Features of Motion Around Global Monopole in Asymptotically
dS/AdS Spacetime

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Abstract

Abstract: In this paper, we study the motion of test particle and light around the Global Monopole in asymptotically dS/AdS spacetime. The motion of a test particle and light in the exterior region of the global monopole in dS/AdS spacetime has been investigated. Although the test particle’s motion is quite different from the case in asymptotically flat spacetime, the behaviors of light(null geodesic) remain unchanged except a energy(frequency) shift. Through a phase-plane analysis, we prove analytically that the existence of a periodic solution to the equation of motion for a test particle will not be altered by the presence of cosmological constant and the deficit angle, whose presence only affects the position and type of the critical point on the phase plane. We also show that the apparent capture section of the global monopole in dS/AdS spacetime is quite different from that in flat spacetime.

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1. Introduction

Various of kinds of topological defects associated with the spontaneous symmetry breaking (SSB) are very interesting objects, whose only plausible production site is a cosmological phase transition wherein they are produced by the Kibble mechanism. Domain walls are two-dimensional topological defects, and strings are one-dimensional defects. Point-like defects also arise in some theories which undergo SSB, and they appear as monopoles. Global monopole, which has divergent mass in flat spacetime, is one of the most important above mentioned defects. The property of the global monopole in curved spacetime, or equivalently, its gravitational effects, was firstly studied by Barriola and Vilenkin. When one considers the gravity, the linearly divergent mass of global monopole has an effect analogous to that of a deficit solid angle plus that of a tiny mass at the origin. Harari and Lousto, and Shi and Li have shown that this small gravitational potential is actually repulsive. A new class of cold stars, addressed as D-stars (defect stars) have been proposed by Li et.al. One of the most important features of such stars, comparing to Q-stars, is that the theory has monopole solutions when the matter field is abscent, which makes the D-stars behave very differently from the Q-stars. On the other hand, there has been a renewed interest in AdS spacetime due to the theoretical speculation of AdS/CFT correspondence, which state that string theory in anti-de Sitter space (usually with extra internal dimensions) is equivalent to the conformal field theory in one less dimension. Recently, the holographic duality between quantum gravity on de Sitter (dS) spacetime and a quantum field theory living on the past boundary of dS spacetime was proposed and the vortices in dS spacetime was studied by Ghezelbash and Mann. Many authors conjectured that the dS/CFT correspondence bear a lot of similarities with the AdS/CFT correspondence, although some interpretive issues remain. The monopole and dyon solution in gauge theories based on the various gauge group have been found. However, in flat space there can not be static soliton solution in the pure Yang-Mills theory. The presence of gravity can supply attractive force which binds non-Abelian gauge field into a soliton. The cosmological constant influence the behavior of the soliton solution significantly. In asymptotically Minkowski spacetime the electric components are forbidden in static solution. If the spacetime includes the cosmological constant, forbidding the electric components of the non-Abelian gauge fields fail,
thus allowing dyon solutions. A continuum of new dyon solutions in the Einstein-Yang-Mills theory in asymptotically AdS spacetime have been investigated [16], which are regular everywhere and specified with their mass, and non-Abelian electric and magnetic charges. Similarly, the presence of cosmological constant affects the behavior of the global monopole remarkably. If the spacetime is modified to include the positive cosmological constant, the gravitation field of global monopole can be attractive in contrast to the same problem in asymptotically Minkowski or AdS spacetime.

In a previous paper by Li and Hao [17], it is shown that the property of the global monopole in asymptotically dS/AdS spacetime is very different from that in the asymptotically flat spacetime. That is, the mass of the monopole might be positive in dS spacetime if the cosmological constant is greater than a critical value. In this paper, we will discuss the intriguing astrophysical effects of global monopoles in some detail. That is, the behavior of test particle and null geodesics in the exterior spacetime of the global monopole in asymptotically dS/AdS spacetime. With the aid of phase-plane analysis, we carefully analyze the motion of light and a test particle around a global monopole in dS/AdS spacetime and show that the behavior is quite different from the case for ordinary object in asymptotically flat spacetime.

2. The Global Monopole in Asymptotically dS/AdS Spacetime

In this section, we briefly introduce the global monopole in dS/AdS spacetime and one may refer to Ref. [17] for a more completed description. The Lagragian for the global monopole is

\[ L = \frac{1}{2} \partial_\mu \phi^a \partial^\mu \phi^a - \frac{1}{4} \lambda^2 (\phi^a \phi^a - \sigma_0)^2 \]  

(1)

where \( \phi^a \) is the triplet of Goldstone field and possesses a internal O(3) symmetry. When the symmetry breaks down to U(1), there will exist topological defects known as monopole. The configuration describing monopole solution is

\[ \phi^a = \sigma_0 f(\rho) \frac{x^a}{\rho} \]  

(2)

where \( x^a x^a = \rho^2 \) and \( a = 1, 2, 3 \).

When \( f \) approaches unity at infinity, we will have a monopole solution. The static
spherically symmetric metric is

\[ ds^2 = B(\rho)dt^2 - A(\rho)d\rho^2 - \rho^2(d\theta^2 + \sin^2\theta d\varphi^2) \]  

(3)

In dS/AdS spacetime, the equation of motion of the monopole field and the Einstein equation are

\[ \frac{1}{A}f'' + \frac{2}{Ar} + \frac{1}{2B}B'f' - \frac{2}{r^2}f - \lambda^2(f^2 - 1)f = 0 \]  

(4)

\[ G_{\mu\nu} + \beta g_{\mu\nu} = \kappa T_{\mu\nu} \]  

(5)

where \( \beta \) is the cosmological constant and \( \kappa = 8\pi G \). dS and AdS spacetime corresponds to the cases that \( \beta \) is positive or negative respectively. By introducing the dimensionless parameters \( r = \sigma_0 \rho \) and \( \epsilon^2 = \kappa \sigma_0^2 \), the Einstein equation can be formally integrated and solution are as following

\[ A^{-1}(r) = 1 - \epsilon^2 + \frac{\beta}{3\sigma_0^2}r^2 - \frac{2G\sigma_0 M_A(r)}{r} \]  

(6)

\[ B(r) = 1 - \epsilon^2 + \frac{\beta}{3\sigma_0^2}r^2 - \frac{2G\sigma_0 M_B(r)}{r} \]  

(7)

where

\[ M_A(r) = 4\pi\sigma_0 \exp[-\Delta(r)] \times \int_0^r dy \exp[\Delta(y)] \{ f^2 - 1 + y^2 \left[ \frac{\lambda^2}{4}(f^2 - 1)^2 + (1 - \epsilon^2 + \frac{\beta}{3\sigma_0^2}y^2)f' \right] \} \]  

(8)

\[ M_B(r) = M_A(r) \exp[\tilde{\Delta}(r)] + \frac{r(1 - \epsilon^2 + \frac{\beta}{3\sigma_0^2}r^2)}{2}\{ 1 - \exp[\tilde{\Delta}(r)] \} \]  

(9)

In which

\[ \Delta(r) = \frac{\epsilon^2}{2} \int_0^r dy y f'^2 \]  

(10)

and

\[ \tilde{\Delta}(r) = \epsilon^2 \int_{\infty}^r dy y f'^2 \]  

(11)
Next, we discuss the behavior of these functions in asymptotically dS/AdS spacetime. A global monopole solution \( f \) should approach unity when \( r \gg 1 \). If this convergence is fast enough then \( M_A(r) \) and \( M_B(r) \) will also quickly converge to finite values. Therefore, we can find the asymptotic expansions:

\[
f(r) = 1 - \frac{3\sigma_0^2}{\beta + 3\lambda^2\sigma_0^2} \frac{1}{r^2} - \frac{9[2\beta\epsilon^2\sigma_0^4 + 3(2\epsilon^2 - 3)\lambda^2\sigma_0^6]}{2(2\beta - 3\lambda^2\sigma_0^2)(\beta + 3\lambda^2\sigma_0^2)^2} \frac{1}{r^4} + O\left(\frac{1}{r^6}\right)
\]  

(12)

\[
M_A(r) = M_A + \frac{4\pi\sigma_0}{r} + O\left(\frac{1}{r^3}\right)
\]

(13)

\[
M_B(r) = M_A(r)(1 - \frac{\epsilon^2}{r^4}) + \frac{4\pi\sigma_0(1 - \epsilon^2)}{r^3} + O\left(\frac{1}{r^7}\right)
\]

(14)

where \( M_A(\beta, \epsilon^2) \equiv \lim_{r \to \infty} M_A(r) \), which is a function dependent on \( \beta \) and \( \epsilon^2 \).

We now investigate the motion of light and test particle around a global monopole in asymptotically dS/AdS spacetime. Since the effective mass \( M_A(r) \) approaches very quickly its asymptotic value, it is a good approximation to take it as the constant \( M \), unless we were interested in a test particle moving right into the core of the monopole. Therefore, we consider the geodesic equations in the metric (3) with

\[
A(r)^{-1} = B(r) = 1 - \epsilon^2 + \beta r^2 - \frac{2GM}{r}
\]

(15)

where, for convenience, we have rescaled \( \beta \) and \( M \) as \( \beta = \frac{\beta}{3\sigma_0} \) and \( M = M\sigma_0 \) respectively.

3. The Behavior of Null Geodesic outside the Global Monopole in dS/AdS spacetime

The orbit of light outside of the monopole core can be obtained by solving the geodesic equation

\[
\frac{d^2x^\rho}{d\tau^2} + \Gamma^\rho_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0
\]

(16)

where \( \tau \) is the affine parameter. But by using the fact that \( g_{ab} \frac{dx^a}{d\tau} \frac{dx^b}{d\tau} = 0 \) for null geodesics and the constant of motion

\[
E = B(r) \frac{dt}{d\tau}
\]

(17)

and

\[
L = r^2 \frac{d\varphi}{d\tau}
\]

(18)
one can, considering the motion is confined on the $\theta = \frac{\pi}{2}$ plane, easily obtain the equation for geodesics as [19]:

$$\frac{1}{2} \dot{r}^2 + \frac{1}{2} B(r) \left( \frac{L^2}{r^2} \right) = \frac{1}{2} E^2$$  (19)

Introducing the effective potential

$$V_{eff} = \frac{1}{2} B(r) \left( \frac{L^2}{r^2} \right) = \frac{L^2 \beta}{2} + \frac{L^2 (1 - \epsilon^2)}{2 r^2} - \frac{L^2 G M}{r^3}$$  (20)

the geodesics of light becomes the same as a test particle with unit mass moving in the effective potential Eq. (20). It is not difficult to found that the effective potential has a maximum

$$V_{max} = \frac{L^2 \beta}{2} + \frac{L^2 G M (1 - \epsilon^2)^3}{2 (3 G M)^3}$$  (21)

at

$$r = r_c = \frac{3 G M}{1 - \epsilon^2}$$  (22)

For convenience, we will choose $G = 1$ in the following discussion. When

$$\frac{1}{2} E^2 = V_{max}$$  (23)

we will have

$$b_{crit} = \frac{3^{3/2} M}{\sqrt{27 M^2 \beta + (1 - \epsilon^2)^3}}$$  (24)

where $b_{crit}$ is the critical value of $b$, which is the generalization of the apparent impact parameter and is defined as $b = \frac{L}{E}$. In the case that the mass of the monopole is positive, that is, the spacetime is asymptotically de Sitter and the cosmological constant is greater than a critical value, the capture cross section of the monopole will be

$$\sigma_c = \pi b_{crit}^2 = \frac{27 \pi M^2}{27 M^2 \beta + (1 - \epsilon^2)^3}$$  (25)

It is not difficult to prove that the path of light has a turning point at the largest radius, $R_0$, where $\frac{dr}{d\phi} |_{r=R_0} = 0$. Using Eqs. (17)-(19), one can obtain the relation between $R_0$ and $b$ as

$$R_0 = 2 b \sqrt{\frac{1 - \epsilon^2}{3 (1 - b^2 \beta)}} \cos \left\{ \frac{1}{3} \arccos \left[ -\left( \frac{3}{1 - \epsilon^2} \right)^{3/2} \frac{M}{b} \sqrt{1 - b^2 \beta} \right] \right\}$$  (26)

Obviously, $R_0 = b$ when $M = \epsilon = \beta = 0$. In order to compute the deflect angle of the light, we rewrite the Eqs. (17)-(19) as

$$\frac{d\phi}{dr} = \left[ r^4 b^{-2} - r (r - \epsilon^2 r + \beta r^3 - 2M) \right]^{-1/2}$$  (27)
The change of light when passing a monopole should be $\Delta \varphi = \varphi_\infty - \varphi_{-\infty}$. Considering the symmetry, we have

$$\Delta \varphi = 2 \int_{R_0}^{\infty} \frac{dr}{[r^4b^{-2} - r(r - \epsilon^2r + \beta r^3 - 2M)]^{1/2}}$$

The deflection of light up to the first order of $M$ ($M$ is small in the unit $G = 1$) is given by (See Fig.1 for the definition of $\delta \varphi$):

$$\delta \varphi = \Delta \varphi - \pi \approx M \frac{\partial(\Delta \varphi)}{\partial M} \bigg|_{M=0} = \frac{4M}{(1 - \epsilon^2)^{3/2}R_0}$$

In terms of the apparent impact parameter $b$, the Eq.\ref{eq:29}

$$\delta \varphi = \frac{2M\sqrt{3(1 - b^2\beta)}}{b(1 - \epsilon^2)^2 \cos\left\{\frac{1}{3} \arccos\left[-\left(\frac{3 - \epsilon^2}{1 - \epsilon^2}\right)^{3/2} \frac{M}{b} \sqrt{1 - b^2\beta}\right]\right\}}$$

One can easily find that when setting $\beta = \epsilon = 0$, the above Eqs\ref{eq:29}, \ref{eq:30} will reduce to the well known form for schwarzschild spacetime. From Eq.\ref{eq:30}, we can find that for a beam of light with a specific $b$ or $L/E$, its deflecting behavior will be significantly influenced by the presence of the deficit angle $\epsilon^2$ and the cosmological constant $\beta$, which could be employed as an characteristic feature of the monopole in dS/AdS spacetime.
On the other hand, from Eq. (19), if we redefine $E$ as $\widetilde{E}^2 = E^2 - L^2 \beta$, the effects of the cosmological constant on the motion of light will be ascribed to a shift of its energy. But, as we will show later, this property does not hold true for the timelike test particles.

4. The Motion of Timelike Particles Around a Global Monopole in dS/AdS spacetime

From Eqs. (16)-(18), the equation of motion for a timelike test particle can be expressed as

$$\left( \frac{L}{r^2} \frac{dr}{d\varphi} \right)^2 = E^2 - \mu^2 B(r) - \frac{L^2}{r^2} B(r)$$  \hspace{1cm} (31)

where $\mu = \frac{\mu}{\sigma_0}$ and $E = \frac{E}{\sigma_0}$ are the rescaled mass and energy of the test particle. Introducing $\chi = \frac{1}{r}$, substituting it into Eq. (31) and then differentiating the equation with respect to $\varphi$, one will obtain the following equation

$$\frac{d^2 \chi}{d\varphi^2} = \frac{\gamma}{\chi^3} + \frac{1}{p} - (1 - \epsilon^2) \chi + \alpha \chi^2$$  \hspace{1cm} (32)

where $\alpha$, $p$ and $\gamma$ are three dimensionless parameters defined as:

$$\alpha = 3GM$$  \hspace{1cm} (33)
$$\frac{1}{p} = \frac{GM \mu^2}{L^2}$$
$$\gamma = \frac{\mu^2 \beta}{L^2}$$

Noting that Eq. (32) is a nonlinear differential equation, it can be integrated formally as

$$\varphi - \varphi_0 = \int_{\chi_0}^{\chi} \frac{d\chi}{\sqrt{\int_{\chi_0}^{\chi} 2 \left[ \frac{\gamma}{\chi^3} + \frac{1}{p} - (1 - \epsilon^2) \chi + \alpha \chi^2 \right] d\chi + \chi_0^2}}$$  \hspace{1cm} (34)

where, $\chi_0$ is the initial value of $\frac{d\chi}{d\varphi} |_{\varphi=\varphi_0}$. However, it is impossible to obtain the exact expression by integrating the above equation. In the following, we will gain some qualitative property of the system with the aid of phase-plane analysis without solving the equation numerically. To do so, we introduce two parameters as $x = \chi$ and $y = \frac{d\chi}{d\varphi}$ and the autonomous system corresponding to Eq. (32) will be

$$\frac{dx}{d\varphi} = f(x, y) = y$$
$$\frac{dy}{d\varphi} = g(x, y) = \frac{\gamma}{x^3} + \frac{1}{p} - (1 - \epsilon^2) x + \alpha x^2$$  \hspace{1cm} (35)
Now, we prove the existence of periodic solution. According to the well known Bendixson’s criterion [18], the equation of motion will have periodic solution if the divergence of the functional vector of the autonomous system is vanishing, i.e., $\nabla \cdot (f, g) = 0$. It is obvious that the functional vector $(f, g)$ corresponding to the Eqs. (35) satisfies the criterion and therefore indicates that Eq. (32) has a periodic solution. One can also found that the periodic solution exists when $\gamma = \epsilon = 0$ which is the case of an ordinary star in asymptotically flat spacetime. This shows that the presence of the cosmological constant and deficit angle will not exclude the existence of periodic solution from the equation of motion.

Next, we analyze the critical points on the phase plane. The critical point is $(x_0, 0)$, where $x_0$ satisfies

$$\frac{\gamma}{x_0^3} + \frac{1}{p} - (1 - \epsilon^2)x_0 + \alpha x_0^2 = 0 \quad (36)$$

To analyze the type of the critical point, we firstly linearize the Eq. (35) and then do the translation $x = x - x_0$. Thus the linearized equations should be:

$$\frac{dx}{d\varphi} = y \quad (37)$$
$$\frac{dy}{d\varphi} = \delta x$$

where

$$\delta = -\frac{3\gamma}{x_0^4} - (1 - \epsilon^2) + 2\alpha x_0 \quad \text{(38)}$$

Using Eqs. (36) and (38) could be rewritten as

$$\delta = \frac{3}{px_0} - 4(1 - \epsilon^2) + 5\alpha x_0 \quad \text{(39)}$$

The eigenvalues corresponding to the system of equations will be

$$\lambda_{1,2} = \pm \sqrt{\delta} \quad \text{(40)}$$

The types of the critical point could be classified according to the eigenvalues as following:

I. when $\delta > 0$, we have $\lambda_1 < 0 < \lambda_2$, which indicates that the critical point is an unstable saddle point. Considering the Eq. (38), this case will correspond to the condition that

1. $\alpha > 0$ and $\Delta < 0$.
2. $\alpha > 0$, $\Delta > 0$ and $x_0 > \frac{2(1-\epsilon^2)+\sqrt{\Delta}}{5\alpha}$ or $0 < x_0 < \frac{2(1-\epsilon^2)-\sqrt{\Delta}}{5\alpha}$. 


\[ \alpha < 0, \Delta > 0 \text{ and } \frac{2(1-\epsilon^2)-\sqrt{\Delta}}{5\alpha} < x_0 < \frac{2(1-\epsilon^2)+\sqrt{\Delta}}{5\alpha}, \text{ where } \Delta = (1-\epsilon^2)^2 - \frac{15\alpha}{4p}. \]

II. when \( \delta < 0 \), we have two pure imaginary eigenvalues \( \lambda_{1,2} = \pm i|\delta| \), which indicates that the critical point is stable center. Considering the Eq. (39), this case will correspond to the condition that

1. \( \alpha < 0 \) and \( \Delta < 0 \).
2. \( \alpha > 0, \Delta > 0 \) and \( \frac{2(1-\epsilon^2)-\sqrt{\Delta}}{5\alpha} < x_0 < \frac{2(1-\epsilon^2)+\sqrt{\Delta}}{5\alpha} \).
3. \( \alpha < 0, \Delta > 0 \) and \( x_0 > \frac{2(1-\epsilon^2)+\sqrt{\Delta}}{5\alpha} \) or \( 0 < x_0 < \frac{2(1-\epsilon^2)-\sqrt{\Delta}}{5\alpha} \).

What we need to point out is that the case \( \alpha < 0 \) is possible because the mass of the global monopole could be negative.

III. when \( \delta = 0 \), we have \( \lambda_{1,2} = 0 \), which, together with the form of the autonomous system Eqs. (37), indicates that the motion is uniformly on the lines \( y = \text{Constants} \) and all the the points on the lines \( y = \text{Constants} \) are balanced positions. In Fig.2, Fig.3 and Fig.4, we show the phase graph for different initial values and different parameters.

Hitherto, we consider only the qualitative property of the equation of motion of test particle in the exterior region of a global monopole in dS/AdS spacetime. One can simply perform some numerical calculation under different parameters so as to get the solutions. But this is beyond the scope of this paper and is not the very purpose of this paper. In the following, we will study the precession of the test particle around the global monopole. Noting that the cosmological constant is generally very small, the term \( \frac{\gamma \chi^3}{\chi} \) could be neglected when \( \chi = r^{-1} \) is very large, which agrees with our notion that the presence of cosmological will not change the property of Einsteinian gravity at the scale of solar system. Therefore, when we consider the behavior of a test particle around a global monopole, we can neglect the influence from the cosmological constant term. So, the equation of motion for a test particle now reduces to

\[
\frac{d^2\chi}{d\varphi^2} = \frac{1}{p} - (1-\epsilon^2)\chi + \alpha\chi^2
\]

(41)

It is obvious that when one neglects the higher order term \( \chi^2 \) and sets \( \epsilon = 0 \), the resulting orbit of the test particle obtained from the above equation will be the case attained through Newtonian Mechanics

\[
\chi = \frac{1}{p}(1 + e \cos \varphi)
\]

(42)

where \( e \) is the eccentricity. In order to get the solution of Eq.(41), as done in Ref.([20]), we
FIG. 2: the phase graph when $\alpha = 0.10$, $\gamma = 0.01$, $\epsilon = 0.01$ and $p = 11$

FIG. 3: the phase graph when $\alpha = 0.10$, $\gamma = 0.00$, $\epsilon = 0.01$ and $p = 11$
decompose $\chi$ as $\chi = \chi_0 + \chi_1$. Then we have

$$\chi_0 = \frac{1}{(1 - \epsilon^2)p}[1 + \epsilon \cos(\sqrt{1 - \epsilon^2}\varphi)]$$  \hspace{1cm} (43)

$$\chi_1 = \frac{\alpha e}{(1 - \epsilon^2)p^2}(\sqrt{1 - \epsilon^2}\varphi) \sin(\sqrt{1 - \epsilon^2}\varphi)$$  \hspace{1cm} (44)

Obviously, when $\frac{\alpha}{p} \varphi \ll 1$, we can write the solution as:

$$\chi = \chi_0 + \chi_1 \simeq \frac{1}{p}\{1 + \epsilon \cos[\sqrt{1 - \epsilon^2}(1 - \frac{\alpha}{(1 - \epsilon^2)p})\varphi]\}$$  \hspace{1cm} (45)

Now, it is straightforward to estimate the precession of a test particle when it rotate one loop around the global monopole. It will take the test particle $\frac{2\pi}{\sqrt{1 - \epsilon^2}}$ to complete one loop. The precession after one loop will be

$$\delta\varphi = \frac{6\pi GM}{a(1 - \epsilon^2)(1 - \epsilon^2)}$$  \hspace{1cm} (46)

where $a = \frac{p}{1 - \epsilon^2}$. Comparing this result with the familiar result for the precession around sun, one can find that the there would be an modification. This modification can be more clear if we rewrite Eq.(46) as

$$\delta\varphi = \frac{6\pi GM}{a(1 - \epsilon^2)} + \frac{6\pi GM \epsilon^2}{a(1 - \epsilon^2)}$$  \hspace{1cm} (47)

This show that the test particle around the global monopole will have an extra precession $\frac{6\pi GM \epsilon^2}{a(1 - \epsilon^2)}$ than that around a ordinary star.

4. Discussion

In this paper, we study the motion of test particle and light around the global monopole in dS/AdS spacetime. We show that the null geodesics and timelike geodesics behave very differently in the presence of cosmological constant and deficit angle. The motion of light is not drastically changed except a factor of $(1 - \epsilon^2)$ and a shift of its energy, which consequently lead to different relation between the apparent impact parameter and the deflect angle.

However, the behavior of timelike geodesics has been affected significantly when there are cosmological constant and deficit angle. By using the phase-plane analysis, we investigate the qualitative property of the dynamical equation governing the motion of a test particle around the global monopole in asymptotically dS/AdS spacetime. We prove that the equation of
motion possesses periodic solution. This property is not altered by the cosmological constant and deficit angle, which, however, affect the position of the critical point and its type on the phase plane. The conditions under which the critical point is stable center and unstable saddle point respectively have been given too. The precession of the test particle around a global monopole has been investigated and the influence of deficit angle on the precession has been manifested.

It is very interesting to consider a further generalization of this model by requiring that the global monopole possess a U(1) charge. This could be realized by coupling to the monopole field with a scalar field with local U(1) symmetry. For details, see Ref.\([6, 7]\). In this case, the effective potential(20) becomes

\[
V_{\text{eff}}(r) = \begin{cases} 
\left(\frac{1}{2}(1 - \epsilon^2 + \beta r^2 - \frac{2GM}{r} + \frac{Q^2}{r^2}) \frac{L^2}{r^2} + 1\right), & \text{for time-like geodesics} \\
\left(\frac{1}{2}(1 - \epsilon^2 + \beta r^2 - \frac{2GM}{r} + \frac{Q^2}{r^2}) \frac{L^2}{r^2}\right), & \text{for null geodesics}
\end{cases}
\]  

If we set \(\epsilon = 0\), this will reduce to the Reissner-Nordström metric in asymptotically dS/AdS spacetime, which has been thoroughly studied in Ref.\([21]\). The introduction of the deficit angle affects the asymptotically dS/AdS Reissner-Nordström spacetime in a similar fashion as it does on the asymptotically dS/AdS Schwarzschild spacetime as we discussed in the former part of this paper and we will not repeat it again.

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