Numerical simulation of wooden beams fracture under impact

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Abstract. In this paper the fracture of wooden (pine) beams under low-speed impact of steel ram tester is numerically investigated, considered speed is ranging between 10-30 m/s. The behavior of wood is described within phenomenological approach taking into account anisotropy of elastic and strength properties. The behavior of the steel ram tester is modelled by the elastoplastic medium. Numerical modeling is carried out by a finite elements method in three-dimensional statement. Fracture propagation in wood is under study.

1. Introduction
Impact action of solid bodies in a wide range of kinematic and geometric conditions is a complex task of mechanics. Difficulties connected with theoretical studies of the process of materials destruction and deformation under impact action using analytical approaches makes one to introduce a number of simplifying hypotheses that cannot always reflect the real situation. Therefore it should be considered that the leading role in studying events connected with high-speed interaction of solid bodies is currently given to experimental and numerical studies.

Research in materials destruction while impact actions demonstrate that due to change in impact conditions the fracture mechanisms are also changed [1]. Experiments demonstrate that in number of cases final destruction is defined by a combination of several mechanisms. However the experiments do not allow tracing the sequence, time of impact and contribution of the different fracture mechanisms. Furthermore, fractures obtained at the initial stages of process cannot always be identified while analysis of the final materials destruction. Therefore numerical simulation is relevant when studying impact interaction [2].

2. Major equations of mathematical model
The system of equations describing the time-dependent adiabatic motion of a compressible medium in an arbitrary coordinate system (\(i=1,2,3\)) includes the following equations:

- equation of continuity

\[
\frac{\partial \rho}{\partial t} + \text{div} \rho \mathbf{v} = 0
\]  

(1)

- equation of motion
\[ \rho a^k = \nabla_i \sigma^{ki} + F^k, \]  
\[ (2) \]

where
\[ a^k = \frac{\partial v^k}{\partial t} + v^j \nabla_j v^k \]  
\[ (3) \]

\[ \nabla_i \sigma^{ki} = \sigma^{ki}_{,i} + \sigma^{m}_{,i} \Gamma^k_{mi} + \sigma^{ki} \Gamma^m_{mi} \]  
\[ (4) \]

- equation of energy
\[ \frac{dE}{dt} = \frac{1}{\rho} \sigma^{ij} e_{ij}, \]  
\[ (5) \]

where \( \rho \) - density of the media; \( \vec{v} \) - velocity vector; \( a^k \) - components of acceleration vector; \( F^k \) - components of the nodal forces vector; \( \Gamma^k_{ij} \) - Christoffel symbols; \( \sigma^{ij} \) - contravariant components of symmetric stress tensor; \( E \) - specific internal energy; \( e_{ij} \) - components of the symmetric strain velocity tensor:
\[ e_{ij} = \frac{1}{2} \left( \nabla_i \vec{v}_j + \nabla_j \vec{v}_i \right). \]  
\[ (6) \]

3. Constitutive equations of isotropic metal materials

The behavior of isotropic metal materials is considered within elastoplastic model.

The stress tensor is represented in the form of the sum of deviatoric \( S^{ki} \) and spherical part \( P \):
\[ \sigma^{ki} = -P \delta^{ki} + S^{ki}, \]  
\[ \text{where } \delta^{ij} \text{ - Kronecker symbol.} \]

Assuming that for media true is the principle of minimum work of true stresses on increments of plastic deformations, the connection of tensor component of strain rate and stress deviator will be recorded in the form of:
\[ 2G \left( e_{ij} - \frac{1}{3} e_{kk} \delta^{ij} \right) = \frac{DS^{ij}}{Dt} + \lambda S^{ij}, \]  
\[ (\lambda \geq 0) \]  
\[ (7) \]

Here
\[ \frac{DS^{ij}}{Dt} = \frac{dS^{ij}}{dt} - S^{ik} \omega_{jk} - S^{jk} \omega_{ik}, \]

where \( \omega_{ij} = \frac{1}{2} \left( \nabla_i \vec{v}_j - \nabla_j \vec{v}_i \right), G \) - shear modulus.

Parameter \( \lambda = 0 \) under elastic deformation and under plastic deformation \( (\lambda > 0) \) determined by the help of a condition of Mises: \( S^{ij} S_{ij} = \frac{2}{3} \sigma_d^2 \) \( \sigma_d \) - dynamic yield stress. Pressure in the material was calculated from the Mie-Grüneisen equation as a function of the specific internal energy \( E \) and density \( \rho \):
\[ P = \sum_{n=1}^{3} K_n \left( \frac{V_0}{V} - 1 \right)^n + K_0 \rho E \]  
\[ (8) \]

where, \( K_0, K_1, K_2, K_3 \) - material constants.
4. Constitutive equations of anisotropic materials

To describe the behavior of anisotropic (orthotropic) body the following conditions and restrictions are accepted:

1. The body is solid (continuous media);
2. The cross dimensions of structural (reinforcing) elements are small in comparison with the body sizes, i.e. the medium is quasi-homogeneous;
3. Relationships between increments of stress tensor components and tensor components of strain rate tensor linear.

Therefore the relations between components of stress tensor and strain rate tensor before fracture to orthotropic body can be given as:

\[
\begin{align*}
\frac{d\sigma_{11}}{dt} &= C_{11}e_{11} + C_{12}e_{22} + C_{13}e_{33}, \\
\frac{d\sigma_{22}}{dt} &= C_{22}e_{11} + C_{23}e_{22} + C_{33}e_{33}, \\
\frac{d\sigma_{33}}{dt} &= C_{33}e_{11} + C_{23}e_{22} + C_{32}e_{33}, \\
\frac{d\sigma_{12}}{dt} &= C_{44}e_{12},
\end{align*}
\]

where, \( C_{ij} \) – elastic constants.

Elastic constants \( C_{ij} \) can be expressed by means of shear modulus \( G_{ij} \), Young's modulus: \( E_i \) and Poisson's ratios \( \nu_{ij} \):

\[
\begin{align*}
C_{11} &= \frac{1}{E_2 A} \left( \frac{1}{E_3} - \frac{v_{23}^2}{E_2} \right); \\
C_{12} &= \frac{1}{E_2 A} \left( \frac{v_{23}^2}{E_3} - \frac{v_{12}}{E_1} \right); \\
C_{13} &= \frac{1}{E_2 A} \left( \frac{v_{23}}{E_3} - \frac{v_{31}}{E_1} \right); \\
C_{22} &= \frac{1}{E_3 A} \left( \frac{1}{E_1} - \frac{v_{31}^2}{E_3} \right); \\
C_{23} &= \frac{1}{E_3 A} \left( \frac{v_{31}^2}{E_1} - \frac{v_{23}}{E_2} \right); \\
C_{33} &= \frac{1}{E_3 A} \left( \frac{v_{23}^2}{E_1} - \frac{v_{31}}{E_2} \right); \\
C_{44} &= G_{12}; \\
C_{55} &= G_{23}; \\
C_{66} &= G_{13}; \\
A &= \frac{1}{E_1 E_2 E_3} (1 - 2\nu_{12} \nu_{23} \nu_{31} + \frac{E_1^2}{E_3^2} + \frac{E_2^2}{E_3^2} + \frac{E_3^2}{E_2^2})^{-1};
\end{align*}
\]

Knowing nine sizes of technical constants in the main axes of symmetry of orthotropic material, one can determine the value of any constant in any direction.

5. Fracture model of anisotropic material under dynamic loadings

Defining the transition of the barrier material to complete fracture the Hoffman criterion was used [3] that takes into account the different strength structural properties of the material in compression and tensile actions:

\[
f(\sigma) = C_1(\sigma_2 - \sigma_3)^2 + C_2(\sigma_3 - \sigma_1)^2 + C_3(\sigma_1 - \sigma_2)^2 + C_4\sigma_1 + C_5\sigma_2 + C_6\sigma_3 + C_7\sigma_4^2 + C_8\sigma_5^2 + C_9\sigma_6^2 \geq 1,
\]

where coefficients \( C_1 - 9 \) are determined from the following equations:
\[ C_1 = \left[ (Y^tY^c)^{-1} + (Z^tZ^c)^{-1} - (X^tX^c)^{-1} \right] / 2; \]
\[ C_2 = \left[ (X^tX^c)^{-1} + (Z^tZ^c)^{-1} - (Y^tY^c)^{-1} \right] / 2; \]
\[ C_3 = \left[ (X^tX^c)^{-1} + (Y^tY^c)^{-1} - (Z^tZ^c)^{-1} \right] / 2; \]
\[ C_4 = (X^t - X^c)^{-1}; \quad C_7 = S_{yz}^{-2}; \]
\[ C_5 = (Y^t - Y^c)^{-1}; \quad C_8 = S_{zx}^{-2}; \]
\[ C_6 = (Z^t - Z^c)^{-1}; \quad C_9 = S_{xy}^{-2}. \]

(15)

Here $X^t$, $X^c$ - ultimate tensile and ultimate compressive strength along the X-axis respectively, $Y^t$, $Y^c$ - ultimate tensile and ultimate compressive strength along the Y-axis, $Z^t$, $Z^c$ - ultimate tensile and ultimate compressive strength along the Z-axis, $S_{xy}$, $S_{yz}$, $S_{zx}$ - ultimate shear strength along the corresponding axes. It is supposed that fracture of anisotropic materials in the conditions of intensive dynamic loadings occurs as follows [4]:

1. If failure criterion is broken under compression ($e_{kk} \leq 0$), it is considered that material loses properties of anisotropy, and its behavior is described by hydrodynamic model, and the material retains only compressive strength; stress tensor becomes in this case spherical $\mathbf{\sigma} = -P$ and its value is determined from the relation [5]:

\[ P = \left[ \exp \left( 4b \frac{V_0 - V}{V_0} \right) - 1 \right] \rho_0 c_0^2 \frac{4b}{D}, \]

(16)

where $\rho_0$ - the initial density, $c_0$ and $b$ - the coefficients of the shock adiabat;

\[ D = c_0 + bu_m, \]

$u_m$ - mass velocity, $V_0$ and $V$ - initial and current specific volumes.

2. If failure criterion is broken under tension ($e_{kk} > 0$), the material is considered completely destructed, and components of stress tensor are set equal to zero ($\sigma_{ij} = 0$). Numerical simulation is carried out by finite element method [6].

6. Results and discussion

The process of interaction of steel ($\rho_0 = 2710$ kg/m$^3$, $\sigma_d = 310$ MPa) ram tester is considered, corresponding 400g in weight, speed range of 10-30 m/s ($\rho_0 = 2710$ kg/m$^3$, $\sigma_d = 310$ MPa) with a barrier from glued wood (pine) thickness of 60 mm. The density of the barrier material $\rho_0 = 580$ kg/m$^3$, the elastic and strength properties have the following values: $E_x = 11700$ MPa, $E_y = 500$ MPa, $E_z = 620$ MPa, $X^t = 84$ MPa, $X^c = 46$ MPa, $Y^t = 30$ MPa, $Y^c = 46$ MPa, $Z^t = 30$ MPa, $Z^c = 46$ MPa, $S_{xy} = 30$ MPa, $S_{yz} = 30$ MPa, $S_{zx} = 30$ MPa.

Figures 1 - 2 show the calculated configuration of the ram tester and the barrier and gradation of grey color show values of relative level of fracture $V_r/V_i$ in barrier. The values $V_i$ and $V_r$ are given to the nodes of the computational grid, $V_r$ - is a volume of the elements connecting in node in which the
condition of fracture (14) was satisfied, $V_t$ – the total amount of the elements forming this node. Value $V_r/V_t=1$ corresponds to complete fracture of material in the node of the computational grid. Resulting from the impact action on the barrier annular area of fracture occurs on a front surface along the perimeter of the ram tester. This destruction area spreads to the back surface along the barrier thickness. Figure 3 demonstrates changes in the time of speed of the mass center of ram tester for various initial speeds of interaction which illustrate intensity of ram tester braking. Penetration of a barrier isn't observed up to speed 30m/s.

Figure 1. Configuration of the projectile and target, isolines of relative volume of fracture in barrier $v_0=15$ m/s.

Figure 2. Configuration of ram tester and barrier, isolines of relative volume of fracture in barrier $v_0=20$ m/s.
7. Conclusion

The developed model describes behavior of brittle-fractured orthotropic material, transferring into model of brittle-fractured transtropic or isotropic material at equality of properties in the directions $X$ and $Y$, or in all directions. It allows investigating the behavior of a wide class of materials with various properties using the same model. Wood and structural elements obtained from it can also be described well by the suggested model thus allowing its broad application in studying the behavior of similar materials under dynamic loading. The model does not restrict the criterion of strength, its choice is defined by appropriateness and given strength properties for particular material.

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