A study of junior high school students reasoning skill in mathematics

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Abstract. Mathematical reasoning skill is an ability that must be possessed by each student because it involves the ability to think logically and systematically. There are two types of tests to measure students’ skills in mathematics TIMSS and PISA. The purpose of this study is to investigate students reasoning skills to solve the tests that are similar to TIMSS and PISA model. The data were collected using 6 questions similar to TIMSS and PISA test from 27 Junior High School students. The data were analyzed through data reduction, data presentation, and data verification, to comprehensively measure students’ abilities for analyzing, generalizing, synthesizing, justifying, and solving a non-routine problem. The results show that there are students have good reasoning skills to analyze, to generalize, to synthesize, to justify, and to solve a non-routine problem (62.69%, 66.67%, 48.15%, 24.07%, and 3.70 respectively). There are some factors influence the different level of students reasoning skills including their ability to identify information in the question, use mathematical arguments to explain the answer and identify the strategy to solve problems.

1. Introduction

There is a strong relationship between reasoning skills and mathematics [1]. Reasoning skills have existed in all levels and topics of mathematical learning [2]. Moreover, through mathematics learning, students are able to develop their reasoning skills because people need this ability to study mathematics [3]. The New Jersey Mathematics Curriculum Framework (NJMCF) [4] also reveals that reasoning skill serves as an adhesive component to combine and cement all skills related to mathematics. Mullis, Martin, and Foy [5] revealed that this reasoning skill is closely related to each individual’s process of logical and systematic thinking. Furthermore, a person with a good reasoning ability is capable to solve a non-routine problem encountering [5]. Therefore, reasoning skill is very important for individuals who learn and work using mathematics.

The reasoning ability is important in mathematics. It also positively serves as a good predictor of students’ achievement in mathematics. It is caused by the achievement of Indonesian students in mathematics, especially in Junior High School, which is still low. For instance, the students’ accomplishment in the national examination is insufficient [6]; and the results of an international survey by the Programme for International Student Assessment (PISA) [7] and Trends in International Mathematics and Science Study (TIMSS) [8] also reveal that students’ mathematics achievements are...
insufficient. The test and questionnaire which is used to measure students’ mathematics achievement also consists of items that purportedly measure learners’ ability in reasoning skill. For example, the TIMSS assessment provides a cognitive domain including knowing, applying, and reasoning. Based on the national examination results and the international assessment of PISA and TIMSS indicates that students in Indonesia have some difficulties in using their reasoning ability to solve mathematics’ problems.

The reasoning is the line of thought adopted to produce affirmations and to reach conclusions in task-problem solving [2]. The reasoning might also be interpreted as a process for making conclusions based on some evidences and assumptions [9]. The reasoning ability is the ability of students to give reasons and to prove their conclusions by using a scientific procedure [10]. Ersoy and Bal-İncebacak [11] define reasoning skill as analyzing problems to solve it, presenting ideas and making suggestions to support the idea, testing the suggestion when it is available, and deciding the accurateness of the result. Therefore, reasoning skill is the individuals’ ability to provide a logic conclusion based on some valid piece of evidence and systematic way to solve a problem. The reasoning skill consists of some components including, analyzing, generalizing, synthesizing justifying, and solving non-routine problems [5].

The student's mathematics achievement in the national examination, PISA, and TIMSS indicates that they have some problems using their reasoning abilities to solve problems. Therefore, it is crucial to identify the influential factors of the students’ low achievement in mathematics, providing insight for further learners’ mathematics competencies development [12]. The results might help teachers and students to find out what is the most difficult component of the reasoning skill where students would have problem to master it. The teachers also can develop a good curriculum and teaching material if they able to identify the learners’ problems, creating more meaningful classroom activities for students [13]. Likewise, when teachers can understand the way how students solve a problem, they can design more effective and contextual learning for students [14, 15, 16]. The innovative learning is designed to enhance conceptual understanding towards the students concerning the difficult indicators [12]. This study focuses on the identification of student difficulties in using reasoning skills to solve problems.

Identifying how student’s reasoning skill in solving problems can be done by asking students to explain why they made some mistakes when solving problems. For example, sometimes students choose the correct problem-solving strategy, but they miscalculate and finally have a wrong result; or they stop the process of problem-solving through a systematic process before finding the final result. If the students solve a problem without using a scientific way, known as the error in the reasoning process, it fosters an in-depth investigation to recognize the way students think [15].

Therefore, this study aims to describe how the reasoning skills of junior high school students in Indonesia. It utilizes TIMSS and PISA tests to explore students’ reasoning skills. The test of TIMSS model was developed to assess two domains, namely content and cognitive [5]. The content domain adapted in this study includes number and geometry, while the cognitive domains are the reasoning. The PISA test was developed to assess the students’ ability to solve real problems so that the content of the problem in the PISA problem is related to the contextual problem. The results of this study might provide a description of the students’ difficulties in learning mathematics and how students reasoning skill in problem-solving based on PISA and TIMSS standards. In addition, the results of this study can be used as a reference material for further study.

2. Methods
This study is a descriptive study. It is intended to explore the reasoning skills of high school students in solving the problem of TIMSS and PISA tests model. The subjects of this study were 27 junior high school students who had diverse abilities.

In this study, the researchers developed six questions adapted from the TIMSS and PISA test model. The six questions consist of 2 essays and 4 multiple choice questions. When completing the tests, the students were asked to describe their calculation or procedures in solving the problem. The test consists of 1 item of question to measure students analysis skill, 1 question to measure students
synthesis skill, 2 items of question to measure students ability to provide scientific reasons, 1 item of question to measure students ability to generalize the conclusion, and 1 item of question to measure students skill in solving non-routine problems. The six items question will contain reasoning indicators according to TIMSS, as shown in table 1.

| Phases         | Description                                                                 |
|----------------|-----------------------------------------------------------------------------|
| Analyze        | Determine and describe or use relationships between variables or objects in mathematical situations and make valid inferences from a given information |
| Generalize     | Extend the domain to which the result of mathematical thinking and problem solving is applicable by restating results in more general |
| Synthesize     | Combine mathematical procedures to establish results, and combine results to produce a further result |
| Justify        | Provide a justification for the truth or falsity of a statement by reference to mathematical results of properties |
| Solve Non-routine problems | Solve problems set in mathematical or real-life contexts where target students are unlikely to have encountered closely similar item and apply mathematical procedures in unfamiliar or complex contexts |

The data were analyzed using the Miles and Huberman method [17] which consists of: (1) data reduction, (2) data presentation, and (3) verification and conclusions. Data interpretation is carried out simultaneously with data presentation activities. During the whole process of data analysis, the researchers comprehensively interpreted the data of students’ achievement in the test and interviewed them right after the test completed. Finally, the researchers provided conclusions and recommendations based on the results of data analysis, providing a description of students reasoning skill to solve TIMSS and PISA model test.

3. Results and Discussion
There were five indicators of students’ ability on reasoning skills analyzed in this study such as students’ ability to analyze, to generalize, to synthesize, to justify, and to solve a non-routine problem. Based on the data analysis, the results show that there are students who correctly, almost correct, and incorrectly answer the questions in the test (figure 1).

![Figure 1. The result of Reasoning Test](image)

Figure 1 shows the result of the mathematical reasoning test. The results show that the students have differences ability in reasoning skill competency. There were 62.96% students have a correct reasoning in the phases are to analyze, 66.67% students have a correct reasoning in the phases are to generalize, 48.15% students have a correct reasoning in the phases are to synthesize, 24.07% students have a correct reasoning in the phases are to justify, and 3.70 % students have a correct reasoning in
the phases are to solve non-routine problem. Details of students’ ability on each indicator of reasoning skills are described in the following paragraph.

3.1. Analyze

The first question given to students is a test to measure students analytical skills which are counted as one of the indicators of reasoning skill in TIMSS. This item is a TIMSS test model with a number content. The results show that 62.96% of students are able to analyze the problem correctly, while 37.04% of students are not incorrectly do the analysis (figure 1). Students who are correctly doing the analysis made several multiplication combinations of four numbers to find the greatest multiplication results. In general, they do trial and error to find the right answer. The data examples revealed that the students did trial and error to find the correct answer which is shown in Figure 2.

On the contrary, the group who made mistakes in doing the analysis, most of the students immediately place the arrangement of numbers so that they get multiplication results. The results of the multiplication are not the biggest. It indicates that students misunderstood the item of question to find the greatest multiplication results. They only understand that being asked to place numbers into the boxes provided then multiply them. Examples of incorrect student answers are shown in Figure 3. The analysis of problems used in this study are still limited to analyze the relationship between the provided information and the drawn conclusions. So that it is still limited to analyze procedures in mathematics.

3.2. Generalize

The second question given to students is a test to measure the students’ generalizing skill. This test is a TIMSS test model with statistical content. The results show that 66.67% of students can make generalizations correctly, while 33.33% of students are incorrect (figure 1). The group of students who correctly make a generalization, mostly made it by looking for patterns formed from the graph provided. From the pattern, they can conclude that the increase in sales of lemon cola is 20 per year and the increase in sales of soft drinks is 10 per year. From this pattern, they can estimate the number of sales in a given year. Most students have found the pattern, so it is correct to make generalizations. Determining patterns has contributed to a better understanding of the structure of mathematics and helps in making generalizations [15]. Examples of correct answers are shown in Figure 4.
Meanwhile, there were students who did not make mistake in generalizing, there are some who did not even understand the purpose of the problem so they did not generalize it. In addition, there were also those who made mistakes when determining the pattern formed in the provided graph. Mistakes in making these patterns, make students experience errors in drawing conclusions. Examples of incorrect student work are found in Figure 5.

3.3. Synthesize
The third question given to students is a test to measure the ability of students to synthesize. This test was a TIMSS test model with geometric content. The results show that 48.15% of students can synthesize correctly, 22.22% of students almost do the synthesize correctly, and 29.63% of students are incorrect in synthesizing (figure 1). Students who are correct in doing synthesis can combine various information in the question to solve the given problem. The students combine the information by drawing cubes, tubes, and circles to determine how many the comparisons are. The figures provided in this test facilitates students to understand the purpose of the test and to give the correct answer. It indicates that visual tools can provide useful insight for students to achieve a better understanding of mathematical concepts [18]. Examples of the correct answers are shown in Figure 6. In addition, there are some students who give a number in tubes, cubes, and circles to make comparisons. The students transform geometric shapes into numbers so that it can be easier to calculate. Examples of student answers which answering the transforming forms into numerical forms are shown in Figure 7.

![Figure 6. Correct Synthesize](image1)
![Figure 7. Correct Synthesize](image2)

Students who are almost correct to answer the test combine information correctly; however, they are incapable to draw the right conclusions. Furthermore, the reason students who incorrectly did the synthesis test were caused by the failure in finding the connection between the informations that are provided. As a result, they cannot properly answer the question.

3.4. Justify
The fourth and fifth questions given to students is a test to measure the students’ ability to justify a problem. The results show that 24.07% of students can justify correctly, 11.11% of students justify it almost correctly and 64.81% of students justify it incorrectly (figure 1). The test to measure students’ skill to justify is geometry. In the fourth question, students were asked to give reasons why the triangle shown a blunt triangle. Many students made mistakes because they do not know in what conditions the triangle is categorize as a blunt triangle. That is the reason of why these students do not master the concept of a blunt triangle. For this reason, mathematical concepts must be understood by students so that mathematics are not just numbers, calculations, symbols and formulas [19]. Examples of incorrect answers can be seen in Figure 8.
The fifth question is also a level 4 geometry test in PISA. In this question, there were no students who correctly answered the question. The test consisted a question where the students need to design a fence that can be installed in the yard. Almost all students do not use mathematical reasons to answer this question. They should calculate the maximum perimeter fence design as large as the length of the fence that is available. They can increase the height of the fence because the height of the fence is not taken into account in this matter. But almost all students do not use it, they only see the shape of the fence. According to the students the fence had to be straight and could not be winding. Therefore, they only give reasons based on the shape of the fence. This is likely because students knowledge of geometry and visual perception has not yet accommodated, so they do not use mathematical arguments to explain the answer [15]. Lack of knowledge about geometry makes it difficult for students to manipulate various geometric shapes [20]. In this problem, students ability to understand various forms of manipulation in the fence design will be needed. In addition, students find it difficult to identify information that is useful in the matter [19]. Examples of incorrect student work results can be shown in Figure 9.

3.5. Solve Non-Routine Problems
The test number six is given to students to measure their ability to solve a non-routine problem. The results of the test revealed that 3.70% of students can solve non-routine problems correctly and 96.29% of students are doing a mistake in solving non-routine problems (graph 1). This question is a level 3 geometry test of PISA. If students are correctly answering the question number six, students can get the biggest profit at the same price, by buying two large martabak (thick folded crepe filled with spices and pieces of meat). This is obtained by measured the surface area of each martabak then multiplying 3 for small martabak and 2 for large martabak. So, it was found that by paying the same price, it would be better to buy 2 large martabak because the surface area is wider.

Students who incorrectly answered the question were caused by the failure of considering the surface area of martabak. Most students only multiply the number of martabak with the price of each martabak. Thus, two big martabak and three small martabak have the same price. In addition, some students also only calculate the diameter length. Two large martabak and three small martabak have the same diameter. Therefore, they assume that buying 2 large martabak with 2 small martabak has no different. Examples of incorrect student answer can be seen in Figures 10 and 11.
The ability of students to solve non-routine problems is very insufficient. This is likely because students have difficulty identifying information that is useful in the question [19]; leading to an incorrect strategy to solve the problem. In this test, 96% of students did not use information on the length of the diameter of the martabak to solve the problem. Length of diameter, is important information from the problem that can be used to solve problems. In addition, students also find it difficult to transform visual expressions into algebraic expressions [21]. Where students actually understand that a larger surface martabak is more profitable. But they did not calculate the surface area by means of mathematics.

4. Conclusion
The reasoning skill component based on a TIMSS standard consists of 5 components including analyzing, generalizing, synthesizing, justifying, and solving non-routine problems. The results of this study show that (1) 62.96% students have a correct reasoning in the phases of analyzing, while 37.04% of students have an incorrect reasoning in the phases of analyzing; (2) 66.67% of students have a correct reasoning in the phases of generalizing, while 33.33% of students have an incorrect reasoning in the phases of generalizing; (3) 48.15% students have a correct reasoning in the phases are to synthesize, 22.22% of students have an almost correct reasoning in the phases of synthesizing, and 29.63% of students have an incorrect reasoning in the phases of synthesizing; (4) 24.07% of students have a correct reasoning in the phases are to justify, 11.11% of students have an almost correct reasoning in the phases of justifying and 64.81% of students have an incorrect reasoning in the phases of justifying; (5) 3.70% of students have a correct reasoning in the phases are to solve non-routine problems and 96.29% of students have an incorrect reasoning in the phases are to solving non-routine problems.

Based on these results, the factors that fostering the students reasoning skill’s insufficient are (1) students unable to identify information that is provided in the question; (2) In geometry, students do not use mathematical arguments to explain the answer; and (3) the students unable to identify the best strategy to solve problem. Therefore, to optimize students reasoning abilities can be done by overcoming the difficulties that are faced by students.

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