Research Article

Control for Isokinetic Exercise with External Disturbance

Xinwei Guo,1 Yixuan Sheng,2 Jinbao Liu,2 and Honghai Liu2

1Department of Industrial Design, School of Mechanical Engineering, University of Science and Technology Beijing, Beijing 100084, China
2State Key Laboratory of Mechanical System and Vibration, School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai 200240, China

Correspondence should be addressed to Honghai Liu; honghai.liu@icloud.com

Received 24 February 2022; Accepted 16 March 2022; Published 9 May 2022

Academic Editor: Bin Xu

Copyright © 2022 Xinwei Guo et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Isokinetic exercise is considered as one of the most effective ways for rehabilitation and muscle strength enhancement. As rehabilitation equipment, the isokinetic exercise device assists isokinetic movement using a single fixed axis and relevant joint adapters, and it requires high stability and safety performances to cooperate with human. In this paper, a brushless DC motor (BLDCM) was adopted to make a study of isokinetic control. Fractional-order fuzzy PID (FOFPID) controller is an effective controller for the nonlinear system to realize isokinetic control according to reference speed and irregular disturbance (load torque). The model of the BLDCM and FOFPID controller system is built in MATLAB/Simulink to simulate the velocity response of the controller, and a comparison between the FOFPID controller and PID controller is made to verify the system stability.

1. Introduction

Relevant research studies of muscular strength enhancement are extremely meaningful for sports and rehabilitation areas to improve the competitive performance and physical fitness [1, 2]. Some isokinetic, isometric, and isotonic physical training could be used with the equipment to achieve the ideal effect of overcoming resistance to exercise muscle strength [3–5]. According to some research studies of muscle contractions, isokinetic exercise is considered as the safest and reliable muscular exercise mode; meanwhile, isokinetic dynamometry is regarded as one of the most credible muscle strength assessments [6]. Hence, isokinetic exercise is increasingly applied for people of dyskinesia and athletes to recover motion ability or enhance muscular strength. To maintain a constant velocity, the isokinetic exercise is accomplished with the assistance of an isokinetic exercise device. At present, isokinetic exercise device mainly aims at “single-joint” configurations which drive a single fixed axis with different adapters corresponding to each joint assembling on it. The joint angular velocity is constant and preset during isokinetic movement while the resistance moment provided by the isokinetic device is equal to joint torque and changes following joint torque [7]. Because of the resistance moment and reciprocating motion provided by the isokinetic exercise device, the isokinetic control under external torque disturbance needs to meet high robustness and reliability performance to achieve stability requirement and security interaction with experimenters. As one of the rehabilitation instruments, isokinetic exercise device has a broad development prospect and potential for hemiplegic patients and athletes. This paper mainly considered the study of isokinetic control to enhance the stability and lay the foundation of isokinetic exercise device development.

Direct current motor (DC motor) has been widely used in industries for several years, owing to its advantages of stability, fast response, high efficiency, simple control system, etc. Brushless DC motor (BLDCM) has evolved from DC motor driven from pulse width modulation (PWM) inverters which overcomes the disadvantages of speed, lifetime, and noise [8, 9]. For isokinetic control application, the system is nonlinear and BLDCM needs a more complicated controller to achieve constant speed control compared with traditional DC motor. Many researchers have dedicated themselves to study effective control strategies to increase the capability of speed tracking under disturbance.
and sudden torque change [10, 11]. As a simple and useful controller, conventional proportional-integral-derivative (PID) controller is applied in industrial control systems generally because of its performance of high adaptation and simplicity [12]. However, its low-precision, poor anti-jamming ability and fixed parameters make PID controller fail to satisfy the high-performance requirements in some industrial applications especially for the systems with high disturbances [13]. Therefore, various speed controllers have been studied on the basis of conventional and intelligent control strategies. Xia et al. discussed switching-gain adaptation current control to achieve high performance with changing disturbance in dynamic and static processes. The result showed that the general disturbance could be eliminated in different operating stages and the controller improved the steady-state accuracy effectively [14]. Mandel and Weiss used a resonant controller in which resonant frequencies could be adjusted on the basis of motor speed to reduce torque ripple. According to the experimental tests, the controller showed the ability of tracking variable speed command and restraining the influence of cogging torque [15]. Wang et al. designed a quadratic single neuron (QSN) adaptive PID controller which could follow the reference speed successfully with sudden load disturbance [16]. Sivari et al. proposed a novel bacterial foraging algorithm-optimized online adaptive neuro-fuzzy inference system (ANFIS) controller which performs better set point tracking and has better learning parameter tuning ability and time domain specifications [11]. Xu et al. proposed a novel real-time planning method for robot to simultaneously avoid obstacle and track the target [17]. For external disturbance, many researchers used appropriate control strategies according to their inherent properties to overcome the perturbation, such as active disturbance rejection control [18], neural network-based adaptive impedance control [19], repetitive control [20], and so on [21].

In terms of various intelligent control techniques, fuzzy logic control (FLC) is a robust, intelligent, and adaptive control strategy for complex nonlinear dynamic systems [22]. As the promotion of intelligent control strategies, FLC is developing towards the trend of self-adaptive with other controllers, such as fuzzy PID control [23], neuro-fuzzy control [24], fuzzy predictive control [25], and so on. Fractional-order fuzzy PID (FOFPID) controller is a combination of fuzzy controller and PID controller with noninteger orders in differential and integral parts. From some relevant research studies, the efficiency and robustness could be enhanced distinctly by combining fractional-order operator. Kumar and Rana applied nonlinear adaptive FOFPID controller for a 2-link planar rigid robotic manipulator with payload, and the results clearly revealed a good trajectory tracking performance [26]. Kumar et al. proposed a self-tuned robust FOFPID controller for a nonlinear active suspension system of a quarter car, and the simulation study showed a better performance by providing comfort ride during hard constraint [27]. Although the previous research studies of FOFPID presented better control capability, there are less research studies of isokinetic control with periodic velocities and irregular external load torque.

In this paper, a FOFPID controller was applied to control the isokinetic movement of BLDCM. Section 2 introduces the mathematical model of BLDCM and the simulation model in MATLAB/Simulink. FOFPID controller for isokinetic control is proposed in Section 3. Section 4 presents the simulation results and makes a comparison between FOFPID controller and PID controller. The discussion and conclusion are given in Section 5.

### 2. Model of BLDCM System

#### 2.1. Mathematical Model of BLDCM System

The mathematical model of BLDCM system would be established under the condition of reasonable simplification. The following assumptions are proposed. (1) Stator has star connection, three-phase winding is completely symmetrical, and the model is working in two-phase conduction and three-phase six states. (2) The magnetic circuit of the motor is unsaturated during operation without eddy current and hysteresis loss. (3) The air gap is uniform and the magnetic field is a square wave. (4) The stator current and rotor magnetic field are symmetrical. (5) The armature windings are uniformly and continuously distributed on the inner surface of the stator. (6) The armature effect and cogging effect are ignored [28, 29].

Under the above assumptions, the voltage balance equation of three-phase winding could be obtained by the Kirchhoff voltage law (KVL):

\[
\begin{bmatrix}
    u_A \\
    u_B \\
    u_C \\
\end{bmatrix} = 
\begin{bmatrix}
    R & 0 & 0 \\
    0 & R & 0 \\
    0 & 0 & R \\
\end{bmatrix} \begin{bmatrix}
    i_A \\
    i_B \\
    i_C \\
\end{bmatrix} + \begin{bmatrix}
    L - M \\
    L - M \\
    L - M \\
\end{bmatrix} \begin{bmatrix}
    e_A \\
    e_B \\
    e_C \\
\end{bmatrix} + \begin{bmatrix}
    \frac{d}{dt}i_A \\
    \frac{d}{dt}i_B \\
    \frac{d}{dt}i_C \\
\end{bmatrix},
\]

where \(u_A, u_B, u_C\) refer to stator phase winding voltage (V); \(i_A, i_B, i_C\) refer to stator phase winding current (A); \(e_A, e_B, e_C\) refer to stator phase winding electromotive force (V); \(R\) is the motor phase resistance (\(\Omega\)); \(L\) is self-inductance of each phase winding (H); and \(M\) is the mutual inductance between each two-phase winding (H).
The correlation between the stator winding input power and the generated electromagnetic torque is described by the following equation:

$$e_{A}i_{A} + e_{B}i_{B} + e_{C}i_{C} = T\omega.$$  (2)

Hence, the electromagnetic torque of BLDCM could be formulated as follows:

$$T = \frac{e_{A}i_{A} + e_{B}i_{B} + e_{C}i_{C}}{\omega},$$  (3)

where $T$ is the electromagnetic torque (Nm) and $\omega$ is the angular velocity of motor (°/s).

The electromagnetic torque of BLDCM is generated by the interaction between the current in the stator winding and the magnetic field generated by the rotor magnetic steel. From (3), it could be known that electromagnetic torque is proportional to the magnetic field and phase current and inversely proportional to the rotational angular velocity.

The motion equation could be expressed as

$$T - T_{L} = J\frac{d\omega}{dt},$$  (4)

where $T_{L}$ is the load torque and $J$ is the moment of inertia of motor.

For the isokinetic movement process, the motor angular velocity $\omega$ gradually increases during acceleration stage, then remains constant during isokinetic stage, and decreases during deceleration stage. The electromagnetic torque $T$ (resistance moment) is equal to joint torque (load torque). Hence, the isokinetic motion state equations could be expressed as

$$\begin{align*}
\frac{d\omega}{dt} &= a > 0 \text{ acceleration stage} \\
\frac{d\omega}{dt} &= 0 \text{ isokinetic stage} \\
\frac{d\omega}{dt} &= -a < 0 \text{ deceleration stage}
\end{align*}$$  (5)

where $\omega$ is the preset joint angular velocity and $a$ is the acceleration of acceleration and deceleration stages.

2.2. Modelling BLDCM in MATLAB/Simulink. The BLDCM system for isokinetic control consists of a BLDCM module, reference current module, PWM current controller module, voltage inverter module, and FOFPID controller module. The modular modelling schematic of the BLDCM system is shown in Figure 1. The system consists of two control parts: speed control and torque control. The FOFPID controller was applied for speed control, and a conventional PID controller was used for torque control. The application target of the BLDCM system is to simulate the isokinetic movement stage during isokinetic exercise with the preset speed and load torque by the experimenter. To validate the robustness and stability performance of the FOFPID controller and BLDCM system during isokinetic movement under disordered joint torque, the system model was built in MATLAB/Simulink as shown in Figure 2.

3. Isokinetic Control System of BLDCM

In this study, a fractional-order fuzzy PID controller, which combines the fractional-order calculus, fuzzy logic control, and conventional PID control, is proposed to control the speed of the BLDCM system for tracking the reference speed command under irregular load torque. The FOFPID controller has evolved from fuzzy PI and fuzzy PD controllers with self-tuning and adaptive capabilities [30]. The characteristic of the FOFPID controller is using noninteger order instead of integer order integrator and differentiator operators. In the subsequent sections, the design of the FOFPID controller is presented.

3.1. Fuzzy PI and Fuzzy PD Controllers. The basic structures of the FOFPID controller are fuzzy PI and fuzzy PD controllers, which could provide proportional, integral, and derivative parameters and fuzzy logic to realize adaptive control for the system. The algorithms of fuzzy PI and fuzzy PD controllers could be expressed as [26]

$$u_{PI}(t) = K_{fi}^{f}e(t) + K_{i}^{f} \int e(t)dt,$$

$$u_{PD}(t) = K_{fp}^{f}e(t) + K_{d}^{f} \frac{de(t)}{dt},$$

where $K_{fp}^{f}$ is the proportional gain of fuzzy control modulation, $K_{i}^{f}$ is the integral gain of fuzzy control modulation, and $K_{d}^{f}$ is the derivative gain of fuzzy control modulation.

3.2. Design of FOFPID Controller. The FOFPID controller has evolved from fuzzy PID with noninteger order calculus. It consists of fractional calculus, fuzzy logic control, and two separate PI and PD controllers, and the structure of the FOFPID controller is shown in Figure 3. The structure only has two input variables, and fuzzy PI and fuzzy PD controllers share the same rule base to perform control actions. The controller has six parameters, including four scaling factors $[K_{u}, K_{r}, K_{s}, K_{d}]$ and two fractional orders $[\mu, \lambda]$. Deviation $e$ and its variation of deviation $\frac{de}{dt}e$ are the two input parameters of fuzzy controller which are used to determine fuzzy rules. Two input scaling factors $[K_{u}, K_{r}]$ are the common input gains of FOFPID controller, modulating the input parameters to meet ranges of fuzzy membership functions, and another two scaling factors $[K_{s}, K_{d}]$ are fuzzy PI controller gain and fuzzy PD controller gain, respectively. Fractional calculus is applied into derivative and integral parts through fractional orders $\mu$ and $\lambda$, as the distinction of fuzzy FID and FOFPID. For the fuzzy PID controller, $\mu$ and $\lambda$ are equal to 1, whereas $\mu$ and $\lambda$ are fractional in the FOFPID controller.
The FOFPID controller could be expressed as [23]

\[ K_u e + K_r \frac{d^\mu}{dt^\mu} e = K_d u_{PD} + K_i \frac{1}{s} u_{PI}, \]

(7)

where \( \frac{d^\mu}{dt^\mu} \) is the differentiator, \( \frac{1}{s} \) is the integrator, \( u_{PD} \) is the control action of the fuzzy PD controller, and \( u_{PI} \) refers to control actions of the fuzzy PI controller.

Formulating fuzzy control rules is the key of designing a fuzzy controller which includes selecting the linguistic variables of input and output parameters, defining the fuzzy subset of fuzzy variables, and establishing the control rules of fuzzy controller [31]. The rule table of the fuzzy controller is a set of fuzzy condition statement with several linguistic variables. Choosing appropriate linguistic variables to describe input and output parameters could make the establishment of control rules more accurate, and defining the fuzzy subset is to determine the shape of membership functions. Figure 4 shows the membership functions of input and output of fuzzy logic control. Symmetrical triangular membership functions were adopted for three inputs factors, including negative (N), zero (Z), and positive (P) variables, while singleton consequents were adopted for five output factors, including NB (negative big), NM (negative medium), ZO (zero), PM (positive medium), and PB (positive big) variables.

Based on the membership functions of inputs and outputs, the fuzzy control rules of fuzzy PI and fuzzy PD controllers are described as follows:

Rule 1: if \( K_u e \) is N and \( K_r \frac{d^\mu}{dt^\mu} e \) is N, then \( u_{PD} \) is NB.
Rule 2: if \( K_u e \) is N and \( K_r \frac{d^\mu}{dt^\mu} e \) is Z, then \( u_{PD} \) is NM.
Rule 3: if \( K_u e \) is N and \( K_r \frac{d^\mu}{dt^\mu} e \) is P, then \( u_{PD} \) is ZO.
Rule 4: if \( K_u e \) is Z and \( K_r \frac{d^\mu}{dt^\mu} e \) is N, then \( u_{PD} \) is NM.
Rule 5: if \( K_u e \) is Z and \( K_r \frac{d^\mu}{dt^\mu} e \) is Z, then \( u_{PD} \) is ZO.
Figure 4: (a) Input membership functions. (b) Output membership functions.

Table 1: Six parameters of the FOFPID controller.

| Parameter | Value |
|-----------|-------|
| $K_u$     | 15.172|
| $K_r$     | 6.489 |
| $K_i$     | 12.239|
| $K_d$     | 1.011 |
| $\mu$     | 0.348 |
| $\lambda$ | 0.624 |
Figure 5: Simulation results under 30°/s. (a) Elbow joint torque. (b) Comparison between reference speed, speed response of the FOFPID controller, and speed response of the PID controller. (c) Error between reference speed and speed response of the FOFPID controller. (d) Error between reference speed and speed response of the PID controller.

Figure 6: Continued.
Figure 6: Simulation results under 90°/s. (a) Elbow joint torque. (b) Comparison between reference speed, speed response of the FOFPID controller, and speed response of the PID controller. (c) Error between reference speed and speed response of the FOFPID controller. (d) Error between reference speed and speed response of the PID controller.

Figure 7: Continued.
Rule 6: if $K_u e$ is Z and $K_r d/\text{d}t e$ is P, then $U_{PD}/U_{PI}$ is PM.

Rule 7: if $K_u e$ is P and $K_r d/\text{d}t e$ is N, then $U_{PD}/U_{PI}$ is ZO.

Rule 8: if $K_u e$ is P and $K_r d/\text{d}t e$ is Z, then $U_{PD}/U_{PI}$ is PM.

Rule 9: if $K_u e$ is P and $K_r d/\text{d}t e$ is P, then $U_{PD}/U_{PI}$ is PB.

Six tuned gains of the FOFPID controller ({$K_u$, $K_r$, $K_i$, $K_d$} and {$\mu$, $\lambda$}) which were adopted to control speed of the simulation model are listed in Table 1.

4. Simulation Results

In this section, the simulation results and result analysis would be presented to explore the stability of the BLDCM system using the FOFPID controller.

4.1. Reference Data Collection. Reference speed and load torque are the two input parameters which were collected from the Isomed2000 isokinetic device. The experiment is focused on the isokinetic movement of the elbow joint, and the elbow adapter is assembled on the fixed axis. The experimenter was seated on the seat with the right hand holding the handle of adapter and was required to perform maximum voluntary torque (MVT) during the elbow joint isokinetic movement. The movement is a kind of elbow joint flexion and extension movement. Eight sets of experimental data were collected under the different joint angular velocities of 30, 45, 60, 75, 90, 120, 150, and 180°/s, respectively, and the experimenter finished five groups of reciprocating motions for each preset speed. Each group includes an elbow flexion and extension process which could be considered as motor forward rotation and reverse rotation. Isokinetic movement is divided into three stages: acceleration stage, isokinetic stage, and deceleration stage. Under different reference speeds (joint angular velocities), the training effects are various and the stability is also affected differently by load torque. The fluctuations of velocities and joint torque were recorded by sensors of the isokinetic exercise device.

4.2. Simulation Results. For the simulation experiment in Simulink, the FOFPID controller was applied to control the reference speed of the BLDCM; meanwhile, a PID controller was used to control the load torque of the system. According to the simulation model in Figure 2, the control results under different reference speeds could be obtained separately. Figures 5–7 show the following under angular velocities of 30, 90, and 180°/s: (a) the elbow joint torque resisting the resistance movement; (b) the comparison between reference speed, speed response of the FOFPID controller, and speed response of the PID controller; (c) error between the reference speed and the FOFPID controller speed response; (d) error between the reference speed and the PID controller speed response. These three figures could represent the variation tendency of joint torque and speed response from low speed to high speed.

For isokinetic movement, the joint torque is related to the muscle force of the experimenter and the preset speed of the equipment. In the experiment, the load torque is the elbow joint torque under MVT. The torque values in Figures 5(a), 6(a), and 7(a) show irregular periodic fluctuations and have spikes between two isokinetic stages especially in the high-speed conditions. This is because the experimenter should follow the speed as quick as possible and switch the directions rapidly. Thus, the torque gained a peak value when the motor went from forward to reverse or from reverse to forward.
(acceleration stages and deceleration stages) where the accelerations are quite large when the angular velocities are high relatively. The load torque in isokinetic stages remains in a relatively stable state compared with other two stages. These load torque values are the input disturbances of the BLDCM system.

The comparison between different reference speeds, speed response of the FOFPID controller, and speed response of the PID controller provides an intuitive presentation of controller’s tracking characteristics. Figures 5(b), 6(b), and 7(b) show a good tracking performance of the FOFPID controller and PID controller that all the speed responses under 30, 90, and 180°/s joint angular velocities could follow the reference speeds successfully. To compare the stability of the FOFPID controller and PID controller, the errors between reference speed and FOFPID controller speed response are described in Figures 5(c), 6(c), and 7(c), and the errors between reference speed and PID controller speed response are given in Figures 5(d), 6(d), and 7(d). These errors revealed the tracking ability during the simulation processes. It is observed that the errors of the FOFPID controller under three angular velocities have uniform error ranges around ±0.2°/s, while the errors of the PID controller have obvious fluctuations along with the changes of speed and torque. Also, comparing the errors of the FOFPID controller with the errors of the PID controller, the stability of the FOFPID controller is better than that of the PID controller for the smaller fluctuation ranges. In Figure 7(c), there exists some abnormal value which exceeds the error range mainly caused by the sudden peak values of load torque during acceleration stages and deceleration stages.

In order to reflect the performance of the FOFPID controller and PID controller more intuitively, data statistics and distribution results of errors are shown in Figures 8 and 9 which display the statistics and distribution results of eight speed errors. The small boxes for each velocity are the main data distribution regions which are limited between the upper interquartile and lower interquartile. Two segments above and under the boxes are upper adjacent and lower adjacent to intercept extreme outliers. The red plus signs which extend to the top and bottom directions are the mild outliers. From Figure 8, the distribution of errors under eight angular velocities remains between ±0.5°/s, while the error distribution in Figure 9 shows an increasing trend. This demonstrates that the FOFPID controller could maintain the stability of the BLDCM system regardless of the velocity change or torque disturbance; however, the stability of the PID controller would decrease as the preset velocity increases. Although some mild outliers exist in Figure 8, all the mild outliers were produced during acceleration stages and deceleration stages as shown in Figure 7(c), where the deviations tend to occur but have small impact on the system especially for isokinetic stages. Meanwhile, the upper adjacent, median,
5. Conclusion

The simulation experiment in this paper is to explore the stability of isokinetic control of the BLDCM system and realize isokinetic control of the BLDCM system in MATLAB/Simulink environment. As a nonlinear system, the FOFPID controller was applied to implement the isokinetic control based on the BLDCM system. This study of isokinetic control discussed the stability performance of the BLDCM system under different preset speeds and resistance moments according to the comparison between the FOFPID controller and PID controller. The biggest challenge is to track the preset speed under continuous and various load torques. From the simulation results in Section 4, the FOFPID controller has better stability performance compared with the PID controller which could maintain the system errors around ±0.5°/s for all joint angular velocities.

According to the comparison between reference speed and FOFPID controller speed response, the proposed controller could obtain a better dynamic performance under various angular velocities and complex load torque. In the future, the proposed controller needs to be further optimized to enhance the stability and robustness by improving the parameters and membership functions of variables and further considering the characteristics of external disturbance. Meanwhile, the simulation results should be experimentally validated to verify the effectiveness of the proposed controller in real experiments.

Data Availability

The data used to support the findings of this study are included within the article.

Disclosure

An earlier version of this paper has been presented as conference in 2019 IEEE 9th Annual International Conference on Cyber Technology in Automation Control and Intelligent Systems (CYBER).

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This study was supported by the National Natural Science Foundation of China (nos. 51575338, 51575407, 51475427, and 61733011) and the Fundamental Research Funds for the Central Universities (17 JCYB03).

References

[1] T. J. Suchomel, S. Nimphius, and M. H. Stone, “The importance of muscular strength in athletic performance,” Sports Medicine, vol. 46, no. 10, pp. 1419–1449, 2016.
[2] M. B. Undheim, C. Cosgrave, E. King et al., “Isokinetic muscle strength and readiness to return to sport following anterior cruciate ligament reconstruction: is there an association? A systematic review and a protocol recommendation,” British Journal of Sports Medicine, vol. 49, no. 20, pp. 1305–1310, 2015.
[3] L. Wang, Z. Zhang, Y. Yu, and G. Chen, “Study on the control of variable resistance for isokinetic muscle training system,” Technology and Health Care, vol. 25, pp. 45–52, 2017.
[4] K. A. Taylor, J. D. Wiles, D. A. Coleman, P. Leeson, R. Sharma, and J. M. O’Driscoll, “Neurohumoral and ambulatory haemodynamic adaptations following isometric exercise training in unmedicated hypertensive patients,” Journal of Hypertension, vol. 37, 2018.
[5] T. Bohman, M. Tegern, A. Halvarsson, I. Broman, and H Larsson, “ Concurrent validity of an isokinetic lift test used for admission to the Swedish Armed Forces,” Journal of Sports Science, vol. 13, no. 11, Article ID e0207054, 2018.
[6] Z. Dvir, Isokinetics: Muscle Testing, Interpretation and Clinical Applications, Elsevier-Churchill Livingstone, Edinburgh, Scotland, 2nd edition, 2004.
[7] G. J. Davies, A Compendium of Isokinetics in Clinical Usage and Rehabilitation Techniques, S Publisher, La Crosse, WI, USA, 4th edition, 1992.
[8] N. H. Kim, O. Yang, and M. H. Kim, “BLDC motor control algorithm for industrial applications using a general purpose processor,” J. Power Electron.vol. 7, no. 2, pp. 132–139, 2007.
[9] M. Nerat and D. Vranic, “A novel fast-filtering method for rotational speed of the BLDC motor drive applied to valve actuator,” IEEE, vol. 21, no. 3, pp. 1479–1486, 2016.
[10] W. Lee, J. H. Kim, W. Choi, and B. Sarioglu, “Torque ripple minimization control technique of high-speed single-phase brushless DC motor drive for electric turbocharger,” IEEE Transactions on Vehicular Technology, vol. 67, no. 9, 2018.
[11] T. S. Sivarani, S. Joseph Jawhar, C. Ages Kumar, and K. Prem Kumar, “Novel Bacterial Foraging-Based ANFIS for Speed Control of Matrix Converter-Fed Industrial BLDC Motors Operated under Low Speed and High torque,” Neural Computing and Applications, vol. 29, 2016.
[12] K. J. Astrom and T. Haggulund, PID Controllers: Theory, Design and Tuning, International Society for Measurement and Control, North Carolina, NC, USA, 2nd edition, 1995.
[13] S. Thamizmani and S. Narasimman, “Design of fuzzy PID controller for brushless DC motor,” Int J Emerg Res Manage Technol, vol. 9359, pp. 68–78, 2014.

Table 2: Statistics of error upper boundary, lower boundary, and median under eight speeds.

| Speed (°/s) | Upper adjacent | Median | Lower adjacent |
|------------|---------------|--------|---------------|
|            | FOFPID        | PID    | FOFPID        | PID            |
| 30         | 0.456         | 0.593  | -2.6e-03      | 4.9e-05        |
| 45         | 0.340         | 0.630  | -4.5e-03      | -7.2e-03       |
| 60         | 0.404         | 0.718  | -1.1e-02      | -1.6e-03       |
| 75         | 0.409         | 0.675  | 4.0e-03       | -6.2e-03       |
| 90         | 0.449         | 0.879  | -6.0e-03      | -1.5e-02       |
| 120        | 0.496         | 1.006  | -2.3e-03      | 1.5e-02        |
| 150        | 0.464         | 1.599  | 1.0e-02       | -1.5e-03       |
| 180        | 0.518         | 1.792  | -9.8e-04      | -1.3e-02       |
[14] C. Xia, G. Jiang, W. Chen, and T. Shi, "Switching-gain adaptation current control for brushless DC motors," IEEE Transactions on Industrial Electronics, vol. 63, no. 4, pp. 2044–2052, 2016.
[15] Y. Mandel and G. Weiss, “Adaptive internal model-based suppression of torque ripple in brushless DC motor drives,” Systems Science & Control Engineering, vol. 3, no. 1, pp. 162–176, 2015.
[16] X. Wang, T. Fu, and X. Wang, "BLDC motor control system based on quadratic single neuron adaptive PID algorithm,” International Journal of Advanced Pervasive and Ubiquitous Computing, vol. 6, no. 1, pp. 12–25, 2017.
[17] Z. Xu, X. Zhou, H. Wu, X. Li, and S. Li, "Motion planning of manipulators for simultaneous obstacle avoidance and target tracking: an RNN approach with guaranteed performance,” IEEE Transactions on Industrial Electronics, vol. 69, no. 4, pp. 3887–3897, 2022.
[18] J. Dong, D. Li, and Y. Zhang, "Active disturbance rejection control for TITO systems,” in Proceedings of the 2020 IEEE 9th Data Driven Control and Learning Systems Conference (DDCLS), IEEE, Xi’an, China, July 2020.
[19] Z. Xu, X. Li, S. Li, H. Wu, and X. Zhou, "Dynamic neural networks based adaptive optimal impedance control for redundant manipulators under physical constraints,” Neurocomputing, vol. 471, no. 2, 2021.
[20] Q. Zhang, H. Guo, Y. Liu et al., "Robust plug-in repetitive control for speed smoothness of cascaded-PI PMSM drive,” Mechanical Systems and Signal Processing, vol. 163, no. 8, Article ID 108090, 2022.
[21] X. Li, X. Zhou, Z. Xu, and G. Tang, "Chaos-based second-order bas for multimodal function optimization,” Filomat, vol. 34, no. 15, pp. 5159–5172, 2020.
[22] R. H. Abiyev, O. Kaynak, T. Alshanableh, and F. Mamedov, "A type-2 neuro-fuzzy system based on clustering and gradient techniques applied to system identification and channel equalization,” Applied Soft Computing, vol. 11, no. 1, pp. 1396–1406, 2011.
[23] K. S. Tang, K. F. Kim Fung Man, G. Guanrong Chen, and S. Kwong, "An optimal fuzzy PID controller,” IEEE Transactions on Industrial Electronics, vol. 48, no. 4, pp. 757–765, 2001.
[24] K. Kiguchi, T. Tanaka, and T. Fukuda, "Neuro-fuzzy control of a robotic exoskeleton with EMG signals,” IEEE Transactions on Fuzzy Systems, vol. 12, no. 4, pp. 481–490, 2004.
[25] P. Muthukumar, P. Balasubramaniam, and K. Ratnavelu, "T-S fuzzy predictive control for fractional order dynamical systems and its applications,” Nonlinear Dynamics, vol. 86, no. 2, pp. 751–763, 2016.
[26] V. Kumar and K. P. S. Rana, "Nonlinear adaptive fractional order fuzzy PID control of a 2-link planar rigid manipulator with payload,” Journal of the Franklin Institute, vol. 354, Article ID S0016003216304227, 2016.
[27] V. Kumar, K. P. S. Rana, J. Kumar, and P. Mishra, "Self-tuned robust fractional order fuzzy PD controller for uncertain and nonlinear active suspension system,” Neural Computing and Applications, vol. 30, 2016.
[28] H. C. Lovatt, M. L. McClelland, and J. M. Stephenson, "Comparative Performance of singly salient reluctance, switched reluctance, and induction motors,” in Proceedings of the The Eighth International Conference on Electrical Machines and Drives, pp. 361–365, Cambridge, UK, September 1997.
[29] P. Crnosija, R. Krishnan, and T. Bjazic, "Transient performance based design optimization of PM brushless DC motor drive speed controller,” in Proceedings of the IEEE International Symposium on Industrial Electronics, 2005. ISIE 2005, vol. 3, pp. 881–886, Dubrovnik, Croatia, June 2005.
[30] P. Mishra, V. Kumar, and K. P. S. Rana, "A fractional order fuzzy PID controller for binary distillation column control,” Expert Systems with Applications, vol. 42, no. 22, Article ID S0957417415004686, 2015.
[31] A. Katbab, "Fuzzy logic and controller design—a review,” in Proceedings of the Southeastcon’95: Visualize the Future, pp. 443–449, Raleigh, NC, USA, March 1995.