Novel Fractional-Order Model Predictive Control: State-Space Approach

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ABSTRACT This paper deals with a novel approach to the fractional-order model predictive control in state space. Except well-known fractional-order models of processes (plants) with arbitrary (real) order of the derivatives in fractional differential equations a new fractional performance index (cost function) and fractional control action are considered. Such combined approach to the model predictive control provides more degrees of freedom and incorporates fractional-order dynamics into the control in the form of memory due to the property of the fractional-order operator. An illustrative example of this approach is presented.

INDEX TERMS Model predictive control (MPC), fractional calculus, fractional-order model, fractional derivative, fractional-order cost function, fractional integral, constrains, linear quadratic regulator (LQR).

I. INTRODUCTION This article describes a new approach to an advanced method of process control known as model predictive control (MPC). The MPC was developed in the 1980s as a method useful for chemical and metallurgy plants in industry. Recently, it has been widely used also in power electronics industry for power system balancing control, food processing industry, and aerospace applications. Generally, the MPC is based on dynamic models of the process obtained by system identification [1]. However, it is well known that almost all models are wrong (inaccurate) but some of them are useful. It depends on model formulation, number of model parameters and used identification technique. The models used in the MPC should precisely represent the behavior of complex dynamical systems because the traditional control methods, as for example PID controller, are not suitable due to the fact that MPC is based on iterative, finite-horizon optimization of a plant model. Besides the plant model, the MPC uses the current states of the process, and the process variable targets and limits to calculate future changes in the dependent variables. Moreover, at each time step the current plant states are measured and a cost function minimizing control strategy is numerically computed for a relatively short time horizon in the future, known as a prediction horizon [2]. Then we apply the first value of the computed control sequence and at the next time step we get the system state and re-compute control action again. The prediction horizon keeps being shifted forward and for this reason the MPC is sometimes called receding horizon control. Although this control approach is not optimal, in practice it gives very good results. Thus, the MPC solves the optimization problem in smaller time windows instead the whole horizon and hence can obtain a suboptimal solution. It is the main difference between the MPC and LQR, where a fixed time window (horizon) is used and we obtain (optimal) solution for the whole time horizon.

One of the possible ways how to improve the quality of the MPC is incorporate fractional-order dynamics in the MPC. Basically, there are three options in the control algorithm where it can be used:

- fractional-order model of the process,
- fractional-order cost function over the receding horizon,
- fractional-order operations in control action.

The above suggestions can be implemented in the optimization algorithm minimizing the cost function with constraints (low and high limits) for inputs, states, and outputs in order to obtain novel MPC strategy.

This article is organized as follows. Section 1 brings the motivation to the model predictive control. In Section 2 the fundamentals of the fractional calculus are described. Section 3 provides new fractional-order model predictive...
control definition together with the new fractional-order performance index definition, new fractional-order control action and constraints. In Section 4 an illustrative example is presented. Section 5 concludes this article with some additional comments.

II. FRACTIONAL CALCULUS FUNDAMENTALS

A. FRACTIONAL-ORDER DERIVATIVE/INTEGRAL

The fractional calculus is a generalization of integration and differentiation to unified non-integer order operator $D^\gamma$. The standard notation for denoting the left-sided fractional-order operator (integral for $\gamma < 0$ and derivative for $\gamma > 0$) of a function $f(t)$ defined in the interval $[a, t]$ is $D^\gamma_t f(t)$, with $\gamma \in \mathbb{R}$. There exist various definitions of the fractional-order derivatives and integrals (operators). In this paper, the Caputo’s, and the Grünwald–Letnikov definitions are used [5]. Both mentioned definitions are equivalent for a wide class of the functions often used in control theory.

The Caputo’s definition (CD) of the fractional derivatives can be written as [5]:

$$D^\gamma_t f(t) = \frac{1}{\Gamma(m - \gamma)} \int_a^t f^{(m)}(\tau) (t - \tau)^{\gamma - m + 1} d\tau,$$

where $\Gamma(\cdot)$ is Gamma function, and $m - 1 < \gamma < m$, $m \in \mathbb{N}$. The initial conditions for this definition are well defined because of integer-order differentiation $f^{(m)}$ in the kernel.

The Grünwald–Letnikov definition (GLD) is given as follows [3]–[5]:

$$D^\gamma_t f(t) = \lim_{h \to 0^+} \frac{1}{h^{\gamma}} \sum_{k=0}^{\lfloor z \rfloor} (-1)^k \binom{\gamma}{k} f(t - kh),$$

where $\lfloor z \rfloor$ is the floor function, i.e. the greatest integer smaller than $z$, and where

$$\binom{\gamma}{k} = \frac{\Gamma(\gamma + 1)}{\Gamma(k + 1)\Gamma(\gamma - k + 1)}$$

are the binomial coefficients; $\binom{\gamma}{0} = 1$.

B. IMPLEMENTATION TECHNIQUE

By using a short memory principle proposed by Podlubny for numerical calculation of the fractional derivatives and integrals [5], the relation for approximation of the fractional-order derivative/integral derived from GLD (2) has the form:

$$D^\gamma_t f(t) \approx \frac{1}{h^{\gamma}} \sum_{k=0}^{N(t)} w_k^{(\gamma)} f(t - kh),$$

where for the lower moving limit $a = t - L_n$ the relation $N(t)$ is given

$$N(t) = \min \left\{ \left\lfloor \frac{a}{h} \right\rfloor, \left\lfloor \frac{L_n}{h} \right\rfloor \right\},$$

where operation $\lfloor \cdot \rfloor$ means an integer part.

For using to the GLD (2) the binomial coefficient $w_k^{(\gamma)}$ can be computed as follows [5], [6]:

$$w_k^{(\gamma)} = 1, \quad w_k^{(\gamma)} = \left(1 - \frac{1 + \gamma}{k}\right) w_{k-1}^{(\gamma)}.$$

Regarding the memory length $L_n$ we have to take into account that for determining this length for required accuracy $\epsilon$ the following inequality can be used [5]:

$$L_n \geq \left( \frac{M}{\epsilon |\Gamma(1 - \gamma)|} \right)^{1/\gamma}$$

where

$$M = \max \{f(t)\}. \quad [0, \infty)$$

Obviously, for this calculation simplification in form of memory length $L_n$, we pay a penalty in the form of certain inaccuracy in solution. It can be easily implemented in Matlab.

III. NOVEL FRACTIONAL-ORDER MODEL PREDICTIVE CONTROL

In order to implement the fractional-order model predictive control (FOMPC) strategy, the basic structure shown in Fig. 1 is considered. A model is used to predict the future plant outputs, based on the past and current values and on the proposed optimal future control actions. These actions are calculated by the optimizer taking into account the cost function as well as the constraints (see e.g. [7], [8]).

![FIGURE 1. Basic structure of the FOMPC.](image-url)
For instance, in [11], [12] the authors described an improved approach of the extended non-minimal state space fractional-order model predictive control and tested it on the temperature model of an industrial heating furnace. In [13], model predictive control is applied to drive autonomously a gasoline-propelled vehicle at low speeds with using the beneficial characteristics of the fractional predictive formulation. In [14], model predictive control scheme for constrained fractional-order discrete time systems is proposed. Its application to arrhythmic medicine with fractional-order pharmacokinetics was described in [15]. In [16] a robust model predictive control of uncertain fractional thermal system was designed.

A. FRACTIONAL-ORDER STATE SPACE MODEL
A usual fractional-order linear time-invariant (LTI) model in state space can be written as (see e.g. [17]–[19]):

\[
\begin{align*}
0D_t^\alpha x(t) &= Ax(t) + Bu(t), \\
y(t) &= Cx(t),
\end{align*}
\]

(9)

where \( x \in \mathbb{R}^n, u \in \mathbb{R}^r \) and \( y \in \mathbb{R}^p \) are the state, input and output vectors of the system, and \( A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times r}, C \in \mathbb{R}^{p \times n}, D \in \mathbb{R}^{p \times r} \), and \( \alpha = [\alpha_1, \alpha_2, \ldots, \alpha_n]^T \) are the fractional orders, \( \alpha \in \mathbb{R} \). If \( \alpha_1 = \alpha_2 = \ldots = \alpha_n = \alpha \), system (9) is called a commensurate-order system, otherwise it is an incommensurate-order system.

To propose the model structure for the MPC, we will use the derivative of the control as its inputs signal, while maintaining the same output. As a result, an integrator is embedded into the design model [20]. Moreover, due to the principle of receding horizon control, where the output of the plant is required for prediction of control, we assume that the input cannot affect the output at the same time and therefore the matrix \( D = 0 \) in plant model (9). Such augmented fractional-order LTI model of the system can be represented by the following new state-space model

\[
\begin{align*}
0D_t^\alpha x(t) &= Ax(t) + B_0 D_t^\lambda u(t), \\
y(t) &= Cx(t).
\end{align*}
\]

(10)

Note that the fractional-order augmented state-space model (10) contains also the fractional \( \lambda \)-order derivative of the control signal as its input, and its output remains the same.

It is important to investigate the controllability and observability of the augmented fractional-order model (10). Similarly to the conventional observability and controllability concept, the controllability is defined as follows: System (10) is \textit{controllable} on \([t_0, t_{final}]\) if the controllability matrix [21]:

\[
C_a = [B | AB | A^2B | \ldots | A^{n-1}B]
\]

has rank \( n \).

The observability is defined as follows: System (10) is \textit{observable} on \([t_0, t_{final}]\) if the observability matrix [21]:

\[
O_a = \begin{bmatrix}
C \\
CA \\
CA^2 \\
\vdots \\
CA^{n-1}
\end{bmatrix}
\]

has rank \( n \).

A minimal realization is both controllable and observable.

B. FRACTIONAL-ORDER PERFORMANCE INDEX
A concept of the usual optimization performance index was generalized to fractional-order one for signals and systems [22]. This approach was already used, for example, in optimal control problem [23], LQR based PID controller design [24], as model predictive frequency control of the islanded microgrid [25], and brings better results than traditional one.

Consider a moving time window which restriction from \( t_i \) to \( t_i + T_p \), and the time variable within this window is denoted \( \tau \). The integer-order cost function for the continuous-time predictive control with the fractional-order control action \( 0D_t^\alpha u(\tau) \) has the form:

\[
J_I = \int_0^{T_p} (x(t_i + \tau | t_i))^T Q x(t_i + \tau | t_i)
\]

\[
+ 0D_t^\alpha u(\tau)^T R 0D_t^\alpha u(\tau) d\tau,
\]

(11)

where the initial state information is given as \( x(t_i) \) and \( \lambda \in \mathbb{R} \). The optimal performance is based on the selection of weight matrices \( Q \succeq 0 \) and \( R \succeq 0 \) in the cost function \( J_I \).

Applying the idea of the fractional performance index to (11), we obtain the following new fractional-order cost function

\[
J_F = \int_0^{T_p} [D^{1-\mu}(x(t_i + \tau | t_i))^T Q x(t_i + \tau | t_i)
\]

\[
+ 0D_t^\alpha u(\tau)^T R 0D_t^\alpha u(\tau)] d\tau,
\]

(12)

where \( \mu \in \mathbb{R} \) is the real order of the fractional-order integral, which defines the fractional-order cost function. The optimal control \( 0D_t^\alpha u(\tau) \) is found by minimizing the cost function \( J_F \) for the same restriction of weight matrices \( Q \) and \( R \).

C. FRACTIONAL-ORDER CONTROL ACTION
Instead of the modeling the control signal \( u(t) \), the continuous-time FOMPC will target the fractional-order derivative of the control signal, which will satisfy the property (see e.g. [20])

\[
\int_0^\infty 0D_t^\alpha u(t)^2 dt < \infty,
\]

(13)

for external constant input signals.

Using the augmented fractional-order state space model (10), where the input is \( 0D_t^\alpha u(t) \), the closed-loop control system is

\[
0D_t^\alpha x(t) = (A - BK_{mpc})x(t).
\]

(14)
From the equation (14) the closed-loop eigenvalues of the predictive control system can be calculated. The state feedback control is then given as

\[ 0D_1^\alpha u(t) = -K_{mpc}x(t) = -[K_x \ K_y] \begin{bmatrix} 0D_1^\beta x(t) \\ y(t) - r(t) \end{bmatrix}, \] (15)

where \( r(t) \) is a set point signal, and the error signal is \( y(t) - r(t) \). The fractional-order \( \beta \in \mathbb{R} \) is an additional controller parameter together with the constants \( K_x \) and \( K_y \).

To obtain the control law and the action of the integral control, we need to integrate the result of (15) as follows

\[ u(t) = 0D_1^{-\alpha}(-K_{mpc}x(t)). \] (16)

In Fig. 2 the block diagram of the FOMPC is depicted. We can see that it has embedded the above integral action.

**IV. ILLUSTRATIVE EXAMPLE**

Let us consider the following example of a fractional-order system to illustrate the proposed methodology. The plant is represented by the following fractional differential equation [5], [6], [17]:

\[ 0.8D_1^{2.2}y(t) + 0.5D_1^{0.9}y(t) + y(t) = u(t), \] (17)

where \( 0D_1^\alpha \) is the CD, with its state-space representation expressed as:

\[ 0D_1^\alpha x(t) = Ax(t) + Bu(t), \]
\[ y(t) = Cx(t), \] (18)

with \( \alpha_1 = 0.9 \) and \( \alpha_2 = 1.3 \), and where system matrices are:

\[ A = \begin{bmatrix} 0 & 1 \\ -1.25 & -0.625 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1.25 \end{bmatrix}, \quad C = [1 \ 0]. \]

For the numerical solution of the fractional differential equation (17) and the state space representation (18) the approximation (4) with time step \( h = 0.001 \) sec, and the full memory length was used.

In Fig. 3 a unit step response of the system (17) for zero initial conditions and simulation time 60 sec is depicted.

**FIGURE 2.** Block diagram of the FOMPC.

**FIGURE 3.** Step response of system (17) for \( u(t) = 1 \).
I. Petráš: Novel Fractional-Order MPC: State-Space Approach

In Fig. 5 the state-space trajectories of the system (18) under the LQR control with the controller parameter $K_{lqr} = [2.316, 1.489]$, initial conditions $x_1(0) = 0$ and $x_2(0) = 1$, are depicted. However, we did not consider any constraints.

In addition, we will design the MPC of the form (15). However, we will consider the orders $\lambda = \beta = 0$ and the prediction horizon $T_p = 10$ sec in (12) with $\mu = 1$. Thus we obtain a classical integer-order MPC. Then with $Q = C^T C$ and $R = 0.1$ it has the values $K_{mpc} = [5.437, 2.200, 0.857]$, where $K_x = [5.437, 2.200]$ and $K_y = [0.857]$. The closed-loop eigenvalues for the controller parameter $K_{mpc}$ are $E_{1,2} = -1.687 \pm 2.279i$.

In Fig. 6 the state-space trajectories of the system (18) under the MPC control with the controller parameter $K_{mpc} = [5.437, 2.200, 0.857]$, prediction horizon $T_p = 10$ sec, initial conditions $x_1(0) = 0$ and $x_2(0) = 1$, are depicted. We did not consider any constraints, the same as it was in the LQR case.

Both controllers, the LQR and the MPC, deliver comparable results. In this particular case, the MPC controller provides a slightly better performances, namely less overshoot and settling time. Moreover, the MPC controller provides a bit greater stability measure derived from the eigenvalues.

In order to demonstrate the main benefits of the novel approach presented in this article, the other control parameters as well as the constraints will be considered. The desired system states and inputs are subject to the following constraints:

$$-0.5 \leq x_k \leq 1.5, \quad -1 \leq u_k \leq 1, \quad k = 1, 2. \quad (19)$$

To preserve the stability, we will not consider the output constraints in this particular example.

Taking into account the dynamics of the controlled system (18), let us consider the sampling interval, i.e. calculation step $h = 1$ sec, prediction horizon $T_p = 10$ sec, and simulation time 60 sec. The FOMPC of the form (15) with the parameters $\lambda = 0.9$ and $\beta = 0.95$ and the same values...
It could be a problem, for instance, for the industrial devices where an optimization algorithm should be implemented too in devices. Implementation methods were described in [26]. Moreover, an explicit MPC optimization can also be implemented in real time even on very simple devices such as PLC by using an idea for reducing the required memory size of the optimization algorithm with help of method proposed in [27].

\[ Q = C^T C \text{ and } R = 0.1 \text{ in (12) with } \mu = 0.9 \text{ the controller gain matrix has values } K_{\text{mpc}} = [1.156, 1.291, 2.156], \]

where \( K_X = [1.156, 1.291] \) and \( K_Y = [2.156] \). The closed-loop eigenvalues for the controller parameter \( K_{\text{mpc}} \) are \( E_{1,2} = -1.119 \pm 1.200i \). The reference value \( r(t) \) is equal to 1.

In Fig. 7 the output trajectory of system (18) under the FOMPC control with the controller parameter \( K_{\text{mpc}} = [1.156, 1.291, 2.156] \), \( \lambda = 0.9 \) and \( \beta = 0.95 \), prediction horizon \( T_p = 10 \text{ sec} \), initial conditions \( x_1(0) = 0, x_2(0) = 1 \) and \( y(0) = 0 \), are depicted. As we can observe the solution (output) is optimal, i.e. almost ideal output.

In Fig. 8 the corresponding control signal trajectory of the system (18) under the FOMPC with constraints is shown.

V. CONCLUSION

In this article the novel fractional-order model predictive control in state space is introduced. For the first time in new fractional-order performance index (12) like programmable logic controllers (PLC) due to the limitations on memory storage and microprocessor speed. Some very useful notes and a list of references on design and implementation methods were described in [26]. Moreover, an explicit MPC optimization can also be implemented in real time even on very simple devices such as PLC by using an idea for reducing the required memory size of the optimization algorithm with help of method proposed in [27].

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I. Petráš: Novel Fractional-Order MPC: State-Space Approach

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