On the Interplay of Monopoles and Chiral Symmetry Breaking in Non-Compact Lattice QED

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Abstract

Non-compact lattice QED is simulated for various numbers of fermion species $N_f$ ranging from 8 through 40 by the exact Hybrid Monte Carlo algorithm. Over this range of $N_f$, chiral symmetry breaking is found to be strongly correlated with the effective monopoles in the theory. For $N_f$ between 8 and 16 the chiral symmetry breaking and monopole percolation transitions are second order and coincident. Assuming powerlaw critical behavior, the correlation length exponent for the chiral transition is identical to that of monopole percolation. This result supports the conjecture that monopole percolation “drives” the nontrivial chiral transition. For $N_f$ between 20 and 32, the monopoles experience a first order condensation transition coincident with a first order chiral transition. For $N_f$ as large as 40 both transitions are strongly suppressed. The data at large $N_f (N_f \gtrsim 20)$ is interpreted in terms of a strongly interacting monopole gas-liquid transition.
I. INTRODUCTION

The interplay of monopole and fermion dynamics has been studied both analytically and computationally in quantum field theory for some time. In the context of Grand Unified model building, the existence of monopole solutions of the field equations and the subsequent interactions of the monopole with the theory’s fermions has led to several interesting phenomena including the existence of exotic fermion condensates and the Callan-Rubakov effect [1]. The Dirac condition plays a crucial role in these discussions and in some cases it guarantees that qualitatively new, non-perturbative effects occur.

It is, therefore, of some interest when monopoles play a role in a broader context. In particular, various models studied in lattice gauge theory afford new glimpses into monopole physics, since the second quantized field theory of monopoles becomes accessible. A classic example is the confinement-deconfinement transition in abelian lattice gauge theory which is now understood to be driven by monopole condensation [2]. It came as some surprise, however, when effective monopoles were discovered in non-compact lattice QED [3], and the percolation transition of these objects was seen to be the same as four dimensional bond percolation [4]. The lattice itself allows such objects to have finite action, but one would expect naively that they would decouple in the theory’s continuum limit. But this depends on the scaling properties of the monopole percolation transition. When non-compact lattice QED is coupled to fermions in the traditional, explicitly gauge invariant fashion of Schwinger (through U(1) phases) one finds that the monopole percolation and the chiral symmetry transitions are coincident [5]. Even more tantalizing is the fact that the correlation length exponents for both transitions also coincide, so monopoles could survive the continuum limit of the chiral transition and the chiral transition itself may represent an interacting continuum field theory in four dimensions. Needless to say, it will prove extraordinarily hard to establish such a scenario purely numerically [6].
It is the purpose of this paper to elucidate the interplay of monopole and fermion dynamics in non-compact lattice QED by studying the system’s phase transitions as $N_f$, the number of fermion species, is varied. Since the fermion-monopole interaction strength is determined by the Dirac quantization condition, $N_f$ is the only natural variable available. We are not in a position to make a quantitative study of each theory’s continuum limit. All our results will be of a semi-quantitative or even qualitative sort. We think that they are interesting and have content, nonetheless. Our lattice sizes will be modest (typically $10^4$) as will our bare fermion masses (typically of order 0.05 and greater in lattice units). It turned out that finite size effects are particularly large when $N_f$ is large so smaller bare fermion masses cannot be sensibly studied on $10^4$ lattices. This fact will also force our chiral symmetry breaking studies to be done rather far from the chiral limit, and therefore they are not as quantitative as one might hope.

Since this paper continues a large body of work discussed in detail elsewhere, it will rely on definitions and formulas presented before [3,4,5]. The reader should consult those references for background. Quantities such as the chiral condensate $\langle \bar{\psi}\psi \rangle$, the monopole susceptibility $\chi$ and the monopole percolation order parameter $M$ should be familiar and will not be reviewed here. Equation of State, scaling laws and critical indices should also be familiar to the reader. Our notation will be the same as past publications.

We begin with an overview of our results. For $N_f$ equal 2 and 4 we will rely on past, more quantitative studies [3]. The $N_f = 8$ and 16 simulations were done on $10^4$ lattices, with selected simulations on $12^4, 14^4$ and $16^4$ lattices to check for finite size effects. Accurate studies of the chiral condensate showed that only bare fermion masses greater or equal to 0.05 (in lattice units) are free of finite size effects. This result implies that some past studies of lattice QED at large $N_f$ were not under quantitative control [7]. Data taken at bare fermion masses of .05, .06, .07, .08, .09 and .10, and at couplings ranging from $\beta = .21$ to .14 in steps of $\Delta \beta = .005$, are consistent with the hypothesis that there is a chiral transition at $\beta_c = .17(1)$ with powerlaw critical singularities. The critical indices for the chiral transition are consistent with those measured more precisely for the $N_f = 2$ and 4 theories previously.
Measurements of the monopole percolation observables also indicate a second order phase transition at essentially the same coupling, $\beta_m = .180(5)$, with critical indices characteristic of conventional four dimensional bond (or site) percolation. The coincidence of the chiral and monopole percolation transitions has been noted before in the $N_f = 2$ and 4 theories $[3]$. Even more intriguing than this is the fact that both transitions may share the same correlation length index $\nu$. Precise measurements of monopole percolation in four dimensions have strongly suggested the exact result $\nu = \frac{2}{3}$ $[4]$. Measurements of the chiral equation of state presented here for $N_f = 8$ and elsewhere for $N_f = 2$ and 4 give the critical indices $\delta = 2.2(1)$ and $\gamma = 1.0(1)$. If we assume that the critical point has powerlaw singularities with conventional properties, then the critical indices should satisfy the hyperscaling relations, and $\delta$ and $\gamma$ determine all of them. The hyperscaling relations read,

$$2 - \alpha = d\nu$$

$$2\beta_{mag}\delta - \gamma = d\nu$$

$$\beta_{mag} = \frac{\nu}{2}(d - 2 + \eta)$$

(1.1)

$$2\beta_{mag} + \gamma = d\nu$$

Then the values $\delta = 2.2$ and $\gamma = 1$ imply $\beta_{mag} = .83, \nu = .67, \alpha = -.67, \eta = .50$ and $\Delta \equiv \beta_{mag}\delta = 1.83$. The intriguing result of this exercise is that measurements of the chiral exponents and the hypothesis of hyperscaling predict that the correlation length exponents $\nu$ of both transitions are identical. This implies that the monopoles are relevant degrees of freedom at the chiral transition and since they scale identically, the monopoles should survive in the continuum limit of the chiral transition. It might be accurate to say that monopole percolation “drives” the chiral transition and the chiral transition defines an interacting, continuum field theory because it “inherits” the non-mean-field correlation length critical index $\nu = \frac{2}{3}$ from monopole percolation. We shall see in the text through analysis of our measurements of the order parameter $\langle \overline{\psi}\psi \rangle$ that this interpretation fits the computer simulation data very well. However, other hypotheses, such as the possibility that the chiral transition is described by a logarithmically trivial Nambu Jona-Lasinio model, might fit the
data adequately as well. It would require considerably more computer power to separate the
monopole picture of the transition from other possibilities just on the basis of numerical fits.
One reason for this difficulty is the fact that finite size effects grow large as \( N_f \) increases
and small bare fermion masses close to the chiral limit cannot be simulated on lattices of
practical proportions like \( 10^4 \) or even \( 16^4 \). For this reason the emphasis in this paper will be
different, although elsewhere the \( N_f = 2 \) and \( 4 \) models are being simulated on even larger
lattices with even better statistics.

We consider the \( N_f = 12, 16, 20, 24, 32 \) and \( 40 \) models here, and measure \( \langle \bar{\psi}\psi \rangle \) and
monopole observables to see if the correlation between these observables persist at all \( N_f \).
We shall see that while the character of the transitions change qualitatively as \( N_f \) increases,
the two transitions remain strongly correlated. To avoid finite size effects we were forced to
simulate a relatively large bare fermion mass \( m = 0.10 \) on \( 10^4 \) lattices. Therefore, many of
our conclusions are just qualitative. Luckily, qualitative changes were seen in the data as
\( N_f \) varied so the study remained useful. We found that as \( N_f \) was increased from \( 8 \) to \( 24 \),
the chiral and monopole percolation transitions both shifted to stronger critical couplings
but they remained coincident and apparently second order. However, at \( N_f \approx 24 \) and
\( m = 0.10 \) both transitions displayed jumps suggesting first order behavior. (The reader
should be careful not to overlook our caveats expressed above in these remarks—simulations
at smaller \( m \) and larger lattices are really necessary to make such statements.) At \( N_f = 24 \),
the chiral condensate \( \langle \bar{\psi}\psi \rangle \) and the monopole percolation order parameter \( M \) display
“discontinuous” jumps between couplings \( \beta = 0.08 \) and \( .085 \). Increasing \( N_f \) even further to
32 simply enhances the sizes of the apparent discontinuities. The fact that the character
of the transition for \( N_f \) between \( 8 \) and \( 24 \) is different from that for \( N_f \) between \( 24 \) and
40 is supported by measurements of the monopole concentration (density). For small \( N_f \)
where the monopole transition is percolation, the concentration of monopoles is expected to
be small and smooth through the transition. The simulations show this clearly. However,
our simulations at \( N_f = 24 \) and especially \( N_f = 32 \) show that the concentration jumps
“discontinuously” at the chiral transition, strongly suggesting a first order transition between
a dilute “gaseous state” of monopoles and a fairly dense “liquid” state. For example, at
\( N_f = 32 \) the monopole concentration jumps from 0.10 (approx.) at \( \beta = 0.055 \) to 0.34 at \( \beta = 0.050 \). Since the transition shows up in the monopole concentration it is accurate to call it a “condensation” transition. Since \( c = 0.34 \) is a substantial density, it is very tantalizing to view the transition as a first order gas-liquid transition. Apparently increasing \( N_f \) effects the monopole core energy and/or the monopole-monopole interactions and thereby induces a gas-liquid transition. It would be interesting to complement the computer results with some analytic calculations.

Of course this physical picture of the transition needs further substantiation. One element of it that we could test here was the expectation that if fermion-induced forces were effecting the monopole dynamics and leading to a gas-liquid transition, then if \( N_f \) were taken truly large free monopoles would never appear in the system. In fact, we confirmed this point at \( N_f = 40 \). The simulation showed that the monopole concentration and the monopole percolation susceptibility and the monopole percolation order parameter all remained strongly suppressed and flat as the coupling varied. The chiral condensate was similarly suppressed. However, the plaquette showed strong dependence on the coupling \( \beta \) suggesting that a transition remains in the model with a divergent specific heat, but it is unrelated to the monopole or chiral properties of the model. The small values of \( M \) and \( c \) indicate that the monopoles remain bound in tight pairs for all coupling \( \beta \). Under these circumstances one would not expect them to induce chiral symmetry breaking and our simulations are consistent with that fact.

The phase diagram (\( N_f \) vs. \( \beta \)) that we are advocating here agrees qualitatively with that of Azcoiti and collaborators [8]. Their work emphasizes the theory’s specific heat, while ours emphasizes monopole and chiral dynamics. We believe that they are two views of the same physics and the qualitative features we are interested in and can deal with fairly reliably are identical.

The body of this paper is organized as follows. In Sec. 2 the \( N_f = 8 \) theory is discussed in detail. In Sec. 3 we turn to the \( N_f = 12, 16, 20 \) and 24 data, and show that the \( N_f = 24 \)
data displays monopole condensation. In Sec. 4 we turn to the $N_f = 32$ and 40 data which shows that for truly large $N_f$ the monopole and chiral activities in the theory are strongly suppressed.

II. THE $N_f = 8$ SIMULATION

We used our Hybrid Monte Carlo code for non-compact QED to explore the eight flavor, $N_f = 8$, model just as we studied the $N_f = 4$ case more quantitatively in an earlier publication. The reader should consult our extensive $N_f = 2$ and 4 studies for details of the algorithm and the definitions of various chiral and monopole observables [3,4,5]. Since this paper is looking for qualitative trends and is a contribution in a long series we will not repeat formulas, definitions and past observations. Rather, our emphasis will be on results, plots and an emerging physical picture.

To gain some understanding of the chiral transition at $m = 0.0$ we measured $\langle \bar{\psi} \psi \rangle$ for bare fermion masses ranging from $0.03$ to $0.10$ and couplings $\beta = 1/e^2$ ranging from $0.20$ to $0.14$. The data is shown in Table 1. Several hundred trajectories of the Hybrid Monte Carlo code were required to achieve the statistical accuracy indicated in the table. Since the lattice size is relatively small, $10^4$, we must be careful about finite size effects in the data, especially at small values of $m$. Therefore, we did limited simulations on $12^4, 14^4$ and $16^4$ lattices. The data is shown in Tables 2, 3 and 4. Comparing Tables 1 and 2 we see evidence on the weak coupling side of the chiral transition, $\beta = 0.20 - 0.18$, for numerically significant finite size effects for the lowest fermion mass, $m = 0.03$. Over this range of parameters, $\langle \bar{\psi} \psi \rangle$ is relatively suppressed on the smaller lattice, which is the expected finite size/finite temperature effect. However, comparing Tables 1-4 we see that the finite size effects are within our statistical error bars at $m = 0.05$, except perhaps at the weakest coupling $\beta = 0.200$. Therefore, in the analysis that follows only the $10^4$ data for $m$ ranging from $0.05$ to $0.10$ will be used.

We will assume that the chiral transition is well-described by a second order phase transition with powerlaw singularities. Other hypotheses could be tried here and some
would probably be fairly successful since our data covers only relatively large \( m \) values and because every fitting hypothesis is accompanied by several free parameters. We will pursue the powerlaw hypothesis here because it is simple and because it is definitely appropriate for monopole percolation [4]. Given this, the data should satisfy the equation of state (EOS),

\[
\frac{\langle \psi \psi \rangle}{m^{\delta}} = f\left(\frac{\Delta \beta}{\langle \psi \psi \rangle}^{\frac{1}{\beta_{mag}}}\right)
\]  

(2.1)

where \( \delta \) and \( \beta_{mag} \) are the usual critical indices, \( \Delta \beta = \beta_c - \beta \), and this form of the EOS has been used extensively elsewhere. Eq. (2.1) simplifies at the critical point and reduces to the scaling law of the order parameter \( \langle \psi \psi \rangle \) as the symmetry breaking field \( m \) is turned on,

\[
\langle \psi \psi \rangle = A m^{\frac{1}{\delta}}, \beta = \beta_c
\]  

(2.2)

We found that Eq.(2.2) is a particularly effective way to determine \( \delta \) and \( \beta_c \) which then can be used in the EOS to find the universal scaling function \( f \) and the critical index \( \beta_{mag} \) away from the transition. In Fig. 1 we plot \(-1/\ln(\langle \psi \psi \rangle)\) vs. \(-1/\ln(m)\) for the data of Table 1 (\( m \geq 0.05 \)). At \( \beta = \beta_c \) these lines should be straight with the slope \( \frac{1}{\delta} \) and should pass through the origin. We see that this hypothesis works well at \( \beta_c = .17 \) in Fig. 1 for \( m = .05 - .10 \). The lower masses are subject to finite size effects, as discussed above, and must be discarded. The slope of the \( \beta_c = .17 \) line in Fig. 1 gives \( \delta = 2.2(1) \). Powerlaw fits of the \( \beta = .17 \) data to Eq. (2.2) are excellent indicating that the numerical evidence for the powerlaw hypothesis is perfectly consistent with the numerical data.

We note that the result \( \delta = 2.2(1) \) is consistent with the results found at \( N_f = 2 \) and 4, assuming powerlaw singularities at the critical point. Those data also gave the susceptibility index \( \gamma = 1.0(1) \) which by the hyperscaling relation \( \beta_{mag} = \gamma/(\delta - 1) \) predicted \( \beta_{mag} = .83(7) \), the magnetic critical index. This motivated us to try the EOS Eq. (2.1) with \( \beta_{mag} = .83 \). The result is shown in Fig. 2. The data fall on a universal scaling function \( f \) rather well, although the quality of the data and the resulting universal curve are not comparable to our \( N_f = 2 \) and 4 results which came from larger lattices and smaller values of \( m \). However, if we compare the EOS for \( N_f = 8 \) in Fig. 2 to the analogous figures in the \( N_f = \)
2 and 4 publications, we see that even the universal function \( f \) as well as the critical indices \( \delta \) and \( \beta_{\text{mag}} \) are consistent with their independence of \( N_f \). One interpretation of this result is that monopole percolation drives each transition, as discussed in the Introduction above, and fermion feedback does not effect the percolation critical behavior as long as \( N_f \) is not too large. More evidence for this scenario will be presented below when monopole observables are presented and analyzed. There is no doubt, however, that other more mundane explanations of these systematics could be presented. For example, it could be that all the \( N_f \neq 0 \) theories are logarithmically trivial and have the scaling properties of Nambu Jona-Lasinio (NJL) models. If this hypothesis is true, the reason for the deviation of \( \delta \) and \( \beta_{\text{mag}} \) from their mean-field values of 3 and 1/2 is the presence of scale-breaking logarithms in the NJL equation of state. The limited accuracy of our \( N_f = 8 \) makes it pointless to pursue alternative fits here–given a few new parameters as would occur in NJL fits, this could certainly be done. Rather we shall investigate just qualitative features of the models with higher \( N_f \) and accumulate additional evidence for strong correlations between the chiral and monopole activities in each model. This will then provide “supporting, circumstantial evidence” for the monopole-driven-chiral transition physical picture we are developing.

Next we accumulated monopole percolation data for the \( N_f = 8 \) theory on a \( 10^4 \) lattice at various \( m \). The data for \( \chi \), the monopole susceptibility, and \( M \), the monopole percolation order parameter, are presented in Table 5. We see that the peak of the susceptibility \( \chi \) occurs at a coupling between .175 and .180 for \( m = .03 \), and it moves to slightly weaker coupling, .19, as \( m \) increases to .10. Our estimate of \( \beta_c = .17 \) for the chiral transition refers, of course, to the \( m = 0 \) chiral limit. So, within uncertainties due to finite size effects, the chiral and monopole percolation transitions are coincident, as we found with better numerical control for \( N_f = 2 \) and 4. It is important to determine if the peaks in \( \chi \) on the \( 10^4 \) lattice are indicative of a real transition. To obtain some evidence for this result and to measure some critical indices, we repeated the measurements summarized in Table 5 on \( 10^4, 12^4, 14^4 \) and \( 16^4 \) lattices at \( m = .05 \). The data is given in Table 6. We see that the peaks grow with \( L \). According to finite size scaling, the peak heights should grow as,
\( \chi_{\text{max}}(L) \sim L^{\gamma_{\text{mon}}/\nu_{\text{mon}}} \) (2.3)

where \( \gamma_{\text{mon}} \) and \( \nu_{\text{mon}} \) are the susceptibility and correlation length exponents for the monopole transition. In addition, the order parameter at the coupling \( \beta_{\text{max}} \) where \( \chi \) peaks for each \( L \) should scale to zero as,

\[ M(\beta_{\text{max}}(L)) \sim L^{-\beta_{\text{mon}}/\nu_{\text{mon}}} \] (2.4)

We test these scaling predictions in Fig. 3 and find that the data supports powerlaw scaling with the indices,

\[ \frac{\gamma_{\text{mon}}}{\nu_{\text{mon}}} = 2.25(3) \]
\[ \frac{\beta_{\text{mon}}}{\nu_{\text{mon}}} = 0.875(80) \] (2.5)

These are exactly the critical indices of ordinary four dimensional percolation. Four dimensional percolation indices satisfy hyperscaling relations and Eq. (2.5) then predicts \( \nu_{\text{mon}} = \frac{2}{3} \). This is the correlation length scaling index discussed in the Introduction. Its coincidence with the correlation length exponent of the chiral transition is crucial to the monopole-percolation-driven-chiral-transition physical picture.

In summary, our \( 10^4 \) numerical results are compatible with the idea that the \( N_f = 8 \) chiral transition is physically indistinguishable from the \( N_f = 2 \) and 4 chiral transitions. If we assume powerlaw critical singularities, then the physical picture of monopole percolation driving the chiral transition is also defensible because the couplings of the transitions coincide as do their correlation length indices.

**III. MONOPOLE CONDENSATION AT \( N_f = 24 \)**

We next increased \( N_f \) in our Hybrid Monte Carlo code and simulated the \( N_f = 12, 16, 20 \) and 24 models on \( 10^4 \) lattices with \( m = 0.10 \). A relatively large bare fermion mass was chosen to control finite size effects. The relatively large symmetry-breaking field will smooth out the chiral transition and make quantitative investigations impossible. However,
qualitative changes in the dynamics of the model will be seen. The reader should understand, however, that we cannot predict the precise $N_f$ values where qualitative changes occur. More simulations at smaller bare fermion masses on larger lattices will be needed for that.

The simulation data for the average plaquette $P$, the chiral condensate $\langle \bar{\psi}\psi \rangle$ and the monopole percolation order parameter $M$ are shown in Fig. 4 for $N_f = 12$ and 16. The transition region between small $\langle \bar{\psi}\psi \rangle$ (or $M$), and large $\langle \bar{\psi}\psi \rangle$ (or $M$) shifts toward stronger coupling and the transition sharpens, somewhat. The shift toward stronger coupling is a consequence of dynamical fermion screening and has been seen in many contexts before. In Fig. 5, we show the same plots for $N_f = 20$ and 24. Now there are suggestions that for each $N_f$ the order parameters $\langle \bar{\psi}\psi \rangle$ and $M$ jump at the same coupling from small to larger values. This is particularly persuasive for $N_f = 24$ where we see signs of discontinuities at $\beta = 0.105(5)$. Perhaps this qualitative effect is more visual in Fig. 6 where the $N_f = 24$ and $N_f = 8$ data for $m = 0.10$ are plotted and we have added the monopole concentration (density) “c” to the list of observables. The chiral condensate, monopole concentration and average plaquette each appear to jump for $\beta = 0.0825(25)$. Certainly for strong coupling, $\beta < 0.0825$, their slopes are much greater than their slopes at weak coupling, $\beta > 0.0825$. By contrast, the same set of observables are smooth in the plot of the $N_f = 8$ data. Of course, there is a transition in the $N_f = 8$ data, but it does not show up clearly at relatively large values of $m$, except in the monopole percolation observables $M$ and $\chi$. In fact, we plot the monopole percolation susceptibilities $\chi$ for the $N_f = 8$ and 24 theories at $m = 0.10$ in Fig. 7. Strong peaks are seen for both $N_f$ values with the width of the $N_f = 24$ peak considerably reduced, again indicating the relative sharpness of the $N_f = 24$ transitions.

Perhaps the clearest indication that the dynamics of the $N_f = 24$ model is qualitatively different from the $N_f = 8$ case comes from the monopole concentration. As seen in Tables 7 and 8, of $N_f = 8$ and $N_f = 24$ data at $m = 0.10$ on $10^4$ lattices, the monopole concentration “jumps” in the $N_f = 24$ case while it is perfectly smooth through the percolation transition in the $N_f = 8$ case. This hints at the fact that the monopoles are condensing in the
$N_f = 24$ theory and the ground state for $\beta < .0825(25)$ is a monopole condensate, perhaps resembling the strong coupling, confining vacuum of the compact $U(1)$ lattice QED model. Since the monopole concentration is small for $\beta > .0825(25)$ and jumps to a distinctly larger value for $\beta < .0825(25)$, we may be seeing signs of a first-order gas-liquid monopole condensation transition. The monopole activation energy is proportional to $\frac{1}{\beta} = \beta$ and it is relatively small here compared to the small $N_f$ models. As the coupling is increased through $0.0825(25)$ a first order monopole condensation transition into a monopole liquid is triggered where a relatively dense monopole ensemble is produced. It would be interesting to study the monopole dynamics through correlation functions in this condensed state and compare them to similar simulations in pure compact QED.

IV. MONOPOLE AND CHIRAL SUPPRESSION AT $N_f = 40$

In this survey of $N_f$, we next turned to the $N_f = 32$ model. The data is presented in Table 9 (for $m = 0.10$ and $10^4$ lattices, as usual) and it is plotted in Fig. 8. Jumps are seen in all observables for $N_f = 32$ at a coupling $\beta = .05125(125)$. On the strong coupling side of the transition, $M$, $\langle \bar{\psi}\psi \rangle$ and $c$ are saturated. The average plaquette has also jumped at $\beta = .05125(125)$, and is growing rapidly in the strong coupling phase. A first-order monopole condensation transition is very apparent.

We finally increased $N_f$ to 40 in order to see the effects of extreme fermion screening. Table 10 and Fig. 8 resulted—the monopole and chiral observables are almost completely suppressed! Throughout the entire range of couplings $\langle \bar{\psi}\psi \rangle$ remains near its weak coupling value. Both of the percolation observables, $\chi$ and $M$, are strongly suppressed and are slightly smaller at $\beta = .01$ than at $\beta = .02$. The average plaquette $P$ rapidly increases over this range of $\beta$, however, probably indicative of a persistent specific heat anomaly, as discussed more quantitatively by Azcoiti and [5]. Our interest in this result is again the strong correlation between the monopole and chiral observables. The fact that they are both deeply suppressed, even while the average plaquette indicates considerable "disorder"
in the ground state, is supportive of the physical picture which contends that the effective monopoles are essential in the model’s chiral dynamics at all $N_f$.

V. CONCLUDING REMARKS

In this survey of $N_f$ we have found that chiral and monopole dynamics are strongly correlated in every case.

A. Small $N_f$

The monopole transition is a second order percolation transition without condensation. If the chiral transition is assumed to be characterized by powerlaw singularities, satisfying hyperscaling, then it was coincident with monopole percolation and the correlation length indices of the two transitions were identical.

B. Intermediate $N_f$

The monopole transition becomes a first order condensation phenomenon. The chiral transition is coincident and also first order.

C. Large $N_f$

The monopole and chiral observables are strongly suppressed, and there are no transitions in these quantities. The average plaquette, is rapidly varying as a function of coupling nonetheless.

In summary, it may be worthwhile to pursue some aspects of the dynamics found here in more detail. The nature of the chiral transition for small $N_f$ is a primary goal, since it may define an interacting field theory which is strongly coupled at short distances. The nature of the field theory and the role of effective monopoles in it would be interesting to
understand. The monopole condensate at intermediate $N_f$ and its “liquid” properties would be interesting to clarify through correlation functions.
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VI. TABLE CAPTIONS

1. Chiral Condensate data for $N_f = 8$ theory on $10^4$ lattice at various couplings $\beta$ and bare fermion masses $m$.

2. Same as Table 1 except on a $12^4$ lattice.

3. Same as Table 1 except on a $14^4$ lattice.

4. Same as Table 1 except on a $16^4$ lattice.

5. Same as Table 1 except for monopole susceptibility $\chi$ and order parameter $M$.

6. Monopole percolation measurements in $N_f = 8$ theory at $m = .05$ on $L^4$ lattice for $L = 10, 12, 14$ and 16.

7. $N_f = 8$ data for $m = 0.10$ on a $10^4$ lattice.

8. Same as Table 7 except $N_f = 24$.

9. Same as Table 7 except $N_f = 32$. 
10. Same as Table 7 except $N_f = 40$. 
TABLE I. Chiral Condensate, $N_f = 8$, $L = 10$

| $\beta/m$ | .03         | .04         | .05         | .06         | .07         | .08         | .09         | .10         |
|-----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| .200      | .1066(5)    | .1396(7)    | .1677(7)    | .1947(6)    | .2148(7)    | .2368(8)    | .2545(7)    | .2739(7)    |
| .195      | .1139(7)    | .1468(9)    | .1778(8)    | .2056(7)    | .2270(8)    | .249(1)     | .269(1)     | .2835(7)    |
| .19       | .1246(6)    | .160(1)     | .1903(8)    | .2160(7)    | .2388(8)    | .260(1)     | .280(1)     | .2958(7)    |
| .185      | .1370(7)    | .173(1)     | .2036(8)    | .2290(8)    | .252(1)     | .274(1)     | .293(1)     | .3098(8)    |
| .18       | .147(1)     | .188(1)     | .2179(8)    | .247(1)     | .268(1)     | .288(1)     | .305(1)     | .3232(8)    |
| .175      | .167(1)     | .209(1)     | .236(1)     | .261(1)     | .286(1)     | .304(1)     | .319(1)     | .3364(9)    |
| .17       | .181(1)     | .219(1)     | .256(1)     | .279(1)     | .302(1)     | .321(1)     | .335(1)     | .352(1)     |
| .165      | .203(1)     | .242(1)     | .274(1)     | .300(1)     | .322(1)     | .335(1)     | .353(1)     | .369(1)     |
| .16       | .232(1)     | .265(1)     | .297(1)     | .320(1)     | .340(1)     | .358(1)     | .371(1)     | .382(1)     |
| .155      | .262(1)     | .293(1)     | .323(1)     | .341(1)     | .360(1)     | .377(1)     | .386(1)     | .399(1)     |
| .15       | .291(2)     | .319(1)     | .344(1)     | .364(1)     | .380(1)     | .394(1)     | .407(1)     | .417(1)     |
| .145      | .324(2)     | .349(1)     | .371(1)     | .386(1)     | .405(1)     | .416(1)     | .425(1)     | .434(1)     |
| .14       | .355(2)     | .377(1)     | .398(1)     | .412(1)     | .424(1)     | .435(1)     | .444(1)     | .452(1)     |
### TABLE II. Chiral Condensate, $N_f = 8, L = 12$

| $\beta/m$ | .03     | .05     |
|-----------|---------|---------|
| .200      | .1123(5)| .1723(8)|
| .190      | .1280(8)| .1916(8)|
| .185      | .1388(7)| .2030(6)|
| .180      | .1516(7)| .2185(6)|
| .175      | .1657(7)| .2344(8)|
| .170      | .1824(8)| .2520(7)|
| .160      | .229(1) | .2949(9)|
| .150      | .289(1) | .345(1) |
| .140      | .355(1) | .396(1) |

### TABLE III. Chiral Condensate, $N_f = 8, L = 14$

| $\beta/m$ | .05     |
|-----------|---------|
| .185      | .2032(4)|
| .180      | .2188(4)|
| .170      | .2356(5)|

### TABLE IV. Chiral Condensate, $N_f = 8, L = 16$

| $\beta/m$ | .05     |
|-----------|---------|
| .185      | .2043(3)|
| .180      | .2184(3)|
| .170      | .2340(4)|
TABLE V. Monopole Data, $N_f = 8, L = 10$

| $\beta$ | $m = 0.03$ | $m = 0.04$ | $m = 0.05$ | $m = 0.06$ | $m = 0.10$ |
|---------|------------|------------|------------|------------|------------|
|        | $\chi$     | $M$        | $\chi$     | $M$        | $\chi$     | $M$        | $\chi$     | $M$        | $\chi$     | $M$        |
| .20    | 19.75(16)  | .041(1)    | 21.98(22)  | .045(1)    | 23.45(32)  | .051(1)    | 25.15(27)  | .053(1)    | 34.69(56)  | .088(2)    |
| .195   | 24.1(2)    | .050(1)    | 26.48(30)  | .055(1)    | 28.55(30)  | .060(1)    | 30.89(46)  | .078(2)    | 45.4(9)    | .125(3)    |
| .19    | 30.9(4)    | .071(1)    | 34.00(50)  | .076(2)    | 36.44(52)  | .084(2)    | 40.35(61)  | .099(3)    | 51.6(1.5)  | .221(6)    |
| .185   | 39.7(7)    | .106(3)    | 41.58(74)  | .123(3)    | 48.4(97)   | .137(4)    | 48.11(111) | .180(5)    | 37.7(1.97) | .383(6)    |
| .18    | 48.5(9)    | .152(4)    | 55.53(14)  | .180(4)    | 49.6(1.8)  | .266(6)    | 45.6(2.2)  | .330(6)    | 19.94(1.6) | .535(5)    |
| .175   | 46.0(1.8)  | .298(6)    | 42.3(1.9)  | .351(6)    | 31.11(1.99)| .434(6)    | 27.18(1.7) | .473(6)    | 8.31(98)   | .652(3)    |
| .17    | 28.1(2.3)  | .457(6)    | 18.6(1.4)  | .519(5)    | 11.12(66)  | .586(3)    | 9.04(74)   | .623(3)    | 3.70(10)   | .747(2)    |
| .165   | 13.8(1.1)  | .579(4)    | 7.67(52)   | .649(3)    | 5.45(17)   | .692(2)    | 4.08(12)   | .727(2)    | 2.11(3)    | .810(1)    |
| .16    | 5.19(17)   | .709(2)    | 3.99(11)   | .736(2)    | 2.94(6)    | .772(2)    | 2.28(3)    | .798(1)    | 1.33(2)    | .859(1)    |
| .155   | 2.48(3)    | .789(1)    | 2.09(3)    | .809(1)    | 1.58(2)    | .837(1)    | 1.32(1)    | .855(1)    | .078(1)    | .899(1)    |
| .15    | 1.46(2)    | .848(1)    | 1.19(2)    | .867(1)    | 0.95(1)    | .883(1)    | 0.81(1)    | .895(1)    | 0.50(1)    | .927(6)    |
| .145   | 0.82(1)    | .897(1)    | 0.69(1)    | .908(1)    | 0.55(1)    | .920(1)    | 0.032(1)   | .948(4)    |             |             |
| .14    | 0.50(1)    | .927(1)    | 0.42(1)    | .937(1)    | 0.32(1)    | .946(1)    | 0.21(1)    | .963(3)    |             |             |

TABLE VI. Monopole Observable Scaling, $N_f = 8$

| $\beta/L$ | 10   | 12   | 14   | 16   |
|-----------|------|------|------|------|
| $\chi$    | $M$  | $\chi$ | $M$  | $\chi$ | $M$  | $\chi$ | $M$  |
| .20       | 25.8(3) | .0267(2) |      |       |       |       |       |
| .19       | 48.0(6)  | .058(1)  |      |       |       |       |       |
| .185      | 66(1)   | .105(3)  | 83(1) | .076(2) | 104(2) | .061(1) |       |
| .18       | 50(2)   | .266(6)  | 75(3) | .228(5) | 107(3) | .197(4) | 142(4) | .180(3) |
| .173      | 42(3)   | .409(4)  | 40(3) | .415(4) | 40(2)  | .412(2) |       |       |
| .17       | 11.9(4) | .575(2)  |      |       |       |       |       |       |
| .16       | 2.89(4) | .773(1)  |      |       |       |       |       |       |
| .15       | .937(8) | .886(1)  |      |       |       |       |       |       |
| .14       | .359(4) | .945(1)  |      |       |       |       |       |       |
| $\beta$ | $P$   | $\langle \bar{\psi} \psi \rangle$ | $\chi$ | $M$   | $c$   |
|-------|-------|----------------------------------|-------|-------|-------|
| .21   | .961(2) | .2550(8)                        | 21.9(2) | .046(1) | .1104(2) |
| .205  | .972(2) | .2643(8)                        | 27.0(3) | .060(1) | .1093(2) |
| .20   | .983(2) | .2739(7)                        | 34.7(6) | .088(2) | .1163(2) |
| .195  | .993(2) | .2835(7)                        | 45.4(9) | .125(3) | .1232(2) |
| .19   | 1.005(2) | .2958(7)                        | 52(2)   | .221(6) | .1309(2) |
| .185  | 1.023(2) | .3098(7)                        | 38(2)   | .383(6) | .1396(3) |
| .18   | 1.040(2) | .3232(8)                        | 20(2)   | .535(5) | .1482(2) |
| .175  | 1.110(2) | .3364(9)                        | 8.3(9)  | .652(3) | .1573(2) |
| .17   | 1.151(2) | .352(1)                         | 3.7(1)  | .747(2) | .1682(2) |
| .165  | 1.180(2) | .369(1)                         | 2.11(3) | .810(1) | .1791(2) |
| .16   | 1.196(2) | .382(1)                         | 1.33(2) | .859(1) | .1900(2) |
| .155  | 1.271(2) | .399(1)                         | .78(1)  | .899(1) | .2033(2) |
| .15   | 1.310(2) | .417(1)                         | .50(1)  | .9270(6) | .2163(2) |
| .145  | 1.372(2) | .434(1)                         | .32(1)  | .9483(4) | .2299(2) |
| .14   | 1.464(2) | .452(1)                         | .21(1)  | .9639(3) | .2451(2) |
### TABLE VIII. $N_f = 24$ Data

| $\beta$ | $P$    | $\langle \bar{\psi}\psi \rangle$ | $\chi$  | $M$    | $c$    |
|---------|--------|----------------------------------|---------|--------|--------|
| .12     | .799(1)| .183(1)                          | 11.2(1) | .019(1)| .0730(2)|
| .11     | .845(1)| .193(1)                          | 14.8(2) | .029(2)| .0847(2)|
| .10     | .915(1)| .211(1)                          | 25.1(3) | .053(2)| .1002(2)|
| .095    | .955(1)| .222(1)                          | 34.2(7)| .088(3)| .1089(2)|
| .09     | 1.019(1)| .238(1)                       | 50(2)   | .197(7)| .1211(3)|
| .085    | 1.097(1)| .258(1)                       | 26(3)   | .476(7)| .1356(3)|
| .08     | 1.336(2)| .329(1)                       | 1.36(3) | .853(2)| .1794(5)|
| .07     | 2.77(1) | .516(2)                          | .0076(5)| .9983(2)| .3354(8)|

### TABLE IX. $N_f = 32$ Data

| $\beta$ | $P$    | $\langle \bar{\psi}\psi \rangle$ | $\chi$  | $M$    | $c$    |
|---------|--------|----------------------------------|---------|--------|--------|
| .09     | .7366(6)| .1524(4)                          | 8.29(4) | .0024(6)| .052(1)|
| .08     | .8100(7)| .1586(4)                          | 9.68(5) | .0102(9)| .061(1)|
| .07     | .934(1) | .1687(5)                          | 11.90(8)| .0218(9)| .073(1)|
| .06     | 1.276(2)| .1867(6)                          | 20.2(3) | .045(1) | .092(1)|
| .0575   | 1.509(2)| .1954(9)                          | 29.2(9) | .061(4) | .100(1)|
| .055    | 1.692(1)| .2070(7)                          | 41.7(9) | .146(6) | .110(1)|
| .0525   | 2.151(1)| .2339(7)                          | 17.4(9) | .509(6) | .130(1)|
| .05     | 4.639(4)| .509(1)                           | .0048(2)| .9989(1)| .335(1)|
| .045    | 5.315(4)| .516(2)                           | .0047(2)| .9989(1)| .342(1)|
| .04     | 6.127(2)| .513(2)                           | .0048(2)| .9989(1)| .343(1)|
| .03     | 8.305(5)| .517(1)                           | .0045(2)| .9989(1)| .344(1)|

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| $\beta$ | $P$    | $\langle \psi \psi \rangle$ | $\chi$  | $m$    | $c$    |
|-------|--------|----------------------------|--------|--------|--------|
| .05   | 3.042(4)| .1522(5)                   | 10.6(1)| .012(1)| .063(1)|
| .04   | 4.275(4)| .1567(4)                   | 11.9(1)| .024(1)| .069(1)|
| .03   | 7.648(5)| .1672(4)                   | 15.2(2)| .036(1)| .075(1)|
| .02   | 11.956(6)| .1686(5)                  | 15.9(2)| .037(1)| .089(1)|
| .01   | 23.207(7)| .1692(5)                  | 15.5(2)| .034(1)| .111(2)|

TABLE X. $N_f = 40$ Data
VII. FIGURE CAPTIONS

1. $-1/\ln(m) \text{ vs. } -1/\ln(\bar{\psi}\psi)$ plot showing critical behaviour at $\beta = .17$.

2. Equation of State for $N_f = 8$ theory.

3. Scaling plots, Eq. (2.3) and (2.4), of monopole percolation quantities for the $N_f = 8$ theory.

4. Chiral condensate $\langle \bar{\psi}\psi \rangle$, monopole percolation order parameter $M$ and average plaquette $P$ for $N_f = 12$ and 16 theories.

5. Same as Fig. 4 except $N_f = 20$ and 24.

6. Same as Fig. 4 except $N_f = 8$ and 24, but the monopole concentration $c$ is shown as well.

7. Monopole percolation susceptibility plots for $N_f = 8$ and 24.

8. Same as Fig. 6 except $N_f = 32$ and 40.
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