Unusual quantum magnetic-resistive oscillations in a superconducting structure of two circular asymmetric loops in series

V. I. Kuznetsov and A. A. Firsov

Institute of Microelectronics Technology and High Purity Materials, Russian Academy of Sciences, Chernogolovka, Moscow Region 142432, Russia

(Dated: May 3, 2014)

We measured both quantum oscillations of a rectified time-averaged direct voltage \( V_{rec}(B) \) and a dc voltage \( V_{dc}(B) \) as a function of normal magnetic field \( B \), in a thin-film aluminum structure of two asymmetric circular loops in series at temperatures below the superconducting critical temperature \( T_c \). The \( V_{rec}(B) \) and \( V_{dc}(B) \) voltages were observed in the structure biased only with an alternating current (without a dc component) and only with a direct current (without an ac component), respectively. The aim of the measurements was to find whether interaction (nonlinear coupling) exists between quantum magnetic-resistive states of two loops at a large distance from each other. The distance between the loop centers was by an order of magnitude longer than the Ginzburg-Landau coherence length \( \xi(T) \). At such distance, one would not expect to detect any interaction between the quantum states of the loops. But we did find such an interaction. Moreover, we found that \( V_{dc}(B) \) functions (like \( V_{rec}(B) \) ones) can be used to describe the quantum states of the loops.

PACS numbers: 74.78.Na, 74.25.F, 74.40.Gh, 73.40.Ei, 85.25.-j

I. INTRODUCTION

Asymmetric superconducting loops without tunnel contacts are interesting for a prospective technological application and fundamental studies of unusual quantum magnetic-field-dependent oscillations of both rectified direct voltage and critical superconducting currents. A superconducting asymmetric circular loop and the asymmetric loops in series are very efficient rectifiers of ac voltages.

Rectification effect can be interpreted as follows. If a superconducting loop is threaded with a magnetic flux \( \Phi \) and biased by an alternating sinusoidal current (with a zero dc component) and if, in addition, the sum of bias ac and magnetic-filed-periodically-dependent loop circulating current exceeds the critical current value in a certain loop part, then an alternating voltage, with the period equal to the current period, appears across the loop. In a strictly symmetric circular loop, the time-averaged value of alternating voltage is zero because of the positive voltage corresponding to certain half-periods is cancelled out by the negative voltage corresponding to other half-periods. In a superconducting asymmetric circular loop, the difference between circulating current densities in two semi-loops disturbs the symmetry between positive and negative voltages, and a nonzero time-averaged (rectified) direct voltage \( V_{rec}(B) \) appears as a function of magnetic field \( B \).

\[ V_{rec}(B) \] voltage oscillates with the period \( \Delta B = \Phi_0/S \), here \( \Phi_0 \) is the superconducting magnetic flux quantum and \( S \) is the effective loop area. In a single asymmetric loop, \( V_{rec}(B) \) is the sign-alternating function of \( B \). \( V_{rec}(B) \) voltage changes its sign in fields corresponding to integer and half-integer values of a normalized magnetic flux \( \Phi/\Phi_0 \).

\( V_{rec}(B) \) oscillations are quite unusual. They radically differ from oscillations in the Little-Parks (LP) effect.

As compared to the LP oscillations, in low fields, the amplitude of the \( V_{rec}(B) \) oscillations can reach a giant magnitude that can be calculated by the expression \( I_c R_N/2\pi \) (for a bias sinusoidal current). Here, \( I_c \) is the critical superconducting current in the zero field and \( R_N \) is the structure resistance in the normal state. The maximum amplitude of voltage oscillations \( V_{rec}(B) \) corresponds to the maximum amplitude \( \Delta R \) (from peak to peak) of resistance oscillations that can reach \( R_N \). For aluminum loops, the \( \Delta R \) amplitude derived from \( V_{rec}(B) \) oscillations can exceed the amplitude of magnetoresistance oscillations expected on the basis of the LP effect more than by one order of magnitude.

\( V_{rec}(B) \) oscillations reach their extreme values (maxima and minima) at \( \Phi/\Phi_0 \) close to \( \pm(n+1/4) \), where \( n \) is an integer, while \( R(B) \) oscillations in the LP effect reach their extreme values at \( \Phi/\Phi_0 = n + 1/2 \).

In a single asymmetric circular loop and identical asymmetric loops in series, a nonzero rectified voltage \( V_{rec}(B) \) appears because of a difference between critical superconducting currents \( I_{c+}(B) \) and \( I_{c-}(B) \) measured for arbitrarily positive and negative half-waves of a bias ac.

\( I_{c+}(B) \) and \( I_{c-}(B) \) oscillations were unexpectedly found to be unusual. It was revealed that in low fields the curves \( I_{c+}(B) \) and \( I_{c-}(B) \) are shifted from one another with a \( \pi \) phase shift (corresponds to magnetic-field shift of \( \Phi_0/2 \)). Note that according to contemporary theoretical conceptions this incomprehensible \( \pi \) phase shift can be hardly expected in the studied asymmetric structures.

Moreover, like \( V_{rec}(B) \) oscillations these \( I_{c+}(B) \) and \( I_{c-}(B) \) oscillations reach their extreme values at \( \Phi/\Phi_0 \) close to \( \pm(n+1/4) \). Some other striking features of \( V_{rec}(B) \), \( I_{c+}(B) \), and \( I_{c-}(B) \) oscillations were also found in asymmetric loops.

Unlike LP oscillations, the unusual quantum \( V_{rec}(B) \),
$I_{c+}(B)$, and $I_{c-}(B)$ oscillations\cite{14} in an asymmetric circular loop cannot be explained in the framework of the simple Ginzburg-Landau (GL) quasi-one-dimensional model\cite{15}, using only the requirement of superconducting fluxoid quantization\cite{16}. Thus, experimental studies of the unusual quantum oscillations\cite{7,8} in simple superconducting circular-asymmetric structures generate many unsolved questions.

Besides the rectification of ac voltage, an asymmetric loop is interesting for other applications. Earlier\cite{17,18,19}, it was assumed that the quantum states of a single superconducting asymmetric circular loop placed in the normal magnetic field $B$ at $T$ below $T_c$ can be described by the oscillating superconducting circulating current of the loop $I_R(B)$. In order to determine the time-averaged quantum states, the loop was periodically switched from superconducting state to resistive state and back by a bias alternating current (without a dc component) with an amplitude close to the critical current value. As a result of the multiple switching, the rectified direct voltage $V_{rec}(B)$ appears in the loop.\cite{11} At certain values of the magnetic field, $V_{rec}(B)$ can be directly proportional to $I_R(B)$ in a single asymmetric circular loop.\cite{11} Therefore, measurements of $V_{rec}(B)$ can allow the readout of the time-averaged quantum states of the loop.\cite{11} Moreover, the oscillating $V_{rec}(B)$ voltage recorded at the different current values can describe quantum magneto-resistive states of the loop (the states depend both on the magnetic field and the bias ac). Quantum magneto-resistive states of the two directly connected asymmetric circular loops\cite{12} can be described by quantum magneto-resistive states of each loop and coupling between the states of the loops.

In addition, an asymmetric loop of high-resistance material with an extremely small wall narrowing could be used as an element of a superconducting flux qubit\cite{20} with quantum phase-slip centers\cite{21,22}. Two successive loops of this kind could be an analog of two successive flux qubits.

For certain technological applications, it is necessary to know both the strength and mechanism of coupling between quantum states of loops. Earlier, an interaction (nonlinear coupling) was revealed between quantum magnetic-resistive states of two different superconducting directly connected asymmetric circular loops forming a figure-of-eight-shaped structure\cite{23,24}. To determine the quantum state of each loop and coupling between the loops, rectified voltage $V_{rec}(B)$ was measured in a figure-of-eight-shaped structure pierced with a magnetic flux and biased with a low-frequency current (without a dc component) and with an amplitude close to critical, at $T$ slightly below $T_c$.\cite{23,24} Possible mechanisms of the interaction between the loops can be magnetic coupling and electro-dynamic coupling through a bias ac.

We assume that electro-dynamic coupling between two successive loops that occurs through a bias ac is a nonlocal phenomenon with the nonlocal superconducting length\cite{25,26} close, by the order of magnitude, to the Ginzburg-Landau coherence length $\xi(T)$\cite{27}. Note that the nonlocal length estimated from Ref.\cite{27} can reach the value several times exceeding $\xi(T)$.

Nonlinear coupling between quantum magnetic-resistive states of two asymmetric loops in series should become weaker with an increasing distance between them. It can be assumed that the most long-scaled electro-dynamic coupling should almost disappear if the spacing between loop centers increases to $10\xi(T)$.

The aim of this work was to find the largest distance between the loops at which the coupling between quantum magnetic-resistive states of the loops would still occur. For this purpose, we experimentally studied the quantum magnetic-resistive behavior of two different superconducting aluminum asymmetric circular loops connected in series with a wire of a length (Fig. 1) close to the penetration depth of a nonuniform electric field into a superconductor\cite{28} $\Lambda_E$.

Like the authors of Refs.\cite{29,30}, we measured the rectified direct voltage $V_{rec}(B)$ in the structure (Fig. 1) versus normal magnetic field $B$ and a bias sinusoidal low-frequency current (without a dc component) at temperatures $T$ slightly below $T_c$ in order to determine the quantum magnetic-resistive states of two loops in series and the coupling between the loop states. In addition, we measured a dc voltage $V_{dc}(B)$ as a function of $B$ and bias dc (without an ac component) through the structure.

One more goal was to test whether $V_{dc}(B)$ oscillations can be used to describe the quantum magnetic-resistive states of an asymmetric structure. We also made a comparison of $V_{rec}(B)$ and $V_{dc}(B)$ oscillations.

\section{II. Samples and Experimental Procedure}

A 45 nm thick structure of two loops connected in series was fabricated by thermal sputtering of aluminum onto a silicon substrate using the lift-off process of electron-beam lithography. Figure 1 displays a scanning electron microscopy (SEM) image of the structure. It consists of two different successive asymmetric circular loops with the distance between the loop centers $L = 12.5$ \textmu{}m. The average widths of all narrow and wide wires in the sample central part are $w_n = 0.22$ \textmu{}m and $w_w = 0.41$ \textmu{}m, respectively. The circular asymmetry permits the observation of nonzero rectified voltage $V_{rec}(B)$ in the structure\cite{31,32}. The minimum area of the loop is the in-

FIG. 1: SEM image of the structure. The scale bar: 2 \textmu{}m.
ternal area within the inner loop border. The minimum areas of the larger and smaller loops are $S_{L_{\text{min}}} = 11.57 \, \mu m^2$ and $S_{S_{\text{min}}} = 6.34 \, \mu m^2$, respectively. From the structure geometry, the average areas of the larger and smaller loops are $S_{L_{\text{avg}}} = 13.93 \, \mu m^2$ and $S_{S_{\text{avg}}} = 7.92 \, \mu m^2$, respectively.

The critical superconducting temperature $T_c = 1.355 \pm 0.001 \, K$ was determined from the midpoint of superconducting transition $R(T)$ in a zero field. The total normal-state resistance measured between two vertical wires at $T = 4.2 \, K$ is $R_N = 32 \, \Omega$. The ratio of room-temperature to the helium-temperature resistance is $R_{300}/R_{1.2} = 2$. Sheet resistance is $R_S = 0.69 \, \Omega$, the resistivity then is $\rho = 3.105 \times 10^{-8} \, \Omega \cdot m$. In the expression \[ l = (6 \pm 2) \times 10^{-16} \, \Omega \cdot m^2, \] we determine the electron mean free path $l = 19 \, nm$. The superconducting coherence length of pure aluminum at $T = 0$ is $\xi_0 = 1.6 \, \mu m$. Hence, the structure is a "dirty" superconductor, because $l < \xi_0$. Therefore, for this structure the temperature-dependent superconducting G-L coherence length at temperatures slightly below $T_c$ is determined from the expression \[ \xi(T) = \xi(0)(1-T/T_c)^{-1/2}, \] where $\xi(0) = 0.85(\xi_0 l)^{1/2} = 0.15 \, \mu m$. In the studied temperature range, $\xi(T) = 0.85 - 1.2 \, \mu m$. For this structure, the theoretical estimation results in the penetration depth of a nonuniform electric field into a superconductor, \[ \Lambda_E \approx 10 \, \mu m \] in the experimental temperature range. So, $\Lambda_E$ is larger than $\xi(T)$ by an order of magnitude.

Two types of four-probe measurements of voltage oscillations versus magnetic field normal to the substrate surface were performed in the structure (Fig. 1). In the first case, a rectified time-averaged direct voltage $V_{rec}(B)$ appeared in the structure biased by a sinusoidal current (without a dc component) $I_{ac}(t) = I_{ac} \sin(2\pi \nu t)$, with the amplitude $I_{ac}$ close to the critical current in the zero field $I_c$ at frequencies $\nu$ from 1 to 10 MHz at $T$ slightly below $T_c$. The experimental procedure was similar to that in Refs. 1,2,8. The $V_{rec}(B)$ voltage was measured at a slowly varying magnetic field. The $V_{rec}(B)$ voltage was the time-averaged value of the alternating voltage $V_{ac}(B,t)$ over a time interval $\Delta t$, i.e. $V_{rec}(B) = \frac{1}{\Delta t} \int_0^{\Delta t} V_{ac}(B,t) dt$. The condition $\Delta t > 20 \Delta t_I$ was valid (in the period of bias ac).

In the second case, a dc voltage $V_{dc}(B)$ was measured when a bias direct current $I_{dc}$ (without an ac component) passed through the structure. As well as $V_{rec}(B)$ curves, $V_{dc}(B)$ data were obtained at different dc values, close to the $I_c(T,B = 0)$ at $T$ slightly below $T_c$.

In addition, we measured a dc voltage $V_{dc}(I)$ as a function of the bias dc at different magnetic-field values.

III. RESULTS AND DISCUSSION

A. Main results

The measured $V_{rec}(B)$ and $V_{dc}(B)$ oscilations are shown in Figs. 2 and 3, respectively. The values of the direct $I_{dc}$, alternating $I_{ac}$ and critical currents $I_c$ together with the temperature $T$ are given in the figures. For a detailed study, fast Fourier transformations (FFT) of the $V_{rec}(B)$ and $V_{dc}(B)$ functions were obtained using 4096 uniformly of distributed points in magnetic fields from $-25$ to +25 G (Gauss = $10^{-4}$ Tesla). The Fourier spectra of these functions exhibit a variety of peaks of different magnitudes and frequencies (insets of Figs. 2 and 3).

The frequency $f$ has the meaning of a value inversely proportional to a certain period of voltage oscillations, i.e. $f = 1/\Delta B$. The great number of various frequencies in the spectra indicates the presence of various periodic magnetic-field-dependent responses of the structure.

Fundamental frequencies of the smaller and larger loops are

$$ f_S = 1/\Delta B_S = S_S/\Phi_0, \quad f_L = 1/\Delta B_L = S_L/\Phi_0, \quad (1) $$

respectively. Here $\Delta B_S$ and $\Delta B_L$ are the periods of voltage oscillations in the loops, and $S_S$ and $S_L$ are effective loop areas, with $S$ and $L$ relating to the smaller and larger loops, respectively. The expected fundamental frequencies corresponding to averaged geometrical areas of the loops are $f_{S_{\text{avg}}} = 0.31 \, G^{-1}$ and $f_{L_{\text{avg}}} = 0.56 \, G^{-1}$, corresponding to the minimum areas of the smaller and larger loops.

Figure 2 presents the rectified direct voltage $V_{rec}(B)$ versus magnetic field through the structure. The structure was biased by a sinusoidal current with the frequency $\nu = 4.023 \, kHz$ (without a dc component) and a current amplitude $I_{ac}$ close to critical $I_c$ at $T$ slightly below $T_c$.

Figure 3 shows a dc voltage $V_{dc}(B)$ through the structure biased with direct current (without any ac component) at $T$ slightly below $T_c$. The FFT spectra of both $V_{rec}(B)$ and $V_{dc}(B)$ functions exhibit peaks (labeled as $S$ and $L$) localized near the fundamental frequencies $f_S = 0.31 - 0.33 \, G^{-1}$ and $f_L = 0.56 - 0.57 \, G^{-1}$, corresponding to the smaller and larger loops. These obtained $f_S$ and $f_L$ frequencies are close to the corresponding minimum expected fundamental $f_{S_{\text{min}}}$ and $f_{L_{\text{min}}}$ frequencies of the loops [insets of Figs. 2(a), 3(b), 3(c), 3(b), and 3(a)]. This means that the effective loop areas are close to the corresponding minimum areas of the loops.

The spectral peaks labeled 2S at $f_{S_{2S}} = 2f_S$ correspond to the second higher harmonics of $f_S$ frequency [insets of Figs. 2(b), 3(c), and 3(a)].

Apart from fundamental $f_S$ and $f_L$ frequencies, the spectra exhibit the difference frequency $f_{\Delta} = f_L - f_S$ [insets of Figs. 2(a), 3(b), 3(d) and 3(a) - lower curve, 3(b) and 3(c)] and the summation frequency $f_{\Sigma} = f_L + f_S$.
earlier, \( f \) function (nonlinear coupling) between the loops. Earlier, summation frequency \( V \) peaks corresponding to the insets of Figs. 2 and 3, the symbols \( \Delta \) and \( \Sigma \) denote and summation of the fundamental frequency of the smaller loop, respectively. Symbols \( S, L, \Delta, \Sigma, \) and \( 2S \) show the spectral regions corresponding to the fundamental frequencies of the smaller and larger loops, difference and sum of these frequencies, and second higher harmonics of the fundamental frequency of the smaller loop, respectively. Symbols \( A, C, B, \) and \( D \) mark the regions corresponding to the extra low frequencies \( f_A = 0.048 \) – 0.060 G\(^{-1}\), \( f_C = 0.119 \) – 0.141 G\(^{-1}\), their difference \( f_B = f_C - f_A = 0.072 \) – 0.081 G\(^{-1}\), and their sum \( f_D = f_C + f_A = 0.167 \) – 0.201 G\(^{-1}\), respectively. Linear combinations of symbols \( S, L, \Delta, \Sigma, A, C, B, \) and \( D \) denote various combination frequencies. For example, the sum \( S + A \) near a peak shows that the peak corresponds to the summation frequency \( f_S + f_A \).

[The rest of the text and figures are not transcribed due to the limitation of space.]
loops into $V_{rec}(B)$ and $V_{dc}(B)$ oscillations vary with external parameters. The resistive response of the smaller loop $S$ is often larger than that of the larger loop $L$ [insets of Figs. 2(a)-(d) and 3(a)]. At $T$ slightly below $T_c$, a larger response of the larger loop $L$ was observed as compared with the smaller loop response $S$ [insets of Figs. 3(b) - the lower curve and 3(c)].

With changing external parameters, dips were observed instead of peaks at certain frequencies. For example, dips are observed near the $f_S$ and $f_L$ frequencies [upper curve in the inset of Fig. 3(b)] instead of peaks $S$ and $L$ [lower curve in the inset of Fig. 3(b)]. A dip $S$ at $f_S$ is also seen in the inset of Fig. 3(c). The dip in the inset of Fig. 2(d) labeled $2S$ corresponds to the second higher harmonics of the $f_S$ frequency. The inset of Fig. 3(c) exhibits a dip $\Sigma$ near the summation frequency $f_S$.

Instead of the spectral peak $\Delta$ observed at the frequency close to $f_A$ at current $I_d = 0.71$ $\mu$A [inset of Fig. 3(a)], a dip $\Delta$ appears at $I_d = 0.81$ $\mu$A [inset of Fig. 3(a), upper curve]. Both curves [Fig. 3(a)] were measured at the same temperature. With increasing current a dip $\Delta - A$ [inset of Fig. 3(a)] appears instead of the extra low-frequency spectral peak $\Delta - A$ [inset of Fig. 3(a)].

We believe that dips arise because of the nonlinear coupling of loop (wire) oscillations to give the product of two or more periodic signals with certain frequencies (amplitude modulation). In an ordinary case, an amplitude modulation is a periodic change in the amplitude of high frequency carrier oscillations by a low modulating frequency. Instead of the original frequencies, the spectrum of this product would contain the difference and summation frequencies. For example, the upper spectrum in the inset of Fig. 3(b) exhibits a dip at the frequency $f_L$ and two side satellite peaks at frequencies $f_{L-A}$ and $f_{L+A}$ instead of the peak at the $f_L$. At the same time, apart from the side satellites at $f_{L-B}$ and $f_{L+B}$, Fig. 2(a) shows a peak at the frequency $f_L$ suggesting the presence of the carrier signal in the spectrum.

So, we consider that both the $V_{rec}(B)$ and $V_{dc}(B)$ oscillations can allow us to determine the quantum magneto-resistant states of the 2-loop structure and the coupling between the quantum states.

In order to obtain a supplementary information, we measured $V_{dc}(I)$ curves (not presented here) in different magnetic fields for two opposite directions of the bias dc sweep. In low fields, a large hysteresis of the $V_{dc}(I)$ curves is observed even at temperatures sufficiently close to $T_c$ and low currents $I_d < 1$ $\mu$A. We believe that the hysteresis is due to a quasiparticle overheating that can be caused by the energy dissipation in thermally activated phase-slip centers (TAPSCs). TAPSC generates a quasiparticle imbalance and consequently, a nonuniform electric field in a nonequilibrium region with the size close to $2\Lambda_E$.

FIG. 3: (Color online) [(a)-(c)] $V_{dc}(B)$ curves measured at the parameters shown in the figure. Field values are given in $G = 10^{-4}$ T. Insets: solid lines correspond to FFT spectra of the curves, dash-dot-dot lines present spectra of other $V_{dc}(B)$ curves (not shown in the figure) measured at the parameters shown in the insets of Figs. 3(a) and 3(b). Regions corresponding to certain frequencies are marked with symbols and their combinations the way used in Fig. 2. From the insets, extra low frequencies are equal: $f_A = 0.050 - 0.062$ G$^{-1}$, $f_C = 0.120 - 0.130$ G$^{-1}$, $f_B = 0.070 - 0.088$ G$^{-1}$, and $f_D = 0.160 - 0.187$ G$^{-1}$.
B. Nonlinear coupling, combination frequencies, heating effects

The difference and sum of fundamental frequencies in FFT spectra suggest that nonlinear coupling does exist between quantum magnetic-resistive states of the successive loops. Let consider now how nonlinear coupling between loops arises and how combination frequencies appear. Nonlinear coupling between directly connected loops can be due to magnetic inductive coupling between loops and electrodynamic interaction through a bias current. However, magnetic coupling cannot explain the great number of frequencies observed in the oscillation spectra, e.g. the sum of loop fundamental frequencies. We assume that in the case of successive loops, the inductive coupling between loops would become much weaker as the distance between the loops increases. Then, the interaction through a bias current should predominate over the inductive coupling in the studied 2-loop structure.

Quantum magnetic-resistive properties of a single loop depend both on the loop circulating current and the bias current. A fraction of the bias current passing through the single loop becomes magnetic-field-dependent, oscillating with a period equal to that of the loop circulating current oscillations ($\Delta B_S = 1/f_S$ for the smaller loop and $\Delta B_L = 1/f_L$ for the larger loop). Because of the finite spatial change of superconducting order parameter, the nonlocal the $V_{rec}(B)$ or $V_{dc}(B)$ oscillations can be expected to appear on a wire part located outside of the loop at a distance equal to the nonlocal superconducting length. As shown experimentally and theoretically in Refs.13-15, the nonlocal superconducting length is close to the $\xi(T)$ length.

If the length of a wire connecting two successive loops is shorter than $\pi\xi(T)$, then it can be expected that some fraction of the oscillating current leaving one of the loops could also pass through the other loop in the 2-loop structure. As a result, current (voltage) oscillations with the fundamental frequency $f_S$ corresponding to the smaller loop could be amplitude modulated by the oscillations with the fundamental frequency $f_L$ corresponding to the larger loop and vice versa. So, the oscillating current fraction passing through both loops becomes dependent on both loop oscillating currents. As a result, a nonlinear coupling occurs between quantum magneto-resistive states of the loops. The amplitude modulation i.e. the multiplication of one oscillating signal by another oscillating signal should be expected to lead to the appearance of combination frequencies.

Note that if the distance between the two loops is several times larger than $\pi\xi(T)$, then one can hardly expect to observe the coupling between the quantum states of the loops. In the 2-loop structure, the distance between the loop centers is close to $13\xi(T)$, therefore the coupling can be expected to disappear. Nevertheless, we found the coupling.

Here we provide an explanation for the unexpected coupling. We consider that the 2-loop structure is in a nonequilibrium state with a nonequilibrium length $\Lambda_E$. Then, weak $V_{rec}(B)$ or $V_{dc}(B)$ oscillations can be expected to appear on a wire part located outside of the loop at the distance several times larger than $\xi(T)$. In addition, the weak coupling between the quantum states of two loops due to a nonlocal effect can be expected to be observed in the 2-loop structure.

In a nonequilibrium state at distances between the loop centers close to $\Lambda_E$, the coupling can be still noticeable whereas the inductive coupling between the loops should almost disappear. Therefore, it is most likely that the coupling of quantum magnetic-resistive states of both loops mainly occurs through a common bias ac (dc) owing to the nonlocal effect.

Let us speculate how the quasiparticle overheating can effect on the quantum magneto-resistive behavior of the 2-loop structure. As a result of the quasiparticle overheating (an increase in the effective local quasiparticle temperature) of the structure nonequilibrium region, an effective nonequilibrium length $\Lambda_E$ should be expected to increase. Since the effective resistance of the region is directly proportional to $\Lambda_E$, then, one would expect that the amplitude of the magneto-resistive oscillations would be increased. Moreover, we believe that the quasiparticle overheating can result in an increase in the coupling between the loops. If the overheating would be very strong and the quasiparticle temperature would exceed $T_c$ in the immediate vicinity of the $2\Lambda_E$ region, then the nonequilibrium region should be expected to transform to the normal-state region with a total size larger than $2\Lambda_E$ and with an increased resistance. Moreover, the strong overheating periodically driven by the field would result in the giant amplitude of the quantum oscillations that is due to the switching between a state close to the normal state and a state close to the superconducting state. So, we assume that the overheating not only doesn’t weaken quantum oscillations, but even can strengthen the oscillations. Moreover, it is possible that the overheating can even result in an increase in the loop coupling.

C. $V_{rec}(B)$ oscillations

The amplitude of $V_{rec}(B)$ oscillations is the function of both a bias ac amplitude $I_{ac}$ and a magnetic field. As was noted above, in low fields $V_{rec}(B)$ oscillations in a single asymmetric circular loop (in a system of identical loops in series) are of unusual character. For example, they have a giant amplitude at a bias ac amplitude $I_{ac}$ close to the critical value. In the studied structure, the behavior of $V_{rec}(B)$ oscillations is also unusual in a certain region of low fields. In this region, the oscillation amplitude is maximum and almost independent of the magnetic field [Fig. 2(d)]. Outside the region, the oscillations first drastically decrease with increasing field and then smoothly fade in high fields [Fig. 2(d)].

In low fields, the weak magnetic-field dependence of
the oscillation amplitude is probably caused by the overheating of the nonequilibrium region.

In a general case, in low magnetic fields, the \( V_{\text{rec}}(B) \) oscillations cannot be described in the framework of the simple GL quasi-one-dimensional theory if only the requirement of superconducting fluxoid quantization is used.

The maximum value of \( V_{\text{rec}}(B) \) oscillations in the structure as well as and the maximum magneto-resistive response of a single asymmetric loop are determined by that how close the resistive state of the structure part (loop) is to the midpoint of the superconducting-normal (\( S-N \)) transition.\( ^{18} \)

On transition from a state close to the superconducting state to a state more close to the midpoint of the \( S-N \) transition, the amplitude of \( V_{\text{rec}}(B) \) oscillations increases in fields close to zero (Figs. 2(a) and 2(b)). When the condition \( I_{\text{ac}} \approx I_c \) holds, these oscillations reach their maxima in fields close to zero, with the state corresponding to the midpoint of the \( S-N \) transition being realized (Fig. 2(d)). When \( I_{\text{ac}} > I_c \), a state more close to the normal realizes (Fig. 2(c)). The oscillations also reach their maxima in fields close to zero. The oscillation amplitude, however, decreases (Fig. 2(c)). The magneto-resistive response of the smaller loop \( S \) dominates over the larger loop response \( L \) at the parameters of Fig. 2. The spectral peak corresponding to the larger loop \( L \) practically disappears with increasing current (inset of Fig. 2(c)).

D. Comparison of \( V_{\text{rec}}(B) \) and \( V_{\text{dc}}(B) \) oscillations

1. Let compare the \( V_{\text{rec}}(B) \) and \( V_{\text{dc}}(B) \) oscillations and their spectra in the 2-loop structure (Fig. 1). We found both the \( V_{\text{rec}}(B) \) and \( V_{\text{dc}}(B) \) voltages measured in the structure under applied ac (with a zero dc component) and dc (with a zero ac component) respectively, can give information about the structure quantum magneto-resistive states and nonlinear coupling between the loops.

Although the \( V_{\text{rec}}(B) \) and \( V_{\text{dc}}(B) \) functions differ fundamentally, their spectra contain a similar set of frequencies. The spectra show quantitative and relative differences between periodic magnetic-field responses of the structure biased with ac (with a zero dc component) and dc (with a zero ac component) at the same values of \( T \) and bias current \( I_{\text{ac}}(I_{\text{dc}}) \). The response of the smaller loop often dominant over the one of the larger loop when \( V_{\text{rec}}(B) \) was measured (Fig. 2). During \( V_{\text{dc}}(B) \) measurements, a response of any of the loops could be dominating at certain values of \( T \) and \( I_{\text{dc}} \). Moreover, unlike \( V_{\text{rec}}(B) \) oscillations, the \( V_{\text{dc}}(B) \) ones reach their maxima in low fields at somewhat smaller currents at the same \( T \) (Figs. 2 and 3).

2. Now compare the \( V_{\text{rec}}(B) \) and \( V_{\text{dc}}(B) \) oscillations in the structure with \( V_{\text{rec}}(B) \) oscillations in the figure-of-eight-shaped structure of Ref.\( ^{17} \). Unlike the spectra of \( V_{\text{rec}}(B) \) oscillations in the figure-of-eight-shaped structure, the spectra of \( V_{\text{rec}}(B) \) and \( V_{\text{dc}}(B) \) oscillations in the structure virtually do not contain higher harmonics of loop fundamental frequencies expect for the second harmonics of \( f_s \). Moreover, the spectra of \( V_{\text{rec}}(B) \) and \( V_{\text{dc}}(B) \) oscillations in the structure exhibit extra peaks at low frequencies and peaks corresponding to extra combination frequencies. The difference is most appears due to another geometry of the structure as compared to that in Ref.\( ^{2} \).

It is seen that \( V_{\text{rec}}(B) \) and \( V_{\text{dc}}(B) \) oscillations can be dependent on the field direction. The magnetic asymmetry (parity and oddness violations of \( V_{\text{dc}}(B) \) and \( V_{\text{rec}}(B) \) functions) considerably exceeds an experimental error. The asymmetry practically disappears at high bias current values (Fig. 2(c)). A similar magnetic asymmetry was first observed in the figure-of-eight-shaped structure.\( ^{2} \) We speculate that a possible reason for the asymmetry is in an increase in the thermo-dynamical instability in the structure (the system of the TAPSCs) that can be caused by the quasiparticle overheating. A careful experimental study of the quantum oscillations in a single asymmetric loop would provide us with clues for an acceptable explanation of the asymmetry.

3. Compare the \( V_{\text{rec}}(B) \) and \( V_{\text{dc}}(B) \) oscillations and their spectra in the structure with LP oscillations in a hollow thin-walled superconducting cylinder. In a low field region, the oscillations in the structure and those in a single asymmetric loop are of unusual character. The striking difference of \( V_{\text{dc}}(B) \) oscillations in the structure from LP oscillations is clearly seen in Fig. 3(c). In a low field region, the peak-to-peak amplitude of \( V_{\text{dc}}(B) \) oscillations can reach a value close to the total voltage through the whole structure in the normal state, i.e. \( V_{\text{dc}}(B) = R_N I_{\text{dc}} \). This means that the transition of the structure as a whole from the state close to superconducting to the state close to normal and back can occur at certain field and current values. The \( V_{\text{dc}}(B) \) oscillations more resemble abrupt jumps between the superconducting and normal states as magnetic field changes.

We believe that the great amplitude of the \( V_{\text{dc}}(B) \) oscillations can be due to the periodical magnetic-field-dependent overheating of the nonequilibrium region by TAPSC up to the effective quasiparticle temperature slightly above \( T_c \) and then by a following cooling of the overheated region to the temperature slightly below \( T_c \).

IV. Conclusion

We found an unexpected interaction (nonlinear coupling) between quantum magnetic-resistive states of two superconducting loops in series with a very large distance between the loop centers close to the penetration depth of a nonuniform electric field into a superconductor \( \Delta_E > > \xi(T) \). Note that according to the present-day studies such an interaction should not be expected in the superconducting aluminum structure of two asymmetric circular loops (Fig. 1).

To detect the interaction we measured both quantum
oscillations of a rectified time-averaged direct voltage \( V_{\text{rec}}(B) \) and a dc voltage \( V_{\text{dc}}(B) \) in the 2-loop structure. The \( V_{\text{rec}}(B) \) and \( V_{\text{dc}}(B) \) curves were recorded for the structure biased only with an alternating current (without a dc component) and only with a direct current (without an ac component), respectively, with the maximum current values close to critical and \( T \) slightly below \( T_c \).

Detailed analysis of the oscillations shows that Fourier spectra of \( V_{\text{rec}}(B) \) and \( V_{\text{dc}}(B) \) functions contain the sum and difference of the loop fundamental frequencies, which implies an interaction (nonlinear coupling) between the quantum states of two loops. We believe the coupling most likely realizes through a common bias ac (dc) due to nonequilibrium nonlocal effects. The large value of the nonequilibrium length \( \Lambda_E \) to nonequilibrium nonlocal effects. The large value of the nonequilibrium length \( \Lambda_E \) in the structure allow us to observe the coupling between the quantum states of successive loops at a considerable distance between them.

Quasiparticle overheating of the structure should be expected to increase in the effective nonequilibrium length and the loop coupling. Moreover, the quasiparticle overheating would result in a great increase in the oscillation amplitude.

Earlier in Refs.17–8, it was suggested that measurements of a rectified voltage \( V_{\text{rec}}(B) \) can be used to determine quantum magneto-resistive states of an asymmetric circular loop and two directly connected asymmetric circular loops. In this work, we found that measurements of a dc voltage \( V_{\text{dc}}(B) \) can be also used to describe the quantum states of the asymmetric structure.

The spectra of \( V_{\text{rec}}(B) \) and \( V_{\text{dc}}(B) \) oscillations are complicated. Apart from the fundamental frequencies of both loops, summation and difference fundamental frequencies, the spectra contain low and extra combination frequencies. The extra combination frequencies are linear combinations of the loop fundamental frequencies and low frequencies.

The \( V_{\text{rec}}(B) \) and \( V_{\text{dc}}(B) \) oscillations in the structure radically differ from the LP oscillations.

Quasiparticle overheating of the structure should be taken into consideration when the \( V_{\text{rec}}(B) \) and \( V_{\text{dc}}(B) \) oscillations are analyzed.

Further investigations would help to elucidate the nature of the \( V_{\text{rec}}(B) \) and \( V_{\text{dc}}(B) \) oscillations.

V. ACKNOWLEDGMENTS

We thank Yu. Khanin and P. Shabelnikova for technical help. This work was financially supported in the frame of the program of fundamental investigations of DNIT RAS “Organization of computations on new physical principles” and the program of RAS Presidium “Quantum Macrophysics” (section “Mesoscopics”).

* Electronic address: kvi@ipmt-hpm.ac.ru

1. S. V. Dubonos, V. I. Kuznetsov, I. N. Zhilyaev, A. V. Nikulov, and A. A. Firsov, JETP Lett. 77, 371 (2003).
2. V. L. Gurtovoi, S. V. Dubonos, S. V. Karpiv, A. V. Nikulov, and V. A. Tulin, JETP 105, 262 (2007).
3. V. L. Gurtovoi, S. V. Dubonos, A. V. Nikulov, N. N. Osipov, and V. A. Tulin, JETP 105, 1157 (2007).
4. W. A. Little and R. D. Parks, Phys. Rev. Lett. 9, 9 (1962); M. Tinkham, Phys. Rev. 129, 2413 (1963).
5. R. Tidecks, Current-Induced Non-equilibrium Phenomena in Quasi-One-Dimensional Superconductors (Springer Tracts in Modern Physics, Vol. 121) (Springer-Verlag, Berlin-Heidelberg, 1990).
6. M. Tinkham, Introduction to Superconductivity (McGraw-Hill, New York, 1975).
7. V. I. Kuznetsov, A. A. Firsov, S. V. Dubonos, Phys. Rev. B 77, 094521 (2008).
8. V. I. Kuznetsov, A. A. Firsov, S. V. Dubonos, and M. V. Chukalina, Bulletin of the Russian Academy of Sciences: Physics 71, 1083 (2007).
9. J. E. Mooij and C. J. P. M. Harmans, New Journal of Physics 7, 219 (2005).
10. O. V. Astafiev, L. B. Ioffe, S. Kafanov, Yu. A. Pashkin, K. Yu. Arutyunov, D. Shahar, O. Cohen, and J. S. Tsai, Nature (London) 484, 355 (2012).
11. A. D. Zaikin, D. S. Golubev, A. van Otterlo, and G. T. Zimanyi, Phys. Rev. Lett. 78, 1552 (1997).
12. A. Bezryadin, C. N. Lau, and M. Tinkham, Nature (London) 404, 971 (2000); C. N. Lau, N. Markovic, M. Bockrath, A. Bezryadin, and M. Tinkham, Phys. Rev. Lett. 87, 217003 (2001).
13. N. E. Israeloff, F. Yu, A. M. Goldman, and R. Bojko, Phys. Rev. Lett. 71, 2130 (1993).
14. C. Strunk, V. Bruyndoncx, V. V. Moshchalkov, C. Van Haesendonck, Y. Bruynseraede, and R. Jonckheere, Phys. Rev. B 54, R12 701 (1996).
15. K. Yu. Arutyunov, J. P. Pekola, A. B. Pavolotski, and D. A. Presnov, Phys. Rev. B 64, 064519 (2001).
16. D. Y. Vodolazov, and F. M. Peeters, Phys. Rev. B 85, 024508 (2012).
17. B. I. Ivlev and N. B. Kopnin, Sov. Phys. Usp. 27, 206 (1984).
18. K. Yu. Arutyunov, D. A. Presnov, S. V. Lotkhov, A. B. Pavolotski, and L. Rinderer, Phys. Rev. B 59, 6487 (1999).
19. V. V. Schmidt, The Physics of Superconductors (Eds. P. Muller and A. V. Ustinov, Springer-Verlag, Berlin-Heidelberg, 1997).
20. V. I. Kuznetsov, A. A. Firsov, S. V. Dubonos, Bulletin of the Russian Academy of Sciences: Physics 71, 1081 (2007).