Phase diagram of a dilute fermion gas with density imbalance

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We map out the phase diagram of a dilute two-component atomic fermion gas with unequal populations and masses under a Feshbach resonance. As in the case of equal masses, no uniform phase is stable for an intermediate coupling regime. For majority component heavier, the unstable region moves towards the BEC side. When the coupling strength is increased from the normal phase, there is an increased parameter space where the transition is into the FFLO state. The converse is true if the majority is light.

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Laser trapped cold dilute fermionic gas [1] opens up a new era to study the superfluid properties. Through the Feshbach resonance, the effective interaction between fermions can be varied over a wide range such that the ground state can be tuned from weak-coupling BCS superfluid to a strong-coupling Bose-Einstein condensation (BEC) regime. Recent experiments [2] on $^6$Li atoms with imbalance spin populations further provide another way to probe the superfluid properties with mismatched Fermi surfaces.

The phase diagram of this imbalanced fermion system has been studied near the crossover region for pairing with equal masses [3]. Here we would like to extend our investigation to unequal masses between the two spin species. Especially, we examine the instabilities towards the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase [4] and phase separated phase. At equal masses, the instability of FFLO state occurs earlier for low density differences but the phase separation reaches first for large density differences. For increasing majority mass, the region where FFLO occurs first is found to increase. For majority heavier with mass ratio of 6.6 (between $^{40}$K and $^6$Li), practically FFLO occurs first for all population differences.

We first consider the uniform phase of two fermion species (spin $\sigma = \uparrow$ and $\downarrow$) and mass ($m_\sigma$) in the generalized BCS mean field approximation, generalizing [5] to the case of unequal populations. The excitation spectrum for each spin is [6]

$$E_{\uparrow,\downarrow}(k) = \pm \left[ \frac{k^2}{4m_\sigma}x + h \right] + \sqrt{\xi(k)^2 + \Delta^2}, \quad (1)$$

where $x = (m_\uparrow - m_\downarrow)/(m_\uparrow + m_\downarrow)$, $\mu \equiv (\mu_\uparrow + \mu_\downarrow)/2$, $h \equiv (\mu_\uparrow - \mu_\downarrow)/2$, $\xi(k) = \hbar^2k^2/(4m_\sigma) - \mu$ and the reduced mass $m_\sigma$. The scattering between fermions is short range and can be modelled as a s-wave effective interaction characterized by the corresponding scattering length $a$. The pairing field $\Delta$ is then determined by

$$-\frac{m_\sigma}{2\pi\hbar^2a} \Delta = \Delta \int \frac{d^3k}{(2\pi)^3} \left[ \frac{1 - f(E_\uparrow) - f(E_\downarrow)}{E_\uparrow + E_\downarrow} - \frac{2m_\sigma}{\hbar^2k^2} \right], \quad (2)$$

where $f$ is the Fermi function. We then solve the pairing field at fixed total density $n = n_\uparrow + n_\downarrow$ and density difference $n_d = n_\uparrow - n_\downarrow \geq 0$ [6]. The stable homogeneous superfluid phase must satisfy both the superfluid density and $\partial h/\partial n_d$ are positive [6].

Our results for $m_\uparrow/m_\downarrow = 6.6$ (the majority is heavier) is shown in Fig. 1. As in the equal mass case we found that the uniform phase is unstable in the intermediate coupling regimes (between the dotted and dot-dashed lines). Compared with the equal mass case, this unstable region moves toward the BEC side if the majority species...
FIG. 2: Instability from normal state to FFLO (dashed lines) and phase separation (solid line) for imbalanced fermion system with $m_\uparrow/m_\downarrow = 0.15$. Notations are the same as Fig 1.

is heavier.

In the above, the dotted line is where we first find a solution to eq (2). However, since this $\Delta \neq 0$ solution has negative superfluid density or $\partial h/\partial n_d$ (or both), the system is unstable towards a state with finite pairing momentum (FFLO state) or phase-separation. To find the first physical instability from the normal state towards these phases, we (i) solve the Cooper problem for finite wave-vector $q$, and (ii) find the smallest coupling where the completely paired superfluid state and the normal phase has the same free energy. These results are also shown in Fig 1. We found that FFLO occurs earlier for almost entire finite $n_d$ for this mass ratio. However, a more detailed calculation is required to determined the possible ground state in the rest of the shaded area.

For the case with the majority is lighter, the story is quite different. In Fig 2, we plot the phase diagram for $m_\uparrow/m_\downarrow = 0.15$. Compared with the equal mass case, the unstable region moves to the BCS side. FFLO instability occurs first only for small $n_d/n$. For the rest of the parameter space, phase separation occurs first.

We comment here that our results are more complete compared with another preprint [7], which did not include our FFLO and phase separation lines.

Other details will be reported elsewhere. For a discussion of the phase diagram exactly at resonance, see [8].

In summary, we have investigated the phase diagram in an imbalanced fermion system with unequal masses. The phase diagram is significantly different from its equal mass counterpart.

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