Abstract: We investigate the different contributions to the in-plane flow of $K^+$ mesons observed recently by the FOPI collaboration in the reaction $\text{Ni} + \text{Ni}$. Due to the kinematics of the three body phase space decay the flow of the kaons produced in baryon-baryon interactions is smaller than that of the baryons in the entrance channel. On the contrary, in $\pi N$ interactions the flow of the sources and of the kaons are identical. Therefore the total kaon flow depends on the relative number of $\Delta N \rightarrow K^+$ and $\pi N \rightarrow K^+$ reactions and hence on the lifetime of the $\Delta$, in addition to the already known dependence on the potential interaction of the kaons with the nuclear environment.

I. INTRODUCTION

The production of K mesons in heavy ion collisions is presently one of the most challenging topics in nuclear physics. At beam energies below or close to the threshold (in NN collisions $E_{\text{thres}} = 1.583$ GeV) we observe a strong enhancement of the kaon production as compared to the extrapolation of pp collisions. Detailed investigations have shown that most of the K's are created in two step processes via an intermediate $\Delta$ or $\pi$ and are produced at a density well above the normal nuclear matter density. This triggered the conjecture that K’s may be of use as a messenger of the high density zone.

What makes a straightforward analysis complicated are several problems:

• The nucleons as well as the K’s and $\Lambda$’s interact with each other via static and momentum dependent interactions. Hence particle properties like the particle mass are modified.

• This modification of the particle properties changes the thresholds of the relevant strange particle production cross sections $NN \rightarrow \Lambda KN$, $NN \rightarrow \Sigma KN$, $NN \rightarrow NNKK$ as well as of the the rescattering reactions.

• Most of the K’s are produced in a collision in which a nuclear resonance or its decay product is involved. Hence the lifetime of a resonance in nuclear matter becomes important. If the life time $\tau$ is short as compared to $\lambda(\rho)/ < v >$, where $\lambda$ is the density dependent mean free path of the nuclear resonance and $< v >$ is its average velocity, the resonance decays before it encounters another nucleon for creating a kaon. Then only the decay product, i.e. the $\pi$ can create a kaon. If the opposite is the case, the dominant production channel will be Resonance + N $\rightarrow \Lambda(\Sigma)NK$.

It is the purpose of this article to investigate the influence of these in medium properties on the production of K’s, especially on the observed in-plane flow which has been proposed as a signature of the relative strength of the scalar and vector part of the interaction of the K’s with its hadronic environment.
For this purpose we employ simulations of the heavy ion reaction with the Quantum Molecular Dynamics (QMD) approach. This approach is described in ref [4]. In addition we have implemented the kaon producing cross sections \( \pi B \to YK \) and \( BB \to BYK \) with \( B = N, \Delta \) and \( Y = \Lambda, \Sigma \). We have displayed in fig. 1 the important cross section with a \( \Lambda \) in the final state. Similar fits to the (fewer) available data for the reaction with a \( \Sigma \) in the final state have also been made and are included in the calculation. The energy range of importance is shown in fig. 2 where we display the number of elementary collisions as a function of \( \sqrt{s} \) in the simulation. The relevance of the different channels is given by the number of collisions times the kaon production cross section. We see that \( \pi N, \Delta N \) and NN collision contribute all to the kaon production in a non negligible way. We see as well that the deviations of our parameterization from data at high \( \sqrt{s} \) is not of importance for the beam energy considered here.

After being produced the K’s move in a potential created by the nuclear environment which has the form [5]:

\[
\begin{align*}
\mathbf{u}_{K}^{opt} &= \sqrt{(k - g_v \Sigma_v)^2 + m_K^2 + m_K g_s \Sigma_s + g_v \Sigma_v^0} - \sqrt{k^2 + m_K^2} \\
\sqrt{\mathcal{S}} &= \sqrt{\mathcal{S}_0} \ (\text{GeV})
\end{align*}
\]

(1)

and rescatter with the nucleons. The cross section of the rescattering of the produced kaons is a parametrisation of the results presented in [6].

II. KAON’S: MESSENGER FROM THE HIGH DENSITY ZONE?

Naively one expects that K’s are created in the high density zone of the reaction. Because we are at subthreshold energies the kaon is produced easiest if there is a nuclear resonance in the entrance channel, which - due to its higher mass as compared to a nucleon - needs only a smaller relative momentum with
respect to its scattering partner to overcome the threshold. The shorter its mean free path, the more probable the resonance encounters a nucleon before it disintegrates. Because $\lambda \propto \frac{1}{\sigma \rho}$, the kaon production via resonances appears preferably at high densities. When created, the kaon is still surrounded by nuclear matter and may encounter a collision with nucleons while the system disintegrates. The cross section for rescattering of $\approx 12\text{mb}$ is small but nonnegligible. Such collisions may destroy the information about the high density zone which is carried by the K’s.

Fig 2. displays the distribution of nuclear densities at the places where the K’s are produced. We see that even for a system as small as Ni+Ni 80% of the K’s are produced at a density above normal nuclear matter density and 60% do not rescatter at a density lower than normal nuclear matter density. Hence K’s can really be regarded as messenger from the high compressed phase of the reaction. It is the only meson which has this property at the energy considered in this article.

III. IN-PLANE FLOW OF THE K’S

Recently it has been speculated [3,7] that the in-plane flow of the K’s is directly related to the relative strength of the scalar and vector potential of K’s in nuclear matter. Therefore it is interesting to discuss in detail the origin of the in-plane flow of the K’s, which is much lower than the in-plane flow of the nucleons measured in the same reaction.

Before discussing the kaon flow it is, however, necessary to verify that the simulations reproduce the nucleon flow although, as we will see later, both are only very loosely connected. The experimental nucleon in-plane flow as com-
FIG. 3. Density distribution of the kaon production and of the last collisions of the kaons with the surrounding nucleons.

pared to the simulation is displayed in fig. 4. We see that the soft momentum dependent interaction \[4\] reproduces the experimental flow as well as the less realistic static soft equation of state.

Fig.5 displays the time evolution of the in-plane flow of different classes of protons. The average directed in-plane flow is defined by

\[
p_{x_{\text{dir}}} = \frac{1}{N} \sum_{i=1}^{N} p_{x}^{i} \cdot \text{sign}(y_{\text{cm}}^{i})
\]

(2)

where \(y_{\text{cm}}^{i}\) is the rapidity of the nucleon \(i\) in the nucleus-nucleus center of mass system and \(p_{x}^{i}\) is the momentum of the nucleon \(i\) in the direction of the impact parameter. The calculation has been done at an impact parameter of 2 fm.

The averaged directed in-plane flow of all nucleons is given by a line. We see that it reaches asymptotically 80 MeV/c. For the dotted line we have counted only those nucleons which have passed a density of \(\rho/\rho_0 \geq 2\). As already found in ref. \[8\] the in-plane flow of nucleons which have passed the high density zone is considerably smaller than that of all nucleons. If we include in the calculation of \(p_{x_{\text{dir}}}^{i}\) only those nucleons, which have been involved in a collision, in which a kaon has been produced (dashed line), we observe about the same in-plane flow as for those nucleons which have passed the high density zone. This agreement is compatible with the fact that K’s are produced in the high density zone as shown in fig.1. More interesting than the in-plane flow of the protons is the in-plane flow of the sources in which the K’s are produced according to phase space, i.e. isotropically:

\[
p_{x_{\text{dir}}} = \frac{1}{M} \sum_{i=1}^{M} (p_{x}^{1i} + p_{x}^{2i}) \cdot \text{sign}(y_{\text{cm}}^{1i} + y_{\text{cm}}^{2i}).
\]

(3)
Here the sum runs over all collisions in which a kaon is produced. 1 and 2, respectively, mark the two hadrons which scatter in these collisions. The result is displayed as the dashed-dotted line. This in-plane flow has about the same size as that for the nucleons involved in a kaon producing collision and is not twice as large as one may think naively. Hence the in-plane directed source velocity $p_{\text{dir}}^x / \text{mass}$ is about half as large as the directed in-plane velocity of the participating nucleons.

The reason for this astonishing result is displayed on the left hand side of fig. 6 where we show the rapidity distribution of the kaon producing sources as well as that of the $\Lambda$’s and of the K’s at the moment of their creation (i.e. before rescattering) in the reaction baryon+baryon $\rightarrow \Lambda K N$. We see that - due to kinematical reasons - most of the sources are centered very close around midrapidity in the nucleus-nucleus center of mass system. Hence it happens quite often that one of the reaction partners has a negative and the other a positive rapidity. In this case the average $p_x$ of the both nucleons points into opposite directions and the vector sum becomes small. This explains the low value of the in-plane flow of the source velocity. The three body phase space decay $\Lambda K N$ broadens the rapidity distribution of the sources considerably. This lowers the in-plane flow of the K’s a second time. At a given rapidity we find K’s from sources with quite different rapidities and consequently with quite different in-plane source velocities. Hence the in-plane velocity of those K’s is an average value of the in-plane velocities of the different sources. Because the in-plane velocity changes sign at mid rapidity, the average value is very small.

This is seen in fig. 6 (left) where the rapidity distribution and the directed in-plane velocity of sources, $\Lambda$’s and K’s are displayed. Fig. 6 (right) displays...
the same quantities for the channel $\pi N \rightarrow \Lambda K$. Here the in-plane velocity of the sources is of about the same size as that for the 3-body channel. However, having only two particles in the exit channel, this reaction does not spread the in-plane velocity as far in rapidity as a three body decay. Hence finally K’s coming from a $\pi N$ collision have a larger in-plane flow than those from a baryon baryon collision.

In conclusion we see that the small value of the in-plane flow of the K’s as compared to that of the nucleons is expected even if one neglects completely any potential or collisional interaction of the K’s with the nuclear environment. It is caused by two processes: nucleons which pass the high density zone of the reaction have a much smaller average flow than the average nucleon and the production of K’s according to phase space transports the K’s far away in rapidity space with respect to the rapidity of the center of mass of the collision in which it is produced. Thus the in-plane velocity is smeared out.

**IV. THE ROLE OF THE LIFETIME OF THE $\Delta$ RESONANCE**

Most of the collisions in which a kaon is produced involve either directly a $\Delta$ or the $\pi$ produced by its disintegration. The relative fraction depends on the lifetime of the $\Delta$. For a long time it has been thought that the life time of a $\Delta$ with a given mass $m_\Delta$ depends on the phase space for decay by.

$$\tau = \frac{1}{\Gamma}; \quad \Gamma \propto \int d^3 p_\pi d^3 p_N | < i | M | f > |^2 \delta^4 (p_i - p_f)$$

(4)

Usually one assumed that the matrix element is independent of the mass of the resonance and therefore the mass dependence is given by phase space.
Only recently it has been realized [10] that this formula is only valid for a well prepared resonance which has the Breit-Wigner distribution of their mass. In simulations we are confronted with a different situation. The variance of the energy of the incoming particles is zero in BUU and small as compared to the variance of the resonance in QMD simulations. Therefore the center of mass energy - which corresponds to the resonance mass - is determined with a very good precision and its variance is much smaller than the variance of the Breit Wigner distribution which characterizes the resonance. In this case the lifetime is given by the more general formula [10]

$$\tau = \frac{d\delta}{dE} = \frac{\Gamma/2}{(E - E_0)^2 + \Gamma^2/4}.$$  

Whereas in the former case (eq.4) the lifetime increases with decreasing mass of the $\Delta$ due to the smaller phase space, in the latter case (eq.5) the lifetime has its maximum at the center of the resonance and approaches zero at energies well above or below. Fig.7 compares the lifetimes presently used in the different numerical programs with that calculated with the above equation. The influence of the different assumptions on the $\Delta$ lifetime on the kaon flow will become increasingly important if one lowers the beam energy because more and more $\Delta$’s will be produced close to the threshold due to the limited available energy. At the reaction considered here the correct equation increases the average lifetime.
FIG. 7. Lifetime of the $\Delta$ resonance as a function of its mass as used in different simulation programs as compared to the formula of a Breit-Wigner resonance.

of the $\Delta$’s as compared to the former approaches because due to the sufficient energy the $\Delta$’s are created preferably at the center of the resonance.

FIG. 8. Influence of the lifetime of the $\Delta$ resonance in central collisions on the in-plane flow of kaons. A soft momentum dependent equation of state is used and the experimental cut has been applied.

Fig.8 displays the influence of the different lifetimes of the $\Delta$ resonance on the kaon in-plane flow. We compare the flow calculated by using the $\Delta$-lifetime as derived by Huber et al. [11] ($\Gamma_H$) with that obtained by eq.6 ($\Gamma_{PS}$) and
FIG. 9. The influence of the different potential parts of the K’s in central collisions on the in-plane flow of kaons. A soft momentum dependent equation of state is used and the experimental cut has been applied.

that obtained by the parameterization of $\tau = \frac{d\delta}{dE} (\Gamma_D)$ from data of [12]. For these calculations we have set $\vec{\Sigma}_v = 0$. We see that the differences for the in-plane flow are considerable what offers the possibility to study the $\Delta$-lifetime in medium.

How our calculation compares with experiment is seen in fig.9 where we display the data of the FOPI collaboration [13] in comparison with different calculations. For this purpose we have filtered our data. The limitation to particles with a transverse momentum larger than 250 MeV/c decreases the statistics by a large factor. For $\Gamma_H$ we display calculations in which we set $\Sigma_s$ and $\Sigma_V^0$, respectively, to zero and compare them with the result obtained by including and excluding the total static potential interaction. A more realistic $\Delta$-lifetime as compared to [7] and the addition of the momentum dependent part of the vector potential balance each other at that energy. Thus finally we come close to the results of Ko [7]. Finally, we display in fig. 10 the influence of the vector part of the vector potential on the in-plane flow of kaons. The temporal vector potential represents the case when we take only the term $\Sigma_V^0$ of the potential into account (i.e. $\vec{\Sigma}_v = 0$). The additional term $\vec{\Sigma}_v$ increases the flow of kaons. This is easy to understand: We have seen (fig.3) that all kaons come from the high density fireball region where due to the symmetry of the system the momentum distribution of the baryons is symmetric with respect to the nucleus-nucleus center of mass system. Assuming that $\Sigma_{\mu}$ is proportional to the baryon current $j_{\mu}$ the scalar product in the optical potential, $\vec{k} \cdot \vec{\Sigma}_v$, is zero. Therefore for symmetric systems the optical potential is reduced to

$$u_{\text{opt}}^K = \sqrt{k^2 + (g_v \vec{\Sigma}_v)^2 + m_K^2} + m_K g_s \Sigma_s + g_v \Sigma^0_v - \sqrt{k^2 + m_K^2}$$

(6)
The $\Sigma_v^2$ term has the same sign as $\Sigma_v^0$ and depends in the same way on the local baryon density. It enhances therefore the action of $\Sigma_v^0$ term as seen in fig. 10. We would like to mention that in ref. [14] an opposite result has been found.

**FIG. 10.** The influence of the different vector potential parts of the K’s in central collisions on the in-plane flow of kaons. A soft momentum dependent equation of state is used and the experimental cut has been applied.

In conclusion we have found that the low value of the in-plane flow of the kaons as compared to nucleons has little to do with the interaction of the kaons after their creation. It is a consequence of the kinematics before and at the time point of the creation. The potential interaction of the kaons with the nuclear environment as well as the life time of the nuclear resonances and the rescattering modifies the directed in-plane kaon momentum. In addition, the value obtained in the simulation is modified by the experimental cuts.

Therefore it is premature to interpret the kaon flow as a signal for strength of one or the other of these processes. Because each of the above mentioned processes is interesting in itself exciting physics lie ahead of us on the way to interpret the data on the kaon flow.

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