ON THE NUMBER OF STABILIZER SUBGROUPS IN A FINITE GROUP ACTING ON A MANIFOLD

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Abstract. If a finite \( p \)-group \( G \) acts continuously on a compact topological manifold \( M \) then, with some bound \( C \) depending on \( M \) alone, \( G \) has a subgroup \( H \) of index at most \( C \) such that the \( H \)-action on \( M \) has at most \( C \) stabilizer subgroups. This result plays a crucial role in the proof of a deep conjecture of Ghys [5].

1. Introduction

It is a basic question how the topology of a manifold restricts the algebraic and geometric complexity of finite group actions on the manifold. The following deep conjecture of Ghys points in this direction.

Conjecture 1.1. For each compact differentiable manifold \( M \) there is a constant \( I \) such that every finite subgroup \( G \) of the diffeomorphism group of \( M \) has a nilpotent subgroup of index at most \( I \).

Originally, about twenty years ago, Ghys conjectured something stronger: the group \( G \) should contain an abelian subgroup of bounded index. That was disproved in [6], so Ghys modified the conjecture to the one above. The conjecture (in a more general form, for the homeomorphism group of topological manifolds) has been proved recently in [5]. One of the main technical tools used in [5] is a weaker version of Theorem 1.3 below. Theorem 1.3 is likely to have a number of further applications, see [4] and [12] discussed below.

Classical Smith theory gives strong restrictions on the topology of fixed point submanifolds, and Borel’s Fixed Point Formula describes locally the possible ways how these submanifolds can meet at any single point. To complete this picture, our paper focuses on bounds on the number of stabilizer subgroups. As Smith theory and Borel’s formula, our results apply only to actions of finite \( p \)-groups.

Definition 1.2. Let \( G \) be a group acting on a set \( X \). Denote by \( G_x \) the stabilizer subgroup of an element \( x \in X \). For subgroups \( H \leq G \), we denote by \( X^H \) the fixed point subset of \( H \). We will study the set of
all stabilizer subgroups:

\[ \text{Stab}(G, X) = \{ G_x \mid x \in X \}. \]

Our main theorem is the following.

**Theorem 1.3.** Let \( M \) be a topological manifold such that \( H_*(M; \mathbb{Z}) \) is finitely generated (as an abelian group). Then there is a number \( C \) depending on \( \dim(M) \) and \( H_*(M; \mathbb{Z}) \) with the following property. For every prime \( p \), every finite \( p \)-group \( G \) acting continuously on \( M \) has a characteristic subgroup \( H \leq G \) of index at most \( C \) containing the center of \( G \) such that

\[ |\text{Stab}(H, M)| \leq C. \]

**Remark 1.4.** In this theorem, it is necessary to consider a subgroup \( H \). Indeed, let \( p \) be a fixed prime, \( n > 0 \) an arbitrary natural number, \( T^p = \{ (z_1, \ldots, z_p) \in \mathbb{C}^p : z_1 \cdots z_p = 1, |z_i| = 1 \text{ for all } i \} \) a \((p - 1)\)-dimensional torus. The subgroup \( H = \{ (z_1, \ldots, z_p) \in T^p : (z_i)^p = 1 \text{ for all } i \} \) of \( T^p \) acts on \( T^p \) by left translations, the cyclic group \( A \) of order \( p \) acts both on \( T^p \) and \( H \) by cyclic permutations of the coordinates \( z_i \). These actions generate an action of the semidirect product \( G = A \rtimes H \) on \( T^p \). This action has \( p^{(p-1)n} \) different stabilizer subgroups, the conjugates of \( A \) in \( G \), however, each of these intersects the subgroup \( H \triangleleft G \) in the trivial group.

Classical Smith theory allows one to bound the cohomology of fixed point submanifolds, and their complements (see Proposition 2.5). Combining this with Theorem 1.3, we can bound the cohomology of many other interesting submanifolds. For example, the following corollary, which is used in [5], provides a technique to reduce a statement on arbitrary actions to the study of free actions.

**Corollary 1.5.** Let \( M \) be a topological manifold such that \( H_*(M; \mathbb{Z}) \) is finitely generated (as an abelian group). Then there is a number \( \tilde{C} \) depending on \( \dim(M) \) and \( H_*(M; \mathbb{Z}) \) with the following property. For every prime \( p \), every finite \( p \)-group \( G \) acting continuously on \( M \) has a characteristic subgroup \( H \leq G \) of index at most \( \tilde{C} \) containing the center of \( G \) such that if \( K \) is minimal among the subgroups in \( \text{Stab}(H, M) \), then the subset \( \tilde{M}^K = \{ x \in M \mid H_x = K \} \) is open and

\[ \dim H_*(\tilde{M}^K; \mathbb{F}_p) \leq \tilde{C}. \]

**Remark 1.6.** If \( M \) is connected and the subgroup \( H \leq G \) given by the corollary acts effectively on \( M \), then \( \{1\} \) is the unique minimal element
Remark 1.7. Similar bound holds for all subgroups $K \in \text{Stab}(H, M)$. In general, $\hat{M}^K$ may not be a manifold, just a disjoint union of cohomology manifolds, and we cannot control its singular homology. Instead, we can bound its Borel-Moore homology with compact support:

$$\dim H^*_c(\hat{M}^K; \mathbb{F}_p) \leq \hat{C}.$$ 

The above results have many important applications. Theorem 1.3 and a variation of Corollary 1.8 play a crucial role in the proof of the Ghys conjecture [5]. Further applications of Theorem 1.3 are presented in [4]. Finally, Theorem 1.3 is a key ingredient in the proof of the almost fixed point property for compact topological manifolds with non-zero Euler characteristic in [12]. The latter means that if $M$ is a compact manifold with nonzero Euler characteristic then there exists a constant $C$ with this property: for any continuous action of a finite group $G$ on $M$ there exists a point $x \in M$ such that the index of $G_x$ in $G$ is at most $C$. Note that, besides Theorem 1.3, the proof of the previous result uses the validity of Ghys’s original conjecture for homeomorphism groups of compact manifolds with nonzero Euler characteristic.

Recall that the rank of a finite group $G$ is the minimal integer $r$ such that every subgroup $H$ of $G$ is $r$-generated. A result of Mann and Su [11] gives an upper bound on the rank of an elementary abelian $p$-group $G$ acting effectively on a compact manifold $M$ in terms of the dimension and the $\mathbb{F}_p$-cohomology of $M$. In order to prove Theorem 1.3 we generalize this result to not necessarily compact manifolds and to arbitrary finite groups $G$.

**Theorem 1.8.** Let $G$ be a finite group acting continuously and effectively on a topological manifold $M$ such that $H_*(M; \mathbb{Z})$ is finitely generated. Then the rank of $G$ is bounded in terms of $\dim(M)$ and $H_*(M; \mathbb{Z})$.

Theorem 1.3 is about actions of finite $p$-groups. One is tempted to ask the following far reaching question:

**Question 1.9.** Let $M$ be a compact topological manifold. Can one find a bound $\hat{C}(M)$ such that each finite group $G$ acting continuously on $M$ has a subgroup $H \leq G$ of index at most $\hat{C}(M)$ such that

$$|\text{Stab}(H, M)| \leq \hat{C}(M)?$$

This question is probably substantially more difficult than Theorem 1.3 and might be out of reach with the tools used in this paper.
Nevertheless, this question is answered affirmatively in [13] for the n-dimensional torus or, more generally, for closed oriented n-manifolds admitting a continuous map of nonzero degree onto an n-dimensional torus.

The structure of the paper is the following. In section 2 we summarize some facts from Smith theory, on equivariant cohomology, and on group cohomology that will be used later. Section 3 is devoted to the proof of Theorem 1.8. The proof of Theorem 1.3 is divided into two parts. First we give a topological proof for elementary abelian p-groups in section 4, then the case of arbitrary p-groups is settled inductively in section 5. The last section contains the proof of Corollary 1.5 and Remark 1.7.

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2. Preliminaries

Throughout this paper, p always denotes a prime number, \( \mathbb{F} \) will denote an arbitrary field, \( \mathbb{F}_p \) the field of p elements. A d-dimensional cohomology manifold \( M \) over \( \mathbb{F}_p \) will be called shortly a \( \text{cm}_p \) or \( d \text{-cm}_p \). The dimension of a \( \text{cm}_p \) \( M \) will be denoted by \( \dim_p(M) \). If, in addition, \( M \) is equipped with an (effective) continuous action of the group \( G \), then \( M \) is called briefly an (effective) \( G \rtimes \text{cm}_p \) or \( G \rtimes d \text{-cm}_p \). Cohomology will always mean sheaf cohomology.

2.1. Orientation sheaf. Every \( \text{cm}_p \) \( M \) comes with an orientation sheaf \( \mathcal{O}_p \) (see [8 Definition V-9.1] for a definition), or with more precise notation \( \mathcal{O}_{p,M} \), if we need to emphasize \( M \). If \( \tilde{M} = \bigcup_{i=1}^{k} M_i \) is the disjoint union of the \( \text{cm}_p \)'s \( M_i \), not necessarily of the same dimension, then the orientation sheaf \( \mathcal{O}_{p,\tilde{M}} \) of \( \tilde{M} \) is defined as the sheaf whose restriction onto \( M_i \) is \( \mathcal{O}_{p,M_i} \) for all \( i \).

Many of our constructions involving the orientation sheaf will be functorial with respect to open embeddings due to the following simple proposition.
Proposition 2.1. For any open embedding $f: N \to M$ of a $\mathcal{cm}_p N$ into a $\mathcal{cm}_p M$, there is a natural isomorphism

$$\mathcal{O}_{p,N} \cong f^* \mathcal{O}_{p,M}.$$ 

Definition 2.2. If $G$ is a discrete group, $X$ is a topological space equipped with a continuous action of $G$, or shortly a $G$-space, and $R$ is a commutative unital ring, then a sheaf of $R$-modules $\pi: \mathcal{A} \to X$ on $X$ is a $G$-sheaf if we are given a continuous action $\tau$ of $G$ on the sheaf space $\mathcal{A}$ for which $\tau_g(\mathcal{A}_x) = \mathcal{A}_{gx}$ and the restriction of $\tau_g|_{\mathcal{A}_x}: \mathcal{A}_x \to \mathcal{A}_{gx}$ is a module isomorphism for every $g \in G$ and $x \in X$.

The most important examples of $G$-sheaves in this paper are the orientation sheaves of cohomology manifolds with a group action.

Proposition 2.3. For a $G \curvearrowright \mathcal{cm}_p M$, the action of $G$ on $M$ can be lifted in a unique natural way to a $G$-action on the sheaf space of $\mathcal{O}_{p,M}$, providing a natural $G$-sheaf structure on $\mathcal{O}_{p,M}$.

Proof. Follows immediately from the functoriality of the orientation sheaf. □

Lemma 2.4. If a $d$-$\mathcal{cm}_p$ $M$ has $C$ connected components, then $C = \dim H^d_c(M, \mathcal{O}_p)$.

Proof. By Poincaré duality [3, V-9.2], $H^d_c(M, \mathcal{O}_p)$ is isomorphic to the Borel-Moore homology $H^d_0(M, \mathbb{F}_p)$ with compact support, and the latter is isomorphic to the free $\mathbb{F}_p$-module generated by the connected components of $M$, see [3, V-5.14]. This implies the equality. □

2.2. Smith theory. Smith theory studies the fixed point set $F$ of a $\mathbb{Z}_p$-action on a $\mathcal{cm}_p M$. The connected components of $F$ are known to be $\mathcal{cm}_p$’s, and a theorem of Smith and Floyd provides a bound on $\dim H^*_c(F, \mathcal{O}_p)$ and $\dim H^*_c(M \setminus F, \mathcal{O}_p)$ in terms of $\dim H^*_c(M, \mathcal{O}_p)$. These results extend to arbitrary $p$-group actions by a simple induction.

Proposition 2.5. Let $G$ be a finite $p$-group, and $M$ a $G \curvearrowright \mathcal{cm}_p$. Then each connected component $F$ of $M^G$ is a $\mathcal{cm}_p$, $\dim_p(F) \leq \dim_p(M)$, and we have an isomorphism

$$(1) \quad \mathcal{O}_{p,M}|_F \cong \mathcal{O}_{p,F}$$

which is natural with respect to open embeddings. If $M$ is connected and the $G$-action is non-trivial then $\dim_p(F) \leq \dim_p(M) - 1$.

Furthermore, we have the inequalities

$$(2) \quad \dim H^*_c(M^G; \mathcal{O}_p) \leq \dim H^*_c(M; \mathcal{O}_p),$$

$$(3) \quad \dim H^*_c(M \setminus M^G; \mathcal{O}_p) \leq 2 \dim H^*_c(M; \mathcal{O}_p).$$
Proof. First we prove the statements on \( F \) and inequality (2) by induction on \(|G|\).

Assume first that \( G \cong \mathbb{Z}_p \). Theorem V-20.1 in [3] implies that \( F \) is a homology manifold over \( \mathbb{F}_p \) of dimension \( \dim_p(F) \leq \dim_p(M) \), and gives us an isomorphism (1) which depends only on the choice of a generator of \( G \). Once such a generator is fixed, the isomorphisms corresponding to that generator behave in a natural way with respect to open embeddings. It is stated after the proof of [3, Theorem V-20.2], that \( F \) is a \( \text{cm}_p \) as well.

Next we prove that if \( M \) is connected and the \( G \)-action is nontrivial then \( \dim_p(F) \neq \dim_p(M) \). Suppose, on the contrary, that \( \dim_p(F) = \dim_p(M) \). Then \( F \) is open by [3, Corollary V-16.19], and it is closed as well. As \( M \) is connected and \( F \neq \emptyset \), we must have \( F = M \). This contradicts the effectiveness of the action.

Finally, inequality (2) for \( G = \mathbb{Z}_p \) is guaranteed by the Smith–Floyd theorem [3, Theorem II-19.7].

Next we do the induction step. Choose a subgroup \( A \cong \mathbb{Z}_p \) in the center of \( G \). If \( F \) is a connected component of \( M^G \), and \( N \) is the connected component of \( M^A \) containing \( F \), then \( N \) and \( GN \subseteq M^A \) are \( \text{cm}_p \)'s and we have \( \dim_p(N) = \dim_p(GN) < \dim_p(M) \) by the induction hypothesis applied to the \( A \)-action on \( M \). As the action of \( A \) on \( GN \) is trivial, there is a canonical \( G/A \) action on \( GN \) and \( F \) is a connected component of \( (GN)^{G/A} \). Applying the induction hypothesis again to this \( (G/A) \)-action, we see that our statements hold for \( F \). Observe that the recursive construction of the isomorphism (1) depends on the choice of a composition series of \( G \), and a choice of a generator in each composition factor, but if these data are fixed for \( G \), then the resulting isomorphisms will behave naturally with respect to open embeddings.

As for inequality (2), the induction hypothesis yields

\[
\dim H^*_c(M^G; \mathcal{O}_p) = \dim H^*_c((M^A)^{G/A}; \mathcal{O}_p) \leq \dim H^*_c(M^A; \mathcal{O}_p) \\
\leq \dim H^*_c(M; \mathcal{O}_p).
\]

Inequality (3) follows from inequality (2) by the long exact sequence of the pair \((M, M \setminus M^G)\) for cohomology with compact support (see [3, II-10.3]).

2.3. Equivariant cohomology. Here we briefly recall the definition of equivariant cohomology with compact support, denoted by \( H^*_G,\text{c}(M; -) \), group cohomology, denoted by \( H^*(G; -) \), and some basic tools for their computation. Our main goal is to give a proof of the following three propositions, that will be needed later.
Proposition 2.6. Let $G$ be a finite $p$-group, $M$ a $G \curvearrowright \mathfrak{cm}_p$. Assume that the fixed point sets of the normal subgroups $H_1, \ldots, H_k$ of $G$ are pairwise disjoint. If $F = \bigcup_{i=1}^k M^{H_i}$ is their union, and $U = M \setminus F$, then we have a long exact sequence

$$\cdots H^d_{G,c}(U; \mathcal{O}_{p,U}) \to H^d_{G,c}(M; \mathcal{O}_{p,M}) \to H^d_{G,c}(F; \mathcal{O}_{p,F}) \to H^{d+1}_{G,c}(U; \mathcal{O}_{p,U}) \cdots,$$

which is functorial for $G$-equivariant open embeddings.

Proposition 2.7. Let $G$ be a finite group, and $M$ a $G \curvearrowright \mathfrak{cm}_p$.

(a) If the action of $G$ is free, then

$$H^*_c(M; \mathcal{O}_{p,M}) \cong H^*_c(M/G; \mathcal{O}_{p,M/G}).$$

(b) If $G = K \times L$ and the stabilizer of each point of $M$ is the subgroup $L$, then

$$H^*_c(M; \mathcal{O}_{p,M}) \cong H^* (L; \mathbb{F}_p) \otimes H^*_c(M/G; \mathcal{O}_{p,M/G}).$$

These isomorphisms are natural transformations for $G$-equivariant open embeddings.

Proposition 2.8. Let $G$ be a finite group, and $M$ a $G \curvearrowright \mathfrak{cm}_p$. Then there exists a spectral sequence

$$E^2_{i,j} = H^{i+j}_c(M; \mathcal{O}_p), \quad E^2_{i,j} = H^i(G; H^j_c(M; \mathcal{O}_p)),$$

which is functorial in $M$ with respect to $G$-equivariant open embeddings.

Actually, it will be more convenient to formulate and prove these propositions in a more general form. Here are the details.

The following construction is due to A. Borel (see [2, Section IV-3.1]). Let $G$ be a topological group and choose a universal principal $G$-bundle $EG \to BG$. If $X$ is a $G$-space, then there is an associated bundle $(EG \times X)/G \to BG$ with fiber $X$, the total space $X_G = (EG \times X)/G$ of which is called the homotopy quotient of $X$. Assuming $G$ is a compact Lie group, Borel approximates $EG$ by a compact $N$-universal principal bundle. Using that he obtains an approximation $X_G^{(N)}$ of $X_G$, and studies its cohomology groups (or rather their limits as $N \to \infty$). These are called the equivariant cohomology groups of $X$. In this paper, we use the version which uses cohomology with compact support, $H^*_c(\tilde{X}_G, -)$. However, it is more convenient for us to use a different (but equivalent) approach due to Grothendieck [7].

Suppose now that $G$ is a discrete group. If we are given a $G$-sheaf $\mathcal{A}$ of $R$-modules on the $G$-space $X$, then we can consider the $R$-module $\Gamma^G_c(\mathcal{A})$ of $G$-equivariant sections of $\mathcal{A}$ with compact support. The functor $\Gamma^G_c(-)$ is left exact, so it gives rise to the right derived functors $R^i\Gamma^G_c(-)$. 
Definition 2.9 (Grothendieck [7, Section 5.7]). The \( i \)-dimensional equivariant sheaf cohomology of the \( G \)-sheaf \( A \) of \( R \)-modules on the \( G \)-space \( X \) with compact support is the \( R \)-module
\[
H^i_{G,c}(X; A) = R^i\Gamma^G_c(A).
\]

We’ll need another derived functor, also defined by Grothendieck.

Definition 2.10. Let \( X \) be a \( G \)-space, and \( f: X \to X/G \) be the quotient map. For a \( G \)-sheaf \( A \) on \( X \) let \( f^*_G(A) \) denote the subsheaf of \( G \)-invariant elements of the sheaf \( f_*(A) \). The functor \( f^*_G \) is left exact, \( R^i f^*_G \) denotes its \( i \)-th derived functor.

The notion of group cohomology is an important special case of equivariant cohomology.

Definition 2.11. Let \( G \) be a discrete group, \( V \) an \( RG \)-module. Let \( V \) be the \( G \)-sheaf on a one point space \( \{p\} \), with stalk \( V \) at \( p \), where \( G \) acts on the stalk via the \( RG \)-module structure. Then the group cohomology of \( G \) with coefficients in \( V \) is
\[
H^*(G; V) = H^*_{G,c}(\{p\}; V).
\]

The cohomology ring\(^1\) of abelian groups is well-known. We collect here some useful facts.

Proposition 2.12. Let \( G \) be an elementary abelian \( p \)-group of rank \( r \).

(a) For \( p = 2 \), we have \( H^*(G; \mathbb{F}_2) = \mathbb{F}_2[t_1, \ldots, t_r] \) with \( \deg(t_i) = 1 \) for all \( i \).

(b) If \( p \geq 3 \), then \( H^1(G; \mathbb{F}_p) \) is naturally isomorphic to \( \text{Hom}(G, \mathbb{F}_p) \), the Bockstein homomorphism \( \beta^1: H^1(G; \mathbb{F}_p) \hookrightarrow H^2(G; \mathbb{F}_p) \) is injective, and
\[
H^*(G; \mathbb{F}_p) = \Lambda^* \left( H^1(G; \mathbb{F}_p) \right) \otimes S^* \left( \beta^1(H^1(G; \mathbb{F}_p)) \right)
\]
where, for a vector space \( V \) over \( \mathbb{F}_p \), \( \Lambda^*(V) \) and \( S^*(V) \) denote the Grassmann algebra and the symmetric algebra of \( V \). In particular, for all \( d \), we have a natural isomorphism
\[
H^d(G; \mathbb{F}_p) = \bigoplus_{l+2s=d} \Lambda^l \left( \text{Hom}(G; \mathbb{F}_p) \right) \otimes S^s \left( \text{Hom}(G; \mathbb{F}_p) \right).
\]

(c) For all \( p \), we have
\[
\dim H^d(G; \mathbb{F}_p) = \binom{d + r - 1}{d} = \binom{d + r - 1}{r - 1},
\]

\(^1\)Grothendieck used the notation \( H^*_G(X; G, A) \) for this group.

\(^2\)In this paper, we neither define, nor use the ring structure, but it is easier to describe these cohomology groups in terms of rings.
and
\[ \dim H^d(G; \mathbb{F}_p) = \frac{d + 1}{r + d} \dim H^{d+1}(G; \mathbb{F}_p). \]

(d) For any finite dimensional \( \mathbb{F}_p \)-module \( V \), we have
\[ \dim H^d(G; V) \leq \dim H^d(G; \mathbb{F}_p) \dim(V). \]

Proof. The cohomology ring of \( \mathbb{Z}_p \) is described in \([2, \text{IV-2.1(3)}]\). Statements (a), (b), (c) for \( G = \mathbb{Z}_p^d \) follow from the Künneth formula.

To prove (d) we recall that any short exact sequence of \( \mathbb{F}_p \)-modules
\[ 0 \rightarrow W' \rightarrow W \rightarrow W'' \rightarrow 0 \]
gives rise to a long exact sequence
\[ 0 \rightarrow H^0(G; W') \rightarrow H^0(G; W) \rightarrow H^0(G; W'') \rightarrow \]
\[ \rightarrow H^1(G; W') \rightarrow H^1(G; W) \rightarrow H^1(G; W'') \rightarrow \ldots, \]
which implies the inequality
\[ \dim H^i(G; W) \leq \dim H^i(G; W') + \dim H^i(G; W''). \]

It is well-known that a finite \( p \)-group has only unipotent representations in characteristic \( p \), hence \( V \) has a filtration \( V = V_d > V_{d-1} > \ldots > V_0 = 0 \) such that the induced \( G \)-action on \( \text{gr} V \) is trivial. Then the previous inequality yields
\[ \dim H^i(G; V_i) \leq \dim H^i(G; V_{i-1}) + \dim H^i(G; V_i/V_{i-1}) \]
\[ = \dim H^i(G; V_{i-1}) + \dim H^i(G; \mathbb{F}_p), \]
and a simple induction completes the proof. \( \square \)

Now we construct an equivariant analogue of the fundamental long exact cohomology sequence associated to a closed subspace of a space.

**Lemma 2.13.** Let \( G \) be a finite group, \( \mathcal{I} \) be an injective \( G \)-sheaf of \( R \)-modules over the \( G \)-space \( X \). Then the stalk \( \mathcal{I}_x \) of \( \mathcal{I} \) at \( x \) is an injective \( RG_x \)-module for any \( x \in X \).

**Proof.** By \([7, \text{Proposition 5.1.2}] \) we can embed \( \mathcal{I} \) into an injective \( G \)-sheaf of the form \( \mathcal{J} = \prod_{x \in X} \mathcal{J}(x) \), where each \( \mathcal{J}(x) \) is a skyscraper sheaf at \( x \) with values in some injective \( RG_x \)-module. Since \( \mathcal{I} \) is injective, it must be a direct summand, hence the stalk \( \mathcal{I}_x \) is a direct summand of \( \mathcal{J}_x \). Therefore \( \mathcal{I}_x \) is injective as an \( RG_x \)-module. \( \square \)

**Definition 2.14** \([8, \text{I-2.6}] \). Let \( A \subseteq X \) be a locally closed subset of \( X \), \( \mathcal{A} \) a sheaf of \( R \)-modules on \( A \). The **extension** \( \mathcal{A}^X \) of \( \mathcal{A} \) onto \( X \) by zero is the sheaf on \( X \) determined uniquely by the conditions that its restriction onto \( A \) is the sheaf \( \mathcal{A} \), while its restriction onto \( X \setminus A \) is the 0 sheaf on \( X \setminus A \).
If $X$ is a $G$-space and $A$ is $G$-invariant, then extension by 0 is an exact functor from the category of $G$-sheaves of $R$-modules on $A$ to the category of $G$-sheaves of $R$-modules on $X$.

**Lemma 2.15.** Let $G$ be a finite group, $X$ be a locally compact Hausdorff $G$-space, $A \subseteq X$ a $G$-invariant closed or open subset, $\mathcal{I}$ an injective $G$-sheaf on $A$. Then $H^n_{G,c}(X, \mathcal{I}^X) = 0$ for $n > 0$.

**Proof.** Let $f : X \to Y = X/G$ be the quotient map. By [7, Section 5.7], there is a spectral sequence $I^{i,j}_2 = H^i_c(Y, R^j f^*_c(\mathcal{I}^X))$ with second page $I^{i,j}_2 = H^i_c(Y, R^j f^*_c(\mathcal{I}^X))$.

By [7, Theorem 5.3.1] $R^j f^*_c(\mathcal{I}^X)$ is a sheaf over $Y$, the stalk of which at $y = f(x)$ is $H^i_c(G_x, \mathcal{I}^X_x)$. Since $\mathcal{I}^X_x = \mathcal{I}_x$ is injective for $x \in A$ by Lemma 2.13 and $\mathcal{I}^Y_x = 0$ for $x \notin A$, $R^j f^*_c(\mathcal{I}^Y)$ is the 0 sheaf for $j > 0$. This implies that the spectral sequence degenerates at the second page and $H^n_{G,c}(X, \mathcal{I}^X) = H^n_c(Y, f^*_c(\mathcal{I}^X)) = H^n_c(Y, f^*_c(\mathcal{I}^Y))$.

As $A$ is $G$-invariant and either open or closed in $X$, $B = f(A)$ is open or closed in $Y$. Thus, according to [3, Theorem II-10.1], we have

$$H^n_c(B, f^*_c(\mathcal{I})) = 0$$

As $\mathcal{I}$ is injective, $f^*_c(\mathcal{I})$ is flabby by [7, Proposition 5.1.3 and its Corollary], which implies $H^n_c(B, f^*_c(\mathcal{I})) = 0$ for $n > 0$. □

**Lemma 2.16.** Let $G$ be a finite group, $X$ be a locally compact Hausdorff $G$-space, $A \subseteq X$ a $G$-invariant closed or open subset. Then for any $G$-sheaf of $R$-modules on $A$, we have a natural isomorphism

$$H^*_c(X, A^X) \cong H^*_c(A, A)$$

**Proof.** The functors $H^*_c(A, -)$ and $H^*_c(X, (-)^X)$ are cohomological $\partial$-functors in the sense of [7, Section 2.1] with naturally isomorphic functors in degree 0

$$H^0_{G,c}(A, -) \cong H^0_{G,c}(X, (-)^X)\cong (1 + G)^\ast$$

Since $H^*_c(A, -)$ consists of the right derived functors of the left exact functor $\Gamma^G_c(-)$, it is a universal $\partial$-functor. On the other hand, Lemma 2.15 together with [7, Proposition 2.2.1] imply that $H^*_c(X, (-)^X)$ is also a universal $\partial$-functor, hence it must be naturally isomorphic to $H^*_c(A, -)$. □
Proposition 2.17. Let $G$ be a finite group, $\mathcal{A}$ a $G$-sheaf on a locally compact Hausdorff $G$-space $X$, $F \subseteq X$ a $G$-invariant closed subset of $X$, $U = X \setminus F$. Then we have a long exact sequence

$$\cdots H^d_{G,c}(U; \mathcal{A}|_U) \to H^d_{G,c}(X; \mathcal{A}) \to H^d_{G,c}(F; \mathcal{A}|_F) \to H^{d+1}_{G,c}(U; \mathcal{A}|_U) \cdots$$

Proof. The sheaves $\mathcal{A}_U = (\mathcal{A}|_U)^X$ and $\mathcal{A}_F = (\mathcal{A}|_F)^X$ can be included in a short exact sequence $0 \to \mathcal{A}_U \to \mathcal{A} \to \mathcal{A}_F \to 0$ of $G$-sheaves on $X$, from which the cohomology functor $H^*_G$ produces a long exact sequence

$$\cdots H^d_{G,c}(X; \mathcal{A}_U) \to H^d_{G,c}(X; \mathcal{A}) \to H^d_{G,c}(X; \mathcal{A}_F) \to H^{d+1}_{G,c}(X; \mathcal{A}_U) \cdots$$

in a natural way. The modules $H^i_{G,c}(X; \mathcal{A}_U)$ and $H^i_{G,c}(X; \mathcal{A}_F)$ in this sequence can be replaced by the isomorphic modules $H^i_{G,c}(U; \mathcal{A}|_U)$ and $H^i_{G,c}(F; \mathcal{A}|_F)$, respectively, as a consequence of Lemma 2.16. □

Proof of Proposition 2.20. First note that the statement makes sense as by Proposition 2.23 the connected components of $F$ are $\mathfrak{m}_p$'s, so $F$ has an orientation sheaf, and all appearing orientation sheaves have a natural $G$-sheaf structure by Proposition 2.3. Applying Proposition 2.17 to the $G$-sheaf $\mathcal{A} = \mathcal{O}_{p,M}$ we obtain our claim since $\mathcal{O}_{p,M}|_U = \mathcal{O}_{p,U}$, and, by Proposition 2.25, $\mathcal{O}_{p,M}|_F \cong \mathcal{O}_{p,F}$. □

Proposition 2.18. Let $X$ be a Hausdorff space, $K$ a finite group acting continuously and freely on $X$, and denote by $f: X \to Y = X/K$ the quotient map. Let $\mathcal{A}$ be a $K$-sheaf of $\mathbb{F}$-vector spaces on $X$, where $\mathbb{F}$ is a field. Let $L$ be another group and $\mathcal{M}$ be an $\mathbb{F}L$-module. Let $L$ act trivially both on $X$ and $\mathcal{A}$. Then $G = K \times L$ acts naturally on $X$ and on the sheaf $\mathcal{A} \otimes_\mathbb{F} \mathcal{M}$, and for all $n \geq 0$, there are isomorphisms

$$H^n_{G,c}(X; \mathcal{A} \otimes_\mathbb{F} \mathcal{M}) \cong \bigoplus_{p+q = n} H^p_c(Y; f^*_K \mathcal{A}) \otimes_\mathbb{F} H^q(L; \mathcal{M}),$$

which are natural transformations with respect to both $\mathcal{A}$ and $\mathcal{M}$.

Proof. For simplicity we introduce the notation $\mathcal{F} = f^*_K(\mathcal{A})$. As $K$ acts freely and $L$ acts trivially on the Hausdorff space $X$, the functor $f^*_K$ is an equivalence between the category of $G$-sheaves of $\mathbb{F}$-vector spaces on $X$ and the category of $L$-sheaves of $\mathbb{F}$-vector spaces on $Y$. Hence we have a natural isomorphism

$$H^*_G(X; \mathcal{B}) \cong R^\mathcal{G}H^*(\mathcal{B}) \cong R^\mathcal{K}H^*(f^*_Y(\mathcal{B})) = H^*_L(Y; f^*_K(\mathcal{B}))$$

for any $G$-sheaf $\mathcal{B}$ of $\mathbb{F}$-vector spaces on $X$. In particular,

$$(5) \quad H^*_G(X; \mathcal{A} \otimes_\mathbb{F} \mathcal{M}) \cong H^*_L(Y; \mathcal{F} \otimes_\mathbb{F} \mathcal{M}).$$
Let $\mathcal{I}$ be an injective sheaf of $\mathbb{F}$-vector spaces on $Y$ (with trivial action of $L$ on $\mathcal{I}$), and $\mathcal{J}$ an injective $\mathbb{F}L$-module. Let $\iota: Y \to Y$ be the identity map. By [7, Theorem 5.3.1], at each point $y \in Y$ we have
$$(R^j \iota^*(\mathcal{I} \otimes_{\mathbb{F}} \mathcal{J}))_y = H^j(L; \mathcal{I}_y \otimes_{\mathbb{F}} \mathcal{J}) = \mathcal{I}_y \otimes_{\mathbb{F}} H^j(L; \mathcal{J}) = 0 \quad \text{for all } j > 0.$$  
This implies that
$$R^j \iota^*(\mathcal{I} \otimes_{\mathbb{F}} \mathcal{J}) = \begin{cases} \mathcal{I} \otimes_{\mathbb{F}} \mathcal{J} & \text{for } j = 0, \\ 0 & \text{for } j > 0. \end{cases}$$

Since tensor product with the vector space $\mathcal{J}^L$ commutes with sheaf cohomology,
$$H^i_c(Y; \mathcal{I} \otimes_{\mathbb{F}} \mathcal{J}^L) \cong H^i_c(Y; \mathcal{I}) \otimes_{\mathbb{F}} \mathcal{J}^L = 0 \quad \text{for all } i > 0.$$  
Combining with the previous formula we obtain that
$$H^i_c(Y; R^j \iota^*(\mathcal{I} \otimes_{\mathbb{F}} \mathcal{J})) = 0 \quad \text{whenever } i + j > 0.$$
By [7, Equation (5.7.4)], there is a spectral sequence
$$I^i_j \Rightarrow H^{i+j}_{c}(Y; \mathcal{I} \otimes_{\mathbb{F}} \mathcal{J}), \quad I^i_2 = H^{i}_c(Y; R^j \iota^*(\mathcal{I} \otimes_{\mathbb{F}} \mathcal{J})),$$
which implies
$$H^n_{L,c}(Y; \mathcal{I} \otimes_{\mathbb{F}} \mathcal{J}) = 0 \quad \text{whenever } n > 0.$$  
Finally let $\mathcal{F}^\bullet$ be an injective resolution of the sheaf $\mathcal{F}$, and $\mathcal{J}^\bullet$ be an injective resolution of the module $\mathcal{M}$. By the above calculation, the complex $\mathcal{F}^\bullet \otimes_{\mathbb{F}} \mathcal{J}^\bullet$ is a $\Gamma^L_c$-acyclic resolution of $\mathcal{F} \otimes_{\mathbb{F}} \mathcal{M}$, hence
$$H^n_{L,c}(Y, \mathcal{F} \otimes_{\mathbb{F}} \mathcal{M}) \cong H^n(\Gamma^L_c(\mathcal{F}^\bullet \otimes_{\mathbb{F}} \mathcal{J}^\bullet)) \cong H^n(\Gamma^L_c(\mathcal{F}^\bullet) \otimes_{\mathbb{F}} (\mathcal{J}^\bullet)^L).$$

The Künneth formula for complexes of vector spaces implies that
$$H^n_{L,c}(Y, \mathcal{F} \otimes_{\mathbb{F}} \mathcal{M}) \cong \bigoplus_{i+j=n} H^i(\Gamma^L_c(\mathcal{F}^\bullet)) \otimes_{\mathbb{F}} H^j(L; \mathcal{M}).$$
This isomorphism and (5) proves the proposition. Naturality is straightforward to check. □

Proof of Proposition 2.7. If the previous proposition is applied to $X = M$, $\mathcal{A} = O_{p,M}$, and $\mathcal{M} = \mathbb{F}_p$, we obtain statement (b). All we need is the obvious isomorphism $f^*_K(O_{p,M}) \cong O_{p,M/G}$. When $G = K$ and $L$ is a trivial group, case (b) reduces to case (a). □
Proof of Proposition 2.8. The spectral sequence $E^{i,j}_t$ is a special case of Grothendieck’s second spectral sequence $II^{i,j}_t$ introduced in [7, Section 5.7]. We remark that this spectral sequence is a variant of Borel’s spectral sequence (see [3, Theorem IV-9.2]). □

3. Bound on the rank of finite topological transformation groups

In this section, we prove Theorem 1.8. The following lemma reduces the general case of finite transformation groups to that of elementary abelian ones.

Lemma 3.1 (Halasi, Podoski, Pyber, Szabó [8, Corollary 1.8]). If every elementary abelian subgroup of a finite group $G$ has rank at most $d$, then $G$ has rank at most $\frac{1}{2}d^2 + 2d + 1$. □

The dimensions of the cohomology vector spaces $H^*_c(M; \mathcal{O}_p)$ of a manifold $M$ are controlled by the homology group $H_*(M; \mathbb{Z})$.

Lemma 3.2. If $M$ is a manifold such that $\dim H_*(M; \mathbb{Q}) = B$ is finite and $H_*(M; \mathbb{Z})$ has $\tau$ torsion elements, then $\dim H^*_c(M; \mathcal{O}_p) \leq B + 2\tau$ for all primes $p$.

Proof. By the Poincaré duality [3, Theorem V-9.2], it is enough to prove that $\dim H_*(M; \mathbb{Z}_p) \leq B + 2\tau$.

By the universal coefficient theorem, there is a split natural short exact sequence

\[ 0 \to H_*(M; \mathbb{Z}) \otimes \mathbb{Z}_p \to H_*(M; \mathbb{Z}_p) \to \text{Tor} \left( H_*(M; \mathbb{Z}), \mathbb{Z}_p \right) \to 0 \]

for all primes $p$, where

\[ \text{Tor} \left( H_*(M; \mathbb{Z}), \mathbb{Z}_p \right) = \{ h \in H_*(M; \mathbb{Z}) : ph = 0 \}, \]

which has at most $\tau$ elements. Moreover, $H_*(M; \mathbb{Z}) \otimes \mathbb{Z}_p$ is an elementary abelian $p$-group of rank at most $\tau + B$. This implies that $\dim H_*(M; \mathbb{Z}_p) \leq B + 2\tau$. □

We recall Borel’s Fixed Point Formula [2, Theorem XIII-4.3].

Proposition 3.3 (Borel). Let $M$ be a first countable $d$-cm$_p$ for some prime $p$, and let $G$ be an elementary abelian $p$-group acting continuously and effectively on $M$. Let $x \in M^G$ be a fixed point of $G$. For a subgroup $H \leq G$, denote by $d(H)$ the dimension of the connected component of $x$ in $M^H$. Then

\[ d - d(G) = \sum_H (d(H) - d(G)), \]

where $H$ runs through the subgroups of $G$ of index $p$. 

Lemma 3.4. Let $G$ be an elementary abelian $p$-group, and $M$ be a connected effective $G$-$d$-$\text{cm}_p$. For a maximal stabilizer subgroup $H \in \text{Stab}_{\max}(G, M)$, let $F$ be a connected component of $M^H$. Then $\text{rk}(H) \leq d$, and $F$ has an $H$-invariant open neighbourhood $U \subseteq M$ such that

(a) there is a complete flag of subgroups

$$
\{1\} = H_0 \subsetneq H_1 \subsetneq \cdots \subsetneq H_{\text{rk}(H)} = H, \quad \text{rk}(H_i) = i,
$$

all $H_i$ belonging to $\text{Stab}(H, U) \subseteq \text{Stab}(G, M)$,

(b) there are at most $d!$ such complete flags in $\text{Stab}(H, U)$.

Proof. By Proposition 2.5, $F$ is a $\text{cm}_p$. For a subgroup $K \leq G$, let $Z(K)$ be the union of those connected components of $M^K$ which are disjoint from $F$. By Proposition 2.5, each connected component of $M^K$ is a $\text{cm}_p$, hence $M^K$ is locally connected, and $Z(K)$ is closed in $M$. If $h \in H$, then $h(Z(K)) = Z(hKh^{-1})$.

Define the $H$-invariant open neighbourhood $U$ of $F$ by

$$
U = M \setminus \bigcup_{K \leq G} Z(K).
$$

If $x \in U$, then $x \notin Z(G_x)$ implies that $M^{G_x}$ intersects $F$ at a point $y$. Then we have $(G_x \cup H) \subseteq G_y$, and $H \in \text{Stab}_{\max}(G, M)$ yields $G_x \leq G_y = H$. Thus, $\text{Stab}(H, U) = \text{Stab}(G, U) \subseteq \text{Stab}(G, M)$.

If $K \leq H$, then $M^K \setminus Z(K)$ is the connected component of $M^K$ containing $F$, and $U^K$ is an open subset of $M^K \setminus Z(K)$, therefore, $U^K$ is a $\text{cm}_p$ and $U^K \supseteq F$.

Assume that we have $H_x \leq H_y$ for the points $x, y \in U$. Then $M^{H_x} \subseteq M^{H_y}$, and consequently $M^{H_y} \setminus Z(H_y)$ is a proper closed subset of $M^{H_x} \setminus Z(H_x)$, which is a connected $\text{cm}_p$, and $M^{H_y} \setminus Z(H_y)$ does not cover the point $x$. This gives that

$$
\dim_p(U^{H_y}) < \dim_p(U^{H_x})
$$

by [2 Corollary I-4.6].

To prove (ii), we construct $H_i$ by downward induction on $i$. We begin with $H_{\text{rk}(H)} = H$ as required. Suppose that for some $i \geq 1$, the subgroup $H_i$ of rank $i$ is already constructed. Let $x \in M$ be a point whose stabilizer $E_x$ is $H_i$. Applying Proposition 3.3 to $x$ we obtain that there are nearby points $y \in M$ whose stabilizer $H' = H_y$ is a subgroup of $H_i$ of rank $i - 1$, and the number of such subgroups $H'$ is between 1 and $d - \dim_p(U^{H_i})$. Choose one of these subgroups for $H_{i-1}$ to complete the induction step.

The sequence

$$
\dim_p(U^{H_0}) > \dim_p(U^{H_1}) > \cdots > \dim_p(U^{H_{\text{rk}(H)}})
$$

is already constructed. Let $y \in M$ be a point whose stabilizer $E_y$ is $H_i$. Applying Proposition 3.3 to $y$ we obtain that there are nearby points $z \in M$ whose stabilizer $H' = H_z$ is a subgroup of $H_i$ of rank $i - 1$, and the number of such subgroups $H'$ is between 1 and $d - \dim_p(U^{H_i})$. Choose one of these subgroups for $H_{i-1}$ to complete the induction step.
is strictly decreasing, hence \( \text{rk}(H) \leq d - \dim_p(F) \leq d \).

Finally, we have seen above that for any fixed subgroup \( H_i \), there are at most \( d - \dim_p(U^{H_i}) \leq d - \dim_p(F) - \text{rk}(H) + i \) possible choices for \( H_{i-1} \). Thus, the total number of complete flags in \( \text{Stab}(H, U) \) is at most

\[
\prod_{i=1}^{\text{rk}(H)} (d - \dim_p(F) - \text{rk}(H) + i) \leq d!,
\]

as claimed. \( \square \)

The following lemma is a variation of [11, Theorem 2.3].

**Lemma 3.5.** For all integers \( d, B > 0 \), there is an integer \( f(d, B) \) with the following property. Let \( G \) be an elementary abelian \( p \)-group, and let \( M \) be a \( G \)-\( d \)-CW complex. Assume that the \( G \)-action is free, and that \( B = \dim H^*_c(M; \mathcal{O}_p) < \infty \). Then the rank of \( G \) is at most \( f(d, B) \).

**Proof.** We show that \( f(d, B) = B^2 + d(B(d+1) - 1) \) is a good choice for \( f \). By assumption, \( |\text{Aut}(H^*_c(M; \mathcal{O}_p))| < p^{B^2} \), hence \( G \) has a subgroup \( \tilde{G} \) of rank \( \tilde{r} \geq \text{rk}(G) - B^2 \) which acts trivially on \( H^*_c(M; \mathcal{O}_p) \). If \( \tilde{G} = \{1\} \), then \( \text{rk}(G) \leq B^2 \), and we are done. Assume that \( \tilde{G} \neq \{1\} \).

Consider the spectral sequence \( E^i_{ij} \) of Proposition 2.8 for the \( \tilde{G} \)-action on \( M \). By assumption, \( E^i_{ij} = 0 \) for \( j > d \), and Proposition 2.7 implies that \( E^{i,j}_\infty = 0 \) for \( i + j > d \). Lemma 2.3 implies that \( H^*_c(M; \mathcal{O}_p) \neq 0 \). Let \( s \) be the smallest integer such that \( H^*_c(M; \mathcal{O}_p) \neq 0 \). Let \( d \) be the smallest integer such that \( H^*_c(M; \mathcal{O}_p) \neq 0 \). Let \( k \) be the smallest integer such that \( \dim H^*_c(M; \mathcal{O}_p) \neq 0 \).

Then

\[
\dim E^{d+1,s}_2 \leq \sum_{k=1}^{d-s} \dim E^{d-k,s+k}_2,
\]

so by the definition of \( E_2 \) (see Proposition 2.8) and by Proposition 2.12 (c), we obtain

\[
\dim H^{d+1}(\tilde{G}; \mathbb{Z}_p) \leq \dim H^{d+1}(\tilde{G}; H^*_c(M; \mathcal{O}_p)) \leq \sum_{k=1}^{d-s} \dim H^{d-k}(\tilde{G}; \mathbb{Z}_p) \cdot \dim H^{s+k}_c(M; \mathcal{O}_p) \leq (d - s) \dim H^d(\tilde{G}; \mathbb{Z}_p) \cdot B \leq dB \frac{d + 1}{\tilde{r} + d} \dim H^{d+1}(\tilde{G}; \mathbb{Z}_p).
\]

As \( H^{d+1}(\tilde{G}; \mathbb{Z}_p) \neq 0 \) by Proposition 2.12, we have \( \tilde{r} \leq dB(d+1) - d \). Then \( \text{rk}(G) \leq \tilde{r} + B^2 \) implies the lemma. \( \square \)

**Theorem 3.6.** For all integers \( d, B > 0 \), there is an integer \( r(d, B) \) with the following property. Let \( G \) be an elementary abelian \( p \)-group, and \( M \) be an effective \( G \)-\( d \)-CW complex such that \( \dim H^*_c(M; \mathcal{O}_p) \leq B \). Then the rank of \( G \) is at most \( r(d, B) \).
Proof. We show that the function \( r(d, B) = \log(B!) + B(f(d, B) + d) \) satisfies the requirement, where \( f(d, B) = B^2 + d(B(d + 1) - 1) \) is the function defined in Lemma 3.5.

Let \( M_1, \ldots, M_C \) be the connected components of \( M \). Lemma 2.3 yields that \( C \leq B \). \( G \) permutes the components of \( M \), let \( \overline{G} \leq G \) be the subgroup of those elements which map each component \( M_i \) into itself. Then \( |G/\overline{G}| \leq C! \leq B! \).

Let \( \overline{G}_i \) be the image of \( G \) in the homeomorphism group of \( M_i \). The group \( \overline{G}_i \) acts effectively on \( M_i \), and since the action of \( G \) on \( M \) is effective, the natural homomorphism \( \pi_i : \overline{G} \to \overline{G}_1 \times \cdots \times \overline{G}_C \) is injective.

Choose a maximal stabilizer \( H_i \in \text{Stab}_{\text{max}}(\overline{G}_i, M_i) \) for each \( 1 \leq i \leq C \) and denote by \( F_i \) the fixed point set \( M_i^{H_i} \).

The quotient group \( \overline{G}_i/H_i \) acts freely on \( F_i \). Proposition 2.5 implies that \( \dim \overline{H}_c(F_i; \mathcal{O}_p) \leq \dim \overline{H}_c(M_i; \mathcal{O}_p) \leq B \), so Lemma 3.5 gives \( \text{rk} \overline{G}_i/H_i \leq f(d, B) \).

Moreover, \( \text{rk} H_i \leq d \) by Lemma 3.4 (a), thus, we have

\[
\text{rk} G \leq \text{rk} \overline{G} + \sum_{i=1}^C (\text{rk} \overline{G}_i/H_i + \text{rk} H_i) \leq \log(B!) + B(f(d, B) + d).
\]

This proves the theorem. \( \square \)

By Lemma 3.1 and Lemma 3.2, Theorem 3.6 completes the proof of Theorem 1.8.

4. PROOF OF THEOREM 1.3 FOR ELEMENTARY \( p \)-GROUPS

Definition 4.1. For a topological space \( X \), let \( \text{Homeo}(X) \) denote the group of all homeomorphisms of \( X \) to itself.

Lemma 4.2. Let \( M \) be a \( d \)-complex such that \( B = \dim H_c(M; \mathcal{O}_p) < \infty \). Let \( r \) be a natural number. Then there is a number \( C_0(d, B, r) \) (independent of \( p \)) with the following property. If \( E < \text{Homeo}(M) \) is an elementary abelian \( p \)-subgroup of rank \( r \) such that for every \( x \in M \) the stabilizer \( E_x \) is either trivial or isomorphic to \( \mathbb{Z}_p \), then we have

\[
|\text{Stab}(E, M)| \leq C_0(d, B, r).
\]

Proof. Set

\[
C_0(d, B, r) := B \sum_{i=0}^{d+1} \dim H^i(E; \mathbb{F}_p) = B \sum_{i=0}^{d+1} \binom{i + r - 1}{r - 1} = B \binom{r + d + 1}{r}.
\]

Let \( \{L_1, \ldots, L_k\} \) be the elements of \( \text{Stab}(E, M) \) isomorphic to \( \mathbb{Z}_p \). Define \( M_i = M^{L_i} \) and \( M^* = \bigcup_i M_i \). Since \( E \) acts freely on \( M \setminus M^* \), the
inclusion $M^* \hookrightarrow M$ induces an isomorphism

$$H^{d+1}_{E,c}(M^*; \mathcal{O}_p) \cong H^{d+1}_{E,c}(M; \mathcal{O}_p)$$

by Proposition 2.6 and Proposition 2.7 (a), so we have

$$\dim H^{d+1}_{E,c}(M^*; \mathcal{O}_p) = \dim H^{d+1}_{E,c}(M; \mathcal{O}_p).$$

The existence of a spectral sequence converging to $H^*_{E,c}(M; \mathcal{O}_p)$, with second page $E_2^{ij} = H^i(E; H^j_c(M; \mathcal{O}_p))$, (see Proposition 2.8, and Proposition 2.12 (d) imply that

$$(8) \quad \dim H^{d+1}_{E,c}(M; \mathcal{O}_p) \leq B \sum_{i=0}^{d+1} \dim H^i(E; \mathbb{F}_p) = C_0(d, B, r).$$

The fixed point sets $M_1, \ldots, M_r$ are disjoint, for if some point $x$ belonged to $M_i \cap M_j$ with $i \neq j$, then $E_x$ would contain $L_iL_j \cong \mathbb{Z}_p^2$, a contradiction. For each $i$, the group $L_i$ acts trivially on $M_i$ and $E/L_i$ acts freely on $M_i$. So the connected components of the quotient $N_i := M_i/(E/L_i)$ are $\mathbb{Z}_p$'s and hence $N_i$ carries an orientation sheaf $\mathcal{O}_p$. Applying Proposition 2.7 (b), we have

$$H^{d+1}_{E,c}(M^*; \mathcal{O}_p) = \bigoplus_{i=1}^{k} H^{d+1}_{E,c}(M^*_i; \mathcal{O}_p) \cong \bigoplus_{i=1}^{k} H^{d+1}_{L_i,c}(N^*_i; \mathcal{O}_p)$$

$$\cong \bigoplus_{i=1}^{k} \bigoplus_{u+v=d+1} H^u(L_i; \mathbb{F}_p) \otimes H^v_c(N^*_i; \mathcal{O}_p).$$

Let $N'_i$ be a connected component of $N_i$, and let $s = \dim N'_i$. Then $H^s_c(N'_i; \mathcal{O}_p) \neq 0$. One can further decompose the above expression for $H^{d+1}_{E,c}(M^*; \mathcal{O}_p)$ in terms of contributions from each connected component of $N_i$, and one of the summands is $H^{d+1-s}(L_i; \mathbb{F}_p) \otimes H^s_c(N'_i; \mathcal{O}_p)$. Since $L_i \cong \mathbb{Z}_p$ and $s \leq d$ we have $H^{d+1-s}(L_i; \mathbb{F}_p) \cong \mathbb{F}_p$, so

$$H^{d+1-s}(L_i; \mathbb{F}_p) \otimes H^s_c(N'_i; \mathcal{O}_p) \neq 0,$$

and hence $H^{d+1}_{E,c}(M^*_i; \mathcal{O}_p) \cong H^{d+1}_{L_i,c}(N_i; \mathcal{O}_p) \neq 0$. It follows that

$$\dim H^{d+1}_{E,c}(M^*; \mathcal{O}_p) \geq k.$$

Combining this with $(8)$ we obtain $k \leq C_0(d, B, r).$ $\square$

**Lemma 4.3.** Let $M$ be a $d$-$\mathbb{Z}_p$ such that $B = \dim H^*_c(M; \mathcal{O}_p) < \infty$. Then there is a number $C_1(d, B)$ (independent of $p$) such that for every elementary abelian $p$-subgroup $E < \text{Homeo}(M)$ we have

$$|\text{Stab}(E, M)| \leq C_1(d, B).$$
Proof. Let us first prove that for any natural number \( r \), there is a number \( C_1(d, B, r) \) (independent of \( p \)) such that for every elementary abelian \( p \)-subgroup \( E < \text{Homeo}(M) \) of rank \( r \), we have \( |\text{Stab}(E, M)| \leq C_1(d, B, r) \).

We use induction on \( r \).

The case \( r = 1 \) is obvious. Suppose that \( r = 2 \), so let \( E < \text{Homeo}(M) \) be isomorphic to \( \mathbb{Z}_p^2 \). Since \( E \) acts effectively on \( M \), it also acts effectively on \( M \setminus M^E \), and hence we can identify \( E \) with a subgroup of \( \text{Homeo}(M \setminus M^E) \). By Proposition 2.5, \( \dim H^*_c(M \setminus M^E; \mathbb{Q}_p) \leq 2B \).

Finally, the stabilizers of the action of \( E \) on any point of \( M \setminus M^E \) are either trivial or isomorphic to \( \mathbb{Z}_p \). Hence by Lemma 4.2 we have \( |\text{Stab}(E, M \setminus M^E)| \leq C_0(d, 2B, 2) \).

Since \( \text{Stab}(E, M \setminus M^E) \) contains at most one element (namely, \( E \), in case \( M^E \neq \emptyset \)) we conclude in this case \( |\text{Stab}(E, M)| \leq C_0(d, 2B, 2) + 1 \).

We now prove the induction step. Assume that \( r \geq 3 \) and that the existence of the constant \( C_1(d, B, r - 1) \) has already been proved. Let \( E < \text{Homeo}(M) \) be an elementary abelian \( p \)-group of rank \( r \). It will be convenient to look \( E \) as an \( r \)-dimensional vector space over \( \mathbb{F}_p \). Let \( \text{Stab}_{\geq 2}(E, M) = \{ F \in \text{Stab}(E, M) \mid \dim F \geq 2 \} \).

We will first bound the size of \( \text{Stab}_{\geq 2}(E, M) \). Choose hyperplanes \( H_1, \ldots, H_r < E \) such that \( H_1 \cap \cdots \cap H_r = \{0\} \). For every \( j \geq 2 \) let \( S_j = \{ F \in \text{Stab}_{\geq 2}(E, M) \mid F \cap H_1 \not\leq H_1 \cap H_j \} \).

It is clear that any \( F \in S_j \) satisfies \( F = (F \cap H_1) + (F \cap H_j) \). Hence each \( F \in S_j \) is uniquely characterized by its intersections with \( H_1 \) and \( H_j \). Since we have \( F \cap H_i \in \text{Stab}(H_i, M) \) for \( i = 1, j \), the induction hypothesis implies that \( |S_j| \leq C_1(d, B, r - 1)^2 \).

But \( \text{Stab}_{\geq 2}(E, M) = \bigcup_{j \geq 2} S_j \); indeed, if \( F \in \text{Stab}_{\geq 2}(E, M) \) does not belong to \( \bigcup_{j \geq 2} S_j \), then \( F \cap H_1 \) is contained in \( H_1 \cap H_2 \cap \cdots \cap H_r = \{0\} \), which implies \( \dim F \cap H_1 = 0 \), a contradiction. It follows that \( |\text{Stab}_{\geq 2}(E, M)| \leq (r - 1)C_1(d, B, r - 1)^2 \).

Now let \( M^*_1 = M \setminus \bigcup_{F \in \text{Stab}_{\geq 2}(E, M)} M^F \).
Applying repeatedly inequality (3), we obtain
\[ \dim H^*_c(M^*_1; O_\rho) \leq 2^{(r-1)C_1(d, B, r-1)^2} B. \]
We have
\[ \text{Stab}(E, M) = \text{Stab}(E, M^*_1) \cup \text{Stab} \geq 2 \text{Stab}(E, M), \]
and the stabilizers of the action of $E$ on $M^*_1$ are either trivial or isomorphic to $Z_p$. Hence by Lemma 4.2 the size of $\text{Stab}(E, M)$ is at most
\[ C_1(d, B, r) := C_0(d, 2^{(r-1)C_1(d, B, r-1)^2} B, r) + (r-1)C_1(d, B, r-1)^2. \]
This finishes the induction step, so the proof of the existence of the constants $C_1(d, B, r)$ is now complete.

\[ \square \]

5. PROOF OF THEOREM 1.3 FOR GENERAL $p$-GROUPS

Our proof is inductive. In order to make the induction working, we prove a slightly more general statement:

**Theorem 5.1.** Let $M$ be a $d$-$\text{cm}_p$ such that $B = \dim H^*_c(M; O_\rho) < \infty$. Then there is a number $\tilde{C} = \tilde{C}(d + B)$ (independent of $p$) with the following property. Each finite $p$-group $G \leq \text{Homeo}(M)$ has a characteristic subgroup $H \leq G$ of index at most $\tilde{C}$ containing the center $Z(G)$ such that
\[ |\text{Stab}(H, M)| \leq \tilde{C}. \]

**Proof.** The proof is by induction on $d$ and $B$. If $d = 0$, then $|M| \leq B$, hence $|G| \leq B!$ and $|\text{Stab}(G, M)| \leq 2^{B!}$, so the statement holds.

Now assume that $d > 0$, and the statement holds for $\text{cm}_p$'s of dimension $< d$, and for each $d$-$\text{cm}_p$ $N$ with $\dim H^*_c(N; O_\rho) < B$.

Let $M$ be as in the statement. To complete the induction step, we need three claims:

**Claim A.** There is a number $C_2(d + B)$ (independent of $p$) such that every finite $p$-group $G \leq \text{Homeo}(M)$ has a characteristic subgroup $H \leq G$ of index at most $C_2(d + B)$, containing $Z(G)$, such that
\[ \left| \left\{ K \in \text{Stab}(H, M) \mid K \cap Z(G) \neq \{1\} \right\} \right| \leq C_2(d + B). \]

**Proof.** Let $E \leq Z(G)$ be the subgroup of elements of order $p$. First we pick a subgroup $L \in \text{Stab}(E, M)$ different from $\{1\}$.
Denote by $M_s^L$ the union of the $s$-dimensional connected components of $M^L$. By assumption $M^L \neq M$, hence if $M_s^L \neq \emptyset$, then either $s < d$ and $\dim H_c^*(M_s^L; \mathcal{O}_p) \leq \dim H_c^*(M^L; \mathcal{O}_p) \leq B$ by Proposition \[2.5\] or $s = d$ and $\dim H_c^*(M_s^L; \mathcal{O}_p) < \dim H_c^*(M; \mathcal{O}_p) = B$.

In both cases the induction hypothesis applies to $M_s^L$ and the image of $G$ in $\text{Homeo}(M_s^L)$, and gives us a subgroup $H_s \leq G$ of index at most $\tilde{C}(d + B - 1)$ containing $Z(G)$ such that

$$\left| \text{Stab}(H_s, M_s^L) \right| \leq \tilde{C}(d + B - 1).$$

Let $H$ be the intersection of all subgroups of $G$ of index at most $\tilde{C}(d + B - 1)$ containing $Z(G)$. It is characteristic, and setting

$$S_{H,L} = \{ K \in \text{Stab}(H, M) \mid K \cap E = L \} \subseteq \bigcup_{s=0}^{d} \{ K \cap H \mid K \in \text{Stab}(H_s, M_s^L) \},$$

we have

$$|S_{H,L}| \leq (d + 1)\tilde{C}(d + B - 1).$$

Moreover, by Theorem \[1.8\] the rank of $G$ is at most $r(d, B)$, so by \[10\] Corollary 1.1.2 the number of subgroups of $G$ of index at most $\tilde{C}(d + B - 1)$ is bounded by

$$x(d + B) = \tilde{C}(d + B - 1)^2 \left( \tilde{C}(d + B - 1)! \right)^{r(d,B)-1},$$

and the index of $H$ in $G$ is at most $\tilde{C}(d + B - 1)^{x(d+B)}$. Since

$$\left\{ K \in \text{Stab}(H, M) \mid K \cap Z(G) \neq \{1\} \right\} = \bigcup_{L \in \text{Stab}(E, M), L \neq \{1\}} S_{H,L},$$

Lemma \[4.3\] implies the claim. \[\square\]

**Claim B.** If $p > C_2(d + B)$, then for each finite $p$-subgroup $G < \text{Homeo}(M)$, every stabilizer $K \in \text{Stab}(G, M)$ with $|K| > 1$ intersects $Z(G)$ non-trivially:

$$K \cap Z(G) \neq \{1\}.$$

**Proof.** We prove this by induction on $|G|$. To begin with, the claim holds if $G$ is abelian. For the induction step we assume that the claim holds for all groups smaller than $G$.

Suppose there is a subgroup $K \in \text{Stab}(G, M)$ with $K \cap Z(G) = \{1\}$.

Let $Z_0 = \{1\} \leq Z_1 \leq Z_2 \leq \ldots$ denote the upper central series of $G$, i.e. $Z_{k+1}/Z_k = Z(G/Z_k)$ for all $k \geq 0$. Since $G$ is a $p$-group, the union of these subgroups is $G$. In particular, one of them contains $K$. 

Let $t$ be the largest index such that $K \cap \mathcal{Z}_t = \{1\}$. By assumption $t \geq 1$. Choose an element $h \in K \cap \mathcal{Z}_{t+1}$ of order $p$. Let $L$ be the subgroup generated by $\mathcal{Z}_t$ and $h$. By construction $\langle h \rangle = K \cap L \in \text{Stab}(L, M)$.

Since $L/\mathcal{Z}_t$ is central in $G/\mathcal{Z}_t$, its pre-image $L$ must be normal in $G$. Since $\mathcal{Z}_t/\mathcal{Z}_{t-1} = \mathcal{Z}(G/\mathcal{Z}_{t-1})$ and $L/\mathcal{Z}_t$ is cyclic, we obtain that $L/\mathcal{Z}_{t-1}$ is abelian, and $G/\mathcal{Z}_{t-1}$ is non-abelian. In particular, $L \neq G$.

By the induction assumption the claim is valid for $L$, and combining this with Claim A we obtain a subgroup $\tilde{L} \leq L$ of index less than $p$ such that $|\text{Stab}(\tilde{L}, M)| \leq C_2(d + B) + 1$ (the +1 stands for the subgroup $\{1\}$). But $L$ is a $p$-group, hence $\tilde{L} = L$, and

$$|\text{Stab}(L, M)| \leq C_2(d + B) + 1$$

Since $L$ is normal, all $G$-conjugates of the subgroup $\langle h \rangle$ belong to $\text{Stab}(L, M)$, hence there are at most $C_2(d + B)$ such conjugates.

On the other hand, the number of $G$-conjugates of any subgroup of $L$ is either 1, or divisible by $p$. Since $p$ is large, we obtain that $\langle h \rangle$ is $G$-invariant, i.e., it is normal in $G$. Since $\langle h \rangle$ is a cyclic group of order $p$, the size of its automorphism group is not divisible by $p$. This implies the $G$ acts trivially on $\langle h \rangle$. In other words, $|\langle h \rangle| \leq \mathcal{Z}(G)$, a contradiction. This completes the induction step. $\square$

The following is a result of Alperin [11] (see also [9, Satz 12.1]):

**Proposition 5.2.** If $E$ is a subgroup of a $p$-group $G$, maximal subject to being normal abelian and of exponent at most $p^n$, then any element of order at most $p^n$ which centralizes $E$ lies in $E$, unless perhaps $p = 2$ and $n = 1$.

**Claim C.** If $p \leq C_2(d + B)$, then each finite $p$-subgroup $G < \text{Homeo}(M)$ has a characteristic subgroup $H \leq G$ of index at most $C_3 = C_3(d + B)$, containing $\mathcal{Z}(G)$, such that

$$|\text{Stab}(H, M)| \leq C_2(d + B).$$

**Proof.** By Theorem 3.6 the rank of $G$ is bounded by $r(d, B)$.

If $p = 2$, then we set $n = 2$, otherwise we set $n = 1$. Let $E \leq G$ be a subgroup maximal subject to being normal abelian and of exponent $p^n$. By assumption, $|E| \leq C_2(d + B)^{2r(d, B)}$, hence the index of the centralizer $\mathcal{C}_G(E)$ is also bounded by some function $C_3(d + B)$.

The center of $\mathcal{C}_G(E)$ contains $\mathcal{Z}(G)$ and $E$. By Proposition 5.2 we know that in $\mathcal{C}_G(E)$ each element of order $p$ is contained in $E$. In particular, if $K \leq \mathcal{C}_G(E)$ and $K \neq \{1\}$, then $K \cap E \neq \{1\}$. 
Applying Claim A to the group $C_G(E)$ we obtain a subgroup $\tilde{H} \leq C_G(E)$ of bounded index, containing $Z(G)$ and $E$, such that $|\text{Stab}(\tilde{H}, M)|$ is bounded. Then $|G : \tilde{H}| = |G : C_G(E)| \cdot |C_G(E) : \tilde{H}|$ is bounded.

Finally, let $H \leq \tilde{H}$ be the intersection of all subgroups of $G$ containing $Z(G)$ whose index is $|G : \tilde{H}|$. It is characteristic, by [10, Corollary 1.1.2] it has bounded index, it contains $Z(G)$, and $|\text{Stab}(H, M)| \leq |\text{Stab}(\tilde{H}, M)|$. This proves the claim. □

5.1. Completing the proof of Theorem 5.1. Claim A gives us a subgroup $H \leq G$ of bounded index, containing $Z(G)$, such that
$$\left| \left\{ K \in \text{Stab}(H, M) \mid K \cap Z(G) \neq \{1\} \right\} \right| \leq C_2(d + B).$$

Since $Z(G) \leq Z(H)$, depending on the value of $p$, either Claim B or Claim C completes the induction step. □

Proof of Theorem 1.3. As $\dim H_c^*(M; \mathcal{O}_p)$ is bounded in terms of $H_c^*(M; \mathcal{O}_p)$ by Lemma 3.2, the statement follows from Theorem 5.1. □

6. Proof of Corollary 1.5

We need some topological information about the fixed point structure of finite $p$-group actions.

Lemma 6.1. Let $M$ be a topological space whose connected components are cm$p$-s such that $B = \dim H_c^*(M; \mathcal{O}_p) < \infty$. Let $H$ be a finite $p$-subgroup acting continuously on $M$. For each subset $S \subseteq \text{Stab}(H, M)$ and each subgroup $K \leq H$, we define the following subsets of $M$:
$$F_S = \bigcup_{L \in S} M^L \quad \text{and} \quad U^K_S = M^K \setminus F_S.$$
Then
$$\dim H_c^*(U^K_S; \mathcal{O}_p) \leq 2^{|S|} B.$$  

Proof. We argue by induction on the size of $S$. For $S = \emptyset$ the statement holds by Proposition 2.5 (2). Assume now that it holds for some $S$, and consider the subset $S' = S \cup \{L\}$ for some $L \in \text{Stab}(H, M) \setminus S$. Then
$$U^K_S = U^K_S \setminus M^L = U^K_S \setminus U^{(K,L)}_S.$$  

The long exact sequence for cohomology with compact support corresponding to the closed subset $U^{(K,L)}_S \subseteq U^K_S$ (see [3, II-10.3]), Proposition 2.5 [1], and the induction hypothesis imply that
$$\dim H_c^*(U^K_S; \mathcal{O}_p) \leq \dim H_c^*(U^K_S; \mathcal{O}_p) + \dim H_c^*(U^{(K,L)}_S; \mathcal{O}_p) \leq 2^{|S|} B + 2^{|S|} B = 2^{|S'|} B.$$  

□
This completes the induction step.

\textbf{Corollary 6.2.} Let $M$ be a $d$-cm$_p$ such that $B = \dim H^*_c(M; \mathcal{O}_p) < \infty$ and $G$ a finite $p$-subgroup acting continuously on $M$. Then $G$ has a characteristic subgroup $H \leq G$ of index at most $\check{C} = \check{C}(d, B)$ containing the center of $G$ such that for each $K \in \text{Stab}(H, M)$, the connected components of the locally closed subset $\check{M}^K = \{ x \in M \mid H_x = K \}$ are cm$_p$-s and

$$\dim H^*_c(\check{M}^K; \mathbb{F}_p) = \dim H^*_c(\check{M}^K; \mathcal{O}_p) \leq \check{C}.$$ 

\textit{Proof.} For any subgroup $K \leq G$, the set $\check{M}^K$ is an open subset of $M$, hence its connected components are cm$_p$-s by Proposition 2.5. Then $\dim H^*_c(M; \mathbb{F}_p) = \dim H^*_c(M; \mathcal{O}_p)$ by the Poincaré duality (which is valid with Borel-Moore homology with compact support, see [3, V-9.2]).

With the notation of Theorem 5.1, let $\check{C} = \max_{d' \leq d} \check{C}(d' + B)$, and let $H$ be the intersection of all subgroups of $G$ of index at most $\check{C}$ containing the center of $G$. This is a characteristic subgroup of $G$. By Theorem 5.1 the rank of $G$ is bounded, so the index of $H$ in $G$ is also bounded in terms of $d$ and $B$.

We apply Lemma 6.1 to a subgroup $K \in \text{Stab}(H, M)$ and the subset $S = \{ L \in \text{Stab}(H, M) \mid L > K \}$. In this case $U^*_S = \check{M}^K$ and $|S| < |\text{Stab}(H, M)| \leq \check{C}$, so we obtain a bound on $\dim H^*_c(\check{M}^K; \mathcal{O}_p)$ depending on $d$ and $B$. \hfill $\square$

\textit{Proof of Corollary 1.5 and Remark 1.7.} By Lemma 3.2 $\dim H^*_c(M; \mathcal{O}_p)$ is bounded in terms of $H^*_c(M; \mathbb{Z})$. Then Corollary 6.2 gives us a subgroup $H$ such that $\dim H^*_c(\check{M}^K; \mathbb{F}_p) = \dim H^*_c(\check{M}^K; \mathcal{O}_p)$ is bounded in terms of $\dim(M)$ and $H_*^*(M; \mathbb{Z})$ for all $K \in \text{Stab}(H, M)$. If $K$ is minimal in $\text{Stab}(H, M)$ then $\check{M}^K$ is an open submanifold in $M$, hence its Borel-Moore homology with compact support is isomorphic to its singular homology. \hfill $\square$

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