Generalized second law of thermodynamics in presence of interacting DBI essence and other dark energies

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In the present work we investigated the validity of the generalized second law (GSL) of thermodynamics in presence of interaction between DBI-essence and other three candidates of dark energy namely modified Chaplygin gas, hessence, tachyonic field and new agegraphic dark energy. It has been observed that the GSL breaks down in presence of the interactions. However, the event horizon remains to be an increasing function of time.

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I. INTRODUCTION

One of the most alluring observational discoveries of the past decade has been that the expansion of the universe is speeding up rather than slowing down [1]. An accelerating universe is strongly suggested by observations of type Ia high redshift supernovae provided these behave as standard candles. The case for an accelerating universe is further strengthened by the discovery of Cosmic Microwave Background (CMB)[2] and galaxy power spectrum for large scale structures [3]. These studies indicate that in spatially flat isotropic universe, about two-thirds of the critical energy density seems to be stored in a so called dark energy (DE) component with enough negative pressure responsible for the currently cosmic accelerating expansion [4]. Strength of this acceleration depends on the theoretical model employed while interpreting the data. A wide range of scenarios have been proposed to explain this acceleration but most of them cannot explain all the features of the universe or they have so many parameters that they are difficult to fit. The models which have been discussed widely in literature are those which consider vacuum energy (cosmological constant) as DE, introduce a fifth element and dub it quintessence or scenarios named phantom with $\omega < -1$, where $\omega = p/\rho$ is a parameter of state [5].

An approach to the problem of DE arises from the holographic principle that states that the number of degrees of freedom related directly to entropy scales with the enclosing area of the system [5]. Based on the holographic principle proposed by Fischler and Susskind [6] several others have studied holographic model for dark energy (HDE) [7]. The holographic energy density is given by $\rho_\Lambda = 3c^2 M_p^2 L^{-2}$ in which $c$ is a free dimensionless parameter and the coefficient 3 is for convenience. As an application of the holographic principle in cosmology, it was shown that the consequence of excluding those degrees of freedom of the system which will never be observed by the effective field theory gives rise to IR cut-off $L$ at the future event horizon. Thus, in a universe dominated by DE, the future event horizon will tend to a constant of the order $H_0^{-1}$, the present Hubble radius [5].

In 1973, Bekenstein [8] assumed that there is a relation between the event of horizon and the thermodynamics of a black hole, so that the event of horizon of the black hole is a measure of the entropy of it. This idea has been generalized to horizons of cosmological models, so that each horizon corresponds to an entropy. Thus the second law of thermodynamics was modified in the way that in generalized form, the sum of all time derivative of entropies related to horizons plus time derivative of normal entropy must be positive, i.e. the sum of entropies must be increasing function of time. Davies [9] investigated the validity of generalized second law (GSL) for the cosmological models which departs slightly from de Sitter space. However, it is only natural to associate an entropy to the horizon area as it measures our lack of knowledge about what is going on beyond it. Setare [10] investigated the validity of the generalized second law of thermodynamics for the quintom model of dark energy. Setare [7] considered the interacting holographic model of dark energy to investigate the validity

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of the generalized second law of thermodynamics in a non-flat (closed) universe enclosed by the event horizon measured from the sphere of the horizon $L$. Setare and Shafei [5] showed that for the apparent horizon the first law is roughly respected for different epochs while the second law of thermodynamics is respected, while for $L$ as the system’s IR cut-off the first law is broken and the second law is respected for the special range of the deceleration parameter.

There have been many works aimed at connecting the string theory with inflation. Various ideas in string theory based on the concept of branes have proved themselves fruitful. Scenarios where the inflation is interpreted as the distance between two branes moving in the extra dimension along a warped throat have given rise to many interesting studies [11]. One area which has been well explored in recent years, is inflation driven by the open string sector through dynamical Dp-branes. This is the so-called DBI (Dirac-Born-Infeld) inflation [12, 13, 14], which lies in a special class of K-inflation models. Martin and Yamaguchi [13] introduced a scalar field model where the kinetic term has a DBI form and considered that the dark energy scalar field is a DBI scalar field, for which the action of the field can be written as

$$S_{DBI} = - \int d^4 x a^3(t) \left[ T(\phi) \sqrt{1 - \dot{\phi}^2 / T(\phi)^2} + V(\phi) - T(\phi) \right]$$  \hspace{1cm} (1)$$

where $T(\phi)$ is the tension and $V(\phi)$ is the potential. To obtain a suitable evolution of the Universe an interaction is often assumed such that the decay rate should be proportional to the present value of the Hubble parameter for good fit to the expansion history of the Universe as determined by the Supernovae and CMB data [15]. These kind of models describe an energy flow between the components so that no components are conserved separately. There are several work on the interaction between dark energy (tachyon or phantom) and dark matter [16], where phenomenologically introduced different forms of interaction term.

The present paper is an extension of the paper of the present authors Chattopadhyay and Debnath (2010) [Ref 17]. In the said paper [17], the interactions between different candidates of dark energy and DBI-essence model were considered. For the sake of simplicity in the presentation, the interactions discussed in reference [17] have been described briefly and subsequently the GSL has been considered. Organizations of the present paper is as follows: in section II we have given a brief introduction to the generalized second law (GSL) of thermodynamics with respect to cosmology; in section III we have considered the GSL in the interacting DBI-essence and modified Chaplygin gas (MCG); in section IV GSL has been viewed in presence of interacting DBI-essence and hessence; in section V we have investigated GSL in presence of interacting DBI-essence and tachyonic field.

II. SECOND LAW OF THERMODYNAMICS

In the present work, we consider the universe as a flat FRW universe and take into account that the accelerating universe has a future event horizon $R_h$, which is also named as cosmological horizon [11]. The radius of observer’s event horizon is given by [5]

$$R_h = a \int_t^\infty \frac{dt}{a} = a \int_a^\infty \frac{da}{Ha^2}$$ \hspace{1cm} (2)$$

To study the generalized second law (GSL) through the universe under the interaction between tachyonic field and scalar (phantom) field we would examine the nature of the derivative of the normal entropy $S$ in presence of interaction. It is a proven fact that for phantom dominated universe $\dot{S} > 0$ and for a quintessence dominated universe $\dot{S} < 0$ [10]. Our target is to answer the question: Is $\dot{S} > 0$ under the interaction between DBI-essence and other dark energies?. The expression for normal entropy using the first law of thermodynamics is

$$TdS = dE + PdV = (P + \rho)dV + Vd\rho$$ \hspace{1cm} (3)$$

Also we know that
\[ H^2 = \frac{1}{3} \rho \] (4)

and

\[ \dot{H} = -\frac{1}{2} (P + \rho) \] (5)

Using \( V = \frac{4}{3} \pi R_h^3 \) in equation (3) we get

\[ T dS = -2 \dot{H} dV + V d\rho = -8\pi R_h^2 \dot{H} dR_h + 8\pi R_h^3 dH \] (6)

From equation (6), it can be obtained that

\[ \dot{S} = \frac{8\pi \dot{H} R_h^2}{T} \] (7)

If the horizon entropy is taken to be \( S_h = \pi R_h^2 \), we get

\[ \dot{S} + \dot{S}_h = \frac{8\pi \dot{H} R_h^2}{T} + 2\pi R_h \dot{R}_h \geq 0 \] (8)

Taking the temperature \( T = \frac{1}{2\pi R_h} \) and using \( \dot{R}_h = H R_h - 1 \) we can write

\[ \dot{S} + \dot{S}_h = \dot{S}_X = 16\pi^2 \dot{H} R_h^3 + 2\pi R_h (H R_h - 1) \geq 0 \] (9)

In the subsequent sections, we would investigate the validity of the equation (9) in various interacting situations.

### III. GSL IN PRESENCE OF INTERACTING DBI-ESSENCE AND MCG

We consider a spatially flat isotropic and homogeneous universe in the FRW model whose metric is given by

\[ ds^2 = dt^2 - a^2(t)(dr^2 + r^2(d\theta^2 + \sin^2 d\phi^2)) \] (10)

where, \( a(t) \) is the scale factor. The Einstein field equations are given by (choosing \( 8\pi G = c = 1 \))

The energy conservation equation is given by

\[ \dot{\rho} + 3\frac{a}{\dot{a}}(\rho + p) = 0 \] (11)

If we consider a model consisting of two component mixture, the an interaction term needs to be introduced. In a two-component model, we replace \( \rho \) and \( p \) of equation (11) by \( \rho_{\text{total}} \) and \( p_{\text{total}} \) where

\[ \rho_{\text{total}} = \rho_D + \rho_X \] (12)

\[ p_{\text{total}} = p_D + p_X \] (13)
where \( \rho_D \) and \( p_D \) denote the density and pressure for the DBI-essence. The terms \( \rho_X \) and \( p_X \) denote the density and pressure corresponding to the other dark energies. Therefore, we get

\[
\frac{3 \dot{a}^2}{a^2} = (\rho_D + \rho_X) \tag{14}
\]

\[
6 \frac{\dot{a}}{a} = -[(\rho_D + \rho_X) + 3(p_D + p_X)] \tag{15}
\]

\[
(\dot{\rho}_D + \dot{\rho}_X) + 3 \frac{\dot{a}}{a}[(\rho_D + \rho_X) + (p_D + p_X)] = 0 \tag{16}
\]

Assuming gravity to obey four-dimensional general relativity with a standard Einstein-Hilbert Lagrangian, the density and pressure for DBI-essence are read as [15]

\[
\rho_D = (\gamma - 1)T(\phi_D) + V_D(\phi_D) \tag{17}
\]

\[
p_D = \left(\frac{\gamma - 1}{\gamma}\right)T(\phi_D) - V_D(\phi_D) \tag{18}
\]

where, \( \phi_D \) denotes the field for DBI-essence and the quantity \( \gamma \) is reminiscent of the usual Lorentz factor given by

\[
\gamma = \frac{1}{\sqrt{1 - \frac{\dot{\phi}_D^2}{T(\phi_D)}}} \tag{19}
\]

Since we are considering two-component model, we consider the interaction term \( 3H\delta\rho_X \) and we can write the conservation equations as

\[
\dot{\rho}_D + 3H(\rho_D + p_D) = 3H\delta\rho_X \tag{20}
\]

\[
\dot{\rho}_X + 3H(\rho_X + p_X) = -3H\delta\rho_X \tag{21}
\]

where, \( H = \frac{\dot{a}}{a} \) is the Hubble parameter, \( \delta \) is the interaction parameter and rest of the symbols are as explained earlier. The pressure and density of MCG are given by

\[
p_{ch} = A\rho_{ch} - \frac{B}{\rho_{ch}^{\alpha}} \tag{22}
\]

and

\[
\rho_{ch} = \left[\frac{B}{1 + A} + \frac{C}{a^{3(1+\alpha)(1+A)}}\right]^{\frac{1}{1+\alpha}} \tag{23}
\]

In the reference [17] it was obtained under interaction that

\[
\rho_{ch} = \left(\frac{B}{1 + A + \delta} + \frac{C}{t^{3m(1+\alpha)(1+A)}}\right)^{\frac{1}{1+\alpha}} \tag{24}
\]
Figs. 1 and 2 show the variation of $\dot{S}_X$ against $z$ in presence of interaction between DBI-essence and MCG ($\delta = 0.05$) and in the case of a mixture of the two dark energies without interaction ($\delta = 0$).

\[
\dot{\phi}_D^2 = 2\sqrt{\frac{n-1}{n}} \left[ \frac{m}{t^2} + \left( \frac{B}{1 + A + \delta} + \frac{C}{t^{3n(1+\alpha)(1+\alpha)}} \right) \frac{\dot{\phi}_H^2}{\phi_H^2} \left( B - (1 + A) \left( \frac{B}{1 + A + \delta} + \frac{C}{t^{3n(1+\alpha)(1+\alpha)}} \right) \right) \right]
\]

(25)

Using expressions (24) and (25), we calculated $\rho_{total}$ and $p_{total}$ and subsequently calculated $H$ and $\dot{H}$ using equations (4) and (5). Afterwards, $H$ is used in equation (2) for event horizon $R_h$ which is finally used to generate $\dot{S}_X$ of equation (9). In figure (1) we have presented the variation of $\dot{S}_X$ with the redshift $z = 1 - \frac{t}{n}$ in the interacting situation, i.e. with $\delta \neq 0$. It is apparent from figure (1), that $\dot{S}_X$ is always remaining in the negative level. This indicates a violation of the GSL as indicated in equation (9). In figure (2), we have plotted the change of $\dot{S}_X$ against $z$ in the situation of non-interaction i.e. $\delta = 0$. Here also, it is observed that the GSL is not valid.

IV. GSL IN PRESENCE OF INTERACTING DBI-ESSENCE AND HESSENCE

Energy density and pressure in the case of hessence are given by [17]

\[
\rho_H = \frac{1}{2}(\dot{\phi}_H^2 - \phi_H^2 \dot{\theta}^2) + V_H(\phi_H)
\]

(26)

\[
p_H = \frac{1}{2}(\ddot{\phi}_H^2 - \phi_H^2 \dot{\theta}^2) - V_H(\phi_H)
\]

(27)

where, $\dot{\theta} = \frac{Q}{\phi_H \phi_H}$. 

Now, taking $T(\phi_D) = m\phi_D^2$ and scale factor $a(t) = t^n$. From the field equations it can be found that

\[
\frac{n}{t^2} = \frac{1}{2} \left( \frac{\phi_H^2}{\phi_H^2} - \frac{Q^2}{\phi_H^2} + m\phi_D^2 \right)
\]

(28)

Under interaction, the forms of the potentials are found as [17]

\[
V_H = \frac{1}{4} t^{-3n\delta} \left[ (3n - 1)Q t^{3n(-2+\delta)} \left( \frac{4\delta}{2 - \delta} + \frac{t^{-2+6n}(2 - 3n(2 + \delta))}{3n\delta - 2} \right) \right] + C_1
\]

(29)
Figs. 3 and 4 show the variation of $\dot{S}$ against $z$ in presence of interaction between DBI-essence and hessence ($\delta = 0.05$) and in the case of a mixture of the two dark energies without interaction ($\delta = 0$).

and

\[ V_D = \frac{2n^2 \left( 2 + 3n\delta + \sqrt{n^{-1} \left( 1 - 3n\delta \right)} \right)}{mt^2(2 + 3n\delta)} + C_2 t^{3n\delta} \quad (30) \]

Like the earlier case, here we calculated the terms involved with $\dot{S}_X$ using the above expressions and plotted its evolution with the redshift $z$ in both interacting (figure 3) and non interacting (figure 4) cases. In both of the cases it is observed that $\dot{S}_X$ remains negative and hence GSL is not valid.

V. GSL IN PRESENCE OF INTERACTING DBI-ESSENCE AND TACHYONIC FIELD

The energy density $\rho_T$ and pressure $p_T$ for tachyonic field are given by

\[ \rho_T = \frac{V_T(\dot{\phi}_T)}{\sqrt{1 - \dot{\phi}_T^2}} \quad (31) \]

\[ p_T = -V_T(\dot{\phi}_T) \sqrt{1 - \dot{\phi}_T^2} \quad (32) \]

Under the interaction, the potentials for tachyonic field and DBI-essence are obtained as [17]

\[ V_T = e^{\frac{3n1^{-m}}{m}} t^{-m+3n(1+\delta)} \quad (33) \]

\[ V_D = \frac{1}{2} \left( \frac{6n}{t^2} + 4k_1 t^{k_2} + (3 - 4k_1) \sqrt{\frac{k_1}{k_1 - 1}} t^{k_2} + \frac{e^{\frac{3n1^{-m}}{m}} t^{-m+3n(1+\delta)}(3 + t^m)}{\sqrt{t^m}} \right) \quad (34) \]

Like the earlier case, here we calculated the terms involved with $\dot{S}_X$ using the above expressions and plotted its evolution with the redshift $z$ in both interacting (figure 5) and non interacting (figure 6) cases. In both of the cases it is observed that $\dot{S}_X$ remains negative and hence GSL is not valid.
Figs. 5 and 6 show the variation of $\dot{S}_X$ against $z$ in presence of interaction between DBI-essence and tachyonic field ($\delta = 0.05$) and in the case of a mixture of the two dark energies without interaction ($\delta = 0$).

VI. GSL IN PRESENCE OF INTERACTING DBI-ESSENCE AND NEW AGEGRAPHIC DARK ENERGY

The so-called agegraphic dark energy model was proposed in Cai (2007) [18], where the energy density is given by

$$\rho_q = \frac{3n^2m_p^2}{T^2}$$

(35)

where,

$$T = \int \frac{da}{Ha}$$

(36)

If we consider a flat FRW universe containing the agegraphic dark energy and pressureless-matter, the corresponding Friedman equation becomes

$$H^2 = \frac{1}{3m_p^2}(\rho_m + \rho_q)$$

(37)

Introducing the fractional energy densities $\Omega_i = \rho_i/3m_p^2H^2$ we can get

$$\Omega_q = \frac{n^2}{H^2T^2}$$

(38)

and the equation-of-state parameter $\omega_q = p_q/\rho_q$ is given by

$$\omega_q = -1 + \frac{2}{3n}\sqrt{\Omega_q}$$

(39)

A new agegraphic dark energy model was proposed in the reference [19], where the energy density $\rho_A$ is given as

$$\rho_A = \frac{3n^2m_p^2}{\eta^2}$$

(40)
Figs. 7 and 8 show the variation of $\dot{S}_X$ against $z$ in presence of interaction between DBI-essence and new agegraphic dark energy ($\delta = 0.05$) and in the case of a mixture of the two dark energies without interaction ($\delta = 0$).

where

$$\eta = \int \frac{dt}{a}$$

(41)

Thus, $\dot{\eta} = 1/a$. The corresponding fractional energy density is given by

$$\Omega_A = \frac{n^2}{H^2\eta^2}$$

(42)

In the present work, we would consider an interaction between DBI-essence and new agegraphic dark energy. To do so, we consider equations (14) and (15) and in the present case $\rho_X = \rho_A$ and $p_X = p_A$. Considering the interaction the equation-of-state parameter and the potential are obtained as [17]

$$\omega_A = -1 - \delta + \frac{2}{3n} \sqrt{\Omega_A}$$

(43)

$$V_D = \frac{1}{2} \left( \frac{6k_1}{t^2} + (1 - t^k_2) \left( 4k_3 - \sqrt{\frac{k_3}{k_3 - 1}}(4k_3 - 3) \right) - \frac{3}{k_1}(k_1 - 1)^2(-2 + 3k_1(2 + \delta))n^2m_p^2t^2(k_1 - 1) \right)$$

(44)

The $\dot{S}_X$ using the above expressions and plotted its evolution with the redshift $z$ in both interacting (figure 7) and non interacting (figure 7) cases. In both of the cases it is observed that $\dot{S}_X$ remains negative and hence GSL is not valid.

VII. CONCLUDING REMARKS

In the present paper we have considered total entropy as the entropy of a cosmological event horizon plus the entropy of the quintom fluid. We have investigated the validity of GSL in some interacting situations. In all of the cases we have observed that the time derivative of the total entropy is remaining at the negative level. This means that the total entropy is a decreasing function of time in the interacting situations considered in this paper. This means that the GSL breaks down. In an earlier work, [20] has shown that the GSL breaks down in the situation of interaction between holographic dark energy and dark matter. The present work proves
the breaking down of GSL in different other interactions between candidates of dark energies. However, in all the cases, we have seen that $R_h$ is always positive. This means that time derivative of future event horizon is positive and it indicates that future event horizon is increasing with time. However, total entropy is decreasing under interaction. It should be further noted that even in the case of non-interaction between two candidates of dark energies the GSL is breaking down.

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