Constraints from Global Symmetries on Radiative Corrections to the Higgs Sector

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Abstract

We discuss the implications of global symmetries on the radiative corrections to the Higgs sector. We focus on two examples: the charged Higgs mass in the minimal supersymmetric model and the Higgs couplings to vector boson pairs. In the first case, we find that in the absence of squark mixing a global SU(2) × SU(2) symmetry protects the charged Higgs mass from corrections of $\mathcal{O}(g^2 m_t^4 / m_W^2)$. In the second case, it is the custodial symmetry which plays an analogous role in constraining the fermion-mass dependence of the radiative corrections.
1. Introduction

Global symmetries play an important role in analyzing the radiative corrections of the tree-level parameters of a theory. Often, a theory will possess a “natural” tree-level relation – i.e., a relation among tree-level parameters which is attributable to some underlying symmetry. In this case, radiative corrections to this relation must be finite; moreover, the nature of the underlying symmetry can provide information of the order of magnitude of these corrections. As an example, in the Standard Model (SM) the so-called global *custodial* SU(2) symmetry\(^{[1]}\) plays a crucial role in the analysis of the radiative corrections to the \(\rho\)-parameter. One of the most important implications of this global SU(2) symmetry is the screening theorem of the Higgs boson\(^{[2]}\).

The purpose of this paper is to make use of global symmetries in the analysis of the radiative corrections to the Higgs sector. The study of such radiative corrections in the minimal supersymmetric model (MSSM) has recently received much attention. One-loop effects have been found which significantly modify the tree-level predictions for the Higgs masses of the MSSM and give rise to important phenomenological consequences\(^{[3,4]}\). For the light neutral CP-even Higgs mass, radiative corrections involving loop contributions from top quarks and their supersymmetric partners induce a substantial squared mass shift of \(\mathcal{O}(g^2m_t^4/m_W^2)\). However, for the charged Higgs squared mass, the radiative corrections are not so important because (in the absence of squark mixing) only one-loop corrections of \(\mathcal{O}(g^2m_t^2)\) are induced. In this paper, we shall show that these results follow easily by studying the implications of the underlying global symmetries of the Higgs potential.

In section 2, we make use of the global symmetries of the Higgs potential to analyze the radiative corrections to the charged Higgs mass in the MSSM. In particular, we will see that due to an approximate extended *custodial* symmetry of the Higgs potential, radiative corrections of \(\mathcal{O}(g^2m_t^4/m_W^2)\) never arise. In section 3, we study in a similar way the one-loop effects to the couplings of the Higgs bosons.
to a pair of vector bosons. We shall demonstrate that the custodial SU(2) symmetry plays a similar role to that in the radiative corrections to the $\rho$-parameter.

2. Radiative corrections to the charged Higgs mass

One of the relations that supersymmetry (SUSY) imposes on the Higgs potential is the mass sum-rule\cite{5}

$$m_{H^\pm}^2 = m_{A^0}^2 + m_W^2. \quad (1)$$

Because SUSY is not an exact symmetry of nature, eq. (1) only holds at tree-level and is modified by radiative corrections. On dimensional grounds, one might naively expect that the radiative corrections to eq. (1) should depend quadratically on some large mass scale in the problem. Specifically, the largest contribution expected would come from loops of superpartners whose masses are of the order of the SUSY breaking scale, $M_{SUSY}$. However, such contributions are certainly absent at one-loop for physical observables\cite{6}. Specifically, all one-loop corrections that grow as $M_{SUSY}^2$ can be absorbed in the redefinition of the mass-squared parameters of the Higgs potential. In contrast, whereas these mass-squared parameters are all independent, the scalar self-couplings are related by SUSY. Therefore, we do not have enough freedom to absorb all the effects of the superpartners. Since these effects can only show up in dimensionless parameters, these will depend at most logarithmically on $M_{SUSY}$. Note that decoupling does not apply when the mass of a heavy particle, $M$, can be made large by increasing a dimensionless parameter (e.g., the masses of the fermions and the Higgs boson in the SM). In that case, one-loop corrections to eq. (1) of $O(M^2)$ can show up.

The recent experimental result that $m_t > 91$ GeV\cite{7} suggests that radiative corrections to eq. (1) due to the loop contributions of top quarks and top squarks should be the dominant corrections. Naively, one expects one-loop corrections of order $h_t^2 m_t^2 \sim g^2 m_t^4 / m_W^2$ where $h_t$ is the top Yukawa coupling. Nevertheless, explicit calculation shows that in the absence of squark mixing, the leading radiative corrections are only of order $g^2 m_t^2 [a]$. 
To explain this result, we first analyze the two-doublet Higgs potential before imposing SUSY. Let \( \Phi_1 \) and \( \Phi_2 \) denote two Higgs doublets with hypercharges \( Y = 1 \). The most general renormalizable and \( SU(2)_L \times U(1)_Y \) gauge invariant Higgs potential is given by

\[
V(\Phi_1, \Phi_2) = m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 - (m_{12}^2 \Phi_1^\dagger \Phi_2 + h.c.) + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2
+ \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2} \left[ \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + h.c. \right],
\]

(2)

where a discrete symmetry \( \Phi_2 \to -\Phi_2 \) has been imposed on the dimension-four terms. This discrete symmetry guarantees the absence of flavor changing neutral current \( \text{[8]} \). It will be convenient to represent \( \Phi_1 \) by a real four vector, \( i.e., \)

\[
\Phi_1 = \left( \begin{array}{c} \phi_1^+ \\ \phi_1^0 \\ \phi_2^0 \\ \phi_1^+ \end{array} \right) = \left( \begin{array}{c} 
\phi_3 + i\phi_4 \\ 
\phi_1 + i\phi_2 \\
0 \\
0 
\end{array} \right) \to \Phi_1 = (\phi_1, \phi_2, \phi_3, \phi_4).
\]

(3)

Since the MSSM Higgs sector automatically conserves CP at tree-level, we henceforth make this assumption. The physical spectrum of the model consists in two charged Higgs bosons (\( H^\pm \)) and three neutral ones: two CP-even (\( h^0 \) and \( H^0 \)) and one CP-odd (\( A^0 \)). The masses of the \( A^0 \) and \( H^\pm \) are related by

\[
m_{H^\pm}^2 = m_{A^0}^2 + \frac{2m_W^2}{g^2} (\lambda_5 - \lambda_4).
\]

(4)

Consider the limit where \( m_{12} = \lambda_4 = \lambda_5 = 0 \). In this limit the global symmetry of the Higgs potential of eq. (2) is enlarged to \( O(4)_1 \times O(4)_2 \). Here, we find convenient to choose the symmetry transformations such that \( \Phi_1 \) transforms as a 4-vector under both \( O(4)_1 \) and \( O(4)_2 \), whereas \( \Phi_2 \) transforms as a 4-vector under \( O(4)_1 \) and as a singlet under \( O(4)_2 \). When the scalar fields develop vacuum expectation values (VEVs), \( \langle \Phi_i \rangle = (v_i, 0, 0, 0) \), the \( O(4)_1 \times O(4)_2 \) symmetry breaks down to \( O(3)_1 \times O(3)_2 \) which is locally isomorphic to \( SU(2) \times SU(2) \). Three of the six Goldstone bosons produced can be associated with the breakdown of \( SU(2)_L \times U(1)_Y \to U(1)_{EM} \); these will be “eaten” when the \( Z \) and \( W^\pm \) bosons acquire mass. The other three Goldstone bosons are the \( A^0 \) and the \( H^\pm \). If the
broken symmetries corresponding to the $A^0$ and $H^\pm$ Goldstone bosons are symmetries of the full theory (prior to symmetry breaking), then it would follow that $m_{A^0} = m_{H^\pm} = 0$ to all orders in perturbation theory. In general, this will not be the case, in which case the $A^0$ and $H^\pm$ are pseudo-Goldstone bosons (i.e., they would acquire a calculable mass due to radiative corrections).

Consider first the coupling of Higgs bosons to third generation quarks. In supersymmetric theories, the coupling quark doublets to Higgs doublets is such that $\Phi_1$ couples exclusively to $b_R$ and $\Phi_2$ couples exclusively to $t_R$. We assume this coupling pattern in the following. In the limit $h_b = 0$,

$$L_Y = -h_t \left( \bar{t}_L \tilde{b}_L \right) i\tau_2 \Phi_2^* t_R + h.c. \quad (5)$$

Since the global symmetry of this term is $SU(2)_L \times U(1)_Y \times O(4)_2$, $t$-loop radiative corrections will not induce a mass terms for the $A^0$ and the $H^\pm$. In the case of the MSSM, SUSY imposes the following condition on the parameters of the two Higgs doublet potential of eq. (2)\[5\]

$$\lambda_1 = \lambda_2 = \frac{1}{8}(g^2 + g'^2), \quad \lambda_3 = \frac{1}{4}(g^2 - g'^2), \quad \lambda_4 = -\frac{1}{2}g^2, \quad \lambda_5 = 0. \quad (6)$$

According to the above argument, the $A^0$ and $H^\pm$ must be massless to all orders of perturbation theory in the limit of $m_{12} = g = 0$. That is, $t$-loop corrections to the mass sum-rule [eq. (1)] must go to zero in this limit. It follows that corrections of $O(h_t^2 m_t^2)$ to eq. (1) must cancel out. In fact, each term in the one-loop radiative corrections to eq. (1) must depend quadratically on either $m_{A^0}$, $m_W$ or $m_b$.

However, in the SUSY model, we must also consider the squark sector since radiative corrections of $O(h_t^2 m_t^2)$ can also arise from top squark loops. Assuming that there is no $\tilde{t}_L - \tilde{t}_R$ mixing, we find in the limit of $h_b = g = 0$

$$L_{stop} = L(\Phi_1^i \Phi_1, \Phi_2^i \Phi_2, \tilde{Q}_i \tilde{Q}, \tilde{U}_i \tilde{U}) + h_t^2 \left| \tilde{Q}_i i\tau_2 \Phi_2^i \right|^2, \quad (7)$$
where
\[
\tilde{Q} = \begin{pmatrix} \tilde{t}_L \\ \tilde{b}_L \end{pmatrix} \quad \text{and} \quad \tilde{U} = \tilde{t}_R^*.
\] (8)

These terms are also SU(2)\(_L\) \(\times\) U(1)\(_Y\) \(\times\) O(4)\(_2\) invariant and, therefore, corrections of \(\mathcal{O}(h_t^2 m_t^2)\) cannot arise from this sector either. Finally, if \(\tilde{t}_L - \tilde{t}_R\) is present, we have new terms given by
\[
\mathcal{L}_{mix} = -\mu h_t \tilde{Q}^\dagger (i \tau_2 \Phi_1^*) \tilde{U}^* + h_t A_U \tilde{Q}^\dagger (i \tau_2 \Phi_2^*) \tilde{U}^* + h.c.
\] (9)

which are not invariant under the global O(4)\(_2\) symmetry. Thus, top squark loops involving the interactions of eq. (9) can induce corrections to eq. (1) of \(\mathcal{O}(h_t^2 m_t^2)\).

Notice that in the limit \(\mu = 0\) the terms in eq. (9) restore the O(4)\(_2\) symmetry and, although we still have a \(\tilde{t}_L - \tilde{t}_R\) mixing \((A_U \neq 0)\), no corrections of \(\mathcal{O}(h_t^2 m_t^2)\) can arise. This results are in agreement with the explicit one-loop radiatively corrected charged Higgs mass obtained in the literature\(^4\).

Let us now analyze the corrections to eq. (1) from other sectors of the theory. First, we consider the two Higgs doublet potential [eq. (2)] before imposing SUSY, where now we take the limit \(\lambda_4 = \lambda_5\). In this limit the Higgs potential is only O(4)\(_1\) invariant. After spontaneous symmetry breaking (SSB), the residual symmetry, O(3)\(_1\) \(\sim\) SU(2), is the so-called custodial symmetry\(^\star\) which is responsible for the relation \(m_{W}^2 = m_{Z}^2 \cos^2 \theta_W\). Setting \(\lambda_4 = \lambda_5\) in eq. (4) yields
\[
m_{H^\pm}^2 = m_{A_0}^2.
\] (10)

Radiative corrections to this relation will only come from sectors of the theory not invariant under the global custodial symmetry. The custodial SU(2) symmetry is

\(^\star\) The one-loop top squark corrections for large \(M_{SUSY}\) are in fact of \(\mathcal{O}\left[\mu^2 h_t^2 (m_{\tilde{t}_L}^2 - m_{\tilde{b}_L}^2)/m_{\tilde{t}_L}^2 \right]\). However, if \(\mu\) and the diagonal soft-supersymmetry breaking squark masses are of the same order, we have that \(\mu^2 (m_{\tilde{t}_L}^2 - m_{\tilde{b}_L}^2)/m_{\tilde{t}_L}^2 \sim m_t^2\) resulting in a top squark correction of \(\mathcal{O}(h_t^2 m_t^2)\).
an approximate symmetry of the minimal supersymmetric Higgs potential \( \lambda_4 = -g^2/2 \sim \lambda_5 = 0 \). In the limit \( g \to 0 \), i.e., \( \lambda_4 = \lambda_5 \), eq. (10) must hold to all orders in the Higgs self-interactions. We conclude that the only non-vanishing correction to eq. (1) from Higgs self-interactions must be proportional to \( g^2 \).

Finally, let us consider the Higgs-gauge boson interactions. They derive from the scalar kinetic term

\[
\mathcal{L}_{\text{kin}} = \sum_{i=1}^{2} \frac{1}{2} \text{tr} \left \{ (D^\mu M_i) \dagger (D_\mu M_i) \right \},
\]

(11)

where

\[
D_\mu M_i = \partial_\mu M_i + \frac{1}{2} ig \tau \cdot \cdots W_\mu M_i - \frac{1}{2} ig' B_\mu M_i \tau_3,
\]

(12)

and

\[
M_i = (i \tau_2 \Phi_i^* \Phi_i) \equiv \begin{pmatrix} \phi_0^+ \\ -\phi_i^- \\ \phi_0^0 \\ \phi_i^0 \end{pmatrix}.
\]

(13)

In the limit \( g' = 0 \), the kinetic term is invariant under the global \( \text{SU}(2)_L \times \text{SU}(2)_R \sim O(4)_1 \) transformation,

\[
M_i \rightarrow L_M R^\dagger, \quad \tau \cdot W \rightarrow L \tau \cdot W L^\dagger.
\]

(14)

After the neutral Higgs fields acquire VEVs the residual symmetry of eq. (11) [for \( g' = 0 \)] is \( \text{SU}(2)_{L+R} \) which is the custodial symmetry described above. Therefore, corrections to eq. (4) are expected to be of \( \mathcal{O}[m_\pm^2 \lambda_4 - \lambda_5] \) for small custodial breaking. In the MSSM it means corrections to \( m_{H^\pm}^2 \) of \( \mathcal{O}(g^2 m_{W}^2) \). When the factor \( \text{U}(1)_Y \) is gauged, the presence of \( \tau_3 \) in eq. (12) explicitly breaks the custodial symmetry and corrections to \( m_{H^\pm}^2 \) of \( \mathcal{O}(g'^2 m_{W}^2) \), where \( m_H \) is the largest Higgs mass, can be generated. However, in the MSSM, the Higgs masses can only be made substantially larger than \( m_Z \) increasing the soft \( m_{12}^2 \) mass-squared parameter.† Therefore, one-loop corrections of \( \mathcal{O}(g'^2 m_{H}^2) \) must cancel in the large \( m_H \) limit.

† This is not the case of the SM or a non-supersymmetric two Higgs doublet model. In these cases, the Higgs masses can be made large by increasing the self-couplings \( \lambda_i \).
We end this section with a comment concerning the natural relation given in eq. (4) which relates Higgs masses and the combination \((\lambda_4 - \lambda_5)\) of Higgs self-couplings. In principle, \((\lambda_4 - \lambda_5)\) can be measured independently of the masses. Then, one can discuss finite radiative corrections to eq. (4). The analysis is identical to the one presented above in the case of the MSSM. Specifically, one-loop corrections terms to eq. (4) can be of \(\mathcal{O}(g^2 m_t^2)\) or \(\mathcal{O}[m_W^2(\lambda_4 - \lambda_5)]\). In particular, no \(\mathcal{O}(g^2 m_t^4/m_W^2)\) corrections can be generated at one-loop. Since these corrections arise from the violation of custodial symmetry, the size of these corrections can be constrained by the \(\rho\)-parameter (\(\rho \equiv m_W^2/m_Z^2 \cos^2 \theta_W\)) whose deviation from 1 also reflects the presence of custodial symmetry violating terms. It is well known that the size of \(m_t\) (or \(h_t\)) is limited via this constraint. However, the dependence of the \(\rho\)-parameter on \((\lambda_4 - \lambda_5)\) can only occur at the two-loop level and probably cannot provide a useful constraint.

3. One-loop effective HVV vertices

The trilinear HVV vertices, where \(H\) refers generically to any Higgs boson and \(V\) to any vector boson, are of interest for the phenomenology of the Higgs bosons. The HVV vertices can provide an important production mechanism for Higgs bosons at future colliders. Furthermore, the decay \(H \rightarrow VV\) can be used as a clear signature of the \(H\). In Higgs sectors consisting in only doublets, HVV vertices are absent at tree-level for the CP-odd and charged Higgs bosons\(^9\). This is the primary reason why the \(A^0\) and \(H^\pm\) may be difficult to find at future hadron colliders. The one-loop induced \(H^\pm W^\mp Z\) and \(A^0 VV\) vertices in the MSSM have been calculated in ref. [10] and refs. [11,12] respectively. The primary contributions to the respective amplitudes arise from a virtual heavy quark pair. In the case of the \(H^\pm W^\mp Z\) vertex, the contribution of a heavy quark doublet \((u, d)\) grows quadratically with the quark mass for \(m_u \neq m_d\). However, this leading contribution vanishes exactly if the heavy quarks in the doublet are mass-degenerate. Scalar and gauge bosons contributions are found to be rather small due to large cancellations among different diagrams. As we shall see, such results are a consequence of the
global custodial symmetry. For simplicity, we will consider a sector with only two Higgs doublets. The analysis, however, can be easily generalize to multi-doublet models.

Let us begin by assuming that the global SU(2)$_L +$R symmetry defined by eqs. (14) with $L = R$ is an exact symmetry of our theory even after SSB. Let us also work in the limit $g' = 0$. In this case, the most general form for the one-loop effective $HVV$ vertices is given by

$$\mathcal{L}_{HVV} = \sum_j O^{\mu\nu}_j \sum_{i=1}^2 \mu_{ij} \text{tr} \left\{ M_i \tau \cdot W_\mu \tau \cdot W_\nu \right\} + h.c. ,$$

(15)

where $O^{\mu\nu}_j = (g^{\mu\nu}, \partial^\mu \partial^\nu, \partial^\mu \partial^\nu, \epsilon^{\mu\nu\rho\sigma} \partial_\rho \partial_\sigma)$ and $\mu_{ij}$ are complex constants of dimension $d = 4 - \text{dim}\{O^{\mu\nu}_j\}$. Using eq. (13), eq. (15) can be written as

$$\mathcal{L}_{HVV} \propto \sum_j O^{\mu\nu}_j \sum_{i=1}^2 \text{Re} \mu_{ij} \left[ W^3_\mu W^3_\nu + W^+_\mu W^-_\nu \right] \text{Re} \phi^0_i .$$

(16)

It is then clear that only the CP-even fields $h^0$ and $H^0$ couple to a pair of gauge bosons. Thus the $A^0VV$ and $H^\pmVV$ vertices will only be generated if the custodial symmetry is violated.

Let us analyze the quark-Yukawa sector and, in particular, its custodial limit. In a general model with two Higgs doublets, there are two possible ways to couple the Higgs to the quarks in a manner consistent with the discrete symmetry $\Phi_2 \rightarrow -\Phi_2$:

Case I: Quarks couple only to the first Higgs doublet $\Phi_1$.

Case II: $\Phi_2$ couples to $u_R$ and $\Phi_1$ couples to $d_R$. 

In case I, the quark-Yukawa interactions are SU(2)$_L \times$SU(2)$_R$ invariant if $h_u = h_d \equiv h$,

$$\mathcal{L}_Y = -h \left( \bar{u}_L \, \bar{d}_L \right) M_1 \begin{pmatrix} u_R \\ d_R \end{pmatrix} + h.c. ,$$

(17)
where the relevant transformation laws are
\[
\Psi_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix} \rightarrow L \Psi_L, \tag{18}
\]
\[
\Psi_R \equiv \begin{pmatrix} u_R \\ d_R \end{pmatrix} \rightarrow R \Psi_R, \tag{19}
\]

\[M_1 \rightarrow L M_1 R^\dagger.\tag{20}\]

When the neutral scalars develop VEVs, the symmetry is broken down to SU(2)$_{L+R}$.

The custodial limit, therefore, corresponds to the limit $m_u = m_d$. Thus, we shall need a large mass splitting within the quark doublet to generate $A^0VV$ and $H^\pmVV$ vertices that are phenomenologically relevant.

We now turn to case II (which is the quark-Higgs interactions required by the MSSM). If we define the transformation law of the scalar fields according to eq. (14), we find that the quark Yukawa sector is not SU(2)$_L \times$SU(2)$_R$ invariant, even in the limit $h_u = h_d$. However, by making the following redefinition,
\[
\Phi_1 \rightarrow \frac{1}{h_d} \Phi_1, \quad \Phi_2 \rightarrow \frac{1}{h_u} \Phi_2, \tag{20}
\]
the quark Yukawa sector can be written by
\[
\mathcal{L}_Y = - \begin{pmatrix} \bar{u}_L \bar{d}_L \end{pmatrix} \begin{pmatrix} i \tau_2 \Phi_2^* \Phi_1 \end{pmatrix} \begin{pmatrix} u_R \\ d_R \end{pmatrix} + h.c., \tag{21}
\]
which is SU(2)$_L \times$SU(2)$_R$ invariant if the scalar fields transform as
\[M_{21} \equiv (i \tau_2 \Phi_2^* \Phi_1) \rightarrow L M_{21} R^\dagger.\tag{22}\]

After SSB the quark mass term is given by
\[
\mathcal{L}_m = - \begin{pmatrix} \bar{u}_L \bar{d}_L \end{pmatrix} \begin{pmatrix} h_u v_2 & 0 \\ 0 & h_d v_1 \end{pmatrix} \begin{pmatrix} u_R \\ d_R \end{pmatrix} + h.c. \tag{23}
\]
This term is SU(2)$_{L+R}$ invariant only if $h_u v_2 = h_d v_1$, i.e., $m_u = m_d$. Then, the effective $HV V$ vertices in the SU(2)$_{L+R}$ custodial limit (i.e., take $L = R$) are given
by

\[ \mathcal{L}_{HVV} = \sum_j \mu_j O_j^{\mu\nu} \, \text{tr} \{ M_{21} \tau \cdot W_\mu \tau \cdot W_\nu \} + h.c. \]

\[ \propto \sum_j O_j^{\mu\nu} (W_\mu^3 W_\nu^3 + W_\mu^+ W_\nu^-) \left[ \text{Im} \mu_j \text{Im}(\phi_2^0 - \phi_1^0) + \text{Re} \mu_j \text{Re}(\phi_2^0 + \phi_1^0) \right] . \]

(24)

Note that this differs from eq. (16) due to the new scalar transformation law [eq. (22) instead of eq. (14)]. From eq. (24), we see that the \( A^0VV \) vertex can be generated even in the custodial limit. Note however that the \( H^\pm W^\mp Z \) vertex is still absent in the same limit. When we turn on the U(1)\(_Y\) gauge interactions, new trilinear \( HWB \) and \( HBB \) vertices can be generated\(^*\). Notice, however, that the above conclusions are still valid up to terms of \( \mathcal{O}(m_W^2/E^2) \) where \( E \) is the energy of the vector bosons. This can be seen using the equivalence theorem\(^{[13]}\) which states that the vector bosons can be replaced by their corresponding Goldstone bosons \((G)\) in processes with \( E \gg m_W \). Proceeding as before, it is possible to show that demanding custodial invariance in the quark-Yukawa interactions, the \( HGG \) vertices \((H = A^0, H^\pm \) for the case I and \( H = H^\pm \) for the case II) are zero.

In order to estimate the contribution of the Higgs self-interactions and Higgs-gauge interactions to the \( HVV \) vertices,\(^†\) we can make use of the same arguments of the previous section, \( i.e., \) these contributions are expected to be small in the MSSM where the custodial SU(2) symmetry is slightly violated. Moreover, the contribution of the Higgs-gauge interactions must vanish in the limit \( m_{h^0} = m_{H^0} \). This can be seen by noting that the kinetic scalar term is invariant under the rotation of the Higgs doublets:

\[ \mathcal{L}_{\text{kin}} = \sum_{i=1}^{2} (D_\mu \Phi_i) \dagger (D_\mu \Phi_i) = \sum_{i=1}^{2} (D_\mu \Phi_i') \dagger (D_\mu \Phi_i') , \]

(25)

\(^*\) In the case of the \( H^\pm W^\mp B \) vertex, the virtual quark-loop contribution does not yield a term that grows quadratically in the quark mass at one-loop\(^{[10]}\).

\(^†\) In fact, gauge and Higgs loops do not contribute to \( A^0VV \) vertices to all orders in perturbation theory\(^{[12]}\).
where $\Phi_i'$ are defined such as $\langle \Phi_1' \rangle = v \equiv \sqrt{v_1^2 + v_2^2}$ and $\langle \Phi_2' \rangle = 0$. In the limit $m_{h^0} = m_{H^0}$, we have

$$
\Phi_1' = \left( v + \frac{1}{\sqrt{2}} (h^0 + i G^0) \right), \quad \Phi_2' = \left( \frac{1}{\sqrt{2}} (H^0 + i A^0) \right),
$$

(26)

so that $\Phi_1'$ and $\Phi_2'$ do not mix with each other. Since $\langle \Phi_2' \rangle = 0$, the kinetic term of $\Phi_2'$ is still $SU(2)_L \times U(1)_Y$ invariant after SSB. Thus, $\Phi_2'VV$ vertices cannot be generated.

4. Conclusions

We have analyzed the radiative corrections to the charged Higgs mass in the MSSM and to the $HVV$ vertices by making use of approximate global symmetries of the theory with two Higgs doublets.

In the analysis of the charged Higgs mass, we have shown that one-loop radiative corrections from top quarks and top squarks cannot be of $O(g^2 m_t^4 / m_W^2)$ in the absence of $\tilde{t}_L - \tilde{t}_R$ mixing. This has been accomplished by analyzing the limit of $g = m_{12} = h_b = 0$ where the Higgs potential possesses a global $O(4) \times O(4)$ symmetry. In this limit the charged Higgs boson is a pseudo-Goldstone boson associated with the breakdown $O(4) \times O(4) \rightarrow O(3) \times O(3)$. By studying the global symmetry properties of the other sectors of the theory, the dependence of $m_{H^\pm}^2$ on the model parameters can be ascertained.

In the analysis of the one-loop effects to the trilinear $HVV$ vertices, we have shown that a custodial $SU(2)$ symmetry plays a crucial role. The appropriate definition of the $SU(2)$ symmetry depends on two possible choices for the pattern of Higgs-fermion couplings. In the first case, the CP-odd Higgs and charged Higgs couplings to $VV$ generated at one-loop are zero if the theory is custodial invariant. In the second case only the charged Higgs couplings to $VV$ are zero in this limit.

To evaluate the order of magnitude of the radiative corrections to such $A^0VV$ and $H^\pm W^\mp Z$ vertices, we have studied the custodial limit of the different sectors of the
theory. In the MSSM, one learns why in the limit $g' = 0$ the $H^\pm W^\mp Z$ vertex is the only $HVV$ vertex that does not receive contributions from a heavy degenerate fermion doublet. Moreover, due to the approximate invariance of the Higgs potential and the Higgs-gauge interactions under the custodial $SU(2)$, contributions from the gauge/Higgs sectors of the theory to the $H^\pm W^\mp Z$ vertex must be very small.

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