Scaling Of Turbulence In The Atmospheric Surface-Layer: Which Anisotropy?

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Abstract. This paper aims to provide an insight into the fundamental relationships between large and small scale wind velocity fluctuations within the boundary layer through careful analysis of measuring mast wind velocities. The measuring mast was in a wind farm on top of a mountain (with steep inclines of about 30°) on an island surrounded by the sea which meant the horizontal mean flow fluctuations were dominated by buoyancy forces and vertical shears at large scales (above 500m). Thus using a variety of methods including spectral, integrated spectral, integrated cospectral and multifractal analysis we were able to clearly dispel the relevance of 2D turbulence and give on the contrary some credence to the multifractal anisotropic model.

1. Introduction

The topic of wall-bounded turbulent flows has received continuous attention since the formulation of the boundary layer concept. Although significant experimental work has been carried out over the past decade on wall-bounded turbulence, many of the outstanding issues remain open. New experiments, driven by the desire to generate data at high Reynolds numbers, have led to new questions related to scaling and the role of the largest scale motions. A recent paper (1) combines the outputs of international cooperative research on high Reynolds number wall-bounded turbulence and highlights the key issues that need to be resolved, e.g. the existence of a logarithmic sublayer, validity of the locally isotropic turbulence hypothesis and the relations between inner and outer scaling. The authors particularly promote the idea that “extracting a theory by sifting through the data more carefully is the missing element”. In response to this, our paper discusses a possibility to explain the observable scaling behaviour of atmospheric turbulence at low altitudes with the help of an anisotropic multifractal model (2).

2. Data Description and Pre-processing

We had available to us six-months (from 16/11/2002 to 15/05/03) of wind velocity and temperature measurements from a wind farm test site subject to wake turbulence effects. The wind farm was in the North of Corsica (France), 3km from the sea on the East and West and 4km on the North. The site has an annual mean wind velocity of about 7.6m/s at 40m. There
are 20 turbines in total with 13 (Ersa site) situated along the crest of Torricella and 7 (Rogliano site) along the crest of Petraggine. The altitudes of the crests range from 480 to 520m with a 30° incline across most of the distance. All of the turbines have a hub height of 60m and are positioned 117m apart at the Ersa site and 136m apart at the Rogliano site. The measurements came from three 3D sonic anemometers with a 10Hz data output rate. The anemometers were positioned at 22, 23 and 43m on a mast in the centre of three concentric turbines at the Ersa site. The first anemometer at 22m was positioned directly on the mast. The second, at 23m, was positioned at the end of a horizontal pole with length 2.5m and azimuth 134°. The highest mast at 43m was positioned on a 3m pole on top of the mast.

When using data from devices not positioned directly on top of the mast (those at 22 and 23m), it was necessary to take into account the possibility of the interaction of the wind with the mast, thus destroying the quality of the measurements. To check for this problem we took data with daily mean wind passing directly through the mast (48 of the 102 days) and did a cross comparison at different heights. We observed large numbers of anomalous small fluctuations in the vertical component (high frequency noise through spectral representation) being measured at 22m. This is likely due to the vertical fluctuations being much smaller in magnitude making the measurements increasingly sensitive to disturbances at small scales. It is important to note that this was not observed at 23 and 43m thus aiding our confidence in the quality of data at these heights and our observations thus from.

Although confident our data was free of physical interference, corrupt and missing data files made it difficult to have long runs of continuous error free (clean) data. Out of the 181 days of data only 10 of the days were time continuously clean. For non-time continuous data (independent samples) there were 161 days of clean data. Note that the requirement for clean data at all three heights reduced the number of independent samples, at for example 43m, from 161 to 102 days.

3. Spectral Analysis

3.1. Overview

A spectral representation was used to determine the overall scaling behaviour of our data. This is because a random field is scaling when its spectrum follows a power law of the form \( E(\omega) \propto \omega^{-\beta} \) (see (3)) where \( E \) is a function of frequency, \( \omega \), in Fourier space and \( \beta \) is the often called “spectral slope” estimated by plotting the spectra on a log-log graph (see section 3.2 for distributions of \( \beta \) for the data). The (co) spectrum of two fields (which are identical for the spectrum) is the real part of the scalar product of their Fourier transforms. The Fourier transforms were computed using the fast Fourier transform (FFT) algorithm (see sections 3.3 and 3.4).

With the use of the FFT algorithm we were restricted to data of sizes \( 2^n \) where \( n \leq \log_2(N_s) \) and \( N_s \) is the sample size. Thus, given the longest time continuous sample was 10 days, the maximum range of scales achievable was of about 6 orders of magnitude. While a spectral representation of long runs of data is indispensable to evaluate the overall scaling behaviour and its limitations, sample averaged estimates are used to define the spectral exponents more precisely (see section 3.3). Since averaging requires more than one sample, given such a large discontinuous dataset, it was important to choose a suitable subsample size, \( N_{ss} \), to obtain the most amount of information from the data. For the majority of this study we focused on analyses with \( N_{ss} = 2^{19} \) (section 3) with a brief discussion on the benefits of a larger subsample (\( N_{ss} = 2^{22} \)) in section 4.

3.2. Probability Distributions of Spectral Slopes \( \beta \)

The spectral analyses showed similar scaling behaviour, consisting of three subranges divided by two breaks, for all three velocity components \( u, v, w \) and temperature \( \theta \). The first two subranges, \( R_{HF} \) and \( R_{MF} \), over high and mid frequencies respectively, were partially in agreement with
Kolmogorov’s -5/3 law of locally isotropic turbulence. As described in (3), the exponent will define an inertial subrange for all three velocity components adjoined by a -1 power law (at sufficiently high Reynolds numbers as discussed in (1)), obtained from dimensional analysis of the logarithmic sublayer, over smaller wave numbers and frequencies. A 3D inertial range was observed but only up to between 1 and 100 seconds at which the vertical component diverged from the scaling of the horizontal components and temperature and remained dissimilar until the third subrange, $R_{LF}$, at low frequencies. The adjoining -1 power law was observed for all three components and temperature but as mentioned before the length and position varied depending not only on the component but on the day.

More specifically for the positions of high to mid frequency breaks, $X$, we observed variations between 5 and 100 seconds and for the positions of mid to low frequency changes in subrange scaling, $Y$, we observed variations between 10 minutes and 2 hours. The change in position of the breaks in scaling are likely due to the changes in wind direction however it was difficult to see correlation because of reasons later discussed in section 3.4.

Because the position and length of each subrange varied greatly for each sample, it was necessary to also calculate $\beta$ over varying positions and lengths and not simply over the whole range. A simple algorithm determined the position of the breaks based on the minimum and maximum of $\Delta \beta = \beta_{n+1} - \beta_n$ over the range $i$ of $E(\omega_i) \approx \omega_i^{\beta_n}$ where $i = 2^n,...,2^{n+\Delta n}$ and $n = 1,...,\log_2(N_s - \Delta n)$. The value $\Delta n = 5$ was found to be the most appropriate compromise between the best fit, $R^2$, and the loss of information at the sample bounds.

The following probability density functions (PDF)s of $\beta$ consist of two types of plot. Those where the distributions do not differ significantly for horizontal and vertical (Figures 1 and 2, one plot per figure in blue) and those that do (Figure 3, two plots compared per figure with horizontal in red and vertical blue).

**Figure 1.** PDF of spectral slopes for high frequency subrange $R_{HF} = [0.2s : X]$ with $X$ varying between 5 and 100 secs. Mean $\beta = 1.21$ ($u, v$ and $w$ at 23 and 43m).

**Figure 2.** PDF of spectral slopes for low frequency subrange $R_{LF} = [Y : 0.1 \times 2^{19}s]$ with $Y$ varying between 10 mins and 2 hrs. Mean $\beta = 2.45$ ($u, v$ and $w$ at 23 and 43m).
Figure 3. PDF of spectral slopes for mid frequency subrange $R_{MF} = [X : Y]$. Mean $\beta = 1.05$ for horizontal components in red and mean $\beta = 0.59$ for vertical component in blue ($u$, $v$ and $w$ at 23 and 43m).

Some of the spectral exponents for the horizontal components in Figure 3 (over 10% of the values) were comparable with those of the high frequency subrange in Figure 1. This suggested there were days where the scaling was observable up to longer time scales. Thus following this observation the data was filtered based on $\beta$ for the mid frequency range, $R_{MF}$, as discussed in the next section.

3.3. Averaged Spectra
We averaged the spectra of the extreme case mid frequency scaling behaviour to obtain better estimates of the spectral exponent. For a fair comparison we needed an equal number of

Figure 4. Energy spectra of $u$, $v$ and $w$ at 43m averaged over 11 unperturbed days. The dashed, dotted and solid lines are the smoothed interpolation (using three-point moving average method) of the spectra.

Figure 5. Energy spectra of $u$, $v$ and $w$ at 43m averaged over 11 perturbed days. As in Figure 4 the dashed, dotted and solid lines are the smoothed interpolation of the spectra.
subsamples for each case. We found the bounds $\beta \geq 1.20$ and $\beta \leq 0.80$, which we define simply as unperturbed and perturbed scaling respectively (see Figures 4 and 5 for the reasoning behind this), gave a suitable representation of the extreme case behaviour (11 days for each case). Note the filtering of the spectral exponent to select these days was only applied to the horizontal $u$-component at 43m.

Our results confirmed unique scaling over small scales with $1.21 \leq \beta \leq 1.34$ for all three velocity components up to between 15 and 50 seconds at which the scaling of the vertical $w$-component changes to an adjoining -1 power law subrange with $0.34 \leq \beta \leq 0.65$ in agreement with wall-bounded theory. Spectral slopes being lower than $5/3$ and 1 could be understood with the intermittency correction (see section 4). Such high intermittency corrections were particularly relevant for our case study due to the increased likelihood of small fluctuations from the wind turbines and complex terrain.

The horizontal velocity components $u$ and $v$ continued to scale, almost identically, up to between $10^2$ and $10^3$ seconds before a departure from the scaling regime was seen. Figure 4 shows a spectral exponent (mean $\beta = 2.28$ over $R_{LF}$) consistent with Bolgiano-Obukhov theory (7; 8) that predicts a power law of $-11/5$ for a buoyancy force subrange i.e. $\beta = 2H + 1$ with $H = 5/3$ for vertical shears. This is not the case for the perturbed days whose spectral exponent (mean $\beta = 2.99$ over $R_{LF}$) is closer to that of the Lumley-Shur law or 2D turbulence spectral exponent of -3 (see section 5 for a more in depth discussion on this topic).

### 3.4. Integrated Spectra

Figures 6 and 7 display the integrated spectra of all three velocity components for perturbed and unperturbed days. The main interest in presenting the data this way was the clarity with which the positions of the breaks in the scaling (defined by the positions of the peaks and troughs in energy) could be seen and compared with the positions of the breaks in Figures 4 and 5. Such clear separations in the scaling allowed us to obtain estimates of the integral length scales using the empirically derived formulae

$$L_u = 10.3z, \quad L_v = 7.5z \quad \& \quad L_w = 0.5z,$$  \hspace{1cm} (1)

as suggested in (4). Tables 1 and 2 show the estimates of the characteristic velocity $U_w$ and the relative frequencies $\Delta t'_u$ and $\Delta t'_v$ derived from $\Delta t_w$.

**Table 1.** Table of estimates derived from length scale coefficients of (4) using $\Delta t_w$ at 43m.

|        | $L_w$ [m] | $\Delta t_w$ [s] | $U_w$ [m/s] | $\Delta t'_u$ | $\Delta t'_v$ |
|--------|-----------|------------------|-------------|---------------|---------------|
| unperturbed | 21.5    | 50               | 0.43        | 1,050         | 750           |
| perturbed | 21.5    | 15               | 1.43        | 316           | 226           |

**Table 2.** Table of estimates derived from length scale coefficients of (4) using $\Delta t_w$ at 23m.

|        | $L_w$ [m] | $\Delta t_w$ [s] | $U_w$ [m/s] | $\Delta t'_u$ | $\Delta t'_v$ |
|--------|-----------|------------------|-------------|---------------|---------------|
| unperturbed | 11.5    | 30               | 0.38        | 636           | 454           |
| perturbed | 11.5    | 8                | 1.44        | 168           | 120           |
Figure 6. Integrated energy spectra of $u$, $v$ and $w$ at 43m averaged over 11 unperturbed days. The dashed, dotted and solid lines are the same interpolation as for Figures 4 and 5. The arrows correspond to $\Delta \tau_{u,v}^t$ calculated from the length scales derived in (4) (see Table 1).

Figure 7. Integrated energy spectra of $u$, $v$ and $w$ at 43m averaged over 11 perturbed days. The dashed vertical lines correspond to the integrated spectral peaks over the mid frequency subrange.

In addition, given we have the characteristic length scales and velocity we can estimate a Reynolds number of about $10/1.5 \times 10^{-5} \sim 10^6$. This estimate confirms that the investigated wind field exhibits fully developed turbulence and remains consistent with the Reynolds numbers of the boundary layer experiments summarised in (1).

We can see from Figure 6 that the change in scale of the horizontal and vertical wind components seems to be in good agreement with the semi-theoretical results of (4) obtained for the atmospheric surface layer. On the contrary, Figure 7 demonstrates that the $-1$ power law appears much earlier than the predicted values defined by Eq. (1). A possible explanation for this could be the wake turbulence effects attributed to the turbines. As underlined in (5) large fluctuations in the wind during the trial period of the wind farm often led to interruptions in the functioning of the turbines during days when either strong or weak winds were being registered. Two examples of contrasting wind speed and direction occurred on October 26th, 2002 and April 26th, 2003 where very strong Westerly winds meant every turbine was operating and very weak South-Eastery winds meant every turbine had to be stopped for each of the days respectively. Both events took place at the Ersa site. Given the very low characteristic velocities $U_w \sim 0.4$ (see Tables 1 and 2) coincide with the better scaling of the horizontal velocity components $u$ and $v$ it is possible this may have been due to the stopping of the turbines in events similar to those aforementioned.

Given this result one would expect to see strong correlation between mean wind speeds, direction and scaling. This however is not the case (correlation coefficient less than 0.5). One possible explanation is that although we have 11 days of extreme behaviour at each end of the spectrum the other 80 days consist of “mixed” periods of functioning. What this means is that by example on April 28th, 2003 all of the turbines on the site were shut down up to 9h30 due to very weak South-Eastery winds. Then at 12h20 due to a much stronger South-Eastery wind 10 of the 20 turbines began to function. This goes some way into explaining why there is no clear correlation.
3.5. Integrated Cospectra
A condition of the applicability of the coefficients derived in (4) is that the structures of the surface-layer turbulence respect the statistical mirror symmetry with respect to the \((x, z)\) plane i.e. when the direction of the horizontal \(u\)-component coincides with the direction of the mean wind \(E(\omega) = \{E_{uv}; E_{vw}; E_{v\theta}\} = 0\)

![Figure 8](image1.png)  ![Figure 9](image2.png)

**Figure 8.** Energy cospectra of \(u\) and \(v\) and \(w\) at 43m averaged over 11 unperturbed days.  **Figure 9.** Energy cospectra of \(u\) and \(v\) and \(w\) at 43m averaged over 11 perturbed days.

Although some of our data and analyses agreed with the length scales of (4) our cospectra are not compatible with the classical theory. As illustrated by Figure 8, we found that the cospectrum, \(E_{uv}\), returned values that were of the same order as the previously calculated spectra over the inertial range and were therefore not neglectable. Furthermore scaling was present the cospectra \(E_{uv}\) similar to the scaling of the integrated spectra (see Figures 6 and 8 for comparison). This demonstrated that the direction of the horizontal \(u\)-component of velocity did not coincide with the direction of the mean wind. In fact the mean wind was seemingly directed in the South-East or North-West directions explaining the strong correlation between \(u\) and \(v\) components. Winds in this direction have the least influence from wake effects on the mast. In comparison, Figure 7 displays very strong fluctuations of the cospectrum over the same frequencies that combined with the characteristic velocity \(U_w \sim 1.5\) m/s for perturbed days (see Tables 1 and 2), gave a range of scales comparable with the height of the turbines and the associated scales of wake-created coherent structures.

In addition to the cospectrum of the horizontal components we looked at the correlation of the other velocity components and temperature. Figures 10 and 11 display the behaviour of the corresponding covariance, which is the buoyancy flux, \(E_{w\theta}\), in Fourier space. The inverse FFT of \(E_{w\theta}\) from the data in these figures was positive. This meant we were observing buoyancy forces thus confirming the presence of anisotropic scaling i.e. that we have vertically dominating shears at large scales and horizontally dominating shears at smaller scales (up to the scales of a few centimetres).

The physical reasoning for this lies in the topographical features of the wind farm. Because the wind farm is close to the sea, strong convective forces drive atmospheric structures vertically. Large structures intercept with the mountain and are pushed by prevailing winds upwards against the side of the mountain and across the face of the mast. Since the mountain is surrounded by such a steep slope we would expect this feature to be prevalent throughout
all of our data at large scales. With a characteristic velocity $U_w \sim 0.5$ for unperturbed days (Tables 1 and 2) the departure from Kolmogorovs scaling gave a rough estimate of 500m. This is a large scale that is indeed compatible with the dominating height of the area.

4. Multifractal Analysis

We have seen from Figures 1 and 2 that we have unique scaling defining an inertial range for small scales where the spectral exponent, $\beta$, varies between 1.21 and 1.34 and is thus lower than the expected spectral exponent $5/3$ predicted by Kolmogorov. The difference corresponds to the intermittency correction (3) that, as discussed below, is due to very high heterogeneity of the mean field for atmospheric turbulence. Its increase implies an increase of wind extremes which the expected spectral exponent $5/3$ predicted by Kolmogorov. The difference corresponds to the small scales where the spectral exponent, $\beta$.

We have seen from Figures 1 and 2 that we have unique scaling defining an inertial range for all of our data at large scales. With a characteristic velocity $U_w \sim 0.5$ for unperturbed days (Tables 1 and 2) the departure from Kolmogorovs scaling gave a rough estimate of 500m. This is a large scale that is indeed compatible with the dominating height of the area.

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$$K(q) = \frac{C_1}{\alpha - 1} (q^\alpha - q)$$

where $q$ is the order of moment and $\alpha$ and $C_1$ are the multifractal parameters defining the degree of multifractality and the inhomogeneity of the mean field respectively. They are estimated with the use of the double trace moment (DTM) method based on the following relation

$$K(q, \eta) = K(q\eta) - qK(\eta) = \eta^\alpha K(q, 1).$$

Normally $\alpha$ is obtained by fixing $q$ and obtaining the slope of $|K(q, \eta)|$ as a function of $\eta$ on a log-log graph. Alternatively, given we know the power law relation between the flux and the velocity i.e. $\varepsilon \propto \Delta V^3$, we can inversely fix $\eta = 1, 3$ for $\varepsilon$ and $\Delta V$ respectively and obtain $\alpha$ as the slope of $K_{\Delta V}$ and $K_\varepsilon$ as a function of $q$ also on a log-log graph where

$$K_{\varepsilon}(q, 1) = \frac{C_{1,\varepsilon}}{\alpha - 1} (q^\alpha - q), \quad K_{\Delta V}(q, 3) = 3^{-\alpha} \frac{C_{1,\Delta V}}{\alpha - 1} (q^\alpha - q).$$
It is then elementary to derive the following relation
\[ \alpha = \log_3 \left( \frac{K \varepsilon}{K \Delta V} \right). \]  
(5)

Figure 12. PDF of parameter \( \alpha \) over high frequencies in red and low frequencies in blue for unperturbed days. Mean \( \alpha = 1.78 \) and 1.75 for high and low frequencies respectively (\( u \) and \( v \) at 22, 23 and 43m).

Figure 13. PDF of parameter \( C_1 \) over high frequencies in red and low frequencies in blue for unperturbed days. Mean \( C_1 = 0.14 \) and 0.1 for high and low frequencies respectively (\( u \) and \( v \) at 22, 23 and 43m).

Figure 14. Scaling moment function over high frequency subrange for \( \Delta u \) where \( \Delta_, \odot \) and \( \Box \) are the function at 22, 23 and 43m respectively.

Figure 15. Plot of lower frequency scaling for \( N_{ss} = 2^{22} \). Mean \( \beta = 2.03 \) with intermittency correction \( K(2) = 0.22 \) for \( u \), \( v \) and \( w \) at 43m.
Figures 12 and 13 display PDFs of the multifractal parameters $\alpha$ and $C_1$ of the energy flux estimated at 22, 23 and 43m for the 11 days (33 samples) of unperturbed scaling. They show that unperturbed data correspond to a stronger multifractality with a lower heterogeneity of the mean field. A dispersion of universal multifractal parameters indicates the mean $\alpha = 1.5$, estimated over the full data (306 samples), is within the lower 80% limit of the unperturbed estimates.

For high frequency ranges the mean multifractality index of $\alpha = 1.78$ and the mean inhomogeneity of the mean field $C_1 = 0.14$. For low frequency ranges $\alpha = 1.75$ and $C_1 = 0.10$. The strong multifractality of the data results in the strong non-linearity of the scaling moment function, as illustrated by Figure 14 for the horizontal $u$-component of the wind velocity at each of the three height measurements. For the large scale range $R_{LF}$ in Figure 12 there were values of $\alpha$ that exceed the maximum of 2. These high values could be explained by either bad or limited scaling. Another explanation could be an inappropriate flux as discussed in section 5. In fact the two issues are closely related.

The PDF of the spectral exponents estimated over large scales (Figure 2) illustrates the difficulty in distinguishing the type of scaling law. In particular the Bolgiano-Obukhov -11/5 and Lumley-Shur -3 laws. Since the integrated spectra (Figures 6 and 7) clearly dismissed the idea of 2D turbulence, the spectral estimates could have being producing values in and around -3 simply because they were too sensitive to the limited length of data. This is confirmed in Figure 15 which displays a much clearer scaling behaviour over the large scales due to the use of longer data samples which results in a much better agreement with Bolgiano-Obukhov -11/5 law (mean $\beta = 2.03$ with intermittency correction $K(2) = 0.22$ for $u$, $v$ and $w$ at 43m).

5. The Multifractal Anisotropic Model

To take into account the dominant role of the vertical motion of large scale atmospheric structures, one may consider that the buoyancy force variance flux, $\phi$, plays the same role as the energy flux, $\varepsilon$, in 3D turbulence but only along the vertical (2). This is contrary to the classical “buoyancy subrange” that postulates an isotropic turbulence (7; 8) with two different (horizontal and vertical) scaling regimes. This corresponds to the coupled sets of scaling equations (2; 6):

$$\frac{\Delta V(\Delta x)}{\Delta V(\Delta z)} \overset{d}{=} \frac{(\varepsilon(\Delta x))^{1/3} \Delta x^{1/3}}{(\phi(\Delta z))^{1/5} \Delta z^{3/5}} \implies (\varepsilon(\Delta x))^{1/3} \approx (\phi(\Delta z))^{1/5} \text{ when } \Delta x^{1/3} \approx \Delta z^{3/5} \quad (6)$$

where $\Delta V(\Delta x)$ and $\Delta V(\Delta z)$ denote the horizontal and vertical shears of the horizontal wind respectively and the symbol $d$ means equality in probability distribution.

Because the scaling fluctuations of both fluxes are not neglected (due to their explicit scale dependency) we can define anisotropic scaling (as defined by the anisotropic multifractal model (2)) at all significant scales instead of two isotropic regimes, separated by a scaling break. This means the iso-shear surfaces will be ellipsoids rather than spheres and that the horizontal and vertical extents of the atmospheric structures will be equal only at the sphero-scale which is generally of the order of 10-20 centimetres. If the multifractality of two fluxes remain the same, the multifractal anisotropic model predicts that both weak and mean events will have codimensions that are in the same ratio as the corresponding degrees of non-conservation of the mean field:

$$\frac{C_{1,\varepsilon}}{C_{1,\phi}} = \frac{H_{1,\varepsilon}}{H_{1,\phi}} = \frac{5}{9} \quad (7)$$

where $C_{1,\varepsilon}$ is the codimension for the energy flux over high frequency ranges, $R_{HF}$, and $C_{1,\phi}$ is the codimension for the buoyancy force variance flux over low frequency ranges, $R_{LF}$. Remember that the codimensions are for the horizontal components on unperturbed days only. In analogy.
to Eq. 4, Eq. 6 implies $C_{1,\phi} = (5/3)^\alpha C_1 \approx 0.25$ when using the mean values $\alpha \approx 1.8$ and $C_1 = 0.1$ estimated for the energy flux over large scales (Figures 12 and 13). The ratio of the mean estimates $C_{1,\epsilon}/C_{1,\phi} = 0.14/0.25 = 0.56$ gives the predicted value 0.555.

6. Conclusion

The aim of this study was to explore the scaling behaviour of atmospheric velocity and temperature measurements in a wind farm test site subject to wake turbulence effects. Two or three scaling subranges were identified depending on the direction of the mean wind. We started from the investigation of possible relations between wind velocity scaling breaks and associated theories of turbulence in the atmospheric surface-layer. Once we verified that the investigated wind field exhibited fully developed turbulence it was possible to use multifractal methods to deal with the strong intermittency. For days with no interaction with the turbines the multifractal anisotropic model was fully validated. These preliminary results encourage a more extensive sifting through of the data for the future development of new theories for the atmospheric surface-layer.

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References

[1] Marusic, I., McKeon, B. J., Monkewitz, P. A., Nagib, H. M., Smits, A. J. & Sreenivasan, K. R. 2010 Wall-bounded turbulent flows at high Reynolds numbers: Recent advances and key issues Phys. Fluid., 22, 065103.
[2] Schertzer, D. & Lovejoy, S. 1984 On the Dimension of Atmospheric motions. In: T. Tatsumi (Editor), Turbulence and Chaotic phenomena in Fluids, Amsterdam, Elsevier Science Publishers B. V., pp. 505-512.
[3] Monin, A. S. & Yaglom, A. M. 1975 Statistical Fluid Mechanics, Cambridge, MIT-Press, Vol. 2, pp. 874.
[4] Kader, B. A., Yaglom, A. M., & Zubkovskii, S. L. 1989 Spatial Correlation Functions of Surface-Layer Atmospheric Turbulence in Neutral Stratification, Bound.-Lay. Meteorol. 47, pp. 233-249.
[5] Faggio, G. & Jolin, C. 2003 Suivi ornithologique sur le parc d’éoliennes d’Ersa- Rogliano (Haute Corse) - Rapport final-SIIF/AAPNR-GOC, 100p.
[6] Lazarev, A., Schertzer, D., Lovejoy, S. & Chigirinskaya, Y. 1994 Unified multifractal atmospheric dynamics tested in the tropics: part II, vertical scaling and generalized scale invariance, 115-123.
[7] Bolgiano, R. 1959 Turbulent spectra in a stably stratified atmosphere, J. Geophys. Res. 64, 2226.
[8] Obukhov, A. N. 1959 Effect of Archimedian forces on the structure of the temperature field in a temperature flow, Sov. Phys. Dokl. 125, 1246.
[9] Pinus, N. Z., Reiter, E. R., Shur, G. N. & Vinnichenko, N. K. 1967 Power spectra of turbulence in the free atmosphere, Tellus 19, 206.