Interacting Dark Energy: Decay into Fermions

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A dark energy component is responsible for the present stage of acceleration of our universe. If no fine tuning is assumed on the dark energy potential then it will end up dominating the universe at late times and the universe will not stop this stage of acceleration. On the other hand, the equation of state of dark energy seems to be smaller than -1 as suggested by the cosmological data. We take this as an indication that dark energy does indeed interact with another fluid (we consider fermion fields) and we determine the interaction through the cosmological data and extrapolate it into the future. We study the conditions under which a dark energy can dilute faster or decay into the fermion fields. We show that it is possible to live now in an accelerating epoch dominated by the dark energy and without introducing any fine tuning parameters the dark energy can either dilute faster or decaying into fermions in the future. The acceleration of the universe will then cease.

I. INTRODUCTION

A dark energy component is probably responsible for the present stage of acceleration of our universe [1,2]. Perhaps the most appealing candidate for dark energy is that of a scalar field, quintessence [3], which can be either a fundamental particle or a composite particle [4]. Since dark energy dilutes slower than matter we expect it to dominate the universe at late times if no fine tuning is assumed on the dark energy potential $V$ and the universe will not stop the present stage of acceleration. However, this conclusion can be overcome by assuming a dark energy interaction [5].

On the other hand, the equation of state of dark energy seems to be smaller than minus one as suggested by the cosmological data [1,2]. In general fluids with $w < -1$ give many theoretically problems such as stability issues or wrong kinetic terms as phantom fields [11]. However, interacting dark energy [6]-[9],[5] is a very simple and attractive option which we will use in this letter. The interaction must be quite weak since dark energy particles have not been produced in the accelerator and because the dark energy has not decayed into lighter (e.g. massless) fields such as the photon.

It was suggested to determine the interaction between dark energy and this other fluid through the cosmological observations and extrapolate it into the future. Doing so, it was shown within a general framework that the dark energy dilutes exponentially fast and matter prevails in the end [5].

In this letter we take the dark energy to interact with fermion fields. It is tempting to take the fermions as neutrinos [8] since the order of magnitude of the energy of dark energy and neutrinos is similar. Furthermore, if the mass of neutrinos is larger than 0.8eV, as implied by the Heidelberg-Moscow experiment, then the dark energy cannot be a cosmological constant [9]. However, in this letter we do not assume the fermions to be neutrinos.

We take the interaction between dark energy and fermions through a field dependent mass $M = f(\phi)$, where $M$ is the mass of the fermions [8]. The function $f(\phi)$ plays an important roll in the evolution of the dark energy. The total potential $V_T$ of the scalar field has a contribution of the original scalar potential $V(\phi)$ and the interaction term $f(\phi)$. We calculate the conditions for acceleration and we show that the naive slow roll conditions $V_T' / V_T < 1$ and $V_T'' / V < 1$ do not imply acceleration. We determine the requirements under which a dark energy can dilute faster or decay into the fermion fields. We obtain that only in the case where the mass of the dark energy, given by $m^2 = V'' + \rho_\phi f'' / f$, is dominated by the interaction term can the dark energy decay. We also show that it is possible to have dark energy diluting faster than the fermion fields.

This letter is organized as follows. In section II we present our assumptions about dark energy in the absence of interactions while in section III we present the dynamical equations of dark energy coupled to fermions via a fields dependent mass. In section IV we present the generic requirements for a particle decay and in section V we set the conditions for dark energy to decay in the future. In section VI we briefly show the generic evolution of two coupled fluids and we discuss the evidence for dark energy decay. Finally, in section VII we study the possibility of dark energy diluting or decaying into fermions and we then present our conclusions in VIII.

II. DARK ENERGY

We will consider in this letter that the dark energy is given in terms of a scalar field with lagrangian

$$L(\phi) = \frac{1}{2} \dot{\phi}^2 - V(\phi),$$

where $V(\phi)$ is only a function of $\phi$. We define the energy density and pressure of the scalar field as

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi),$$

where $\dot{\phi}^2$ is the kinetic energy of the scalar field and $V(\phi)$ is the potential energy. The equation of state of dark energy is given by $w$. We will consider that $w = -1$ and $w = -\frac{1}{3}$ for the two cases of dark energy.

In section II we present our assumptions about dark energy and the interaction term $f(\phi)$. We show that the dark energy can dilute faster or decay into the fermion fields. We obtain that only in the case where the mass of the dark energy, given by $m^2 = V'' + \rho_\phi f'' / f$, is dominated by the interaction term can the dark energy decay. We also show that it is possible to have dark energy diluting faster than the fermion fields.
i.e. without any interaction with other fields, and

$$w_{\phi} \equiv \frac{p_{\phi}}{\rho_{\phi}} = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)}. \quad (3)$$

Clearly $w_{\phi} \geq -1$ at all times if $\rho_{\phi} \geq 0$. For $V$ to inflate the slow roll constrains to the potential must be satisfied,

$$|V' / V| \ll 1, \quad |V'' / V| \ll 1, \quad (4)$$

where a prime denotes derivative w.r.t. $\phi$, i.e. $V' \equiv dV/d\phi$. We do not want to introduce any arbitrary dark energy scale so we will assume that the slow roll conditions are satisfied also in the future and therefore $V$ is a runaway potential and tends to zero at $\phi \to \infty$. From eqs.(4) we see that $V'$ and $V''$ also approach zero at late times and $V'$ is therefore negative. Inflation occurs in general for $\phi \geq 1$ and the mass $m \approx H$.

III. INTERACTING DARK ENERGY

The interaction term between two types of particles have two important contributions to the relative ratio of energy densities of these particles. The first one has a classical interpretation and it is due to the cosmological evolution of the different energy densities. This evolution will be determined the dynamics which includes the potential $V$ and the interaction term $L_{\text{int}}$ and is calculated by solving the equation of motion of the lagrangian in a FRW metric. The second consequence of the interaction term $L_{\text{int}}$ is to allow for a particle decay, which is due to quantum physics. Both effects are relevant for determining the ratio of energy densities.

A. Interacting Dark Energy with Fermions

The interaction between dark energy and fermions $\psi$ can be achieved by taking a Yukawa type interaction \cite{8}

$$L_{\text{int}} = -f(\phi) \bar{\psi} \psi \quad (5)$$

with a function $f$ that depends on the scalar field $\phi$. This term gives an effective field dependent mass to $\psi$

$$M = f(\phi). \quad (6)$$

In principle the function $f$ is an arbitrary semipositive definite function of $\phi$, $f(\phi) \geq 0$. However, it is useful to assume that it is a monotonic function, otherwise not much can be said about $f$.

The evolution of the energy density of the fermions $\rho_{\psi}$ can be determined using the Fermi-Dirac distribution and taking into account that the mass $M$ is a function of the scalar field $\phi$, the evolution of $\psi$ and $\phi$ are given by \cite{8}

$$\dot{\rho}_{\psi} + 3H \rho_{\psi}(1 + w_{\psi}) = (1 - 3w_{\psi}) \rho_{\psi} \frac{\dot{f}}{f} \quad (7)$$

$$\ddot{\rho}_{\phi} + 3H \dot{\rho}_{\phi} + V' = -(1 - 3w_{\psi}) \rho_{\psi} \frac{\dot{f}}{f} \quad (8)$$

with $V(\phi)$ the potential for the scalar field $\phi$ in the absence of interaction and $f' = df/d\phi$. The solution to eq.(7) is

$$\rho_{\psi} = \rho_{\psi} a^{-3(1+w_{\psi})} \left( \frac{f}{f_0} \right)^{1-3w_{\psi}} = \tilde{\rho}_{\psi} \left( \frac{f}{f_0} \right)^{1-3w_{\psi}} \rho_{\psi} a^{-3(1+w_{\psi})} \quad (9)$$

where from now on the subscript $o$ is present time. In this case the fermion fluid no longer redshifts as $a^{-3(1+w_{\psi})}$ since the evolution of $f(\phi)$ also contributes to the redshift. Eq.(9) reduces to $\rho_{\psi} = \rho_{\psi} a^{-3} f/f_0$ when $w_{\psi} = 0$ which was used in [8]. The quantity $\tilde{\rho}_{\psi}$ is independent on $\phi$. We see that for radiation $w_{\psi} = 1/3$ the interaction term is $\delta \equiv (1 - 3w_{\psi}) \rho_{\psi} \dot{f}/f = 0$ and $\rho_{\psi} = \tilde{\rho}_{\psi} \sim a^{-4/3}$, consistent with having a fermion field with a constant (vanishing) mass. From now on we will set $w_{\psi} = 0$ for simplicity but the generalization is straightforward.

Eqs.(7) and (8) can be rewritten in terms of $\rho_{\psi}$, defined in eq.(2), and $\rho_{\psi}$ as

$$\dot{\rho}_{\psi} + 3H \rho_{\psi}(1 + w_{\psi}) = \rho_{\psi} \frac{\dot{f}}{f} = \delta(t), \quad (10)$$

$$\dot{\rho}_{\phi} + 3H \rho_{\phi}(1 + w_{\phi}) = -\rho_{\psi} \frac{\dot{f}}{f} = -\delta(t) \quad (11)$$

where we have used $\dot{f} = f' \phi$. The interaction term is defined by $\delta \equiv \rho_{\psi} \dot{f}/f$ is a function of time and we could solve eqs.(10) without making reference to the scalar field $\phi$. We can write eqs.(10) and (11) as

$$\dot{\rho}_{\psi} = -3H \rho_{\psi}(1 + w_{\psi eff}) \quad (12)$$

$$\dot{\rho}_{\phi} = -3H \rho_{\phi}(1 + w_{\phi eff})$$

with the effective equation states given by

$$w_{\psi eff} = w_{\psi} - \frac{\delta}{3H \rho_{\psi}} = w_{\psi} - \frac{f' \phi}{3H f} \quad (13)$$

$$w_{\phi eff} = w_{\phi} + \frac{\delta}{3H \rho_{\phi}} = w_{\phi} + \frac{\rho_{\psi} f' \phi}{\rho_{\phi} 3H f}. \quad (14)$$

For $\delta > 0$ we have $w_{\phi eff} > w_{\phi}$ and the fluid $\rho_{\phi}$ will dilute faster then without the interaction term while $\rho_{\psi}$ will dilute slower since $w_{\psi eff} < w_{\psi}$.

The complete evolution of $\rho_{\phi}$ and $\rho_{\psi}$ depends on the effective equation of state parameters defined in eqs.(13). Which fluid dominates at late time will depend on which effective equation of state is smaller. The difference in eqs.(13) is

$$\Delta w_{eff} \equiv w_{\psi eff} - w_{\phi eff} = \Delta w - \Upsilon \quad (15)$$

with

$$\Upsilon = \frac{\dot{f}}{3H f} \left( \frac{\rho_{\phi} + \rho_{\psi}}{\rho_{\phi}} \right). \quad (16)$$

While the sum gives the constraint

$$\Omega_{\psi} w_{\psi eff} + \Omega_{\phi} w_{\phi eff} = \Omega_{\psi} w_{\psi} + \Omega_{\phi} w_{\phi}. \quad (17)$$
Clearly the relevant quantity to determine the relative growth is given by $\Gamma$ and if $\Gamma > \Delta w$ we have $\Delta w_{eff} < 0$ and $\rho_\phi$ will dominate the universe at late times while for $\Gamma < \Delta w$ we have $\Delta w_{eff} > 0$ and $\rho_\phi$ will prevail.

B. Effective Potential $V_T$

Form eq.(8) we see that the effective (total) potential $V_T$ and its derivative w.r.t $\phi$ are given by

$$V_T \equiv V + \rho_\phi = V + \tilde{\rho}_\phi \frac{f(\phi)}{f_0}$$  \hspace{1cm} (18)

$$V'_T = V' + \rho_\phi \frac{f'}{f} = V' + \tilde{\rho}_\phi \frac{f''}{f_0}$$  \hspace{1cm} (19)

where we have used eqs.(9) in the last equality of eqs.(18) and (19). The mass of $\phi$ is given by

$$m^2 = V''_T = V'' + \rho_\phi \frac{f''}{f} = V' + \tilde{\rho}_\phi \frac{f''}{f_0}.$$  \hspace{1cm} (20)

Using eq.(19) we can rewrite eq.(8) as

$$\ddot{\phi} + 3H \dot{\phi} + V'_T = 0.$$  \hspace{1cm} (21)

While the derivative of the effective potential $V'_T$ is negative the field $\phi$ will evolve to larger values since $\phi > 0$. However, a minimum of $V_T$ can be reached if $V'_T = 0$ which requires $f' > 0$, since $V' < 0$ by hypothesis, with $V' = -\rho_\phi f'/f$. Taking the time derivative $V'_T = V''_T + \partial_t V'_T = 0$ one obtains [10]

$$\ddot{\phi} = -\frac{3HV}{m^2} = \frac{3H \rho_\phi f'}{m^2 f} = \frac{3H \rho_\phi f'}{m^2 f_0}.$$  \hspace{1cm} (22)

Notice that even in the case with $V'_T = 0$ we have $\dot{\phi} > 0$, i.e. the $\phi$ grows with time even at the minimum of $V_T$, and we also have $\dot{\phi} = f' \dot{\phi} > 0$. The mass $m$ given by eq.(20) becomes [10]

$$m^2 = V'' = \rho_\phi \frac{f''}{f} = \rho_\phi \frac{f''}{f_0} (1 + \frac{f'^2}{f_0^2} \tilde{\rho}_\phi \frac{V}{V})$$  \hspace{1cm} (23)

with $\tilde{\Gamma} \equiv V'' / V / V''$ ($\tilde{\Gamma}_m \geq 1$ if the field $\phi$ is tracking [3]) and we used $V' = -\rho_\phi f'/f$. Approximating $V \simeq \rho_\phi$ we get

$$m^2 / H^2 \simeq 3 \Omega_\psi f'' / f \left(1 + \frac{f'^2}{f_0^2} \tilde{\Gamma}_m \frac{\Omega_\psi}{\Omega_\phi}\right).$$  \hspace{1cm} (24)

Solving eq.(21) with $V'_T \equiv 0$ (i.e. the usual slow roll condition $\ddot{\phi} \ll 3H \dot{\phi}$ is no longer satisfied) gives $\ddot{\phi} + 3H \dot{\phi} = 0$ with a solution $\phi \sim a^{-3}$. Since $\rho_\phi$ redshifts as $a^{-3}$ we see from eq.(22) that

$$k \equiv \frac{\dot{\phi}}{\rho_\phi} = \frac{3HF}{m^2 f_0} = \frac{f \sqrt{\rho_\phi(1+w_\phi)}}{\rho_\phi}.$$  \hspace{1cm} (25)

with $k$ a constant and we have used from eq.(3) $\dot{\phi}^2 = \rho_\phi(1+w_\phi)$ and $\rho_\phi = \rho_\phi f_0 / f$. By taking $\Omega_\phi = \rho_\phi / 3H^2, \Omega_\psi = \rho_\psi / 3H^2$, eq.(25) with $M = f$ gives

$$\frac{1}{k^2} \frac{M^2}{H^2} = \frac{3 \Omega_\psi^2}{\Omega_\phi(1+w_\phi)}.$$  \hspace{1cm} (26)

Dividing eq.(24) by eq.(26) we get

$$\frac{m^2}{M^2} = \frac{(1+w_\phi) \Omega_\phi}{\Omega_\psi f} \left(1 + \frac{f'^2}{f''} \tilde{\Gamma}_m \frac{\Omega_\psi}{\Omega_\phi}\right).$$  \hspace{1cm} (27)

At present time with $\Omega_\phi = 0.7, \Omega_\psi = 0.3, w_\phi = -0.9$ we have $\Omega_\psi / \Omega_\phi(1+w_\phi) \simeq 1.3$ and $M^2 / H^2 k^2 \simeq 5$.

IV. DECAY WIDTH $\Gamma$

The conditions for a particle to decay is that its lifetime $\tau = 1/\Gamma$, where $\Gamma$ is the decay width, is smaller than the life of the universe given by $t \propto 1/H$,

$$\frac{\Gamma}{H} > 1$$  \hspace{1cm} (28)

and that the energy condition (in the rest frame of the decaying particle)

$$m^2 \geq \Sigma_i M_i^2$$  \hspace{1cm} (29)

must be satisfied, where $m$ is the mass of the decaying particle and the sum is over the product particles with mass $M_i$.

We assume an interaction term $L_{int} = -f(\phi) \bar{\psi} \psi$ as in eq.(5) with $f$ not necessarily a positive power law function of $\phi$, as for example $h \propto e^{\phi}$ or $h \propto 1/\phi^2$. If we expand $f$ in a Taylor series around a time $t = t_c$, $\phi = \phi(t_c)$, we have $h = f_c + f'_c \delta \phi + (1/2) f''_c \delta \phi^2 + ...$, with $f' = df / d\phi$ and $f_c = f(\phi_c)$, then the interaction term gives an effective coupling

$$-L_{int} = f(\bar{\psi} \psi)$$  \hspace{1cm} (30)

between two fermions and $q$ quantum scalar fields $\phi$, with $q = 1, 2, ...$. The first term in eq.(30) gives the mass of the fermion while the second gives an interaction between one scalar field and two fermions. This term is responsible for the scalar decay. At the time $t_c$ the fermion field gets a mass given by $M = f_c$. Since the potential of $\phi$ might be a run away potential, i.e. the minimum is at $\phi \rightarrow \infty$, the expansion point $\phi_c$ is a function of time and the mass $M$ and couplings $f_c, f'_c$ are also functions of time.

If $f = h \dot{\phi}$, with $h$ constant, the decay rate of $\phi$ into two fermions is given by $\Gamma = h^2 m / 8\pi$ and generalizing it to an arbitrary function $f(\phi)$ we obtain

$$\Gamma = \frac{f'^2 m}{8\pi}.$$  \hspace{1cm} (31)
The dark energy scalar field will decay if $\Gamma/H > 1$ and
$m^2 > 2M^2$ which implies that
\[
\frac{\Gamma^2}{H^2} = \frac{f''m^2}{(8\pi)^2H^2} = \frac{3f''}{(8\pi)^2} \left( \frac{V''}{3H^2} + \Omega_\psi \frac{f''}{f} \right) > 1 \tag{32}
\]
or
\[
\frac{m^2}{2M^2} = \frac{1}{2} \left( \frac{V''}{f^2} + \rho_\psi \frac{f''}{f^3} \right) > 1 \tag{33}
\]
with $m^2 = V'' + \rho_\psi f''/f$ as defined in eq.(20).

V. GENERIC CONDITIONS

The interaction between dark energy and $\rho_\psi$ must be such that it allows the universe to accelerate recently. Therefore the dark energy scalar field must not decay before present day. However, the dark energy may decay in the future and the universe would then stop accelerating. We have therefore different sets of conditions: for $t$ smaller than $t_o$, present day $t = t_o$ and for times $t > t_o$.

A. Conditions for acceleration

For the universe to accelerate in the absence of any interaction the slow roll conditions in eq.(4) must be satisfied. However, once the interaction is turned on the slow roll conditions change.

For generality purpose we take the energy density of the universe as $\rho_\phi + \rho_\psi + \rho_{nim}$, with $\rho_{nim}$ a non-interacting matter with $w_{nim} = 0$. The Hubble parameter is then
\[
3H^2 = \rho_\phi + \rho_\psi + \rho_{nim} = \frac{1}{2} \dot{\phi}^2 + V(\phi) + \rho_\psi \frac{f(\phi)}{f_0} + \rho_{nim} = \frac{1}{2} \dot{\phi}^2 + V_T + \rho_{nim}. \tag{34}
\]

Using eqs.(10) and (11) it is easy to see that the universe accelerates if
\[
\frac{\ddot{a}}{a} = H^2 + \dot{H} = -\frac{1}{6} \left( \rho_m + \rho_\psi + \rho_\phi + 3\rho_\phi \right)
\]
\[
= -\frac{1}{6} \left( \rho_m + 2\rho_\phi^2 + V_T - 3V \right) \tag{35}
\]
\[
= -\frac{1}{6} \left( \rho_m + 2\rho_\phi^2 + \rho_\psi - 2V \right)
\]
is positive. A positive acceleration gives the constraint
\[
\frac{1}{2} \rho_{nim} + \rho_\psi + \dot{\phi}^2 < V. \tag{36}
\]

Clearly $V$ must be larger than $(\rho_m + \rho_\psi)/2$ and $\rho_\phi > (\rho_m + \rho_\psi)/2$. Eq.(36) gives the usual constraint $\Omega_\phi w_\phi < -1/3$, even though $\rho_\psi$ is also a function of $\phi$. So as long as $\Omega_\phi < 1/3$, or $\Omega_{nim} + \Omega_\psi > 2/3$, there will be no acceleration. For recent times $\rho_\psi > \rho_\phi$ but well before that we will have the dark energy as subdominant energy density $\rho_\psi \ll \rho_m \equiv \rho_{nim} + \rho_\phi$.

We would like to emphasize that the ”naive” conditions on the total scalar potential $V_T$
\[
\left| \frac{V'}{V_T} \right|, \quad \left| \frac{V''}{V_T} \right| < 1 \tag{37}
\]
do not imply an accelerating epoch. The reason is that only the potential $V$ and not $V_T$ enters with a negative sign (within the brackets) in eq.(35). The proof is simple, just take the limiting case $V = V' = V'' = 0$, in this case $V_T = \rho_\psi$ and $V_T'/V_T = f'/f, V_T''/V_T = f''/f$ can be both much smaller than one but condition in eq.(36) is not satisfied and therefore the universe does not accelerate.

B. Conditions for $t < t_o$

For dark energy not to decay before present day we need the constraints in eqs.(32) and (33), $\Gamma/H < 1$ or $m^2 > 2M^2$, not to be satisfied simultaneously, therefore we need
\[
\frac{\Gamma^2}{H^2} = \frac{f''m^2}{(8\pi)^2H^2} = \frac{3f''}{(8\pi)^2} \left( \frac{V''}{3H^2} + \Omega_\psi \frac{f''}{f} \right) < 1 \tag{38}
\]
or
\[
\frac{m^2}{2M^2} = \frac{1}{2} \left( \frac{V''}{f^2} + \rho_\psi \frac{f''}{f^3} \right) < 1 \tag{39}
\]
Clearly the interaction term $f(\phi)$ and its derivatives play the crucial roll. The term $V''/3H^2$ in eq.(38) is much smaller than one by hypothesis so the inequality requires $3f'' f^4/8(8\pi)^2 < 1/\Omega_\phi$.

C. Present day $t = t_o$

At present day we know that the universe is dominated by dark energy with
\[
\Omega_{\phi o} = 0.73, \quad \Omega_{mo} = 0.27 \tag{40}
\]
and dark matter $\Omega_m$ includes non-interacting dark matter $\rho_{nim}$ and interacting dark matter $\rho_{im}$, i.e. $\rho_m = \rho_{nim} + \rho_{im}$. In our case the interacting dark matter is given by fermions, $\rho_{im} = \rho_\psi$. For simplicity we could take $\rho_m = \rho_\psi, \rho_{nim} = 0$.

The cosmological data for a constant equation of state of a dark energy, assuming no interacting with other fluids, is [7, DE]
\[
< w_{ap} > = -1.04 \pm 0.06. \tag{41}
\]
The equation of state of state $w_\phi$ (given by eq.(3)), is $w_\phi \geq -1$ but an apparent $w_{ap} < -1$, as defined in eq.(56), can be obtained due to the interaction between
with \( \Delta \) and the fermion fields. Since \( w_{app} \) is smaller than -1 we take this as an indication of dark energy interaction.

At present time \( \rho_\phi > \rho_0 \) and using the condition for acceleration in eq.(36) we need
\[
\frac{\rho_\phi}{H^2} < 2
\]  
(42)
where we have taken for simplicity \( \rho_{nim} = 0 \) and the slow roll conditions in eq.(4), which imply that \( \dot{\phi}^2/V \ll 1 \).

D. Conditions for dark energy decay for \( t > t_o \)

Dark energy will stop dominating the universe if either \( \rho_\phi \) redshifts slower than \( \rho_0 \), due to the interaction terms \( f(\phi) \), or if the dark energy particle decays into the fermion fields.

Dark energy density will dilute compared to the fermion energy density if
\[
\Upsilon = \frac{\dot{f}}{3Hf} \left( \frac{\rho_\phi + \rho_\psi}{\rho_\phi} \right) > \Delta w. 
\]  
(43)
Dark energy decay will take place if \( \Gamma > 0 \) and \( \rho_\phi \) dominating the universe. In this case the interaction term \( \delta \) is subdominant and the evolution of \( \rho_\phi \) and \( \rho_\psi \) is the usual one, i.e. \( \rho_\phi \propto a^{-3(1+w_\phi)} \) and \( \rho_\psi \propto a^{-3(1+w_\psi)} \).

VI. FLUID EVOLUTION

The complete evolution of \( \rho_\phi \) and \( \rho_\psi \) depends on the effective equation of state parameters defined in eqs.(13). The difference in eqs.(13) is [5]
\[
\Delta w_{eff} \equiv w_{\phi eff} - w_{\phi eff} = \Delta w - \Upsilon 
\]  
(44)
with \( \Delta w \equiv w_\phi - w_\psi \) and
\[
\Upsilon = \frac{\delta}{3H} \left( \frac{\rho_\phi + \rho_\psi}{\rho_\phi} \right). 
\]  
(45)
If we take the ratio \( y \equiv \rho_\phi/\rho_\psi = \Omega_\phi/\Omega_\psi \) the derivative of \( y \) w.r.t. time is [5]
\[
\dot{y} = 3Hy [\Delta w - \Upsilon] . 
\]  
(46)

The value for \( y \) is constraint to \( 0 \leq y \leq \infty \) with \( y = 0 \) for \( \rho_\phi = 0 \) and \( y = \infty \) for \( \rho_\psi = 0 \). Clearly from eq.(46) we see that the evolution of \( y \) depends on the sign of \( \Delta w - \Upsilon \). Clearly the relevant quantity to determine the relative growth is given by \( \Upsilon \) and if \( \Upsilon > \Delta w \) we have \( \Delta w_{eff} < 0 \) and \( \rho_\phi \) will dominate the universe at late times while for \( \Upsilon < \Delta w \) we have \( \Delta w_{eff} > 0 \) and \( \rho_\phi \) will prevail. For no interaction \( \delta = 0 \) and \( \Upsilon = 0 \) gives \( \Delta w_{eff} = \Delta w > 0 \) and \( \rho_\phi \) dominates at late times. If \( \Upsilon = \Delta w \) then \( w_{\phi eff} = w_{\phi eff} \) and the ratio of both fluids \( \rho_\psi/\rho_\phi \) will approach a constant value, and if the universe is dominated by \( \rho_\phi \) it will not change, i.e. \( \rho_\phi \propto a^{-3(1+w_\phi)} \) and \( \rho_\psi \propto a^{-3(1+w_\psi)} \).

1. Non Interaction solution: \( \Delta w > \Upsilon \)

If \( (w_\phi - w_\psi) > \Upsilon \) then \( \dot{y} \) is positive and\( y \) will increase, i.e. \( \rho_\psi \) will dilute faster than \( \rho_\phi \), and we will end up with \( \rho_\psi \) dominating the universe. In this case the interaction term \( \delta \) is subdominant and the evolution of \( \rho_\phi \) and \( \rho_\psi \) is the usual one, i.e. \( \rho_\phi \propto a^{-3(1+w_\phi)} \) and \( \rho_\psi \propto a^{-3(1+w_\psi)} \).

2. Finite solution: \( \Delta w = \Upsilon \)

A solution to eq.(46) with \( y \) constant and \( \rho_\psi \neq 0 \), \( \rho_\phi \neq 0 \) is only possible if \( \delta/H \rho_\phi \) is positive and constant since \( \Upsilon = \delta(1+y)/H \rho_\phi \) must be constant and positive, taking \( \Delta w = w_\psi - w_\phi \) constant. Let us take \( \delta = C H \rho_\phi \) with \( C \) a positive constant, and from eq.(46) for \( \dot{y} = 0 \) we get a stable value of \( y \) given by [5]
\[
y = \frac{\rho_\phi (a/a_i)}{\Omega_\phi} = \frac{C}{1 + \frac{w_\psi - w_\phi}{3}} - 1 . 
\]  
(48)
It is easy to see that the solution \( y \) is stable since from eq.(46) the fluctuation \( \delta y = y - y_k \) first order behaves as \( \delta y/\delta y = -C H y_s \) (\( CH \) is positive by hypothesis) giving \( \delta y \to 0 \). The evolution of \( \rho_\phi \) is given by [5]
\[
\rho_\phi = \rho_{\phi i} \left( \frac{a}{a_i} \right)^{3(1+w_\phi) - C} = \rho_{\phi i} \left( \frac{a}{a_i} \right)^{3(1+w_\phi) - C(1+w_\psi)}/(1 + y_s) . 
\]  
(49)
where we have used that \( C = 3(w_\psi - w_\phi)/(1 + y_s) \). Since \( C \) is positive the solution in eq.(49) dilutes faster than the non interacting solution \( \rho_\phi \propto a^{-3(1+w_\phi)} \) and with an effective equation of state \( w_{\phi eff} = w_\phi + C/3 = (w_\psi + y_s w_\phi)/(1 + y_s) > w_\phi \).

3. Interacting solution: \( \Delta w < \Upsilon \)

For \( \rho_\phi \) to dominate the universe we need \( y \ll 1 \) at late time and the (interaction) term \( \Upsilon \) should dominate over \( \Delta w \). A simple example is when \( A = \delta/\rho_\phi \) is constant and positive. In this case the evolution of \( y \) is given by
\[
y = y_i \left( \frac{a}{a_i} \right)^{3(w_\psi - w_\phi)} e^{-A \tau} \to 0 
\]  
(50)
at late times for any value of \( w_\psi - w_\phi \). Now, let us take \( A \) as a constant decay rate \( \Gamma \).

I) Constant decay Rate \( \Gamma = A = \delta/\rho_\phi \)

If the mean lifetime of a particle is given by a constant \( \tau \) then we can expect it to decay into lighter fields. In the case \( \Gamma = 1/\tau \) constant the constrain \( \Gamma > 3H \) will be satisfied at some time \( t \sim \tau \) (or equivalently at \( \Gamma/H \sim 1 \)).

For a constant \( \Gamma \) the interaction term is \( \delta = \Gamma \rho_\phi \) and the solution to eq.(11) using (14) is simply
\[
\rho_\phi = \rho_{\phi i} \left( \frac{a}{a_i} \right)^{-3(1+w_\phi)} e^{-\Gamma \tau} . 
\]  
(51)
For small $t$, i.e. for $3H(1 + w_ψ) \gg \Gamma$, one has the usual redshift in the absence of any interaction $\rho_ψ \propto a^{-3(1 + w_ψ)} \propto 1/t^2$ while for large $t$, i.e. $3H(1 + w_ψ) \ll \Gamma$, one has an exponential decrease $\rho_ψ \propto e^{-\Gamma t}$. Since $\Gamma$ is constant, then clearly $\Gamma \rho_ψ/\rho_φ$ will not be constant and the solution to eq.(11) will be quite different to that of $\rho_φ$.

Let as take the ansatz $\rho_ψ = \rho_ψ o a^{-3(1 + w_ψ)} + g(t) \rho_φ$ taking the time derivative we have

$$\dot{\rho}_ψ = -3H(1 + w_ψ)\rho_ψ + \dot{\rho}_ψ + g\dot{\rho}_φ = -3H(1 + w_ψ)\rho_ψ + (3H\Delta w_ψ - \Gamma g + \dot{g}) \rho_φ$$

Eq. (52) is valid for any functional form of $\Gamma$ and not necessarily a constant. However, a simple solution can be found when $\Gamma$ is constant for small $t$. The solution $g = g(t)$ giving $\Gamma \simeq (3\Delta w H t + 1)\dot{g}$, constant, and using for the dominant energy density $H t = 2/3(1 + w_ψ)$ we have $\Gamma = \dot{g}(1 + 2w_ψ - w_φ)/(1 + w_ψ)$. The solution to eq.(52) is then

$$\rho_ψ = \rho_ψ o a^{-3(1 + w_ψ)} + q \rho_φ \Gamma t$$

with $q = (1 + w_ψ)/(1 + 2w_ψ - w_φ)$ (if we have matter decaying into radiation, $w_ψ = 0$ and $w_φ = 1/3$, then $q = 3/5$). Eq.(54) shows that the fluid redshifts as usual for $\Gamma t \ll 1$, i.e. $\rho_ψ \propto a^{-3(1 + w_ψ)}$, where the first term in eq.(54) dominates. At $t \approx t_o(\Gamma t_o)^{-q}$ the second term starts to dominate giving $\rho_ψ \propto \rho_φ t \propto t^{-1} \propto a^{-3(1 + w_ψ)/2}$, with the fluid $\rho_φ$ decaying into $\rho_ψ$ at around $\Gamma t \approx 1$ when $\rho_ψ$ starts decreasing exponentially fast. For $\Gamma t \gg 1$ the fluid $\rho_ψ$ will dominate the universe redshifting again as $\rho_ψ \propto a^{-3(1 + w_ψ)}$.

We have seen that a constant decay rate $\Gamma$ gives an exponentially suppressed energy density $\rho_ψ$. However, a constant decay rate is not realistic. The decay rate $\Gamma$ for a scalar field decaying into fermions, given in eq.(31), is a function of $m$ and $f'$ and both are in general field and time dependent so $\Gamma$ will not be constant.

A. Apparent Equation of State

An interesting result of the interaction between dark energy "DE" with other particles is to change the apparent equation of state of dark energy [6]-[10]. An observer that supposes that DE has no interaction sees a different evolution of DE as an observer that takes into account for the interaction between DE and another fluid. This effect allows to have an apparent equation of state $w < -1$ for the "non-interaction" DE [10] even though the true equation of state of DE is larger than -1.

Let as take the energy density $\rho = \rho_φ + \rho_ψ = \rho_{DE} + \tilde{\rho}_ψ$. The energy densities $\rho_φ, \rho_ψ$ are given by eqs.(11) and (10) and these two fluid interact via the $\delta$ term. On the other hand the energy densities $\rho_{DE}$ and $\tilde{\rho}_ψ$ do not interact with each other by hypothesis and therefore we have $\tilde{\rho}_ψ = -3H (we have taken $w_ψ = 0$) and $\dot{\rho}_{DE} = -3H \rho_{DE}(1 + w_{ap})$, i.e.

$$\tilde{\rho}_ψ = \rho_ψ o a^{-3}$$

$$\rho_{DE} = \rho_{DE} o a^{-3(1 + w_{ap})}$$

if $w_{ap}$ is constant. It was pointed out that the apparent equation of state $w_{ap}$ can take values smaller than -1 and it is given by [10]

$$w_{ap} = \frac{w_φ}{1 - x}$$

and we have used eq.(9). We see from eq.(56) that for $\rho_ψ < \tilde{\rho}_ψ$, i.e. $f(t) < f_o(t_o)$ for $t < t_o$, we have $x > 0$ and $w_{ap} < w_φ$ which allows to have a $w_{ap}$ smaller than -1 in the past. However, for $f > f_o$ with $x < 0$ and $w_{ap} > w_φ$ for $t > t_o$, The observational evidence shows that $f$ is at present time growing function of $\phi$, i.e. $f' > 0$.

B. Evidence for Dark Energy Decay

The cosmological observations prefer an equation of state $w < -1$ for dark energy. In principal a $w < -1$ for a fluid is troublesome since it has instabilities and causality problems. However, as seen in section VI A this can be an optical effect due to the interaction between dark energy with other particles as for example fermions. It was shown that for an effective $w < -1$, interpreted as an interaction between dark energy and another fluid, can be a signal for dark energy decay in the future with the universe no longer accelerating [5].

Lets us expand $f$ around present time $t_o$ $f = f_o + \tilde{f}_o \delta t$ and we keep only the first order term. Now, from eq.(56) $x$ can be written as [5]

$$x = -\frac{\tilde{\rho}_ψ}{\rho_ψ} \left( \frac{f}{f_o} - 1 \right) \simeq -\frac{\tilde{\rho}_ψ}{\rho_ψ} \frac{f_o}{f_o} \delta t = -\frac{\rho_ψ}{\rho_φ} \frac{f_o}{f} \delta t$$

with $\tilde{\rho}_ψ = \rho_ψ f_o/f$, $\delta t \equiv t - t_o$. For $t < t_o$ we have $x > 0$ since $f_o > 0$. The interaction term $\delta = \rho_ψ / f$, is now [5]

$$\delta = -\rho_ψ \frac{x}{\delta t}$$

where we have taken $\tilde{f} = f_o$, consistent with the approximation taken for $f$. Let us take the simple phenomenological ansatz for $x$ given as $x = \beta (a_o - a) = -\beta a$ with $\beta$ a constant to be determined by observations. The SN1a data are in the range $1 > a > 2/5$, i.e. for a redshift $0 < z < 1.5$, and the best fit solution has an average
equation of state $<w> \approx -1.1$ [1]. Taking the average of $w_{app} = w/\beta$ we have

$$<w_{app}> = \frac{\int_{a_1}^{a_2} w_{app} \frac{da}{a}}{\int_{a_1}^{a_2} \frac{da}{a}} = \frac{w \log[1 - \beta(1 - a_1)]}{\beta(a_1 - 1)}$$

(59)

where we have used $x = -\beta a$ and $a_0 = 1$. As an example let us take $<w_{app}> = -1.1$, as suggested by the observations $[1, 2], [9]$, and $w_\phi = -0.9, w_\psi = 0, a_1 = 2/5$. In this case we obtained from eq.(59) the value $\beta = 0.56$ [5]. Using the ansatz $x = -\beta a$ with $\beta a = aH\delta t$ then eq.(58) becomes [5]

$$\delta = \beta H \rho_\psi, \quad \frac{\dot{f}}{f} = \beta a H \text{ and } \Upsilon = \frac{\beta a}{3} \left(1 + \frac{\rho_\psi}{\rho_\phi}\right).$$

(60)

Using eq.(11) with the interaction term given in eq.(60) we get an energy density [5]

$$\rho_\phi = \rho_\psi a^{-3(1+w_\phi)}e^{-\beta(a-1)}$$

(61)

which shows that $\rho_\phi$ dilutes as $a^{-3(1+w_\phi)}$ for $a \ll a_0 = 1$ and $\rho_\phi$ is exponentially suppressed for $a \gg 1$. In this case $3\Delta w < \Upsilon$ and $\Upsilon \to \infty$ at late times, as seen from eq.(60).

VII. DARK ENERGY DISAPPEARANCE

The decay rate $\Gamma = f''m^2/8\pi$, the mass $m$ and $M$ are a field and time dependent quantities. If we take $f$ as a monotonic function, since $f''$ at present time is positive (cf. section VI B), then $\dot{f}$ should be an increasing function of $\phi$ for all values of $\phi$. This implies that the mass of the fermion $M = f$ increases at all times. So, if the mass of the scalar field $m$ vanishes asymptotically then $m/M$ will be smaller than one at some point in the future but larger in the past. So, we would expect on general grounds that the condition $m^2 > 2M^2$ would be easily satisfied in the past. Therefore we require $\Gamma/H < 1$ and from eq.(38) we need

$$\frac{3f''}{8\pi} \frac{f^4}{f^2} < 1$$

(62)

where we have taken $\rho_\psi \gg \rho_\phi$ ($\Omega_\psi = 1$) in the far past.

A. Dark Energy Dilution

As we have seen in section VI dark energy dilution, $\rho_\phi \gg \rho_\psi$ at late times, will only take place if $\Upsilon > \Delta w$ with

$$\Upsilon = \frac{\dot{f}}{3fH} \left(1 + \frac{\rho_\psi}{\rho_\phi}\right).$$

(63)

Let us take $\dot{f}/fH = (\alpha/f)(df/da)$ with the ansatz $f = f_0 a^\alpha$. We then have $f''/fH = q$, $\Upsilon = q/3$ and the interaction term $\delta = q\rho_\psi H$. For $q > q_0 \equiv 3\Delta w$ we have $w_{eff} < w_{eff}$ and $\rho_\psi$ will dominate while for $q < q_0$ then $\rho_\phi$ will end up dominating the universe. Furthermore, if $f$ grows faster than power law then $\Upsilon \to \infty$ and $\rho_\phi$ will dominate at late times. this is the case discussed in section VI B as sen from eq.(60). For $f$ growing slower than power law we have $\Upsilon \to 0$ and $\rho_\phi$ will end up dominating the universe.

The limiting case is for $\rho_\phi/\rho_\phi$ constant and a solution is obtained for $\Upsilon = q/3 = \Delta w$. This case is equivalent as in section VI 2 with $\rho_\phi \leftrightarrow \rho_\phi$ and $q = C$. From eq.(48) we have

$$\frac{\rho_\psi}{\rho_\phi} = \frac{\Omega_\psi}{\Omega_\phi} = \frac{3\Delta w}{q} - 1$$

(64)

(see discussion below eq.(48)).

Now, we consider again the interaction term $f(\phi)$ as a function of a scalar field $\phi$ with $\dot{f} = f''\phi$. The evolution of $\phi$ is determined by eq.(7) and the sign of $V_T$ (c.f. eq.(63)) and $\rho_\phi$ will dilute slower than $\rho_\phi$ (even slower than without the interaction).

1. Case: Positive Derivative $V_T > 0$

First, let us consider the possibility that $V_T > 0$, which implies from eq.(21) that $\phi < 0$. In this case since $V' < 0$ then $f' > 0$ and $f'' \psi$ is negative giving a negative $V_T$ (c.f. eq.(63)) and $\rho_\phi$ will dilute slower than $\rho_\phi$ (even slower than without the interaction).

2. Case: Negative Derivative $V_T < 0$

For $V_T < 0$ we have $|V'| > \rho_\phi f'/f$ and from eq.(8) we can approximate

$$\dot{\phi} \approx -V'/3H.$$ 

(65)

Taking, for simplicity, $\rho_\phi + \rho_\psi = 3H^2$ in eq.(63) we get

$$\Upsilon = \frac{H}{f} \frac{\dot{\phi}' V'}{\rho_\phi f} > \frac{-f' V'}{3 \rho_\psi} > \Delta w.$$ 

(66)

The condition $V_T < 0$ implies $f'/f < |V'|/\rho_\phi$ and together with the slow roll conditions $\rho_\phi > V > |V'|$ and eq.(66) we get the constraint

$$\frac{\Omega_\phi}{\Omega_\psi} > \frac{f'}{f} > 3\Delta w$$

(67)

which sets an upper value

$$\Omega_\psi < \frac{1}{1 + f'/f} < \frac{1}{1 + 3\Delta w}.$$ 

(68)

If we take for example $w_\psi = 0$ and $w_\phi \approx -1$ we have $\Delta w = 1$ and $\Omega_\psi < 1/4$. Therefore, we conclude that in the case $V_T < 0$ it is not possible for $\rho_\phi$ to dominate the universe.
3. Case: \( V'' = 0 \)

As we have seen, while the derivative of the effective potential \( V' \) is negative the field \( \phi \) evolves to larger values since \( \dot{\phi} > 0 \). However, a minimum of \( V'' = 0 \) can be reached if \( V'' = 0 \), which requires \( f' > 0 \) since \( V'' < 0 \) by hypothesis, and \( V'' = -\rho f'/f \). Taking the time derivative \( V'_t = 0 \) one obtains with

\[
\dot{\phi} = -\frac{3HV'}{m^2} = \frac{3H\rho f'}{m^2f_o} \quad (69)
\]

(c.f. eq.(22)) which is still positive, i.e. even tough \( V'' = 0 \) the field \( \phi \) still grows with time. The reason being that \( \rho \) is a function of time (through \( a(t) \)). Calculating \( \Gamma = H\dot{\phi}f' - \rho \phi > \Delta w \) and using eq.(69) and \( 1 > |V'|/V = \rho f'/fV \) we get the constraint

\[
\frac{\Omega_\phi}{\Omega_f} > \frac{f'}{f} > \frac{m^2}{3H^2}\Delta w \quad (70)
\]

which sets an upper value

\[
\Omega_\phi < \frac{1}{1 + (m^2/3H^2)\Delta w}. \quad (71)
\]

Comparing \( \dot{\phi} \) form eqs.(65) (labeled \( \dot{\phi}_1 \)) and (69) (labeled \( \dot{\phi}_2 \)) we see that the ratio \( \phi_1/\phi_2 = m^2/9H^2 \) is exactly that in the denominator of eqs.(68) and (71). For \( \phi_1/\phi_2 = m^2/9H^2 \ll 1 \) the evolution of \( \phi \) is faster in the case \( V'' = 0 \) than for \( V'' < 0 \). Now, if \( m^2/3H^2 \geq 1 \) and taking \( \Delta w = 1 \) we have \( \Omega_\phi \leq 1/2 \). However, if \( m^2/3H^2 \ll 1 \) then the constraint in eq.(71) becomes \( \Omega_\phi \approx 1 \) allowing for the fermion fields to dominate at late times.

It is easy to see from eq.(70) that in the limit \( \Omega_\phi \to 0 \) we need \( f'/f \to 0 \) and \( m^2/3H^2 \to 0 \). From eq.(23) the condition \( m^2/3H^2 \to 0 \) implies that \( f''/f \to 0 \) since \( \Omega_\phi = 1 - \Omega_\phi = 1 \) and \( V''/V \ll 1 \) by hypothesis.

### B. Dark Energy Decay

The conditions for dark energy to decay into fermions given in eqs.(32) and (33) are constraints on the dark energy mass \( m^2 = V'' + \rho \phi f''/f \). Let us consider the two possible cases \( V'' \geq \rho \phi |f''/f| \) and \( V'' \leq \rho \phi |f''/f| \) separately.

1. \( \text{Case: } m^2 \simeq V'' \), i.e. \( V'' \geq \rho \phi |f''/f| \).

Using the slow roll hypothesis on \( V \) given in eqs.(4) we can set the upper limit \( \rho \phi > V > V'' > \rho \phi |f''/f| \) which gives the constraint

\[
\frac{\Omega_\phi}{\Omega_f} > \frac{|f''|}{f} \quad (72)
\]

If we take \( \Omega_\phi + \Omega_\psi = 1 \) then eq.(72) gives an upper limit \( \Omega_\phi < 1/(1 + |f''/f|) \) and in the case \( \Omega_\phi \to 0 \), \( \Omega_\phi \to 1 \) we require \( |f''/f| \to 0 \).

The constrain in eq.(32) gives \( V'' > H^2/f^{r4}(8\pi)^2 \) and using \( H^2 > V/3 \) and \( V > V'' \) we obtain

\[
\frac{f''}{f^4} > \frac{1}{24\pi} \quad (73)
\]

while for condition in eq. (33) we have

\[
1 > 2\frac{f^2}{V''} \geq 0. \quad (74)
\]

Since \( V'' \to 0 \) by hypothesis, eq.(74) implies that \( f \to 0 \) at late times, i.e. \( f \) is a decreasing function at large \( t \), since \( M = f \geq 0 \). Eq.(74) can only be satisfied if \( f'' \) is negative, i.e. \( f \) is a decreasing function. However, at present time the dark energy interacting must have \( f''/f > 0 \) if \( w < -1 \), as seen in section (VI B). So, unless \( f \) increases at present time and than it starts decreasing at a later stage (i.e. it is no longer a monotonic function of \( \phi \)) the conditions for a dark energy decay into fermions with \( V'' > \rho \phi f''/f \) cannot take place. However, \( f \) will no longer be a monotonic function of \( \phi \).

2. \( \text{Case: } m^2 \simeq \rho \phi f''/f, \text{ i.e. } \rho \phi f''/f \geq V'' \).

Let us now take the case \( m^2 \simeq \rho \phi f''/f > V'' \), which implies that

\[
0 \leq \frac{f''}{f^{r4}} < \frac{\rho \phi}{f} = \frac{\rho \phi}{f_o} \propto a^{-3} \quad (75)
\]

where we used eq.(9). Eq.(75) shows that at late times (with \( a \to \infty \)) we have \( V''/f'' \to 0 \), i.e. \( V'' \) must decrease faster than \( f'' \). Using \( m^2 \simeq \rho \phi f''/f \) and condition in eq.(33) we get in this case the constraint

\[
0 \leq \frac{f}{f''} < \frac{\rho \phi}{2f} = \frac{\rho \phi}{2f_o} \propto a^{-3} \quad (76)
\]

where we have used again eq.(9). Once again we see that at \( a \to \infty \) that \( f/f'' \to 0 \) at late times. Finally, the constraint in eq.(32), with \( m^2 \simeq \rho \phi f''/f \) and \( 3H^2 > \rho \phi \), gives

\[
\frac{f''}{f} > \frac{1}{3(8\pi)^2}. \quad (77)
\]

Since from eq.(76) \( f/f'' \) must vanish at late times, eq.(77) is satisfied as long as \( f^{r4} \) does not go to zero faster than \( f/f'' \).

### C. Examples

To have dark energy decaying the conditions \( V''/f'' \to 0, f/f'' \to 0 \) and \( f^{r4}/f > 1/3(8\pi)^2 \) must be satisfied. A simple example is

\[
f = f_o e^{\alpha \phi^2} \quad (78)
\]
with $\alpha$ a positive constant. Since $\phi$ grows with time, so does $f, f', f''$ and the constraint $V''/f'' \ll 1$ and $f''f''/f > f^2 > 1/3(8\pi)^2$ are satisfied trivially and the condition
\begin{equation}
\frac{f''}{f} = 2\alpha + 4\alpha^2\phi^2 \gg 1
\end{equation}
is also satisfied for $\phi \to \infty$. Clearly, this potential will not work for dark energy dilution.

Dark energy diluting needs $f'/f \to 0$ and $f''/f \to 0$ and an example is
\begin{equation}
f = f_0 \phi^\alpha
\end{equation}
with $\alpha \geq 2$. In this case $f'/f \sim 1/\phi$ and $f''/f \sim 1/\phi^2$ and $\phi$ grows with time.

D. Summary

To conclude, we have seen that if the conditions on the interaction term $f'/f \to 0$ and $f''/f \to 0$ are satisfied then we can have a dark energy diluting faster than the energy density of fermions. In this case we will have $m^2/H^2 \ll 1$. On the other hand, for dark energy to decay into fermion fields we require that dominant part of the scalar mass is given by the interaction term $m^2 \sim \rho_\phi f''/f \gg V''$. In this case, the conditions to be satisfied are given by eqs. (75), (76) and (77), i.e. $V''/f'' \to 0$, $f/f'' \to 0$ and $f''f''/f > 1/3(8\pi)^2$.

The condition for dark energy redshifting faster than fermions $f''/f \ll 1$ and the condition for dark energy decay into fermions $f/f'' \ll 1$ cannot be simultaneously meet. Clearly form eq.(36) if the universe is dominated by $\rho_\psi$ than it will not accelerate. We have also seen simple examples of the interaction term $f$ which require no fine tuning of any parameter.

VIII. CONCLUSIONS

A dark energy component is responsible for the present stage of acceleration of our universe. If no fine tuning is assumed on the dark energy potential it is easy to see that since it dilutes slower than the other fluids, e.g. matter, then it will end up dominating the universe at late times and the universe will not stop this stage of acceleration anymore.

In this letter we have studied the possibility that the universe will stop accelerating and that dark energy decays into fermion fields. The interaction between dark energy and fermions is given through a fields dependent mass $M = f(\phi)$ of the fermion fields.

Furthermore, the fact that the equation of state of dark energy seems to be smaller than minus one as suggested by the cosmological data can be an indication that dark energy does indeed interact with other fluids. We take this fluid to be fermions. Using the observational data we can determine present day value of the interacting function $f(\phi)$ and its derivative and we can then extrapolate the result into the future.

The interaction term $f(\phi)$ plays an important role in the evolution of the dark energy. We determine the conditions under which a dark energy can dilute faster than the fermion fields or it can decay into these fields. We obtained that only in the case where the mass of the dark energy, given by $m^2 = V'' + \rho_\phi f''/f$, is dominated by the interaction term can the dark energy decay. While for dark energy diluting faster then the fermion fields the conditions $f'/f \rightarrow 0$ and $f''/f \rightarrow 0$ must be satisfied.

We have shown that naive slow roll conditions on the effective potential $V_T$ do not imply an accelerating epoch. The condition needed is that the scalar potential $V$ dominates the universe (c.f. eq.(36))

We have seen, therefore, that it is indeed possible to live now in an accelerating epoch dominated by the dark energy and without introducing any fine tuning parameters the dark energy can either dilute faster or decaying into fermions in the future. The acceleration of the universe will then cease.

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