Can the energy dependence of elliptic flow reveal the QGP phase transition?

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Abstract

Ideal hydrodynamic simulations are performed to compute the evolution with collision energy of hadron spectra and elliptic flow between AGS and LHC energies. We argue that viscous effects should decrease with increasing energy, improving the applicability of ideal fluid dynamics at higher energies. We show that the increasing radial flow at higher energies pushes the elliptic flow to larger transverse momenta, leading to a peaking and subsequent decrease of the elliptic flow at fixed $p_T$ with increasing collision energy, independent of whether or not there is a phase transition in the equation of state.

1. Introduction and summary

Ideal hydrodynamic simulations of the expansion stage of the hot and dense fireballs created in relativistic heavy-ion collisions predict a non-monotonic collision energy dependence of the ($p_T$-integrated) elliptic flow $v_2(p_T)$ [1]. The softening of the equation of state (EOS) at the quark-hadron phase transition leads to a predicted reduction of $v_2$ at RHIC energies, down from SPS energies, followed by another increase towards LHC energies. This effect is not seen in experiment [2] which shows instead a monotonic increase of $v_2$ with $\sqrt{s}$. This is now understood as a failure of the ideal fluid picture during the late hadron gas stage which is highly viscous and inhibits the buildup of elliptic flow [3,4]. Both viscous hydrodynamics [5] and hydro+cascade hybrid algorithms [3,4] reproduce qualitatively the experimentally observed monotonic beam energy dependence of the integrated elliptic flow. Viscosity, in particular its strong increase in the hadronic phase, thus washes out the phase transition signature in the integrated elliptic flow excitation function.

The PHENIX Collaboration observed that the $p_T$-differential elliptic flow $v_2(p_T)$, on the other hand, when plotted at fixed $p_T$ as a function of $\sqrt{s}$, shows signs of saturation at RHIC energies [6]. This has been interpreted as a possible remnant of the non-monotonic energy dependence predicted by hydrodynamics, signalling the softening of the EOS near $T_c$ and, possibly, even the existence of a critical end point (CEP) in the QCD phase diagram [7]. The apparent contradiction between a monotonically rising $v_2(\sqrt{s})$ as observed by NA49 and STAR [2] and a saturation within increasing $\sqrt{s}$ of the differential elliptic flow $v_2(p_T)$ is resolved by the observation that also the radial flow increases monotonically with $\sqrt{s}$, leading to flatter $p_T$-spectra at higher energies and thus pushing the hydrodynamically generated momentum anisotropy, which is reflected in $v_2(p_T)$, to larger transverse momenta.

Within a hydrodynamic picture of the collision fireball’s collective evolution, the monotonic increase with $\sqrt{s}$ of radial flow is a simple and unavoidable consequence of energy conservation,
the existence of a phase transition in the QCD phase diagram. A systematic analysis of the hadron $p_T$-spectra and $v_2(p_T)$ as functions of collision energy [8] shows that a non-monotonic $\sqrt{s}$-dependence of the elliptic flow $v_2(p_T)$ at fixed $p_T$, first rising from AGS to low SPS energies but then falling again towards RHIC and the LHC, is a generic consequence of the evolution of radial flow and, as such, cannot be used unambiguously as evidence in support or against the existence of the quark-hadron phase transition. To make this point is the purpose of this contribution. When searching for a clear QCD phase transition signature (in particular for the CEP), one has to look elsewhere.

2. Ideal fluid dynamics from RHIC to LHC

The analysis of Ref. [8] which is reported here is based on ideal relativistic fluid dynamics (IRFD). As already discussed above, IRFD is not perfect at RHIC energies and becomes increasingly worse at lower energies, due to the growing dynamical role played by the highly viscous hadron gas stage. At higher energies, the role of the hadronic phase decreases since more and more of the finally observed collective flow (in particular its anisotropy in non-central collisions) is generated already during the quark-gluon plasma (QGP) stage. The specific shear viscosity $\eta/s$ of the QGP (where $s$ is its entropy density) is known to be very small, of the order of at most a few times the KSS [9] bound $\eta/s = 1/4\pi$ [10]. For fixed $\eta/s$, viscous effects in heavy-ion collisions are largest at early times, due to the large initial expansion rate from approximately boost-invariant longitudinal expansion. At any given early time $\tau$ (before the onset of significant transverse expansion, $\tau \ll R/c_s$, where $R$ is the transverse fireball radius and $c_s$ is the sound speed), viscous effects are controlled by the ratio of times scales $L_\tau = \frac{1}{\sqrt{s}} T_\tau$, where $T_\tau = \eta/(sT)$ is the sound attenuation length and $1/\tau$ is the longitudinal expansion rate [11]. In perturbative QCD, the dimensionless specific shear viscosity $\eta/s$ is expected to increase only logarithmically with $T$ [12]. Hence, $T_\tau$ is expected to decrease, leading (at the same $\tau$) to smaller viscous effects on hydrodynamic flow. Correspondingly, the validity of the IRFD approach should improve from RHIC to LHC.

To extrapolate from lower to higher collision energies, we assume that thermalization occurs earlier at higher densities, i.e. at constant product $T_0\tau_0 = \text{const}$. Using entropy conservation in IRFD, we can relate the final charged multiplicity to $T_0$ and $\tau_0$ as follows: $dN_{ch}/dy \sim dS/dy = \tau_0 \int d^2 x_\perp s(x_\perp, \tau_0) \sim s_0\tau_0 \sim T_0^3$, where $s_0 \sim T_0$ is the peak value of the entropy density at $\tau_0$ in central collisions. Combining both conditions we see that, starting from well-established initial conditions for 200 A GeV Au+Au collisions at RHIC [8], the initial thermalization time $\tau_0$ and peak entropy density $s_0$ scale as $\tau_0 \sim \left(\frac{dN_{ch}}{dy}\right)^{-1/2}$, $s_0 \sim \left(\frac{dN_{ch}}{dy}\right)^{1/2}$. The value of $\frac{dN_{ch}}{dy}$ for Pb+Pb at LHC energies cannot be predicted by hydrodynamics, but will be measured on the first day of LHC Pb-beam operation. We therefore present our results as a function of $\frac{dN_{ch}}{dy}$ or, equivalently, of $s_0$. In [8], the range $s_0 \leq 270 \text{ fm}^{-3}$ ($\frac{dN_{ch}}{dy} \leq 1200$) was explored; central 200 A GeV Au+Au collisions at RHIC correspond to $s_0 = 117 \text{ fm}^{-3}$ and $\frac{dN_{ch}}{dy} = 685$.

3. Results

The left panel of Fig. 1 shows the $p_T$-spectra of thermally emitted pions and protons (resonance decay contributions not included) as they evolve from low AGS to LHC energies. The flattening effects of increasing radial flow are clearly visible, especially for protons where strengthening radial flow leads to a yield reduction at low $p_T$ in spite of the increasing total proton multi-
Energy conservation and hydrodynamic behavior during the fireball expansion stage lead to increased radial flow from RHIC to LHC and correspondingly to flatter $p_T$- and $m_T$-spectra, and the bottom part of the right panel of Fig. 1 shows that (for $\sqrt{s} \geq 10$ GeV) this decrease of $v_2^{\text{fixed } p_T}(\sqrt{s})$ is monotonic, and that it holds for all $p_T$ values in the range $p_T \lesssim 1$ GeV. Plotted logarithmically, the slope of this decrease is steeper for protons than for pions (not shown in Fig. 1), reflecting the stronger radial flow effects on the heavier protons.

The radial flow induced decrease of $v_2^{\text{fixed } p_T}(\sqrt{s})$ is independent of the behavior of the $p_T$-integrated elliptic flow, shown in the upper part of the right panel of Fig. 1 for thermally emitted (dashed) and all pions (including resonance decays, solid). The integrated elliptic flow shows the well-documented non-monotonic behavior of IRFD [1], featuring a decrease between top AGS and RHIC energies caused by the softening EOS near the quark–hadron phase transition, followed by an increase above $\sqrt{s} > 100$ GeV caused by the stiffening of the EOS in the QGP phase. The bottom panel shows that, at fixed $p_T$, the differential elliptic flow continues to decrease while the integrated $v_2$ increases; these tendencies persist to the highest values of $\sqrt{s}$ where it is known that the elliptic flow fully saturates in the QGP phase, and that its finally observed value is therefore insensitive to the QCD phase transition and to the details of the conversion of quarks and gluons to hadrons. In this $\sqrt{s}$-region, it is obvious that the decrease of $v_2^{\text{fixed } p_T}(\sqrt{s})$ is unrelated to the softening of the EOS near $T_c$, and has therefore nothing at all to do with the phase transition.

4. Conclusions

Energy conservation and hydrodynamic behavior during the fireball expansion stage lead to increased radial flow from RHIC to LHC and correspondingly to flatter $p_T$- and $m_T$-spectra,
especially for heavy hadron species. As shown in Ref. [8] this causes baryon/meson ratios to continue to increase with both $p_T$ and $m_T - m_0$ at LHC energies, as they do at RHIC. The slope of this increase as a function of transverse kinetic energy $m_T - m_0$ is almost the same at LHC and RHIC, but as a function of $p_T$ the baryon/meson ratios increase with smaller slope at LHC than at RHIC, due to overall flatter $p_T$-spectra.

In ideal relativistic fluid dynamics (IRFD), the $p_T$-integrated elliptic flow of pions and charged hadrons increases about 10-15% from RHIC to LHC energies; accounting additionally for viscous effects at RHIC (mostly of hadronic origin) that weaken or disappear at the LHC, the corresponding increase is about 25%. At the same time, the differential elliptic flow at fixed $p_T$, $v_2^{\text{fixed } p_T}(\sqrt{s})$, decreases from RHIC to LHC. This decrease is driven by an increase in radial flow which pushes the momentum anisotropy to larger $p_T$; it does not depend on a phase transition in the EOS. Combined with the increase of $v_2^{\text{fixed } p_T}(\sqrt{s})$ at low $\sqrt{s} < 10 \text{ GeV}$, this leads to a non-monotonic $\sqrt{s}$-dependence of $v_2^{\text{fixed } p_T}$ that is generic, caused by the interplay between radial flow and freeze-out, and not unambiguously associated with a phase transition in the QCD EOS. Although the analysis presented here was based on IRFD, the interplay between radial flow and freeze-out is a general principle that controls the buildup of elliptic flow also in real fluids. The observed non-monotonic energy dependence of $v_2^{\text{fixed } p_T}$ is therefore robust, and (like variations of the EOS) inclusion of viscous effects is expected to only change the energy where $v_2^{\text{fixed } p_T}$ peaks, but not the fact that it peaks. The search for QCD phase transition signatures, in particular for the predicted critical end point connecting a first order transition at high baryon density to a smooth cross-over transition at RHIC, cannot be based on this non-monotonic energy dependence of fixed-$p_T$ elliptic flow.

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