CFD investigation of internal elbow pipe flows in laminar regime

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Abstract. Elbow pipes are crucial parts of many fluid transport systems in the oil and gas industry. The curved shape of such pipes induces centrifugal forces on the internal flow, ultimately affecting the flow velocity and creating pressure differences within the elbow. The present study aims to investigate the effects of the curvature ratio of an elbow pipe on the internal pipe flow using three-dimensional numerical simulations. For laminar flows, the simulations are based on four Reynolds numbers ranging from 200 to 2000 and three curvature ratios of $Ro=5.6, 11.2$ and $22.4$. A mesh convergence study is carried out for 3 meshes with increasing resolution. The results based on the optimal mesh is then compared with the published experimental and numerical results for validation. Once the validation is confirmed, further simulation and analysis are performed for each combination of curvature ratio and Reynolds number. The results reveal that there is flow separation due to the centrifugal forces induced by the curved shape. It is also shown that secondary flows consisting of symmetrical helical vortices called Dean vortices are generated. The intensity of this secondary flow is shown to increase with the increasing Dean number.

1. Introduction
In multiple engineering applications, such as the oil and gas industry, fluid transport is an essential part of the process. The extracted oil and gas are transported by pipelines from the subsea reservoirs to topside. After the fluid is extracted, it travels within a series of pipes which are connected together by different pipe fitting types, namely elbow, tee, and Y-junctions. Due to the different geometries of the pipes, they are subjected to different pressure and velocity variations within the internal flow. This results in high and low-pressure regions in the flow, which may create a high load on the pipe wall. These factors must be taken into consideration in the design process of such fittings and pipes. With the ever-increasing demand for energy, methods that can improve planning prior to well operation will improve the economic feasibility of the pipe systems. Many studies focusing on flow within pipes and, specifically, elbow pipes, have been conducted. An experimental study on turbulent flow in a 90-degree bent pipe was performed by Sudo et al. (1997). The curvature ratio of the elbow was $Ro = 2D$ where $D = 104$ mm is the diameter of the pipe. An inlet pipe with a length of $100D$ was connected to the elbow, allowing the flow to be fully developed. Measurements were taken for air entering the pipe with a mean velocity of $8.7$ m/s, corresponding to a Reynolds number of $Re = 60000$. This value was also used as a reference in the previous work conducted by Enayet et al. (1982) and Azzola et al. (1986). Nicolaou and Zaki (2016) used numerical simulations to investigate both laminar and turbulent flows within curved pipes. Their investigations included pipe flow simulations for the laminar case at $Re = 1000$ and the turbulent case at $Re = 10000$. The pipe geometry was created with a diameter of $D =$
0.01 m, inlet length of \(D\), outlet length of \(2D\) and curvature radius of \(R_c = 2.8D\). The finite volume formulation used in Rosenfeld, Kwak and Vinokur (1991) were adopted in the numerical simulations. Another numerical investigation of laminar flows within elbow pipes was conducted by Inthavong (2018) for the purposes of studying particle deposition in human airways. Eight diameters ranging from 0.005 to 0.100 m, nine bend radii ranging from 0.006 m to 0.070 m and three different Reynolds numbers of 400, 1000 and 2000 were investigated in this study. A geometry was created consisting of a straight inlet pipe of \(5D\), a straight outlet pipe of \(3D\) and an elbow with a curvature ratio of 5.6. Numerical simulations were carried out for 48 combinations of these diameters, curvature ratios and Reynolds numbers.

The aforementioned previous studies have explored both laminar and turbulent flow. The present study will only focus on laminar flow. A detailed grid convergence with three different grid resolutions and a validation study are conducted. The results from multiple simulations are then presented to discuss the effects of different elbow geometries and Reynolds numbers on the flow fields. Finally, conclusions are given.

2. Methodology

2.1 Flow model

The flow in the present study is assumed to be incompressible and viscous. Numerical analysis is performed for laminar flows; and three-dimensional Navier-Stokes equations are solved. These equations include continuity and moment conservation equations. With these assumptions established, the governing equations for laminar flow are given by:

\[
\nabla \cdot \mathbf{u} = 0 \quad (1)
\]

\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} \quad (2)
\]

where \(\mathbf{u}\) is the velocity vector, comprised of the three vector components, \(\nabla\) is the gradient operator, \(\rho\) is the fluid density, \(p\) is the pressure of the fluid and \(\nu\) is the kinematic viscosity. Due to the incompressibility of the fluid flow, the density and viscosity of the fluid are constant.

2.2 Computational domains and boundary conditions

The simpleFoam solver in the open-source CFD code OpenFOAM is adopted in this study. It is a steady-state solver for incompressible flows. The term SIMPLE stands for the algorithm of Semi-Implicit Method for Pressure Linked Equations.

The computational domain includes a straight inlet pipe, an elbow and a straight outlet pipe. The geometries of the pipe for the baseline study are as Table 1

| Parameter            | Value |
|----------------------|-------|
| Bend curvature       | 90°   |
| Inner diameter, D    | 1 m   |
| Inlet length         | 50D   |
| Outlet length        | 20D   |
| Curvature ratio, \(R_o\) | 5.6   |

The effects of increasing the curvature ratio will be further studied. As the elbow is the main part of this study, different values of the curvature ratio, \(R_o\) will be investigated. The curvature ratio is given by

\[
R_o = \frac{R_c}{R_p} \quad (3)
\]

where \(R_p\) is the pipe radius and \(R_c\) is the radius of the elbow curved part. The value of \(R_p\) remains constant in each case of this study, while \(R_c\) will be changed according to \(R_o\). Other geometrical
properties such as inlet and outlet length are kept the same for each case. Twelve cases are created by the combination of four different Reynolds numbers of $Re = 200, 500, 1000, 2000$, and three different curvature ratios of $R_o = 5.6, 11.2, 22.4$. Figure 1 shows the geometry used as a baseline for geometry and mesh creation of subsequent cases.

The boundary conditions are defined as follows:

- **Inlet**: In the streamwise direction, a parabolic velocity profile entering the flow domain is given as $u_x = 2u_{avg}[1 - \frac{r}{R_p}]^2$ with $u_{avg} = 1\text{m/s}$ and $u_y = u_z = 0$ ($r$ is the coordinates in the radial direction of the pipe cross-section with $r = 0$ at the centre of the cross-section). The pressure at this boundary is defined as zero normal gradient.

- **Wall**: The no-slip condition is prescribed for the velocities at the wall of the pipe. The pressure is set as zero normal gradient at the pipe wall.

- **Outlet**: The velocity is set as zero normal gradient and the pressure is set to be zero.

### 2.3 Convergence and validation study

The purpose of the convergence study is to evaluate whether increasing the mesh resolution significantly changes the simulated results. By observing the deviation between the results of different mesh resolutions, the optimal mesh resolution can be selected and utilized for the further study of the elbow part. Using the optimal mesh allows for the balance between the accuracy of the simulation and the computational resources used to perform the simulations. Three meshes are created with the increasing resolution. The number of cells is increased by at least 30% between meshes as shown in Table 2. This convergence study is carried out for the highest investigated Reynolds number of $Re = 2000$ which is a laminar flow according to Pantokratoras (2016) and a curvature ratio of $R_c = 5.6$. An example of the mesh at the cross-section of the pipe and at the elbow part is shown in Figure 2.

### Table 2 The three different meshes resolutions used for convergence study

| Mesh   | No. of cells |
|--------|--------------|
| Mesh 1 | 1413733      |
| Mesh 2 | 2321088      |
| Mesh 3 | 3406543      |

Figure 3 shows the velocity profiles at specified diameters located at the bend outlet. The left plot corresponds to the velocity profile at $90^\circ$ of the bend parallel to the symmetry plane (A-A diameter as shown in Figure 2), while the right plot shows the profile at the diameter perpendicular to the symmetry.
plane (B-B diameter as shown in Figure 2). The graphs exhibit areas where the different mesh resolutions clearly deviate from each other. Mesh 1 deviates from Meshes 2 and 3. However, the deviation between Meshes 2 and 3 is relatively smaller. Based on these results, the decision was made to proceed with validation using the grid resolution of Mesh 2. Once the convergence study is complete, the results obtained using the optimal mesh are validated against the experimental data or other published numerical results. For validation purposes, the velocity profiles at the A-A and B-B diameters are compared to the data reported in Nicolaou and Zaki (2016) in Figure 4. The results shown in Figure 4 are for $Re = 1000$, $R_o = 5.6$ and $D = 0.01m$ using the grid resolution of Mesh 2. The shape of the profile shows relatively good agreement with the previously published results. There is some obvious deviation in certain areas of the pipe, which may have occurred due to the usage of a longer inlet pipe than the one used in the study of Nicolaou and Zaki (2016). The peak values of the profile near the pipe wall have been well captured.

![Figure 2](image1.png)

**Figure 2.** An example of the mesh at the cross section (left) at the outlet of the elbow part and the elbow part (right)

![Figure 3](image2.png)

**Figure 3.** Convergence study of three different mesh sizes. a) illustrates the velocity profile along the A-A diameter and b) the velocity profile along the B-B diameter in the streamwise direction
3. Results and discussion

In this section, the post-processed results from the simulations of the laminar flow inside the elbow pipes are presented. Twelve simulations were carried out for all combinations of Reynolds numbers and curvature ratios. The values of Reynolds numbers used in the present study are $Re = 200, 500, 1000$ and 2000. The curvature ratios $Ro = 5.6, 11.2$ and 22.4 are used for the flows at each of the Reynolds number values. Velocity contour plots have been created to illustrate the behaviour of the flows. The velocity contour plots shown in Figure 5 show the velocity magnitudes at a cross section of the symmetry plane, accompanied by circular cross sections at the 90-degree outlet of the elbow. The circular cross section velocity contour plots show the secondary fluid flow induced by the centrifugal force.

Figure 5 illustrates the contour plots of all combinations of $Re$ and $Ro$ investigated in the present study. The elbows are split along the symmetry plane (XY-plane) to reveal the contour plots in the middle section of the pipe. The contour shows a fully developed laminar flow within the inlet straight pipe. The flow velocity is 0 at the walls and gradually increases to a maximum value of approximately 2m/s at the middle, which is in accordance with the expected parabolic shape. It is observed that after the flow enters the elbow part, a separation of the flow occurs due to the curvature of the elbow. The velocity region towards the inner curve of the elbow is slower relative to the region towards the outer curve. This behaviour is resulted from the centrifugal force of the flow within the curved pipe. Furthermore, the flow separation is more obvious with the increasing $Re$ by a presence of an obvious shear layer, as shown in Figure 5 at $Re = 1000$ and 2000. From the elbow outlet cross sections (XZ-plane), a swirling pattern occurs. By increasing the curvature ratio, the intensity of the swirling is reduced. However, when the Reynolds number increases, the swirling becomes more significant. As the Dean number (Dean 1928) defined as $Re/\sqrt{Ro}$ increases, the swirling gets more significant. The opposite effect is observed when the Dean number decreases. Less significant swirling can be seen in the cases with lower Dean numbers compared to the cases with higher Dean numbers.

Figure 6 shows the pressure contours at $Re = 2000$. It clearly illustrates the lower pressure region generated by the curvature of the pipe. This pressure difference contributes to the separation of the high velocity flow close to the outer curve and lower velocity flow close to the inner curve as shown in Figure 5. At the upper part of the pipe a high-pressure region is created, which may be due to the blockage effect of the curved pipe wall to the flow. As $Ro$ increases, this pressure difference becomes smaller.
Figure 5. Contour plots of velocity magnitudes of the laminar flows at different Reynolds numbers within the middle of elbows of different curvature ratios. Circular cross sections at 90-degrees (outlet of elbow) are also shown for each case.

Figure 6. The pressure contour plots for $Re = 2000$ and (a) $R_o = 5.6$; (b) $R_o = 11.2$; (c) $R_o = 22.4$. 
The streamwise velocity profiles at the 90-degree cross section of the pipe have been plotted for comparison. Figure 7 illustrates the velocity profiles at diameter A-A in the streamwise direction (y-direction) at all investigated Reynolds numbers and curvature ratios. It is shown that the initially parabolic velocity profile along the inlet is deformed by the effects of the curved elbow. The degree of deformation is low at the lowest Reynolds number of Re = 200, compared to the highest Reynolds number of Re = 2000. It is also observed that a greater curvature ratio gives a less deformed shape of the initial parabolic shape, than a lower curvature ratio. There are velocity peaks close to the outer curve of the pipe. Figure 8 shows the streamwise velocity profiles extracted from the B-B diameter as shown in Figure 2. These profiles are symmetrical about the center line of the cross section, which is evidence of the two symmetric helical swirls occurring due to the centrifugal forces. It is clear that the swirling motion indicated by the peak values close to the wall is more significant at greater values of Re and lower values of \( R_o \).

**Figure 7.** The Streamwise velocity profiles along the A-A diameter as shown in Figure 2 at the 90 degree for different Reynolds numbers and curvature ratios
Figure 8. The streamwise velocity profiles along the B-B diameter as shown in Figure 2 for different Reynolds number and curvature ratios

4. Conclusion
In the present study, numerical investigations on laminar flow at different Reynolds numbers within the elbow part with varying curvature ratios have been performed. Mesh convergence study is conducted for different mesh resolutions to support the selection of the optimal mesh. The investigated Reynolds number of the laminar flow are in the range of 200 to 2000. Elbows with curvature ratios of 5.6, 11.2 and 22.4 are used for the simulations at each Reynolds number. The results from these numerical simulations provide predictions of the streamwise flow as well as the secondary flow induced by the centrifugal force of the curved pipe. The post-processed velocity contour plots at the cross sections at the outlet of the elbow part illustrate that the Dean vortices of the secondary flows are generated. It is shown that the intensity of the Dean vortices increases with the increasing Dean number. Furthermore, the pressure contour plots show a low-pressure region is generated towards the inner curve of the elbow, as well as a high-pressure region at the outer curve which to the blocking effects to the internal flow of the elbow part. The low-pressure region moves further downstream as the curvature ratio increases.

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