Stabilization of the extra dimension size in RS model by bulk Higgs field

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Abstract. An extension of the Standard Model is considered, which is built on the basis of a stabilized Randall-Sundrum model with two branes. The stabilization of the extra dimension size is achieved with the help of a five-dimensional Higgs field, which plays the role of the Goldberger-Wise field. The stabilization makes the radion massive, and all the fermion fields, which are assumed to be localized on the TeV brane, get their masses due to the interaction with the boundary value of the Higgs field. The gauge invariance of the theory demands that the electroweak gauge fields also live in the bulk. The equations of motion for the background field configurations and for the field fluctuations against a background solution are obtained. The interaction of the bulk Higgs field with the multidimensional gauge field is studied and possible values of the model parameters are estimated.

1. Introduction
In the framework of the Randall-Sundrum model we consider two branes with tension interacting with gravity in a five-dimensional space-time \cite{1}. The extra dimension forms the orbifold $S^1/Z_2$ and the branes are located at its fixed points. The background metric is warped, providing a solution to the hierarchy problem of the gravitational interaction.

In case the interbrane distance is arbitrary the model contradicts already the classical gravity, predicting the existence of a “strongly” coupled massless scalar mode, the radion. In order to solve this problem the Goldberger-Wise field is introduced into the model \cite{2,3}. It is a 5D scalar field with a bulk potential and additional potentials on the branes. This allows one to stabilize the extra dimension size, making the radion massive and the model phenomenologically acceptable.

The idea of this report is to stabilize the interbrane distance by a two-component complex scalar field propagating in the bulk and carrying the same representation of the gauge group $SU(2) \times U(1)$ as the usual Higgs field. It will act as the Higgs field on the brane, where our world is supposed to be located, providing the spontaneous symmetry breaking. Thus, we introduce an object, which can be called the bulk Higgs field.

The question about the stabilization of the extra dimension size by the bulk Higgs field was previously raised in papers by L. Vecchi \cite{4} and M. Geller et al. \cite{5}. In the last paper a perturbative solution was found, whereas the gauge fields were not considered. Since the gauge group acts in all the five-dimensional space-time, the gauge invariance of the theory necessarily demands the presence of the corresponding bulk gauge fields. We find a non-perturbative...
background solution for gravity and the bulk Higgs field and take these gauge fields into account in the linearized theory.

2. Equations of motion for background fields

Let us consider gravity interacting with two branes, a two-component scalar field \( \phi = \left( \phi_1 \phi_2 \right) \) and bulk gauge fields \( A_M \) and \( B_M \) in a five-dimensional space-time \( E = M_4 \times S^1/Z_2 \). The action of the model can be written as follows:

\[
S = S_g + S_\phi + S_{gauge} + S_{brane+SM},
\]

where the gravitational action \( S_g \) is given by

\[
S_g = 2M^3 \int d^4x \int dyR\sqrt{g},
\]

\( S_\phi \) stands for the action of the two-component complex scalar field,

\[
S_\phi = M \int d^4x \int dy \left[ (D_M \phi)^+ D^M \phi - V(\phi^+ \phi) \right] \sqrt{g},
\]

the action \( S_{gauge} \) of the gauge fields is determined by

\[
S_{gauge} = - \int d^4x \int dy \left[ \frac{1}{4p^2} A^a_{MN} A^{aMN} + \frac{1}{4q^2} B_{MN} B^{MN} \right] \sqrt{g},
\]

and \( S_{brane+SM} \) is the action of the branes and the Standard Model,

\[
S_{brane+SM} = - \int_{y=0} d^4x \lambda_1 (\phi^+ \phi) \sqrt{-g} + \int_{y=L} d^4x \left[ -\lambda_2 (\phi^+ \phi) + L_{SM-HP} (\phi, \phi^+) \right] \sqrt{-g}.
\]

Here \( D_M \) is the covariant derivative, \( p \) and \( q \) are the gauge coupling constants and \( L_{SM-HP} \) is the Lagrangian of the Standard model without the Higgs potential and the electroweak gauge field Lagrangian.

A background solution is sought in the standard form, which preserves the Poincaré invariance in any four-dimensional subspace \( y = \text{const} \):

\[
ds^2 = \gamma_M dx^M dx^N = e^{-2A(y)\eta_{\mu\nu}dx^\mu dx^\nu - dy^2}, \quad \phi(x,y) = \tilde{\phi}(y),
\]

\( A_\mu(x,y) = 0, \quad A_4(x,y) = A_4(y), \quad B_\mu(x,y) = 0, \quad B_4(x,y) = B_4(y). \)

Varying the action, we arrive at the equations of motion for background fields:

\[
\frac{1}{2} \left( \phi'' + V + \frac{\lambda_1}{M} \delta(y) + \frac{\lambda_2}{M} \delta(y-L) \right) = 2M^2 \left( 3A'' - 6A' \right),
\]

\[
12M^2 (A')^2 + \frac{1}{2} (V - \phi'') = 0,
\]

\[
\frac{dV}{d\phi} + \frac{1}{M} \frac{d\lambda_1}{d\phi} \delta(y) + \frac{1}{M} \frac{d\lambda_2}{d\phi} \delta(y-L) = \phi'' - 4A' \phi',
\]

\[
\frac{dV}{d\phi^+} + \frac{1}{M} \frac{d\lambda_1}{d\phi^+} \delta(y) + \frac{1}{M} \frac{d\lambda_2}{d\phi^+} \delta(y-L) = \phi'' - 4A' \phi',
\]

\[
A_M = B_M = 0.
\]

The requirement of the Poincaré invariance leads to the vanishing of the background gauge fields. Thus, they do not affect the background solution for the metric and the Higgs field.
3. Equations for the field fluctuations

Suppose we have a solution to these equations. In order to construct the linearized theory we represent the metric and the scalar field in the form “background solution + fluctuation”:

\[ g_{MN}(x, y) = \gamma_{MN}(y) + \frac{1}{\sqrt{2M^2}} h_{MN}(x, y), \quad \phi(x, y) = \phi_0(y) + f(x, y). \]  

The vector fields can be treated as fluctuations, because their vacuum values are zero.

Substituting this representation into the Lagrangian of the model and keeping only the terms of the second order in the deviations, one derives the so-called second variation Lagrangian for the field fluctuations against a vacuum solution. Varying the action built with this Lagrangian we obtain the equations of motion for the fluctuations. Choosing a vacuum solution \( \phi_0(y) \) having only a real lower component \( v(y)/\sqrt{2} \), which breaks the gauge group \( SU(2) \times U(1) \) to \( U(1)_{em} \), and passing to the gauge, where the scalar field \( f(x, y) \) also has only a real lower component, we get the equations for the scalar degrees of freedom in the following form:

1. \( \mu \nu \)-component of the metric fluctuation:

\[
\frac{1}{4} \left( \partial_\mu \partial_\nu \tilde{h} - 2 \partial_\mu \partial_\nu h_{44} \right) + \frac{1}{4} \gamma_{\mu\nu} \left( -\partial_\sigma \partial^{\sigma} \tilde{h} + 2 \partial_\sigma \partial^{\sigma} h_{44} + \frac{3}{2} \partial_\sigma \partial_\tau h_{\tau\sigma} \right) - \\
- \frac{1}{2} \gamma_{\mu\nu} A' \left( 4 \partial_\mu \tilde{h} + 3 \partial_\nu h_{44} \right) + \frac{1}{2} \gamma_{\mu\nu} (A')^2 \left( 12 h_{44} + \tilde{h} \right) - \\
- \frac{1}{4} \gamma_{\mu\nu} A'' \left( \tilde{h} + 6 h_{44} \right) + \frac{1}{\sqrt{2M^2}} \gamma_{\mu\nu} \left[ \phi_0^+ f' + \left( \phi_0'^+ - 4A'\phi_0^+ \right) f \right] = 0; \tag{14}
\]

2. \( \mu 4 \)-component of the metric fluctuation:

\[
\frac{3}{4} \partial_\mu \partial_\nu \tilde{h} - 3 A' \partial_\mu h_{44} + \sqrt{\frac{2}{M^2}} \phi_0^+ \partial_\mu f = 0; \tag{15}
\]

3. 44-component of the metric fluctuation:

\[
\frac{3}{4} \partial_\nu \partial^\mu \tilde{h} + 3 A' \partial_\nu h_{44} + \frac{1}{2M^2} V h_{44} + \sqrt{\frac{2}{M}} \left[ \phi_0^+ f' - \frac{1}{2} \left( \frac{dV}{d\phi} f + f^+ \frac{dV}{d\phi^+} \right) \right] = 0; \tag{16}
\]

4. equation for the field \( f \):

\[
M \left( \partial_M \partial^M f + 4 A' f' + \frac{d^2 V}{d\phi^+ d\phi} f + f^+ \frac{d^2 V}{d\phi^+ d\phi^+} f \right) - \\
- \frac{1}{\sqrt{2M^2}} \left[ \frac{1}{2} \phi_0' \left( \partial_\mu \tilde{h} + \partial_4 h_{44} \right) + (\phi_0'^+ - 4A'\phi_0^+) h_{44} \right] + \\
+ \left( \frac{d^2 \lambda_1}{d\phi^+ d\phi} f + f^+ \frac{d^2 \lambda_1}{d\phi^+ d\phi^+} \right) \delta(y) + \left( \frac{d^2 \lambda_2}{d\phi^+ d\phi} f + f^+ \frac{d^2 \lambda_2}{d\phi^+ d\phi^+} \right) \delta(y - L) + \\
+ \frac{1}{\sqrt{2M^2}} \left[ \frac{d\lambda_1}{d\phi^+} \delta(y) + \frac{d\lambda_2}{d\phi^+} \delta(y - L) \right] h_{44} = 0. \tag{17}
\]

Here \( \tilde{h} = \gamma^{\mu\nu} h_{\mu\nu} \). A remarkable result is that the fluctuations of the scalar components of the gauge fields, namely \( A_4 \) and \( B_4 \), vanish due to the equations of motion and the gauge condition choice. Hence there are no extra light scalars, which allows one to correctly reproduce the electroweak sector of the Standard Model in the effective four-dimensional theory.
4. Radion-like interactions of the Higgs boson
Since the Higgs field plays the stabilizing role of the Goldberger-Wise field, the Higgs boson here is the radion at the same time. The radion as a separate particle does not exist. Therefore the Higgs boson inherits the interaction with the energy-momentum tensor, which is intrinsic to the radion. The interaction term can be written as [6]

$$S \supset -\frac{1}{\sqrt{8M^3}} \int dx \int_{-L}^{L} dy T^{MN} h_{MN} \sqrt{\gamma} = \frac{1}{\sqrt{8M^3}} \int dx \int_{-L}^{L} dy \left(-\frac{1}{2} T^\mu_{\mu} - T_{44}\right) e^{-2A} g.$$ (18)

where $g = e^{-2A(y)} h_{44}(x, y)$. The field $g$, in fact, represents the Higgs field $f$ because $h_{44}$ and $f$ are connected by the gauge condition choice and thereby represent one degree of freedom.

Let us consider the interaction with the bulk vector field $Z_M$. Its Lagrangian has the form: $L_Z = -\frac{1}{4} Z_{MN} Z^{MN} + \frac{1}{2} \phi_1^+ \phi_0 Z_M Z^M$. Performing the mode decomposition, $Z_\mu (x, y) = \sum_n z^n_\mu (x) Z_n(y)$ (where the gauge condition $Z_4(x, y) = 0$ is chosen), one can obtain the interaction term as follows:

$$S \supset \frac{1}{\sqrt{8M^3}} \sum_n \int dx \int_{-L}^{L} dy \left(\frac{1}{4} \eta^{\rho\sigma} z^n_\rho (x) z^n_\sigma (x) - z^n_\mu(x) \sum_m z^n_m(x) B_{nm} + n^2 Z_n z^n_\mu(x) z^n_\nu(x) \right) \eta^{\mu\nu} e^{-2A} Z^2_n(y) g,$$ (19)

where $B_{nm} = \int_{-L}^{L} dy e^{-2A} \phi^+_0 \phi_0 Z_n(y) Z_m(y)$, $Z^n_\mu(x, y) = z^n_\mu(x) Z_n(y)$. Note that the interaction Lagrangian turns out to be non-diagonal, which means that the KK number non-conservation takes place. Moreover, the couplings of the excited bulk states to the Higgs boson are not characterized by one coupling constant, as in the case of brane localized fields, but depend on the overlap integrals of their wave functions squared and the Higgs wave function. A similar interaction Lagrangian can be obtained for the W-boson field.

5. Specific example of a non-perturbative background solution
Let us choose an ansatz:

$$V = \frac{1}{4} \frac{dW}{d\phi} \frac{dW}{d\phi^+} - \frac{1}{24M^2} (W(\phi^+ \phi))^2, \quad A'(y) = \text{sign}(y) \frac{1}{24M^2} W(\phi^+ \phi),$$ (20)

$$\phi'(y) = \text{sign}(y) \frac{1}{2} \frac{dW}{d\phi^+}, \quad \phi'^+(y) = \text{sign}(y) \frac{1}{2} \frac{dW}{d\phi^+}, \quad W = 24M^2 k - 2u\phi^+ \phi,$$

$$\lambda_1(\phi^+ \phi) = MW(\phi^+ \phi) + \beta_1 \left(\phi^+ \phi - \frac{v_1^2}{2}\right)^2, \quad \lambda_2(\phi^+ \phi) = -MW(\phi^+ \phi) + \beta_2 \left(\phi^+ \phi - \frac{v_2^2}{2}\right)^2.$$ 

Here $k, u, \beta_{1,2}$ and $v_{1,2}$ are the model parameters. Note that the potential $\lambda_2$ chosen in this simplest form has the same structure as the standard Higgs potential.

The corresponding solution looks like

$$\phi(y) = \left(\frac{u}{\sqrt{2}} e^{-u(|y| - L)}\right), \quad A(y) = k (|y| - L) + \frac{v^2}{96M^2} \left(e^{-2u(|y| - L)} - 1\right).$$ (21)

Here $v = 246 \, \text{GeV}$ is the vacuum value of the Higgs field. In this case all the fermion fields localized on the brane will get the same masses as in the interaction with the usual Higgs field.

The interbrane distance is defined by the boundary conditions for the scalar field and is expressed in terms of the model parameters by the relation: $L = \frac{1}{u} \ln \left(\frac{u}{v}\right)$. The size of the extra dimension becomes stabilized.
6. Estimate of the model parameters

Taking the mode decomposition of the previously defined field $g$ we obtain the equations for $g_n$ with the boundary conditions on the branes:

$$\frac{d}{dy} \left( \frac{e^{2A}}{(\phi_2')^2} g_n' \right) - \frac{e^{2A}}{6M^2} g_n = -\mu_n^2 g_n e^{4A},$$

$$\left( \frac{1}{4M} \frac{d^2 \lambda_1}{d \phi_2^2} - \phi_2' \right) g_n' + \mu_n e^{2A} g_n = 0, \quad \left( \frac{1}{4M} \frac{d^2 \lambda_2}{d \phi_2^2} + \phi_2' \right) g_n' - \mu_n e^{2A} g_n = 0.$$

Here $\phi_2$ is the lower component of a background solution for the Higgs field. These equations define the mass spectrum of the infinite Kaluza-Klein tower of the field $\phi$.

In the case $uL \ll 1$ we can get the following expression for the mass of the lowest excitation of the scalar field, which is identified with the Higgs boson [7]:

$$m_H^2 = \frac{e^{2A} \beta \phi_2^2 nM^2 + uM}{4M^2 + uM},$$

where $u \sim 1.76 \text{ TeV}, \phi_1 = 345 \text{ TeV}, k \sim 186 \text{ TeV}, L = 0.2 \text{ TeV}^{-1} \approx 2 \cdot 10^{-18} \text{ cm}$. The coupling of the Higgs boson to the energy-momentum tensor of the Standard Model, given by the expression $\epsilon_H = -\sqrt{\frac{k}{2M^2}}$, turns out to be of the order of $1 \text{ TeV}^{-1}$. It must significantly affect the properties of the Higgs boson in this model. The next excitations of the field $\phi$ have masses of the order of hundreds of TeV and cannot be observed at the existing colliders.

7. Conclusion

In the present paper we have shown that the extra dimension size in the Randall-Sundrum model can be stabilized by means of the bulk field that acts simultaneously as the Higgs field on the brane and provides spontaneous symmetry breaking. The background equations of motion are found, as well as the equations of motion for the field fluctuations against a background solution. An important point is that the bulk electroweak gauge fields that necessarily appear in the model do not give rise to extra light scalars in the effective four-dimensional theory. The Higgs boson here is the radion at the same time, so it interacts with the energy-momentum tensor. These interactions with the bulk fields are studied. An important result is that the coupling constants of these interactions depend on a particular excitation and there is no KK number conservation in such interactions. A specific example of a non-perturbative vacuum solution is found. Based on it, the values of the model parameters are estimated, which give the correct value of the Higgs boson mass in the approximation of a small deviation of the metric of the stabilized model from the metric of the unstabilized one.

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References

[1] Randall L and Sundrum R 1999 Large mass hierarchy from a small extra dimension Phys. Rev. Lett. 83 3370-3
[2] Goldberger W D and Wise M B 1999 Modulus stabilization with bulk fields Phys. Rev. Lett. 83 4922-5
[3] DeWolfe O et al. 2000 Modeling the fifth dimension with scalars and gravity Phys. Rev. D 62 046008
[4] Vecchi L 2011 A natural hierarchy and a low new physics scale from a bulk Higgs J. High Energy Phys. 1111 102
[5] Geller M, Bar-Shalom S and Soni A 2014 Higgs-radion unification: radius stabilization by an SU(2) bulk doublet and the 126 GeV scalar Phys. Rev. D 89(9) 095015
[6] Osaki C, Hubisz J and Lee S J 2007 Radion phenomenology in realistic warped space models Phys. Rev. D 76 125015
[7] Boos E E et al. 2006 Physical degrees of freedom in stabilized brane world models Mod. Phys. Lett. A 21 1431