Conversion efficiency in a resonant Josephson effect mixer

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The RCSJ (Resistor-Capacitor Shunted Junction) model provides us an analytical formula for the conversion efficiency for a resonant Josephson effect mixer. An external rf signal is applied to a long Josephson junction operating in the fluxon oscillating regime. The sin-Gordon equation is used to investigate the dynamics of the phase-locking between the fluxon oscillations and the external signal. We obtain that it is possible to have conversion efficiency greater than unity.

Introduction

A Josephson junction having one physical dimension larger than the Josephson penetration depth can support periodic motion of magnetic flux quanta. The evidence for this is the existence of singularities in the current-voltage (I-V) characteristic of the junction, known as zero-field steps (ZFS). Thus, a long Josephson junction (LJJ) dc biased on a ZFS emits electromagnetic radiation whose frequency is proportional to the geometric length of the junction. Due to the low dynamic resistance of a ZFS, this radiation has a very narrow linewidth and it is very interesting for potential practical applications as local oscillator for a superconducting mixer: the narrow linewidth can reduce the mixer-noise very much.

When an external signal is applied to the junction a new singularity in the I-V curve appears, known as phase-locking step: the junction is said to be in resonance with the external signal. Phase-locking of a long Josephson junction (LJJ) to an external rf drive has been investigate in several publications by numerical simulation, theory, and experiments. Already, theoretical results have shown that some of the observed experimental and numerical features can be reproduced by means of analytical approaches, based on sine-Gordon fluxon dynamics.

In this paper we will also make use of the Resistor-Capacitor Shunted Junction (RCSJ) model. With this model conversion efficiencies larger than unity, and in very good agreement with the experiments, have been found for mixers build with small Josephson junctions but it was never applied to LJJ.

The purpose of this paper is to apply the RCSJ model to a resonant Josephson effect mixer to obtain an analytical formula for the conversion efficiency.

Conversion gain in a Josephson effect mixer

The conversion efficiency, $G$, for a Josephson effect mixer is defined as:

$$G = \frac{P_{\text{if}}}{P_s},$$

where $P_{\text{if}}$ is the power delivered to the room-temperature amplifier and $P_s$ is the power from the small signal source. In the RCSJ model $G$ can be conveniently written in the form:

$$G = C_{\text{if}} \frac{R_{\text{dyn}}}{R} \chi^2,$$

where $C_{\text{if}}$ is the output coupling efficiency (we will only consider the ideal case, $C_{\text{if}} = 1$), $R_{\text{dyn}}$ is the
inverse slope of the I-V curve at the bias point, $R$ is the shunt resistance of the junction, and $\chi$ is a dimensionless parameter:

$$\chi = \frac{\partial(\Delta I/I_c)}{\partial \left[(P_{rf}/RI_c)^2\right]^{1/2}}. \quad (3)$$

In equation (3), $\Delta I$ is the height of the rf induced step on the I-V curve in presence of the $P_{rf}$ power, and $I_c$ is the critical current of the junction. Since $P_{rf} = I_{rf}^2R$ and $I_c$ is a constant, we find:

$$\chi = \frac{\partial(\Delta I)}{\partial I_{rf}}. \quad (4)$$

Inserting (3) in (2) we can easily calculate the values of $G$ once we have estimated $R_{dyn}$ and $\Delta I$ from the I-V curve of the junction.

We want to apply this theory to a resonant Josephson effect mixer and demonstrate how it is possible to have conversion efficiency larger than unity.

Dynamics of a long Josephson junction

To modelling a long Josephson junction we consider the forced sine-Gordon system:

$$\phi_{tt} - \phi_{xx} + \sin \phi + \alpha \phi_t = \gamma + \eta \sin \omega t, \quad (5)$$

with the boundary conditions:

$$\phi_x(0, t) = \phi_x(l, t) = 0. \quad (6)$$

In the (3) and (4) the length is normalised to the Josephson penetration depth, $\lambda_J$, the time is normalised to the inverse of the Josephson plasma frequency, $\Omega_J$, and the currents are normalised to the critical current, $I_c$.

In equation (3) we assumed the Ohmic resistance of the junction and $\gamma$ represents the bias current. $\eta$ is the rf current and $\omega$ is the frequency of the rf signal. The junction is uniformly rf-driven by the term $\eta \sin \omega t$. This can be experimentally achieved in the overlap-geometry. In equation (4) $l$ is the normalised length of the junction.

For $\eta = 0$ the system (3)-(4) exhibits fluxon oscillations over the spatial interval of length $l$, whose frequency is the inverse of the time that the fluxon spends to cover a distance $2l$. The I-V curve of the junction will show the characteristic Zero Field Steps.

For $\eta \neq 0$ the system (3)-(4) shows current steps due to the interaction between the external radiation and the internal oscillations. The height of the step $\Delta I$ for a given value of $\eta$ is called phase-locking range. A very important feature of the system (3)-(4) is that the dependence of the phase-locking range upon the externally applied rf-current amplitude, $\eta$, is linear. If $\Delta I$ is the phase-locking range, we have that:

$$\Delta I = \frac{1}{\sqrt{2}} \alpha \eta R_p, \quad (7)$$

where

$$\frac{1}{R_p} = \frac{\partial I(\gamma, \eta = 0)}{\partial V}$$

is the inverse of the slope of the I-V curve without external radiation applied, i.e. the slope of the Zero Field Step, at the point where the phase-locking step appears.

The basic idea beyond the equation (7) is [3]: A long junction can be viewed (see Figure 1) as the parallel combination of the dynamical resistance of the junction when biased on a Zero Field Step, $R_p$, and the Ohmic resistance $1/\alpha$. Since usually we have that $R_p \ll 1/\alpha$.

The parallel combination of these two resistances and the current generator, $\eta \sin(\omega t)$, can be approximated by a voltage generator of amplitude $\eta R_p \sin(\omega t)$ applied to the resistance $1/\alpha$.

Equation (3) and (4) where spatially discretized and integrated in time with a fourth-order Runge-Kutta method. The following numerical results were obtained for a junction with $l = 4$ and $\alpha = 0.1$. This means that the junction is four times longer than $\lambda_J$, and that the current leakages are about 10% with respect to the critical current, $I_c$.

The I-V curves in Figure 2 were obtained with a bias increment $\delta \gamma = 10^{-4}$. For each increment, $\delta \gamma$, we let the routine run for a transient of 1000 rf cycles to stabilise the fluxon oscillations in the junction. After this transient the average value of the dc voltage, $\langle \phi_t \rangle$, was taken over 1024 rf cycles. The dc voltage across the junction is instantaneously
Given by the second Josephson equation, that in our normalised units, is:

$$\phi_t = V.$$

In Figure 2(a) the I-V curves obtained from the system (1)-(2) with the parameters given before, are shown. We chose the bias point on the Zero Field Step corresponding to the Josephson plasma frequency $\Omega_J = 1.35$. The phase-locking steps will then appear at

$$V = \langle \phi_t \rangle = \Omega_J.$$

In Figure 2(b) we show a comparison between the heights of the phase-locking steps as evaluated from the Figure 2(a) (dots) and the equation (7) (straight line). The agreement is very good as previous works show [3, 6–10].

### Conversion gain for a resonant Josephson mixer

From the equations (4) and (6) we derive the following general expression for the conversion efficiency in a Josephson effect mixer as a function of the position $V$ on the I-V curve:

$$G(V) = \frac{R_{dyn}(V, \eta)}{R(V)} \left[ \frac{\partial (\Delta I)}{\partial I_{rf}} \right]^2,$$

(8)

where we assume the output coupling efficiency, $C_{if}$, to be unitary. $R$ represents the shunt resis-
tance to the junction, and
\[
\frac{1}{R_{dyn}(V, \eta)} = \frac{\partial I(\gamma, \eta)}{\partial V}.
\] (9)

For a resonant Josephson effect mixer, from equations (4) and (7), we see:
\[
\chi = \frac{\partial (\Delta I)}{\partial I_{rf}} = \frac{\partial (\Delta I)}{\partial \eta} = \frac{1}{\sqrt{2} \alpha R_p},
\] (10)
i.e., that \( \chi \) is a constant depending only on the bias point on the Zero Field Step through the dynamical resistance \( R_p \) at the point where the phase-locking step will appear:
\[
\frac{1}{R_p} = \left( \frac{\partial I(\gamma, \eta = 0)}{\partial V} \right)_{V = \Omega_J}. \] (11)

This case is very interesting especially because for a non-resonant Josephson mixer \( \chi \) has a very complicated behaviour, while the (10) is a simply linear relation. This simplicity is a remarkable property of the resonant Josephson effect mixer that we are studying in this paper.

In the equation (8) \( R \) is the shunt resistance to the junction which is the parallel resistance between the ohmic resistance, \( 1/\alpha \) and dynamic resistance of the Zero Field Step \( R_d(V) \), where
\[
\frac{1}{R_d(V)} = \frac{\partial I(\gamma, \eta = 0)}{\partial V}. \] (12)
The approximation made in Figure 1 is still valid because we look at a small part of the I-V curve around the bias point. We have then that:
\[
\frac{1}{R} = \frac{1}{R_d(V)} + \alpha \approx \frac{1}{R_d(V)} \left( R_d(V) \ll \frac{1}{\alpha} \right) \]. (13)

The final expression for the conversion efficiency for a resonant Josephson effect mixer therefore is:
\[
G = \frac{R_{dyn}(V, \eta)}{R_d(V)} \chi^2 = \frac{\Delta I(V, \eta)}{R_d(V)} \left( \frac{1}{\sqrt{2} \alpha R_p} \right)^2 \]. (14)

Results

In order to evaluate the conversion efficiency from equation (14), we have to calculate the dynamic resistances in that equation. Attention must be paid to this because an inaccurate estimate of these parameters can give rise to big inaccuracy in the conversion efficiency that we want to study in this paper. We evaluated the dynamical resistances by differentiating the eighth-order polynomial curve fitting of the I-V curve points spaced by a \( 10^{-4} \) current step. The results are shown in Figure 3. In Figure 3(a) we look at the branch of I-V curve on the left with respect to the phase-locking step. In (b) we look at the branch of I-V curve on the right with respect to the phase-locking step. We observe from (a) that it is possible to obtain a conversion efficiency bigger than unitary for a bias point close to the phase-locking step.

Figure 3: Conversion efficiency for a resonant Josephson effect mixer. The junction is driven by an external signal whose current amplitude is given by \( \eta \). In (a) we look at the branch of I-V curve on the left with respect to the phase-locking step. In (b) we look at the branch of I-V curve on the right with respect to the phase-locking step. We observe from (a) that it is possible to obtain a conversion efficiency bigger than unitary for a bias point close to the phase-locking step.
From Figure 3 we also observe that:

For $\Omega$ and $\eta$ given, the conversion gain is larger where the I-V curve is flatter. Indeed, there we have a larger dynamic resistance, $R_{\text{dyn}}(V, \eta)$, and consequently a larger gain.

For $\Omega$ and $\eta$ given, the conversion gain is larger for a bias on the branch of the I-V curve to the left of the phase-locking step. This is easily explained by observing that the I-V curve of the junction is generally flatter (giving a bigger dynamical resistance, $R_{\text{dyn}}(V, \eta)$) on the left branch.

For $\Omega$ given, the conversion gain is larger for larger values of $\eta$. Again, a larger $\eta$ gives a flatter shape to the I-V curve.

Conclusions

We have studied the conversion efficiency for a resonant Josephson effect mixer and we found a theoretical analytic expression for it. From the fundamental equations for a LJJ (long Josephson junction) we have obtained an estimate for the conversion efficiency: it is possible to have a conversion efficiency exceeding units.

The necessary condition set by the sine-Gordon equation is that the junction should be uniformly rf driven which can be experimentally accomplished in the overlap geometry. The calculations where also performed in the limit of an ideal coupling between the junction and the if circuit ($C_{\text{if}} = 1$).

We conclude that a resonant Josephson effect mixer can have very good performances. They can be explained as a consequence of the resonant nature of the long Josephson junctions: The linewidth of the signal is in fact very small (1 KHz at 10 GHz, 1 MHz at 100 GHz) and this decreases the noise in the mixer. The noise is in fact due mostly to down conversions of frequencies in the linewidth of the mixer signal, and it decreases as the linewidth shrinks: Even very small signals can be detected with reasonably good conversion efficiencies.

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