The operational reality of the quantum state and of its reduction

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Is the quantum state real? Does the reduction of the state of a quantum system by a measurement on another system, entangled with it and distant, constitute a real disturbance? As is well acknowledged, there is a lack of consensus among physicists over such ontological questions concerning quantum mechanics (QM). Here we argue that it is possible, surprisingly, to address such questions on the basis of only operational considerations. More generally, we show that the operational perspective itself provides an intrinsic interpretation of QM. The theory’s syntax supplies its semantics, as it were. This interpretational framework, which we call “signaling ontology”, builds on the simple insight that unconditional quantum state reduction, namely the state change of a measured quantum system due to a nonselective measurement, can be used for signaling locally, and that this signal provides a natural, operational criterion for the reality of the state reduction. This argument is formalized in the framework of generalized probability theories and extended to multipartite systems. Among other consequences of this approach, the two questions posed above at the beginning are testably answered in the affirmative. In particular, a steering inequality is proposed whose violation, together with a local signaling condition, provides an experimental test for the reality of the remote disturbance referenced in the second question. Accordingly, quantum correlations that meet this criterion are nonlocal in an operationally real sense. A further ramification of our work is an elucidation of the contrast between quantum no-signaling and relativistic signal locality.

I. INTRODUCTION

Given a pair of mutually entangled and geographically separated systems A and B, can the remote preparation of the state of B by a measurement on A be said to really disturb system B? Does the quantum state correspond to reality? Even though a century has elapsed since the founding of quantum mechanics (QM), there is still no consensus among physicists as to how to interpret it, meaning that they disagree on what QM means, and what the mathematical theory of QM says about reality [1, 2]. In specific, there are no unequivocal answers to the questions posed above. With reference to the first question, although quantum information processing tasks such as quantum teleportation, quantum dense coding and remote state preparation are suggestive of some sort of distant influence, their standard formulation remains mute on whether this influence corresponds to a real disturbance. Bell’s theorem [3] only indicates a nonlocal disturbance in any hidden-variable (HV) model for the quantum violation of Bell’s theorem [4] only indicates a nonlocal disturbance in any hidden-variable (HV) model for the quantum violation of Bell’s theorem [4–6], and the many-worlds interpretation [7] and the Beltrametti-Bugajski ontological model [8]. The Pusey-Barrett-Rudolph (PBR) theorem [9] provides support for a ψ-ontic interpretation in the ontological framework of Ref. [10], under certain assumptions [11].

The ψ-epistemic interpretations may further be classified into two groups: (a) the realist ψ-epistemic interpretations, wherein the observer’s knowledge is about the microstate of an underlying ontology; (b) the Copenhagenesque ψ-epistemic interpretation, wherein the observer’s knowledge or belief is about measurement outcomes, rather than about an underlying reality. The latter are largely in the spirit of the founding fathers of QM, such as Bohr, Heisenberg, Pauli et al., who established the standard Copenhagen interpretation [12]. Among ψ-epistemic interpretations of this kind we may count the works of Refs. [13–15].

Although earlier considered as rather abstract and unsettleable, yet thanks to recent advances in quantum information theory and quantum foundations, the problem of the interpretation of QM has enjoyed a revival over the past decade, and may indeed become urgent in times to come [16]. What is at stake is the clear separation of the subjective and objective elements of QM, which is obviously important for its deeper understanding, and would be crucial if QM turns out to be an approximation of a more fundamental theory. A theoretical line of research in quantum foundations quite independent of the study of interpretations of QM, and indeed quite opposite in spirit, is its operational formulation as a generalized probability theory (GPT) [17–19]. A GPT prescribes the outcome probabilities for inputs in different configurations in a blackbox setting, leaving out unobservable mathematical abstractions such as the Hilbert space, complex phase, etc. and the details of the physical realization of systems. If a physical theory is likened to a language, then the operational formulation of the theory corresponds to the theory’s “syntax”, stripped of the “semantics”. Under the GPT program, QM is studied as a special case within the family

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of more general GPTs that also feature the various nonclassical properties such as measurement disturbance, uncertainty, nonlocality, etc [12–22]. The basic aim of the program is to understand natural principles that would single out QM within this framework and thereby shed light on the question of what makes QM special in Nature. A related line of research constitutes efforts at “quantum reconstruction”, which aims to derive QM from operational or information theoretic principles [26–28].

Prima facie, the GPT approach would appear to be incapable of shedding any light on the interpretation of QM, because by design operational considerations seem to be antithetical to questions regarding the nature of reality underlying the quantum probabilities and other nonclassicalities.

Even so, it turns out, surprisingly, as we show here, that a natural and intrinsic interpretation of QM is indeed furnished by the operational perspective. This interpretation is shown to provide an operational notion of reality to pure quantum states and their reduction under measurement. This will enable us to answer both the questions posed at the beginning of this section in the affirmative. Owing to the operational motivation adopted here, it is convenient to couch this interpretational approach in the framework of GPTs. Accordingly, the key results that emerge from this approach are applicable to any nonclassical GPT in the considered framework. We are thus led to the somewhat unexpected standpoint that for a nonclassical physical theory, its operational formulation itself provides its ontology and interpretation, or somewhat picturesquely, the “syntax” of the theory furnishes its “semantics”. In the context of QM, this operationally-inspired interpretation is parsimonious, being devoid of ontological paraphernalia such as multiple worlds, a preferred basis, or a space of ontic elements underlying the operational states.

Our key idea to construct an ontology using only operational arguments is to begin by establishing the reality of measurement-induced alteration or reduction of the operational state in a GPT. We consider a thought experiment where the state disturbance produced by a nonselective measurement is detected using another, incompatible measurement, so that the choice of whether or to perform the first measurement provides a method to send a signal. This signaling potential, suitably formalized, forms our operational criterion to infer the reality of the state reduction due to the measurement. This criterion is extended in a natural way to address the reality of the remote reduction of a distant, entangled system caused by a local measurement on a given system. In view of the restriction imposed by no-signaling, this criterion is formulated as a condition on the quantity of the signal corresponding to the global reduction of the composite system. As a natural application, the above observations are used to infer the operational reality of the pure quantum state. We call this approach to infer the reality of a state or its reduction as signaling ontology.

The article is arranged as follows. In Section III a brief overview of the framework of non-signaling GPTs is given. This framework is quite broad, and encompasses almost all known examples of relevant operational theories in the literature, including the operational formulation of QM itself. Section II addresses the issue of determining the operational reality of state reduction in a nonclassical GPT and the wider conceptual implications thereof. Our first result, on the operational reality of unconditional state reduction of a known state of a system in QM or a nonclassical GPT, is presented in Section II A. The argument is specialized to the case of multipartite systems subjected to the measurement of a subsystem in Section II B. The implications of these results for possible HV models or extensions of QM or of a GPT are pointed out in Section II C. The result for the reality of state reduction form the basis for our next result, which is an operational argument for the reality of any pure state in QM or in a nonclassical GPT, and presented in Section IV. Section V considers the problem of determining the reality of the reduction of an unknown state by verification of the violation of a single-system inequality.

Section VI addresses the question of whether the remote state reduction of an entangled system by a local measurement on its entangled partner system constitutes a real disturbance. To this end, Section VI A introduces a steering-like statistical inequality testable in QM or in a GPT. Section VI B develops a criterion to decide the reality of the remote disturbance of an entangled particle, a phenomenon we term “operationally real nonlocality”. We argue in Section VI C that the experimental violation of the above inequality together with the satisfaction of a local signaling condition, entails fulfillment of the above criterion for the reality of the remote state reduction. An experimental test in QM for operationally real nonlocality is described in Section VII. The relationship of this type of nonlocality to Einstein-Podolsky-Rosen (EPR) steering and Bell nonlocality is pointed out in Section VIII. We argue in Section IX that the reality of remote state reduction brings to the fore a fundamental distinction between relativistic signal locality and quantum no-signaling, and suggests that the latter should be understood as the consequence of a consistency condition in a class of GPTs, rather than as a basic principle inspired by relativistic signal locality. The conceptual implication of operationally real nonlocality for the well-known EPR paradox [23] is discussed in Section X, where it is pointed out that our result represents a different response to the paradox than the historical responses due to Bohr [20] and Bell [3]. Finally, we present our conclusions and related discussions in Section XI.

II. FRAMEWORK: GENERALIZED PROBABILITY THEORIES

The GPT framework arose as an attempt to represent and understand QM as an abstract mathematical object, independent of the details of the physical realization, and wherein one adopts an operational perspective and is concerned with how inputs and output probabilities of a system are related. In the GPT framework, an operational state \( \phi \) represents an equivalence class of preparations that are operationally indistinguishable under all possible measurements of a given GPT \( \mathcal{F} \). State \( \phi \in \Sigma \), the (convex) set of all such states of a system in a given theory. Set \( \Sigma \subset V \), a linear vector space.
Pure states correspond to extremal points of set $\Sigma$, and constitute the set denoted $\partial \Sigma$. A measurement $x$ can be considered as a set of complete, mutually exclusive 1-bit measurements—
or, effects $e^{a|x}$—associated with the outcomes $a$. The effect $e^{a|x}$ is an affine functional that maps every state $\varphi \in \Sigma$ to a probability distribution $\rho(a|x, \varphi) = e^{a|x}(\varphi)$. Associated with the “yes” outcome of the effect is the post-measurement state denoted $\hat{\varphi}^{a|x}$ such that $e^{a|x}(\hat{\varphi}^{a|x}) = 1$. The effects can be represented as vectors in $V^*$, the linear vector space dual to $V$. The set of all mutually exclusive effects of a measurement satisfies the completeness requirement $\sum_a e^{a|x} = u$, where $u$ is the unit measure, which is a special effect defined so that the probability $u(\varphi) = 1$ for any normalized state. This ensures that the probabilities $e^{a|x}(\varphi)$ over all these effects sum to 1 for any state $\varphi$. The unit measure is the GPT analog of the identity (null) measurement in QM, which assigns measure 1 to all normalized states. The unnormalized states $\hat{\varphi}$ of a system form a convex positive cone in $V$. An extremal effect is one that can indicate a pure state or a subspace of states with certainty. A measurement comprised of extremal effects is said to be sharp. Sharp measurements are the GPT counterparts of projective measurements in QM.

Given a bipartite state $\varphi_{AB}$, the marginal state of Alice $\varphi_A^{\text{max}}$ is the GPT analogue of the reduced density operator obtained via partial tracing over $B$ in QM, and is defined so that $(e^{a|x} \otimes u_B)(\varphi_{AB}) = e^{a|x}(\varphi^{\text{max}}_A)$. Denoting Alice’s inputs (resp., outputs) by $x, x'$ (resp., $a, a'$), and Bob’s inputs (resp., outputs) by $y, y'$ (resp., $b, b'$), the no-signaling principle for the GPT is the requirement that

$$\forall y \neq y' P(a|x, y') = P(a|x, y) \equiv P(a|x), \quad (1a)$$

$$\forall x' \neq x P(b|x', y) = P(b|x, y) \equiv P(b|y), \quad (1b)$$

i.e., that outcome probability of any party is independent of the other’s input. For a bipartite system satisfying the assumptions of no-signaling and tomographic locality [22], the set $\Sigma_{AB}$ of all bipartite states lies between (according to a natural ordering) the minimal tensor product $\Sigma_A \otimes_{\text{min}} \Sigma_B$ and the maximal tensor product $\Sigma_A \otimes_{\text{max}} \Sigma_B$. Here, $\Sigma_A \otimes_{\text{min}} \Sigma_B$ denotes the convex hull of all product states of system $A$ and $B$, i.e., the collection of all possible separable states (having the form $\sum_j p_j \omega^j_A \otimes \omega^j_B$, where $\{p_j\}$ denotes a valid probability distribution), and $\Sigma_A \otimes_{\text{max}} \Sigma_B$ denotes the set of all joint states that yield a valid probability under the action of any product effect and furthermore induce valid conditional states of the single system $A$ or $B$. State $\varphi$ is entangled if $\varphi \notin \Sigma_A \otimes_{\text{min}} \Sigma_B$.

Throughout this article, any theory considered will be a GPT within the framework described in this Section. It is broad enough to encompass operationalQM and many other theories of interest, such as Spekkens’ toy theory, gdit theory, etc.

III. REALITY OF STATE REDUCTION

A familiar yet admittedly puzzling feature of QM is state (vector) reduction, the state change caused by the measurement of an observable, conditioned on reading out the measurement outcome. Thereby the measured system’s state is discontinuously and unpredictably altered, randomly disturbing the value of a conjugate observable [3, 4]. The inability to directly observe such a reduction (or, “collapse”) process, not to mention its discontinuous nature, has led to the (in)famous quantum measurement problem, which asks how (or whether) the wavefunction objectively collapses. By contrast, the unitary dynamics of QM is continuous and reversible, and presents no such conceptual difficulty. The different interpretations of QM don’t agree on whether the collapse is a real, physical phenomenon or merely an epistemic epiphenomenon. This confusion also seems related in part to the fact that QM features (preparation) uncertainty, which bounds the predictability of the measurement outcomes when one of two or more incompatible observables is measured. If it turns out that the uncertainty can be attributed at least in part to the observer’s lack of some information, then the measurement may merely play the epistemic role of eliminating that lack, rather than produce a real disturbance. An analogous duality of reversible dynamics and irreversible measurement dynamics holds for any non-classical GPT, a feature ultimately connected with the non-simpliciality of the state space $\Sigma$ of the theory [25].

A. Detectable disturbance

Against the above backdrop, our observation is that uncondition quantum state reduction of a system constitutes a real disturbance in an operational sense. We recollect that (conditional) state reduction corresponds to a selective measurement, wherein a system initially prepared in state $\varphi$ is reprepared in the unnormalized post-measurement state $M_{a|x}(\varphi)$, conditioned on obtaining outcome $a$ upon measurement $x$.

The state reduction is said to be “unconditional” if the state change of the measured system is not conditioned on the measurement outcome, and thus is averaged over all possible outcomes. The unconditional state reduction corresponds to a non-selective measurement, which refers to a measurement that is performed but not read. The non-selective post-measurement operational state due to measurement $x$ is given by $M_x(\varphi) \equiv \sum_a M_{a|x}(\varphi)$.

The point of departure for the present work may be given by the following method of signaling in a nonclassical GPT. Alice chooses to perform or not to perform a certain measurement $x$ on system $A$ prepared in state $\varphi$ known to Bob, and then sends $A$ to him. Suppose the measurement, if performed, operationally disturbs the particle in a nontrivial way, i.e.,

$$M_x(\varphi) \neq \varphi. \quad (2)$$

Alice can use this fact to signal Bob by transmitting system $A$ to him. Her communication consists of a state, or “symbol”, drawn from measurement alphabet $\mathcal{A}_{A \to A} = \{\varphi, M_x(\varphi)\}$. If Eq. (2) is the case, then by measuring the received system suitably Bob can determine (probabilistically) whether or not she had measured.

That Alice is able to signal him through her measuring or non-measuring action shows that the state reduc-
tion produced by her measurement corresponds to an intrinsic change of the system. Accordingly, the transformation \( \varphi \rightarrow M_x(\varphi) \) is not merely an update to the observer’s knowledge of the system, but instead constitutes an objective alteration to the measured system. The basic tenet of signal- ing ontology, the framework of the operationally inspired interpretation advanced here, is that Alice’s signaling potential in the above thought experiment provides a natural basis to infer that the state reduction due to measurement \( x \) is real in an operational sense. This argument works in any nonclassical operational theory, whether QM or any other nonclassical GPT that features measurement disturbance, and provides a novel but simple way to bridge the epistemological gap between the operational perspective and the ontology of a theory.

As a simple example, let \( \varphi \) be the quantum state \( |0\rangle \) (+1 eigenstate of Pauli \( \sigma_Z \)), and Alice may or may not choose to measure \( x := \sigma_X \), before sending the qubit to Bob. In this case, \( M_x(\varphi) = \sum_\pi \pi \rho \pi \), where \( \pi \rho \pi \) are the projectors of \( \sigma_X \). The measurement alphabet is \( \mathcal{A}_{A \rightarrow \sigma} = \{ |0\rangle, |\frac{1}{2}\rangle \} \). Measuring the received qubit again in the basis \( \sigma_Z \), Bob can determine her action probabilistically, by checking whether the probability to obtain outcome +1 is 1 or \( \frac{1}{2} \). The difference between these two outcome probabilities would be one way to quantify the corresponding signal \( \Xi_{A \rightarrow A} \) from Alice to Bob. Alternatively, it can be quantified by the Holevo bound \( \chi = S(\frac{1}{2}|0\rangle + |\frac{1}{2}\rangle \langle 0|) - \frac{1}{2} = 0.31 \) bits, which provides a (non-tight) upper bound on the accessible information \( I_{\sigma | A} \). Here \( S(\rho) \equiv -\text{Tr} \rho \log_2(\rho) \) is the von Neumann entropy. This non-vanishing signal indicates that the state reduction due to Alice’s measurement of \( x \) is operationally real. This terminology may seem somewhat oxymoronic considering that one would expect to associate reality with HV-ontological phenomena, which the operational perspective manifestly eschews.

In a classical theory, measurements can only remove classical ignorance about the system and can in principle be performed without disturbing the system in a way that leads to a signal of the above kind. Thus, classical state reduction has at best only epistemic significance and cannot be operationally real.

**Definition 1** (Reality of state reduction). Let \( \Xi_{A \rightarrow A} \) represent any quantification of the signal transmitted using the measurement alphabet \( \mathcal{A}_{A \rightarrow A} = \{ \varphi, M_x(\varphi) \} \), where \( \varphi \) is a known operational state and \( x \) a measurement in a GPT. If

\[
\Xi_{A \rightarrow A} > 0,
\]

then the state reduction due to measurement \( x \) constitutes an operationally real disturbance of the system.

This criterion for reality does not invoke a HV-ontological framework, but is based on natural, operational considerations. In the qubit signaling example mentioned above, the state reduction caused by Alice’s measurement of \( \sigma_X \) is real according to Definition 1. The nonclassical GPT in which the above reality criterion is applicable can be quite simple. In the hierarchical scheme of Ref. [44], it suffices for the GPT to feature two or more incompatible measurements, even if it lacks “higher” nonclassical features such as contextuality and nonlocality.

Compatibility of two measurements \( x \) and \( y \) implies that there exists an operational joint measurement \( M \) with two-index outcomes denoted \( m_{a,b} \) such that the outcome probabilities for measuring \( x \) and \( y \) can be obtained as marginals of \( M \) [33]. That is, we must have \( \forall \varphi \psi(b|y, \varphi) = \sum_a P(m_{a,b}|M, \varphi) \) and \( \forall \varphi P(a|x, \varphi) = \sum_b P(m_{a,b}|M, \varphi) \). If no such \( M \) exists, then \( x \) and \( y \) are incompatible.

**Theorem 1.** In a nonclassical GPT, the unconditional state reduction of a measured system, if nontrivial, corresponds to an operationally real disturbance of the system.

**Proof.** Let system \( A \) prepared in state \( \varphi \) be subjected to measurement \( x \), leading to the system’s non-trivial state reduction in the sense of Eq. [2]. Let \( y \) be a measurement that is incompatible with \( x \). We consider “protocol \( \mathcal{D} \)”, wherein Alice freely chooses to either measure system \( A \) with \( x \) or not, and then sends \( A \) to Bob. Define the signal by the dependence of Bob’s outcome probability on whether or not Alice priorly measured \( x \):

\[
\Xi_{A \rightarrow A} \equiv \max_b \left[ e^{b|u}(\varphi) - e^{-b|u}[M_x(\varphi)] \right].
\]

If \( \Xi_{A \rightarrow A} = 0 \), this implies that the outcome probabilities \( p(b|y, \varphi) \) for input \( y \) are unaffected if \( x \) was pre-measured. In turn, this would entail that \( x \) and \( y \) are compatible for we can construct the joint measurement \( M \) given by \( M(m_{a,b}|x, y, \varphi) := v(a|x, \varphi) \psi(b|y, \varphi) \) for this state, simply by first measuring \( x \) and then \( y \), and noting the respective outcome probabilities. Therefore, if \( y \) is chosen incompatible with \( x \), then we expect \( \Xi_{A \rightarrow A} > 0 \) and by Definition 1 the state reduction constitutes an operationally real disturbance.

Even so, for the given state, the particular choice of \( y \) may yield a vanishing signal. In that case, one can always find another suitable \( y' \). If no such alternative measurement is available, then \( M_x(\varphi) \) would be operationally indistinguishable from \( \varphi \), contradicting the assumption of nontriviality Eq. [2]. \( \square \)

Theorem 1 sets the flavor for the framework of signaling ontology, wherein ontological conclusions can be reached by purely operational considerations of signaling scenarios. In the quantum context, Theorem 1 supports the standpoint that the (non-selective) wavefunction collapse must be a real phenomenon, as against being merely a subjective collapse in the observer’s knowledge.

**Example 1.** Suppose Alice has qubit \( A \) that is part of a two-qubit system \( AB \) prepared in the state

\[
|\psi(\theta)|_{AB} = \cos(\theta)|00\rangle_{AB} + \sin(\theta)|11\rangle_{AB}, \quad \theta \in [0, \frac{\pi}{2}]
\]

known to Bob. Alice chooses to perform or not to perform measurement \( x \) on particle \( A \) and then forwards \( A \) to Bob. If \( x := \sigma_X \), then the measurement alphabet corresponding to her yes/no choices is \( \mathcal{A}_{A \rightarrow A} = \{ |\frac{1}{2}\rangle, \cos^2(\theta)|0\rangle \langle 0| + \sin^2(\theta)|1\rangle \langle 1| \} \). The symbols of the alphabet are probabilistically distinguishable, so that signal \( \Xi_{A \rightarrow A} > 0 \), and thus by
Theorem\cite{1} we may infer the reality of the state reduction of $A$ under Alice’s measurement. However, if $x := \sigma_Z$, which is the eigenbasis of $A$’s reduced state $\rho_A$, then Bob is unable to make that inference. Furthermore, if $\theta = \frac{\pi}{4}$, so that the state $|\psi(\theta)\rangle_{AB}$ is maximally entangled, then for any $x$, the state reduction cannot be indicated to be real by Theorem\cite{1}.

In Example\cite{2} even if $\theta = \frac{\pi}{2}$ in Eq. \ref{eq:psi_example}. Theorem\cite{1} can be used to infer the reality of the disturbance produced by Alice’s measurement if Bob can perform two-qubit measurements. This is considered in Section IV.B below.

B. Multipartite systems

Alice and Bob share the bipartite state

$$\varphi_{AB} \equiv \sum_{\lambda} q(\lambda)\varphi_{AB}^{x,\lambda}.$$  \hspace{1cm} (6)

with $\sum_{\lambda} q(\lambda) = 1$ and $\varphi_{AB}^{x,\lambda}$ being pure states. Here $\lambda$ labels operational (not ontological) pure bipartite states known to state preparer Charlie. If $\varphi_{AB}^{x,\lambda}$ is entangled, then conditioned on Alice measuring $x$ and obtaining outcome $a$, the assemblage of unnormalized states is given by

$$\varphi_{AB}^{a|x} \equiv \sum_{\lambda} q(\lambda)p(a|x,\lambda)\varphi_{AB}^{a|x,\lambda} \otimes \varphi_{B}^{a|x,\lambda},$$  \hspace{1cm} (7)

where $p(a|x,\lambda) = [e^{a|x} \otimes u_B]\varphi_{AB}^{x,\lambda}$ with $u_B$ being the unit effect in the space of state $B$. The non-selective post-measurement state is $(M_x \otimes I_B)\varphi_{AB} \equiv \sum_{\lambda} \varphi_{AB}^{a|x,\lambda}$, where $I_B$ denotes the identity operation on state space $\Sigma_B$.

Consider an entanglement-assisted version of protocol $\mathcal{D}$, wherein Alice chooses to measure or to not measure $x$ on particle $A$. She then forwards it to Bob, who measures the correlations between $A$ and $B$, possibly using a joint measurement. The symbols of her communication correspond to the measurement alphabet $\mathcal{A}_{AB} = \{\varphi_{AB},(M_x \otimes q)\varphi_{AB}\}$. We denote by $\mathcal{E}_{A\rightarrow AB}$ any quantification (entropic, probabilistic, etc.) of the signal corresponding to this communication. The reality condition Eq. \ref{eq:reality_cond} specialized to the present case is:

$$\mathcal{E}_{A\rightarrow AB} > 0.$$  \hspace{1cm} (8)

The above protocol is equivalent to the one where Alice holds the joint system $AB$, which she forwards to Bob after measuring $A$. Therefore if Eq. \ref{eq:reality_cond} holds, so that Bob can determine whether or not Alice measured $x$ based on a joint measurement on the composite system $AB$, then by Definition\cite{1} we conclude that the (global) state reduction of the system $AB$ due to her local measurement on $A$ is real.

The state reduction of the composite system $AB$ due to a subsystem measurement is nontrivial if

$$\varphi_{AB} \neq (M_x \otimes I_B)\varphi_{AB},$$  \hspace{1cm} (9)

which represents a particular specialization of condition Eq.\cite{2}. Since state $\varphi$ is entangled, from Eq. \ref{eq:reality_cond} we have

$$[M_x \otimes I_B]\varphi_{AB} = \sum_a p(a|x,\lambda)\varphi_{AB}^{a|x,\lambda} \otimes \varphi_{B}^{a|x,\lambda} \neq \varphi_{AB}.$$  \hspace{1cm} (9)

since by assumption of preparation, $\varphi_{AB}$ lies outside the minimal tensor product $\mathcal{E}_{\text{min}}$, whilst the l.h.s lies within it. In other words, the disturbance of the state $\varphi_{AB}$ is in general nontrivial. Thus, there exists in principle a joint measurement strategy (incompatible with $x \otimes I_B$) to probabilistically distinguish between the symbols of the alphabet $\mathcal{A}_{A\rightarrow AB}$ and thereby infer Eq. \ref{eq:reality_cond}, namely, $\mathcal{E}_{A\rightarrow AB} > 0$. The reality of the state reduction produced by Alice’s measurement then follows from Definition\cite{1}.

This result can be interpreted as saying that the collapse of the quantum correlation between $A$ and $B$ in state $\varphi_{AB}$ into a classical correlation between them represents a real change of the composite system $AB$. It is straightforward to generalize this to tripartite and larger systems, with Alice’s measurements being made on one or more subsystems or jointly on all subsystems.

Example 2. Suppose $|\psi(\theta)\rangle_{AB}$ is given by Eq. \ref{eq:psi_example} and $x := \sigma_Z$. The measurement alphabet for the communication from Alice to Bob is $\{\rho_{AB}^{\psi(\theta)}(\theta) \equiv |\psi(\theta)\rangle_{AB}\langle \psi(\theta)|, \rho_{AB}^{\psi(\theta)}(\theta) = \cos^2(\theta)\{00\langle 00 | + \sin^2(\theta)\{11\langle 11|\}\}$. That $\rho_{AB}^{\psi(\theta)}(\theta) \neq \rho_{AB}^{\psi(\theta)}(\theta)$ implies Eq. \ref{eq:reality_cond} for this situation for a suitable two-qubit measurement strategy of Bob. By Definition\cite{1} this implies the operational reality of the global state reduction of the composite system $AB$.

The global reduction of the state of system $AB$ does not imply that the state change of $A$ individually or $B$ individually is real. We would require other signaling-ontological arguments to reach such a conclusion, which are considered later below. In the case of system $A$, one can invoke the local signaling condition Eq. \ref{eq:reality_cond}, as was done earlier in Example\cite{2}. The question of the reality of the remote disturbance of $B$ in this context is considered later in Section IV.C.

C. Implications for HV-ontological models or extensions of the GPT

A simple yet fundamental fact about a non-vanishing signal $\mathcal{E}_{A\rightarrow A}$ or $\mathcal{E}_{A\rightarrow AB}$, as observed in protocol $\mathcal{D}$, is that cannot be banished in a non-signaling HV-ontological model or extension of QM or a GPT. In the quantum context in particular, therefore, the operational reality of the state reduction as inferred above is robust in sense that it is independent of whether QM is exactly true or an approximation of a deeper theory. In any realist epistemic model of a GPT, if it exists, the operational signal $\mathcal{E}_{A\rightarrow A}$ or $\mathcal{E}_{A\rightarrow AB}$ cannot be represented by an underlying joint probability distribution (JPD) over HVs and thus will remain as a signal even on the ontic level.

A simple illustration of this idea can be given using Spekkens’ toy theory\cite{3}. This observation is a bit ironic inasmuch as the toy theory was advanced in support of an epistemic (to be precise, realist $\psi$-epistemic) interpretation of QM, and presents a case for interpreting nonclassical features such as interference, superposition, etc. in the toy theory epistemically.

Consider the fragment of QM obtained by restricting to the three Pauli measurements. This may be represented as an op-
eral toy theory \( \mathcal{T}_Q \) having three measurements \( \sigma^X_t \), \( \sigma^Y_t \) and \( \sigma^Z_t \), each of which has two "eigenstates" or maximum information states, and these are the only (six) pure states of \( \mathcal{T}_Q \). If the measured state is an eigenstate of the measurement, then the outcome can be deterministically predicted, whereas if it is not, then the outcome can be predicted with probability \( \frac{1}{2} \). Accordingly, uncertainty Eq. (17) is characterized by \( \nu = \frac{1}{2} \) in this theory. Measuring a non-eigenstate alters the state to an eigenstate of the measurement.

The toy theory admits an ontological model, wherein any pure state of a single system is the uniform mixture of two out of four ontic states, \( 1_t \equiv (1, 0, 0, 0), 2_t \equiv (0, 1, 0, 0), 3_t \equiv (0, 0, 1, 0) \) and \( 4_t \equiv (0, 0, 0, 1) \). The three toy Pauli observables and their corresponding eigenstates are as follows:

\[
\begin{align*}
\sigma^X_t &: \{ x^+ \equiv \frac{1}{2} (1_t + 2_t), x^- \equiv \frac{1}{2} (3_t + 4_t) \}, \\
\sigma^Y_t &: \{ y^+ \equiv \frac{1}{2} (1_t + 3_t), y^- \equiv \frac{1}{2} (2_t + 4_t) \}, \\
\sigma^Z_t &: \{ z^+ \equiv \frac{1}{2} (2_t + 3_t), z^- \equiv \frac{1}{2} (1_t + 4_t) \}. 
\end{align*}
\] (11)

This entails that the ontic support for certain pairs of states overlap (e.g., \( x^+ \) and \( y^- \)), meaning that these eigenstates do not correspond to reality in the framework of Ref. [12].

An instance of state reduction based signaling in this theory is as follows. Charlie prepares particle \( A \) in an eigenstate of \( \sigma^X_t \) and sends it to Alice. She either doesn’t perform any measurement or measures it with \( \sigma^Y_t \), thereby re-preparing it in one of the eigenstates of \( \sigma^Z_t \). Alice sends the measured particle \( A \) to Bob, who checks by measuring \( \sigma^Z_t \) whether he can verify Charlie’s preparation. By this method, Alice can signal Bob with strength \( \mathcal{S}_{A \rightarrow B} = \frac{1}{2} \) per Eq. (3). By Theorem (3) the unconditional state reduction due to Alice’s measurement is operationally real.

Now consider constructing a putative JPD \( P^*_{AB}(a, b; x, y) \) for the above protocol \( \mathcal{D} \) in the toy theory, where \( x = 0 \) (resp., \( x = 1 \)) corresponds to non-measurement (resp., measuring \( \sigma^Y_t \)). Denote the marginals on Bob’s side by \( P^*_B(b|x = 0) = \sum_a P^*_{AB}(a, b|0, y) \). The fact that the signal \( \mathcal{S}_{A \rightarrow B} \) in Eq. (3) is non-vanishing implies that

\[ P^*_B(b|x = 0) \neq P^*_B(b|x = 1), \]

i.e., there is no well-defined, unique marginal. Thus, in fact, there can be no such JPD \( P^*_{AB} \), and the signal persists on the ontic level. In other words, though the initial and final states can be modelled epistemically, yet the measurement-induced transformation between the two states cannot be epistemic, in that the observer’s ignorance is now over a different set of ontic states.

To expand on this, we observe that the measurement produces an equiprobable transformation of \( x^+ \) to \( z^+ \) or \( z^- \). These two transformations require, respectively, the ontic transition:

\[
\begin{align*}
2_t &\rightarrow 3_t, \\
1_t &\rightarrow 4_t. 
\end{align*}
\] (12)

Therefore, no matter what the measurement outcome is, at least one of the initial pair of ontic states must be switched under the measurement.

As another instance of real reduction in Spekkens’ toy theory, in the context of Example (2) let the state shared by Alice and Bob be \( \phi_{AB} := \frac{1}{2} (a_1 \otimes 0_2 + 1_t \otimes 1_2 + 2_t \otimes 3_t \otimes 3_t) \), one of the four entangled states permitted by the theory, and further let \( x := \sigma^X_t \otimes I_B \). The corresponding measurement alphabet is \( \mathcal{A}_{A \rightarrow AB} = \{ \phi_{AB}, \frac{1}{2} (x^+ \otimes x^+ + x^- \otimes x^-) \} \). Since the symbols of the alphabet are operationally distinguishable, we have \( \mathcal{S}_{A \rightarrow AB} > 0 \), entailing that the global state reduction is operationally real by Definition (3).

A subtlety is worth pointing out here. It turns out that one can write down the joint measurement strategy of any two of the measurements in \( \mathcal{T}_Q \). For instance, for \( \sigma^X_t \) and \( \sigma^Y_t \), the joint measurement \( M^{XY}(j, k) \) must satisfy:

\[
\begin{align*}
M_X^{Y(+) +} &= (0, 1, 0, 0), \\
M_X^{Z(+) +} &= (1, 0, 0, 0), \\
M_X^{Z(-) +} &= (0, 0, 1, 0), \\
M_X^{Z(-) -} &= (0, 0, 0, 1). 
\end{align*}
\] (15)

Thus, for example, \( \sigma^X_t^+ = \sum_k M_{XZ}(+, k) \) and \( \sigma^Z_t^- = \sum_j M_{XZ}(j, -) \). Even so, the measurements \( \sigma^X_t \) and \( \sigma^Z_t \) are incompatible, since \( M_{XZ}(j, k) \) is not an element of the effect space of \( \mathcal{T}_Q \). Therefore, we are justified in invoking Theorem (4) to deduce that the state reductions of \( \mathcal{T}_Q \) are operationally real. Similar arguments hold for \( M_{XY}(j, k) \) and \( M_{YZ}(j, k) \).

IV. OPERATIONAL REALITY OF THE QUANTUM STATE

It seems natural to expect that if the measurement-induced reduction of an operational state \( \varphi \) is real, then the state itself, or at least an aspect of it, must be real. Analogous to Eq. (2), a conditional state reduction (or, simply, state reduction) is said to be nontrivial if

\[
\varphi \neq M_{A|x}(\varphi), \tag{14}
\]

for some outcome \( a \). Consider a qubit prepared in the mixed state \( \rho = \sum_j |p_j \rangle \langle j| \) (where \( p_j > 0 \) and \( \sum_j p_j = 1 \)), that is, a mixture that is diagonal in the computational basis. Measuring \( x := \sigma_Z \) and reading the outcome, Alice may obtain a nontrivial state reduction, say \( \rho \rightarrow p_b |0 \rangle \). However, obviously this state reduction cannot be the basis of signaling in a protocol \( \mathcal{D} \), since the corresponding unconditional state reduction is trivial in that it fails to satisfy Eq. (2). This observation shows that a mixed state is at least partly epistemic.

It also suggests that if any nontrivial reduction of a state \( \varphi \), no matter how small the reduction is, entails a signal (i.e., the corresponding unconditional reduction is also nontrivial), then we would interpret such \( \varphi \) as real, i.e., as an intrinsic, objective property of the system in question. These considerations lead to the following natural, operational criterion for the reality of a state in the framework of signaling ontology:

**Definition 2.** A state \( \varphi \) in a GPT \( \mathcal{T} \) is operationally real if its every nontrivial state reduction \( M_{A|x}(\varphi) \) entails signaling.

**Theorem 2.** Any pure state \( \varphi \) of a nonclassical GPT \( \mathcal{T} \) is operationally real.
Proof. We need to show that for a pure state, every nontrivial reduction entails a non-vanishing signal \( \Xi_{A \rightarrow A} \). That is, contrapositively,

\[ \Xi_{A \rightarrow A} = 0 \implies \mathcal{M}_a(x) = \varphi \quad (15) \]

for any outcome \( a \) that can be realized under any measurement \( x \). The l.h.s of the implication in Eq. (15) entails the triviality of the unconditional state reduction, i.e., \( \mathcal{M}_x(\varphi) = \varphi \), and accordingly, for any \( y \), we should have

\[ p(b|y, \varphi) = \sum_a p(a|x, \varphi)p(b|y, \varphi|x) \quad (16) \]

in view of Eq. (3). There are precisely two situations under which Eq. (16) holds. To state these generally, let \( \varphi \) be a mixture \( \varphi := \sum_j \gamma_j |\psi_j\rangle \) over pure states \( |\psi_j\rangle \).

The first situation is one where the reduction due to the measurement of \( x \) produces a distinct but operationally indistinguishable ensemble. In other words, \( \varphi \) and \( \mathcal{M}_x(\varphi) \) correspond to distinct but equivalent ensembles. The existence of such ensembles is directly related to the non-simpliciality of the GPT since they imply that its pure states are not linearly independent [13]. Such indistinguishable ensembles are necessarily mixed, and therefore ruled out by the proviso of purity.

The other situation is where the state reduction does not alter the ensemble. Here, we should have \( \varphi := \sum_j \gamma_j |\psi_j\rangle \), with \( \gamma_j \) being some fixed probability distribution. In this case, the r.h.s of Eq. (16) evaluates to

\[ \sum_j \gamma_j p(b|y, |\psi_j\rangle) = p(b|y, \sum_j \gamma_j |\psi_j\rangle) = p(b|y, \varphi), \]

the l.h.s of Eq. (16). By assumption \( \varphi \) is pure, and therefore it must have the form \( \hat{\varphi}^{k|x} \) for a specific \( k \). But this would mean that the state reduction is trivial and the r.h.s of the implication in Eq. (15) follows. Since this holds for any measurement \( x \), thus by Definition 4 pure state \( \varphi \) of a system is operationally real. \( \square \)

By Theorem 2 the pure operational states of a nonclassical GPT correspond to reality. That is, the probabilities associated with pure states are indicated to be objective. In the context of QM, Theorem 2 can be interpreted as asserting that the quantum superposition of single systems and the quantum correlations of multipartite systems are operationally real. It is worth noting is that in our approach, the reality of state reduction precedes that of the state.

Theorem 2 entails a \( \psi \)-ontic interpretation of the quantum state. In the process, there is no appeal to constructs that lie outside of QM proper, such as multiple worlds or a space of sub-quantum ontic states. Instead, it is based on inferences that follow naturally from just the operational aspect of QM. In that sense, the present interpretation may be considered as intrinsic to QM. A surprising element in this approach is that this interpretation of QM emerges as the offspring from the unlikely marriage between the operational and ontological perspectives on QM.

Interestingly, applying Theorem 2 to the pure states \( x^\pm \) and \( z^\pm \) of Spekkens’ toy theory leads to the inference that they are operationally real, even though they are epistemic in the HV framework of [12], as noted earlier. This discrepancy is due to the different concepts of reality in these two approaches.

V. VERIFIABLE DISTURBANCE: SINGLE SYSTEMS

Protocol \( \mathcal{D} \) (in Theorem 11) for establishing the reality of state reduction works only if the state \( \varphi \) is known. If Bob does not know the initial state \( \varphi \), then she cannot signal him as he cannot determine whether or not Alice’s measurement altered the state. However, we now show that Bob can use another protocol to verify the state reduction due to her measurement, and thereby infer its operational reality retrospectively. This we shall call this verification scheme as “protocol 7”, described later below.

Incompatibility can be accompanied with a (preparation) uncertainty relation, though this is not necessary—gdt theory being a case in point [23]. For the remaining article, unless otherwise mentioned, we shall restrict ourselves to nonclassical GPTs that do feature an uncertainty relation. This restriction is not a problem, as our ultimate focus is on QM, which does feature an uncertainty relation. Various formulations of the uncertainty principle are possible, such as one based on standard deviation, or one on entropy, or probability, or fine-grained uncertainty [20], etc.

Let \( p(y) \equiv \max_z p(b|y, \varphi) \) for an operational state \( \varphi \). Given measurements \( y_0 \) and \( y_1 \), a version of the uncertainty relation is

\[ p(y_0) + p(y_1) \leq \nu, \quad (17) \]

where the bound \( \nu \) (with \( 1 \leq \nu \leq 2 \)) holds for any given state \( \varphi \) and characterizes the measure of certainty in the given theory. A nonclassical GPT can have \( \nu < 2 \) for certain pair of measurements, whilst \( \nu = 2 \) for any pair of observables in classical theory. For qubits, \( \nu = 1 + \frac{1}{\sqrt{2}} \approx 1.7 \), when \( y_0 := \sigma_x \) and \( y_1 := \sigma_z \). The optimal states that saturate this bound are the eigenstates of \( \sigma_x \pm \sigma_z \).

Another version of the uncertainty relation would be

\[ p(y_0) + p(y_1) + p(y_2) \leq \nu_3, \quad (18) \]

which expresses the joint uncertainty of the measurements \( y_0, y_1, \) and \( y_2 \) for any state \( \varphi \). If \( \nu_3 < 3 \), then not all of the three measurements can be predicted with certainty. For classical theory, \( \nu_3 = 3 \) for any triple of sharp measurements. In the case of qubits, let

\[ y_0 := \sigma_z, \quad y_1 := \cos(\eta)\sigma_z + \sin(\eta)\sigma_x, \quad y_2 := \cos(\eta)\sigma_z - \sin(\eta)\sigma_x, \quad (19) \]

with \( \eta \in [0, \frac{\pi}{2}] \). Then, the minimum value of \( \nu_3 \) is \( \frac{3}{2} \), which occurs when \( \eta = \frac{\pi}{4} \), and this bound is saturated for the eigenstates of \( \sigma_z \) and for \( |\zeta(\alpha)\rangle \equiv \cos(\alpha)|0\rangle + \sin(\alpha)|1\rangle \) with \( \alpha \in \{ \frac{\pi}{4}, \frac{3\pi}{4} \} \).
The verification protocol $\mathcal{V}$ works as follows: Charlie prepares a particle $A$ in a pure state ensemble $\{q(\lambda), \varphi_\lambda\}$ with $\sum_\lambda q(\lambda) = 1$. He sends $A$ to Alice, who measures it randomly with one of the two incompatible measurements $x_0$ or $x_1$, and records her corresponding outcome $a_0$ or $a_1$. Her choices of $x_j$ are uncorrelated with Charlie’s preparation. Next, she forwards the measured particle $A$ to Bob, and also classically communicates to Bob the measurement data $(x_j, a_j)$. Bob checks by performing a corresponding measurement $y_j$ on the particle $A$ that Alice sent, as to whether he can verify her claim. In particular, if $y_j = x_j$, Bob checks whether his outcome $b_j = a_j$. More generally, he checks over many such trials whether the conditional uncertainty relation

$$p(y_j|x_j) + p(y_j|x_j) \leq \upsilon$$

(20)

is violated. Here, $p(y_j|x_j)$ represents the uncertainty in $y_j$, conditioned on Alice’s measurement and classical communication about it. Eq. (20) may be considered as a GPT analogue of the uncertainty relation in the presence of quantum memory [5]. The conditional analogue of the uncertainty relation Eq. (18) can also be used in place of Eq. (20) and appears later in Section VI. In the present context, it is simpler to use Eq. (20). For the verification protocol $\mathcal{V}$, the following holds.

**Theorem 3.** The violation of inequality Eq. (20) implies that the state reduction due to measurements $x_j$ corresponds to an operationally real disturbance of the measured system.

**Proof:** Charlie is assumed to prepare the state $\varphi = \sum_\lambda q(\lambda) \varphi_\lambda$ not known to Alice and Bob. Conditioned on Alice’s measuring $x$ and obtaining outcome $a$, the assemblage of unnormalized states of Bob is $\hat{\varphi}^{a|x} = \sum_\lambda q(\lambda) p(a|x, \varphi_\lambda) \hat{\varphi}_\lambda^{a|x}$, where the tilde indicates non-normalization. If Alice’s measurement does not re-prepare the state of Charlie’s particles, then the forward-particle remains in the original state $\varphi$, and as Alice’s measurements are uncorrelated with Charlie’s preparation, the conditional uncertainties $p(y_j|x_j)$ reduce to their unconditioned counterparts $p(y_j)$, and hence uncertainty Eq. (19) will govern Bob’s outcomes. Therefore, if Bob can verify the violation of inequality Eq. (20), then at least one of Alice’s measurements should have re-prepared $\varphi$ and thereby nontrivially disturbed the initial state, i.e., Eq. (20) holds—$\mathcal{M}_x(\varphi) \neq \varphi$. We now show that this re-preparation constitutes an operationally real reduction of the state.

Consider a complementary scheme to the above protocol, wherein Alice forwards her measured particles to Dave, rather than to Bob. Charlie reveals to Dave the preparation information of $\varphi_\lambda$, and Dave must find out whether Alice measured $x_0$ or $x_1$. Dave can do this, since the post-measurement mixtures satisfy

$$\mathcal{M}_x(\varphi_\lambda) \equiv \sum_{a_0} p(a_0|x_0, \varphi_\lambda) \hat{\varphi}_\lambda^{a_0|x_0}$$

(21)

if Eq. (20) is violated. To show this, suppose to the contrary that

$$\mathcal{M}_x(\varphi_\lambda) \equiv \sum_{a_1} p(a_1|x_1, \varphi_\lambda) \hat{\varphi}_\lambda^{a_1|x_1} \equiv \mathcal{M}_x(\varphi_\lambda)$$

(22)

Applying the effect $\epsilon_{a_j|x_0}$ to the mixtures on the two sides, we find:

$$p(a_0|x_0, \varphi_\lambda) = \sum_{a_0} p(a_0|x_0, \varphi_\lambda) \hat{\varphi}_\lambda^{a_0|x_0} = \sum_{a_1} p(a_0|x_0, \varphi_\lambda) \hat{\varphi}_\lambda^{a_1|x_j} \equiv p(a_0|x_0, \varphi_\lambda)$$

(23a)

$$= \sum_{a_0} p(a_0|x_1, \varphi_\lambda) \hat{\varphi}_\lambda^{a_0|x_1} = \sum_{a_1} p(a_0|x_1, \varphi_\lambda) \hat{\varphi}_\lambda^{a_1|x_1} \equiv p(a_0|x_1, \varphi_\lambda)$$

(23b)

$$\equiv \sum_{a_1} p(a_0, \varphi_\lambda) a_1|x_1, \varphi_\lambda)$$

(23c)

where $p(a_0, \varphi_\lambda) = \epsilon_{a_j|x_0}$ is the joint probability distribution (JPD) that reproduces $p(a_0|x_0, \varphi_\lambda)$ or $p(a_0|x_1, \varphi_\lambda)$ as marginals. To go from Eq. (23a) to (23b), applying effect $\epsilon_{a_j|x_0}$ to both sides of Eq. (23c), we find $p(a_0|x_1, \varphi_\lambda) = \epsilon_{a_j|x_0}(\mathcal{M}_x(\varphi_\lambda)) \equiv p(a_0|x_1, \varphi_\lambda)$, i.e., the probability to find $a_0$ is independent of whether $x_0$ had been measured earlier; and furthermore, $\varphi_{a_0|x_0}$ being re-prepared as an “eigenstate” of measurement $x_1$, the outcome probability for any subsequent measurement is unaffected by conditioning on the initial state $\varphi_\lambda$, so that $p(a_0|x_0, \varphi_\lambda) = p(a_0|x_0, \varphi_\lambda)$.

Therefore, if Eq. (23) holds good for arbitrary ensembles $\{q(\lambda), \varphi_\lambda\}$, then $x_0$ and $x_1$ will be compatible, contrary to assumption. Therefore, Eq. (23) must be the case, which allows Dave to obtain a signal from Alice in the complementary protocol if Bob can verify the violation of Eq. (20) in the original protocol. By definition [10] the signaling entails that the state reduction due to Alice’s measurement intervention constitutes a real disturbance of the initial state.

Equation (23) does not imply that both $\mathcal{M}_x(\varphi) \neq \varphi$ and $\mathcal{M}_x(\varphi) \neq \varphi$ simultaneously. Thus, strictly speaking, the violation of Eq. (20) only entails that at least one of the two measurements $x_0$ and $x_1$ disturbed the system. However, as the preparations $\varphi_\lambda$ are uncorrelated with either measurement, then on average the reality of reduction can be attributed to both measurements.

Essentially, the proof uses the conceptual device of a complementary protocol in order to validate the operational reality of the state reduction indicated by the violation of Eq. (20) in the original protocol.

The proof of Theorem 3 requires the existence of Charlie’s classical record of the pure state ensemble $\{q(\lambda), \varphi_\lambda\}$. If the system prepared by Charlie is entangled with another system, then no such record will exist, and the marginal states $\varphi_\lambda$ will be mixed states. The argument of Theorem 3 still works, provided Eq. (20) holds.

**VI. VERIFIABLE DISTURBANCE: MULTIPARTITE SYSTEMS**

Theorems 2 and 3 only provide sufficient but not necessary conditions for the reality of state reductions. There are instances suggestive of an operationally real reduction of the state, but these theorems are unable to indicate their reality. We will refer to such cases as “operationally-real indeterminate” (or, OR indeterminate). One such instance is in Example 4 where the state reduction of system $A$ in the joint state
$|\psi(\theta)\rangle_{AB}$ is operationally real in the limit $\theta \to \frac{\pi}{2}$, but can’t be so indicated at $\theta = \frac{\pi}{4}$ by those theorems. Indeed, from Example 2 we see that the global real disturbance of the joint system $AB$ inferred via the signal $\Xi_{A→AB}$ is maximal when $\theta = \frac{\pi}{4}$, which is, again, suggestive (but not definitive) of the reality of subsystem $A$’s reduction.

To return to the first question posed in the Introduction (Section II), the remote reduction of $B$ due to the measurement of its entangled partner $A$, provides another instance where the phenomenon in question is OR indeterminate within the scope of results discussed thus far. In this Section, we will develop further analytical tools to settle the question of reality of the remote reduction of $B$. Of course, this reduction is subject to no-signaling, i.e., $\Xi_{A→B} = 0$. If this weren’t so, then we would readily agree that particle $B$ is really disturbed. Interestingly, in Example 3 we note that in the case $|\psi(\theta = \frac{\pi}{4})\rangle_{AB}$ the state reduction of $A$ and that of $B$ are identical for any outcome of measuring $\sigma_X$ and $\sigma_Z$, in that the initial mixed state and the final specific pure state of $A$ and $B$ are identical. Thus, any operational argument that succeeds in inferring the reality of the state reduction of $A$ would entail that the reduction of $B$ is real!

A. A state-dependent steering-type inequality

Consider the verification protocol $\mathcal{V}'$ (Section III) adapted to two correlated systems, which we call $\mathcal{V}'^*$. Unlike in protocol $\mathcal{V}'$, here the measurements $x_j$ and $y_j$ are performed on two different systems $A$ and $B$, rather than as sequential measurements on the same system. For a reason to be clarified later, the conditional uncertainty inequality that we shall use here will be based on uncertainty relation Eq. (18) rather than Eq. (17). Thus, in place of Eq. (20), we will use:

$$p(y_0|x_0) + p(y_1|x_1,a_1) + p(y_2|x_1,\bar{a}_1) \leq v_3.$$  

This works analogously to inequality Eq. (20), but with different settings. Alice has only two measurement choices: $x_0$ and $x_1$, whilst Bob has three. If Alice measures $x_0$, then Bob measures $y_0$. If her measurement is $x_1$, then corresponding to her outcome $a_1$ (resp., $\bar{a}_1$), Bob measures $y_1$ (resp., $y_2$).

In protocol $\mathcal{V}'^*$, Charlie prepares an entangled state $\phi_{AB}$ of two particles, $A$ and $B$. He sends particle $A$ to Alice, and $B$ to Bob. Alice measures $x_j$ on particle $A$ and classically communicates to Bob the classical pair $(x_j, a_j)$, but not the non-classical particle $A$ itself. After receiving her classical communication, Bob checks whether inequality Eq. (24) holds, by performing a suitable measurement $y_j$ on particle $B$. Suppose Alice’s communication merely tries to unveil a pre-existing ensemble on Bob’s side, i.e., the post-measurement assemblage is of the form

$$\phi^{a_jx}_{AB} = \sum_\lambda q(\lambda)p(a|x,\lambda)\phi^{a_jx}_A \otimes \phi^{j}_B,$$  

rather than the more general form Eq. (7). Then, the conditional uncertainty inequality Eq. (24) reduces to the single-system uncertainty relation Eq. (18), and thus could not lead to the violation of Eq. (24). Thus, its violation guarantees a genuine reduction or repreparation of $B$’s state, i.e.,

$$\phi^{a_jx}_{B} = \sum_\lambda q(\lambda)p(a|x,\lambda)\phi^{a_jx}_{B},$$  

in view of Eq. (7), whereby the assemblage of the remotely reduced states of $B$ show explicit dependence on Alice’s measurement. However, it is still not clear whether or not this violation entails the reality of this remote repreparation. This is addressed later in Sections VII B and VII C.

Example 3 below shows that this inequality is sufficient to obtain maximal violation with a non-maximally entangled state, whereas Eq. (20) (adapted to the two-particle scenario) would require maximal entanglement.

Example 3. In Eq. (24), we set $x_0 = y_0 := \sigma_Z$, $x_1 := \sigma_X$, $y_1 := \sin(2\theta)\sigma_X + \cos(2\theta)\sigma_Y$ and $y_2 := \sin(2\theta)\sigma_X - \cos(2\theta)\sigma_Y$. For these settings of Bob, which are of the form Eq. (17), we have $v_3 = \frac{\pi}{2}$. Consider state $|\psi(\theta)\rangle$ in Eq. (5), with $0 < \theta < \frac{\pi}{4}$, which is maximally entangled at $\theta = \frac{\pi}{4}$. This state under the above settings entails a violation of Eq. (24) up to its algebraic maximum of 3 for any $\theta$ in the above range. The inequality reduces to Eq. (20) for $\theta = \frac{\pi}{4}$.

The margin of maximal violation over the local bound $v$ in the case of Eq. (20) is given by $\frac{\pi}{4}$, but not the non-maximally entangled state $\phi_{AB}$, the use of inequality Eq. (24) is preferable to that of Eq. (20).

The maximal violation in Example 3 requires varying the measured state according to the measurement settings $y_j$, or vice versa. If (say) the measurement $\theta$ is taken to be $\frac{\pi}{4}$ but the state is the maximally entangled state $|\psi(\pi/4)\rangle$, then the l.h.s. of Eq. (24) is $2 + \frac{\pi}{4}$. Though not maximal, it nevertheless violates the local bound $\frac{\pi}{4}$.

B. Operationally real nonlocality

In Example 3 the entangled state of the system $AB$ experiences a global, real reduction due to the measurement on system $A$. This follows from the fact that $\Xi_{A→AB} > 0$ (Eq. (8)), as discussed in Example 2. In the process, both systems $A$ and $B$ individually experience a state reduction, but it is not evident that the individual system reductions are real. As to system $A$, the reality of its reduction can be checked via Definition 1, but this criterion cannot be used for system $B$ since no-signaling requires that $\Xi_{A→B} = 0$. Therefore, a signaling-ontological argument for the reality of the state reduction of $B$ would require a workaround.

To this end, our approach would be to first ensure the operational reality of the reduction of system $A$ via the local signaling condition (Eq. (9))

$$\Xi_{A→A} > 0,$$
and then to study to what extent the global signal $S_{A\rightarrow AB}$ can exceed the local signal $S_{A\rightarrow A}$ and still remain naturally consistent with the idea of the remote reduction of system $B$ not being operationally real.

To begin with, if $B$'s reduction is real, then including it should enhance the total signal beyond the local signal, i.e.

$$S_{A\rightarrow AB} > S_{A\rightarrow A}. \quad (27)$$

For any reasonable definition of the total signal, we should have $S_{A\rightarrow AB} \geq S_{A\rightarrow A}$. Thus our above requirement is for this inequality to hold strictly.

However, condition Eq. $\Rightarrow$ cannot be sufficient. In the general form of post-measurement two-system assemblage $\Rightarrow$, Alice is able to remotely prepare (or, steer) Bob's ensemble at $B$ by means of her measurement. It turns out that even unsteerable states $\phi_{AB}$, for which the post-measurement assemblage possesses the form Eq. $\Rightarrow$, corresponding to purely classical correlations between $A$ and $B$, can lead to the signal excess Eq. $\Rightarrow$. Therefore, a further necessary condition for certifying the operational reality of the remote reduction of $B$ is that the correlations should certify that Alice truly reprepared or steered Bob's ensemble. The violation of Eq. $\Rightarrow$ provides this certification.

Therefore, fulfilling the above three conditions, namely satisfaction of both Eqs. $\Rightarrow$ and $\Rightarrow$, and furthermore the violation of Eq. $\Rightarrow$ or similar inequality certifying remote steering by Alice, jointly signify that the remote reduction of Bob's system by Alice's measurement on system $A$ augments the total signal $S_{A\rightarrow AB}$ in a way that could not be accounted for by a pre-existing hidden states on Bob's side. This provides a natural sufficient condition for the reality of the reduction of $B$ in the signaling ontology approach.

It turns out that the last two of the above three conditions are not independent. The unsteerable correlations of the type Eq. $\Rightarrow$ form a convex set, and the signal $S_{A\rightarrow AB}$ is a convex function. Thus, there is a largest value of the $S_{A\rightarrow AB}$ attainable by correlations of the type Eq. $\Rightarrow$, which we denote by $\zeta$. Therefore, these two conditions can be equivalently expressed by the requirement that

$$S_{A\rightarrow AB} > \zeta. \quad (28)$$

Obviously, $\zeta > S_{A\rightarrow A}$. Thus, Eq. $\Rightarrow$ is a stronger condition than Eq. $\Rightarrow$, and expresses the fact that the global signal $S_{A\rightarrow AB}$ exceeds the local signal $S_{A\rightarrow A}$ by an amount not explainable in terms of a pre-existing ensemble of $B$, and therefore that the global signal excess is due to genuine remote reduction of system $B$.

These considerations can be crystallized to the following criterion.

**Definition 3.** Given system $AB$ prepared in the joint state $\phi_{AB}$ and subjected subsequently to the measurement of $A$, if the state reduction of system $A$ is operationally real (Eq. $\Rightarrow$ is satisfied) and the global signal exceeds the unsteerable bound $\zeta$ (Eq. $\Rightarrow$ is satisfied), then the reduction of system $B$ also corresponds to a real disturbance in the operational sense.

This is in the spirit of Definition $\Rightarrow$ in that reality is ascribed to the state change if it can be witnessed in a signaling scenario. In this case, not only is the remote state reduction of particle $B$ not epistemic, but it enhances the total signal $S_{A\rightarrow AB}$ more than can be accounted for by any pre-existing arrangement of states of $B$.

Satisfaction of Eq. $\Rightarrow$ is necessary as part of Definition $\Rightarrow$ because fulfilling Eq. $\Rightarrow$ by itself only provides a criterion for the reality of the global reduction. With the inclusion of the local signaling at $A$, the signal excess $S_{A\rightarrow AB} - S_{A\rightarrow A}$ provides an independent criterion for judging the reality of the state reduction of $B$. In particular, the signaling ontological perspective is that if Eq. $\Rightarrow$ is satisfied, then this excess is too large to be explainable unless $B$ is really disturbed. Furthermore, as in the context of local state reduction (Section III), so does Eq. $\Rightarrow$ hold a HV-ontological significance in the context of remote state reduction, as will be discussed in Section VII.

The kind of real remote reduction indicated by Definition $\Rightarrow$ will be referred to as “operationally real nonlocality”, or OR nonlocality. In the following subsection, we will find that the violation of Eq. $\Rightarrow$ implies the satisfaction of the global signaling condition Eq. $\Rightarrow$. Moreover, a non-maximally entangled state $\phi_{AB}$ is required, as will be shown, to fulfill the requirement of Eq. $\Rightarrow$, while the use of inequality Eq. $\Rightarrow$ can still allow a maximal violation of the inequality.

**C. Inequality for operationally real nonlocality**

The marginal state of a joint state $\phi_{AB}$, denoted $\phi_A^{\mu}$, is the GPT equivalent of the reduced state in QM, and defined so that $(\epsilon_A \otimes \mu_B)\phi_{AB} = \epsilon_A(\phi_A^{\mu})$ for any effect $\epsilon_A$ pertaining to system $A$ $\Rightarrow$. The maximally mixed state is characterized by the requirement that its “overlap” with every pure state is a constant. A pure state $\phi_{AB}$ is non-maximally entangled if the marginal states $\phi_A^{\mu}$ and $\phi_B^A$ are not maximally mixed. As clarified below, a necessary condition for OR nonlocality is non-maximal entanglement.

**Theorem 4.** Given an entangled state $\phi_{AB}$ in a GPT, the violation of inequality Eq. $\Rightarrow$, together with the local signaling condition Eq. $\Rightarrow$, implies that the state reduction of subsystem $B$ caused by a measurement on subsystem $A$ corresponds to an operationally real disturbance of $B$.

**Proof.** It is convenient to break up the proof into three parts, as follows. First, we establish that the violation of Eq. $\Rightarrow$ implies that Alice’s measurements $x_0$ and $x_1$ are incompatible with each other (Part 1). This, along with the non-maximality of the entanglement of state $\phi_{AB}$, is necessary for satisfying the local signaling Eq. $\Rightarrow$, as shown in Part 2. Finally, we establish that the global signaling requirement Eq. $\Rightarrow$ is fulfilled when Eq. $\Rightarrow$ is violated (Part 3).

**Part 1:** The violation of Eq. $\Rightarrow$ implies that $x_0$ and $x_1$ are incompatible. To show this, suppose to the contrary that $x_0$ and $x_1$ are jointly measurable. Then, by definition, the outcome function $\nu(a_j|x_j, \lambda)$ for either measurement should...
be derivable as the marginal statistics of a JPD over $x_0$ and $x_1$, i.e.,

$$p(a_j|x_j, \lambda) = \sum_{a_i} p(a_0, a_i|x_0, x_1, \lambda)$$  \hspace{1cm} (29)

where $a \equiv a + 1 \mod 2$. The assemblage of the reduced states of $B$, given by Eq. (26), entail in view of no-signaling that for any given $\lambda$

$$\sum_{a_0} p(a_0|x_0, \lambda)\varphi^a_B|x_0, \lambda = \sum_{a_1} p(a_1|x_1, \lambda)\varphi^a_B|x_1, \lambda.$$  \hspace{1cm} (30)

If follows from Eqs. (29) and (30) that for all states $\varphi_{AB}$

$$\sum_{a_0, a_1} p(a_0, a_1|x_0, x_1, \lambda)(\varphi^a_B|x_0, \lambda - \varphi^a_B|x_1, \lambda) = 0.$$  \hspace{1cm} (31)

This can hold true in general only if for each $\lambda$, we have $\varphi^a_B|x_0, \lambda = \varphi^a_B|x_1, \lambda$, i.e., the local scenario where Eq. (26) reduces to:

$$\varphi^a_B|x = \sum_{\lambda} p(\lambda)p(a|x, \lambda)\varphi^a_B.$$  \hspace{1cm} (32)

where states $\varphi^a_B$ are classically correlated with $A$, and $p(a|x, \lambda) = \exp\{b^a|x(\varphi^a_B)\}$. (This can arise for example when $\varphi^a_B$ has the product form $\varphi^A_B \otimes \varphi^B_B$.)

Thus, the operational disturbance on the composite system $\mathcal{A}_B$ due to measurement on $A$ is real in the signaling ontological sense by Definition 7.

Denote the l.h.s of Eq. (24) by $\upsilon_{\text{exp}}$, which is larger than $\upsilon_3$ if Eq. (24) is violated in a certain experiment. Given the violation, if Alice applied effect $e_{A}^{j|x_j}$ for some $j$, then $B$ is prepared in the state $\varphi^{a_j|x_j}_B$ such that the following "state discrimination conditions" are satisfied: $e^k|y_k(\varphi^{a_j|x_j}_B) = \frac{k}{2} \equiv p^k$ on average, whereas $e^k|y_k(\varphi^{a_j|x_j}_B)$ is $\geq p^k$ by virtue of constraint Eq. (18).

The fidelity $F$ between the remotely prepared states $\varphi^{a_j|x_j}_B$ and $\varphi^{a_k|x_k}_B$ ($j \neq k$) on average can be estimated as the ratio

$$F \approx \frac{p^k - p}{p^k} = \frac{1}{2} \left(\frac{3\upsilon_3 - \upsilon_{\text{exp}}}{\upsilon_{\text{exp}}} - 1\right).$$  \hspace{1cm} (34)

If Alice’s measurement does not genuinely re-prepare Bob’s state, i.e., Eq. (25) is the joint post-measurement state, then
is constrained by the local uncertainty relation Eq. \( (38) \), and \( \nu_\text{exp} \) is at most \( \nu_3 \), so that \( F = 1 \) in Eq. \( (39) \). As Alice steers Bob’s state to a larger degree, the violation \( \nu_\text{exp} \) reaches the algebraic maximum of 3, and correspondingly fidelity \( F \) falls monotonically. This implies that under the violation of Eq. \( (24) \), on average the pair of states \( \varphi_A^{a_j|x_j,\lambda} \otimes \varphi_B^{a_j|x_j,\lambda} \) and \( \varphi_A^{a_j|x_j,\lambda} \otimes \varphi_B^{a_j|x_j,\lambda} \) \( (j \neq k) \) are mutually more distinguishable than the pair of states \( \varphi_A^{a_j|x_j,\lambda} \otimes \varphi_B^{a_j|x_j,\lambda} \) and \( \varphi_A^{a_j|x_j,\lambda} \otimes \varphi_B^{a_j|x_j,\lambda} \) \( (j \neq k) \), for \( j \neq k \), for any \( \varphi_B^j \) not reprepared by Alice’s measurement. Thus, the possibility to remotely steer Bob’s ensemble improves the global signal \( \Sigma_{A \to AB} \), or distinguishability, beyond that possible for any correlations of the type Eq. \( (25) \), which encompasses those that saturate the bound in Eq. \( (28) \). The exact degree of enhancement of distinguishability would depend on the specific structure of the GPT, but a consequence is Eq. \( (28) \), namely, \( \Sigma_{A \to AB} > \varsigma \), fulfilling the global signal excess requirement Eq. \( (27) \). \( \square \)

Maximally entangled states cannot satisfy the local signaling condition Eq. \( (1) \) required by Definition \( [3] \). We can thus identify OR nonlocality with correlations due to non-maximally entangled states that violate an inequality of the type Eq. \( (24) \), such that Alice’s settings for the violation allow to verify the local signaling. An example here would be the correlations that violate Eq. \( (24) \) maximally (Example 3), which also lead to a non-vanishing local signal (Example 4).

An OR nonlocal theory supports the existence of Alice’s certificate (the classical information \( (x, a) \)) by which Bob can verify that she really disturbed his particle in an operational sense. By contrast, in a signaling theory, Bob would be able to unilaterally (i.e., without the help of a certificate) attribute the state reduction of his particle to Alice’s distant measurement action. Assemblages of the type Eq. \( (27) \), that may lead to local signaling but not to remote-steerable correlations, correspond to the operational principle of locality stronger than no-signaling, which we may term “OR locality”. Classical correlations are OR local.

To demonstrate OR nonlocality using the quantum state \( |\psi(\theta)\rangle \) in Example 3 it is necessary that \( \theta \neq \frac{\pi}{2} \) (or an odd multiple thereof) for satisfying the local signal requirement Eq. \( (3) \). Such a state, which violates inequalities of the type Eqs. \( (20) \) and \( (24) \), and yet cannot be indicated as operationally real, is called OR-indeterminate nonlocal, rather than “OR local”. Perhaps, a future expansion of the concept of OR nonlocality may be able to encompass them.

The set \( \Sigma_{\text{ORL}} \) of “OR local” states is convex because the unsteerability condition imposes a convex constraint on the set. Eqs. \( (20) \) and \( (24) \) are (not necessarily tight) witnesses that separate the set \( \Sigma_{\text{ORL}} \) from certain OR nonlocal states.

\[ B \]

\[ \nu_\text{exp} \]

\[ F = 1 \]

\[ (38) \]

\[ \nu_3 \]

\[ (39) \]

\[ \varphi_A^{a_j|x_j,\lambda} \otimes \varphi_B^{a_j|x_j,\lambda} \]

\[ \varphi_A^{a_j|x_j,\lambda} \otimes \varphi_B^{a_j|x_j,\lambda} \]

\[ (j \neq k) \]

\[ \varphi_A^{a_j|x_j,\lambda} \otimes \varphi_B^{a_j|x_j,\lambda} \]

\[ (j \neq k) \]

\[ (28) \]

\[ \Sigma_{A \to AB} > \varsigma \]

\[ (27) \]

\[ \Sigma_{A \to AB} \]

\[ (1) \]

\[ (24) \]

\[ (28) \]

\[ (27) \]

\[ (24) \]

\[ (20) \]

\[ (24) \]

\[ \theta \neq \frac{\pi}{2} \]

\[ (3) \]

\[ \varphi_A^{a_j|x_j,\lambda} \otimes \varphi_B^{a_j|x_j,\lambda} \]

\[ (20) \]

\[ (24) \]

\[ \Sigma_{\text{ORL}} \]

\[ |\psi(\theta)\rangle \]

\[ \theta \neq \frac{\pi}{2} \]

\[ (20) \]

\[ (24) \]

\[ \Sigma_{\text{ORL}} \]

\[ \text{VII. EXPERIMENTAL TEST} \]

For an experimental implementation of protocol “7” in QM, it is suitable to use the pure state \( |\psi(\theta)\rangle \) and the measurement settings of Example 3 above, with \( \theta = \frac{\pi}{2} \), which guarantees the minimal uncertainty bound for this family of observables, and thus make it more resistant to experimental noise. Therefore, the inequality tested would be:

\[ p(M^+|\sigma_X, +1) + p(M^-|\sigma_X, -1) \leq \frac{3}{2}, \]  

\[ (35) \]

where \( p(M^+|\sigma_X, \pm 1) \) denotes Bob’s uncertainty for measurement \( M^\pm \) conditioned on Alice measuring \( \sigma_X \) and obtaining outcome \( \pm 1 \). Here, \( M^\pm \equiv \frac{\sqrt{2}}{2} \sigma_X \pm \frac{1}{2} \sigma_Z \), and the input state is

\[ |\psi(\pi/6)\rangle \equiv \frac{\sqrt{3}}{2} |00\rangle + \frac{1}{2} |11\rangle, \]  

\[ (36) \]

which yields the maximal violation of Eq. \( (35) \) of the algebraic maximum of 3.

Ideally, the local signaling condition Eq. \( (3) \) is certified by the above violation of Eq. \( (35) \). In practice, to improve the experimental confidence level, the experiment may additionally verify that Eq. \( (7) \) holds by a subsequent measurement of qubit \( A \) on Alice’s side after she has measured it.

For arbitrary \( \theta \), the measurement alphabet for this qubit is \( \{\exp(i\theta)|0\rangle, \{\exp(i\theta)|1\rangle \} \langle 1|, \langle 0|/2\} \). Signal \( \Sigma_{A \to A} \) can be quantified according to Eq. \( (33) \):

\[ \Sigma_{A \to A}(\theta) \equiv \max_c |P(\sigma_Z = c|x_0) - P(\sigma_Z = c|x_0 = \sigma_X)| \]

\[ = \max \{|\cos^2(\theta) - \frac{1}{2}|, |\sin^2(\theta) - \frac{1}{2}|\}. \]  

\[ (37) \]

For the present experiment, this yields \( \Sigma_{A \to A}(\pi/6) = \frac{1}{4} \).

Alternatively, \( \Sigma_{A \to A} \) may be quantified by the Holevo bound \( \chi_{A \to A} \equiv S(\sum p_j S(\rho_j)) - \sum p_j S(\rho_j) \), where \( \rho_j \) are taken to be the symbols of the alphabet \( \Sigma_{A \to A} \) and we assume that Alice chooses \( \sigma_X \) and \( \sigma_Z \) with equal probability. Then, \( \chi_{A \to A} = h \left( \frac{1}{2}\cos^2(\theta) + \frac{1}{4} \right) - \frac{1}{4} (h(\cos^2(\theta)) + 1) \). For \( \theta = \frac{\pi}{6} \), one finds \( \chi_{A \to A} \approx 0.049 \), satisfying the reality condition Eq. \( 3 \).

\[ \text{VIII. RELATION TO EPR STEERING AND BELL NONLOCALITY} \]

OR nonlocality is closely related to remote steering based on the single-system uncertainty relation or a linear steering inequality \( [39, 40] \) in the context of GPTs \( [41, 42] \). Indeed, Eqs. \( (20) \) and \( (24) \) may be considered as steering inequalities, but a few points merit notice here. First, the standard scenario for steering \( [43, 44] \) is different from that for OR nonlocality. The latter is understood as part of the broader phenomenon of the operational reality encompassing single systems also, and not just multipartite correlations. Furthermore, OR nonlocality provides a criterion to decide whether a remote state rescaling corresponds to a real disturbance, whereas steering remains mute on such ontological issues. Finally, OR nonlocality requires that, additionally, Eq. \( 3 \) should be satisfied, implying that it is stronger, and only a subset of steerable states are indicated to be OR nonlocal. The state \( |\psi(\theta)\rangle \) in Eq. \( 3 \) with \( \theta = \frac{\pi}{2} \), which is maximally steerable in the standard steering scenario, is OR-indeterminate.
nonlocal, since it fails to satisfy Eq. 3. As another example, the entangled states of Spekkens’ toy theory (discussed in Section III C) are all maximally steerable in the sense of violating Eq. 20 to its algebraic limit of 2, but fail the reality condition Eq. 3.

OR locality implies local-realism, but not vice versa. Specifically, in the OR local correlation Eq. 25, the states \( \varphi^A_B \) should be operational states of the GPT, whereas under local-realism they may be any HV state. As a result, there can be Bell local states that can be OR nonlocal. An example here in the case of QM would be the Werner-like state

\[
W(f) \equiv f |\psi(\theta)\rangle \langle \psi(\theta)| + (1 - f) \frac{1}{4} \tag{38}
\]

with noise parameter \( f \in [0, 1] \) and \( \theta = \frac{\pi}{4} \), with measurement settings as in Example 5. If \( f > f_{\text{OR}} \equiv \frac{1}{2} \), the l.h.s. of Eq. 20 violates the relevant bound of \( \vartheta_x := \frac{1}{2} \). (This does not mean that states with \( f < f_{\text{OR}} \) necessarily lack OR nonlocality, for Eq. 23 is not shown to be optimal.) But this is immaterial for our present purpose. In the range \( f \in \left[ \frac{1}{2}, 1 \right] \), \( \exists_{A \rightarrow A} \geq 0 \), entailing that \( W(f) \) in Eq. 20 is OR nonlocal in this range. On the other hand, from the positivity under partial transpose (PPT) criterion \[34\], we find that \( W(f) \) in Eq. 20 is entangled if \( f > f_{\text{E}} \equiv \frac{1}{\sqrt{3}} \approx 0.57 \). That \( f_{\text{E}} < f_{\text{OR}} \) is a reflection of the fact that entanglement is necessary but not sufficient for OR nonlocality. Using the two-qubit nonlocality criterion \[34\], we find that the state state \( W(f) \) is Bell nonlocal for \( f > \frac{1}{\sqrt{3}} \approx 0.57 \). Thus, in the range \( f \in \left[ \frac{1}{3}, \frac{2}{3} \right] \), the state \( W(f) \) is OR nonlocal but Bell local. Even so, Bell nonlocality is not strictly stronger than OR nonlocality. This is because of states such as \( |\psi(\pi/4)\rangle \), which though (maximally) Bell nonlocal, are not indicated to be OR nonlocal (but instead are OR-indeterminate nonlocal) as they fail the local signaling requirement Eq. 3 for any settings \( x_j \).

It is important to stress that even for OR-nonlocal but Bell-local states, such as \( W(f) \) with \( \frac{1}{4} < f < \frac{1}{\sqrt{3}} \), the experimental correlations that lead to certifying OR nonlocality cannot be explained by a non-signaling HV ontological model, by virtue of the signaling conditions Eqs. 3 and 25. To clarify this point, let the OR-nonlocal but Bell-local correlations correspond to the JPD \( P_{AB}(a, b | x, y) \) that pre-decides the outcome for any combination of Alice’s and Bob’s inputs. Let us assume that this JPD corresponds to a specific HV state \( \zeta_{AB}(x, y) \). This would entail that the same HV state \( \zeta_{AB}(x, y) \) is sent to Dave (on the ontic level) in the complementary protocol of Part 3 in the proof of Theorem 3 (page 11), no matter whether Alice measured \( x_0 \) or \( x_1 \). Thus, the assumption could not account for the fact that Dave can (probabilistically) determine (in view of Eqs. 3 and 25) whether Alice measured \( x_0 \) or \( x_1 \). Therefore, the entanglement \( W(f) \) of the state \( \zeta_{AB}(x, y) \) corresponding to the case where Alice measured \( x_0 \) must be different from the HV state \( \zeta_{AB}(x = x_0, y) \) corresponding to that where she measured \( x_1 \).

It follows that Bell locality has a different significance here than in the standard Bell scenario. In the latter, the existence of a JPD can indeed be interpreted as corresponding to a specific HV state \( \zeta_{AB}(x, y) \). But such a correspondence is not possible in the context of OR nonlocality. Furthermore, we note that the satisfaction of Eq. 3 implies that the marginal probability distribution of Dave \( P_D \) is such that \( P_D(c(z, a) = 0) \neq P_D(c(z, a) = 1) \), and that thus there is no such Alice-Dave JPD \( P_{AB}'(a, c | x, z) \). In turn, this entails that the a JPD \( P_{AB}'(a, b, c | x, y, z) \) doesn’t exist. Thus, as in the case of single systems (Section III C), here too the operational reality of the disturbance of the remote system is robust with respect to a HV modelling or an extension of the theory, even when the correlations between \( A \) and \( B \) are Bell-local.

The Werner-like state example discussed above to demarcate OR nonlocal correlations from others, such as steering, can be used to recast the relations between different correlations from a signaling perspective. In the context of Example 3 the measurement alphabet pertaining to the case when Alice’s (measured) qubit \( A \) and Bob’s unmeasured qubit \( B \) are analyzed in the complementary protocol is

\[
\Psi_{AB} = \{ \rho_{AB} = (\cos^2(\theta))|00\rangle \langle 00| + \sin^2(\theta)|11\rangle \langle 11| \}, \quad P_{AB}^{(X)} = \sqrt{1 - |\langle 11| |00\rangle|^2}, \quad \rho_{AB} = \sqrt{1 - |\langle 11| |00\rangle|^2} \theta \cos^2(\theta) |11\rangle \langle 11|.
\]

The quantum state \( \zeta_{AB}(x) \) may be quantified analogous to Eq. 37 via Dave’s probability to obtain identical outcomes, i.e., by \( \sum_{c|x} P_{\sigma_Z \otimes \sigma_Z = +1} |x_0 = \sigma_Z\rangle - \sum_{c|x} P_{\sigma_Z \otimes \sigma_Z = +1} |x_0 = \sigma_X\rangle \), which is \( \frac{1}{2} \). An upper bound on \( c \) in this method can be obtained by mixing state Eq. 39 with the maximally mixed state \( \frac{1}{4} \). One finds that the critical fraction below which inequality Eq. 35 is not violated is given by \( f_{\text{OR}} = \frac{1}{3} \). In this case, the alphabet \( \Psi_{AB}^{1/2} \) is obtained by diluting both symbols of \( \Psi_{AB} \) with an admixture of \( \frac{1}{2} \). This provides an estimate on \( c \) to be \( \frac{3}{4} \). In conjunction with the result for the local signaling in Section VIII, these results are consistent with our expectations that under a violation of Eq. 37, the following hierarchy holds: \( \exists_{A \rightarrow AB} > c > \exists_{A \rightarrow A} \).

Similarly, the signal \( \exists_{A \rightarrow AB}(\theta) \) represents the Holevo bound with the states \( \rho_j \) given by the symbols of the alphabet \( \Psi_{AB}^{1/2} \), evaluates to \( \chi_{AB} = \hat{h} \left( \frac{1}{2} \left[ -4 \cos^2(\theta) + 5 \right] \right) + \hat{h} \left( \frac{1}{2} \left[ 4 \cos(\theta) - 5 \right] \right) - \frac{1}{2} \log(\cos^2(\theta)) \), where \( \hat{h}(p) \equiv -p \log(p) \). For \( \theta = \frac{\pi}{4} \), one finds \( \chi_{AB} \approx 0.46 \). As above, we can estimate \( c \) in this quantification of the signal, by the Holevo bound for the alphabet \( \Psi_{AB}^{1/2} \), which turns to be \( \chi_{AB}^{1/2} = \frac{1}{2} \hat{h}(1/2) + \hat{h} \left( \frac{1}{2} \left[ 4 \cos(\theta) - 5 \right] \right) - \frac{1}{2} \log(\cos^2(\theta)) \), an expression which for a sufficiently entangled state is intermediate between \( \chi_{AB} \) and \( \chi_{AB} \) as expected. In particular, for the experiment at hand, we obtain this estimate on \( c \) to be about \( \chi_{AB}^{1/2}(\pi/6) = 0.162 \). These entropic results are consistent with our expectations that under a violation of Eq. 37, the hierarchy \( \exists_{A \rightarrow AB} > c > \exists_{A \rightarrow A} \) should hold.

IX. DERIVING NO-SIGNALING

The fact of OR nonlocality makes the no-signaling principle somewhat surprising. If we don’t have an argument for the reality of the remote disturbance of Bob’s system \( B \), then
the principle seems natural. Thus, OR nonlocality elicits the question of why Bob can only verify, but not directly detect (as in protocol \(\mathcal{V}\)) this disturbance, despite its reality. It might be argued that the possibility of direct inference is forbidden by the no-signaling principle, which finds its basis in relativistic signal locality.

Indeed, in the context of deriving QM in a GPT framework or from information theoretic axioms (e.g., Bell), no-signaling is usually assumed axiomatically in this spirit. However, quantum no-signaling and relativistic signal locality are distinct principles, having rather different flavors. The former is related to the tensor product structure of the state space, and prohibits nonlocal signaling, but has no explicit association with light’s speed or any other specific numerical value of speed. By contrast, the latter is tied to the speed of light being an invariant under Lorentz transformations and is essentially a prohibition on superluminal signaling. Evidently, these two no-go requirements belong to different frameworks. In point of fact, quantum nonlocality in even non-relativistic QM is non-signaling.

Moreover, from a general GPT perspective, a "price" that must be paid for requiring no-signaling is a large reduction in the dimension of the correlations. Let \(\mathcal{X}\) (resp., \(\mathcal{Y}\)) denote the set of Alice’s (resp., Bob’s) inputs and \(\mathcal{A}\) (resp., \(\mathcal{B}\)) denote the set of Alice’s (resp., Bob’s) outputs. In Eq. \((\text{13})\), fix the input \(x = X\) of Alice. Given her output \(a\), one obtains \(|\mathcal{Y}| - 1\) independent constraints corresponding to different \(y\)’s, and this holds for each of the \(|\mathcal{A}| - 1\) values that can be independently assigned to \(a\) (minus 1 for normalization). Thus, the total number of constraints forbidding signaling from Bob to Alice are \(|\mathcal{X}|(|\mathcal{Y}| - 1)(|\mathcal{A}| - 1)\). One derives analogously for Eq. \((\text{13})\), which yields \(|\mathcal{Y}|(|\mathcal{X}| - 1)(|\mathcal{B}| - 1)\) further constraints. Therefore, the total number of no-signaling constraints is the sum of these two expressions:

\[
\mathcal{D} = |\mathcal{X}|(|\mathcal{Y}| - 1)(|\mathcal{A}| - 1) + |\mathcal{Y}|(|\mathcal{X}| - 1)(|\mathcal{B}| - 1),
\]

which represents the decrease in the dimension of the correlations allowed.

In contrast to a nonclassical GPT, it is quite natural and not surprising that the no-signaling condition is satisfied by an OR-local GPT such as classical theory.

These arguments suggest that no-signaling shouldn’t be considered as a fundamental principle, but as a consequence of other intuitive principles. In the context of efforts at an axiomatic reconstruction of QM, this would mean that it is preferable to derive no-signaling from other postulates in a GPT framework, rather than assume it axiomatically for agreement with relativistic signal locality.

Here we submit that no-signaling is a consequence of the reasonable and natural requirement that the properties of single-systems should be consistent with those of reduced systems. To see how this is the case, notice that the violation of Eq. \((\text{23})\) requires Alice’s classical communication. If the theory were signaling, and in particular if there is a breakdown of Eq. \((\text{13})\), then Bob would find the violation of Eq. \((\text{23})\) happening even without Alice’s classical communication. But this would mean that Bob can observe a violation of the uncertainty principle Eq. \((\text{15})\) at \(B\) merely by virtue of Alice’s distant measurement on \(A\). It follows that if the single-system uncertainty of the state of particle \(B\) holds, irrespective of whether \(B\) is a single system or entangled, then no-signaling is necessary.

X. REVISITING THE EPR ARGUMENT

The argument for OR nonlocality can be construed as a response to the historic EPR paradox \([20]\), but one that is distinct from the well known responses due to Bohr \([10]\) and Bell \([3]\). The EPR argument can be rephrased briefly as follows. According to it, a system’s property possesses physical reality if its value can be predicted “without in any way disturbing” the system. EPR argue that the quantum uncertainty of two complementary observables implies either (a) that the two lack simultaneous physical reality, or (b) that QM is incomplete, being unable to predict them simultaneously.

If assumption (a) is the case, then under the maximal violation of Eq. \((\text{20})\) in the context of two maximally entangled and geographically separated particles, the act of measuring particle \(A\) imparts EPR-reality to the corresponding observables of \(B\), i.e., measuring \(A\) disturbs \(B\). But EPR reject the possibility of this disturbance on the grounds (essentially in the spirit of relativistic signal locality) that the local measurements on \(A\) cannot produce any real change at \(B\). Therefore, according to this line of argument, assumption (b) must be the case, and the measurement on particle \(A\) merely reveals the (pre-existing) value of \(B\)’s observable. Thereby, EPR conclude that QM is incomplete.

The operational concept of a remote disturbance at \(B\) that is non-signaling and yet real in a verifiable way, eludes the scope of EPR-reality. Thus, from the perspective of signaling ontology, the concept of operational reality is broader than EPR-reality, and it is not necessary to reject option (a) above. Consequently, the EPR inference of QM being incomplete does not follow.

Bell’s theorem \([3]\) refutes the final conclusion of the EPR paradox (namely, the incompleteness of QM), showing that any completion of QM would involve a nonlocal disturbance in the purported completed theory. In Bohr’s response \([20]\) to the EPR paradox, the local measurement on \(A\) is argued to exert a non-mechanical influence on particle \(B\), in effect disputing the EPR criterion for physical reality in the context of QM. Our approach goes farther with the idea of such an influence, by arguing that it is real in an operational sense. Bohr’s non-mechanical influence has been identified with quantum discord \([16]\), whereas OR nonlocality is stronger than remote steering, as noted in Section VIII.

OR nonlocality suggests that Lorentzian spacetime does not encompass the entirety of the operational reality indicated by QM, i.e., there is no causal story or mechanism in Lorentzian spacetime that can account for the idea that remote state reduction is operationally real. Ironically, this suggests that not QM, but special relativity, is incomplete, in stark contrast to the conclusion drawn by EPR. Ref. \([47]\) expresses a similar view as ours, albeit on different grounds.

On the question of “completing” special relativity to ac-
commodate OR nonlocality, one may consider supplementing it with additional causal structure along the lines of [43–50]. We believe that such explorations may point to new ways in which quantum information and quantum matter are related, and may be relevant to ongoing efforts to unify general relativity and QM. That this unification has eluded a solution despite decades of intense investigation suggests that a major revision of the conceptual foundations of space, time and QM is in order.

XI. CONCLUSIONS AND DISCUSSIONS

Since the founding of QM, physicists have remained divided about the interpretation of QM and what we learn of reality through it. In specific, the nature of the quantum wave function $\psi$, and of its collapse under measurement, have remained debated. The essential question here has been whether $\psi$ is ontic (an objective state of reality in Nature) or epistemic (a subjective state the observer’s knowledge of Nature) or, inadvertently, some scrambling of the two types of interpretation. Needless to mention, it is important to disambiguate the subjective and objective elements of the problem for a deeper understanding of QM.

A novel approach, which we call “signaling ontology”, was presented here that uses the admittedly unexpected instrumentality of the operational perspective itself to decide such ontological questions. This approach enabled us to advance an interpretation of QM that is intrinsic in the sense that it does require any extraneous constructs outside QM proper, but instead relies only on operational facts about QM. Prima facie, it seems almost oxymoronic that solely operational arguments, which by design avoid reference to any reality underlying the quantum probabilities, can lead to ontological conclusions.

Our key idea is to invoke the signaling potential of the unconditional state vector reduction, the alteration of the state consequent to a system’s nonselective measurement, as a natural indicator of reality of the state vector reduction in the case of single systems, and then extend this idea in a natural way to that of subsystems in the case of multipartite systems. Building on this, we show that quantum states are also real according to this operational criterion. As such, the reality of state reduction is used to indicate the operational reality of the quantum or GPT state, rather than the other way. Here we are led naturally to a $\psi$-ontic interpretation of the quantum state, wherein the quantum probabilities of pure states are entirely objective. This observation, as it were, evokes the idea that physical reality is more “thoughtlike” and less “bricklike” than suspected so far. Expressed differently, the quantum wave function is no less real than the quantum matter it governs.

A $\psi$-ontic interpretation must arguably contend with two difficulties [1]. The first concerns the matter of “wavefunction collapse”, whose discontinuity doesn’t fit in well with the otherwise continuous unitary quantum dynamics. By contrast, in a $\psi$-epistemic approach, collapse reduces to an innocuous Bayesian update. The other difficulty is the “excess baggage problem”, which refers to the fact that a wavefunction, if real, must contain an infinite and exponentially scaling surplus of information over that accessible by a measurement [51]. Again, this excess poses no conceptual problem if the quantum state is interpreted epistemically.

We now discuss these two issues in general, and how signaling ontology addresses them, in particular. With regard to the former problem, the study of GPTs makes it clear why there should be two kinds of dynamics: a reversible dynamics, analogous to quantum unitary evolution, and an irreversible measurement disturbance dynamics, analogous to wavefunction collapse. In the GPT framework, this dichotomy is a natural consequence of generalizing classical probability theory. The core idea seems to be that the state space of a nonclassical GPT is non-simplicial, which entails multiple pure state decompositions of a mixed state. In turn, this multiplicity entails an uncontrollable and random alteration of the GPT state during measurement, to ensure that the different equivalent decompositions remain indistinguishable [24].

Perhaps, the non-simpliciality is a reflection of an optimization in Nature to enhance the cardinality $|\partial \Sigma|$ of the set $\partial \Sigma$ of pure states at any given dimension $\dim(\Sigma)$ of the state space $\Sigma$. A classical GPT corresponds to a simplex where $|\partial \Sigma| = \dim(\Sigma) + 1$, whereas a nonclassical GPT corresponds to a polytope where $|\partial \Sigma|$ is larger [24]. Furthermore, Ref. [1] shows how the existence of an infinite number of pure states, connected by continuous reversible transformations, forms the point of departure of QM from classical probability theory. From this reading, the wavefunction collapse is simply a manifestation of the richness of the quantum state space.

A resolution in a similar spirit may be advanced for the excess baggage problem. In mathematical logic, Gödel famously showed that every consistent formalization $\mathcal{F}$ of arithmetic is incomplete in the sense that there is a theorem expressible in it but not provable in $\mathcal{F}$, or simply put, that there are more arithmetic truths than are arithmetically provable. In this light, the $\psi$-ontic excess baggage, although not manifestly related to logical self-reference on which Gödel’s theorem is based, could be regarded as a similar “embarrassment of riches”, indicative of the richness of nonclassicality, rather than a problem (in this connection, cf. [53–54]). In the same vein, propositional logic, which is Gödel-complete in the sense that all statements expressible in it are provable in it, would be the analog of classical theory.

In the context of the state reduction of a distant system by local measurement on another (entangled) system, no-signaling prohibits the direct application of the above criterion for reality of state reduction of single systems. Instead, the reality of the remote state reduction is based on whether the global signaling due to the joint reduction of the composite system is boosted beyond the limit of what can be explained by some pre-arranged distribution of states of the distant system. This criterion is formulated as the violation of a steering inequality along with the satisfaction of a local signaling condition. Remote state reduction indicated in this way is referred to as operationally real (OR) nonlocality. Unlike Bell-nonlocality, which is essentially an ontological
argument, OR-nonlocality is motivated by only operational arguments. It is stronger than EPR steering but incomparable with Bell nonlocality.

No infringement of relativistic signal locality is implied by OR nonlocality, which only violates an operational principle of locality stronger than no-signaling. All the same, our result provides a sharper formulation of the tension between quantum nonlocality and special relativity. One foundational ramification thereof, relevant in the context of the reconstruction of QM, is the novel perspective that no-signaling cannot be considered as a basic principle, but instead as a consequence of other principles, in particular, the requirement of consistency between reduced and single-system properties. This inverts the theme explored in a number of works, where no-go results are derived on the basis of no-signaling; e.g., quantum limits on cloning [6], discrimination of non-orthogonal states [59], etc.

In early 20th century, the philosopher Bergson criticized the conceptual and metaphysical framework of relativity [52]. He argued that relativity is an epistemological rather than a physical theory and only deals with the data based on clocks and their behavior in different reference frames, rather than with time on its deepest philosophical level, and that the theory fails to distinguish these two different conceptions of time. The Bergsonian standpoint is compatible with the concept of OR nonlocality. Specifically, we may suggest that the failure of relativistic spacetime to encompass operational reality (Section X) indicates that relativistic signal locality is an epistemological limitation rather than a true bound on the scope of real influence.

Our results potentially provide insights into the quantum information structure of spacetime, an emerging important area of research, and thereby help in the elusive unification of QM and gravitation. In particular, it suggests a different route to quantum gravity than if the quantum state were interpreted epistemically. If QM turns out to be an approximate version of a deeper theory, then in view of our result that the quantum state is real, it would not be amiss to consider whether Hilbert space itself should be discretized in this deeper theory. We note that this discretization would make little sense in an epistemic interpretation of the quantum state, where it is a subjective entity. The reality of state reduction in our approach suggests that an experimentally testable intrinsic time-scale can be associated with the process, which could potentially be an important direction for future experimental research. This problem, again, will be meaningless in a ψ-epistemic interpretation, where at best a time scale can be associated with intrumetal limitations.

Among ψ-ontic approaches to interpreting QM, the signal ontological interpretation is closer in spirit to objective wave-function collapse models. Indeed, the present approach can be supplemented seamlessly with a Ghirardi-Rimini-Weber-like dynamical reduction [68] or a Penrosian gravitationally induced collapse [5]. Signaling ontology in conjunction with the latter may provide yet another possible road to quantum gravity through the present work.

The status of reality of an operational reduction of the quantum state, and thereby the signaling ontological interpretation of QM, was shown to be robust in the sense that it is not affected even in an extension of QM. Generally, other interpretations of QM are not expected to go through intact under such an extension [5]. Finally, our result on OR nonlocality can be construed as a different response to the EPR paradox than the historical responses due to both Bohr [30] and Bell [3].

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