Coherence as witness for quantumness of gravity

Ahana Ghoshal, Arun Kumar Pati, Ujjwal Sen
Harish-Chandra Research Institute, HBNI, Chhatnag Road, Jhunsi, Allahabad 211 019, India

We propose an interferometric set-up that utilizes the concept of quantum coherence to provide quantum signatures of gravity. The gravitational force comes into nontrivial play due to the existence of an extra mass in the set-up that transforms an incoherent state to a coherent state. The implication uses the fact that quantum coherence at a certain site cannot be altered by local actions at a separate site. The ability to transform an incoherent state to a coherent one in the presence of gravitational field provides a signature of quantumness of gravity.

I. INTRODUCTION

Even though quantum theory has been successfully formalized for strong, weak, and electromagnetic fields, there has been a long issue about how to unify quantum theory with gravity. There are many controversies and curiosities that surround the question of quantum aspects of gravity [1–3]. In this respect, one remembers the experiment in 1975 by R. Colella, A. W. Overhauser and S. A. Werner, with a neutron beam, split by an interferometer [4]. It was an interference experiment where the phase of the neutron wave function was changed by the gravitational potential. For further experiments in this direction, see [5–7]. It has been argued that gravity is purely classical in their experiments. For discussions on quantum aspects of gravity and possible ramifications, see [8–10].

The ubiquitous feature of quantum theory is superposition and entanglement. Entanglement is a phenomenon that is observed in systems of two or more parts, and is a clear signature of the quantum nature of the system [11]. In this respect, it has been shown that “classically mediated gravitational interaction between two gravitationally coupled resonators cannot create entanglement” [12]. Models which support the relativistic semiclassical theory of gravity were given in Ref. [13]. (See also [14].) One of the central dogmas of entanglement theory is that local quantum operations and classical communication (LOCC) cannot create entanglement [11]. If any state is unentangled initially, then LOCC can create only a separable state out of it. So, if any unentangled state results in an entangled state after the action of any field, then the field is definitely a quantum entity. Refs. [15, 16] proposed a thought experiment, which have been called the BMV effect, in which two massive particles were sent through two interferometers. At the end of the interferometry, they found that the phase of the neutron wave function was changed by the gravitational potential. From this result, they concluded that the gravitational interaction between the two particles must be of quantum nature, as it can create entanglement. A simple analysis of these aspects is found in Ref. [17]. See also [18].

Just like entanglement, quantum coherence is also an exclusively quantum phenomenon. Both quantum coherence and quantum entanglement arise from superposition. See Ref. [19, 20] for a formal definition and further ramifications of the concepts around quantum coherence. This should not be confused with the “coherent states” in quantum optical systems [21].

In this paper, we show that gravitational interaction between two particles can create a quantum coherent state with respect to some basis, while in absence of one of the particles, there occurs an incoherent state with respect to the same basis. We quantify the amount of coherence created by using two distance-based measures of the same. Thus, we argue that the ability to transform an incoherent state into a coherent one in the presence of a gravitational field is a signature of quantumness of gravity.

II. A MASSIVE PARTICLE SENT THROUGH A BEAM SPLITTER

In this section, we consider gravitational interaction acting between two components of the same particle. Consider a mass $M$ moving through a beam splitter, by which the particle is split in two spatially separated components $|L\rangle$ and $|R\rangle$ as shown in Fig. 1. The distance of the centres of the two components is $D$ and $|L\rangle$ and $|R\rangle$ are orthogonal states i.e.,
We can assume that each part is a localized Gaussian wave packet with width, $\Delta x \ll D$, as in [15]. So, we can assume that the entire system is in the state,

$$\langle L | R \rangle = 0.$$  

Therefore, energy of each component will be the gravitational potential energy each component of the particle due to the other. The energy where

$$E_L = E_R = E = \frac{GM^2}{D}.$$  

Since physical spacetime geometry can be in superposition of macroscopically distinct configurations [17], at time $t = \tau$, the state will be

$$\langle \psi(t = \tau) \rangle = \frac{1}{\sqrt{2}}(e^{-i\frac{E_L}{\hbar}} | L \rangle | g_L \rangle + e^{-i\frac{E_R}{\hbar}} | R \rangle | g_R \rangle).$$  

As the masses of the two components of the particle are considered to be same, we have $|g_L \rangle = |g_R \rangle$. The state at time $t = \tau$ is

$$\langle \psi(t = \tau) \rangle = \frac{1}{\sqrt{2}}(\langle L \rangle | g_L \rangle + | R \rangle | g_R \rangle),$$  

where we can now neglect the overall phase. The two components of the particle are brought back together at $B_2$. Hence, the wave function, then, is,

$$\langle \psi(t = \tau) \rangle = \frac{1}{\sqrt{2}}(\langle L \rangle | g_L \rangle + | R \rangle | g_R \rangle).$$  

If the gravitational field is traced out, the particle state at time $t = \tau$ will be

$$\langle \psi_1 \rangle = \frac{1}{\sqrt{2}}(\langle L \rangle + | R \rangle).$$  

So, the corresponding density matrix is

$$\rho_1 = \langle \psi_1 \rangle \langle \psi_1 \rangle^\dagger = \frac{1}{2}(\langle L \rangle + | R \rangle)(\langle L \rangle + | R \rangle).$$  

Now, we will compute the coherence of this state with respect to the basis $\{ \langle \pm | R \rangle \sqrt{2} \}$. Here, we will consider two measures of coherence defined in Ref. [20].

Quantum coherence of the state of a quantum system is the existence of off-diagonal terms in the density matrix of the state. An understanding of its presence was known since the beginnings of quantum mechanics, and was known to the reason for several phenomena including interference. However, the modern theory of quantum coherence is relatively new [19, 20], and has also partially fed the interesting stream of research on resource theories. It is clear that whether a quantum state possesses coherence depends on the choice of basis.

There are many ways in which one can quantify coherence, and we choose two such instances.

- **Relative entropy of coherence**: Let $\hat{\rho}$ be a density matrix written in some basis. The relative entropy of coherence of $\hat{\rho}$ in that basis is

$$C_{\text{rel. ent.}}(\hat{\rho}) = S(\hat{\rho}_{\text{diag}}) - S(\hat{\rho}),$$  

where $S(\cdot)$ is the von Neumann entropy of its argument and $\hat{\rho}_{\text{diag}}$ is the state obtained from $\hat{\rho}$ by removing the off-diagonal elements.

- **$l_1$-norm coherence**: The $l_1$-norm coherence of $\hat{\rho}$ in a given basis is defined as

$$C_{l_1}(\hat{\rho}) = \sum_{i \neq j} | \langle \rho_{i,j} \rangle |,$$  

i.e., it is the sum of the moduli of all nonzero non-diagonal elements of $\hat{\rho}$, where $\hat{\rho}$ is expressed in the given basis. In our case, $\rho_1$ is diagonal in the basis $\{ \langle \pm | R \rangle \sqrt{2} \}$. So, $C_{l_1}(\rho_1) = 0$.

We therefore see that both the measures of coherence indicate that we have obtained an incoherent state with respect to the basis $\{ \langle \pm | R \rangle \sqrt{2} \}$. We will next see what happens if an extra mass is present in this above experiment.

### III. A MASSIVE PARTICLE SENT THROUGH A BEAM SPLITTER WITH AN EXTRA MASS RUNNING PARALLEL TO THE SPLIT PARTICLE

In this section, we consider a set-up which has all the components of the preceding one, but has an additional feature. In the current set-up, a particle of mass $m$ moves in parallel to the split components of the particle of mass $M$. The experimental set-up is described in Fig. 2. The initial state of the entire system is

$$\langle \psi(t = 0) \rangle = | m \rangle \otimes \frac{1}{\sqrt{2}}(\langle L \rangle + | R \rangle) \otimes | \bar{g} \rangle.$$  

Here, $| \bar{g} \rangle$ is the quantum state of the gravitational field due to the mass $m$ on the components of mass $M$. As the distances of the two components ($L$ and $R$) of mass $M$ are different from the mass $m$, carried by $AC$, the quantum states of the gravitational field are different in the $L$ and $R$ channels. Now, the gravitational potential energies on each component of $M$, due to the existence of the extra mass, are

$$E_L = \frac{GMm}{d} \quad \text{and} \quad E_R = \frac{GMm}{d+D}.$$  

Here, we have ignored the gravitational potential energy on one component of the particle due to the other. As in the previous case, it will introduce an overall phase, which we can neglect. Since the metric in different branches of the interferometer represent distinct spacetimes, at time $t = \tau$, the state will be

$$\langle \psi(t = \tau) \rangle = \frac{1}{\sqrt{2}}(e^{-i\phi_L}| m \rangle | L \rangle | \bar{g}_L \rangle + e^{-i\phi_R}| m \rangle | R \rangle | \bar{g}_R \rangle),$$  

Where

$$\phi_L = k_{\perp}A_L \text{ and } \phi_R = k_{\perp}A_R,$$  

and $A_L$ and $A_R$ are the diameters of the two components.


As in the previous case, we trace out the gravitational field, to then, tracing out the degrees of freedom of the particle carried by the channel \( AC \), we get the state of the particle of mass \( M \), as

\[
|\tilde{\psi}_2\rangle = \frac{1}{\sqrt{2}} (|\psi_2\rangle + e^{i\Delta \phi} |R\rangle).
\]

The four elements of the density matrix are

\[
\rho_{11} = \cos^2\left(\frac{\Delta \phi}{2}\right),
\]

\[
\rho_{12} = \frac{i}{2} \sin(\Delta \phi),
\]

\[
\rho_{21} = -\frac{i}{2} \sin(\Delta \phi),
\]

\[
\rho_{22} = \sin^2\left(\frac{\Delta \phi}{2}\right).
\]

So, \( \tilde{\rho}_2 \) is a non-diagonal matrix in the \( \{ \frac{(|L\rangle \pm |R\rangle)}{\sqrt{2}} \} \) basis. Hence, \( \tilde{\rho}_2 \) is coherent in this basis.

- **Relative entropy of coherence**: The relative entropy of coherence of \( \tilde{\rho}_2 \) is

\[
C_{\text{rel. ent.}}(\tilde{\rho}_2) = S(\tilde{\rho}_{2\text{dia}}) - S(\tilde{\rho}_2) = -(1 + \cos(\Delta \phi)) \log_2 \left[ \cos\left(\frac{\Delta \phi}{2}\right) \right] - (1 - \cos(\Delta \phi)) \log_2 \left[ \sin\left(\frac{\Delta \phi}{2}\right) \right].
\]

- **l_1-norm coherence**: The \( l_1 \)-norm coherence of \( \tilde{\rho}_2 \) is

\[
C_{l_1}(\tilde{\rho}_2) = |\frac{i}{2} \sin(\Delta \phi)| + | - \frac{i}{2} \sin(\Delta \phi)| = \sin(\Delta \phi).
\]

This can be re-written as

\[
\Delta \phi = \frac{\alpha M m}{m_P^2} \pi/2,
\]

where \( \alpha = (\tau D c)/(d(d + D)) \) is a dimensionless parameter and \( m_P \) is the Planck mass given by \( m_P^2 = \hbar c/G \). With suitable choice of the parameters, it is possible to make \( \Delta \phi = \pi/2 \), whereby we have a maximal coherence of unity in the output.

The following comment is in order here. Quantum coherence is basis-dependent quantity. Therefore, the numbers obtained are altered if we change the basis. In particular, if we choose the basis \( \{ |L\rangle, |R\rangle \} \), then the outputs in both the situations considered (i.e., in Figs. 1 and 2) will have nonzero quantum coherence. However, the particular basis that we have chosen provides a stark zero versus nonzero contrast between the two situations.
IV. CONCLUSION

In this paper, we have seen that the presence of the gravitational force due to a mass in the neighborhood of a particle split by a beam splitter can create a coherent state with respect to some basis, while the absence of the mass will give an incoherent state with respect to the same basis. It is to be noted that a local action at a certain site cannot create coherence at a separated site. Therefore, the increase of coherence must have been created due to the gravitational interaction between the two particles. Our result is based on two assumptions, viz. (i) in the perturbative regime, Newtonian theory holds, and (ii) superposition of distinct spacetimes is possible. Since controlling entanglement is a difficult task, we hope that our scheme can be tested with relative ease, in comparison to those in Refs. [15, 16].

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