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Lifetime estimations from RHIC Au+Au data

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Abstract: We discuss a recently found family of exact and analytic, finite and accelerating, 1+1 dimensional solutions of perfect fluid relativistic hydrodynamics to describe the pseudorapidity densities and longitudinal HBT-radii and to estimate the lifetime parameter and the initial energy density of the expanding fireball in Au+Au collisions at RHIC with \( \sqrt{s_{NN}} = 130 \text{ GeV} \) and 200 GeV colliding energies. Our novel method fixes several oversimplifications in Bjorken’s famous initial energy density estimate. When compared to similar estimates at the LHC energies, the results indicate a surprising and non-monotonic dependence of the initial energy density on the energy of heavy ion collisions.

Keywords: relativistic hydrodynamics; quark-gluon plasma; longitudinal flow; rapidity distribution; pseudorapidity distribution; HBT-radii; initial energy density

1. Introduction

Relativistic hydrodynamics of nearly perfect fluids is the current paradigm in analyzing soft particle production processes in high energy heavy ion collisions. The development of this paradigm goes back to the classic papers of Fermi from 1950 [1], Landau from 1953 [2] and Bjorken from 1982 [3], that analyzed the statistical and collective aspects of multiparticle production in high energy collisions of elementary particles and atomic nuclei.

Although hydrodynamical relations in the double-differential invariant momentum distribution and in the parameters of the Bose-Einstein correlation functions of hadron-proton collisions were observed at \( \sqrt{s} = 22 \text{ GeV} \) colliding energies as early as in 1998 [4], relativistic hydrodynamics of nearly perfect fluids became the dominant paradigm for heavy ion collisions only after the White Papers of the four RHIC experiments, BRAHMS [5], PHENIX [6], PHOBOS [7] and STAR [8]. These papers summarized the results of the first four years of data-taking at Brookhaven National Laboratory’s Relativistic Heavy Ion Collider (BNL’s RHIC) as a circumstantial evidence for creating a nearly perfect fluid of strongly coupled quark-gluon plasma or quark matter in \( \sqrt{s_{NN}} = 200 \text{ GeV} \) Au+Au collisions.

Recently, the STAR Collaboration significantly sharpened this result by pointing out that the fluid created in heavy ion collisions at RHIC is not only the most perfect fluid ever made by humans but also the most vortical fluid as well [9]. In addition, the PHENIX collaboration demonstrated that the domain of validity of the hydrodynamical paradigm extends to p + Au, d + Au and \(^3\text{He} + \text{Au} \) collisions [10] and also to as low energies as \( \sqrt{s_{NN}} = 19.6 \text{ GeV} \) [11]. So by 2018, the dominant paradigm of analyzing the collisions of hadron-nucleus and small-large nucleus collisions also shifted to the domain of relativistic hydrodynamics, as reviewed recently by ref. [12]. Recent results by the ALICE Collaboration at LHC suggested that enhanced production of multi-strange hadrons in high-multiplicity proton-proton collisions may signal the production of a strongly coupled quark-gluon plasma not only in high energy proton/deuteron/Helium+ nucleus but also in hadron-hadron collisions [13]. These results
are re-opening the door to the application of the tools of relativistic hydrodynamics in hadron-hadron collisions too, however the applicability of the hydrodynamical paradigm in these collisions is currently under intense theoretical and experimental scrutiny. Although more than 20 years passed since hydrodynamical couplings were found in the double-differential invariant momentum distribution and in the parameters of the Bose-Einstein correlation functions of hadron-proton collisions [4], this field is still a subject of intense debate, where the picture that protons at high colliding energies behave in several ways similarly to a small nucleus [14] are gaining popularity but are rather far from being a dominant view at present.

The theory and applications of relativistic hydrodynamics to high energy collisions was reviewed and detailed recently in ref. [15], with special attention also to the role of analytic solutions, but with a primary interest in the hydrodynamical interpretation of azimuthal oscillations in the transverse momentum spectra, and its relation to the transverse hydrodynamical expansion. In contrast, the focus of our manuscript is to gain a deeper understanding of the longitudinal expansion dynamics of high energy heavy ion and proton-proton collisions. As reviewed recently in ref. [16], the longitudinal momentum spectra of high energy collisions can a posteriori be very well described by exact solutions of 1+1 dimensional solutions of perfect fluid hydrodynamics, the central theme of our current investigation.

Let us also stress that this manuscript is the fourth part of a manuscript series, that is a straightforward continuation of investigations published in refs. [17–19]. The first part of this series [17] gives an introductory overview of the special research area of 1+1 dimensional exact solutions of relativistic hydrodynamics, defines the notation that we also utilize in the current work, and summarizes a recently found new class of exact solutions of perfect fluid relativistic hydrodynamics in 1+1 dimensions. This new family of exact solutions, the CKCJ solution was discovered by Csörgő, Kasza, Csanád and Jiang and presented first in ref [16].

The first part of this manuscript series [17] connects the CKCJ solution to experimentally measured quantities by evaluating the rapidity and pseudorapidity density distributions. The second part of this manuscript series [18] presents exact results for the initial energy density in high energy collisions that at the same time represent an apparently fundamental correction to the famous Bjorken estimate of initial energy density [3] in high energy collisions. The third part of this manuscript series [19], evaluates the Bose-Einstein correlation functions and in a Gaussian approximation it determines the so-called Hanbury Brown - Twiss or HBT-radii in the longitudinal direction from the CKCJ solution in order to determine the life-time of the fireball created in high-energy collisions.

In this work, corresponding to the fourth part of this manuscript series, we started to investigate the excitation function of the parameters of the initial state, as reconstructed with the help of the CKCJ solution [16]. The results demonstrate the advantage of analytic solutions in understanding the longitudinal dynamics of fireball evolution in high energy heavy ion collisions.

This work is following up refs. [17–19] by presenting a new method to evaluate the lifetime and the initial energy density of high energy heavy-ion collisions, utilizing the recently found CKCJ family of solutions [17–19] to describe their longitudinal expansion and the corresponding measurable, the pseudorapidity density distributions. Namely, in this paper we show new fit results for Au+Au at \( \sqrt{s_{NN}} = 130 \) GeV and Au+Au at \( \sqrt{s_{NN}} = 200 \) GeV collisions at RHIC. Through these calculations we are able to estimate the acceleration and the effective temperature of the medium at freeze-out. In this paper, we utilize these results to determine the lifetime of the fireball by fitting the longitudinal HBT-radii data of Au+Au at \( \sqrt{s_{NN}} = 130 \) GeV and Au+Au at \( \sqrt{s_{NN}} = 200 \) GeV collisions, simultaneously \( dN/d\eta \) and the slope of the transverse momentum spectra are also described. In ref. [18] we presented a new exact formula of the initial energy density derived from our new family of perfect fluid hydrodynamic solutions. In this new formula all the unknown parameter can be determined by fits to pseudorapidity densities and longitudinal radii, except the initial proper time \( \tau_0 \). As a consequence, this new method makes it possible to describe the initial proper time dependence of the initial energy density of the thermalized fireball in high-energy heavy-ion collisions, for predominantly 1+1 dimensional expansions.
2. New, exact solutions of perfect fluid hydrodynamics

The equations of relativistic perfect fluid hydrodynamics express the local conservation of entropy and four-momentum:

\[ \partial_\mu (\sigma u^\mu) = 0, \]
\[ \partial_\nu T^{\mu\nu} = 0, \]

where \( \sigma \) is the entropy density, the four velocity is denoted by \( u^\mu \) and normalized as \( u^\mu u_\mu = 1 \), and \( T^{\mu\nu} \) stands for the energy-momentum four tensor of perfect fluids:

\[ T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu - p g^{\mu\nu}. \]

The metric tensor is \( g^{\mu\nu} = \text{diag}(1, -1, -1, -1) \), the pressure is denoted by \( p \), and the energy density by \( \epsilon \). The entropy density \( \sigma \equiv \sigma(x) \), the energy density \( \epsilon \equiv \epsilon(x) \), the temperature \( T \equiv T(x) \), the pressure \( p \equiv p(x) \), the four-velocity \( u^\mu \equiv u^\mu(x) \) and the four-momentum tensor \( T^{\mu\nu} \equiv T^{\mu\nu}(x) \) are fields, i.e. they are functions of the four coordinate \( x \equiv x^\mu = (t, r) = (t, r_x, r_y, r_z) \). This set of equations provides five equations for six unknown fields, and it is closed by an equation of state (EoS). In this manuscript we assume that the energy density \( \epsilon \) is proportional to the pressure \( p \) with a constant, temperature independent proportionality factor:

\[ \epsilon = \kappa p. \]

For zero baryochemical potential, \( \kappa \) can be expressed by the speed of sound:

\[ c_s = \sqrt{\frac{\partial p}{\partial \epsilon}} = 1/\sqrt{\kappa}. \]

2.1. The CKCJ solution

An exact and analytic, finite and accelerating, 1+1 dimensional solution of relativistic perfect fluid hydrodynamics was recently found by Csörgö, Kasza, Csanád and Jiang (CKCJ) [16] as a family of parametric curves. The thermodynamic parameters as the entropy density \( \sigma \) and the temperature \( T \) are functions of the longitudinal proper time \( \tau \) and the space-time rapidity \( \eta_x \):

\[ (\tau, \eta_x) = \left( \sqrt{t^2 - r_z^2}, \frac{1}{2} \ln \frac{t + r_z}{t - r_z} \right). \]
The fluid rapidity $\Omega$ is assumed to be independent of the proper time. The four-velocity is chosen as $u^\mu = (\cosh(\Omega), \sinh(\Omega))$, consequently the three-velocity is $v_z = \tanh(\Omega)$. The new class of solutions can be summarized by the following equations:

$$
\eta_x(H) = \Omega(H) - H, \\
\Omega(H) = \frac{\lambda}{\sqrt{\lambda - 1} \sqrt{\kappa - \lambda}} \arctan \left( \sqrt{\frac{\kappa - \lambda}{\lambda - 1}} \tanh(H) \right), \\
\sigma(\tau, H) = \sigma_0 \left( \frac{\tau_0}{\tau} \right)^\lambda \mathcal{V}_s(s) \left[ 1 + \frac{\kappa - 1}{\lambda - 1} \sinh^2(H) \right]^{-\frac{\lambda}{2}} , \\
T(\tau, H) = T_0 \left( \frac{\tau_0}{\tau} \right)^\frac{\lambda}{2} \mathcal{T}(s) \left[ 1 + \frac{\kappa - 1}{\lambda - 1} \sinh^2(H) \right]^{-\frac{\lambda}{2}} , \\
\mathcal{T}(s) = \frac{1}{\mathcal{V}_s(s)} , \\
s(\tau, H) = \left( \frac{\tau_0}{\tau} \right)^{\lambda - 1} \sinh(H) \left[ 1 + \frac{\kappa - 1}{\lambda - 1} \sinh^2(H) \right]^{-\lambda/2} ,
$$

where $H$ is the parameter of the solution, $\lambda$ is the acceleration parameter, $s(\tau, H)$ stands for the scale variable, and $\mathcal{V}_s(s)$ is an arbitrary positive definite scaling function for the entropy density. The integration constants $\sigma_0$ and $T_0$ stand for $\sigma(\tau_0, H = 0)$ and $T(\tau_0, H = 0)$, where the initial proper time is denoted by $\tau_0$. The space-time rapidity dependence appears only through the parameter $H$, which is the difference of the fluid rapidity $\Omega$ and the coordinate rapidity $\eta_x$. The CKCJ solutions are limited to a cone within the forward light-cone around midrapidity. The domain of validity of these solutions in space-time rapidity is described in details in ref. [16].

3. Observables

This section presents the results of the observables that are derived from the CKCJ solution. The pseudorapidity density and the longitudinal HBT-radii together are the keys to determine the lifetime parameter of the fireball. In this manuscript we do not detail the derivation of these quantities, see refs. [18,19] for more details.

3.1. Pseudorapidity density

To obtain the pseudorapidity density, we embedded these 1+1 dimensional solutions to the 1+3 dimensional space-time and assumed, that the freeze-out hypersurface is pseudo-orthogonal to the four velocity. As detailed in refs. [16,17] the analytic expression of the rapidity distribution was calculated by an advanced saddle-point integration to yield

$$
\frac{dN}{dy} \approx \frac{dN}{dy} \bigg|_{y=0} \cosh^{-\frac{1}{2} n(\kappa) - 1} \left( \frac{y}{\alpha(1)} \right) \exp \left( -\frac{m}{T_{\text{eff}}} \left[ \cosh^{n(\kappa)} \left( \frac{y}{\alpha(1)} \right) - 1 \right] \right). 
$$

In this equation $y = \frac{1}{2} \ln \left( \frac{E + \sqrt{E^2 - p_x^2}}{p_y} \right)$ stands for the rapidity, the four-momentum is defined as $p^\mu = (E, p_x, p_y, p_z)$ with $E = \sqrt{m^2 + p^2}$, where the modulus of the three-momentum is $p = \sqrt{p_x^2 + p_y^2 + p_z^2}$ and $m$ stands for the particle mass. The slope parameter of the transverse momentum spectrum is denoted as $T_{\text{eff}}$, denoting the effective temperature, and

$$
\alpha(x) = \frac{2\lambda - \kappa}{\lambda - \kappa}.
$$
By a saddle-point integration of the double differential spectra \[16,17\] the pseudorapidity density is calculated as a parametric curve, where the parameter is the rapidity \(y\):

\[
\left( \eta_p(y), \frac{dN}{d\eta_p}(y) \right) = \left( \frac{1}{2} \log \frac{p(y) + p_2(y)}{p(y) - p_2(y)} \right) \frac{p(y) dN}{E(y) dy}.
\]

(15)

The pseudorapidity is denoted by \(\eta_p = \frac{1}{2} \ln \left( \frac{p + p_2}{p - p_2} \right)\). The average of the momentum space variable \(q\) is indicated by \(q\), as given in details in ref. \[17\]. This formula includes 4 fit parameter, namely the average speed of sound \(c_s = 1/\sqrt{\kappa}\), the acceleration parameter \(\lambda\), the effective temperature \(T_{\text{eff}}\) and the normalization at midrapidity. When comparing to measured \(dN/d\eta_p\) data, \(\kappa\) is taken from lattice or experimental data, \(T_{\text{eff}}\) is also taken from the experimentally determined transverse momentum spectra, so from fits to the rapidity or pseudorapidity distributions, one obtains the shape and the normalization parameters \(\lambda\) and \(dN/d\eta_p\) respectively. See \[16,17\] for further details and applications.

3.2. Longitudinal HBT-radii

As discussed in \[20\], for a 1+1 dimensional relativistic source, in a Gaussian approximation the relative momentum dependent part of the two-particle Bose-Einstein correlation function is characterized by (generally mean pair momentum dependent) longitudinal Hanbury-Brown Twiss (HBT) radii. The general expression in the longitudinally co-moving system (LCMS), where the mean momentum of the pair has zero longitudinal component, reads \[20\] as

\[
R_{\text{long}}^2 = \cosh^2(\eta_x^f) \tau_f^2 \Delta \eta_x^2 + \sinh^2(\eta_x^f) \Delta \tau^2.
\]

(16)

Here \((\tau_f, \eta_x^f)\) stands for the Rindler coordinates of the main emission point of the source in the \((t, r_z)\) plane, which are derived from a saddle-point calculation of the rapidity density, and including \(\tau_f\), the lifetime parameter of the fireball. The \(\Delta \tau\) and \(\Delta \eta_x\) characteristic sizes define the main emission region around the saddle-point at \((\tau_f, \eta_x^f)\). For \(R_{\text{long}}\) measurements at midrapidity, where \(\eta_x^f \approx 0\), this formula can be simplified as

\[
R_{\text{long}} = \tau_f \Delta \eta_x.
\]

(17)

If the emission function of particles with \(y = 0\) can be well approximated by a Gaussian shape, with \(\Delta \eta_x\) being the width of the space-time rapidity distribution of these particles with vanishing momentum-space rapidity, the longitudinal HBT-radii at \(y = 0\) can be derived from the CKCJ solutions as we have already shown in ref. \[19\]:

\[
R_{\text{long}} = \tau_f \Delta \eta_x \approx \frac{\tau_f}{\sqrt{\lambda(2\lambda - 1)}} \sqrt{\frac{T_f}{m_T}}
\]

(18)

where the transverse mass of the particles is denoted by \(m_T = \sqrt{m^2 + p_T^2}\), and \(T_f\) stands for the freeze-out temperature that can be extracted from the analysis of the transverse momentum spectra. Note that in general \(T_f < T_{\text{eff}}\) as the effective temperature of the transverse momentum spectra contains not only the freeze-out temperature but also radial flow effects.

Importantly, eq. (18) is found to depend on the acceleration parameter \(\lambda\) that characterizes the CKCJ solutions, and to be independent of the \(\kappa\) parameter, that characterizes the speed of sound and the Equation of State. It is interesting to note that this result corrects the \(R_{\text{long}} \approx \tau_f \sqrt{T_f/m_T}\) approximation of Csörgő, Nagy and Csanád (CNC) in refs. \[21,22\], obtained in the \(\kappa \rightarrow 1\) limit. The effect of the accelerating trajectories is corrected by eq. (18) by a \(\lambda \rightarrow \sqrt{\lambda(2\lambda - 1)}\) transformation in the CNC estimate for \(R_{\text{long}}\) refs. \[21,22\]. In the boost invariant limit (\(\lambda \rightarrow 1\)), the longitudinal radii of the CKCJ solution and the CNC solution both reproduce the same Makhlin - Sinyukov formula of ref. \[23\].
4. Fit results

To extract $\tau_f$, the lifetime parameter of the fireball, we fit the pseudorapidity density to extract the shape parameter $\lambda$, and the longitudinal HBT-radius of the CKCJ solution to the experimental data, selected to be the $\sqrt{s_{NN}} = 130$ GeV and 200 GeV Au+Au collisions where the pseudorapidity density, the slope parameter of the transverse momentum spectra and the HBT radii are known to be measured in the same, 0-30% centrality class. In both of these reactions, the PHENIX collaboration published their two-pion correlation results in the 0-30% centrality class, consequently we fitted our hydrodynamic model to the pseudorapidity density data of PHOBOS experiment from ref. [24], that we averaged in this 0-30% centrality class. The centrality dependence of the effective temperature can be estimated with the help of an empirical relation,

$$T_{\text{eff}} = T_f + m\langle u_T \rangle^2. \quad (19)$$

This relation has been derived for a three-dimensionally expanding, ellipsoidally symmetric exact solution of fireball hydrodynamics with a temperature dependent speed of sound, as a model for non-central heavy ion collisions, in ref. [25], but only for a single kind of hadron that had a fixed mass $m$. Recently this relation has also been re-derived for a non-relativistic, 3 dimensionally expanding fireball that hadronizes to a mixture of various hadrons that have different masses $m_i$ just like a mixture of pions, kaons and protons in ref. [26]. This derivation included a lattice QCD EoS in the quark matter or strongly interacting quark gluon plasma phase [26].

The slope of the affin-linear function in eq. (19) is the square of the radial flow at the average radius, which has a significant centrality dependence, while $T_f$ is interpreted as the freeze-out temperature, its value is empirically found from the analysis of the transverse momentum spectrum and it is approximately independent of the particle type, the centrality and the center of mass energy of the collision. Figure 1 illustrates the linear fits to the effective temperature of charged pions, charged kaons, and protons, antiprotons for PHENIX Au+Au at $\sqrt{s_{NN}}=130$ GeV and 200 GeV data, and suggests that the $T_f$ value that is consistent with this interpretation of the transverse momentum spectra is $T_f \approx 175$ MeV for both $\sqrt{s_{NN}} =130$ and 200 GeV Au+Au collisions, indeed independently of particle mass, centrality and center of mass energy of the collision. Given that several theoretical expectations suggests a lower value for the freeze-out temperature, we have also investigated if $T_f = 140$ MeV can be assumed for the extraction of the life-time of the reaction. Lower freeze-out temperatures correspond to larger life-times for the same datasets according to eq. (18), and we shall see subsequently that longer life-times correspond to larger initial energy densities for the same pseudorapidity distributions, so in what follows we use the shorter life-times obtained with larger freeze-out times but summarize the estimated freeze-out times in Table 1 so that the interested reader can also evaluate the corrections of a less conservative results for the initial energy densities.
Figure 1. The mass and centrality dependence of the effective temperature of charged pions and kaons, as well as protons and anti-protons in $\sqrt{s_{NN}} = 130$ GeV and $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions. For each centrality classes, the linear fits are shown by continuous lines. $T_f$ is fixed to 175 MeV in both cases, although at 130 GeV, a slightly higher $T_f \approx 180$ MeV is preferred by the data.

If the effective temperature is allowed to be a free fit parameter in the $dN/d\eta_p$ fits, we have found, that the fit results are in agreement with the estimated centrality dependence of $T_{eff}$. We fixed the speed of sound to $c^2_s = 0.1$ in accordance to the experimental results of the PHENIX collaboration [27]. Since the CKCJ solution is valid in a limited pseudorapidity interval, we fitted the $dN/d\eta_p$ of the CKCJ solution to data in the $[-2.5, 2.5]$ pseudorapidity interval. The fits and the best fit parameters are shown on Figure 2.

As can be seen on the right panel of Fig. 1, for Au+Au at $\sqrt{s_{NN}}=200$ GeV, $T_{eff}$ data are available in the 0-5 % and 40-50 % centrality classes, but they were not directly measured in the 0-30 % centrality class. Note that the $T_{eff} = 203$ MeV value that we obtained from fitting the $dN/d\eta_p$ data at $\sqrt{s_{NN}}=200$ GeV for the 0-30 % centrality class is just in between the measured values for the 0-5 % and 40-50 % centrality classes. However, for the same system at $\sqrt{s_{NN}}=130$ GeV, $T_{eff}$ data are measured also in the 5-15 % and the 15-30 % centrality classes – although these are not shown on the left panel of Fig. 1. We evaluated the inverse slope for the 0-30 % centrality class by averaging the fit of the $T_{eff} = T_f + m(\langle uT \rangle)^2$ relation over the 0-5 %, 5-15 % and 15-30 % centrality classes for the pion mass. It is important for the current study, that $T_{eff}$, $dN/d\eta_p$ and $R_{long}$ be determined in the same centrality class of the same colliding system at the same energy. These new results continue the trend that we saw earlier in ref. [16,17]: the lower the collision energy, the greater the acceleration parameter.
Figure 2. Fits of the pseudorapidity density with the CKCJ hydro solution \[16\], to PHOBOS Au+Au data at $\sqrt{s_{NN}} = 130 \text{ GeV} \[24\]$ (left) and $\sqrt{s_{NN}} = 200 \text{ GeV} \[24\]$ (right) in the 0-30 % centrality class. The speed of sound is $c_s^2 = 1/\kappa = 0.1$, fixed in both cases.

Since we have obtained the effective temperature $T_{\text{eff}}$ and the acceleration parameter $\lambda$ from the transverse momentum spectra and the pseudorapidity density data, the number of fit parameters in eq. (18), i.e. in our new formula for the longitudinal radii is reduced to one: the remaining free parameter is the lifetime parameter $\tau_f$. Our fits to the $R_{\text{long}}$ data are illustrated in Figure 3. The systematic variation of the freeze-out time or life-time parameter $\tau_f$ with the variation of the assumed value of the freeze-out temperature $T_f$ is indicated in Table 1. Given that longer life-times yield larger initial energy density estimates for the same pseudorapidity distribution, we evaluated the initial energy densities for the more conservative values i.e. for shorter lifetimes, corresponding to $T_f = 175 \text{ MeV}$, which value is also consistent with the systematics of the transverse momentum spectra as indicated on Figure 1.

Figure 3. Fits of the longitudinal HBT-radii with the CKCJ hydro solution \[16\], to PHENIX and STAR Au+Au data at $\sqrt{s_{NN}} = 130 \text{ GeV} \[28\]$ (left) and to PHENIX Au+Au data at $\sqrt{s_{NN}} = 200 \text{ GeV} \[29\]$ (right) in the 0-30 % centrality class, for a fixed centrality, colliding energy and colliding system.

With the help of these values of the lifetime parameter $\tau_f$ we are ready to calculate the the initial energy density of the expanding medium as a function of its initial proper time, $\tau_0$. This consideration serves the topic of the next section.
Table 1. Freeze-out proper-times $\tau_f$ extracted from the transverse mass dependent longitudinal HBT radii using eq. (18) of the CKCJ exact solutions, assuming different values for the freeze-out temperature $T_f$.

| $\sqrt{s_{NN}}$ | $130 \text{ GeV}$ | $200 \text{ GeV}$ |
|----------------|------------------|------------------|
| $T_f [\text{MeV}]$ | 140 | 175 | 140 | 175 |
| $\tau_f [\text{fm/c}]$ | 13.2 ± 0.6 | 11.8 ± 0.5 | 11.3 ± 0.4 | 10.2 ± 0.3 |

5. Initial energy density

In ref. [18] we have already shown an exact calculation of the initial energy density of the expanding fireball by utilizing the CKCJ hydrodynamic solution. We have found that Bjorken’s famous estimate [3], denoted by $\epsilon_0^{Bj}$, is corrected by the following formula, if the rapidity distribution is not flat and if the CKCJ solution is a valid approximation for the longitudinal dynamics:

$$\epsilon_0(\kappa, \lambda) = \epsilon_0^{Bj}(2\lambda - 1) \left(\frac{\tau_f}{\tau_0}\right)^{\lambda \left(1 + \frac{1}{\kappa}\right)^{-1}},$$

(20)

where Bjorken’s (under)estimate for the initial energy density reads as

$$\epsilon_0^{Bj} = \frac{\langle E_T \rangle}{S_{\perp}} \frac{dN}{d\eta_p} \bigg|_{\eta_p=0}.$$  

(21)

In Bjorken’s estimate $S_{\perp}$ is the overlap area of the colliding nuclei, and $\langle E_T \rangle$ is the average of the total transverse energy. In eq. (20) the $(2\lambda - 1) \left(\frac{\tau_f}{\tau_0}\right)^{\lambda^{-1}}$ factor takes into account the shift of the saddle-point corresponding to the point of maximum emittivity for particles with a given rapidity, and it takes into account also the change of the volume element during the non-boost-invariant expansion. In addition to that, we have found an unexpected and in retrospective rather surprising feature of the energy density estimate from the CKCJ solution, as detailed in ref. [18]. Due to the EoS dependent correction factor, in the boost-invariant limit, corresponding to $\lambda = 1$ and to the lack of acceleration, our estimate does not reproduce the Bjorken estimate for the initial energy density. However, in this boost-invariant limit the CKCJ result for the initial energy density corresponds to Sinyukov’s result from ref. [30].

This difference between Bjorken’s and our estimate for the initial energy density, that exists even in the boost invariant $\lambda = 1$ limit, is related to the work, which is done by the pressure during the expansion stage. If the pressure is vanishing, $p = 0$, it corresponds to the equation of state of dust, and is obtained in the $1/\kappa \to 0$ limit of our calculations. If $\lambda \to 1$ and $1/\kappa \to 0$, our estimate for the initial energy density reproduces Bjorken’s estimate [18]. In summary, the Bjorken estimate of the initial energy density has to be corrected not only due to the lack of boost-invariance in the measured rapidity distributions, but also because it neglects that fraction of the initial energy, which is converted to the work done by the pressure during the expansion of the volume element in the center of the fireball [18]. Bjorken was actually aware of this $p = 0$ approximation in his original paper [3], which is a reasonable approximation if $p \ll \epsilon$ and if one aims to get an order of magnitude estimations, that is precise within a factor of 10. However, at the present age of complex and very expensive accelerators and experiments, an order of magnitude or factor of 10 increase in initial energy density is less, than what can be gained by moving from the RHIC collider top energy of $\sqrt{s_{NN}} = 200 \text{ GeV}$ for Au+Au collisions to the $\sqrt{s_{NN}} = 2.76 \text{ TeV}$ for Pb+Pb collisions at LHC. Although the rise in the center of mass energy is a factor 100 when going from RHIC to LHC,
the increase in the initial energy density of the order of 2 only, as demonstrated recently for example in ref. [31] and summarized in Table 2. Note that the conjectured values of the $\kappa$ dependent estimates of the initial energy density, $\varepsilon_{\text{conj}}(\kappa, \lambda)$ values were based in ref. [31] on a conjectured $\kappa$ dependence of the initial energy density, given in ref. [32] as follows:

$$\varepsilon_{0}^{\text{CNC}}(\lambda) = \varepsilon_{0}^{Bj}(2\lambda - 1) \left( \frac{f_{\kappa}}{\tau_{0}} \right)^{\lambda - 1}, \quad \varepsilon_{0}^{\text{conj}}(\kappa, \lambda) = \varepsilon_{0}^{\text{CNC}}(\lambda) \left( \frac{f_{\kappa}}{\tau_{0}} \right)^{(\lambda - 1)(1 - \frac{1}{\lambda})}. \quad (22)$$

The first of the above equations is an exact, proper-time dependent result for $\varepsilon_{0}^{\text{CNC}}(\lambda)$, the initial energy density in the CNC solution of refs. [21,22,32]. However, this eq. (22) lacks the dependence of the initial energy density on the equation of state parameter $\kappa$. Nevertheless it contains two $\lambda$-dependent prefactors as compared to the Bjorken estimate, and so it allows for the possibility of a non-monotonic dependence of the initial energy density on the energy of the collision, if the shape and the transverse energy density both change monotonously but non-trivially with $\sqrt{S_{NN}}$.

The second estimate of the initial energy density, eq. (23) was a conjecture, assumed at times when the CKCJ solution was not yet known. This conjecture was obtained under the condition that it reproduces the an exact CNC result in the $\kappa \rightarrow 1$ as well as in the $\lambda \rightarrow 1$ limits [32]. As compared to the CNC estimate, this conjecture contains a proper-time dependent prefactor, that generalized the CNC estimate under the requirement that it has to reproduce the Bjorken estimate in the $\lambda \rightarrow 1$ limit for all values of the parameter of the Equation of State $\kappa$. This is reflected by the vanishing of the exponent of the second proper-time dependent factor in the $\lambda \rightarrow 1$ limiting case. However, by now we know that this requirement is not valid as the Bjorken formula has a so far largely ignored but also equation of state dependent prefactor. This is due to an actual $\kappa$ dependent (but neglected) term in the initial energy density in the boost-invariant $\lambda \rightarrow 1$ limiting case. Fortunately the conjecture was numerically reasonably good, as it contained the dominant and fastly rising $(2\lambda - 1)(\tau_{f}/\tau_{0})^{\lambda - 1}$ prefactor already, that increases with increasing values of $\lambda$ that corresponds to decreasing energies and increasing deviations from an asymptotic boost-invariance. Table 2 summarizes the Bjorken, the CNC exact and conjectured initial energy densities, and the CKCJ exact initial densities for $\sqrt{S_{NN}} = 130$ and 200 GeV, 0-30% Au+Au collisions in the first two lines. The last line of the same table also indicates the Bjorken, the CNC and the CNC conjectured result for the initial energy density for $\sqrt{S_{NN}} = 2.76$ TeV Pb+Pb collisions at CERN LHC, assuming the value of $\tau_{f}/\tau_{0} = 10$. The first column of this Table 2 indicates that the Bjorken initial energy density increases monotonically with increasing $\sqrt{S_{NN}}$. The second column indicates that the acceleration parameter $\lambda$ dependent correction terms generate a non-monotonous energy dependence for the initial energy density already in the CNC model fit results, because the evolution of the shape of the pseudorapidity distribution is more dramatic at lower energies, so the shape parameter $\lambda$ dependent terms in eq. (22) create a non-monotonic behaviour, that is inherited by the more-and-more refined calculations, corresponding to the conjecture and to the CKCJ solution indicated in the last two columns of the same Table 2. Given that a more precise evaluation of the life-time parameter of 2.76 TeV Pb+Pb collisions needs more detailed analysis of this reaction with the help of the CKCJ solution, that goes beyond the scope of the current manuscript, the last cell in Table 2 is left empty, to be filled in by future calculations.

In what follows, let us demonstrate, how we got to this result ie how one can improve on Bjorken’s estimate for the initial energy density, using straightforward fits to the already published data and relying on exact results from 1+1 dimensional relativistic hydrodynamics.

Let us use the fit parameters determined in Section 4 to the evaluation of our new, exact result for the initial energy density and for a comparison of it with Bjorken’s estimate for the initial energy density, as a function of the initial proper time $\tau_{0}$. In Figure 4 we compare eq. (20) to $\varepsilon_{0}^{Bj}$ in Au+Au $\sqrt{S_{NN}} = 130$ GeV...
Table 2. Initial energy density estimation from [31] by Bjorken’s formula $\varepsilon_{0}^{Bj}$, and the conjectured values of $\varepsilon_{0}^{conj}(\kappa, \lambda)$, evaluated for $\tau_{0} = 1 \text{ fm}/c$. These values also indicate the non-monotonic behaviour of the initial energy density as a function of $\sqrt{s_{NN}}$, the center of mass energy of colliding nucleon pairs.

| System         | $\varepsilon_{0}^{Bj}$ [GeV/fm$^3$] | $\varepsilon_{0}^{CNC}(\lambda)$ | $\varepsilon_{0}^{conj}(\kappa, \lambda)$ | $\varepsilon_{0}(\kappa, \lambda)$ |
|----------------|-----------------------------------|----------------------------------|-------------------------------------------|----------------------------------|
| Au+Au at 130 GeV, 6-15 % | 4.1 ± 0.4                         | 14.8 ± 2.2                      | 11.2 ± 1.8                                | 11.9 ± 0.5                       |
| Au+Au at 200 GeV, 6-15 %  | 4.7 ± 0.5                         | 12.2 ± 2.3                      | 9.9 ± 1.6                                 | 9.8 ± 0.4                        |
| Pb+Pb at 2.76 TeV, 10-20 % | 10.1 ± 0.3                        | 14.1 ± 0.5                      | 13.3 ± 0.6                                |                                 |

and $\sqrt{s_{NN}} = 200 \text{ GeV}$ collisions. The normalization at midrapidity $\frac{dN}{d\eta_{p}}|_{\eta_{p}=0}$, the acceleration parameter $\lambda$ as well as the effective temperature $T_{\text{eff}}$ are determined by fits shown on Figures 1 and 2, while the value of the lifetime parameter $\tau_{f}$ is taken from fits illustrated in Figure 3. The normalization parameter $\frac{dN}{d\eta_{p}}|_{\eta_{p}=0}$ corresponds to all the final state particles, and many of them are emitted from the decays of resonances, while the hydrodynamical evolution is restricted to describe the production of directly emitted particles that mix with the short-lived resonance decays whose life-time is comparable to the 1-2 fm/c duration of the freeze-out process. This thermal part of hadron production can be calculated from the exact solutions of relativistic hydrodynamics, but they should be corrected for the contribution of the decays of the long-lived resonances. According to the Core-Halo model [33], particle emission is divided into two parts. The Core corresponds to the direct production, which includes the hydrodynamic evolution and the short-lived resonances that decay on the time-scale of the freeze-out process. This part is responsible for the hydrodynamical behavior of the HBT radii and for that of the slopes of the single-particle spectra, a behaviour that is successfully describing the data shown in Figures 1, 3 as well as the shape of the pseudo-rapidity distributions on Fig. 2. On the other hand, the halo part consists of mostly particles, predominantly pions, emitted from the decays of long lived resonances. Exact hydrodynamic solutions, such as the CKCJ solutions can make predictions only for the time evolution of the core. According to the core-halo model of ref. [33] the normalization parameter of the pseudo-rapidity density, $\frac{dN}{d\eta_{p}}|_{\eta_{p}=0}$ can be corrected by a measurable core-halo correction factor, to get the contribution of the core from Bose-Einstein correlation measurements:

$$
\frac{dN_{\text{core}}}{d\eta_{p}}|_{\eta_{p}=0} = \sqrt{\lambda_{HBT}^{*}} \frac{dN}{d\eta_{p}}|_{\eta_{p}=0},
$$

(24)

where the newly introduced the intercept of the two-pion Bose-Einstein correlation function, $\lambda_{s}^{HBT}$ is taken from measurements of Bose-Einstein correlation functions. In principle this correction factor is transverse mass and pseudorapidity dependent. However, its transverse mass dependence is averaged out in the pseudorapidity distributions so we take its typical value at the average transverse mass, at midrapidity. In addition, the pseudorapidity dependence of this intercept parameter is, as far as we are aware of, not determined in Au+Au collisions at RHIC, given that the STAR and the PHENIX measurements of the Bose-Einstein correlation functions were performed at mid-rapidity, as both detectors identify pions around mid-rapidity only. In the core-halo model, its value is given as $\lambda_{s}^{HBT} = N_{\text{core}}^{2}/N^{2}$ and its average value is measured through the HBT or Bose-Einstein correlation functions for Au+Au $\sqrt{s_{NN}}=130 \text{ GeV}$ [28] and $\sqrt{s_{NN}}=200 \text{ GeV}$ [29] as well. The size of the overlap area of the colliding nuclei, $S_{\perp}$ is estimated...
by the Glauber calculations of [34], and the average thermalized transverse energy is estimated from the nearly exponential shape of the transverse momentum spectra in $m_T - m$ as follows:

$$\langle E_{T}^{th} \rangle = (m + T_{\text{eff}}) \left(1 + \frac{T_{\text{eff}}^2}{(m + T_{\text{eff}})^2}\right).$$

(25)

This estimate includes only the thermalized energy, which estimate corresponds to the CKCJ solution estimate after being embedded into 1+3 dimensions. Correspondingly, the energy density present in high $p_T$ processes is neglected by this formula and its applications yield a lower limit of the initial energy density. The measured values of the transverse energy production at mid-rapidity, however, include the effects of high transverse momentum processes from perturbative QCD. The difference between the two kind of initial energy densities, the thermalized source and the full initial energy density is illustrated as the difference between the left and the right panels of Figure 5.

Figure 4. Initial energy density estimates from the CKCJ solution are shown with solid lines and compared to Bjorken’s estimate, indicated with dashed lines, as a function of the initial proper time. The parameters of the left panel correspond to fit results of the CKCJ solution to PHOBOS Au+Au pseudorapidity density data in the 0-30 % centrality class both at $\sqrt{s_{NN}} = 130$ GeV (left panel) at $\sqrt{s_{NN}} = 200$ GeV (right panel).
Figure 5. Location of the 0-30 % central Au+Au collisions on the $(\lambda, dE_T/d\eta_p)$ diagram for $\sqrt{s} = 130$ and 200 GeV. The color code indicates the contours of constant initial energy densities, evaluated for the realistic $\kappa = 10$ equation of state, corresponding to the measured speed of sound $c_s^2 \simeq 0.1$ in these reactions. Conservatively, these contours are evaluated for a $\tau_f/\tau_0 = 10$ ratio of the final over initial proper-time. The left panel indicates the thermalized energy density while the right panel indicates the pseudorapidity density of all transverse energy, including energy in non-thermal, high transverse momentum processes.

Figure 6. Initial energy density estimates from CKCJ analytic solution are show with blue solid lines and compared to the initial energy estimate from a numerical calculation of Bozek and Wyskiel [35], indicated with red markers, as a function of the initial proper time. The curves of the CKCJ analytic solution correspond to the solid red curves in the right panel of Fig. 4. The left panel shows the comparison with logarithmic vertical scale and the right panel shows the same with linear vertical scale.

In Figure 4, one can see, that Bjorken’s formula underpredicts the initial energy density for both collision energies and that it predicts higher initial energy densities for higher collision energies, because the pseudorapidity density of the average transverse energy, $d\langle E_T \rangle/d\eta_p$ is a monotonously increasing function of the colliding energy $\sqrt{s_{NN}}$ for a given centrality class. However, an unexpected and really surprising feature is also visible on these plots: namely, our formula finds that for the lower collision energies of $\sqrt{s_{NN}} = 130$ GeV, the initial energy densities are higher, due to the larger acceleration and work effects, than at the larger colliding energies of $\sqrt{s_{NN}} = 200$ GeV! This unexpected behaviour is caused by an interplay of two different effects. Although the midrapidity density is increasing monotonically with increasing colliding energy, that would increase the initial energy densities with increasing energy of the collision, the acceleration and the related work effects decrease with increasing energies. Although both $dE/d\eta_p|_{\eta_p=0}$ and $\lambda$ change as a monotonic function of $\sqrt{s_{NN}}$, $\varepsilon_0$ is a non-trivial function of both $\lambda$ and...
\( \frac{dE}{d\eta} \big|_{\eta=0} \), so that the net effect is a decrease of the initial energy density with increasing colliding energies. Thus the CKCJ corrections of the Bjorken estimation have more significant effects in lower collision energy because of the greater acceleration and longer lifetime of the fireball.

This feature is clearly illustrated on both the left and the right panels of Fig. 5. The projection of the two data points to the vertical axis indicates that the transverse energy production is a monotonically increasing function with increasing \( \sqrt{s_{NN}} \) for Au+Au collisions in the same centrality class. This feature is independent of the usage of the thermalized energy density (left panel) or the total available energy density (right panel). The projection of the two points to the horizontal axis is also indicating a monotonously changing/flattening shape of the (pseudo)rapidity distribution with increasing \( \sqrt{s_{NN}} \). However, the contours of constant initial density are non-trivially dependent on both the vertical and the horizontal axes, and the location of the two points on the two-dimensional plot with respect to the contour lines indicates, that the initial energy density in \( \sqrt{s_{NN}} = 200 \) GeV Au+Au collisions is actually less than the initial energy density in the same reactions but at lower, 130 GeV colliding energies, in the same 0-30% centrality class.

The astute reader may think at this point that such a non-monotonic behaviour of the initial energy density as a function of the colliding energy may be due just to the 1+1 dimensional nature of the adopted CKCJ solution i.e. the neglect of the effects of transverse expansion on the transverse energy density and also due to the neglect of a possible temperature dependence of the speed of sound.

To cross-check and clarify these questions, in Fig. 6 we compared the CKCJ initial energy density estimate of Au+Au collisions at \( \sqrt{s_{NN}}=200 \) GeV in the 0-30% centrality class to the 1+3 dimensional numerical solution of the equations of relativistic hydrodynamics by Bozek and Wyskiel (BW) [35]. This numerical result was shown to reproduce the pseudorapidity density of charged particles in \( \sqrt{s_{NN}} = 200 \) GeV Au+Au collisions, using lattice QCD equation of state, with a temperature dependent speed of sound. Both of the analytic and numerical estimations are corrected for the decays of (long-lived) resonances, so Fig. 6 compares only the thermalized initial energy densities from the CKCJ analytic solution and from the BW numerical solution. The estimation of the analytic but 1 + 1 dimensional CKCJ solution is surprisingly similar to the numerical but 1 + 3 dimensional calculations as far as initial energy densities are concerned. The similarity of the evolution of the energy density in the center of the fireball in these two different hydrodynamical solution may be just a coincidence, given that ref. [35] did not detail the centrality dependence of the time evolution of the energy density. However, the centrality dependence is expected to influence the overall normalization predominantly, so the similar shape of the time dependence of the energy density in the CKCJ solution and in the calculations of ref. [35] may also indicate that the collective dynamics in the center of the fireball is predominantly longitudinal and that transverse flow effects do not substantially modify the pseudorapidity density of transverse energy at mid-rapidity, as conjectured by Bjorken in ref. [3].

Although the centrality classes and the estimations of the EoS dependence of the initial energy densities are somewhat different in the current work as compared to that of ref. [31], that paper provides an additional possibility for a cross-check. The results are summarized in Table 2. It is important to realize, that in ref. [31] the critical energy densities as well as their non-monotonic dependence on \( \sqrt{s_{NN}} \) are within errors the same as in the exact calculations presented in this paper, that can be seen from a comparison of the last two columns of Table 2, where the results in the last column correspond to on Fig. 4.

6. Summary

In this manuscript, we started to evaluate the excitation functions of the main characteristics of high energy heavy ion collisions in the RHIC energy range. The initial energy density and the lifetime of these reactions was estimated with the help of a novel family of exact and analytic, finite and accelerating, 1+1 dimensional solutions of relativistic perfect fluid hydrodynamics, found recently by Csörgő, Kasza, Csanád and Jiang [16]. With this new solution we evaluated the rapidity and the pseudorapidity densities
and demonstrated, that these results describe well the pseudorapidity densities of Au+Au collisions at $\sqrt{\text{NN}} = 130$ GeV and $\sqrt{\text{NN}} = 200$ GeV in the 0-30% centrality class.

From fits to these pseudorapidity distributions, we determined the acceleration parameter $\lambda$ as well as the effective temperature, the slope of the transverse momentum distribution $T_{\text{eff}}$. With the help of these values, we reduced the number of free fit parameters to one in the analytic expression from the CKCJ solution that describes the transverse mass dependence of the longitudinal HBT-radii. We have determined the freeze-out proper time $\tau_f$ from transverse mass dependent fits to PHENIX and STAR $R_{\text{long}}$ measurements. We have found that $\tau_f = 11.8 \pm 0.5$ fm at $\sqrt{\text{NN}} = 130$ GeV and $\tau_f = 10.2 \pm 0.3$ fm at $\sqrt{\text{NN}} = 200$ GeV Au+Au collisions in the 0-30% centrality class. These values were utilized to evaluate the initial energy densities, as a function of the initial proper time, for both collision systems. We compared our new, exactly calculated formula to Bjorken’s estimate and the results were quite surprising. The CKCJ solution and the corresponding data analysis finds higher initial energy densities in $\sqrt{\text{NN}} = 130$ GeV Au+Au collisions in the 0-30% centrality class as compared to the same kind of collisions at larger, $\sqrt{\text{NN}} = 200$ GeV colliding energies. Given that at the LHC energies the initial energy densities increase, as indicated on Table 2, our results suggest that the initial energy density is a non-monotonic function of the colliding energy at the RHIC energy range of $\sqrt{\text{NN}} \leq 200$ GeV, as the observed decrease of the initial energy density from $\sqrt{\text{NN}} = 130$ GeV to 200 GeV collisions is an indication of a non-monotonic behaviour, that has a limited range. It is thus important and urgent to map out the dependence of the initial energy density of high energy heavy ion collisions on the colliding energy for all currently available data sets.

The detailed evaluation of the excitation function of the initial energy density may also become a new tool to search for the critical point of the QCD phase diagram, given that in the vicinity of this critical point, several observables may behave in a non-monotonic manner, including life-time related observables, as recently pointed out in ref. [36].

In order to refine our the resolution of our “femtoscope” further, and in order to deepen our understanding, the generalizations of the CKCJ solution to the cases of i) a temperature dependent speed of sound, ii) a one plus three dimensional expansion, iii) viscous solutions with shear and bulk viscosity terms are being explored at the time of closing this manuscript. Further comparisons to numerical calculations may also help to clarify or refine our results, that depend critically on the description of the finite longitudinal extension, namely on the shape of the pseudorapidity distribution, where the CKCJ solution does an excellent job.

References
1. E. Fermi, Prog. Theor. Phys. 5, 570 (1950).
2. L. D. Landau, Izv. Akad. Nauk Ser. Fiz. 17, 51 (1953).
3. J. D. Bjorken, Phys. Rev. D 27, 140 (1983).
4. N. M. Agababyan et al. [EHS/NA22 Collaboration], Phys. Lett. B 422, 359 (1998) [hep-ex/9711009].
5. I. Arsene et al. [BRAHMS Collaboration], Nucl. Phys. A 757, 1 (2005) [nucl-ex/0410020].
6. K. Adcox et al. [PHENIX Collaboration], Nucl. Phys. A 757, 184 (2005) [nucl-ex/0410003].
7. B. B. Back et al., Nucl. Phys. A 757, 28 (2005) [nucl-ex/0410022].
8. J. Adams et al. [STAR Collaboration], Nucl. Phys. A 757, 102 (2005) [nucl-ex/0501009].
9. L. Adamczyk et al. [STAR Collaboration], Nature 548, 62 (2017) [arXiv:1701.06657 [nucl-ex]].
10. C. Aidala et al. [PHENIX Collaboration], arXiv:1805.02973 [nucl-ex], published in Nature, Dec. 10, 2018.
11. C. Aidala et al. [PHENIX Collaboration], Phys. Rev. Lett. 120, no. 6, 062302 (2018) [arXiv:1707.06108 [nucl-ex]].
12. J. L. Nagle and W. A. Zajc, Ann. Rev. Nucl. Part. Sci. 68, 211 (2018) [arXiv:1801.03477 [nucl-ex]].
13. J. Adam et al. [ALICE Collaboration], Nature Phys. 13, 535 (2017) [arXiv:1606.07424 [nucl-ex]].
14. F. Nemes, T. Csergő and M. Csanád, Int. J. Mod. Phys. A 30, no. 14, 1550076 (2015) [arXiv:1505.01415 [hep-ph]].
15. R. Derradi de Souza, T. Koide and T. Kodama, Prog. Part. Nucl. Phys. 86, 35 (2016) [arXiv:1506.03863 [nucl-th]].
16. T. Csergő, G. Kasza, M. Csanád and Z. Jiang, Universe 4 (2018) 69 [arXiv:1805.01427 [nucl-th]].
17. T. Csörgő, G. Kasza, M. Csanád and Z. F. Jiang, arXiv:1806.06794 [nucl-th].
18. G. Kasza and T. Csörgő, arXiv:1806.11309 [nucl-th].
19. T. Csörgő and G. Kasza, arXiv:1810.00154 [nucl-th].
20. T. Csörgő and B. Lörstad, Phys. Rev. C 54, 1390 (1996)
21. T. Csörgő, M. I. Nagy and M. Csanád, Phys. Lett. B 663, 306 (2008) [nucl-th/0605070].
22. M. I. Nagy, T. Csörgő and M. Csanád, Phys. Rev. C 77, 024908 (2008)
23. A. N. Makhlin and Y. M. Sinyukov, Z. Phys. C 39 (1988) 69.
24. B. Alver et al. [PHOBOS Collaboration], Phys. Rev. C 83 (2011) 024913 [arXiv:1011.1940 [nucl-ex]].
25. T. Csörgő, S. V. Akkelin, Y. Hama, B. Lukács and Y. M. Sinyukov, Phys. Rev. C 67, 034904 (2003) [hep-ph/0108067].
26. T. Csörgő and G. Kasza, Universe 4 (2018) no.4, 58 [arXiv:1801.05716 [nucl-th]].
27. A. Adare et al. [PHENIX Collaboration], Phys. Rev. Lett. 98, 162301 (2007) [nucl-ex/0608033].
28. K. Adcox et al. [PHENIX Collaboration], Phys. Rev. Lett. 88 (2002) 192302 [nucl-ex/0201008].
29. S. S. Adler et al. [PHENIX Collaboration], Phys. Rev. Lett. 93 (2004) 152302 [nucl-ex/0401003].
30. M. I. Gorenstein, Y. M. Sinyukov and V. I. Zhdanov, Phys. Lett. 71B (1977) 199. doi:10.1016/0370-2693(77)90777-8
31. J. Ze-Fang, Y. Chun-Bin, M. Csanád and T. Csörgő, Phys. Rev. C 97 (2018) no.6, 064906 [arXiv:1711.10740 [nucl-th]].
32. T. Csörgő, M. I. Nagy and M. Csanád, J. Phys. G 35, 104128 (2008) [arXiv:0805.1562 [nucl-th]].
33. T. Csörgő, B. Lörstad and J. Zimányi, Z. Phys. C 71 (1996) 491 [hep-ph/9411307].
34. M. L. Miller, K. Reygers, S. J. Sanders and P. Steinberg, Ann. Rev. Nucl. Part. Sci. 57 (2007) 205 [nucl-ex/0701025].
35. P. Bozek and I. Wyskiel, Phys. Rev. C 79 (2009) 044916 [arXiv:0902.4121 [nucl-th]].
36. R. A. Lacey, Phys. Rev. Lett. 114, no. 14, 142301 (2015) [arXiv:1411.7931 [nucl-ex]].

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