Permeability Measurement up to 30 GHz of a Magnetically Isotropic Thin Film Using a Short-Circuited Coaxial Line

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In this study, the high frequency permeability ($\mu$) and ferromagnetic resonance (FMR) phenomena of a thin film with a strong perpendicular magnetic anisotropy and in-plane magnetically isotropic properties was measured using the short-circuited coaxial line technique; the analyzed sample had a toroidal shape. A field method was used for the background correction, where a strong magnetic bias field was applied and removed. However, when using a short-circuited coaxial line, the $\mu=1$ condition cannot be achieved beyond a few ten GHz frequencies, whereas ferromagnetic resonance (denoted as FMR2) occurred because of the insufficient bias field. This resonance was compensated using the Landau-Lifshitz-Gilbert (LLG) equation, and the net $\mu f$ properties without the bias field (denoted as FMR1) up to 30 GHz successfully extracted. Finally, a good agreement between the experimental results and the calculations based on the assumption of a magnetic multi-domain structure in FMR1 was achieved.

Keywords: magnetic thin film, perpendicular anisotropy, ferromagnetic resonance, FMR, LLG, permeability measurement, GHz band, short-circuited coaxial line, wideband measurement

1. Introduction

Nowadays, the development of wireless local area network, car collision avoidance radar, and automatic car driving test system, among others, has led to the application of magnetic materials at several 10 GHz bands. With this trend, the demand for measuring the high frequency permeability of magnetic thin films in this band region has been increasing accordingly.

We have already reported such measurement results for the 10-30 GHz band using short-circuited microstrip line test fixtures. Improving the present fixtures is crucial for performing measurements at higher frequencies. On the other hand, a short-circuited coaxial line can also be used, without the need to consider the external noise due to a perfectly shielded structure. Furthermore, the output connector of the vector network analyzer (VNA) is always coaxial, avoiding drastic mode changes and allowing the suppression of a higher-order wave. Because of this structural advantage, the permittivity ($\varepsilon$) and permeability ($\mu$) of bulk materials have been investigated in several studies by measuring the transmitted and reflected waves simultaneously.

However, these simultaneous measurements can generate some errors, the method is not suitable for materials with large frequency dispersions of $\varepsilon$ and $\mu$, and there are no reports on permeability measurements based on the short-circuited coaxial line technique, which is the topic of this study.

Amorphous Co–Zr–Nb films and Fe–Ni films with uniaxial anisotropy are typical magnetic thin films exhibiting high permeability at high frequencies, but their natural resonant frequencies are limited to few-gigahertz bands. In our group, nanograniual films with a higher natural resonant frequency were prepared by sputtering. In this study, a Co$_{90}$SiO$_{40}$ film was selected as the target material because (1) it has a higher resonant frequency above 10 GHz and (2) it is in-plane isotropic, which is favorable for a coaxial line with a rotational symmetry.

The lumped-element approximation of a short-circuited transmission line and the field method are usually adopted for the zero correction. The latter requires the $\mu=1$ condition for the sample which is obtained by applying a strong external bias magnetic field. However, this condition cannot be achieved for the coaxial line because the bias magnetic field is straight-line-like and the microwave field is circular-like, so that some parts certainly cross between both of them. Here we also discuss the problems of applying the field method to the coaxial line configuration.

2. Measurement Procedure

2.1. Test fixture

Figure 1 displays a schematic cross sectional view of our short-circuited coaxial line test fixture with a mounted sample. A 5 mm straight line is connected to the transform adaptor between the Sub Miniature Type A (SMA) and the Amphenol Precision Connector-7 mm (APC-7) connectors. Figure 2 shows the time-domain reflectometry characteristics of the test fixture without sample: the characteristic impedance is kept within 50 ± 1 $\Omega$ until the short end.

The rated upper limited frequency of an APC-7 connector is 18 GHz, and the commercial calibration kit could not be used. Hence, we realized an open-short-load calibration kit for up to 30 GHz. The reference plane for calibration is the APC-7 side of the transform adaptor, as shown in Fig.1.
The magnetic toroid-shaped thin film, with 1.5 μm thickness, deposited on a 0.5 mm thick glass substrate (Fig. 3), is loaded at the short end. The sample is mounted so that the film comes into contact with the short end and the substrate faces the microwave source side, as shown in Fig. 4.

When thickness d = 0.5 mm and permittivity ε = 6 of the substrate, the electric length is \( d/\lambda = 1.22 \text{ mm} \) that is less than half of \( \lambda/4 = 2.5 \text{ mm} \) where \( \lambda \) is free space wavelength of 30 GHz. So, the lumped-element approximation enough holds.

2.2. Field method

Permeability can be derived from the reflection parameter (S11) measured with the VNA using the short-circuited coaxial line.

First, a glass substrate without a magnetic thin film is inserted into the test fixture, an external strong bias magnetic field, \( S_{11} \) is measured, with and without strong bias magnetic field, which is applied in-plane of the thin film. This procedure allows the detection of the signal that carrying the information about the permeability of the magnetic thin film.

Finally, a signal related to this \( \mu \) can be picked out by subtracting the abovementioned background signal from a signal including the magnetic permeability of this film.

3. Effective Permeability Evaluation

3.1. Lumped-element approximation

In general, when a measurement angular frequency is \( \omega = 2\pi f \), the input admittance \( Y \) of the short-circuited transmission line with electrical length \( l \), filled with a medium having \( \mu \) and \( \varepsilon \) is expressed as follows:

\[
Y = \frac{1}{j\omega L_0} + \left(1.15/3\right)j\omega C_l,
\]

where, \( L = 166.9 \text{ nH/m} \) and \( C = 66.67 \text{ pF/m} \) are the inductance and capacitance per unit length of 50 Ω transmission line in the air.

Equation (1) is applicable to both partially and uniformly filled cases. In the former case, the effective permeability (\( \mu_e \)) and permittivity (\( \varepsilon_e \)) are used instead of \( \mu \) and \( \varepsilon \). The equation holds within ±5 % error range until \( 2(\varepsilon_e \mu_e)^{0.5} l \) = 1.4 rad. 9) Here, we introduce new parameters:

\[
L_e = L_l, \quad C_e = \left(1.15/3\right) C_l.
\]

Hence, Eq. (1) can be rewritten as follows:

\[
Y_e = 1/(j\omega L_e) + j\omega C_e,
\]

meaning that the equivalent circuit is a parallel \( L_e C_e \) circuit, as shown in Fig.5.

The reference plane to analyze is placed on the substrate surface, where the microwave impinges. Therefore, \( l \) is given by the sum of the thickness of

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**Fig. 1** Cross section of the short-circuited coaxial fixture loading a toroidal shaped thin film sample.

**Fig. 2** TDR characteristics of the short-circuited coaxial test fixture without the sample.

**Fig. 3** Front and cross sectional views of the magnetic toroid-shaped thin film on a glass substrate.

**Fig. 4** Schematic configuration of the short-circuited coaxial line and the magnetic toroid-shaped thin film sample.

Second, the substrate coated with the magnetic thin film is inserted into the coaxial line. Two \( S_{11} \) values are measured, with and without strong bias magnetic field, which is applied in-plane of the thin film. This procedure allows the detection of the signal that carrying the information about the permeability of the magnetic thin film.

Finally, a signal related to this \( \mu \) can be picked out by subtracting the abovementioned background signal from a signal including the magnetic permeability of this film.
substrate \((d)\) and the magnetic thin film \((t)\): however, when the latter is ignored, \(l = d\). When the imaginary part \(\varepsilon''\) of the effective complex permeability of the substrate is so small to be negligible, \(C_i\) becomes \(C_i = \varepsilon' C_o\) as shown in Fig. 6(a).

3.2. Determination of \(C_i\) and the coupling coefficient

The substrate permittivity of \(\varepsilon = 6\), measured by a different way, was used to calculate \(C_i = \varepsilon C_o\).

The coupling coefficient \((\eta)\), which is the ratio between the thin magnetic film volume and the whole volume of a toroidal sample, is given by

\[
\eta = t / (d + t) \cong t / d = 0.0015 / 0.5 = 0.03. \tag{4}
\]

Its value is very small but can be obtained relatively correctly, allowing also the absolute value measurement. In our study, \(d = 0.5\) mm and \(t = 1.5\) \(\mu\)m were used.

3.3. Derivation of the effective permeability \(\mu'' - j\mu''\)

The effective magnetic permeability is derived using the field method from the input admittances for the equivalent circuits shown in figures 6 and 7 in the usual way: please refer to previous papers\(^{13,10,11}\) for details.

Attention should be paid to the use of magnetic materials in the transform adaptor between SMA and APC-7, so the magnetic signal is observed by the field method even in an empty fixture. When both substrates without and with the magnetic thin film are individually loaded into the fixture, each signal contains also this magnetic signal. The signal from the magnetic thin film alone is obtained by subtracting the substrate one from that of the film-substrate system; this procedure can be described using numerical formulas.

First, when considering the background effective complex permeability \(\mu_b = \mu_b' - j\mu_b''\), \(L_o\) becomes \(\mu_b' L_o\) and the series resistance \(\omega \mu_b' L_o\) is introduced, as shown in Fig. 6(a). When a strong bias magnetic field \(H_{ext}\) is applied to this, the effective magnetic permeability changes into \(\mu_{b0} = \mu_b' - j\mu_b''\), corresponding to what is shown in Fig. 6(b). The effective susceptibility \((\mu_b'' - \mu_{b0}'')\) is obtained by comparing the measured input admittances for the two circuits of Fig. 6.

Next, when considering the glass substrate coated with the magnetic thin film, the whole effective permeability is given by the sum of the background effective permeability and the thin film permeability: \(\mu_b + \mu\). When applying a strong bias magnetic field, this becomes \(\mu_{b0} + \mu_b\). These two cases are represented in Fig. 7(a) and (b), respectively. The effective susceptibility, that is, \((\mu_{b0} + \mu_b) - (\mu_{b0} - \mu_b)\) is obtained by comparing both admittances.

The subtraction of the results of Fig. 6 from those for Fig. 7 leads to \(1 + \chi\), and, consequently, to

\[
1 + \chi = 1 + (\mu_b + \mu) - (\mu_{b0} + \mu_b) - (\mu_{b0} - \mu_b) = 1 + \mu - \mu_b = 1 + \mu - \mu_b - \mu_b^* - \mu_b' - \mu_b'' = \mu_b', \tag{5}
\]

where \(1 + \chi\) is the measured permeability. This corresponds to the vertical axis of Fig. 12.

The effective permeability \(1 + (\mu_{b0} - \mu_b)\) of the magnetic thin film can be picked out. The condition of \(\mu_b = 1\) is realized in the usual field method, so that \(\mu = 1 + \chi\), allowing the \(\mu_b\) calculation.

However, the \(\mu_b = 1\) condition can be avoided for the following reason. The direction of the microwave magnetic field \((H_{b0})\) to the bias magnetic field \((H_{ext})\) in the field method for a toroidal sample is considered, as shown in Fig. 8. The microwave magnetic field acts uniformly along the circumference direction inside the toroidal-shaped sample, whereas \(H_{ext}\) acts along the one-way straight one, as shown in Fig. 8. Therefore, \(H_{ext}\) is parallel or antiparallel to \(H_{b0}\) on \(a'\) and \(c'\) parts, so that the signals from those parts are rarely detected.

On the other hand, both fields are normal on the \(b'\) and \(d'\) parts, and this partial signal is clearly detected.

As in this experiment, the thin film does not saturate even if a magnetic field of \(H_{ext} = 5240\) Oe is applied. On the contrary, the ferromagnetic resonance occurs by \(H_{ext}\) around 24 GHz. At that frequency, the value of \(\mu''\) becomes very large due to the resonance so the imaginary part \((\mu'' - \mu_{b0}'')\) of the equation (5) is observed as negative because \(\mu''\) is almost zero.

![Fig. 5 Equivalent circuit of coaxial test fixture.](image)

![Fig. 6 Equivalent circuits for the glass substrate without the sample and loaded in the test fixture, (a) without and (b) with the bias magnetic field.](image)

![Fig. 7 Equivalent circuits for the magnetic thin film loaded in the test fixture, (a) without and (b) with the bias magnetic field.](image)

![Fig. 8 Schematic of bias magnetic field and RF magnetic field in a toroidal sample.](image)
4. Magnetic Thin Film Preparation

A nanogranular film of Co_{52-(SiO_{2})_{48}} was fabricated using the radiofrequency (RF) magnetron sputtering method with the conditions: the targets of Co chips (5 × 5 mm) and SiO_{2} (ϕ=50 mm), the input power of 200 W, in the atmosphere of Ar gas (pressure 0.22 Pa).

The fabricated film’s crystalline structure and organization were determined using transmission electron microscopy energy dispersive spectroscopy. As shown in Fig. 9, the structure consisted of a slender Co grains with 3-5 nm diameter and 5-10 nm length, arranged normal to plane in amorphous SiO_{2} matrix; these phases appeared as separate, and Co exhibited a random-hexagonal-closed-packed structure.

The static B-H curve was measured in a vibrating sample magnetometer. The magnetization curve is shown in Fig. 10: its shape remained unchanged even when the direction of the static magnetic field was changed in the plane. From the in-plane magnetization curve, saturation magnetization \(4\pi M_s\) and perpendicular magnetic anisotropy field \(H_k\) were decided to be 8.9 kG and 4.1 kOe, respectively. The coercivity of a hysteresis perpendicular to the film plane was about 210 Oe, which is quite smaller than that of the hard magnetic film.

5. Results and Discussion

5.1. Bias magnetic field influence on the background

The experimental results of the effective complex permeability for the glass substrate in the short-circuited coaxial line, alone and coated with the thin film, are shown in Fig. 11.

Fig. 9 Cross-sectional image and diffraction pattern of Co_{52-(SiO_{2})_{48}} thin film.

Fig. 10 Magnetization curve of Co_{52-(SiO_{2})_{48}} magnetic thin film.

Fig. 11 Frequency dependences of effective complex permeability, \(1+\mu^r\mu_s\) and \(1+\mu^i\mu_s\) of the substrate with film and \(1+\mu^r\mu_s\) of it without film.

Fig. 12 Measured \(\mu^r\mu_s\) curve of magnetic thin film with 1.5 μm thickness on 0.50 mm t substrate by the field method (5.2kOe)

Fig. 13 Measured \(\mu^r\mu_s\) curve of magnetic thin film with 1.5 μm thickness on 0.50 mm t substrate using field method with various \(H_{ext}\) of 700-9200 Oe.

Fig. 14 Field dependences of the FMR frequencies and the maximum imaginary value \(\mu^i\) of FMR1.
Both bias magnetic fields were 5.2 kOe in the field method, with the dashed lines corresponding to \( \mu_b \), and the solid lines corresponding to \( \mu_b + \mu_f \). Both permeability values at 2GHz increases more than ten times compared to those of at 10 GHz, probably because the magnetic property of the transform adaptor between SMA and APC-7 was detected. Very large background signal were recorded, but a little difference was observed when expanding the magnification beyond 2 GHz, which is our objective signal of the magnetic thin film. Four spike-like noises (at 19, 23, 26.5, and 28 GHz) were observed.

The rated frequency of APC-7 is 18GHz, which relates to the cutoff frequency \( f_c \) of TE_{11} mode as a higher harmonic and is given as follows 12):
\[
f_c = \frac{c}{2 \pi} \sqrt{(a + b)^2}\]
where, \( c \) is the speed of light, and \( a \) and \( b \) are the inner and outer diameters, respectively, of the coaxial line.

The \( f_c \) is calculated as 19 GHz for APC-7, which corresponds to the first spike noise in Fig. 11.

Hereinafter, these spike-like noises will be omitted from permeability profile.

Figure 12 shows the results of subtracting the signals of the glass substrate alone from those of the one coated with the magnetic thin film, which correspond to the permeability signals of the latter that were detected with a relatively low noise in spite of a large background signal.

A resonant peak (denoted as FMR1), looking like natural resonance, was observed at around 13 GHz, but a large minus peak of \( \mu_f \) (FMR2) was also observed at 24 GHz.

In the measurement principle of the adopted field method, the state applied by the strong bias magnetic field is assumed to be the reference state (\( \mu = 1 \)); hence, in case of some absorption in this state, \( \mu'' \) is observed as a minus. This phenomenon corresponds to the minus value of \( \mu'' \) in case of \( \mu'' > \mu'' \) in the equation (5).

Then, the maximum \( \mu'' \) of FMR1 was denoted as \( \mu_f \) max, and we examined how its value was influenced by the \( H_{ext} \) strength.

Changes in the \( \mu'' \) property of the film by \( H_{ext} \) with 700–9200 Oe are indicated in Fig. 13. Both the FMR1 and FMR2 peaks appeared at 3 kOe, and the resonant frequency of FMR2 linearly increased with the \( H_{ext} \) strength.

5.2. Derivation of the anisotropic field \( H_k \) of the thin film at a high frequency

The results shown in Fig. 13 are plotted in Fig. 14; the resonant frequency of 13 GHz of FMR1 remained unchanged even when the magnetic field strength was changed. As discussed in detail in Section 5.3, FMR1 was definitely regarded as natural resonance of the thin film. Its maximum \( \mu_f \) max value increased straightforwardly and became constant at about 4.1 kOe which corresponds to the saturation by the bias magnetic field.

The main concern was the magnetic field dependence of FMR2. This peak appeared at 1.5 GHz and \( H_{ext} = \sim 2.2 \) kOe; and when the magnetic field strength was increased, it moved monotonously toward the high frequency side. It was detected mostly at the b’ and d’ parts shown in Fig. 8 and corresponds to the FMR by the bias magnetic field.

The resonance frequency \( (f_f) \) of FMR2 is given as follows 13) (see the Appendix).
\[
f_f = \frac{\mu_f (H_{ext} - H_k + 4\pi M_s)}{2 \pi (H_{ext} - H_k + 4\pi M_s)}\]
The curve calculated on the basis of this equation, which well agreed with the measured frequency dependence of FMR2, is plotted as a dotted line in Fig. 14. In this case, \( 4\pi M_s = 8.9 \) kG, \( H_k = 3.4 \) kOe, and \( (2\pi) = 3.1 \) GHz/kOe \( (g = 2.21) \) were used; this \( H_k \) is smaller than that derived from the static magnetization curve (4.1 kOe) shown in Fig. 10. The value of \( H_k = 3.4 \) kOe was used for the following analysis.

5.3. Calculation of the \( \mu'' \) curve of FMR2

The film saturated by the bias magnetic field had a single domain structure magnetized in-plane. In this case, an FMR curve can be calculated by the following equation derived from the Landau-Lifshitz-Gilbert (LLG) formula. It is defined that the x-axis is parallel to the microwave magnetic field in the film plane, which is perpendicular to the x’ axis of the bias magnetic field in the film plane, the y’ axis was vertical to the film plane.

Then, the demagnetization factor can be expressed as \( N_x = N_y = 0 \) and \( N_z = 1 - H_k/4\pi M_s \) (see the Appendix). The
perpendicular magnetic anisotropy field $H_k$ reduced $N_c$. 

$$\chi' = \mu_{oi} - 1 = \omega_0 \alpha_0 \alpha_1 (1 + \alpha^2) + \omega_0 \alpha_0 \alpha_2 (1 + \alpha^2) / CC$$

$$\chi'' = \omega_0 \alpha_0 \alpha_2 (1 + \alpha^2) / CC$$

where $CC = [\alpha_0 \alpha_2 (1 + \alpha^2)] / CC'$

Equations assumed a periodic checkered magnetic domain structure repeated alternatively. We measured $CC$ single magnetic domain and always has numerous $kOe$, $H_k = 3.4 kOe$ and $K = 0.47$ were used.

Using Field method with application of maximum $H_{ext}$ and the y-axis was normal to the x-axis in-plane. (8a) and (8b) can be used in the same way. Then, we measured $H_{ext}$ up to 9.2 kOe and obtained similar to those shown in Fig. 16. Fig. 16 can be explained using an LLG equation.

The compared results well agreed below the natural resonance frequency of 15 GHz. When a measurement error was considered, they also qualitatively agreed at frequencies higher than 15 GHz. However, when comparing them precisely, a discrepancy was observed since $\mu'$ did not approach zero at higher frequencies, as predicted by the calculation.

However, this phenomenon is physically reasonable. Magnetostatic and spin waves are easily excited as a higher mode because the supplied microwave energy has a high level and, reflecting this influence, the loss increases at bands higher than resonant frequency. This phenomenon is usually observed in FMR experiments.

Thus, the saturation magnetization, the anisotropic magnetic field, and the bias magnetic field can help quantitatively explain the magnetic behavior of the

If the magnetic domain size is sufficiently small, the demagnetization field will not appear along the x-axis and we will have $N_x = 0$.

The in-plane demagnetization field is strongly influenced by the neighboring magnetic domain. The x-direction component of the high frequency magnetization synchronizes with $h_{rf}$ so that the magnetic charge does not appear on the magnetic domain wall (y-z plane), and we have $N_y = 0$.

The magnetization along the y-direction was an issue. A large magnetic charge appeared on the domain wall (z-x plane) because of the Smit and Wijn effect and introduced a large demagnetization field. We had to estimate a large $N_y$ because it would have allowed the calculation of the $\mu$-$f$ curves of the remaining magnetization. Here, we had to substitute $H_k$ with $H_{ext}$ in Eq. (9).

Figure 17 shows the results calculated using $4\pi M_s = 8.9 kG$, $x = 0.11$, $H_{ext} = 5.24 kOe$, $H_k = 3.4 kOe$ and $K = 0.47$ were used.

In Fig. 16, the corrected $\mu$-$f$ curve is plotted that the measured $\mu$ in Fig. 12., compensated by the calculated $\mu_f$ of FMR2 in Fig. 15. FMR2 disappeared and the $\mu$ and $\mu_f$ values below 15 GHz rose a little. To confirm the validity of this correction (Fig. 16), we measured $\mu_f$ by using Field method with application of maximum $H_{ext}$ up to 9.2 kOe and obtained similar to those shown in Fig. 16.

5.4. Calculation of the $\mu$-$f$ curve of FMR1

Next, we discuss how the result for FMR1 shown in Fig. 16 can be explained using an LLG equation.

The magnetic domain structure of the remaining magnetization state was considered for FMR1. The state of the vertical magnetization cannot exist as a single magnetic domain and always has numerous magnetic domain structures repeated alternatively. We assumed a periodic checkerered magnetic domain structure, as shown in Fig. 18(b).

This periodic structure can be represented by one magnetic domain, if the magnetic circumstances surrounding one domain could be considered. Equations (8a) and (8b) can be used in the same way. Then, we assumed that the x-axis was vertical to the film plane and parallel to $H_k$, the x-axis was parallel to $h_{rf}$ in-plane, and the y-axis was normal to the x-axis in-plane.

Fig. 17 Comparison between calculated and the experimental $\mu$-$f$ curve of FMR1 at the bias field of 5.2 kOe.

Fig. 18 Schematic domain structures of thin film with perpendicular magnetic anisotropy. (a) with and (b) without bias magnetic field.
Table 1 Comparison between natural resonance (FMR1) and compulsory FMR (FMR2).

|                | FMR1 | FMR2 |
|----------------|------|------|
| $\alpha$       | 0.65 | 0.64 |
| $K$            | 0.72 | 0.47 |
| $N_x$          | 0    | 0    |
| $N_z$          | 0.21 | 0.47 |
| Magnification factor $K$ (Theoretical value) | (1) | (0.64) |
| Relaxation factor $\alpha$ | 0.16 | 0.11 |
| Resonance      | Natural | Ferromagnetic |

prepared thin film near 30 GHz using values from other experiments.

The compulsory ferromagnetic resonance (FMR) phenomenon FMR2 due to the bias magnetic field, beside the natural resonance FMR1 of the thin film material, was observed using this experiment method. As a result, we could specify the FMR relaxation coefficient $\alpha$.

The analytical method discussed here contains substantial information, natural resonance and ferromagnetic resonance, and is expected to expand its application area more.

6. Conclusion

In this study, we measured the magnetic permeability up to 30 GHz of an in-plane isotropic nanogranular thin film with perpendicular magnetic anisotropy using a short-circuited coaxial line test fixture. The specimen's geometry was toroidal.

A field method was used for the zero correction: a strong external bias magnetic field was applied to ensure the $\mu=1$ condition. However, this condition was not achieved in the short-circuited coaxial line. In addition to the natural resonance of the magnetic thin film, a compulsory FMR phenomenon caused by the bias magnetic field was also observed.

We contrived how to compensate the FMR by calculating the susceptibility, taking the perpendicular anisotropy field into account. This calculations process was carried out on the basis of the phenomenological theory of FMR (i.e., the LLG equation): the FMR relaxation coefficient could also be derived by fitting the experimental results to the theoretical curve.

The FMR1 and FMR2 results are summarized in Table 1. The relaxation coefficient of FMR2 was quite smaller than that of FMR1. The $K$ factor was not 0.64 for FMR2, but the $K_{FMR2}/K_{FMR1}$ ratio became 0.47/0.72 = 0.65, which closely resembles 0.64. If about 70% of the whole thin magnetic film acted along the theoretical prediction, this agreement makes sense, thereby, insisting the justice of our analytical way.

From the abovementioned results, we can conclude that the high frequency magnetic permeability of the prepared thin film with perpendicular magnetic anisotropy was measured within a frequency band up to 30 GHz using a short-circuited coaxial line and considering the magnetic field strength for the zero correction and the film magnetic parameters.

However, this analytical way is restricted to the area where the condition of a single magnetic domain should be required when applying a strong bias magnetic field. Materials with a high magnetic coercive force cannot be analyzed, whereas soft magnetic thin films allow FMR experiments even at a frequency as high as 30 GHz.

Appendix

Resonance condition and complex susceptibility of an in-plane magnetized thin film having perpendicular anisotropy

In general, the complex susceptibility $\chi = \chi' - j\chi''$ and the resonance condition of a magnetic thin film, magnetized along the $x$ axis by $H_{ext}$ and excited by $hrf$ along the $x$ axis, are expressed as same as Eq. (8a), (8b), and (8c).

The resonance condition is achieved when Eq. (8c) becomes minimum, where $\alpha = 0$ leads to Eq. (8c) = 0, resulting the next equation:

$$\alpha^2 = A_x A_y |(\omega - (N_x - N_y)\omega_0)| |(\omega - (N_y - N_z)\omega_0)|$$  \(\text{(A1)}\)

In case of a film without magnetic anisotropy, $N_x = N_y = 0$, $N_y = 1$, and the resonance condition is

$$\alpha^2 = \omega_0 (\omega_0 + \omega_0)$$  \(\text{(A2)}\)

The coordinate axes are shown in Fig.A-1.

The film system including the perpendicular anisotropic constant $K_u$, shown in Fig.A-2, possesses magnetic free energy that is expressed as follows:

$$E = -(1/2)4\pi M_s H_{ext} + K_u \cos^2 \theta$$  \(\text{(A3)}\)

where $\theta$ is the angle slightly deviated from the $x$ axis.

$H_{ext}$ is the demagnetizing field expressed as $H_{ext} = -N_y 4\pi M_s \sin \theta$.

Eventually, the free energy becomes

$$E = -(1/2) (4\pi M_s)^2 [(N_x - 2K_u/(4\pi M_s))^2 \sin^2 \theta$$  \(\text{(A4)}\)

When $N_x = 1$, the effective demagnetization factor $N_{vo}$ along the $y$’direction can be written as follows:

$$N_{vo} = 1 - H_k/(4\pi M_s)$$  \(\text{(A5)}\)

$$H_k = 2K_u/(4\pi M_s)$$  \(\text{(A6)}\)

The resonance condition is written as follows:

$$\alpha^2 = \omega_0 (\omega_0 - \omega_k + \omega_0)$$  \(\text{(A7)}\)

$$\omega_k = \gamma H_{ext}$$  \(\text{(A8)}\)

Fig.A-1

Without anisotropy

$N_x = N_y = 0$, $N_{vo} = 1$

Fig.A-2

With perpendicular anisotropy

$N_x = N_y = 0$, $N_{vo} = 1 - H_k/(4\pi M_s)$
When the perpendicular anisotropy is induced, the resonance frequency decreases.

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