In-plane dipole coupling anisotropy of a square ferromagnetic Heisenberg monolayer

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In this study we calculate the dipole-coupling-induced quartic in-plane anisotropy of a square ferromagnetic Heisenberg monolayer. This anisotropy increases with an increasing temperature, reaching its maximum value close to the Curie temperature of the system. At $T = 0$ the system is isotropic, besides a small remaining anisotropy due to the zero-point motion of quantum mechanical spins. The reason for the dipole-coupling-induced anisotropy is the disturbance of the square spin lattice due to thermal fluctuations (‘order-by-disorder’ effect). For usual ferromagnets its strength is small as compared to other anisotropic contributions, and decreases by application of an external magnetic field. The results are obtained from a Heisenberg Hamiltonian by application of a mean field approach for a spin cluster, as well as from a many-body Green’s function theory within the Tyablikov-decoupling (RPA).

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I. INTRODUCTION

The investigation of the magnetic properties of ferromagnetic ultrathin films is a field of intense current interest. In this contribution we determine the magnetic in-plane anisotropy of a square two-dimensional (2D) Heisenberg ferromagnet in presence of the magnetic dipole interaction. In 2D magnets the action of magnetic anisotropies is twofold. First, they induce a long range dipole interaction. In 2D magnets the action of magnetic in-plane anisotropies is twofold. Secondly, the anisotropies determine the easy and hard axes of the magnetization with respect to the lattice frame. For a ferromagnetic thin film the dipole coupling prefers an in-plane direction of the magnetization (shape anisotropy, demagnetizing field). Except for very small thicknesses thin films are usually in-plane magnetized. The corresponding in-plane easy axis depends on the lattice symmetry of the film face. For a rectangular (110) monolayer the dipole interaction induces a uniaxial in-plane anisotropy, the easy axis being the in-plane axial direction with the smaller lattice constant, as can easily be shown by calculating the corresponding lattice sums.

On the other hand, for a square monolayer the dipole coupling, although itself not rotational invariant, exhibits a continuous energy degeneracy for classical spins at $T = 0$. Thus, due to the Mermin-Wagner theorem a long range magnetic order is not expected at finite temperatures. However, as has been shown by Monte Carlo calculations and interacting spin wave theory, a magnetic ordering and a critical temperature exists for dipole coupled spins on a square lattice, since the magnetic excitations are not continuously degenerate. In this case a quartic in-plane anisotropy is present, the corresponding easy axes being the edges of the square lattice. In other words, the density of states and thus the entropy depends on the magnetic direction within the lattice. This phenomenon is an example of the ‘order-by-disorder’ effect in frustrated magnets. In these systems thermal fluctuations or lattice disturbances partly remove frustrations, and a collective magnetic ordering emerges at finite temperatures.

Whereas the existence of a finite magnetization and a quartic anisotropy at finite temperatures for dipole coupled spins on a square lattice has been proven, the strength and the temperature behavior of this anisotropy has not been determined yet. Of particular interest is whether the dipole-coupling-induced quartic anisotropy can be measured, and how this anisotropy compares with the single-ion quartic lattice anisotropy due to the spin-orbit coupling. In this study we will address these questions with the help of a mean field approximation, taking into account a square spin cluster with different numbers of spins (Oguchi-theory), as outlined in Sec.II.

Furthermore, we apply a many-body Green’s function theory within the Tyablikov-decoupling (RPA) by considering collective magnetic excitations (spin waves) and interactions between them, which are in particular important for 2D magnetic systems. Our results are summarized and discussed in Sec.III.

II. THEORY

The free energy $F(T, \phi)$ is determined within a Heisenberg-type Hamiltonian as a function of the temperature $T$ and the in-plane angle $\phi$. A square (001) ferromagnetic monolayer is considered, which is spanned by the $xz$-plane, the $z$-direction refers to $\phi = 0$. The isotropic nearest neighbor Heisenberg exchange coupling $J$, the magnetic dipole-dipole interaction, and an external magnetic field $B$ between localized Heisenberg spins...
\[ \mathcal{H} = \frac{J}{2} \sum_{\langle i,j \rangle} S_i S_j - g \mu_B \sum_i \mathbf{B} \cdot \mathbf{S}_i \]
\[ + \frac{(g \mu_B B)^2}{2} \sum_{\langle i,j \rangle} \frac{1}{r_{ij}^3} \left[ r_{ij}^2 S_i S_j - 3 \left( r_{ij} \cdot S_i \right) \left( r_{ij} \cdot S_j \right) \right], \tag{1} \]

with \( g \) the Landé factor and \( \mu_B \) the Bohr magneton. The spin quantum number is assumed to be \( S = 1/2 \). The lattice vector between sites \( i \) and \( j \) is given by \( r_{ij} \), with \( r_{ij} = |r_{ij}| \) the distance. All spins are assumed to be aligned parallelly (ferromagnetic order), the magnetization \( S_r \) aligned parallelly (ferromagnetic order), the magnetization \( \mathbf{m}_i(T) = S_r(T) = (0,0,m(T)) \) is directed along the magnetic field \( \mathbf{B} = (0,0,B) \).

To account for a varying angle of the magnetization with respect to the lattice frame, the lattice is rotated by the in-plane angle \( \phi \) with respect to the \( z \) -axis, yielding the rotated lattice vectors:
\[ r'_{ij} = \begin{pmatrix} r_{ij} \sin \phi + r^x_{ij} \cos \phi \\ r^x_{ij} \cos \phi - r_{ij} \sin \phi \end{pmatrix}. \tag{2} \]

Clearly, the only source in Eq. (1) for a possible in-plane anisotropy is the double scalar product of the dipole interaction. Within the single-spin (Bragg-Williams-) mean field approximation, the dipole coupling yields an additional contribution to the molecular field. The resulting single-spin Hamiltonian reads

\[ \mathcal{H}^{(1)} = - \left( m(T) + w S(0,0) m(T) + g \mu_B B \right) S^z_1 \]
\[ + \frac{1}{2} m^2(T) \left( q J + w S(0,0) \right), \tag{3} \]

with \( q = 4 \) the number of nearest neighbors (coordination number), \( w = (g \mu_B B)^2 / a_0^3 \) the strength of the dipole interaction, \( a_0 \) the lattice constant, \( S(0,0) \sim 4.51681 \) the corresponding lattice sum, \( \mathbf{m}(T) = (S^z_1) \) the magnetization. Thus, within this approximation the dipole interaction is isotropic for a square monolayer and does not depend on the in-plane angle \( \phi \).

Our approach will now be improved by applying the so-called Oguchi method. A number \( N > 1 \) of neighboring spins is considered, the interactions between the \( N \) spins in this cluster are treated exactly. The remaining lattice is coupled to the spin cluster by a molecular field. To conserve the symmetry of the square lattice, we take into account only square-shaped spin clusters, the three smallest possible clusters are characterized by \( N = 4, 9, \) and 16. Note that the resulting cluster Hamiltonians \( \mathcal{H}^{(N)} \) are usually non-diagonal, i.e. contributions proportional to the \( S^z \)- and \( S^y \)-spin components may be present. Therefore, as a further approximation we consider only the diagonal elements. This approach guarantees that the magnetization is directed along the external magnetic field. As an example, the Hamiltonian for the four-spin cluster \( (N = 4) \) reads:

\[ \mathcal{H}^{(4)} = -(J - w) \left( S^+_1 S^-_2 + S^+_2 S^-_1 + S^+_3 S^-_4 + S^+_4 S^-_3 \right) - \frac{w}{2^{3/2}} \left(S^+_1 S^+_2 + S^+_3 S^+_4 \right) \]
\[ -3 w \left[ \sin^2 \phi (S^+_1 S^-_2 + S^+_3 S^-_4) + \cos^2 \phi (S^+_1 S^+_3 + S^+_2 S^-_4) + \frac{2 \cos \phi \sin \phi}{2^{1/2}} (S^+_1 S^-_4 - S^-_1 S^+_4) \right] \]
\[ - \left( 2 J m(T) + g \mu_B B \right) \left( S^+_1 + S^-_2 + S^+_3 + S^-_4 \right) - w m(T) \left( S^+_1 + S^-_3 \right) \left( S(0,0) - 1 - \frac{1}{2^{5/2}} (1 - 6 \cos \phi \sin \phi) \right) \]
\[ - w m(T) \left( S^+_2 + S^-_4 \right) \left( S(0,0) - 1 - \frac{1}{2^{5/2}} (1 - 6 \cos \phi \sin \phi) \right) + 4 J m^2(T) + 2 w m^2(T) \left( S(0,0) - 1 - \frac{1}{2^{5/2}} \right). \tag{4} \]

The partition function for this case is given by:
\[ Z^{(4)}(T, B, \phi) = \sum_{S_1^z, S_2^z, S_3^z, S_4^z} \exp(-\beta \mathcal{H}^{(4)}), \tag{5} \]
and the (average) magnetization by:
\[ m(T, B, \phi) = \frac{1}{4 \ Z^{(4)}(T, B, \phi)} \times \]
\[ \sum_{S_1^z, S_2^z, S_3^z, S_4^z} \left( S^+_1 + S^-_2 + S^+_3 + S^-_4 \right) \exp(-\beta \mathcal{H}^{(4)}), \tag{6} \]

with \( \beta = 1/k_B T \) and \( k_B \) the Boltzmann constant. The corresponding expressions for the other investigated sizes are rather lengthy and not shown here. The difference of the free energy \( F(T, B, \phi) = -k_B T \ln Z(T, B, \phi) \) between the diagonal \( (\phi = \pi/4) \) and the axial directions \( (\phi = 0) \) yields the in-plane anisotropy \( K_{4, dip}(T,B) \), which will be calculated as a function of the temperature \( T \) and the external magnetic field \( B \).

It is important to mention that the mean field theory as described in the preceding subsection does not fulfill the Mermin-Wagner-theorem for the isotropic 2D Heisenberg magnet. In this case the long range magnetic order becomes unstable against collective magnetic excitations with long wavelengths. Thus, the consideration of
these spin waves is very important for the magnetic behavior of a ferromagnetic monolayer. For the determination of the dipole-coupling-induced in-plane anisotropy we apply in addition a many-body Green’s function theory to the Hamiltonian Eq. (3), after the lattice is rotated by the in-plane angle $\phi$, Eq. (4). We consider the Green’s functions $G_{ij}^{\alpha} = \langle \langle \hat{S}^\alpha_i, \hat{S}^\alpha_j \rangle \rangle$, which are solved within the Tyablikov-decoupling. Since interactions between spin waves are partly taken into account, the magnetic properties can be determined up to the Curie temperature by this method. The respective formalism is described in detail in Sec.III. For comparison, we determine the corresponding quantities also by considering the Holstein-Primakoff approximation, which is valid at low temperatures.

### III. RESULTS AND DISCUSSION

By application of the described methods we calculate the effective dipole-coupling-induced in-plane magnetic anisotropy $K_{4,dip}(T,B)$ for a square ferromagnetic monolayer as a function of the temperature $T$ and the applied magnetic field $B$. A finite value indicates that this anisotropy is caused by the magnetic dipole coupling, since other sources of magnetic anisotropies are not present in the Hamiltonian Eq. (3).

For the strength of the dipole interaction we choose $(g\mu_B)^2/a_0^6 = w = 0.01 \text{ J}$, which is appropriate for 3d-transition metal ferromagnets. $K_{4,dip}(T,B)$ is given in units of the energy difference between the perpendicular and the in-plane magnetization (demagnetizing energy) $E_{demag}(0) = (3/2) w S^2 (0) m^2 (0) \approx 2 \pi w S^2$ for a ferromagnetic monolayer at $T = 0$.

The free energy $F(T,\phi)$ exhibits a four-fold symmetry as a function of the in-plane angle $\phi$. The easy magnetic axes are directed along the edges ($\phi = 0, \pi/2$, etc.), and the hard axes along the diagonals ($\phi = \pi/4, 3\pi/4$, etc.) of the square lattice as obtained in[4]. The quartic anisotropy $K_{4,dip}(T,B)$ is depicted in Fig.1 as a function of the temperature, calculated by the Oguchi approach. Three different sizes of the spin cluster are considered ($N = 4, 9, 16$), as well as different magnetic field strengths $B$. The Curie temperature $T_C(N)$ decreases with an increasing $N$, since additional spin correlations are considered. As can be seen, $K_{4,dip}(T,B)$ increases with an increasing temperature, reaching a maximum at $T_C$. This temperature behavior is in striking contrast to the corresponding behavior of other anisotropic contributions. Usually the effective anisotropies decrease with an increasing temperature, and vanish at $T_C$ for $B = 0$. $K_{4,dip}(T,B)$ exhibits a cusp at $T_C$ and decreases for temperatures $T > T_C$. By increasing the size $N$ of the spin cluster, $K_{4,dip}(T,B)$ becomes larger. An application of a magnetic field reduces $K_{4,dip}(T,B)$, and the cusp near $T_C$ changes to a maximum. For comparison, in Fig.1 we show $K_{4,dip}(T,B)$ for $B = 0$ and $N = 4$ by considering also non-diagonal elements in the cluster Hamiltonian $H^{(4)}$, cf. Sec.II. A decrease of $T_C$ and an increase of $K_{4,dip}(T,B)$ by $\sim 10\%$ is obtained with respect to the calculations which consider diagonal elements only.

In addition, we have calculated $K_{4,dip}(T,B)$ by applying the many-body Green’s function theory as outlined in[4] and in the Appendix. For $w = 0.01 \text{ J}$ the Curie temperature is calculated to be $k_BT_C^{RPA} = 0.373 \text{ J}$. In Fig.2(a,b) we present the resulting magnetization $m(T,B)$ and anisotropy $K_{4,dip}(T,B)$ as functions of the temperature and the magnetic field. In accordance with the results obtained from the Oguchi approach, $K_{4,dip}(T,B)$ increases with an increasing temperature. Due to the consideration of quantum mechanical spins a finite value of $K_{4,dip}(T,B)$ is already present at $T = 0$, resulting from the zero-point spin motion (quantum fluctuations). Note that the presence of the dipole coupling causes a non-saturated magnetization $m(T = 0) < S = 1/2$, cf. In Fig.2 also the results as obtained from the Holstein-Primakoff approximation are shown, which neglects interactions between spin waves. The corresponding magnetization $m(T,B)$ and anisotropy $K_{4,dip}(T,B)$ at low temperatures are close to the results as obtained from the RPA. At elevated temperatures $T \gtrsim T_C/3$ the Holstein-Primakoff approxima-
The effective single-ion quartic anisotropy term is no longer valid.

As has already been obtained from Monte Carlo calculations and interacting spin wave theory, the dipole-coupling-induced in-plane anisotropy is caused by magnetic fluctuations. This order-by-order effect is thus controlled by the magnetic entropy of the system. An energetic influence should become apparent at $T = 0$ already within a mean field approximation. Besides the effects resulting from the zero-point motion, the terms of the dipole interaction dependent on the in-plane angle $\phi$ cancel for a strictly square magnetic lattice, i.e. for a fully saturated ferromagnetic state. At finite temperatures thermal fluctuations disturb this square periodicity, causing an effective quartic in-plane magnetic anisotropy, which reflects the underlying spatial lattice symmetry. The single-spin mean field theory conserves the periodicity of the square magnetic lattice, since all other spin operators are replaced by uniform expectation values. Thus, within this approximation one obtains $K_{4,dip}(T, B) = 0$ for all temperatures. On the other hand, within the Oguchi approach the interactions between a number of spins are treated exactly, considering a few collective magnetic excitations with short wavelengths ranging over several lattice constants, and resulting in a finite value for $K_{4,dip}(T, B)$ for $T > 0$. The many-body Green’s function theory takes into account spin waves with all possible wavelengths. Since $K_{4,dip}(T, B)$ is maximal at elevated temperatures, $K_{4,dip}(T, B)$ is caused by magnetic fluctuations in particular with short wavelengths, which become excited in this temperature range.

Within the Oguchi approach we calculate a maximum value of $\mathcal{K}_{4,dip}(T, B)$ of the order of 0.1% of the demagnetizing energy $E_{\text{demag}}(0)$. On the other hand, $K_{4,dip}(T, B)$ as calculated from the Green’s function method is more than a magnitude larger, namely $K_{4,dip}(T, B) \sim 0.01 E_{\text{demag}}(0)$ near $T_C$. Similarly, the action of a magnetic field $B$ on the anisotropy $K_{4,dip}(T, B)$ is calculated to be much more pronounced within the Green’s function theory than within the Oguchi approach, cf. Figs.1 and 2. The reason for the strong differences between the results obtained from the two methods is that the former one considers collective magnetic excitations with long wavelengths. These are known to have strong effects on the magnetic properties of ultrathin films. In this case the magnetic field acts merely on spin blocks, i.e. correlated regions of neighboring spins characterized by the short range order parameter resulting in a much larger magnetic response. For instance, the induced magnetization in a ferromagnetic trilayer has been determined to be an order of magnitude larger by consideration of spin waves. The collective magnetic excitations are most pronounced for a single magnetic layer, the zero-point motion is strongest for the spin number $S = 1/2$. Note that the theoretical methods applied in the present study are much less demanding than Monte Carlo simulations.

The obtained dipole-coupling-induced in-plane anisotropy is small as compared to other (effective) anisotropies, since we have considered interaction strengths appropriate for 3$d$- transition metal ferromagnets. A small value for $K_{4,dip}(T, B)$ has been conjectured in. If a ten times larger ratio $w/J$ between the dipole coupling strength and the exchange interaction is assumed, the ratio $K_{4,dip}(T, B)/E_{\text{demag}}(0)$ increases roughly by the same factor. Since usually the effective anisotropies decrease with an increasing temperature, $K_{4,dip}(T, B)$ might become observable at elevated temperatures. For comparison, we show in Figs.1 and 2 also the effective single-ion quartic in-plane anisotropy $K_{4,so}(T)$ resulting from the spin-orbit interaction. This quantity is calculated with the help of a thermodynamic perturbation theory. By assuming its strength to be $K_{4,so}(0) = 0.01 E_{\text{demag}}(0)$, and adapting corresponding Curie temperatures, $K_{4,so}(T)$ becomes comparable to $K_{4,dip}(T, B)$ at $T/T_C \sim 0.8 - 0.9$. In this temperature range the total quartic in-plane anisotropy should increase again, or exhibit an in-plane magnetic reorientation. As mentioned, this dipole-coupling-induced anisotropy should be more apparent for ferromagnetic (001) thin films with a large $w/J$- ratio.

We like to comment on the fact that a finite anisotropy is obtained for $T > T_C$. Usually the effective anisotropy as observed for a collectively ordered ferromagnetic state vanishes above $T_C$. However, a vanishing effec-
tive anisotropy for $T > T_C$ does not indicate that the anisotropy as present in the Heisenberg Hamiltonian disappears. A single (paramagnetic) spin is still subject to e.g. the single-ion uniaxial anisotropy $K_2$ also if the net magnetization is zero. The resulting free energy difference between the easy and the hard magnetic directions ('paramagnetic anisotropy') behaves as $\propto K_2^2/k_BT$ for $K_2 \ll k_BT$, and is rather small if $K_2$ is small as compared to the exchange interaction $J \propto k_BT$. With regard to the present study, a finite value of $K_{4,dip}(T, B = 0)$ for $T > T_C$ reflects the dipole-coupling-induced anisotropy of a spin block.

Note that the free energy $F(T, B)$ as obtained from the Green’s function method yields unphysical results for temperatures near and above $T_C$. As can be shown for large temperatures the free energy $F(T, B)$ as calculated by this method does not approach the value $-k_BT \ln 2$, which is the free energy of a single, non-interacting $S = 1/2$ spin. This unphysical behavior results in a still increasing $K_{4,dip}(T, B)$ for $T > T_C$, cf. Fig.2(b). Nevertheless, we expect that the Green’s function approach reflects the correct behavior of $K_{4,dip}(T, B)$ for $T < T_C$, since its behavior is corroborated by the Oguchi method and the Holstein-Primakoff approach.

We have considered a square monolayer only. A similar effect is expected also for thicker films with a square (001) face, or for a hexagonal (111) thin film. In addition, a corresponding effect may emerge for a three-dimensional cubic lattice, for which the dipole interaction cancels exactly. At elevated temperatures, however, caused by thermal fluctuations the frustration due to this periodicity will be lifted. The dipole coupling then induces a cubic magnetic anisotropy with easy axes directed along e.g. the edges of the cubic lattice. Also is this case an increasing anisotropy with an increasing temperature is expected. To obtain an anisotropic contribution caused by the magnetic dipole interaction via the order-by-disorder effect, we emphasize that the lattice symmetry must be strictly cubic or square. A lattice distortion e.g. due to magnetostrictive effects will probably blur this effect.

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Appendix: Many-body Green’s function theory

The following commutator Green’s functions in the frequency space are taken into account: $G_{ij}^\alpha(\omega) = \langle \langle S_i^\alpha; S_j^\alpha \rangle \rangle_{\omega}$, $\alpha = +, -, z$. By considering the Hamiltonian Eq.4, these Green’s functions are solved in the usual way by the equation of motion. The higher order Green’s functions are approximated by the Tyablikov-decoupling (RPA), e.g. $\langle \langle S_i^z S_j^z; S_l^z \rangle \rangle \simeq \langle S_i^z \rangle \langle \langle S_j^z; S_l^z \rangle \rangle$, resulting in a system of linear equations for the $G_{ij}^\alpha(\omega)$. By application of a Fourier transformation into the 2D wave vector space $k_0 \equiv k$, one obtains $G^2(k, \omega) = 0$, if $\langle S_i^+ \rangle = \langle S_i^- \rangle = 0$. The remaining two Green’s functions $G^\pm(k, \omega)$ are easily evaluated:

$$G^+(k, \omega) = 2m(T) \frac{\omega + a(k)}{\omega^2 - \varepsilon^2(k)}$$

$$G^-(k, \omega) = 2m(T) \frac{b(k)}{\omega^2 - \varepsilon^2(k)}$$

with $m(T) = \langle S_i^+ \rangle$. The magnon dispersion relation $\varepsilon(k)$ is given by

$$\varepsilon^2(k) = a^2(k) - b^2(k)$$

$$a(k) = g\mu_B B + m(T) \left( J(0) - J(k) \right)$$

$$+ w \frac{m(T)}{2} \left[ S(0, 0) + S(k_x, k_y) \right] \left( 1 - \frac{3}{2} \cos^2 \phi \right) + S(k_x, k_z) \left( 1 - \frac{3}{2} \sin^2 \phi \right) + 3T(k_z, k_z) \cos \phi \sin \phi$$

$$+ S(k_x, k_z) \left( 1 - \frac{3}{2} \sin^2 \phi \right) + 3T(k_z, k_z) \cos \phi \sin \phi$$

$$b(k) = \frac{3}{2} \frac{w m(T)}{2} \left[ S(k_x, k_z) \cos^2 \phi + S(k_x, k_z) \sin^2 \phi - 2T(k_z, k_z) \cos \phi \sin \phi \right]$$

$$J(k) = 2J \left( \cos k_x a_0 + \cos k_y a_0 \right)$$

is the Fourier transform of the exchange interaction, with $a_0$ the lattice constant. The oscillating lattice sums are defined by

$$S(k_x, k_z) = \sum_{l,n=-\infty}^{+\infty} \frac{l^2}{(l^2 + n^2)^{3/2}} \exp(-ik_x a_0 l - ik_z a_0 n)$$

$$T(k_x, k_z) = \sum_{l,n=-\infty}^{+\infty} \frac{l n}{(l^2 + n^2)^{3/2}} \exp(-ik_x a_0 l - ik_z a_0 n)$$

where the terms with $l = n = 0$ have to be omitted. The following expressions are valid for the spin quantum number $S = 1/2$. The magnetization $m(T)$ is obtained from the spectral theorem:

$$m(T, \phi) = \frac{1}{2} - \frac{1}{N} \sum_k \langle S_i^- S_i^+ \rangle(k)$$

$$= \frac{1}{2} - \frac{m(T)}{N} \sum_k \left( \frac{a(k)}{\varepsilon(k)} \coth x - 1 \right)$$

with $x = \varepsilon(k)/2k_BT$. The summation extends over the first Brillouin zone, $N$ denotes the number of k-points. By calculating the expectation value $\langle S_i^+ S_i^- \rangle$ with the help of $G^+(k, \omega)$ we obtain for the internal energy $E(T, \phi)$ per spin:

$$E(T, \phi) = E_0 + \frac{m(T, \phi)}{2N} \sum_k \left( e_1(k) \frac{a(k)}{\varepsilon(k)} \coth x - 1 \right) + \left( e_2(k) \frac{b(k)}{\varepsilon(k)} + \varepsilon(k) \right) \coth x - a(k) + b(k)$$

$$= \left( e_1(k) \frac{a(k)}{\varepsilon(k)} + e(k) \right) \coth x - a(k) + b(k)$$
with the denotations
\[ E_0 = -\frac{1}{2} g \mu_B B - \frac{1}{8} \left( J(0) + w S(0,0) \right), \quad (15) \]
\[ e_1(k) = g \mu_B B + \frac{1}{2} \left( J(0) - J(k) \right) + \frac{w}{2} \left( S(0,0) + S(k_z,k_z) + S(k_x,k_z) \right), \quad (16) \]
\[ e_2(k) = \frac{1}{2} J(k) - \frac{w}{2} \left( S(k_z,k_x) + S(k_x,k_z) \right). \quad (17) \]

Finally, the free energy \( F(T, \phi) \) is calculated from a temperature integral over \( E(T, \phi) \). The Holstein-Primakoff approximation is obtained by replacing the magnetization \( m(T) \) on the right side of Eq. (13) by its saturation value \( S = 1/2 \).

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