Cosmology and entanglement

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In this paper we present the problem of quantum to classical transition of quantum fluctuations during inflation and in particular the question of evolution of entanglement. After a general introduction, three specific very recent works are discussed in some more detail drawing some conclusion about the present status of these researches.

Keywords: entanglement, primordial fluctuations, inflationary cosmology, decoherence

I. INTRODUCTION

In the last years cosmology rapidly developed thanks to observational inputs about far supernovas, large scale structures and cosmic background radiation.

A new paradigm has emerged where big bang model has been complemented by dark matter, accelerated expansion, inflation, the so called LCDM (inflationary Cold Dark Matter model plus cosmological constant).

This new paradigm presents an interesting observational support, even if the available data would be probably considered at most as presumptive evidences in other fields, and consistency with numerical simulation on structure generation.

Nevertheless, this paradigm could be by far much weaker than usually assumed in cosmologists community since most of the very bases of it lack of a demonstrated physical theory supporting them, for example dark matter is not yet identified (and also alternatives related to a modified gravity exist), dark energy related to accelerated expansion is still rather "obscure", etc.

Even the first element of this paradigm, inflation, not only still lacks of a confirmed physical theory describing it, but also has various problems to be solved at its very basis. The most important is probably the transition from

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quantum to classical fluctuations.

In little more detail the primordial spectrum of perturbations, which represent the seed for the developing of structures in the universe, is usually assumed to be described by a classical distribution function. However, they are supposed to derive from generation of short wave length quantum fluctuations of the inflaton field. Thus, the transition from a quantum to a classical distribution must be justified. In particular, as emphasized recently, this transition must be considered keeping into account the entanglement of vacuum, indeed quantum fluctuations cannot be regarded as stochastic fluctuations as long as the system is entangled, in principle even violations of Bell inequalities could be measured in this situation. On the other hand entanglement among modes is an unavoidable consequence of evolution equation. This requires to specifically analyze the evolution of some entanglement measure and not only of other parameters used to quantify classicality, as previously done (for further criticisms to these previous works see also). As a further argument on the relevance of entanglement, not mentioned previously as far as we now, we would like also to hint at the fact that quantum field theory vacuum is expected to be in an entangled state leading to violation of some Bell inequality.

Here we will review the most recent progresses about this specific problem and present some personal consideration. We will try to avoid to enter technical details, trying to emphasize main assumptions and limits of the presented studies from a quantum mechanical point of view.

The outline of the paper is the following, in section II we introduce the concept of entanglement and its quantification, in section III we epitomize the ideas about quantum to classical systems transition. Then some recent models for quantum to classical perturbations transition are summarized: in section IV Nambu’s model, in section V Campo-Parentani one and in section VI De Unánue–Sudarsky model. Finally, in section VII we draw some conclusion.

II. SOMETHING ABOUT ENTANGLEMENT

Entanglement was defined by Schrödinger "the characteristic trait of quantum mechanics". A system of many particles is defined as entangled when the global wave function cannot be factorized in single particle wave functions. This means that the system must be considered as a whole and a measurement on one subsystem instantaneously influences the following measurements on other subsystems (it can be rigorously demonstrated that this does not allow superluminal transmission of information).

This property has been at the very basis of many of the discussions about foundations of quantum mechanics (QM) and now it is seen as a resource for developing quantum technologies or a tool for studying phase transitions.
(see and ref.s therein). Thus a huge literature exits on this field. For our purposes we limit to consider the case of entanglement of continuous variables states.

Thus, we consider $n$ bosonic modes described by annihilation operators $a_n$ satisfying usual commutation relations. Then one defines the phase space variables:

$$q_n = (a_n + a_n^\dagger)/2$$  

$$p_n = (a_n - a_n^\dagger)/(2i)$$  

$$S = (q_1, p_1, ..., q_n, p_n)$$

and a covariance matrix:

$$V_{ij} = 1/2[(\{S_i, S_j\}) - \langle S_i \rangle \langle S_j \rangle]$$

$\{,\}$ being the anticommutator.

Defined the symplectic matrix

$$\Omega = \begin{pmatrix}
0 & 1 & ... & 0 & 0 \\
-1 & 0 & ... & 0 & 0 \\
0 & 0 & ... & 0 & 1 \\
0 & 0 & ... & -1 & 0
\end{pmatrix}$$

for physical systems one has

$$V + \frac{i}{2} \Omega \geq 0$$

On the other hand a generic bipartite mixed state is separable if its density matrix $\rho$ is of the form

$$\rho = \sum_i (\rho_i^A \otimes \rho_i^B) w_i$$

where $\sum_i w_i = 1$, $w_i > 0$ and where $\rho_i^A$ $\rho_i^B$ are the density matrices of subsystems A and B. Every non-separable state is called entangled.

A general criterium for separability does not exist yet. Anyway, for a gaussian state (i.e. a state whose Wigner function is Gaussian) it is the Simon’s one

$$\Lambda \Lambda^T + \frac{i}{2} \Omega \geq 0$$
where we have defined $\Lambda = \text{diag}(1, -1, \ldots, 1, -1)$.

The matrix $AV\Lambda^T$ can be diagonalized by an appropriate symplectic transformation, e.g. to $\text{diag}(\lambda_+, \lambda_+, \lambda_-, \lambda_-)$ for a two modes gaussian state.

Considering two modes gaussian states, in terms of its symplectic eigenvalues the condition reads

$$\lambda_- \geq 1/2$$ (8)

On the other hand various measures of entanglement have been proposed\cite{17,48}. One has to mention that the hierarchy of entanglement is entanglement-measure dependent\cite{23,48} and that all of them present both advantages and disadvantages\cite{48}. Among them one that presents various useful properties is negativity, defined as (denoting $||A||_{tr} \equiv \text{Tr}[\sqrt{A^\dagger A}]$)

$$N(\rho) \equiv 1/2(||\rho^{T_A}||_{tr} - 1)$$ (9)

where $T_A$ denotes the partial transposition of subsystem $A$. Often also the related logarithmic negativity $E_N \equiv \ln||\rho^{T_A}||_{tr}$ is used. For a bipartite gaussian system in terms of symplectic eigenvalues one has:

$$E_N = -\min[\log_2(2\lambda_-), 0]$$ (10)

that is a decreasing function of the smallest partially transposed symplectic eigenvalue $\lambda_-$, that therefore completely qualifies and quantifies the entanglement of a two-mode gaussian state.

### III. HINTS AT THE DECOHERENCE PROBLEM

The problem we are considering is the transition from quantum states to classical ones during universe evolution, i.e. the developing of decoherence: this is connected to a long debated problem at the foundations of quantum mechanics pertaining the transition from a microscopic quantum world to a macroscopic classical one (the so called macro-objectivation problem), that indeed did not found yet a conclusive solution\cite{18,19,20}.

This problem derives from the fact that Schrödinger equation is linear and thus requires that a macroscopic system interacting with a superposition state originates an entangled state including the macroscopic system as well.

In order to introduce, in a very synthetic way, the question let us consider a macroscopic system described by the state $|\Phi_0 \rangle$ that interacts with the (microscopic) states $|\chi_1 \rangle$ and $|\chi_2 \rangle$. The interaction, lasting a time interval $\Delta t$, is described by a linear evolution operator $U(\Delta t)$ and leads to
\[ |\Phi_0\rangle|\chi_1\rangle \rightarrow U(\Delta t)[|\Phi_0\rangle|\chi_1\rangle] = |\Phi_1\rangle|\chi_1\rangle \] (11)

and

\[ |\Phi_0\rangle|\chi_2\rangle \rightarrow |\Phi_2\rangle|\chi_2\rangle \] (12)

where the states \( |\Phi_1\rangle \) and \( |\Phi_2\rangle \) represent the state of the macroscopic system after interaction.

If \( |\Phi_0\rangle \) interacts with the superposition state

\[ a|\chi_1\rangle + b|\chi_2\rangle \] (13)

because of linearity of the evolution equation, one has

\[ |\Phi_0\rangle[a|\chi_1\rangle + b|\chi_2\rangle] \rightarrow [a|\Phi_1\rangle|\chi_1\rangle + b|\Phi_2\rangle|\chi_2\rangle] \] (14)

that is an entangled state involving the macroscopic system as well.

Of course at macroscopic level one does not observe superpositions of different macroscopic states, thus one expects that at some point the wave function "collapses" to a defined state, i.e. only one state in the superposition survives: in the previous example the classical system is either in the situation described by \( |\Phi_1\rangle \), with probability \( |a|^2 \), or in the one described by the state \( |\Phi_2\rangle \), with probability \( |b|^2 \). However, this request must be justified more precisely.

Different answers to macro-objectivation problem have emerged. A first possibility is to split the world into a macroscopic one following classical mechanics and a microscopic one following quantum mechanics (substantially the one adopted by the Copenhagen school). However this solution, even if perfectly useful for all practical calculation of quantum processes, does not really solve the problem since it does not answer to the fundamental question of how and when the quantum world becomes classical.

Therefore, various different ideas have been considered for explaining/understanding decoherence at macroscopic level, without reaching for any of them a general consensus in the physicists community\(^ {19} \). Among them (without any pretence to be exhaustive) one can mention: the many universes models\(^ {24} \) (each state is realized in non-communicating universes), modal interpretations (one tries to attribute definite values at certain sets of observables)\(^ {25,26,37} \), decoherence and quantum histories schemes\(^ {28,29,30,31} \) (coherence is preserved and classical level, but unobservable), transactional interpretation\(^ {32} \), 'informational' interpretation (QM concerns "information" not reality)\(^ {33,34} \), dynamical reduction models (where a non-linear modification of Schrödinger equation is introduced)\(^ {35,36,37} \), reduction by consciousness (wave function collapse happens at observer level)\(^ {40} \) and many others (see for example\(^ {18,41,42,43,44,61} \) and Refs therein).
Let us just epitomize a little more in detail the schemes that can have some interest in our discussion.

In general the considerations on quantum to classical transition\cite{5,6,7,10} in cosmology adopt the idea that quantum mechanics is valid without any change at every scale. Here, the evolution is always unitary and the interacting macroscopic states remain entangled, but the interference cannot be observed at macroscopic level since the macroscopic states rapidly become orthogonal. In particular when environmental degrees of freedom are included the density matrix of the system under analysis is obtained after having traced off the environment degrees of freedom: since environment states rapidly become orthogonal one is left with a statistical mixture for the system. This is the "decoherence" framework\cite{28,29,30,31}. Various model and physical examples have been considered for demonstrating this scenario. Of course, it has the advantage that quantum mechanics is assumed to be valid as it is without any change, wave function collapse is simply eliminated from the discussion as "not needed". Somehow this scheme may be reconnected to "many worlds" one. Nevertheless, many physicists studying foundations of quantum mechanics are not satisfied by this model\cite{29,46,47}. The main objections are that:

- linear superpositions of macroscopic states occur anyway, to replace them with statistical mixtures is only a trick "valid for most of practical purposes".
- in quantum mechanics the correspondence between statistical ensembles and statistical operators is infinitely many to one. Different statistical mixtures (also containing macroscopic states superpositions) correspond to the same statistical operator.

Thus, decoherence scheme, as admitted by the same proposers\cite{48}, remain not clearly justified.

On the other hand, macro-objectivation problem simply does not exist in Hidden Variable Models (HVM)\cite{14}, since in this case the specification of the state by using state vectors is insufficient, there are further parameters (the hidden variables), which we ignore, for characterizing the physical situation: in this case the physical system is always in a well specified state univocally determined by the value of the hidden variables. One must mention that if Bell inequality tests strongly disfavor local HVM\cite{14}, they do not concern non-local HVM, as de Broglie-Bohm or Nelson’s model, or high energy (Planck) scale HVM\cite{14}.

As an example of non-local HVM, in dBB framework the hidden variable is the position of the particle that evolves with Hamilton-Jacobi equations including also a "quantum potential" related to the wave function ($\Psi = R(x,t) \cdot \exp[iS(x,t)/\hbar]$), evolving with standard quantum equation:

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0$$ (15)
The presence of the quantum potential $Q = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial R^2}$ implies that the trajectory is "instantaneously" affected by any change in the system where the wave function is non-zero (the non-locality). In principle in dBB the evolution is the same as in standard quantum mechanics, but the single particle is never in a superposition, the QM probabilistic structure derives from the ignorance of the hidden variables values (i.e. the positions), values that even in principle cannot be determined (otherwise superluminal signalling would be possible). Generalization to relativistic case is not trivial (e.g. see and ref.s therein).

Rather different are HVM at large scales. Here the idea is that at large energy (e.g. Planck) scales physical systems (including gravity) are described by a deterministic theory (non-unitary evolution), but at smaller scales we have loss of information (for dissipation): "quantum states" are equivalence classes of the deterministic states, the loss information are the hidden variables. The evolution of these equivalence classes of states becomes unitary. Present tests of Bell inequalities do not exclude these models since not pertaining the "correct" degrees of freedom. Of course in these models (for the moment far from reaching a definite theory) "decoherence" at large energy scale should be properly treated using the models themselves.

Finally, in dynamical reduction models the wave function collapse is introduced by modifying the quantum evolution equation with a non-linear term (the collapse can eventually be related to Quantum Gravity, see also Ref. for cosmological implications of modifications of QM related to QG).

For example in one of the seminal models the wave function suddenly randomly collapses according to

$$\Psi(x_1, ..., x_N) j(x - x_i)/R$$

$$j(x - x_i) = A \exp[-(x - x_i)^2/(2a)^2] |R(x)|^2 = \int dx_1...dx_N |\Psi(x_1, ..., x_N) j(x - x_i)|^2$$

where $x_i$ is the specific coordinate of the $i$th particle of the system (the one undergoing the collapse). The probability of the collapse is given, for each particle, by $1/\tau$, where $\tau$ can be fixed to be $\approx 10^{15}$ s $\approx 10^8$ years; the constant $a$ should be $a \approx 10^{-7}$ m and the collapse center $x$ is randomly chosen with probability distribution $|R(x)|^2$.

These collapse models usually comport small (not observable with present experiments) violation of energy conservation in non-relativistic versions, but these become difficult to tame in extensions to relativistic case. Obviously in these models decoherence should be kept into account by considering the modified evolution equation and could take a long time before happening.

As mentioned, almost all the studies about transition to classical perturbations are inside the decoherence scheme,
however one should keep in mind that alternative schemes as dynamical reduction models or HVM could give different scenarios. As far as we know this point was considered only in a couple of papers of Sudarsky et al.\textsuperscript{8,12} where a specific wave function collapse model was studied in relation to the generation of seeds of cosmic structures (presented in section VI).

\textbf{IV. NAMBU’S APPROACH}

As a first example of a recent study on the transition from a quantum to a classical spectrum, we consider Ref.\textsuperscript{5}. He considers a real massless scalar field $\phi$ in a de Sitter universe (i.e. with no ordinary matter content but with a positive cosmological constant), corresponding to a metric and a Lagrangian:

$$ds^2 = a(\eta)^2(-d\eta^2 + dx^2), \quad a = \frac{-1}{H\eta}$$

$$-\infty < \eta < 0, \quad L = \int d^3x \frac{1}{2\sqrt{-g}} \left( -\frac{1}{2}g^{\mu\nu}\partial_\mu \phi \partial_\nu \phi \right)$$

where $\eta \equiv \int \frac{dt}{a(t)}$ is conformal time and $H$ the Hubble constant.

In terms of the conformally rescaled variable $q = a\phi$ one has the usual equation of motion

$$q'' - \frac{a''}{a} q - \partial^2 q = 0$$

(19)

A discrete unidimensional lattice model of the scalar field, that regularizes ultraviolet divergences, is adopted. The quantized canonical variables are represented as:

$$q_j = \frac{1}{\sqrt{N}} \sum_{K=0}^{N-1} (f_k a_k + f_k^* a_{N-k}^\dagger) e^{i\theta_{kj}}$$

$$p_j = -i/\sqrt{N} \sum_{K=0}^{N-1} (g_k a_k - g_k^* a_{N-k}^\dagger) e^{i\theta_{kj}} \quad \theta_k = \frac{2\pi k}{N}$$

(20)

with the usual commutation relations $[a_k, a_j^\dagger] = \delta_{k,j}, \ldots$.

Then the Bunch-Davis\textsuperscript{63} vacuum (i.e. the one with only positive frequencies in asymptotic past) is assumed:

$$f_k = \frac{1}{2\omega_k} \left( 1 + \frac{1}{i\omega_k \eta} \right) e^{-i\omega_k \eta}, g_k = \sqrt{\frac{\omega_k}{2}} e^{-i\omega_k \eta}$$

(21)

In order to reduce to a bipartite entanglement the next step is introducing block variables pertaining two spatial regions $A, B$ (containing $n$ lattice sites):

$$q_{A(B)} = \frac{1}{\sqrt{n}} \sum_{j \in A(B)} q_j$$

(22)
Finally, the covariance matrix is evaluated and the separability criterion is applied. The fact to limit the calculation to two regions implies a non-unitary evolution.

The numerical simulation (on a 100 sites lattice) shows that the logarithmic negativity, initially different from 0 pointing out an entanglement between regions A and B (when the initial distance between the borders of the two regions is zero), always goes to zero (separable states) when grows (reaching zero essentially when the physical size of each region exceeds the Hubble horizon length).

For the sake of completeness, we can mention that also other "classicality" tests are considered, as positivity of P-function and identity of P and Wigner functions (for a discussion on the hierarchy of classicality measures for gaussian states see Ref. 64).

In summary, inside Nambu’s model a transition from quantum to classical perturbations happens. Nevertheless, this result, albeit interesting, relies on certain approximations. First of all, as the same author mentions, he considers the Bunch-Davis vacuum. Thus the results concerns this specific choice; in particular what would happen for a non-gaussian initial state remains unknown. The analysis of Lesgourgues et al. shows that for an initial non-gaussian state one has a transition to a semiclassical behaviour in the sense that the Wigner function, albeit remaining non positive in some regions, becomes concentrated near classical trajectories. This can also be rephrased by saying that the quantum field becomes a highly squeezed state, for which the non-commutativity between conjugated variables can be neglected (for analogous conclusion for gaussian state see also 10). However, this analysis is limited since it does not consider entanglement and, in absence of an entanglement criterium for non-gaussian states, this cannot be done even in principle.

Besides this point, other weak points are present in Nambu’s model. We would like just to mention:

- the non-unitary evolution due to limiting the analysis to regions A and B must be considered carefully. Entanglement evolution of systems in a bath can presents non-trivial aspects as a ”revival” of entanglement after this had disappeared.
- the simplification of a monodimensional model on lattice.

V. CAMPO-PARENTANI MODEL

First of all Campo and Parentani mention the problems of previous approaches (substantially those in ref. 10). The main identified problems are

- the use of a master equation, since it is neither clear how to properly define it (due to renormalization) nor how
to trace over ”unobservables” degrees of freedom.
- the introduction of an ”environment”.

They argue to solve these problems by using Green function method, since, on the one hand, this overcomes the use of master equations and, on the other hand, allows defining a clear intrinsic coarse graining procedure. Respect to Ref.\textsuperscript{5}, less emphasis on the necessity of strictly considering the entanglement is given, even if then they state that ”the quantum to classical-transition thus occurs when this [between modes of opposite wave vectors] entanglement is lost”.

The physical situation they consider\textsuperscript{6,7} is exactly the one presented in the previous paragraph: study of the evolution of entanglement starting from the Bunch-Davis vacuum\textsuperscript{63}. Again the covariance matrix and its evolution are evaluated and a coarse graining is defined (through a truncation of the hierarchy of Green functions). The Von Neumann entropy, $S = -tr[\rho \ln(\rho)]$ (valuing 0 for a pure state), associated with the coarse graining gaussian two modes reduced density matrices is used as measure of entanglement (in order to motivate this choice, the authors show that $S(n) \simeq ln(n)$ (n being the average particles number) at the threshold of separability deriving form Simon’s criterium. On the other hand it is shown that the criterium of separability depends on the choice of canonical variables.

They focus on curvature perturbations $\zeta$ (defined as\textsuperscript{66} $g_{ij} \simeq a^2(\eta) \delta_{ij} e^{2\zeta}$) relating the growth in entropy to the covariance matrix of $\zeta$. A little more in detail in linear approximation the evolution of $\zeta$ is determined by the Lagrangian:

$$L = \frac{1}{8\pi G} \int d^3x a^3 \left[ \frac{\partial H}{\partial t} - (\partial \zeta/\partial t)^2 - (\nabla \zeta)^2/a^2 \right]$$

(23)

The free vacuum is assumed to be the Bunch-Davis one:

$$(i\partial_\tau - q) \left( a \sqrt{\frac{-\partial H/\partial t}{H^2}} \zeta_q \right) \rightarrow 0, \quad \frac{q}{aH} \rightarrow \infty$$

(24)

where $\zeta_q$ is the Fourier mode with comoving wave vector $q$

Then the covariance matrix ($\pi_{-q}$ being the momentum conjugate)

$$C = \frac{1}{2} Tr[\rho[V, V^\dagger]] \quad V = \begin{pmatrix} \zeta_q \\ \pi_{-q} \end{pmatrix}$$

(25)

is evaluated by the Green functions of reduced density matrices (that are gaussian when using a coarse graining truncating Green function hierarchy at order 2) $G(t, t', q) \delta^3(q - q') \equiv \frac{1}{2} Tr[\rho^{red}_{q_{-q}}(\zeta_q(t), \zeta_{-q'}(t'))]$, being $C_{1,1} = G(t, t, q), \ldots$.

The entropy of $\rho^{red}_{q, -q}$ is related to the determinant of the covariance matrix by $S = ln[det(C)]$. 
After having derived the equation for the evolution of $S$, this is applied to different situations. One considered case is multifield inflation, in particular the two field model:

$$L = \int d^3x \sqrt{-g} \left[ \frac{R}{16\pi G} - \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi \partial_\nu \phi + e^{2b(\phi)} \partial_\mu \chi \partial_\nu \chi) - V(\phi, \chi) \right]$$

(26)

Defined the field $\sigma$

$$\partial_t \sigma^2 = \partial_t \phi^2 + e^{2b} \partial_t \chi^2$$

(27)

with ($\Phi$ being the gravitational potential)

$$ds^2 = -(1 + 2\Phi) dt^2 + a^2(1 - 2\Phi) \delta_{ij} dx^i dx^j$$

(28)

one has

$$\zeta = \Phi + \frac{H}{\partial_t \sigma} \delta \sigma$$

(29)

Another analyzed case is a single inflaton field model plus a matter field $\sigma$ (in a fundamental representation of $O(N)$), whose quadratic part of the Lagrangian describing the free evolution is:

$$L = \frac{1}{2} \sum_{n=1}^{N} \int d^3x a^3 \left[ \frac{d\sigma_n}{dt}^2 - \frac{1}{a^2} (\nabla \sigma_n)^2 \right]$$

(30)

for a minimal coupling to gravity.

Without entering into the details of the calculation, the conclusions are that in the first case entropy grows at a high rate, both in presence of an anomalous kinetic term ($b(\phi) \neq 0$) or in its absence ($b(\phi) = 0$), reaching at the end of inflation a value largely above the separability threshold $S_T = 4N_{end}$ ($N_{end}$ being the number of e-folds from horizon exit to the end of inflation). Besides the authors argue that this result can be extended also to inflationary models with many fields at least when the kinetic term is in the canonic form. On the other hand the single field model does not show evidences of decoherence since von Neumann entropy of reduced density matrix remains small.

For the sake of completeness, we can also mention that, similarly to Ref.\textsuperscript{5}, other criteria of classicality are considered as well, as broadness of Wigner function (i.e. the Wigner function is broad enough to be the Husimi representation of some normalizable density matrix), existence of a P-representation, etc. showing how these lead to different thresholds: this induces the authors to state that there is no intrinsic threshold of decoherence at which quantum–to–classical transition occurs.

For what concerns the generality of these results, one has to mention that the first two considerations about Nambu’s model (on gaussian vacuum and coarse graining) completely keep their validity here as well.
VI. DE UNÁNUE–SUDARSKY MODEL

In Ref. 8 De Unáñue–D. Sudarsky consider a specific wave function collapse model, improving a precedent scheme 12.

The starting point is always to consider inflation with scalar field with a Lagrangian like Eq. 18, a metric like Eq. 28 and a Bunch-Davis vacuum Eq. 21.

Then a collapse is introduced: before the collapse there are no metric perturbations, it is only after the collapse that the gravitational perturbations appear, i.e the collapse of each mode represents the onset of inhomogeneity and anisotropy at the scale represented by the mode.

Of course introducing the collapse solves completely the problem of quantum to classical transition: after the collapse the Universe is in a defined single state (and of course entanglement has disappeared).

The specific collapse a certain time in a state $|\Omega\rangle$ is such that one finds for the expectation values of the variable defined in Eq. 20

$$
\langle q^R_k \rangle_{\Omega} = x_k \Lambda_k \cos \Theta_k, \quad \langle p^R_k \rangle_{\Omega} = x_k \Lambda_k k \sin \Theta_k
$$

(31)

where $x_k$ is a random variable, $\Lambda_k$ is the major semi-axis of the ellipse corresponding to the boundary of the region in phase space where the Wigner function is larger than 1/2 its maximum value, $\Theta_k$ is the angle between that axis and the $f^R_k$ axis.

From this expressions one calculates the fluctuations $\langle \delta \phi_k \rangle$. The perturbation spectrum is then compared with what needed by present numerical simulations of evolution of structures. The authors remark that this analysis is just preliminary and more refined studies are needed. Their results show how the power spectrum is sensible to the choice of the collapse model, being in agreement with what expected for certain choices and clearly in disagreement for others: thus different models could be selected by comparison with observational data.

VII. CONCLUDING REMARKS

In conclusion our opinion is that a clear demonstration of the transition to classical from quantum perturbations at the end of inflation is still lacking.

Since, for the considerations expressed in the introduction, one should unavoidably consider entanglement of vacuum, we have limited a specific analysis to the most recent papers where this was done, even if most of the considerations expressed about these models also pertain previous studies 10. A first point to be mentioned is that the results are model dependent. Then, all of them consider gaussian states, but this is a limiting hypothesis that has no motivation.
These two problems already sustain our conclusion. But the situation is even worse than that, since in almost all the studies about this phenomenon, a decoherence model for quantum-classical transition was assumed, but in principle completely different results could derive if other models would be applied (as dynamical reduction one or HVM). A first attempt in this sense was presented in the last section, where a scheme considering a specific wave function collapse model was presented. However, of course this scheme is just a first step of this possible analysis and it is pertaining a very specific framework.

Thus, in summary, in our opinion, the transition to classical from quantum perturbations remains (and probably will remain for a long time up to when we will not dispose of a well proven theory at the scale of inflation) one of the unsolved problem of present cosmological models.

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67. some doubts remaining due to unsolved detection loophole (or in the case of ions being the measurements not separated)\(^\text{14}\).
68. see Ref.\(^\text{49}\) and refs therein for some alternative point of view.
69. A similar situation happens also for ”pre-quantum” theory of Adler.\(^\text{62}\).