) \frac{k^2}{\omega_3^2}$ | $-1/\sqrt{6}$ | $0$ |
| $A_S^2 S_M \to A_1^2 P_A$ | $-\frac{\sqrt{2}}{18} (\frac{1}{m} + \frac{2}{m_s}) \omega_3 (1 - \frac{k^2}{12\omega_3^2} + \frac{k^4}{24\omega_3^2})$ | $1/\sqrt{3}$ | $0$ |
| $A_S^2 S_M \to A_2^2 P_M$ | $-\frac{\sqrt{2}}{108\omega_3} [(\frac{1}{m} + \frac{2}{m_s}) \frac{k^2}{\omega_3^2} - \frac{k^4}{6\omega_3^2}]$ | $-1/\sqrt{6}$ | $0$ |
| $A_S^2 S_M \to A_4^2 P_M$ | $-\frac{\sqrt{2}}{162\omega_3} [(\frac{1}{m} - \frac{2}{m_s}) \frac{k^2}{\omega_3^2} - \frac{k^4}{8\omega_3^2}]$ | $-1/\sqrt{6}$ | $0$ |

where we have defined $\hat{T}_a = \hat{T}_{35} + \hat{T}_{53}$, which correspond to nondiagonal transitions.

Taking into account the configurations mixing effects and the contributions of the five-quark components, we need to calculate 36 transition amplitudes for each decay. For the diagonal transitions ($3q \to 3q$ and $5q \to 5q$) the calculations are similar to that in Refs. [15, 49, 53]. Explicit calculations of the nondiagonal ($N - D$) electromagnetic transitions elements in our approach are similar to the one in Ref. [42] for the $\gamma^* N \to N(1535)$ process. Amplitudes for $\gamma^* \Lambda(1116) \to \Lambda(1405)$ and $\gamma^* \Sigma^o(1194) \to \Lambda(1405)$ are given in Tables III and IIII respectively.

Notice that, in Tables III and IIII we have defined

$$C_{35} = \langle \varphi_{00}(\vec{r}_1) \varphi_{00}(\vec{r}_2) | \varphi_{00}(\vec{r}_1) \varphi_{00}(\vec{r}_2) \rangle = \left( \frac{2\omega_3 \omega_5}{\omega_3^2 + \omega_5^2} \right)^3,$$

which is the orbital overlap integral factor in the matrix elements of the nondiagonal transitions. Here, $\omega_3$ and $\omega_5$ are the oscillator frequencies for the $qqq$ and $qqqqq\bar{q}$ systems, respectively.

Finally, the radiative decay width of $\Lambda(1405)$ in terms of the helicity amplitudes $A_{1/2}$ at
TABLE III: Helicity amplitude $A_{1/2}^\gamma$ for electromagnetic transition $\gamma^*\Sigma^0 \rightarrow \Lambda(1405)$. Note that the full amplitudes in columns 2 to 4 are obtained by multiplying each term by the following expressions: $\sqrt{\frac{2\pi a}{K}} A_{3q}^\gamma A_{1q}^\gamma \exp\{-\frac{k^2}{2m^2}\}$ for $3q \rightarrow 3q$, $\sqrt{\frac{2\pi a}{K}} A_{5q}^\gamma A_{1q}^\gamma \exp\{-\frac{k^2}{3m^2}\}$ for $5q \rightarrow 5q$, and $\sqrt{\frac{2\pi a}{K}} A_{3q}^\gamma A_{5q}^\gamma C_{35} \exp\{-\frac{3k^2}{20m^2}\}$ for $N - D$.

|          | $3q \rightarrow 3q$ | $5q \rightarrow 5q$ | $N - D$ |
|----------|---------------------|---------------------|---------|
| $\Sigma_8^2 S_S \rightarrow \Lambda_2^2 P_A$ | $-\frac{1}{2}\frac{\omega}{m} \left(1 + \frac{k^2}{2\omega_3}\right)$ | $-\frac{1}{16}$ | $-\frac{1}{2\sqrt{3}}$ |
| $\Sigma_8^2 S_S \rightarrow \Lambda_2^2 P_M$ | $-\frac{1}{4}\frac{\omega}{m} \left(2 + \frac{k^2}{3\omega_3}\right)$ | $-\frac{3}{16\sqrt{2}}$ | $\frac{1}{2\sqrt{6}}$ |
| $\Sigma_8^2 S_S \rightarrow \Lambda_4^4 P_M$ | $\frac{\sqrt{3}k^2}{36m\omega_3}$ | $-\frac{3}{16\sqrt{2}}$ | $\frac{1}{2\sqrt{6}}$ |
| $\Sigma_8^2 S'_S \rightarrow \Lambda_2^2 P_A$ | $\frac{\omega_3}{6m} \left[(1 + \frac{k^2}{2\omega_3}) - (1 - \frac{k^2}{3\omega_3}) \frac{k^2}{\omega_3}\right]$ | $-\frac{1}{16}$ | $0$ |
| $\Sigma_8^2 S'_S \rightarrow \Lambda_2^2 P_M$ | $-\frac{1}{4m} \left[(1 - \frac{k^2}{6\omega_3}) - \frac{2\omega_3}{3} \left(1 + \frac{k^2}{3\omega_3}\right)\right]$ | $-\frac{3}{16\sqrt{2}}$ | $0$ |
| $\Sigma_8^2 S_M \rightarrow \Lambda_2^2 P_M$ | $\frac{\sqrt{7}\omega_3}{6m} \left(1 - \frac{k^2}{12\omega_3} + \frac{k^2}{4\omega_3}\right)$ | $-\frac{1}{16}$ | $0$ |
| $\Sigma_8^2 S_M \rightarrow \Lambda_2^2 P_M$ | $\frac{\sqrt{7}\omega_3}{2m} \left(1 - \frac{k^2}{12\omega_3}\right)$ | $-\frac{3}{16\sqrt{2}}$ | $0$ |
| $\Sigma_8^2 S_M \rightarrow \Lambda_4^4 P_M$ | $-\frac{\sqrt{27} k^4}{2m\omega_3}$ | $-\frac{3}{16\sqrt{2}}$ | $0$ |

the real photon point is $[53]$.

\[\Gamma_{Y\gamma} = \frac{k^2 m_Y}{\pi M} |A_{1/2}(Q^2 = 0)|^2.\]  

(27)

B. Formalism for strong decay

In the chiral constituent quark model, the coupling of the light quarks ($u, d, s$) to the octet of light pseudoscalar mesons takes the form

\[\mathcal{L}_{Mqq} = i \frac{g^q_A}{2f_M} \bar{\psi}_q \gamma^5 \gamma_\mu \partial^\mu m_a \lambda_a \psi_q.\]  

(28)

Here, $g^q_A$ denotes the axial coupling constant for the constituent quarks, $f_M$ is the decay constant of meson $M$ ($\pi, K$). $\psi_q$ is the quark field and $m_a$ the meson field. Finally, $\lambda_a$s are the $SU(3)$ Gell-Mann matrices. Combination of Eq. (28) with the representation

\[m_a \lambda_a = \sqrt{2} \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\ -\frac{1}{\sqrt{2}} \pi^- & \frac{1}{\sqrt{6}} \eta + K^0 & \frac{1}{\sqrt{2}} \eta \\ K^- & -\frac{1}{\sqrt{2}} \pi^0 & -\sqrt{\frac{2}{3}} \eta \end{pmatrix}.\]  

(29)
leads to the following quark-meson-quark chiral coupling in the momentum space

\[ \mathcal{L}_{Mqq} = \frac{g_A q}{2f_M} \bar{\psi}_q \gamma_5 \gamma_\mu k^\mu X_M^q \psi_q , \]  

(30)

where \( X_M^q \) is the flavor operator for emission of meson \( M \) from the corresponding quark \( q \)

\[ \begin{align*}
X_{\pi^0}^q &= \lambda_3 , \\
X_{K^+}^q &= \frac{1}{\sqrt{2}}(\lambda_4 + \lambda_5) , \\
X_{K^0}^q &= -\frac{1}{\sqrt{2}}(\lambda_6 - i\lambda_7) , \\
X_\eta^q &= \cos\theta \lambda_8 - \sin\theta \mathcal{I}, \\
X_\eta' &= \cos\theta \lambda_8 + \sin\theta \mathcal{I},
\end{align*} \]

(31-34)

with \( \mathcal{I} \) the unit operator in the \( SU(3) \) flavor space.

Within the non-relativistic approximation, we can get the baryon-meson-baryon coupling in the chiral constituent quark model

\[ \begin{align*}
\hat{T}_d^M &= \sum_i \frac{g_A q}{2f_M} \phi_z^i \begin{pmatrix}
(1 + \frac{k_0}{2m_f})k_M - \frac{k_0}{2m_f}q_i \\
-\sqrt{2}\frac{k_0}{2m_f}q_i - \\
-\sqrt{2}\frac{k_0}{2m_f}q_i + \\
-1 + \frac{k_0}{2m_f}k_M + \frac{k_0}{2m_f}q_i
\end{pmatrix} \phi_z X_M^i , \\
\hat{T}_{53}^M &= -\sum_i \frac{g_A q}{2f_M} (m_i + m_f) \phi_z^i \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix} \phi_z X_M^i , \\
\hat{T}_{35}^M &= -\sum_i \frac{g_A q}{2f_M} (m_i + m_f) \phi_z^i \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix} \phi_z X_M^i .
\end{align*} \]

(35-37)

Here \( m_i \) and \( m_f \) are the initial and final constituent masses of the quark which emits a meson, and \( \mu = m_i m_f/(m_i + m_f) \). \( k_0 \) and \( k_M \) denote the energy and magnitude of the three-momentum of the final meson in the centre-of-mass frame of the initial baryon. Note that we have taken the meson three-momentum to be in the \( z \)-direction, \( k_M = (0, 0, k_M) \), and it is related to the masses of the initial and final hadrons

\[ k_M = \{[M_i^2 - (M_f + M_M)^2][M_i^2 - (M_f - M_M)^2]\}^{1/2}/2M_i . \]

(38)

The transition amplitudes are obtained by the calculations of the following matrix elements

\[ T^M = \langle \Lambda(1405), \frac{1}{2}|(\hat{T}_d^M + \hat{T}_a^M)|Y, \frac{1}{2} \rangle , \]

(39)

where we have defined \( \hat{T}_a^M = \hat{T}_{35}^M + \hat{T}_{53}^M \).

Tables IV and V give the transition amplitudes for strong decay channel. We note that none of the diagonal transitions of the five-quark components contributes to the transition
TABLE IV: Transition amplitudes of the $\Lambda(1405) \to \Sigma(1194)\pi$ decay. Note that the full amplitudes in columns 2 and 3 are obtained by multiplying each term by the following expressions: 

\[
\frac{q}{2\pi} A_3^{\pi} A_3^{\Sigma} \omega_3 \exp\{-\frac{k^2}{6\omega_3}\} \text{ for column } 3q \to 3q, \text{ and } \frac{q}{2\pi} A_3^{\pi} A_3^{\Sigma} m C_{55} \exp\{-\frac{3k^2}{2\omega_3}\} \text{ for column } N - D. \text{ Here } k \text{ denotes the } \pi \text{ three-momentum magnitude } k_\pi, \text{ and } k_0 \text{ the energy of the } \pi \text{ meson.}
\]

|       | 3q → 3q | N-D |
|-------|----------|-----|
| $\Lambda_1^2 P_A \to \Sigma_8^2 S_S$ | $-\frac{1}{3\sqrt{6}}[(1 + \frac{k_0}{6m}) \frac{k^4}{\omega_3} - 3 \frac{k_0}{m}]$ | $\frac{1}{\sqrt{6}}$ |
| $\Lambda_1^2 P_M \to \Sigma_8^2 S_S$ | $-\frac{1}{3\sqrt{6}}[(1 + \frac{k_0}{6m}) \frac{k^4}{\omega_3} - 3 \frac{k_0}{m}]$ | $-\frac{1}{2\sqrt{6}}$ |
| $\Lambda_1^2 P_M \to \Sigma_8^2 S'_S$ | $-\frac{2}{3\sqrt{6}}[(1 + \frac{k_0}{6m}) \frac{k^4}{\omega_3} - 3 \frac{k_0}{m}]$ | $-\frac{2}{\sqrt{6}}$ |
| $\Lambda_1^3 P_A \to \Sigma_8^2 S'_S$ | $-\frac{1}{3\sqrt{2}}[3 \frac{k_0}{m} + (1 + \frac{k_0}{6m}) \frac{k^2}{\omega_3} - (1 + \frac{k_0}{6m}) \frac{k^4}{6\omega_3}]$ | 0 |
| $\Lambda_1^3 P_M \to \Sigma_8^2 S'_S$ | $-\frac{1}{9\sqrt{2}}[3 \frac{k_0}{m} + (1 + \frac{k_0}{6m}) \frac{k^2}{\omega_3} - (1 + \frac{k_0}{6m}) \frac{k^4}{6\omega_3}]$ | 0 |
| $\Lambda_1^3 P_M \to \Sigma_8^2 S'_M$ | $-\frac{2}{9\sqrt{2}}[3 \frac{k_0}{m} + (1 + \frac{k_0}{6m}) \frac{k^2}{\omega_3} - (1 + \frac{k_0}{6m}) \frac{k^4}{6\omega_3}]$ | 0 |
| $\Lambda_1^3 P_A \to \Sigma_8^2 S_M$ | $-\frac{1}{9}[3 \frac{k_0}{m} + (1 + \frac{k_0}{6m}) \frac{k^2}{\omega_3} - (1 + \frac{k_0}{6m}) \frac{k^4}{6\omega_3}]$ | 0 |
| $\Lambda_1^3 P_M \to \Sigma_8^2 S_M$ | $-\frac{1}{9}[3 \frac{k_0}{m} + (1 + \frac{k_0}{6m}) \frac{k^2}{\omega_3} - (1 + \frac{k_0}{6m}) \frac{k^4}{6\omega_3}]$ | 0 |
| $\Lambda_1^3 P_M \to \Sigma_8^2 S_M$ | $-\frac{1}{9}[3 \frac{k_0}{m} + (1 + \frac{k_0}{6m}) \frac{k^2}{\omega_3} - (1 + \frac{k_0}{6m}) \frac{k^4}{6\omega_3}]$ | 0 |

amplitudes. Those null values can easily be understood, noticing that the spin configurations for $\Lambda(1405)$ and for the octet baryons are taken to be $[22]_S$, for which the total spin is $S = 0$, and there are no spin-independent terms in the diagonal transition operator (which is not the case in the electromagnetic transition operator). However, the configuration mixing effects might be significant.

Finally, following Eq. (39), the strong decay width for $\Lambda(1405) \to (\Sigma(1194)\pi)^o$ reads

\[
\Gamma_{\Lambda(1405)\to(\Sigma\pi)^o} = \frac{3}{4\pi} \frac{E'}{M} |\vec{k}_\pi|^2 |T\pi|^2, \tag{40}
\]

where $E'$ is the energy of the final $\Sigma$ hyperon

\[
E' = \frac{M^2 - m_{\pi}^2 + m_{\Sigma}^2}{2M}. \tag{41}
\]

In addition, taking the hadronic level Lagrangian for the $\Lambda(1405)BM$ coupling, with $B \equiv \Sigma, N$ and $M \equiv \pi, K$, to be of the following form:

\[
\mathcal{L}_{\Lambda(1405)BM} = i \frac{f_{\Lambda(1405)BM}}{m_M} \bar{\psi}_B \gamma_\mu \partial^\mu \phi_M X_M \psi_{\Lambda(1405)} + h.c., \tag{42}
\]

the transition coupling amplitude reads $f_{\Lambda(1405)BM}(M_{\Lambda(1405)} - m_B)/m_M$. Comparing the
TABLE VI: Transition amplitudes of the $\Lambda(1405) \to K^- p$ decay. Note that the full amplitudes in columns 2 and 3 are obtained by multiplying each term by the following expression: $\frac{g}{f_{K\pi}} A_{3q'} A_{3q} \omega_3 \exp\{-\frac{k_{\omega_3}^2}{2}\}$ for column 3$q' \to 3q$, and the factors $\frac{g}{f_{K\pi}} A_{3q'} A^*_{3q} (m + m_s) C_{35} \exp\{-\frac{4k_{\omega_3}^2}{2m}\}$ for column $N - D$. Here $k$ denotes the three-momentum magnitude $k_K$, and $k_0$ the energy of the $K$ meson.

| $3q \to 3q$ | $N - D$ |
|-----------------|-----------------|
| $\Lambda^g_P \to N^g S_S$ | $-\frac{1}{\sqrt{6}} \left[ (1 + \frac{k_0}{2m} - \frac{k_0}{6\mu}) \frac{k_0^2}{\omega_3^2} - \frac{3k_0}{2\mu^2} \right] - \frac{1}{2\sqrt{6}}$ |
| $\Lambda^g_M \to N^g S_S$ | $\frac{1}{\sqrt{6}} \left[ (1 + \frac{k_0}{2m} - \frac{k_0}{6\mu}) \frac{k_0^2}{\omega_3^2} - \frac{3k_0}{2\mu^2} \right] - \frac{1}{4\sqrt{3}}$ |
| $\Lambda^g_P \to N^g S_S$ | $0 - \frac{1}{4\sqrt{3}}$ |
| $\Lambda^g_P \to N^g S_S'$ | $-\frac{1}{3\sqrt{2}} \left( \frac{3k_0}{4\mu} + (1 + \frac{k_0}{2m} + \frac{k_0}{4\mu}) \frac{k_0^2}{\omega_3^2} - (1 + \frac{k_0}{2m} - \frac{k_0}{6\mu}) \frac{k_0^4}{6\omega_3^2} \right)$ |
| $\Lambda^g_M \to N^g S_S'$ | $\frac{1}{3\sqrt{2}} \left( \frac{3k_0}{4\mu} + (1 + \frac{k_0}{2m} + \frac{k_0}{4\mu}) \frac{k_0^2}{\omega_3^2} - (1 + \frac{k_0}{2m} - \frac{k_0}{6\mu}) \frac{k_0^4}{6\omega_3^2} \right)$ |
| $\Lambda^g_P \to N^g S_M$ | $0$ |
| $\Lambda^g_M \to N^g S_M$ | $\frac{1}{9} \left( \sqrt{4} \right) \left( 1 + \frac{k_0}{2m} - \frac{k_0}{6\mu} \right) \frac{k_0^2}{\omega_3^2} - (1 + \frac{k_0}{2m} - \frac{k_0}{6\mu}) \frac{k_0^4}{6\omega_3^2}$ |
| $\Lambda^g_P \to N^g S_M$ | $\frac{1}{9} \left( \sqrt{4} \right) \left( 1 + \frac{k_0}{2m} - \frac{k_0}{6\mu} \right) \frac{k_0^2}{\omega_3^2} - (1 + \frac{k_0}{2m} - \frac{k_0}{6\mu}) \frac{k_0^4}{6\omega_3^2}$ |
| $\Lambda^g_M \to N^g S_M$ | $0$ |

latter expression to the results obtained in the chiral quark model, one gets

$$\frac{f_{\Lambda(1405)BM}}{m_M} = \frac{\langle [\hat{T}_d^M + \hat{T}_3^M + \hat{T}_5^M] \rangle}{M_{\Lambda(1405)} - m_B}.$$  \hspace{1cm} (43)

IV. NUMERICAL RESULTS AND DISCUSSION

Using the formalism developed in the previous section, here we present our numerical results for both electromagnetic and strong decays. Those results have been obtained with no adjustable parameters. In Table VII we give the input parameters used in our calculations and comment on the adopted values.

TABLE VII: The input values used in this manuscript for non vanishing five-quark probability ($P_{5q} \neq 0$). Here, $m \equiv m_u = m_d$. For $P_{5q} = 0$ we used $m = 340$ MeV. Values in columns 1 to 6 are in MeV.

| $m$ | $m_s$ | $\omega_3$ | $\omega_5$ | $f_K$ | $g_A^q$ | $A_{3q}$ | $A_{3q}^g$ | $A_{3q}^{g*}$ | $A_{3q}^{g*}$ |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 290    | 430    | 340    | 600    | 530    | 340    | 113    | 0.82   | $\sqrt{0.80}$ | $\sqrt{0.20}$ | $\sqrt{0.55}$ | $\sqrt{0.45}$ |

Since we have introduced the five-quark components in the baryons, The constituent
quark masses are slightly different from those used in the traditional constituent quark models. We take $m_u = m_d = 290$ MeV and $m_s = 430$ MeV, as suggested in Ref. 43 in order to reproduce the mass of the proton with 20% five-quark components, and to investigate successfully the electromagnetic transitions $\gamma^* N \to N^*(1535)$ and the strong decays of $N(1535)$. Values for the oscillator parameters $\omega_3$ and $\omega_5$ come also from this latter Reference.

The probability of five-quark components in proton leading to $A_{5q}^N = \sqrt{0.20}$ (see e.g. Ref. 43) is also used for the lowest mass hyperons, $A_{5q}^\Lambda$ and $A_{5q}^\Sigma^0$. Then the probabilities for $3q$ components are obtained within the used truncated Fock space, implying $(A_{3q}^\Lambda)^2 + (A_{3q}^\Sigma^0)^2 = 1$. For the $\Lambda(1405)$, our numerical results reported below (see sec. IV) favor $A_{3q}^{\Lambda*} = \sqrt{0.45}$, and hence, $A_{3q}^{\Sigma*} = \sqrt{0.55}$.

In Table VI, $g_A^q$ denotes the axial coupling constant for the constituent quarks, and its extracted phenomenological values are in the range $0.70 - 1.26$. Here, we have taken $g_A^q = 0.82$, which differs slightly from its value (0.88) in Ref. 55, due to the fact that we have introduced the five-quark components.

Finally, for the decay constants of mesons, the empirical values are used ($f_\pi = 93$ MeV and $f_K = 113$ MeV).

A. Radiative decays of $\Lambda(1405)$

Helicity amplitudes $A_{1/2}^\Lambda$ and $A_{1/2}^{\Sigma^0}$ for the electromagnetic transitions $\gamma \Lambda(1116) \to \Lambda(1405)$ and $\gamma \Sigma(1194) \to \Lambda(1405)$ at the real photon point are given in Tables VII and VIII, respectively, showing that the configurations mixing effects are very important, and the diagonal transitions between the five-quark components also have non negligible contributions to the helicity amplitudes. Moreover, as we can see in the nondiagonal ($N - D$) columns, those transitions between the three- and five-quark components in $Y_{S}^2 S_{S}$ and $\Lambda(1405)^2 P_{A}$ contribute significantly to the helicity amplitudes $A_{1/2}^Y$: about 27% to $A_{1/2}^\Lambda$ and 24% to $A_{1/2}^{\Sigma^0}$.

Table IX shows our results for the radiative decay widths of $\Lambda(1405)$, employing Eq. (27). Column A contains the results obtained without five-quark admixture, i.e. $P_{5q} = 0\%$, columns B, C, D, and E correspond to $P_{5q} = 25\%, 45\%, 75\%$ and $100\%$, respectively. The $\Lambda(1405) \to \Lambda \gamma$ channel shows a significant sensitivity ($\approx 30\%$) to the five-quark components, roughly in the range $20\% \lesssim P_{5q} \lesssim 50\%$. For the $\Lambda(1405) \to \Sigma \gamma$ decay, in going from $P_{5q} = 0\%$
to $P_{5q} = 25\%$, the decay width increases by roughly $36\%$, and drops down with the increasing $P_{5q}$ faster than the width for $\Lambda(1405) \to \Lambda \gamma$ decay.

TABLE VII: Results for the helicity amplitude $A^\Lambda_{1/2}$ (in GeV$^{-1/2}$) for electromagnetic transition $\gamma \Lambda \to \Lambda(1405)$.

|                      | $3q \to 3q$ | $5q \to 5q$ | N-D | total        |
|----------------------|-------------|-------------|-----|-------------|
| $\Lambda^2 S_S \to \Lambda^2 P_A$ | 0.050       | 0.013       | 0.024 | 0.087       |
| $\Lambda^2 S_S \to \Lambda^2 P_M$ | -0.027      | -0.005      | 0.011 | -0.021      |
| $\Lambda^2 S_S \to \Lambda^4 P_M$ | 0.011       | -0.004      | 0.009 | 0.016       |
| $\Lambda^2 S'_S \to \Lambda^2 P_A$ | -0.023      | 0.015       | 0    | -0.008      |
| $\Lambda^2 S'_S \to \Lambda^2 P_M$ | -0.005      | -0.020      | 0    | -0.025      |
| $\Lambda^2 S'_S \to \Lambda^4 P_M$ | 0.002       | -0.011      | 0    | -0.009      |
| $\Lambda^2 S_M \to \Lambda^2 P_A$ | -0.019      | 0.008       | 0    | -0.011      |
| $\Lambda^2 S_M \to \Lambda^2 P_M$ | -0.003      | -0.013      | 0    | -0.016      |
| $\Lambda^2 S_M \to \Lambda^4 P_M$ | 0           | -0.053      | 0    | -0.053      |

TABLE VIII: Results for the helicity amplitude $A^\Sigma_{1/2}$ (in GeV$^{-1/2}$) of electromagnetic transitions $\gamma \Sigma \to \Lambda(1405)$.

|                      | $3q \to 3q$ | $5q \to 5q$ | N-D | total        |
|----------------------|-------------|-------------|-----|-------------|
| $\Sigma^2 S_S \to \Lambda^2 P_A$ | -0.120      | -0.035      | -0.050 | -0.205      |
| $\Sigma^2 S_S \to \Lambda^2 P_M$ | -0.073      | -0.041      | 0.021 | -0.093      |
| $\Sigma^2 S_S \to \Lambda^4 P_M$ | 0.017       | -0.031      | 0.017 | 0.003       |
| $\Sigma^2 S'_S \to \Lambda^2 P_A$ | 0.038       | -0.027      | 0    | 0.011       |
| $\Sigma^2 S'_S \to \Lambda^2 P_M$ | 0.278       | -0.353      | 0    | -0.075      |
| $\Sigma^2 S'_S \to \Lambda^4 P_M$ | 0.002       | -0.093      | 0    | -0.091      |
| $\Sigma^2 S_M \to \Lambda^2 P_A$ | 0.046       | -0.019      | 0    | 0.027       |
| $\Sigma^2 S_M \to \Lambda^2 P_M$ | -0.001      | -0.107      | 0    | -0.108      |
| $\Sigma^2 S_M \to \Lambda^4 P_M$ | 0           | -0.204      | 0    | -0.204      |

Table[X] summarizes the widths for the electromagnetic decay of the $\Lambda(1405)$ reported by several authors. One of the early extractions of those quantities is due to Burkhardt
TABLE IX: Results for the radiative decays widths of $\Lambda(1405) \to \Lambda(1116)\gamma$ ($\Gamma_{\Lambda\gamma}$), $\Lambda(1405) \to \Sigma(1194)\gamma$ ($\Gamma_{\Sigma\gamma}$) (in keV), and their ratio $R = \Gamma_{\Sigma\gamma}/\Gamma_{\Lambda\gamma}$.

| A  | B  | C  | D  | E  |
|----|----|----|----|----|
| $P_{5q}$ (\%) | 0  | 25 | 45 | 75 | 100 |
| $\Gamma_{\Lambda\gamma}$ | 91 | 122| 123| 104| 56 |
| $\Gamma_{\Sigma\gamma}$ | 164| 223| 212| 164| 73 |
| $R$       | 1.8| 1.8| 1.7| 1.6| 1.3 |

and Lowe [57], motivated by the advent of reliable $K^-p$ atom data [58] published in late 80’s. Since then, those results have been introduced in PDG, and are considered by some authors as ”data”, though Burkhardt and Lowe state clearly in their paper the highly phenomenological character of their investigation, e.g. ”There is some degree of arbitrariness in assigning values to the individual coupling constants required to calculate radiative decays”. In other words, at the present time there are no reference values for those widths and various calculations put forward the relative importance of mechanisms considered in each approach. Moreover, given that the $\Lambda(1405)$ is 27 MeV below the $K^-p$ threshold, in kaonic atom only the upper tail of that resonance intervenes.

Predictions for both channels decay widths (Table X) may vary by two orders of magnitude from one approach to another, making any conclusive comparisons pointless in the absence of data. Landberger [63] suggested that the predicted ratio $R = \Gamma_{\Sigma\gamma}/\Gamma_{\Lambda\gamma}$ might be more instructive. Inspection of that ratio for different approaches (fourth column in Table X) allows us to distinguish three ranges: $R \gtrsim 1.0$ (present work and Refs. [57, 59, 60]), $0.4 \lesssim R \lesssim 0.6$ (Refs. [16, 18, 20, 25, 57, 60, 61]), and $R \lesssim 0.3$ (Refs. [18, 19, 25, 62, 64]).

Our model gives $R=1.7$, almost 29% larger than that obtained with algebraic model [59], but about two times smaller than the ratio given by a very recent coupled channels unitary chiral perturbation theory ($U\chi PT$) [60]. This latter generates 2 poles corresponding to the nominal $\Lambda(1405)$, resulting in two different radiative decay widths. The low-energy pole leads to $R=4.56$, with $\Gamma_{\Lambda\gamma}=16$ keV, compatible with the value extracted within the above mentioned isobar model [57]. However, that model leads to a ratio compatible, within 1-$\sigma$, with both $\approx 1.2$ and $\approx 0.5$, so within the first two ranges. The results for $R \gtrsim 1.0$ lead then to two series with respect to the width $\Gamma_{\Lambda\gamma} \approx 100$ keV (present work and Ref. [59]) and $\approx 20$
TABLE X: Radiative decay widths (in keV) of the $\Lambda(1405) \to \Lambda\gamma$, $\Sigma\gamma$ decays in different approaches, and the corresponding ratios $R = \Gamma_{\Sigma\gamma}/\Gamma_{\Lambda\gamma}$.

| Approach      | $\Gamma_{\Lambda\gamma}$ (keV) | $\Gamma_{\Sigma\gamma}$ (keV) | $R$  | Reference                        |
|---------------|---------------------------------|---------------------------------|------|----------------------------------|
| $\chi$QM      | 123                             | 212                             | 1.72 | Present work, with $P_{5q}=45\%$ |
| $\chi$QM      | 168                             | 103                             | 0.61 | Yu et al. [20]                   |
| Algebric model| 117                             | 156                             | 1.33 | Bijker et al. [59]               |
| $U\chi$PT     | 16                              | 73                              | 4.56 | Geng et al. [60]                 |
|               | 65                              | 33                              | 0.51 | Geng et al. [60]                 |
| $U\chi$PT     | 19                              | 113                             | 5.95 | Doring et al. [78]               |
|               | 83                              | 55                              | 0.66 | Doring et al. [78]               |
| Bonn CQM      | 912                             | 233                             | 0.26 | Van Cauteren et al. [19]         |
| NRQM          | 143                             | 91                              | 0.64 | Darewych et al. [16]             |
| NRQM          | 154                             | 72                              | 0.47 | Kaxiras et al. [18]              |
|               | 200                             | 72                              | 0.36 | Kaxiras et al. [18]              |
| RCQM          | 118                             | 46                              | 0.39 | Warns et al. [61]                |
| MIT bag       | 60                              | 18                              | 0.30 | Kaxiras et al. [18]              |
|               | 17                              | 3                               | 0.18 | Kaxiras et al. [18]              |
| Chiral bag    | 75                              | 2                               | 0.03 | Umino - Myhrer [62]              |
| Soliton       | 40                              | 17                              | 0.43 | Schat et al. [25]                |
|               | 44                              | 13                              | 0.30 | Schat et al. [25]                |
| Isobar model  | $27 \pm 8$ 10 $\pm 4$          | 0.37 $\pm 0.18$                |      | Burkhardt - Lowe [57]            |
|               | $27 \pm 8$ 23 $\pm 7$          | 0.85 $\pm 0.36$                |      | Burkhardt - Lowe [57]            |

keV [57, 60], while $\Gamma_{\Sigma\gamma}$ varies by two orders of magnitude.

The higher-energy pole in the $U\chi$PT [60] comes out in the second range $0.4 \lesssim R \lesssim 0.6$. It is worth noticing that various quark model based approaches [16, 18, 20, 61] predict ratios in the same interval, and three of them [16, 18, 61] give close enough predictions for both $\Gamma_{\Lambda\gamma}$ and $\Gamma_{\Sigma\gamma}$. It is however known that those approaches fail in describing the $\Lambda(1405)$. In the last part of this section we will come back to the recent chiral quark approach [20].

Moreover, in the MIT bag model [18] there are two $J^P = 1/2^-$ $\Lambda$ states at 1364 MeV and 1446 MeV, leading to $R= 0.18$ and 0.30, respectively, with decay widths much smaller.
than those predicted by quark models, but closer to the Soliton models \[18, 25\]. A more advanced chiral approach \[62, 64\] including gluon exchange mechanism, predicts a larger width for $\Gamma_{\Lambda\gamma}$ (75 keV), but that for $\Gamma_{\Sigma\gamma}$ shrinks down to 2 keV. That work reproduces well enough the total width of $\Lambda(1520)$, but underestimates that for $\Lambda(1405)$.

Now, we would like to proceed to more detailed comparisons between our results set and that reported by Yu et al. \[20\], also within a chiral quark approach. Here, we need to go back to Eqs. (2) to (5). Table XI summarizes the state assignments used in the present work and those in Ref. \[20\], showing that in this latter work all resonances have been replaced by the lowest mass relevant baryon. The drawback of that approximation on numerical results is presented below. The hereafter called hybrid model (HM) results are obtained using our

| State | Baryon | Baryon |
|-------|--------|--------|
| Present work | Ref. \[20\] |
| $\Lambda^2 P_A$ | $\Lambda^*(1405)$ | $\Lambda^*(1405)$ |
| $\Lambda^2 P_M$ | $\Lambda^*(1670)$ | $\Lambda^*(1405)$ |
| $\Lambda^4 P_M$ | $\Lambda^*(1800)$ | $\Lambda^*(1405)$ |
| $\Lambda^2 S_S$ | $\Lambda(1116)$ | $\Lambda(1116)$ |
| $\Lambda^2 S_{S'}$ | $\Lambda^*(1600)$ | $\Lambda(1116)$ |
| $\Lambda^2 S_M$ | $\Lambda^*(1810)$ | $\Lambda(1116)$ |
| $\Sigma^2 S_S$ | $\Sigma(1193)$ | $\Sigma(1193)$ |
| $\Sigma^2 S_{S'}$ | $\Sigma^*(1660)$ | $\Sigma(1193)$ |
| $\Sigma^2 S_M$ | $\Sigma^*(1770)$ | $\Sigma(1193)$ |
| $N^2 S_S$ | $N(938)$ | $N(938)$ |
| $N^2 S_{S'}$ | $N^*(1440)$ | $N(938)$ |
| $N^2 S_M$ | $N^*(1710)$ | $N(938)$ |

The resulting widths are reported in Table XII. The width $\Gamma_{\Lambda\gamma}$ increases by about 15% in going from for pure $3q$ configuration to $P_{5q} \lesssim 45\%$, while $\Gamma_{\Sigma\gamma}$ almost doubles, and the ratio $R$ increases rather smoothly. Although the ratio (0.6) found for $P_{5q} = 0\%$ is very close to that obtained by Yu et al. \[20\], there are about 25% discrepancies among the widths. We will come back to this point.
TABLE XII: Same as Table IX but for hybrid model (using our formalism with resonance assignments of Ref. 20).

|       | A | B | C | D | E |
|-------|---|---|---|---|---|
| \(P_{0q}(\%)\) | 0 | 25 | 45 | 75 | 100 |
| \(\Gamma_{\Lambda\gamma}\) | 119 | 141 | 134 | 105 | 47 |
| \(\Gamma_{\Sigma\gamma}\) | 77 | 158 | 169 | 158 | 102 |
| \(R\) | 0.6 | 1.1 | 1.3 | 1.5 | 2.2 |

TABLE XIII: Numerical results for the helicity amplitude \(A_{\Lambda_{1/2}}^\gamma\) (in GeV\(^{-1/2}\)) for electromagnetic transition \(\gamma\Lambda \rightarrow \Lambda(1405)\), with our results (2nd column), those from the hybrid model (\(HM\), 3rd column), and from Yu et al. 20 (last column).

| \(\Lambda^\gamma_{S}S \rightarrow \Lambda_{A}^\gamma P\) | total | \(HM\) | Ref. [20] |
|-----------------------------------------------|-------|--------|------------|
| \(\Lambda^\gamma_{S}S \rightarrow \Lambda_{P_{A}}^\gamma P\) | \(-0.070\) | 0.087 | 0.087 |
| \(\Lambda^\gamma_{S}S \rightarrow \Lambda_{P_{M}}^\gamma P\) | 0.062 | -0.032 | -0.021 |
| \(\Lambda^\gamma_{S}S \rightarrow \Lambda_{P_{A}}^\gamma P\) | -0.004 | 0.013 | 0.016 |
| \(\Lambda^\gamma_{S}S \rightarrow \Lambda_{P_{A}}^\gamma P\) | 0.030 | -0.006 | -0.008 |
| \(\Lambda^\gamma_{S}S \rightarrow \Lambda_{P_{M}}^\gamma P\) | -0.035 | -0.012 | -0.025 |
| \(\Lambda^\gamma_{S}S \rightarrow \Lambda_{P_{M}}^\gamma P\) | -0.002 | 0.006 | 0.009 |
| \(\Lambda^\gamma_{S}S \rightarrow \Lambda_{P_{M}}^\gamma P\) | -0.021 | -0.019 | -0.011 |
| \(\Lambda^\gamma_{S}S \rightarrow \Lambda_{P_{M}}^\gamma P\) | -0.008 | -0.015 | -0.016 |
| \(\Lambda^\gamma_{S}S \rightarrow \Lambda_{P_{M}}^\gamma P\) | -0.002 | -0.009 | -0.053 |

In Tables XIII and XIV we report our results for helicity amplitudes for each state, including those obtained using the hybrid model, and compare them with values found in Ref [20]. Notice that there is an overall sign difference between our conventions and those used in Ref [20]. The first observation is that the state assignments of Ref [20], affect almost all the amplitudes for \(\gamma^*\Lambda \rightarrow \Lambda(1405)\), bringing them closer to those in Ref [20]. Then, the fact that the hybrid model and Ref. [20] produce different results for the decay width can be attributed on the one hand to small differences in some of the amplitudes and on the other hand to the input values.

The situation is very different for the \(\gamma^*\Sigma \rightarrow \Lambda(1405)\) transition (Table XIV). Although
TABLE XIV: Results for the helicity amplitude $A_{1/2}^\Sigma$ (in GeV$^{-1/2}$) of electromagnetic transitions $\gamma\Sigma \rightarrow \Lambda(1405)$. Columns are as in Table XIII.

|            | total   | $HM$   | Ref. [20] |
|------------|---------|--------|-----------|
| $\Sigma_2^3 S_S \rightarrow \Lambda_1^2 P_A$ | -0.205  | -0.205 | -0.216    |
| $\Sigma_2^3 S_S \rightarrow \Lambda_2^2 P_M$ | -0.093  | -0.146 | -0.202    |
| $\Sigma_2^3 S_S \rightarrow \Lambda_4^2 P_M$ | 0.003   | -0.032 | 0.007     |
| $\Sigma_2^3 S'_S \rightarrow \Lambda_1^2 P_A$ | 0.011   | 0.018  | 0.196     |
| $\Sigma_2^3 S'_S \rightarrow \Lambda_2^2 P_M$ | -0.075  | -0.014 | 0.109     |
| $\Sigma_2^3 S'_S \rightarrow \Lambda_4^2 P_M$ | -0.091  | -0.043 | 0.004     |
| $\Sigma_2^3 S_M \rightarrow \Lambda_1^2 P_A$ | 0.027   | 0.047  | -0.074    |
| $\Sigma_2^3 S_M \rightarrow \Lambda_2^2 P_M$ | -0.108  | -0.076 | 0.005     |
| $\Sigma_2^3 S_M \rightarrow \Lambda_4^2 P_M$ | -0.204  | -0.074 | 0.003     |

The $HM$ results show significant deviations from our original values, they also differ very significantly from values reported in Ref [20]. The main explanation for that feature might be due to a sign difference in their expression for $\Phi_{\Sigma^o}^\rho$ (Eq. (A1) in that reference) and $|\Sigma^o\rangle_\rho$ in the present manuscript (Eq. (A5)). This observation explains, at least partly, the differences between the values found for $\Gamma_{\Sigma\gamma}$ in the result coming from hybrid model and those reported in Ref [20]. Results from this latter work, after correcting the sign, might allow more conclusive comparisons with our findings.

At this point, and having discussed results compiled in Table XI, the main firm message is that decay widths measurements are mandatory in identifying the most reliable approaches. In the meantime, comparisons among outputs from those works with other observables constitute an alternative way to progress. Accordingly, in the next Section we concentrate on the strong channels decay.

B. Strong decay of $\Lambda(1405)$

Using the formalism developed in Sec. III.B and transition amplitudes reported in Tables IV and V, here we present our numerical results.

The transition amplitudes for $\Lambda(1405) \rightarrow \Sigma(1194)\pi$ and $\Lambda(1405) \rightarrow K^- p$ are given in
Tables [XV] and [XVI] respectively.

**TABLE XV:** Results for the amplitudes of the $\Lambda(1405) \rightarrow \Sigma(1194)\pi$ decay. Amplitudes for $5q \rightarrow 5q$ transitions vanish (see Sec. [II.B]).

| $3q \rightarrow 3q$ N-D total | $3q \rightarrow 3q$ N-D total |
|------------------------------|------------------------------|
| $\Lambda^2_1 P_A \rightarrow \Sigma^2_8 S_S$ | 0.736 0.384 1.120 |
| $\Lambda^2_8 P_M \rightarrow \Sigma^2_8 S_S$ | 0.287 -0.229 0.058 |
| $\Lambda^4_8 P_M \rightarrow \Sigma^2_8 S_S$ | 0.491 -0.204 0.287 |
| $\Lambda^2_1 P_A \rightarrow \Sigma^2_8 S'_S$ | -0.722 0 -0.722 |
| $\Lambda^2_8 P_M \rightarrow \Sigma^2_8 S'_S$ | 0.001 0 -0.001 |
| $\Lambda^4_8 P_M \rightarrow \Sigma^2_8 S'_S$ | -0.228 0 -0.228 |
| $\Lambda^2_1 P_A \rightarrow \Sigma^2_8 S_M$ | -0.511 0 -0.511 |
| $\Lambda^2_8 P_M \rightarrow \Sigma^2_8 S_M$ | -0.051 0 -0.051 |
| $\Lambda^4_8 P_M \rightarrow \Sigma^2_8 S_M$ | -0.016 0 -0.016 |

**TABLE XVI:** Results for the amplitudes of the $\Lambda(1405) \rightarrow K^- p$ decay. Amplitudes for $5q \rightarrow 5q$ transitions vanish (see Sec. [II.B]).

| $3q \rightarrow 3q \ N - D$ total | $3q \rightarrow 3q \ N - D$ total |
|------------------------------|------------------------------|
| $\Lambda^2_1 P_A \rightarrow N^2_8 S_S$ | 1.478 -0.824 0.654 |
| $\Lambda^2_8 P_M \rightarrow N^2_8 S_S$ | -0.878 -0.228 -1.106 |
| $\Lambda^4_8 P_M \rightarrow N^2_8 S_S$ | 0 -0.198 -0.198 |
| $\Lambda^2_1 P_A \rightarrow N^2_8 S'_S$ | 0.745 0 0.745 |
| $\Lambda^2_8 P_M \rightarrow N^2_8 S'_S$ | -0.140 0 -0.140 |
| $\Lambda^4_8 P_M \rightarrow N^2_8 S'_S$ | 0 0 0 |
| $\Lambda^2_1 P_A \rightarrow N^2_8 S_M$ | 0.082 0 0.082 |
| $\Lambda^2_8 P_M \rightarrow N^2_8 S_M$ | -0.363 0 -0.363 |
| $\Lambda^4_8 P_M \rightarrow N^2_8 S_M$ | -0.194 0 -0.194 |

The nondiagonal terms, wherever relevant, play significant roles in both decay channels. For the $\Lambda(1405) \rightarrow \Sigma(1194)\pi$ transition (Table [XV]) the effect turns out to be constructive for the first transition, $\Lambda^2_1 P_A \rightarrow \Sigma^2_8 S_S$, enhancing its dominant character. For the two
other transitions the destructive combinations of those terms with pure $3q$ transitions lead to almost vanishing contribution from $\Lambda^2_P M \rightarrow \Sigma^2 S_S$, suppressed by a factor of 2, the magnitude of $\Lambda^4_P M \rightarrow \Sigma^2 S_S$ transition amplitude.

For the $\Lambda(1405) \rightarrow K^- p$ decay (Table XVI), the dominant term in pure $3q$ transition, $\Lambda^4_P N \rightarrow N^2 S_S$, gets reduced by more than 50% due to the nondiagonal term, while the magnitude of the second transition, $\Lambda^2_P M \rightarrow N^2 S_S$, increases by 20%. Finally, the nondiagonal terms attribute a significant role to the $\Lambda^4_P M \rightarrow N^2 S_S$ transition, otherwise vanishing in pure $3q \rightarrow 3q$ scheme.

Using those transition amplitudes, we now move to numerical results for decay width and coupling constants. In Table XVII, we give the numerical results with $P_{5q} = 0\%, 25\%, 45\%, 75\%$ and $100\%$ in columns A, B, C, D and E, respectively. By comparing results in columns A and B, we observe very significant effects arising from the nondiagonal terms discussed above.

| $P_{5q}$ (%) | 0  | 25 | 45 | 75 | 100 |
|-------------|----|----|----|----|-----|
| $\Gamma_{\Sigma\pi}$ (MeV) | 24 | 47 | 50 | 45 | 23  |
| $f_{\Lambda(1405)\Sigma\pi}/m_\pi$ | 3.0 | 4.1 | 4.3 | 4.1 | 2.9 |
| $f_{\Lambda(1405)K^- p}/m_K$ | 11.3 | 7.4 | 5.4 | 1.9 | -4.1 |

The most striking result is the predicted values for the width of $\Lambda(1405) \rightarrow \Sigma\pi$ decay. While a pure $3q$ constituent quark model underestimates that observable by a factor of 2, introduction of five-quark components in $\Lambda(1405)$ with $P_{5q} \approx 50\%$, leads to excellent agreement with the value, $50 \pm 2$, reported in PDG [37], and coming from Ref. [65]. This latter work, published by Dalitz and Deloff in 1991, is an impulse approximation approach fitting a subset of data from Ref. [66], and discarding the only other data set [67] available at that time.

In Table XVIII we summarize the relevant works on $\Gamma_{\Lambda(1405)\rightarrow \Sigma\pi}$. Recent data obtained at COSY by Zychor et al. [68] give a decay width of about 60 MeV, and a recent [69] phenomenological analysis leads to $40 \pm 8$. Two other formalisms, based on Bethe-Salpeter coupled-channels [70] and chiral quark model [71], find values compatible with the findings.
by Dalitz and Deloff \[65\]. Our result is also in line with those reported values. Width determined within a unitary chiral perturbation theory \[72\] suggests a smaller value, within a double-pole picture of Λ(1405). Very recently Akaishi et al. \[73\], using a variational treatment, questioned that picture and advocated a single-pole nature for that resonance.

So, within our work with \(P_{5q}=45\%\), the Λ(1405) resonance appears to favor a mixed structure of the three- and five-quark components.

**TABLE XVIII: Results for the Σπ decay width of Λ(1405).**

| Approach                                | \(\Gamma_{\Lambda(1405)\rightarrow(\Sigma\pi)^0}\) | Reference                      |
|-----------------------------------------|-----------------------------------------------|--------------------------------|
| \(\chi QM\)                            | 50                                            | Present work with \(P_{5q}\) 45\% |
| Bethe-Salpeter coupled-channels         | 50±7                                          | Garcia-Recio et al. \[70\]     |
| \(U\chi PT\)                           | 38                                            | Magas et al. \[72\]           |
| \(\chi QM\) coupled-channels potential model | 48                                            | Zhong - Zhao \[71\]          |
| COSY experiment                         | \(\approx 60\)                                | Zychor et al. \[68\]         |
| K-matrix                                | 50±2                                          | Dalitz - Deloff \[65\], PDG \[37\] |

Finally, our results for the \(\Lambda(1405)\Sigma\pi\) and \(\Lambda(1405)K^-p\) couplings (Table \text{XVII} reported without including isospin factors, show significantly different dependence on the structure of Λ(1405), namely, in going from a pure \(3q\) configuration to an admixture of the three- and five-quark components, the coupling \(f_{\Lambda(1405)\Sigma\pi}\) gets increased by roughly 30\%, while \(f_{\Lambda(1405)K^-p}\) decreases by about 40\%.

**V. SUMMARY AND CONCLUSIONS**

Within an extended chiral constituent quark model, we investigated the three- and five-quark structure of the \(S_{01}\) resonance \(\Lambda(1405)\). The wave functions for this resonance and the octet baryons in our approach were reported explicitly. We derived the electro-excitation helicity amplitudes for \(\gamma^*\Lambda(1116)\rightarrow\Lambda(1405), \gamma^*\Sigma^0(1194)\rightarrow\Lambda(1405)\) processes, as well as transition amplitudes for the \(\Lambda(1405)\rightarrow\Sigma(1194)\pi, K^-p\) decays. Using those amplitudes, we gave expressions for the electromagnetic and strong decays widths, namely, \(\Gamma_{\Lambda(1405)\rightarrow Y\gamma}\), with \(Y \equiv \Lambda(1116), \Sigma(1194)\) and \(\Gamma_{\Lambda(1405)\rightarrow(\Sigma\pi)^0}\), with \((\Sigma\pi)^0 \equiv \Sigma^0\pi^0, \Sigma^+\pi^-, \Sigma^-\pi^+\).
The numerical values computed using those expressions were presented and the dependence of various decay widths on the percentage of the five-quark components were investigated and compared with other sources. For the photo-excitation helicity amplitudes $A_{1/2}^{\Lambda}$, we found good agreements with the only set of published results by Yu et al. [20], using their approximations. For the $A_{1/2}^{\Sigma}$, a seemingly sign problem in that paper did not allow us to proceed to meaningful comparisons. We also examined the situation with respect to the decay widths $\Gamma_{\Lambda(1405)\to \Lambda\gamma}$, $\Gamma_{\Lambda(1405)\to \Sigma\gamma}$ and their ratio. We argued that large discrepancies among a dozen works [16, 18, 20, 25, 57, 59, 62, 64] devoted to that topic render it impossible to make any conclusive comparisons. Then, among the quantities investigated here, the only firm ground is offered by the experimental results for the $\Lambda(1405)\to \Sigma(1194)\pi$ decay width ($\Gamma_{(\Sigma\pi)^0}$). Our formalism, embodying about 45% of five-quark components in the $\Lambda(1405)$ resonance and 20% in the octet baryons, allows reproducing $\Gamma_{(\Sigma\pi)^0} = 50\pm2$ reported in PDG and endorsed OUR other findings, especially with respect to the electromagnetic decay widths.

Our work hence favors a mixed structure of the three- and five-quark components in the $\Lambda(1405)$ resonance, with $[31]_{X_{FS}}[4]_{X}[211]_{F}[22]_{S}$ scheme for the orbital-flavor-spin configuration of the four-quark subsystem. This configuration allows the presence of the $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ components in $\Lambda(1405)$, while as shown by An et al. [42, 45], that configuration rules out the $u\bar{u}$ and $d\bar{d}$ components in $N(1535)$. Moreover, the probability of the five-quark components in $N(1535)$ turns out to be in the same range as that of $\Lambda(1405)$, making the $N(1535)$ heavier than $\Lambda(1405)$. In consequence, with respect to the five-quark components in baryons, our results complementing those published on the Roper [40, 41] and the first $S_{11}$ resonances [42, 43], allows us to put forward an explanation for the mass ordering of the $N(1440)$, $\Lambda(1405)$, and $N(1535)$ resonances. Those issues have also been investigated in lattice QCD approaches [33, 74], an effective linear realization chiral $SU_L(2) \times SU_R(2)$ and $U_A(1)$ symmetric Lagrangian [73], and concisely reviewed in [76].

Finally, we wish to underline the importance of the mixing mechanism resulting from the present study. The presence of three- and five-qurak components in $\Lambda(1405)$ leads to nondiagonal terms arising from transitions among those components ($qqq \leftrightarrow qqq\bar{q}\bar{q}$). In the case of photo-excitation helicity amplitudes, we find larger effects due to those transitions than contributions from five-quark components. For the strong channels, not getting any contributions from those pure five-quark components, the nondiagonal terms turn out again
to be crucial, increasing by about a factor of 2 the width for the \( \Lambda(1405) \rightarrow \Sigma(1194)\pi \) decay and bringing it into agreement with the data. Comparable effects due to the mixing mechanism have also been reported for the electromagnetic transition \( \gamma^* N \rightarrow N(1535) \), and the radiative and strong decays of the Roper resonance \([40, 41]\). This may reveal a new mechanism for the decay properties of baryons, i.e. \( q\bar{q} \rightarrow \gamma^*, \pi, K \) transitions have significant contributions to the baryon resonance decays.

Appendix A: Wave functions for the three quark components

1. Flavor wave functions

The flavor wave functions for the baryons considered in this paper are as follows:

\[
|\Lambda\rangle_a = \frac{1}{\sqrt{6}}\{ |uds\rangle + |dsu\rangle + |sud\rangle - |usd\rangle - |sdu\rangle \}, \quad (A1)
\]

\[
|\Lambda\rangle_\rho = \frac{1}{2\sqrt{3}}\{ |usd\rangle - |dsu\rangle - |sud\rangle + |sdu\rangle + 2|uds\rangle - 2|dus\rangle \}, \quad (A2)
\]

\[
|\Sigma^0\rangle_\lambda = -\frac{1}{2\sqrt{3}}\{ |usd\rangle + |dsu\rangle + |sud\rangle + |sdu\rangle - 2|uds\rangle - 2|dus\rangle \}, \quad (A3)
\]

\[
|\Lambda\rangle_\lambda = \frac{1}{2}\{ |usd\rangle + |sud\rangle - |sdu\rangle - |dsu\rangle \}, \quad (A4)
\]

\[
|\Sigma^0\rangle_\rho = \frac{1}{2}\{ |usd\rangle + |dsu\rangle - |sud\rangle - |sdu\rangle \}, \quad (A5)
\]

\[
|p\rangle_\lambda = \frac{1}{\sqrt{6}}\{ 2|uud\rangle - |duu\rangle - |udu\rangle \}, \quad (A6)
\]

\[
|p\rangle_\rho = \frac{1}{\sqrt{2}}\{ |udu\rangle - |duu\rangle \}. \quad (A7)
\]

2. Spin wave functions

The spin-orbital coupled wave function read

\[
X_a = -\frac{1}{2}|\frac{1}{2}, \frac{1}{2}\rangle_\lambda(\rho, 0) + \sqrt{2}|\frac{1}{2}, -\frac{1}{2}\rangle_\lambda(\rho, +1) + |\frac{1}{2}, -\frac{1}{2}\rangle_\rho(\lambda, 0) - \sqrt{2}|\frac{1}{2}, \frac{1}{2}\rangle_\rho(\lambda, +1), \quad (A8)
\]

\[
X_\lambda = -\frac{1}{2}|\frac{1}{2}, \frac{1}{2}\rangle_\lambda(\lambda, 0) + \sqrt{2}|\frac{1}{2}, -\frac{1}{2}\rangle_\lambda(\lambda, +1) + |\frac{1}{2}, -\frac{1}{2}\rangle_\rho(\rho, 0) - \sqrt{2}|\frac{1}{2}, \frac{1}{2}\rangle_\rho(\rho, +1), \quad (A9)
\]

\[
X_\rho = |\frac{1}{2}, -\frac{1}{2}\rangle_\rho(\lambda, 0) - \sqrt{2}|\frac{1}{2}, -\frac{1}{2}\rangle_\rho(\lambda, +1) + |\frac{1}{2}, \frac{1}{2}\rangle_\lambda(\rho, 0) - \sqrt{2}|\frac{1}{2}, \frac{1}{2}\rangle_\lambda(\rho, +1), \quad (A10)
\]

\[
X'_\lambda = \sqrt{3}|\frac{3}{2}, \frac{3}{2}\rangle_\lambda(\lambda, -1) - \sqrt{2}|\frac{3}{2}, \frac{1}{2}\rangle_\lambda(\lambda, 0) + |\frac{3}{2}, -\frac{1}{2}\rangle_\lambda(\lambda, 1), \quad (A11)
\]

\[
X'_\rho = \sqrt{3}|\frac{3}{2}, \frac{3}{2}\rangle_\rho(\rho, -1) - \sqrt{2}|\frac{3}{2}, \frac{1}{2}\rangle_\rho(\rho, 0) + |\frac{3}{2}, -\frac{1}{2}\rangle_\rho(\rho, 1), \quad (A12)
\]
with \( \lambda, 0 = q_{\lambda, z}, (\lambda, +1) = -\frac{1}{\sqrt{2}} (q_{\lambda, x} + iq_{\lambda, y}), (\rho, 0) = q_{\rho, z} \) and \((\rho, +1) = -\frac{1}{\sqrt{2}} (q_{\rho, x} + iq_{\rho, y}).\)

The spin wave functions are
\[
\frac{1}{2} \frac{1}{2}\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle), \frac{1}{2} \frac{1}{2}\rangle = \frac{1}{\sqrt{6}} (|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle - |\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle), \tag{A13}
\]
\[
\frac{1}{2}, -\frac{1}{2}\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle), \frac{1}{2} \frac{1}{2}\rangle = -\frac{1}{\sqrt{6}} (|\downarrow\downarrow\rangle - |\downarrow\uparrow\rangle - |\uparrow\uparrow\rangle), \tag{A14}
\]
\[
\frac{3}{2}, \frac{3}{2}\rangle = |\uparrow\uparrow\rangle, \frac{3}{2} \frac{1}{2}\rangle = \frac{1}{\sqrt{3}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\uparrow\rangle), \tag{A15}
\]
\[
\frac{3}{2}, -\frac{1}{2}\rangle = \frac{1}{\sqrt{3}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\uparrow\rangle). \tag{A16}
\]

3. Orbital wave functions

Here we employ the harmonic oscillator wave functions
\[
\Phi_{\lambda}\Phi_{\rho} = \frac{\sqrt{\pi}}{\pi^{3/2} \omega_3^3} \exp\left\{-\frac{q_\lambda^2 + q_\rho^2}{2 \omega_3^2}\right\}, \tag{A17}
\]
\[
\Phi_{000} = \frac{1}{(\pi \omega_3^3)^{3/2}} \exp\left\{-\frac{q_\lambda^2 + q_\rho^2}{2 \omega_3^2}\right\}, \tag{A18}
\]
\[
\Phi_{200} = \frac{1}{\sqrt{3}(\pi \omega_3^3)^{3/2}} \left(3 - \frac{q_\lambda^2 + q_\rho^2}{\omega_3^2}\right) \exp\left\{-\frac{q_\lambda^2 + q_\rho^2}{2 \omega_3^2}\right\}, \tag{A19}
\]
\[
\Phi_{200} = \frac{2}{\sqrt{3} \pi^{3/2} \omega_3^3} \left(\Phi_{\rho} \cdot \Phi_{\lambda}\right) \exp\left\{-\frac{q_\lambda^2 + q_\rho^2}{2 \omega_3^2}\right\}, \tag{A20}
\]
\[
\Phi_{200} = \frac{1}{\sqrt{3} \pi^{3/2} \omega_3^3} \left(q_\rho^2 - q_\lambda^2\right) \exp\left\{-\frac{q_\lambda^2 + q_\rho^2}{2 \omega_3^2}\right\}. \tag{A21}
\]

Appendix B: Wave functions for the five quark components

1. Flavor and spin couplings

The decomposition of the flavor-spin configuration \( [31]_{FS}[211]_{F}[22]_{S} \) is
\[
[31]_{FS} = \frac{1}{\sqrt{2}} \{[211]_{F1}[22]_{S1} + [211]_{F2}[22]_{S2}\}, \tag{B1}
\]
\[
[31]_{FS} = \frac{1}{2} \{-\sqrt{2}[211]_{F3}[22]_{S2} + [211]_{F2}[22]_{S2} - [211]_{F1}[22]_{S1}\}, \tag{B2}
\]
\[
[31]_{FS} = \frac{1}{2} \{[211]_{F1}[22]_{S2} + [211]_{F2}[22]_{S2} + \sqrt{2}[211]_{F3}[22]_{S1}\}, \tag{B3}
\]
and that for \([4]_{FS}[22]_{F}[22]_{S}\)
\[
[4]_{FS} = \frac{1}{\sqrt{2}} \{[22]_{F1}[22]_{S1} + [22]_{F2}[22]_{S2}\}. \tag{B4}
\]
2. Flavor wave functions

The flavor wave functions for $|22\rangle_F$ in the $uuds\bar{s}$ component

\begin{align}
|22\rangle_{F1} &= \frac{1}{\sqrt{24}} \{2|uuds\rangle + 2|uusd\rangle + 2|dsuu\rangle + 2|sduu\rangle - |duus\rangle - |udsu\rangle - |sudu\rangle - |usdu\rangle - |suud\rangle - |usdu\rangle - |suud\rangle - |usdu\rangle - |suud\rangle \}, \\
|22\rangle_{F2} &= \frac{1}{\sqrt{8}} \{ |udus\rangle + |sudu\rangle + |dusu\rangle + |usud\rangle - |duus\rangle - |usdu\rangle - |sudu\rangle \}. 
\end{align}

(B5)

(B6)

The flavor wave functions for $|211\rangle_F$ in the $uuds\bar{s}$ component

\begin{align}
|211\rangle_{F1} &= \frac{1}{4} \{2|uuds\rangle - 2|uusd\rangle - |duus\rangle - |sudu\rangle - |suud\rangle + |sudu\rangle + |usud\rangle + |udsu\rangle \}, \\
|211\rangle_{F2} &= \frac{1}{\sqrt{48}} \{ 3|uds\rangle - 3|duus\rangle + 3|suud\rangle - 3|usdu\rangle + 2|dsuu\rangle - 2|sduu\rangle - |susd\rangle + |sudu\rangle + |dusu\rangle - |usdu\rangle \}, \\
|211\rangle_{F3} &= \frac{1}{\sqrt{6}} \{ |sudu\rangle + |udsu\rangle + |dsuu\rangle - |susd\rangle - |dusu\rangle - |usdu\rangle - |sudu\rangle \}. 
\end{align}

(B7)

(B8)

(B9)

All of the other flavor wave functions which are used in this paper are obtained by applying the lowering operator in the $SU(3)$ flavor space to the above functions.

3. Spin wave functions

Expressions for the spin wave functions $|22\rangle_S$ are

\begin{align}
|22\rangle_{S1} &= \frac{1}{\sqrt{12}} \{ 2|\uparrow\uparrow\downarrow\downarrow\rangle + 2|\downarrow\downarrow\uparrow\uparrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\uparrow\downarrow\rangle - |\downarrow\uparrow\downarrow\uparrow\rangle - |\uparrow\downarrow\uparrow\uparrow\rangle \}, \\
|22\rangle_{S2} &= \frac{1}{2} \{ |\uparrow\downarrow\uparrow\downarrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\downarrow\uparrow\rangle - |\uparrow\downarrow\uparrow\downarrow\rangle \}. 
\end{align}

(B10)

(B11)

4. Orbital wave functions

The orbital wave function of the five-quark components in $\Lambda(1405)$ reads

\begin{align}
[4]_X \Psi(\kappa_i) = \frac{1}{\pi^3 \omega_0^6} \exp\{-\sum_i \frac{\kappa_i^2}{2\omega_0^6} \}. 
\end{align}

(B12)
The color-orbital coupled wave function for the five-quark components in the octet baryons is

$$\psi_C(\{\vec{\kappa}_i\}) = \frac{1}{\sqrt{3}} \left\{ [211]C_1 \varphi_{01m}(\vec{\kappa}_1) \varphi_{000}(\vec{\kappa}_2) \varphi_{000}(\vec{\kappa}_3) - [211]C_2 \varphi_{000}(\vec{\kappa}_1) \varphi_{01m}(\vec{\kappa}_2) \varphi_{000}(\vec{\kappa}_3) + [211]C_3 \varphi_{000}(\vec{\kappa}_1) \varphi_{000}(\vec{\kappa}_2) \varphi_{01m}(\vec{\kappa}_3) \right\} \varphi_{000}(\vec{\kappa}_4).$$

(B13)

Here $[211]C_i$ denote the three color configurations, $\varphi_{0lm}(\vec{\kappa}_i)$ the harmonic orbital wave function with the quantum number $nlm$ and the oscillator frequency $\omega_5$. Notice that the $\vec{\kappa}_i (i = 1, 2, 3)$ generate the 3 configurations of $[31]_X$ in Eqs. (17)-(19).

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