Analysis of Breaking Mode of FRP Reinforced Concrete Beam and Calculation of Bearing Capacity

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Abstract: The application of FRP (Fiber Reinforced Polymer) in modern civil engineering is one of the new ways to solve the problem of steel corrosion. Because the stress-strain relationship of FRP bars is characterized by linear elastic characteristics, the structural properties of FRP reinforced concrete members are quite different from those of ordinary reinforced concrete members, and further research is needed. Based on the results of experiments and finite element analysis, this paper uses modern reinforced concrete structural analysis methods, combined with China's current norms, to comprehensively analyse the FRP reinforced concrete beam's mechanical characteristics, failure modes, fracture morphology and deformation characteristics, etc. The calculation method of the bearing capacity of the normal section of the curved member was systematically studied, and the calculation and design method for the characteristics of the FRP rib was established.

1. Instruction
In recent years, Japan, Germany, the United States, Canada, Switzerland and other countries have carried out extensive, in-depth, systematic theoretical and experimental research on the application technology of FRP bars in civil engineering. Some countries have issued corresponding FRP reinforced concrete members design codes. However, China's research on FRP reinforced concrete members[1-4] started relatively late. At present, there is no systematic specification for FRP reinforced concrete members design, which greatly affects the promotion and application of FRP ribs in China.

Based on the experimental and finite element analysis results and modern reinforced concrete structural design theory, combined with the current Chinese norms and FRP reinforced concrete beam deformation and failure characteristics, the calculation method of the normal section bearing capacity of FRP reinforced concrete flexural members is studied, and establishes a calculation and design method suitable for practical engineering.

2. FRP reinforced concrete beam breaking mode
FRP reinforced concrete beams have linear changes due to the stress-strain relationship of FRP bars. It is not possible to simply model the three types of failures: reinforced concrete, super-gluten and less reinforced. From the above experimental research, it is found that there are two types of normal cross-section failure modes that are most likely to occur:

(1) Before the concrete crushing damage, the FRP rib is pulled beyond the allowable tensile strain and reaches the ultimate tensile strain, which is called the failure mode I. This failure mode is very dangerous for the FRP reinforced concrete structure. This approach should be avoided.

(2) Before the FRP reaches the allowable tensile strain, the concrete in the nip is crushed first, that
is, the FRP fails to reach the allowable tensile strain or is broken when the beam is broken, which is called the failure mode II. Destruction Mode II is also a kind of brittle failure, but it is relatively safe compared to Mode I. Moreover, structural safety can be effectively improved by arranging steel bars in the concrete compression zone or by incorporating an appropriate amount of steel fibers. Of course, the development of a HFRP rib with excellent ductility is the most effective way to solve the problem of brittle failure.

3. Calculation of Bearing Capacity

3.1 Failure Mode I Calculation of Bearing Capacity (Tensile Failure)

When $\rho_f = \frac{A_f}{bh_0} \leq \rho_{fh}$, the tensioned FRP ribs first undergo tensile damage. In this paper, only the beam bearing capacity of concrete strength grade less than or equal to C50 is deduced. For high-strength concrete, a similar method can be used to derive. It is assumed from the flat section that the FRP rib has a section curvature of $\phi_u = \frac{\epsilon_{fu} (h_0 - x_c)}{c_{ux}}$ when it is subjected to tensile failure and a compressive strain of $\epsilon_{c} = y \cdot \phi_u$ from the neutral axis $y$, thereby further introducing

$$ y = \epsilon_{c} (h_0 - x_c) / \epsilon_{fu} $$

(1)

$$ dy = \frac{h_0 - x_c}{\epsilon_{fu}} d\epsilon_{c} $$

(2)

Assuming that the FRP rib is subjected to tensile failure, the compressive strain at the edge of the concrete is $\epsilon_{c}^{'}$, and the resultant compressive stress of the concrete in the compression zone is:

$$ C_c = \int_{0}^{\epsilon_{c}^{'}} \sigma_c (\epsilon_c) bdy = \frac{b(h_0 - x_c)}{\epsilon_{fu}} \int_{0}^{\epsilon_{c}^{'}} \sigma_c (\epsilon_c) d\epsilon_c $$

(3)

The distance between the resultant force $C_c$ and the neutral axis:

$$ y_{c} = \frac{\int_{0}^{\epsilon_{c}^{'}} \sigma_c (\epsilon_c) bdy}{C_c} = \frac{\int_{0}^{\epsilon_{c}^{'}} \sigma_c (\epsilon_c) dy}{\epsilon_{fu}} \frac{h_0 - x_c}{\epsilon_{fu}} \int_{0}^{\epsilon_{c}^{'}} \sigma_c (\epsilon_c) d\epsilon_c $$

(4)

$\epsilon_{fu}$ is the ultimate tensile strain of the FRP rib. Generally speaking, under the premise that the FRP reinforced concrete beam meets the minimum reinforcement ratio, under the limit of bearing capacity, the compressive strain of the concrete under compression has exceeded its peak compressive strain. According to the "Concrete Structure Design Code" (GB50010-2002), Section 7.1.2 gives the following formula for the compressive stress-strain relationship of concrete:

$$ \epsilon_{c} \leq \epsilon_{0}, \sigma_c = f_c \left[1 - \left(\frac{\epsilon_{c}}{0.002}\right)^2\right] $$

$$ \epsilon_{0} \leq \epsilon_{c} \leq \epsilon_{cu}, \sigma_c = f_c $$

(5)

Bring equation (5) into equation (3), (4) to be available:

$$ C_c = \frac{b(h_0 - x_c)}{\epsilon_{fu}} f_c \left(\epsilon_{c}^{'} - \frac{2}{3000}\right) $$

(6)

$$ y_{c} = \frac{(0.5\epsilon_{c}^{e^2} - 7.34 \times 10^{-7})}{(\epsilon_{c}^{'} - 6.67 \times 10^{-4})} \frac{h_0 - x_c}{\epsilon_{fu}} $$

(7)
According to the balance condition of the force on the section, the tensile force of the FRP rib $T_f = C_f$. Bring equation (6)–(8) into the available:

$$\frac{b(h_0-x_c)}{\varepsilon_{fu}} f_c (\varepsilon'_{c} - \frac{2}{3000}) - f_{fu} A_f = 0$$

By the moment balance condition:

$$M_u = f_{fu} A_f (h_0 + y_c - x_c)$$
$$\phi_u = \varepsilon_{fu} \left( h_0 - x_c \right)$$

### 3.2 Failure Mode II Calculation of Bearing Capacity (Compressive Failure)

When $\rho_f = \frac{A_f}{b h_0} > \rho_{fb}$, the concrete at the edge of the compression zone is destroyed first, and the FRP rib has not yet reached the tensile strength. Similar to reinforced concrete beams, the equivalent rectangular stress map method is used according to the flat section assumption. Equivalent principle: the equivalent stress and force position of the concrete compression zone after the equivalent, as shown in Figure 1.:

![Figure 1. Failure mode II calculation diagram](image_url)

From the equilibrium condition of the force on the section,

$$\alpha_f f_c b x - A_f E_f \varepsilon_f = 0$$

$$\varepsilon_f = \varepsilon_{cu} \left( h_0 - x_c \right) / x_c$$
$$x = \beta \cdot x_c$$

By the moment balance condition:

$$M_u = \varepsilon_f E_f A_f \left( h_0 - \frac{x}{2} \right)$$
$$\phi_u = \varepsilon_{cu} / x_c$$
In the formula, $\alpha$ and $\beta$ are the same as ordinary reinforced concrete flexural members.

As mentioned above, considering that the failure of FRP reinforced concrete beams is brittle failure, the authors have improved the ductility of the members by configuring compression bars in the compression zone during the test. For the double-ribbed rectangular section, the following bearing capacity formula is derived according to the aforementioned assumptions:

1) Destruction mode I

From the equilibrium condition of the force on the section,

$$\frac{b(h_0 - x_c)}{E_{fu}} f_c(\varepsilon' - \frac{2}{3000}) + E_s \varepsilon_s' A_s - f_{fu} A_f = 0$$  

By the moment balance condition:

$$M_u = f_{fu} A_f \left(h_0 + y_c - x_c\right) + E_s \varepsilon_s' A_s \left(x_c - y_c - \varepsilon_s'\right)$$

2) Destruction mode II

From the equilibrium condition of the force on the section,

$$\alpha f_c b x + E_s \varepsilon_s' A_s - E_f \varepsilon_f A_f = 0$$

By the moment balance condition:

$$M_u = E_f \varepsilon_f A_f \left(h_0 - \frac{x}{2}\right) + E_s \varepsilon_s'A_s' \left(x - \varepsilon_s'\right)$$

or

$$M_u = \alpha f_c b x \left(h_0 - \frac{x}{2}\right) + E_s \varepsilon_s'A_s' \left(h_0 - \varepsilon_s'\right)$$

It should be noted that for the above formulas, when the strain $\varepsilon'$ of the pressed steel bar reaches $f_y / E_s$, the stress of the pressed steel bar reaches the yield point, and thereafter the stress of the pressed steel bar takes the yield strength $f_y'$.

In the engineering, when the failure mode I design is adopted, the bearing capacity of the FRP reinforced concrete beam mainly depends on the tensile strength of the FRP rib. In order to improve the safety reserve, the tensile strength of the FRP bars can be nominal. Generally speaking, the nominal yield strength is taken as 70%-80% of the ultimate strength, 80% is taken here, and 70% is recommended in engineering design. In the case of compression failure, the bearing capacity of the beam is mainly determined by the compressive strength and ultimate compressive strain of the concrete under the condition of constant reinforcement ratio. When the test results are predicted by the above formula, the compressive strength According to China's concrete structure design specifications, $f_{cu} = 3.1430 \times 10^3 - (f_{cu,k} - 50) \times 10^5 \leq 0.0033$. When designed according to compressive failure, in order to ensure that members are subjected to compressive failure, the reinforcement ratio should still meet the requirement of $\rho_f \geq \alpha \rho_{fb}$, and the coefficient $\alpha$ is greater than 1. Refer to ACI-440[5] code of the United States for 1.4. The calculated value is in good
agreement with the experimental value.

4. Conclusions

(1) There are two failure modes when FRP reinforced concrete beams are damaged in normal section: failure mode I (pull failure) and failure mode II (pressure failure). Both failure modes are brittle failure. From the perspective of safety, the failure mode II should be used as much as possible. In order to improve the ductility of the component, an appropriate amount of steel bars can be placed in the compression zone of the concrete beam, and a double rib design is adopted.

(2) The boundary reinforcement ratio $\rho_{\text{fb}}$ derived from the flat section assumption can be used in engineering design, but considering the brittleness of FRP bars and the difference between the bond properties of FRP bars and concrete and steel bars, and referring to ACI 440 code of the United States, nominal yield strength is recommended for the tensile strength of FRP bars. At the same time, when designing according to failure mode II, $\rho_{\text{f}} \geq 1.4 \rho_{\text{fb}}$ should be made to ensure that the components are damaged by pressure.

(3) According to the test results and related specifications, the normal section bearing capacity of FRP bars is deduced, which is in good agreement with the test results.

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