Baryon Asymmetry, Inflation and Squeezed States.

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Abstract

We use the general formalism of squeezed rotated states to calculate baryon asymmetry in the wake of inflation through parametric amplification. We base our analysis on a B and CP violating Lagrangian in an isotropically expanding universe. The B and CP violating terms originate from the coupling of complex fields with non-zero baryon number to a complex background inflaton field. We show that a differential amplification of particle and anti-particle modes gives rise to baryon asymmetry.

1 Introduction

Baryon asymmetry is one of the challenging problems in the standard model of cosmology \cite{1, 2}. It is characterized by the ratio $\eta = \frac{n_b}{n_\gamma}$, where $n_b$ is the number of baryons and $n_\gamma$ is the number of photons in the universe. The present value of the asymmetry in the universe is $\eta \simeq 10^{-10}$. Three conditions postulated by Sakharov \cite{3} are sufficient to guarantee baryon asymmetry. These are (i) Baryon number (B) violation (ii) charge and parity (CP) violation (iii) and the presence of nonequilibrium processes. These conditions are satisfied at very high temperatures, i.e. at Grand Unified Theory (GUT) scales. Thus the first theory of baryogenesis proposed was that of GUT baryogenesis \cite{1, 4, 5}. In GUT baryogenesis, baryon asymmetry is generated through the decay of baryon number violating bosons known
generically as X bosons. The general feature of GUT baryogenesis is that the characteristic energy scale for the processes with B-nonconservation is extremely high, close to the Planck scale, $M_{GUT} = 10^{16} GeV$, thus, it would be kinematically difficult to produce these Bosons in a thermal environment. At temperatures higher than the GUT scale, the rate of their production would be smaller than of the expansion of the universe and any generated baryon asymmetry is presumed to be wiped out by this expansion. If the temperature of the universe could always be smaller than the GUT scale, the corresponding, thermally produced X-bosons would never have been abundant and their role in baryogenesis would be negligible. Furthermore, together with B violating interactions that produce asymmetry in GUTS, there should be proton decay and neutron antineutron oscillations, which have, unfortunately, not been detected experimentally. Since then, many other scenarios of baryogenesis have been developed with baryon non-conservation at much lower energies. One such theory is electroweak baryogenesis where essential physical processes take place at an energy scale of around 100 GeV. The problem with electroweak baryogenesis is that the CP violation that generates the asymmetry is at least 12 orders in magnitude smaller than the observed asymmetry. One proposed solution to overcome these difficulties with baryogenesis models is to consider baryon asymmetry in the context of inflation. It is believed that an inflationary phase precedes the hot phase of cosmological evolution. In the inflationary phase, the energy density is dominated by the contribution from scalar field(s) called inflaton(s) [7]. A phase transition from an inflationary phase to a thermal phase may provide, in principle, the large temperatures needed for baryogenesis. During inflation, all matter and radiation are inflated away. Thus when inflation is over, there must be a reheating process, when the energy stored in the inflaton(s) is converted into thermal radiation. There are several possibilities for this reheating process. The inflaton may decay perturbatively into light particles, which thermalize eventually. There are also non-perturbative models of reheating known as “preheating” models. These are of two types. In one type of model the process of parametric resonance [8, 9] may greatly enhance the production of particles [10]. In another, a rapid quench in the motion of the inflaton may lead to a spinodal instability and results in fast tachyonic preheating [11]. The preheating process is a far from equilibrium process forcing the universe into a non-equilibrium state after the end of inflation and before thermalization takes place. Thus it fulfills the third criterion for baryogenesis. It has been shown that some baryon-number violating processes similar to sphaleron transitions take place at preheating [12, 13], as well as tachyonic preheating [14]. An interesting feature of the preheating scenario is that the production of particles with masses greater than the inflaton mass is possible. In particular, the non-thermally produced particles could include the baryon number violating gauge and Higgs bosons of grand unified theories (GUTs) or right-handed neutrinos, resurrecting the possibility of GUT-scale baryogenesis or leptogenesis.

Preceding all of this, Papastamatiou and Parker [15] proposed a simple model in which
the matter-antimatter asymmetry created by a baryon and CP violating Lagrangian in an isotropically expanding universe is found by a perturbative expansion. Taking an initial state in which no particles participating in the asymmetric interaction are present, they found that matter-antimatter asymmetry is produced during a stage when the radius of the universe is small with respect to its present value. In this work, we have adapted their work to consider the asymmetry that might be created in a more realistic universe that is initially inflating and then enters a rapid preheating phase either by parametric resonance or tachyonic preheating. Furthermore we use a more realistic source of CP violation, namely, a complex time varying background field which we can identify with a classical inflaton field, like in ref. [17]. Preheating is nothing but particle creation in the oscillating background, or one with spinodal instability (tachyonic preheating). Although particle creation or annihilation generally occurs in a time-dependent background, the number of created particles grows exponentially either when the background is periodic in time or when the coupling constant changes suddenly. This results in far from equilibrium particle production and there is an amplification of long wavelength modes. Therefore, if we add the ingredients of CP and B violation, there are chances for generating the matter-antimatter asymmetry of the universe in these eras. Recently, Ref. [17, 18] appeared in which the authors discuss the generation of the baryon asymmetry during reheating. In one of these [18] baryon asymmetry was generated by considering particle creation in the interaction of scalar fields with an oscillating inflaton background. In both these papers, baryon asymmetry was evaluated perturbatively, and the perturbation theory limited the produced asymmetry. In this paper, the first of a series, we extend the work in [17, 18] to include both expansion and non-perturbative effects and look for baryon asymmetry in the wake of inflation through parametric amplification. We calculate the effect of CP violation without resorting to perturbation theory. This allows us to compute non-perturbative out of equilibrium generation of baryon asymmetry. We do so by considering the Lagrangian of Papastamatiou and Parker in an expanding inflationary universe and treat particle production in the general formalism of squeezed rotated states.

2 The Formalism

The most general Lagrangian density with a baryon non-conserving interaction in an expanding inflationary cosmology has been given in [15, 17] as

\[ L = \sqrt{-g}[g^{\mu\nu}\partial_\mu \phi^* \partial_\nu \phi + g^{\mu\nu}\partial_\mu \psi^* \partial_\nu \psi - (m_\phi^2 + \xi_\phi R)\phi^* \phi - (m_\psi^2 + \xi_\psi R)\psi^* \psi - g^{\mu\nu}\partial_\mu \eta^* \partial_\nu \eta - (m_\eta^2 + \xi_\eta R)\eta^* \eta - V(\eta) - \lambda(\eta^2 \phi^* \psi + \eta^2 \psi^* \phi)], \]

(1)

where, \( m_{\phi,\psi} \) are masses of fields \( \phi \) and \( \psi \). \( \phi \) is a complex scalar field that carries a baryon number 1 and \( \psi \) is a complex scalar field that carries a baryon number 0 (which can be
treated as an antilepton field). The field $\eta$ is a minimally coupled complex inflaton field, $R$ is the Ricci scalar and $\xi$ is curvature coupling ($\xi = \frac{1}{6}$ gives the traditional conformal coupling of scalar fields to gravity). The sole purpose of the inflaton field $\eta$ is to provide for inflation and therefore, it is treated as a background classical field while quantum excitations are considered for the other scalar fields. Our prime interest in this paper is to explore the effects of the inflaton field and the expansion of the Universe on the baryon asymmetry. To this end, we consider the background gravitational field to be that of a standard FRW cosmology with an expansion parameter $a(t)$. This Lagrangian can be thought of as an effective Lagrangian which contains terms that violate baryon number by means of a complex field which carries a baryon number interacting with the background inflaton field. This results in a net asymmetry between the produced particles and antiparticles represented by these complex fields, which gives rise to a net baryon asymmetry [15]. It is this particle-antiparticle asymmetry that we now proceed to calculate.

In order to see the salient features of baryogenesis, we simplify the Lagrangian density through the following assumptions. The inflaton field $\eta$ is a classical homogeneous time dependent complex background field, whose temporal evolution is determined by a Euler Lagrange equation of motion, with the inflaton potential $V(\eta)$ in the Friedman-Robertson-Walker metric $ds^2 = dt^2 - a^2(t)dx^2$, given by

$$\ddot{\eta} + 3\frac{\dot{a}}{a}\dot{\eta} + \frac{\partial V}{\partial \eta} = 0.$$  (2)

For this paper, we consider the minimal coupling to the background gravitational field ($\xi_\phi = \xi_\psi = \xi_\eta = 0$). The action is given by

$$S = \int d^3x dt a^3(t)\left(\frac{1}{2}(\dot{\phi}^*\dot{\phi}) - \frac{1}{2a^2(t)}(\nabla \phi^*)(\nabla \phi) + \frac{1}{2}(\dot{\psi}^*\dot{\psi}) - \frac{1}{2a^2(t)}(\nabla \psi^*)(\nabla \psi) - m_\phi^2\phi - m_\psi^2\psi - \lambda(\eta^2\phi^*\psi + \eta^2\psi^*\phi)\right).$$  (3)

Applying a Legendre transformation we can write the Hamiltonian as:

$$H = \int d^3x dt a^3[t\left(\frac{1}{a^6}\pi_\phi \pi_\phi + \frac{1}{a^2}\nabla \phi^*\nabla \phi + \frac{1}{a^6}\pi_\psi \pi_\psi + \frac{1}{a^2}\nabla \psi^*\nabla \psi + m_\phi^2\phi + m_\psi^2\psi + \lambda(\eta^2\phi^*\psi + \eta^2\psi^*\phi)\right),$$  (4)

where, $\pi_\phi$ and $\pi_\psi$ are the canonical momenta of the $\phi$ and $\psi$ fields. We canonically quantize this Hamiltonian. Since we are treating the inflaton field as a classical complex background
field we quantize only the $\phi$ and $\psi$ field by introducing the mode expansions

$$
\psi(x^\mu) = \int d\tilde{k} [a^\psi_k e^{-i\tilde{k} \cdot x} + b^{\dagger \psi}_k e^{i\tilde{k} \cdot x}],
$$

(5)

$$
\psi^*(x^\mu) = \int d\tilde{k} [a^{\dagger \psi}_k e^{i\tilde{k} \cdot x} + b^{\psi}_k e^{-i\tilde{k} \cdot x}],
$$

(6)

$$
\phi(x^\mu) = \int d\tilde{k} [a^\phi_k e^{-i\tilde{k} \cdot x} + b^{\dagger \phi}_k e^{i\tilde{k} \cdot x}],
$$

(7)

$$
\phi^*(x^\mu) = \int d\tilde{k} [a^{\dagger \phi}_k e^{i\tilde{k} \cdot x} + b^{\phi}_k e^{-i\tilde{k} \cdot x}],
$$

(8)

here

$$
k \cdot x = k_\mu x^\mu = \omega t - k_i x_i,
$$

(9)

$$
d\tilde{k} = \frac{d^3kd\tilde{t}}{(2\pi)^3 \sqrt{\omega k^2}},
$$

(10)

and it is clear from the line element given above, the signature of the metric is $(+,-,-,-)$. The quantized Hamiltonian is given as

$$
H = \int d^3k \[ \frac{\omega_\psi}{a^3} \dot{a}^\psi_k a^\psi_k + \omega_\phi \dot{a}^{\dagger \phi}_k a^\phi_k + \frac{\lambda \eta^2 a^3}{2\sqrt{\omega_\phi \omega_\psi}} a^{\dagger \psi}_k a^\psi_k + \frac{\lambda \eta^2 a^3}{2\sqrt{\omega_\phi \omega_\psi}} a^{\dagger \phi}_k a^\phi_k \\
\frac{\omega_\psi}{a^3} b_k^{\dagger \psi}_k b^{-\psi}_k + \omega_\phi b_k^{\dagger \phi}_k b^{-\phi}_k + \frac{\lambda \eta^2 a^3}{2\sqrt{\omega_\phi \omega_\psi}} b^{-\psi}_k b^{\dagger \psi}_k + \frac{\lambda \eta^2 a^3}{2\sqrt{\omega_\phi \omega_\psi}} b^{-\phi}_k b^{\dagger \phi}_k \\
+ \frac{\lambda \eta^2 a^3}{2\sqrt{\omega_\phi \omega_\psi}} [a_k^{\dagger \psi}_k b^{-\psi}_k + b_k^{\dagger \psi}_k a^{-\psi}_k] + \frac{\lambda \eta^2 a^3}{2\sqrt{\omega_\phi \omega_\psi}} [a_k^{\dagger \phi}_k b^{-\phi}_k + b_k^{\dagger \phi}_k a^{-\phi}_k],
$$

(11)

where

$$
\frac{\omega_\phi^2}{a^6} = k^2 + m_\phi^2,
$$

(12)

$$
\frac{\omega_\psi^2}{a^6} = k^2 + m_\psi^2,
$$

(13)

here $\frac{k}{a}$ is the physical wave (co-wave) number of the mode, $k$ is the comoving wave number.

We define the following

$$
\frac{\Omega_\psi^2(t)}{a^6} = \frac{\omega_\psi^2}{a^6} + \sqrt{\frac{\omega_\psi}{\omega_\phi}} \lambda \eta^2(t),
$$

(14)
\[ \frac{\Omega_\phi^2(t)}{a^6} = \frac{\omega_\phi^2}{a^6} + \sqrt{\frac{\omega_\phi}{\omega_\psi}} \eta^2(t), \]

and write the Hamiltonian as

\[ H = \int d^3k \left[ \frac{\omega_\phi}{a^3} a_k^\dagger a_k^\phi + \frac{\omega_\psi}{a^3} a_k^\dagger a_k^\psi + \frac{\omega_\phi}{a^3} \left[ \frac{\Omega_\phi^2(t)}{\omega_\phi^2} - 1 \right] a_k^\dagger a_k^\phi + \frac{\omega_\phi}{a^3} \left[ \frac{\Omega_\phi^2(t)}{\omega_\phi^2} - 1 \right] a_k^\dagger a_k^\psi \right] + \frac{\omega_\phi}{a^3} \left[ \frac{\Omega_\phi^2(t)}{\omega_\phi^2} - 1 \right] a_k^\dagger a_k^\phi + \frac{\omega_\phi}{a^3} \left[ \frac{\Omega_\phi^2(t)}{\omega_\phi^2} - 1 \right] a_k^\dagger a_k^\psi \right]^{(15)} \]

To see the symmetries of this Hamiltonian, which will aid us in its diagonalization, we define the operators:

\[ N_1 = a_k^\dagger a_k^\psi, \quad N_2 = a_k^\dagger a_k^\phi, \quad N_3 = b_k^\dagger b_k^\psi, \quad N_4 = b_k^\dagger b_k^\phi, \]

\[ J_+ = a_k^\dagger a_k^\phi, \quad J_- = a_k^\dagger a_k^\phi, \quad J_0 = \frac{1}{2}(N_1 - N_2), \]

\[ M_+ = b_k^\dagger b_k^\psi, \quad M_- = b_k^\dagger b_k^\psi, \quad M_0 = \frac{1}{2}(N_3 - N_4), \]

satisfying the su(2) algebra

\[ [J_+, J_-] = -2J_0, \quad [J_+, J_0] = -J_+, \quad [J_-, J_0] = J_, \]

\[ [M_+, M_-] = -2M_0, \quad [M_+, M_0] = -M_+, \quad [M_-, M_0] = M_- \]

(19)

(20)

and

\[ K_+ = a_k^\dagger b_k^\phi, \quad K_- = b_k^\dagger a_k^\phi, \quad K_0 = \frac{1}{2}(N_1 + N_3 + 1), \]

\[ L_+ = a_k^\dagger b_k^\psi, \quad L_- = b_k^\dagger a_k^\psi, \quad L_0 = \frac{1}{2}(N_2 + N_4 + 1), \]

satisfying the su(1,1) algebra

\[ [K_+, K_-] = 2K_0, \quad [K_+, K_0] = K_+, \quad [K_-, K_0] = -K_. \]
\[ [L_+, L_-] = 2L_0, \quad [L_+, L_0] = L_+, \quad [L_-, L_0] = -L_- \]  

In terms of these generators we can write the Hamiltonian as:

\[
H = \int d^3k \left[ \frac{2\omega_\psi}{a^3} K_0 + \frac{2\omega_\phi}{a^3} L_0 + \frac{\omega_\psi(t)}{a^3} \right] \left[ \Omega_\psi(t) - 1 \right] (J_+ + M_+) + \frac{\omega_\phi}{a^3} \left[ \Omega_\phi(t) - 1 \right] (J_- + M_-) + \frac{\omega_\psi}{a^3} \left[ \Omega_\psi(t) - 1 \right] (K_+ + L_+) + \frac{\omega_\phi}{a^3} \left[ \Omega_\phi(t) - 1 \right] (K_- + L_-),
\]

The su(1,1) and su(2) symmetries of H are now manifest. We can diagonalize this Hamiltonian by using the following unitary transformation:

\[
H' = U^\dagger(R_2)U^\dagger(R_1)HU(R_1)U(R_2),
\]

where

\[
U(R_1)U(R_2) = \text{exp}[\theta(J_+e^{2\xi} + J_-e^{2\xi})] \text{exp}[\theta(M_+e^{2\xi} + M_-e^{2\xi})],
\]

and operator \( U(R_1) \) provides the well known transformation relations:

\[
U^\dagger(R_1) \begin{pmatrix} a_k^\psi \\ a_k^\phi \end{pmatrix} U(R_1) = \begin{pmatrix} \cos(\theta) & e^{2\xi}\sin(\theta) \\ -e^{-2\xi}\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} a_k^\psi \\ a_k^\phi \end{pmatrix} = \begin{pmatrix} A_k \\ B_k \end{pmatrix},
\]

while \( U(R_2) \) provides the relation

\[
U^\dagger(R_2) \begin{pmatrix} b_k^- \phi \\ b_k^- \psi \end{pmatrix} U(R_2) = \begin{pmatrix} \cos(\theta) & e^{2\xi}\sin(\theta) \\ -e^{-2\xi}\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} b_k^- \phi \\ b_k^- \psi \end{pmatrix} = \begin{pmatrix} C_k \\ D_k \end{pmatrix},
\]

and the angle \( \theta \) is determined from the relation \( \tan(\theta) = \sqrt{\omega_\phi/\omega_\psi} \) so that \( \cos(\theta) = \sqrt{\omega_\psi/(\omega_\phi + \omega_\psi)} \) and \( \sin(\theta) = \sqrt{\omega_\phi/(\omega_\phi + \omega_\psi)} \). The inflaton field is \( \eta = |\eta|e^{i\xi} \) and \( \eta^* = |\eta|e^{-i\xi} \).

After the rotations\(^1\) the Hamiltonian assumes the form:

\[
H' = U^\dagger(R_2)U^\dagger(R_1)HU(R_1)U(R_2) = \int d^3k \left( \omega_\phi + \omega_\psi \right) \frac{a^3}{[\beta(t)][A_k^\dagger A_k + C_k^\dagger C_k]} + \alpha(t)[B_k^\dagger B_k + D_k^\dagger D_k] + \frac{\omega_\psi}{a^3} \left[ \Omega_\psi(t) - 1 \right] [A_k^\dagger C_k^\dagger + D_k B_k] + \frac{\omega_\phi}{a^3} \left[ \Omega_\phi(t) - 1 \right] [B_k^\dagger D_k^\dagger + C_k A_k].
\]

\(^1\)This gives us the quantum optical analogy which we are exploiting in this work, as it is analogous to the effect of a lossless beam splitter. \( r = \cos(\theta) \) and \( t = \sin(\theta) \) are the reflection and transmission coefficients and the phase shift \( \xi \) is between the reflected and transmitted fields.
This Hamiltonian has a $\text{su}(1,1) \times \text{su}(1,1)$ dynamical symmetry seen by defining:

\[
D_{1+} = A_k^\dagger C_{-k}, \quad D_{1-} = C_{-k} A_k, \quad D_{10} = \frac{1}{2}(A_k^\dagger A_k + C_{-k} C_{-k} + 1),
\]

\[
D_{2+} = B_k^\dagger D_k^\dagger, \quad D_{2-} = D_k B_{-k}, \quad D_{20} = \frac{1}{2}(B_k^\dagger B_{-k} + D_k^\dagger D_k + 1),
\]

satisfying the $\text{su}(1,1)$ algebras

\[
[D_{1+}, D_{1-}] = 2D_{10}, \quad [D_{1+}, D_{10}] = D_{1+}, \quad [D_{1-}, D_{10}] = -D_{1-},
\]

\[
[D_{2+}, D_{2-}] = 2D_{20}, \quad [D_{2+}, D_{20}] = D_{2+}, \quad [D_{2-}, D_{20}] = -D_{2-}.
\]

Once again this symmetry is manifest through rewriting the Hamiltonian as

\[
H' = \int d^3k \frac{2(\omega_\phi + \omega_\psi)}{a^3} \left[ [\beta(t)D_{10} + \alpha(t)D_{20}] \right.
\]

\[
+ \frac{\omega_\psi}{a^3} \left[ \frac{\Omega_\psi^2(t)}{\omega_\psi^2} - 1 \right] [D_{1+} + D_{2-}] + \frac{\omega_\phi}{a^3} \left[ \frac{\Omega_\phi^2(t)}{\omega_\phi^2} - 1 \right] (D_{2+} + D_{1-})],
\]

where:

\[
2\alpha = 1 - \frac{\lambda |\eta|^2 a^6}{2\omega_\phi \omega_\psi},
\]

\[
2\beta = 1 + \frac{\lambda |\eta|^2 a^6}{2\omega_\phi \omega_\psi}.
\]

Since an $\text{su}(1,1)$ symmetry implies diagonalizability through a squeezing (Bogolubov) transformation from the quantum optical analogies that we are applying, we see here that a product of two squeezing transformations will provide us with a diagonal Hamiltonian. The squeezing transformations we use are given by the following product of squeezing operators

\[
S(\zeta_1)S(\zeta_2) = \exp[\zeta_1 D_{1+} - \zeta_1^* D_{1-}] \exp[\zeta_2 D_{2+} - \zeta_2^* D_{2-}],
\]

where, $\zeta_1 = r_1 \exp[i \gamma_1]$ and $\zeta_2 = r_2 \exp[i \gamma_2]$ are the squeezing parameters.

The operators $S(\zeta_1)$ and $S(\zeta_2)$ provide the relevant Bogolubov transformations for the Hamiltonian given in equation (29) in terms of the creation and annihilation operators $A_k, B_k, C_k$ and $D_k$ etc.

\[
A_s(k,t) = \mu_1 A_k + \nu_1 C_{-k}^\dagger,
\]

\[
A_s^\dagger(k,t) = \mu_1^* A_k^\dagger + \nu_1^* C_{-k},
\]

\[
B_s(k,t) = \mu_2 B_k + \nu_2 D_k^\dagger,
\]

\[
B_s^\dagger(k,t) = \mu_2^* B_k^\dagger + \nu_2^* D_k.
\]
where \( \mu_1 = \cosh(r_1) = \frac{2\alpha(t)}{(4\alpha(t)-1)^{\frac{1}{2}}} \), \( \nu_1 = e^{-i\gamma_1} \sinh(r_1) = e^{-i\gamma_1} \frac{2\alpha(t)-1}{(4\alpha(t)-1)^{\frac{1}{2}}} \), \( \mu_2 = \cosh(r_2) = \frac{2\beta(t)}{(4\beta(t)-1)^{\frac{1}{2}}} \) and \( \nu_2 = e^{-i\gamma_2} \sinh(r_2) = e^{-i\gamma_2} \frac{2\beta(t)-1}{(4\beta(t)-1)^{\frac{1}{2}}} \).

It is to be noted that obvious mode(k) dependence has been suppressed and will be made explicit when required. Thus the final diagonalized Hamiltonian is

\[
H_f = \int d^3k \frac{(\omega_\phi + \omega_\psi)}{2\alpha^3} (4\alpha(t)-1)^{\frac{1}{2}} [A_1^+(k,t)A_1(k,t)+1] + \frac{(\omega_\phi + \omega_\psi)}{2\alpha^3} (4\beta(t)-1)^{\frac{1}{2}} [B_1^+(k,t)B_1(k,t)+1].
\]  

(42)

Let \( |0(t), 0(t)\rangle \) be the vacuum state of \( H_f(t) \). Then, the su(1,1) symmetry implies that its evolution is generated by the product of squeezing operators \( S(\zeta_1) S(\zeta_2) \) given above. We can write this in terms of the generators of the su(1,1) × su(1,1) symmetry as:

\[
|0(t), 0(t)\rangle = e^{\int \frac{d^3k}{(2\pi)^3} \zeta_1(D_1^+-D_1^-)\zeta_2(D_2^+-D_2^-)}|0(k), 0(k)\rangle
\]  

(43)

where \( |0(k), 0(k)\rangle \), vacuum state for \( H \) is defined as \( a^\phi_k|0(k), 0(k)\rangle = 0, a^\psi_k|0(k), 0(k)\rangle = 0, b^\phi_k|0(k), 0(k)\rangle = 0, \) and \( b^\psi_k|0(k), 0(k)\rangle = 0 \).

For a given Lie algebra, a Baker-Campbell-Hausdorff formula can be used to simplify the exponentials of operators in (43) and is called disentangling in quantum optics [25]. The disentangling formula for su(1,1) is

\[
e^{\zeta_1(D_1^+-D_1^-)}e^{\zeta_2(D_1^+-D_1^-)}|0(k), 0(k)\rangle = e^{\gamma_{1k}D_1^+}e^{\eta'_{1k}D_1^-}e^{\gamma_{2k}D_2^+}e^{\eta'_{2k}D_2^-}|0(k), 0(k)\rangle
\]  

(44)

where, \( \gamma_{1k} = \tanh(\zeta_{1k}) \) and \( \eta' = 2\ln(\cosh(\zeta_{1k})) = -\ln(1 - |\gamma_{1k}|^2) \), \( \gamma'_{1k} = \gamma_{1k}^* \), \( \gamma_{2k} = \tanh(\zeta_{2k}) \) and \( \eta' = 2\ln(\cosh(\zeta_{2k})) = -\ln(1 - |\gamma_{2k}|^2) \), \( \gamma'_{2k} = \gamma_{2k}^* \) and the modes are independent. This allows us to express the time evolution of the state as follows [25]

\[
|0(t), 0(t)\rangle = \frac{1}{\cosh(\zeta_1(t))\cosh(\zeta_2(t))} \prod_k \exp(\tanh(\zeta_1(t))D_{1+})\exp(\tanh(\zeta_2(t))D_{2+})|0(k), 0(k)\rangle.
\]  

(45)

This is just a generalized su(1,1) squeezed state [26].

QFT in curved space times has taught us the importance of Bogolubov transformations and consequent population of one vacuum with particles of another vacuum at an earlier time. Here, we see this phenomenon with respect to both the squeezing (su(1,1)) symmetries. We see that the vacuum at times(t) is populated with particles and anti-particles with respect to vacuum at t=0.

The number of particles and anti-particles can be calculated by the relationship between the
creation and annihilation operators of the initial quanta $a_k^\psi$, $b_k^\psi$, $a_k^\phi$ and $b_k^\phi$, before reheating with the final creation and annihilation operators $A_s$ and $B_s$ given by

$$A_s(k,t) = (\mu_1 \cos(\theta))a_k^\psi + (\nu_1 \sin\theta e^{2i\xi})b_k^\phi + (\mu_1 \sin\theta e^{2i\xi})a_k^\phi + (\nu_1 \cos(\theta))b_k^\psi,$$  \hspace{1cm} (46)

$$B_s(k,t) = (\mu_2 \cos(\theta))a_k^\psi - (\nu_2 \sin\theta e^{-2i\xi})b_k^\phi + (\mu_2 \sin\theta e^{-2i\xi})a_k^\phi + (\nu_2 \cos(\theta))b_k^\psi.$$  \hspace{1cm} (47)

Note that it is the $\phi$ field which carries the baryon number. The number of baryons at time $t$ is then given by

$$N_B(t) = \sum_k \langle A_s\dagger(k,t)A_s(k,t) \rangle = \sum_k |\nu_{k1}|^2,$$  \hspace{1cm} (48)

while the number of anti-baryons is given by

$$N_{\overline{B}}(t) = \sum_k \langle B_s\dagger(k,t)B_s(k,t) \rangle = \sum_k |\nu_{k2}|^2.$$  \hspace{1cm} (49)

Therefore we have a baryon asymmetry which is given by

$$N_B(t) - N_{\overline{B}}(t) = \sum_k (|\nu_{k1}|^2 - |\nu_{k2}|^2),$$  \hspace{1cm} (50)

$$= \sum_k \left[ \frac{\lambda a^6|\eta|^2}{(\omega_{\phi}\omega_{\psi} - \lambda|\eta|^2a^6)} \right]^2 - \left[ \frac{\lambda a^6|\eta|^2}{(\omega_{\phi}\omega_{\psi} + \lambda|\eta|^2a^6)} \right]^2.$$  \hspace{1cm} (51)

where the $k$ dependence is through $\omega_{\phi}, \omega_{\psi}$.

For $\lambda \ll 1$, i.e the perturbative limit we recover the results of Papastamatiou and Parker, the asymmetry being zero to order $\lambda$, and non zero at order $\lambda^2$ and above.

### 3 Evolution Of Asymmetry Parameter:

In order to get some exact results and numerical values for the parameter providing the baryon asymmetry of the universe, we consider a (quite realistic) situation where we can evaluate the Bogolubov coefficients exactly.

To this end, we consider the time evolution of wave function under the action of the Hamiltonian $H_f$ given by (42).
We go over to the coordinate representation, by defining the operators

\[ A_s(k,t) = e^{i \int \frac{\Omega_+}{a^3} dt} \left( \frac{\Omega_+}{a^3} \Pi_A(k,t) + iP_{\Pi_A}(k,t) \right), \]  

\[ A_s^\dagger(k,t) = e^{-i \int \frac{\Omega_+}{a^3} dt} \left( \frac{\Omega_+}{a^3} \Pi_A(k,t) - iP_{\Pi_A}(k,t) \right), \]  

\[ B_s(k,t) = e^{i \int \frac{\Omega_-}{a^3} dt} \left( \frac{\Omega_-}{a^3} \Pi_B(k,t) + iP_{\Pi_B}(k,t) \right), \]  

\[ B_s^\dagger(k,t) = e^{-i \int \frac{\Omega_-}{a^3} dt} \left( \frac{\Omega_-}{a^3} \Pi_A(k,t) - iP_{\Pi_B}(k,t) \right). \]  

\( H_f \) then reduces to the following:

\[ H_f(t) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} \left[ (\frac{\Omega_+}{a^6})^2 \Pi_A^2(k,t) + P_{\Pi_A}(k,t) + (\frac{\Omega_-}{a^6})^2 \Pi_B^2(k,t) + P_{\Pi_B}(k,t) \right], \]  

where

\[ (\frac{\Omega_+}{a^6})^2 = \frac{(\omega_\phi + \omega_\psi)^2}{4a^6} (4\alpha(t) - 1), \]  

\[ (\frac{\Omega_-}{a^6})^2 = \frac{(\omega_\phi + \omega_\psi)^2}{4a^6} (4\beta(t) - 1). \]  

The time evolution of a wave function \( \chi(t) \) under the action of a Hamiltonian \( H(t) \) is simply

\[ H(t)\chi(t) = i \frac{d}{dt} \chi(t), \]  

From the form of \( H_f \) given above, it is clear that it is the direct sum of two independent Hamiltonian \( H_A(t) \) and \( H_B(t) \) for each of the \( A_s \) and \( B_s \) modes. Therefore, the wave function \( \chi(t) \) for the Hamiltonian \( H_f \) is just the sum of two wavefunctions \( \chi_A(t) \) and \( \chi_B(t) \) where they evolve independently. Fourier decomposing the wave function \( \chi_A(t) \) and \( \chi_B(t) \) and the Hamiltonian \( H_A(t) \) and \( H_B(t) \), we write
\[ \chi_A(t) = \int \frac{d^3 k}{(2\pi)^3} \chi_A(k, t); \quad H_A(t) = \int \frac{d^3 k}{(2\pi)^3} H_A(k, t) \]
\[ \chi_B(t) = \int \frac{d^3 k}{(2\pi)^3} \chi_B(k, t); \quad H_B(t) = \int \frac{d^3 k}{(2\pi)^3} H_B(k, t), \]

and each mode evolves as.

\[ H_A(k, t) \chi_A(k, t) = \frac{i}{d} \chi_A(k, t), \]
\[ H_B(k, t) \chi_B(k, t) = \frac{i}{d} \chi_B(k, t). \]

Since the Hamiltonians \( H_A \) and \( H_B \) are time dependent harmonic oscillators, in the coordinate space representation \((\Pi_A, P_A)\) and \((\Pi_B, P_B)\) the wave functions \( \chi_A(k, t) \) and \( \chi_B(k, t) \) can be represented by gaussian wave functions.

\[ \chi_A(k, t) = L_A(t) e^{-\frac{W_A(t) \Pi_A^2}{2}} \chi_A(k, 0), \]
\[ \chi_B(k, t) = L_B(t) e^{-\frac{W_B(t) \Pi_B^2}{2}} \chi_B(k, 0), \]

A time derivative gives

\[ i \frac{\partial}{\partial t} \chi_A(k, t) = \left( \frac{i}{L_A} \Pi_A - \frac{i}{P_A} W_A(k, t) \right) \chi_A(k, t), \]

similarly for \( \chi_B(k, t) \) while the evolution equation (62) gives

\[ i \frac{\partial}{\partial t} \chi_A(k, t) = \frac{1}{2a^6} \left[ \Omega_A^2 \Pi_A^2 - \frac{\partial^2}{\partial \Pi_A^2} \right] \chi_A(k, t), \]

A similar equation holds for \( \chi_B(k, t) \).

This gives us

\[ W_A(t) = -i a^3 \left( \frac{\dot{\chi}_A(k, t)}{\chi_A(k, t)} \right), \]

implying

\[ \ddot{\chi}_A(k, t) + \frac{3}{a} \dot{\chi}_A(k, t) + \frac{\Omega_A^2}{a^6} \chi_A(k, t) = 0. \]
The equations satisfied by the two wave functions for each mode are

\[
\ddot{\chi}_A(k, t) + 3\frac{\dot{a}}{a}\dot{\chi}_A(k, t) + \frac{\Omega_\phi^2}{a^6}\chi_A(k, t) = 0, \tag{71}
\]

\[
\ddot{\chi}_B(k, t) + 3\frac{\dot{a}}{a}\dot{\chi}_B(k, t) + \frac{\Omega_\psi^2}{a^6}\chi_B(k, t) = 0. \tag{72}
\]

Our aim in this paper has been to calculate the baryon asymmetry of the universe for the specific model we have chosen. To this end, we have now obtained the time evolution equations for the modes of the wave functions of our baryons and anti-baryons. To solve these equations, we shall choose the universe to be FRW as mentioned earlier.

The metric of our FRW is chosen to be conformally flat.

\[
d\tilde{s}^2 = a(\tau)^2(d\tau^2 - dx^2). \tag{73}
\]

by employing the scaled time

\[
d\tau = a(t)^{-1}dt. \tag{74}
\]

The equations of motion for the wave functions \(\chi_A(k, t)\) and \(\chi_B(k, t)\) given above can be transformed into ones that resemble a harmonic oscillator with time dependent frequencies.

\[
\frac{1}{a^2(\tau)}\frac{d^2\chi_A(k, \tau)}{d\tau^2} + \frac{2}{a^3(\tau)}\frac{da}{d\tau}\frac{d\chi_A(k, \tau)}{d\tau} + \frac{\Omega_\phi}{a^6}\chi_A(k, \tau) = 0, \tag{75}
\]

\[
\frac{1}{a^2(\tau)}\frac{d^2\chi_B(k, \tau)}{d\tau^2} + \frac{2}{a^3(\tau)}\frac{da}{d\tau}\frac{d\chi_B(k, \tau)}{d\tau} + \frac{\Omega_\psi}{a^6}\chi_B(k, \tau) = 0. \tag{76}
\]

A change of variable

\[
u_A = a\chi_A \tag{77}
\]

and similarly for \(\chi_B(k, \tau)\) transforms (75), (76) to

\[-u_A'' + (E - V_1(\eta\tau))u_A = 0, \tag{78}\]

and

\[-u_B'' + (E + V_2(\eta\tau))u_B = 0, \tag{79}\]

where prime denotes differential with respect to \(\tau\) and

\[E = -\frac{(\omega_\phi + \omega_\psi)^2}{4a^6} \tag{80}\]

\[V_1(\eta\tau) = -\frac{1}{a(\tau)d\tau^2} - [A|\eta|^2], \tag{81}\]

13
\[ V_2(\eta \tau) = \frac{1}{a(\tau)} \frac{d^2 a}{d\tau^2} - |A|^2. \]  

(82)

where \( A = \frac{\Lambda(\omega_\phi+\omega_\psi)^2}{8\omega_\phi \omega_\psi} \), and \( \omega_\phi \), \( \omega_\psi \) are given by the equation (12) and (13).

These forms of the equations suggest interpretations in two ways. Treating \( \tau \) as a spatial variable, they are Schroedinger like equations with \( E = \omega_\phi^2(\tau) \). This allows the calculation of reflection and transmission coefficients over potential barriers provided by \( V_1(\eta \tau) \) and \( V_2(\eta \tau) \) terms. On the other hand, they are equations for time dependent harmonic oscillators with time dependent frequencies (parametric oscillators).

In particle creation problems, the potential barrier reflection \( (R) \) and transmission \( (T) \) coefficients can be related to the squeezing parameter \( (r) \) through \( \sinh^2(r) = \nu^2 = \frac{R}{T} \). We shall use this relationship for the calculation of the squeezing-parameter-dependent number operator \( N(k) = |\nu|^2 \) involved in the evolution of wave functions for the particles and anti-particles in our FRW background.

Following previous work \[9\] we assume an oscillating inflaton background

\[ \eta = \Lambda \sin(m \tau), \]  

(83)

where \( \Lambda = |\Lambda|e^{i \xi} \) is complex and \( m \) is the inflaton mass. Substituting the value of \( \eta \) in eq(78) and (79) we get the following differential equations,

\[ u''_A + \left( -\frac{a''}{a} + E - \frac{B}{2} + \frac{B}{2} \cos(2m\tau) \right) u_A = 0, \]  

(84)

and

\[ u''_A + \left( -\frac{a''}{a} + E + \frac{B}{2} - \frac{B}{2} \cos(2m\tau) \right) u_A = 0, \]  

(85)

where \( B = |\Lambda|^2 A \).

These are exact equations for the evolution of the baryon and anti-baryon fields in an expanding universe. Since the number of baryons and anti-baryons are related to the reflection and transmission coefficients we see from the above that there will be an asymmetry, since the baryons encounter a potential barrier whereas the anti-baryons encounter a potential well.

To analyze these equations for the baryon asymmetry, we first consider the case of constant expansion \( \left( \frac{a''}{a} = 0 \right) \).

We see then that equations (84) and (85) are Mathieu equations

\[ u''_A + (E - \frac{B}{2} + \frac{B}{2} \cos(2m\tau)) u_A = 0, \]  

(86)
and

\[ u''_A + (E + B - \frac{B}{2} \cos(2m\tau))u_A = 0, \quad (87) \]

From the theory of Mathieu equations and parametric resonance we see that the frequency

\[ \omega^2_k = \frac{2k^2 + m_\phi^2 + m_\psi^2 + 2\sqrt{k^2 + m_\phi^2}\sqrt{k^2 + m_\psi^2}}{4} = \left(\frac{n}{2}\omega\right)^2, \quad (88) \]

must be half integer multiples of a lowest frequency \( \omega \). The resonance for the lowest frequency occurs for

\[ \omega^2_k - \left(\frac{n}{2}\omega\right)^2 \equiv \Delta_n, \quad (89) \]

where \( \Delta_n \) is the width of the the instability band of Mathieu equations. For the values of \( \omega_k \) in this instability band, we assume a broad band resonance such that the Mathieu equation has instability bands within which parametric resonance can occur. We shall select the first instability region as a broad resonance band. The frequency in a broad resonance band is slowly varying, thus

\[ \left| \frac{\omega'_k}{\omega_k} \right| \ll \omega. \quad (90) \]

Since \( \omega \propto \frac{k}{a} \) this implies that

\[ \frac{a'}{a} \ll \omega. \quad (91) \]

Thus \( \frac{a''}{a} \) being put to zero is a justifiable approximation.

Now we make another approximation.

We will assume that \( m_\phi^2, m_\psi^2 \ll \omega_k^2 \). Thus, to a good approximation,

\[ \omega \propto \frac{k}{a} \quad (92) \]

and we find

\[ u''_A + (E - B \sin^2(m\tau))u_A = 0, \quad (93) \]

and

\[ u''_B + (E + B \sin^2(m\tau))u_B = 0, \quad (94) \]

where

\[ E = k^2, \quad A = \lambda, \quad B = \lambda|\Lambda|^2 \quad (95) \]
In the region of broad resonance we replace the oscillating potential near its zeros with an asymptotically flat potential of the form

$$|\eta|^2 = |\Lambda|^2 \sin^2(m\tau) \simeq 2|\Lambda|^2 \tanh^2\left(\frac{m(\tau - \tau_i)}{\sqrt{2}}\right). \quad (96)$$

In each instability region, we have

$$u''_A + (k^2 - 2\lambda|\Lambda|^2 \tanh^2\left(\frac{m(\tau - \tau_i)}{\sqrt{2}}\right))u_A = 0, \quad (97)$$

and

$$u''_B + (k^2 + 2\lambda|\Lambda|^2 \tanh^2\left(\frac{m(\tau - \tau_i)}{\sqrt{2}}\right))u_B = 0. \quad (98)$$

We will calculate the transmission and reflection coefficients of the above equations, which will then provide the amount of particle production for each case.

The differential equations (97) can be solved through the following definitions

$$\kappa_1^2 = \frac{k^2 - (2\lambda|\Lambda|^2)}{\rho^2}, \quad \rho^2 = \frac{m^2}{2} \quad (99)$$

along with a change of variable given by:

$$y = \rho(\tau - \tau_i), \quad (100)$$

we get the following differential equation

$$\frac{d^2u_1}{dy^2} + \left[\kappa_1^2 + \frac{(2\lambda|\Lambda|^2)}{\rho^2} \sech^2(y)\right]u_1 = 0. \quad (101)$$

The transmission [27] for this barrier is calculable and in terms of physically known values is

$$|T|^2 = \frac{\sinh^2(\pi\kappa_1)}{\left(\cos(\pi\sqrt{\frac{2|\Lambda|^2}{\rho}} + \frac{1}{4})\right)^2 + \sinh^2(\pi\kappa_1)}. \quad (102)$$

The amount of particle production $n_{1k}$ is given as

$$n_{1k} = |\nu_{1k}|^2 = \frac{1 - T}{T} = \left(\frac{\cos(\pi\sqrt{\frac{2|\Lambda|^2}{\rho}} + \frac{1}{4})}{\sinh(\pi\kappa_1)}\right)^2 \quad (103)$$
Now for the anti-particle modes the differential equations (98) governing the evolution of their wave functions can also be solved exactly. Defining

$$\kappa_2^2 = \frac{k^2}{\rho^2},$$

(104)

and a change of variable as before,

$$y = \rho(\tau - \tau_i)$$

(105)

gives the following reduced differential equation

$$\frac{d^2 u_2}{dy^2} + \left[\kappa_2^2 + \frac{(2\lambda|\Lambda|^2)}{\rho^2}\tanh^2(y)\right] u_2 = 0.$$  

(106)

The amount of particle production for $B_s(k, t)$ modes can be calculated from the transmission coefficient and is given by

$$n_{2k} = |\nu_{2k}|^2 = \left(\frac{\cosh\left(\pi \sqrt{\frac{2\lambda|\Lambda|^2}{\rho^2} + \frac{\kappa_2^2}{2}}\right)}{\sinh\left(\pi \sqrt{\frac{2\lambda|\Lambda|^2}{\rho^2} + \kappa_2^2}\right)}\right)^2.$$  

(107)

We plot the co-wave number dependent number of baryons and anti-baryons in figure 1 and figure 2. for various values of the parameter $\frac{2\lambda|\Lambda|^2}{m^2}$. As expected, from the differential equations satisfied by the mode functions, we see that because the baryons see a potential barrier and the anti-baryons see a potential well, there is a differential amplification of particle and anti-particle modes leading to an asymmetry which survives with time. Therefore, from our model we have generated a baryon asymmetry in an entirely non-perturbative fashion. The approximations made are only at the end of derivations and have been used to illustrate the methodology. It is to be noticed that the equations (84-85) are exact for an oscillating inflaton field in an FRW Universe. It is entirely possible to solve these equations to incorporate the effects of the expansion of the Universe on the generation of baryon asymmetry. Since we are relying on parametric resonance for enhancement/suppression of the particle/anti-particle creation, the restriction to the lowest instability band of the Mathieu equation is a reasonable one. The total integrated baryon asymmetry of the universe is given by

$$N_B - N_{\overline{B}} = \int_0^\infty dk k^2 n_{1k} - \int_0^\infty dk k^2 n_{2k}.$$  

(108)
Figure 1: shows the variation of particles (dashed line) and anti particles (solid lines) for $\frac{2\lambda |\Lambda|^2}{m^2} = .5$ as a function of comoving wave number $kbar = \frac{k}{a}$.

Figure 2: shows the variation of particles (dashed line) and anti particles (solid lines) for $\frac{2\lambda |\Lambda|^2}{m^2} = .25$ as a function of comoving wave number $kbar = \frac{k}{a}$.
4 Conclusions.

Sakharov's requirements for the generation of a baryon asymmetry in the Universe have been the starting point of our paper. Adapting the model of [15], to the currently prevalent models of inflation and preheating through a background classical complex scalar inflaton field [17], we have found equations for the evolution of baryon/anti-baryon modes in an expanding FRW metric. We have been able to derive the general evolution equations in an entirely non-perturbative fashion. This has been possible through an analogy with quantum optics and its methods. In particular, we have found that at any particular time, the Hamiltonian for the quantum modes has $su(2)$ and $su(1, 1)$ symmetries. Quantum optical techniques allowed us to make these symmetries explicit and thereafter to diagonalize the Hamiltonian. The diagonalization procedure further allowed us to relate the rotated and squeezed operators to the traditional Bogoliubov transformations between the mode function at an earlier time to those at later times. The approximations that we have utilized at the end to find explicit variation of the number of particles/anti-particles created as a function of the co-moving wave number are only therefore illustrative. It is entirely possible to carry out the calculations without these approximations. Of course, we will then not have the luxury of the vast literature on the solutions of the Mathieu equations and instability regions allowing for parametric resonant solutions will have to be looked for. On the plus side, the full equations (84-85) allow the possibility of including the contributions to the baryon asymmetry coming from the expansion of the Universe. The centerpiece of our effort are the equations [78] and [79], which are exact and show the role played by the expansion factor and the inflaton field on the evolution of the particle and antiparticle modes. These can be solved for any type of inflationary scenario and will be the subject of future investigations.

References

[1] S. Weinberg, Gravitation and Cosmology (J Wiley, New York, 1972).
[2] E. Kolb and M. Turner, The Early Universe, (Addison-Wesley Publishing Company, Redwood City, California, 1990).
[3] A. Sakharov Pis’ma Z. Eksp. Tero. Fiz. 5, 32 (1967); English translation: JETP Lett. 5, 24 (1967).
[4] M. Yoshimura, Phys. Rev. Lett. 41, 281 (1978).
[5] D. V. Nanopoulous and S. Weinberg, Phys. Rev. D20, 2484 (1979).
[6] A. D. Dolgov, “Baryogenesis, 30 years after,” arXiv:hep-ph/9707419.

[7] A. H. Guth, Phys. Rev. D23, 347 (1981).

[8] R. H. Brandenberger, Braz. J. Phys. 31, 131 (2001) arXiv:hep-ph/0102183.

[9] Y. Shtanov, J. Traschen and R. H. Brandenberger, Phys. Rev. D51, 5438 (1995).

[10] L. Kofman, A. Linde, A. A. Starobinsky, Phys. Rev. D56, 3258 (1997).

[11] G. Felder, J. Garcia-Bellido, P. B. Greene, L. Kofman, A. Linde and I. Tkachev, Phys. Rev. Lett. 87, 011601 (2001) hep-ph/0012142.

[12] M. Dine and A. Kusenko, Rev. Mod. Phys. 76, 1 (2004) arXiv:hep-ph/0303065.

[13] J. Garcia-Bellido, “Tachyonic preheating and spontaneous symmetry breaking,” arXiv:hep-ph/0106164.

[14] J. I. Skullerud, J. Smit and A. Tranberg, Nucl. Phys. Proc. Suppl. 129, 771 (2004) arXiv:hep-lat/0309046.

[15] N. J. Papastamatiou and L. Parker, Phys. Rev. D19, 2283 (1979).

[16] R. Rangarajan, Pramana. 53, 1061 (1999).

[17] R. Rangarajan, D. V. Nanopoulous, Phys. Rev. D65, 063511 (2001).

[18] K. Funakubo, A. Kakuto, S. Otsuki and F. Toyoda, Prog. Theor. Phys. 105, 773 (2001) arXiv:hep-ph/0010266.

[19] K. R. S. Balaji and R. H. Brandenberger, Phys. Rev. Lett. 94, 031301 (2005) arXiv:hep-ph/0407090.

[20] I. Affleck and M. Dine Nucl.phy. B249, 361 (1985).

[21] M. Riazuddin, “Particle aspects of cosmology and baryogenesis,” arXiv:hep-ph/0302020.

[22] B. A. Bambah and C. Mukku, Annals of Physics. 54, 314 (2004).

[23] B. A. Bambah and C. Mukku, Phys. Rev. D70, 034001 (2004).

[24] B. Bambah and C. Mukku, “Charged vs. neutral particle creation in expanding universes: A quantum field theoretic treatment,” arXiv:hep-th/0307286.
[25] A. M. Perelomov, *Generalized Coherent States and their Applications*, (Springer-Verlag, Berlin, 1986).

[26] V.V. Dodonov, *J. Opt. B: Quantum Semiclass. Opt.* **4** R1-R33 (2004).

[27] S. Flugge, *Practical Quantum Mechanics, Vol. I and Vol. II*, (Springer, New York 1976).

[28] D. Campo and R. Parentani Phys Rev **D67** (2003) 103522