Resonant transport through midgap states in voltage-biased Josephson junctions of $d$-wave superconductors

Tomas Löfwander, Göran Johansson, Vitaly Shumeiko, Göran Wendin, and Magnus Hurd

Chalmers University of Technology and Göteborg University, S-412 96 Göteborg, Sweden

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We study theoretically the ac Josephson effect in voltage biased planar junctions of $d$-wave superconductors. For some orientations of the superconductors a current peak is found at finite voltage in the current-voltage characteristics. We pick out the relevant physical processes and write down an analytical formula for the current which clearly shows how the midgap state acts as a resonance and produces the peak. We present a possible explanation for the zero-bias conductance peak, recently found in experiments on grain boundary junctions of high-temperature superconductors, in terms of resonant transmission through midgap state of quasiparticles undergoing multiple Andreev reflections. We note that within our framework the zero-bias conductance peak appears in rather transparent Josephson junctions of $d$-wave superconductors.

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1. Introduction

The controversy regarding the symmetry of the order parameter in high-temperature superconductors (HTS) seems today, to some extent, have been resolved in favour of the $d_{x^2−y^2}$-wave ($d$-wave) symmetry [1, 2, 3]. Since current transport through superconducting junctions is sensitive to both magnitude and phase of the order parameter, experiments on Josephson junctions can measure the intrinsic phase of the $d$-wave order parameter. Therefore, a great deal of attention has been directed towards the Josephson effect in HTS junctions. Both the dc [4, 5, 6, 7] and ac [8, 9, 10, 11, 12] effects have been studied.

An important discovery was that at surfaces and interfaces of $d$-wave superconductors a midgap state (MGS) may be formed [8], which affects the current transport. A quasiparticle reflected at the interface will sense different order parameters before and after the scattering event since the momentum is changed. When there is a sign difference between the order parameters before and after scattering, a zero-energy bound state is formed (MGS). In normal metal-$d$-wave superconductor (N|$d$) junctions the MGS gives rise to a zero-bias conductance peak (ZBCP), established both theoretically [13, 14, 15] and experimentally [16, 17, 18, 19]. Previously, it has been shown that MGS gives rise to a current peak at finite voltage in both $s$-wave superconductor-$d$-wave superconductor ($s|d$) and $d$-wave superconductor-$d$-wave superconductor ($d|d$) junctions, resulting in negative
Fig. 1. The model system we are considering. The orientations of the two $d$-wave superconductors are given by the angles $\alpha_1$ and $\alpha_2$. An electron-like quasiparticle is incident on the junction at the angle $\theta$. We have here introduced short normal regions on each side of the barrier. The strength of the barrier can be tuned from the ballistic to the insulating case.

It is not straightforward to predict the effects in the $s|d$ and $d|d$ junctions. This is the case because the mechanisms involved in the current transport are more complicated when both electrodes are superconducting. In a voltage biased junction between two superconductors, the phase difference over the junction is time-dependent according to the Josephson relation $\dot{\varphi} = \frac{2eV}{\hbar}$. This makes the scattering in the junction inelastic: a quasiparticle incident on the junction at energy $E$ produces a scattering state with sideband energies $E_n = E + n\hbar\omega$, where $V$ is the voltage over the junction and $n$ is an integer.

The knowledge about current transport through voltage biased $s$-wave $s|s$ junctions has greatly increased in recent years. Using a Landauer-Büttiker scattering method (extended to include superconducting electrodes) it has become possible to understand $s|s$ junctions with arbitrary transparency of the insulator in terms of multiple Andreev reflections. The theory has also been extended to include $d$-wave superconductors. This theory is used in the present paper in order to study the effects of MGS in voltage biased superconducting junctions.

The rest of the paper is organized in the following way. In Chapter 2 we briefly describe the model and how to calculate the current-voltage (IV) characteristics. In Chapter 3, the scattering problem is solved analytically taking into account only processes giving rise to the current peak. We study the $s|d$ junction (where the peak is most pronounced) and compare the current contribution from the processes responsible for the current peak with the complete (numerically calculated) IV-curve. In Chapter 4 we present a qualitative discussion of how a ZBCP may appear in voltage biased $d|d$ junctions. This study was motivated by recent experiments. In Chapter 5 we summarize the paper.

2. The model

The system we are considering is a planar junction between $d$-wave superconductors as shown in Fig. 1. The superconducting gap function depends on the quasiparticle’s direction of propagation $\theta$. The differential conductance. The main purpose of this paper is to analyze the results of Refs. [8] and [10] in terms of the physical processes giving rise to the peak at finite voltage.
We assume that the superconductors have pure \(d\)-wave symmetry and let \(\Delta(\theta) = \Delta_0 \cos[2(\theta - \alpha)]\), where \(\alpha\) is the orientation of the superconductor relative to the interface normal. For an \(s\)-wave superconductor, \(\Delta(\theta) = \Delta_s\).

We neglect surface roughness in our treatment of the junction region, modelling the barrier by the potential \(V(x) = H\delta(x)\). The reflection and transmission amplitudes of the barrier are therefore angle-dependent: \(r(\theta) = Z/(i\cos\theta - Z)\) and \(t(\theta) = i\cos\theta/(i\cos\theta - Z)\), where the dimensionless parameter \(Z = 2mH/h^2\) describes the transparency of the junction \[27\]. A small value of \(Z\) corresponds to a transparent junction and a large value corresponds to a tunnel junction. The limiting cases are \(Z = 0\) (the ballistic junction - no scattering) and \(Z = \infty\) (two uncoupled electrodes).

Having only specular reflections implies that the quasiparticle’s momentum along the junction \(k_y\) is a conserved quantity. This means that the current is an average over all injection angles where the current contribution from each injection angle can be calculated separately. The difference from a junction between \(s\)-wave superconductors is that we here must take into account the angle dependence of the \(d\)-wave gap. In particular, we must remember that the gap changes after normal reflection at the barrier.

Self-consistency of the gap is important in junctions of \(d\)-superconductors \[11\]. It has been shown in the tunnelling limit that including self-consistency of the gap produces bound states with non-zero energy along with the MGS. The length of the normal regions introduced on each side of the barrier in Fig. \[3\] could be thought of as a model of the possible suppression of the \(d\)-wave gap near the interface. Since we in this paper focus on effects of MGS, which is due to the intrinsic sign of the \(d\)-wave order parameter only, we neglect these effects and put the length of the normal regions to zero. We therefore assume a step-like behaviour of the gap-functions:

\[
\Delta = \begin{cases} \\
\Delta_1(\theta), & x < 0 \\
\Delta_2(\theta)e^{i\phi_0}, & x > 0.
\end{cases}
\]

Since the overall phase is unimportant we can here choose \(\Delta_1\) real and let the phase of the right-hand superconductor be equal to the phase-difference \(\phi_0\) over the junction.

The time-dependent Bogoliubov-de Gennes equation describing anisotropic superconductors is solved for a voltage biased junction, modeled as described above, by the method described in detail in Ref. \[10\]. We obtain the scattering states by matching ansatz wavefunctions at the NS interfaces and at the barrier for all injection angles \(\theta\) and energies \(E\). The current is a sum over contributions from all scattering states, where each contribution is found by inserting the wavefunction into the current formula. The sum is in the end turned into an integration over injection angle and energy.

We choose to calculate the current in the normal region to the left of the barrier. The wave function in this region is

\[
\Psi^e_L = \sum_n \left( a_n^\sigma e^{i\mathbf{k}^e\cdot\mathbf{x}} + d_n^\sigma e^{i\mathbf{k}^e\cdot\mathbf{x}} \right) e^{-i(E_n + a\phi_L)x},
\]

where the side-band energy is \(E_n = E + neV\). The index \(\sigma = \{e^+, h^+, h^-, e^-\}\) labels the four types of incoming quasiparticles: electron-like and hole-like quasiparticles injected from the left and right electrodes. Here is \(\mathbf{k}^e/h\) the wave vector of electrons and holes respectively. In our two-dimensional problem we have \(k = (k_x, k_y) = k(\cos\theta, \sin\theta)\) and \(\bar{k} = (-k_x, k_y) = k(\cos\theta, \sin\theta)\), where \(\bar{\theta} = \pi - \theta\) is the angle after normal reflection. The wave function coefficients \(a, b, c,\) and \(d\) in Eq. \[2\] are found by solving the matching equations.

The current per \(ab\)-plane for a junction of width \(L_y\) is

\[
\frac{I}{\sigma_0} = \frac{1}{4D} \int_{-\pi/2}^{\pi/2} d\theta \cos\theta \int_{-\infty}^{\infty} dE f(E) \sum_\sigma j^\sigma(\theta, E),
\]

where \(D\) is the diffusion constant and \(f(E)\) is the Fermi function.
where \( \sigma_0 = 2e k_F L_B D / \pi \hbar \) is the normal state conductance, \( D = \int d\theta D \cos \theta / 2 \) is the normal state transparency averaged over all angles \( \theta \) (\( D = |t(\theta)|^2 \)), \( E_F \) is the Fermi energy, and \( f(E) = 1/[1 + \exp(E/k_B T)] \) is the Fermi distribution function. In Eq. (3) we sum over electron-like and hole-like quasiparticles injected from the left and right reservoirs described by the index \( \sigma \).

The current density appearing in Eq. (3) is expressed in terms of the wavefunction coefficients in \( \Psi_L^\sigma \):

\[
j^\sigma(\theta, E) = N^\sigma(\theta, E) \sum_n |a_n^\sigma|^2 - |d_n^\sigma|^2 + |b_n^\sigma|^2 - |c_n^\sigma|^2.
\]

In order to collect contributions with the same momentum along the junction \( k_n \), we inject the four quasiparticles \( \sigma \) at four different angles with the following four gaps: \( \Delta^\sigma = \{ \Delta_1, \Delta_2, \Delta_2, \Delta_2 \} \) suppressing the dependence on \( \theta \). A consequence of this is that four density of states \( N^\sigma(\theta, E) = \Theta(|E| - |\Delta^\sigma(\theta)|)|E|/\sqrt{E^2 - \Delta^\sigma(\theta)^2} \) appear in Eq. (3).

3. Midgap state resonance

Consider an \( N|d_{\pi/4} \) junction biased at arbitrary small voltage \( V \) and an electron incident on the junction from the normal metal side at the Fermi energy. Due to MGS, the electron will be Andreev reflected with unit probability, independent of the strength of the interface barrier. In energy space, this can be thought of as resonant transmission through MGS: the electron at energy \( E \) is turned into a hole with energy \( E + 2eV \) with unit probability. In the Andreev reflection process a charge \( 2e \) is transferred into the superconductor (forming a Cooper pair) and a current flows through the junction. This current flow through MGS is the explanation of the ZBCP seen in many experiments \([1, 2, 9, 18, 19]\).

The advantage of the above description of current flow through MGS is that it can be generalized to the case when the normal metal is changed to a superconductor. In a junction between two superconductors an incident quasiparticle will create both electrons and holes in the normal region, which will undergo multiple Andreev reflections. Because of the voltage drop over the normal region each particle changes its energy by \( eV \) each time it passes the normal region. In this way a scattering state with amplitudes at all sideband energies \( E_n \) is built up \([22]\). If we consider a quasiparticle incident from the left superconductor at energy \( E \), Andreev reflections at the right superconductor will take place at the odd sideband energies \( E_{2n+1} \), while Andreev reflections at the left superconductor will take place at the even sideband energies \( E_{2n} \). When e.g. the right electrode is a \( d \)-wave superconductor, oriented in such a way that MGS is formed, a transmission resonance is produced in energy space: the probability of transmission from energy \( E_{2n} \) to energy \( E_{2n+2} \) is unity if \( E_{2n+1} = 0 \).

The resonant transport through MGS described above results in current peaks in the IV-characteristics of junctions containing \( d \)-wave superconductors, as previously reported in Refs. \([8\) and \([9]\). Since the current peak is most pronounced in the \( s|d_{\pi/4} \) junction we will here concentrate on this particular junction.

3.1. Midgap state resonance in the \( s|d_{\pi/4} \) junction

Generally, it is possible to express the dc current in superconducting junctions as a sum over contributions from different \( n \)-particle processes, where \( n \) is an integer. In a single scattering state, originating from a quasiparticle incoming at energy \( E \), the \( n \)-particle contribution to the dc current is \( n \cdot I_n^\sigma \), where \( I_n^\sigma \) is the outgoing probability current at energy \( E = E + neV \) \([26]\). In this section we will focus on the resonant two-particle process involving the MGS and derive the expression for
its contribution to the current explicitly, without using the concept of probability current. To be specific we will calculate the dc current contribution from quasiparticles incoming from the left, at energy \( E \approx -eV \), that are Andreev reflected through the MGS on the right hand side, at \( E_1 \approx 0 \) and then leave the normal region to the left at energy \( E_2 \approx eV \) (see Fig. 2). This two-particle process involves one Andreev reflection, implying a net current transport of \( 2e \), as we will also see in the final expression for the current.

In order to solve the scattering problem analytically, we divide the energy axis into three parts: the part in between the injection point and the exit point, the part below the injection point, and finally the part above the exit point. We denote these three parts by I, II, and III, as shown in Fig. 2. The problem of calculating the scattering state can be mapped onto the problem of calculating the wavefunction for tunneling through a one-dimensional multi-barrier structure (in energy space), as drawn in Fig. 3. The structure of the scattering state is better illustrated by Fig. 3 which also makes clear the way the matching of the ansatz wave functions is done and how resonances may appear. From now on, the discussion will therefore be connected to Fig. 3.

The coefficients in part I are coupled by a scattering matrix \( S_{20} \):

\[
\begin{pmatrix}
\begin{array}{c}
d_0^\sigma \\
b_0^\sigma
\end{array}
\end{pmatrix}
= S_{20}
\begin{pmatrix}
\begin{array}{c}
a_0^\sigma \\
c_1^\sigma
\end{array}
\end{pmatrix}
= \begin{pmatrix}
\begin{array}{cc}
r_M & d_M \\
\bar{d}_M & \bar{r}_M
\end{array}
\end{pmatrix}
\begin{pmatrix}
\begin{array}{c}
a_0^\sigma \\
c_1^\sigma
\end{array}
\end{pmatrix},
\]

(5)

where the reflection amplitude \( \bar{r}_M \) is related to the other amplitudes by \( \bar{r}_M = -r_M^*\bar{d}_M/d_M^* \). By matching inside region I, we find the amplitudes for reflection and transmission through MGS: \( d_M = |t|^2A_{21}/(1 - A_{21}\bar{A}_{21}|r|^2) \), \( d_M = |t|^2A_{21}/(1 - A_{21}\bar{A}_{21}|r|^2) \), \( r_M = r(1 - A_{21}\bar{A}_{21})/(1 - A_{21}\bar{A}_{21}|r|^2) \), and \( \bar{r}_M = r^*(1 - A_{21}\bar{A}_{21})/(1 - A_{21}\bar{A}_{21}|r|^2) \). Here we have introduced the amplitude \( A_{1n} = A_i(E_n, \theta) \) for Andreev reflection at energy \( E_n \), which is defined in terms of the BCS coherence factors \( u \) and \( v \) as

\[
A_i(E, \theta) = \frac{v_i(E, \theta)}{u_i(E, \theta)} = \frac{E - \text{sgn}(E)\sqrt{E^2 - \Delta_i(\theta)^2}}{\Delta_i(\theta)}, \quad |E| > |\Delta_i(\theta)|,
\]
A\textsubscript{i}(E, \theta) = \frac{v\textsubscript{i}(E, \theta)}{u\textsubscript{i}(E, \theta)} = \frac{E - i\sqrt{\Delta\textsubscript{i}(\theta)^2 - E^2}}{\Delta\textsubscript{i}(\theta)}, \quad |E| < |\Delta\textsubscript{i}(\theta)|, \quad (6)

where |u\textsubscript{i}|^2 + |v\textsubscript{i}|^2 = 1 and \(i = 1, 2\) refers to the left and right superconductors respectively. The barred amplitudes \(\bar{A}\) are calculated at the angle \(\bar{\theta} = \pi - \theta\).

The amplitudes for reflection from energies outside regions I, II, and III, all the way to \(\pm\infty\) in energy (taking into account all possible processes) can be collected into \(r_0\) and \(r_2\) respectively.

The exact numerical values of \(r_0\) and \(r_2\) are determined by solving the matching equations for the complete scattering state by the method described in Ref. [10]. Without solving these rather complex equations analytically, we can write down the following formal relations between coefficients within part II and III respectively:

\[b_0^\sigma = r_0 c_0^\sigma, \quad d_2^\sigma = r_2 a_2^\sigma.\] \[a_0^\sigma = J \delta_{\sigma e^{-}} + A_{1,0} b_0^\sigma, \quad c_0^\sigma = J \delta_{\sigma h^{-}} + A_{1,0} d_0^\sigma, \quad a_2^\sigma = A_{1,2} b_2^\sigma, \quad c_2^\sigma = A_{1,2} d_2^\sigma,\] \[ (8) \]

where \(J = [u_1(E)^2 - v_1(E)^2]/u_1(E)\) is the amplitude for injection into the normal region from the left superconductor. The \(\delta\)-functions in Eq. (8) are included because we do the matching for incoming electron-like (\(\sigma = e^{-} \Rightarrow \delta_{\sigma e^{-}} = 1\)) and hole-like (\(\sigma = h^{-} \Rightarrow \delta_{\sigma h^{-}} = 1\)) quasiparticles separately. The two contributions are summed up in the end in the current formula. The contributions to the two-particle current from particles injected from the right superconductor will be proportional to \(D^2\) since the trajectories are non-resonant (they do not hit MGS since it only appears at the surface of the right superconductor). We therefore neglect these contributions to the current in the present discussion. The structure of the above matching equations is drawn in Fig. 3b.

Solving Eqs. (5)-(8), we get the following expression for the current density by inserting the
obtained coefficients $a_n, b_n, c_n,$ and $d_n$ ($n = 0, 2$) into Eq. (8):

$$\sum_{\sigma} j^\sigma(\theta, E) \approx \sum_{\sigma=e^{-}, h^{-}} j^\sigma(\theta, E) = 2(1 - A_{1,0}^2)(1 - A_{1,2}^2)|G_{20}|^2 \approx 2(1 + |r_2|^2 A_{1,2}^2)(1 + |r_0|^2 A_{1,0}^2),$$

(9)

where we introduced

$$G_{20} = \frac{d_M}{(1 - A_{1,2}^2 r_2 r_M)(1 - A_{1,0}^2 r_0 r_M) - A_{1,0}^2 A_{1,2}^2 r_0 r_2 d_M d_M}. \quad (10)$$

The factors in the current density given in Eq. (9) can be interpreted in a suggestive way (see also the more general discussion in Ref. [20]). The factor of two appears since we are considering a two particle process (transfer of the charge $2e$). The factor $(1 - A_{1,0}^2)$ is the probability to enter the normal region at energy $E_0$ and the factor $(1 - A_{1,2}^2)$ is the probability to exit at energy $E_2$. The propagator $G_{20}$ is taking us from the injection point up to the exit point. The factor $(1 + |r_2|^2 A_{1,2}^2)$ adds the two possible ways of exiting the normal region. First, we may go out directly when we come from below (giving the term 1). Second, we may Andreev reflect at $E_2$, go further up and be reflected back (in region III) and then go out at $E_2$ (giving the term $|r_2|^2 A_{1,2}^2$). The last factor $(1 + |r_0|^2 A_{1,0}^2)$ adds the contributions from injected electron-like quasiparticles (giving the term 1) and hole-like quasiparticles (giving the term $|r_0|^2 A_{1,0}^2$). The reason for getting the extra factor $|r_0|^2 A_{1,0}^2$ for the hole-like quasiparticles is that the injection is downwards (into the coefficient $c_0$ as seen in Eq. (9)) meaning that the particle must be reflected at negative energy, and then be Andreev reflected at $E_0$ before going up in energy and out from the normal region at $E_2$.

Eqs. (8) and (10) describe the current density from quasiparticles incoming from the left electrode at energy $E \approx -eV$, resonantly transmitted through MGS, leaving the normal region into the left electrode at energy $E_2 \approx \epsilon V$. This process is not possible for voltages below $\epsilon V = \Delta_s$, because then either $E$ or $E_2$ is in the gap of the left superconductor. For voltages well above $\epsilon V = \Delta_s$ we may approximate the propagator $G_{20}$ by the bare transmission amplitude $d_M$, since $A_{1,0} \approx A_{1,2} \approx 0$. For the particular orientation $\alpha = \pi/4$, $|d_M|^2$ is equal to $1/(1 + E^2/\Gamma^2)$, i.e. a Breit-Wigner resonance of width $\Gamma = D \Delta(\theta)/[2(1 - D)]$. The total contribution to the current, after integration over energy, is then $I \propto \Gamma$, i.e. $I$ is proportional to $D$, which is of the same order of magnitude as the single particle current. This means that the main current (current proportional to $D$) has an onset at $\epsilon V = \Delta_s$, due to the resonant two-particle process. Fig. 4 confirms this picture: we see that the resonant two-particle process (solid line) gives the main contribution to the total current (dot-dashed line) for voltages between $\Delta_s < \epsilon V < \Delta_0$. For voltages $\epsilon V > 2\Delta_s$ the single particle contribution is noticeable. We note in passing that the above effect is similar to the effect of a Breit-Wigner resonance in the normal region of an s/s junction [28].

Close to the main current onset at $\epsilon V = \Delta_s$, the magnitude of the Andreev reflection amplitudes $|A_{1,0}|$ and $|A_{1,2}|$ are both close to unity. This together with the reflections from $\pm \infty$ in energy (given by $r_0$ and $r_2$) give rise to an additional resonance in $G_{20}$ (the denominator gives rise to a singularity at the gap edge). This boundary resonance is quite weak and does not change the order of magnitude of the current on its own, since it is connected to the divergency in the superconducting density of states which is integrable. At the onset ($\epsilon V = \Delta_s$) the boundary resonance overlaps with the MGS resonance. The overlap broadens the bare transmission amplitude which results in a current peak. The boundary resonance is due to the reflection from $\pm \infty$ in energy, so by removing them by hand (putting $r_0 = r_2 = 0$) we find that the current peak disappears and is replaced by a smooth onset (the dashed line in Fig. 4).

We note that the N$|d_m/4$ case is easily reached by letting $\Delta_s \rightarrow 0$. In this limit the current onset will be at $\epsilon V = 0$ and the current peak disappears since we lose the boundary resonances ($r_0$ and $r_2 \rightarrow 0$) when $\Delta_s \rightarrow 0$). The onset at $\epsilon V = 0$ result in a ZBCP [3]-[13].
Comparing the current contribution from the two-particle process (solid line) with the complete IV-curve (dot-dashed line), we see that the peak is due to this particular process. For comparison, we include the current (dashed line) from the bare MGS resonance (letting $r_0 = r_2 = 0$). Taking into account the reflections $r_0$ and $r_2$ gives rise to boundary resonances, which at the voltage $eV = \Delta_s = 0.2\Delta_0$ overlap with (and therefore broaden) the MGS resonance and produce the current peak.

4. Zero-bias conductance peak in the $d_{\alpha}|d_{-\alpha}$ junction

In recent experiments on grain boundary junctions of HTS a ZBCP has been found [20, 21]. Within our model a ZBCP can only be found in rather transparent $d_{\alpha}|d_{-\alpha}$ junctions as previously reported [10]. Generally, the current contribution from a process of order $n$ is (without resonances) proportional to the transparency $D$ of the junction to the power of $n$ [24, 25, 26]. In the limit of small bias voltage, the current contributions are from high-order processes, meaning that the current is extremely small without any resonances. This picture is changed by the MGS resonance.

In Fig. 4, we have drawn the map of the scattering state contributing to the current at small bias voltage. The injected quasiparticles undergo multiple Andreev reflections and exit the normal region above the gap. At some particular voltage, the lowest order process contributing to the current, will contain $2n$ Andreev reflections and MGS will be reached, after $n$ Andreev reflections, at the sideband energy $E_n$. The path from the injection point up to MGS consists of $n$ passings through the barrier. In Fig. 4, we have collected this path into an effective first-order path with a barrier of height $D^n$. The same is done for the path from MGS up to the exit point. The map in Fig. 4a is analogous to the map in Fig. 3a, meaning that we should expect a Breit-Wigner resonance of width $\Gamma_{\text{eff}} \propto D^n$, broadened by the boundary resonances. In Fig. 4b, the middle region (corresponding to region 1 in the discussion of the $s|d_{\pi/4}$ junction) is collected into a scattering matrix. We can from this picture immediately write down a formal expression for the current, equivalent to Eqs. (9) and (10), in terms of the effective reflection and transmission amplitudes through MGS ($r_{\text{eff}}, t_{\text{eff}}, d_{\text{eff}}$, and $\delta_{\text{eff}}$), and the reflection amplitudes from $\pm\infty$ in energy ($r_0$ and $r_2$). Since the process under consideration is of order $2n$, the expression for the current will contain the factor $2n$. Due to the MGS resonance, the current density will therefore contain peaks of height $2n$ and width $\Gamma_{\text{eff}}$, which after the integration over energy give rise to an enhanced current at small voltage resulting in a ZBCP.
Fig. 5. Mapping of the $2n$ particle process involving MGS contributing to the current in the high transparency $d_\alpha |d_{-\alpha}$ junction at small voltage. The $n$ barriers on each side of MGS shown in (a) are in (b) collected into effective barriers with transparency $D^n$. The resonant transport through MGS can then be described by an effective scattering matrix as drawn in (c). The bare transmission resonance (described by $d_{eff}$) is broadened by boundary resonances (due to the reflections $r_0$ and $r_{2n}$ at the boundaries) producing an enhanced current at small voltage, i.e a ZBCP.

Since the peak width scales with $D^n$ (neglecting broadening effects due to boundary resonances), the current contribution will be considerable only in junctions with high transparency [10].

5. Summary

We have discussed the effects of the midgap state on current transport through voltage-biased Josephson junctions of $d$-wave superconductors and demonstrated that in connection with multiple Andreev reflection the MGS acts as a transmission resonance in energy space. Depending on the orientation of the $d$-wave superconductors and the transparency of the junction, resonant transport through MGS influences the current in different ways. In low transparency $s|d_{\pi/4}$ junction the MGS resonantly enhances the two-particle current, which gives the main contribution to the current for voltages between $\Delta_s < eV < \Delta_0$. At the onset an overlap of boundary resonances, connected to the peaks in the density of states, and the MGS resonance results in a current peak. In the same way, the MGS resonantly enhances the two-particle current in $d|d$-junctions which, depending on the orientations of the superconductors, results in a current peak at finite voltage [8]. Since the gap has nodes in both electrodes there is no clear onset of the resonant current. In addition, a smooth background contribution from the single-particle current at all voltages makes the effect less pronounced compared to the $s|d_{\pi/4}$ junction. The relation between our results and experiments was discussed in Ref. [10].
In high transparency $d_\alpha|d_{-\alpha}$ junctions, there are resonant $2n$-particle processes involving the MGS, corresponding to tunneling through an effective symmetric double barrier structure in energy space. These symmetric processes give a huge contribution to the current, resulting in finite current at low voltages, as in ballistic s/s-junctions. This may explain the zero-bias conductance peaks seen in experiments on grain boundary junctions of HTS superconductors. [20, 21].

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