Magnetic structures propagating in non-equilibrium relativistic plasma of pulsar wind nebulae

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Abstract. The kinetic model of highly non-equilibrium relativistic electron-positron plasma is used to study dynamical magnetic structures in pulsar wind nebulae (PWNe). The evolution equation which describes a propagation of a long-wavelength magnetosonic type perturbation of small but finite amplitude is derived. The wavelength is assumed to be longer than the scattering length of the background positrons and electrons. The rates of scattering of electrons and positrons by the stochastic magnetic field fluctuations are distinguished but the difference is supposed to be small compared with the gyrofrequencies of particles. The phase velocity, the dissipation rate and the dispersion length of the magnetic pulse are studied as the functions of plasma parameters and the scattering rates of electrons and positrons. The model being confronted to observations can help to determine the pulsar wind composition, particle distribution and non-thermal pressure in PWNe.

1. Introduction
Investigations of physical mechanisms responsible for conversion of energy of a magnetized rotating neutron star into a pulsar wind (PW) and for particle acceleration at the termination shock (TS) of a PW are of great interest for the high energy astrophysics [1, 2]. The studies of these phenomena require modeling of highly non-equilibrium processes in relativistic magnetized plasma of pulsar wind nebulae (PWNe). An important issue here is to determine the values of the plasma parameters, like its pressure and particle energy distribution.

One of the most intriguing problems here is the composition of the relativistic PW. The PW plasma is thought to be predominately consisted of pairs, but the ions also may present. The question of the ion component presence in the PWs is important for understanding of physics of these objects. Because the particle energy in the upstream flow is determined by the Lorentz-factor of the flow, even small by the number density fraction of ions can affect the processes, responsible for the energy conversion in the wind. If the ratio of the ion concentration to the pair concentration exceeds the ratio of the electron mass to the ion mass, then the ion component contains the predominant fraction of the PW energy [3]. The presence of ions can affect the particle acceleration at the TS and, as a consequence, the observed spectra of emission [3].

An adequate interpretation of the observed dynamic structures in PWNe allows to put some constraints on relativistic plasma composition, particle distribution function and pressure [4–6]. The observed structures in PWN also may shed light on the nature of the giant gamma-ray flares in the Crab nebula [7]. Owing to the predominately synchrotron nature of the nebular emission, the observed structures may be related to the propagation of magnetic field perturbations.
The kinetic consideration of the magnetic field perturbation propagation in PWN plasma allows to derive an evolution equation, which describes the local dynamics of the structures. The obtained parameters of this equation describing the features and the evolution of the structures should depend on the plasma parameters and the parameters of particle scattering by the stochastic magnetic field fluctuations. In section 2 we provide a sketch of the procedure of the evolution equation derivation. We present the coefficients of this equation — the velocity of propagation, the dissipation rate, the dispersion coefficient. In sections 3-4 we consider the dependencies of these coefficients on the plasma parameters and the scattering rates. We discuss possible constraints on the plasma parameters, which could be imposed via the comparison of the observations with the model results using the obtained relations. In section 5 the presented results are summarized.

2. Local dynamics of magnetic structures

As like as in [8], we consider a propagation of a local magnetic field perturbation. Due to locality, the one-dimensional analysis is performed: a plane wave propagating along the Ox axis across the mean quasi-stationary magnetic field \( \mathbf{B}_0 \parallel O\mathbf{z} \) is studied. The system of kinetic equations for the distribution functions \( f_p \) and \( f_e \) of plasma components — positrons and electrons — is treated (hereafter index \( p \) marks functions related to positrons, \( e \) — to electrons). The regime of the strong scattering of particles by the stochastic magnetic field fluctuations is considered, which is expressed via the non-zero collision operators. These are taken in the relaxation time approximation with different scattering frequencies \( \nu \) fluctuations is considered, which is expressed via the non-zero collision operators. These are taken in the relaxation time approximation with different scattering frequencies \( \nu_1 \) and \( \nu_2 \) in the form \(-\nu_1 (f_p - f_p^{iso}) + \nu_2 (f_e - f_e^{iso})\) in the equation for the positron component and in the same form but with the opposite sign in the equation for the electron component. Here \( f_{\alpha}^{iso} \), \( \alpha = (p, e) \) denotes the isotropic part of the distribution function of the component \( \alpha \).

The difference between the typical frequencies of scattering is introduced in order to take into account possible influence of the admixture of an ion component in the PWN plasma on the processes studied here. For instance, in [3] the authors presented the model of acceleration of pairs by the resonant cyclotron absorption of high harmonics of the ion-cyclotron waves (ICWs) emitted by ions. These waves propagate in the pair-ion plasma across the magnetic field and have an electrostatic component [3]. They are elliptically polarized and left-handed, and the direction of their electric field vector rotation matches the direction of the positrons’ gyration [3]. Due to that, the positrons absorb the ICWs more effectively than the electrons [3]. A similar mechanism can affect the scattering of pairs by the stochastic electromagnetic field fluctuations in PWN plasma. This may cause the difference between the typical frequencies of scattering for positrons and electrons, which are thought to be equal in pure pair plasma. Thus, in order to build a simplified model of studied processes in the pair-ion plasma, we introduce different scattering frequencies. This is the only manifestation of the ion component influence in the discussed model: ions are not included explicitly here.

We consider the processes, whose typical frequencies \( \omega \) are small in comparison with \( \nu_{1,2} \leq \Omega = eB_0/m_e\gamma c \) (here \( e \) is the positron charge, \( m_e \) is the mass of an electron, \( c \) is the light velocity, \( \gamma \) — the Lorentz-factor of the particle). We suppose hereafter that \( \nu_{1,2} \) are proportional to the gyrofrequency \( \Omega \):

\[
\omega \ll \nu_{1,2} = a_{1,2}\Omega \leq \Omega
\]

The solution of the system of kinetic equations is sought in the form of

\[
f_{\alpha} = f_0 + \delta f_{\alpha0} + \delta f_{\alpha x} \sin \theta \cos \phi + \delta f_{\alpha y} \sin \theta \sin \phi + \delta f_{\alpha z} \cos \theta
\]

where \( \alpha = p, e \). Function \( f_0 \) is a non-perturbed (isotropic in the momentum space) distribution function, taken to be equal for positrons and electrons, \( \delta f_{\alpha0} \) is the perturbation of the isotropic
part of the distribution function, and the other terms give the anisotropic perturbation. Here $\theta$ and $\phi$ are the polar and the azimuthal angles in the momentum space.

In the considered case the transverse propagation corresponds to the wave mode with the dispersion equation $c^2 k^2 / \omega^2 = \epsilon_{yy} - \epsilon_{xy} \epsilon_{yx} / \epsilon_{xx}$, where $\epsilon_{ij}$ are the components of the dielectric permittivity tensor, $k$ is the wave number. The electric field of this wave mode is polarized in the $Oxy$ plane.

In the framework of the kinetic approach one has to calculate the current response on the electromagnetic field perturbation to get the evolution equation. This current response $j$ should be substituted into the Maxwell equation for $\textbf{rot} \; \textbf{B}$. In the geometry under consideration the $y$-component of this equation is of interest, thus, the $j_y$ component has to be derived. The derivation procedure is similar to the presented in [8] one.

The long-wavelength limit $ck / \nu_1,2 \ll 1$ is considered, thus, one can expand the components of the dielectric permittivity and conductivity tensors in the power series in $ck / \Omega$ and retain only a few terms of the lowest orders. The dispersion and dissipative effects are of great interest for the dielectric permittivity and conductivity tensors in the power series in $ck / \nu_1,2$. The derivation procedure is similar to the presented in [8] one.

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At the first stage of the derivation the system of kinetic equations is linearized, and their Fourier transforms (FTs) are taken. After the substitution of (2) into the derived equations, one can obtain system of eight algebraic equations for $f_{\alpha \lambda \omega k}$, where $\alpha = (p, e)$, via multiplying of the equations on 1, $\sin \theta \cos \phi$, $\sin \theta \sin \phi$ and $\cos \theta$ and integration of the result over angles. The solution of this system provides the FT of the distribution function response on the electromagnetic field perturbation. This allows to evaluate the components of the linear current response FT and therefore the components of the $\sigma_{ij}$ and $\epsilon_{ij}$ tensors.

We suppose here that $\delta \nu^2 = (\nu_1 - \nu_2)^2 / \Omega^2 \ll 1$. While deriving the solution of the dispersion equation and the current response we expand them into the power series in $\delta \nu^2$ and keep only the lowest-order terms of zeroth and first orders. Also, all calculations are performed in the ultrarelativistic limit. The solution of the general dispersion equation is sought in the form:

$$\omega = Vk - i \tilde{\chi} k^2 - \mu k^3$$

(3)

Using (1) and the condition $ck \ll \nu_1,2$, one can perform the third-order expansions of $\sigma_{ij}$ mentioned above. After substitution of the results in the dispersion equation one can substitute them into the equation for $\omega$ (3). After disregarding of all the terms of the fourth and elder orders of $ck / \Omega$ and equating of the coefficients at each power of $ck / \Omega$ to zero, the equations for the coefficients of (3) can be obtained. The derived equations for the phase velocity $V$, the dissipation coefficient $\tilde{\chi}$, and the dispersion coefficient $\mu$ are:

$$\frac{V^2}{c^2} = u^2 = \frac{1}{3} + \frac{U_0^2}{3} + \frac{2}{3} \left(1 + \frac{2 \beta}{1 + 2 \beta} - 2 \beta \delta \nu^2\right)$$

(4)

$$\tilde{\chi} = X_0 \frac{c^2}{2u^2} U_0 \left( U_0^2 - \delta \nu^2 u^2 c^2 \frac{c^4}{e^2} \left(2 + \frac{c^2}{c^4}\right) \right)$$

(5)

$$\mu = A_0 \frac{c^3}{2u^4} U_0^4 \left[ 1 - 4a^2 \left(1 - \frac{1}{3u^2}\right) + \delta \nu^2 \left(6 - 2 \frac{c^2}{e^2} - \frac{c^4}{e^4}\right) + 8a^2 \delta \nu^2 \left(\frac{c^2}{e^2} - \frac{2}{3}\right) \right] + 2a^2 \delta \nu^2 \frac{c^3}{e^2} A_0 \frac{U_0^4}{u^4} \left[ \frac{c^4}{e^4} - \frac{U_0^4}{u^4} \left(3 - 2 \frac{c^2}{e^2}\right) \right] + V d^2_\chi$$

(6)

where $d^2_\chi$, $U_0^2$, $X_0$ and $A_0$ are defined as follows:
\[ d^2_x = \delta v^2 X_0^2 \frac{c^2}{3u^4} \left[ \left( 2u^2 - \frac{1}{3} \right) \left( \frac{1}{3u^2} - \frac{2c^2}{c^2} \right) - u^2 \left( 1 - \frac{c^2}{c^2} \right)^2 \right] - \]

\[ - \frac{\chi X_0}{18u^4} \left[ 1 + 6u^2 - 27u^4 - \delta v^2 (2 + 15u^2 - 81u^4) + \delta v^2 \left( 1 - \frac{c^2}{c^2} \right)^2 (27u^4 - 3u^2) \right] - (7) \]

\[ \frac{\delta v^2 X_0}{9u^4} \left( 1 - \frac{c^2}{c^2} \right) (1 + 9u^2 - 54u^4) - \frac{\chi^2}{2c^4u^2} \]

\[ U_0^2 = \frac{2}{3} \left( 1 + 2\beta \right)^2 - 2\beta \delta v^2; \quad X_0 = 10a \frac{\omega_p^2}{\omega_B^2} \langle \gamma^2 \rangle U_0^2; \quad A_0 = 6 \frac{\omega_p^2}{\omega_B} \langle \gamma^3 \rangle U_0^2 \quad (8) \]

Here \( a = (\nu_1 + \nu_2) / 2\Omega \), \( \omega_p = (4\pi e^2 n_0 / m_e)^{1/2} \) is the plasma frequency, \( n_0 = \int f_0 d^3p \) is the equilibrium number density of all kinds of pairs, where integration is performed over the momentum space, and \( \omega_B = eB_0 / mc \). The angle brackets denote the averaging over the distribution function: \( \langle (\ldots) \rangle = \int (\ldots) f_0 d^3p / n_0 \). The typical velocity \( c_A \) — the ultrarelativistic analogue of the Alfven velocity — is defined as \( c^2 / c_A^2 = 1 + 2\beta \), where \( \beta = 8\pi P / B_0^2 \) and \( P \) is the particle pressure. We also introduce the dispersion length \( a_D = |\mu| / V \).

Using the obtained equations for conductivities, one can derive the FT of the linear part of the current response. To express it only via the magnetic field of the perturbation \( b \) one may use the FTs of the Maxwell equations.

At the second stage the nonlinear correction to the current response is evaluated. The procedure is analogous to the discussion in [8] one. Again the system of partial differential equations is reduced to the algebraic system via the Green functions evaluations and the expansions into series over small \( \omega / \Omega, \omega / \nu_{1,2} \) parameters, such that only the terms of the first non-vanishing order in the non-linear current response are kept. Finally, after the substitution of the sum of the linear and non-linear current responses into the \( y \)-component of the Maxwell equation for \( \text{rot} \ B \), one can obtain the evolution equation for the magnetic field perturbation. We present it in the dimensionless form:

\[ \partial_x h + \partial_{\xi} h + \frac{\mu}{|\mu|} \partial_{\xi}^2 h + \lambda h \partial_{\xi} h = \chi \partial_{\xi}^2 h \quad (9) \]

Here \( h = b / B_0 \), while \( \lambda \) and \( \chi = \chi / Va_D \) — dimensionless coefficients; \( \tau = Vt / a_D \), \( \xi = x / a_D \) — dimensionless time and coordinate. Coefficient \( \lambda \) is a cumbersome functional of \( f_0 \) depending on \( B_0 \) and on the frequencies \( \nu_{1,2} \).

3. The phase velocity of structures and the pressure of plasma

The dependence of the phase velocity \( (4) \) on the ratio of the particle pressure to the magnetic field pressure \( \beta = 8\pi P / B_0^2 \) is of interest for the physics of PWN plasma. In Figure 1 we show the values of the phase velocity \( V \) at various \( \beta \). The results are presented for \( \delta v = 0 \) and \( \delta v = 0.4 \). As one can see from (4), the dependence of the phase velocity on \( \delta v \) is monotonic for \( 0 \leq \delta v \leq 1 \) and for small \( \delta v^2 \ll 1 \) is not very strong, which is demonstrated in Figure 1.

The velocity of propagation of a small amplitude perturbation described by (9) is mainly defined by the phase velocity. From (4) and Figure 1 one can see that the value of \( V \) significantly decreases with the growth of \( \beta \). For \( \beta \to 0 \) it tends to the limit \( V = c \), while at large \( \beta \) it approaches to \( c / \sqrt{3} \). This sensitivity of the phase velocity to the particle pressure can allow to impose the constraints on the PWN plasma pressure via the comparison of the results of the modeling with the observed velocities of propagation of dynamic structures.
Figure 1. The phase velocity $V$ for various values of $\beta = 8\pi P/B_0^2$ at $\delta\nu = 0$ and $\delta\nu = 0.4$.

4. The dissipation coefficient and the ion component of plasma
The dependence of the dissipation coefficient (5) on $\delta\nu = (\nu_1 - \nu_2)/\Omega$ also may help to get some constraints on the plasma parameters. This is illustrated in Figure 2, were the values of the dimensionless dissipation coefficient $\chi$ at various $\nu_1$,2 are presented. Figures show the values of $\chi$ at various $\nu_1 = \nu_2 + \delta\nu/\Omega$: $\delta\nu$ is varied, while $\nu_2$ and $\Omega$ are fixed.

Figure 2. The value $\chi_{(-2)} = \chi/10^{-2}$, where $\chi$ is the dimensionless dissipation coefficient, at various $\nu_1$ for $\beta = 8\pi P/B_0^2 = 10$, $B_0 = 200\mu$G. The energy distribution is the broken power-law with the indices $s_1 = 1.35$ for $100 \leq \gamma \leq \gamma_b = 7.7 \times 10^5$, $s_2 = 2.2$ for $\gamma_b < \gamma \leq \gamma_{max} = 10^9$. The value of $\nu_2$ is fixed, the red points at each curve mark the values of $\chi$ for $\nu_1 = \nu_2$.

One can see that the increasing of the absolute value of $\delta\nu$ provides remarkable variation of $\chi$. Indeed, in the case of $\nu_2 = 0.3\Omega$ (Figure 2a) the increasing of $\nu_1$ by $0.3\Omega$ gives growth of $\chi$ by a fourth (in comparison with the value for $\nu_1 = \nu_2 = 0.3\Omega$), while for the shift with $\delta\nu = -0.2$ one finds a 1.7 times reduction of $\chi$. For the case of $\nu_2 = 0.1\Omega$ (Figure 2b, dot-dashed grey line) increasing of $\nu_1$ by $0.3\Omega$ leads to twofold increase of $\chi$, and the tenfold reduction of $\nu_1$ (down to $\nu_1 = 0.01\Omega$) results in nearly twofold decrease of $\chi$. For the cases of smaller values of $\nu_2$ (see Figure 2b) increasing of $\nu_1/\Omega$ by $\delta\nu = 0.2 - 0.3$ provides even more significant variations of $\chi$: it becomes $3 - 4$ times larger in the case of $\nu_2 = 0.03\Omega$ (dashed green line) and about 10 times larger in the case of $\nu_2 = 0.01\Omega$ (solid blue line).
The present consideration is limited by the approximation $\delta \nu^2 \ll 1$, but even in the framework of this treatment one can find significant variations of $\chi$ caused by the deflection of $\delta \nu$ from zero. Thus, if the influence of the ion component on the scattering processes shortly described in Section 2 can cause sufficient variations of the frequency of scattering of positrons, $\chi$ can be significantly affected. The value of $\chi$ determines the character of evolution of the magnetic field perturbation, described by (9): the evolution of its profile depends on the relation of the dispersion and dissipative terms [9]. Also, the dissipation coefficient, obviously, defines the characteristic distance of damping of the perturbation. The synthetic synchrotron images of the models of nebulae with the considered magnetic structures giving the spatial distributions of the synchrotron emission intensity of nebular particles should reflect such manifestations. On the other side, the observations reveal dimming and deceleration of the wisps [4]. Due to these reasons, modeling of magnetic structures, propagating in PWN plasma, and comparison of the synthetic synchrotron images with the observations may help to get observational constraints on the composition of plasma. This may give an answer to the question of presence of ions in the PWs. The crucial issue here is how large the shift of the frequency of scattering due to the influence of the ions can be. It requires modeling of the processes of scattering.

5. Conclusions
In this paper the kinetic model of local dynamics of magnetic field perturbation in highly non-equilibrium relativistic PWN plasma is briefly considered. The analysis of the obtained equations for quantities defining the features and the evolution of magnetic structures is presented. The dependencies of these parameters on the pressure of PWN plasma and the parameters of scattering of pairs by the stochastic magnetic field fluctuations are treated. These dependencies can allow to get some observational constraints on the PWN plasma parameters. In particular, the dependence of the phase velocity of the perturbations on the particle pressure of plasma can provide constraints on the plasma pressure. Also, on the base of the mechanism of acceleration of pairs in the pulsar wind presented in [3], we suppose that the presence of the ion component in the pulsar wind can affect the parameters of scattering of pairs. If the typical frequencies of scattering are affected strongly enough, the substantial change of the dissipation coefficient value can occur. Thus, the constraints on the particle pressure and the composition of the PWN plasma could be imposed via the comparison of the simulated spatial distributions of the nebular synchrotron emission intensity with the observations. In order to implement this for the problem of plasma composition, the modeling of the processes of scattering by the stochastic magnetic field fluctuations is required. Thus, the kinetic modeling of the dynamical structures, like wisps, can help to answer some questions about the PWN plasma parameters important for the hot issues of mechanisms responsible for the energy conversion in the relativistic PWs.

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