FIXED POINT THEOREMS FOR WEAK CONTRACTION IN INTUITIONISTIC FUZZY METRIC SPACE

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Abstract. The notion of weak contraction in intuitionistic fuzzy metric space is well known and its study is well entrenched in the literature. This paper introduces the notion of \((\psi, \alpha, \beta)\)-weak contraction in intuitionistic fuzzy metric space. In this contrast, we prove certain coincidence point results in partially ordered intuitionistic fuzzy metric spaces for functions which satisfy a certain inequality involving three control functions. In the course of investigation, we found that by imposing some additional conditions on the mappings, coincidence point turns out to be a fixed point. Moreover, we establish a theorem as an application of our results.

1. Introduction

The concept of fuzzy set was introduced in 1965 by Zadeh [16]. Since then, with a view to utilize this concept in topology and analysis, several authors have extensively developed the theory of fuzzy sets along with their applications. In 1986, with similar endeavor, Atanassov [1] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [16]. Using the idea of intuitionistic fuzzy sets, Alaca et al. [2] defined the notion of intuitionistic fuzzy metric space and with the help of continuous \(t\)-norms and continuous \(t\)-conorms, Park [13] modified the concept as a generalization of fuzzy metric space due to Kramosil and Michalek [8].

In this paper we prove certain coincidence point results in partially ordered intuitionistic fuzzy metric spaces for functions which satisfy a
certain inequality involving three control functions. Throughout the paper $M_d$ and $N_d$ stands for fuzzy metric associated with metric $d$.

**Definition 1.1.** [14] A t-norm is a binary operation $\ast$ on $[0, 1]$ satisfying the following conditions:

(i) $\ast$ is continuous, commutative and associative;
(ii) $a \ast 1 = a \forall a \in [0, 1]$;
(iii) $a \ast b \leq c \ast d$, whenever $a \leq c$ and $b \leq d$, $\forall a, b, c, d \in [0, 1]$.

**Example 1.2.** Examples of t-norms are $a \ast b = ab$ and $a \ast b = \min\{a, b\}$.

**Definition 1.3.** [14] A t-conorm is a binary operation $\circ$ on $[0, 1]$ satisfying the following conditions:

(i) $\circ$ is continuous, commutative and associative;
(ii) $a \circ 0 = a \forall a \in [0, 1]$;
(iii) $a \circ b \leq c \circ d$, whenever $a \leq c$ and $b \leq d$, $\forall a, b, c, d \in [0, 1]$.

**Example 1.4.** Examples of t-conorm are $a \circ b = a + b - ab$ and $a \circ b = \max\{a, b\}$.

The concepts of triangular norms (t-norms) and triangular conorms (t-conorms) was originally introduced by Menger [12] in his study of statistical metric spaces and further studied by Schweizer and Sklar [14]. The following definition of intuitionistic fuzzy metric space is due to Park [13].

**Definition 1.5.** The 5-tuple $(X, M, N, \ast, \circ)$ is said to be an intuitionistic fuzzy metric space if $X$ is an arbitrary set, $\ast$ is a continuous t-norm, $\circ$ is continuous t-conorm and $M, N$ are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following axioms: for all $x, y, z \in X$ and $s, t > 0$

(i) $M(x, y, t) + N(x, y, t) \leq 1$;
(ii) $M(x, y, t) > 0$;
(iii) $M(x, y, t) = 1$ if and only if $x = y$;
(iv) $M(x, y, t) = M(y, x, t)$;
(v) $M(x, y, t) \ast M(y, z, s) \leq M(x, z, s + t)$;
(vi) $M(x, y, \cdot) : (0, \infty) \to (0, 1]$ is continuous;
(vii) $N(x, y, t) > 0$;
(viii) $N(x, y, t) = 0$ if and only if $x = y$;
(ix) $N(x, y, t) = N(y, x, t)$;
(x) $N(x, y, t) \circ N(y, z, s) \geq N(x, z, s + t)$;
(xi) $N(x, y, \cdot) : (0, \infty) \to (0, 1]$ is continuous.
Then \((M, N)\) is called an intuitionistic fuzzy metric on \(X\). The functions \(M(x, y, t)\) and \(N(x, y, t)\) denotes the degree of nearness and the degree of non-nearness between \(x\) and \(y\) with respect to \(t\), respectively.

The following remark gives the relation between fuzzy metric and intuitionistic fuzzy metric space [11].

**Remark 1.6.** Every fuzzy metric space \((X, M, \ast)\) is an intuitionistic fuzzy metric space of the form \((X, M, 1 - M, \ast, \diamond)\) such that \(t\)-norms \(\ast\) and \(t\)-conorms \(\diamond\) are associated, that is, \(x \diamond y = 1 - ((1 - x) \ast (1 - y))\) for all \(x, y \in X\).

**Remark 1.7.** In intuitionistic fuzzy metric space \((X, M, N, \ast, \diamond)\), \(M(x, y, \cdot)\) is non-decreasing and \(N(x, y, \cdot)\) is non-increasing for all \(x, y \in X\).

**Example 1.8.** Let \(X = \mathbb{N}\). Define \(a \ast b = \max\{0, a + b - 1\}\) and \(a \diamond b = a + b - ab\) for all \(a, b \in [0, 1]\) and let \(M, N\) be fuzzy sets on \(X^2 \times (0, \infty)\) as follows:

\[
M(x, y, t) = \begin{cases} \frac{x}{y} & \text{if } x \leq y, \\ \frac{y}{x} & \text{if } y \leq x, \\
\end{cases} \quad N(x, y, t) = \begin{cases} \frac{y-x}{y} & \text{if } x \leq y, \\ \frac{x-y}{x} & \text{if } y \leq x.
\end{cases}
\]

for all \(x, y \in X\) and \(t > 0\). Then \((X, M, N, \ast, \diamond)\) is an intuitionistic fuzzy metric spaces.

**Remark 1.9.** Note that in the above example, \(t\)-norm \(\ast\) and \(t\)-conorm \(\diamond\) are not associated and there exists no metric \(d\) on \(X\) satisfying

\[
M(x, y, t) = \frac{t}{t + d(x, y)} \quad N(x, y, t) = \frac{d(x, y)}{t + d(x, y)}
\]

where \(M(x, y, t)\) and \(N(x, y, t)\) are as defined in the above example. Also note that the above function \((M, N)\) is not an intuitionistic fuzzy metric with \(t\)-norm and \(t\)-conorm defined as \(a \ast b = \min\{a, b\}\) and \(a \diamond b = \max\{a, b\}\).

Alaca et al. [2] also introduced the notion of Cauchy sequences in intuitionistic fuzzy metric spaces and proved the well known fixed point theorems of Banach [4]. Edelstein [5] extended to intuitionistic fuzzy metric spaces with the help of Grabiec [6].

**Definition 1.10.** [2] A sequence \(\{x_n\}\) in an intuitionistic fuzzy metric space converge to \(x \in X\) if, for each \(t > 0\), \(\lim_{n \to \infty} M(x_n, x, t) = 1\) and \(\lim_{n \to \infty} N(x_n, x, t) = 0\); and sequence \(\{x_n\}\) is called convergent.
Definition 1.11. [2] A sequence \( \{x_n\} \) in an intuitionistic fuzzy metric space is said to be Cauchy if and only if for each \( r \in (0, 1) \) and \( t > 0 \) there exists \( n_0 \in \mathbb{N} \) such that \( M(x_n, x_m, t) > 1 - r \) and \( N(x_n, x_m, t) < r \) for all \( n, m \geq n_0 \).

Definition 1.12. An intuitionistic fuzzy space \((X, M, N, *, \odot)\) is said to be complete if and only if every Cauchy sequence in \( X \) is convergent.

Note that metric space \((X, d)\) is complete if and only if intuitionistic fuzzy metric space \((X, M_d, N_d, *, \odot)\) is complete.

Turkoglu et al. [15] introduced the concept of compatible maps and compatible maps of types \((\alpha)\) and \((\beta)\) in intuitionistic fuzzy metric spaces and gave some relations between the concepts of compatible maps. The formal definition of concept introduced in [15] is as follows:

Definition 1.13. A pair of self-mappings \((f, g)\) of an intuitionistic fuzzy metric space \((X, M, N, *, \odot)\) is said to be compatible if
\[
\lim_{n \to \infty} M(fgx_n, gfx_n, t) = 1 \quad \text{and} \quad \lim_{n \to \infty} N(fgx_n, gfx_n, t) = 0
\]
for every \( t > 0 \) whenever \( \{x_n\} \) is a sequence in \( X \) such that \( \lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = z \) for some \( z \in X \).

In view of Definition 1.13, the following fact is evident.
If \((f, g)\) is a compatible pair and \( z \) is a coincidence point of \( f \) and \( g \), then \( fgz = gfz \).

Definition 1.14. [7] Two self mappings \( f \) and \( g \) are said to be weakly compatible if they commute at coincidence points.

Definition 1.15. [17] A partial order is a binary relation ”\( \preceq \)” over a set which is reflexive, antisymmetric, and transitive. A set equipped with a partial order is called a partially ordered set.

Definition 1.16. \((g-\text{non decreasing Mapping}) [7]\) Suppose \((X, \preceq)\) is partially ordered set and \( f, g : X \to X \) are mapping of \( X \) to itself, \( f \) is said to be \( g\)-non-decreasing if for \( x, y \in X \), \( gx \preceq gy \) implies \( fx \preceq fy \).

In 1984, Khan et al. [9] employed the idea of altering distance in metric fixed point results. An altering function is a control function employed to alter the metric distance between two points enabling one to deal with relatively new classes of fixed point problems. The involvement of altering distance sometimes requires special techniques as the triangular inequality does not remain directly applicable. Using the control function considerable work have been done (see [10] ).
Definition 1.17. An altering distance function is a function \( \psi : [0, \infty) \to [0, \infty) \) which is continuous monotone non-decreasing with \( \psi(0) = 0 \).

Before going to main sections, we need the following classes of functions, which will be used in sequel. We denote by \( \Psi \) the set of all functions \( \psi : [0, \infty) \to [0, \infty) \) satisfying

\[
(i_\psi) \; \psi \text{ is continuous and monotone non-decreasing}, \\
(ii_\psi) \; \psi(t) = 0 \text{ if and only if } t = 0;
\]

and by \( \Theta \) we denote the set of all functions \( \alpha : [0, \infty) \to [0, \infty) \) such that

\[
(i_\alpha) \; \alpha \text{ is bounded on any bounded interval in } [0, \infty), \\
(ii_\alpha) \; \alpha \text{ is continuous at } 0 \text{ and } \alpha(0) = 0.
\]

Recently, Beg et al. [3] introduced the concept \((\phi, \psi)-\text{weak contraction}\) in intuitionistic fuzzy metric space and proved some results using the altering distance function. The formal definition of \((\phi, \psi)-\text{weak contraction}\) in intuitionistic fuzzy metric space is as follows:

Definition 1.18. Let \((X, M, N, \ast, \diamond)\) be an intuitionistic fuzzy metric space and \(T, f : X \to X\) be two mappings. The mapping \(T\) is called intuitionistic \((\phi, \psi)-\text{weak contraction}\) with respect to \(f\) if there exist a function \(\psi : [0, \infty) \to [0, \infty)\) with \(\psi(r) > 0\) for \(r > 0\) and \(\psi(0) = 0\) and the altering distance function \(\phi\) such that

\[
\phi\left(\frac{1}{M(Tx, Ty, t)} - 1\right) \leq \phi\left(\frac{1}{M(fx, fy, t)} - 1\right) - \psi\left(\frac{1}{M(fx, fy, t)} - 1\right)
\]

and

\[
\phi(N(Tx, Ty, t)) \leq \phi(N(fx, fy, t)) - \psi(N(fx, fy, t))
\]

holds for every \(x, y \in X\) and each \(t > 0\). If the mapping \(f\) is the identity mapping, then the mapping \(T\) is called intuitionistic \((\phi, \psi)-\text{weak contraction}\).

Inspired, from the above idea of the intuitionistic \((\phi, \psi)-\text{weak contraction}\), here, we define the concept of intuitionistic \((\psi, \alpha, \beta)-\text{weak contraction}\), where \(\psi \in \Psi\) and \(\alpha, \beta \in \Theta\). The formal definition of new notion is as follows:

Definition 1.19. Let \((X, M, N, \ast, \diamond)\) be an intuitionistic fuzzy metric space and \(f, g : X \to X\) be two mappings. The mapping \(f\) is called intuitionistic \((\psi, \alpha, \beta)-\text{weak contraction}\) with respect to \(g\) if there exist
functions $\psi, \alpha$ and $\beta$, $\psi \in \Psi$ and $\alpha, \beta \in \Theta$ such that

\begin{equation}
\psi\left(\frac{1}{M(fx, fy, t)} - 1\right) \leq \alpha\left(\frac{1}{M(gx, gy, t)} - 1\right) - \beta\left(\frac{1}{M(gx, gy, t)} - 1\right)
\end{equation}

and

\begin{equation}
\psi(N(fx, fy, t)) \leq \alpha(N(gx, gy, t)) - \beta(N(gx, gy, t))
\end{equation}

holds for every $x, y \in X$ and for all $t > 0$. If $g$ is the identity mapping, then the mapping $f : X \rightarrow X$ is called intuitionistic $(\psi, \alpha, \beta)$-weak contraction.

In this paper we prove certain coincidence point and fixed point results in partially ordered intuitionistic fuzzy metric spaces for functions which satisfy a certain inequality using Definition 1.19.

2. MAIN THEOREMS

**Theorem 2.1.** Let $(X, \preceq)$ be a partially ordered set and $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space. Let $f : X \rightarrow X$ be $(\psi, \alpha, \beta)$-weak contraction with respect to $g : X \rightarrow X$ and $f(X) \subseteq g(X)$. Also assume that $f$ is $g$-non-decreasing, $g(X)$ is closed, $g$ is continuous and for all $x, y \in X$, $gx \preceq gy$. Further, $\psi \in \Psi$ and $\alpha, \beta \in \Theta$ such that for all $s, t \geq 0$

\begin{equation}
\psi(s) \leq \alpha(t) \Rightarrow s \leq t,
\end{equation}

and for any sequence $\{t_n\}$ in $[0, \infty)$ with $t_n \rightarrow t > 0$,

\begin{equation}
\psi(t) - \lim_{n \rightarrow \infty} \alpha(t_n) + \lim_{n \rightarrow \infty} \beta(t_n) > 0.
\end{equation}

Also, for any non-decreasing sequence $\{x_n\}$ in $X$ converges to $z$,

\begin{equation}
x_n \preceq z \text{ for all } n \geq 0.
\end{equation}

If there is a point $x_0 \in X$ such that $gx_0 \preceq fx_0$, then $f$ and $g$ have a coincidence point.

**Proof.** Choose a point $x_0 \in X$ such that $gx_0 \preceq fx_0$, since range of $g$ contains the range of $f$ one may find $x_1 \in X$ such that $gx_1 = fx_0$. This gives $gx_0 \preceq fx_0 = gx_1$ and $fx_0 \preceq fx_1$ as $f$ is $g$-non-decreasing. Continuing in this way we can construct the sequence $\{x_n\}$ such that

\begin{equation}
f x_n = g x_{n+1} \forall \ n \geq 0
\end{equation}
whence

\[ gx_0 \leq fx_0 = gx_1 \leq fx_1 = gx_2 \leq fx_2 \]
\[ = gx_3 \leq fx_3 \ldots \leq fx_{n-1} = gx_n \leq fx_n = gx_{n+1} \leq \ldots \]

(7)

Clearly, if the terms \( x_{n-1} = x_n \), then there is a coincidence point of \( f \) and \( g \), so nothing to prove. Therefore, we must assume that \( x_{n-1} \neq x_n \) for all \( n \geq 1 \), which implies that

\[ M(f \, x_{n-1}, \, f \, x_n, \, t) \neq 1 \quad \forall \, n \geq 1, \forall \, t > 0. \]

and

\[ N(f \, x_{n-1}, \, f \, x_n, \, t) \neq 0 \quad \forall \, n \geq 1, \forall \, t > 0. \]

Substituting \( x = x_n \), and \( y = x_{n+1} \) in Definition 1.19, and using (6) with (7), for all \( n \geq 1, t > 0 \), gives us

\[ \psi \left( \frac{1}{M(f \, x_n, \, f \, x_{n+1}, \, t)} - 1 \right) \leq \alpha \left( \frac{1}{M(g \, x_n, \, g \, x_{n+1}, \, t)} - 1 \right) - \beta \left( \frac{1}{M(g \, x_n, \, g \, x_{n+1}, \, t)} - 1 \right) \]
\[ = \alpha \left( \frac{1}{M(f \, x_{n-1}, \, f \, x_n, \, t)} - 1 \right) - \beta \left( \frac{1}{M(f \, x_{n-1}, \, f \, x_n, \, t)} - 1 \right) \]

(10)

and

\[ \psi(N(f \, x_n, \, f \, x_{n+1}, \, t)) \leq \alpha(N(g \, x_n, \, g \, x_{n+1}, \, t)) - \beta(N(g \, x_n, \, g \, x_{n+1}, \, t)) \]
\[ = \alpha(N(f \, x_{n-1}, \, f \, x_n, \, t)) - \beta(N(f \, x_{n-1}, \, f \, x_n, \, t)). \]

(11)

In view of Inequality (10), for all \( t > 0 \) and \( n \geq 1 \), we have

\[ \psi \left( \frac{1}{M(f \, x_n, \, f \, x_{n+1}, \, t)} - 1 \right) \leq \alpha \left( \frac{1}{M(f \, x_{n-1}, \, f \, x_n, \, t)} - 1 \right) \]

using, Inequality (3), the expression turns out to be

\[ \frac{1}{M(f \, x_n, \, f \, x_{n+1}, \, t)} - 1 \leq \frac{1}{M(f \, x_{n-1}, \, f \, x_n, \, t)} - 1. \]

Let us take

\[ a_n(t) = \frac{1}{M(f \, x_n, \, f \, x_{n+1}, \, t)} - 1. \]
Notice that the sequence \( \{a_n(t)\} \), is monotonically decreasing for all \( t > 0 \) and, consequently, there exists \( r_1(t) \geq 0 \) such that

\[
\lim_{n \to \infty} a_n(t) = \lim_{n \to \infty} \left( \frac{1}{M(f_{x_n}, f_{x_{n+1}}, t)} - 1 \right) = r_1(t)
\]

In similar vein, using Inequality (11) for all \( t > 0 \) and \( n \geq 1 \), we obtain

\[
\psi(N(f_{x_n}, f_{x_{n+1}}, t)) \leq \alpha(N(f_{x_{n-1}}, f_{x_n}, t))
\]

and from (3), the expression becomes

\[
N(f_{x_n}, f_{x_{n+1}}, t) \leq N(f_{x_{n-1}}, f_{x_n}, t).
\]

Again, let us take

\[
b_n(t) = N(f_{x_n}, f_{x_{n+1}}, t)
\]

Clearly, the sequence \( \{b_n(t)\} \) is also monotonically decreasing for all \( t > 0 \) and hence, there exists \( r_2(t) \geq 0 \) such that

\[
\lim_{n \to \infty} b_n(t) = \lim_{n \to \infty} (N(f_{x_n}, f_{x_{n+1}}, t)) = r_2(t).
\]

Now we want to show that \( r_1(t) \) and \( r_2(t) \) are both zero. To do this, firstly, we take the limit supremum on both sides of (10), then for all \( t > 0 \),

\[
\psi(r_1(t)) \leq \overline{\lim}(a_n(t)) + \overline{\lim}(-\beta(a_n(t))) \\
\leq \overline{\lim}(a_n(t)) - \underline{\lim}(\beta(a_n(t)))
\]

(since \( \overline{\lim}(-\beta(a_n(t))) = -\lim(\beta(a_n(t)), \beta(a_n(t)), \text{being non-negative.})\)

It follows that \( r_1(t) = 0 \) for all \( t > 0 \), due to (4) and (12).

In the similar vein, taking limit supremum on both sides of (11) we obtain, for all \( t > 0 \),

\[
\psi(r_2(t)) \leq \overline{\lim}(b_n(t)) + \overline{\lim}(-\beta(b_n(t))) \\
\leq \overline{\lim}(b_n(t)) - \underline{\lim}(\beta(b_n(t)))
\]

(since \( \overline{\lim}(-\beta(b_n(t))) = -\lim(\beta(b_n(t)), \beta(b_n(t)), \text{being non-negative.})\)

Again, it implies that \( r_2(t) = 0 \) for all \( t > 0 \), due to (4) and (13)

Therefore, from (12) and (13) for all \( t > 0 \), we have

\[
\lim_{n \to \infty} M(f_{x_n}, f_{x_{n+1}}, t) = 1
\]

and

\[
\lim_{n \to \infty} N(f_{x_n}, f_{x_{n+1}}, t) = 0.
\]
It is clear that for arbitrary \(0 < \lambda < 1\) we can find \(L = L(\lambda)\) such that for all \(n \geq L(\lambda)\), (by (14))

\[
M(f_{x_{n-1}}, f_{x_n}, \lambda) > (1 - \lambda)
\]

Now, we shall prove that \(\{f_{x_n}\}\) is a Cauchy sequence in intuitionistic fuzzy metric space. Suppose on contrary, \(\{f_{x_n}\}\) is not Cauchy then there exist some \(s > 0\), and \(0 < \epsilon < 1\), for which we can find two sub sequences, \(\{f_{x_m(k)}\}\) and \(\{f_{x_n(k)}\}\) of \(\{f_{x_n}\}\) such that

\[
n(k) > m(k) > k
\]

and

\[
M(f_{x_m(k)}, f_{x_n(k)}, s) \leq (1 - \epsilon).
\]

By taking \(n(k)\) to be the smallest integer corresponding to \(m(k)\) for which (18) is satisfied, we have that

\[
M(f_{x_m(k)}, f_{x_n(k)-1}, s) > (1 - \epsilon),
\]

Choose \(\lambda\) in such a manner that \(0 < \lambda < s\), then by (16), (17) and (18), it follows that for all \(k > L(\lambda)\),

\[
(1 - \epsilon) \geq M(f_{x_m(k)}, f_{x_n(k)}, s)
\]

\[
\geq M(f_{x_m(k)}, f_{x_n(k)-1}, s - \lambda) * M(f_{x_n(k)-1}, f_{x_n(k)}, \lambda)
\]

\[
\geq M(f_{x_m(k)}, f_{x_n(k)-1}, s - \lambda) * (1 - \lambda) \text{ (by (16) and (17))}
\]

(20)

Take the function \(h(t) = \inf_{k \geq N(\lambda)} M(f_{x_m(k)}, f_{x_n(k)}, t)\).

Also \(h(s) = \inf_{k \geq N(\lambda)} M(f_{x_m(k)}, f_{x_n(k)-1}, s) \geq (1 - \epsilon)\) (by (19)). Since \(M(x, y, .)\) is continuous and monotonically increasing in the third variable, this implies that \(h(t)\) is continuous and monotonically increasing. Now,

\[
h(s - \lambda) = \inf_{k \geq N(\lambda)} M(f_{x_m(k)}, f_{x_n(k)-1}, s - \lambda) \geq 1 - \epsilon - g(\lambda)
\]

where

\[
g(\lambda) \to 0, \quad \text{as} \quad \lambda \to 0
\]

Combining (20) and (21), we obtain

\[
(1 - \epsilon) \geq M(f_{x_m(k)}, f_{x_n(k)}, s) \geq h(s - \lambda) * (1 - \lambda)
\]

\[
(1 - \epsilon - g(\lambda)) * (1 - \lambda)
\]
Taking $\lambda \to 0$ in the above inequality, using (22) and the continuity of $*$, we obtain

\begin{equation}
\lim_{k \to \infty} M(f x_{m(k)}, f x_{n(k)}, s) = 1 - \epsilon.
\end{equation}

Again for any choice of $0 < \lambda < \frac{s}{2}$ for all $k \geq L(\lambda),

\begin{equation}
(1 - \epsilon) \geq M(f x_{m(k)}, f x_{n(k)}, s) \ (\text{by (18)})
\end{equation}

\begin{equation}
\geq M(f x_{m(k)}, f x_{m(k)} - 1, \lambda) * M(f x_{m(k)} - 1, f x_{n(k)} - 1, s - 2 \lambda) *
\end{equation}

\begin{equation}
M(f x_{n(k)} - 1, f x_{n(k)}, \lambda)
\end{equation}

\begin{equation}
\geq M(f x_{m(k)} - 1, f x_{n(k)} - 1, s - 2 \lambda) * (1 - \lambda) * (1 - \lambda) \ (\text{by (16) and (17)})
\end{equation}

Taking the function $h_1(t) = \lim_{k \to \infty} M(f x_{m(k)} - 1, f x_{n(k)} - 1, t), t > 0$.

Since $M(x, y, .)$ is continuous, it follows that $h_1(t)$ is a continuous and from (24), we get

\begin{equation}
(1 - \lambda) * (1 - \lambda) * h_1(s - 2 \lambda) = (1 - \lambda) * (1 - \lambda) *
\end{equation}

\begin{equation}
\lim_{k \to \infty} M(f x_{m(k)} - 1, f x_{n(k)} - 1, s - 2 \lambda)
\end{equation}

\begin{equation}
\leq (1 - \epsilon).
\end{equation}

Proceeding $\lambda \to 0$ in the Inequality (24), and using the continuity of $h_1$, we obtain

\begin{equation}
\lim_{k \to \infty} M(f x_{m(k)} - 1, f x_{n(k)} - 1, s) \leq (1 - \epsilon).
\end{equation}

Again, let us write

\begin{equation}
h_2(t) = \lim_{k \to \infty} M(f x_{m(k)} - 1, f x_{n(k)} - 1, t), t > 0
\end{equation}

The arguments are analogous to those used above, by which we get that $h_2(t)$ is a continuous function. It is clear that, for all $k \geq L(\lambda)$

\begin{equation}
M(f x_{m(k)} - 1, f x_{n(k)} - 1, s + \lambda) \geq M(f x_{m(k)} - 1, f x_{m(k)}, \lambda) *
\end{equation}

\begin{equation}
M(f x_{m(k)}, f x_{n(k)} - 1, s)
\end{equation}

\begin{equation}
\geq (1 - \lambda) * (1 - \epsilon) \ (\text{by (16), (17) and (19)})
\end{equation}

Taking $\lambda \to 0$ in the above inequality, and using the continuity of $*$, it is easy to see that

\begin{equation}
\lim_{k \to \infty} M(f x_{m(k)} - 1, f x_{n(k)} - 1, s) \geq (1 - \epsilon).
\end{equation}
In light of the inequalities (26) and (29) we found that
\( \lim_{k \to \infty} M(f x_m(k)^{-1}, f x_n(k)^{-1}, s) = (1 - \epsilon). \)

Let us take
\( s_k = \frac{1}{M(f x_m(k)^{-1}, f x_n(k)^{-1}, s)} - 1 \)

Consequently, from (30) the expression becomes
\( \lim_{k \to \infty} s_k = \frac{\epsilon}{1 - \epsilon} \)

Using (7) and (17), we get \( g x_m(k) \preceq g x_n(k). \)

On substituting \( x = x_m(k), y = x_n(k), \) in the Definition 1.19, for all \( k \geq 1, \) we obtain
\( \psi\left( \frac{1}{M(f x_m(k), f x_n(k), s)} - 1 \right) \leq \alpha\left( \frac{1}{M(g x_m(k), g x_n(k), s)} - 1 \right) - \beta\left( \frac{1}{M(g x_m(k), g x_n(k), s)} - 1 \right) \)
\( = \alpha\left( \frac{1}{M(f x_m(k)^{-1}, f x_n(k)^{-1}, s)} - 1 \right) - \beta\left( \frac{1}{M(f x_m(k)^{-1}, f x_n(k)^{-1}, s)} - 1 \right) \)
\( \leq \alpha(s_k) - \beta(s_k). \) (from (31))

Taking limit supremum as \( k \to \infty \) in (33) using the continuity of \( \psi \)

\( \psi\left( \frac{\epsilon}{1 - \epsilon} \right) \leq \lim_{k \to \infty} \alpha(s_k) + \lim_{k \to \infty} (-\beta(s_k)) \)
\( = \lim_{k \to \infty} \alpha(s_k) - \lim_{k \to \infty} \beta(s_k) \) [since \( \beta(s_k)'s \) are positive]

Consequently, \( \epsilon = 0 \) using (4), (32) and (35) a contradiction to the hypothesis.

Thus, \( M(f x_n, f x_m, t) > 1 - r, \) (or \( M(f x_n, f x_m, t) = 1 - r_1 \) (say), where \( r_1 < r \)). Now it remains to show that \( N(f x_n, f x_m, t) < r. \) Due to Definition 1.5

\[ M(f x_n, f x_m, t) + N(f x_n, f x_m, t) \leq 1 \]

that is \( 1 - r_1 + N(f x_n, f x_m, t) \leq 1, \) it follows that \( N(f x_n, f x_m, t) \leq r_1 < r. \) This implies that \( \{f x_n\} \) is Cauchy sequence in intuitionistic
fuzzy metric space and hence \( \{fx_n\} \) is convergent. Since \( g(X) \) is closed and \( fx_n = gx_{n+1} \) for all \( n \geq 0 \), so there exists \( z_1 \in X \) such that
\[
(36) \quad \lim_{n \to \infty} gx_n = \lim_{n \to \infty} fx_n = gz_1.
\]
Finally, our claim is that \( z_1 \) is a coincidence point of \( f \) and \( g \). Since \( \{gx_n\} \) is a non-decreasing sequence in \( X \), thus in view of (5) and (36), for all \( n \geq 0 \) we get
\[
(37) \quad gx_n \preceq gz_1.
\]
Putting \( x = x_n \) and \( y = z_1 \) in the Definition 1.19, and by virtue of (6) together with (37) gives us
\[
\psi\left(\frac{1}{M(gx_{n+1}, fz_1, t)} - 1\right) = \psi\left(\frac{1}{M(fx_n, fz_1, t)} - 1\right) \leq \alpha\left(\frac{1}{M(gx_n, gz_1, t)} - 1\right) - \beta\left(\frac{1}{M(gx_n, gz_1, t)} - 1\right).
\]
Taking \( n \to \infty \) in the above inequality, using (36), the continuities of \( \psi \) and \( M \), continuities of \( \alpha, \beta \) at zero, and the fact that \( \alpha(0) = 0 = \beta(0) \), we obtain that for all \( t > 0 \)
\[
\psi\left(\frac{1}{M(gz_1, fz_1, t)} - 1\right) = \alpha(0) - \beta(0) = 0
\]
it follows that
\[
\frac{1}{M(gz_1, fz_1, t)} - 1 = 0, \text{ for all } t > 0,
\]
or equivalently,
\[
M(gz_1, fz_1, t) = 1, \text{ for all } t > 0
\]
that is,
\[
(38) \quad fz_1 = gz_1
\]
Therefore, \( z_1 \) is a coincidence point of \( f \) and \( g \). Hence the proof. \( \blacksquare \)

Now at this stage one may naturally ask; what additional conditions should be imposed on the mappings \( f \) and \( g \) so that coincidence point become the fixed point? In the following theorem we answer to this question:

**Theorem 2.2.** If in the Theorem 2.1 it is additionally assumed that
(i) For a coincidence point \( z_1 \) of the mappings \( f \) and \( g \)
\[
(39) \quad gz_1 \preceq gz_1;
\]
(ii) The mappings $f$ and $g$ are compatible, then the coincidence point $z_1$ (obtained in Theorem 2.1) become the common fixed point of $f$ and $g$.

**Proof:** According to the assumption for the coincidence point $z_1$ (obtained in (38)) $gz_1 \preceq ggz_1$. Also, $fgz_1 = gfz_1$ as $f$ and $g$ are compatible.

Let us take

\[ w = gz_1 = f z_1. \]  

In view of assumption (39), clearly

\[ gz_1 \preceq ggz_1 = gw. \]

Then

\[ fw = fgz_1 = gfz_1 = gw. \]

Now, if $z_1 = w$, then $z_1$ is common fixed point, due to (40) and if $z_1 \neq w$ then, by Definition 1.19 together with (40), (41) and (42) we get

\[ \psi(\frac{1}{M(gz_1, gw, t)} - 1) = \psi(\frac{1}{M(fz_1, fw, t)} - 1) \]

\[ \leq \alpha(\frac{1}{M(gz_1, gw, t)} - 1) - \beta(\frac{1}{M(gz_1, gw, t)} - 1). \]

For $t > 0$, consider the sequence \( \{t_n\} \), where

\[ t_n = \frac{1}{M(gz_1, gw, t)} - 1, \text{ for all } n \geq 1, \]

clearly,

\[ t_n \to 0, \text{ as } n \to \infty \]

Then, (4) together with (45), leads us to a contradiction unless

\[ t_n = 0 \]

Consequently, for all $t > 0$,

\[ M(gz_1, gw, t) = 1 \]

that is,

\[ gz_1 = gw \]

From (40), (42) and (47) we conclude that

\[ w = gw = fw. \]

Hence the proof is seen to be complete.  

\[ \blacksquare \]
In our next theorem we omit the order condition (5) of Theorem 2.1 in whose place the continuity of \( f \) and the compatibility of the pair \((f, g)\) is assumed.

**Theorem 2.3.** Let \((X, \preceq)\) be a partially ordered set and \((X, M, N, \ast, \odot)\) be a complete intuitionistic fuzzy metric space. Let \( f : X \to X \) be \((\psi, \alpha, \beta)\)-weak contraction with respect to \( g : X \to X \), \( f \) is continuous and \( f(X) \subseteq g(X) \). Also assume that \( f \) is \( g \)-non-decreasing, \( g \) is continuous, \( g(x) \) is closed and for all \( x, y \in X \), \( gx \preceq gy \). Further, \( \psi \in \Psi \) and \( \alpha, \beta \in \Theta \) such that for all \( s, t \geq 0 \)

\[
\psi(s) \leq \alpha(t) \Rightarrow s \leq t,
\]

and for any sequence \( \{t_n\} \) in \([0, \infty)\) with \( t_n \to t > 0 \),

\[
\psi(t) - \lim_{n \to \infty} (\alpha(t_n) + \lim\beta(t_n)) > 0.
\]

If there is a point \( x_0 \in X \) such that \( gx_0 \preceq fx_0 \) and the pair \((f, g)\) is compatible, then \( f \) and \( g \) have a coincidence point.

**Proof:** Following the methodology of Theorem 2.1 one may easily get (36) that is,

\[
\lim_{n \to \infty} gx_n = \lim_{n \to \infty} fx_n = g z_1 = \overline{z}. \quad \text{(say)}
\]

From the triangular inequality in intuitionistic fuzzy metric space for \( t > 0 \), we have

\[
M(g\overline{z}, fgx_n, t) \geq M(g\overline{z}, gfx_n, \frac{t}{2}) \ast M(gfx_n, fgx_n, \frac{t}{2})
\]

and

\[
N(g\overline{z}, fgx_n, t) \leq N(g\overline{z}, gfx_n, \frac{t}{2}) \odot N(gfx_n, fgx_n, \frac{t}{2}).
\]

Using continuities of \( f, g, M, N, \ast, \odot \) and compatibility of the pair \((f, g)\) together with (51), we obtain

\[
M(g\overline{z}, fz, t) \geq 1 \ast 1 = 1,
\]

and

\[
N(g\overline{z}, fz, t) \geq 0 \odot 0 = 0.
\]

This implies that,

\[
g\overline{z} = fz
\]

that is, \( \overline{z} \) is a coincidence point of \( f \) and \( g \). \( \blacksquare \)

Of course here also it is expected that whether coincidence point obtained in Theorem 2.3 can become a fixed point of the mappings \( f \).
and $g$ by imposing certain conditions. In the regard we have the following theorem.

**Theorem 2.4.** If in the Theorem 2.3 it is additionally assumed that
\begin{equation}
 gz \preceq ggz,
\end{equation}
for a coincidence point $z$ of $f$ and $g$, then the coincidence point $z$ becomes the fixed point.

**Proof:** The desired result can be proved by the argument analogous to those used in Theorem 2.2 except that the compatibility of the pair $(f, g)$ is already included in the assumption of the Theorem 2.3. \hfill \blacksquare

From the foregoing analysis, a more general formulation of the results arises by relaxing the any one of the conditions, (viz., 5 and continuity of $f$) which are closely related to Theorems 2.1–2.4, it seems to us that the above mentioned conditions together with the continuity of the functions in the class $\Theta$, plays an important role. Henceforth, the following result is a stronger version of the above theorems.

**Theorem 2.5.** Let $(X, \preceq)$ be a partially ordered set and $(X, M, N, *, \circ)$ be a complete intuitionistic fuzzy metric space. Let $f : X \to X$ be intuitionistic $(\psi, \alpha, \beta)$-weak contraction with respect to $g : X \to X$ and $f(X) \subseteq g(X)$. Also assume that $f$ is $g$-non-decreasing, $g$ is continuous, $g(X)$ is closed and for all $x, y \in X$, $gx \preceq gy$. Further, $\psi \in \Psi$, $\alpha, \beta \in \Theta$ and $\alpha, \beta$ are continuous such that for all $s, t \geq 0$
\begin{equation}
 \psi(s) \leq \alpha(t) \Rightarrow s \leq t,
\end{equation}
and for all $t > 0$,
\begin{equation}
 \psi(t) - \alpha(t) + \beta(t) > 0
\end{equation}
Also assume that any one of the following
(i) $f$ is continuous,
(ii) for any non-decreasing sequence $\{x_n\}$ in $X$ converges to $z$,
\begin{equation}
 x_n \preceq z \text{ for all } n \geq 0.
\end{equation}
holds. If there is a point $x_0 \in X$ such that $gx_0 \preceq fx_0$, then $f$ and $g$ have a coincidence point. Furthermore, if
\begin{equation}
 gz \preceq ggz
\end{equation}
for a coincident point $z$ of $f$ and $g$, then $f$ and $g$ have a common fixed point in $X$. 


Proof: Using the assumption (continuity of $\alpha, \beta$), it is not difficult to see that the inequality (4) is reduces to (55). The rest proof of the result follows by applications of Theorems 2.3 and 2.4 in the case of condition (i) and by application of the Theorem 2.1 and 2.2 in the case of condition (ii).

Of course, in any of the results reported above, still we do not have any result which provides the information about the uniqueness of the fixed point. So our next approach is to determine such a result. To do so, it is necessary to put some additional assumptions in Theorem 2.1 and 2.3 which ensure the existence of a unique fixed point.

Theorem 2.6. In addition to the hypothesis of Theorem 2.1 it is assumed that for every $x, y \in X$ there exists $u \in X$ such that $fx$ and $fy$ are comparable to $fu$, then there exists a unique common fixed point.

Proof. In light of Theorem 2.1, we know that the set of coincidence points of $f$ and $g$ is non-empty. Let us suppose that $x$ and $y$ be coincidence points of $f$ and $g$, that is, $fx = gx$ and $fy = gy$. Clearly, from the assumption, there exists $u \in X$ such that $fu$ is comparable with $fx$ and $fy$. Put $u_0 = u$ and choose $u_1 \in X$ so that $gu_1 = fu_0$. Continuing the process analogous to those used in the proof of the Theorem 2.1, we can inductively define sequence $\{gu_n\}$, where $gu_{n+1} = fu_n$ for all $n \geq 0$. Due to comparability of the terms, let us suppose that $gu_1 \preceq gx$, now our aim is to show that $gu_n \preceq gx$ for each $n \in \mathbb{N}$. To do this, we shall make use of mathematical induction. Clearly, the relation is true for $n = 1$. We presume that $gu_n \preceq gx$ holds for some $n > 1$. Since $f$ is $g$-nondecreasing, we get

$$gu_{n+1} = fu_n \preceq fx = gx.$$ 

Let, for $t > 0$,

$$R_n = \frac{1}{M(gx, gu_n, t)} - 1.$$ 

Then, from the Definition 1.19 and $gu_n \preceq gx$, we have, $\psi(R_{n+1}) \leq \alpha(R_{n+1}) - \beta(R_{n+1})$, which yields $\psi(R_{n+1}) \leq \alpha(R_n)$ due to the fact that $\beta \geq 0$, it follows that $R_{n+1} \leq R_n$ for all positive integer $n$ (by (3)). In other words, $\{R_n\}$ is a monotonically decreasing sequence.

Then, following the proof technique of the Theorem 2.1, we have

$$\lim_{n \to \infty} \left( \frac{1}{M(gx, gu_n, t)} - 1 \right) = 0$$
that is,
\[
\lim_{n \to \infty} (M(gx, gu_n, t)) = 1
\]
Similarly, we show that
\[
\lim_{n \to \infty} \left( \frac{1}{M(gy, gu_n, t)} - 1 \right) = 0
\]
that is,
\[
\lim_{n \to \infty} (M(gy, gu_n, t)) = 1.
\]
In the similar vein,
Let, for \( t > 0 \),
\[
R'_n = N(gx, gu_n, t)
\]
In view of Definition 1.19 and \( gu_n \leq gx \) we have, \( \psi(R'_{n+1}) \leq \alpha(R'_{n+1}) - \beta(R'_{n+1}) \), now using the above procedure \( \{R'_n\} \) is a monotonically decreasing sequence.
Clearly following the proof technique of Theorem 2.1, we have
\[
\lim_{n \to \infty} (N(gx, gu_n, t)) = 0
\]
In the similar vein, one can obtain
\[
\lim_{n \to \infty} (N(gy, gu_n, t)) = 0
\]
Now we shall make use of the continuity of \( * \) and \( \diamond \). For \( t > 0 \), we have
\[
M(gx, gy, t) \geq M(gx, gu_n, t/2) * M(gu_n, gy, t/2) \rightarrow 1 * 1 = 1 \text{ as } n \to \infty
\]
and
\[
N(gx, gy, t) \leq N(gx, gu_n, t/2) \diamond N(gu_n, gy, t/2) \rightarrow 0 \diamond 0 = 0 \text{ as } n \to \infty.
\]
The above inequalities implies that
\[
(57) \quad gx = gy.
\]
Thus the claim holds.
Also \( gx = fx \) and compatibility of \( g \) and \( f \), gives
\[
(58) \quad ggx = gfx = fgx.
\]
Let us take
\[
(59) \quad gx = z.
\]
Clearly, from (58), we obtain
\[
gz = fz.
\]
Thus $z$ is a coincidence point of $g$ and $f$ and form (57) with $y = z$ it follows that

$$g x = g z.$$  

Using (59)

$$z = g z.$$  

(60)

From (59) and (60), we get $z = g z = f z$. Therefore, $z$ is a common fixed point of $g$ and $f$. Finally, it remains to show the uniqueness of common fixed point of $g$ and $f$. Towards this, assume that $z_1$ is another common fixed point of $g$ and $f$. Using (57) we have $z_1 = g z_1 = g z = z$, this implies that $z = z_1$. Hence the common fixed point of $g$ and $f$ is unique.

\[ \square \]

Similarly we have the following result.

**Theorem 2.7.** If in addition to the hypothesis of Theorem 2.3 it is assumed that for every $x, y \in X$, there exists $u \in X$ such that $f x$ and $f y$ are comparable to $f u$, then $f$ and $g$ have a unique common fixed point.

**Theorem 2.8.** If in any of the Theorems 2.6 and 2.7 the condition of the compatibility between $f$ and $g$ is replaced by the commuting condition between $f$ and $g$, then the conclusions of either of the theorems are valid.

**Proof.** The proof follows from the observation that commuting condition implies compatibility condition between two mappings. \[ \square \]

The results reported above are illustrated with the help of two examples given below.

**Example 2.9.** Let $X = [0, \infty)$ and $(X, \preceq)$ is a partially ordered set with the natural ordering of real numbers, that is, $x \preceq y$ if and only if $x \leq y$. Let $t$-norm is $a \ast b = \min\{a, b\}$, $t$-conorm is $a \odot b = \max\{a, b\}$ and

$$M(x, y, t) = \frac{t}{t + |x - y|}; \quad N(x, y, t) = \frac{|x - y|}{t + |x - y|}$$

for all $x, y \in X$ and $t > 0$. Clearly, $(X, M, N, \ast, \odot)$ is a complete intuitionistic fuzzy metric space. Let $f, g : X \to X$ be given respectively by the formulas

$$f(x) = \begin{cases} \frac{x}{2} & 0 \leq x < 1, \\ \frac{x}{3} & x \geq 1, \end{cases} \quad g(x) = x.$$
Let for all $s \geq 0$, the functions $\psi, \alpha, \beta : [0, \infty) \rightarrow [0, \infty)$ be given respectively by the formulas

$$\psi(s) = s; \quad \alpha(s) = s; \quad \beta(s) = \frac{s}{2}.$$ 

$$\psi(s) = s; \quad \alpha(s) = s; \quad \beta(s) = \begin{cases} 0 & s = 0, \\ \frac{s}{2} & 0 < s < 1 \\ \frac{s}{4} & s \geq 1, \end{cases}$$

It is easy to verify that $\alpha, \beta \in \Theta$ for any bounded interval in $[0, \infty)$, $\psi \in \Psi$, $f$ is discontinuous and $fX \subseteq gX$. Now, it remains to show that $f$ is $(\psi, \alpha, \beta)$-weak contraction with respect to $g$. Towards this, we examine the inequalities (1) and (2). For all $x, y \in [0, \infty)$ and $t > 0$, (1) reduces to

$$\Rightarrow \psi\left(\frac{|f(x) - f(y)|}{t}\right) \leq \alpha\left(\frac{|g(x) - g(y)|}{t}\right) - \beta\left(\frac{|g(x) - g(y)|}{t}\right),$$

where

$$f(x) - f(y) = \begin{cases} \frac{x-y}{2}, & 0 \leq x, y < 1, \\ \frac{x-y}{3x-2y}, & x, y \geq 1, \\ \frac{x-y}{6}, & 0 \leq x < 1, y \geq 1; \quad g(x) - g(y) = (x - y). \end{cases}$$

Substituting these values in (61), one can verify that Inequality (1) satisfied, moreover the inequality (2) is also satisfied, due to the fact that $N = 1 - M$. Thus, for the above choices of $\psi, \alpha$ and $\beta$, all the assumptions of Theorem 2.1 are satisfied and the point $0$ is coincident point of $f$ and $g$.

Note that $(f, g)$ is a compatible pair of mappings and condition (39) is also holds. Hence, Theorem 2.2 is also applicable to this example. Therefore coincidence point $z = 0$ become a common fixed point of $f$ and $g$.

Example 2.10. Let $X = [0, \infty)$ and $(X, \leq)$ is a partially ordered set with the natural ordering of real numbers, that is, $x \leq y$ if and only if $x \leq y$. Let $t$-norm is $a \ast b = \min\{a, b\}$, $t$-conorm is $a \diamond b = \max\{a, b\}$ and

$$M(x, y, t) = \frac{t}{t + |x - y|}; \quad N(x, y, t) = \frac{|x - y|}{t + |x - y|}$$

for all $x, y \in X$ and $t > 0$. Clearly, $(X, M, N, \ast, \diamond)$ is a complete intuitionistic fuzzy metric space. Let $f, g : X \rightarrow X$ be given respectively by
the formulas $fx = \frac{x}{3}$ and $gx = x$ for all $x \in X$. Let for all $s \geq 0$, the functions $\psi, \alpha, \beta : [0, \infty) \to [0, \infty)$ be given respectively by the formulas

$$
\psi(s) = s; \quad \alpha(s) = s; \quad \beta(s) = \frac{s}{2}.
$$

It is easy to verify that $\alpha, \beta \in \Theta$ for any bounded interval in $[0, \infty)$, $\psi \in \Psi$, $fX \subseteq gX$ and $(f, g)$ is a compatible pair of mappings. Choose a sequence $t_n = \frac{1}{n} + 1$ in $X$, then the Inequality (4) is also satisfied. Now, it remains to show that $f$ is $(\psi, \alpha, \beta)$-weak contraction with respect to $g$. Towards this, we examine the inequalities (1) and (2). For all $x, y \in [0, \infty]$ and $t > 0$, (1) reduces to

$$
\Rightarrow \psi\left(\frac{|f(x) - f(y)|}{t} \right) \leq \alpha\left(\frac{|g(x) - g(y)|}{t} \right) - \beta\left(\frac{|g(x) - g(y)|}{t} \right)
$$

Since $f(x) - f(y) = \frac{1}{3}(x - y)$ and $g(x) - g(y) = (x - y)$, substituting these values in (62),

$$
\psi\left(\frac{1}{3t}|(x - y)|\right) \leq \alpha\left(\frac{1}{t}|(x - y)|\right) - \beta\left(\frac{1}{t}|(x - y)|\right)
$$

$$
\frac{1}{3t}|(x - y)| \leq \frac{1}{2t}|(x - y)|
$$

clearly it holds for all $x \leq y$ and $0 < t < 1$.

The inequality (2) satisfied, due to the fact that $N = 1 - M$. Thus, for the above choices of $\psi, \alpha$ and $\beta$, all the assumptions of Theorem 2.1 are satisfied and the point $0$ is coincident point of $f$ and $g$. Moreover, Theorem 2.2, 2.3 and 2.4 are also applicable to this example. Here $z = 0$ is a coincidence point as a well as common fixed point of $f$ and $g$.

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