Non-Critical String Theory Formulation of Microtubule Dynamics and Quantum Aspects of Brain Function

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Abstract

Microtubule (MT) networks, subneural paracrystalline cytoskeletal structures, seem to play a fundamental role in the neurons. We cast here the complicated MT dynamics in the form of a 1 + 1-dimensional non-critical string theory, thus enabling us to provide a consistent quantum treatment of MTs, including enviromental friction effects. We suggest, thus, that the MTs are the microsites, in the brain, for the emergence of stable, macroscopic quantum coherent states, identifiable with the preconscious states. Quantum space-time effects, as described by non-critical string theory, trigger then an organized collapse of the coherent states down to a specific or conscious state. The whole process we estimate to take $O(1\text{sec})$, in excellent agreement with a plethora of experimental/observational findings. The microscopic arrow of time, endemic in non-critical string theory, and apparent here in the self-collapse process, provides a satisfactory and simple resolution to the age-old problem of how the, central to our feelings of awareness, sensation of the progression of time is generated.

ACT-09/95
CTP-TAMU-24/95
ENSLAPP-A-524/95
May 1995

\hspace{1cm}\diamond On leave from P.P.A.R.C. Advanced Fellowship, Dept. of Physics (Theoretical Physics), University of Oxford, 1 Keble Road, Oxford OX1 3NP, U.K.
1 Introduction

The interior of living cells is structurally and dynamically organized by cytoskeletons, i.e. networks of protein polymers. Of these structures, MicroTubules (MT) appear to be the most fundamental. Their dynamics has been studied recently by a number of authors in connection with the mechanism responsible for dissipation-free energy transfer. Recently, Hameroff and Penrose have conjectured another fundamental rôle for the MT, namely being responsible for quantum computations in the human brain, and, thus, related to the consciousness of the human mind. The latter is argued to be associated with certain aspects of quantum theory that are believed to occur in the cytoskeleton MT, in particular quantum superposition and subsequent collapse of the wave function of coherent MT networks. While quantum superposition is a well-established and well-understood property of quantum physics, the collapse of the wave function has been always enigmatic. We propose here to use an explicit string-derived mechanism - in one interpretation of non-critical string theory - for the collapse of the wave function, involving quantum gravity in an essential way and solidifying previous intuitively plausible suggestions. It is an amazing surprise that quantum gravity effects, of order of magnitude $G_N^{1/2} m_p \sim 10^{-19}$, with $G_N$ Newton’s gravitational constant and $m_p$ the proton mass, can play a rôle in such low energies as the eV scales of the typical energy transfer that occurs in cytoskeletons. However, as we show in this article, the fine details of the MT characteristic structure indicate that not only is this conceivable, but such scenaria lead to order of magnitude estimates for the time scales entering conscious perception that are close enough to those conjectured/“observed” by neuroscientists, based on completely different grounds.

To understand how quantum space-time effects can affect conscious perception, we mention that it has long been suspected that large scale quantum coherent phenomena can occur in the interior of biological cells, as a result of the existence of ordered water molecules. Quantum mechanical vibrations of the latter are responsible for the appearance of ‘phonons’ similar in nature to those associated with superconductivity. In fact there is a close analogy between superconductivity and energy transfer in biological cells. In the former phenomenon electric current is transferred without dissipation in the surface of the superconductor. In biological cells, as we shall discuss later on, energy is transferred through the cell without loss, despite the existence of frictional forces that represent the interaction of the cell with the surrounding water molecules. Such large scale quantum coherent states can maintain themselves for up to $O(1 \text{ sec})$, without significant environmental entanglement. After that time, the state undergoes self-collapse, probably due to quantum gravity effects. Due to quantum transitions between the different states of the quantum system of MT in certain parts of the human brain, a sufficient distortion of the surrounding space-time occurs, so that a microscopic (Planck size) black hole is formed. Then collapse is induced, with a collapse time that depends on the order of magnitude of the number $N$ of coherent microtubulins. It is esti-
mated that, with an $N = O[10^{12}]$, the collapse time of $O(1\text{ sec})$, which appears to be a typical time scale of conscious events, is achieved. Taking into account that experiments have shown that there exist $N = 10^8$ tubulins per neuron, and that there are $10^{11}$ neurons in the brain, it follows that that this order of magnitude for $N$ refers to a fraction $10^{-7}$ of the human brain, which is very close to the fraction believed responsible for human perception.

The self-collapse of the MT coherent state wave function is an essential step for the operation of the MT network as a quantum computer. In the past it has been suggested that MT networks processed information in a way similar to classical cellular automata (CCA) \[9\]. These are described by interacting Ising spin chains on the spatial plane obtained by filing open and flattening the MT cylindrical surface. Distortions in the configurations of individual parts of the spin chain can be influenced by the environmental spins, leading to information processing. In view of the suggestion \[2\] on viewing the conscious parts of the human mind as quantum computers, one might extend the concept of the CCA to a quantum cellular automaton (QCA), undergoing wave function self-collapses due to (quantum gravity) environmental entanglement.

An interesting and basic issue that arises in connection with the above rôle of the brain as a quantum computer is the emergence of a direction in the flow of time (arrow). The latter could be the result of successive self-collapses of the system’s wave function. In a recent series of papers \[4\] we have suggested a rather detailed mechanism by which an irreversible time variable has emerged in certain models of string quantum gravity. The model utilized string particles propagating in singular space-time backgrounds with event horizons. Consistency of the string approach requires conformal invariance of the associated $\sigma$-model, which in turn implies a coupling of the backgrounds for the propagating string modes to an infinity of global (quasi-topological) delocalized modes at higher (massive) string levels. The existence of such couplings is necessitated by specific coherence-preserving target space gauge symmetries that mix the string levels \[4\].

The specific model of ref. \[4\] is a completely integrable string theory, in the sense of being characterised by an infinity of conserved charges. This can be intuitively understood by the fact that the model is a $(1+1)$-dimensional Liouville string, and as such it can be mapped to a theory of essentially free fermions on a discretized world sheet (matrix model approach \[10\]). A system of free fermions in $(1 + 1)$ dimensions is trivially completely integrable, the infinity of the conserved charges being provided by appropriate moments of the fermion energies above the Fermi surface. Of course, formally, the precise symmetries of the model used in ref. \[4\] are much more complicated \[11\], but the idea behind the model’s integrability is essentially the above. It is our belief that this quantum integrability is a very important feature of theories of space-time associated with the time arrow. In its presence, theories
with singular backgrounds appear consistent as far as maintainance of quantum coherence is concerned. This is due to the fact that the phase-space density of the field theory associated with the matter degrees of freedom evolves with time according to the conventional Liouville theorem

$$\partial_t \rho = -\{\rho, H\}_{PB}$$

as a consequence of phase-space volume-preserving symmetries. In the two-dimensional example of ref. [4], these symmetries are known as $W_\infty$, and are associated with higher spin target-space states [11]. They are responsible for string-level mixing, and hence they are broken in any low-energy approximation. If the concept of ‘measurement by local scattering experiments’ is introduced [4], it becomes clear that the observable background cannot contain such global modes. The latter have to be integrated out in any effective low-energy theory. The result of this integration is a non-critical string theory, based on the propagating modes only. Its conformal invariance on the world sheet is restored by dressing the matter backgrounds by the Liouville mode $\phi$, which plays the role of the time coordinate. The $\phi$ mode is a dynamical local world-sheet scale [4], flowing irreversibly as a result of certain theorems of the renormalization group of unitary $\sigma$-models [12]. In this way time in target space has a natural arrow for very specific stringy reasons.

Given the suggestion of ref. [2] that space-time environmental entanglement could be responsible for conscious brain function, it is natural to examine the conditions under which our theory [4] can be applied. Our approach utilizes extra degrees of freedom, the $W_\infty$ global string modes, which are not directly accessible to local scattering ‘experiments’ that make use of propagating modes only. Such degrees of freedom carry information, in a similar spirit to the information loss suggested by Hawking [13] for the quantum-black-hole case. For us, such degrees of freedom are not exotic, as suggested in ref. [14], but appear already in the non-critical String Universe [1, 15], and as such they are considered as ‘purely stringy’. In this respect, we believe that the suggested model of consciousness, based on the non-critical-string formalism of ref. [4], is physically more concrete. The idea of using string theory instead of point-like quantum gravity is primarily associated with the fact that a consistent quantization of gravity is at present possible only within the framework of string theory, so far. However, there are additional reasons that make advantageous a string formalism. These include the possibility of construction of a completely integrable model for MTs, and the Hamiltonian representation of dynamical problems with friction involved in the physics of MTs. This leads to the possibility of a consistent (mean field) quantization of certain soliton solutions associated with the energy transfer mechanism in biological cells.

According to our previous discussion emphasizing the importance of strings, it is imperative that we try to identify the completely integrable system underlying MT networks. Thus, it appears essential to review first the classical model for energy
transfer in biological systems associated with MT. This will allow the identification of the analogue of the (stringy) propagating degrees of freedom, which eventually couple to quantum (stringy) gravity and to global environmental modes. As we shall argue in subsequent sections, the relevant basic building blocks of the human brain are one-dimensional Ising spin chains, interacting among themselves in a way so as to create a large scale quantum coherent state, believed to be responsible for preconscious behaviour in the model of [2]. The system can be described in a world-sheet conformal invariant way and is unitary. Coupling to gravity generates deviations from conformal invariance which lead to time-dependence, by identifying time with the Liouville field on the world sheet. The situation is similar to the environmental entanglement of ref. [16, 17]. Due to this entanglement, the system of the propagating modes opens up as in Markov processes [18]. This leads to a dynamical self-collapse of the wave function of the MT quantum coherent network. In this way, the part of the human brain associated with consciousness generates, through successive collapses, an arrow of time. The interaction among the spin chains, then, provides a mechanism for quantum computation, resembling a planar cellular automaton. Such operations sustain the irreversible flow of time.

The structure of the article is as follows: in section 2 we discuss a model used for the physical description of a MT, and in particular for a simulation of the energy transfer mechanism. The model can be expressed in terms of a 1 + 1-dimensional classical field, the projection of the displacement field of the MT dimers along the tubulin axis. There exists friction due to interaction with the environment. However, the theory possesses travelling-wave solitonic states responsible for loss-free transfer of energy. In section 3, we give a formal representation of the above system as a 1 + 1-dimensional $c = 1$ Liouville (string) theory. The advantage of the method lies in that it allows for a canonical quantization of the friction problem, thereby yielding a model for a large-scale coherent state, argued to simulate the preconscious state. There is no time arrow in the above system. In section 4, we discuss our mechanism of introducing a dynamical time variable with an arrow into the system, by elevating the above $c = 1$ Liouville theory to a $c = 26$ non-critical string theory, incorporating quantum gravity effects. Such effects arise from the distortion of space-time due to abrupt conformational changes in the dimers. Such a coupling leads to a breakdown of the quantum coherence of the preconscious state. Estimates of the collapse times are given, with the result that in this approach conscious perception of a time scale of $\mathcal{O}(1 \text{ sec})$, is due to a $10^{-7}$ part of the total brain. In section 5 we briefly discuss growth of a MT network in our framework, which would be the analogue of a non-critical string driven inflation for the effective one-dimensional universe of the MT dimer degrees of freedom. We view MT growth as an out-of-equilibrium one-dimensional spontaneous-symmetry breaking process and discuss the connection of our approach to some elementary theoretical models with driven diffusion that could serve as prototypes for the phenomenon. Conclusions and outlook are presented in section 6. We discuss some technical aspects of our approach in two Appendices.
2  Physical Description of the Microtubules

In this section we review certain features of the MT that will be useful in subsequent parts of this work. MT are hollow cylinders (cf Fig 1) comprised of an exterior surface (of cross-section diameter 25 nm) with 13 arrays (protofilaments) of protein dimers called tubulins. The interior of the cylinder (of cross-section diameter 14 nm) contains ordered water molecules, which implies the existence of an electric dipole moment and an electric field. The arrangement of the dimers is such that, if one ignores their size, they resemble triangular lattices on the MT surface. Each dimer consists of two hydrophobic protein pockets, and has an electron. There are two possible positions of the electron, called $\alpha$ and $\beta$ conformations, which are depicted in Fig. 2. When the electron is in the $\beta$-conformation there is a $29^\circ$ distortion of the electric dipole moment as compared to the $\alpha$ conformation.

In standard models for the simulation of the MT dynamics, the ‘physical’ degree of freedom - relevant for the description of the energy transfer - is the projection of the electric dipole moment on the longitudinal symmetry axis ($x$-axis) of the MT cylinder. The $29^\circ$ distortion of the $\beta$-conformation leads to a displacement $u_n$ along the $x$-axis, which is thus the relevant physical degree of freedom. This way, the effective system is one-dimensional (spatial), and one has a first indication that quantum integrability might appear. We shall argue later on that this is indeed the case.

Information processing occurs via interactions among the MT protofilament chains. The system may be considered as similar to a model of interacting Ising chains on a triangular lattice, the latter being defined on the plane stemming from filing open and flattening the cylindrical surface of Fig. 1. Classically, the various dimers can occur in either $\alpha$ or $\beta$ conformations. Each dimer is influenced by the neighboring dimers resulting in the possibility of a transition. This is the basis for classical information processing, which constitutes the picture of a (classical) cellular automaton.

The quantum computer character of the MT network results from the assumption that each dimer finds itself in a superposition of $\alpha$ and $\beta$ conformations [4]. There is a macroscopic coherent state among the various chains, which lasts for $O(1 \text{sec})$ and constitutes the ‘preconscious’ state. The interaction of the chains with (stringy) quantum gravity, then, induces self-collapse of the wave function of the coherent MT network, resulting in quantum computation.

In what follows we shall assume that the collapse occurs mainly due to the interaction of each chain with quantum gravity, the interaction from neighboring chains being taken into account by including mean-field interaction terms in the dynamics of the displacement field of each chain. This amounts to a modification of the effective potential by anharmonic oscillator terms. Thus, the effective system under study is two-dimensional, possessing one space and one time coordinate. The precise
meaning of ‘time’ in our model will be clarified when we discuss the ‘non-critical string’ representation of our system.

Let \( u_n \) be the displacement field of the \( n \)-th dimer in a MT chain. The continuous approximation proves sufficient for the study of phenomena associated with energy transfer in biological cells, and this implies that one can make the replacement

\[
u_n \rightarrow u(x, t)
\]

with \( x \) a spatial coordinate along the longitudinal symmetry axis of the MT. There is a time variable \( t \) due to fluctuations of the displacements \( u(x) \) as a result of the dipole oscillations in the dimers. At this stage, \( t \) is viewed as a reversible variable. The effects of the neighboring dimers (including neighboring chains) can be phenomenologically accounted for by an effective double-well potential

\[
U(u) = -\frac{1}{2}Au^2(x, t) + \frac{1}{4}Bu^4(x, t)
\]

with \( B > 0 \). The parameter \( A \) is temperature dependent. The model of ferroelectric distortive spin chains of ref. \[20\] can be used to give a temperature dependence

\[
A = -|cost|(T - T_c)
\]

where \( T_c \) is a critical temperature of the system, and the constant is determined phenomenologically \[19\]. In realistic cases the temperature \( T \) is very close to \( T_c \), which for the human brain is taken to be the room temperature \( T_c = 300K \). Thus, below \( T_c \) \( A > 0 \). The important relative minus sign in the potential \[3\], then, guarantees the necessary degeneracy, which is necessary for the existence of classical solitonic solutions. These constitute the basis for our coherent-state description of the preconscious state.

Including a phenomenological kinetic term for the dimers, each having a mass \( M \), one can write down a Hamiltonian \[19\]

\[
H = kR_0^2(\partial_x u)^2 - M(\partial_t u)^2 - \frac{1}{2}Au^2 + \frac{1}{4}Bu^4 + qEu
\]

where \( k \) is a stiffness parameter, \( R_0 \) is the equilibrium spacing between adjacent dimers, \( E \) is the electric field due to the ‘giant dipole’ representation of the MT cylinder, as suggested by the experimental results \[19\], and \( q = 18 \times 2e \) (\( e \) the electron charge) is a mobile charge. The spatial-derivative term in \( \[3\] \) is a continuous approximation of terms in the lattice Hamiltonian that express the effects of restoring strain forces between adjacent dimers in the chains \[19\].
The effects of the surrounding water molecules can be summarized by a viscous force term that damps out the dimer oscillations,

\[ F = -\gamma \partial_t u \]  

with \( \gamma \) determined phenomenologically at this stage. This friction should be viewed as an environmental effect, which however does not lead to energy dissipation, as a result of the non-trivial solitonic structure of the ground-state and the non-zero constant force due to the electric field. This is a well known result, directly relevant to energy transfer in biological systems \[8\]. The modified equations of motion, then, read

\[ M \frac{\partial^2 u}{\partial t^2} - kR_0^2 \frac{\partial^2 u}{\partial x^2} - Au + Bu^3 + \gamma \frac{\partial u}{\partial t} - qE = 0 \]  

According to ref. \[8\] the importance of the force term \( qE \) lies in the fact that eq (7) admits displaced classical soliton solutions with no energy loss. The solution acquires the form of a travelling wave, and can be most easily exhibited by defining a normalized displacement field

\[ \psi(\xi) = \frac{u(\xi)}{\sqrt{A/B}} \]  

where,

\[ \xi \equiv \alpha(x - vt) \quad \alpha \equiv \sqrt{\frac{|A|}{M(v_0^2 - v^2)}} \]  

with

\[ v_0 \equiv \sqrt{k/MR_0} \]  

the sound velocity, of order 1 km/sec, and \( v \) the propagation velocity to be determined later. In terms of the \( \psi(\xi) \) variable, equation (7) acquires the form of the equation of motion of an anharmonic oscillator in a frictional environment

\[ \psi'' + \rho \psi' - \psi^3 + \psi + \sigma = 0 \]  

\[ \rho \equiv \gamma v \sqrt{M|A|(v^2 - v_0^2)^{-\frac{3}{2}}} \quad \sigma = q\sqrt{|B|A|^{-3/2}}E \]  

which has a unique bounded solution \[19\]

\[ \psi(\xi) = a + \frac{b - a}{1 + e^{\frac{-b \xi}{\sqrt{2}}} \sigma} \]  

with the parameters \( b, a \) and \( d \) satisfying:

\[ (\psi - a)(\psi - b)(\psi - d) = \psi^3 - \psi - \left( \frac{q\sqrt{B|A|^{-3/2}}}{E} \right) \]  

Thus, the kink propagates along the protofilament axis with fixed velocity

\[ v = v_0\left[1 + \frac{2\gamma}{9d^2Mv_0^2}\right]^{-\frac{1}{2}} \]
This velocity depends on the strength of the electric field $E$ through the dependence of $d$ on $E$ via (13). Notice that, due to friction, $v \neq v_0$, and this is essential for a non-trivial second derivative term in (11), necessary for wave propagation. For realistic biological systems $v \simeq 2\text{m/sec}$. With a velocity of this order, the travelling waves of kink-like excitations of the displacement field $u(\xi)$ transfer energy along a moderately long microtubule of length $L = 10^{-6}\text{m}$ in about

$$t_T = 5 \times 10^{-7}\text{sec}$$

(15)

This time is very close to Frohlich’s time for coherent phonons in biological systems. We shall come back to this issue later on.

The total energy of the solution (12) is easily calculated to be (19)

$$E = \int_{-\infty}^{+\infty} dx H = \frac{2\sqrt{2} A^2}{3B} + \frac{kA}{B} + \frac{1}{2}M^*v^2 \equiv \Delta + \frac{1}{2}M^*v^2$$

(16)

which is conserved in time. The ‘effective’ mass $M^*$ of the kink is given by

$$M^* = \frac{4}{3\sqrt{2}} \frac{MA_0}{R_0B}$$

(17)

The first term of equation (16) expresses the binding energy of the kink and the second the resonant transfer energy. In realistic biological models the sum of these two terms dominate over the third term, being of order of $1eV$ (13). On the other hand, the effective mass in (17) is of order $5 \times 10^{-27}\text{kg}$, which is about the proton mass ($1\text{GeV}$) (!). As we shall discuss later on, these values are essential in yielding the correct estimates for the time of collapse of the ‘preconscious’ state due to our quantum gravity environmental entangling. To make plausible a consistent study of such effects, we now discuss the possibility of representing the equations of motions (11) as being derived from string theory.

Before closing we mention that the above classical kink-like excitations (12) have been discussed so far in connection with physical mechanisms associated with the hydrolysis of GTP (Guanosine-ThreePhosphate) tubulin dimers to GDP (Guanosine-DiPhosphate) ones. Because the two forms of tubulins correspond to different conformations $\alpha$ and $\beta$ above, it is conceivable to speculate that the quantum mechanical oscillations between these two forms of tubulin dimers might be associated with a quantum version of kink-like excitations in the MT network. This is the idea we put forward in the present work. The novelty of our approach is the use of Liouville (non-critical) string theory for the study of the dynamics involved. This is discussed in the next section.
3  Non-Critical (Liouville) String Theory Representation of a MT

It is important to notice that the relative sign (+) between the second derivative and the linear term in $\psi$ in equation (11) are such that this equation can be considered as corresponding to the tachyon $\beta$-function equation of a $(1+1)$-dimensional string theory, in a flat space-time with a dilaton field $\Phi$ linear in the space-like coordinate $\xi$ [21],

$$\Phi = -\rho \xi$$

Indeed, the most general form of a ‘tachyon’ deformation in such a string theory, compatible with conformal invariance is that of a travelling wave [22] $T(x')$, with

$$x' = \gamma_v (x - vt) ; \quad t' = \gamma_v (t - vx)$$

$$\gamma_v \equiv (1 - v^2)^{-1/2}$$

(19)

where $v$ is the propagation velocity. As argued in ref. [22], these translational invariant configurations are the most general backgrounds, consistent with a unique factorisation of the string $\sigma$-model theory on a Minkowski space-time $G_{\mu\nu} = \eta_{\mu\nu}$

$$S = \frac{1}{4\pi \alpha'} \int d^2 z \left[ \partial X^\mu \partial X^\nu G_{\mu\nu}(X) + \Phi(X) R^{(2)} + T(x) \right]$$

(20)

into two conformal field theories, for the $t'$ and $x'$ fields, corresponding to central charges

$$c_v = 1 - 24v^2 \gamma_v^2 ; \quad c_{x'} = 1 + 24\gamma_v^2$$

(21)

In our case (11), the rôle of the space-like coordinate $x'$ is played by $\xi$, and the velocity $v$ is the velocity of the kink. The velocity of light in this effective string model is replaced by the sound velocity $v_0$ (10), and the friction coefficient $\rho$ is expressed in terms of the central charge deficit (21)

$$\rho = \sqrt{\frac{1}{6} (c(\xi) - 1)} = 2\gamma$$

(22)

The space-like ‘boosted’ coordinate $\xi$, thus, plays the rôle of space in this effective/Liouville mode string theory.

The important advantage of formulating the MT system as a $c = 1$ string theory, lies in the possibility of casting the friction problem in a Hamiltonian form. To this end, we now make some comments on the various non-derivative terms in the tachyon potential $V(T) \equiv U(\psi)$ in the target-space effective action [23]. Such terms contributre higher order non-derivative terms in the equations of motion (11). The term linear in $\psi$ is fixed in string theory and normalized (with respect to the second derivative term ) as in (11) [22]. The higher order terms are polynomials in $\psi$ and their coefficients can be varied according to renormalization prescription [23].
The general structure of the tachyon effective action in the target space of the string is, therefore, of the form \[23\]
\[ \mathcal{L} = e^{-\Phi} \sqrt{G} \left[ (T - c)^2 + \hat{L}(\nabla T, \nabla \Phi) \right] \]
where $G_{\mu \nu}$ is a target space metric field, and $c$ is a constant. The linear term is the only universal term, showing the impossibility of finding a stable tachyon background in bosonic string theory, unless it is time dependent.

In our specific model, we have seen that a term cubic in $\psi$ in the equation of motion (11), with a relative minus sign as compared to the linear term, was responsible for the appearance of a kink-like classical solution. Any change in the non-linear terms would obviously affect the structure of the solution, and we should understand the physical meaning of this in our biological system. To this end, we consider a general polynomial in $T$ equation of motion for a static tachyon in $(1+1)$ string theory
\[ T''(\xi) + 2\gamma T'(\xi) = P(T) \]
where $\xi$ is some space-like co-ordinate and $P(T)$ is a polynomial of degree $n$, say. The ‘friction’ term $T'$ expresses a Liouville derivative, since the effective string theory of the displacement field $u$ is viewed as a $c = 1$ matter non-critical string. In our interpretation of the Liouville field as a local scale on the world sheet it is natural to assume that the single-derivative term expresses the non-critical string $\beta$-function, and hence is itself a polynomial $R$ of degree $m$
\[ T'(\xi) = R(T) \]
Using Wilson’s exact renormalization group scheme, we may assume that $R(T) = a_2 T + a_4 T^2$, where $a_2, a_4$ are related to the anomalous dimension and operator product expansion coefficients for the tachyon couplings. The compatibility conditions for the existence of bounded solutions to the equation (24), then, imply the form [24]
\[ P(T) = A_1 + A_2 T + A_3 T^2 + A_4 T^4, \]
with $A_i = f_i(a_2, a_3), i = 1, \ldots 4$. Indeed such equations have been shown [24] to lead to kink-like solutions,
\[ T(\xi) = \frac{1}{2a_4} \{ \text{sgn}(a_2 a_4) a_2 \tanh[\frac{1}{2} a_2 (x - ut)] - a_2 \} \]
where the velocity $u = \frac{A_4 - 3a_2 a_3}{a_4}$. This fact expresses for us a sort of universal behaviour for biological systems. This shows the existence of at least one class of schemes which admit kink-like solutions of the same sort as the ones of Lal [8] for energy transfer without dissipation in cells.

The importance of solutions of the form (26) lies in the fact that they can be derived from a Hamiltonian and, thus, can be quantized canonically [25]. They are connected to the solitons (12) by a Renormalization Scheme change on the world-sheet of the effective string theory, reproducing (11). This amounts to the possibility...
of casting friction problems, due to the Liouville terms, into a Hamiltonian form. This is quite important for the quantization of the kink solution, which will provide one with a concrete example of a large-scale quantum coherent state for the pre-conscious state of the mind. In a pure field-theoretic setting, a quantization scheme has been discussed in ref. [26], using a variational approach by means of squeezed coherent states. There is a vast literature on soliton quantization using approximate WKB methods. We selected this method for our purposes here, because it yields more accurate results than the usual WKB methods of soliton quantization [27], and it seems more appropriate for our purposes here, due to its direct link with coherent ground states. We shall not give details on the derivation but concentrate on the results. We refer the interested reader to the literature [26]. A brief description of the method is provided in Appendix B.

The result of such a quantization was a modified soliton equation for the (quantum corrected) field $C(x,t)$ [26]

$$\partial_t^2 C(x,t) - \partial_x^2 C(x,t) + M^{(1)}[C(x,t)] = 0 \quad (27)$$

with the notation

$$M^{(n)} = e^{\frac{1}{2}(G(x,x,t) - G_0(x,x))} \frac{\partial^2}{\partial z^2} U^{(n)}(z) \mid_{z=C(x,t)} ; \quad U^{(n)} \equiv \frac{d^n U}{dz^n} \quad (28)$$

Above, $U$ denotes the potential of the original soliton Hamiltonian, and $G(x,y,t)$ is a bilocal field that describes quantum corrections due to the modified boson field around the soliton. The quantities $M^{(n)}$ carry information about the quantum corrections, and in this sense the above scheme is more accurate than the WKB approximation [26]. The whole scheme may be thought of as a mean-field-approach to quantum corrections to the soliton solutions. For the kink soliton (26) the quantum corrections (27) have been calculated explicitly in ref. [26], thereby providing us with a concrete example of a large-scale quantum coherent state.

The above results on a consistent quantization of the soliton solutions (26), derived from a Hamiltonian function, find a much more general and simpler application in our Liouville approach [28, 29]. To this end, we first note that the structure of the equation (24), which leads to (26), is generic for Liouville strings, with arbitrary targets. If we view the Liouville mode as a local scale of the renormalization group on the world-sheet [4, 30], one can easily show that for the coupling $g^i$ of any non-marginal deformation $V_i$ of the $\sigma$-model [1], the following Liouville renormalization group equation holds [31, 4]

$$\ddot{g}^i + Qg^i = -\beta^i = -G^{ij}\partial_i C'[g] \quad ; \quad Q^2 = \frac{C[g] - 25}{6} + \ldots \quad (29)$$

where the dot denotes differentiation with respect to the renormalization group Liouville scale $t$, and $\ldots$ denote terms removable by redefinitions of the couplings.

\footnote{For a concise review of the formalism and relevant notation see Appendix A.}


$g^i$ (renormalization scheme changes). The tensor $G^{ij}$ is an inverse metric in field space[12]. Notice in (29) the role of the non-criticality of the string ($Q \neq 0$) as providing a source of friction [4] in the space of fields $g^i$. The non-vanishing renormalization group $\beta$-functions play the role of generalized forces. The functional $C[g]$ is the Zamolodchikov $c$-function, which is constructed out of a particular combination of components of the world-sheet stress tensor of the deformed $\sigma$-model [12, 4].

The existence of friction terms in (29) implies a statistical description of the temporal evolution of the system using classical density matrices $\rho(g^i, t)$ [4]

$$\partial_t \rho = -\{\rho, H\}_{PB} + \beta^i G^{ij} \frac{\partial \rho}{\partial p_j}$$  \hspace{1cm} (30)

where $p_i$ are conjugate momenta to $g^i$, and $G^{ij} \propto \langle V_i V_j \rangle$ is a metric in the space of fields $\{g^i\}$ [12]. The non-Hamiltonian term in (30) leads to a violation of the Liouville theorem (1) in the classical phase space $\{g^i, p_j\}$, and constitutes the basis for a modified (dissipative) quantum-mechanical description of the system [4], upon quantization.

In string theory, summation over world sheet surfaces will imply quantum fluctuations of the string target-space background fields $g^i$ [4, 28]. Canonical quantization in the space $\{g^i\}$ can be achieved, given that the necessary Helmholtz conditions [25] can be shown to hold in the string case [28]. The important feature of the string-loop corrected (quantum) conformal invariance conditions is that they can be derived from a target-space action [32], which schematically can be represented as [25, 28]

$$S = -\int dt \int_0^1 d\tau g^i E_i(t, \tau g, \tau \dot{g}, \tau \ddot{g}) + total \ derivatives$$

$$E_i(t, g, \dot{g}, \ddot{g}) \equiv G_{ij}(\ddot{g}^j + Q \dot{g}^j + \beta^j)$$  \hspace{1cm} (31)

where the tensor $G_{ij}$ is a (quantum) metric [12] in theory space $\{g^i\}$. It is characterized by a specific behaviour [29] under the action of the renormalization group operator $D \equiv \partial_t + \dot{g}^i \partial_i$, $DG_{ij} = Q G_{ij} = \langle V_i V_j \rangle + \ldots$  \hspace{1cm} (32)

where the ... denote diffeomorphism terms in $g$-space, that can be removed by an appropriate scheme choice [29].

In this way, a friction problem (29) can be mapped non-trivially onto a canonically quantized Hamiltonian system, in similar spirit to the solitonic point-like field theory discussed in section 3. The quantum version of (31) reads [4]

$$\partial_t \hat{\rho} = i[\hat{\rho}, \hat{H}] + i\beta^i G_{ij}[\dot{g}^i, \hat{\rho}]$$  \hspace{1cm} (33)


where the hat notation denotes quantum operators, and appropriate quantum ordering is understood (see below). We note that the equation (33) implies that $\partial_t \rho$ depends only on $\rho(t)$ and not on a particular decomposition of $\rho(t)$ in the projections corresponding to various results of a measurement process. This automatically implies the absence of faster-than-light signals during the evolution.

The analogy with the soliton case, discussed previously, goes even further if we recall the fact that energy is conserved in average in our approach of time as a renormalization scale. Indeed, it can be shown that the temporal change of the energy functional of the string particle, $\mathcal{H}$, obeys the equation

$$\partial_t \mathcal{H} \propto \mathcal{D} < \Theta(z, \bar{z}), \Theta(0) > = 0 \quad (34)$$

where $\Theta$ denotes the trace of the world-sheet stress tensor; the vanishing result is due to the renormalizability of the $\sigma$-model on the world-sheet surface, which, thus, replaces time-translation invariance in target space.

However, the quantum energy fluctuations $\delta E \equiv [<< \mathcal{H}^2 >> -(<< \mathcal{H} >>)^2]^\frac{1}{2}$ are time-dependent:

$$\partial_t (\delta E)^2 = -i << \beta^i, \mathcal{H} \beta^j G_{ji} >> = << \beta^j G_{ji} \frac{d}{dt} \beta^i >> \quad (35)$$

Using the fact that $\beta^i G_{ij} \beta^j$ is a renormalization-group invariant quantity, we can express (35) in the form

$$\partial_t (\delta E)^2 = - << Q^2 \beta^i G_{ij} \beta^j >> = - << Q^2 \partial_t C >> \leq 0 \quad (36)$$

We, thus, observe that, for $Q^2 > 0$ (supercritical strings), the energy fluctuations decrease with time for unitary string $\sigma$-models [12].

Before closing this section we wish to make an important remark concerning quantum ordering in (33). The quantum ordering is chosen in such a way so that energy and probability conservation, and positivity of the density matrix are preserved in the quantum case. Taking into account that in our string case $\beta^j G_{ij} = \sum_n C_{ij1 ... jn} g^{ji} ... g^{jn}$, with the expansion coefficients appropriate vertex operator correlation functions [4], it is straightforward to cast the above equation into a Lindblad form [33]

$$\dot{\rho} \equiv \partial_t \rho = i[\rho, \mathcal{H}] - \{B^\dagger B, \rho\}_+ + 2B\rho B^\dagger \quad (37)$$

where the ‘environment’ operators $B, B^\dagger$ are defined appropriately as ‘squared roots’ of the various partitions of the operator $\beta^j G_{ij} ... g^i$. This form may have important consequences in the case one considers a wave-function representation of the density matrix. Indeed, as discussed in ref. [34], (37) implies a stochastic diffusion equation for a state vector, which has important consequences for the localization of the wave-function in a quantum theory of measurement. We stress, however, that our approach based on density matrices (37) and renormalizability of the string $\sigma$-model is more general than any approach assuming state vectors.
4 Quantum Gravity and Breakdown of Coherence in the String Picture of a Microtubule

Above we have established the conditions under which a large-scale coherent state appears in the MT network, which can be considered responsible for loss-free energy transfer along the tubulins. As a result of conformational (quantum) transitions of the tubulin dimers, there is an abrupt distortion of space-time. Formally this is expressed by coupling the $c = 1$ string theory to two-dimensional quantum gravity. This elevates the matter-gravity system to a critical $c = 26$ theory. Such a coupling, then, causes decoherence, due to induced instabilities of the kink quantum-coherent ‘preconscious state’, in a way that we shall discuss below. As the required collapse time of $O(1\text{ sec})$ of the wave function of the coherent MT network is several orders of magnitude bigger than the energy transfer time $t_T$, the two mechanisms are compatible with each other. Energy is transferred during the quantum-coherent preconscious state, in $10^{-7}$ sec, and then collapse occurs to a certain (classical) conformational configuration. In this way, Frohlich’s frequency associated with coherent ‘phonons’ in biological cells is recovered, but in a rather different setting.

We now proceed to describe the precise mechanism for the breakdown of coherence, once the system couples to quantum gravity. First, we discuss an explicit way of dynamical creation of black holes in two-dimensional string theory through collapse of tachyonic matter $T(x, t)$. This is a procedure that can happen as a result of quantum fluctuations of various excitations. Consider the equations of motion for the graviton and dilaton fields obtained by imposing conformal invariance in the model (20) to order $\alpha'$, ignoring the non-universal tachyon terms in the potential. It can be shown that a generic solution for the graviton deformation has the form

$$ds^2 = -(1 + \int_{-\infty}^{\infty} dx'[(\partial_x T)^2 + (\dot{T})^2] - \int_{-\infty}^{t} dt' \dot{T} \partial_x T) dt^2 + \{1 + \int_{-\infty}^{t} dt' \dot{T} \partial_x T\} dx^2 \quad (38)$$

Consider now an incoming localized wavepacket of the form ($a \equiv \text{const}$)

$$T = e^{-x} \frac{a}{\cosh[2(x + t)]} \quad (39)$$

It becomes clear from (38) that there will be an horizon, obtained as a solution of the equation derived by imposing the vanishing of the coefficient of the $dt^2$ term. Thus, for late times $t \to \infty$, the resulting metric configuration is a two-dimensional static black hole

$$ds^2 = -(1 - \frac{4}{3} a^2 e^{-2x}) dt^2 + (1 - \frac{4}{3} a^2 e^{-2x})^{-1} dx^2 \quad (40)$$
and the whole process describes a dynamical collapse of matter. The energy of the collapsing wavepacket gives rise to the ADM mass \( \frac{4}{3}a^2 \) of the black hole \[36\]. It is crucial for the argument that there is no part of the wave-packet reflected. Otherwise the resulting ADM mass of the black hole will be the part of the energy that was not reflected. In realistic situations, the black hole is only virtual, since low-energy matter pulses are always reflected in two-dimensional string theories, as suggested by matrix-model computations \[37\]. Indeed, if one discusses pulses which undergo total reflection \[35\],

\[
T(x, t) = e^{-\frac{x}{a^2}} \eta(x, t) \quad ; \quad \eta(x, t) = \frac{a}{\cosh(2(x + t))} + \frac{a}{\cosh(2(x - t))}
\]

then, it can be easily shown that there is a transitory period where the space time geometry looks like a black hole \[40\], but asymptotically in time one recovers the linear dilaton (flat) vacuum \[21\]. The above example \[41\] gives a generic way of a (virtual) dynamical matter collapse in a two-dimensional stringy space-time, of the type that we encounter in our model for the brain, as a result of quantum conformational changes of the dimers. In the case of MT, the replacement \( x \rightarrow \xi \), where \( \xi \) is the boosted coordinate in \( 12 \), is understood.

The important point to notice is that the system of \( T(\xi) \) coupled to a black hole space-time \[40\], even if the latter is a virtual configuration, it cannot be critical (conformal invariant) non-perturbatively if the tachyon has a travelling wave form. The factorisation property of the world-sheet action \[20\] in the flat space-time case breaks down due to the non-trivial graviton structure \[40\]. Then a travelling wave cannot be compatible with conformal invariance, and renormalization scale dressing appears necessary. The gravity-matter system is viewed as a \( c = 26 \) string \[36, 4\], and hence the renormalization scale is time-like \[21\]. This implies time dependence in \( T(\xi, t) \).

A natural question arises whether there exist a deformation that turns on the coupling \( T(\xi) \) which is exactly marginal so as to maintain conformal invariance. To answer this question, we first note that there is an exact conformal model \[37\], a Wess-Zumino \( SL(2, R)/U(1) \) coset theory, whose target space has the metric \[40\]. The exactly marginal deformation of this black hole background that turns on matter, couples necessarily the propagating tachyon \( T(\xi) \) zero modes to an infinity of higher-level string states \[38\]. The latter are classified according to discrete representations of the \( SL(2, R) \) isospin, and together with the propagating modes, form a target-space \( W_{\infty} \)-algebra \[4, 11\]. This coupling of massive and massless modes is due to the non-vanishing Operator Product Expansion (O.P.E.) among the vertex operators of the \( SL(2, R)/U(1) \) theory \[38\]. The model is completely integrable, due to an infinity of conserved charges in target space \[4\] corresponding to the Cartan subalgebra of the infinite-dimensional \( W_{\infty} \) \[11\]. This integrability persists quantization \[39\], and it is very important for the quantum coherence of the string black hole space-time \[4\]. Due to the specific nature of the \( W_{\infty} \) symmetries, there is no
information loss during a stringy black hole decay, the latter being associated with instabilities induced by higher-genus effects on the world-sheet [40, 4]. The phase-space volume of the effective field theory is preserved in time, *only if* the infinite set of the global string modes is taken into account. This is due to the string-level mixing property of the $W_\infty$ - symmetries of the target space.

However, any local operation of measurement, based on local scattering of propagating matter, such as the functions performed by the human brain, will necessarily break this coherence, due to the truncation of the string deformation spectrum to the localized propagating modes $T(\xi)$. The latter will, then, constitute a subsystem in interaction with an *environment* of global string modes. The quantum integrability of the full string system is crucial in providing the necessary couplings. This breaking of coherence results in an arrow of time/Liouville scale, in the way explained briefly above [4]. The black-hole $\sigma$-model is viewed as a $c = 26$ critical string, while the travelling wave background is a non-conformal deformation. To restore criticality one has to dress $T(\xi)$ with a Liouville time dependence $T(\xi, t)$ [4]. From a $\sigma$-model point of view, to $O(\alpha')$, a non-trivial consistency check of this approach for the black hole model of ref. [36] has been provided in ref. [4]. We stress once more that the Liouville renormalization scale now is time-like, in contrast with the previous string picture of a $c = 1$ matter string theory, representing the displacement field $u$ alone before coupling to gravity.

By viewing the time $t$ as a local scale on the world sheet, a natural identification of $t$ will be with the logarithm of the area $A$ of the world sheet, at a fixed topology. As the non-critical string runs towards the infrared fixed point the area expands. In our approach to Liouville time [4], the actual flow of time is opposite to the world sheet renormalization group flow. This is favoured by a bounce interpretation of the Liouville flow due to specific regularization properties of Liouville correlation functions [4, 41]. This implies that we may set $t \propto -lnA$, with $A$ flowing always towards the infrared $A \to \infty$. In this way, a *Time Arrow* is implemented automatically in our approach, without requiring the imposition of time-asymmetric boundary conditions in the analogue of the Hartle-Hawking state. In this respect, our theory has many similarities to models of conventional dissipative systems, whose mathematical formalism [42, 43] finds a natural application to our case.

With this in mind, one can examine the properties of the correlation functions of $V_i, A_N = < V_{i_1} \ldots V_{i_N} >$, and hence the issue of coherence breakdown. In critical string theory such correlators correspond to scattering amplitudes in the target-space theory. It is, therefore, essential to check on this interpretation in the present situation. Since the correlators are mathematically formulated on fixed area $A$ world-sheets, through the so-called fixed area constraint formalism [44, 45], it is interesting to look for possible $A$-dependences in their evolution. In such a case their interpretation as target-space scattering amplitudes would fail. Indeed, it
has been shown \[29\] that there is an induced target space $A$-dependence of the regularized correlator $A_N$, which, therefore, cannot be identified with a target space $S$-matrix element, as was the case of critical strings \[46\]. Instead, one has non-factorisable contributions to a superscattering amplitude \[S \neq SS^\dagger\], as is usually the case in open quantum mechanical systems, where the fundamental building blocks are density matrices and not pure quantum states\[5\]. For completeness, we describe in Appendix A some formal aspects of this situation, based on results of our approach to non-critical strings \[4\].

It is this sort of coherence breakdown that we advocate as happening inside the part of the brain related to consciousness, whose operation is described by the dynamics of (the quantum version of) the model \(5\). The effective two-dimensional substructures, that we have identified above as the basic elements for the energy transfer in MT, provide the necessary framework for coupling the (integrable) stringy black hole space-time \(40\) to the displacement field $u(x,t)$. This allows for a qualitative description of the effects of quantum gravity on the coherent superposition of the preconscious states.

One can calculate in this approach the off-diagonal elements of the density matrix in the string theory space $u^i$, with now $u^i(t)$ representing the displacement field of the $i$-th dimer. In a $\sigma$-model representation \(20\), this is the tachyon deformation. The computation proceeds analogously \[4\] to the Feynman-Vernon \(16\) and Caldeira and Leggett \(17\) model of environmental oscillators, using the influence functional method, generalized properly to the string theory space $2u^i$. The general theory of time as a world-sheet scale predicts \[16, 17\] the following expression for the reduced density matrix\[16, 17\] of the observable states:

\[
\rho(u_I, u_F, t) \rho_S(u_I, u_F, t) \simeq e^{-N \int_0^t \int_{\tau-\epsilon}^{\tau+\epsilon} d\tau' \int_{\tau-\epsilon}^{\tau+\epsilon} d\tau'' \frac{(u(\tau'-\epsilon)-u(\tau''+\epsilon))^2}{(\epsilon^2)^2}} \\
\simeq e^{-N \int_0^t \int_{\tau-\epsilon}^{\tau+\epsilon} d\tau' \int_{\tau-\epsilon}^{\tau+\epsilon} d\tau'' \beta^{\epsilon}G_{ij}(S_0) \beta^{\epsilon}j} \simeq e^{-DN(u_I-u_F)^2+...} \tag{42}
\]

where the subscript “S” denotes quantities evaluated in conventional Schrödinger quantum mechanics, and $N$ is the number of the environment atoms\[4\] interacting with the background $u^i$. For $(1,1)$ operators, that we are interested in, the structure of the renormalization group $\hat{\beta}$-functions is $\hat{\beta}^i = \epsilon u^i + \beta^{ui}$, where $\beta^{ui} = O(u^2)$ and $\epsilon \rightarrow 0$ is the anomalous dimension. Recalling \[4\] the pole structure of the Zamolodchikov metric, $G_{ij} = \frac{1}{\epsilon}G_{ij}^{(1)} + \text{regular}$, one finds that the dominant contribution to the exponent $K$ of the model \[12\] comes from the $\epsilon$-term in $\hat{\beta}^i$ and the pole term in $G_{ij}$:

\[
K = N \int_0^t \hat{\beta}^i G_{ij} \hat{\beta}^j d\tau \simeq 2N \int_0^t u^i G_{ij}^{(1)} \beta^{ui} d\tau + O(\epsilon) \tag{43}
\]

\[2\]Throughout this work, as well as our previous works on the subject \[4\], we assume that such a space exists, and admits \[12\] a metric $G_{ij}$. Indications that this is true are obtained from perturbative calculations in $\sigma$-model, to which we base our belief.
Assuming slowly varying $u^i$ and $\beta^i$ over the time $t$, this implies that the off-diagonal elements of the density matrix would decay exponentially to zero, within a collapse time of order [6]

$$t_{\text{coll}} = \frac{1}{N}(O[\beta^u G^{(1)}_{ij} w^j])^{-1}t_s \quad (44)$$

in fundamental string units $t_s$ of time. The superscript (1) in the theory space metric denotes the single residue in, say, dimensional regularization on the world sheet [4, 30]; $N$ is the number of (coherent) tubulin dimers in interaction with the given dimer that undergoes the abrupt conformational change. Here the $\beta^u$-function is assumed to admit a perturbative expansion in powers of $\lambda^2 s^{-2}X^2$, in target space, where the fundamental string unit of length is defined as

$$\lambda_s = \left(\frac{\hbar c}{v_0^2} \right)^{1/2} \quad (45)$$

where $v_0$ is the sound velocity [13], of order $1\text{km/sec}$ [19]. We work in a system of units where the light velocity is $c = 1$, and we use as the scale where quantum gravity effects become important, the grand unified string scale, $M_{\text{gus}} = 10^{18}\text{GeV}$, or in length $10^{-32}\text{cm}$, which is 10 times the conventional Planck scale. This is so, because our model is supposed to be an effective description of quantum gravity effects in a stringy (and not point-like) space-time. This scale corresponds to a time scale of $t_{\text{gus}} = 10^{-42}\text{sec}$. We now observe that, to leading order in the perturbative $\beta$ function expansion in (44), any dependence on the velocity $v^2$ disappears in favour of the scales $M_{\text{gus}}$ and $t_{\text{gus}}$. It is, then, straightforward to obtain a rough estimate for the collapse time

$$t_{\text{col}} = O \left(\frac{M_{\text{gus}}}{E^2 N} \right) \quad (46)$$

where $E$ is a typical energy scale in the problem. Thus, we estimate that a collapse time of $O(1\text{sec})$ is compatible with a number of coherent tubulins of order

$$N \simeq 10^{12} \quad (47)$$

provided that the energy stored in the kink background is of the order of $eV$. This is indeed the case of the (dominant) sum of binding and resonant transfer energies $\Delta \simeq 1eV$ [16] at room temperature in the phenomenological model of ref. [19]. This number of tubulin dimers corresponds to a fraction of $10^{-7}$ of the total brain, which is pretty close to the fraction believed to be responsible for human perception on the basis of completely independent biological methods.

An independent estimate for the collapse time $t_{\text{col}}$, can be given on the basis of point-like quantum gravity quantum gravity theory, assuming that the latter exists, either per se, or as an approximation to some string theory. One incorporates quantum gravity effects by employing wormholes [17] in the structure of space time, and then applies the calculus of ref. [8] to infer the estimates of the collapse time.
In that case, one evaluates the off-diagonal elements of the density matrix in real configuration space $x$, which should be compared to that in string theory space (12). The result of ref. [6] for the time of collapse induced by the interaction with a ‘measuring apparatus’ with $\mathcal{N}$ units is

$$t'_{\text{coll}} \simeq \frac{1}{\mathcal{N}} (M_{\text{gas}}/m)^3 \frac{1}{m^3(\delta x)^2}$$

(48)

where $m$ is a typical mass unit in the problem. The fundamental unit of velocities in (48) is provided by the velocity of light $c$, since the formula (48) refers to generic four-dimensional space-time effects. In the case of the tubulin dimers, it is reasonable to assume that the pertinent moving mass is the effective mass $M^*$ (17) of the kink background (13). This will make contact with the microscopic model above. To be specific, (17) gives $M^* \simeq 3m_p$, where $m_p$ is the proton mass. This makes plausible the rather daring assumption that the nucleons (protons, neutrons) themselves inside the protein dimers are the most sensitive constituents to the effects of quantum gravity. This is a reasonable assumption if one takes into account that the nucleons are much heavier than the (conformational) electrons. If true, this would really imply, then, that elementary particle scales come into play in brain functioning. In this picture, then, $\mathcal{N}$ is the number of tubulin dimers, and moreover in our case $\delta x = O[4nm]$, since the relevant displacement length in the problem is of order of the longitudinal dimension of each conformational pocket in the tubulin dimer. Thus, substituting these in (18) one derives the result $\mathcal{N} = N \simeq 10^{12}$, for the number of coherent dimers that induce a collapse within $O(1 \text{ sec})$.

It is remarkable that the final numbers agree between these two estimates. If one takes into account the distant methods involved in the derivation of the collapse times in the respective approaches, then one realizes that this agreement cannot be a coincidence. Our belief is that it reflects the fundamental rôle of quantum gravity in the brain function.

At this stage, it is important to make some clarifying remarks concerning the kind of collapse that we advocate in the framework of non-critical string theory. In the usual model of collapse due to quantum gravity [4], one obtains an estimate of the collapse of the off-diagonal elements of the density matrix in configuration space, but no information is given for the diagonal elements. In the string theory framework of ref. [1] the collapse (12) also refers to off-diagonal elements of the string density matrix, but in this case the configuration space is the string background field space. In this case the off-diagonal element collapse suffices, because it implies localization in string background space, which means that the quantum string chooses to settle down in one of the classical backgrounds, which in the case at hand is the solitonic background discussed in section 2.

Of course, the important question ‘which specific background is selected by the above process ?’ cannot be answered unless a full string field theory dynamics is
developed. However, we believe that our approach \[4\] of viewing the selection of a critical string ground state as a generalized ‘measurement’ process in string theory space might prove advantageous over other dynamical methods in this respect.

5 Growth (Dynamical Instability) of a Microtubule Network and Liouville Theory

The above considerations are valid for MT networks whose size of individual MT is larger than a certain critical size \[19\]. Kink-like excitations, that were argued to be crucial for the physics of the conscious functions of the brain, cannot form for small microtubules. The question, therefore, arises whether the non-critical effective string theory framework described above is adequate for describing the growth process associated with the formation of a MT network. This phenomenon is physically and biologically very interesting since these structures are known to be the only ones so far that exhibit the so-called ‘dynamical instability growth’ \[48\]. This is an out-of-equilibrium process according to which an individual MT can switch randomly between an ‘assembly state’ (+), in which the MT grows with a speed \(v_+\), and ‘diassembly state’ (−), in which the MT shrinks with velocity \(v_-\). Recently there have been attempts to construct simple one-dimensional theoretical models with diffusion \[49\] that can describe qualitatively the above phenomenon. An interesting feature of these models, relevant to our framework, is that for a certain range of their parameters exhibit a phase transition to an unbounded growth state \[3\]. In the case of MT networks, it is known experimentally that a ‘sawtooth’ behaviour in the time-dependence of the size of a MT (Fig. 3), occurs as a result of hydrolysis of GTP nucleotides bound to the tubulin proteins (i.e. the transformation GTP → GDP) \[48\]. The hydrolysis is responsible for providing the necessary free energy for the conformational changes of the tubulin dimers: the dynamical instability phenomenon pertains to polymerization of the GTP tubulin, while the GDP tubulin stays essentially unpolymerized. In view of quantum oscillations between the two conformations of the tubulin, and the above different behaviour of polymerization, it is natural to conjecture that quantum effects may play a rôle in the MT growth process.

\[3\] This situation may also be viewed from a spontaneous-symmetry breaking point of view along the lines of ref. \[50\], which in one dimension can occur when the system is out of equilibrium. This second point of view is more relevant to our non-critical string approach, which as we explained earlier is an out-of-equilibrium process. The symmetry breaking can be exhibited easily by looking at one-dimensional models with driven diffusion of say two species of particles corresponding to the (+) and (-) conformational states of the MT growth. In the broken phase there is a difference between the ‘currents’ corresponding to the (+) or (-) states. Under certain plausible conditions \[54\], associated with formation of droplets of the ‘wrong sign’ in any given configuration of the above states, the system can switch between (+) and (-) states with a switching time that depends on dynamical parameters. For a certain range of these parameters the switching time is of order \(e^N\), where \(N\) is the size of the system, thereby implying spontaneous symmetry breaking in the thermodynamic limit where \(N \to \infty\).
Thus, our Liouville theory prepresentation of the effective degrees of freedom $u$ involved in the model of [19], invented to explain classical aspects of the hydrolysis of $\text{GTP} \rightarrow \text{GDP}$, might, in principle, be able of explaining qualitatively the ‘sawtooth’ behaviour of Fig. 3, even before the formation of kinks. To this end, we remark that in Liouville dynamics, with the Liouville scale identified with the target time $t$, the inherent non-unitarity (in the world-sheet) of the Liouville mode implies that the central charge $Q$ of the theory flows with the scale in such a way that near fixed points it oscillates a bit before settling down. Indeed, for a non-critical string with running central charge $C[g, t]$, $t$ is the Liouville scale/time, the following second order equation (local in target space-time) holds near a fixed point of the Renormalization Group Flow [31]

$$\ddot{C}[g, t] + Q[g, t] \dot{C}[g, t] \leq 0 \text{ for } C \geq 25 ; \quad Q^2[g, t] = \frac{1}{3}(C[g, t] - 25) \quad (49)$$

This is a local phenomenon in target space-time. Globally, there is a preferred direction in time along which the entropy of the system increases [12, 4].

The small oscillations of $C$ in (49) may be attributed to the double direction of Liouville time that arises as a result of imaginary parts (dynamical instabilities) appearing due to world-sheet regularization by analytic continuation of non-critical string correlation functions [11, 3] (Fig. 4) This point is discussed briefly in Appendix A.

Along each direction of Liouville time in Fig. 4 there is an associated variation of $Q[g, t]$ and $C[g, t]$, and the volume of the ‘one-dimensional universe’ of MT either increases or decreases [28]. Thus, as a result of the oscillations of the Liouville dynamics (49) a ‘sawtooth’ behaviour of MT, which expresses the result of polymerization of GTP tubulin, can be qualitatively explained within the framework of non-critical Liouville dynamics. The analogy of the above situation to a stochastic expansion of the Universe (inflation) in non-critical string theory as discussed in ref. [28] should be pointed out.

It should be noted at this stage that in this effective framework the origin of non-criticality of the subsystem of tubulin dimers is left unspecified. Quantum Gravity fluctuations appear on an equal footing with the environment of the nucleating solvant that surrounds the MT in their physical environment. The distinction can be made once a detailed description of the environment is given, which of course would specify the form of the target metric coupled to the matter system of tubulins. As we have discussed in previous sections, in the quantum Gravity case this is achieved by the exactly marginal operators of the $\text{SL}(2, R)/U(1)$ conformal field theory that describes space-time singularities in one-dimensional strings [36]. The latter involve global (non-propagating) string modes which cannot be detected by localized scattering experiments [4]. On the other hand, it seems likely that an exact conformal field theory that describes the nucleating solvant does not exist. However,
the effective Liouville string that describes the embedding of a MT in it, and the associated environmental entanglement, is obtained by a simple Liouville dressing of the model discussed in section 3. The information about the environment is hidden in the form of the target metric that couples to the system. The fact that both the formation of an MT via the dynamical instability phenomenon, and the quantum gravity effects on MT, involve conformational changes of the tubulin supports the above point of view. Of course the strength of the dynamical instability in case the latter is due to quantum gravity fluctuations will be much more suppressed as compared to the hydrolysis case. In that case, there will not be sufficient time for complete polymerization of the GTP tubulin conformation. In such a case one would probably expect ‘sudden’ ‘sawtooth’ peaks of the tip of the MT whose magnitude will be affected by the order of quantum gravity entanglement. The growth in this case will be bounded.

It should be mentioned that recently there have been some experiments claiming such length fluctuations [51], in the case of carbon nanotubes. The authors of ref. [51] claimed that they observed such changes in the length of the tubes, which they attributed to sudden jumps of the respective wavefunctions, according to the approach of Ghirardi Rimini and Weber [52]. In this respect, one should think of repeating the tests but with isolated MT [53]. We should stress, however, that the above discussion is at this stage highly speculative and even controversial, given that there appear to be conventional explanations of this phenomenon in carbon nanotubes [54]. Of course such conventional explanations do not exclude the possibility of future observations of quantum-gravity induced bounded growth in MT, along the lines sketched above.

We close this section by mentioning that in realistic situations the growth process of an MT network is not unlimited. After a critical length is exceeded, the growth is saturated and eventually stops. The formation of kink excitations [12] might be important for this auto-regulation of the MT growth [19]. For more details on the conjectural role of the kinks in this growth-control mechanism we refer the reader to the literature [19, 49].

The above situation should be compared with the limitations on the (stochastic) Universe expansion in the non-critical-string-driven inflationary scenario of ref. [28]. There, it can be shown that within our framework of identifying the Liouville field with the target time, the average density $\langle\langle \delta_s \rangle\rangle \equiv Tr(\rho_\delta)$ of the non-critical strings that drive the inflationary scenario obey an equation of the form [28, 55]

$$\partial_t \langle\langle \delta_s \rangle\rangle = -aQ \langle\langle \delta_s \rangle\rangle + bQ^3 \langle\langle \delta_s \rangle\rangle$$

(50)

where $\alpha, b$ are positive quantities, computable in principle in the Liouville-string framework [28]. The first term in (50) is due to the (exponential) expansion of the string-Universe volume, and the second term corresponds to the regeneration of
strings via breaking of large strings whose size exceeds that of the Hubble horizon \[28\]. This second term comes from the diffusion due to the non-quantum mechanical terms in the equation (33). At early times the diffusion term balances the string depletion effects of the first term and the uniform density condition for inflation (universe exponential expansion) is satisfied. As the time elapses, however, the depletion term in (50) dominates, the Universe’s expansion is diminished gradually, and eventually stops. This is the case when the non-equilibrium non-critical string approaches its (critical-string) equilibrium state. Hence, in our case one may view (50) as an effective model for the temporal evolution of the density of tubulin dimers. Then, one can understand, at least qualitatively, the above limitations of the MT growth process by the presence of the kinks, since the latter can be associated with an equilibrium ground state of the effective string theory describing a MT.

6 Conclusions

We have presented an effective model for the simulation of the dynamics of the tubulin dimers in the brain. We have used an effective (1 + 1)-dimensional string representation to study the dynamics of a detailed mechanism for energy transfer in the biological cells. We argued how it can give rise to a large-scale coherent state in the dimer lattice. Such a state is obtained from quantization of kink solitonic states that transfer energy through the cell without dissipation. The quantization became possible through the freedom that string theory offers, enabling one to cast dynamical problems with friction in a Hamiltonian form. The collapse phenomenon in our approach does not require the existence of a wave-function, and it is induced by the formation of microscopic black holes (singularities) in the effective one-dimensional space-time of the tubulin chains. This is achieved by the dynamical collapse of pulses of the displacement field of the MT dimers. The pulses are a result of abrupt conformational changes ($\alpha \leftrightarrow \beta$) that sufficiently distort the surrounding space-time. In this sense, the situation is similar but not identical, to the ‘sudden hits’ that a particle’s wave function suffers occasionally (every $10^9$ years) in the model of quantum measurement of Girardi, Rimini and Weber \[52\]. However, contrary to these conventional theories, our stringy approach to gravity-induced collapse \[4\] incorporates automatically an irreversible flow of time for specifically stringy reasons, and energy conservation. When applied to the model of MT, our approach implies a collapse time of $\mathcal{O}(1 \text{ sec})$, which is obtained by the interaction of a tubulin dimer with a fraction of $10^{-7}$ of the total number of tubulin dimers in the brain. This number is fairly close to the fraction of the brain that neuroscientists believe responsible for human perception. This is a very strong indication that the above ideas, although speculative at this stage, might be relevant for the discovery of a physical model for consciousness and its relation to the irreversible flow of time. In addition, our model predicts damped (microscopic) quantum-gravity-induced oscillations of the
length of isolated MTs, which are due to the different properties of the two tubulin conformations under polymerization (phenomenon of bounded dynamical-instability growth).

There are certain formal aspects of our effective model, namely its two-dimensional structure and its complete (quantum) integrability, that might turn out to be important features for the construction of realistic soluble models for brain function. The quantum integrability is due to generalized infinite-dimensional symmetry structures ($W$-symmetries and its generalizations) which are strongly linked with issues of quantum coherence and unitary evolution in phase-space. Such symmetries are related to global non-propagating modes of the effective string theory, which do not decouple from the propagating (observed) modes in the presence of (microscopic) space-time singularities. It will be interesting to understand further the physical rôle of such structures in the models of MT. At present they appear as an environment of fundamental string modes that are physical at Planck scales. However, such structures, may admit a less-ambitious physical meaning, associated with fundamental biological structures of the brain. We should stress that the entire picture of non-critical string we have described above, which is a model-independent picture as far as environmental operators are concerned, could still apply in such cases, but simply describing purely biological environmental entanglement of the conscious part of the brain, the latter being described as a completely integrable model. We do hope to come back to these issues in the near future.
Acknowledgements

It is a pleasure to acknowledge discussions with J. Ellis. We also acknowledge useful discussions and correspondence with S. Hameroff. N.E.M. acknowledges informative communications with H. Rosu on quantum jumps and experimental observations of dynamical instabilities in nanotubes. The work of N.E.M. is supported by a EC Research Fellowship, Proposal Nr. ERB4001GT922259. That of D.V. N. is partially supported by D.O.E. Grant DEFG05-91-GR-40633.
Appendix A

Extracts from Non-Critical (Liouville) String Theory and Time as the Liouville scale

In this appendix we shall comment on the non-factorizability of the induced target-space $S$-matrix for matter scattering in a non-conformal string background. This reflects information leakage as a result of the non-critical character of the string. Although for our purposes primarily we shall be interested in a specific string background, that of a stringy black hole, however in this section our discussion will be kept as general as possible with the aim of demonstrating the generality of our scheme.

Consider a conformal field theory on a two-dimensional world sheet, described by an action $S[g^*]$. The $\{g^*\}$ are a set of space-time backgrounds. The theory is perturbed by a deformation $V_i$, which is not conformal invariant

$$S[g] = S[g^*] + \int d^2 z g^i V_i$$

(51)

The couplings $g^i$ correspond to world-sheet renormalization group $\beta$-functions

$$\beta^i = (h_i - 2)(g^i - (g^*)^i) + c^i_{jk}(g^j - (g^*)^j)(g^k - (g^*)^k) + \ldots$$

(52)

expressing the scale dependence of the non-conformal deformations. The operator product expansion coefficients are defined as usual by coincident limits in the product of two vertex operators $V_i$

$$\lim_{\sigma \to 0} V_j(\sigma)V_k(0) \simeq c^i_{jk} V_i(\frac{\sigma}{2}) + \ldots$$

(53)

where the completeness of the set $\{V_i\}$ is assumed.

Coupling the theory (51) to two-dimensional quantum gravity restores the conformal invariance at a quantum level, by making the gravitationally-dressed operators $[V_i]_\phi$ exactly marginal, i.e. ensuring the absence of any covariant scale dependence with respect to the world-sheet metric $\gamma_{\alpha \beta}$. Below we simply outline the basic results, used in our approach here.

One rescales the world-sheet metric

$$\gamma_{\alpha \beta} = e^\phi \hat{\gamma}_{\alpha \beta}$$

(54)

with $\hat{\gamma}$ is kept fixed, and then one integrates over the Liouville mode $\phi$. The measure of such an integration can be expressed in terms of the fiducial metric $\hat{\gamma}$ by means of a determinant which is the exponential of the Liouville action. The final result for the gravitationally-dressed matter theory is then

$$S_{L-m} = S[g^*] + \frac{1}{4\pi\alpha'} \int d^2 z \partial_\alpha \phi \partial^\alpha \phi - QR(2) + \lambda^i(\phi)V_i$$

(55)
where $\alpha'$ is the Regge slope for the world-sheet theory (inverse of the string tension). The gravitational dressing of the operators follows from the requirement of restoring the conformal invariance of the theory, at any given order in the coupling-constant expansion. For instance, to order $O(g^2)$ the gravitationally-dressed coupling $\lambda(\phi)$ are given by [44, 56]:

$$\lambda^i(\phi) = g^i e^{\alpha_i \phi} + \frac{\pi}{Q \pm 2\alpha_i} c_{jk}^i g^j g^k e^{\alpha_i \phi} + \ldots$$  \hspace{1cm} (56)

with

$$Q = \sqrt{\left|\frac{25 - c}{3}\right|} ; \quad \alpha_i^2 + \alpha_i Q = s q n (25 - c) (h_i - 2)$$  \hspace{1cm} (57)

and $c$ is the (constant) central charge of the non-critical string. From the quadratic equation for $\alpha_i$ only the solution

$$\alpha_i = -\frac{Q}{2} + \sqrt{\frac{Q^2}{4} - (h_i - 2)}$$  \hspace{1cm} (58)

for $c \geq 25$, is kept due to the Liouville boundary conditions.

In ref. [4] we made an extra assumption, as compared to the above standard Liouville dynamics. We identified the field $\phi$ with a dynamical local scale on the world sheet. This induces extra counterterms in the world-sheet renormalized action. Consistency of the scheme required that the Liouville $\beta$ functions are identical with the flat space renormalization coefficients upon the replacement $g^i \to \lambda(\phi)^i$.

The type of operators that we are interested in this work, are such that $h_i = 2$ but $c_{jk} \neq 0$. In the language of conformal field theory this means that these operators are $(1, 1)$ but not exactly marginal. From (56), then, one obtains the simple relation

$$\frac{d\lambda^i(\phi)}{dt_p} = \beta^i$$  \hspace{1cm} (59)

where the time $t_p$ is related to the Liouville mode $\phi$ as

$$t_p = -\frac{1}{\alpha Q} \ln A \quad ; \quad A \equiv \int d^2 z \sqrt{\gamma} e^{\alpha \phi(z, \bar{z})} \quad ; \quad \alpha = -\frac{Q}{2} + \frac{1}{2} \sqrt{Q^2 + 8}$$  \hspace{1cm} (60)

with $A$ the world-sheet area. In the local scale formalism of ref. [4] $Q$ is given by

$$Q = \sqrt{\left|\frac{25 - C[g, \phi]}{3}\right|} + \frac{1}{2} \beta^i G_{ij} \beta^j$$  \hspace{1cm} (61)

where $C[g, \phi]$ is the Zamolodchikov $C$-function [12], which reduces to the central charge $c$ at a fixed point of the flow. The extra terms in (61), as compared to (57), are due to the local character of the renormalization group scale [4]. Such terms may always be removed by non-standard redefinitions of $C[g, \phi]$. The quantity $G_{ij}$ is related to divergencies of the two-point functions $<V_i V_j>$ and hence to Zamolodchikov metric [12, 4].


From the renormalization-group structure (56) one obtains close to a fixed point

\[ \dddot{\lambda}(\phi) + Q \dot{\lambda} = -\beta^i = -G^{ij} \partial_i C[\lambda, \phi] \quad (62) \]

where the dot denotes differentiation with respect to the Liouville local scale \( \phi \).

For the \( C[g, \phi] \) (local in target space-time) one obtains near a fixed point

\[ \dddot{C}[g, t] + Q[g, t] \dot{C}[g, t] \leq 0 \quad \text{for} \quad C \geq 25 \quad ; \quad Q^2[g, t] = \frac{1}{3} (C[g, t] - 25) \quad (63) \]

The small oscillations of \( C[\lambda, \phi] \), before it settles down to a fixed point, are due to the ‘non-unitary’ world-sheet contributions of the Liouville mode \( \phi \); however globally in target space-time there is a monotonic change of the degrees of freedom of the system, as discussed in detail in [4].

These considerations can be understood more easily if one looks at the correlation functions in the Liouville theory, viewing the Liouville field as a local scale on the world sheet. Standard computations yield for an \( N \)-point correlation function among world-sheet integrated vertex operators \( V_i \equiv \int d^2z \sqrt{\hat{\gamma}} e^{\alpha \phi} \)

\[ A_N \equiv \langle V_{i_1} \ldots V_{i_N} \rangle_{\mu} = \Gamma(-s) \mu^s \langle (\int d^2z \sqrt{\hat{\gamma}} e^{\alpha \phi})^s \dot{V}_{i_1} \ldots \dot{V}_{i_N} \rangle_{\mu=0} \quad (64) \]

where the tilde denotes removal of the Liouville field \( \phi \) zero mode, which has been path-integrated out in (64). The world-sheet scale \( \mu \) is associated with cosmological constant terms on the world sheet, which are characteristic of the Liouville theory [44]. The quantity \( s \) is the sum of the Liouville anomalous dimensions of the operators \( V_i \)

\[ s = -\sum_{i=1}^{N} \frac{\alpha_i}{\alpha} - \frac{Q}{\alpha} \quad ; \quad \alpha = -\frac{Q}{2} + \frac{1}{2} \sqrt{Q^2 + 8} \quad (65) \]

The \( \Gamma \) function can be regularized (for negative-integer values of its argument) by analytic continuation to the complex-area plane using the the Saaschultz contour of Fig. 4. This yields the possibility of an increase of the running central charge due to the induced oscillations of the dynamical world sheet area (related to the Liouville zero mode). This is associated with the oscillatory solution (63) for the Liouville central charge. On the other hand, the bounce interpretation of the infrared fixed points of the flow, given in refs. [41, 4], provides an alternative picture of the overall monotonic change at a global level in target space-time.

The above formalism also allows for an explicit demonstration of the non-factorizability of the superscattering matrix associated with target-space interactions in non-critical string theory. This was very important for our purposes in the context of the collapse of the wave-function as a result of quantum entanglement due to quantum gravity fluctuations.

To this end, one expands the Liouville field in (normalized) eigenfunctions \( \{ \phi_n \} \) of the Laplacian \( \Delta \) on the world sheet

\[ \phi(z, \bar{z}) = \sum_n c_n \phi_n = c_0 \phi_0 + \sum_{n \neq 0} \phi_n \quad \phi_0 \propto A^{-\frac{1}{2}} \quad (66) \]
with $A$ the world-sheet area, and

$$\Delta \phi_n = -\epsilon_n \phi_n \quad n = 0, 1, 2, \ldots \quad (\phi_n, \phi_m) = \delta_{nm} \quad (67)$$

The result for the correlation functions (without the Liouville zero mode) appearing on the right-hand-side of eq. (64) is, then

$$\tilde{A}_N \propto \int \Pi_{n \neq 0} c_n \exp \left( -\frac{1}{8\pi} \sum_{n \neq 0} \epsilon_n c_n^2 - \frac{Q}{8\pi} \sum_{n \neq 0} R_n c_n + \sum_{n \neq 0} \alpha_i \phi_n(z_i)c_n \right) d^{2}\xi \frac{\sqrt{\hat{\gamma}e^{\alpha} \sum_{n \neq 0} \phi_n c_n}}{s} \quad (68)$$

with $R_n = \int d^{2}\xi R^{(2)}(\xi) \phi_n$. We can compute (68) if we analytically continue $[57]$ $s$ to a positive integer $s \rightarrow n \in \mathbb{Z}^+$. Denoting

$$f(x, y) = \sum_{n, m \neq 0} \frac{\phi_n(x)\phi_m(y)}{\epsilon_n} \quad (69)$$

one observes that, as a result of the lack of the zero mode,

$$\Delta f(x, y) = -4\pi \delta^{(2)}(x, y) - \frac{1}{A} \quad (70)$$

We may choose the gauge condition $\int d^{2}\xi \sqrt{\gamma} \phi = 0$. This determines the conformal properties of the function $f$ as well as its ‘renormalized’ local limit $[58]$

$$f_R(x, x) = \lim_{x \rightarrow y} (f(x, y) + \ln d^{2}(x, y)) \quad (71)$$

where $d^{2}(x, y)$ is the geodesic distance on the world sheet. Integrating over $c_n$ one obtains

$$\tilde{A}_{n+N} \propto \exp \left[ \frac{1}{2} \sum_{i, j} \alpha_i \alpha_j f(z_i, z_j) \right] + \frac{Q^2}{128\pi^2} \int \int R(x)R(y)f(x, y) - \frac{Q}{8\pi} \sum_{i} \alpha_i \int \sqrt{\hat{\gamma}} R(x)f(x, z_i) \right) \quad (72)$$

We now consider infinitesimal Weyl shifts of the world-sheet metric, $\gamma(x, y) \rightarrow \gamma(x, y)(1 - \sigma(x, y))$, with $x, y$ denoting world-sheet coordinates. Under these, the correlator $A_N$ transforms as follows $[29]$

$$\delta \tilde{A}_N \propto \left[ \sum_{i} h_i \sigma(z_i) + \frac{Q^2}{16\pi} \int d^{2}\xi \sqrt{\gamma} \sigma(x) \right] + \frac{1}{A} \left[ \sum_{i} \alpha_i \int d^{2}\xi \sqrt{\gamma} \sigma(x) \int d^{2}\xi \sigma(x) \tilde{f}_R(x, x) + \frac{Q}{8\pi} \sum_{i} \alpha_i \int d^{2}\xi \sqrt{\gamma} \sigma(x) \tilde{f}_R(x, x) \right] - \frac{1}{2} \sum_{i} \alpha_i \tilde{f}_R(z_i, z_i) \int d^{2}\xi \sqrt{\gamma} \sigma(x) - \frac{Q}{16\pi} \sum_{i} \alpha_i \int d^{2}\xi \sqrt{\gamma} \sigma(x) \tilde{f}_R(x, x) \sigma(y) \right] \tilde{A}_N \quad (73)$$
where the hat notation denotes transformed quantities, and the function $G(x,y)$ is defined as

$$G(z,\omega) \equiv f(z,\omega) - \frac{1}{2}(f_R(z,z) + f_R(\omega,\omega)) \quad (74)$$

and transforms simply under Weyl shifts. We observe from (73) that if the sum of the anomalous dimensions $s \neq 0$ (‘off-shell’ effect of non-critical strings), then there are non-covariant terms in (73), inversely proportional to the finite-size world-sheet area $A$. In general, this is a feature of non-critical strings wherever the Liouville mode is viewed as a local scale of the world sheet. In such a case, the central charge of the theory flows continuously with time/scale $t$, as a result of the Zamolodchikov $c$-theorem [12]. In contrast, the screening operators yield quantized values [21]. This induced time ($A$-) dependence of the correlation function $A_N$ implies the breakdown of their interpretation as factorisable $S$-matrix elements.

In our framework, the effects of the quantum-gravity entanglement induce such $A$-dependences in correlation functions of the propagating matter vertex operators of the string [4], corresponding to the displacement field $u(x,t)$ of the MTs. To this end, we first note that the physical states in such completely integrable models fall into representations of the $SL(2,R)$ target symmetry, which are classified by the non-compact isospin $j$ and its third component $m$ [59]. There is a formal equivalence of the physical states between the flat-space time (1 + 1)-dimensional string and the black-hole model [4], which confirms the point of view [32, 4] that the flat-space (1 + 1)-dimensional string theory is the spatially- and temporally- asymptotic limit of the $SL(2,R)/U(1)$ black hole. The existence of discrete (quasi-topological, non-propagating) Planckian modes in the two-dimensional string theory leads to selection rules [62] in the number $N$ of the scattered propagating degrees of freedom, according to the intermediate-exchange state:

Asymptotic correspondence ($\epsilon_\phi (p) \equiv$ Liouville energy (momentum)):

$$j \rightarrow \epsilon_\phi, \quad m \rightarrow \frac{3p}{2\sqrt{2}}$$

Asymptotic Kinematics:

$$\sum_{i=1}^{N-1} p_i = \frac{N-2}{\sqrt{2}} \quad ; \quad p_N = -\frac{N-2}{\sqrt{2}}$$

Selection Rules:

$$j = \frac{1}{3} m - 1 + \frac{1}{2}(N-2), \quad j \geq \frac{1}{4}(N-5) \quad (75)$$

Such rules are obtained by imposing the Liouville energy $\epsilon_\phi$ and momentum $p$ conservation, leading to $s = 0$, with $s$ the sum of Liouville anomalous dimensions as defined earlier. Obviously, if the exchange state is an (off-shell) propagating

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4Originally, there were claims [59] that there are extra states in the black hole models, as compared to the flat-space time string; however, later on it has been shown that such states can be either gauged away [40] or boosted [61], and so they disappear from the physical spectrum.
mode, belonging to the continuous representation of $SL(2, R)$, i.e. $j \in \mathbb{R}, j \geq -\frac{1}{2}$, there are no restrictions on $N$, and a conventional $S$-matrix amplitude can be defined as the residue of the Liouville amplitudes with respect to the single poles in $s$. However, in two space-time dimensions graviton excitations are discrete, corresponding to string-level-one representations of $SL(2, R)$. Hence, once non-trivial quantum-gravity fluctuations are considered in our approach, which in two dimensions are black-hole backgrounds, one has to take into account discrete on-shell exchange modes in the Liouville correlation functions. Such states represent excited states of the (virtual) black-holes, created by the collapse of the propagating matter modes $u(x,t)$, as described in section 4. In two-dimensional string theory black holes are like particles, the difference being their topological nature. Such modes constitute, in our case, ‘the consciousness degrees of freedom’, which cannot be measured by local scattering experiments. Integrating them out in the ‘mind’, implies a time arrow as described in ref. Indeed, in the correlation functions, as a result of Liouville energy conservation, one of the modes is necessarily discrete. If we suppress such modes, and consider only external propagating modes, accessible to physical scattering processes, then it is evident that $s \neq 0$. According to our previous analysis, then, this implies world-sheet-area dependence of the correlation functions. In this picture, we also note that Quantum-Gravitational fluctuations of singular space-time form, corresponding to higher-genus world-sheet effects, have been argued to be represented collectively by world-sheet instanton-anti-instanton deformations in the stringy $\sigma$-model. It is known that such configurations are responsible for a non-perturbative breakdown of the conformal invariance of the $\sigma$-model. Using a dynamical (world-sheet) renormalization-group scale (Liouville mode) to represent all such non-conformal invariant effects, and identifying it with the target time, one, then, arrives at non-factorisable superscattering operators, as described above.

Notice that the precise microscopic nature of the environmental operators is not essential as long as the latter imply a conformal anomaly. There are general consequences of this conformal anomaly, including dynamical collapse of the string theory space to a certain configuration, as discussed in section 4. A similar situation occurs in ordinary quantum mechanics of open systems. Once a stochastic framework using state vectors is adopted for the description of environmental effects, there will always be localization of the state vector in one of its channels, irrespective of the detailed form of the environment operators. It should be noted that stochasticity is a crucial feature of our approach too. This follows from the stochastic nature of the renormalization group in two-dimensions. This stochasticity was argued to play an important role in the MT growth, discussed in section 5.

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5 The on-shell condition imposes algebraic relations among $j$ and $m$ for such modes, involving the string-level number due to the Virasoro constraints. This, in turn, implies restrictions to the number $N$ of scattered particles in such cases.
Appendix B

Variational Approach to Soliton Quantization via Squeezed Coherent States

It is the purpose of this appendix to discuss briefly the formalism leading to the quantization of the solitonic states discussed in section 2.

One assumes the existence of a canonical second quantized formalism for the \((1 + 1)\)-dimensional scalar field \(u(x, t)\), based on creation and annihilation operators \(a_k^\dagger, a_k\). One then constructs a squeezed vacuum state

\[
|\Psi(t)\rangle = N(t)e^{T(t)}|0\rangle ; \quad T(t) = \frac{1}{2} \int \int dx dy u(x)\Omega(x, y, t)u(y)
\]

where \(|0\rangle\) is the ordinary vacuum state annihilated by \(a_k\), and \(N(t)\) is a normalization factor to be determined. \(\Omega(x, y, t)\) is a complex function, which can be splitted in real and imaginary parts as

\[
\Omega(x, y, t) = \frac{1}{2}[G^{-1}_0(x, y) - G^{-1}(x, y, t)] + 2i\Pi(x, y, t)
\]

\[
G_0(x, y) = <0|u(x)u(y)|0>
\]

The squeezed coherent state for this system can be then defined as

\[
|\Phi(t)\rangle \equiv e^{iS(t)}|\Psi(t)\rangle ; \quad S(t) = \int_{-\infty}^{+\infty} dx [D(x, t)u(x) - C(x, t)\pi(x)]
\]

with \(\pi(x)\) the momentum conjugate to \(u(x)\), and \(D(x, t), C(x, t)\) real functions. With respect to this state \(\Pi(x, t)\) can be considered as a momentum canonically conjugate to \(G(x, y, t)\) in the following sense

\[
<\Phi(t)| - i\frac{\delta}{\delta\Pi(x, y, t)}|\Phi(t)\rangle = -G(x, y, t)
\]

The quantity \(G(x, y, t)\) represents the modified boson field around the soliton.

To determine \(C, D, \) and \(\Omega\) one applies the Time-Dependent Variational Approach (TDVA) according to which

\[
\delta \int_{t_1}^{t_2} dt <\Phi(t)|(i\partial_t - H)|\Phi(t)\rangle = 0
\]

where \(H\) is the canonical Hamiltonian of the system. This leads to a canonical set of (quantum) Hamilton equations

\[
\dot{D}(x, t) = -\frac{\delta H}{\delta C(x, t)} ; \quad \dot{C}(x, t) = \frac{\delta H}{\delta D(x, t)}
\]

\[
\dot{G}(x, y, t) = \frac{\delta H}{\delta \Pi(x, y, t)} ; \quad \dot{\Pi}(x, y, t) = \frac{\delta H}{\delta G(x, y, t)}
\]
where the quantum energy functional $H$ is given by \[26\]

$$H \equiv <\Phi(t)|H|\Phi(t)> = \int_{-\infty}^{\infty} dx \mathcal{E}(x) \quad (82)$$

with

$$\mathcal{E}(x) = \frac{1}{2} D^2(x, t) + \frac{1}{2} (\partial_x C(x, t))^2 + M^{(0)}[C(x, t)] +$$

$$\frac{1}{8} < x|G^{-1}(t)|y > + 2 < x|\Pi(t)G(t)\Pi(t)|y > + \frac{1}{2} \lim_{x \to y} \nabla_x \nabla_y < x|G(t)|y > \quad (83)$$

$$\frac{1}{8} < x|G_0^{-1}|y > - \frac{1}{2} \lim_{x \to y} \nabla_x \nabla_y < x|G_0(t)|y > \quad (84)$$

where we use the following operator notation in coordinate representation $A(x, y, t) \equiv <x|A(t)|y>$, and

$$M^{(n)} = e^{\frac{1}{2}(G(x,x,t)-G_0(x,x))} \frac{\partial^2}{\partial x^2} U^{(n)}(z)|_{z=C(x,t)} \quad ; \quad U^{(n)} \equiv d^n U/dz^n \quad (85)$$

Above, $U$ denotes the potential of the original soliton Hamiltonian, $H$. Notice that the quantum energy functional is conserved in time, despite the various time dependences of the quantum fluctuations. This is a consequence of the canonical form \[(81)\] of the Hamilton equations.

Performing the functional derivations in \[(81)\] one can get

$$\dot{D}(x, t) = \frac{\partial^2}{\partial x^2} C(x, t) - M^{(1)}[C(x, t)]$$

$$\dot{C}(x, t) = D(x, t) \quad (86)$$

which after elimination of $D(x, t)$, yields the modified (quantum) soliton equation \[(27)\]. We note that the quantities $M^{(n)}$ carry information about the quantum corrections, and in this sense the above scheme is more accurate than the WKB approximation \[(27)\]. The whole scheme may be thought of as a mean-field-approach to quantum corrections to the soliton solutions.

In our string framework, then, these point-like quantum solitons can be viewed as a low-energy approximation to some more general ground state solutions of a non-critical string theory, formulated in higher genera on the world sheet to account for the quantum corrections. We have not worked out in this work the full string-theory representation of the relevant quantum coherent state. This will be an interesting topic to be studied in the future, which will allow for a rigorous study of the effects of the global string modes on the collapse of the quantum-coherent preconscious state.
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Figure Captions

Figure 1 - Microtubular Arrangement: (a) the structure of a Microtubule (MT), (b) cross section of a MT, (c) two neighboring dimers along the direction of a MT axis.

Figure 2 - The two conformations $\alpha$ and $\beta$ of a MT dimer. Transition (switching) between these two conformational states can be viewed as a quantum-mechanical effect. Quantum-Gravity entanglement can cause the collapse of quantum-coherent states of such conformations, which might arise in a MT network modelling the preconscious state of the human brain.

Figure 3 - Illustration of the phenomenon of ‘dynamical instability’ of a MT network: (a) unbounded ‘sawtooth’ growth (b) bounded ‘sawtooth’ growth. Dotted lines show the average over many MT with the same dynamical parameters.

Figure 4 - (a) Contour of integration in the analytically-continued (regularized) version of $\Gamma(-s)$ for $s \in \mathbb{Z}^+$. This is known in the literature as the Saalschutz contour, and has been used in conventional quantum field theory to relate dimensional regularization to the Bogoliubov-Parasiuk-Hepp-Zimmermann renormalization method, (b) schematic representation of the evolution of the world-sheet area as the renormalization group scale moves along the contour of fig. 4(a).