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Nonlocal Buckling Analysis of Composite Curved Beams Reinforced with Functionally Graded Carbon Nanotubes

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Abstract: This work deals with the size-dependent buckling response of functionally graded carbon nanotube-reinforced composite (FG-CNTRC) curved beams based on a higher-order shear deformation beam theory in conjunction with the Eringen Nonlocal Differential Model (ENDM). The material properties were estimated using the rule of mixtures. The Hamiltonian principle was employed to derive the governing equations of the problem which were, in turn, solved via the Galerkin method to obtain the critical buckling load of FG-CNTRC curved beams with different boundary conditions. A detailed parametric study was carried out to investigate the influence of the nonlocal parameter, CNTs volume fraction, opening angle, slenderness ratio, and boundary conditions on the mechanical buckling characteristics of FG-CNTRC curved beams. A large parametric investigation was performed on the mechanical buckling behavior of FG-CNTRC curved beams, which included different CNT distribution schemes, as useful for design purposes in many practical engineering applications.

Keywords: buckling; Galerkin method; nanocomposites; nonlocal elasticity theory

1. Introduction

The reinforcement of nanocomposites with the introduction of carbon nanotubes (CNTs) as filler beside a polymeric matrix is well known to improve the potential applications of a structure in some fields of mechanics and electronics. Indeed, in recent decades, CNTs reinforced nanocomposites have been increasingly studied in the scientific community because of their remarkable properties [1–9]. CNTs are made of graphene sheets as it is the thinnest material in the world. Therefore, the use of CNTs with very small dimensions cannot disregard the possibility of size-dependent behavior of materials, especially at a nanoscale. This represents a challenging aspect to consider during the evaluation of the structural behavior of nanomaterials. To overcome this issue, a large variety of methods and strategies have been proposed in the literature, including laboratory tests, molecular dynamics-based simulations, and non-classical mathematical methods [10–19]. Among them, experimental tests and molecular dynamics simulations, however, are typically expensive and time-consuming, which has led to find an attention to use theoretical and numerical models for approaching similar problems. In this framework, Eringen [20,21] proposed a size-dependent model in which the size-dependent behavior is considered by introducing one small-scale nonlocal parameter. However, this approach considers only the softening enhancement of the size-dependence in nanostructured systems. Bouafia et al. [22] analyzed the bending and vibration response of FG nanobeams via a nonlocal quasi-3D...
theory. Shahsavari et al. [23] studied the forced vibration of viscoelastic graphene sheet under the moving load using a nonlocal refined plate theory. Ganapathi et al. [24] studied the vibrations of curved nanobeams via a nonlocal higher-order theory based on a finite element approach. For the first time, a guided wave propagation analysis of porous nanoplates was performed by Karami et al. [25] using the differential constitutive nonlocal model of Eringen in conjunction with the first-order shear deformation theory. The elastic stability response of curved nanobeams was analyzed by Polit et al. [26] using a nonlocal higher-order shear deformation theory employed in a finite element context. A further application of the nonlocal higher-order theory can be found in the work of Ganapathi and Polit [27] for the numerical study of the bending and buckling response of curved nanobeams, including the thickness stretching effect. For the first time, the shear buckling analysis of porous nanoplates was presented by Shahsavari et al. [28] using a nonlocal quasi-3D plate theory. A different single variable shear deformable nonlocal theory was applied instead, by Shimpi et al. [29], for the static analysis of rectangular micro/nanobeams subjected to a transverse loading, whereas a comprehensive study of the CNTs reinforced composite plates was presented by Karami et al. [30] by applying a nonlocal second-order shear deformable theory.

In a context where curved structures like beams or tubes play a remarkable role in many nanotechnology applications because of their engineering properties (i.e., high strength/stiffness to weight ratios), various size-dependent investigations of reinforced curved beams, tubes, and shells have been carried out in literature [31–41], including different theoretical or computational strategies.

In the current work, the buckling response of CNT reinforced composite curved beams was investigated through the constitutive equations of the nonlocal elasticity, while originally employing the Galerkin method. A continuum model of the nanobeam was also considered based on a higher-order refined theory of beams, which included the shear deformation effects without any proper introduction of shear correction factors. The nonlocal governing equations of the CNT reinforced curved size-dependent beams are here described by means of the Hamiltonian principle, which has been written in a variational form, and they are solved numerically for simply-supported and clamped boundary conditions. After evaluating the accuracy of the proposed method using the available literature, we represent the main results based on a large parametric investigation aimed to studying the influence of boundary conditions, opening angles, CNT distribution patterns, volume fractions, and nonlocal parameters on the critical mechanical buckling force, which is useful for the structural analysis and design of composite curved nanostructures.

The paper is organized as follows. Following the introduction section, we describe the basic fundamentals of the size-dependent problem in Section 2, while the considered solution strategy is presented in Section 3. Afterwards, Section 4 presents the numerical results of a large parametric investigation, useful for design purposes for many engineering applications. Finally, concluding remarks are summarized in Section 5.

2. Size-Dependent Problem

2.1. Basic Fundamentals

In this section, we consider the nonlocal model of Eringen [20], which is based on the following stress-strain relations:

\[ \sigma_{ij} = \int_V \alpha(|x' - x|) \tau_{ij}(x') \, dV' \]  

\[ \tau_{ij} = \int_V \alpha(|x' - x|) \tau_{aij}(x') \, dV' \]  

(1)

\[ \sigma_{ij} \text{ and } \tau_{ij} \text{ being the local and nonlocal stress tensors, together with the following differential equations typically defined for a size-dependent behavior of nanostructure systems:} \]

\[ (1 - (\epsilon_0 a)^2 \nabla^2) \sigma_{ij} = C_{ijkl} \epsilon_{kl} \]

(2)

where \( \nabla^2 \) is the Laplacian operator.
Let us consider a CNTRC curved beam with length \( L \) and thickness \( h \), as shown in Figure 1. Two different distributions of CNTs are here considered, namely a uniform distribution (UD) and a non-uniform functionally graded (FG) distribution, along the thickness direction of the curved beam (Figure 2), whereby the CNTs are added as filler beside the matrix for the reinforcement purposes. Hence, the effective material properties of CNTRC curved beams are defined, based on the Mori–Tanaka micromechanical scheme and the rule of mixture, as follows [42]:

\[
E_{11} = \eta_1 V_{CNT} E_{11}^{CNT} + V_m E^m \tag{3}
\]

\[
\frac{\eta_2}{E_2} = V_{CNT} \frac{E_{22}^{CNT}}{E_2} + V_m \frac{E^m}{E_2} \tag{4}
\]

\[
\frac{\eta_3}{G_{12}} = V_{CNT} \frac{G_{12}^{CNT}}{G_{12}} + V_m \frac{G^m}{G_{12}} \tag{5}
\]

In the previous relations, \( E_{11}^{CNT}, E_{22}^{CNT}, G_{12}^{CNT} \) are the Young moduli and shear modulus of CNT; \( E^m, G^m \) refer to the mechanical properties for the matrix; and \( V_{CNT} \) and \( V_m \) denote the volume fractions of the CNT and matrix, respectively, such that:

\[
V_{CNT} + V_m = 1 \tag{6}
\]

The CNTs efficiency parameters \( \eta_j \) in Equations (3)–(5) must be determined before computing the effective material properties of the structure. Thus, we estimate the CNT efficiency parameters \( \eta_1 \) and \( \eta_2 \) by comparing the Young’s moduli \( E_{11}^{CNT} \) and \( E_{22}^{CNT} \) for the CNTRCs, as obtained by the rule of mixtures, with those given by Han and Elliott [43]. In Table 1, the mechanical properties with a
clear good agreement between the molecular dynamics and the rule of mixture are summarized after a proper selection of $\eta_1$ and $\eta_2$. Moreover, the effective Poisson’s ratio and mass density are expressed as

$$v_{12}^* = V_{CNT}^* v_{12} + V_m v_m$$

$$\rho = V_{CNT}^* \rho_{CNT} + V_m \rho_m$$

where $v_{12}^*$, $\rho_{CNT}$ stand for the Poisson’s ratio and mass density of the CNT; and $v_{12}^*$, $\rho_{CNT}$ refer to the Poisson’s ratio and mass density of the matrix, respectively. The selected distribution schemes for CNTs along the thickness direction can be expressed analytically as [42]:

$$V_{CNT} = \begin{cases} V_{CNT}^* (UD) \\
(1 + \frac{z}{h})V_{CNT}^* (FG) \end{cases}$$

where:

$$V_{CNT}^* = \frac{w_{CNT}}{w_{CNT} + (\rho_{CNT}/\rho_m) - (\rho_{CNT}/\rho_m)w_{CNT}}$$

and $w_{CNT}$ is the mass fraction of the CNTs.

### Table 1.
Mechanical properties for a Poly{[(m-phenylenevinylene)-co-[(2,5-dioctoxy-p-phenylene)vinylene]]} (PmPV)/CNT composites reinforced by (10,10) SWCNT at the temperature $T = 300$ K.

| $V_{CNT}$ (MD [43]) | Rule of Mixture |
|---------------------|-----------------|
| $E_{11}$ (GPa)      | $E_{11}$ (GPa)  |
| $E_{22}$ (GPa)      | $E_{22}$ (GPa)  |
| $\eta_1$            | $\eta_2$        |
| 0.11                | 94.8            |
| 0.14                | 120.2           |
| 0.17                | 145.6           |

In what follows, we include the interactions among the CNTs and the matrix, while ignoring the effects of strains at general points of the nanocomposite on the stresses at a reference point. Thus, to avoid any possible inaccuracy related to the above-mentioned approximation, it is referred to the presence of nonlocal parameters as required by the Eringen Nonlocal Differential Model (ENDM) to predict the size-dependent behavior of nanostructure systems.

#### 2.2. Displacement Field and Strain

According to the refined beam theory, the curved beam is modeled as a continuum model with its displacement field defined as [44]:

$$u_\theta(\theta, r, t) = (1 + \frac{z}{R})u(\theta, t) + \frac{z}{R} \left( \frac{\partial w_b(\theta, t)}{\partial \theta} \right) + \frac{f(z)}{R} \left( \frac{\partial w_s(\theta, t)}{\partial \theta} \right)$$

$$w_r(\theta, r, t) = -w_b(\theta, t) - w_s(\theta, t)$$

where $u$ is the tangential mid-plane displacement, $w_b$ and $w_s$ are the bending and shear components of the radial displacement, respectively; and $f(z)$ is the shape function defined as:

$$f(z) = \frac{\hbar}{\pi} \left( \frac{\sinh(\frac{\pi z}{h}) - z}{\cosh(\frac{\pi z}{h})} \right)$$

It is interesting to note that the shape function in Equation (13) satisfies the stress-free boundary conditions on the top and bottom surfaces of the beam without using any shear correction factor. The non-zero strain field related to the displacement components is:

$$\varepsilon_x = \varepsilon_x^0 + zk^b + f(z)k^s, \quad \gamma_{xz} = g(z)(\gamma_{xz}^0)$$

$$\varepsilon_x^0 + zk^b + f(z)k^s, \quad \gamma_{xz} = g(z)(\gamma_{xz}^0)$$

$$\varepsilon_x^0 + zk^b + f(z)k^s, \quad \gamma_{xz} = g(z)(\gamma_{xz}^0)$$
where:
\[
\varepsilon^0_x = \frac{1}{R}(-w_b - w_s + \frac{\partial u}{\partial \theta}), \quad k_x^b = \frac{1}{R^2} \left( \frac{\partial u}{\partial \theta} + \frac{\partial^2 w_b}{\partial \theta^2} \right), \quad k_x^s = f(z) \left( \frac{\partial^2 w_s}{\partial \theta^2} \right), \quad \gamma^0_{xx} = -\frac{\partial w_s}{R \partial \theta}
\]
(15)

and \(g(z) = f'(z)\).

2.3. Governing Equations

The equations of motion for the stability of composite curved beams can be derived from the Hamilton’s principle:
\[
\int_0^t \delta(U + V) dt = 0
\]
(16)

where \(U\) and \(V\) refer to the strain energy and work done by external forces, respectively. The variational form of the strain energy is expressed as:
\[
\delta U = \int \sigma_{ij} \delta \varepsilon_{ij} dV = \int \left( \sigma_{xx} \delta \varepsilon_{xx} + \tau_{xz} \delta \gamma_{xz} \right) dV
\]
(17)

where:
\[
(N, M_b, M_s) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z, f(z)) \sigma_{xx} dz, \quad Q = \int_{-\frac{h}{2}}^{\frac{h}{2}} g(z) \tau_{xz} dz
\]
(18)

Accordingly, the work done by the applied forces takes the following form:
\[
\delta V = \int_0^L N_b \left( \frac{\partial (w_b + w_s)}{\partial \theta} \right) d\theta
\]
(19)

\(N_b\) is the applied tangential force here. By substituting Equations (17), (19) into Equation (16) and integrating by parts with respect to space and time variables, the equations of motion in terms of the displacement components of the curved beam can be obtained as:

\[
-\frac{\partial N}{\partial \theta} - \frac{1}{R} \frac{\partial M_b}{\partial \theta} = 0
\]
(20)

\[
\frac{\partial^2 M_b}{\partial \theta^2} - N - \frac{N}{R} \frac{\partial^2 (w_b + w_s)}{\partial \theta^2} = 0
\]
(21)

\[
\frac{\partial^2 M_s}{\partial \theta^2} - \frac{\partial Q}{\partial \theta} - \frac{N}{R} \frac{\partial^2 (w_b + w_s)}{\partial \theta^2} = 0
\]
(22)

Now, the constitutive equations of the nonlocal refined curved beam are introduced as follows:
\[
\sigma_{xx} - \mu \frac{\partial^2 \sigma_{xx}}{\partial \theta^2} = E \varepsilon_{xx}
\]
(23)

\[
\tau_{xz} - \mu \frac{\partial^2 \tau_{xz}}{\partial \theta^2} = G \gamma_{xz}
\]
(24)

where \(\mu = (\epsilon_0 a)^2\). By the combination of Equations (2)–(21), (23), (24), we get to the following relations for the curved beam:
\[
N - \mu \frac{\partial^2 N}{\partial \theta^2} = \left( \frac{A}{R}(-w_b - w_s + \frac{\partial u}{\partial \theta}) + \frac{B}{R^2} \left( \frac{\partial u}{\partial \theta} + \frac{\partial^2 w_b}{\partial \theta^2} \right) + \frac{B_s}{R^2} \frac{\partial^2 w_s}{\partial \theta^2} \right)
\]
(25)
where: 

\[ (A, B, B, D, D_s, H_s) = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(1, z, f(z), z^2, z f(z), f^2(z)) dz \]

\[ A_s = \int_{-\frac{h}{2}}^{\frac{h}{2}} 8^2(z) G dz \]

Upon rearrangement, we get to the following governing equations of the beam in terms of displacement components:

\[ \frac{A}{R} \left( \frac{\partial^2 w_b}{\partial \theta^2} - \frac{\partial w_b}{\partial \theta} \right) + \frac{B}{R^3} \left( \frac{\partial^2 w_b}{\partial \theta^2} - \frac{\partial w_b}{\partial \theta} + \frac{2 \partial^3 u}{\partial \theta^3} + \frac{\partial^4 w_b}{\partial \theta^4} \right) + \frac{B}{R} \left( \frac{\partial^3 u}{\partial \theta^3} + \frac{\partial^4 w_b}{\partial \theta^4} + \frac{D_s}{R^3} \frac{\partial^4 w_b}{\partial \theta^4} - \frac{A}{R} \left( -w_b - \frac{\partial w_b}{\partial \theta} \right) \right) = 0 \]

\[ \frac{B}{R^3} \left( \frac{\partial^2 w_b}{\partial z^2} - \frac{\partial^2 w_b}{\partial \theta^2} \right) + \frac{D_s}{R^3} \left( \frac{\partial^2 w_b}{\partial \theta^2} + \frac{\partial^3 w_b}{\partial \theta^3} + \frac{D_s}{R^3} \frac{\partial^4 w_b}{\partial \theta^4} - \frac{A}{R} \left( -w_b - \frac{\partial w_b}{\partial \theta} \right) \right) = \frac{\partial^2 w_b}{\partial \theta^2} + \frac{\partial^3 u}{\partial \theta^3} + \frac{\partial^4 w_b}{\partial \theta^4} \]

3. Solution Methodology

The Galerkin method is here employed to solve the equations of motion for functionally graded carbon nanotube-reinforced composite (FG-CNTRC) curved beams with simply-simply (S-S) supports, clamped-simply (C-S) supports, and clamped-clamped (C-C) supports, respectively:

Simply-supports (S):

\[ w_b = w_s = M = 0 \text{ at } x = 0, L \]

Clamped-supports (C):

\[ u = w_b = w_s = 0 \text{ at } x = 0, L \]

Assuming the following expansion for the displacement field:

\[ u(\theta) = \sum_{n=1}^{\infty} U_n \frac{\partial F_m(\theta)}{\partial \theta} \]

\[ w_b(\theta) = \sum_{n=1}^{\infty} W_{bn} F_m(\theta) \]

\[ w_s(\theta) = \sum_{n=1}^{\infty} W_{sn} F_m(\theta) \]
and by introducing the Equations (34)–(36) into Equations (31)–(33), the following set of relations can be obtained:

\[
\mathbf{K} \begin{bmatrix} U_n \\ W_{bn} \\ W_{sn} \end{bmatrix} = 0 \tag{37}
\]

in which \( \mathbf{K} \) represents the stiffness matrix. The admissible function \( F_m \) is selected in the following as the beam eigenfunction, i.e.,

- S-S: \( F_m = \sin \left( \frac{mn}{R} \theta \right) \)
- C-S: \( F_m = \sin \left( \frac{mn}{R} \theta \right) \left[ \cos \left( \frac{mn}{R} \theta \right) - 1 \right] \)
- C-C: \( \sin^2 \left( \frac{mn}{R} \theta \right) \)

To obtain the critical buckling force, we must enforce the determinant of the stiffness matrix equal to zero. This parameter will be quantified in nondimensional form in the next parametric analysis, namely:

\[
N_{cr} = N_b \frac{R^2}{E_M h^3} \tag{38}
\]

4. Numerical Results

The procedure proposed in the previous section is here applied to study the size-dependent buckling behavior of FG-CNTRC curved beams. The higher-order shear deformation beam theory is also applied to model the nanobeam, whereby the size-dependent effect is considered by means of the application of the Eringen nonlocal differential model. Thus, the buckling phenomena of the nanostructure are solved mathematically via the Galerkin method for different boundary conditions. The parametric study presented in this work analyzes the sensitivity of the size-dependent buckling response of FG-CNTRC curved beams reinforced with CNTs to some mechanical parameters (i.e., the nonlocal parameter and the nanotube volume fraction), as well as to some geometrical parameters, (namely, the opening angle, slenderness ratio, and the CNT distribution schemes). The preliminary focus of the investigation was on the accuracy of the proposed method to compute the critical buckling load, whose results are summarized in Table 2 in nondimensional form for an S-S beam, while varying the nonlocal parameter \( \mu \). Based on a comparative evaluation between our predictions and those obtained by Reddy [45], Aydogdu [46], and Eltaher [47], a very good match was observed, which confirms the accuracy of the proposed formulation for similar problems.

| \( \mu \) | Reddy [45] | Aydogdu [46] | Eltaher [47] | Present |
|---|---|---|---|---|
| 0 | 9.8696 | 9.8696 | 9.86973 | 9.80601 |
| 1 | 8.9830 | 9.6319 | 8.98312 | 8.92692 |
| 2 | 8.2426 | 9.4055 | 8.24267 | 8.19176 |
| 3 | 7.6149 | 9.1894 | 7.61499 | 7.56846 |
| 4 | 7.0761 | 8.9830 | 7.07614 | 7.03246 |

Next, we discuss about the size-dependence of the buckling load for FG-CNTRC curved beams with different boundary conditions (see Tables 3–11 and Figures 3–5), together with results for UD-CNTRC counterparts, for a direct comparison. Unless otherwise stated before, the length of the curved beam is fixed at \( L = 20 \), whereby a Poly{[m-phenylenevinylene]-co-[2,5-dioctoxy-p-phenylene vinylene]} is selected as matrix (henceforth labeled as PmPV), with Poisson’s ratio \( \nu^m = 0.34 \), elastic modulus \( E^m = 2.1 \text{ GPa} \), and temperature \( T = 300 \text{ K} \). As reinforcement phase, instead, we select an armchair (10, 10) SWCNTs, with elastic moduli \( E_{11}^{\text{CNT}} = 5.6466 \text{ TPa} \), \( E_{22}^{\text{CNT}} = 7.080 \text{ TPa} \), and Poisson’s ratio \( \nu^{\text{CNT}} = 0.175 \).
Table 3. Nondimensional critical buckling load for simply-simply (S-S) CNTRC curved beams with \(L/h = 10\), \(\alpha = \pi/3\).

| \(V_{\text{CNT}}^*\) | \(\mu = 0\) | \(\mu = 0.5\) | \(\mu = 1\) | \(\mu = 1.5\) | \(\mu = 2\) | \(\mu = 3\) |
|---------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| UD-CNTRC            |           |           |           |           |           |           |
| 0.11                | 12.5642   | 12.4111   | 12.2617   | 12.1158   | 11.9734   | 11.6983   |
| 0.14                | 14.3533   | 14.1784   | 14.0077   | 13.8411   | 13.6783   | 13.3641   |
| 0.17                | 19.6015   | 19.3626   | 19.1295   | 18.9019   | 18.6797   | 18.2505   |
| FG-CNTRC            |           |           |           |           |           |           |
| 0.11                | 10.1067   | 9.9835    | 9.8633    | 9.7459    | 9.6314    | 9.4101    |
| 0.14                | 11.7678   | 11.6244   | 11.4844   | 11.3478   | 11.2144   | 10.9567   |
| 0.17                | 15.6689   | 15.4780   | 15.2916   | 15.1097   | 14.9321   | 14.5890   |

Table 4. Nondimensional critical buckling load for clamped simply (C-S) CNTRC curved beams with \(L/h = 10\), \(\alpha = \pi/3\).

| \(V_{\text{CNT}}^*\) | \(\mu = 0\) | \(\mu = 0.5\) | \(\mu = 1\) | \(\mu = 1.5\) | \(\mu = 2\) | \(\mu = 3\) |
|---------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| UD-CNTRC            |           |           |           |           |           |           |
| 0.11                | 123.0005  | 119.3204  | 115.8541  | 112.5835  | 109.4924  | 103.7931  |
| 0.14                | 153.2617  | 148.6762  | 144.3571  | 140.2818  | 136.4303  | 129.3288  |
| 0.17                | 189.3549  | 183.6894  | 178.3532  | 173.3182  | 168.5597  | 159.7857  |
| FG-CNTRC            |           |           |           |           |           |           |
| 0.11                | 120.5401  | 116.9336  | 113.5366  | 110.3314  | 107.3022  | 101.7169  |
| 0.14                | 150.8821  | 146.3677  | 142.1157  | 138.1037  | 134.3120  | 127.3207  |
| 0.17                | 185.4017  | 179.8545  | 174.6296  | 169.6998  | 165.0406  | 156.4498  |

Table 5. Nondimensional critical buckling load for clamped-clamped (C-C) CNTRC curved beams with \(L/h = 10\), \(\alpha = \pi/3\).

| \(V_{\text{CNT}}^*\) | \(\mu = 0\) | \(\mu = 0.5\) | \(\mu = 1\) | \(\mu = 1.5\) | \(\mu = 2\) | \(\mu = 3\) |
|---------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| UD-CNTRC            |           |           |           |           |           |           |
| 0.11                | 251.6676  | 239.8324  | 229.0603  | 219.2143  | 210.1798  | 194.1748  |
| 0.14                | 316.4206  | 301.5402  | 287.9965  | 275.6171  | 264.2581  | 244.1351  |
| 0.17                | 386.8498  | 368.6573  | 352.0990  | 336.9642  | 323.0770  | 298.4749  |
| FG-CNTRC            |           |           |           |           |           |           |
| 0.11                | 249.5883  | 237.8509  | 227.1678  | 217.4031  | 208.4433  | 192.5705  |
| 0.14                | 314.4762  | 299.6872  | 286.2267  | 273.9234  | 262.6343  | 242.6349  |
| 0.17                | 383.5269  | 365.4907  | 349.0746  | 334.0698  | 320.3018  | 295.9111  |

Table 6. Effect of the slenderness ratio \(L/h\) on the nondimensional critical buckling load for S-S CNTRC curved beams with \(\alpha = \pi/3\), \(V_{\text{CNT}}^* = 0.14\).

| \(L/h\) | \(\mu = 0\) | \(\mu = 0.5\) | \(\mu = 1\) | \(\mu = 1.5\) | \(\mu = 2\) | \(\mu = 3\) |
|---------|-----------|-----------|-----------|-----------|-----------|-----------|
| UD-CNTRC|           |           |           |           |           |           |
| 10      | 14.3533   | 14.1784   | 14.0077   | 13.8411   | 13.6783   | 13.3641   |
| 20      | 25.2537   | 24.9459   | 24.6456   | 24.3524   | 24.0660   | 23.5132   |
| 30      | 29.4450   | 29.0862   | 28.7360   | 28.3941   | 28.0603   | 27.4156   |
| 50      | 32.1881   | 31.7938   | 31.4130   | 31.0393   | 30.6744   | 29.9697   |
| FG-CNTRC|           |           |           |           |           |           |
| 10      | 11.7678   | 11.6244   | 11.4844   | 11.3478   | 11.2144   | 10.9567   |
| 20      | 18.4256   | 18.2010   | 17.9819   | 17.7680   | 17.5591   | 17.1557   |
| 30      | 20.6574   | 20.4057   | 20.1600   | 19.9202   | 19.6860   | 19.2337   |
| 50      | 22.0620   | 21.7931   | 21.5307   | 21.2746   | 21.0244   | 20.5414   |
Table 7. Effect of the slenderness ratio $L/h$ on the nondimensional critical buckling load for C-S CNTRC curved beams with $\alpha = \pi/3$, $V_{\text{CNT}}^* = 0.14$.

| $L/h$ | $\mu = 0$ | $\mu = 0.5$ | $\mu = 1$ | $\mu = 1.5$ | $\mu = 2$ | $\mu = 3$ |
|-------|-----------|-------------|-----------|-------------|-----------|-----------|
| UD-CNTRC | 10 | 153.2617 | 148.6762 | 144.3571 | 140.2818 | 136.4303 | 129.3288 |
| | 20 | 575.5663 | 558.3455 | 542.1252 | 526.8208 | 512.3568 | 485.6873 |
| | 30 | 1247.1406 | 1209.8265 | 1174.6804 | 1141.5187 | 1110.1779 | 1052.3904 |
| | 50 | 3353.4099 | 3253.0768 | 3158.5732 | 3069.4054 | 2985.1338 | 2829.7502 |
| FG-CNTRC | 10 | 150.8821 | 146.3677 | 142.1157 | 138.1037 | 134.3120 | 127.3207 |
| | 20 | 564.4312 | 547.5436 | 531.6372 | 516.6288 | 502.4446 | 476.2911 |
| | 30 | 1228.5852 | 1191.8263 | 1157.2031 | 1124.5348 | 1093.6603 | 1036.7326 |
| | 50 | 3327.6844 | 3228.1210 | 3134.3424 | 3045.8586 | 2962.2334 | 2808.0419 |

Table 8. Effect of the slenderness ratio $L/h$ on the nondimensional critical buckling load for C-C CNTRC curved beams with $\alpha = \pi/3$, $V_{\text{CNT}}^* = 0.14$.

| $L/h$ | $\mu = 0$ | $\mu = 0.5$ | $\mu = 1$ | $\mu = 1.5$ | $\mu = 2$ | $\mu = 3$ |
|-------|-----------|-------------|-----------|-------------|-----------|-----------|
| UD-CNTRC | 10 | 316.4206 | 301.5402 | 287.9965 | 275.6171 | 264.2581 | 244.1351 |
| | 20 | 1230.3705 | 1172.5095 | 1119.8461 | 1071.7102 | 1027.5419 | 949.2954 |
| | 30 | 2714.6098 | 2586.9490 | 2470.7560 | 2364.5519 | 2267.1018 | 2094.4639 |
| | 50 | 7393.5474 | 7045.8487 | 6729.3838 | 6440.1251 | 6174.7087 | 5704.5098 |
| FG-CNTRC | 10 | 314.4762 | 299.6872 | 286.2267 | 273.9234 | 262.6343 | 242.6349 |
| | 20 | 1218.4960 | 1161.1934 | 1109.0383 | 1061.3669 | 1017.6249 | 940.1336 |
| | 30 | 2691.1353 | 2564.5784 | 2449.3901 | 2344.1045 | 2247.4971 | 2076.3521 |
| | 50 | 7355.7738 | 7009.8515 | 6695.0035 | 6407.2225 | 6143.1622 | 5675.3655 |

Table 9. Effect of the opening angle $\alpha$ on the nondimensional critical buckling load for S-S CNTRC curved beams with $L/h = 10$, $V_{\text{CNT}}^* = 0.14$.

| $\alpha$ | $\mu = 0$ | $\mu = 0.5$ | $\mu = 1$ | $\mu = 1.5$ | $\mu = 2$ | $\mu = 3$ |
|---------|-----------|-------------|-----------|-------------|-----------|-----------|
| UD-CNTRC | $\pi/4$ | 28.3891 | 28.0431 | 27.7055 | 27.3759 | 27.0540 | 26.4325 |
| | $\pi/3$ | 14.3533 | 14.1784 | 14.0077 | 13.8411 | 13.6783 | 13.3641 |
| | $\pi/2$ | 4.5393 | 4.4840 | 4.4300 | 4.3773 | 4.3258 | 4.2264 |
| | $2\pi/3$ | 1.4001 | 1.3830 | 1.3664 | 1.3501 | 1.3342 | 1.3036 |
| FG-CNTRC | $\pi/4$ | 23.3784 | 23.0935 | 22.8155 | 22.5440 | 22.2790 | 21.7672 |
| | $\pi/3$ | 11.7678 | 11.6244 | 11.4844 | 11.3478 | 11.2144 | 10.9567 |
| | $\pi/2$ | 3.6889 | 3.6440 | 3.6001 | 3.5573 | 3.5155 | 3.4347 |
| | $2\pi/3$ | 1.1279 | 1.1141 | 1.1007 | 1.0876 | 1.0748 | 1.0501 |

Table 10. Effect of the opening angle $\alpha$ on the nondimensional critical buckling load for C-S CNTRC curved beams with $L/h = 10$, $V_{\text{CNT}}^* = 0.14$.

| $\alpha$ | $\mu = 0$ | $\mu = 0.5$ | $\mu = 1$ | $\mu = 1.5$ | $\mu = 2$ | $\mu = 3$ |
|---------|-----------|-------------|-----------|-------------|-----------|-----------|
| UD-CNTRC | $\pi/4$ | 172.4731 | 167.3128 | 162.4523 | 157.8662 | 153.5319 | 145.5402 |
| | $\pi/3$ | 153.2617 | 148.6762 | 144.3571 | 140.2818 | 136.4303 | 129.3288 |
| | $\pi/2$ | 139.5855 | 135.4092 | 131.4754 | 127.7638 | 124.2560 | 117.7882 |
| | $2\pi/3$ | 134.8634 | 130.8284 | 127.0277 | 123.4417 | 120.0526 | 113.8035 |
| FG-CNTRC | $\pi/4$ | 168.1669 | 163.1354 | 158.3962 | 153.9246 | 149.6986 | 141.9064 |
| | $\pi/3$ | 150.8821 | 146.3677 | 142.1157 | 138.1037 | 134.3120 | 127.3207 |
| | $\pi/2$ | 138.6057 | 134.4586 | 130.5525 | 126.8670 | 123.3838 | 116.9614 |
| | $2\pi/3$ | 134.3841 | 130.3633 | 126.5672 | 123.0029 | 119.6258 | 113.3990 |
Table 11. Effect of the opening angle $\alpha$ on the nondimensional critical buckling load for C-C CNTRC curved beams with $L/h = 10$, $V_{CNT}^* = 0.14$.

| $\alpha$   | $\mu = 0$   | $\mu = 0.5$ | $\mu = 1$   | $\mu = 1.5$ | $\mu = 2$   | $\mu = 3$   |
|------------|-------------|-------------|-------------|-------------|-------------|-------------|
| UD-CNTRC   |             |             |             |             |             |             |
| $\pi/4$    | 337.8116    | 321.9253    | 307.4660    | 294.2497    | 282.1228    | 260.6394    |
| $\pi/3$    | 316.4206    | 301.5402    | 287.9965    | 275.6171    | 264.2581    | 244.1351    |
| $\pi/2$    | 301.1614    | 286.9986    | 274.1080    | 262.3256    | 251.5144    | 232.3618    |
| $2\pi/3$   | 295.8488    | 281.9358    | 269.2726    | 257.6981    | 247.0776    | 228.2628    |
| FG-CNTRC   |             |             |             |             |             |             |
| $\pi/4$    | 334.3510    | 318.6274    | 304.3162    | 291.2353    | 279.2327    | 257.9693    |
| $\pi/3$    | 314.4762    | 299.6872    | 286.2267    | 273.9234    | 262.6343    | 242.6349    |
| $\pi/2$    | 300.3202    | 286.1970    | 273.3424    | 261.5930    | 250.8119    | 231.7128    |
| $2\pi/3$   | 295.4045    | 281.5124    | 268.8683    | 257.3111    | 246.7066    | 227.9201    |

Figure 3. Critical buckling load versus slenderness ratio for different volume fractions and distribution patterns of CNTs: (a) UD pattern, (b) FG pattern. ($\mu = 2 \text{ nm}^2$, $L = 20$, $\alpha = \pi/3$).

Figure 4. Critical buckling load versus slenderness ratio for different nonlocal parameters and distribution patterns of CNTs: (a) UD pattern, (b) FG pattern. ($L = 20$, $\alpha = \pi/3$, $V_{CNT}^* = 0.14$).
Figure 5. Critical buckling load versus slenderness ratio for different opening angles and distribution patterns of CNTs: (a) UD pattern, (b) FG pattern. (µ = 1 nm², L = 20, V^*_{CNT} = 0.14).

More specifically, Tables 3–5 evaluate the effect of the volume fraction and distribution patterns of CNTs on the nondimensional critical buckling load of the composite curved beams for S-S, C-S, and C-C CNTRC curved beams, respectively, while \( L/h = 1 \) and \( \alpha = \pi/3 \) are considered. By exploiting the numerical results in Tables 3–5 comparatively, it is worth noting that clamped nanostructures yield the maximum buckling load, while S-S beams get the lowest buckling values. Moreover, an increment in the volume fraction of CNTs \( V^*_{CNT} \) significantly raises the buckling load of both UD- and FG-CNTRCs, with its behavior also affected by the nonlocality \( \mu \). More specifically, a rise in nonlocality reduces the buckling load of CNTRC curved beams because of the stiffness-softening mechanisms characterizing the nanostructure. The sensitivity of the buckling response to the volume fraction of CNTs is also plotted in Figure 3 versus the slenderness ratio \( L/h \), for a C-C boundary condition and different distributions of CNTs (namely a UD pattern in Figure 3a and an FG pattern in Figure 3b).

Based on Figure 3, it is worth to note that the monotone behavior of the critical buckling load increases for development in slenderness ratios \( L/h \), especially for the higher values of the volume fraction of CNTs \( V^*_{CNT} \).

In addition, Tables 6–8 summarize the results of the nondimensional critical buckling load for different \( L/h \) ratios and nonlocal parameters \( \mu \), while considering a S-S, C-S, and C-C composite curved beams reinforced with CNTs, respectively. It is clear that the highest sensitivity of the buckling response of curved beams to the length-to-thickness ratio is obtained for C-C boundary conditions, followed by C-S, and S-S supports, respectively. Moreover, the highest value of the critical load is always reached in size-dependent composite curved beams with \( \mu = 0 \), whereby as \( \mu \) increases, the buckling load decreases, independently of the selected \( L/h \) ratios and CNTs distributions. A meaningful sensitivity of the response to the boundary conditions is also detected due to an expectable variation in the structural stiffness of the composite curved beams. Furthermore, Figure 4 illustrates the double effect of the nonlocal parameter and the slenderness ratio \( L/h \) on the nondimensional critical buckling load of CNTRC curved beams for fixed C-C boundary conditions and different CNTs distribution patterns (namely, a UD pattern in Figure 4a and an FG pattern in Figure 4b).

It is worth noting that the moderately thick CNTRC curved beam with \( L/h = 10 \) features the lowest critical buckling load. This last one increases as the length-to-thickness ratio \( L/h \) is increased, both in UD and FG-CNTRC curved beams. Another key aspect related to the sensitivity of the response with the nonlocal parameter is that the impact is more pronounced for higher values of \( L/h \), (or equivalently to a lower thickness of the curved beam for a fixed length).

The effect of the opening angle and the nonlocal parameter on the nondimensional critical buckling load is shown in Tables 9–11 for S-S, C-S, and C-C CNTs reinforced composite curved beams, respectively. By exploiting comparatively, the results can be found that an increasing value of the opening angle decreases the buckling load whose value is also affected by the selected boundary condition. The results are obtained far from a size-dependence of the structure. It means that the
buckling load of size-dependent and independent response of curved beams decreases by increasing the opening angle for each boundary conditions.

The double effect of the opening angle and slenderness ratio is finally emphasized in Figure 5 for each CNT reinforcement patterns, while considering a fixed C-C boundary condition. Based on this last plot, it is clearly visible that the higher sensitivity of the response for thick CNTs reinforced curved beams (i.e., for \( L/h = 50 \)) compared to thin structures.

5. Conclusions

The size-dependent buckling of FG-CNTRC curved beams was investigated within the framework of a refined beam theory and Eringen nonlocal differential model. The CNTs distributions were considered uniform and graded through the thickness direction, and the material properties were estimated using the rule of mixtures. The Galerkin method was also employed to obtain the critical buckling load of FG-CNTRC curved beams for different boundary conditions. The effects of the nonlocal parameter, CNT volume fraction, slenderness ratio, opening angle, boundary conditions, and CNTs distribution scheme on the critical buckling load of FG-CNTRC curved beams were discussed in detail. Based on the numerical results, the following concluding remarks can be summarized:

An increase in CNT volume fraction leads to an increase in the critical buckling load for both UD- and FG-CNTRC curved beams.

A UD of CNTs in composite curved beams yields higher values of the critical buckling load compared to an FG distribution of CNTs.

An increase in the opening angle leads to a lower value of the critical buckling load for both UD- and FG-CNTRC curved beams.

The highest values of the critical buckling load of FG-CNTRC curved beams is obtained for completely clamped C-C boundary conditions, due to an increase in structural stiffness compared to simply supported boundary conditions.

Using nonlocality phenomena, the critical buckling load of FG-CNTRC curved beam decreases. Moreover, the effect of the nonlocal parameter in curved beams with higher slenderness ratios is more pronounced, if compared to lower slenderness ratios.

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