Design with Uncertainty: The Role of Future Options for Infrastructure Integration

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Abstract: Decision making for effective infrastructure integration is challenging because the performance of long-lasting facilities is often difficult to foresee or well beyond the designer’s control. We propose a new approach for integrating the construction/retrofitting of two or more types of facilities. Infrastructure integration has many perceived benefits, but practitioners also express serious doubts, particularly when it comes to civil engineering works. To substantiate this approach, we test all of the major options for integrating a ground source heat pump system with the construction/retrofitting of an archetypal office building. We use actual data from the United Kingdom, which represent a middle-of-the-road setting among major developed countries. The model highlights the sensitivity of the range of cost-effective solutions to the embedding of future options. The findings point to a clear need for appropriate standards for managing infrastructure integration. We expect this kind of model to find increasing applications among infrastructure complexes, particularly as cities become denser and more multifunctional.

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1 INTRODUCTION

In this article, we develop and test a new modeling approach for realistically assessing the costs and benefits of integrating the construction/retrofitting of two or more different types of infrastructure. Infrastructure integration is attracting increasing attention across different sectors, from building works to energy, transport, water, waste, and even information and communication technology, due to the potentially enormous benefits in cutting whole life-cycle costs, reducing key resource consumption, and improving service quality. However, integrative planning and design also face serious challenges that arise from hitherto little understood conflicts in scheduling construction and coordinating service provision. Infrastructure investors and designers (in both the public and private sectors) often have reasons to doubt whether infrastructure integration brings real net benefits, particularly in sectors where the construction process is already complex, the expected service life is long, key cost parameters are very uncertain, and the decision-making process is slow—circumstances that are particularly common in civil engineering works (Versikari and Soderqvist, 2003, April; Ellingwood, 2005; Langdon, 2007; INNOTRACK, 2009). Furthermore, even when such integration is an option, the medium- to long-term uncertainties are rarely considered rigorously within
the methodological framework; this is due, at least in part, to a lack of knowledge of how the uncertainties affect the optimality of the options chosen (Duthie et al., 2011).

Considering these challenges, we developed a new model that extends the “future options” approach to flexible design, as put forward by Ellingham and Fawcett (2006) and De Neufville and Scholtes (2011). The model is designed to assess the costs and benefits of embedding construction and retrofitting options in a realistic and robust way, allowing users to better integrate different types of infrastructure. In particular, the model is designed to address the challenges arising from construction process conflict, long life cycles, and significant uncertainties in costs, input prices, and other external conditions.

We hypothesize that incorporating flexibility in the design of integrated infrastructure complexes (e.g., buildings and utilities) can significantly increase the opportunities for overcoming uncertainties and, therefore, the value of investments and assets over their life cycles. We focus on cases with long life cycles, that is, where assets are expected to last around 50–100 years. Over their long service life, assets naturally face external conditions that deviate from what was known or assumed by the planner or designer at the outset; for instance, the recent drop in oil and gas prices was not anticipated by many decision makers in the previous decade. If, at the time of conception, a designer ignores the possibility that external conditions may evolve unexpectedly over time, then a substantial risk arises that the designed object will become inadequate (Martani, 2015). For events that depend on uncertain elements, predictions cannot be made in a deterministic manner. In these cases, approaches that take uncertainty in consideration, such as probabilistic approaches (Castillo and Alvarez, 1990), have to be used.

The issue of design optimization has been investigated frequently in structural engineering, particularly with regard to bridges (e.g., weight, shape, and cost optimization). The consolidated approaches used in this field are based on neural networks (Aldwaik and Adeli, 2014; Sirca and Adeli, 2005), fuzzy logic (Sarma and Adeli, 2000; Fisco and Adeli, 2011; Baroth et al., 2011), and evolutionary computing (Kociecki and Adeli, 2015). However, there is an apparent lack of studies of long-term cost optimization concerning the integration of energy infrastructure and buildings, in spite of the critical role this is believed to play in energy efficiency and reducing carbon emissions.

To ensure, as far as possible, that an infrastructure asset will meet a set of given needs not only under the conditions known at the design stage but also over the asset’s longlife time, a common practice is to take into consideration the most likely or average conditions. Despite the undoubtable merit of incorporating the time dimension in the design process, the traditional approach has proven to be problematic as it can lead to the “flaw of averages” (FoA), as opposed to the “law of averages” (Savage, 2009). The FoA refer to a mistaken assumption that appraising a project based on approximated average conditions gives a correct result; this way of thinking only applies in the few, exceptional cases when all relevant relationships are linear. The FoA can cause significant loss of value for an infrastructure project because focusing on the “average” or most probable situation often leads to the neglect of real variations (both risks and opportunities) over time. Contrary to probabilistic performance-based design, the future options approach does not aim to accommodate the most probable future needs. There is an emerging consensus that designing while anticipating multiple sets of possible future conditions is a better way of overcoming uncertainties in engineering design (De Neufville and Scholtes, 2011). Here, the most cost-effective method is not necessarily to create designs that can cope with all eventualities from the outset—in fact, it is often not feasible to do so—but to embed in the design potential options that can be called into service during the working life of the facility: a fast, easy, and economical way to adapt to changing conditions. The aim of this article is to extend this concept from individual engineering design projects to planning the integration of two or more different infrastructures.

The rest of this article is laid out as follows: Section 2 outlines the concepts and terms used in future options modeling, followed by the main steps taken in our approach. In Section 3, we describe our case study and how we applied the model to it, referencing the steps set out in Section 2. In Section 4, we outline and discuss our results, and in Section 5, we conclude.

## 2 METHODS AND GENERIC MODEL

Because future options models are still relatively new and therefore, uncommon, and, as far as we know, this is the very first effort to consider future options models in urban energy infrastructure integration, we begin by defining the main concepts and terms.

**Alternatives and options.** Although “alternatives” and “options” are often used interchangeably, the terms have more precise meanings in this case. When choosing between two or more possible solutions, a designer is deciding between alternatives. When he/she defers this choice to a later date, he/she is establishing options that can be called into service in the future (Ellingham and Fawcett, 2006). The purpose of establishing and calling
Uncertainty and risk. Although uncertainty is the state of lacking information about the nature, likelihood, and consequences of an event, risk in this context refers to the effect that uncertainty has on achieving objectives (Gigerenzer, 2002; International Organization for Standardization, 2009). There are two main types of uncertainties: epistemic, which arises from imperfect knowledge, and intrinsic which is attributable to natural variabilities and randomness (Ang and Tang, 1975; Kiureghian and Ditlevsen, 2009; Jahani et al., 2014). Epistemic uncertainty can be estimated, treated, and reduced through better theories, data, and modeling (De Finetti, 1990). By contrast, variables that have intrinsically high uncertainty cannot. For infrastructure planners and designers, external conditions such as energy prices or the long-term effects of climate change can be considered epistemic uncertainties.

The future options approach is an effort to address such uncertainties and any significant risks associated with them via a new epistemic framework. This framework can complement and work alongside a wide range of existing models that aim to reduce epistemic uncertainties, such as interval modeling (Change et al., 2001), Bayesian modeling (Cheung and Beck, 2010; Yin et al., 2010; Yuen and Mu, 2011), chaos theory (Schoefs and Yanez-Godoy, 2011), evidence theory (Dixon and Rilett, 2002; Zavadskas and Vaidogas, 2009), fuzzy modeling (Faturechi and Miller-Hooks, 2014; Zhang et al., 2014), Monte Carlo simulations Jahani et al., 2014), and gray system theory (Tseng et al., 2015).

It appears that the methods that deal with large uncertainties, such as energy price volatility and long-term effects of climate change are the least well developed in urban infrastructure planning and design. Therefore, we focus in this article on addressing such uncertainties. More specifically, we aim to define whether—and with what degree of flexibility—it is appropriate to integrate different infrastructures, based on total financial costs over a life cycle. As far as we are aware, this is the first attempt at incorporating future options when dealing with a highly uncertain but important option of urban energy infrastructure. The core problem of infrastructure integration can thus be translated into a modeling process where the decision maker faces uncertainties in both external conditions and interactions among the components of infrastructure complexes.

In the sections below, we outline the four steps that we use in our approach.

2.1 Defining alternative design and investment strategies

First, we define alternative designs and/or investment strategies. For major infrastructure investment, alternative designs must be thoroughly tested to enable robust decision making (Castillo et al., 2015). For projects where future demand is expected to rise, there are typically three generic alternatives (Fawcett and Hughes, 2014): (1) scale the project for the high demand forecast (high cost with risk of overinvestment); (2) scale the project for immediate needs only (low cost with risk of underinvestment); and (3) scale the project for immediate needs and incorporate flexibility for upgrading if and when need arises (potentially with an intermediate level of cost). The future options approach (3) allows the minimization of today’s irrevocable decisions and leaves as many decisions open as possible. It should be noted that while flexibility is attractive in principle, it should not be considered the best-performing choice a priori; this is because flexibility usually has an associated cost.

2.2 Recognizing and modeling long-term uncertainties

When designing a piece of energy supply infrastructure, it is necessary for the owners and managers to predict the outcomes of interventions in building energy supply as well as to have sufficient insight into how energy generation and supply will evolve (Lee et al., 2013). The projections or forecasts of contextual conditions that influence design decisions are a major source of long-term uncertainty. As forecasts are “always wrong” (De Neufville and Scholtes, 2011), such uncertainties need to be handled better. Recognizing and modeling such uncertainties begins with identifying the external conditions that could significantly affect the core objectives of the infrastructure facility; for renewable energy supply systems, for example, these would include energy prices and regulatory policies.

2.3 Defining future options triggers

For those alternatives that consider future options, there is the need to define the conditions under which the options are called into service. These are known as “triggering conditions” and may be case specific; however, for financial analysis they are usually related to the trade-offs between the expected savings resulting from calling the option into service and the risks associated with embedding the option in the design at the outset. Fawcett et al. (2012) point out that there is a difference between calling upon the option at the first moment when it may be justified...
and identifying the best moment to implement it to maximize the benefits; this creates complexity as the first moment may not be optimal. The first-moment approach compares the value of the trigger variable(s) with a prespecified threshold value. By contrast, the best-moment approach depends on knowing both the immediate and life-cycle savings. Indeed, in some cases where external conditions are volatile—as with energy prices—it makes sense to examine the entire life-cycle cost to determine when implementing the option would maximize the savings. In the context of ground source heat pumps (GSHPs), we add one more eventuality: procrastination in decision making. This occurs when the managers of the building and GSHP asset have neither the time nor the inclination to identify the best moment or fail to call the option into service even when the best moment is known to them.

2.4 Addressing uncertainties through simulation

For highly uncertain external inputs, Monte Carlo simulation is a standard method used to generate test cases in sufficiently large numbers to minimize the variance in model estimations; here, the uncertain conditions are represented using random sampling from explicitly defined probability distributions (Marseguerra and Zio, 2013). The probability distribution of the uncertain variables incorporates any knowledge, expectations, and beliefs in an explicit way (Martani et al., 2013). A key advantage of testing randomly sampled test cases—as opposed to inputting the model average values—is avoiding the FoA.

Notably, although Monte Carlo simulation is widely used in studying individual engineering projects, there are surprisingly few such applications in planning and designing infrastructure integration. Here, because there are often a considerably larger number of variables to be considered, the number of test cases needed grows exponentially with the number of pieces of infrastructure involved. As a result, the number of tests performed for such complex tests is alarmingly low.

As we have deliberately kept our test example simple for verification purposes, computation time is not a particular issue. However, as the number of pieces of infrastructure involved grows, computation may become an issue. For more complex cases, it would be necessary to seek computationally more efficient methods than Monte Carlo simulation. In this article, we take advantage of a simple case study to implement all of the tests in the Monte Carlo simulation, making the approach easy to grasp. We then test a sample of the scenarios using Latin hypercube sampling (LHS), which has the potential to reduce the test runs required so that more complex models can be tested easily across the construction industry. We test those input variables that have a reasonably certain range of variability (as a result of either reduced epistemic uncertainty or policy/regulatory control) using a selection of the most relevant values for decision making.

The four steps outlined above are summarized in a flowchart in Figure 1. First, the contextual conditions that determine the long-term costs of an integrated infrastructure complex are used as inputs to define the boundary conditions. We differentiate between three types of variables: (1) variables that are fairly well known to the designers and investors, which are inputted directly into the model; (2) inputs that vary within reasonably narrow and well-known bounds, which are represented through carefully selected values that are informative for decision making; and (3) inputs that have near-random variability, which are initially fed into the simulation as Monte Carlo test cases, after which repeat runs are made using LHS. Next, we define three generic design approaches: (1) All Initial, representing the implementation of the basic design and options all in one go, with all capacities operating from the beginning; (2) No Initial, where only the basic design is implemented and operated, with no regard for any future options; and (3) Flexible Initial, which is a design with a degree of flexibility embedded. For this last approach, a trigger strategy is defined so that the embedded options may be called into service during the infrastructure’s lifetime if and when the trigger conditions are met. Of course, more than one variant of Flexible Initial may be tested. The total number of test runs is a product of the number of simulations required to obtain a stable mean for the Monte Carlo/LHS draws multiplied by the number of sensitivity tests with the selected values for the variables with narrow variability bounds.

3 MODEL APPLICATION

In this article, we develop a case study model based on realistic data and the application of infrastructure integration. The case study is kept as simple as possible to demonstrate the key features of the approach in a manner that is easy to grasp. We study the integration of a GSHP installation with an archetypal office building in a relatively dense urban area. Although take-up is still exceptionally slow, GSHP is among the most important options in decarbonizing building energy use, particularly in urban areas (DECC, 2012). In dense urban areas where recent population and employment growth has been significant, incorporating GSHP has proved to be a major challenge. Following a number of unsuccessful pilot projects in the United Kingdom to install and
operate GSHP systems in high-profile office buildings, there is a widely held belief that GSHP systems are not cost-effective, particularly given the recent volatility in energy prices. Outside a handful of flagship green building proposals, few businesses currently intend to implement GSHP. Even in the context of green buildings, there is ongoing debate on whether GSHPs represent the best value for achieving sustainable building energy outcomes. Nevertheless, many businesses and developers are deeply concerned by the likely future rise in energy prices as well as climate-impacting carbon dioxide emissions and wish to incorporate as many sustainable energy solutions as possible in their future investments, where there are robust financial justifications for the cost of doing so.

Given the high levels of uncertainty over energy prices, engineering system performance, and government funding constraints, decision making for long-lasting investments such as GSHP systems is challenging. Indeed, the decision to install and operate GSHP systems is subject to the combined uncertainties across these factors. Furthermore, the long-term nature of the investment makes it necessary to foresee how its design and installation may adapt to different potential circumstances (Martani et al., 2014).

To demonstrate the capabilities of our future options approach, we have parameterized an infrastructure integration model for a GSHP–office building development. We use actual data from the United Kingdom to test the impact of energy prices, building energy loads, and GSHP system performance, given the partial and fragmented initial knowledge of these aspects and the long asset life cycle. We incorporate unavoidable uncertainties and randomness in the external boundary
conditions to test the effectiveness of embedding future options.

Although the building in our case study is hypothetical, both the plot of land and building dimensions are typical of those in new developments in U.K. cities. The plot measures 8,100 m² (90 m × 90 m). We assume a high net job density of 100,000 employees/km², which implies that the building will host 810 employees within 16,200 m² of floor space at 20 m²/employee. Of the three archetypal building forms—tower, mini-blocks, and court—we opt for the court configuration because of its low energy potential and benign daylighting impact in densely populated areas (Martin and March, 1972). It is also an increasingly popular form in new urban developments in the United Kingdom. This particular building form translates in our case study to a five-story building with a footprint of 3,744 m².

The GSHP system in our case study requires 203 boreholes (the use of a closed system has been assumed and the boreholes will have associated pipework) of 150 m deep (calculation based on CIBSE, 2008). This equates to a total cost of £1,800,000 (calculation based on Kavanaugh and Rafferty, 1997; GSHP Association, 2011), including £1,200,000 for boreholes and £600,000 for heat pumps (HPs) (source: calculation based on HeatPumpsDirect.co.uk).

In our model, the total cost of integrating GSHP can be defined as:

\[
T_c = C_{\text{initial}} + \sum \frac{1}{(1+i)^{yn}} C_{\text{retrofitting}} - \sum \frac{1}{(1+i)^{yn}} RHI + \sum \frac{1}{(1+i)^{yn}} C_{\text{operating}} \tag{1}
\]

where \( C \) is the cost, \( i \) is the discount rate, \( yn \) refers to the years when the associated costs are incurred and \( \Sigma \) denotes the summation of all costs within the same category over the life of the structure (Adeli and Sarma, 2006). The cost terms in the right-hand side of (1) are the costs in the year they actually occur. The \( 1/(1+i) \) \( yn \) factor is used to convert the cost into its present value discounted by the discount rate \( i \) for period \( yn \) (Sarma and Adeli, 2000). According to Tietz (1987), a discount rate of 3% above the rate of inflation is considered an appropriate value for such infrastructure investments, although businesses may typically set their own discount rates for asset investments that vary between 0% and 12%. Operating costs \( (C_{\text{operating}}) \) are not fixed values; instead, they depend on both the design strategy and uncertain external conditions. For this reason, two approaches are developed to compute \( C_{\text{operating}} \): with GSHP and without GSHP. When the GSHP system is not installed, the operating costs \( (C_{\text{operatingNoGSHP}}) \) are computed as follows:

\[
C_{\text{operatingNoGSHP}} = \sum \frac{1}{(1+i)^{yn}} ((D_{\text{cool}}, C_{\text{elec}}) + (D_{\text{heat}}, C_{\text{gas}})) \tag{2}
\]

where \( D_{\text{cool}} \) and \( D_{\text{heat}} \) are the demand for cooling and for heating respectively, while the \( C_{\text{elec}} \) and \( C_{\text{gas}} \) the cost of electricity and gas respectively.

Where the GSHP system is installed, the operating costs \( (C_{\text{operatingGSHP}}) \) are computed as follows:

\[
C_{\text{operatingGSHP}} = \sum \frac{1}{(1+i)^{yn}} \left( \frac{(D_{\text{cool}} + D_{\text{heat}}), C_{\text{elec}}}{COP} \right) \tag{3}
\]

where COP is the coefficient of the system performance and represents the kWh produced by the GSHP for each kWh of electricity consumed. Typically, COP is greater than one and is often in the range of two to four.

Equations (2) and (3) are based on the assumption that traditional systems in the United Kingdom include heating via gas and cooling via electricity. Instead, when the GSHP is installed, both heating and cooling switch to the GSHP, which consumes only electrical power; thanks to \( \text{COP} > 1 \).

In Figure 2, the generic method outlined in Section 2 is applied to our case study. In this application, the nearrandom variables on which the uncertainties are modeled consist of gas and electricity prices, heating and cooling loads, and the COP for the GSHP system, which is subject to particular soil conditions and construction quality. The variables that have narrow and more firmly known bounds include the discount rate for net-present-value computations and the level and availability of renewable heat incentives (RHI).

For Flexible Initial, we test the two extremes of triggering: (1) the first and best is a trigger that considers the most cost-effective year (assuming building managers are able to estimate potential costs) over a 20-year period from the outset in terms of the life-cycle costs; (2) the second and worst accepts that the trigger is activated at random (considering the future option that boreholes were built into the design) over the 20-year period. We believe that all other situations are likely to fall between these two situations.

The simulations are performed over a 50-year period for life-cycle analysis, and the number of Monte Carlo draws are determined such that the discrepancy between the cumulative mean and the true mean (which is defined as the mean obtained after 1 million draws) is less than 0.1%. This ensures that the cost estimates are well within the margin of error for investment decisions.
Three alternatives are formulated in line with the generic typology (Figure 3).

**Alternative 1: All Initial.** The entire GSHP system is installed when the office building is constructed (conventional cooling and heating—including hot water—systems are included as part of the building cost in all the scenarios; under Alternative 1, the conventional systems form a necessary back-up). This consists of a total investment in the GSHP system of £1,760,000 (£1,200,000 for 203 boreholes and £560,000 for HPs); 76 boreholes costing £450,000 are placed under the foundations of the building and 127 boreholes costing £750,000 are placed in the remaining grounds. Here, a maximum number of boreholes are installed at the lowest cost as the underground construction work takes place prior to the foundations being laid. The GSHP supplies all of the building’s cooling and heating needs (including hot water).

**Alternative 2: No Initial.** GSHP is not integrated when constructing the building. This scenario precludes the installation of a full-scale GSHP system, although it may be possible to implement a reasonably full-scale system at a later stage through directional drilling, as in shale gas fracking (the eventuality that directional drilling will become cheap enough for GSHP applications is among the many uncertainties not tested in this article; such uncertainties could be tested in this model as further alternatives). Under this scenario, the building uses a conventional system with gas for heating (including hot water) and electricity for cooling.

**Alternative 3: Flexible Initial.** A smaller initial investment is made to create 122 boreholes under the
Fig. 3. Alternative designs (green boreholes: installed with building construction; red boreholes: installed after the option trigger).

building’s foundations at a cost of £721,200. If the GSHP option is called upon at a later date, a further 81 boreholes may be installed in the remaining grounds (at a cost of £478,800, translated into prices for that construction year); the GSHP equipment, which costs £560,000, is only purchased when the option is called into service. Before GSHP is adopted, the building’s energy is supplied by conventional systems, as in the No Initial scenario.

3.2 Recognition and modeling of long-term uncertainties

Table 1 summarizes all of the main parameters that are required for defining the model tests. Some of the inputs in Table 1 are fixed because of design choices made, whereas others are variables with an assumed mean and a function of the associated standard deviation (d). The mean is derived from external projections, whereas the deviation is defined as either a function of time (f) only, which means that it increases over time (e.g., energy prices), or both a function of time and a coefficient of technological development (c), which means that as time passes, technology also evolves (e.g., COPs). There are also situations where the deviation is defined as a function of user circumstances (u; e.g., loading hours for heating and cooling).

Here, we define probability distributions for the three types of variables that have a strong influence (Baroth et al., 2011) on the total cost of the infrastructure in the long term (as computed in Equation 1) and are highly uncertain over a 50-year operating lifespan: (1) gas and electricity prices, (2) demand for heating and cooling loads, and (3) the coefficient of GSHP performance (COP). For each of the variables, we define a probability distribution for each of the 50 years. A widening of the variability range represents a lowering degree of confidence in the projections over time (De Finetti, 1990). The annual mean values for gas and electricity prices are taken from National Grid (2011) projections, which suggest that the probability spread will get progressively wider, although there is a distinct tendency for the entire range to shift gradually toward higher prices; this is not unreasonable given the long-term constraints of nonrenewable sources (Figures 4 and 5).

The heating and cooling loads are derived from estimates reported on the GI Energy and ASHRAE Web sites (2015), where the annual loads are sampled from the probability distributions (see top of Figure 6). As ASHRAE is a U.S. database, we use the values for the city of Seattle, considering its similarity to Southern England in terms of climate conditions. The range of possible values for the COP is given according to the general soil characteristics of London, in line with research carried out at the Department of Engineering of the University of Cambridge. The probability distributions for the COP shift gradually upwards over time to the high end of the range, in line with expected technical progress (see bottom of Figure 6).

In addition, a further set of variables is examined through sensitivity tests: RHI subsidies for GSHP users (ICAX Web site, 2014; DECC, 2013), the discount rate and the additional engineering costs for calling upon the GSHP future option. In line with infrastructure planning and design terminology, we define each unique combination of input variables as a scenario. Each scenario is simulated under each of the four
Table 1
Input values, sources, and deviations from the mean

| Inputs                                      | Values          | Values source                      | Deviation (D) from the mean value |
|--------------------------------------------|-----------------|------------------------------------|-----------------------------------|
| Land surface                               | 8,100 m²        | Assumption                         | /                                 |
| Building surfaces                          | 16,200 m²       | Assumption                         | /                                 |
| Occupancy rate                             | 20 m²/pers.     | Assumption                         | /                                 |
| GSHP system dimension                      | 203 Boreholes   | Computation                        | /                                 |
| Max. no. of bars under the building        | 122 Boreholes   | Design choice                      | /                                 |
| Max. no. of bars in the gardens            | 81 Boreholes    | Design choice                      | /                                 |
| Renewable heat incentives (RHI)            | 9 p/kWh produced| Given (ICAX Web site, 2014; DECC, 2013) | /                                 |
| Retrofitting costs                         | £0              | Assumption                         | /                                 |
| Discount rate                              | 6%              | Assumption                         | /                                 |
| Coefficient of performance (COP)           | Variable        | Interval of possibilities for London | d = F(t, c)                       |
| Gas price                                  | Variable        | Mean: projections (National Grid, 2011) | d = F(t)                         |
| Electricity price                          | Variable        | Mean: projections (National Grid, 2011) | d = F(t)                         |
| Loading hours (LH) for heating             | Variable        | Mean: projections (GI Energy, 2014; ASHRAE, 2014) | d = F(u)                        |
| Loading hours (LH) for cooling             | Variable        | Mean: projections (GI Energy, 2014; ASHRAE, 2014) | d = F(u)                        |

3.3 Defining the triggers

We define the basic trigger as an overall positive net financial saving, which is in turn defined as the difference between the net saving on energy expenditure by using GSHP (compared to using the conventional systems), the capital cost of GSHP, and any renewable energy subsidies. The savings are defined in terms of net present value, which corresponds to the savings of operating GSHP over the period, discounted back to Year 0 (i.e., 2015); the capital costs are similarly discounted.

We assume the following construction and operating timelines: investment decisions are made in Year 0; the building is designed and constructed in Years 1–5; and the building and installations become operational in Year 6. This applies to all tests. For any future options, the feasible trigger years are from Year 6 to Year 26. After Year 26, it is deemed too late to call upon the GSHP option.

Model simulations are run for each of the 20 triggerable years, and the year that results in the highest positive total net present value (NPV) is selected as the trigger year for the Flexible Initial scenario. If the net savings are not positive in any of the 20 years, then the GSHP option is not triggered; however, in such a situation the initial costs for embedding the option are still included in the costs. The RHI is assumed to be available for this scenario.

Three further Flexible Initial scenario variants are considered. The first is a variant where, for whatever reason, the GSHP option is called into service at random during the 20-year period. The RHI is assumed to be available for this variant, but instead of assuming the option is implemented in year of least cost (i.e., assuming perfect foresight), we define the net NPV as the average over the 20-year triggerable period. This variant is therefore named as “Flexible Initial (average cost).” The second variant is the same as the first except that the RHI is only available at 25% of the current DECC offer. The third variant is the same as above except no RHI is available.

3.4 Model tests

Monte Carlo simulations are run until the test results are stable. Each simulation is defined through: (1) sampling from the probability distributions of the highly uncertain variables; (2) computing the capital and operating costs for each year; (3) for option-embedding alternatives, checking the triggerable years to determine
Fig. 4. Gas price assumptions, National Grid (2011). The top line of the bottom matrix reports a discrete sequence for possible gas prices over the next 50 years (in £/kWh). Each line that follows shows the yearly probability distribution (in percentage) of the prices. To clarify, the upper box presents an enlargement of the probability distribution over the first 15 prices for the initial 5-year period.

if and when the option is triggered; and (4) recording the test run for building up the cumulative costs and savings profiles for the scenario.

In step (1), a distinction has to be made between variables. Although for energy prices and COP, a random number is sampled for each year in order to associate it with the yearly probability distribution, the approach used is different for loading hours. The loading hours for both heating and cooling from the second year of operations are assumed to vary within ±10% of the first-year values. The degree of variation within this range is again simulated randomly using Monte Carlo sampling. The reason for this is that it is very improbable that a user will have totally different consumption within the spectrum of possibility of a probability distribution from one year to another. Instead, it would be reasonable to assume that the value of loading hours for the first year represents an energy consumption behavior profile for a particular tenant, who is likely to occupy the building for a number of years and therefore set a user pattern that will not vary by more than ±10% each year.

In other words, each test run can be seen as representing one distinct user pattern, where the user profile and system efficiency take on particular characteristics when the building is in use.

For each design/investment alternative, the number of Monte Carlo test runs is determined by the stability of the mean outturn costs, which are in turn calculated as the percentage discrepancy between a pseudo-true mean (after running 1 million Monte Carlo draws, offering a wide safety margin) and the cumulative mean of the Monte Carlo simulations. We settle arbitrarily on a discrepancy of 0.1%, which we consider robust enough for making investment decisions. In all scenarios, the Monte Carlo simulation reaches this cut-off point well before 30,000 draws, and for comparability, we run all scenarios with 30,000 simulations (Figure 7).

3.5 Sensitivity tests for variables with narrow bounds

The sensitivity tests are defined by modifying the variables one at a time. The scope of the analysis is
to define the impact of all of the major factors that influence the trigger strategy and GSHP operating costs on the total NPV of the costs/savings. Factors include: (1) varying the underlying mean energy prices from the National Grid projections to 50% price levels for gas and electricity, which represents more recent trends in the energy markets; (2) retaining or removing RHIs; (3) varying the discount rates from 6% to 0.5%, 3%, and 12% (this is due to the wide spread of discount rates currently used by different investors and the difficulty forecasting their evolution over the next 50 years); (4) considering an additional nuisance-abating cost of £200,000 for installing the GSHP system under Flexible Initial (i.e., managing the engineering works at a later date when the building is already in use); and (5) considering the possibility of using an alternative method for sampling the values of variables. For the sake of computational efficiency, we test the LHS method as an alternative to Monte Carlo, which may be necessary in complex infrastructure integration projects.

To retrofit GSHP heating at a later stage, it is also necessary to have a low-temperature heat emitter, which costs more than a regular unit. However, this has not been considered here as an additional retrofitting cost. This is because low-temperature heat emitters are generally encouraged for energy efficiency purposes, and we therefore assume that such heaters are installed across all scenarios. Table 2 provides a list of the sensitivity tests undertaken.

4 RESULTS AND DISCUSSION

An extensive range of model tests shows that the building energy costs (including any GSHP installations) for the four scenarios fall within a fairly narrow range in the U.K. case study. This fits the middle-of-the-road setting in the United Kingdom in terms of energy costs and RHIs. This compares to the U.S. case at one extreme, where gas and electricity prices are low, with few incentives for GSHP, and the German case at the other, where energy prices are high, with good incentives and strong technical know-how for GSHP. Under Sensitivity Test A, which applies the National Grid’s energy price projections, a discount rate of 6% and the RHI subsidies, the All Initial scenario (total NPV building energy costs of £3.06 million) outperforms the No Initial scenario (£4.25 million) in terms of costs saved. This indicates that the RHI level in the United
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Fig. 6. Top: Probability distribution assumptions (in percentage) over the loading hours for heating (including hot water) and cooling. The top line of the matrix reports a discrete sequence of the possible loading hours for the building based on data from GI Energy and ASHRAE. Bottom: COP assumptions for the GSHP system from a sample of 6 years between Year 1 and the end of the triggerable period (i.e., 20 years since the opening of the building). The top line of each matrix reports a discrete sequence of the possible COPs for London ground conditions. The second row in each data block shows the annual probability distribution (in percentage).

Kingdom may have been chosen judiciously for GSHP installations. The Flexible Initial (least-cost) scenario outperforms both scenarios, and the All Initial scenario in particular, by a considerable margin, with a total NPV cost of £2.41 million (Figure 8). If the trigger year is decided at random over the 20-year triggerable period, the Flexible Initial (average-cost) scenario results in costs (£3.30 million) that are moderately higher than the All Initial scenario but still significantly lower than under the No Initial scenario. If the RHI is reduced to 25% of its current value, then the Flexible Initial scenario becomes financially unattractive (£4.05 million) and not significantly different from the No Initial scenario.

To investigate the variabilities of the results systematically, 11 further scenarios (B–L) are tested. The headline inputs for these sensitivity tests are compared in Table 2, while the results are summarized in Table 3.

As in Sensitivity Test A, the sensitivity tests treat No Initial and All Initial as fixed scenarios with no recourse to trigger optional investments. If the discount rate is lowered to 3% or 0.5%, Flexible Initial (least-cost) remains the approach with the lowest costs, even though the cost of all solutions increases significantly because of the changed basis for NPV calculations. On the other hand, if the discount rate reaches 12%, the cost of the GSHP investment becomes a much more significant share of the costs compared to the GSHP savings over time; in this case, the No Initial scenario is the clear winner. The most topical result is that the Flexible Initial (least-cost) scenario tends to be resilient to a reduction in energy prices and the removal of RHI and thus remains the lowest-cost scenario when gas or electricity prices fall by 50% and RHI is not accounted for, so long as the timing of the trigger is closely monitored to allow judicious decision making.
Fig. 7. Progressive stabilization of results (in percentage) over 30,000 simulations.

Table 2
Sensitivity factors (values in £1,000): Test A is the default condition

| Sensitivity tests | Retrofitting costs | Electricity price | Gas price | RHI | Discount rate |
|-------------------|--------------------|-------------------|-----------|-----|---------------|
| A                 | £0                 | 100%              | 100%      | Yes | 6%            |
| B                 | £0                 | 100%              | 100%      | Yes | 0.5%          |
| C                 | £0                 | 100%              | 100%      | Yes | 3%            |
| D                 | £0                 | 100%              | 100%      | Yes | 12%           |
| E                 | £0                 | 50%               | 100%      | Yes | 3%            |
| F                 | £0                 | 100%              | 50%       | Yes | 3%            |
| G                 | £0                 | 100%              | 100%      | No  | 3%            |
| H                 | £0                 | 50%               | 50%       | Yes | 3%            |
| I                 | £0                 | 50%               | 50%       | No  | 3%            |
| J                 | £0                 | 50%               | 50%       | No  | 12%           |
| K                 | £0                 | 50%               | 50%       | No  | 0.5%          |
| L                 | £200               | 100%              | 100%      | Yes | 6%            |

As mentioned above, an additional cost of £200,000 is assumed to be required to exercise the options in scenario L, for example, to cover nuisance-abating costs such as minimizing the disruption caused by construction work and grounds for the duration of the engineering works. Even considering this cost, the Flexible Initial (least-cost) scenario is still the most cost-efficient overall.

In addition to the sensitivity analysis scenarios listed in Table 3, a further set of tests was performed using LHS (Huntington and Lyrintzis, 1998) in place of the Monte Carlo method to evaluate the performance of different sampling methods. We defined a broad-brush LHS strategy with only four equal intervals, which radically lowers the number of test runs to 256 with three types of random variable. However, the results from these broad-brush LHS tests produced the same rankings as the Monte Carlo tests, with the Flexible Initial (least-cost) scenario being identified as the best solution, with a NPV cost of £2.80 million (compared to £2.41 million under the Monte Carlo simulation); this was followed by All Initial, with £2.83 million (compared to £3.05 million under Monte Carlo), and No Initial, with £4.17 million (compared to £4.25 million under Monte Carlo).

Refining the LHS method further (for instance, by introducing hierarchical LHS; see Vofechovský, 2015) should be considered in future studies of infrastructure integration involving more pieces of infrastructure. An additional element that could also be taken into consideration when running the simulations is the additional cost of national low-carbon emission policies under the No Initial scenario. Many countries, such as the United Kingdom, require new building designs to meet carbon emission targets through energy-saving features and/or renewable technologies. As such, if a GSHP system is not invested in initially, other green measures may have to be employed, resulting in additional costs not considered in our analysis.
Table 3
Summary of headline results: Tests A–L (values in £1,000)

| Tests | No Initial | All Initial | Flexible Initial |
|-------|------------|-------------|------------------|
|       | Mean       | Standard Deviation | Mean       | Standard Deviation | Mean       | Standard Deviation | Mean       | Standard Deviation |
| A     | 4.245      | 997          | 3.054          | 525          | 2.41        | 191           | 3.302        | 492          | 4.046        | 604          |
| B     | 14.837     | 3.478        | 8.568          | 1.838        | 6.091       | 716           | 7.537        | 1.183        | 10.472       | 1.631        |
| C     | 7.898      | 1.853        | 4.881          | 975          | 3.645       | 359           | 4.882        | 754          | 6.397        | 983          |
| D     | 1.682      | 396          | 1.844          | 210          | 1.511       | 85            | 1.901        | 255          | 2.125        | 288          |
| E     | 4.242      | 928          | 2.398          | 507          | 1.857       | 279           | 2.724        | 312          | 3.788        | 467          |
| F     | 7.605      | 1.852        | 4.881          | 975          | 3.922       | 616           | 5.238        | 824          | 6.303        | 981          |
| G     | 7.898      | 1.853        | 6.609          | 1.029        | 5.512       | 636           | 6.375        | 976          | n.a.         | n.a.         |
| H     | 3.949      | 926          | 2.398          | 507          | 1.824       | 271           | 2.63         | 311          | 3.694        | 465          |
| I     | 3.949      | 926          | 4.126          | 514          | 3.608       | 400           | 4.049        | 518          | n.a.         | n.a.         |
| J     | 841        | 198          | 1.885          | 110          | 1.412       | 157           | 1.489        | 148          | n.a.         | n.a.         |
| K     | 7.419      | 1.739        | 6.4            | 966          | 5.524       | 700           | 6.403        | 865          | n.a.         | n.a.         |
| L     | 4.245      | 997          | 3.055          | 525          | 2.551       | 191           | 3.388        | 492          | 4.131        | 604          |

5 CONCLUSIONS

In the middle-of-the-road setting considered in our study, the financial case for integrating a GSHP system with an office building appears to have very narrow margins in terms of net benefits. Our analysis shows that the setting of RHI appears to have been made with GSHP in mind, giving GSHP a narrow lead in terms of cost effectiveness under a range of practical circumstances, especially when discount rates are kept low. We find that carefully designed flexible future options have a very important role to play in both expanding the range of cost-effective applications and providing significantly improved financial performance for flagship organizations spearheading green buildings, even when the costs are somewhat higher than those of conventional energy solutions.

Our tests show that flexible options do not necessarily perform the best in terms of cost-efficiency. This is particularly the case when external boundary conditions are volatile or exercising future options involves additional costs or management barriers. Careful design and systematic modeling that take full account of the main uncertainties influencing financial and technical performance are therefore of critical importance. More specifically, embedded options that require a relatively low level of initial investment are likely to be the most effective. In the context of GSHP systems, this implies that the form and footprint of new buildings could make a very great difference to the life-cycle costs of...
prospective applications. Building plan configurations that allow for subsequent GSHP installations may be advantageous in lowering retrofitting costs, although this approach may need to be tempered by employing other sustainability applications in the initial design. All of the above implies that appropriate standards in these emerging areas of infrastructure planning and design are urgently required. New technologies, such as directional drilling, could also be a relevant consideration in the future, particularly if they become more affordable.

Our model provides a systematic assessment of the key dimensions of decision making when considering infrastructure integration. It highlights how sensitive the range of cost-effective solutions is to the setting of RHIs, discount rates, the technical performance of GSHP systems and proper life-cycle asset management for interdependent pieces of infrastructure. The generic model developed in this article can also be extended to cover the integration of other types of infrastructure integration where financial and technical performance are subject to interdependencies during the construction and operation phases and where very uncertain external conditions play a prominent role.

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