π-phase and Spontaneous Supercurrent induced by Pseudo-ferromagnetic Junction in a Spin-polarized Superfluid Fermi Gas

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Abstract. We theoretically investigate a possible spontaneous current state in a polarized superfluid Fermi gas confined in a toroidal trap. When one puts a weak nonmagnetic potential barrier in this system, this barrier is known to be magnetized in the sense that some of excess ↑-spin atoms are localized around it (where the ↑-spin describes the majority component of the polarized Fermi gas). Using this unique property, we show that this magnetized barrier, or the pseudo-ferromagnetic junction, induces a spontaneous current circulating along the toroidal trap. While the ordinary supercurrent state appears as a metastable state, this spontaneous current state is realized as the most stable state, originating from the phase twist of the superfluid order parameter by the magnetized potential barrier.

1. Introduction
The coexistence of superconductivity and ferromagnet has been extensively discussed in condensed matter physics. A possible way to examine this coexistence is to fabricate a ferromagnetic junction in a superconductor [1]. In this superconductor/ferromagnet/superconductor-junction, the penetration of the superconducting order parameter into the ferromagnet (proximity effect) enables us to study how superconducting properties are affected by a ferromagnet. In particular, the so-called π-phase is realized, where the superconducting order parameter changes its sign across the junction. This phenomenon is quite different from the case of a superconductor/insulator/superconductor-junction [2], where such a phase modulation does not occur in the ground state. In superconductivity, the π-phase has been experimentally realized.

In a previous paper [3], we showed that the π-phase can be realized in a superfluid Fermi gas with population imbalance [4, 5] (N↑ > N↓, where Nσ is the number of Fermi atoms in the hyperfine states described by pseudospin σ =↑, ↓). When a weak nonmagnetic barrier is embedded in this system, this barrier potential is magnetized in the sense that some excess ↑-spin atoms are localized around it, working like a ferromagnetic junction. Indeed, the π-phase state is realized as the ground state under a certain condition [3]. Since the π-phase modulates the phase of the superfluid order parameter by π, when it is realized in a toroidal trap [6], we expect a finite superflow associated with this phase twist. While the ordinary supercurrent state...
is a metastable state, the spontaneous current state appears as the ground state. In Ref. [7], we have confirmed this expectation in a simple one-dimensional (1D) ring system.

In this paper, we extend our previous work for a 1D ring system [7], to include effects of a finite width of the ring trap in the radial direction. Within the mean-field theory at \( T = 0 \), we examine the stability of the spontaneous current state induced by the magnetized potential barrier, consisting of localized excess \( \uparrow \)-spin atoms. We note that this extension is important in considering a real toroidal trap system to realize this interesting supercurrent state. On the other hand, although a real Fermi gas is a continuum system, to simply calculate spatial variations of physical quantities, we use a lattice model. However, the presence of the assumed lattice is not essential for the stability of the spontaneous current discussed in this paper. For simplicity, we take \( \hbar = 1 \), and the lattice constant is taken to be unity throughout this paper.

2. Formulation

We consider a polarized two-component Fermi gas, described by pseudospin \( \sigma = \uparrow, \downarrow \). To include effects of a finite width of a toroidal trap in a simple manner, we consider a model two-dimensional (2D) attractive Hubbard model, described by the Hamiltonian,

\[
H = -t \sum_{\langle i,j \rangle, \sigma} [\hat{c}^\dagger_{i,\sigma} \hat{c}_{j,\sigma} + \text{h.c.}] - U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow} + \sum_{i, \sigma} [V_i - \mu_{\sigma}] \hat{n}_{i,\sigma}.
\] (1)

Here, \( \hat{c}^\dagger_{i,\sigma} \) is the creation operator of a Fermi atom with pseudospin \( \sigma \) at the \( i \)-th site. The \( i \)-th lattice is at the spatial position \( (i_x, i_y) \) in an \( L_x \times L_y \) square lattice. The periodic boundary condition is imposed in the \( x \)-direction so as to describe a toroidal trap. \( t \) is a nearest-neighbor hopping, and the summation \( \langle i, j \rangle \) is taken over the nearest-neighbor pairs. \( U(>0) \) is an on-site interaction, and \( \hat{n}_{i,\sigma} = \hat{c}^\dagger_{i,\sigma} \hat{c}_{i,\sigma} \) is the number operator at the \( i \)-th site. \( \mu_{\sigma} \) is a pseudospin-dependent chemical potential to adjust the population imbalance. The nonmagnetic potential \( V_i \) consists of a harmonic trap \( V_{\text{trap}} \) for the confinement in the \( y \)-directions and a weak barrier potential \( V_{\text{barrier}} \) at \( i_x = L_x/2 \), having the form

\[
V_i = V_{\text{trap}} \left( i_y - \frac{L_y}{2} \right)^2 + V_{\text{barrier}} \exp \left[ - \left( \frac{i_x - L_x/2}{\ell} \right)^2 \right],
\] (2)

where \( \ell \) describes the width of the potential barrier.

We treat (1) within the mean-field theory at \( T = 0 \). The ground state energy is given by [8, 7]

\[
E_G = \sum_{\sigma} E_{j,\sigma} \theta(-E_{j,\sigma}) + \sum_{\sigma} \mu_{\sigma} N_{\sigma} + \sum_i \left[ V_i - \mu_{\downarrow} - U \langle \hat{n}_{i,\uparrow} \rangle - E_i \downarrow + U \langle \hat{n}_{i,\uparrow} \rangle \langle \hat{n}_{i,\downarrow} \rangle + |\Delta_i|^2 / U \right],
\] (3)

where \( \Delta_i = U \langle \hat{c}_{i,\uparrow} \hat{c}_{i,\downarrow} \rangle \) is the superfluid order parameter, and \( E_{j,\sigma} \) is the Bogoliubov excitation spectrum. Determining \( \Delta_i, \langle \hat{n}_{i,\sigma} \rangle, \) and \( \mu_{\sigma} \), self-consistently [8], we energetically compare the \( \pi \)-junction solution with the current free (0-junction) solution.
3. Spontaneous supercurrent in a 2D toroidal trap

Figure 2 shows the $\pi$-junction solution in a 2D toroidal trap. The particle density $\langle \hat{n}_{i\downarrow} \rangle$ of the minority $\downarrow$-spin component is suppressed around the potential barrier, where the excess $\uparrow$-spin atoms are localized, as shown in panels (a) and (b). One may regard this polarized region as a pseudo-ferromagnetic junction [3]. This magnetization remarkably suppresses the magnitude $|\Delta_i|$ of the superfluid order parameter, as shown in panel (c). The magnetized barrier (ferromagnetic junction) also modulates the phase $\theta_i$ of $\Delta_i$ (See panel (d)), which is characteristic of the $\pi$-junction. Because of the single-valueness of $\Delta_i$, this phase twist in the ferromagnetic junction leads to the spatial variation of $\theta_i$ outside of the ferromagnetic junction, as shown in panel (d).

Although we also obtain the 0-junction solution shown in Fig. 3, the energy of this state ($E_G^0$) is larger than that of the $\pi$-junction state ($E_G^\pi$), as $E_G^0 = -178.5326t > E_G^\pi = -178.5337t$. We briefly note that the energy difference between them is actually much smaller than $E_G^\pi$ or $E_G^0$, because the origin of this energy difference is, not a bulk effect, but a surface effect around the junction, namely, the phase modulation across the junction.

Figure 4 shows the spontaneous supercurrent density $\mathbf{j}_i = (j_i^x, j_i^y)$ in the $\pi$-junction solution,
which clearly shows that the phase modulation shown in Fig. 2(d) induces a spontaneous superflow. We also find that the supercurrent density is large in the trap center ($i_y = 6.5$). Panel (a) also indicates that $j_{x}^i$ in the trap center is slightly enhanced in the ferromagnetic junction (magnetized region). In the present case, the current density satisfies the continuous condition $\nabla \cdot j = 0$ [7], so that the total current passing through a surface perpendicular to the x-direction is conserved in Fig. 4. Because of this, the local enhancement of $j_{x}^i$ seen in panel (a) induces the $y$-component $j_{y}^i$ of the supercurrent, as shown in panel (b). However, apart from this local effect, our results indicate that the spontaneous current state is more stable than the current-free 0-junction state, even when the finite width of the toroidal trap is taken into account.

We briefly note that the spontaneous current in Fig. 4 does not contradict with the Bloch’s theorem [9], stating the absence of a finite total current in any thermodynamically stable states. As shown in our previous paper [7], this theorem does not hold in the present case when the magnitude of the current in the $x$-direction is $O(L_x^0)$. For more detail, we refer to [7].

4. Summary
To summarize, we have discussed the spontaneous current state in a superfluid Fermi gas confined in a toroidal trap. We examined how our previous results for the simplest 1D ring system are altered, when we include effect of a finite width of the trap in the radial direction. We show that, apart from the detailed spatial variation of the current density, the stability of the spontaneous current state is essentially unaffected by the finite width of the toroidal trap. Since a real toroidal trap always has a finite width, our results would be useful for the study toward the realization of the $\pi$-phase, as well as the spontaneous superflow, in cold Fermi gases.

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[8] To save space, for the details of the derivation of (3), as well as the way of self-consistent calculations for $(\Delta_i, \langle \hat{\rho}_{i,\sigma} \rangle, \mu_0)$, we refer to our previous paper[7].
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