Radiative capture cross sections: challenges and solutions

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Abstract

Radiative capture reactions are one of the main inputs in stellar modelling. In numerous situations, the low energies at which these reactions occur in stars are not accessible with present experimental techniques. Electron screening is one of the major causes of problems in separating the bare cross sections from the screened ones. Indirect experimental techniques have been proposed and are now one of the main tools to obtain these cross sections. I discuss the electron screening problem and the latest progresses obtained with indirect methods.

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1 Introduction

Present studies in nuclear astrophysics are focused on the opposite ends of the energy scale for nuclear reactions: (a) the very high and (b) the very low relative energies between the reacting nuclei. Projectiles with high bombarding energies produce nuclear matter at high densities and temperatures. This is the main goal at the RHIC accelerator at the Brookhaven National Laboratory and also the main subject discussed in this Workshop. One expects that matter produced in central nuclear collisions at RHIC for \( \sim 10^4 \) GeV/nucleon of relative energy, and at the planned Large Hadron Collider at CERN, will undergo a phase transition and produce a quark-gluon plasma. One can thus reproduce conditions existent in the first seconds of the universe and also in the core of neutron stars. At the other end of the energy scale are the low energy reactions of importance for stellar evolution (see figure 1). A chain of nuclear reactions starting at \( \sim 10-100 \) keV leads to complicated phenomena like supernovae explosions or the energy production in the stars.

Nuclear astrophysics at low energies requires the knowledge of the reaction rate \( R_{ij} \) between the nuclei \( i \) and \( j \). It is given by \( R_{ij} = n_i n_j \langle \sigma v \rangle / (1 + \delta_{ij}) \), where \( \sigma \) is the cross section, \( v \) is the relative velocity between the reaction partners, \( n_i \) is the number density of the nuclide \( i \), and \( \langle \rangle \) stands for energy average.

In our Sun the reaction \(^7\text{Be}(p, \gamma)^8\text{B}\) plays a major role for the production of high energy neutrinos originated from the \( \beta \)-decay of \(^8\text{B}\). These neutrinos come directly from center of...
the Sun and are an ideal probe of the Sun’s structure. Long ago, Barker has emphasized that an analysis of the existing experimental data yields an S-factor for this reaction at low energies which is uncertain by as much as 30%. This situation has changed recently, mainly due to the use of radioactive beam facilities.

The reaction $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ is extremely relevant for the fate of massive stars. It determines if the remnant of a supernova explosion becomes a *black-hole or a neutron star*. It is argued that the cross section for this reaction should be known to better than 20%, for a good modelling of the stars. This goal has not yet been achieved.

Both the $^7\text{Be}(p, \gamma)^8\text{B}$ and the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reactions cannot be measured at the energies occurring inside the stars (approximately 20 keV and 300 keV, respectively). Direct experimental measurements at low energies are often plagued with low-statistics and large error bars. Extrapolation procedures are often needed to obtain cross sections in the energy region of astrophysical relevance. While non-resonant cross sections can be rather well extrapolated to the low-energy region, the presence of continuum, or subthreshold resonances, complicates these extrapolations. Numerous radiative capture reactions pose the same experimental problem.

Approximately half of all stable nuclei observed in nature in the heavy element region about $A > 60$ is produced in the r–process. This r–process occurs in environments with large neutron densities which lead to $\tau_n \ll \tau_\beta$. The most neutron–rich isotopes along the r–process path have lifetimes of less than one second; typically $10^{-2}$ to $10^{-1}$ s. Cross sections for most of the nuclei involved are hard to measure experimentally. Sometimes, theoretical calculations of the capture cross sections as well as the beta–decay half–lives are the only source of the nuclear physics input for r–process calculations. For nuclei with about $Z > 80$ beta–delayed fission and neutron–induced fission might also become important.

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**Figure 1:** The two ends of the energy scale of nuclear reactions with interest for nuclear astrophysics.
2 The Electron Screening Problem

Besides the Coulomb barrier, nucleosynthesis in stars is complicated by the presence of electrons. They screen the nuclear charges, therefore increasing the fusion probability by reducing the Coulomb repulsion. Evidently, the fusion cross sections measured in the laboratory have to be corrected by the electron screening when used as inputs of a stellar model. This is a purely theoretical problem as one cannot reproduce the interior of stars in the laboratory. Applying the Debye-Hückel, or Salpeter’s, approach[^4], one finds that the plasma enhances reaction rates, e.g., \(^3\)He(\(^3\)He, 2p)\(^4\)He and \(^7\)Be(p, \(\gamma\))\(^8\)B, by as much as 20%. This does not account for the dynamic effect due to the motion of the electrons (see, e.g.,[^5][^6]).

A simpler screening mechanism occurs in laboratory experiments due to the bound atomic electrons in the nuclear targets. This case has been studied in great details experimentally, as one can control different charge states of the projectile+target system in the laboratory[^7][^8][^9][^10][^11]. The experimental findings disagree systematically by a factor of two with theory. This is surprising as the theory for atomic screening in the laboratory relies on our basic knowledge of atomic physics. At very low energies one can use the simple adiabatic model in which the atomic electrons rapidly adjust their orbits to the relative motion between the nuclei prior to the fusion process. Energy conservation requires that the larger electronic binding (due to a larger charge of the combined system) leads to an increase of the relative motion between the nuclei, thus increasing the fusion cross section. As a matter of fact, this enhancement has been observed experimentally. The measured values are however not compatible with the adiabatic estimate[^7][^8][^9][^10][^11]. Dynamical calculations have been performed, but they obviously cannot explain the discrepancy as they include atomic excitations and ionizations which reduce the energy available for fusion. Other small effects, like vacuum polarization, atomic and nuclear polarizabilities, relativistic effects, etc., have

[^4]: Applied the Debye-Hückel, or Salpeter’s, approach
[^5]: See, e.g.,
[^6]: See, e.g.,
[^7]: The stopping cross section of protons on H-targets. The dotted line gives the energy transfer by means of nuclear stopping, while the solid line is the result for the charge-exchange stopping mechanism[^17]. The data points are from the tabulation of Andersen and Ziegler[^16].
Figure 3: The stopping cross section of He$^+$ ions on He-targets. The dashed line is the extrapolation from the Andersen-Ziegler tables. The solid line is a calculation of ref. [19]. The data points are from ref. [18]. The theoretical calculations do not include the nuclear stopping.

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also been considered [12]. But the discrepancy between experiment and theory remains [12, 11].

A possible solution of the laboratory screening problem was proposed in refs. [14, 15]. Experimentalists often use the extrapolation of the Andersen-Ziegler tables [16] to obtain the average value of the projectile energy due to stopping in the target material. The stopping is due to ionization, electron-exchange, and other atomic mechanisms. However, the extrapolation is challenged by theoretical calculations which predict a lower stopping. Smaller stopping was indeed verified experimentally [11]. At very low energies, it is thought that the stopping mechanism is mainly due to electron exchange between projectile and target. This has been studied in ref. [17] in the simplest situation; proton+hydrogen collisions (see figure 2). Two-center electronic orbitals were used as input of a coupled-channels calculation. The final occupation amplitudes were projected onto bound-states in the target and in the projectile. The calculated stopping power was added to the nuclear stopping power mechanism, i.e. to the energy loss by the Coulomb repulsion between the nuclei. The obtained stopping power is proportional to $v^\alpha$, where $v$ is the projectile velocity and $\alpha = 1.35$. The extrapolations from the Andersen-Ziegler table predict a larger value of $\alpha$. Although this result seems to indicate the stopping mechanism as a possible reason for the laboratory screening problem, the theoretical calculations tend to disagree on the power of $v$ at low energy collisions. For example, ref. [18] found $S \sim v_p^{3.34}$ for protons in the energy range of 4 keV incident on helium targets. This is an even larger deviation from the extrapolations of the Andersen-Ziegler tables.

Recently, we have performed [19] a calculation of the stopping power in atomic He$^+$/He collisions using the two-center molecular orbital basis (MO). The advantage of the use of (MO) basis over the method of ref. [20] is that the numerical calculation converges more rapidly at small nuclear distances. The disadvantage is that it converges slower at larger
nuclear distances. However this can be easily corrected by using a linear combination of atomic orbitals (LCAO) at large distances. A much smaller basis than in ref. [20] is needed in both situations. One also has to account for the level crossing problem in the adiabatic collision model. This has been done in the Landau-Zenner approximation [21]. It has been shown that the level crossing is practically diabatic, i.e., as if no level repulsion were present [19]. The result of this calculation is shown in figure 3. The agreement with the data from ref. [18] at low energies is excellent. However, the theoretical calculations do not include the nuclear recoil. The agreement with the data disappears completely if the nuclear recoil is included. In fact, the unexpected ”disappearance” of the nuclear recoil was also observed in ref. [27] (see figure 4). This seems to violate a basic principle of nature, as the nuclear recoil is due to the Coulomb repulsion between the projectile and the target atoms [16]. This effect should always be present in the experimental data.

We are faced here with a notorious case of obscurity in nuclear astrophysics. The disturbing conclusion is that as long as we cannot understand the magnitude of electron screening in stars or in the atomic electrons in the laboratory, it will be even more difficult to understand color screening in a quark-gluon plasma, an important tool in relativistic heavy ion physics (e.g., the J/Ψ suppression mechanism) [22].

3 Radioactive Beam Facilities and Indirect Methods

Transfer reactions are a well established tool to obtain spin, parities, energy, and spectroscopic factors of states in a nuclear system. Experimentally, (d, p) reactions are mostly used due to the simplicity of the deuteron. Variations of this method have been proposed by several authors. For example, the Trojan Horse Method was proposed in ref. [23] (see also [24]) as a way to overcome the Coulomb barrier. If the Fermi momentum of the particle $x$ inside $a = (b + x)$ compensates for the initial projectile velocity $v_a$, the low energy reaction $A + x = B + c$ is induced at very low (even vanishing) relative energy between $A$ and $x$. Successful applications of this method has been reported recently [25]. Figure 5 shows the
Figure 5: S-factor for the reaction $^7\text{Li}(p,\alpha)\alpha$ obtained by the measurement of the cross section for the reaction $^2\text{He}(\text{Li},\alpha\alpha)n$ with the analysis based on the Trojan-Horse method. The solid curve is a theoretical fit to the direct measurement data for $^7\text{Li}(p,\alpha)\alpha$ assuming a screening energy of 350 eV. The dashed curve is the extrapolation of the S-factor to low energies, assuming no screening effect.

astrophysical S-factor for the reaction $^7\text{Li}(p,\alpha)\alpha$ obtained by the measurement of the cross section for the reaction $^2\text{He}(\text{Li},\alpha\alpha)n$ with the analysis based on the Trojan-Horse method. One sees that the problems related to the atomic electron screening are not present in the experimental data. This clearly shows, for the first time, the advantage of using this technique over the direct measurements, as one avoids the treatment of the mysterious screening problem.

Recently the stripping reactions have been demonstrated to be a useful tool to deduce spectroscopic factors in many reactions of relevance for nuclear astrophysics. The method is also free of the screening problem.

At low energies the amplitude for the radiative capture cross section is dominated by contributions from large relative distances of the participating nuclei. Thus, what matters for the calculation of the direct capture matrix elements are the asymptotic normalization coefficients (ANC). This coefficient is the product of the spectroscopic factor and a normalization constant which depends on the details of the wave function in the interior part of the potential. The normalization coefficients can be found from peripheral transfer reactions whose amplitudes contain the same overlap function as the amplitude of the corresponding astrophysical radiative capture cross section. This idea was proposed in ref. and many successful applications of the method have been obtained. For example, the astrophysical S-factor for the reaction $^{13}\text{N}(p,\gamma)^{15}\text{O}$ obtained by using the asymptotic normalization coefficient technique (see figure 6). Only the S-factor for the capture to the $1/2^-$ ground state is shown. The technique also allows for the measurement of S-factors to excited states.

Charge exchange induced in $(p,n)$ reactions are often used to obtain values of Gamow-Teller matrix elements which cannot be extracted from beta-decay experiments. This approach relies on the similarity in spin-isospin space of charge-exchange reactions and $\beta$-decay operators. As a result of this similarity, the cross section $\sigma(p,n)$ at small momentum transfer $q$ is closely proportional to $B(GT)$ for strong transitions. As shown in ref.,
Figure 6: The astrophysical S-factor for the reaction $^{13}\text{N}(p, \gamma)^{15}\text{O}$ obtained by using the asymptotic normalization coefficient technique $^{31}$. Only the S-factor for the capture to the $1/2^-$ ground state is shown. The technique also allows for the measurement of S-factors to excited states.

for important GT transitions whose strength are a small fraction of the sum rule the direct relationship between $\sigma(p, n)$ and $B(GT)$ values fails to exist. Similar discrepancies have been observed $^{34}$ for reactions on some odd-A nuclei including $^{13}\text{C}$, $^{15}\text{N}$, $^{35}\text{Cl}$, and $^{39}\text{K}$ and for charge-exchange induced by heavy ions $^{35}$ $^{36}$.

The (differential, or angle integrated) Coulomb breakup cross section for $a+A \rightarrow b+x+A$ can be written as $\sigma_{\pi\lambda}(\omega) = F_{\pi\lambda}(\omega) \cdot \sigma_{\gamma}(\omega)$, where $\omega$ is the energy transferred from the relative motion to the breakup, and $\sigma_{\gamma}(\omega)$ is the photo nuclear cross section for the multipolarity $\pi\lambda$ and photon energy $\omega$. The function $F_{\pi\lambda}$ depends on $\omega$, the relative motion energy, and nuclear charges and radii. They can be easily calculated $^{37}$ for each multipolarity $\pi\lambda$. Time reversal allows one to deduce the radiative capture cross section $b+x \rightarrow a+\gamma$ from $\sigma_{\pi\lambda}(\omega)$. This method was proposed in ref. $^{38}$. It has been tested successfully in a number of reactions of interest for astrophysics ($^{39}$ and references therein). The most celebrated case is the reaction $^7\text{Be}(p, \gamma)^8\text{B}$. It has been studied in numerous experiments in the last decade. For a recent compilation of the results obtained with the method, see e.g. ref. $^{40}$ (see also figure 7). They have obtained an $S_{17}(0)$ value of 19.0 eV.b which is compatible with the value commonly used in solar model calculations $^{41}$. To achieve the goal of applying this method to many other radiative capture reactions (for a list, see, e.g. $^{39}$), detailed studies of dynamic contributions to the breakup have to be performed, as shown in refs. $^{42}$ $^{43}$. The role of higher multipolarities (e.g., E2 contributions $^{44}$ $^{45}$ $^{46}$ in the reaction $^7\text{Be}(p, \gamma)^8\text{B}$) and the coupling to high-lying states $^{52}$ has also to be investigated carefully.

In the later case, a recent work has shown that the influence of giant resonance states is small (see figure 8). Studies of the role of the nuclear interaction in the breakup process is also essential to determine if the Coulomb dissociation method is useful for a given system $^{53}$.

In summary, radioactive beam facilities have opened a new paved way to disclosure many unknown features of reactions in stars and elsewhere in the universe.

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Figure 7: The astrophysical S-factor for the reaction $^{7}\text{Be}(p,\gamma)^{8}\text{B}$ at low energies. The data points from direct measurements (Weizmann) are from ref. [47]. The GSI data were obtained by using the Coulomb dissociation method [48, 49]. The solid curve is a calculation from ref. [50] and the dashed curve is a calculation from refs. [44, 51].

Figure 8: Energy dependence of the Coulomb breakup cross section for $^{8}\text{B} + \text{Pb} \rightarrow \text{p} + ^{7}\text{Be} + \text{Pb}$ at 84 MeV/nucleon. First-order perturbation calculations (PT) are shown by the solid curve. The dashed curve is the result of a CDCC calculation including the coupling between the ground state and the low-lying states with the giant dipole and quadrupole resonances [52]. The data points are from ref. [13].
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