Meson mass modification in strange hadronic matter

Subrata Pal, Song Gao, Horst Stöcker, Walter Greiner
Institut für Theoretische Physik, J.W. Goethe-Universität, D-60054 Frankfurt am Main, Germany

Abstract

We investigate in stable strange hadronic matter (SHM) the modification of the masses of the scalar ($\sigma, \sigma^*$) and the vector ($\omega, \phi$) mesons. The baryon ground state is treated in the relativistic Hartree approximation in the nonlinear $\sigma$-$\omega$ and linear $\sigma^*$-$\phi$ model. In stable SHM, the masses of all the mesons reveal considerable reduction due to large vacuum polarization contribution from the hyperons and small density dependent effects caused by larger binding.

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The study of the properties of hadrons, in particular the light vector mesons ($\omega$, $\rho$, $\phi$), has recently attracted wide interest both experimentally and theoretically. Recent experiments from the HELIOS-3 [1] and the CERES [2] collaborations indicate a significant amount of strength below the $\rho$-meson peak. This has been interpreted [3] as the decrease of the $\rho$-meson mass in the medium. In an effective chiral model based on the symmetries of QCD, Brown and Rho [4] predicted an approximate scaling law for the in-medium decrease of the masses of the mesons and nucleons. On the other hand, in the quantum hadrodynamic (QHD) model [5,6] based on structureless baryons, the meson masses are found to increase when particle-hole excitations from the nucleon Fermi sea are considered. By also including the particle-antiparticle excitations from the Dirac sea, the meson masses has been, however, shown to decrease [7–9].

So far all investigations of in-medium meson mass modification has been done in the nuclear matter environment. However, presently there is a growing interest about the possibility of bound strange matter. In analogy to the stable strange quark matter [10], strange hadronic matter (SHM) composed of nucleons and hyperons has been speculated [11] to be absolutely stable or at least metastable with respect to weak hadronic decays. It is expected that such strange matter may possibly be created at RHIC and LHC [12]. It is therefore worth investigating the properties of meson masses in the strange baryonic matter environment. This study is not only of interest in itself but could serve as a signal of the formation of SHM in heavy ion collisions. In this letter, in a complete self-consistent calculation within the framework of the QHD model, we examine the mass modifications of both, the scalar and the vector mesons in SHM.

Let us consider the composition of SHM which is stable with respect to particle emission. The analysis of level shifts and widths of $\Sigma^-$ atomic level suggest [13] a well-depth of $\Sigma$ in nuclear matter of $U^{(N)}_{\Sigma} \approx 20 – 30$ MeV. While for strong processes, $\Sigma N \rightarrow \Lambda N$ and $\Sigma \Lambda \rightarrow \Xi N$, the energy released is $Q \approx 78$ and 52 MeV, respectively. Consequently, systems involving $\Sigma$’s are unstable with respect to these strong decays. Analysis of binding energies of $\Lambda$, and emulsion experiments with $K^-$ beams [13] yields well-depths of $\Lambda$ and $\Xi$ in nuclear matter of $U^{(N)}_{\Lambda,\Xi} \approx 28 – 30$ MeV. However, in strong decays, $\Lambda \Lambda \rightleftarrows N \Xi$, the system can be stable since $Q \approx 25$ MeV. Therefore, we consider here only the set of baryon species $B \equiv \{N, \Lambda, \Xi\}$, which can constitute stable SHM.

Up to now all investigations of meson mass modification has been performed in the simple linear Walecka model with nucleons only. Considering here nonlinear self-interactions of the scalar field $\sigma$ in the QHD model, the total Lagrangian is

$$\mathcal{L} = \sum_B \bar{\psi}_B \left(i \gamma^\mu \partial_\mu - m_B + g_{\sigma B} \sigma - g_{\omega B} \gamma^\mu \omega^\mu \right) \psi_B + \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - m_{\sigma}^2 \sigma^2 \right) + U(\sigma) - \frac{1}{4} \omega^\mu \omega^\nu + \frac{1}{2} m_{\omega}^2 \omega_\mu \omega^\mu + \delta \mathcal{L}, \tag{1}$$

where the summation is over all baryon species $B \equiv \{N, \Lambda, \Xi\}$. The scalar self-interaction $U(\sigma) = g_\sigma \sigma^3/3 + g_3 \sigma^4/4$ yields sufficient flexibility to the effective Lagrangian of the model and they are also necessary for a good reproduction of nuclear ground state properties, in particular the compressibility. The term $\delta \mathcal{L}$ contains renormalization counterterms. This model is not able to reproduce the observed strongly attractive $\Lambda \Lambda$ interaction. The situation can be remedied by introducing two additional meson fields, the scalar meson $f_0(975)$ (de-
noted as \( \sigma^* \) hereafter) and the vector meson \( \phi(1020) \) which couple only to the hyperons (\( Y \)). The corresponding (linear) Lagrangian is

\[
\mathcal{L}^{YY} = \sum_{B=\Lambda, \Xi} \overline{\psi}_B \left( g_{\sigma^* B} \sigma^* - g_{\phi B} \gamma_\mu \phi^\mu \right) \psi_B \\
+ \frac{1}{2} \left( \partial_\mu \sigma^* \partial^\mu \sigma^* - m_{\sigma^*}^2 \sigma^* \sigma^* \right) + \frac{1}{4} \phi_{\mu\nu} \phi^{\mu\nu} + \frac{1}{2} m_{\phi}^2 \phi \phi^\mu. \tag{2}
\]

The self-consistent propagator in the medium for baryon species \( B \) can be written as sum of the Feynman \( [G^F_B(k)] \) and density-dependent \( [G^D_B(k)] \) parts:

\[
G_B(k) = G^F_B(k) + G^D_B(k) \\
= \left( \gamma^\mu k^*_\mu + m_B^* \right) \frac{1}{k^2 - m_B^2 + i\eta} + \frac{i\pi}{E^*_B(k)} \delta(k^*_0 - E^*_B(k)) \theta(k_{F_B} - |k|). \tag{3}
\]

The momentum and energy of baryon \( B \) are \( k^*_\mu = (k_0 - g_{\omega B} \omega_0 - g_{\phi B} \phi_0, \mathbf{k}) \) and \( E^*_B(k) = [k^2 + m_B^2]^{1/2} \), and \( k_{F_B} \) are the Fermi momenta. The scalar mean fields shift the mass \( m_B \) of the baryons both in the Fermi and Dirac sea to

\[
m_B^* = m_B - g_{\sigma B} \sigma - g_{\sigma^* B} \sigma^*. \tag{4}
\]

In mean field theory (MFT), the total energy of the system is generated by the presence of all baryons in the occupied Fermi seas. In contrast, in the relativistic Hartree approximation (RHA), the effect of the infinite Dirac sea is also included. The renormalized total energy density in RHA is given by

\[
\varepsilon_{\text{RHA}} = \varepsilon_{\text{MFT}} + V_B + V_\sigma, \tag{5}
\]

where \( \varepsilon_{\text{MFT}} \) is the usual MFT energy density. The contribution from vacuum fluctuation of all the baryons is given by

\[
V_B = -\sum_B \frac{I_B}{8\pi^2} \left[ m_B^4 \ln \left( \frac{m_B}{m_B} \right) + m_B^3 (m_B - m_B^*) - \frac{7}{2} m_B^2 (m_B - m_B^*)^2 \\
+ \frac{13}{3} m_B (m_B - m_B^*)^3 - \frac{25}{12} (m_B - m_B^*)^4 \right], \tag{6}
\]

where \( I_B = 2I + 1 \) is the isospin degeneracy of the baryon \( B \). The mass shift of the Dirac sea from \( m_B \) to \( m_B^* \) produces the vacuum fluctuation contribution. The renormalized nonlinear \( \sigma \)-meson contribution is

\[
V_\sigma = \frac{m_\sigma^4}{(8\pi)^2} \left[ (1 + \lambda_1 + \lambda_2)^2 \ln(1 + \lambda_1 + \lambda_2) - (\lambda_1 + \lambda_2) \\
- \frac{3}{2} (\lambda_1 + \lambda_2)^2 - \frac{1}{3} \lambda_1^2 (\lambda_1 + 3\lambda_2) + \frac{1}{12} \lambda_1^4 \right], \tag{7}
\]

where \( \lambda_1 = 2g_2 \sigma/m_\sigma^2 \) and \( \lambda_2 = 3g_3 \sigma^2/m_\sigma^2 \).

The meson propagator in the baryonic medium can be computed by summing over bubbles which consists of repeated insertions of the lowest order one-loop proper polarization
part. This is equivalent to relativistic random phase approximation (RPA). Since both, scalar \((\sigma, \sigma^*)\) and vector \((\omega, \phi)\) mesons are present in our model, it is essential to include scalar-vector mixing which is a pure density dependent effect. Therefore it is convenient to define a full scalar-vector meson propagator \(D_{ab}\) in the form of a \(5 \times 5\) matrix with indices \(a, b\) ranging from 0 to 4, where 4 corresponds to the scalar meson and 0 to 3 the components of vector meson. Moreover, the (strangeness violating) scalar-vector coupling between the strange and nonstrange mesons are prohibited by invoking the OZI rule. We therefore consider couplings between \(\sigma - \omega\) and \(\sigma^* - \phi\) separately. We shall present explicitly the calculations only for the \(\sigma - \omega\) propagator; the expressions for \(\sigma^* - \phi\) propagator would follow similarly. Dyson’s equation for the full \(\sigma - \omega\) propagator \(D\) can be written in matrix form as

\[
D = D^0 + D^0 \Pi D ,
\]

(8)

where \(D^0\) is the lowest order \(\sigma - \omega\) meson propagator

\[
D^0 = \begin{pmatrix}
D^0_{\mu\nu} & 0 \\
0 & \Delta^0
\end{pmatrix} ,
\]

(9)

expressed in terms of the noninteracting \(\sigma\) and \(\omega\)-meson propagators

\[
\Delta^0(q) = \frac{1}{q_\mu^2 - m_\sigma^2 + i\eta} ,
\]

(10)

\[
D^0_{\mu\nu}(q) = \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{m_\omega^2} \right) D^0(q) ,
\]

(11)

\[
D^0(q) = \frac{-1}{q_\mu^2 - m_\omega^2 + i\eta} ,
\]

(12)

where \(q_\mu^2 \equiv q_0^2 - \mathbf{q}\) is the four-momentum carried by the meson. Note that the effect of nonlinear \(\sigma\)-interaction (the boson loops) is to replace the bare meson mass \(m_\sigma^2\) in Eq. (10) by \(\tilde{m}_\sigma^2 = m_\sigma^2 + \partial^2 U(\sigma)/\partial\sigma^2 = m_\sigma^2 + 2g_2\sigma + 3g_3\sigma^2\) (see Ref. [14]).

The polarization insertion of Eq. (8) is also a \(5 \times 5\) matrix

\[
\Pi = \left( \begin{array}{cc}
\sum_B \Pi^B_{\mu\nu}(q) & \sum_B \Pi_{\mu}^{(M)B}(q) \\
\sum_B \Pi_{\nu}^{(M)B}(q) & \Pi^B_{\sigma}(q) + \sum_B \Pi^B_{\omega}(q)
\end{array} \right) ,
\]

(13)

where each entries in Eq. (13) is summed over all the baryons, except for the pion loop \(\Pi^B_{\pi}(q)\). The effect of relatively large \(\sigma\)-width has been accounted by including the contribution of pion loop to \(\Pi^B_{\sigma}\); the in-medium modification of the pion loop is neglected. The pion propagator \(\Delta_\pi\) is obtained from Eq. (10) with \(\tilde{m}_\sigma^2\) replaced by \(m_\pi^2\). The pion loop polarization contribution to the \(\sigma\)-meson is

\[
\Pi^B_{\pi}(q) = \frac{3}{2} g_{\sigma\pi} m_\pi^2 \int \frac{d^4 k}{(2\pi)^4} \Delta_\pi(k) \Delta_\pi(k + q) .
\]

(14)

In terms of the baryon propagator the lowest order \(\sigma, \omega\) and \(\sigma - \omega\) (mixed) polarizations for the baryon loop \(B\) are respectively given by
\[ \Pi^B_\sigma(q) = -ig^2_\sigma B \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ G_B(k) G_B(k + q) \right] , \] (15)

\[ \Pi^B_\mu\nu(q) = -ig_\omega B \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ G_B(k) \gamma_\mu G_B(k + q) \gamma_\nu \right] , \] (16)

\[ \Pi^{(M)B}_\mu(q) = ig_\sigma B g_\omega B \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ G_B(k) \gamma_\mu G_B(k + q) \right] . \] (17)

As in the case for the baryon propagator (Eq. (3)) the above polarization insertions (except \( \Pi^\pi_\sigma \)) can be expressed as the sum of Feynman (F) part and density-dependent (D) part, i.e. \( \Pi^B = \Pi^{B(F)} + \Pi^{B(D)} \). The finite D-part has the form \( G^D \times G^D \times G^F + G^F \times G^D \) which describes particle-hole excitations and also includes the Pauli blocking of \( BB \) excitations. Since the polarizations of the D-part are defined in Ref. [15], we will not refer them here.

The divergent F-part of the polarization insertions can be rendered finite by adding appropriate counterterms to the Lagrangian of Eq. (1). For the \( \sigma \), each of the baryons loops and the \( \pi \)-loop have to be renormalized separately. For any baryon loop contribution to the \( \sigma \) the usual counterterm Lagrangian [5,6] is used

\[ \delta \mathcal{L}_\sigma = \sum_{l=2}^4 \frac{\alpha_l}{l!} \sigma^l + \frac{\zeta_\sigma}{2} \partial_\mu \sigma \partial^\mu \sigma . \] (18)

The coefficients \( \alpha_2 \) and \( \zeta_\sigma \) can be obtained by imposing the condition that the propagator in vacuum \( \left( m^*_B = m_B \right) \) reproduces the “physical” properties of \( \sigma \)-meson [15]:

\[ \Pi^B_\sigma^{(RF)}(q^2_\mu; m^*_B = m_B) = \frac{\partial}{\partial q^2_\mu} \Pi^B_\sigma^{(RF)}(q^2_\mu; m^*_B = m_B) = 0 \quad \text{at} \quad q^2_\mu = m^2_\sigma . \] (19)

The renormalized \( \sigma \)-meson self-energy for a baryon loop is

\[ \Pi^B_\sigma^{(RF)}(q) = \frac{g^2_\sigma B^2}{2\pi^2} \left[ \frac{m^2_\sigma - q^2_\mu}{4} + 3m_B \left( m^*_B - m_B \right) + \frac{9}{2} \left( m^*_B - m_B \right)^2 \right. \]

\[ - \frac{3}{2} \int_0^1 dx \left( m^*_B - q^2_\mu x(1-x) \right) \ln \frac{m^2_\sigma - q^2_\mu x(1-x)}{m^2_B - m^2_\sigma x(1-x)} \]

\[ - \frac{3}{2} \left( m^*_B - m^2_B \right) \int_0^1 dx \ln \left( 1 - \frac{m^2_\sigma}{m^2_B} x(1-x) \right) . \] (20)

For the \( \pi \)-loop we employ the renormalization condition in free space, \( \Pi^\pi_\sigma^{(RF)}(q^2_\mu) = 0 \) at \( q^2_\mu = m^2_\sigma \) [15]. We obtain finally

\[ \Pi^\pi_\sigma^{(RF)}(q) = \frac{3g^2_\pi m^2_\pi}{32\pi^2} \int_0^1 dx \ln \frac{m^2_\pi - q^2_\mu x(1-x)}{m^2_\pi - m^2_\sigma x(1-x)} . \] (21)

For the \( \omega \), only a wavefunction counterterm \( \mathcal{L}_\omega = \zeta_\omega \omega_\mu \omega^{\mu\nu} / 4 \) is required to make the F-part \( \Pi^{B(F)}_\mu\nu \) finite. The renormalized \( \omega \) self-energy is \( \Pi^{B(RF)}_\mu\nu = -g_\mu \omega^\mu_{\nu} + q_\mu q_\nu \Pi^{B(RF)} \).
Employing the renormalization condition in vacuum, \( \Pi^{(RF)}(q^2; m^*_B = m_B) = 0 \) at \( q^2 = m^2_\omega \), we obtain
\[
\Pi^{(RF)}_{\mu\nu}(q) = \frac{g^2_B I_B}{2\pi^2} q^2 \int_0^1 dx \, x(1-x) \ln \frac{m^2_B - q^2 x(1-x)}{m^2_B - m^2_\omega x(1-x)}.
\] (22)

As mentioned before, the mixed part \( \Pi^{(M)}_{\mu\nu} \) of Eq. (17) does not contribute to vacuum polarization.

The solution of Dyson’s equation, Eq. (8), is \( D = D^0/(1 - D^0 \Pi) \). By defining the dielectric function \( \varepsilon \) as
\[
\varepsilon = \det \left( 1 - D^0 \Pi \right) = \varepsilon_T^2 \varepsilon_L^2,
\] (23)

the poles of the \( \sigma-\omega \) propagator which define the respective meson masses in the medium are now contained in the dielectric function when \( \varepsilon = 0 \). By taking \( q = (0, 0, q) \) where \( q = |q| \), we obtained in Eq. (23) the transverse and longitudinal dielectric functions defined as
\[
\varepsilon_T = \left( 1 + D^0 \Pi_T \right),
\] (24)
\[
\varepsilon_L = \left( 1 - \Delta^0 \Pi_\sigma \right) \left( 1 - D^0 \Pi_L \right) + \frac{q^2_\mu}{q^2} \Delta^0 D^0 \left( \Pi_0^{(M)} \right)^2,
\] (25)

where the polarization insertion of Eq. (16) is now split into transverse \( \Pi_T = \Pi_{11} = \Pi_{22} \) and longitudinal \( \Pi_L = \Pi_{00} - \Pi_{33} \) components. Because of baryon current conservation, only the 0-th component of the mixed part \( \Pi^{(M)} \) survives. Note that in Eqs. (24) and (25) the polarizations represent those which have been summed over all the baryons (see Eq. (13)). The eigencondition for determining the collective excitation spectrum (i.e. finding the effective meson masses) is equivalent to searching for the zeros of the dielectric function.

In particular, for a given three-momentum transfer \( q \equiv |q| \), the “invariant mass” of a meson \( \sigma \) or \( \omega \) is \( m^*_m = \sqrt{q^2_0 - q^2} \), where \( q_0 \) is obtained from the condition \( \varepsilon = 0 \). In the present study of meson mass modifications in the medium, we restrict ourselves to the meson branch in the time-like region \( (q^2_\mu > 0) \).

Since the propagation of the strange \( \sigma^*-\phi \) mesons are decoupled from that of the non-strange \( \sigma-\omega \) mesons, we may follow the same procedure as given in Eqs. (8)-(25) to obtain the effective masses of the strange mesons. In particular, a dielectric function (Eq. (23)) is obtained but with the masses of the mesons and their couplings to the baryons correspond to the \( \sigma^* \) and \( \phi \). Moreover, since a linear \( \sigma^* \) interaction is used in this case, \( \tilde{m}^2_\sigma \) in Eq. (10) should be replaced by the bare meson mass \( m^2_\sigma \).

The renormalization conditions in RHA in Eq. (6) is imposed at \( q^2_\mu = 0 \), while those used in Eq. (19) for \( \sigma \) and \( \sigma^*-\)mesons are at \( q^2_\mu = m^2_\sigma \) and \( q^2_\mu = m^2_\sigma^* \), respectively. This difference yields an additional term in the vacuum fluctuation energy \([13]\), so the total energy density for SHM is
\[
\mathcal{E} = \mathcal{E}_{\text{RHA}} + \sum_B \frac{a_{\sigma B} + a_{\sigma^* B}}{4\pi^2} I_B m^2_B (m^*_B - m_B)^2,
\] (26)

where \( \mathcal{E}_{\text{RHA}} \) is the RHA energy of Eq. (5) and
\[ a_{\sigma B} = \frac{m^2_{\sigma}}{4m^2_B} + \frac{3}{2} \int_0^1 dx \ln \left[ 1 - \frac{m^2_{\sigma}}{m^2_B} x(1-x) \right], \]  
and a similar expression for \( a_{\sigma^* B} \) for \( \sigma^* \); for nucleons \( a_{\sigma^* B} = 0 \).

The field equations are obtained by minimizing the energy density \( E \) with respect to that field. At a given baryon density \( n_B \) and strangeness fraction \( f_S = (n_\Lambda + 2n_\Xi)/n_B \), the set of field equations are solved self-consistently in conjunction with the chemical equilibrium condition \( 2\mu_\Lambda = \mu_N + \mu_\Xi \) due to the reaction \( \Lambda\Lambda \leftrightarrow N\Xi \). The chemical potential of a baryon species \( B \) is
\[
\mu_B = \left[ k_B^m + m^2_B \right]^{1/2} + g_\omega B \omega_0 + g_\phi B \phi_0.
\]
The four saturation properties of nuclear matter (NM): density \( n_0 = 0.16 \text{ fm}^{-3} \), binding energy \( E/B = -16 \text{ MeV} \), effective nucleon mass \( m_N^*/m_N = 0.78 \) and compression modulus \( K = 300 \text{ MeV} \) are used to fix the nucleon coupling constants \( g_{\sigma N} \), \( g_{\omega N} \) and the parameters \( g_2 \) and \( g_3 \) of the \( \sigma \) self-interaction. The coupling constants for pure NM without hyperons are shown in Table I. When hyperons are included, i.e. for SHM, they will contribute to \( E \) from their vacuum fluctuations \( V_B \) even if their Fermi states are empty. This entails a redetermination of the coupling constants for the nucleons. The \( V_B \) depends on the effective strangeness mass, which in turn depends on the scalar-baryon coupling constants (see Eqs. (4) and (6)). Therefore, the \( \sigma \) and \( \sigma^* \) couplings to the hyperons (\( Y \)) should be predetermined. For this purpose, we adopt the SU(6) model, i.e. \( g_{\sigma \Lambda}/g_{\sigma N} = 2/3, g_{\sigma \Xi}/g_{\sigma N} = 1/3 \) for the \( \sigma-Y \) couplings, and \( g_{\sigma^* \Lambda}/g_{\sigma N} = \sqrt{2}/3, g_{\sigma^* \Xi}/g_{\sigma N} = 2\sqrt{2}/3 \) for the \( \sigma^*-Y \) couplings. Note that the nucleons do not couple to strange mesons, i.e. \( g_{\sigma^* N} = g_{\phi N} = 0 \). The coupling constants of nucleons for the SHM are given in Table I. The couplings of \( \omega \) to the hyperons can be obtained from the well-depth of \( Y \) in saturated NM: \( U_Y^{(N)} = g_{\omega Y} \sigma^N - g_{\omega \sigma Y} \sigma^0 \) with \( U_\Lambda^{(N)} = 30 \text{ MeV} \) and \( U_\Xi^{(N)} = 28 \text{ MeV} \). The \( \phi - Y \) couplings are obtained by fitting them to a well-depth \( U_\Lambda^{(\Xi)} \approx 40 \text{ MeV} \), for a \( \Lambda \) or \( \Xi \) in a \( \Xi \) "bath" with \( n_\Xi \simeq n_0 \). To determine the couplings of the pion to scalar mesons, \( g_{\sigma \pi} \) and \( g_{\sigma^* \pi} \), we adjust them to reproduce the widths of \( \sigma \) and \( \sigma^* \) in free space, i.e. \( \Gamma_\sigma^0 = -3 \pi \Gamma_\sigma^{(RF)}/m_\sigma \) at \( q^2 = m_\sigma^2 \). For a conservative estimate we consider \( \Gamma_\sigma^0 = 300 \text{ MeV} \) and \( \Gamma_{\sigma^*} = 70 \text{ MeV} \).

The stability of strange hadronic matter may be explored by considering its binding energy defined as \( E/B = E/n_B = \sum_i Y_i m_i \), where the abundance \( Y_i = n_i/n_B \). In Fig. 1, we present the binding energy \( E/B \) as a function of baryon density \( n_B \) at various strangeness fractions \( f_S \). With increasing \( f_S \), the binding energy of SHM is found to increase and the saturation point is shifted to higher density. This is a consequence of the opening of new (strangeness) degrees of freedom in the form of hyperons. At high densities for large \( f_S \).

**TABLE I.** The nucleon-meson coupling constants in RHA obtained by reproducing the nuclear matter saturation properties (see text). The results are for nuclear matter (NM) with strangeness fraction \( f_S = 0 \) and for strange hadronic matter (SHM) which includes also the vacuum polarization of hyperons. All the couplings are dimensionless except \( g_2 \) which has the dimension of \( \text{fm}^{-1} \). The bare hadronic masses are \( m_N = 939 \text{ MeV} \), \( m_\Lambda = 1116 \text{ MeV} \), \( m_\Xi = 1313 \text{ MeV} \), \( m_\sigma = 550 \text{ MeV} \), \( m_\omega = 783 \text{ MeV} \), \( m_{\sigma^*} = 975 \text{ MeV} \), and \( m_\phi = 1020 \text{ MeV} \).

|          | \( g_{\sigma N} \) | \( g_{\omega N} \) | \( g_2 \) | \( g_3 \) |
|----------|--------------------|--------------------|----------|----------|
| NM       | 8.060              | 8.498              | 13.962   | 1.273    |
| SHM      | 7.917              | 8.498              | 9.471    | 15.017   |
values when the Fermi energy of nucleons exceeds the effective masses of the hyperons minus their associated interaction, the conversion of nucleons at its Fermi surface to hyperons of increasingly larger mass is favored. This conversion lowers the Fermi energy of the nucleons leading to enhanced binding. A stable SHM with a maximum binding of $E/B = -38.46$ MeV is obtained for a large $f_S \approx 1.35$ at a relatively high density $n_B \approx 3n_0$. A further increase of $f_S$ enforces the Fermi energy of the hyperons to increase resulting in the decrease of binding. This finding is quite similar to that obtained in the quark-meson coupling model [10].

The variation of baryon effective masses $m_B^*$ is shown in Fig. 2 as a function of density $n_B$ for varying strangeness $f_S$. It is observed that $m_B^*$ decreases with increasing $n_B$, with $m_N^*$ having the largest decrease rate and $m_{\Xi}^*$ the smallest at each $f_S$ value. Furthermore, with increasing $f_S$ the effective nucleon mass increases while the effective masses of hyperons decrease at any density. This effect stems from decrease of the nonstrange meson fields $\sigma$ and $\omega$ and increase of the strange meson fields $\sigma^*$ and $\phi$ with increasing $f_S$. Since the nucleons couple only to $\sigma$ and $\omega$, $m_N^*$ is increased. The hyperons, however, do couple to all the meson fields resulting in a decrease of their effective masses with increasing $f_S$, especially for $\Xi$ which has the strongest coupling to the strange mesons.

In Fig. 3, we display the “invariant mass” $m_n^* = \sqrt{q_0^2 - q^2}$ of nonstrange $\sigma$-meson, $m_{\sigma}^*$, (left panels) and $\omega$-meson, $m_{\omega}^*$, (right panels) as a function of baryon density $n_B$ for different strangeness fractions $f_S$. We consider first the features observed for $m_\sigma^*$ for small three-momentum transfer $q = |\mathbf{q}| = 1$ MeV (top-left panel). For $f_S = 0$, $m_\sigma^*$ is found to decrease with increasing $n_B$ for small values of $n_B \leq n_0$. This reduction is caused due to two competing effects. The vacuum polarization which leads to reduction of $m_\sigma^*$ dominates over the density dependent dressing of the meson propagator which causes an increase in $m_\sigma^*$. In fact, this reduction can be traced back to the corresponding reduction of $m_N^* < m_N$ in the medium (see Eq. (20)). However, the decrease in the scalar polarization depends on $m_B^2$, $m_B^2$, and $m_\omega^2$ in a complicated fashion. For densities above $n_0$, the density-dependent (D) part becomes increasingly dominant resulting in increase of $m_\sigma^*$. In SHM with $f_S \neq 0$, the vacuum polarization contribution from the hyperons and the nucleons causes a considerable suppression of $m_\sigma^*$ at low densities. At higher densities $m_\sigma^*$ increases, however, the rate of increase above $\sim n_0$ becomes smaller with increasing $f_S$. An explanation to this effect is as follows. The D-part of $\sigma$ propagator is primarily determined by nucleons to which it has the strongest coupling. The increase of $f_S$ causes an increase in the Fermi momenta of the hyperons while that of the nucleons decrease. Consequently, with increasing $f_S$ the decreasing D-part of the nucleons results in a slower rate of increase of $m_\sigma^*$ with $n_B$.

At small value of $q = 1$ MeV, the (density dependent) effect of scalar-vector mixing is negligible (see Eq. (25)). On the other hand, for large values of $q = 500$ MeV (central-left panel) and $q = 1$ GeV (bottom-left panel), the mixing is more effective. (For the latter value, the present model may have been stretched to the extremes of applicability.) It gives rise to a repulsion which again leads to decrease of $m_\sigma^*$ at high density. With increasing $f_S$, the onset of this decrease or the peak positions in the figure are found to shift to higher densities. This is a manifestation of the shift of saturation value $E/B$ to higher $n_B$ as $f_S$ increases (see Fig. 1).

For small values of $q = 1$ MeV, the transverse and longitudinal invariant $\omega$-meson mass, $m_{\omega_T}^*$ and $m_{\omega_L}^*$ are practically identical as evident in Fig. 3 (top-right panel). In contrast
to $\sigma$ mass, the vacuum polarization effect is much stronger (see Eq. (22)) and the density dependent effect is much weaker for the heavier $\omega$ mass. This causes for $f_S = 0$ a substantial decrease of $m_\omega^*$ up to $n_B \approx 2n_0$, and a subsequent small increase with density. The reduction is more pronounced for SHM where the total vacuum polarization effect is stronger and the D-part (mainly controlled by N) is weaker. In fact, for $f_S = 1.5$, $m_\omega^*$ decreases considerably even up to large densities. As a consequence, the crossing between $m_\sigma^*$ and $m_\omega^*$ at $q = 1$ MeV is shifted to lower nuclear matter density and finally increases with density attaining values higher than $m_{\omega_T}^*$.

At large values of $q = 500$ MeV and 1 GeV, the longitudinal mass $m_{\omega_L}^*$ (solid line) and transverse mass $m_{\omega_L}^*$ (dashed line) get well separated. It is found that $m_{\omega_L}^*$ is reduced near nuclear matter density and finally increases with density attaining values higher than $m_{\omega_T}^*$. As in the $q = 1$ MeV case, the reduction in mass is stronger for SHM. Because of strong repulsion from the mixing at these high three-momentum values, $m_\sigma^*$ and $m_{\omega_L}^*$ never cross each other.

In Fig. 4 the “invariant mass” of the strange $\sigma^*$-meson, $m_{\sigma^*}$, (left panels) and $\phi$-meson, $m_\phi^*$, (right panels) are shown as a function of baryon density $n_B$ for different strangeness fractions. In this case the masses are determined by the hyperons as nucleons do not couple to the strange mesons. As for the nonstrange mesons, the masses $m_{\sigma^*}$ and $m_\phi^*$ in general decreases with increasing $n_B$. However, the decrease is more enhanced over the large density range explored here. This arises due to large vacuum fluctuation contribution from the hyperons, and, in particular, a small density-dependent part of the hyperons stemming from the large binding in SHM. At high densities $n_B \geq 4n_0$, in contrast to nonstrange meson masses, the strange meson masses has higher values for larger $f_S$. This reversal in behavior for the $m_{\sigma^*}$ and $m_\phi^*$ results from the increase of the Fermi momenta and hence the D-part of the hyperons for large $f_S$. The scalar mass $m_{\sigma^*}$ however becomes sensitive to high values of $q$ and drops at $f_S = 1.5$ below that for $f_S = 0.5$.

We have investigated the meson mass modification in strange hadronic matter as a function of baryon density for a fixed strangeness fraction. The results correspond to a metastable SHM. For a given $n_B$, an absolute stable SHM could be obtained by determining the absolute minimum of the binding energy $E/B$ as a function of $f_S$. In this situation, we have found that the masses of the mesons in SHM undergo a drastic reduction over a wide range of density. Besides the large vacuum part, the minimum in the energy per baryon $E/B$ at each density enforces a smallest possible Fermi momentum and hence the density-dependent for all the baryons.

It is worth mentioning here that in the linear Walecka model, it has been demonstrated \cite{5,6,17} that a self-consistent inclusion of exchange (Fock) term has an insignificant contribution to binding energy when the two parameters of this model, $g_{\sigma N}$ and $g_{\omega N}$, are renormalized to reproduce the same NM saturation properties of baryon density and binding energy. Even the predicted values of the effective nucleon mass, $m_N^*$, and incompressibility, $K$, in this Hartree-Fock calculation are almost identical to the Hartree results \cite{5}. However, the applicability of the linear Walecka model to moderate and high density phenomena can be misleading if the two parameters $m_N^*$ and $K$ at $n_0$ are not under control. In fact, it was shown \cite{18} that in the nonlinear Walecka model when the four parameters, $g_{\sigma N}$, $g_{\omega N}$, $g_2$, and $g_3$ (the latter two are from the $\sigma$ self-interaction) are fitted to the same NM saturation properties of $n_0$, $E/B$, $m_N^*$, and $K$ (as in our calculation), nearly all the properties obtained in the relativistic Hartree calculation differ from the mean field results only by $\approx 3\%$ even at
\( n = 10n_0 \). (This is in contrast to the linear Walecka model results.) It is therefore expected that by further inclusion of exchange terms (in all the baryonic sectors) and pseudoscalar mesons in the present nonlinear Walecka model, and performing a self-consistent calculation (with the parameters renormalized to the same NM saturation properties), the results for \( E/B \) and \( m_B^* \) will be practically unaltered from the Hartree calculation. Consequently, exchange corrections from the nucleonic and strangeness sectors should also have an insignificant contribution and therefore neglected in the present study of the modification of meson masses in the medium.

In summary, we have investigated the masses of baryons (\( N, \Lambda, \Xi \)) and, in particular, the nonstrange (\( \sigma, \omega \)) and strange (\( \sigma^*, \phi \)) mesons in stable strange hadronic matter. The ground state properties of the SHM in the relativistic Hartree approximation is obtained by using a nonlinear \( \sigma-\omega \) and linear \( \sigma^* - \phi \) Lagrangians. With increasing strangeness fraction, the effective mass of the nucleons increases while that of the hyperons decreases. The masses of all the mesons reveal a considerable reduction over a wide density range with increasing strangeness. This may be attributed to a large contribution from the vacuum polarization of the hyperons which causes the decrease in the meson masses at small densities. The larger binding for the SHM and therefore a smaller density-dependent part helps to reduce meson masses at high densities.

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FIG. 1. Binding energy $E/B$ versus baryon density $n_B$ for strange hadronic matter with various strangeness fraction $f_S$. 
FIG. 2. Effective masses of nucleons (top panel), lambda (central panel) and cascade (bottom panel) in strange hadronic matter with strangeness fraction $f_S$ from 0 to 1.5 in steps of 0.5.
FIG. 3. The in-medium masses of $\sigma$-meson (left panels) and $\omega$-meson (right panels) as a function of baryon density for various $f_S$ with $q = |q| = 1, 500, \text{ and } 1000 \text{ MeV}$. The $\sigma$-meson masses are
for $f_S$ from 0 to 1.5 in steps of 0.5. In the right panels, the solid and dashed lines correspond to longitudinal and transverse $\omega$-meson masses, respectively.
FIG. 4. Same as Fig. 3 but for the $\sigma^*$-meson (left panels) and $\phi$-meson (right panels).