Degrees-of-Freedom of the MIMO Three-Way Channel with Node-Intermittency
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Abstract

The characterization of fundamental performance bounds of many-to-many communication systems in which participating nodes are active in an intermittent way is one of the major challenges in communication theory. In order to address this issue, we introduce the multiple-input multiple-output (MIMO) three-way channel (3WC) with an intermittent node and study its degrees-of-freedom (DoF) region and sum-DoF. We devise a non-adaptive encoding scheme based on zero-forcing, interference alignment and erasure coding, and show its DoF region (and thus sum-DoF) optimality for non-intermittent 3WCs and its sum-DoF optimality for (node-)intermittent 3WCs. However, we show by example that in general some DoF tuples in the intermittent 3WC can only be achieved by adaptive schemes, such as multi-hop or decode-forward relaying. This shows that non-adaptive encoding is sufficient for the non-intermittent 3WC and for the sum-DoF of intermittent 3WCs, but adaptive encoding is necessary for the DoF region of intermittent 3WCs. Our work contributes to a better understanding of the fundamental limits of multi-way communication systems with intermittency and the impact of adaptation therein.

Index Terms

Degrees-of-freedom, three-way channel, intermittency, multiple-input multiple-output (MIMO), interference alignment, zero forcing, relaying.

I. INTRODUCTION

In multi-way communication scenarios multiple nodes communicate with each other, each acting as a source, a destination, and possibly a relay at the same time. This mode of communication is especially important for future systems employing full-duplex and device-to-device (D2D) communication, [2]–[4]. It is an important technique for efficient resource utilization that is expected to gain more prominence in future communication systems, especially with the rise of mesh networks, e.g., in industrial and vehicular networks.
Several multi-way communication scenarios have been studied in the literature, [5]. It started with the two-way channel which was first studied by Shannon in [6] and subsequently in [7]–[10] for instance. Then, the scope was extended to two-way networks where two groups of nodes communicate with each other in a two-way fashion [11]–[13], and two-way relay networks where two nodes communicate with each other in a two-way fashion via a relay node [14]–[17]. This line of research has been further extended to multi-way networks where multiple nodes communicate with each other in a multi-way fashion, each node being a source and a destination at the same time [18]–[22], and to multi-way relay networks where the same communication as in multi-way networks takes place via a relay node [23]–[34].

A. The Three-Way Channel with Intermittency

In this work, we focus on the multiple-input multiple-output (MIMO) three-way channel (3WC) which can be described as follows: Consider a system consisting of three terminals communicating with each other in a multi-way fashion, e.g., two D2D user terminals and a base station (BS) where the D2D users communicate with each other while exchanging signals with the BS (control signals or data). This 3WC is an extension of Shannon’s two-way channel [6] and has been studied in [20]–[22].

Therein, it is assumed that the three nodes are connected all the time (Fig. 1a). In practice however, connectivity might be intermittent. For instance, a pair of D2D users, commonly chosen to be nearby users [3], [4], might be both disconnected from the BS due to shadowing (Fig. 1b). Future generations of mobile communication systems (e.g., using mmWaves, where line-of-sight (LOS) propagation will become dominant) are expected to be heavily susceptible to this type of shadowing. In another scenario, the D2D users might operate in an underlay mode over a resource block used by the BS to communicate to cellular users (CUs). The BS connects to the D2D users whenever it does not communicate with a CU, and disconnects from the D2D users otherwise (see Fig. 1c). Note that D2D users are chosen so that they cause/receive negligible interference (relative to the desired signals) to/from other nodes using the same resource block [35], [36]. In both cases the two links from the D2D users to the BS are jointly intermittent, i.e., both are available or blocked at the same time. We call this node intermittency and call the BS an intermittent node (from the D2D users’ perspective). Motivated by these scenarios, we study the 3WC with this type of intermittency. The impact of intermittency in various networks was studied in [37]–[40] for instance.
Fig. 2: A D2D pair sharing the same resources with a cellular user (CU), where the BS communicates with the D2D pair part of the time [a] and with the CU the rest of the time [b]. The D2D pair is far enough from both the BS and the CU and thus causes/receives negligible interference (dotted).

B. Degrees-of-Freedom and Intermittency

In many previously mentioned works, the focus is generally on the capacity of the studied network. Since finding the capacity is elusive in most cases, some works (and also this work) focus on the degrees-of-freedom (DoF) which provide a good capacity approximation at high signal-to-noise ratio (SNR) [41], thereby highlighting the interaction between the signals of the different nodes while diminishing the impact of noise. This is of interest since state-of-the-art wireless communication systems operate in a regime where they are essentially interference-limited rather than noise-limited.

The concept of intermittency as a form of channel impairment also fits well with the philosophy of DoFs: With increasing SNR, signal components ‘harden’ in the sense that some allow (almost) noise- and interference-free communication at a rate that scales with SNR on a logarithmic scale, while others are hopelessly buried in uncancelable interference and are therefore useless. This effect is exactly what the DoF perspective captures as it essentially counts available Shannon-Hartley communication ‘units’. Intermittency is the channel impairment that goes well together with this ‘all or nothing’ perspective, capturing the notion that some signal dimensions might become useless due to stochastic processes in the channel, such as shadowing of dominant LOS components.

C. Scope of this Work

Here, we study the impact of node-intermittency on the DoF region and sum-DoF, i.e., the capacity scaling versus SNR in a dB scale, of a full-duplex MIMO 3WC. In this MIMO network, each node generally has two independent messages, each intended for one of the remaining two nodes. We pay particular attention to the necessity (or the lack thereof) of adaptive encoding for reaching DoF region/sum-DoF. Adaptive encoding enables cooperative communication schemes by allowing the transmit signal of a node to depend on its previously received signals, which can be interpreted as a form of feedback. In contrast, with non-adaptive encoding transmit signals depend only on the messages to be sent, and cooperation (e.g., in the form of
relaying) is excluded. The issue of (non-)adaptive encoding has been studied for other networks earlier in [9]–[11] for instance.

We assume that nodes have causal knowledge of the intermittency state of adjacent links. This can be obtained by estimating the connectivity from the receive signals (e.g., signal strength). Note that in some cases the intermittency state can be known without the need for estimation, occasionally even ahead of time, e.g., in the scenario of D2D/CU/BS users, where the BS knows its scheduling of users in advance and can anticipate when it will not be able to receive the D2D user signals.

D. Outline and Overview of Results

After introducing the details of the system model in Section II and highlighting the main results in Section III, we examine the (node-)intermittent 3WC in Section IV. We devise a non-adaptive encoding scheme based on zero-forcing (ZF), interference alignment (IA) and erasure coding (EC) and derive its achievable DoF region and sum-DoF. We present the genie-aided converse techniques used to derive DoF region and sum-DoF upper bounds, both under non-adaptive and adaptive encoding. We conclude that for the intermittent 3WC the presented non-adaptive scheme is sum-DoF optimal, so adaptation is not necessary to achieve sum-DoF. Then, we provide examples of adaptive relaying schemes that can achieve a DoF region point that no non-adaptive scheme can achieve. This shows that adaptive schemes can achieve strictly larger DoF regions, and therefore adaptation is required to achieve the DoF region of intermittent 3WCs. To complete the picture, we examine the non-intermittent 3WC as special case of intermittent 3WCs in Section V. We show that for the non-intermittent 3WC the presented scheme is DoF region optimal (and thus also sum-DoF optimal) and therefore adaptive encoding is not required. This reveals an interesting interplay between intermittency and adaptation. In Section VI we provide conclusive remarks and directions for future research.

E. Notation

Throughout the paper, we use $x^n_i := (x_{i,1}, \ldots, x_{i,n})$ for some index $i$, and $x^n_{i,\ell} := (x_{i,\ell}, \ldots, x_{i,n})$. We use regular letters to denote scalar-valued quantities, boldface letters to denote vector- and matrix-valued quantities; lowercase letters for scalar and vector values (e.g., realizations of random variables), uppercase letters for matrix values and for random variables. The $N \times N$ identity matrix is denoted $I_N$, the $N \times N$ all-zero matrix is $0_N$. We write $X \sim \mathcal{CN}(0, Q)$ to indicate that $X$ is a multivariate complex Gaussian random variable with zero mean and covariance matrix $Q$, and $S \sim \text{Bern}(\tau)$ to indicate that $S$ is Bernoulli distributed with $\Pr[S = 1] = \tau$ and $\Pr[S = 0] = 1 - \tau =: \bar{\tau}$. We write $x^+$ to denote $\max\{0, x\}$ for some $x \in \mathbb{R}$, and $H^\dagger, H^\top, H^H$, and $\text{span}(H)$ to denote the Moore-Penrose pseudo-inverse, the transpose, the Hermitian transpose, and the subspace spanned by the columns of the matrix $H$. By $\log(x)$ we denote the logarithm of $x$ to base 2, by $i \rightarrow j$ the communication from node $i$ to node $j$ one-way, and by $i \leftrightarrow j$ both $i \rightarrow j$ and $j \rightarrow i$. By $p_X(x)$ we denote the probability density function (PDF) of random variable $X$.

II. THE MIMO THREE-WAY CHANNEL WITH NODE-INTERMITTENCY

Throughout this section we assume $i, j, k \in \{1, 2, 3\}$ and mutually distinct, and $n$ is the number of channel accesses. The MIMO 3WC with node-intermittency is comprised of three terminals 1, 2 and 3 communicating
with each other in full-duplex mode over a shared medium. Each node $i$ has two messages $w_{ij}$ and $w_{ik}$ ($w_i := (w_{ij}, w_{ik})$) to be delivered to the remaining nodes $j$ and $k$, and two messages $\hat{w}_{ji}$ and $\hat{w}_{ki}$ to be decoded from the receive signals (Fig. 3). Each message $w_{ij}$ is a realization of the random variable $W_{ij}$. All random variables $W_{ij}$ are independent. In the (node-)intermittent 3WC one link is always available and two links are jointly intermittent with probability of being available $\tau$ (Fig. 1b and 3).

Our objects under investigation (sum-DoF and DoF region of the intermittent 3WC – both are introduced in the sequel) depend on how the numbers of antennas at each node relate to each other, i.e., whether the intermittent node has most, second most, or least antennas. To reduce the number of cases one has to analyze, yet study the system without loss of generality (w.l.o.g.), two symmetries come to mind that can be exploited: Either a) fix a certain node to be intermittent (e.g., node 1 is intermittent), and investigate all possible relations among the numbers of antennas, or b) fix a relation between the numbers of antennas, and allow any one of the three nodes to be intermittent. The respective remaining cases follow by renaming. Preliminary work [1] fixed node 1 to be intermittent and further assumed a relation on the numbers of antennas, which is not w.l.o.g. We fix node 1 to be intermittent, but allow for any combination of numbers of antennas, which is approach a) and w.l.o.g.

Node $i$ is equipped with $M_i$ antennas that are used for reception and transmission simultaneously. We assume channel accesses of the nodes are synchronized and time-discretized: At time instance $\ell$ the transmit signal $x_{i,\ell} \in \mathbb{C}^{M_i}$ is a realization of a random vector $X_{i,\ell}$ satisfying the power constraint
\begin{equation}
\sum_{\ell=1}^{n} \mathbb{E}[\|X_{i,\ell}\|^2_2] \leq nP.
\end{equation}

The receive signals $y_{i,\ell} \in \mathbb{C}^{M_i}$ are (Fig. 4)
\begin{equation}
\begin{bmatrix}
y_{1,\ell} \\
y_{2,\ell} \\
y_{3,\ell}
\end{bmatrix} =
\begin{bmatrix}
0_{M_1} & s_{\ell}H_{21} & s_{\ell}H_{31} \\
s_{\ell}H_{12} & 0_{M_2} & H_{32} \\
s_{\ell}H_{13} & H_{23} & 0_{M_3}
\end{bmatrix}
\begin{bmatrix}
x_{1,\ell} \\
x_{2,\ell} \\
x_{3,\ell}
\end{bmatrix} +
\begin{bmatrix}
z_{1,\ell} \\
z_{2,\ell} \\
z_{3,\ell}
\end{bmatrix}.
\end{equation}

$H_{ij} \in \mathbb{C}^{M_i \times M_j}$ represents the channel matrix from node $i$ to node $j$ and is constant over time and known to all nodes in advance. The elements of these matrices are drawn independently from the same continuous distribution, such that $\text{rank}(H_{ij}) = \min(M_j, M_i)$ almost surely. $z_{i,\ell} \in \mathbb{C}^{M_i}$ is a realization of the noise process $Z_{i,\ell} \sim \mathcal{CN}(0, \sigma^2 I_{M_i})$ independent and identically distributed (i.i.d.) with respect to (w.r.t.) $\ell$, and $s_{\ell} \in \{0,1\}$ is a realization of the intermittency state process $S_{\ell}$. $S_{\ell} \sim \text{Bern}(\tau)$ is assumed to be i.i.d. w.r.t. $\ell$. The state sequence $s^n$ is causally known at all nodes (i.e., in time instance $\ell$ all nodes know $s^{\ell-1}$). All random variables $H_{ij}$, $Z^n$ and $S^n$ are mutually independent. We denote $\rho := \frac{P}{n}$ and call it SNR (signal-to-noise-power-ratio) throughout the paper.

The messages $W_{ij}$ are each uniformly distributed over $W_{ij} = \{1, \ldots, |W_{ij}(\rho)|\}$. Using an encoding function $E_{i,\ell}$ node $i$ constructs $x_{i,\ell}$ either from $(w_{ij}, w_{ik})$ (non-adaptive encoding) or from $(w_{ij}, w_{ik}, y_{i,\ell}^{\ell-1})$ (adaptive encoding). After $n$ transmissions (where $n$ is the code length), node $i$ decodes its desired messages using a decoding function $F_i$ to obtain $(\hat{w}_{ji}, \hat{w}_{ki}) = F_i(s^n, y^n, w_{ij})$. Transmission is considered successful if all messages are recovered successfully ($w_{ij} = \hat{w}_{ij}$), otherwise an error is reported. The average over all messages of the error probability is denoted by $P_{e,n}$.
Fig. 3: Every node $i$ in the MIMO 3WC with node-intermittency sends messages $w_{ij}$ and $w_{ik}$ to nodes $j$ and $k$, respectively (with $i,j,k \in \{1,2,3\}$ mutually distinct). W.l.o.g. node 1 is assumed to be intermittent.

Fig. 4: AWGN channel model of the shared medium in the MIMO 3WC with node-intermittency, with channel inputs $X_i$ and outputs $Y_i$, channel gains $H_{ij}$, noise $Z_i$ and intermittency state $S$ (time index $\ell$ omitted)

A rate tuple

$$R(\rho) := (R_{12}(\rho), R_{13}(\rho), R_{21}(\rho), R_{23}(\rho), R_{31}(\rho), R_{32}(\rho)) \in \mathbb{R}^6_+$$

(3)

with $R_{ij}(\rho) = \frac{\log(\|W_{ij}(\rho)\|_2)}{n}$ is said to be achievable if there exists a sequence of encoder-decoder pairs for increasing code length $n$, where $P_{e,n} \to 0$ as $n \to \infty$. The capacity region $\mathcal{C}(\rho)$ is the set of all achievable rate tuples.

The DoF region $\mathcal{D}$ is the set of achievable DoF tuples

$$d := (d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32}) \in \mathbb{R}^6_+$$

(4)
defined as in [42]. Roughly speaking, if a rate tuple \( R(\rho) \) as a function of \( \rho \) is achievable, i.e., \( R(\rho) \in C(\rho) \) for all \( \rho > 0 \), then the DoF tuple \( d \) with \( d_{ij} = \limsup_{\rho \to \infty} \frac{R_{ij}(\rho)}{\log(\rho)} \) is achievable. We denote DoF regions with \( \mathcal{D} \) and use suitable subscripts when further restricting assumptions apply (e.g., under non-adaptive encoding). We define the corresponding sum-DoF as \( d_{\text{sum}} = \max_{d \in \mathcal{D}} \sum_{i,j \in \{1,2,3\}, i \neq j} d_{ij} \). The DoF perspective only captures rate contributions that are non-vanishing relative to \( \log(\rho) \) as we let \( \rho \to \infty \). It neglects vanishing rate portions \( f(\rho) \) that grow sublinear in \( \log(\rho) \) and are therefore \( o[\log(\rho)] \), i.e., where \( \lim_{\rho \to \infty} \frac{f(\rho)}{\log(\rho)} = 0 \).

III. MAIN RESULTS

In this section, we summarize the main results of this paper. We denote the DoF region of the intermittent 3WC under adaptive encoding by \( \mathcal{D}^1_\infty \), the DoF region of the intermittent 3WC under non-adaptive encoding by \( \mathcal{D}^1_X \) and the sum-DoF by \( d_{\text{sum}}^1 \). Obviously, \( \mathcal{D}^1_R \subseteq \mathcal{D}^1_{\infty} \). We denote the DoF region of the non-intermittent 3WC by \( \mathcal{D}^1_N \) and the sum-DoF by \( d_{\text{sum}}^N \). In Section [IV-A] we devise a non-adaptive encoding scheme whose achievable DoF region we denote by \( \mathcal{D}^1_{\text{IB}} \). We start with this achievable DoF region given in the following theorem.

**Theorem 1** (DoF Region Inner Bound for Node-Intermittent 3WC). All DoF tuples \( d \in \mathcal{D}^1_{\text{IB}, X} \) satisfying the following set of inequalities are achievable in the node-intermittent 3WC using non-adaptive encoding:

\[
\begin{align*}
\max\{d_{12} + d_{13}, d_{21} + d_{31}\} & \leq \tau M_1 \\
\max\{d_{21} + \tau d_{23}, d_{12} + \tau d_{32}\} & \leq \tau M_2 \\
\max\{d_{31} + \tau d_{32}, d_{13} + \tau d_{23}\} & \leq \tau M_3 \\
\max\{d_{12} + d_{13} + \tau d_{23}, d_{21} + d_{31} + \tau d_{32}\} & \leq \tau \max\{M_1, M_3\} \\
\max\{d_{12} + d_{13} + \tau d_{32}, d_{21} + d_{31} + \tau d_{23}\} & \leq \tau \max\{M_1, M_2\} \\
\max\{d_{12} + d_{31} + \tau d_{32}, d_{21} + d_{13} + \tau d_{23}\} & \leq \tau \max\{M_2, M_3\} \\
\min\{d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32}\} & \geq 0
\end{align*}
\]

Therefore, \( \mathcal{D}^1_{\text{IB}, X} \) constitutes an inner bound on the DoF region of the node-intermittent 3WC, such that

\[ \mathcal{D}^1_{\text{IB}, X} \subseteq \mathcal{D}^1_X \subseteq \mathcal{D}^1. \]

**Proof.** This theorem follows by the construction in Section [IV-A].

Subsequently we investigate whether this non-adaptive scheme is optimal. For the intermittent 3WC it turns out to be sum-DoF optimal, establishing the sum-DoF of the intermittent 3WC and the fact that adaptive encoding is not necessary to achieve it.

**Theorem 2** (Sum-DoF of Node-Intermittent 3WC). A non-adaptive encoding scheme achieves the sum-DoF of the node-intermittent 3WC given by

\[ d_{\text{sum}}^1 = 2\tau \min\{M_2, M_3\} + 2\tau (M_1 + M_2 + M_3 - \min\{M_1, M_2, M_3\} - \max\{M_1, M_2, M_3\}). \]

**Proof.** The theorem follows from achievability and converse results developed in Sections [IV-B1] and [IV-B2] (Lemmas [1] and [2]).

From a DoF region perspective however it turns out that non-adaptive schemes cannot be optimal, and therefore the presented scheme is not DoF region optimal in the intermittent 3WC.

**Theorem 3** (Necessity of Adaptive Encoding for DoF Region of Node-Intermittent 3WC). Adaptive encoding is required to achieve the DoF region of the node-intermittent 3WC, i.e.
\[
D^I \setminus D^I_A \neq \emptyset
\]

*Proof.* The theorem follows from an upper bound on \(d_{31}\) under non-adaptive encoding, presented in Section [IV-C1] and counterexamples of adaptive schemes exceeding this bound, devised in Section [IV-C2].

Theorems 2 and 3 show that non-adaptive encoding is sufficient to achieve sum-DoF, but not sufficient to achieve the DoF region of the intermittent 3WC. This is particularly interesting in light of the fact, that non-adaptive encoding is sufficient to achieve the DoF region of the non-intermittent 3WC, which the presented non-adaptive scheme does.

**Theorem 4** (DoF Region of Non-Intermittent 3WC). The DoF region of the non-intermittent 3WC \(D^N\) (with \(M_1 \geq M_2 \geq M_3\) w.l.o.g.) is given by

\[
\begin{align*}
\max\{d_{12} + d_{13} + d_{23}, d_{12} + d_{13} + d_{32}\} &\leq M_1 \\
\max\{d_{21} + d_{31} + d_{32}, d_{21} + d_{31} + d_{23}\} &\leq M_1 \\
\max\{d_{21} + d_{13} + d_{23}, d_{12} + d_{31} + d_{42}\} &\leq M_2 \\
\max\{d_{31} + d_{32}, d_{13} + d_{23}\} &\leq M_3 \\
\min\{d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32}\} &\geq 0
\end{align*}
\]

and achievable using a non-adaptive encoding scheme.

*Proof.* The theorem follows from Lemmas 5 and 6 in Sections [V-A] and [V-B].

This complements earlier work [18] that characterized the sum-DoF of the non-intermittent 3WC and showed its achievability using non-adaptive encoding. Our results show that intermittency does not decrease sum-DoF as long as node 1 has the smallest number of antennas. Otherwise, intermittency does affect sum-DoF, which is affine-linearly increasing in \(\tau\).

Fig. 5 and 6 plot sum-DoF of non-intermittent and node-intermittent 3WC against \(M_3\), for different values of \(\tau\) and \(M_2\), respectively, in a scenario where node 1 has the largest number of antennas. The resulting graphs are piecewise linear with a change in slope at \(M_3 = M_2\). For \(M_3 < M_2\) the sum-DoF of the 3WC is a constant depending only on \(M_2\), for \(M_3 > M_2\) it is linear in \(M_3\). The slope of the sum-DoF of the intermittent 3WC is proportional to \(\tau\) for \(M_3 < M_2\) and proportional to \(\tau\) for \(M_3 > M_2\).

The sum-DoF of the intermittent 3WC with \(0 < \tau < 1\) is a convex time-sharing combination of the two extreme cases \(\tau = 0\) and \(\tau = 1\) (Fig. 5). Note, that such a time-sharing combination is the best any non-adaptive coding scheme can achieve, when each node knows the intermittency state ahead of time and codes optimally for the respective state in each time instance \(\ell\). The achievability scheme presented in this work does not use intermittency state information at all, yet achieves the same sum-DoF, averaging out intermittency state through
erasure coding. This shows that intermittency state information is dispensable for the non-adaptive scheme in this case. Furthermore, note that the sum-DoF of the 3WC is larger than the sum-DoF of the intermittent 3WC (Fig. 6).

In the sequel we provide detailed derivations and proofs of our main results highlighted in this section.
Fig. 7: Rationale behind the aliases $s_{1,\ell} := 1, s_{2,\ell} := s_{3,\ell} := s_{\ell}, \tau_1 := 1, \tau_2 := \tau_3 := \tau$ introduced in Section IV-A, treating every link $j \leftrightarrow k$ as potentially intermittent with intermittency state variable $s_{i,\ell}$ (marginally distributed as Bern($\tau_i$)) allows to derive general expressions for, e.g., $y_{i,\ell}$ based on $x_{j,\ell}, x_{k,\ell}, s_{j,\ell}$ and $s_{k,\ell}$, independent of whether $i = 1, i = 2$ or $i = 3$.

IV. NODE INTERMITTENCY

In this section we first introduce a non-adaptive ZF/IA/EC-based scheme and derive its achievable sum-DoF and DoF region. We then show, using enhanced genie-aided bounds for both adaptive and non-adaptive encoding, that this scheme is sum-DoF optimal, but not DoF region optimal. Furthermore, no non-adaptive scheme can be DoF region optimal, since tighter outer bounds hold for non-adaptive schemes, that however can be exceeded by adaptive schemes, as we show by examples of multi-hop and decode-forward relaying. This establishes that adaptive encoding is necessary from a DoF region perspective, but non-adaptive encoding is sufficient to achieve sum-DoF.

A. A Non-Adaptive Scheme Based on ZF, IA and EC

In this subsection we present a non-adaptive transmission scheme based on ZF, IA and EC that provides an inner bound on sum-DoF and DoF region of 3WCs. Throughout this section we continue to assume $i, j, k \in \{1, 2, 3\}$ and mutually distinct. For notational simplicity (capturing the symmetries inherent in the model thereby avoiding case distinctions), we introduce the following aliases that will be resubstituted towards the end of the section:

$$ s_{1,\ell} := 1, s_{2,\ell} := s_{3,\ell} := s_{\ell}, \tau_1 := 1, \tau_2 := \tau_3 := \tau \quad (5) $$

The idea behind these aliases is visualized in Fig. 7. Using these aliases we can, for instance, find a general expression for the receive signal $y_{i,\ell}$ of node $i$ in terms of the transmit signals $x_{j,\ell}$ and $x_{k,\ell}$ of nodes $j$ and $k$, and the intermittency states $s_{k,\ell}$ and $s_{j,\ell}$ (marginally distributed as Bern($\tau_k$) and Bern($\tau_j$), respectively) of the links $j \leftrightarrow i$ and $k \leftrightarrow i$, respectively. The fact that $2 \leftrightarrow 3$ is not intermittent, and $1 \leftrightarrow 2$ and $1 \leftrightarrow 3$ are jointly intermittent, is accounted for by the appropriate resubstitution at due time.

1) Encoding: Each node splits each message $w_{ij}$ into $w_{ij}^{[ZF]}$ and $w_{ij}^{[IA]}$ to be sent via zero-forcing and interference alignment, respectively. At node $i$, the four messages $w_{ij}^{[q]}$ ($q \in \{ZF, IA\}$) are encoded into codewords $x_{ij}^{[q]}$ with symbols $x_{ij,\ell}^{[q]} \in \mathbb{C}^{a_{ij}^{[q]}}$ each, for some vector lengths $a_{ij}^{[q]} \in \mathbb{N}_0$. The symbols of these
codewords are chosen i.i.d. \( \mathcal{C}N(0, p_i I_{q[i]}) \) respectively, where \( p_i \) is the power. The power constraints on \( x_i^n \) stated in Section II are satisfied by choosing
\[
p_i = \frac{P}{a_{ij}[ZF] + a_{ij}[IA] + a_{ik}[ZF] + a_{ik}[IA]}.
\]
For encoding, the codes are designed to employ EC to be able to tolerate a certain number of symbol erasures (e.g., caused by intermittency), by not using all codeword symbols for net user data, but deliberately adding some redundancy. The rates and DoFs are thereby reduced accordingly. While for the non-intermittent 3WC this additional layer of EC is not required, it is made use of for intermittent 3WCs to cope with intermittency.

2) Transmission: At time \( \ell \), node \( i \) sends
\[
x^n_{i,\ell} = \sum_{q \in \{ZF, IA\}} V_{ij}^{[q]} x_{ij,\ell}^{[q]} + V_{ik}^{[q]} x_{ik,\ell}^{[q]} \tag{7}
\]
where \( V_{ij}^{[q]} \in \mathbb{C}^{M_i \times a_{ij}^{[q]}} \) are pre-coding matrices with unit-norm column vectors. Zero-forcing is achieved by choosing the \( V_{ij}^{[ZF]} \) such that
\[
H_{ik} V_{ij}^{[ZF]} = 0. \tag{8}
\]
Such matrices \( V_{ij}^{[ZF]} \) exist if node \( i \) has enough antennas to send \( a_{ij}^{[ZF]} \) streams to node \( j \) without interfering with node \( k \), i.e.
\[
a_{ij}^{[ZF]} \leq (M_i - M_k)^+. \tag{9}
\]
To avoid any overlap of the different transmit signal subspaces, we require furthermore that
\[
a_{ij}^{[ZF]} + a_{ij}^{[IA]} + a_{ik}^{[ZF]} + a_{ik}^{[IA]} \leq M_i. \tag{10}
\]
3) Decoding: Node $i$ receives (Fig. [3])

$$y_{i,\ell} = s_{k,\ell}H_{ji}x_{j,\ell} + s_{j,\ell}H_{ki}x_{k,\ell} + z_{i,\ell} \quad (11)$$

$$= \sum_{q \in \{ZF, IA\}} \left[ s_{k,\ell}H_{ji}V_{ji}^{[q]}x_{j,\ell} + s_{j,\ell}H_{ki}V_{ki}^{[q]}x_{k,\ell} \right] + G_i \left[ a_{i,j}^{[IA]} + G_i \right] + z_{i,\ell} \quad (12)$$

with $G_i = [s_{k,\ell}H_{ji}V_{ji}^{[IA]} s_{j,\ell}H_{ki}V_{ki}^{[IA]}]$. Note that the terms $H_{ji}V_{ji}^{[ZF]}x_{j,\ell}^{[ZF]}$ and $H_{ki}V_{ki}^{[ZF]}x_{k,\ell}^{[ZF]}$ vanished due to the zero-forcing condition (9). The first summand represents the four desired signals from $j \leftrightarrow i$ and $k \rightarrow i$, the second summand represents occasional interference from $j \leftrightarrow k$, and the third summand is noise.

To decode a desired signal $x_{i,j}^{[q]}$, node $i$ zero-forces the remaining signals by multiplying $y_{i}^{T}$ with a suitable post-coder $T_{ji}^{[q]} \in \mathbb{C}^{q_{j,i} \times M_{i}}$ with unit-norm row vectors satisfying:

$$T_{ji}^{[q]} T_{ji}^{[q]^H} = I_{a_{i,j}^{[q]}} \quad (13)$$

$$T_{ji}^{[q]} \left[ H_{ji}V_{ji}^{[IF]} H_{ki}V_{ki}^{[IF]} \right] = 0 \quad (14)$$

$$\text{rank}(T_{ji}^{[q]} H_{ji}V_{ji}^{[IF]}) = a_{i,j}^{[q]} \quad (15)$$

Here, (14) ensures zero-forcing of the remaining three messages and the interference and (15) ensures post-coding without loss of meaningful signal dimensions. The existence of such post-coders $T_{ji}^{[q]}$ is guaranteed as long as the columns of

$$\left[ H_{ji}V_{ji}^{[IF]} H_{ji}V_{ji}^{[IF]} H_{ki}V_{ki}^{[IF]} H_{ki}V_{ki}^{[IF]} G_i \right] \quad (16)$$

are linearly independent. Let $\gamma_i$ be the dimension of $\text{span}(H_{ji}V_{ji}^{[IF]}) \cap \text{span}(H_{ki}V_{ki}^{[IF]})$. Then, the dimension of $\text{span}(G_i)$ is $a_{i,j}^{[IF]} + a_{k,j}^{[IF]} - \gamma_i$, and the above linear independence is possible almost surely if we choose

$$a_{i,j}^{[IF]} + a_{k,j}^{[IF]} + a_{i,j}^{[IF]} + a_{k,j}^{[IF]} - \gamma_i \leq M_i. \quad (17)$$

To minimize the impact of interference, we choose the pre-coders $V_{ji}^{[IF]}$ such that all $\gamma_i$ are maximized, i.e., we ‘maximally’ align the interference subspaces at the receivers. The $\gamma_i$ cannot be chosen arbitrarily large, the dimension of the intersection of the interference subspaces ($\gamma_i$) is upper bounded by the dimensions of the interference subspaces ($a_{i,j}^{[IF]}$ and $a_{k,j}^{[IF]}$). Furthermore, $\gamma_i$ needs to be smaller than the dimension of $\text{span}(H_{ji}) \cap \text{span}(H_{ki})$, which is $(M_j + M_k - M_i)^+)$ almost surely. Therefore, we require that

$$\gamma_i \leq \min\{a_{i,j}^{[IF]}, a_{k,j}^{[IF]}, (M_j + M_k - M_i)^+\}. \quad (18)$$

After post-coding, node $i$ is left with the signals

$$y_{j,\ell}^{[q]} = s_{k,\ell}T_{ji}^{[q]}H_{ji}V_{ji}^{[IF]}x_{j,\ell} + T_{ji}^{[q]}z_{i,\ell}. \quad (19)$$

The resulting channel is an erasure-Gaussian MIMO channel with erasure probability $\tau_k$, i.e., a channel whose output is

$$Y_{j}^{[q]} = S_k T_{ji}^{[q]} H_{ji} V_{ji}^{[IF]} X_{j}^{[IF]} + T_{ji}^{[q]} Z_{i} \quad (20)$$
with random variables $S_k \sim \text{Bern} (\tau_k)$, $X_{ji}^{[q]} \sim \mathcal{CN} (0, \sigma^2 I_{M_i})$ and $Z_i \sim \mathcal{CN} (0, \sigma^2 I_{M_i})$. We treat this as a channel with state known causally to the receiver [43, ch. 7.4], such that for large $n$, the achievable rate over this channel is the mutual information between $X_{ji}^{[q]}$ and $(Y_{ji}^{[q]}, S_k)$:

$$I(X_{ji}^{[q]}, Y_{ji}^{[q]} | S_k) = I(X_{ji}^{[q]}, Y_{ji}^{[q]} | S_k = 1) + \tau_k I(X_{ji}^{[q]}; Y_{ji}^{[q]} | S_k = 0)$$

Due to (6) and (15), this leads to a DoF of $\tau_k a_{j|i}^{[ZF]}$, and the code that achieves it is an EC.

4) Achievable DoF Region: For $n$ large, $i \to j$ has a total of $\tau_k a_{j|i}^{[ZF]} + \tau_k a_{j|i}^{[IA]}$ DoFs per channel use,

$$d_{ij} = \tau_k a_{j|i}^{[ZF]} + \tau_k a_{j|i}^{[IA]}.$$  (24)

Collecting (9), (10), (17) and (18) as well as non-negativity of every $a_{j|i}^{[q]}$, we obtain:

$$a_{j|i}^{[ZF]} + a_{j|i}^{[IA]} + a_{k|i}^{[ZF]} + a_{k|i}^{[IA]} \leq M_i$$  (25)

$$a_{j|i}^{[ZF]} + a_{j|i}^{[IA]} + a_{k|i}^{[ZF]} + a_{k|i}^{[IA]} + (a_{j|k}^{[IA]} + a_{k|j}^{[IA]}) - \gamma_i \leq M_i$$  (26)

$$a_{j|i}^{[ZF]} \leq (M_i - M_k)^+$$  (27)

$$\gamma_i \leq \min\{a_{j|k}^{[IA]}, a_{k|j}^{[IA]}, (M_j + M_k - M_i)^+\}$$  (28)

$$0 \leq a_{j|i}^{[q]}.$$  (29)

Using (24), we obtain:

$$\tau_j d_{ij} + \tau_k d_{ik} \leq \tau_j \tau_k M_i$$  (30)

$$\tau_j \tau_i d_{ji} + \tau_k \tau_i d_{ki} + \tau_j \tau_k d_{kj} + \tau_j \tau_k a_{j|k}^{[ZF]} - \tau_j \tau_k a_{j|k}^{[ZF]} - \tau_j \tau_k \gamma_i \leq \tau_j \tau_k M_i$$  (31)

$$\tau_i (\gamma_i + a_{j|k}^{[ZF]}) \leq d_{jk}$$  (32)

$$\gamma_i \leq (M_j + M_k - M_i)^+$$  (33)

$$a_{j|i}^{[ZF]} \leq (M_i - M_k)^+$$  (34)

$$\min\{a_{j|i}^{[ZF]}, d_{ij}, \gamma_i\} \geq 0$$  (35)

Instantiating these constraints for every possible combination of $i, j, k \in \{1, 2, 3\}$ mutually distinct, substituting all $\tau_i$ from (5), collecting the resulting bounds and eliminating redundant bounds, finally yields (for both $(i, j) \in \{(2, 3), (3, 2)\}$):

$$d_{12} + d_{13} \leq \tau M_1$$  (36)

$$d_{4} + \tau d_{17} \leq \tau M_1$$  (37)

$$d_{21} + d_{31} + \tau d_{23} + \tau d_{32} - \tau a_{23}^{[ZF]} - \tau a_{32}^{[ZF]} - \tau \gamma_1 \leq \tau M_1$$  (38)

$$d_{11} + \tau d_{16} + d_{17} + d_{17} - \tau a_{16}^{[ZF]} - \tau a_{17}^{[ZF]} - \tau \gamma_1 \leq \tau M_1$$  (39)

$$0 \leq (\gamma_1 + a_{16}^{[ZF]}) \leq d_{17}$$  (40)

$$0 \leq \tau (\gamma_1 + a_{17}^{[ZF]}) \leq d_{17}.$$  (41)
\[ 0 \leq \tau(\gamma_i + a_{ij}^{[ZF]}) \leq d_{ij} \quad (42) \]
\[ 0 \leq \gamma_i \leq (M_2 + M_3 - M_1)^+ \quad (43) \]
\[ 0 \leq \gamma_i \leq (M_1 + M_3 - M_1)^+ \quad (44) \]
\[ 0 \leq a_{ij}^{[ZF]} \leq (M_1 - M_1)^+ \quad (45) \]
\[ 0 \leq a_{ij}^{[ZF]} \leq (M_1 - M_1)^+ \quad (46) \]
\[ 0 \leq a_{ij}^{[ZF]} \leq (M_1 - M_1)^+ \quad (47) \]

In order to resolve the \((.)^+\) expressions, we do the following for each of the twelve cases of \(M_i \geq M_j + M_k \geq M_j \geq M_k \) and \(M_j + M_k \geq M_i \geq M_j \geq M_k \) (for all possible combinations \(i, j, k \in \{1, 2, 3\}\) mutually distinct):

1) Instantiate the \((.)^+\) expressions under the respective assumption on the numbers of antennas, therefore some of the \(a_{ij}^{[ZF]}\) and \(\gamma_i\) will be forced to zero.

2) Perform Fourier-Motzkin’s elimination to remove all remaining \(a_{ij}^{[ZF]}\) and \(\gamma_i\) and obtain the achievable DoF region.

We then combine the resulting achievable DoF regions into the following compact formulation:

\[ \max\{d_{12} + d_{13}, d_{21} + d_{31}\} \leq \tau M_1 \quad (48) \]
\[ \max\{d_{21} + \tau d_{23}, d_{12} + \tau d_{32}\} \leq \tau M_2 \quad (49) \]
\[ \max\{d_{31} + \tau d_{32}, d_{13} + \tau d_{23}\} \leq \tau M_3 \quad (50) \]
\[ \max\{d_{12} + d_{13} + \tau d_{23}, d_{21} + d_{31} + \tau d_{32}\} \leq \tau \max\{M_1, M_3\} \quad (51) \]
\[ \max\{d_{12} + d_{13} + \tau d_{32}, d_{21} + d_{31} + \tau d_{23}\} \leq \tau \max\{M_1, M_2\} \quad (52) \]
\[ \max\{d_{12} + d_{31} + \tau d_{32}, d_{21} + d_{13} + \tau d_{23}\} \leq \tau \max\{M_2, M_3\} \quad (53) \]
\[ \min\{d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32}\} \geq 0 \quad (54) \]

This region is achievable for tuples \(d\) with non-negative integer entries. Tuples with non-negative real entries (such as the corner points of the region) are first approximated by non-negative rationals which then can be achieved using symbol extension, as in \(\[44\].

The set of all DoF tuples \(d\) satisfying constraints \(\{48\} \text{ to } \{54\}\) is denoted by \(D_{IB,FX}^I\). This proves Theorem 1.

What is the rationale behind the DoF region inner bound \(\{48\}\) to \(\{54\}\)?

The first three inequalities constrain the sum-DoF of outbound and inbound streams at each node, similar to cut-set bounds. The next three inequalities follow this rule: For each of the three links \(1 \leftrightarrow 2, 1 \leftrightarrow 3\) and \(2 \leftrightarrow 3\) there are two DoF variables, one for each direction (i.e., \(d_{12}\) and \(d_{21}\), etc.). For each link pick one direction. There are eight such combinations. Whenever a combination contains both DoF variables that occur in a node’s outbound or inbound sum-DoF constraint, the number of antennas at this node appears in the \(\max\{\cdot\}\) operator at the right side of the inequality. This means that whenever a node’s index appears two times as left or two times as right index, this node’s index appears also in the \(\max\{\cdot\}\) on the right side.

As can be seen above, there are six cases where each case applies to two nodes each, while for the third node it does not, because the third node’s index appears once as left and once as right index. There are two cases missing altogether, \(d_{12} + d_{31} + \tau d_{23}\) and \(d_{21} + d_{13} + \tau d_{32}\), where the indices of all three nodes appear
once as left and once as right index. Depending on numbers of antennas, this achievable DoF region yields four bounds for the largest and two bounds for the second largest node from the inequalities (51) to (53), and two bounds for the third largest node from (48) to (50). The remaining bounds from (48) to (50) are inactive due to the tighter bounds from (51) to (53).

Although we proved that the above region is achievable, it is still useful to provide a ‘recipe’ which describes how a specific DoF tuple can be achieved. To obtain the actual allocation of signal dimensions \(a_{ij}^{[q]}\) for a DoF tuple \(d\) satisfying (48) to (54), proceed as follows: First, use as many ZF resources as possible. Only once the ZF dimensions are exhausted, assign IA resources and align as much of the resulting interference as possible. Throughout the process, account for redundancy required by EC to be able to tolerate intermittency. As an example (Fig. 9), assume \((M_1, M_2, M_3, \tau) = (5, 7, 4, 0.5)\). There is one ZF dimension \(1 \to 2\), three ZF dimensions \(2 \to 1\) and two ZF dimensions \(2 \to 3\), all other communication cannot be zero-forced. We try to achieve \(d = (0.5, 0, 0.5, 4, 0, 4)\). To transmit on average half a symbol per channel access full-duplex over the intermittent \(1 \leftrightarrow 2\), we use a rate \(\frac{1}{2}\) EC over one available ZF dimension in each direction \((a_{12}^{[ZF]} = a_{21}^{[ZF]} = 1)\), the remaining two ZF dimensions \(2 \to 1\) remain unused. IA is not required for \(1 \leftrightarrow 2\) \((a_{12}^{[IA]} = a_{21}^{[IA]} = 0)\). No communication \(1 \leftrightarrow 3\) takes place \((a_{13}^{[ZF]} = a_{31}^{[IA]} = a_{13}^{[ZF]} = a_{31}^{[IA]} = 0)\). To transmit four symbols per channel access \(2 \to 3\), we use the two available ZF dimensions for two of them \((a_{23}^{[ZF]} = 2)\), and two IA dimensions \((a_{23}^{[IA]} = 2)\) that occupy a two-dimensional interference subspace at node 1. To transmit four symbols per channel access \(3 \to 2\), we use four IA dimensions \((a_{32}^{[IA]} = 4)\), since ZF is not possible \((a_{32}^{[ZF]} = 0)\). All of them create interference at node 1, but this four-dimensional interference subspace can be aligned with the two-dimensional interference subspace caused by \(2 \to 3\) \((\gamma_1 = 2)\). As a result, four of the five receive dimensions at node 1 are interference of \(2 \leftrightarrow 3\) communication, while the remaining was used for zero-forced and erasure-coded communication \(2 \to 1\). At nodes 2 and 3 no interference is caused, such that no interference alignment takes place \((\gamma_2 = \gamma_3 = 0)\).

**B. Sum-DoF**

The devised non-adaptive encoding scheme based on ZF, IA and EC is sum-DoF optimal in the intermittent 3WC, as we will prove in this section.

1) Lower Bounds: Using (36) to (47), we derive a lower bound on the sum-DoF of the intermittent 3WC with intermittent node 1. For each of the six cases of \(M_i \geq M_j \geq M_k\) (for all possible combinations \(i, j, k \in \{1, 2, 3\}\) mutually distinct), we solve the linear program maximizing sum-DoF using, e.g., the simplex algorithm.

**Definition 1.** We denote

\[
d^d_{\text{sum}, \text{LB}, \mathbf{X}} := \max_{d \in \mathcal{D}_{\text{in}, \mathbf{X}}} \left[ d_{12} + d_{13} + d_{21} + d_{23} + d_{31} + d_{32} \right].
\]

The resulting lower bounds for each of the different cases are listed in Table I and are condensed into a single expression in the following lemma:

**Lemma 1** (Sum-DoF Lower Bound for Node-Intermittent 3WC). The sum-DoF

\[
d^d_{\text{sum}, \text{LB}, \mathbf{X}} = 2\tau \min\{M_2, M_3\} + 2\tau(M_1 + M_2 + M_3 - \min\{M_1, M_2, M_3\} - \max\{M_1, M_2, M_3\}) \leq d^d_{\text{sum}}
\]
is achievable in the node-intermittent 3WC and therefore constitutes a lower bound on the sum-DoF of the node-intermittent 3WC.

2) Upper Bounds: We first motivate the converse techniques used throughout this section and in the sequel provide rigorous proofs. The general approach for upper bounding the sum-DoF of the intermittent 3WC is as follows: Partition the DoF sum \( d_{ij} + d_{ik} + d_{ji} + d_{jk} + d_{ki} + d_{kj} \) into two partial sums \( d_{ij} + d_{kj} + d_{ki} \) and \( d_{ik} + d_{jk} + d_{ji} \) (Fig. 10), where \( w_{ij}, w_{kj}, w_{ki} \) are to be decoded by node \( j \) and \( w_{ik}, w_{jk}, w_{ji} \) are to be decoded by node \( k \). There are three such partitions, and the partition is fully determined by choosing which node takes the role of node \( i \). While nodes \( j \) and \( k \) function exclusively as source and sink in any one of the partial sums, node \( i \) is an intermediary node in both.

Have an imaginary genie provide just enough side information to node \( j \) and \( k \) (hence the name ‘genie-aided’ bound), respectively, such that they can recover the receive signal \( \mathbf{y}_i^n \) of node \( i \), then (assuming existence of a suitable coding scheme) nodes \( j \) and \( k \) can decode the additional messages \( w_{ki} \) and \( w_{ji} \), respectively. The details of this decoding process and the required side information will be presented in due course. Here we
Fig. 10: Partition the DoF sum $d_{ij} + d_{ik} + d_{ji} + d_{jk} + d_{ki} + d_{kj}$ into two partial sums $d_{ij} + d_{kj} + d_{ki}$ to be decoded at node $j$ (a) and $d_{ik} + d_{jk} + d_{ji}$ to be decoded at node $k$ (b) where nodes $j$ and $k$ are provided enough side information such that they can recover the receive signal $y^n_i$ of node $i$ and from it decode the one message not originally intended for them (additional $d_{ki}$ and $d_{ji}$ DoFs, respectively).

only remark, that the side information can serve for the following four purposes (some cases might not require some of the types of side information):

1) An additional message is required such that node $j$ (or $k$) can decode the additional message $w_{ki}$ (or $w_{ji}$) from the recovered $y^n_i$, because decoding requires knowledge of $w_i$, and node $j$ (or $k$) only knows $w_{ij}$ (or $w_{ik}$) from decoding its own receive signal $y^n_j$ (or $y^n_k$).

2) Node $j$ (or $k$) might be incapable of capturing enough information about $y^n_i$ because $M_j$ (or $M_k$) is small. In this case, additional measurements about $y^n_i$ (or alternatively about the unknown ‘ingredient’ of interest, $x^n_i$ or $x^n_j$, respectively) need to be provided by side information.

3) Node $j$ (or $k$) might be incapable of capturing enough information about $y^n_i$ because of intermittency. In this case, additional measurements about $y^n_i$ for those time instances $\ell$ where $s_\ell = 0$ need to be provided by side information.

4) For rather technical reasons a noise correction signal is required to accurately recover $y^n_i$. However, the information contained in this signal about the three desired messages in question scales only $o[\log(\rho)]$.

Assuming reliable communication, the partial DoF sum of each three messages is necessarily upper bounded by a mutual information expression, using Fano’s inequality. Adding the two resulting bounds yields an upper bound on the sum-DoF. The challenge with this approach is two-fold: a) provide as little side information as possible to the respective nodes, b) use tight bounding when expanding the mutual information expression from Fano’s inequality.

For the intermittent 3WC the partition that yields the tightest upper bound on the sum-DoF depends on the numbers of antennas. The node with largest number of antennas should take the role of the intermediary node $i$ (Fig. 11). The order among the remaining two nodes decides about which side information to give to which node, to compensate for insufficient number of antennas or intermittency. We prove upper bounds for the three cases $M_1 \geq M_2 \geq M_3$, $M_2 \geq M_1 \geq M_3$ and $M_2 \geq M_3 \geq M_1$, the remaining three cases go by renaming node 2 and 3.

a) Case 1: $M_1 \geq M_2 \geq M_3$: For the case where $M_1$ is the largest number of antennas, we develop two partial sums around nodes 3 and 2 (Fig. 11a and 11b), respectively. We start with the bound around node 2 (Fig. 11b), as it requires less side information and is therefore simpler to argue, and in the sequel extend the
Fig. 11: Partition of sum-DoF into partial sums for case $M_1 \geq M_2 \geq M_3$ (a)/(b) and cases $M_2 \geq M_1 \geq M_3$ and $M_2 \geq M_3 \geq M_1$ (c)/(d).

Fig. 12: Decoding of $w_{31}$ at node 2 by iterative reconstruction of $y^n_{1}$ from a-priori knowledge $w_2$ and observations $(y^n_2, s^n)$ using side information $(w_{13}, z^n_{\text{corr}})$ (grey box: a-priori knowledge and channel output, dashed boxes: side information): successively obtain $x_{1,\ell}$, cancel its effect from $y_{2,\ell}$ to obtain a noisy version of $x_{3,\ell}$, and combine this with $x_{2,\ell}$ and $z_{\text{corr},\ell}$ to finally obtain $y_{1,\ell}$; repeat for next $\ell$.

basic technique to develop the bound around node 3 (Fig. 11a), which requires more side information and is therefore slightly more involved.

Which side information does node 2 need to be able to recover $y^n_{1}$ and decode $w_{31}$ from it, assuming a scheme allowing every node to decode its desired messages with high probability? We present a suitable process depicted in Fig. 12. At the end of the transmission, node 2 has $w_2$, $y^n_2$, $s^n$ and $x^n_{3}$, as shown on the top left of the figure. Using the decoder $F_2$ it can decode $(\hat{w}_{12}, \hat{w}_{32}) = (w_{12}, w_{32})$ with high probability. Assume we provide $w_{13}$ as side information, so that node 2 can decode messages intended for node 1 using $F_1$ as soon as it obtains $y^n_{1}$, as shown on the bottom left of the figure. Node 2 can now obtain $x_{1,1}$ from $w_1$ using $E_{1,1}$. 
From \((s_1, y_{2,1})\) it can obtain \(H_{32} x_{3,1} + z_{2,1}\) using \(x_{1,1}\). Since \(H_{32}\) is a tall matrix, a noisy version of \(x_{3,1}\) can be obtained from \(H_{32} x_{3,1} + z_{2,1}\) using the pseudo-inverse \(H_{32}^\dagger\), i.e.,

\[
H_{32}^\dagger (H_{32} x_{3,1} + z_{2,1}) = x_{3,1} + H_{32}^\dagger z_{2,1}.
\]

Using the noisy version of \(x_{3,1}\), node 2 can obtain a noisy version of \(y_{1,1}\), i.e.,

\[
s_1 \left( H_{21} x_{2,1} + H_{31} (x_{3,1} + H_{32}^\dagger z_{2,1}) \right) = s_1 H_{21} x_{2,1} + s_1 H_{31} x_{3,1} + s_1 H_{31} H_{32}^\dagger z_{2,1}.
\]

Given a suitably formed noise correction term \(z_{\text{corr},1} := z_{1,1} - s_1 H_{31} H_{32}^\dagger z_{2,1}\) as side information, node 2 can finally obtain \(y_{1,1}\), i.e.,

\[
y_{1,1} = s_1 H_{21} x_{2,1} + s_1 H_{31} x_{3,1} + s_1 H_{31} H_{32}^\dagger z_{2,1} + z_{\text{corr},1}.
\]

Then, this reconstruction cycle repeats for the next \(\ell = 2, ..., n\), where all previous \(y_{1,\ell-1}\) are used to obtain \(x_{1,\ell}\) using \(E_{1,\ell}\). From the four abstract types of side information introduced before, only two are required for the reconstruction and subsequently for this bound: a message and a noise correction term. No side information to compensate for insufficient number of antennas or intermittency is required.

In a nutshell, side information \(w_{13}\) and \(z_{\text{corr}}\), is provided to node 2 by a genie, defined as

\[
Z_{\text{corr},\ell} := Z_{1,\ell} - S_{\ell} H_{31} H_{32}^\dagger Z_{2,\ell}.
\]

Since the scheme ought to be reliable, we bound the sum rate of \(w_{12}, w_{32}\) and \(w_{31}\) using Fano’s inequality:

\[
n(R_{12} + R_{32} + R_{31} - e^{(1)}_{n}) \leq I(W_{12} W_{32} W_{31}; W_{2} Y_{2}^n S_n \overline{W}_{13} Z_{\text{corr}}^n )
\]

\[
\begin{align*}
&\text{(a)} \quad I(W_{12} W_{32} W_{31}; Y_{2}^n | W_{13} W_{2} S_n Z_{\text{corr}}^n ) \\
&\text{(b)} \quad \sum_{\ell=1}^{n} I(W_{12} W_{33}; Y_{2,\ell} | Y_{2}^{\ell-1} S_n W_{13} W_{2} Z_{\text{corr}}^n ) \\
&\quad = \sum_{\ell=1}^{n} [h(Y_{2,\ell} | Y_{2}^{\ell-1} S_n W_{13} W_{2} Z_{\text{corr}}^n ) - h(Y_{2,\ell} | Y_{2}^{\ell-1} S_n W_{13} W_{2} Z_{\text{corr}}^n )] \\
&\text{(c)} \quad \sum_{\ell=1}^{n} [h(Y_{2,\ell} | S_{\ell}) - h(Y_{2,\ell} | S_{\ell} Z_{\text{corr},\ell} X_{1,\ell} X_{3,\ell})] \\
&\text{(d)} \quad \sum_{\ell=1}^{n} [h(Y_{2,\ell} | S_{\ell}) - h(Y_{2,\ell} | S_{\ell} Z_{\text{corr},\ell} X_{1,\ell} X_{3,\ell})] \\
&= \sum_{\ell=1}^{n} I(Z_{\text{corr},\ell} X_{1,\ell} X_{3,\ell}; Y_{2,\ell} | S_{\ell}) \\
&\text{(e)} \quad \sum_{\ell=1}^{n} [I(X_{1,\ell} X_{3,\ell}; Y_{2,\ell} | S_{\ell}) + I(Z_{\text{corr},\ell} Y_{2,\ell} | S_{\ell} X_{1,\ell} X_{3,\ell})] \\
&\text{(f)} \quad \sum_{\ell=1}^{n} \left[ I(X_{1,\ell} X_{3,\ell}; Y_{2,\ell} | S_{\ell}) + I(Z_{\text{corr},\ell} Z_{2,\ell} | S_{\ell}) \right]
\end{align*}
\]

These steps are justified as follows:

(a) \((W_{12}, W_{3})\) is independent of \((W_{13}, W_{2}, S_n, Z_{\text{corr}}^n)\)
(b) Chain rule for mutual information
(c) Conditioning reduces entropy
(d) \( Y_{2,\ell} \) is independent of \( (Y_2^{\ell-1}, S^{\ell-1}, S_{\ell+1}^{\ell}, Z_{\text{corr},\ell+1}^n, Z_{\text{corr},\ell+1}^n, W_1, W_2, W_3) \) given \( (S_{\ell}, X_{1,\ell}, X_{3,\ell}) \)
(e) \( Z_{\text{corr},\ell} \) is independent of \( (X_{1,\ell}, X_{3,\ell}) \) and \( Y_{2,\ell} \) given \( (X_{1,\ell}, X_{3,\ell}) \) is only noise with a mean \( Z_{2,\ell} + \text{const} \)
(f) \( (X_{1,\ell}, X_{3,\ell}) \) is a MIMO channel with min\{\( M_1 + M_3, M_2 \)\} = \( M_2 \) DoFs if \( s_\ell = 1 \), and min\{0 + \( M_3, M_2 \)\} = \( M_3 \) DoFs if \( s_\ell = 0 \)

Dividing both sides by \( n \log(p) \) and letting \( p, n \to \infty \) we obtain

\[
d_{12} + d_{32} + d_{31} \leq \tau M_2 + \tau M_3.
\]

We turn to the second partial sum, developed around node 3 (Fig. 11a). This bound is slightly more involved, as an additional type of side information is required which compensates for the small number of antennas \( M_3 \).

Which side information does node 3 need to be able to recover \( y_3^n \) and decode \( w_{21} \) from it, assuming a scheme allowing every node to decode its desired messages with high probability? A suitable process is depicted in Fig. 13 At the end of the transmission, node 3 has \( w_3, y_3^n, s^n \) and \( x_3^n \), as shown on the top left of the figure. Using the decoder \( F_3 \) it can decode \( (\hat{w}_{13}, \hat{w}_{23}) = (w_{13}, w_{23}) \) with high probability. Assume we provide \( w_{12} \) as side information, so that node 3 can decode messages intended for node 1 using \( F_1 \) as soon as it obtains \( y_1^n \), as shown on the bottom left of the figure. Node 3 can now obtain \( x_{1,1} \) from \( w_1 \) using \( E_{1,1} \). From \( (s_1, y_3,1) \) it can obtain \( H_{23}x_{2,1} + z_{3,1} \) using \( x_{1,1} \). Assume we ‘virtually’ increase the number of antennas at node 3 so that it can fully observe \( x_{2,1} \) whenever node 1 can, by providing \( \tilde{y}_{3,1} = \tilde{H}_{23}x_{2,1} + \tilde{z}_{3,1} \) as side information if \( s_1 = 1 \), with \( \tilde{H}_{23} \in \mathbb{C}^{(M_2-M_3)\times M_2} \) such that \( \text{rank}[\tilde{H}_{23}] = M_2 \). Note that if \( s_1 = 0 \) then \( x_{2,1} \) does not contribute to \( y_{3,1} \). In this case, \( x_{2,1} \) does not need to be reconstructed, and therefore no side information to compensate for insufficient number of antennas is required. We define shortcuts to group receive signal \( y_3^n \) and side information \( \tilde{y}_{3,1} \) into a joint signal \( \tilde{y}_{3,1} \), i.e.,

\[
\tilde{y}_{3,1} := \begin{bmatrix} y_{3,1}^1 \\ y_{3,1}^n \end{bmatrix}, \quad \tilde{H}_{23} := \begin{bmatrix} H_{23} \\ \hat{H}_{23} \end{bmatrix}, \quad \tilde{z}_{3,1} := \begin{bmatrix} z_{3,1}^1 \\ \hat{z}_{3,1} \end{bmatrix}.
\]

A matrix \( \tilde{H}_{23} \) satisfying \( \text{rank}(\tilde{H}_{23}) = M_2 \) exists almost surely and it allows to obtain \( x_{2,1} + \tilde{H}_{23}^{-1}\hat{z}_{3,1} \) if \( s_1 = 1 \). Assume we provide \( z_{\text{corr},1} = z_{3,1} - s_1 H_{23}\tilde{H}_{23}^{-1}\hat{z}_{3,1} \) as side information. Then node 3 can obtain \( y_{1,1} \) from \( z_{\text{corr},1} \) if \( s_1 = 0 \), and from \( x_{1,1}, x_{2,1} + \tilde{H}_{23}^{-1}\hat{z}_{3,1} \) and \( z_{\text{corr},1} \) if \( s_1 = 1 \), i.e.,

\[
y_{1,1} = s_1 H_{21}(x_{2,1} + \tilde{H}_{23}^{-1}\hat{z}_{3,1}) + s_1 H_{31}x_{3,1} + z_{\text{corr},1} = s_1 H_{21}x_{2,1} + s_1 H_{31}x_{3,1} + z_{1,1}.
\]

Using \( (y_{1,1}, w_1) \) and the encoder \( E_{1,2} \) node 3 can obtain \( x_{1,2} \) and the cycle repeats, for \( \ell = 2, ..., n \). Finally, node 3 obtains \( y_3^n \), and decodes \( w_{21} \) from \( (y_3^n, w_1, s^n) \). Side information to compensate for intermittency is not required.

In a nutshell, side information \( w_{12}, \tilde{y}_{3,1} \) and \( z_{\text{corr}} \) is provided to node 3 by a genie, following the definitions

\[
\hat{Y}_{3,\ell} := S_\ell(\hat{H}_{23}X_{2,\ell} + \hat{Z}_{3,\ell}), \quad Z_{\text{corr},\ell} := Z_{1,\ell} - S_\ell(\hat{H}_{21}\hat{H}_{23}^{-1}\hat{Z}_{3,\ell}),
\]

where

\[
\hat{H}_{23} := \begin{bmatrix} H_{23} \\ \hat{H}_{23} \end{bmatrix}, \quad \tilde{H}_{23} \in \mathbb{C}^{(M_2-M_3)\times M_2} \quad \text{s.t.} \quad \text{rank}(\tilde{H}_{23}) = M_2,
\]

\[
\hat{Z}_{3,\ell} \sim \mathcal{CN}(0, \sigma^2 I_{M_2-M_3}), \quad \hat{Z}_{3,\ell} := \begin{bmatrix} \hat{Z}_{3,\ell}^1 \\ \hat{Z}_{3,\ell}^n \end{bmatrix}, \quad \hat{Y}_{3,\ell} := \begin{bmatrix} \hat{Y}_{3,\ell}^1 \\ \hat{Y}_{3,\ell}^n \end{bmatrix}.
\]
Fig. 13: Decoding of $w_{21}$ at node 3 by iterative reconstruction of $y_3^n$ from a-priori knowledge $w_3$ and observations $(y_3^n, s^n)$ using side information $(w_{12}, \tilde{y}_3^n, z_{\text{corr}})$ (grey box: a-priori knowledge and channel output, dashed boxes: side information): successively obtain $x_{1,\ell}$, cancel its effect from $y_{2,\ell}$ (using side information $\tilde{y}_3^n$) to obtain a noisy version of $x_{2,\ell}$, and combine this with $x_{3,\ell}$ and $z_{\text{corr}}$ to finally obtain $y_{1,\ell}$; repeat for next $\ell$.

Since the scheme ought to be reliable, we again bound the sum rate of $w_{13}$, $w_{23}$ and $w_{21}$ using Fano’s inequality, and following similar steps as before (for details see appendix A) we obtain

$$d_{13} + d_{23} + d_{21} \leq \tau M_2 + \tau M_3.$$  \hfill (56)

The two bounds on partial DoF sums are combined into a sum-DoF upper bound for the case $M_1 \geq M_2 \geq M_3$:

**Proposition 1** (Sum-DoF of Node-Intermittent 3WC for $M_1 \geq M_2 \geq M_3$). 

$$d_{\text{sum}}^1 \leq 2\tau M_2 + 2\tau M_3$$

**Proof.** The proposition follows by adding (55) and (56). \hfill \Box

**b) Case 2:** $M_2 \geq M_1 \geq M_3$: Since $M_2$ is the largest number of antennas, we develop two partial sums around nodes 3 and 1 (Fig. 11c and 11d), respectively.

The reasoning around node 3 in this case proceeds in close analogy to the bound around node 3 in the previous case, just with 1 and 2 interchanged. We provide as side information $w_{21}$ (to allow for decoding using $F_2$), $\tilde{y}_3^n$ (to compensate for small number of antennas $M_3$) and $z_{\text{corr}}$ (a noise correction), defined as

$$\tilde{Y}_{3,\ell} := S_\ell(\tilde{H}_{13}X_{1,\ell} + \tilde{Z}_{3,\ell}), \quad Z_{\text{corr},\ell} := Z_{2,\ell} - S_\ell(\tilde{H}_{12}\tilde{H}_{13}^{-1}Z_{3,\ell}),$$

where

$$\tilde{H}_{13} := \begin{bmatrix} H_{13} \\ H_{13} \end{bmatrix}, \quad \tilde{H}_{13} \in \mathbb{C}^{(M_1 - M_3) \times M_1} \quad \text{s.t. rank}(\tilde{H}_{13}) = M_1,$$

$$\tilde{Z}_{3,\ell} \sim \mathcal{CN}(0, \sigma^2 I_{M_1 - M_3}), \quad \tilde{Z}_{3,\ell} := \begin{bmatrix} Z_{3,\ell} \\ \tilde{Z}_{3,\ell} \end{bmatrix}, \quad \tilde{Y}_{3,\ell} := \begin{bmatrix} Y_{3,\ell} \\ \tilde{Y}_{3,\ell} \end{bmatrix}.$$

With this information, node 3 can construct $x_{2,1}$ from $w_2$ using $E_{2,1}$ (since $w_{23}$ is a message intended for node 3 and assumed to have been decoded from $y_3^n$ using $F_3$, and $w_{21}$ is side information), obtain a noisy version
of $x_{1,1}$ from channel output $y_{3,1}$ and side information $y_{3,1}$ as necessary for $y_{2,1}$ (i.e., if $s_1 = 1$), combine all relevant signals into $y_{2,1}$, encode $u_2$ using $E_{2,2}$ and $y_{2,1}$ to obtain $x_{2,2}$, and continue this cycle for the next $\ell = 2, \ldots, n$ until $y_{2}^{n}$ is complete, from which $w_{1,2}$ can be decoded with the help of $u_2$ using $F_2$.

After similar steps as before (see appendix [B] for details) we obtain

$$d_{13} + d_{23} + d_{12} \leq \tau M_1 + \tau M_3. \tag{57}$$

We turn to the second partial sum, developed around node 1 (Fig. [11d]), where node 1 should be enabled to decode $w_{32}$. The main difference to the previous cases is that the link $3 \leftrightarrow 2$ is always available, while the link $3 \leftrightarrow 1$ is intermittent. Therefore, $y_{1,\ell}$ does not contain information about $x_{3,\ell}$ if $s_\ell = 0$; this needs to be compensated for by side information, here $y_1^{n}$ defined as

$$y_{1,\ell} := s_\ell (H_{31} x_{3,\ell} + \tilde{z}_{1,\ell}),$$

which provides measurements of $x_{3,\ell}$ for those time instances where node 1 is intermittent, i.e., $s_\ell = 0$. This is an instance of the fourth type of side information, that for previous bounds was not necessary, namely side information that compensates for intermittency. Given this side information and the customary side information (a message, to allow for decoding, and a noise correction signal), the reconstruction proceeds in analogy to the previous cases. In a nutshell, the genie provides $w_{23}$, $y_1^{n}$ and $z_{\text{corr}}^{n}$ to node 1, defined as

$$Y_{1,\ell} := y_{1,\ell} - \sum_{i=1}^{\ell-1} (H_{31} x_{3,i} + Z_{i,\ell}) = Z_{2,\ell} - H_{32} H_{31}^\dagger (S_\ell Z_{1,\ell} + \tilde{S}_\ell \tilde{Z}_{1,\ell}),$$

with

$$\tilde{Z}_{1,\ell} \sim \mathcal{CN}(0, \sigma^2 I_{M_1}), \quad \tilde{Z}_{1,\ell} := (Z_{1,\ell}, \tilde{S}_\ell \tilde{Z}_{1,\ell}), \quad \tilde{Y}_{1,\ell} := (Y_{1,\ell}, \tilde{Y}_{1,\ell}).$$

At the end of the transmission, node 1 has $w_{1}$, $y_1^{n}$, $s^n$ and $x_1^n$. It decodes $(w_{21}, w_{31})$ from $y_1^n$ using its decoder $F_1$, and gets $w_{23}$ from side information. It generates $x_{2,1}$, then uses its channel output $y_{1,1}$ (if $s_1 = 1$) or side information $y_{1,1}$ (if $s_1 = 0$) to obtain a noisy version of $x_{3,1}$, and with it $y_{2,1}$. From there the cycle repeats, until $y_2^n$ is obtained and $w_{32}$ can be decoded.

After similar steps as before (see appendix [C] for details) we obtain

$$d_{21} + d_{31} + d_{32} \leq \tau M_1 + \tau M_3. \tag{58}$$

**Proposition 2** (Sum-DoF of Node-Intermittent 3WC for $M_2 \geq M_1 \geq M_3$).

$$d_{\text{sum}}^1 \leq 2\tau M_1 + 2\tau M_3.$$

**Proof.** The proposition follows by adding (57) and (58). \hfill $\square$

c) **Case 3:** $M_2 \geq M_1 \geq M_1$: Since $M_2$ is still the largest number of antennas, we again develop two partial sums around nodes 3 and 1 (Fig. [11c] and [11d], respectively. The only difference to the previous case is that this time $M_3 \geq M_1$, therefore the number of antennas at node 1 needs to be augmented ‘virtually’ to obtain sufficient measurements of $x_1^n$, while node 3 remains unchanged.

We turn to the partial sum around node 3 and provide as side information $w_{23}$ and $z_{\text{corr}}^{n}$ with

$$Z_{\text{corr},\ell} := Z_{2,\ell} - S_\ell (H_{12} H_{13}^\dagger Z_{3,\ell}).$$
Note that $x_{1,\ell}$ contributes to $y_{2,\ell}$ only if $s_\ell = 1$. In these instances, node 3 has sufficient information about $x_{1,\ell}$ from $y_{3,\ell}$. If $s_\ell = 0$, node 3 does not have information about $x_{1,\ell}$, but $x_{1,\ell}$ does not contribute to $y_{2,\ell}$ anyhow, so node 3 does not need additional side information in these cases. Therefore, with the given side information, node 3 can construct $x_{2,1}$ from $w_2$. obtain a noisy version of $x_{1,1}$ from channel output $y_{3,1}$ as necessary for $y_{2,1}$ (i.e., if $s_1 = 1$), generate $y_{2,1}$, and continue this cycle for the next $\ell = 2, \ldots, n$ until $y_{2}^n$ is complete, from which $w_{12}$ can be decoded with the help of $w_2$.

After similar steps as before (see appendix D for details) we obtain

$$d_{13} + d_{23} + d_{12} \leq M_3. \tag{59}$$

We turn to the second partial sum, developed around node 1 (Fig. 11d), where node 1 should be enabled to decode $w_{32}$. Again the main difference to the previous cases is that the link $3 \leftrightarrow 2$ is always available, while the link $3 \leftrightarrow 1$ is intermittent. Therefore, $y_{1,\ell}$ does not contain information about $x_{3,\ell}$ if $s_\ell = 0$; this effect of intermittency needs to be compensated for by side information, here $\tilde{y}_{1}^n$ (a formal definition follows).

Furthermore, the number of antennas at node 1 needs to be increased to fully capture $x_{3,\ell}$, here accomplished by side information $\hat{y}_{1}^n$ (a formal definition follows). In addition, the genie provides the message $w_{23}$ and noise correction $z_{\text{corr}, 1}$ to node 1. In a nutshell, side information variables are defined as

$$\tilde{Y}_{1,\ell} := \tilde{S}_{\ell}(H_{31}X_{3,\ell} + \tilde{Z}_{1,\ell}), \quad \hat{Y}_{1,\ell} := \hat{H}_{31}X_{3,\ell} + \hat{Z}_{1,\ell}, \quad Z_{\text{corr}, \ell} := Z_{2,\ell} - H_{12}\hat{H}_{31}^{-1}\left[\begin{array}{c} S_{1,\ell}\tilde{S}_{\ell} + \tilde{Y}_{1,\ell} \\ \hat{Y}_{1,\ell} \end{array}\right],$$

with auxiliary variables

$$\tilde{Z}_{1,\ell} \sim \mathcal{CN}(0, \sigma^2 I_{M_1}), \quad \hat{Z}_{1,\ell} \sim \mathcal{CN}(0, \sigma^2 I_{M_3-M_1}),$$

$$\hat{H}_{31} := \frac{H_{31}}{H_{31}}, \quad \tilde{H}_{31} \in \mathbb{C}^{(M_3-M_1) \times M_3} \quad \text{s.t.} \quad \text{rank}(\tilde{H}_{31}) = M_3,$$

$$\hat{Y}_{1,\ell} := (Y_{1,\ell}, \hat{Y}_{1,\ell}, \tilde{Y}_{1,\ell}), \quad Z_{1,\ell} := (Z_{1,\ell}, \tilde{S}_{\ell} \tilde{Z}_{1,\ell}, \hat{Z}_{1,\ell}).$$

At the end of the transmission, node 1 has $w_1, y_{1}^n, s^n$ and $x_{1}^n$. It decodes $(w_{21}, w_{31})$ using $F_1$, and gets $w_{23}$ from side information. It generates $x_{2,1}$ using $\hat{e}_{2,1}$, then uses side information $\hat{y}_{1,1}$ and channel output $y_{1,1}$ (if $s_1 = 1$) or side information $\hat{y}_{1,1}$ and $\hat{y}_{1,1}$ (if $s_1 = 0$) to obtain a noisy version of $x_{3,1}$, and with it $y_{2,1}$ using $z_{\text{corr}, 1}$. From there the cycle repeats, until $y_{2}^n$ is obtained and $w_{32}$ can be decoded using $F_2$.

After similar steps as before (see appendix E for details) we obtain

$$d_{21} + d_{31} + d_{32} \leq M_3. \tag{60}$$

**Proposition 3** (Sum-DoF of Node-Intermittent 3WC for $M_2 \geq M_3 \geq M_1$).

$$d_{\text{sum}}^1 \leq 2M_3$$

**Proof.** The proposition follows by adding (59) and (60). \hfill \Box

**Lemma 2** (Sum-DoF Upper Bound for Node-Intermittent 3WC).

$$d_{\text{sum}}^1 \leq 2\tau \min\{M_2, M_3\} + 2\tau(M_1 + M_2 + M_3 - \min\{M_1, M_2, M_3\} - \max\{M_1, M_2, M_3\})$$

**Proof.** The lemma follows by Propositions 1, 2, and 3 and symmetry of node 2 and 3. \hfill \Box
Using the achievability and converse results developed in Sections IV-B1 and IV-B2, we establish the sum-DoF of the intermittent 3WC for which non-adaptive encoding is sufficient, i.e. 
\[
d_{\text{sum,LB,3WC}}^d = 2\tau \min\{M_2, M_3\} + 2\tau (M_1 + M_2 + M_3 - \min\{M_1, M_2, M_3\} - \max\{M_1, M_2, M_3\}) = d_{\text{sum}}^d.
\]

This proves Theorem 2.

C. DoF Region

In this section we first derive an upper bound on \(d_{31}\) under the assumption of non-adaptive encoding. We then show adaptive schemes that can exceed this bound, namely multi-hop and decode-forward relaying. This proves that some DoF region points are only achievable by adaptive encoding schemes, hence adaptive encoding is in general required to achieve the DoF region of the intermittent 3WC.

1) Upper Bound on \(d_{31}\) under Non-Adaptive Encoding: Node 1 is able to decode \(w_{31}\) from its channel output \((y^n_1, s^n)\) and a-priori knowledge \(w_1\) with high probability. We further provide \(w_2\) as side information and bound the rate of \(w_{31}\) using Fano’s inequality:

\[
n(R_{31} - \varepsilon_n) \leq I(W_{31}; W_1 Y_1^n S^n | \widehat{W}_2)
\]

\[
= I(W_{31}; Y_1^n | W_1 W_2 S^n)
\]

\[
= \sum_{\ell=1}^{n} I(W_{31}; Y_{1,\ell} | Y_1^{\ell-1} W_1 W_2 S^n)
\]

\[
\leq \sum_{\ell=1}^{n} [h(Y_{1,\ell} | Y_1^{\ell-1} W_1 W_2 S^n) - h(Y_{1,\ell} | Y_1^{\ell-1} W_1 W_2 W_{31} S^n)]
\]

\[
\leq \sum_{\ell=1}^{n} [h(Y_{1,\ell} | W_2 S_{\ell}) - h(Y_{1,\ell} | W_2 S_{\ell} X_{2,\ell} X_{3,\ell})]
\]

\[
= \sum_{\ell=1}^{n} I(X_{2,\ell} X_{3,\ell}; Y_{1,\ell} | W_2 S_{\ell})
\]

\[
= \sum_{\ell=1}^{n} [I(X_{2,\ell}; Y_{1,\ell} | W_2 S_{\ell}) + I(X_{3,\ell}; Y_{1,\ell} | X_{2,\ell} W_2 S_{\ell})]
\]

\[
\leq n \tau M_3 \log(\rho) + n \tau o[\log(\rho)]
\]

These steps are justified as follows:

(a) \(W_{31}\) is independent of \((W_1, W_2, S^n)\)

(b) Chain rule for mutual information

(c) Conditioning reduces entropy

(d) \(Y_{1,\ell}\) is independent of \((Y_1^{\ell-1}, S^{\ell-1}, S_{\ell+1}^n, W_1, W_{31})\) given \((S_{\ell}, X_{2,\ell}, X_{3,\ell})\)

(e) \(X_{2,\ell}\) is independent of \(Y_{1,\ell}\) given \((W_2, S_{\ell})\) because \(x_{2,\ell} = E_{2,\ell}(w_2)\) under the assumption of non-adaptive encoding.
Fig. 14: All messages except \( w_{31} \) are fixed to 0, node 2 supports node 3 in communicating \( w_{31} \) to node 1

\((f)\) \( X_{3,t} \rightarrow Y_{1,t} \) given \( X_{2,t} \) is a MIMO channel with maximum DoF \( M_3 \) and 0 for \( s_t = 1 \) and \( s_t = 0 \), respectively

The following lemma provides the desired upper bound.

**Lemma 3** (Upper Bound on \( d_{31} \) under Non-Adaptive Encoding). The achievable DoFs of non-adaptive encoding schemes are restricted by

\[
d_{31} \leq \tau M_3. \tag{62}
\]

**Proof.** We divide both sides of (61) by \( n \log(\rho) \) and take \( \rho, n \to \infty \).

2) Adaptive Schemes Achieving \( d_{31} > \tau M_3 \): We assume all messages are fixed to 0, except for \( w_{31} \), which node 3 wants to convey to node 1, potentially with the help of node 2 (Fig. 14).

If \( \tau M_2 \geq M_3 \), multi-hop relaying can be used to achieve \( M_3 \) DoFs: Assume a block-Markov transmission scheme over \( B \) blocks consisting of \( n \) channel accesses each. In each of the first \( B - 1 \) blocks, \( nM_3 \) symbols are transmitted \( 3 \rightarrow 2 \). In the final block \( B \) node 3 remains silent, as does node 2 in block \( b = 1 \). In each block \( b \geq 2 \), node 2 retransmits the message it received in the previous block \( b - 1 \), using EC to protect against intermittency. If \( \tau M_2 \geq M_3 \), \( 2 \rightarrow 1 \) can carry all \( nM_3 \) symbols it received in the previous block. Node 1 uses backwards-decoding to successively cancel the interference from \( 3 \rightarrow 1 \) and decode the messages.

While multi-hop relaying achieves \( d_{31} \) larger than \( \tau M_3 \) if \( \tau M_2 \geq M_3 \), this condition can be relaxed: Consider using decode-forward relaying at node 2. This can be used to achieve \( \min\{\tau M_1, \tau (M_2 + M_3), M_3\} \) DoFs and outperforms any non-adaptive scheme as soon as \( M_1 > M_3 \). We derive the achievable DoF for \( d_{31} \) based on the well-known lower bound for decode-forward relaying \([45]\):

\[
C \geq \max_{\rho \rho_3, \rho_2} \min \{ I(X_3; X_2; Y_1, S), I(X_3; Y_2 S | X_2) \} \\
\geq \min \{ I(X_3; X_2; Y_1 | S), I(X_3; Y_2 | X_2) \} \quad \text{with } X_2, X_3 \text{ Gaussian} \\
= \min \{ \tau \min \{ M_1, (M_2 + M_3) \}, M_3 \} \log(\rho) + o(\log(\rho)) \\
= \min \{ \tau M_1, \tau (M_2 + M_3), M_3 \} \log(\rho) + o(\log(\rho)) \tag{63}
\]

The following lemma provides the desired lower bound.

**Lemma 4** (Lower Bound on \( d_{31} \) under Adaptive Encoding). Decode-forward relaying achieves

\[
d_{31} \geq \min \{ \tau M_1, \tau (M_2 + M_3), M_3 \}. \tag{64}
\]

**Proof.** We divide both sides of (63) by \( \log(\rho) \) and take \( \rho \to \infty \).
Note that if $\tau M_2 > \tau M_3$ and $\tau M_1 > M_3$, then we can transmit at $M_3$ DoF from node 3 to node 1 using this adaptive scheme, compensating all the the loss due to intermittency.

We proved in Lemma 3 that the DoF region point

$$d = (0, 0, 0, d_{31, A}, 0) \quad d_{31, A} := \min\{\tau M_1, \tau (M_2 + M_3), M_3\}$$

is not achievable for any non-adaptive encoding scheme if $M_1 > M_3$, while we showed in Lemma 4 that there exist adaptive schemes that achieve it. This proves Theorem 3 which states that adaptive encoding is in general required to achieve the DoF region of the intermittent 3WC.

Theorems 2 and 3 show that non-adaptive encoding is sufficient to achieve sum-DoF, but not sufficient to achieve the DoF region of the intermittent 3WC. This is particularly interesting in light of the next section, where we show that adaptive encoding is not beneficial in the non-intermittent 3WC even from a DoF region perspective.

V. No Intermittency

The sum-DoF of the non-intermittent 3WC was investigated in [18]. We present the DoF region of the non-intermittent 3WC and show that the non-adaptive encoding scheme presented in Section IV-A is sufficient to achieve it; therefore, adaptive encoding is neither required from a sum-DoF nor from a DoF region perspective in the non-intermittent 3WC. The non-intermittent 3WC is a special case of the intermittent 3WC with $\tau = 1$.

We may assume without loss of generality $M_1 \geq M_2 \geq M_3$.

A. Achievability

From (48) to (54) we obtain with $\tau = 1$:

$$d_{12} + d_{13} + d_{23} \leq M_1 \quad (65)$$
$$d_{12} + d_{13} + d_{32} \leq M_1 \quad (66)$$
$$d_{21} + d_{31} + d_{32} \leq M_1 \quad (67)$$
$$d_{21} + d_{31} + d_{23} \leq M_1 \quad (68)$$
$$d_{21} + d_{13} + d_{23} \leq M_2 \quad (69)$$
$$d_{12} + d_{31} + d_{23} \leq M_2 \quad (70)$$
$$d_{31} + d_{32} \leq M_3 \quad (71)$$
$$d_{13} + d_{23} \leq M_3 \quad (72)$$
$$\min\{d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32}\} \geq 0 \quad (73)$$

**Definition 2.** We denote the set of all DoF tuples $d$ satisfying constraints (65) to (73) by $\mathcal{D}_{\text{NIB}, A}^N$.

**Lemma 5** (DoF Region Inner Bound for Non-Intermittent 3WC). All $d \in \mathcal{D}_{\text{NIB}, A}^N$ are achievable in the non-intermittent 3WC. Therefore, $\mathcal{D}_{\text{NIB}, A}^N$ constitutes an inner bound on the DoF region of the non-intermittent 3WC, i.e.

$$\mathcal{D}_{\text{NIB}, A}^N \subseteq \mathcal{D}^N.$$
Proof. The lemma follows by construction in Section IV-A.

B. Converses

Besides cut-set outer bounds [46] on pairs of DoFs (involving two messages either intended for or originating at a certain node), we use tighter genie-aided outer bounds [18], [19], [21] on triplets of DoFs (involving two messages either intended for or originating at a certain node, and one message exchanged between the remaining two nodes):

\[ R_{13} + R_{23} \leq M_3 \log(\rho) + o[\log(\rho)] \] (74)
\[ R_{31} + R_{32} \leq M_3 \log(\rho) + o[\log(\rho)] \] (75)
\[ R_{21} + R_{31} + R_{32} \leq \min\{M_1, M_2 + M_3\} \log(\rho) + o[\log(\rho)] \] (76)
\[ R_{21} + R_{31} + R_{23} \leq \min\{M_1, M_2 + M_3\} \log(\rho) + o[\log(\rho)] \] (77)
\[ R_{12} + R_{32} + R_{13} \leq M_1 \log(\rho) + o[\log(\rho)] \] (78)
\[ R_{13} + R_{23} + R_{12} \leq M_1 \log(\rho) + o[\log(\rho)] \] (79)
\[ R_{12} + R_{32} + R_{31} \leq M_2 \log(\rho) + o[\log(\rho)] \] (80)
\[ R_{13} + R_{23} + R_{21} \leq M_2 \log(\rho) + o[\log(\rho)] \] (81)

Note that if \( M_2 + M_3 \leq M_1 \), then (76) and (77) are redundant given (75) and (81), therefore \( \min\{M_1, M_2 + M_3\} \) can be replaced with \( M_1 \) in (76) and (77). Dividing these bounds by \( \log(\rho) \) and taking \( \rho \to \infty \) yields to the following DoF region outer bounds:

\[ d_{21} + d_{31} + d_{32} \leq M_1 \] (82)
\[ d_{21} + d_{31} + d_{23} \leq M_1 \] (83)
\[ d_{12} + d_{32} + d_{13} \leq M_1 \] (84)
\[ d_{13} + d_{23} + d_{12} \leq M_1 \] (85)
\[ d_{12} + d_{32} + d_{31} \leq M_2 \] (86)
\[ d_{13} + d_{23} + d_{21} \leq M_2 \] (87)
\[ d_{13} + d_{23} \leq M_3 \] (88)
\[ d_{31} + d_{32} \leq M_3 \] (89)

Note that (86) and (87) can also be obtained from (55) and (56), respectively, with \( \tau = 1 \).

Definition 3. We denote the set of all DoF tuples \( d \) satisfying constraints (82) to (89) by \( \mathcal{D}_\text{N}^\text{OB} \).

Lemma 6 (DoF Region Outer Bound for Non-Intermittent 3WC). None of the \( d \notin \mathcal{D}_\text{N}^\text{OB} \) are achievable in the non-intermittent 3WC. Therefore, \( \mathcal{D}_\text{N}^\text{OB} \) constitutes an outer bound on the DoF region of the non-intermittent 3WC, i.e.

\[ \mathcal{D}^\text{N} \subseteq \mathcal{D}_\text{OB}^\text{N}. \]

Proof. The lemma follows by the DoF region outer bounds (82) to (89).
C. DoF Region and Sum-DoF of Non-Intermittent 3WC

The previous achievability and converse results establish the DoF region (and thus sum-DoF) optimality of non-adaptive schemes, rendering adaptive encoding dispensable, i.e.

$$D_{IB,\mathbf{x}}^N = D^N = D_{GB}^N.$$ 

This proves Theorem 4. From the DoF region of the 3WC and using the sum-DoF of the intermittent 3WC, we reproduce the sum-DoF of the 3WC given in [18]:

**Corollary 1** (Sum-DoF of Non-Intermittent 3WC).

$$d_{sum, LB, \mathbf{x}}^N = 2M_2 = d_{sum}^N$$

**Proof.** The statement follows as corollary of Theorems 2 and 4. \(\square\)

VI. CONCLUSION

We introduced the MIMO 3WC with node-intermittency and studied its DoF region and sum-DoF. In particular, we devised a non-adaptive encoding scheme based on zero-forcing, interference alignment and erasure coding, and showed its DoF region (and thus sum-DoF) optimality for non-intermittent 3WCs and its sum-DoF optimality for node-intermittent 3WCs. This shows that adaptive encoding is not required in those cases. However, we showed by example that in general there are DoF region points in the node-intermittent 3WC that can only be achieved by adaptive schemes, such as multi-hop or decode-forward relaying, making adaptive encoding a necessity. Our work contributes to a better understanding of the necessity of adaptive schemes such as relaying in multi-way communications with intermittency.

As remarked in the introduction, node-intermittency is only one of a multitude of practically relevant intermittency scenarios. Links might be intermittent independently of each other, e.g., moving objects passing by only interrupt the link between the two D2D users from time to time, while the other links remain intact (Fig. 15a). Or all links being intermittent, but independently of each other, and with different probabilities (Fig. 15b). Here, we speak of link intermittency and intermittent links. Intermittent 3WCs with other intermittency models are interesting directions for future research.
APPENDIX

A. Sum-DoF Upper Bound for Intermittent 3WC with \( M_1 \geq M_2 \geq M_3 \) (Part II)

Since the scheme ought to be reliable, we bound the sum rate of \( w_{13}, w_{23} \) and \( w_{21} \) using Fano’s inequality:

\[
n(R_{13} + R_{23} + R_{21} - \varepsilon_n) \leq I(W_1; Y_{3,\ell}^{n} S^{n_1} W_1 Y_{3}^{n} Z_{\text{corr}}^{n})
\]

\[
= I(W_1 W_2; Y_{3,\ell}^{n} S^{n_1} W_1 W_2 Z_{\text{corr}}^{n})
\]

\[
= \sum_{\ell=1}^{n} I(W_1 W_2; Y_{3,\ell}^{n} | X_{1,\ell}^{n} X_{2,\ell})
\]

\[
\leq \sum_{\ell=1}^{n} I(Z_{\text{corr},\ell}; X_{1,\ell}^{n} X_{2,\ell}) + I(Z_{\text{corr},\ell}; Y_{3,\ell})
\]

These steps are justified as follows:

(a) \((W_{13}, W_2)\) is independent of \((W_{12}, W_3, S^n, Z_{\text{corr}}^n)\)

(b) Chain rule for mutual information

(c) Conditioning reduces entropy

(d) \(Y_{3,\ell}\) is independent of \((Y_{3}^{l-1}, S^{l-1}_1, S^n, Z_{\text{corr},l}^{n}, Z_{\text{corr},l+1}^{n}, W_1, W_2, W_3)\) given \((S_{\ell}, X_{1,\ell}, X_{2,\ell})\)

(e) \(Z_{\text{corr},\ell}\) is independent of \((X_{1,\ell}, X_{2,\ell})\) and \(Y_{3,\ell}\) given \((X_{1,\ell}, X_{2,\ell})\) is only noise with a mean \( \hat{Z}_{3,\ell} + \text{const} \)

(f) \((X_{1,\ell}, X_{2,\ell}) \rightarrow (Y_{3,\ell}, \hat{Y}_{3,\ell})\) is a MIMO channel with \( \min\{M_1 + M_2, M_3 + (M_2 - M_3)\} = M_2 \) DoFs if \( s_\ell = 1 \) and \( \min\{M_1 + M_2, M_3 + 0\} = M_3 \) DoFs if \( s_\ell = 0 \)

Dividing both sides by \( n \log(\rho) \) and letting \( \rho, n \rightarrow \infty \) we obtain

\[
d_{13} + d_{23} + d_{21} \leq \tau M_2 + \tau M_3.
\]

B. Sum-DoF Upper Bound for Intermittent 3WC with \( M_2 \geq M_1 \geq M_3 \) (Part I)

Since this scheme ought to be reliable, we bound the sum rate of \( w_{13}, w_{23} \) and \( w_{12} \) using Fano’s inequality:

\[
n(R_{13} + R_{23} + R_{12} - \varepsilon_n) \leq I(W_1; Y_{3,\ell}^{n} S^{n_1} W_1 Y_{3}^{n} Z_{\text{corr}}^{n})
\]

\[
= I(W_1 W_2; Y_{3,\ell}^{n} | W_2 W_3 S^{n_1} Z_{\text{corr}}^{n})
\]
\[
\begin{align*}
&\sum_{\ell=1}^{n} I(W_1W_{23}; \hat{Y}_{3,\ell} | \hat{Y}_{3}^{\ell-1}S^nW_2W_3Z_{\text{corr}}) \\
&= \sum_{\ell=1}^{n} \left[ h(\hat{Y}_{3,\ell} | \hat{Y}_{3}^{\ell-1}S^nW_2W_3Z_{\text{corr}}) - h(\hat{Y}_{3,\ell} | \hat{Y}_{3}^{\ell-1}S^nW_1W_2W_3Z_{\text{corr}}) \right] \\
&\leq \sum_{\ell=1}^{n} \left[ h(\hat{Y}_{3,\ell} | S_\ell) - h(\hat{Y}_{3,\ell} | \hat{Y}_{3}^{\ell-1}S^nW_1W_2W_3Z_{\text{corr}}, X_{1,\ell}X_{2,\ell}) \right] \\
&= \sum_{\ell=1}^{n} I(Z_{\text{corr},\ell}X_{1,\ell}X_{2,\ell}; \hat{Y}_{3,\ell} | S_\ell) \\
&\leq \sum_{\ell=1}^{n} \left[ I(X_{1,\ell}X_{2,\ell}; \hat{Y}_{3,\ell} | S_\ell) + I(Z_{\text{corr},\ell}; \hat{Y}_{3,\ell} | S_\ell) \right] \\
&\overset{(c)}{=} \sum_{\ell=1}^{n} \left[ I(X_{1,\ell}X_{2,\ell}; \hat{Y}_{3,\ell} | S_\ell) + I(Z_{\text{corr},\ell}; \hat{Z}_{3,\ell} | S_\ell) \right] \\
&\overset{(f)}{\leq} n [\tau M_1 + \tau M_3] \log(\rho) + no \log(\rho)
\end{align*}
\]

These steps are justified as follows:

(a) \((W_1, W_{23})\) is independent of \((W_21, W_3, S^n, Z_{\text{corr}}^n)\)

(b) Chain rule for mutual information

(c) Conditioning reduces entropy

(d) \(\hat{Y}_{3,\ell}\) is independent of \((\hat{Y}_{3}^{\ell-1}, S_\ell^{\ell-1}, S_{\text{corr}}^{\ell-1}, Z_{\text{corr},\ell}; W_1, W_2, W_3)\) given \((S_\ell, X_{1,\ell}, X_{2,\ell})\)

(e) \(Z_{\text{corr},\ell}\) is independent of \((X_{1,\ell}, X_{2,\ell})\) and \(\hat{Y}_{3,\ell}\) given \((X_{1,\ell}, X_{2,\ell})\) is only noise with a mean \(\hat{Z}_{3,\ell} + \text{const}\)

(f) \((X_{1,\ell}, X_{2,\ell}) \rightarrow (Y_{3,\ell}, \hat{Y}_{3,\ell})\) is a MIMO channel with \(\min\{M_1 + M_2, M_3 + (M_1 - M_3)\} = M_1\) DoFs if \(s_\ell = 1\), and \(\min\{M_1 + M_2, M_3 + 0\} = M_3\) DoFs if \(s_\ell = 0\)

Dividing both sides by \(n \log(\rho)\) and letting \(\rho, n \to \infty\) we obtain

\[d_{13} + d_{23} + d_{12} \leq \tau M_1 + \tau M_3\]

C. Sum-DoF Upper Bound for Intermittent 3WC with \(M_2 \geq M_1 \geq M_3\) (Part II)

We bound the sum rate of \(w_{21}, w_{31}\) and \(w_{32}\) using Fano’s inequality:

\[n(R_{21} + R_{31} + R_{32} - e^{(2)}_n) \leq I(W_1W_{31}W_{32}; Y_1^n S^n W_{23} \hat{Y}_1^n Z_{\text{corr}}^n)\]

\[\overset{(a)}{=} I(W_1W_{31}W_{32}; \hat{Y}_1^n | W_{1}^{23} S^n Z_{\text{corr}}^n)\]

\[\overset{(b)}{=} \sum_{\ell=1}^{n} I(W_2 W_3; \hat{Y}_{1,\ell} | \hat{Y}_{1}^{\ell-1} S^n W_{23} Z_{\text{corr}}^n)\]

\[= \sum_{\ell=1}^{n} \left[ h(\hat{Y}_{1,\ell} | \hat{Y}_{1}^{\ell-1} S^n W_{23} Z_{\text{corr}}^n) - h(\hat{Y}_{1,\ell} | \hat{Y}_{1}^{\ell-1} S^n W_1 W_2 W_3 Z_{\text{corr}}^n) \right] \]

\[\overset{(c)}{\leq} \sum_{\ell=1}^{n} \left[ h(\hat{Y}_{1,\ell} | S_\ell) - h(\hat{Y}_{1,\ell} | \hat{Y}_{1}^{\ell-1} S^n W_1 W_2 W_3 Z_{\text{corr}}^n) \right] \]
These steps are justified as follows:

(a) \((W_{21}, W_3)\) is independent of \((W_1, W_{23}, S^n, Z^n_{\text{corr}})\)

(b) Chain rule for mutual information

(c) Conditioning reduces entropy

(d) \(\hat{Y}_{1,\ell}\) is independent of \((Y_{1,\ell-1}, S_{\ell-1}^{\ell-1}, S^n_{\ell+1}, Z^n_{\text{corr}}, Z^n_{\text{corr},\ell+1}, W_1, W_2, W_3)\) given \((S_\ell, X_{2,\ell}, X_{3,\ell})\)

(e) \(Z^n_{\text{corr},\ell}\) is independent of \((X_{2,\ell}, X_{3,\ell})\) and \(\hat{Y}_{1,\ell}\) given \((X_{2,\ell}, X_{3,\ell})\) is only noise with a mean \(\tilde{Z}_{1,\ell} + \text{const}\)

(f) \((X_{2,\ell}, X_{3,\ell}) \rightarrow Y_{1,\ell}\) is a MIMO channel with \(\min\{M_2 + M_3, M_1\} = M_1\) DoFs if \(s_\ell = 1\), and 0 DoFs if \(s_\ell = 0\); \((X_{2,\ell}, X_{3,\ell}) \rightarrow \hat{Y}_{1,\ell}\) is a MIMO channel with 0 DoFs if \(s_\ell = 1\) (because then \(\hat{Y}_{1,\ell} = 0\)), and \(\min\{0 + M_3, M_1\} = M_3\) DoFs if \(s_\ell = 0\) (because then \(Y_{1,\ell}\) is noise, and \(\hat{Y}_{1,\ell}\) is independent of \(X_{2,\ell}\))

Dividing both sides by \(n \log(\rho)\) and letting \(\rho, n \rightarrow \infty\) we obtain

\[
d_{21} + d_{31} + d_{32} \leq \tau M_1 + \tau M_3.
\]

D. Sum-DoF Upper Bound for Intermittent 3WC with \(M_2 \geq M_3 \geq M_1\) (Part I)

We bound the sum rate of \(w_{13}, w_{23}\) and \(w_{12}\) using Fano's inequality:

\[
n(R_{13} + R_{23} + R_{12} - \varepsilon_n^{(1)}) \leq I(W_{13}W_{23}W_{12}; Y_3^n, S^n, W_1^n, W_2^n, W_3^n, Z^n_{\text{corr}}) = I(W_{13}W_{23}W_{12}; Y_3^n, S^n, Z^n_{\text{corr}})
\]

\[
= \sum_{\ell=1}^{n} I(W_{1,\ell}W_{2,\ell}; Y_3^{\ell-1}S^nW_{21}\overrightarrow{W_3^nZ^n_{\text{corr}}})
\]

\[
= \sum_{\ell=1}^{n} [h(Y_{3,\ell} | Y_3^{\ell-1}S^nW_{21}\overrightarrow{W_3^nZ^n_{\text{corr}}}) - h(Y_{3,\ell} | Z^n_{\text{corr},\ell}X_{1,\ell}X_{2,\ell})]
\]

\[
= \sum_{\ell=1}^{n} [h(Y_{3,\ell} | S_\ell) - h(Y_{3,\ell} | S_\ell Z^n_{\text{corr},\ell}X_{1,\ell}X_{2,\ell})]
\]

\[
= \sum_{\ell=1}^{n} I(Z^n_{\text{corr},\ell}X_{1,\ell}X_{2,\ell}; Y_{3,\ell} | S_\ell)
\]
These steps are justified as follows:

(a) \( (W_1, W_2) \) is independent of \((W_{21}, W_3, S^n, Z^n_{\text{corr}})\)

(b) Chain rule for mutual information

(c) Conditioning reduces entropy

(d) \( Y_{3, \ell} \) is independent of \((Y_{3}^{\ell-1}, S^{\ell-1}, S^n_{\ell+1}, Z^n_{\text{corr}, \ell+1}, W_1, W_2, W_3)\) given \((S_\ell, X_1, X_2, X_\ell)\)

(e) \( Z_{\text{corr}, \ell} \) is independent of \((X_1, X_2, X_\ell)\) and \(Y_{3, \ell}\) given \((X_1, X_2, X_\ell)\) is only noise with a mean \(Z_{3, \ell} + \text{const}\)

(f) \((X_1, X_2, X_\ell) \sim Y_{3, \ell}\) is a MIMO channel with \(\min\{M_1 + M_2, M_3\} = M_3\) DoFs if \(s_\ell = 0\) (because \(X_1, X_\ell\) is independent of \(Y_{3, \ell}\))

Dividing both sides by \(n \log(\rho)\) and letting \(\rho, n \to \infty\) we obtain

\[
d_{13} + d_{23} + d_{12} \leq M_3.
\]

E. Sum-DoF Upper Bound for Intermittent 3WC with \(M_2 \geq M_3 \geq M_1\) (Part II)

We bound the sum rate of \(w_{21}, w_{31}\) and \(w_{32}\) using Fano’s inequality:

\[
n(R_{21} + R_{31} + R_{32} - \varepsilon_n^{(3)}) \leq I(W_{21}W_{31}W_{32}; W_1Y^n_1S^n W_3Y^n_2Z^n_{\text{corr}})_{\text{side information}}
\]

\[
= I(W_{21}W_{31}W_{32}; \hat{Y}^n_1 | W_3^n Z^n_{\text{corr}})
\]

\[
= n \sum_{\ell=1}^{n} I(W_{21}W_{3}; \hat{Y}_{1, \ell} \infty 1 | \hat{Y}^{\ell-1}_1 S^n W_3 Z^n_{\text{corr}}) - h(\hat{Y}_{1, \ell} | \hat{Y}^{\ell-1}_1 S^n W_3 Z^n_{\text{corr}}) - h(\hat{Y}_{1, \ell} | \hat{Y}^{\ell-1}_1 S^n W_3 Z^n_{\text{corr}})
\]

\[
\leq \sum_{\ell=1}^{n} I(Z_{\text{corr}, \ell} X_{2, \ell} X_{3, \ell} \infty 1, \hat{Y}_{1, \ell} | S_{\ell})
\]

\[
= \sum_{\ell=1}^{n} I(Z_{\text{corr}, \ell} X_{2, \ell} X_{3, \ell} \infty 1, \hat{Y}_{1, \ell} | S_{\ell})
\]

\[
= \sum_{\ell=1}^{n} I(Z_{\text{corr}, \ell} X_{2, \ell} X_{3, \ell} \infty 1, \hat{Y}_{1, \ell} | S_{\ell})
\]

\[
\leq n [M_3] \log(\rho) + n \log(\rho)
\]
These steps are justified as follows:

(a) $(W_{21}, W_3)$ is independent of $(W_1, W_{23}, S^n, Z^n_{\text{corr}})$

(b) Chain rule for mutual information

(c) Conditioning reduces entropy

(d) $\hat{Y}_{1,\ell}$ is independent of $(\hat{Y}_{1,\ell-1}, S_{\ell-1}, S^n_{\text{corr},\ell+1}, Z^n_{\text{corr},\ell+1}, W_1, W_2, W_3)$ given $(S_{\ell}, X_{2,\ell}, X_{3,\ell})$

(e) $Z_{\text{corr},\ell}$ is independent of $(X_{2,\ell}, X_{3,\ell})$ and $\hat{Y}_{1,\ell}$ given $(X_{2,\ell}, X_{3,\ell})$ is only noise with a mean $\hat{Z}_{1,\ell} + \text{const}$

(f) $(X_{2,\ell}, X_{3,\ell}) \rightarrow (Y_{1,\ell}, \hat{Y}_{1,\ell}, \hat{Y}_{1,\ell})$ is a MIMO channel with $\min\{M_2 + M_3, M_1 + 0 + (M_3 - M_1)\} = M_3$ DoFs if $s_t = 1$, and $\min\{M_2 + M_3, 0 + M_1 + (M_3 - M_1)\} = M_3$ DoFs if $s_t = 0$

Dividing both sides by $n \log(\rho)$ and letting $\rho, n \rightarrow \infty$ we obtain

$$d_{21} + d_{31} + d_{32} \leq M_3.$$

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