Same-diff?
Part II: A compendium of similarities between gauge transformations and diffeomorphisms

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Abstract

How should we understand gauge-(in)variant quantities and physical possibility? Does the redundancy present in gauge theory pose different interpretational issues than those present in general relativity? Here, I will assess new and old contrasts between general relativity and Yang-Mills theory, in particular, in relation to their symmetries. I will focus these comparisons on four topics: (i) non-locality, (ii) conserved charges, (iii) Aharonov-Bohm effect, and (iv) the choice of representational conventions of the field configuration. In a companion paper, I propose a new contrast and defend sophistication for both theories.

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1 Introduction

Same-diff [noun]: an oxymoron, used to describe something as being the same as something else. Often used as an excuse for being wrong. (Urban dictionary).

Diff: A common abbreviation for “diffeomorphism”. E.g. Diff(M) is the group of diffeomorphisms of the (differentiable) manifold M.

1.1 Roadmap

In the study of physical possibility in the context of modern physics, general relativity and Yang-Mills theory are especially relevant since the former best represents the importance of diffeomorphism and the latter does so for gauge symmetry. In the accompanying paper, Gomes (2021c), I describe what I take to be the most perspicuous understanding of these symmetries; and I introduce one important conceptual distinction between them. In this paper, I will assess other attempts at distinguishing the two types of symmetry and conclude that they fail.

I will focus my analysis on comparisons between diffeomorphisms and gauge symmetries that have not been given much attention in the literature thus far. This will helpfully narrow the scope of the paper. But even in this limited scope, aiming for completeness requires some artificial separation of the topics to be discussed. That is also unavoidable: both gauge and general relativity are rich, multi-faceted theories, and may be compared along different axes.

In Section 2, I begin by criticizing the more obvious, and more concrete, attempts to conceptually distinguish diffeomorphisms in general relativity from gauge transformations in Yang-Mills theory. This Section considers a disparate array of attempts and requires a few “gear-changes”. More specifically, I will discuss:

(1) the representation of each type of symmetry in the Hamiltonian, or Dirac, analysis of constraints,
(2) the relation between the symmetries and charge conservation;
(3) the comparison between the Aharonov-Bohm effect in gauge theory and general relativity.

Regarding (1) there is a distinction to be drawn in the context of Dirac constraint analysis, between, on one side, certain diffeomorphisms, and, on the other, the remaining diffeomorphisms and gauge symmetries. Namely, the diffeomorphisms whose generators act by shuffling points in time are of a different type than both those shuffling points in space and gauge transformations—which are of the same type. I take this discrepancy to be part and parcel of the infamous Problem of Time (cf. Kuchar (2011)), and to lie outside the scope of this paper (which is already quite broad!). So, to sum up about (1): the spatial diffeomorphisms and the gauge symmetries are found to be similar in every important way.
Regarding (2), the main question is whether there are quasi-local symmetry-invariant conserved charges that are associated to the symmetries. The answer is that there are in electromagnetism, but there aren’t (generically) in either general relativity or in non-Abelian Yang-Mills theories. Therefore the difference here is really between Abelian and non-Abelian symmetries (namely, those that generate commuting or non-commuting transformations, respectively), and not between diffeomorphisms and gauge symmetries.

Regarding (3), the obvious suggested difference is that in general relativity a “phase difference” is acquired infinitesimally along each of the trajectories, while in the Aharonov-Bohm effect the acquired phase is strictly a global concept: one can only relate it to closed loops, i.e. one cannot pinpoint ‘where’ in the trajectory the phase is acquired. I will show that this interpretation involves a simultaneous misconstrual of phase acquisition by a charged particle and of the gravitational analogue for the Aharonov-Bohm effect.

Next, in Section 3, I tackle a less well-known attempt to distinguish the types of symmetries, formulated by Healey (2007). Healey’s argument is functionalist, in the sense of Lewis’ idea of uniquely specifying an item as the occupant or filler of a functional role (D. Lewis, 1970, 1972). He says that we can distinguish individual representatives of the metric—e.g. one particular representative of the metric tensor field amongst all of the isomorphic copies—by stipulating extra conditions on it. And he argues that one cannot similarly distinguish amongst gauge-equivalent representatives of the same physical possibility. I will reconstruct Healey’s argument for such “gauge-exceptionalism” and show that it is misguided: it only arises if we unduly, and arbitrarily, restrict the tools we take to be available for the functional definition.

In the last section, Section 4, I will first provide another overview of what we have achieved (Section 4.1) Then I marshall a multitude of arguments in defence of gauge redundancy (Section 4.2); for it has many explanatory and pragmatic virtues.

2 Three natural comparisons assessed

The accompanying paper Gomes (2021c) focuses on formal differences between the symmetry structures of general relativity and Yang-Mills theory. This Section will focus on three more concrete comparisons between diffeomorphisms and gauge symmetries that will naturally come to mind for anyone thinking about the issue.

My argument requires brief expositions of symmetries, for both Yang-Mills theory and general relativity. In Section 2.1 I undertake this brief exposition.

Section 2.2 compares the symmetries in the constrained Hamiltonian formalism. The symmetries are shown to be similar in certain important respects; in particular, in their relationship to a (non-signalling) type of non-locality. But here we also find the seed of the robust difference between the symmetries found in Gomes (2021c): a conceptual difference in the transformation properties of the canonical variables in the two theories.

The second type of comparison, investigated in Section 2.3, is about the relation between symmetries and charges. It is often remarked that diffeomorphisms are unlike other symmetries of nature, since we cannot obtain from them a physical, conserved charge. That is: generically, we cannot associate to a diffeomorphism symmetry a physical quantity in a spacetime region whose change in time is solely due to a flux through the region’s spatial boundaries. While I agree that such an obstruction to the relation between symmetries and charges exists for diffeomorphisms, I will show that they also exist for any non-Abelian Yang-Mills theory.

The third comparison, investigated in Section 2.4, is about the Aharonov-Bohm effect, in gauge theory. There are a few arguments here. One is that the Aharonov-Bohm effect presents a sui generis type of physical under-determination. Resolving this under-determination, it has been argued, requires an eliminativist approach to the theory, unlike the under-determination
posed by the ‘hole argument’ in general relativity. I will consider gravitational analogues of the Aharonov-Bohm effect, and dispel most of the distinctions that have been attributed to the Aharonov-Bohm effect.

2.1 Symmetries

I leave a detailed exposition of the symmetries of Yang-Mills theory and general relativity to Gomes (2021c). Here I will make the following assumption: in the language of category theory, symmetries are to form a groupoid, with the objects of the category being the elements of the space of models, \( \mathcal{M} \), and the maps, or arrows, being isomorphisms in the category-theoretic sense.\(^1\)

**Symmetries of general relativity** In vacuo, we take the basic variables of general relativity to be covariant symmetric tensors of rank two, \( g_{ab} \), on a smooth manifold \( M \). The models of the theory will thus be given by the doublets \( \langle M, g_{ab} \rangle \). We will say that \( \langle M, g_{ab} \rangle \) and \( \langle M, h_{ab} \rangle \) are symmetry-related iff in the category of smooth manifolds, there is an automorphism \( f \in \text{Diff}(M) \), such that \( h_{ab} = f^*g_{ab} \). We write this symmetry relation as:

\[
\langle M, g_{ab} \rangle \sim \langle M, f^*g_{ab} \rangle.
\]

Symmetry-related models equally satisfy or fail to satisfy the dynamical equations (in vacuo):

\[
G_{ab} = 0,
\]

where \( G_{ab} \) is the Einstein tensor. If we are not in vacuum, the right-hand side of (2.2) is not zero, but \( T_{ab} \), the energy-momentum tensor (scaled by the appropriate coupling constant), but the equations are still preserved for an appropriate extension of the isomorphism relation to the matter fields.

Thus we identify the symmetry group as \( G := \text{Diff}(M) \), which acts on the space of Lorentzian metrics over \( M \), namely, \( \mathcal{M} = \text{Lor}(M) \). It is also useful to represent the local, infinitesimal action of diffeomorphisms. Namely, for a one-parameter family of diffeomorphisms \( f_t \in \text{Diff}(M) \), such that \( f_0 = \text{Id} \), we write the flow of \( f_t \) at \( t = 0 \) as the vector field \( X^a \).

Then, infinitesimally we obtain:

\[
\frac{d}{dt} \bigg|_{t=0} f^*_tg_{ab} \equiv \mathcal{L}_Xg_{ab} = \nabla(aX_b),
\]

where \( \mathcal{L}_X \) denotes the Lie derivative along \( X^a \) and \( \nabla \) is the Levi-Civita covariant derivative associated to \( g_{ab} \) and we use round brackets to denote index-symmetrization.

**Symmetries of Yang-Mills theory** Here I will take the basic kinematic variables of Yang-Mills theory as: \( A = A^I_a \text{d}x^a\tau_I \), where \( \tau_I \in \mathfrak{g} \) is a Lie-algebra basis, and \( A^I_a \in C^\infty(M) \).\(^2\) Here the gauge transformations act infinitesimally, for a Lie-algebra valued function \( \xi^a \in C^\infty(M, \mathfrak{g}) \), as

\[
\delta_{\xi}A^I_a = \nabla_a\xi^I + [A_a, \xi]^I = D_a\xi^I,
\]

A groupoid is a category in which every arrow is an isomorphism, in the abstract category-theoretic sense of ‘isomorphism’, i.e. every arrow has an inverse. The further assumption, that formal symmetries are represented as groups (which could be infinite-dimensional), labeled \( G \), such that, given the space of models of a theory, \( \mathcal{M} \), there is an action of \( G \) on \( \mathcal{M} \), a map \( \Phi : G \times \mathcal{M} \to \mathcal{M} \), that preserves the action functional, holds for the covariant Lagrangian version of both Yang-Mills theories and general relativity, and it holds for the Hamiltonian version (in which \( \mathcal{M} \) is phase space) of Yang-Mills theory, but it does not hold for the Hamiltonian version of general relativity; there we have only a groupoid structure (see Blohmann et al. (2013)).

\(^1\) A more perspicuous picture of the kinematical content of the Yang-Mills field is through a principal connection, \( \omega \), which is a Lie-algebra valued one-form on a special manifold, viz a principal fiber bundle \( P \). \( P \) is a smooth manifold that admits a free, proper action of the Lie group \( G \) and is such that \( P/G \simeq M \) (cf. Blohmann et al. (2013)).
where $D_a(\bullet) = \nabla_a(\bullet) + [A_a, \bullet]$, the gauge-covariant derivative, is defined to act on Lie-algebra valued functions.

We define the Lie-algebra valued curvature two-form as:

$$F_{ab} = \nabla_{[a} A_{b]} + [A_a, A_b], \quad (2.5)$$

where square brackets denote anti-symmetrization of indices. The equations of motion, in vacuum are:

$$D^a F_{ab} = 0, \quad (2.6)$$

and they are preserved by the symmetries generated by (2.4).

### 2.2 The symmetry generators (constraints) and their locality

Gauge symmetries are often said to be a necessary evil: if we are to obtain an unconstrained, local description of the physics, we need to include redundancy, it is said. Let us investigate the meaning of this folklore.

First, since the formalism is slightly unfamiliar to philosophers of physics, let us quickly review, in the simple case of a non-relativistic mechanical system, how constraints arise in Hamiltonian mechanics, and their broad relation to symmetries and under-determination; (for a more complete treatment, see (Henneaux & Teitelboim, 1992, Ch. 1), and for philosophical introductions, Wallace (2002) and (Gomes & Butterfield, 2021, Section 2)).

In the Lagrangian formulation of mechanics, one is given a Lagrangian $L(q^\alpha(t), \dot{q}^\alpha(t))$, and obtains, from the least action principle $\delta S = 0$, the Euler–Lagrange equations:

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}^\alpha} = \frac{\partial L}{\partial q^\alpha}. \quad (2.7)$$

If we use the chain rule for the $\frac{d}{dt}$ derivative, we get from (2.7):

$$\ddot{q}^\beta \frac{\partial^2 L}{\partial q^\beta \partial \dot{q}^\alpha} + \dot{q}^\beta \frac{\partial^2 L}{\partial q^\beta \partial q^\alpha} = \frac{\partial L}{\partial q^\alpha}. \quad (2.8)$$

From this equation it becomes clear that the accelerations are uniquely determined by the positions and velocities if and only if the matrix $M_{\alpha\beta} := \frac{\partial^2 L}{\partial q^\beta \partial q^\alpha}$ is invertible. If it isn’t, our system possesses some kind of redundancy in its description. As we know, this is indicative of gauge symmetries.

Here, we seek a representation of symmetries in the Hamiltonian formalism. Thus, one replaces $\frac{\partial L}{\partial \dot{q}^\alpha} =: p_\alpha$, and the question whether the accelerations are determined by the positions and velocities is translated to the question whether the momenta $p_\alpha$ are invertible, as functions of the velocities. If $M_{\alpha\beta} = \frac{\partial p_\beta}{\partial q^\alpha}$ is not invertible, there are constraints among the $p_\beta$, which we write as $\Phi^I(p_\alpha, q^\beta) = 0$, where $I$ parametrizes the constraints.\(^3\)

When these constraints are conserved by the equations of motion and are compatible amongst themselves—in the jargon of the Dirac algorithm: are first-class—they correspond, by (Gomes, 2021c, Section 3)). The action of $G$ on $P$ determines orbits of the group, and $\omega$ respects this structure by being equivariant with respect to translation along the orbit. Given a local section of $P$, namely a smooth injective map $s : M \rightarrow P$ that intersects the orbits of the group $G$ only once, then the gauge potential is the local spacetime representative of $\omega$ according to $s$, i.e. it is the pull-back $A := s^* \omega$. Here, for simplicity, we are taking $A$ to be defined by a global section, which requires the extra assumption that $P$ is simple enough to be covered by a single chart. Clearly, $I$ are Lie-algebra indices and $a$ are spacetime indices. We take $\{dx \otimes \tau\}$ as the basis for sections of the vector bundle $\Gamma(T^*M \otimes g)$.

\(^3\)Here we assume that the rank of $M_{\alpha\beta}$ is constant, and that the ensuing constraints obey regularity conditions. See (Henneaux & Teitelboim, 1992, Ch. 1.1.2).
(the converse of) Noether’s theorem, to symmetries of the system. Indeed, some of the great advantages of the Hamiltonian formalism are: (i) that the symmetries are not postulated, but, given a few extra assumptions, algorithmically identified; and (ii) the relation between constraints and symmetries is very straightforward. Namely, the symmetries act on any quantity through the Poisson bracket between that quantity and the symmetry’s corresponding generator, which is just the constraint.

In a bit more detail: as scalar functions on phase space the constraints \( \Phi^I \) (for each \( I \)), have (differential geometric) gradients, \( d\Phi^I \), which are in one-one correspondence with vector fields \( X_{\Phi^I} =: X_I \) due to the symplectic structure of phase space. Namely, given the symplectic form \( \omega \) (a closed, non-degenerate two-form on phase space), we can define vector fields from one-forms (and vice-versa). Applying this definition to \( d\Phi^I \) we obtain the associated vector fields \( X_I \) defined by \( \omega(X_I, \bullet) = d\Phi^I \). The key idea is that, just as the flow specified by the Hamiltonian function conserves energy, these vector fields associated to \( \Phi^I \) are tangential to, and so preserve, the intersection of all the constraint- and energy-surfaces. That is, in a less geometric (and maybe more familiar) language: they not only commute with the Hamiltonian and conserve energy, but also conserve the charges associated with the constraints. From the more geometrical viewpoint, it is easy to check that, on the constraint surface, \( \omega \) is degenerate, and the \( X_I \) form its (integrable) kernel. This is the origin of gauge symmetry in the Hamiltonian formalism (cf. Earman (2003) for a more comprehensive philosophical treatment).

In Section 2.2.1, we will see how the canonical symmetries of both Yang-Mills theory and general relativity are alike. Namely, both emerge from elliptic differential constraints. These constraints are responsible for a (mild) form of non-locality, in the sense that the allowed values of the fields at a point depend on the values of the field a finite distance away.

In Section 2.2.2, we will focus on the symmetries that are generated by the constraints. Here we will see a qualitative difference between the actions of these symmetries on the canonical momenta of the two theories.

### 2.2.1 The similarities between canonical spatial diffeomorphisms and gauge transformations

In the case of general relativity, the configuration variables are spatial, i.e. Riemannian metrics, \( h_{ij} \), and their conjugate momenta are denoted by \( \pi^{ij} \). There are two sets of constraints that emerge: one is associated to “refoliations” of spacetime, namely, redefinitions of the surfaces of simultaneity, and the other is associated to diffeomorphisms that map each leaf to itself. Here, I will set aside the generator of refoliations, due to several interpretational difficulties, including whether it really generates a bona-fide symmetry or not (the infamous ‘Problem of Time’, cf. Isham (1992); Kuchar (2011)).

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4The relation to Poisson brackets is given by: \( \{ f, h \} = df(X_h) = \omega(X_f, X_h) \), for \( f, h \in C^\infty(\mathcal{P}) \). (Here, as usual, \( df(X) \) is the contraction between 1-forms and vectors; and \( df(X) \) is equal to \( X(f) \) i.e. the directional derivative of a scalar function \( f \) along \( X \)).

5Geometrically, given a spacelike foliation of a globally hyperbolic spacetime by leaves which are surfaces of simultaneity, we have \( M \simeq \Sigma \times \mathbb{R} \), where \( \Sigma \) is a three-dimensional manifold, and \( h_{ab} \) corresponds to the pull-back of the spacetime metric \( g_{ab} \) onto its leaves, \( \Sigma \), and the momenta are essentially the extrinsic curvature of the leaves. In the Yang-Mills case, \( A_I^i \) corresponds to the pull-back of \( A^i_{\mu} \), and the conjugate momenta, \( \pi^{ij}_I \), is essentially the curvature contrasted with the normal to the foliation, \( F_{\mu \nu}^I \).

6But I have two cursory remarks. The first is that, unlike the spatial diffeomorphisms, which arise without further restrictions or qualifications from the Einstein-Hilbert Lagrangian, one can only establish a map between the spacetime refoliations and a Hamiltonian constraint if one restricts the domain of this map to spacetimes that satisfy (some of) the equations of motion (Lee & Wald, 1990). This seems like an important fact, but it also seems to concern the nature of time and symmetry, not the nature of diffeomorphisms: thus supporting my here setting aside the refoliations. The second remark is that, in the case of general relativity, determining whether the constraint surfaces and their intersections actually form a regular submanifold of the phase space of general relativity is a non-trivial matter. As far as I understand the current state of the problem, this
The generator of spatial diffeomorphisms, is:

\[ \nabla_i \pi^i_j = p^j, \quad (2.9) \]

where \( \nabla \) is the Levi-Civita covariant derivative intrinsic to the surface, associated to \( h_{ij} \), and \( p^j \) is the momentum density of the fields that source the gravitational field. This is a set of elliptic equations, meaning they have more spatial than time derivatives.

Applying the same procedure to Yang-Mills theory, we obtain a constraint of a very similar form to (2.9): the Gauss constraint

\[ D^i E^j_i = \rho^j, \quad (2.10) \]

where \( \rho^j \) is the (Lie-algebra valued) current density and \( D_i \) is the spatial version of the gauge-covariant derivative of (2.4). Equation (2.10) is of precisely the same character as the momentum constraint in general relativity (2.9); in particular, both are elliptic equations.

In practice, ellipticity means that boundary value problems require only the ‘instantaneous’ state of the field, i.e. do not also require the field’s velocity. Moreover, the solution of these equations exist on each slice; it does not correspond to the propagation of a field, as would a solution to a hyperbolic equation.

Indeed, elliptic equations, are, to a certain extent, non-local. For instance, in electromagnetism, to determine the allowed values of e.g. the electric field \( E^i \) at a point \( x \), we need to know the value of the electric field on the boundary of a small region surrounding \( x \), and we need to know the distribution of charges in this region, and not just at \( x \).\(^7\) Note that this goes beyond the well-established denial of pointillisme Butterfield (2006), for it does not amount just to the requirement that the value of a quantity at \( x \) depends on other values, infinitesimally distant from \( x \).\(^8\)

The solution of the momentum constraints in general relativity proceeds very much in the same fashion;\(^9\) it similarly implies a mild form of non-locality.

The idea that gauge symmetries are the price we pay for local representations can then be characterized as follows. A Lagrangian, such as the Yang-Mills Lagrangian, that employs gauge symmetries, ensures that the ensuing theory will possess a Gauss-type local conservation law (Gomes et al., 2021, Section 4), (Strocchi, 2013, Ch 7), (Strocchi, 2015, Section 3). As the above argument illustrates, such local, differential conservation laws are tantamount to the elliptic differential constraints on the dynamical variables and to the (mild) degree of non-locality mentioned above.

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\(^7\)In practice, one splits the electric field into a radiative and a Coulombic part, \( E^i = E^i_{\text{rad}} + \partial^i \varphi \), where \( \varphi \) is a Coulombic potential and \( \partial_i E^i_{\text{rad}} = 0 \) with \( n_i E^i_{\text{rad}} = 0 \), where \( n^i \) is the normal to the boundary of the region. One then uses a Green’s function to solve the Poisson equation, \( \nabla^2 \varphi = \rho \) with boundary conditions given by \( \partial_n \varphi = E_n \), the flux of the electric field. This decomposition decouples the value of the electric field at the boundary from the choice of \( E^i_{\text{rad}} \). Lastly, \( E^i_{\text{rad}} \) is symplectically orthogonal to the pure gauge part of the gauge potential, i.e. terms like \( \partial_i \xi \), where \( \xi \) represents the change of gauge. Thus the “pure-gauge” component of the gauge potential is dual (i.e. has an invertible Poisson bracket on the constraint surface) to the remaining part of the electric field, that is involved in the (mild) non-locality due to the Gauss constraint. These ideas are elaborated further in (Gomes & Butterfield, 2021).

\(^8\)For instance, a smooth vector field would violate pointillisme in this milder sense and yet, not being subject to any elliptic constraint, would be independently specifiable at \( x \) and at the boundary of a spatial region surrounding \( x \).

\(^9\)But with the added difficulty due to the coupled Hamiltonian constraint. Due to this complication, instead of a radiative, unconstrained degree of freedom that is just transverse (like \( \partial_i E^i_{\text{rad}} = 0 \)), one has to solve the initial value constraints. Here the standard way (cf. (York, 1971)), also has a tracelessness condition, so that the unconstrained momenta degrees of freedom are the transverse-traceless momenta, \( \pi^T_T \), obeying both \( h^{ab} \pi^T_T = 0 \) and \( \nabla^T \pi^T_T = 0 \).
In sum: certain constraints are non-local in the sense that we are not free to simultaneously specify the values of corresponding physical quantities completely independently at spatially distant points. In the Hamiltonian theory, the non-locality can be, in a well-defined sense (cf. footnote 7 and preceding text), put into direct correspondence with the gauge symmetry in question.

2.2.2 The difference between canonical spatial diffeomorphisms and gauge transformations

By imposing constraints one can recover the symmetries. That is, both (2.9) and (2.10) represent an infinite set of constraints: one per spatial point. A linear combination of these constraints is thus given by an integral where the constraints are multiplied, or ‘smeared’, by an appropriate coefficient function. And such linear combinations generate, through the action of the Poisson bracket (cf. footnote 4 and preceding text), infinitesimal diffeomorphisms and gauge transformations, respectively.

In other words, given the foliation of spacetime into \( \Sigma \times \mathbb{R} \) (cf. footnote 5), let \( \xi^I \in C^\infty(\Sigma, \mathfrak{g}) \) be a smooth Lie-algebra valued function and \( X^I \in \mathfrak{X}(\Sigma) \) be a smooth spatial vector field. Then with \( \xi \) and \( X \) giving the smearing, we obtain that the action of the vacuum constraints on the (doublet forming the) canonical pair ((\( h, p \)) and (\( A, E \)), respectively) is:

\[
\left\{ h_{ij}(x), \pi^{ij}(x) \right\}, \int dx' X^k \partial_k \pi_{lk}(x') = (\mathcal{L}_X h_{ij}(x), \mathcal{L}_X \pi^{ij}(x)) \quad (2.11)
\]

and

\[
\left\{ A^I(x), E^I(x) \right\}, \int dx' \xi_j \partial^j E^I_{J}(x') = (\mathcal{D}_I \xi^I(x), [E^I, \xi^I](x)) \quad (2.12)
\]

as we would expect from (2.4).

But there is a difference between the two transformation laws of the canonical variables, that is important. The difference is that the transformation of the momenta, \( E \), of the Yang-Mills potential is algebraic, or non-derivative, in a way that the transformation of the gravitational momenta, \( \pi \), is not. For instance, if the electric field vanishes at \( x \), so will its flow under the symmetry generator; but a vanishing gravitational momentum at \( x \) implies no such vanishing for its flow under the symmetry. This algebraic property of the electric field is merely the Hamiltonian version of the equivariance property of the gauge curvature; and it constitutes the main difference that I advocate between the two sets of transformations Gomes (2021c) (where the difference is dubbed \( \Delta \)).

2.3 Quasi-local charges

The second comparison that I would like to address concerns the relation between symmetry and conserved charges. It is generally accepted that diffeomorphism invariance implies that the stress-energy tensor of matter derived from a Lagrangian must be covariantly divergence-free; I agree.\(^{11}\) What is more contentious is whether this counts as a “local conservation” of stress-energy and, if not, whether this would distinguish diffeomorphisms from other symmetries.

\(^{10}\)We will denote the canonical Poisson brackets as \( \{ \cdot, \cdot \} \), which, when applied to the fundamental phase space conjugate variables of general relativity, \( (h_{ij}(x), \pi^{ij}(y)) \), yields: \( \{ h_{ij}(x), \pi^{kl}(y') \} = \delta^{[k}_{[i} \delta^{l]}_{j]} \delta(x, x') \); when applied to the fundamental phase space conjugate variables of Yang-Mills theory, yields: \( \{ A^I_j(x), E_{J}^{I}(x') \} = \delta^{I}_{I} \delta^{I}_{J} \delta(x, x') \). It should be noted that we use the standard inner product on the Lie algebra: in this basis \( \delta^{IJ} \), which thus does not distinguish between lower or upper indices.

\(^{11}\)More generally, one can show that local symmetries imply, through Noether’s second theorem, that the equations of motion for the field will obey a conservation law corresponding to the conservation of charges; cf. Gomes et al. (2021).
of nature, such as gauge symmetries of Yang-Mills theories. The folklore is that it does not count as a local conservation law and thus that diffeomorphisms thereby differ from from gauge symmetries.

We can here take Curiel (n.d.) to express the folklore:\footnote{I only single out Curiel since he puts the issue in his usual clear-thinking style, and not because I believe he has an unusually wrong opinion (I do not). As I said: he only here expresses an opinion that is widespread even among the thoughtful, well-informed, specialists.}

Killing fields are symmetries of individual solutions, but that, I think, is all one can say about [the] symmetries of the theory [i.e. general relativity]. The theory does not seem to me to have any symmetries in the standard sense of the term as it is used elsewhere in physics. (That, inter alia, is why I think that trying to cast it as a Yang-Mills theory is doomed to failure.) [...] In the other cases where we apply Noether’s second theorem and talk about symmetries and conserved quantities, those conserved quantities are conserved in the very strong sense that one can write down local continuity equations that can be integrated to yield global conservation laws. One cannot do any of this based on the fact that a tensor in a generic curved spacetime is covariantly divergence-free. So something that looks and acts a little bit like a symmetry yields something that looks and acts a little bit like a conservation law. I find the differences more striking than the similarities.

The role of this Section will be essentially to unpack and criticize this quotation; and, by so doing, to efface the distinction Curiel draws, at least between general relativity and non-Abelian Yang-Mills theory.

First, let us find a relation between local symmetries and conservation, a la Noether’s second theorem. Suppose that the action for a given theory in spacetime takes the following form:

\[
S = \int d^4x (\mathcal{L}_g + \mathcal{L}_m); \quad (2.13)
\]

where I will assume that all of the contributions of the matter fields to the Lagrangian are confined to the second component, \(\mathcal{L}_m\), of the total Lagrangian density, and that both terms are individually diffeomorphism invariant. I will also assume that the energy-momentum tensor for the matter fields is given by:

\[
T^{ab} = \frac{\delta \mathcal{L}_m}{\delta g_{ab}}. \quad (2.14)
\]

There is an important physical feature of the gravitational field underlying equation (2.14): that the metric is nowhere vanishing and that it couples to every field. These two properties ensure that from the invariance under diffeomorphism of the Lagrangian and the transformation properties of the metric, we are able to deduce local conservation laws for the matter fields.

Then, assuming that infinitesimal diffeomorphisms generate the symmetries of the theory, the corresponding infinitesimal variation of the metric is \(\delta g_{ab} := \mathcal{L}_X g_{ab} = \nabla_{(a} X_{b)}\), for \(X^a\) a vector field. Thus, discounting boundary terms, we obtain, from a variation of the action functional (2.13), setting \(G^{ab} = \frac{\delta \mathcal{L}_g}{\delta g_{ab}}\):

\[
0 = \delta S = \int d^4x ((G^{ab} + T^{ab})\mathcal{L}_X g_{ab}), \quad (2.15)
\]

and after integration by parts, \(\nabla^a T_{ab} = 0\), even without imposing the equations of motion, i.e. the Einstein equations \(G_{ab} = T_{ab}\). The problem Curiel alludes to is that these local conservation laws cannot be integrated to yield global conservation laws.
For in order to obtain a symmetry-invariant quantity from the local conservation law, we must contract $\nabla^a T_{ab}$ with another auxiliary vector field $X$—in order to obtain a scalar—and integrate by parts. Namely:

$$\int d^4 x \sqrt{g} X^b \nabla^a T_{ab} = - \int d^4 x \sqrt{g} \nabla^a X^b T_{ab} + \oint \sqrt{h} d^3 x n^a X^b T_{ab} = 0. \quad (2.16)$$

(here $\oint \sqrt{h}$ is the integral density at the boundary, which is a closed manifold). Now supposing that $\nabla^a X^b = 0$, we get a physical quantity in a spacetime region whose change in time is solely due to a flux through the region’s spatial boundaries. For example, if the integration region extends spatially to infinity, where we assume $T_{ab} = 0$, then the second equation says that the spatial integral of $X^a T_{ab}$ is conserved. That is, it takes the same value on an initial and on a final Cauchy surface, $\Sigma_1$ and $\Sigma_2$, respectively:

$$\int_{\Sigma_1} \sqrt{h} d^3 x X^a T_{ab} = \int_{\Sigma_2} \sqrt{h} d^3 x X^a T_{ab}.$$

(2.17)

Vector fields for which $\nabla^a X^b = 0$ are called Killing fields. For example: from the above argument, for a time-like Killing vector field $\partial_t$, one gets conservation of energy.

But if $\nabla^a X^b \neq 0$, the volume integral on the rhs of (2.16) does not vanish, and there is no such conservation law.

And Curiel is also correct that, in electromagnetism, Noether’s theorem guarantees that the symmetry gives rise to a density whose rate of change in time is solely due to a flux through the region’s spatial boundaries. That is, for electromagnetism, the local conservation laws can be integrated without obstruction. Namely, from $\partial^a F_{ab} = j_b$ (equation (2.5), for the Abelian case), and taking the divergence we obtain $\partial^a j_a = 0$. Integrating this equation,

$$\int \partial^a j_a = \oint n^a j_a = 0.$$

(2.18)

Again, assuming that the current density vanishes at spatial infinity we obtain that the spatial integral of $j_0 := \rho$ is conserved in time.

Here, notice the conservation law needs no special condition like the auxiliary vector field obeying $\nabla^a X^b = 0$. So it would seem that at least Abelian Yang-Mills theory has a very different relation between symmetries and conserved charges.

However, the non-Abelian theory behaves, in this respect, exactly like general relativity. Namely, the local covariant conservation law is:

$$D_a J^a_I = 0 \quad (2.19)$$

We can integrate $D_a J^a_I$ against any Lie algebra-valued scalar $\xi \in C^\infty(M, g)$:

$$\int (D_a J^a_I) \xi^I = \oint n_a J^a_I \xi^I = \int J^a_I D_a \xi^I = 0. \quad (2.20)$$

But this is a bona fide regional conservation law only if the $\xi^I$’s are such that $D_a \xi^I = 0$; for otherwise the integral will not generically reduce to a boundary flux. As with general relativity, generic configurations have no such ‘stabilizers’: generically there are no solutions to $D_a \xi^I = 0$, just as there are no solutions to the Killing equations, $\nabla^a X^b = 0$. In other words, to obtain physically significant conservation laws from the local conservation laws guaranteed by Noether’s second theorem, $X^a$ and $\xi^I$ must have some physical meaning.\(^\text{13}\)

\(^\text{13}\)Indeed, only symmetries that are related to stabilizers deserve the label ‘global symmetries’, in the sense of
In sum, both general relativity and non-Abelian Yang-Mills theories lack a generic, symmetry-invariant definition of constancy. In both types of theory, it is only certain configurations that admit the non-trivial automorphisms that implicitly define such ‘covariantly constant generators of transformations’.

This establishes some caveats to our usual understanding of the Noether conserved currents. Such caveats are acknowledged in the physics literature, but their more radical consequences are usually left unsaid. Ultimately, the very concept of a non-Abelian regionally conserved charge, like the concept of regionally conserved energy-momentum in general relativity, only makes sense over ‘uniform’ backgrounds—in terms of \( D_\alpha \xi^\alpha = 0 \) and \( \nabla_{(a} X_{b)} = 0 \).

2.4 The Aharonov-Bohm effect as an argument for eliminativism

In this Section, I will describe why the Aharonov-Bohm effect cannot be used to draw important distinctions between the symmetries of general relativity and of Yang-Mills theory. In Section 2.4.1 I describe the effect, in Section 2.4.2 I discuss the gravitational analog of the effect, and whether the analogy still leaves room for a salient distinction between the symmetries of the two theories.

2.4.1 The Aharonov-Bohm effect

To investigate the physical significance of the gauge potential, Aharonov and Bohm proposed an electron interference experiment, in which a beam is split into two branches which go around a solenoid and are brought back together to form an interference pattern.\(^{14}\) This solenoid is perfectly shielded, so that no electron can penetrate inside and detect the magnetic field directly.

The experiment involves two different set-ups—solenoid on or off—which produce two different interference patterns. As the magnetic flux in the solenoid changes, the interference fringes shift. And yet, in both set-ups, the field-strength (i.e. the magnetic field) along the possible paths of the charged particles is zero. So, the general outline of the experiment is: (a) the observable phenomena change when the current in the solenoid changes; and (b) the electrons that produce the phenomena are shielded from entering the region of non-zero magnetic fields; so (c) if we rule out unmediated action-at-a-distance, whatever physical difference accounts for the change must be located outside the solenoid.

Thus, to explain the different patterns, one must either conjecture a non-local action of the field-strength upon the particles, or regard the gauge potential as carrying ontic significance. Taking this second stance, the Aharonov-Bohm effect shows that the gauge potential cuts finer physical distinctions than the field-strength tensor can distinguish. How much finer?

\(^{14}\)Aharonov & Bohm’s work was conducted independently of the work by Ehrenberg & Siday (1949) who proposed the same experiment with a different framing in a work that did not receive much attention at the time. According to Hiley (2013), the effect was discovered “at least three times before Aharonov and Bohm’s paper”; with the first being a talk by Walter Franz, which described a similar experiment in a talk in 1939.
We can simplify our treatment and imagine an electrostatic situation, considering only the spatial configuration of the fields. In this case we identify the purely spatial component of the field-strength tensor with the magnetic field. Supposing the electron takes the paths $\gamma_1$ and $\gamma_2$ around the solenoid, we can infer from the amount of the shift that there is a field-dependent contribution to the relative phase of electron paths that pass to the left and to the right of the solenoid, given by:

$$e^{i\Delta} = \exp \left( i \oint_{\gamma_1 \circ \gamma_2} A \right).$$  \hspace{1cm} (2.21)

A gauge transformation $A \rightarrow A + d\phi$ will not affect (2.21), since the difference is the integral of an exact form—$d\phi$—over a manifold without boundary, $\gamma_1 \circ \gamma_2 \simeq S^1$, and so must vanish. Thus the phase difference $\Delta$ cares only about the gauge-equivalence class of $A$.\(^{16}\)

To find out more precisely what physical information the equivalence classes of the gauge potential carry that goes beyond that encoded by the curvature,\(^{17}\) we suppose that the underlying spatial manifold has a non-trivial topology, in the sense of a non-trivial de Rham cohomology $H^1(M) := \text{Ker} \, d^1 / \text{Im} \, d^0 \neq 0$, where $d^1$ is the exterior derivative operator acting on the space of 1-forms, and $d^0$ is that same operator acting on smooth functions (or 0-forms). Then there are distinct equivalence classes $[A^1] \neq [A^2]$ that can nonetheless correspond to the same electric and magnetic field. More precisely, there are potentials $A^1, A^2$ such that: $A^1 = A^2 + C$ where $dA^1 =: F^1 = F^2 := dA^2$, and so $dC = 0$, and yet $C \neq d\phi$ (for any $\phi \in C^\infty(M)$). This implies $A^1$ and $A^2$ are not related by a gauge-transformation and so are not in the same gauge-equivalence class. Their local physical, or gauge-invariant content, represented by $F$, matches, and yet they differ globally, or in their global gauge-invariant content. (See (Belot, 1998, Sec 4) for a more thorough philosophical analysis of this paragraph’s discussion).

In other words, $[A]$ carries a local physical component—expressed in the magnetic field, or in the spatial part of the field-strength tensor, $F$—and a non-local one: expressed in the cohomological content of $C$. So, we take the field-strength tensor to capture the local, gauge-invariant, dynamical content of the gauge potential. But it doesn’t exhaust the non-local physical content of the gauge potential.

The Aharonov-Bohm effect confirms that the distinction between the equivalence classes, $[A^1] \neq [A^2]$, that relies only on the non-local part, is empirically significant: thus implying that the electric and magnetic fields, and the field-strength tensor as well, as not the sole bearers of ontic significance.

### 2.4.2 Gravitational analogies and disanalogies

In light of the Aharonov-Bohm effect, do we need to recalibrate our attitudes towards gauge symmetry? First, it is clear that the effect causes no trouble for the non-eliminativist, ‘sophisticated approach to symmetry-related models’. The name derives from sophisticated substantivalism: a label of an anti-hacceicistic attitude towards the diffeomorphism symmetry of general relativity. It has been extended to an anti-quidditist attitude towards the gauge symmetries.

\(^{15}\)In units for which $e/\hbar c = 1$.

\(^{16}\)As made clear in the accompanying paper, Gomes (2021c), this means the phase cares only about the principal connection $\omega$, not about how we represent it on spacetime (cf. footnote 2). And, since the magnetic field vanishes outside the solenoid in both situations, the connection $\omega$ is different in the two situations, although the curvature of that connection is the same, viz. zero.

\(^{17}\)In electromagnetism, the curvature encodes all the local gauge-invariant degrees of freedom of the potential. In the non-Abelian theory, traces of products of the curvature encode the local gauge-invariant degrees of freedom.
of Yang-Mills theory in Dewar (2017). While the sophisticated approach is not eliminativist, it still awards physical significance to those and only those quantities that are gauge-invariant (as discussed in (Gomes, 2021c, Section 3.3)); and the phase is such a quantity.

Nonetheless, the effect has spurred eliminativists about gauge: in particular, those that endorse the holonomy interpretation of gauge theory. The holonomy formalism, as we will discuss further in Section 4.2, takes as basic variables complex-valued loops on the spacetime manifold. These loops obey certain composition laws that match those of the phases on the left-hand-side of (2.21) (more precisely, it matches the composition laws the phases would obey if their base curves were composed). There are several explanatory deficits of the holonomy interpretation, which I will leave for discussion in Section 4. For now, I will just question whether the Aharonov-Bohm effect is truly a distinctive feature of gauge theory as opposed to general relativity. If it is not, it should serve equally well—or equally poorly!—as a motivation for eliminativism in general relativity.

There are by now several treatments of the analogues of the Aharonov-Bohm effect within general relativity (cf. Anandan (1977); Dowker (1967); Ford & Vilenkin (1981)). All hands agree that non-local effects of gravitational curvature can arise already at the classical level; (this is another distinction that I deem only peripheral to the topic of this paper). The treatment here is closest in spirit to both (Dowker, 1967; Ford & Vilenkin, 1981). But unlike those papers, I will not exhibit solutions of the Einstein equations that can incorporate the essential features of the set-up; and unlike (Anandan, 1977) I am also not interested in the experimental set-up required to verify this effect. The morals that I will draw are also similar in spirit to those of (Weatherall, 2016, Section 5).

As a first approximation we would like to find, in general relativity, two physically distinct situations in which the curvature remains zero in the entire region declared ‘accessible’ to the system under investigation.

The geometric curvature is defined analogously to the gauge curvature: parallel propagate a vector around a loop and check whether it comes back to the original or has been rotated; the curvature measures this rotation, for an infinitesimal loop. In a similar two-dimensional setting, the following two situations are closely analogous to the two distinct situations of the Aharonov-Bohm effect, i.e. solenoid on or off:

(i) the parallel propagation of a vector along $\gamma_1$ and $\gamma_2$, in Euclidean or Minkowski space; and

(ii) before parallel propagating the vectors along the two curves, pick out a point between the two curves and ‘cut out’ a wedge from the spacetime, encompassing an angle $\theta$, and then stitch spacetime back together along the edges of the wedge.

This second situation creates a cone, with a singular curvature at its apex, whose value depends on $\theta$. In the first, but not the second situation, the vector will come back to itself, unrotated. In the second situation, there will be a relative rotation, depending on $\theta$.

The singular curvature between the paths will affect the interference properties of a coherent beam of particles such as neutrons, or indeed, of any system whose state has a vector...
component, e.g. an axis of rotation of a gyroscope. Thus, for example, in situations like (ii), neutrons which traverse a region of space where the curvature is identically zero are nonetheless capable of detecting the effects of curvature in a far away region of space-time, i.e. at the conical singularities.

Lastly, we can also impute to the gravitational case a cohomological understanding of the non-local effect. For there is a straightforward extension of the usual de Rham cohomology to flat vector bundles (see e.g. (Gomez, 2004, Ch. 5)), and thus we can, much like in the electromagnetic case, attribute the difference of the two gravitational situations to different spin connections whose associated curvature is identically zero in the accessible regions, but which have different cohomological contributions. So, as far as symmetries go, the analogy with general relativity seems again very tight.

One often repeated objection here is that for each choice of gauge potential, the profile of the phase gained along the trajectory will look different (Healey, 2004, Section 6), (Healey, 2007, Ch. 2). Thus there can be no physically significant local accrual of a phase.

And yes, it is true that there is a type of gravitational Aharonov-Bohm effect, essentially based on proper time, and is thus incrementally accrued. This is the type of effect that one would obtain for the phase difference of a spinless particle. But this type of effect is less interesting, as it does not require closed loops: it only requires the selection of particular points along each trajectory (cf. (Healey, 2004, Section 6)). An analogy between the gravitational and the gauge Aharonov-Bohm effects requires the use of vectors (or sections of vector bundles) in each case.

And once we have restricted the analogy, the argument about local accrual cuts both ways. For we should note that the dependence on a choice of gauge only occurs if we use a local spacetime representative $A$ of the global principal connection $\omega$ (see footnote 2). That is, if we use only the connection to parallel transport the phase, there will be no dependence on a choice of gauge, since $\omega$ is invariant under passive gauge transformations, cf. (Gomes, 2021c, Section 3.3.2). Nonetheless, using $\omega$ won’t help ‘localize’ the accrual of relative rotation: if the particle’s phase is being parallel transported, its phase is just constant: parallel transport after all is what defines a constancy of phase across the value spaces over different points (usually called, ‘the fibers’, see (Gomes, 2021c, Section 3)). Similarly, the direction of a spacetime vector being parallel transported along a curve is either just constant or ill-defined, in the sense of being dependent on the choice of coordinates used to describe the rotation.

So, my point is that in both cases the effect is holistic: although the total effect is measurable, there simply is no fact of the matter as to how this effect comes about as the result of small, locally accrued differences. There are just facts about parallel transport that are evinced only globally, or rather, only when the paths reconverge.

A second objection is based on (Anandan, 1993) and is highlighted by Healey (2004, Secs. 6-7) as the main difference between the gravitational and the gauge potential-based Aharonov-Bohm effect. The objection focuses on one sense in which the vector rotation can be construed as locally accrued. Namely, since tangent vectors are ‘soldered onto’ spacetime, the angle between the parallel transported vector and the tangent to the curve is locally accrued. Thus, for example, if the vector was just tangent to one of the trajectories, the accrued difference would be a function of the total intrinsic acceleration (and thus related to the difference in

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21This caveat may be also related to the importance of quantum physics in the original Aharonov-Bohm proposal: in the gauge potential case, we require a closed loop to observe the effect, open trajectories will not do. That is because there is a single charge that is in a quantum superposition along the two trajectories. Otherwise, we could make sense of a phase difference in a gauge-invariant way for open paths as follows: given two charges, with associated wavefunctions $\psi_1(x_1), \psi_2(x_2)$, respectively at points $x_1 \in \gamma_1$ and $x_2 \in \gamma_2$, we can compare the phases of the two in a gauge-invariant way by transporting along the segment of $\gamma := \gamma_1 \circ \gamma_2$ that connects them. Namely, we transport the phase of $\psi_1(x_1)$ along $\gamma$ to $x_2$ and compare it with the phase of $\psi_2$: this is an invariant (since $\psi_1(x_1) \exp(i \int_{x_1}^{x_2} A) \psi_2^{-1}(x_2)$ is gauge-invariant).
total elapsed proper times).\footnote{So I take the angle between the parallel transported vector and the tangent to the curve to be locally accrued even if that angle vanishes. In this point I depart from Weatherall (2016, Section 5)’s kindred criticism (of Healey’s argument for a disanalogy). For Weatherall argues that a reference of constancy is only meaningful if the particle paths are geodesic, which would constrain the angle to vanish throughout motion. Thus, he says, there would be no local accrual. But there is a difference between local accrual not being well-defined and a vanishing accrual. Moreover, I also believe one could have a meaningful notion of constancy even for non-geodesics, e.g. for an accelerating rocket.}

But this second objection cuts ice only in the simplified two-dimensional treatment I gave above, and is discarded when more detail is added. That is, to assess the relative rotation of the spin of a particle such as that of the neutron, i.e. to assess the relative rotation of polarization vectors, we must use Fermi-Walker transport. In other words, we are calculating a type of Thomas precession, which is about the rotation of a spatial vector (a 3-vector), i.e. the rotation in the plane orthogonal to the timelike trajectory of the particle. Of course, the angle between a polarization 3-vector and the tangent to the curve is also constantly zero; there is no coordinate-independent way to locally measure the rotation of the polarization vector. To put it differently: comparing the parallel transported 4-vector to the tangent to the trajectory allows us to locally determine the evolution of one degree of freedom of the 4-vector; but this still leaves open how the remaining polarization degrees of freedom—three if the particle is massive, two if it is massless—evolve along the trajectory. For these, all we can do is compare a relative rotation upon the reconvergence of the paths, and thus the qualitative analysis presented above still holds.

One disanalogy between the gravitational and the gauge Aharonov-Bohm effects remains: about shielding. We can, in the laboratory, easily shield magnetic sources from the paths of the electron. Shielding gravitational sources, however, is not so easy:\footnote{See Beig & Chruściel (2017); Carlotto & Schoen (2016) for some interesting shielding results in perturbative and non-perturbative gravity, respectively.} for that, we may need more esoteric gravitational objects, such as cosmic strings. But I take this feature to not germaine to the comparison. For it is of course due to the particular dynamics of the two theories—electromagnetism is not gravity!—and not due to the character of their symmetries.

In sum, apart from differences that are due to dynamical features of gravity (such as, the absence of particles with negative mass, that would allow shielding), there seems to be no conceptual distinction between the Aharonov-Bohm effect and parallel transport around a conical defect in general relativity. In both cases, one cannot, by surveying a neighborhood of the particles’ trajectories, infer whether they will experience a relative shift when they are once again reunited. But a mystery arises only if we assign undue significance to the coordinate choices used to evaluate global rotations. Thus one cannot use the effect to advocate eliminativism for gauge theories but not for general relativity.

### 3 Healey against localized gauge potential properties

In this Section and Section 4.2 I take issue with two arguments in Healey’s (otherwise outstanding!) book on the philosophical interpretation of gauge theories. They both occur in his Chapter on classical gauge theories (Chapter 4); and they are both directed at an interpretation of the theories that Healey calls ‘the localized gauge potential properties view’.

Healey’s second argument is brief; and occurs in (Healey, 2007, Section 4.4), devoted to defending his own preferred view of gauge, the ‘holonomy interpretation’. My disagreement with that argument is best postponed to Section 4.2.

The present Section will respond to Healey’s first argument. For Healey takes the ‘localized gauge potentials properties view’ to postulate that the gauge potential $A$ at a spacetime point
"x" represents a physically real property of, or at, "x. This is, in essence, a quidditist viewpoint, by which properties and relations have a nature that outstrips their patterns of instanitision in objects and co-instantiation with each other. The viewpoint has been defended, for instance in (Arntzenius, 2012, Ch. 6). And (Healey, 2007, Section 4.2) is devoted to assessing, indeed rebutting, this view. And although I also reject this quidditist view, I will disagree with Healey’s arguments, which are distinctively philosophical.

Healey’s main argument against the localized gauge potential properties view is that it suffers from a massive or radical under-determination—of a kind familiar in philosophy, especially associated with the labels ‘multiple realizability’, and ‘permutation argument’. More problematically, Healey argues that this threat of under-determination is unique to gauge theories.

In broad lines, my reply will be that the under-determination is avoided by a ‘structuralist’ construal of the properties in question, that is as tenable as—no more dubious or controversial than—the ‘structuralist’ construal of spacetime points (that Healey himself endorses). Since I have already argued at length in favor of these views in the accompanying paper (see (Gomes, 2021c, Section 2 and Section 3)), one could reasonably leave it at that. But in making his argument, Healey touches on an interesting topic: that of specifying, amongst the infinitely many physically equivalent representatives, a particular spacetime distribution of the gauge potentials or of the metric. As I understand Healey, he posits that this specification is easy for the metric, but impossible for the gauge potential. And with this alleged contrast, I will disagree.

I will start in Section 3.1 by laying out Healey’s argument in more detail and furnishing what I take to be the more interesting challenge that can be read into his argument. Then I will give my answer in Section 3.2.

3.1 Healey’s argument from functional roles

To spell out his first argument in more detail, I will now indulge in a bit of ‘Healey exegesis’. Healey admits that within a theory there may be many terms for unobservable items; but unobservability by itself is not condemning, since we can employ D. Lewis (1970, 1972)’s ideas about simultaneously specifying several theoretical items that are, in some sense, problematic, by their each uniquely satisfying some description (usually called “functional role”) that can be formulated in terms of less problematic items. Lewis’s ideas allow us to fix what such theoretical (or, less broadly, unobservable) terms in a theory refer to, without having a prior interpretation of those terms, by describing how they fit in a pattern of better understood (or, less broadly, observable) items.

In Lewis’s framework, functional roles such as the one Healey discusses here usually involve a binary division of our theory’s vocabulary: a ‘troublesome’ part, that we denote with a T, and an ‘okay’ part, that we denote with an O, whose members’ reference is already fixed (so T is less well understood than O). Each T-terms is to be specified by satisfying a certain pattern of relations, whereas the O-vocabulary is assumed to be already interpreted.24 So the leading idea of functional definition (and here, of identifying particular realizers for the gauge potential by their patterns of instantiation) is to use O and T-terms to jointly fix the reference of the T-terms.

But Healey argues that, in a quidditist interpretation of gauge theories, this Lewisian strategy is bound to be plagued by under-determination. We will see the details of his arguments in a moment. I will give to responses: one that is a straightforward rejection of quidditism,

24Here is Lewis describing O and T-terms: ‘T-term’ need not mean ‘theoretical term’, and ‘O-term’ need not mean ‘observational term’ [...] ‘O does not stand for ‘observational’. [...] They are just any old terms’ (D. Lewis, 1972, p. 250). So despite the letters ‘T’ and ‘O’, Lewis’ proposals are not only about the theory-observation distinction. See Butterfield & Gomes (2020) for a recent analysis of functionalism as a species of reduction.
and which I deem to be less interesting. But I also want to give another response, that rebuts
a more interesting construal of the details of Healey’s arguments.

The straightforward challenge. I illustrate this challenge with Healey’s chosen example, based
on a toy-theory of coloured quarks (Healey, 2007, p. 94). In a world in which this theory is
true, particles can have one of three colours: red, green and blue. Although colours figure in
the laws of the toy theory, these laws are invariant under colour-permutation symmetry. One
way to realize this symmetry is to have the particles be dynamically confined in colour-neutral
combinations of red, green and blue. Healey thus supposes that in this world it is a law of
nature that red, green and blue always occur colour-neutrally and that there is no further
distinction between the colour-carrying particles; once they occur in certain colour-neutral
combinations, not even their locations can differ.

Healey then argues, correctly, that in this set-up the terms ‘green’, ‘red’ and ‘blue’ are
referentially indeterminate. For $x_1$ could stand for any colour, as long as $x_2$ and $x_3$ stand for
the remaining two colours. Thus the individual colour terms cannot directly refer.

This challenge fits comfortably into the debate we have already encountered in this Section’s
preamble: namely, about whether to conceive a property as having an intrinsic nature indepen-
dent of its patterns of association with other properties (quidditism), or not (anti-quidditism).
And there is an analogous philosophical debate about objects, rather than properties, i.e.
about whether an object has an intrinsic identity (or ‘thisness’: haecceitas), independent of its
properties and its relations to other objects. Healey himself rehearses this debate and says
(as most authors do) that the best response to under-determination, for someone who be-
lieves that spacetime points are objects, is to take an anti-haecceitist view of spacetime points’
individuation.

In that jargon, the structuralist response against under-determination is easy to state: if all
the properties of a certain kind are each exhausted by their each filling a certain theoretical role
(of course, with different roles for different properties)—in other words: if each property has
no further “intrinsic nature”—then permuting which properties fill which roles make no sense.
There is no such permutation. Similarly, in the general-relativistic case, if we specify points by
their chronogeometric relations, then permuting which points fill which roles makes no sense:
and the same type of argument applies for the gauge potential and gauge symmetries.

In sum, I agree with Healey about the problem: a literal understanding of symmetry-related
gauge potential distributions across spacetime as being physically distinct is untenable. But
I disagree about what is the best solution: I advocate sophistication and Healey eliminativism
(see Section 2.4.2 and footnote 18 for an explanation about the jargon). In other words, for
general relativity as much as for gauge theory, a literal interpretation of models would give
rise to ‘an infinity of distinct distributions’ describing a given physical world. The response to
this underdetermination that I rehearsed above is simply to take a sophisticated interpretation,
which judges all of these different distributions to be physically equivalent, so that no re-
dundancy of physical properties would arise. I find this counter-argument simple and convincing,
and applicable to both general relativity and Yang-Mills theories.

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25(Jacobs, 2021, Section 6) also treats this argument of Healey (2007) (condemning the under-determination
of the gauge potential), and gives essentially this type of anti-haecceitist response. See (Gomes, 2021c, Section
3.3.3).

26But it is tenable if we consider only passive gauge transformations, i.e. changes of trivialization of the
principal bundle $P$ (see footnote 2). For different choices of the gauge potential are only different choices of
coordinate representations of the same underlying $\omega$, and, equivalently, of the same section $\Gamma$ of the Atiyah-Lie
algebroid, which is a vector bundle over spacetime which does not employ a trivialization of the bundle. See
(Gomes, 2021c, Secs. 3.3.3 and 3.4) for a brief description of $\Gamma$ and for the importance of focusing on invariance
under passive transformations, respectively.
The interesting challenge  If one endorses quidditism, as David Lewis did, there is still a second line of response to the threat of physical underdetermination: to appeal to patterns of facts of “geography” to break the underdetermination. As described by D. K. Lewis (2009):

Should we worry about symmetries, for instance the symmetry between positive and negative charge? No: even if positive and negative charge were exactly alike in their nomological roles, it would still be true that negative charge is found in the outlying parts of atoms hereabouts, and positive charge is found in the central parts. O-language has the resources to say so, and we may assume that the postulate mentions whatever it takes to break such symmetries. Thus the theoretical roles of positive and negative charge are not purely nomological roles; they are locational roles as well. [my italic]

Based on his use of Lewis’s ideas, I interpret Healey as saying that one can functionally specify a spacetime metric, but cannot specify a gauge potential. More precisely, here is Healey’s argument that the functionalist methodology applies so as to single out spacetime metrics, but not to single out gauge potentials:

The idea seems to be to secure unique realization of the terms […] in face of the assumed symmetry of the fundamental theory in which they figure by adding one or more sentences [namely, S] stating what might be thought of as “initial conditions” to the laws of that theory. These sentences would be formulated almost exclusively in what Lewis calls the O-language—i.e. the language that is available to us without the benefit of the term-introducing theory T. But they would also use one or more of the [symmetry-related] terms […] to break the symmetry of how these terms figure in T. They would do this by applying further constraints [namely, S] that must be met by the denotations of these terms in order that S&T be true. Those constraints would then fix the actual denotation of the […] [symmetry-related terms] in T so that, subject to these further constraints, T is uniquely realized. […] [But] The gauge symmetry of the theory would prevent us from being able to say or otherwise specify which among an infinity of distinct distributions so represented or described is realized in that situation. This is of course, not the case for general relativity. (Healey, 2007, p. 93) [my italics]

Why is this “of course not the case for general relativity”? And why does Healey see a contrast between general relativity and gauge theory? These questions are central for this paper.

For I do not see in this entire passage an attempt to draw a distinction between anti-quidditism for gauge and anti-haecceitism for gravity, per se. I believe the more interesting interpretation of this passage is as an attempt to address other questions, about the use of the theories. I think the interesting question being alluded to here is whether we can use features of the world around us to single out a unique model of the theory, or a model with unique features. This other question is interesting because, in practice, we do select some models over others when we represent a given physical situation, and therefore in using the theory we must ‘break the symmetry’ between all of the models. Lewis takes this breaking to justify a type of quidditism; but I do not: I think it is solely based on pragmatic issues, to do with the use of theory.

In more detail, suppose that we cannot find a perspicuous interpretation of general relativity that includes just the symmetry-invariant quantities as part of the basic syntax of the theory (cf. (Gomes, 2021c, Section 2.2.2 and footnotes 21 and 47)). And suppose we agree, as I have argued in that paper, that the best we can then do is to keep all of the symmetry-related representations on a par. Then we are faced with a mystery: in practice we do select particular
representatives over others, whether we take them represent different physical possibilities or not. No particular choice is mandatory, but each must be based on something: physical features, indexicals, ostention, etc.

Thus I take the more interesting interpretation of Healey’s passage here to be that this ‘singling out’ of particular models is possible for gravity but not for gauge theory. If this were so—if there was literally nothing we could base our choices of gauge potential representative on—we would be more motivated to seek out a pure structural formulation of gauge theory than of gravity.²⁷

But I will argue that having some physical “hook” with which to choose representatives does not imply that we are breaking the symmetry at a fundamental level. Different choices of representational conventions would be equally capable of representing a given state of affairs; some may be just more cumbersome than others, or they obscure matters for the purposes at hand, even while they may shine light on complementary aspects of that state of affairs. And in this sense, we can shift our focus to different features of the world, according to our interest, and thereby single out different representative models models—within both general relativity and Yang-Mills theory.

3.2 Refuting the distinction

We are now ready to rebut Healey’s argument from functional roles.

As we have seen, the argument brings an interesting question to the fore: under what conditions could we be justified in choosing for the gauge potential one spacetime distribution over another? Selecting such a representative involves a tension between: (i) a structural construal of physical properties—as ones that are invariant under the symmetries in question—and (ii) selecting unique representative distributions of the gauge potential, among the infinite representatives of the same situation. At first sight, these two requirements, (i) and (ii), are inimical, if not contradictory, for (i) implies we can have no physical guidance for accomplishing (ii)!

Below I will show that we can construct a particular representative of the gauge potential as fulfilling a given role, and explicitly check that such a notion is invariant under the permutations of properties. In more detail, I will first show that (1) intra-theoretic resources enable us to pick out gauge representatives; and then show that (2) indexicals make no difference for the unique specification of the metric.

Starting with (1), I will resolve the tension between (i) and (ii) with explicit examples; by, in Healey’s words: ‘breaking the symmetries’, by providing ‘further constraints’, such that the we fix the denotation of ‘a section s’, or, equivalently, a particular gauge-potential, A, that is, a particular spacetime representation of the principal connection ω (see footnote 2). In Gomes (2021b) it is argued that such choices are what Wallace (2019) dubs ‘a representational convention’.

Note to begin with that we are justified in including in our O-vocabulary all the ‘locational roles’, which describe contingent, happenstantial facts about ‘where and when’ specified events happen; and which I will loosely interpret as ‘referring to spacetime’. Thus I free myself to include in the O-vocabulary, and thereby use in the specification of the roles, the differential geometry of spacetime.

I will first expound the functional roles in the case of the gauge potential, and then draw the analogies with the metric.

²⁷And indeed, later in the book Healey uses this distinction as a motivation for seeking a different, symmetry-invariant ontology of gauge theory, based on holonomies, which I will criticize in Section 4.2.
In the simple example of electromagnetism, we require the model to satisfy certain relations among the parts of the field. For example, an explicitly Lorentz-covariant choice, advocated by Mattingly (2006) is (in vacuo):

\[ F(A) := \partial^\mu A_\mu = 0. \]  

(3.1)

Here the value of the connection \( \omega \) is fixed: all we are trying to do is to determine a particular section \( s \) (seen as a submanifold of \( P \); see footnote 2). The only extra constraints that we have imposed in this equation, namely, that the spacetime divergence of the particular representative of \( \omega \) vanishes, use only the \( O \)-vocabulary that Healey would grant us, and therefore should qualify as providing ‘actual denotation’. We can also explicitly display in \( O \)-vocabulary the projection of an arbitrary representative of \( \omega \) into a representative satisfying (3.1).

Thus, given any representative of any equivalence class, \( A \), we define:

\[ h(A)_a := A_a - i\partial_a(\Box^{-2}\partial^b A_b), \]  

(3.2)

where \( A_a \) is any 1-form and \( \Box^{-1} \) is a propagator, the inverse operator to the d’Alembertian \( \Box := \partial^\mu \partial_\mu \). The choice is not physically restrictive: given any \( A \), we can translate it along the fibers, looking for a representation of the field that satisfies equation (3.1).

And it follows from this construction that the determination of the representative is structural, in the sense that, though \( h(A)_a \) is, like \( A \), a Lie-algebra valued one-form, \( h(A)_a \) is a gauge-invariant functional of \( A \). That is, for a gauge-related \( A'_a = A_a + \partial_a \chi \), we obtain \( h(A')_a = h(A)_a \), and therefore \( h(A)_a \) can be construed as a structural property of the field. Moreover, it is an exhaustive property, since each equivalence class (or physical world, according to the theory) will project to a single \( h(A)_a \).

In other words, comparing this with Healey’s example in the straightforward challenge: \( h \) are directly analogous to the colour-neutral quantities, and the coloured particles are directly analogous to the gauge potential \( A \). Within a single world, or physical situation, or equivalence class \( [A] \), permuting among the infinity of distinct representing distributions, namely, permuting among the corresponding \( A \)’s, makes no difference to \( h(A) \), just as permuting the colours makes no difference to colour neutral combinations of particles in the toy example. The properties of \( h(A) \) are exhausted by its filling a certain theoretical role, or pattern of instantiation, reflecting anti-quidditism.

Thus, the representational convention: ‘find a representation of the electromagnetic potential that is divergence-free’, \( \partial \) \( \Box \) (3.1), is a gauge-invariant specification. Nonetheless, it successfully pins down representation for the electromagnetic potential. This construction thus explicitly contradicts the letter of Healey’s under-determination argument in the quotation above.

One could still ask what conditions could possibly suggest a choice such as (3.1). The answer is that different pragmatic virtues can motivate different choices. Of course, the representational convention cannot be empirically mandated, since, by assumption, the symmetries leave all empirical matters invariant, in both the gravitational and the gauge cases. The choice of convention is rather a pragmatic matter: the choice of a uniquely specifying functional

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28Thus, in the Euclidean, or spatial setting, such an \( h(A) \) captures the radiative component of the potential. This example is merely illustrative, as in the Lorentzian setting this gauge-fixing is not complete. But in the Hamiltonian setting footnote 7, lays out a bona-fide example.

29For any \( A \), \( h(A)_a \) is a potential that is related to \( A \) by a gauge-transformation, namely \( g(A) = \Box^{-2}\partial^b A_b \). Thus to constructively find the section \( s \) for which \( s^*\omega \) satisfies our condition, we can start by guessing one section \( s' \), with \( s'^*\omega =: A' \) and, by writing \( g(A') := g_s(\omega) \), we find that the transformation required to go from \( s' \) to the section selected by our functional role is: \( s = g_s(\omega) \circ s' \) for any \( s' \). Of course we still have the freedom to change the section \( s \), but a different section would not satisfy the original condition that uniquely specified \( s \).
role such as (3.1) can, even should, be one of mere preference for particular representations of the field; it is at most suggested by being suitable to a given physical situation. It is a pragmatically-guided choice of "coordinates".

Similar choices occur in any theory with symmetries. For instance, in the more familiar case of special relativity, we may choose Lorentz frames that are adapted to some phenomenon under study: e.g. "co-moving with a rocket". Singling out this choice of frame does not imply that another frame is less capable of describing the goings-on in the laboratory inside the rocket, just that it may be more cumbersome to do so. Or even simpler: in \( \mathbb{R}^3 \), it is wise to choose polar coordinates for a cylindrically symmetric problem.

In the gauge case, other pragmatic criteria are in play. For example, the choice of (3.1) is explicitly Lorentz covariant (and has its virtues thoroughly extolled by Mattingly (2006), who argues for that it should be considered as Maudlin (1998)’s “ONE TRUE GAUGE”). But we may want to highlight the helicity degrees of freedom of the theory, in which case we would use temporal gauge. Or again, we might choose to split the electric field into one component that is purely ‘electrostatic’—or rather, due solely to a Coulombic potential—and another that is purely radiative. Such a split for the electric field corresponds, through symplectic duality, to a Coulomb gauge for the spatial vector potential: the gauge potential splits into a term that is “pure gauge” and one that is radiative (or in Coulomb gauge). See Gomes & Butterfield (2021) for a thorough analysis of this choice).

Besides, this discussion applies equally for the metric. When we gauge-fix the representational conventions for the metric (say using harmonic gauge), we functionally specify a single metric to represent the geometry by the use of some extra condition \( S \), and this specification employs the same (differential geometric) tools and enjoys the same properties as the gauge version.

And this flexibility is also explanatory, or at least is able to shed light on important physical features. Just like it is easy to explain the Larmor effect by a Lorentz boost between different frames, the ability to choose different conventions, or gauge-fixings, makes it easy to explain that a given process in quantum electrodynamics involves just two physical polarisation states and that it is Lorentz invariant. In both the special relativistic and the gauge scenarios, two different ‘frames’—for quantum field theory, the temporal and Lorentz gauge—are necessary to explain two different aspects of a given phenomenon.

This concludes the first part (1) of my response to what I called the ‘interesting challenge’: showing that intra-theoretic resources enable us to choose representational conventions and pick out representatives in all kinds of theories with symmetries. Now on to the matter of indexicals.

(2): Here is one possible objection: “Fine”, you, or Healey, might say, “we use the same tools to specify local representatives of both types of fields (amongst all of their symmetry-related models), but I can use indexicals to specify a metric and I cannot do the same to specify gauge potentials”.

The idea here is in effect that you could specify the metric along your own worldline. But the idea is misguided, for to specify the entire metric you will still need to specify a linear frame along your worldline, and how do you do that? You must use relations to other objects, fields, or features of the metric field itself (such as anisotropy directions). All of these are just other (types of) conjunct of the ‘functional role’ specifying the (particular representative of) the metric, of the same sort that could be appealed to for the gauge potential.

Besides, you could do something similar with electromagnetic interactions: this is what we do when we interpret ‘photons’ that propagate and hit our retinas as purely radiative, i.e. as

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30 As briefly described in footnote 7, this choice is interesting in the context of the (mild) form of non-locality discussed in Section 2.2.1: the radiative component of the electric field is independent of any value of the electric field at the boundary of the region in discussion.
having no polarization along the time axis or along the direction of their motion. \(^{31}\) We could also indexically fix the gauge by resorting to the charged matter fields. In the spirit of the opening passage of Yang and Mills’ original paper Yang & Mills (1954), I could say: “what is trapped in this tiny (sub-atomic) box is a proton and not a neutron”. And if, even after all this, you still insist that indexicals somehow apply only to spacetime quantities and external properties, and not to internal properties such as charge, I would say that this exceptionalism would clearly beg the question.

Thus, under closer scrutiny, the distinction suggested by Healey (2007) crumbles.

4 Summing up

I will divide this concluding Section into three parts. In 4.1 I will summarize and reflect on what has been argued thus far. In Section 4.2, I will buttress the formal arguments in favor of sophistication made in the accompanying paper Gomes (2021c) with more concrete/pragmatic considerations, that show us the many drawbacks of eliminativism and the benefits of redundant representation. In 4.3, I briefly conclude.

4.1 Summary

Although I have focused on general relativity as the best-known diffeomorphism-invariant theory, I have kept out of the discussion those idiosyncrasies of the theory that are of a more dynamical nature. Thus, putting these issues aside, in Section 2 I could find no salient difference between gauge symmetries and diffeomorphisms in the three topics considered: (i) as to the constraints, a difference exists but it is due to the problem of time (Section 2.2); (ii) as to the association between symmetries and conserved charges, a difference exists but it is due to an Abelian vs. non-Abelian nature of the symmetries (Section 2.3); and (iii) as to the Aharonov-Bohm effect (Section 2.4), although there may be practical difficulties involved in ‘shielding’ any source of gravitational curvature, I could find no salient difference: the rotation of the electron phase (respectively, vector) along an open path is either zero or has an arbitrary dependence on the section (resp. coordinates). In both cases the shift in rotation can be invariantly measured only for closed paths, and then it carries non-local information.

Healey brings up two different distinctions than those studied in Section 2. He articulates the conceptual analogy between the under-determination argument for gauge and the hole argument. Although the analogy is close, mathematically as well as philosophically (for general relativity also can be formulated as a gauge theory, see e.g. (Baez & Munian, 1994, Part III, Ch. 5) and (Bleecker, 1981, Ch. 8)) Healey sees what he considers an important disanalogy with general relativity, and it is this disanalogy that I rejected in section 3.

And although Healey’s target is a form of quidditism—a target I also shoot down—his arguments attempt to draw a distinction from general relativity that I found unconvincing. For Healey believes that one can use D. Lewis (1970, 1972)’s ideas about functionalism to fix a particular representative of the metric, i.e. a particular local distribution of properties; but one cannot do the same for the gauge potential. If this were so, it would indeed point to a salient difference between spacetime diffeomorphisms and gauge transformations: the latter but not the former would leave us no handle with which to fix representation. Thus gauge under-determination would be of a more problematic sort, and eliminativism—the attempt to formulate a theory with a syntax that never uses gauge-variant terms—would become better

\(^{31}\) There is here a slight awkwardness in the physics lingo. The ‘photon field’ is taken as \(A_{\mu}\), but a ‘photon’ is taken to be radiative, usually written \(p^\mu A_{\mu} = 0\).
motivated. But the truth is that in neither theory are there general physical distinctions that select one or the other spatial distribution of the field, be it the metric or the gauge potential. This raised a conundrum, since we often select one representation over another: but what warrants such a choice?

I then resolved the conundrum by specifying roles for the gauge potential to fill (loosely called ‘functional’ in Section 3). The key to the resolution is to note that the ‘functional’ role—e.g. highlighting the ‘radiative’ features of the photon field—operates as a projection map in the space of models, \( \mathcal{M} \): all symmetry-related distributions project down to the same representative that fills one such role. Consistently, the choice of role is not physically mandated; it is based on pragmatic, contingent, or user-centric criteria, that operate similarly in both general relativity and gauge theory.

### 4.2 The unifying power of gauge

In the accompanying paper Gomes (2021c) I argue that sophistication, in the sense of Section 2.4.2 and footnote 18, is equally conceptually transparent in both general relativity and gauge theory. Now I will give negative arguments, about what is lost with the opposing doctrine of eliminativism. I hope these arguments convince the reader that excising gauge redundancy from our theory would incur significant explanatory deficit.

Gauge theories have found enormous use in modern physics. They illustrate the immense value that a symmetry, or a redundancy of description, has for theory-building. The underlying reason for their usefulness, is that their symmetries consistently combine different parts of the system: the charges and the fields they interact with. Once we have empirically conserved charges, we have an associated theoretical rigid (global) symmetry, related to this charge through Noether’s first theorem. By localizing this symmetry, we are assured that the dynamics of any force coupling to this charge will be automatically compatible with the original charge conservation.\(^{32}\) Thus local symmetries are useful for theory-building.

One may still feel that gauge redundancies are like Wittgenstein’s ladder: they are useful in the course of constructing our theories, but thereafter may be thrown away in favour of a non-redundant description of reality, i.e. descriptions formulated using exactly as many variables as there are degrees of freedom in the system studied.

But I would reply to this: on the contrary, the ladder is invaluable. This is shown by all the other arguments that militate against eliminativism for the gauge potential—and for gauge symmetry by proxy—and may not so easily be discarded; there are, in fact, many ‘gauge-arguments’, and here I will rehearse a few of them.

This Section thus presents a reply to the second argument by Healey (in Healey, 2007, Ch. 4.4), alluded to in Section 3. In the course of defending his holonomy interpretation, Healey says that the localized gauge potential properties do not, so to speak, earn their living—that is: earn our believing in them—in the way that a theoretical posit should. For, as he puts it, they have no unifying power.\(^{33}\) And with this I will of course disagree. For it will be the job of this Section to show that indeed they do earn their living in the way indicated by Healey’s phrase ‘unifying power’.

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\(^{32}\)Although it is well-known that Noether’s first theorem implies charge conservation, and that the second theorem implies relations between the theory’s equations of motion, this further point—that making a rigid symmetry malleable enforces compatibility between charge conservation and the dynamics of the corresponding fields—was first emphasized in Gomes et al. (2021).

\(^{33}\)He says: “Localized gauge potential properties do not unify a classical Yang-Mills gauge theory, because of their severely limited role within the theory. They are unobservable, even given the theory; and they do not lead to predictions of new phenomena. No justifiable principle of inference to the best explanation should saddle us with them, no matter how much evidence supports the theory.” (Healey, 2007, 121-122)
Indeed, one might think that the difference between the two types of symmetry transformations found in Gomes (2021c), labeled \( \Delta \), can be pressed into service against redundancy. For \( \Delta \) says that non-Abelian Yang-Mills theories can be formulated using a mathematical structure, called the Atiyah-Lie algebroid (see footnote 26), in which the curvature represents all local degrees of freedom of the theory in a gauge-invariant manner. Thus one may imagine that, by employing just the curvature of the connection in an Atiyah-Lie formalism, we could rid the formalism of gauge symmetry at last. In the Abelian case, we need not even resort to the Atiyah-Lie formalism to make this claim, and so, for the purposes of this discussion, we can set it aside.

But there are two problems with this. First, the curvature represents only, and all, those physical degrees of freedom that are localizable. As we saw in Section 2.4, there are properties pertaining to the parallel transport of internal and external quantities that are non-local and cannot be easily encapsulated by the curvature. Thus, while we may have a good reason to distinguish the gauge symmetries from the diffeomorphisms, we do not have warrant to eliminate the connections, and their gauge-related counterparts, from the formalism.

Second, even in the absence of something like the Aharonov-Bohm effect, the curvature cannot be articulated as a primitive, local quantity: it must make reference to the gauge potentials. For instance, if we really believe the physical world embodies Lorentz invariance, we would also like our theories to exhibit explicit Lorentz covariance. This is not possible with local, gauge-invariant fields, such as the standard electric and the magnetic fields, even in the Abelian case. In fact, this difficulty, noticeable in the ‘moving magnet and conductor problem’ (as in the opening of Einstein’s 1905 paper), led to the creation of special relativity.

Thus a unified, Lorentz covariant formulation of the electric and magnetic field necessarily employs \( F_{\mu \nu} \). And \( F_{\mu \nu} \) is not just an arbitrary 2-form, but one that necessarily satisfies—satisfies independently of any other contingent fact, such as the metric or the matter distribution—the Bianchi identities. These constraints are enforced by representing the curvature as a function of the gauge potential. One could try to represent the theory without constraints and symmetries, but that is a difficult task, if we are to keep other pragmatic criteria, such as kinematical locality. As can be seen clearly in the Hamiltonian formalism (cf. Section 2.2), constraints encode symmetries, and thus finding a formalism that is genuinely symmetry-invariant without the imposition of further constraints is highly non-trivial. And a unified, Lorentz covariant formulation of the electric and magnetic field necessarily employs \( F_{\mu \nu} \). And \( F_{\mu \nu} \) is not just an arbitrary 2-form, but one that necessarily satisfies—satisfies independently of any other contingent fact, such as the metric or the matter distribution—the Bianchi identities. These constraints are enforced by representing the curvature as a function of the gauge potential. One could try to represent the theory without constraints and symmetries, but that is a difficult task, if we are to keep other pragmatic criteria, such as kinematical locality. As can be seen clearly in the Hamiltonian formalism (cf. Section 2.2), constraints encode symmetries, and thus finding a formalism that is genuinely symmetry-invariant without the imposition of further constraints is highly non-trivial. Thus, in order to find primitive, gauge-invariant quantities, one should also aim to unshackle them from any constraint.

Another attempt to get rid of gauge variance (the one favoured by Healey (2007)), adopts gauge-invariant but non-local fundamental variables for the theory. The most common version in an attempt to excise mention of the gauge potential from the formalism, we could try to include another term in the Lagrangian with the use of a Lagrange-multiplier, a term whose equations of motion would recover the Bianchi identity. Formally, this is easy enough to do:

\[
S[\lambda, F] := \int F_{ab} F_{ab}^d + \lambda^{[abc]} \partial_c F_{ab].
\]

But in fact, this implicitly invokes the gauge potential that we are trying to avoid. For note that, in the previous equation, \( \lambda^{[abc]} \) can be rewritten as \( \lambda^{abc} = \epsilon^{abcd} \lambda_d \) by the Hodge-duality between three-forms and one-forms. Therefore, after integration by parts (we assume, as usual, that \( F \) falls off quickly and there are no boundary contributions to the integral) we can rewrite the Lagrange multiplier term as: \( F_{ab} \epsilon^{abcd} \partial_c \lambda_d = F \wedge * d\lambda \), where \( \lambda \) is a one-form. Re-inserting this identity in (4.1), the new equations of motion for \( F \) now yield precisely \( F = d\lambda \). Re-insertion of this identity into the action recovers the standard Maxwell action, without the Lagrange multiplier term, but with the trivial notational substitution of \( A \) by \( \lambda \). The general lesson here is that constraints cannot be so easily eliminated. And the non-Abelian theory is even more inimical to a local, gauge-invariant representation: we cannot even write equations of motion without the explicit appearance of the gauge potential, for, in the non-Abelian case, the covariant derivative of the curvature, figuring in the (e.g. vacuum) equations of motion \( D^a F_{ab} = 0 \), includes commutation with the gauge potential.
adopts a holonomy formulation: a holonomy basis of gauge-invariant quantities associates to each loop in spacetime a phase, namely, the one we found in our treatment of the Aharonov-Bohm effect, in equation (2.21). The holonomy interpretation goes beyond interpreting the holonomies as integrals of the gauge potential: it promotes the ontic status of these quantities, so that they should no longer be thought of as derivative from the gauge potentials (or from the connection form), but as primitive.

But the holonomy formalism carries many explanatory deficits in comparison to the formulation of the theory on the bundle. For instance, since the basis of primitive holonomies is vastly overcomplete, it obeys certain constraints, which characterize the composition properties of loops. And, as far as I can see, these composition properties can only be derived by reference to the original, gauge potential variable, A. Without appeal to A, these composition properties must be postulated ex nihilo, and, being quite unnatural, they leave an explanatory gap.35

This type of explanatory reliance on the theory with more symmetry is in fact a common issue with relationist approaches. To give a simple, oft-repeated example: in relational particle dynamics, even if the vastly overcomplete set of inter-particle separations are somehow taken as primitives, they are not independent. They must obey constraints—e.g., the triangle inequality—which can be either posited ab initio, or, more naturally, arise from the dimensionality (and geometry) of the (substantial) space in which they are embedded (see (Belot, 2003, Section 6)). In short, the reduced picture is too deeply rooted in the formulation of the theory before reduction to have any conceptual transparency on its own.

Note, moreover, that, if a holonomy interpretation were satisfactory for describing the gauge-invariant ontology of the non-Abelian theory, it should also be considered satisfactory to describe diffeomorphism-invariant ontology of general relativity.36

As is well-known, locality provides another reason for keeping gauge redundancy (cf. Section 2.4). Even apart from the Aharonov-Bohm phases, there is no denying that the Gauss constraint is a sign of some kind of non-locality—encoded in the Coulombic mode of the electric field—as do other differential initial value constraints (see Section 2.2). For e.g., the Gauss constraint implies that by simultaneously measuring the electric field flux on all of a large surface surrounding a charge distribution, and integrating, we can ascertain the total amount of charge inside the sphere at the given instant. Underlying this non-locality is the fact that the set of invariant quantities of the whole universe does not equal the union of the sets of invariant quantities of a mutually exclusive, jointly exhaustive partition of the universe into subsystems. Gauge theories involve a type of holism, or non-separability (cf. Gomes (2019a, 2021a); Gomes & Riello (2021), and references therein). In its quantum version, the non-locality implies the total Hilbert space of possible states is not factorizable, even setting aside the fully quantum sources of entanglement such as appear in EPR-pairs.37 And indeed, this non-locality

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35In the general non-Abelian case, the full gauge-invariant content of the theory can be reconstructed from Wilson loops, W(γ). And there is no homomorphism from the composition of loops to the composition of Wilson loops (as there is in the Abelian case). That is, it is no longer true that W(γ1 ◦ γ2) ̸= W(γ1)W(γ2) holds. This is due solely to the presence of the trace in the Wilson loop. The general ‘composition constraints’—named after Mandelstam—come from generalizations of the Jacobi identity for Lie algebras, and depend on N for SU(N)-theories; e.g. for N = 2, they apply to three paths and are: W(γ1)W(γ2)W(γ3) − 1 2(W(γ1γ2)W(γ3) + W(γ2γ3)W(γ1) + W(γ1γ3)W(γ2)) + 1 2(W(γ1γ2γ3) + W(γ1γ3γ2)) = 0. Taking holonomies as primitive would thus require us to postulate one such relation—which seem quite ad hoc—per group. Equally worrisome is the fact that we cannot explicitly write the dynamical equations of motion of the theory in the Wilson loop (or holonomy, in the Abelian case) basis.

36Using the vielbein formalism for general relativity, we could similarly find a map from loops into the structure group of the theory (in the vielbein case, the Lorentz group, and not the charge group G). This is essentially the starting point for certain approaches to the project of quantizing gravity, such as ‘loop quantum gravity’ (see e.g. Thiemann (2003)) which, in this version, has notorious problems with implementing dynamics.

37This type of holism, or non-locality is indeed a well-known issue for theories with elliptic initial value
is apparent when we try to compose regions: the composition of the gauge-invariant content of each region can only be articulated by employing the redundant variables, or the vertical directions in the principal bundle (see (Gomes, 2019a; Gomes & Riello, 2021, Section 6.3)). As argued in (Gomes, 2019a), ‘forgetting’ the symmetry-variant structure of subsystems is fine when we consider subsystems in themselves, but causes trouble when we consider the ‘gluing’ of subsystems. Accordingly, Gomes (2019a) advocates “external sophistication and internal reduction”: that is, in order to glue subsystems together, we need to pay attention to their symmetry-variant features.\footnote{As is apparent also in the holonomy formulation, here too, the underlying point is the totality of gauge-invariant quantities is not freely recombines. In (Jacobs, 2020, Ch. 5), this failure is seen as one of the main obstacles to any fully gauge-invariant formulation of the theory: “the claim that only invariant quantities are fundamental faces two crucial issues: our theories are not expressed in terms of those quantities, and even if they were, such quantities fail the condition of free recombination. Both are reasons to suspect that invariant quantities are not fundamental.” (Jacobs, 2020, p. 141) Here I take this as less evidence against explicit gauge-invariance, but more as a statement about the fundamental holism of gauge theory Gomes (2021a); or, in the nomenclature of Myrvold (2010), it is a statement about the incomensurability between “patchy-separability” and gauge-invariance.}

Finally, I want to emphasize the usefulness of ‘choosing gauges’ in the sense of Section 3.2—namely: gauge-invariantly specifying the representative of a physical situation, based on explanatory and pragmatic criteria—and note that if we had no such choice, we would likely lose enormous explanatory ability. In the words of Tong (2018, p. 1):

At the perturbative level, the [gauge] redundancy allows us to make manifest the properties of quantum field theories, such as unitarity, locality, and Lorentz invariance, that we feel are vital for any fundamental theory of physics but which teeter on the verge of incompatibility. If we try to remove the redundancy by fixing some specific gauge, some of these properties will be brought into focus, while others will retreat into murk. By retaining the redundancy, we can flit between descriptions as is our want, keeping whichever property we most cherish in clear sight.

4.3 Conclusion

The lesson of this and the accompanying paper is that there are many similarities and only one robust dissimilarity between gauge transformations and diffeomorphisms. In particular, I see no smoking gun to validate eliminativism for gauge while endorsing sophistication for diffeomorphisms.

Thus we can understand the ontological commitments of both theories as structural: one describes chronogeometric relations, in a well-understood sense, and the other describes the parallel transport of all sorts of charges that figure in the standard model, in an equally well-understood sense.

As I hope to have shown here, this understanding of gauge theories also scores points for consilience with the rest of theoretical physics. This is no small feat; it is a trait that should weigh heavily in our interpretative preferences. In light of these similarities, it seems appropriate to end with the wise words of Belot (2003, p. 218):

But this much, I suppose, is uncontentious: judgments about the interest and correctness of interpretations of theories which are (in the strictest sense) false must rest ultimately upon judgments about the extent to which various interpretations
of a given theory contribute to, and integrate smoothly with, our understanding of the world. Here the following sorts of considerations play a role: background metaphysical commitments and hopes; judgments about the relative perspicuity of various alternative formulations of the theory that we are interested in, and about the links between variant formulations and competing interpretations; and considerations—operating at the technical, conceptual, and metaphysical levels—that arise when we consider how our theory is related to neighboring theories, both more and less fundamental.

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