1. INTRODUCTION

After a century of development, during which the microscopic foundations of transport kinetics were established as one of the great achievements of modern physics, the last decade and a half has seen fresh, indeed revolutionary, progress in the understanding of electrical conduction. This progress has never been so rapid as in the mesoscopic realm. It can be said that, in the place of more traditional microscopic perspectives, a succession of novel, simpler, and more succinct approaches have arisen to bolster our notions of mesoscopic transport physics.

The guiding spirit of this new philosophy is rightly identified as Rolf Landauer. Other pioneers, such as Büttiker, Beenakker, and Imry, have been foremost in deepening and extending Landauer’s phenomenological insights into their final, contemporary form [1-5]. The astonishing success of these highly influential mesoscopic models elicits two questions:

(i) what is the relationship between the “new” viewpoint of electron transport and the established, microscopic one?

(ii) Are there reasons to continue placing confidence in the “old” microscopic theory as the source of fresh insights into mesoscopics?

Our review proposes answers to these questions. In the first instance, an objective assessment of the principles underlying the physics of mesoscopic transport leads us to state – with some assurance – that conservative microscopic methods remain the only reliable basis for solving mesoscopic problems. For example, Fermi-liquid theory [6, 7] readily accounts for some major and puzzling experimental results in quantum point contacts. Second, a fully conserving approach lets one predict new effects to be sought out in new experiments, using available methods and device structures. These effects are not addressed by recently developed phenomenologies.

The credibility of any mesoscopic model, new or old, hinges above all upon its respect for the conservation laws. A mesoscopic theory that violates these basic
statements does not make physical sense. It hardly matters if such a description were to
claim for itself some special sort of compactness, simplicity, and intuitive appeal. If it cannot secure conservation, nothing else can commend it.  

The centrality of energy, charge, and number conservation [6] will form the connecting thread of our review. In Section 2 we briefly recall the recent history of mesoscopics, setting the stage for our microscopically based critique; this analysis rests on the close relationship between microscopic conservation and the open-boundary conditions dominant in mesoscopic electron transport. In Sec. 3 we apply that knowledge to an exemplary case: the conductance of a one-dimensional quantum point contact (QPC). We will see how Landauer’s celebrated expression for quantized conductance emerges straight from the standard Kubo-Greenwood theory [8-10] applied to an open mesoscopic wire, in which elastic and dissipative scattering are both vital elements. The theme is filled out, in Sec. 4, in our kinetic description of nonequilibrium current fluctuations (noise) of a driven quantum point contact. Noise and conductance are intimately linked: each manifests the two-body electron-hole dynamics at the heart of metallic conduction [6, 11]. We discuss the resolution of a long-standing experimental problem involving the excess noise of a QPC [12, 13], which other models have been unable to address. This has far-reaching implications for the way in which measurements of fluctuation phenomena are interpreted to reveal the microscopic details of low-dimensional electron transport. In Sec. 5 we foreshadow some novel, testable consequences of the physics of the preceding Section. Finally, we gather our thoughts in Sec. 6.

2. LANDAUER THEORY: A SHORT HISTORY

In 1957, Rolf Landauer published a different and – to some of the leading transport gurus of the epoch – subversive interpretation of metallic resistivity [14]. Landauer envisioned the current, rather than the electromotive voltage, as the stimulus by which resistance is manifested [15]. The measured voltage is simply the macroscopic effect of the carriers’ inevitable encounters with the localized scattering centers within a conductor. Around any such scatterer the carrier flux resembles a phenomenological “diffusive” flow, set up by the density difference between the upstream and downstream electron populations. In this purely passive scenario, energy dissipation does not enter.

The Landauer theory describes electron transport in an environment of scatterers that are purely elastic. As such it is not able to address the dynamical mechanisms of energy dissipation that characterize transport. Yet the Landauer theory, like any other description of conductance, must satisfy the fluctuation-dissipation (Johnson-Nyquist) theorem at some level. If it did not do so, the theorem must be force-fit – by hand – to the model. The fluctuation-dissipation theorem implies that dissipation is always present whenever there is resistance. It is an inescapable element in every theory of conductance, be it microscopic or phenomenological. Landauer’s conceptual model assigns no role at all to inelastic dissipation in its handling of scattering-mediated conductance.

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1 One recalls Einstein’s sharpened form of Occam’s Razor: “Everything should be made as simple as possible, but not simpler.”
The Landauer formula for the conductance $G$ of a quantum point contact is

$$G = \frac{2e^2}{h} T.$$  \hspace{1cm} (1)

This version of the formula prescribes the outcome of a “two-point” measurement; that is, the bulk source and drain leads (needed to supply current to the QPC) are the points between which the voltage as well as the current is measured. For an ideal ballistic conductor the transmission coefficient $T$ is unity. In the presence of coherent backscattering off the contact, or of inelastic scattering – or both – the conductance is nonideal since $T < 1$. Note, however, that the Landauer model provides no guidance for computing $T$ in the case of coherent elastic scattering. Moreover the obvious possibility of inelastic scattering remains permanently out of scope, by construction.

At this point we slightly anticipate what is to come. Eq. (1) conceals a tantalizing conundrum: how, exactly, can a driven system shed its excess energy in the Landauer picture? A mesoscopic channel with conductance given by Eq. (1) should dissipate the electrical power supplied to it at the rate

$$P = IV = \frac{2e^2}{h} T V^2.$$  \hspace{1cm} (2)

Yet, to drive this dissipation, there is no dynamical mechanism that can be identified within the object $T$, central to the accepted picture. How can this be? The process of transmission is supposed to be purely coherent (elastic). Elastic collisions conserve energy; they are nondissipative.

This means that Eq. (2) can never be more than an added-on hypothesis in the Landauer view of transport; one that is devoid of supporting reason beyond the obviously heuristic observation that finite conductances dissipate finite electrical energy. Therefore, because of the complete absence of a microscopic nexus between the Landauer formula and the fluctuation-dissipation relation, Eq. (2), there is an enormous conceptual gap that needs healing. Such a gap cannot be closed within the scheme’s internal logic. To resolve the dilemma we have no option but to go back to the pre-Landauer understanding of quantum transport.

For all the succinctness and empirical success of Landauer’s conception, by his own recounting [16] the theory languished until its rediscovery and reinterpretation by a fresh, bold generation of transport physicists. The main early objection seemed to be his emphasis on the localized action of the scattering impurities in resisting current flow, at a time when the theoretical dogma held that local effects could never be individually probed; all that one could (and should) do was to compute a spatial average over ensembles of samples, with a spread of scattering-center distributions [17].

All this changed radically in the ’eighties, with the advent of truly mesoscopic sample fabrication. It was now possible to study not bulk, and hence coarsely grained assemblies, but individual samples with individual spatial arrangements of scatterers. Moreover, the phase coherence of the carriers could now be preserved over the much shorter lengths of the samples, bringing to the fore the effects of quantum transmission.
Landauer’s criticism of bulk averaging was entirely justified; that particular approximation is no longer meaningful for truly mesoscopic structures. However, the breakdown of bulk averaging is, as we have said, at a very coarse-grained level. It does not impact upon the fundamental structure of statistical mechanics with its microcanonical averaging in configuration space, over identical runs for the system’s response (or, alternatively, over a set of identical copies of the device: ergodicity). The whole of quantum kinetics is built on the microscopic ensemble procedure, which is universally applicable at all length scales and dimensionalities [18].

The applicability of statistical mechanics and kinetics to mesoscopic transport is clearly not subject to Landauer’s restricted argument against bulk spatial averaging. His reconsideration of its limitations carries no physical reason to bypass – let alone supplant – traditional microscopic methods in favor of more conjectural treatments. One such traditional method, which we will apply below in a new experimental context, is the Landau-Silin quantum-transport equation for metallic electrons,

Despite the circumscribed nature of Landauer’s critique, there seems to be an informal but widespread perception that his quasi-diffusive picture (dressed in a single-particle interpretation of coherent transmission) supersedes, as it were, the collective efforts of kinetic theorists through the preceding century of research.\(^2\) Granted, the old microscopic ways are too labor-intensive to meet modern demands for a prolific research output. Further, the folklore has it that microscopics can, at best, only confirm and enshrine Landauer’s far more compact phenomenology. In any case, it is now believed that mesoscopic transport occurs exclusively by elastic coherent transmission, to all intents dissipation-free (even when the fluctuation-dissipation theorem explicitly says otherwise). It is as if the reality of inelastic processes beyond that horizon were suddenly inoperative.

The point is not to dwell on the achievements of the past, nor to speculate on the durability of present phenomenologies (that is a suitable case for Occam’s Razor). Our aim is quite practical. The minimum requirement for transport models is that they be conserving. Physical credibility must be tested, not by fashion or expediency, but by the same objective criteria against which theories have always been – and will always be – judged. Let us put the new mesoscopics to the test.

Do quasi-diffusive models genuinely bring a novel understanding to mesoscopics, an understanding that could not be reached through the canonical and conservative methods of kinetic theory? Do quasi-diffusive models genuinely conform to the conservation laws that apply to open mesoscopic conductors? The answers to these issues may surprise some readers.

3. THE LANDAUER FORMULA, WITHOUT PHENOMENOLOGY

In the literature there are already numerous attempts to derive and hence justify Landauer’s phenomenological conductance formula [1, 4]. This is natural, given

\(^2\) Such an impression was never put about by Landauer who, in the abstract for the reprint of his seminal 1957 paper in J. Math. Phys. 37, 5269 (1996), interpolated this forthright observation: “...[The] IBM Journal of Research and Development, is not all that easily located in 1996. As a result the frequent citations to it often assign content to that paper which does not agree with reality.”
that the empirical success of the formula is as compelling as its detailed microscopic underpinning is unclear. Its true underpinning will be exhibited below.

The best-known derivations of Landauer’s result mostly start with the many-body Kubo-Greenwood conductance formula, which is our starting point as well. However, those derivations tend to rely on the same assumptions about boundary conditions (and the absence of recognizably dissipative processes) that were made by Landauer himself. As reputedly microscopic confirmations, they beg the question of logical circularity and their dependence on the results that they aim to prove.

The key to mesoscopic transport theory is knowing how to handle, in a fully conserving way, the physical interactions between the open device and the macroscopic environment, mediated by its interfaces. Before turning to the microscopic Kubo-Greenwood theory and its straightforward recovery of Landauer’s result, we recall some basic facts about the boundary conditions for an open electronic conductor.

In mesoscopic transport the role of the macroscopic metallic leads connected to the sample is paramount. The leads, or reservoirs, will

(a) confine all electric fields, be they external or inbuilt, to a well-defined region comprising the device and its interfaces with the leads (namely, the connected region where carriers are appreciably disturbed by the applied current);

(b) ensure the permanent neutrality of the device with its interfaces, irrespective of the applied current (this is the consequence of strong Thomas-Fermi screening by the conduction electrons in the leads); and

(c) pin the asymptotic state, to which the driven carriers in the active volume will tend, to the permanently stable unchanging (equilibrated and electrically neutral) local distribution in each reservoir.

These conditions, almost self-evident for metallic conduction, are common to all transport models including those of Landauer type. They also correspond to the laboratory set-up required in all measurements of conductance.

\textit{a. The Kubo-Greenwood Formula}

Under the open-boundary conditions above, it was proved by Sols [10] that the Kubo-Greenwood conductance formula holds in a form absolutely identical to the well known case of a closed electronic conductor with periodicity. (An independent but physically equivalent proof was given by Magnus and Schoenmaker [19]). There is a crucial proviso: the global gauge invariance (charge conservation) of the open-system Kubo-Greenwood formula is guaranteed \textit{if and only if} an external generator actively injects and extracts the external current that energizes the open mesoscopic device.

The physical need for active sources and sinks of current in the problem is, then, mandated by charge conservation. It is a fundamental result that should be compared with the very different requirement of the Landauer model for an open conductor. There, it is assumed that a phenomenological mismatch of carrier densities in the leads “drives”, entirely passively, the diffusive-like transfer of electrons from the nominally high-density source lead to the nominally low-density drain lead (while the quantum transmission of the individual carriers regulates the net diffusion rate).
The hypothesis of quasi-diffusive transport stands in clear contrast with condition (c) above. We note that this already hints at some internal contradiction, since (c) involves the asymptotic neutrality of the leads. That transport-independent condition is still required for the Landauer model to remain electrically stable.

The Kubo-Greenwood conductivity is

$$\sigma(t) = \frac{ne^2}{m^*} \int_0^t C_{vv}(t) dt,$$

where the velocity-velocity correlation function is

$$C_{vv}(t) = \frac{\langle v(t)v(0) \rangle}{\langle v(0)^2 \rangle} \sim \exp(-t/\tau_m);$$

the expectations are taken in the equilibrium state (giving linear response) over the full many-body density matrix for the assembly of active carriers in the channel\(^3\), and \(\tau_m\) characterizes the dominant decay of the correlations. The decay parameter includes, on an equal footing, the microscopic contribution from every physically relevant collision mechanism \([8, 9]\). In the long time limit,

$$\sigma \to \frac{ne^2\tau_m}{m^*}.$$  \(\text{(5)}\)

This is the venerable Drude conductivity.

In one dimension, appropriate to a QPC, the density in terms of Fermi velocity is \(n = 2m^*v_F/\pi\hbar\). The conductance over a sample of length \(L\) then becomes

$$G \equiv \frac{\sigma}{L} = \frac{2m^*v_F e^2}{\pi\hbar m^* L} \tau_m = \frac{2e^2}{\hbar} \left( \frac{2v_F}{L} \tau_m \right) = \frac{2e^2}{\hbar} T_{KG},$$

in which the transmission coefficient \(T_{KG}\) (KG for Kubo-Greenwood) is now proportional to the ratio of the effective scattering length \(v_F \tau_m\) to length \(L\).

All of the dissipative, many-body scattering effects have been embedded within \(\tau_m\), as well as all the one-body coherent potential and elastic impurity scattering. The interface physics is incorporated into the microscopic KG conductance formula, as fully and directly as the physics of the device itself.\(^4\)

There is nothing in Eq. (6) to forestall its conformity with the Landauer formula. To be sure, Eq. (1) is about to appear as a particular case. Nevertheless, unlike the received derivations of the latter, the KG formula does not call for hand-waving

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\(\text{3}\) The expectations in \(C_{vv}\) are determined by the electron-hole excitations in the structure. In its very essence \(C_{vv}\) is a correlated many-body quantity, impossible to represent in pure terms of independent single-particle properties.

\(\text{4}\) In contrast with the Landauer formula, there is no need to graft the fluctuation-dissipation theorem onto this formula as a heuristic appendage; the open-system conductivity equation (3) is the theorem. Therefore no “supplementary hand-waving” \([15]\) – no heuristics of any sort – is required to make Eqs. (3)–(6) mesoscopically legitimate. They are valid from within.
Figure 1. An ideal, uniform ballistic wire. Its diffusive leads (S, D) are at equilibrium. A paired source and sink of current $I$ at the boundaries drive the transport. The boundary regions, separated by distance $L$, are loci for both inelastic and elastic scattering. Local charge clouds (shaded), induced by the influx and efflux of $I$, generate the resistivity dipole potential $V = E(I)L$ between D and S.

supplementations that favor elastic processes over the inelastic ones that are equally important in real physical situations. We now demonstrate just how vital inelastic scattering is for the Landauer formula itself, by constructing its correct microscopic origins.

b. The Landauer Formula Redux

Figure 1 displays the nonequilibrium transport of current through a conductor, in the present case a quantum point contact. Note the arrangement whereby an external generator imposes current flow, while the outlying macroscopic reservoirs remain permanently undisturbed at their fixed, equilibrium electron densities (and their fixed local chemical potentials). That is,

- the current through an open conductor cannot be a function of the asymptotic states in the leads, and vice versa.

The immunity of the lead states to transport is the general consequence of global charge conservation in any open device [10]. It is to be contrasted with the very different boundary condition invoked by phenomenological prescriptions: current is the quasi-diffusive flow of carriers between reservoir leads when their nominal chemical potentials, and hence their nominal densities, become mismatched in the presence of a source-drain voltage. Such a statement would clearly contradict global gauge invariance, since it posits a direct functional link between the presence of the current and the mismatch in reservoir populations, set up to sustain it.

The existence of macroscopically large reservoirs outside the active region sets limits on both the elastic and inelastic mean free path (MFP). These will be finite in range even if the channel’s mesoscopic core is ballistic and perfectly collisionless. The presence of defects in the leads put an upper bound on the elastic MFP. More to the point, the dynamical effect of the external sources and drains for current sets the same essential bound on the inelastic and elastic MFP. When nonequilibrium effects, say phonon emission, set in as the current is increased, the inelastic MFP will become small compared with the elastic one, which remains practically unchanged.

We suppose that the ballistic channel has an operational length $L$. This is identifiable with its elastic MFP. If the characteristic mean speed of its carriers is
$v_{av}$, then the elastic collision time is

$$\tau_{el} = \frac{L}{v_{av}}. \quad (7a)$$

On the other hand, in a ballistic conductor the inelastic scattering time cannot exceed $\tau_{el}$ as a limiting value:

$$\tau_{in} \leq \frac{L}{v_{av}}. \quad (7b)$$

The two types of scattering are spatially coextensive, but temporally independent in their action. Therefore Matthiessen’s rule holds, giving

$$\frac{1}{\tau_m} = \frac{1}{\tau_{in}} + \frac{1}{\tau_{el}}. \quad (8)$$

When the form for $\tau_m$ is substituted in Eq. (6), the Kubo-Greenwood form for the transmission factor $T_{KG}$ becomes

$$T_{KG} = \left(\frac{2v_F}{L}\tau_{el}\right) \frac{\tau_{in}}{\tau_{el} + \tau_{in}} \equiv \frac{v_F}{v_{av}} \frac{2\zeta}{1 + \zeta}, \quad (9a)$$

where the collision-time ratio is

$$\zeta = \frac{\tau_{in}}{\tau_{el}} \leq 1. \quad (9b)$$

The result of Eq. (9) is shown in Fig. 2 overleaf. The conductance of a QPC, in this case consisting of two well separated one-dimensional subbands, exhibits the classic Landauer quantization in units of $G_0 = 2e^2/h$ as the density of mobile electrons in the QPC is swept upwards, and the chemical potential $\mu$ successively crosses the subband thresholds $\varepsilon_1$ and $\varepsilon_2$. There are two cases of interest.

(a) **Classical limit.** When the density in one of the subbands $i = 1, 2$ is very low we have $\mu - \varepsilon_i \ll k_B T$ and $v_{av} \to \sqrt{2m^*k_B T}$, the thermal velocity of Maxwellian particles. Eq. (5) together with (9a) gives the conductance contribution in subband $i$

$$G_i = G_0 \frac{hn}{4m^*v_{th}} \frac{2\zeta}{1 + \zeta} \sim e^{-(\varepsilon_i - \mu)/k_B T}. \quad (10)$$

This contribution vanishes as the subband becomes depleted.

(a) **Degenerate limit.** When the subband electron density is high, we have $\mu - \varepsilon_i \gg k_B T$ and $v_{av} \to v_F$. Then

$$G_i = G_0 \frac{2\zeta}{1 + \zeta}. \quad (11)$$

The kinematic ratio $v_F/v_{av}$ goes to unity, and the conductance reaches a plateau exactly as for the phenomenological Landauer formula.
Figure 2. Conductance of a one-dimensional quantum point contact, computed with the fully microscopic Kubo-Greenwood model of Eqs. (6)–(9). We show $G$ scaled to the Landauer quantum $G_0$, as a function of chemical potential $\mu$ in units of thermal energy [20]. $G$ exhibits strong shoulders as $\mu$ successively crosses the subband energy thresholds (here at $\varepsilon_1 = 5k_B T$ and $\varepsilon_2 = 17k_B T$). Well above each threshold, the subband electrons are strongly degenerate and the conductance tends to a well defined quantized plateau. Much below each threshold, the population and its contribution to $G$ vanish as $e^{-(\varepsilon_i - \mu)/k_B T}$. Solid line: $G$ in the ideal ballistic regime, for which the collision-time ratio $\tau_{\text{in}}/\tau_{\text{el}}$ is unity. Chain line: non-ideal case for $\tau_{\text{in}}/\tau_{\text{el}} = 0.75$. Note how the increased inelastic scattering brings down the plateaux. Dotted line: the case of $\tau_{\text{in}}/\tau_{\text{el}} = 0.5$. The departure from ideality is now substantial.

For perfect ballistic transport in a degenerate QPC (a highly idealized limiting case) one has $\zeta = v_F/v_{\text{av}} = \tau_{\text{KG}} = 1$. When $\zeta$ is at its maximum of unity, the elastic and inelastic scattering times are exactly matched. Each contributes equally to the visible transport behavior, yielding the original Landauer result:

$$G_i = G_0.$$  

Thus his formula is recovered strictly from the fundamental and much more general KG description, for the case of an open one-dimensional ballistic channel.

Our microscopic derivation of the Landauer formula, Eq. (1), makes no appeal to Landauer’s phenomenological premises. Up to now, these assumptions have been regarded as indispensable to the physics of conductance quantization. We stress that Eq. (1) emerges naturally and directly from a fully conserving microscopic treatment. This starts with the Kubo-Greenwood formula underpinned by strict boundary-condition requirements, imposed upon the physics by the gauge invariance of an open electrical conductor.

What now of the conundrum we met in the previous Section? The conceptual dilemma that has always confronted the Landauer picture, namely how to conjure dissipation out of a strictly dissipationless conductance, simply vanishes away. For,
the detailed physical processes that lead directly to inelasticity and dissipation are fully included in the microscopic Eq. (6) side by side with the elastic processes, as a matter of course.

Unlike the Landauer formula, the KG formula tells us how to compute the conductance in explicit terms of the microscopic electron-hole pair excitations of the system. That is because Eq. (3) embodies the fluctuation-dissipation theorem, in which the magnitude of the excitations (setting the loss rate) is modified by the effects of all sources of scattering on the propagating electron-hole pair. For an open device, it follows that dissipative and elastic mechanisms both contribute to the structure of the current fluctuations, which themselves fix the total collision time $\tau_m$, the transmissivity $T_{KG}$, and finally the conductance $G$ as measured in the laboratory.

Microscopically it makes little sense to rank either one of the two modes of scattering, inelastic and elastic, as more “physical” than the other in some subjective way. It makes even less sense to neglect inelastic scattering altogether in mesoscopics, as is centrally assumed in diffusive-like approaches. To the contrary we have demonstrated that

- within a strictly conserving framework, both elastic and inelastic scattering are needed to for the proper (many-body) description of transport in an open mesoscopic structure.

Figure 2 further shows that increasing the inelasticity in the system ($\zeta < 1$) makes the ballistic QPC nonideal. While preserving the quantized-step character of $G$ as a function of electron density, the size of the steps undershoots the “perfect” value $G_0$. Observationally, however, one cannot tell from such plots alone whether the transmissivity $T_{KG}$ really falls below unity because stronger inelasticity overcomes the elastic processes, or (conceivably) because coherent backscattering reduces the elastic relaxation time $\tau_{el}$ itself while inelasticity plays no role at all. The phenomenological understanding of nonideality advances the latter alternative.

From the microscopic standpoint, we have already seen that there are powerful reasons, based on the conservation laws, to assert the first possibility over the second. To uniquely resolve the issue of nonideal transport in a QPC requires a deeper look at carrier dynamics as revealed by studies of current fluctuations, and the noise that provides their experimental signature. We now turn to the microscopic behavior of fluctuations in a quantum point contact.

4. QPC NOISE: NEW RESULTS FROM KINETIC ANALYSIS

We can easily quantify the open boundary conditions (a)–(c) of Sec. 3. First: the active region, made up of the driven channel and its disturbed interfaces with the equilibrated leads (the interfaces are also the loci of current injection and removal), occupies a finite volume $\Omega$. Metallic screening of the fields internal to $\Omega$ means that it remains independent of the externally applied current $I$. Second: let the total number of mobilized electrons in $\Omega$ be $N$. Since global neutrality holds, the total electronic charge $-eN$ within $\Omega$ is always compensated by its positive ionic background, regardless of how much current is applied. Hence
\[
\frac{d \Omega}{d I} = 0 = \left. \frac{\partial N}{\partial \mu} \right|_\mu. \tag{12}
\]

\textit{a. Boundary Conditions on Mesoscopic Fluctuations}

In terms of the local distribution of electrons \( f_k(r, t) \) in wavevector space \( k \) and real space \( r \), Eq. (12) immediately implies that

\[
\int_\Omega d^\nu r \int \frac{2d^\nu k}{(2\pi)^\nu} f_k(r, t) = N = \int_\Omega d^\nu r \int \frac{2d^\nu k}{(2\pi)^\nu} f_{eq}^k(r) \tag{13}
\]

for all \( I \) and all times \( t \), and where \( f_{eq}^k(r) \) is the equilibrium distribution within \( \Omega \). The dimensionality of the system is \( \nu \). Note that Eqs. (12) and (13) do \textit{not} mean that \( N \) is fixed once and for all, and cannot be changed. If, for example, we alter the thermodynamic conditions so that the chemical potential changes by \( \delta \mu \), the local electron distribution in the neutral leads will change its density accordingly to \( n(\mu + \delta \mu) \) (the leads’ positive background, of course, must also change to compensate). It follows that both sides of Eq. (13) are altered by an identical amount:

\[
\int_\Omega d^\nu r \int \frac{2d^\nu k}{(2\pi)^\nu} \delta f_k(r, t) = \delta N = \int_\Omega d^\nu r \int \frac{2d^\nu k}{(2\pi)^\nu} \delta f_{eq}^k(r). \tag{14}
\]

This is the perfect-screening sum rule [6], expressed for an open system bounded by its large metallic leads. Global neutrality (gauge invariance) guarantees it.

Equation (14) has a striking corollary for the dynamic fluctuations of an open mesoscopic conductor, and it is this result that holds the key to our new interpretation of quantum-point-contact noise and our new predictions for it. Recall that the local, mean-square number fluctuation in the free electron gas is given by

\[
\Delta f_{eq}^k(r) \equiv k_B T \frac{\partial f_{eq}^k}{\partial \mu}(r) = f_{eq}^k(r)(1 - f_{eq}^k(r)).
\]

Denoting by \( \Delta f_k(r, t) \) the corresponding mean-square number fluctuation in the driven channel, our key result is that

\[
\int_\Omega d^\nu r \int \frac{2d^\nu k}{(2\pi)^\nu} \Delta f_k(r, t) = k_B T \frac{\partial N}{\partial \mu} = \int_\Omega d^\nu r \int \frac{2d^\nu k}{(2\pi)^\nu} \Delta f_{eq}^k(r). \tag{15}
\]

The consequences of the gauge-invariant boundary conditions for the fluctuations themselves – and thus the noise – now become clear for a mesoscopic device: Eq. (15) means that

- the total fluctuation strength within the active region of an open mesoscopic channel is perfectly independent of the applied current.
b. Compressibility Sum Rule for Open Systems

In the physics of the closed electron gas, the compressibility sum rule is a well-known identity [6]; all viable models of such a system must satisfy it. We present it here in a new form, generalized to the open electron gas of a mesoscopic conductor, such as a QPC to which we will apply it.

Recall the thermodynamic definition of the electron-gas compressibility $\kappa$:

$$\kappa \equiv \Omega \frac{\partial N}{N^2 \partial \mu} \bigg|_{\Omega,T} = \frac{\Omega}{Nk_B T} \frac{\Delta N}{N},$$

(16)

where $\Delta N$ is the total mean-square fluctuation over the region $\Omega$. The compressibility is the inverse of the average energy density of the system. If it is large, the system fluctuates readily. If $\kappa$ is small, the system is stiff and its fluctuations are suppressed.

Empirically, $\kappa$ can be obtained from measurements of the sound velocity in the electron gas. Eq. (16) relates its value to the (calculable) magnitude of the microscopic fluctuations of the carriers. Note the close analogy with the fluctuation-dissipation theorem. There, the empirical conductance $G$ is related to the magnitude of the current fluctuations in the system.

We know that the constraints imposed by the open-boundary conditions, Eqs. (13) to (15), mean that every quantity on the right-hand side of the compressibility relation is independent of the current $I$. The open-system compressibility sum rule follows directly:

- in an open mesoscopic channel, the electronic compressibility is perfectly independent of the applied current and is given by Eq. (16).

This statement has profound implications for the proper description of mesoscopic transport. Since it is a strict identity, which originates in the charge-conserving structure of an open channel together with its interfaces, the generalized compressibility sum rule has the same canonical importance as the fluctuation-dissipation theorem.

To be meaningful and credible, a model of mesoscopic transport and noise must satisfy not only the fluctuation-dissipation theorem, but the nonequilibrium compressibility sum rule as well. We will not discuss here how the Landauer-like theories of mesoscopic noise fare with regard to this principle; interested readers can find our analysis in Reference [21]. Such theories do not respect the compressibility sum rule. Therefore the basis for their conclusions about fluctuation properties is questionable.

c. General Consequences for Noise

Equipped with the knowledge of Eq. (16), we can make some powerful inferences about the structure of the fluctuations in a quantum point contact, when it is driven away from equilibrium. In the limit of low density in the channel, the carriers are classical. Then $\Delta N \rightarrow N$ and

$$\kappa \rightarrow \kappa_{cl} = \frac{1}{n k_B T}.$$  

(17a)

In the opposite limit, the electrons are highly degenerate. Since $n = 2m^* v_F / \pi \hbar \propto$
\[
\sqrt{\mu - \varepsilon_i} \text{ for subband } i, \text{ then}
\]

\[
\kappa \rightarrow \kappa_{\text{cl}} \left( \frac{k_B T}{n} \frac{\partial n}{\partial \mu} \right) = \kappa_{\text{cl}} \frac{k_B T}{2(\mu - \varepsilon_i)} \ll \kappa_{\text{cl}}. \tag{17b}
\]

Not surprisingly the Pauli exclusion principle strongly inhibits the scale of the fluctuations as the compressibility becomes negligible, in comparison with a classical system at the same density.

This tells us that degeneracy must also have an enormous effect on the scale of the noise exhibited by a QPC (or any mesoscopic device, for that matter). At high density the noise must scale as in Eq. (17b), no matter how hard the channel is driven. That constraint is enforced by the compressibility sum rule. We therefore expect that the expression for nonequilibrium noise in the QPC must always carry a proportionality to the ratio \( \kappa / \kappa_{\text{cl}} \). A noise formula that behaved otherwise would immediately advertise its violation of the compressibility sum rule [21].

\textit{d. Nonequilibrium Noise of a QPC}

Our treatment of QPC transport and noise begins with the Boltzmann-Landau-Silin transport equation [7] for electrons in a one-dimensional channel. Since a qualitative appreciation of the sum-rule behavior is enough to interpret the results, we merely sketch the basic makeup of the theory and point readers to the full technical details in Refs. [11], [13], and [22].

In the leads, outside the active region \(-L/2 < x < L/2\), the electrons are permanently undisturbed. Within the leads, their distribution function \( f_k(x, t) \) is displaced from equilibrium by the current injected at the source boundary, and extracted at the drain. the nonequilibrium function satisfies

\[
\left[ \frac{\partial}{\partial t} + v_k \frac{\partial}{\partial x} + \frac{eE}{\hbar} \frac{\partial}{\partial k} \right] f_k(x, t) = -W_k[f(t)]. \tag{18}
\]

The driving field \( E \) is due to the resistivity dipole [15] between the source and drain interfaces, set up in response to the perturbing external current; see Fig. 1. The collision term \( W[f(t)] \) contains all of the scattering effects at the microscopic level, be they single- or multi-particle. In the spirit of Boltzmann, Landau, and Silin it is represented as a functional of the single-particle distribution.

We parametrize \( W[f(t)] \) in terms of the inelastic and elastic collision rates \( 1/\tau_{\text{in}} \) and \( 1/\tau_{\text{el}} \), in such a way that Eq. (18) remains exactly conserving [22]. We then solve the equation for \( f_k(x, t) \) and its fluctuation counterpart

\[
\Delta f_k(x, t) = k_B T \frac{\delta f_k(x, t)}{\delta \mu} \equiv \int_{-L/2}^{L/2} dx' \int \frac{2dk'}{2\pi} \frac{\delta f_k(x, t)}{\delta f^\text{eq}_{k'}(x')} \Delta f^\text{eq}_{k'}(x'). \tag{19}
\]

Eq. (19) contains the variational Green function \( \delta f_k(x, t) / \delta f^\text{eq}_{k'}(x') \). This calculable object has all of the information needed to construct the current-current correlator that determines the noise spectrum [11, 22].

We present in Fig. 3 the analytic results of our nonequilibrium noise model, based on strictly conservative kinetics. We show both the QPC conductance \( G \),
Figure 3. Left scale: excess thermal noise $S^{xs}$ of a ballistic QPC at source-drain voltage $V = 9k_B T/e$, normalized to the Johnson-Nyquist level at ideal conductance, and plotted as a function of chemical potential \[22\]. Right scale: the corresponding two-point conductance $G$. At the subband crossing points of $G$, the excess noise peaks. Noise is high at the crossing points, where subband electrons are classical, and low at the plateaux where subband degeneracy is strong. Much more than the conductance, $S^{xs}$ is sensitive to the scattering-time ratio $\zeta = \tau_{\text{in}}/\tau_{\text{el}}$. There is a dramatic lowering of the upper noise peak (chain and dotted lines) as the inelasticity in the second subband is made stronger. Dashed line: ideal excess noise, including shot noise, predicted by the Landauer-Büttiker model \[5\] and corresponding to our full line ($\zeta_1 = \zeta_2 = 1$). The estimated noise is much smaller.

whose form is identical to that given by Eq. (6) with (9a), and the excess noise of the channel. This represents the excitation strength of the current fluctuations over and above the Johnson-Nyquist equilibrium contribution $S_{\text{JJ}} = 4Gk_B T$. At current $I$ and source-drain voltage $V = I/G$, the expression for excess noise at fixed $V$ is

$$S^{xs}(V, \mu) = \left(\frac{2e^2 V^2}{m^* L^2}\right) \frac{\kappa}{\kappa_{\text{cl}}} G \left(\frac{\tau_{\text{in}}^2}{\tau_{\text{in}}^2 + 2\tau_{\text{el}} \tau_{\text{in}}^2} \tau_{\text{el}} + \tau_{\text{in}} - \frac{\tau_{\text{el}}^2 \tau_{\text{in}}^2}{(\tau_{\text{el}} + \tau_{\text{in}})^2}\right).$$  \(20\)

To interpret Fig. 3 we need first to note the second and third factors on the right-hand side, $\kappa/\kappa_{\text{cl}}$ and $G$. The excess noise is plotted as a function of chemical potential; that is, the channel density goes from being practically depleted to where both subbands in the model are fully degenerate.

How do these two factors behave in the transition from classical to strongly quantum regimes for the electrons? As we saw above, the compressibility ratio reaches its maximum of unity in the classical limit, and decreases monotonically as the density rises, vanishing as $k_B T/2(\mu - \varepsilon_i)$ in the degenerate limit. At the same time, the conductance rises monotonically from its exponentially small value $\sim e^{-(\varepsilon_i - \mu)/k_B T}$ to its maximum Landauer value of $G_0$ per filled subband. At each threshold $\mu \approx \varepsilon_i$, ...
the two countervailing trends combine to generate the strong peaks seen in the Figure.

Next we consider the last factor in Eq. (20). It is a function of the inelasticity ratio $\zeta = \tau_n/\tau_{el}$, and a highly responsive one at that. As $\zeta$ is decreased for the upper subband, the conductance shows nonideal behavior just as in Fig. 2. However, the corresponding suppression of the second noise peak is dramatic. In practical terms this means the noise is an \textit{extremely sensitive predictive marker} of the inelastic processes inside the QPC, much more so than the shot-noise based prediction of the phenomenological models [5], which reflects only the change in $G$.

Before applying our model to an experimental situation, we go back to the question of whether changes in $G$ are truly governed by inelastic scattering via $\zeta$ or whether, as required by the Landauer theory, it depends on elastic backscattering alone to modify $\tau_{el}$ while dissipative effects never enter the picture.

Also in Fig. 3 we plot the ideal, shot-noise based prediction for the excess noise prescribed by the Landauer-Büttiker (LB) theory of QPC fluctuations [5]. At the relatively large source-drain voltage used to calculate our results, the LB prediction is far smaller than our ideal result (topmost set of peaks). Only for enhanced inelasticity, $\zeta < 1$, does our fully conserving kinetic calculation of $S^{xs}$ begin to come down to the LB curve; at small driving voltages the situation is no different [22]. This graphic comparison is quite apart from the fact that the LB model fails to respect the compressibility sum rule [21], as shown by the absence of $\kappa$ anywhere in the Landauer-Büttiker noise formula [5].

e. \textit{Comparison with Experiment}

In 1995, a significant set of QPC noise measurements was published by Reznikov, Heiblum, Shtrikman, and Mahalu [12]. This experiment was distinguished by the fact that, for the first time, such noise measurements were performed for a range of fixed values of the current as well as for fixed source-drain voltages, as is more customary (and for which the outcome accords semi-quantitatively with Fig. 3 above).

Figure 4 reproduces the fixed-current noise plots from Ref. [12]. On the basis of the LB formula, one would have anticipated a \textit{strictly monotonic} behavior in $S^{xs}$, complementary to the rising stepwise form of $G$ as the gate voltage applied to the mesoscopic channel sweeps its electron density upwards into the degenerate limit. No monotonic falloff is seen in the experimental data.

Unequivocally, the Reznikov et al. experiment at fixed values of QPC current tells us that the Landauer-Büttiker noise model – along with its many emulators – does not work. If it did, the dramatic peak features of Fig. 4 would have been predicted. Something essential is evidently missing from the phenomenological treatments. It is the vital action of the compressibility sum rule, whose noise signature we have already prefigured and analyzed in Fig. 3 above.

Just such a family of constant-current noise maxima results from our conserving kinetic theory for the excess noise [13]. We show our calculation of the noise peaks in Fig. 5. They are in excellent accord with the structures measured by Reznikov et al. around the subband threshold.\footnote{Neither the LB model, nor our presently simplified one, reproduces the mea-}
Figure 4. Nonequilibrium current noise in a quantum point contact at 1.5K, after the measurements of Reznikov et al., Ref. [12] as a function of gate bias, at fixed source-drain current values. The gate bias allows the electron density in the channel to be swept from near-depletion to full degeneracy. Dotted line: the most widely adopted theoretical noise model [5] typically produces a strictly monotonic noise signal at the first subband energy threshold. That model totally fails to predict the very strong noise peaks actually observed at threshold.

What generates the noise maxima in the case of Fig. 5? The indispensable role of compressibility is as before: the ratio $\kappa/\kappa_{cl}$ moves smoothly from unity at low density, to vanishing values at large $n$. Interestingly, in the case of constant current, the competing effect that leads to the threshold peaks is different to that for constant source-drain voltage, where $G$ filled that role. Here, it is the last factor in Eq. (20), which as we saw in Fig. 3 is extremely sensitive to inelasticity.

In an experimental situation where the energy $eV = eI/G$, available to drive the carriers, gets larger and larger as the density goes down, we must expect inelastic phonon emission to become dominant. Hence the inelastic relaxation rate $1/\tau_{in}(I, \mu)$ increases markedly, and the inelasticity parameter $\zeta(I, \mu)$ drops fast. Let us recast Eq. (20) at fixed $I$:

\[
S^{xs}(I, \mu) = \frac{2e^2I^2\tau_{cl}^2}{m^*L^2} \frac{\kappa}{\kappa_{cl}} \frac{\zeta(I, \mu)^2}{G(I, \mu)} \left(1 + \frac{2}{1 + \zeta(I, \mu)} - \frac{1}{(1 + \zeta(I, \mu))^2}\right). \tag{21}
\]
Figure 5. Excess hot-electron noise at 1.5K in a QPC at its first subband threshold, computed with our strictly conserving Eq. (21), as a function of chemical potential (referred to the first threshold) at fixed levels of source-drain current. (See our Ref. [13]). There is close quantitative affinity of our peaks to the experimentally observed first-threshold maxima in Fig. 4 of Ref. [12]. The dotted line at 100nA shows the Landauer-Büttiker prediction [5] based on Eq. (6). This should be compared with the chain line based on Eq. (21).

Omitting details of our calculation (see Ref. [13]), we discuss this form qualitatively. Attention falls on the behavior of those quantities that depend strongly on the field, and hence on $I$. The elastic relaxation rate $1/\tau_{\text{el}}$ is relatively immune to field effects; it remains as in Eq. (7a).

1. “Pinchoff” limit. (This is when a large negative bias on the control gate depletes the QPC channel.) Since $G \sim n\zeta$ while $\kappa/\kappa_{\text{cl}} = 1$, we have that

$$S_{xs}(I, \mu) \sim I^2 \zeta(I, \mu)/n.$$  

When the field-induced inelasticity is very strong, $\zeta$ falls faster than the density. The net effect is to cause the excess noise to go down.

2. Degenerate limit. Well above the threshold, the conductance is near-ideal as the energy $eV$ is less than the Fermi energy $\mu - \varepsilon_1$. Then $\zeta \approx 1$ and we recover the same asymptotic behavior as before, where $S_{xs}$ is dominated by $\kappa \sim 1/\mu - \varepsilon_1$, and therefore falls off at high density.

There must be a crossover between these two extremes, and we see it precisely in the region of transition between classical and quantum degenerate regimes. That transition occurs at threshold.

To end our discussion, we present in Figs. 6 and 7 the results of our model for constant $V$. Fig. 6 shows the noise, while Fig. 7 shows the conductance of a QPC. Our calculation of $S_{xs}(V, \mu)$ fits the fixed-voltage data of Ref. [12] quite well, including the quasi-linear dispersion of the threshold peak heights, as functions...
Figure 6. Hot-electron excess noise calculated for a quantum point contact at 1.5K, for fixed values of source-drain driving voltage $V$ (going upwards, the values are 0.5, 1, 1.5, 2, and 3meV). Normalization of the noise is to the thermal value $2eI_{th} \equiv 2G_0k_B T$. The peak heights rise monotonically and just less than linearly with $V$. Nonideality from the inelastic scattering is strong enough to suppress the thermal peaks (refer to Fig. 3), making them quasilinear in the voltage. The maxima agree well with experimental results (see Fig. 2, Ref. [12]). Our fully conserving, nonequilibrium kinetic computation shows that quasilinear dispersion of the noise maxima with $V$ is not unique to shot noise.

Figure 7. The total QPC conductance $G$ corresponding to Fig. 6. Our results are comparable to the measurements of Reznikov et al. [12] (see their Fig. 2). Our choice of subband spacing, $7k_B T$, approximately equals the shoulder width of the steps as noted for the experimental plots of $G_s$. The step-like size of $G$ decreases monotonically with the applied voltage (as in Fig. 6 but reading downwards from 0.5 to 3meV). There is progressively greater loss of ideality as $eV$ exceeds the subband gap energy $\varepsilon_2 - \varepsilon_1 = 0.9\text{meV}$, and inelastic phonon emission sets in.
of $V$. While such dispersion is popularly considered as unique to shot noise, we emphasize that there is no shot noise in Eqs. (20) and (21). Our microscopically derived excess noise formula describes so-called hot-electron noise which is an entirely different quantity, thermodynamically, from shot noise [11]. Had we been describing shot noise, we would not have seen the governing influence of the compressibility on the results. The evidence for the role of the compressibility sum rule is in Fig. 4, the outcome of a real experiment. Shot noise cannot account for it; hot-electron noise does, fully.

5. FUTURE APPLICATIONS

A potentially rich field of investigation lies within the structure of the inelasticity $\zeta$ and its mode of interaction with the compressibility sum rule, leading to the striking form of the excess noise spectrum. Inelasticity is not only a function of experimental control parameters such as current or applied voltage. Far more important is its dependence on both materials and device geometry.

As an example of what can be done with the tools provided by Eq. (20) and our supporting kinetic machinery, consider the structural differences between a QPC channel embedded in a heterojunction substrate, as in the Reznikov et al. experiment [12], and a suspended carbon nanotube. Both are essentially one-dimensionally confined channels. However, in the former case, inelastic phonon emission couples the carriers to a bath of three-dimensional lattice excitations. In the latter case both the electrons and the phonons are one-dimensional.

We therefore predict qualitatively different forms of hot-electron noise behavior from experiments done on suspended nanotubes. Such noise measurements have already been performed [23], though not at fixed current as far as we are aware. Because the behavior of $\zeta$ will be quite different, it should be interesting to compare $S_{xx}$ measured for suspended tubes, with other tubes intimately contacted to, or even embedded in, a surrounding matrix.

Finally we note that the electronic compressibility $\kappa$ is a strong function of the electron-electron correlations in a metallic electron system [6]. Such correlations are enhanced at low temperatures and low densities. The information on electron correlations, conveyed by the compressibility, should then be available through studies of nonequilibrium noise in structures where those correlations become significant. Comparisons could be made between, say, sound-velocity data and noise data.

6. SUMMARY

Our review has covered much territory. We began with some background to contemporary developments in mesoscopic transport, and raised a number of delicate points which modern phenomenological approaches do not address. We posed two questions: do such philosophies of transport (and noise) still fall short in their theoretical treatment of electron motion at short scales? If so, is there still something novel to learn from the body of knowledge firmly in place well before Landauer?

We then focused on the microscopic derivation of the Landauer conductance formula, paying close attention to the need to integrate charge conservation consistently
into the dynamical description of an open mesoscopic channel. We found that those boundary conditions that are uniquely consistent with the fluctuation-dissipation theorem (to which the Kubo-Greenwood formula is equivalent) require active sources and sinks to carry the external current into and out of the driven device [10, 19]. These are not the boundary conditions invoked by quasi-diffusive phenomenologies of transport.

Nevertheless, the Landauer conductance formula emerged very naturally from the microscopic Kubo-Greenwood theory applied to a quantum point contact. We identified the role of active dissipation of electrical energy, effected by the inevitable presence of inelastic scattering in the leads, as a key ingredient in the microscopic derivation. This resolves the strange cognitive gap in quasi-diffusive models, whereby a purely coherent – nondissipative – description of conductance must unaccountably “produce” dissipative energy loss not on the basis of physics, but solely to save the appearance of the fluctuation-dissipation relation within Landauer’s theory.

At this point we were able to answer the two initial questions in the specific context of transport. First, Landauer-like approaches to mesoscopic transport, albeit necessary and highly successful correctives to inappropriate bulk averaging at small scales, remain microscopically incomplete. Second, their completion comes only through the standard microscopic theory of electrically open systems, unaided by any superfluous phenomenology. The new ingredient here is the explicit appearance of incoherent, inelastic scattering.

Noise was the next phenomenon examined. The very same canonical boundary conditions, responsible for the microscopically consistent derivation of the Landauer formula, turned out to constrain the electron-hole pair fluctuations arising in the conductive channel. The constraint is in the form of the compressibility sum rule, itself a consequence of global charge conservation. The constraint is so strong that it compels the noise of a degenerate channel to scale with the ratio of the thermal bath energy to the Fermi energy no matter how hard the system is driven by an applied current.

A novel outcome of the compressibility sum rule emerged from our detailed kinetic-theoretical treatment of nonequilibrium noise in a QPC. The interplay of compressibility and the scattering dynamics within the channel led to a characteristic series of noise peaks, which we were able to match finely to the measurements of Reznikov et al. [12]. With the help of the compressibility sum rule we have successfully explained [13] the remarkable noise features observed at fixed levels of the driving current. Such structures have no explanation in terms of more phenomenological, quasi-diffusive noise models [5].

We have revealed the marked, and previously unsuspected, influence of the conserving sum rules in the phenomena surrounding mesoscopic electron motion. This is particularly true of the compressibility sum rule which, beyond QPCs, may be expected to play an equally pre-eminent role over the whole stretch of mesoscopic transport and noise problems, and at scales yet smaller [24]. A range of novel experimental effects can now be theoretically discussed and sought out in appropriately planned observations. Noise in carbon nanotubes is only one of a potentially wide set of options to explore.

Questions about the microscopic standing of modern phenomenologies are not
unprecedented. They have been repeatedly raised within the mesoscopics commu-
nity (and repeatedly avoided) from the earliest times that the coherent-cum-diffusive
picture was taken up in a major way. Our own considerations strike out in new and
different directions, but they are also consonant with a whole family of other critiques.
Aside from the impressive formal contributions by Sols [10] and the IMEC group [19,
25], readers may wish to refer to elegant papers by Fenton [26] and Kamenev and
Kohn [27], and to the overview of established mesoscopic theories contained in the
recent review by Agraït et al. [24].

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