Joint Beam Training and Positioning For Intelligent Reflecting Surfaces Assisted Millimeter Wave Communications

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Abstract—Intelligent reflecting surface (IRS) offers a cost effective solution to link blockage problem in mmWave communications, and the prerequisite of which is the accurate estimation of (1) the optimal beams for base station/access point (BS/AP) and mobile terminal (MT), (2) the optimal reflection patterns for IRSs, and (3) link blockage. In this paper, we carry out beam training design for IRSs assisted mmWave communications to estimate the aforementioned parameters. To acquire the optimal beams and reflection patterns, we firstly perform random beamforming and maximum likelihood estimation to estimate angle of arrival (AoA) and angle of departure (AoD) of the line of sight (LoS) path between BS/AP (or IRSs) and MT. Then, with the estimate of AoAs and AoDs, we propose an iterative positioning algorithm that achieves centimeter-level positioning accuracy. The obtained location information is not only a fringe benefit but also enables us to cross verify and enhance the estimation of AoA and AoD, and facilitates the prediction of blockage indicator. Numerical results show the superiority of our proposed beam training scheme and verify the performance gain brought by location information.

I. INTRODUCTION

Millimeter-wave (mmWave) band, ranging from 30GHz to 300GHz, has attracted great interests from both academia and industry for its abundant spectrum resources [1], [2]. The Wi-Fi standard IEEE 802.11ad runs on the 60GHz (V band) spectrum with data transfer rates of up to 7 Gbit/s [3], [4]. In 3GPP Release 15, 24.25-29.5GHz and 37-43.5GHz, as the most promising frequencies for the early deployment of 5G millimeter wave systems, are specified based on a TDD access scheme [5]. The millimeter scale wavelength, on one hand, renders massive antennas integratable on an antenna array with portable size [6]. and, on the other hand, results in severe free space path loss especially for non-line-of-sight (NLoS) paths. Directional transmission enabled by beamforming techniques is an energy efficient transmission solution to compensate for the path loss in mmWave communications [7]. By properly adjusting the phase shifts of each antenna elements, it concentrates the emitted energy in a narrow beam between transmitter and receiver. However, the directional link is easily blocked by obstacles like human bodies, walls, and furniture, attributed to the millimeter scale wavelength [8]. Once LoS path is blocked, it is highly possible that the blocked link cannot be restored no matter how the beam direction is adjusted, as the NLoS paths are not strong enough to serve as a qualified alternative link. Channel measurement campaigns reveal that power of the LoS component is about 13dB higher than the sum of power of NLoS components [9]. Therefore, blockage is the biggest hindrance to the large scale applications of mmWave band in mobile communication systems.

Recently, intelligent reflecting surface (IRS) [10]–[13], a.k.a. reconfigurable intelligent surface (RIS) [14], [15], large intelligent surface (LIS) [16], passive (intelligent) reflectors/mirrors [17]–[19], or programmable metasurface [20]–[22], is proposed as an energy-effective and cost-effective hardware structure for future wireless communications. IRS is essentially a new type of electromagnetic structure which is typically designed by deliberately arranging a set of sophisticated passive scatterers or apertures in a regular array to achieve the desired ability for guiding and controlling the flow of electromagnetic waves [23]. Current applications of IRS to wireless communications can be categorized into two types, namely IRS modulator and IRS “relay”. In [20]–[22], amplitude/phase modulations over IRS are investigated. Through controlling the reflection coefficient of IRS, the incident carrier wave from a feed antenna can be digitally modulated without requiring high-performance radio frequency (RF) chains. A more extensive application of IRS is IRS “relay”, in which the radiated power from BS/AP towards IRS is reflected to MT via intelligently managing the phase shifters on IRS [19]–[19]. It is noteworthy that the rationale behind IRS “relay” and conventional amplify-and-forward (AF) relay is significantly different. AF relay firstly receives signal and then re-generates and re-transmits signal. In contrast, IRS only reflects the ambient RF signals as a passive array and bypasses conventional RF modules such as power amplifier, filters, and ADC/DAC [11]. Hence, IRS “relay” incurs no additional power consumption and is free from thermal noise introduced by RF modules. In this sense, IRS can be regarded as a smart “mirror” that enables us to change the paradigm of wireless communications from adjusting to wireless channel to changing wireless channel [14], [24]. As an active way to make wireless channel better, IRS “relay” assisted wireless communications have attracted great interests from researchers. In [10], IRS is applied to mmWave communications to provide effective reflected paths and thus enhance signal coverage. In [12], [15], [17], joint optimization of the transmit beamforming by active antenna array at the BS/AP and reflect beamforming by passive phase shifters at the IRS is carried out. In [18], empirical studies are performed to analyze the capability of signal coverage.
enhancement for IRSs assisted mmWave MIMO at 28GHz. In [19], the reconfigurable 60GHz IRS is designed, implemented and deployed to strengthen mmWave connections for indoor networks threatened by blockage. The objective of the work is to validate IRS’s capability to address link blockage problem in mmWave communications, and beam training design is not investigated. Although extensive analytical and empirical studies have been done on IRSs assisted wireless communications in the aforementioned literature, these work either assume the availability of channel state information (CSI) or accurate measurement of BS/AP, MT and IRS’s position and direction.

Current study of channel parameter estimation of IRSs assisted wireless communications either focuses on non-mmWave frequency band or are based on an assumption of hardware upgrade. In [11], a practical transmission protocol and channel estimation are firstly proposed for an IRS-assisted orthogonal frequency division multiplexing (OFDM) system under frequency-selective channels. In [13], by exploiting the channel correlation among different users, a channel estimation scheme with reduced training overhead is proposed. Specifically, with a typical user’s reflection channel vector, estimation of the other users’ reflection channel vector can be simplified as the estimation of a multiplicative coefficient. In [16], to facilitate channel estimation of IRSs assisted link over mmWave band or LoS dominated sub-6GHz band, an upgrade of IRS’s structure is proposed to add a small number of channel sensors to sense and process incident signal. Although [16] is intended to mmWave band, the proposed compressive sensing and deep learning algorithms are incompatible to current structure of IRS which is without channel sensors.

Due to the deployment of multiple IRSs, beam training of IRSs assisted mmWave communications requires much heavier training overhead than traditional mmWave communications. Also, as the purpose of IRSs is to anti blockage and expand coverage, an accurate prediction of blockage is essential to beam selection by BS/AP. In addition, the lack of RF chains results in the inability of IRSs to sense signal, which further complicates beam training for the paths assisted by IRSs. These three features jointly render traditional beam training methods [25, 26] incompetent in IRSs assisted mmWave communications. Despite the aforementioned new challenges of integrating IRSs to mmWave communications, a notable advantage is that the estimation of path parameters, e.g., AoA/AoD and blockage indicator, can be cross verified, thanks to the relatively large number of deployed IRSs. Specifically, three accurate estimates of AoA/AoD, associated with other essential information, e.g., direction of arrays, can yield the location of MT, and the location of MT will in turn reproduce the path parameters. In this way, the path parameters of IRSs assisted mmWave MIMO can be enhanced according to their geometric relationship. To estimate the channel parameters of IRSs assisted mmWave communications, we have made the following contributions in this paper:

- We propose a simple and flexible beam training method for IRSs assisted mmWave MIMO by breaking it down into several mathematically equivalent sub-problems, and we further perform random beamforming and maximum likelihood (ML) estimation to jointly estimate AoA and AoD of the dominant path in each sub-problem. The proposed scheme does not require feedback from MT at training stage, and thus can be performed in a broadcasting manner. Hence, the required training overhead will not increase over MT number.
- We prove the uniqueness of the AoA and AoD estimated by beam training with random beamforming. We further study the impact of training length, and we prove that larger training length almost surely results in smaller pairwise error probability of AoA, AoD pair.
- By sorting the reliability of the estimated AoA, AoD pairs, we propose an iterative positioning algorithm to estimate the location of MT, and, through numerical analysis, we show that the algorithm achieves centimeter-level positioning accuracy.
- With the estimated position of MT, we propose to cross verify and enhance the estimation of path parameters, i.e., AoA and AoD, according to their geometric relationship. We further propose an accurate method of blockage prediction by comparing the ML estimate of pathloss and MT position based estimate of pathloss.

Numerical results show the superiority of our proposed beam training scheme and verify the performance gain brought by location information.

The rest of the paper is organized as follows. Section II introduces the system model. In Section III, we break down the beam training design of IRSs assisted mmWave communications. In Section IV, we propose beam training with random beamforming. In Section V, we study the interplay between positioning and beam training. In Section VI, numerical results are presented. Finally, in Section VII, we draw the conclusion.

Notations: Column vectors (matrices) are denoted by bold-face lower (upper) case letters, ||·|| denotes the Frobenius norm of a vector or a matrix, denotes Hadamard product. Subtraction and addition of the cosine θ = [θ, θ+, θ0] mod 2 is the indicator of blockage of the LoS path, and δBM,i, θBM,i and φBM,i are channel gain, cosine
of AoA, and cosine of AoD of the $l$-th path, respectively. The parameters $\{\zeta_{\text{LoS}}, \delta_1, \phi_{BM,1}, \phi_{BM,1}\}$ characterize LoS path, which are of particular interest to us in mmWave communications. According to [14], the path gain of LoS is
\[
\delta_1 = \frac{\lambda}{4\pi d_{BM}},
\]
where $\lambda$ is the wavelength, and $d_{BM}$ is the distance between BS and MT. Further, the steering vectors are given by
\[
\begin{align*}
a_M(\theta_{BM,l}) &= [1, e^{j\pi \theta_{BM,1}}, \ldots, e^{j\pi (N_M-1)\theta_{BM,1}}]^T \\
a_B(\phi_{BM,l}) &= [1, e^{j\pi \phi_{BM,1}}, \ldots, e^{j\pi (N_B-1)\phi_{BM,1}}]^T
\end{align*}
\]
where $N_B$ is the number of antennas of BS/AP, $N_M$ is the number of antennas of MT.

We also assume that IRSs adopt ULA antenna structure. Thus, the channel response of the reflected path from BS to MT assisted (reflected) by the $i$-th IRS is
\[
\mathbf{H}_{BR,M} = \zeta_{V\text{LoS},i} \delta_{BR,M} \mathbf{a}_M(\theta_{R,M}) \mathbf{a}_R^H(\phi_{R,M})
\]
\[
= \zeta_{V\text{LoS},i} \delta_{BR,M} (\tilde{\mathbf{g}}_i) \mathbf{a}_M(\theta_{R,M}) \mathbf{a}_R^H(\phi_{R,M})
\]
(2)

where $\zeta_{V\text{LoS},i} \in \{0, 1\}$ is the indicator of blockage of the path reflected by the $i$-th IRS and $\delta_{BR,M} = \frac{\lambda}{d_{BR,M}}$. In which $\xi$ is reflection loss.

Fig. 1. Illustration of IRSs assisted mmWave communications

The steering vector $\mathbf{a}_{R,i}(\phi_{R,M})$ is given by
\[
\mathbf{a}_{R,i}(\phi_{R,M}) = [1, e^{j\pi \phi_{R,1}}, \ldots, e^{j\pi (N_{R,i}-1)\phi_{R,1}}]^T
\]
(4)

where $N_{R,i}$ is the number of passive reflectors of the $i$-th IRS.

Hence, the channel response between BS and MT with the assistance of $N_{IRS}$ IRSs is represented as
\[
\mathbf{H} = \mathbf{H}_{BM} + \sum_{i=1}^{N_{IRS}} \gamma_i \mathbf{H}_{BR,M} = \\
\zeta_{\text{LoS}} \delta_1 \mathbf{a}_M(\theta_{BM,1}) \mathbf{a}_B^H(\phi_{BM,1}) + \sum_{i=2}^L \delta_i \mathbf{a}_M(\theta_{BM,i}) \mathbf{a}_B^H(\phi_{BM,i})
\]
\[
\sum_{i=1}^{N_{IRS}} \gamma_i \zeta_{V\text{LoS},i} \delta_{BR,M}(\tilde{\mathbf{g}}_i) \mathbf{a}_M(\theta_{R,M}) \mathbf{a}_R^H(\phi_{R,M})
\]
VLoS component

where
\[
\gamma_i = \begin{cases} 
1, & \text{when the } i\text{-th IRS is activated} \\
0, & \text{when the } i\text{-th IRS is deactivated}
\end{cases}
\]
indicates the activation status of the $i$-th IRS and $\gamma_i$ can be configured by BS/AP.

When beam pattern of the reflection vector $\tilde{\mathbf{g}}_i$, is omnidirectional, IRS works as a scatterer that diffuses the energy radiated from BS. When $\tilde{\mathbf{g}}_i = \mathbf{a}_R(\phi_{BM,1} \ominus \theta_{BR,1})$, IRS works as a “mirror” that builds a virtual LoS (VLoS) path between BS and MT, and thus the energy from BS will be concentrated on MT, and $\phi_{BM,1} \ominus \theta_{BR,1}$ is termed as the optimal reflection angle of the $i$-th VLoS path. We can categorize channel components of $\mathbf{H}$ into three types as in Eq. (5), namely LoS path component, VLoS path component, and NLoS path component. LoS path component is the direct path between BS and MT, VLoS path component consists of the paths between BS and MT reflected by IRSs, and NLoS path component consists of the paths between BS and MT reflected by scatterers, e.g., walls, human bodies, and etc.

III. BREAKDOWN OF BEAM TRAINING FOR IRSs ASSISTED MMWAVE MIMO

As NLoS path component usually varies fast and its weight to the channel is marginal especially in mmWave band, we are more interested in LoS path and VLoS paths. Hence, beam training of IRSs assisted mmWave MIMO intends to estimate (1) the optimal reflection angle $\phi_{R,M} \ominus \theta_{BR,1}$ of IRSs and (2) the path parameters $\{\zeta_{BM,1}, \delta_{BM,1}, \theta_{BM,1}, \phi_{BM,1}\}$ of the LoS path and $\{\zeta_{BR,M}, \delta_{BR,M}(\tilde{\mathbf{g}}_i), \theta_{R,M}, \phi_{R,M}\}$ of the VLoS paths. For conventional mmWave communications, training overhead can be significantly reduced by exploiting the sparse nature of mmWave channel [26], [27]. However, with the assistance of IRSs, the sparse channel of mmWave band is artificially converted into rich scattering channel. The increased scattering effect, together with the unknown optimal reflection angle, jointly complicates the process of beam training. In this section, to make the over-complicated problem tractable, we propose to break down beam training of IRSs assisted mmWave MIMO into two sub-problems, and we will further show that the two sub-problems are mathematically equivalent.

At first, it is noteworthy that AoA/AoD of the LoS path between IRSs and BS/AP can be accurately pre-measured, since both IRSs and BS/AP are pre-configured. Thus, $\theta_{BR,1}$,
Fig. 2. Two steps of beam training with random beamforming in IRSs assisted mmWave communications

and $\phi_{BR}$ are used as prior knowledge hereafter. Then, beam training of IRSs assisted mmWave MIMO is carried out in the following two steps as illustrated in Fig. 2.

**Step 1.** De-activate all the IRSs, and estimate the parameters $(\delta_{BM,1}, \theta_{BM,1}, \phi_{BM,1})$ of LoS path

To estimate the parameters, measures of channel are collected via Tx/Rx random beamforming in BS/AP side and MT side, i.e.,

$$
y = \sqrt{P_T}m^H H_{BM} f + m^H w = \sqrt{P_T} \zeta_{LoS} \delta_{BM,1} m^H a_M(\theta_{BM,1}) a_B^H(\phi_{BM,1}) f + \sum_{i=2}^L \sqrt{P_T} \delta_{BM,i} m^H a_M(\theta_{BM,i}) a_B^H(\phi_{BM,i}) f + m^H w
$$

where $P_T$ is transmit power, $w \sim CN(0, \sigma_w^2 I_{N_M})$ is the zero-mean complex Gaussian additive noise, $s = 1$ is the pilot signal sent by the user, $f$ and $m$ are transmit random beamforming vector at BS/AP side and receive random beamforming vector at MT side respectively, and the entries of $f$ and $m$ are phase-only complex variables with invariable amplitude [28], i.e.,

$$
f = \frac{1}{\sqrt{N_B}} (e^{j\pi \theta_1}, e^{j\pi \theta_2}, \ldots, e^{j\pi \theta_{NB}})^T $$

$$
m = \frac{1}{\sqrt{N_M}} (e^{j\pi \sigma_1}, e^{j\pi \sigma_2}, \ldots, e^{j\pi \sigma_{NM}})^T $$

$\theta_{nB}$ is the phase shift value of the $n_B$-th analog phase shifter in BS/AP side, $\sigma_{nM}$ is the phase shift value of the $n_M$-th analog phase shifter in MT side.

As NLoS paths are much weaker than LoS path in mmWave band, i.e., $\delta_{BM,l}(l = 2, \ldots, L)$ are small compared to $\delta_{BM,1}$, we are very less likely to build an effective communication link via NLoS paths. Hence, the AoA, AoD pair that we are interested in is merely $(\zeta_{LoS}, \delta_{BM,1}, \theta_{BM,1}, \phi_{BM,1})$, and the term $\nu$ will be treated as interference. Considering the small scale and randomness of $\delta_{BM,l}(l = 2, \ldots, L)$, we assume that $\nu$ follows complex Gaussian distribution for the simplicity of analysis. Then, the beam training problem for IRSs assisted mmWave MIMO communications is formulated as the estimation of $(\zeta_{LoS}, \delta_{BM,1}, \theta_{BM,1}, \phi_{BM,1})$ from the following received signal

$$
y = \sqrt{P_T} \zeta_{LoS} \delta_{BM,1} m^H a_M(\theta_{BM,1}) a_B^H(\phi_{BM,1}) f + \nu + m^H w $$

Adding the subscript $n$ to $y$ to denote the received signal in the $n$-th time slot, we have

$$
y_n = \sqrt{P_T} \zeta_{LoS} \delta_{BM,1} m_n^H a_M(\theta_{BM,1}) a_B^H(\phi_{BM,1}) f_n + \nu_n + m_n^H w
$$

where $b(\theta_{BM,1}, \phi_{BM,1}) \triangleq vec(a_M(\theta_{BM,1}) a_B^H(\phi_{BM,1}))$.

To estimate AoA and AoD, $N$ channel measurements are to be collected and concatenated, and its vector form is derived as

$$
y = \sqrt{P_T} \zeta_{LoS} \delta_{BM,1} D b(\theta_{BM,1}, \phi_{BM,1}) + \nu + w $$

where

$$
y = [y_1, y_2, \ldots, y_N]^T $$

$$D = [f_1 \otimes m_1, f_2 \otimes m_2, \ldots, f_N \otimes m_N]^T $$

$$\nu = [\nu_1, \nu_2, \ldots, \nu_N]^T \sim CN(0, \sigma_n^2 I_N) $$

$$w = [m_1^H w, m_2^H w, \ldots, m_N^H w]^T $$

Since

$$E(\bar{w}(\iota) \bar{w}^*(\kappa)) = E(m^H w \bar{w}^H m) = \sigma_w^2, $$

$$E(\bar{w}(\iota) \bar{w}^*(\kappa)) = E(m^H \bar{w} \bar{w}^H m) = 0, \forall \iota \neq \kappa $$

the covariance of the equivalent noise $w$ is thus $E(ww^H) = \sigma_w^2 I_N$. Let $\nu \triangleq \nu + w$, as $\nu$ and $w$ are independent of each other, we have $w \sim CN(0, (\sigma_w^2 + \sigma_n^2) I_N)$.

Based on the above analysis, beam training for the link between BS/AP and MT is summarized as follows.

**Sub-problem 1:** How to accurately estimate the parameter set $(\zeta_{LoS}, \delta_{BM,1}, \theta_{BM,1}, \phi_{BM,1})$ from $y$.

**Step 2.** Activate the $i$-th IRS, de-activate the rest IRSs, and estimate the parameters $(\delta_{BR,i}, \theta_{BR,i}, \phi_{BR,i})$ of the $i$-th VLoS path. Repeat the above process for the rest IRSs.

As $\phi_{BR}$ is known, with the transmit beamforming vector $f = a_M(\theta_{BM,1})/\sqrt{N_B}$, BS/AP is able to concentrate its power towards IRSs via transmit beamforming. Simultaneously, IRS performs

1A good random beamforming codebook can be derived offline by high performance computers, and they will be pre-configured in BS/AP, IRS and MT side.

2Although we assume that $\nu$ follows Gaussian distribution in theoretical analysis, the channel model to be applied in numerical simulation is still the cluster based model as in [1].
passive random reflection and MT performs receive random beamforming, the received signal at MT side is written as

\[
y = \sqrt{P_{Tx}} m^H (H_{BM} + H_{BR,M}) a_B(\phi_{BR}) + m^H \bar{w}
\]

\[
= N_B P_{Tx} \zeta_{\text{LoS},M} m^H a_M(\phi_{R,M}, \theta_{BR}) + m^H \bar{w}
\]

\[
+ \sum_{l=1}^{L} P_{Tx} \zeta_{\text{LoS},l} m^H a_M(\phi_{R,M}) a_B(\phi_{BR}) \frac{N_B}{N_B} + m^H \bar{w}
\]

(9)

The interference terms \( \nu_1 \) and \( \nu_2 \) are insignificant due to (1) the small NLoS path coefficients \( \delta_l \) in mmWave band, (2) the spatial filtering impact, i.e., \( a_B^H(\theta_{BR}) a_B(\phi_{BR}) \approx 0 \), \( l = 1, 2, \cdots, L \) for \( \phi_{BR} - \theta_{MB, l} \frac{N_B}{N_B} \).

Similar to (5), by concatenating \( N \) channel measurements, we have

\[
y = \sqrt{N_B} P_{Tx} \zeta_{\text{LoS},M} D B(\phi_{R,M}, \phi_{R,M}, \theta_{BR}) + \nu_1 + \nu_2 + w
\]

(10)

where

\[
D = [g_1 \otimes m^*_1, g_2 \otimes m^*_2, \cdots, g_N \otimes m^*_N]^T
\]

Based on the above analysis, beam training for the reflected path between BS/AP and MT assisted by the \( i \)-th IRS is summarized as follows.

**Sub-problem 2:** How to accurately estimate the parameter set \( (\zeta_{\text{LoS},M}, \delta_{BR,M}, \theta_{R,M}, \phi_{R,M}, \theta_{BR}) \) from \( y \).

**Remark 1.** We can find that Sub-problem 1 and Sub-problem 2 are mathematically equivalent. Owing to the flexible control over IRS, we are capable to decompose the complicated non-sparse channel estimation problem of IRSs assisted mmWave MIMO into a set of equivalent sub-problems of beam training design.

**IV. Beam Training With Random Beamforming**

In this section, ML estimation method is applied to estimate the path parameters \( (\delta, \theta, \phi) \) of LoS/VLoS paths from channel measurements sampled by random Rx/Tx beamforming. Furthermore, the feasibility of random beamforming based beam training is verified.

**A. Maximum Log-likelihood Estimation of \( (\delta, \theta, \phi) \)**

For conciseness of expression, we write the unified model of sub-problem 1 and sub-problem 2 as

\[
y = \zeta \delta D B(\theta, \phi) + n
\]

(11)

where \( \zeta \) is the indicator of blockage, \( \delta \) is equivalent path gain (\( \delta = \sqrt{P_{Tx}} \delta_{BM,1} \) or \( \delta = \sqrt{P_{Tx}} N_B \delta_{BR,M} \)), \( \theta \) is cosine AoA, \( \phi \) is equivalent cosine AoD (\( \phi = \phi_{BM,1} \) or \( \phi = \phi_{R,M} \theta_{BR} \)), and \( b(\theta, \phi) = \text{vec}(a_B^H(\theta) a_{BR}^H(\phi) ) \).

It is noteworthy that estimation of \( (\delta, \theta, \phi) \) should be performed merely when \( \zeta = 1 \), as the measurement vector \( y \) given that \( \zeta = 0 \) contains no information about \( (\delta, \theta, \phi) \). Therefore, we estimate the parameters \( (\delta, \theta, \phi) \) through maximizing log-likelihood function under the assumption that \( \zeta = 1 \), i.e.,

\[
(\hat{\delta}, \hat{\theta}, \hat{\phi}) = \arg \max_{\delta, \theta, \phi} \mathcal{L}(\delta, \theta, \phi)
\]

(12)

where

\[
\mathcal{L}(\delta, \theta, \phi) = \log P(\gamma | \zeta = 1, \delta, \theta, \phi)
\]

\[
= - N \log \pi - N \log \sigma^2 - \frac{\| y - \delta D B(\theta, \phi) \|^2}{\sigma^2}
\]

(13)

and the conditional probability is

\[
P(\gamma | \zeta, \delta, \theta, \phi) = \frac{1}{\pi^N \det(\sigma^2 I_N)^{l/2}} e^{-\frac{(y - \hat{\zeta} D B(\theta, \phi))^H(y - \hat{\zeta} D B(\theta, \phi))}{\sigma^2}}
\]

(14)

1) **Estimation of \( \delta \):** Before the derivation of \( \hat{\delta}, \hat{\theta}, \hat{\phi} \), we should find the expression of \( \delta \). To this end, we ignore terms independent thereof and set

\[
\frac{\partial \mathcal{L}(\delta, \theta, \phi)}{\partial \delta} = 0
\]

(15)

Expanding Eq. (15), we have

\[
2 Re \left\{ (D B(\theta, \phi))^H(y - \delta D B(\theta, \phi)) \right\} = 0
\]

(16)

From Eq. (16), the optimal \( \hat{\delta} \) is derived as

\[
\hat{\delta} = \frac{b^H(\theta, \phi) D B^H(\theta, \phi)}{\| D B(\theta, \phi) \|^2}
\]

(17)

2) **Estimation of \( \theta \) and \( \phi \):** Next, we will jointly estimate \( \theta \) and \( \phi \). Substituting Eq. (17) into Eq. (13), we have

\[
\mathcal{L}(\delta, \theta, \phi)
\]

\[
= - N \log \pi - N \log \sigma^2 - \frac{\| y - D B(\theta, \phi) b^H(\theta, \phi) D B^H(\theta, \phi) \|^2}{\sigma^2}
\]

(18)

Since

\[
\| y - D B(\theta, \phi) b^H(\theta, \phi) D B^H(\theta, \phi) \|^2
\]

\[
= y^H(I - \frac{D B(\theta, \phi) b^H(\theta, \phi) D B^H(\theta, \phi)}{\| D B(\theta, \phi) \|^2}) y
\]

(19)

the beam training problem is formulated as

\[
P_1 : \max_{\theta, \phi} \left\{ \frac{b^H(\theta, \phi) D B^H(\theta, \phi) y}{\| D B(\theta, \phi) \|^2} \right\}^2
\]

s.t. \(-1 \leq \theta < 1 \)

\(-1 \leq \phi < 1 \)

P1 is a non-convex problem. However, as there are only two real-valued variables to be estimated, a simple but efficient two-step algorithm can be readily applied to solve P1. To
facilitate the development of the two-step algorithm, we firstly derive the partial derivatives of the objective function as follows. Let \( g(\theta, \phi) \triangleq \left\| \mathbf{b}^H(\theta, \phi) \mathbf{D}^H \mathbf{y} \right\|^2_2 \), the derivative of \( g(\theta, \phi) \) with respect to \( \theta \) is
\[
\frac{\partial g(\theta, \phi)}{\partial \theta} = \frac{\partial b^H(\theta, \phi) \mathbf{D}^H \mathbf{y} \mathbf{D} \mathbf{b}(\theta, \phi)}{\partial \theta} - \mathbf{b}^H(\theta, \phi) \mathbf{D}^H \mathbf{y} \mathbf{D} \mathbf{b}(\theta, \phi) \frac{\partial \mathbf{D}(\theta, \phi)}{\partial \theta} \frac{\partial \mathbf{b}(\theta, \phi)}{\partial \theta} \mathbf{D}^H \mathbf{b}(\theta, \phi)
\]
Repeat the above operations over the rest \( N_{pk} - 1 \) maxima derived in Step 1, and select the best one as \((\hat{\theta}, \hat{\phi})\). Then, the exact value of the estimated path gain \( \hat{\delta} \) can be subsequently obtained by substituting \((\hat{\theta}, \hat{\phi})\) into (17).

**Remark 2.** A notable advantage of the proposed scheme is that it does not need feedback at random beamforming stage, which enables BSAP and IRSs to broadcast its pilot signal. Therefore, its training overhead does not increase over the number of MTs.

### B. Uniqueness of The Estimated AoA and AoD Pair

To delve into the effectiveness of beam training with random beamforming, conditions under which \((\theta, \phi)\) can be accurately estimated from the measurement signal \(y\) are studied in the ideal scenario without noise or interference.

Firstly, two definitions of uniqueness are introduced as follows.

1. **Uniqueness of measurement signal representation**, namely
   \[
   y = \delta \mathbf{D}(\theta, \phi) \neq \delta \mathbf{D}(\tilde{\theta}, \tilde{\phi}), \quad \forall \tilde{\delta} \in \mathbb{C}, \forall (\tilde{\theta}, \tilde{\phi}) \neq (\theta, \phi)
   \]

2. **Uniqueness of estimated AoA and AoD pair**, namely
   \[
   \left\| \frac{\mathbf{b}^H(\theta, \phi) \mathbf{D}^H \mathbf{y}}{\left\| \mathbf{D}(\theta, \phi) \right\|_2} \right\|_2 > \left\| \frac{\mathbf{b}^H(\tilde{\theta}, \tilde{\phi}) \mathbf{D}^H \mathbf{y}}{\left\| \mathbf{D}(\tilde{\theta}, \tilde{\phi}) \right\|_2} \right\|_2, \quad \forall (\tilde{\theta}, \tilde{\phi}) \neq (\theta, \phi)
   \]

Uniqueness of measurement signal representation means that any AoA, AoD pair \((\theta, \phi)\) that differs from \((\theta, \phi)\) cannot construct the measurement signal \(y\). It is an inherent property of the sampling method, which is primarily determined by \(\mathbf{D}\). By contrast, uniqueness of the estimated AoA and AoD depends on both sampling method and estimation method. It indicates that AoA, AoD pair can be accurately estimated from the measurement signal \(y\) using a specific estimation method.

In the following Theorem, we will study the relationship between the above two types of uniqueness.

**Theorem 1.** As long as uniqueness of measurement signal representation is satisfied, ML method is capable to accurately estimate the AoA, AoD pair.

**Proof.** See Appendix A.

According to Theorem 1, the uniqueness of AoA and AoD estimation is equivalent to the uniqueness of measurement signal representation, which means we just need to investigate the conditions on which uniqueness of measurement signal representation can be achieved.

Before studying the sensing matrix \(\mathbf{D}\), we will observe the signal space of channel response. The vectorized response of LoS path, namely \(h = \delta \mathbf{D}(\theta, \phi)\), is a high dimensional \((N, N)\)-dimensional variable that is characterized by \((\delta, \theta, \phi)\), and we define the signal space of \(h\) as
\[
S \triangleq \{ \delta \mathbf{b}(\theta, \phi) | \delta \in \mathbb{C}, -1 \leq \theta, \phi < 1 \}
\]
S is a nonlinear \(k\)-dimensional \((k = 3)\) submanifold of \(\mathbb{C}^{N, N}\) with the parameters \((\delta, \theta, \phi)\). As \(\mathbf{b}(\theta, \phi)\)
is the Kronecker product of two array steering vectors, \( S \) is indeed the so-called *array manifold* \([31]\). Thus, one channel realization \( \Phi \) with the parameters \((\delta, \theta, \phi)\) can be seen as a point in the array manifold. The dimensionality \( k \) can be interpreted as an “information level” of the signal, analogous to the sparsity level in compressive sensing problems \([29, 32, 33]\). In \([29]\), it is proved that signals obeying manifold models can also be recovered from only a few measurements, simply by replacing the traditional compressive sensing model of sparsity with a manifold model for \( h \). The above statement is supported by Lemma 1.

**Lemma 1.** For a random orthoprojector \( \Phi \in \mathbb{C}^{M \times N} \), the following statement

\[
(1-\epsilon)\sqrt{\frac{M}{N}} \leq \frac{\|\Phi h_1 - \Phi h_2\|_2^2}{\|h_1 - h_2\|_2^2} \leq (1+\epsilon)\sqrt{\frac{M}{N}},
\]

\( \forall h_1, h_2 \in S, h_1 \neq h_2 \) \quad (27)

holds with high probability, when dimensionality \( M \) of the projected low-dimensional space is sufficient\(^1\) where \( h_1 \in S, h_2 \in S, h_1 \neq h_2, 0 < \epsilon < 1 \) is the isometry constant \([29]\).

**Remark 3.** \( \|h_1 - h_2\|_2^2 \) is the Euclidean distance between two points \( h_1, h_2 \) on the manifold, and \( \|\Phi h_1 - \Phi h_2\|_2^2 \) is the Euclidean distance between the projected points \( \Phi h_1, \Phi h_2 \) on the image of \( S \) (namely \( \Phi S \)). The isometry constant \( \epsilon \) measures the degree that the pairwise Euclidean distance between points on \( S \) is preserved under the mapping \( \Phi \). Apparently, Lemma 1 indicates that \( \|\Phi h_1 - \Phi h_2\|_2^2 > 0 \) is satisfied with high probability, as it is a weaker condition than Lemma 1.

Although the sensing matrix \( M \) is not necessarily an orthoprojector, via singular value decomposition, it can be decomposed as \( M = \Psi \Lambda \Phi \), where \( \Psi \in \mathbb{C}^{M \times M} \), \( \Lambda \in \mathbb{C}^{M \times M} \), and \( \Phi \in \mathbb{C}^{M \times N} \). Then, we have \( \|D h_1 - D h_2\|_2^2 = \|\Lambda \Phi h_1 - \Lambda \Phi h_2\|_2^2 \), where \( \Phi \) is indeed the orthoprojector, and \( \Lambda \) is a diagonal matrix with non-zero elements that scales the component in each dimension. \( \|\Phi h_1 - \Phi h_2\|_2^2 > 0 \) implies \( \|D h_1 - D h_2\|_2^2 > 0 \), which is equivalent to \( D h_1 \neq D h_2 \), namely, \( \delta_1 D b(\theta_1, \phi_1) \neq \delta_2 D b(\theta_2, \phi_2), \forall (\delta_1, \theta_1, \phi_1) \neq (\delta_2, \theta_2, \phi_2) \). Thus, it is easy to find that \( D b(\theta_1, \phi_1) \neq \mu D b(\theta_2, \phi_2), \forall (\theta_1, \phi_1) \neq (\theta_2, \phi_2), \forall \mu \in \mathbb{C} \), where \( \mu = \frac{\delta_2}{\delta_1} \).

To conclude, the randomly generated sensing matrix \( M \) has a large probability to guarantee the uniqueness of ML based joint AoA and AoD estimation.

### C. On The Impact of Training Length \( N \)

Theorem 1 indicates that, with random beamforming, Eq. (25) holds with high probability. In other words, in noiseless scenario, the distance gap between the highest peak (global optimum) and other peaks (other local optima) exist with high probability. However, in practice, corrupted by noise and interference, the highest peak may (1) shift to its adjacent points, or (2) be transcended and replaced by other peaks. Error Type 1 incurs mild AoA, AoD estimation error followed by power loss of an acceptable level; Error Type 2 incurs significant AoA, AoD estimation error followed by beam misalignment. Apparently, we would like to avoid Error Type 2.

To study the estimation error, the pairwise error probability (PEP) of any two parameter sets \((\theta, \phi)\) and \((\bar{\theta}, \bar{\phi})\) is derived in the following theorem.

**Theorem 2.** The PEP \( P_e((\theta, \phi) \rightarrow (\bar{\theta}, \bar{\phi})) \) that \((\theta, \phi)\) is mistaken as \((\bar{\theta}, \bar{\phi})\) in relatively high SNR regime can be approximated as

\[
P_e((\theta, \phi) \rightarrow (\bar{\theta}, \bar{\phi})) \approx Q \left( \frac{\|\delta\|_2^2}{2\sigma^2} - \frac{d^2(D, \theta, \phi, \bar{\theta}, \bar{\phi})}{\|D b(\theta, \phi)\|_2^2} \right)
\]

where

\[
d^2(D, \theta, \phi, \bar{\theta}, \bar{\phi}) \triangleq \|D b(\theta, \phi)\|_2^2 - \frac{b^H(\bar{\theta}, \bar{\phi}) D^H D b(\theta, \phi)}{\|D b(\theta, \phi)\|_2^2}
\]

**Proof.** See Appendix B.

Theorem 2 indicates that PEP is inversely proportional to \(d^2(D, \theta, \phi, \bar{\theta}, \bar{\phi})\). To build the connection between PEP and training length \( N \), Proposition 1 is derived.
Proposition 1. $d^2(D_N, \theta, \phi, \bar{\theta}, \bar{\phi})$ is monotonically increasing over training length $N$, where $D_N = [D_{N-1}^H d_N]^H$, i.e.,

$$d^2(D_N, \theta, \phi, \bar{\theta}, \bar{\phi}) \geq d^2(D_{N-1}, \theta, \phi, \bar{\theta}, \bar{\phi})$$

and the equality holds only if

$$b^H(\theta, \phi)d_ND_N^H b(\theta, \phi) = b^H(\theta, \phi)d_ND_N^H b(\theta, \phi) = b^H(\theta, \phi)d_ND_N^H b(\theta, \phi)$$

(29)

Proof. See Appendix C.

To verify Proposition 1, we plot the contour of $g(\theta, \phi)$ with different training lengths in noiseless scenario in Fig. 3. We set $\delta = 1$, $\theta = 0$, $\phi = 0$. As can be seen that the gap between the first and the second peaks increases over training length, and the value of which is given in Table I. In addition, we can find that position of the first peak is invariant to training length and remains the same as the actual AoA, AoD pair, while position of the second peak varies. This verifies the uniqueness of ML based joint AoA, AoD estimation.

Remark 4. According to Proposition 1, with random beamforming, the PEP probability of an erroneous estimate $(\bar{\theta}, \bar{\phi})$ being mistaken as the authentic parameters $(\theta, \phi)$ decreases almost surely over training length $N$. Therefore, an appropriate $N$ can guarantee a satisfying accuracy of parameter estimation in scenarios with different SNR and interference levels.

V. INTERPLAY BETWEEN POSITIONING AND BEAM TRAINING

In IRSs assisted mmWave MIMO system, BS/AP and IRSs, with their positions and array directions being known by all the MTs, can be seen as anchor nodes or beacons. The AoDs derived at beam training stage enable MT to estimate its own position. Hence, IRSs assisted mmWave MIMO system is endowed with the capability of high-accuracy localization. The acquired position information is not only a fringe benefit, but also in turn facilitates beam training. The interplay between beam training and indoor positioning is explained as follows. AoD estimate of the unblocked reliable links can yield the position of MT, and the position of MT, associated with anchor positions and anchor directions, can improve the precision of AoD/AoA estimation and assist in the decision of blockage indicator $\zeta$.

A. RELIABILITY OF THE ESTIMATED AOA, AOD PAIR $(\hat{\theta}, \hat{\phi})$

To be concise, we treat BS/AP and IRSs as identical anchor nodes. The $\eta = 1$-st anchor is BS/AP and the rest $N_{IRS}-1$ anchors ($\eta = 2, 3, \cdots, N_{IRS}+1$) are IRSs. Although we have already obtained $N_{IRS}-1$ sets of path parameters $(\hat{\delta}_\eta, \hat{\theta}_\eta, \hat{\phi}_\eta)$, we should be aware that the estimation is performed under the assumption that $\zeta_\eta = 1$. In practice, LoS and VLoS paths may suffer from blockage (namely $\zeta_\eta = 0$) by moving obstacles, which will jeopardize the estimation of $(\hat{\delta}_\eta, \hat{\theta}_\eta, \hat{\phi}_\eta)$. Other than blockage, insufficient training length or low SNR may incur Error Type 2 of joint AoA and AoD estimation, which is defined in Section IV.C.

Therefore, it is essential to select the trustworthy parameters as the input of positioning algorithm. To this end, we introduce the metric – residual signal power ratio $\varpi_\eta$, to measure the reliability of $(\hat{\delta}_\eta, \hat{\theta}_\eta, \hat{\phi}_\eta)$, i.e.,

$$\varpi_\eta = \frac{\|y_{\eta} - \hat{\delta}_\eta D_b(\hat{\theta}_\eta, \hat{\phi}_\eta)\|^2_2}{\|y_{\eta}\|^2_2}$$

(31)

Recall that $(\hat{\delta}_\eta, \hat{\theta}_\eta, \hat{\phi}_\eta)$ are obtained by minimizing $\|y_{\eta} - \hat{\delta}_\eta D_b(\theta, \phi)\|^2$, the yielded estimate $(\hat{\delta}_\eta, \hat{\theta}_\eta, \hat{\phi}_\eta)$ will thus always result in $\|y_{\eta} - \hat{\delta}_\eta D_b(\theta, \phi)\|^2 \leq \|y_{\eta}\|^2$. Therefore, the range of $\varpi_\eta$ is $\varpi_\eta \in [0, 1]$.

Since the dominant component of mmWave channel is LoS path, the reconstructed signal $\delta_\eta D_b(\theta, \phi)$ should account for the majority of the received signal $y$ given that the parameters $(\delta_\eta, \theta_\eta, \phi_\eta)$ are accurate and residual signal power ratio $\varpi_\eta$ should be smaller. Conversely, when blockage or Error Type 2 occurs, the parameters $(\delta_\eta, \theta_\eta, \phi_\eta)$ are heavily biased, and thus $\varpi_\eta$ should be larger. Following the above heuristics, anchors’ reliability can be sorted.

B. AoD Based Positioning

1) Geometric Relationship Between AODs and MT Position:

We denote the index set of the reliable links as $\mathcal{N}$, position coordinates of the $\eta$-th anchor as $p_\eta$, ULA direction of the $\eta$-th anchor as $e_\eta$. Note that $p_\eta, e_\eta$ are known by MTs. The direction vector of the LoS path between MT and the $\eta$-th anchor is $\|p - p_\eta\|_2$, where $p$ is the position of MT. Thus, the geometric relationship between AoDs and MT position is expressed as

$$\hat{\phi}_\eta = \frac{(p - p_\eta)^T e_\eta + \varepsilon_\eta}{\|p - p_\eta\|_2}, \quad \eta \in \mathcal{N}$$

(32)

where $\hat{\phi}_\eta$ is the estimate of cosine AoD of the $\eta$-th link derived in beam training stage, $\phi_\eta(p)$ is the actual cosine AoD that is dependent on position $p$, and $\varepsilon_\eta$ is estimation error. For illustrative purposes, a typical scenario of IRSs assisted mmWave communications is shown in Fig. 4.

2) Taylor Series Method for AoD Based Positioning: In the ideal case, when $\varepsilon_\eta = 0$, we have $\hat{\phi}_\eta = \phi_\eta(p)$. The equation $\phi_\eta(p) = \frac{(p - p_\eta)^T e_\eta}{\|p - p_\eta\|_2}$ corresponds to a right circular cone. There are 3 unknown variables of MT’s position coordinates, thus the minimum sufficient number of unblocked links to estimate the 3-D position of MT is $|\mathcal{N}| = 3$, which is the intersection of the three right circular cones. As IRSs are cost-effective compared with conventional mmWave devices, they can be massively installed with minimal effort. We can expect that IRSs assisted mmWave with a large number of delicately placed IRSs is capable to guarantee $|\mathcal{N}| \geq 3$ unblocked links with high probability.
In practice, estimation error $\epsilon_{\eta}$ cannot be zero. To estimate the 3-D position $p = (x, y, z)^T$, least square criterion is adopted, i.e.,

$$\min_p \xi_p(p) \triangleq \sum_{\eta \in \mathcal{N}} \left( \hat{\phi}_\eta - \phi_\eta(p) \right)^2$$

(s.t. $p \in S$)

where $S$ is the position range of indoor MT, e.g., the 3-D space of lecture hall. As the objective function $\xi_p(p)$ is non-convex, it is non-trivial to derive the analytical solution to this problem. Fortunately, Taylor-series estimation method is capable to effectively solve a large class of position-location problems [34]. Starting with a rough initial guess, the Taylor-series estimation method iteratively improves its guess at each step by determining the local linear least-sum-squared-error correction [34]. In AoD based positioning, with the initial position guess $\hat{p}$, the following approximation can be obtained through Taylor series expansion by neglecting $m$-th order terms ($m \geq 2$), i.e.,

$$\phi_\eta(p) \approx \phi_\eta(\hat{p}) + (p - \hat{p})^T \frac{\partial \phi_\eta(p)}{\partial p} \bigg|_{p=\hat{p}}$$

(34)

where the first order derivative is denoted as

$$\frac{\partial \phi_\eta(p)}{\partial p} = \left[ \frac{\|p - p_{\eta}\| - \|\hat{p} - p_{\eta}\|}{\|p - p_{\eta}\|_2} \right] e_{\eta}$$

(35)

Substituting (37) into (32), we have

$$\hat{\phi}_\eta - \phi_\eta(\hat{p}) \approx \frac{\partial \phi_\eta(p)}{\partial p} \bigg|_{p=\hat{p}} (p - \hat{p}) + \epsilon_\eta, \ \eta \in \mathcal{N}$$

(36)

Its matrix form is written as

$$\Delta_\phi \approx A^T \Delta_p + \epsilon$$

(37)

where

$$\Delta_\phi = \left[ \hat{\phi}_1 - \phi_1(\hat{p}), \ldots, \hat{\phi}_{\mathcal{N}} - \phi_{\mathcal{N}}(\hat{p}) \right]^T$$

and

$$\Delta_p = \left[ \frac{\partial \phi_\eta(p)}{\partial p} \bigg|_{p=\hat{p}}, \ldots, \frac{\partial \phi_{\mathcal{N}}(p)}{\partial p} \bigg|_{p=\hat{p}} \right]$$

(38a)

(38b)

On the basis of (37), the Taylor series method for AoD based positioning is summarized in Algorithm 1.

3) Reliable Link Set $\mathcal{N}$: An intuitive method to construct the set of reliable links is to select $|\mathcal{N}|$ links with the $|\mathcal{N}|$ smallest $\varphi_{\eta}$ to avoid unreliable AoDs resulted from blockage and Error Type 2 of joint AoA, AoD estimation. However, it is non-trivial to determine the exact value of $|\mathcal{N}|$. Although $|\mathcal{N}| = 3$ anchors are theoretically sufficient to yield the position of MT in the ideal noiseless case, more anchors are desirable in practice for positioning algorithm to enhance the accuracy of position estimation.

To utilize as many reliable anchors as possible, the following strategy is proposed to iteratively construct the reliable link set $\mathcal{N}$. Firstly, we sort the anchors in ascending order according to $\varphi_{\eta}$. Then, starting from $|\mathcal{N}| = 3$ anchors, we iteratively increase the number of anchors used for positioning in Algorithm 1, and by the end of each iteration, we calculate the cost $\frac{\xi(p|\mathcal{N})}{|\mathcal{N}|}$. Finally, we select the output corresponding to the largest $|\mathcal{N}|$ that satisfies $\frac{\xi(p|\mathcal{N})}{|\mathcal{N}|} \leq \xi_{th}$ as the estimated position of MT, where $\xi_{th}$ is a preset threshold.

C. Parameter Estimation With The Aid of MT Position

With the estimated position $\hat{p}$, channel parameters can be refined according to the geometric relationship.

1) AoD Estimation: With $\hat{p}$, AoD estimation is updated by

$$\bar{\phi}_\eta = \frac{(\hat{p} - p_{\eta}) e_{\eta}}{\|\hat{p} - p_{\eta}\|_2}, \ \eta \in \{1, 2, \cdots, N_{\text{RIS}} + 1\}$$

(39)

2) AoA Estimation: To estimate AoA, the direction of ULA in MT’s side is essential. Therefore, we firstly find the least

---

4 An appropriate $\xi_{th}$ can be obtained by carrying out a great number of Monte Carlo experiments offline. In our numerical experiment, we find that $\sqrt{\xi_{th}} = 0.005$ results in a good performance.
Therefore, we will use the estimates of AoA and AoD refined cross verified by multiple anchors and are thus more reliable.

Note that \( \mathcal{N} \) can be derived in the iterative process according to Section V. A. 3.

The objective function of (40) can be rewritten in matrix form as

\[
\min_{\mathbf{e}_{MT}} \xi_0(\mathbf{e}_{MT}) = \| \mathbf{P}^T \mathbf{e}_{MT} \otimes \hat{\theta} \|^2_2
\]

(41)

where \( \mathbf{P} = \left[ \frac{(\mathbf{p} - \mathbf{p}_{n_1})}{\| \mathbf{p} - \mathbf{p}_{n_1} \|^2_2}, \ldots, \frac{(\mathbf{p} - \mathbf{p}_{n_k})}{\| \mathbf{p} - \mathbf{p}_{n_k} \|^2_2} \right] \), \( \hat{\theta} = [\hat{\theta}_{n_1}, \ldots, \hat{\theta}_{n_k}]^T \) and \( \mathcal{N} = \{ n_1, \ldots, n_k \} \). The optimization problem can be solved via projected gradient descent method [35], in which we iteratively update \( \mathbf{e}_{MT} \) as follows.

\[
d_{MT,i+1} = e_{MT,i} - \lambda \frac{\partial \xi_0(\mathbf{e}_{MT})}{\partial \mathbf{e}_{MT}} \bigg|_{\mathbf{e}_{MT}=\mathbf{e}_{MT,i}}
\]

(42)

where \( \lambda \) is step size and \( \frac{\partial \xi_0(\mathbf{e}_{MT})}{\partial \mathbf{e}_{MT}} = \mathbf{P}^T \mathbf{e}_{MT} \otimes \hat{\theta} \).

Finally, with \( \hat{\mathbf{e}}_{MT} \) yielded by projected gradient descent method, AoA estimation is updated by

\[
\hat{\theta}_n = \frac{(\hat{\mathbf{p}} - \mathbf{p}_n)^T \hat{\mathbf{e}}_{MT}}{\| \mathbf{p} - \mathbf{p}_n \|^2_2}
\]

(43)

3) Prediction of Blockage: As a prerequisite of our proposed blockage prediction method, we firstly introduce the estimation of \( \hat{\delta}_n \), which is dependent on the values of \( (\hat{\theta}_n, \hat{\phi}_n) \). Note that the parameter estimate obtained in Section IV by ML estimation is under the assumption that \( \zeta_n = 1 \), while it is probable that \( \zeta_n = 0 \) in fact. It would be misleading in the estimation of \( \delta_n \) by directly substituting \( (\hat{\theta}_n, \hat{\phi}_n) \) into (17).

Therefore, we will use the estimates of AoA and AoD refined by position to assist the estimation of \( \delta_n \) and \( \zeta_n \), as they are cross verified by multiple anchors and are thus more reliable.

Substituting \( (\hat{\theta}_n, \hat{\phi}_n) \) into (17), we have

\[
\hat{\delta}_n = \frac{\mathbf{b}^H(\hat{\theta}_n, \hat{\phi}_n) \mathbf{D}^H \mathbf{y}}{\| \mathbf{D} \mathbf{b}(\hat{\theta}_n, \hat{\phi}_n) \|^2_2} = \frac{\mathbf{b}^H(\hat{\theta}_n, \hat{\phi}_n) \mathbf{D}^H \mathbf{D} \mathbf{b}(\hat{\theta}_n, \hat{\phi}_n) + \mathbf{b}^H(\hat{\theta}_n, \hat{\phi}_n) \mathbf{D}^H \mathbf{n}}{\| \mathbf{D} \mathbf{b}(\hat{\theta}_n, \hat{\phi}_n) \|^2_2}
\]

(44)

where \( f(\hat{\theta}_n, \hat{\phi}_n) \triangleq \frac{\mathbf{b}^H(\hat{\theta}_n, \hat{\phi}_n) \mathbf{D}^H \mathbf{D} \mathbf{b}(\hat{\theta}_n, \hat{\phi}_n)}{\| \mathbf{D} \mathbf{b}(\hat{\theta}_n, \hat{\phi}_n) \|^2_2} \) and \( \sigma^2_n = \frac{\sigma^2_{\|} + \sigma^2_{\phi}}{\| \mathbf{D} \mathbf{b}(\hat{\theta}_n, \hat{\phi}_n) \|^2_2} \). Thus, we have

\[
\delta_n = \left\{ \begin{array}{ll}
\delta_n f(\hat{\theta}_n, \hat{\phi}_n) + \bar{n}, & \zeta_n = 1 \\
\bar{n}, & \zeta_n = 0
\end{array} \right.
\]

(45)

Theoretically, with the knowledge of \( \delta_n \), \( f(\hat{\theta}_n, \hat{\phi}_n) \) and \( \sigma^2_n \), the decision of \( \zeta_n \) can be made by comparing the probabilities of \( \delta_n \) conditioned on \( \zeta_n = 0 \) and \( \zeta_n = 1 \). However, accurate estimation of \( f(\hat{\theta}_n, \hat{\phi}_n) \) is challenging in practice. With respect to \( \delta_n \), its amplitude \( |\delta_n| \) is estimable from the distance of MT, while its phase cannot be accurately estimated from the distance, as it is very sensitive to distance estimation error and may be affected by random initial phase of local oscillator in transmitter side.

Alternatively, a heuristic method is proposed to decide blockage indicator by comparing the pathloss estimated from \( (\hat{\theta}, \hat{\phi}) \) and pathloss estimated from \( \mathbf{p} \), i.e.,

\[
10\log_{10} \frac{1}{|\delta(\mathbf{p})|^2} - 10\log_{10} \frac{1}{|\delta(\hat{\mathbf{p}})|^2} \leq P_{L_{th}}
\]

(46)

where

\[
|\delta(\mathbf{p})| = \left\{ \begin{array}{ll}
\sqrt{-\frac{4\pi}{d_{BM}}} & n = 1 \\
\sqrt{-\frac{4\pi}{d_{BM}}} & n = 2, \ldots, N_{IRS+1}
\end{array} \right.
\]

BS/AP to MT distance \( d_{BM} \) and IRS to MT distance \( d_{R_{n,M}} \) are attainable from \( \mathbf{p} \), and \( P_{L_{th}} \) is the preset threshold of pathloss distance. (In numerical simulations, we set \( P_{L_{th}} = 60 \) dB).

VI. NUMERICAL RESULTS

In this section, we numerically study the performance of the proposed joint beam training and positioning scheme for IRS-assisted mmWave MIMO.

A. Settings of Numerical Experiment

We assume that IRSs-assisted mmWave MIMO system is deployed in an indoor scenario, e.g., lecture hall, and the length, width and height of which are 20 meters, 20 meters and 5 meters, respectively. The rest system parameters are listed in Table I. For simplicity, we assume that AoA, AoD of NLoS paths follow uniform distribution, i.e., \( \theta_{BM,l}, \phi_{BM,l} \sim U(0, 2\pi) \), \( l = 2, \ldots, L \), and path coefficient follows complex Gaussian distribution, i.e., \( \delta_{l} \sim C\mathcal{N}(0, \sigma^2_{\delta}) \), \( l = 2, \ldots, L \) and \( 10\log_{10} \sigma^2_{\delta} = 20 \) dB. We model user (MT holder) as a cube with its length, width and height being 0.6m, 0.4m and 1.7m, respectively. We denote position of the MT held by user

| Parameter | Value |
|-----------|-------|
| Operating frequency | 28GHz |
| Tx power of BS/AP | [0, 30)dBm |
| Noise power | -84dBm |
| Number of NLoS paths | 0, 4 |
| Altitude of MT | 1.2, 1.4 meters |
| Direction of IRSs’ ULA | (0, 1, 0) |
| Size of obstacles | 0.6 × 0.4 × 1.7 meters |
| Number of users | 20, 50, 100 |
| Number of IRSs | 4 |

TABLE II. Simulation Parameters
as \((x, y, z)\), where \(x, y, z\) follow uniform distribution, i.e., 
\(x, y \sim U(-10, 10)\) and \(z \sim U(1.2, 1.4)\). Users are uniformly 
distributed in the lecture hall under the non-overlapping 
constraint. For a typical MT, the other MT holders is its potential 
obstacles, and thus the blockage probability increases with user 
density.

To gain insights into the relationship between user density 
and blockage probability, Fig. 5 is presented where there are 12 
IRSs deployed, which means a total of 13 LoS/VLoS links are 
available. From the Fig. 5 we can see that when the number of 
MTs is 20, more than 50\% of channel realizations experience 
no link blockage, the largest number of blocked links is 4, and 
the percentage of which is less than 5\%; when the number of 
MTs is 50, more than 80\% of channel realizations experience 
less than 3 blocked links, the largest number of blocked links 
is 7, and the percentage of which is less than 1\%; when 
the number of MTs is 100, more than 80\% of channel realizations 
experience less than 5 blocked links, the largest number of 
blocked links is 9, and the percentage of which is almost 
negligible. Note that when there exists at least 1 unblocked 
link, uninterrupted communication over mmWave band can be 
guaranteed, and when there exist at least 3 unblocked links, 
positioning algorithm can be performed to locate MT and 
meanwhile enhance parameter estimation.

B. Performance of Beam Training With Random Beamforming

In Fig. 6, we study the mean squared error (MSE) performance 
of the estimated AoA and AoD, which fundamentally 
determines the accuracy of beam alignment and positioning. 
Cramér-Rao bound (CRB) is adopted as the benchmark. Since 
the estimation of \((\delta, \theta, \phi)\) is part of the joint estimation 
of \((\delta, \theta, \phi)\), CRBs of \(\theta\) and \(\phi\) are obtained as the last two diagonal 
elements of the inverse of Fisher information matrix w.r.t. 
\((\delta, \theta, \phi)\). The detailed derivation of CRB is omitted, as it 
follows the standard procedure. When the training length is 
16, to study the performance limit of joint AoA and AoD 
estimation, we firstly measure the MSE of AoA and AoD 
when the channel is with merely LoS path. As can be seen 
from Fig. 6(a) that, from 0dBm to 6dBm the empirical MSE 
of both AoA and AoD is significantly higher than CRB, and the 
performance gap gradually turns to be constant from 6dBm 
to above. It indicates that, from 0dBm to 6dBm the joint 
estimation of \((\theta, \phi)\) experiences Error Type 2 as mentioned 
in Section IV. C, in which the estimated AoA and AoD pair 
is far apart from their authentic values, and from 6dBm to 
above only Error Type 1 happens, in which the estimation 
error is mild and tightly lower bounded by CR bound. In 
practice, NLoS path’s impacts on beam training cannot be 
overlooked. Therefore, we further carry out the simulation of 
random beamforming based beam training in LoS channel model and LoS + NLoS channel model, 
where NLoS path number is 4, training length is \(N = 8, 16\).

Fig. 5. Blockage rate with different user densities

![Fig. 5](image-url)

Fig. 6. MSE performance of AoA/AoD estimated by random beamforming 
based beam training in LoS channel model and LoS + NLoS channel model, 
where NLoS path number is 4, training length is \(N = 8, 16\).

![Fig. 6](image-url)
is not satisfying in low SNR regimes. Through case analysis, \( N = 4 \log_2 \) hierarchical beam sweeping reduces the training overhead to \( N \) performance is severely degraded within the transmit power 

\[ N \]

\[ N \]

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beamforming. This is because location information is derived by multiple anchors, and AoA/AoD refinement according to geometric relationship means that the estimation is cross verified. It is noteworthy that the performance enhancement is more significant when the training length is \( N = 8 \), from which we find the potential to reduce training length of beam training with the aid of location information. Another notable point is that AoA refined by location information is always worse than AoD refined by location information. This is because the direction vector \( \mathbf{e}_{MT} \) is derived from estimation in (40), while the direction vectors of anchors \( \mathbf{e}_i \) are well known.

### VII. Conclusion

In this paper, beam training for IRSs assisted mmWave communications is studied. By breaking down beam training for IRSs assisted mmWave MIMO into several mathematically equivalent sub-problems, we perform random beamforming and maximum likelihood estimation to derive the optimal beam of BS/AP and MT and the optimal reflection pattern of IRSs. Then, by sorting the reliability of the estimated AoA, we find the potential to reduce training length of beam training with the aid of location information. Another notable point is that AoA refined by location information is always worse than AoD refined by location information. This is because the direction vector \( \mathbf{e}_{MT} \) is derived from estimation in (40), while the direction vectors of anchors \( \mathbf{e}_i \) are well known.

### APPENDIX A

#### PROOF OF THEOREM 1

In the noiseless scenario where \( y = \mathbf{D}b(\theta, \phi) \), according to Cauchy-Schwarz inequality, we have

\[
\frac{\|b^H(\hat{\theta}, \hat{\phi})D^H b(\theta, \phi)\|_2}{\|Db(\theta, \phi)\|_2} \leq \|Db(\theta, \phi)\|_2
\]  

(47)

Then, the proof of Eq. (25) is reduced to prove that

\[
\frac{\|b^H(\hat{\theta}, \hat{\phi})D^H b(\theta, \phi)\|_2}{\|Db(\theta, \phi)\|_2} \neq \|Db(\theta, \phi)\|_2
\]  

(48)

namely \( Db(\theta, \phi) \neq \mu Db(\tilde{\theta}, \tilde{\phi}) \), \( \forall \mu \in \mathbb{C}, \forall (\theta, \phi) \neq (\tilde{\theta}, \tilde{\phi}) \), which is mathematically equivalent to Eq. (24).

### APPENDIX B

#### PROOF OF THEOREM 2

The PEP is written as

\[
P_e(\theta, \phi) \rightarrow (\hat{\theta}, \hat{\phi})
\]

\[
= Pr\left( \frac{\|b^H(\hat{\theta}, \hat{\phi})D^H y\|_2^2}{\|Db(\theta, \phi)\|_2^2} < \frac{\|b^H(\hat{\theta}, \hat{\phi})D^H y\|_2^2}{\|Db(\theta, \phi)\|_2^2} \right)
\]

\[
= Pr\left( -\frac{\|b^H(\hat{\theta}, \hat{\phi})D^H y\|_2^2}{\|Db(\theta, \phi)\|_2^2} + \frac{\|b^H(\hat{\theta}, \hat{\phi})D^H y\|_2^2}{\|Db(\theta, \phi)\|_2^2} - 2Re\left\{ \delta n^H Db(\theta, \phi) \right\}
\]

\[
+ 2Re\left\{ \delta n^H Db(\theta, \phi) \mathbf{H}(\hat{\theta}, \hat{\phi}) D^H \mathbf{H}(\hat{\theta}, \hat{\phi}) Db(\theta, \phi) \right\}
\]

\[
> \|\delta Db(\theta, \phi)\|_2^2 - \frac{\|\delta Db(\hat{\theta}, \hat{\phi})D^H Db(\theta, \phi)\|^2}{\|Db(\theta, \phi)\|_2^2}
\]

\[
\approx Pr\left( N_1 > \|\delta Db(\theta, \phi)\|_2^2 - \frac{\|\delta Db(\hat{\theta}, \hat{\phi})D^H Db(\theta, \phi)\|^2}{\|Db(\theta, \phi)\|_2^2} \right)
\]  

(49)

where

\[
N_1 = 2Re\left\{ -\delta n^H Db(\theta, \phi) + \delta n^H Db(\tilde{\theta}, \tilde{\phi}) \mathbf{H}(\hat{\theta}, \hat{\phi}) D^H Db(\theta, \phi) \right\}
\]

and \( Re\{\} \) is the real part of a complex number. Eq. (49) is obtained by neglecting the component \( -\frac{\|b^H(\theta, \phi)D^H y\|_2^2}{\|Db(\theta, \phi)\|_2^2} \) in high SNR regime. Since \( N_1 \) is a Gaussian random variable, we have

\[
N_1 \sim \mathcal{N}(0, 2\sigma^2 |\delta|^2 \left( \frac{\|Db(\theta, \phi)\|_2^2 - \|b^H(\hat{\theta}, \hat{\phi})D^H Db(\theta, \phi)\|^2}{\|Db(\theta, \phi)\|_2^2} \right))
\]

According to the definition of Q function, Eq. (28) is obtained.
APPENDIX C

PROOF OF PROPOSITION 1

Firstly, we write the expression of $d^2(D_n, \theta, \phi, \tilde{\theta}, \tilde{\phi})$ as

$$d^2(D_n, \theta, \phi, \tilde{\theta}, \tilde{\phi}) = \left\| D_n b(\theta, \phi) \right\|^2 + \left| \frac{b^H(\theta, \phi) D_n b(\theta, \phi)}{\left\| D_n b(\theta, \phi) \right\|^2} \right|^2$$

Thus

$$d^2(D_n, \theta, \phi, \tilde{\theta}, \tilde{\phi}) - d^2(D_n-1, \theta, \phi, \tilde{\theta}, \tilde{\phi}) = b^H(\theta, \phi) d_{n-1}^H b(\theta, \phi) + \frac{b^H(\theta, \phi) D_n b(\theta, \phi)}{b^H(\theta, \phi) D_{n-1} b(\theta, \phi)} - b^H(\theta, \phi) D_{n-1} b(\theta, \phi) - b^H(\theta, \phi) D_{n-1} b(\theta, \phi)$$

For the purpose of conciseness, let

$$a = b^H(\tilde{\theta}, \tilde{\phi}) d_n b(\theta, \phi)$$
$$b = b^H(\tilde{\theta}, \tilde{\phi}) d_n b(\theta, \phi)$$
$$c = b^H(\tilde{\theta}, \tilde{\phi}) D_{n-1} b(\theta, \phi)$$
$$d = b^H(\tilde{\theta}, \tilde{\phi}) D_{n-1} b(\theta, \phi)$$

As $b^H(\tilde{\theta}, \tilde{\phi}) d_n$ and $d_n^H b(\theta, \phi)$ are numbers, rather than vectors, we have

$$b^H(\theta, \phi) d_n^H b(\theta, \phi) = \frac{\left| b^H(\tilde{\theta}, \tilde{\phi}) d_n^H b(\theta, \phi) \right|^2}{\left| b^H(\theta, \phi) d_n^H b(\theta, \phi) \right|^2} = \left\| b^H(\tilde{\theta}, \tilde{\phi}) d_n b(\theta, \phi) \right\|^2$$

Then

$$d^2(D_n, \theta, \phi, \tilde{\theta}, \tilde{\phi}) = \frac{|b|^2 + |d|^2}{\bar{a} + \bar{c}} - \frac{|b + d|^2}{\bar{a} + \bar{c}}$$

Equality holds when $a = b = 0$, namely

$$b^H(\tilde{\theta}, \tilde{\phi}) d_n^H b(\theta, \phi) = b^H(\tilde{\theta}, \tilde{\phi}) D_{n-1} b(\theta, \phi)$$
$$b^H(\theta, \phi) d_n^H b(\theta, \phi) = b^H(\theta, \phi) D_{n-1} b(\theta, \phi)$$

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