Geometry-related magnetic interference patterns in long SNS Josephson junctions

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We have measured the critical current dependence on the magnetic flux of two long SNS junctions differing by the normal wire geometry. The samples are made by a Au wire connected to W contacts, via Focused Ion Beam assisted deposition. We could tune the magnetic pattern from the monotonic gaussian-like decay of a quasi 1D normal wire to the Fraunhofer-like pattern of a square normal wire. We explain the monotonic limit with a semiclassical 1D model, and we fit both field dependences with numerical simulations of the 2D Usadel equation. Furthermore, we observe both integer and fractional Shapiro steps. The magnetic flux dependence of the integer steps reproduces as expected that of the critical current $I_c$, while fractional steps decay slower with the flux than $I_c$.

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Introduction – A non-dissipative supercurrent can be transmitted between two superconductors (S) through a nanometer-thin insulating layer (I), when a phase difference is imposed. Superconducting correlations can also penetrate into a micrometer-long, non-superconducting coherent metal (N) over lengths greater than $\xi_s$, the superconducting coherence length in N. A supercurrent can then flow through long $(L > \xi_s)$ SNS junctions, provided that the phase coherence is preserved in the normal wire. The supercurrent amplitude only depends on the normal metal length $L$ and diffusion coefficient $D$ (which set the characteristic Thouless energy $E_{\text{Th}} = \hbar D/L^2$) and on the normal metal resistance $R_N$. The supercurrent thus reflects the transport mechanisms in the normal wire, and is affected by the interference and diffraction phenomena present in the normal metal as a result of the phase coherence.

We have studied the behavior of the supercurrent in long SNS junctions in a perpendicular magnetic field. In a previous work, we observed that the maximum supercurrent, the critical current $I_c$, monotonously decreased with the magnetic field following a quasi-gaussian dependence. This behavior is different from the interference Fraunhofer pattern usually observed, for example, in SIS junctions, short SNS junctions $(L \leq \xi_s)$, magnetic SFS Josephson junctions, etc. This difference is induced by the different aspect ratios of short weak links and long SNS junctions. Indeed, in SIS junctions the thickness of the junction is limited to a few Angstroms, to permit the tunneling of Cooper pairs; similarly, in SFS junctions the magnetic layer thickness has to be shorter than a few tens of nanometers, so that Cooper pairs are not broken by the internal exchange field. In contrast to the wide and short SIS and SFS junctions, SNS junctions offer the interesting possibility to explore a broad range of aspect ratios, since the length of the normal metal is only limited by the phase coherence length, which can be as long as a few microns at low temperature. In this paper we explore different geometries of long SNS junctions. Both monotonic and non-monotonic $I_c(B)$ dependences have been observed before, but we show for the first time that we can tune the $I_c(B)$ curve from an interference pattern to a quasi-gaussian monotonic dependence by varying the normal metal’s aspect ratio.

Monotonic $I_c(B)$ dependences have been observed in ballistic long SNS junctions, formed by a normal bidimensional InAs electron gas connected to superconducting Nb contacts larger than the London screening length. Their magnetic field dependence resulted from the screening currents in the Nb. On the contrary, the diffusive junctions investigated in the present work are contacted by thin disordered superconducting wires in which the magnetic field screening is negligible. We show that the geometry dependent magnetic field decay can be explained taking into account only the interferences between Andreev pairs’ trajectories in the normal metal. As a reminder, we first consider the case of a wide short SIS junction. A magnetic field in the SIS junction plane penetrates in the insulating layer of thickness $d$ and in the superconductors nearby over a length $\lambda_L$, the London penetration length. In a magnetic field $\vec{B} = -B\hat{z}$ of vector potential $\vec{A} = B\hat{y}\hat{z}$, the phase shift of the Cooper pairs tunneling at different points of the junction width is (Fig. 1c):

$$\theta(y) = \frac{2\pi}{\Phi_0} \int_{-\lambda_L}^{d+\lambda_L} A_x dy = \frac{2\pi}{\Phi_0} \Phi(y)$$

where $\Phi(y)$ is the flux through the surface $S_y = (d + 2\lambda_L) y$, and $\Phi_0 = \hbar/(2e)$ is the quantum flux. The current is obtained by integrating over the junction surface the supercurrent density $j = j_c \sin(\delta + \theta)$, taking into account both the superconducting phase difference between the contacts $\delta$ and the phase due to the vector potential. The critical current dependence on the magnetic field $B$ is

$$I_c(B) = I_{c0} \left(1 - \frac{B}{B_{c0}}\right)^n$$

where $I_{c0}$ is the critical current at zero field, $B_{c0}$ is the upper critical magnetic field, and $n$ is an exponent that depends on the geometry of the junction.

In the present work, we investigate the case of long SNS junctions, where the magnetic field is applied along the long direction of the junction. The phase shift $\theta(y)$ is given by

$$\theta(y) = \frac{2\pi}{\Phi_0} \int_{-\lambda_L}^{d+\lambda_L} A_x dy = \frac{2\pi}{\Phi_0} \Phi(y)$$

where $\Phi(y)$ is the flux through the surface $S_y = (d + 2\lambda_L) y$, and $\Phi_0 = \hbar/(2e)$ is the quantum flux. The current is obtained by integrating over the junction surface the supercurrent density $j = j_c \sin(\delta + \theta)$, taking into account both the superconducting phase difference between the contacts $\delta$ and the phase due to the vector potential. The critical current dependence on the magnetic field $B$ is

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flux $\Phi = B(d+2\lambda_L)w$, follows the well-known Fraunhofer pattern, a diffraction pattern created by the interference between the ballistic trajectories over the junction width:

$$I_c = I_c(0) \left| \frac{\Phi_0}{\pi} \sin \left( \frac{\pi \Phi_0}{\Phi} \right) \right|$$ (2)

If we now consider SNS junctions, we expect strong differences between junctions containing a short and wide or a long and narrow normal wire: in a short wide wire the phase difference between the trajectories comes from the phase distribution along the junction width, while in a narrow long wire, the phase of each trajectory is accumulated along the junction length.

The samples – We have fabricated long SNS junctions where a Au normal wire links two superconducting W contacts. First, a 50 nm thick Au wire is drawn by e-beam lithography and deposited onto a SiO$_2$ substrate. We use 99.9999% pure gold, with a content in magnetic impurities (Fe) smaller than 0.1 ppm. This insures a long phase coherence length $L_\Phi \sim 10 \mu m$ below 50 mK, measured in a separated weak localisation experiment. We then deposit the superconducting contacts in a Focused Ion Beam (FIB). After slightly etching the Au wire with the FIB to remove possible impurities on its surface, we inject a metallo-organic vapor of tungsten hexacarbonyl over the sample. This vapor is decomposed by the focused Ga$^+$ ion beam, and a disordered W alloy is deposited on the substrate. The wires produced are composed of tungsten, carbon and gallium in varying proportions (in our case, the atomic concentrations are roughly 30% W, 50% C and 20% Ga). The superconducting critical temperature of the wires produced is $T_c \sim 4 K$, an order of magnitude higher than the bulk $T_c$ of W. This could be due to the inclusion of Ga, which is itself a superconductor with $T_c = 1 K$. The critical magnetic field of the wires is also strikingly high: at 1 K, $H_c = 7 T$. The W wires are 200 nm wide and 100 nm thick. The dependence of the superconducting properties of these wires on the deposition conditions have been investigated in detail in W. Li et al.[7] The superconducting gap as well as the Abrikosov flux lattice have been studied by STM experiments.[8] The investigation of proximity induced superconductivity in metallic nanowires contacted by FIB has also been recently performed.[8] The advantages of this technique are the deposition of the material of practically any shape and size, without any mask, and the good quality of the interface created. The main disadvantage is the Ga contamination of about 250 nm around the deposited wires. The long SNS junctions created by FIB-assisted deposition are comparable to junctions created with more standard fabrication methods. We thus recovered the general results for the voltage vs. current curves and the temperature dependence of the critical current.[17]

To investigate the influence of the geometry on the $I_c(\Phi)$ dependence, we have designed two samples with different aspect ratios: sample WAu-Sq is 1.2 $\mu m$ long and 1.75 $\mu m$ wide, with an aspect ratio $L/w = 0.7$, while sample WAu-N is 1.53$\mu m$ long and 0.34 $\mu m$ wide, with an aspect ratio $L/w = 4.5$ (Fig. 1(a),(b)).

The $I_c(\Phi)$ normalised curves for samples WAu-N and WAu-Sq are shown in Fig. 2 and 3 respectively. They illustrate the important role of the aspect ratio: sample WAu-N displays a quasi-gaussian decay of the critical current, while sample WAu-Sq displays oscillations which recall a Fraunhofer pattern.

Semiclassical model – To explain the behavior of sam-
The authors do not suppose a purely sinusoidal current-phase relation to start with, but calculate the complete $I(\delta)$ relation of a long SNS junction. However, the wide junction limit corresponds exactly to the Fraunhofer pattern obtained in the case of a sinusoidal current-phase relation.

**Experimental results** – In agreement with the prediction of J.C. Cuevas et al. for the aspect ratio $L/w = 4.5$, junction WAu-N does not exhibit any oscillating pattern.
Shapiro steps – We have measured the differential resistance \(dV/dI\) when irradiating the junctions with microwaves from an antenna. We have observed Shapiro steps, in the form of microwave-induced zero resistance dips at \(V = \hbar/2n f\). They result from the resonance of the ac voltage at frequency \(f\) induced by the microwaves, with the current oscillations at frequency \(f_J = 2eV/h\), due to the ac Josephson effect at finite voltage \(V\) (Fig. 4(a),(b)). In addition to the Shapiro steps at integer \(n\), we find fractional Shapiro steps at \(n=1/2, n=1/3, n=1/4\). Both integer and fractional steps can be observed at voltages larger than the Thouless energy \((E_{Th}/e = 5.3\mu V)\), and for frequencies larger than the minigap \((\Delta/h = 4\text{ GHz})\); the voltage was directly deduced from the measured \(V/I\) curves. Fractional Shapiro steps appear in SNS junctions as a consequence of a non-sinusoidal current-phase relation. The additional harmonics in the current-phase relation can be generated by multiple coherent Andreev reflections (MAR), when the coherence length is much longer than the \(N\) length, or by non-equilibrium effects. Fractional Shapiro steps reflect the behavior of each harmonic individually: the step at \(n=1/2\), for example, is generated by the second harmonic and is proportional to its amplitude, having the same dependence on field and temperature. We have studied the magnetic field dependence of both integer and fractional steps amplitudes. As expected, the field dependence of the integer Shapiro steps follows roughly that of the zero-th order step, the critical current (Fig. 4(c)). This is not the case however for the fractional steps: in Fig. 4 we show that the decay with the normalised flux of step \(n=1/2\) is slower than that of the critical current. If this fractional Shapiro step was due to the MAR at equilibrium, the field dependence would show a periodicity half that of the critical current, corresponding to the double length covered by the Andreev pairs, and the zeros of the Fraunhofer pattern should be found at multiples of \(\Phi_0/2\). This is evidently not the case, so that the supplementary harmonic in the current-phase relation may be traced back to non-equilibrium effects, as already suggested in P. Dubos et al.\(^\text{13}\) and F. Chiodi et al.\(^\text{10}\) where the effect of the magnetic field at
the lowest temperatures was shown to increase the total critical current in the out of equilibrium SNS junction.

Conclusions – We have measured the $I_c(\Phi)$ curves for two different geometries of long SNS junctions. The samples are made by a Au wire connected to W contacts, via FIB-assisted deposition. We have observed a monotonic gaussian-like decay for a quasi 1D normal wire, in contrast to the Fraunhofer-like interference pattern of a square normal wire. We explain the monotonic limit with a semiclassical 1D model, and fit both field dependences with numerical simulations of the 2D Usadel equation. Moreover, we have observed both integer and fractional Shapiro steps and their dependence in magnetic field. While integer steps follow as expected the field dependence of the critical current, fractional steps decay slower than $I_c$. This is incompatible with equilibrium MAR-originated steps, but may be explained as an out-of-equilibrium effect.

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