Microlensing in a Prolate All-Macho Halo

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Abstract

It is widely believed that dark matter halos are flattened, that is closer to oblate than prolate. The evidence cited is based largely on observations of galaxies which do not look anything like our own and on numerical simulations which use ad hoc initial conditions. Given what we believe to be a “reasonable doubt” concerning the shape of dark Galactic halo we calculate the optical depth and event rate for microlensing of stars in the LMC assuming a wide range of models that include both prolate and oblate halos. We find, in agreement with previous analysis, that the optical depth for a spherical (E0) halo and for an oblate (E6) halo are roughly the same, essentially because two competing effects cancel approximately. However the optical depth for an E6 prolate halo is reduced by $\sim 35\%$. This means that an all-Macho prolate halo with reasonable parameters for the Galaxy is consistent with the published microlensing event rate.

1. Introduction

The announcements by the MACHO (Alcock et al. 1993) and EROS (Aubourg et al. 1993) collaborations of candidate microlensing events towards the Large Magellanic Cloud (LMC) generated tremendous excitement by suggesting that the identity of one contribution to the dark Galactic halo had been found. Yet despite two years of additional observations and intense analysis the implications of the microlensing results remain unclear. The event rate toward the LMC, as determined by the MACHO collaboration for example, is significantly below what one would expect if Machos are the dominant component of the “standard” Galactic halo but significantly above what one would expect if the lenses are from known stellar populations. The best guess is that 5%-30% of the halo is in Machos. An all-Macho halo cannot be ruled out though this would require a model for the Galaxy that is only marginally acceptable (Alcock et al. 1995a,b, Gates, Gyuk, & Turner 1995a,b).

Attempts to derive an accurate and robust estimate for the mass density of Machos in our Galaxy are hampered by statistical uncertainties due to the small number of events and systematic uncertainties in the detection efficiency for the experiments. But even if these difficulties could be overcome we would be left with important uncertainties in modelling our Galaxy. In particular the parameters which describe our Galaxy, such as the local rotation speed and the rotation speed far from the Galactic center, are not well constrained.

Perhaps the most important unknown in Galactic models is the shape of the halo. The conventional wisdom is that halos are flattened. With this in mind a number of researchers have calculated the optical depth for oblate versus spherical model halos (Sackett & Gould 1993; Frieman & Scoccimarro 1994; Alcock et al. 1994a; Gates, Gyuk & Turner 1995a,b). The optical depth to microlensing toward the LMC is found to be roughly the same for an E6 (axial ratio $q = 0.4$)
oblate halo and an E0 \((q = 1.0)\) spherical halo. (Following standard notation \(a > b > c\) refer to the three semi-axis lengths for a triaxial halo. For a spheroidal halo, two of these lengths are equal and the shape of the halo is specified by a single parameter \(q\) defined to be the ratio of the symmetry axis to the equatorial axis.) This is essentially because two competing effects, one geometric and the other related to the central density, cancel approximately. The general conclusion is that the total mass in Machos is constrained tightly by the microlensing data while the mass fraction is model-dependent (Alcock et al. 1995a,b; Gates, Gyuk, & Turner 1995b). (This conclusion is based on analysis in which other halo parameters such as the thickness of the disk are also varied.)

Here we consider a broad spectrum of models ranging from E6 prolate \((q = 2.5)\) to E6 oblate. In each model the symmetry axis is chosen to be perpendicular to the disk. For a fixed rotation curve, the central density decreases as we increase \(q\) and this leads to a decrease in the optical depth. The optical depth for an E6 prolate halo, for example, is reduced relative to the optical depth for either an E6 oblate or spherical halo by \(\sim 35\%\). This is enough to change significantly the conclusions based on the MACHO data. In particular an all-Macho halo with reasonable halo parameters is now allowed. Moreover our models provide an example where the total mass in Machos inferred from the data is different from that found assuming either an oblate or spherical halo.

Prolate halos are unconventional to say the least. The overriding view of researchers in the field is that halos are spherical \((a \sim b \sim c)\), oblate \((a \sim b > c)\), or triaxial \((a > b > c)\) (Rix 1995, Sackett 1995). Evidence for the shape of the dark Galactic halo comes mainly from dynamical modelling of metal-poor halo stars. Observations constrain \(q_s\), the axial ratio for the (metal-poor) stellar halo, to be \(0.6 \lesssim q_s \lesssim 0.8\) (Gilmore, Wyse, Kuijken 1989). The simplest assumption, that the dark Galactic halo has similar kinematical properties, implies \(q < 1\), i.e., an oblate dark Galactic halo. However, the relation between \(q\) and \(q_s\) may not be so simple. van der Marel (1991) has derived this relation under the assumption that the halo stars are in hydrostatic equilibrium (i.e., obey the Jeans equations). The results indicate that this relation is extremely sensitive to assumptions made about the local velocity ellipsoid of the halo stars. Indeed the axial ratio for the dark matter can be much larger than that for the stellar halo and in some cases greater than 1.

There is compelling evidence from observations of other galaxies that oblate halos exist. Observations of rare polar-ring galaxies (eg. Sackett et al. 1994) find flattened halos with \(0.3 \lesssim c/a \lesssim 0.6\). As well observations of X-ray emitting gas around ellipticals, used to trace the gravitational potential of the halos around these galaxies, also find flattened halos (Franx, van Gorkum & de Zeeuw 1994). However, there is no reason to expect all dark halos to have the same shape especially when the visible components of galaxies are so different. Of particular interest would be observations of spiral galaxies such as our own. At present the only such observations are of flaring HI gas layers (Olling 1995). The most detailed observations of this type are for the spiral galaxy NGC 4244 with the result \(0.2 \lesssim c/a \lesssim 0.8\). Clearly more observations of spirals are be needed before any definite conclusions are to be drawn.

Theoretical evidence in favour of oblate halos comes largely from numerical simulations. In fact the halos found in dissipationless simulations are triaxial with nearly prolate halos being more common than oblate ones. One must consider the effects of dissipational matter in order to see why oblate halos are favoured. Katz & Gunn (1991) simulate the gravitational collapse of constant density spherical perturbations consisting of both dark matter and gas. These are isolated perturbations (i.e., no external tidal effects) and so angular momentum must be put in by hand. This is done by imposing solid-body rotation on the initial perturbations. The simulations show that halos that would be triaxial or prolate in the absence of a gas component are nearly oblate once the gas is included. However it is not known whether these results are due to the very special...
initial conditions used and in particular whether the conclusions would be different if the rotation axes for the gas and dark matter were misaligned initially. Indeed the gas and dark matter may have collapsed at different times therefore experiencing different tidal torques. As an interesting aside, Katz & Gunn (1991) find that the angular momentum axes often become misaligned through the course of simulations as angular momentum is transferred from the dark matter to the disk.

Dubinski (1994) models the effects of dissipational matter by growing a disk in the center of a dark halo that is the output from a cosmological N-body simulation. The symmetry axis of the disk is aligned with the short axis of the halo which in turn coincides roughly with the rotation axis of the halo. The result, that an initially prolate-triaxial halo evolves into an oblate-triaxial one, is not unexpected given the set-up for the simulation. But again we are lead to wonder whether the conclusions would be different if the rotation axis were misaligned initially.

Clearly, there is a need for numerical simulations that model self-consistently both the dark matter and the gas with enough resolution to study individual galaxies and large-scale effects. Steps in this direction have been made by Evrard, Summers & Davis (1994). While the small-scale resolution is not fantastic their results do suggest that halos become rounder in the presence of dissipational matter. But in the end the halos are still, on average, more prolate than oblate.

Finally it is worth pointing out that all of the simulations described above assume that galaxies form in a hierarchical clustering scenario such as the Cold Dark Matter model. Clearly if Machos are the Galactic dark matter then these ideas would require serious revision.

2. Optical Depth to Microlensing from Prolate Halos

The dark Galactic halo is modelled as a cored isothermal spheroid (see, e.g., Sackett & Gould 1993). The density profile is given by

\[ \rho(R, z) = \frac{v_\infty^2}{4\pi G} \frac{1}{R_c^2 + R^2 + z^2/q^2} f(q) \] (1)

where \( z \) is the distance from the equatorial plane, \( R \) is the distance from the \( z \) axis, \( R_c \) is the core radius, and

\[ f(q) = \begin{cases} \frac{\sqrt{1-q^2}}{q \arccos q} & q < 1 \\ 1 & q = 0 \\ \frac{\sqrt{q^2-1}}{q \cosh^{-1} q} & q > 1 \end{cases} \] (2)

\( v_\infty \) is the asymptotic circular speed due to the halo alone. The tightest constraints on \( v_\infty \) come from the local circular speed \( v_c \). \( v_c \) receives nontrivial contributions from the disk and bulge and therefore \( v_\infty \) is model dependent. We use \( v_\infty = 200 \text{ km/sec} \) for the standard Galactic model. Lower values for \( v_\infty \) require a heavier disk and/or bulge but are still allowed by the observations.

The optical depth \( \tau_{\text{LMC}} \) is defined as the probability that the light from a star in the LMC will be amplified via gravitational microlensing by a factor \( A \geq 1.34 \). For this to occur, a Macho must pass within a distance \( R_E = (4GML(D - L)/c^2 D)^{1/2} \) of the line of sight to the star (eg. Paczynski
1986, Griest 1991). Here $D$ and $L$ are the distance to the background star and Macho respectively, $M$ is the mass of the Macho, and $R_E$ is the Einstein radius. The optical depth is given by

$$
\tau_{\text{LMC}} = \int_0^D dL \frac{4\pi GL(D - L)}{c^2 D} \rho(L)
$$

where $\rho(L)$ is the mass density in Machos along the lines of sight to the LMC. The optical depth is given by

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where $\rho(L)$ is the mass density in Machos along the lines of sight to the LMC. The uncertainties in the optical depths allowed by the MACHO group’s data are difficult to pin down, but following Gates, Gyuk & Turner 1995, we take $2.0 \times 10^{-7}$ as a 2σ upper bound (95% confidence level) of $\tau_{\text{LMC}}$.

Figure 1 gives $\tau_{\text{LMC}}$ as a function of $q$ for $R_c = 5 \text{kpc}$ and three values of $v_\infty$, with the sun taken to be 8.5 kpc from the galactic center. The physical effects at work are easy to understand. As the halo becomes more oblate the constraint that the rotation curve remain largely unchanged requires an increase in the density of the halo by the factor $f(q)$. This tends to increase the optical depth. At the same time our line of sight to the LMC is passing through less of the halo and so there is a decrease in the optical depth. These two factors lead to the turnover in $\tau_{\text{LMC}}$ with the maximum occurring for $q \simeq 0.4$ depending on the value of $v_\infty$. No “reasonable model” ($0.4 \lesssim q \lesssim 2.5$) is consistent with $v_\infty = 200 \text{km/sec}$. However an all-Macho E6 prolate halo can fit the observations with $v_\infty = 160 \text{km/s}$. This value for $v_\infty$ is comfortably allowed by observations of our Galaxy provided one assumes appropriate masses for the for disk and bulge. Note that if we limit ourselves to oblate halos then we must choose $v_\infty \lesssim 135 \text{km/sec}$ which is extremely low though not entirely ruled out.

The MACHO group (Alcock et al. 1995a,b) argue that the LMC microlensing results provide a model-independent constraint on the mass in Machos within 50 kpc of the Galactic center. To be precise, it is $v_\infty$ that is constrained. The small variance in the optical depth across the range of oblate models means that the only way to reduce the optical depth is to reduce $v_\infty$. That is, for a given optical depth, there is a very small spread in $v_\infty$ that is allowed. This is what is meant by a model-independent constraint on the circular speed due to Machos. However, if the range of models is extended to include prolate models, then this constraint is no longer model-independent. For a given optical depth, there can now be a range of flattenings allowed corresponding to different models of the Galaxy each with a different $v_\infty$.

### 3. Expected Number of Events

It has been pointed out (Griest 1991) that $N_{\text{exp}}$, the expected number of events for their experiment, is more important than the optical depth when comparing predictions with observations. $N_{\text{exp}}$ is given by

$$
N_{\text{exp}} = E \int_0^\infty \frac{d\Gamma}{d\tilde{t}} \epsilon(\tilde{t}) \, d\tilde{t},
$$

where $\tilde{t}$ is the event duration, $\epsilon(\tilde{t})$ is the detection efficiency, and $E$ is the total “exposure” given in units of “star yr”. $d\Gamma/d\tilde{t}$ is the differential event rate where the total event rate $\Gamma$ has units “events/star/yr”. $\Gamma$ depends on the velocity distribution and mass function of the Machos as well as their mass density. For simplicity we assume that all of the Machos have the same mass $M$. In addition we assume a Maxwellian velocity distribution centered on $v_c = 200 \text{km/sec}$ so that the distribution function $f(r, v) \propto \rho(r) \exp \left(-v^2/v_c^2\right)$. We then have (Griest 1991, Alcock et al. 1995b)
\[
\frac{d\Gamma}{dt} = \frac{32}{\ell^4 m v^2} \int_0^D \rho(L) R_E^4(L) \exp\left(-\frac{4R_E^2(L)}{\ell^2 v^2}\right) dL. \tag{5}
\]

Figure 2 is a plot of \(N_{\text{exp}}\) as a function \(M\) for three models: E6 oblate, E0, and E6 prolate, using this Maxwellian distribution. A rough fit to the published expected efficiencies of the MACHO collaboration (Alcock et al. 1995a) is used for \(\epsilon(\hat{t})\) and all other parameters are the same as for Figure 1. It can be seen that the oblate(E6) and spherical models predict nearly identical event rates, while the predicted event rate for a prolate (E6) halo is substantially reduced. More importantly, the excluded range (i.e. \(N_{\text{exp}} \geq 7.7\)) has been reduced such that an all-Macho halo consisting of brown dwarfs is not ruled out.

The Maxwellian distribution described above does not satisfy the Vlasov equation and so technically cannot describe a realistic galaxy. Recently several authors have attempted to address this problem by using the so-called power-law models for the Machos. When taken alone these model halos, which are constructed from simple power-law functions of the energy and angular momentum, are guaranteed to be solutions of the Vlasov equation. This is no longer the case when the other components of the galaxy are included. It has been pointed out that at a distance halfway to the LMC the disk contributes less than 4% to the circular velocity (Evans & Jijina 1994). However statistically speaking well over half of the lensing events are likely to occur at a closer distance (e.g. Kan-Ya et al. 1995, Roulet al 1994). The 4% figure therefore represents a lower limit to the error incurred in neglecting the disk. The net effect is to underestimate systematically the velocities of the Machos. This leads to a lower than expected event rate along with a shift towards longer timescales. Figure 12 of Alcock et al. 1995b shows \(d\Gamma/d\hat{t}\) for the standard halo with a Maxwellian velocity distribution along with the corresponding spherical power-law halo. The power-law halo has a slightly lower event rate along with a shift towards longer timescales as expected from the discussion above. For an oblate halo the lenses are, on average, closer to us than for the spherical case and so the error that comes from neglecting the disk is even greater. This explains quantitatively Figure 2 of Evans & Jijina (1994) where the lensing rate for an E6 halo appears to be slightly lower and shifted to a longer timescale as compared to a spherical halo.

The great advantage of the power-law halo models is that the distribution functions are analytic making lensing calculations relatively simple. There are however indications that they may not correspond to realistic halos. In particular, while the equipotential surfaces are spheroidal, the density profiles are dimpled at the poles for \(q \lesssim 0.85\) (Evans 1993). Models which correctly take into account the gravitational couplings between disk, bulge, and halo have recently been constructed (Kuijken & Dubinski 1995). In addition these models have more reasonable density profiles. The models are not analytic and so lensing calculations are more complicated (though still straightforward). Moreover the range in \(q\) for these models is fairly limited and only includes a small fraction \((q \lesssim 1.1)\) of the prolate halos. It is for these reasons that we have taken the simplest approach wherein velocities are Maxwellian.

\section{4. Conclusions and Discussion}

Our primary result is that the conclusions one draws from the published data on microlensing towards the LMC depend sensitively on the model one chooses for the Galactic halo. In particular, an all-Macho prolate halo is still consistent with the data. Conversely, a spherical or oblate all-Macho halo is all but ruled out. Few theorists or observers believe that halos are prolate. However
a careful review of the evidence cited in favour of flattened halos suggests that a prolate spheroid is still a viable model for our Galaxy’s halo.

It should be stressed that the reduction in the predicted microlensing rates for model prolate halos is independent of the shape of the rotation curve. That is, for a given rotation curve, an oblate or spherical model halo will have a higher density of Machos and therefore a higher predicted microlensing event rate. Even if the shape of the Galactic rotation curve were known precisely, the fraction of dark matter composed of Machos, as determined from experiments like MACHO and EROS, would remain uncertain until more information about the shape of our halo could be obtained.

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Figure Captions

Figure 1: Plot of optical depth to the LMC, $\tau_{LMC}$ vs. flattening parameter $q$ for $v_{\infty}$=200,160,135 km/s. The solid line at $\tau_{LMC} = 2 \times 10^{-7}$ represents a $2\sigma$ upper bound from observations.

Figure 2: Plot of expected number of events, $N_{exp}$ vs. mass of lenses for three halo models: E0, E6 oblate and E6 prolate.
