DOES THE THERMAL DISK INSTABILITY OPERATE IN ACTIVE GALACTIC NUCLEI?

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ABSTRACT

We examine all possible stationary, optically thick, geometrically thin accretion disk models relevant for active galactic nuclei (AGN) and identify the physical regimes in which they are stable against the thermal-viscous hydrogen ionization instability. Self-gravity and irradiation effects are included. We find that most if not all AGN disks are unstable. Observed AGN therefore represent the outburst state, although some or all quasars could constitute a steady population having markedly higher fueling rates than other AGN. This finding has important implications for the AGN mass supply and for the presence of supermassive black holes in nearby spirals.

Subject headings: accretion, accretion disks — galaxies: active — instabilities

1. INTRODUCTION

Accretion disks are a nearly ubiquitous feature of close binary systems, and their presence is widely invoked in models of active galactic nuclei (AGN). A major feature of the disks in binaries is the thermal-viscous instability driven by hydrogen ionization (Meyer & Meyer-Hofmeister 1982; Smak 1982). It is now commonly accepted that this instability drives outbursts in cataclysmic variables and soft X-ray transients. Lin & Shields (1986) showed by a local stability analysis that this instability can also operate in accretion disks thought to be present around supermassive black holes in AGN. They concluded that these disks were unstable at radii \( \approx 10^{15} \text{ to } 10^{16} \text{ cm} \) where the surface temperature is several thousand degrees. The expected characteristic timescale for this instability is \( 10^{-4} \text{ to } 10^{-7} \text{ yr} \).

Because of its generic nature, the ionization instability plays a dominant role in characterizing the observed behavior of the host systems. In the binary context, attempts to understand the precise conditions (e.g., the mass of the accreting object or the accretion rate) under which it occurs have been at least partially successful (e.g., Smak 1982; van Paradijs 1996; King, Kolb, & Burderi 1996; King, Kolb, & Szuszkiewicz 1997 and references therein). These studies show that self-irradiation of the disk by the central X-ray source has a determining effect on the disk stability if the accreting object is compact, as in soft X-ray transients (see below). Delineating the stable and unstable disk regions is equally important for AGN. If the instability is present in AGN disks, the suppression of central accretion in the quiescent state means that we can identify only the outburst states of unstable systems as AGN. Two important consequences follow: (1) quiescent AGN must appear as quite normal galaxies, and (2) the average mass fueling rate in many, if not all, AGN is much lower than implied by their current luminosities. This in turn limits the masses that their central black holes are expected to reach.

The idea of intermittent activity in AGN was already suggested by Shields & Wheeler (1978). They noticed that the fueling problem could be solved if active nuclei store mass during quiescence, and this mass then feeds the hole for a shorter period of intense activity. The thermal-viscous hydrogen ionization instability found to operate in AGN accretion disks (Lin & Shields 1986) is capable of triggering such behavior. Clarke & Shields (1989), Mineshige & Shields (1990), Cannizzo & Reiff (1992), and Cannizzo (1992) studied the full range of black hole masses and accretion rates in order to determine the observational consequence of the instability for the AGN population. Siemiginowska & Elvis (1997) attempted to reproduce the observed luminosity function, assuming that this mechanism operates in all AGN.

Our aim here is to decide if the ionization instability still operates in AGN when irradiation effects are included. As we have seen, irradiation is central to the discussion of disk stability in soft X-ray transients. Further, irradiation is often thought to dominate the disk emission (e.g., Collin-Souffrin 1994). For both reasons it is vital to include it in any attempt to decide the disk stability. The actual form of the instability when irradiation is included is outside the scope of our paper. Siemiginowska, Czerny, & Kostyunin (1996) have performed studies for particular black hole masses and accretion rates, with assumed forms of irradiation.

There is a simple criterion for the instability to appear: the disk must contain regions with effective temperature \( T_\text{eff} \) close to the value \( T_\text{H} \) at which hydrogen is ionized. In practice, \( T_\text{H} \) depends on the density and may be quite different in different environments; we shall consider a range of values in this paper. However, the criterion is not easy to use in this form, since one does not in general know the radial distribution of the accretion rate and thus the run of \( T_\text{eff} \) in a time-varying disk. Accordingly, one usually uses the criterion in an indirect form: a disk with a given constant accretion rate \( M \) is self-consistently steady only if \( T_\text{eff} > T_\text{H} \) throughout it. If the criterion fails we may expect outbursts, although the precise nature of these will depend, for example, on the detailed behavior of the disk viscosity.

This version of the stability criterion is easy to apply. Since \( T_\text{eff} \) always decreases with disk radius \( R \) in a steady disk, the condition is most stringent at the outer disk radius \( R_\text{out} \), so we need apply it only there. If the disk’s only source of energy is local viscous dissipation, we have

\[
[T_\text{eff}(R)]^4 = \frac{3GM}{8\pi R^3 \sigma} f ,
\]
(e.g., Frank, King, & Raine 1992; all symbols are explained after eqs. [2]–[9]), and the criterion is simply $T_{\text{eff}}(R_{\text{out}}) > T_H$. In a binary system we can estimate $R_{\text{out}}$ with reasonable accuracy as 70% of the Roche lobe radius of the accreting star, and the problem is now well determined. Using this approach, Smak (1982) successfully divided outbursting cataclysmic variables (dwarf novae) from the persistent systems (novalikes). The extension to low-mass X-ray binaries (LMXBs) is complicated by the fact that the dominant heat source for the disk is not local viscous dissipation (eq. [1]) but irradiation by the central X-rays. The instability is similarly suppressed if the disk surface temperature given by irradiation exceeds $T_H$ (Tuchman, Mineshige, & Wheeler 1990). Provided that due account is taken of this, one can again successfully divide the outbursting systems (soft X-ray transients) from the persistent systems (van Paradijs 1996; King et al. 1997). The key feature, as in the unirradiated case, is that the edge temperature of the disk can be simply expressed in terms of $M$ and $R_{\text{out}}$ without any need to solve for the full internal disk structure. In both the cataclysmic variable and LMXB cases, there are important consequences for the study of the binary evolution (e.g., King et al. 1996), which gives a connection between $M$, $M$, and $R_{\text{out}}$.

The extension of this approach to AGN is more complicated; here the outer edge of the disk is no longer determined by the simple Roche lobe condition that holds in binaries but by the requirement that the disk becomes locally self-gravitating (see eq. [13]). This condition requires a knowledge of the disk density at the outer edge, so we are now required to solve the full global structure of the steady disk to find $R_{\text{out}}$.

Thus we examine all possible stationary, optically thick, geometrically thin disk models relevant for AGN. If these correspond to stable states, AGN disks may be globally steady and require fueling at the currently inferred central accretion rates. If not, they will be the outburst states, and the required fueling rates will be lower than the current central accretion rate.

2. GLOBAL DISK STRUCTURE

As explained above, to apply the stability criterion we need the self-gravity radius and thus the global structure of steady disks. We assume these to be optically thick and geometrically thin. If we exclude the region close to the central object and consider total disk luminosities $L \lesssim 0.2L_\odot$ ($L_\odot$ being the Eddington luminosity), these approximations are justified, and we may parameterize all possible disk structures by $M$, $M$, and the viscosity parameter $\alpha$ (see below). Our approach exploits only hot radiative regimes of the disk, for which a vertically averaged structure is a good approximation. The algebraic system describing a Shakura-Sunyaev disk is

$$\rho = \Sigma H^{-1},$$

$$H = c_s R^{3/2} G^{-1/2} M^{-1/2},$$

$$c_s = P^{1/2} \rho^{-1/2},$$

$$P = \frac{k}{\mu m_p} \rho T + \frac{4\sigma}{3c} T^4,$$

$$\sigma T^4 = \frac{3\pi GM M}{8\pi R^3} f + Q_{\text{abs}}$$

(see, e.g., Frank et al. 1992), in the extreme hypothesis that the flux absorbed from the external radiation $Q_{\text{abs}}$ (see eq. [14]) is reprocessed at the center of the disk, or in the other extreme case of an external flux reprocessed in a thin layer located just below the disk photosphere

$$\sigma T^4 = \frac{3\pi GM}{8\pi R^3} f + Q_{\text{eq}},$$

$$\tau = \Sigma \kappa,$$

$$\Sigma = \frac{1}{3\pi} M v^{-1} f,$$

$$v = \frac{2}{3} \alpha H c_s \rho$$

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2.1. Disk with No Irradiation

In general, we can divide an optically thick disk into four different regimes, namely,

$$P_g \ll P_{\text{rad}}, \quad \kappa_{\text{es}} \gg \kappa_{\text{abs}} \quad \text{inner region (a)},$$

$$P_g \gg P_{\text{rad}}, \quad \kappa_{\text{es}} \gg \kappa_{\text{abs}} \quad \text{middle region (b)},$$

$$P_g \ll P_{\text{rad}}, \quad \kappa_{\text{es}} \ll \kappa_{\text{abs}} \quad \text{middle* region (b*),}$$

$$P_g \gg P_{\text{rad}}, \quad \kappa_{\text{es}} \ll \kappa_{\text{abs}} \quad \text{outer region (c).}$$

The names of the regions correspond to those used by Shakura & Sunyaev (1973); region $b^*$ was not included in their paper. We have found that this region exists in disks with $\alpha \gtrsim 3 \times 10^{-5}$. Which regions are actually present in a given disk depends on $M$, $M$, and $\alpha$.

Shakura & Sunyaev used only the Rosseland mean for free-free absorption

$$\kappa_{\text{abs}} = 6.2 \times 10^{22} \rho T^{-3.5} \text{ cm}^2 \text{ g}^{-1}.$$
Cannizzo & Reiff (1992) and Huré et al. (1994b). In fact, many local considerations are unaffected by the change in opacities, because most quantities depend only very weakly on them (typically as $\sim k_{\text{abs}}^{1/3}$). However, the global structure of the disk is very severely affected by the use of the wrong opacities, since these determine where the various regions $a-c$ match to each other. This is illustrated in Figure 1, where the borders $R_{ab}$ and $R_{bc}$—between regions $a$ and $b$ and between $b$ and $c$, respectively—are given as functions of $M$ for two different opacity approximations. The mass of the central black hole is taken as equal to $10^8 M_\odot$ and parameter $x = 0.001$. The solid line for $R_{bc}$ is obtained using a power law fitted to the recently compiled solar abundance parameter $\alpha$.

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$$k_{\text{abs}} = 9 \times 10^{24} \rho T^{-3.5} \text{ cm}^2 \text{ g}^{-1}. \quad (11)$$

The dotted line gives the $R_{ab}$ values if the opacities are given by equation (10). The filled squares are taken from the numerical calculation (Suszskiewicz, Malkan, & Abramowicz 1996), where Cox & Stewart (1970) opacity tables were used. The values of $R_{ab}$ are the same in both cases, since in regions $a$ and $b$ electron scattering is a dominant source of opacity. We list all relevant expressions in Appendix A.

The disk is terminated at the radius where self-gravity becomes important. This is evaluated using the stability criterion for a differentially rotating disk

$$Q_T = c_s \Omega / \pi \kappa \Sigma \geq 1 \quad (12)$$

(Toomre 1964), where $\kappa$ is the angular velocity. The condition $Q_T = 1$ defines a self-gravity radius

$$R_{sg} = (M / \pi \rho)^{1/3}. \quad (13)$$

Knowing the gas density from the given accretion disk model, we can find the outer edge of the disk. The disk can terminate in any of the regions mentioned above, and the appropriate density structure must be used. After simple algebra one can find appropriate formulae for the self-gravity radius for every region in the disk. We give them for all regions and for both $x$ and $\beta$ disks in Appendix C. We compared our results with the self-gravity radii calculated by Huré et al. (1994a) and found reasonable agreement. Many formulae for the self-gravity radius used in the literature simply assume that this radius is always located in region $a$, although there is no particular reason to believe this. Region $a$ can also be terminated by the change from radiation to gas pressure before the density reaches the self-gravitating value. In order to determine in which region, $a$, $b$, $b^*$ or $c$, a disk actually ends, it is sufficient to compare the dimension of each region with the appropriate self-gravity radius for given $M$, $M$, and $x$. The results are shown in Figure 2.

### 2.2. Irradiated Disks

If irradiation dominates the heating, the disk can extend to much larger radii before self-gravity starts to be important. A similar case was considered by Ko & Kallman (1991), but their results are affected by the use of the opacity form (eq. [10]). The structure of the irradiated disk can be found from equations (2)-(9). Viscous energy generation is now negligible in comparison with the flux absorbed from the external radiation, $Q_{\text{irr}}$. For photons emitted radially from a central source, $Q_{\text{irr}}$ has the form

$$Q_{\text{irr}} = \frac{\eta M c^2 (1 - \chi)}{4 \pi R^2} \frac{dH(R)}{dR}, \quad (14)$$

where $\eta (\sim 0.1)$ is the efficiency of the accretion process, $\chi$ (reasonable values are between 0.1 and 0.9) is the albedo of the disk, and $d[H(R)]/dR$ takes into account the projected surface area of the disk normal to the radiation flux. If the central emission is not radial but from the inner disk, $Q_{\text{irr}}$ is multiplied by a factor $H/R$, because this source is foreshortened (see King et al. 1997). We shall not consider this case here, because we shall find that, even with the favorable assumption (eq. [14]), irradiation is never crucial in deciding the stability of an AGN disk. We strongly suspect that this is true also for other possible ways in which a disk can be illuminated (for example, by an extended corona or a jet). However, to show this requires additional assumptions about these poorly understood configurations, and we leave this for further investigation. For low luminosities and low central masses, the disk structure is altered by the presence of irradiation. This is shown in Figure 2, where the part of the $M - M$ plane for which the disk is terminated in an irradiated region is called $C^+$. The thickness of the disk in the irradiated part does not vary with radius as $H \propto R^{0.9/2}$ (the nonirradiated case) but as $H \propto R^{4/3}$, if the irradiated flux is absorbed and reprocessed in the center of the disk, or as $H \propto R^{0.9}$, if it is absorbed and reprocessed in a thin layer located just below the photosphere (see, e.g., Vrtilek et al. 1990). Here we discuss the intermediate case ($H \propto R^{4/3}$) between the nonirradiated and the one with absorption and reprocessing of the external radiation in a thin layer just below the disk photosphere. Irradiation effects are relevant in zone $c$ if

$$\frac{L}{L_\odot} = \frac{M}{M_\odot} \leq 7 \times 10^{-5} \chi^{3/2} M_8^{-7/2} (1 - \chi)^{5/2}. \quad (15)$$
Fig. 2.—Uniquely determined global disk structure for a given set of parameters (α, Ṁ, and M). Region A contains all models in which self-gravity truncates the disk in a region where the pressure is dominated by radiation and the opacity by electron scattering (Shakura-Sunyaev region a). Region B contains the models in which the self-gravity radius terminates the disk in the region where the pressure is dominated by gas pressure and opacity by electron scattering (Shakura-Sunyaev region b). Region B* is the regime in which the self-gravity radius occurs in the region where radiation pressure dominates, but the main source of opacity is absorption rather than scattering. Region C contains models with the outer radii determined by self-gravity in the region where gas pressure dominates and opacity is given by absorption (Shakura-Sunyaev region c). In all regions, A, B, B*, and C, the main source of heating is due to the viscous energy dissipation. In region C the main heating process is irradiation by the central source instead. The size of this region depends on the albedo of the disk, χ. It is shown here for two particular values of χ: 0.1 and 0.9. In every region the stability properties have been determined, both for α and β disks, for two values of hydrogen ionization temperature $T_\text{HI}$ (left panel, $T_\text{HI} = 6500$ K; right panel, $T_\text{HI} = 2000$ K) and for different values of α parameter (left panel, $α = 10^{-4}$, $α = 10^{-3}$, $α = 10^{-2}$; right panel, $α = 10^{-3}$, $α = 10^{-2}$, $α = 10^{-1}$). The shaded zones show the location of the α disks models in which the ionization instability does not operate. The hatched zones on the left panel show the location of the β disk models in which the ionization instability is suppressed. For $T_\text{HI} = 2000$ K (right panel) the stability properties for α and β disks are the same, so the shading shows the stable region for both cases.
in zone b if
\[
\frac{M}{M_e} \leq 1 \times 10^{-16} \alpha^4 M_8^{-10}(1 - \chi)_c^{6.9}, \tag{16}
\]
in zone b* (for \( \alpha \) disks) if
\[
\frac{M}{M_e} \geq 3 \times 10^2 \alpha^{-1} M_8^{1/2}(1 - \chi)_c^{-5/2}, \tag{17}
\]
and in zone b+ (for \( \beta \) disks) if
\[
\frac{M}{M_e} \geq 1 \times 10^7 \alpha^{-3} M_8^{1/2}(1 - \chi)_c^{-13/2}. \tag{18}
\]
Here \( L_{\text{th}} = 1.5 \times 10^4 M_8 \) ergs s\(^{-1}\) is the Eddington luminosity, \( M_8 = 2.6 \times 10^8 M_8 \) g s\(^{-1}\), \( M_8 = M/10^8 M_\odot \), and \((1 - \chi)_c = (1 - \chi)/0.1\). The thin disk approximation we use here requires \( M/M_e \leq 0.2\). The vertical thickness of the disk in region \( a \) is constant, and it therefore cannot be irradiated by photons emitted radially from a central source, because it is all shadowed (see eq. [14]). The situation will be, of course, quite different in the case of irradiation by a corona or emission scattered above the disk.

3. DISK STABILITY

Armed with the values of the disk self-gravity radius from the previous sections, we can now check the simple criterion for stable disk accretion. We consider values \( T_{\text{th}} = 6500 \) or 2000 K, corresponding to the extremes of what is normally claimed for AGN disks (Lin & Shields 1986; Clarke 1988; Clarke & Shields 1989; Mineshige & Shields 1990; Cannizzo 1992).

Accordingly, we identify the stable regimes in regions A, B, B* C, and C* for both \( \alpha \) and \( \beta \) disks (Fig. 2). Here the \( M-M \) plane is divided into regions A, B or B*, C, and C*, consisting of all models in which the disk is terminated by self-gravity in regions \( a, b, b*, \) and \( c \), respectively. The Eddington limit and the limit of validity for the thin disk approximation, namely, \( L \leq 0.2L_{\text{th}} \), are also shown.

Unlike in the case of LMXBs, even in the extreme case of irradiation with very low albedo (90% of the incident radiation absorbed by the disk), irradiation never stabilizes the disk. To see this, we note that the criterion for stability in the irradiated disk in zone \( c \) is

\[
7 \times 10^{-5} \alpha^3 M_8^{-7/2}(1 - \chi)_c^{5/2} \geq \frac{M}{M_e}.
\]

\[
\geq 2.6 \alpha^{17/26} M_8^{-17/26}(1 - \chi)_c^{-9/26} T_{\text{th}}^{2000}, \tag{19}
\]

where \([T_{\text{th}}]^{2000} = T_{\text{th}}/2000\). From this criterion and from the requirement that \( M/M_e \leq 0.2 \), it follows easily that irradiation is unable to stabilize the disk in zone \( c \). Equivalent criteria for zone \( b, b* \) lead to the same conclusion. In the case of irradiation affecting only a thin layer located just below the photosphere, the same conclusion is obtained.

4. DISCUSSION

Our aim in this study was to investigate whether the thermal-viscous ionization instability operates in AGN in the presence of irradiation. We have studied stationary, optically thick, geometrically thin disks in the range of accretion rates and central black hole masses for which these models are self-consistent. It is worth mentioning here that advection-dominated optically thin disks can in principle coexist in some particular regions of the parameter space, but which type of the solution will be actually chosen in nature is still an open question. We used a very simple analytic criterion to determine the stability of each model; if the disk is hot enough for hydrogen to be completely ionized everywhere, all the way out until its self-gravity radius, the ionization instability cannot operate. We identify such hot regions in the relevant parts of \( M-M \) plane and show them as gray (for \( \alpha \) disks) and hatched (for \( \beta \) disks) areas in Figure 2. Unlike other authors (Clarke & Shields 1989; Mineshige & Shields 1990; Cannizzo 1992), we consider only the upper stable branch of the whole cycle, where our method is appropriate. A major advantage of our approach is that we do not need a complicated discussion of the limit cycle. This method proved successful in similar studies of accretion disks in X-ray binaries. We gave careful consideration to the opacities used in our calculations. There are only small differences between results using opacities from Mazzitelli (1989) and Cox (1970). However, differences appear when using simple fitting formulae such as equation (10) instead of equation (11) (see Fig. 1); it is important to check carefully that a particular fit found in the literature is appropriate for the range of temperatures and densities used in a given problem.

Another result of our study is that for \( \alpha \gtrsim 0.003 \), the region between region \( a \) and \( c \) differs from the standard Shakura-Sunyaev region \( b \). We denote it \( b* \). It is radiation-pressure-dominated, but the main source of opacity is true absorption. We have confirmed the existence of this region in numerical calculations of global disk structure performed using the Cox & Stewart (1970) opacity tables. It is interesting that \( b* \) is stable against disk instabilities triggered by radiation pressure (Pringle 1976): while irradiated it might significantly change its properties.

In Figure 3 we compare our results with those based on detailed studies of the outburst cycle over the parameter space considered by various authors. The dotted lines are from Mineshige & Shields (1990), the dotted-dashed lines are from Clarke & Shields (1989), the long-dashed lines are from Cannizzo (1992), and the bold lines are from this paper. The short-dashed line gives the Eddington limit. Our results for nonirradiated disk are in good agreement with those obtained previously. Our main result, quite contrary to the case of close binaries, is that irradiation does not change the borders between unstable and stable (partially or completely ionized) regions. In other words, irradiation by a central point source is unable to stabilize the whole disk out to its self-gravity radius. An important reason for this is that one of the effects of such irradiation is to move the self-gravity radius even farther out from the central black hole. The irradiated disk structure for low-luminosity, low-mass objects differs from that of the equivalent disks without irradiation (regions C* in Fig. 2). Thus the actual appearance of the ionization instability might well be affected. This can be studied only by detailed calculations of thermal limit cycles in the presence of irradiation.

We see from Figure 2 that, in general, AGN disks will be subject to the ionization instability even if they are irradiated by a central point source. For typical AGN luminosities, corresponding to central accretion rates \( \lesssim 10^{-2} M_\odot \) yr\(^{-1}\), we see from Figure 2 that it is inconsistent to assume that the disk is stable. Since central accretion (and thus, e.g., X-ray emission) is suppressed in the quiescent state, all observed AGN must presumably be identified as...
such only in their outburst states (which last $\gtrsim 10^3$ yr). Thus AGN currently observed to have central accretion rates below the stability limits $\sim 10^{-1} - 10^{-2} M_\odot$ yr$^{-1}$ shown in Figure 2 must actually have considerably lower fueling rates. Even rather brighter observed AGN need not be steady systems but may simply represent the outburst states of unstable disk with fueling rates below the stability limits.

As pointed out in the Introduction, if most AGN disks are unstable, then in quiescence these systems must be indistinguishable from normal galaxies. Moreover, the mass fueling rates needed to power AGN must be much lower than implied by their current luminosities. If the duty cycle for the outburst can be made short enough ($\lesssim 10^{-2}$), no fueling rates greater than about $10^{-2} M_\odot$ yr$^{-1}$ would be needed in AGN. This would also remove the problem that the remnant black holes are predicted to have excessively high masses if accretion is continuous (Cavaliere & Padovan 1988).

Alternatively, since most quasars have observed central accretion rates above the stability limits in Figure 2, some or all of them could have steady disks. This group would then form a separate class with much higher fueling rates $\dot{M} \sim 0.1 - 1 M_\odot$ yr$^{-1}$. It is not easy to decide between these possibilities by looking at detailed properties of the individual systems, because outbursting disks rapidly take on a quasi-steady surface density profile (see Cannizzo 1993; this property is well known in the context of cataclysmic variables, where the persistent systems—novalike variables—look like dwarf novae in permanent outburst). A complicating feature is that many of the objects with high steady fueling rates would be subject to the radiation-pressure (Lightman-Eardley) instability.

We conclude that many (if not all) AGN represent the outburst state of a thermal-viscous disk instability. We should then consider candidates for the quiescent state. It is tempting to suggest that this may comprise most or all "normal" spirals. Galaxies such as our own could therefore harbor moderately massive $(10^6 - 10^8 M_\odot)$ black holes in their nuclei.

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APPENDIX A

SIZE OF THE REGIONS IN NONIRRADIATED $\alpha$ AND $\beta$ DISKS

Here and in Appendices B and C we give expressions for all characteristic radii used in our paper. They are given in a form of two equalities. In the first equality, for generality, we retain a dependence on the electron scattering opacity coefficient $\kappa_{es} = 0.2(1 + X)$ cm$^2$ g$^{-1}$, where $X$ is the hydrogen content by mass (in this paper we use $X = 0.7$) and on $\kappa_0$, which is a constant coefficient used in our fitting procedure:

$$\kappa_{abs} = \kappa_0 \rho T^{-3.5} \text{cm}^2 \text{g}^{-1}.$$  

The best fit to the solar-abundance opacities (Mazzitelli 1989) was obtained with $\kappa_0 = 9 \times 10^{24}$. Moreover, still in the first equality, $\dot{M}$ should be in g s$^{-1}$ and $M$ in g in order to get the radii in cm. In the second equality, in order to make the use of our formulae more easily, we introduced the following quantities:

$$\dot{M}_{-1} = \frac{M}{0.1 M_e}, \quad \text{and} \quad M_s = \frac{M}{10^8 M_\odot},$$

where $M_e = 2.6 \times 10^{26} M_\odot$ g s$^{-1}$. All the coefficients in front of these quantities are given in cm.

Note that for $\alpha$ less than 0.003 only region $b$ is present in a disk, and for $\alpha$ greater than 0.003 only region $b^*$ is present.

**Fig. 3.—** $\dot{M}$-$M$ plane for AGN disks, divided into three distinct regions according to the degree of hydrogen ionization: neutral, partially ionized, and completely ionized. We take $\alpha = 0.1$. The box with thick solid lines defines the domain discussed in this paper, and the horizontal line inside it shows the accretion rate above which the disks are completely ionized. Previous studies of the $\dot{M}$-$M$ plane are shown by different lines: Clarke & Shields (1989; dot-dashed line), Mineshige & Shields (1990; dotted line), Cannizzo (1992; long-dashed line). The short-dashed line represents the Eddington limit. See text for full details.
\[ R_{sc} = 2.8224 \times 10^2 k_0^{-2/3} k_{es}^{4/3} M^{2/3} M_1^{1/3} = 7.9 \times 10^{15} \mathcal{M}^{2/3} M_8, \]
\[ R_{ac} = 1.5089 \times 10^{-5} k_0^{6/5} a^{4/5} M^{14/15} M_1^{1/3} = 4.1 \times 10^{16} a^{4/5} M_8^{14/15} M_8^{11/15}, \]
\[ R_{ab} = 2.6602 \times 10^{-17} k_{es}^{18/21} a^{2/21} M_6^{16/21} M_1^{1/3} = 1.4 \times 10^{16} a^{2/21} M_6^{16/21} M_8^{3/21}. \]  
(20)

For \( \alpha \) disks,
\[ R_{ab} = 3.7322 \times 10^{-7} k_0^{-16/45} k_{es}^{10/9} a^{2/45} M^{32/45} M_1^{1/3} = 1.0 \times 10^{16} a^{2/45} M_8^{32/45} M_8^{27/45}. \]

For \( \beta \) disks,
\[ R_{ab} = 4.1353 \times 10^{-6} k_{es}^{58/51} k_0^{-20/51} a^{2/51} M^{36/51} M_1^{1/3} = 1.0 \times 10^{16} a^{2/51} M_8^{36/51} M_8^{53/51}. \]

APPENDIX B

SIZE OF THE REGIONS \( c \) AND \( c^+ \), \( b \) AND \( b^+ \), AND \( b^* \) AND \( b^{**} \) OF IRRADIATED DISKS

\[ R_{cc^+} = 2.5706 \times 10^{-28} k_0^{2/45} a^{4/45} M^{-2/15} M_1^{11/9} = 2.4 \times 10^{18} a^{4/45} M_8^{-2/15} M_8^{11/9}, \]
\[ R_{bb^+} = 2.0215 \times 10^{-30} k_{es}^{-2/21} a^{2/21} M^{-4/21} M_1^{9/7} = 4.1 \times 10^{18} a^{2/21} M_8^{-4/21} M_8^{9/7}. \]
(21)

For \( \alpha \) disks,
\[ R_{bb^{**}} = 8.8331 \times 10^{-23} k_0^{-16/45} a^{2/45} M^{-2/5} M_1^{13/9} = 3.6 \times 10^{18} a^{2/45} M_8^{-2/5} M_8^{17/45}. \]

For \( \beta \) disks,
\[ R_{bb^{**}} = 3.9392 \times 10^{-22} k_0^{-20/51} a^{2/51} M^{-22/51} M_1^{25/17} = 3.8 \times 10^{18} a^{2/51} M_8^{-22/51} M_8^{53/51}. \]

APPENDIX C

SELF-GRAVITY RADIUS FOR EACH REGION IN NONIRRADIATED AND IRRADIATED DISKS

Region \( c \) (for both \( \alpha \) and \( \beta \) disks):
\[ R_{sg} = 3.9687 \times 10^{12} k_0^{2/15} a^{28/45} M^{-22/45} M_1^{1/3} = 1.8 \times 10^{17} a^{28/45} M_8^{-22/45} M_8^{-7/45}. \]  
(22)

Region \( b \) (for both \( \alpha \) and \( \beta \) disks):
\[ R_{sg} = 8.0781 \times 10^{10} k_0^{2/9} a^{14/27} M^{-8/27} M_1^{1/3} = 1.1 \times 10^{17} a^{14/27} M_8^{-8/27} M_8^{1/27}. \]  
(23)

Region \( b^* \) for \( \alpha \) disks:
\[ R_{sg} = 6.5485 \times 10^{-12} k_0^{8/15} a^{22/45} M^{2/45} M_1^{1/3} = 1.0 \times 10^{17} a^{22/45} M_8^{2/45} M_8^{7/45}. \]  
(24)

Region \( b^* \) for \( \beta \) disks:
\[ R_{sg} = 1.7057 \times 10^{2} k_0^{14/13} a^{22/39} M^{-10/39} M_1^{1/3} = 1.4 \times 10^{17} a^{22/39} M_8^{-10/39} M_8^{1/13}. \]  
(25)

Region \( a \) for \( \alpha \) disks:
\[ R_{sg} = 4.6733 \times 10^{-9} k_{es}^{2/3} a^{2/9} M^{9} M_1^{1/3} = 2.6 \times 10^{16} a^{2/9} M_8^{2/9} M_8^{1/7}. \]  
(26)

Region \( a \) for \( \beta \) disks:
\[ R_{sg} = 7.6518 \times 10^{-2} k_0^{16/12} a^{13/18} M^{16} M_1^{1/3} = 4.5 \times 10^{16} a^{13/18} M_8^{16} M_8^{1/2}. \]  
(27)

Region \( c^+ \):
\[ R_{sg} = 1.3615 \times 10^{20} k_0^{1/6} a^{13/18} M^{-5/9} M_1^{1/6} = 1.1 \times 10^{17} a^{13/18} M_8^{-5/9} M_8^{7/18}. \]  
(28)

Region \( b^+ \):
\[ R_{sg} = 1.3118 \times 10^{25} k_{es}^{1/3} a^{2/3} M = 3.1 \times 10^{16} a^{2/3} M_8^{-1/3} M_8^{-1/3}. \]  
(29)

Region \( b^{**} \) (for \( \alpha \) disks):
\[ R_{sg} = 8.0208 \times 10^{14} k_0^{8/3} a^{14/9} M^{10/9} M_1^{1/3} = 2.1 \times 10^{13} a^{14/9} M_8^{10/9} M_8^{-11/9}. \]  
(30)

Region \( b^{**} \) (for \( \beta \) disks):
\[ R_{sg} = 7.2272 \times 10^{18} k_0^{4/5} a^{14/15} M^{-2/15} M_1^{1/3} = 1.4 \times 10^{16} a^{14/15} M_8^{-2/15} M_8^{-3/5}. \]  
(31)

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