Testing anthropic reasoning for the cosmological constant with a realistic galaxy formation model

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ABSTRACT
The anthropic principle is one of the possible explanations for the cosmological constant (Λ) problem. In previous studies, a dark halo mass threshold comparable with our Galaxy must be assumed in galaxy formation to get a reasonably large probability of finding the observed small value, \( P(<\Lambda_{\text{obs}}) \), though stars are found in much smaller galaxies as well. Here we examine the anthropic argument by using a semi-analytic model of cosmological galaxy formation, which can reproduce many observations such as galaxy luminosity functions. We calculate the probability distribution of Λ by running the model code for a wide range of Λ, while other cosmological parameters and model parameters for baryonic processes of galaxy formation are kept constant. Assuming that the prior probability distribution is flat per unit Λ, and that the number of observers is proportional to stellar mass, we find \( P(<\Lambda_{\text{obs}}) = 6.7\% \) without introducing any galaxy mass threshold. We also investigate the effect of metallicity; we find \( P(<\Lambda_{\text{obs}}) = 9.0\% \) if observers exist only in galaxies whose metallicity is higher than the solar abundance. If the number of observers is proportional to metallicity, we find \( P(<\Lambda_{\text{obs}}) = 9.7\% \). Since these probabilities are not extremely small, we conclude that the anthropic argument is a viable explanation, if the value of Λ observed in our universe is determined by a probability distribution.

Key words: cosmological parameters – cosmology: theory – galaxies: formation

1 INTRODUCTION
The early observational indications for non-vanishing cosmological constant, \( \Lambda \) (Efstathiou et al. 1990; Fukugita et al. 1990; Yoshii 1993; Krauss & Turner 1995; Ostriker & Steinhardt 1995), were further strengthened by type Ia supernova observations (Riess et al. 1998; Perlmutter et al. 1999), and established by the WMAP data of the cosmic microwave background radiation (Spergel et al. 2003), leading to the standard ΛCDM model that are consistent with many further high-precision observational tests until now (see e.g. Frieman et al. 2008; Weinberg et al. 2013, for reviews). The cosmological constant is interpreted as the vacuum energy density, but theoretically natural values expected by particle physics are larger than the observed value \( \Lambda_{\text{obs}} \) by a factor of at least \( \sim 10^{55} \) (see e.g. Weinberg 1989; Carroll 2001; Sahni 2002; Caldwell & Kamionkowski 2009, for theoretical reviews). The discrepancy becomes even much worse if we assume that the natural value of the cosmological constant is set by the Planck energy density, which is \( 10^{123} \) times larger than the energy density corresponding to \( \Lambda_{\text{obs}} \). This is the so-called smallness problem. Furthermore, \( \Lambda \) is not exactly zero but has a finite value, and we are living in a very special epoch when the energy densities of matter and vacuum become comparable in the long history of the universe. This is the coincidence problem. Many approaches have been proposed, ranging from a new field called dark energy to modification of the gravity theory, but there is no satisfactory solution yet for this problem.

One approach to this problem is using the anthropic principle (Barrow & Tipler 1986; Weinberg 1987, see also Banks 1985). If \( \Lambda \ll -\Lambda_{\text{obs}} \), the universe would have collapsed much earlier than the present epoch (Barrow & Tipler 1986). On the other hand, if \( \Lambda \) is positively too large, gravitational condensation of matter is suppressed...
by an accelerated expansion of the background universe, and hence no galaxy or intelligent life is formed (Weinberg 1987). Such an idea is supported by some theories about very early universe suggesting a possibility that there is an ensemble of many multiverses and \( \Lambda \) varies among different multiverses. It is reasonable to expect a nearly flat prior probability distribution \( p(\Lambda) \) per unit \( \Lambda \), because \( \Lambda \) of a habitable universe must be smaller than the theoretically natural scale \( \Lambda_{\text{obs}} \) by many orders of magnitude. If \( p(\Lambda) \) is non-zero at \( \Lambda = 0 \), and \( dp/d\Lambda \sim \Lambda^{-1} \), \( p(\Lambda) \) would be essentially constant within the range of \( \Lambda \) for a habitable universe. Then not only the smallness but also the coincidence problem is solved because the probability of observing an absolute value smaller than \( |\Lambda| \) scales as \( \infty |\Lambda| \).

The anthropic argument may not simply work if not only \( \Lambda \) but also other physical constants or quantities are varied in different multiverses (Tegmark & Rees 1998; Aguirre 2001; Graesser et al. 2004), but in this work we consider the simplest scenario that only \( \Lambda \) changes in a flat universe. Recently, it has been shown that only \( \Lambda \) changes with a flat \( p(\Lambda) \) among different homogeneous patches of the universe created by inflation, if the theory of gravity is extended to allow any inhomogeneous initial conditions about space-time and matter, in contrast to general relativity in which the four constraints in the Einstein field equations limit the possible initial conditions (Totani 2016).

If we know the expected number density of observers in a universe with the cosmological constant \( \Lambda \), \( n(\Lambda) \), we can calculate the probability distribution \( P(\Lambda) \) per unit \( \Lambda \) for an observer realized in a universe, as

\[
P(\Lambda) = \frac{n(\Lambda)p(\Lambda)}{\int_0^n n(\Lambda)p(\Lambda)d\Lambda}, \quad (1)
\]

where \( n(\Lambda) \) is a comoving density scaled to an early epoch when the effect of \( \Lambda \) is negligible, to correct the difference of late expansion factor by changing \( \Lambda \). Here we consider only \( \Lambda \geq 0 \), because the extension of \( n(\Lambda) \) to the negative range is dependent on the formation time scale of an intelligent observer, which is highly uncertain. If it takes \( \sim 5 \text{ Gyr} \) (appearance of human being after the formation of the Earth), \( n(\Lambda) \) should rapidly drop below \( \Lambda \lesssim -\Lambda_{\text{obs}} \), and hence ignoring the rather narrow range of \( -\Lambda_{\text{obs}} \lesssim \Lambda < 0 \) does not seriously affect the probability calculation (Weinberg 1996, see also Peacock 2007). The probability to observe \( \Lambda \) smaller than the observed value is then

\[
P(<\lambda_{\text{obs}}) = \int_0^{\lambda_{\text{obs}}} P(\Lambda)d\Lambda. \quad (2)
\]

If this is not small, the anthropic principle can be a viable solution to the cosmological constant problem.

Previous studies indeed show that \( P(<\lambda_{\text{obs}}) \) is not extremely small (typically 1–10\%), based on the structure formation theory of the universe with cold dark matter (CDM) (Efstathiou 1995; Martel et al. 1998; Garriga et al. 2000; Peacock 2007). In these studies the total dark matter mass included in collapsed dark haloes was used as an estimator of \( n(\Lambda) \), and it was calculated analytically by formulations like the Press-Schechter theory, but baryonic physics related to formation of galaxies (e.g., gas cooling, star formation, supernova feedback, starbursts by galaxy mergers, metal production and chemical evolution) were not considered, though it should also be important to estimate \( n(\Lambda) \). Furthermore, these studies assumed a minimum mass threshold for dark haloes that can harbor life, with a value similar to that of our Galaxy, to get a probability \( P(<\lambda_{\text{obs}}) \) that is not extremely small. However, such a treatment is clearly ad hoc, and we know that the mass distribution of galaxies extends to dwarf galaxies that are smaller than our Galaxy by a factor of 1000.

In this work, we calculate \( n(\Lambda) \) in a wide range of \( \Lambda \) by using a semi-analytic model of galaxy formation in the framework of cosmological structure formation in a CDM universe. In such a model, the baryonic processes of galaxy formation mentioned above are taken into account, and model parameters are determined to reproduce a variety of observed data including luminosity functions, galaxy number counts, and several empirical relationships like the Tully-Fisher relation (see e.g. Baugh 2006, for a review). The aim of this work is to examine whether a reasonably large \( P(<\lambda_{\text{obs}}) \) is obtained without an ad hoc galaxy mass threshold, when we take into account realistic physical processes of galaxy formation. It is known that the faint-end slope of galaxy luminosity function is flatter than that of dark haloes, and supernova feedback is believed as the primary mechanism working preferentially in low mass haloes to reduce the amount of stars. This would have a similar effect to the mass threshold in the previous studies, but a quantitative computation is necessary, which is the distinctive feature of this work.

We first calculate \( n(\Lambda) \) assuming that the number of life systems in a galaxy, \( N_{\text{life}} \), is proportional to stellar mass of the galaxy, \( M_{*} \). However, formation of a star is not a sufficient condition of a habitable system for an observer. Clearly, we do not expect formation of a terrestrial planet or life around a zero metallicity star. Then there should be a metallicity dependence for the probability of finding an observer in a stellar system. It is known that massive galaxies generally have high metallicities, and this trend is reproduced by galaxy formation models. Then high-metallicity preference of life formation may also work as an effective threshold in galaxy mass, and hence help the anthropic argument of \( \Lambda \). Metallicity evolution of galaxies is calculated in the galaxy formation model used in this work, and we will also study this effect quantitatively.

The outline of this paper is as follows. In Section 2, we describe the galaxy formation model that we use, and methods of calculating \( n(\Lambda) \) in universes with different \( \Lambda \). Section 3 presents our main results, followed by a summary in Section 4. We adopt the "Planck+WP" values reported in Table 2 of Planck Collaboration (2014) for the cosmological parameters of the present universe that we observe.

2 METHODS

2.1 Semi-analytic modeling of galaxy formation

In this work we use a semi-analytic galaxy formation model of Nagashima & Yoshii (2004, hereafter NY04), called the Mitaka model. A semi-analytic modeling of cosmological galaxy formation starts from producing a mock catalog of dark matter haloes and their merger histories in the past. A mock sample is constructed by producing many dark haloes with various masses at an output redshift. The merger history of a dark halo (so-called merger tree) is generated by...
a Monte-Carlo calculation based on the extended Press-Schechter (PS) theory. Then they are summed up with an weight so that their number density obeys to the dark halo mass function. In NY04, the halo mass function was calculated by a fitting formula to \( N \)-body simulation results, which is different from the analytic PS mass function by a factor of at most 1.7. It is highly uncertain whether this fitting formula holds for a value of \( \Lambda \) quite different from that of the standard \( \Lambda \)CDM model, and such a high precision correction is not crucial in this work. Therefore we simply use the PS mass function in this work. We produce about 100 Monte Carlo realisations of dark halo merger trees within a dark matter mass interval of 0.1 dex at an output redshift.

It should be noted that, instead of using the Monte-Carlo approach, another way of generating a mock halo catalog is to directly sample haloes and their merger histories from a cosmological \( N \)-body simulation (e.g., Makiya et al. 2016). Though this gives a more accurate halo mass function and merger histories, it is necessary to run many \( N \)-body simulations for various values of \( \Lambda \) for our purpose, which would be an unrealistically high computational cost.

Baryonic processes such as star formation, supernova feedback, and galaxy mergers are then calculated in these haloes. There are phenomenological model parameters for various baryonic processes, and they are determined to reproduce the observed luminosity functions and cold gas mass fraction of galaxies at \( z = 0 \). Predictions of this model are in agreement with not only various properties of \( z = 0 \) galaxies, but also those at high redshifts, such as cosmic star formation history and luminosity functions of Lyman break galaxies and Lyman \( \alpha \) emitters (Kashikawa et al. 2006; Kobayashi et al. 2007, 2010). It should be noted that these model parameters are for physical processes inside collapsed dark haloes, and they are dependent only on physical properties of a halo (e.g., mass and size), without direct dependence on cosmological parameters. Therefore we fix the baryonic model parameters when we try various values of \( \Lambda \).

### 2.2 Estimating the Amount of Observers, \( n(\Lambda) \)

The cosmological parameters affect the results of the Mitaka model by the dark halo mass function and their merger histories. It should be noted that the standard cosmological parameters, such as \( H_0 \), \( \Omega_M \), \( \Omega_{\Lambda} \), and \( \sigma_8 \) which are inputs to the Mitaka model, depend on the definition of the present time, i.e., \( z = 0 \). In this work the flat geometry is always assumed after inflation in the early universe, according to the standard paradigm. As argued in Sec. 1, we assume that only \( \Lambda \) is changed in different multiverses, and hence all other physical quantities do not change in the early universe when \( \Lambda \) is negligible. Here we define the initial epoch by the cosmic time slightly after the recombination, \( t_1 = 4.7 \times 10^5 \) yr, corresponding to \( z = 1000 \) in our universe. We calculate physical densities of photons (\( \rho_c \)), baryons (\( \rho_b \)), and dark matter (\( \rho_M \)), and the amplitude of the matter density fluctuation (\( \sigma_8 \)) at \( t = t_1 \) by solving the Friedmann equation and the linear perturbation theory with the cosmological parameters of our universe. Then these quantities, i.e., \( \rho_c(t_1), \rho_b(t_1), \rho_M(t_1), \) and \( \sigma_8(t_1) \), are set to be the same among different multiverses.

We need to consider about an output age of the universe that is appropriate to calculate \( n(\Lambda) \). The age of the present universe (13.8 Gyr), the age of the Sun (4.6 Gyr), and the main-sequence life time of the Sun (10 Gyr) are all similar. A typical time scale for a life to evolve into an intelligent observer may also be similar, though we know only one example. Ideally, we should integrate over time all stars that can harbor an intelligent life, and here we estimate this by the total stellar mass density at an age of the universe when the majority of cooled gas is already converted into stars and hence significant more star formation is not expected in future. In the present universe, the age 13.8 Gyr roughly satisfies this condition, because the peak of cosmic star formation (\( z \sim 1–3 \)) has already passed. If we consider a \( \Lambda \) value larger than the observed value, gravitational collapses of dark haloes occur only in earlier epochs, and hence we can use the same age. In the limit of \( \Lambda \rightarrow 0 \), though larger mass dark haloes would be formed at later epochs, star formation is not expected in haloes that are much more massive than our Galaxy. This is because the gas cooling time becomes much longer than the age of the universe, due to high temperature and low density, as seen in warm-hot intergalactic medium and intracluster medium in the present universe. Therefore we calculate the number of observers \( n(\Lambda) \) at a fixed age of \( t_0 = 15 \) Gyr from the Big Bang for any value of \( \Lambda \), and choose of a different output age will be tested later. This defines the zero point of redshift \( z \) for a given value of \( \Lambda \), and we can calculate cosmological parameters of \( H_0, \Omega_M, \Omega_{\Lambda}, \) and \( \sigma_8 \) at \( t = t_0 \) by evolving the universe from \( t = t_1 \) to \( t_0 \). These are used as inputs to the Mitaka model calculation.

The shape of power spectrum \( P(k) \) of linear matter density fluctuation as a function of the comoving wavenumber \( k \) is necessary as another input to the Mitaka model. Here we consistently use the \( P(k) \) shape of our universe at \( z = 0 \) calculated by the code CAMB (Lewis et al. 2000) for any value of \( \Lambda \), because the shape of \( P(k) \) in the linear regime does not evolve significantly after baryon density fluctuation catches up that of dark matter, which occurs shortly after the recombination. To calculate dark halo mass function, their virial radii, and merger histories by the PS and extended PS theories, we need to compute key quantities of the spherical collapse model: \( \delta_0 \), the fractional overdensity extrapolated linearly to a time when a spherically symmetric fluctuation collapses to a dark halo, and \( \theta_* \), the non-linear overdensity (= \( 1 + \delta \)) of a collapsed and virialized object. We calculated these for various values of \( \Lambda \) by analytic formulations given in Nakamura & Suto (1997) for the case of a flat universe. Calculated dark halo mass functions are shown in the top panel of Figure 1 for some values of \( \Lambda \).

From the outputs by the Mitaka model, we can compute cosmic densities of various quantities (e.g., galaxy number or stellar mass) at the fixed output age of \( t_0 \). However, if we change \( \Lambda \), it also changes the expansion factor \( a_0/a_i \) from \( t = t_i \) to \( t_0 \). For calculation of \( n(\Lambda) \) we need to consider an amount in a fixed physical volume defined at the initial epoch, which is not affected by \( \Lambda \). Therefore, we calculate \( n(\Lambda) \) from the comoving densities scaled to the initial epoch of \( t = t_i \), which is denoted with a subscript “i”, e.g., \( \rho_{*i} \equiv \rho_{*}(t_0)(a_0/a_i)^3 \) for stellar mass density.

Then we can calculate \( P(\Lambda) \) from \( n(\Lambda) \) and the prior distribution \( p(\Lambda) \). We assume a flat distribution for \( p(\Lambda) \) only in \( \Lambda \geq 0 \), which is reasonable as discussed in §1 and assumed in previous studies (e.g., Martel et al. 1998).
3 RESULTS

3.1 Effects of Baryonic Processes

First we calculate $n(\Lambda)$ only using dark halo masses, without baryonic physics such as star formation, for comparison with previous studies. Figure 2 shows the expected number of observers $n(\Lambda)$ (top panel) and the probability distribution per unit $\ln \Lambda$, $AP(\Lambda)$ (bottom panel), assuming that $n(\Lambda)$ is proportional to the total dark matter mass of collapsed haloes calculated by the PS mass function at $t_0 = 15$ Gyr. Here, we introduced a mass threshold $M_{DM,1h}$ for dark haloes and only haloes more massive than this are taken into account. Results for several values of $M_{DM,1h}$ are shown in the figure, and the median of $P(\Lambda)$ and the probability $P(<\Lambda_{obs})$ are summarized in Table 1. If we set the threshold close to our Galaxy, $M_{DM,1h} = 5 \times 10^{11} M_\odot$ (Xue et al. 2008), we find the probability $P(<\Lambda_{obs}) = 3.4\%$ that is not extremely small, in agreement with Martel et al. (1998). If we take smaller thresholds, the probability $P(<\Lambda_{obs})$ rapidly decreases to 0.58\% for $M_{DM,1h} = 5 \times 10^6 M_\odot$.

The main result of this paper, i.e., the probability distribution calculated by the galaxy formation model assuming that the number of life systems $N_{life}$ is proportional to stellar mass $M_*$ in a galaxy (and hence $n(\Lambda)$ proportional to the co-moving stellar mass density, $\rho_*$), but without any threshold about dark halo mass, is also shown in Fig. 2. Compared to the results by the PS mass function only, we find that $n(\Lambda)$ drops faster with increasing $\Lambda$ even than the case of the largest $M_{DM,1h} = 5 \times 10^{12} M_\odot$. When $\Lambda$ is large, only small mass haloes can collapse at early epochs. Therefore this result indicates that there must be a mechanism in the galaxy formation model to suppress star formation in low mass dark haloes. Indeed, the supernova feedback is commonly incorporated in galaxy formation models, and it suppresses star formation in low mass haloes where interstellar gas is easily expelled by the energy/momentum input from supernovae. This is essential to make the faint-end of galaxy luminosity functions flatter than that of PS mass function and to match the observed data. This can be seen quantitatively in Fig. 1, where we present dark halo mass functions and galaxy stellar mass functions for some values of $\Lambda$. We find that the median of $\Lambda/\Lambda_{obs}$ in $P(\Lambda)$ to be 11 and the probability $P(<\Lambda_{obs})$ = 6.7\%, which is not very small without an ad hoc dark halo mass threshold.

To further examine the effect of mass threshold in the galaxy formation model, we show $n(\Lambda)$ and $P(\Lambda)$ using the galaxy formation model but assuming that only galaxies more massive than a stellar mass threshold $M_{*,1h}$ can harbor an observer, in Figure 3. This plot shows that changing $M_{*,1h}$ has little effects. The probability $P(<\Lambda_{obs})$ increases only slightly to 8.0\% for $M_{*,1h} = 10^{11} M_\odot$, compared with the no threshold case. This confirms that the amount of stars in low mass galaxies is not significant in the galaxy formation model thanks to the supernova feedback. To show this quantitatively, the fraction $f(>M_*)$ of cosmic stellar mass contained in galaxies whose stellar masses exceed $M_*$ is shown in the bottom panel of Figure 1. When $\Lambda/\Lambda_{obs} = 100$, only small galaxies of $M_* \leq 10^9 M_\odot$ can be formed, but the stellar mass fraction contained in such galaxies is small in the $\Lambda = \Lambda_{obs}$ universe because of the feedback.

3.2 Dependence on Metallicity and Output Age

It is reasonable to expect that metallicity affects the formation of observers, because earth-like planets are composed of heavy elements and known life systems on the earth need various heavy elements. A strength of using the galaxy for-
show the results of introducing a metallicity threshold for formation of a habitable planet, and the other is assuming that the number of life systems in a galaxy is proportional to $N_{\text{life}} \propto M_* Z$ (and hence $n(\Lambda)$ proportional to the comoving density of $M_* Z, \rho_{*Z}$), where $Z$ is metallicity (mass fraction of heavy elements in all baryonic matter).

Figure 4 and Table 2 show the results of introducing a metallicity threshold $Z_{\text{th}}$, under which no life is assumed to exist. The dependence on $Z_{\text{th}}$ is not strong; $P(<\Lambda_{\text{obs}})$ is slightly increased from 6.7% to 9.0% by increasing $Z_{\text{th}}$ from zero to $Z_{\odot}$. This implies that the stellar mass in low metallicity galaxies is not a significant fraction. This is indeed expected from the well-known mass-metallicity relation (Garnett 2002; Tremonti 2004; Savaglio 2005; Erb et al. 2006; Lee et al. 2006); metallicity is tightly correlated with stellar mass of galaxies, and the stellar mass in low mass (i.e., low metallicity) galaxies is not a significant fraction in the universe (see the previous section). The result for the case of $N_{\text{life}} \propto M_* Z$ is also shown in Figure 4. We find $P(<\Lambda_{\text{obs}}) = 9.7\%$, which is the largest among all the calculations tried in this work, though the increase from the case of no metallicity effect (6.7%) is modest.

Finally, we check the dependence of our results on the output age of the galaxy formation model, $t_0$, for which 15 Gyr was adopted in our standard calculation. In Table 3 we show the results for $t_0 = 10, 15, 20$ Gyr, for the models assuming $N_{\text{life}} \propto M_*$ or $M_* Z$. The dependence on $t_0$ is indeed small, well within theoretical uncertainties. This confirms that our results are not significantly changed by a choice of $t_0$ in a reasonable range.

### Table 1. The median of the probability distribution $P(\Lambda)$ and the probability of finding $\Lambda$ smaller than the observed value, $P(<\Lambda_{\text{obs}})$, as a function of the mass threshold of dark haloes, $M_{\text{DM,th}}$. These results assume that the number of observers is proportional to the amount of dark matter mass in collapsed haloes, without taking into account galaxy formation.

| $M_{\text{DM,th}} [M_\odot]$ | $5 \times 10^8$ | $5 \times 10^9$ | $5 \times 10^{10}$ | $5 \times 10^{11}$ | $5 \times 10^{12}$ |
|------------------------|----------------|----------------|----------------|----------------|----------------|
| median $\Lambda/\Lambda_{\text{obs}}$ | 197 | 112 | 55.6 | 27.3 | 10.9 |
| $P(<\Lambda_{\text{obs}})$ [\%] | 0.58 | 0.97 | 1.9 | 3.4 | 7.7 |

### Figure 3. The same as Fig. 2, but for the results using the galaxy formation model ($N_{\text{life}} \propto M_*$) with various values of stellar mass threshold $M_{*,\text{th}}$.

### Figure 4. The same as Fig. 3, but showing dependence on metallicity. Models assuming a metallicity threshold $Z_{\text{th}}$ for an observer to exist are shown for some values of $Z_{\text{th}}$. Another model assuming that the number of observers $N_{\text{life}}$ is proportional to $M_* Z$ in a galaxy is also shown.

### Table 2. The same as Table 1, but showing the dependence on the metallicity threshold $Z_{\text{th}}$ in the calculations using the galaxy formation model, assuming $N_{\text{life}} \propto M_*$ in galaxies whose metallicity is larger than $Z_{\text{th}}$.

| $Z_{\text{th}} [Z_{\odot}]$ | 0 | 0.1 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|------------------------|---|-----|-----|-----|-----|-----|-----|
| median $\Lambda/\Lambda_{\text{obs}}$ | 11 | 9.7 | 9.4 | 9.3 | 9.3 | 9.2 | 9.1 |
| $P(<\Lambda_{\text{obs}})$ [\%] | 6.7 | 7.3 | 7.5 | 7.7 | 8.0 | 8.4 | 9.0 |

### Table 3. The same as Table 1, but showing the dependence on the output age $t_0$, for the case of using the galaxy formation model assuming that the number of observers $N_{\text{life}}$ is proportional to either $M_*$ or $M_* Z$ in a galaxy.

| $t_0$ [Gyr] | 10 | 15 | 20 |
|------------------------|-----|-----|-----|
| Models assuming $N_{\text{life}} \propto M_*$ | 12 | 11 | 12 |
| $P(<\Lambda_{\text{obs}})$ [\%] | 6.3 | 6.7 | 6.2 |

| $t_0$ [Gyr] | 10 | 15 | 20 |
|------------------------|-----|-----|-----|
| Models assuming $N_{\text{life}} \propto M_* Z$ | 9.3 | 8.4 | 9.1 |
| $P(<\Lambda_{\text{obs}})$ [\%] | 8.5 | 9.7 | 9.2 |

### 4 SUMMARY

In this work we examined the anthropic argument to explain the cosmological constant problem by a semi-analytic model of cosmological galaxy formation, assuming that an observable universe is created with a variable value of $\Lambda$ obeying to a nearly flat prior probability distribution per unit $\Lambda$, while any other physical parameter does not change. The galaxy formation model used here produces a mock catalog of dark matter haloes and their merger history by Monte-
Carlo simulations based on the structure formation theory in the ΛCDM universe. Various astrophysical processes such as gas cooling, star formation, and supernova feedback are phenomenologically modeled to produce galaxies in dark haloes with physical quantities such as stellar mass and metallicity. Astrophysical model parameters have been determined to reproduce various observed data, and they are assumed not to change for different Λ.

Assuming that the number of observers $N_{\text{life}}$ is proportional to stellar mass $M_\odot$ in a galaxy, we find a median in the probability distribution $P(\Lambda)$ to be $\Lambda/\Lambda_{\text{obs}} = 11.0$, and the probability of finding $\Lambda \leq \Lambda_{\text{obs}}$ to be $P(\Lambda < \Lambda_{\text{obs}}) = 6.7\%$. It should be noted that we obtained this result without introducing any galaxy mass threshold, which is in contrast to previous results based only on the formation history of dark matter haloes. Using the PS formalism and assuming that the number of observers is proportional to dark matter mass of collapsed haloes, we confirmed the previous results that a mass threshold close to our Galaxy halo must be assumed to get a probability that is not extremely small: $P(\Lambda < \Lambda_{\text{obs}}) = 3.4\%$ for $M_{\text{DM,obs}} = 5 \times 10^{11} M_\odot$, though there exist much smaller galaxies. If we take a smaller threshold of $5 \times 10^8 M_\odot$, the probability reduces to 0.58%. Our result using the galaxy formation model can be understood by the supernova feedback taken into account in the model; a significant fraction of dark matter mass is distributed in small mass haloes, but star formation in such haloes is suppressed by the feedback.

We also tested the possibility that the number of observers depends on metallicity of galaxies. Introducing a metallicity threshold does not change the probability $P(\Lambda < \Lambda_{\text{obs}})$ significantly; it increases from 7.3 to 9.0% for $Z_{\text{th}} = 0.1$ to 1.0 $Z_\odot$. This is because low metallicity galaxies are generally small galaxies (the mass-metallicity relation), and such galaxies do not include a significant fraction of stellar mass in the universe due to the supernova feedback. If we assume that the number of observers is proportional to $N_{\text{life}} \propto M_\odot Z$ of galaxies, we found $P(\Lambda < \Lambda_{\text{obs}}) = 9.7\%$.

We conclude that a reasonable estimate of the probability to find a small Λ as observed is 7–10%, which is not extremely small, based on a realistic model of galaxy formation. Therefore the anthropic argument is a viable explanation for the cosmological constant problem. If, in future, a convincing theory is established by fundamental physics predicting that only Λ is variable with a flat prior distribution when a universe is created, the anthropic argument may become the leading candidate for the solution of the cosmological constant problem.

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