Application of the multiplicative cascade model for the
description of the seismic regime and for the seismic hazard
assessment

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Abstract. Seismic process is usually considered as a realization of the regime of self-organizing
criticality (SOC-model). This model meets, however, definite problems, besides it gives only a
statistical description of the seismic regime. The multiplicative cascade model (MCM) treats the
seismic regime as an assemblage of randomly developing episodes of avalanche-like relaxation,
occurring in a set of similar metastable sub-systems. In the simplest linear variant without
memory the MCM is defined by the flow of events and by two parameters characterizing the
hierarchical structure and the level of metastability of a geophysical medium. An advantage of
such approach consists in a clear physical sense of character of the model. The MCM model
$b$-value behavior is compared with the complex of typical anomalies revealed in result of
examination of a large earthquake generalized vicinity (LEGV). The LEGV vicinity is
constructed of earthquakes falling into the zones of influence of a large number (300, 500, or
1000) of largest earthquakes. The construction and examination of LEGV gives possibility to
increase radically the available statistics, crucially diminish a random component of the seismic
regime, and in result to reveal the typical features of pre- and post-shock seismic activity in
details. The combined use of MCM and LEGV methods gives possibility to interpret a few
features of the seismic regime and to suggest the typical scenario of the fore- and aftershock
regimes of seismicity. The possibilities of application of the obtained results for earthquake
prediction are discussed.

1. Introduction
Seismic regime is usually considered as an example of realization of the regime of self-
organizing criticality - the SOC-model [Bak et al., 1988]. However, the analogue between
the critical phenomena and the seismic process is not well satisfying [Ben-Zion, 2008; Rodkin,
2011]. The critical phenomena (the second order phase transitions, for example) proceed
without discharge or absorption of energy; this is their fundamental peculiarity that in many
respects determines the specific features of the critical behavior. But during earthquakes a huge
amount of energy is released. Thus, the consideration of the regime of seismicity in the terms
of the SOC-model is not quite satisfactory.

For quantitative statistic modeling of regime of seismicity the Epidemic Type Aftershocks-
Sequences (ETAS) model is used [Ogata, 1988; and many others]. This model is based on the
empirical laws of the seismic regime – namely the Gutenberg-Richter and the Omory laws.
Such statistical description of the seismic regime is very fruitful; however, it does not result in
an essential progress in the understanding of the physics of seismic process. So, the consideration of seismicity in the terms of the SOC and ETAS models only is incomplete, and other approaches appear to be required.

As a possible variant of a more physical approach to the examination of the seismic regime the multiplicative cascade model (MCM) describing the seismicity in the terms of hierarchy of the medium, the memory of the system, and the degree of disequilibrium of processes going in the geophysical medium is used [Rodkin et al., 2008; Rodkin, 2011; Rodkin, Tikhonov, 2016]. The MCM model treats the seismic process as an assemblage of avalanche-like episodes of relaxation, occurring randomly in a set of uniform metastable sub-systems. Here the simple linear variant of the MCM model without memory is used. This model is defined by a flow of events and by two parameters, characterizing hierarchy and the degree of metastability of the system under examination.

The relations obtained in the MCM model are compared with the typical anomaly revealed in result of the examination of the generalized vicinity of a large earthquake. This comparison permits to interpret the typical behavior occurring in the vicinity of a large earthquake in terms of the MCM model. The experience and perspectives of the use of the MCM model in earthquake prediction are discussed.

2. The MCM model

The seismic regime is treated in terms of the MCM model as an assemblage of the episodes of avalanche-like relaxation, occurring randomly in a set of statistically identical metastable sub-systems.

Let us briefly describe the MCM model in terms of recurrent relations (some longer description for a continuous case is presented in [Rodkin et al., 2008]). Let an ongoing random event (here earthquake) of quantity (energy) \( X_i \) in some moment \( t_i \) can continue its development with probability \( p \) or to be ended with probability \( 1-p \). In a case if the process interrupts on \( i \)-th step, the quantity of the event (earthquake energy) is considered equal \( X_i \) reached at this step.

In a case when the process of relaxation of metastable sub-system continues, we suggest that the quantity of the event (energy) \( X_{i+1} \) in the next moment of time \( t_{i+1} \) grows up to the value

\[
X_{i+1} = r \times X_i
\]

where \( r \) is random parameter with average value more than 1. Let the nonessential initial (on the first step) value of quantity of the earthquakes is equal to \( X_0 \).

In scheme (1) the probability of interrupting of the process on n-th stage and correspondingly obtaining of the value \( X = X_0 \times r^n \) is equal to \( (1-p) \times p^n \). From this we observe that the tail of the function of distribution \( F(X_n > X) \) is described by the relationship: \( (1-F(X)) = p^{\log(X)/\log(r)} \).

From here we have

\[
\log(1-F(X)) = \log(p)/\log(r) \times \log(X).
\]

and, thus, we receive a power dependence for the tail of the distribution function \( (1-F(X)) \) from \( X_n \), as it takes place for the distribution of the of seismic moment and seismic energy values.

The used scheme treats the development of earthquake as a process of sequential transition to higher hierarchical levels. At constant parameter \( r \) value we have a discrete-hierarchical distribution of the energy values of earthquakes. With a growth of random spread in values \( r \),
the step-by-step character of model distributions becomes smoother, and at the limit we receive a monotonic distribution with quasi-rectilinear graph of the events’ recurrence in coordinates \( \{\log(X), \log(N)\} \). A slope of graph of recurrence is equal in this case

\[
b = \log(1/p)/\log(r),
\]

where \( b \) characterizes the power distribution of quantities \( X \) and has a meaning similar to the \( b \)-value in the Gutenberg-Richter law (for the energy or seismic moment of earthquakes).

In terms of the MCM model the \( b \)-value is defined by two parameters, where one (\( r \)) characterizes a hierarchy of the medium, and the second (\( p \)) gives a probability of a continuation of an avalanche-like relaxation in metastable subsystems, so it characterizes the degree of metastability of the ongoing process.

It should be added that the MCM model permits to imitate all well-known features of the seismic regime (see [Rodkin, 2011; Rodkin, Tikhonov, 2016; Sherman et al., 2017] for details). In the presented above simple linear variant without memory MCM model permits besides the reproducing of the Gutenberg–Richter law to imitate a tendency of a correlation of strong earthquakes occurrences with periods of lower \( b \)-values. For the more complex linear variant with a memory the MCM model permits to imitate the Omory law for aftershock sequence and the tendency of an existence of a quasi-periodical seismic cycle. In the non-linear variant the MCM model can imitate the existence of characteristic earthquakes and the bend down of the earthquake recurrence plot in the strongest events domain [Sherman et al., 2017]. Thus, the MCM model permits to imitate all reliably known features of the seismic regime. From here it seems that the MCM model reliably can be treated as a possible alternative of the ETAS model.

The equation (3) will be compared below with the typical anomaly revealed in the generalized vicinity of a large earthquake.

3. The generalized vicinity of a large earthquake

The seismic regime occurring in the vicinity of individual large earthquakes can differ significantly from the common patterns believed to be typical of fore- and aftershock sequences. Large earthquake generalized vicinity (LEGV) was constructed and examined to confirm and to detail the typical features of fore- and aftershock behavior [Rodkin, 2008; 2012; Rodkin, Tikhonov, 2016]. The LEGV is constructed as a sum of vicinities of individual earthquakes falling into the spatial-temporal vicinity of a large number (300, 500 or 1000) of largest earthquakes. The spatial size of the vicinity of a particular large earthquake is defined as follows. According to [Kasahara, 1981; Sobolev and Ponomarev, 2003], the radius of a zone of aftershock activity of an earthquake of magnitude \( M \) is equal to

\[
R(km) = 10^{0.5M-1.9}
\]

The described below results of the LEGV examination involve data for earthquakes occurring no more than 3R distance from the hypocenter of at least one of any of the used main large earthquakes.

The construction of the time vicinity of a large earthquake was based on the result [Smirnov and Ponomarev, 2004] that the duration of a cycle of seismic failure only slightly depends on an earthquake magnitude. Having this in mind, in constructing the time vicinities of large earthquakes of different magnitudes, one can use the method of epoch superposition without time domain scaling.
Similar method of stacking in time of fore- and aftershock sequences triggered by different main shocks was used in [Molchan et al. 1999; Narteau et al., 2005; et al.] to confirm a tendency of a decrease in b-values in the vicinity of large earthquake, in examination of parameters of aftershock sequences, and in a few other cases. The LEGV approach differs from that used in [Molchan et al. 1999; Narteau et al., 2005] by a larger number (up to one thousand) of united vicinities of large events, and by a spatial scaling of vicinities of different main shocks according to the relation (4). As a result the LEGV earthquakes are much more numerous, the results become more statistically valid, and character of fore- and aftershock cascades and a number of other fore- and aftershock anomalies can be seen in LEGV clearer.

Most of the anomalies revealed in LEGV could be expected from the earlier findings, but the character of their change with approaching the moment of the generalized large event (GLE) was not identified before. As an example, the character of change of the b-value in a foreshock domain is shown at Figure 1, where the Global Centroid Moment Tensor (GCMT) catalog data with restriction $M_w \geq 5.4$ were used and vicinities of 500 strongest events were combined. The errors are estimated using the boot-strap numerical simulation procedure. The same tendency of change of b-values in a foreshock domain was found when other catalogs and other magnitude limitations were used.

The relation of the b-value estimation by the maximum-likelihood technique [Utsu, 1965; Aki, 1965] is the following

$$b = \log(e)/(M_{av} - M_c),$$

(5)

where $M_{av}$ is an average magnitude in a sample. From relation (5) it can be expected that the mean $M_w$ value would increase while approaching the moment of the generalized main event in LEGV in a similar manner as a b-value decreases. This is seen in Figure 2.

The type of change of anomaly that can be seen in Figs. 1 and 2 was found to be typical of the LEGV. It has a form

$$A = c + d \log(|\Delta t|),$$

(6)
where $A$ – parameter value with an anomaly, $c, d$ – coefficients that can differ in different cases, $|\Delta t|$ - the time interval from/to the generalized main shock (GLE). This anomaly was found [Rodkin, 2008; 2012; Rodkin and Tikhonov, 2016] to be typical in LEGV for fairly different parameters: mean $M_w, M_b$, and $(M_w-M_b)$ values, $b$-values, apparent stress $\sigma_a$ values, fractal dimension $D$ value, mean depth (H, km), half-duration of the seismic process, and a few other parameters. Further this typical pattern of the foreshock anomaly will be compared with the model behavior obtained in the MCM model.

4. Discussion

Comparing relations (3) and (6) it is easy to see that the character of development of the anomaly is the same; $b$-value in (3) has a type of an anomaly (6) if probability $p$ of a transition of a process of fracturing to a higher rank $p \rightarrow 1$, when $|\Delta t| \rightarrow 0$. This conclusion appears to be quite natural and logical that supports the fruitfulness of the use of the MCM and LEGV approaches.

For the further consideration we will need relations connecting fractal dimension $D$ value and parameters of hierarchy of different systems. As it was done in the review [Chen Yanguang, 2017] introduce the parameters of hierarchy for numbers of elements of a given system $R_n$, and for sizes $R_L$ of elements of a system

$$ R_n = \frac{N_{m+1}}{N_m}, \quad (7a) $$

$$ R_L = \frac{L_{m+1}}{L_m}, \quad (7b) $$

where $m$ denotes the rank ($m=1, 2, \ldots$), $N_m$ is the number of the fractal elements of a given size $L_m$, $N_1$ and $L_1$ are the number and length of the initiator (rank =1, $N_L =1$). From [Chen Yanguang, 2017], the fractal dimension of such system $D$ is equal to:

$$ D = \log(R_n) / \log(R_L) \quad (8) $$

The well-known relation [Aki, 1981], connecting $D$ and $b$ values for 3-dimension media is the following:

$$ D = 2 \times b \quad (9) $$

This relation appears to be valid for small-size earthquakes [Legrand, 2002]. It may be considered as the result of a certain consistency between the structure of failures and the structure of heterogeneities of the medium, when the energy distribution of earthquakes is stable and controlled by the size distribution of heterogeneities of the medium. In this case, the probability of the failure of different-size heterogeneities proves to be constant. Such a situation is typical for the background seismicity, but it is sharply violated in transient modes [Smirnov 2003, Smirnov and Ponomarev 2004].

Parameters of size hierarchy $R_L$ has the same sense as parameter $r$ of hierarchy in the MCM model, see (1) - (3). From here, denote $r$ as $R_L$ and will have:

$$ b = \log(1/p) / \log(R_L), \quad (10) $$

$$ D = \log(N_{m+1}/N_m) / \log(R_L) $$
Easy to see that $D/b$ relation can be constant only at stationary conditions when $p$ is constant. Thus, in terms of MCM model and LEGV approach we have got a conclusion that relation (9) can be valid only at stationary conditions; earlier this result was obtained from the examination of empirical data [Smirnov 2003, Smirnov and Ponomarev 2004].

5. Conclusion
The presented results testify evidently for the usefulness of MCM and LEGV methods of examination of the seismic regime.

The possibility of interpretation of a few phenomena of the seismic regime in terms of the MCM and LEGV methods is a convincing reason to use these methods for an earthquake prediction. The probability $p$ of transition of a seismic failure to a higher rank appears to be a characteristic very closely connected with probability of occurrence of strong earthquakes. The preliminary attempts of using of the anomaly of probability $p$ value increase for the strong earthquake prediction were carried out in [Rodkin & Tikhonov, 2012] and in a number of conference reports. The results were encouraging: episodes of growth of $p$ values occurring in connection with the strong earthquakes occurrence were found.

However, as can be seen from relation (3), from the formal point of view the anomaly in $p$ probability value is factually nothing more, then the well-known anomaly of a decrease in $b$-value. The existence of the $b$-value anomaly was confirmed repeatedly, but its use for the earthquake prediction was not fruitful enough. By analogue, the anomaly of growth of $p$ value also hardly has a chance to be highly effective. Thus, this anomaly can be fruitful in practice probably only if using in a complex with other anomalies.

Note, that a complex of anomalies revealed in the LEGV gives possibility to construct an ideal scenario of the precursor behavior. The real seismic regime can be monitored calculating the conditional current distance from the ideal scenario of the strong earthquake preparation.

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