Model predictive control paradigms for fish growth reference tracking in precision aquaculture

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A B S T R A C T

In precision aquaculture, the primary goal is to maximize biomass production while minimizing production costs. This objective can be achieved by optimizing factors that have a strong influence on fish growth, such as feeding rate, temperature, and dissolved oxygen. This paper provides a comparative study of four model predictive control (MPC) strategies for fish growth reference tracking under a representative bioenergetic growth model in precision aquaculture. We propose to evaluate four candidate MPC formulations for fish growth reference tracking based on the receding horizon. The first MPC formulation tracks a desired fish growth trajectory while penalizing the feed ration, temperature, and dissolved oxygen. The second MPC optimization strategy directly optimizes the feed conversion ratio (FCR), which is the ratio between food quantity and fish weight gain at each sampling time. This FCR-like optimization strategy minimizes the feed while maximizing the predicted growth state’s deviation from the given reference growth trajectory. The third MPC formulation includes a tradeoff between the growth rate trajectory tracking, the dynamic energy, and food cost. The last MPC formulation aims to maximize the fish growth rate while minimizing the costs. Numerical simulations that integrate a realistic bioenergetic fish growth model of Nile tilapia (Oreochromis niloticus) are illustrated to examine the comparative performance of the four proposed optimal control strategies. A sensitivity analysis is conducted to study the robustness of these MPC strategies with respect to the effect of the prediction horizon, the regularization term, and the additive input disturbances to the process. The obtained results show great potential and flexibility to meet the fish farmers’ needs depending on the type of fish, selling price, culture duration, and feed cost.

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1. Introduction

Aquaculture is one of the largest and fastest-growing food production sectors in the world and is likely to become the primary source of seafood in the future [1]. As commercial fish production continues to increase, both its impact and reliance on protein sources provided by ocean fisheries are likely to expand. To mitigate these impacts, adequate growth models are relevant for efficient aquaculture management, as they provide an optimized protocol for feeding and monitoring fish welfare throughout the grow-out cycle from stocking through harvesting [2]. Feeding and water quality are crucial components to balance productivity and enlarge fish health. Increasing the amount of nutrient affects wasted food in the water, leading to highly polluted water, resulting in diseases and stress [3]. The cost of fish feeding is usually around 40–50 percent of the total production cost [4,5]. Therefore, optimizing feeding and water quality play an essential role in managing the total aquaculture operational cost and estimating the state of the fish in a tank. The optimal control of the aquaculture system by considering the economic aspects was investigated with several approaches such as optimization techniques. Some of these approaches focus on improving the economics and management of the aquaculture systems taking into consideration the market variation and the best harvesting time [6,7]. However, these works focus more on building a general economic framework and did not consider all the biological
parameters of the fish growth model. Other works studied the influence of different factors such as mortality rate and fish price on the harvest to derive the best feeding schedule \([8,9]\). Thus, there is a pressing need to develop precision aquaculture techniques that improve fish farming efficiency by optimizing feeding protocols \([10]\). Moreover, efficient control strategies that can be implemented in practice are highly needed to respond to various challenges related to the practical aspects of aquaculture systems for different fish species.

Modern aquaculture systems can benefit from integrating emerging technologies and theories from multiple research disciplines such as marine science and optimal control theory. In integrating control systems, classical feedback approaches are not suitable to most feeding regimes due to the scheduled nature of the feed ration and biological constraints of the aquaculture environment. Hence, the control problem is generally derived as an optimization problem targeting the desired growth trajectory subject to some constraints such as food quantity, environmental parameters, and economic factors, as illustrated in Fig. 1. The integration of new technology-based solutions and policies may help to promote sustainable aquaculture production. Currently, there are no examples of closed-loop precision fish farming systems, which include the different components of observing the fish for decision making \([11]\). Therefore, the aquaculture industry and researchers aspire to develop strategies that optimize biomass production by monitoring and controlling factors which influence fish growth.

In this paper, model predictive control (MPC) strategies are investigated \([12–18]\). The main advantage of the MPC control is its ability to predict the behavior of controlled variables and take the right control action sequences that will optimize a predefined cost function over a prediction horizon. This cost function can optimize a quantity implying the state’s dynamics and the constrained inputs, such as minimize the tracking error of a reference state. It can also optimize an external quantity such as the operating costs or any other measurable metric that is not directly related to the system dynamics. To the best of the authors’ knowledge, MPC-based on the receding-horizon framework for reference growth trajectory has not been fully investigated in the aquaculture system.

This study aims to provide a comparative assessment of four candidate MPC formulations to track a desired growth rate trajectory accounting for specific economic considerations and handling inputs constraints at each sampling time in an aquaculture environment. The optimal control problem studied aims at tracking a desired fish growth reference trajectory while penalizing the manipulated inputs in an aquaculture environment under a representative bioenergetic fish growth model. The first MPC formulation is based on receding-horizon that minimizes the tracking error while penalizing the feed ration, temperature, and dissolved oxygen. The second receding-horizon approach is formulated as the ratio between food quantity and fish weight gain or the feed conversation ratio, which defines the fish efficiency in converting feed mass into increased body mass. The third MPC formulation maintains a tradeoff between reference growth trajectory tracking and dynamic energy and feeding price. The final MPC formulation aims to maximize the fish growth rate while minimizing the costs.

The outline of the rest of the paper is as follows. Section 2 describes the bioenergetic fish growth model of Nile tilapia \((Oreochromis niloticus)\), which includes the anabolism and catabolism growth coefficients. Section 3 formulates the MPC problem; the four candidate MPC formulations are presented. Section 4 presents the obtained results and discusses the findings for the four candidate MPC formulations and their sensitivity analysis, followed by a conclusion in Section 5.

2. Fish growth modeling

A representative two-term bioenergetic fish growth model that captures the dominant growth factors, including adequate fish size, feed ration, and water temperature, is proposed in this work. The bioenergetic model is obtained from the dynamic energy budget. It presents a mechanistic basis for understanding an organism’s energetic used to model the mass and energy flow through the fish from the uptake to usage for maintenance, reproduction, growth, and excretion \([19–21]\), as illustrated in Fig. 2. The model is expressed in terms of energy fluxes between the organism and the environment. It constitutes valuable tools in the early stage of an aquaculture activity to carry the capacity of a system before installing new farms \([22,23]\) estimate production and feeding ration \([24]\), or to optimize integrated multitrophic aquaculture systems \([25]\).

According to Ursin’s work \([26]\), the fish growth model in both recirculating aquaculture systems and marine cages can be expressed as the difference between anabolism and catabolism \([27–30]\). In this paper, a bioenergetic growth model is adopted for Nile tilapia cultured in fertilized marine ponds, incorporating available information in pond dynamic and fish physiology. The model includes the effects of different parameters such as water temperature, body size, un-ionized ammonia (UIA), dissolved oxygen (DO), photoperiod, and food availability \([28]\). Thus, the growth rate model of Nile tilapia is described as the difference between anabolism and catabolism \([28]\)

\[
\frac{dw}{dt} = \underbrace{\Psi(f, T, DO)\nu(UIA)u^m}_\text{anabolism} - \underbrace{k(T)w^n}_\text{catabolism},
\]

where \(\Psi(f, T, DO)\) \((g^{1-m} \cdot \text{day}^{-1})\) and \(\nu(UIA)\) are the coefficients of anabolism and \(k(T)\) \((g^{1-n} \cdot \text{day}^{-1})\) is the coefficient of fasting catabolism expressed as

\[
\Psi(f, T, DO) = \eta_f b_f (1 - a)\tau(T)\sigma(DO),
\]

and

\[
k(T) = k_{\text{min}} \exp\left(j(T - T_{\text{min}})\right).
\]
where $v$ follows of the growth model [28]. Table 1 summarizes the nomenclature and the main parameters of the growth model [28].

The rate of growth (1) can be expressed in a compact form as follows

$$\frac{d w}{dt} = g(w, f, T, DO, UIA),$$

where $w \in \mathbb{W} \subset \mathbb{R}$ denotes the state and $u = [u_1, u_2, u_3]^T$ is the input vector. $u \in \mathbb{U} \subset \mathbb{R}^3$ describes the manipulated control input vector of the fish growth model that depends on the feeding rate, temperature, and dissolved oxygen, respectively. Theunionized ammonia function $v(UIA)$ is considered to be known and constant over the prediction horizon. The set of admissible input values $\mathbb{U}$ is compact. The relative feeding rate $f$ is formulated as the ratio between the daily ration $r$ and the maximal daily ration $R$ as follows

$$f = \frac{r}{R}$$

The function $g : \mathbb{W} \times \mathbb{U} \rightarrow \mathbb{W}$ is locally Lipschitz on $\mathbb{W} \times \mathbb{U}$. The measurement growth state (4) is synchronously sampled at the current sampling time defined as $t_k = ks$ where $k \in \mathbb{Z}^+$ is a positive integer and $\varepsilon > 0$ is the sampling period.

3. Optimization Formulations

The bioenergetic fish growth model is highly nonlinear with multi-inputs, as defined in Eq. (4). The MPC provides the benefits of efficiently handling the multi-inputs of the growth model (4), considering all the constraints and non-linearity and re-optimizing an $N$-step control sequence at each time step. The MPC strategies studied in this work control optimally the inputs to track the desired fish growth reference trajectory or maximize the production while minimizing the energy feed costs to improve the economic profitability. Fig. 3 illustrates a schematic of the proposed MPC strategies for fish growth reference trajectory tracking. The general optimal control problem can be formulated as a minimization with a finite-time prediction horizon as follows

$$\min_{w(t)} \int_{t_k}^{t_{k+N}} L(\tilde{t}, \tilde{w}, w^d) dt$$

subject to

$$u(t) \leq u(t) \leq u_{\text{max}}, \forall t \in [t_k, t_{k+N}]$$

$$\Delta u(t_k) = u(t_k) - u(t_{k-1})$$

$$w^d_0 \leq \tilde{w}(t) \leq w^d_{\text{end}}, \forall t \in [t_k, t_{k+N}]$$

$$\tilde{w}(t_k) = w(t_k), \tilde{w}(0) = w(t_0)$$

where $N$ is the prediction horizon of this MPC, $\tilde{w}$ is the predicted state trajectory over the prediction horizon $[t_k, t_{k+N}]$ and $w(t_k)$ is the state measurement obtained at time $t_k$. $w^d$ is the desired reference live-weight growth trajectory. $\Delta u$ is the control increment. Thestage cost $L(\cdot)$ can represent the tracking error performance or an economic measure of the fish growth rate with a regularization parameter term $\lambda$. $w^d_0$ and $w^d_{\text{end}}$ are the desired initial and maximal fish weight constraints, respectively.

The potential growth rate profile $w^d$ is based on experimental data analysis and describes the rate achieved by a specific strain that satisfies all the nutritional requirements. $L(\varepsilon)$ represents the set of piecewise constant functions described by the sampling period $\varepsilon$. The first control action $u(t_k)$ of the MPC (5) is implemented, and then the MPC horizon is rolled again over the next time step. Throughout the sampling period $[t_k, t_{k+N}]$, the first control action is applied in a sampled-and-hold fashion. At each sampling time $t_k$, the optimal solution to this optimization problem, which is defined for $[t_k, t_{k+N}]$ is denoted by $u^*(t | t_k), i = 1, \ldots, m$.

The terminal cost $\ell_f(\tilde{w}, w^d)$ can be chosen to be a Lyapunov function [31]. For the sake of simplicity, we define the terminal cost $\ell_f(\tilde{w}, w^d)$ as follows

$$\ell_f(\tilde{w}, w^d) = (\tilde{w} - w^d)^2.$$

The four MPC optimization formulations are described as follows.

1. Reference Trajectory Tracking, including Feeding and Energy Consumption Minimization (MPC1): In this approach, the MPC strategy minimizes the growth rate tracking error while penalizing the food and energy quantities. We denote this cost function $J_{\text{MPC1}}$.

2. Feed Conversion Ratio (FCR) (MPC2): In this approach, the MPC optimization strategy optimizes the FCR as a standard metric to assess the aquaculture systems. We define this metric as the ratio between food quantity and fish weight.
Fig. 3. Model predictive control framework.

3. Reference Trajectory Tracking, including Economic Profitability Feeding, and Energy Consumption Minimization (MPC1): In this approach, the MPC formulation tracks a given reference growth trajectory while reducing the economic profit and the total costs related to the feed and the electrical energy used for heating and oxygenation. This cost function is called $J_{\text{MPC1}}$.

4. Production Maximization and Costs Minimization Strategy (MPC2): In this approach, the MPC formulation tends to maximize the fish production without tracking any predefined trajectory while reducing the economic profit and the total costs related to the feed and the electrical energy used for heating and oxygenation. This cost function is called $J_{\text{MPC2}}$.

The two first MPC formulations, which are the reference growth trajectory tracking including feed, and energy consumption minimization $J_{\text{MPC1}}$ and the feed conversion ratio (FCR) $J_{\text{MPC2}}$ strategies consider objective functions that are not directly related to the economic cost.

3.1. Reference Trajectory Tracking, Food and Energy Consumption Minimization Strategy (MPC1)

The first MPC formulation tracks the desired fish growth trajectory while minimizing the feed ration, temperature, and dissolved oxygen. We formulate the MPC optimization problem for this strategy as follows:

$$\min_{u(t)} J_{\text{MPC1}} = \int_{t_k}^{t_k+N} \ell_1(\tilde{w}(\tau), w^d(\tau), u(\tau)) \, d\tau$$

$$+ \ell_T(\tilde{w}(t_k+N), w^d(t_k+N))$$

s.t. $\tilde{w}(t) = g(\tilde{w}(t), u(t))$

$$u_{\min} \leq u(t) \leq u_{\max}, \quad \forall t \in [t_k, t_k+N]$$

$$\Delta u(t_k) = u(t_k) - u(t_{k-1})$$

$$\tilde{w}(t_k) = u(t_k)$$

$$\tilde{w} = u(t_k)$$

where the stage cost $\ell_1$ is defined as follows:

$$\ell_1(\tilde{w}(\tau), w^d(\tau), u(\tau)) = \frac{\| \tilde{w}(\tau) - w^d(\tau) \|^2}{2} + \lambda \|u(\tau)\|^2,$$

and $\lambda$ is a positive regularization term to assess the control inputs preference. The parameter $\lambda$ is tuned empirically such that a good compromise between tracking error performance and fast tracking response is achieved over the entire prediction horizon.

3.2. Feed Conversion Ratio (FCR)-like Optimization Strategy (MPC2)

Fish feeding is an essential component of fish farming; the reduction of the feed conversion ratio (FCR) helps in more efficient use of fishmeal, energy consumption, and fish oil, which has primarily been achieved through improved management. The FCR describes the quantity of feed used to the fish organisms under satisfactory conditions for its development. In this approach, we propose a (FCR)-like cost function that optimizes the ratio between the mass of the food eaten and the mass body gain at each sampling time while regulating the growth around the desired growth reference trajectory. The cost function is defined as follows:

$$\min_{u(t)} J_{\text{MPC2}} = \int_{t_k}^{t_k+N} \ell_2(\tilde{w}(\tau), w^d(\tau), u(\tau)) \, d\tau$$

$$+ \ell_T(\tilde{w}(t_k+N), w^d(t_k+N))$$

s.t. $\tilde{w}(t) = g(\tilde{w}(t), u(t))$

$$u_{\min} \leq u(t) \leq u_{\max}, \quad \forall t \in [t_k, t_k+N]$$

$$\Delta u(t_k) = u(t_k) - u(t_{k-1})$$

$$\tilde{w}(t_k) = u(t_k)$$

$$\tilde{w} = u(t_k)$$

where $\ell_2$ represents the FCR-like cost function defined as

$$\ell_2(\tilde{w}(\tau), \tilde{w}, u(\tau)) = \frac{\Delta F}{\tilde{w}}$$

$$= \frac{u(t) \tilde{w}(\tau) - \tilde{w} \tilde{w}(\tau)}{\tilde{w}(\tau)}.$$

with $\tilde{w}$ is the measured growth rate state obtained at time $t_k$, $\Delta F$ is the feeding quantity, $\lambda$ is the regularization term, $u(t)$ is the relative feeding rate, $\tilde{w}$ is the maximal daily ration, and $\Delta \tilde{w}(t_k) = \tilde{w}(t_k) - \tilde{w}(t_{k-1})$ is the growth rate defining by the difference between the predicted weight and the measured growth rate state obtained at time $t_k$. $\Delta \tilde{w}(\tau)$ is sampled synchronously at time instants $t_k$ and $t_{k-1}$ over the entire prediction horizon. MPC formulated as an optimal feeding strategy provides a good indicator of farming efficiency, economic and environmental performance since this index successfully minimizes the deviation of the growth rate performance and the use of feeding resources supplied.

3.3. Reference Trajectory Tracking, Economic Profitability Food and Energy Consumption Minimization Strategy (MPC3)

The third MPC formulation minimizes the economic profit and the energy consumption costs, including the feeding, heating, and oxygenation of the aquaculture environment system. The MPC optimization problem for this strategy is defined as follows:

$$\min_{u(t)} J_{\text{MPC3}} = \int_{t_k}^{t_k+N} \ell_3(\tilde{w}(\tau), w^d(\tau), u(\tau)) \, d\tau$$

$$+ \ell_T(\tilde{w}(t_k+N), w^d(t_k+N))$$

s.t. $\tilde{w}(t) = g(\tilde{w}(t), u(t))$

$$u_{\min} \leq u(t) \leq u_{\max}, \quad \forall t \in [t_k, t_k+N]$$

$$\Delta u(t_k) = u(t_k) - u(t_{k-1})$$

$$\tilde{w}(t_k) = u(t_k)$$

$$\tilde{w} = u(t_k)$$
where $\ell_3$ represents the stage cost associated with an economic profitability term defined as follows

$$
\ell_3(\tilde{w}(\tau), w^d(\tau), u(\tau)) = B_1(\tilde{w}(\tau) - w^d(\tau))^2 \text{ tracking error cost} \\
+ B_2 u_1^2(\tau)^2 \text{ feeding cost} \\
+ B_3 u_2^2(\tau)^2 \text{ heating cost} \\
+ B_4 u_3^2(\tau)^2 \text{ oxygenation cost}
$$

For this comparative study, the weight $B_1$ is considered constant over the prediction horizon. The cost weights $B_2$, $B_3$, and $B_4$ vary with the time and account for the price of the feeding, heating, and oxygenation resources.

The cost weights $B_1$, $B_2$, $B_3$ and $B_4$ of this third MPC optimization strategy are explicitly defined as follows

$$
\ell_3(\tilde{w}(\tau), w^d(\tau), u(\tau)) = (P_1\tilde{w}(\tau) - w^d(\tau))^2 \\
+ \lambda \left[ P_1 R_{u_1}(\tau)^2 \\
+ \beta_1 \left( \frac{P_c L_m N \Delta u_2(\tau)}{3600} \\
+ \beta_2 \left( 24 P_p P_{\max} u_3(\tau) \right)^2 \right) \right].
$$

where $\lambda$ is a regularization term, $P_1$ is the fish selling price per kg [32], $P_f$ is the fish food price per kg, $u_1 = f$ represents the feeding rate, and $R$ is the maximal daily ration. $\beta_1$ and $\beta_2$ are regularization terms defining the heater’s daily operation duration ratio and the air pump, respectively. $P_c$ is the electricity price per kWh, $c_p$ is the specific heat of the tank water, $L$ is the tank volume in liters, $m = 1$ is the water mass, and $\Delta u_2 = \Delta T$ represents the temperature difference in °C [33]. $P_{\max}$ is the maximal electrical power of the air pump, and $u_3 = DO$ represents the dissolved oxygen level.

### 3.4. Production maximization and costs minimization strategy (MPC$^4$)

The MPC formulation maximizes the fish growth, without tracking a predefined trajectory in the stage cost, while minimizing the feed ration, temperature, and dissolved oxygen. We formulate the MPC optimization problem for this strategy as follows

$$
\min_{u(t)} J_{MPC}^{4} = \int_{t_k}^{t_{k+N}} \ell_4(\tilde{w}(\tau), u(\tau)) \, d\tau \\
+ \ell_1(\tilde{w}(t_k), u^d_k) \\
\text{s.t.} \\
\tilde{w}(t) = g(\tilde{w}(t), u(t)) \\
u_{\min} \leq u(t) \leq u_{\max}, \quad \forall t \in [t_k, t_{k+N}] \\
w^d_0 \leq \tilde{w}(t) \leq w^d_{\text{end}}, \quad \forall t \in [t_k, t_{k+N}] \\
\tilde{w}(t_k) = w(t_k)
$$

where the stage cost $\ell_4$ is defined as follows

$$
\ell_4(\tilde{w}(\tau), u(\tau)) = -P_f \tilde{w}(\tau) + \lambda \left[ P_1 R_{u_1}(\tau) \\
+ \beta_1 \left( \frac{P_c L_m N \Delta u_2(\tau)}{3600} \\
+ \beta_2 \left( 24 P_p P_{\max} u_3(\tau) \right)^2 \right) \right].
$$

The parameter $\lambda$ is tuned empirically such that a good compromise between fish production quantity and cost is achieved over the entire prediction horizon.

### 4. Numerical simulations

This section presents a comparative analysis of the four proposed candidate MPC formulations using the fish growth model and interprets the obtained results. The parameters of the Nile tilapia growth model are set based on the values provided in [28]. Besides, the potential growth reference tracking profile $w^d$ is based on experimental data analysis and describes the rate achieved by a specific strain that satisfies all the nutritional requirements [34]. The predicted growth trajectory is constrained to $w^d_0 = 5.57$ gram and $w^d_{\text{end}} = 520$ gram. The four MPC formulations are implemented using the Open Optimal Control Library [35]. Table 3 summarizes the parameter’s value of the cost functions used in the numerical results, and the MPC parameters constraints are given in Table 3.

To compare the four MPC optimization strategies’ performance, we consider the tracking error based on the mean squared error (MSE), feed conversion ratio, profit, and profit percentage as performance evaluation metrics to assess the aquaculture systems. These metrics are defined as follows

$$
\text{MSE} = \frac{1}{n} \sum_{i=1}^{n} (w_i - w^d_i)^2,
$$

where $w$ and $w^d$ are the current fish weight and the desired fish weight, respectively. $n$ is the number of culture days.

$$
\text{FCR} = \frac{\text{Total feed quantity (kg)}}{\text{End fish weight (kg) - Start fish weight (kg)}},
$$

and

$$
\text{Profit percentage} = \frac{\text{Revenue} - \text{Total costs}}{\text{Total costs}}
$$

### 4.1. Effect of horizon length on the performance and computation time of the MPC formulations

The prediction horizon plays an essential role in the optimization problem of the different MPC formulations. Fig. 4 shows that the overall performance increases with the increase of the prediction horizon. Specifically and for most of the formulations, increasing the prediction horizon helps increase profitability and
improve the FCR. It is worth mentioning that the shadow area, in Fig. 4 and Fig. 9, represents the standard deviation the each metric with respect to the variation of the parameter $\lambda$. Moreover, the solid curve represents the average performance of that same metric.

Overall, the third MPC3 strategy achieves the best average performance for the tracking error performance and computational cost. Consequently, the prediction horizon length of $N = 4$ days is used for all the following simulations for this comparative study, as shown in the next section.

4.2. Effect of additive input disturbances on the performance of the MPC formulations

As the used model is not considering all the real dynamics, which are highly affecting the aquaculture environment in real practice. For instance, as the fish tank gets bigger, the temperature will vary from a location to another in a closed system. In addition, fish will not be feed equally as we will need to use more than one single feeder. Therefore, additive feeding and temperature process disturbances are taken into account in the nominal system model (1). Thus, the feeding and heating systems are contaminated with Gaussian noises with zero mean. These disturbances are added to the daily feeding ration $u_1 = f + f_o$ with $f_o \sim \mathcal{N}(0, 0.1 \times R)$ and to the tank temperature $u_2 = T + T_o$ with $T_o \sim \mathcal{N}(0, 2^\circ \text{C})$. We simulate this performance by adding zero-mean Gaussian noise to the feeding and temperature control actuators over each integration step.

As the performance of the MPC formulations is highly dependent on horizon length and the regularization term, the average performance of the MPC controller, for $N = 4$ days and $\lambda = [0.001, 0.1, 1, 1.2]$, is used to compare their performances using the predefined evaluation metrics. Table 4 presents a quantitative comparison of the average performance of the different MPC formulations with and without additive input disturbances. It shows that the MPC1 controller gives the second best FCR with the lowest tracking error and with the second best economic profitability. This result is expected as MPC1 formulation does not consider directly the profitability aspect. MPC2 gives the best average performance across all the metrics. This second MPC formulation shows great potential as it does not include any direct economic parameter. This might justify its use to evaluate

**Fig. 4.** Effect of the horizon length on the four MPC formulations for different regularization terms: $\lambda \in [0.001, 1.2]$.

**Table 4**

Average performance comparison of the four candidate MPC formulations with and without additive input disturbances ($N = 4$ days, $\lambda \in [0.001, 1.2]$).

| Additive disturbances | Controller type | Tracking error (MSE) | Elapsed Time (s) | Fish Weight (gram) | Feeding (gram) | Feed Cost (USD) | Heating Cost (USD) | Oxygenation Cost (USD) | Total Cost (USD) | Revenue (USD) | Profit (USD) | Profit rate (%) | FCR |
|-----------------------|-----------------|----------------------|-----------------|------------------|---------------|----------------|-------------------|---------------------|-----------------|--------------|-------------|----------------|-----|
| No                    | MPC₁            | 0.02                 | 64.35           | 425.69           | 850.70        | 344.28         | 10.21             | 3.66                | 358.15          | 510.83       | 152.68      | 44.70         | 1.37 |
|                       | MPC₂            | 0.10                 | 56.66           | 431.48           | 931.52        | 372.61         | 11.43             | 6.65                | 390.69          | 517.78       | 127.09      | 35.29         | 1.37 |
|                       | MPC₃            | 0.32                 | 52.26           | 412.13           | 765.30        | 306.12         | 11.28             | 3.36                | 320.76          | 494.56       | 173.79      | 54.24         | 1.34 |
|                       | MPC₄            | 20.76                | 61.83           | 433.33           | 6728.60       | 2691.44        | 11.58             | 11.13               | 2714.15         | 520.00       | −2194.16    | −80.85        | 4.46 |
| Yes                   | MPC₁            | 0.02                 | 64.83           | 426.53           | 926.53        | 370.61         | 11.42             | 3.78                | 385.81          | 511.83       | 126.01      | 33.19         | 1.42 |
|                       | MPC₂            | 0.02                 | 82.65           | 426.98           | 923.24        | 329.30         | 12.20             | 5.02                | 346.52          | 512.37       | 165.85      | 48.09         | 1.33 |
|                       | MPC₃            | 0.32                 | 61.96           | 411.65           | 789.11        | 315.64         | 14.00             | 11.13               | 2734.79         | 520.00       | −2214.80    | −80.99        | 4.51 |
|                       | MPC₄            | 20.66                | 48.18           | 433.33           | 6780.02       | 2712.01        | 11.58             | 11.21               | 2734.79         | 520.00       | −2214.80    | −80.99        | 4.51 |
the fish growth rate in real aquaculture practice. However, the MPC³ formulation could achieve high production, high economic profitability and the lowest FCR. Thanks to its ability to minimize the tracking error while keeping an eye on reducing the costs. The MPC³ reaches the highest economic cost–benefit ratio with an acceptable tracking error, demonstrating its superiority over the other controllers. On the other hand, the MPC⁴ formulation shows the worst economic profitability and tracking performance. However, the MPC⁴ could achieve the desired fish market size earlier (twice faster than the other formulation as shown in Fig. 8). Thus, fish will be fed for a long time to keep them alive, indicating that this MPC formulation can get the fastest growth rate. It worth mention that the fish growth rate should be well monitored to avoid health issues due to fish overfeeding.

Figs. 5, 6, 7 and 8 illustrate the four MPC optimization performances without additive input disturbances. The first MPC strategy provides the best tracking of given fish growth, representing a healthy growth profile to track, specifically in the early stage of the fish age where the mortality ratio is high. The second MPC strategy seems to have the best compromise between the two abilities as it can achieve a good tracking performance with an acceptable economic profit. Moreover, the second MPC strategy reflects an existing metric in the aquaculture sector as a metric, which can be interpreted easily. Finally, the third and fourth MPC optimization can be used for commercial uses, where the final profit and fish growth rate are the most significant concerns. Besides, the third MPC strategy can help manage other costs streams as long as they are measurable such as fish vaccination, medical treatment, and the fish feed price variation.

4.3. Effect of the regularization term in the MPC strategies

The previously shown results demonstrate that the cost function of the MPC formulations plays a key role in the achieved performance. However, the choice of the parameter’s value or weights is crucial and needs to be analyzed. Therefore, a sensitivity analysis of the regularization parameter λ is performed for all the MPC formulations. In this experiment, a range of values is considered $\lambda = [0.001, 0.01, 0.1, 1, 1.2]$. Fig. 9 illustrates the effect of the regularization term $\lambda$ in the MPC formulations for different horizon lengths. For each shadow area, the minimum and maximum value of regularization term $\lambda$ correspond to the area’s lower and upper limit, respectively, and the respective curve is their average.

Overall, Fig. 9 shows that increasing the regularization term $\lambda$ improves most of the MPC formulation slightly except for MPC³ that decreases the performance significantly. This is because the tracking error increases while limiting the final attainable fish weight up to the quarter. It worth mention that the MPC² formulation is the most sensitive strategy, which might be due to the direct impact of $\lambda$ on the FCR-like formulation and on reducing the maximal daily ratio (see Eq. (10)).
Fig. 9. Effect of the regularization term $\lambda$ in the MPC formulations for different horizon lengths $N \in [4, 40]$: (a) without additive input disturbances and (b) with additive input disturbances.

Table 5
Comparative analysis of the four MPC formulations.

| Keys feature | tracking | Profitability maximization | FCR minimization |
|--------------|----------|-----------------------------|------------------|
| MPC\(^1\)    | ✓✓✓      | ✓                           | ✓               |
| MPC\(^2\)    | ✓✓✓      | ✓                           | ✓               |
| MPC\(^3\)    | ✓✓✓      | ✓                           | ✓               |
| MPC\(^4\)    | ✓✓✓      |                            |                |

4.4. Comparative performance analysis

To compare the best performance of each of the four controllers, a comparative score is proposed considering three components: the tracking error, the profitability, and the feed conversion ratio as follows

$$
\text{Score} = \frac{1}{3} \left( \frac{\epsilon + \text{MSE}}{\epsilon + \text{MSE}_{\text{min}}} + \frac{\text{Pr}(\%)}{\text{Pr}(\%)_{\text{max}}} + \frac{\text{FCR}_{\text{min}}}{\text{FCR}} \right),
$$

where MSE and MSE\(_{\text{min}}\) are the tracking error and its minimal achieved value by all MPC Formulations, respectively. $\epsilon = 10^{-3}$ is a positive constant to regularize the division and avoid the zero denominator. Pr(\%) and Pr(\%)\(_{\text{max}}\) are the profit percentage and the maximal achieved profit percentage, respectively. FCR and FCR\(_{\text{min}}\) are the feed conversion ratio and the minimal achieved feed conversion ratio, respectively.

Table 5 illustrated the usefulness of each MPC formulation depending on the desired goal in terms of the previously defined comparative score. Moreover, Table 6 shows more details of the highest score achieved by each of the MPC formulations.

4.5. Limitations and future work

In this work, the MPC strategies show great potential and flexibility to respond to different needs such as fast growth rate, fish weight trajectory tracking, and higher economic profitability. However, the proposed work still needs further improvement, especially when implemented in real aquaculture practice. The first challenge will be to estimate the fish weight, which can be estimated using recent artificial intelligence (AI) and computer vision techniques such as object segmentation combined with object tracking. It worth mention that the type and number of used cameras will definitely affect the estimation accuracy. In this case, adding another model-based estimator such as the Kalman filter can help to improve the results. The second challenge is to ensure that the controlled set-points (feeding, heating, and oxygenation) do not affect fish health and integrate this parameter into the control strategy. This is a difficult task because the fish health assessment is still done using manual inspection and needs periodic lab diagnosis and tests. One possible solution can be to use computer vision to track fish motion, speed, and habits before, during, and after the feeding time to extract signs indicating health issues. By solving these challenges, the real implementation of the closed-loop system can open the door toward building smart control systems for the aquaculture environment.

5. Conclusion

In this work, four different MPC control strategies are proposed for fish growth control in the aquaculture environment. These MPC strategies can be used for the following purposes: (1) fish growth trajectories tracking, (2) feed and energy consumption reduction, (3) Feed Conversion Ratio (FCR) reduction, and (4) economic profit maximization. The obtained results show that the proposed MPC strategies can successfully meet these targets, which gives the fish farmer more flexibility to choose the most suitable method as a baseline control strategy for his own needs depending on the type of fish, selling price, culture duration, feed cost, etc. For instance, the MPC\(^1\) approach presents the best
tracking ability, but it consumes an average feed quantity compared to the other MPC strategies. The MPC2 or FCR-like strategy achieves the best compromise between good tracking and good profitability. The MPC3 strategy shows the best economic profit with lower tracking ability. However, the MPC3 strategy reaches the fastest growth rate. A sensitivity analysis of the proposed MPC formations is provided to study the effect of the prediction horizon, the regularization term, and the additive input disturbances. The proposed approach can be further improved by using artificial intelligence and computer vision techniques to assess fish weight estimation, which helps in its implementation in a real aquaculture environment.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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