Design of rotor profile of a new roots vacuum pump

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Abstract—This paper presents a new rotor profile of roots vacuum pump composed of arc, involute and cycloid. It’s whole tooth profile which is described exactly by combining the meshing principle of gears with parameter equation. The performance of meshing of two rotors is improved without profile interference and packet gas, which are the problems of conventional involute profiles. The work provides new theoretical basis for digital design of new rotor profile, its meshing analysis, dynamic emulation, kinematics and dynamics analysis.

1. INTRODUCTION
The rotor is the core problem of the roots vacuum pump, the rationality of the design of the rotor type line will directly related to the merits of the pump suction performance [1,2]. The meshing segments of the traditional involute rotor are all involutes, and the addendum and dedendum are arcs. But the arc segment of this kind of tooth shape is larger, and when the instantaneous point meshing is carried out, it has an adverse effect on the stability of gas flow and the area utilization coefficient of tooth shape [3]. Based on the deficiency of the above rotor profile, a new tooth profile is proposed, which can greatly reduce the arc segment of the addendum and dedendum.

2. ESTABLISHMENT OF PARAMETER EQUATION OF NEW ROTOR PROFILE

2.1. New rotor profile analysis
The unilateral tooth profile is composed of arc AB-extension inner cycloid BC-involute CD-extension outer cycloid DE-arc EF, as shown in figure 1. The radius of the arc segment is equal to the height of the addendum and the dedendum, and the length from E to G on the cycloid is exactly equal to the height of the addendum. Sufficient and necessary conditions for the determination of radius of rolling circle: firstly, ensure that the G spot on the rolling circle falls outside G at the end of addendum cycloidal meshing; secondly, ensure that the tangential point P1 of the rolling circle and the pitch circle is also where the normal line passes through the meshing point D at the end of the involute. At the same time, the DP1 is also the length of the meshing segment when the top of the involute addendum meshes. PC on the tooth profile is the involute tooth root, PD is the tooth top curve.

2.2. Determination of parametric equation of profile
(1) Arc AB section
AB segment is the dedendum arc, and when the rolling circle rolls round from the dedendum point H to the horizontal axis H’, its parametric equation is:

\[
\begin{align*}
x &= r - r \cos \theta_2 \\
y &= r \sin \theta_2
\end{align*}
\]

(1) $\theta_2 = 6^\circ \sim 0^\circ$

(2) Extension inner cycloid BC segment

As shown in figure 1, the inner and outer cycloidal circles have the same radius. When O\(_1\) rolls outside the pitch circle at the top of the tooth, O\(_2\) rolls inside the pitch circle at the root of the tooth. The initial position of rolling circle O\(_2\) is 11° up the horizontal axis, OP\(_2\) is the center line of the two wheels engaged at that time, and nodal point P\(_2\) is the intersection point of T\(_1\)C and the pitch circle at that time, while the pitch circle is tangent to the rolling circle here and starts rolling, then the parametric equation of the inner cycloid is extended:

\[
\begin{align*}
x &= (r_i - r_c) \cdot \cos \theta_2 - L_c \sin \left( \frac{\lambda - (90 - 11)}{} \cdot \pi / 180 \right) \\
y &= (r_i - r_c) \cdot \sin \theta_2 + L_c \cos \left( \frac{\lambda - (90 - 11)}{} \cdot \pi / 180 \right) \\
\theta_1 &= 10^\circ \sim 0^\circ
\end{align*}
\]

(2)

(3) Involute CD

CD segment is involute, and its parametric equation is:

\[
\begin{align*}
x &= r_c [\sin \theta + (1 + \theta) \cos \theta] \\
y &= r_c [\cos \theta + (1 + \theta) \sin \theta]
\end{align*}
\]

(3)

Addendum segment $\theta = 0^\circ \sim 34^\circ$,

dedendum segment $\theta = 0^\circ \sim 34^\circ$

(4) Extension outer cycloid DE segment

DE segment is the external cycloid of the addendum, and its parametric equation is:

\[
\begin{align*}
x &= (r_i + r_c) \cdot \sin \theta_2 + L_c \cos \left( \frac{\lambda - (90 + 11)}{} \cdot \pi / 180 \right) \\
+ (r_i / r_c + 1) \cdot (11 - \theta_2) \cdot \pi / 180 \\
y &= (r_i + r_c) \cdot \cos \theta_2 + L_c \sin \left( \frac{\lambda - (90 + 11)}{} \cdot \pi / 180 \right) \\
+ (r_i / r_c + 1) \cdot (11 - \theta_2) \cdot \pi / 180 \\
\theta_1 &= 10^\circ \sim 0^\circ
\end{align*}
\]

(4)

(5) Arc EF section
EF segment is an arc with G' point as the center and G'F as the radius. Its parametric equation is as follows:

\[
\begin{align*}
    x &= r \cdot \sin \theta \\
    y &= r \cdot \cos \theta + r \cdot \sin \theta \\
\end{align*}
\]

\( \theta = 6^\circ - 0^\circ \)  \( \text{(5)} \)

3. Comparison of area utilization coefficients

3.1. Area utilization coefficient of traditional involute

The area utilization coefficient is the ratio of the area contained between the outer surface of a rotor and the inner surface of a cylinder to the inner circle area of a cylinder, represented by \(C\), which represents the effective utilization of the cylinder space.

\[
C = \frac{2B}{\pi D^2} = \frac{\pi D^2 - 4S}{\pi D^2} = 1 - \frac{4S}{\pi D^2}
\]

\(B\) - the projection of the end face of the maximum volume formed by the rotor and the cylinder.

The key to find the area utilization coefficient is to find the rotor sectional area, as shown in figure 2:

\[S = S_1 + S_2 + S_3\]

According to the polar coordinate parameter equation and area integral formula of the involute:

\[
S_1 = \frac{1}{2} \int_0^{\pi} \left[ \frac{r}{\cos \alpha} - \frac{1}{\cos \alpha} \right] d\alpha
\]

\(\text{(8)}\)

\[
S_1 = \frac{1}{2} \int_0^{\pi} (rr \cdot \cos t + r^2) d\alpha
\]

\(\text{(9)}\)

\[
S_1 = \frac{1}{2} \int_0^{\pi} (rr \cdot \cos t - r^2) d\alpha
\]

\(\text{(10)}\)

Substituting the parameters into the formula, \(\alpha = 50.46^\circ\), \(C = 0.52\)

![Figure 2. Sectional area of blade end face](image)

3.2. Area utilization coefficient of the new rotor profile

(1) The area \(S_t(\alpha)\) of the cycloid line BC.

The arc length HQ of the circle itself rotated is equal to the arc length \(P_2Q\) it rolled on the base circle, and the radius of the circle is \(r_c\), as shown in figure 3.

Parametric equation of moving point H(x,y) :

\[
\begin{align*}
    x &= \overline{OO_2} \cdot \cos \alpha + \overline{O_2H} \cdot \cos(\phi - \alpha) = \\
    (r_1 - r_c) \cdot \cos \alpha + r_c \cdot \cos[\left( r_2 / r_2 - 1 \right) \cdot \alpha] \\
    y &= \overline{OO_2} \cdot \sin \alpha - \overline{O_2H} \cdot \sin(\phi - \alpha) = \\
    (r_1 - r_c) \cdot \sin \alpha - r_c \cdot \sin[r_2 / r_2 - 1 \cdot \alpha]
\end{align*}
\]

\(\text{(11)}\)
The area of the shaded part in figure 3 is:

\[ S_1(\alpha) = \frac{1}{2} \int_{r_0}^{r} (x \, dy - y \, dx) \]  

(12)

Figure 3. Cross-sectional area corresponding to the inner and outer cycloid of the new rotor profile

(2) Area \( S_2(\alpha) \) corresponding to the cycloid

Similarly, the parametric equation of moving point \( G(x,y) \) is:

\[
\begin{align*}
  x &= \overline{OG} \cdot \cos \alpha + r_0 \cdot \cos(\pi - \phi - \alpha) = \\
  &= (r_0 + r) \cdot \cos \alpha - r_0 \cdot \cos[(r_0 / r_0 + 1) \cdot \alpha] \\
  y &= \overline{OG} \cdot \sin \alpha - \overline{OG} \cdot \sin(\pi - \phi - \alpha) = \\
  &= (r_0 + r) \cdot \sin \alpha - r_0 \cdot \sin[(r_0 / r_0 + 1) \cdot \alpha]
\end{align*}
\]

(13)

The area of the shaded part in figure 3 is:

\[ S_2(\alpha) = \frac{1}{2} \int_{r_0}^{r} (x \, dy - y \, dx) \]  

(14)

(3) Area \( S_2 \) corresponding to the involute, area \( S_1 \) and area \( S_3 \) corresponding to the arc

The formulas of \( S_1, S_2 \) and \( S_3 \) are the same as those of traditional involute

(4) the total area of the new rotor profile is shown in figure 4 is:

\[ S = S_1 + S_2 + S_3 + S_1(\alpha) + S_2(\alpha) \]  

(15)

Substituting the parameters into the formula, \( C=0.61 \)

Figure 4. Cross-sectional area of the new rotor profile
4. ANALYSIS OF THE CHARACTERISTICS OF THE NEW ROTOR PROFILE

The new rotor profile adopts a meshing cycloidal profile, which can make a good transition between the involute profile and the cycloid profile, avoiding the unsteady gas flow caused by the large arc segment of the involute profile [4]. As can be seen from the above calculation results, the area utilization coefficient of the new line is 17.3% higher than the traditional involute type, and the pump efficiency is also improved.

The coincidence degree ($\varepsilon_{\alpha}$) is an important index to measure the transmission continuity and transmission load uniformity of the gear[5]. According to the formula of reference [6], the traditional linearity $\varepsilon_{\alpha}=0.52$; the modified profile $\varepsilon_{\alpha}=0.61$. The coincidence degree of the new rotor profile is 26% higher than that of the traditional profile, which not only improves the stability of the rotor drive, but also improves the bearing capacity of the rotor drive.

5. CONCLUSION

(1) The area utilization coefficient and coincidence degree of the new rotor profile are greatly improved compared with the traditional profile, which not only improves the efficiency of the pump and solves the problem of profile interference in the traditional involute profile, but also improves the stability of the rotor drive and the bearing capacity of the rotor drive.

(2) The complete tooth profile of the new rotor profile is accurately and truly described by combining the gear meshing principle with the parametric equation. It provides a new theoretical basis for the digital design, meshing analysis, meshing process simulation, and kinematics and dynamics analysis of the new rotor profile.

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