Gluon fragmentation into S-wave heavy quarkonium

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Received: date / Revised version: date

Abstract. Fragmentation is the dominant production mechanism for heavy hadronic bound states with large transverse momentum. We numerically calculate the initial $g \to H(QQ)$ fragmentation functions (FFs) using the nonrelativistic QCD factorization approach. Our analytical expression of FFs depends on both the momentum fraction $z$ and the transverse momentum of the gluon, and contains most of the kinematical and dynamical properties of the process. Specifically, using the perturbative QCD we present the FF for a gluon to split into S-wave charmonium meson $H_c$ to leading order in the QCD coupling constant.

1 Introduction

Heavy quarkonia, as the bound states of a heavy quark and antiquark are the simplest particles when the strong interactions are concerned. Heavy quarkonium production in antiquark are the simplest particles when the strong interaction scheme is normally suppressed. Fragmentation refers to the process of a parton with high transverse momentum which subsequently decays to form a jet containing the expected hadron [2]. The $QQ$ pair is created with a separation of order $1/m_Q$, where $m_Q$ stands for the mass of the heavy quark $Q(=c,b)$. The lowest states in the charmonium ($\eta_c, J/\psi$) and bottomonium ($T$) systems have typical radius that are significantly smaller than those of hadrons containing light quarks. They have simple internal structures, consisting primarily of a nonrelativistic quark and antiquark, so in recent years a great deal of theoretical effort has been focused on the nonrelativistic QCD factorization approach [3] to calculate the quarkonium production rates.

Generally, according to the factorization theorem of QCD [4] the production of heavy quarkonium $H$ in the typical scattering process of $A + B \to H(k_T) + X$, can be expressed as

$$d\sigma = \sum_{a,b,c} \int_0^1 dx_a \int_0^1 dx_b \int_0^1 dz f_{a/A}(x_a, \mu) f_{b/B}(x_b, \mu) \times$$
$$\frac{d}{d\ln \mu^2} D_{i\to H}(z, \mu) = \sum_j \int_z^1 \frac{dy}{y} P_{ij}(\frac{z}{y}, \mu) D_{j\to H}(y, \mu),$$

where $\mu$ is a factorization scale, $a$ and $b$ are incident partons in the colliding initial hadrons $A$ and $B$ respectively, $f_{a/A}$ and $f_{b/B}$ are the parton distribution functions at the scale $\mu$, $c$ is the fragmenting parton (either a gluon or a quark) and $X$ stands for the unobserved jets. Here, $D_{c\to H}(z, \mu)$ is the fragmentation function (FF) at the scale $\mu$ which can be obtained by evolving from the initial FF $D_{c\to H}(z, \mu_0)$ using the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) renormalization group equations [5].

In production of heavy quarkonia with sufficiently large transverse momentum $k_T$, the dominant mechanism is actually fragmentation while the direct leading order production scheme is normally suppressed. Fragmentation refers to the process of a parton with high transverse momentum which subsequently decays to form a jet containing the expected hadron [2]. The $QQ$ pair is created with a separation of order $1/m_Q$, where $m_Q$ stands for the mass of the heavy quark $Q(=c,b)$. The lowest states in the charmonium ($\eta_c, J/\psi$) and bottomonium ($T$) systems have typical radius that are significantly smaller than those of hadrons containing light quarks. They have simple internal structures, consisting primarily of a nonrelativistic quark and antiquark, so in recent years a great deal of theoretical effort has been focused on the nonrelativistic QCD factorization approach [3] to calculate the quarkonium production rates.

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$$d\sigma(a + b \to c + X) D_{c\to H}(z, \mu),$$

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In [6] in the framework of $e^-e^+$ annihilation and afterward was proved in a more general way in Ref. [4]. The importance of FFs is for the model independent predictions of the cross sections at the LHC in which a hadron is detected in the outgoing productions as a colorless bound state.

In [6], using the perturbative QCD we calculated the initial scale FF for c-quark to split into S-wave D-meson considering the effect of hadron mass. There, we compared our result with the current well-known phenomenological models and we also compared the FF with experimental data from BELLE and CLEO. Our result was in good consistency with the other ones.

In Ref. [7] we studied the importance of gluon FF by accounting the effect of gluon fragmentation on the scaled-
energy distribution \((x_B)\) of bottom-flavored hadrons \(B\) inclusively produced in top-quark decays in the standard model \((t \to bW^+ (+g) \to BW^+ + X)\) and in the general two Higgs doublet model \((t \to bH^+ (+g) \to BH^+ + X)\). We found that gluon fragmentation leads to an appreciable reduction in the partial decay width at low values of \(x_B\) and for higher values of \(x_B\) the NLO result is practically exhausted by the \(b \to B\) contribution.

In this paper we calculate the gluon FF into the S-wave heavy quarkonium \(H\left(D^H_H(z, \mu_0)\right)\) by calculating a specific physical process in perturbative QCD in the finite momentum frame of the fragmenting gluon. Specifically, we focus on the S-wave charmonium \(H_c\) (i.e. \(\eta_c, J/\psi\)) as a heavy quarkonium and present our result for the \(D^H_H(z, \mu_0)\)-FF, where the \(z\) is defined in the Lorentz boost invariant forms as \[10\]

\[
\begin{align*}
  z &= \frac{E^H + k_L^H}{E^g + k_L^g}, \quad (3) \\
  z &= \frac{k^g \cdot k^H}{(k^g)^2}, \quad (4) \\
  z &= \sqrt{M^2_{H^+} + (k_L^g)^2} / E^g, \quad (5) \\
  z &= \frac{E^H + k_L^H}{E^g + k_L^g}, \quad (6) \\
  z &= \frac{k_L^H}{k_L^g}. \quad (7)
\end{align*}
\]

In the above equations we take the \(z\)-axis along the momentum of outgoing meson and \(E^g, k_L^g, E^H\) and \(k_L^H\) are the energies and longitudinal components \((z\)-components\) of the four-momenta of the fragmenting gluon and the produced heavy quarkonium \(H\), respectively. The first definition \((3)\) is the usual light-cone form. The first and second definitions of \(z\) are hard to be employed in the application of the gluon FFs, because they involve the longitudinal momentum of the resulting heavy quarkonium. Instead, usually the non-covariant definitions \((\text{Eqs. \,(5)-(7)}\) are used approximately, which are convenient for the finite momentum frame. In \([10]\), authors analyzed the uncertainties induced by different definitions of \(z\) in the application of gluon to heavy quarkonium FF. They calculated the initial \(g \to J/\psi\) FF by calculating the differential cross section of process \(g + q \to g + q^* (\to J/\psi + g + g)\). They showed that the FFs have strong dependence on the gluon momentum \(k\), and when \(|k| \to \infty\) these FFs approach to the FF in the light-cone definition \((3)\) and large uncertainties remain while the non-covariant definitions of \(z\) are employed.

The FFs are related to the low energy part of the hadron production and they consist of the nonperturbative aspects of QCD. There are two main approaches for evaluating the initial scale FFs. In the first approach called the phenomenological approach, these functions are extracted from experimental data analysis instead of theoretical calculations. This scheme is explained in more detail in section 2. The second approach is based on the fact that the FFs for mesons containing, at least, a heavy quark can be calculated theoretically using perturbative QCD \((pQCD)\) \([11, 12, 13, 14, 15]\). We employ this approach to drive an exact analytical form of FF for the transition of \(g \to H_c\).

This paper is organized as follows. In Sec. 2 we explain the phenomenological approach to calculate the FFs and introduce some well-known phenomenological models. In Sec. 3 the theoretical approach to calculate the FFs is introduced in detail. We discuss the use of \(pQCD\) in calculating the fragmentation of a charm quark into the heavy charmonium \(H_c\) and in Sec. 4 our conclusion is summarized.

2 Phenomenological determination of FFs

The FFs describe hadron production probabilities from the initial partons and their importance is for model independent predictions of the cross sections and decay rates at the LHC. They can also be applied to detect the internal structure of the exotic hadrons using the differences between the disfavored and favored FFs \([16]\). One of the main approaches to evaluate the FFs, which is normally called the phenomenological approach, is based on the experimental data analyzing. In this approach, the FFs are mainly determined by hadron production data analysis of \(e^- e^+\) annihilation, lepton-hadron deep inelastic scattering (DIS) and hadron-hadron scattering processes by working either in Mellin-N space \([17, 18]\) or in \(x\)-space \([19, 20, 21]\). Among these methods, the FFs are mainly determined by hadron production data of \(e^- e^+\) annihilation, because there are more accurate data for this process.

In this approach, according to Collin’s factorization theorem \([4]\) the cross section of hadron production in the \(e^- e^+\) annihilation is expressed by the convolution of partonic hard-scattering cross sections \((e^- e^+ \to qq (+g))\), and the nonperturbative FFs \(D^H_H(z, Q^2)\), describing the transition of a parton into an outgoing hadron \(H\),

\[
\frac{d}{dz} \sigma(e^- e^+ \to HX) = \sum_{i=g,u,d,s,\ldots} C_i(z, \alpha_s) \otimes D^H_H(z, Q^2),
\]

where, \(C_i(z, \alpha_s)\) are the Wilson coefficient functions based on the partonic cross sections which are calculated in the perturbative QCD \([22, 23]\), and the convolution integral is defined as \(f(x) \otimes g(x) = \int f(y) g(x/y) dy\). Here, \(z = 2E_H / \sqrt{s}\) is the fragmentation parameter where \(E_H\) is the energy of observed hadron and \(s = Q^2\) is the squared center-of-mass energy. In fact, the fragmentation parameter \(z\) refers to the energy fraction of process which is taken away by the detected hadron.

In the phenomenological scheme, the FFs are parameterized in a convenient functional form at the initial scale \(\mu_0\) in each order, i.e. leading order (LO) and next-to-leading order (NLO). The initial scale \(\mu_0\) is different for partons and the initial FFs are evolved to the experimental \(\mu\).
points by the DGLAP equations (24). The FFs are parameterized in terms of a number of free parameters which are fixed by an $\chi^2$ analysis of the $e^+e^- \to H + X$ data at the scale $\mu^2 = m$. In (23), using this scheme we presented a new functional form of $\pi^+ / K^+$ FFs up to next-to-leading order through a global fit to single-inclusive electron-positron annihilation data. The situation is very similar to the one for determination of PDFs.

Various phenomenological models like Peterson model (24), Lund model (25), Cascade model (26) etc., have been developed to describe the fragmentation processes. In (27), authors reported the nonperturbative $B$-hadron FFs that were determined at NLO in the ZM-VFN scheme through a joint fit to $e^+e^-$-annihilation data taken by ALEPH (28) and OPAL (29) at CERN LEP1 and by SLD (30) at SLAC SLC. Specifically, the power ansätze $D(z, \mu_F^{(n)}) = Nz^n(1-z)^\beta$ with three free parameters was used as the initial condition for the $b \to B$ FF at $\mu_F^{(n)} = 4.5$ GeV, while the gluon and light-quark FFs were generated via the DGLAP evolution. The fit yielded $N = 4684.1$, $\alpha = 16.87$, and $\beta = 2.628$. In Ref. (21), authors determined the FFs for $D^0$, $D^+$ and $D^{*+}$ mesons by fitting the data from the BELLE, CLEO, ALEPH, and OPAL Collaborations in the modified minimal-subtraction ($\overline{MS}$) factorization scheme by considering the model suggested by Bowler (31), as $D^H_q(z, \mu_0) = Nz^{-1+\gamma}(1-z)^\alpha e^{-\gamma/z}$. At the scale $\mu = m_c = 1.5$ GeV, the Bowler model is taken for the $c$-quark FF, while the FFs of the light quarks $q = (u, d, s)$ and the gluon are set to zero. Then these FFs are evolved to higher scales using the DGLAP equations at NLO. In Figs. 1 and 2, the behavior of $g \to H (= B, D^\pm)$ FFs at the scales $\mu = 10.52$ GeV and $\mu = m_c = 91.2$ GeV are shown. The scale of $\mu = 10.52$ GeV, which is much close to the production threshold of D-mesons, has been set as the center-of-mass energy of $e^+e^-$ annihilation by the Belle and the CLEO Collaborations (32).

3 Theoretical determination of FFs

The second current approach for calculating the FFs is based on the fact that the FFs for hadrons containing a heavy quark can be computed theoretically using perturbative QCD (pQCD) (11,12,13,14,15). The first theoretical attempt to explain the production procedure of hadrons containing a heavy quark or a heavy antiquark was made by Bjorken (33) by using a naive quark-parton model (QPM). He construed that the inclusive distribution of heavy hadron should peak nearly at $z = 1$, where $z$ stands for the scaled energy variable. This property is mainly important for heavy quarks for which the peak of heavy quark FF occurs closer to $z = 1$. In following, Peterson (24) proposed the popular form of FF which manifestly behaves as $(1-z)^2$ at large $z$ values, using a nonrelativistic quantum mechanical parton model. The perturbative QCD approach was followed by Suzuki (34), Ji and Amiri (35). While in this approach Suzuki calculates the heavy FFs using a Feynman diagram similar to that in Fig. 1. Here, we focus on gluon fragmentation into a heavy quarkonium considering a special example: $g \to H_c(= \eta_c, J/\psi)$, and drive an exact analytical form of $D^H_q(z, \mu_0)$ using the Suzuki’s approach which embeds most of the kinematical and dynamical properties of the process. Our result can be directly used for the bottomonium state ($T$-system) with some simple replacements.

In a hadron collider, a charmonium meson with large transverse momentum $k_T$ can either be produced directly at large $k_T$ or can be produced indirectly by the decay of a $B$ meson or a higher charmonium state with large $k_T$. A typical Feynman diagram which contributes to the production of the charmonium state at the order-$\alpha_s^3$ is shown in Fig. 3a and the order-$\alpha_s^4$ radiative corrections to this process is shown in Fig. 3b. In most regions of phase space,
The virtual gluons in Fig. 3 are off their mass shells by values of order \( k_T \), and the contribution from this diagram is suppressed relative to the diagram in Fig. 3(b) by a power of the strong coupling constant \( \alpha_s(k_T) \).

Using the Suzuki’s approach which is based on the perturbative QCD scheme, we obtain the analytical form of FF for gluon to split into the charmonium state considering the Feynman diagram for \( g \to H(\bar{c}c) + g \) in the order of \( \alpha_s^4 \). This diagram along with the spins and the four-momenta of meson and partons is shown in Fig. 4.

Following Ref. 36, we adopt the infinite momentum frame where the fragmentation parameter in the usual light-cone form (3) is reduced to the more popular form \( z = E^H/E^g = \bar{P}_0/P_0 \), where we also may write the parton energies in terms of the initial gluon energy \( E^g \).

Considering the four-momenta in Fig. 4, we start with the definition of FF introduced in Refs. 36,57 as

\[
D_{g \to H_s}(z, \mu) = \int d^3p d^3k d^3k' |T_M|^2 \delta^3(k' + p - k - p'),
\]

where the average probability amplitude squared \(|T_M|^2\) is calculated as \( \sum_i |T_{M,i}|^2 / (1 + 2s_i) \) in which the summation is going over the spins and colors and \( s_i \) is the initial gluon spin. The probability amplitude to split a gluon into the meson \( (T_M) \) is expressed as the convolution of the hard scattering amplitude \( T_H \) which is, in essence, the partonic cross section to produce a heavy quark-antiquark (\( QQ \)) pair with certain quantum numbers, and the process-independent distribution amplitude \( \Phi_M \), i.e.,

\[
T_M = \int [dx_i] T_H(x_i, Q^2)\Phi_M(x_i, Q^2),
\]

where \( [dx_i] = dx_1dx_2\delta(1 - x_1 - x_2) \). The short-distance coefficient \( T_H \) can be calculated as perturbation series in the strong coupling constant \( \alpha_s \). The long-distance distribution amplitude \( \Phi_M \) which contains the bound state nonperturbative dynamic of produced meson, is the probability amplitude for a \( QQ \) pair to evolve into a particular heavy quarkonium state. The probability amplitude \( \Phi_M \) is related to the mesonic wave function \( \Psi_M \) by

\[
\Phi_M(x_i, Q^2) = 2(2\pi)^3\delta^2\left( \sum_{j=1}^2 q_{L,j} \right) \prod_{i=1}^2 \frac{d^2q_{L,i}}{(2\pi)^3} \times \Psi_M(x_i, q_{L,i}) \delta(q_{L,i}^2 < Q^2),
\]

where, \( \delta(x) \) is the Heaviside step function and \( q_{L,i} \) stands for the transverse momentum of constituent quarks. A simple nonrelativistic wave function is given as \( 38 \)

\[
\Psi_M(x_i, q_{L,i}) = \frac{(128\pi^3 b^5 M)^{1/2}}{x_1^2 x_2^2 \{M^2 - \frac{m_1^2 + q_{L,1}^2}{x_1} - \frac{m_2^2 + q_{L,2}^2}{x_2} \}^{3/2}},
\]

where \( M \) is the meson mass and \( b \) is the binding energy of the mesonic bound state. Working in the infinite-momentum frame we integrate over \( q_{L,i} \) for \( 0 \leq q_{L,i}^2 < \infty \) where \( q_{L,i} \) stands for either \( q_{L,1} \) or \( q_{L,2} \). The integration yields

\[
\Phi_M(x_i, Q^2) = \frac{(128\pi b^5 M)^{1/2}}{16\pi^2(x_1 + x_2)(m_1^2 x_2 + m_2^2 x_1 - x_1 x_2 M^2)^{3/2}}.
\]
which grows rapidly at $x_1 = 1 - x_2 = m_1/M$ when $M$ is set to $m_1 + m_2$. Therefore this function can be estimated as a delta function \[ \delta(x_1 - \frac{m_1}{m_1 + m_2}). \] In conclusion, the probability amplitude for a S-wave pseudoscalar meson at large $Q^2$, reads

\[ \Phi_M \approx \frac{f_M}{2\sqrt{3}} \delta(x_1 - \frac{m_1}{m_1 + m_2}), \]  

(15)

where $f_M$ refers to the decay constant for the meson. The delta-function form is convenient for our assumption where we ignore the relative motion of quark and antiquark and thus the constituent quarks are emitted collinearly with each other and they have no transverse momentum. However, as in Ref. [40] mentioned, the squared coupling constant. Performing an average over the initial collinearly with each other and they have no transverse momentum. However, as in Ref. [40] mentioned, the squared coupling constant. Performing an average over the initial

\[ \sum_{\text{initial}} \frac{1}{2} + \frac{1}{2} \]  

(16)

where, using the feynman rules the transition amplitude $M$ reads

\[ M = \frac{g_\epsilon^2C_F}{(k + k')^2 - m_c^2} \langle \bar{u}(k,s_1)\Gamma\nu(p,s_2) \rangle. \]  

(17)

Here, $\Gamma = \epsilon^{\mu\nu}(k + k' + m_c)\epsilon^{\nu}(p,s)$ where $\epsilon$ is the polarization vector of gluon, $C_F$ is the color factor and $g_\epsilon$ is the strong coupling constant. Performing an average over the initial spin states and a sum over the final spin states, the mean amplitude squared reads

\[ |M|^2 = \frac{1}{1 + 2s_1} \sum_s M^* M \]  

(18)

where, using $\sum_s e^{\mu\nu}(p,s)e^{\nu}(p,s) = -g^{\mu\nu}$ we have

\[ \tilde{\omega} = \sum_s L^{\mu\nu}L_{\mu\nu} = \sum_{s_1,s_2}^{12} \text{Tr}[\Gamma(y - m_c)\Gamma(y + m_c)] = 32(p.k)(k'k') - 32m_c^2(p.k) - 32m_c^2(p.k') - 64m_c^2(k.k') - 64m_c^2. \]  

(19)

To obtain the FF for an unpolarized meson, considering [10],[19] we have

\[ D_{g\rightarrow H_c}(z,\mu_0) = \frac{m_c^4g_\epsilon^4C_F^2}{24} \frac{1}{12} \int \frac{\tilde{\omega}d^3k'}{[(k + k')^2 - m_c^2]^2} \times \int \frac{d^3k}{k_0p_0k'_0} \int \frac{d^3p_3d^3p_4}{D_0p_0} \]  

(20)

where $D_0 = p_0 + k_0 + k'_0 - p'_0$ is the energy denominator. To perform the phase space integration we consider the following integral

\[ \int \frac{d^3p_3d^3p_4}{D_0p_0} = \frac{[m_c^2 + (k + k')^2 + 2p_3(k + k')]}{[m_c^2 + (k + k')^2 + 2p_3(k + k')]} \]  

(21)

where, considering [9] one has

\[ (k + k')^2 = m_c^2 + \frac{k_0^2}{M^2} \frac{2}{2(1 - z)} + \frac{1}{2} - 1], \]  

(22)

We also write

\[ \int \frac{d^3k'}{D_0p_0} \approx k'_0f(z, (k'_0^2)), \]  

(23)

where, for simplicity, we replaced the transverse momentum integration by its average value \( \langle k'_0^2 \rangle \), which is a free parameter and can be specified experimentally. Putting all in [10] we obtain the fragmentation function as

\[ D_{g\rightarrow H_c}(z,\mu_0) = \frac{N_z}{F(z, (k'_0^2))} \times \left\{ \frac{[z^2k_0^2 + M^2(1 - z)^2]}{Mz(1 - z)} \right\}^{1 - 6m_c^2}, \]  

(24)

where,

\[ F(z, (k'_0^2)) = \left[ \frac{2m_c^2}{M^2} + \frac{k_0^2}{M^2} \frac{z}{1 - z} + \frac{1}{2} - 1 \right]^2 \times \left[ \frac{k_0^2}{M^2} \frac{z}{1 - z} + \frac{1}{2} - 1 \right]^2, \]  

(25)

and $N$ is proportional to $(\pi C_F\alpha_s f_M)^2$ but it is related to the normalization condition $\int_{1}^{4} D_{g\rightarrow H_c}(z,\mu_0)dz = 1$ [35].

In general, fragmentation function $D_{g\rightarrow H_c}$ depends on both the fragmentation parameter $z$ and the factorization scale $\mu$. The function [24] should be regarded as a model for the gluon FF at the scale $\mu_0$ of order $2m_c$. For values of $\mu$ much larger than $\mu_0$, the obtained FF should be evolved from the scale $\mu_0 = 2m_c$ to the desired scale $\mu$ using the DGLAP equation [42].

In Fig. [5] the FF of $g \rightarrow H_c$ at the starting scale $\mu_0 = 2m_c = 3$ GeV is shown. The behavior of $D_{g\rightarrow H_c}$ is shown for different values of the transverse momentum of the gluon. Note that one of the uncertainties in determination of FFs is due to the freedom in the choice of scaling variable, i.e. the covariant and non-covariant definitions presented in Eqs.(3)-(7). As a comparison, our result shown in Fig. [5]
is in reliable consistency with the result presented in Fig. 3 of Ref. [10] when the third definition of fragmentation parameter (Eq. (5)) is applied, i.e. \( z = \frac{M_H^2 + (k_L^H)^2}{E^9} = \frac{E^H}{E^9} \). Since there are no phenomenological results for the \( g \rightarrow H_c \) FF, then at the moment it is not possible for us to compare our result phenomenologically, but the behavior of our obtained FF in comparison with the \( g \rightarrow B \) and \( g \rightarrow D^+ \) FFs, shown in Figs. 1 and 2 assures one to rely on our result.

4 Conclusion

The dominant production mechanism for heavy quarkonium at high transverse momentum is fragmentation; the production of a high energy parton followed by its splitting into S-wave charmonium states to leading order in \( \alpha_s \). We used a different approach, Suzuki’s approach, in getting them from the normal analytic calculation in the literatures [10,12] and found good agreement with the result in [10] when use the normal definition of the fragmentation parameter, i.e. \( z = \frac{E^H}{E^9} \). Finally, although the fragmentation function obtained in this work is schematically for charmonium, in fact it can be directly applied to the S-wave bottomonium state \( \Upsilon \) except that \( m_c \) is replaced by \( m_b \) and the decay constant \( f_M \) is the appropriate constant for the bottomonium mesons.

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