Charged Balanced Black Rings in Five Dimensions

Kirsten Schnüll
MPIA Heidelberg, Königstuhl 17, D-69117 Heidelberg
E-mail: schnuelle@mpia.de

Abstract. Balanced black ring solutions of pure Einstein-Maxwell theory in five dimensions are presented here. Those solutions are asymptotically flat, and their tension and gravitational self-attraction are balanced by the repulsion due to rotation and electrical charge. Hence they are free of conical singularities and possess a regular horizon which exhibits the topology $S^1 \times S^2$ of a torus. The global charges and the horizon properties of the solutions are discussed, and it is shown that they satisfy a Smarr relation. These black ring solutions are constructed numerically, and are restricted to the case of black rings with a rotation in the direction of the $S^1$.

1. Introduction
When looking for a theory that unifies all fundamental interactions including gravity, string theory is one of the most promising candidates. For mathematical consistency however, it requires spacetime to consist of more than four dimensions. Thus, higher-dimensional black holes have received much interest in recent years, opening up the possibility of direct observation in future high energy collisions.
The presence of extra dimensions can affect the properties of black holes dramatically: In four-dimensional spacetimes, black holes are restricted to have a spherical horizon topology, and are uniquely characterized by their global charges. In five dimensions however, topologies like black strings and black rings can be found. In a certain parameter range, spherical black holes and black rings coexist, so black hole uniqueness is violated in five dimensions.

2. The Vacuum Black Ring
A fascinating development was the discovery of the black ring as a topologically different solution of the five-dimensional Einstein-Maxwell equations by Emparan and Reall [2,3]. This is a black hole with ring horizon topology $S^1 \times S^2$ and the metric:

$$ds^2 = -\frac{F(y)}{F(x)} \left( dt + CR \frac{1 + y}{F(y)} d\psi \right)^2 + \frac{R^2}{(x-y)^2} F(x) \left[ -\frac{G(y)}{F(y)} d\psi^2 + \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)} d\phi^2 \right],$$

$$F(\xi) = 1 + \lambda \xi, \quad G(\xi) = (1 - \xi^2)(1 + \nu \xi), \quad C = \sqrt{\lambda(\lambda - \nu)} \frac{1 + \lambda}{1 - \lambda}.$$

Static solutions always suffer from the presence of a conical singularity, preventing the ring from collapsing under its tension and gravitational self-attraction. Their horizon is not smooth however (Fig. 1(a)), so no regular solutions exist. When including rotation, the tension and gravitational self-attraction can be balanced by the centrifugal repulsion, leading to balanced solutions without conical singularity (Fig. 1(b)).

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Figure 1. Spatial sections of the static (a) and rotating (b) black ring described by surfaces of constant $y$. The $\psi$ part of the metric is suppressed.

3. Charged Balanced Black Rings – Action and Ansatz
A charged stationary black ring solution of pure Einstein-Maxwell theory is not known in closed from yet. However, we have constructed such a solution numerically.

From the five-dimensional Einstein-Maxwell action
\[
S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} (R - F_{ab} F^{ab}),
\]
we obtain the Einstein-Maxwell equations
\[
G_{ab} = 2T_{ab}, \quad \nabla_b F^{ab} = 0,
\]
with the curvature scalar $R$, the five-dimensional Newton constant $G_5$ and the field strength tensor $F_{ab} = \partial_a A_b - \partial_b A_a$, where $A_a$ denotes the gauge potential. $G_{ab}$ and $T_{ab}$ represent the Einstein tensor and the energy-stress tensor, respectively. For the numerical work, we choose an ansatz in isotropic coordinates
\[
ds^2 = -f_0(r, \theta) dt^2 + \frac{1}{f_1(r, \theta)} (dr^2 + r^2 d\theta^2) + f_2(r, \theta) d\psi^2 + f_3(r, \theta) (d\varphi - \frac{\omega(r, \theta)}{r} dt)^2,
\]
where
\[
A\alpha(r, \theta) dx^\alpha = A_0(r, \theta) dt + A_\varphi(r, \theta) d\varphi.
\]
Here, the rotation occurs in the direction of the $S^1$ of the ring. The resulting system of seven coupled non-linear elliptic partial differential equations is solved numerically.

4. Charged Balanced Black Rings – Results
The solutions depend on four parameters, i.e. the horizon radius $r_0$, the horizon angular velocity $\Omega_H$, the charge parameter $\alpha$ and the parameter $b$ representing a rough measure for the radius of the $S^1$.

In order to analyze the physical properties of the solution, it is convenient to work with dimensionless quantities, i.e. the scaled horizon area $a_H$ and the scaled temperature $t_H$ as well as the scaled squared angular momentum $j^2$ and the scaled charge $q$
\[
a_H = \frac{3}{16} \sqrt{\frac{3}{\pi (G_5 M)^{3/2}}} A_H, \quad t_H = T_H \sqrt{G_5 M}, \quad j^2 = \frac{27\pi}{32G_5 M^3}, \quad q = \frac{Q}{M}.
\]
We find that all solutions satisfy a Smarr relation

\[ M = \frac{3}{16\pi G_5} \kappa A_H + \frac{3}{2} \Omega_H J + \Phi_H Q, \]

where \( \kappa = 2\pi T_H \) is the surface gravity.

In the static case as well as for generic values of the rotation parameter, the solutions suffer from a conical deficit. By adjusting the parameter values, however, we have succeeded in obtaining regular solutions. Our solutions agree excellently with the static analytical solution by...
Yazadjiev [4] and the neutral analytical solution by Emparan and Reall [3]. In comparison with the neutral rotating solutions, we observe that for the electrically charged balanced solutions an increasing value of the scaled charge $q$ makes the conical deficit vanish for lower values of the rotation parameter $\Omega_H$ and hence for lower values of the scaled squared angular momentum $j^2$. The phase diagram is shown in Fig. 2(b).

While the gyromagnetic ratio $g = \frac{2M M_\phi}{j^2}$ for weakly charged rings has been found to be $g = 3$, in accordance with Ortaggio and Pravda [6], a negative deviation from this value has been observed for larger magnitudes of the electric charge (Fig. 3(a)), similar to the case of black holes with a spherical horizon topology ([7]).

The ergosurface of the balanced solutions moves farther away from the horizon with increasing scaled charge (Fig. 3(b)).

For further reading confer the peer-reviewed article of this work ([1]).

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