Generalization of Lagrangian of electro-weak interaction to the octonionic algebra

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Abstract

In the article it is considered the extension of Weinberg-Salam theory from SU(2) group to the octonionic algebra. The extended octonionic algebra is used as particle wave function instead of spinors on su(2). It is shown, that leads to appearance of a new neutral massive vector $C$ and $E$-bosons (peculiar property of the $E$-boson is that it does not interact with matter), and two charged massive vector $D$ and $D^*$ bosons.

Introduction

The progress in extending physical theory to octonionic algebra is related with developing of works [1] and [2], where useful matrix representation of non-associative algebra was suggested. The tax for using matrix view was an unusual multiplication law. Therefore, according to Feynman interpretation of quantum particle motion in extern field, although there is no charge, during the particle movement its condition alternates and the alternation is caused by non-associative interaction nature.

Perhaps, the first time octonionic algebra was applied to the question in work [3], where octonionic science of quantum mechanics was introduced.

Some scientists connect extended algebra with extended space-time interpretation [4]. Also, a range of scientists regard the extension as deriving new properties in the network of electromagnetic theory (e.g. in [6]).

The interpretation of this new type of interaction, in terms of fields, could be found in [7]. Meanwhile, the research over the relation of the new types of interaction with the conventional ones, as far as the author is concerned, has not ever taken place.

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1 Octonionic algebra

The doubling of quaternionic algebra leads, in particular, to octonionic algebra \( \mathbb{O} \), which is a linear space over the field of real numbers \( \mathbb{R} \), for any \( \varnothing \) from octonionic algebra \( \mathbb{O} \) has a linear representation

\[
\varnothing = \sum_{j=0}^{7} \alpha_j e^j, \quad \alpha_j \in \mathbb{R}, j = 0, 1, 2, \ldots, 7.
\]

The multiplication is defined through generatrices \( e^k \in \mathbb{O} \). In particular,

\[
(e^0)^2 = 1, \quad (e^j)^2 = -1, \quad j = 1, 2, \ldots, 7,
\]

so the first component is a real number while the others are to be considered as complex units.

Provided we denote \( \hat{e}^k = e^{k+4}, k = 1, 2, 3 \), the multiplication on octonionic algebra is defined in the following way \cite{2} \((i, j, k = 1, 2, 3)\):

\[
e^i e^j = -\delta^{ij} e^0 + \varepsilon^{ijk} e^k \\
\hat{e}^i \hat{e}^j = -\delta^{ij} e^0 - \varepsilon^{ijk} e^k \\
e^i \hat{e}^j = -\delta^{ij} e^4 - \varepsilon^{ijk} \hat{e}^k \\
e^i e^4 = \hat{e}^i, \quad e^4 \hat{e}^i = e^i, \tag{1}
\]

where the entirely antisymmetrical about its indexes permutation symbol is introduced \( \varepsilon^{123} = 1 \). It is easy to ensure, that the multiplication law for generatrices leads to non-associative algebra. Also,

\[
\{e^i, e^j, e^k\} = (e^i e^j) e^k - e^i (e^j e^k) = 2\varepsilon^{ijkl} e^l, \quad i, j, k, l = 1, 2, \ldots, 7, \tag{2}
\]

where \( \varepsilon^{ijkl} \) is entirely antisymmetrical symbol, which is equal to the unit for the following expressions:

\[
1247, \quad 1265, \quad 2345, \quad 2376, \quad 3146, \quad 3157, \quad 4567. \tag{3}
\]

Octonionic algebra cannot be represented by matrices with traditional multiplication rule, but the special multiplication rule can be introduced, which admits such representation. In the capacity of these matrices one unit matrix corresponding to the unit element \( e^0 \) can be chosen along with seven matrices \( \tilde{\Sigma}_k \) for which the multiplication law \( * \) is introduced as follows \((i, j, k = 1, 2, \ldots, 7)\):

\[
\tilde{\Sigma}_i * \tilde{\Sigma}_j = -\delta^{ij} + \varepsilon^{ijk} \tilde{\Sigma}_k, \tag{4}
\]

where entirely antisymmetrical symbol \( \varepsilon^{ijk} \) is not null if only

\[
\varepsilon^{123} = \varepsilon^{145} = \varepsilon^{176} = \varepsilon^{246} = \varepsilon^{257} = \varepsilon^{347} = \varepsilon^{365} = 1. \tag{5}
\]
Instead of matrices \( \tilde{\Sigma}^{k}, k = 1, 2, \ldots, 7 \) it is more convenient to use \( \Sigma^{k} = i \tilde{\Sigma}^{k}, k = 1, 2, \ldots, 7 \), for which, as it follows from (4), multiplication can be defined as follows:

\[
\Sigma^{i} \ast \Sigma^{j} = \delta^{ij} + i \varepsilon^{ijk} \Sigma^{k}, \quad i, j, k = 1, 2, \ldots, 7.
\]

Introduce the matrices \( \Sigma^{i} \) as it is in [1] \( i = 1, 2, 3 \):

\[
\Sigma^{0} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \Sigma^{i} = \begin{pmatrix} 0 & \sigma^{i} \\ i \sigma^{i} & 0 \end{pmatrix}, \quad \Sigma^{4} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \Sigma^{4+i} = \begin{pmatrix} 0 & -\sigma^{i} \\ \sigma^{i} & 0 \end{pmatrix},
\]

where \( \sigma^{i}, i = 1, 2, 3 \) are Pauli matrices:

\[
\sigma^{1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^{2} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^{3} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\]

Along with \( \Sigma^{i} \) matrices lets introduce abstract matrices \( \mathcal{O} \supseteq \mathcal{O} \) according to the rule, claiming any abstract matrix \( o \in \mathcal{O} \) looks like:

\[
o = \begin{pmatrix} \lambda I & A \\ B & \xi I \end{pmatrix},
\]

where \( A, B \) are complex matrices, \( (2 \times 2) \), \( \lambda, \xi \) are complex functions and \( I \) is a unit matrix \( (2 \times 2) \).

The abstract matrices are summed up as follows:

\[
o + o' = \begin{pmatrix} \lambda I & A \\ B & \xi I \end{pmatrix} + \begin{pmatrix} \lambda' I & A' \\ B' & \xi' I \end{pmatrix} = \begin{pmatrix} (\lambda + \lambda') I & A + A' \\ B + B' & (\xi + \xi') I \end{pmatrix}.
\]

Let’s define the multiplication law for the abstract matrices in the following way:

\[
o \ast o' = \left( \begin{pmatrix} \lambda I & A \\ B & \xi I \end{pmatrix} \right)^{+} \ast \left( \begin{pmatrix} \lambda' I & A' \\ B' & \xi' I \end{pmatrix} \right) = \begin{pmatrix} (\lambda \lambda' + \frac{1}{2} \text{tr}(AB')) I & \lambda A' + \xi' A + \frac{i}{2} [B, A'] \\ \lambda' B + \xi B' - \frac{i}{2} [A, B'] & (\xi \xi' + \frac{1}{2} \text{tr}(BA')) I \end{pmatrix}.
\]

It is easy to ensure, that with multiplication law defined (11), the matrices \( \Sigma^{i}, i = 1, 2, \ldots, 7 \) belong to octonionic algebra.

Hermitian conjugation on abstract matrix algebra is introduced as follows:

\[
o^{+} = \begin{pmatrix} \lambda I & A \\ B & \xi I \end{pmatrix}^{+} = \begin{pmatrix} \lambda^{*} I & B^{+} \\ A^{+} & \xi^{*} I \end{pmatrix},
\]

where \( \lambda^{*}, \xi^{*} \) are complex conjugated functions and \( A^{+}, B^{+} \) are Hermitian conjugated matrices. From the definition of Hermitian conjugation it follows that the matrices introduced above \( \Sigma^{k}, k = 0, 1, \ldots, 7 \) are Hermitian, and matrices \( \Sigma^{k}, k = 1, 2, \ldots, 7 \) are anti-hermitian.

Obviously, we do not leave abstract matrices space after applying addition as well as multiplication commands introduced above to abstract matrices.
2 State vector on octonionic algebra

Because of unusual multiplication law on octonionic algebra it is impossible to define the result of multiplication matrix and row as we had it with standard “smart” multiplication law. That is why we shall use not vectors-rows but matrices $\mathbf{\Psi}(x)$ from abstract matrix algebra, specified above:

$$
\mathbf{\Psi}(x) = \frac{1}{\sqrt{2}} \begin{pmatrix}
\lambda(x) I & A(x) \\
B(x) & \xi(x) I
\end{pmatrix}.
$$

(13)

Let’s call the square of the norm of state vector $\mathbf{\Psi}(x)$ as the number

$$
||\mathbf{\Psi}(x)||^2 = \frac{1}{2} \text{tr}(\begin{pmatrix}
\lambda^* I & B^+ \\
A^+ & \xi^* I
\end{pmatrix} \begin{pmatrix}
\lambda I & A \\
B & \xi I
\end{pmatrix} ) = \\
= \lambda^* \lambda + \xi^* \xi + \frac{1}{2} \text{tr}(B^+ B + A^+ A).
$$

(14)

Thus, on the algebra of extended matrix representations of octonions, the complex scalar fields octet could be introduced. $\varphi_1(x): \lambda(x), \xi(x), \varphi_1(x), \varphi_2(x), i = 1, 2, 3, j = 1, 2, 3, \ldots, 6$:

$$
\varphi_1(x) = \frac{1}{\sqrt{2}} \begin{pmatrix}
\lambda(x) I & 0 \\
0 & 0
\end{pmatrix}, \quad \varphi_2(x) = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 & 0 \\
0 & \xi(x) I
\end{pmatrix},
$$

$$
\varphi_{2+i}(x) = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 & \varphi_{1+i}(x) \sigma^i \\
0 & 0
\end{pmatrix}, \quad \varphi_{5+i}(x) = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 & 0 \\
\varphi_{1+i}(x) \sigma^i & 0
\end{pmatrix}
$$

(15)

(there is no addition on $i$ index) or their linear combinations.

Further, we shall consider the potential energy on state vectors

$$
V(\mathbf{\Psi}) = -m^2 \mathbf{\Psi} \ast \mathbf{\Psi} + \frac{f}{4} (\mathbf{\Psi} \ast \mathbf{\Psi})^2,
$$

(16)

where $m$ and $f$ are some constants.

By the infinitesimality of state vector $\mathbf{\Psi}(x)$ (denote the vector as $\delta \mathbf{\Psi}(x)$), we mean the infinitesimality of its norm, i.e. if (14) then all matrix elements $\delta \mathbf{\Psi}(x)$ are close to zero. In this occasion by potential energy variation we mean the following value:

$$
\delta V(\mathbf{\Psi}) = -m^2 \delta \mathbf{\Psi} \ast \mathbf{\Psi} - m^2 \mathbf{\Psi} \ast \delta \mathbf{\Psi} + \frac{f}{2} \delta \mathbf{\Psi} \ast |\mathbf{\Psi}|^2 \mathbf{\Psi} + \frac{f}{2} \mathbf{\Psi} \ast |\mathbf{\Psi}|^2 \delta \mathbf{\Psi} = \\
= \delta \mathbf{\Psi} \ast (-m^2 \mathbf{\Psi} + \frac{f}{2} |\mathbf{\Psi}|^2 \mathbf{\Psi}) + (-m^2 \mathbf{\Psi} + \frac{f}{2} |\mathbf{\Psi}|^2 \mathbf{\Psi}) \delta \mathbf{\Psi}.
$$

(17)

By condition partial derivative of function $\mathbf{\Psi}(x)$ and $\mathbf{\dot{\Psi}}(x)$ we mean the variations (17).

The smallest value of potential energy $V(\mathbf{\Psi})$ is indicated when

$$
\frac{\partial V}{\partial \mathbf{\Psi}} = 0 \quad \frac{\partial V}{\partial \mathbf{\dot{\Psi}}} = 0,
$$

(18)
which with \(m, f > 0\), as is seen from (17), gives a stable equilibrium
\[
|\Psi_0|^2 = \frac{2m^2}{f}.
\]

Let's introduce Lagrangian of the field \(\Psi(x)\), the latter is considered as Higgs field in the approach being developed.
\[
L = \text{tr}(\partial_\mu \bar{\Psi} \gamma^\mu \Psi + m^2 \bar{\Psi} \gamma^0 \Psi - \frac{f}{4}(\bar{\Psi} \gamma^5 \Psi)^2).
\]  
(19)

Consider what happens with the field \(\Psi(x)\) near the minimum of potential energy as following
\[
\Psi(x) = \frac{1}{2\sqrt{2}}(\frac{2m}{\sqrt{f}} + \sigma(x) + \Theta^k(x) \cdot i\Sigma^k) \begin{pmatrix} 0 & i\sigma^3 \\ 0 & I \end{pmatrix},
\]  
(20)

under supposition, that \(\sigma(x)\) and \(\Theta^k(x)\) are some real functions (here and further the repeating indexes \(k = 1, 2, \ldots, 7\) are implied to sum up, if other is not mentioned), so it is easy to ensure
\[
||\begin{pmatrix} 0 & i\sigma^3 \\ 0 & I \end{pmatrix}||^2 = 4.
\]

3 Generalization of the electro-weak interactions

Lagrangian to octoninic algebra

In Minkovskian space \((M_4)\), where the interval \(ds^2\) is defined as
\[
ds^2 = dt^2 - dx^2 - dy^2 - dz^2 = dx_\mu dx^\mu = dt^2 - dl^2 = \eta_{\mu\nu} dx^\mu dx^\nu
\]
(21)

\((\mu, \nu = 0, 1, 2, 3\), here summing up on the indexes of a different height is implied with metric tensor of Minkovsky space \(\eta_{\mu\nu}\)\) free Dirac equation for a spinor fields \(\psi(x)\) and \(\bar{\psi}(x)\) with mass of \(m\) looks like [8]:
\[
(i\gamma^\mu \overrightarrow{\partial}_\mu - m)\psi(x) = 0 \quad \overline{\psi}(x)(i\overrightarrow{\partial}_\mu \gamma^\mu + m) = 0,
\]  
(22)

where \(\overrightarrow{\partial}_\mu \psi(x) = \partial\psi/\partial x^\mu = \psi_\mu\). Upper and lower indexes of tensors sink and raise respectively in \(M_4\) by mutually inverse tensors \(\eta_{\mu\nu}\) and \(\eta^{\mu\nu}\). \(\overline{\psi}(x) = \psi(x)\gamma^0, x \in M_4, \gamma^\mu, \mu = 0, 1, 2, 3\) are Dirac matrices:
\[
\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad i = 1, 2, 3.
\]  
(23)

In the contemporary theory the interaction of a particle with an extern field is believed to be in minimum way, which means the derivative merely lengthens because of interacting field member
\[
\partial_\mu \rightarrow \partial_\mu + iA_\mu
\]  
(24)
Here the field variable $A_\mu$ is introduced. Modern perspective to field interactions is based on the fact that the interactions comply the intern theory symmetries, which are visualized in the field structure $A_\mu$ provided the interaction is introduced as it is done above. It is known from the Maxwell electro-magnetic theory that it is Abel symmetry that corresponds with electro-magnetic field. In work [9] it is shown the technique of introducing field variables to satisfy the given symmetry. In such way, the incorporating of electro-magnetic field $A_\mu$ for particles with charge "$-e$" with minimum connection

$$A_\mu = -eA_\mu \cdot I, \quad I = 1$$

leads to Dirac equation in an extern electro-magnetic field

$$(i\gamma^\mu(\partial_\mu - ieA_\mu) - m)\psi(x) = 0$$

$$\bar{\psi}(x)(i(\partial_\mu - ieA_\mu)\gamma^\mu + m) = 0$$

(Here the structure unit $I$ is introduced, which in our matter is merely equal to 1.)

The union of electro-magnetic and weak interaction through the minimum connection in the standard Weinberg-Salam theory on group $SU(2) \times U(1)$ with fields $A^{(1)}_\mu = A^k_\mu, k = 1, 2, 3$ and $A^{(2)}_\mu = B_\mu$ and charges $g^k = g - SU(2)$ group constant – symmetry and $g^{(0)}$ – Abel symmetry constant:

$$A_\mu = A^{(1)}_\mu + A^{(2)}_\mu = gA^{(1)}_\mu T^{(1)} + g^{(0)}A^{(2)}_\mu T^{(2)} = \frac{i}{2}g\sigma^k A^k_\mu + \frac{i}{2}g^{(0)}Y B_\mu,$$

$$T^{(1)} = \frac{\sigma^k}{2}, \quad T^{(2)} = Y,$$

therefore the Dirac equation in extern field for fermions $\psi(x)$, with hypercharge $Y$ (in particular in case of leptons doublet $Y = -1$, and in case of singlet $Y = -2$) looks like [10]:

$$(i\gamma^\mu(\partial_\mu + \frac{i}{2}g\sigma^k A^k_\mu + \frac{i}{2}g^{(0)}Y B_\mu) - m)\psi(x) = 0$$

$$\bar{\psi}(x)(i(\partial_\mu - \frac{i}{2}g\sigma^k A^k_\mu - \frac{i}{2}g^{(0)}Y B_\mu) + m) = 0$$

The Lagrangian of free electro-weak field looks like [11]:

$$L = -\frac{1}{2}\text{tr} G_{\mu\nu}G^{\mu\nu} - \frac{1}{4} F_{\mu\nu}F^{\mu\nu}$$

There, the first member of Lagrangian $(k = 1, 2, 3)$

$$-\frac{1}{2}\text{tr} G_{\mu\nu}G^{\mu\nu} = -\frac{1}{2}\text{tr}((\partial_\mu A_\nu - \partial_\nu A_\mu - ig(A_\mu A_\nu - A_\nu A_\mu)) \cdot (\partial^\mu A^\nu - \partial^\nu A^\mu - ig(A^\nu A^\mu - A^\mu A^\nu))),$$

$$A_\mu = A^k_\mu \sigma^k / 2$$
responds for non-Abel symmetry, while the second member
\[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} (\partial_\mu B_\nu - \partial_\nu B_\mu) (\partial^\mu B^\nu - \partial^\nu B^\mu) \] (31)
responds for Abel one.

The spinor equation ψ(x) forms to (26) in case of singlet particle condition (electron) when substituted \( Y = -2 \) and \( g^0 = e, g^k = 0 \). It is known from weak interactions theory the relation between charges \( g^k \) \( SU(2) \times U(1) \) charges model and charge \( e \) from Dirac theory [11].

In the current work the effort to abstract from common way of introducing interacting field as the field, corresponding with certain symmetry, and it is suggested to introduce the field as characteristic of algebraic structure of the interaction. In this approach the existence of a symmetry, e.g. \( SU(2), SU(3) \) is a corollary of field algebraic structure. Thus, we agree the generalization of Dirac equation to interacting field is a generalization to octonion fields and looks like:
\[ (i\gamma^\mu (\partial_\mu + \frac{i}{2} q^a A^a_\mu \Sigma^a) - m) \Psi(x) = 0 \]
\[ \bar{\Psi}(x) (i(\partial_\mu - \frac{i}{2} q^a A^a_\mu \Sigma^a) \gamma^\mu + m) = 0, \] (32)
where \( q^a \) is a charge of the spinor \( \Psi \), defined in (13), which interacts with the field \( A^a_\mu(x), \Sigma^a, a = 0, 1, \ldots, 7 \) are Hermitian generatrixes of octonionic algebra.

(Note, that in (32) matrices \( \Sigma^a, a = 0, 1, 2 \ldots, 7 \) act to spinor intern index \( \Psi(x) \), therefore it cannot be multiplied to Dirac matrices using standard matrix multiplication.)

Consider the doublet of left-polarized leptons \( L(x) \) (we shall restrict the class by using only electron sector, consisted of electron \( e(x) = e^-(x) \) and electron neutrino \( \nu = \nu_e(x) \)), singlet of right-polarized electron \( e_R = e_R^-(x) \) and octet of scalar mesons \( \varphi^a(x) \) [20]:
\[ L = \begin{pmatrix} (\alpha_1 \nu + \alpha_2 e) I \\ B_1 \nu(x) + B_2 e(x) \end{pmatrix}, \quad R(x) = e_R^-(x), \quad \varphi^a(x), i = 0, 1, \ldots, 7, \] (33)
where
\[ \frac{1}{2}(1 + \gamma^5) L = \Psi_L, \quad \frac{1}{2}(1 - \gamma^5) R = \Psi_R, \]
\[ \frac{1}{2}(1 - \gamma^5) e_R = R \quad \gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3. \] (34)

Matrices \( A_1, A_2 \) and \( B_1, B_2 \) look like \( A_i = \alpha_0 I + \sum_{k=1}^3 \alpha^k_i \sigma^k \) and \( B_i = \beta_0 I + \sum_{k=1}^3 \beta^k_i \sigma^k \) with numeric constants still unknown \( \alpha^k_i \) and \( \beta^k_i, i = 1, 2, k = 0, 1, 2, 3. \) Numbers \( \alpha_{1,2} \) and \( \beta_{1,2} \) are still unknown too.

Let’s define the Lagrangian, localized on octonion algebra, as follows \( (a, a') = 0, 1, \ldots, 7) \)
\[ L_{oct.} = \text{tr} \left( -\frac{1}{16} F^{(a)}_{\mu\nu} * F^{(a')\mu\nu} + \right) \]
provided charged potential \( A \) so we come to \((0)\) after a random violation of symmetry, chosen corresponding calibration, come to

\[
\Psi_\varphi(x) = \Psi_0 = \frac{m}{\sqrt{2f}} \left( \begin{array}{c} 0 \\ i\sigma^3 \end{array} \right),
\]

(36)

so we come to \((a = 0, 1, \ldots, 7)\):

\[
L_{\text{act.}} = \text{tr}(-\frac{1}{16} F^{(a)}_{\mu\nu} F^{(a)\mu\nu} + \frac{1}{4} g_0 q_\alpha A^a_\mu A^{(a)\mu\nu} \bar{\Psi}_0 \ast \Sigma^a \ast \bar{\Sigma}^a \ast \Psi_0 +
+i \frac{1}{2} (\bar{\Psi} \ast \gamma_\mu \partial^\mu L - \partial^\mu \overline{\bar{\Psi} \bar{\gamma}_\mu} \ast L) + i \frac{1}{2} (\bar{R} \ast \partial^\nu \gamma_\mu R - \partial^\nu \overline{\bar{R} \bar{\gamma}_\mu} \ast R) -
q^a A^{(a)} \bar{\Psi} \ast \gamma_\mu \Sigma^a \ast L - q^0 A^{(0)} \bar{R} \bar{\gamma}_\mu R -
-h \bar{\Psi}_0 R - h \bar{R} \Psi_0 \ast L + \frac{m^2}{f})
\]

(37)

In the standard Weinberg-Salam \( SU(2) \times U(1) \) theory of weak interaction after a random violation of symmetry, chosen corresponding calibration, come to Lagrangian

\[
L_{\text{ST}} = -\frac{1}{4} \text{tr}(F^{(a)}_{\mu\nu} F^{(a)\mu\nu}) + \frac{g^2 m^2}{2f} A^a_\mu A^{(a)\mu} - \frac{g g^{(1)} m^2}{f} A^a_\mu B^\mu -
\]

(38)

\[
+ \frac{g^{(1)}}{2} \overline{\overline{\nu}} L \gamma_\mu B^\mu \nu_L + \frac{g^{(1)}}{2} \overline{\overline{\nu}} L \gamma_\mu B^\mu e_L + \frac{g^{(1)}}{2} \overline{\overline{\nu}} L \gamma_\mu A^{(a)\mu} e_L - \frac{g^{(1)}}{2} \overline{\overline{\nu}} L \gamma_\mu A^{(a)\mu} e_L +
- \frac{g^{(1)}}{2} \overline{\overline{\nu}} L \gamma_\mu (A^{(1)} - i A^{(2)}) - \frac{g^{(1)}}{2} \overline{\overline{\nu}} L \gamma_\mu (A^{(1)} + i A^{(2)}) -
+ i \frac{1}{2} (\overline{\overline{\nu}} L \gamma_\mu \partial^\mu e_L - \partial^\mu \overline{\overline{\nu}} L \gamma_\mu e_L) +
\]

(39)

It is known about this Lagrangian that it diagonalized by fields \( A^a_\mu(x)\), provided charged \( W^+\), \( W^-\) and neutral \( Z^0\) bosons and electromagntetical vector-potential \( A_\mu\) are introduced:

\[
A^3_\mu = Z^0_\mu \cos \theta + A_\mu \sin \theta, \quad B_\mu = A_\mu \cos \theta - Z^0_\mu \sin \theta,
\]

(39)
\[
A_1^\mu = \frac{1}{\sqrt{2}} (W_\mu \cos \theta + \overline{W}_\mu \sin \theta), \quad A_2^\mu = \frac{i}{\sqrt{2}} (W_\mu \cos \theta - \overline{W}_\mu \sin \theta) \quad (40)
\]

We shall show now the solution for posed problem exists by presenting partial solution.

Equality

\[
\frac{i}{2} (\mathcal{L} \ast \gamma_\mu \partial^\mu L - \partial^\mu \mathcal{L} \gamma_\mu \ast L) + \frac{i}{2} (\mathcal{R} \ast \gamma_\mu \partial^\mu R - \partial^\mu \mathcal{R} \gamma_\mu \ast R) =
\]

\[
= \frac{i}{2} (\mathcal{L} \gamma_\mu \partial^\mu e_L - \partial^\mu \mathcal{L} \gamma_\mu e_L) + \frac{i}{2} (\mathcal{R} \gamma_\mu \partial^\mu \nu_L - \partial^\mu \mathcal{R} \gamma_\mu \nu_L) +
\]

\[
+ \frac{i}{2} (\mathcal{L} \gamma_\mu \partial^\mu e_R - \partial^\mu \mathcal{L} \gamma_\mu e_R)
\]

is provided by constraint \( R = e_R(x) \) and

\[
L = \frac{c_0}{\sqrt{2}} \begin{pmatrix} 0 \\ (\frac{2}{3} i \sigma_1 + \frac{1}{3} \sigma_2) e(x) + (y_0 I + \frac{257}{32} \sigma_1 + \frac{5}{3} \sigma_2 + \frac{257}{32} \sigma_3) e(x) \end{pmatrix}_L
\]

\[
\mathcal{L} = L^+ \gamma_0,
\]

\[
\mathcal{L} = \frac{c_0}{\sqrt{2}} \begin{pmatrix} 0 \\ (\frac{2}{3} i \sigma_1 + \frac{1}{3} \sigma_2) e(x) + (y_0 I - \frac{2304}{257} \sigma_1 - \frac{5}{3} \sigma_2 - \frac{9}{16} \sigma_3) e(x) \end{pmatrix}_L
\]

It is easy to ensure then

\[
\text{tr} \mathcal{L} \ast \Sigma^1 \ast L = c_0^2 (\mathcal{L} e_L + \mathcal{L} \nu_L)
\]

\[
\text{tr} \mathcal{L} \ast \Sigma^2 \ast L = c_0^2 (-\mathcal{L} e_L + \mathcal{L} \nu_L)
\]

\[
\text{tr} \mathcal{L} \ast \Sigma^3 \ast L = c_0^2 (\mathcal{L} \nu_L - \mathcal{L} e_L)
\]

\[
\text{tr} \mathcal{L} \ast L = c_0^2 (\frac{257}{32} \mathcal{L} \nu_L + (y_0^2 + \frac{5729}{2304}) \mathcal{L} e_L)
\]

(42)

Notice, that the order of multiplication does not matter when valuating the trace \( (42) \). Given \( y_0^2 = \frac{257}{32} - \frac{5729}{2304} \) and \( c_0^2 = \frac{2304}{5729} \), we come to

\[
\text{tr} \mathcal{L} \ast L = \mathcal{L} \nu_L + \mathcal{L} e_L
\]

Equality

\[
q^{(0)} (A_\mu^0 \mathcal{L} \gamma_\mu \ast L + A_\mu^0 \mathcal{R} \gamma_\mu R) =
\]

\[
= \frac{q^{(1)}}{2} (\mathcal{L} \gamma_\mu B^\mu \nu_L + \mathcal{L} \gamma_\mu B^\mu \nu_L + \mathcal{R} \gamma_\mu B^\mu e_R)
\]

is achieved provided \( q^{(0)} e_0^2 = -q^{(1)} \) and \( A_\mu^0 = B^\mu \).

Equality

\[
\text{tr} (q^1 A^\mu \Sigma^1 + q^2 A^\mu \Sigma^2 + q^3 A^\mu \Sigma^3) \ast \gamma_\mu L =
\]

\[
= \frac{q^1}{2} \mathcal{L} \gamma_\mu e_L (A^\mu_1 - i A^\mu_2) + \frac{q^2}{2} \mathcal{L} \gamma_\mu \nu_L (A^\mu_1 + i A^\mu_2) -
\]
\[-\frac{g}{2} \tau_L \gamma_\mu A^{\mu 3} e_L + \frac{g}{2} \tau_L \gamma_\mu A^{\mu 3} \nu_L \]

is guaranteed when \( q^{(k)} c^3_0 = g, A^{\mu k} = A^{\mu k}, k = 1, 2, 3. \)

Equality
\[
\hat{h} L \ast \Psi_0 R + \hat{h} R \ast \Psi_0 L = \frac{\sqrt{2} \hat{h} m}{\sqrt{f}} (\tau_L e_R + \tau_R e_L) \tag{43}
\]
is attained if \( \hat{h} c_0 / \sqrt{2} = h. \)

Thus, the Lagrangian is offered, given on generalized algebra of Cally octaves, and which in particular case derives to the Lagrangian of a standard theory of electro-weak interactions, with condition function looking like:
\[
L = \frac{c_0}{\sqrt{2}} \begin{pmatrix}
0 & (2 i \sigma^1 - 2 \sigma^2) \nu(x) + (y_0 I + \frac{2 i}{3} \sigma^1 + \frac{2}{3} \sigma^2 + i \sigma^3) e(x) \\
- \frac{i}{8} \sigma^1 + \frac{1}{8} \sigma^2 & (0 - \frac{2 i}{3} \sigma^1 - \frac{1}{3} \sigma^2 + \frac{1}{16} \sigma^3) e(x)
\end{pmatrix}_L = \frac{c_0}{\sqrt{2}} \begin{pmatrix}
0 & 2 i \sigma^1 - 2 \sigma^2 \\
- \frac{i}{8} \sigma^1 + \frac{1}{8} \sigma^2 & 0
\end{pmatrix} \nu_L + \begin{pmatrix}
0 & y_0 I + \frac{2 i}{3} \sigma^1 + \frac{2}{3} \sigma^2 + i \sigma^3 \\
- \frac{3 i}{8} \sigma^1 - \frac{1}{8} \sigma^2 + \frac{1}{16} \sigma^3 & 0
\end{pmatrix} e_L.
\]

Notice, the offered Lagrangian on generalized octonionic algebra does not differ from the Lagrangian in \([38]\), given \([38]\) \( a = 0, \ldots, 7 \) on chosen condition function \( [37] \).

4 Entire Lagrangian of theory

Thus, consider the integrated Lagrangian after symmetry violation, that looks like (before fields diagonalization)
\[
L_{\text{act.}} = \text{tr}( - \frac{1}{16} F_{\mu \nu}^\alpha F^{\mu \nu} (\alpha) + \frac{1}{4} q^\alpha q^{\alpha'} A_{\mu}^\alpha A^{\mu (\alpha')} \ast \Psi_0 \ast \Sigma^{\alpha} \ast \Sigma^{\alpha'} \ast \Psi_0 + \\
\frac{i}{2} (\bar{L} * \gamma_\mu \partial^\mu L - \partial^\mu \bar{L} \gamma_\mu * L) + \frac{i}{2} (\bar{R} * \partial^\mu \gamma_\mu R - \partial^\mu \bar{R} \gamma_\mu * R) - \\
- q^\alpha A^{\mu} \bar{L} * \gamma_\mu \Sigma^{\alpha} \ast L - q^0 A^{(0)} \bar{R} \gamma_\mu R - \\
- h \bar{L} \ast \Psi_0 R - h \bar{R} \ast \Psi_0 L) + \frac{m^4}{f} \tag{44}
\]

The Lagrangian, along with fields corresponding to electro-weak interaction, has four additional fields \( A^{(k)}, k = 4, \ldots, 7, \) which correspond to non-associative parts of Lagrangian. The Lagrangian is considered on non-associative octonionic algebra, therefore it is non-associative. The Lagrangian can be decomposed to two expressions, associative and non-associative, according to the rule:
\[
A = a_1 a_2 a_3 = \frac{1}{2} ((a_1 a_2) a_3 + a_1 (a_2 a_3)) + \frac{1}{2} ((a_1 a_2) a_3 - a_1 (a_2 a_3)) = A_{\text{as.}} + A_{\text{nas.}}.
\]

Three sigma matrices trace equals zero, therefore associative and non-associative parts of four matrices trace we shall define as follows:
\[
\text{tr} \{ \Sigma^a, \Sigma^b, \Sigma^c, \Sigma^d \} = \text{tr} (\Sigma^a \{ \Sigma^b, \Sigma^c, \Sigma^d \} - \{ \Sigma^a, \Sigma^b, \Sigma^c \} \Sigma^d) = 8 \varepsilon^{abcd}. \tag{45}
\]
The non-associative part (44) looks like:

\[
\left( \frac{1}{16} \text{tr} F\!^{(a)} \rho \equiv_{\mu \nu} A^{a} \right)_{\text{nass.}} = \eta^{\lambda \delta} \eta^{\mu \nu} \varepsilon^{abcd} A^{a}_{\lambda} A^{b}_{\mu} A^{c}_{\nu} A^{d}_{\delta}.
\]

\[
(q^{(5)} A^{\mu(5)} \mathcal{T} \equiv_{\gamma \mu} \Sigma^{5} \equiv_{L} )_{\text{nass.}} = g_{5} A^{\mu(5)} \left( \frac{5i}{4} \sqrt{g} \gamma_{\mu} e - \frac{5i}{4} \sqrt{g} L \gamma_{\mu} \nu_{L} \right)
\]

\[
(q^{(6)} A^{\mu(6)} \mathcal{T} \equiv_{\gamma \mu} \Sigma^{6} \equiv_{L} )_{\text{nass.}} = g_{6} A^{\mu(6)} \left( \frac{3}{2} \sqrt{g} L \gamma_{\mu} e_{L} + \frac{5}{4} \nu_{L} \gamma_{\mu} \nu_{L} \right)
\]

\[
g_{7} A^{\mu(7)} \mathcal{T} \equiv_{\gamma \mu} \Sigma^{7} \equiv_{L} = 0
\]  \hspace{1cm} (46)

At the same time, associative part of the last three expressions equals zero. The field \( A^{a}_{\mu} \) has only associative part:

\[
(q^{(4)} A^{\mu(4)} \mathcal{T} \equiv_{\gamma \mu} \Sigma^{4} \equiv_{L} )_{\text{nass.}} = g_{4} A^{\mu(4)} \left( \kappa_{1} \nu_{L} \gamma_{\mu} e_{L} - \kappa_{2} \sqrt{g} L \gamma_{\mu} e_{L} \right)
\]  \hspace{1cm} (47)

(Here normalization constants \( \kappa_{1} \) and \( \kappa_{2} \) are approximately equal to eight and seven respectively and the designation is introduced \( q^{(k)} e^{2}_{0} = g_{k}, k = 4, \ldots, 7 \).)

After summing up those expressions we come to non-associative part of the entire Lagrangian:

\[
L_{\text{nass.}} = -\eta^{\lambda \delta} \eta^{\mu \nu} \varepsilon^{abcd} A^{a}_{\lambda} A^{b}_{\mu} A^{c}_{\nu} A^{d}_{\delta} - \frac{3}{2} g_{6} A^{\mu(6)} \sqrt{g} L \gamma_{\mu} e_{L} +
\]

\[
- \frac{5}{4} \left( g_{6} A^{\mu(6)} + ig_{5} A^{\mu(5)} \right) \sqrt{g} L \gamma_{\mu} e_{L} - \frac{5}{4} \left( g_{6} A^{\mu(6)} - ig_{5} A^{\mu(5)} \right) \sqrt{g} L \gamma_{\mu} \nu_{L}. \hspace{1cm} (48)
\]

Therefore the entire Lagrangian on non-associative algebra consists of parts and looks like:

\[
L_{\text{oct.}} = L_{\text{nass.}} + L_{\text{nass.}} = -\frac{1}{4} \text{tr} (F^{\rho(\nu) F^{\rho(\nu)}(a)} + g_{2} m^{2} A^{a}_{\rho} A^{\rho(a)} - \frac{g g^{(1)} m^{2}}{f} A^{a}_{a} B^{a} -
\]

\[
+ \frac{g^{(1)}}{2} \sqrt{g} \nu_{L} \gamma_{\mu} B^{\mu} \nu_{L} + \frac{g^{(1)}}{2} \sqrt{g} \nu_{L} \gamma_{\mu} B^{\mu} \nu_{L} + \left( \frac{g}{2} \sqrt{g} \nu_{L} \gamma_{\mu} e_{L} - \frac{g}{2} \sqrt{g} \nu_{L} \gamma_{\mu} e_{L} \right) A^{a}(A^{a} - i A^{a}) -
\]

\[
+ \frac{i}{2} (\sqrt{g} \nu_{L} \gamma_{\mu} \nu_{L} \gamma_{\mu} e_{L}) + \frac{i}{2} (\sqrt{g} \nu_{L} \gamma_{\mu} \nu_{L} \gamma_{\mu} e_{L}) +
\]

\[
+ \frac{i}{2} \left( \nu_{R} \gamma_{\mu} \nu_{R} e_{R} - \nu_{R} \gamma_{\mu} \nu_{R} e_{R} \right) + g^{(1)} \nu_{R} \gamma_{\mu} B^{\mu} e_{R} + \frac{\sqrt{g_{m}^{2}}}{\sqrt{f}} \left( \nu_{R} e_{R} + \nu_{R} e_{R} \right) +
\]

\[
+ \frac{m^{4}}{f} - g_{4} A^{\mu(4)} \left( \kappa_{1} \nu_{L} \gamma_{\mu} e_{L} - \kappa_{2} \sqrt{g} L \gamma_{\mu} e_{L} \right) -
\]

\[
- \eta^{\lambda \delta} \eta^{\mu \nu} q^{a} q^{b} q^{c} q^{d} \varepsilon^{abcd} A^{a}_{\lambda} A^{b}_{\mu} A^{c}_{\nu} A^{d}_{\delta} - \frac{3}{2} g_{6} A^{\mu(6)} \sqrt{g} L \gamma_{\mu} e_{L} -
\]

15
\[ -\frac{5}{4}(g_6 A^\mu(6) + ig_5 A^\mu(5)) \mathcal{F}_L \gamma_\mu \epsilon_L - \frac{5}{4}(g_6 A^\mu(6) - ig_5 A^\mu(5)) \mathcal{F}_L \gamma_\mu \epsilon_L. \]  

(49)

The entire Lagrangian presents here with associative and non-associative parts separated.

It seems the obtained expressions reminds the expression in the standard model, \( SU(2) \), provided new negative-charged \( D \) (it is vice versa when speaking of standard model concerning \( W^- \)-bosons) and positive-charged \( \ast D \)-bosons with one charge constant \( g_a \) are introduced: \( g_a A = g_6 A^\mu(6) + ig_5 A^\mu(5) \) and \( g_a \ast A = g_6 A^b(6) - ig_5 A^b(5) \), but it is easy to see from (48) such representation faces contravention second member of equation (48). In fact, the proposed approach to define condition in unusual way, using matrix, allows to solve the problem. It is because when deriving the expression \( 2 \frac{3}{4} g_6 A^\mu(6) \mathcal{F}_L \gamma_\mu \epsilon_L \) in intermediate stage of valuation the cross members existed, which were successfully reduced on the next stages and therefore we can rewrite

\[ 2g_6 A^\mu(6) \mathcal{F}_L \gamma_\mu \epsilon_L = \]

\[ = (g_6 A^\mu(6) + ig_5 A^\mu(5)) \mathcal{F}_L \gamma_\mu \epsilon_L + (g_6 A^\mu(6) - ig_5 A^\mu(5)) \mathcal{F}_L \gamma_\mu \epsilon_L \]  

(50)

After such modifications the non-associative Lagrangian part renews to

\[ L_{\text{nas.}} = -\frac{\eta}{4} \eta^\mu \nu q^a q^b q^c q^d \epsilon_{abcd} A^a A^b A^c A^d \frac{3}{4} g_a (\ast D^\mu + \ast \ast D^\mu) \mathcal{F}_L \gamma_\mu \epsilon_L - \]

\[ - \frac{5}{4} g_a D^\mu \mathcal{F}_L \gamma_\mu \epsilon_L - \frac{5}{4} g_a \ast D^\mu \mathcal{F}_L \gamma_\mu \epsilon_L. \]  

(51)

Therefore the generalized Lagrangian has additional free part:

\[ L_{\text{add}} = -\frac{1}{2} \left( \partial_\mu \ast D^\nu - \partial_\nu \ast D^\mu \right) \left( \partial^\mu \ast D_\nu - \partial^\nu \ast D_\mu \right) + \frac{g_a m^2}{f} \ast D^\mu D_\mu - \]

\[ - \frac{1}{4} \left( \partial_\mu C^\nu - \partial_\nu C^\mu \right) \left( \partial^\mu C_\nu - \partial^\nu C_\mu \right) + \frac{g_a^2 m^2}{2f} C^\mu C_\mu - \]

\[ - \frac{1}{4} \left( \partial_\mu E^\nu - \partial_\nu E^\mu \right) \left( \partial^\mu E_\nu - \partial^\nu E_\mu \right) + \frac{g_a^2 m^2}{2f} E^\mu E_\mu - \]

\[ - \frac{3}{4} g_a (D^\mu + \ast \ast D^\mu) \mathcal{F}_L \gamma_\mu \epsilon_L - \frac{5}{4} g_a D^\mu \mathcal{F}_L \gamma_\mu \epsilon_L - \frac{5}{4} g_a \ast D^\mu \mathcal{F}_L \gamma_\mu \epsilon_L. \]  

(52)

As it follows from the latter, it is natural to expect omitting two oppositely charged bosons simultaneously, imitating neutral current. (It is noteworthy, there is no resembling neutral neutrino current in the Lagrangian (51).)
5 Conclusion

The result of the Lagrangian introduced is the appearance of new neutral massive \( C \)–current and one new massive charged \( D \)–current. Unfortunately, the proposed theory, in the frameset of a model in the Minkowskian space, does not offer any exact values of new bosons masses.

The important issue is the appearance of a massive neutral particle, \( E \)–boson, which does not interact with matter, but interact with \( A_\mu^{(a)} \) fields by means of non-associative connection.

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