Asymmetric Orientifolds, Brane Supersymmetry Breaking and Non-BPS Branes

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Abstract

A new class of six-dimensional asymmetric orientifolds is considered where the orientifold operation is combined with T-duality. The models are supersymmetric in the bulk, but the cancellation of the tadpoles requires the introduction of brane configurations that break supersymmetry. These can be described by D7-brane anti-brane pairs, non-BPS D8-branes or D9-brane anti-brane pairs. The transition between these different configurations and their stability is analysed in detail.

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1. Introduction

In the course of the last two years various attempts at constructing non-supersymmetric open string models have been undertaken [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17]. These models always describe consistent string compactifications, but the stability of the resulting theories is often difficult to establish. In particular, the theories often develop tachyonic modes in certain regions of the moduli space. This indicates an instability of the system to decay into another, quite possibly supersymmetric, configuration.

Non-supersymmetric tachyon-free models in various space-time dimensions have been constructed using a variety of different approaches. In one approach one starts with the ten-dimensional (non-supersymmetric) tachyonic type 0B string theory and performs a suitable orientifold projection that removes the closed-string tachyon [2]. The various projections correspond to inequivalent choices of the Klein bottle amplitude consistent with the constraints of [18]. Tadpole cancellation then requires the introduction of D-branes, whose open string spectrum is also non-supersymmetric; in the resulting theory supersymmetry is therefore broken in the bulk as well as on the branes. It is also possible to consider compactifications of these models [14,15,16], where the orientifold projection is combined with an action of the space-time group.

An alternative way to break supersymmetry is via Scherk-Schwarz compactifications [19]. In the simplest case of a circle compactification, higher dimensional fields are allowed to be periodic up to an R-symmetry transformation. Modular invariance suggests a suitable extension of this mechanism to closed strings [20] and then, via orientifolds, to open strings [3] as well. As a result supersymmetry is broken both in the bulk and on the branes†, and different cosmological constants are generated both in the bulk and on various branes. An interesting variant of Scherk-Schwarz compactifications involving asymmetric $\mathbb{Z}_2$ orbifold projections [21] leads to a non-supersymmetric spectrum with Fermi-Bose degeneracy at all mass levels. Non-abelian gauge symmetries can be generated via orientifolds [5,7]; this leads to models where supersymmetry is preserved on the branes (at lowest order), but is absent in the bulk.

In a different approach, named in [10] brane supersymmetry breaking, supersymmetry is broken only in the unoriented open-string sector where different orientifold planes and D-branes are suitably combined. In the simplest ten-dimensional case [8] the orientifold

† Actually, in the M-theory construction, supersymmetry is still present at the massless level, although it is broken for the massive modes.
projection involves $O_-$ planes with positive tension and positive R-R charge instead of the more familiar $O_+$ with negative tension and negative R-R charge. The projection on the closed string spectrum is insensitive to the particular orientifold plane involved and therefore yields the standard closed sector of the type I string, but the cancellation of the massless R-R tadpoles now requires the introduction of anti-branes with negative charge and positive tension. Thus, the orientifold projection for the open-string bosons is reverted and leads to a $USp(32)$ gauge group while the spinors are still in the antisymmetric representation, consistently with anomaly cancellation. As a result, a positive cosmological constant is generated on the branes thus reflecting the impossibility to cancel the NSNS tadpole. In this construction the choice of the types of orientifold planes is optional and two different open-string spectra (the supersymmetric $SO(32)$ and the non-supersymmetric $USp(32)$) can both be consistently tied to a single closed-string spectrum. In lower dimensional models, however, brane supersymmetry breaking is often demanded by the consistency of the construction [10] and represents a natural solution [15] to old problems in four-dimensional open-string model building [22]. Further non-supersymmetric deformations affecting only the open-string sector involve a background magnetic field [23].

Finally, in all (supersymmetric and non-supersymmetric) orientifold models one has the additional option to add pairs of branes and antibranes consistently with tadpole cancellations [24,14,17,15]. As a result the bulk is not affected, while supersymmetry (if present) is broken on the branes where a cosmological constant is generated. Using similar deformations, quasi-realistic theories with three generations in the standard model, left-right symmetric extensions of the standard model or Pati-Salam gauge groups have been constructed [14,17]. Depending on the concrete model, the string scale can be in the TeV range, or at an intermediate scale; the latter is typical for models featuring gravity mediated supersymmetry breaking.

For models containing parallel branes and anti-branes of the same dimension, the stability of the resulting configuration is problematic. In particular, the system develops a tachyon (and thus becomes unstable) if the branes and anti-branes come close together. In order to obtain a stable configuration it is therefore necessary to remove the moduli that describe the relative distance between the brane and the anti-brane. This can partially be achieved by considering so-called fractional D-branes which are trapped at the fixed points of some orbifold. However, the theory typically still contains bulk moduli that describe the separation between the different fixed points, and it is therefore necessary to remove
those moduli as well. In all examples that have been studied so far, this could only be achieved dynamically, and the details were out of reach of concrete computations.

In this paper we study a six dimensional orientifold, where we combine the world-sheet parity reversal with T-duality, or rather T-duality together with the symmetric reflection of two directions.‡ (Similar models were also considered in [16].) As we shall show, the Klein-bottle amplitude only leads to a twisted R-R sector tadpole in our case. This is of significance since it implies that the tadpoles can not be canceled by any supersymmetric brane configuration: every BPS brane carries untwisted RR charge, but since the entire D-brane configuration must have vanishing untwisted R-R charge, it will necessarily also involve anti-branes and therefore break supersymmetry. Thus one should expect that the resulting non-supersymmetric theory (for a suitable brane configuration) is stable, and this is indeed what we shall find.

In fact we shall find different brane configurations that cancel the tadpoles (as well as the six-dimensional gravitational anomaly), and we shall be able to understand how they can decay into one another. The simplest solution consists of fractional D7-brane anti-brane pairs where the D7-branes and the anti-D7-branes are localized at different fixed points of the underlying orbifold.* If the underlying torus is an orthogonal torus at the $SU(2)^4$ point without any $B$-field, the ground state of the string between the brane anti-brane pairs is either massless or massive. Most of the bulk moduli that describe the shape of the torus are removed by the orientifold projection (T-duality is only a symmetry for a specific class of torii). However, certain shear deformations remain, and if the torus is deformed in this way, an open string tachyon develops in the string between some D7-brane anti-brane pair. This tachyon indicates the instability of the brane anti-brane system to decay into a non-BPS D8-brane with magnetic flux. The resulting configuration of non-BPS D8-branes can be described in detail and it also cancels the tadpoles (as well as the irreducible gravitational and gauge anomalies); as far as we are aware, this is the first time that non-BPS D-branes have naturally appeared in the tadpole cancellation of an orientifold model.

The configuration of non-BPS D8-branes still contains massless scalars in the open string spectrum, and these can indeed become tachyonic if another bulk modulus is turned on. The system then decays into a configuration involving D9-branes and anti-D9-branes

¶ These two descriptions are T-dual to one another.

* The orientifold group is $\mathbb{Z}_4$, and it contains a $\mathbb{Z}_2$ orbifold.
where both branes and anti-branes carry magnetic flux of appropriate type. This can also be constructed in detail, and indeed cancels the tadpoles as well as the irreducible anomalies. The configuration is stable in a certain domain of the moduli space. However, if the torus is tilted sufficiently, yet another tachyonic mode appears in the open string spectrum, and the system decays into a configuration of diagonal D7-brane anti-brane pairs; this final configuration appears to be stable.

The paper is organized as follows. We begin in section 2 by describing the model and the Klein bottle amplitude. Sections 3-6 deal with various brane configurations that cancel the R-R tadpoles. In section 7 we discuss the stability of the different configurations, and their deformations into one another. Finally, section 8 contains some conclusions. We have included an appendix where the more technical material referring to the construction of boundary and crosscap states is discussed.

2. The definition of the model and the Klein bottle amplitude

The model that we shall discuss in this paper is the asymmetric orientifold of Type IIB string theory compactified on a 4-torus, whose coordinates are labeled by $x_6, \ldots, x_9$. The orientifold group is $\mathbb{Z}_4$, and it is generated by $\Omega \Theta_4$, where $\Omega$ denotes the standard Type IIB orientifold, and $\Theta_4$ describes T-duality of the 4-torus. For the orthogonal self-dual $SU(2)^4$ torus with vanishing internal B-field, T-duality is equivalent to the asymmetric $\mathbb{Z}_2$ operation $I_4^L$. In the following we shall discuss mostly this case, and we shall then use the notation $I_4^L$ for T-duality.

The orientifold group contains a $\mathbb{Z}_2$ orbifold subgroup that is generated by $I_4$; thus the theory is equivalently described as a $\mathbb{Z}_2$ orientifold of Type IIB on K3. The $\Omega \Theta_4$ symmetry fixes the metric of the $T^4$ completely, whereas the six independent values of the internal $B$-field are free parameters.

It is actually more convenient to consider the theory that is obtained from the above after T-duality in the $x_7$ and $x_9$ directions, say. If we denote by $\mathcal{R}$ the (symmetric) reflection in these two coordinates, the theory in question is then described by

$$\text{Type IIB on } SU(2)^4 \frac{\text{Type IIB on } SU(2)^4}{\{(1 + I_4) + \Omega \Theta_4 \mathcal{R}(1 + I_4)\}}. \quad (2.1)$$

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Under this T-duality, some of the B-field moduli of the original model are mapped to geometric moduli of (2.1). In fact, looking at the massless modes that survive the projection, one finds that infinitesimally the following deformations are allowed

$$\delta G = \begin{pmatrix} 0 & \delta g_{67} & 0 & \delta g_{69} \\ \delta g_{67} & 0 & \delta g_{78} & 0 \\ 0 & \delta g_{78} & 0 & \delta g_{89} \\ \delta g_{69} & 0 & \delta g_{89} & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & \delta b_{68} & 0 \\ 0 & 0 & 0 & \delta b_{79} \\ -\delta b_{68} & 0 & 0 & 0 \\ 0 & -\delta b_{79} & 0 & 0 \end{pmatrix}.$$  (2.2)

For simplicity we shall mainly consider the moduli that preserve the decomposition of $T^4$ as $T^4 = T^2 \times T^2$, where the first $T^2$ has coordinates $x_6, x_7$, and the second has coordinates $x_8, x_9$. (These moduli correspond to $g_{67}$ and $g_{89}$.) Let us determine to which global deformation of the torus these moduli correspond to. The action of the various symmetries on the complex structure $U$ and the Kähler structure $T$ of $T^2$ is given as

$$\Omega : (U, T) \mapsto (U, -\overline{T}),$$
$$\mathcal{R} : (U, T) \mapsto (-\overline{U}, -\overline{T}),$$
$$\Theta_4 : (U, T) \mapsto (-1/U, -1/T),$$

so that the combined action is

$$\Omega \mathcal{R} \Theta_4 : (U, T) \mapsto (1/U, -1/T).$$  (2.3)

Thus $\Omega \mathcal{R} \Theta_4$ leaves a given $T^2$ invariant provided that $T = i$ and $|U|^2 = 1$; this gives indeed rise to a one-dimensional moduli space.

Every two-torus can be described as the quotient of the complex plane, $T^2 = \mathbb{C}/\Lambda$, where $\Lambda$ is a lattice. The tori that have a complex structure $U$ satisfying $|U| = 1$ are characterized by the property that $\Lambda$ is generated by the basis vectors

$$e_1 = \frac{1}{R}, \quad e_2 = \frac{\kappa}{R} + iR,$$  (2.4)

where $\kappa^2 + R^4 = 1$. (More precisely $U$ is a phase, $U = \exp(i\phi)$, and $\phi$ is the angle between $e_1$ and $e_2$.) Furthermore, $T = i$ implies that the B-field vanishes. The torus is shown in figure 1.

We can write the left- and right-moving momenta as

$$p_L = \frac{1}{i\sqrt{U_2 T_2}} \left[ U m_1 - m_2 - T (n_1 + U n_2) \right]$$
$$p_R = \frac{1}{i\sqrt{U_2 T_2}} \left[ U m_1 - m_2 - T (n_1 + U n_2) \right],$$

and we can thus directly determine the action of $\Omega \mathcal{R} \Theta_4$ on them; this leads to

$$\begin{align*}
\Omega \mathcal{R} \Theta_4 : \left\{ \begin{array}{l}
 p_L \rightarrow -iU \overline{p_R} \\
 p_R \rightarrow iU \overline{p_L}.
\end{array} \right.
\end{align*}$$  (2.5)

Note, that the relation $(\Omega \mathcal{R} \Theta_4)^2 = I_4$ holds in general, even for $U \neq i$. 

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2.1. The Klein bottle amplitude and the crosscap state

First we study the Klein-bottle amplitude for the model (2.1) defined on the $SU(2)^4$ torus. We want to determine the correct crosscap states from the Klein bottle amplitude, which can be directly computed in the loop channel; this is given by

$$\mathcal{K} = 8C \int_0^\infty \frac{dt}{t^4} \text{Tr}_{1,I^4} \left( \frac{\Omega R I^L_4 + \Omega \bar{R} I^R_4}{4} P_{GSO} e^{-2\pi t (L_0 + \bar{L}_0)} \right), (2.8)$$

where $C = V_6/(8\pi^2\alpha')^3$ and the momentum integration over the non-compact directions has already been performed. In the untwisted sector we get

$$\text{Tr}_1(\ldots) = \frac{f_3^4 f_4^4 - f_4^4 f_3^4 - f_2^4 f_0^4 + f_0^4 f_2^4}{f_1^4 f_2^4} (2.9)$$

with argument $q = \exp(-2\pi t)$. The various $f$-functions are defined by

$$f_0(q) = \sqrt{2} q^{\frac{1}{12}} \prod_{n=0}^{\infty} (1 - q^{2n}) = 0,$$

$$f_1(q) = q^{\frac{1}{12}} \prod_{n=1}^{\infty} (1 - q^{2n}),$$

$$f_2(q) = \sqrt{2} q^{\frac{1}{12}} \prod_{n=1}^{\infty} (1 + q^{2n}),$$

$$f_3(q) = q^{-\frac{1}{12}} \prod_{n=1}^{\infty} (1 + q^{2(n-1)}),$$

$$f_4(q) = q^{-\frac{1}{12}} \prod_{n=1}^{\infty} (1 - q^{2(n-1)}),$$

Figure 1 The shape of the torus.
where we have used the notation of [25]. In the \( I_4 \) twisted sector the trace vanishes, as the action of T-duality \( I_4^{L} \) on the sixteen fixed points is given by the traceless matrix [26]

\[
M = \bigotimes_{i=1}^{4} \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 1 \\
1 & -1
\end{pmatrix}.
\]

After a modular transformation to the tree channel, writing \( t = \frac{1}{4l} \) and using the transformation rules of the \( f_i \) functions,

\[
\begin{align*}
  f_0(e^{-\pi/l}) &= f_0(e^{-\pi l}), \\
  f_1(e^{-\pi/l}) &= \sqrt{l} f_1(e^{-\pi l}), \\
  f_2(e^{-\pi/l}) &= f_4(e^{-\pi l}), \\
  f_3(e^{-\pi/l}) &= f_3(e^{-\pi l}),
\end{align*}
\]

we get

\[
\tilde{\mathcal{K}} = \mathcal{C} \int_{0}^{\infty} dl \ 128 \frac{f_4^4f_2^4 - f_4^4f_0^4 - f_2^4f_3^4 + f_0^4f_4^4}{f_4^4f_2^4}
\]

with argument \( q = \exp(-2\pi l) \). It is immediate from this expression that (2.13) only contains a twisted sector tadpole. This is in contrast to ordinary \( \mathbb{Z}_2 \)-orientifolds, where only the untwisted sector propagates between the two crosscap states. The fact that in our case only the twisted sector, \( g = I_4 \), is allowed to flow between the two crosscap states is a consequence of the relation \( g = (\Omega R I_4^{L})^2 = (\Omega R I_4^{R})^2 \). The relevant crosscap states, \( |\Omega RI_4^{L,R}\rangle \) are characterized by the equation

\[
\left( \mathcal{X}^\mu(\sigma,0) - \mathcal{R} I_4^{L,R} \mathcal{X}^\mu(\sigma + \pi,0) \right) |\Omega RI_4^{L,R} \rangle = 0
\]

together with a similar condition for the world-sheet fermions; the solution to these equations is constructed in the appendix.

Under \( I_4^{L} \) (or \( I_4^{R} \)) the two different crosscap states, \( |\Omega RI_4^{L} \rangle \) and \( |\Omega RI_4^{R} \rangle \), are mapped into one another. This follows from the identity

\[
I_4^{L}(\Omega RI_4^{L,R}) = (\Omega RI_4^{R,L})I_4^{L}.
\]

Thus the physical crosscap state is the sum of \( |\Omega RI_4^{L} \rangle \) and \( |\Omega RI_4^{R} \rangle \). Also, since \( I_4^{L} \) (or \( I_4^{R} \)) maps the sixteen fixed points non-trivially into one another, the crosscap state must involve coherent states in different twisted sectors; the most symmetric solution involves then all sixteen fixed points equally, and this is what we shall consider in the following. Finally, the crosscap state is constrained by the condition that the overlap with itself reproduces the
tree level Klein bottle amplitude (2.13). This requires that it involves components both from the twisted NS-NS and the twisted R-R sector, and that we have

$$\langle C_L | e^{-i H_{\text{cl}}} | C_R \rangle = 0,$$

where $|C_L\rangle$ and $|C_R\rangle$ denote the total crosscap states. A solution to all of these constraints is given by

$$|C_L\rangle = (|L_1\rangle + |L_3\rangle) \otimes (|L_1\rangle + |L_3\rangle) + (|L_1\rangle + |L_3\rangle) \otimes (|L_4\rangle - |L_2\rangle)$$

$$+ (|L_4\rangle - |L_2\rangle) \otimes (|L_1\rangle + |L_3\rangle) + (|L_4\rangle - |L_2\rangle) \otimes (|L_4\rangle - |L_2\rangle),$$

$$|C_R\rangle = (|R_1\rangle + |R_2\rangle) \otimes (|R_1\rangle + |R_2\rangle) + (|R_1\rangle + |R_2\rangle) \otimes (|R_4\rangle - |R_3\rangle)$$

$$+ (|R_4\rangle - |R_3\rangle) \otimes (|R_1\rangle + |R_2\rangle) + (|R_4\rangle - |R_3\rangle) \otimes (|R_4\rangle - |R_3\rangle).$$

Here

$$|L_i\rangle \otimes |L_j\rangle = |\Omega R I^L_{ij}\rangle_{\text{NSNS},T(ij)} + |\Omega R I^L_{ij}\rangle_{\text{RR},T(ij)},$$

where $|\Omega R I^L_{ij}\rangle$ is defined in the appendix, and the twisted sector denoted by $T(ij)$ is localized at the fixed point $T_i$ of the $T^2$ with coordinates $x_6, x_7$, and at the fixed point $T_j$ of the $T^2$ with coordinates $x_8, x_9$; on each $T^2$ we denote the different fixed points by

$$T_1 = \left(0, 0\right), \quad T_2 = \left(\frac{1}{2}, 0\right), \quad T_3 = \left(0, \frac{1}{2}\right), \quad T_4 = \left(\frac{1}{2}, \frac{1}{2}\right).$$

The notation for $|R_i\rangle \otimes |R_j\rangle$ is analogous.

Both $|C_L\rangle$ and $|C_R\rangle$ can be thought to consist of four parallel O7-planes that ‘stretch’ between four fixed points each (and fill the uncompactified space). For example $(|L_1\rangle + |L_3\rangle) \otimes (|L_1\rangle + |L_3\rangle)$ defines an O7-plane that is localized at $x_6 = x_8 = 0$ and ‘stretches’ between the four fixed points with $x_7 = 0, 1/2$ and $x_9 = 0, 1/2$; similar statements also hold for the other terms in (2.17).

The $\mathcal{N} = (0, 1)$ supersymmetric massless spectrum consists of the supergravity multiplet, 11 tensor multiplets and 10 hypermultiplets. There exist various configuration of D-branes that cancel the twisted sector tadpoles from the Klein bottle and therefore cancel the anomalies from the closed string sector. These configurations will be discussed in turn.
3. The D7-brane antibrane configuration

The simplest configuration of branes that cancels the tadpoles consists of D7-branes and anti-branes that are arranged in the same way as the O7-planes. The branes in question are so-called ‘fractional’ branes whose boundary states have components in the untwisted as well as the twisted sectors. In particular, this implies that the branes are stuck at the fixed planes. Since the Klein bottle amplitude does not have any untwisted R-R tadpoles, we have to introduce D7-branes and D7-antibranes ($\overline{D7}$) in pairs.

The boundary states of the D7-branes are schematically described by (see for example [27] for an introduction into these matters)

$$|D7\rangle = (|U,NS\rangle + |U,R\rangle) + (|T,NS\rangle + |T,R\rangle)$$, \hspace{1cm} (3.1)

where $U$ denotes the untwisted sector and $T$ the twisted sector. The normalization factors for the untwisted and twisted sector parts have to be determined by loop channel-tree channel equivalence. Under the action of $\Omega R I_4 L$, a $D7$-brane is mapped to a $\overline{D7}$ brane that is orthogonal to the former. In particular this implies that the tree exchange between a $D7$-brane and a $\overline{D7}$ brane vanishes. The boundary states for the antibranes, $\overline{D7}$, are of the form

$$|\overline{D7}\rangle = (|U,NS\rangle - |U,R\rangle) - (|T,NS\rangle - |T,R\rangle)$$, \hspace{1cm} (3.2)

and thus the open string that stretches between a $D7$ and a $\overline{D7}$ brane has the opposite GSO projection. (It also has the opposite $I_4$ projection.) In order to achieve a local cancellation of the tadpoles the charge assignment for the various twisted sector ground states must be chosen as for the O7-planes in (2.17).

We also have to guarantee that the total configuration is invariant under $I_4^{L,R}$, i.e. under T-duality. Under this operation, spatial distances are exchanged with Wilson lines which in turn are related to the relative signs of the twisted (R-R) charges at different fixed points. A consistent choice for the $D7$-branes and $\overline{D7}$-branes is described in table 1 (see also figures 2 and 3). (The last column in table 1 describes the action on the Chan-Paton factors for the term in the open string with the insertion of $I_4$; this can be determined from the given boundary states by world-sheet duality.)
Table 1: D7-branes for model I.

| brane   | location | Wilson line | twisted sector | (γI₄) |
|---------|----------|-------------|----------------|-------|
| D7₁     | x₆ = 0, x₈ = 0 | θ₇ = 0, θ₀ = 0 | (T₁ + T₃)(T₁ + T₃) | I     |
| D7₂     | x₆ = 0, x₈ = ½ | θ₇ = 0, θ₀ = ½ | (T₁ + T₃)(T₁ - T₂) | I     |
| D7₃     | x₆ = ½, x₈ = 0 | θ₇ = ½, θ₀ = 0 | (T₄ - T₂)(T₁ + T₃) | I     |
| D7₄     | x₆ = ½, x₈ = ½ | θ₇ = ½, θ₀ = ½ | (T₄ - T₂)(T₄ - T₂) | I     |
| D7₁     | x₇ = 0, x₉ = 0 | θ₆ = 0, θ₈ = 0 | (T₁ + T₂)(T₁ + T₂) | I     |
| D7₂     | x₇ = 0, x₉ = ½ | θ₆ = 0, θ₈ = ½ | (T₁ + T₂)(T₄ - T₃) | I     |
| D7₃     | x₇ = ½, x₉ = 0 | θ₆ = ½, θ₈ = 0 | (T₄ - T₃)(T₁ + T₂) | I     |
| D7₄     | x₇ = ½, x₉ = ½ | θ₆ = ½, θ₈ = ½ | (T₄ - T₃)(T₄ - T₃) | I     |

Given the branes described in table 1, it is straightforward to compute the annulus amplitude in the tree channel, and the result is

\[
\tilde{A} = C \int_{0}^{\infty} dl \, N^2 \left\{ 2 \left( \frac{f^3 g - f^4 g}{f^1 g} \right)_{\text{NSNS}, \text{U}} \left( \Theta^2_{0,1} + \Theta^2_{1,1} \right)^2 - 8 \left( \frac{f^8 g - f^8 g}{f^1 g} \right)_{\text{RR}, \text{U}}^{\text{RR}, \text{U}} \Theta^2_{0,1} \Theta^2_{1,1} 
+ 8 \left( \frac{f^4 f^4 - f^4 f^4 - f^4 f^4 + f^0 f^4}{f^1 f^4} \right)_{\text{NSNS-RR}, \text{T}}^{\text{NSNS-RR}, \text{T}} + 8 \left( \frac{f^4 f^4 - f^4 f^4}{f^1 f^2} \right)_{\text{NSNS}, \text{U}} - 8 \left( \frac{f^4 f^4 - f^4 f^4}{f^1 f^3} \right)_{\text{NSNS}, \text{T}} \right\},
\]

(3.3)
where the argument is $q = \exp(-2\pi t)$, and we have used the standard definition for

$$\Theta_{j,k}(q^2) = \sum_{m \in \mathbb{Z}} q^{2k(m+j)^2}.$$  \hfill (3.4)

In the above, we have assumed that we have the same number, $N$, of each of the D-branes. We should note that the massless untwisted R-R-tadpole vanishes and that the fourth term in (3.3) does not give rise to any massless tadpole, either. Upon world-sheet duality, writing $t = 1/(2l)$, the annulus amplitude becomes

$$A = C \int_0^\infty \frac{dt}{t^4} \frac{N^2}{2} \left\{ \left( \frac{f_3^8 - f_4^8 - f_2^8 + f_0^8}{f_1^8} \right) (\Theta_{0,1}^4 + \Theta_{1,1}^4) \right. \right.$$

$$+ \left. \left( \frac{f_3^8 + f_4^8 - f_2^8 - f_0^8}{f_1^8} \right) 2 (\Theta_{0,1}^2 \Theta_{1,1}^2) + 4 \left( \frac{f_3^4 f_4^4 - f_3^4 f_2^4 + f_2^4 f_0^4 + f_0^4 f_4^4}{f_1^4 f_1^4} \right) \right. \right.$$

$$+ \left. \left( \frac{f_3^4 f_2^4 - f_2^4 f_3^4}{f_1^4 f_1^4} \right) - 4 \left( \frac{f_4^4 f_3^4 - f_0^4 f_3^4}{f_1^4 f_1^4} \right) \right\},$$  \hfill (3.5)

where the argument is $q = \exp(-\pi t)$. Here we have used that the $\Theta_{j,1}$ functions transform as

$$\Theta_{0,1}(e^{-2\pi/\tau}) = \sqrt{\frac{\tau}{2}} \left( \Theta_{0,1}(e^{-2\pi\tau}) + \Theta_{1,1}(e^{-2\pi\tau}) \right),$$

$$\Theta_{1,1}(e^{-2\pi/\tau}) = \sqrt{\frac{\tau}{2}} \left( \Theta_{0,1}(e^{-2\pi\tau}) - \Theta_{1,1}(e^{-2\pi\tau}) \right).$$  \hfill (3.6)

The first term in (3.5) arises from open strings stretched between parallel $D7$-branes or parallel $\overline{D7}$-branes, without the insertion of $I_4$ in the trace. The second term comes from
open strings stretched between parallel $D7$ and $\overline{D7}$ branes without $I_4$ insertion. The third term is due to parallel D-branes with $I_4$ insertion in the trace, the fourth term is from orthogonal D-branes without $I_4$ insertion and finally the fifth term comes from orthogonal branes with $I_4$ insertion.

The Möbius strip amplitude is determined, in the tree channel, by the overlap between the total crosscap state and the different $D7$-brane states. The different contributions include for example

$$\int_0^\infty dt \langle \Omega I_4^L|e^{-lt}|D7_{1,4}\rangle = -C \int_0^\infty dt 4N \left( \frac{f_4^4 f_2^4 - f_4^4 f_0^4 - f_2^4 f_3^4 + f_0^4 f_3^4}{f_4^4 f_3^4} \right)$$

(3.7)

$$\int_0^\infty dt \langle \Omega I_4^R|e^{-lt}|D7_{1,4}\rangle = 0,$$

where the argument is $q = i\exp(-2\pi t)$. Adding all these terms together, we find that the contribution of the (twisted) R-R sector is given by

$$\tilde{\mathcal{N}} = C \int_0^\infty dt 32N \left( \frac{f_2^4 f_3^4 - f_3^4 f_0^4 + f_3^4 f_4^4 - f_0^4 f_3^4}{f_1^4 f_3^4} \right)_{RR,T}.$$  \hspace{1cm} (3.8)

Note that (3.8) is indeed real, as under complex conjugation one has

$$\left( f_3(iq) \right)^* = e^{-\pi i} f_4(iq), \quad \left( f_4(iq) \right)^* = e^{-\pi i} f_3(iq).$$  \hspace{1cm} (3.9)

In order to determine the massless spectrum of the open strings we have to determine the Möbius strip amplitude in loop channel for the different combinations separately, and we find that

$$\int_0^\infty \frac{dt}{t^4} \text{Tr}_{11,44} \left( \frac{\Omega I_4^L + \Omega I_4^R}{4} P_{GSO} e^{-2\pi t L_0} \right)$$

$$= \frac{N}{2} \int_0^\infty \frac{dt}{t^4} \left( \frac{f_3^4 f_4^4 - f_3^4 f_0^4 + f_3^4 f_4^4}{f_3^4 f_4^4} + f_4^4 f_2^4 - f_2^4 f_0^4 - f_2^4 f_3^4 + f_0^4 f_2^4 \right),$$

(3.10)

with argument $q = i\exp(-\pi t)$, so that for $D7$-branes both the NS and the R sector are symmetrized. For the two pairs of $\overline{D7}$-branes we obtain

$$\int_0^\infty \frac{dt}{t^4} \text{Tr}_{22,33} \left( \frac{\Omega I_4^L + \Omega I_4^R}{4} P_{GSO} e^{-2\pi t L_0} \right)$$

$$= \frac{N}{2} \int_0^\infty \frac{dt}{t^4} \left( \frac{f_3^4 f_0^4 - f_3^4 f_2^4 + f_3^4 f_4^4}{f_3^4 f_4^4} - f_4^4 f_2^4 - f_2^4 f_4^4 + f_0^4 f_2^4 \right),$$

(3.11)
so that the NS sector is now antisymmetrized whereas the R-sector is still symmetrized. The change of sign between $M$ and $\tilde{M}$ is due to

$$f_3 \rightarrow f_4, \quad f_4 \rightarrow e^{-i\pi/4} f_3$$

(3.12)

under the $P = TST^2 S$ transformation (that relates the tree and the loop channel Möbius strip amplitude).

All three tree-level diagrams do not have a massless untwisted R-R tadpole, and therefore only the twisted (massless) R-R tadpole needs to be canceled. It follows from (2.13), (3.3) and (3.8) that this requires

$$8 N^2 - 64 N + 128 = 8 (N - 4)^2 = 0,$$

(3.13)

so that we need four D-branes of each kind. On the other hand, we can not cancel one untwisted and one twisted NS-NS-tadpole; this will therefore lead to a shift in the background via the Fischler-Susskind mechanism [28].

The massless spectrum in the closed string sector consists of the $N = (0,1)$ supergravity multiplet in addition to 11 tensor multiplets and 10 hypermultiplets, and the massless spectrum arising from the different open strings is listed in table 2.

| spin | states |
|------|--------|
| (2, 2) | $U(4) \times U(4) \times U(4) \times U(4)$ |
| (2, 1) | $2 \times [(\text{adj}, 1, 1, 1) + (1, \text{adj}, 1, 1) + (1, 1, \text{adj}, 1) + (1, 1, 1, \text{adj})]$ |
| (1, 1) | $2 \times [(4, \bar{T}, 1, 1) + (\bar{T}, 4, 1, 1) + (4, 1, \bar{T}, 1) + (\bar{T}, 1, 4, 1)] +$ |
| | $2 \times [(1, 4, 1, \bar{T}) + (1, \bar{T}, 1, 4) + (1, 1, 4, \bar{T}) + (1, \bar{T}, 4, 1)]$ |
| (1, 2) | $[(10 + \bar{10}, 1, 1, 1) + (1, 10 + \bar{10}, 1, 1) + (1, 1, 10 + \bar{10}, 1) + (1, 1, 1, 10 + \bar{10})]$ |
| (1, 1) | $2 \times [(10 + \bar{10}, 1, 1, 1) + (1, 1, 10 + \bar{10}, 1) + (1, 1, 1, 10 + \bar{10})]$ |
| (2, 1) | $[(4, 4, 1, 1) + (\bar{T}, \bar{T}, 1, 1) + (4, 1, 4, 1) + (\bar{T}, 1, \bar{T}, 1)] +$ |
| | $[(1, 4, 1, 4) + (1, \bar{T}, 1, \bar{T}) + (1, 1, 4, 4) + (1, 1, \bar{T}, \bar{T})]$ |
| (1, 2) | $[(4, 1, 1, 4) + (\bar{T}, 1, 1, \bar{T}) + (1, 4, 4, 1) + (1, \bar{T}, \bar{T}, 1)]$ |
| (1, 1) | $2 \times [(4, 1, 1, 4) + (\bar{T}, 1, 1, \bar{T}) + (1, 4, 4, 1) + (1, \bar{T}, \bar{T}, 1)]$ |

Table 2: Massless open string spectrum for model I (orthogonal $D7$-branes).
It is worth mentioning that the sector of open strings starting and ending on the same D-brane does not contain any scalar moduli. This implies that the D7-branes are not allowed to move off the fixed points. This is basically a consequence of the fact that all four fractional branes carry the same twisted R-R charge; pairs of fractional branes can move off a fixed point, but only if they carry the opposite twisted charge.

The distance between the different branes and anti-branes is such that the ground state ‘tachyon’ is either massless or massive; in particular, the open string spectrum therefore does not contain any actual tachyons. Since the branes and antibranes are fixed to lie on the fixed points, their distance is determined in terms of the radii of the underlying torus. However, these radii are not moduli any more since the theory is only well-defined for a self-dual torus, and the configuration is (at least at this level) stable.

The non-supersymmetric spectrum of table 2 also contains $N_+ = 256$ massless fermions of $(2, 1)$ chirality, and $N_- = 144$ massless fermions of $(1, 2)$ chirality, giving rise to $\Delta N = N_+ - N_- = 112$; this is precisely what is needed to cancel the non-factorizable gravitational anomaly. Moreover, the configuration of table 2 is also free of irreducible gauge anomalies, consistently with the R-R tadpole cancellation [29].

The arrangement of $D7$ and $\overline{D7}$ branes listed in table 1 is not the only possible configuration of parallel $D7$-branes and anti-branes that cancels the R-R-tadpole. In fact, we can exchange the roles of some of the branes and anti-branes, and consider instead the brane configuration described in table 3. (This configuration differs from that described in table 1 by the property that $D7_2$ is now a brane, whereas $D7_4$ is now an anti-brane.)

| brane  | location  | Wilson line | twisted sector | $(\gamma I_4)$ |
|--------|-----------|-------------|----------------|---------------|
| $D7_1$ | $x_6 = 0$, $x_8 = 0$ | $\theta_7 = 0$, $\theta_9 = 0$ | $(T_1 + T_3)(T_1 + T_3)$ | $I$ |
| $D7_2$ | $x_6 = 0$, $x_8 = \frac{1}{2}$ | $\theta_7 = 0$, $\theta_9 = \frac{1}{2}$ | $(T_1 + T_3)(T_4 - T_2)$ | $-I$ |
| $\overline{D7}_3$ | $x_6 = \frac{1}{2}$, $x_8 = 0$ | $\theta_7 = \frac{1}{2}$, $\theta_9 = 0$ | $(T_4 - T_2)(T_1 + T_3)$ | $I$ |
| $\overline{D7}_4$ | $x_6 = \frac{1}{2}$, $x_8 = \frac{1}{2}$ | $\theta_7 = \frac{1}{2}$, $\theta_9 = \frac{1}{2}$ | $(T_4 - T_2)(T_4 - T_2)$ | $-I$ |
| $\tilde{D7}_1$ | $x_7 = 0$, $x_9 = 0$ | $\theta_6 = 0$, $\theta_8 = 0$ | $(T_1 + T_2)(T_1 + T_2)$ | $I$ |
| $\tilde{D7}_2$ | $x_7 = 0$, $x_9 = \frac{1}{2}$ | $\theta_6 = 0$, $\theta_8 = \frac{1}{2}$ | $(T_1 + T_2)(T_4 - T_3)$ | $-I$ |
| $\tilde{D7}_3$ | $x_7 = \frac{1}{2}$, $x_9 = 0$ | $\theta_6 = \frac{1}{2}$, $\theta_8 = 0$ | $(T_4 - T_3)(T_1 + T_2)$ | $I$ |
| $\tilde{D7}_4$ | $x_7 = \frac{1}{2}$, $x_9 = \frac{1}{2}$ | $\theta_6 = \frac{1}{2}$, $\theta_8 = \frac{1}{2}$ | $(T_4 - T_3)(T_4 - T_3)$ | $-I$ |

**Table 3:** $D7$-branes for model II.
This modification only changes the annulus amplitude which, in the tree channel, now becomes
\[
\tilde{A} = C \int_0^\infty \frac{dl}{N^2} \left\{ 2 \left( \frac{f_3^8 - f_1^8}{f_1^8} \right) \left( \Theta_{0,1}^2 + \Theta_{1,1}^2 \right)^2 
- 4 \left( \frac{f_2^8 - f_0^8}{f_1^8} \right) \left( \Theta_{0,1}^2 \Theta_{0,1} \right) + 8 \left( \frac{f_4^4 f_2^4 - f_4^4 f_0^4 - f_2^4 f_0^4 + f_0^4 f_4^4}{f_1^4 f_2^4} \right) \left( NSNS_{-RR,T} \right) 
+ 8 \left( \frac{f_4^4 f_2^4 - f_4^4 f_3^4}{f_1^4 f_2^4} \right) \left( NSNS_{U} \right) \right\}.
\]

In particular, the contribution of the massless R-R sector states in (3.14) is unmodified compared to (3.3), and therefore N = 4 still cancels the R-R tadpoles. The massless spectrum of this model is described in table 4.

| sector | spin | states |
|--------|------|--------|
| ii     | (2, 2) | U(4)×U(4)×U(4)×U(4) |
|       | (2, 1) | 2 × [(adj, 1, 1, 1) + (1, adj, 1, 1) + (1, 1, adj, 1) + (1, 1, 1, adj)] |
| i(i + 2) | (1, 1) | 2 × [(4, 1, 4, 1) + (4, 1, 4, 1) + (1, 4, 1, 4) + (1, 4, 4, 1)] |
| i̅i    | (1, 2) | [(10 + 10, 1, 1, 1) + (1, 10 + 10, 1, 1) + (1, 1, 10 + 10, 1) + (1, 1, 1, 10 + 10)] |
|      | (1, 1) | 2 × [(10 + 10, 1, 1, 1) + (1, 10 + 10, 1, 1) + (1, 1, 6 + 6, 1) + (1, 1, 1, 6 + 6)] |
| i̅(i + 2) | (2, 1) | [(4, 1, 4, 1) + (4, 1, 4, 1) + (1, 4, 1, 4) + (1, 4, 4, 1)] |
| i̅(i̅ + 3) | (1, 1) | 2 × [(4, 1, 1, 4) + (4, 1, 1, 4) + (1, 4, 4, 1) + (1, 4, 4, 1)] |

**Table 4:** Massless open string spectrum for model II (orthogonal D7-branes).

Again, the massless fermion spectrum cancels the non-factorizable gravitational anomaly of the closed string spectrum, and ensure the vanishing of irreducible gauge anomalies as well.

The two D7-brane models are actually closely related (see also [11] for a similar construction). Suppose we add to the second theory four bulk D7-brane anti-brane pairs that are parallel to the \(x_6 - x_8\) plane, together with their images under \(\Omega R_I^L\), i.e. together with another four D7-brane anti-brane pairs in the bulk that are parallel to the \(x_7 - x_9\) plane. We can then consider moving the anti-branes to the fixed planes at \(x_6 = 0, x_8 = 1/2\) and
$x_7 = 0, x_9 = 1/2$, respectively, while moving the branes to the fixed planes at $x_6 = x_8 = 1/2$ and $x_7 = x_9 = 1/2$, respectively. A bulk brane carries twice the untwisted R-R charge of a fractional brane, and therefore precisely changes the sign of the untwisted R-R charge (if it is of opposite sign). Thus the above operation transforms model II into model I.

4. Non-BPS D8-branes

Both of the above models contain massless scalars in the $i(i + 2)$ sector. This sector consists of open strings that stretch between a D7-brane and a parallel anti-D7-brane, for example between $D7_1$ and $\overline{D7}_3$. The ‘tachyonic’ ground state from the NS sector is therefore invariant under the GSO-projection; its zero momentum and winding component is removed by the orbifold projection, but states with non-trivial winding and momentum survive. For the actual configuration that we are considering (where the brane and the anti-brane are separated by a finite distance along the $x_6$ direction and where they carry a relative Wilson line in the $x_7$ direction), the lowest lying physical state is in fact massless. This suggests that the corresponding scalars describe a marginal transformation along which the D7-anti-D7-brane system can be deformed into a non-BPS D8-brane that fills the space between the two D7-branes [30,31,32,33] (see also [34,35,27] for a review of these matters).

Actually, the relevant non-BPS D8-brane is not a conventional non-BPS D-brane since the charge distribution at the end-points cannot be described in terms of constant Wilson lines. (This is basically a consequence of the fact that the two D7-branes, $D7_1$ and $\overline{D7}_3$, out of which the D8-brane forms do not have the same Wilson line in the $x_7$ direction.) In fact, the non-BPS D8-brane into which the system decays carries a non-trivial magnetic flux. One way to see this is to observe that the charge distribution requires that the Wilson line in the $x_7$ direction depends nontrivially on $x_6$, i.e.

$$A_6 = \frac{1}{2}, \quad A_7 = x_6, \quad A_9 = 0,$$

(4.1)

and therefore that the magnetic flux $F_{67} = \partial_6 A_7 - \partial_7 A_6 = 1$. Alternatively, the correct charge distribution can be described by the superposition of a conventional non-BPS D8-brane (where all eight twisted R-R charges are $+$) together with a non-BPS D6-brane that stretches along $x_9$ (and is localised at $x_6 = \frac{1}{2}, x_7 = x_8 = 0$), both of whose twisted R-R charges are $-$. Since the magnitude of the twisted R-R charge at the end of a non-BPS D6-brane is twice that of a non-BPS D8-brane, the total charge of the configuration agrees
then with that of the D7-brane anti-brane configuration. On the other hand, the open string between the D6-brane and the D8-brane contains a tachyon, and the system decays (presumably) into a non-BPS D8-brane with magnetic flux.

It is not difficult to describe the non-BPS D8-brane with magnetic flux in terms of boundary states. Let us consider the configuration that is relevant to the previous discussion: it is localised at \( x_8 = 0 \), and has magnetic flux \( F_{67} = 1 \). The boundary conditions for the internal directions are then

\[
\begin{align*}
\partial_\sigma X_6 + \partial_\tau X_7 &= 0, \\
\partial_\sigma X_7 - \partial_\tau X_6 &= 0, \\
\partial_\sigma X_8 &= 0, \\
\partial_\tau X_9 &= 0,
\end{align*}
\]

and the exponential of the bosonic oscillators is of the form

\[
|B\rangle = \exp\left(\sum_n -\frac{1}{n} (\alpha_n^6 \bar{\alpha}_n^7 - \alpha_n^7 \bar{\alpha}_n^6 - \alpha_n^8 \bar{\alpha}_n^8 + \alpha_n^9 \bar{\alpha}_n^9)\right) |k, \omega\rangle
\]

and similarly for the fermions. Under \( \Omega R I^L_4 \) this is mapped to

\[
|\tilde{B}\rangle = \exp\left(\sum_n -\frac{1}{n} (\alpha_n^6 \bar{\alpha}_n^7 - \alpha_n^7 \bar{\alpha}_n^6 - \alpha_n^8 \bar{\alpha}_n^8 + \alpha_n^9 \bar{\alpha}_n^9)\right) |k, \omega\rangle.
\]

The last boundary state describes a non-BPS D8-brane that is localised at \( x_9 = 0 \), and that has magnetic flux \( F_{67} = -1 \). This is indeed the appropriate non-BPS D8-brane into which the combination of \( \tilde{D}7_1 \) and \( \tilde{D}7_3 \) can decay.

Schematically speaking, the boundary state of the whole non-BPS D8-brane has the form

\[
|D8\rangle = (|U, NS\rangle + |T, R\rangle),
\]

where again the normalization is fixed by world-sheet duality to the loop channel

\[
A = C \int_0^\infty dt \frac{dt}{t^4} \Tr \left( \frac{1 + (-1)^F I_4}{4} e^{-2\pi t L_0} \right).
\]

In order to cancel the twisted sector tadpoles we need four such non-BPS D8-branes with parameters as shown in table 5, where again, \( \tilde{D}8_i \) is the image of \( D8_i \) under \( \Omega R I^L_4 \).
Table 5: Non-BPS D8-branes.

| brane | location | Wilson line | twisted sector | $F_{67}$ |
|-------|----------|-------------|----------------|---------|
| $D8_1$ | $x_8 = 0$ | $\theta_9 = 0$ | $(T_1 + T_3)(T_1 + T_3) + (T_4 - T_2)(T_1 + T_3)$ | 1 |
| $D8_2$ | $x_8 = \frac{1}{2}$ | $\theta_9 = \frac{1}{2}$ | $(T_1 + T_3)(T_4 - T_2) + (T_4 - T_2)(T_4 - T_2)$ | 1 |
| $\tilde{D}8_1$ | $x_9 = 0$ | $\theta_8 = 0$ | $(T_1 + T_2)(T_1 + T_2) + (T_4 - T_3)(T_1 + T_2)$ | $-1$ |
| $\tilde{D}8_2$ | $x_9 = \frac{1}{2}$ | $\theta_8 = \frac{1}{2}$ | $(T_1 + T_2)(T_4 - T_3) + (T_4 - T_3)(T_4 - T_3)$ | $-1$ |

As was explained in some detail in [36], when dealing with branes with background gauge fields two issues need special attention. Firstly, the zero-mode spectrum of the open strings between two parallel D-branes with background gauge flux changes to

$$M^2 = \frac{|r + Us|^2}{U_2} \frac{T_2}{|n + Tm|^2},$$

(4.7)

where in our case $U = T = i$, $n = m = 1$ and $n = -m = 1$, respectively. Thus compared to a D8-brane without magnetic flux one gets an extra factor of one-half for the zero mode spectrum

$$M^2 = \frac{r^2 + s^2}{2},$$

(4.8)

leading to an extra factor of $\sqrt{2}$ in the normalization of the D8-brane boundary states. In the overlap of two non-BPS D8-branes with $F = 1$ and $F = -1$ (for which the winding and momentum sum is absent), one therefore obtains an extra multiplicity of two, implying that in loop channel every state in this open string sector is two-fold degenerate. Both effects can easily be seen in the T-dual picture involving branes at angles. A D-brane with non-trivial gauge flux is generally mapped to a brane wrapping around rational cycles of the $T^2$ different from the two fundamental ones; apparently, this changes the zero mode spectrum. The second effect is due multiple intersection points of D-branes intersecting at angles $[37, 38, 39]$ (see also [40]). In our case the T-dual D-branes stretch along the main and the off-diagonal of the T-dual torus. Therefore it is evident that we really get an extra factor of two for open strings between a non-BPS brane with $F = 1$ and a non-BPS brane with $F = -1$.‡

‡ Using the Poisson resummation formula, the factor of one-half in the zero mode spectrum leads to an extra factor of two for the normalisation of the tree-channel zero mode contribution; this requires an additional factor of $\sqrt{2}$ for the boundary state.
Upon a modular transformation this becomes in loop channel for two such branes are orthogonal to each other, the twisted sector contribution vanishes.

states for two branes with opposite magnetic fields. Since the twisted sector ground states for two branes with identical magnetic fields, and the last term is the untwisted part of the overlap of the boundary states for two branes with opposite magnetic fields. Since the twisted sector ground states for two such branes are orthogonal to each other, the twisted sector contribution vanishes. Upon a modular transformation this becomes in loop channel

\[ A = C \int_0^\infty dt N^2 \left\{ \left( \frac{f_2^8 f_3^8 - f_1^8 f_3^8}{f_1^8} \right) \left( \sum_{m \in \mathbb{Z}} e^{-\pi tm^2} \right)^2 \left[ \left( \sum_{n \in \mathbb{Z}} e^{-\pi tn^2} \right)^2 \right. \right. \\
\left. \left. + \left( \sum_{n \in \mathbb{Z}} (-1)^n e^{-\pi tn^2} \right)^2 \right] \right\} + 8 \left( \frac{f_2^4 f_4^4 - f_3^4 f_4^4}{f_1^4 f_2^4} \right). \]  

The first two terms arise from the overlap of the boundary states for two branes with identical magnetic fields, and the last term is the untwisted part of the overlap of the boundary states for two branes with opposite magnetic fields. Since the twisted sector ground states for two such branes are orthogonal to each other, the twisted sector contribution vanishes.

The annulus amplitude becomes in tree channel

\[ \tilde{A} = C \int_0^\infty dl N^2 \left\{ \left( \frac{f_3^8 - f_4^8}{f_1^8} \right) \left( \sum_{m \in \mathbb{Z}} e^{-\pi lm^2} \right)^2 \left[ \left( \sum_{n \in \mathbb{Z}} e^{-\pi ln^2} \right)^2 \right. \right. \\
\left. \left. + \left( \sum_{n \in \mathbb{Z}} (-1)^n e^{-\pi ln^2} \right)^2 \right] \right\} \right. \] 

where we have used the Poisson resummation formula,

\[ \sum_{m \in \mathbb{Z}} e^{-\pi l(m/R)^2} = \frac{R}{\sqrt{l}} \sum_{n \in \mathbb{Z}} e^{-\pi (nR)^2/l}. \] 

Note, that in (4.10) the contribution for two branes with opposite magnetic fields and \((-1)^F I_4\) insertion vanishes identically. This implies that \((-1)^F I_4\) acts with opposite signs on the two-fold degenerate ground states in this sector. Using the crosscap states defined in the appendix we can determine the tree-channel Möbius amplitude

\[ \tilde{M} = C \int_0^\infty dl 32 N \left( \frac{f_2^4 f_3^4 f_3 f_4 - f_0^4 f_2^4 f_4 f_3}{f_1^4 f_2^4 f_3 f_4} + \frac{f_2^4 f_2^4 f_4 f_3 - f_0^4 f_3^4 f_3 f_4}{f_1^4 f_3^4 f_4 f_3} \right) \] 

with argument \(q = i \exp(-2\pi l)\). The tadpole cancellation condition is therefore as before in (3.13), i.e. we need \(N = 4\) non-BPS D8-branes of each kind. In loop channel, the Möbius amplitude is then

\[ M = C \int_0^\infty dt \frac{N}{4} \left( e^{i\frac{\pi}{2}} \frac{f_2^4 f_3^4 f_3 f_4 - f_0^4 f_2^4 f_4 f_3}{f_1^4 f_2^4 f_3 f_4} + e^{-i\frac{\pi}{2}} \frac{f_2^4 f_3^4 f_3 f_4 - f_0^4 f_4^4 f_4 f_3}{f_1^4 f_4^4 f_3 f_4} \right). \]
Taking into account that \((-1)^F I_4\) acts with opposite signs on the two-fold degenerate ground states of the open string between a brane with \(F = 1\) and one with \(F = -1\), and that at the massless level the loop channel Möbius amplitude \([1,13]\) vanishes we derive the massless open string spectrum presented in table 6.

| sector | spin | states |
|--------|------|--------|
| \(ii\) | (2, 2) | \(U(4) \times U(4)\) |
|        | (2, 1) | \(4 \times [(\text{adj}, 1) + (1, \text{adj})]\) |
| \(i(i + 1)\) | (1, 1) | \(2 \times [(4, \overline{4}) + (\overline{4}, 4)]\) |
| \(\tilde{ii}\) | (1, 2) | \([(10 + \overline{10}, 1) + (1, 10 + \overline{10})]\) |
|        | (2, 1) | \([(6 + \overline{6}, 1) + (1, 6 + \overline{6})]\) |
|        | (1, 1) | \(2 \times [(10 + \overline{10}, 1) + (1, 10 + \overline{10}) + (6 + \overline{6}, 1) + (1, 6 + \overline{6})]\) |
| \(i(i + 1)\) | (2, 1) | \([(4, 4) + (\overline{4}, \overline{4})]\) |
|        | (1, 2) | \([(4, 4) + (\overline{4}, \overline{4})]\) |
|        | (1, 1) | \(4 \times [(4, 4) + (\overline{4}, \overline{4})]\) |

**Table 6:** Massless open string spectrum on non-BPS D8-branes.

The spectrum of massless fermions cancels again the anomaly; in order for this to work, it is important that the extra multiplicities arising from double intersection points in the \(D8-\overline{D8}\) sector is taken into account. This provides an independent confirmation of our claim that the non-BPS D8-branes carry magnetic flux.

5. The configuration with \(D9-\overline{D9}\) branes

The spectrum in table 6 contains massless scalars in the (12) sector. They arise from open strings that stretch between the non-BPS D8-branes localised at \(x_8 = 0\), and those that are localised at \(x_8 = \frac{1}{2}\). These massless scalars describe the marginal deformation along which the non-BPS D8-branes can be deformed into a \(D9-\overline{D9}\) brane pair.

The \(D9-\overline{D9}\) branes into which the system can decay have to carry magnetic flux in both the \(x_6-x_7\) and the \(x_8-x_9\) directions in order to reproduce again the correct twisted R-R sector charges. The corresponding boundary states are then characterised by the
equations

\[ \partial_{\sigma} X_6 + F_{67} \partial_{\tau} X_7 = 0, \]
\[ \partial_{\sigma} X_7 - F_{67} \partial_{\tau} X_6 = 0, \]
\[ \partial_{\sigma} X_8 + F_{89} \partial_{\tau} X_9 = 0, \]
\[ \partial_{\sigma} X_9 - F_{89} \partial_{\tau} X_8 = 0. \] 

(5.1)

For \( F_{67} = F_{89} = +1 \), the exponential of the bosonic oscillators is then of the form

\[ |B\rangle = \exp \left( \sum_n -\frac{1}{n} (\alpha_n^6 \tilde{\alpha}_n^7 - \alpha_n^7 \tilde{\alpha}_n^6 + \alpha_n^8 \tilde{\alpha}_n^9 - \alpha_n^9 \tilde{\alpha}_n^8) \right) |k,\omega\rangle \] 

(5.2)

and similarly for the fermions. Under the action of \( \Omega RI_4^T \) this boundary state is mapped to

\[ |\tilde{B}\rangle = \exp \left( \sum_n -\frac{1}{n} (\alpha_n^6 \tilde{\alpha}_n^7 - \alpha_n^7 \tilde{\alpha}_n^6 + \alpha_n^8 \tilde{\alpha}_n^9 - \alpha_n^9 \tilde{\alpha}_n^8) \right) |k,\omega\rangle. \] 

(5.3)

This corresponds then to a \( D9 \)-brane state with magnetic flux \( F_{67} = F_{89} = -1 \).

Again schematically, the boundary states of the \( D9 \)-brane and the \( \bar{D9} \) brane have the form

\[ |D9\rangle = (|U,NS\rangle + |U,R\rangle) + (|T,NS\rangle + |T,R\rangle) \]
\[ |\bar{D9}\rangle = (|U,NS\rangle - |U,R\rangle) - (|T,NS\rangle - |T,R\rangle) , \] 

(5.4)

where the normalisations are determined by world-sheet duality. The open string loop amplitude of the open string stretched between the \( D9 \) and the \( \bar{D9} \) brane is then

\[ A = C \int_0^\infty \frac{dt}{t^4} \text{Tr} \left( -\frac{1}{2} - \frac{1}{2} \left| -1 \right|^F \frac{1}{2} - \frac{1}{2} e^{-2\pi t L_0} \right). \] 

(5.5)

In particular, since the \( I_4 \) projection appears now with the opposite sign, the ground state tachyon is removed. Under \( \Omega RI_4^T \) the two boundary states (5.4) are now mapped to

\[ |\tilde{D9}\rangle = (|U,NS\rangle - |U,R\rangle) - (|T,NS\rangle - |T,R\rangle) \]
\[ |\bar{\tilde{D9}}\rangle = (|U,NS\rangle + |U,R\rangle) + (|T,NS\rangle + |T,R\rangle) . \] 

(5.6)

In particular, \( \tilde{D9} \) is an anti-brane and \( \bar{\tilde{D9}} \) a brane (since \( \Omega R \) maps a \( D9 \)-brane into an anti-brane and vice versa). Thus we are led to consider a configuration of \( D9 \)-branes and anti-branes as described in table 7.
| brane | twisted sector | $\mathcal{F}_{67}$ | $\mathcal{F}_{89}$ |
|-------|----------------|-----------------|-----------------|
| $D9$  | $(T_1 + T_3)(T_1 + T_3) + (T_4 - T_2)(T_1 + T_3)$ + $(T_1 + T_3)(T_4 - T_2) + (T_1 - T_2)(T_4 - T_2)$ + $(T_1 + T_3)(T_4 - T_2) + (T_4 - T_2)(T_4 - T_2)$ | 1 | 1 |
| $\overline{D9}$ | $(T_1 + T_3)(T_1 + T_3) + (T_1 - T_2)(T_1 + T_3)$ + $(T_1 + T_3)(T_4 - T_2) + (T_4 - T_2)(T_4 - T_2)$ | -1 | -1 |
| $\tilde{D9}$ | $(T_1 + T_2)(T_1 + T_2) + (T_4 - T_3)(T_1 + T_2)$ + $(T_1 + T_2)(T_4 - T_3) + (T_4 - T_3)(T_1 + T_2)$ + $(T_1 + T_2)(T_4 - T_3) + (T_4 - T_3)(T_4 - T_3)$ | -1 | -1 |

Table 7: $D9-\overline{D9}$ branes.

The annulus amplitude in tree channel then becomes

$$\tilde{A} = \mathcal{C} \int_0^\infty dl \ N^2 \left\{ 2 \left( \frac{f_3^8 - f_3^8}{f_3^8} \right)_{\text{NSNS, U}} \left( \sum_{r,s \in \mathbb{Z}} e^{-2\pi l^2 \frac{r^2 + s^2 - 2\kappa r s}{\sqrt{1 - \kappa^2}}} \right)^2 - 8 \left( \frac{f_2^4 f_3^4 - f_0^4 f_4^4}{f_1^4 f_4^4} \right)_{\text{RR, T}} + 8 \left( \frac{f_3^4 f_4^4 - f_3^4 f_3^4 - f_2^4 f_0^4 + f_2^4 f_0^4}{f_1^4 f_2^4} \right) \right\}, \quad (5.7)$$

where we have introduced, for later convenience, $\kappa = \kappa_{67} = \kappa_{89}$ to denote the tilt of the $x_6 - x_7$ and the $x_8 - x_9$ torus. Using the crosscap states defined in the appendix we can also determine the tree-channel Möbius amplitude

$$\tilde{M} = \mathcal{C} \int_0^\infty dl \ 64 N \left( \frac{f_2^4 f_3^4 f_4^4 - f_0^4 f_3^4 f_3^4}{f_1^4 f_3^4 f_4^4} \right), \quad (5.8)$$

and thus read off the tadpole cancellation condition. This turns out to be the same as (3.13), independent of $\kappa$, and we thus have to choose $N = 4$. The massless spectrum depends on the other hand on $\kappa$ (as we shall discuss below); for $\kappa = 0$ it is shown in table 8. As before, the massless fermions cancel the non-factorizable gravitational anomaly.

$\dagger$ For $\kappa \neq 0$, the crosscap states are modified in the obvious way; this does not affect the Klein bottle amplitude however.
In the previous sections we have described a number of different tadpole cancelling configurations that are related to each other by standard deformations. There exists one other interesting brane distribution that is not so obviously related to these configurations, but that is relevant for the stability analysis of the theory. This configuration consists of \( D7 \)-branes and anti-branes that stretch diagonally across the tori. If we denote by \( y_1 \) and \( y_2 \) the coordinate along the main diagonal of the two \( T^2 \)s, this brane configuration is described in table 9 (see also figure 4).

| \( \text{brane} \) | \( \text{location} \) | \( \text{Wilson line} \) | \( \text{twisted sector} \) |
|-----------------|-----------------|-----------------|-----------------|
| \( D7 \)        | \( y_1 = y_2 = 0 \) | \( \theta_1 = \theta_2 = 0 \) | \( (T_1 + T_4)(T_1 + T_4) \) |
| \( \overline{D7} \) | \( y_1 = y_2 = \frac{1}{2} \) | \( \theta_1 = \theta_2 = \frac{1}{2} \) | \( (T_3 - T_2)(T_3 - T_2) \) |

Table 9: \textit{Diagonal} \( D7-\overline{D7} \) \textit{branes.}

From the point of view of the boundary states, it is actually not surprising that this configuration also cancels the tadpoles. The total crosscap state \( |C\rangle = |C_L\rangle + |C_R\rangle \) only has twisted R-R charge at eight of the sixteen corners, since the contributions from \( |C_L\rangle \) and \( |C_R\rangle \) cancel at the other eight fixed points. (This can be directly seen from (2.17).) Where the contributions add, the charge is twice as large as before, and we therefore expect that
we shall need eight D7-brane-anti-brane pairs of the above type. This is indeed correct:
the tree channel annulus amplitude of the above brane system is

\[
\tilde{A} = C \int_0^\infty dl \ N^2 \left\{ \frac{1}{2} \left( \frac{1 - \kappa}{1 + \kappa} \right) \left( \frac{f_3^8 - f_4^8}{f_1^8} \right) \right\}_{\text{NSNS,U}} \left[ \left( \sum_{r \in \mathbb{Z}} e^{-2\pi l \sqrt{\frac{1 + \kappa}{1 + \kappa} r^2}} \right)^4 + \left( \sum_{r \in \mathbb{Z}} (-1)^r e^{-2\pi l \sqrt{\frac{1 + \kappa}{1 + \kappa} r^2}} \right)^4 \right] \\
- \frac{1}{2} \left( \frac{1 - \kappa}{1 + \kappa} \right) \left( \frac{f_2^8 - f_0^8}{f_1^8} \right) \left[ \left( \sum_{r \in \mathbb{Z}} e^{-2\pi l \sqrt{\frac{1 + \kappa}{1 + \kappa} r^2}} \right)^4 - \left( \sum_{r \in \mathbb{Z}} (-1)^r e^{-2\pi l \sqrt{\frac{1 + \kappa}{1 + \kappa} r^2}} \right)^4 \right] \\
+ 2 \left( \frac{f_3^4 f_2^4 - f_4^4 f_0^4 - f_3^4 f_3^4 + f_0^4 f_4^4}{f_1^4 f_4^4} \right) \left\{ \right\}_{\text{NSNS-R,R,T}}
\]

(6.1)

where we have again performed the calculation for general tilt parameter \( \kappa \). On the other hand, the contribution to the twisted R-R tadpole of the tree channel Möbius amplitude is

\[
\tilde{M} = C \int_0^\infty dl \ 32N \left( \frac{\vartheta [1/2]^2 \vartheta [0/4]^2}{\eta^6 \vartheta [0/1/4]^2} \right)_{\text{RR,T}}
\]

(6.2)
with
\[
\frac{\partial \left[ \alpha \beta \right]}{\eta} (q) = e^{2\pi i \alpha \beta} q^{\frac{\alpha^2}{\pi r} - \frac{1}{2}} \prod_{n=1}^{\infty} \left( \left( 1 + q^{n-\frac{1}{2}+\alpha} e^{2\pi i \beta} \right) \left( 1 + q^{n-\frac{1}{2}-\alpha} e^{-2\pi i \beta} \right) \right)
\] (6.3)
and
\[
\eta(q) = q^{rac{1}{24}} \prod_{n=1}^{\infty} (1 - q^n). \] (6.4)

Thus the tadpole cancellation condition becomes
\[
2 N^2 - 32 N + 128 = 2 (N - 8)^2 = 0.
\] (6.5)

This requires indeed \( N = 8 \) D7-branes and \( \overline{D7} \)-branes. For \( \kappa = 0 \), the massless open string spectrum that follows from this configuration is given in table 10.

| sector | spin | states |
|--------|------|--------|
| \( \overline{ii} \) | (2, 2) | \( \text{Sp}(8) \times \text{SO}(8) \) |
|          | (2, 1) | \( 2 \times [(28, 1) + (1, 28)] \) |
| \( \overline{\overline{ii}} \) | (1, 1) | \( 8 \times (8, 8) \) |

**Table 10:** Massless open string spectrum on diagonal D7-\( \overline{D7} \) branes.

In this case there are 112 fermions of spin (2, 1), and this cancels indeed the non-factorizable gravitational anomaly. There also exists the configuration for which the \( D7 \)-branes and anti-branes stretch along the off-diagonal; the analysis of this case is identical to the above.

### 7. Regimes of stability

In the previous sections we have discussed a number of different tadpole cancelling configurations, all of which are free of tachyons at the point in moduli space where the torus is an orthogonal \( SU(2)^4 \) torus. In this section we want to explore which of these configurations is stable at a more general point in moduli space; we shall only consider a one-parameter subspace of the six-dimensional moduli space of tori (that was described in section 2), but it is clear that at least the essential arguments and observations will hold more generally.

Let us consider the deformation of the torus where we tilt both the (67) and the (89) torus, and let us, for simplicity, assume that \( \kappa = \kappa_{67} = \kappa_{89} \). As we increase \( \kappa \), the distance
between a brane at \( x_i = 0 \), and an anti-brane at \( x_i = 1/2 \) is reduced; in particular, if the ‘tachyonic’ open string between these two branes is massless for \( \kappa = 0 \), it will become tachyonic for \( \kappa \neq 0 \). This is precisely what happens for the (13) string of the original orthogonal \( D7-\overline{D7} \) brane system, and therefore the marginal perturbation along which this system can be deformed into a system of non-BPS D8-branes becomes relevant. The argument also applies to the \( i(i + 1) \) sector of the non-BPS D8-brane system, and the marginal deformation of this system into the \( D9-\overline{D9} \) brane system becomes also relevant. Thus, for \( \kappa \neq 0 \), either of the configurations described in sections 3 and 4 decays into the \( D9-\overline{D9} \) brane configuration described in section 5

\[
D7 - \overline{D7} \text{ (Section 3)} \rightarrow \text{non-BPS-D8 (Section 4)} \rightarrow D9 - \overline{D9} \text{ (Section 5).} \quad (7.1)
\]

On the other hand, from the point of view of the \( D9-\overline{D9} \) system, the massless scalars in the \( i(i + 1) \) sector of table 8 (that describe the marginal deformation back to the non-BPS D8-brane system) become massive for \( \kappa \neq 0 \). Indeed, the mass formula for the KK states on each \( T^2 \) is given by

\[
M^2 = \frac{T_2}{|n + T m|^2} \left( \frac{|r - Us|^2}{U_2} - \frac{1}{2} \right) = \frac{r^2 + s^2 - 2\kappa rs}{2\sqrt{1 - \kappa^2}} - \frac{1}{2}. \quad (7.2)
\]

For \( \kappa = 0 \), the massless states in the \( i(i + 1) \) sector correspond to states with \( r = \pm 1, s = 0 \) or \( r = 0, s = \pm 1 \). If \( \kappa \neq 0 \), these states become indeed massive. (This also follows directly from the loop amplitude

\[
A = C \int_0^\infty \frac{dt}{t^4} N^2 \left\{ \left( \frac{f_3^8 - f_2^8}{f_1^8} \right) \left( \sum_{r,s \in \mathbb{Z}} e^{-\pi t \frac{r^2+s^2-2\kappa rs}{\sqrt{1-\kappa^2}}} \right)^2 - 4 \left( \frac{f_3^4 f_3^4 - f_0^4 f_2^4}{f_1^4 f_2^4} \right) + 4 \left( \frac{f_3^4 f_2^4 - f_4^4 f_0^4 - f_3^4 f_4^4 + f_0^4 f_4^4}{f_1^4 f_4^4} \right) \right\}, \quad (7.3)
\]

that can be obtained by a modular transformation from the tree channel amplitude \((5.7)\).)

For \( 0 < \kappa < 3/5 \), the open string spectrum of the \( D9-\overline{D9} \) brane system is tachyon free, and this should imply that the configuration is indeed stable. At \( \kappa = 3/5 \), however, the states with \( (r, s) = \pm (1, 1) \) become massless, and for \( 3/5 < \kappa < 1 \), in fact tachyonic. This implies that the stable configuration in this domain is described by another system. Intuitively, these tachyons arise because for \( \kappa > 3/5 \), the D9-branes are stretched too much along the main diagonal direction of the torus, and it becomes energetically preferable to

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decay into two non-BPS D8-branes that stretch along the off-diagonal direction. The corresponding brane configuration can be constructed, but it is also always unstable to decay into the diagonal D7-brane system that we described in section 6. Thus we find that the stable configuration for \(3/5 < \kappa < 1\) is described by diagonal \(D7\)-branes and anti-branes!

Conversely, we can analyze the stability of the diagonal \(D7\)-brane system for all values of \(\kappa\). The mass formula for KK and winding states for the off-diagonal branes on each \(T_2\) is given by

\[
M^2 = \frac{r^2 + s^2}{2} \sqrt{\frac{1 + \kappa}{1 - \kappa}} - \frac{1}{2}.
\] (7.4)

For \(\kappa \geq 0\) the system is therefore non-tachyonic and the massless states in the \(i\bar{i}\) sector become massive for \(\kappa > 0\). For \(\kappa \leq 0\) the open string between the \(D7\) and the \(\overline{D7}\) brane becomes tachyonic and the system decays into a \(D7\) and a \(\overline{D7}\) along the off-diagonal.

It is worth mentioning that the system does not develop a marginal deformation (let alone a relevant deformation) for \(\kappa = 3/5\); this suggests that the configuration of diagonal \(D7\) branes and anti-branes is actually stable for all values of \(\kappa\), and that the \(D9\)-brane anti-brane system is only metastable (for \(|\kappa| < 3/5\)).

Finally, we have to address the question of what the effect of the dynamically generated potential for \(\kappa\) is. In the \(D9\)-brane configuration, the NS-NS tadpoles do not depend on \(\kappa\) (as follows from (5.7)), and the first \(\kappa\) dependent contribution to the potential arises at one loop. Since there exist NS-NS tadpoles, we are not really sitting in a string theory vacuum and the background fields get modified by the Fischler-Susskind mechanism. Nevertheless, since the \(\kappa\) dependence of \(\Lambda_{1-loop}\) only arises via the Kaluza-Klein and winding modes, we are confident that qualitative features of the \(\kappa\) dependence can be reliably extracted from the one-loop partition functions computed above. In particular we compute

\[
\Lambda_{1-loop}(\kappa) - \Lambda_{1-loop}(0) = -A(\kappa) + A(0),
\] (7.5)

where in fact the contributions from the torus, the Klein-bottle and the Möbius strip vanish and only the first term in (7.3) contributes. Numerically evaluating (7.5) yields the curve depicted in figure 5.

It follows from this result that to one-loop, \(\kappa = 0\) is a stable minimum. It is separated by a finite potential barrier from the configuration of diagonal \(D7\)-branes (into which it can decay at \(\kappa = 3/5\)).
Figure 5 $\Lambda_{1\text{-loop}}(\kappa) - \Lambda_{1\text{-loop}}(0)$ for the D9-brane system.

For the configuration of diagonal D7-branes, the NS-NS tadpole depends on $\kappa$, thus giving rise to a tree level contribution to the potential (see figure 6)

$$V(\Phi, \kappa) \sim e^{-\Phi} c \sqrt{\frac{1 - \kappa}{1 + \kappa}} N + V_{1\text{-loop}} + \ldots.$$  \hspace{1cm} (7.6)

The potential is minimized in the singular limit $\kappa = 1$; this simply expresses the fact that the tension of the D7-branes pulls the two sides of each $T^2$ together. However, the point $\kappa = 1$ is infinitely far away in moduli space and does not represent an actual decay mode. Moreover, it might also happen that higher loop or non-perturbative contributions to the
potential stabilize $\kappa$ at a finite value $0 \leq \kappa < 1$. The actual form of these contributions is however beyond computational control.

8. Conclusion

In this paper we have constructed a new kind of asymmetric orientifold in six dimensions which is supersymmetric in the bulk and non-supersymmetric on the branes. Tadpole cancellation naively led to the introduction of pairs of fractional $D7$ and $\overline{D7}$ branes, which were localized on different fixed points, thus preventing the development of tachyons. However, by turning on some of the closed string moduli the configuration of $D7$-$\overline{D7}$-branes became unstable and via non-BPS D8-branes eventually decayed into pairs of $D9$-$\overline{D9}$ branes with magnetic flux. For this configuration we computed the one-loop cosmological constant and found that the system is stabilized on the $SU(2)^4$ torus. However, this configuration is only metastable, and it is separated by a finite energy barrier from the stable system consisting of diagonal D7-branes and anti-branes.

One of the lessons of this analysis is that models that are tachyon-free at first sight (such as the configuration described in section 3) can in fact be highly unstable, and may
well decay into more exotic brane configurations. In fact, in order to get control over the stable configurations, it is important to analyze the various closed string moduli in some detail. As far as we are aware, the present paper is the first example where this has been done to any degree.

Most of the configurations that we found were at criticality in the sense that some of the tachyonic modes were precisely massless. Non-supersymmetric configurations at criticality sometimes lead to precise Bose-Fermi degeneracy in the open string spectrum [11]. However, for the class of configurations that we considered, here, this did not occur; it would be interesting to find a model where Bose-Fermi degeneracy is realized in the supersymmetry breaking open string sector.

Finally, if we compactify the six-dimensional model on a further $T^2$ and T-dualise this torus, we obtain a four-dimensional model, for which the latter torus can be made large, thus leading to some brane world scenario. It would therefore be interesting to generalize our approach directly to four-dimensional models.

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Appendix A. The crosscap states
Let us recall from (2.14) that the crosscap states $|\Omega RI_4^{L,R}\rangle$ satisfy the equation

$$\left(X^\mu(\sigma,0) - R_4^{L,R} X^\mu(\sigma + \pi,0)\right)|\Omega RI_4^{L,R}\rangle = 0,$$

(A.1)
where $\mu = 2, \ldots, 9$. In terms of the oscillator modes, this can be rewritten as

$$
(\alpha^m_r \pm \epsilon_m e^{-i\pi r} \tilde{\alpha}^m_r) |\Omega R I^L_4, \eta\rangle = 0, \quad \text{for } m \in \{6, 7, 8, 9\}
$$

(A.2)

$$
(\alpha^m_r \mp \epsilon_m e^{i\pi r} \tilde{\alpha}^m_r) |\Omega R I^L_4, \eta\rangle = 0, \quad \text{for } m \in \{6, 7, 8, 9\}
$$

(A.2)

$$
(\alpha^\mu_n + (-1)^n \tilde{\alpha}^\mu_n) |\Omega R I^L_4, \eta\rangle = 0, \quad \text{for } \mu \in \{2, 3, 4, 5\},
$$

where

$$
\epsilon_m = \begin{cases} 
  +1 & \text{for } m = 6, 8 \\
  -1 & \text{for } m = 7, 9.
\end{cases}
$$

(A.3)

The upper sign in (A.2) corresponds to $|\Omega R I^L_4\rangle$, whereas the lower sign refers to $|\Omega R I^R_4\rangle$.

The first two conditions in (A.2) are compatible only for $r \in \mathbb{Z} + 1/2$, confirming our general observation that the crosscap state is a coherent state in the $I_4$ twisted sector.

The crosscap conditions (A.2) also give rise to a relation among the zero-modes which, as expected, can only be solved trivially. The conditions that arise for the fermionic modes are similar

$$
\left(\psi^m_r \pm i \epsilon_m \eta e^{-i\pi r} \tilde{\psi}^m_r\right) |\Omega R I^L_4, \eta\rangle = 0, \quad \text{for } m \in \{6, 7, 8, 9\},
$$

(A.4)

$$
\left(\psi^\mu_r + i \eta e^{-i\pi r} \tilde{\psi}^\mu_r\right) |\Omega R I^L_4, \eta\rangle = 0, \quad \text{for } \mu \in \{2, 3, 4, 5\},
$$

where, as usual, $\eta = \pm 1$ gives rise to the different spin structures. The solution to these equations is given by

$$
|\Omega R I^L_4, \eta\rangle = \mathcal{M} \exp \left( - \sum_{\mu=2}^5 \sum_{n \in \mathbb{Z}} \frac{(-1)^n}{n} \alpha^\mu_{-n} \tilde{\alpha}^\mu_{-n} + \sum_{m \in \{6,8\}} \sum_{r \in \mathbb{Z} + \frac{1}{2}} \frac{e^{\pm i\pi r}}{r} \alpha^m_r \tilde{\alpha}^m_r 
\right.
$$

$$
+ \sum_{m \in \{7,9\}} \sum_{r \in \mathbb{Z} + \frac{1}{2}} \frac{e^{\mp i\pi r}}{r} \alpha^m_r \tilde{\alpha}^m_r
$$

$$
+ \frac{1}{2} \left[ \sum_{\mu=2}^5 \sum_{r} e^{-i\pi r} \psi^\mu_{-r} \tilde{\psi}^\mu_{-r} \mp \sum_{m \in \{6,8\}} \sum_{r} e^{-i\pi r} \psi^m_{-r} \tilde{\psi}^m_{-r} \right] |T_{L,R}, \eta\rangle,
$$

(A.5)

where the overall normalization $\mathcal{M}$ is determined by worldsheet duality. The moding of the fermions $\psi^\mu_r$ and $\tilde{\psi}^\mu_r$ depends on whether we are considering the twisted NS-NS or the twisted R-R sector of the theory.

The ground states in the twisted sectors are constrained by the conditions that arise from the fermionic zero modes in (A.4). In the twisted R-R sector, the theory has only
fermionic zero modes in the directions unaffected by the orientifold, and therefore the standard argument applies (see for example [25]). On the other hand, there are fermionic zero modes for \( \psi^m_0 \) with \( m = 6, 7, 8, 9 \) in the twisted NS-NS sector, and they give rise to the conditions
\[
\psi^m_+|T_L, +\rangle = 0 \quad \text{for} \ m = 6, 8
\]
\[
\psi^m_-|T_L, +\rangle = 0 \quad \text{for} \ m = 7, 9
\]
and similarly for \( T_R \),
\[
\psi^m_-|T_R, +\rangle = 0 \quad \text{for} \ m = 6, 8
\]
\[
\psi^m_+|T_R, +\rangle = 0 \quad \text{for} \ m = 7, 9.
\]
Here we have defined
\[
\psi^m_\pm = \frac{1}{\sqrt{2}} \left( \psi^m_0 \pm i \tilde{\psi}^m_0 \right).
\]
For the IIB orbifold under consideration, the GSO-projection in the twisted NS-NS sector is given by
\[
\frac{1}{4} \left( 1 + (-1)^F \right) \left( 1 + (-1)^{\tilde{F}} \right),
\]
where the two operators \((-1)^F\) and \((-1)^{\tilde{F}}\) are defined by
\[
(-1)^F = \prod_{m=6}^{9} \sqrt{2} \psi^m_0 = \prod_{m=6}^{9} (\psi^m_+ + \psi^m_-)
\]
\[
(-1)^{\tilde{F}} = \prod_{m=6}^{9} \sqrt{2} \tilde{\psi}^m_0 = \prod_{m=6}^{9} (\psi^m_+ - \psi^m_-).
\]
Thus if we define
\[
|T_L, -\rangle = \psi^6_- \psi^7_+ \psi^8_- \psi^9_+ |T_L, +\rangle,
\]
from which it follows that
\[
|T_L, +\rangle = \psi^6_+ \psi^7_- \psi^8_+ \psi^9_- |T_L, -\rangle,
\]
we have that
\[
(-1)^F |T_L, \pm\rangle = |T_L, \mp\rangle,
\]
\[
(-1)^{\tilde{F}} |T_L, \pm\rangle = |T_L, \mp\rangle,
\]
and therefore
\[
(|T_L, +\rangle + |T_L, -\rangle)
\]
is a GSO-invariant state. Since the GSO-operators act in the standard way on the oscillator exponential, this implies that

$$|\Omega RI_4^L\rangle \equiv \left( |\Omega RI_4^L, +\rangle + |\Omega RI_4^L, -\rangle \right) \quad (A.15)$$

is GSO-invariant. The analysis for $\Omega RI_4^R$ is identical since the comparison of $(A.6)$ with $(A.7)$ implies that we can define $|T_R, +\rangle = |T_L, -\rangle$ and $|T_R, -\rangle = |T_L, +\rangle$. This analysis applies separately for each twisted sector of the theory.

The actual crosscap states also have to be invariant under the orientifold projection $\Omega RI_4^L$ (which generates the whole orientifold group). Since these crosscap states are effectively $O7$-planes, the invariance under $\Omega R$ is familiar. In order to understand this more explicitly (compare [42] for a similar analysis), we recall that $\Omega$ acts on the fermionic modes as

$$\Omega \psi^m r \Omega^{-1} = \tilde{\psi}^m r$$

so that

$$\Omega \tilde{\psi}^m r \Omega^{-1} = -\psi^m r$$

$$\Omega \psi^m_+ \Omega^{-1} = -i\psi^m_+$$

$$\Omega \tilde{\psi}^m_- \Omega^{-1} = +i\tilde{\psi}^m_-$$. \quad (A.17)

If we denote by $|C9, \eta\rangle$ the ground state that satisfies (A.4) without $\epsilon_m$, then $\Omega$ is defined to satisfy $\Omega |C9, \eta\rangle = |C9, \eta\rangle$. Since $|T_L, +\rangle = \psi^7_+ \psi^9_+ |C9, +\rangle$, it then follows that $|T_L, \eta\rangle$ has eigenvalue $-1$ under the action of $\Omega$, i.e. $\Omega |T_L, \eta\rangle = -|T_L, \eta\rangle$. On the other hand, $\mathcal{R}$ acts on the ground states as

$$\mathcal{R} = 4\psi^7_0 \tilde{\psi}^7_0 \psi^9_0 \tilde{\psi}^9_0 = - (\psi^7_+ + \psi^7_-)(\psi^9_+ - \psi^9_-)(\psi^9_+ + \psi^9_-)(\psi^9_+ - \psi^9_-)$$

and thus $\mathcal{R} |T_L, \eta\rangle = -|T_L, \eta\rangle$. This implies that $\Omega \mathcal{R}$ leaves $|T_L, \eta\rangle$ invariant. Again, the action on the oscillator states is trivial, and therefore also $|\Omega RI_4^L\rangle$ is invariant under $\Omega \mathcal{R}$. The same argument obviously also applies to $|\Omega RI_4^R\rangle$. 

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