The straintronic spin-neuron

Ayan K Biswas\textsuperscript{1}, Jayasimha Atulasimha\textsuperscript{2} and Supriyo Bandyopadhyay\textsuperscript{1}

\textsuperscript{1}Department of Electrical and Computer Engineering, Virginia Commonwealth University, Richmond, Virginia 23284, USA
\textsuperscript{2}Department of Mechanical and Nuclear Engineering, Virginia Commonwealth University, Richmond, Virginia 23284, USA

E-mail: sbandy@vcu.edu

Received 3 April 2015, revised 19 May 2015
Accepted for publication 27 May 2015
Published 26 June 2015

Abstract
In artificial neural networks, neurons are usually implemented with highly dissipative CMOS-based operational amplifiers. A more energy-efficient implementation is a ‘spin-neuron’ realized with a magneto-tunneling junction (MTJ) that is switched with a spin-polarized current (representing weighted sum of input currents) that either delivers a spin transfer torque or induces domain wall motion in the soft layer of the MTJ to mimic neuron firing. Here, we propose and analyze a different type of spin-neuron in which the soft layer of the MTJ is switched with mechanical strain generated by a voltage (representing weighted sum of input voltages) and term it \textit{straintronic spin-neuron}. It dissipates orders of magnitude less energy in threshold operations than the traditional current-driven spin neuron at 0 K temperature and may even be faster. We have also studied the room-temperature firing behaviors of both types of spin neurons and find that thermal noise degrades the performance of both types, but the current-driven type is degraded much more than the straintronic type if both are optimized for maximum energy-efficiency. On the other hand, if both are designed to have the same level of thermal degradation, then the current-driven version will dissipate orders of magnitude more energy than the straintronic version. Thus, the straintronic spin-neuron is superior to current-driven spin neurons.

Keywords: spin neuron, neural networks, straintronics, nanomagnets

(Some figures may appear in colour only in the online journal)

1. Introduction
The building blocks of neural computing architectures are ‘neurons’ usually connected to each other and to external stimuli through programmable ‘synapses’. The transfer function of a neuron can be expressed as

\[ O = f \left( \sum_i w_i x_i + b \right) \]  \hspace{1cm} (1)

where \( f \) is some nonlinear function, \( w_i \)-s are programmable weights of synapses, \( x_i \)-s are the input signals (representing dendrites), \( b \) is a fixed bias and \( O \) is the output (representing a neuron’s axon). In threshold operations, \( f \) mimics a Heaviside unit step function whose value is 1 if the argument is positive and 0 otherwise.

Neural networks have myriad topologies, such as cellular neural network [1], feed-forward network [2], convolutional neural network [3], hierarchical temporal memory [4], etc. However, the basic unit of computing i.e., the neuron, remains more or less invariant across all topologies and its operation is governed by equation (1). In a conventional neural network, CMOS operational amplifiers carry out the threshold operation of equation (1) [5] and dissipate exorbitant amounts of energy. To a large extent, this has stymied the progress of neural computing. Alternate implementations to lower the energy dissipation have been proposed in recent years [6–8] and utilize a magneto-tunneling junction (MTJ) whose soft layer is an anisotropic nanomagnet with two stable magnetization orientations. Input variables are encoded in spin-polarized currents that are summed with variable weights to produce a net spin polarized current which is driven through the nanomagnet. When the net current exceeds a
threshold value, the magnetization of the soft layer rotates from one stable orientation to the other, thereby changing the resistance of the MTJ abruptly. This implements the threshold firing behavior of a neuron. These types of artificial neurons have been termed ‘spin-neurons’ and unlike CMOS-based neurons, they are ‘non-volatile’ since the final state of the neuron can be stored in the magnetization state of the nanomagnet (and therefore the resistance of the MTJ) after the device is powered down. In [8], the soft layer of the MTJ is a nanomagnet possessing perpendicular magnetic anisotropy (see figure 3(a) of [8]) and it is switched with spin-polarized current generated via the giant spin Hall effect [9] which induces domain wall motion. This type of spin neurons belongs to the general class of (spin-polarized) current driven artificial neurons.

In this paper we propose and analyze a different type of spin-neuron implemented with MTJs having soft layers that are magnetostrictive or multiferroic nanomagnets and whose magnetizations are flipped with mechanical stress/strain generated by a voltage. We call them ‘straintronic spin-neurons’ and they are voltage-driven as opposed to current-driven. This has the advantage of further reducing the energy dissipation during firing. Switching of multiferroic nanomagnets with voltage-generated stress has been proposed and/or demonstrated by many groups [10–14] and is particularly useful for writing bits in non-volatile memory [15–20]. It can be also harnessed for logic applications [12, 21–23] and results in exceptionally low dissipation. Here, we propose it for neural applications. We compare the energy-efficiency of a straintronic spin neuron with that of a traditional current-driven spin neuron and show that the former is more energy efficient. Finally, since magnetization dynamics is vulnerable to thermal noise, we study the operation of spin-neurons at room temperature in the presence of thermal noise and compare that with 0 K operation to assess the degree of thermal degradation. As expected, thermal noise has a deleterious effect on the threshold behavior and seriously degrades the abruptness of the firing action. The degradation is far worse for the current-driven type than for the straintronic type.

2. Straintronic spin neuron

Figure 1 shows the schematic of a straintronic spin-neuron with programmable synapses. Inputs $x_i$-s and the bias $b$ are in the form of voltages $V_i$-s and $b$, the latter being realized with a constant current source $i = I (R_1 || R_2 || \cdots || R_N)$. The voltage appearing at node $P$ is dropped across the piezoelectric layer underneath the (shorted) contact pads $A$ and $X$. It is a weighted sum of input voltages and bias, and is given by

$$V_P = \sum_{i=1}^{N} w_i V_i + b,$$

where $w_i = \frac{R_i || R_{i+1} || \cdots || R_N}{R_1 || \cdots || R_{i-1} || R_{i+1} || \cdots || R_N || r_i + r_f}$

and $b = I (R_1 || R_2 || \cdots || R_{i-1} || R_{i+1} || \cdots || R_N || r_f)$

$$V_o = I_s R_{MTJ} = f(V_P) = f\left(\sum_{i=1}^{N} w_i V_i + b\right) = f\left(\sum_{i=1}^{N} w_i x_i + b\right)$$

$$\Rightarrow O = f\left(\sum_{i=1}^{N} w_i x_i + b\right)$$

Figure 1. Schematic of a straintronic spin-neuron implementing a step transfer function. The artificial synapses are realized with the passive resistors $r_1 \cdots r_N$. The primed coordinate axes system, as well as the polar ($\theta'$) and azimuthal ($\phi'$) angles, used to define the magnetization vector orientation are shown. The unprimed coordinate axis is obtained by rotating clockwise about the $x$-axis by 45°.

The MTJ in figure 1 is the central unit of the neuron. It has a hard nanomagnet, a spacer layer and a soft magnetostrictive nanomagnet in contact with the piezoelectric. All nanomagnets are shaped like elliptical disks. A bias magnetic field $B$ in the plane of the soft nanomagnet directed along its minor axis makes the magnetization orientation of the soft nanomagnet bistable, with the two stable directions shown as $\Psi_1$ and $\Psi_2$ which subtend an angle of $\sim 90°$ between them [15]. The hard nanomagnet is implemented with a synthetic antiferromagnet and its two stable magnetization orientations are roughly along its major axis because of the extremely high shape anisotropy that this nanomagnet possesses. The hard nanomagnet is placed such that its major axis is collinear with one of the stable magnetization orientations of the soft nanomagnet (say $\Psi_1$), resulting in a ‘skewed MTJ stack’ where
the major axes of the two nanomagnets are at an angle. The hard nanomagnet is then magnetized permanently in the direction that is anti-parallel to $\Psi_0$. Thus, when the soft nanomagnet is in the stable state $\Psi_s$, the magnetizations of the hard and soft layers of the MTJ are mutually anti-parallel, resulting in high MTJ resistance, while when the soft nanomagnet is in the other stable state $\Psi_p$, the magnetizations of the two layers are roughly perpendicular to each other, resulting in lower MTJ resistance. The ratio of the high-to-low MTJ resistances is approximately $1/(1 - \eta_1 \eta_2)$, where the $\eta$-s are the spin injection/detection efficiencies at the interfaces of the spacer with the two nanomagnets. We assume that at room temperature, $\eta_1 = \eta_2 \approx 70\%$ [24] and therefore the resistance ratio will be roughly 2:1. Higher resistance ratios exceeding 6:1 have been demonstrated at room temperature [25], but we will be conservative and assume the ratio to be 2:1.

The electrodes A and A$'$ are placed on the piezoelectric layer such that the line joining their centers is parallel to $\Psi_s$ and hence also to the major axis of the hard nanomagnet. Their lateral dimensions are of the same order as, and the inter-electrode separation is 1–2 times, the PZT thin film thickness. When voltages are applied between the electrode pair and the (grounded) conducting substrate, electric fields are generated underneath the electrode pads in the PZT layer as shown in figure 1. They produce out-of-plane compressive strain and in-plane tensile strain or vice versa, depending on the polarity of the voltage in the PZT layer below the electrodes [26]. These strain fields interact and produce biaxial strain between the electrodes (tensile along the line joining the electrodes and compressive in the perpendicular direction, or vice versa) [26], which is almost completely transferred to the magnetostrictive soft magnet since the latter’s thickness is much smaller than that of the strained PZT layer.

If the magnetostriction coefficient of the soft magnet material is positive (e.g. terfenol-D), then sufficient compressive stress resulting along the line joining the electrodes centers, i.e. in the direction of $\Psi_s$, will rotate the soft layer’s magnetization to $\Psi_p$, while sufficient tensile stress will keep the magnetization aligned along $\Psi_s$. The situation will be opposite if the magnetostriction coefficient of the soft magnet is negative (e.g. cobalt). Since, for either sign of the magnetostriction coefficient, the sign of the stress (compressive or tensile) depends on the polarity of the voltage applied between the electrodes and the grounded substrate, the magnetization of the soft magnet can be aligned along either of the two stable orientations $\Psi_s$ and $\Psi_p$ at will by simply choosing the voltage polarities (of the inputs and bias voltages).

There is a minimum stress (compressive/tensile) required to switch the magnetization of the soft nanomagnet of the MTJ from one orientation (say, $\Psi_s$) to the other (say, $\Psi_p$) because the two stable states are separated by an energy barrier [15, 18, 19, 23] that needs to be overcome by stress to make the switching occur. At 0 K temperature, this feature gives rise to a sharp threshold in the switching behavior and makes it possible to mimic the sudden firing behavior of a neuron. The energy barrier (and hence the threshold stress) depends on the permanent magnetic field $B$ and the size and shape of the soft nanomagnet, if we ignore dipole coupling with the hard nanomagnet and any magneto-crystalline anisotropy (the magnets are assumed to be amorphous). These parameters determine the effective in-plane energy barrier between the two stable magnetization states $\Psi_s$ and $\Psi_p$ that must be overcome by stress to switch the magnetization of the soft nanomagnet from one state to the other and therefore the MTJ resistance. The minimum stress needed for switching (also called the ‘critical stress’) at 0 K can be found by equating the stress anisotropy energy to the effective in-plane energy barrier.

The critical stress gives rise to a critical voltage $V_c$ for switching at 0 K. When the total voltage $V_P$, appearing at node $P$, due to all weighted inputs and the bias, exceeds the critical voltage, the MTJ resistance switches abruptly because the soft layer’s magnetization rotates from one stable orientation to the other. If we bias the MTJ with a constant current source $I_P$ as shown in figure 1, then the voltage across the MTJ is $V_{MTJ} = I_P R_{MTJ}$ where $R_{MTJ}$ is the MTJ resistance that has a nonlinear dependence on $V_P$ (abrupt switching when $V_P$ exceeds $V_c$). Because there is virtually no electric field in the PZT layer directly underneath the magnet, we can ignore any potential drop in this region and write the output voltage in figure 1 as $V_0 \approx V_{MTJ} = I_P R_{MTJ}$ which has a nonlinear dependence on $V_P$ and hence exhibits a threshold behavior. Using equation (2), we can now write

$$V_0 = f(V_P) = f\left(\sum_{i=1}^{N} w_i V_i + b\right),$$

(3)

which replicates the neural behavior.

3. Simulation of neuron firing with the Landau–Lifshitz–Gilbert (LLG) equation

We will design our straintronic spin-neuron by choosing the dimensions of the nanomagnet and the magnitude of the permanent magnetic field to select the critical voltage $V_c$, and therefore the firing threshold. We choose a soft nanomagnet of major axis $a = 100$ nm, minor axis $b = 42$ nm and thickness $d = 16.5$ nm, which ensures that it has a single ferromagnetic domain [27]. When stress is applied on the soft nanomagnet by applying the voltage $V_P$ at the electrodes A and A$'$, which is the weighted sum of the inputs and bias, the magnetization vector of the soft nanomagnet experiences a torque that makes it rotate and ultimately switch the MTJ resistance. The torque depends on the shape anisotropy energy of the soft nanomagnet, the permanent magnetic field and the stress.

The shape anisotropy energy is a function of the time-dependent magnetization orientation during switching and
can be written as

\[ E_{ab}(t) = E_{a1}(t)\sin^2 \theta'(t) + E_{a2}(t)\sin 2\theta'(t) + \frac{\mu_0}{4} \Omega M_s^2 \left( N_{d-yy} + N_{d-zz} \right) \]

\[ E_{a1}(t) = \left( \frac{\mu_0}{4} \right) \Omega M_s^2 \left( 2N_{d-xx} - N_{d-yy} - N_{d-zz} \right) \cos^2 \phi'(t) \]

\[ E_{a2}(t) = \left( \frac{\mu_0}{4} \right) \Omega M_s^2 \left( N_{d-zz} - N_{d-yy} \right) \sin \phi'(t), \]

(4)

where \( \theta'(t) \) and \( \phi'(t) \) are, respectively, the instantaneous polar and azimuthal angles of the soft nanomagnet’s magnetization vector in the primed reference frame shown in figure 1. The unprimed reference frame is such that the \( z \)-axis coincides with the soft nanomagnet’s easy axis and \( y \)-axis with the hard axis. The primed reference frame is obtained by rotating anticlockwise about the \( x \)-axis by 45°. The quantity \( M_s \) is the saturation magnetization of the soft nanomagnet, \( N_{d-xx}, N_{d-yy} \) and \( N_{d-zz} \) are the demagnetization factors that can be evaluated from the nanomagnet’s dimensions [28], \( \mu_0 \) is the permeability of free space, and \( \Omega = (\pi/4)abcd \) is the soft nanomagnet’s volume.

The permanent magnetic field contributes an additional term to the potential energy of the soft nanomagnet given by

\[ E_{m}(t) = \frac{1}{\sqrt{2}} M_s \Omega B \left[ \cos \theta'(t) - \sin \theta'(t) \sin \phi'(t) \right]. \]

The locations of the total potential energy \( E_{ab}(\theta(t), \phi(t)) + E_{m}(\theta(t), \phi(t)) \) minima (in the absence of stress) determine the stable magnetization orientations of the soft nanomagnet and are found by minimizing the total potential energy with respect to \( \theta(t), \phi(t) \). In our case, the two stable orientations (the two degenerate energy minima) turn out to be \( \Psi_{1\theta} \) (\( \theta = \theta_0 = 46.9°, \phi = \phi_0 = 90° \)) and \( \Psi_{2\theta} \) (\( \theta = \theta_0 = 133°, \phi = \phi_0 = 90° + \phi_0 \)) with an angular separation of 86.1° (figure 1) between them when \( B = 0.14 \text{T} \).

The in-plane energy barrier separating these two energy minima is 73.1 kT at room temperature resulting in a tiny probability \( e^{-73.1 \sim 10^{-32}} \) of the neuron firing spontaneously at room temperature by switching between the stable states \( \Psi_{1\theta} \) and \( \Psi_{2\theta} \) per attempt. With an attempt frequency of \( 10^{15} \text{ Hz}, \) the mean time between successive spontaneous firing (switching of the MTJ) would then be \( 10^{-15} \times 10^{32} = 10^{17} \text{s} = 3 \times 10^7 \text{centuries}. \)

When voltages are applied at the electrodes A and A', the resulting stress produced in the soft nanomagnet contributes a stress anisotropy energy to the soft nanomagnet’s potential energy. Although the generated strain is biaxial, we will approximate it as uniaxial strain to somewhat compensate for the fact that not 100% of the strain in the PZT will be transferred to the soft nanomagnet. With this assumption, the stress anisotropy energy is written as

\[ E_{st}(t) = -\frac{3}{2} \lambda_s \varepsilon(t) Y \Omega \cos^2 \theta'(t), \]

(6)

where \( \lambda_s \) is the magnetostriction coefficient of the soft nanomagnet, \( Y \) is the Young’s modulus, and \( \varepsilon(t) \) is the strain generated by the applied voltage \( V_p(t) \) at the instant of time \( t \).

The total potential energy of a stressed nanomagnet at any instant of time \( t \) is therefore

\[ E(t) = E(\theta'(t), \phi'(t)) = E_{ab}(t) + E_{m}(t) + E_{st}(t). \]

(7)

We follow the standard procedure to derive the time evolution of the polar and azimuthal angles of the magnetization vector of the soft nanomagnet in the rotated coordinate frame under the actions of the torques due to shape anisotropy, stress anisotropy and magnetic field by solving the LLG equation:

\[ \frac{d\mathbf{m}(t)}{dt} = -\alpha \left[ \mathbf{m}(t) \times \frac{d\mathbf{m}(t)}{dt} \right] - \frac{1}{\mu_0 M_s \Omega} \tau_{ax}(t), \]

(8)

where \( \alpha \) is the Gilbert damping coefficient which depends on the soft nanomagnet’s material, \( \gamma \) is the gyromagnetic ratio (a universal constant) and \( \tau_{ax} \) is the total torque acting on the magnetization vector and is given by

\[ \tau_{ax}(t) = -\mathbf{m}(t) \times \left( \frac{\partial E}{\partial \theta} \hat{\theta} + \frac{1}{\sin \theta} \frac{\partial E}{\partial \phi} \hat{\phi} \right) \]

\[ - \frac{1}{\sqrt{2}} M_s \Omega \cos \phi'(t) \hat{\theta} \]

\[ - \left( E_{a1}(t) \sin 2\theta'(t) + 2E_{a2}(t) \cos 2\theta'(t) \right) \]

\[ - \frac{1}{\sqrt{2}} M_s \Omega \left( \sin \phi'(t) \cos \theta'(t) + \sin \theta'(t) \right) \]

\[ + (3/2) \lambda_s \varepsilon(t) Y \Omega \sin 2\theta'(t) \hat{\phi}, \]

(9)

where \( \mathbf{m}(t) \) is the normalized magnetization vector, quantities with caretts are unit vectors in the original frame of reference, and

\[ E_{a1}(t) = \frac{\mu_0}{4} M_s^2 \Omega \left( N_{d-yy} + N_{d-zz} \right) \sin 2\phi'(t) \]

\[ - 2N_{d-xx} \sin 2\phi'(t) \]

\[ E_{a2}(t) = \frac{\mu_0}{2} M_s^2 \Omega \left( N_{d-zz} - N_{d-yy} \right) \cos \phi'(t). \]

At non-zero temperatures, there is an additional torque acting on the magnetization vector owing to thermal noise. The procedure for finding this torque has been described in [29] and is not repeated here.

Solution of the LLG equation (8) yields the magnetization of the stressed soft nanomagnet as a function of time and steady state is achieved when the magnetization no longer changes appreciably with time. This yields the switching time (time elapsed before reaching steady-state) and the energy dissipation for any given stress. The procedure for finding these quantities in the presence of thermal fluctuations requires solving the stochastic LLG equation and is described in [29] and [30].
We assume that the soft nanomagnet is made of terfenol-D which has the following material parameters: saturation magnetization $M_s = 8 \times 10^5$ A m$^{-1}$, magnetostriction coefficient $\lambda_s = 60 \times 10^{-5}$, Young’s modulus $Y = 80$ GPa and Gilbert damping coefficient $\alpha = 0.1$ [31–33].

We solve the LLG equation (8) at 0 K (and its stochastic version at 300 K) to find the steady-state orientation of the magnetization vector of the soft nanomagnet as a function of stress in the nanomagnet and therefore as a function of the sum-total voltage applied at the electrode pairs. The stress is varied between −50 and +50 MPa. By following the procedure in [26], we compute that an electric field of 37.5 kV m$^{-1}$ is required to produce a stress of 1 MPa in the PZT, which we assume is fully transferred to the soft magnet. Therefore, assuming that stress is linearly proportional to the voltage, the corresponding voltages for ±50 MPa are ±187.5 mV if the PZT film’s thickness is 100 nm.

At the critical stress value (or critical voltage), the soft layer’s magnetization vector changes abruptly from its initial stable orientation to the other causing the angle between the magnetizations of the soft and hard layer to change from $\beta_L = 180^\circ - 86.1^\circ = 93.9^\circ$ to $\beta_H = 180^\circ$, or vice versa. This will cause the MTJ resistance to change by a factor of $(1 + \eta_1\eta_2\cos(93.9^\circ))/(1 - \eta_1\eta_2) \sim 1.9$ if we assume $\eta_1 = \eta_2 = 0.7$.

Figure 2 shows the ratio $R(V)/R_L$ as a function of voltage $V$ applied at the contact pads, where $R(V)$ is the MTJ resistance at a voltage $V$ and $R_L$ is the MTJ resistance in the low-resistance state. If the MTJ is initially in the high resistance state, a compressive stress (positive voltage) is required to drive it to the low resistance state, whereas if it is initially in the low resistance state, a tensile stress (negative voltage) is required to drive it to the high resistance state because we have assumed the soft magnet material to be terfenol-D which has positive magnetostriction. Thus, the critical voltage to switch from high-to-low resistance is $+93.15$ mV and the critical voltage to switch from low-to-high resistance is $-93.15$ mV, which produces the appearance of a ‘hysteresis’ in the characteristic in figure 2. This hysteretic behavior indicates that the device can also be used as a non-volatile memory. Note that the transitions are not completely abrupt even at 0 K temperature because sub-critical stress that is slightly lower than the critical stress can cause some rotation of the soft magnet’s magnetization vector and hence change the MTJ resistance perceptibly. This is the reason for the ‘rounded corners’.

The energy dissipated during a firing event is the sum of the $CV^2$ dissipation associated with charging the capacitance of the electrodes $A$ and $A'$, and the internal dissipation in the soft magnet due to Gilbert damping. We neglect any dissipation due to the current sources assuming that the currents are small and the current sources are turned on only when the inputs arrive. Here, $C$ is the capacitance of the electrodes and $V$ is the voltage at the electrodes. The capacitance is determined from the areas of the electrodes, the dielectric constant of PZT and the PZT film thickness. It is found to be 0.88 fF.

Therefore, the capacitance charging dissipation will be $2 \times (1/2)CV^2 = 0.093 15^2 \times 0.88$ fF = 7.63 aJ (there are two electrodes $A$ and $A'$). The dissipation due to Gilbert damping at 0 K was found to be 1.2 aJ from the solution of the LLG equation as described in [29] and [34]. Therefore, the total dissipation during a firing event is 8.83 aJ. This is ~450 times lower than that reported (4 fJ) in [8] for current-driven spin neurons when the input voltage level was 100 mV and ~45 times lower than that (0.4 fJ) when the input voltage level was 10 mV (our input voltage level is ~93 mV). Note that the synapse resistances $R_1 \cdots R_N$ can be set very high (without making the charging RC time constant too long because numerous synapse resistors will appear in parallel along the charging path) and hence the dissipation in them can be neglected. The switching (firing) delay is found to be 1.37 ns at the threshold voltage level.

4. Current-driven spin neuron based on spin transfer torque (STT) switched MTJ

In this section, we discuss a spin-neuron implemented with the same type of MTJ as above, except this time the soft magnet is not switched with strain, but with a spin polarized current delivering a STT. The inputs are therefore not voltages, but currents. We choose the material to be CoFeB with in-plane anisotropy which has a Gilbert damping constant of ~0.004 [35], much lower than that of terfenol-D. This material has in-plane anisotropy if the magnet thickness exceeds a few nm [36]. STT-driven spin neurons built with magnets exhibiting perpendicular anisotropy have been studied in [8] and [37].

Note that we chose two different materials—terfenol-D for the straintronic neuron and CoFeB for the current driven neuron—because we wish to optimize both for minimal energy dissipation and then make a fair comparison. Terfenol-D has a very large magnetostriction coefficient and is hence
beneficial for a straintronic neuron. A current driven neuron does not benefit from large magnetostriction, but benefits from small Gilbert damping, which is why we chose CoFeB for it.

Figure 3 shows the schematic of a STT-based current-driven spin neuron consisting of an MTJ stack that is not skewed. Both magnets are elliptical in shape and each has two stable magnetization orientations along the major axis of the ellipse. Their major axes are collinear.

The hard magnet is permanently magnetized along one of its stable orientations. The soft magnet’s magnetization can be either parallel (low-resistance state) or anti-parallel (high-resistance state) to that of the hard magnet. The high-to-low resistance ratio in this case is \(1 + \eta_1 \eta_2 / (1 - \eta_1 \eta_2) = 2.9\), assuming once again that \(\eta_1 = \eta_2 = 0.7\) [24]. This corresponds to a tunneling magnetoresistance ratio, or TMR, of \(\sim 200\%\).

A negative potential applied between the hard and soft layers will align the magnetization of the soft layer parallel to that of the hard layer (low MTJ resistance), while a positive potential will make it anti-parallel (high MTJ resistance). These potentials cause a spin polarized current to be injected into or extracted from the soft layer which brings about the magnetization switching by exerting a STT on the magnetization vector [38–41]. For consistency, the soft layer’s dimensions are chosen such that the in-plane shape anisotropy energy barrier is 71.7 kT, close to that of the soft magnet in the straintronic spin neuron. This is ensured by choosing the major axis \(a = 50\) nm, minor axis \(b = 32\) nm and thickness \(d = 5\) nm [28]. The critical switching current density is proportional to the shape anisotropy energy barrier [42] and is fixed since the latter is fixed at 71.7 kT. Changing the nanomagnet dimensions will not change it. Contemporary STT often uses magnets of smaller thickness (\(\sim 2\) nm) but that is because of the need to have perpendicular anisotropy. We consider only in-plane anisotropy and hence we do not need such ultrathin soft layers.

In the absence of spin polarized current, the soft nanomagnet has only shape anisotropy energy and its potential energy at any instant of time \(t\) is given by

\[
E_{\text{shape}}(t) = E_{\text{ss}}(t) \sin^2 \theta(t) + \frac{\mu_0 M_s^2}{2} N_{d-\text{zz}}
\]

\[
E_{\text{ss}}(t) = \left(\frac{\mu_0}{2}\right) N_{d-\text{yy}} \cos^2 \phi(t)
\]

\[
+ N_{d-\text{zz}} \sin^2 \phi(t) - N_{d-\text{zz}},
\]

(10)

where \(\theta(t)\) and \(\phi(t)\) are again, respectively, the instantaneous polar and azimuthal angles of the magnetization vector, and \(M_s\) is the saturation magnetization which is \(\sim 10.4 \times 10^5\) A m\(^{-1}\) [43] for amorphous CoFeB.

The torque on the magnetization vector at any time \(t\) due to shape anisotropy can be expressed as

\[
\tau_{\text{ss}}(t) = -\mathbf{m}(t) \times \left(\frac{\partial E(t)}{\partial \theta(t)} \dot{\theta} + \frac{1}{\sin \theta(t)} \frac{\partial E(t)}{\partial \phi(t)} \dot{\phi}\right)
\]

\[
= E_{\phi s}(t) \sin \theta(t) \dot{\theta} - E_{\phi s}(t) \sin 2 \theta(t) \dot{\phi},
\]

(11)

where \(\mathbf{m}(t)\) is the normalized magnetization vector and \(E_{\phi s}(t) = \frac{\mu_0 M_s^2}{2} \left(N_{d-\text{yy}} - N_{d-\text{zz}}\right) \Omega \sin 2 \phi(t)\).

Passage of the spin-polarized current \(I_s\) through the nanomagnet generates a STT on the magnetization vector given by [44]

\[
\tau_{\text{st}}(t) = s \left[ b \sin(\zeta - \theta(t)) \dot{\phi} - c \sin(\zeta - \theta(t)) \dot{\theta} \right].
\]

(12)

where \(s = (\gamma/2e) I_s\) is the spin angular deposition per unit time, \(\chi\) is the spin polarization of the current, \(b\) and \(c\) are coefficients of the out-of-plane and in-plane components of the STT. We assume \(\chi = 70\%\) (assuming 70% spin injection efficiency), and \(b\) and \(c\) are 0.3 and 1, respectively. The current is passed perpendicular to the plane of the magnet as shown in figure 3. The quantity \(\zeta\) is the angle subtended by the direction of spin polarization with the \(z\)-axis and it is either \(0^\circ\) or \(180^\circ\).

The LLG equation and its stochastic version are solved again to extract the STT-induced magnetization switching behavior of the soft nanomagnet in the absence (0 K) and presence (300 K) of thermal noise.

\[
\frac{d\mathbf{m}(t)}{dt} = -\alpha \left( \mathbf{m}(t) \times \frac{d\mathbf{m}(t)}{dt} \right) = -\frac{|\mathbf{m}(t)|^2}{\mu_0 M_s \Omega} \left( \tau_{\text{st}}(t) + \tau_{\text{ss}}(t) \right).
\]

(13)

First, we determine the current required to switch the soft layer (from the LLG simulations) and find it to be 127.13 \(\mu\)A (the corresponding current density is 10.11 MA cm\(^{-2}\)) for a current pulse of duration 10 ns. [45] also assumed a current pulse duration of 10 ns and found the switching current density to be 4 MA cm\(^{-2}\) for a barrier height of 42 kT in perpendicular magnetic anisotropy (our barrier height is 71.7 kT). [46] considered a CoFeB soft nanomagnet with in-plane anisotropy barrier of \(\sim 57\) kT (calculated from the reported
dimensions of 170 nm × 60 nm × 1.4 nm using [28] and found the switching current density to be ∼5.5 MA cm⁻² for a switching time of 10 ns.

Figure 4 shows the 0 K temperature transfer characteristic (or firing behavior) of the MTJ when the total input current I has been varied between −200 μA and +200 μA. Note that rounded corners are more ‘square’ here because a subthreshold current (unlike subthreshold voltage) hardly rotates the magnetization of the soft magnet and hence does not alter the MTJ resistance perceptibly.

If we consider a typical resistance-area (RA) product of 2.1 Ω μm² for the MTJ in the high resistance state [47] and a TMR ratio of 200%, then the high-state MTJ resistance for our chosen dimensions becomes 1671 ohms and the low-state MTJ resistance 557 ohms. Energy dissipation \( E_R \) due to the current passing through MTJ’s tri-layered structure is given by [48]

\[
E_R = \int_0^T dt \left[ R_P + (R_{AP} - R_P) \left( \frac{1 - \cos \theta(t)}{2} \right) \right],
\]

where \( T \) is the switching time, \( R_P \) and \( R_{AP} \) are the MTJ resistances in the parallel (low) and anti-parallel (high) state, and \( \theta(t) \) is the angle between the magnetization states of the soft layer and the hard layer at time \( t \). This dissipation turns out to be ∼0.26 pJ. The LLG simulation showed that the neuron takes 14 ns to switch. The dissipation due to Gilbert damping in the soft magnet is a mere ∼0.48 aJ, which is negligible.

The total dissipation reported in [8] for current-driven spin neurons that use magnets with perpendicular anisotropy is 0.4 fJ. We used magnets with in-plane anisotropy. There are at least two reasons why [8] could have reported a 650 times lower dissipation compared to what we found (0.26 pJ). First, their critical current density was 4 MA cm⁻² which is 2.5 times less than ours. Presumably, the lower current density is due to the fact that the energy barrier between stable magnetization states in their device might have been only ∼40 kT which is what we estimate following the procedure in [49] and [36]. Ours was 71.7 kT. The critical current density scales with the energy barrier height; for example, [50] reported a critical current density of 8.7 MA cm⁻² for a barrier height of 67 kT. Additionally, in [8], the soft layer thickness was only 2 nm to maintain perpendicular anisotropy, whereas our thickness was 5 nm and the critical current density increases with the soft layer thickness [43]. These two factors increased the current density in our case and caused a higher dissipation. Second, and more importantly, [8] utilized the spin Hall effect to inject/extract spin polarized current from the magnets which would allow passing the charge current parallel to the heterointerface between the spacer and the magnets. This would allow the current to avoid going through the highly resistive spacer and decrease the RA product of the MTJ considerably compared to ours. These two factors might be the cause for the 650 times lower dissipation figure reported in [8] compared to what we find. Even then, the lower dissipation reported in [8] is still 45 times higher than that encountered in the straintronic spin neuron. If we carry out the comparison between similar designs (in-plane anisotropy magnets, similar energy barrier heights to maintain similar resilience to thermally induced random firing), then the difference is even more stark; the straintronic spin neuron is 29 445 times more energy-efficient and yet 10 times faster. The current-driven spin neuron, however, has one small advantage; it has a 50% higher on/off ratio of the MTJ resistance because the angular separation between the two stable orientations of the soft magnet in the MTJ is ∼180° for the current-driven spin neuron and ∼90° for the straintronic spin neuron.
The room temperature firing characteristics are found by solving the stochastic LLG equation which will generate a distribution of characteristics since each one is slightly different in the presence of thermal noise. Each characteristic will show slightly different switching threshold and slightly different switching delay.

Figure 5 shows the $R(V)/R_L$ versus $V$ characteristic of the straintronic spin neuron at 300 K when switching from high- to low-resistance state (the characteristic for switching from low- to high-resistance state will be qualitatively similar and hence not shown). As many as 10,000 switching trajectories were simulated from the stochastic LLG equation and make up this plot. The average switching time was 0.61 ns when the voltage appearing at node $P$ in figure 1 was above the 0 K threshold voltage of 93.15 mV. Clearly, the threshold has been significantly broadened at room temperature, indicating that there is a significant probability of pre-mature firing (firing before reaching the threshold defined at 0 K) because of random thermal torque, as well as failure to fire at or slightly beyond the threshold for the same reason. The likelihood of false firing is a matter of concern which calls for further investigation of thermal degradation.

One measure of the threshold broadening is the ratio $\Delta V/V_{th}$, where $V_{th}$ is the average voltage at which the neuron fires and $\Delta V$ is the standard deviation. For the straintronic spin neuron studied, this quantity turns out to be 16.5%. Of course this can be reduced by increasing $V_{th}$ (by making the soft nanomagnet more shape-anisotropic to increase the in-plane shape anisotropy energy), but this will also increase the energy dissipation which varies roughly as $V_{th}^2$ (because the dissipation is dominated by the $CV^2$ loss). Let us say that 1% broadening is acceptable. Then we will have to increase $V_{th}$ 16.5-fold (assuming that it does not change $\Delta V$), resulting in an energy dissipation of $8.83 \, \text{aJ} \times 16.5^2 = 2.4 \, \text{fJ}$. This is still much less than what is encountered in CMOS-based implementations (0.7 pJ reported in [8]).

The same thermal broadening is not only present in a current-driven spin neuron, but it is much worse there. Figure 6 shows $R(I)/R_L$ versus $I$ for the current-driven spin neuron where $I$ is the driving current. Here, 5000 switching trajectories were simulated to produce this plot because simulation of 10,000 trajectories became computationally prohibitive (because of the longer simulation time). The average switching time was now 7.26 ns when the driving current was above the 0 K threshold current of 127.13 $\mu$A. The quantity $\Delta I/I_{th}$ turns out to be 69.92% for the simulated neuron. Had we been able to simulate more trajectories, we might have found the broadening to be even worse. Once again, the relative broadening can be reduced by increasing $R_h$ (by increasing the shape anisotropy of the soft nanomagnet) but of course at the cost of increasing dissipation since the latter varies roughly as $R_h^2$ (because the dissipation is dominated by the $I^2R$ loss). Once again, if 1% broadening is acceptable, then we will need to increase the threshold 69-fold (assuming this does not change $\Delta I$), resulting in an energy dissipation of $0.26 \, \text{pJ} \times 69^2 = 1.23 \, \text{nJ}$. This would make it 1768 times worse than CMOS-based neurons and therefore the viability of STT-based spin neurons at room temperature is questionable. Even if better design (use of spin Hall effect, perpendicular anisotropy magnets, etc) can reduce the energy dissipation by two orders of magnitude, it will still provide little advantage over CMOS implementations. In contrast, the straintronic spin neuron is still ~290 times more energy-efficient than its CMOS-based counterpart at room temperature.

6. Conclusion

In conclusion, we have proposed and analyzed a straintronic spin-neuron that is orders of magnitude more energy-efficient than the usual current-driven spin neuron and also faster. The primary obstacle they both face is the significant broadening of the firing threshold at room temperature. For the current-driven spin neuron, it may not be possible to mitigate this problem without increasing the energy dissipation to the point where it is no longer superior to CMOS-based implementations. Fortunately, for the straintronic spin neuron, it may still be possible to mitigate this problem while retaining a significant energy advantage over CMOS.

Acknowledgments

This work was supported by the US National Science Foundation under grants ECS-1124714 and CCF-1216614. JA would also like to acknowledge the NSF CAREER grant CCF-1253370.
Appendix

The broadening in figure 6 for current-driven spin neuron due to thermal noise at room temperature can be reduced if we choose a different soft material with larger Gilbert damping coefficient ($\alpha$). Since the switching current density is approximately linearly proportional to $\alpha$, the energy dissipation will increase quadratically. In order to illustrate this, we chose a hypothetical material which is identical to CoFeB in all respects, except its Gilbert damping coefficient $\alpha$ is 0.1 and carried out the stochastic LLG simulations. The switching current turned out to be 1.44 mA for a current pulse duration of 10 ns, but the broadening ($\Delta I/I_n$) was reduced to 11.25%. However, the extremely large switching current results in energy dissipation of 23.8 pJ which is clearly prohibitive. This case study shows us that although thermal degradation may be countered with increased energy dissipation, the price to be paid in energy may be prohibitive.

Programmable synapses: In this paper, our synapses have fixed weights because they are implemented with passive resistors. Programmable synapses are required for more versatile renditions of neural computing such as Spike time dependent plasticity (STDP) models of neural networks [51–54] that are popular for Hebbian learning [55]. For STDP models, the synapse weight should be programmed by a spike signal, e.g. it should be a function of a time-integrated spike current. The time integrated spike current is a charge that could be stored in a capacitor which then applies a voltage across a piezoelectric layer that generates strain and rotates the magnetization of a magnetostrictive magnet that is elastically coupled to the piezoelectric layer. The magnetostrictive magnet could be the soft layer of a MJT whose resistance is thus programmed by the time integrated spike signal, resulting in the appropriate programmable synapse. This is a different topic and would be treated elsewhere.

References

[1] Chua L and Yang L 1988 Cellular neural networks: applications *IEEE Trans. Circuits Syst.* **35** 1273
[2] Ueda M, Kaneko Y, Nishitani Y and Fuji E 2011 A neural network circuit using persistent interfacial conducting heterostructures *J. Appl. Phys.* **110** 086104
[3] Simard P, Steinkraus D and Platt J C 2003 Best practices for convolutional neural networks applied to visual document analysis *Proc. Int. Conf. on Document Analysis and Recognition (ICDAR)* vol 3 p 958
[4] George D and Hawkins J 2009 Towards a mathematical theory of cortical micro-circuits *PLOS Comput. Biol.* **5** 1000532
[5] Lippmann R 1987 An introduction to computing with neural nets *IEEE. ASSP Mag.* **2** 4
[6] Datta S, Salahuddin S and Behin-Acin B 2012 Non-volatile spin switch for boolean and non-boolean logic *Appl. Phys. Lett.* **101** 252411
[7] Sharad M, Augustine C, Panagopoulos G and Roy K 2012 Spin-based neuron model with domain-wall magnets as synapse *IEEE Trans. Nanotech.* **11** 843
[8] Sharad M, Fan D and Roy K 2013 Spin-neurons: a possible path to energy-efficient neuromorphic computers *J. Appl. Phys.* **114** 234906
[9] Liu L, Pai C-F, Li Y, Tseng H W, Ralph D C and Buhrman R A 2012 Spin-torque switching with the giant spin hall effect of tantalum *Science* **336** 555
[10] Zavaliche F, Zhao T, Zheng H, Straub F, Cruz M P, Yang P-L, Hao D and Ramesh R 2007 Electrically assisted magnetic recording in multiferroic nanostructures *Nano Lett.* **7** 1586
[11] Britlinger T, Lim S-H, Baloch K H, Alexander P, Qi Y, Barry J, Melngailis J, Salamanca-Riba L, Takeuchi I and Cumings J 2010 *In situ* observation of reversible nanomagnetic switching induced by electric fields *Nano Lett.* **10** 1219
[12] Atulasimha J and Bandyopadhyay S 2010 Bennett clocking of nanomagnetic logic using multiferroic single domain nanomagnets *Appl. Phys. Lett.* **97** 173105
[13] Buzzi M, Chopdekar R, Hockel J L, Bur A, Wu T, Pilet N, Warnicke P, Carman G P, Heyderman L J and Nolting F 2013 Single domain spin manipulation by electric fields in strain coupled artificial multiferroic nanostructures *Phys. Rev. Lett.* **111** 027204
[14] DSouza N, Salehi-Fashami M, Bandyopadhyay S and Atulasimha J 2014 Strain induced clocking of nanomagnets for ultra low power boolean logic arXiv:1405.1388[cond-mat.mes-hall]
[15] Tiercelin N, Dusch Y, Preobrazhensky V and Pernod P 2011 Magnetoelastic memory using orthogonal magnetization states and magnetoelastic switching *J. Appl. Phys.* **109** 07D726
[16] Pertsev N A and Kohlstedt H 2009 Magnetic tunnel junction on a ferroelectric substrate *Appl. Phys. Lett.* **95** 163503
[17] Roy K, Bandyopadhyay S and Atulasimha J 2013 Binary switching in a symmetric potential landscape *Sci. Rep.* **03** 3038
[18] Biswas A K, Bandyopadhyay S and Atulasimha J 2014 Energy-efficient magnetoelastic non-volatile memory *Appl. Phys. Lett.* **104** 232403
[19] Biswas A K, Bandyopadhyay S and Atulasimha J 2014 Complete magnetization reversal in a magnetostrophic nanomagnet with voltage generated stress: a reliable energy-efficient non-volatile magnetoelastic memory *Appl. Phys. Lett.* **105** 072408
[20] Wang J J, Hu J M, Ma J, Zhang J X, Chen L Q and Nan C W 2014 Full 180° magnetization reversal with electric fields *Sci. Rep.* **04** 7507
[21] Fashami M S, Roy K, Atulasimha J and Bandyopadhyay S 2011 Magnetization dynamics, Bennett clocking and associated energy dissipation in multiferroic logic *Nanotechnology* **22** 155201
[22] Fashami M S, Atulasimha J and Bandyopadhyay S 2012 Magnetization dynamics, throughput and energy dissipation in a universal multiferroic nanomagnetic logic gate with fan-in and fan-out *Nanotechnology* **23** 105201
[23] Biswas A K, Atulasimha J and Bandyopadhyay S 2014 An error-resilient non-volatile magneto-elastic universal logic gate with ultralow energy-delay product *Sci. Rep.* **4** 7553
[24] Sali S, Wang R, Jiang X, Shelby R M, Parkin S S P, Bank S R and Harris J S 2005 Temperature independence of the spin-injection efficiency of a MgO-based tunnel spin injector *Appl. Phys. Lett.* **87** 262503
[25] Ikeda S, Hayakawa J, Ashizawa Y, Lee Y M, Miura K, Hasegawa H, Tsunoda M, Matsukura F and Ohno H 2008 Tunnel magnetoresistance of 604% at 300 K by suppression of Ta diffusion in CoFeB/MgO/CoFeB pseudo-spin-valves annealed at high temperature *Appl. Phys. Lett.* **93** 082508
[26] Cui J, Hockel J L, Nordeen P K, Pisani D M, Lian C y, Carman G P and Lynch C S 2013 A method to control...
magnetism in individual strain-mediated magnetoelectric islands Appl. Phys. Lett. 103 232905
[27] Cowburn R P, Koltsos D K, Adeyeye A O, Welland M E and
Tricker D M 1999 Single-domain circular magnetonanomats
Phys. Rev. Lett. 83 1042
[28] Beleggia M, Graef M D, Millev Y T, Goode D A and
Rowlands G 2005 Demagnetization factors for elliptic
cylinders J. Phys. D: Appl. Phys. 38 3333
[29] Roy K, Bandyopadhyay S and Atulasimha J 2012 Energy
dissipation and switching delay in stress-induced switching of
multiferroic nanomagnets in the presence of thermal
fluctuations J. Appl. Phys. 112 023914
[30] Scholz W, Schrefl T and Fidler J 2001 Micromagnetic
simulation of thermally activated switching in fine particles
J. Magn. Magn. Mater. 233 296
[31] Abbundi R and Clark A E 1977 Anomalous thermal expansion
and manetostriiction of single crystal Tb0.25Dy0.75Fe2 IEEE
Trans. Magn. 13 1519
[32] Ried K, Schnell M, Schatz F, Hirscher M, Ludescher B,
Sigle W and Kronmüller H 1998 Crystallization behaviour and
magnetic properties of magnetostriuctive TbDyFe films
Phys. Status Solidi A 167 195
[33] Kellogg R and Flatau A 2008 Experimental investigation of
terfenol-Ds elastic modulus J. Intell. Mater. Syst. Struct. 19 583
[34] Lyyty T V, Denisov S I, Peletsky A Y and Binns C 2015 Energy
dissipation in single-domain ferromagnetic nanomagnets: Dynamical approach Phys. Rev. B 91 054425
[35] Ishama S, Mizukami S, Naganuma H, Oogane M, Ando Y and
Miyazaki T 2014 Gilbert damping constants of TbCoFeB/MgO(Ta) thin films measured by optical detection of
precessional magnetization dynamics Phys. Rev. B 89
174416
[36] Ikeda S, Miura K, Yamamoto H, Mizunuma K, Gan H D,
Endo M, Kanai S, Hayakawa J, Matsukura F and Ohno H
2010 A perpendicular-anisotropy CoFeB-MgO magnetic
tunnel junction Nat. Mater. 9 721
[37] Sharad M, Fan D, Aitken K and Roy K 2014 Energy-efficient
non-boolean computing with spin neurons and resistive
memory IEEE Trans. Nanotechnol. 13 23
[38] Słonczewski J 1996 Current-driven excitation of magnetic
multilayers J. Magn. Magn. Mater. 159 L1
[39] Ralph D C and Siles M D 2008 Spin transfer torques J. Magn.
Magn. Mater. 320 1190
[40] Kubota H et al 2008 Quantitative measurement of voltage
dependence of spin-transfer torque in MgO-based magnetic
tunnel junctions Nat. Phys. 4 37
[41] Rowlands G E et al 2011 Deep subnanosecond spin torque
switching in magnetic tunnel junctions with combined
plane and perpendicular polarizers Appl. Phys. Lett. 98
102509
[42] Diao Z, Li Z, Wang S, Ding Y, Panchula A, Chen E,
Wang L-C and Hwu Y 2007 Spin-transfer torque switching in
magnetic tunnel junctions and spin-transfer torque random
access memory J. Phys.: Condens. Matter 19 165209
[43] Hayakawa J, Ikeda S, Lee Y M, Sasaki R, Meguro T,
Matsukura F, Takahashi H and Ohno H 2005 Current-driven
magnetization switching in CoFeB/MgO/CoFeB magnetic
tunnel junctions Japan. J. Appl. Phys. 44 L1267
[44] Salahuddin S, Datta D and Datta S 2008 Spin transfer torque as
a non-conservative pseudo-field arXiv:0811.3472
[45] Amiri P K et al 2011 Switching current reduction using
perpendicular anisotropy in CoFeB/MgO magnetic tunnel
junctions Appl. Phys. Lett. 98 112507
[46] Tomasello R, Pulaifio V, Azzerboni B and Finocchio G 2014
Switching properties in magnetic tunnel junctions with
interfacial perpendicular anisotropy: micromagnetic study
IEEE Trans. Magn. 50 7100305
[47] Isogami S, Tsuboza M, Komagaki K, Sunaga K, Uehara Y,
Sato M, Miyajima T and Takahashi M 2008 In situ heat
treatment of ultrathin mgo layer for giant magnetoresistance
ratio with low resistance area product in CoFeB/MgO/
CoFeB magnetic tunnel junctions Appl. Phys. Lett. 93
192109
[48] Carpentieri M, Ricci M, Burrascano P, Torres L and
Finocchio G 2012 Wideband microwave signal to trigger
fast switching processes in magnetic tunnel junctions
J. Appl. Phys. 111 07C909
[49] Lee K, Sapan J J, Kang S H and Fullerton E E 2011
Perpendicular magnetization of CoFeB on single crystal
MgO J. Appl. Phys. 109 123910
[50] Ikeda S, Hayakawa J, Lee Y M, Matsukura F, Ohno H and
Hanyu T 2007 Magnetic tunnel junctions for spintronic
memories and beyond IEEE Trans. Electron Devices 54 991
[51] Bi G-q and Poo M-m 1998 Synaptic modifications in cultured
hippocampal neurons: dependence on spike timing, synaptic
strength, and postsynaptic cell type J. Neurosci. 18 10464
[52] Favero M, Cangiano A and Busetto G 2014 Hebb-based rules
of neuro-plasticity: are they ubiquitously important for the
refinement of synaptic connections in development
Neuroscientist 20 8
[53] Aoki T and Aoyagi T 2006 A possible role of incoming spike
synchrony in associative memmory model with STDP
learning rule Prog. Theor. Phys. Suppl. 161 152
[54] Sengupta A, Al Azim Z, Feng X and Roy K 2015 Spin–orbit
torque induced spike-timing dependent plasticity Appl. Phys.
Lett. 106 093704
[55] Hebb D 1949 The Organization of Behavior (New York: Wiley)

10