Abstract

We present a new variant of the Växjö interpretation: contextualistic statistical realistic. Basic ideas of the Växjö interpretation-2001 are essentially clarified. We also discuss applications to biology, psychology, sociology, economy,...

The first version of the Växjö interpretation of quantum mechanics was presented in [1], see also [2], after the conference “Quantum Theory: Reconsideration of foundations”, Växjö, June-2001, on the basis of numerous exciting discussions with participants. I was really surprised, that in spite of the formal acceptance of the official Copenhagen interpretation, many people (having top-qualification in quantum physics) still have doubts of various kinds and many of them are still looking for a realistic interpretation. This dream about quantum realism was the main stimulus for my attempt to present a new version of the realistic interpretation of quantum mechanics. The main problem was to create such an interpretation in which realism...
would coexist with a rather strange (people like to say “nonclassical”) behaviour of quantum probabilities – *Born’s rule and interference of probabilities.*

In 2000 it was demonstrated [5], see also [6]-[10], that Born’s rule and interference of probabilities can be (quite easily) derived in the realistic framework. The only thing that should not be neglected is *contextuality of probabilities* – dependence of probabilities on complexes of physical conditions (physical contexts). My investigations were induced by interest to frequency probability theory, see R. von Mises [20], see also [21]. In the frequency approach probabilities directly depend on collectives (random sequences) which are associated with concrete complexes of physical conditions. This probability theory is contextual from the very beginning.

Since 2001 I organized a series of conferences on foundations of probability and quantum mechanics and through intensive discussions (and, in particular, hard critique of some my colleagues, see [25], [26]) my ideas on interpretations of quantum mechanics are now essentially clearer.

First of all I understood the difference between my contextualism and

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1This problem was well known to founders of quantum theory, see, for example, the correspondence between A. Einstein and E. Schrödinger (see [3] for English translation and comments, see also [4]). E. Schrödinger did not like the Copenhagen interpretation; in particular, he created his cat just to demonstrate absurdness of this interpretation. Unfortunately, people practically forgot about this, see [4] for detail. We also recall that Schrödinger’s cat was just a modification of Einstein’s example “involving a charge of gunpowder in a state of unstable chemical equilibrium”, see [4] (letter of Einstein to Schrödinger, 8 August 1935, see [3], p.78). But neither Einstein nor Schrödinger could combine a realistic ensemble model with quantum statistics. In particular, Schrödinger wrote to Einstein that he will accept the realistic statistical interpretation of quantum mechanics if the interference of probabilities would be explained, see [4]. Consequently the Copenhagen interpretation preserved its status of the official quantum ideology until present time.

2And this is one of advantages of my contextual statistical approach, cf. G. Mackey [11], A. Lande [12], G. Ludwig [13], K. Kraus [14], see also P. Busch, M. Grabowski, P. Lahti [15], S. Gudder [16], and A. Holevo [17]. In our approach everything is trivial: complex amplitudes are constructed automatically on the basis of the formula of total probability with interference term, see [6]-[10]. Moreover, besides the ordinary complex Hilbert space representation, there exists the hyperbolic one and mixed hyper-trigonometric, see [10]. I emphasize that the study of former approaches, especially investigations of A. Lande [12] and G. Mackey [11], were very important for me. However, the starting point were the books of P. Dirac [18] and R. Feynman [19] in which they paid attention to the mystery of quantum interference of probabilities.

3See http://www.msi.vxu.se/aktuellt/konferens/index.html and [22]-[24].
Bohr’s contextualism, see Remark 1.

Then I understood the difference between my contextualism and contextualism of operational (empiristic) interpretation.

Another new issue is understanding of the role of special reference observables which are used in a concrete model for probabilistic representation contexts (e.g., in classical and quantum physical models we use the position and momentum observables).

Finally, it became clear that, in fact, I discussed [1], [2] not an interpretation of quantum mechanics, but a model – statistical and contextual – of physical reality. The corresponding interpretation of quantum mechanics is obtained automatically for those models in which statistical data can be represented by complex amplitudes. This is the Växjö interpretation of quantum mechanics: contextualistic statistical realistic interpretation.

By starting not from the formalism of quantum mechanics (calculus of probabilities in a complex Hilbert space), but from the general contextual statistical model of reality, we get the possibility to apply contextual statistical methods in many domains of science: biology, psychology, sociology, economy,... In some special cases we can use even the quantum probabilistic formalism. Such new applications of powerful mathematical methods developed in quantum theory can induce revolutionary changes in many sciences. But for us the quantum formalism is not the starting point. We should start with the general Växjö model of reality (physical, biological, psychological, social,...) and then test statistical data to find an appropriative mathematical formalism. In general there are no reasons to hope to obtain the complex quantum-like representation. For example, there might appear models in which data cannot be represented by complex amplitudes, but by hyperbolic ones. In this case we should use the formalism of hyperbolic quantum mechanics, [10].

1. Realism of contexts

We start with the basic definition:

**Definition 1.** A physical context $C$ is a complex of physical conditions.

In principle, the notion of context can be considered as a generalization of a widely used notion of preparation procedure [15]. I prefer to use contextualistic and not preparation terminology. By using the preparation terminology we presuppose the presence of an experimenter preparing physical systems.

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4In particular, our approach implies that quantum mechanics is not complete.
for a measurement. By using the contextualistic terminology we need not appeal to experimental preparations, experimenter should appear only on the stage of a measurement. Moreover, context need not be macroscopic. Of course, there exist experimental contexts – preparation procedures. However, in general contexts are not coupled to preparation procedures. I consider contexts as elements of physical reality which exist independently of experimenters. This is the cornerstone of my contextualistic viewpoint to physics (quantum as well as classical):

**Contexts are elements of reality**

To construct a concrete model $M$ of reality, we should fix some set of contexts $C$, see definition 2.

**Remark 1.** (Copenhagen and Växjö contextualisms) Bohr’s interpretation of quantum mechanics is considered as contextualistic, see [28] for detailed analysis. However, we should sharply distinguish two types of contextualism: Copenhagen and Växjö contextualisms. For N. Bohr “context” had the meaning “context of a measurement”. For example, in his answer to the EPR challenge N. Bohr pointed out that position can be determined only in context of position measurement. For me “context” has the meaning a complex of physical conditions. As was underlined, a context is an element of physical reality and it has no direct relation to measurements (or existence of experimenters at all). For example, there exist contextualistic statistical models which cannot be represented in a complex Hilbert space – so called hyperbolic quantum-like models, [10]. Moreover, a Bohrian measurement context is always macroscopic, our context – a complex of physical conditions – need not be macroscopic.

**2. Observables**

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5We use the notion “elements of physical reality” in common sense. There is no direct coupling with the EPR sufficient condition for values of physical observables to be elements of physical reality, see [27]. Moreover, in general the Växjö model need not contain physical systems. Thus even the formulation of the question: “Can values of observables be considered as objective properties of physical systems?” – is in general meaningless. We shall come to the problem of reality of quantum observables as observables on physical systems (i.e., classical or EPR reality) in section 11. There we shall present the Växjö model completed by physical systems.

6We remark that so far we do not speak about an interpretation of quantum mechanics. We are presenting an approach to modeling of physical reality. The quantum representation is possible only for some class of models, $M_{\text{quantum}}$. The class $M_{\text{quantum}}$ is a very special subclass of the class of contextualistic statistical models.
Suppose that there is fixed a set of observables \( O \) such that any observable \( a \in O \) can be measured under a complex of physical conditions \( C \) for any \( C \in \mathcal{C} \).

There can be in principle defined other observables on contexts \( \mathcal{C} \) which do not belong to the system \( O \), but they will define another contextual model of reality, see definition 2.

We remark that our general Växjö-representation of reality does not contain physical systems, cf. footnote 5. At the moment we do not (and need not) consider observables as observables on physical systems. It is only supposed that if a context \( C \) is fixed then for any instant of time \( t \) we can perform a measurement of any observable \( a \in O \).

We do not assume that all these observables can be measured simultaneously; so they need not be compatible. The sets of observables \( O \) and contexts \( \mathcal{C} \) are coupled through

**Axiom 1:** For any observable \( a \in O \), there are well defined contexts \( C_\alpha \) corresponding to \( \alpha \)-filtrations: if we perform a measurement of \( a \) under the complex of physical conditions \( C_\alpha \), then we obtain the value \( a = \alpha \) with probability 1. It is supposed that the set of contexts \( \mathcal{C} \) contains filtration-contexts \( C_\alpha \) for all observables \( a \in O \).

**3. Probabilistic representation of contexts**

**Axiom 2:** There are defined contextual probabilities \( P(a = \alpha/C) \) for any context \( C \in \mathcal{C} \) and any observable \( a \in O \).

At the moment we do not fix a definition of probability. Depending on a choice of probability theory we can obtain different models. For any \( C \in \mathcal{C} \), there is defined the set of probabilities:

\[
E(O, C) = \{ P(a = \alpha/C) : a \in O \}
\]

We complete this probabilistic data by \( C_\alpha \)-contextual probabilities:

\[
D(O, C) = \{ P(a = \alpha/C), ..., P(a = \alpha/C_\beta), P(b = \beta/C_\alpha), ... : a, b, ... \in O \}
\]

(we remark that \( D(O, C) \) does not contain the simultaneous probability distribution of observables \( \mathcal{O} \)). Data \( D(O, C) \) gives a probabilistic image of the context \( C \) through the system of observables \( \mathcal{O} \). Probabilities \( P(a = \alpha, ... \in O) \)
\( \alpha / C_\beta \), \ldots play the role of structural constants of a model. We denote by the symbol \( D(O, C) \) the collection of probabilistic data \( D(O, C) \) for all contexts \( C \in \mathcal{C} \). There is defined the map:

\[
\pi : \mathcal{C} \to D(O, C), \quad \pi(C) = D(O, C). \tag{1}
\]

In general this map is not one-to-one. Thus the \( \pi \)-image of contextualistic reality is very rough: not all contexts can be distinguished with the aid of probabilistic data produced by the class of observables \( O \).

Mathematically such probabilistic data can be represented in various ways. In some special cases it is possible to represent data by complex amplitudes. A complex amplitude (wave function) \( \phi \equiv \phi_{D(O, C)} \) is constructed by using a formula of total probability with cos-interference term, see [6]-[10] for extended exposition. In this way we obtain the probabilistic formalism of quantum mechanics. In other cases it is possible to represent data by hyperbolic amplitudes\(^8\) and we obtain the probabilistic formalism of “hyperbolic quantum mechanics,” [9], [10].

4. Contextualistic statistical model (Växjö model)

**Definition 2.** A contextualistic statistical model of reality is a triple

\[
M = (\mathcal{C}, O, D(O, C)) \tag{2}
\]

where \( \mathcal{C} \) is a set of contexts and \( O \) is a set of observables which satisfy to axioms 1,2, and \( D(O, C) \) is probabilistic data about contexts \( \mathcal{C} \) obtained with the aid of observables \( O \).

We call observables belonging the set \( O \equiv O(M) \) reference of observables. Inside of a model \( M \) observables belonging \( O \) give the only possible references about a context \( C \in \mathcal{C} \).

5. Realistic interpretation of reference observables

Our general model can (but, in principle, need not) be completed by some interpretation of reference observables \( a \in O \). By the Växjö interpretation reference observables are interpreted as properties of contexts:

“If an observation of \( a \) under a complex of physical conditions \( C \in \mathcal{C} \) gives the result \( a = \alpha \), then this value is interpreted as the objective property of the context \( C \) (at the moment of the observation).”

\(^8\)Such amplitudes are constructed by using a formula of total probability with cosh-interference term (“hyperbolic interference”), see [10].
As always, a model is not sensitive to interpretation. Therefore, instead of the realistic Växjö interpretation, we might use the Bohrian measurement-contextualistic interpretation, see Remark 1. However, by assuming the reality of contexts it would be natural to assume also the reality of observables which are used for the statistical representation of contexts. Thus we use the realistic interpretation both for contexts and reference observables. This is Växjö realism.

6. On the role of reference observables

Reader has already paid attention that reference observables play the special role in our model. I interpret the set $\mathcal{O}$ as a family of observables which represent some fixed class of properties of contexts belonging $\mathcal{C}$. For example, such a family can be chosen by some class of cognitive systems $Z_{\text{cogn}}$ — “observers” — which were interested only in the $\mathcal{O}$-properties of contexts $\mathcal{C}$ (and in the process of evolution they developed the ability to “feel” these and only these properties of contexts). The latter does not mean that observables $\mathcal{O}$ are not realistic. I would like just to say that observers $\tau \in Z_{\text{cogn}}$ use only observables $\mathcal{O}$.

We remark again that there can exist other properties of contexts $\mathcal{C}$ which are not represented by observables $\mathcal{O}$. The same set of contexts $\mathcal{C}$ can be the basis of various models of contextual reality: $M_i = (\mathcal{C}, \mathcal{O}_i, D(\mathcal{O}_i, \mathcal{C})), i = 1, 2, \ldots$. For example, such models can be created by various classes of cognitive systems $Z_{\text{cogn},i}$.

Moreover, we may exclude the spiritual element from observables. By considering “observation” as “feeling” of a context $\mathcal{C}$ by some system $\tau$ we need not presuppose that $\tau$ is a cognitive system. Such a $\tau$ can be, e.g., a physical system (e.g. an electron) which “feel” a context $\mathcal{C}$ (e.g., electromagnetic-context).

Remark 2. (Number of reference observables) In both most important physical models – in classical and quantum models – the set $\mathcal{O}$ of reference observables consists of two observables: position and momentum. I think that this number “two” of reference observables plays the crucial role (at least in the quantum model).

7. Växjö model outside physics

Our contextual statistical realistic models of reality can be used not only in physics, but in any domain of natural and social sciences. Instead of complexes of physical conditions, we can consider complexes of biological,
social, economic,... conditions – contexts – as elements of reality. Such elements of reality are represented by probabilistic data obtained with the aid of reference observables (biological, social, economic,...).

In the same way as in physics in some special cases it is possible to encode such data by complex amplitudes. In this way we obtain representations of some biological, social, economic,... models in complex Hilbert spaces. We call them complex quantum-like models. These models describe the usual cos-interference of probabilities.

Thus, when we speak, e.g., about a quantum-like mental model, this has nothing to do with quantum mechanics for electrons, photons, ... contained in the brain, see [29] for detail. A quantum-like mental model is a contextualistic probabilistic model of brain and nothing more, []. There were found (at least preliminary) experimental evidences that in psychology there can be obtained quantum-like (i.e., represented by complex probability amplitudes) statistical data, see [30]; such data also can be generated by some games, [31] (which have been called “quantum-like games” in [31]).

8. Choice of a probability model

As was mentioned, any Växjö model $M$ should be combined on some concrete probabilistic model describing probabilistic data $\mathcal{D}(\mathcal{O, C})$. Of course, the Kolmogorov measure-theoretical model dominates in modern physics. However, this is not the only possible model for probability, see [21]. In particular, I strongly support using of the frequency model [20], [21]. Here we shall use this model to describe probabilistic data. It does not mean that other models which are used in physics cannot be combined with some Växjö models. Of course, such a combination is not straightforward, see [8] on the use of the contextual extension of the Kolmogorov model. We now present the frequency probabilistic description of data $\mathcal{D}(\mathcal{O, C})$ for some $C \in \mathcal{C}$.

9. Frequency description of probability distributions

By taking into account Remark 2, we consider a set of reference observables $\mathcal{O} = \{a, b\}$ consisting of two observables $a$ and $b$. We denotes the sets of values (“spectra”) of the reference observables by symbols $X_a$ and $X_b$, respectively.

Let $C$ be some context. In a series of observations of $b$ (which can be infinite in a mathematical model) we obtain a sequence of values of $b$:

$$x \equiv x(b/C) = (x_1, x_2, ..., x_N, ...), \quad x_j \in X_b.$$  \hspace{1cm} (3)
In a series of observations of \( a \) we obtain a sequence of values of \( a \):

\[
y \equiv y(a/C) = (y_1, y_2, ..., y_N, ...), \quad y_j \in X_a.
\]  

(4)

We suppose that the principle of statistical stabilization for relative frequencies holds true and the frequency probabilities are well defined:

\[
p^b(\beta) \equiv P_x(b = \beta) = \lim_{N \to \infty} \nu_N(\beta; x), \quad \beta \in X_b;
\]  

(5)

\[
p^a(\alpha) \equiv P_y(a = \alpha) = \lim_{N \to \infty} \nu_N(\alpha; y), \quad \alpha \in X_a.
\]  

(6)

Here \( \nu_N(\beta; x) \) and \( \nu_N(\alpha; y) \) are frequencies of observations of values \( b = \beta \) and \( a = \alpha \), respectively (under the complex of conditions \( C \)).

Let \( C_\alpha, \alpha \in X_a \), be contexts corresponding to \( \alpha \)-filtrations, see Axiom 1. By observation of \( b \) under the context \( C_\alpha \) we obtain a sequence:

\[
x^\alpha \equiv x(b/C_\alpha) = (x_1, x_2, ..., x_N, ...), \quad x_j \in X_b.
\]  

(7)

It is also assumed that for sequences of observations \( x^\alpha, \alpha \in X_a \), the principle of statistical stabilization for relative frequencies holds true and the frequency probabilities are well defined:

\[
p^{b/a}(\beta/\alpha) \equiv P_{x^\alpha}(b = \beta) = \lim_{N \to \infty} \nu_N(\beta; x^\alpha), \quad \beta \in X_b.
\]  

(8)

Here \( \nu_N(\beta; x^\alpha), \alpha \in X_a \), are frequencies of observations of value \( b = \beta \) under the complex of conditions \( C_\alpha \). We obtain probability distributions:

\[
P_x(\beta), \quad P_y(\alpha), \quad P_{x^\alpha}(\beta), \quad \alpha \in X_a, \beta \in X_b.
\]  

(9)

We can repeat all previous considerations by changing \( b/a \)-conditioning to \( a/b \)-conditioning. We consider contexts \( C_\beta, \beta \in X_b \), corresponding to selections with respect to values of the observable \( b \) and the corresponding collectives \( y^\beta \equiv y(a/C_\beta) \) induced by observations of \( a \) in contexts \( C_\beta \). There can be defined probabilities \( p^{a/b}(\alpha/\beta) \equiv P_{y^\beta}(\alpha) \). Combining these data with data (8) we obtain

\[
D(O, C) = \{p^a(\alpha), p^b(\beta), p^{b/a}(\beta/\alpha), p^{a/b}(\alpha/\beta) : \alpha \in X_a, \beta \in X_b\}
\]

This data gives a statistical contextual image of reality based on reference observables \( a \) and \( b \). As was remarked, there exist various mathematical
methods for encoding of data $D(O, C)$, e.g., in some cases by complex amplitudes – complex quantum-like representations.

10. **Representation in a complex Hilbert space**

Let $M$ be a contextualistic statistical model such that $O$ contains only two observables $a$ and $b$. For any context $C \in C$, by using statistical data $D(a, b, C)$ we can compute a quantity $\lambda(\beta/\alpha, C), \alpha \in X_a, \beta \in X_b$, see [6]-[10]. This quantity was called in [6]-[10] a *measure of statistical disturbance* (of the $b$-observable by the $a$-observations under the context $C$). If

$$|\lambda(\beta/\alpha, C)| \leq 1$$

for all $\alpha \in X_a, \beta \in X_b$, then data $D(O, C)$ can be represented (by using the formula of total probability with interference term) by a complex amplitude $\phi_C$ or in the abstract framework by an element the unit sphere $U_1$ of the complex Hilbert space $H$. Denote the family of all contexts which satisfy to (10) by the symbol $C^{tr}$. We have the map:

$$J : C^{tr} \rightarrow U_1$$

We emphasize that $J$ is determined by the reference observables $a$ and $b$. Thus (11) is a Hilbert space representation of contexts determined by these concrete reference observables. The map $J$ is not one to one. Thus by representing contexts by complex amplitudes we lose a lot of information about contexts.

The map (11) induces [8] a map:

$$L : O \rightarrow L(H),$$

where $L(H)$ is the set of self-adjoint operators. Probability distributions of operators $\hat{a} = L(a)$ and $\hat{b} = L(b)$ (calculated by using quantum Hilbert space framework) in the state $\phi_C$ coincide with $p^a(\alpha)$ and $p^b(\beta)$.

If for a context $C$ we find that

$$|\lambda(\beta/\alpha, C)| \geq 1$$

then $C$ can be represented by a hyperbolic amplitude.

11. **Systems, ensemble representation**

We now complete the contextualistic statistical model by considering systems $\omega$ (e.g., physical or cognitive, or social...) Systems are also elements
of reality. In our model a context $C \in \mathcal{C}$ is represented by an ensemble $S_C$ of systems which have been interacted with $C$. For such systems we shall use notation:

$$\omega \leftrightarrow C$$

The set of all (e.g., physical or cognitive, or social) systems which are used to represent all contexts $C \in \mathcal{C}$ is denoted by the symbol $\Omega \equiv \Omega(\mathcal{C})$. Thus we have a map:

$$C \rightarrow S_C = \{\omega \in \Omega : \omega \leftrightarrow C\}.$$  \hspace{1cm} (14)

This is the ensemble representation of contexts. We set

$$S \equiv S(\mathcal{C}) = \{S : S = S_C, C \in \mathcal{C}\}.$$

The ensemble representation of contexts is given by the map $I : \mathcal{C} \rightarrow S$

Reference observables $\mathcal{O}$ are now interpreted as observables on systems $\omega \in \Omega$. In principle, we can interpret values of observables as objective properties of systems. Oppositely to the very common opinion, such models (with realistic observables) can have nontrivial quantum-like representations (in complex and hyperbolic Hilbert spaces) which are based on the formula of total probability with interference terms.

Probabilities are defined as ensemble probabilities, see [21].

**Definition 3.** The ensemble representation of a contextualistic statistical model $M = (\mathcal{C}, \mathcal{O}, D(\mathcal{O}, \mathcal{C}))$ is a triple

$$S(M) = (S, \mathcal{O}, D(\mathcal{O}, \mathcal{C}))$$  \hspace{1cm} (15)

where $S$ is a set of ensembles representing contexts $\mathcal{C}$, $\mathcal{O}$ is a set of observables, and $D(\mathcal{O}, \mathcal{C})$ is probabilistic data about ensembles $S$ obtained with the aid of observables $\mathcal{O}$.

12. **Algebraic structure on the set of reference observables**

We do not assume the presence of any algebraic structure on $\mathcal{O}$. Even if these observables take values in some set endowed with an algebraic structure, e.g., in $\mathbb{R}$, we do not assume that this structure induces (in the standard way) the corresponding algebraic structure on $\mathcal{O}$. If $a, b \in \mathcal{O}$ and take values in $\mathbb{R}$ it does not imply that $d = a + b$ is well defined as observable on every
context $C \in \mathcal{C}$. In the general contextual approach it is very clear why we cannot do this. If $a$ and $b$ are not compatible, then we cannot measure they simultaneously under a context $C$ at the fixed instant of time and form $d = a + b$. But a reader may say:

“You use the realistic interpretation of the reference observables in a model $M$. Thus one can form the sum $d = a + b.$”

a). By the realistic contextualistic interpretation, $a(t)$ and $b(t)$ are objective properties of a context $C$ at the instant of time $t$. There is defined $d(t) = a(t) + b(t)$.

b). By the realistic interpretation of the model with systems, $a(\omega)$ and $b(\omega)$ are objective properties of a system $\omega$. There is defined $d(\omega) = a(\omega) + b(\omega)$.

However, this is the ontic or “hidden sum” and the representation (12) cannot be extended to such sums. Quantum theory cannot say us anything about $d = a + b$ as pointwise observable. Of course, we can define the sum of operators $\hat{d} = \hat{a} + \hat{b}$, but in general this operator would represent not the ontic observable $d$, but another observable $d_{\text{quant}}$. Observables $d$ and $d_{\text{quant}}$ can have different probability distributions! (see [8]). Nevertheless (and this seems to be crucial in using of quantum theory), averages of these observables coincide:

$$\langle d_{\text{quant}} \rangle = \langle \hat{d} \rangle = \langle d \rangle$$

(16)

This is a consequence of linearity of both quantum (Hilbert space) and classical probabilistic averages and the coincidence of probability distributions of reference observables and they representatives in the Hilbert space.

13. Realist and empirisist interpretations of quantum mechanics

We emphasize again that up to now we have not been considering an interpretation of quantum mechanics. There was proposed a contextualistic statistical model of physical reality. Sometimes this model can be mathematically described by using the formalism of classical mechanics, sometimes quantum, sometimes hyperbolic and so on. However, it is useful to discuss relation of our model to models of physical reality corresponding to various interpretations of quantum mechanics. Here we follow to P. Busch, M. Grabowski, P. J. Lahti [15], de Muynck, De Baere, and Martens [32] and L. Ballentine [33].

13.1. Empirisist interpretation. In this interpretation the formalism of quantum mechanics does not describe reality as such. It only serves to calculate probabilities (relative frequencies) of certain phenomena that can
be interpreted as corresponding to the results of a quantum measurement. The probabilities are conditioned on certain procedures, to be interpreted as quantum mechanical preparation procedures. Thus, the wave function or density operator can be interpreted as symbolizing a preparation procedure; in the same way a hermitian operator describes symbolically a quantum mechanical measurement. Wave function and hermitian operator are not thought to correspond to something existing in microscopic reality. They are just labels of (macroscopic) instruments that can be found in the laboratory. QM is thought to describe only (cor)relations of preparation acts and measurement phenomena. It is also important for our further considerations to underline that in an empirisist interpretation of QM the eigenvalues of the hermitian operator do not play a significant role, because these eigenvalues do not correspond to properties of the microscopic object. The empirisist interpretation has achieved great popularity because its antimetaphysical flavor: physics must be about observables only, and about nothing else. Hence, in this interpretation neither the wave function nor the observable must be taken as a property of the microscopic object system.

13.2. Realist interpretation. By this interpretation values of physical observables are considered as objective properties – properties of objects (physical systems).

13.3. Växjö interpretation: realism of contexts. In the Växjö approach quantum mechanics (as a physical theory) is a particular contextualistic statistical model of reality in which the probabilistic data $D(O, C)$ can be encoded by complex amplitudes. This point of view to quantum formalism induces the Växjö interpretation of quantum mechanics. This is a contextualistic statistical realistic interpretation of quantum mechanics. And Växjö realism is realism of contexts and reference observables.

The Växjö interpretation of quantum mechanics is quite close to the empirisist interpretation. The crucial difference is that by the Växjö interpretation quantum mechanics is about reality – reality of contexts, and not about preparation and measurement procedures. Contexts exist independently of our measurement activity and values of reference observables $a \in O$ are objective properties of contexts. The space-scale does not play any role, because the description of reality is purely probabilistic. Quantum probabilistic behavior is a consequence of complementarity of information for reference observables. Such complementarity of information can take place at microscopic as well as macroscopic scales and, moreover, not only in physics, but in any domain of natural and social sciences.
We remark that by considering context as an element of reality we eliminated the important difference between realist and empirisist interpretations – the wave function is considered as a description of the result of preparation rather than as a symbolic representation of the preparation itself. If a model $M$ has a quantum(-like) complex representation then the wave function represents context – a complex of physical (or biological,...) conditions.

13.4. Växjö interpretation: realism of contexts, systems and observables. Let us now consider the completed Växjö model which contains physical systems, contexts are represented by ensembles of systems. Physical observables are considered as objective properties of systems. As well as for general contextualistic model, quantum mechanics (as a physical theory) is about a rather special class of contexts $\mathcal{C}^\text{tr}$ such the probabilistic data $\mathcal{D}(\mathcal{O}, \mathcal{C}^\text{tr})$ can be encoded by complex amplitudes. The only difference is that probabilities are defined as ensemble probabilities. This interpretation of quantum mechanics is very close to the well known ensemble interpretation which was strongly supported by A. Einstein, see introduction; L. Ballentine called it the statistical interpretation, see [33]. A difference is that in our model we start with reality of contexts which can be (but need not be) represented by ensembles.

But this is not the main difference. The main difference is that we did not start at all with an interpretation of one special mathematical formalism, calculus of probabilities in complex Hilbert spaces. We started with a general contextual statistical model of reality and then demonstrated that some special contexts can be represented by quantum-like complex amplitudes. Interpretation of such amplitudes follows automatically from the basic contextual statistical model.

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