The Structure and Stability of Massive Hot White Dwarfs

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Abstract

We investigate the structure and stability against radial oscillations, pyconuclear reactions, and inverse β-decay of hot white dwarfs. We consider the fluid matter to be made up of nucleons and electrons confined in a Wigner–Seitz cell surrounded by free photons. It is considered that the temperature depends on the mass density considering the presence of an isothermal core. We find that the temperature produces remarkable effects on the equilibrium and radial stability of white dwarfs. The stable equilibrium configuration results are compared with those for white dwarfs estimated from the Extreme Ultraviolet Explorer survey and the Sloan Digital Sky Survey. We derive masses, radii, and central temperatures for the most massive white dwarfs according to the surface gravity and effective temperature reported by the surveys. We note that these massive stars are in the mass region where general relativity effects are important. These stars are near the threshold of instabilities due to radial oscillations, pyconuclear reactions, and inverse β-decay. Regarding the radial stability of these stars as a function of the temperature, we find that it decreases with the increment of central temperature. We also find that the maximum-mass point and the zero eigenfrequencies of the fundamental mode are determined at the same central energy density. Regarding low-temperature stars, pyconuclear reactions occur in similar central energy densities, and the central energy density threshold for inverse β-decay is not modified. For $T_c \lesssim 10^8$ K, the onset of radial instability is attained before pyconuclear reaction and inverse β-decay.

Unified Astronomy Thesaurus concepts: White dwarf stars (1799); Compact objects (288)

1. Introduction

1.1. Equilibrium Configuration of White Dwarfs

According to the theory of evolution, stars that leave the main sequence with masses below $\sim 10 M_\odot$ end up as white dwarfs (Shapiro & Teukolsky 1983; Weidemann & Koester 1983). These stars start their lives at high temperatures, which build them up as a core—usually composed of oxygen, helium, and carbon—surrounded by an envelope that could be rich in hydrogen (Dufour et al. 2008).

One of the first theoretical studies about white dwarfs based on temperature was developed by Marshak (1940). Considering a star constituted by an isothermal core composed of degenerate matter and an envelope whose temperature distribution depends on the energy generation rate and that is made up of nondegenerate matter, he obtained an estimate of the quantity of hydrogen (in mass percentage) in the envelope of the stars Sirius B and Eridani B. At low densities, the change from degenerate to nondegenerate matter is a perfect environment in which to implement temperature distribution and energy transport (see, e.g., Koester 1972), since this is associated with the effect of the electron temperature.

Usually, in the envelope, it is considered that the energy transport mechanism is realized by radiation or by convection. In the first case, energy transport by radiation is produced by photons, and some models associate luminosity with small nuclear reactions in the envelope, such as $\pi$−$\pi$ and CNO cycles (see, e.g., Bethe & Critchfield 1938; Bethe 1939; Bethe & Marshak 1939). In the second case, energy transport by convection appears to be due to the temperature difference between the core and the envelope. This process has been considered in some white dwarf studies, for instance, those by van Horn (1970), Böhm (1968, 1970), Koester (1972), Fontaine & van Horn (1976), and Hubbard & Wagner (1970). In both cases, energy transport provides a temperature distribution that affects the equilibrium configuration of white dwarfs.

The temperature influence on the structure of white dwarfs has been investigated under diverse conditions. For example, in the Newtonian framework, to optimize the temperature effects, Vavrukh & Smerechynskyi (2012) assumed the thermal energy is proportional to the kinetic energy of the electrons. Considering the Fermi–Dirac equation of states (EOS)—with the Sommerfeld (1928) expansion—the authors found that the static equilibrium configurations derived from their model are within the results estimated by observational data.

A generalization of the work developed in Vavrukh & Smerechynskyi (2012) was published by de Carvalho et al. (2014). Inspired by Rotondo et al. (2011), who once generalized the Feynman–Metropolis–Teller treatment of compressed matter to the case of finite temperature in a Fermi–Dirac EOS, de Carvalho et al. investigated the white dwarf equilibrium configurations at finite temperatures. They found that the correction in the lattice has more influence at low masses. The authors deduced that the onset of the inverse β-decay instability is not altered for temperatures $T \lesssim 10^8$ K; however, higher temperatures could have significant influences on pyconuclear reaction rates within white dwarfs. In addition, they reported that the presence of temperature increases the electric field on the surface of the core of these objects. Based on the EOS derived in de Carvalho et al. (2014), Boshkayev et al. (2016) analyzed the equilibrium configuration of rotating white dwarfs at finite temperatures. They reported that the impact of finite temperatures is relevant in low-mass white dwarfs, as in the estimation of the radii of these objects. Comparing their results with observational data from Sloan Digital Sky Survey (SDSS)
Data Release 4 (Tremblay et al. 2011; Należyty & Madej 2004; Koester & Kepler 2019; Madej et al. 2004), the authors determined that their model can be used to illustrate some white dwarfs detected from the SDSS.

1.2. On the Stability of White Dwarfs

In the study of white dwarfs, an interesting physical property to analyze is their stability against some perturbations. In this sense, it is important to study and analyze how this aspect changes against small radial perturbations, pycnonuclear reactions, and inverse $\beta$-decay since it could give some information about the conditions that allow the existence of white dwarfs.

Since the analysis of the stability of compact objects against small radial perturbations developed in Chandrasekhar’s seminal works (Chandrasekhar 1964a, 1964b), several articles have investigated the radial stability of white dwarfs at zero and nonzero temperatures, without taking into account the Wigner–Seitz cell. A few of them are described as follows. At zero temperatures: In Meltzer & Thorne (1966) the periods and e-folding of the lowest three normal radial pulsations of white dwarf equilibrium configurations at the end point of thermonuclear evolution were investigated. The authors found that at mass densities of $2.5 \times 10^8 < \rho < 1.3 \times 10^9$ [g cm$^{-3}$], there are metastable white dwarfs with e-folding time $\geq 10^{10}$ yr, which corresponds to Hubble time. The radial stability of zero-temperature white dwarfs was also investigated taking into account different values of the mean molecular weight per electron, $\mu_e$, in Wheeler et al. (1968) and accounting for two different adiabatic indices in Chanmugam (1977). These works found a limit to the radial stability for completely degenerate white dwarfs at central mass densities $\rho_c \lesssim 10^{10}$ [g cm$^{-3}$]. At nonzero temperatures: White dwarfs were analyzed by using an analytical approximated EOS for the relativistic Fermi gas (Bisnovaty–Kogan 1966). In that work, Bisnovaty–Kogan found the critical mass of isothermal white dwarfs depends on the central density. Using Chandrasekhar’s equation for radial oscillations, Baglin (1966) reported that the relativistic effect can affect the stability of white dwarfs with low temperatures. Moreover, the author showed that radial instability is attained in lower energy densities compared to those obtained from the classical framework.

Since high-density matter in compact objects must be in chemical equilibrium against nuclear reactions, it is also important to investigate white dwarfs’ stability against both pycnonuclear reactions and inverse $\beta$-reactions.

Pycnonuclear reactions, as schematically expressed by

$$\frac{3}{2} Y + \frac{1}{2} Y \rightarrow \frac{3}{2} \ K,$$

have been studied for white dwarfs by Gasques et al. (2005). The authors developed a phenomenological formalism for pycnonuclear reaction rates between identical nuclei and applied it to the carbon fusion reaction. They also found a limit for carbon burning of $T \sim (4-15) \times 10^8$ [K] for $\rho \lesssim 3 \times 10^9$ [g cm$^{-3}$] and of mass density $\rho \sim (3-5) \times 10^9$ [g cm$^{-3}$] for $T \sim 10^9$ [K]. This type of reaction could be interpreted as an event preceding a Type Ia supernova explosion (Niemeyer & Woosley 1997; Hillebrandt & Niemeyer 2000; Han & Podsiadlowski 2004; Liu et al. 2013; Baron 2014).

An inverse $\beta$-reaction is an instability due to the decay of atom $A(N, Z)$ into $A(N, Z-1)$, $N$ being the mass number and $Z$ the atomic number. This type of reaction has been investigated in the context of white dwarfs, e.g., by Rotondo et al. (2011) and by Mathew & Nandy (2017). In the first article, in the context of general relativity, the authors determined that the inverse $\beta$-reaction occurs above the threshold density estimated for white dwarfs. In the second work, it was found that the heavier the atom element, the lower the instability threshold density.

1.3. Our Aim

In this article, in the framework of general relativity, we study the static structure configuration and stability against small radial perturbations, pycnonuclear reactions, and inverse $\beta$-decay of white dwarfs with finite temperatures. Inspired by previous studies (Timmes & Arnett 1999; de Carvalho et al. 2014; Boshkayev et al. 2016), we model the EOS for hot white dwarfs taking into account a Wigner–Seitz cell composed of electrons and nucleons (Salpeter 1961) surrounded by free photons. We consider that these stars are constituted by an isothermal core, made up of degenerate matter, and an envelope, made up of nondegenerate matter, whose temperature distribution depends on the mass density (see Böhm 1968; Shapiro & Teukolsky 1983). This consideration is reasonable since energy transport occurs by conduction in the core, making the temperature almost constant, and radiation and convection create a temperature distribution in the envelope. For this model, the static equilibrium configurations are investigated through the numerical integration of the Tolman (1939) and Oppenheimer & Volkoff (1939) (TOV) equation. We compare our results for the hot white dwarf structure with those for observable white dwarfs from the Extreme Ultraviolet Explorer (EUV) and SDSS (Vennes et al. 1997; Madej et al. 2004; Należyty & Madej 2004; Tremblay et al. 2011; Koester & Kepler 2019). We estimate the mass, radius, and central temperature analyzing the effective temperature and gravity for massive stars with $M/M_\odot \geq 1.33$ from Vennes et al. (1997). For some central temperatures and surface gravities $\log(g/g_\odot) > 4.4$, we obtain an equation that connects the mass with the surface gravity and effective temperature. Furthermore, we investigate the eigenfrequencies of radial oscillations using the Chandrasekhar (1964a) pulsation equation. We correlate the behavior of radial eigenfrequency oscillations as a function of the star mass and central temperature with the existence of hot and massive white dwarfs. We also study the dependence of some physical white dwarf properties such as the fluid pressure, energy density, mass, radius, and fundamental-mode eigenfrequency on the temperature.

This article is organized as follows: In Section 2 the EOS is described. Section 3 presents both the stellar equilibrium equations and the radial stability equations and their boundary conditions. Section 4 describes the pycnonuclear reactions and inverse $\beta$-decay inside white dwarfs. The results are presented in Section 5. Finally, we conclude in Section 6. Throughout this article we consider the units $c = 1 = G$, where $c$ and $G$ represent the speed of light and the gravitational constant.
2. The EOS

The first assumption about the matter that makes up white dwarfs came from Chandrasekhar’s works (Chandrasekhar 1931, 1935) with the theory of degenerate electron gas. These pioneering works have been extended to include electrostatic energy (Auluck & Mathur 1959) and to insert Thomas–Fermi deviations from a uniform charge distribution of the electrons and the exchange energy and spin–spin interactions between the electrons (Salpeter 1961), which were used to determine white dwarfs’ mass and radius in Hamada & Salpeter (1961). According to Hamada & Salpeter (1961), the electron density of stars is affected by these implementations, decreasing Chandrasekhar’s mass limit. General relativity and temperature effects were investigated concerning the stability of white dwarfs in Binovoy–Kogan (1966), where a critical density and temperature as a function of the star mass were obtained. In that work, it was already concluded that the critical density for the electron capture reaction responsible for the neutralization of white dwarfs should appear before the threshold density of the general relativity instability.

The stability of white dwarfs with Salpeter’s correction was studied by Wheeler et al. (1968) considering pycnonuclear reactions and electron capture reactions, thermonuclear processes, and radial oscillations. The effects of the magnetic field and rotation in white dwarfs were investigated by Ostriker & Hartwick (1968), who found that moderated magnetic fields can increase the radius of white dwarfs. Another approach for the high-density matter EOS was proposed by Feynman et al. (1949) and used to investigate the radial oscillation and stability of white dwarfs by Chanmugam (1977). Lai & Shapiro (1991) in the beginning of the 1990s also investigated the effect of strong magnetic fields in the degenerate EOS, and in particular, in the Baym–Pethick–Sutherland EOS (Baym et al. 1971) used in the neutron star crust. An important review that takes into account all the progress made up to the 1990s concerning white dwarfs’ structure and EOS and also the physical processes in the nondegenerate envelope was written by Koestler & Chanmugam (1990).

A comprehensive review regarding the EOS of white dwarfs and neutron stars was done in Balberg & Shapiro (2000), where condensed matter at extreme densities was discussed. Recently, the Feynman et al. (1949) approximation has been applied in many works on white dwarfs, such as in investigations of white dwarf matter (Bertone & Ruffini 2000; Ruffini 2000), relativistic corrections (Rotondo et al. 2011), and the inclusion of temperature in corrections (de Carvalho et al. 2014). Inspired by Lai & Shapiro’s (1991) work on neutron stars, white dwarfs with magnetic fields have been investigated using electron–ion interactions with a body-centered cubic lattice coordinate system (Otoniel et al. 2019), and also with the addition of face-centered cubic, simple cubic, and hexagonal close-packed lattices (Chamel et al. 2014). Since the EOS used by Salpeter allows us to obtain white dwarf static equilibrium configurations similar to the ones obtained using the other EOSs reported in the previously mentioned works, we have decided to use Salpeter’s approach. In such a way, the total pressure that supports the white dwarf against collapse and the total energy density are, respectively, considered as follows:

\[ P = P_L + P_R + P_i + P_e, \]  
\[ \varepsilon = \varepsilon_L + \varepsilon_R + \varepsilon_i + \varepsilon_e, \]

where \( P \) and \( \varepsilon \) with subscripts \( L, R, i, \) and \( e \) indicate, respectively, the pressure and density of the lattice (with Salpeter (1961) correction), radiation, nucleons, and electrons (Timmes & Arnett 1999). The numerical method to obtain the energy density and pressure contributions of Equations (2) and (3) is explained in detail in Section 5.1.

We consider these stars to be constituted of an isothermal core, made up of degenerate matter, and an envelope, made up of nondegenerate matter, whose temperature distribution depends on the mass density of the form (Böhm 1968; Shapiro & Teukolsky 1983)

\[ T / \rho^{2/3} = \text{constant}. \]  

This temperature–density distribution is valid only for densities below the threshold of degeneracy. The temperature–density distribution obtained from envelope models of carbon white dwarfs by Kritcher and collaborators (Kritcher et al. 2020) is different from the relation in Equation (4). We test the temperature–density profiles of Kritcher et al. (2020) in our EOS and the results are very similar to the ones obtained in our work with Equation (4). In fact, the sensitivity in the EOS of these temperature–density profiles is quite low since in the envelope the ranges of temperatures and densities are quite small compared to the core ones.

3. Stellar Equilibrium Equations and Radial Perturbation Equations

3.1. The Energy Momentum Tensor and the Background Line Element

The fluid that makes up white dwarfs is depicted by the perfect energy momentum tensor, which can be represented in the form

\[ T_{\mu\nu} = (p_0 + \varepsilon_0) u_\mu u_\nu + p_0 g_{\mu\nu}. \]  

\( u_\mu \) and \( g_{\mu\nu} \) stand for the fluid’s four-velocity and the metric tensor, respectively.

The unperturbed line element employed to investigate the equilibrium configuration of hot white dwarfs is of the form

\[ ds^2 = -e^{\nu_0}dt^2 + e^{\lambda_0}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \]  

where \( (t, r, \theta, \phi) \) are the Schwarzschild-like coordinates.

The variables of the fluid \( p_0 \) and \( \varepsilon_0 \), and of the metric \( \nu_0 \) and \( \lambda_0 \) are functions of the coordinates \( t \) and \( r \). When radial stability against small radial perturbations is investigated, following the Chandrasekhar method, the aforementioned functions with subscript 0 can be divided into the form (Chandrasekhar 1964a)

\[ f_0(t, r) = f(r) + \delta f(t, r). \]  

\( f(r) \) denotes the physical quantities of the fluid and the unperturbed metric functions. \( \delta f(t, r) \) represents the Eulerian perturbations that depend on the coordinates \( t \) and \( r \).

3.2. Stellar Equilibrium Equations

The stellar equilibrium equations employed to investigate the configuration of white dwarfs in a static regime, i.e., in an unperturbed system, \( \delta f(t, r) = 0 \), are placed as follows:

\[ \frac{dm}{dr} = 4\pi\varepsilon r^2, \]
\[
\frac{dp}{dr} = -(p + \varepsilon) \left(4\pi rp + \frac{m}{r^2}\right) e^\lambda, \quad (9)
\]

\[
\frac{d\nu}{dr} = -\frac{2}{p + \varepsilon} \frac{dp}{dr}, \quad (10)
\]

with

\[
e^\lambda = \left(1 - \frac{2m}{r}\right)^{-1}. \quad (11)
\]

As usual, the function \(m\) represents the mass within a sphere of a radius \(r\). Equation (9) is known as the hydrostatic equilibrium equation, also called the TOV equation.

After deriving the EOS, Equations (2) and (3), with the aim of looking for equilibrium solutions, we solve simultaneously the stellar equilibrium equations (Equations (8)–(10)). This set of equations is integrated from the center \((r = 0)\) to the star’s surface \((r = R)\). The initial conditions at the center of the star are

\[
m(0) = 0, \quad \varepsilon(0) = \varepsilon_0, \quad T(0) = T_0, \quad and \quad \nu(0) = \nu_0. \quad (12)
\]

The star’s surface is found when

\[
P(R) = 0, \quad (13)
\]

and consequently \(T(R) = 0\). At this point, the interior solution connects smoothly to the spacetime outside the star. This indicates that the interior and exterior metric functions are related as follows:

\[
e^{\nu(R)} = e^{-\lambda(R)} = 1 - \frac{2M}{R}, \quad (14)
\]

where \(M\) represents the total mass of the star. In addition, this relation provides the boundary condition for the functions \(\nu\) and \(\lambda\) at the star’s surface.

### 3.3. Radial Perturbation Equations

To investigate the stability of white dwarfs against small radial perturbations, it is necessary to determine the radial oscillation equations. For such an aim, first, the Eulerian perturbations must be derived. Once the nonzero four-velocity components are defined, the fluid and spacetime variables are decomposed into the form presented in Equation (7). After replacing these definitions and decompositions in the field equations, the Eulerian perturbations are found just keeping the first-order terms.

The radial pulsation equation is derived taking into account the linearized form of the stress energy tensor conservation and the Eulerian perturbations, and considering that the perturbed quantities have a time dependence \(e^{i\omega t}\), where \(\omega\) is the eigenfrequency. This constant parameter is determined through the equality (Chandrasekhar 1964a; Baglin 1966)

\[
\omega^2 = \frac{Z}{D}. \quad (15)
\]

The functions \(Z\) and \(D\) are, respectively, determined by the equations

\[
Z = 4 \int_0^R e^{\nu + \lambda/2} r^3 dp \, dr + 8\pi \int_0^R e^{\nu + 3\lambda/2} p (p + \varepsilon) r^4 dr
\]

\[
+ 9 \int_0^R e^{\nu + \lambda/2} \Gamma pdr - \int_0^R e^{\nu + \lambda/2} r^4 \left[\frac{dp}{dr}\right]^2 dr,
\]

where \(\Gamma\) represents the adiabatic index and it is determined by the condition

\[
\Gamma = \frac{d \log p}{d \log \varepsilon}. \quad (18)
\]

Equation (15) helps us to discriminate stable equilibrium solutions from unstable ones. This is made possible by analyzing the eigenfrequency values \(\omega\).

### 4. Pycnonuclear Reactions and Inverse $\beta$-decay

#### 4.1. Pycnonuclear Reactions

The possible occurrence of pycnonuclear reactions in dense stellar matter (Yakovlev et al. 2005) creates a constraint in white dwarfs’ chemical composition. These reactions are almost independent of temperature and appear even at zero temperature. A rigorous approach to calculating these nuclear reactions was realized by Salpeter & van Horn (1969), who established a ratio of the temperature-dependent pycnonuclear rate to the zero-temperature rate. For a pure-carbon core (Otoniel et al. 2019), where \(^{12}_{\text{C}} + ^{12}_{\text{C}} \rightarrow ^{24}_{\text{Mg}}\), this reaction rate can be written as (Salpeter & van Horn 1969)

\[
\frac{R_{\text{pyc}}(T)}{R_{\text{pyc}}(0)} - 1 = a_1 \lambda^{1/2} \left[1 + a_2 e^{b_1 \beta/2}\right]^{-1/2} \times \exp \left[b_2 \beta^2 + \lambda^{1/2} a_3 e^{b_1 \beta/2} \left[1 - a_4 e^{b_1 \beta/2}\right]\right],
\]

where \(R_{\text{pyc}}(0)\) represents the pycnonuclear reaction at zero temperature, and \(a_1, a_2, a_3, a_4, b_1, b_2\) are model-dependent dimensionless constants.

The time to complete atomic nuclear fusion is obtained from previous studies (Gasques et al. 2005; Boshkayev et al. 2013; Otoniel et al. 2019):

\[
\tau_{\text{pyc}} = \frac{n_N}{R_{\text{pyc}}}. \quad (20)
\]

As considered in Otoniel et al. (2019), we employ a pycnonuclear reaction time \(\tau_{\text{pyc}} = 10\) [Gyr], which corresponds to an upper limit where pycnonuclear reactions are extremely slow (Chugunov et al. 2007). For white dwarfs, a limit of \(\rho_{\text{pyc}}(10\text{[Gyr]})\) corresponds to a maximum mass density for stable stars against pycnonuclear reactions. Through these times, we can estimate the mass density where these reactions appear, \(\rho_{\text{pyc}}(\tau_{\text{pyc}})\). For white dwarfs, the limit of \(\rho_{\text{pyc}}(10\text{[Gyr]})\) corresponds to a maximum density for stable stars, since from this point pycnonuclear reactions begin to appear.
4.2. Inverse $\beta$-decay

It is known that the matter inside white dwarfs may experience instability against the inverse $\beta$-decay process,

$$A(N, Z) + e^- \rightarrow A(N + 1, Z - 1) + \nu_e.$$  \hspace{1cm} (21)

Due to this process, atomic nuclei become more neutron-rich, and as a consequence, the electron energy density and pressure are reduced thus leading to a softer EOS (Gamow 1939; Shapiro & Teukolsky 1983). Since we are considering a nucleus of $^{12}$C, the instabilities are reached at energies higher than $e_{\beta}^Z = 13.370$ [MeV] (see Rotondo et al. 2011; de Carvalho et al. 2014; Otoniel et al. 2019).

5. Equilibrium and Stability of Hot White Dwarfs

5.1. Numerical Method

Due to the partial degeneracy considered in this model, the pressure and energy of electrons appearing on the EOS (Equations (2) and (3)) are solved numerically by means of the adaptive quadrature method. Through this numerical method, we reproduce the results reported in Vavruk & Smerechynskyi (2012), which resolved the same EOS through the Sommerfeld approximation, and the results found in de Carvalho et al. (2014) and Boshkayev et al. (2016), where the authors investigated the equilibrium of white dwarfs with a constant finite temperature.

Once the EOS is defined, both the stellar equilibrium equations, Equations (8)-(10), and the radial stability equations, Equations (16) and (17), are integrated from the center $(r = 0)$ toward the surface of the spherical object $(r = R)$ through the Runge–Kutta fourth-order method for different values of $\varepsilon_c$ and $T_c$ and a trial value for $\nu_c$.

The numerical solution of the stellar structure equations begins with the initial conditions (12) at $r = 0$. Once $\varepsilon_c$, $T_c$, and $\nu_c$ are given, the integration proceeds from the center toward the star’s surface where $P(R) = 0$. Nonetheless, if after the integration the condition of Equation (14) is not satisfied, $\nu_c$ is corrected through a Newton–Raphson iteration scheme until it fulfills this condition. Thus, the zero fluid pressure determines the total radius $R = R_{\odot}$ and total mass $M = m(R)$.

After the coefficients $p$, $\varepsilon$, $m$, $\lambda$, and $\nu$ are determined for each $\varepsilon_c$, $T_c$, and correct $\nu_c$, the radial stability equations are integrated from the center to the surface of the star. After the integration, the eigenfrequency squared is found by Equation (15).

5.2. Influence of Temperature on the Fluid Pressure, Energy Density, and Mass of the Star

With the purpose of observing the EOS behavior, Figure 1 plots the change of the fluid pressure against the energy density for different central temperatures. The energy density employed goes from $10^3$ [g cm$^{-3}$] to $10^{11}$ [g cm$^{-3}$].

In Figure 1, in all cases presented, it can be observed that the pressure decays monotonically with the energy density. Moreover, the effects of central temperature are noted in the graph. At low energy densities, the pressure has a slight growth with central temperature. This increment is associated with the increase of the radiation pressure, and the pressure of the nucleons.

At the central energy density $10^4$ [g cm$^{-3}$] and central temperature interval $[10^4, 10^7]$ [K], we find fluid pressures similar to those reported by de Carvalho et al. (2014); namely, we derive $p_c$ in the range $[1 \times 10^{19}, 2 \times 10^{19}]$ [erg cm$^{-3}$]. This result can be understood to be due to the fact that the temperature distribution is constant in the aforementioned range of $\varepsilon$ and $T_c$, and the treatment assumed in this work (Salpeter correction) and the one employed by Carvalho et al. (Feynman–Metropolis–Teller treatment) are similar. In addition, the central pressures found are lower than the one derived by Boshkayev et al. (2016). This could be associated with the fact that we are considering the Wigner–Seitz lattice correction.

The star mass as a function of the radial coordinate is presented in Figure 2 for a white dwarf with a total mass of $1.37 M_{\odot}$ and several central temperatures. From this figure, we conclude that for a fixed star mass, the effect of the temperature is still important for very massive white dwarfs: when the temperature increases, the star radius increases, and this effect is more pronounced for central temperatures $T_c \geq 10^8$ [K]. For the extreme case of a central temperature $T_c = 10^8$ [K] the radius increases by 135% as compared to $T_c = 10^4$ [K] due to the nucleons’ pressure. This result was not observed for near-Chandrasekhar-mass white dwarfs with finite temperatures in a previous study (Boshkayev et al. 2016), since this thermal pressure was not taken into account. This is an important observation since very massive white dwarfs, with larger radii
than the ones derived for low temperatures \((T_c \lesssim 10^4 \, [K])\), are an indication of lower surface gravity compared to the ones observed for cold white dwarfs (since \(g \sim 1/r\)), and of high central temperature. In summary, it is important to consider the mass–radius relation obtained at finite temperature in the case of very massive white dwarfs, in order to obtain the correct mass and star radius from the observed surface gravity and effective temperature values.

5.3. Equilibrium Configurations of Hot White Dwarfs

5.3.1. Hot White Dwarf Equilibrium Configuration Sequence

The gravitational mass, in solar masses \(M_\odot\), as a function of the central energy density is plotted in Figure 3 for different central temperatures. It is considered at central energy densities within the interval \(10^5 \, [g \, cm^{-3}] \leq \varepsilon_c \leq 10^7 \, [g \, cm^{-3}]\). The filled triangles placed over the curves mark the points where the maximum masses are attained.

In Figure 3, in all central temperatures considered, we note that the curves present two branches. In the first one, the mass grows monotonically with \(\varepsilon_c\) until it reaches its maximum value \((M_{\text{max}}/M_\odot)\); after this point, the curve turns downward for the mass starts to decay with the growth of \(\varepsilon_c\). The central energy density used to determine the maximum-mass points coincides with the \(\varepsilon_c\) employed to find the eigenfrequency of the fundamental mode \(\omega = 0\). This indicates that the maximum-mass point divides regions constituted by stable equilibrium configurations from regions established by unstable ones. Thus, regions made up of stable and unstable equilibrium configurations against small radial perturbations are differentiated through the inequalities \(dM/d\varepsilon_c > 0\) and \(dM/d\varepsilon_c < 0\), respectively. These conditions are necessary and sufficient to recognize a stable region from an unstable one.

Equilibrium configurations with energy densities lower than those considered in curves with \(T_c \gtrsim 5 \times 10^7 \, [K]\) are also analyzed. As in previous references (de Carvalho et al. 2014; Boshkayev et al. 2016), it is found that the total mass decreases with the increment of the central energy density. Moreover, these equilibrium configurations have an eigenfrequency of the fundamental mode close to zero \((\omega \sim 0)\).

The behavior of the total mass as a function of the radius is plotted in Figure 4 for a few different central temperatures. The filled triangles in pink over the curves indicate the maximum-mass points. The largest total radii shown in each curve are derived from the respective minimum central energy density value considered in each curve of Figure 3. As already mentioned, for lower central energy densities than those mentioned, the eigenfrequency of the fundamental mode is close to zero. Moreover, in the figure, some observational results obtained from the catalogs in Tremblay et al. (2011), Należyty & Madej (2004), Koester & Kepler (2019), Vennes et al. (1997), and Madej et al. (2004) are, respectively, marked with gray triangles, purple circles, green diamonds, orange squares, and blue hexagons.

The central temperature’s influence on the total mass and radius is noted in Figure 4. In all curves, the mass grows monotonically with the diminution of the total radius until it reaches \(M_{\text{max}}/M_\odot\). After this point, the \(M(R)\) curves turn upward as the masses start to decrease with the radii’s diminution. In the figure, a large group of white dwarfs detected is placed below \(M = 1.3 \, M_\odot\) and \(T_c = 10^6 \, [K]\) and a small group is located in higher masses and central temperatures. These results could indicate that the mass of a white dwarf is associated with the central temperature in its core, i.e., for higher central temperatures, more massive white dwarfs are found. The growth of the mass with the temperature could be understood by noting that some factors that compose the total fluid pressure—the pressures coming from the radiation and nucleons—increase with \(T_c\). This increment in the pressure helps to support more mass against collapse. Near the maximum-mass limit, we notice that some of the white dwarfs observed are within the range of masses with high central temperature. In particular, they present a small gravity value compared to low-temperature white dwarfs.

Comparing our results with the ones reported by Boshkayev et al. (2016) for a star made up of \(^{12}\text{C}\) and having \(\mu_e = 2\), with a central energy density of \(10^6 \, [g \, cm^{-3}]\) and temperature \(T = 10^6 \, [K]\), we find that the mass of the star is smaller by 7\% and the radius is larger by 23\%, and for a temperature of \(T = 10^7 \, [K]\), what is derived is nearly the same total mass and a
radius greater by 34%. As can be seen, the mass derived in this work is in good agreement with the one reported by Boshkayev et al. (2016), unlike the total radius, which differs significantly. The radii derived are larger due to the inclusion of the nucleons’ pressure, thermal energy, lattice corrections, and electron energy contributions in the EOS. In the maximum total mass range, we find that the white dwarfs’ masses depend on central temperature, unlike the ones published in Boshkayev et al. (2016), where the total masses stay independent of the temperature. For instance, for \( T_c = 10^9 \text{[K]} \), we find a maximum mass value of 1.40 \( M_\odot \) and a respective total radius of 1400 [km], while Boshkayev et al. found a white dwarf with a mass of 1.43 \( M_\odot \) and a radius of 1000 [km]. This difference is associated with the nucleons’ pressure effects, which are still important for massive white dwarfs, and not only for low-mass ones. It is the first time that the nucleons’ pressure is taken into account in a study of massive white dwarfs; it was previously only considered in the analysis of low-mass white dwarfs (de Carvalho et al. 2014).

The gravity at the white dwarf surface versus the gravitational mass is shown in Figure 5 for different central temperatures. The filled triangles over the curves mark the maximum-mass points. Observational results obtained from the catalogs in Tremblay et al. (2011), Należyty & Madej (2004), Koester & Kepler (2019), Vennes et al. (1997), and Madej et al. (2004) are, respectively, indicated by gray triangles, purple circles, green diamonds, orange squares, and blue hexagons.

The results reported in Figures 4 and 5 are important in the cooling study of hot \( T_c > 10^7 \text{[K]} \) and very massive white dwarfs \( M > 1.37 M_\odot \). In the common cooling process, these stars shrink, thus keeping constant the baryonic mass (Althaus et al. 2010) and increasing their densities. In our curves with constant gravitational mass, this would occasion them to run from a stable region to an unstable one (according to the threshold of instabilities due to radial oscillations, pycnocnuclear reactions, and inverse \( \beta \)-decay; see Section 5.4). This evolutionary instability could explain the star collapse and justify an interesting mass limit for observable white dwarfs. Besides, as we consider the gravitational mass and not the baryonic one for this analysis, such collapse would not occur if they lose enough gravitational mass, thus allowing a stable cooling. To obtain a robust conclusion regarding this possible mechanism originating Type Ia supernova explosions further investigations are needed.

In Table 1, the central temperatures \( T_c \) used and the maximum white dwarf mass values and their respective radii and central energy densities are shown. It is found that for central temperatures in the range \( 10^4 < T_c \lesssim 10^7 \text{[K]} \), near the maximum total mass, white dwarf masses remain nearly constant. At this range, the total pressure stays nearly constant with the increment of temperature, since the electron pressure decays with \( T_c \) and the radiation pressure and the pressure of nucleons do not contribute considerably to the white dwarf’s structure. In contrast, for \( T_c \gtrsim 10^7 \text{[K]} \), an increase in total mass is observed. At this central temperature range, the contributions of \( P_e \) and \( P_N \) produce considerable effects on the white dwarf’s structure. Thus, the white dwarf’s mass at \( T_c \gtrsim 10^7 \text{[K]} \) is larger than \( M/M_\odot \) at \( 10^4 < T_c \lesssim 10^7 \text{[K]} \). On the other hand, in all cases analyzed here, at the maximum masses, it is found that the respective total radii and central energy densities increase and decrease with temperature, respectively.

5.3.2. The Procedure to Identify Central Temperatures in Massive White Dwarfs

The existence of some very massive white dwarfs has been shown by Vennes et al. (1997). According to their analysis, they obtained the effective temperature and gravity and used a fitting method to estimate the mass of such stars. In order to verify our model, we use the effective temperature and gravity reported in Vennes et al. (1997) to fit our curves. In our model, the effective temperature is obtained using the Stefan–Boltzmann law in the photons’ luminosity, which depends on the elements’ mass contributions (see Shapiro & Teukolsky 1983).

In Table 2, we report the data from Vennes et al. (1997) for some observational white dwarfs with \( M_{\text{He}} = 10^{-4} M_\odot \) and their mass, radius, central temperature, and \( \alpha_0 \), with \( \alpha \) being a dimensionless parameter that relates the effective temperature,
gravity, and central temperature by means of the following relation (Koester 1976):

\[
\frac{T_{\text{eff}}^4}{g} = 2.05 \times 10^{-10} \alpha T_c^4.
\]

The index \(\alpha\) takes the value of 2.56 in Koester (1976).

From the results in Table 2, we can note that our mass presents values in the same range as the ones reported in Vennes et al. (1997), except for WD 1659 + 440 since Vennes et al. (1997) found a mass of 1.41 ± 0.04 \(M_\odot\) for it and we obtain 1.33 ± 0.01 \(M_\odot\). This is due mainly to general relativity effects, which are more remarkable at large total masses. For all stars analyzed, we note that our radius results are within the range of the values reported in Koester (1972), \(\alpha = 2.50\), and Koester (1976), \(\alpha = 2.56\). The central temperature related to these massive white dwarfs suggests that some of them like EQ J0443 - 037, EQ J0916 - 197, and EUVE J1535 - 77.4 can have a high central temperature.

In Table 3, we analyze the same white dwarfs shown in Table 2 but considering \(M_{\text{He}} = 10^{-2}\) and \(M_{\text{H}} = 10^{-4}\). The results in mass and radius are very similar to the ones found in Vennes et al. (1997), except for WD 0346 - 011. This is due to the effects of general relativity being more remarkable at large total masses. For the stars in Table 3, we find \(\alpha = 2.54 \pm 0.02\). This is very similar to the one reported in Koester (1976) and used by Boshkayev et al. (2016), i.e., \(\alpha = 2.56\). The central temperatures reported in this table are lower than the ones reported in Table 2, due to the photons’ luminosity, which increases with the hydrogen contribution. In Tables 2 and 3, we find that all massive observable white dwarfs have masses \(M \lesssim 1.35 \, M_\odot\). In the cooling process, these stars would not run from stable to unstable regions. On the contrary, they are likely to cool in the common white dwarf evolutionary track.
In Figure 6 we show the mass as a function of radius for some temperatures. The dashed curves are the same ones obtained in Figure 5 considering a TOV relativistic equation. The continuous curves are obtained using the Newtonian formulation (Carvalho et al. 2018). The green points represent the source WD 1659 + 440 with a helium envelope and the pink ones represent WD 0346 – 011 with a helium and hydrogen envelope. The unfilled points are obtained according to Vennes et al. (1997) and the filled ones are obtained with our fitting.

From Figure 6, as obtained in Carvalho et al. (2018) for cold white dwarfs, at the range of maximum white dwarf masses, for a fixed central temperature $T_c$, we obtain smaller masses at the general relativity scope than in the Newtonian formulations. In addition, when the central temperature is increased, the total mass stays closer to the masses of WD 1659 + 440 and WD 0346 – 011. On the other hand, at a fixed total mass, we find a smaller total radius in the relativistic case than in the classical one.

We propose an equation that relates mass and gravity values according to the central temperature. For such an equation, we fit the curves in Figure 5 in Fourier second-order equations to obtain the relation

$$M(g) = \frac{k_0 \left( \log \left( \frac{g}{g_\odot} \right) \right)^2 + k_1 \left( \log \left( \frac{g}{g_\odot} \right) \right) + k_2 \log \left( \frac{g}{g_\odot} \right) + k_3}{g_\odot}, \quad (23)$$

where $g_\odot$ is the Sun’s surface gravity; $k_0$, $k_1$, and $k_2$ are parameters in solar masses $M_\odot$; and $k_3$ is a dimensionless constant. This relation is valid for surface gravity values $\log \left( \frac{g}{g_\odot} \right) \geq 4.4$. The fits implemented have $R$-squared = 1. As shown in Table 4, the parameters $k_0$, $k_1$, $k_2$, and $k_3$ depend on the central temperature $T_c$. For different $T_c$, the new numerical values for the parameters from Equation (23) can be obtained by interpolating the curves $M(g)$ from Figure 5.

### 5.4. Stability of Hot White Dwarfs

#### 5.4.1. Stability of Hot White Dwarfs against Small Radial Perturbations

The very massive white dwarfs reported in Vennes et al. (1997) have masses in the range of instabilities caused by radial oscillations, inverse $\beta$-decay, and pycnonuclear reactions. This, with their central temperatures that reach very high values, near $10^8$ [K], made us realize the importance of investigating the stability of hot white dwarfs.

The behavior of the eigenfrequency squared with the central energy density $\omega^2(\varepsilon_c)$ and with the total mass $\omega^2(M/M_\odot)$ is plotted on the top and bottom panels of Figure 7, respectively, for some central temperatures. All curves $\omega^2(\varepsilon_c)$ and $\omega^2(M/M_\odot)$ show Gaussian behavior, where the heights of the curves’ peaks are, respectively, found in the range of central energy densities $1.0 \times 10^8 \lesssim \varepsilon_c \lesssim 5.0 \times 10^{10}$ [g cm$^{-3}$] and total masses $1.20 \lesssim M/M_\odot \lesssim 1.40$. From the curves $\omega^2(M/M_\odot)$, it can be seen that the points of maximum mass are reached at $\omega^2 = 0$. In addition, it is important to say that the curves $\omega^2(\varepsilon_c)$ derived for $T_c \leq 10^6$ [K] are similar to the ones reported by Wheeler et al. (1968) and Chanmugam (1977).

The influence of temperature on radial stability can also be observed in Figure 7. In both panels of the figure, it can be noted that the increment of $T_c$ decreases the squared eigenfrequency of the fundamental mode; this indicates that hotter white dwarfs will have lower stability. In fact, in our fitting for the stars reported in Tables 2 and 3 we also calculate their fundamental eigenfrequency. For white dwarfs composed of $M_{16} = 10^{-4} M_\odot$, we find $2.5 \leq \omega^2 \leq 4.4$ [rad$^2$/s$^{-2}$] and for those made up of $M_{14} = 10^{-4} M_\odot$ and $M_{16} = 10^{-2} M_\odot$, we find $2.8 \leq \omega^2 \leq 4.6$ [rad$^2$/s$^{-2}$]. Furthermore, most of the observable white dwarfs reported in previous studies (Vennes et al. 1997; Madej et al. 2004; Należyty & Madej 2004; Tremblay et al. 2011; Koester & Kepler 2019), i.e., white dwarfs with masses within $0.3 \leq M/M_\odot \leq 1.3$, have an eigenfrequency of oscillation in the interval $0 < \omega^2 \leq 4.5$ [rad$^2$/s$^{-2}$].

### Table 4

Central Temperatures and the Parameter Values Appearing in Equation (23)

| $T_c$ (K) | $k_0 (M_\odot)$ | $k_1 (M_\odot)$ | $k_2 (10 M_\odot)$ | $k_3$ |
|----------|-----------------|-----------------|-------------------|-------|
| $1.0 \times 10^6$ | $-0.355$ | $5.400$ | $-1.442$ | $-2.187$ |
| $2.0 \times 10^6$ | $0.347$ | $5.293$ | $-1.402$ | $-2.162$ |
| $5.0 \times 10^7$ | $-0.333$ | $5.076$ | $-1.277$ | $-1.845$ |
| $1.0 \times 10^8$ | $-0.324$ | $4.894$ | $-1.137$ | $-1.414$ |

### Figure 7

Square of eigenfrequency $\omega^2$ as a function of the central energy $\varepsilon_c$ and of the total mass, shown on the panels at the top and bottom, respectively, for some central temperatures.
5.4.2. Stability of Hot White Dwarfs against Pycnonuclear Reactions and Inverse β-decay

Recently, Otoniel and collaborators (Otoniel et al. 2019) discussed the observation that the threshold central energy density, at which pycnonuclear reactions occur, is obtained by taking into account $\tau_{\text{pvc}} = 10$ [Gyr]. In our model, at zero temperature, pycnonuclear reactions occur at the threshold density of $9.56 \times 10^9$ [g cm$^{-3}$], being a very close value to the one derived in Otoniel et al. (2019). In contrast to that for pycnonuclear reactions, the threshold of central density for inverse β-decay instabilities is $3.52 \times 10^9$ [g cm$^{-3}$], close to the ones derived in Rotondo et al. (2011) and Otoniel et al. (2019).

The behavior of the total mass with the central energy density, at large total masses, is shown in Figure 8 for some central temperatures. In the figure, we present the thresholds where instabilities occur against pycnonuclear reactions and inverse β-decay, in the gray shaded regions, and the places where the radial instability begins, marked by pink filled triangles.

Table 5 shows the threshold energy density values for instability against pycnonuclear reactions, inverse β-decay, and radial oscillations for four central temperature values. From the results, we can note that the increase of temperature more remarkably affects stability against small radial perturbations. Moreover, for central temperatures higher than $1 \times 10^8$ [K], radial stability is attained before the ones produced by pycnonuclear reactions and inverse β-decay.

6. Conclusions

In this article, the static equilibrium configuration and stability against small radial perturbations, pycnonuclear reactions, and inverse β-decay in white dwarfs with a finite temperature are studied. Following previous studies (Tolman 1939; Timmes & Arnett 1999; Rotondo et al. 2011), with respect to the matter within white dwarfs, we take into account that it is made up of nucleons and electrons confined in a Wigner–Seitz cell surrounded by free photons. Moreover, with the purpose of obtaining null radiation pressure at the star’s surface, a temperature distribution is considered in the nondegenerate envelope. We assume both temperature dependence on the mass density and the existence of an isothermal degenerate core. The static configurations under analysis have spherical symmetry and are connected smoothly with the Schwarzschild exterior spacetime. The hydrostatic configuration and stability are investigated for different central energy densities $\varepsilon_c$ and central temperatures $T_c$.

We find that the necessary central temperature to influence the static structure and radial stability of very massive white dwarfs is approximately $T_c = 10^7$ [K]. For a fixed central energy density, we find that white dwarfs’ radius and total mass grow with the increase of the central temperature (for $T_c > 10^7$ [K]). The temperature effects in the static equilibrium configurations are in concordance with the ones obtained in the study of low-mass white dwarfs with a finite temperature reported in de Carvalho et al. (2014) but are different from the ones obtained in Boshkayev et al. (2016) for very massive white dwarfs. Analyzing the mass and radius according to the central temperature reported in this work, we can understand that hot ($T_c > 10^7$ [K]) and massive white dwarfs ($M > 1.37 M_\odot$) could collapse. In this sense, further studies of the cooling of massive white dwarfs considering general relativity and instabilities due to radial perturbations, pycnonuclear reactions, and inverse β-decay are needed to conclude if they are likely to cool or collapse.

For some massive white dwarfs, we derive the mass and radius by fitting both the gravity and effective temperature reported by the observation. This is done by assuming some white dwarfs are composed of $M_{16e} = 10^{-4} M_\odot$, and other ones are made up of $M_1 = 10^{-4} M_\odot$ and $M_{16e} = 10^{-3} M_\odot$. The results show that some white dwarfs could have central temperatures above $5 \times 10^7$ [K]. Besides, from our results, we note that the masses and radii are in the same range as those of the white dwarfs reported in Vennes et al. (1997), with the exception of EUVE J0003 + 43.6 and WD 1659 + 440. For these two stars, we find masses below the ones reported in Vennes et al. (1997). This is associated with the relativistic effects on these two massive stars. Furthermore, considering general relativity effects, we derive an equation that facilitates finding mass values from surface gravity and effective temperature values for observable massive white dwarfs with surface gravity $\log \left( \frac{g}{g_\odot} \right) > 4.4$.

We have conducted a novel study concerning the stability of white dwarfs with a finite temperature. For a central energy density interval, we find that the radial stability of white dwarfs diminishes with the increment of the central temperature. Moreover, we find that the maximum mass and zero eigenfrequencies of oscillation are attained at the same central energy density. This points out the fact that in a system of equilibrium star configurations at finite temperature, the regions

| $T_c$ (K) | $\varepsilon_{\text{pvc}}$ ($10^9$ g cm$^{-3}$) | $\varepsilon_{\beta}$ ($10^9$ g cm$^{-3}$) | $\varepsilon_{\text{fi}}$ ($10^9$ g cm$^{-3}$) |
|-----------|---------------------------------------------|---------------------------------------------|---------------------------------------------|
| 0.1       | 0.878                                       | 3.515                                       | 1.744                                       |
| 1.0       | 0.878                                       | 3.515                                       | 1.622                                       |
| 5.0       | 0.876                                       | 3.515                                       | 1.118                                       |
| 10        | 0.874                                       | 3.515                                       | 0.675                                       |
formed by stable and unstable white dwarfs can be recognized by the inequalities $dM/d\varepsilon_\gamma > 0$ and $dM/d\varepsilon_\delta < 0$, respectively. On the other hand, unlike the threshold energy density of instability for inverse $\beta$-decay, which does not change with temperature, the energy density threshold of instability for pycnonuclear fusion reactions is reduced when temperature in the stellar interior is present. Besides, for central temperatures higher than $1 \times 10^9$[K], we determine that instability due to small radial perturbations occurs before those produced by pycnonuclear reactions and inverse $\beta$-decay.

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