Dissipation-assisted quantum gates with cold trapped ions

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It is shown that a two-qubit phase gate and SWAP operation between ground states of cold trapped ions can be realised in one step by simultaneously applying two laser fields. Cooling during gate operations is possible without perturbing the computation and the scheme does not require a second ion species for sympathetic cooling. On the contrary, the cooling lasers even stabilise the desired time evolution of the system. This affords gate operation times of nearly the same order of magnitude as the inverse coupling constant of the ions to a common vibrational mode.

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For many years now ion trapping technology has been one of the standard techniques for investigating quantum phenomena with single particles. Paul traps or more complex structures have been used to create chains of several hundred ions [1]. Many schemes for implementing quantum gates using trapped ions have been proposed. Some of them require cooling into the vibrational ground state [2–5]. The feasibility of this has already been experimentally demonstrated [6]. Other schemes can even be implemented with “hot” ions [7] and have been applied to entangle up to four ions [8] and to observe violation of Bell’s inequality with single atoms [9].

This paper proposes an alternative scheme for realising a phase gate and SWAP operation with cold trapped ions in one step by simultaneously applying two laser fields. Compared with other schemes [2,4], the experimental effort for quantum computing can thus be greatly reduced. Ideally, the system remains during the whole gate operation in the ground state of a common vibrational mode. Thus cooling of this mode does not perturb the computation. On the contrary, it is shown that cooling can even improve the gate fidelity significantly. This affords gate durations of nearly the same order of magnitude as the inverse coupling constant of the ions to the common vibrational mode and there is no need to include a different ion species for sympathetic cooling [10] in the trap. In addition, the scheme only requires good control over one Rabi frequency. The size of the coupling strength of the ions to the vibrational mode does not enter the effective time evolution of the qubits.

Each qubit is obtained from the atomic ground states |0⟩ and |1⟩ of one ion, while the common vibrational mode is cooled down to its ground state |0\text{vib}\rangle. To establish coupling between qubits, we use a metastable state |2⟩ and a strong laser field with Rabi frequency Ω2 detuned by the phonon frequency ν, as shown in Figure 1. The phase gate that adds a minus sign to the amplitude of the qubit state |01⟩ then requires in addition only a weak laser pulse with the Rabi frequency Ω coupling resonantly to the 0-2 transition of ion 1. Let us denote the creation operator of a single phonon in the mode as b† and introduce the coupling constant of the ions to the common vibrational mode as g2 = 1/2 g1 Ω2. (Here η is the Lamb-Dicke parameter characterising the steepness of the trap.) The Hamiltonian of the system within the dipole and the rotating wave approximation and in the interaction picture with respect to the free Hamiltonian is then given by

\[ H = \sum_{i=1}^{2} i h g_{2}|1\rangle\langle 2|b^\dagger + \frac{1}{2} h \Omega |0\rangle_1|2\rangle + \text{h.c.} \]  

(1)

Here the Lamb-Dicke regime and the condition \( g_2^2 \ll \nu^2 \) have been assumed, as in [2].

![Figure 1](#)

**FIG. 1.** Level scheme for implementation of a phase gate. A strong laser field with detuning \( \nu \) establishes coupling between the ions via a common vibrational mode. In addition, a laser pulse individually addressing the 0-2 transition of ion 1 is required.

Let us first assume that the coupling constant \( g_2 \) is a few orders of magnitude larger than the Rabi frequency \( \Omega \). Then there are two different time scales in the system and the time evolution can be calculated to very good approximation by adiabatic elimination. To do so, the amplitudes of the state with \( n \) phonons in the vibrational mode and the ions in |ij⟩ is denoted as \( c_{ij} \). Only the coefficients of the qubit states, \( c_{00}, c_{001}, c_{010} \) and \( c_{011} \), and of the entangled state |0a⟩ with |a⟩ = (|12⟩ − |21⟩)/√2 change slowly in time. Their time evolution is given by

\[ \dot{c}_{000} = -\frac{i}{2} \Omega c_{020}, \]

\[ \dot{c}_{001} = \frac{i}{2\sqrt{2}} \Omega (c_{0a} - c_{0\bar{a}}), \]
\[ \dot{c}_{0a} = \frac{1}{2\sqrt{2}} \Omega c_{001} \]
\[ \dot{c}_{010} = \dot{c}_{011} = 0. \] (2)

Setting the derivatives of all other coefficients equal to zero yields \( c_{020} = c_{03} = 0 \). For \( \Omega \ll g_2 \), the time evolution of the system (2) can thus be summarised in the effective Hamiltonian
\[ H_{\text{eff}} = -\frac{1}{2\sqrt{2}} \hbar \Omega \left[ |001\rangle/0a| + \text{h.c.} \right]. \] (3)

If the duration of the laser pulse equals \( T = 2\sqrt{\pi}/\Omega \), then the resulting evolution is the desired phase gate. As the effective Hamiltonian \( H_{\text{eff}} \) and the gate operation time \( T \) are independent of the coupling constant \( g_2 \), the proposed scheme is widely protected against fluctuations of this system parameter.

Deviations from the time evolution (3) arise because population accumulates unintentionally in the states \( |010\rangle \) and \( |011\rangle \). In first order in \( \Omega/g_2 \) one has
\[ c_{110} = -\frac{\Omega}{2g_2} c_{000}, \quad c_{111} = -\frac{i\Omega}{4g_2} c_{001} \] (4)
and the fidelity of the proposed phase gate equals
\[ F(T, |\psi\rangle) = 1 - \frac{\Omega^2}{4g_2^2} \left[ |c_{000}|^2 + \frac{1}{4} |c_{001}|^2 \right]. \] (5)

Figure 2 compares this fidelity with the exact solution resulting from numerical integration of the time evolution (1). Good agreement is only found for \( \Omega < 0.1 \Omega_2 \). For somewhat larger Rabi frequencies, in general fidelities worse than the result predicted by (5) are obtained due to nonadiabaticity. Thus the fidelity is above 99% for all initial states if \( \Omega < 0.1 \Omega_2 \) is chosen. This corresponds to gate operation times \( T > 90/\Omega_2 \). In the following we aim at enlarging the parameter regime for which the fidelity is at least 99% [11].

**FIG. 2.** The fidelity of a single phase gate as a function of \( \Omega/g_2 \) for the initial qubit states \( |00\rangle \) (a,b) and \( |01\rangle \) (c,d). The curves (a,c) result from an exact solution of the time evolution (1) while the curves (b,d) result from (5). For the states \( |10\rangle \) and \( |11\rangle \) optimal fidelities \( F \equiv 1 \) are obtained.

Dominating error source in the scheme is heating. To avoid this, the gate should be performed fast. Therefore we assume in the following that \( \Omega \) is of nearly the same size as the coupling constant \( g_2 \). As can be seen from above, increasing \( \Omega \) leads to the population of unwanted states. To reduce the error rate of the scheme one could therefore measure the population of states with \( n > 0 \) at the end of each gate. Under the condition that no population is found in these states, the system gets projected back onto the subspace with \( n = 0 \) and the fidelity of the prepared state increases. In case of the detection of an error, the gate failed and the whole computation has to be repeated.

To realise such an error detection measurement one could use a laser field that couples the atomic ground state \( |1\rangle \) with detuning \( \nu \) to an auxiliary state \( |3\rangle \) and another laser that excites the atomic 1-2 transition with the same detuning. To assure that populating level 3 leads to the emission of many photons, an even stronger laser should couple \( |3\rangle \) to a rapidly decaying fourth level which decays into \( |3\rangle \) with a high spontaneous decay rate. Gate failure leads thus to an effect which is known as a “macroscopic light period” [12], i.e., fast Rabi oscillations between level 3 and 4 accompanied by photon emission at a high rate (for details see [13]). This can easily be detected and the whole computation can be repeated, if necessary.

**FIG. 3.** Level scheme of a dissipation-assisted phase gate with cold trapped ions. In addition to the basic setup shown in Figure 1, a strong laser field couples the state \( |1\rangle \) with detuning \( \nu \) to an auxiliary atomic state \( |3\rangle \). Populating this level leads to the emission of photons with a rate \( \Gamma_3 \).

However, such an error detection measurement would take much longer than the inverse of the coupling constants of the ions to the vibrational mode and much longer than \( T \). Hence, error detection does not help to decrease the gate operation time for a given minimum fidelity \( F \). To shorten the gate duration without reducing the fidelity more than predicted by (5), we propose that the desired time evolution (3) be stabilised during the gate performance by using dissipation. This is achieved by continuously applying the laser fields, proposed for the implementation of error detection, as shown in Figure 3. For simplicity, the strong laser coupling to the fourth level and spontaneous decay from this level have been combined into a single decay rate assigned to the
metastable state \([3]\). Let us denote this spontaneous decay rate as \(\Gamma_3\), while \(g_3 = \frac{1}{2}\Omega\Gamma_3\) is the coupling strength of the atomic 1-3 transition to the vibrational mode. In the following, \(g_3 \sim g_2\) and \(10g_2 < \Gamma_3 < 100g_2\) is assumed.

More general, any process that indicates whether the phonon mode is excited or not can serve as an error detection measurement. This applies to ground state cooling [14] because populating the vibrational mode leads to the emission of photons at a high rate while no emission takes place if the ions are in the vibrational ground state. Thus level 3 and the additional strong laser field in Figure 3 could as well be replaced by the cooling laser setup. Indeed, continuous cooling can improve the fidelity of the performed gate operation. However, for simplicity, the continuous read out of the phonon mode is in the following modeled as shown in Figure 3.

Basic mechanism of the improved scheme is that observing for emitted photons implements a (conditional) no-photon time evolution, thus resulting in continuous damping of the population in unwanted states. As long as the amplitudes \(c_{20}\) and \(c_{0s}\) are negligible, the time evolution of the other states resembles the desired phase gate, as can be seen from (2). In addition, we show that the population that now accumulates in the states \([110]\) and \([111]\) is about the same as predicted for an adiabatic process and the fidelity of the phase gate coincides with \(F(T,|\psi\rangle)\) given in (5) to a very good approximation. The price one has to pay for this improvement of the precision of the gate is that photon emission might occur with a small probability. Then the gate operation would have failed.

To describe the time evolution of the system under the condition of no photon emission, we use in the following the Schrödinger equation with the conditional Hamiltonian \(H_{\text{cond}}\). As predicted by the quantum jump approach [15], the norm of a vector developing with this non-Hermitian Hamiltonian decreases in general with time and

\[
P_{0}(T,|\psi\rangle) = \| U_{\text{cond}}(T,0) |\psi\rangle \|^2 \tag{6}
\]

is the probability of no photon emission in \((0,T)\) if \(|\psi\rangle\) is the initial state of the system. For the level configuration shown in Figure 3 the conditional Hamiltonian equals

\[
H_{\text{cond}} = \sum_{i=1}^{2} \sum_{j=2}^{3} i\hbar g_j \langle j|b|1\rangle + \frac{\hbar}{2} |\psi\rangle \langle\psi| \frac{1}{2} g_2 \Gamma_3 \tag{7}
\]

Because of the different time scales of the scheme, the no-photon time evolution of the system can again be calculated by adiabatic elimination. This yields the same effective Hamiltonian as in (3). In first order in \(\Omega/g_2\), population accumulates unintentionally in the states \([020]\), \([110]\), \([030]\), \([0s]\) \([111]\), \([013]\) and \([031]\) and it is

\[
(c_{0s}, c_{111}, c_{013}, c_{031}) = \left(- \frac{i\Omega}{g_2} \left( \frac{\sqrt{2}g_3^2}{g_2} \frac{1}{2} \frac{g_3}{\Gamma_3} \Gamma_3 \right) c_{001},
\right.
\]

\[
\left. (c_{020}, c_{110}, c_{030}) = - \frac{i\Omega}{g_2} \left( \frac{g_3^2}{2g_2^2 \Gamma_3} \frac{1}{2} \frac{g_3}{\Gamma_3} \Gamma_3 \right) c_{000} \right). \tag{8}
\]

To optimise the fidelity, \(\Gamma_3\) should be much larger than \(g_3\) so that all coefficients proportional \(g_3/\Gamma_3\) become negligible. In this case, \(F(T,|\psi\rangle)\) becomes the fidelity calculated in (5) to a very good approximation.

That this is indeed the case is shown in Figure 4 which results from a numerical solution of the time evolution (7). As expected, the dissipation channel continuously introduced in the system stabilises the desired time evolution and corrects for errors resulting from the nonadiabaticity of the scheme if \(\Omega\) becomes of about the same order of magnitude as \(g_2\). The gate fidelity (5) applies now to a much wider parameter regime. Fidelities above 99 % are obtained if \(\Omega < 0.18\ g_2\) is chosen. However, for too big spontaneous decay rates \(\Gamma_3\), the damping of unwanted amplitudes becomes ineffective which is why \(\Gamma_3 < 100\ g_2\) has been assumed.

![Fidelity of a single phase gate under the condition of no photon emission as a function of \(\Omega/g_2\) for the initial qubit states \(|\psi\rangle = |00\rangle\) (a,b) and \(|\psi\rangle = |01\rangle\) (c,d). Deviations from the time evolution with \(g_3 = \Gamma_3 = 0\) are continuously damped away with \(g_3 = g_2\) and \(\Gamma_3 = 20\ g_2\) (a,c) and the fidelity is close to the theoretically predicted fidelity (5) assuming adiabaticity (b,d).

Using the coefficients \(c_{0s}\) and \(c_{020}\) given in (8) and the differential equations that govern the time evolution of the qubit states and the entangled state \(|0a\rangle\) (the same as (2)), the unnormalised state of the system at the end of the gate operation under the condition of no photon emission can be calculated up to first order in \(\Omega/g_2\). Its norm squared equals the gate success rate (6) and

\[
P_{0}(T,|\psi\rangle) = 1 - 2\sqrt{2} \frac{\Omega g_2}{g_2^2 \Gamma_3} \left[ |c_{000}|^2 + |c_{001}|^2 \right]. \tag{9}
\]

This is in good agreement with the numerical results shown in Figure 5. For example, for \(\Omega < 0.1g_2\) and \(\Gamma_3 = 20\ g_2\) one has \(P_{0}(T,|\psi\rangle) > 95\%\), independent of
the initial state of the system. The probability for photon emission during the gate operation is of the order of \( \Omega/g_2 \) and for \( \Omega g_2^2/(g_3^2 \Gamma_3) \ll 1 \) close to unity. Gate failure might be a bit more likely than for quantum error detection (see Figure 5). However, no additional time is required which would increase the sensitivity of the scheme with respect to heating.

Another two-qubit gate that can easily be realised with the same experimental setup is SWAP operation. This gate exchanges the states of two qubits without the corresponding ions having to change their places physically. Compared with the above phase gate, its implementation does not require individual laser addressing. To realise SWAP operation, the laser field with Rabi frequency \( \Omega \) does not need to be individually addressed. To realise SWAP operation, the laser field with Rabi frequency \( \Omega \) does not need to be individually addressed.

Compared with the above phase gate, its implementation requires which would increase the sensitivity of the scheme with respect to heating.

**FIG. 5.** Success rate of a single phase gate as a function of \( \Omega/g_2 \) for the initial qubit states \( \psi = |00\rangle \) (a,b) and \( \psi = |01\rangle \) (c,d) for the case of continuous monitoring of the system (a,c) and the case of only a single error detection measurement at the end of the gate (b,d).

\[ H_{\text{cond}} = \sum_{i=1}^{2} \sum_{j=2}^{3} \hbar g_{ij} |i\rangle \langle j| b^\dagger + \frac{1}{2} \hbar \Omega |0\rangle \langle 2| + \text{h.c.} \]

\[ -\frac{1}{2} \hbar \Gamma_3 |3\rangle \langle 3| . \]

Proceeding as above leads to the effective Hamiltonian

\[ H_{\text{eff}} = \frac{1}{2 \sqrt{2}} \hbar \Omega \left[ -|001\rangle \langle 0a| + |010\rangle \langle 0a| + \text{h.c.} \right] . \]

and if \( T = 2\pi/\Omega \), then the time evolution of the system exchanges the amplitudes of the qubit states \( |01\rangle \) and \( |10\rangle \) of the initial state. Deviations of the fidelity from unity are, for the same parameter regime as considered above, about the same size as for the proposed phase gate since implementation of the two gates is very similar. The same applies to the improvement of the gate precision that can be achieved with the help of dissipation and to the gate success rates.

Summarising, we have shown that dissipation can be used to construct fast, simple and precise gates for quantum computing. As an example we discussed the implementation of a two-qubit phase gate and SWAP operation with cold trapped ions. Here the parameter regime has been chosen such that auxiliary decay channels, resulting from continuous cooling of the ions, stabilise the desired adiabatic time evolution of the system and improve the fidelity of the gate operation significantly. The more general regime where strong ground state cooling allows the implementation of arbitrary single laser pulse gates will be discussed elsewhere [13].

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[1] G. Birkl, S. Kassner, and H. Walther, Nature **357**, 310 (1992); M. Drewsen, C. Brodersen, L. Hornekaer, J. S. Hangst, and J. P. Schiffer Phys. Rev. Lett. **81**, 2878 (1998).

[2] J. I. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995).

[3] C. Monroe, D. Leibfried, B. E. King, D. M. Meekhof, W. M. Itano, and D. J. Wineland, Phys. Rev. A **55**, R2489 (1997).

[4] D. Jonathan, M. B. Plenio, and P. L. Knight, Phys. Rev. A **62**, 042307 (2000).

[5] For a recent review see M. Šašura and V. Bužek, J. Mod. Opt. **49**, 1593 (2002).

[6] C. Monroe, D. Meekhof, B. E. King, W. M. Itano, and D. J. Wineland, Phys. Rev. Lett. **75**, 4714 (1995); C. F. Roos, D. Leibfried, A. Mundt, F. Schmidt-Kaler, J. Eschner, and R. Blatt, Phys. Rev. Lett. **85**, 5547 (2000).

[7] J. F. Poyatos, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **81**, 1322 (1998); A. Sørenson and K. Mølmer, Phys. Rev. Lett. **82**, 1971 (1999); S. Schneider, D. F. V. James, and G. J. Milburn, J. Mod. Opt. **47**, 499 (2000); D. Jonathan and M. B. Plenio, Phys. Rev. Lett. **87**, 127901 (2001).

[8] C. A. Sackett, D. Kielpinski, B. E. King, C. Langer, V. Meyer, C. J. Rowe, Q. A. Turchette, W. M. Itano, D. J. Wineland, and C. Monroe, Nature **404**, 256 (2000).

[9] M. A. Rowe, D. Kielpinski, V. Meyer, C. A. Sackett, W. M. Itano, C. Monroe, and D. J. Wineland, Nature **409**, 791 (2001).

[10] D. Kielpinski, B. E. King, C. J. Myatt, C. A. Sackett, Q. A. Turchette, W. M. Itano, C. Monroe, D. J. Wineland, and W. H. Zurek, Phys. Rev. A **61**, 032310 (2000); G. Morigi and H. Walther, Eur. Phys. J. D **13**, 261 (2001).

[11] For specific values of \( \Omega \), for example \( \Omega = 0.8 g_2 \), fidelities above 96% are achieved even if \( \Omega \) is about the same size as \( g_2 \).

[12] H. G. Dehmelt, Bull. Am. Phys. Soc. **20**, 60 (1975).

[13] A. Beige, W. Lange, and H. Walther (in preparation).

[14] See for example G. Morigi, J. Eschner, and C. H. Keitel, Phys. Rev. Lett. **85**, 4485 (2000).

[15] G. C. Hegerfeldt, Phys. Rev. A **47**, 449 (1993).