A STUDY OF MOUNTAIN WAVE ACROSS 2-D OROGRAPHIC BARRIERS FOR VARIABLE WIND

1. Teixeira et al. (2004) developed an analytical model to predict the surface drag exerted by gravity waves on an isolated axisymmetric mountain with velocity profile that varies slowly with height. Recently Teixeira and Miranda (2004) modified Teixeira et al. (2004) model to calculate the gravity wave drag and surface pressure perturbation analytically exerted by a stratified flow over a 2-D mountain ridge with same velocity profile. Therefore our motive of study is to extend this study to evaluate mountain drag, energy flux and surface pressure perturbation for orographic barriers of Indian region.

Mountain wave problem addressing properties of mountain waves over Indian region was firstly studied by Das (1964), Sarker (1965, 1966 & 1967), Sarker et al. (1978). Later Dutta (2001) have studied the mountain drag and energy flux across 2-D profile of Western Ghats of India, he has shown that plateau part of Ghats does not have any impact on the drag and flux. Dutta et al. (2002) analytically evaluated 3-D expression of streamline displacement and vertical velocity for Western Ghats and Khasi Jayantia hills. Dutta (2003) has developed model to compute mountain drag and energy flux for realistically varying wind and also computed these for some cases of Western Ghats and Khasi-Jayantia hills. Recently Dutta and Naresh (2005) studied fluxes of momentum and energy generated by mountain waves over Assam-Burma hills of India and shown the impact of valley between the ridges on the mountain drag and energy flux. Very recently Naresh et al. (2005) have shown the effect of Coriolis force on the mountain drag and energy flux across the profile of Khasi Jayantia hills of India.

The aim of the present study is to develop a mathematical model to obtain the analytical expressions for mountain drag, energy flux and surface pressure perturbation for wind which varies with height across Western Ghats as well as Assam-Burma hills of India using the analytical model of Teixeira and Miranda (2004).

2. The mathematical model - The surface pressure perturbation [Eqn. (10) of Teixeira and Miranda 2004] is given by

$$\rho(z=0) = i\rho_0 N U_0 \left[ 1 + \frac{i U'_0}{2 N} - \frac{1}{8} \left( \frac{U_0''^2}{N^2} + 2 \frac{U_0 U'_0}{N^2} \right) \right] \hat{h}(k)$$

(2.1)

where $N^2 = -\frac{g}{\rho_0} \frac{d\rho}{dz}$ is Brunt-Väisälä frequency, $\rho_0$ is mean density, $U_0$ is the unperturbed surface wind velocity, $U'_0$ and $U''_0$ are the first order and second order derivative of $U_0$, $\hat{h}(k)$ is the Fourier transform of the profile of orographic barrier $\hat{h}(k)$.

By inverse Fourier transform of (2.1), we have

$$p'(z=0) = i\rho_0 N U_0 \left[ 1 + \frac{i U'_0}{2 N} - \frac{1}{8} \left( \frac{U_0''^2}{N^2} + 2 \frac{U_0 U'_0}{N^2} \right) \right] \int_{-\infty}^{\infty} \hat{h}(k) e^{ikz} \, dk$$

(2.2)

The expression of mountain drag [Eqn. (11) of Teixeira and Miranda 2004] is

$$D = 2\pi\rho_0 N U_0 \left[ 1 - \frac{1}{8} \left( \frac{U_0''^2}{N^2} + 2 \frac{U_0 U'_0}{N^2} \right) \right] \int_{-\infty}^{\infty} k \hat{h}(k) \hat{h}^*(k) \, dk$$

(2.3)

where $\hat{h}^*(k)$ is the complex conjugate of $\hat{h}(k)$.

Again the expression of Energy flux at surface is given by

$$E = -2\pi i U_0 \int_{-\infty}^{\infty} k \rho(z=0) \hat{h}^*(k) \, dk$$

(Dutta, 2001) (2.4)
Finally substituting \( \dot{p}(z = 0) \) from Eqn. (2.1) into Eqn. (2.4) for real solution of energy flux

\[
E = 2\pi \rho_0 U_0^2 \left[ 1 - \frac{1}{8} \left( \frac{U_0^2}{N^2} + 2 \frac{U_0 U_0^*}{N^2} \right) \right] \int_{-\infty}^{\infty} k h(k) \hat{h}^* (k) dk
\]

(2.5)

3. Surface pressure perturbation, mountain drag and energy flux across Assam-Burma hills of India - The 2-D profile of Assam-Burma hills is

\[
h(x) = \frac{a^2 b_1}{a^2 + x^2} + \frac{a^2 b_2}{a^2 + (x - d)^2}
\]

(De, 1973) (3.1)

where, \( a = 20.0 \) km, \( b_1 = 0.9 \) km, \( b_2 = 0.7 \) km and \( d = 55.0 \) km.

The Fourier transform of Eqn. (3.1) is

\[
\hat{h}(k) = ae^{-ak} \left( b_1 + b_2 e^{-ik} \right)
\]

(3.2)

and

\[
\hat{h}(k) \hat{h}^* (k) = a^2 e^{-2ak} \left[ b_1^2 + b_2^2 + 2h_1 b_2 \cos(2k) \right]
\]

(3.3)

The expression of surface pressure perturbation [Eqn. (2.2)] using Eqn. (3.2) becomes

\[
p'_A(z = 0) = \rho_0 aNU_0 \left[ 1 + \frac{i}{2} \frac{U_0'}{N} - \frac{1}{8} \left( \frac{U_0'^2}{N^2} + 2 \frac{U_0 U_0^*}{N^2} \right) \right] \int_{-\infty}^{\infty} \left( h_1 + b_2 e^{-ik} \right) e^{-i(\omega t - kx)} dk
\]

Integrating \( p'(z = 0) \) for real solution, we have

\[
p'_A(z = 0) = -\rho_0 NU_0 \left[ 1 + \frac{i}{2} \frac{U_0'}{N} - \frac{1}{8} \left( \frac{U_0'^2}{N^2} + 2 \frac{U_0 U_0^*}{N^2} \right) \right] \left[ \frac{b_1 a (x - i\omega) + b_2 a [(x - d) - i\omega]}{a^2 + x^2} \right]
\]

Now to find analytical expression of mountain drag, substitute Eqn. (3.3) into Eqn. (2.5), we get

\[
D_A = 2\pi a^2 \rho_0 NU_0 \left[ 1 - \frac{1}{8} \left( \frac{U_0'^2}{N^2} + 2 \frac{U_0 U_0^*}{N^2} \right) \right] \int_{-\infty}^{\infty} k \left( b_1^2 + b_2^2 + 2b_2 b_2 \cos(2k) \right) e^{-2ak} dk
\]

The mountain drag \( D_A \) for real solution becomes

\[
D_A = \frac{1}{2} \pi a^2 \rho_0 NU_0 \left[ 1 - \frac{1}{8} \left( \frac{U_0'^2}{N^2} + 2 \frac{U_0 U_0^*}{N^2} \right) \right] \left[ \left( b_1^2 + b_2^2 \right) + 8a^2 b_2 b_2 \left( 4a^2 - d^2 \right) \right]
\]

(3.5)

Eqn. (3.5) is the analytical expression for mountain drag across Assam- Burma hills, when wind varies with height.

Now to find the energy flux across Assam-Burma hills substitute Eqn. (3.3) into Eqn. (2.5), the corresponding analytical expression for energy flux for real solution becomes

\[
E_A = \frac{1}{2} \pi a^2 \rho_0 NU_0^2 \left[ 1 - \frac{1}{8} \left( \frac{U_0'^2}{N^2} + 2 \frac{U_0 U_0^*}{N^2} \right) \right] \left[ \left( b_1^2 + b_2^2 \right) + 8a^2 b_2 b_2 \left( 4a^2 - d^2 \right) \right]
\]

(3.6)
Now mountain drag and energy flux due to valley between the ridges of Assam- Burma hills from equations (3.5) and (3.6) as done by Dutta and Naresh (2005) may be written as

\[
D_{AV} = 4\pi\rho_0 NU_0 a^2 b_1 b_2 \left[ 1 - \frac{1}{8} \left( \frac{U_0'^2}{N^2} + 2 \frac{U_0'U_0''}{N^2} \right) \right] \left( \frac{4a^2 - d^2}{4a^2 + d^2} \right)^2
\]

and

\[
E_{AV} = 4\pi\rho_0 NU_0^2 a^2 b_1 b_2 \left[ 1 - \frac{1}{8} \left( \frac{U_0'^2}{N^2} + 2 \frac{U_0''}{N^2} \right) \right] \left( \frac{4a^2 - d^2}{4a^2 + d^2} \right)^2
\]

Now we have chosen similar type of profiles for dated 24 December 2004 and 26 December 2004, which may be favourable cases for the occurrence of lee waves, whose profiles are shown in Fig. 1(b) and Fig. 2(b) respectively.

Next our aim is to calculate $P_A$, $D_A$ and $E_A$ for dated 24 December 2004 and 26 December 2004, so for dated 24 December 2004, we have

\[
P_A'(z = 0) = - \frac{(366x + 6732)}{400 + x^2} - \frac{(285x - 10439)}{400 + (x - 55)^2}
\]

$D_A = 67715 \text{ N/m}$ and $E_A = 69809 \text{ W/m}$

and for dated 26 December 2004, we have

\[
P_A'(z = 0) = - \frac{(1879x + 1461)}{400 + x^2}
\]

$D_A = 210328 \text{ N/m}$ and $E_A = 441690 \text{ W/m}$

De (1971) discussed the existence of lee waves on dated 9 January 1967 and 14 February 1967, whose profile are shown in Fig. 1(a) & Fig. 2(a) respectively.
4. Surface pressure perturbation, Mountain drag and energy flux across Western Ghats - Analytical expression for profile of Western Ghats is

\[ h(x) = \frac{a^2H}{a^2 + x^2} + b \tan^{-1} \frac{x}{a} \] (Sarker et al., 1978) \hspace{1cm} (4.1)

where, \( a = 18.0 \text{km} \), \( H = 0.52 \text{km} \), \( b = \frac{2}{\pi} \times 0.35 \text{km} \).

By Fourier transform of Eqn. (4.1)

\[ \hat{h}(k) = \left[ aH - i \frac{b}{k} \right] e^{-ak} \] \hspace{1cm} (4.2)

and

\[ \hat{h}(k)\hat{h}^*(k) = \left[ a^2H^2 + \frac{b^2}{k^2} \right] e^{-2ak} \] \hspace{1cm} (4.3)

As done in case of Assam-Burma hills, similarly the surface pressure perturbation for Western Ghats of India [Eqn. (2.5)] using Eqn. (4.2) becomes

\[ p'_{\text{w}}(z = 0) = -\rho_0NU_0 \left[ 1 + \frac{i}{2} \frac{U'_0}{N^2} \left( \frac{U'_0}{N^2} + 2 \frac{U_0U'_0}{N^2} \right) \right] \frac{aH(x - ia)}{a^2 + x^2} \]

\[ = -\rho_0NU_0 \left[ 1 - \frac{1}{8} \left( \frac{U'_0}{N^2} + 2 \frac{U_0U'_0}{N^2} \right) \right] \frac{aHx}{a^2 + x^2} - \frac{U'_0}{N} \frac{a^2H}{a^2 + x^2} \] \hspace{1cm} (4.4)

\( p'_{\text{w}}(z = 0) \) is expression of surface pressure perturbation for Western Ghats of India, which is
independent on the plateau part of the Western Ghats. Both the parts of the normalized surface pressure perturbation for Western Ghats independent on the plateau part of the Western Ghats.

Similarly mountain drag across Western Ghats by substitute Eqn. (4.3) into Eqn. (2.3) becomes

\[
D_w = \frac{1}{2} \pi \rho_0 N U_0 H^2 \left[ 1 - \frac{1}{8} \left( \frac{U_0'}{N^2} + 2 \frac{U_0^*}{N^2} \right)^2 \right] \] (4.5)

Eqn. (4.5) is expression of mountain drag, which is independent on the plateau part as well as half width of the Western Ghats.

For energy flux, substituting Eqn. (4.3) into Eqn. (2.5)

\[
E_W = \frac{1}{2} \pi \rho_0 NH^2 U_0' \left[ 1 - \frac{1}{8} \left( \frac{U_0'}{N^2} + 2 \frac{U_0^*}{N^2} \right)^2 \right] \] (4.6)

If we assume that wind is constant with height in equations (4.5) and (4.6), then our results will reduce to similar results as obtained by Dutta (2001).

Now our aim is to evaluate \( P_w \), \( D_w \) and \( E_w \) using realistic wind profile of Santacruz of dated 05 August 2005 and 08 August 2005 as given in Fig. 3 and Fig. 4 respectively.

So for dated 05 August 2005, we have

\[
P_w'(z = 0) = \frac{8.6238x + 116.6}{324 + x^2}
\]

\( D_w = 45820 \text{N/m} \) and \( E_w = 188780 \text{W/m} \)

and for dated 08 August 2005, we have

\[
P_w'(z = 0) = \frac{8.08x + 112.6}{324 + x^2}
\]

\( D_w = 32460 \text{N/m} \) and \( E_w = 100626 \text{W/m} \)
5. Results and discussions - We have derived the analytical expressions for surface pressure perturbation (Eqn. 3.4), mountain drag (Eqn. 3.5) and energy flux (Eqn. 3.6) for 2-D profile of Assam-Burma hills for variable wind. Similar analytical expressions from Eqn. (4.4) to Eqn. (4.6) also have been obtained for western ghats.

For constant wind velocity, mountain drag (Eqn. 3.5) and energy flux (Eqn. 3.6) reduce into following results

\[
\begin{align*}
D_A &= \frac{1}{2} \pi \rho_0 \frac{N U_0}{\rho} \left[ b_1^2 + b_2^2 \right] + 8 a^2 b_1 b_2 \left( \frac{4a^2 - d^2}{4a^2 + d^2} \right) \\
E_A &= \frac{1}{2} \pi \rho_0 \frac{N U_0}{\rho} \left[ b_1^2 + b_2^2 \right] + 8 a^2 b_1 b_2 \left( \frac{4a^2 - d^2}{4a^2 + d^2} \right)
\end{align*}
\]

These results are the same as obtained by Dutta and Naresh (2005)

Using above results into equations (3.5) and (3.6) respectively, we get

\[
\begin{align*}
\frac{D_A}{D_A} &= \frac{E_A}{E_A} = \frac{D_A}{E_A} = \frac{E_A}{E_A} = .98 \\
&= \frac{D_A}{D_A} = \frac{E_A}{E_A} = \frac{D_A}{E_A} = \frac{E_A}{E_A} = .98
\end{align*}
\]

for 26 December 2004

In Similar way normalized mountain drag for realistic wind profile of Santacruz may be written as

\[
\begin{align*}
\frac{D_W}{D_W} &= \frac{E_W}{E_W} = .92135 \\
&= \frac{D_W}{D_W} = \frac{E_W}{E_W} = .863362
\end{align*}
\]

for 05 August 2005

Thus values of normalized mountain drag and energy flux is near to one for both the profiles of hills in SW monsoon.

As in equations (4.5) and (4.6) a factor ‘b’ for plateau part does not appear, so we may say that plateau part of the Western Ghats does not contribute towards the generation of the mountain drag and energy flux, which is conformity with the earlier findings of Dutta (2001). Also normalized pressure perturbation (Eqn. 4.4) is independent on the plateau part of the Western Ghats.

Normalized pressure perturbation (equations 3.4 and 4.4) contains two parts, first part is antisymmetric with respect to hills and second part is symmetric with respect to hills. The symmetric parts of normalized pressure perturbation depend on the shear of surface wind. Thus for the constant wind velocity case Eqn. (3.4) reduces to

\[
P_0(z = 0) = \frac{\rho_0}{\rho_0} \left( \frac{N U_0}{\rho} \right) \left( \frac{U_0^*}{N^2} \right) - \frac{ab_1 x}{a^2 + x^2} - \frac{ab_1 (x-d)}{a^2 + (x-d)^2}
\]

Thus in \( P_0(z = 0) \), there is no symmetric part in the above expression.

References

Das, P. K., 1964, “Lee waves over large Circular Mountain”, Indian J. Met. & Geophys., 15, 4, 547-554.

De, U. S., 1973, “Some studies of mountain waves”, Ph. D. Thesis submitted to Banaras Hindu University.
THE STUDY OF CYCLONIC DISTURBANCES OVER INDIAN SEAS DURING 1991 - 2004

1. Indian coasts around Bay of Bengal and Arabian Sea experience severe weather associated with cyclonic storms every year. Therefore the prediction of such storms with respect to their formation, movement and rainfall assumes great importance. Krishna Rao and Jagannathan (1953) studied the frequency of depressions and cyclonic storms which crossed the east coast of India, south of the latitude 16° N during October to December in the period 1906-1949 and their contributions to the northeast monsoon rainfall over Tamil Nadu. Rai Sarkar (1955) studied the frequency of cyclonic disturbances (depressions and cyclonic storms) crossing each one-degree latitude-longitude square in the Bay of Bengal for the period 1890-1950. Chellappa and Seshadri (1981) using the data for the period 1891 - 1970 enumerated the number of cyclonic storms that crossed four coastal segments on the eastern part of India. Month-wise distribution of storms and severe cyclonic storms crossing the coast of Andhra Pradesh was presented in the paper. Jenamani and Dash (2005) studied the characteristics of monsoon disturbances for different phases of El-Nino.

The present study is based on the data of last 14 years (1991 - 2004) and aims at calculating the number of cyclonic disturbances crossing each 3° latitudinal strip on the Indian coasts. In this study, the characteristics of cyclonic disturbances, namely their speeds, distance traveled before and after crossing the coast have been studied. The intensity of rainfall and wind speed in various sectors of the cyclonic disturbances have also been analyzed. The data of last 35 years (1970 - 2004)