Questioning corona—a study and research path

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Abstract

We present a so-called ‘study and research path’, designed for advanced high school teaching in the context of a master’s thesis project, along with an outline of the mathematical and didactic background. The aim of the paper is to present and discuss an adaptation of Chevallard’s ‘paradigm of questioning the world’ to currently prevailing institutional conditions and in particular to show how mixed mathematics may be revived at a time of crisis, such as the coronavirus outbreak during which the specific design was developed.

1. Introduction

March 2020 witnessed a seldom moment of sudden, worldwide attention to one question, being universally viewed as essential to the well-being of humanity and its societies:

\[ Q_0: \text{What can be done to stop the spread of COVID-19?} \]

One could certainly formulate \( Q_0 \) in many different ways, leaving open whether anything can be done, etc. Notwithstanding such nuances, governments were under strong pressure to come up with swift answers. In many societies, such as the authors’, a central part of the answer given was to ask the citizens to stay at home, except for the workers needed to sustain critical institutions such as hospitals and supermarkets.

Danish universities and schools were asked to function, as best they could, via the Internet. This is where the present essay begins, with the first author being in the midst of a half-year graduate research project, constituting the final piece of her MSc degree in mathematics. Such a degree is the path to become a secondary school teacher in Denmark. At the University of Copenhagen, students interested in this career path may choose to do their master’s thesis in the didactics of mathematics. This typically involves developing and experimenting a more or less advanced design for secondary mathematics teaching, based on a thorough preliminary analysis of the mathematical content involved and a no less stringent analysis of the observations of student mathematical activity during the experiment. The second author has supervised such projects since 1996, including the project just mentioned.
By the end of February 2020, the first author had finalized the design and preliminary analysis of a ‘study and research path’ (see the next section) for upper secondary school and was ready to experiment it from early March. Only the very first session was realized before all schools closed. What to do? With no prospect of realizing the data collection as planned, we agreed on compensating for the lack of experimentation by extending the theoretical parts of the thesis, to involve the design and preliminary analysis of two related study and research paths. The first one was already constructed and concerned the (few) mandatory subjects in combinatorics in Danish high school, essentially the addition and multiplication principles in counting and binomial coefficients. The idea was to study these matters while investigating a wider question, ‘how to choose a secure password’. The new, second, study and research path was to be somewhat more ambitious, both in terms of the mathematical contents and the question investigated—essentially $Q_0$ above, with a deliberate emphasis on aspects of the question that could be amenable to mathematical answers.

The purpose of this essay is to share the main outcomes of this second part of the project, which we propose as an instance of the paradigm of ‘questioning the world’ (Chevallard, 2015) motivated and forced upon us by the COVID-19 epidemic. We first outline the didactic framework (including the notion ‘study and research paths’) and some of the scholarly background. Then we present and analyse the design developed. To conclude, we briefly discuss the perspectives for relating mathematics education with burning societal questions.

2. Didactic framework: mixed mathematics through study and research paths

In his seminal lecture to the twelfth International Congress on Mathematical Education, Chevallard (2015, p. 186) argued the need to **revive the epistemological spirit of mixed mathematics, although without any cultural arrogance, but with the political and social will necessary to revitalize the idea that mathematics is for us, human beings, a solution, not a problem.**

Here, the idea of ‘mixed’ mathematics—which goes back at least four centuries (Brown, 1991)—reflects that a problem or question is studied through an integrated effort of mathematical inquiry and other forms of inquiry, such as more or less systematical experiments; classical mechanics is often cited as a historical example. This idea can be opposed to the later idea of ‘applied’ mathematics, which is certainly more familiar today: it refers to viewing pure mathematics as a source of ‘tools’ which can be packed and delivered for use in non-mathematical investigations in other disciplines—while, in these investigations, mathematical activity is restricted to ‘application’. This division of labour may be reasonable in highly specialized scholarship, but when transferred uncritically to the didactic realm, the result may appear highly unreasonable, as elements of ‘pure’ mathematics get taught to a general population with no sight of what it may be good for. Indeed, the *visiting of mathematical works has become increasingly unsuited to people’s needs and wants, up to the point that there currently exists a widespread belief that mathematical knowledge is something one can almost altogether dispense with* (ibid., p. 177). As a result, curves illustrating the potential scenarios for the spread of COVID-19 are displayed in our media as more or less impenetrable signs of a knowledge reserved for experts, not as models that educated populations could relate to and might even question, given their impact on everyone’s lives.

By contrast, the **paradigm of questioning the world** (ibid), pursued by the research programme known as the anthropological theory of the didactic (see Chevallard, 2019) for well over a decade, aims at reinserting mathematics in a wider project of general education: asking *questions* and seeking *answers*
which are important to individuals, to societies and to humanity at large. We will now briefly outline one practical (and, at the same time, theoretical!) notion which this programme proposes and which has, in particular, been invested in the first author’s master’s project.

This is the notion of study and research path (Chevallard, 2006; Winsløw et al., 2013). Viewed as an activity, it can be represented in various ways: by a so-called Herbartian schema (Chevallard, 2019, p. 100) or as a so-called QA-tree (Winsløw et al., 2013, and figures appearing later in this essay). The last representation is somewhat simpler and, therefore, reveals less (in this paper, we even omit the answers for simplicity). It emphasizes the non-linear dynamics in which questions and answers are produced—or could be proposed—by a study community. The process starts from a generating question (often named \( Q_0 \)) for which this community seeks answers, or at least partial answers, and which appears not only meaningful but also of some importance to the community. It can be safely assumed that the example of such a question, which was proposed at the entry of this essay, holds this quality for many people, including pupils, at the time this is written. It should be noted that with this and many other examples of important questions, one cannot expect all answers to result from a single discipline or school subject. Indeed, we might expect many important elements will have to come from medicine in some form, such as efficient immunization programmes. This is one of the main assets—and challenges—of working with study and research paths: working with ‘big’ questions usually requires one to step out of the comfort zone of a single specialty. The fact that health specialists display the aforementioned ‘curves’ to explain why certain restrictions are imposed on entire populations certainly leads us to believe that mathematics is somehow a part of the solution.

Thus, in general, a study and research path begins with examining what we do know about the larger question, e.g. from media (including news media but also accessible sources on the internet). These media, together with our own experience and first ideas, then prompt us to pose derived questions \( Q_1, Q_2 \) and so on. Some of these may already be answered in accessible media (these could be named \( A_k \)) while others appear as intractable as the first one. Reducing the first questions to special cases, reformulating them or otherwise producing new derived questions based on these investigations make the tree grow further. In the process, questions or answers appear which pertain, at least to some extent, to mixed mathematics. Certainly, mathematics teachers will be alert to where it does—not to cancel the general inquiry and just focus on these parts, but to encourage and support further examination of how these parts may enlighten the larger inquiry. In the case of our example, there is little doubt such opportunities will occur, as we shall now see. We consider that any study community will pursue a question under certain conditions and constraints and that it is perfectly legitimate to dwell on parts that are of particular relevance or importance to that community. In the case of upper secondary mathematics students and their teacher, that may well mean that one dwells on answers with a strong mathematical component, as long as the generating question, together with other relevant parts, is kept firmly in mind. To prepare our presentation of a study and research path (SRP) on a question that is derived from \( Q_0 \), we first briefly present some main elements of ‘mixed mathematics’ that it could be designed to enable students to study.

### 3. Mixed mathematics of relevance to \( Q_0 \)

During the first weeks of the corona pandemic, many people learned, through the media, of two somewhat bell-shape ‘curves’ which were explained by experts and politicians to describe the possible spread of the virus and used to justify wide-ranging political decisions. Both curves come from the so-called SIRD-model (susceptible-infected-recovered-dead), whose origin goes back to around 1930 (e.g. Kermack & McKendrick, 1927). The curves show how the number of sick (infected, but not recovered) people
develops over time according to this model. A first public step to answer $Q_0$ above thus seems to be to pose the derived question.

$Q_1$: How does the number of sick people develop over time?

and of course also

$Q_2$: How deadly is the virus (what fraction of infected people die)?

Both questions are highly intricate, as the number of infected people is not easy to estimate, even with a reliable diagnostic test; for COVID-19, it also appears that people with other medical conditions are more likely to die, so that the cause of death may not be unique.

### 3.1 The classical model

We now briefly look at the mathematics behind those curves: the SIRD-model. As the name says, the model involves four possible states of the members of a population: susceptible (people who can get the disease), infected (people who have it), recovered and dead (from the disease we look at). The number of people in each of these states at time $t$ is modelled as continuous functions $S$, $I$, $R$ and $D$. The model is then the non-linear system of differential equations

$$\frac{dS}{dt} = -\beta IS; \quad \frac{dI}{dt} = \beta IS - \gamma I - \mu I; \quad \frac{dR}{dt} = \gamma I; \quad \frac{dD}{dt} = \mu I$$

where $\beta$, $\gamma$ and $\mu$ are positive constants: $\beta$ is a measure of how infectious the disease is, $\gamma$ the rate at which infected people recover (and become immune) and $\mu$ the rate at which they die. If nobody dies from the disease ($\mu = 0$), one gets the slightly simpler SIR model.

The meaning of these equations is certainly within reach of high school mathematics in Denmark, and the numeric solutions can be graphed using CAS-systems frequently used there. Running the model in some sense enables to produce rough answers to $Q_1$ and $Q_2$, namely the function $I$ and the constant $\mu$, respectively. Even if we assume the model is adequate in a given context, there are many unknowns to be determined from data before the system above can be solved, and $\mu$ is certainly one of them.

The constant $\beta$ is important, at least qualitatively: from the first equation $\frac{dS}{dt} = -\beta IS$ we see that it expresses how quickly susceptible people are infected, as the rate of change is assumed proportional to $IS$. This is justified by assuming that the number of ‘meetings’ between a susceptible and an infected person is proportional to $IS$ and that each such meeting results in infection with a fixed probability. The first assumption is not realistic unless all infected and susceptible have some possibility of meeting each other; in a large population spread over long distances, this is hardly the case. Therefore, this is one point where the classical model is unrealistic, and a main point of this paper is to point out alternatives, and show how they can be approached in study and research paths for high school students.

Before we go into that, let us return briefly to $Q_0$. Assuming the SIRD-model, a simple answer is to reduce the constant $\beta$ (to zero, if possible). From the discussion above, it follows that $\beta$ hides a product of two probabilities: the probability for an infected and a susceptible person to meet, and the probability that infection occurs at a meeting. The last factor is inherent to the disease; what one can try to reduce is therefore the first factor, the probability of meeting. Indeed, this is what containment strategies, deployed in many countries throughout the world, try to do. Isolation of all members of the population (1 by 1) clearly gives $\beta = 0$, independently of the disease, and is clearly unrealistic. More refined techniques to reduce meeting probabilities are naturally needed and to some extent implemented. As already mentioned, it is not realistic that the any pair of people in the population are equally likely
to meet, and many strategies in fact go in the direction of making this distribution of probabilities even less homogenous, for instance by isolating people who have been identified as infected.

### 3.2 More recent modelling

Models of more recent date consider populations as complex networks through which an infectious disease spreads (e.g. Newman, 2002). Each individual is represented by a vertex; if the individuals are ‘related’, an edge connects the corresponding vertices. The number of edges connected to a vertex is the vertex degree, and the whole graph ‘maps’ the passageways of the disease spread. The classical SIRD model assumes ‘full mixing’ of the population; numerical computation allows replacing this by adapting the classical model to networks with a particular (vertex) degree distribution and even to vary the probability of disease-causing contacts between connected vertices (to reflect that relations among people may be more or less close).

These models are particularly relevant when considering possible countermeasures to an epidemic. Quarantine, close down of institutions and vaccination all work by removing some edges—in the case of vaccination, all edges from the immunized vertices. This can divide the network into disconnected components and limit the outbreak of an infectious disease.

Vaccination, if possible, is a desirable solution, while it may take time and be costly. However, the entire population does not need to be vaccinated if the network is broken up sufficiently (herd immunity) (Newman, 2010, p. 604). An important question is how to do this best, and we believe students would be interested in it without much preparation, at the time this is written:

**Q3:** Once we have a vaccine for COVID-19, who should be vaccinated first?

Extensive research on immunization strategies has been conducted to answer this question (e.g. Cohen et al., 2003; Liu & Miao, 2009). Three important strategies are random immunization, targeted immunization and acquaintance immunization. The simplest strategy is random immunization. However, as human contact networks are heterogeneous, i.e. display a broad degree distribution (Barrat & Vespignani, 2008), it is unlikely to be optimal. Some individuals have many human contacts (higher degrees) and thus high odds of spreading the disease. In targeted immunization, the highest-degree vertices are removed (vaccinated). However, this strategy requires knowledge of all vertex degrees, which renders targeted immunization practically impossible.

Acquaintance immunization is a compromise between the two and calls for vaccination of neighbours of random vertices. One would think that this strategy is as ineffective as random immunization. However, acquaintance immunization is based on the fact that the mean number of friends of individuals is less than or equal to the mean number of friends of friends. This ‘friendship paradox’ was first formulated by Feld (1991). Specifically, the mean degree, ⟨k⟩, is bounded by the mean degree of neighbours, ⟨k⟩FF. In fact, ⟨k⟩FF = ⟨k⟩ + σ^2/⟨k⟩, where σ^2 is the degree variance. Given the high variance of degree distributions in human networks, the friendship paradox implies the relative effectiveness of acquaintance immunization.

The three strategies have also been investigated through simulations (numerical calculations) based on specific networks with various degree distributions (e.g. Cohen et al., 2003; Liu & Miao, 2009), confirming that acquaintance immunization often significantly outperforms random immunization. This is also supported by empirical evidence, such as the study by Christakis and Fowler (2012), who monitored the 2009 outbreak of H1N1 (swine flu) among randomly selected Harvard students and friends nominated by the random group.
4. Study and research path on $Q_3$: design

We now outline the design of a study and research path, with $Q_3$ as the generating question. This is clearly a question that could lead to a genuine activity of mixed mathematics. Our choice of this more limited generating question is also related to a need to be able to predict the mathematics involved in the main answers, at least under current high school conditions such as time pressure and monodisciplinary teaching. Of course, this gives the activity some flavour of ‘visiting works’. In particular, we aim at students identifying, formulating and proving the friendship paradox and explaining how it can be used in immunization strategies. The students should experience how a seemingly non-mathematical statement is formalized and proved and how mixed mathematics involving graphs can be used to answer $Q_3$—in particular, to learn about different simple strategies and compare their efficiency and feasibility. At the same time, the SRP involves ‘questioning the world’ as $Q_3$ is perfectly genuine and the students are given full autonomy to propose initial answers, derived questions and so on, which are subsequently used in the more directed activity involving the friendship paradox.

The design consists in two 90 min lessons during which the students will work in groups (Figs 1 and 2) and a written assignment (Fig. 3). In the lessons, the teacher initiates the path and directs ‘conferences’ where students present their first takes, based on web searches and discussions. The groups are supposed to work autonomously in between conferences, while the teacher directs the choice of (or even helps formulating) questions to retain for further work. The lessons help the students gain familiarity with $Q_3$ and some of the derived questions and quick answers that can be generated or found rather easily. In the written assignment, students are invited to make a more precise analysis of some of the points found in the lessons. The aim of this overall design is thus to link the last part, where more precise mathematical models are worked on, with the broad initial questions and what can be generated from it through (mostly) oral discussion.

5. Study and research path on $Q_3$: a priori analysis

In Fig. 2, a rough map is presented of the questions and answers that we anticipate could likely occur in the initial work with the generating question, $Q_3$, during the two lessons. Some of the questions are asked by the teacher if needed, to prepare for the written assignment.

Lesson 1 begins with the teacher’s introduction of the generating question. Then, the generating question is discussed in groups, with the explicit task to pose ‘subquestions’ that (if answered) could help answer $Q_3$. The students have access to the Internet. At conference 1, the groups present their subquestions. The teacher directs the discussion and could pose further subquestions like ‘is it necessary to vaccinate the whole population?’, in order to bring forward the idea of herd immunity. This is an important motivational factor for the students, since herd immunity is the ultimate goal of immunization strategies. The teacher illustrates the effect of vaccination using a simple drawn network, and how disease spread is inhibited when the network is divided into separate components. In this setting, she introduces the graph-theoretical vocabulary (vertex, edge, degree). This vocabulary is used in many online resources on the friendship paradox. Based on the presented network, the teacher asks, ‘Who should be vaccinated in this network?’ to ensure a first glimpse of targeted immunization is discussed. The teacher facilitates a discussion of the advantages and disadvantages of random and targeted immunization.

At the end of conference 1, the teacher asks, ‘What if we choose random people and ask them to point out a friend to be vaccinated? Would this be better than the random strategy?’ Maybe the students have already posed this question in some form so that the teacher only needs to focus attention on it. The following group work is expected to remain at an intuitive level and the students will likely have different
‘opinions’ and feel uncertain about the question, given the counter-intuitiveness of the ‘friend’ strategy. The groups will share their hypotheses, and it is likely that many think the strategies are equally effective.

At the end of conference 2, the teacher asks, ‘Is the mean number of friends of friends in a network larger than the mean number of friends?’ ($Q_1$). The students are asked to investigate this with a given network, which is the same as the one given in the written assignment (Fig. 3).

The students will possibly consider a few people and investigate whether these have fewer friends than their friends have. This will not answer $Q_1$, but it will make the students familiar with the dataset and lead to the question of what information they need to answer the question $Q_n$ as stated in Fig. 2. Some students may be able to calculate the mean number of friends and mean number of friends of friends at this point without further study ($Q_q$ and $Q_r$). Others may search the Internet: typing ‘mean number of friend and mean number of friends of friends’ or ‘does a person’s friend have more friends than the person does’ they could find the ‘Friendship paradox’ page of Wikipedia (2020). Here they learn more about the statement and may continue their search using its name. This can lead to YouTube videos such as ‘The Friendship Paradox: Why Your Friends Are More Popular Than You (and Why that Matters)’ (Star, 2019). Based on material like this, it is likely that at least some groups will produce a table corresponding to the given network (Table 1) and calculate that the mean number of friends of individuals is $12/6 = 2$ and the mean number of friends of friend is $30/12 = 2.5$.

At conference 3, the students present their work. If their calculations are correct, the mean number of friends of individuals and the mean number of friends of friends only differ by 0.5. Therefore, the students may be uncertain and possibly some students will ask whether it is just a coincidence. Otherwise, at the end of the conference, the teacher will bring up the question whether the ‘Friendship Paradox’ holds for arbitrary networks, as a more precise variant of $Q_1$. Top hits on Google when typing ‘friendship paradox proof’ are (Talwalkar, 2012) and (Wirth, 2018) along with similar pages; some of these will likely be
Fig. 2. SRP diagram. Q3 is the generating question. The dotted black questions are posed by the teacher if the students do not pose them. The green questions are expected be posed by the students, based on web searches and preceding questions. The blue ones are asked in the assignment.

Table 1. Number of friends and friends of friends

|       | Number of friends | Number of friends of friends | Mean number of friends of friends |
|-------|-------------------|-----------------------------|----------------------------------|
| Anne  | 2                 | 6                           | 3                                |
| Bob   | 4                 | 7                           | 1.75                             |
| Carl  | 1                 | 4                           | 4                                |
| Dolly | 2                 | 5                           | 2.5                              |
| Eric  | 1                 | 2                           | 2                                |
| Fred  | 2                 | 6                           | 3                                |
| Total | 12                | 30                          |                                  |
**Once we have a vaccine for COVID-19, who should be vaccinated first?**

Describe the three immunization strategies random immunization, targeted immunization and acquaintance immunization and discuss their advantages and disadvantages.

In the description of acquaintance immunization, include the friendship paradox. Based on the below dataset, perform calculations to formulate a hypothesis. Provide a proof.

When are the mean number of friends of individuals and the mean number of friends of friends equal? Construct three networks (high, low and zero variance), and compare the mean number of friends and the mean number of friends of friends in these. Do most people have fewer friends than their friends have? Is this always true?

Explain the connection between the friendship paradox and acquaintance immunization. In which of your networks do you think acquaintance immunization is most effective?

Can you think of other strategies or improvements of the three already discussed?

**Data**

- Anne is friends with Fred and Bob.
- Bob is friends with Anne, Carl, Dolly and Fred.
- Carl is friends with Bob.
- Dolly is friends with Bob and Eric.
- Eric is friends with Dolly.
- Fred is friends with Anne and Bob.

Fig. 3. Assignment given to the students after the completion of the two lessons.

found by all groups. It is expected that the students with some effort can follow the proofs on these pages. The proof on these pages relies on the formula $\text{Var}(X) = E((X - E(X))^2) = E(X^2) - (E(X))^2$ for a stochastic variable $X$, from which the identity $\langle k \rangle_{FF} = \langle k \rangle + \langle \frac{\sigma^2}{k} \rangle$ can be obtained. The students’ success in the proving activity will depend highly on their prior knowledge concerning mean and variance, since these formulas are applied in the proofs (in certain streams of Danish high school, one could simply posit that the moment formula for variance had been encountered prior to this SRP). Given the time the students are given to both find resources and work on the material found on the Internet, it is not certain that any of the groups will have complete a proof during the following group session. However, some may be able to present relevant parts and at least share their resources at conference 4. All students will later complete the proofs while working on the written assignment. During the fourth conference, the teacher ensures that first explanations of the friendship paradox are discussed and then facilitates a discussion of the paradox as an immunization strategy $(Q_p)$.

To launch the last group work, the teacher asks, ‘How can we compare the effectiveness of the different strategies?’ $(Q_y)$, and introduces an online simulation tool (Brockmann, 2018), which acts as a milieu
Fig. 4. Screenshots from (Brockmann, 2018). (a) The network of disease spread without vaccination. (b) Random immunization: vaccinating 38% at random. The red individuals represent a large component, wherein the disease spreads. (c) Targeted immunization: vaccinating the 38% with highest-degree vertices. (d) Acquaintance immunization: choosing 38% at random and vaccinating one of their acquaintances.

(Chevallard, 2019, p. 100) in which the students can perform experiments. The applet displays a network of 200 individuals (vertices) connected in one component. One chooses a fraction of the population and applies one of three strategies (random, targeted and acquaintance immunization). Thus, the students can visualize how the strategies work (Fig. 4) and ‘see’, for instance, that a large fraction of the population should be vaccinated to reach herd immunity in the case of random immunization.

In the applet, one can choose Erdős–Rényi or Barabási–Albert networks (Newman, 2010). These types of networks are not explained in detail, but the teacher can inform the students that the latter one has higher variance than the former (this is in fact also explained on the webpage). Thus, the students see how the variance affects strategy effectiveness: the effect of acquaintance immunization is largest in high-variance networks (see Fig. 5). In conference 5, the teacher ensures that this is connected to the expression from the friendship paradox: \( \langle k \rangle_{FF} = \langle k \rangle + \sigma_k^2 / \langle k \rangle \). The applet is to some extent a ‘black box’ for students, as they have no knowledge of the mathematical model that underlies the simulations. However, the students will get an intuitive sense of how the different strategies relate to type of the network.

After the lessons, the students are given a written assignment (Fig. 3) to finalize their work at home. From the expression \( \langle k \rangle_{FF} = \langle k \rangle + \sigma_k^2 / \langle k \rangle \), the students could derive that the mean number of friends of individuals and the mean number of friends of friends are equal if and only if the degree variance is zero \( (Q_u) \). The students will have to create such a network, and also networks with high/low variance, to calculate the two quantities for these networks. This gives further insight to how the variance of the network affects the means. Further, the students are asked whether most people have fewer friends than their friends have \( (Q_w) \)—this is a common misinterpretation (also present in some common webpages) which is important to address. The students may simply answer this question with a simple counterexample. Lastly, the students are asked to think of other immunization strategies or improvements
to those already discussed. The students can look for strategies online or think of some themselves. Again, the Internet provides many ideas, although accounting for their mathematical treatment is likely much beyond what can be expected from high school students.

The SRP of the present design is relatively directed and end-targeted given the clearly stated intention with the teaching. A more open SRP could leave more space for autonomous study and research which would be more in line with Chevallard’s paradigm of ‘questioning the world’. However, a less open approach is specifically chosen in order to accommodate the institutional settings present in high schools in Denmark (and many other countries). For instance, the possibilities of a teaching design are limited by time constraints and curriculum demands. Thus, the presented SRP represents a way to incorporate elements from the paradigm of questioning the world while still considering the setting wherein the teaching is to take place.

6. Concluding remarks

The master’s thesis project could not proceed to actual experimentation and a posteriori analysis of the proposed SRP, due to the very same epidemic that motivated its construction based on $Q_0$ and more specifically $Q_3$. Thus, the present essay has no data to present. However, we think that it is worthwhile to present the theoretical design for two reasons:

- It illustrates how ‘mixed mathematics’ may be pursued in a didactic study which, even if the empirical data is missing, shows the potential of directed questioning (in the form of a SRP) that links high school mathematics to burning issues in society;
- It proposes a more general approach to go beyond ‘toy questions’ posed only as an excuse to pose exercises, as the main purpose of the lessons is to familiarize students with burning questions and some (widely available) answers, while relegating a few more technical parts of the SRP to written assignments to be worked on after the lessons.

Of course, a much more open SRP could be imagined, based on $Q_3$ or even $Q_0$. These would involve a rich array of questions and answers to be explored, many of which have little or no mathematical aspects. The present design proposes and exemplifies an approach which is, in some sense, a compromise between the aspirations of the paradigm of questioning the world and the conditions currently available in mathematics teaching in upper secondary schools and consists of a first oral phase of exploring the question, followed by an answer-directed written assignment which is, however, clearly linked to derived questions from the first phase, and to the generating question.
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