POWER TAILS OF ELECTRIC FIELD DISTRIBUTION FUNCTION IN 2D METAL-INSULATOR COMPOSITES

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The 2D "Swiss-cheese" model of conducting media with round insulator inclusions is studied in the 2nd order of inclusion concentration and near the percolation threshold. The electric field distribution function is found to have power asymptotics for fields much exceeding the average field, independently on the vicinity to the threshold, due to finite probability of arbitrary proximity of inclusions. The strong field in the narrow necks between inclusions results in the induced persistent anisotropy of the system. The critical index for noise density is found, determined by the asymptotics of electric field distribution function.

Introduction

The distribution function of electric field is an important characteristics of a conducting mixture. It determines such properties as noise, electric breakdown and nonlinear dielectric permeabilities. Its long-range tails are usually considered as a result of the electric field amplification due to the vicinity to the percolation threshold [1], [2], [3].

The purpose of the present study is to find the distribution function for the electric field F in the conducting phase. The 2D model of conducting media containing randomly distributed round voids with density n and radius a is considered. This system is interesting both as a real object of nanoelectronics [4] and as an exact solvable model. Without inclusions the electric field is uniform. The field around a solitary smooth void is limited, so it does not produce the field, much exceeding the average field. The strong field appears in the 2nd order of void concentration only.

We shall study both 2nd order of concentration and the field distribution at the percolation threshold.

Potential distribution around two holes.

The distribution of field in a conducting media is described by the solution of the continuity equation \( \nabla \mathbf{j} = 0 \) for current \( \mathbf{j} = \sigma \mathbf{F} = \sigma \nabla \varphi \), where \( \sigma \) is the conductivity of the conductor. The boundary conditions on the void boundaries require vanishing of the normal component of current density \( \sigma \mathbf{n} \nabla \varphi \) (\( \mathbf{n} \) is the normal to the circle). The electrostatic problem of field distribution is two-dimensional.

Since the asymptotics of the field distribution is determined by narrow necks between two holes, we turn our attention on the media, containing two round holes only (Fig. 1). We assume that the x axis goes through circles centers. The conformal mapping of system of Fig.1 to the band \( 1/R_0 < Rew < R_0 \) is given by the function.
\[ w = \frac{z + b}{z - b}. \]  

where \( z = x + iy \), \( 2\delta \) is the distance between the holes with radii \( a \),

\[ b = \sqrt{2a\delta + \delta^2}, \quad R_0 = \frac{\sqrt{2a + \delta + \sqrt{\delta}}}{\sqrt{2a + \delta - \sqrt{\delta}}} \]

We should solve the problem of current flow with conditions \( \varphi \sim \text{Im} \left( F_0 z \right) \) for \( z \to \infty \), where \( F_0 \) is the average electric field. The corresponding potential in \( w \)-plane is the dipole potential with asymptotic behavior \( \varphi \sim \text{Im} \left( \frac{2F_0 b}{w-1} \right) \), for \( w \to 0 \).

The dipole potential in the ring between two circles is determined by the sum of its images in circles:

\[ \varphi = 2F_0 b \quad \text{Im} \left( \frac{1}{w-1} + \sum_{n=1}^{\infty} \frac{1}{wR_0^{2n} - 1} - \sum_{n=1}^{\infty} \frac{w}{R_0^{2n} - w} \right) \]

The strong field results from the close approaching of two circles, \( \delta \ll a \). In this limit (2) transforms:

\[ \varphi = \pi F_0 a \quad \text{Im} \left( \cot \frac{\pi a}{z} \right) \]

The last extreme limit means the absence of percolation between contacting circles if \( \delta \to 0 \). This leads to the gap of potential between up and down sides of the circles near the points of tangency:

\[ U = 2\pi F_0 a \]

If the width of constriction is small but finite, the potential gap is distributed along the length of constriction \( \sqrt{2a\delta} \). The field in the vicinity of constriction is

\[ F_y = \frac{U}{\pi} \sqrt{\frac{2\delta}{a} \frac{1}{2\delta + y^2/a}} \]

The maximal value of field \( F_{max}(\delta) = F_0 \sqrt{2a/\delta} \) diverges with \( \delta \to 0 \). Another component of field, \( F_x \), remains finite if \( \delta \to 0 \) and is insufficient below.

**Breakdown-induced training.**

The divergency of field and current density can lead to overheating and melting of narrow necks. If we assume that breakdown is determined by the critical density of heat emission \( Q_c \), all places where \( \sigma F^2 > Q_c \) will be overheated. It means that some number of overheated places occurs in any weak field \( F_0 \) independently on the field strength. So the application of field irreversibly change the electrical properties of system. Moreover, the direction of field tends to the residual anisotropy of a system.

We shall consider this effect in the framework of swiss-cheese system with rare, randomly distributed round voids. The residual change of linear conductivity in a low field originates from the breakdown of narrowest necks. The criterion of breakdown is \( F_{max}(\delta) \geq F_c = \sqrt{Q_c/\sigma} \).
The contribution of necks between pairs of voids which have x-orientation and widths between \( \delta \) and \( \delta + d\delta \) to the average conductivity is

\[
\Delta \sigma_{yy} = 16\pi \sqrt{2}(na)^2 \sigma \frac{\sqrt{\delta \, d\delta}}{a},
\]

\[
\Delta \sigma_{xx} = 0.
\]  

(6)

One should average (6) over \( \delta \) and the direction of pair of voids. The value \( \delta \) is limited by the maximum \( \delta_{\text{max}} \) determined by the situation when the field in the neck exceeds the breakdown field \( F_c = F_0 \sqrt{\frac{\delta_{\text{max}}}{a}} \cos \theta \), where \( \theta \) is the angle between the external field and the direction of axis, connecting centers of circles. The resulting change of conductivity tensor is

\[
\Delta \sigma_{xx} = -\frac{128\sqrt{2}}{45}(na)^2 \left(\frac{|F_0|}{F_c}\right)^3 \sigma
\]

\[
\Delta \sigma_{yy} = 4\Delta \sigma_{xx}.
\]  

(7)

(8)

The direction \( y \) is determined along the training field.

The change of longitudinal conductivity appears to be 4 times higher than transversal. The equation (8) shows the non-analytical dependence on the field. It should be noted that the training, being a nonlinear effect in applied field, nevertheless means the change of linear conductivity.

**Distribution function of field for pairs of circles.**

The asymptotics of distribution function for high field is determined by rare approaching of circles, when the distance \( \delta \) between them is much less than the radius and mean distance \( (\pi n)^{-1/2} \).

The probability of field to have value \( F \), \( P_{\delta,U}(F) \) for the fixed distance \( \delta \) and potential gap \( U \) is determined by area where \( F \) is between \( F \) and \( F + dF \). By means of (3) we find:

\[
P_{\delta,U}(F) = \left(\frac{2\delta}{\pi^2}\right)^{3/4} \frac{1}{a^{1/4}} \frac{U^2}{F^{5/2}(U - F\pi\sqrt{2a\delta})^{1/2}}
\]  

(9)

This result should be integrated by \( \delta \) with the density of pairs \( 4\pi n^2 a \, d\delta \), distance of which is within the range \( 2\delta \) and \( 2(\delta + d\delta) \). The result of averaging on \( \delta \) and the direction of axis \( y \) is

\[
P(F) = \frac{128}{3\pi}(\pi na)^2 \frac{F_0^5}{F^6}
\]  

(10)

The equation (10) gives power long-range tail of the electric field distribution function \( P(F) \sim F^{-6} \).

The long-range distribution tail gives the divergency of high order power of the local field, starting from 6th power and does not affect the density of \( 1/f \) noise, determined by 4th power of \( F \).

The existence of power tail results from the microgeometry and is not connected with the vicinity to the percolation threshold. Thus such power tails are universal property of 2D system both far and near threshold. They result from the possibility
for 2D system to have a finite potential gap $U$ on infinitely small distance, connected with the necessity of current to flow around tangent circles on the finite distance to equilibrate the potential gap. On the other hand, the shortest path between two infinitely close points along the conducting phase in the 3D case is always small and the field near close pair of inclusions is limited. Hence, no long-range tail exists in 3D system of non-intersecting spheres and only exponential tail $P(F)$ is possible in the intersecting inclusions 2D or 3D models determined by rare large coupling of inclusions.

**Percolational situation.**

The power-law tails due to formula (10) are determined by very small part of total current while most part of current flows around both disks. Hence in the percolational limit $na^2 \sim 1$ the contribution retains, described by random necks inside the conducting phase. Formula (10) may be applied to this case also (except for numerical coefficient) if one changes $na^2 \rightarrow 1$.

The key bonds tend to additional possibility for local field amplification near the threshold.

The distribution function of voltages in lattice models was studied on the metal side of percolation threshold [1, 6]. It was found that this function has a long-range tail, $P(U) \sim \exp((\ln(U/U_0))^2/C)$, which is much weaker than any power-like one. The distribution function in insulator phase, found below, exhibits power behavior. In the Swiss-cheese model narrow necks, insufficient for current flow may be considered as almost insulating phase, so these tails are reproducing themselves in the contribution from narrow necks to metal phase distribution function long tail.

To estimate the insulator phase distribution function tail we shall use the evident formula for energy:

$$\frac{\langle \varepsilon F^2 \rangle}{8\pi} = \varepsilon_e \frac{\langle F \rangle^2}{8\pi}$$

where $\varepsilon$ is the permittivity of the insulator. Just below percolation threshold the effective permittivity $\varepsilon_e$ diverges like $\tau^{-q}$, producing divergency of $\langle F^2 \rangle$ if the system comes to the percolation threshold $\tau = 0$, while the mean field $\langle F \rangle$ is limited. These two facts do not contradict if the distribution function behaves like $1/F^{3-r}$, where $0 < r < 1$.

Near the percolation threshold one should expect the scaling behavior of a distribution function, both above and below the threshold. The characteristic scale for voltage on single bond $U = Fa$ is determined by the maximal voltage $U_m$ in a percolation cell. This voltage can be estimated by assumption, that full voltage across the cell with size $L_c$ is applied to one disconnected bond: $U_m \sim F_0 L_c \sim F_0 a \tau^{-\nu}$. Hence the distribution function for voltages is

$$p(U) = \frac{1}{F_0 a} (F_0 a/U)^{3-r} f(U/F_0 a \tau^\nu)$$

The expression for exponent $r = q/\nu$ is determined by equation (11). According to known values of critical exponents in 2D case $r = 0.9$. This result is consistent with numerical result of [7] but slightly deviates from estimation based on the assumption, that asymptotics $p(U)$ is determined by the number of single-disconnected
bond per a percolation cell \( \left[ 8 \right] \) \( N_{sd} \sim \tau^{-1} \): \( p(U_m) \sim \frac{N_{sd}}{U_m L_c} \sim \tau^{3\nu - 1} \). Like so called nodes-links-blobs model we shall disregard small deviations from 1 of critical exponents for effective conductivity \( \sigma_e \sim \tau^t \) and effective permittivity \( \varepsilon_e \sim \tau^{-q} \).

The far tail for electric field distribution function in the Swiss-cheese model on the metal side of percolation transition is determined by narrow necks on the infinite cluster. The strongest voltage is expected from a narrow neck on a branch of infinite cluster clumped to the backbone in two points with distance \( L_c \) between them. The width of the neck is supposed so small, that it resistance exceeds the backbone resistance and current through it is negligible. Hence the typical width is limited by \( \delta \lesssim \tau^{2t} \). Such necks may be considered as insulating for the problem of current flow. For the neck with the potential gap has order of magnitude \( U = F_0 L_c \). Integrating the distribution function of field \( \left[ 9 \right] \) together with the distribution function of voltages we find the distribution function of field in the vicinity of percolation threshold

\[
P(F) \sim \tau^{-(t+3\nu)} \frac{F_0^5}{F^6} \quad \text{for} \quad F \gtrsim F_0 \tau^{-(t+\nu)} \quad (13)
\]

\[
P(F) \sim \tau^{(5t+\nu)/2} \frac{F_0^{3/2}}{F^{5/2}} \quad \text{for} \quad F \lesssim F_0 \tau^{-(t+\nu)}. \quad (14)
\]

Comparison of formulae \( (13) \), \( (14) \) and \( (10) \) shows that the validity of \( (14) \) is limited by an inequality \( F_0 \tau^{-(5t+\nu)/7} \lesssim F \lesssim F_0 \tau^{-(t+\nu)} \) where the contribution of single-disconnected bonds prevails. For lower fields \( F \lesssim F_0 \tau^{-(5t+\nu)/7} \) the asymptotics of distribution function is given by the contribution of narrow necks inside metal phase \( (10) \).

The amplification of field distribution function tails near the percolation threshold leads to enhancement of current noise. The integral noise in disordered media is determined by mean 4th power of field, according to \( \left[ 4 \right] \):

\[
C_e = \frac{\langle \sigma^2 F^4 \rangle}{(\langle F \rangle \langle \sigma F \rangle)^2}. \quad (15)
\]

The substitution of \( (13, 14) \) to \( (15) \) gives the critical exponents for integral noise:

\[
C_e \sim \tau^{-2(t+\nu)}. \quad (16)
\]

Concluding remarks.

Let us summarize the obtained results.

We found the field distribution around pair of circular insulating inclusions into the conducting phase. It was used to find the second virial correction on the density of inclusions to the distribution function of field. The distribution function was found to have the power-like long range tail, determined by close locations of inclusions. The same configurations result in the training of medium, that is, an irreversible modification of material by a weak electric field, which destroys the narrow necks and produces the persistent anisotropy.

It was demonstrated, that the tail of distribution function is enhanced near the percolation threshold. The field distribution function was employed to find
the current noise in the system. It was found that it is amplified of the near the percolation threshold. The main sources of this amplification are the narrow necks belonging to the percolation cluster backbone.

The results of the present article have close tights to the system dimensional-ity. They are determined by the possibility of existence of a finite potential gap between two close points in 2D, in contradiction with 3D system. This conclusion is independent on the model of inclusions we considered.

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References

[1] L. de Arcangelis, S. Redner, A. Coniglio, Phys. Rev. 34 B (1986) 4656.
[2] S. Feng, B.I. Halperin, P.N. Sen, Phys. Rev. 35 B (1987) 197.
[3] D. Sornet, J. Physique, 48 (1987) 1843.
[4] E.M.Baskin, G.M.Gusev, Z.D.Kvon et al.,JETP Lett. 55 (1992) 678.
[5] M.A. Lavrent’ev, B.V. Shabat, Methods of theory of analytical functions, (Nauka, Moscow, 1965)
[6] L. de Arcangelis, S. Redner, A. Coniglio, Phys. Rev. 31 B (1985) 4725.
[7] S.S. Manna, B.K. Chakrabarty, Phys. Rev. B, 36,(1987),4078.
[8] H.E. Stanley, J. Phys. A10 (1977) L211.
[9] R. Rammal, C. Tannous, A.M. Treblay, Phys. Rev. A31, (1985), 2662.
Figure 1. System with 2 round holes.