Tidal disruptions of rotating stars by a supermassive black hole

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Abstract

We study tidal disruption events of rotating stars by a supermassive black hole in a galactic nucleus by using a smoothed-particle hydrodynamics (SPH) code. We compare mass infall rates of tidal-disruption debris of a non-rotating and of a rotating star when they come close to the supermassive black hole. Remarkably the mass distribution of debris bound to the black hole as a function of specific energy shows clear difference between rotating and non-rotating stars, even if the stellar rotation is far from the break-up limit. The debris of a star whose initial spin is parallel to the orbital angular momentum has a mass distribution which extends to lower energy than that of non-rotating star. The debris of a star with anti-parallel spin has a larger energy compared with a non-rotating counterpart. As a result, debris from a star with anti-parallel spin is bound more loosely to the black hole and the mass-infall rate rises later in time, while that of a star with a parallel spin is tightly bound and falls back to the black hole earlier. The different rising timescales of mass-infall rate may affect the early phase of flares due to the tidal disruptions.

In the Appendix we study the disruptions by using a uniform-density ellipsoid model which approximately takes into account the effect of strong gravity of the black hole. We find the mass infall rate reaches its maximum earlier for strong gravity cases because the debris is trapped in a deeper potential well of the black hole.

I. INTRODUCTION

Observational evidences have been accumulating that most of large galaxies have a supermassive black hole (SMBH) of mass $M_{BH}$ in the range of $10^6 \leq M_{BH}/M_\odot \leq 10^{10}$ at their centers. Some of these SMBHs show continuous cataclysmic activities known as active galactic nuclei (AGN). AGN are thought to occur in a environment where gas is supplied to SMBH in sufficiently high rates \[^1\]. In an opposite environment with little gas supply to SMBH, transient activities are still possible with a star or gas cloud occasionally accreting to SMBH. A tidal disruption event (TDE) occurs when the star or the gas cloud passes close enough to be disrupted by the tidal force of SMBH\[^2-4\]. Around the periastron of the object’s orbit, the tidal force of SMBH exceeds the self-gravity of the object and tears it apart.

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A part of disrupted debris then accretes onto SMBH and radiates mainly in ultra-violet to soft X-ray band.

Although details of hydrodynamic and radiation processes of the disruptions are not fully known, a simple model predicts an important aspect of the light curves of the TDEs [5–8]. A star passing by a SMBH whose periastron distance is smaller than the tidal radius is instantaneously disrupted. The fluid elements of the disrupted object follow Keplerian orbits around the black hole which are determined by the total specific energy $E$. The orbital period $T$ is related to $E$ by

$$E = -\frac{1}{2} \left( \frac{2\pi G M_{\text{BH}}}{T} \right)^{\frac{4}{3}},$$

(1)

when $E$ is negative (i.e., the fluid element is bound to the black hole). Differentiating Eq. (1) with respect to $T$ and multiplying it by the differential mass distribution of the fluid elements as a function of $E$, $dM/dE$, we have

$$\frac{dM}{dT} = \frac{dM}{dE} \frac{dE}{dT} = \frac{(2\pi G M_{\text{BH}})^{\frac{4}{3}}}{3} \frac{dM}{dE} T^{-\frac{5}{3}},$$

(2)

The fluid elements return to the point close to the periastron after the time interval of $T$. They collide each other to form a disk, in which the gas accretes in a time scale much shorter than the orbital period. Thus we expect the fall-back rate $dM/dT$ to be proportional to the luminosity of the event. If the mass distribution $dM/dE$ is flat, Eq. (2) shows the luminosity changes in time as $\sim T^{-5/3}$. The numerical simulations actually shows that $dM/dE$ have a flat profile around $E = 0$ (see e.g. [8]). Thus the time evolution of luminosity for the later stage of the TDEs is simply represented by the $T^{-5/3}$-law. This is observed in X-ray [9] and in optical band [10].

The ratio of the tidal radius $R_t$ to the Schwarzschild radius $R_s$ of the central black hole is

$$\frac{R_t}{R_s} = 24 \left( \frac{M_{\text{BH}}}{10^6 M_\odot} \right)^{-\frac{4}{3}} \left( \frac{R_\star}{R_\odot} \right) \left( \frac{M_\star}{M_\odot} \right)^{-\frac{1}{3}},$$

(3)

by which we see a TDE of solar type star occurs in strong gravity region of the spacetime for $M_{\text{BH}} \sim 10^6 M_\odot$. Thus general relativistic effects have been taken into account in recent modeling of TDEs [11–13]. One of the main issues is observational consequence of such strong gravity effect as dragging of inertial frame [14].

In the early phase of the TDEs, the light curve may not necessarily follow the simple model above. In fact, [15] examined the effect of the internal structure of the incident star...
on the TDEs (see also [16]). They show that the higher concentration of density in the star results in the slower rise of the light curve. Further observational signatures of more realistic stellar structures are examined by [17], in which a time evolution of chemical composition in tidal disruption flare is discussed.

Structure of the incident star is also affected by stellar spin. Centrifugal force comes into the hydrostatic balance between self-gravity and pressure gradient. Observationally non-negligible fractions of early type stars have large angular momentum so that the stellar structure significantly deviates from spherical symmetry [18]. Although the event rate of tidal disruptions for rotating stars is not known, [19] investigates the event rate of TDE’s as a function of black hole mass as well as mass of different type of stars for different histories of star formations. It is shown the rate for stars more massive than $1M_\odot$ is rather small and the TDE events are dominated by the stars with the smaller masses. Considering the observed stellar surface velocity tends to be slow for late-type stars [20, 21], possibly due to angular momentum loss by stellar wind or magnetic breaking, there may not be much chance of observing TDE of rapidly rotating early type stars. Still it may be useful to study the possible outcome of the stellar rotation in the TDEs. Although MOSFiT [22] can generate light curves for TDEs with wide parameter ranges, it does not take into account the stellar rotation. TDE light curves would not be determined only by the fall-back rate [23, 24], however the fall-back rate is still an important property for TDEs. In this paper we examine the effect of stellar rotation on the mass infall rate after the tidal disruption by using a non-linear hydrodynamic simulation as well as a simple ellipsoidal model of a star. As in precedent works, we evaluate the energy of the tidally disrupted debris as a proxy of the mass infall rate by assuming its Keplerian motion around SMBH.

The paper is organized as follows. In Sec.II we briefly introduce our setup of hydrodynamic simulations. In Sec.III we present our main results. Finally in Sec.IV we summarize our results and make some remarks. In Appendix, we study effects of strong gravity of supermassive black hole on the tidally disrupted debris by using a simple homogenous ellipsoidal model of stars.
II. NUMERICAL SIMULATIONS

A. Numerical simulation code

We perform three-dimensional smooth particle hydrodynamic (SPH) simulations by using numerical code developed in [25] (see also [26, 27]). Cartesian coordinate is used in the code. Wendland C2 kernel for the SPH kernel interpolation [28, 29] is adopted. The number of neighbor particles of a given particle is about 120 (3D). The artificial viscosity proposed by [30] is used. The viscosity from shear motion is suppressed by the Balsara switch [31]. Self gravity among SPH particles is computed with adaptive gravitational softening [32]. The SPH and self-gravity calculations are optimized on distributed-memory systems, using FDPS [33, 34] and explicit AVX instructions [35, 36].

B. Rotating stars

We compute uniformly rotating stellar models by using a numerical code [37] based on Hachisu’s self-consistent method [38]. The code computes stationary and axisymmetric rotating stellar equilibria on the spherical polar-coordinate grids. The models need to be projected onto the Cartesian coordinate grids to prepare the initial stellar models for the numerical simulations. Although the projection is mathematically straightforward, the initial models produced on the finite grids require relaxation runs in the SPH code before performing simulations of tidal disruptions. A projected model on the Cartesian grid is evolved by the SPH code until we have a stationary state. Due to the numerical viscosity in the code, entropy is produced in the model during the relaxation run which lasts two to three dynamical timescales of the star. In Fig.1 we plot the values of \( p/\rho^{\gamma} \) as a function of \( \rho \), which is a proxy of entropy distribution. For each fluid element the value is computed after the relaxation procedure. For the polytropic fluid used to construct the equilibrium model, the value would be constant. The deviation of it from being constant results from the fact that the code suffers from numerical dissipations. Nevertheless the fluids composing the relaxed models for non-rotating and rotating cases are identified when the profiles of \( p/\rho^{\gamma} \) overlap as in Fig.1.

In our numerical simulations we focus on stellar models with \( \gamma = 5/3 \) and the mass of \( 4.0M_\odot \). The stellar rotation is parametrized by the so-called ‘T/W’ value, which is the
FIG. 1. Comparison of $p/\rho\gamma$ as a function of density $\rho$ after the SPH relaxation process. The original models of stationary star have polytropic index $N = 1.5$. The constant $K$ is the polytropic constant of the equilibrium model which is used to construct the initial data by the SPH relaxation.

The ratio of kinetic energy to the gravitational energy $[38]$. We choose the star to have a polar-to-equatorial axis ratio of 0.95, which corresponds to $T/W = 8.3 \times 10^{-3}$. The rotational velocity at the equator of the stars amounts to 300 km s$^{-1}$.

### III. RESULTS

A black hole with mass $M_{\text{BH}} = 10^6 M_\odot$ is placed at the coordinate origin. We assume the mass of stars is negligibly small compared to that of the black hole, thus we fix the position of the black hole. In this approximation the center of the mass of the star moves along a conic section. We choose the orbit of the center of the mass to be a parabola whose periastron distance is half the tidal radius $R_t$. The initial distance of the star from the black hole is twice as far as $R_t$. We performed simulations with stars whose rotational axis is perpendicular to the orbital plane. The model is named as ‘parallel spin’ when the spin angular momentum of the star is parallel to that of the orbit. When the spin angular
FIG. 2. Snapshot of SPH particle distributions projected onto the orbital plane. In each panel, the black hole is at the origin. The top left panel (a) is for the non-rotating stellar model. The top right (b) and the bottom (c) correspond to the stars with parallel and anti-parallel spins respectively. Dimensionless parameter $T/W$ of initial stellar spin is $8.3 \times 10^{-3}$. The contour map corresponds to the mass density. From the innermost to the outermost region, the corresponding density is as follows: $-1.5 < \log_{10} \rho < 0.0$, $-2.0 < \log_{10} \rho_0 < -1.5$, $-2.5 < \log_{10} \rho_0 < -2.0$, $-3.0 < \log_{10} \rho_0 < -2.5$, $-3.5 < \log_{10} \rho_0 < -3.0$, $-4.0 < \log_{10} \rho_0 < -3.5$ and $-4.5 < \log_{10} \rho_0 < -4.0$, where $\rho_0 \equiv \rho/1g \text{ cm}^{-3}$.

momentum is parallel to the orbital one but with a different sign, it is called 'anti-parallel spin' case. We also performed a simulation with an initially non-rotating star (‘zero spin’ case).

In Fig. 2 the distributions of SPH particles after the stars passed the periastron are shown. The distribution of fluid elements are viewed from the direction perpendicular to the orbital plane. The orbital angular momentum is pointing toward us. In this figure, particles off the orbital plane are projected onto the plane. The snapshots are for the time lapse $t = 2t_0$, where $t_0$ is the time it takes for a star to reach its periastron. The figure shows tidally
elongated structures of stellar fluid. A remarkable difference is that the distribution in
the parallel spin star is less compact than the zero spin case. For the anti-parallel spin
model, on the other hand, the distribution is more compact than that of the zero spin
case. The expansion and contraction of the particles distributions compared to the zero
spin case result in the difference in the mass distribution per specific energy as is seen in
Fig\textsuperscript{3}. Here differential mass of fluid per unit mechanical (kinetic plus gravitational) energy
d\textit{dM/dE} is plotted as a function of specific mechanical energy. The fluid elements having
negative specific energy are regarded as being bound to the central black hole after the
disruption. They accrete to the black hole and flare up. The plateau around the zero energy
corresponds to the fluid elements which are loosely bound to the black hole. Because of
their flat profile of \textit{dM/dE} and the long orbital period (see Eq.(11)), they regulate the late-
time canonical behaviour of the TDE events (”\textit{T}\textsuperscript{−5/3}- law” as seen in Eq.(2)). Beyond the
edge of the plateau the distribution of \textit{dM/dE} gradually decreases as the specific energy
decreases. Fluid elements in this tail are deeply bound to the black hole and accrete faster
after the disruption. They are expected to determine the early rises of the TDE flares. It
is remarkable in this context that the parallel spin star has the most extended low energy
tail among the three cases, while the anti-parallel spin case has the shortest tail as seen in
Fig\textsuperscript{3}. It reflects the fact that parallel spin case has the most extended debris by the tidal
disruption and has more fluid elements strongly bound to the potential of the black hole.
The anti-parallel spin star, on the other hand, has the most compact debris and the fluid
elements is weakly bound to the black hole.

We see a difference of mass infall rate for the three cases as a consequence of the difference
in the debris distributions. As \textit{dM/dE} appears in the mass infall rate of Eq.(2), we directly
compare the rate for these cases (Fig\textsuperscript{4}). As seen in the figure, the late time behaviour of
mass infall rate scales as power law of elapsed time \textit{t} with the expected index \textsuperscript{−5/3}. In the
early phase the mass infall rate for the parallel spin case rises fastest and the peak infall rate
is the highest among three cases. The anti-parallel spin case is the slowest in rise of mass
infall rate and the lowest at its peak. This behavior is expected from Figs.2 and 3 since the
parallel spin case has more fluid elements being trapped deep in the potential well of the
black hole and the fluid elements of the anti-parallel spin case is more tightly bound to each
other.

The physics behind the difference is rather simple. When we see the star in a comoving
FIG. 3. Mass distribution of TDE debris as a function of mechanical energy of fluid element. Long-dashed line is for an initially non-rotating star of $4M_\odot$. The dashed-dotted line is for a spinning star whose spin angular momentum is parallel to the orbital angular momentum while the solid line corresponds to the star whose spin angular momentum is anti-parallel to the orbital angular momentum. The rotation of the model is parameterized by $T/W = 8.3 \times 10^{-3}$ or normalized rotational angular frequency of $\omega/\sqrt{\pi G \rho c} = 0.13$. All the model is polytropic with $N = 1.5$.

(accelerating) frame with it, the stellar fluid suffers from inertial forces due to the orbital motion. The centrifugal and Euler forces are canceled out, thus we are left with the Coriolis force acting on the stellar fluid. If the star is spinning with the angular velocity $\vec{\omega}$, the Coriolis force acting on the fluid at $\vec{r}$ (the position vector measured from the stellar center) is $2(\vec{r} \times \vec{\omega}) \times \vec{\Omega}$, where $\vec{\Omega}$ is the instantaneous orbital angular velocity of the center of mass of the star. The ratio of the Coriolis force to the centrifugal force from the stellar spin is thus $2\Omega/\omega$. In our case the ratio is at least $O[10]$. Moreover the direction of the Coriolis force is such that it makes the star expand when the stellar spin is parallel, while the force make the star contract when the spin is anti-parallel.
IV. SUMMARY AND REMARKS

We studied influences of stellar spin on tidal disruption of a star by a supermassive black hole. By using SPH simulations we compared the tidally-disrupted debris of stellar model with the same mass but with different spins. We found the distribution of debris, when the stellar spin is initially anti-parallel to its orbital angular momentum, is more compact than that of a non-rotating star. On the other hand the debris distribution is less compact for a star with parallel spin. This leads to the difference in the initial rise of the mass-infall rate of debris. It occurs earlier than the non-rotating case for a parallel spin star whose angular momentum is aligned to the orbital one. It occurs later for an anti-parallel spin case. whose spin direction is opposite to the orbital one. The difference comes from the Coriolis force acting on the stellar fluid. The force tends to bind the fluid together when the spin is anti-parallel to the orbital angular momentum, while it tends to tear it apart when the spin is parallel to the orbit. The force is at least an order of magnitude larger.

FIG. 4. Mass infall rate computed from $dM/dE$ profile in Fig. Solid line is for a zero spin star, while dashed line is for a parallel spinning star and dashed-dotted is for a anti-parallel spinning star.
than that of the centrifugal force by the stellar rotation itself. It should be noted that the mechanism we see here may be also applied to totally different systems. For instance, a planet or planetesimal may be tidally disrupted at its close encounter to a star. A parallel spin planet to its orbit is more susceptible to tidal disruption than an anti-parallel spin one. A planet with an anti-parallel spin is more likely to survive in its close encounter to the central star. Another case of interest is a tidal disruption of a dwarf satellite galaxy by a larger one. The disruption is more likely to occur if the rotation of the dwarf is aligned to its orbital angular momentum. Finally, a binary of stars or black holes orbiting around a massive black hole may be bound tighter if the angular momentum of the binary system is anti-aligned to the orbital angular momentum of the binary around the massive black hole. A breakup of a binary is more likely to occur if the binary motion is in the same direction as that of the orbit of the binary around a massive black hole.

Appendix: Ellipsoidal (Affine) model

In this appendix we investigate the difference in TDE mass infall rate arising from the stellar rotation by using a simple ellipsoidal (Affine star) model \[39\]. The model has been used to study tidal interactions of stars orbiting around supermassive black holes as well as quasi-equilibrium configurations of close binary stars \[40\]. We follow a simplified version of Affine star model formulated by Usami and Fujimoto \[41\]. A star is approximated to be homogeneous, compressible and uniformly rotating while the other component (SMBH) is regarded as a point source of gravity. We assume the mass of the star \(m_\star\) is much smaller than that of the black hole \(M_{\text{BH}}\). An orbit of the star around the black hole is expressed by a conic section. We introduce a Cartesian coordinate centred at the black hole whose z-axis is perpendicular to the orbital plane of the star. Then the geometrical center of the star is expressed as \(\vec{R} = (R(t) \cos \varphi(t), R(t) \sin \varphi(t), 0)\). The equilibrium figure of the star is assumed to be ellipsoidal, which enables the profiles of pressure and scalar potential of self-gravity to be expressed by quadratic forms of the Cartesian coordinate centred at the geometrical center of the star. Three principal axes of the ellipsoidal star \(a_1, a_2, a_3\) are functions of time. We denote \(a_1\) as the semi-major axis and \(a_3\) is the principal axis perpendicular to the orbital plane. We have \(a_1 \geq a_2 \geq a_3\). We consider the case in which the stellar spin angular momentum is perpendicular to the orbital plane. The vorticity of
the stellar fluid $\lambda(t)$, thus, has only the component perpendicular to the plane. Introducing the angle $\theta(t)$ of the principal axis $a_1$ relative to $\vec{R}$, the relative angular frequency $\Omega(t)$ of the $a_1$ axis to $\vec{R}$ is $\Omega = \frac{\omega}{dt} + \frac{d\theta}{dt}$.

Momentum conservation of the star leads to the following equations.

$$
\frac{d^2 a_1}{dt^2} = a_1(\Omega^2 + \lambda^2) + 2a_2\Omega \lambda - 2a_1 A_1 + 2\frac{p_c}{\rho a_1} + \frac{G M_{BH} a_1}{R^3}(3 \cos^2 \theta - 1),
$$

(A.1)

$$
\frac{d^2 a_2}{dt^2} = a_2(\Omega^2 + \lambda^2) + 2a_1\Omega \lambda - 2a_1 A_2 + 2\frac{p_c}{\rho a_2} + \frac{G M_{BH} a_2}{R^3}(3 \sin^2 \theta - 1),
$$

(A.2)

$$
\frac{d^2 a_3}{dt^2} = -2a_3 A_3 + 2\frac{p_c}{\rho a_3} - \frac{G M_{BH} a_3}{R^3}
$$

(A.3)

Conservation of angular momentum and vorticity is written as,

$$
\frac{d\Omega}{dt} = \frac{1}{a_1^2 - a_2^2} \left[ -2 \left( a_1 \frac{da_1}{dt} - a_2 \frac{da_2}{dt} \right) \Omega - 2 \left( a_1 \frac{da_2}{dt} - a_2 \frac{da_1}{dt} \right) \lambda - \frac{3G M_{BH} \sin 2\theta}{2R^3}(a_1^2 + a_2^2) \right]
$$

(A.4)

and

$$
\frac{d\lambda}{dt} = \frac{1}{a_2^2 - a_1^2} \left[ -2 \left( a_2 \frac{da_1}{dt} - a_1 \frac{da_2}{dt} \right) \Omega - 2 \left( a_2 \frac{da_2}{dt} - a_1 \frac{da_1}{dt} \right) \lambda - \frac{3G M_{BH} \sin 2\theta}{2R^3}a_1 a_2 \right]
$$

(A.5)

where $\rho(t)$ and $p_c(t)$ is the uniform density and the central pressure. $A_1$ to $A_3$ are defined as [42]:

$$
A_1 = \pi G \rho \frac{2a_1 a_3}{a_1^2 \sin^3 \Phi \sin^2 \Theta} \left[ F(\Theta, \Phi) - E(\Theta, \Phi) \right],
$$

(A.6)

$$
A_2 = \pi G \rho \frac{2a_2 a_3}{a_1^2 \sin^3 \Phi \sin^2 \Theta \cos^2 \Theta} \left[ E(\Theta, \Phi) - F(\Theta, \Phi) - \frac{a_3}{a_2} \sin^2 \Theta \sin \Phi \right],
$$

(A.7)

$$
A_3 = \pi G \rho \frac{2a_2 a_3}{a_1^2 \sin^3 \Phi \cos^2 \Theta} \left[ a_2 \sin \Phi - E(\Theta, \Phi) \right],
$$

(A.8)

where $\sin \Theta \equiv \sqrt{(a_1^2 - a_2^2)/(a_1^2 - a_3^2)}$ and $\cos \Phi \equiv a_3/a_1$. $E(\Theta, \Phi)$ and $F(\Theta, \Phi)$ are incomplete elliptic integrals of the second and the first kind [42].

We assume the star is composed of ideal gas whose equation of state is written as

$$
p_c = \frac{k_B \tau_c}{\mu m_\text{H}} \rho,
$$

(A.9)

where $k_B$, $m_\text{H}$, $\mu$ are the Boltzmann’s constant, the atomic mass unit and the mean molecular weight (to be fixed as 0.5). $\tau_c$ is the central temperature. It should be noticed that the profiles of pressure and temperature are quadratic in the Cartesian coordinate [43].
FIG. 5. Mass distribution of tidally-disrupted star as a function of specific energy. The specific energy $E$ is normalized by $GM_\odot/R_\odot$, while the mass is normalized by $M_\odot$.

We assume the fluid configuration changes adiabatically. Thus the first law of thermodynamics leads to

$$\frac{1}{\tau_c} \frac{d\tau_c}{dt} = - \sum_{i=1}^{3} \frac{1}{a_i} \frac{da_i}{dt}. \tag{A.10}$$

Eqs. (A.1), (A.2), (A.3), (A.4), (A.5), (A.10) are numerically integrated for given orbital parameters $R(t)$ and $\varphi(t)$. Here we choose a parabolic orbit around the black hole. Far from the periastron we put the initial star which is an axisymmetric spheroid and whose spin axis is perpendicular to the orbital plane.

After the periastron passage, we assume the star tidally disrupted. We compute the total specific energy $E$ of fluid element of the star and compute its mass distribution $dM/dE$. The mass distribution translates to the mass infall rate $dM/dT$ by Eq. (2).

For an illustration, we compare a zero spin star model of $m_*=4M_\odot$ with a rapidly rotating star of the same mass. The ratio of the semi-minor to the semi-major axis is 0.95 for the rotating star (the so-called $T/|W|$ parameter, the ratio of kinetic energy to gravitational energy, is $1.36 \times 10^{-2}$. Angular frequency of the model is $4.2 \times 10^{-5}$ Hz, which corresponds to
FIG. 6. Mass infall rate $dM/dT$ (Eq. (2)) is plotted as a function of infall time $T$ (in units of day). The infall rate is in units of $M_{\odot} s^{-1}$.

the surface velocity of 67 kms$^{-1}$). We fix the average radius $r_{av} = (a_1a_2a_3)^{1/3}$ of the initial stars. The initial central temperature of rotating star is $\tau_c = 9.7 \times 10^6 K$, while that of the zero spin star is $1.0 \times 10^7 K$. The periastron distance of the orbit is $2(M_{BH}/m_\star)^{1/3}R_\odot$ which corresponds to 87% of the tidal radius of the star. We compute $dM/dE$ and $dM/dT$ when the orbital phase is $\varphi = \pi/6$ after the periastron. The cases with the spin axis parallel and anti-parallel to the orbital angular momentum are compared (Fig.5, Fig.6).

Since the ratio of the tidal radius to the Schwarzschild radius scales as $M_{BH}^{-2/3}$, the ratio may be close to unity for $M_{BH} = 10^6 M_{\odot}$. For the Newtonian models above, the ratio is 1/9. Thus the general relativistic effect may be important in the tidal disruption processes considered here. We assess the effect by introducing the modified gravitational potential (pseudo-Newtonian, or Paczyński-Wiita potential. See [45]) to mimic the strong gravity around the central black hole. In Fig.7 we compare the Newtonian (N) and pseudo-Newtonian (pN) cases for parallel (P) and anti-parallel (A) spins. The axis ratio of the star is 0.95 and the periastron distance is 12 times the Schwarzschild radius. In each case of parallel or anti-parallel spin, the mass spreads in wider energy range for the pseudo-
FIG. 7. Comparison of Newtonian and pseudo-Newtonian potential of the central black hole. 'N' stands for Newtonian, ‘pN’ for pseudo-Newtonian, ‘P’ for parallel spin, and ‘A’ for anti-parallel spin.

Newtonian model than for the corresponding Newtonian case. As a result, the mass infall rate rises earlier for the pseudo-Newtonian potential (Fig.8). It reflects the fact that the gravity and the tidal force of the pseudo-Newtonian case are stronger than those of the Newtonian case. The tidally-disrupted fluid elements bound to the black hole fall deeper in the potential well and result in the earlier rise of the mass infall rate.

In Fig.9 we plot as a function of dimensionless parameter $T/W$ the time $T_{\text{max}}$ between the periastron passage and the maximum of mass infall rate. We see that neglecting the strong gravity of the central black hole overestimates $T_{\text{max}}$. It is reasonable since the strong gravity tends to bind more mass to the central objects. However, rotation of a star is also not to be neglected if $T/W$ parameter is $\sim \mathcal{O}[10^{-2}]$. The difference of $T_{\text{max}}$ between parallel and anti-parallel spins is smaller for stronger gravity of pseudo-Newtonian potential.
FIG. 8. Mass infall rate $dM/dT$ for the model in Fig. 7.

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FIG. 9. The elapsed time between the disruption and the maximum of mass infall rate, $T_{\text{max}}$ (in units of day), as a function of dimensionless parameter $T/W$. 'N' is for Newtonian potential, while 'pN' is for pseudo-Newtonian potential for the central black hole. 'P' and 'A' are parallel and anti-parallel spin cases.

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