Indefinite causal order with fixed temporal order for electrons and positrons

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Abstract We provide an analysis of indefinite causal orders in relativistic quantum mechanics based on the electron–positron picture of Feynman involving negative energy electrons ‘moving backward in time’ (i.e., involving a retrocausal link). We show that genuine implementations of the paradigmatic quantum switch, but here violating some causal inequalities and with a fixed temporal order, become possible in extreme external electromagnetic field conditions.

Keywords Indefinite causal order · Feynman’s quantum electrodynamics · Retrocausality · No-signaling theorem

1 General introduction

As it was famously stated by R.P. Feynman [1], the superposition principle is really at the core of quantum mechanics. Remarkably, in the last decade this principle has been extended to the classical notion of causality leading to the definition of superposed or indefinite causal order (ICO) of quantum events [2–5]. ICOs have been intensively studied for their potential applications as resources in information processing (e.g., [6–8] for a review, see [9]) and more recently quantum thermodynamics [10,11]. At a fundamental level ICO offer motivating perspectives for understanding the connections between quantum mechanics and general relativity as well as for their potential unification [2,12–14].

Among the systems that have been proposed to illustrate the concept of ICO the quantum SWITCH (QS) is paradigmatic due to its simplicity [15]. The basic idea of QS is to consider an interferometric situation where two unitary operations \(\hat{A}\) and \(\hat{B}\) acting on a target system with state \(\psi_0 \in \mathcal{H}^T\) are applied sequentially in the order \(\hat{A}\) before \(\hat{B}\) or \(\hat{B}\) before \(\hat{A}\) depending on the state of a control system \(\phi_0\) or \(\phi_1 \in \mathcal{H}^C\). In the case where the control system is initially in the state \(\frac{1}{\sqrt{2}}(\phi_0 + \phi_1)\) the final entangled system in \(\mathcal{H}^C \otimes \mathcal{H}^T\) becomes \(\frac{1}{\sqrt{2}}(\phi_0 \hat{A}\hat{B} + \phi_1 \hat{B}\hat{A})\psi_0\) that exemplifies ICO. Moreover, after a subsequent projection on the control states \(\phi_{\pm} = \frac{1}{\sqrt{2}}(\phi_0 \pm \phi_1)\) we end up with the states \(\frac{1}{2}\phi_{\pm}[\hat{A}, \hat{B}]_{\pm}\psi_0\), where \([\hat{A}, \hat{B}]_{\pm} = \hat{A}\hat{B} \pm \hat{B}\hat{A}\), defining pure ICOs [4]. QS has been extensively

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studied both theoretically and experimentally \cite{16–19} but some controversies remain about the role of time versus cause in its physical implementation. Indeed, the ideal process sketched in Fig. 1a relies on a two-events sequence \( A \prec (causes) \ B \) or \( B \prec A \). Moreover, its physical realization involves four space-time regions and not two as sketched in Fig. 1b. Fundamentally this implementation of QS is not different from the ‘unfolded’ configuration of Fig. 1c that is just a standard interferometer with four different unitaries (\( \hat{A} \) being equivalent to \( \hat{A}' \) and \( \hat{B} \) to \( \hat{B}' \)) and for this reasons the recent tabletop implementations of QS have been criticized \cite{20}. However, it has been stressed that any attempt to operationally distinguish the times of interaction \( t_A \) from \( t_{A'} \) or \( t_B \) from \( t_{B'} \) would break the coherence needed for observing ICO by providing a ‘which-order’ information \cite{12,16,19} and altering the ‘time-delocalization’ \cite{21}. This therefore is enough to solve the ‘controversy’. Moreover, it has also been speculated than a superposition of two quantized states of the gravitational field could lead to a genuine version of the QS involving two localized space-time events \cite{14}.

The goal of this work is to provide a different strategy relying on quantum field theory and more precisely on Feynman’s theory of electrons and positrons in which an antiparticle is considered as a particle going backward in time \cite{22–24}. Importantly, this alternative solution to the controversy (which does not invalidate or contradict the operational solution discussed before) does not require a curved space-time but only a flat Minkowski’s one. Moreover, as we show this is equivalent to demonstrate some form of retrocausality in quantum mechanics. We show that this can still be done safely in the context of quantum field theory without violating a no-signaling theorem that would otherwise enter in conflict with microcausality and special relativity. In turn, we show that our finding allows us to violate causal inequalities \cite{3,25} that are usually considered as being satisfied for QS and more generally with quantum mechanics \cite{3,26–28}. Whereas this looks at first contradictory we show how a consistent picture emerges out and leads to a better understanding of causal relations in relativistic quantum mechanics.

2 Retrocausality and Feynman diagrams

2.1 The scattering problem with negative energy waves

As it is known, the modern story of quantum electrodynamics (QED) started with the Dirac electron–hole picture for interpreting the negative energy solutions of the relativistic Dirac electron equation. Motivated by Pauli’s exclusion principle Dirac introduced a bottomless energy sea with all negative energy states occupied for justifying the non observation of negative energy waves \cite{29}. Moreover, Feynman subsequently developed a over-all space-time view where he justified the absence of negative energy waves by a acute causal analysis of relativistic Green’s functions and propagators used in scattering processes \cite{22}. For instance, writing \( S(x - x') \) the Green function solution of

\[
(i \gamma^\mu \partial_\mu - m)S(x - x') = i \delta^4(x - x'),
\]

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Feynman imposed the solution

\[
\begin{align*}
    iS_F(x - x') &= \sum_{n, E_n > 0} w_n(x) \bar{w}_n(x') \text{ for } t < t' \\
    iS_F(x - x') &= -\sum_{n, E_n < 0} w_n(x) \bar{w}_n(x') \text{ for } t > t' \\
\end{align*}
\]  

(2)

where \( w_n(x) = w_n(x) e^{-iE_nt} \) are usual bispinor plane wave solutions of Dirac’s equation \((i\gamma^\mu \partial_\mu - m)\psi(x) = 0\) in vacuum (\(\gamma^\mu\) are the standard \(4 \times 4\) Dirac’s matrices). Compared to the ‘natural’ retarded Green function

\[
S_{ret.}(x - x') = \sum_n w_n(x) \bar{w}_n(x') \theta(t - t').
\]  

(3)

Feynman’s choice does not propagate negative energy waves \((E_n < 0)\) into the future direction but only positive one \((E_n > 0)\); therefore, prohibiting unsuitable scattering to negative energy levels. The price to pay is the presence of negative energy waves ‘scattered’ into the past direction.

This leads to the interpretation of negative energy electron waves going backward in time as positive energy positron waves going forward in time. Physically, of course this is not a time-machine in the sense of a time traveler through a worm-hole (as defined by general relativity). Here it is the causal connection defined by the Green function \(S_F(x - x')\) that implies a backward in time interaction from point \(x'\) to point \(x\) (if \(t' > t\)). Moreover, from this point of view information is indeed sent into the past and can have a physical effect (as we show in this work). It is in that specific sense that we must interpret the sentence scattering into the past direction or backward in time motion.

Every thing is actually non-ambiguous and rigorous as clearly explained in the seminal Feynman articles [22].

Feynman’s method is extensively used in introductory textbooks on QED where we generally emphasize the intuitive character of the results but let a rigorous justification of the rules to a more elaborated formalism based on quantum fields and second quantization (compare for example [30] and [31]). It is often overlooked that Feynman aim was not to substitute a one electron picture (with a single electron making multiple zigzags in time) to the electron–hole theory. Better, he justified rigorously his approach using the scattering matrix formalism within Dirac’s hole theory (see the appendix in [22]) that is rigorously equivalent to the usual second quantized formulation. Furthermore, the physical consequences of taking \(S_F(x - x')\) has been generally underestimated and the implications on causality mostly neglected.
To illustrate the problem consider Feynman’s second order diagram shown in Fig. 2a where an electron of electric charge $e$ is scattered two times by an external electromagnetic potential $V^\mu(x)$ located in two space-time regions $A$ and $B$. In a first quantized picture the positive energy initial wave $\psi_0(x)$ evolves as $\psi(x) = \psi_0(x) + \psi^{(1)}(x) + \psi^{(2)}(x) + \ldots$ where the second order term reads

$$\psi^{(2)}(x) = \int_{\Omega_A} \int_{\Omega_B} d^4x_A d^4x_B \chi(x, x_A, x_B) + \ldots$$

with integrations over hypervolumes $\Omega_{A,B}$ such that

$$\chi = e^2 S_F(x - x_A)\nu(x_A) S_F(x_A - x_B)\nu(x_B)\psi_0(x_B)$$

and $\nu(y) := \gamma\mu V^\mu(y)$. Very often processes like Eq. 5 are considered as virtual but this has not necessarily to be so as it was emphasized by Feynman [24]. This holds true because $S_F(x - x') = -(i\gamma^\mu\partial_\mu + m I)D_F(x - x')$ where $D_F(x - x')$, Feynman’s scalar Green function solution of $(\partial^2 + m^2)D_F(x - x') = \delta^4(x - x')$, is given by

$$D_F(x) = \frac{\delta(x^2)}{4\pi} - \frac{m}{8\pi s} H_1^{(2)}(ms)$$

(with $s = \sqrt{t^2 - x^2}$ inside the light-cone and $s = -i\sqrt{x^2 - t^2}$ outside). Outside the light-cone $D_F(x)$ dies off exponentially but inside it is dominated by propagating waves associated with the Hankel function $H_1^{(2)}(ms)$ [24] that can have a physical effect even far away from the interaction zones. Moreover, in the case shown in Fig. 2a the two space-time volumes $\Omega_{A,B}$ are disjoint and if $\Omega_A$ is in the absolute past of $\Omega_B$ (i.e., inside its past light-cone) then we have a retrocausal influence from $B$ to $A$ driven by negative energy waves propagating backward in time (as explained before this motion backward in time is associated with the specific propagator $S_F(x - x')$). A priori, this could be used to send a signal into the past since the field $V^\mu(x_B)$ could be monitored after the interaction at $x_A$ occurred.

To avoid paradoxes and problems with causality we stress that the one electron wave function $\psi(x)$ is actually a probability amplitude defined relatively to the amplitude $C_v$ of a vacuum field remaining a vacuum under the influence of a potential [22]. This vacuum to vacuum amplitude reads $C_v = \langle \emptyset | \hat{U}(T, 0) | \emptyset \rangle$ where $\hat{U}(T, 0)$ is the scattering unitary matrix operator (between the initial time $t = 0$ and the final time $t = T \rightarrow +\infty$) and $| \emptyset \rangle$ defines the filled Dirac sea. Interestingly $\psi(x)$ can be defined as a weak value [32] linked to the two-state formalism [33]:

$$\psi(x) := \langle \emptyset ; T | \hat{\Psi}(x)\hat{F}^\dagger | \emptyset \rangle / \langle \emptyset ; T | \emptyset \rangle,$$

where $\hat{\Psi}(x)$ is a fermionic field operator in the Heisenberg representation $^2$, $\hat{F}^\dagger = \int d^3x \hat{\Psi}^\dagger(x) \psi_0(x)$ is a creation operator for the mode $\psi_0(x)$ and $| \emptyset ; T = U(0, T) | \emptyset \rangle$ is a backward in time evolving quantum state that approaches $| \emptyset \rangle$ at time $t = T$. Again here the notion of backward in time motion is defined through the unitary operator. Moreover [22,30], relative scattering amplitudes for an electron ending in a positive energy mode $w_n(x)$ are given by

$$r_n = \int d^3x w_n^\dagger(x) \psi(x, T).$$

The absolute probability amplitudes are thus given by $r_n C_v$ due to the presence of vacuum. Using this formalism we obtain, in agreement with hole theory, a transition probability $P_n = P_v |r_n|^2$ with $P_v = |C_v|^2$. Importantly, the probabilities of all alternatives must sum to one:

$$1 = P_v + \sum_n P_n + \sum P(1e^- + \text{at least 1, pair})$$

\[\psi^{(2)}(x)\] also includes $\int_{\Omega_A} \int_{\Omega_A} d^4x_A d^4x_A' \chi(x, x_A, x_A') + \int_{\Omega_B} \int_{\Omega_B} d^4x_B d^4x_B' \chi(x, x_B, x_B')$ that we omitted here

\[^2\text{We have } \hat{\Psi}(x) = \hat{U}(0, t)\hat{\Psi}(x)\hat{U}(t, 0) \text{ and in the electron–hole picture this is a pure annihilation operator [22]}\]
where we have to include all contributions where electron-positron pairs can be created from vacuum [22]. It is the presence of these pairs interfering with the incident electron wave scattered by the field that preserves causality and prohibits a backward in time signaling (i.e., driven by the Green propagator $S_F(x - x')$).

It is important to stress that all these issues were already known in 1949 and discussed by Feynman in his article [22] as well as in some lectures [23, 24]. In [24] commenting the physical meaning of the causal propagator $S_F(x - x')$ and the zigzag diagram of Fig. 2a for a single electron he wrote that ‘Pauli invented this thought experiment [with some shutters and mirrors replacing our temporal cavities] after he had thought the idea was wrong.’ But actually it is not. Often the objection made to Feynman’s approach is that it is an intuitive formulation ‘awaiting’ for a more rigorous justification and indeed such a justification was given by Dyson [34] based on a different second quantized picture (i.e., more in agreement with the work of Schwinger and Tomonaga). However, in [22] Feynman provided a justification using the second quantized theory of Dirac. It is clear that the zigzag diagram looks counter-intuitive but it moreover necessary to recover the full relativistic covariance of the theory. Furthermore, as stressed in [22–24], the theory fully agrees with Pauli’s exclusion principle and no contradiction with a charge superselection rule occurs.

2.2 A temporal interferometer for electrons/positrons

In the following, we examines this issue for an idealized system that is treated analytically and will subsequently be used in a QS implementation. We start with a normalized bispinor plane wave in a mode volume $V$:

$$\psi_{E,p}(t, x) = \sqrt{\frac{E + m}{2E V}} \left( \begin{array}{c} \chi \\ \sigma \frac{p}{E + m} \chi \end{array} \right) e^{i p \cdot x} e^{-i E t}$$

with $E = \sqrt{p^2 + m^2}$, and $\chi = \begin{pmatrix} a \\ b \end{pmatrix}$ a constant spinor with $\chi^\dagger \chi = 1$. This wave with positive energy is impinging on a temporal beam splitter located in region $B$ at $t = t_B$ (see Figs. 2(b-f)). The temporal beam splitter at $B$ (respectively, $A$) is a space-time domain limited by two hyperplanes at times $t = t_B,A$ and $t = t_B,A + \tau$ ($\tau$ is a delay) and where exists a constant magnetic potential $A$ vanishing outside the domain. These temporal beam splitters are mathematically equivalent to temporal Fabry–Perot cavities: The two temporal boundaries are analogous to the two interfaces of a usual Fabry–Perot cavity but here in time not in space! The analogy is useful for calculating the transmission and reflection coefficients.

We stress that the words ‘transmission’, ‘reflection’ (like ‘scattering’) are usually considered to discuss propagation through a time-like boundary as an interface between two media. Here we consider space-like boundaries and still use the same concept. This is fully justified since the formal and mathematical development shows a clear equivalence and analogy between these two different physical scattering processes (this is derived in 5). As shown in Fig. 2b, the wave can be reflected as a negative energy electron $r e^{-i 2E t_B} \psi_{-E,p}(t, x)$ propagating backward in time (again the meaning is clear from the point of view of the Green function $S_F(x - x')$ implying a retrocausal link). Here, $r := r_{tot}(E, E')$ is the total Fresnel reflection coefficient for an electron with positive energy moving forward in time, and the phase term $e^{-i 2E t_B}$ occurs because $r$ is defined for a cavity located between time $t = 0$ and $t = \tau$ as shown in Fig. 3. The wave is directed into the direction of a second identical temporal beam splitter located at time $t_A$, where it is again reflected forward in time as $r' r e^{-i 2E(t_B - t_A)} \psi_{E,p}(t, x)$ ($r' := r_{tot}(E, E') = -r$ is the reflection coefficient for an electron with negative energy $-E$ moving backward in time in agreement with the

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3 The zigzag diagram represents indeed a single particle wave function and it could look at first as if we have three particles (two particles and one antiparticle) at times between the beginning and end of the process. However, this picture doesn’t alter the ‘one particle interpretation of the wave function $\psi(x)$ of Eq. 7 during scattering processes where we are only interested to what is happening asymptotically before and after the interaction and where only one particle is present in the diagram of Fig. 2a.

4 The mode volume $V$ is associated with the finite spatial extension of a quasi manochromatic wavepacket

5 See the supplementary material for this article available online.

6 $E' = \sqrt{q^2 + m^2}$ is the energy of the electron mode inside the temporal beam splitter where the wave vector is $q = p - eA$. 

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Fig. 3 The temporal beam splitter acting as a temporal Fabry–Perot cavity for a single electron with incident positive (a) or negative (b) energy. The method to compute the Fresnel coefficients of this cavity is to add the different contributions associated with multiple reflections and transmissions. The method is similar to the one used in optics but here the cavity is located in time not in space.

use of $S_F(x - x')$ implying retrocausal links). The accumulated phase $e^{-i2E(t_B - t_A)}$ is reminiscent of Feynman’s zigzag diagram [22]. As shown in Fig. 2c, there is a second channel where the electron is directly transmitted by the $B-$cavity as a positive energy wave $t\psi_E, P(t, x)$.

The Fabry–Perot cavities/beam splitters are characterized by four coefficients $r, r', t, t'$ that are calculated by usual means (see lengthy technical details in 5**). In Fig. 3, we show the two configurations where the wave is either (a) incident from the region $t < 0$ with a positive energy, or (b) from the region $t > \tau$ with a negative energy (Fig. 3 considers a cavity confined between time $t = 0$ and $t = \tau$). The usual method for calculating the Fresnel coefficients is based on an iterative approach often applied in optics and related to Feynman diagrammatic. In particular, we have for the reflectivity $R = |r|^2 = |r'|^2 = R'$ and transmission $T = |t|^2 = |t'|^2 = T'$ with $R + 1 = T$. The numerical properties of the temporal beam splitters are considered in more details in 5**. In this context it is interesting to observe that physically our temporal beam splitter is an idealization corresponding to infinite slopes at times $t = 0$ and $t = \tau$. Infinite slopes actually means that the electric field $E(t) = -\delta A(t)$ equals $E(t) = -A\delta(t)$ near the first space-like interface $t = 0$ and $E(t) = +A\delta(t - \tau)$ near the second space-like interface where $A$ is the constant magnetic potential value in the cavity. This is of course an idealization but we must have $|E| \simeq |A|/\varepsilon$ near $t = 0$ and $t = \tau$ where $\varepsilon \ll \tau$. In other words, we have $|eE| \gg |eA|/\tau$ and consequently if
Once more, the ‘intuitive’ method is completely rigorous and does not lead to contradictions.

\[ m \tau \sim 1 \] (a condition that is required to get large values of \( R > 1 \) as shown numerically in \(^{5}**\) and as needed in this work) we have the constraint

\[ |E| \gg \frac{m^2}{|e|} \quad (11) \]

which is the limit obtained by Schwinger for pair productions in a constant electric field \(^{35}\). This limit for electron/positron pair,  \( \frac{m^2}{|e|} \approx 10^{+18} \text{eV.m}^{-1} \) is an extremely intense field currently unattainable with our technology but supposed to be possible during extreme astrophysical/cosmological events.

The probability for the electron to be reflected (Fig. 2b) is given by \( P_{\text{reflec.}} = P_{v}|rr^\prime|^2 = P_{v}R^2 \) and the one to be transmitted (Fig. 2c) is \( P_{\text{trans.}} = P_{v}|t|^2 = P_{v}(1 + R) \) where \( P_{v} \) is the vacuum to vacuum probability. To have normalization of the probability (as in Eq. 9) we must include two other Feynman diagrams shown in Figs. 2d,e associated with an additional pair creation in either \( A \) or \( B \). The two diagrams interfere and a relative minus sign is introduced to satisfy Pauli’s exclusion principle. The additional probability term is thus \( P_{e^{+}1\text{pair}} = P_{v}|rr^\prime r - r^\prime t^\prime|^2 = P_{v}R|rr^\prime - t^\prime|^2 = P_{v}R^{5**} \). The total probability is the sum

\[ P_{\text{tot.}} = P_{v}(R^2 + 1 + 2R) = P_{v}(1 + R)^2. \quad (12) \]

To prove that \( P_{\text{tot.}} = 1 \) we have to evaluate \( P_{v} \). This is done by considering the 3 alternatives where in absence of the incoming electron (i) nothing happens with probability \( P_{v} \), (ii) a pair is created at \( B \) with probability \( P_{v}|r|^2 = P_{v}R \), or (iii) a pair is created at \( A \) with probability \( P_{v}|t|^2 = P_{v}RT \). The sum of (i-iii) gives

\[ 1 = P_{v}(1 + R + RT) = P_{v}(1 + R)^2 \quad (13) \]

and, therefore, we have the normalization \( P_{\text{tot.}} = 1 \) as required. Note that Feynman’s scattering method has been previously applied to solve the famous Klein paradox \(^{36}\) or the related Schwinger pair production problem \(^{37}\). Once more, the ‘intuitive’ method is completely rigorous and does not lead to contradictions.

To see the impact of the previous analysis on causality we consider the interferometer of Fig. 4a where a single electron (\( E > 0 \)) emitted by the source \( S \) and impinging on a beam splitter \( BS \) is following one of the three paths (i) \( BS < A < D_1 \), (ii) \( BS < M < B < A < D_1 \), or (iii) \( BS < M < B < D_3 \) with \( M \) a mirror and \( D_1 \) detectors. Paths (i) and (ii) interfere and by introducing a phase shift \( e^{ix} \) in (ii) we can modulate the intensity at \( D_1 \) through a retrocausal link involving the cavities \( B, A \). The probability of path (iii) to detect an electron at \( D_3 \) and no particle at \( D_{1,2} \) is given by \( P(D_3, \triangle D_{1,2}) = P_{v}T/2 \) where \( P_{v} \) is as before given by Eq. 13. Paths (i) and (ii) lead to the modulated probability of finding an electron in gate \( D_1 \) and no particle in gates \( D_{2,3} \), i.e.

\[ P_{\theta}(D_1, \triangle D_{2,3}) = \frac{P_{v}}{2}|t - ie^{i(x-2E(\tau-A))}r|^2 \]
with visibility \( V = \frac{2 \sqrt{\gamma}}{t + R^2} \leq 1 \) and phase \( \theta = \chi - 2E(t_B - t_A) + 2 \arg [r] - \arg [r] \) as we show in \(^{5**}\).

This interference illustrates the retrocausal mechanism associated with the presence of zigzag Feynman diagrams in Figs. 2 and 4. In particular the phase can always be tuned to \( \theta_\pm = \pm \pi/2 \) and if \( V = 1 \) (implying the ‘golden ratio’-reflectivity \( R_0 = \frac{1 + \sqrt{5}}{2} \simeq 1.62 \)) we have \( P_{\theta_\pm}(D_1, \varnothing D_{2,3}) = 0 \). Note that the value of \( R_0 \) requires a strong electromagnetic field as mentioned before. Therefore, in this regime the presence of the phase shifter at time \( t \simeq t_B \) strongly retrocausally influences the dynamics of the particle at time \( t_A < t_B \).

Remarkably, this feature which is a direct consequence of Feynman formalism cannot be used to send a signal back to the past. Indeed, Eq. 14 is associated with processes where a single incident electron coming from the source \( S \) is scattered by the interferometer and is subsequently registered in \( D_1 \). However, we neglected ‘3 particles’ Feynman’s diagrams where a pair of particle-antiparticle is created either at \( A \) or \( B \) and interfere with the incident one. Similar to those shown in Fig. 2d,e we obtain three additional interfering Feynman’s graphs that yield the probability for finding an electron at \( D_1 \) and a pair of particle at \( D_{2,3} \) (see \(^{5**}\)):

\[
P_0(D_1, D_2, D_3) = \frac{P_0 R}{2}[2 + R - 2\sqrt{\gamma}\sin \theta].
\] (15)

The \( \theta \)-oscillating term in Eq. 15 exactly compensates the one in Eq. 14 so that the full probability to detect one electron at \( D_1 \) (i.e., obtained by summing Eqs. 14,15) is independent of \( \theta \). An observer at \( D_1 \) not knowing what happens at \( D_{2,3} \) will thus detect a constant probability. This is a form of non-signaling theorem (similar to the one involved in discussing Bell’s theorem \(^{38,39}\)) protecting quantum mechanics from backward in time information transfer. This is imposed by local commutativity and microcausality in QED where local measurements of quantities \( \hat{M}_1 \) and \( \hat{N}_2 \) made at points 1 and 2 must commute, i.e., \( [\hat{M}_1, \hat{N}_2]_\varnothing = 0 \), if they can be connected by a space-like hypersurface passing through 1 and 2. Here, this is so if detectors \( D_1 \) and \( D_{2,3} \) are located on such a space-like hypersurface.

Moreover, like for Bell’s theorem, correlations and postselections are key and can be used to retrodict a backward influence. For this purpose, consider the following game where a fair quantum coin allows us to select randomly between the case \( \theta_- \) and \( \theta_+ \) (with \( V = 1 \)). A single electron is sent through the interferometer and the agent having access to the outcomes at \( D_1 \) must guess the result of the coin tossing and therefore \( \theta \). Calling \( a = \pm \pi/2 \) the guess we introduce an average Gain

\[
\langle G \rangle = \sum_{a,\theta} \delta_{a,\theta} P(a, \theta),
\] (16)

where the joint probability \( P(a, \theta) \) depends on information available to the agent. Classically, i.e., without the retrocausal channel, the agent could not use the data at \( D_1 \) to infer \( \theta \) and therefore she would have to toss a coin to guess yielding the classical bound

\[
\langle G \rangle_{\text{clas.}} \leq \frac{1}{4} \sum_{a,\theta} \delta_{a,\theta} = \frac{1}{2}.
\] (17)

Moreover, as we show in \(^{5**}\), in our retrocausal quantum model we obtain

\[
\langle G \rangle_{\text{quant.}} = \frac{3}{4} - \frac{R_0}{4(1 + R_0)^2} \simeq 0.69 > \frac{1}{2}.
\] (18)

Note that all data and correlations have been used and our causal inequality Eq. 17 is clearly violated. Furthermore, by postselecting only on those events where a single detection at \( D_1 \) occurs and none at \( D_{2,3} \) we can increase the gain to reach the maximal value \( \langle G \rangle_{D_1, \varnothing D_{2,3}} := \sum_{a,\theta} \delta_{a,\theta} P(\theta|D_1, \varnothing D_{2,3}) = 1 \) (see \(^{5**}\) for details).

It is important to realize that the results discussed before are far from being obvious. For instance: Already at a classical level and independently of our QED proposal an agent \( A \) at time \( t_A \) trying to guess the result of a quantum coin tossing done at time \( t_B > t_A \) can apparently lead to a retrodictive/retrocausal scenario if we only consider

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a subset of the data where the predictions of $\mathcal{A}$ agree with the results of $\mathcal{B}$. In our scenario what is remarkable is that the probability $P_B(D_1, \varnothing D_{2,3})$ for a single electron reaching $D_1$ actually depends on the phase $\theta$ selected at a time $t_B > t_A$. This feature is in our formulation a direct consequence of the retrocausal Feynman channel needed for evaluating the single-electron wave function $\psi(x)$ as defined by Eq. 7. But once again, like for Bell’s theorem showing nonlocality in the analysis of correlations here the retrocausality is only available if we consider the correlations between the detectors since $P(D_1)$ obtained by summing $P_B(D_1, \varnothing D_{2,3})$ with $P_B(D_1, D_2, D_3)$ is independent of $\theta$.

It is important to observe that Feynman’s description involving electrons with negative energy moving backward in time (driven by the specific choice for the Green propagator $S_F(x - x')$) is not the only one that could be used. Alternatively, we could go back to the original Dirac see picture where the single electron created initially in source $S$ is moving on top of the filled Dirac’s sea $|\varnothing\rangle$. The initial state is $\tilde{F}^\dagger|\varnothing\rangle$ and during the interaction with the interferometer this many-particle state will evolve nonlocally to create an influence of $D_1$ on the detectors $D_2, D_3$. The probabilities $P_B(D_1, \varnothing D_{2,3}), P_B(D_1, D_2, D_3)$ and $P(D_3, \varnothing D_{1,2})$ are given by the same formula as before but now the retrocausal influence associated with the zigzag path is replaced by a non local one\(^7\). Retrictively this will look as retrocausality. Again, this explanation agrees with results concerning Bell’s theorem where to explain the violation of Bell’s inequality one must relax one of the assumption associated with local-causality. Nonlocality and retrocausality appear as alternatives in this discussion.

We stress that the relation between Bell’s theorem nonlocality and retrocausality in quantum mechanics is well established (for good reviews see for example [40–43]). The key idea to use retrocausality in connection with Feynman QED goes back to Costa de Beauregard in the 1950s [44,45] and this has been associated with interesting interpretations of quantum mechanics like the transactional interpretation of Cramer [46]. In the same context, several important studies have been inspired by Feynman work to develop a time symmetric version of QED (e.g., the works of Davies [47,48] and Hoyle [49]) in relation with Feynman’s path integrals. All these ideas strongly motivated the present work focussing on causality issues in QED. Moreover, the idea of using some form of scattering in time involving boundary conditions in the past and future is also discussed in the work of Schulman [50] and most importantly here in the work of Aharonov, Vaidman and coworkers [32,33,51,52] where a two-state/two-time formalism has been developed based on the concept of weak value. As we mentioned after Eq. 7 it is remarkable that the wave function in Feynman’s formulation of QED is actually a kind of weak value involving a two-time state. This emphasizes the importance of this formulation for understanding causal issues in QED. Finally in relation with the two-time formalism we mention that retrocausal connections with particles ‘going backward in time’ have bee used in the context of Bohmian mechanics where particles follow trajectories doing zig-zag in space-time [53].

3 Discussion concerning indefinite causal orders and perspectives

Going back to the issue of ICO, the previous analysis allows us to develop genuine versions of QS involving only two times regions $\mathcal{A}, \mathcal{B}$ as required in Fig. 1(a). In the example of Fig. 4(b), a single electron emitted by the source $S$, moving through an interferometer and reaching a detector $D_2$ or $D_3$ follows either a ‘normal’ path $BS_1 < M_2 < \mathcal{A} < B < M_3 < D_{2,3}$ or the zig-zag one $BS_1 < M_1 < B < \mathcal{A} < M_4 < D_{2,3}$. Unitaries $\hat{U}_A, \hat{U}_B$ acting on the bi-spinor quantum state of the electron $\psi_0$ are inserted in the paths joining $\mathcal{A}, B$ and the interference at $D_2$ and $D_3$ leads to ICO. The probability for detecting a single electron at $D_2$ and nothing at $D_{3,1}$ is

$$P(D_2, \varnothing D_{3,1}) = \frac{P}{4} \sum_s \| t^2 C_s A < B - r^2 e^{i\xi} C_s B < A \|^2$$

(19)

\(^7\) As examples of other alternative quantum formalisms we could consider a more usual quantum field formalism for the scattering matrix or use Feynman path integrals. In the end the probabilities obtained in Eqs. 14 and 15 are associated with observables and are independent of the method considered.
Fig. 5 Sketch of a QS involving a retrocausal path involving temporal beam splitters \( A, B \) and unitaries \( \hat{U}_A, \hat{U}_B \) (compare with Fig. 4b). Note that the two counterpropagating paths between the cavities \( A \) and \( B \) are superposed in Fig. 4b

with \( \xi = \chi - 2E(t_B - t_A) \) and here \( P'_0(1 + R)^4 = 1 \) because we need two counterpropagating electron modes. The amplitudes \( C^A_{s \prec B}, C^B_{s \prec A} \) (\( s = \pm \frac{1}{2} \) are spin states) depend on the order of the unitaries \( \hat{U}_A, \hat{U}_B \) applied on \( \psi_0 \) as we show in 5**.

Remarkably, unlike previous proposals this QS exemplifies ICO but with definite time order. This is more clearly visible if we draw a causal sketch like the one of Fig. 1 but here with the time along the vertical axis. The electron can move along the blue retrocausal path (associated with the control state \( \phi_0 \)) or along the red ‘normally-causal’ path (associated with the control state \( \phi_1 \)). Interactions of the single electron with devices located in the interferometer are made locally and there is no ambiguity concerning the notion of time in this implementation of the QS. This shines some new lights concerning the distinction between time and causal order in relativistic quantum mechanics. More precisely, going back to the introduction we see that the operational definition of time used in the QS involves the distinction between \( \hat{A} \hat{B} \psi_0 \) and \( \hat{B} \hat{A} \psi_0 \) for noncommuting operators applied on a target state \( \psi_0 \). This operational time has in principle no reason to be identical to the ‘physical’ time associated with external clock. Our proposal involving retrocausal Feynman diagram shows that if local operations \( \hat{A}_{t,A}, \hat{B}_{t,A} \) are tagged or labeled by a clock-time \( t_A, t_B \) we can still define a QS leading to ICO with well defined clock-time. Of course, this does not invalidate the results obtained previously (e.g.,[16–19]) where the possibility to define experimentally the which order information is prohibited (if we do not want to destroy the coherence in the QS). Similarly, the present proposal doesn’t conflict with proposals involving quantum gravity [2,12–14]. Better, it would be interesting to see the connections between the different scenarios since high electromagnetic fields are supposed to exist near black holes where quantum mechanics (and perhaps quantum gravity) could play an important role.

Moreover, our analysis also impacts the discussion of causal inequalities that are usually non violated with QS [3,25,27] at least if we assume no-backward causation [54]. But this is precisely the condition that we relax here and as a consequence causal inequalities can be violated even with a definite time order.

Consider for example the standard bipartite ‘guess your neighbor’s input’ (GYNI) game [3,25,55] where Alice and Bob with their respective (input) quantum bits with results \( x, y \) (and probabilities \( P(x) = P(y) = \frac{1}{2} \)) try to guess each other’s input. The gain in this game with outputs \( a, b \) is thus [25]

\[
\langle G \rangle = \frac{1}{4} \sum_{a,b,x,y} \delta_{a,y} \delta_{b,x} P(a, b|x, y),
\]

where the conditional probability \( P(a, b|x, y) \) depends on causal rules. Suppose that Alice’s lab \( A \) is in the past light cone of Bob’s lab \( B \) we know that Alice can communicate with certainty her input \( x \) to Bob (i.e., \( b = x \)) but Bob cannot and Alice must guess \( y \) using a random number so that \( P(a, b|x, y) \leq \delta_{b,x} \frac{1}{2} \) leading to the causal bound \( \langle G \rangle_{\text{caus.}} \leq \frac{1}{2} \). However, if Bob uses a single electron with a well controlled bispinor state \( \psi_0 \) he can send information backward in time to Alice by acting unitarily on \( \psi_0 \) and using a zigzag channel like the one of Fig. 2b (Alice would have to analyze the bispinor state). As we saw the probability of this single electron channel requiring
postselection is $P_{\text{reflec.}} = P_e R^2$ and we now have $P(a, b|x, y) = \delta_{b, x} \delta_{a, y} P_{\text{reflec.}}$ yielding

$$
\langle G \rangle_{\text{retrocaus.}} = P_{\text{reflec.}} = \frac{R^2}{(1 + R)^2}.
$$

Furthermore, if $R \geq R_1 := 1 + \sqrt{2}$ we have $\langle G \rangle_{\text{retrocaus.}} \geq \frac{1}{2}$ violating the causal inequality. By studying numerically $^5$ $R$ as a function of the external potential $A$ and input particle energy $E$ we can reach the value $\langle G \rangle_{\text{retrocaus.}} \simeq 0.98$. We stress that in this work values for $R \gtrsim R_0, R_1$ require very strong external field beyond the Schwinger limit $^{35} 10^{18}$ V/m during times of the order of the Compton period $\sim m^{-1} 5^{**}$. As mentioned before, in the present knowledge these fields could be attainable in extreme physical phenomena like supernova or blackholes.

At a more pragmatic level it is interesting to point out that QED with electrons and positrons can be partially mimicked using condensed matter physics where electron/hole pairs replace electron/positron pairs. For example, with graphene $^{56}$ we can define a two-dimensional Dirac’s equation for electrons where the velocity of light is replaced by the Fermi velocity $v_F \simeq 8 \times 10^5$ m/s. Klein tunneling associated with the Klein paradox has been discussed in this context $^{56}$. Moreover, as we mentioned the Klein paradox was consistently analyzed using the Feynman approach $^{36}$. Therefore, we can speculate that the results we obtained in the present work could be implemented at low energy as well using state-of-the-art condensed matter physics with graphene or other exotic materials.

To conclude this analysis we can go back to the controversy discussed in the introduction of this article. The usual approach for discussing ICO is to consider that the QS can be implemented without postselection. To define the switch, and to perform any protocol that witnesses its indefinite order, one has to retain the control system and make measurements on it, keeping all the outcomes. Here we do not follow this strategy since we strongly rely on correlations and still we can define operations that have similar properties as in the usual QS. The strategy used in our work is however different from the postselection discussed extensively in $^{33,57,58}$ that do not consider retrocausal relativistic propagators like $S_F(x - x')$. Moreover, we stressed that we must be careful when using these correlations to avoid problems with violation of non-signaling and microcausality. This should be compared with other remarkable results in quantum mechanics such as the quantum delayed-choice quantum eraser $^{59,60}$ or entanglement swapping $^{61}$ or more recently delayed-choice causal order $^{58}$ that however, unlike our work, don’t rely on the ‘retrocausal’ Feynman propagator $S_F(x - x')$. Still the different approaches are apparently showing quite similar things: Retrocausality can be invoked to explain such quantum experiments (e.g., using the transactional interpretation $^{46}$). Moreover, in these last examples postselection is a key element (whereas in our proposal correlations ‘à la Bell’ are more central): Filtering data in a specific way can indeed be interpreted as an evidence for retrocausal links. However, as it is also well known e.g., with the delayed-choice experiment (see the discussions in $^{60,62,63}$) such a backward in time causality is not necessary and we can invoke retrodiction without requiring retrocausality. Further work should be done to consider the relations between the different methods of approaching retrocausality in quantum mechanics.

We point out that the definition of the wave function $\psi(x)$ used in Eq. 7 relies on the notion of weak value advocated by Aharonov and Vaidman $^{32}$. Indeed, such a weak value associated with an operator $\hat{A}(t)$ at time $t$ and written in the Heisenberg picture reads $A_w(t) := \langle F | \hat{A}(t) | I \rangle / \langle F | I \rangle$, where $| I \rangle$ is an initial state and $| F \rangle$ a final state. This idea of involving two states $| I, F \rangle$ to define physical quantities has been recently advocated in $^{33}$ to translate and understand results obtained with the process matrix formalism $^{3}$ (and playing a fundamental role in many discussions concerning ICO in terms of two-state correlations involving $| I, F \rangle$. Clearly, this suggests a strong link between our methodology also involving a Feynmanian two-state description adapted to relativistic physics and other methods such as the process matrix formalism. We believe that this motivates further studies in this direction.

Declarations

Conflict of interest The author declares no competing interest and no conflict of interest for this work.
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