1. Introduction

The study of an artificial kidney in hemodialysis has attracted many researchers in the past decades. When the two natural kidneys present in humans are damaged, an artificial kidney, a dialysis machine, is used during hemodialysis to provide significant continuous blood purification and improve the quality of life of the kidney for patients. The main impact on a patient’s well-being using an artificial kidney in hemodialysis is that it improves adequacy of life and prolongs survival [1]. A dialysis machine is used in which the patient’s blood is exposed to one side of a membrane of a large surface area, on the other side of which is a fluid into which unwanted waste materials in the blood can pass through natural diffusion [2]. The artificial kidney is a man-made device. It is then made by hollow fiber and contains approximately 10000 hollow fibers, each with an inner diameter of about 200 μm when wet, the membrane thickness of about 20 – 45 μm, the length of about 160 – 125 mm, pore area of 1.5 m², and mean pore size of 5 – 10 nm [3]. An artificial kidney can be applied in biomedical engineering, such as the design of blood pumping machines, the design of peristaltic plasma pumping, and the design of modern artificial kidney machines. Neurologists and engineers who can operate biomedical machines and provide good services to patients with kidney problems in different communities were first considered. A medical study on how to improve renal kidney services in Sub-Saharan African countries, particularly in Tanzania [4], showed that the number of clinicians and experts in kidney problems increased to 14 in...
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2018 from one in 2006. Furthermore, Somji et al. [5] conducted a nonexperimental study on dialysis centers and embedded factors in patients with end-stage chronic kidney disease receiving hemodialysis. Different models were formulated to study more about how waste products and toxins can be filtered out of the blood to improve the life of patients, the difference between Newtonian and non-Newtonian fluid flow models of an artificial heart valve, [6]. There is an effective method of mass transfer through a dialyzer with different empirical approaches, [7].

In the recent past, nanotechnology has played an important role in biomedical fields by dissolving nanoparticles in the base fluid to improve the thermal properties of the base fluid, [8]. Fluids with suspended nanoparticles, or nanometre-sized particles, are said to be nanofluids. Nanofluids are liquids or base fluids such as blood, water, or oils that contain a suspension of nanoparticles that give thermo-physical properties such as thermal conductivity, thermal diffusivity, viscosity, and convective heat transfer coefficients. Nanoparticles are made up of metals, oxides, nitrides, or carbides having very small diameters (< 100 nm). In the targeted field of use, nanofluids have a variety of applications such as designing blood pumping machines, peristaltic plasma pumping, using gold nanoparticles in thermo-therapy, and getting rid of cancer cells. Various studies on the use of nanofluids have been carried out, such as [9] who developed a flow problem model for heat transfer and flow of nanofluids through a porous medium in the presence of a magnetic field. Recent advancements in the thermal stability, machining, solar energy, and biomedical fields used nanofluids [10]. A mathematical simulation model of a convective boundary layer of a nanofluid along an inclined angle plate considering the presence of a magnetic field was developed [11]. A mathematical model governed a chemically reactive porous flat vertical plate with a second-order slip condition of natural convective nanofluid flow [12]. However, a mathematical analysis of the thermal and mass of species of nanofluid flow along a thin membrane exponentially stretching sheet with partial slip was developed [13], a review of energy transfer through free convection using nanofluids in cavities by the influence of magnetic field [14]. Keller-box numerical investigation of micropolar nanofluid flows across an inclined surface [15]. A mathematical model for hydromagnetic nanofluid flow along a semipermeable membrane with stretching hollow tubes was developed [16].

Nanofluid flow problems past a porous stretching cylinder have received much attention in recent years because of their application in biomedical engineering. For example, blood dialysis in an artificial kidney, blood flow in the capillaries, the flow of blood in oxygenators, the design of filters in transpiration, cooling boundary layer control, and gaseous diffusion. A number of studies have been done with stretching cylinder, a numerical study on hydromagnetic Casson nanofluid flow with heat and mass properties over a porous stretching cylinder [17]. A mathematical model governing MHD nanofluid flow in non-Darcian Forchheimer past a porous stretching cylinder is analyzed [18]. We analyzed heat transfer characteristics on MHD nanofluid over-stretching cylinder [19]. A mathematical model governing the hydromagnetic mass transfer of Casson nanofluid over a thin permeable stretching cylinder considering the energy of isothermal radiation was developed [20]. The effects of heat dissipation on MHD nanofluid over a porous stretching cylinder through Darcy–Forchheimer medium were determined [21].

For more than a century, scientists and engineers have made great efforts to enhance the poor thermal conductivity of conventional fluids. In 1873, Maxwell proposed the idea of using metallic particles to enhance the thermal conductivity of materials. Maxwell presented a theory for the effective conductivity of slurries by dispersing millimeter or micrometer-sized particles with a size between 0.1 μm and 100 μm in liquids. Metallic nanoparticles proposed to improve the thermal conductivity of the bio-fluid must have the following factors: nanoparticle concentration, small nanoparticle size, cylindrical nanoparticle shape, and type of nanoparticle such as metallic particles, [22]. We analyzed studies with the effect of thermo-conductivity on nanofluid flow past stretching plates, such as the study on unsteady MHD slip fluid flow of non-Newtonian nanofluid using Steven’s power law model over a moving surface with temperature-dependent thermal conductivity [23]. This study explores the effects of combined thermal conductance and viscosity on Casson nanofluid flow with radiation and velocity infiltrate, [24]. Nanoparticles are dissolved in the base fluids to improve thermo-conductivity with various effective nondimensional parameters [25].

Various studies have not been done on hydromagnetic nanofluid convective flow past a porous stretching cylinder in an artificial kidney with the effect of thermo-conductivity. The goal of the present investigation is to study the effects of nanoparticles in an electrically conducting nanofluid in order to improve the thermal conductivity past a porous stretching cylinder during hemodialysis in an artificial kidney.

2. Mathematical Model Formulation

A mathematical model governing the effects of thermo-conductivity on hydromagnetic nanofluid convective flow past porous stretching cylinder in an artificial kidney is established. A laminar flow of an incompressible electrically conducting fluid past a porous stretching cylinder subjected to a magnetic field applied in the direction perpendicular to the fluid flow is considered. An artificial kidney has a cylindrical shape with hollow tubes; each tube has two compartments, one for patient’s blood and the other one for a washing fluid called dialysis solution. A thin membrane with hollow fibers separates these two parts. From the flow problem geometry, Figure 1, the cylinder has an axis of symmetry z-axis, any point in the cylinder is defined by three dimension coordinates. The projection of the perpendicular distance of the position vector from the z-axis on the rz-plane is r, the height of the point element from the rz-plane is z, and the projection angle from the θ -axis is θ. Therefore, in cylindrical polar coordinate, the coordinates are r, θ and z. The base fluid flows upward against the gravitational force
due to a constant pressure gradient $\nabla P$ which is transmitted through the hollow fiber membrane of the artificial kidney by the stretching material with uniform velocity $w_z$, and the solutes flow through the second conduit as waste products.

2.1. Flow Governing Equations. The conservation of mass, momentum, energy, and concentrations equations that govern the flow of an electrically conducting nanofluid past a porous stretching cylinder with the effect of thermo-conductivity are given in cylindrical polar coordinates as follows:

$$\frac{1}{r} \frac{\partial}{\partial r} (r w_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (w_\theta) + \frac{\partial}{\partial z} (w_z) = 0,$$

$$\frac{\partial w_z}{\partial t} + w_r \frac{\partial w_z}{\partial r} + w_\theta \frac{\partial w_z}{\partial \theta} + w_z \frac{\partial w_z}{\partial z} = \nabla \cdot \left( \rho \mathbf{T} \right) = \nabla \cdot \left( \rho \mathbf{F} \right) = \nabla \cdot \left( \rho \mathbf{u} \right)$$

$$= \left( \frac{1}{r} \frac{\partial}{\partial r} (r w_r) + \frac{\partial w_z}{\partial z} \right) + \frac{\partial^2 w_z}{\partial z^2} - \frac{\nabla \cdot (\mathbf{u} \times \mathbf{B})}{\mu} + \frac{\nabla \cdot (\mathbf{u} \mathbf{B})}{\mu} + \frac{\partial}{\partial z} \left( \frac{\partial \mathbf{u}}{\partial z} \mathbf{B} \right) + \frac{\partial}{\partial r} \left( \frac{\partial \mathbf{u}}{\partial r} \mathbf{B} \right) + \frac{\partial}{\partial \theta} \left( \frac{\partial \mathbf{u}}{\partial \theta} \mathbf{B} \right),$$

$$\frac{\partial}{\partial t} \left( \frac{\partial C}{\partial z} \right) + w_r \frac{\partial C}{\partial r} + w_\theta \frac{\partial C}{\partial \theta} = D_m \frac{\partial^2 C}{\partial z^2} + \frac{\partial^2 C}{\partial r^2} + \frac{\partial^2 C}{\partial \theta^2} + \alpha_{nf} \left( \frac{\partial^2 C}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C}{\partial r} \right) \right).$$

Initial and boundary conditions $w_r = 0$, $w_z = 0.5 \text{ms}^{-1}$, $C = 0$, $T = 37 \text{K}$, at $0 \leq r \leq R_2$, $L = R_2 - R_1$ and $R_1 \leq r \leq R_2$, when $t = 0$.

$$w_z = N w_z(z),$$

$$w_r = w_0,$$

$$\lim_{r \to R_2} w_r(R_2, z, t) = 0,$$

$$\lim_{r \to R_1} C(R_2, z, t) = C_{R_2},$$

$$\lim_{r \to R_2} T(R_2, z, t) = T_{R_2},$$

where $w_z(z)$ is the stretching velocity, $w_0 > 0$ is the suction velocity, $w_0 > 0$ is the injection velocity, $C_{R_2}$ is the surface concentration, $T_{R_2}$ is the surface temperature, $w_r$ is the velocity component in radial direction, $w_z$ is the velocity component in the axial direction, and $C$ is the concentration of solute such as urea and creatinine.

$N$ is the number of hollow fibers that every hollow fiber is assumed to have the same flow rate at the blood entrance in an artificial kidney, $r$ denotes the inner radius of hollow fiber, $L$ is the characteristics length, and $w_1 (> 0)$ is the initial stretching rate of the porous stretching plate.

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}},$$

$\kappa_{nf}$ is the thermal diffusivity along the porous medium and $\kappa_{nf}$ is the effective thermal conductivity of nanofluid.

$\rho_{nf}$ is the density of nanofluid and $(C_p)_{nf}$ specific heat capacity at constant pressure of nanofluid. $\nu$ is the kinematic viscosity and it is given as $\nu = \mu_{nf}/\rho_{nf}$, $w_r$, $w_\theta$, $w_z$ are the radial velocity, velocity in theta direction, and axial velocity. $B_0$ is the magnetic field applied perpendicularly to the permeable and impermeable plates. $D_m$ is the mass diffusion coefficient.
2.2. Nondimensionalization of the Flow Governing Equations. The flow governing equations of the problem are transformed to a nondimensional form; this makes sure that the flow governing equations are reduced to make it less complex so that can be solved much easily and the results are applicable to other geometrical similar configuration in the same set of flow boundary conditions of the problem.

In order to transform the equation of motion (momentum), the equation of energy, and the equation of concentration into their respective nondimensional form, the following analysis is carried out defining first the nondimensional parameters:

\[
\begin{align*}
 w_0 &= \frac{w_0}{w_r}, \\
 w_1^* &= \frac{w_r}{w_r}, \\
 w_2^* &= \frac{w_z}{w_r}, \\
 r^* &= \frac{r}{L}, \\
 R_1^* &= \frac{R_1}{L}, \\
 R_2^* &= \frac{R_2}{L}, \\
 t^* &= \frac{w_r t}{L}, \\
 z^* &= \frac{z}{L}, \\
 T^* &= \frac{T - T_{R_1}}{T_{R_1} - T_{R_2}}, \\
 C^* &= \frac{C - C_{R_1}}{C_{R_2} - C_{R_1}},
\end{align*}
\]

where \(w_r, L, C, \text{and} T\) are the characteristics velocity, length, concentration, and temperature.

\[
\frac{\partial w_1^*}{\partial t^*} + w_1^* \frac{\partial w_1^*}{\partial r^*} + w_2^* \frac{\partial w_1^*}{\partial z^*} = \frac{1}{Re} \left( \frac{1}{r} \frac{\partial w_2^*}{\partial r^*} + \frac{\partial^2 w_1^*}{\partial r^* \partial r^*} + \frac{\partial^2 w_1^*}{\partial z^* \partial z^*} \right)
\]

\[- \lambda_p w_2^* + M w_2^* + G_{\nu} T^* + G_{ce} C^*,
\]

(5)

\[
\frac{\partial T^*}{\partial t^*} + w_1^* \frac{\partial T^*}{\partial r^*} + w_2^* \frac{\partial T^*}{\partial z^*} = \frac{1}{PrRe} \left( \frac{1}{r} \frac{\partial T^*}{\partial r^*} + \frac{\partial^2 T^*}{\partial r^* \partial r^*} + \frac{\partial^2 T^*}{\partial z^* \partial z^*} \right)
\]

\[+ \frac{2 Ec}{Re} \left[ \left( \frac{\partial w_1^*}{\partial r^*} \right)^2 + \left( \frac{\partial w_1^*}{\partial z^*} \right)^2 \right] + \frac{Ec}{Re} \left[ \left( \frac{\partial w_2^*}{\partial r^*} \right)^2 + 2 \left( \frac{\partial w_2^*}{\partial r^*} \right) \left( \frac{\partial w_2^*}{\partial z^*} \right) + \left( \frac{\partial w_2^*}{\partial z^*} \right)^2 \right] + H_j \frac{w_2^*}{\partial z^*},
\]

(6)

\[
\frac{\partial C^*}{\partial t^*} + w_1^* \frac{\partial C^*}{\partial r^*} + w_2^* \frac{\partial C^*}{\partial z^*} = \frac{Sc}{Re} \left( \frac{1}{r} \frac{\partial C^*}{\partial r^*} + \frac{\partial^2 C^*}{\partial r^* \partial r^*} + \frac{\partial^2 C^*}{\partial z^* \partial z^*} \right)
\]

\[+ \frac{Sr}{Re} \left( \frac{1}{r} \frac{\partial T^*}{\partial r^*} + \frac{\partial^2 T^*}{\partial r^* \partial r^*} + \frac{\partial^2 T^*}{\partial z^* \partial z^*} \right),
\]

\[w_1^* = 0,
\]

\[w_2^* = 0.5ms^{-1},
\]

\[C^* = 0,
\]

\[T^* = 37K, \text{at} 0 \leq r^* \leq \frac{R_1}{L},
\]

\[L = R_2 - R_1 \text{and} \frac{R_1}{L} \leq r^* \leq \frac{R_2}{L},
\]

\[w_2^* = N \left( \frac{w_2^*}{w_r} \right),
\]

\[w_1^* = w_0 w_r, \text{for any value of} z,
\]

\[z^* = \infty,
\]

\[w_2^* = \frac{1}{1},
\]

\[w_1^* = \frac{1}{1},
\]

\[C^* = \frac{1}{1},
\]

\[T^* = 1,
\]

\[\lim_{r^* \to (R_1/L)} w_1^*(\frac{R_2}{L}, z, t) = 0,
\]

\[\lim_{r^* \to (R_1/L)} C^*(\frac{R_2}{L}, z, t) = 1,
\]

\[\lim_{r^* \to (R_1/L)} T^*(\frac{R_2}{L}, z, t) = 1, \text{at} r^* = \left( \frac{R_1}{L} \right) \text{when} r^* > 0.
\]

(8)

2.3. Numerical Methods. There are a class of numerical techniques for solving differential equations by approximating derivatives with finite differences that applies
Crank–Nicolson techniques. The numerical scheme will be implemented in MATLAB.

The finite difference equation for the equation of motion (4) along the z-direction is given by

\[
\frac{w_{zij}^{m+1} - w_{zij}^m}{\Delta t} + \frac{w_{zij}^m (w_{zij}^{m+1} - w_{zij}^m + w_{zij,j}^m - w_{zij,j}^m)}{2(\Delta r)} + \frac{w_{zij}^m (w_{zij,j}^{m+1} - w_{zij,j}^m + w_{zij}^m - w_{zij,j}^m)}{2(\Delta z)} = \frac{1}{Re} \left[ \frac{1}{r} \left( \frac{w_{zij}^{m+1} - w_{zij}^m}{2(\Delta r)} \right) \right] + \frac{1}{Re} \left[ \frac{1}{r} \left( \frac{w_{zij,j}^{m+1} - w_{zij,j}^m}{2(\Delta r)} \right) \right] + \frac{1}{Re} \left[ \frac{1}{r} \left( \frac{w_{zij}^{m+1} - w_{zij}^m}{2(\Delta r)} \right) \right] + \frac{1}{Re} \left[ \frac{1}{r} \left( \frac{w_{zij,j}^{m+1} - w_{zij,j}^m}{2(\Delta r)} \right) \right]
\]

(9)

solving for \(w_{zij}^{m+1}\) in equation (9) to get

\[
w_{zij}^{m+1} = \left[ \left( \frac{w_{zij}^m - (w_{zij} \Delta t/2(\Delta r)) - (w_{zij}^{m+1} + w_{zij}^m - w_{zij,j}^m) + (\Delta t/2Re(\Delta r)) (w_{zij}^m - w_{zij} - w_{zij,j}^m)}{1 + (w_{zij}^{m+1} - (\Delta t/2(\Delta z))) + (\Delta t/2Re(\Delta z)) (w_{zij}^m - w_{zij} - w_{zij,j}^m)} \right) \right]
\]

(10)

The finite difference equation for the equation of motion (10) along r-direction is given by

\[
\frac{w_{rij}^{m+1} - w_{rij}^m}{\Delta t} + \frac{w_{rij}^m (w_{rij}^{m+1} - w_{rij}^m + w_{rij,j}^m - w_{rij,j}^m)}{2(\Delta r)} + \frac{w_{rij}^m (w_{rij,j}^{m+1} - w_{rij,j}^m + w_{rij}^m - w_{rij,j}^m)}{2(\Delta z)} = \frac{1}{Re} \left[ \frac{1}{r} \left( \frac{w_{rij}^{m+1} - w_{rij}^m}{2(\Delta r)} \right) \right] + \frac{1}{Re} \left[ \frac{1}{r} \left( \frac{w_{rij,j}^{m+1} - w_{rij,j}^m}{2(\Delta r)} \right) \right] + \frac{1}{Re} \left[ \frac{1}{r} \left( \frac{w_{rij}^{m+1} - w_{rij}^m}{2(\Delta r)} \right) \right] + \frac{1}{Re} \left[ \frac{1}{r} \left( \frac{w_{rij,j}^{m+1} - w_{rij,j}^m}{2(\Delta r)} \right) \right]
\]

(11)

solving for \(w_{rij}^{m+1}\) in equation (11) to get
The finite difference equation for the equation of energy (6) is given by

\[
\begin{align*}
\frac{T_{i,j}^{m+1} - T_{i,j}^m}{\Delta t} + &w_{r,i}^m \left( \frac{T_{i,j}^{m+1} + T_{i,j} - T_{i+1,j}^m}{2(\Delta r)} \right) + w_{z,i}^m \left( \frac{T_{i,j}^{m+1} - T_{i-1,j}^m}{2(\Delta z)} \right) \\
= &\left[ \frac{1}{RePr} \left( \frac{T_{i+1,j}^{m+1} - T_{i,j}^{m+1} + T_{i,j} - 2T_{i+1,j}^m}{2(\Delta r)} \right) \right] + \left[ \frac{1}{RePr} \left( \frac{T_{i,j+1}^{m+1} - 2T_{i,j}^{m+1} + T_{i,j} - 2T_{i,j+1}^m + T_{i,j-1}^m}{2(\Delta z)} \right) \right] \\
+ &\frac{1}{RePr} \left[ \frac{T_{i+1,j+1}^{m+1} - 2T_{i+1,j}^{m+1} + T_{i,j+1}^{m+1} - 2T_{i,j}^{m+1} + T_{i,j-1}^m}{2(\Delta z)^2} \right] \\
+ &\frac{2Ec}{Re} \left( \frac{w_{r,i,j}^m + w_{z,i,j}^m}{2(\Delta r)} \right)^2 + \frac{Ec}{Re} \left( \frac{w_{r,i,j}^m - w_{z,i,j}^m}{2(\Delta r)} \right)^2 \\
+ &\frac{2Ec}{Re} \left( \frac{w_{r,i,j}^m + w_{z,i,j}^m}{2(\Delta z)} \right)^2 + H J \left( \frac{w_{r,i,j}^m + w_{z,i,j}^m}{2(\Delta z)} \right) \frac{2Ec}{Re} \left( \frac{w_{r,i,j}^m - w_{z,i,j}^m}{2(\Delta z)} \right)^2 \end{align*}
\]

(13)

Solving for \( T_{i,j}^{m+1} \) in equation (13) to get

\[
\begin{align*}
T_{i,j}^{m+1} = &\left[ \frac{T_{i,j}^m - \left( w_{r,i,j}^m \Delta t/2(\Delta r) \right) \left( -T_{i,j}^{m+1} + T_{i,j}^m - T_{i,j-1}^m \right) \left( -T_{i,j}^{m+1} + T_{i,j}^m - T_{i,j+1}^m \right) \left( -T_{i,j}^{m+1} + T_{i,j}^m - T_{i,j-1}^m \right) \right] + \left( \Delta t / 2 RePr(\Delta r) \right) \left( -T_{i,j}^{m+1} + T_{i,j}^m - T_{i,j+1}^m \right) \left( -T_{i,j}^{m+1} + T_{i,j}^m - T_{i,j-1}^m \right) \right] \\
+ &\left( Ec \Delta t / 4 Re(\Delta r)^2 \right) \left( w_{r,i,j}^m - w_{z,i,j}^m + w_{r,i,j}^m \right)^2 + \left( Ec \Delta t / 4 Re(\Delta r)^2 \right) \left( w_{r,i,j}^m + w_{z,i,j}^m \right)^2 + \left( Ec \Delta t / 2 Re(\Delta r)^2 \right) \left( w_{r,i,j}^m - w_{z,i,j}^m \right)^2 + \left( Ec \Delta t / 2 Re(\Delta r)^2 \right) \left( w_{r,i,j}^m + w_{z,i,j}^m \right)^2 \\
+ &\left( H J / 2 Re \right) \left( w_{r,i,j}^m + w_{z,i,j}^m \right)^2 \left( w_{r,i,j}^m - w_{z,i,j}^m \right)^2 + \left( H J / 2 Re \right) \left( w_{r,i,j}^m + w_{z,i,j}^m \right)^2 \left( w_{r,i,j}^m - w_{z,i,j}^m \right)^2 \left( w_{r,i,j}^m + w_{z,i,j}^m \right)^2 \left( w_{r,i,j}^m - w_{z,i,j}^m \right)^2 \end{align*}
\]

(14)

Similarly, the finite difference equation for the equation of concentration (8) is given by
\[
\frac{C_{i+1,j}^{m+1} - C_{i,j}^{m}}{\Delta t} + w_{r,j}^m \left( \frac{C_{i+1,j}^{m+1} - C_{i-1,j}^{m+1} + C_{i,j}^{m} - C_{i,j-1}^{m}}{2(\Delta r)} \right) + w_{z,j}^m \left( \frac{C_{i+1,j}^{m+1} - C_{i-1,j}^{m+1} + C_{i,j}^{m} - C_{i,j+1}^{m}}{2(\Delta z)} \right) = \frac{1}{\text{Sc} \Re} \left[ \frac{C_{i+1,j}^{m+1} - C_{i,j}^{m+1} + C_{i,j}^{m} - C_{i-1,j}^{m}}{2(\Delta r)} \right] + \frac{1}{\text{Sc} \Re} \left[ \frac{C_{i+1,j+1}^{m+1} - 2C_{i,j+1}^{m+1} + C_{i,j+1}^{m} - 2C_{i,j}^{m} + C_{i+1,j}^{m}}{2(\Delta z)^2} \right] + \frac{\text{Sr}}{\text{Re}} \left[ \frac{T_{i+1,j+1}^{m+1} - 2T_{i,j+1}^{m+1} + T_{i,j}^{m+1} - T_{i,j-1}^{m+1}}{2(\Delta r)^2} \right]
\]

(15)

solving for \(C_{i,j}^{m+1}\) in equation (15) to get

\[
C_{i,j}^{m+1} = \frac{\left[ C_{i,j}^m - \left( w_{r,j}^m \Delta t / 2(\Delta r) \right) \left( -C_{i+1,j}^m + C_{i-1,j}^m \right) - \left( w_{z,j}^m \Delta t / 2(\Delta z) \right) \left( -C_{i,j+1}^m + C_{i,j-1}^m \right) + \left( \Delta t / 2(\Delta r) \right) \text{Sc} \Re \left( -C_{i+1,j+1}^m + C_{i,j+1}^m - 2C_{i,j}^m + C_{i,j-1}^m \right) \right]}{1 + \left( w_{r,j}^m \Delta t / 2(\Delta r) \right) + \left( w_{z,j}^m \Delta t / 2(\Delta z) \right) + \left( \Delta t / (\Delta r)^2 \text{Sc} \Re \right) + \left( \Delta t / (\Delta z)^2 \text{Sc} \Re \right) + \left( \Delta t / (\Delta x)^2 \text{Sc} \Re \right)}
\]

(16)

3. Discussion of the Results

From the findings, the effects of various nondimensional parameters such as Reynolds number, magnetic field parameter, permeability parameter, Grashof number, Prandtl number, Eckert number, Joule heating parameter, Schmidt number, and Soret number on velocity, temperature, and concentration profiles are effectively discussed.

From Figure 2(a), it is observed that an increase in Reynolds number (Re) results in an axial velocity \(\left( W_x \right)\) of the layers to decrease exponentially from the center of the hollow fibers in the blood compartment to the walls of the dialysis solution compartment in an artificial kidney. This is due to the friction that results in the resistance of the nanofluid against the motion produced on the wall adjacent to it. Since Reynolds number is the ratio of inertial force to viscous force. From Figure 2(b), it is observed that an increase in Reynolds number (Re) results in radial velocity \(\left( W_r \right)\) of nanofluid to decreases exponentially along the hollow fiber walls of the artificial kidney. It is observed from Figure 2(c) that an increase in Reynolds number (Re) leads to an increase of the temperature \(\left( T \right)\) profile in the direction of the flow of the nanofluid at the center of the hollow fiber at the radial distance \(r = 0.945\), the inner radius of the hollow tube in the blood compartment. This is because a higher laminar kinetic energy in the nanofluid particles increases along the axis symmetry of the flow of the electrically conducting nanofluid inside the blood compartment in an artificial kidney but decreases as it moves periphery near the wall of the stretching cylinder of an artificial kidney of the dialysis solution compartment and from Figure 2(d) it is observed that an increase in Reynolds number (Re) leads to an increase in concentration \(\left( C \right)\) profile as it approaches the radial distance \(r = 0.945\), the inner radius of the hollow fibers in an artificial kidney but decreases as it moves periphery near the wall of the stretching cylinder in an artificial kidney of the dialysis solution compartment. The concentration of solutes in nanofluid is higher at the center of the flow in the blood compartment but decreases as it moves near the wall of the membrane in the dialysate compartment because not all waste products will diffuse through the pores of the hollow fibers some have bigger size molecules like protein and blood cells to diffuse through membrane by diffusion method.

It is noted from figure 3(a) that as the permeability parameter \(\left( \lambda_p \right)\) increases there is a decrease in axial velocity \(\left( W_x \right)\); this is because permeability parameter \(\left( \lambda_p \right)\) is inversely proportional to the normal permeability number \(\left( K_n \right)\) leads the flow to decelerate and figure (b) shows that the radial velocity \(\left( W_r \right)\) is found to increase with an increase in permeability parameter \(\left( \lambda_p \right)\); this is because the velocity is an increasing function of permeability parameter \(\left( \lambda_p \right)\). An increase in the porosity of hollow fibers in an artificial kidney allows nanofluid and other solutes to pass through after blood filtration during hemodialysis in an artificial kidney, the higher the porosity, the higher the permeability that acts...
as the drag force of the permeability. It is observed from the Figure 3(c) that as the permeability parameter ($\lambda_p$) increases the temperature profile increases and decreases as it approaches the surface of the dialysis solution compartment when the radial distance is $r = 0.945$, the temperature of nanofluid can be determined by the boundary layer velocity rather than thermal boundary layer when the temperature of nanofluid at the impermeable stretching plate in an artificial kidney and that of the dialysis solution compartment varies. From Figure 3(d) as permeability parameter ($\lambda_p$) increases concentration profile increases in the direction of electrically conducting nanofluid, at the center of the hollow fiber in the blood compartment and decreases as it approaches near the surface of the stretching wall of the dialysis solution compartment when radial distance is at $r = 0.945$. Therefore, the concentration gradient between blood and dialysate helps the diffusion of solutes to move through the pores of hollow fibers in an artificial kidney to the dialysis solution compartment near the surface of the stretching cylinder.

It is noted from Figure 4(a) that as the magnetic field parameter increases the axial velocity ($W_z$) profile decreases, because of the force called the Lorentz force generated by the magnetic field applied in opposite direction to the flow direction of the electrically conducting nanofluid which leads to a decrease in velocity. Figure 4(b) shows that the radial velocity ($W_r$) profile increases as the magnetic field increases. This is because the magnetic field is applied in the direction of radial distance. It is observed from Figure 4(c) that an increase in the magnetic field parameter leads to an increase in the temperature profile in the blood compartment and decreases exponentially when it reaches a radial distance of 0.945 as it approaches the surface of the
hollow fiber tubes stretching from the dialysis solution compartment in an artificial kidney. The metallic nanoparticles used in the study improve thermal conductivity that increases fluid temperature as a result of the interaction of the atomic ions of the metallic particles. Lorentz force generated by magnetic force has the tendency to slow the motion of the nanofluid in the boundary layer and increases its temperature profiles and Figure 4(d) shows that an increase in the magnetic field parameter leads to an increase in the concentration profile for the blood compartment and decreases exponentially when it reaches a radial distance of 0.945 as it approaches the dialysis solution compartment in an artificial kidney. Decreasing the velocity of the nanofluid tends to slow the movement of solutes and other waste products from the blood during hemodialysis, causing a decrease in concentration.

From Figure 5(a), it is observed that as the Grashof number due to the temperature gradient (GrT) increases, the axial velocity ($W_z$) profile also increases, as the Grashof number is the ratio between buoyant to viscous forces acting on the nanofluid in the velocity boundary layer. The role of the Grashof number is to describe the effects of natural convection currents due to temperature change of the nanofluid or cooling of the surface. Figure 5(b) shows that an increase in Grashof number due to temperature change (GrT) leads to a radial velocity ($W_r$) profile near the surface of the hollow fibers in an artificial kidney. Therefore, such an increase in Grashof number due to temperature change

Figure 3: Effects of permeability parameter ($\lambda_p$) on axial velocity ($W_z$), radial velocity ($W_r$), temperature ($T$), and concentration ($C$) profiles.
(GrT) does not have a significant decrease in the radial velocity profile ($W_r$). It is observed that in Figure 5(c) as the Grashof number due to temperature increases, the temperature profile also decreases along the axis symmetry of the nanofluid flow and then increases exponentially. This is because the nanofluid temperature within the boundary layer is enhanced by fluid temperature and Figure 5(d) shows that as Grashof number due to temperature increases leads concentration profile to decrease along the axis symmetry of the nanofluid flow and then sharply increases exponentially. Natural convectional currents help bringing more mass transfer of solutes from the blood compartment to the dialysis solution compartment of the hollow tubes in an artificial kidney and hence concentration profiles increase.

In Figure 6(a), it is observed that an increase in Grashof number due to the change in concentration ($GrC$) leads to an increase in axial velocity ($W_z$) profile, which means that the increase of the Grashof number due to the concentration leads to an increase of the buoyancy force. An increase in Grashof number leads to a decreased viscous force which opposes the flow of nanofluid. Figure 6(b) shows that an increase in the Grashof number due to a change in concentration results in decrease in the radial velocity ($W_r$) profile; this shows that there is no significant increase in radial velocity ($W_r$) profile when Grashof number due to the change in concentration increases. In Figure 6(c) it is noted that an increase in Grashof number due to a change in concentration results in a slight decrease in the temperature profile in the blood compartment but as it approaches the
surface of the stretching cylinder in the dialysis solution compartment increases exponentially. The fact is that an increase in Grashof number due to concentration gradient results in a decrease in the thickness of the thermal boundary layer leading to a decrease in the temperature profile. From Figure 6(d) it is noted that an increase in Grashof number due to change in concentration results in a slight decrease in the concentration profile in the blood compartment but as it approaches the surface of the stretching cylinder in the dialysis solution compartment increases exponentially. The concentration profile decreases then increases as it approaches the dialysis solution compartment of the stretching surface in an artificial kidney due to an increase in buoyancy force acting over the nanofluid.

It is observed from Figure 7(a) that an increase in the Prandtl number (Pr) results in an increase in axial velocity (W_z). Physically, the Prandtl number is the ratio between momentum diffusivity and the thermal diffusivity of the nanofluid properties. This implies an increase in Prandtl number, heat diffuses relatively slowly compared to the velocity of the nanofluid. The decrease in thermal diffusivity leads to a velocity boundary layer thickness past the porous stretching cylinder decreasing when the Prandtl number increases. From Figure 7(b) it is noted that an increase in the Prandtl number (Pr) results in an increase in the radial velocity profile (W_r). It is noted that as the Prandtl number increases, there is no significant change in the velocity since the changes in the Prandtl number are approximately too
small to cause the nanofluid radial velocity to increase significantly. Figure 7(c) shows that increasing the Prandtl number causes an increase in the temperature profile. It is due to the fact that thermal diffusion takes place at a slower rate compared to the diffusivity of kinematic viscosity diffusivity. Nanoparticles dissolved in the base fluid improve the thermal conductivity of base fluid by increasing thermal boundary layer thickness.

It is observed from Figure 8(a) that as the Eckert number (Ec) increases, so does the axial velocity ($W_z$) profile increases. Since Eckert number is the ratio between the kinetic energy and thermal energy in the nanofluid flow. When thermal heating increases, it leads to a rise in the velocity of the nanofluid particles. Figure 8(b) shows that as the Eckert number (Ec) increases, so does the radial velocity ($W_r$) profile decreases. There is no significant increase in Eckert number with the velocity profile of solutes in the radial direction, since there is no entry of temperature into the equation of motion in the radial direction. Figure 8(c) shows that temperature ($T$) profile increases as Eckert number (Ec) increases. Physically, Eckert number is the change in kinetic energy in the nanofluid flow into the internal energy by work done against the viscosity of the nanofluid. When thermal heating increases, this leads to an increase in the temperature of the nanofluid particles. Figure 8(d) shows that, as the
concentration ($C$) profile increases the Eckert number ($Ec$) increases. There is a significant increase in the concentration profile since there is a mutual relationship between the Eckert number and concentration gradient as the equation of concentration has an entry effect of temperature. Changes in temperature and kinetic energy of the particles translate into mass transfer of solutes from nanofluid. Physically, the Joule heating parameter is the ratio between the amount of heat released from an electrical resistor to its resistance and the charge passing through it to produce thermal energy. An increase in the temperature of the nanofluid leads to an increase in the kinetic energy of the nanofluid particles that results in an axial velocity profile increasing. Pressure gradients created at the bottom wall of the pipe in the direction of flow results in an increased velocity distribution of the nanofluid particles. Figure 9(b) shows that as Joule heating increases, so does radial velocity ($W_r$) increases. An increase in Joule heating has a minor effect on radial velocity because the equation of motion in the radial direction has no temperature entry. Figure 9(c) shows that as Joule heating increases temperature profile increases. The generated heat increases the temperature distribution inside the hollow tubes of the blood compartment in the artificial kidney. Figure 9(d) shows that as Joule heating increases concentration profile increases. The pressure gradient modified for nanofluid particles creates the velocity distribution inside the hollow tubes of the blood compartment to keep the mass flow rate of solutes. From Figure 10(a), it is observed that with an increase in Schmidt number ($Sc$), there is a slight decrease in the axial

**Figure 7:** Effects of Prandtl number ($Pr$) on axial velocity ($W_z$), radial velocity ($W_r$), temperature ($T$), and concentration ($C$) profiles.
velocity \((W_z)\). Physically, the Schmidt number is the ratio of kinematic viscosity diffusivity to mass diffusivity of solutes in a nanofluid. Schmidt number increases the nanofluid viscosity with results in an increase in the velocity boundary layer thickness and so reduces shear stress and however decreases the flow of the nanofluid during hemodialysis in an artificial kidney. It is noted that in Figure 10(b) with an increase in Schmidt number \((Sc)\), there is a slightly insignificant decrease in the radial velocity \((W_r)\). Figure 10(c) shows that with an increase in Schmidt number, there is an increase in temperature in the pipe of the blood compartment but as it approaches the surface of the hollow tubes of the dialysis solution compartment, temperature decreases exponentially. Figure 10(d) shows that with an increase in Schmidt number, there is a decrease in concentration. Schmidt number can be used to characterize the nanofluid flow of mass transfer analysis, i.e., momentum and mass diffusion convection processes. When Schmidt number is large, kinematic viscosity across a nanofluid is much more effective than solute particles with small diffusivity and becomes less relevant at the surface of the porous stretching cylinder in an artificial kidney [26]. This is why the concentration boundary layer of an artificial kidney is relatively thin as the Schmidt number increases near the end of the stretching cylinder of an artificial kidney.

From Figure 11(a), it is observed that as the Soret number \((Sr)\) increases, the axial velocity \((W_z)\) profile increases. Physically, the Soret number is the ratio of temperature difference to the concentration gradient. Nanoparticles dissolved in the base fluid results in the increase in thermal conductivity of an electrically conducting nanofluid, which helps to increase the kinetic energy of the

Figure 8: Effects of the Eckert number \((Ec)\) on axial velocity \((W_z)\), radial velocity \((W_r)\), temperature \((T)\), and concentration \((C)\) profiles.
particles and however the axial and radial velocity of nanofluid increases in the boundary layer thickness as the Soret number increases. Figure 11(b) shows that as the Soret number ($S_r$) increases radial velocity ($W_r$) profiles increase. Figure 11(c) shows that as the Soret number ($S_r$) increases temperature ($T$) profile decreases in the direction of nanofluid flow and as it is approaching the wall of the hollow tube in the dialysis solution compartment, the temperature profile increases. As the value of the Soret number increases, the value of temperature difference increases. Figure 11(d) shows that as the Soret number ($S_r$) increases, the concentration ($C$) profile increases also. An increase in Soret number affects the temperature gradient of nanofluid during purification of blood in an artificial kidney; this signifies the improvement of mass diffusion of solutes from blood to the dialysis solution through a porous stretching cylinder in the artificial kidney hence increasing the in concentration profile at the wall surface membrane in the artificial kidney.

It is observed from Figure 12(a) that, as time ($t$) decreases, the axial velocity ($W_z$) profile increases. This is to the fact that when nanoparticles dissolved in the base fluid during hemodialysis result in an increase in the kinetic energy of the particles, however, leading to the increase in the nanofluid velocity past porous stretching cylinder as the time decreases. It is noted from Figure 12(b) that, as the time ($t$) decreases, the radial velocity ($W_r$) profile increases. This is to the fact that, solutes move faster near the wall surface of the stretching cylinder of the hollow fibers in an artificial kidney during dialysis as time decreases. It is noted from Figure 12(c) that, as time ($t$) decreases, the temperature ($T$) profile decreases.
profile increases. This is to the fact that when nanoparticles dissolved in the base fluid during hemodialysis result in an increase in the thermo-conductivity of the particles; hence, thermal boundary layer thickness increases that rises the temperature of the nanofluid past porous stretching cylinder. From Figure 12(d) it is observed that, as time $t$ decreases, concentration $C$ profile increases. This is to the fact that the concentration gradient of the blood and dialysate enhances the faster movement of the solutes from blood to the dialysis solution compartment as time decreases.

The wall skin friction coefficient can be calculated from axial and radial velocity profiles. The laminar flow of an incompressible nanofluid has low wall skin friction that improves the nanofluid movement through the wall surface of the hollow fibers in an artificial kidney. The wall skin friction coefficients in axial and radial direction are given as

$$C_{sz} = \frac{\mu_{nf}(w_{Rz}/(1/2)\rho_{nf}w_{Rz})}{2L} \left[ \frac{\partial w_z^*}{\partial r^*} \right]_{r^*=0}$$

$$C_{s^*} = \frac{2\mu_{nf}(w_{Rz}/(1/2)\rho_{nf}w_{Rz})}{L} \left[ \frac{\partial w_z^*}{\partial r^*} \right]_{r^*=0},$$

where $Re = (L\rho_{nf}w_{Rz}/\mu_{nf})$.

Then, equation (17) is reduced to

$$C_{sz} = \frac{2}{Re} \left[ \frac{\partial w_z^*}{\partial r^*} \right]_{r^*=0}. \quad (18)$$

Again,
\[ C_{sr} = \frac{-2\mu_{nf} \left( w_{Rz}/L \right) \left[ \partial w_z^* / \partial r^* \right]_{r^*=0}}{(1/2)\rho_{nf} w_{Rz}^2} \]  

where \( \text{Re} = (L \rho \omega_{Rz}/\mu_{nf}) \)

Then, equation (19) is reduced to

\[ C_{sr} = \frac{2}{\text{Re}} \left[ \frac{\partial w_z^*}{\partial r^*} \right]_{r^*=0} \]  

The wall mass transfer rate is determined from concentration profiles. It is given by Sherwood number past a porous stretching cylinder of hollow fibers in an artificial kidney.

\[ S_{hm} = \frac{-\left( D_{m}/L \right) \left( C_{R1} - C_{R2} \right) \left[ \partial C^* / \partial r^* \right]_{r^*=0}}{\left( D_{m}/L \right) \left( C_{R1} - C_{R2} \right)} \]  

\[ = \frac{\partial C^*}{\partial r^*} \bigg|_{r^*=0}. \]  

The rate of heat transfer is determined from temperature profiles and it is given by the Nusselt number.

\[ N_{u_w} = \frac{-\left( k_{nf}/L \right) \left( T_{R1} - T_{R2} \right) \left[ \partial T^* / \partial r^* \right]_{r^*=0}}{\left( k_{nf}/L \right) \left( T_{R1} - T_{R2} \right)} \]  

\[ = \frac{\partial T^*}{\partial r^*} \bigg|_{r^*=0}. \]
Effects of Reynolds number (Re), magnetic field (M), permeability parameter ($\lambda_p$), Joule heating parameter ($HJ$), Eckert number ($Ec$) and time ($t$) on skin friction coefficient ($C_s$), heat transfer ($Sh_m$), and mass transfer ($Nu_w$) are given as follows.

From Table 1,

(i) It is observed that as Re increases both $C_s$ and $C_r$ decreases in magnitude, due to the fact that, the laminar flow of an incompressible nanofluid has low skin friction between fluid and wall surface of hollow fibers in an artificial kidney. $Sh_m$ and $Nu_w$ decrease in magnitude as Re increases; this shows that there is conduction and mass transfer at the boundary layers.

(ii) An increase in $M$ results in an increase in the magnitude of $C_s$, due to Lorentz force that is applied perpendicular to the direction flow of an electrically conducting nanofluid but decreases in $C_r$ due to the fact that the magnetic field is applied in the direction flow of the radial distance. It is observed that as $M$ increases, the magnitudes of $Sh_m$ and $Nu_w$ decrease, due to a decrease in thermal and concentration boundary layers that improve heat and mass transfer of solutes from blood to dialysis solution.

(iii) It is observed that as $\lambda_p$ increases results to an increase in $C_s$ but decreases in $C_r$. Sherwood and Nusselt numbers decrease in magnitude due to a
decrease in thermal and concentration boundary layer thickness that allows the mass transfer of solutes through pores of a thin membrane of hollow fibers in an artificial kidney from blood to dialysis solution compartment.

(iv) It is observed that as the $Ec$ increase $C_{sz}$ decreases but $C_{sz}$ increases by increasing velocity boundary layer thickness. Sherwood and Nusselt number decreases in magnitude as $Ec$ increases.

(v) It is noted that an increase in $H_f$ results to decrease in $C_{sz}$ but an increase in magnitude of $C_{sz}$. Sherwood number and Nusselt number decreases in magnitude as Joule heating parameter increases. Heat distribution in the nanofluid of the hollow fibers due to viscous dissipation improves the mass transfer of solutes from blood to the dialysis solution compartment.

(vi) It is observed that as $t$ decreases both $C_{sz}$ and $C_{sz}$ decreases but $Sh_m$ and $Nu_w$ increases due to the application of nanoparticles in the base fluid that improves nanofluid thermo-conductivity by improving the mass transfer of solutes from blood during filtration.

4. Conclusion

From the study of hydromagnetic nanofluid convective flow past a porous stretching cylinder with the effect of thermo-conductivity, an efficient artificial kidney is developed by improving an optimal artificial kidney and hemodialysis conditions using simulation by reducing the number of hollow fibers from 10,000 hollow fibers, 10 permeability parameter values and flow rates at the entrance and outlets of the dialysis solution compartment of 200 – 400 mil/min and 400 – 800 mil/min used in literature [2] to 100 hollow fibers having a permeability parameter value of 90 values since nanoparticles dissolved in the base fluid improves the thermo-conductivity of nanofluid. A sensor to detect all dissolved nanoparticles in the base fluid so that none of the particles will remain in the blood after blood purification has to be included in the design of an optimal artificial kidney. Flow rates at the entrance and outlets of the dialysis solution compartment have been increased to 400 – 800 mil/min and 700 – 1200 mil/min. The following results have been noticed as it saves substantive time and experimental cost of the hemodialysis session when varying different nondimensional parameters with velocity, temperature and concentration profiles.

(i) The results of thermo-conductivity, when Reynolds number, permeability parameter, magnetic parameter, Eckert number, Schmidt number increase, axial velocity decreases, but when Grashof number due to temperature, Grashof number due to concentration, Prandtl number, Joule heating, Soret number increase, leads to an increase in axial velocity of nanofluid at the center of blood compartment.

(ii) The results of thermo-conductivity with an increase in Reynolds number, Grashof number due to temperature, Grashof number due to concentration, Eckert number, Schmidt number, there is a decrease in the radial velocity while an increase in permeability parameter, magnetic parameter, Prandtl number, Joule heating, Soret number leads to an increase in the radial velocity of nanofluid at the center of the blood compartment.

(iii) It is observed that an increase in Reynolds number, Eckert number, Schmidt number, permeability parameter, magnetic parameter, Prandtl number, and Joule heating results in an increase in temperature while an increase in Grashof number due to temperature, Grashof number due to concentration, Soret number leads to decrease in the temperature of nanofluid at the center of blood compartment with the results in thermo-conductivity of nanofluid.

(iv) It is observed that an increase in Reynolds number, Eckert number, permeability parameter, magnetic parameter, Prandtl number, Joule heating, and Soret number results in an increase in concentration while an increase in Grashof number due to temperature, Grashof number due to concentration, Schmidt number leads to decrease in the concentration of nanofluid at the center of blood compartment.

(v) Using nanoparticles that are dissolved in the base fluid helps improve dialysis session of hemodialysis during the filtration of a patient’s blood in an artificial kidney from a minimum of six hours to two hours. However, it helps a good number of patients with kidney problems from different dialysis centers, especially in Sub-Saharan countries to receive the service of the filtration of blood.

Table 1: Variation of various nondimensional parameters on skin friction, heat, and mass transfer.

| Re  | M  | $H_f$ | $Ec$ | $t$ | $C_{sz}$ | $Sh_m$ | $Nu_w$ |
|-----|----|------|------|-----|---------|-------|-------|
| 0.02| 10.5| 90   | 20   | 0.3 | 2.45918e+05 | 3.42263e+01 | -1.26345e+07 | -1.99508e+06 |
| 0.03| 10.5| 90   | 20   | 0.3 | 1.94594e+05 | 2.62293e+01 | -1.503368e+07 | -2.23001e+06 |
| 0.02| 11.5| 90   | 20   | 0.3 | 2.46994e+05 | 3.42155e+01 | -1.27244e+07 | -2.00502e+06 |
| 0.02| 10.5| 100  | 20   | 0.3 | 2.56489e+05 | 3.41217e+01 | -1.35202e+07 | -2.09327e+06 |
| 0.02| 10.5| 90   | 21   | 0.3 | 2.45044e+05 | 3.42358e+01 | -1.27385e+07 | -2.01106e+06 |
| 0.02| 10.5| 90   | 20   | 0.4 | 2.41264e+05 | 3.43302e+01 | -1.50590e+07 | -2.42248e+06 |
| 0.02| 10.5| 90   | 20   | 0.3 | 2.60816e+05 | 4.08138e+02 | -7.00017e+06 | -1.28864e+06 |
| 0.02| 10.5| 90   | 20   | 0.3 | 2.60000e+00 | 0.00000e+00 | 0.00000e+00 | 0.00000e+00 |
**Nomenclature**

\(e\): Unit electric charge (C)

\(E\): Electric field (V)

\(J\): Current density (Am\(^{-2}\))

\(H\): Magnetic field intensity vector (Am\(^{-1}\))

\(\rho_e\): Charge density (cm\(^{-3}\))

\(B\): Magnetic field vector (Wbm\(^{-2}\))

\(M\): Magnetic field parameter

\(D_{mp}\): Mass diffusivity coefficient (m\(^2\)s\(^{-1}\))

\(C\): Concentration vector (kgm\(^{-3}\))

\(Re\): Reynolds number parameter

\(P_i\): Force due pressure (Nm\(^{-2}\))

\(W\): Velocity Vector of a fluid (ms\(^{-1}\))

\(w_z\): Axial velocity of the nanofluid in z-direction (ms\(^{-1}\))

\(w_i\): Radial velocity of solutes through porous plate in r-direction (ms\(^{-1}\))

\(i, j, k\): Component unit vectors in the x, y, and z directions, respectively

\(u, v, w\): Components of velocity vector \(\overline{W}\) (ms\(^{-1}\))

\(u^*, v^*, w^*\): Dimensionless velocity components

\(r, \theta, z\): Dimensional cylindrical coordinates with position vector \(\overline{r}\) (m)

\(r^*, \theta^*, z^*\): Dimensionless cylindrical polar coordinates

\(F\): Fluid body force (Nm\(^{-3}\))

\(g\): Gravitational force (kgm\(^{-2}\))

\(K_p\): Darcy permeability (m\(^2\))

\(D\): Displacement current density (cm\(^{-2}\)).

\(F_e\): Electromagnetic force (kgm\(^{-3}\)s\(^{-2}\)).

\(Pr\): Prandtl number

\(\lambda_p\): Permeability parameter

\(Gr_T\): Temperature Grashof number

\(Gr_C\): Concentration Grashof number

\(C_f\): Skin friction coefficient

\(Sh_m\): Sherwood number

\(Nu_w\): Nusselt number

\(Ec\): Eckert number

\(H_j\): Joule heating parameter

\(Sc\): Schmidt number

\(Sr\): Soret number

\(F_e\): Electromagnetic force (kgm\(^{-3}\)s\(^{-2}\))

\((D/Dt)\): Material derivative

\((\partial/\partial t) + (\partial/\partial x) + (\partial/\partial y) + (\partial/\partial z)\)

\(L\): Dimensional length between plates of hollow fibers in an artificial kidney (m)

\(C\): Dimensional concentration of solutes through hollow fibers (kg\(^{-3}\))

\(C_{f}\): Dimensional concentration at the stretching plate (kgm\(^{-3}\))

\(C_{R_i}\): Dimensional concentration of the solutes at impermeable plate (kgm\(^{-3}\))

\(T\): Absolute temperature of the nanofluid in an artificial kidney (K)

\(T_{R_i}\): Absolute temperature at the stretching plate in an artificial kidney (K)

\(T_{R_i}\): Absolute temperature at the impermeable plate in an artificial kidney (K)

\(c\): Stretching rate constant (s\(^{-1}\))

\(C_p\): Specific heat capacity at constant pressure (Jkg\(^{-1}\)K\(^{-1}\))

\(\rho_n\): Nano fluid density (kgm\(^{-3}\))

\(\rho_{nf}\): Coefficient of viscosity of nanofluid (kgm\(^{-3}\)s\(^{-1}\))

\(\nu\): Kinematic viscosity (m\(^2\)s\(^{-1}\))

\(\beta_T\): Volumetric coefficient of thermal expansion (K\(^{-1}\))

\(\beta_C\): Coefficient of thermal expansion due to concentration gradient (K\(^{-1}\))

\(\phi\): Viscous dissipation function

\([\phi = (\partial V/\partial x_i + \partial V/\partial x_j)](s^{-1})\)

\(\sigma\): Electrical conductivity, (Ω\(^{-1}\)m\(^{-1}\))

\(\mu_\|\): Magnetic permeability (Hm\(^{-1}\))

\(V\): Gradient operator

\(([\partial^2/\partial x^2] + (\partial^2/\partial y^2) + (\partial^2/\partial z^2))\)

\(\alpha\): Coefficient of thermal expansion due to concentration gradient (K\(^{-1}\))

\(\tau\): Viscous resistance function

\(\Omega\): Viscous resistance function

\(\mu(\phi)\): Viscous resistance function

**Data Availability**

No data were used to support this study.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest in this article.

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