INTERVAL-VALUED IFAT-IDEALS OF AT-ALGEBRA

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Abstract. Background: The interval-valued IFset to AT-ideals on AT-algebras investigated. We apply the concept interval-valued intuitionistic AT-ideal, and investigate some of their properties. Materials and Methods: Moreover, Cartesian product of IFAT-ideal, and related properties. It was extended to IFideal by many literatures studies. Several researchers are studied the IFKU-ideals in KU- and intuitionistic image. Results: AT-ideals and AT- ideals fuzzy in AT-algebras defined in this study. IFAT-sub algebra showed. In interval-valued IFAT-ideal on AT-algebras. IF- sub algebra or (fuzzy AT-ideal) on AT-algebras. Conclusion: It introduced a Cartesian give to two interval-valued IFAT-ideal on AT-algebras and investigate some important the structure

Keywords: AT-algebras, fuzzy AT-sub algebra and fuzzy AT-ideals on AT-algebra, interval-valued IFAT-ideal of AT-algebras.

1. Introduction
L.A. Zadeh [1] explained an idea to the subset as a way to represent the principle of relativity in the real physical world. G. Takeuti and S. Titanti in [2] defined the IF(IF) sets and introduced the IFset theory. The idea of the IFsubset was introduced [3] and it was extended to IFideal by Basnet and Ramesh [4, 5]. Several researchers like S.M. Mostafa, [6] studied the IFKU-ideal in KU-algebra and fuzzy intuitionistic of KU-ideal in KU-algebra. Notion of interval-valued IFAT-subalgebra. In [7] interval-valued IFAT-ideal on AT-algebras and properties are found of its. The cartesian product to two interval-valued IFAT-ideal on AT-algebras and investigate some important the structure of AT.

2. PRELIMINARIES
Definitions recalled which used in the later sections. The AT-algebra is an important branch [8-11].
Definition 1.1. ([7]). Let (X;*,O) be an algebra of type (2,O)
(AT2): O* x= x,
(AT3): x*O= O.
binary relation " ≤ " by : x ≤ y
Lemma 1.2.([7]). AT-algebra(X,O )
x≤ y implies that y * z≤ x * z ,
z * x ≤z*y implies that x ≤ y( left cancellation law)
Example 1.3.([7]). X = {O, 1, 2, 3} in "*" is
(X; O) is AT

Definition 1.4. ([7]). (X;*,O) be AT. S called AT-subalgebra (X).

Definition 1.5. ([7]). A nonempty subset I,(X;*,O) called an AT-ideal

\[(\text{ATI2})\quad x * (y *z) \in I \text{ and } y \in I\]

Proposition 1.6. ([7]). Every AT-optimize to AT-algebra(X;*,O) is an AT-subalgebra.

Definition 1.7. [1]. (X;*,O) nonempty subset 
\[\mu: X \rightarrow [0,1]\].

Definition 1.8. ([7]). (X;*,O) AT-algebra, subset \(\mu\ AT\)-subalgebra (X)

Definition 1.9. ([7]). (X;*,O) be an AT-algebra, \(\mu\) in X is AT-ideal: x, y, z \(\in X\),
\[(\text{FAT1})\quad \mu (O) \geq \mu (x)\]

Proposition 1.10. ([7]). Every fuzzy AT-optimize to AT

Definition 1.11: \(f:(X;*,0) \rightarrow (Y;*,0')\) mapping an AT-algebra, \(I \subseteq X\) and \(J \subseteq Y\). image I of \(X\), \(f(I) = \{f(x) \mid x \in I\}\), image J of \(Y\): \(f^{-1}(J) = \{x \in X \mid f(x) \in J\}\). \(\{x \in X \mid f(x) \in J\}\).

Remark 1.12 [8]. An interval number is \(\tilde{a} = [a^-,a^+]\), where \(0 \leq a^-\leq a^+\leq 1\). i.e.,\(I = [0,1]\). Let \(D[0,1] : l = [0,1]\), \(D[0,1] = \{\tilde{a} = [a^-, a^+] \mid a^-\leq a^+, \text{ for } a^-, a^+\in I\}\).

Definition 1.13 [8]. \(\tilde{a}\) is a fuzzy AT-subalgebra (X;*,O), \(x, y \in X\).

Definition 1.14. ([7]). Every fuzzy AT-optimize to AT

Definition 1.15. ([7]). \(\tilde{a}\) is a fuzzy AT-subalgebra (X;*,O), \(x, y \in X\).

Definition 1.16. ([7]). A X interval AT-algebra (X;*,O), \(x, y \in X\).

Definition 1.17. ([7]). i-v subset \(A = \{(x, \mu_A(x))\}, x \in X\) in AT-algebra (X;*,O):
\[\text{Theorem 1.18. ([8]).} A = \{(x, \mu_A^- (x)), x \in X\} \text{ in AT-algebra}
\]
\[\text{Theorem 1.19. ([8]).} \text{Let } (X;*,O) \text{be an AT. The nonempty set}
\]
i-v level AT-optimize to \(A\).

Definition 1.20. ([11]). (X;*,O) an AT-algebra is an antifuzzy AT-subalgebra

Definition 1.21. ([11]). (X;*,O) AT-algebra, a subset is an antifuzzy AT,
\[(\text{FAT1})\quad \mu (0) \leq \mu (x)\],

Proposition 1.22. [7]. Every of antifuzzy AT-ideal AT-algebra X is an antifuzzy AT-subalgebra.

Definition 1.23. [8]. A in a nonempty \(\mu_A: X \rightarrow [0,1]\) and \(v_A: X \rightarrow [0,1]\).

Remark 1.24. [10]. If intuitionistic AT, \(A\) then
\[x \in X. \text{ Now } \mu_A \text{ is named fuzzy subset while } \nu = \mu_A^c \text{ is the complement of } \mu_A.
\]

Definition 1.25. ([10,8]). Intuitionistic fuzzy subset of AT-algebra X. A is IFAT-subalgebra (X) if:
\[x, y \in X\],
That mean \(\mu_A\) is a fuzzy AT

Example 1.26. ([10,8]). Let X = \(\{0, 1, 2, 3\}\) in which "is:

|   | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| * | 0 | 1 | 2 | 3 |
| 0 | 0 | 1 | 2 | 3 |
| 1 | 0 | 0 | 2 | 3 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
\[ (2) = 0.6 > 0.2 = \mu_A (1) = \mu_A (3) = 0.1 < 0.7 = \nu_A (1) = \nu_A (3) . \]

Proposition 1.27. ([10,8]). Every IFAT-subalgebra of AT-algebra X satisfies the inequalities \( \mu_A (O) \geq \mu_A (x) \)

Definition 1.28. ([10,8]). \( t \in [0,1] \) and a subset \( \mu \).

Theorem 1.29. ([10,8]). An IF subset AT-algebra X if and only if, \( t \in [0,1] \).

Definition 1.30. ([10,8]). \( t \in [0,1] \) and a subset \( \mu \).

Theorem 1.31. ([10,8]). An IF subset AT-algebra X if and only if, \( t \in [0,1] \).

Definition 1.32. ([8]). IF AT-optimize to AT-algebra (X), A is an IFAT-ideal (X).

Proposition 1.33. ([10,8]). An IF subset AT-algebra X if and only if, \( t \in [0,1] \).

Theorem 1.34. ([10,8]). An IF subset AT-algebra X if and only if, \( t \in [0,1] \).

Definition 1.35. ([7]). \( f : (X;*,0) \rightarrow (Y;*,0) \) homomorphism we define new in X by, \( x \in X \).

Definition 1.36. ([2,7]). A, B fuzzy. Cartesian \( X \times X \rightarrow [0,1] \), \( A \times B (x,y) = \{ A (x), B (y) \} \)

Definition 1.37. ([2,14]). \( A \times B = (X \times X, \{ A \mu \} B ) \) such that \( A \mu \times B \mu \).

Definition 1.38. ([10,8]). \( A = \{ x \in X \}, B = \{ x \in X \} \)

Definition 1.39. ([7,15]). IF subsets A over X \( A = \{ x \in X \} \rightarrow D[0,1] \)

Remark 1.40. ([2]). If an interval-valued IF subset A in a nonempty set X, i.e. \( \mu_A (x) = 1 - \mu_A^c (x) \)

Definition 1.41. ([2]): \( D_1, D_2 \in D[0,1] \).

3. INTERVAL-VALUED

The notions of interval-valued IFAT on AT-algebra. We study and prove some important properties and theorem.

Definition 2-1 \( \{(x, \mu_A (x), \nu_A (x)) | x \in X \} \) an intuitionistic subset AT-algebra. Interval-valued IFAT

That mean \( \mu_A \) IFAT-subalgebra \( \nu_A \) is an antifuzzy AT.

Example 2.2. X = {0, 1, 2, 3} in which \( * \) is:

\[
\begin{array}{cccc}
0 & 1 & 2 & 3 \\
0 & 0 & 1 & 2 \\
1 & 0 & 0 & 2 \\
2 & 0 & 0 & 0 \\
3 & 0 & 0 & 0 \\
\end{array}
\]

\( (X;0) \) is an AT-algebra.

Proposition 2.3. Every interval AT

Proof: by proposition (2.5), \( \mu \) A (x) = \( \mu \) A (x).

Definition 2.7. A = \( \{ \mu_A , \nu_A \} \).

Hence \( A_1 \cap A_2 \) interval fuzzy AT-subalgebra (X).

Corollary 2.9. \( \{ A_i | i=1,2,3,\ldots \} \) interval-valued

Definition 2.10. Interval IF subset A = \( \{ \mu_A , \nu_A \} \) IFAT

Theorem 2.10. Interval IF subset A = \( \{ \mu_A , \nu_A \} \) IFAT

Proof - A = \( \{ \mu_A , \nu_A \} \) interval-valued AT-subalgebra (X).
Proposition 2.11. \( A=(\tilde{\mu}_A, \tilde{\nu}_A) \) interval-valued IFAT-subalgebra (X), then the sets \( I_{(\tilde{\mu}_A)} \) is fuzzy \( \text{AT-subalgebra} (X) \) and \( I_{(\tilde{\nu}_A)} \) is anti-fuzzy AT-subalgebra (X).

Theorem 2.12. \( B \) a nonempty to AT-algebra. \( A=(\tilde{\mu}_A, \tilde{\nu}_A) \) interval-valued:
\[
\lambda, \tau, \gamma, \delta \in [0,1], \quad \lambda \geq \tau \quad \text{and} \quad \gamma \leq \delta \quad \text{and} \quad \lambda + \gamma \leq 1; \tau + \delta \leq 1. 
\]
another \( A \) is an interval-valued fuzzy AT-B an AT (X). Proof: A interval-valued IFAT-subalgebra (X) \( \tilde{\mu}_A(x*y) \geq \tau \min\{\tilde{\mu}_A(x), \tilde{\mu}_A(y)\} \)
\[
= \tau \min\{\gamma, \gamma\} = \gamma . \text{Thus } x*y \in B, \text{therefore } B \text{ is an AT-subalgebra from } X .
\]

Reciprocally, assume that \( B \) is AT-subalgebra from X and let \( x, y \in X \). study two cases.

Proof: A be interval If subset

4. INTERVAL-VALUED

Notion of interval AT-ideals on AT-algebra and we study its properties when the interval-valued IFAT-algebra is replace with interval-valued fuzzy AT-optimize to AT-algebra (X).

That mean \( \mu_A \) interval-valued AT-ideal.

5. CARTESIAN PRODUCT OF INTERVAL-VALUED IFAT-IDEAL

A new notion called cartesian product of interval-valued IFAT-ideals

Remark 5.1. \[10\]. \( X \) and \( Y \) be AT-algebras

interval-valued IFAT-ideals (X), then \( A \times B \) is interval-valued IFAT-ideal (X).

Example 5.9: Let \( X_1=\{0,1,2\} \) and \( X_2=\{0,1,2,3\} \) are AT-algebras by

\[
\begin{array}{c|cccc}
* & 0 & 1 & 2 & 3 \\
\hline
0 & 0 & 1 & 2 & 3 \\
1 & 0 & 0 & 2 & 3 \\
2 & 0 & 0 & 0 & 0 \\
3 & 0 & 0 & 0 & 0 \\
\end{array}
\]

We defined interval valued fuzzy-subalgebra \( A_1=(\tilde{\mu}(A_1), \tilde{\nu}(A_1)): X_1 \rightarrow [0,1] \) by \( \tilde{\mu}(A_1)(0)=[0.7,0.8], \tilde{\nu}(A_1)(0)=[0.3,0.4] \), \( \tilde{\mu}(A_1)(1)=[0.5,0.6] \) and \( \tilde{\nu}(A_1)(1)=[0.3,0.4,0.5] \).

\( \tilde{\mu}(A_2)(2)=[0.5,0.6] \) and \( \tilde{\nu}(A_2)(2)=[0.3,0.4,0.5] \).

\( \tilde{\nu}(A_1)(0)=[0.3,0.35] \).

\( \tilde{\mu}(A_2)(3)=[0.2,0.3] \).

\( \tilde{\nu}(A_2)(3)=[0.4,0.5] \).

By routine calculation \( A_1 \times A_2 = \langle \mu_{(A_1 \times A_2)}, \nu_{(A_1 \times A_2)} \rangle \) is interval valued intuitionistic - Optimize to AT-algebra.

Proof: \( (x_1,2,3,\ldots,x_n) \in \prod_{(i=1)}^n X_i \)

6. CONCLUSION

The interval-valued put to AT-ideals on AT-algebras. The concept interval-valued AT-ideal and fundamental properties to AT-algebrae explained and the picture of interval-valued and investigate some properties. The notion of Cartesian product of IFAT-ideal on AT-algebras and investigate are calculated.
The idea of the intuitionistic subset was studied. A notion of interval IFAT-sub algebra introduced. In interval-valued IFAT-ideal on AT-algebras. Cartesian get of 2 interval-valued IFAT on AT-algebras and investigate of some important of the structure of AT-algebras.

Acknowledgments
The author acknowledges the financial support of the Kufa University, Iraq. The author is grateful to Dr. B. Almayahi, Kufa University for assisting us.

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