Parameter determination for the Mini-Oscillator Model of the Viscoelastic Material

HUANG Zhi-cheng¹, WU Nan-xing*, WANG Xing-guo¹, Li zelun²

¹Jingdezhen Ceramic Institute, Jingdezhen 333001, China
²College of Mechanical and Dynamic Engineering, Chongqing University of Science and Technology
huangwu555@sina.com

Abstract. The viscoelastic composite structure is widely used in the vibration and noise suppression of thin-walled components. The vibration analysis of viscoelastic composite structures must involve the constitutive equation of viscoelastic materials. The form of the constitutive equation of viscoelastic material has a decisive influence on the dynamic analysis process of viscoelastic composite structures. Since the constitutive relation of the viscoelastic material changes with time, frequency and temperature, the analysis of the dynamic characteristics of the viscoelastic composite structure is greatly complicated. The mini-oscillator model considers the frequency-dependent properties of viscoelastic materials. Therefore, it is widely used in the dynamic analysis of composite structures. Aims at the need of viscoelastic material passive vibration control for viscoelastic composite structures, a method for determining the parameters of mini-oscillator model is proposed. The method obtains the viscoelastic material mini-oscillator model parameters by parameter fitting by the measured viscoelastic material complex modulus data in the frequency domain or other viscoelastic material damping model expressions obtained from experimental data. The results are compared with fractional derivative model. The results show that the mini-oscillator model can correctly describe stress-strain relationship of viscoelastic material and the parameter fitting method proposed in this paper is accurate and effective.

1. Introduction
The viscoelastic material is composed of a high molecular polymer, which has both a large viscosity and elasticity. When subjected to alternating stress, the strain of the viscoelastic material is not proportional to the stress as the elastic material, but lags behind the stress. The stress-strain curve is approximated by an elliptical hysteresis loop as shown in Figure 1, resulting in dissipation of energy, which corresponds to material damping [1]. Due to its good damping properties, the viscoelastic material and the elastic material are bonded together to construct a viscoelastic composite material, which can suppress the vibration and noise of the structure well. Therefore, viscoelastic materials are widely used in vibration damping and noise reduction of engineering structures.

An equation describing the stress-strain-temperature relationship of a material is called a constitutive equation. The vibration analysis of viscoelastic composite structures must involve the constitutive equations and forms of viscoelastic materials. The form of constitutive equations of viscoelastic materials has a decisive influence on the dynamic analysis process of viscoelastic composite structures. Since the constitutive relation of the viscoelastic material changes with time, frequency and temperature, the analysis of the dynamic characteristics of the viscoelastic structure or
the viscoelastic composite structure is greatly complicated. For the same material, different forms of relaxation modulus function are used, and the constitutive equations of viscoelastic materials will also appear in different specific forms. To this end, scholars have proposed a variety of viscoelastic material constitutive models, such as typical and commonly used complex constant modulus model\textsuperscript{[2]}, standard rheological model\textsuperscript{[3]}, fractional derivative model\textsuperscript{[4-7]} and mini-oscillator model\textsuperscript{[8-10]}. The true mechanical behavior of viscoelastic materials is related to frequency. The parameters in the complex constant modulus model are constant, regardless of their frequency dependence. Standard rheological model requires a wide range of frequencies to obtain the performance parameters of viscoelastic materials, which can lead to practical difficulties. The fractional derivative model is difficult to transform into the time domain, and the calculation is large when the vibration analysis of the structure is performed. The error introduced in the solution process is too large. This is the inherent characteristic of the fractional derivative, which limits the application of the model. The Mini-Oscillator model can be combined with finite element method, which results in its widely application. The Mini-Oscillator model of viscoelastic material has many parameters and is difficult to determine. A curve fitting method is used to determine the model parameters and verified to solve this problem.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{stress_strain_curve.png}
\caption{The stress-strain curve of viscoelastic material}
\end{figure}

2. Mini-Oscillator model
The Mini-Oscillator model represents the complex shear modulus function of the viscoelastic material as an algebraic sum of a series of Mini-Oscillator sub-items. It was proposed by Golla and Hughes and was modified by McTavish, so it is also known as the GHM model. The schematic diagram of the principle is shown in Figure 2.

In the GHM model, the complex shear modulus function of the viscoelastic material in the Laplace domain can be expressed:

\[ sG(s) = G^\infty 1 + \sum_{k=1}^{N} \alpha_k \frac{s^2 + 2 \zeta \omega_k s}{s^2 + 2 \zeta \omega_k s + \omega_k^2} \]  \hspace{1cm} (1)

Where, the \( G^\infty \) is the steady state value of the relaxation function, \( s \) is the Laplacian operator. \( N \) is the number of terms of the micro-vibrator, which is determined by the frequency dependence of the complex shear modulus. \( \{\alpha_k, \omega_k, \zeta\} \) are three positive constants. Obviously, if the \( N \)-order Micro-oscillator is chosen, the model has \( 3N+1 \) parameters that need to be determined. These parameters can be obtained by curve fitting of experimental data.
3. Parameter determination method of GHM model

The mechanical properties of viscoelastic materials are generally more convenient to measure in the frequency domain, so the parameters of the GHM model can be determined by curve fitting the data measured in the frequency domain. The curve fitting expression is the expression of the GHM model in the frequency domain.

Let \( s = j\omega \) in equation (1), one can get

\[
G^\prime (i\omega) = G^\prime \left[ 1 + \sum_{k=1}^{N} \frac{\alpha_k}{(i\omega)^2} + \frac{2\xi_k\omega_k}{(i\omega)^2} + \frac{\omega_k^2}{\omega^2} \right] = G^\prime_k(\omega) + iG^\prime_k(\omega) \tag{2}
\]

The parameters of the GHM model can be determined by curve fitting the measured complex modulus data in the frequency domain by equation (2). Obviously this is a nonlinear optimization problem with constraints in the complex plane. The Matlab software can be used to write the corresponding optimization program to find the parameters of the GHM model. The optimization objective function can be expressed as

\[
F(x) = \sum_{i=1}^{M} \left[ G^\prime(x, \omega_i) - G_0(\omega_i) \right]^2 = \min
\]

Where, \( G^\prime(x, \omega_i) \) is the GHM model to be determined, \( G_0(\omega_i) \) is the measured modulus of viscoelastic material in the complex frequency domain, \( M \) is the measured number of complex modulus, \( x \) is the GHM model parameter, and whose expression is

\[
x_i = G^\prime, x_2 = \alpha_1, x_3 = \alpha_2, \ldots x_{N+1} = \alpha_N, x_{N+2} = \xi_1, \ldots x_{3N+3} = \xi_2, \ldots
\]

\[
x_{2N+1} = \frac{G^\prime_1}{G^\prime}, x_{2N+2} = \xi_1, x_{2N+3} = \xi_2, \ldots x_{3N+1} = \xi_N, \quad x_i > 0, i = 1, 2, 3 \cdots \]

Solving the above optimization problem and one can determine the parameters of viscoelastic material GHM model.

4. Numerical Simulation

Reference [1] fitted a three-parameter fractional derivative model of a viscoelastic material in the frequency domain based on the measured data and verified it by experiment. The expression of the three-parameter fractional derivative model is

\[
G^\prime(s) = \left( 3.6275 + 0.2091 s^{0.6275} \right) \times 10^5 \tag{5}
\]

Substituting Equation (5) into Equation (3), curve fitting it by the method described above, one can obtained the GHM model parameters. 3 mini-oscillators are used to fit the curve, and that is, in equation (2), the parameters to be determined are 10, and the results are shown in Table 1. Figure 3(a)
is the curve fitting result of real part. Figure 3(b) is the curve fitting result of imaginary part. Figure 3(c) is the curve fitting result of error.

| Parameters | $k = 1$ | $k = 2$ | $k = 3$ |
|------------|---------|---------|---------|
| $G^\infty$ | 5.7618e5 |
| $\alpha_k$ | 24.3411 | 2.1611 | 10.9283 |
| $\zeta_k$  | 180.3360 | 88.4707 | 4.4230 |
| $\hat{\omega}_k$ | 4.1598e6 | 1.3464e5 | 4.0512e5 |

![Fig.3 (a)](image-url) The real part fitting of the Mini-Oscillator model

![Fig.3 (b)](image-url) The imaginary part fitting of the Mini-Oscillator model
It can be seen from the fitting results of Fig. 3 that the GHM model can well simulate the modulus of the viscoelastic material when three micro vibrators are used. The fitting accuracy of the real part and the imaginary part is high, and the relative error is less than 3% in a wide frequency band (100 Hz-500 Hz). When the frequency is less than 20HZ, the fitting result is distorted. Regardless of the real part fitting or the imaginary part fitting, the high frequency range is better than the low frequency range, and the imaginary part fitting is better than the real part fitting.

5. Conclusion
This paper analyzes the method of calculating the parameters of viscoelastic material Mini-Oscillator model by curve fitting. The numerical simulation results show that the parameter determination method of the Mini-Oscillator model is correct and effective. The Mini-Oscillator model can also simulate the frequency-dependent properties of viscoelastic material parameters well. It is feasible to use the Mini-Oscillator model for the study of the dynamic properties of viscoelastic materials.

Acknowledgements
This research is supported by Natural Science Foundation of China (11862007, 51565020), Key Project of Jiangxi Youth Science Foundation of China (20171ACB21047), Key Science and Technology Projects of Jiangxi Education Department of China (GJJ160864) and Jingdezhen Science and Technology Bureau Project (20182GYZD011-02).

References
[1] Park CH. Vibration control of plates with active constrained layer damping [D]. Washington: The School of Engineering of the Catholic University, 1996.
[2] Scanlan R H. Linear damping models and causality in vibrations. Journal of Sound and Vibration, 1970, 13(4): 499-503.
[3] Lesieutre G A, Bianchini E. Time domain modeling of linear viscoelasticity using anelastic displacement fields. Journal of Vibration and Acoustics, 1995, 117(4): 424-430.
[4] Bagley R L, Torvik J. Fractional calculus-a different approach to the analysis of viscoelastically damped structures. AIAA journal, 1983, 21(5): 741-748.
[5] Bagley R L, Torvik P J. Fractional calculus in the transient analysis of viscoelastically damped structures. AIAA journal, 1985, 23(6): 918-925.
[6] Bagley R L, Torvik P J. On the fractional calculus model of viscoelastic behavior. Journal of
Rheology, 1986, 30(1): 133-155.

[7] Bagley R L, Calico R A. Fractional order state equations for the control of viscoelastically damped structures. Journal of Guidance Control and Dynamics, 1991, 14(2): 304-311.

[8] Golla D F, Hughes P C. Dynamics of viscoelastic structures-a time-domain, finite element formulation. ASME J. Appl. Mech, 1985, 52(4): 897-906.

[9] Mctavish D J, Hughes P C. Finite Element Modeling of Linear Viscoelastic Structures-The GHM Method, 1992, 13-15.

[10] Mctavish D J, Hughes P C. Modeling of linear viscoelastic space structures. Journal of Vibration and Acoustics, 1993, 115(1): 103-110.

[11] Chen qian, Zhu demao. The Form of Constitutive Equation of Viscoelastic Materials in Vibration Analysis of Composite Structures [J]. Journal of Applied Mechanics, 1987, 4(1): 39-51.