Ab initio calculations of charge symmetry breaking in the $A = 4$ hypernuclei

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We report on ab initio no-core shell model calculations of the mirror $\Lambda$ hypernuclei $^3\Lambda\text{H}$ and $^3\Lambda\text{He}$, using the Bonn-Jülich leading-order chiral effective field theory hyperon-nucleon potentials plus a charge symmetry breaking $\Lambda$-$\Sigma^0$ mixing vertex. In addition to reproducing rather well the $0^+_{\text{exc}}$ and $1^+_{\text{exc}}$ binding energies, these four-body calculations demonstrate for the first time that the observed charge symmetry breaking splitting of mirror levels, reaching hundreds of keV for $0^+_{\text{exc}}$, can be reproduced using realistic theoretical interaction models, although with a non-negligible momentum cutoff dependence. Our results are discussed in relation to recent measurements of the $B_{\Lambda}$ mirror hypernuclei level diagram. The $0^+_{\text{exc}}$ $\Lambda$ separation energies $B_{\Lambda}$, loosely termed $\Lambda$ binding energies, are from emulsion work [9], and the $1^+_{\text{exc}}$ $B_{\Lambda}$ values follow from $\gamma$-ray measurements of the excitation energies [11].

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Introduction. Charge symmetry in hadronic physics is broken in QCD by the up-down light quark mass difference and by the up and down quark QED interactions. Recent lattice QCD+QED simulations of octet baryon mass differences within isospin multiplets, such as the neutron-proton mass difference $\Delta_{np}$ which vanishes in the limit of charge symmetry, account nicely for the observed charge symmetry breaking (CSB) pattern in the lowest-mass nonstrange as well as strange baryon spectrum [1]. A comparable level of precision in reproducing theoretically CSB effects in the baryon-baryon interaction is lacking [2]. In practice, introducing two charge-dependent contact interaction terms in chiral effective field theory (EFT) applications, one is able at next-to-next-to-leading order (N3LO) to account quantitatively for the low-energy nucleon-nucleon ($NN$) scattering parameters [3]. For strangeness $S = -1$, however, given that low-energy $\Lambda p$ cross sections are poorly known and $\Lambda n$ scattering data do not exist, the available chiral EFT hyperon-nucleon ($YN$) interactions [4,5] do not include charge-dependent interaction terms. Potentially unique information on CSB in the $\Lambda N$ interaction and in $\Lambda$ hypernuclei is provided by the large $\Lambda$ separation-energy difference $\Delta B_{\Lambda}^{S=0} = 350 \pm 60$ keV [6] in the $A = 4$ mirror hypernuclei ground states (g.s.) and the apparently negligible difference $\Delta B_{\Lambda}^{S=1}$ in the $\Lambda$ excited states [7], see Fig. 1. Here, $\Delta B_{\Lambda}^{S} \equiv B_{\Lambda}^{(3\Lambda\text{H})} - B_{\Lambda}^{(3\Lambda\text{He})}$. The recent precise measurement of the $^3\Lambda\text{H} \rightarrow ^3\Lambda\text{He} + \pi^-$ decay at the Mainz Microtron (MAMI) [8] reaffirms a substantial CSB g.s. splitting $\Delta B_{\Lambda}^{S=0} = 270 \pm 95$ keV, which is consistent with the emulsion value cited above. Note that $\Delta B_{\Lambda}^{S=0}$ is considerably larger than the $\approx 70$ keV assigned to CSB splitting in the mirror core nuclei $^3\text{H}$ and $^3\text{He}$ [9].

Dalitz and von Hippel [10] suggested that the SU(3) octet $\Lambda_I=0$ and $\Sigma_I=1$ hyperons are admixed in the physical $\Lambda$ hyperon, thereby generating a CSB direct $\Lambda N$ potential $V_{\text{CSB}}$ that consists of isovector meson exchanges, notably a long-range one-pion exchange (OPE) component. Although these exchanges are forbidden in the $\Lambda N$ channel by the strong interactions (SI), they do contribute strongly to the $\Lambda N \leftrightarrow \Sigma N$ coupling potential. Quite generally, the matrix element of $V_{\text{CSB}}$ arising from $\Lambda - \Sigma^0$ mixing is related to the SI $I_{NY} = 1/2$ matrix element $\langle N\Sigma|V_{\text{SI}}|N\Lambda\rangle$ by [11]

$$\langle N\Lambda|V_{\text{CSB}}|N\Lambda\rangle = -0.0297 \frac{1}{\sqrt{3}} \langle N\Sigma|V_{\text{SI}}|N\Lambda\rangle, \quad (1)$$

where $\tau_{Nz} = \pm 1$ for protons and neutrons, respectively, and the space-spin structure of this $N\Sigma$ state is taken identical with that of the $N\Lambda$ state embracing the $\Lambda$ mixing vertex. In addition to reproducing rather well the $0^+_{\text{exc}}$ $\Lambda$ separation energies $B_{\Lambda}$, loosely termed $\Lambda$ binding energies, are from emulsion work [9], and the $1^+_{\text{exc}}$ $B_{\Lambda}$ values follow from $\gamma$-ray measurements of the excitation energies $E_{\gamma}$ [11].

![Fig. 1: $^3\text{H}^\Lambda$ and $^3\text{He}^\Lambda$ mirror hypernuclei level diagram. The $0^+_{\text{exc}}$ $\Lambda$ separation energies $B_{\Lambda}$, loosely termed $\Lambda$ binding energies, are from emulsion work [9], and the $1^+_{\text{exc}}$ $B_{\Lambda}$ values follow from $\gamma$-ray measurements of the excitation energies $E_{\gamma}$ [11].](image-url)
CSB scale coefficient 0.0297 in \( \Omega \) follows from the \( \Lambda - \Sigma^0 \) mass-mixing matrix element \( \frac{1}{\sqrt{3}} (\Delta \Sigma^0 \Sigma^+ - \Delta_{np}) = 1.14(5) \) MeV \( \Omega \) and has been used in all previous CSB works listed below. A visualization of Eq. (1) is provided by the CSB \( \Lambda N \) interaction diagram of Fig. 2, where the \( V_{\Lambda N - \Sigma N} \) blob represents any SI isovector meson exchange or contact term such as introduced in chiral EFT models \( \Omega \).

Precise four-body calculations using the Nijmegen soft-core realistic meson exchange \( \Lambda N \) interaction models NSC97e,f \( \Omega \), which include charge-dependent interactions induced by \( \Lambda - \Sigma^0 \) mixing and meson mixings, produced at most 30% of the observed CSB g.s. splitting \( \Delta B_{\Lambda}^{4} \Omega = 1.4(18) \). Below we comment on this insufficiency. More recent Nijmegen \( \Omega \) or quark-cluster \( \Omega \) models have not been used in four-body studies. With SI \( \Lambda N \leftrightarrow \Sigma N \) potential energy contributions of order 10 MeV \( \Omega \), and with a CSB scale of order 3%, Eq. (1) could yield CSB contributions of order 300 keV. Reproducing the observed CSB splitting poses a challenge for microscopic \( \Lambda N \) interaction models.

In this Letter we report on detailed ab initio no-core shell model (NCSM) calculations of the \( A = 4 \Lambda \) hypernuclei that employ the SI Bonn-Jülich LO chiral EFT \( \Lambda N \) interaction potentials \( \Omega \), plus a CSB \( \Lambda - \Sigma^0 \) mixing interaction potential \( V_{\Sigma \Lambda} \) generated by applying Eq. (1) to each one of the \( \Lambda N \leftrightarrow \Sigma N \) \( V_{\Sigma \Lambda} \) components in this LO version. CSB meson mixings, with negligible contributions in the \( A = 4 \Lambda \) hypernuclei \( \Omega \), are disregarded here. In addition to reproducing reasonably well the \( g_{\Lambda} \), and \( 1_{\text{exc}} \) binding energies, these four-body calculations establish for the first time as large CSB splittings \( \Delta B_{\Lambda}^{4} \) as suggested by experiment, see Fig. 1, although with a non-negligible cutoff dependence. We also discuss possible implications to the recent Bonn-Jülich-Munich NLO chiral EFT \( \Lambda N \) interaction model \( \Omega \).

Methodology. The nuclear NCSM technique used in the present four-body calculations employs realistic two-body and three-body model interactions and is formulated in a translationally invariant Jacobi-coordinate harmonic-oscillator (HO) basis \( \Omega \). Antisymmetrization with respect to nucleons is exercised in order to satisfy the Pauli principle. The resulting Hamiltonian is diagonalized in finite four-body HO bases, admitting all HO excitation energies \( \hbar \omega \), \( N \leq N_{\text{max}} \), up to \( N_{\text{max}} \) HO quanta. Extrapolated energy values \( E(\omega) \), \( N_{\text{max}} \rightarrow \infty \), are obtained by fitting an exponential function to \( E(N_{\text{max}}, \omega) \) sequences in the vicinity of the variational minima with respect to the HO basis frequency \( \omega \). The reliability of such extrapolations is then reflected in the independence of \( E(\omega) \) of the frequency \( \omega \).

This NCSM technique, extended recently to light hypernuclei \( \Omega \), is applied here to the \( A = 4 \Lambda \) mirror hypernuclei using chiral N3LO \( \Lambda N \) and N2LO \( \Lambda NN \) interactions \( \Omega \), respectively, both with a momentum cutoff of 500 MeV. These together with the Coulomb interaction reproduce the binding energies of the \( A = 3 \) core nuclei. For the \( \Lambda N \) coupled-channel potentials \( V_{\text{SI}} \), we use the Bonn-Jülich LO chiral EFT SU(3)-based model with cutoff momenta \( \Lambda \) from 550 to 700 MeV \( \Omega \) plus \( V_{\text{CSB}} \) evaluated from \( V_{\text{SI}} \) by using Eq. (1). Baryon mass differences within isomultiplets are incorporated. The reported calculations consist of fully converged \( 3 \)\( H \) and \( 4 \)\( He \) binding energies, and \( 3 \)\( \langle H, \frac{4}{3} \text{He} \rangle 0_{\Pi}^{\text{*}} \) and \( 1_{\text{exc}} \) binding energies extrapolated to infinite model spaces from \( N_{\text{max}} = 18(14) \) for \( J = 0(1) \). The \( \Lambda NN \) interaction is excluded from the calculations reported here, in order to save computing time, after verifying that its inclusion makes a difference of only a few keV in the calculation of the CSB splittings \( \Delta B_{\Lambda}^{4} \) for both \( J = 0, 1 \).

Results. The cutoff dependence of \( \Lambda \) separation energies in both \( A = 4 \Lambda \) mirror hypernuclei, obtained from NCSM calculations with LO chiral EFT coupled-channel \( \Lambda N \) potentials \( \Omega \) and \( V_{\text{CSB}} \) from Eq. (1), is shown in Table I. We used \( N_{\text{max}} \rightarrow \infty \) extrapolated binding-energy values for the \( \frac{4}{3} \text{He} \) and \( \frac{4}{3} \text{He} J = 0(1) \) levels at fixed \( \hbar \omega = 30(32) \) MeV, which is where the absolute variational minima occur for \( A = 550 \) and 600 MeV. For higher values of \( \Lambda \) the four-body absolute variational minima occur at slightly higher \( \hbar \omega \) values. Although the spread of \( B_{\Lambda}^{4} \) values for a given cutoff momentum is of the order of 100 keV, it is considerably smaller and in fact marginal for the CSB splittings \( \Delta B_{\Lambda}^{4} \) on which we focus here, as demonstrated by Fig. 4 below.

The \( \Lambda \) separation energies listed in Table I show a
moderate cutoff dependence for the $0^+_g$ mirror levels and a stronger dependence for the $1^+_s$ mirror levels, with mean values for their charge-symmetric (CS) averages given by $\overline{B}^\text{CS}(0^+_g)=2.39^{+0.18}_{-0.12}$ MeV and $\overline{B}^\text{CS}(1^+_s)=1.16^{+0.59}_{-0.39}$ MeV which compare well with the CS-averaged experimental values derived from the last column in Table I. Furthermore, considering NCSM $N_{\text{max}} \to \infty$ extrapolation uncertainties, our CS-averaged $B_J$ values are in fair agreement with those reported in other four-body calculations using CS LO $YN$ chiral EFT interactions \cite{14,13,25,24}. A detailed analysis of calculational uncertainties will be given elsewhere.

Shown in Fig. 3 by solid lines is the cutoff momentum dependence of the $0^+_g \to 1^+_s$ excitation energies $E_x$ formed from the $B_A$ values listed in Table I for both $A=4$ mirror hypernuclei. As observed in several few-body calculations of $s$-shell hypernuclei \cite{26,29}, $E_x$ is strongly correlated with the $\Lambda N \leftrightarrow \Sigma N$ coupling potential which in the present context, through $\Lambda - \Sigma^0$ mixing, gives rise to CSB splittings of the $A=4$ mirror levels.

Figure 3 demonstrates a steady rise of both $E_x(^3\text{He})$ and $E_x(^4\text{He})$ as a function of the cutoff momentum $\Lambda$, with a CS-averaged value $\overline{E}_x^\text{CS}=1.23^{+0.35}_{-0.32}$ MeV compared to $1.25\pm 0.02$ MeV deduced from the two $\gamma$-ray energies shown in Fig. I. A steady rise is also observed in the difference $\Delta E_x^\text{CS}$ with a mean value $380^{+140}_{-130}$ keV compared to $320\pm 20$ keV, again from Fig. I. In agreement with previous calculations \cite{14,13}, residual CSB contributions of up to 30 keV from electromagnetic mass differences, mostly of $\Sigma$ hyperons, and from the increased Coulomb repulsion in the $^3\text{He}$ core of $^4\Lambda\text{He}$, survive upon switching off $V_{\text{CSB}}$, as demonstrated by the slight difference between the upper and lower dashed lines in the figure.

In Fig. 4 we show the $\hbar \omega$ dependence of separation-energy differences $\Delta B_A^J$ between $^3\Lambda\text{He}$ and $^4\Lambda\text{He}$ for $0^+_g$ (upper curve) and for $1^+_s$ (lower curve) on the HO $\hbar \omega$ in ab initio NCSM calculations using LO chiral EFT coupled-channel $YN$ potentials with cutoff momentum $A=600$ MeV \cite{4} plus $V_{\text{CSB}}$ derived from these SI potentials using Eq. I. Results for other values of $\Lambda$ are shown at $\hbar \omega=30(32)$ MeV for $J=0(1)$.

**Discussion.** To understand the CSB pattern Eq. 4 for the $A=4$ hypernuclei, we note that the SI $\Lambda N \leftrightarrow \Sigma N$ coupling potential in the LO chiral EFT $YN$ model of Ref. 4 consists of a pseudoscalar (PS) meson exchange, dominated by OPE, plus two $s$-wave interaction contact terms (CT) of which the $^3S_1$ CT is negligible and the $^1S_0$
CT is large. In a zeroth-order single-particle description of the $A=4$ hypernuclei, and using Eq. (4), these $\Lambda N \leftrightarrow \Sigma N$ coupling-potential components contribute to the CSB separation-energy differences as follows:
\[
\Delta B_A^{J=0} = \frac{3}{2} C_\pi - \frac{1}{2} C_0, \quad \Delta B_A^{J=1} = \frac{1}{2} C_\pi + \frac{1}{2} C_0, \quad (4)
\]
with $C_\pi = C_{\pi}^{CT} + C_{\pi}^S$ the sum of contributions to the triplet ($S=1$) and singlet ($S=0$) matrix elements from CT and from OPE. The $\vec{\sigma}_Y \cdot \vec{\sigma}_N$ spin dependence of $C_\pi^S$ leads in this approximation to a very large $S_B$ dominated by OPE. Dominance of $\Lambda N$ to the recently published NLO chiral EFT $Y N$ interaction model to generate sizable CSB splitting of the $0^+_{g.s.}$ mirror levels. However, the opposite-sign values of $\Delta B_A^{J=0}$ are large with respect to the near degeneracy observed for the $1^+_{exc}$ mirror levels, even when updated values from the latest MAMI measurement are considered (30).

In contrast to the ability of the LO chiral EFT $Y N$ interaction model to generate sizable CSB g.s. splittings $\Delta B_A^{J=0}$ owing to a dominant $1^+_S \Lambda N \leftrightarrow \Sigma N$ coupling-potential CT, the $\Lambda N \leftrightarrow \Sigma N$ coupling potential in NSC97 models is dominated by a $3^+_S - 3^+_D$ tensor component which is ineffective in generating a large CSB contribution when used in the right-hand side of Eq. (5). The reason is that the $1^+_S \Lambda N$ states on the left-hand side, in the case of NSC97, are dominated by purely s-wave channels (14). The NSC97 $1^+_S \Lambda N \leftrightarrow \Sigma N$ coupling-potential contribution that replaces $C_\pi^0$ in Eq. (5) is too weak to generate on its own a sizable $\Delta B_A^{J=0}$. A detailed account of this item will be given elsewhere. It is tempting to speculate on the $A=4$ CSB separation-energy differences $\Delta B_A^{J}$ anticipated from applying Eq. (1) to the recently published NLO chiral EFT $Y N$ interaction (3). The $\Lambda N \leftrightarrow \Sigma N$ coupling-potential contact terms differ considerably in NLO from those in LO, with a very large $C_{\pi}^{CT}$ that dominates in NLO over $C_{\pi}^{CT}$, and with a new $3^+_S - 3^+_D$ CT. It is fair to assume that PS one- and two-meson exchange contributions in NLO are still dominated by OPE. Dominance of $C_{\pi}^{CT}$ over all other allowed contributions would result of negative values of $\Delta B_A^{J}$, with $\Delta B_A^{J=0} \approx 3$ times as large as $\Delta B_A^{J=1}$; this would disagree with the observed positive value for $\Delta B_A^{J=0}$, see Fig. (1) confirmed also by the new MAMI measurement (8). We note, furthermore, that the NLO version underestimates the $A=4$ hypernuclear g.s. separation energy, with $B_A^{CS} \approx 1.5-1.6 \text{ MeV}$ (17), compared to $\approx 2.2 \text{ MeV}$ from Fig. (1). Three-body $Y N N$ interaction terms introduced in higher-order versions in order to recover the missing g.s. attraction might provide additional source of CSB in $\Lambda$ hypernuclei. However, expecting that the dominant $Y N N$ terms correspond to $\Sigma^* (1385) N N$ intermediate states (31) and realizing that, unlike $\Sigma^0, \Sigma^0(3^+)$ cannot mix with $\Lambda^0( \frac{1}{2}^+)$ to generate CSB, these $Y N N$ interaction terms will not produce as strong CSB as evaluated here using Eq. (1), which is based on the Dalitz–von Hippel $\Lambda^0 - \Sigma^0$ mixing mechanism (10). It is therefore questionable whether the NLO version (5) offers an advantage over the LO version (4) for $\Lambda$ hypernuclei, given also that both provide comparably reasonable fits to the low-energy $Y N$ scattering data.

Summary and outlook. In conclusion, we have presented the first CSB $ab$ initio calculation in hypernuclei with chiral EFT coupled-channel $Y N$ interactions, showing that the LO version (4) is capable of producing a large CSB $0^+_{g.s.}$ splitting $\Delta B_A^{J=0} \approx 180 \pm 130 \text{ keV}$. This is consistent with a g.s. splitting of $270 \pm 95 \text{ keV}$ reported by the MAMI experiment (8). Our NCSM calculation reproduces quantitatively and with weak cutoff dependence the $0^+_{g.s.}$ binding energies of the $A=4$ mirror hypernuclei, whereas the $1^+_{exc.}$ binding-energy calculation, which is known to be numerically more challenging (14), displays a strong cutoff dependence. The calculated CSB $1^+_{exc.}$ splitting is of opposite sign to that of the $0^+_{g.s.}$ splitting and fairly large: $\Delta B_A^{J=1} \approx -200 \pm 30 \text{ keV}$, with a weak cutoff dependence. While the latest results from MAMI suggest a smaller negative CSB splitting of $-83 \pm 94 \text{ keV}$ for the $1^+_{exc.}$ mirror levels (30), the measurement systematic uncertainty is still too large to rule out the prediction of the LO version.

In future work it would be of great interest to apply the CSB generating equation (1) in $ab$ initio calculations of the $A=4$ mirror hypernuclei using the recent NLO EFT version (3), and also to readjust the $\Lambda N \leftrightarrow \Sigma N$ contact terms in NLO by imposing the accurate CSB datum $E_{\gamma} (\Lambda^0 (\frac{3}{2}^+) \rightarrow \frac{1}{2}^+ \text{He}) - E_{\gamma} (\frac{1}{2}^+ \text{H}) = 0.32 \pm 0.02 \text{ MeV}$, so it is reproduced in four-body calculations with as weak cutoff dependence as possible. Another natural follow-up would be to extend these CSB calculations in LO and NLO to $p$-shell hypernuclei. Recent shell model calculations (11), using a schematic $\Lambda N \leftrightarrow \Sigma N$ coupling-potential model, suggest that CSB splittings of g.s. mirror levels in $p$-shell hypernuclei decrease in size with respect to $A = 4$, and perhaps even reverse sign, in rough agreement with old emulsion data (3). Such extensions of the present work pose a valuable theoretical challenge to the microscopic understanding of strange nuclear systems.

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