Yang-Mills thermodynamics: The confining phase

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Abstract

We summarize recent nonperturbative results obtained for the thermodynamics of an SU(2) and an SU(3) Yang-Mills theory being in its confining (center) phase. This phase is associated with a dynamical breaking of the local magnetic center symmetry. Emphasis is put on an explanation of the involved concepts.
**Introduction.** This is the last one in a series of three papers giving an abbreviated presentation of nonperturbative concepts and results for the thermodynamics of an SU(2) or an SU(3) Yang-Mills theory as obtained in [1, 2, 3]. Here we discuss the confining or center phase.

The three unexpected results for the confining phase are the spin-1/2 nature of the massless (neutral, Majorana) and massive (charged) excitations, the Hagedorn nature of the transition from the confining to the preconfining phase, and the exact vanishing of the pressure and energy density of the ground state in the confining phase of an SU(2) or SU(3) Yang-Mills theory.

The first result clashes with the perception about bosonic glueballs being the observable excitations of pure SU(3) Yang-Mills theory at zero temperature. This statement seems to be supported by lattice simulations [4] and by analysis based on the QCD-sum-rule method [5]. We have discussed in [1] why lattice simulations of pure SU(2) and SU(3) Yang-Mills theory run into a severe finite-size problem at low temperatures and thus are unreliable. QCD spectral sum rules [6], on the other hand, assume the existence of a lowest resonance with finite coupling to the currents of a given production channel (these currents are formulated as local functionals of the fundamental fields in the QCD Lagrangian). The resonance’s properties are determined subsequently by assuming the analyticity of the associated correlation function in the external momentum and by appealing to an operator product expansion in the deep euclidean region. Analyticity, however, must break down across two phase boundaries (deconfining-preconfining, preconfining-center) provided that the effects arising due to the deviation from the thermodynamical limit can, on a qualitative level, be neglected in the production process. As a consequence, the QCD sum rule method is probably unreliable for the investigation of the spectrum of a pure SU(2) and SU(3) Yang-Mills theory. (We hasten to add that the situation is different for real-world hadronic resonances because the dynamical mixing of pure SU(3) and pure dual SU(3) gauge theories may restore a quasi-analytical behavior of the relevant correlation functions [1]. After all the overwhelming phenomenological successes of QCD sum rules are a lot more than coincidence. The term ‘dynamical mixing’ includes the occurrence of the fractional Quantum Hall effect [7, 8] which renders quarks to be emerging phenomena, for a discussion see [1].)

The second result – the Hagedorn nature of the transition to the truly confining phase – was suspected to occur for the real-world strong interactions a long time ago [9, 10]. Subsequently performed lattice simulations seemed to exclude a Hagedorn transition (diverging partition function above the critical point) even in the case of a pure SU(2) or SU(3) Yang-Mills theory [12]. Again, this a consequence of the lattice’s failure to properly capture the infrared physics in thermodynamical simulations at low temperature, for an extended discussion see [1].

The third result, namely the exact vanishing of the ground-state pressure and energy density of the Yang-Mills theory at zero temperature, commonly is used as a normalization assumption in lattice computations [11] and not obtained as a dynamical result. In [1] we have shown the absoluteness, that is, the gravitational mea-
surability of the finite and exactly computable energy-momentum tensor associated with the ground-state in the deconfining and preconfining phases: An immediate consequence of the fact that these ground states are determined by radiatively protected BPS equations. (Since these equations are first-order as opposed to second-order Euler-Lagrange equations the usual shift ambiguity in the corresponding potentials is absent.) Recall, that the finiteness of the ground-state energy density and pressure in the deconfining and preconfining phases arises from averaged-over interactions between and radiative corrections within solitonic field configurations. Being (euclidean) BPS saturated, classical configurations in the deconfining phase the latter are free of pressure and energy density in isolation. The same applies to the massless, interacting magnetic monopoles which, by their condensation, form the ground state in the preconfining phase. In the confining phase configurations that are free of pressure and energy density do also exist (single center-vortex loops). In contrast to the other phases propagating gauge field fluctuations are, however, absent in the confining phase. Only contact interactions occur between the center solitons, which, however, do not elevate the vanishing energy density of the isolated soliton to a finite value for the ensemble. The proof for this relies on computing the curvature of the potential for the spatial coarse-grained center-vortex condensate at its zeros and by comparing this curvature with the square of the maximal resolution that is allowed for in the effective theory [1], see below.

The outline of this paper is as follows: First, we discuss the occurrence of isolated, instable, that is, contracting and collapsing center-vortex loops in the preconfining phase. From the evolution of the magnetic coupling constant in this phase we conclude that center-vortex loops become stable, particle-like excitations at the deconfining-confining phase boundary. Second, we point out the spin-1/2 nature of these particles, and we derive a dimensionless parameter with discrete values describing the condensate of pairs of single center-vortex loops after spatial coarse-graining. A discussion of the creation of center-fluxes (local phase jumps of the vortex condensate) by the decay of the monopole condensate in the preconfining phase is given. Third, we construct potentials for the vortex condensates which, in their physical effects, are uniquely determined by the remaining local symmetry and by the positive semi-definiteness of the energy density: Particle creation by local phase jumps of the order parameter may only go on so long as the energy density feeding into their creation is nonvanishing. Fourth, we discuss in detail the remarkable result that for SU(2) and SU(3) Yang-Mills dynamics the confining phase’s ground-state energy density is exactly nil. In particular, we stress the fact that radiative corrections to the tree-level result are entirely absent. Fifth, we give an estimate for the density of static fermion states and thus establish the Hagedorn nature of the transition from the confining to the preconfining phase. Finally, we summarize our results in view of its implications for particle physics and for cosmology.

**Instable center-vortex loops in the preconfining phase.** Here we discuss the SU(2) case only, results for SU(3) follow by simple doubling. The ground-state of the
preconfining phase is a condensate of magnetic monopoles peppered with instable
defects: closed magnetic flux lines whose core regions dissolve the condensate locally
and thus restore the dual gauge symmetry $U(1)_D$ (for $SU(3)$: $U(1)^2_D$). It was shown
in [1] that the magnetic flux carried by a given vortex-loop solely depends on the
charge of the monopoles and antimonopoles contributing to the explicit magnetic
current inside the vortex core. Thus the various species of vortex-loops, indeed, are
mapped one-to-one onto the nontrivial center elements of $SU(2)$ or $SU(3)$: They de-
serve the name center-vortex loops. In the magnetic phase, center-vortex loops are,
however, instable as we show now. To derive the classical field configuration associ-
ated with an infinitely long vortex line one considers an Abelian Higgs model with
no potential and a magnetic coupling $g$. (We need to discuss the energy-momentum
tensor of the solitonic configuration relative to the ground state obtained by spatially
averaging over instable vortex loops. Thus we need to subtract the temperature
dependent ground-state contribution which is reached far away from the considered
vortex core as a result of the applicable spatial coarse-graining, see [1] for details.)
The following ansatz is made for the static dual gauge field $a^D_\mu$ [13]:

$$a^D_4 = 0, \quad a^D_i = \epsilon_{ijk} \hat{r}_j e_k A(r)$$

where $\hat{r}$ is a radial unit vector in the $x_1x_2$ plane, $r$ is the distance from the vortex
core, and $e$ denotes a unit vector along the vortex' symmetry axis which we choose
to coincide with the $x_3$ coordinate axis. No analytical solution with a finite energy
per vortex length is known to the system of the two coupled equations of motion
honouring the ansatz (1) and the Higgs-field decomposition $\varphi = |\varphi|(r)\exp[i\theta]$. An
approximate solution, which assumes the constancy of $|\varphi|$, is given as

$$A(r) = \frac{1}{gr} - |\varphi|K_1(g|\varphi|r) \rightarrow \frac{1}{gr} - |\varphi|\sqrt{\frac{\pi}{2g|\varphi|r}} \exp[-g|\varphi|r], \quad (r \to \infty).$$

In Eq. (2) $K_1$ is a modified Bessel function. Outside the core region the isotropic
pressure $P_v(r)$ in the $x_1x_2$ plane is, up to an exponentially small correction, given
as

$$P_v(r) = -\frac{1}{2} \frac{\Lambda_\beta^3 \beta}{g^2r^2}.$$  

Notice that we have substituted the asymptotic value $|\varphi| = \sqrt{\frac{\Lambda_\beta^3 \beta}{2\pi}}, \quad (\beta \equiv \frac{1}{T}),$ as
it follows from the spatially coarse-grained action in the preconfining (or magnetic)
phase [11 14]. Notice also the minus sign on the right-hand side of Eq. (3): The
configuration in Eq. (2) is static due to its cylindrical symmetry but highly instable
w.r.t. bending of the vortex axis. In particular, the pressure inside a center-vortex
loop is more negative than outside causing the soliton to contract, and, eventually,
to dissolve. Bending of the vortex axis occurs because there are no isolated magnetic
charges in the preconfining phase which could serve as sources for the magnetic flux.
An equilibrium between vortex-loop creation by the spatially and temporally corre-
related dissociation of large-holonomy calorons and vortex-loop collapse is responsible
for the negative pressure of the ground state in the preconfining phase. The typical core-size $R$ of a center-vortex loop evaluates as $R \sim \frac{1}{m_D}$ and its energy as $E_v \sim \frac{g}{\beta} \sqrt{\frac{A_g}{2\pi}}$. (This takes into account an estimate for $\varphi$’s gradient contribution to the total energy of the soliton.)

Notice that core-size $R$, energy $E_v$, and pressure $P_v(r)$ of a center-vortex vanish in the limit $g \to \infty$. This situation is reached at the critical temperature $T_{c,M}$ where the magnetic coupling diverges in a logarithmic fashion: $g \sim -\log(T - T_{c,M})$ [1].

At $T_{c,M}$ the creation of single center-vortex loops at rest with respect to the heat bath (i) does not cost any energy and (ii) entails the existence of stable and massless particles. The latter do, in turn, condense pairwise into a new ground state.

**Pairwise condensation of single center-vortex loops: Ground-state decay and change of statistics.** We consider a static, circular contour $C(x)$ of infinite radius – an $S_1$ – which is centered at the point $x$. In addition, at finite coupling $g$ we consider a system $S$ of two single center-vortex loops, 1 and 2, which both are pierced by $C(x)$ and which contribute opposite units of center flux $F_{v_1} = \frac{2\pi}{g} = \oint_{C(x)} dz_\mu a_D^{\mu} = -F_{v_2}$ through the minimal surface spanned by $C(x)$. Depending on whether 1 collapses before or after 2 or whether 1 moves away from $C$ before or after 2 the total center flux $F$ through $C$’s minimal surface reads

$$F = \begin{cases} \pm \frac{2\pi}{g} & \text{(either 1 or 2 is pierced by } C(x)) \\ 0 & \text{(1 and 2 or neither 1 nor 2 are pierced by } C(x)) \end{cases}.$$  

(4)

The limit $g \to \infty$, which dynamically takes place at $T_{c,M}$, causes the center flux of the isolated system $S$ to vanish and renders single center-vortex loops massless and stable particles. The center flux of the isolated system $S$ does no longer vanish if we couple $S$ to the heat bath. Although 1 and 2 individually are spin-1/2 fermions the system $S$ obeys bosonic statistics. (Both, 1 and 2, come in two polarizations: the projection of the dipole moment, generated by the monopole current inside the core of the center-vortex loop, onto a given direction in space either is parallel or antiparallel to this direction.) Thus, assuming the spatial momentum of 1 and 2 to vanish, the quantum statistical average flux reads

$$\lim_{g \to \infty} F_{th} = 4\pi F \int d^3p \delta^{(3)}(p) n_B(\beta|2E_v(p)|) = 0, \pm \frac{8\pi}{\beta|\varphi|} = 0, \pm 4 \lambda_{c,M}^{3/2}.$$  

(5)

According to Eq. (5) there are finite, discrete, and dimensionless parameter values for the description of the macroscopic phase

$$\Gamma \Phi |\Phi(x) \equiv \lim_{g \to \infty} \exp[i \left\langle \oint_{C(x)} dz_\mu a^{\mu}_D \right]\].$$  

(6)
associated with the Bose condensate of the system $S$. In Eq. (6) $\Gamma$ is an undetermined and dimensionless complex constant. Notice that taking the limit of vanishing spatial momentum for each single center-vortex loop is the implementation of spatial coarse-graining. This coarse-graining is performed down to a resolution $|\Phi|$ which is determined by the (existing) stable solution to the equation of motion in the effective theory, see below.

For convenience we normalize the parameter values given by $\lim_{g \to \infty} F_{th}$ as $\hat{\tau} \equiv \pm 1, 0$.

Coarse-grained action and center jumps. To investigate the decay of the monopole condensate at $T_{c,M}$ (pre- and reheating) and the subsequently emerging equilibrium situation, we need to find conditions to constrain the potential $V_C$ for the macroscopic field $\Phi$ in such a way that the dynamics arising from it is unique. Recall that at $T_{c,M}$ the dual gauge modes of the preconfining phase decouple. Thus the entire process of fermionic pre- and reheating in the confining phase is described by spatially and temporally discontinuous changes of the modulus (energy loss) and phase (flux creation) of the field $\Phi$. Since the condensation of the system $S$ renders the expectation of the ’t Hooft loop finite (proportional to $\Phi$) the magnetic center symmetry $Z_2$ (SU(2)) and $Z_3$ (SU(3)) is dynamically broken as a discrete gauge symmetry. Thus, after return to equilibrium, the ground state of the confining phase must exhibit $Z_2$ (SU(2)) and $Z_3$ (SU(3)) degeneracy. This implies that for SU(2) the two parameter values $\hat{\tau} = \pm 1$ need to be identified while each of the three values $\hat{\tau} = \pm 1, 0$ describe a distinct ground state for SU(3). Let us now discuss how either one of these degenerate ground states is reached. Spin-1/2 particle creation proceeds by single center vortex loops being sucked-in from infinity. (The overall pressure is still negative during the decay of the monopole condensate thus facilitating the in-flow of spin-1/2 particles from spatial infinity.) At a given point $\mathbf{x}$ an observer detects the in-flow of a massless fermion in terms of the field $\Phi(\mathbf{x})$ rapidly changing its phase by a forward center jump (center-vortex loop gets pierced by $C(\mathbf{x})$) which is followed by the associated backward center jump (center-vortex loop lies inside $C(\mathbf{x})$). Each phase change corresponds to a tunneling transition inbetween regions of positive curvature in $V_C$. If a phase jump has taken place such that the subsequent potential energy for the field $\Phi$ is still positive then $\Phi$’s phase needs to perform additional jumps in order to shake off $\Phi$’s energy completely. This can only happen if no local minimum exists at a finite value of $V_C$. If the created single center-vortex loop moves sufficiently fast it can subsequently convert some of its kinetic energy into mass by twisting: massive, self-intersecting center-vortex loops arise. These particles are also spin-1/2 fermions: A $Z_2$ or $Z_3$ monopole, constituting the intersection point, reverses the center flux [15], see Fig.[1].

If the SU(2) (or SU(3)) pure gauge theory does not mix with any other preconfining or deconfining gauge theory, whose propagating gauge modes would couple to the $Z_2$ (or $Z_3$) charges, a soliton generated by $n$-fold twisting is stable in isolation and possesses a mass $n \Lambda_C$. Here $\Lambda_C$ is the mass of the charge-one state (one
self-intersection). After a sufficiently large and even number of center jumps has occurred the field $\Phi(x)$ settles in one of its minima of zero energy density. Forward - and backward tunneling inbetween these minima corresponds to the spontaneous on-shell generation of a massless, single center-vortex loop of zero momentum. In a WKB-like approximation one expects the associated euclidean trajectory to have a large action which, in turn, predicts large suppression. We conclude that tunneling between the minima of zero energy density is forbidden.

Let us summarize the results of our above discussion: (i) the potential $V_C$ must be invariant under magnetic center jumps $\Phi \rightarrow \exp[\pm i\pi]\Phi$ (SU(2)) and $\Phi \rightarrow \exp[\pm \frac{2\pi}{3}]\Phi$ (SU(3)) only. (An invariance under a larger continuous or discontinuous symmetry is excluded.) (ii) Fermions are created by a forward - and backward tunneling corresponding to local center jumps in $\Phi$’s phase. (iii) The minima of $V_C$ need to be at zero-energy density and are all related to each other by center transformations, no additional minima exist. (iv) Moreover, we insist on the occurrence of one mass scale $\Lambda_C$ only to parameterize the potential $V_C$. (As it was the case for the ground-state physics in the de - and preconfining phases.) (v) In addition, it is clear that the potential $V_C$ needs to be real.

**SU(2) case:**

A generic potential $V_C$ satisfying (i),(ii), (iii), (iv), and (v) is given by

$$V_C = \overline{v_C} v_C \equiv \left( \frac{\Lambda_C^3}{\Phi} - \Lambda_C \Phi \right) \left( \frac{\Lambda_C^3}{\Phi} - \Lambda_C \Phi \right). \quad (7)$$

The zero-energy minima of $V_C$ are at $\Phi = \pm \Lambda_C$. It is clear that adding or subtracting powers $(\Phi^{-1})^{2l+1}$ or $\Phi^{2k+1}$ in $v_C$, where $k, l = 1, 2, 3, \cdots$, generates additional zero-energy minima, some of which are not related by center transformations (violation of requirement (iii)). Adding $\Delta V_C$, defined as an even power of a Laurent expansion in $\Phi \Phi$, to $V_C$ (requirements (iii) and (v)), does in general destroy property (iii).
possible exception is

$$\Delta V_C = \lambda \left( \Lambda_C^2 - \Lambda_C^{-2(n-1)} (\Phi \Phi)^n \right)^{2k}$$

where $\lambda > 0; k = 1, 2, 3, \cdots; n \in \mathbb{Z}$. Such a term, however, is irrelevant for the description of the tunneling processes (requirement (ii)) since the associated euclidean trajectories are essentially along U(1) Goldstone directions for $\Delta V_C$ due to the pole in Eq. (7). Thus adding $\Delta V_C$ does not cost much additional euclidean action and therefore does not affect the tunneling amplitude in a significant way. As for the curvature of the potential at its minima, adding $\Delta V_C$ does not lower the value as obtained for $V_C$ alone. One may think of multiplying $V_C$ with a positive, dimensionless polynomial in $\Lambda_C^{-2} \Phi \Phi$ with coefficients of order unity. This, however, does not alter the physics of the pre- and reheating process. It increases the curvature of the potential at its zeros and therefore does not alter the result that quantum fluctuations of $\Phi$ are absent after relaxation.

SU(3) case:

A generic potential $V_C$ satisfying (i), (ii), (iii), (iv), and (v) is given by

$$V_C = \overline{v_C} \equiv \overline{\left( \left( \frac{\Lambda_C^2}{\Phi} - \Phi^2 \right) \left( \frac{\Lambda_C^2}{\Phi} - \Phi^2 \right) \right)}.$$ (9)

The zero-energy minima of $V_C$ are at $\Phi = \Lambda_C \exp \left[ \pm \frac{2\pi i}{3} \right]$ and $\Phi = \Lambda_C$. Again, adding or subtracting powers $(\Phi^{-1})^{3l+1}$ or $(\Phi)^{3k-1}$ in $v_C$, where $l = 1, 2, 3, \cdots$ and $k = 2, 3, 4, \cdots$, violates requirement (iii). The same discussion for adding $\Delta V_C$ to $V_C$ and for multiplicatively modifying $V_C$ applies as in the SU(2) case. In Fig. 2 plots of the potentials in Eq. (7) and Eq. (9) are shown. The ridges of negative tangential curvature are classically forbidden: The field $\Phi$ tunnels through these ridges, and
a phase change, which is determined by an element of the center $Z_2$ (SU(2)) or $Z_3$ (SU(3)), occurs locally in space. This is the afore-mentioned generation of one unit of center flux.

No vacuum energy after relaxation. The action describing the process of relaxation of $\Phi$ to one of $V_C$’s minima is
\[
S = \int d^4x \left( \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2} V_C \right).
\]

Once $\Phi$ has settled into $V_C$’s minima $\Phi_{\text{min}}$ there are no quantum fluctuations $\delta \Phi$ to be integrated out anymore. Let us show this: Writing $\Phi = |\Phi| \exp \left[ i \frac{\theta}{\Lambda_C} \right]$, we have
\[
\frac{\partial^2 V_C(\Phi)}{|\Phi|^2} \bigg|_{\Phi_{\text{min}}} = \frac{\partial^2 V_C(\Phi)}{|\Phi|^2} \bigg|_{\Phi_{\text{min}}} = \begin{cases} 8 \ (\text{SU}(2)) \\ 18 \ (\text{SU}(3)). \end{cases}
\]

Thus a potential fluctuation $\delta \Phi$ would be harder than the maximal resolution $|\Phi_{\text{min}}|$ corresponding to the effective action Eq. (10) that arises after spatial coarse-graining. Thus quantum fluctuations are already contained in the classical configuration $\Phi_{\text{min}}$. The cosmological constant in the confining phase of an SU(2) or SU(3) Yang-Mills theory vanishes exactly. Again, adding the term $\Delta V_C$ of Eq. (8) to the potentials in Eqs. (7) and (9) or performing the above multiplicative modification does not lower the value for the curvature as obtained in Eq. (11) and therefore does not change this result.

Estimate for density of states, Hagedorn nature of the transition. That the transition from the confining to the preconfining phase is of the Hagedorn nature is shown by an estimate for the density of massive spin-1/2 states. The multiplicity of massive fermion states, associated with center-vortex loops possessing $n$ self-intersections, is given by twice the number $L_n$ of bubble diagrams with $n$ vertices in a scalar $\lambda \phi^4$ theory. In [16] the minimal number of such diagrams $L_{n,\text{min}}$ was estimated to be
\[
L_{n,\text{min}} = n! 3^{-n}.
\]

The mass spectrum is equidistant. That is, the mass $m_n$ of a state with $n$ self-intersections of the center-vortex loop is $m_n \sim n \Lambda_C$. If we only ask for an estimate of the density of static fermion states $\rho_{n,0} = \tilde{\rho}(E = n \Lambda_C)$ of energy $E$ then, by appealing to Eq. (12) and Stirling’s formula, we obtain [1]
\[
\rho_{n,0} > \frac{\sqrt{8\pi}}{3\Lambda_C} \exp[n \log n] \left( \log n + 1 \right) \quad \text{or} \quad \tilde{\rho}(E) > \frac{\sqrt{8\pi}}{3\Lambda_C} \exp\left[ E \Lambda_C \log \frac{E}{\Lambda_C} \right] \left( \log \frac{E}{\Lambda_C} + 1 \right). \tag{13}
\]

Eq. (13) tells us that the density of static fermion states is more than exponentially increasing with energy $E$. The partition function $Z_\Phi$ for the system of static fermions
thus is estimated as
\[
Z_\Phi > \int_{E^*}^{\infty} dE \, \tilde{\rho}(E) \, n_F(\beta E) \, \sqrt{\frac{8 \pi}{3 \Lambda_C}} \int_{E^*}^{\infty} dE \, \exp\left[\frac{E}{\Lambda_C}\right] \exp[-\beta E],
\]
(14)

where \(E^* \gg \Lambda_C\) is the energy where we start to trust our approximations. Thus \(Z_\Phi\) diverges at some temperature \(T_H < \Lambda_C\). Due to the logarithmic factor in the exponent arising in estimate Eq. (13) for \(\tilde{\rho}(E)\) we would naively conclude that \(T_H = 0\). This, however, is an artefact of our assumption that all states with \(n\) self-intersections are infinitely narrow. Due to the existence of contact interactions between vortex lines and intersection points this assumption is the less reliable the higher the total energy of a given fluctuation. (A fluctuation of large energy has a higher density of intersection points and vortex lines and thus a larger likelihood for the occurrence of contact interactions which mediate the decay or the recombination of a given state with \(n\) self-intersections.) At the temperature \(T_H\) the entropy wins over the Boltzmann suppression in energy, and the partition function diverges. To reach the point \(T_H\) one would, in a spatially homogeneous way, need to invest an infinite amount of energy into the system which is impossible. By an (externally induced) violation of spatial homogeneity and thus by a sacrifice of thermal equilibrium the system may, however, condense densely packed (massless) vortex intersection points into a new ground state. The latter’s excitations exhibit a power-like density of states and thus are described by a finite partition function. This is the celebrated Hagedorn transition from below.

**Summary in view of particle physics and cosmology.** The confining phase of an SU(2) and SU(3) pure Yang-Mills theory is characterized by a condensate of single center-vortex loops and a dynamically broken, local magnetic \(Z_2\) (SU(2)) and \(Z_3\) (SU(3)) symmetry: No massless or finite-mass gauge bosons exist. Single center-vortex loops emerge as massless spin-1/2 particles due to the decay of a monopole condensate. A fraction of zero-momentum, single center-vortex loops subsequently condenses by the formation of Cooper-like pairs. Protected from radiative corrections, the energy density and the pressure of this condensate is precisely zero in a thermally equilibrated situation. The spectrum of particle excitations is a tower of spin-1/2 states with equidistant mass levels. A massive state emerges by twisting a single center-vortex loop hence generating self-intersection point(s). This takes place when single center-vortex loops collide. The process of mass generation thus is facilitated by converting (some of) the kinetic energy of a single center-vortex loop into the (unresolvable) dynamics of a flux-eddy marking the self-intersection point, see Fig. (1). Due to their over-exponentially increasing multiplicity heavy states become unstable by the contact interactions facilitated by dense packing. In a spatially extended system (such as the overlap region for two colliding, heavy, and ultrarelativistic nuclei) there is a finite value in temperature, comparable to
the Yang-Mills scale $\Lambda_C$, where a given, spatially nonhomogeneous perturbation induces the condensation of vortex intersections. This is the celebrated (nonthermal) Hagedorn transition.

The existence of a Hagedorn-like density of states explains why in an isolated system, governed by a single SU(2) Yang-Mills theory, the center-flux eddy in a spin-1/2 state with a single self-intersection appears to be structureless for external probes of all momenta with one exception: If the externally supplied resolution is comparable to the Yang-Mills scale $\Lambda_C$, that is, close to the first radial excitation level of a BPS monopole [17] then the possibility of converting the invested energy into the entropy associated with the excitation of a large number of unstable and heavy resonances does not yet exist. As a consequence, the center of the flux eddy – a BPS monopole – is excited itself and therefore reveals part of its structure. For an externally supplied resolution, which is considerable below $\Lambda_C$, there is nothing to be excited in a BPS monopole [17] and thus the object appears to be structureless as well.

There is experimental evidence [18, 19, 20] that this situation applies to charged leptons being the spin-1/2 states with a single self-intersection of SU(2) Yang-Mills theories with scales comparable with the associated lepton masses [1]. The corresponding neutrinos are Majorana particles (single center-vortex loops) which is also supported by experiment [21]. The weak symmetry SU(2)$_W$ of the Standard Model (SM) is identified with SU(2)$_e$ where the subscript $e$ refers to the electron. The important difference compared with the SM is that the pure SU(2)$_e$ gauge theory by itself provides for a nonperturbative breakdown of its continuous gauge symmetry in two stages (deconfining and preconfining phase) and, in addition, generates the electron neutrino and the electron as the only stable and apparently structureless solitons in its confining phase: No additional, fundamentally charged, and fluctuating Higgs field is needed to break the weak gauge symmetry. The confining phase is associated with a discrete gauge symmetry – the magnetic center symmetry – being dynamically broken.

As far as the cosmological-constant problem is concerned the state of affairs is not as clear-cut as it may seem. Even though each pure SU(2) or SU(3) gauge theory does not generate a contribution to the vacuum energy in its confining phase one needs to include gravity, the dynamical mixing of various gauge-symmetry factors, and the anomalies of emerging global symmetries in the analysis to obtain the complete picture on the Universe’s present ground state. We hope to be able to pursue this program in the near future. Notice that today’s ground-state contribution due to an SU(2) Yang-Mills theory of scale comparable to the present temperature of the cosmic microwave background is small as compared to the measured value [22]. This SU(2) Yang-Mills theory masquerades as the U(1)$_Y$ factor of the Standard Model within the present cosmological epoch.
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