Are Fresnel filtering and the angular Goos–Hänchen shift the same?

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Abstract
The law of reflection and Snell’s law are among the tenets of geometrical optics. Corrections to these laws in wave optics are respectively known as the angular Goos–Hänchen shift and Fresnel filtering. In this paper we give a positive answer to the question of whether the two effects are common in nature and we study both effects in the more general context of optical beam shifts. We find that both effects are caused by the same principle, but have been defined differently. We identify and discuss the similarities and differences that arise from the different definitions.

Keywords: beam deflection, critical angle, total internal reflection

(Some figures may appear in colour only in the online journal)

1. Introduction

It is not uncommon in physics that the same effect is discovered independently by different people and, as a consequence, may be known under two or more different names owing to differences in language and scientific tradition. A prominent and relevant example for this paper is Snell’s law, which in French speaking countries is more commonly known as Descartes’ law, to remind us that it was René Descartes who first published it in modern times [1] (although Ibn Sahl, Willebrord Snellius and others derived the result before him [2]).

The deviation from the predictions of Snell’s law in wave optics is known as ‘Fresnel filtering’ (FF) [3] and describes a correction to the angle of refraction, which is particularly prominent for focused beams incident close to the critical angle of total internal reflection. The origin of this effect is easily understood from the following simple picture (see figure 1): assuming a symmetric angular spectrum and that the central wavevector of the focused beam be incident exactly at the critical angle, all plane waves of the incident beam with an angle of incidence larger than the critical angle are fully reflected. On the other hand, all plane waves incident below critical incidence are transmitted. The magnitude of the effect is then the difference between the angle of refraction expected from geometrical or ray optics and the far field angle of the transmitted field. If the incident angular spectrum is large, so is the transmitted spectrum, which explains intuitively why the effect of FF depends on the angular width of the incident beam.

Independently from FF a family of optical beam shift effects has been studied, mainly on reflection [4]. In contrast to FF, which has been discovered in the context of two-dimensional scalar fields in optical microcavities [5, 6], optical beam shifts have been explored in all three dimensions. Confined within the plane of incidence, and therefore comparable to the 2D fields in microcavities, are the spatial and angular Goos–Hänchen shift (GH) [7, 8], and it is the latter which concerns us here as a related effect to FF. In its simplest form the angular GH shift pertains to generic incidence angles, that is not for the critical angle or the Brewster angle, although these special cases have been studied too [9–11]. Using the same simple picture as above, but for generic incidence (see figure 2), the angular GH shift...
Figure 1. Schematic of FF at critical incidence. A light beam is incident from the region of higher refractive index. The central wavevector impinges on the interface precisely at the critical angle of total internal reflection. All plane waves of the incident angular spectrum (green gradient) with larger angles of incidence (crossed) are fully reflected, whereas plane waves with smaller angles of incidence are transmitted (blue gradient). The gradient indicates the amplitude of the angular spectrum. Precisely at the critical angle the transmittance is zero, which is why the transmitted beam has the maximum shifted away from grazing transmission. For clarity the reflected beam is shown as arrows only.

can be explained as a weighting in the Fourier spectrum of the incident beam upon reflection, which leads to a shift of the mean angle in the reflected beam. Unsurprisingly, the magnitude of the angular GH shift also depends on the angular spectrum.

One difference between the two effects is based on the different definitions as peak emission angle of the transmitted field and mean angle of the transmitted spectrum. Another distinction between the FF effect and the angular GH shift is rooted in their importance for the different scientific communities. For example, the presence of regular, triangular-type resonator modes in chaotic microcavities of spiral shape could only be explained by the FF effect at near-critical reflection [12]. Such quasi-scarred resonances [14, 15] had been observed in wave calculations and, implementing the FF effect into an extended ray dynamics, could now be understood in terms of ray optics. (Notice that implementing the FF effect alters the ray optics from Hamiltonian to non-Hamiltonian [12, 13], an interesting feature that is, however, beyond the scope of the present paper.) This example of FF in quasi-scarred modes illustrates nicely two properties of the effect: (i) The FF effect is noticeable especially around the critical angle, which, given a refractive index of \( n = 2 \) inside the cavity, happened to be the inner angle of the triangular-type quasi-scarred modes. (ii) The FF effect depends on the resonance wavenumber, implying a slight rotation between adjacent resonances in order to adjust the resonance geometry to the slight change in the FF correction.

In beam optics, on the other hand, the angular shift had been found to occur in the same setting as the spatial GH shift, but for incidence below the critical angle [16, 17]. As the angular GH shift is a deflection of the beam’s propagation direction, the displacement of the beam caused by this effect scales with the distance from the interface. This is why in situations where both spatial and angular GH shifts occur, for example in metallic reflection [18], the contribution of the angular shift to the overall displacement will become dominant when looking sufficiently far down the beam [19]. Recently, it has been found that for higher-order Laguerre–Gaussian beams, or more generally any vortex beam, the total spatial shift can be explained by a mixing of the spatial and angular shifts of the fundamental beam [20–22].

FF and the angular GH shift are both counterparts to the spatial GH shift. Either as two independent directions in phase space [23], or as the real and imaginary parts of the same complex quantity, namely the logarithmic derivative of the reflection coefficients [24, 9, 8, 25]. As such the formulas for the angular and spatial GH shift on reflection are

\[
\Delta_{GH}^{\text{GH}} = \sigma_2 \text{Re} \left( \frac{r'}{r} \right)_{\theta_0} \quad \text{and} \quad D_{GH}^{\text{GH}} = -\frac{1}{k} \text{Im} \left( \frac{r'}{r} \right)_{\theta_0},
\]

where \( r \) is the reflection coefficient [27, 28] and the prime denotes the derivative with respect to the incidence angle at the central angle \( \theta_0 \). These shifts have their natural units, given by the angular width or variance \( \sigma_2 \) for the angular shift and the inverse of the wavenumber \( k \) for the spatial shift.
In this paper we focus on the differences and similarities between the angular GH shift and FF. The arrangement of the paper follows our logical outline of the argument. In the first part of section 2 we derive an explicit expression for the angular GH shift on transmission for generic incidence and compare it with the known formula for the corresponding shift on reflection. In the second part we find an explicit expression for critical incidence and compare it with the known results from FF at critical incidence. Section 3 contains derivations of analytical formulas for FF at a generic angle of incidence, which we use for a concluding comparison in a discussion in section 4.

2. Angular GH on transmission

Our derivation for the expression of the angular GH shift in transmission relies on the concept of the virtual beam, as introduced in [25]. However, whereas for the case of reflection the virtual beam is obtained by a specular reflection of every plane wave, that is the reflection coefficient is \( r = \pm 1 \) throughout, for transmission the virtual refracted beam is obtained by applying Snell’s law for every plane wave according to the local angle of incidence. The spatial and angular shifts on transmission are then the differences to the virtual reflected beam due to the inclusion of the appropriate transmission coefficient.

In 2D we can treat the polarization orthogonal to the plane of incidence (s or TM) and parallel to it (p or TE) separately\(^4\), and the incident and virtual transmitted beam are the same for both polarizations [27]; the only place where the polarization enters is the choice of the appropriate transmission coefficient. We adopt the notation of a reduced refractive index \( n = n_2/n_1 \), where \( n_1 \) is the index of the medium in which the incident beam and reflected beam propagate. Using \( x \) for the spatial coordinate along the interface and \( z \) for the coordinate normal to it, we parametrize the incident beam by means of the angle of incidence \( \theta \) for each plane within the spectrum \( \sigma (\theta) \)

\[
\psi_i = \int d\theta \sigma (\theta)e^{i k (x \sin \theta - z \cos \theta)}, \tag{2}
\]

The virtual transmitted beam is constructed by changing the direction of every plane wave in accordance with Snell’s law \( \sin \theta = n \sin \tau \) while maintaining the parametrization of the beam by \( \theta \) (see figure 3):

\[
\psi_v = \int d\theta \sigma (\theta)e^{i k (x \sin \theta - z \sqrt{n^2 - \sin^2 \theta})}. \tag{3}
\]

If the incident beam is in the optically thicker medium, the virtual beam includes evanescent waves. For the calculation of the angular GH shift, however, we only include propagating plane waves, which can be enforced by setting the upper bound of the integration to the critical angle \( \theta_c \). The real transmitted beam is obtained from the virtual beam by inserting the appropriate transmission coefficients for the s(TM) and p(TE) polarization [28]

\[
t_s = \frac{2 \cos \theta}{\cos \theta + \sqrt{n^2 - \sin^2 \theta}} \quad (\text{TM}), \tag{4}
\]

\[
t_p = \frac{2 n \cos \theta}{n^2 \cos \theta + \sqrt{n^2 - \sin^2 \theta}} \quad (\text{TE}), \tag{5}
\]

which yields

\[
\psi_v = \int d\theta \sigma (\theta)t_v(\theta)e^{i k (x \sin \theta - z \sqrt{n^2 - \sin^2 \theta})}. \tag{6}
\]

where \( \sigma = s(\text{TM}), p(\text{TE}) \).

Following the treatment in [25] we calculate the mean angle of the transmitted beam \( \psi_v \) as the centroid of the transmitted angular spectrum. Unlike for reflection, however, we can observe a deviation from the laws of geometrical optics even for the virtual beam. Because the incident and transmitted beam are in two different media, the intensity is no longer simply proportional to the modulus of the amplitude [28]. To account for this distinction we introduce a virtual transmittance \( V = n |\cos \tau / \cos \theta| \), where \( \tau \equiv \sin^{-1}(\sin \theta/n) \) denotes the angle of transmitted plane waves. As we only consider propagating waves in transmission, all the angles and the transmission coefficients are real, so we can ignore the moduli. This also implies that there will be no spatial GH shift in this setting. The mean angle for the virtual transmitted beam is thus calculated as the centroid of the virtual transmitted spectrum:

\[
\tau_v = \langle \tau (\theta) \rangle_v = \frac{\int d\theta |\sigma (\theta)|^2 V(\theta) \tau (\theta)}{\int d\theta |\sigma (\theta)|^2 V(\theta)} \tag{7}
\]
Assuming that the spectrum \( \sigma \) of the incident beam is symmetric and narrowly concentrated around the central plane wave with incidence \( \theta_0 \), we expand all \( \theta \) dependent quantities by setting \( \theta = \theta_0 + \delta \), with \( \delta \) being small. Identifying \( \tau (\theta_0) \equiv \tau_0 \), we write explicitly

\[
V(\theta_0 + \delta) \approx n \cos \theta_0 \cos \theta_0 \left[ 1 + \delta (\tan \theta_0 - \tau' |_{\theta_0} \tan \tau_0) + \cdots \right],
\]

\[
\tau (\theta_0 + \delta) \approx \tau_0 + \delta \tau' |_{\theta_0} + \frac{\delta^2}{2} \tau'' |_{\theta_0} + \cdots,
\]

where the prime denotes derivatives with respect to \( \theta \). On substitution of these expansions into (7) we can identify the zeroth order in \( \delta \) with Snell’s law for the central plane wave, while the first correction term is of order \( \delta^2 \) as the first-order terms vanish on integration due to symmetry. Collecting all terms of order \( \delta^2 \) yields an approximate expression for the mean angle of the virtual beam

\[
\tau_v \approx \tau_0 + \left[ \tau' |_{\theta_0} (\tan \theta_0 - \tau' |_{\theta_0} \tan \tau_0) + \frac{1}{2} \tau'' |_{\theta_0} \right] (\delta^2),
\]

where the equality follows from the properties of \( \tau (\theta) \) and \( \langle \delta^2 \rangle \) is the second spectral moment or angular variance of the incident beam calculated as

\[
\langle \delta^2 \rangle = \int d\theta |\sigma (\theta_0 + \delta)\rangle^2 \delta^2 / \int d\theta |\sigma (\theta_0 + \delta)\rangle^2.
\]

The second-order derivative in (10) is not present in other derivations of the angular GH shift on transmission (and it does not occur for reflection [25, 22]). Our approach thus differs from known formulas [4, 20]. We support our results by the excellent agreement with a direct numerical calculation of the virtual shift as defined in (7) (see figure 4). The virtual angular GH shift is thus given by

\[
\Delta_v^{GH} = \tau_v - \tau_0.
\]

This deviation from Snell’s law stems from the asymmetric refraction of the plane waves: whereas the incident spectrum is symmetrically distributed around the central plane wave \( \theta_0 \), refraction introduces an asymmetry which leads to a shift or deflection of the mean angle even for the virtual transmitted beam.

For the real transmitted beam we have to exchange the virtual transmittance \( V \) with the real transmittance \( T_\alpha \), \( \alpha = s, p \) around \( \theta_0 \)

\[
T_\alpha (\theta_0 + \delta) \approx n \cos \theta_0 \cos \theta_0 \left[ 1 + \delta \left( 2 \frac{\tau'}{\tau_0} + \tan \theta_0 \right. \\
- \tau' |_{\theta_0} \tan \tau_0 + \cdots \right],
\]

where the prime denotes derivatives with respect to \( \theta \). The mean angle of the real transmitted beam is therefore approximately

\[
\tau_\alpha \approx \tau_0 + \left[ 2 \frac{\tau'}{\tau_0} - \frac{1}{2} \tau'' |_{\theta_0} \right] (\delta^2),
\]

\[
\alpha = s (TM), p (TE).
\]

The intrinsic angular GH shift is therefore the difference between the real and virtual transmitted angle \( \Delta_\alpha^{GH} = \tau_\alpha - \tau_v \) for \( \alpha = s, p \), which yields

\[
\Delta_\alpha^{GH} (\theta_0) = 2 \frac{\tau'}{\tau_0} \left| \frac{\tau'}{\tau_0} \right| (\delta^2) = \frac{2 \cos \theta}{\sqrt{n^2 - \sin^2 \theta}} \left| \frac{\tau'}{\tau_0} \right| (\delta^2) \quad \alpha = s, p.
\]
the sum of (12) and (17) as
\[
\Delta V^G + \Delta a^G = \left( \tau |_{\delta_0}^\prime \right|_{\delta_0} \left( \delta^2 \right) \right) \left( \delta^2 \right)
\]
which will be compared to the FF in section 3.

Of course, the distinction between the angular deflection for the virtual beam and the real transmitted beam is to some degree artificial and a measurement of the mean angle would give a result corresponding to (18). However, actual measurements are often differential, say for different angle would give a result corresponding to (18). However, some degree artificial and a measurement of the mean for the virtual beam and the real transmitted beam is to

A general spectrum a meaningful interpretation in terms of moments of the spectrum is no longer possible in this form, but for a Gaussian beam with spectrum \(\sigma(\delta) = \exp[-(k w_0)^2 (\theta_0 - \delta)^2/4]\), the integrals can be calculated. The angular GH shift for transmission at critical incidence is given by

\[
(\Delta V^G + \Delta a^G)|_{\theta_0} = - \frac{2^{1/4}}{\tan \theta_c} \frac{1}{\sqrt{\Gamma(3/4) \sqrt{k w_0}}},
\]
where \(\Gamma\) is the Gamma function [26].

Comparing the result with Tureci and Stone [3] (see section 3) we find the same dependence on the square root of the width of the beam, but the proportionality factor is different. However, the difference is fairly small, as the factor \(2^{1/4} \approx 0.97\) and therefore close to unity. In their original paper Tureci and Stone state a FF effect of around 30° at critical incidence for a refractive index of \(n = 1/1.56\) and a scaled beam width of \(k w_0 = 8.82\). Evaluating (21) for these parameter values gives an angular deflection of about 29° which is in line with the different factors.

This observation is interesting as the difference in the definition of FF and angular GH as peak and mean angle is of particular importance at critical incidence, as only half of the incident spectrum is transmitted. The transmitted spectrum is therefore no longer Gaussian, which would enhance the difference between the peak and mean angle. A fuller account of the shifts at the critical angle which could explain this behavior is in preparation.

In figure 5 we show a comparison between our formula (21) and a numerical calculation of (20) for two different refractive indices. The agreement is slightly worse than for general incidence, but nevertheless excellent. The fact that the shift is negative and does not depend on the incident
polarization suggest that the angular GH shift at critical incidence is predominantly given by the virtual shift.

3. Fresnel filtering

In this section, we study the beam shift defined for the field, and not for the intensity as in section 2. The deviation from Snell’s law for the field amplitude has been studied by Tureci and Stone, who named this effect ‘Fresnel filtering’ (FF) [3]. To make this paper self-contained, we briefly review their theory. We consider a situation where a Gaussian beam with central incidence angle \( \theta_0 \) is propagating from a thicker medium with refractive index \( n_1 \) to a medium with refractive index 1, such that \( n = 1/n_1 \), and scattered by an infinite planar interface dividing the two media. We adopt the coordinate systems \((x_i, z_i)\) and \((x, z)\) attached to the incident and refracted beams, as shown in figure 3. Within the coordinate system \((x_i, z_i)\), the incident Gaussian beam with a beam waist \( w_0 \) is given by

\[
\psi_i(x_i, z_i) = \frac{E_0 w_0}{w(z_i)} \exp \left[ -\left( \frac{x}{w(z_i)} \right)^2 + ikz_i \right],
\]

where \( w(z_i) = w_0^2 - i\frac{2z_i}{\kappa} \). We assume that the position of the beam waist is located at the interface.

By virtue of the angular spectrum representation, this beam can be expressed as a superposition of plane waves

\[
\psi_i(x_i, z_i) = \frac{k w_0 E_0}{2\sqrt{\pi}} \int ds \sigma(s) \exp \left[ ik (x_i \sin \delta + z_i \cos \delta) \right],
\]

where \( \delta \) depends on \( s \) via \( \sin \delta = s + \cos \delta = \sqrt{1 - s^2} \). The angular spectrum \( \sigma \) is given by \( \sigma(s) = \exp(-k w_0 s^2/2) \). The transmitted field is then given by

\[
\psi_a(x, z) = \frac{k w_0 E_0}{2\sqrt{\pi}} \int ds t_a(s) \sigma(s) \times \exp \left[ ik (x \sin \epsilon + z \cos \epsilon) \right],
\]

for \( a = s(TM), p(TE) \) polarization. Here, \( \delta \) and \( \epsilon \) are related through \( \sin(\theta_0 + \delta) = n \sin(\theta_0 + \epsilon) \), which also defines \( \epsilon \) as a function of \( s \), and \( t_a(s) \) is the appropriate Fresnel transmission coefficient (4), (5) with \( \theta(s) = \theta_0 + \delta(s) \). On using the polar coordinates \( x_i = \rho \sin(\tau - \tau_0) \) and \( z_i = \rho \cos(\tau - \tau_0) \) the refracted beam (24) can be rewritten as

\[
\psi_a(\rho, \tau) = \frac{k w_0 E_0}{2\sqrt{\pi}} \int ds t_a(s) \sigma(s) \times \exp \left[ ik \rho \cos(\tau - \tau_0 - \epsilon) \right],
\]

where \( \epsilon \) depends on \( s \) via Snell’s law. In the far field, that is in the limit \( \rho \to \infty \), the integrand is highly oscillatory and can be evaluated using the saddle-point method. As pointed out by Tureci and Stone, the relevant saddle point is where the cosine in the integrand takes its maximum. This relates the free variable \( \tau \) to \( \tau_0 + \epsilon \) and via Snell’s law also to \( \theta_0 + \delta \). For a given \( \tau \) the saddle point \( s_0 \) is thus determined by solving \( \sin[\theta_0 + \delta(s_0)] = n \sin \tau \) for \( s_0 \). Upon expansion of the exponent, and subsequent integration, the transmitted field is given by

\[
\psi_a(\rho, \tau) \approx \frac{k w_0 E_0}{\sqrt{\pi}} t_a(s_0) \sigma(s_0) J(\tau; s_0) e^{ik\rho},
\]

in the far field. The term \( J(\tau; s_0) \) is defined as

\[
J(\tau; s_0) = \frac{\cos \tau}{\cos \theta} \frac{1}{\sqrt{1 - s_0^2}},
\]

Equation (26) describes the transmitted field as a function of \( \tau \) and for a saddle point \( s_0 \) which, for a given \( \tau \), is fully determined by the incidence angle \( \theta_0 \) and the refractive index \( n \).

From (26), one can estimate the peak position of the refracted beam for certain cases. First, we consider the case of critical incidence (i.e. \( \theta_0 = \theta_c \)), which has already been treated by Tureci and Stone [3]. We put \( \tau = \pi/2 - \epsilon \), with \( \epsilon \) being a small parameter. From \( \sin[\theta_0 + \delta(s_0)] = n \cos \epsilon \), we have

\[
s_0 \approx -\frac{n^2}{2\sqrt{1-n^2}},
\]

Inserting this into \( t_a(s_0) \) and \( J(\tau; s_0) \), we get for both the p(TE) and s(TM) polarization the same proportionality in the leading order in \( \epsilon \)

\[
t_a(s_0) \sigma(s_0) J(\tau; s_0) \propto \epsilon \exp \left[ -\frac{n^2}{4(1-n^2)} \left( \frac{k w_0}{2} \right)^2 \epsilon^4 \right].
\]

The different proportionality factor for \( p \) and \( s \) polarization does not affect the maximum of this expression. Finding the maximum determines the peak position of the electric field as \( \pi/2 - \epsilon \); the magnitude of the FF is then given by the difference between this peak position and the angle expected from Snell’s law:

\[
\Delta_a^{FF} = -\left( 1 - \frac{n^2}{n^2} \right)^{1/4} \frac{2}{k w_0} = -\frac{2}{\tan \theta_c} \frac{1}{\sqrt{k w_0}}.
\]

This is Tureci and Stone’s original result and shows that the angular shift scales as \( (kw_0)^{-1/2} \) at critical incidence. A comparison with the corresponding expression for the angular GH shift at critical incidence (21) is shown in figure 5.

For generic incidence Tureci and Stone did not give an explicit analytic result which could be compared to the sum \( \Delta_v^{GH} + \Delta_a^{GH} \) (see (12) and (17)). In the remainder of this section we therefore derive an expression for the magnitude of the Fresnel filtering for generic incidence. Let us assume that the incident beam is narrow and the incident angle is far from the critical angle (i.e. \( \delta \ll \theta_c - \theta_0 \)), so that the profile of the refracted beam can be expected to be well approximated by a Gaussian distribution. Assuming that the refracted beam is also narrow, we put \( \tau = \tau_0 + \epsilon \), with \( \epsilon \) being a small parameter. Inserting this into \( \sin[\theta_0 + \delta(s_0)] = n \sin \tau \), we have \( s_0 \approx (\cos \tau_0 / \cos \theta_0) n \epsilon \). Thus, the Gaussian distribution \( \sigma(s_0) \) is written as

\[
\sigma(s_0) \approx \exp \left[ -\left( \frac{\cos \tau_0 k w_0}{\cos \theta_0} \right)^2 n^2 \epsilon^2 \right].
\]
Now, let us consider how the functions $J(\tau; s_0)$ and $t_a(s_0)$ shift the above Gaussian distribution. To do this, we expand $\ln J(\tau; s_0)$ and $\ln t_a(s_0)$ in terms of $\epsilon$ as follows:

\[
\ln J = \delta^{(0)} + \delta^{(1)} \epsilon + \delta^{(2)} \epsilon^2 + \ldots,
\]

\[
\ln t_a = \delta^{(0)}_a + \delta^{(1)}_a \epsilon + \delta^{(2)}_a \epsilon^2 + \ldots.
\]

Then, we have

\[
\ln t_a(s_0)\sigma(s_0)J(\tau; s_0) = \left[\delta^{(2)}_a + \delta^{(2)} \right. \left. + (\delta^{(1)}_a + \delta^{(1)}) \epsilon + \ldots\right]
\]

Ignoring the terms higher than $\epsilon^2$ and completing the square on the right hand side, we can rewrite the above equation as

\[
\ln t_a(s_0)\sigma(s_0)J(\tau; s_0) \approx C - \frac{\left(\frac{\cos \theta_0 n k w_0}{\cos \theta_0} \right)^2}{2} \epsilon^2 \ln \left(\frac{\cos \theta_0 n k w_0}{\cos \theta_0} \right)^2 \epsilon^2
\]

\[
\times \left(\epsilon - \Delta^{\text{FF}} \right)^2,
\]

where $C$ is a constant and for $k w_0 \gg 1$. Here, $\Delta^{\text{FF}}$ is given by

\[
\Delta^{\text{FF}} = 2 \frac{\left(\frac{\cos \theta_0}{\cos \theta_0} \right)^2}{\frac{\left(\frac{1}{n k w_0} \right)^2}{2}} (\delta^{(1)}_a + \delta^{(1)}).
\]

Equation (35) tells us that the peak position of the refracted field is not a plane wave, but a spatially localized beam of light. Both effects are based on a modification or “filtering” of the angular spectrum, and are therefore of common nature. Nevertheless the results can be quite different, as shown in figure 4, where the formulas for FF and the angular GH shift predict deflections in opposite directions.

4. Discussion

In this paper we have highlighted the similarities and differences between Fresnel filtering (FF) and the angular Goos–Hänchen shift (GH) on transmission: two related effects which give rise to a deviation from Snell’s law if the incident field is not a plane wave, but a spatially localized beam of light. Both effects are based on a modification or “filtering” of the angular spectrum, and are therefore of common nature. However, different assumptions in the definition of the two effects gives rise to a difference in the magnitude of the deflections. To examine these differences we have extended the existing theories for FF and the angular GH shift. For the former we have derived an analytical expression for the deviation at generic incidence, which is not for the special case of critical incidence, and for the latter we have found an approximate analytic result for the special case of critical incidence.

Before we discuss the results in detail we clearly state the differences in definition. FF has been defined as the difference between the peak transmission angle in the far field and the angle expected from Snell’s law, whereas the angular GH shift on transmission compares Snell’s law with the mean angle. As a consequence, FF uses the amplitude transmission coefficient, whereas the angular GH shift depends on the transmittance, that is transmission coefficient for the energy or intensity.

In light of these differences it may seem surprising to find any similarities, but of course amplitude and intensity, near field and far field are related and we do find terms common to both formulas. Most prominently perhaps the logarithmic derivative of the transmission coefficient in equations (17) and (36), which is a familiar term from reflection. Interestingly, it is not this shared term which explains the very similar result for critical incidence. At this special angle the deviation from Snell’s law calculated in both theories is almost identical, and the remaining difference can be explained as the difference between the mean and peak emission angles. In terms of the competing terms in (18) and (36), however, it is not the common term which is dominant at the critical angle, but, somewhat surprisingly, the ‘virtual’ shift $\Delta^{\text{GH}}$ or the $\delta^{(1)}$ term.

For generic incidence the magnitudes of the angular GH shift and FF can be quite different, though of course the precise difference depends on the angle of incidence. Our choice of $\theta_0$ showcases the differences. Angles of incidence closer to critical incidence lead to smaller differences. As the logarithmic derivative of the transmission coefficient is common to both, it is again the ‘virtual’ term which is responsible for the difference. In the case of the FF this term is always negative (for $n < 1$), and larger in magnitude, which explains the overall negative shift. For the angular GH shift the virtual term is also always negative if $n < 1$, but smaller than the derivative term, which is why the shift turns negative for larger angles of incidence. The fundamental difference may be found in the distinction between amplitude and field, which raises the question whether the difference between the angular GH shift and FF has its roots in the difference between the electric field and the displacement field. An answer to this question, however, is outside the scope of this paper and deserves its own exposition.

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