$B-L$ Neutrinos

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Abstract

Neutrino masses and mixings are analyzed in terms of left-handed fields and a $6 \times 6$ complex symmetric mass matrix $M$ whose singular values are the neutrino masses. An angle $\theta_\nu$ characterizes the kind of the neutrinos, with $\theta_\nu = 0$ for Dirac neutrinos and $\theta_\nu = \pi/2$ for Majorana neutrinos. At $\theta_\nu = 0$ baryon-minus-lepton number is conserved. If $\theta_\nu \approx 0$, the six neutrino masses coalesce into three nearly degenerate pairs. Thus the tiny mass differences exhibited in the solar and atmospheric neutrino experiments are naturally explained by the approximate conservation of $B-L$. Neutrinos are nearly Dirac fermions.

This $B-L$ model leads to these predictions: neutrinos oscillate mainly between flavor eigenfields and sterile eigenfields, and so the appearance of neutrinos and antineutrinos is suppressed; neutrinos may well be of cosmological importance; in principle the disappearance of $\nu_\tau$ should be observable; and $0\nu\beta\beta$ decay is suppressed by an extra factor of $10^{-5}$ and so will not be seen in the Heidelberg/Moscow, IGEX, GENIUS, or CUORE experiments.

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The current wisdom on neutrinos is that the seesaw mechanism [1] forces their masses to be very small. This paper presents a rather different explanation of the experimental facts based upon the approximate conservation of baryon-minus-lepton number, $B - L$: If $B - L$ is almost conserved, then the six two-component neutrino fields form three nearly Dirac neutrinos, and the six neutrino masses coalesce into three nearly degenerate pairs.

If there are three right-handed neutrinos, then there are six left-handed fields, the three left-handed flavor eigenfields $\nu_e, \nu_\mu,$ and $\nu_\tau$ and the charge conjugates of the three right-handed neutrinos. The neutrino mass matrix is then a $6 \times 6$ complex symmetric matrix $\mathcal{M}$ which admits a singular-value decomposition $\mathcal{M} = U M V^\dagger$. The singular values are the six neutrino masses $m_j$, and the unitary matrix $V^\dagger$ describes the neutrino mixings.

An angle $\theta_\nu$ is introduced that describes the kind of the neutrinos. Dirac neutrinos have $\theta_\nu = 0$, and Majorana neutrinos have $\theta_\nu = \pi/2$. If all Majorana mass terms vanish, that is if $\theta_\nu = 0$, then the standard model conserves $B - L$, which is a global $U(1)$ symmetry. It is therefore natural in the sense of ’t Hooft [2] to assume that $\theta_\nu \approx 0$ so that this symmetry is only slightly broken. The neutrinos then are nearly Dirac fermions and their masses coalesce into three pairs of almost degenerate masses. Thus the approximate conservation of $B - L$ explains the tiny mass differences seen in the solar and atmospheric neutrino experiments without requiring the neutrino masses to be absurdly small. If one sets $\theta_\nu \approx 0.003$, suppresses inter-generational mixing, and imposes a quark-like mass hierarchy, then one may fit the essential features of the solar, reactor, and atmospheric neutrino data with otherwise random mass matrices $\mathcal{M}$ in the eV range. Thus neutrinos easily can have masses that saturate the cosmological bound of about 8 eV. Moreover because neutrinos are almost Dirac fermions, neutrinoless double-beta decay is suppressed by an extra factor $\sim \sin^2 \theta_\nu \sin^2 \phi_\nu \lesssim 10^{-5}$, where $\phi_\nu$ is a second neutrino angle, and is very slow, with lifetimes in excess of $2 \times 10^{27}$ years.

This $B - L$ model of neutrino masses and mixings leads to these predictions about future experiments: The three flavor neutrinos oscillate mainly into the conjugates of the right-handed fields, which are sterile. Thus all experiments that look for the appearance of neutrinos will yield small or null
signals, like those of LSND and KARMEN. Secondly because neutrino masses are not required to be nearly as small as the solar and atmospheric mass differences might suggest, neutrinos may well be an important part of hot dark matter. Thirdly if a suitable experiment can be designed, it should be possible to see the tau neutrino disappear. Fourthly, the rate of neutrinoless double-beta decay is suppressed by an extra factor \( \sim \sin^2 \theta_\nu \sin^2 \phi_\nu \lesssim 10^{-5} \) and hence will not be seen in the Heidelberg/Moscow, IGEX, GENIUS, or CUORE experiments.

Masses and Mixings

Because left- and right-handed fields transform differently under Lorentz boosts, they cannot mix. It is therefore convenient to write the action exclusively in terms of two-component, left-handed fields. The two-component, left-handed neutrino flavor eigenfields \( \nu_e, \nu_\mu, \nu_\tau \) will be denoted \( \nu_i \), for \( i = e, \mu, \tau \). The two-component, left-handed fields that are the charge conjugates of the putative right-handed neutrino fields \( n_{re}, n_{r\mu}, n_{r\tau} \) will be denoted \( n_i = -i \sigma \sigma^2 n^\dagger \) for \( i = e, \mu, \tau \), where \( \sigma^2 \) is the second Pauli spin matrix.

The six left-handed neutrino fields \( \nu_i, n_i \) for \( i = 1, 2, 3 \) can have three kinds of mass terms: The fields \( \nu_i \) and \( n_j \) can form the Dirac mass terms \( iD_{ij} \nu_i \sigma^2 n_j - iD_{ij}^* n^\dagger_j \sigma^2 \nu^\dagger_i \); in a minimal extension of the standard model, the complex numbers \( D_{ij} \) are proportional to the mean value in the vacuum of the neutral component of the Higgs field. The fields \( n_i \) and \( n_j \) can form the Majorana mass terms \( iE_{ij} n_i \sigma^2 n_j - iE_{ij}^* n^\dagger_j \sigma^2 n^\dagger_i \), which break \( B - L \). Because these mass terms connect right-handed neutrino fields, which are sterile, they do not affect neutrinoless double-beta decay, at least in leading order. Within the standard model, the complex numbers \( E_{ij} \) are simply numbers; in a more unified theory, they might be proportional to the mean values in the vacuum of neutral components of Higgs bosons. The fields \( \nu_i \) and \( \nu_j \) can form the Majorana mass terms \( iF_{ij} \nu_i \sigma^2 n_j - iF_{ij}^* n^\dagger_j \sigma^2 \nu^\dagger_i \), which break \( SU(2)_L \otimes U(1)_Y \) and \( B - L \). Because these mass terms connect left-handed neutrino fields, they potentially drive neutrinoless double-beta decay. In a minimal extension of the standard model, the complex numbers \( F_{ij} \) might be proportional to the mean values in the vacuum of the neutral component of a new Higgs triplet \( h_{ab} = h_{ba} \).
Since $\sigma^2$ is antisymmetric and since any two fermion fields $\chi$ and $\psi$ anticommute, it follows that $\chi\sigma^2\psi = \psi\sigma^2\chi$ and $\chi^\dagger\sigma^2\psi^\dagger = \psi^\dagger\sigma^2\chi^\dagger$, which implies that the $3 \times 3$ complex matrices $E$ and $F$ are symmetric $E^\dagger = E$ and $F^\dagger = F$ and that $iD_{ij}\sigma^2\nu_i = iD_{ij}\nu_i\sigma^2\nu_j$. Thus if we introduce the $6 \times 6$ matrix

$$\mathcal{M} = \begin{pmatrix} F & D \\ D^\dagger & E \end{pmatrix}$$

and the (transposed) six-vector $N^\dagger = (\nu_e, \nu_\mu, \nu_\tau, n_e, n_\mu, n_\tau)$ of left-handed neutrino fields, then we may gather the mass terms into the matrix expression

$$i\frac{2}{\mathcal{M}} N^\dagger \mathcal{M} \sigma^2 N - i\frac{2}{\mathcal{M}^\dagger \mathcal{M}} \sigma^2 N^\dagger.$$

(2)

The complex symmetric mass matrix $\mathcal{M}$ is not normal unless the positive hermitian matrix $\mathcal{M} \mathcal{M}^\dagger$ is real because $[\mathcal{M}, \mathcal{M}^\dagger] = 2i \mathfrak{Im} (\mathcal{M} \mathcal{M}^\dagger)$. When the mass matrix $\mathcal{M}$ is real, it may be diagonalized by an orthogonal transformation. In general $\mathcal{M}$ is neither real nor normal; but like every matrix, it admits a singular-value decomposition [3]

$$\mathcal{M} = U M V^\dagger$$

(3)

in which the $6 \times 6$ matrices $U$ and $V$ are both unitary and the $6 \times 6$ matrix $M$ is diagonal, $M = \text{diag}(m_1, m_2, m_3, m_4, m_5, m_6)$, with singular values $m_j \geq 0$, which will turn out to be the masses of the six neutrinos.

The free, kinetic action density of a two-component left-handed spinor $\psi$ is $i\psi^\dagger (\partial_0 - \vec{\sigma} \cdot \nabla) \psi$. Thus by including the mass terms (2), one may write the free action density of the six left-handed neutrino fields $N$ as

$$\mathcal{L}_0 = iN^\dagger (\partial_0 - \vec{\sigma} \cdot \nabla) N + i\frac{2}{\mathcal{M} \sigma^2 N - i\frac{2}{\mathcal{M}^\dagger \mathcal{M} \sigma^2 N^\dagger}$$

(4)

from which follow the equations of motion for $N$

$$(\partial_0 - \vec{\sigma} \cdot \nabla) N = \mathcal{M}^\dagger \sigma^2 N^\dagger$$

(5)

and $N^\dagger$

$$(\partial_0 + \vec{\sigma} \cdot \nabla) \sigma^2 N^\dagger = -\mathcal{M} N.$$

(6)

Applying $\partial_0 + \vec{\sigma} \cdot \nabla$ to the field equation (5) for $N$ and then using the field equation (6) for $N^\dagger$, we find that

$$(\Box - \mathcal{M}^\dagger \mathcal{M}) N = 0,$$

(7)
in which we used the symmetry of the matrix $\mathcal{M}$ to write $\mathcal{M}^*$ as $\mathcal{M}^\dagger$.

The singular-value decomposition $\mathcal{M} = U M V^\dagger$ allows us to express this equation (7) in the form

$$\left( \Box - M^2 \right) V^\dagger N = 0,$$

(8)

which shows that the singular values $m_i$ of the mass matrix $\mathcal{M}$ are the neutrino masses and that the eigenfield of mass $m_j$ is

$$\nu_{m_j} = \sum_{i=1}^{6} V^*_{ij} N_i.$$ (9)

The vector $N_m$ of mass eigenfields is thus $N_m = V^\dagger N$, and so the flavor eigenfields $N$ are given by $N = V N_m$. In particular, the three left-handed fields $\nu_i$ for $i = e, \mu, \tau$ are linear combinations of the six mass eigenfields, $\nu_i = \sum_{j=1}^{6} V_{ij} \nu_{m_j}$ and not simply linear combinations of three mass eigenfields.

### Experimental Constraints

The four LEP measurements of the invisible partial width of the $Z$ impose [4] upon the number of light neutrino types the constraint $N_\nu = 2.984 \pm 0.008$. The amplitude for the $Z$ production of two neutrinos $\nu_{m_j}$ and $\nu_{m_k}$ to lowest order is $A(\nu_{m_j}, \nu_{m_k}) \propto \sum_{i=1}^{3} V^*_{ik} V_{ij}$, and therefore the x-section for that process is $\sigma(\nu_{m_j}, \nu_{m_k}) \propto |\sum_{i=1}^{3} V^*_{ik} V_{ij}|^2$. The LEP measurement of the number $N_\nu$ of light neutrino species thus implies that the sum over the light-mass eigenfields is

$$\sum_{j,k \text{ light}} |\sum_{i=1}^{3} V^*_{ik} V_{ij}|^2 = 2.984 \pm 0.008.$$ (10)

This constraint on the $6 \times 6$ unitary matrix $V$ is quite well satisfied if all six neutrino masses are light. For in this all-light scenario, the sum is

$$\sum_{j,k=1}^{6} \sum_{i=1}^{3} \sum_{i'=1}^{3} V^*_{ik} V_{ij} V^*_{i'k} V^*_{i'j} = \sum_{i=1}^{3} \sum_{i'=1}^{3} \delta_{ii'} \delta_{ii'} = \sum_{i=1}^{3} 1 = 3 \simeq 2.984 \pm 0.008.$$ (11)
If the Hubble constant in units of $100 \text{ km/sec/Mpc}$ is $h \simeq 0.65$, then the conservative upper bound on the neutrino component of hot dark matter, $\Omega_\nu \lesssim 0.2$, implies \[5, 6\] that the sum of the masses of the light, stable two-component neutrinos that interact weakly is bounded by

$$
\sum_{j \text{ light}} m_j \lesssim 8 \text{ eV}.
$$

(12)

The lowest-order amplitude for a neutrino $\nu_i$ to be produced by a charged lepton $e_i$, to propagate with energy $E$ a distance $L$ as some light-mass eigenfield of mass $m_j \ll E$, and to produce a charged lepton $e_i'$ is

$$
A(\nu_i \rightarrow \nu_i') \propto \sum_{j \text{ light}} V^*_{i,j} V_{i',j} e^{-\frac{im^2_j L}{2E}}.
$$

(13)

The lowest-order amplitude for the anti-process, $\bar{\nu}_i \rightarrow \bar{\nu}_i'$, involves the complex conjugate of the matrix $V$

$$
A(\bar{\nu}_i \rightarrow \bar{\nu}_i') \propto \sum_{j \text{ light}} V^*_{i',j} V_{i,j} e^{-\frac{im^2_j L}{2E}}.
$$

(14)

To lowest order the corresponding probabilities are

$$
P(\nu_i \rightarrow \nu_i') \propto \sum_{j,j' \text{ light}} V^*_{i,j} V_{i',j'} V_{i,j'}^* V_{i',j} \exp \left( \frac{i(m^2_j - m^2_{j'})L}{2E} \right) \tag{15}
$$

and

$$
P(\bar{\nu}_i \rightarrow \bar{\nu}_i') \propto \sum_{j,j' \text{ light}} V^*_{i',j} V_{i,j'} V^*_{i,j} V_{i',j} \exp \left( \frac{-i(m^2_j - m^2_{j'})L}{2E} \right) \tag{16}
$$

If all six neutrinos are light, then in the limit $L/E \rightarrow 0$ these sums are $\delta_{ii'}$.

If for simplicity we stretch the error bars on the Chlorine experiment and average over one year, then the solar neutrino experiments, especially Gallex and SAGE, see a diminution of electron neutrinos by a factor of about one-half:

$$P_{\text{sol}}(\nu_e \rightarrow \nu_e) \simeq \frac{1}{2}, \tag{17}
$$

which requires a pair of mass eigenstates whose squared masses differ by at least $\sim 10^{-10} \text{ eV}^2$ \[5\]. The reactor experiments, Palo Verde and especially Chooz, imply that these squared masses differ by less than $\sim 10^{-3} \text{ eV}^2$ \[5\].

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The atmospheric neutrino experiments, Soudan II, Kamiokande II, IMB-3, and especially SuperKamiokande, see a diminution of muon neutrinos and antineutrinos by about one-third:

\[ P_{\text{atm}}(\nu_\mu \to \nu_\mu) \simeq \frac{2}{3}, \quad (18) \]

which requires a pair of mass eigenstates whose squared masses differ by \(10^{-3} \text{eV}^2 \lesssim |m_j^2 - m_k^2| \lesssim 10^{-2} \text{eV}^2 \) [5].

**The \( B - L \) Model**

When the Majorana mass matrices \( E \) and \( F \) are both zero, the action density (4) is invariant under the \( U(1) \) transformation \( N' = e^{i\theta G} N \) in which the \( 6 \times 6 \) block-diagonal matrix \( G = \text{diag}(I, -I) \) with \( I \) the \( 3 \times 3 \) identity matrix. The kinetic part of (4) is clearly invariant under this transformation. The mass terms are invariant only when the anti-commutator

\[ \{ \mathcal{M}, G \} = 2 \begin{pmatrix} F & 0 \\ 0 & -E \end{pmatrix} = 0 \quad (19) \]

vanishes.

This \( U(1) \) symmetry is the restriction to the neutrino sector of the symmetry generated by baryon-minus-lepton number, \( B - L \), which is exactly conserved in the standard model. A minimally extended standard model with right-handed neutrino fields \( n_{ri} \) and a Dirac mass matrix \( D \) but with no Majorana mass matrices, \( E = F = 0 \), also conserves \( B - L \). When \( B - L \) is exactly conserved, \( i.e., \) when \( D \neq 0 \) but \( E = F = 0 \), then the six neutrino masses \( m_j \) collapse into three pairs of degenerate masses because the left-handed and right-handed fields that form a Dirac neutrino have the same mass.

Suppose this symmetry is slightly broken by the Majorana mass matrices \( E \) and \( F \). Then for random mass matrices \( D, E, \) and \( F \), the six neutrino masses \( m_j \) will form three pairs of nearly degenerate masses as long as the ratio

\[ \sin^2 \theta_\nu = \frac{\text{Tr}(E^\dagger E + F^\dagger F)}{\text{Tr}(2D^\dagger D + E^\dagger E + F^\dagger F)} \quad (20) \]
Figure 1: The six neutrino masses are plotted against the parameter \( \sin \theta_{\nu} \) for a set of random \( 6 \times 6 \) mass matrices.

is small. For a generic mass matrix \( M \), the parameter \( \sin^2 \theta_{\nu} \) lies between the extremes \( 0 \leq \sin^2 \theta_{\nu} \leq 1 \) and characterizes the kind of the neutrinos. The parameter \( \sin^2 \theta_{\nu} \) is zero for purely Dirac neutrinos and unity for purely Majorana neutrinos.

Let us now recall 't Hooft’s definition [2] of naturalness: It is natural to assume that a parameter is small if the theory becomes more symmetrical when the parameter vanishes. In this sense it is natural to assume that the parameter \( \sin^2 \theta_{\nu} \) is small because the minimally extended standard model becomes more symmetrical, conserving \( B - L \), when \( \sin^2 \theta_{\nu} = 0 \).

In Fig. 1 the six neutrino masses \( m_j \) are plotted for a set of mass matrices \( \mathcal{M} \) that differ only in the parameter \( \sin \theta_{\nu} \). Apart from \( \sin \theta_{\nu} \), every other parameter of the mass matrices \( \mathcal{M} \) is a complex number \( z = x + iy \) in which \( x \) and \( y \) were chosen randomly and uniformly on the interval \([-1 \text{ eV}, 1 \text{ eV}]\). It is clear in the figure that when \( \sin^2 \theta_{\nu} \approx 0 \), the six neutrino masses \( m_j \) coalesce into three nearly degenerate pairs. Although the six masses of the neutrinos are in the eV range, they form three pairs with very tiny mass differences when \( \sin^2 \theta_{\nu} \approx 0 \).
Thus the very small mass differences required by the solar and atmospheric experiments are naturally explained by the assumption that the symmetry generated by $B - L$ is broken only slightly by the Majorana mass matrices $E$ and $F$. This same assumption implies that neutrinos are very nearly Dirac fermions and hence explains the very stringent upper limits [5] on neutrinoless double-beta decay. Because the masses of the six neutrinos may lie in the range of a few eV, instead of being squashed down to the meV range by the seesaw mechanism, they may contribute to hot dark matter in a way that is cosmologically significant. This $B - L$ model with $\sin^2 \theta_\nu \approx 0$ is the converse of the seesaw mechanism.

If $\sin^2 \theta_\nu = 0$, then there are three purely Dirac neutrinos, and the mixing matrix $V$ is block diagonal $V = \text{diag}(u^*, v)$ in which the $3 \times 3$ unitary matrices $u$ and $v$ occur in the singular-value decomposition of the $3 \times 3$ matrix $D = u m v^\dagger$. If these three Dirac neutrinos are also light, then unitarity implies that the sum of the normalized probabilities is unity $\sum_{i'=e}^\tau P(\nu_i \to \nu_{i'}) = 1$.

If $\sin^2 \theta_\nu = 1$, then this sum is also unity by unitarity because in this case the mixing matrix for the six purely Majorana neutrinos is also block diagonal $V = \text{diag}(v_F, v_E)$. But if there are six light, nearly Dirac neutrinos, then each neutrino flavor $\nu_i$ will oscillate both into other neutrino flavor eigenfields and into sterile neutrino eigenfields. In this case this sum tends to be roughly a half $\sum_{i'=e}^\tau P(\nu_i \to \nu_{i'}) \simeq \frac{1}{2}$ as long as $\sin^2 \theta_\nu$ is small but not infinitesimal.

Because of this approximate, empirical sum rule [7] for $i = e$ and $\mu$, the only way in which the probabilities $P_{\text{sol}}(\nu_e \to \nu_i)$ and $P_{\text{atm}}(\nu_\mu \to \nu_i)$ can fit the experimental results (17) and (18) is if inter-generational mixing is suppressed so that $\nu_e$ oscillates into $n_e$ and so that $\nu_\mu$ oscillates into $n_\mu$. In other words, random mass matrices $M$, even with $\sin \theta_\nu \approx 0$, produce probabilities $P_{\text{sol}}(\nu_e \to \nu_e)$ and $P_{\text{atm}}(\nu_\mu \to \nu_\mu)$ (suitably averaged respectively over the Earth’s orbit and over the atmosphere) that are too small. The probabilities $P_{\text{sol}}(\nu_e \to \nu_e)$ and $P_{\text{atm}}(\nu_\mu \to \nu_\mu)$ do tend to cluster around $(\frac{1}{2}, \frac{2}{3})$ as required by the experiments when inter-generational mixing is severely repressed, that is if the singly off-diagonal matrix elements of $D, E,$ and $F$ are suppressed by 0.05 and the doubly off-diagonal matrix elements by 0.0025.

It is possible to relax the factors that suppress inter-generational mixing to 0.2 and 0.04 and improve the agreement with the experimental constraints (17) and (18) (while satisfying the CHOOZ constraint) provided that one also
requires that there be a quark-like mass hierarchy. The points in Fig. 2 were generated by random mass matrices $\mathbf{M}$ with $\sin \theta_\nu = 0.003$ by using CKM-suppression factors of 0.2 and 0.04 and by scaling the $i,j$-th elements of the mass matrices $E, F, D$ by the factor $f(i) \ast f(j)$ where $\vec{f} = (0.2, 1, 2)$. Thus the mass matrix $\mathbf{M}$ has the $\tau, \tau$ elements that are larger than its $\mu, \mu$ elements and $\mu, \mu$ elements that in turn are larger than its $e, e$ elements. The clustering of the probabilities $P_{\text{sol}}(\nu_e \to \nu_e)$ and $P_{\text{atm}}(\nu_\mu \to \nu_\mu)$ around $(\frac{1}{2}, \frac{2}{3})$ in Fig. 2 shows that the experimental results (17) and (18) are satisfied. The vector $\vec{f}$ was tuned so as to nearly saturate the cosmological upper bound (12) of about 8 eV.

In this scatter plot, every parameter of each of the 10000 matrices $\mathbf{M}$ is a complex number $z = x + iy$ with $x$ and $y$ chosen randomly and uniformly from the interval $[-1\text{eV}, 1\text{eV}]$. The solar neutrinos are taken to have an energy of 1 MeV, and the probability (15) is averaged over one revolution of the Earth about the Sun. The atmospheric neutrinos are taken to have an energy of 1 GeV, and the probability (15) is averaged over the atmosphere weighted by $\sec \theta_\nu^2$ in the notation of Fisher et al. [5]. The thousands of singular-value decompositions were performed by the LAPACK [3] driver subroutine ZGESVD [8].

Neutrinoless double beta decay occurs when a right-handed antineutrino emitted in one decay $n \to p + e^- + \bar{\nu}_e$ is absorbed as a left-handed neutrino in the another decay $\nu_e + n \to p + e^-$. To lowest order these decays proceed via the Majorana mass term $-i F_{ee}^* \nu^\dagger_e \sigma^2 \nu^\dagger_e$. Let us introduce a second angle $\phi_\nu$ defined by

$$
\sin^2 \phi_\nu = \frac{\text{Tr}(F^\dagger F)}{\text{Tr}(E^\dagger E + F^\dagger F)}.
$$

We have seen that we may fit the experimental data (17) and (18) by assuming that $\sin \theta_\nu \simeq 0.003$ and by requiring the mass matrices $E, F, D$ to exhibit quark-like mass hierarchies with little inter-generational mixing. Under these conditions the rate of $0\nu\beta\beta$ decay is limited by the factor

$$
|F_{ee}|^2 \lesssim \sin^2 \theta_\nu \sin^2 \phi_\nu m_{\nu_e}^2,
$$

in which $m_{\nu_e}$ is the heavier of the lightest two neutrino masses. Thus the rate of $0\nu\beta\beta$ decay is suppressed by an extra factor $\sim \sin^2 \theta_\nu \sin^2 \phi_\nu \lesssim 10^{-5}$ resulting in lifetimes $T_{1/2}^{0\nu\beta\beta} > 2 \times 10^{27} \text{ yr}$. The $B-L$ model therefore explains why neutrinoless double-beta decay has not been seen and predicts that the
Figure 2: The probabilities $P_{\text{sol}}(\nu_e \rightarrow \nu_e)$ and $P_{\text{atm}}(\nu_\mu \rightarrow \nu_\mu)$ for 10000 random mass matrices $\mathcal{M}$ all with the parameter $\sin \theta_\nu = 0.003$, with inter-generational mixing suppressed, and with a quark-like mass hierarchy.

Conclusions

The standard model slightly extended to include right-handed neutrino fields exactly conserves $B - L$ if all Majorana mass terms vanish. It is therefore natural [2] to assume that the Majorana mass terms are small compared to the Dirac mass terms. A parameter $\sin^2 \theta_\nu$ is introduced that characterizes the relative importance of these two kinds of mass terms. When this parameter is very small, then the neutrinos are nearly Dirac and only slightly Majorana. In this case the six neutrino masses $m_j$ coalesce into three pairs of nearly degenerate masses. Thus the very tiny mass differences seen in the solar and atmospheric neutrino experiments are simply explained by the natural assumption that $\sin \theta_\nu \simeq 0.003$ or equivalently that $B - L$ is almost conserved. In these experiments the probabilities $P_{\text{sol}}(\nu_e \rightarrow \nu_e)$ and
$P_{\text{atm}}(\nu_\mu \rightarrow \nu_\mu)$ are respectively approximately one half and two thirds. One may fit these probabilities with random mass matrices in the eV range by requiring the neutrino mass matrices $E, F,$ and $D$ to exhibit quark-like mass hierarchies with little inter-generational mixing.

This $B - L$ model leads to these predictions:

1. Because $\sin^2 \theta_\nu \approx 0$ and because inter-generational mixing is suppressed, neutrinos oscillate mainly into sterile neutrinos of the same flavor and not into neutrinos of other flavors. Hence rates for the appearance of neutrinos, $P(\nu_i \rightarrow \nu_{i'})$ with $i \neq i'$, are very low as shown by LSND and KARMEN.

2. The assumption that $\sin^2 \theta_\nu$ is very small naturally explains the very small differences of squared masses seen in the solar and atmospheric experiments without requiring that the neutrino masses themselves be very small. Thus the neutrinos may very well saturate the cosmological bound, $\sum_j m_j \lesssim 8 \text{ eV}$. In fact the masses associated with the points of Fig. 2 do nearly saturate this bound. Neutrinos thus may well be an important part of hot dark matter.

3. The disappearance of $\nu_\tau$ should in principle be observable.

4. In the $B - L$ model, the rate of neutrinoless double-beta decay is suppressed by an extra factor $\sim \sin^2 \theta_\nu \sin^2 \phi_\nu \lesssim 10^{-5}$ resulting in lifetimes greater than $2 \times 10^{27}$ yr. Thus the current and upcoming experiments Heidelberg/Moscow, IGEX, GENIUS, and CUORE will not see $0\nu\beta\beta$ decay.

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References

[1] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity* (ed. by P. van Nieuwenhuisen and D. Z. Freedman, North-Holland, 1979), p. 315, and R. N. Mohapatra and G. Senjanovic, *Phys. Rev. Lett.* 99, 412 (1980).

[2] G. ’t Hooft, in *Recent Developments in Gauge Theories* (proc. 1979 Cargèse Summer Institute, ed. by G. ’t Hooft et al., (Plenum Press, 1980), p. 135. See also H. Georgi, *Weak Interactions and Modern Particle Theory* (Benjamin/Cummings, 1984), p. 127.

[3] E. Anderson, et al., *LAPACK Users’ Guide* (3d ed., SIAM, Philadelphia, PA, 1999) online at http://www.netlib.org/lapack/lug/lapack_lug.html.

[4] C. Caso et al., *E. Phys. J.* C3, 1 (1998) and 1999 off-year update http://pdg.lbl.gov/.

[5] P. Fisher, B. Kayser, and K. S. McFarland, hep-ph/9906244 and *Ann. Rev. Nucl. Part. Sci.* 49 (1999, in press).

[6] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Redwood City, CA, 1990), p. 123.

[7] K. Cahill, hep-ph/9912416.

[8] All of LAPACK including the subroutine ZGESVD are freely available from http://www.netlib.org/lapack/.