Strong Electromagnetic Waves Propagation in an Electron-Positron-Ion Plasma

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Abstract

The main purpose of this work is the investigation of the influence of electron-positron-ion on the electrostatic wakefields that are derived by intense, short electromagnetic wave packets in a three component unmagnetized plasma. The equations that describes the derived electrostatic wakefield are given in the form of system of nonlinear differential equations. The solution of that system was carried out numerically and the results are displayed graphically.
Introduction

Recently, the nonlinear propagation of electromagnetic waves in electron-positron plasma has attracted the interest of researchers\[1\] \[2\] and\[3\]. Due to the fact that the electron-positron plasma are found in Van Allen Belts, near polar cap of pulsar, in the active galactic nuclei, as well as in the early universe.

When plasma becomes so hot that it becomes relativistic the temperature $T$ of the plasma exceeds the rest mass energy of electrons $mc^2 = 0.5 \text{ MeV}$. In this relativistic regime the processes of electron-positron pair creation and annihilation become important; $2\gamma \leftrightarrow e^+ + e^-$. In relativistic temperature the electrons (and positrons) energy $E_e$ far exceeds the rest mass energy so that electrons and positrons behave kinematically similar to photons and come into equilibrium with nearly equal population.

In 1990 T. Tajima and T. Taniuti \[4\] are interested in the process of electron-positron plasma creation and annihilation occurs in relativistic; unmagnetized plasma at high temperature of the plasma exceeds the rest mass energy of the electrons. They are emphasize the dominant population of electrons and positrons and neglect ions effect.

In \[6\], it was shown that, the positrons can be used to probe the particle transport in Tokamak plasma. This new diagnostic technique could also be useful in studying transport in other magnetic confinement devices such as reversed-field pinches and magnetic mirrors. Electron-positron pair production can also be possible during intense short laser pulse propagation in plasma. When picosecond electromagnetic pulse having intensity beyond $10^{21}$ W/cm$^2$ interact with plasma, the plasma electrons can acquire oscillatory velocity ($V_{os} = eE_o/mc$) higher than the speed of light $c$. Here $E_o(W_o)$ is the amplitude (frequency) of the $EM$ radiation, $c$ & $m$ are respectively the magnitude of the electron charge and the rest mass of the electron.

Several authors have suggested that when the oscillatory electron energy $E_{os} = mc^2(1 + V^2_{os})^{1/2}$ exceeds $3mc^2$ then these electrons can create an electron-positron pairs in the presence of background positive ions. And also it has been shown that powerful short laser pulsar can generate a large amplitude electrostatic wave (Wakefield) in a plasma in which the electron plasma frequency $\omega_{pe}$ is much smaller than $\omega_o$. Since the amplitude of the excited wakefield can be very high ($e\phi/m_o c^2 \equiv \Phi \gg 1$, where $\phi$ is the electrostatic potential ), the plasma electrons also acquire the longitudinal energy $E_L = m_o c^2(1 + P^2_\parallel/(m_o c^2)) \gg 3m_o c^2$, where $P_\parallel$ is the relativistic momentum in the wakefield. Thus, there is the possibility of creating electron-positron pairs in those regions in which wakefields and short $EM$ pulses are localized. Since the lifetime $\tau_p > \omega_p^{-1}$ of the positron in such a plasma is sufficiently long.

We shall have a plasma that is an admixture of electrons, positrons and positive ions. A three-component electrons-positron-ion plasma can indeed be created in laboratory plasma. V. I. Berezhiani, and et. al. \[3\] have investigated the influence of electron-positron-ion plasma on the electrostatic wakefield that are driven by intense, short $EM$ wave packets in a three component unmagnetized
plasma. They found that in contrast to pure electron-ion plasma, the presence of positrons significantly reduce both the amplitude and the wavelength of wakefields. In view of their investigation, it is suggestive that the production of electron-positron pairs by wakefields in laboratory and cosmic plasma ought to be reconsidered because the presence of pairs may affect the wakefield that is responsible for accelerating electrons to very high energy.

The important of three component admixture plasma had led to much related theoretical investigations. Of these, Rizzato [7], investigated the possibility of high-frequency EM localization in cold unmagnetized plasmas made up of electrons, positrons and ions. It was shown that such a possibility depends on the concentration of particles, on the velocity of the solitons and on the angle between the direction of modulation and the direction of the fast spatial dependence. In particular in the case of pure electron-positron plasmas Rizzato was shown that no localized solution is possible which is in contrast with results derived by Mofiz et. al. (1985) from a similar model. Also it was shown that low frequency magnetic fields may appear in the case of oblique modulation and that both the amplitude of the magnetic field and the amplitude of the solitary wave are very sensitive functions of the angle between the direction of modulation and fast spatial dependence.

Berezhiani, et. al. [1] analyzed the nonlinear interaction of an arbitrarily large amplitude circularly polarized EM wave with an unmagnetized electron-positron plasma. taking into account relativistic particle-mass variation as well as large-scale density perturbation created by radiation pressure. It was found that the interaction is governed by an equation for the electromagnetic wave envelope, which is coupled with a pairs of equations describing fully nonlinear longitudinal plasma motion. This results should be useful for the understanding of nonlinear photon motion in cosmic plasmas, such as those found in the early universe and active galactic nuclei.

Mofiz [5] studied the nonlinear propagation of Langmiur wave in a hot ultrarelativistic electron-positron plasma. It was shown that, in a dense ultrarelativistic electron-positron plasma electrostatic modes with wavelength greater than the plasma Debye length produce Langmiur Solitons that are spiky in nature. One essential feature of these solitons is that they can not form an energy flow toward smaller size, since they can not merge with each other. To merge, the soliton would have to give energy to sound waves, which do not exist in electron-positron plasma. In the one dimensional case the only process accruing in a soliton gas is nonlinear interaction with electron and positron, i. e. nonlinear Landau damping, which stops the solitons without changing its amplitude. The stopping length is of order of the soliton width, which is very small for a dense plasma. Thus the soliton will be stopped in a short time, of the order of their creation. these spiky short duration Langmiur solitons might berelated to the pulsar radiation and with its microstructures.

In [8], V.I.Berezhiani, M.Y. El-Ashry and U.A. Mofiz are investigated the propagation of intense electromagnetic radiation in an admixture of unmagnetized electron-positron-ion plasma analytically. It was shown that electromagnetic radiation of arbitrary amplitude in presence of heavy ions, in contrast to
the case of pure electron-positron plasma, may be localized with the generation of humped ambipolar potential in the plasma, i.e. the driving field intensity creates intense soliton in the plasma with the generation of double hump ambipolar potentials. With the increase of the value of \( \epsilon (=n_{op}/n_{oe}) \), they found that the tendency of converging a single hump soliton to a double hump one. In their investigation they neglected the ion dynamics. Consideration of ion dynamics, of course, may effect the localization phenomenon. This effect will be studied here.

Now, the vector potential \( \mathbf{A} \) for circularly polarized EM wave can be expressed as:

\[
\mathbf{A} = A_\perp + A_z
\]

and

\[
A_\perp = a(z, t) \, e^{i(kz - \omega t)} + \text{c.c.}
\]

\[
A_z = \dot{a}(z, t) \, e^{i(kz - \omega t)} + \text{c.c.}
\]

where:

\[
a(z, t) = a(z - Vt)(\mathbf{x} + i\mathbf{y})
\]

\[
\dot{a}(z, t) = \dot{a}(z - Vt)\mathbf{z}
\]

and \( a \) and \( \dot{a} \) are real functions.

To describe the admixture of plasma made up of electrons, positrons and ions, we use Maxwell equations, in which the fields are expressed in terms of the potentials, as described above. Accordingly, using Poissons equation, we may obtain the following equations for the potentials:

\[
\frac{\partial^2}{\partial t^2} A_\perp - c^2 \nabla^2 A_\perp = 4\pi c J - c \frac{\partial}{\partial t} \nabla \phi \tag{1}
\]

and:

\[
\nabla^2 \phi = -4\pi \rho \tag{2}
\]

here, \( \rho \) and \( \mathbf{J} \) are the charge and current densities, and given by:

\[
\rho = \sum_\alpha e_\alpha n_\alpha
\]

\[
\mathbf{J} = \sum_\alpha e_\alpha n_\alpha \mathbf{V}_\alpha
\]

where \( \alpha \) indicates the particle species \( \alpha (=e, p, i \text{ for electrons, positrons, and ions, respectively}) \), \( e_\alpha \) and \( n_\alpha \) are the charge and density of the corresponding particle \( \alpha \). We shall consider the case in which the admixture equilibrium state is characterized by \( n_{oe} = n_{op} + n_{oi} \), where \( n_{o\alpha} \), is the equilibrium density of the particle \( \alpha \).

The relativistic equations of motion of different particles of unmagnetized plasma admixture is written as:

\[
\frac{\partial}{\partial t} p_\alpha + m_\alpha c^2 \nabla \gamma_\alpha = e_\alpha \left[ -\nabla \phi - \frac{1}{c} \frac{\partial A_\perp}{\partial t} \right] \tag{3}
\]
where
\[ P_\alpha = m_\alpha \gamma_\alpha V_\alpha \]

and
\[ \gamma_\alpha = \left( 1 - \frac{v^2_\alpha}{c^2} \right)^{-1/2} = \left( 1 + \frac{P^2_\alpha}{m^2_\alpha c^2} \right)^{1/2} \]

where, \( m_\alpha \) is the rest mass of the particle \( \alpha \).

The continuity equation for the particle \( \alpha \) is:
\[ \frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha V_\alpha) = 0 \quad (4) \]

We are looking for localized one-dimensional solution of this system of equation for a circularly polarized EM wave, where the vector potential \( A \) can be expressed as:
\[ A = \frac{1}{2} (\hat{x} + i\hat{y}) A(\zeta) e^{i(kz - \omega t)} + c.c. \]

here, \( \zeta = z - Vt \).

Analyzing the equation of motion and the wave equations into longitudinal (parallel to the direction of the wave propagation) and a transverse (perpendicular plan) parts, then the transverse part of the equation of motion is immediately integrated to give:
\[ P_{\alpha \perp} = -\frac{e_\alpha}{c} A_\perp \quad (5) \]

where the constant of integration is set equal to zero, since the particles were assumed to be immobile at infinity where the field is zero.

Meanwhile, the longitudinal part of the equation of motion takes the following form:
\[ \frac{\partial P_{\alpha z}}{\partial t} + m_\alpha c^2 \frac{\partial}{\partial z} \left( 1 + \frac{P^2_{\alpha \perp} + P^2_{\alpha z}}{m^2_\alpha c^2} \right)^{1/2} = -\frac{e_\alpha}{c} \frac{\partial \phi}{\partial z} \quad (6) \]

And, accordingly, equations (5), and (6) may be rewritten as:
\[ \left( \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) A_\perp = -\frac{4\pi e}{c} (n_p V_p + n_i V_i - n_e V_e) \]

\[ \frac{\partial^2 \phi}{\partial z^2} = -4\pi e (\delta n_p + \delta n_i - \delta n_e) \quad (7) \]

Now, we consider only the electron; but everything stated below applies equally well, of course, to positron and ion.

We have:
\[ \frac{\partial}{\partial t} = \frac{\partial}{\partial \zeta} \frac{\partial \zeta}{\partial t} = -v_\alpha \frac{\partial}{\partial \zeta} \]

where
and also
\[ \frac{\partial}{\partial z} = \frac{\partial}{\partial \zeta} \frac{\partial \zeta}{\partial z} = \frac{\partial}{\partial \zeta} \]
then, equation (6) reduced to
\[ -v_g \frac{\partial}{\partial \zeta} P_{ez} + mc^2 \frac{\partial}{\partial \zeta} \sqrt{1 + \frac{P_{ez}^2 + P_{e\perp}^2}{m^2c^2}} = e \frac{\partial \phi}{\partial \zeta} \]

Now, we can integrate both sides with respect to \( \zeta \), yield:
\[ -v_g P_{ez} + mc^2 \sqrt{1 + \frac{P_{ez}^2 + P_{e\perp}^2}{m^2c^2}} = e\phi + C \]
where, \( C \) is the constant of integration, which could be determine from the boundary conditions. Here, \( P_{e\perp}, P_{ez} \to 0 \) and \( \phi \to 0 \) when \( \zeta \to 0 \), we get: \( C = mc^2 \), and accordingly:
\[ -\left( \frac{v_g}{c} \right) \left( \frac{P_{ez}}{mc} \right) + \sqrt{1 + \frac{P_{ez}^2}{m^2c^2} + \frac{P_{e\perp}^2}{m^2c^2}} = \frac{e\phi}{mc^2} + 1 \]  
(9)

Let:
\[ \Gamma_{az} = \frac{P_{az}}{mc^2} \]
\[ U_g = \frac{v_g}{c} \]
\[ \Psi_\alpha = -\frac{P_{e\perp}}{m_\alpha c^2} \]
\[ \Phi_\alpha = -\frac{e_\alpha \phi}{m_\alpha c^2} \]
but for simplicity we will drop here the subscript \( e \); i.e. \( \Phi_e = \Phi \) and \( \Psi_e = \Psi \)

Equation (9) becomes
\[ -U_g \Gamma_{ez} + \sqrt{1 + \Gamma_{ez}^2 + \Psi^2} = \Phi + 1 \]

We note here that the began value of \( \Gamma_{ez} \) is considered to be zero, then the above equation yields:
\[ \Gamma_{ez} = U_g (\Phi + 1) \gamma_g^2 \]
\[ \gamma_g^2 \sqrt{(\Phi + 1)^2 - \frac{1 + \Psi^2}{\gamma_g^2}} \]  
(10)

where \( \gamma_g = \frac{1}{\sqrt{1-U_g^2}} \).
If we return back to the continuity equation; we get:

\[-v_g \frac{\partial n_e}{\partial \zeta} + n_e \frac{\partial v_{ez}}{\partial \zeta} = 0\]

by integrating:

\[-v_g n_e + n_e v_{ez} = C\]

here, \(v_{ez} \rightarrow 0\) when \(n_e \rightarrow n_{oe}\), then, \(C = -v_g n_{oe}\), and accordingly:

\[n_e = \frac{U_g n_{oe} \sqrt{1 + \Gamma_{ez} + \Psi^2}}{U_g \sqrt{1 + \Gamma_{ez} + \Psi^2} - \Gamma_{ez}}\]

but; \(\delta n_e = n_e - n_{oe}\), hence

\[\delta n_e = \frac{n_{oe} \Gamma_{ez}}{U_g \sqrt{1 + \Gamma_{ez} + \Psi^2} - \Gamma_{ez}}\] (11)

From equations (10) and (11), we get:

\[\delta n_e = n_{oe} \gamma_g^2 \left( \frac{U_g (1 + \Phi)}{\sqrt{(1 + \Phi)^2 - \frac{1+\Psi^2}{\gamma_g^2}}} - 1 \right)\] (12)

here, we should note that:

\[\Phi_e = \Phi\]
\[\Phi_p = -\Phi\]
\[\Phi_i = -\frac{m}{M} \Phi\]

where \(M\) is the mass of an ion. and that:

\[\Psi_e = \Psi\]
\[\Psi_p = -\Psi\]
\[\Psi_i = -\frac{m}{M} \Psi\]

By follows similar steps; for positron and ion; we can get the following:

\[\delta n_p = n_{op} \gamma_g^2 \left( \frac{U_g (1 - \Phi)}{\sqrt{(1 - \Phi)^2 - \frac{1+\Psi^2}{\gamma_g^2}}} - 1 \right)\] (13)

and

\[\delta n_i = n_{oi} \gamma_g^2 \left( \frac{U_g (1 - \frac{m}{M} \Phi)}{\sqrt{(1 - \frac{m}{M} \Phi)^2 - \frac{1+\Psi^2}{\gamma_g^2}}} - 1 \right)\] (14)

6
The equation (8) can be rewritten as:

\[ \frac{\partial^2}{\partial z^2} \Phi = \frac{4\pi e^2}{mc^2} (\delta n_e - \delta n_p - \delta n_i) \]  

(15)

Substituting equations (12), (13) and (14) in equation (15):

\[ \frac{\partial^2}{\partial z^2} \Phi = \left( \frac{\omega^2}{c^2} \right) \gamma_g^2 U_g \left[ \frac{U_g(1 + \Phi)}{\sqrt{(1 + \Phi)^2 - \frac{1 + \Psi^2}{\gamma_g^2}}} \right] \]

\[ - \epsilon_1 \left( \frac{U_g(1 - \Phi)}{\sqrt{(1 - \Phi)^2 - \frac{1 + \Psi^2}{\gamma_g^2}}} \right) \]

\[ - \epsilon_2 \left( \frac{U_g(1 - \frac{m}{M} \Phi)}{\sqrt{(1 - \frac{m}{M} \Phi)^2 - \frac{1 + \frac{m}{M} \Psi^2}{\gamma_g^2}}} \right) \]

\[ - 1 - \epsilon_1 - \epsilon_2 \]

But \( 1 - \epsilon_1 - \epsilon_2 = 1 - \frac{n_{op}}{n_{ac}} - \frac{n_{oe}}{n_{ac}} = \frac{n_{ac} - n_{op} - n_{oi}}{n_{ac}} \) and \( n_{ac} = n_{op} + n_{oi} \); we get, 

\( 1 - \epsilon_1 - \epsilon_2 = 0 \), hence:

\[ \frac{\partial^2}{\partial z^2} \Phi = \left( \frac{\omega^2}{c^2} \right) \gamma_g^2 U_g \left[ \frac{U_g(1 + \Phi)}{\sqrt{(1 + \Phi)^2 - \frac{1 + \Psi^2}{\gamma_g^2}}} \right] \]

\[ - \epsilon_1 \left( \frac{U_g(1 - \Phi)}{\sqrt{(1 - \Phi)^2 - \frac{1 + \Psi^2}{\gamma_g^2}}} \right) \]

\[ - \epsilon_2 \left( \frac{U_g(1 - \frac{m}{M} \Phi)}{\sqrt{(1 - \frac{m}{M} \Phi)^2 - \frac{1 + \frac{m}{M} \Psi^2}{\gamma_g^2}}} \right) \]

\[ - 1 - \epsilon_1 - \epsilon_2 \]

If we let \( \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial \eta^2} = \frac{\omega^2}{c^2} \frac{\partial^2}{\partial \eta^2} \), we can write:

\[ \frac{\partial^2 \Phi}{\partial \eta^2} = \gamma_g^2 U_g \left[ \frac{(1 + \Phi)}{\sqrt{(1 + \Phi)^2 - \frac{1 + \Psi^2}{\gamma_g^2}}} \right] \]

\[ - \epsilon_1 \left( \frac{1 - \Phi}{\sqrt{(1 - \Phi)^2 - \frac{1 + \Psi^2}{\gamma_g^2}}} \right) \]

\[ - \epsilon_2 \left( \frac{1 - \frac{m}{M} \Phi}{\sqrt{(1 - \frac{m}{M} \Phi)^2 - \frac{1 + \frac{m}{M} \Psi^2}{\gamma_g^2}}} \right) \]

(16)
Equation (7) can be written as:

$$\left( \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) A_\perp =$$

$$\frac{4\pi e^2}{mc^2} \left( \frac{n_e}{\gamma_e} + \frac{n_p}{\gamma_p} + \frac{mn_i}{M\gamma_i} \right) A_\perp$$

By following similar steps; the last equation can be written as:

$$\frac{\partial^2 \Psi}{\partial \eta^2} = U_g \gamma_g^2 \Psi \left( \frac{1}{(1 + \Phi)^2 - \frac{1+\Psi^2}{\gamma_g^2}} \right)$$

$$+ \frac{\epsilon_1}{\sqrt{(1 - \Phi)^2 - \frac{1+\Psi^2}{\gamma_g^2}}}$$

$$+ \frac{(m/M)\epsilon_2}{\sqrt{(1 - \frac{m}{M}\Phi)^2 - \frac{1+\Psi^2}{\gamma_g^2}}}$$

(17)

Equation (16) and equation (17) are coupled together and forming a system of nonlinear second order differential equations. It is rather difficult to obtain an analytical solution of them. However, in the following, we present numerical solutions of equation (16) and equation (17). Taking some typical parameters:

$$\frac{1}{\gamma_g} = 0.12$$

$$\frac{m}{M} = \frac{1}{1834}$$

$$U_g = \sqrt{1 - 0.12}$$

and by using Taylor method, we have obtained the wakefield profiles at different values of $\epsilon_1$ and $\epsilon_2$. The results are displayed in the following figures:
Fig1-a: The wakefield profiles for $\Psi$ at $\epsilon_1 = 0.3, \epsilon_2 = 0.7$

Fig1-b: The wakefield profiles for $\Phi$ at $\epsilon_1 = 0.3, \epsilon_2 = 0.7$
Fig2-a: The wakefield profiles for $\Phi$ at $\epsilon_1 = 0.4$, $\epsilon_2 = 0.6$

Fig2-b: The wakefield profiles for $\Phi$ at $\epsilon_1 = 0.4$, $\epsilon_2 = 0.6$
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