QCD estimate of the long-distance effect in $B \rightarrow K^*\gamma$

A. Khodjamirian$^a$, R. Rückl$^{a,b}$

$^a$ Institut für Theoretische Physik, Universität Würzburg, D-97074 Würzburg, Germany
$^b$ Max-Planck-Institut für Physik, Werner-Heisenberg-Institut, D-80805 München, Germany

G. Stoll$^c$, D. Wyler$^{c,d}$

$^c$ Institut für Theoretische Physik, Universität Zürich, CH-8057 Zürich, Switzerland
$^d$ Physics Department, Technion, Haifa 32000, Israel

Abstract

We suggest to use operator product expansion and QCD sum rule techniques to estimate the long-distance contribution to the amplitude of the exclusive rare decay $B \rightarrow K^*\gamma$. In contrast to a phenomenological description in terms of $\psi$ resonances converting into a photon, the virtual charm quark loop interacting with soft gluons is represented by a new quark-gluon-photon operator. The matrix element of this operator is calculated in the same approximation as the matrix element of the leading magnetic penguin operator. The overall correction is found to be small, not more than about 5%.

---

1 on leave of absence from the Yerevan Physics Institute, 375036 Yerevan, Armenia
1 Introduction

Rare decays of $B$ mesons such as $B \to K^*\gamma$ discovered at CLEO \cite{1}, proceed mainly through loops, and are therefore an important tool to study new physics, once the standard model contributions are sufficiently understood. The generally accepted calculational procedure is to use an effective Hamiltonian

$$H_{eff} = 4 \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \sum_{i=1}^{8} C_i(\mu) O_i(\mu)$$  \hspace{1cm} (1)$$

with suitable operators $O_i$ and Wilson coefficients $C_i$, renormalized at the scale $\mu$. The task is then to evaluate the Wilson coefficients and the matrix elements of the operators between the initial and final hadronic states, in the case at hand, between $B$ and $K^*$. The coefficients $C_i$ can be calculated perturbatively. A comprehensive review of the current status of the Hamiltonian (1) can be found in \cite{2}. In order to obtain the exclusive matrix elements $\langle K^* | O_i | B \rangle$, some hadronic model or nonperturbative method must be invoked.

In leading logarithmic approximation, only the matrix element of the dominant $O_7$ operator is needed. It has been calculated using various methods \cite{3,4,5,6,7}. However, there remains a rather strong $\mu$ dependence. It can be reduced by next-to-leading order calculations of the $C_i$ and a corresponding higher order in $\alpha_s$ determination of the matrix elements. For example, in the case of the inclusive reaction $b \to s\gamma$ one has to calculate the two-loop-diagrams involving the operator $O_2$. Here, the next-to-leading analysis has essentially been completed \cite{8,9,10,11}. Generally, it is assumed that a similar approach is adequate for exclusive transitions. The corresponding diagrams are shown in Fig. 1.

When considering the penguin-like loops in these diagrams, a perturbative calculation seems adequate for gluon virtualities $|k^2| \geq m_b^2$ and even when $m_c^2 \leq |k^2| \leq m_b^2$. Furthermore, if the photon is emitted from the $b$ or $s$ quark or from the spectator quark (Figs. 1b,d), the integration over the $c$-quark loop produces a factor $k^2$ which cancels the denominator of the gluon propagator, effectively suppressing low-virtuality gluons. However, when the $c$–quark loop converts into a photon (Fig. 1a,c), the gluon may have a characteristic virtuality $|k^2| \ll 4 m_c^2$. This case is beyond perturbation theory. The role of these long-distance penguins in $B \to K^*\gamma$ illustrated graphically in Fig. 2 will be the main concern of this paper.

Their effect has been considered by many authors on a phenomenological level. The possibility extensively discussed in the literature \cite{12} is to model the transition $B \to M\gamma$, $M$ being a light meson, by a sum over the nonleptonic decays $B \to V^* M$ followed by the transition $V^* \to \gamma$ where $V^*$ is an appropriate virtual vector state such as the $J/\psi$. Since at zero momentum squared $p^2$ of the photon, the characteristic distance to the physical threshold $p^2 = M_{\psi}^2$ is rather large, not only the ground state $J/\psi$ but also excited $\psi$ states must be included. Unfortunately, the transverse parts of these amplitudes are poorly known and the extrapolation from $p^2 = M_{\psi}^2$ to $p^2 = 0$ is not straightforward. Therefore, for a real photon, the vector-resonance description of the virtual $\bar{c}c$ loop seems uncertain, calling for a more reliable or at least different estimate.

As a new approach to the problem, we suggest to use operator product expansion (OPE) and QCD sum rules \cite{13}. Several important circumstances make these methods exceptionally suitable. Firstly, the fact that the $\bar{c}c$ loop is far off–shell, which is a disadvantage for $\psi$ insertion, turns out to be a major advantage here. It means that we must keep only a few terms in the OPE. Secondly, the QCD sum rule method provides a physically very intuitive parametrization of the interaction of the $\bar{c}c$ loop with soft gluons in terms of vacuum conden-
sates. Thirdly, and most importantly, we will be able to compare the long-distance effect in $B \rightarrow K^{*}\gamma$ with the contribution of the dominant magnetic penguin operator obtained in one and the same calculational framework. Of course, as with any low energy effective method, there are some unavoidable theoretical uncertainties. One of them is the question of double counting in the perturbative and nonperturbative contributions. As usual, we assume that the former are small in the region where the latter dominate, that is at $|k^2| \ll 4m_c^2$.

There are other nonperturbative effects, such as those illustrated in Fig. 3, which involve radiative matrix elements of the four-fermion operators $O_{1,2}$. These contribute already at tree level and have been estimated [14, 15] using QCD sum rules on the light-cone. They are unimportant for the $b \rightarrow s$ transitions due to CKM suppression (Fig. 3a), and also relatively small for $b \rightarrow d$ transitions (Fig. 3b).

The paper is organized as follows. In Sect. 2, we consider the contribution of the operator $O_2$ to the exclusive $B \rightarrow K^{*}\gamma$ amplitude and single out the term responsible for soft gluon emission from the $c$-quark loop. With the help of OPE, this term is expressed by a composite operator involving quark, gluon and photon fields. In Sect. 3, we derive a QCD sum rule for the $B \rightarrow K^{*}\gamma$ matrix element, including the new operator. The most important nonperturbative contributions of gluon, quark-gluon, and quark condensates up to dimension 6 to the relevant correlation function are calculated in Sect. 4. Our numerical results are presented in Sect. 5 and our conclusions in Sect. 6.

## 2 Long-distance contribution of the operator $O_2$

In the framework of the effective Hamiltonian (1), the exclusive $B \rightarrow K^{*}\gamma$ decay amplitude is given by

$$A(B \rightarrow K^{*}\gamma) = \langle K^{*}(q) \gamma(p) | \mathcal{H}_{\text{eff}} | B(p+q) \rangle =$$

$$= \frac{4G_F}{\sqrt{2}} V_{cb} V_{cs}^{*} \langle K^{*}(q) | [C_7 O_7 + i\epsilon^\mu \sum_{i \neq 7} C_i \int d^4x \ e^{ipx} T\{j_{\mu}^{em}(x)O_i(0)\}] | B(p+q) \rangle ,$$

(2)

where the four-momenta of the $B$ and $K^{*}$ meson, and the photon are denoted by $p+q$, $q$, and $p$, respectively, while $\epsilon_\mu$ is the photon polarization four-vector. The electromagnetic current is given by

$$j_{\mu}^{em} = \sum_{q=u,d,s,c,b} e_q \bar{q} \gamma_\mu q ,$$

(3)

where $e_q$ are the quark electric charges.

In (2), we have isolated the operator $O_7$, as it is the only operator which contains the photon field at the tree level:

$$O_7 = \frac{e m_b}{16\pi^2} \bar{s}_L \sigma_{\mu\nu} b_R F^{\mu\nu} ,$$

(4)

with $q_{R,L} = \frac{1}{2}(1 \pm \gamma_5)q$, $F^{\mu\nu}$ being the photon field tensor, and $e = \sqrt{4\pi\alpha}$ being the electromagnetic coupling. Note that throughout the paper we put $m_s = 0$. The other operators generate radiative decays when combined with the electromagnetic interaction of quarks with the photon. In this paper, we restrict ourselves to the four-quark operators

$$O_1 = (\bar{c}_L \gamma_\mu c_L)(\bar{s}_L \gamma^\mu b_L) ,$$

(5)

$$O_2 = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L) ,$$

(6)
relying on the fact that the other operators have much smaller short-distance coefficients. To our knowledge, an analysis of long-distance effects in $B \to K^*\gamma$ generated by $O_{3-6}$ and $O_8$ has never been done and is certainly desirable. Perturbative contributions of $O_8$ have been estimated in the relativistic quark model [10] and found to be small.

For $O_2$, the relevant diagrams contributing to the amplitude (3) are shown in Fig. 1. The analogous graphs for the operator $O_1$ vanish for a real photon. After Fierz transformation, the operator $O_2$ reads:

$$O_2 = \frac{1}{3}O_1 + \frac{1}{2}\tilde{O}_1,$$

$$(7)$$

where $\Gamma_\mu = \gamma_\mu(1 - \gamma_5)$. It is only the operator $\tilde{O}_1$ which plays a role. The effective colour transfer in the weak vertex is compensated by a gluon exchange between the $c$–quark loop and other quarks in the process. The photon emission from the $c$–quark loop is therefore a purely nonfactorizable effect.

Keeping in (2) only the operators $O_7$ and $\tilde{O}_1$ and retaining in $j^{e\mu}$ only the $c$–quark part, the decay amplitude simplifies to

$$A(B \to K^*\gamma) = 4G_F\sqrt{2}V_{cb}V_{cs}^\ast\langle K^*(q) | \left[ C_7O_7 + C_2O_F \right] | B(p + q) \rangle.$$  (9)

In this expression, one encounters a product of $\bar{c}\gamma_\mu c$ and $\bar{c}\Gamma_\rho^{\lambda a}c$ currents generating a $c$-quark loop. When the photon is on-shell, the $c$-quark pair is far off-shell. In this situation, we expect that the OPE for the $c$–quark loop in terms of local operators is adequate.

The first term of this expansion is simply a gluon field operator. The corresponding Wilson coefficient is represented by the diagrams of Fig. 4. It can easily be calculated using the fixed-point ($x = 0$) gauge which in the approximation needed reads

$$A_\mu^a(x) = 1/2x^\tau G^a_{\tau\mu}(0).$$

The OPE then results in

$$\int e^{ipx}T\{\bar{c}(x)\gamma_\mu c(x), \bar{c}(0)\Gamma_\rho^{\lambda a}c(0)\} = C_{\alpha\beta\mu\rho}g\tilde{G}^{\alpha\alpha\beta} + \ldots$$

with the short-distance coefficient ($p^2 = 0$),

$$C_{\alpha\beta\mu\rho} = -\frac{i}{48\pi^2m_c^2} \left( p_\alpha p_\beta g_{\beta\mu} + p_\alpha p_\mu g_{\beta\rho} \right),$$

and $\tilde{G}^{\alpha\alpha\beta} = 1/2\epsilon^{\alpha\beta\tau\lambda}G^{\tau\lambda}_{2\gamma}$. Note that in the diagrams of Fig. 4 only the axial part of $\Gamma_\rho^{\lambda a}$ contributes, whereas the contribution of the vector part vanishes due to Furry’s theorem. The higher dimension operators in the expansion (11) denoted by the ellipses contain derivatives of the gluon field and/or additional gluon field operators. They are neglected here. Substituting the expansion (11) into (9), and putting $e_c = (2/3)e$ yields

$$A(B \to K^*\gamma) = 4 \left( \frac{G_F}{\sqrt{2}} \right) V_{cb}V_{cs}^\ast\langle K^*(q) | \left[ C_7O_7 + C_2O_F \right] | B(p + q) \rangle.$$  (13)
where
\[ O_F = -\frac{e}{288\pi^2 m_c^2} \tilde{G}^a \frac{\lambda^a}{2} g G_{\alpha\beta} D^\rho F^{\alpha\beta} b \]  
(14)
is a new effective operator describing the soft gluon interaction sketched in Fig. 2.

Before turning to the estimate of the matrix element of \( O_F \), a comment on the accuracy
of approximation (11) is in order. A similar approximation was used in [17] in estimating
the nonfactorizable contributions to \( B \to D\pi \) (see also [18]). In this analysis, the criteria
for the applicability of OPE are discussed in detail. Following the same line of arguments, we
notice that the next-to-leading operator in (11) will contain a derivative of the gluon field
\( D^\rho \tilde{G} \) multiplied by the photon momentum \( p_\alpha \) and an extra factor of order of \( 1/4m_c^2 \). The overall
suppression factor will thus be of order of \( \mu_p p_0 / 4m_c^2 \) where \( \mu_p \) is a characteristic hadronic
scale, and \( p_0 \simeq m_b/2 \) is the photon energy. Assuming \( \mu_p \simeq 300 \text{ MeV}, m_b \simeq 5 \text{ GeV} \) and
\( m_c \simeq 1.3 \text{ GeV} \) one expects a 10% effect. From this very rough estimate the truncation of (11)
after the first term appears justifiable. On the other hand, one clearly cannot apply the same
approximation to light-quark loops.

Next, we write the matrix element of the magnetic penguin operator in the form
\[ \langle K^*(q) | O_7 | B(p + q) \rangle = \frac{e m_b}{32\pi^2} F (i\epsilon_{\alpha\beta\sigma\tau} q^\tau - 2g_{\sigma\alpha} q_\beta) e_{K^*}^\sigma F^{\alpha\beta}. \]  
(15)
where \( e_{K^*} \) is the polarization vector of the \( K^* \) and \( F_{\mu\nu} = i(e_\nu p_\mu - e_\mu p_\nu) \). In the above, the relation
\[ \sigma_{\mu\nu} \gamma_5 = -\frac{i}{2} \epsilon_{\mu\nu\alpha\beta} \sigma^{\alpha\beta} \]  
(16)
has been used, which leaves only one independent invariant amplitude \( F \). Similarly to (15),
we parametrize the matrix element (13) of the operator \( O_F \) in the form
\[ \langle K^*(q) | O_F | B(p + q) \rangle = -\frac{e}{288\pi^2 m_c^2} (iL \epsilon_{\alpha\beta\sigma\tau} q^\tau - 2\tilde{L} g_{\sigma\alpha} q_\beta) e_{K^*}^\sigma F^{\alpha\beta}. \]  
(17)
Note that the invariant amplitudes \( L \) and \( \tilde{L} \) have dimension \( \text{GeV}^3 \) and that, in general, \( \tilde{L} \neq L \).
Finally, substituting (17) and (15) in (13) one gets
\[ A(B \to K^*\gamma) = \left( \frac{e G_F V_{tb} V_{ts}^*}{8\sqrt{2}\pi^2} \right) \left[ -i \epsilon_{\alpha\beta\sigma\tau} q^\tau \left( -C_7 m_b F + \frac{C_2}{9m_c^2} L \right) + 2g_{\sigma\alpha} q_\beta \left( -C_7 m_b F + \frac{C_2}{9m_c^2} \tilde{L} \right) e_{K^*}^\sigma F^{\alpha\beta}. \]  
(18)
While the amplitude proportional to \( F \) has been investigated previously, the long-distance
corrections proportional to \( L \) and \( \tilde{L} \) are new.

3 QCD sum rule for the \( B \to K^*\gamma \) amplitude

Estimates for the matrix element (15) already exist in the framework of local [4, 5] and light-
cone QCD sum rules [6]. The additional matrix element (17) can in principle be calculated
in the same framework with the same accuracy. The light-cone approach to this matrix element
involves quark-antiquark-gluon wave functions of the \( K^* \) meson. Since these wave functions
are not yet sufficiently known, we shall estimate (17) using the local OPE. Following [4, 5], we consider the three-point correlation function

$$T_{\alpha\beta}^\nu(p, q) F^{\alpha\beta} = -\frac{e}{32\pi^2} \int d^4 x d^4 y \exp \left[-i(p+q)x + i q y\right] \langle 0 | T \left\{ \bar{u}(y) \gamma^\nu s(x)\right\} |0\rangle F^{\alpha\beta} ,$$

(19)

interpolating the complete matrix element in (13), with the $B$ and $K^*$ mesons replaced by appropriate generating currents. Moreover, for the operators $O_T$ and $O_F$ we have substituted the explicit expressions (4) and (14). Isolating in (19) the tensor structures (15) and (17) we write

$$T_{\alpha\beta}^\nu F^{\alpha\beta} = \frac{e}{32\pi^2} \left( -T i\epsilon_{\alpha\beta\nu} q^\nu F^{\alpha\beta} + 2\bar{T} q^\beta F_{\nu\beta} + ... \right) ,$$

(20)

where the invariant amplitudes $T$ and $\bar{T}$ are functions of the two independent variables $(p+q)^2$ and $q^2$.

Inserting in the r.h.s. of (19) the complete sets of hadronic states carrying the $B$ and $K^*$ quantum numbers one obtains the double dispersion relation

$$T((p+q)^2, q^2) = \frac{m_B^2 m_{K^*} f_B f_{K^*}}{m_B(m_B^2 - (p+q)^2)(m_{K^*}^2 - q^2)} \left( C_T m_b F - \frac{C_2}{9m_c^2} \tilde{L} \right)$$

$$+ \int ds_1 ds_2 \frac{\rho_h(s_1, s_2)}{(s_1 - (p+q)^2)(s_2 - q^2)}$$

(21)

where we have used (15) and (17) together with

$$\langle B \mid \bar{b} \gamma_5 u \mid 0 \rangle = \frac{m_B^2}{m_b} f_B ,$$

(22)

$$\langle 0 \mid \bar{u} \gamma_\nu s \mid K^* \rangle = m_{K^*} f_{K^*} e_{K^*\nu} .$$

(23)

The integral over the double spectral density $\rho_h$ in (21) represents the contributions of excited $B$ and $K^*$ states as well as of continuum contributions. Possible subtraction terms are omitted for brevity. The analogous dispersion relation for $\bar{T}$ is obtained from (21) by the replacements $L \rightarrow \tilde{L}$ and $\rho_h \rightarrow \tilde{\rho}_h$.

Furthermore, at $p^2 = 0$ and at large spacelike values of $(p+q)^2$ and $q^2$, the amplitudes $T$ and $\bar{T}$ can be evaluated in QCD in terms of vacuum expectation values of certain local operators $\Omega_d$:

$$T_{QCD}((p+q)^2, q^2) = \sum_d \left[ C_T m_b T_d((p+q)^2, q^2) - \frac{C_2}{9m_c^2} U_d((p+q)^2, q^2) \right] \langle \Omega_d \rangle .$$

(24)

A similar representation holds for $\bar{T}_{QCD}$ with $U_d$ replaced by $\tilde{U}_d$. The index $d$ refers to the dimension of the operators. In perturbation theory, only the $d = 0$ unit operator contributes to (24). The nonperturbative effects are accumulated in terms with $d > 0$. In this analysis, we take into account $\langle \Omega_3 \rangle = \langle \bar{q} q \rangle$, $\langle \Omega_4 \rangle = \langle \alpha_s^2 G_{\mu\nu} G^{\mu\nu} \rangle$, $\langle \Omega_5 \rangle = m_\sigma^2 \langle \bar{q} q \rangle$, and $\langle \Omega_6 \rangle = \langle \bar{q} q \rangle^2$. These are the standard parametrizations for gluon and quark-gluon condensate densities. For the four-quark condensate the vacuum insertion approximation [12] is used.
Following the usual procedure, one equates (21) and (24), and replaces the double spectral density \( \rho_h \) in (24) by the double imaginary part of the amplitude \( T_{QCD} \). Finally, the Borel transformation

\[
\hat{B}_{M^2} f(Q^2) \equiv \lim_{Q^2 \to \infty, n \to \infty, Q^2/n = M^2} \left( \frac{d^{n+1}}{dQ^2} \right)^n f(Q^2) \equiv f(M^2) \tag{25}
\]

with respect to the variables \((p + q)^2\) and \(q^2\) is applied, in order to suppress exponentially the contribution of higher states and to get rid of subtraction terms in the dispersion relations. The second invariant amplitude \( \tilde{T} \) is treated in the same way.

This yields the sum rule

\[
C_7 m_b F - \frac{C_2}{9m_c^2} L = \frac{m_b}{M_B^2 m_K^* f_B f_{K^*}} e^{m_{b}/M_1^2 + m_{K^*}/M_2^2} e^{s_1/M_1^2 - s_2/M_2^2} \times \left\{ \frac{1}{\pi^2} \int \int \{ s_0^B s_0^{K^*} \} ds_1 ds_2 \left[ C_7 m_b I_{m_1} I_{m_2} T_0(s_1, s_2) - \frac{C_2}{9m_c^2} I_{m_1} I_{m_2} U_0(s_1, s_2) \right] e^{-s_1/M_1^2 - s_2/M_2^2} + \sum_{d \neq 0} \left[ C_7 m_b T_d(M_1^2, M_2^2) - \frac{C_2}{9m_c^2} U_d(M_1^2, M_2^2) \right] \langle \Omega_d \rangle \right\} . \tag{26}
\]

where the threshold parameters \( s_0^B \) and \( s_0^{K^*} \) indicate the subtraction of higher state contributions. A second sum rule is obtained from (26) by replacing \( L \to \tilde{L} \) and \( U_d \to \tilde{U}_d \).

In the approximation considered, the above sum rule can obviously be divided into two separate sum rules, one for \( F \) (terms involving \( T_d \)) and one for \( L \) (terms involving \( U_d \)). The sum rule for \( F \) have already been analyzed \[3\] including terms up to \( d = 6 \), in lowest order in \( \alpha_s \). In the following, we calculate the sum rule for the long-distance corrections \( L \) and \( \tilde{L} \) in the same approximation.

### 4 Calculation of the long-distance amplitudes

We need to calculate the coefficients \( U_d \) and \( \tilde{U}_d \) up to \( d = 6 \). The coefficients \( U_0, \tilde{U}_0, U_3 \) and \( \tilde{U}_3 \) are of order \( \alpha_s \) as is obvious from Fig. 5. Since the \( O(\alpha_s) \) corrections to the sum rule for \( F \) (exemplified in Fig. 5a,d) are not taken into account in \[3\] we put \( U_0 = \tilde{U}_0 = U_3 = \tilde{U}_3 = 0 \) in (26). A systematic inclusion of these effects would require two-loop calculations and a proper treatment of operator mixing. Thus, to the requested accuracy, we have

\[
L = \frac{m_b}{M_B^2 m_{K^*} f_B f_{K^*}} e^{m_{b}/M_1^2 + m_{K^*}/M_2^2} \left\{ U_4(M_1^2, M_2^2) \left( \frac{\alpha_s}{\pi} C_{\mu\nu}^a G_{\mu\nu} \right) \right. \\
+ U_5(M_1^2, M_2^2) m_b^2 \langle \bar{q}q \rangle + U_6(M_1^2, M_2^2) \langle \bar{q}q \rangle^2 \right\} , \tag{27}
\]

and a similar expression for the amplitude \( \tilde{L} \) with \( U_{4,5,6} \) replaced by \( \tilde{U}_{4,5,6} \).

In order to calculate the Wilson coefficients \( U_4 \) and \( \tilde{U}_4 \), one has to contract all quark fields in the second term of the correlation function \[19\]. One of the three resulting quark propagators is substituted by the first order expression in the external gluon field \( A_{\mu}^a \), while for the other two propagators the free approximation is used. This yields the diagrams of Fig. 6a, b, c. The calculation of these diagrams is performed in the Fock-Schwinger gauge \[10\]. Furthermore, when expressing the result in form of a double dispersion integral in the
variables \((p + q)^2\) and \(q^2\) one may omit all terms which depend only on one of these variables since such terms vanish after Borel transformation. The double dispersion contribution of the diagram Fig. 6c vanishes altogether. The result is

\[ U_4 = U_4^{(Fig.6a)} + U_4^{(Fig.6b)} , \]  

with

\[ U_4^{(Fig.6a)} = \frac{-m_b}{48} \int_{m_b^2}^{\infty} ds_1 \int_0^{s-m_b^2} ds_2 \left[ \frac{1}{s_1 - (p + q)^2} \frac{1}{s_2 - q^2} \right] \],

\[ U_4^{(Fig.6b)} = \frac{m_b^3}{48} \int_{m_b^2}^{\infty} ds_1 \int_0^{s-m_b^2} ds_2 \frac{1}{(s_2 - q^2)(s_1 - s_2)} \left[ \frac{1}{s_1 - (p + q)^2} + \frac{1}{s_2 - q^2} \right] , \]

and

\[ \tilde{U}_4 = \tilde{U}_4^{(Fig.6a)} + \tilde{U}_4^{(Fig.6b)} , \]

with

\[ \tilde{U}_4^{(Fig.6a)} = -U_4^{(Fig.6a)} , \quad \tilde{U}_4^{(Fig.6b)} = U_4^{(Fig.6b)} . \]

The coefficients \(U_5\) and \(\tilde{U}_5\) of the \(d = 5\) quark-gluon condensate term are obtained from Fig. 6d which involves the vacuum average of the \(\bar{u}uG\) operator. The analogous diagram, involving \(\bar{s}sG\) depends only on \(q^2\) and can therefore be dropped. One finds

\[ U_5 = \tilde{U}_5 = -\frac{1}{12} \frac{m_b^2}{(m_b^2 - (p + q)^2)(-q^2)} . \]

There are two \(d = 6\) contributions to the correlation function \([19]\). One of them originates from expanding \(\bar{u}(y)u(x)\) at \(x = y = 0\). In vacuum insertion approximation, the vacuum average of the resulting operator \(\bar{u}\nabla_{\rho}uG^a_{\mu\nu}\) is expressed in terms of \(\langle \bar{q}q \rangle^2\) according to the formula given in \([19]\). The corresponding coefficients derived from Fig. 6d read

\[ U_6^{(Fig.6d)} = \tilde{U}_6^{(Fig.6d)} = \frac{4\pi\alpha_s m_b}{27} \frac{1 - m_b^2/q^2}{(-q^2)(m_b^2 - (p + q)^2)} . \]

The second \(d = 6\) contribution comes from the four-quark condensate. The relevant diagrams are shown in Fig. 6e and 6f. The coefficients are

\[ U_6^{(Fig.6e,f)} = 0 , \]

\[ \tilde{U}_6^{(Fig.6e,f)} = \frac{4\pi\alpha_s m_b}{9} \frac{1 - m_b^2/q^2}{(-q^2)(m_b^2 - (p + q)^2)} . \]

In total, we have

\[ U_6 = U_6^{(Fig.6d)} , \]

\[ \tilde{U}_6 = \tilde{U}_6^{(Fig.6d)} + \tilde{U}_6^{(Fig.6e,f)} . \]

After double Borel transformation \([23]\) of the above expressions for \(U_d\) and \(\tilde{U}_d\) in \((p + q)^2\) and \(q^2\), and substitution in \([27]\) and in the analogous expression for \(\tilde{L}\), we obtain the final sum rules

\[ L = \frac{m_b}{m_{K^*} m_B^2 f_{BJK^*}} \exp \left( \frac{m_B^2}{M_1^2} + \frac{m_{K^*}^2}{M_2^2} \right) . \]
With (39) and (40) we find for the relative magnitude of the long-distance corrections in (18): \[ r = \left( \frac{C_2}{-9C_7} \right) \frac{L}{m_b m_c^2 F} \approx 0.02 \div 0.04 , \]

\[ \times \left\{ \frac{m_b}{48} \frac{\alpha_s \langle GG \rangle}{\pi} \int_{m_b^2 / m_1^2}^{\infty} d\tau \exp(-\tau) \left[ \frac{m_b^2}{\tau} - \frac{M_1^4}{M_1^2 + M_2^2} \left( 1 - \frac{M_1^2}{\tau M_1^2} \right) \left( 1 + \frac{M_2^2}{\tau M_1^2} \right) \right] \right\} - \left[ \frac{m_b^2 \langle \bar{q}q \rangle}{12} - \frac{4\pi \alpha_s \langle \bar{q}q \rangle^2 m_b}{27} \right] \left( 1 + \frac{m_b^2}{M_2^2} \right) \exp \left( -\frac{m_b^2}{M_2^2} \right) \right\} , \] 

\[ \tilde{L} = \frac{m_b}{m_{K^*} m_B f_B f_{K^*}} \exp \left( \frac{m_B^2}{M_1^2} + \frac{m_{K^*}^2}{M_2^2} \right) \times \left\{ \frac{m_b}{48} \frac{\alpha_s \langle GG \rangle}{\pi} \int_{m_b^2 / m_1^2}^{\infty} d\tau \exp(-\tau) \left[ \frac{m_b^2}{\tau} + \frac{M_1^4}{M_1^2 + M_2^2} \left( 1 - \frac{M_1^2}{\tau M_1^2} \right) \left( 1 + \frac{M_2^2}{\tau M_1^2} \right) \right] \right\} - \left[ \frac{m_b^2 \langle \bar{q}q \rangle}{12} - \frac{16\pi \alpha_s \langle \bar{q}q \rangle^2 m_b}{27} \right] \left( 1 + \frac{m_b^2}{M_2^2} \right) \exp \left( -\frac{m_b^2}{M_2^2} \right) \right\} . \] 

\[ (37) \]

\[ (38) \]

5 Numerical estimates

For the numerical estimate of the amplitudes \( L \) and \( \tilde{L} \) we take the values of the physical parameters also used in [3], that is \( m_b = 4.8 \) GeV, \( \langle \frac{\alpha_s}{\pi} \langle GG \rangle \rangle = 0.012 \) GeV\(^4\), \( \langle \bar{q}q \rangle = -(240 \) MeV\(^3\) at the scale \( \mu = 1 \) GeV, \( m_{b0}^2 = 0.8 \) GeV\(^2\), \( \Lambda_{QCD}^{(4)} = 230 \) MeV, \( f_{K^*} = 210 \) MeV, and \( f_B = 140 \) MeV. The latter value results from the corresponding two-point sum rule without \( O(\alpha_s) \) corrections and should not be confused with the physical value of \( f_B \) which is larger. We also use the same interval of the Borel mass \( M_1 \), \( 7 \leq M_1^2 \leq 10 \) GeV\(^2\), keeping the ratio \( M_1^2 / M_2^2 = 3 \) fixed. The characteristic scale at which the amplitudes are estimated by the sum rules is of order of the average Borel parameter that is, of order of the virtuality of the quarks in (19). In [3], the scale \( \mu_B = (M_1 M_2)^{1/4} \simeq 2 \) GeV is chosen for evaluation of \( F \). Neglecting an inessential evolution of the quark-gluon condensate and of the product \( \alpha_s \langle \bar{q}q \rangle^2 \) from \( \mu = 1 \) GeV to \( \mu = \mu_B \), the sum rules (37) and (38) yield:

\[ L = 0.55 \pm 0.1 \) GeV\(^3\), \]

\[ \tilde{L} = 0.70 \pm 0.1 \) GeV\(^3\). \] 

(39)

The theoretical uncertainty quoted here reflects only the variation of \( L \) and \( \tilde{L} \) with the Borel masses \( M_1 \) and \( M_2 \) in the accepted interval. In contrast to [3], we have neglected the finite strange quark mass effect which however makes no difference numerically. The dominant contributions in (37) and (38) come from the quark-gluon condensate (75 \%) and gluon condensate (15-20 \%), while the \( d = 6 \) terms do not exceed 10\%. The deviation of \( \tilde{L} \) from \( L \) which is numerically not very significant is generated partly by the gluon condensate and partly by the four-quark condensate.

For comparison, we quote the corresponding prediction for the penguin amplitude (at \( \mu = \mu_B \)) given in [3]:

\[ F = 0.9 \pm 0.1 \]. \] 

(40)

With (39) and (40) we find for the relative magnitude of the long-distance corrections in (18):

\[ r = \left( \frac{C_2}{-9C_7} \right) \frac{L}{m_b m_c^2 F} \simeq 0.02 \div 0.04 , \]
\[ \tilde{r} = \left( \frac{C_2}{-9C_7} \right) \frac{\bar{L}}{m_b m_c^2 F} \simeq 0.03 \div 0.05, \]  
(41)

Apart from \( m_b = 4.8 \text{ GeV} \) and \( m_c = 1.3 \text{ GeV} \) we have used here \( C_2 = 1.14 \) and \( C_7 = -0.30 \), that is the leading-logarithmic approximation of the short-distance coefficients at \( \mu \simeq m_b \) [2]. Since both estimates (39) and (40) are obtained by the same method and in the same approximation, one can hope that many of the uncertainties cancel in the above ratios.

6 Conclusion

In this paper, we have estimated the long-distance effect in \( B \to K^*\gamma \) due to interactions of the virtual \( c \)-quark loop with soft gluons. Using OPE we have derived a new quark-gluon-photon operator \( O_F \) given in (14) which is responsible for the effect. Furthermore, we have obtained a QCD sum rule for the matrix element \( \langle K^*\gamma | O_F | B \rangle \) taking into account the most important nonperturbative contributions. Quantitatively, the correction is found to increase the magnetic penguin amplitude by a few percent. This result indicates that long-distance effects do not play an important role in the exclusive decay \( B \to K^*\gamma \).

The estimate presented here is not free from theoretical uncertainties. They are partly due to the lack of knowledge of the perturbative \( O(\alpha_s) \) corrections to the Wilson coefficients. Related to that is the ambiguity in the choice of the scale \( \mu \). In addition, there are uncertainties inherent to the short-distance sum rules. It would therefore be desirable to repeat the calculation using light-cone sum rules for a cross-check. However, the latter approach requires a better understanding of the higher twist light-cone wave functions of the \( K^* \) meson.

When this work was completed, the paper [20] appeared in which the nonperturbative correction to the inclusive \( B \to X_s\gamma \) rate due to soft gluon emission by the \( c \)-quark loop is estimated. The responsible effective operator obtained in [20] coincides with our operator (14). After revision of [20], the effect predicted for the inclusive rate is similar in size to the correction found here for the exclusive channel \( B \to K^*\gamma \), however, opposite in sign.

7 Acknowledgements

One of the authors (A.K.) is grateful to P. Ball for a useful discussion concerning her work [3]. The work of A.K. and R.R. is supported by the Bundesministerium für Bildung und Forschung (BMBF) under contract 05 7WZ91P(0) and by the EC program HCM under contract CHRX-CT93-0132. The work of G.S. and D.W is supported by Schweizerischer Nationalfond.
References

[1] R. Ammar et al. (CLEO Collab.), *Phys. Rev. Lett.* **71** (1993) 674.

[2] G. Buchalla. A.J. Buras and M.E. Lautenbacher, *Rev. Mod. Phys.* **68** (1996) 1125.

[3] T. Altomari, *Phys. Rev.* **D37** (1988) 677;
N. Isgur and M. Wise, *Phys. Rev.* **D42** (1990) 2388;
D. Wyler, *Nucl. Phys. (Proc. Suppl.)* **7A** (1989) 358;
A. Ali, T. Ohl and T. Mannel, *Phys. Lett.* **B298** (1993) 195.

[4] C.A. Dominguez, N. Paver and Riazuddin, *Phys. Lett.* **B214** (1988) 459;
T.M. Aliev, A.A. Ovchinnikov and V.A. Slobodeniuk, *Phys. Lett.* **B237** (1990) 569;
S. Narison, *Phys. Lett.* **B327** (1994) 354;
P. Colangelo, C.A. Dominguez, G. Narduli and N. Paver, *Phys. Lett.* **B317** (1994) 354.

[5] P. Ball, preprint hep-ph/9308244 (unpublished).

[6] A. Ali, V.M. Braun and H. Simma, *Z. Phys.* **C63** (1994) 437.

[7] D.R. Burford et al. (UKQCD Collab.), *Nucl. Phys.* **B447** (1995) 425.

[8] A. Ali and C. Greub, *Phys. Lett.* **B361** (1995) 146.

[9] N. Pott, *Phys. Rev.* **D54** (1996) 938.

[10] C. Greub, T. Hurth and D. Wyler, *Phys. Rev.* **D54** (1996) 3350.

[11] K. Chetyrkin, M. Misiak and M. Münz, Preprint ZU-TH 24/96, TUM-HEP-263/96, hep-ph/9612313.

[12] J. M. Soares, *Phys. Rev.* **D51** (1995) 3518;
E. Golowich and S. Pakvasa, *Phys. Rev.* **D51** (1995) 1215;
N. Deshpande, X.-G. He and J. Trampetic, *Phys. Lett.* **B367** (1996) 362;
D. Atwood, B. Blok and A. Soni, *Int. J. Mod. Phys.* **A11** (1996) 3743.

[13] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Nucl. Phys.* **B147** (1979) 385, 448.

[14] A. Khodjamirian, G. Stoll and D. Wyler, *Phys. Lett.* **B358** (1995) 129.

[15] A. Ali and V. Braun, *Phys. Lett.* **B359** (1995) 223.

[16] C. Carlson and J. Milana, *Phys. Rev.* **D51** (1995) 4950.

[17] B. Blok and M. Shifman, *Nucl. Phys.* **B389** (1993) 534.

[18] I. Halperin, *Phys. Lett.* **B349** (1995) 548.

[19] B.L. Ioffe and A.V. Smilga, *Nucl. Phys.* **B216** (1983) 373.

[20] M.B. Voloshin, preprint TPI-MINN-96/30-T, hep-ph/9612483.
Figure 1: Diagrams contributing to the $B \to K^*\gamma$ amplitude generated by the operator $O_2$ (black circle).

Figure 2: Interaction of the virtual $c$-quark loop with soft gluons.

Figure 3: Photon emission by quarks at long-distances in (a) $B \to K^*\gamma$ and (b) $B \to \rho\gamma$. 
Figure 4: Diagrams contributing to the operator product expansion of the $c$-quark loop.

Figure 5: Diagrams determining the $O(\alpha_s)$ corrections to the Wilson coefficients of the three-point correlation function (19). The $\bar{s}b$ vertex involves $O_7$, while the $\bar{s}bG$ vertex involves $O_F$.

Figure 6: Diagrams yielding the coefficients (a-c) $U_4$, $\bar{U}_4$, (d) $U_5$, $\bar{U}_5$, and (d-f) $U_6$, $\bar{U}_6$ in the sum rules for $L$ and $\bar{L}$. 