How does nonequilibrium activity modify the approach to a glass? This is an important question, since many experiments reveal the near-glassy nature of the cell interior, remodeled by activity. However, different simulations of dense assemblies of active particles, parametrized by a self-propulsion force, \( f_0 \), and persistence time, \( \tau_p \), appear to make contradictory predictions about the influence of activity on characteristic features of glass, such as fragility. This calls for a broad conceptual framework to understand active glasses; here, we extend the random first-order transition (RFOT) theory to a dense assembly of self-propelled particles. We compute the active contribution to the configurational entropy through an effective model of a single particle in a caging potential. This simple active extension of RFOT provides excellent quantitative fits to existing simulation results. We find that whereas \( f_0 \) always inhibits glassiness, the effect of \( \tau_p \) is more subtle and depends on the microscopic details of activity. In doing so, the theory automatically resolves the apparent contradiction between the simulation models. The theory also makes several testable predictions, which we verify by both existing and new simulation data, and should be viewed as a step toward a more rigorous analytical treatment of active glass.

### Active Systems

Active systems, consisting of particles that convert energy supplied to it into mechanical work, are a fascinating class of driven nonequilibrium systems (1, 2). The range of systems that fall under this ambit include living systems, from cells and their motor-cytoskeletal extracts to collections of cells constituting tissues, which utilize chemical energy to perform biological function (3), and synthetic systems, such as magnetic colloidal beads (4), light-activated colloidal swimmers (5), and vertically vibrated grains (6, 7). So far, theoretical and experimental studies have focused on dilute collections of active particles, however the situation in cells and tissues lie at the other extreme—for instance, the material within a cell and a collection of confluent cells within a tissue are both at high densities and require activity to facilitate either molecular rearrangements (8) or collective cell movement (9–14); thus, it is of interest to study the effect of activity in systems where the dynamics are slow and multiparticle correlations are significant and that approach a collective jammed or glassy state.

Motivated by this, there have been a number of simulation models (15–28) of dense assemblies of stochastically driven particles, parametrized by a self-propulsion force of magnitude \( f_0 \) and an orientational persistence time \( \tau_p \). Each of these models characterizes the statistics of activity in a slightly different way, however their predictions on the variation of signature glassy behavior, such as fragility, on the extent of activity have been divergent. Unlike in the case of dilute collections of active particles, there is no analytically calculable framework to understand the dynamics of active particles in the dense limit. There have been attempts to extend the usual mode-coupling theory (MCT) to include active self-propulsion (15, 16, 25, 29), however such theories are not applicable to the glassy dynamics at high densities where relaxation typically occurs via activated processes (30, 31).

The random first-order transition (RFOT) theory (32–36) has been remarkably successful (37–39) in describing a glassy system both above and below the regime where activated processes dominate the dynamics. RFOT theory generalizes the theory of first-order crystallization transition to that of freezing to a disordered structure (34, 36), describing the system in terms of a mosaic of local aperiodic domains (SI Appendix, Fig. S1) and a mismatch energy at the interface between domains (37–39). Including activity within this basic RFOT picture presents a major challenge (SI Appendix). Here we propose an extension of the RFOT theory to an active system, which should be viewed as a first step toward a more rigorous analytical treatment of active glass. Even so, the theory presented here makes several testable predictions, regarding how the active parameters affect glassy behavior: Whereas \( f_0 \) always inhibits glassiness, the effect of \( \tau_p \) is more subtle and depends on the microscopic details of activity. This not only helps reconcile the apparently divergent results of the simulation models discussed above but also makes predictions that we verify with numerical simulations (Materials and Methods).

### Significance

Understanding how activity affects the physics of dense glassy suspensions is of fundamental interest in a variety of cellular and tissue contexts. However, current simulations of dense active systems make qualitatively different predictions about the influence of activity on characteristic features of glass, such as fragility. Acknowledging the need for a broad theoretical framework, we extend random first-order transition (RFOT) theory to active glasses, based on an effective independent-particle treatment. We find that the analytic model predicts that the behavior of the active glass is strongly influenced by the microscopic details of activity. This not only resolves the apparent contradictions posed by previous studies but provides a number of testable predictions, some of which we verify using numerical simulations.

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Active RFOT: General Formalism

The RFOT theory for passive glasses describes the activated reconfigurations of such glasses through a mechanism similar to nucleation (32–34, 37, 40). Consider a domain of volume \( v \) and length scale \( R \approx v^{1/3} \), in \( d \) dimension (SI Appendix, Fig. S1). The RFOT theory posits that the driving force for reconfiguration of the mosaics is the configurational entropy density, \( s_c(\Phi, T) \), which is a function of the potential energy density \( \Phi \) at temperature \( T \) (36, 41). We can write the free-energy gain of the system due to such reconfiguration as

\[
\Delta F(R) = -\Omega_d R^d T s_c(\Phi, T) + S_d R^d \tilde{\gamma},
\]

where \( \Omega_d \) is the volume and \( S_d \) the surface of a region with unit radius in dimension \( d \) and \( \tilde{\gamma} \) is the surface mismatch energy density. The driving force responsible for the reconfiguration of a domain of size \( R \) is obtained by differentiating Eq. 1 with respect to \( R \). The critical nucleation size \( \ell^* \) is determined by balance. If the cavity size \( R > \ell^* \), then the volume contribution drives the reconfiguration of the mosaic. On the other hand, if \( R < \ell^* \), the surface contribution dominates and the state within the cavity remains the same. Thus, the system consists of mosaics of correlated domains with a typical length-scale \( \ell^* \). Note that this length-scale may be different from dynamic length-scales associated with domains of correlated motion (11, 42, 43).

In extending the conventional RFOT theory to active systems that are manifestly out of equilibrium, we must first find a physically reasonable definition of the configurational entropy density \( s_c \) for such systems (SI Appendix). The configurational entropy of passive thermal systems is usually defined in terms of the multiplicity of the local minima of the potential energy (“inherent structures”) whose basins of interaction are visited by the system at a particular temperature \( T \). We continue to use the same definition of the configurational entropy for active systems. Particle configurations in a steady state of active systems can be used as starting points of a minimization procedure to locate a set of inherent structures characteristic to the steady state being considered. The multiplicity of inherent structures obtained this way would be different from that of the inherent structures obtained for an identical system without activity at the same temperature.

Therefore, the temperature dependence of the configurational entropy (defined as the fragility of an active system) would be different from that for its passive counterpart.

To proceed further, we need to evaluate the active corrections to the configurational entropy, which we do within a Langevin dynamics of active particles (see SI Appendix for details). Since the typical persistence time of the active force is larger than the time scale of thermal fluctuations, the effects of activity, to leading order, can be accommodated in a renormalized potential energy density, \( \Phi = \Phi + \delta \Phi \), whose form \( \delta \Phi = -N^{-1} \sum_i (\mathbf{f}_i \cdot \mathbf{x}_i) \) depends on the precise model of activity (44, 45). Here, \( \mathbf{x}_i \) is the position vector of particle \( i \), \( \mathbf{f}_i \) is the active force parametrized by a self-propulsion force of magnitude \( \mathbf{f}_i \) and an orientational persistence time \( \tau_\alpha \), and \( \langle \cdot \cdot \cdot \rangle \) denotes an average over a steady-state distribution of the particle configurations (SI Appendix).

Expanding the active configurational entropy about its value for the passive system, we get

\[
s_c(\Phi) = \left. s_c(\Phi) + \delta \Phi \frac{\partial s_c(\Phi)}{\partial \Phi} \right|_{\Phi=\Phi_0} + \ldots,
\]

where the leading correction involves the derivative of the configurational entropy evaluated at zero active force \( \mathbf{f}_i \). We write this correction term as \( \kappa_\alpha \delta \Phi \), where we have introduced an active fragility parameter, \( \kappa_\alpha \), that quantifies the sensitivity of the configurational entropy to changes in active force. We assume, for simplicity, that \( \kappa_\alpha \) does not depend on the temperature. In addition, to arrive at an active extension of RFOT, we must also evaluate the active surface contribution. Here we simply take it to be \( \gamma(\mathbf{f}_0, \tau_\gamma, T) \) (Eq. 1).

Since the Kauzmann temperature \( T_K \) is defined as the temperature where the configurational entropy \( s_c(\Phi, T) \) of the passive system vanishes (46), we take, following the prescription in refs. 34 and 36, \( s_c(\Phi, T) = \Delta C_p (T - T_K) / T_K \) close to but above \( T_K \), where \( \Delta C_p \) is the jump in specific heat from the liquid to the crystalline state (46). Further, we assume, as in the passive glass (47), that the temperature dependence of \( \gamma \) is linear: \( \gamma = \xi(T - T_K) \), where \( \xi \) is a function of the active parameters alone. As in the passive glass, the detailed temperature dependence of \( \gamma \) does not change the qualitative results because the change in configurational entropy plays the major role in governing the dynamics (47).

Defining \( D \equiv c_\gamma \xi T_K / \Delta C_p \), where \( c_\gamma = (\delta s_c / \delta \mathbf{f}_0, d \mathbf{f}_0 / d T) \), we obtain, close to \( T_K \),

\[
\ell^* = \left[ \frac{D}{(T - T_K) + \frac{T_K c_\gamma \kappa_\alpha \delta \Phi}{\Delta C_p}} \right]^{1/(d - \theta)}.
\]

This expression is similar in spirit to what Wang and Wolynes proposed for active network materials (48). Note that while in principle, \( D \) depends on the active parameters, \( \mathbf{f}_0, \tau_\gamma \), for simplicity we will consider it to be independent of the activity.

The relaxation dynamics of the system is characterized by the reconfiguration of a domain of length-scale \( \ell^* \). The typical potential energy barrier height for the relaxation of the system is \( \Delta(\ell^*) \sim \ell^* \xi \), where \( \psi \) is a scaling exponent. Then the relaxation time of the system is \( \tau = \tau_0 \exp \left( \frac{\Delta \mathbf{f}_0 \ell^* \xi}{T} \right) \), where \( \Delta \mathbf{f}_0 \) is an energy scale and \( \tau_0 \) a microscopic time scale that depend on \( T \), interparticle interactions, and the active parameters. If, following refs. 33, 36, we take \( \psi = \theta = d/2 \), and assuming \( \Delta \mathbf{f}_0 = \kappa \mathbf{f}_0 \), we obtain, close to \( T_K \),

\[
\ln \left( \frac{\tau}{\tau_0} \right) = \frac{E}{(T - T_K) + \frac{T_K \kappa_\alpha \delta \Phi}{\Delta C_p}}.
\]

where \( E = \kappa D = c_\gamma \xi T_K / \Delta C_p \). The alternative suggestion (42), \( \theta = d - 1, \psi = 1 \), would change the expression for \( \ell^* \) but lead to the same expression for \( \tau \). Therefore, within this active extension of the RFOT, all one needs is the activity correction \( \kappa_\alpha \delta \Phi \) to the configurational entropy \( \kappa \kappa_\alpha \delta \Phi \). The sign of this correction is important because it tells us whether the presence of activity increases or decreases the configurational entropy. We have carried out test simulations (see SI Appendix for details) to check how the average energy of inherent structures in the model of active glass considered in ref. 19 depends on the strength of activity \( \mathbf{f}_0 \) (SI Appendix, Fig. S2). We find that the average energy of inherent structures increases as \( \mathbf{f}_0 \) is increased at a fixed temperature. This observation implies that the configurational entropy also increases with \( \mathbf{f}_0 \)—that is, the sign of the activity correction is positive. This conclusion is consistent with the results of recent work by Preisler and Dijkstra (49). A positive entropy correction implies that the temperature at which \( \tau \) diverges for an active system is lower than \( T_K \)—that is, increasing the active force decreases the glass transition temperature.

Active RFOT: Application to Self-Propelled Particle Models

We will now explicitly evaluate the active correction in Eq. 2, for a system of self-propelled particles (1, 2, 17, 19), described by the following overdamped dynamics:

\[
\partial_t x_i = -\mu \nabla \phi_i + \mathbf{f}_i(t) + \mathbf{f}_i(t),
\]

where \( x_i \) is the position of the \( i \)th particle, \( \mu \) is the particle mobility, \( \phi_i = \sum_{j \neq i} \phi(\mathbf{x}_i, \mathbf{x}_j) \) is the instantaneous many-body potential experienced by particle \( i \), and \( \mathbf{f}_i(t) \) and \( \mathbf{f}_i(t) \) are the active and
thermal noises, respectively. To calculate the active correction to the configurational entropy (SI Appendix), we use an approximate theory where we replace the many-particle dynamics (Eq. 5) by the overdamped dynamics of a single self-propelled particle with friction \( \gamma \), caged by the other particles, whose effect is represented as an effective confining harmonic potential of strength \( k \) (29, 50, 51). We then calculate the correction to the configuration entropy \( \kappa_s (F^* x) \), where the angular bracket is an average over the steady-state probability distribution of the caged active particle. Details of this “mean-field” approximation are provided in SI Appendix. We consider two different models of active forces that have been widely used in the simulation literature and show that they give fundamentally different behavior as a function of the orientational persistence time \( \tau_p \), although the dependence on the magnitude of the active force \( f_0 \) is the same for both models. These opposite effects of the propulsion force and the persistence time on the glass transition in active glasses, which were reported in simulations in refs. 19, 26, were a puzzle and find a natural resolution in our active RFOT theory. We further discuss the distinction between these two models and their phenomenological relevance in SI Appendix.

**Model 1.** The active noise \( \mathbf{f}^* \) with fixed amplitude \( f_0 \) has zero mean and a shot-noise temporal correlation:

\[
\langle \mathbf{f}^*_i(0) \cdot \mathbf{f}^*_i(t) \rangle = f_0^2 \exp[-t/\tau_p].
\]

This is the realization of the active noise considered in the simulation studies of ref. 19. For this model of active forces, the active RFOT calculation detailed in SI Appendix gives for the correction to the configurational entropy appearing in Eqs. 3 and 4,

\[
\kappa_s \delta \Phi = \frac{\kappa_s (\gamma f_0)}{1 + (k/\gamma) \tau_p},
\]

thus leading to

\[
\ell^* = \left[ \frac{D}{(T - T_K) + Hf_0^2 / (1 + G \tau_p)} \right]^{2/\gamma}
\]

\[
\ln \left[ \frac{\tau}{\tau_0} \right] = \frac{E}{(T - T_K) + Hf_0^2 / (1 + G \tau_p)},
\]

where \( H = \tau_0 \kappa_s k / 2 \kappa_x , G = k / \gamma \), and as before, \( \theta = \psi = d/2 \). Eq. 8 shows that both \( f_0 \) and \( \tau_p \) decrease the effective Kauffman temperature \( T_K^{eff} = T_K - Hf_0^2/\tau_p(1 + G \tau_p) \), defined as the temperature when \( \ell^* \) and \( \tau \) diverge. In other words, as we increase either \( f_0 \) or \( \tau_p \), the system shows glassy behavior at lower \( T \) compared with the corresponding passive system—that is, both \( f_0 \) and \( \tau_p \) promote fluidization (or suppress glassiness) for small \( \tau_p \). This prediction with respect to \( f_0 \) is consistent with many simulation studies (16, 17, 19, 20, 43).

**Model 2.** The active noise \( \mathbf{f}^* \) with fixed single-particle effective temperature \( T_{sp} \) is an Ornstein–Uhlenbeck process (52), with correlations

\[
\langle \mathbf{f}^*_i(0) \cdot \mathbf{f}^*_i(t) \rangle = (T_{sp}/\tau_p) \exp[-t/\tau_p].
\]

This is the realization of the active noise considered in the simulation studies of ref. 26. For this model of active forces, the active RFOT calculation detailed in SI Appendix gives for the correction to the configurational entropy appearing in Eqs. 3 and 4,

\[
\kappa_s \delta \Phi = \frac{\kappa_s (T_{sp}/\gamma)}{1 + (k/\gamma) \tau_p},
\]

thus leading to

\[
\ell^* = \left[ \frac{D}{(T - T_K) + HT_{sp}/(1 + G \tau_p)} \right]^{2/\gamma}
\]

\[
\ln \left[ \frac{\tau}{\tau_0} \right] = \frac{E}{(T - T_K) + HT_{sp}/(1 + G \tau_p)},
\]

with the same expressions for \( H \) and \( G \) as above.

While the effect of \( T_{sp} \) is same as that of \( f_0^2 \) in Model 1, the effect of changing \( \tau_p \) is entirely different. At a fixed \( T_{sp} \), an increase \( \tau_p \) now makes the system show glassy behavior at higher \( T \)—that is, \( \tau_p \) promotes glassiness. This prediction with respect to \( \tau_p \) is consistent with another set of simulations (26).

Thus, what was perceived as contradictory results reported in simulations, namely that increasing \( \tau_p \) may either promote fluidization or glassiness, finds a natural resolution within our active RFOT framework. As demonstrated, the trends should indeed depend on the microscopic details of activity.

We now discuss other predictions that emerge from our active RFOT theory of models 1 and 2 and verify their consistency with our simulations of an active glass (Materials and Methods) and ref. 26, respectively.

**Active RFOT Confronts Simulations—Model 1.** We now perform molecular dynamics (MD) simulations of a glass former driven by active propulsion forces in three dimensions (Materials and Methods) and make a quantitative comparison with our theoretical predictions of model 1. We first tune \( f_0 \) at fixed \( \tau_p \) and plot \( \ell^* \) and \( \tau \) as a function of \( T \) in Fig. 1A and B, respectively. For fixed \( T \), we see that \( \ell^* \) and \( \tau \) decrease as we increase \( f_0 \). Next, we calculate the modification to the glass transition temperature \( T_g \) by the activity. We define \( T_g \) as the temperature where the relaxation time of the system increases beyond the threshold value of \( \tau/\tau_0 = 10^6 \). From Eq. 9 we see that \( T_g \) gets modified due to activity as \( T_g = E/(6 \ln 10) + T_K - Hf_0^2/\tau_p(1 + G \tau_p) \), similar to \( T_{sp}^{eff} \). Using this definition, we compare our theory with the MD simulation data (see Materials and Methods and ref. 19), by rewriting Eq. 9 as

\[
\ln \left[ \frac{\tau}{\tau_0} \right] = \frac{E}{(T - T_K) + f_0^2 \Lambda},
\]

where \( \Lambda = Hf_0^2/(1 + G \tau_p) \) is a constant, since \( \tau_p \) is held fixed. Analysis of our simulation data gives \( \tau_0 = 0.135 \) and \( T_K = 0.3 \). We obtain the values of the other two constant parameters by fitting our analytical expression to one particular dataset, at \( f_0 = 1.50 \) (fitting to the data for other values of \( f_0 \) works equally well), and obtain \( E = 1.55 \) and \( \Lambda = 0.1 \). The Angell plot (53) shown in Fig. 1C demonstrates the excellent agreement between active RFOT theory and simulation data. We emphasize here that the theoretical lines are not individual fits to the simulation data, since all of the plots of Eq. 14 use the same constant parameter values obtained from one initial fit. Comparing Eq. 14 to the Vogel–Fulcher–Tammann-form \( \ln (\tau/\tau_0) = 1/(T/T_{sp}^{eff} - 1)^{-1} \), we obtain for the fragility parameter

\[
m = \frac{T_K}{E} - \frac{f_0^2 \Lambda}{E}.
\]

We find that this differs from the simulation data by a scale factor of \( \sim 1.25 \) (this is true even in the passive limit); however, it is remarkable that our simple theory captures the functional dependence on \( f_0 \) correctly (Fig. 1D).

This leads to an important result, namely when \( \tau_p \) is fixed and the active force \( f_0 \) increases, the behavior of \( \tau \) becomes closer to Arrhenius law: The system becomes a stronger glass former (53) with increasing activity.
We next fix \( f_0 \) and tune activity by changing \( \tau_p \). The active RFOT theory of model 1 gives explicit formulae for \( \tau^* \) (Eq. 8), \( \tau \) (Eq. 9), and the fragility parameter \( m = T_K / E - H \tau_p^2 / (1 + G \tau_p) \). This predicts that the relaxation time decreases for increasing \( \tau_p \), while the dynamics becomes independent of \( \tau_p \) when \( \tau_p \gg 1 / G \). We have tested these predictions by obtaining \( \tau \) as a function of \( \tau_p \) at different values of \( f_0 \) from our simulations (Materials and Methods). The agreement is good for low \( f_0 \) and \( \tau_p \) (Fig. 2A) and become systematically worse at larger activity— for example, at large \( \tau_p \) for \( f_0 = 1.0 \) and 2.0.

We next compare the theoretical predictions for the fragility parameter with our simulations (Fig. 2B). We again see fair agreement for small \( f_0 \) and \( \tau_p \) and a clear deviation between theory and simulation data for larger \( f_0 \) and \( \tau_p \). Within model 1, both \( \tau \) and \( m \) decrease as \( \tau_p \) increases: Increasing \( \tau_p \) fluidizes the system and makes it a stronger glass former.

Many experimental and numerical studies of glass have shown that the relaxation of density fluctuations in supercooled liquids is well approximated by a stretched exponential \( \exp (-t / \tau)^{\beta} \), where \( \beta \) is the stretch exponent [Kohlrausch law (54)]. Using our simulations, we can measure the change in the \( \beta \) exponent due to activity. A fit to the computed density correlation function \( Q(t) \) (SI Appendix, Eq. S32) shows that \( \beta \) increases with both \( f_0 \) and \( \tau_p \) (Fig. 2C), with the change \( \Delta \beta \) scaling as \( f_0^2 \tau_p \) (Fig. 2D). In Discussion, we provide a rationalization for this.

**Active RFOT Confronts Simulations—Model 2**

We first tune \( T_0 \) at fixed \( \tau_p \). The active RFOT theory for model 2 shows that both \( \tau \) (Eq. 13) and fragility parameter \( m = T_K / E - H \tau_p^2 / (1 + G \tau_p) \) decrease with increasing \( \tau_p \). For this case, model 2 shows the same qualitative behavior as model 1. We then fix \( T_0 \) and change activity by tuning \( \tau_p \). We find that at fixed \( T \), \( \tau^* \) increases monotonically with \( \tau_p \) (Fig. 3A). Thus, we predict that \( \tau_p \) promotes glassy behavior in the sense that larger \( \tau_p \) drives the system toward the glassy state. Using the active RFOT theory for model 2, we obtain the effective Kauzmann temperature, \( T_K^\sigma \), and the glass transition temperature, \( T_g \). The Angell plot (53) (Fig. 3B) \( \tau \) versus \( T_0 / T \) at different values of \( \tau_p \), shows a systematic departure from Arrhenius behavior as \( \tau_p \) increases:

Here, fragility increases with \( \tau_p \).

Since the simulations of ref. 26 with which we would like to compare our results were done in the athermal limit, we have to take \( T \to 0 \) in Eq. 13. In this athermal limit, the fit parameters are \( E, H, G \), and \( \tau_0 \) (note that \( T_K \sim 0.3 \) is obtained from simulations of the passive system and hence is not a fit parameter). We find that with \( E = 1.255, G = 3.801, \) and \( H = 1.0 \), we obtain a good fit as long as we choose \( \tau_0 \) to depend on \( \tau_p \) (Fig. 3C).

Indeed, this dependence of \( \tau_0 \) on \( \tau_p \) was reported in the simulations of ref. 26; this is replotted in SI Appendix, Fig. S3 (40) for completeness. We emphasize that the parameters \( E, G, \) and \( H \) are obtained from one set of data in Fig. 3C and use the same values for the rest of the plots.

Our active RFOT theory makes a prediction for the dependence of the fragility parameter on \( \tau_p \): \( \tau_m = T_K / E - H \tau_p^2 / (1 + G \tau_p) \), at fixed \( T_0 > 0 \), the fragility is at first insensitive to \( \tau_p \) and then increases before saturating to the passive value \( T_K / E \) (Fig. 3D). Irrespective of this detailed behavior, the point is that this system becomes more fragile as \( \tau_p \) increases. This is opposite to the behavior of model 1 (at small \( \tau_p \)). This is
\( \text{Results for model 2 when } \tau_p \text{ is the control parameter. (A) } t^* \text{ as a function of } T \text{ for different } \tau_p \text{ with } T_0 = 0.01, \text{ using Eq. } (12). \text{ We define } T_0 \text{ when } \tau_0/T_0 \text{ becomes 10 and plot log}(t^*/\tau_0) \text{ as a function of } T_0/T \text{ for } \tau_0 = 0.64, \text{ using Eq. } (13). \text{ As } \tau_0 \text{ increases, the curves depart further from the Arrhenius behavior; the system becomes more fragile with increasing } \tau_0. \text{ (C) Relaxation time of the system as a function of } 1/T_0 \text{ at different } \tau_p \text{ when } T = 0 \text{ (athermal limit), allowing us to make comparisons with simulation data } (26) \text{ (see text for discussion of fit parameters). (D) } \text{Our active RFOT theory for model 2 predicts that, for fixed } T_0 > 0, \text{ the fragility parameter is at first insensitive to } \tau_p \text{ and then rises before saturating to the passive value } T_0/E. \)
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