Small Area Estimation with Bivariate Hierarchical Bayes (HB) Approach to Estimate Monthly Average per Capita Expenditure of Food and Non-Food Commodities in Province of Bali

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Abstract. Small Area Estimation (SAE) is an indirect method that has been widely used for estimating parameters in a small area or small domain by borrowing strength of predictor variables from census or registration. This study uses Hierarchical Bayes (HB) method under the univariate and bivariate Fay-Herriot (FH) model to estimate monthly average per capita expenditure of food and non-food commodities for each district level in Province of Bali in 2014. Then estimation results from both models will be compared. The bivariate FH model is expected to increase the accuracy of the results of estimation by taking into account correlation between two types of expenditure rather than perform univariate estimation separately. Thirteen predictor variables from the administrative record of village data (PODES 2014) are included in each model as factors that affect these two types of expenditure. From the result, there are three variables that have significant effect on food expenditure, both in univariate and bivariate FH model. While, for non-food expenditure both model show different result on significant variables. Based on the results of the performance comparison, the best model is bivariate FH model since it has smaller Mean Square Prediction Error (MSPE), Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RMSE) value than univariate FH models. In addition, the bivariate FH model produces shorter 95% credible interval of estimated values. These conditions indicate that jointly modeling can improve the accuracy of estimation. Bivariate FH also produces significant improvement in adjusted $R^2$ value. Finally, the mapping result shows the same pattern for two types of expenditure. The highest monthly average per capita expenditure is more localized in the southern districts of Bali. While the lowest expenditure is more localized in the eastern and western districts of Bali.

Keywords: Monthly Average per Capita Expenditure, Food and Non-Food Commodities, Small Area Estimation (SAE), Bivariate Fay-Herriot Model, Hierarchical Bayes (HB)

1. Introduction

Consumption expenditure is one of the important data as a measure of equity among regions. If this expenditure is specified as food and non-food expenditure, it can be used to evaluate the standard of living and the level of welfare of the population [1]. This is in accordance with Engel's Law which states that if tastes are not different, the percentage of food expenditure decreases with increasing income. That is, the higher the level of welfare of the population, the higher the non-food expenditure.
compared to food expenditure. Data related to these expenditures originated from the National Socio-Economic Survey (Susenas) conducted by the Statistics Indonesia (BPS), where the estimation is designed only for regencies/cities. Whereas government programs for poverty eradication are directly targeting the poor people. Therefore, the consumption expenditure data is expected to reach smaller areas, until village levels.

However, the addition of survey samples cannot be done because of the constraints of costs, time and human resources. While estimation by utilizing the results of the existing survey will produce a fairly large variant because of the inadequate number of samples. As an alternative is the use of Small Area Estimation (SAE) as an indirect estimation method by linking the estimated results of the survey with the predictor variables of the census or registration results for estimates to smaller areas [2].

Various developments have been carried out in order to improve the accuracy of the SAE estimation results by using a multivariate model to estimate two or more population parameters at once. The research by [3] with the Hierarchical Bayes (HB) method and [4] with the Empirical Best Linear Unbiased Prediction (EBLUP) method compared the estimation results of the multivariate model with the univariate model. The results of the two studies showed that the use of multivariate models could increase the accuracy of the estimation results since it considers the correlation between response variables.

Based on the formulation of the problem above, this study uses the univariate and bivariate Fay-Herriot (FH) model with the HB method to estimate monthly average per capita expenditure of food and non-food commodities in each district in Province of Bali, 2014 then performs comparison of the estimation results from two models to get the best model.

2. Theoretical Review

2.1. Small Area Estimation (SAE)

There are two types of models in SAE based on the type of unit observed, i.e the basic area level model and the basic unit level model. This study uses basic area level model in SAE that first used by [5]. The first level is the direct estimation model,

\[ \hat{\theta}_i = \theta_i + \epsilon_i \]  

where \( \hat{\theta}_i \) is the result of direct estimation for \( i \)th area, \( i = 1,2, ..., m \), \( \theta_i \) is the parameter of interest at the \( i \)th area and the sampling error for \( i \)th area, \( \epsilon_i \sim (0, \sigma^2) \). Then at the second level there is a linking model that connects \( \theta_i \) parameter with a set of predictor variables of the census or registration results, \( x_i = [x_{i1} \ x_{i2} \ ... \ x_{ip}]^T \), i.e:

\[ \theta_i = x_i^T \beta + u_i \]  

Where \( \beta = [\beta_1 \ \beta_2 \ ... \ \beta_p]^T \) is the regression coefficients and the random effect for \( i \)th area \( u_i \sim (0, \sigma_u^2) \). The FH model obtained by substituting equation (2) to equation (1) and get the following linear mixed model:

\[ \hat{\theta}_i = x_i^T \beta + u_i + \epsilon_i \]  

2.2. Hierarchical Bayes (HB) Method

In this method, each parameter that will be estimated in the model has a prior distribution, that according to [6] shows the uncertainty of the parameter value before considering information from the observed data. Thus creating a hierarchical arrangement of these parameters. If \( \theta \) is a parameter that will be estimated, then the prior distribution of this parameter need to be determined, i.e \( f(\theta | a) \). Then the parameters of that prior distribution, \( a \), also have a prior distribution, \( f(a | b) \) (Figure 1).
The next process is to determine the joint distribution, the posterior distribution and the full conditional distribution of each parameter in the model. The estimation process is carried out by the Markov Chain Monte Carlo (MCMC) method with Gibbs Sampling algorithm by generating samples of each parameter from the corresponding full conditional distribution for $h + H$ iteration, where $h$ iterations for burnin period and $H$ iteration for posterior analysis or estimation of each parameter, with a certain value of thin (lag) for each chain. So the estimate value for $\theta$ parameters is the posterior mean, $\hat{\theta} = E[\theta|y]$, and the variance of estimate value is the posterior variance, $V(\hat{\theta}) = V[\theta|y]$.

We use three different method for checking convergence, i.e by visually checking the trace plot, autocorrelation plot and density plot, calculate the percentage of Monte Carlo error (MCE) to the standard deviation of posterior distribution where percentage under 5 percent indicates convergent condition [7] and calculating factor scale reduction (Rhat) by [8] where convergent condition indicated by the value of Rhat that reach to 1.

3. Results And Discussion

The two response variables used in this study are transformed (natural logarithm/ln) expenditure of food and non-food consumption obtained from National Socio-Economic Survey (Susenas) Province of Bali, 2014. These response variables are assumed follow multivariate normal distribution since from Mardia test [9] the null hypothesis is rejected. The 13 predictor variables obtained from the administrative record of village data (PODES 2014) of Province Bali are presented below:

| Predictor Variable                                      |
|--------------------------------------------------------|
| $X_1$ District’s main topography: 0 – Flatland, 1 – Valley and slope/top of the mountain |
| $X_2$ Average number of household members               |
| $X_3$ Proportion of villages with primary income from the agricultural sector |
| $X_4$ Proportion of family as electricity users         |
| $X_5$ Proportion of family live in slums area           |
| $X_6$ Ratio of elementary school per 1,000 population   |
| $X_7$ Ratio of junior high school per 1,000 population   |
| $X_8$ Ratio of medical labour per 1,000 population      |
| $X_9$ Proportion of malnutrition over the past 3 years  |
| $X_{10}$ Proportion of residents that receive health insurance card |
| $X_{11}$ Ratio of poor certificate per 1,000 population |
| $X_{12}$ Proportion of people with disabilities        |
| $X_{13}$ Ratio of micro and small industries per 1000 population |

3.1. SAE Using Univariate FH Model with HB Method

The hierarchical structure of the parameters in the univariate FH model, for expenditure of food or non-food commodities, is presented in Directed Acyclic Graph (DAG) Figure 2.
According to the DAG of univariate FH model above, $\hat{\theta}_i | \beta, u_i, \tau_{\varepsilon_i} \sim N(x_i^T \beta + u_i, \tau_{\varepsilon_i}^{-1})$; \( i = 1, 2, ..., m \) and the likelihood function is:

$$f(\hat{\theta} | \beta, u_i, \tau_{\varepsilon_i}) = \prod_{i=1}^{m} \frac{1}{\sqrt{2\pi \tau_{\varepsilon_i}}} e^{-\frac{\tau_{\varepsilon_i}(\hat{\theta}_i - (x_i^T \beta + u_i))^2}{2\tau_{\varepsilon_i}}}$$

(4)

and prior specification are:

(i). $\beta_k \sim N(\mu_{\beta_k}, \tau_{\beta_k}^{-1})$, with $\mu_{\beta_k}$ and $\tau_{\beta_k}$ obtain from multiple linear regression.

(ii). $u_i | \tau_u \sim N(\mu_u, \tau_u^{-1})$, with $\mu_u = 0$.

(iii). $\tau_u \sim G(a_u, b_u)$, with parameter $a_u = b_u = 0.01$.

(iv). $\tau_{\varepsilon_i} \sim G(a_{\varepsilon}, b_{\varepsilon})$, with parameter $a_{\varepsilon} = b_{\varepsilon} = 0.01$.

where $G(a_u, b_u)$ and $G(a_{\varepsilon}, b_{\varepsilon})$ are Gamma distribution. That prior specification for $\beta_k$ is different from [10], that use flat prior $f(\beta_k) \propto 1$.

The estimation process uses MCMC method with Gibbs Sampling algorithm by drawing sample from the full conditional distribution of each parameter in the model. The number of chain use in the drawing sample are two chains, with the number of iteration for burn-in period ($h$) are 1,000 iterations, number of iteration for posterior analysis ($H$) are 20,000 iterations and thin is 1, for each chain. This setting make the Markov chain already in convergent condition.

**Table 2.** Estimation Results of Univariate FH Model – Food Commodities

| Parameter | Mean  | sd    | MC error       | 2.50% | 97.50% | MCE/sd (%) | Rhat |
|-----------|-------|-------|----------------|-------|--------|------------|------|
| $\beta_0$ | 12.68 | 0.6791| 0.003919       | 11.35 | 14.02  | 0.58       | 1.00 |
| $\beta_1$ | 0.0589 | 0.0701| 4.30E-04       | -0.0789 | 0.1964 | 0.61       | 1.00 |
| $\beta_2$ | -0.0333 | 0.0430 | 2.49E-04     | -0.1178 | 0.05076 | 0.58       | 1.00 |
| $\beta_3$ | -0.3858 | 0.105 | 6.29E-04      | -0.5911 | -0.1798 | 0.60       | 1.00 |
| $\beta_4$ | 0.863 | 0.6757 | 0.003837       | -0.4651 | 2.189  | 0.57       | 1.00 |
| $\beta_5$ | 6.29  | 4.182 | 0.0209         | -1.91  | 14.61  | 0.50       | 1.00 |
The estimation results from univariate FH model for food commodities (Table 2) shows that the percentage of MCE to the posterior standard deviation are already below 5 percent and the value of factor scale reduction (Rhat) close to 1 for all parameters indicate the convergent condition. There are four variables that have significant effect to the response variable, i.e proportion of villages with primary income from the agricultural sector, ratio of poor certificate per 1.000 population and small industries per 1000 population with negative effect and proportion of people with disabilities and ratio of micro with positive effect to the response variable.

Table 3. Estimation Results of Univariate FH Model – Non-food Commodities

| Parameter | Mean  | sd    | MC error | 2.50% | 97.50% | MCE/sd (%) | Rhat |
|-----------|-------|-------|----------|-------|--------|------------|------|
| β_0       | 12.21 | 1.573 | 0.01083  | 9.146 | 15.33  | 0.69       | 1.00 |
| β_1       | 0.1462| 0.1542| 0.00124  | -0.1577| 0.447  | 0.80       | 1.00 |
| β_2       | 0.1819| 0.08975| 7.27E-04 | 0.005454| 0.359  | 0.81       | 1.00 |
| β_3       | -0.639| 0.2128| 0.001546 | -1.051 | -0.2113| 0.73       | 1.00 |
| β_4       | 1.051 | 1.554 | 0.01073  | -2.027 | 4.071  | 0.69       | 1.00 |
| β_5       | 3.965 | 8.267 | 0.05759  | -12.39 | 20.16  | 0.70       | 1.00 |
| β_6       | -0.2917| 0.4215| 0.00331  | -1.135 | 0.5144 | 0.79       | 1.00 |
| β_7       | -0.9802| 1.634 | 0.0123   | -4.168 | 2.269  | 0.75       | 1.00 |
| β_8       | 0.03216| 0.04627| 3.50E-04 | -0.05803| 0.1237| 0.76       | 1.00 |
| β_9       | -3.066| 1751  | 12.23    | -6450 | 445.1  | 0.70       | 1.00 |
| β_10      | -0.191| 0.233 | 0.001761 | -0.6543| 0.2594 | 0.76       | 1.00 |
| β_11      | -0.00446| 0.001653| 1.35E-05 | -0.00775| -0.00127| 0.82       | 1.00 |
| β_12      | 32.9  | 26.31 | 0.2015   | -19.87 | 83.77  | 0.77       | 1.00 |
| β_13      | -0.00452| 0.00285| 1.90E-05 | -0.01016| 0.001042| 0.67       | 1.00 |

D(θ) = 52.9140  pD = 79.9000
D(θ) = -26.9860  DIC = 132.8150

Note:  significant variable

The estimation of univariate FH model for non-food commodities (Table 3) shows different result compare to univariate FH model for food commodities. There are just three predictor variables that have significant effect to the response variable, i.e proportion of villages with primary income from the agricultural sector and ratio of poor certificate per 1.000 population that have negative effect and average number of household members that has positive effect to the response variable.
3.2. SAE Using Bivariate FH Model with HB Method

The bivariate FH model is the extension of FH model when there are two response variables, the expenditure of food and non-food consumption. The model is

\[ \tilde{\theta} = X^T B + u + \epsilon \]  \hspace{1cm} (5)

or in the matrix form

\[ \begin{bmatrix} \tilde{\theta}_{11} \\ \tilde{\theta}_{21} \\ \vdots \\ \tilde{\theta}_{m1} \end{bmatrix} = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1p} \\ 1 & x_{21} & \cdots & x_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{mp} & \cdots & x_{mp} \end{bmatrix} \begin{bmatrix} \beta_{10} & \beta_{20} \\ \beta_{11} & \beta_{21} \end{bmatrix} + \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \\ \vdots & \vdots \\ u_{m1} & u_{m2} \end{bmatrix} + \begin{bmatrix} \epsilon_{11} \\ \epsilon_{21} \\ \vdots \\ \epsilon_{m1} \end{bmatrix} \]

with the DAG presented below.

![Figure 3. DAG of Bivariate FH Model - Food and Non-Food Commodities](image)

According to the DAG of bivariate FH model above, \( \tilde{\theta}_i | B, u, \tau_{\epsilon i} \sim N_2(x_i^T B + u, \tau_{\epsilon i}^{-1}) \); \( i = 1, 2, \ldots, m \) and the likelihood function is:

\[ f(\tilde{\theta} | B, u, \tau_{\epsilon i}) = \prod_{i=1}^{m} \left( \frac{1}{2\pi} \right)^{n/2} \exp \left( -\frac{1}{2} (\tilde{\theta}_i - (x_i^T B + u)) \tau_{\epsilon i}^{-1} (\tilde{\theta}_i - (x_i^T B + u)) \right) \]  \hspace{1cm} (6)

and prior specification are:

(i). \( \beta_{1k} \sim N(\mu_{\beta_{1k}}, \tau_{\beta_{1k}}^{-1}) \) and \( \beta_{2k} \sim N(\mu_{\beta_{2k}}, \tau_{\beta_{2k}}^{-1}) \), with \( \mu_{\beta_{1k}}, \tau_{\beta_{1k}}, \mu_{\beta_{2k}} \) and \( \tau_{\beta_{2k}} \) obtain from multiple linear regression, same with specification of the univariate FH model.

(ii). \( u_i | \tau_u \sim N_2(\mu_u, \tau_u^{-1}) \), with \( \mu_u = [0 \ 0]^T \).

(iii). \( \tau_{\epsilon i} \sim W(R_u, v_u) \), with the scale matrix \( R_u = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \) and the degree of freedom \( v_u = 57 \), as number of district.

(iv). \( \tau_{\epsilon j} \sim W(R_e, v_e) \), with the scale matrix \( R_e = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \) and the degree of freedom \( v_e = 57 \).

where \( W(R_u, v_u) \) and \( W(R_e, v_e) \) are the Wishart distribution.

### Table 4. Estimation Results of Bivariate FH Model

| Parameter | Mean  | sd    | MC error | 2.50% | 97.50% | MCE/sd (%) | Rhat |
|-----------|-------|-------|----------|-------|--------|------------|------|
|           | (1)   | (2)   | (3)      | (4)   | (5)    | (6)        | (7)  |
| Food Commodities |       |       |          |       |        |            |      |
| \( \beta_{10} \) | 12.69 | 0.6591| 0.02152  | 11.43 | 13.99  | 3.27       | 1.01 |
| \( \beta_{11} \) | 0.06152 | 0.0616 | 0.000331 | -0.05865 | 0.183 | 0.54       | 1.00 |
| \( \beta_{12} \) | -0.03692 | 0.0386 | 0.000324 | -0.1119 | 0.03873 | 0.84       | 1.00 |
| \( \beta_{13} \) | -0.3817 | 0.09412 | 0.000524 | -0.5668 | -0.1955 | 0.56       | 1.00 |
| \( \beta_{14} \) | 0.8426 | 0.6554 | 0.02162 | -0.4558 | 2.105 | 3.30       | 1.01 |
| \( \beta_{15} \) | 6.628 | 3.785 | 0.01881 | -0.8146 | 14.07 | 0.50       | 1.00 |
| \( \beta_{16} \) | -0.2932 | 0.181 | 0.001124 | -0.6485 | 0.06002 | 0.62       | 1.00 |
### Parameter Estimates

| Parameter | Mean   | sd     | MC error  | 2.50%   | 97.50%  | MCE/sd (%) | Rhat |
|-----------|--------|--------|-----------|---------|---------|------------|------|
| $\beta_{17}$ | -0.2649 | 0.7012 | 0.003429  | -1.625  | 1.112   | 0.49       | 1.00 |
| $\beta_{18}$ | -0.01009 | 0.02086 | 0.000122  | -0.05092 | 0.03095 | 0.58       | 1.00 |
| $\beta_{19}$ | -1112   | 810.9  | 4.266     | -2717   | 467.2   | 0.53       | 1.00 |
| $\beta_{110}$ | 0.01279 | 0.09859 | 0.00048   | -0.1806 | 0.2055  | 0.49       | 1.00 |
| $\beta_{111}$ | -0.0022 | 0.000741 | 3.97E-06  | -0.00366 | -0.00075 | 0.54       | 1.00 |
| $\beta_{112}$ | 34.2    | 11.1   | 0.05322   | 12.4    | 55.91   | 0.48       | 1.00 |
| $\beta_{113}$ | -0.00371 | 0.001274 | 6.53E-06  | -0.00621 | -0.00121 | 0.51       | 1.00 |
| $\beta_{20}$ | 12.39   | 1.289  | 0.06366   | 9.938   | 14.88   | 4.94       | 1.00 |
| $\beta_{21}$ | 0.1099  | 0.1066 | 0.000833  | -0.09931 | 0.3211  | 0.78       | 1.00 |
| $\beta_{22}$ | 0.1781  | 0.06616 | 0.000897  | 0.0482  | 0.3068  | 1.36       | 1.00 |
| $\beta_{23}$ | -0.5549 | 0.1658 | 0.001665  | -0.8802 | -0.229  | 1.00       | 1.00 |
| $\beta_{24}$ | 0.9222  | 1.278  | 0.06303   | -1.559  | 3.352   | 4.93       | 1.00 |
| $\beta_{25}$ | 4.972   | 6.436  | 0.03821   | -7.618  | 17.54   | 0.59       | 1.00 |
| $\beta_{26}$ | -0.2928 | 0.3212 | 0.003612  | -0.9323 | 0.333   | 1.12       | 1.00 |
| $\beta_{27}$ | -1.329  | 1.202  | 0.008516  | -3.682  | 1.013   | 0.71       | 1.00 |
| $\beta_{28}$ | 0.03056 | 0.03584 | 0.000344  | -0.03974 | 0.1011  | 0.96       | 1.00 |
| $\beta_{29}$ | -2884   | 1393   | 8.456     | -5574   | -143.3  | 0.61       | 1.00 |
| $\beta_{210}$ | -0.1669 | 0.1683 | 0.000997  | -0.4995 | 0.1621  | 0.59       | 1.00 |
| $\beta_{211}$ | -0.00503 | 0.001272 | 8.12E-06  | -0.00751 | -0.00255 | 0.64       | 1.00 |
| $\beta_{212}$ | 24.85   | 19.38  | 0.1176    | -13.29  | 62.64   | 0.61       | 1.00 |
| $\beta_{213}$ | -0.00352 | 0.00222 | 1.37E-05  | -0.00784 | 0.000834 | 0.62       | 1.00 |

\[ D(\theta) = -140.2780 \quad D(\hat{\theta}) = -214.8350 \quad p_D = 74.5570 \quad DIC = -65.7210 \]

Note: **significant variable**

The bivariate FH model need more iteration and thin setting to reach convergent condition since this model is more complex than univariate model. The number of chain are two, with the total number of iteration are 1,010,000 iterations that 10,000 iterations for burn-in period (h) and set the thin value to 50 since slow decay of autocorrelation plot for several parameters. This setting produces same iteration with univariate FH model, i.e 20,000 iterations for posterior analysis (H), for each chain.

The estimation results of the bivariate FH model show the similarity of predictor variables that have a significant effect on the response variable, especially for food commodities. Whereas for non-food commodities there is an one additional variable that has a significant effect, i.e proportion of malnutrition over the past 3 years.

#### 3.3. Comparison of Estimation Results and Selection of the Best Model

First, we will compare the point estimation from univariate and bivariate FH model. Figure 4 shows that visually the point estimation of univariate and bivariate FH model are very similar, especially for food commodities in Figure 4a. While for non-food commodities, the point estimation is quite different for several districts in Figure 4b.
Figure 4. Point Estimation of Univariate vs. Bivariate FH Model for Food (a) and Non-food Commodities (b).

In the SAE with HB method, the posterior variance is a proxy for Mean Square Error (MSE) or efficiency. To see more objectives, the efficiency of estimation results produced by univariate and bivariate FH model, in Figure 5 we plot the Coefficient of Variation (CV) of univariate FH model against bivariate FH model. Almost all the CV plot for food commodities and all for non-food commodities are below the diagonal line, indicating that bivariate FH model can produce more precise estimate than univariate FH model.

Figure 5. Coefficient of Variation of Univariate and Bivariate FH Model for Food (a) and Non-food Commodities (b).

The comprehensive visualization for comparison of estimation results from both models are presented in 95% credible interval of estimate value which is sorted by number of samples in each district from the smallest to the largest (Figure 6). Generally, the bivariate FH model can produce shorter 95% credible intervals, especially for non-food commodities. In addition, it can be seen that in this study, the number of samples in each district turned out to have no effect in minimizing posterior variance values. This happens because the specifications on the FH model that the variance of sampling error are different for each district.
Figure 6. The 95% Credible Interval of Univariate and Bivariate FH Model for Food (a) and Non-food Commodities (b).

Finally, we will find the best model by comparing the value of MSPE, MAPE, RMSE and adjusted $R^2$ from both model. From the Table 5 below, the best model is bivariate FH model since it produces smaller MSPE, MAPE and RMSE than univariate FH model. Furthermore, the bivariate FH model has higher value of adjusted $R^2$ than univariate FH model.

| Commodity   | Criteria | Univariate FH | Bivariate FH |
|-------------|----------|---------------|--------------|
| Food        | MSPE     | 0.0263        | 0.0127*      |
|             | MAPE(%)  | 0.9710        | 0.6960*      |
|             | RMSE     | 0.1390        | 0.0624*      |
|             | $AdjustedR^2$ | 64.4011      | 92.8186**    |
| Non-food    | MSPE     | 0.1200        | 0.0226*      |
|             | MAPE(%)  | 1.9345        | 0.9044*      |
|             | RMSE     | 0.3399        | 0.1072*      |
|             | $AdjustedR^2$ | 49.4171      | 94.9639**    |

Note: *) smallest, **) highest

3.4. Mapping of Estimation Results from The Best Model
Before mapping the estimation results, the transformation from ln to rupiah must be carried out first.

The mapping results for food and non-food commodities in Figure 7 show the same pattern that the districts with the highest group of monthly average per capita expenditure are more localized in the southern districts of Bali, i.e in Denpasar Utara (57), Denpasar Selatan (54), Kuta (17) and Kuta
Selatan (16) for food commodities and in Denpasar Selatan (54) for non-food commodities. While the lowest group of expenditure are more localized in the eastern and western districts of Bali.

4. Conclusion
In the estimation process, the bivariate FH model needs more iteration since it is more complex than univariate FH model. The best model is bivariate FH model since it can produce more efficient estimation results than univariate FH model. The mapping of estimation results with best model, bivariate FH model, shows that the districts with the highest group of expenditure are more localized.
in the southern of Bali. On the other hand, the lowest group of expenditure are more localized in the eastern and western of Bali. This condition applies to both food and non-food commodities.

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