Signature inversion in axially deformed $^{160,162}$Tm

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The microscopic analysis of experimental data in $^{160,162}$Tm is presented within the two-quasiparticle-phonon model. The model includes the interaction between odd quasiparticles and their coupling with core vibrations. The coupling explains naturally the attenuation of the Coriolis interaction in rotating odd-odd nuclei. It is shown that the competition between the Coriolis and neutron-proton interactions is responsible for the signature inversion phenomenon.

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Progress in the development of large arrays of $\gamma$-detectors led to observations of various phenomena in rotating nuclei. In many cases rotational states can be characterised by the signature quantum number $r = e^{-i\alpha \pi}$ which defines the admissible spin sequence for a rotational band according to the relation $I = \alpha + 2n (n = 0, 1, \ldots)$. It is associated with the $D_2$ spatial symmetry of non-rotational degrees of freedom of a nuclear system [1]. In particular, in experimental data we observe two signature partner bands with $r = +1 (\alpha = 0)$ and $r = -1 (\alpha = 1)$ in even-mass nuclei and $r = \pm 1 (\alpha = \pm \frac{1}{2})$ in odd-mass nuclei ($\alpha = -\frac{1}{2} \equiv \frac{3}{2}$), which are separated by a signature splitting energy. The band, which is lower in energy, is called the favored band. It is expected that the signature splitting should increase with a growing of the angular momentum $I$. However, for some nuclei the signature splitting decreases with spin and, furthermore, the unfavoured band becomes lower in energy.
than the favoured one. This phenomenon is called the signature inversion and attracts the experimental and theoretical attention, since it is not completely understood.

For example, the low-spin signature inversion of the $\pi(h_{11/2}) \otimes \nu(i_{13/2})$ band has been studied systematically in odd-odd nuclei of the $A \sim 160$ region \cite{2}. The analysis within the Cranked Shell Model (CSM) \cite{3} suggests that the signature inversion could be only observed in the region of $62 < Z < 70$ and the phenomenon is a consequence of the triaxiality. However, these predictions are not consistent with subsequent observations of the low-spin signature inversion in nuclei with $Z = 71, 73$ \cite{2}. The signature inversion can be seen also in bands different from $\pi(h_{11/2}) \otimes \nu(i_{13/2})$. Since the density of neutron-proton (n-p) two-quasiparticle configurations is large in the low-lying part of the spectrum, one expects their strong mixing via residual interactions of different nature. It was proposed that the competition between the Coriolis interaction (CI) and the n-p interaction of an odd neutron and an odd proton may be responsible for the signature inversion \cite{4,5}. On the other hand, the coupling of odd quasiparticles with vibrational excitations of an even–even core could be important as well. Vibrational admixtures are essential elements for the description of energy spectra and transition probabilities in odd-odd nuclei and it was demonstrated in numerous calculations in the microscopic quasiparticle-plus-phonon model \cite{6,7}. We suggest that the interplay between the coupling of external nucleons with core vibrations, the n–p interaction between external nucleons, and the Coriolis interaction create an important mechanism of the signature inversion phenomenon. To this aim, we use the microscopic two-quasiparticle+phonon+rotor model. To distinguish between the considered mechanism and the one created by the triaxiality, the analysis has been done for two isotopes $^{160,162}Tm$. Using the self-consistent cranking Hartee-Fock approach with the Skyrme III interaction (HF+Skyrme) \cite{8}, we obtained very small equilibrium $\gamma$ deformations ($\gamma \sim 4^\circ$) for these nuclei in the considered region of the angular momentum $I \leq 28\hbar$. In addition, the equilibrium axial quadrupole deformation is quite stable: the quadrupole momentum $Q_{20}$ changes from 1314 (1399) $fm^2$ to 1268 (1375) $fm^2$ in $^{160}Tm(162Tm)$ in the range of values of the rotational frequency 0.1-0.25 MeV.
The low-lying states in odd-odd deformed nuclei can be described within the adiabatic approximation of a separation of intrinsic, non-rotational degrees of freedom and rotational ones, i.e., with the Hamiltonian $H = H_{\text{rot}} + H_{\text{intr}}$. These states were extensively investigated theoretically for a long time \[7\], however, the main attention was paid to the Gallagher-Moszkowski (GM) splitting \[9\] and the Newby shift \[10\]. Since the $\gamma$-deformation is negligible for considered nuclei, we use the Hamiltonian of the axially symmetric rotor model \[1\]

$$H_{\text{rot}} = \frac{\hbar^2}{2J} \left[ (I^2 - \hat{I}_3^2) - (\hat{I}_+ \hat{j}_- + \hat{I}_- \hat{j}_+) + \frac{1}{2}(\hat{j}_+ \hat{j}_- + \hat{j}_- \hat{j}_+) \right]$$

(1)

where the first term is a pure rotational term, the second term represents the Coriolis interaction and the last one is the centrifugal interaction. Here $J$ is the moment of inertia of an odd-odd nucleus, $\hat{I}_3$ is the projection of the angular momentum ($\vec{I}$) on the symmetry axis, $\hat{I}_\pm = \hat{I}_1 \pm i \hat{I}_2$, and $\hat{j}_\pm = \hat{j}_1 \pm i \hat{j}_2$. The intrinsic angular momentum $\vec{j} = \vec{j}_n + \vec{j}_p$ is a vector coupling of single-particle angular momenta of an odd neutron and odd proton. Angular momentum is a good quantum number and the model has the advantage in comparison with the CSM at low spin region for deformed nuclei. The intrinsic part $H_{\text{intr}}$ consists of an axially deformed mean field $H_{\text{sp}}$, a short-range residual interaction $H_{\text{pair}}$ (a monopole pairing), the n-p interaction $H_{\text{np}}$ between an odd neutron and an odd proton, and of a long-range residual interaction, $H_{\text{res}}$, taken in the form of the iso-scalar and iso-vector multipole decomposition

$$H_{\text{res}} = -1/2 \sum_{\lambda \mu \geq 0} \tau \tau' (\kappa_0^{(\lambda \mu)} + \tau' \kappa_1^{(\lambda \mu)}) Q_{\lambda \mu}^{(\tau)} Q_{\lambda' \mu}^{(\tau')} .$$

(2)

Here $Q_{\lambda \mu}^{(\tau)}$ is a symmetrised multipole operator with a multipolarity $\lambda$ and projection $\mu$. The index $\tau = -1$ and $+1$ corresponds to neutron and proton systems, respectively. Using the Bogoliubov transformation from single-particle operators ($a_\nu, a_\nu^\dagger$) to quasiparticle ones ($\alpha_\nu, \alpha_\nu^\dagger$) and the random phase approximation (RPA), the intrinsic Hamiltonian $H_{\text{intr}}$ can be represented in the form $H_{\text{intr}} = H_{\text{core}} + H_{nO} + H_{pO} + H_{np}$. The term $H_{\text{core}}$ generates quasiparticle and phonon (vibrational) excitations of an even–even core, $H_{nO}(H_{pO})$ describes the coupling of odd neutron (proton) quasiparticles with the core vibrations. Ex-
plicit expressions for all terms involved in $H_{\text{intr}}$ are given in [7]. The eigenvalue problem of the full Hamiltonian can be solved in the basis of the symmetrised wave functions $|I^\pi MK\varrho\rangle \sim (D^I_{MK} + (-1)^I D^I_{M-K}\hat{R}_1)|\psi_\varrho(K^\pi)\rangle$ [8]; $\varrho$ is the additional quantum number characterising the intrinsic state. The intrinsic wave function $|\psi_\varrho(K^\pi)\rangle$ corresponds to the intrinsic energy $\eta_\varrho K$, i.e., $H_{\text{intr}}|\psi_\varrho(K^\pi)\rangle = \eta_\varrho K|\psi_\varrho(K^\pi)\rangle$. For $K^\pi = 0^\pm$ the function $|I^\pi MK\varrho\rangle$ is the eigenvector of the signature operator $\hat{R}_1 = \exp(-i\pi\hat{J}_1)$. Consequently, the rotational band with $K^\pi = 0^\pm$ splits into the band with positive signature states ($\alpha = 0$) and even values of $I$ and the band with negative signature states ($\alpha = 1$) and odd values of $I$. First, we solve the RPA equations to determine structure and energies of two-quasiparticle phonons $O^\dagger_{\lambda\mu}$ describing the low-lying vibrational states of an even-even core. Second, we solve the variational problem for the intrinsic Hamiltonian $H_{\text{intr}}$. As a result, we obtain the amplitudes $C^\varrho_{\nu_n\nu_p}$ of neutron-proton two-quasiparticle components and the amplitudes $D^\varrho_{\lambda\mu\nu_n\nu_p}$ of the coupling of two-quasiparticle components with core vibrations in the intrinsic wave function $|\psi_\varrho(K^\pi)\rangle$:

$$|\psi_\varrho(K^\pi)\rangle = \left( \sum_{\nu_n\nu_p} C^\varrho_{\nu_n\nu_p}\alpha^\dagger_{\nu_n}\alpha^\dagger_{\nu_p} + \sum_{\nu_n\nu_p\lambda\mu} D^\varrho_{\lambda\mu\nu_n\nu_p}\alpha^\dagger_{\nu_n}\alpha^\dagger_{\nu_p} O^\dagger_{\lambda\mu} \right)$$

(3)

Finally, we diagonalise the full Hamiltonian $H$ in which the intrinsic and rotational terms are coupled by the Coriolis interaction.

We remind the reader that in odd–odd nuclei one of the two-quasiparticle components $\alpha^\dagger_{\varrho_n}\alpha^\dagger_{\varrho_p}|\rangle$ with the corresponding quantum number $K = |K_{\varrho_n} \pm K_{\varrho_p}|$ dominates in low-lying intrinsic states $|\psi_\varrho(K)\rangle$ [7]. Two intrinsic states with $K_1 = K_{\varrho_n} + K_{\varrho_p}$ and $K_2 = |K_{\varrho_n} - K_{\varrho_p}|$ are similar in structure (which means that amplitudes $C^\varrho_{\nu_n\nu_p}$ and $D^\varrho_{\lambda\mu\nu_n\nu_p}$ are similar) and they form the well-known GM doublet with the corresponding GM splitting energy $\Delta E^{(GM)}_{\varrho=\varrho_n,\varrho_p} = \eta_\varrho K = K_{\varrho_n} + K_{\varrho_p} - \eta_\varrho K = |K_{\varrho_n} - K_{\varrho_p}|$. Moreover, for the case of $K^\pi = 0^\pm$, one can define the Newby shift $\Delta E^{(N)}_{\varrho K=0} = \eta_\varrho K = |K_{\varrho N} = 0| - \eta_\varrho K = |K_{\varrho N} = 0^\pm|$ that determines the energy shift between two rotational bands with the same internal structure but with different quantum numbers $\alpha$. In other words, the Newby shift is the signature splitting energy for the signature partners with $K^\pi = 0^\pm$ at the beginning of the signature splitting.
As discussed above, we estimated the equilibrium deformation in the HF+Skyrme approach. To solve the RPA equations with density dependent forces for rotating nuclei, especially for the odd-odd system, is quite difficult. This problem is still in its infancy and it needs a dedicated study. To carry out the numerical analysis for $^{160,162}$Tm, the single-particle mean field $H_{sp}$ is approximated by the Nilsson Hamiltonian with the parameters taken from [11]. All shells up to $N = 7$ are included for neutrons and protons, respectively. The deformation parameters which are similar to the HF+Skyrme estimations, the neutron and proton pairing gaps for ground states obtained with the use of the Strutinsky method are taken from [12]. According to the experimental systematics [13], in the vicinity of the proton and neutron Fermi levels for the nuclei with $Z = 69, 71$($N = 90, 92$) and with $N = 91, 93$($Z = 68, 70$) there are the following sequences of single-particle states increasing with energy:

**protons**: ...$3/2[411], 7/2[523], 1/2[411], 7/2[404], 1/2[541], 5/2[402], 9/2[514], ...$

**neutrons**: ...$1/2[660], 1/2[400], 1/2[530], 3/2[532], 3/2[402], 3/2[651], 3/2[521], 5/2[642], 5/2[523], 11/2[505], ...$

We reproduce these sequences and the mean field Fermi levels for the $^{160,162}$Tm are $1/2[411]$ and $3/2[521]$ for protons and neutrons, respectively. The BCS approximation is used to fix the number of protons and neutrons. The quadrupole and octupole multipoles have been taken as a residual long-range interaction of the core. The corresponding strength constants $\kappa^{2\mu}, \kappa^{3\mu}$ are fitted in order to reproduce experimental quadrupole and octupole one-phonon energies of the $^{158}$Er (the core for $^{160}$Tm) and $^{160}$Er (the core for $^{162}$Tm). The residual n-p interaction $H_{np}$ is taken in the form of $\delta$-force, i.e. $H_{np} = \delta(\vec{r}_p - \vec{r}_n)(u_0 + u_1 \vec{\sigma}_p \vec{\sigma}_n)$. The parameters $u_0 = -3.504$ MeV and $u_1 = -0.876$ MeV have been chosen from the systematics of the GM splitting and Newby shifts for rare-earth nuclei [4,13]. All solutions of the RPA equations up to 1 MeV have been taken into account at the diagonalisation procedure of the full Hamiltonian $H$ for each angular momentum and parity quantum numbers.
The signature inversion in both nuclei is observed in the negative parity bands. To describe the experimental data in $^{160}$Tm we included six negative parity bands in the diagonalisation procedure. The inertial parameters $\hbar^2/2J$ (see Eq.(1)) of each band have been considered as variational parameters. The optimal inertial parameter value is 11.2 keV for the ground rotational band and we used $\hbar^2/2J \sim 11$ keV for all other bands. The calculations reproduce surprisingly well the experimental signature inversion at $I \sim 18\hbar$ (see Fig.1a) in the negative parity band. This band starts at $I = 8\hbar$ as a band built on the intrinsic state with the largest p-n two-quasiparticle state $K^{\pi} = 0^- (\pi 1/2[411] \otimes \nu 1/2[530])$. The structure of this intrinsic state is following: $(\pi 1/2[411] \otimes \nu 3/2[532]) \otimes Q_{22}^+(17\%), ((\pi 3/2[411] \otimes \nu 1/2[530]) \otimes Q_{22}^+(4\%), ((\pi 5/2[413] \otimes \nu 1/2[521]) \otimes Q_{22}^+(3\%), ((\pi 1/2[411] \otimes \nu 5/2[532]) \otimes Q_{22}^+(1\%)). The position of the signature inversion point depends on the relative values of the Newby shift and the strength of the CI between this band and its GM partner band built on the $K^{\pi} = 1^- (\pi 1/2[411] \otimes \nu 1/2[530])$ state. If we are limited by the independent quasiparticle approach, it is necessary to introduce the attenuation factor for the Coriolis interaction to reproduce the experimental data in the vicinity of the inversion point. Thanks to collective phonon components in the intrinsic wave functions, Eq.(3), the strength of the Coriolis interaction is reducing upon $\sim 25\%$. Consequently, the phonon components fix the strength of the CI self-consistently. The exact position of the signature inversion point can be obtained by the fitting of the value of the Newby shift, $\Delta E^{(N)}$. The calculated value $\Delta E^{(N)} = 120$ keV is closed to the optimal value $\Delta E^{(N)} = 135$ keV of the Newby shift for this band.

Six negative parity bands are involved in the CI mixing calculations for $^{162}$Tm. The optimal values for the inertial parameters are similar to the ones of the $^{162}$Tm. The calculated energy difference $[E(I) - E(I - 1)]/2I$ for negative parity yrast states is compared with the experimental data in Fig.1b. The low spin region of the negative parity yrast rotational band (up to the spin $I \sim 6\hbar$) is built on the intrinsic state with the largest two-quasiparticle component $K^{\pi} = 1^- (\pi 1/2[411] \otimes \nu 3/2[521])$. In the region $I > 6\hbar$ (up to the spin $I \sim 28\hbar$ where we stopped our calculations) the yrast band is built on the $K^{\pi} = 0^- (\pi 1/2[411] \otimes \nu 1/2[530])$. 
state. Again, the vibrational admixtures decrease naturally the strength of the Coriolis interaction. The experimental signature inversion point at $I \sim 16\hbar$ is well reproduced in our calculations. The calculated Newby shift $\Delta E^{(N)}$ is $\sim 100$ keV which is slightly larger than the fitted Newby shift $\Delta E^{(N)} = 88$ keV. It seems that the adiabatic approximation and our configuration space constitute a reasonable approach even at high spins. However, it should be necessary to compare the results with estimations within the cranking Hartree-Fock-Bogoliubov+RPA approach to make a final conclusion. Since the main question is the underlying mechanism of the signature inversion, we believe that the model reproduces well enough the important features of the phenomenon.

Let us discuss in detail the physical mechanism of the signature inversion in both nuclei. The largest component, $K^\pi = 0^-(\pi 1/2[411] \otimes \nu 1/2[530])$, of the favoured band contains the proton quasiparticle state from the $d_{3/2}$ sub-shell and the neutron quasiparticle state from the $h_{9/2}$ sub-shell. While the sign of the projections of the proton $\vec{s}_p$ and neutron $\vec{s}_n$ spins onto the symmetry axes (z-axis) is the same, the sign of the z-projection of the proton intrinsic angular momentum $\vec{j}_p$ is opposite to the sign of the one of the neutron intrinsic angular momentum $\vec{j}_n$. The opposite situation holds for the corresponding GM partner $K^\pi = 1^-(\pi 1/2[411] \otimes \nu 1/2[530])$: the sign of the z-projections of the neutron and proton angular momenta is the same, while the z-projections of the neutron and proton spins have the opposite signs. The configuration $K^\pi = 0^-(\pi 1/2[411] \otimes \nu 1/2[530])$ with the same sign of the z-projections of the intrinsic angular momenta $\vec{j}_n$ and $\vec{j}_p$ has the lowest excitation energy due to the n-p interaction and this is consistent with the empirical GM rule. The CI has a tendency to lower the energy of the n-p state where the intrinsic angular momenta $\vec{j}_n = \vec{l}_n + \vec{s}_n$ and $\vec{j}_p = \vec{l}_p + \vec{s}_p$ are aligned in parallel ($K^\pi = 1^-$ state). Therefore, when the lower partner of the GM splitting has the same sign of the z-projections of the neutron $\vec{s}_n$ and proton $\vec{s}_p$ spins but the signs of the z-projections of the intrinsic angular momenta $\vec{j}_n$ and $\vec{j}_p$ are opposite, the competition between the n-p interaction and the CI could lead to the crossing of the GM partners and, consequently, to the signature inversion for a particular spin.
This mechanism can be illustrated in a simple two-level model. The matrix of the full Hamiltonian for the GM doublet ($K^\pi = 0^-$ and $K^\pi = 1^-$ bands) coupled by the CI (expressions for the matrix elements are given in [15]) can be written as

$$
\begin{pmatrix}
A + \frac{h^2}{2J} I(I+1) - (-1)^I a & \frac{h^2}{2J} [c - (-1)^I b] \sqrt{I(I+1)} \\
\frac{h^2}{2J} [c - (-1)^I b] \sqrt{I(I+1)} & B + \frac{h^2}{2J} [I(I+1) - 1]
\end{pmatrix}
$$

(4)

The term in the upper left corner of the matrix corresponds to the unperturbed GM partner with the lowest energy ($K^\pi = 0^-$ band). The term in the lower right corner corresponds to the unperturbed GM partner with a highest energy ($K^\pi = 1^-$ band). In Eq.(4), we use the following notation: $A = -\Delta E^{(GM)}/2$, $B = \varepsilon_1 + \Delta E^{(GM)}/2$, $\Delta E^{(GM)}$ is the GM splitting, $a$ includes a contribution of the n-p interaction (a half of the Newby shift, $\Delta E^{(N)}/2$), and the centrifugal interaction which is smaller than $\Delta E^{(N)}/2$. The quantities $|b| < |c|$ are generated by the CI. The term $\varepsilon_1$ is the energy difference between the contributions of the centrifugal interaction in the non-perturbed $K^\pi = 0^-$ and $K^\pi = 1^-$ bands. For small $I$, the n-p interaction is dominant and the CI can be neglected. Consequently, the matrix (4) has diagonal non-zero matrix elements only, and we obtain for yrast states

$$
\Delta E(I) = E(I) - E(I-1) \approx \begin{cases} 
\frac{h^2}{2J} I - 2a & \text{for } I \text{ even,} \\
\frac{h^2}{2J} I + 2a & \text{for } I \text{ odd.}
\end{cases}
$$

(5)

For large $I$, the CI is dominant, the n-p interaction and the centrifugal interaction can be neglected, and we have

$$
\Delta E(I) = E(I) - E(I-1) \approx \begin{cases} 
\frac{h^2}{2J} I + b & \text{for } I \text{ even,} \\
\frac{h^2}{2J} I - b & \text{for } I \text{ odd.}
\end{cases}
$$

(6)

for yrast states. From Eqs.(5) and (6) it follows that if $a > 0$, $b > 0$, ($a < 0$, $b < 0$), the yrast band consists of the states with even (odd) values of the angular momentum in the low-spin region, while the odd (even) spin states form the yrast band in the high-spin region. Signs of $a$, $b$, $c$ are determined by the signs of the intrinsic matrix elements of the n-p interaction, the Coriolis interaction, and the centrifugal interaction. Consequently, the result depends on the structure of the intrinsic wave functions, Eq.(3), for both $K^\pi = 0^-$ and $K^\pi = 1^-$ bands.
In conclusion, the signature inversion in negative parity bands of $^{160,162}\text{Tm}$ is described in the microscopic model which includes the coupling of quasiparticle excitations with core vibrations. Vibrational components of rotational states lead to the attenuation of the Coriolis interaction which is crucial for a correct description of the signature inversion point. The competition between the Coriolis interaction and the neutron-proton interaction between odd quasiparticles explains the mechanism of the signature inversion in $\pi(d_{3/2}) \otimes \nu(d_{9/2})$ band of odd-odd axially deformed rotating nuclei. It is different, for example, from the signature inversion mechanism in the negative parity band in $^{72}\text{Br}$, which is due to the onset of the triaxiality [10]. These two mechanisms complement each other and, in principle, should be included as main ingredients of the model of the signature inversion. The measurement of electromagnetic transitions could provide a more detailed understanding of the contribution of phonon components in the structure of excited states of odd-odd nuclei and their role in the signature inversion phenomenon.

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[1] A. Bohr and B.R. Mottelson, *Nuclear Structure*, Vol.2 (Benjamin, New York, 1975).

[2] Y.Liu, Y.Ma, H.Yang, S.Zhou, Phys. Rev.C 52, 2514 (1995).

[3] R.Bengtsson, H Frisk, F.R.May, J.A.Pinston, Nucl.Phys A415,189 (1984).

[4] P.B. Semmes and I. Ragnarsson, Proc. Int. Conf.on High -Spin Physics and Gamma-Soft Nuclei, Pittsburgh, 1990 (World Scientific, Singapore, 1991), p.500.

[5] L.L.Riedinger, H.Q.Jin, W.Reviol, J.-Y.Zhang, R.A.Bark, G.B.Hagemann, P.B.Semmes, Prog.Part.Nucl.Phys. 38, 251 (1997).
[6] V.G. Soloviev, *Theory of Complex Nuclei* (Nauka, Moscow; translation Pergamon, Oxford, 1976).

[7] A.K. Jain, R.K. Sheline, D.M. Headly, P.C. Sood, D.G. Burke, I. Hrivnáčová, J. Kvasil, D. Nosek, R.W. Hoff, Rev. Mod. Phys. **70**, 843 (1998).

[8] J. Dobaczewski and J. Dudek, Comp. Phys. Comm. **102**, 166 (1997); **102**, 183 (1997).

[9] G.J. Gallagher and S.A. Moszkowski, Phys. Rev. **111**, 1282 (1958).

[10] N.D. Newby, Phys. Rev. **125**, 2063 (1962).

[11] A.K. Jain, R.K. Sheline, P.C. Sood, and K. Jain, Rev. Mod. Phys. **62**, 303 (1990).

[12] [http://t2.lanl.gov](http://t2.lanl.gov).

[13] [http://www.nndc.bnl.gov](http://www.nndc.bnl.gov).

[14] J.P. Boisson, R. Piepenbring, W. Ogle, Phys. Rep. **26**, 99 (1976).

[15] A.K. Jain, J. Kvasil, R.K. Sheline, R.W. Hoff, Phys. Rev. **C40**, 432 (1989).

[16] C. Plettner, I. Ragnarsson, H. Schnare, R. Schwengner, L. Käubler, F. Dönaub, A. Algora, G. de Angelis, D.R. Napoli, A. Gadea, J. Eberth, T. Steinhardt, O. Thelen, M. Hausmann, A. Müller, A. Jungclaus, K.P. Lieb, D.G. Jenkins, R. Wadsworth, and A.N. Wilson, Phys. Rev. Lett. **85**, 2454 (2000).

**Figure Caption**

Fig.1 The energy difference \((E(I) - E(I - 1))/2I\) (keV/\(\hbar\)) vs \(I(\hbar)\) for the negative parity yrast bands in the odd-odd nuclei: (a) \(^{160}\text{Tm}\), (b) \(^{162}\text{Tm}\). The signature inversion points are shown by arrows. Full triangles correspond to the result of calculations, empty triangles correspond to the experimental data from [2][3]. Lines connecting symbols are used to guide the eye.
\[
\frac{E(I) - E(I-1)}{2I} \text{ (keV/\(\hbar\))}
\]

(a) \(^{160}\text{Tm}\)

(b) \(^{162}\text{Tm}\)