Terahertz metamaterial beam splitters based on untraditional coding scheme

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Abstract: Terahertz waves have attracted considerable research interest in recent years because of their potential applications in diverse fields. As an important device to control terahertz waves, beam splitters with greater flexibility and higher degrees of freedom are highly desirable. In order to obtain higher degrees of freedom in beam splitting, 2-bit or higher-bit coding elements are usually introduced into metamaterial beam splitters based on the coding theory. In this work, a new “offset” coding scheme using only the 1-bit coding elements of “0” and “1” is presented, and the period of coding for beam splitting can be a non-integer multiple of the length of a single unit rather than only its integer multiples. Therefore, more beam-splitting degrees of freedom can be obtained, and the design strategy is experimentally verified. We believe that the new coding scheme will also be of significance in radar cross section reduction and flexible wave control.

1. Introduction

Because of their special spectral position in the electromagnetic spectrum, terahertz (THz) waves have a number of unique properties [1] and find important applications in spectroscopy, imaging, biomedicine, astronomy, security control, communications and other fields [2–4]. Being a vital device to control THz waves, beam splitters with greater flexibility and higher degrees of freedom are in great demand. However, due to the lack of suitable natural materials with the required electromagnetic response at THz frequencies, previous studies routinely use electromagnetic metamaterials to manipulate THz waves [5–17]. Metamaterials, with macroscopic artificial subwavelength units, enable electromagnetic responses that cannot be found in natural materials and are widely used in many fields, including wave phase modulation [18–20], polarization control [21–23], perfect absorption [24–26], wave focusing and holography [27–32].

To increase the ability to perform real-time control of electromagnetic waves and improve the construction of multifunctional devices, the method to construct “metamaterial bytes” was proposed by Giovampaol and Engheta in 2014 [33]. In the same year, Cui et al. put forward the concept of coding metamaterials [34], where the basic unit cells are associated with digital states and a link between the physical material world is established with the digital world. Furthermore, a single metamaterial can achieve different...
functions through digital or programmable control, thus realizing digital and real-time control of electromagnetic waves [34]. In coding metamaterials, the function is typically achieved by designing two units with “0” and “π” phase responses to simulate the “0” and “1” digital states, and to employ digital control to realize real-time control [35,36].

In previous reports, in order to obtain higher angular degrees of freedom in beam splitting, two methods are typically adopted: The first method is to combine 1-bit coding elements according to the addition theorem and use coding elements of 2-bit or higher-bit, which can produce anomalous single-beam scattering more flexibly and enable continuous control of the beam to arbitrary directions [37,38]. In the second approach, a so-called “super unit cell” is usually adopted, which is generated by a subarray of the same basic unit cells with a size \( N^*N \) [36]. The period length designed in this conventional way can only be an integer multiple of the length of a single unit cell. In addition, the number of split beams is mostly two and four, which is limited and the degrees of freedom are few. In this paper, a new design method of “offset” coding is proposed, which makes it possible to obtain more beam splitting degrees of freedom by using only the 1-bit coding elements “0” and “1”. For instance, the number of split beams can be increased to six and eight. In previous studies, the structures used are mainly of two types, “stripes” and “chessboard”. By contrast, a larger number of structures are enabled by the new coding scheme. Even with the same number of split beams, new angles different from previous designs can be obtained. Therefore, the “offset” coding arrangement provides a new strategy for designing more flexible beam splitters.

2. Theoretical analysis and element design

The beam splitting angle can be obtained according to the generalized Snell’s law [39]:

\[
n_i \sin \theta_i - n_1 \sin \theta_1 = \frac{\lambda_0}{2\pi} \frac{d\phi}{dx},
\]

(1)

where \( n_i \) (\( n_1 \)) is the refractive index of the incident (refraction) medium, \( \theta_i \) (\( \theta_1 \)) represents the incident (refraction) angle, \( \lambda_0 \) is the vacuum wavelength, and \( d\phi/dx \) is the phase gradient along the interface between the two media.

In previous 1-bit coding scenarios [34], a “010101... /010101...” or “010101....../ 101010......” sequence is usually employed for beam splitting. The part before the stroke is the coding sequence along the \( x \) or \( y \) direction, and that after the stroke is the coding sequence of the next row or column parallel to the chosen direction. The beam splitting angle of the first coding sequence can be obtained by the formula \( \theta = \sin^{-1}(\lambda_0/\Gamma) \) [36] derived from (1), where \( \Gamma \) represents the physical length of one period of the gradient phase distribution. The directions of beam splitting are the same as the directions of phase change. For the second coding sequence, the direction (\( \theta, \varphi \)) of the anomalously reflected waves can be expressed as [40]:

\[
\theta = \sin^{-1} \left( \lambda_0 \sqrt{\frac{1}{H_x^2} + \frac{1}{H_y^2}} \right),
\]

(2)

\[
\varphi_{1,2} = \pm \tan^{-1} \left( \frac{D_y}{D_x} \right), \quad \varphi_{3,4} = \pi \pm \tan^{-1} \left( \frac{D_x}{D_y} \right),
\]

(3)

where \( H_x \) and \( H_y \) represent the physical lengths of one period of the coding sequence along the \( x \) and \( y \) directions, respectively; \( D_x \) and \( D_y \) are the side lengths of one single coding element and in most cases are equal. \( \varphi_1, \varphi_2, \varphi_3, \varphi_4 \) are the values of \( \varphi \) for each split beam. Therefore, under a specific operating wavelength and with the same coding bits, how to change the period \( \Gamma \) becomes the key factor to increase the degrees of freedom in beam splitting.

Different from previously reported arrangements of the periodic phase gradient change in one direction of the two-dimensional plane and repeated or fixed arrangements of the phase
gradient change in the other vertical direction, the “offset” arrangement proposed here is a new arrangement in which the phase gradient changes periodically in one direction and the coding sequence is translated in the next row or column in the other direction. This will be discussed below in detail.

Figure 1(a) shows, as an example, a conventional coding sequence “00011000111... / 00111000111...” in the x/y directions, while Fig. 1(b) is the “offset” coding sequence which is obtained by translating each previous row of the conventional coding sequence to the left by two units. By this arrangement, the period $\Gamma$ is no longer restricted to being only an integer multiple of a single coding element length, thus being able to increase the degrees of freedom of the beam splitting angle. However, if the “offset” coding sequence is applied, the phase gradient direction is no longer along the x/y directions, a “saw tooth” shape is observed at the junction between the areas with different phases. Therefore, the period $\Gamma$ does not completely share the same position and length (as in Fig. 1(a)) in the direction perpendicular to the phase gradients. Rather, there is offset to the right or left side with a change of the “saw tooth” shape. It can be seen that there is no formula directly corresponding to this “offset” coding scheme. Fortunately, the change of the period $\Gamma$ is periodic and can be predicted. Therefore, in order to facilitate calculation, it is necessary to convert this unconventional arrangement into an equivalent conventional coding scheme. As illustrated in Fig. 1(c), the red dashed lines are used to divide different phase gradient areas as equally as possible, which makes the “offset” arrangement equivalent to the conventional arrangement. The equivalent structure is obtained as in Fig. 1(d). Thus, the angle between the direction of the equivalent structure and the y axis is angle $\alpha_1$, where $\alpha_1 \approx 26.6^\circ$ (arctan0.5 from Fig. 1(c)). The equivalent period $\Gamma^*$ can then be obtained according to the Pythagorean theorem through $\alpha_1$ and the hypotenuse ($3p$ in this case). Then, the results of beam splitting can be easily calculated by using the formulas described above. Figure 1(e) shows a schematic diagram for $\theta$ and $\varphi$ in the Cartesian coordinate system to determine the direction of the deflected beam to be discussed later.

**Fig. 1.** (a) Conventional coding “00011000111... /00111000111...” sequence, with “0” elements in blue and “1” elements in yellow. (b) Unconventional “offset” coding sequence. (c) Equivalent modeling of “offset” sequence. (d) Equivalent structure of “offset” sequence for (b). (e) Schematic diagram of $\theta$ and $\varphi$ in the Cartesian coordinate system.
In order to realize the “offset” coding sequence, the chosen medium is an all-dielectric construction of rectangular-shaped silicon pillars on a silicon substrate \( n_{Si} = 3.45 \) [22]. The schematic of a unit cell is shown in Fig. 2(a). The all-dielectric structure is easy to process and design, and it can effectively reduce the ohmic loss associated with metallic metamaterials. The results reported below are all for transmission coding devices based on this structure. Based on the commercially available software CST Microwave Studio, numerical simulations are carried out by varying the side lengths of the pillars \( l_x \) and \( l_y \), while the height of the pillar \( h = 200 \mu m \) and the periods \( p_x = p_y = p = 150 \mu m \) are fixed. Here, the target frequency in the whole work is 1.0 THz. When the incident electromagnetic wave is \( x \)-polarized, the difference in the transmission phase is obtained in Fig. 2(b), from which the following parameters are determined: for the selected “0” element, \( l_x = 75 \mu m \) and \( l_y = 75 \mu m \), and for the “1” element \( l_x = 50 \mu m \) and \( l_y = 93.5 \mu m \). When the incident electromagnetic wave is \( y \)-polarized, one only needs to switch the lengths of \( l_x \) and \( l_y \) for the “1” element.

**Fig. 2.** (a) Schematic of rectangular-shaped silicon coding element. (b) Transmission phase difference corresponding to “0” and “1” elements for \( x \)-polarized incidence

### 3. Results and discussion

#### 3.1. Increase of angular degrees of freedom for beam splitting

When we design beam splitters based on the coding metamaterial theory, how to achieve higher angular degrees of freedom in beam splitting is a crucial point. In Fig. 3(a), a conventional beam splitting coding scheme is illustrated. Given the formula \( \theta = \sin^{-1}(\lambda_0/\Gamma) \), when the wavelength \( \lambda_0 \) is kept constant, there is a one-to-one correspondence between the beam splitting angle \( \theta \) and the period length \( \Gamma \). In previous research, e.g. [38], normally \( n = m \) is presumed, and the values of \( \Gamma \) are restricted to \( 2p, 4p, 6p, \ldots \), where \( p \) is the length of a single unit. Thus, the period \( \Gamma \) can only be an even multiple of the length of a single unit. When \( n \neq m \), similar conclusions can be drawn. For example, if the sequence is “001001001....../001001001......”, then the values of \( \Gamma \) are \( 3p, 5p, 7p, \ldots \), odd multiples of the unit length. All the analysis shows that for the conventional beam splitting method, the period length \( \Gamma \) can only be an integer multiple of the length of a single unit \( p \). However, if an “offset” sequence is made, as shown in Fig. 1(b), \( \Gamma^o = \frac{6\sqrt{5}}{5} p \), which is \( 3p \cos \alpha_1 \) as determined from Fig. 1(d), and the theoretical value of \( \theta \) is 48.2°, while the theoretical value of \( \varphi \) is 116.6°. CST Microwave Studio is used to verify the theory. The incident wave is selected as \( x \)-polarization and the propagation direction is \(+z\). This setting is used in all the following simulations. The result of far-field beam splitting is shown in Fig. 3(b), where the derivation of the calculated angles of \( \theta \) and \( \varphi \) from the theoretical ones is about 1°, fully corroborating our analysis. Other similar coding sequences can also be manipulated with a corresponding “offset” scheme.
Fig. 3. (a) Conventional coding sequence. (b) Simulation results of field intensity as a function of angle for the structure in Fig. 1(b) with peak values determined to be at $\theta = 47^\circ$ and $\varphi = 117^\circ$. (c) “Chessboard” coding sequence. (d) Novel “offset” coding sequence producing four split beams and calculation of equivalent structure. (e) 3D far-field scattering pattern of the structure in (d).

When the number of the split beams needed is four, the traditional way is to use the “chessboard” structure (Fig. 3(c)) for the two-dimensional coding. Accordingly, the value range of $\Gamma$ is limited. A novel construction based on the “offset” arrangement outlined above is illustrated in the upper panel of Fig. 3(d). As can be seen, the structure consists of four coding areas with mirror symmetry with respect to the $x$ and $y$ axes. The fourth quadrant is used to illustrate how to calculate the equivalent period $\Gamma^*$ in the lower panel of Fig. 3(d). Here, the angle between the direction of the equivalent structure and the $y$ axis is angle $\alpha_2$, where $\alpha_2 = 45^\circ$. The equivalent period can be obtained as $\Gamma^* = 3\mu\cos\alpha_2 = \frac{3\sqrt{2}\mu}{2}$ and the theoretical value of $\theta$ is $70.5^\circ$. The other three quadrants share the same $\Gamma^*$ and $\theta$. The simulation results of the 3D far-field scattering patterns of the structure are shown in Fig. 3(e), where $\theta = 69^\circ$. The central lobe is due to the directly transmitted wave.

Because of polarization sensitivity of the chosen all-dielectric construction of rectangular-shaped silicon pillars on a silicon substrate, different sequence arrangements of the sequence “001100110011....../001100110011......” under $x$ and $y$ polarizations with different offset directions can be combined as an anisotropic two-dimensional coding structure, as schematically illustrated in Fig. 4(a). The anisotropic structure can be used to verify the feasibility of the new “offset” arrangement. Basic coding elements with polarization sensitivity are obtained by changing the lengths of $l_x$ and $l_y$. For a “0/0” element (where the first letter stands for $x$ polarization and the latter for $y$ polarization), $l_x = 75 \mu m$ and $l_y = 75 \mu m$. For a “1/0” element, $l_x = 50 \mu m$ and $l_y = 93.5 \mu m$. For a “0/1” element, $l_x = 93.5 \mu m$ and $l_y = 50 \mu m$. For a “1/1” element, $l_x = 57.5 \mu m$ and $l_y = 57.5 \mu m$. The simulated far-field beam splitting results are shown in Fig. 4(b). Given the fact that the experiment system used to verify our design cannot measure
all the variables at the same time, for comparison purposes the simulation here is to keep $\varphi$ fixed and obtain the relative intensity as a function of $\theta$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig4}
\caption{(a) Anisotropic “offset” coding sequence. White elements: $x$-polarization $\rightarrow$ “0” element and $y$-polarization $\rightarrow$ “0” element; green elements: $x$-polarization $\rightarrow$ “1” element and $y$-polarization $\rightarrow$ “0” element; blue elements: $x$-polarization $\rightarrow$ “0” element and $y$-polarization $\rightarrow$ “1” element; red elements: $x$-polarization $\rightarrow$ “1” element and $y$-polarization $\rightarrow$ “1” element. (b) Simulation results for $x$-polarized incidence (at $\varphi = 45^\circ$) with output peak at $\theta = 44^\circ$ and $y$-polarized incidence (at $\varphi = 135^\circ$) with output peak at $\theta = 43^\circ$.

Metamaterial samples for beam splitting based on the new coding scheme are fabricated by optical lithography followed by deep reactive ion etching. The scanning electron microscopy image of one structure corresponding to Fig. 4(a) is shown in Fig. 5(a). A fiber laser-based THz time-domain spectroscopy system [41] is utilized to characterize the samples. The splitting angles $\theta$ of the sample are measured for different parameters $\varphi$ of $x$- or $y$-polarized THz wave. The THz signal is measured in the time domain and converted into the frequency domain by fast Fourier transform. Figure 5(b) shows the measured results for $x$- and $y$-polarized incidence. The measured angles are in good agreement with calculated values based on the measured sample parameters. The difference between the measured intensity distribution and the simulation can be attributed to the fabrication error in the sample parameters.

3.2. Increase of the number of beam splitting

Increase of the angular degrees of freedom in beam splitting is not the only advantage that the “offset” coding scheme can offer. One can envisage the design of beam splitting devices with new functions by rearranging and combining the “offset” sequence and the conventional sequence or one “offset” sequence with another “offset” sequence.
Fig. 5. (a) Scanning electron microscopy image of sample. (b) Experiment results of x-polarized incidence (at $\varphi = 45^\circ$) with peak at $\theta = 44^\circ$, and experiment results of y-polarized incidence (at $\varphi = 135^\circ$) with peak at $\theta = 43^\circ$. The corresponding simulations are given in Fig. 4(b).

For the design with six split beams (Fig. 6(a)), we consider using structures with different “offset” directions in area 1 or 2 so that two split beams can be obtained in different diagonal directions for each structure individually. Then, by combining areas 1 and 2 alternatively along the direction of the y axis, the structure can be seen as a conventional coding sequence in the x direction and thus two addition split beams are obtained in this direction. With the four split beams in the diagonal directions, there are six split beams in total. Similarly, for the design with eight split beams (Fig. 6(c)), we consider obtaining four split beams in the axis directions by using areas 1 and 2 and obtaining another four split beams in the diagonal directions by using areas 3 and 4. In total, eight split beams are obtained. The simulation results of 3D far-field scattering patterns of the two structures are given in Figs. 6(b) and 6(d), respectively. The angles of each beam splitting structure can be obtained by the equivalent method described above. In Fig. 6(a), the actual size of the structure used in the simulation is 32*32, while it is 31*31 in Fig. 6(c).

Fig. 6. (a) The “offset” coding sequence with six split beams. (b) 3D far-field scattering pattern simulation results of Fig. 6(a). (c) The “offset” coding sequence with eight split beams. (d) 3D far-field scattering pattern simulation results of Fig. 6(c).
4. Conclusion

A novel “offset” coding arrangement for metamaterial devices is proposed and demonstrated. The period length in the new coding scheme is no longer restricted to be an integer multiple of the unit length, enabling the design of beam splitters with more flexibility and higher degrees of freedom. Furthermore, the new designed “offset” coding scheme provides strategies for devices from which a larger number of beam splitting can be obtained and shows significant advantages in reducing RCS and manipulating the THz waves.

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Disclosures

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