Formation of cosmological brane defects

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Abstract

We study cosmological formation of D-term strings, axionic strings, domain walls and Q-balls in braneworld models of the Hanany-Witten type. For the D-term strings, we show that the strings are the daughter branes extended between mother branes. We show that the domain walls can be produced by conventional cosmological phase transitions. In this case, the formation of the domain walls is induced by the continuous deformation of the branes, which means that they are not created as daughter branes. First we consider classical configurations of the axionic strings and the domain walls, then we investigate the quantum effect of the brane dynamics. We also study brane Q-balls and show how they can be distinguished from conventional Q-balls.
1 Introduction

Although there were great successes in quantum field theory, we still have no consistent scenario in which the quantum gravity is included. The most promising scenario in this direction will be string theory where the consistency is ensured by the requirement of additional dimensions and supersymmetry. The idea of large extra dimension\[1\] may solve or weaken the hierarchy problem. In this case, denoting the volume of the \(n\)-dimensional compact space by \(V_n\), the observed Planck mass is obtained by the relation 
\[
M_p^2 = M_*^{n+2}V_n,
\]
where \(M_*\) denotes the fundamental scale of gravity. The standard model fields are expected to be localized on a wall-like structure, and the graviton propagates in the bulk. The natural embedding of this picture in the string theory context is realized by a brane construction. Inflation with such a low fundamental scale is still an interesting topic\[2, 3, 4\]. Other cosmological issue such as baryogenesis with low fundamental scale is discussed in ref.\[5, 6, 7\], where cosmological defects play important roles. Constructing models for the particle cosmology where non-static brane configurations (such as brane defects and Q-balls\[8\]) are very important. We are expecting that future cosmological observations will reveal the evolution of the Universe, which might also reveal the physics beyond the standard model. To know what kind of brane defects are allowed in the evolution of the Universe, we need to understand how they are formed (and disappeared) in the history of the Universe. In the original scenario for brane inflation\[9\], the inflationary expansion is driven by the potential between branes and anti-branes evolving in the bulk space of the compactified dimensions. The end of inflation is induced by the brane collision where the brane annihilation proceeds through tachyon condensation\[10\]. During brane inflation, tachyon is trapped in the false vacuum. Then the tachyon starts to condensate after inflation, which may result in the formation of the daughter branes.

The production of cosmological defects after brane inflation is discussed in ref.\[11, 12\], where it is concluded that cosmic strings are copiously produced but the domain walls are negligible. In ref.\[13\], however, it is discussed that all kinds of defects can be produced and the conventional problems of cosmic domain walls and monopoles should arise. Later in ref.\[14, 15, 16\], the brane production is reexamined and the conclusion was different from \[11, 12\] and \[13\]. In ref.\[11, 12\], it is discussed that the effect of compactification
is significant for the defect formation due to tachyon condensation. It must be useful to make a brief review of the previous arguments about the cosmological formation of brane defects. Their argument is that since the compactification radius is small compared to the horizon size during inflation, any variation of a field in the compactified direction is suppressed. Then the daughter brane wraps the same compactified dimensions as the mother brane. As a result, the codimensions of the daughter branes lie within the uncompactified space. Since the number of the codimension must be even, the defect is inevitably a cosmic string. Moreover, in ref.[14], it is pointed that the analysis does not fully account for the effect of compactification, since the directions transverse to the mother brane is not considered. The effect of the RR fields extended to the compactified dimensions is discussed in ref.[14]. The result is that the creation of the gradients of the RR fields in the bulk of the compactified space is costly in energy, so that the creation of the daughter brane is suppressed if it does not fill all the compactified dimensions. In this case, it was concluded that the production of cosmic strings requires efficient mechanisms, and monopoles and domain walls are not produced after brane inflation.\(^2\)

In this paper, however, we show explicit examples where the production of cosmic strings is realized by the formation of daughter branes that are extended between splitting mother branes. It should be noted that we are not considering a counter example of the mechanism of tachyon condensation. The problem of the RR field is avoided, since the length of the extended daughter brane vanishes when it is formed. The tension of the fully extended daughter brane matches to the tension of the D-term string in the effective Lagrangian.\(^3\) Moreover, we also show that other cosmological defects, such as domain walls and Q-balls, can be produced after brane inflation. In our model, it is natural to

\(^2\)Another kind of defects, which are parameterized by the positions of the branes, were constructed in ref.[17] and later in ref.[18]. In ref.[17], brane is replaced by a domain wall that is embedded in the higher-dimensional spacetime, so that one can see what happens in the core. Then the position of a brane in the fifth dimension is used to parameterize the cosmic string in the effective four-dimensional spacetime. The brane is shown to be smeared in the core, so that it resolves the anticipated singularity. Then in ref.[18], the relative positions between branes are used. In ref.[18], these defects are called incidental brane defects. In these field-theoretical constructions, branes are replaced by domain walls or vortices embedded in the higher-dimensional spacetime.

\(^3\)In our next paper[19], we consider another type of angled brane inflation and solve the problem of the $\theta$-dependence of the string tension.
think that the domain walls and the Q-balls are not produced by the brane creation.\footnote{Of course it is not impossible to think that such domain walls are the daughter branes being extended between vacuum branes. In our case, however, the cosmological evolution of the brane configuration suggests that they are not produced by the creation of daughter branes. Cosmological formation of domain walls and monopoles, which is induced by the creation of daughter branes, will be discussed in ref.\cite{20}. In this case, the production of the branes extended between branes is crucial.} Domain walls are corresponding to the spatial deformations of the vacuum branes, which can be formed by the thermal effect\cite{21} or brane oscillation after inflation. Thermal effect can induce attractive forces between branes. Then the observer in the four-dimensional spacetime sees the restoration of the corresponding symmetry. The spontaneous breaking of the symmetry triggers the formation of the cosmological defects, which is parameterized by the relative position between branes.

In Section 2, we begin with a short review of a model\cite{22} for brane inflation due to the Hanany-Witten\cite{23} type brane dynamics. We show how the extended branes are produced by the brane dynamics after brane inflation. Although our result seems to be contradicting to the previous arguments about daughter brane production, we stress that we are not considering a counter example of the mechanism of tachyon condensation. We think it is not difficult to understand how one can avoid the serious criteria given in ref.\cite{11, 14}. The extended branes are formed after brane inflation. The correspondence between the extended brane and the D-term string in the effective action is examined. Then, the formation of axionic strings and domain walls is discussed in Section 3. Unlike the D-term strings, these defects are formed by the spatial deformations of the vacuum branes. The domain wall that we are considering in this paper is different from the usual BPS domain walls\cite{24} in SQCD and MQCD. In the effective action, we add a soft mass for the adjoint scalar field in $N = 2$ SYM, which introduces a shallow potential on the coulomb branch. In the brane counterpart, we are considering $D4$-branes separated by a weak repulsive force between them. Thus our defect configurations are not stable in the supersymmetric limit. Our discussions in this paper compensate the analysis in ref.\cite{17} and \cite{18}, in which defects were constructed in the classical brane configurations. It should be noted that the conventional BPS domain walls in MQCD are not suitable for our argument, because they cannot exist in the classical limit. We also consider brane Q-ball\cite{8}, which is the configuration of branes in motion. Our conclusions and discussions
are given in Section 4.

## 2 D-term string produced after brane Inflation

In this section, we discuss the formation of D-term strings. We show explicitly why it is possible to produce daughter branes that are extended between mother branes.

### 2.1 4D effective Lagrangian and D-term strings

The form of the 4D hybrid potential for the P-term inflation model is

$$
V = \frac{g^2}{2} \left[ \left( |\Phi_1|^2 + |\Phi_2|^2 \right) |\Phi_3|^2 + |\Phi_1|^2|\Phi_2|^2 + \frac{1}{4} \left( |\Phi_1|^2 - |\Phi_2|^2 + \frac{2\xi}{g} \right)^2 \right] \quad (2.1)
$$

Here the two complex scalar fields $\Phi_1$ and $\Phi_2$ form a quaternion of the hypermultiplet, which is charged under the $U(1)_X$ group. The complex scalar field $\Phi_3$ appears from the $N = 2$ vector multiplet. $\xi$ is the Fayet-Iliopoulos (FI) term for the $U(1)_X$ symmetry. When the vacuum expectation value (vev.) of $|\Phi_3|$ is large, the vev of the hyper multiplets vanish and the $|\Phi_3|$ direction becomes flat. In this case, $\Phi_3$ is the inflaton of hybrid inflation, whose potential is lifted by the logarithmic corrections. The trigger field is $\Phi_2$, which roll toward true vacuum and reheat the Universe at the end of inflation.

The D-term strings are formed at the end period of inflation. Note that $\Phi_3$ takes large value during inflation, which stabilizes the potential for $\Phi_1$ and $\Phi_2$ to make these fields to stay at their origin. At the end of inflation, when $\Phi_3$ becomes smaller than a critical value, the potential is destabilized and the trigger field starts to roll down the potential. The D-term string is formed at the end of inflation, by the spontaneous breakdown of the $U(1)_X$ symmetry. It is easy to calculate the scaling property of the tension of the D-term strings from the potential in eq. (2.1), which becomes

$$
T_{D-string} = \frac{2\pi \xi}{g}. \quad (2.2)
$$

The BPS conditions for the D-term strings are examined in [26], in which the D-term strings are shown to satisfy the BPS conditions in supergravity.
2.2 D2 string in NS5-D4/D6-NS5 model

The D-term strings that are produced after P-term inflation have the tension of the form (2.2), and satisfy the BPS conditions. The possible brane counterpart is the D2-brane, which is extended between the split D4-branes.\(^5\) From the viewpoint of the effective Lagrangian, the production of the D-term strings after inflation is not suppressed. However, as is noted in ref.\([11, 12, 14]\), the effect of compactification seems to be significant for the defect formation due to tachyon condensation. Since the compactification radius should be small compared to the horizon size during inflation, any cosmological variation of a field in the compactified direction must be suppressed. If there is no other mechanism that induces variation of the field, the daughter brane must wrap the same compactified dimensions as the mother brane. As a result, the codimensions of the daughter branes seem to lie within the uncompactified space. Moreover, in ref.\([14]\), it is pointed that the analysis does not fully account for the effect of compactification, since the directions transverse to the mother brane is not considered. Then in ref.\([14]\), the effect of the RR fields that are extended to the compactified dimensions is discussed. Their conclusion is that the creation of the gradients of the RR fields in the bulk of the compactified space is costly in energy, so that the creation of the daughter brane is suppressed even if they wrap the same compactified dimensions as the mother brane.

Is it really impossible to produce extended branes by the daughter brane production? Let us see more detail. If the initial perturbation of the tachyon condensation produces the seed for the $D2$ branes on the world volume of the mother brane, it might wrap the same compactification space as the mother brane. The dotted line in the left picture in fig.2 denotes the “seed” for the $D2$ brane. To be more precise, a careful treatment of the effective action\([27]\) shows that the eigenfunction of the tachyonic mode is localized at the intersection. Since the mechanism of this localization is different from the Kibble mechanism, the “seed” for the $D2$ brane can be localized at the intersection. From the string perspective, it seems obvious that the brane creation is due to the local dynamics of the open strings at the D4-D6-D4 intersection of the splitting D4 branes. As the recombination proceeds, the $D2$ brane is pulled out from the mother $D4$ brane, and

\(^5\)See fig.1
finally becomes extended between split mother branes. In this case, the problem of the RR field is avoided since the length of the extended $D2$ brane vanishes at the time when it is pulled out from the mother brane. Of course, it costs energy to pull $D2$ branes out from the $D4$ branes, however in this case the cost is paid by the repulsive force between the splitting $D4$ branes. Our conclusion is consistent with the analysis of the effective action, where the production of the D-term strings is not suppressed.

Let us examine if the tension of the fully extended brane matches to the result obtained from the effective Lagrangian. The brane splitting in the brane dynamics corresponds to the spontaneous breaking of the $U(1)_X$ symmetry in the 4D effective Lagrangian. We take the distance $\Delta L$ as in fig.1. Then the tension of the string that corresponds to the extended $D2$-brane becomes

\[ T_{D2} = \frac{\Delta L}{g_s (2\pi)^2 \alpha'^2}. \]  

(2.3)

Here $g_s$ and $l_s$ are the string coupling and the string length. We have defined that $\alpha' = l_s^2$. Following ref.[22], the gauge coupling constant in the effective four-dimensional Lagrangian is given by the formula,

\[ g^2 = (2\pi)^2 g_s l_s L. \]  

(2.4)

where $L$ is given in fig.1. As a result, the tension of the string in the effective action, which corresponds to the D2-brane, is given by

\[ T_{D2} = \frac{\Delta L}{\alpha' g^2 L}. \]  

(2.5)

The above result matches to the tension of the D-term string, since the Fayet-Iliopoulos term in the effective Lagrangian is given by[22]

\[ \xi = \frac{\Delta L / L}{g^2 2\pi \alpha'}, \]  

(2.6)

where $\Delta L \ll L$ is assumed. Thus we can confirm that the tension of the D-term string matches to the extended D2-brane, $T_{D2} = 2\pi \xi / g = T_{D-string}$. 
3 Domain walls, axionic strings and brane Q-balls from brane dynamics

The defect formation in the brane world does not always correspond to the production of the lower-dimensional branes. For example, the position of a brane in the fifth dimension can be used to generate cosmic strings\textsuperscript{[17]}. Later in ref.\textsuperscript{[18]}, it is shown that the relative position of the branes in the higher-dimensional bulk space can fluctuate to form such brane defects. Explicit examples for domain walls, strings and monopoles are shown in ref.\textsuperscript{[18]}. These defects are formed by the deformations of the vacuum branes. In ref.\textsuperscript{[17]}, the branes are replaced by the domain walls that are embedded in the higher-dimensional spacetime, so that one can see what happens in the core. Then it is shown that the branes are smeared in the core, so that the anticipated singularity is resolved. Therefore, the field-theoretical construction is useful to understand the inner structure of these defects. However, if one considers the field-theoretical construction, it is quite difficult to include the quantum effect of the brane dynamics. Therefore, it is important to formulate these types of brane defects in the Hanany-Witten type brane dynamics, so that we can investigate the quantum effect of the brane dynamics. We think it is easy to understand that these defects are produced during the usual cosmological evolutions of the Universe. Although it is not impossible to regard that the domain walls are the daughter branes being extended between mother branes, from the cosmological viewpoint these domain walls are formed by the spatial deformations of the vacuum branes during the usual cosmological evolution of the Universe.

First, we briefly describe our basic ideas for the incidental brane defects in the classical brane configuration. We consider parallel branes of codimension 2, and assume that there is a weak repulsive force between them. We also assume that the potential for the distance between branes is stabilized at a distance.\textsuperscript{6} Then we can consider a configuration in which the branes are located on a circle, as is depicted in fig.\textsuperscript{8}. Here we start from the classical arguments. Seeing fig.\textsuperscript{8} it seems natural to consider the following configurations.

- “Domain wall that interpolates between the two degenerated vacua.”

\textsuperscript{6}The origin of the repulsive force is a weak supersymmetry breaking. The stabilization at a distance is possible if there are higher dimensional terms suppressed by the cut-off scale.
A domain wall is produced by the simple permutation between two branes. The permutation is shown in fig.4.

• “Strings being formed by the rotation of their relative positions.”
  The configuration is shown in fig.5.

• “Brane Q-balls”
  In this case, unlike the above setups for axionic strings and domain walls, the true vacuum is needed to be placed at the origin. The brane Q-ball is the configuration of the branes in motion, where the branes are rotating around each other.

In ref.[18], from the field-theoretical construction (i.e., the branes are constructed as the embedded defects in the higher-dimensional bulk), we have constructed several types of incidental brane defects. The brane Q-ball was discussed in ref.[8], in which the simple brane anti-brane system was considered. As we have discussed above, the field-theoretical construction is convenient to see the structure in the defect core. In ref.[17], the field-theoretical construction is used to show how the expected singularity in the core is resolved by the smearing brane. On the other hand, in the field-theoretical construction, it is difficult to examine the quantum effect, which is induced by the brane dynamics. Therefore, it is important to investigate the quantum effect of the brane dynamics for the configurations of these brane defects. In this section, we try to construct the above-mentioned defects in the setups of the Hanany-Witten type, where the quantum effect of the brane dynamics is shown to play important roles for cosmological strings and domain walls. Our arguments compensate the previous results in ref.[8, 17, 18]. Our classical setup of the Hanany-Witten model is depicted in fig.6.

Let us consider a global $U(1)$ symmetry, which is depicted in fig.5. The $U(1)$ symmetry is the classical symmetry of the effective four-dimensional action, but will be broken by the quantum effect. Therefore, if a global string is constructed by the $U(1)$ symmetry, it should turn out to be a junction (or boundary) of the domain walls. However, as far

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7One can expect that the flat potential is lifted by a soft mass. One can also consider a small $\mu$-term for the adjoint field.

8Although the basic idea for the brane Q-ball is not different from the previous discussions in ref.[8], the situation in this paper is more realistic than the simple brane anti-brane pair that is discussed in ref.[8]
as one is considering the classical configurations of the field-theoretical construction, it is quite difficult to examine the quantum effect. Another example is the domain wall that is induced by the permutation between branes. As far as we are considering only the classical brane configurations, the permutation between branes seems to generate another degenerated vacuum\cite{18}. In this section, however, we show that such permutations become trivial if the quantum effect of the brane dynamics is properly included.\footnote{From the viewpoint of the effective four-dimensional action, the triviality of the permutation is originated from the gauge fixing, which is unclear in its classical counterpart of the brane dynamics. See appendix for more detail.} Then we discuss about brane Q-balls. In the usual four-dimensional theory, one can construct Q-balls if

- There is a flat potential that is lifted by a small perturbation.
- There is an effective $U(1)$ global symmetry.

Our model satisfies the above criteria. The brane Q-balls are conceptually different from the conventional Q-balls\cite{8}. We show how one can distinguish the conventional Q-balls from the brane Q-balls.

We show that cosmological domain walls, axionic strings and Q-balls can be produced by the usual brane dynamics after brane inflation. We stress here that it is quite natural to expect cosmological formation of these defects in the conventional scenarios of the evolution of the Universe.

\section{3.1 Brane Dynamics}

The analysis of the four-dimensional effective action is discussed in appendix A. We pay attention to the discussions in this section so that they become parallel to the analysis in appendix A.

Here we consider brane configurations of the $3+1$ dimensional $N = 2$ supersymmetric Yang-Mills (SYM), and the brane counterpart of the cosmological defects. The starting point of our discussion is the brane configuration of type IIA string theory consisting of solitonic (NS) five-branes and $N_c$ D4-branes. Following ref.\cite{29}, we consider the object:

\begin{equation}
\text{NS5} \quad (x^0, x^1, x^2, x^3, x^4, x^5)
\end{equation}
\(D4 \ (x^0, x^1, x^2, x^3, x^6), \)  

(3.1)

which is schematically sketched in fig.6. Here the gauge coupling of the 3 + 1 dimensional gauge theory is given by:

\[
\frac{1}{g^2} = \frac{L}{g_s l_s}. \tag{3.2}
\]

If supersymmetry is not broken, there are no forces between D4-branes, and the field that parameterizes the brane distance represents the flat direction of the effective four-dimensional Lagrangian. However, in more realistic situations, supersymmetry must be broken. Here we consider a simple assumption that a tiny breaking of supersymmetry induces weak repulsive force between the D4-branes. In this case, the thermal effect can stabilize the potential between branes, which induce the restoration of the corresponding symmetry in the four-dimensional effective action\[^{[18, 21]}\]. After symmetry breaking, D4-branes are placed on a circle, as is depicted in fig.3 \(^{[10]}\) The elements (a_i’s) in the effective theory correspond to the position \((x^4, x^5)\) of the endpoints of the D4-branes on NS5.

Let us first consider a naive permutation between the two D4-branes. We stress here that it is important to understand why the naive permutations in the classical configuration (see fig.4) do not induce any physical domain walls. It should be noted that it is rather hard to understand the reason from the classical configuration. As we have discussed for the effective action, the quantum effect is described by the deformation of the moduli space. The vacuum of the MQCD coulomb branch\[^{[29]}\] is represented by the curve, which takes the same form as eq. (A.7). From the viewpoint of the brane dynamics, the curve represents a M5-brane. In this case, one can see that the above-mentioned permutation in the classical configuration becomes a trivial symmetry of the quantum configuration, which works on the two parts of the identical M5-brane. This result is a triumph of MQCD, as it cannot be obtained from the classical argument of the brane dynamics. Moreover, as is already discussed in ref.\[^{[24]}\], defects are the important probes for the examination of the consistency between SQCD and MQCD. In our case, since the results obtained from the quantum brane dynamics are consistent with SQCD for the axionic strings and the domain walls, one can understand that our results present alternative proofs for the consistency between SQCD and MQCD in a rather exotic situation.

\[^{10}\text{Here we have assumed that the higher dimensional terms stabilize the potential at a distance, as we have considered in the discussion about four-dimensional theory in appendix A.}\]
Although the permutations do not induce any physical domain wall, the axionic string and $Z_{N_c}$ domain walls are still the physical objects of the quantum brane dynamics.\textsuperscript{11} We have included a small soft mass that destabilizes the coulomb branch, which is so small that it does not ruin the equations for the curve. The classical $U(1)_R$ symmetry is broken by curving the left and the right fivebranes to

\begin{align}
  s_L &= -N_c R_{10} \log v \\
  s_R &= N_c R_{10} \log v,
\end{align}

where $s = x^6 + ix^{10}$, and $R_{10}$ is the radius of the compactified $x^{10}$. Because of the fact that $\text{Im} s = x^{10}$ lives on a circle of radius $R_{10}$, there is the residual discrete symmetry $Z_{N_c}$. The domain walls are produced by the spontaneous breaking of $Z_{N_c}$. Similar to the usual cases, our axionic strings are formed at the scale where the spontaneous breaking of the $U(1)_R$ symmetry is induced by $\langle \phi \rangle \neq 0$. Then a dynamical potential is generated at lower energy scale, which breaks $U(1)_R$ to the discrete symmetry. At this time, the axionic string becomes a junction (or boundary) of the $Z_{N_c}$ domain walls.\textsuperscript{12}

Here we should note how one can circumvent the criteria given in ref.\textsuperscript{[11, 12, 14, 15, 16]}, in which the formation of cosmological domain walls is discussed to be negligible. In our case, although it is not impossible to think that the domain walls are the daughter branes being extended between vacuum branes, cosmological consideration suggests that they are formed by the continuous deformations of the vacuum branes, therefore they are not produced by tachyon condensation. The actual construction of the $Z_{N_c}$ domain walls is straightforward. Since the supersymmetry breaking is weak in this case, thermal effect can stabilize the potential between branes. At the beginning when the temperature is higher than the dynamical scale, the branes are placed on top of each other. The corresponding gauge symmetry is restored in the effective action. Then the branes start to fall apart at low temperature, with spatial fluctuations of their positions as is anticipated by the conventional Kibble mechanism. The domain walls are described by the fivebranes that interpolate between the two adjacent vacua. Let us consider a domain wall, which looks

\textsuperscript{11} Unlike the usual BPS domain walls in $N = 1$ SQCD, these defects are unstable in the supersymmetric limit. On the other hand, the usual BPS domain walls cannot appear in the classical configuration.

\textsuperscript{12} Cosmological considerations of the stability of the usual BPS domain walls are already discussed in ref.\textsuperscript{[11]}. 
like one vacuum of the theory for \( z \to -\infty \) and looks like another vacuum for \( z \to \infty \).

Here we denote these two vacua by the index “A” and “B”, and \( z \) is one of the three spatial coordinates. Then the domain wall is described by the fivebrane that has two adjacent vacua on its boundaries, morphing one to the other along the \( z \) direction. If the vacuum states were described by the fivebranes of the form \( R^4 \times \Sigma \), where \( \Sigma \) was a Riemann surface embedded in the extra dimensions \( Y \equiv R^5 \times S^1 \), the domain walls are described by the fivebranes of the form \( R^3 \times D \), where \( R^3 \) is the four-dimensional spacetime without \( z \), and \( D \) is a three-surface in the seven manifold \( Y' \equiv R_z \times Y \), where \( R_z \) is the copy of the spatial \( z \) direction. Near \( z \to -\infty \), \( D \) should look like \( R_z \times \Sigma_A \). On the other side, near \( z \to \infty \), \( D \) should look as \( R_z \times \Sigma_B \). The defect is described by the continuous deformation of the existing M-theory fivebrane, morphing from one side to the other along the \( z \) direction. The cosmological formation of such defects is already discussed in ref. [18], by using the field-theoretical construction.

As we have stated above, in addition to the conventional brane defects that are formed by brane creation, one should consider another kind of brane defects that are formed by the continuous deformations of the branes if one wants to consider generic cases of cosmological scenarios. Since the two kinds of brane defects can be produced by the same process, one must deal with the mixture of these defects in the actual analysis of the brane Universe.

Finally, we consider the brane Q-balls in the Hanany-Witten type brane dynamics. Conventional Q-balls in the four-dimensional theories are already discussed by many authors [30]. A brane-counterpart of the Q-ball is discussed in [8] for brane anti-brane pair. Here we consider the brane Q-balls in the Hanany-Witten type brane dynamics. Our discussions are parallel to the analysis of the effective action in appendix A. Unlike the above discussions for axionic strings and domain walls, since the brane Q-balls are constructed deep inside the classical region, the configuration of the brane Q-ball is not affected by the quantum effect. However, as is discussed in [8], there is a crucial difference between Q-balls in the effective action and brane Q-balls. In the case of brane anti-brane pair, the decay mode of the brane Q-ball is dominated by the radiation into the bulk when the charge of the brane Q-ball exceeds a critical value. In the followings, we derive the critical charge in the Hanany-Witten type brane dynamics. We assume that
the supersymmetry breaking is dominated by the conventional supergravity mediation at large $< \phi >$. In the cases of the gravity mediation, the potential is schematically given by$^{13}$

$$V(\phi) = m_{3/2} |\phi|^2 \left(1 + K \log \frac{|\phi|^2}{M_*^2}\right).$$

(3.4)

Then the Q-balls have the properties,

$$R_Q \simeq |K|^{-1/2} m_{3/2}^{-1}, \quad \omega \simeq m_{3/2}$$

$$|\phi_Q| \simeq |K|^{3/4} m_{3/2} Q^{1/2}, \quad E_Q \simeq m_{3/2} Q,$$

(3.5)

When the branes are rotating, the acceleration of the rotating branes in the bulk (which is denoted by $a_b$) generates the radiation into the bulk of the form$^{32}$,

$$\left|\frac{dE_Q}{dt}\right| \simeq \frac{1}{8\pi} (\kappa_4 T_{D4} V_{D4})^2 a_b^2,$$

(3.6)

where $\kappa_4$ and $V_{D4}$ are the four-dimensional gravitational coupling and the spatial volume of the D4-brane, which becomes $V_{D4} \simeq \frac{4\pi}{3} R_Q^3 \times L$ in our model. Here $L$ is the length of the D4-brane in the compactified direction. $T_{D4}$ is the tension of the D4-brane, which is given by the usual formula, $T_{D4} \simeq M_5^5$. Here we consider the acceleration of the rotating brane that is given by $a_b \simeq (|\phi_Q|/M_*) \omega^2$. On the other hand, the conventional decay mode has been studied in ref.$^{33}$, which is given by;

$$\left|\frac{dQ}{dt}\right| \leq \frac{\omega^2 4\pi R_Q^2}{192\pi^2}.$$  

(3.7)

From eq.(3.5) and eq.(3.7), one can obtain;

$$\left|\frac{dE_Q}{dt}\right| \leq \frac{m_{3/2} \omega^4 4\pi R_Q^2}{192\pi^2}.$$  

(3.8)

From eq.(3.6) and (3.8), we can derive the critical charge

$$Q_c \simeq 10^{-2} \times \frac{K^{1/2} m_{3/2}^2}{\kappa_4^2 L^2 M_*^6}.$$  

(3.9)

The above result suggests that the decay of the brane Q-ball is always dominated by the efficient radiation into the bulk. The existence of the critical charge is crucial in the...
analysis of realistic cosmological models. If the actual gauge symmetry of the Universe is
due to the brane dynamics of the Hanany-Witten type, Q-balls should have their brane
counterpart, which **can be distinguished from the conventional Q-balls even if**
the **four-dimensional effective action looks the same**.

Since we are considering large Q-balls, we should mention the relation between the
scales of the compactified space and large $\phi_Q$. In the above example, the gravitino mass
is given by $m_{3/2} \simeq \Lambda_{\text{susy}}^2/M_p$, where $\Lambda_{\text{susy}}$ denotes the scale of the supersymmetry break-
ing. Although the charge of the Q-ball becomes quite large in conventional cosmological
scenarios, the distance between rotating branes, which is denoted by the scalar field $\phi_Q$,
do not exceed the compactified radius. In this case, the suppression factor of $m_{3/2}$ in
eq(3.5) is crucial. Therefore, because of the factor $m_{3/2}$ in eq.(3.5), we can safely embed
the configuration of brane Q-balls in the above brane model.

4 Conclusions and Discussions

In this paper, we have considered the formation of D-term strings, axionic strings, domain
walls and Q-balls in the Hanany-Witten type brane dynamics. Here we summarize our
conclusions.

- For the D-term string, we have considered D2-brane that is stretched between the
splitting D4-branes. Contrary to the previous arguments, the production of the
extended D2-branes is not suppressed. Our arguments are general, because the
brane collision with a huge kinetic energy will inevitably induce chaotic process of
the production/annihilation and the recombination of the branes, which makes it
possible to produce many kinds of extended daughter branes. Further discussions
of this topic is given in ref.\[19, 20].

- We have considered the production of axionic strings and domain walls. In our case,
the defects are not produced by the tachyon condensation but are formed by the
spatial deformations of the mother branes. The parameter of the deformation is the
position of the branes in the compactified space. These defects are first constructed
in the classical brane configuration, and then lifted to MQCD. We have shown
that quantum effect is crucial for axionic strings and domain walls. On the other hand, since the defects are the non-trivial excitations of the system, they are also important in examining the consistency between SQCD and MQCD [24].

- We have discussed brane Q-balls in the Hanany-Witten brane dynamics. We have found that there is a distinguishable difference between brane Q-balls and conventional Q-balls.

In our future work [20], it is shown that monopoles and domain walls can be produced by daughter brane creations, which are extended between mother branes. For the cosmic strings, in our next paper [19], the consideration of the extended daughter brane is used to solve the long-standing problem of the $\theta$-dependence. **In either case, the production of the extended brane is crucial for the cosmological defect formation.** We show in ref. [20] that the cosmological process that is required for the creation of the monopoles and the domain walls is the same as the one required for the formation of incidental brane defects. **Then, the actual cosmological relics after brane inflation are the mixture of the two kinds.** The cosmological evolution of such brane defects is quite interesting and deserves further discussions.

5 Acknowledgment

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A Four-dimensional $N=2$ SYM and coulomb branch

$N = 2$ supersymmetric gauge theory with the gauge group $SU(N_c)$ is written by $N = 1$ supersymmetric vectormultiplet and a chiral superfield in the adjoint representation [29].

$$\mathcal{L}_{vec} = \text{Im} \text{Tr} \left[ \tau \left( \int d^4\theta \Phi^\dagger e^{-2V} \Phi + \int d^2\theta W_\alpha W^\alpha \right) \right]$$

where the trace runs over the gauge group, and $\tau = \frac{\theta}{2\pi} + \frac{i}{g^2}$ is the complex coupling. In components, the bosonic part of the superfield $\Phi$ includes the potential

$$V \sim Tr[\phi^\dagger, \phi]^2,$$
which has flat directions. The Lagrangian (A.1) is invariant under the $U(1)_R$ symmetry $\Phi \to e^{2i\alpha_R} \Phi (e^{-i\alpha_R} \theta)$ that is a consequence of the classical conformal invariance. Here we assume that the Fayet-Iliopoulos D-term is zero. The vacuum expectation value of the complex scalar field $\phi$, which is the lowest component of the adjoint superfield $\Phi$, can always be rotated by a gauge transformation, to lie in the Cartan subalgebra of $SU(N_c)$. Namely, one can always rotate the vev to be in the form

$$ < \phi > = \sum_{i=1}^{N_c} a_i H_i = \text{diag}[a_1, a_2, ..., a_{N_c}] .$$

(A.3)

Up to gauge transformation, the D-flatness condition $[\phi^\dagger, \phi] = 0$ is satisfied when

$$ \sum_{i=1}^{N_c} a_i = 0 .$$

(A.4)

The elements of the $SU(N_c)$, which act non-trivially on the Cartan subalgebra, are the elements of the Weyl group, isomorphic to the permutation group $S_{N_c}$. We assume that there is a mechanism that destabilizes the origin of the potential, and the elements are placed along a circle.\footnote{For example, one can assume that the supersymmetry is broken in a hidden sector, and the breaking of supersymmetry is induced by the soft term for the field $\Phi$,}

$$ V_{\text{soft}} = -\frac{1}{2} m_{\text{soft}}^2 \text{Tr}|\phi|^2 .$$

(A.5)

which destabilizes the coulomb branch. Because of the destabilized potential, $\phi$ develops large vacuum expectation value.
$U(1)_R \to Z_{4N_c}$. Since the superfield $\Phi$ is charged 2 under the $U(1)_R$ symmetry, the effective symmetry of the field $\phi$ is $Z_{2N_c}$. The spontaneous breaking of the discrete symmetry is the origin of the domain walls. However in the present model, the $N_c$ domain walls are produced by the spontaneous breaking of the discrete $Z_{N_c}$ symmetry, not by the discrete symmetry $Z_{2N_c}$. To see the mechanism of the domain wall formation, we briefly review the structure of the coulomb branch. We are assuming that the soft supersymmetry breaking is so weak that the usual analysis on $N = 2$ coulomb branch is still a good approximation. The curve $^{29}$ is described by,

$$t^2 + P_n(v)t + 1 = 0. \quad (A.7)$$

Here $P_n(v)$ is the polynomial of the form

$$P_n(v) = v^n + u_2v^{n-2} + \ldots + u_n, \quad (A.8)$$

where $u_i$'s are the order parameters of the theory, which are usually given by the formula $u_i = Tr \Phi_i$. For large $v$, the above equations behave like

$$t_{\pm} \simeq v^{\pm N_c}, \quad (A.9)$$

where $t_{\pm}$ denotes the two roots of $t$. The $U(1)_R$ symmetry rotates $v$ that has the $U(1)_R$ charge of 2. However, the remaining $Z_{2N_c}$ symmetry of $v$ is not the symmetry of $(A.9)$. Considering the discrete rotation of the form

$$v' \rightarrow ve^{\frac{2\pi i}{2N_c}k}$$

$$k = 1, 2, \ldots, 2N_c, \quad (A.10)$$

the $v'^{\pm N_c}/v^{\pm N_c} = -1$ element does not keep $(A.9)$ invariant. Thus the symmetry of $(A.9)$ becomes $Z_{N_c}$, which is spontaneously broken in $(A.8)$. The spontaneously broken discrete symmetry $Z_{N_c}$ induces $N_c$ domain walls.

Here we consider a rather trivial question. Why the domain walls are not induced by the permutation between $a_i$'s? Although the reason is easily understood if one considers the gauge fixing, we will show another answer, without mentioning the gauge fixing. One may think that our arguments are circumbendibus, however they are useful for our later discussions about the brane dynamics. In the quantum space of eq. $(A.8)$, the permutation corresponds to the permutation between the $N_c$ roots $a_i$. Of course, the permutation is the
exact symmetry of the curve (A.8), which is never broken. Thus in the quantum moduli space, the permutation cannot induce domain walls, because the permutation symmetry is trivial.

Finally, we consider the Q-balls in this four-dimensional effective Lagrangian. Unlike the setups for axionic string and domain walls, the origin of the flat direction is needed to be the global minimum. We assume that the potential is slightly lifted by a weak supersymmetry breaking, or a small $\mu$-term such as $\mu T\Phi^2$, so that the origin of $\phi$ becomes the global minimum. There remains an effective $U(1)_R$ global symmetry, which is later broken to $Z_{4N_c}$ by the quantum effect. In such situations, the Q-balls can be produced if the potential that breaks the global $U(1)_R$ symmetry is shallow.\footnote{The situation is similar to the conventional QCD axion.} The following potential will be generated for the adjoint field $\phi$, \begin{equation} \label{eq:potential} V_{adj} = M_1^4 \log \left( 1 + \frac{|\phi|^2}{M_1^2} \right) + m_{3/2}^2 |\phi|^2 \left( 1 + K \log \frac{|\phi|^2}{M_*^2} \right), \end{equation} where $m_{3/2}$ is the gravitino mass, and $K$ is determined by the one-loop corrections. Here $M_1$ is determined by the mechanism of the supersymmetry breaking (which is not fixed in our discussion), and $M_*$ is the renormalization scale. Using the conventional notations $\phi = \phi_Q e^{i\omega t}$, where $\phi_Q$ denotes the vacuum expectation value of the field $\phi$ in the core of the Q-ball, one can find for the large Q-ball, \begin{align} R_Q &\simeq |K|^{-1/2} m_{3/2}^{-1/2}, \quad \omega \simeq m_{3/2} \nonumber \\
\phi_Q &\simeq |K|^{3/4} m_{3/2} Q^{1/2}, \quad E_Q \simeq m_{3/2} Q, \end{align} (A.12) for $\phi_Q > \sqrt{2} M_1^2 / m_{3/2}$, and \begin{align} R_Q &\simeq M_1^{-1} Q^{1/4}, \quad \omega \simeq M_1 Q^{-1/4} \\
\phi_Q &\simeq M_1 Q^{1/4}, \quad E_Q \simeq M_1 Q^{3/4}, \end{align} (A.13) for $\phi_Q < \sqrt{2} M_1^2 / m_{3/2}$. Here $Q$, $R_Q$ and $E_Q$ are the charge, the radius and the energy of the Q-ball. We consider the brane counterpart of the above-mentioned Q-balls in the latter half of section 3.
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Figure 1: A schematic representation of brane inflation in ref. [22]. The splitting of the D4-brane is due to the rotation between the D4-brane and the D6-brane, which induces the constant Fayet-Iliopoulos term in the effective Lagrangian. Dotted line on the D6-brane denotes the D2-brane, which corresponds to the D-term string in the effective Lagrangian.
Figure 2: Brane inflation ends when the $D4$ brane splits on the $D6$ brane. The dotted line in the left picture is the seed for the $D2$ brane. To be more precise, a careful treatment of the effective action$^{[27]}$ shows that the eigenfunction of the tachyonic mode is localized at the intersection. Since the mechanism of this localization is different from the Kibble mechanism, the “seed” for the $D2$ brane can be localized at the intersection. Then the $D2$ brane is pulled out from the mother brane when the mother $D4$ brane splits on the $D6$ brane.

Figure 3: Each dot correspond to the position of the branes in the codimension 2 space. In the effective four-dimensional Lagrangian, the dots represent $a_i$'s.
Figure 4: Permutation between the two branes “3” and “4”. If the two vacua are physically distinct, the domain wall, which interpolates between them, is produced.

Figure 5: The $U(1)_R$ rotation of the adjoint scalar field is shown in the left picture. If the windings are formed in the three-dimensional space, they form strings as is depicted in the right picture.
Figure 6: The brane configuration in Section 3. The parallel $N_c$ D4-branes are stretching between the two NS5-branes. The endpoint of a D4-brane on the NS5-brane is on the $(x^4, x^5)$-plane. In more realistic models, these configurations are assumed to be embedded in the compactified space.