Optimal Motion Planning of Connected and Automated Vehicles at Signal-Free Intersections with State and Control Constraints

Mohamad Hafizulazwan Mohamad Nor* and Toru Namerikawa**

Abstract: This paper presents the optimal motion planning problem for connected and automated vehicles (CAVs) to cross a conflict area at an intersection with state and control constraints. First, we formulate the scheduled merging (or crossing) time for all CAVs as a mixed integer linear programming (MILP) problem where the merging time is solved frequently. Second, we formulate the optimal motion planning problem so that the CAVs can achieve their scheduled merging time as well as minimizing the energy consumption. Since we solve the motion planning problem analytically, not all the solutions are feasible to comply with the frequently updated merging time. To solve this problem, we propose a feasibility enforcement period (FEP). Then, we validate the proposed solution through simulation, and the results show that even the merging time is frequently updated, the CAVs can still achieve the merging time with a minimal deviation. Besides, our proposed framework also shows a significant improvement in terms of travel time as compared to the conventional one.

Key Words: connected and automated vehicles (CAVs), autonomous intersections, optimal scheduling, optimal motion planning.

1. Introduction

The increasing number of vehicles on the roadway every year may be seen as the cause of traffic congestion, especially in urban areas to increase. Nevertheless, [1] reported that the primary sources of bottlenecks are intersections and merging roadways. In the United States, traffic congestion had caused a passenger to spend an extra 42 hours on the road yearly and wasted 11.7 billion liter of fuel (which is caused by unnecessary acceleration or deceleration of the vehicles due to the congestion) in 2014 [2]. Besides, it is reported in [3] that about 40% of accidents in the United States are related to intersections, which are similar to the percentage of intersection-related accidents happened in the European Union [4]. For solving the problems as mentioned earlier, many research efforts have been conducted especially in providing better coordination for the movement of vehicles at the intersections. This includes improving the signal phase and timing plans of the existing physical traffic lights. Recently, many efforts had been made to explore the benefits of connected and automated vehicles (CAVs) technology where no traffic lights are required (we call this strategy as autonomous intersection hereafter). The CAVs technology can provide much faster response time compared to human-driven vehicles and capable of controlling their speed precisely. Therefore, the problems mentioned above tend to be solved more effectively and efficiently.

From the literature, we can categorize the autonomous intersection strategy into two general frameworks that are commonly utilized in this area of research. First is the framework with both scheduling and motion planning, and the second one is the motion planning framework only. For the first framework, the CAVs are depending on the assigned merging time (finding the appropriate merging time is known as a scheduling problem) to cross the intersection zone, and a controller will provide some inputs (acceleration/deceleration) to the CAV (finding the appropriate input to the CAV is known as motion planning problem) to comply with the scheduled merging time. Note that what we mean by the merging time is the time for each CAV to enter the conflict area at the intersection. On the other hand, the second framework is only depending on the position of the other CAVs to cross the intersection, and this framework mimics more like human behavior. In this paper, we are focusing on the first framework, and thus, all the references provided in the following paragraph will be related to the first framework only.

There are several approaches to formulate the scheduling problem where the main objective is either to minimize the travel time or to guarantee the safety of all CAVs or both. For example, a reservation-based approach (heuristic) was proposed in [5] where the CAV needs to reserve a space-time block in the conflict area. Scheduling based on first-in-first-out (FIFO) order was utilized in [6] where the objective is to minimize the position gap between the CAVs. In [7]–[9], they considered the optimal time for each CAV to enter the conflict area where their scheduling problem were formulated based on mathematical optimization tools. Formulation based on conflict point concept were utilized in [10],[11]. In [12], the time for the CAVs to enter the conflict area were regularly swapped with other CAVs in the intersection area so that minimum time to enter the conflict area can be achieved. On the other hand, some approaches are proposed and utilized to formulate the motion planning problem to reach the scheduled merging time while minimizing fuel consumption. Note that

* Graduate School of Science and Technology, Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama 223-8522, Japan
** Department of System Design Engineering, Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama 223-8522, Japan
E-mail: mhaifizulazwan@nl.sd.keio.ac.jp, namerikawa@sd.keio.ac.jp
(Received June 17, 2019)
(Revised October 30, 2019)
there is a monotonic relationship between the vehicle’s acceleration and fuel consumption [13]. Therefore, by optimizing the acceleration of each vehicle, we can have the benefits in reducing fuel consumption. Some existing approaches for the optimal motion planning problem are an analytical solution, e.g., Hamiltonian analysis [14]–[17], model predictive control (MPC) [18], and discrete optimization [19]. The objective of this paper is to have both optimality in scheduling and motion planning. Some works considered the same objective for example in [12],[15],[18]–[22] but we did not find any work which combines mixed integer linear programming (MILP) and Hamiltonian analysis to formulate the scheduling and motion planning problems respectively.

In our previous work [23], we proposed the merging time scheduling based on MILP to address the FIFO based scheduling problem in [14], and the same optimal motion planning approach is utilized to reach the scheduled merging time, i.e., Hamiltonian analysis. However, no state and control constraints, as well as continuous arrival of the CAVs at the intersection, were considered in our previous work. For considering the state and control constraints in the Hamiltonian analysis, the existing solution in [14] cannot be directly applied. This is because, in our case, the scheduled merging time is regularly updated (the scheduled merging time in [14] is fixed), and this will not always provide a feasible solution in the analysis. To solve this problem, we need to ensure the state and control values are within the maximum and minimum during the update time. Therefore, we propose a feasibility enforcement period (FEP) which slows down or speeds up the CAV if the state or control value lies at the maximum or minimum during the update time. In addition, [14] only considered low traffic environment in their simulation. The content of this paper has been partially presented in our prior conference publication [24]. In this paper, we improve and provide a detailed formulation of the preceding publication.

This paper is organized as follows. In Section 2, we explain the modeling frameworks and discuss two problem formulations, which are optimal merging time scheduling and optimal motion planning problems. Also, we discuss the analytical solution for the optimal motion planning problem with the state and control constraints using Hamiltonian analysis. In Section 3, we provide simulation results to evaluate the proposed solution and compare it in terms of travel time and fuel consumption. Finally, concluding remarks are provided in Section 4.

2. Problem Formulation

Before further explaining, we summarize the list of abbreviations used in this paper in Table 1.

| Abbreviations | Definition |
|---------------|------------|
| CAVs          | Connected and Automated Vehicles |
| FIFO          | First-In-First-Out |
| MILP          | Mixed Integer Linear Programming |
| CZ            | Control Zone |
| MZ            | Merging Zone |
| ID            | Identification |
| ACC           | Adaptive Cruise Control |
| FEP           | Feasibility Enforcement Period |
| FEZ           | Feasibility Enforcement Zone |

Figure 1 shows four-movement, i.e., southbound, \(X'\), northbound, \(X''\), westbound, \(O'\), and eastbound, \(O''\), intersection model that we consider in this paper. The set of movements, \(M\), are grouped into two phases which are Phase \(X\), \(\phi_X = \{X', X''\}\) and Phase \(O\), \(\phi_O = \{O', O''\}\) (see the right-hand side of Fig. 1) and each phase consists a set of non-conflicting movements. The intersection also has a control zone (CZ), which is depicted in a blue circle, and this is the region where the coordinator can reach the CAVs via communication. The length of this CZ is denoted as \(L\) and is measured from the entry of the CZ, which is zero, to the entry of the merging zone (MZ) which is \(L\). The MZ (also known as conflict area) is the region where the lateral collision between vehicles from different phases (\(\phi_X \neq \phi_O\)) can potentially occur. The shape of the MZ is assumed to be a square of side \(S\). As mentioned previously, we will obtain the optimal solution for both merging time scheduling and motion planning. Therefore, to simplify the presentation of ideas and to reduce the complexity of the problem, only one lane is considered for each movement, and no lane changing and turning (left or right) are allowed. We also assume that all vehicles are CAVs.

For the model of each CAV \(i = 1, 2, ..., N(t)\), it is considered as a second order linear differential equation given as

\[
\begin{bmatrix}
\dot{p}_i \\
\dot{v}_i \\
0 \\
0 \\
1
\end{bmatrix} = \begin{bmatrix}
0 & 1 & p_i(t) & v_i(t) & 0 \\
0 & 0 & 1 & 1 & u_i(t)
\end{bmatrix},
\]

(1)

where \(N(t)\) is the total CAVs subscribed to the coordinator at time \(t\) (the detail of the subscription process is discussed in Section 2.2), \(t_0 \in \mathbb{R}^+\) is the current time, \(p_i(t) \in [0, L + S]\) is the position, \(v_i(t)\) is the speed, and \(u_i(t)\) is the control input (acceleration/deceleration). Note that the current position, \(p_i(t_0)\), and speed, \(v_i(t_0)\), are given. As can be noticed from the CAV model in (1), only the longitudinal movement of the CAV is considered. Since the CAV is not allowed to turn right or left, thus the CAV model in (1) should be sufficient, i.e., only longitudinal movement of the CAV is considered.

For the system in (1), we have to define the control input and speed constraints considering actuator saturation and so that the CAV is within a given admissible range respectively. Typically, these constraints are defined as follows:

\[
u_{i,\text{min}} \leq u_i(t) \leq u_{i,\text{max}}, \quad \text{and} \quad 0 \leq v_{\text{min}} \leq v_i(t) \leq v_{\text{max}} \quad \forall t \in [t_0, t_f]
\]

(2)
where $u_{min}, u_{max}$ are the minimum and maximum inputs for each CAV $i$, and $v_{min}, v_{max}$ are the minimum and maximum speed limits respectively, and $t_f$ is the time that CAV $i$ leaves the MZ (we also call it as final time). In this paper, we assume that all CAVs are the same and thus we can set $u_{min} = u_{max}$ and $u_{max} = u_{max}$.

Rear-end collision can potentially happen if two consecutive CAVs are traveling on the same movement (e.g., CAV 4 and CAV 5 in Fig. 1). Therefore, to ensure the absence of the rear-end collision, we can define a safety distance between the two CAVs:

$$s_i(t) = p_i(t) - p_j(t) \geq \delta, \quad \forall t \in [t_0, t_f^i], \quad \forall m_i, m_j \in M, \quad m_i = m_j,$$

(3)

where $m_i \in M$ is the movement of CAV $i$, $k$ is the preceding vehicle of CAV $i$, (mathematically can be described as $k = \max \{ j : m_i = m_j, j = 1, \ldots, i - 1 \} < i$) and $\delta$ is the minimum safety distance to avoid rear-end collision between CAV $i$ and CAV $k$. From the optimal merging time scheduling formulation in Section 2.2, we will show that we can satisfy the constraint (3) at $t = t_m$ only where $t_m$ is the time for CAV $i$ to enter the MZ (see Fig. 2 for the clear difference between $t_m$ and $t_f^i$). To satisfy the constraint (3) for all $t \in [t_0, t_f^i]$, we need to include the constraint in the optimal motion planning formulation in Section 2.3. However, including the constraint will make the problem difficult to be solved because of the inequality. Another alternative that we can take to satisfy (3) for all $t \in [t_0, t_f^i]$ is by incorporating a car-following subroutine such as adaptive cruise control (ACC) into the controller. For considering ACC, many problems need to be addressed, such as the feasibility in the solution of merging time scheduling and the accuracy to achieve the merging time. Therefore, we will activate the ACC only when the CAV is inside the MZ so that we can guarantee the constraint (3) for all $t \in [t_m, t_f^i]$. To satisfy (3) for all $t \in [t_0, t_f^i]$, we will leave the problem in the future research. Note that, after the CAV leaves the MZ, we can let the human take over the control.

A lateral collision might happen inside the MZ between two CAVs traveling from different phases, for example, CAV 2 and CAV 3 in Fig. 1. Before explaining the strategy on how this collision can be avoided, first, we provide the following definition:

**Definition 1.** For each CAV $i$, we define the set $\Gamma_i$ that includes all the time instants when a lateral collision involving CAV $i$ is possible:

$$\Gamma_i \triangleq \{ t | t \in [t_m, t_f^i] \}.$$  

(4)

From the above definition, one of the strategies to avoid the lateral collision is to prevent the two CAVs $i, j \in N(t), i \neq j$ to be inside the MZ at the same time. This strategy can be formalized as the following constraint

$$\Gamma_i \cap \Gamma_j = \emptyset$$

(5)

where $\emptyset$ is the empty set. In some cases, the size of MZ might be large and the constraint in (5) might not be realistic. To solve this problem, we can modify the safety headway time to avoid the lateral collision for the two CAVs to enter the MZ in Section 2.2.

In the following subsection, we discuss the formulation of optimal merging time scheduling for each CAV to cross the intersection. For simplicity of notation in the remainder of this paper, we will write $p_i(t_0) \equiv p_i^0, v_i(t_0) \equiv v_i^0, v_i(t_m) \equiv v_i^m, v_i(t_f^i) \equiv v_i^f$ where $v_i(t_m)$ and $v_i(t_f^i)$ are merging and final speed respectively.

### 2.2 Optimal Merging Time Scheduling Formulation

When a new CAV arrived at the entry of the CZ, it sends a subscription request to the coordinator to announce its presence and exchanges the following information set until the CAV unsubscribes (leaves the MZ) from the coordinator.

**Definition 2.** The information set for each CAV $i$ is defined as

$$Y_i \triangleq \{ m_i, p_i, v_i, t_f^i \},$$

(6)

where $m_i \in M$ is the CAV’s movement (from $m_0$, the coordinator will know the phase of the CAV). Note that when the subscription is successful, the coordinator assigns the ID $i = N(t) + 1$ to this newly subscribed CAV. In some cases, there will be two or more CAVs arrive at the CZ at the same time. To deal with this problem, the coordinator can assign the ID based on the movement of each CAV, $m_i$. In this paper, we prioritized the movement in an anti-clockwise direction starting from $O''$ and ending at $X''$. This means that if two or more CAVs arrive at the CZ at the same time, then the CAV with the most priority movement will be numbered as $i = N(t) + 1$, the following CAV will be numbered as $i = N(t) + 2, etc.$

The first three parameters in the information set (6) are used by the coordinator to compute $t_m$ for each CAV $i$ at every execution time, $t_e$. To compute $t_m$, the coordinator may need to re-sort the ID of the subscribed CAVs (it is okay for not to re-sort the ID, but for ease of explanation of the formulation later, we assume the coordinator re-sorts the ID). In this case, we can sort the ID based on the CAV’s distance to the MZ, i.e., the nearest CAV to the MZ will be numbered first. Note that the coordinator only computes $t_m$ for all CAVs which are still subscribing to the coordinator (not yet leaving the MZ). Because of this, once a CAV left the MZ, the ID will be eliminated and the ID of the CAVs that are still subscribed to the coordinator is dropped by 1 which means that the total number of subscribed CAVs also is reduced to $N(t) - 1$ (see Fig. 1).
The primary goal of obtaining the optimal \( t^m_i \) is to minimize the travel time of all CAVs and thus, reducing the traffic congestion at the intersection. In this paper, we modify the optimal merging time scheduling formulation based on MILP from our previous work [23]. For safety reason, we provide the following assumption.

**Assumption 1.** The coordinator can obtain the information set (6) for each CAV \( i \) via infrastructure-to-vehicle (I2V) communication without errors or delays.

Before formulating the problem of obtaining the optimal \( t^m_i \), some constraints need to be considered first.

1) **Feasible merging time:** The speed and control input for each CAV \( i \) is restricted as in (2); therefore, there is also a restriction for the value of \( t^m_i \). To avoid assigning \( t^m_i \) beyond the reachability of the CAV, we provide the following constraint

\[
t^m_i \geq t^m_{i\text{min}},
\]

where \( t^m_{i\text{min}} \) is the minimum time (lower bound of \( t^m_i \)) for each CAV \( i \) to enter the MZ, and it can be derived from the basic equation of motion [8] as follows

\[
t^1_i = \min \left\{ \frac{v_{\text{max}} - v_i^0}{u_{\text{max}}}, \frac{v^m_{i0} - v_i^0}{u_{\text{max}}} \right\},
\]

\[
t^2_i = \max \left\{ \frac{v_i^0}{v_{\text{max}}} - \frac{v^m_{i0} - v_i^0^2}{2u_{\text{max}}v_{\text{max}}}, 0 \right\},
\]

\[
t^m_{i\text{min}} = t_0 + t^1_i + t^2_i,
\]

where \( D_i = L - p_i^0 \) is the distance of each CAV \( i \) to the MZ at current time \( t_0 \), and \( v^m_{i0} = \sqrt{(v_i^0)^2 + 2u_{\text{max}}D_i} \) is the maximum merging speed that the CAV can reach before entering the MZ. To clearly describe (8), we provide Fig. 3.

![Fig. 3 Minimum merging time of each CAV i.](Image)

From the above figure, \( t^1_i \) is the time for the CAV to reach the maximum speed or below before entering the MZ. If the CAV can reach the maximum speed before entering the MZ, then the first terms in \( t^2_i \) is used to determine the cruising time with the maximum speed until it enters the MZ. On the other hand, if the CAV cannot achieve the maximum speed before entering the MZ, then \( t^2_i \) becomes zero. Note that, we do not introduce the maximum merging time (upper bound of \( t^m_i \)) in (7) to avoid the infeasibility in the solution of \( t^m_i \). Instead, we may force the CAV to come to a stop before entering the MZ if the assigned \( t^m_i \) exceeds the maximum merging time and proceeds back to cross the MZ once \( t^m_i \) has arrived. Planning the CAV to stop at the entry of the MZ and moving back to cross the MZ will be addressed in the future research.

2) **Avoiding rear-end collision at MZ:** As stated before, two types of collision avoidance need to be addressed, and one of them is the rear-end collision. By imposing the following constraint, we can guarantee the absence of rear-end collision at the entry of MZ for two consecutive CAVs traveling on the same movement, namely,

\[
i^m_i - i^m_j \geq h_R
\]

\[\forall m_i, m_j \in M, \quad m_i = m_j,\]

(9)

where \( h_R \) is the safety headway time to avoid rear-end collision. We can see the relation between the constraints (9) and (3) from \( p_i (i^m_i - p_i (i^m_j) = L + v_{\text{max}}(i^m_i - i^m_j) - L \geq \delta \), and rearranging this equation, we can obtain \( t^m_i - i^m_j \geq \delta/v^m_j \) where \( v^m_j \) is the speed for the preceding CAV \( k \) to enter the MZ. In our case, obtaining \( v^m_j \) is difficult, and even we can obtain it, \( v^m_j \) might become zero (i.e., stop at the entry of MZ) in some cases which will lead \( \delta/v^m_j \) to infinity. Because of this factor, we use fixed safety headway time \( h_R \) in (9) instead of \( \delta/v^m_j \) in this merging time scheduling formulation.

3) **Avoiding lateral collision at MZ:** As stated in (5), we can avoid the lateral collision by allowing only one CAV to be inside the MZ at a time. This can be done by imposing the following constraint

\[
i^m_i - i^m_j \geq h_L
\]

\[\forall \phi_i, \phi_j \in \phi, \quad \phi_i \neq \phi_j,\]

(10)

where \( h_L \) is the safety headway time to avoid the lateral collision between CAV \( i \) and CAV \( j \), and it needs to be properly selected so that we can satisfy (5). If the size of MZ is large, then we can relax (5) and adjust \( h_L \) accordingly. We can see the relation between (10) and (5) from \( i^m_i \geq i^m_j \Rightarrow i^m_i = i^m_j + S/v^m_j \) (this condition satisfies (5)). Same in 2), \( v^m_j \) cannot be easily calculated and will make \( S/v^m_j \) go to infinity if \( v^m_j \) becomes zero, and thus, we use \( h_L \) instead of \( S/v^m_j \). The same relation can be obtained for \( i^m_j \).

Note that, the OR logic in (10) enables us to determine the sequence for the CAVs to cross the intersection, i.e., either the CAV \( i \) or CAV \( j \) crosses first. Unfortunately, the OR combination of the inequalities has discontinuity and is not linear. To solve this problem, we can convert the OR combination to AND combination of the inequalities using the big-M method [25] as follows:

\[
i^m_i - i^m_j + M_{\text{big}} b_i \geq h_L
\]

AND

\[
i^m_j - i^m_i + M_{\text{big}} (1 - b_i) \geq h_L
\]

\[\forall \phi_i, \phi_j \in \phi, \quad \phi_i \neq \phi_j,\]

(11)

where \( M_{\text{big}} \) is an arbitrarily large constant number and \( b_i, (i = 1, 2, ..., \text{no. of constraints}) \) is a binary variable which is also a decision variable. From (11), if \( b_i \) is zero, the first inequality holds true if \( i^m_i - i^m_j \geq h_L \), and the second inequality \( i^m_j - i^m_i \geq h_L - M_{\text{big}} \) is redundant and always holds true if the value of \( M_{\text{big}} \).
is large enough. If \( b_i \) is zero and both of the inequalities in (11) hold true, then \( \text{CAV}_j \) crosses the MZ first instead of \( \text{CAV}_i \). On the other hand, if \( b_i \) is one, the first inequality \( t_{m_j}^i - t_{m_k}^i \geq b_i - M_{b_{inj}} \) is redundant and always holds true if the value of \( M_{b_{inj}} \) is large enough, and the second inequality holds true if \( t_{m_j}^i - t_{m_k}^i \geq b_i \). If \( b_i \) is one and both of the inequalities in (11) hold true, \( \text{CAV}_i \) crosses the MZ first instead of \( \text{CAV}_j \).

Now we are ready to provide the objective function for the optimal merging time scheduling, and we choose the objective function as below

\[
\min_{T,B} J = \omega_1 \sum_{i=1}^{NCA} t_{m_i}^i + \omega_2 \sum_{i=1}^{NCA} \left| t_{m_i}^i - t_p^i \right|
\]

subject to: (7), (9), (11),

\[
(12)
\]

where \( T \) contains all \( t_{m_i}^i \), i.e., \( T := [t_{m_1}^i, t_{m_2}^i, ..., t_{m_NCA}^i] \), \( B \) contains all \( b_i \), i.e., \( B := [b_1, b_2, ..., b_{NCA}, \text{constraints}] \), \( \omega_1 \) and \( \omega_2 \) are the weights, and \( t_p^i \) is the previously assigned merging time. Instead of just minimizing the total merging time of all CAVs as we did in our previous work [23], we add the second terms to the objective function to minimize the difference between the newly assigned merging time, \( t_{m_i}^i \) and the previously assigned merging time, \( t_p^i \) or \( t_{m_j}^i \) can be used for the newly subscribed CAVs where \( \psi_i \) is the arrival speed at the entry of the CZ. Consequently, this may increase passenger comfort as well as reducing energy consumption.

According to [8], the absolute value sign in (12) cannot be utilized in a linear programming formulation. However, we can solve this problem by adding a slack variable, \( t_{m_i}^{i\text{slack}} = \left| t_{m_i}^i - t_p^i \right| \). To ensure that the slack variable is equal to \( \left| t_{m_i}^i - t_p^i \right| \), we can add two additional constraints, i.e., \( t_{m_i}^{i\text{slack}} \geq (t_{m_i}^i - t_p^i) \) and \( t_{m_i}^{i\text{slack}} \geq -(t_{m_i}^i - t_p^i) \) into the objective function. Then, with the slack variable and the new constraints, we can restate (12) as

\[
\min_{T,B} J = \omega_1 \sum_{i=1}^{NCA} t_{m_i}^i + \omega_2 \sum_{i=1}^{NCA} t_{m_i}^{i\text{slack}}
\]

subject to: (7), (9), (11),

\[
(13)
\]

In every execution time \( t_e \), the MILP problem in (13) can be solved by using various kinds of available MILP solvers. In this research, we use intlinprog function from Matlab optimization toolbox. Besides that, other MILP solvers that worth to try is IBM’s CPLEX optimization toolbox [26] and grouping based strategy as proposed in [27]. Note that, we denote the optimal solution for \( T \) in (13) as \( T^* := \{t_{m_i}^{i\text{slack}}, t_{m_2}^{i\text{slack}}, ..., t_{m_NCA}^{i\text{slack}}\} \) since this solution is used to specify the terminal time for the optimal motion planning problem in the following subsection.

For extending problem (13) into the right and left turning, we can add two more phases into the intersection model instead of considering only two phases as shown in the right-hand side of Fig. 1. In addition, we also have to take into consideration the time for the CAV to exit the MZ when it performs turning so that collision can be avoided [28].

### 2.3 Optimal Motion Planning Formulation

In this subsection, we want to find the optimal control strategies for the motion planning of CAVs to reach the scheduled merging time \( t_{m_i}^{i\text{slack}} \) where the objective is to minimize the energy consumption. In [13], a monotonic relationship between fuel consumption and acceleration was shown. This relationship enables us to minimize the energy consumption (which is the fuel consumption in this case) by only minimizing the acceleration of each CAV. Therefore, the objective function of the optimal control strategy for the motion planning problem can be defined as

\[
\min_0 \frac{1}{2} \int_0^{t_F} u_i^2(t) \, dt,
\]

subject to: (1), (2), \( p_i(t_{m_i}^{i\text{slack}}) = L \), and given \( b_i, p_i, v_i, t_{m_i}^{i\text{slack}} \).

After receiving \( t_{m_i}^{i\text{slack}} \) from the coordinator, (14) can be solved by an onboard computer of each CAV to obtain the optimal acceleration to reach \( t_{m_i}^{i\text{slack}} \) with minimum energy consumption. From the literature, there are two options to solve the problem, which is by utilizing the analytical [14] or numerical [19] solutions (the problem in (14) and the system in (1) need to be discretized first to utilize the numerical solution). Both of these analytical and numerical solutions have some advantages and disadvantages. Since the analytical solution can provide much faster computation time (which is one of the demands in the CAV technology), this motivates us to utilize the analytical solution.

In this paper, the analytical solution that we used to solve (14) is Hamiltonian analysis [14], which is also a standard methodology used in the optimal control problems [29],[30]. To begin the Hamiltonian analysis, we provide the Hamiltonian function for each CAV \( i \) as follows:

\[
\mathcal{H}_i(t, p(t), v(t), u(t)) = \frac{1}{2} m_i^2 \dot{v}_i + \pi_i \cdot v_i + \pi_i \cdot u_i
+ \lambda_1 \cdot (u_i - u_{max}) + \lambda_2 \cdot (u_{max} - u_i)
+ \lambda_3 \cdot (v_i - v_{max}) + \lambda_4 \cdot (v_{max} - v_i),
\]

(15)

where \( \pi_i \) and \( \pi_i \) are the co-states, and \( \lambda^2 \) is a vector of Lagrange multipliers. The Lagrange multiplier is larger than zero if the constraint is active, and it is equal to zero if the constraint is not active. Note that the Lagrange multipliers (the co-states are also Lagrange multipliers) above are used to convert the optimization problem in (14) with the constraints (1) and (2) to the unconstrained optimization problem as in (15). There are a few necessary conditions for the Hamiltonian function in (15) to be optimized. First is the Euler-Lagrange equation which is given by

\[
\frac{\partial \mathcal{H}_i}{\partial p_i} = -\pi_i,
\]

(16)

where, by partial differentiating (15) with respect to \( p_i \) and \( v_i \), as well as using the condition of the Lagrange multipliers described before, we can obtain \( \pi_i \) and \( \pi_i \) as follows:

\[
\pi_i = \begin{cases} \pi_i, & v_i(t) - v_{max} < 0 \\
-v_{max} - v_i(t) < 0, & \pi_i = \lambda_1 \end{cases}
\]

\[
\pi_i = \begin{cases} -\pi_i - \lambda_2, & v_i(t) - v_{max} = 0 \\
-v_{max} - v_i(t) = 0, & \pi_i = \lambda_2 \end{cases}
\]

(18)
The second necessary condition is the condition for optimality, which is
\[
\frac{\partial H}{\partial u_i} = u_i + \pi_i^* + X_i^* + \lambda_i^* = 0. \tag{19}
\]

After providing these conditions, we can solve the optimal control input and states for each CAV \(i\) by piecing together the constrained and unconstrained arcs [14]. Before deriving the solution, the following assumptions should be noted to avoid the infeasibility in the solution.

**Assumption 2.** When a new CAV arrives at the CZ, none of the constraints in (2) is active.

**Assumption 3.** Every time the problem in (15) is resolved, i.e., at every execution time \(t_0 = t_e\), none of the constraints in (2) is active.

From the above assumptions, what we mean by the constraints are not active is when the speed or control input is within the bounded values, e.g., \(v_{min} < v_i(t) < v_{max}\). In [14], \(v_i^{max}\) for each CAV \(i\) is fixed (because they did not re-solve \(v_i^{max}\)) from the time the CAV arrives at the CZ until the time it enters the MZ. Therefore, from Assumption 2 only they can avoid the infeasibility in the solution. To prevent Assumption 2 from being violated, [14],[31] introduced a feasibility enforcement zone (FEZ) which is a zone outside the CZ designed to slow down or to speed up the CAV so that the speed and control input of the CAV are not active before entering the CZ. However, the FEZ (or Assumption 2) alone is not possible in our case, and we need to provide Assumption 3 so that no constraints are active at every execution time, \(t_e\). One way that we can do to avoid Assumption 3 from being violated is to slow down or speed up the CAV if the CAV reaches the maximum or minimum speed at the execution time (we call this as feasibility enforcement period (FEP)) as follows

\[
u_i(t) = \begin{cases} \frac{v_{B,max} - v_i^0}{t_e}, & \text{mod}(t_0, t_e) = 0 \\ \frac{v_{B,\min} - v_i^0}{t_e}, & \text{mod}(t_0, t_e) = 0 \end{cases} \tag{20}
\]

where \(v_{B,max} < v_{max}\) and \(v_{B,\min} > v_{min}\) are the maximum and minimum speeds that the CAV \(i\) needs to achieve before solving (15), and \(\text{mod}(t_0, t_e)\) is the modulus of \(t_0\) divided by \(t_e\). Note that from (20), if the speed constraint is active at the execution time, the CAV needs to achieve \(v_{B,max}\) or \(v_{B,\min}\) first before the next execution time with the input \(u_i(t)\) for all \(t \in [t_0, t_0 + t_e]\). Because of this, we have to set \(v_{B,max}\) and \(v_{B,\min}\) as close as possible to \(v_{max}\) and \(v_{min}\) respectively.

To further describe the FEP, we provide Fig. 4 as an example. From the figure, we can see that the speed of CAV \(i\) at the earliest execution is within \(v_{max}\) and \(v_{min}\), and this means that the solution of (15) is feasible. However, at execution \(t_e + 2\), the speed already became \(v_{max}\). Therefore, the FEP takes place between the execution time \(t_e + 2\) and \(t_e + 3\) so that it can provide the feasibility in the solution of (15) at execution \(t_e + 3\). This is also the same if the speed becomes \(v_{min}\). On the other hand, one may concern about \(t_{min}\) and \(t_{max}\) during the execution time. We stress that this is not a problem where we will show in the solution of (15) that the control input will become zero if the speed is at \(v_{max}\) or \(v_{min}\).

Now, we can turn back our attention to solve (15) to obtain the optimal control input and states for each CAV \(i\) where the solution is the result of different combinations of the following possible arcs. Under Assumptions 2 and 3, we can start the solution from the unconstrained arc.

**CASE 1: No constraints are active**

From the condition of the Lagrange multiplier, we know that the Lagrange multipliers are zero, i.e., \(\lambda_i^* = \lambda_i^t = \lambda_i^f = 0\) if no constraints are active. Therefore from (19), the optimal control input is obtained as

\[
u_i^* = 0, \tag{21}\]

where we can solve \(\pi_i^*\) from (17) and (18) which yield

\[
u_i^* = a_i^t + b_i, \tag{22}\]

where \(a_i\) and \(b_i\) are integration constants. Substituting (22) into the system (1) gives us the optimal speed and position

\[
v_i^*(t) = \frac{1}{2} a_i t^2 + b_i t + c_i, \tag{23}\]

\[
p_i^*(t) = \frac{1}{6} a_i t^3 + \frac{1}{2} b_i^2 + c_i t + d_i, \tag{24}\]

where \(c_i\) and \(d_i\) are integration constants. To solve the constants \(a_i, b_i, c_i,\) and \(d_i\) above, we can use the initial condition \(v_i^0\), the initial and terminal conditions \(p_i^0, p_i^f = L\), as well as the boundary condition of the co-state \(\pi_i^*(t_i^*) = -u_i(t_i^*) = 0\) of the problem (14). Then, we can put (22)-(24) and the conditions mentioned above in the following matrix (in the form of \(AX_i = q_i\)):

\[
\begin{bmatrix} \frac{1}{2} (t_i^0)^3 \\ \frac{1}{2} (t_i^0)^2 \\ \frac{1}{2} (t_i^0) \\ t_i^0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a_i \\ b_i \\ c_i \\ d_i \end{bmatrix} = \begin{bmatrix} p_i(t_0) \\ v_i(t_0) \\ p_i(t_0) \\ \pi_i(t_0) \end{bmatrix}. \tag{25}\]

where \(t_i^0\) is obtained by solving the problem (13). Since the above matrix is solved (by rearranging the above matrix to \(X_i = A_i^{-1} q_i\)) for every \(t > t_0\), we can re-evaluate the four constants as \(a_i(t, p_i(t), v_i(t)), b_i(t, p_i(t), v_i(t)), c_i(t, p_i(t), v_i(t)), d_i(t, p_i(t), v_i(t))\).

**CASE 2: Only control constraint is active**

Since the integration constants in the optimal control input (22) are always being updated at every \(t > t_0\), at one point, it may reach \(u_{max}\). Suppose that the optimal control input reaches \(u_{max}\) at \(t = t_1\) (while \(v_{min} \leq v_i(t) \leq v_{max}\)), then (22) becomes

\[
u_i^* = 0, \tag{21}\]

where we can solve \(\pi_i^*\) from (17) and (18) which yield

\[
u_i^* = a_i^t + b_i, \tag{22}\]

where \(a_i\) and \(b_i\) are integration constants. Substituting (22) into the system (1) gives us the optimal speed and position

\[
v_i^*(t) = \frac{1}{2} a_i t^2 + b_i t + c_i, \tag{23}\]

\[
p_i^*(t) = \frac{1}{6} a_i t^3 + \frac{1}{2} b_i^2 + c_i t + d_i, \tag{24}\]

where \(c_i\) and \(d_i\) are integration constants. To solve the constants \(a_i, b_i, c_i,\) and \(d_i\) above, we can use the initial condition \(v_i^0\), the initial and terminal conditions \(p_i^0, p_i^f = L\), as well as the boundary condition of the co-state \(\pi_i^*(t_i^*) = -u_i(t_i^*) = 0\) of the problem (14). Then, we can put (22)-(24) and the conditions mentioned above in the following matrix (in the form of \(AX_i = q_i\)):

\[
\begin{bmatrix} \frac{1}{2} (t_i^0)^3 \\ \frac{1}{2} (t_i^0)^2 \\ \frac{1}{2} (t_i^0) \\ t_i^0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a_i \\ b_i \\ c_i \\ d_i \end{bmatrix} = \begin{bmatrix} p_i(t_0) \\ v_i(t_0) \\ p_i(t_0) \\ \pi_i(t_0) \end{bmatrix}. \tag{25}\]

where \(t_i^0\) is obtained by solving the problem (13). Since the above matrix is solved (by rearranging the above matrix to \(X_i = A_i^{-1} q_i\)) for every \(t > t_0\), we can re-evaluate the four constants as \(a_i(t, p_i(t), v_i(t)), b_i(t, p_i(t), v_i(t)), c_i(t, p_i(t), v_i(t)), d_i(t, p_i(t), v_i(t))\).
and by substituting (26) into the system (1), we can obtain the new optimal speed and position, namely

$$v_i^*(t) = u_{\text{max}}t + f_i,$$

$$p_i^*(t) = \frac{1}{2}u_{\text{max}}^2t^2 + f_it + e_i \quad \forall t \geq t_i,$$

(27) and (28)

where $f_i$ and $e_i$ are the integration constants and can be computed from the obtained speed and position at $t = t_i$.

**CASE 3: Both control and state constraints are active**

Since the optimal input in (26) keeps to be $u_{\text{max}}$, then the optimal speed in (27) will become $v_{\text{max}}$ at one point. Suppose that the optimal speed becomes $v_{\text{max}}$ at $t = t_2 > t_1$ (while $u_i(t) = u_{\text{max}}$), then (27) becomes $v_i^*(t) = v_{\text{max}}$ and from (1) we have $v_i^* = u_i^* = 0$. For the optimal position, it becomes

$$p_i^*(t) = v_{\text{max}}t + r_i \quad \forall t \geq t_2,$$

(29)

where $r_i$ is the integration constant, and since the Hamiltonian is discontinuous at $t = t_2$, thus $r_i$ can be computed from $t_2$ (time just before $t_2$).

Since (14) has some terminal constraints, the state constraint may become inactive again [14]. Suppose that the optimal speed in (23) becomes active, i.e., $\dot{v}(t) = \dot{v}_{\text{opt}}$ at $t = t_3$, i.e., $\mathcal{H}(v_i(t)) = \mathcal{H}(v_{\text{opt}}(t))$, then (23) becomes $v_i^*(t) = v_{\text{opt}}(t)$ and from (1) we have $v_i^* = u_i^* = 0$. For the optimal position, it becomes

$$p_i^*(t) = v_{\text{opt}}t + r_i \quad \forall t \geq t_3,$$

where $v_{\text{opt}}$ is the FEP at $t_i = t_3$.

**CASE 4: Only state constraint is active**

There is also the case where only the state (speed) constraint is active. Suppose that the optimal speed in (23) becomes active, i.e., $\dot{v}(t) = \dot{v}_{\text{opt}}$ at $t = t_4$ (while $u_{\text{max}} \leq u_i(t) \leq u_{\text{max}}$), then from (1) we have $v_i^* = u_i^* = 0$. For the optimal position, it becomes

$$p_i^*(t) = v_{\text{opt}}t + r_i \quad \forall t \geq t_4,$$

(34)

where $r_i$ is the integration constant, and since the Hamiltonian is discontinuous at $t = t_4$, thus $r_i$ can be computed from $t_4$ (time just before $t_4$). Similar to case 3, since (14) has some terminal constraints, the state constraint may become inactive again. For this case, it follows the same discussion as in case 3 where the optimal control input, speed, and position are given by (31)-(33).

As can be noticed from case 2 to case 4, only maximum control or state constraints are considered. For minimum control or state constraints, the same procedure from case 2 to case 4 can be repeated, which makes up seven cases that need to be considered in total.

To further extend the analytical solution into the right and left turning, we can include the passenger comfort as a function of jerk into the objective function in (14) so that the vehicle can perform the turning smoothly and safely [28].

3. **Simulation Results**

The proposed solution presented previously is simulated in Matlab. In the simulation, we considered the length of CZ, $L = 300$ m and the size of MZ, $S = 20$ m. The safety headway time to avoid the rear-end collision is set to be $h_R = 1$ s while the safety headway time to avoid the lateral collision is set to be $h_L = 2.2$ s. The maximum and minimum speed limits are $13$ m/s and $0$ m/s, respectively. On the other hand, the maximum acceleration is $2$ m/s$^2$, and the minimum acceleration is $-2$ m/s$^2$. By following the provided guide in [8], we chose $M_{\text{big}} = 3000$. The merging time for all CAVs needs to be updated frequently, and thus, we set the execution time, $t_e = 4$ s, which means that the merging time has to be updated at every 4 s. When the distance of the CAV is below $30$ m to the entry of the MZ, we froze the merging time. Besides that, to avoid the infeasibility in the solution of (15), we set $v_{B,\text{max}} = 12.8$ m/s and $v_{B,\text{min}} = 0.2$ m/s. Finally, we chose the weights for the MILP formulation (13) to be $\omega_1 = \omega_2 = 0.5$.

![Fig. 5 Control input of the first 20 CAVs until the entry of MZ.](image-url)

### 3.1 Preliminary

In this subsection, we evaluated the proposed solution for the first 20 CAVs to cross the intersection. The arrival speed for all CAVs is set to be the same, which is $11.1$ m/s.

Figures 5, 6, and 7 show the control input, speed, and position of the 20 CAVs to cross the intersection. From these figures, in particular Fig. 6, we can see the FEP is activated in some CAVs, for example, CAV 1, CAV 2, and CAV 3 (the reader can see the speed went from $v_{\text{max}}$ to $v_{B,\text{max}}$). Even though the FEP is activated, we observed that the CAVs still can reach the assigned merging time without much deviation (it only deviates around 0.01 seconds to 0.03 seconds). On the other hand, we can see from Fig. 7 how the merging time assigned by the coordinator arranged the CAVs to decrease the travel time and, at the same time, avoided the collisions at the MZ. For example, CAV 16 and 18 were instructed to enter the MZ first so that CAV 14, 15, 17, 19, and 20 can enter the MZ in a group where this situation...
3.2 Performance Evaluation

In this subsection, we compared the performance of the proposed solution with the conventional method [14] in low and medium traffic environments in terms of average travel time per vehicle as well as cumulative fuel consumption. We assumed that the vehicle’s arrival rate for each lane is given by a Poisson distribution where the mean for the low and medium traffic environments are 425 vehicles per hour and 850 vehicles per hour respectively. Note that the travel time is obtained from the difference between the merging time and the arrival time of each vehicle at the entry of the CZ. To quantify the impact of the proposed solution on fuel consumption, we utilized the polynomial metamodel from [32]. Instead of considering the same time for the last vehicle, i.e., the 200th vehicle, we examined the random speed with a normal distribution ranging from 7 m/s to 12 m/s. Finally, to compare the proposed and conventional methods fairly, we studied 200 vehicles in total for both low and medium traffic environments.

In future research, we will consider how we can guarantee the absence of rear-end collision from the time the CAV enters the CZ until it leaves the MZ. Besides that, we will try to include the occasional stop and go driving (at the entry of the MZ) strategy as a higher traffic environment does not always allow the CAV to cross the MZ without stopping. Since the FEP helped to decrease the average travel time for each CAV.

4. Conclusion

In this paper, we have addressed the optimal motion planning problem for CAVs to cross a conflict area at the intersection with state and control constraints. First, we formulated the scheduled merging time for all CAVs as a MILP problem where it is solved at every execution time. Second, we formulated the optimal motion planning problem where the objectives are to achieve the scheduled merging time accurately and minimize energy consumption. We have shown that the analytical solution we utilized to solve the optimal motion planning problem did not always have feasible solutions because it was affected by the frequently updated merging time. To solve this issue, we proposed FEP, and the results have shown that the CAVs can still achieve the merging time with small deviation if the FEP is activated. We also have made a comparison between our proposed and the conventional frameworks. From the comparison, it was shown that our proposed framework could provide a significant improvement in terms of travel time.

In future research, we will consider how we can guarantee the performance evaluation between the proposed and conventional methods. Note that because the interval time for vehicle arrival is short for the medium traffic environment, the time for the last vehicle, i.e., the 200th vehicle, to leave the MZ is faster. Therefore, the test duration in Table 3 is shorter compared to the test duration in Table 2. On the other hand, we can see a very small improvement in the average travel time in a low traffic environment but have shown significant improvement in a medium traffic environment. This happened because the coordinator cannot encourage the vehicles to travel in a group because the interval of the arrival time is higher in the low traffic environment. However, this is not a major concern because the average travel time is not so much different if we assume all vehicles travel in maximum speed from the entry of the CZ until the entry of the MZ, i.e., $L/v_{max} = 23.08$ s. As the conventional method also utilized the optimal motion planning to reach the assigned merging time, we cannot see much improvement for the cumulative fuel consumption, particularly in the medium traffic environment. This is because, in our approach, the fuel consumption only can be reduced if we minimize the acceleration for each vehicle. This means even though the average travel time is longer for the conventional method in Table 3, many of the vehicles have to decelerate and then move in constant speed. Clearly, this situation did not consume much fuel consumption unless there is a stop and go driving situation happened.

Table 2 Performance evaluation in low traffic environment.

| Performance evaluation | Conventional | Proposed | Improvement |
|-------------------------|--------------|----------|-------------|
| Test duration (s)       | 470          | 470      | -           |
| Average travel time per vehicle (s) | 26.40 | 26.18 | 0.83% |
| Cumulative fuel consumption (m³) | 0.00308 | 0.00318 | −3.25% |

Table 3 Performance evaluation in medium traffic environment.

| Performance evaluation | Conventional | Proposed | Improvement |
|-------------------------|--------------|----------|-------------|
| Test duration (s)       | 320          | 320      | -           |
| Average travel time per vehicle (s) | 44.93 | 27.47 | 38.86% |
| Cumulative fuel consumption (m³) | 0.00310 | 0.00299 | 3.55% |
is a naive solution, we will also try to explore a better approach such as an event-driven controller to guarantee the optimality of the analytical solution with the frequently updated merging time. Finally, we will extend the intersection model with multiple lanes and include the right and left turning of the vehicles.

Acknowledgments
This work was supported by JSPS KAKENHI Grant Number JP17H03283.

References
[1] R. Margiotta and D. Snyder: An agency guide on how to establish localized congestion mitigation programs, Tech. Rep., U.S. Department of Transportation, 2011.
[2] D. Schrank, B. Eisele, T. Lomax, and J. Bak: 2015 urban mobility scorecard, Tech. Rep., Texas A&M Trans. Inst. College Station, TX, USA, 2015.
[3] C. Eun-Ha: Crash factors in intersection-related crashes: An on-scene perspective, Tech. Rep., U.S. Department of Transportation, 2010.
[4] M. Simon, T. Hermitte, and Y. Page: Intersection road accident causation: A European view, Proc. 21st International Technical Conference on the Enhanced Safety of Vehicles, pp. 1–10, 2009.
[5] K. Dresner and P. Stone: A multiagent approach to autonomous intersection management, Journal of Artificial Intelligence Research, Vol. 31, pp. 591–656, 2008.
[6] Y.J. Zhang, A.A. Malikopoulos, and C.G. Cassandras: Optimal control and coordination of connected and automated vehicles at urban traffic intersections, Proc. American Control Conference (ACC), pp. 6227–6232, 2016.
[7] E.R. Muller, R.C. Carlson, and W.K. Junior: Intersection control for automated vehicles with MILP, IFAC Papers Online, Vol. 49, No. 3, pp. 37–42, 2016.
[8] S.A. Fayazi and A. Vahidi: Mixed-integer linear programming for optimal scheduling of autonomous vehicle intersection crossing, IEEE Transactions on Intelligent Vehicles, Vol. 3, No. 3, pp. 287–299, 2018.
[9] R. Hult, M. Zanon, S. Gros, and P. Falcone: Optimal coordination of automated vehicles at intersections: Theory and experiments, IEEE Transactions on Control Systems Technology, Vol. 27, No. 6, pp. 2510–2525, 2018.
[10] H. Ahn and D. Del Vecchio: Safety verification and control for collision avoidance at road intersections, IEEE Transactions on Automatic Control, Vol. 63, No. 3, pp. 630–642, 2018.
[11] J. Wang, X. Zhao, and G. Yin: Multi-objective optimal cooperative driving for connected and automated vehicles at non-signalised intersection, IET Intelligent Transport Systems, Vol. 13, No. 1, pp. 79–89, 2018.
[12] Y. Zhang and C.G. Cassandras: A decentralized optimal control framework for connected automated vehicles at urban intersections with dynamic resequencing, Proc. IEEE Conference on Decision and Control (CDC), pp. 217–222, 2018.
[13] J. Rios-Torres and A.A. Malikopoulos: Automated and cooperative vehicle merging at highway on-ramps, IEEE Transactions on Intelligent Transportation Systems, Vol. 18, No. 4, pp. 780–789, 2017.
[14] A.A. Malikopoulos, C.G. Cassandras, and Y.J. Zhang: A decentralized energy-optimal control framework for connected automated vehicles at signal-free intersections, Automatica, Vol. 93, pp. 244–256, 2018.
[15] X. Zhao, J. Wang, Y. Chen, and G. Yin: Multi-objective cooperative scheduling of CAVs at non-signalized intersection, Proc. 21st International Conference on Intelligent Transportation Systems (ITSC), pp. 3314–3319, 2018.
[16] J. Han, J. Rios-Torres, A. Vahidi, and A. Sciarretta: Impact of model simplification on optimal control of combustion engine and electric vehicles considering control input constraints, Proc. Vehicle Power and Propulsion Conference (VPPC), pp. 1–6, 2018.
[17] I.A. Ntousakis, I.K. Nikolos, and M. Papageorgiou: Optimal vehicle trajectory planning in the context of cooperative merging on highways, Transportation Research Part C: emerging technologies, Vol. 71, pp. 464–488, 2016.
[18] R. Hult, M. Zanon, S. Gros, and P. Falcone: Energy optimal coordination of autonomous vehicles at intersections, Proc. European Control Conference (ECC), pp. 602–607, 2018.
[19] E.R. Muller, B. Wahlberg, and R.C. Carlson: Optimal motion planning for automated vehicles with scheduled arrivals at intersections, Proc. European Control Conference (ECC), pp. 1672–1678, 2018.
[20] J. Ding, L. Li, H. Peng, and Y. Zhang: A rule-based cooperative merging strategy for connected and automated vehicles, IEEE Transactions on Intelligent Transportation Systems, in print.
[21] W. Zhao, R. Liu, and D. Ngody: A bilevel programming model for autonomous intersection control and trajectory planning, Transportmetrica A: Transport Science, 1–25, in print.
[22] A.I.M. Medina, F. Creemers, E. Lefeber, and N. van de Wouw: Optimal access management for cooperative intersection control, IEEE Transactions on Intelligent Transportation Systems, in print.
[23] M.H. Mohamad Nor and T. Namikawa: Optimal coordination and control of connected and automated vehicles at intersections via mixed integer linear programming, SICE Journal of Control, Measurement, and System Integration, Vol. 12, No. 6, pp. 215–222, 2019.
[24] M.H. Mohamad Nor and T. Namikawa: Optimal control of connected and automated vehicles at intersections with state and control constraints, Proc. IEEE/ASME Advanced Intelligent Mechatronics (AIM), pp. 1397–1402, 2019.
[25] D. Bertsimas and J.N. Tsitsiklis: Introduction to Linear Optimization, Athena Scientific Belmont, MA, 1997.
[26] IBM Knowledge Center website: Cplex for MATLAB, [online] https://www.ibm.com/support/knowledgecenter/, last accessed 4 May 2019.
[27] H. Xue, S. Feng, Y. Zhang, and L. Li: A grouping based cooperative driving strategy for CAVs merging problems, IEEE Transactions on Vehicular Technology, Vol. 68, No. 6, 6125–6136, 2019.
[28] Y. Zhang, A.A. Malikopoulos, and C.G. Cassandras: Decentralized optimal control for connected automated vehicles at intersections including left and right turns, Proc. IEEE 56th Annual Conference on Decision and Control (CDC), pp. 4428–4433, 2017.
[29] D.E. Kirk: Optimal Control Theory: An Introduction, Courier Corporation, 2012.
[30] A.E. Bryson: Applied Optimal Control: Optimization, Estimation and Control, Routledge, 2018.
[31] J. Zhang, A.A. Malikopoulos, and C.G. Cassandras: Optimal control of connected automated vehicles at urban traffic intersections: A feasibility enforcement analysis, Proc. American Control Conference (ACC), pp. 3548–3553, 2017.
[32] M.A.S. Kamal, M. Mukai, J. Murata, and T. Kawabe: Model predictive control of vehicles on urban roads for improved fuel economy, IEEE Transactions on Control Systems Technology, Vol. 21, pp. 831–841, 2012.
Mohamad Hafizulazwan Mohamad Nor

He received the B.Eng. and M.Phil. degrees in electronic systems engineering from the Universiti Teknologi Malaysia (UTM), Kuala Lumpur, Malaysia, in 2015 and 2017, respectively. He is currently a full-time Ph.D. student at Keio University, Kanagawa, Japan. His current research interests include connected and automated vehicles and multi-agent systems.

Toru Namerikawa (Member)

He received the B.E., M.E., and Ph.D. in electrical and computer engineering from Kanazawa University, Japan, in 1991, 1993, and 1997, respectively. From 1994 until 2002, he was with Kanazawa University as an Assistant Professor. From 2002 until 2005, he was with the Nagaoka University of Technology as an Associate Professor, Niigata, Japan. From 2006 until 2009, he was with Kanazawa University again. In April 2009, he joined Keio University, Yokohama, Japan, where he is currently a Professor at Department of System Design Engineering, Keio University. He held visiting positions at Swiss Federal Institute of Technology in Zurich in 1998, University of California, Santa Barbara in 2001, University of Stuttgart in 2008 and Lund University in 2010. He received 2014 Pioneer Technology Award from SICE Control Division and 2017 Outstanding Paper Award from SICE. His main research interests are robust control, distributed and cooperative control, and their application to power network and transportation network systems. He is a member of IEEE CSS and ISCIE.