Modelling Brazil Nut Effect Phenomenon of Boulders on Asteroid Surface

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Abstract. The surface of an asteroid is full of boulders that varies in size. One of the phenomena happening in the asteroid’s surface is a tendency for bigger boulders to come up to the surface. This phenomenon is called Brazil Nut Effect (BNE). Some asteroids, like Itokawa and Eros, are known for having that tendency and BNE is supposed to happen in both of them. This study conducts a modelling of the phenomenon happening on the surface of an asteroid for mimicking the concept of BNE. Physical parameters used in this study follow a model of a common asteroid. The very first step of the modelling is to generate an initial condition of particles. Then, these particles are dropped into the surface of an asteroid model. After these particles reached the dynamical relaxation, the additional force will be added into the system which is represented by some sinusoidal oscillation formulas. Modelling the concept of BNE which only involves collisions of inter-particles and gravity factor indicates that the rising of bigger boulders to come up to the surface is likely to occur. Nevertheless, there are many other parameters that have a role in the low-gravity environment and need to be explored more.

1. Introduction

The surfaces of most asteroids are covered in the pile of boulders and gravels. This pile of boulders and gravels is called regolith. Regolith can be considered as a system of granular particles, or a system that consists of particles in the shape of grains. This kind of system is dynamically different from the other system of particles in the response of some external forces. The granular system will reach the dynamic relaxation in a longer period of time. This can happen because the particles will adjust their position even after the shaking [1].

The mixture of particles of different sizes will segregate themselves based on their sizes. The bigger particles will tend to come up to the surface. This size segregation phenomenon is called Brazil Nut Effect (BNE) [2]. Because of the shaking, the position configuration of the particles will be changed. Smaller particles will be easier to move downward and fill the gap that cannot be filled by the bigger particles. As a result, the bigger particles will move upward when the smaller ones fill the gap downward.

Image taken by Hayabusa spacecraft shows us the detailed picture of the surface of Itokawa. From the image, we know that the surface of Itokawa is full of bigger boulders and there are not...
Table 1. Simulation parameters used in this work.

| Parameter                  | Used Module |
|----------------------------|-------------|
| Gravity                    | Basic       |
| Collision                  | Direct      |
| Boundary                   | Periodic    |
| Integrator                 | Leapfrog    |
| Time-step of integration   | $10^{-4} \text{s}$ |

Table 2. Physical parameters from model asteroid and boulder particles

| Physical parameters                  | Value              |
|--------------------------------------|--------------------|
| Radius of model asteroid             | 320 m              |
| Radius of large boulders             | 2 cm               |
| Radius of small boulders             | 1 cm               |
| Density of the asteroid and the boulders | 280 kg/m$^3$    |
| Coefficient of restitution          | $v^{-0.234}$       |

many crater impacts found there [3]. It is predicted that particles in Itokawa undergo some size sorting process by seismic vibration caused by the impacts. Although it shows the evidence of some size sorting process, Itokawa does not show a complete one. In some areas, we can spot big boulders and fine grains as well.

In this paper, we will conduct a modelling of a phenomenon taking place in the surface of an asteroid mimicking the concept of BNE.

2. Model and Simulation

2.1. REBOUND

REBOUND is an open-source software that can be used for conducting N-body simulation associated with collisions including molecular dynamics and granular flow [4]. This software is written in C and can be run at Linux, Unix, and Mac OS.

The users can choose the parameter of simulation as they need in order to meet with their purpose. The simulation parameters used in this work are listed in Table 1.

2.2. Model Asteroid and the Boulders

In this work, we adopt physical parameters from one of the rubble-pile asteroid, Itokawa. This physical parameters are listed in Table 2.

For this work, we use the total of 1200 particles with 3:37 ratio of bigger and smaller boulders. To obtain the initial coordinates of the boulders, we put the particles in three-dimensional Cartesian coordinates. These particles are distributed to six rows and six columns. The gap among every particles follows Equation 1,

\[
\begin{align*}
  d_x &= r_s + r_l + f \\
  d_y &= r_l + f \\
  d_z &= 2 \times r_l + f
\end{align*}
\]

where $d_x$, $d_y$, $d_z$ are the gaps between particles in three-dimensional coordinates, $r_s$ and $r_l$ are the radius of the small and large boulders, respectively, with $f$ is the maximum value of random
Figure 1. Initial position configuration of the particles after added by random factor. The red and blue dots are representing small and large boulders, respectively.

factor. These equations prevent the particles to be intersected. Random factor is added so that the configuration of particles is no longer perfect in symmetry. The illustration of the configuration is presented in Figure 1.

2.3. Seismic Vibration Simulation

These particles are then dropped into the surface of the model asteroid and when the dynamical relaxation is reached, seismic vibration will be added. Seismic vibration is modelled with ordinary and damped sinusoidal function.

The ordinary sinusoidal function used in this work is written as in Equation 2,

$$z = 2A \sin(4\omega t).$$

where $z$ is the z-coordinate of the model asteroid, $A$ is the amplitude of the vibration that is the radius of the small boulders, $\omega$ is the angular frequency, and $t$ is the time in seconds. The value of $\omega$ used in this work follows [5] and is expressed in Equation 3

$$\omega = 3\sqrt{\frac{g}{A}}$$

with $g$ is the gravitational acceleration of the model asteroid.

Another model of vibration uses a damped sinusoidal function. This kind of function is applied to make the model more realistic. As we know, the amplitude of the seismic vibration
The black dashed vertical line is drawn at $t = 3600$ s.

dwindles over time until it stops. The damping function used for this work is inverse hyperbolic cosine ($\text{acosh}$), so that the modified equation as follows

$$z = k_1 \times (k_2 - \text{acosh}(t)) \sin(4\omega t),$$

where $k_1$ is a damping constant, dan $k_2$ is a translation constant. The value of $k_1$ and $k_2$ used are 0.15 and 9, respectively. This equation and the values of the constant chosen are arbitrary. The main reason for choosing the equation and the constant is to create a vibration that is not damped too fast.

3. Result

The output from REBOUND is a file consisting of the number of boulders, time, and the position of each boulder in three-dimensional Cartesian coordinate. The height evolution of large and small boulders is checked from the position taken from the output.

3.1. Model with Ordinary Sinusoidal Shaking

The height evolution of the boulders using the ordinary sinusoidal function as vibration model is shown in Figure 2. At the beginning of the simulation, all of the large boulders placed on the surface of the asteroid. After the shaking started, the height of large boulders increases slowly. The height of small boulders is relatively constant over the time. When the time reached 4400 s, the upper limit of the large and small boulders increase suddenly. The upper limit of the large boulders becomes higher than the limit of the small ones. This indicates that in this time, the height of some large boulders surpassed the small ones. Nonetheless, not all of the large boulders come up to the surface. So, it can be said that the incomplete form of BNE is reached.

3.2. Model with Damped Sinusoidal Shaking

The height evolution of the boulders using the damped sinusoidal function as vibration model is presented in Figure 3. At the beginning of the simulation, the large boulders are already surpassed the lower limit height of the small ones. When the dynamical relaxation is reached, the surface of asteroid filled by the small boulders only. After the shaking started, the large
boulders come up gradually and then the height becomes constant. The height of the small boulders is relatively constant. This condition lasted until the time of 3800 s. The pattern recurs after that. However, the height of large boulders never exceeded the average height of the small ones in this simulation timescale.

4. Discussion and Conclusion
The vibration of the seismic shaking is modelled by the mathematical function of the ordinary and damped sinusoidal function. We can see the pattern of BNE in these two models of shaking. The model with ordinary sinusoidal function shows a greater effect of shaking than the damped one. The gradually decreasing amplitude of damped shaking slows the BNE down, so the height of large boulders can not surpass the height of the small ones. So it can be stated that the damped shaking will lengthen the timescale of BNE to occur.

This work involves only inter-particles collisions and gravity as the factors that affect BNE phenomenon. With considering both of them, the pattern of BNE can be seen although not all of the large boulders come up to the surface. These two factors can be set as minimum factors for the BNE to occur.

Modelling BNE in an asteroid or another low gravity environment is not that simple. There are many parameters other than inter-particles collision and gravity that need to be explored further.

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