On the problem and algorithms for coordinated optimal management of production and material flows of an enterprise

Y A Mezentsev, N V Baranova and P S Pavlov
Novosibirsk State Technical University, K. Marx Ave. 20, Novosibirsk, Russia

E-mail: baranova.nina.vl@gmail.com

Abstract. The paper presents the problem of controlling the input and output material flows of an industrial enterprise, supplemented by the condition for the choice of sales prices and adjusted in terms of the number of products sold. This economic and mathematical model finds optimal solutions for the management problem at enterprises, namely, the problem of forming a production program according to the criterion of the maximum net profit at the end of the planning period. In the constraints of the model, both production components and constraints on resources and the logic of input and output material flows are systematically taken into account. The considered model and the given control problems are investigated using a unified approach that allows working with logical conditions of any complexity and setting the corresponding formal optimization problems. The results of testing the algorithm on test data close to industrial (real) dimensions are also given.

1. Introduction
As part of the work, a model and an algorithm for finding optimal solutions have been developed, which are based on methods of mathematical programming. Works related to this topic, for the most part, describe various problems: placement problems, supplier selection problems [1], job assignment, inventory management, supply chain management, logistics [2, 3], production [4]. In this study, a comprehensive approach is used to optimize the management of the production assortment and material flows of an industrial enterprise [5].

The rationality of the assortment structure from the point of view of its economic profitability of the planned sales volumes should be assessed even at the stage of its formation. Here it must be said that as such, a general methodology for finding the optimal assortment of a manufacturing enterprise has not yet been developed. By analyzing various sources, the authors have compiled a number of approaches to determining and optimizing the range of products sold. At industrial enterprises or researchers, only a small number of calculation approaches are created and applied, depending on the goals of the tasks and for each individual situation. This suggests that such specific calculation systems are irrational for widespread use.

At the moment, optimization problems are widely used in various areas of production [6]. For example, the article [7] describes the process of finding a solution to the problem of multipurpose optimization of material movement in a logistics network by using a control system based on fuzzy logic, as well as an annealing simulation algorithm and a genetic algorithm.
In articles [8,9] the problem of finding a production plan is solved taking into account the satisfaction of demand.

A wide range of ready-made management solutions for the production processes of the enterprise was created by researchers from the KNRTU-KAI named after A.N. Tupolev [10]. But in this control system, in terms of forming a production program, scenario modeling is used, and not a discrete optimization toolkit.

A group of scientists from Youngstown State University in the USA has been working on the problem of multipurpose optimization of the selection of suppliers and the distribution of orders between them for a long time [11,12]. In their works, they consider one of the alternative decision support systems with several criteria - visual interactive goal programming.

Also, the problem of choosing suppliers is dealt with by the authors of [13]. To solve the problem, they use a Least square-support vector machine in conjunction with continuous general search with alternating neighborhoods.

The existing methods for solving this kind of problems for the most part do not stand up to criticism. Most often, a choice is proposed by means of scoring of suppliers, consumers, quality of service and (or) consistent solution of logistics problems (transport, financial, warehouse) [14]. In the general case, it is impossible to effectively solve the problems considered below using classical models: for determining transportation plans (transport problem), for choosing routes (traveling salesman problem), for selecting suppliers (assignment problem), for optimal inventory level management (Wilson inventory control), etc. [14,15]. The noted approach is also typical of modern works that claim to be systematic [15 - 16].

At the same time, logistics problems do not have the highest estimates of complexity in comparison with other discrete problems of control of production processes [5]. The consequence is a high economic efficiency of using EMM and a relatively low estimate of the structural complexity of the models used, which is primarily characterized by belonging to the NP class and the number of integer variables in control problems.

2. Substantive and formal statement of the problem
It is necessary to provide such a procurement strategy (selection of suppliers, supply volumes taking into account discounts), as well as a pricing policy for sales by consumer groups that maximize the criterion indicator (net income, or the amount of working capital at the end of the planning period), taking into account the restrictions on working capital at the beginning period and storage capacity. The term "procurement strategy" refers to the totality of the planned volumes of purchases of goods for the entire range, selectable prices and discounts from all potential suppliers, determined for each allocated time interval of the planning period. The term "sales strategy" will mean the totality of the planned sales of goods across the entire range for all groups of consumers, determined on the basis of demand for each allocated time interval of the planning period.

It should also be noted here that the duration of the production cycle for the case under consideration is much less than any interval of the planning period.

A more detailed meaningful statement of the problem can be seen in [17].

Controlling influences: selection of suppliers of goods, determination of the volume of purchases for the entire assortment list, transportation, production, storage and sale [5,18,19].

We use the following notations:

$t$ – number of the time interval, with discreteness up to which the model time is determined (further months); $j$ – supplier number ($j = 1, J$), $i$ – product number in the range of supplies ($i = 1, I$), $l$ – consumer type index ($l = 1, L$), $k$ – number of the interval of the discount volume scales ($k = 1, K$),

and demand ($k = 1, K$), $v$ – product number in the range of sales ($v = 1, V$);

$y_{ij}^{(t)}$ – volume of purchases in kind of product $i$ at the supplier $j$ in a month $t$;
\[ C_p(t) \] – product base wholesale price \( i \) at the supplier \( j \) in a month \( t \);

\[ h_{jk}(t) \] – value of the right border of the interval \( k \) scales of volumes of discounts at the supplier \( j \) in a month \( t \);

\[ g_{jk}(t) \] – supplier discount \( j \) in a month \( t \) on the interval \( k \) corresponding scale (percentage);

\[ x_{vt}(t) \] – product sales volume \( v \) consumer type \( l \) in a month \( t \);

\[ p_{vl}(t) \] – unit selling price \( l \) for consumer type \( l \) in a month \( t \) on the interval \( k \) demand functions;

\[ Q(t), \Delta Q(t) \] – size and balance of working capital per month \( t \),

\[ N(t) \] – wages and overheads per month \( t \);

\[ s_{vlk}(t) \] – value of the right border of the interval \( k \) scales of the demand function for goods \( i \) consumer type \( l \) in a month \( t \).

The EMM of optimal management of supplies and sales of heterogeneous products of an enterprise will look like this:

\[
p_{vl}(x_{vl}(t)) = \begin{cases} p_{vl,1}(t), & \text{if } x_{vl}(t) \leq s_{vl,1}(t), \\ a_{vl,1}^0(t) + a_{vl,1}(t)x_{vl}(t), & \text{if } x_{vl}(t) > s_{vl,1}(t), \end{cases} \quad v = 1, V, l = 1, L, t = 1, T, \tag{1}
\]

\[
\sum_{j=1}^{J} \sum_{i=1}^{I} [1 - \tilde{g}_j(t)]C_{i,j}(t)y_{i,j}(t) + \Delta Q(t) = Q(t), \quad t = 1, T, \tag{2}
\]

\[
Q(t) \geq 0, \Delta Q(t) \geq 0, y_{i,j}(t) \geq 0, i = 1, I, j = 1, J, t = 1, T, \tag{3}
\]

\[
0 \leq x_{vl}(t) \leq s_{vl,2}(t), \quad v = 1, V, l = 1, L, t = 1, T, \tag{4}
\]

\[
\sum_{i=1}^{I} x_{vl}(t) = \sum_{i=1}^{I} A_{vl}(t)\sum_{j=1}^{J} y_{i,j}(t), \quad v = 1, V, t = 1, T, \tag{5}
\]

\[
\Delta Q(t+1) - \Delta Q(t) - \sum_{i=1}^{I} \sum_{l=1}^{L} x_{vl}(t)p_{vl}(x_{vl}(t)) + \sum_{i=1}^{I} \sum_{j=1}^{J} [1 - \tilde{g}_j(t+1)]C_{i,j}(t+1)y_{i,j}(t+1) \leq -N(t), \quad t = 1, T \tag{6}
\]

\[
z = \Delta Q(T + 1) \rightarrow \max. \tag{7}
\]

Expressions (1) set constraints on the choice of sales prices, (5) - a constraint on the methods of converting raw materials and components \( Y \) into sold goods \( X \), where \( A \) is the tensor of technological coefficients. (2) - restrictions on the volume of purchases in value terms, taking into account discounts in the month \( t \) for all suppliers. (3) - restrictions on the non-negativity of the size and balance of the working capital and the volume of purchases in kind. (4) - restrictions on demand for each product for all types of consumers in the month \( t \). (6) - sets the dynamics of change in net income and (7) - criterion indicator of efficiency, meaning the value of net income at the end of the planning period.
3. Algorithm for solving the problem
To solve this problem, a special approximate efficient algorithm was found, at the iterations of which another algorithm is used, described in [17].

1. We set the initial value of the number of the step of the algorithm \( q := 0 \). We use the initial data
\[
O_{i, 1}^{1}, Q_{1}^{1}, N(t), i, j, k, h_{i, j, k}(t), p_{i, j, k}(t), s_{i, j, k}(t), i = 1, 1, j, 1, k = 1, 2, k = 1, K, l = 1, L.
\]

2. Let us suppose that
\[
p_{i, j}(x_{i, j}(t)) := a_{i, j}^{0}(t) + a_{i, j}^{1}(t)x_{i, j}(t), \quad i = 1, 1, j, 1, l = 1, L, t = 1, T, \quad k = 1, 2,
\]
forming the original weakened task.

3. We find a solution to the semidefinite programming problem \((2) - (7)\) with discrete variables \( G^{q} \) using the approximate algorithm \([5, 17]\) and the method of following the central path. The optimal solution is denoted by:
\[
Y^{q}, X^{q}, z^{q}, \quad (Y^{q} = \|y_{i, j}^{q}(t)\|, \quad X^{q} = \|x_{i, j}^{q}(t)\|, \quad z^{q} = z(X^{q}, Y^{q})).
\]

4. We will increase the step number of the algorithm \( q := q + 1 \).

5. Checking the optimality conditions. We check the fulfillment of conditions \((1)\). If all conditions are met:
\[
p_{i, j}(x_{i, j}(t)) = \begin{cases} p_{i, j, 1}(t), & \text{if } x_{i, j}^{q+1}(t) \leq s_{i, j, 1}(t), \\ a_{i, j}^{0}(t) + a_{i, j}^{1}(t)x_{i, j}(t), & \text{if } x_{i, j}^{q+1}(t) > s_{i, j, 1}(t), \end{cases} \forall i = 1, 1, j, 1, l = 1, L, t = 1, T, \quad \text{go to item 8, otherwise, the next item.}
\]

6. If \( x_{i, j}^{q+1}(t) \leq s_{i, j, 1}(t) \) and \( p_{i, j}(x_{i, j}(t)) > p_{i, j, 1}(t) \), replace the value of the variable \( x_{i, j}(t) \) fixed value:
\[
p_{i, j}(x_{i, j}(t)) := p_{i, j, 1}(t).
\]

If \( x_{i, j}^{q+1}(t) > s_{i, j, 1}(t) \) and \( p_{i, j}(x_{i, j}(t)) = p_{i, j, 1}(t) \), replace the value of the variable \( x_{i, j}(t) \) linear function
\[
p_{i, j}(x_{i, j}(t)) := a_{i, j}^{0}(t) + a_{i, j}^{1}(t)x_{i, j}(t)
\] and supplement the system of constraints \((2) - (6)\) with the condition \( x_{i, j}(t) \geq s_{i, j, 1}(t) \).

Checks and necessary replacement of estimates are performed for all variables. \( x_{i, j}(t) \forall i = 1, 1, j, 1, l = 1, L, t = 1, T. \)

7. We find a solution to the current problem \( Y^{q}, X^{q}, \quad (Y^{q} = \|y_{i, j}^{q}(t)\|, \quad X^{q} = \|x_{i, j}^{q}(t)\|) \) and go to step 4.

8. At the current step \( q \), an optimal solution to problem \((1) - (7)\) was obtained \( Y^{q}, X^{q}, z^{q} \). Stopping the algorithm.

4. Software implementation and results obtained
The IBM ILOG CPLEX package was used as the main calculator for solving a sequence of relaxed semidefinite programming subproblems generated by the algorithm.

To implement the algorithm and interact with CPLEX tools, the common programming language C \# and the JetBrains Rider 2020.2 development environment were chosen.

The program results in a set of multidimensional arrays of variable values and the obtained value of the objective function.
Table 1 summarizes the calculation results for 10 generated test problems of various dimensions. The dimensions of the tasks were chosen close to the real ones at large enterprises. The initial data for the tests were collected from open sources (namely, wholesale prices of products and discounts (in percent) from suppliers) - websites of trading and manufacturing firms that sell computers and their components. A hypothetical firm with three types of consumers: retail, consumers with a 2% discount, and consumers with a 5% discount was taken as an enterprise for which a solution was sought.

System Configuration: Intel Core i5 8265U @ 1.60 GHz, 16GB DDR4 RAM, Windows 10 Home (version 1903, OS build 18362.1016), Framework: .NET Core 3.1.201.

Columns of Table 1: No - test number, Cont - number of continuous variables, Boolean - number of boolean variables, LineConstr - number of linear constraints, NonlineConstr - number of nonlinear constraints, It - average number of algorithm iterations, and Time - average solution time, in seconds.

| No  | Cont | Boolean | LineConstr | NonlineConstr | It | Time  |
|-----|------|---------|------------|---------------|----|-------|
| 1   | 690  | 120     | 816        | 6             | 1  | 00.25 |
| 2   | 3180 | 120     | 3426       | 6             | 5  | 02.71 |
| 3   | 6180 | 120     | 6426       | 6             | 5  | 03.86 |
| 4   | 12180| 120     | 12426      | 6             | 5  | 08.56 |
| 5   | 24180| 120     | 24426      | 6             | 2  | 06.46 |
| 6   | 48180| 120     | 48426      | 6             | 2  | 09.66 |
| 7   | 60180| 120     | 60426      | 6             | 2  | 13.33 |
| 8   | 96180| 120     | 96426      | 6             | 2  | 21.4  |
| 9   | 120180| 120    | 120426     | 6             | 2  | 30.57 |
| 10  | 192180| 120   | 192426     | 6             | 2  | 51.43 |

5. Conclusion
The obtained results of computational experiments allow us to hope that in most cases the found algorithm finds an exact solution to problems (1) - (7) of arbitrary dimension.

Computational experiments with a software implementation of the algorithm for solving problem (1) - (7), formally unsolvable by exact methods, have shown the high efficiency of the algorithm. The estimation of the algorithm performance is also quite encouraging. This leads to excellent prospects for the implementation of the development, since, as noted above, the search for optimal solutions to the considered management problem is relevant for most of the economic entities.

Acknowledgments
The study was carried out with the financial support of the Russian Foundation for Basic Research within the framework of scientific project No. 19-37-90048. (The reported study was funded by RFBR, project number 19-37-90048) And also with the financial support of the Ministry of Science and Higher Education within the framework of the State Assignment (project No. FSUN-2020-0009). (The research is supported by Ministry of Science and Higher Education of Russian Federation (project No. FSUN-2020-0009).

References
[1] Butusov O B and Dubin M E 2013 Decision support system for choosing a supplier in the supply chain Izvestia of the Moscow State Technical University MAMI 4(1) 15
[2] Parunakjan V and Sizova E 2008 Increase of efficiency of interaction of production and transport in the logistic chains of material traffic of enterprises Transport Problems 3 95-104
[3] Bosov A and Khalipova N 2017 Formation of separate optimization models for the analysis of transportation-logistics systems Eastern European Journal of Advanced Technologies 3(3) 11-20
[4] Maria K et al. 2013 Planning principles in metallurgy Bulletin of the Magnitogorsk State Technical University GI Nosova 5(45)

[5] Mezentsev Yu A 2008 Mathematical models of management of logistics subsystems at enterprises Automation and modern technologies (Moscow Publishing house “Mechanical engineering”) 8 46-55

[6] Arshinsky L and Zhang K 2012 The application of operations research in logistics Information technologies and problems of mathematical modeling of complex systems 10 5-12

[7] Mehrsai A et al. 2013 Using metaheuristic and fuzzy system for the optimization of material pull in a push-pull flow logistics network Mathematical Problems in Engineering 2013

[8] Koriashkina L, Saveliev V and Zhelo A 2017 On Mathematical Models of Some Optimization Problems Arising in the Production of Autoclaved Aerated Concrete Advanced Engineering Forum – Trans Tech Publications 22 173-81

[9] Castro P M, Grossmann I E and Zhang Q 2018 Expanding scope and computational challenges in process scheduling Computers & Chemical Engineering 114 14-42

[10] Avkhadiev R A, Elizarova N Yu and Sabitov Sh R 2014 System of operational management of production processes of the enterprise 1C: MES: cloud production management XII All-Russian meeting on management problems VSPU-2014

[11] Karpak B, Kumcu E and Kasuganti R R 2001 Purchasing materials in the supply chain: managing a multi-objective task European Journal of Purchasing & Supply Management 7(3) 209-16

[12] Karpak B, Kasuganti R R and Kumcu E 1999 Multi-objective decision-making in supplier selection: An application of visual interactive goal programming Journal of Applied Business Research 15 57-72

[13] Branch S T A 2017 New enhanced support vector model based on general variable neighborhood search algorithm for supplier performance evaluation: A case study

[14] Prosvetov G I 2006 Mathematical Methods in Logistics (Moscow: Publishing House of RDL) p 272

[15] Brodetsky GL 2010 System analysis in logistics. Choice under Uncertainty (Moscow: Academia) p 336

[16] Bochkarev AA 2008 Planning and modeling of the supply chain (Moscow: Alfa-Press) p 192

[17] Baranova N V, Mezentsev N V and Baranova Y A 2018 Research and development of an algorithm for solving the problem of control over the input-output material flows of an industrial company CEUR Workshop Proceedings 2098: Optimization Problems and their Applications (OPTA-SCL 2018) (Omsk) 33-44

[18] Mezentsev Yu A and Pavlov P S 2012 On the software implementation of a decomposition algorithm for solving one class of discrete optimization problems with semi-definite relaxation Information Technologies 2(186) (Moscow, Publishing House "New Technologies") 54-59

[19] Mezentsev Yu A and Pavlov P S 2011 Implementation of an algorithm for solving special problems of semidefinite programming using IBM ILOG CPLEX Scientific Bulletin of NSTU (Novosibirsk) 4(45) 25-34