About the Steady Horizontal Circulation of Airship

V T Grumondz¹, N I Morozov¹ and Moung Htang Om¹

¹ Moscow Aviation Institute, Moscow, Russia

E-mail: V.grumondz@gmail.com

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Abstract
In this paper, the questions of the existence and calculation of the steady circulating motion of the airship in the horizontal plane are considered.

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1. Introduction
In recent years, active work has been carried out all over the world on the design of airships and other aircraft, using, in addition to aerodynamic, the aerostatic principle [1–27]. In the design process, an important place belongs to the studies on the dynamics of the flight of an airship, which is very different from the dynamics of flights of aircraft heavier than air. However, until now, the problems of the dynamics of flight of an airship as a new type of aviation equipment remain completely unexplored. This article is devoted to one of the important problems of the airship’s flight dynamics — the study of the existence and algorithms for calculating the parameters of the steady motions of the airship in the horizontal plane and is a natural continuation of the work [27] of one of the authors.

As noted in [27], the airship’s flight dynamics has some similarities with the motion dynamics of an underwater vehicle of a similar shape, completely immersed in water during continuous flow. First of all, this is due to the great influence that the dynamics of the airship have on the value of the aerostatic force and its moments, as well as the magnitude of the attached masses, static moments and moments of inertia. In turn, this is due (as in the case of body motion in a fluid) the proximity of the average density of the airship to the density of the medium in which it moves. Based on these considerations, the differential equations of the spatial motion of the airship were obtained in [27], which we will use in this article.

2. Steady circulating airship motions in the horizontal plane
It’s assumed that the airship is made according to the classical scheme, i.e. has a body in the form of an elongated body of revolution, a cross-shaped tail located in the rear, a nacelle with equipment or a crew, and two rotary screw propellers located symmetrically with respect to the longitudinal plane of symmetry. It will also be assumed that the airship body is an absolutely solid (non-deformable) body and changes in the mass, moments of inertia and attached masses of the airship during the considered period of flight time are negligible \( (m = \text{const}, J = \text{const}, \lambda_o = \text{const}) \). As the beginning of the coordinate system associated with the airship \( Oxyz \), we choose the center of volume of the airship body. This point will be called the pole of the airship. The coordinates of the center of mass of the airship in the system \( Oxyz \) will be denoted as \( x_c, y_c, z_c \), in the future it will be assumed that \( z_c = 0 \). It will also be assumed that the centrifugal moments of inertia \( J_{xy}, J_{yx} \) are negligible.

The general vector equations for the dynamics of the spatial motion of the airship, as in [27], can be described in the following form;

\[
\frac{dQ}{dt} + \hat{Q} \times \mathbf{F} = \frac{dL}{dt} + \hat{L} \times \mathbf{V} = \mathbf{M}. \quad (1)
\]

Where \( \mathbf{Q}, \mathbf{L} \) — the main vector and the main moment of the total momentum of the airship and its ideal (not having viscosity) continuous medium; \( \mathbf{F}, \mathbf{M} \) — the main vector and
the main moment of the external forces acting on the airship, with the exception of inertial forces, already taken into account in the left parts of these equations. The right-hand sides of this system are vectors

\[ \mathbf{F} = \mathbf{G} + \mathbf{A} + \mathbf{P} + \mathbf{\overline{R}}, \quad \mathbf{\overline{M}} = \mathbf{G} \cdot \mathbf{A} + \mathbf{P} + \mathbf{\overline{R}}. \]

Where \( \mathbf{G} \) — weight of the airship taking into account the carrier gas and air inside the hull; \( \mathbf{A} \) — aerostatic force vector; \( \mathbf{P} \) — vector of the resultant forces created by the propulsions; \( \mathbf{\overline{R}} \) — vector of resultant aerodynamic forces; \( \mathbf{G} \cdot \mathbf{A} + \mathbf{P} + \mathbf{\overline{R}} \) — the main moments of gravity, thrust propulsion and aerodynamic forces relative to the pole \( \mathbf{O} \).

When recording the projections of the vectors \( \mathbf{\overline{R}}, \mathbf{\overline{M}} \), the traditional dimensionless aerodynamic coefficients \( c_s, c_r, c_c, m_s, m_r, m_c \) will be used. For the airships of the classical scheme, it is accepted to choose \( l = U^{1/3} \) as a characteristic linear size, and as a characteristic area — value \( S = U^{2/3} \).

It will be assumed that the volume of the instrument (cargo) nacelle and engines is small compared to the volume of the hull, so that the axis \( \alpha \) is the axis of geometric symmetry of the entire airship. Then in the matrix of attached masses, only the following will be nonzero: \( \lambda_{x, i} \), \( i = 1...6, \lambda_{y, i} \).

System (1) must be supplemented with well-known equations of kinematic and geometric relationships, which are not given here.

Under the assumptions made, the system of equations for the steady horizontal circulation of an airship with a constant radius will have the form:

\[
\begin{align*}
-mV \omega_x + (m + \lambda_{x, 0})V \omega_x &= (\rho g U - G) \cos \beta \sin \gamma + c, qS; \\
-myV \omega_y + mV \omega_y &= G_y, \cos \beta \sin \gamma + m qU; \\
m_xV \omega_y - (mx + \lambda_{y, 0})V \omega_y &= -G_x, \cos \beta \sin \gamma + m qU; \\
V_y = V \cos \alpha \cos \beta; V_\gamma = -V \sin \alpha \cos \beta; \omega_y &= \omega_y \frac{tg \beta}{cos \gamma}.
\end{align*}
\]

For aerodynamic coefficients, the following structure will be taken:

\[
\begin{align*}
c_s &= c_s' + c_s'' \beta + \beta \beta + c_s^5 \delta_x + c_s^6 \delta_y; \\
m_s &= m_s' \beta + m_s^0 \delta_x + m_s^1 \delta_y; \\
m_c &= m_c' \beta + m_c^0 \delta_x + m_c^1 \delta_y.
\end{align*}
\]

In system (2) \( \beta, \delta_x, \delta_y, \delta_\gamma \) are unknown functions; \( V, \alpha, \gamma \) must be obtained in advance as a result of solving the equations of longitudinal motion of an airship [27]. Thus, two of the five unknowns must be specified. The sign \( \beta \) can be arbitrarily assigned at the beginning of the calculation, followed by verification of the assumption made. Note that the first and third equations (2) under assumption (3) can be considered independently of the second.

3. Algorithms for calculating horizontal circulation parameters

Let’s consider some possible linear and nonlinear algorithms for calculating the parameters of the steady horizontal circulation of the airship and the corresponding balancing angles.

1. The angle \( \beta \) is small. Given \( \omega_x, \gamma \) (or \( \delta_x, \gamma \)). System (2) can be linearized. From the first and third equations, the balancing angles \( \delta_x, \delta_y \) can be obtained, then from the second — the balancing angle \( \delta_\gamma \). These solutions are as follows:

\[
\begin{align*}
\beta_x &= \frac{1}{\kappa} \left[ c_s^h q_s - m_s^0 q_U \right]; \\
\delta_x &= \frac{1}{\kappa} \left[ m_s^0 q_U \right] - c_s^h q_s; \\
\delta_\gamma &= \frac{1}{\kappa m_{s, 0}^0} \left[ c_s^h m_s^0 q_s + m_s^0 m_s^h q_U - c_s^h m_s^0 q_s - c_s^h m_s^0 q_s \right],
\end{align*}
\]

where

\[
\kappa = \frac{\rho V^2}{2} U \left[ c_s^h m_s^0 - c_s^h m_s^h \right];
\]

\[
q_s = -V \omega_x \left[ m + \lambda_{x, 0} \right] - (\rho g U - G) \gamma; \\
-q\omega_x \frac{U^2}{2} \left[ c_s^h + c_s^h \right]; \\
q_\gamma = -V \omega_x \left[ m \left( 1 + \alpha' \right) + \lambda_{y, 0} \right] - G_y, \cos \beta \sin \gamma + m qU; \\
-\frac{G_y, \cos \beta \sin \gamma + m qU}{2} \left[ m_s^0 + m_s^h \alpha' \right];
\]

In (4), it is possible to additionally take into account the smallness of the terms proportional \( \alpha' \), in comparison with the terms containing \( \alpha \), in the first and zero degrees, but for the future this is not significant. When \( \gamma = 0 \), balancing angles are independent of speed \( V \).

2. The angle \( \beta \) is not small. System (2) is nonlinear. The following two possible algorithms for solving the problem of finding balancing values are possible:

a) Given \( \omega_x, \gamma \) (or \( \delta_x, \gamma \)). Imagine \( \cos \beta \approx 1 - \frac{1}{2} \beta^2 \), we obtain from (2) a system of the following structure:

\[
\begin{align*}
g_{x, 1} \beta_1 + g_{x, 2} \beta_2 + g_{y, 1} \gamma_1 + g_{y, 2} \gamma_2 &= i = 1, 2; \\
h_1 \beta_1 + h_2 \beta_2 &= h_3 \beta_3 + h_4 \beta_4.
\end{align*}
\]

To obtain balancing values \( \beta, \delta_x \), the system of the first two equations \( (i = 1, 2) \) is solved. Then, substituting \( \beta \) those found in the third equation, find \( \delta_2 \). The expressions for the coefficients \( g_{x, 1}, h_1, g_{y, 1}, G_{y, 1} \), as well as the final formulas for the solution, are given in view of the obviousness of the question;

b) Given \( \gamma, \beta \). The system of the first and third equations (2) turns out to be linear in the variables \( \delta_x, \delta_y \) (or \( \omega_x, \omega_y \)).
so that it can be written their explicit expressions. It will done this for the case \( \gamma = 0, |\alpha| \leq 3^\circ, |\beta| \leq 3^\circ, 0 < \beta < 30^\circ \), which is of greatest interest from the point of view of practical operation of the airship. It's obtained

\[
\alpha_1 = \frac{1}{\Delta} (a_1 - b_1), \quad \beta_1 = \frac{1}{\Delta} (a_2 - b_2), \quad \gamma_1 = \frac{1}{\Delta} (a_3 - b_3),
\]

where

\[
a_1 = c_1^{\alpha} + c_1^{\beta} \beta + 2 \mu \cos \beta; \quad a_2 = c_1^{\nu_1} \mu; \quad \mu = \frac{m}{\rho U};
\]

\[
a_3 = m_1^{\alpha} + m_1^{\nu} \beta - 2 \mu \nabla \cos \beta; \quad a_2 = m_1^{\nu};
\]

\[
\nabla = \beta \cos \beta + c_2^1 \mu \cos \nabla \cos \beta U \theta.
\]

Then, from the second equation (2) the required balancing angle of rudder deflection is found:

\[
\delta_2 = - \frac{1}{\beta \cos \beta} \left[ (2 \mu \nabla \cos \beta + m_2^{\nu_1} \mu) \beta + m_2^{\nu} \right].
\]

The radius of the steady circulation is

\[
R_1 = \frac{1}{\beta \cos \beta} \cos \beta U \theta.
\]

4. Conclusions

In the general case, an iterative algorithm for calculating the parameters of the steady circulation of an airship can be represented as follows:

1. The calculation of the parameters \( \alpha_1^{(1)}, \alpha_1^{(2)}, P_1^{(2)} \) of the longitudinal balancing in the first step for given \( V, \phi, \Theta \) and \( c_1 = c_1^{\nu} = \text{const} \).

2. Calculation of balancing values \( \beta_1^{(1)}, \delta_1^{(1)}, \delta_1^{(2)} \) for any of the modifications of the above algorithms.

3. The calculation \( c_1^{(2)} = c_1^{(1)} + \Delta c_1^{(1)}, \alpha_1^{(2)}, \beta_1^{(2)}, \delta_1^{(2)} \) in the second step.

4. Calculation \( \alpha_1^{(2)}, \beta_1^{(2)}, P_1^{(2)} \) in the second step, etc.

With the help of (5), by varying \( \beta \), it is possible to construct the so-called airship controllability diagram as the geometrical place of the points of the plane \( (\delta_2, \delta_1) \), to which steady circulating motions in the horizontal plane may correspond.

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