Remarks on Delta Radiative and Dalitz Decays

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Abstract

Phenomenological expressions are derived for rates of the $\Delta(1232)$ radiative and Dalitz decays, $\Delta(1232) \rightarrow N\gamma$ and $\Delta(1232) \rightarrow Ne^+e^-$. Earlier calculations of these decays are commented.

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The $\Delta(1232)$ resonance is expected to give an important contribution to the dilepton yield in nucleon-nucleon and heavy-ion collisions. In refs. [1–5], expressions are derived for the $\Delta \rightarrow N e^+e^-$ decay rate, which, however, are not equivalent with respect to the kinematical factors. In refs. [2,5–7], the radiative decay $\Delta \rightarrow N\gamma$ is calculated. The results, surprisingly, are also not equivalent. We thus give an independent calculation of these two decays.

In terms of helicity amplitudes, the decay width of a resonance, $R$, decaying into a nucleon, $N$, and a photon, $\gamma^*$, can be written as

$$\Gamma(R \rightarrow N\gamma^*) = \frac{k}{8\pi m_R^2(2J_R + 1)} \sum_{\lambda\lambda'} \left|<\lambda|S|\lambda'\lambda''>\right|^2$$

(1)

where $m_R$ is the resonance mass, $J_R$ its spin, $k$ is the photon momentum in the resonance rest frame, $\lambda$, $\lambda'$, and $\lambda''$ are the resonance, nucleon, and photon helicities, and $<\lambda|S|\lambda'\lambda''>$ the corresponding amplitudes.

For the $\Delta \rightarrow N\gamma^*$ transition, there are three independent helicity amplitudes which can be found in paper by Jones and Scadron [8], eqs.(18). Using these amplitudes, we obtain the $\Delta$ resonance width for decay into a nucleon and a virtual photon:

$$\Gamma(\Delta \rightarrow N\gamma^*) = \frac{\alpha}{16} \frac{(m_\Delta + m_N)^2}{m_\Delta^2 m_N^2} \left((m_\Delta + m_N)^2 - M^2\right)^{1/2}$$

$$\times \left((m_\Delta - m_N)^2 - M^2\right)^{3/2} \left(G_M^2 + 3G_E^2 + \frac{M^2}{2m_\Delta^2}G_C^2\right).$$

(2)

Here, $m_N$ and $m_\Delta$ are the nucleon and $\Delta$ masses, $M^2 = q^2$ where $q_\mu = (\omega, 0, 0, k)$ is the photon four-momentum, $G_M$, $G_E$, and $G_C$ are magnetic, electric and Coulomb transition form factors, as defined in ref. [8], eqs.(15). The normalization conventions are the following:

In order to get the physical amplitudes $<\lambda|S|\lambda'\lambda''> \equiv i e \epsilon^{(\lambda'')}_{\mu}(q) J_\mu(q)$, one needs to multiply the helicity amplitudes of ref. [8] by a factor of $\sqrt{2}\epsilon$, with $\epsilon$ being the electron charge, and the single-spin-flip amplitude $\lambda'' = 0$ by an additional factor $\frac{M}{\omega}$, with $\omega$ being the photon energy in the $\Delta$ rest frame. The photon polarization vectors, $\epsilon^{(\lambda)}_{\mu}(q)$, are normalized by $\epsilon^{(\lambda)}_{\mu}(q)\epsilon^{(\lambda')}_\mu(q)^* = -\delta_{\lambda\lambda'}$. 

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In the limit of the vanishing virtual photon mass, \( M \to 0 \) (real photons), the longitudinal polarization vector equals \( \epsilon^{(0)}_\mu(q) = q_\mu/M + O(M) \). The current conservation implies \( q_\mu J_\mu(q) = 0 \), so \( < 1/2|S| - 1/2 > = O(M) \). The single-spin-flip amplitude is proportional to the Coulomb form factor \( G_C \) (see ref. [8]). The coefficient at the \( G_C^2 \) in eq.(2) has therefore the correct behavior at \( M \to 0 \).

The factorization prescription (see e.g. [9]) allows to find the dilepton decay rate of the \( \Delta \) resonance:

\[
d\Gamma(\Delta \to Ne^+e^-) = \Gamma(\Delta \to N\gamma^*) MT(\gamma^* \to e^+e^-) \frac{dM^2}{\pi M^4},
\]

with

\[
MT(\gamma^* \to e^+e^-) = \frac{\alpha}{3} (M^2 + 2m_e^2) \sqrt{1 - \frac{4m_e^2}{M^2}} \]

being the decay width of a virtual photon into the dilepton pair with the invariant mass \( M \).

The physical \( \Delta(1232) \to N\gamma \) decay rate is given by eq.(2) at \( M = 0 \). The last three equations being combined give the \( \Delta(1232) \to Ne^+e^- \) decay rate.

In ref. [1] the \( \Delta \to Ne^+e^- \) transition is calculated. The width \( \Delta \to N\gamma \) can be extracted from eqs.(10) - (12) of this work. It does not coincide with our eq.(2). The \( M = 0 \) limits of eqs.(4.9) - (4.13) of ref. [2] and of eqs.(3) - (13) of ref. [3] do not coincide with our eq.(2) also. In ref. [3], the physical \( \Delta \to N\gamma \) decay is calculated in the light cone QCD assuming \( F_1 = \sqrt{3/2}g_{\Delta N\gamma} \neq 0 \) and \( F_2 = F_3 = 0 \), with the form factors \( F_i \) defined as in ref. [8], eq.(4), and the coupling constant \( g_{\Delta N\gamma} \) defined as in ref. [4], eq.(3). Using eqs.(15) of ref. [8] and our eq.(2), we obtain an expression for the \( \Delta \to N\gamma \) width, which differs from eq.(13) of ref. [3] (by a factor of 2/3 in the heavy-baryon limit). In ref. [4], an expression is derived for the radiative decay of a spin \( J_R \) baryon resonance. We agree with eq.(2.59) of this work. The results [1,2,5–7] for the \( \Delta \to N\gamma \) decay are distinct from each other.

Our result for the \( \Delta \to Ne^+e^- \) width, eqs.(2) - (4), is distinct from the results of refs. [1,2,5], since we disagree already on the \( \Delta \to N\gamma \) width. In ref. [3], the \( \Delta \to Ne^+e^- \) decay is calculated using the chiral perturbation theory. We reproduce kinematical factors of the
M1 part of the decay width in eq.(2) of ref. [3]. In the soft dilepton limit, \( m_e = 0 \) and \( M \rightarrow 0 \), we agree also with the \( E2 \) part, but disagree with it at finite values of \( M \). Our expression for the \( \Delta \) decay rate differs from that of ref. [4], eqs.(8) and (9). The numerical distinction is, however, small [10]. The results of refs. [1–5] for the \( \Delta \rightarrow Ne^+e^- \) decay are distinct from each other.

The helicity formalism which we used is completely equivalent to the standard technique based on the calculation of traces of products of the projection operators and \( \gamma \)-matrices (see e.g. [11]). We verified analytically that the helicity method and the standard method give for the \( \Delta \) decays the identical results.

The \( \Delta \rightarrow N\gamma^* \) decay amplitude is transverse with respect to the photon momentum and therefore gauge invariant. It is invariant also with respect to the contact transformations of the Rarita-Schwinger fields, since the decaying \( \Delta \) is on the mass shell. There are ambiguities in the effective field theories for interacting spin-3/2 particles, which come into play when spin-3/2 particles go off the mass shell (see e.g. [12]). In the first order of the perturbation, the decay rates of the on-mass-shell spin-3/2 particles are, however, well defined theoretically.

There are no ambiguities in the \( \Delta \rightarrow N\gamma^* \) amplitude. The discrepancies between refs.[1-6] and our paper can probably be attributed to errors in calculations of refs.[1-6].

Notice that in the heavy-baryon limit, \( m_\Delta - m_N \ll m_N \), \( G_M = \frac{2}{3}m^2F_1 \), \( G_E = 0 \), and \( G_C = \frac{2}{3}m^2(F_1 + F_2) \). The photons in this approximation are soft, \( M < m_\Delta - m_N \ll m_N \), so the third term in Eq.(2) can be neglected. The \( M1 \) mode in the heavy-baryon limit appears to be the dominant one.

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\[ ^1\text{The corresponding Maple codes are available upon request} \]
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