Generalized Chaplygin gas model: constraints from Hubble parameter versus Redshift Data

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Abstract

We examine observational constraints on the generalized Chaplygin gas (GCG) model for dark energy from the 9 Hubble parameter data points, the 115 SNLS SNe Ia data and the size of baryonic acoustic oscillation peak at redshift, $z = 0.35$. At a 95.4\% confidence level, a combination of three data sets gives $0.67 \leq A_s \leq 0.83$ and $-0.21 \leq \alpha \leq 0.42$, which is within the allowed parameters ranges of the GCG as a candidate of the unified dark matter and dark energy. It is found that the standard Chaplygin gas model ($\alpha = 1$) is ruled out by these data at the 99.7\% confidence level.

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I. INTRODUCTION

Many astrophysical and cosmological observations, including Type Ia Supernovae (SNe Ia)\cite{1} and cosmic microwave background radiation (CMBR)\cite{2, 3} etc, indicated that the universe is undergoing an accelerating expansion. Many works have being done in order to explain this discovery. Some people attribute the observed acceleration to a possible breakdown of our understanding of the laws of gravitation, thus they attempted to modify the Friedmann equation\cite{4, 5}. However, many more think that the cosmic acceleration is driven by an exotic energy component with the negative pressure in the universe, named dark energy, which at late times dominates the total energy density of our universe and accelerates the cosmic expansion. Up to now there are many candidates of dark energy, such as the cosmological constant $\Lambda$\cite{6}, quintessence\cite{7}, phantom\cite{8} and quintom\cite{9} etc.

Recently an interesting model of dark energy, named the Chaplygin gas, was proposed by Kamenshchik et al\cite{10}. This model is characterized by an exotic equation of state

$$p_{ch} = -\frac{A}{\rho_{ch}^\alpha}$$

with a positive constant $A$ and $\alpha = 1$. Progress has been made toward generalizing these model parameters. In this regard, Bento et al. generalized parameter $\alpha$ from 1 to an arbitrary constant in Ref.\cite{11}, and this generalized model was called the generalized Chaplygin gas (GCG) model and can be obtained from a generalized version of the Born-Infeld action. For $\alpha = 0$ the GCG model behaves like the scenario with cold dark matter plus a cosmological constant.

Inserting the above equation of state of the GCG into the energy conservation equation, it is easy to obtain

$$\rho_{ch} = \rho_{ch0} \left( A_s + \frac{1 - A_s}{a^{3(1+\alpha)}} \right)^{\frac{1}{1+\alpha}},$$

where $\rho_{ch0}$ is the present energy density of the GCG and $A_s \equiv A/\rho_{ch0}^{1+\alpha}$. It is worth noting that, when $0 < A_s < 1$, the GCG model smoothly interpolates between a non-relativistic matter phase ($\rho_{ch} \propto a^{-3}$) in the past and at late times a negative pressure dark energy regime ($\rho_{ch} = -p_{ch}$). As a result of this interesting feature, the GCG model has been proposed as a model of the unified dark matter and dark energy (UDME). Meanwhile, for $A_s = 0$ the GCG behaves always like matter while for $A_s = 1$ it behaves always like a cosmological constant.
The GCG model, thus, has been the subject of great interest and many authors have attempted to constrain this UDME model by using various observational data, such as the SNe Ia \cite{12, 13, 14, 15, 16, 17, 18}, the CMBR \cite{18, 19, 20}, the gamma-ray bursts \cite{21}, the gravitational lensing \cite{14, 17, 22}, the X-ray gas mass fraction of clusters \cite{13, 14, 15}, the large scale structure \cite{18, 23}, and the age of high-redshift objects \cite{24}.

In this paper we shall consider the new observational constraints on the parameter space of the GCG for a flat universe by using a measurement of the Hubble parameter as a function of redshift \cite{25}, the new 115 SNe Ia data released by the Supernova Legacy Survey (SNLS) collaboration recently \cite{27} and the baryonic acoustic oscillation (BAO) peak detected in the large-scale correlation function of luminous red galaxies from Sloan Digital Sky Survey (SDSS) \cite{28}. We perform a combined analysis of three databases and find that the degeneracy between $A_s$ and $\alpha$ is broken. At a 95.4\% confidence level we obtain a strong constraint on the GCG model parameters: $0.67 \leq A_s \leq 0.83$ and $-0.21 \leq \alpha \leq 0.42$, a parameter range within which the GCG model could be taken as a candidate of UDME and the pure Chaplygin gas model could be ruled out.

II. CONSTRAINT FROM THE HUBBLE PARAMETER AS A FUNCTION OF REDSHIFT

Last year, based on differential ages of passively evolving galaxies determined from the Gemini Deep Deep Survey \cite{29} and archival data \cite{30}, Simon et al. \cite{31} gave an estimate for the Hubble parameter as a function of the redshift $z$,

$$H(z) = -\frac{1}{1+z} \frac{dz}{dt},$$

(3)

where $t$ is the time. They obtained 9 data points of $H(z)$ at redshift $z_i$ and used the estimated $H(z)$ to constrain the dark energy potential. Later these 9 data points were used to constrain parameters of holographic dark energy model \cite{32}, parameters of the $\Lambda CDM$, $XC\!DM$ and $\phi CDM$ models \cite{34} and the interacting dark energy models \cite{33}. Here we will use this data to constrain the GCG model.

For a flat universe containing only the baryonic matter and the GCG, the Friedmann equation can be expressed as

$$H^2(H_0, A_s, \alpha, z) = H_0^2E^2(A_s, \alpha, z),$$

(4)
where
\[
E(A_s, \alpha, z) = [\Omega_b(1+z)^3 + (1-\Omega_b)(A_s + (1-A_s)(1+z)^{3(1+\alpha)})]^{1/2},
\]
(5)
\(\Omega_b\) is the present dimensionless density parameter of baryonic matter and \(H_0 = 100h\,\text{Kms}^{-1}\,\text{Mpc}^{-1}\) is present Hubble constant. The Hubble Space Telescope key projects give \(h = 0.72 \pm 0.08\) and the WMAP observations give \(\Omega_b h^2 = 0.0233 \pm 0.0008\). The best fit values for model parameters \(A_s, \alpha\) and constant \(H_0\) can be determined by minimizing
\[
\chi^2(H_0, A_s, \alpha) = \sum_{i=1}^{9} \frac{[H(H_0, A_s, \alpha, z_i) - H_{\text{obs}}(z_i)]^2}{\sigma^2(z_i)}.
\]
(6)
Since we are interested in the model parameters, \(H_0\) becomes a nuisance parameter. We marginalize over \(H_0\) to get the probability distribution function of \(A_s\) and \(\alpha\): \(L(A_s, \alpha) = \int dH_0 P(H_0) e^{-\chi^2(H_0, A_s, \alpha)/2}\), where \(P(H_0)\) is the prior distribution function for the present Hubble constant. In this paper a Gaussian priors \(H_0 = 72 \pm 8\,\text{Kms}^{-1}\,\text{Mpc}^{-1}\) is considered.

In Fig. (1), we show the data of the Hubble parameter plotted as a function of redshift for the case \(H_0 = 72\,\text{Kms}^{-1}\,\text{Mpc}^{-1}\). Fig. (2) shows the results of our statistical analysis for the Hubble parameter data. Confidence contours (68.3%, 95.4% and 99.7%) in the \(A_s-\alpha\) plan are displayed by considering the Hubble parameter measurements discussed above. The best fit happens at \(A_s = 0.82\) and \(\alpha = 0.71\). It is very clear that two model parameters, \(A_s\) and \(\alpha\), are degenerate.

### III. JOINT STATISTICS WITH SDSS BAO AND SNLS SNE IA

Using a large spectroscopic sample of 46,748 luminous red galaxy from the SDSS, last year Eisenstein et al [28] successfully found the size of baryonic acoustic oscillation (BAO) peak and obtained a parameter \(A\), which is independent of cosmological models and for a flat universe can be expressed as
\[
A = \frac{\sqrt{\Omega_m}}{E(z_1)^{1/3}} \left[ \frac{1}{z_1} \int_0^{z_1} \frac{dz}{E(z)} \right]^{2/3},
\]
(7)
where \(z_1 = 0.35\), \(A\) is measured to be \(A = 0.469 \pm 0.017\) and \(\Omega_m\) is the effective matter density parameter given by \(\Omega_m = \Omega_b + (1-\Omega_b)(1-A_s)^{1/(1+\alpha)}\) [14, 15, 26]. Using parameter \(A\) we can obtain the constraint on dark energy models from the BAO. In Fig. (3) we show the constraints from this measurement on the parameter space \(A_s - \alpha\). The best fit happens
at \( A_s = 0.76 \) and \( \alpha = 0.01 \). Although the BAO data constrains efficiently the parameter plane into a narrow strip, parameters \( A_s \) and \( \alpha \) are also degenerate.

However, from Fig. (2, 3) it is interesting to see that possible degeneracies between these parameters may be broken by combining these two kinds of observational data. In Fig. (4) we show the results of such an analysis. The best fit happens at \( A_s = 0.61 \) and \( \alpha = -0.28 \). At the 95.4\% confidence level we obtain \( 0.46 \leq A_s \leq 0.79 \) and \(-0.53 \leq \alpha \leq 0.2 \), a stringent constraint on the GCG. Apparently at the 68\% confidence level the scenario of standard dark energy plus dark matter scenario (i.e. the case of \( \alpha = 0 \)) is excluded.

If further adding the new 115 SNLS Sne Ia data [27], which contains 44 previously published nearby Sne Ia \((0.015 < z < 0.125)\) plus 71 distant Sne Ia \((0.15 < z < 1)\) discovered by SNLS and gives the best fit values, \( A_s = 0.78 \) and \( \alpha = 0.16 \), for the GCG model, we find that a more stringent constraint is obtained, namely, at the 95.4\% confidence level a combination of three databases gives \( 0.67 \leq A_s \leq 0.83 \) and \(-0.21 \leq \alpha \leq 0.42 \) with the best fits \( A_s = 0.75 \) and \( \alpha = 0.05 \). In Fig.(5) we show the 68.3\%, 95.4\% and 99.7\% confidence level contours from these three data sets. It is easy to see that our results are consistent with the standard dark energy plus dark matter scenario at a 68\% confidence level.

IV. CONCLUSION AND DISCUSSION

The constraints on the generalized Chaplygin gas (GCG) model, proposed as a candidate of the unified dark matter-dark energy scenario (UDME), has been studied in this paper. The Hubble parameter as a function of redshift has been used to constrain the parameter space of the GCG model. We find, although the Hubble parameter gives a degeneracy between model parameters \( A_s \) and \( \alpha \), the complementary and interesting constraints on the parameters of the model could be obtained. Combining the new SNLS Sne Ia data and the recent measurements of the baryon acoustic oscillations found in the SDSS, we obtained a very stringent constraint on model parameters of GCG. At the 95.4\% confidence level, we found \( 0.67 \leq A_s \leq 0.83 \) and \(-0.21 \leq \alpha \leq 0.42 \). At addition we find at a 68\% confidence level the combination of these three databases allows the scenario of standard dark energy plus dark matter, although the Hubble parameter plus the SDSS BAO exclude it.

Using the X-ray gas mass fractions of galaxy clusters and the dimensionless coordinate distance of Sne Ia and FRIIb radio galaxies, Zhu [15] obtained, at a 95.4\% confidence
level, $A_s = 0.70^{+0.17}_{-0.17}$ and $\alpha = -0.09^{+0.54}_{-0.33}$. Using the CMBR power spectrum measurements from BOOMERANG and Archeops, together with the SNe Ia constraints, Bento et al. [20] found that $0.74 < A_s < 0.85$, and $\alpha < 0.6$. Apparently these results are comparable with our results in this paper, which are within the allowed parameters ranges of the GCG as a candidate of UDME. However the standard Chaplygin gas model ($\alpha = 1$) is ruled out by these data at the 99.7% confidence level. Meanwhile it is easy to see that at a 68.3% confidence level our result is consistent with the standard dark energy plus dark matter scenario (i.e. the case of $\alpha = 0$), which is also in agreement with what obtained in Ref. [15, 20].

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FIG. 1: The Hubble parameters $H(z)$ as a function of $z$ for the case $H_0 = 72 \text{ km/s/Mpc}$. The solid curve corresponds to our best fit to 9 Hubble parameter data plus SNLS SNe Ia data and SDSS baryonic acoustic oscillation peak with $A_s = 0.75, \alpha = 0.05$. The dotted line and dashed line correspond to $A_s = 1.0$ and $A_s = 0.0$ respectively.
FIG. 2: The 68.3%, 95.4% and 99.7% confidence level contours for $A_s$ versus $\alpha$ from the measurement of Hubble parameter with a Gaussian priors $H_0 = 72 \pm 8 \text{km} \text{s}^{-1} \text{Mpc}^{-1}$. The best fit happens at $A_s = 0.82$ and $\alpha = 0.71$. 
FIG. 3: The 68.3%, 95.4% and 99.7% confidence level contours for $A_s$ versus $\alpha$ from the SDSS baryonic acoustic oscillations. The best fit happens at $A_s = 0.76$ and $\alpha = 0.01$. 
FIG. 4: The 68.3%, 95.4% and 99.7% confidence level contours for $A_s$ versus $\alpha$ from the Hubble parameter data plus the SDSS baryonic acoustic oscillations peak. The best fit happens at $A_s = 0.61$ and $\alpha = -0.28$. 
FIG. 5: The 68.3%, 95.4% and 99.7% confidence level contours for $A_s$ versus $\alpha$ from the Hubble parameter data plus the SDSS baryonic acoustic oscillations peak and the SNLS Sne Ia data. The best fit happens at $A_s = 0.75$ and $\alpha = 0.05$. 