The Low-Frequency Acoustic Scattering Characteristics Study on Underwater Elastic Targets by the Coupled FEM-BEM Method

Weilong Xu1,*, Lijiao Shen1 and Weicai Peng1

1National Key Laboratory on Ship Vibration & Noise, China Ship Develop and Design Center, Wuhan 430064, China.
Email: milo0324@163.com

Abstract. A mathematical model about calculating acoustic scattering of elastic shell with air inside unbounded fluid was developed using a coupled finite element and boundary element technique. The coupling of elastic shell with air and shell with fluid was included in this model. To validate the scheme, the scattering characteristic of spherical shell with air submerged in water was calculated. Comparison of the numerical results with the corresponding analytical solutions shows that they are in good agreement in low frequency. On this basis, the low-frequency acoustic scattering characteristic of underwater elastic target was calculated with this method. The results showed the shell thickness and material have little effect on the target strength in the transverse direction, when the frequency is 100-300 Hz. In the omni-direction, the target strength of titanium alloy shell is higher than that of steel shell, and the larger the shell thickness, the greater the target strength. As the frequency is 200 Hz, the steel shell with a thickness of 28 mm has the phenomenon of acoustic resonance, resulting in a sharp increase in the target strength.

1. Introduction
The investigation of acoustic scattering characteristics of underwater targets has important theoretical value and wide application background. The theory, calculation method and experimental verification of acoustic scattering have been studied extensively and detailed. Through the research, a series of rigorous theories and approximate solutions of acoustic scattering have been put forward, in addition to the in-depth understanding of the acoustic scattering characteristics of underwater target, some prediction models of the scattering characteristics are established from the perspective of engineering application.

Theoretically, the calculation of target acoustic scattering field is to solve a mathematical and physical problem, which is to solve the scattering field that satisfies the wave equation, surface boundary conditions and radiation boundary conditions generated by the acoustic excitation of the target in the three-dimensional fluid space. In order to solve this problem, the research methods of underwater acoustic scattering are mainly divided into theoretical method and approximated method. The theoretical method, which can be divided in to integral equation method and separation of variables method, can be used to investigate the acoustic scattering excitation of the target thoroughly, but only for some targets with regular shape. In practical engineering, the target is generally more complex, so it is difficult to solve the scattered sound field with the theoretical method directly. In terms of approximated method, many methods have been used commonly, including Physical Acoustic method, Statistical Energy Analysis, Finite Element method (FEM)/ Boundary Element method (BEM), Planar Elements method and T-matrix method[1-3]. At the same time, many
prediction models have been developed around the world, such as SUBTAS[4] software developed in Sweden, BASIS and ASBIEM models and AVAST software developed in Canada, and the SONAX[5] software developed by the US Naval Laboratory based on FEM-BEM method. At present, it has become a hot topic to calculate the low frequency acoustic scattering of complex underwater targets with the boundary element theory.

In this paper, the omni-dimensional scattering characteristics of BeTSSi-sub[6] have been studied, based on the method of finite element coupling boundary element. By changing the material and thickness of the elastic shell, the influence of shell material and shell thickness on the target strength of underwater elastic shell is investigated.

2. Basic Theory

2.1. Theory of Boundary Element Method

The boundary element method can be simply described as three processes, first of which is to change the differential equation of the problem into an integral equation on the boundary, then to introduce a finite number of elements on the boundary to discretize the integral equation, finally obtain a system of equations containing only unknown quantities of nodes on the boundary and solve it numerically. Since the BEM transforms the governing equation on the region into the integral equation along the region boundary, it only needs to define the element on the boundary and solve it by combining with the boundary conditions. In this way, the dimension of the problem can be reduced by one dimension and the complex three-dimensional geometric model can be effectively simplified into a two-dimensional figure. Compared with an equally complex, complete three-dimensional finite element model, a boundary element model is smaller, easier to create, easier to test, and easier to process, which can give results in less time. However, coefficient matrix of the equations established by the BEM is dense and generally asymmetric, resulting in increasing computation time, as the computation of matrix element component is larger. Still as the basic solution of the differential operator used by BEM can automatically be satisfied by the boundary conditions of infinite distance, the BEM is especially suitable for the problems of infinite field and semi-infinite field.

When an acoustic wave is incident on an object, the interaction between the structure and the fluid must be taken into account, as long as the interaction cannot be neglected. In this paper, the investigation of acoustic scattering of an elastic body is one of these cases, solved in the basis of the coupling acoustic boundary element theory combined by BEM and FEM.

2.2. Analysis Theory of Coupled Acoustic Boundary Element

When BEM is applied to solve the integral equation, the boundary element needs to discretize the boundary $\Omega$ into many elements and nodes. The sound pressure and normal velocity at any point in each element can be represented by the sound pressure $p_i$ and normal velocity $v_{ni}$ and global matrix form function $N_a$, belonging to the element node, as

$$p(r_a) = N_a \cdot \{p_i\}, \quad r_a \in \Omega$$

(1)

$$v_s(r_a) = N_a \cdot \{v_{ni}\}, \quad r_a \in \Omega$$

(2)

where $N_a$ is a global form function related to sound pressure $p_i$ and normal velocity $v_{ni}$ at $n_a$ node on the boundary.

However, there are still some nodes in the boundary element that do not know the sound pressure and vibration velocity, such as node $b$, namely $r_a=r_b$, then it can be described as

$$A_b \{p_i\} = j\rho_0 \omega B_b \{v_{ni}\}, \quad b=1,2,\ldots,n_a$$

(3)

where the coefficient matrix $A_b$ and $B_b$ are both matrices of $(1 \times n_a)$.

When it comes to coupled boundary element, the distribution of acoustic field and the vibration displacement of the structure are calculated simultaneously. The boundary element grid can be divided into two parts, one is the part coupled with the structure grid, containing nodes of $a_{ni}$, and the other
part is not involved in the coupling, containing nodes of \( a_{n1}, (a_{n1} + a_{n2} = a_n) \), so that the sound pressure \( p(r_a) \) and velocity \( v(r_a) \) can be expressed as

\[
p(r_a) = N_{a1} \cdot \{p_{i1}\} + N_{a2} \cdot \{p_{i2}\}, \quad r_a \in \Omega_a
\]

\[
v_a(r_a) = N_{a1} \cdot \{v_{i1}\} + N_{a2} \cdot \{v_{i2}\}, \quad r_a \in \Omega_a
\]

where \( N_{a1} \) and \( N_{a2} \) are form function related to node \( a_{n1} \) and node \( a_{n2} \) respectively. Since the sound pressure also acts on the structure, it can also cause the vibration of the structure as a load, so the dynamic equation of the mechanism part can be written as

\[
(K_s + j\omega C_s - \omega^2 M_s) \cdot \{u_i\} + L_c \cdot \{p_{i1}\} = \{F_s\}
\]

where \( K_s \), \( C_s \) and \( M_s \) are respectively stiffness matrix, damping matrix and mass matrix of finite element model of structure, \( \{u_i\} \) is the vector of structural displacement, \( \{F_s\} \) is the external load acting on the structure grid, excluding the sound pressure load. \( L_c \cdot \{p_{i1}\} \) is the load applied by sound pressure on the structural grid. \( L_c \) is the coupling matrix of \((n_s \times n_a)\), written as

\[
L_c = -\sum_{c=1}^{n_c} \left[ \int_{\Omega_c} \left( N_s^T \cdot \{n^e\} \cdot N_{a1} \right) \cdot d\Omega \right]
\]

where \( n_{se} \) is the number of cells in a coupled structure grid, \( N_s \) is the form function of the structure grid, \( \{n^e\} \) is the normal direction of the element, and the minus sign means that the normal direction of the element of structure grid is opposite to that of the boundary element.

As it for the coupled BEM, the nodes of the structure grid and the acoustic grid need to coincide at the coupling plane \( \Omega_s \) of structure and acoustic. At the coupling plane \( \Omega_s \), the relationship among the continuity of structure grid and acoustic grid, displacement \( \{u\} \) and the velocity \( \{v\} \) can be written as \( \{v\} = j\omega \{u\} \), so it can be obtained that

\[
\{v_{ni1}\} = j\omega \left( T_s \{u_i\} + T_u \{\widetilde{u}_i\} \right)
\]

where \( \{\widetilde{u}_i\} \) is the known displacement. \( T_s \) is the matrix of \((n_{a1} \times n_s)\) and \( T_u \) is the matrix of \((n_{a1} \times n_u)\), defined as respectively

\[
T_s = -\{n^e\}^T N_s (r_i), \quad i1 = 1, 2, \cdots, n_{a1}
\]

\[
T_u = -\{n^e\}^T N_u (r_i), \quad i1 = 1, 2, \cdots, n_{a1}
\]

where \( \{n^e\} \) is the normal direction of the structural grid, and the minus sign means that the normal direction of the structure grid is opposite to that of the acoustic grid. Substituting equation (8) into equation (3), it can be obtained that

\[
\begin{bmatrix}
A_{11} & A_{12} & p_{i1} \\
A_{21} & A_{22} & p_{i2}
\end{bmatrix}
= 
\begin{bmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{bmatrix}
+ 
\begin{bmatrix}
-\rho_s \omega^2 (T_s \{u_i\} + T_u \{\widetilde{u}_i\}) \\
j\rho_s \omega \nu_{v_{ni2}}
\end{bmatrix}
\]

By combing the structural finite element equation (6) with boundary element equation (11), the coupled equation of structural finite element and boundary element can be obtained as

\[
\begin{bmatrix}
K_s + j\omega C_s - \omega^2 M_s & L_c & 0 \\
\rho_s \omega^2 B_{11} T_s & A_{11} & A_{12} & p_{i1} \\
\rho_s \omega^2 B_{21} T & A_{21} & A_{22} & p_{i2}
\end{bmatrix}
= 
\begin{bmatrix}
F_s \\
F_{v_{ni1}} \\
F_{a_{ni1}}
\end{bmatrix}
\]

(12)
where
\[
\{ F_{a1} \} = -\rho_0 \omega^2 B_{a1} \{ \overline{u} \} + j \rho_0 \omega B_{a2} \{ v_{a2} \} \\
\{ F_{a2} \} = -\rho_0 \omega^2 B_{a2} \{ \overline{u} \} + j \rho_0 \omega B_{a2} \{ v_{a2} \}
\] (13)

(14)

3. Result Analysis

3.1. Analysis of Scattering Characteristics of Elastic Spherical Shell

In order to verify the effectiveness of FEM-BEM coupled method, the scattering characteristics of the elastic spherical shell in unbounded fluid are calculated.

As shown in figure 1, elastic spherical shell is immersed in the water (acoustic velocity \(c_1=1500\) m/s, \(\rho_1=1000\) kg/m³), and inside of the shell is air (\(c_2=340\) m/s, \(\rho_2=1.26\) kg/m³). The shell is made of steel (Young’s modulus \(E=2.06 \times 10^{11}\) Pa, Poisson’s ratio \(\sigma=0.28\), \(\rho=7800\) kg/m³), of which the outer diameter is 1 meter and the thickness is 20 mm. The centre of the sphere is assumed to be ordinate origin in the case of plane wave incidence. The field point of the receiver is 1000 meters away from the spherical center in the backscattering direction, which is the same position of the microphone. The comparison results of the numerical calculation with the classic theoretical solution are shown in Figure 2.

![Figure 1](image1)

**Figure 1.** The acoustic scattering of elastic sphere shell

As can be seen in figure 1, elastic spherical shell is immersed in the water (acoustic velocity \(c_1=1500\) m/s, \(\rho_1=1000\) kg/m³), and inside of the shell is air (\(c_2=340\) m/s, \(\rho_2=1.26\) kg/m³). The shell is made of steel (Young’s modulus \(E=2.06 \times 10^{11}\) Pa, Poisson’s ratio \(\sigma=0.28\), \(\rho=7800\) kg/m³), of which the outer diameter is 1 meter and the thickness is 20 mm. The centre of the sphere is assumed to be ordinate origin in the case of plane wave incidence. The field point of the receiver is 1000 meters away from the spherical center in the backscattering direction, which is the same position of the microphone. The comparison results of the numerical calculation with the classic theoretical solution are shown in Figure 2.

![Figure 2](image2)

**Figure 2.** Scattering characters of elastic sphere shell

As can be seen in figure 2, the numerical solution and analytical solution are in good agreement, preliminarily demonstrates the FEM-BEM coupling method in calculation of underwater target scattering characteristics is feasible.

3.2. Analysis of the Influence of BeTSSI-Sub Shell Material On Scattering Characteristics

According to the submarine specification diagram in reference 6 (see figure 3), a simplified version of the BeTSSI-Sub model was created in COMSOL Multiphysics. As it for the boundary element model, the grid of the maximum cell is required to be less than 1/6 of the wavelength of the calculation frequency, so if the frequency is calculated to a high level, the number of grid cells will increase...
sharply, and the requirements on the computer will also become higher. According to the grid model built in this paper, the maximum calculated frequency is about 400 Hz, as shown in figure 4.

From literature [7], the submarine target strength value change along with the measurement of distance, so in order to get stable and reliable measurement results, the measurement should be conducted in the far field. For the submarine of length $L$, when the incident wavelength is $\lambda$, the measured distance $r$ should be greater than $L^2/\lambda$. Therefore, the position of the acoustic source is set as 1000 meters away from the geometric center of the submarine, same as the position of the receiver. The target strength of the omni-directional scattering of the elastic (steel, Young’s modulus $E=2.06\times10^{11}$ Pa, Poisson’s ratio $\sigma=0.28$, $\rho=7800$ kg/m$^3$) shell and the elastic (titanium alloy, Young’s modulus $E=1.1\times10^{11}$ Pa, Poisson’s ratio $\sigma=0.34$, $\rho=4500$ kg/m$^3$) shell is calculated. The incident from the bow is defined as 0 degree, and the incident from the tail is defined as 180 degree, and the calculated frequencies are respectively 100 Hz, 200 Hz and 300 Hz. The shell thickness is fixed at 28 mm. The target strength comparison of elastic shell (steel) and elastic shell (titanium alloy) are shown in figure 5.

As can be seen in figure 5, when the frequency is 100 Hz, the material properties have little influence on the target strength in the transverse direction, but in the omni-direction, the target strength of the titanium alloy shell is greater than that of the steel shell. It also can be seen at the frequency of 300 Hz, the material properties have little effect on the target strength in the omni-direction. It is worth noting that when the frequency is 200 Hz, the target strength of the steel shell is...
much higher than that of titanium alloy material, which is quite different from others. It is possible that resonance has occurred, which need to be investigated furtherly.

3.3. Analysis of Influence of BeTSSi-Sub Shell Thickness On Scattering Characteristics

In order to investigate the influence of shell thickness on the scattering characteristics of underwater targets, shell thickness of 25mm, 28mm and 35mm are calculated in this paper. The results are shown in figure 6. It can be seen that when the frequency is 100 Hz and 300 Hz, the target strength increases with the increasing of the shell thickness. Still, it can be seen when the frequency is 200 Hz and the shell thickness is 28mm, resonance has occurred, resulting in a sharp increasing of the values.

![Figure 6. Omni-directional scattering of the elastic (steel) BeTSSi-Sub with different shell thickness](image)

4. Conclusions

In this paper, the low-frequency acoustic scattering characteristics of underwater elastic targets had been investigated. It can be seen from the numerical calculation in this paper that the FEM-BEM coupling numerical calculation method combined with commercial software can not only solve the problem of acoustic scattering characteristics of simple elastic targets, but also accurately and successfully predict the target strength of underwater targets with complex structures.

By calculating the target strength of BeTSSi-Sub with different shell materials and different thickness in the low frequency band, it is found that:

(1) When the frequency is between 100 Hz and 300 Hz, the shell material and shell thickness have little influence on the target strength in the transverse direction;
(2) When the frequency is 100 Hz, the target strength of titanium alloy shell is larger than that of steel shell in the omni-direction, while when the frequency is 300 Hz, the target strength are almost the same;
(3) When the frequency is between 100 Hz and 300 Hz, the target strength increases with the increasing of shell thickness in the omni-direction.
(4) It is worth noting that when the frequency is 200 Hz and the shell thickness is 28 mm, resonance has occurred, resulting in a sharp increase in the target strength.

5. References

[1] LI Wei, ZHAO Yao, ZHANG Tao, et al. Computation and analysis of the acoustic scattering field from submerged rigid objects[J]. Technical Acoustics, 2007, 26(5): 844-849
[2] TANG Wei-lin. Calculation of acoustic scattering from object near an interface using physical acoustic method[J]. Acta Acustica, 1999, 24(1).

[3] HASTING F. D., SCHNEIDER J. B., BROSEHAT S.. A finite-difference time-domain solution to scattering from a rough pressure-release surface[J]. Acoust. Soc Am, 1997, 102(6): 3394-3400.

[4] ALMEGRON M., NODIN M.. Acoustics target strength of submarines. PRAO’ S, 1991.

[5] SHIRRON J., Computational simulation of elastic scattering[D]. Naval Research Laboratory. Washington D. C. 2002

[6] NELL C. W., GILROY L. E.. An improved BASIS model for the BeTSSi submarine[R]. DRDC TR 2003-199. Defence R&D Canada. 2003.11.

[7] LIU Bo-sheng. Water acoustics[M]. Harbin: Harbin University Press, 2011.