A novel analytical model of initial fracture pressure for horizontal staged fracturing in fractured reservoir

Lixi Liang | Yi Ding | Xiangjun Liu | Pingya Luo

1State Key Laboratory of Oil and Gas Reservoir Geology and Exploitation, Southwest Petroleum University, Chengdu, Sichuan, China
2Deep Earth Energy Lab, Department of Civil Engineering, Monash University, Melbourne, Australia

Abstract
The horizontal well-staged fracturing technology has been widely used in many oilfields to enhance well production by creating fracture network. Accurate prediction of fracture initiation is significant for successful hydraulic fracturing operation. However, hydraulic fracture initiation in fractured reservoir is highly tricky due to its rich natural fractures and artificial fractures previously created during staged fracturing. These fractures are thought to be major factors in conducting efficient staged fracturing in fractured reservoir. Therefore, in this paper, induced stress from artificial fractures has been analyzed by using two-dimensional analytical model. Besides, based on the assumption that rock in the formation is composed by rock matrix and fracture plane, considering tensile and shear failure type, failure criterion with multiple natural fracture planes has been developed. By combining impacts from natural fractures and artificial fractures, we establish an analytical model to calculate initial fracture pressure for horizontal staged fracturing in fractured reservoir. These results demonstrate that induced stress is associated with artificial fractures’ location, height, and number. This induced stress leads to increment of initial fracture pressure, restricting fracture initiation. Due to natural fractures, there are three types of initiation, which are tensile failure along rock matrix, tensile failure along natural fracture, and shear failure along natural fracture plane. When rock failure is related to natural fracture, initial fracture pressure decreases and is variable with natural fracture plane occurrence. Especially, with large number of natural fractures, only one initiation type exists, which is tensile failure along natural fracture. Additionally, for horizontal borehole, influencing extent of fracture planes is associated with wellbore trajectory. The case study proves the applicability of this analytical model, meaning it can be used to guide horizontal staged fracturing in oilfield.

Keywords
artificial fracture, fracture initiation, fractured reservoir, natural fractures, staged fracturing

1 INTRODUCTION
Hydraulic fracturing is the most common method for reservoir stimulation in petroleum field. Especially in hydraulic fracturing, horizontal staged fracturing technology has been applied into different types of reservoir, showing good application of boosting production. In hydraulic fracturing operation, hydraulic fracture initiation is a vital parameter for hydraulic fracturing, horizontal staged fracturing technology has been applied into different types of reservoir, showing good application of boosting production.
fracturing design and optimization. The prediction of fracture initiation requires a comprehensive understanding of in situ stress state, geomechanical properties, fracturing fluid, and so on. It is thus not easy to acquire accurate prediction. In particular, for horizontal staged fracturing in fractured reservoir, artificial fractures and natural fractures simultaneously affect fracture initiation by changing stress state, rock strength, and failure type. Thus, complexity of initial fracture pressure is highly increased in this condition. Different approaches including analytical, numerical, and experimental studies have been performed by many researchers to analyze the fracture initiation. The initial model of analyzing fracture initiation was built on homogeneity condition for a vertical wellbore. Based on this research, so many scholars extended the investigation to fracture initiation of deviated wellbore and perforation wellbore. These influence factors of fracture initiation, such as wellbore track, rock strength, perforation azimuth, and perforation number, have been fully analyzed. However, these studies are all established on homogeneity situation. Their limitations are obvious while applying into hydraulic fracturing in fractured reservoir with strong anisotropy.

When it comes to fractured reservoir, single weak plane criterion and tensile criterion are frequently employed to calculate fracture initiation. According to these criteria, rock is composed of two parts, that is, rock matrix and fracture plane. Correspondingly, rock could have tensile failure along rock matrix or natural fracture plane. This assumption has been used in lots of analytical models to analyze influence of fracture plane on initiation. Meanwhile, different numerical methods in 2D or 3D structure, like finite element method (FEM) and discrete element method (DEM), have been applied to simulate the fracture initiation with weak planes (porosity, fracture plane, or bedding plane) and different engineering parameters (well trajectory, perforation azimuth, perforation number, multiple-staged fracturing, and so on). Based on these numerical models, theoretical systems of hydraulic fracture initiation, dynamic fracture propagation, induce stress, and stress distribution around wellbore have been established. Except for numerical or analytical model of hydraulic fracturing initiation, Beugelsdijk et al., Peacock and Mann, and Buner et al. used true triaxial test to simulate hydraulic fracturing in the laboratory. By using this true triaxial, hydraulic fracture initiation and propagation can be visually observed. Findings of these researches conclude that natural fracture can make hydraulic fracture more tortuous. Well completion method and in situ state will affect type of fracture initiation.

Even though lots of researches have been done in hydraulic fracture initiation, there are still some problems. Firstly, in fractured reservoir, natural fractures are abundant and have random distribution. This characteristic makes wellbore likely to have an intersection with several fracture planes. Thus, single weak plane criterion cannot precisely express this initiation mechanism. Secondly, artificial fractures will cause reorientation of in situ stresses in hydraulic fracturing area. As a result, fracture initiation will be modified. Currently, influences of natural fracture and artificial fracture have been studies separately, whereas no much attention has been given to this coupling effect from natural fracture and artificial fracture. In this paper, considering the stress reorientation and different initiation mechanism from natural fracture and artificial fracture, analytical solution of initial fracture pressure for horizontal staged fracturing in fractured reservoir has been developed. Under the coupling effect of artificial and natural fracture plane, hydraulic fracturing initiation has been fully investigated.

## 2 | STRESS DISTRIBUTION OF HORIZONTAL BOREHOLE IN STAGED FRACTURING

### 2.1 | In situ stress components around wellbore

To obtain the stress components around wellbore, we assume that the formation is the porous and elastic linear medium. After coordinate transformation, for arbitrary borehole trajectory, stress components can be expressed as:

\[
\begin{align*}
\sigma_r &= \frac{r^2}{r^2} p_i + \sigma_{sy} \left(1 + \frac{3}{4} \frac{r^4}{r^6} - \frac{4}{r^4} \right) \sin 2\theta + \frac{\sigma_{sy} - \sigma_{s}}{2} \left(1 + \frac{3}{4} \frac{r^4}{r^6} - \frac{4}{r^4} \right) \cos 2\theta + \frac{\sigma_{sy} + \sigma_{s}}{2} \left(1 - \frac{r^6}{r^2} \right) + \frac{\sigma_{sy} + \sigma_{s}}{2} \left(1 - \frac{r^6}{r^2} \right) \sin 2\theta + \frac{\sigma_{sy} + \sigma_{s}}{2} \left(1 - \frac{r^6}{r^2} \right) \cos 2\theta + \phi (p_i - p_p) \\
\sigma_\theta &= \frac{r^2}{r^2} p_i + \frac{\sigma_{sy} + \sigma_{s}}{2} \left(1 + \frac{3}{4} \frac{r^4}{r^6} - \frac{4}{r^4} \right) \cos 2\theta - \sigma_{s} \left(1 + \frac{3}{4} \frac{r^4}{r^6} - \frac{4}{r^4} \right) \sin 2\theta + \phi (p_i - p_p) \\
\sigma_z &= \sigma_{s} - 2\nu \left[\sigma_{sy} - \sigma_{s} \frac{r^2}{r^2} \cos 2\theta + 2\sigma_{sy} \frac{r^2}{r^2} \sin 2\theta + \phi (p_i - p_p) \right] \\
\tau_{\theta \phi} &= \sigma_{sy} \left(1 + \frac{2}{r^2} \right) \cos 2\theta + \frac{\sigma_{sy} - \sigma_{s}}{2} \left(1 + \frac{3}{4} \frac{r^4}{r^6} - \frac{4}{r^4} \right) \sin 2\theta \\
\tau_{\phi \theta} &= \sigma_{sy} \left(1 + \frac{2}{r^2} \right) \cos \theta - \sigma_{s} \left(1 + \frac{2}{r^2} \right) \sin \theta \\
\tau_{\phi \phi} &= \sigma_{sy} \left(1 - \frac{r^2}{r^2} \right) \cos \theta + \sigma_{s} \left(1 - \frac{r^2}{r^2} \right) \sin \theta \\
\tau_{\theta \theta} &= \sigma_{sy} \left(1 + \frac{2}{r^2} \right) \cos 2\theta + \frac{\sigma_{sy} - \sigma_{s}}{2} \left(1 + \frac{3}{4} \frac{r^4}{r^6} - \frac{4}{r^4} \right) \sin 2\theta \\
\tau_{\phi \phi} &= \sigma_{sy} \left(1 - \frac{r^2}{r^2} \right) \cos \theta + \sigma_{s} \left(1 - \frac{r^2}{r^2} \right) \sin \theta \\
\tau_{\phi \theta} &= \sigma_{sy} \left(1 + \frac{2}{r^2} \right) \cos \theta - \sigma_{s} \left(1 + \frac{2}{r^2} \right) \sin \theta
\end{align*}
\]

[](https://example.com)
where \(\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{xz}, \) and \(\sigma_{yz}\) are in situ stress components, respectively, MPa; \(\tau_{xy}, \tau_{xz}, \) and \(\tau_{yz}\) are radial, circumferential, and axial stress, respectively, MPa; \(r\) is radius of wellbore, m; \(\rho_p\) is pore pressure, MPa; \(a\) is Biot coefficient; and \(\delta\) is wellbore permeability coefficient.

### 2.2 Induced stress of artificial fractures

In hydraulic fracturing point of horizontal borehole, stress state around borehole is affected by induced stress made by artificial fracture. According to the analytical model proposed by Sneddon, this induced stress can be computed on the basis of two-dimensional plane condition, shown as Figure 1. In this model, the shape of artificial fracture is regarded as ellipse and the whole number of fractures is \(k\). For arbitrary natural fracture \(i\), induced stress on hydraulic fracturing point is shown as Equation 2.

\[
\begin{align*}
\sigma_{ix}(i) & = -p(i) \left( \frac{c(i)^2}{r(i)r_2(i)} \right) \sin \theta(i) \sin \left[ \frac{3}{2} (\theta_1(i) + \theta_2(i)) \right] - p\left( \frac{r(i)}{(r(i)r_2(i))^{0.5}} \right) \cos (\theta(i) - \frac{1}{2} \theta_1(i) - \frac{1}{2} \theta_2(i)) - 1 \right] \\
\sigma_{iy}(i) & = p\left( \frac{r(i)}{(r(i)r_2(i))^{0.5}} \right) \sin \theta(i) \sin \left[ \frac{3}{2} (\theta_1(i) + \theta_2(i)) \right] - p\left( \frac{r(i)}{(r(i)r_2(i))^{0.5}} \right) \cos (\theta(i) - \frac{1}{2} \theta_1(i) - \frac{1}{2} \theta_2(i)) - 1 \right] \\
\sigma_{iz}(i) & = p\left( \frac{r(i)}{(r(i)r_2(i))^{0.5}} \right) \sin \theta(i) \cos \left[ \frac{3}{2} (\theta_1(i) + \theta_2(i)) \right]
\end{align*}
\]

where \(\sigma_{ix}(i), \sigma_{iy}(i), \) and \(\sigma_{iz}(i)\) are normal stress caused by artificial fracture \(i\) in \(x, y, z\) direction, respectively, MPa; \(\sigma_{ix}(i)\) is shear stress caused by artificial fracture \(i\), MPa; and \(p(i)\) is the fluid pressure in artificial fracture \(i\) MPa.

Additionally, \(c(i), r(i), r_1(i), r_2(i), \theta_1(i), \theta_2(i),\) and \(\theta(i)\) are geometrical parameters of artificial fracture \(i\). In term of geometrical relation in Figure 1, these parameters can be obtained:

\[
\begin{align*}
c(i) & = \frac{h(i)}{2} \\
r(i) & = \sqrt{x(i)^2 + z^2}, r_1(i) = \sqrt{x(i)^2 + (c(i) - z)^2}, r_2(i) = \sqrt{x(i)^2 + (c(i) + z)^2} \\
\theta(i) & = \arctan \left( \frac{x(i)}{c(i) - z} \right), \theta_1(i) = \arctan \left( \frac{x(i)}{c(i) - z} \right), \theta_2(i) = \arctan \left( \frac{x(i)}{c(i) + z} \right)
\end{align*}
\]

where \(h(i)\) is height of artificial fracture \(i\), MPa; \(x(i)\) is distance between fracturing point and artificial fracture in \(x\) direction, m; and \(z\) is distance between fracturing point and \(x\) axis, m.

Based on the certain artificial fracture property, Equations 2 and 3 have been applied to illustrate the induced stress, shown as Figure 2. Horizontal coordinate axis means the distance between artificial fracture 1 and hydraulic fracturing point. The artificial fracture spacing is 20 m. It can be found that in the neighborhood of hydraulic fracturing point, artificial fracture can create large induced stress. More artificial fractures exist, stronger induced stress occurs. With increasing distance, this induced stress tends to disappear. These induced stresses in Figure 2 will modify original in situ stresses \((\sigma_v, \sigma_{H}, \sigma_h)\) during horizontal staged fracturing. By using superposition method, in situ stresses with stress interference of artificial fractures \((\sigma'_H, \sigma'_v, \sigma'_z)\) can be written as:

\[
\begin{align*}
\sigma'_H & = \sigma_H + \sum_{i=1}^{n-1} \sigma_{ix}(i) + \sum_{i=1}^{n-1} \sigma_{iy}(i) \\
\sigma'_v & = \sigma_v + \sum_{i=1}^{n-1} \sigma_{iz}(i) \\
\sigma'_z & = \sigma_z + \sum_{i=1}^{n-1} \sigma_{iz}(i) \\
& \quad \text{for } i = 1, 2, 3 \ldots k.
\end{align*}
\]

### 3 Fracture Initiation in Fractured Reservoir

#### 3.1 Mechanism of the influence of natural fractures on initiation

For arbitrary point at the borehole, when its stress state is confirmed, its initiation type is related to mechanical property and structure, shown as Figure 3. The initiation type is associated with initial fracture pressure. With natural fracture, initiation type could be changed, modifying initial fracture pressure. In Figure 3, for homogeneity situation, only one initiation type exists, having tensile failure along rock matrix. When natural fracture exists, there are three types of initiation, including tensile failure mode and shear failure mode. With different failure
types, stress state needs to reach a different level to cause initiation. For instance, with tensile failure along rock matrix, stress state has to overcome tensile strength of rock matrix. With tensile failure along natural fracture, stress state must be larger than normal stress at the natural fracture plane. Besides, when it comes to shear failure along natural fracture, stress state at the natural fracture needs to overcome shear strength of natural fracture (i.e., cohesion and internal frictional angle of natural fracture). In a word, due to different initiations, stress state has to satisfy different requirements, leading to various initial fracture pressures. Therefore, based on these initiation types, corresponding calculation method of initial fracture pressure has been established in the following sections.

### 3.2 | Tensile failure along rock matrix

According to tensile strength criterion, fracture initiation along rock matrix occurs when tensile stress reaches tensile strength of rock matrix. In this case, initial fracture pressure is the same as homogeneity condition, shown as:

\[
\sigma_p - ap_p = S_t, \tag{5}
\]

where \(S_t\) is tensile strength of rock matrix, MPa.

### 3.3 | Tensile failure along natural fracture

The main characteristic of fractured reservoir is rich natural fracture planes, forming intersection between fracture plane and horizontal borehole,\(^\text{30}\) shown as Figure 4. For fracture plane \(i\), its normal direction vector can be written as:

\[
\begin{align*}
n_1(i) &= a_1(i)I + a_2(i)J + a_3(i)K \\
a_1(i) &= -\sin(Dp(i)) \cos(Az(i)) \\
a_2(i) &= \sin(Dp(i)) \sin(Az(i)) \\
a_3(i) &= \cos(Dp(i))
\end{align*}
\tag{6}
\]

where \(n_1(i)\) is the normal direction vector of fracture plane \(i\); \(Dp(i)\) is inclined angle of fracture plane \(i\), degree; and \(Az(i)\) is azimuth angle of fracture plane \(i\), degree.

Spatial relation of principal stress in geostress coordinate system can be expressed as Figure 5. Then, the coordinate transformation is applied to deduce the direction vector of maximum principal stress, shown as:

\[
\begin{align*}
n_2(\sigma_1) &= b_1(\sigma_1)I + b_2(\sigma_1)J + b_3(\sigma_1)K \\
b_1(\sigma_1) &= \cos(\theta) \cos(\Omega) \\
b_2(\sigma_1) &= \sin(\theta) \cos(\Omega) \\
b_3(\sigma_1) &= \sin(\Omega)
\end{align*}
\tag{7}
\]

where \(\sigma_1\) is maximum principal stress, MPa, and \(n_2(\sigma_1)\) is the direction vector of maximum principal stress.

Similarly, direction vector of \(\sigma_2\) and \(\sigma_3\) can be written as:

\[
\begin{align*}
n_2(\sigma_2) &= b_1(\sigma_2)I + b_2(\sigma_2)J + b_3(\sigma_2)K \\
b_1(\sigma_2) &= \sin(\theta+\psi) \sqrt{\cos^2\gamma + \sin^2\gamma \sin^2\Omega} \\
b_2(\sigma_2) &= -\cos(\theta+\psi) \sqrt{\cos^2\gamma + \sin^2\gamma \sin^2\Omega} \\
b_3(\sigma_2) &= -\cos(\Omega) \sin\gamma
\end{align*}
\tag{8}
\]
where \( u \) and \( w \) are expressed as:

\[
\begin{align*}
\psi &= \arctan \frac{\sin \Omega \sin \gamma}{\cos \gamma} \\
\gamma &= \frac{1}{2} \arctan \left( \frac{2 \tau_{\Omega i}}{\sigma_\Omega - \sigma_\eta} \right), \\
w &= \arctan \left( -\frac{\sin \Omega \sin \gamma}{\cos \gamma} \right).
\end{align*}
\]

According to above calculations, the included angle between normal direction of natural fracture \( i \) and principal stresses is shown as:

\[
\cos \beta_i = \frac{n_1(i) \cdot n_2(\sigma_j)}{|n_1(i) \cdot n_2(\sigma_j)|} \quad i = 1, 2, 3 \ldots k \quad j = 1, 2, 3, \quad (12)
\]

where \( \beta_1(i), \beta_2(i), \) and \( \beta_3(i) \) are included angles between normal stress and principal stresses, degree.

By using the relation in Equation 12, the normal stress at natural fracture \( i \) can be written as Equation 13. The fluid pressure in natural fracture needs to overcome the normal stress at natural fracture \( i \) so that the tensile failure can happen. Therefore, the initial fracture pressure of tensile failure along fracture plane \( i \) is shown as Equation 14.

\[
\sigma_n(i) = \sigma_1 \cos^2 \beta_1(i) + \sigma_2 \cos^2 \beta_2(i) + \sigma_3 \cos^2 \beta_3(i) \quad i = 1, 2, 3 \ldots k, \quad (13)
\]

\[
P^t_{\text{incl}}(i) = \sigma_n(i) - \Delta \sigma \quad i = 1, 2, 3 \ldots k, \quad (14)
\]
where \( \sigma_n(i) \) is the normal stress at natural fracture plane \( i \), MPa, and \( P_t^\infty(i) \) is initial fracture of tensile failure along fracture plane \( i \).

### 3.4 Shear failure along natural fracture

Currently, shear failure is built on the basis of Mohr-Coulomb theory, which can be expressed by classic Mohr’s circle. In this paper, since we take account for numerous natural fractures, Mohr’s circle with multiple fracture planes can be established using the superposition method, illustrated as Figure 6. Correspondingly, stress state at fracture plane \( i \) can be written as:

\[
\tau = c_w + \sigma \tan \varphi_w \quad (i = 1, 2, \ldots, k), \tag{15}
\]

where \( c_w \) is the cohesion of natural fracture \( i \), MPa, and \( \varphi_w \) is the internal angle of friction of natural fracture \( i \), degree.

Normal stress and shear stress at fracture plane \( i \) are written as:

\[
\begin{align*}
\tau &= \frac{\sigma_1 - \sigma_3}{2} \sin 2\beta_1(i) \\
\sigma &= \frac{\sigma_1 + \sigma_3 + \sigma_1 - \sigma_3}{2} \cos 2\beta_1(i) \quad (i = 1, 2, \ldots, k) \tag{16}
\end{align*}
\]

Combining Equation 16 into 15, shear failure equation can be obtained, shown as Equation 17. Meanwhile, to have shear failure along fractures happens, initial fracture pressure from failure along fracture plane \( i \) can be determined on the basis of Equation 17.

\[
\begin{align*}
\sigma_1 - \sigma_3 &= \frac{2(c_w + \sigma_3 \tan \varphi_w)}{(1 - \tan \varphi_w \cot \beta_1(i)) \sin 2\beta(i)} \quad (i = 1, 2, \ldots, k) \tag{17}
\end{align*}
\]

\[
\begin{align*}
\beta_{\text{min}}(i) &= \frac{\varphi_w}{2} + \frac{1}{2} \arcsin \left( \frac{\sigma_1 + \sigma_3 + 2c_w \cot \varphi_w \sin \varphi_w}{\sigma_1 - \sigma_3} \right) \\
\beta_{\text{max}}(i) &= \frac{\pi}{2} + \frac{\varphi_w}{2} - \frac{1}{2} \arcsin \left( \frac{\sigma_1 + \sigma_3 + 2c_w \cot \varphi_w \sin \varphi_w}{\sigma_1 - \sigma_3} \right) \\
\beta_{\text{min}}(i) &< \beta_1(i) < \beta_{\text{max}}(i) \quad (i = 1, 2, \ldots, k) \tag{18}
\end{align*}
\]

### 3.5 Confirmation of initial fracture pressure

According to above investigations, there are three types of initiation: (a) tensile failure along rock matrix, (b) tensile failure along natural fracture plane, and (c) shear failure along natural fracture. For each point at the wall of horizontal wall, initial fracture pressures of different initiation types have to be calculated. Subsequently, by performing comparison among all these initial fracture pressures, the minimum value is confirmed, which is the final initial fracture pressure, shown as Equation 19. The location of minimum value is the initiation point, and the whole flow chart of this calculation is illustrated in Figure 7.

\[
P_{\text{init}} = \min \left( P_{\text{init}}^m, P_t^\infty(i), P_s^\infty(i) \right), \quad i = 1, 2, 3 \ldots k, \tag{19}
\]

where \( P_{\text{init}}^m \) is initial fracture pressure from tensile failure along rock matrix, MPa; \( P_t^\infty(i) \) is initial fracture pressure from shear failure along natural fracture, MPa; and \( P_{\text{init}} \) is the final initial fracture pressure, MPa.
As mentioned in the introduction of this paper, various models of initial fracture pressure have been established. In this section, we conduct comparison between our model and models from previous researches. Typical models of initial fracture pressure have been illustrated in Table 1. According to literatures, these models have been widely used, whereas each one has its own limitation. For instance, model No. 1 is built on homogeneity condition, meaning it is not applicable for fractured formation. Model No. 2 includes single natural fracture with tensile initiation, but ignores shear initiation and multiple natural fractures. Model No. 3 combines tensile and shear initiation mode, yet still not include the multiple natural fractures condition. For model No. 4, it is only suitable for staged hydraulic fracturing in homogeneity condition. Despite their limitations, their theories of calculating initial fracture pressure are correct and proved by application.

**FIGURE 6** Illustration of Mohr’s circle with multiple natural fracture planes

**FIGURE 7** Flowchart of calculation of initial fracture pressure
in oilfield. Thus, this comparison can verify our model to some degree. Besides, further verification can be noticed in the application of this paper.

The comparison is demonstrated in Figure 8. Initial fracture pressure of previous models, oilfield data, and input parameters are all obtained from references. It can be found out that calculated results of our model are consistent with previous models and their oilfield data. Deviations between our models and model No. 1 to No 4 are 1.1%, 3.3%, 9.4%, and 3.1%, respectively. Deviations for oilfield data are all under 9%, proving its practicability. In addition, comparing to previous models, our model accounts for more aspects, such as multiple natural fractures, artificial fractures, and tensile-shear failure types, showing better application prospect.

| No. | Description                                                                 | Reference |
|-----|----------------------------------------------------------------------------|-----------|
| 1   | Model with homogeneity and tensile initiation                               | Yu et al  |
| 2   | Model with single natural fracture and tensile initiation                   | Zhao et al|
| 3   | Model with single natural fracture and tensile, shear initiation            | Liu et al |
| 4   | Model with homogeneity, staged fracturing condition, and tensile initiation| Deng et al|

5.2 | The influence of natural fracture occurrence

5.2.1 | The influence of natural fracture

In addition, according to the induced stress equation in Equation 2, fracture height is a significant factor for calculating induced stress. The computed result of initial fracture pressures with different fracture heights is presented in Figure 10. In Figure 10, due to variable fracture heights, effect degree of artificial fracture has clearly change. With increasing height, the increment of initial fracture pressure is becoming stronger. Besides, the effect distance of artificial fracture with relatively small height (60 m) is merely 58 m. The influence of artificial fracture disappears when the distance is over 58 m. In comparison, for artificial fracture with large height (120 m), effect distance is approximately 130 m, much longer than the artificial fracture having small height, indicating stronger influence of artificial fracture with higher height.

In the context of multiple-staged fracturing, numerous artificial fracture planes exist. To calculate the initial fracture pressure in different number of artificial fractures, the artificial fracture spacing and height remain constant, which are 30 and 60 m, respectively. The computed result is illustrated in Figure 11. It can be found that with increasing artificial fracture number, initial fracture pressure grows. Note that in multiple artificial fractures condition, this increment becomes very small and initial fracture pressure tends to reach a stable level. For instance, curves in three and four artificial fractures are very similar. The average difference between initial fracture pressures in three and four artificial fractures is merely 0.62 MPa, which can be ignored in the application.
**FIGURE 8** Comparison of initial fracture pressures from different models.
to homogeneity condition, initial fracture pressure in single fracture plane has clear decline and this average decreasing range is approximately 9.4 MPa.

5.2.2 The influence of natural fracture number

Considering numerous fracture planes in fractured reservoir, we set different number of fracture planes and all natural fracture planes have uniform distribution. Additionally, take one fracture plane as basic plane \( (i = 1) \) and the occurrences of other fracture planes \( (i = 2, 3, 4 \ldots k) \) modify with this basic plane, shown as Equation 20. According to Equation 20, distribution of multiple natural fracture planes has been illustrated in Figure 14. Correspondingly, with this fracture distribution, initial fracture pressure in different fracture number has been calculated, shown in Figures 15-17.

It is noted that inclination and azimuth in Figures 15-17 represent occurrence of base plane. Decline of initial fracture pressure enlarges with increasing natural fracture number. In two and three fracture planes, maximum values are 53.2 and 51.4 MPa, respectively (below the value in homogeneity condition). Meanwhile, the occurrence area with tensile failure

### TABLE 2 Basic inputs for calculation

| No. | Parameters/unit             | Value |
|-----|-----------------------------|-------|
| 1   | Vertical in situ stress/MPa | 67.1  |
| 2   | Maximum horizontal in situ stress/MPa | 60.8 |
| 3   | Minimum horizontal in situ stress/MPa | 51.3 |
| 4   | Radius of wellbore/m        | 0.1   |
| 5   | Biot coefficient            | 0.85  |
| 6   | Pore pressure/MPa           | 30.8  |
| 7   | Poisson ratio               | 0.25  |
| 8   | Porosity/%                  | 5.6   |
| 9   | Tensile strength/MPa        | 7.1   |
| 10  | Fluid pressure in artificial fracture/MPa | 10.4 |

**FIGURE 9** Relation between artificial fracture location and initial fracture pressure

**FIGURE 10** Initial fracture pressure in different height of artificial fracture
**FIGURE 11** Initial fracture pressure in different artificial fracture numbers

![Graph showing initial fracture pressure in different artificial fracture numbers]

**FIGURE 12** Initial fracture pressure in homogeneity condition

![Schematic, failure type distribution, and initial fracture pressure distribution]

**FIGURE 13** Initial fracture pressure in different natural fracture occurrence

![Schematic, failure type distribution, and initial fracture pressure distribution]
along matrix and shear failure along natural fracture is disappeared after natural fracture number reaches two. Thus, in this scenario, fracture initiations all belong to tensile failure along natural fracture plane.

\[
\begin{align*}
\theta_i^\text{in} &= \theta_1^\text{in} + \frac{90}{k-1} \cdot (i-1), \quad \theta_i^\text{az} = \theta_1^\text{az} + \frac{180}{k} \cdot (i-1) \quad k > 1, \quad i = 1, 2, 3, \ldots, k,
\end{align*}
\]

where \( k \) is total number of natural fractures; \( \theta_1^\text{in} \) is the inclination of first plane (base plane), degree; \( \theta_1^\text{az} \) is the azimuth of first plane (base plane), degree; \( \theta_i^\text{in} \) is the inclination of fracture plane \( i \), degree; and \( \theta_i^\text{az} \) is the azimuth of fracture plane \( i \), degree.

5.3 The coupling influence of natural and artificial fractures

In combination with natural fracture and artificial fracture, initial fracture pressure has been calculated, shown as Figures 18 and 19. It can be found that natural fracture and artificial fracture have opposite effects on fracture initiation. That is to say natural fracture and artificial fracture have decreasing and increasing influence on initial fracture pressure, respectively. When the number of natural fracture plane is one, induced stress from artificial fracture not only increases initial fracture pressure, but also changes failure type in some occurrences. For instance, at azimuth 0°-20° and inclination 15°-30°, failure along natural fracture is converted to failure along rock matrix (Figure 17), thus leading to increment of initial fracture pressure and making fracture initiation more difficult. However, for shear failure mode, artificial fracture does not show the ability of causing more shear failure. On the other hand, due to influence of natural fracture, multiple natural fractures are beneficial for tensile failure along natural fracture. As a result, in three natural fractures (Figure 18), even though initial fracture pressure has increment from induced stress, failure type still remains same, that is, tensile failure along natural fracture.

In addition, for certain occurrence, changing range of initial fracture pressure can be calculated on the basis of Equation 21. Considering changing ranges in all occurrences, its average value in variable number of natural and artificial fracture planes can be calculated, shown as Figure 20. As can be seen from Figure 20, with increasing in number of

FIGURE 14 Distribution of natural fracture occurrence

FIGURE 15 Initial fracture pressure in one natural fracture
natural and artificial fractures, changing range of initial fracture pressure tends to become stable. This is because with increasing natural fracture number, failure is completely relied on natural fracture with only tensile failure type, expressing homogeneity feature. Thus, this decreasing caused by natural fracture gradually ceases. Meanwhile, artificial fracture has the impact of elevating the difficulty of initiation, meaning the increment of initial fracture pressure. With increasing artificial plane number, increment of induce stress is gradually small (as shown in Section 5.1) because more artificial planes are located far away from initiation point, having minor effect on initiation. As a result, influence of artificial fracture on initial fracture plane arrives at a constant level with a large number of artificial fractures.

\[
\Delta P(ci, ai) = P_{\text{init}}^m - P_{\text{init}}(ci, ai),
\]

where \(\Delta P(ci, ai)\) is pressure changing range at occurrence \((ci, ai)\), MPa; \(P_{\text{init}}\) is initial fracture pressure at occurrence \((ci, ai)\), MPa; \(ci\) is the inclination of base plane, degree; and \(ai\) is the azimuth of base plane, degree.

5.4 | The influence of horizontal wellbore trajectory

For horizontal wellbore, a different trajectory can affect fracture initiation by changing its stress state.38,39 Hence, we conduct the investigation on fracture initiation in different wellbore azimuth. In the calculation, artificial fracture property is same as that in Section 5.1 and natural fracture occurrence is shown in Table 3. Figure 21 gives initial fracture pressure distribution in different wellbore azimuths. With increasing borehole azimuth, initial fracture pressure decreases. By ignoring artificial fracture (Figure 21A), in one natural fracture plane condition, initial fracture pressure has no decline in some azimuths (0°-20°), suggesting it is not affected by fracture plane. However, when fracture plane number continues to increase, all azimuths show decreasing trend, meanings that natural fracture plane...
FIGURE 18 Initial fracture pressure with artificial and natural fracture (one natural fracture)
**Figure 19**  Initial fracture pressure with artificial and natural fracture (three natural fractures)

**A** Schematic

**B** Failure type distribution

**C** Initial fracture pressure
influence cannot be avoided. Meanwhile, in combination with natural fracture and artificial fracture, the azimuth area of having no influence of natural fracture enlarges to approximately 0°-30° (Figure 20B). The whole initial fracture pressure increases and the difference between initial fracture pressure in homogeneity and fracture plane condition becomes small, indicating induced stress elevates the difficulty of initiation.

6 | APPLICATION

In order to prove the practicability of analytical method of fracture initiation in this paper, field-measured results of MH-4 well have been used as a case study. The depth of fracture reservoir of MH-4 well is in 3108m. Geological data show that there are two natural fracture planes with inclination angles of 12° and 35°, and azimuth angles are 73° and 121°. There are four artificial fractures in horizontal staged fracturing. The geological and engineering parameters of staged fracturing are shown in Table 4.

It can be seen that initial fracture pressure is 48.5 MPa from hydraulic fracturing construction curve, shown as Figure 22A. Computed results of initial fracture pressure in homogeneity, natural fracture and artificial-natural fracture condition are, respectively, calculated, shown in Figure 22B. From Figure 22B, initial fracture pressure in homogeneity is 56.2 MPa and clearly higher than oilfield data, indicating method based on homogeneity is not applicable in fractured reservoir. Meanwhile, only considering the influence of natural fracture, initial fracture pressure has obviously decline and the value is lower than oilfield data. In combination with natural fracture and artificial fracture, the computed initiation pressure is 46.8 MPa, which has good consistency with oilfield value, proving its practicability. This case study demonstrates that to have precise prediction of initial fracture for horizontal staged fracturing in fractured reservoir, coupling effect from natural fracture and artificial fracture needs to be included.

7 | CONCLUSION

Considering coupling effect from artificial fracture and natural fracture, the analytical model of fracture initiation in fractured reservoir for horizontal staged fracturing has been established. The fracture initiation has been fully analyzed using this analytical model. Based on these results presented in this study, following conclusions can be drawn:

1. There are three types of fracture initiation in fractured reservoir, which are tensile failure along rock matrix,
tensile failure along natural fracture plane, and shear failure along natural fracture. The existence of natural fracture can decrease initial fracture pressure by causing failure along natural fracture plane. Different occurrence of natural fracture plane leads to variable initial fracture pressure. Besides, with increasing natural fracture number, decline of initial fracture pressure becomes large and it is more likely to have tensile failure along fracture plane.

2. The artificial fracture will create induced stress for hydraulic fracturing area, thus changing fracture initiation. Artificial fracture is able to elevate initial fracture pressure. This increment gradually disappears with distance. Besides, artificial fracture with large height can have relatively bigger increase range and influence distance. With growing number of artificial fracture, initial fracture pressure rises. When the number continues to grow, increment of initial fracture pressure becomes small and even can be ignored. It is also noticed that the artificial fracture can restrict tensile failure along natural fracture and increase the difficulty of initiation.

3. Fracture initiation is also related to horizontal azimuth. With increasing borehole azimuth, initial fracture pressure shows decreasing trend. In one natural fracture plane condition, initial fracture pressure has no decline in some azimuths. However, in multiple fractures condition, all azimuths declines. Meanwhile, under coupling effect from natural fracture and artificial fracture, the azimuth area of having initiation along natural fracture decreases and initial fracture pressure has clear growth. Application results illustrate initial fracture pressure combining influences from natural and artificial fracture planes is consistent with practical data. Therefore, in order to precisely predict fracture initiation for horizontal staged fracturing in fractured reservoir, natural fracture and artificial fracture are both non-neglected factors.
### TABLE 4  Geological and engineering parameters

| No. | Parameters/unit                  | Value   |
|-----|----------------------------------|---------|
| 1   | Vertical in situ stress/MPa      | 61.2    |
| 2   | Maximum horizontal in situ stress/MPa | 53.5    |
| 3   | Minimum horizontal in situ stress/MPa | 44.9    |
| 4   | Pore pressure/MPa                | 30.8    |
| 5   | Poisson ratio                    | 0.25    |
| 6   | Porosity/%                       | 10.6    |
| 7   | Biot coefficient                 | 0.85    |
| 8   | Inclination angle of wellbore/degree | 89     |
| 9   | Azimuth angle of wellbore/degree | 16      |
| 10  | Radius of wellbore/m             | 0.1     |
| 11  | Tensile strength of rock/MPa     | 7.2     |
| 12  | Artificial fracture height/m     | 24      |
| 13  | Fluid pressure in artificial fracture/MPa | 13.3 |
| 14  | Distance between artificial fracture 1 and hydraulic fracturing zone/m | 45 |
| 15  | Distance between artificial fracture 2 and hydraulic fracturing zone/m | 75 |
| 16  | Distance between artificial fracture 3 and hydraulic fracturing zone/m | 105 |
| 17  | Distance between artificial fracture 4 and hydraulic fracturing zone/m | 135 |

(A) Fracturing construction curve

(B) Initial fracture pressure from different methods
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ORCID

Xiangjun Liu  https://orcid.org/0000-0002-2919-588X

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