Restricted Bernoulli Matrix Factorization: Balancing the trade-off between prediction accuracy and coverage in classification based collaborative filtering

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Abstract

Reliability measures associated with the prediction of the machine learning models are critical to strengthening user confidence in artificial intelligence. Therefore, those models that are able to provide not only predictions, but also reliability, enjoy greater popularity. In the field of recommender systems, reliability is crucial, since users tend to prefer those recommendations that are sure to interest them, that is, high predictions with high reliabilities. In this paper, we propose Restricted Bernoulli Matrix Factorization (ResBeMF), a new algorithm aimed at enhancing the performance of classification-based collaborative filtering. The proposed model has been compared to other existing solutions in the literature in terms of prediction quality (Mean Absolute Error and accuracy scores), prediction quantity (coverage score) and recommendation quality (Mean Average Precision score). The experimental results demonstrate that the proposed model provides a good balance in terms of the quality measures used compared to other recommendation models.

Keywords: recommender systems, collaborative filtering, matrix factorization, reliability.

1 Introduction

In the age of the information society, people's daily lives are conditioned by a large number of cloud services. A clear example of this is the way people consume entertainment products. Today, it is difficult to think about watching a movie without Netflix, listening to a song without Spotify, or playing video games without Microsoft Game Pass. However, these types
of service have a problem: They offer such a large amount of content to their users that customers often feel lost and disoriented and do not know which item to consume. This is known as the information overload problem [24].

Recommender Systems (RSs) [23, 21, 5, 7] are powerful machine learning-based tools that alleviate the problem of information overload. They are also known as filters, since they block information that is irrelevant to users and let pass information that matches the user’s preferences.

The most popular implementation of RSs is Collaborative Filtering (CF) [14, 10, 12, 20]. In CF, recommendations are calculated with respect to explicit or implicit ratings that a set of users give on a set of items. CF algorithms use these ratings to make recommendations based on the ratings of other users similar to a given one. Depending on how the recommendations are computed, CF is divided into three classes. The first one are K-Nearest Neighbors (KNN) based CF [2, 6] computes the recommendations by finding the K most similar users of a given one and selecting the preferred items of these users. In a different style, we find Matrix Factorization (MF) based CF [16, 13] issues the recommendations by learning a low-dimensional latent representation of the matrix that contains the ratings. Finally, Neural based CF [10, 22] creates the recommendations using a Artificial Neural Network (ANN) that extracts the patterns of votes made by users on the items. Graph based models have become very popular in recent days [27, 25, 28].

Current trends in RS research focus on measuring system performance beyond prediction accuracy [26]. One of the most popular measures is reliability [4, 11, 1]. Reliability indicates the system’s certainty in knowing whether a prediction is correct or not. Adding reliability to RSs helps to improve user confidence in them. On the one hand, if the prediction is correct and the reliability is high, user satisfaction increases. On the other hand, if the prediction fails but the reliability is low, the user can forgive the error.

In this paper, we propose a new CF-based MF model that provides reliable predictions: Restricted Bernoulli Matrix Factorization (ResBeMF). This model aims to enhance the performance of CF-based RS that provide reliability linked to their predictions. To achieve this, the RS is approached as a multi-objective optimization problem, where the goal is to find a RS that provides the best quality of predictions without diminishing the quantity of reliable predictions that the system can offer. The rest of the document is structured as follows: Section 2 dives into the related work, Section 3 describes the proposed model, Section 4 shows the experimental results of the proposed model on gold standard datasets in the field of CF, and section 5 presents the conclusions and future work of this research.
2 Related Work

Incorporating reliability into RSs is a hot research topic in recent years. In [11], a reliable RS is constructed by combining the output of content-based filtering and CF using fuzzy cognitive maps. In a different vein, [1] presents a novel RS which is based on three different points of view on reliability measures: (i) a user-based reliability measure used to evaluate the performance of user rating profiles in predicting unseen items, (ii) an item-based reliability measure used to improve low-quality rating profiles, and (iii) a rating-based reliability measure used to evaluate initial predicted ratings. Furthermore, [29] proposes a MF architecture to provide a reliability value for each prediction in any CF based RS. These reliability values show improvements in the quality of prediction and recommendation in different RS.

In addition, the evaluation of reliability measures has also been studied. In [4] two scores are proposed to measure the quality of a reliability measure. Both quality measures are based on the hypothesis that the more suitable a reliability measure is, the better accuracy results it will provide when applied.

The algorithms mentioned above seek to add a quality measure to an existing CF model. However, there exists another trend that aims to create RSs that intrinsically embed reliability into their models. In other words, the output of these models is a tuple (prediction, reliability), instead of just a prediction as in regular methods. To this end, these models change the paradigm on which the RS algorithms are based. Traditionally, predicting the rating of a user $u$ to an item $i$ has been treated as a regression problem, despite the fact that rating scores are usually not represented by continuous values (e.g., in MovieLens, scores are a discrete set of 1 to 5 stars). These new models that provide both the prediction and its associated reliability treat RS as a classification problem.

In this spirit, Bernoulli Matrix Factorization (BeMF) was proposed in [17]. BeMF is a CF-based RS that assumes a Bernoulli distribution for the ratings by representing the known ratings using one hot encoding. However, this model assumes that these binary ratings are pairwise independent. To address this issue, Dirichlet Matrix Factorization (DirMF), presented in [15], assumes a Dirichlet distribution for the ratings that avoids the independence of the ratings. In both cases, the output of the model is a discrete probability distribution that allows us to know the probability that a user $u$ rates an item $i$ with a score $s$.

However, these classification-based CF-methods suffer a dichotomy in their performance that prevents their extension to more general scenarios. In the case of BeMF, by design of the model, the rating scores are treated as Bernoulli (binary) independent variables. This is a very unnatural assumption that disregards some subtle patterns and ignores the information provided by the natural ordering in the scores. Additionally, precisely because we are performing several independent matrix factorizations, the raw output of BeMF is not a probability distribution. Instead, it must be re-normalized to get such probability. This has the effect that the output distributions tend to be very spiky, leading to risky predictions. Hence, if we
restrict ourselves to recommend predictions with only high reliability, the system tends to be very aggressive and gets a high coverage but with a low accuracy.

On the other side of the spectrum, DirMF does treat the different ratings as dependent variables, and the output of the model can be directly understood as a probability distribution in a very flexible class. But this produces the opposite effect of BeMF: the output distributions tend to be very flat. Hence, the system is very conservative and thus, the coverage that we can get with high reliability is very small.

3 Restricted Bernoulli Matrix Factorization

The aim of this work is to balance between both BeMF and DirMF approaches, obtaining a model able to provide accurate predictions with large coverage at the high-reliability regime. For this purpose, we will modify BeMF to force the reliability scores to be dependent on design. As we will show, this choice also produces several matrix factorizations, one for each possible rating as in BeMF, but the training of each factorization affects the other ones. This leads to a constrained (restricted) optimization problem on the Bernoulli approach, leading to a model that we shall call ResBeMF.

To be precise, suppose that we have a set of $U$ users that can rate a collection of $I$ items. Possible ratings that can be assigned form a set $S = \{s_1, \ldots, s_d\}$ (typically the elements of $S$ are numerical values such as in the MovieLens dataset $S = \{1, \ldots, 5\}$). The collection of issued ratings is collected in a $|U| \times |I|$ matrix $R = (r_{u,i})$, where $r_{u,i} \in S$ is the rating that the user $u$ assigned to the item $i$. As is customary in RSs, if the user $u$ did not rate the item $i$, we shall denote $r_{u,i} = \bullet$.

The key idea of ResBeMF is to create, for each user $u$ and item $i$, a discrete probability distribution with support in $S$, $p_{u,i} : S \rightarrow [0, 1]$, such that $p_{u,i}(s)$ is the probability that $u$ would rate $i$ with the score $s$. To construct this distribution, we shall suppose that, for each $s \in S$, each user $u$ is associated with a vector $P^s_u \in \mathbb{R}^k$ and each item $i$ is associated with a vector $Q^s_i \in \mathbb{R}^k$, the so-called latent vectors or hidden factors. Here $k$ is a fixed hyper-parameter of the model called the latent dimensionality. The shall gather all these latent vectors into matrices $P = (P^s_u)_{u,s}$ and $Q = (Q^s_i)_{i,s}$.

In this way, the probability distribution is given by

$$p_{u,i}(s) = p_{u,i}(s \mid P, Q) = \sigma_s (P_u \cdot Q_i).$$

Here, $P_u \cdot Q_i = (P^s_1 \cdot Q^s_1, \ldots, P^s_d \cdot Q^s_d)$ denotes the list of scalar products of the vectors $P^s_u$ and $Q^s_i$ for $s \in S$, and $\sigma_s$ is the $s$-th component of the softmax function on $S$

$$\sigma_s(x_1, \ldots, x_d) = \frac{e^{x_s}}{\sum_{t \in S} e^{x_t}}.$$
Notice that, since \( \sum_{s \in S} \sigma_s(x) = 1 \) for all \( x \in \mathbb{R}^d \), the function \( p_{u,i} : S \to [0,1] \) is actually a probability on \( S \). This is in sharp contrast with other MF-based models such as BeMF \[^{[17]}\].

In order to compute the parameters \( \mathbf{P} \) and \( \mathbf{Q} \), we can consider the likelihood function

\[
\mathcal{L}(\mathbf{P}, \mathbf{Q}) = \prod_{r_{u,i} \neq \bullet} p_{u,i}(r_{u,i} | \mathbf{P}, \mathbf{Q}) = \prod_{R_{u,i} \neq \bullet} \sigma_{r_{u,i}}(P_u \cdot Q_i).
\]

In this way, the log-likelihood function \( \ell(\mathbf{P}, \mathbf{Q}) = \log \mathcal{L}(\mathbf{P}, \mathbf{Q}) \) is

\[
\ell(\mathbf{P}, \mathbf{Q}) = \sum_{r_{u,i} \neq \bullet} \log(\sigma_{r_{u,i}}(P_u \cdot Q_i)).
\]

Since we want to maximize the log-likelihood function, we compute the gradient of this function. Recall that the derivative of the softmax function is

\[
\frac{\partial \sigma}{\partial x_t} = \sigma_s(\delta_{s,t} - \sigma_t),
\]

where \( \delta_{s,t} = 1 \) if \( s = t \) and \( \delta_{s,t} = 0 \) otherwise. Hence, the partial derivative is

\[
\frac{\partial}{\partial P_{u_0}^{s_0}} \ell(\mathbf{P}, \mathbf{Q}) = \sum_{\{i \mid r_{u_0,i} = s_0\}} \frac{\sigma_{s_0}(P_{u_0} \cdot Q_i)(1 - \sigma_{s_0}(P_{u_0} \cdot Q_i))}{\sigma_{s_0}(P_{u_0} \cdot Q_i)} Q_i^{s_0}
\]

\[
- \sum_{\{i \mid r_{u_0,i} \neq s_0, \bullet\}} \frac{\sigma_{r_{u_0,i}}(P_{u_0} \cdot Q_i)\sigma_{s_0}(P_{u_0} \cdot Q_i)}{\sigma_{r_{u_0,i}}(P_{u_0} \cdot Q_i)} Q_i^{s_0}
\]

\[
= \sum_{r_{u_0,i} = s_0} (1 - \sigma_{s_0}(P_{u_0} \cdot Q_i)) Q_i^{s_0} - \sum_{r_{u_0,i} \neq s_0, \bullet} \sigma_{s_0}(P_{u_0} \cdot Q_i) Q_i^{s_0}.
\]

Analogously for \( Q_{i_0}^{s_0} \) we get

\[
\frac{\partial}{\partial Q_{i_0}^{s_0}} \ell(\mathbf{P}, \mathbf{Q}) = \sum_{r_{u,i_0} = s_0} (1 - \sigma_{s_0}(P_u \cdot Q_{i_0})) P_{u}^{s_0} - \sum_{r_{u,i_0} \neq s_0, \bullet} \sigma_{s_0}(P_u \cdot Q_{i_0}) P_{u}^{s_0}.
\]

In this manner, we can optimize these parameters through stochastic gradient ascend with learning rate \( \eta \) with the update rules for a vote \( r_{u,i} = s_0 \)

\[
P_u^{s_0} \leftarrow P_u^{s_0} + \eta (1 - \sigma_{s_0}(P_u \cdot Q_i)) Q_i^{s_0},
\]

\[
Q_{i_0}^{s_0} \leftarrow Q_{i_0}^{s_0} + \eta (1 - \sigma_{s_0}(P_u \cdot Q_i)) P_{u}^{s_0},
\]

\[
P_u^{s} \leftarrow P_u^{s} - \eta \sigma_{s_0}(P_u \cdot Q_i) Q_i^{s_0}, \text{ if } s \neq s_0,
\]

\[
Q_{i_0}^{s} \leftarrow Q_{i_0}^{s} - \eta \sigma_{s_0}(P_u \cdot Q_i) P_{u}^{s_0}, \text{ if } s \neq s_0.
\]

Furthermore, if in addition we consider gaussian priors with zero mean and fixed standard deviation for the parameters \( \mathbf{P} \) and \( \mathbf{Q} \), we then get that the update rules result (see \[^{[17]}\] Section
\[
P_{u}^{s_0} \leftarrow P_{u}^{s_0} + \eta (1 - \sigma_{s_0}(P_{u} \cdot Q_{i}))Q_{i}^{s_0} - \gamma P_{u}^{s_0},
Q_{i}^{s_0} \leftarrow Q_{i}^{s_0} + \eta (1 - \sigma_{s_0}(P_{u} \cdot Q_{i}))P_{u}^{s_0} - \gamma Q_{i}^{s_0},
P_{u}^{s} \leftarrow P_{u}^{s} - \eta \sigma_{s_0}(P_{u} \cdot Q_{i})Q_{i}^{s_0} - \gamma P_{u}^{s}, \text{ if } s \neq s_0,
Q_{i}^{s} \leftarrow Q_{i}^{s} - \eta \sigma_{s_0}(P_{u} \cdot Q_{i})P_{u}^{s_0} - \gamma Q_{i}^{s}, \text{ if } s \neq s_0.
\]

Here \( \gamma \) is another hyper-parameter called the \((L^2)\) regularized of the model.

Once the model has been trained, the optimal parameters \( P \) and \( Q \) has been determined so for each pair of a user \( u \) and an item \( i \) we get a probability distribution \( p_{u,i} : \mathcal{S} \to [0, 1] \). From this information, the most straightforward criterion that can be used to obtain the prediction is the so-called ‘mode criterion’, that is:

- The predicted rating is
  \[
  \hat{r}_{u,i} = \arg \max_{s \in \mathcal{S}} p_{u,i}(s) = \arg \max_{s \in \mathcal{S}} \sigma_{s}(P_{u} \cdot Q_{i}).
  \]

- The reliability in the prediction is
  \[
  \rho_{u,i} = p_{u,i}(\hat{r}_{u,i}) = \sigma_{\hat{r}_{u,i}}(P_{u} \cdot Q_{i}).
  \]

Finally, if we fix a threshold \( 0 \leq \theta \leq 1 \) of reliability, we set \( \hat{r}_{u,i} = \bullet \) provided that the reliability \( \rho_{u,i} < \theta \) (no reliable prediction can be issued).

### 3.1 Algorithmic implementation of ResBeMF

A pseudocode implementation of the proposed method is included in Algorithm 1. The algorithm receives as input the ratings \( R \), the plausible scores \( \mathcal{S} \) and the model hyper-parameters: the number of latent factors \( k \), the number of iterations \( m \), the regularization \( \gamma \) and the learning rate \( \eta \). The algorithm output is made up of the latent factor for users (\( P \)) and item (\( Q \)). The algorithm contains two main loops: the ‘for each’ loop from lines 3 to 21 is used to update users factors, and the ‘for each’ loop from lines 22 to 40 allows us to update item factors. These loops can be computed in parallel for each user or item, respectively. Inside these loops, the statements for update users and items factors are equivalent: first, the gradient of all known ratings of the user or item is accumulated in \( \Delta \) (users) or \( \Phi \) (items), and then the factors are updated proportionally to these gradients and the learning rate \( \eta \).
input: $R, k, \gamma, \eta, m, \mathcal{S}$
output: $P, Q$
1. Initialize $P \leftarrow U(0,1)$, $Q \leftarrow U(0,1)$
2. repeat
   for each user $u$ do
      Initialize $\Delta$ to 0
      for each item $i$ rated by user $u$: $R_{u,i}$ do
         for each possible score $s \in \mathcal{S} = \{s_1, \ldots, s_D\}$ do
            for each $f \in \{1, \ldots, k\}$ do
               if $R_{u,i} = s$ then
                  $\Delta^s_f \leftarrow \Delta^s_f + (1 - \sigma_{R_{u,i}}(P_u \cdot Q_i)) \cdot Q^s_{i,f} - \gamma \cdot P^s_{u,f}$
               else
                  $\Delta^s_f \leftarrow \Delta^s_f - \sigma_{R_{u,i}}(P_u \cdot Q_i) \cdot Q^s_{i,f} - \gamma \cdot P^s_{u,f}$
            end
         end
      end
   end
   for each possible score $s \in \mathcal{S} = \{s_1, \ldots, s_D\}$ do
      for each $f \in \{1, \ldots, k\}$ do
         $P^s_{u,f} \leftarrow P^s_{u,f} - \eta \cdot \Delta^s_f$
      end
   end
   for each item $i$ do
      Initialize $\Phi$ to 0
      for each user $u$ that rated item $i$: $R_{u,i}$ do
         for each possible score $s \in \mathcal{S} = \{s_1, \ldots, s_D\}$ do
            for each $f \in \{1, \ldots, k\}$ do
               if $R_{u,i} = s$ then
                  $\Phi^s_f \leftarrow \Phi^s_f + (1 - \sigma_{R_{u,i}}(P_u \cdot Q_i)) \cdot P^s_{u,f} - \gamma \cdot Q^s_{u,f}$
               else
                  $\Phi^s_f \leftarrow \Phi^s_f - \sigma_{R_{u,i}}(P_u \cdot Q_i) \cdot P^s_{u,f} - \gamma \cdot Q^s_{u,f}$
            end
         end
      end
      for each possible score $s \in \mathcal{S} = \{s_1, \ldots, s_D\}$ do
         for each $f \in \{1, \ldots, k\}$ do
            $Q^s_{i,f} \leftarrow Q^s_{i,f} - \eta \cdot \Phi^s_f$
         end
      end
   end
3. until $m$ iterations

Algorithm 1: ResBeMF model-fitting algorithm
4 Experimental results

This section contains the experimental results carried out to evaluate the proposed ResBeMF model. All experiments have been performed using the Collaborative Filtering for Java (CF4J)\cite{19,18} framework and its source code can be found on GitHub\footnote{https://github.com/KNODIS-Research-Group/resbemf}.

Experimental evaluation has been conducted using the MovieLens\cite{9}, FilmTrust\cite{8} and MyAnimeList datasets. The main parameters of these datasets are shown in table 1. To facilitate the reproducibility of these experimental results, the default train/test splits of these datasets included in CF4J were used.

| Dataset         | Number of users | Number of items | Number of ratings | Number of test ratings | Scores  |
|-----------------|-----------------|-----------------|-------------------|------------------------|---------|
| FilmTrust       | 1,508           | 2,071           | 32,675            | 2,819                  | 0.5 to 4.0 |
| MovieLens 100K  | 943             | 1,682           | 92,026            | 7,974                  | 1 to 5  |
| MovieLens 1M    | 6,040           | 3,706           | 911,031           | 89,178                 | 1 to 5  |
| MyAnimeList     | 69,600          | 9,927           | 5,788,207         | 549,027                | 1 to 10 |

Table 1: Main parameters of the datasets used in the experiments.

During the experiments performed in this section, several baselines have been included to compare the proposed ResBeMF method with other approaches. The selection of these baselines has been made trying to ensure that the two current trends in CF, MF, and ANN are represented. Regarding MF-based models, we have selected our previous works BeMF\cite{17} and DirMF\cite{15}, as, like the proposed method, they are CF methods based on classification. Furthermore, we have included Probabilistic Matrix Factorization (PMF)\cite{16} as it is the most popular implementation of MF-based CF. For the models based on ANN, we have chosen both Multi Layer Perceptron (MLP)\cite{10} and Graph Convolutional Matrix Completion (GCMC)\cite{3}. The former is a simple model that aims to mimic the functioning of MF using ANN. The latter is a much more complex model that addresses the CF problem from the perspective of link prediction on graphs.

4.1 Quality measures

As we stated previously, the output of the ResBeMF model consists of a discrete probability distribution that indicates the probability that a user $u$ will rate an item $i$ with any of the plausible scores $S$. This output allows for tuning the degree of reliability of the model by filtering out those predictions that are unreliable. If a high-reliability threshold is set, the model will be able to compute only a few predictions, but these will be very accurate. If a low-reliability threshold is set, the model will be able to compute many predictions, but these will be less accurate. Therefore, the quality of the model should be measured in terms of two quality measures: the error of the predictions and the ability to compute predictions.
To measure the error of predictions, we can follow two approaches: Consider the problem of predicting the vote as a regression problem, in which case we should use a regression score such as Mean Absolute Prediction (MAE), or consider the problem of predicting the vote as a classification problem, in which case we should use a classification score such as accuracy. Both quality measures offer different perspectives on the performance of the models, and therefore, we will use both. For the sake of completeness, we will briefly review these quality measures, as well as their particular incarnations for our model. For a thorough revision of these metrics, please refer to [30].

We define the MAE of the predictions with reliability greater or equal than a parameter $0 \leq \theta \leq 1$ as

$$\text{MAE}^\theta = \frac{1}{|U|} \sum_{u \in U} \frac{1}{|\hat{R}_u^\theta|} \sum_{i \in \hat{R}_u^\theta} \frac{|r_{u,i} - \hat{r}_{u,i}|}{\max(S) - \min(S)},$$

where $U$ is the set of users and $\hat{R}_u^\theta$ is the set of items rated by the user $u$ in the test split with realiability greater or equal than $\theta$.

Analogously, we define the accuracy with reliability greater or equal than $\theta$ as

$$\text{accuracy}^\theta = \frac{1}{|U|} \sum_{u \in U} \frac{1}{|\hat{R}_u^\theta|} \sum_{i \in \hat{R}_u^\theta} \delta_{r_{u,i}, \hat{r}_{u,i}},$$

where $\delta_{r_{u,i}, \hat{r}_{u,i}}$ is an indicator function equal to 1 if the test rating is equal to the prediction and 0 otherwise.

To measure the capability to compute predictions at threshold $\theta$, we use

$$\text{coverage}^\theta = \frac{1}{|U|} \sum_{u \in U} \frac{|\hat{R}_u^\theta|}{|R_{\text{test}}^u|},$$

where $R_{\text{test}}^u$ is the whole set of items rated by the user $u$ in the test split.

In any case, the main purpose of a RS is to suggest a list of items that may be of interest to a user. Knowing the quality of these lists is crucial to assess the real performance of a RS. To measure the quality of the recommendation lists, we use mean Average Precision (mAP). Before defining mAP, we define the precision for user $u$ with threshold $\theta$ at $N$ best items to be

$$\text{precision}^\theta_{u@N} = \frac{\#\{i \in T(N)_u \mid r_{u,i} \geq \theta\}}{N}$$

where $T(N)_u^\theta$ is the list of top $N$ items recommended to the user $u$ based on the predictions. Generally, this list of recommendations is built by selecting the recommended items with the highest predictions. However, when using classification-based CF, such as ResBeMF, there are many identical predictions, leading to many ties that make it difficult to determine which predictions are the highest. Therefore, in these cases, the sorting is done first based on the mode of the probability distribution generated by the classification based CF, and in case of a tie, based on the mean of this probability distribution.
We also define the average precision with threshold $\theta$ at $N$ items to be

$$\text{AP}_u^\theta@N = \frac{1}{\# \{i \in T(N)_u \mid r_{u,i} \geq \theta \}} \sum_{k=1}^{N} \text{precision}_u^\theta N \cdot \text{rel}(k),$$

where $\text{rel}(k)$ is an indicator function equal to 1 if the recommended item at rank $k$ is relevant (i.e. if its test rating is greater than or equal to $\theta$), and 0 otherwise.

And finally we define $\text{mAP}$ to be the mean of $\text{AP}_u^\theta@N$ among all the users, that is

$$\text{mAP}_u^\theta@N = \frac{1}{|U|} \sum_{u \in U} \text{AP}_u^\theta@N.$$

### 4.2 Multi-objective optimization through hyper-parameter tuning

At this point, observe that the prediction task with reliability is a multi-objective problem: increasing prediction quality reduces prediction capability and vice versa. Hence, hyper-parameters, the only gadgets able to modify the behaviour of a given model, must be used to seek the model that maximizes both prediction quality and prediction capability at the same time.

For this purpose, it is necessary to extend the quality measures defined in section 4.1 as these measures report the quality of the model for a fixed reliability threshold $\theta$. To evaluate the real quality of the model, we must average these measures for different values of $\theta$. As $\theta \in [0, 1]$, we sample $\theta$ in an equidistant partition of the unit interval with $N$ points

$$\theta_k = \frac{k}{N-1}$$

for $k = 0, \ldots, N-1$. For example, if $N = 5$ we have $\theta_0 = 0, \theta_1 = 0.25, \theta_2 = 0.50, \theta_3 = 0.75$ and $\theta_4 = 1.00$. In our experiments, we have fixed $N = 20$. To average the results, we define

$$1 - \text{MAE} = \frac{2}{(N+1)N} \sum_{k=0}^{N} (1 - \text{MAE}^{\theta_k})$$

and

$$\text{coverage} = \frac{2}{(N+1)N} \sum_{k=0}^{N} \text{coverage}^{\theta_k}$$

The tuning of hyper-parameters was carried out by a random search using 5 folds cross-validation. The hyper-parameters tested are listed in table 2. This random search has been carried out for the three algorithms of identical nature evaluated in this study: ResBeMF, BeMF [17], and DirMF [18]. It should be noted that these three methods are MF based CF, they output a discrete probability distribution, and they are based on gradient descent. For this reason, the hyperparameters of these models are the same: number of factors, regularization,
learning rate, and number of iterations. The main difference between these three algorithms lies in the probability distribution underlying the RS ratings.

Figure 1 contains the results of this search on the MovieLens 1M dataset. The figure shows a total of 12 scatter plots: one plot for each of the 4 hyper-parameters of the 3 models evaluated. This figure shows both the Pareto fronts of each model and the influence of each hyper-parameter on these Pareto fronts. We can observe that:

- The regularization is crucial for ResBeMF, since higher values results in accurate predictions but low prediction capability.
- The learning rate has a great influence for BeMF: Better models are obtained when the learning rate is low.
- The number of latent factors modulates the recommendation capability of DirMF.

The results of the same experiment for the MovieLens 100K, FilmTrust and MyAnimeList datasets can be found in A.

Using the result of this search, the Pareto front of the validation error of each model can be obtained. Figure 2 compares the Pareto fronts in the four datasets evaluated.

Table 2: Hyper-parameters evaluated during the random search.
We can observe that the proposed ResBeMF model is the model that provides the best balance between prediction quality and prediction capability. This model is the only one that...
can cover a wide range of coverage values (from 0.5 to 1.0) without reducing the quality of the predictions. On the contrary, BeMF provides the highest prediction quality, but its prediction capacity is lower than ResBeMF since its range of coverage values is much narrower (from 0.6 to 8.5 in the best case). Finally, DirMF gives poor results in terms of both prediction quality and prediction capability.

4.3 Test performance

The previous experiments show good performance of ResBeMF. However, the results shown are the average quality measures of the 5 cross-validation runs. To evaluate the true performance of ResBeMF, we must compare their performance using: (a) the test partition, (b) other models of different nature, and (c) other quality measures.

Figure 3 compares the test scores of the proposed method with respect to other MF based CF methods: BeMF, DirMF and PMF in the selected datasets (see table 1). In this experiment, two new quality measures have been added to those previously used in the validation step: accuracy, available only for classification-based models, and mAP. As the hyper-parameter tuning process executed for ResBeMF, BeMF and DirMF returns multiple solutions (i.e. a Pareto front), the results of these figures for these models are shown as a solid line presenting the average error suffered in test by all solutions of the Pareto front and a shaded area representing 95% confidence interval of these errors. In contrast, PMF does not have a shaded area, since its coverage is always 1.0. Therefore, its hyperparameters have been tuned to minimize only the prediction error in terms of MAE using a random search.

We can observe that DirMF is the model that provides the best prediction (MAE and accuracy) and recommendation (mAP) quality in absolute terms, but its prediction capacity is very low; ResBeMF has the best prediction and recommendation capacity without degrading the quality of the predictions; BeMF is the most balanced model, offering good predictions and recommendation with acceptable coverage; and PMF does not have the best prediction nor recommendation quality, although their prediction capacity is perfect (recall that no filtering is applied). This reinforces the observation that ResBeMF is the MF based model that is the most flexible and customizable model, capable of compelling operations in scenarios of high reliability and high accuracy (and thus low coverage), and in scenarios of low reliability and low accuracy (but in contrast, high coverage).
Figure 3: Test error of MF based CF models.

Figure 4 compares the test scores of the proposed method with respect to other ANN based CF methods: MLP and GCMC. In this case, none of the models exhibits a shaded area. In the case of MLP, similar to what happened with PMF, its parameters have been tuned to minimize the error of the predictions, since the coverage is always 1 as it does not implement any reliability-based filtering criterion. In the case of GCMC, due to the complexity of the model, the fitting times are very high, so the default hyper-parameters proposed in its implementation have been chosen.

https://github.com/riannevdberg/gc-mc
In the results, we can observe a trend similar to that found in the comparison of ResBeMF with other MF models. On the one hand, the proposed model has a lower coverage than MLP, but it improves the quality of predictions and recommendations. On the other hand, ResBeMF has better coverage than GCMC, but the quality of its predictions and recommendations is lower. However, the fitting times of GCMC are several orders of magnitude bigger than for non-ANN based methods, making it unusable for very large datasets.

Figure 4: Test error of ANN based CF models.
5 Conclusions and future work

In this paper, we have introduced ResBeMF, a new MF model that addresses CF as a classification problem. The output of the model is a tuple \( \langle \text{prediction}, \text{reliability} \rangle \), so the unreliable predictions made by the model can be filtered out to increase the accuracy of the model. The hyper-parameter tuning has been treated as a multi-objective optimization problem, trying to discover those hyper-parameters that maximize both the quality and quantity of predictions.

The experimental results conducted demonstrate the proposed model ResBeMF, as the model that offers a better balance between quality and predictive capability. Some state-of-the-art models, such as PMF or MLP, can provide a higher number of predictions at the expense of sacrificing the quality of predictions and recommendations. However, other models, such as DirMF or GCMC, provide higher quality predictions and recommendations, but, in turn, the number of predictions they can make with high reliability is lower. Furthermore, it is interesting to highlight the differences with respect to its predecessor model, BeMF. On average, the quality and predictive capability of both models are very similar; however, the Pareto front of the ResBeMF model is broader, allowing for a better fit of the type of RS we want to use. Therefore, we can customize our RS to have higher predictive quality or better predictive capability.

The techniques applied in this paper open a new horizon of possibilities regarding the use of hyper-parameters to push the Pareto front of the multi-objective optimization problem that confronts accuracy and coverage. In particular, as future work, we propose to introduce new hyper-parameters into CF models associated with regularization coefficients of the loss function. This would allow us to have better control over the balance between quantity of predictions and quality of predictions of the model output, leading to even more flexibility of the models.

Acknowledgements

This work was partially supported by Ministerio de Ciencia e Innovación of Spain under the project PID2019-106493RB-I00 (DL-CEMG) and the Comunidad de Madrid under Convenio Plurianual with the Universidad Politécnica de Madrid in the actuation line of Programa de Excelencia para el Profesorado Universitario.

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A Random Search Results

This appendix contains the random search results for the MovieLens100K (fig. 5), FilmTrust (fig. 6), and MyAnimeList (fig. 7) datasets.

Figure 5: Random search results for parameter tuning in MovieLens 100K dataset.
Figure 6: Random search results for parameter tuning in FilmTrust dataset.
Figure 7: Random search results for parameter tuning in MyAnimeList dataset.