Correlations in the properties of static and rapidly rotating compact stars

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Abstract

Correlations in the properties of the static compact stars (CSs) and the ones rotating with the highest observed frequency of 1122Hz are studied using a large set of equations of state (EOSs). These EOSs span various approaches and their chemical composition vary from the nucleons to hyperons and quarks in \( \beta \)-equilibrium. It is found that the properties of static CS, like, the maximum gravitational mass \( M_{\text{stat max}} \) and radius \( R_{\text{stat 1.4}} \) corresponding to the canonical mass and supramassive or non-supramassive nature of the CS rotating at 1122 Hz are strongly correlated. In particular, only those EOSs yield the CS rotating at 1122Hz to be non-supramassive for which \( \left( \frac{M_{\text{stat max}}}{M_\odot} \right)^{1/2} \left( \frac{10 \text{ km}}{R_{\text{stat 1.4}}} \right)^{3/2} \) is greater than unity. Suitable parametric form which can be used to split the \( M_{\text{stat max}} - R_{\text{stat 1.4}} \) plane into the regions of different supramassive nature of the CS rotating at 1122Hz is presented. Currently measured maximum gravitational mass 1.76\( M_\odot \) of PSR J0437-4715 suggests that the CS rotating at 1122Hz can be non-supramassive provided \( R_{\text{stat 1.4}} \leq 12.4 \text{ km.} \)

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I. INTRODUCTION

The accurate knowledge of the properties of static and rotating compact stars (CSs) are of utmost importance to probe the behaviour of the equation of state (EOS) of superdense matter. Even the accurate information on the maximum gravitational mass $M_{\text{stat}}^\text{max}$ for the static CS and its radius $R_{1.4}^\text{stat}$ with the canonical mass (1.4$M_\odot$), not yet well known, would narrow down the choices for the plausible EOSs to just a few. The newly measured CS mass 1.76$\pm$0.20$M_\odot$ of PSR J0437-4715 [1] is obtained by the precise determination of the orbital inclination angle, the highest measured mass for any known pulsar to date. Recent observations of the thermal emission from the quiescent LMXB X7 in the globular cluster 47 Tuc yield the value of $R_{1.4}$ to be 14.5$^{+1.8}_{-1.6}$ km [2]. The binary pulsars PSR J0737-3039A,B with masses of the individual star being 1.338$M_\odot$ and 1.249$M_\odot$ are plausible candidates for the measurement the moment of inertia due to the spin-orbit coupling effects [3]. It is expected that a reasonably accurate value of $R_{1.4}$ can be deduced from the moment of inertia measurement of PSR J0737-3039A [4].

Rotating CSs discovered until recent past have periods of rotation close to one millisecond. The first millisecond pulsar PSR B1937+214 rotating at the frequency $\nu = 641$Hz was discovered in 1982 [5]. In year 2006, a more rapid pulsar PSR J1748-2446ad rotating at $\nu = 716$ Hz was detected [6]. Such frequencies are too low to affect significantly the structure of CSs with $M > 1M_\odot$ [7], since, these CSs have the Keplerian (mass-shedding) frequencies larger than 1000Hz. Very recent discovery of X-ray transient XTE J1739-285 by Kaaret et al [8] suggests that it contains a CS rotating at 1122Hz. Following this discovery, the structure of the CS rotating with 1122Hz are studied using several EOSs [9]. It is found that, for some of the EOSs, this CS is supramassive, i.e.,

$$\delta M_B = M_{B,\text{max}}^\text{stat} - M_{B,\text{min}},$$

(1)

is less than 0. In the above equation, $M_{B,\text{max}}^\text{stat}$ is the maximum baryonic mass of the static CS and $M_{B,\text{min}}$ is the minimum mass for the CS rotating with 1122Hz for a given EOS.

In the present work we search for the possible correlations in some key properties of static CSs and the ones rotating with the highest observed frequency of 1122Hz. The properties of these CSs are computed for a large set of EOSs, which are constructed using variety of approaches with the chemical compositions ranging from nucleons to hyperons and quarks in $\beta$-equilibrium. Our results suggest that the values of $M_{\text{max}}^\text{stat}$, $R_{1.4}^\text{stat}$, and $\delta M_B$ are strongly correlated. Though, $M_{\text{max}}^\text{stat}$ and $R_{1.4}^\text{stat}$ alone does not show any systematic correlations [10]. We also unmask the underlying feature
of the EOS responsible for the supramassive or non-supramassive nature of the CS rotating with 1122Hz.

II. EQUATIONS OF STATE

In this work we consider 24 different EOSs with $M_{\text{max}}^{\text{stat}} \geq 1.6 M_\odot$ which exceeds the recent mass measurements suggesting only 5% probability that the mass of pulsar PSR J1516+02B is below 1.59 $M_\odot$ [11]. These EOSs are constructed using various approaches which can broadly be grouped into (i) models based on variational approach, (ii) relativistic or non-relativistic mean-field models and (iii) Dirac-Brueckner-Hartree-Fock model. The first group contains EOSs involving neutrons, protons, electrons and muons. The EOSs considered in this group are: BJ-C [12], FPS [13], BBB2 [14], AU, WS, and UU [15], and APR [16]. For the second group we consider the EOSs: O [17], GN3 [18], GM1 [19], TM1 [20], G2 [21], BalbN1H1 [22], GMU110 [23], DH [24], SSK and GSK1 [25], UY, U0 and L0 [26], GM1-H, UQM52, and CFL52. The EOS GM1-H is composed of nucleons and hyperons in $\beta$ equilibrium. The nucleon-meson interaction parameters are taken from the GM1 parameter set whereas hyperon meson couplings are obtained from SU(6) model. The EOS UQM52 involves noninteracting unpaired quark matter, composed of massive $u$, $d$, and $s$ quarks, is based upon the MIT Bag Model of quarks. This EOS has been calculated by using model parameters, bag constant, $B = 52$ MeV/fm$^3$, masses of three quarks, $m_u = m_d = 5.00$ MeV/$c^2$, $m_s = 150$ MeV/$c^2$, and QCD coupling constant $\alpha_c = 0.1$. The Color-Flavor-Locked quark matter equation of state (CFL52) is based upon the free energy as described by [27] and [28], by using pairing gap parameter $\Delta = 100$ MeV. The other model parameters such as bag constant, quark masses, and QCD constant are same as used for UQM52 EOS. In the third group we consider only one EOS: DBHF by Krastev et al. [29].

III. RESULTS AND DISCUSSIONS

The properties of spherically symmetric static and axially symmetric rotating CSs are obtained by solving the Einstein’s equations in 1D and 2D, respectively. The numerical computations are performed by using RNS code written by Stergioulas and Friedman [30]. In Fig. 1 we present the values of $M_{\text{max}}^{\text{stat}}$, $R_{1.4}^{\text{stat}}$, and $\delta M_B$ obtained for several EOSs. We notice that the value of $\delta M_B$ for a given EOS is only weakly correlated with those of $M_{\text{max}}^{\text{stat}}$ as compared to $R_{1.4}^{\text{stat}}$. It seems, larger is
the value of \(R_{1,4}^{\text{stat}}\) smaller will be the \(\delta M_B\). On the other hand, \(\delta M_B\) large means large \(M_{\text{max}}^{\text{stat}}\) but \(R_{1,4}^{\text{stat}}\) small. For instance, \(\delta M_B < 0\) for all those EOSs for which \(R_{1,4}^{\text{stat}} \gtrsim 14\) km. Whereas, \(\delta M_B \approx 0.75 M_\odot\) for the cases with \(M_{\text{max}}^{\text{stat}} > 2 M_\odot\) and \(R_{1,4}^{\text{stat}} \sim 10 - 11\) km. Though \(\delta M_B\) is not correlated well with \(M_{\text{max}}^{\text{stat}}\) or \(R_{1,4}^{\text{stat}}\) alone, but, it might be well correlated with some appropriate combination of \(M_{\text{max}}^{\text{stat}}\) and \(R_{1,4}^{\text{stat}}\). In Table II we summarize the properties of the CS, rotating with 1122Hz, calculated at the maximum circumferential equatorial radius \(R_{\text{eq}}^{\text{max}}\) and the minimum circumferential equatorial radius \(R_{\text{eq}}^{\text{min}}\) for a few selected EOSs. The values of the radius \(R_{\text{eq}}^{\text{max}}\) are determined by the mass shedding instability and that of \(R_{\text{eq}}^{\text{min}}\) are determined by the secular axi-symmetric instability according to turning point theorem [31]. It can be verified from the table that the variations in the gravitational mass,

\[
\delta M = \left| M(R_{\text{eq}}^{\text{max}}) - M(R_{\text{eq}}^{\text{min}}) \right|, 
\]

of the CS rotating at 1122Hz are correlated with \(\delta M_B\) up to some extent. The difference \(\delta M \lesssim 0.1 M_\odot\) when \(\delta M_B\) is negative. For \(\delta M_B > 0\), \(\delta M\) increases with \(\delta M_B\). Therefore, \(\delta M_B\) not only determines whether the CS rotating at 1122Hz is supramassive or not, but, it also gives an estimate about the value of \(\delta M\) for a given EOS.

In Fig. 2 we consider the variations of \(\delta M_B\) with \(\left(\frac{M_{\text{max}}^{\text{stat}}}{M_\odot}\right)^{1/2} \left(\frac{10 \text{ km}}{R_{1,4}^{\text{stat}}}\right)^{3/2}\). This combination of \(M_{\text{max}}^{\text{stat}}\) and \(R_{1,4}^{\text{stat}}\) is analogous to the one derived within the Newtonian approximation to determine the value of the Keplerian frequency. The values of \(\delta M_B\) and \(\left(\frac{M_{\text{max}}^{\text{stat}}}{M_\odot}\right)^{1/2} \left(\frac{10 \text{ km}}{R_{1,4}^{\text{stat}}}\right)^{3/2}\) are well correlated. It is interesting to note that \(\delta M_B > 0\) only if \(\left(\frac{M_{\text{max}}^{\text{stat}}}{M_\odot}\right)^{1/2} \left(\frac{10 \text{ km}}{R_{1,4}^{\text{stat}}}\right)^{3/2}\) is greater than unity. These correlations simply suggest that the high density behaviour of a EOS with respect to its behaviour at low density plays a predominant role in determining whether the CS rotating at 1122Hz is supramassive or not. Since, the \(M_{\text{max}}^{\text{stat}}\) probes densest segment of the EOS whereas, \(R_{1,4}^{\text{stat}}\) probes relatively lower density region of EOS.

We parameterize \(\delta M_B\) in terms of \(M_{\text{max}}^{\text{stat}}\) and \(R_{1,4}^{\text{stat}}\) as,

\[
\frac{\delta M_B}{M_\odot} = a_0 + a_1 \left(\frac{M_{\text{max}}^{\text{stat}}}{M_\odot}\right)^{\alpha} \left(\frac{10 \text{ km}}{R_{1,4}^{\text{stat}}}\right)^{\beta}.
\]

The best fit values of the parameters appearing in Eq. 3 are calculated using the results displayed in Fig. 1. The values of parameters are \(a_0 = -2.75\), \(a_1 = 2.5\), \(\alpha = 0.75\) and \(\beta = 1.56\). In Fig. 3 we plot the results for \(R_{1,4}^{\text{stat}}\) versus \(M_{\text{max}}^{\text{stat}}\) obtained by solving Eq. 3 for fixed values of \(\delta M_B\). These plots can provide us immediately some idea of \(\delta M_B\) once the properties of the static CS like \(M_{\text{max}}^{\text{stat}}\) and \(R_{1,4}^{\text{stat}}\) are known. We also superpose the results shown in Fig. 1 by dividing them in to three
classes depending on the values of the $\delta M_B$. The symbols, triangles, circles and squares represent the values of $R_{1.4}^{\text{stat}}$ and $M_{\text{max}}^{\text{stat}}$ with $\delta M_B$ lie in the range of $-0.5 - 0.0, 0.0 - 0.5$ and $0.5 - 1.5 \, M_\odot$ respectively. It is evident from Fig. 3 that Eqs. 3 can be used to divide the $M_{\text{max}}^{\text{stat}} - R_{1.4}^{\text{stat}}$ plane into the regions with different $\delta M_B$. It is to be noted from Fig. 3 that the current measurement of maximum gravitational mass $1.76 M_\odot$ [1] would set the upper limit on $R_{1.4}^{\text{stat}}$ to be $12.4$ km which corresponds to $\delta M_B = 0$. Interestingly, this upper limit on $R_{1.4}^{\text{stat}}$ is closer to the lower limit of $12.9$ km obtained by analyzing the high quality X-ray spectra from CS in qLMXB X7 [2]. We plot in Fig. 4 the curves for moment of inertia $I_{1.4}^{\text{stat}}$ versus $M_{\text{max}}^{\text{stat}}$ with fixed values of $\delta M_B$. These curves are generated by fitting the values of $\delta M_B$ to the following expression,

$$\frac{\delta M_B}{M_\odot} = a_0' + a_1' \left( \frac{M_{\text{max}}^{\text{stat}}}{M_\odot} \right)^{\alpha'} \left( \frac{I_{1.4}^{\text{stat}}}{I_0} \right)^{\beta'},$$

(4)

where, $I_0 = 10^{45}$ g cm$^{-2}$ and the values of the best fit parameters are $a_0' = -3.25$, $a_1' = 3.25$, $\alpha' = 0.63$ and $\beta' = 0.85$. Similar to the case of $R_{1.4}^{\text{stat}}$, we obtained the upper limit of $I_{1.4}^{\text{stat}} = 1.53 \times 10^{45}$ g cm$^{-2}$ from maximum mass of CS measured to date [1].

IV. SUMMARY

The key properties such as $M_{\text{max}}^{\text{stat}}$ and $R_{1.4}^{\text{stat}}$ of static CS and $\delta M_B$ (Eq. 1) for the CS rotating with the highest observed frequency of $1122$Hz are computed using $24$ diverse EOSs. These EOSs are chosen in a manner that they correspond to a wide variety of approaches and their chemical composition vary from the nucleons to hyperons and quarks in $\beta$-equilibrium. The values of $\delta M_B$ are found to be almost linearly correlated with $\left( \frac{M_{\text{max}}^{\text{stat}}}{M_\odot} \right)^{1/2} \left( \frac{10 \text{ km}}{R_{1.4}^{\text{stat}}} \right)^{3/2}$; a combination of $M_{\text{max}}^{\text{stat}}$ and $R_{1.4}^{\text{stat}}$ analogous to the one popularly used to determine Keplerian frequency. For a given EOS, the CS rotating at $1122$Hz is non-supramassive (i.e., $\delta M_B > 0$) only if $\left( \frac{M_{\text{max}}^{\text{stat}}}{M_\odot} \right)^{1/2} \left( \frac{10 \text{ km}}{R_{1.4}^{\text{stat}}} \right)^{3/2}$ is greater than unity. It is also noticed that the variations in the gravitational mass for the CS rotating with $1122$Hz are up to some extent correlated with the values of $\delta M_B$ (see Table I). In view of these results, it appears that the observation of the rapidly rotating CSs constrain relative behaviour of EOS at high density with respect to it’s behaviour at low or moderate densities. Since, the $M_{\text{max}}^{\text{stat}}$ probes densest segment of the EOS, whereas, $R_{1.4}^{\text{stat}}$ probes relatively lower density region of EOS. The suitable parametric forms for the $\delta M_B$ in terms of $M_{\text{max}}^{\text{stat}}$ and $R_{1.4}^{\text{stat}}$ or $I_{1.4}^{\text{stat}}$ (Eqs. 3 and 4) are also presented. Using these parametric forms, one can divide the $M_{\text{max}}^{\text{stat}} - R_{1.4}^{\text{stat}}$ and $M_{\text{max}}^{\text{stat}} - I_{1.4}^{\text{stat}}$ planes into regions of different $\delta M_B$. Thus, for a given EOS, only the knowledge of the key properties of static CSs can
well estimate a priori the properties of the resulting CS rotating with 1122Hz. Currently measured maximum gravitational mass $1.76M_\odot$ of PSR J0437-4715 suggests that the CS rotating at 1122Hz can be non-supramassive provided $R^{\text{stat}}_{1.4} \lesssim 12.4$ km or equivalently $I^{\text{stat}}_{1.4} \lesssim 1.53 \times 10^{45}$ g cm$^{-2}$. It will be worth while to repeat the present investigations for the CS rotating at higher frequencies.

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TABLE I: The properties of the compact stars, rotating with frequency $\nu = 1122$ Hz, calculated at the maximum and the minimum circumferential equatorial radius $R_{eq}^{max}$ and $R_{eq}^{min}$. The quantities $\delta M$ and $\delta M_B$ for a given EOS are determined by using Eqs. (1 and 2) respectively. The central mass densities $\epsilon_c$ at the $R_{eq}^{max}$ and $R_{eq}^{min}$ are also presented.

| EOS   | $\epsilon_c$ ($10^{15}$ g/cm$^3$) | $M(R_{eq}^{max})$ ($M_\odot$) | $R_{eq}^{max}$ (km) | $\epsilon_c$ ($10^{15}$ g/cm$^3$) | $M(R_{eq}^{min})$ ($M_\odot$) | $R_{eq}^{min}$ (km) | $\delta M$ ($M_\odot$) | $\delta M_B$ ($M_\odot$) |
|-------|---------------------------------|-------------------------------|-------------------|---------------------------------|-------------------------------|-------------------|---------------------|---------------------|
| GMU110| 1.30322                         | 2.071                         | 17.53             | 1.48038                         | 2.053                         | 16.33             | 0.018               | -0.393              |
| GM1-H | 1.25875                         | 2.235                         | 18.03             | 1.56496                         | 2.158                         | 15.23             | 0.074               | -0.328              |
| BalbN1H1 | 1.42497                         | 1.825                         | 16.88             | 3.56904                         | 1.704                         | 9.97              | 0.123               | -0.079              |
| G2    | 1.05609                         | 2.051                         | 17.48             | 1.81037                         | 2.116                         | 13.02             | 0.065               | -0.064              |
| GM1   | 0.86585                         | 2.456                         | 18.55             | 1.53015                         | 2.576                         | 13.77             | 0.120               | 0.007               |
| UQM52 | 1.14801                         | 1.585                         | 16.14             | 2.33042                         | 1.864                         | 11.48             | 0.279               | 0.246               |
| BJ-C  | 1.05732                         | 1.695                         | 16.46             | 2.55674                         | 1.954                         | 10.88             | 0.259               | 0.278               |
| GSK1  | 0.98210                         | 1.788                         | 16.70             | 2.37515                         | 2.089                         | 11.15             | 0.301               | 0.335               |
| O     | 0.78443                         | 2.138                         | 17.81             | 1.69183                         | 2.554                         | 12.72             | 0.416               | 0.461               |
| SSK   | 0.94170                         | 1.684                         | 16.39             | 2.55229                         | 2.127                         | 10.72             | 0.443               | 0.537               |
| FPS   | 0.98885                         | 1.395                         | 15.51             | 3.04073                         | 1.881                         | 9.89              | 0.486               | 0.583               |
| BBB2  | 0.9615                          | 1.476                         | 15.79             | 2.87618                         | 2.002                         | 10.08             | 0.526               | 0.654               |
| DBHF  | 0.7813                          | 1.732                         | 16.54             | 2.19589                         | 2.412                         | 11.24             | 0.680               | 0.861               |
| CFL52 | 0.948                           | 0.846                         | 12.99             | 2.39711                         | 1.990                         | 10.39             | 1.144               | 1.357               |
| AU    | 0.9237                          | 1.143                         | 14.46             | 2.87473                         | 2.215                         | 9.76              | 1.072               | 1.402               |
FIG. 1: (Color online) Values of the maximum gravitational mass $M_{\text{max}}^{\text{stat}}$ for static CSs, radius $R_{1.4}^{\text{stat}}$ for static CSs with mass $1.4M_\odot$ and the difference $\delta M_B$ (Eq. 1) obtained for several EOSs.
FIG. 2: (Color online) Correlations between $\delta M_B$ and \left(\frac{M_{\text{max}}}{M_\odot}\right)^{1/2}(10\text{ km}/R_{1.4}^{\text{stat}})^{3/2}$. This combination of $M_{\text{max}}^{\text{stat}}$ and $R_{1.4}^{\text{stat}}$ is analogous to the one commonly used to determine the Keplerian frequency.
FIG. 3: (Color online) Plots for $R_{1.4}^{\text{stat}}$ versus $M_{\text{max}}^{\text{stat}}$ generated using Eq. [3] for $\delta M_B = -0.5, 0.0$ and $0.5 M_\odot$ as indicated. Different symbols represent the values of $M_{\text{max}}^{\text{stat}}$ and $R_{1.4}^{\text{stat}}$ with $\delta M_B$ lying in the range of $-0.5 - 0.0$ (triangles), $0.0 - 0.5$ (circles) and $0.5 - 1.5$ (squares) $M_\odot$ as also depicted in Fig. [1]
FIG. 4: (Color online) Same as fig. [3] but, values of moment of inertia $I_{1.4}^{\text{stat}}$ are used instead of $R_{1.4}^{\text{stat}}$. 