On the distribution of initial masses of stellar clusters inferred from synthesis models

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Abstract The fundamental properties of stellar clusters, such as the age or the total initial mass in stars, are often inferred from population synthesis models. The predicted properties are then used to constrain the physical mechanisms involved in the formation of such clusters in a variety of environments. Population synthesis models cannot, however, be applied blindly to such systems. We show that synthesis models cannot be used in the usual straightforward way to small-mass clusters (say, \( M < \text{few times } 10^4 \, M_\odot \)). The reason is that the basic hypothesis underlying population synthesis (a fixed proportionality between the number of stars in the different evolutionary phases) is not fulfilled in these clusters due to their small number of stars. This incomplete sampling of the stellar mass function results in a non-gaussian distribution of the mass-luminosity ratio for clusters that share the same evolutionary conditions (age, metallicity and initial stellar mass distribution function). We review some tests that can be carried out a priori to check whether a given cluster can be analysed with the fully-sampled standard population synthesis models, or, on the contrary, a probabilistic framework must be used. This leads to a reassessment in the estimation of the low-mass tail in the distribution function of initial masses of stellar clusters.

Keywords stars: mass function, statistics: methods, galaxies: clusters: general

1 Introduction

The study of astrophysical objects is often limited by our ability to infer their physical properties such as distances, masses or ages, from their observed fluxes. In stellar astrophysics, when the distance to an observed star is known, the luminosity at different wavelengths or spectral energy distribution (SED), \( l_\lambda \), provides constraints on the effective temperature and on its mass-luminosity ratio. This ratio depends mainly on the effective temperature, gravity and the mass of the star. In the case of stellar clusters with known distances, the observed luminosity is the sum of the luminosities of the stars in the cluster, each one with its own mass-luminosity ratio. In a cluster composed by \( N_\star \) stars, this luminosity can be written as

\[
L_{\text{cluster}} = \sum_{i=1}^{N_\star} l_i .
\]

As stated in Eq \( \text{I} \), the integrated luminosity of the cluster does not provide a great deal of information on the stars in the cluster. However, we know that the possible luminosities and spectral shapes of individual stars are in the range defined by stellar evolution, and thus it is possible to group individual stars in representative classes or evolutionary phases \( j \) of luminosity \( l_j \).

Assuming a total number of \( N_{\text{class}} \) stellar evolutionary phases, Eq. \( \text{I} \) can be rewritten as

\[
L_{\text{cluster}} = \sum_{j=1}^{N_{\text{class}}} n_j l_j ,
\]

\[
N_\star = \sum_{j=1}^{N_{\text{class}}} n_j .
\]

The problem now becomes the estimation of the \( n_j \) coefficients in such a way that we can obtain physical...
properties of the cluster from them. At this stage, it is not possible to know the total number of stars in the cluster from its integrated light, nor how many stars are in a given evolutionary phase. However, we can relate the relative number of stars in different evolutionary phases thanks to stellar evolution: the number of stars in a given evolutionary phase is proportional to the amount of fuel that can be consumed in that phase, and therefore with the lifetime \( t_j \) of the stellar evolutionary phase. This is the Fuel Consumption Theorem (Tinsley & Gunn 1976; Renzini & Buzzoni 1986; Marigo & Girardi 2001) which underlies, implicitly or explicitly, any population synthesis method.

The comparison of different evolutionary phases (say, phase \( i \) vs phase \( j \)) provides the number or population ratio \( n_i/n_j \) in the limit of an infinite number of stars in the cluster. Indeed, if the cluster actually has a very large number of stars in all the theoretical evolutionary phases, we have that

\[
\frac{t_i}{t_j} \propto \frac{w_i}{w_j}, \quad (4)
\]

\[
w_k = \lim_{N_* \to \infty} \left( \frac{n_k}{N_*} \right), \quad (5)
\]

These relations hold for the post main sequence evolutionary phases, that is, the populations of the most luminous stars which dominate the total integrated luminosity. Therefore once we explicit the relation between the \( w_i \) coefficients and the most luminous stars, the values of luminosity ratios (i.e. colours) are also fixed. Since the proportionality relations between the stellar evolutionary phases that \textit{would be} present in a cluster depend on the age of the cluster, in general \( w_i = w_i(t) \). This provides a way to estimate the age of the given cluster, for example by the comparison of different colours of the cluster with theoretical predictions.

Note, however, that these relations do not allow us to obtain the total mass of the cluster unless the most luminous stars are also the ones that define the total mass (i.e. the unlikely case where the most massive stars are also the most numerous ones, a hypothesis that is ruled out by the observations). So what is the fraction of the total mass which is responsible for the total luminosity? Since we do not know the actual stellar mass distribution function of the cluster, we have to use a statistical method to describe how many stars with a given initial mass are expected in the cluster: the stellar initial mass function. For simplicity, we assume that all the stars in the cluster have been formed in a single star formation episode, and that there are no other episodes so that we do have a single stellar population (SSP). The integration of the stellar initial mass function over the mass range of initial masses that defines an evolutionary phase \( i \), \( m_i \pm dm_i \), provides the \( w_i \) coefficients that allow us to obtain the mass-luminosity ratio of stellar clusters as a function of age as

\[
\left( \frac{M}{L} \right) = \frac{\sum_{i=1}^{N_{\text{class}}} w_i m_i}{\sum_{i=1}^{N_{\text{class}}} w_i \ell_i} = \lim_{N_* \to \infty} \frac{\sum_{i=1}^{N_{\text{class}}} n_i m_i}{\sum_{i=1}^{N_{\text{class}}} n_i \ell_i}. \quad (6)
\]

The inferred cluster initial mass is then obtained from this implicit mass-luminosity relation through a direct combination of the theoretical mean luminosity

\[
L_{\text{theo}} = \sum_{i=1}^{N_{\text{class}}} w_i \ell_i, \quad (7)
\]

with the inferred mass-luminosity ratio:

\[
M_{\text{inferred}} = L_{\text{theo}} \times \left( \frac{M}{L} \right). \quad (8)
\]

Strictly speaking, this direct comparison provides in fact the expected number of stars in the cluster \(< N >\), and the expected mass \(< M >\) which corresponds to this expected number of stars. It is important to note that an incorrect age estimation also implies an incorrect cluster mass estimation.

Synthesis models provide this mass-luminosity relation for different ages and metallicities\(^1\) for theoretical clusters which contain an infinite number of stars. This deterministic method has been used, rather blindly, to clusters of any mass, and in this case the mass-luminosity relation is a simple function of the age and metallicity \( (M/L = f(t, Z)) \), and the fundamental properties of clusters are inferred by statistical tests such as \( \chi^2 \) fits. This result is also recovered as the mean value of the distribution of the possible mass-luminosities relations, \( (M/L = f(t, Z, M)) \), under a probabilistic framework (Cerviño & Luridiana 2006). In this case, it is necessary to take into account the shape of the distribution and how it varies with the cluster mass when synthesis models results are applied to the analysis of real clusters.

\(^1\)Note that this metallicity refers to the evolutionary tracks and not the metallicity in the stellar atmospheres, which may not be the same.

2 The distributed of the mass-luminosity ratio in stellar clusters and the initial cluster mass function

The deterministic method is only valid, as seen above, in the limit of a very large number of stars popul-
ing most, if not all, the evolutionary phases. The blind application of the method to small clusters results in wrong inferences have been made because the underlying assumptions are violated. A simple illustration is provided by Fig. 1 which shows the mass-luminosity ratio for different cases as a function of the cluster luminosity in the V band. The thick blue line at the top shows the evolution with age of the mass-luminosity ratio and the V magnitude for a $10^7 \, M_\odot$ cluster obtained from the SSP models provided by Girardi et al. (2002). The circle at the top of the line shows the position of a 4 Ma-old cluster and the monotonic decrease in luminosity yields a monotonic increase in the mass-luminosity ratio, a property often used to infer fundamental properties as described above. The thick black line at the bottom of the plot shows the position of 4 Ma-old individual stars from the corresponding isochrone provided by Girardi et al. (2002). Note that, because of the use of the V band, the turn-off point appears to be the brightest with the smaller mass-luminosity ratio. The upper branch are the post-MS stars, while the lower branch provides the locus of the MS stars, down to very low luminosities and hence large mass-luminosity ratios. The shadow region in the middle of the plot is the result of $10^6$ Monte Carlo simulations for 4 Ma-old clusters using the same stellar initial mass function than the SSP models. The mass of each cluster is the result of a random sampling of a power-law initial cluster mass function with slope $\alpha = -1$ covering the cluster mass range between 0.1 and $10^5 \, M_\odot$. The stellar initial mass function has been sampled randomly until the cluster mass of the cluster has been reached.

Let us consider the mass-luminosity relation of individual stars of a given age (bottom thick black line in Fig. 1) isochrone of a 4 Ma-old population of single stars). Obviously, the possible values of the mass-luminosity depends on the particular properties of each star (its age, metallicity and mass; $M/L = f(t, Z, M)$). The figure shows that the range covered by the mass-luminosity relation of individual stars include all the possible mass-luminosity relations of SSP models. It also shows that the stellar mass-luminosity relation defines a natural limit of the mass-luminosity relations obtained in the Monte Carlo simulations of clusters.

When stars are combined to describe stellar clusters (following a given stellar initial mass function), the mass-luminosity relation gradually collapses to a single mass-luminosity ratio. The origin of this evolution in the distribution function of the mass-luminosity ratio is simply explained by the right-hand side of Eq. (10): the actual fraction of stars in a given evolutionary phase $n_i/N$, does not coincide with the theoretical value $w_i$, but fluctuates around it following a multinomial distribution (see Cerviño & Luridiana (2000) for a detailed discussion, and Cerviño & Valls-Gabaud (2002) for a quasi-Poisson formalism). If $N_iw_i$ is the expected number of stars in the $i$-th evolutionary phase, the $n_i$ value of real clusters will be distributed around it, producing variations with respect to the expected total luminosity of the cluster, but almost no variation in its total mass. Equivalently, variations in the number of low-mass stars yield variations in the total mass, but not in the total luminosity. Obviously, the dispersion in the mass-luminosity ratio will be larger for clusters which have a smaller number of stars since these clusters have large relative dispersions in $n_i$ (Cerviño, Luridiana, & Castaño 2000; Cerviño et al. 2002, Cerviño & Valls–Gabaud 2003; Cerviño & Luridiana 2006). Note that the distributed nature of the mass-luminosity relation is a result of the
intimate composition of stellar populations of real stellar clusters. Its physical nature implies that it remains a distribution even in the case of perfect observations performed in perfect telescopes with perfect instruments with no statistical observational errors.

Only when the number of stars in a cluster is large enough (i.e. the cluster is bright enough) the mass-luminosity ratio obtained by SSP models becomes a reliable, unique and well-behaved quantity. In other terms, the assumption of a mass-luminosity relation independent of the cluster mass is only valid for massive clusters, typically with masses larger than $10^5 M_\odot$.

We want to stress that the main issue due to the incomplete initial stellar mass function sampling is that the proportionality between the actual evolutionary phases in the cluster at a given age $t_1$, $n_i(t_1)/n_j(t_1)$, differs from the assumed one in the synthesis models, $w_i(t_1)/w_j(t_1)$. Not only it may well be not fitted by the models, but it could also be close to the proportion $w_i(t_2)/w_j(t_2)$ that corresponds to a different age $t_2$. For example, young clusters without massive stars (due to the sampling of the stellar initial mass function) are systematically best fit by models at older ages because older clusters do not have massive stars. Under the usual assumption of full sampling, the sparse sampling of the IMF in these clusters is wrongly interpreted as a pure evolutionary effect. As the mass-luminosity ratio decreases with age, this effect translates into an overestimation of the initial cluster mass, producing a systematic bias in the cluster mass estimation.

From another perspective, when sampling effects are present, there is more information on the properties of particular stars in the clusters (the effective temperatures and luminosities of individual stars) but there is less information about the global properties of the system (age and cluster masses). We refer to Buzzoni (1993) and Buzzoni (2005) for a more detailed analysis on the information that can be obtained from a stellar population through synthesis models.

In the case of extreme sampling effects, the integrated light does not provide any information about the cluster, and accurate age or mass determinations can only be done taking into account the theoretical probability distribution functions that will produce a distribution of possible physical properties compatible with the observations. The range of physical properties will be larger when the number of stars in the cluster is smaller (the range of stellar mass-luminosity ratio is larger than the range predicted by SSP models), and implies an intrinsic loss of precision in the global properties of the cluster (see Cervino & Luridiana 2007 for a more extended discussion). The only way to estimate precise ages in this situation is to obtain the most detailed information about the number of stars in each evolutionary phase, that is, to analyse the colour-magnitude diagram (i.e. the individual stars) of the clusters (e.g. Pellerin et al. 2006, Hernandez & Valls-Gabaud 2008). Unfortunately, the colour-magnitude analysis is not reliable for obtaining cluster masses, which are controlled by low-luminosity stars.

An alternative choice for a rough estimation of cluster ages is to look for signatures that are only present in a limited temporal range. In the case of young clusters, an example would be to look for young star signatures, such as Wolf-Rayet features or emission lines: the presence of these signatures implies a young cluster, but the absence of these signatures does not imply an old cluster, just the absence of massive stars! Again, this rough age estimation does not provide information about the cluster mass.

2.1 A simple test for sampling effects identification

The most trivial test to identify when sampling effects are essential for an accurate analysis is to use the Lowest Luminosity Limit (LLL) method described in Cervino & Luridiana (2004). The LLL implies that it has no meaning to compare a cluster with synthesis models (in a deterministic way) if the integrated luminosity of the cluster is lower than the luminosity of the most luminous star included in the model. This simple statement restricts the deterministic use of synthesis models to young clusters with masses larger than a few $10^4 M_\odot$ in the optical domain (Cervino & Luridiana 2004), which corresponds to a limiting magnitude of $M_V = -11$. In fact it is just a common-sense requirement: as an example, Zhang & Fall (1999) reject point-like sources fainter than $M_V = -9$ mag in the analysis of clusters in Antennae, since it is the luminosity of single luminous variable stars. However, based on the LLL requirement, not only “point-like” sources but all sources fainter than $M_V = -11$ should be rejected in their analysis. For example, Pessev et al. (2008) show that young (200 Ma $< t <$ 1 Ga) clusters in the LMC do not fulfill the LLL requirements, and therefore the use of synthesis models within a deterministic framework is useless because it may yield wrong results.

2.2 Implications for the initial mass cluster distribution estimation

In a recent pedagogical paper, Fall (2006) gives the relations between luminosity, mass and age distributions of young stellar clusters. A power-law luminosity function for young clusters is directly related to a power-law initial cluster mass function, under the assumption
that the mass-luminosity ratio depends only of the age of the cluster. We have shown that this assumption is only valid for the case of massive clusters. The current controversy on the shape of the initial mass function of clusters, where small differences between a possible power-law or a log-normal distribution are important, depend crucially on clusters with masses around $10^4 \, M_\odot$. As we have shown, this mass range is below the limit of application of synthesis models in a deterministic way, and a probabilistic framework is required for proper results, even though it implies an intrinsic loss of precision.

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