We show that the combined effects of a rotation plus a magnetic field can cause charged pion condensation. We suggest that this phenomenon may yield to observable effects in current heavy ion collisions at collider energies, where large magnetism and rotations are expected in off-central collisions.

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I. INTRODUCTION

The combined effects of rotations and magnetic fields on Dirac fermions are realized in a wide range of physical settings ranging from macroscopic spinning neutron stars and black holes [1], all the way to microscopic anomalous transport in Weyl metals [2]. In any dimensions, strong magnetic fields reorganize the fermionic spectra into Landau levels, each with a huge planar degeneracy that is lifted when a parallel rotation is applied. The past decade has seen a large interest in the chiral and vortical effects and their relationship with anomalies [3] (and references therein).

Perhaps, a less well known effect stems from the dual combination of a rotation and magnetic field on free or interacting Dirac fermions. Recently, it was noted that this dual combination could lead to novel effects for composite fermions at half filling in 1+2 dimensions under the assumption that they are Dirac fermions [4], and more explicitly for free and interacting Dirac fermions in 1+3 dimensions [5–7]. Indeed, when a rotation is applied along a magnetic field, the charge density was observed to increase in the absence of a chemical potential. A possible relationship of this phenomenon to the Chern-Simons term in odd dimensions, and the chiral anomaly in even dimensions was suggested. Recently, there have been few studies along these lines using effective models of the NJL type in 1+3 dimensions, where the phenomenon of charge density enhancement was also confirmed with new observations [6–8].

Current heavy ion collisions at collider energies in non-central collisions involve large angular momenta in the range $10^3 - 10^5 \hbar$ [9, 10]. Recently, STAR reported a large vorticity with $\Omega \sim (9 \pm 1) \times 10^{21}$ s$^{-1}$ $\sim 0.05 m_\pi$, by measuring the global polarization of $\Lambda$ and $\bar{\Lambda}$ in off central AuAu collisions in the Beam Energy Scan program [11]. During the prompt part of the collision, large magnetic fields $B \sim m_\pi^2$ are expected [12]. In this letter, we show that the combined effects of magnetism plus a rotation can induce a pion superfluid phase in off-central heavy ion collision. This superfluid phase maybe at the origin of the large multi-pion correlations reported by ALICE [13], as also suggested by a recent non-equilibrium study [14].

In section II we show how this combination yields a charged pion condensation. In section III, we make an estimate of the amount of pion condensation in current heavy ion collisions at collider energies. Our conclusions are in section V.

II. PION CONDENSATION

In the presence of a fixed magnetic field in the $+z$ direction $B = B \hat{z}$, the charged $\pi^\pm$ pion spectrum is characterized by highly degenerate Landau Levels (LL)

$$E_{np} = (|eB|(2n+1) + p^2 + m_\pi^2)^{\frac{1}{2}}$$

with $p$ the pion momentum along the 3-direction, each with a degeneracy $N = |eB|S/2\pi$ with $S = \pi R^2$ the area of the plane transverse to $B$. We will assume that the magnetic length $l_M = 1/\sqrt{|eB|} \ll R$ for the LL to fit...
This equation is solved by inspection with $\mu$. Therefore, the orbital assignments equal at any temperature conservation requires that the number of allowed to flow in or out, strict charge conservation in dense, provided that charge conservation is enforced. For a fixed and isolated volume $V$, $\Omega = 0$, and $\Omega > 0$ but $(N-2)\Omega < 2m_0$, so that only the $l = N$ state for $\pi^+$ and $l = 0$ state for $\pi^-$ condense. The energy per unit length in the Bose-Einstein condensate (BEC) state is

$$E_{\pi} = -n(N\Omega - 2m_0) + d_N n^2$$

with the Coulomb factor

$$d_N \approx \frac{e^2}{2} \frac{1}{l_M} R = \frac{1}{4\pi} \ln \frac{R}{a} \approx \frac{e^2}{8\pi} \ln N$$

$d_N$ characterizes the electric field energy stored between two charged rings with radius $l_M \sim 1/\sqrt{eB}$ and charge $-e$ ($\pi^-$), and radius $R \gg l_M$ and charge $+e$ ($\pi^+$). The Coulomb self-energy is subleading and omitted. In the ground state, the BEC density $n$ is fixed by minimizing the energy density $E_{\pi}$ in (5), with the result

$$n = \theta(N\Omega - 2m_0) \frac{N\Omega - 2m_0}{2d_N}$$

The rotating $\pi^+$ condensate induces a uniform magnetic field $b_z$ that enhances the applied initial field $B$, and back-reacts on the formation of the charged condensates to order $\alpha = e^2/4\pi$. Indeed, the rotating BEC of $\pi^+$ at $r = R$ generates an azimuthal current

$$J^\theta[n] = \frac{eN}{m_0 r} |f_{0N}|^2 \approx \frac{e^2 B n}{4\pi m_0} \delta(r - R)$$

where $f_{0N}$ is theLLL with angular momentum $l = N$. The corresponding induced magnetic field

$$b_z[n] = \frac{e^2 B n}{4\pi m_0}$$

modifies the applied magnetic field to order $\alpha = e^2/4\pi$ through $B \to B + b_z[n]$. The back-reacted LL problem amounts to the following substitutions for $m_0$ and $N$ within $S$. In the circular gauge the degeneracies of the LL are identified with the eigenstates of the $z$-component of the angular momentum in position space. They are labeled by $l$ which enters the azimuthal wave-function as $e^{il\phi}$ with the restriction $-n \leq l \leq N - n$ where $n$ labels the LL. For the Lowest Landau Level (LLL) with $n = 0$, $l$ has a fixed sign since $0 \leq l \leq N$. After quantization, the angular momentum for positive charged particles is $l$ and for negative charged particles is $-l$. This means that in the LLL, the $\pi^+$ spins along the magnetic field, while the $\pi^-$ spins opposite to the magnetic field as illustrated in Fig. 1.

When a rotation $\Omega$ along the magnetic field is applied, it causes the spectrum to shift linearly. Throughout we will consider the parallel case with $\vec{\Omega} \cdot \vec{B} > 0$ unless specified otherwise. With this in mind, and in the rotating frame

$$E_{np} \to E_{np} - \Omega l_z \equiv E_{np} - j\Omega$$

with $j = 1$ for positively charged pions (particles) and $j = -1$ for negatively charged pions (anti-particles). As a result, the degeneracy of each LL is lifted. In particular, the $\pi^+$ in the LLL splits down and the $\pi^-$ in the LLL splits up as also illustrated in Fig. 1. Since the chargeless pions $\pi^0$ are unaffected by the magnetic field, their rotational shift averages out. Also we note that causality requires $v = \Omega R \leq 1$ [7] which together with the magnetic length constraint (see above) translates to $l_M \ll R < 1/\Omega$.

The mechanism of $\pi^\pm$ splitting by a rotation parallel to a magnetic field in the LLL can cause $\pi^\pm$ pion condensation. Indeed, in the shifted spectrum (2), the combination $\mu = \Omega l$ plays the role of a chemical potential for $\pi^+$ and $-\mu = -\Omega l$ for $\pi^-$, in much the same way as noted for fermionic particles and anti-particles in the LLL [1] [7] [15] [16]. Therefore, when $\mu_N = N\Omega$ apparently exceeds the $\pi^+$ effective mass in the LLL, $m_0 = \sqrt{eB + m_0^2}$, but is still below the $\pi^-$ effective mass in the first LL with $n = 1$, the LLL $\pi^+$ may Bose condense, provided that charge conservation is enforced.

For a fixed and isolated volume $V = SL$ with no charge allowed to flow in or out, strict charge conservation in the co-moving frame is achieved by introducing a charged chemical potential $\mu$, in addition to the induced chemical potential $\Omega l$ by rotation. (For an open volume discussion see [16] and references therein). For the LLL, charge conservation requires that the number of $\pi^\pm$ in $V$ are equal at any temperature

$$\sum_{l=0}^{N-1} \frac{dp}{2\pi} \frac{1}{e^{(E_{np} - l\Omega - \mu)} - 1} = \sum_{l=0}^{N-1} \frac{dp}{2\pi} \frac{1}{e^{(E_{np} + l\Omega + \mu)} - 1}$$

This equation is solved by inspection with $\mu = -\frac{N\Omega}{2}$. Therefore, the orbital assignments $l = N - m$ and $l = m$ for $\pi^+$ and $\pi^-$ in the LLL will have the same occupation number

$$n_{\pi^+}(l = N - m) = n_{\pi^-}(l = m)$$

which together with $\mu = -\frac{N\Omega}{2}$, all $m \leq \frac{N}{2}$ will condense, i.e. $\pi^+$ with $\frac{N}{2} \leq l \leq N$ and $\pi^-$ with $0 \leq l \leq \frac{N}{2}$, and so on.

Now consider the rotating ground state with $T = 0$ and $N\Omega > 2m_0$ but $(N-2)\Omega < 2m_0$, so that only the $l = N$ state for $\pi^+$ and $l = 0$ state for $\pi^-$ condense. The energy per unit length in the Bose-Einstein condensate (BEC) state is

$$E_{\pi} = -n(N\Omega - 2m_0) + d_N n^2$$

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$$n = \theta(N\Omega - 2m_0) \frac{N\Omega - 2m_0}{2d_N}$$

The rotating $\pi^+$ condensate induces a uniform magnetic field $b_z$ that enhances the applied initial field $B$, and back-reacts on the formation of the charged condensates to order $\alpha = e^2/4\pi$. Indeed, the rotating BEC of $\pi^+$ at $r = R$ generates an azimuthal current

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where $f_{0N}$ is the LLL with angular momentum $l = N$. The corresponding induced magnetic field

$$b_z[n] = \frac{e^2 B n}{4\pi m_0}$$

modifies the applied magnetic field to order $\alpha = e^2/4\pi$ through $B \to B + b_z[n]$. The back-reacted LL problem amounts to the following substitutions for $m_0$ and $N$.
\[ m_0^2[n] = m_\pi^2 + eB \left( 1 + \frac{e^2 n}{4\pi m_0} \right) \]
\[ N[n] = N \left( 1 + \frac{e^2 n}{4\pi m_0} \right) \]

The back-reacted density for the \( \pi^\pm \) condensates follows by minimizing the energy per unit length

\[ E[\Omega, n] = -n(N[n]\Omega - 2m_0[n]) + n^2e^2 \left( \frac{eBN}{16\pi m_0^2[n]} + \frac{\ln N[n]}{8\pi} \right) \]

This is the analogue of \( (5) \) with \( d_N = \frac{e^2 \ln N(n)}{8\pi} \), including the additional magnetic energy from the back reaction

\[ \frac{\pi R^2 b_x^2}{2} = \frac{n^2e^4B^2R^2}{32\pi m_0^2[n]} = \frac{e^3BNn^2}{16\pi m_0^2[n]} \]

The true ground state follows by minimizing \( (11) \) with respect to \( n \). Both \( m_0[n] \) and \( N[n] \) are observed to be weakly dependent on the \( n \)-contributions from the back-reaction. We now explore the physical implication of \( (11) \) in heavy ion collisions.

### III. PION BEC IN HEAVY-ION COLLISIONS

Current heavy ion collisions at collider energies are characterized by large angular momenta \( l \sim 10^3 - 10^5 \) \( h \) \([11]\) and large magnetic fields \( B \sim m_\pi^2 \) \([12]\) in off central collisions. Assuming that at chemical freeze-out, \( R \sim 10 \) fm with still \( eB \sim m_\pi^2 \), this would translates to a LL degeneracy \( N = eBR^2/2 \sim (m_\pi \times 10 \text{fm})^2 \sim 100/4 \) and a rotational chemical potential \( \mu_N = N\Omega \sim 1.25 m_\pi \). From the hadro-chemistry analysis, the pion chemical potentials at freeze-out are typically \( \mu_f \sim 0.5 m_\pi \) at RHIC, and \( \mu_f \sim 0.86 m_\pi \) at the LHC \([11]\). With the rotation at finite \( B \), they would translate to \( \mu_\pi = \mu_N + 2\mu_f \sim 1.96 m_\pi \) and \( 2.98 m_\pi \) respectively. Since the threshold of the LLL for the combined \( \pi^\pm \) energy is \( 2\sqrt{2} m_\pi \), charge pion condensation is possible. Using \( (11) \) at finite \( T, \mu_f \), the number of \( \pi^\pm \) pions in the BEC are

\[ N_\pi^\pm = \sum_{n=0}^{\infty} n e^{-\frac{1}{2}(LE[\Omega, n] - 2n\mu_f)} \sum_{n=0}^{\infty} e^{-\frac{1}{2}(LE[\Omega, n] - 2n\mu_f)} \]

For \( L \sim 10 \) fm, \( eB \sim m_\pi^2 \) and \( N \approx 25 \), we show in Fig. 2 the average number of condensed \( \pi^\pm \) for temperatures in the range \( 0.5 m_\pi \leq T \leq 1.5 m_\pi \) and rotations in the range \( 0.04 m_\pi \leq \Omega \leq 0.06 m_\pi \) for the most favorable case with \( \mu_f = 0.86 m_\pi \) at the LHC. It is interesting to note that the ALICE collaboration has recently reported a large coherent emission from multi-pion correlation studies in Pb-Pb collisions \([17]\).

![Figure 2: The mean number of condensed pions \( N_\pi^\pm \) in the range \( 0.04 m_\pi \leq \Omega \leq 0.06 m_\pi \), for \( \mu_f = 0.86 m_\pi \) and \( 0.5 m_\pi \leq T \leq 1.5 m_\pi \).](image)

### IV. CONCLUSIONS

The combined effects of a rotation parallel to a magnetic field yields to pion condensation both in the vacuum and at finite temperature. The \( \pi^+ \) condense at the edge, while the \( \pi^- \) at the center in equal amount when charge conservation is strictly enforced in a closed volume. Since parallel rotations and magnetic fields can be generated in current heavy ion collisions at collider energies, charged pion condensation could be generated if the combined effects survive with considerable strength in the freeze-out phase. Such effects are likely to affect both the flow of charged particles and their number fluctuations. This separation of charged bosons by centrifugation in a magnetic field may also be probed in atomic physics (trapped and cooled atoms), in condensed matter physics (quantum Hall effect) and possibly compact stars (magnestars).

### V. ACKNOWLEDGEMENTS

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### VI. APPENDIX: KLEIN-GORDON EQUATION IN ROTATING FRAME

We now present and explicit derivation of the Klein-Gordon spectrum in a rotating frame. In particular, we will recover the assignment given in \( (2) \) in the infinite volume limit. The finite volume effects will be briefly discussed. We will also show that the appearance of the electric field in the rotating frame cancels out in the pion spectrum. No net force is generated by a frame change.

Consider the metric for a rotating frame in 1+3-dimensions with mostly negative signature \((+, -,-,-)\)

\[ ds^2 = (1 - \Omega^2 \rho^2)dt^2 + 2y\Omega dxdt - 2x\Omega dydt - dr^2 \]
The co-moving frame is defined as $e_a = e_a^\mu \partial_\mu$ with $(e_0, e_1, e_2, e_3) = (\partial_t + y \Omega \partial_x - x \Omega \partial_y, \partial_x, \partial_y, \partial_t)$. Now, consider a constant magnetic field in the $z$-direction $B\hat{z}$ as described by the circular vector potential $A_{R\mu} = B(0, y_R/2, -x_R/2, 0)$ in the rest frame sub-labeled by $R$. The coordinate transformation to the rotating frame $r_R = r, t_R = t, \theta_R = \theta + \Omega t$ allows the re-writing of the vector potential in the rotating frame as

\[ A_\mu = B(-\Omega r^2/2, y/2, -x/2, 0) \tag{15} \]

In the rotating frame there is in addition to the magnetic field $B\hat{z}$, an induced electric field $\vec{E} = \Omega \vec{B}$. This is expected from a Lorentz transformation from the fixed frame with $B\hat{z}$ to the co-moving frame with $B\hat{z}$ and $\vec{E}$.

A charged scalar field $\Pi$ in the rotating frame and subject to the vector potential is characterized by the Lagrangian

\[ \mathcal{L} = \frac{1}{2}(D_\mu + y \Omega D_x - x \Omega D_y)^2 - |D_\mu \Pi|^2 - m_\pi^2 \Pi^2 \tag{16} \]

with the long derivative $D_\mu = \partial_\mu + ieA_\mu$. We now note the identity

\[ D_\mu + y \Omega D_x - x \Omega D_y = \partial_\mu + y \Omega \partial_x - x \Omega \partial_y \tag{17} \]

The electric field following from the Lagrangian cancels out. The co-moving frame corresponds only to a frame change with no new force expected. With this in mind, the equation of motion for the charged field in the rotating frame is

\[ -(\partial_t + y \Omega \partial_x - x \Omega \partial_y)^2 \Pi - D_\mu D_\mu \Pi - m_\pi^2 \Pi = 0 \tag{18} \]

In the infinite volume case, we solve using the algebraic ladder construction with

\[ a = \frac{i}{\sqrt{2eB}}(D_x + iD_y) \]
\[ b = \frac{1}{\sqrt{2eB}}(2\partial + \frac{eB}{2} \hat{z}) \tag{19} \]

Choosing the positive $z$-direction to be that for which $eB$ is positive, yields the operator identities

\[ D_\mu D_\mu + D_\mu D_\mu = eB(2a^\dagger a + 1) \]
\[ L_z = i(-x \partial_y + y \partial_x) = b^\dagger b - a^\dagger a \tag{20} \]

The general stationary solution to the equation $\Pi = e^{ipz - iT} f$ with $f$ solving

\[ (E + \Omega L_z)^2 f = (m_\pi^2 + p^2) f + eB(2a^\dagger a + 1) f \tag{21} \]

The normalizable solutions form a tower of LLL

\[ f_{mn} = \frac{1}{\sqrt{m!n!}} (a^\dagger)^n (b^\dagger)^m f_{00} \]
\[ (E_{mn} + \Omega (m-n))^2 = eB(2n+1) + m_\pi^2 = E_n^2 \tag{22} \]

with $f_{00} \sim e^{-\frac{eB}{2}(z^2 + y^2)}$ as the LLL. For the LL to fit in a volume $V = LS$, we need $m, n \leq N$. The quantized charged field $\Pi$ in the rotating frame is

\[ \Pi = \int \frac{dp}{2\pi} \sum_{nm} \frac{f_{mn}}{\sqrt{2E_n}} (a_{nmp} e^{-iE^+ t + ipz} + b^\dagger_{nmp} e^{iE^- t - ipz}) \tag{23} \]

with the bosonic canonical rules

\[ \left[ b_{nmp}, b^\dagger_{n'm'p'} \right] = \left[ a_{nmp}, a^\dagger_{n'm'p'} \right] = 2\pi \delta_{nn'}\delta_{mm'}\delta(p - p') \]

The particle state created by $a^\dagger_{nmp}$ has energy $E^+ = E_n + \Omega(m-n)$, charge $+e$ and orbital angular momentum in the $z$ direction as $m-n$. The anti-particle state created by $b_{nmp}$ has energy $E^- = E_n + \Omega(m-n)$, charge $-e$ and orbital angular momentum in the $z$ direction as $-m+n$.

Therefore, the energy relationship between the rotating frame and the rest frame is $E = E_R - \Omega L_z$. This is in agreement with the statement that we have set $l = m - n$ and defined $L_z = j\hat{l}$ with $j = +1$ for particle or positive charge state and $j = -1$ for anti-particle or negative charge state.

In a finite volume can be solved using instead the circular wavefunctions with zero boundary conditions

\[ f_l(r, \theta) = e^{ilq_r|l|} e^{-\frac{eB}{2} \Omega q_r} F_l \left( -a, |l| + 1, \frac{eBr^2}{2} \right) \tag{24} \]

where $F_l$ is a hypergeometrical function with the parameter

\[ -a(l) = \frac{1}{2}(|l| - l + 1) - \frac{1}{2eB}((E + \Omega)^2 - p^2 - m_\pi^2) \tag{25} \]

Thus, for positive angular momentum states we have

\[ (E + \Omega)^2 = p^2 + m_\pi^2 + eB(2a(l) + 1) \]
\[ F_l(-a(l), l + 1, \frac{eBR^2}{2}) = 0 \tag{26} \]

The zero of the hypergeometric function fixes $a(l)$, and therefore the LL for a finite volume. For example, for $N = 25$ we have $l = 20$, and $a_{mn}(20) = 0.43$ for which the energy is $E = \sqrt{m_\pi^2 + 1.86eB - 209}$. For $eB = m_\pi^2$, the threshold rotation is $\Omega = \frac{20}{29} \sqrt{eB}$. Note that for $N = 25$, we have $R = \sqrt{50/eB}$ and the luminal constraint is still fulfilled since $\Omega_c R = 0.59 < 1$. For $N =
100 we have \( l = 84 \), and \( a_{\min}(84) = 0.18 \) for which the energy is \( \sqrt{m_2^2 + 1.36eB - 84\Omega} \) with the threshold \( \Omega, R = 0.25 \). For \( N = 1000 \) we have \( l = 935 \) and \( a_{\min}(935) < 0.1eB \). Thus as \( N \) goes to infinity, the state with the lowest energy will approach \( l = N \), and our approximation in the main text becomes more precise. Note that for exactly \( l = N - 1 \), we always have \( a_{\min}(N - 1) = 1 \), and the energy for such state is \( \sqrt{m_2^2 + 3eB - (N - 1)\Omega} \).

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