Data-Driven Predictive Control With Switched Subspace Matrices for an SCR System

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ABSTRACT
Selective catalytic reduction (SCR) systems are distributed systems with strong time-varying parameter characteristics such that an accurate model for it is difficult to establish. Its control task simultaneously achieving high NO\textsubscript{x} conversion efficiency and low NH\textsubscript{3} slip is a typical multi-objective and multi-constraint problem, which is suitable to be solved in the framework of the model predictive control (MPC). However, how to find a data-driven identification method based on the dynamic characteristics of an SCR system and a corresponding MPC method for satisfying its emission requirements remain a formidable challenge. The sufficient identification for the traditional identification method with fixed subspace model requires an excessively high order subspace matrix, such that a degradation in real-time performance is caused and the generality of the method under non-identification conditions is limited. In this paper, utilizing the transient data of the SCR system under the WHTC cycle, a novel identification method for some lower order subspace matrices excited by the segmented data referring to the dynamic of the ammonia coverage ratio is established. A corresponding predictive controller with the switched subspace matrices according to working conditions is designed in order to further improve its real-time performance, generality and robustness. The simulation results show that under the identification condition the proposed predictive controller compared to the traditional method can improve the emissions of NO\textsubscript{x} and NH\textsubscript{3}, that under the non-identification condition the proposed predictive controller can also improve the emissions and its optimization effects have better robustness to uncertainties of the transient cycle, and that the proposed predictive controller saves a significant computation time.

INDEX TERMS
Subspace identification, data-driven predictive control, SCR systems.

I. INTRODUCTION
Selective catalytic reduction (SCR) systems are the most promising aftertreatment technologies for reducing the NO\textsubscript{x} emission [1]. The working principle of SCR systems is that NO\textsubscript{x} exhausted by engines can be reduced by ammonia (NH\textsubscript{3}) generated from a 32.5% aqueous urea solution (AdBlue), and the excess NH\textsubscript{3} is also a harmful emission [2]. Therefore, the key to controlling SCR systems is how to improve the NO\textsubscript{x} conversion rate while ensuring that the NH\textsubscript{3} slip does not exceed the limit. Researchers introduce ammonia coverage rates in the modeling process and balance the NO\textsubscript{x} and NH\textsubscript{3} emissions by optimizing them [3]. That is a typical multi-objective problem with constraints, which is suitable to be solved under the model predictive control (MPC) framework [4]. The working processes of SCR systems involve many complex chemical reactions and transfer processes of heat and mass. SCR systems are distributed systems with strong time-varying parameter characteristics such that an accurate system model for it is difficult to establish [5], [6].

In recent years, large amounts of data can be obtained in modern industries with the development of the computer technology. Modeling through data-driven methods not only becomes possible but also new challenges both in theory...
and applications [7]. These data-based methods get around the difficulty of mechanism modeling. In addition, the MPC strategy takes time-domain constraints into account explicitly and deals with multiple objectives in a somewhat optimal sense, so it has a great potential for the control of an automated machinery [8]. In [9], a data-driven predictive control algorithm with elegantly combining data-driven subspace identification and MPC is proposed.

The data-driven technology has been widely used in industrial occasions such as multidimensional blast furnace system, wind power generation, vapor compression refrigeration cycle and superheated steam temperature system in power plant. In [10], a data-driven model based on Volterra series is developed for multidimensional blast furnace system. Three different low-order Volterra filters are designed to effectively describe the evolution of the studied silicon sequence. In [11], based on the data-driven technology, a predictive controller of wind turbine torque is designed. The engine torque is controlled by the prediction of wind force to change the rotation speed, so as to obtain the best power coefficient and maximize the power. And different data mining algorithms are used to improve the accuracy and robustness properties of the wind speed model. In [12], a controller of vapor compression refrigeration cycle system driven by multivariable data is proposed, which controls the opening of expansion valve to realize the overheating of evaporator and constant pressure. The controller shows satisfactory robustness to the disturbance caused by the change of temperature variable. In [13], a pre-set adaptive predictive controller using the data-driven technology for superheated steam temperature system in power plant is designed, which shows strong anti-interference ability and desirable robustness in off-design system.

Moreover, the data-driven technology has been used in many aspects of vehicle mechanical components control. In [14], a torque coordination controller of hybrid electric vehicle (HEV) drive system based on a data-driven method is proposed. Excellent quality in the dynamic process of HEV mode switching from pure electric to hybrid drive is achieved by the controller, which can significantly improve the driver comfort and economy of HEV. In addition, the latest vehicle operation data collected online can be used to update the subspace prediction model for ensuring the higher performance of the proposed control strategy for a long time. In [15], an air-fuel ratio controller for the engine is designed by using the data-driven technology to achieve small air-fuel ratio error, reasonable injection duration and injection pulse width increment. It remains effective in real-time simulation experiments with interference. In [16], a combustion controller for the diesel engine is designed by using the data-driven technology with the system matrix put forward from the kernel function to realize the real-time coordinated control of power and emission. In [17], a data-driven predictive controller for the starting process of automatic manual transmission (AMT) vehicle is designed to obtain satisfied startup performance by controlling slip speed of the clutch. In [18], a shift controller is designed with the data-driven predictive control technology to improve the shift quality of dual clutch transmission (DCT) vehicles. The controller is robust to the disturbance caused by the uncertainty of working conditions.

Additionally, researchers have made considerable efforts to solve the problems of the modeling and optimal control for SCR systems utilizing the data-driven technology during recent years. In [19], a urea dosing controller of the SNCR denitration system based on the data-driven technology is designed, which strictly controls the concentration of NO\textsubscript{x} at the outlet of the SNCR. The experimental results show that the fluctuation of the concentration of NO\textsubscript{x} at the outlet is relatively gentle, and the ammonia dosing amount and ammonia escape amount are generally reduced. In [20], a data-driven method is used to establish the NO\textsubscript{x} model at the outlet of an SCR system, and an optimal control strategy of urea dosing is proposed, which can ensure the denitration efficiency and reduce the ammonia escape when the working conditions change. In [21], a fixed subspace matrix and a data-driven model using the engine data of the ETC transient cycle are identified, and a urea dosing controller is designed. Thus, Utilizing the experimental data of commercial SCR systems, a data-driven subspace model can be established to avoid the problem of mechanistic modeling and a corresponding MPC method can also solve their problem of emission optimization.

Overall, there are many researches on the data-driven technology applied to SCR systems, most of which are based on the traditional identification method with fixed subspace model, and the dynamic characteristics of SCR systems are rarely considered in the identification process. Compared to the traditional identification method and corresponding MPC method, there are several challenges in the development of a data-driven predictive controller for SCR systems:

1). How to find a data-driven identification method based on the dynamic characteristics of an SCR system and a corresponding MPC method for satisfying its emission requirements remain a formidable challenge.

2). The sufficient identification for the traditional identification method with fixed subspace model requires an excessively high order subspace matrix, such that a degradation in real-time performance is caused and the generality of the method under non-identification conditions is limited.

To address the associated challenges as mentioned above, the primary merits of the design procedure cover the following points:

1). A novel identification method for some lower order subspace matrices excited by the segmented data referring to the dynamic of the ammonia coverage ratio is established.

2). A corresponding predictive controller with the switched subspace matrices according to the working conditions is designed in order to further improve the real-time performance, generality and robustness.

The rest of this paper is organized as follows. Section 1 explains the problem statement of the SCR system. Section 2 presents the traditional data-driven predictive
controller with fixed subspace identification model. Section 3 primarily elaborates the proposed data-driven predictive controller with segmented subspace identification model and shows its optimization results. Section 4 presents the conclusion.

II. PROBLEM STATEMENT OF SCR SYSTEM
SCR systems are complex dynamic systems of chemical reaction with highly nonlinear properties. Their chemical reaction primarily includes the following several processes. The AdBlue (Urea, ) is pyrolyzed to produce ammonia (NH₃, in) and isocyanate (HNCO) at the upstream of an SCR system. Inside the catalyst, one part of NH₃, in is adsorbed on the catalyst substrate (NH₃(ads)), while the other part remains in the gaseous phase (NH₃(g)). Both parts can be transformed into each other, i.e.,

\[
\text{NH}_3(\text{ads}) \leftrightarrow \text{NH}_3(\text{g}). \tag{1}
\]

The NH₃(ads) can catalytically react with the NOₓ(NOₓ, in) generated by the engine, and then they are converted into N₂ and H₂O. When the temperature is higher than 200 degrees Celsius, the SCR system is governed by an Eley-Rideal mechanism [22] consuming NH₃(ads) and gaseous NOₓ.

\[
4\text{NH}_3(ads) + 4\text{NO}_x + z\text{O}_2 \rightarrow 4\text{N}_2 + 6\text{H}_2\text{O}. \tag{2}
\]

The NH₃(ads) can also be oxidized to N₂ and H₂O by the following reaction:

\[
4\text{NH}_3(ads) + 3\text{O}_2 \rightarrow 2\text{N}_2 + 6\text{H}_2\text{O}. \tag{3}
\]

Proceeding through the chemical reaction described above, the remaining NOₓ (NOₓ,out) and NH₃ (NH₃,out) are discharged as pollutants. The ratio of the amount of NH₃(ads) and the ammonia storage capacity is defined as the ammonia coverage ratio \(\theta_{NH3}\).

A schematic diagram of problem statement for an SCR system is shown in Fig. 1. For the SCR system, the primary inputs are NOₓ, in, NH₃, in, EFₘ (the exhaust gas mass flow) and \(T_{engine}\) (the exhaust gas temperature of engine), and the primary outputs are NOₓ,out and NH₃,out, and the primary internal state is \(\theta_{NH3}\). The control task of the SCR system is to simultaneously achieve high NOₓ conversion efficiency (the ratio of (NOₓ, in − NOₓ, out) to NOₓ, in) and low ammonia slip NH₃,out. The task can be achieved through the optimization and tracking control of \(\theta_{NH3}\), which is used to balance the NOₓ and NH₃ emissions [2, 3].

![FIGURE 1. Schematic of the problem statement for the SCR system.](image)

III. TRADITIONAL DATA-DRIVEN PREDICTIVE CONTROLLER
In this section, a traditional data-driven predictive controller with fixed subspace identification model is designed and verified for an SCR system. Firstly, a subspace identification model and an incremental subspace identification model are presented. Secondly, a traditional data-driven predictive controller is proposed. Thirdly, a few simulations are performed to verify the traditional data-driven predictive controller with fixed subspace identification model under the WHTC cycle. Finally, the computation times of the data-driven predictive controllers between the different order subspace matrices are compared and discussed.

A. SUBSPACE IDENTIFICATION MODEL
According to the problem statement of SCR systems, a state-space model with discretized form considered for deriving the subspace matrices is presented as follows:

\[
\begin{align*}
\dot{x}(k+1) &= Ax(k) + B_u u(k) + K d(k), \\
y_c(k) &= C_x x(k), \\
y_b(k) &= C_p x(k),
\end{align*}
\tag{4}
\]

which is a system with four inputs and two outputs, the control input \(u(k) = \text{NH}_3,\text{in}\), the control output \(y^c(k) = \text{NO}_x,\text{out}\), the system constrained output \(y^b(k) = \text{NH}_3,\text{out}\) and the disturbance input \(d(k)\) is constructed as follows:

\[
d(k) = \left[ d^1(k) d^2(k) d^3(k) \right]^T = \left[ \text{NO}_x,\text{in} \ EF_m \ T_{\text{engine}} \right].
\]

The state of the system is \(x(k) \in \mathbb{R}^n\) (\(n\) is the state order), and the matrices \(A, B_u, K, C, C_p\) are the state-space matrices. The data of \(u(k), d(k), y^c(k)\) and \(y^b(k)\) for \(k \in \{0, 1, \ldots, 2i+j-1\}\) are collected through the high-precision model of an SCR system. The prediction horizon and the control horizon are denoted as \(N_p\) and \(N_u\), satisfying \(N_p \geq N_u\). The number of counting elements are denoted as \(j\) with \(i = N_p\). The data block Hankel matrices \(U_p, U_f, Y_p\) and \(Y_f\) are constructed as follows:

\[
U_p = \begin{bmatrix}
  u_0 & u_1 & \cdots & u_{i-1} \\
  d_0^1 & d_1^1 & \cdots & d_{j-1}^1 \\
  d_0^2 & d_1^2 & \cdots & d_{j-1}^2 \\
  d_0^3 & d_1^3 & \cdots & d_{j-1}^3 \\
  u_1 & u_2 & \cdots & u_j \\
  \vdots & \vdots & \ddots & \vdots \\
  u_{i-1} & u_i & \cdots & u_{i+j-2} \\
  d_{i-1}^1 & d_i^1 & \cdots & d_{i+j-2}^1 \\
  d_{i-1}^2 & d_i^2 & \cdots & d_{i+j-2}^2 \\
  d_{i-1}^3 & d_i^3 & \cdots & d_{i+j-2}^3 \\
\end{bmatrix}_{4i \times j},
\]
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The solution is the orthogonal projection of the row space of \( Y_f \) into the row space spanned by \( W_p \) and \( U_f \) as [25]:

\[
L_{wu} = \begin{bmatrix} L_w & L_u \end{bmatrix} = Y_f \begin{bmatrix} W_p & U_f \end{bmatrix}^\dagger = Y_f \begin{bmatrix} W_p^T & U_f^T \end{bmatrix} \begin{bmatrix} W_p & U_f \end{bmatrix} \begin{bmatrix} W_p^T & U_f^T \end{bmatrix}^{-1},
\]

where \( \dagger \) represents the Moore–Penrose pseudoinverse. In terms of (6) and (7), we can obtain subspace matrices \( L_w \) and \( L_u \) as follow:

\[
L_w = [L_{wu}(:,1), \ldots, L_{wu}(:,N_p \times 6)]_{2N_p \times 6N_p},
\]

\[
L_u = [L_{wu}(:,N_p \times 6 + 1), \ldots, L_{wu}(:,N_p \times 10)]_{2N_p \times 4N_p}.
\]

In order to simplify the derivation of the predictive controller and reduce the computation time, the first column of \( \hat{Y}_f \) is only used to predict the future dynamics of the system:

\[
\hat{y}_f = L_w w_p + L_u u_f
\]

where

\[
w_p = \begin{bmatrix} y_p^c & y_p^b \end{bmatrix},
\]

\[
u_p = \begin{bmatrix} u(k-N_u+1) & d^1(k-N_u+1) \\
                        d^1(k-N_u+1) & d^2(k-N_u+1) \\
                        d^2(k-N_u+1) & d^3(k-N_u+1) \\
                        d^3(k-N_u+1) & u(k-N_u+2) \\
                        u(k-N_u+2) & d^1(k-N_u+2) \\
                        d^1(k-N_u+2) & d^2(k-N_u+2) \\
                        d^2(k-N_u+2) & d^3(k-N_u+2) \\
                        d^3(k-N_u+2) & u(k+N_u) \\
                        u(k+N_u) & d^1(k+N_u) \\
                        d^1(k+N_u) & d^2(k+N_u) \\
                        d^2(k+N_u) & u(k+N_u) \\
                        u(k+N_u) & d^1(k+N_u) \\
                        d^1(k+N_u) & d^2(k+N_u) \\
                        d^2(k+N_u) & d^3(k+N_u) \\
                        d^3(k+N_u) & u(k+N_{p-1}) \\
                        u(k+N_{p-1}) & d^1(k+N_{p-1}) \\
                        d^1(k+N_{p-1}) & d^2(k+N_{p-1}) \\
                        d^2(k+N_{p-1}) & u(k+N_{p-1}) \\
                        u(k+N_{p-1}) & d^1(k+N_{p-1}) \\
                        d^1(k+N_{p-1}) & d^2(k+N_{p-1}) \\
                        d^2(k+N_{p-1}) & d^3(k+N_{p-1}) \\
                        d^3(k+N_{p-1}) & u(k+N_p) \end{bmatrix}_{4N_u \times 1},
\]

\[
u_f = \begin{bmatrix} y_f^c & y_f^b \end{bmatrix},
\]

\[
\hat{y}_f = \hat{L}_w \hat{w}_p + \hat{L}_u \hat{u}_f + \hat{L}_d \hat{d}_f.
\]

B. INCREMENTAL SUBSPACE IDENTIFICATION MODEL

In order to guarantee zero steady-state error for the tracking of the system output to the reference input, an integral form is introduced. The Eq. (9) is transformed into an incremental form:

\[
\Delta \hat{y}_f = \Delta \hat{L}_w \Delta \hat{w}_p + L_u^p \Delta d_p + \Delta \hat{L}_u \Delta \hat{u}_f + L_d^p \Delta d_f.
\]
Assuming that the future disturbance input $d_f$ is unchanged in the control horizon, the incremental form of the predicted output sequence at the $k$ sampling time is:

$$
\Delta \hat{y}(k) = \hat{L}_{df} (1 : N_P, :) \begin{bmatrix} \Delta y_p \\ \Delta \hat{u}_p \end{bmatrix} + L^p_{df} (1 : N_P, :) \Delta d_p + L^f_{df} (1 : N_P, :) \Delta d_f + L^f_{ad} (1 : N_P, 1 : N_u) \Delta \hat{u}_f (k),
$$

where

$$
\hat{L}_{df} = \Pi \hat{L}_{w}, \quad L^p_{df} = \Pi L^p_{d}, \quad L^f_{df} = \Pi L^f_{d}, \quad \hat{L}_{ad} = \Pi \hat{L}_{u}.
$$

$$
\Delta y_p = \begin{bmatrix} \Delta y(k - N_P + 1) \\ \Delta y(k - N_P + 2) \\ \vdots \\ \Delta y(k - 1) \end{bmatrix}^T,
$$

$$
\Delta \hat{u}_p = \begin{bmatrix} \Delta u(k - N_P) \\ \Delta u(k - N_P + 1) \\ \vdots \\ \Delta u(k - 1) \end{bmatrix}^T,
$$

$$
\Delta d_p = \begin{bmatrix} \Delta d(k - N_P + 1) \\ \Delta d(k - N_P + 2) \\ \vdots \\ \Delta d(k - 1) \end{bmatrix}^T,
$$

$$
\Delta d_f = \begin{bmatrix} \Delta d(k) 0 \cdots 0 \end{bmatrix}^T,
$$

$$
\Pi = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 1 & 0 & \cdots & 1 \end{bmatrix}_{2N_P \times 2N_P}.
$$

To design a data-driven predictive controller with constraints, the incremental predicted outputs shown in Eq. (11) are accumulated. The predicted control output $\hat{y}_c$ and the predicted constraint output $\hat{y}_b$ are further extracted from $\hat{y}$ as:

$$
\begin{align*}
\Delta \hat{y}_c(k) &= \Gamma_c \Delta \hat{y}(k) \\
\Delta \hat{y}_b(k) &= \Gamma_b \Delta \hat{y}(k),
\end{align*}
$$

where

$$
\Gamma_c = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix}_{N_P \times 2N_P},
$$

$$
\Gamma_b = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}_{N_P \times 2N_P}.
$$

In Eq. (12), the predicted control output and the predicted constraint output are

$$
\begin{align*}
\hat{y}_c(k + 1) &= F_c^c + S_c^c \Delta \hat{u}_p(k) \\
\hat{y}_b(k + 1) &= F_b^b + S_b^b \Delta \hat{u}_f(k),
\end{align*}
$$

where

$$
\begin{align*}
F_c &= y_c(k) + \Gamma_c \hat{L}_{ad} (1 : N_P, :) \begin{bmatrix} \Delta y_p \\ \Delta \hat{u}_p \end{bmatrix} + \Gamma_c L^p_{df} (1 : N_P, :) \Delta d_p + \Gamma_c L^f_{df} (1 : N_P, :) \Delta d_f + \Gamma_c L^f_{ad} (1 : N_P, 1 : N_u) \Delta \hat{u}_f (k),
\end{align*}
$$

$$
\begin{align*}
F_b &= y_b(k) + \Gamma_b \hat{L}_{ad} (1 : N_P, :) \begin{bmatrix} \Delta y_p \\ \Delta \hat{u}_p \end{bmatrix} + \Gamma_b L^p_{df} (1 : N_P, :) \Delta d_p + \Gamma_b L^f_{df} (1 : N_P, :) \Delta d_f + \Gamma_b L^f_{ad} (1 : N_P, 1 : N_u) \Delta \hat{u}_f (k),
\end{align*}
$$

$$
\begin{align*}
S_c^c &= \Gamma_c \hat{L}_{ad} (1 : N_P, 1 : N_u) \Delta \hat{u}_f (k),
\end{align*}
$$

$$
\begin{align*}
S_b^b &= \Gamma_b \hat{L}_{ad} (1 : N_P, 1 : N_u) \Delta \hat{u}_f (k),
\end{align*}
$$

where $F_c$ and $F_b$ are the system free response, and $S_c^c$ and $S_b^b$ are the system control response.

C. TRADITIONAL DATA-DRIVEN PREDICTIVE CONTROLLER WITH INCREMENTED SUBSPACE IDENTIFICATION MODEL

In this subsection, a data-driven predictive controller with incremental subspace identification model is developed to account for the emission objectives and constraints and optimize $\text{NH}_3_{in}$ for the SCR system, as shown in Fig. 2.

![Figure 2](image-url)

**FIGURE 2.** Schematic of data-driven predictive controller for optimizing $\text{NH}_3_{in}$.

The key to controlling SCR systems is how to improve the NO conversion rate while ensuring that the NH3 slip does not exceed the limit. Thus, the optimization problem with the input and output constraints of the SCR system is described as follows:

$$
\min_{\Delta \hat{u}_f(k)} J \left( \hat{y}_c(k), \Delta \hat{u}_f(k), N_P, N_u \right),
$$

where

$$
J = \begin{bmatrix} R_e(k + 1) - F_c - S_c \Delta \hat{u}_f(k) \end{bmatrix}^T \times \begin{bmatrix} R_e(k + 1) - F_c - S_c \Delta \hat{u}_f(k) \end{bmatrix} + \Delta \hat{u}_f(k)^T (\Lambda I) \Delta \hat{u}_f(k),
$$

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The optimization function $J$ consists of two parts: the predictive control output $y_{p}^{c}(k + 1)$ tracks the desired reference value ($R_{c}(k + 1)$); the change rate of the control action $\Delta u(k)$ is as small as possible. Because of the physical constraints of the SCR system, the range of control input $u(k)$ and constraint output $y_{c}^{b}(k)$ are: $u_{\min} = \Delta u_{\min} = 0 \text{ mol/s}$, $u_{\max} = 0.003 \text{ mol/s}$, $\Delta u_{\max} = 0.0005 \text{ mol/s}$, $y_{\min}^{b} = 0 \text{ mol/s}$ and $y_{\max}^{b} = 0.00002 \text{ mol/s}$. The weight factor matrices $\Gamma_{u}$ and $\Gamma_{y}$ are given to trade off the two objectives. The ratio of weight coefficient $\Gamma_{u}$ and $\Gamma_{y}$ is $\Lambda$.

Since problem (15) is a multi-objective optimization problem with constraints [26], [27], the efficient set method can be used to solve it [28]. In order to facilitate the solution by the efficient set method, the relevant constraints can be organized as:

$$A\Delta \tilde{u} \leq b,$$

where

$$A = \begin{bmatrix}
-L_{1} \\
-R \\
-S_{b}^{b} \\
\text{I} \\
\text{R} \\
\text{S}_{b} \\
\end{bmatrix}_{6N_{p} \times N_{u}}, \quad b = \begin{bmatrix}
-L_{1} \\
-L_{2} \\
-L_{3} \\
U_{1} \\
U_{2} \\
U_{3} \\
\end{bmatrix}_{6N_{p} \times 1},$$

$$L_{1} = \begin{bmatrix}
\Delta u_{\min}(k) & \cdots & \Delta u_{\min}(k + q + 1) \\
\Delta u_{\max}(k) & \cdots & \Delta u_{\max}(k + q + 1) \\
\end{bmatrix}_{T},$$

$$U_{1} = \begin{bmatrix}
\Delta u_{\min}(k) & \cdots & \Delta u_{\max}(k + q + 1) \\
\Delta u_{\max}(k) & \cdots & \Delta u_{\max}(k + q + 1) \\
\end{bmatrix}_{T},$$

$$U_{2} = \begin{bmatrix}
\text{I} \\
\text{R} \\
\text{S}_{b} \\
\end{bmatrix}_{T} - F^{b},$$

$$U_{3} = \begin{bmatrix}
\text{I} \\
\text{R} \\
\text{S}_{b} \\
\end{bmatrix}_{T} - F^{b},$$

$$I = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1 \\
\end{bmatrix},$$

$$R = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
1 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
1 & 1 & \cdots & 1 \\
\end{bmatrix}.$$
D. IDENTIFICATION OF SUBSPACE IDENTIFICATION MODEL AND VERIFICATION OF THE TRADITIONAL DATA-DRIVEN PREDICTIVE CONTROLLER

In this section, an identification and verification of subspace identification model is presented and a few simulations are performed to verify the emission optimization effects of the traditional data-driven predictive controller with fixed subspace identification model. The simulation is conducted in MATLAB/Simulink environment utilizing an Intel(R) Core(TM) i7-10700 CPU (2.90GHz).

1) IDENTIFICATION AND VERIFICATION OF SUBSPACE IDENTIFICATION MODEL

The identification data of the SCR system (as shown in Fig. 3) under the WHTC cycle with cold-initial engine are adopted, which come from the high-precision model in our previous work [29]. They consist of four inputs (NO\textsubscript{x, in}, NH\textsubscript{3, in}, T\textsubscript{engine}, EF\textsubscript{M}) and two outputs (NO\textsubscript{x, out}, NH\textsubscript{3, out}), which has 1650 seconds with a sampling time of 0.1 seconds. And, the two outputs are calibrated by the control strategy of the original ECU in accordance with the China/Euro VI emission regulations.

The normalized input and output data are used to derive the \( L_u \) and \( L_w \) matrices by the identification method described above, that is, the subspace identification model is obtained. As shown in Fig. 4, Two sets of matrices \( L_u = 50 \times 100 \), \( L_w = 50 \times 150 \) and \( L_u = 80 \times 160 \), \( L_w = 80 \times 240 \) are derived to compare the predicted accuracy.

As shown in Fig. 5, when the input data are the same, the comparison between the identification data (Real marked by red solid line) and the predicted data of subspace identification model is performed. When the dimension of \( L_u \) is \( 50 \times 100 \) and \( L_w \) is \( 50 \times 150 \), the predicted accuracy of the subspace identification model for NO\textsubscript{x, out} and NH\textsubscript{3, out} are 90.17\% and 74.23\%, respectively. When the dimension of \( L_u \) is \( 80 \times 160 \) and \( L_w \) is \( 80 \times 240 \), the predicted accuracy of the subspace identification model for NO\textsubscript{x, out} and NH\textsubscript{3, out} are 97.84\% and 96.23\%, respectively. Its predicted accuracy can satisfy the test requirements of the control system.
2) VERIFICATION OF THE TRADITIONAL DATA-DRIVEN PREDICTIVE CONTROLLER

In this subsection, a few simulation comparisons of the NOx and NH3 emissions are performed under the WHTC cycle with cold-initial and hot-initial engine, respectively. The emission optimization effects of the traditional data-driven predictive controller with $L_u = 80 \times 160$ and $L_w = 80 \times 240$ are verified through the comparisons with the identification data of NOx_out and NH3_out controlled by the original ECU. As shown in Fig. 6, the emission comparisons between the identification data (NOx_out - exp and NH3_out - exp) and optimized data (NOx_out - opt and NH3_out - opt) controlled by the traditional data-driven predictive controller are performed under the WHTC cycle with cold-initial engine.

As compared to those of the identification data controlled by the original ECU, the average NOx and NH3 emissions of the traditional predictive controller are improved by 75.10% and 66.40%, respectively. The worst computation time of the traditional data-driven predictive controller is approximately 0.016 s with an average 0.0028 s. The worst computation time exceeds the reasonable real-time computation time (0.01 s).

As shown in Fig. 7, the emission comparisons between the identification data and optimized data controlled by the traditional data-driven predictive controller are performed under the WHTC cycle with hot-initial engine to verify its generality and robustness. As compared to those of the original ECU, the average NOx and NH3 emissions of the traditional predictive controller are improved by $-0.93\%$ and $25.79\%$, respectively. The worst computation time of the traditional data-driven predictive controller is approximately 0.0098 s with an average 0.0028 s.

E. COMPARISON AND DISCUSSION OF THE COMPUTATION TIMES

The computation time of the predictive controller is affected by the complexity of the subspace identification method, which is determined by the dimension of the subspace matrices. As shown in Fig. 8, under the WHTC cycle with cold-initial engine the computation times of the traditional data-driven predictive controller are compared, whose subspace matrices $L_w$ and $L_u$ are divided into four different dimensions. When the dimension of $L_u$ is $50 \times 100$ and $L_w$ is $50 \times 150$, the average computation time of the predictive controller saves a $17.86\%$ compared with that of the controller when $L_u$ is $80 \times 160$ and $L_w$ is $80 \times 240$. The reduced
dimension of the subspace matrices will inevitably lead to a decrease in model accuracy. In the following sections, a segmented identification model excited by the segmented data will be discussed to guarantee model accuracy, and a corresponding predictive control method is proposed.

IV. PROPOSED DATA-DRIVE PREDICTIVE CONTROLLER WITH SWITCHED SUBSPACE MATRICES

In this section, a novel identification method for some lower order subspace matrices excited by the segmented data is established, and a corresponding data-driven predictive controller is designed and verified.

A. IDENTIFICATION OF SEGMENTED SUBSPACE MODEL Referring to the Dynamic of the Ammonia Coverage Ratio

In this subsection, subspace matrices are excited by the segmented data referring to the dynamic of the ammonia coverage ratio. For obtaining the segmented data, a tracking controller of the ammonia coverage ratio $\theta_{NH3}^*$ is employed, which controls $NH3_{in}$ for tracking the target of the ammonia coverage ratio $\theta_{NH3}^*$ as shown in Fig. 9.

Under the same working conditions ($NOx_{in}$, $T_{engine}$ and $EF_M$), several sets of segmented data for $NH3_{in}$, $NOx_{out}$ and $NH3_{out}$ are obtained by the controller tracking the different $\theta_{NH3}$. The target $\theta_{NH3}$ are set to 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35 and 0.4, respectively, and 8 sets of segmented data collected are shown in Fig. 10. Subspace matrices $L_u$ and $L_w$ excited under the different $\theta_{NH3}$ are shown in Tab. 1.

B. VERIFICATION OF THE PROPOSED DATA-DRIVEN PREDICTIVE CONTROLLER

In this subsection, based on the previous segmented subspace matrices a data-driven predictive controller with the switched subspace matrices according to the dynamic of the ammonia coverage ratio is designed. The switched regulation of the subspace matrices is described as Eq. (17). The input data collected of the proposed controller includes $\theta_{NH3}$. When $0 < \theta_{NH3} < 0.075$, $L_u$ is assigned $L_{u1}$, and $L_w$ is assigned $L_{w1}$. Similarly, when $\theta_{NH3}$ takes values in
TABLE 1. Subspace matrices $L_u$ with the dimension of $50 \times 100$ and $L_w$ with the dimension of $50 \times 150$ excited under THE different tracking targets of the ammonia coverage ratio.

| $\theta^{\text{NH}_3}$ | $L_u$ | $L_w$ |
|-------------------------|-------|-------|
| 0.05                    | ![Graph](image1) | ![Graph](image2) |
| 0.1                     | ![Graph](image3) | ![Graph](image4) |
| 0.15                    | ![Graph](image5) | ![Graph](image6) |
TABLE 1. (Continued.) Subspace matrices $L_u$ with the dimension of $50 \times 100$ and $L_w$ with the dimension of $50 \times 150$ excited under THE different tracking targets of the ammonia coverage ratio.
As shown in Fig. 11, the emission comparisons between the identification data controlled by the original ECU and optimized data controlled by the proposed data-driven predictive controller under the WHTC cycle with cold-initial engine: (1) NO\textsubscript{X, out}, (2) NH\textsubscript{3, out}, (3) computation time.

Table 1. (Continued.) Subspace matrices $L_w$ with the dimension of $50 \times 100$ and $L_u$ with the dimension of $50 \times 150$ excited under THE different tracking targets of the ammonia coverage ratio.

![Table 1](image)

The proposed data-driven predictive controller with the switched subspace matrices is verified under the WHTC cycle with cold-initial and hot-initial engine, respectively. As shown in Fig. 11, the emission comparisons between the identification data (NO\textsubscript{X, out} = exp and NH\textsubscript{3, out} = exp) and optimized data (NO\textsubscript{X, out} = opt and NH\textsubscript{3, out} = opt) controlled by the proposed data-driven predictive controller are performed under the WHTC cycle with cold-initial engine. As compared to those of identification data controlled by the original ECU, the average NO\textsubscript{X} and NH\textsubscript{3} emissions of the proposed data-driven predictive controller are improved by 81.13% and 66.31%, respectively. The worst computation time of the proposed data-driven predictive controller is approximately 0.0034 s with an average 0.0023 s. Moreover, the average NO\textsubscript{X} and NH\textsubscript{3} emissions of the traditional predictive controller mentioned in section III are improved by 75.10% and 66.40%, respectively. The worst computation time of it is approximately 0.016 s with an average 0.0028 s. As compared to those of the traditional controller, the average NO\textsubscript{X} and NH\textsubscript{3} emissions of the proposed data-driven predictive controller are improved by 6.03% and -0.09%, the worst and average computation time are improved by 0.0126 s and 0.0005 s.

As shown in Fig. 12, the emission comparisons between the identification data and optimized data controlled by the proposed data-driven predictive controller are performed under the WHTC cycle with hot-initial engine to verify its general-ity and robustness. As compared to those of the identification data, the average NO\textsubscript{X} and NH\textsubscript{3} emissions of the proposed data-driven predictive controller are improved by 2.71% and 31.46%, respectively. The worst computation time of it is approximately 0.016 s with an average 0.0028 s. As compared to those of the traditional controller, the average NO\textsubscript{X} and NH\textsubscript{3} emissions of the proposed predictive controller are improved by 3.64% and 5.67%, the worst and average computation time are improved by 0.0067 s and 0.0005 s. Under the non-identification condition of the WHTC cycle...
with hot-initial engine, the proposed data-driven predictive controller can also improve the emissions and computation time, and its optimization effects have better robustness to uncertainties of the transient cycle.

V. CONCLUSION

To simultaneously achieve high NO\textsubscript{x} conversion efficiency and low NH\textsubscript{3} slip, this paper presents a data-driven predictive controller with switched subspace matrices for an SCR system. The segmented identification method for some lower order subspace matrices excited by the segmented data referring to the dynamic of the ammonia coverage ratio is established. The corresponding predictive controller with the switched subspace matrices according to working conditions is designed in order to further improve its real-time performance, generality and robustness. The simulation results show that under the identification condition of the WHTC cycle with cold-initial engine the proposed predictive controller compared to those of the traditional method can improve the average emission amount of NO\textsubscript{x} and NH\textsubscript{3} by 6.03\% and −0.09\% and reduce the worst and average computation time by 0.0126 s and 0.0005 s, that under the non-identification condition of the WHTC cycle with hot-initial engine the proposed predictive controller compared to those of the traditional method can also improve the average emission amount of NO\textsubscript{x} and NH\textsubscript{3} by 3.64\% and 5.67\% and reduce the worst and average computation time by 0.0067 s and 0.0005 s. Our future work will focus on using real-time calculation methods to solve the optimization problems, and performing experimental verification for the control strategy proposed in this paper.

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