Unifying inflation with LambdaCDM epoch in modified f(R) gravity consistent with Solar System tests

Shin’ichi Nojiri
Department of Physics, Nagoya University, Nagoya 464-8602, Japan

Sergei D. Odintsov
Institució Catalana de Recerca i Estudis Avançats (ICREA) and Institut de Ciencies de l’Espai (IEEC-CSIC), Campus UAB, Facultat de Ciencies, Torre C5-Par-2a pl, E-08193 Bellaterra (Barcelona), Spain

We suggest two realistic f(R) and one F(G) modified gravities which are consistent with local tests and cosmological bounds. The typical property of such theories is the presence of the effective cosmological constant epochs in such a way that early-time inflation and late-time cosmic acceleration are naturally unified within single model. It is shown that classical instability does not appear here and Newton law is respected. Some discussion of possible anti-gravity regime appearance and related modification of the theory is done.

PACS numbers: 11.25.-w, 95.36.+x, 98.80.-k

1. It is expected that modified gravity (for a review, see [1]) approach which may be related with string/M-theory [2] may help in the understanding of the dark energy origin. For instance, various modified f(R) gravities have been constructed and applied to the description of the late-time cosmic acceleration [3, 4, 5, 6, 7, 8] with also the check of local tests. Even the form of modified f(R) gravity may be reconstructed from the known universe expansion history [9]. Hence, such gravitational alternative for dark energy may be considered as an alternative gravity theory subject that it passes cosmological bounds and Solar System tests.

Recently, it has been proposed the class of modified gravities with the effective cosmological constant epoch [7, 11, 12, 13, 15]. The very simple versions of such theories [11, 12, 13, 15] with vanishing cosmological constant may describe successfully the ΛCDM epoch and pass the local tests/cosmological bounds. Nevertheless, the early universe epoch is not included there and deviations from Newton law at large scales may occur.

In the present letter we suggest two realistic new models of modified f(R) gravity which may describe the early-time inflation and late-time acceleration in the unified manner, extending the earlier proposal of ref. [4]. Moreover, such theories successfully pass the Solar System tests as well as simplest cosmological bounds and they are free of instabilities. Nevertheless, the first of suggested models may develop the anti-gravity regime, that is why its simple modification to avoid this problem is done. Finally, viable F(G) gravity which also unifies inflation with late-time acceleration and may pass the local tests/cosmological bounds is suggested.

2. The action of general f(R) gravity is given by

\[ S = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} (R + f(R)) . \tag{1} \]

Here f(R) is an arbitrary function. The equation of motion in f(R)-gravity with matter is given by

\[ \frac{1}{2} g_{\mu\nu} F(R) - R_{\mu\nu} F'(R) - g_{\mu\nu} \Box F'(R) + \nabla_\mu \nabla_\nu F'(R) = -\frac{\kappa^2}{2} T_{(m)\mu\nu} . \tag{2} \]

Here \( F(R) = R + f(R) \) and \( T_{(m)\mu\nu} \) is the matter energy-momentum tensor.

Recently an interesting f(R) model has been proposed in [11]. In the model f(R) is given by

\[ f_{HS}(R) = -\frac{m^2 c_1}{c_2 (R/m^2)^n + 1} , \tag{3} \]

which satisfies the condition

\[ \lim_{R \to \infty} f_{HS}(R) = \text{const}, \]
\[ \lim_{R \to 0} f_{HS}(R) = 0 . \tag{4} \]
The second condition means that there could be a flat spacetime solution (vanishing cosmological constant). The estimation of ref. [11] suggests that \( R/m^2 \) is not so small but rather large even in the present universe and \( R/m^2 \sim 41 \). Hence,

\[
f_{HS}(R) \sim -\frac{m^2c_1}{c_2} + \frac{m^2c_1}{c_2} \left( \frac{R}{m^2} \right)^{-n},
\]

which gives an "effective" cosmological constant \(-m^2c_1/c_2\) and generates the late-time accelerating expansion. One can show that

\[
H^2 \sim \frac{m^2c_1\kappa^2}{c_2} \sim (70 \text{km/s} \cdot \text{pc})^2 \sim (10^{-33} \text{eV})^2.
\]

At the intermediate epoch, where the matter density \( \rho \) is larger than the effective cosmological constant,

\[
\rho > \frac{m^2c_1}{c_2},
\]

there appears the matter dominated phase and the universe expands with deceleration. Hence, above model describes the effective ΛCDM cosmology.

Although the model [11] is very successful (see, however, ref. [12] for description of the Newton law at large scales here), the early time inflation is not included there. We suggest the modified gravity model to treat the inflation and the late-time accelerating expansion in a unified way. We now consider simple extention of the model [11] to include the inflation at the early universe. In order to generate the inflation, one may require

\[
\lim_{R \to \infty} f(R) = -\Lambda_i.
\]

Here \( \Lambda_i \) is an effective cosmological constant at the early universe and therefore we assume \( \Lambda_i \gg (10^{-33} \text{eV})^2 \). We may assume \( \Lambda_i \sim 10^{20-38} \). Instead of (8), as in Starobinsky’s model, one may require

\[
\lim_{R \to \infty} f(R) \propto R^2.
\]

In order that the accelerating expansion in the present universe could be generated, let us consider that \( f(R) \) could be a small constant at present universe, that is,

\[
f(R_0) = -2R_0, \quad f'(R_0) \sim 0.
\]

Here \( R_0 \) is current curvature \( R_0 \sim (10^{-33} \text{eV})^2 \). The next condition corresponding to the second one in (4) is:

\[
\lim_{R \to 0} f(R) = 0.
\]

In the above class of models, the universe starts from the inflation driven by the effective cosmological constant [8] at the early stage, where curvature is very large. As curvature becomes smaller, the effective cosmological constant also becomes smaller. After that the radiation/matter dominates. When the density of the radiation and the matter becomes small and the curvature goes to the value \( R_0 \) [10], there appears the small effective cosmological constant [10]. Hence, the current cosmic expansion could start.

An example satisfying [8], [10], and [11] is

\[
f(R) = -\frac{(R-R_0)^{2n+1} + R_0^{2n+1}}{f_0 + f_1 \left\{ (R-R_0)^{2n+1} + R_0^{2n+1} \right\}} = \frac{1}{f_1} + \frac{f_0/f_1}{f_0 + f_1 \left\{ (R-R_0)^{2n+1} + R_0^{2n+1} \right\}}.
\]

Here \( n \) is a positive integer, \( n = 1, 2, 3, \cdots \) and

\[
\frac{R_0^{2n+1}}{f_0 + f_1 R_0^{2n+1}} = 2R_0, \quad \frac{1}{f_1} = \Lambda_i,
\]

that is

\[
f_0 = \frac{R_0^{2n}}{2} - \frac{R_0^{2n+1}}{\Lambda_i} \sim \frac{R_0^{2n}}{2}, \quad f_1 = \frac{1}{\Lambda_i}.
\]
By introducing the auxiliary field $A$ one may rewrite the action (11) in the following form:

$$S = \frac{1}{\kappa^2} \int d^4 x \sqrt{-g} \left\{ \left( 1 + f'(A) \right) (R - A) + A + f(A) \right\} .$$

(15)

From the equation of motion with respect to $A$, it follows $A = R$. By using the scale transformation $g_{\mu\nu} \to e^\sigma g_{\mu\nu}$ with $\sigma = -\ln (1 + f'(A))$, we obtain the Einstein frame action:

$$S_E = \frac{1}{\kappa^2} \int d^4 x \sqrt{-g} \left\{ R - \frac{3}{2} \left( \frac{F''(A)}{F'(A)} \right)^2 g^{\sigma\rho} \partial_\sigma A \partial_\rho A - \frac{A}{F'(A)} + \frac{F(A)}{F'(A)^2} \right\} ,$$

$$V(\sigma) = e^{\sigma} g \left( e^{-\sigma} - e^{2\sigma} f \left( g \left( e^{-\sigma} \right) \right) \right) = \frac{A}{F'(A)} - \frac{F(A)}{F'(A)^2} .$$

(16)

(17)

Here $g \left( e^{-\sigma} \right)$ is given by solving $\sigma = -\ln (1 + f'(A)) = \ln F'(A) \quad \text{as} \quad A = g \left( e^{-\sigma} \right)$. After the scale transformation $g_{\mu\nu} \to e^\sigma g_{\mu\nu}$, there appears a coupling of the scalar field $\sigma$ with the matter. For example, if the matter is the scalar field $\Phi$ with mass $M$, whose action is given by

$$S_\phi = \frac{1}{2} \int d^4 x \sqrt{-g} \left( -g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - M^2 \Phi^2 \right) ,$$

(18)

there appears a coupling with $\sigma$ in the Einstein frame:

$$S_{\phi E} = \frac{1}{2} \int d^4 x \sqrt{-g} \left( -e^{\sigma} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - M^2 e^{2\sigma} \Phi^2 \right) .$$

(19)

The strength of the coupling is of the gravitational coupling $\kappa$ order. Unless the mass of $\sigma$, which is defined by

$$m_\sigma^2 \equiv \frac{1}{2} \frac{d^2 V(\sigma)}{d\sigma^2} = \frac{1}{2} \left\{ \frac{A}{F'(A)} - \frac{4F(A)}{(F'(A))^2} + \frac{1}{F''(A)} \right\} ,$$

(20)

is large, there appears the large correction to the Newton law.

In air on the earth, the scalar curvature could be given by $A \sim R \sim 10^{-50} \text{eV}^2$. On the other hand, in the solar system, we find $A \sim R \sim 10^{-61} \text{eV}^2$. In the model (12) with (13), if $n \geq 3$ in the air or $n \geq 10 \sim 12$ (if $\Lambda_i \sim 10^{20 \sim 38}$) in the solar system, we find

$$f_0 \ll f_1 \left\{ (R - R_0)^{2n+1} + R_0^{2n+1} \right\} \sim f_1 R^{2n+1}$$

(21)

and

$$F(R) = R + f(R) \sim R - \frac{1}{f_1} + \frac{f_0}{f_1 R^{2n+1}} \sim \frac{1}{f_1} .$$

(22)

Furthermore, if $n > 6 \sim 7$ (if $\Lambda_i \sim 10^{20 \sim 38}$) in the air, we find $F'(R) \sim 1$ and

$$m_\sigma^2 \sim -\frac{F(A)}{F'(A)^2} \sim \frac{2}{f_1} \sim 10^{38 \sim 56} \text{eV}^2 ,$$

(23)

which is very large and there is no observable correction to the Newton law. We should note $1 \text{mm} \sim (10^{-4} \text{eV})^{-1}$. On the other hand, in the Solar System, if $n \gg 10 \sim 12$ (if $\Lambda_i \sim 10^{20 \sim 38}$),

$$m_\sigma^2 \sim \frac{1}{2F'(A)} \sim \frac{f_0}{f_1 R^{2n+3}} \sim 10^{239 \sim 295 \sim 10 n} \text{eV}^2 ,$$

(24)

which is large enough since the radius of earth is about $10^7 \text{m} \sim 10^{-14} \text{eV}$ and there is no visible correction to the Newton law.
There may exist another type of instability (so-called matter instability) in \( f(R) \) gravity \[10\] (see, also \[12, 14\]). The instability might occur when the curvature is rather large, as on the planet, compared with the average curvature at the universe \( R \sim \left(10^{-33} \text{eV}\right)^2 \). By multiplying Eq.\((2)\) with \( g^\mu\nu \), one obtains

\[
\Box R + \frac{F''(R)}{F(R)} \nabla_\mu R \nabla^\mu R + \frac{F''(R)R}{3F'(R)} - \frac{2F(R)}{3F'(R)} = \frac{\kappa^2}{6F'(R)} T .
\]  

(25)

Here \( T = T_{(m,p)} \) and \( F^{(n)}(R) = d^n F(R) / dR^n \). We consider a perturbation from the following solution of the Einstein gravity:

\[
R = R_b + R_p , \quad (|R_p| \ll |R_b|) .
\]  

(26)

Note that \( T \) is negative since \( |p| \ll \rho \) on the earth and \( T = -\rho + 3p \sim -\rho \). Then we assume

\[
R = R_b + R_p , \quad (|R_p| \ll |R_b|) .
\]  

(27)

Now one can get

\[
0 = \Box R_p + \frac{F''(R_b)}{F(R_b)} \nabla_\mu R_b \nabla^\mu R_b + \frac{F'(R_b)R_b}{3F'(R_b)} - \frac{2F(R_b)}{3F'(R_b)} - \frac{R_b}{3F'(R_b)}
\]

\[
+ \Box R_p + 2\frac{F''(R_b)}{F'(R_b)^2} \nabla_\mu R_b \nabla^\mu R_p + \Box(R_b)R_p ,
\]

\[
U(R_b) \equiv \left( \frac{F'(R_b)}{F'(R_b)} \frac{F'(R_b)^2}{F'(R_b)^2} \right) \nabla_\mu R_b \nabla^\mu R_b + \frac{R_b}{3} - \frac{F'(R_b)F'(R_b)R_b}{3F'(R_b)^2} - \frac{F'(R_b)F'(R_b)R_b}{3F'(R_b)^2} - \frac{F'(R_b)F'(R_b)R_b}{3F'(R_b)^2} .
\]  

(28)

Since \( \Box R_p \sim -\partial^2_t R_p \), Eq.\((25)\) has the following form:

\[
0 = -\partial^2_t R_p + U(R_b)R_p + \text{const.} .
\]  

(29)

Then if \( U(R_b) \) is positive, \( R_p \) becomes exponentially large as a function of \( t \): \( R_p \sim e^{\sqrt{U(R_b)t}} \) and the system becomes unstable. In the model \[12\] with \[13\], if \( n > 2 \), we find

\[
U(R_b) \sim - \frac{(2n + 3)f_1 R_0^{2n + 2}}{3(2n + 1) f_0} < 0 .
\]  

(30)

Therefore there is no this kind of the instability.

Hence, in the model \[12\], the universe could start from the inflation. As curvature becomes smaller, the effective cosmological constant becomes small and after that the radiation/matter could dominate. When the density of the radiation and, later, of the matter becomes small and the curvature goes to the value \( R_0 \), there could appear the small effective cosmological constant and the late-time accelerating expansion starts.

3. As is clear from \[15\], if \( f'(R) = 1 + f'(R) > 0 \), the square of the effective gravitational coupling becomes negative \( \kappa^2_{\text{eff}} = \kappa^2 / f'(A) \) and theory enters the anti-gravity regime. Nevertheless, if the theory enters such regime at the future, such theory may still be viable. We now check if the model \[12\] could pass the anti-gravity constraint. Since

\[
f'(R) = \frac{(2n + 1)f_0 (R - R_0)^2}{f_0 + f_1 \left\{ (R - R_0)^{2n + 1} + \frac{R_0^{2n + 1}}{f_1} \right\}^2} ,
\]

\[
f''(R) = \frac{2(2n + 1)f_0 (R - R_0)^{2n - 1} \left\{ (R - R_0)^{2n + 1} + f_1(n + 1) (R - R_0)^{2n + 1} \right\}}{(f_0 + f_1 \left\{ (R - R_0)^{2n + 1} + \frac{R_0^{2n + 1}}{f_1} \right\})^3} ,
\]  

(31)

\( f'(R) \) has a maximum at \( R = \tilde{R} \), which satisfies

\[
(R - R_0)^{2n + 1} = \frac{f_0 + f_1 R_0^{2n + 1}}{f_1(n + 1)} \sim \frac{f_0}{f_1(n + 1)} .
\]  

(32)
Then the maximum value of \( f'(R) \) is given by

\[
f'(\tilde{R}) \sim -\frac{f_0}{f_1^{2n+1}} \sim (R_0 f_1)^{-\frac{2}{2n+1}} \ll -1.
\]  

(33)

Here we have used (14). Therefore \( F'(R) < 0 \) and the anti-gravity regime could occur in general. We can consider only the region where \( F'(R) > 0 \) or \( f'(R) > -1 \) without the the anti-gravity problem. From (32), however, one gets

\[
\tilde{R} \sim \left(10^{-33+\frac{4n-32}{2n+1}} \text{eV}\right)^2,
\]  

(34)

which could be small compared with the curvature at the early universe. There are two solutions for the equation \( f'(R) = -1 \). Let denote the larger one by \( R_+ \) and smaller one by \( R_- \). By assuming

\[
f_0 \ll f_1 \left\{ (R_+ - R_0)^{2n+1} + R_0^{2n+1}\right\} \sim f_1 R_+^{2n+1},
\]

(35)

we find

\[
R_+ \sim \left(\frac{(2n+1)f_0}{f_1^2}\right)^{1/(2n+1)} \sim \left(10^{-33+\frac{4n-32}{2n+1}} \text{eV}\right)^2.
\]

(36)

On the other hand, by assuming

\[
f_0 \gg f_1 \left\{ (R_- - R_0)^{2n+1} + R_0^{2n+1}\right\},
\]

(37)

we find

\[
R_- \sim R_0 \sim \left(10^{-33} \text{eV}\right)^2.
\]

(38)

Then \( R_\pm \) could be rather small compared with the scale of the inflation \( (10^{19-25} \text{eV})^2 \). Then the model (12) itself seems to be not viable. It could be possible to replace \( f'(R) \) given from (12) in a region \( R_- < R < R_+ + \epsilon_+ \), where \( \epsilon_\pm \) are small positive constants, with a proper function whose first derivative is always greater than \(-1\). For example,

\[
f(R) = \begin{cases} 
  f_{\text{old}}(R) & R < R_- - \epsilon_- \\
  f_{\text{old}}(R_- - \epsilon_-) + f'_{\text{old}}(R_- - \epsilon_-)(R - R_- + \epsilon_-) & R_- - \epsilon_- < R < R_+ + \epsilon_+ \\
  f_{\text{old}}(R) + f'_{\text{old}}(R_- - \epsilon_-)(R_+ + \epsilon_+ - R_- + \epsilon_-) - f_{\text{old}}(R_+ - \epsilon_+) + f_{\text{old}}(R_- - \epsilon_-) & R > R_+ \epsilon_+ 
\end{cases}
\]

\[f_{\text{old}} \equiv \frac{(R - R_0)^{2n+1} + R_0^{2n+1}}{f_0 + f_1 \left\{ (R - R_0)^{2n+1} + R_0^{2n+1}\right\}},\]

(39)

We now choose \( \epsilon_\pm \) to be \( f'_{\text{old}}(R_- - \epsilon_-) = f'_{\text{old}}(R_+ + \epsilon_+) \). Hence, one obtains \( f'(R) > -1 \) and there is no problem of anti-gravity so such corrected model seems to be quite realistic.

From another side, in order that the model (12) could be viable, there should occur the discrete transition from \( R = R_+ \) and \( R = R_- \). Such a transition might occur as in the usual phase transition with the jump of the value of the order parameter. The properties of such possible transition should be studied in detail.

4. In order to avoid the above anti-gravity problem from the very beginning, we may propose another model given by

\[
f(R) = -f_0 \int_0^R dRe^{-\frac{\alpha R^{2n}}{(R-R_1)^{2n}}} = \frac{R_1^{2n}}{f_0 A_n^2}.
\]

(40)

Here \( \alpha, \beta, f_0, \) and \( R_1 \) are constants. Then by construction, as long as \( 0 < f_0 < 1, f'(R) > -1 \) and therefore \( F'(R) > 0 \). Since

\[
f(R_1) \sim -f_0 \int_0^{R_1} dRe^{-\frac{\alpha R^{2n}}{(R-R_1)^{2n}}} = -f_0 A_n(\alpha) R_1,
\]

\[A_n(\alpha) \equiv \int_0^1 dx e^{-\frac{x^{2n}}{2n}},\]

(41)
and $-f(R_1)$ could be identified with the effective cosmological constant $2R_0$, we find

$$f_0A_n(\alpha)R_1 = R_0.$$  \hspace{1cm} (42)

Note that $A_n(0) = 1$, $A_n(\pm\infty) = 0$, and $A'(x) < 0$. On the other hand, since

$$f(+\infty) \sim \int_0^\infty dRe^{-R\beta\Lambda_i} = -f_0\beta\Lambda_i,$$  \hspace{1cm} (43)

and $-f(+\infty)$ could be identified with the effective cosmological constant at the inflationary epoch, $\Lambda_i$, we find

$$f_0\beta = 1.$$  \hspace{1cm} (44)

Let us now investigate the correction to the Newton law. One gets

$$f''(R) = -f_0e^{-\frac{\beta R_n^2}{2}} \left( -\frac{2nR_1^{2n}}{(R - R_1)^{2n+1}} - \frac{1}{\beta\Lambda_i} \right).$$  \hspace{1cm} (45)

If in air on the earth $n > 3 \sim 4$ (if $\Lambda_i \sim 10^{20-38}$) or in Solar System $n > 10 \sim 12$, we find $m_\sigma^2$ (20) is given by

$$m_\sigma^2 \sim \frac{1}{2F''(R)} \sim \frac{\beta\Lambda_i}{2f_0} \sim \left(10^{10-16} \text{ GeV}\right)^2,$$  \hspace{1cm} (46)

which is very large and positive and therefore the correction to the Newton law is very small.

We now also investigate the instability indicated in [10]. Inside the earth, $U(R)$ (28) has the following form

$$U(R_b) \sim -\frac{F''(0)(R_0)}{3F''(R_0)} \sim -\frac{\beta\Lambda_i}{f_0} < 0.$$  \hspace{1cm} (47)

Here we have assumed $n > 10 \sim 12$. Since $U(R_0)$ is negative, there does not occur such the instability.

Hence, in the model (40), the universe could start from the inflation driven by the effective cosmological constant $-f(+\infty)$ at the early stage, where curvature is large. As curvature becomes small, the effective cosmological constant becomes small too and radiation/matter dominates at the intermediate epoch. When the density of the radiation/matter becomes small and the curvature goes to the value $R_1$, there appears the small effective cosmological constant $-f(R_1)$, so that the accelerating expansion starts. Hence, the realistic modified gravity consistent with Solar System tests and unifying inflation with cosmic acceleration is constructed.

Instead of $f(R)$-gravity, we may consider the $F(G)$-gravity, where the action is given by (16)

$$S = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} (R + F(G)),$$  \hspace{1cm} (48)

and $G$ is the Gauss-Bonnet invariant:

$$G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}.$$  \hspace{1cm} (49)

Note that in $F(G)$-gravity, there are no problems [16] with the Newton law, instabilities and the anti-gravity regime. One may consider the model similar to (12):

$$F(G) = \frac{(G - G_0)^{2n+1} + G_0^{2n+1}}{F_0 + F_1 \left\{ (G - G_0)^{2n+1} + G_0^{2n+1} \right\}} = \frac{1}{F_1} + \frac{F_0/F_1}{f_0 + F_1 \left\{ (G - G_0)^{2n+1} + G_0^{2n+1} \right\}}.$$  \hspace{1cm} (50)

Here $G_0$ corresponds to the present value of the Gauss-Bonnet invariant. Since $F'(G) = 0$ when $G = G_0$ and $G = +\infty$, $F(G)$ becomes almost constant and can be regarded as the effective cosmological constant. As in (13), we may identify $F(\infty)$ as the cosmological constant for the inflationary epoch and $F(R_0)$ as that at the present accelerating era. Then instead of (14), one gets

$$F_0 = \frac{G_0^{2n}}{2} - \frac{G_0^{2n+1}}{\Lambda_i} \sim \frac{G_0^{2n}}{2}, \quad F_1 = \frac{1}{\Lambda_i}. \hspace{1cm} (51)$$

Hence, the universe starts from the inflation driven by the effective cosmological constant $\Lambda_i$ [51] at the early epoch, where curvature and therefore the Gauss-Bonnet invariant $G$ is very large (for the study of cosmological perturbations
in such theory, see [17]). As curvature becomes small, the effective cosmological constant becomes small too and radiation/matter could dominate at the intermediate universe. When the density of the radiation/matter becomes small and \( G \) goes to the value \( G_0 \) in [40], there appears the small effective cosmological constant \( -F(G_0) \), and cosmic acceleration starts.

Thus, we suggested several realistic models of \( f(R) \) and \( F(G) \) gravity which propose natural unification of the early-time inflation and late-time acceleration being consistent with local tests and cosmological bounds. Definitely, more precise local tests/cosmological checks for such theories should be made in future, having in mind that more precise observational data will be available soon.

**Acknowledgements**

The research by S.N. has been supported in part by the Ministry of Education, Science, Sports and Culture of Japan under grant no.18549001 and 21st Century COE Program of Nagoya University provided by Japan Society for the Promotion of Science (15COEG01). The research by S.D.O. has been supported in part by the projects FIS2006-02842, FIS2005-01181 (MEC, Spain), by the project 2005SGR00790 (AGAUR, Catalunya) and by RFBR grant 06-01-00609 (Russia).

**Appendix**

We now consider how the universe can reach the exit of the inflation. For simplicity, we consider the model in (12). In order to compare with usual scenario of the inflation, we work in the scalar-tensor form in the Einstein frame (16). In the epoch of the inflation, the curvature \( R = A \) could be large and we find

\[
f(R) \sim -\frac{1}{f_1} + \frac{f_0}{f_1^2 R^{2n+1}},
\]

and therefore

\[
\sigma \sim \frac{2(n+1) f_0}{f_1^2 A^{2n+2}}, \quad V(\sigma) \sim \frac{1}{f_1} \frac{2(n+1)f_0}{f_1^2} \left( \frac{f_1^2 \sigma}{(2n+1)f_0} \right)^{\frac{2n+4}{n+2}}.
\]

Since \( \sigma \) is now dimensionless, the condition for the slow roll could be given by \( |V'|/V| \ll 0 \). Now we have

\[
\frac{V'}{V} \sim -f_1 \left( \frac{f_1^2 \sigma}{(2n+1)f_0} \right)^{-\frac{n+2}{n}}.
\]

If we start with \( \sigma \sim 1 \), by using (14), we find

\[
\frac{V'}{V} \sim -\left( \frac{R_0^2}{\Lambda_i} \right)^{\frac{n+2}{n-1}},
\]

which could be very small and the slow roll condition could be satisfied. The potential \( V(\sigma) \) in (53) tells that if we start with \( \sigma \sim 1 \), the value of \( \sigma \) increase very slowly and therefore \( R \) becomes smaller. Then \( \sigma \) becomes large enough, the inflation could stop.

**References**

[1] S. Nojiri and S. D. Odintsov, Int. J. Geom. Meth. Mod. Phys. 4, 115 (2007) [arXiv:hep-th/0601213].

[2] S. Nojiri and S. D. Odintsov, Phys. Lett. B 576, 5 (2003) [arXiv:hep-th/0307071].

[3] S. Capozziello, Int. J. Mod. Phys. D 11, 483 (2002); S. Capozziello, S. Carloni and A. Troisi, [arXiv:astro-ph/0303041].

[4] S. M. Carroll, V. Duvvuri, M. Trodden and S. Turner, Phys. Rev. D 70 (2004) 043528.

[5] F. Faraoni, arXiv:gr-qc/0607116, M. Ruggiero and L. Iorio, arXiv:gr-qc/0607093; A. Cruz-Dombriz and A. Dobado, arXiv:gr-qc/0607118; N. Poplawski, arXiv:gr-qc/0610133; A. Brookfield, C. van de Bruck and L. Hall, arXiv:hep-th/0608015; Y. Song, W. Hu and I. Sawicki, arXiv:astro-ph/0610532; B. Li, K. Chan and M. Chu, arXiv:astro-ph/0610794; X. Jin, D. Liu and X. Li, arXiv:astro-ph/0610854; T. Sotiriou and S. Liberati, arXiv:gr-qc/0604006; T. Sotiriou, arXiv:gr-qc/0604028; R. Bean, D. Bernat, L. Pogosian, A. Silvestri and M. Trodden, arXiv:astro-ph/0611321; I. Navarro and K. Van Acoleyen, arXiv:gr-qc/0611127; A. Bustelo and D. Barraco, arXiv:gr-qc/0611149; G. Olmo, arXiv:gr-qc/0612047; F. Brizzone, E. Elizalde, S. Nojiri and S. D. Odintsov, Phys. Lett. B 646, 105 (2007) [arXiv:hep-th/0612220]; B. Li and J. Barrow, arXiv:gr-qc/0707111; T. Rador, arXiv:hep-th/0702081; V. Faraoni, arXiv:gr-qc/0703041; S. Rahvar and Y. Sobouti, arXiv:astro-ph/0704.0680; O. Bertolami, C. Boehmer, T. Harko and F. Lobo, arXiv:0704.1733; S. Carloni, A. Troisi and P. Dunsby, arXiv:0706.0452; 0707.0106; K. Kainulainen, J. Pilolonen and D. Sunhede, arXiv:0704.2729; S. Capozziello and M. Francaviglia, arXiv:0706.1146.
[6] S. Nojiri and S. Odintsov, Gen. Rel. Grav. 36, 1765 (2004) arXiv:hep-th/0308176; Phys. Lett. B599, 137 (2004) arXiv:astro-ph/0403622; P. Wang and X. Meng, arXiv:astro-ph/0406455 arXiv:gr-qc/0311019 M. Abdalla, S. Nojiri and S. D. Odintsov, Class. Quant. Grav. 22, L35 (2005) arXiv:hep-th/0409177; G. Cognola, E. Elizalde, S. Nojiri, S. D. Odintsov and S. Zerbini, JCAP 0502, 010 (2005) arXiv:hep-th/0501096; Phys. Rev. D73, 084007 (2006), arXiv:hep-th/0601008; D. A. Easson, Int. J. Mod. Phys. A 19, 5343 (2004) arXiv:astro-ph/0411209; S. Capozziello, V. Cardone and A. Troisi, arXiv:astro-ph/0501426 G. Allemandi, M. Francaviglia, M. Francaviglia and S. D. Odintsov, arXiv:gr-qc/0504057; G. Allemandi, M. Francaviglia, M. Ruggiero and A. Tartaglia, arXiv:gr-qc/0506123 T. Multamaki and I. Vilja, arXiv:astro-ph/0612775; J. A. R. Cembranos, Phys. Rev. D 73, 064029 (2006) arXiv:gr-qc/0507039; T. Koivisto and H. Kurki-Suonio, arXiv:astro-ph/0509422 T. Clifton and J. Barrow, arXiv:gr-qc/0509059 J. Santiago and J. Weller, arXiv:astro-ph/0510453 M. Amarzguioui, O. Elgaroy, D. Mota and T. Multamaki, arXiv:astro-ph/0510519 I. Brevik, arXiv:gr-qc/0601100 R. Woodard, arXiv:astro-ph/0601672 T. Koivisto, arXiv:0706.0974 T. Faulkner, M. Tegmark, E. Bunn and Y. Mao, arXiv:astro-ph/0612569 G. Cognola, M. Castaldi and S. Zerbini, arXiv:gr-qc/0701138 S. Capozziello and R. Garattini, arXiv:gr-qc/0702075 S. Nojiri, S. D. Odintsov and P. Tretyakov, arXiv:0704.2520 hep-th; M. Movahed, S. Baghram and S. Rahvar, arXiv:0705.0889 astro-ph; L. Amendola and S. Tsujikawa, arXiv:0705.0396 astro-ph; M. Fairbaim and S. Rydbeck, arXiv:astro-ph/0701900 K. Uddin, J. Lidsey and R. Tavakol, arXiv:0705.0232 gr-qc; C. Boehmer, L. Hohenstein and F. Lobo, arXiv:0706.1663 S. Nojiri, S. D. Odintsov and A. Troisi, Phys. Lett. B 639, 135 (2006). [7] S. Fay, S. Nesseris and L. Perivolaropoulos, arXiv:hep-th/0504206 S. Fay, R. Tavakol and S. Tsujikawa, arXiv:astro-ph/0701479 [9] S. Nojiri and S. D. Odintsov, arXiv:hep-th/0611071 [10] A. D. Dolgov, M. Kawasaki, Phys. Lett. B 573, (2003) 1 arXiv:astro-ph/0307285; M. Soussa and R. Woodard, Gen. Rel. Grav. 36, 855 (2004). [11] W. Hu and I. Sawicki, arXiv:0706.1168 [12] S. Nojiri and S. D. Odintsov, arXiv:0706.1378 [13] S. A. Appleby and R. A. Battye, arXiv:0705.3199 [14] V. Faraoni, Phys. Rev. D 74, (2006) 104017 arXiv:astro-ph/0610734; T. Sotiriou, Phys. Lett. B 645, (2007) 389; I. Sawicki and W. Hu, arXiv:astro-ph/0702278 [15] A. Starobinsky, arXiv:0706.2041 [16] S. Nojiri and S. D. Odintsov, Phys. Lett. B 631, (2005) 1 arXiv:hep-th/0508049; S. Nojiri, S. D. Odintsov and O. Cebulnova, arXiv:hep-th/0510183; G. Cognola, E. Elizalde, S. Nojiri, S. D. Odintsov and S. Zerbini, Phys. Rev. D 75, (2007) 086002; I. Brevik and J. Quiroga, arXiv:gr-qc/0610044 [17] B. Li, J. D. Barrow, and D. F. Mota, arXiv:0705.3795