Transverse Enhancement Model and MiniBooNE Charge Current Quasi-Elastic Neutrino Scattering Data

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Abstract. Recently proposed Transverse Enhancement Model of nuclear effects in Charge Current Quasi-Elastic neutrino scattering [A. Bodek, H. S. Budd, and M. E. Christy, Eur. Phys. J. C71 (2011) 1726] is confronted with the MiniBooNE high statistics experimental data. It is shown that the effective large axial mass model leads to better agreement with the data.

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1 Introduction

Charge current quasi-elastic (CCQE) scattering is the most abundant neutrino reaction in oscillation experiments like MiniBooNE (MB) or T2K with a flux spectrum peaked below 1 GeV. Its full understanding is important for detail neutrino oscillation pattern measurements.

Under an assumption that the impulse approximation picture [1] is valid the CCQE reaction both on free and bound nucleons is defined as:

$$\nu + n \rightarrow l^- + p \quad \text{or} \quad \bar{\nu} + p \rightarrow l^+ + n$$ (1)

with $\nu$, $\bar{\nu}$, $l^\pm$, $p$ and $n$ standing for: neutrino, antineutrino, charged lepton, proton and neutron respectively.

A theoretical description of free nucleon target CCQE reaction is based on the conserved vector current (CVC) and the partially conserved axial current (PCAC) hypotheses. The only unknown quantity is then the axial form-factor $G_A(Q^2)$ for which one typically assumes the dipole form $G_A(Q^2) = G_A(0)(1 + Q^2/2M_A^2)^{-2}$ with a free parameter, called the axial mass $M_A$.

The aim of CCQE cross section measurements is to determine the value of $M_A$ and also to validate a nuclear physics input used in theoretical cross section computations. There is a variety of approaches [2] including the Fermi Gas (FG) model implemented in all the neutrino Monte Carlo (MC) events generators.

Measurements of $M_A$ can use an information contained in the shape of the distribution of events in the four-momentum transfer $Q^2$ (strictly speaking in the variable $Q^2_{QE}$, see [3]) which is sensitive enough for precise evaluations of $M_A$. The dependence of the total cross-section on $M_A$ gives an additional input: if the $M_A$ value is increased from 1.03 to 1.35 GeV for $E_\nu > 1$ GeV the cross-section is raised by $\sim 30\%$ (for $E_\nu < 1$ GeV the increase is smaller). Another interesting option to validate models is to compare to high statistics double differential (2D) cross section data (muon kinetic energy and scattering angle) on carbon provided by the MB collaboration [3].

In the past, several measurements of $M_A$ were done on deuterium for which most of nuclear physics complications are absent. Until a few years ago it seemed that the results converge to a value $\sim 1.03$ GeV [4]. There is an additional argument in favor of a similar value of $M_A$ coming from the weak pion-production at low $Q^2$. When put together they suggest the value $M_A = 1.014$ GeV [4]. On the contrary, all (with an exception of the NOMAD experiment) more recent measurements of $M_A$ report much larger values (for a discussion see [5]).

A theoretical mechanism which can explain the $M_A$ value discrepancy comes from the many-body nuclear model proposed 10 years ago [6] based on the ideas of M. Ericson and developed more recently by Martini, Ericson, Chanfray and Martinez (MEChM model). The model predicts a large contribution to the muon inclusive CC cross section from elementary $2p-2h$ and $3p-3h$ excitations leading to multinucleon ejection. The contribution is absent in a free nucleon neutrino reaction and in the MB event selection is treated as CCQE giving rise to effective large $M_A$ value.

A microscopic evaluation of the multinucleon ejection contribution was reported in [7]. The computations were done in the theoretical scheme which was succesfull in describing electron scattering in the kinematical region of QE and $\Delta$ peaks together with the dip region between them. The model was applied to MB 2D cross section data and a fit to the axial mass value was done. In the fitting procedure [8] the authors included an overall 10.7% normalization error. The two-parameter fit gave results: $M_A = 1.077 \pm 0.027$ GeV and for the normalization scale: $\lambda = 0.917 \pm 0.029$. Using the low-momentum cut procedure, as proposed in [9], with $q_{cut} = 400$ MeV the value
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Muon kinetic energy [MeV]

-1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1

2 Transverse Enhancement Model (TEM)

In [11] a new approach to model CCQE scattering on nuclear targets was proposed. The approach is intended to be easy to implement in MC event generators. It assumes that it is sufficient to describe properly an enhancement of the transverse electron QE response function keeping all other ingredients as in the free nucleon target case.

The authors of [11] proposed a transverse enhancement function for the carbon target. For low $Q^2$ its form is determined by the scaling arguments while for high $Q^2$ ($>0.5$ GeV$^2$) it is obtained as a fit to the inclusive electron cross section data from the JUPITER experiment. The

$$M_A = 1.007 \pm 0.034 \text{ GeV}$$

was obtained. There is still another approach to include 2p-2h contribution coming from Meson Exchange Current (MEC) diagrams [10]. It is shown that with the MEC contribution one gets closer to the MB experimental results.

3 Results and discussion

In our numerical analysis we compare predictions from two models:

A) effective large axial mass model (ELAMM) with $M_A = 1.35 \text{ GeV}$ together with the FG model with parameter values as in the MB experimental analysis: $p_F = 220$ MeV and $B = 34$ MeV.

B) TEM with the standard axial mass $M_A = 1.014 \text{ GeV}$ (as used in [11]). We investigate two implementations of the TEM: (i) as in the original paper: without Fermi motion and with Pauli blocking effect introduced by means of the NEUGEN $Q^2$ dependent reduction function; (ii) with the Fermi motion and Pauli blocking implemented via the FG model; we call the model: TEM-FG.

In both models modifications of the standard ($M_A \sim 1.03 \text{ GeV}$) theory are introduced in the $Q^2$ dependent way in agreement with the MB analysis of the 2D distribution of final muons (see Figs 11,12 in [3]).

We produced three samples of $10^6$ events using NuWro MC event generator [12]. We checked that statistical fluctuations are small. Because in the MB data there is a large

prescription to include TE contribution in the numerical computations amounts to rescaling of the magnetic proton and neutron form factors:

$$G^{p,n}_M(Q^2) \rightarrow \sqrt{1 + A Q^2 \exp(-Q^2/B)} G^{p,n}_M(Q^2)$$

where $A = 6 \text{ GeV}^{-2}$ and $B = 0.34 \text{ GeV}^2$.

TEM model offers a chance to explain an apparent contradiction between recent low (MB) and high (NO-MAD) neutrino energy $M_A$ measurements: for energies up to $\sim 700$ MeV the model predicts the CCQE cross section similar to effective large axial mass predictions with $M_A = 1.3 \text{ GeV}$. For higher neutrino energies the TEM cross section becomes smaller and at $E_\nu \sim 5 \text{ GeV}$ corresponds to $M_A \sim 1.15 \text{ GeV}$.

The aim of this paper is to confront the predictions of the TEM model with the MB CCQE data.
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Differential cross section $d\sigma/dQ^2_{QE}$

**Fig. 4.** Differential cross section in $Q^2$

| Model                      | $\chi^2_{2D}$ | $\chi^2_{2D,\text{min}}$ | $\chi^2_{Q^2}$ | $\chi^2_{Q^2,\text{min}}$ |
|----------------------------|---------------|---------------------------|----------------|---------------------------|
| ELAMM                     | 1.03          | 34.1                      | 1.075          | 15.8                      |
| TEM                       | 1.03          | 196.2                     | 1.015          | 22.3                      |
| TEM-FG                    | 1.135         | 133.3                     | 1.08           | 44.0                      |

Table 1. Results for 2D and $Q^2$ fits

Overall flux (normalization) error we introduce a renormalization factor to the $\chi^2$ statistical test defined as [8]:

$$\chi^2(\lambda) = \left( \frac{\lambda^{-1} - 1}{\Delta\lambda} \right)^2 +$$

$$\sum_{i=1}^{137} \left( \frac{d\sigma}{dQ^2_{QE}} \right)_{j}^{\text{exp}} - \lambda \left( \frac{d\sigma}{dQ^2_{QE}} \right)_{j}^{\text{th}} \right)^2$$

with $\Delta\lambda = 0.107$. It means that basically we compare the shapes of two-dimensional distributions of events.

Results are shown in Table 1 in the second and third columns. The number of degrees of freedom is $DOF = 137$ (the number of non-zero bins) $-1 = 136$. We see that both $\chi^2_{TEM,\text{min}}$ and $\chi^2_{TEM-FG,\text{min}}$ are larger than $\chi^2_{ELAMM,\text{min}}$. Additionally, the value $\chi^2_{ELAMM,\text{min}} = \frac{34.1}{136} \approx 0.25$ is much smaller than 1 which suggest that the shape errors were evaluated in a too conservative way.

Figs. 1, 2, and 3 show contributions to $\chi^2_{\text{min}}$ from three models. Contributions in bins are proportional to the area of boxes. If a box is crossed the model prediction is larger than the experimental data. We see that three patterns are rather different in shape and in the case of Figs. 1 and 2 they may indicate that the models do not reproduce the cross section $Q^2$ dependence very well. We will come back to this point later. In all the cases there is a significant deficit of events in the region $\cos \theta_\mu \sim 1$. These are low $Q^2$ events for which it is known that techniques going beyond the FG (like RPA or CRPA) should be used [13].

We made a similar statistical analysis with the $Q^2_{QE}$ differential cross section data. The number of bins is smaller (17) but the uncorrelated relative shape errors are also much smaller. The $\chi^2$ is defined as:

$$\chi^2(\lambda) = \left( \frac{\lambda^{-1} - 1}{\Delta\lambda} \right)^2 +$$

$$\sum_{i=1}^{17} \left( \frac{d\sigma}{dQ^2_{QE}} \right)_{j}^{\text{exp}} - \lambda \left( \frac{d\sigma}{dQ^2_{QE}} \right)_{j}^{\text{th}} \right)^2$$

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Additionally, the value $\chi^2_{ELAMM,\text{min}} = \frac{34.1}{136} \approx 0.25$ is much smaller than 1 which suggest that the shape errors were evaluated in a too conservative way.
three such bins: $T_\phi \in (500, 600)$ MeV and $\cos \theta_\phi \in (-0.1, 0.2)$ with contributions: 9.5, 7.2 and 5.5 (see Fig. 5). We checked that according to TEM events from the selected bins contribute only to two $Q_{QE}^2$ bins: (1.2, 1.5) GeV$^2$ and (1.5, 2) GeV$^2$ (cross sections $5.42 \cdot 10^{-48}$ cm$^2$ and $1.44 \cdot 10^{-47}$ cm$^2$/MeV$^2$ respectively). Fig. 6 shows that these contributions represent a small fraction of the overall cross section in the last $Q_{QE}^2$ bin and the large disagreement in three selected bins is hidden in the averaged $Q_{QE}^2$ analysis. It is clear that in order to get a deep insight into CCQE, the complete 2D data should be analyzed.

We conclude that it seems that the effective large axial mass model leads to better agreement with the MB data. However, one should remember that a real challenge is to provide predictions for the hadronic final states in multinucleon ejection effectively described by either ELAMM or TEM.

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