Research Article

Additive Slack-Based Measure for a Two-Stage Structure with Shared Inputs and Undesirable Feedback

M. Salahi 1,2, A. Jamalian 3, R. Shakouri 4, and K. Asanimoghadam 1,2

1 Department of Applied Mathematics, Faculty of Mathematical Sciences, University of Guilan, P.O. Box 1914, Rasht, Iran
2 Center of Excellence for Mathematical Modeling, Optimization and Combinatorial Computing (MMOCC), University of Guilan, P.O. Box 1914, Rasht, Iran
3 Department of Computer Science, Faculty of Mathematical Sciences, University of Guilan, P.O. Box 1914, Rasht, Iran
4 Department of Computer Engineering, Faculty of Mathematical Sciences, Technical and Vocational University (TVU), Tehran, Iran

Correspondence should be addressed to M. Salahi; salahi.maziar@gmail.com

Received 11 April 2022; Revised 27 May 2022; Accepted 8 June 2022; Published 11 July 2022

The two-stage data envelopment analysis models are among the widely used mathematical programming approaches to evaluate the performance of two-stage structures. In this paper, a two-stage structure with shared inputs and feedback is studied. To reduce undesirable outputs, an additive slack-based measure model is proposed to evaluate the stages and overall efficiencies, while undesirable outputs are weakly disposable. As it does not require determining the weights for combining stages' efficiencies, all Pareto optimal stages' efficiencies can be gained. In addition, the proposed approach can identify desirable outputs from undesirable outputs, thereby avoiding the need for weighting. This advantage from the aspect of multiobjective programming helps internal evaluation of the network model to match priorities of managers. The proposed nonlinear model is reformulated as a second-order cone program, which is a convex optimization problem that can be solved to global optimality. This is a computational improvement over the parametric models in the literature. Furthermore, the proposed model is applied for country-wise and area-wise performance evaluations of a real industrial application dataset in mainland China. Results show that the efficiency of the overall system relies between the efficiencies of the two stages and for all DMUs, the first stage's efficiency scores are always higher than the second stage ones in both evaluations. Also, the Pearson correlation coefficient test results show that the overall efficiency is more correlated with the waste disposal stage. Finally, to show the effect of the decision maker's preference, a detailed sensitivity analysis is performed.

1. Introduction

Data envelopment analysis (DEA), proposed by Charnes et al. [1], is a nonparametric approach to measure the relative efficiency of decision-making units (DMUs). In most basic DEA models, a DMU is considered as a black box and its internal structure is ignored. However, there are extensive applications where DMUs include subunits and ignoring them may lead to incorrect evaluations, see Kao and Hwang [2]. To deal with such systems, network DEA (NDEA) models were first proposed by Charnes et al. [1] and further developed by [3–6]. This approach consider DMUs as a system composed of some stages characterized by their own inputs and outputs and related by intermediate measures [7]. In the sequel, some recent developments of NDEA models are summarized, the interested readers may see Kao [8] for the most NDEA models with their applications.

Nemati et al. [9] considered a partial impact between inputs and outputs in a two-stage network production system. To this aim, they formulated several new models in the DEA framework for calculating aggregate, overall, and subunit efficiencies along with resource usage by production lines. Chen et al. [10] presented a conic relaxation approach for searching the global solution of the NDEA model. Esfandi et al. [11] proposed an additive model to measure the multiperiod efficiency of the two-stage systems under the
constant returns to scale assumption. Based on the computed efficiencies, the new efficiency change indexes of the whole system and the first and second stages between two are also developed. Chen and Zhu [12] successfully applied the additive slack-based measure (ASBM) model of Green et al. [13] to formulate NDEA models from both internal and external perspectives. They further reformulated the proposed ASBM models as second-order cone programs (SOCPs) that are convex optimization problems. An important feature of the ASBM model is adapting the preference of the decision maker by choosing the weights for aggregating individual components in the network structures, which is not the case in the SBM models. Esfandi et al. [14] used a network SBM model to measure the efficiency of a system with a multiperiod two-stage structure and applied it to evaluate bank branches in Iran. Recently, Torabi Golsefid and Salahi [15] presented a conic relaxation approach to solve multiplicative efficiency decomposition for a NDEA structure.

From the view of stage attributes of the production activity, undesirable output's verification in DEA has extended from the early black-box production investigation to multistage research. In black-box activity investigation, the DEA approaches dealing with undesirable outputs, generally include the SBM model of Tone [16] and slack-based inefficiency measure of Fukuyama and Weber [17]. Liu et al. [18] performed a detailed revision about undesirable output investigation in the black-box DEA approach, but when it gets to the multistage process, the former approaches are no longer enforceable. Hence, Liu et al. [19] expanded the radial scale to a network radial analysis model. Cui et al. [20] used a virtual frontier SBM model to show the effect of carbon dioxide emissions on the efficiency of 22 airlines with both strong and weak disposability. Maleki et al. [21] in a two-stage network in presence of undesirable intermediate products and nondiscretionary exogenous inputs, by focusing on the principle of mathematical dominance, proposed new models for evaluating overall and stages' efficiencies. [22] offered a NDEA model using the DEA and Stackelberg game for measuring efficiency of DMUs under undesirable outputs. The authors used goal programming and defined a sort of collaboration between the leader and follower, in order to append the objectives of the administrators in the models. Hu et al. [23] proposed a new two-stage SBM model with undesirable outputs and a feedback variable from an oil industry structure. In another study, Cui et al. [24] represented network interval SBM with strong disposability to measure the efficiencies of 24 global airlines. Yang et al. [25] proposed a dynamic network SBM model to study regional industrial water systems that wastewater is considered as an undesirable intermediate output and total volume of industrial wastewater discharged is supposed to be an undesirable output. Shi et al. [26] proposed a network SBM model with undesirable outputs to evaluate the performance of production process with both series and parallel processes.

In this paper, a two-stage structure including shared inputs and feedback is considered that frequently arises in industrial applications. Some outputs of the first stage are undesirable and are processed to the second stage to obtain desirable resources that are immediately fed back into the first stage as inputs. The SBM NDEA model for such structures needs specific weights of stages' in order to be linearized. This might be a drawback from the decision maker's point view which ignores adapting preferences. To tackle this issue, we propose an ASBM model under the weak disposability of Kuosmanen [27] to reduce undesirable outputs. The contributions of the paper are as follows:

(i) Developing an ASBM model from the internal perspective to evaluate DMUs' performance in the presence of shared inputs and feedback
(ii) The SOCP reformulation of the nonlinear ASBM model which is a convex optimization problem and can be solved efficiently to global optimality
(iii) The application of the proposed model to a real dataset from an industrial application and its sensitivity analysis
(iv) Performing the Pearson correlation coefficient test to show the correlations between the stages' and overall efficiencies

The rest of the paper is organized as follows. In Section 2, the related literature review is given. In Section 3, the production possibility set (PPS) for the two-stage structure is introduced. Then, the ASBM model in the presence of shared inputs and feedback is discussed. Section 4 also discusses the application of the ASBM model for a real dataset of 30 provincial-level regions in China to evaluate the efficiency of industrial production processes in these regions. Finally, conclusions are drawn in Section 5.

2. Literature Review

In a two-stage structure, DMUs may include shared inputs and feedback. In the sequel, some two-stage DEA models dealing with shared inputs, feedback, or both of them are summarized. Zha and Liang [28] offered an approach to deal with shared flow in a two-stage DEA structure, where the shared inputs can be freely allocated among different stages. Chen et al. [10] developed a DEA model for two-stage network processes with nonsplitable shared inputs and presented an additive efficiency decomposition under variable return to scale (VRS). Yu and Shi [29] discussed the efficiency scale of a two-stage system with additional inputs to the second stage and part of intermediate products as the final output. Tavassoli et al. [30] used the SBM two-stage model to evaluate domestic airlines in Iran, in which the workforce is considered as the shared input of both stages. Also, Fathalikhani [31] proposed a two-stage DEA approach considering shared inputs, free intermediate measures, and undesirable outputs that takes into consideration the structure of inputs by differentiating between the inputs assigned to each stage and the inputs shared by the two stages so that parts of outputs from the second stage are wastages that can be fed back as inputs to the first stage. Wu et al. [32] studied the reuse of undesirable intermediate outputs in a two-stage production process with a shared resource. They proposed additive efficiency and noncooperative efficiency measures to illustrate the overall efficiency
of each DMU and the respective efficiency of each sub-DMU. Wu et al. [33] proposed a two-stage DEA model involving feedback and additional exogenous inputs. Chen et al. [10] developed a two-stage DEA model with extra exogenous inputs and shared resources. Hu et al. [34] proposed an integrated bi-level programming and DEA model with a feedback variable to deal with poor output to rank DMUs using a superefficient DEA model. In their structure, they considered capital invested as the shared input and reused water as feedback. Nematizadeh et al. [35] proposed a DEA-based model for a two-stage-feedback structure, they considered capital invested as the shared resources, and feedback factors to evaluate the Chinese high-tech industry.

3. The PPS for a Two-Stage DEA Model with Undesirable Outputs

Suppose there are $n$ DMUs to be evaluated, indexed by $j = 1, \ldots, n$ as given in Figure 1. For each DMU, in the first stage using $m$ inputs $x_{ij}^1 (i = 1, \ldots, m)$, $K$ shared inputs $x_{kj} (k = 1, \ldots, K)$ $T$ undesirable feedback inputs $y_{tj} (t = 1, \ldots, T)$, $D$ undesirable intermediate products $z_{dj} (d = 1, \ldots, D)$, and $S$ final outputs $y_{rj} (r = 1, \ldots, S)$ are produced.

In the second stage, by taking $D$ undesirable intermediate products as inputs, $K$ shared inputs $(1 - \delta_{kj})x_{kj}$ $(k = 1, \ldots, K)$, and $E$ exogenous inputs $x_{ej}^2 (e = 1, \ldots, E)$, it produces $W$ final outputs $y_{rj}^2 (H = 1, \ldots, W)$ and $T$ undesirable feedback. Note that variable $\delta_{kj} \in (0, 1)$ specifies the portion of shared inputs to the two stages. The above-mentioned notations are also summarized in Table 1.

In general, most production operations in the real world are those in which desirable outputs and undesirable outputs are produced together. In this case, desirable outputs are freely or strongly disposable and undesirable outputs are weakly disposable [38, 39]. Therefore, the corresponding DEA technology, $T_v$, exhibiting VRS for our model can be described as follows:

$$T_v = \left\{ \left( Y, \delta_{kj} X, f, (1 - \delta_{kj})X, X^2, Z, Y^2 \right) | \begin{array}{l}
x^1_j \leq \sum_{j=1}^{n} x_{ij}^1 \lambda_1^j,
\delta_{kj} X \geq \sum_{j=1}^{n} \delta_{kj} x_{kj} \lambda_1^j,
 Y \leq \sum_{j=1}^{n} y_{rj} \lambda_1^j,
 Z = \sum_{j=1}^{n} z_{dj} \lambda_1^j,
 f \geq \sum_{j=1}^{n} f_{tj} \lambda_1^j,
 X^2 \geq \sum_{j=1}^{n} x_{ij}^2 \lambda_2^j,
 (1 - \delta_{kj})X \geq \sum_{j=1}^{n} (1 - \delta_{kj}) x_{kj} \lambda_2^j,
 f = \sum_{j=1}^{n} f_{tj} \lambda_2^j,
 Y^2 \leq \sum_{j=1}^{n} y_{rj}^2 \lambda_3^j,
 Z \geq \sum_{j=1}^{n} z_{dj} \lambda_3^j,
 \sum_{j=1}^{n} \lambda_1^j = 1,
 \sum_{j=1}^{n} \lambda_2^j = 1,
 \lambda_1^j, \lambda_2^j, \lambda_3^j \geq 0,
 0 \leq \delta_{kj} \leq 1 \right\}.\right.$$  

In the case of intermediate products and feedback, the quantity produced must be greater than or equal to the amount consumed. In other words,

$$\sum_{j=1}^{n} \lambda_1^j f_{tj} \leq \sum_{j=1}^{n} \lambda_2^j f_{tj},$$

$$\sum_{j=1}^{n} \lambda_1^j z_{dj} \geq \sum_{j=1}^{n} \lambda_2^j z_{dj}.$$  

Now, based on the PPS (1) and by adding slacks, the following constraints are obtained:

$$x_{io} = \sum_{j=1}^{n} x_{ij}^1 \lambda_1^j + s_i^+, \quad \forall i,$$

$$y_{ro} = \sum_{j=1}^{n} y_{rj} \lambda_1^j - s_r^+, \quad \forall p,$$

$$z_{do} = \sum_{j=1}^{n} z_{dj} \lambda_1^j, \quad \forall d,$$

$$f_{fo} = \sum_{j=1}^{n} f_{tj} \lambda_1^j + s_t^+, \quad \forall t,$$

$$\delta_{ko} x_{ko} = \sum_{j=1}^{n} \lambda_1^j \delta_{kj} x_{kj} + s_k^+, \quad \forall k,$$

$$x_{eo}^2 = \sum_{j=1}^{n} \lambda_2^j x_{ej}^2 + s_e^+, \quad \forall e,$$ 

$$z_{do} = \sum_{j=1}^{n} \lambda_2^j z_{dj} + s_d^+, \quad \forall d,$$

$$y_{ho} = \sum_{j=1}^{n} y_{rj} \lambda_2^j - s_h^+, \quad \forall h,$$

$$y_{ho}^2 = \sum_{j=1}^{n} y_{rj} \lambda_2^j - s_h^+, \quad \forall h.$$
\[ f_{to} = \sum_{j=1}^{N} \lambda_j^t f_{tj}, \quad \forall t, \quad (11) \]

\[ (1 - \delta_{k0}) x_{ko} = \sum_{j=1}^{N} \lambda_j^t (1 - \delta_{kj}) x_{kj} + \tilde{s}_k, \quad \forall k, \quad (12) \]

\[ \sum_{j=1}^{n} \lambda_j^t f_{tj} \leq \sum_{j=1}^{n} \lambda_j^t f_{tj}, \quad \forall t, \quad (13) \]

\[ \sum_{j=1}^{n} \lambda_j^t z_{dj} \geq \sum_{j=1}^{n} \lambda_j^t z_{dj}, \quad \forall d, \quad (14) \]

\[ \sum_{j=1}^{n} \lambda_j^t = 1, \quad (15) \]

\[ \max \frac{w_1}{m + T + s + K} \left( \sum_{i=1}^{m} x_{io}^1 - s_i \right) x_{io} + \sum_{k=1}^{K} \frac{\delta_{k0} x_{ko} - \tilde{s}_k}{\delta_{kj} x_{ko}} + \sum_{l=1}^{L} f_{lo} - s_l + \sum_{r=1}^{R} y_{ro} + \tilde{s}_r + \sum_{d=1}^{D} z_{do} - s_d + \sum_{h=1}^{H} \lambda_{hj}^1 y_{hj} + \tilde{s}_h, \quad (19) \]

\[ \text{s.t.} \quad \text{Constraint sets (2) and (3) to (18)} \]

where \( 0 \leq \omega_1 \leq 1 \) is the weight parameter. The objective function in model (19), which is the overall efficiency, is the weighted average of the efficiencies of the two stages. As we see, model (19) in the current form is nonlinear. In the next theorem, it is reformulated as an SOCP that is convex and its global optimum solution can be found efficiently using interior point-based software packages like CVX Grant et al. [40].

**Theorem 1.** Model (19) is equivalent to the following SOCP:

\[ \text{Max} \xi_1 + \sum_{r=1}^{W} \xi_r^2 + \sum_{h=1}^{H} \xi_h^2, \quad (20) \]

\[ \text{s.t.} \quad \sum_{j=1}^{N} \lambda_j^t x_{ij} + \tilde{s}_i = x_{io}, \quad \forall i, \quad (21) \]

\[ \sum_{j=1}^{n} \lambda_j^t f_{tj} + \tilde{s}_t = f_{to}, \quad \forall t, \quad (22) \]

\[ \sum_{j=1}^{n} \lambda_j^t z_{dj} = z_{do}, \quad \forall j, \quad (23) \]

\[ \sum_{j=1}^{n} \lambda_j^t y_{rj} = y_{ro}, \quad \forall r, \quad (24) \]

\[ \sum_{j=1}^{n} \lambda_j^t = 1, \quad (16) \]

\[ L \leq \delta_{kj} \leq U, \quad (17) \]

\[ \lambda_j^1, \lambda_j^2, \tilde{s}_1, \tilde{s}_2, s_r, s_d, s_h, \tilde{s}_h \geq 0, \quad (18) \]

4 The ASBM Model

The ASBM model for DMUs’ evaluation from the internal perspective is as follows:

\[ \sum_{j=1}^{n} \lambda_j^1 x_{ej}^2 + \tilde{s}_e = x_{eo}, \quad \forall e, \quad (25) \]

\[ \sum_{j=1}^{n} \lambda_j^1 y_{hj}^2 - s_h^* = y_{ho}^2, \quad \forall h, \quad (26) \]

\[ \sum_{j=1}^{n} \lambda_j^1 z_{dj} + \tilde{s}_d = z_{do}, \quad \forall d, \quad (27) \]

\[ \sum_{j=1}^{n} \lambda_j^2 f_{tj} = f_{to}, \quad \forall t, \quad (28) \]

\[ \sum_{j=1}^{n} \lambda_j^2 x_{ko} + \tilde{s}_h = x_{ko}, \quad \forall k, \quad (29) \]

\[ \sum_{j=1}^{n} \delta_{kj} x_{ko} + \tilde{s}_k = x_{ko}, \quad \forall k, \quad (30) \]

\[ \sum_{j=1}^{n} \lambda_j^1 f_{tj} \leq \sum_{j=1}^{n} \lambda_j^2 f_{tj}, \quad \forall t, \quad (31) \]

\[ \sum_{j=1}^{n} \lambda_j^1 z_{dj} \geq \sum_{j=1}^{n} \lambda_j^2 z_{dj}, \quad \forall d, \quad (32) \]
Obviously, the objective function and constraints (49) and (50) also become linear as follows:

\[
\begin{aligned}
\sum_{j=1}^{n} \lambda_j^1 &= 1, \\
\sum_{j=1}^{n} \lambda_j^2 &= 1, \\
\lambda_j^1 &= \sum_{k=1}^{K} \tau_{kj} + \alpha_j, \quad \forall j, \\
\lambda_j^2 &= \sum_{k=1}^{K} \sigma_{kj} + \beta_j, \quad \forall j,
\end{aligned}
\]

and constraints (7) and (12) become as follows:

\[
\begin{aligned}
x_{ko} &= \sum_{j=1}^{n} \lambda_j^1 \sigma_{kj} x_{kj} + \bar{s}_k, \quad \forall k, \\
x_{ko} &= \sum_{j=1}^{n} \lambda_j^2 \mu_{kj} x_{kj} + \bar{s}_k, \quad \forall k.
\end{aligned}
\]
Further, for the terms in the objective function (48), let $\xi_1$, $\xi_2$, $\xi_3$ be such that

$$\frac{w_1}{m + T + S + K} \left( \sum_{i=1}^{m} x_{1i} \delta_x + \sum_{k=1}^{r} \sum_{j=1}^{K} (x_{ko} - z_{k}) \right) \leq \xi_1,$$

$$\frac{1 - w_1}{E + D + K + W} \left( \sum_{c=1}^{E} x_{ro} - s_{r} \right) \leq \xi_2,$$

$$\frac{w_1}{m + T + S + K} \left( \sum_{i=1}^{m} x_{1i} \delta_x + \sum_{k=1}^{r} \sum_{j=1}^{K} (x_{ko} - z_{k}) \right) \leq \xi_3.$$

Inequality (58) is linear, and inequality (59) is equivalent to the following:

$$(m + T + S + K)(y_{ro} + s_{r}) \xi_3 \geq \left( \sqrt{w_1 y_{ro}} \right)^2,$$

which is a second-order cone constraint. Similarly, (60) can be written as conic constraints. Thus, model (20) is obtained.
4.1. Case Study and Discussion. The aim of this section is to show the potential applicability of the proposed model. It is applied to a real dataset of 30 provincial-level regions in mainland China that is taken from Wu et al. [32] to evaluate the efficiency of industrial production processes in these regions. The corresponding two-stage structure is given in Figure 2. The first stage is the industrial production subsystem, and the second one is the waste disposal subsystem. In the first stage, labor, capital, energy, and value of products made from wastewater, waste gas, and solid waste (OV) are the inputs. The gross industrial output value (GIOV), wastewater, waste gas, and solid waste are the outputs. Wastewater, waste gas, and solid waste are undesirable intermediate products that are also inputs to the second stage. In the second stage, labor, energy, investment, wastewater, waste gas, and solid waste are the inputs. The output of this stage is OV, which is an input resource that feeds back to the first stage. Also, labor and energy are taken as shared resources for both stages. More details of the inputs and outputs are given in Table 2, and the data are given in Table 3.

The 30 regions can be divided into four major areas (Figure 3): the Eastern, Central, Western, and Northeastern areas [37]. The Eastern area comprises seven coastal provinces (Hebei, Fujian, Jiangsu, Zhejiang, Hainan, Shandong, and Guangdong) and three municipalities (Shanghai, Beijing, and Tianjin). The Central area has six inland provinces (Hubei, Anhui, Shanxi, Jiangxi, Hunan, and Henan). The Western area includes eleven provinces (Guangxi, Qinghai, Yunnan, Guizhou, Shaanxi, Ningxia, Gansu, Xinjiang, Sichuan, Inner Mongolia, and Tibet) and one municipality (Chongqing). Finally, the Northeast area has three provinces (Liaoning, Jilin, and Heilongjiang). Since sufficient data for Tibet were not available, it was excluded from the list.

Both country-wise and area-wise efficiency evaluations are performed. Further, it is assumed that the shared inputs, labor, and energy resources are divided to the waste disposal subsystem (stage 2) by a weight that lies in (0.1, 0.3). In the overall setting also, it is assumed that both stages have the same contribution to the industrial production overall efficiency, so the weights given to the two stages are 0.5.

The country-wise efficiency evaluation results are summarized in Table 4. The results show that Zhejiang, Hainan, and Yunnan are overall efficient regions. The average overall efficiency is 0.640346 and 12 (40%) of the regions have better efficiency scores than the average. Two

\[ E_1 = \frac{w_1}{m + T + S + K} \left( \sum_{i=1}^{m} x_{io} - x_i - \sum_{k=1}^{K} x_{ko} - \pi_k + \sum_{t=1}^{T} f_{to} - s_t + \sum_{r=1}^{S} y_{ro} + \pi_r \right), \]  

\[ E_2 = \frac{1 - w_1}{E + D + K + W} \left( \sum_{c=1}^{C} \frac{x_{co}^2 - s_c}{x_{co}} + \sum_{d=1}^{D} \frac{z_{do} - s_d}{z_{do}} + \sum_{k=1}^{K} x_{ko} - \pi_k + \sum_{h=1}^{H} y_{hj} + \pi_h \right). \]
Figure 3: Four major economic regions of China.

Table 3: Inputs and outputs of industrial production for 30 regions in China.

| DMU | Region      | Labor | Energy | Capital | GIOV | Waste water | Waste gas | Solid waste | Investment | OV  |
|-----|-------------|-------|--------|---------|------|-------------|-----------|-------------|------------|-----|
| 1   | Beijing     | 124.15| 6954   | 22750.58| 13699.84| 8198        | 4750      | 1269        | 1.9026     | 3.43658 |
| 2   | Tianjin     | 148.91| 6818   | 14584.31| 16751.82| 19680       | 7686      | 1862        | 8.32203    | 19.26504 |
| 3   | Hebei       | 344.67| 27531  | 24943.75| 3143.29 | 114232      | 56324     | 31688       | 10.67334   | 107.1801 |
| 4   | Shanxi      | 219.88| 16808  | 18505.94| 12471.33| 49881       | 35190     | 18270       | 23.46763   | 42.63718 |
| 5   | Inner Mongolia | 125.19| 16820  | 14691.38| 13406.11| 39536       | 27488     | 16996       | 11.70925   | 27.23754 |
| 6   | Liaoning    | 401.74| 20947  | 29076.78| 36219.42| 71521       | 26955     | 17273       | 14.25687   | 32.80902 |
| 7   | Jilin       | 139.81| 8297   | 10196.15| 13098.35| 38656       | 8240      | 4642        | 6.2945     | 39.16633 |
| 8   | Heilongjiang| 147.6 | 11234  | 10471.17| 9535.15 | 38921       | 10111     | 5405        | 4.22225    | 32.34714 |
| 9   | Shanghai    | 291.62| 11201  | 27555.88| 30114.41| 36696       | 12969     | 2448        | 4.11153    | 17.03791 |
| 10  | Jiangsu     | 1153.88| 25774 | 66134.06| 92056.48| 263760      | 31213     | 9064        | 15.52205   | 218.9749 |
| 11  | Zhejiang    | 857.58| 16865  | 47282.79| 51394.2 | 72526       | 9812      | 9407        | 5.95067    | 59.34731 |
| 12  | Anhui       | 264.87| 9707   | 15930.28| 18732   | 70971       | 17849     | 9158        | 5.41871    | 36.4491   |
| 13  | Fujian      | 411.75| 9809   | 16058.7 | 21901.23| 124168      | 13507     | 7487        | 4.78486    | 37.50288 |
| 14  | Jiangxi     | 199.16| 6355   | 8637.45 | 13883.2 | 72526       | 9812      | 9407        | 5.95067    | 59.34731 |
| 15  | Shandong    | 931.5 | 34808  | 53761.28| 83851.4 | 208257      | 43837     | 16038       | 36.4491    | 187.1898 |
| 16  | Henan       | 479.27| 21438  | 23467.42| 34995.53| 150406      | 22709     | 10714       | 12.07734   | 74.39088  |
| 17  | Hunan       | 249.97| 15138  | 20894.32| 21623.12| 94593       | 13865     | 6813        | 24.24997   | 82.28357  |
| 18  | Guangdong   | 1568  | 26908  | 62626.9 | 85824.64| 187031      | 24092     | 5456        | 20.90697   | 62.42653  |
| 19  | Guangxi     | 150.51| 7919   | 8516.45 | 9844.13 | 162111      | 4520      | 16522       | 12.78466   | 84.12533  |
| 20  | Hainan      | 12.44 | 1359   | 1621.38 | 1381.25 | 5782        | 1360      | 212         | 0.41153    | 3.16232   |
| 21  | Chongqing   | 146.56| 7856   | 8099.01 | 9143.55 | 45180       | 10943     | 2837        | 6.38182    | 29.13266  |
| 22  | Sichuan     | 351.67| 17892  | 22564.76| 23473.87| 93444       | 20107     | 11239       | 7.00433    | 45.78465  |
| 23  | Guizhou     | 80.3  | 8175   | 5960.13 | 4206.37 | 14130       | 10192     | 8188        | 6.51451    | 17.91425  |
| 24  | Yunnan      | 92.6  | 8674   | 9611.09 | 6464.63 | 30926       | 10978     | 9392        | 10.33956   | 65.45546  |
| 25  | Shaanxi     | 151.08| 8882   | 14688.7 | 11199.84| 45487       | 13510     | 6892        | 25.22795   | 29.34996  |
| 26  | Gansu       | 71.34 | 5923   | 6487.35 | 4882.68 | 15352       | 6252      | 3745        | 13.63106   | 22.41008  |
| 27  | Qinghai     | 20.9  | 2568   | 3053.61 | 1481.99 | 9031        | 3952      | 1783        | 0.97472    | 5.51878   |
| 28  | Ningxia     | 29.04 | 3681   | 3293.16 | 1924.39 | 21977       | 16324     | 2465        | 2.9096     | 10.07503  |
| 29  | Xinjiang    | 60.18 | 8290   | 7911.97 | 5341.9  | 25413       | 9310      | 3914        | 6.67628    | 22.2187   |

Regions
- Non-Study Areas
- Central
- Northeast
- Western
- Eastern
efficient regions, Zhejiang and Hainan, are in the Eastern area, and the Yunnan is in the Western area. Moreover, the average production efficiency (stage 1) is 0.864593 and 20 (66.67%) of the regions have better efficiency scores than the average. In this stage, all regions in the Eastern area, four regions in the Central area (Shanxi, Jiangxi, Henan, and Hunan), five regions in the Western area (Inner Mongolia, Guangxi, Guizhou, Yunnan, and Ningxia), and the Liaoning in the Northeastern area are efficient. The average waste disposal efficiency (stage 2) is 0.416098 and 12 (40%) regions have better efficiency scores than the average. The efficient regions in this stage are the same as the overall ones (Zhejiang, Hainan, and Yunnan). The Pearson correlation coefficients based on efficiencies of Table 4 are given in Table 5. As can be seen, the overall efficiency is meaningfully correlated with the waste disposal efficiency (stage 2). For the efficient regions, that are efficient in both stages, there is no sensitivity to parameter $w$. The thickness of the colored area in each region illustrates the effect of parameter $w$ on efficiency scores of DMUs. The results show that for $w = 1$, 17 regions are efficient, and for any other value of $w$, we only have three efficient regions.

The area-wise efficiency analysis results are summarized in Tables 6–9 for $w = 0.5$. For area 1, it can be seen in Table 6 that Zhejiang and Hainan are efficient regions. The average overall efficiency is 0.722366 and 3 (30%) regions have better efficiency scores than the average. Moreover, the average production efficiency (stage 1) is 0.936746 and 4 (66.67%) regions have better efficiency scores than the average. The average overall efficiency is 0.945318 and 4 (66.67%) regions have better efficiency scores than the average. Moreover, the average production efficiency (stage 1) is 1 and all regions are efficient in this stage. The average waste disposal efficiency (stage 2) is 0.444732 and 3 (30%) regions have better efficiency scores than the average. Moreover, the average production efficiency (stage 1) is 1 and all regions are efficient in this stage. The average waste disposal efficiency (stage 2) is 0.890637 and 2 (33.33%) regions have smaller scores than the average. The efficient regions are the same as the overall evaluation. Table 8 illustrates the results of evaluating 11 regions in the Western area. It shows that Guangxi, Chongqing, Gansu, Yunnan, and Qinghai are overall efficient in this area. The average overall efficiency is 0.852004 and 5 (45.5%) regions have better efficiency scores than the average. Moreover, the average production efficiency (stage 1) is 0.936746 and 9 (81.82%) regions have better efficiency scores than the average. Shaanxi and Xinjiang are the inefficient regions in this stage. The average waste disposal efficiency (stage 2) is 0.767264 and 5 (45.45%) regions have smaller scores than the average. Finally, Table 9 shows the results of 3 regions in the Northeastern area. It indicates that Liaoning is the only overall inefficient region in this area. The average overall efficiency is 0.895132 and 2 (66.67.5%) regions have better efficiency scores than the average. Moreover, the average production efficiency (stage 1) is 1 and all regions are efficient. Also, the average waste disposal efficiency (stage 2) is 0.790264 and efficient regions are the same as the overall evaluation.

Also, in the area-wise approach, we conducted sensitivity analysis on the weight parameter $w$ for the average overall efficiency of areas that are depicted in Figure 5. One can see that the average overall efficiency scores of areas increase when $w$ increases. The thickness of the colored area in Figure 5 shows the effect of parameter $w$ on the average efficiency score of each area. We
find that the average efficiency score in the Central area has the least dependence on $w$. In the first stage, the average score for any value of $w$ is one. In the second stage, the scores have a smaller standard deviation and thus less dependency on $w$. In contrast, average efficiency score in the Eastern area has the most dependency on $w$ and its value has an important effect on the results. It shows that the efficiency score of the waste disposal stage is more important than the production.

Table and Figure 6 also compare the area-wise and country-wise average efficiency scores of each area. The Eastern area has a relatively high average overall efficiency score in the country-wise analysis, and also, has much a larger average efficiency score in stage 1 and stage 2. The Northeastern area has the lowest average overall efficiency score. In the area-wise analysis, the Central area has the highest average industrial production efficiency score, and the Eastern area has the lowest average overall efficiency score. Although the average efficiency score of the Eastern area in the country-wise analysis is much larger than the one of the Central area, results of area-wise analysis show that by eliminating other areas, the average efficiency score of the Central area increases and will be much larger than that of the Eastern area. These findings indicate an unbalanced development in these industrial regions, and the Eastern region has a significant impact on the overall index.
Table 8: Area-wise efficiency evaluation results in the Western area.

| Area       | DMU | Region     | Stage 1 Production | Stage 2 Waste disposal | Overall  |
|------------|-----|------------|--------------------|------------------------|----------|
| Western    | 17  | Inner Mongolia | 1                      | 0.302466               | 0.651233 |
|            | 18  | Guangxi    | 1                        | 1                      | 1        |
|            | 19  | Chongqing  | 1                        | 1                      | 1        |
|            | 20  | Sichuan    | 1                        | 1                      | 0.999995 |
|            | 21  | Guizhou    | 1                        | 0.508518               | 0.754259 |
|            | 22  | Yunnan     | 1                        | 1                      | 1        |
|            | 23  | Shaanxi    | 1                        | 0.688922               | 0.549665 |
|            | 24  | Gansu      | 1                        | 1                      | 1        |
|            | 25  | Qinghai    | 1                        | 1                      | 1        |
|            | 26  | Ningxia    | 1                        | 0.604252               | 0.802126 |
|            | 27  | Xinjiang   | 1                        | 0.615289               | 0.614771 |
|            |     | Mean       | 1                        | 0.936746               | 0.852004 |

Table 9: Area-wise efficiency evaluation results in the Northeastern area.

| Area       | DMU | Region    | Stage 1 Production | Stage 2 Waste disposal | Overall  |
|------------|-----|-----------|--------------------|------------------------|----------|
| Northeastern | 28  | Liaoning  | 1                        | 0.370791               | 0.685395 |
|            | 29  | Jilin     | 1                        | 1                      | 1        |
|            | 30  | Heilongjiang | 1                        | 1                      | 1        |
|            |     | Mean      | 1                        | 0.790264               | 0.895132 |

Figure 5: Sensitivity analysis of parameter \( \omega \) in the area-wise efficiency evaluations. (a) Average efficiency scores in stage 1. (b) Average efficiency scores in stage 2. (c) Average overall efficiency scores.
5. Conclusions and Future Research

Network DEA models are successfully applied for various real-world applications efficiency analyses. In this paper, Green et al.’s (1997) ASBM model is used to develop a new model for a two-stage structure with shared inputs and feedback. An important feature of the proposed model is that unlike the SBM model, it can take a decision maker’s preference into account. Although the proposed ASBM model is nonlinear, it is reformulated as an SOCP problem, which is a convex optimization problem and one can find its global optimal solution efficiently. This is computationally significant as the additive and multiplicative DEA models in the literature apply parametric approach to find the global optimal solution. The proposed model is applied for the country-wise and area-wise efficiency evaluations of a real dataset of 30 provincial-level regions in mainland China. Noticeable efficiency differences are discovered in various areas and the following statements can be concluded:

(i) The system is efficient in overall if only if it is efficient in both stages
(ii) The efficiency of the overall system declines between the efficiencies of the two stages
(iii) For all DMUs, the first stage’s efficiency scores are always higher than the second stage ones in both country-wise and area-wise analyses
(iv) Sensitivity analysis shows the importance of the weight parameter in the evaluation process
(v) Pearson correlation coefficient test results show that the overall efficiency is meaningfully correlated with the waste disposal stage. Thus, the decision makers need to make corresponding adjustments to the waste disposal stage in order to improve its efficiency.

As uncertainty is usually present in the real-world datasets, it has been employed in several DEA models from both robust optimization and fuzzy perspectives, see Hatami-Marbini et al. [41]; Kachouei et al. [42]; Shakouri et al. [43]; Salahi et al. [44]; Toloo and Mensah [45]; Toloo et al. [46]; and references therein. Thus, studying the current model in the presence of uncertainty can be considered as a future research direction. Furthermore, real-world structures may involve three or more stages with shared inputs and feedback Kao [8]; Chen et al. [10]. Thus developing SOCP formulations that are convex and easy to solve compared to the existing parametric models is another interesting future research direction.

Data Availability

The data are taken from the literature and are available upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.
References

[1] A. Charnes, W. W. Cooper, and E. Rhodes, “Measuring the efficiency of decision making units,” European Journal of Operational Research, vol. 2, no. 6, pp. 429–444, 1978.

[2] C. Kao and S.-N. Hwang, “Efficiency measurement for network systems: IT impact on firm performance,” Decision Support Systems, vol. 48, no. 3, pp. 437–446, 2010.

[3] C. Chen and H. Yan, “Network DEA model for supply chain performance evaluation,” European Journal of Operational Research, vol. 213, no. 1, pp. 147–155, 2011.

[4] W. D. Cook, L. Liang, and J. Zhu, “Measuring performance of two-stage network structures by DEA: a review and future perspective,” Omega, vol. 38, no. 6, pp. 423–430, 2010.

[5] S. Lozano, E. Gutiérrez, and P. Moreno, “Network DEA approach to airports performance assessment considering undesirable outputs,” Applied Mathematical Modelling, vol. 37, no. 4, pp. 1665–1676, 2013.

[6] K. Tone and M. Tsutsui, “Network DEA: a slacks-based measure approach,” European Journal of Operational Research, vol. 197, no. 1, pp. 243–252, 2009.

[7] S. M. Mirhedyatian, M. Azadi, and R. Farzipoor Saen, “A novel network data envelopment analysis model for evaluating green supply chain management,” International Journal of Production Economics, vol. 147, pp. 544–554, 2014.

[8] C. Kao, Network Data Envelopment Analysis: Foundations and Extensions, Springer, Berlin, Germany, 2017.

[9] M. Nemati, R. Kazemi Matin, and M. Toloo, “A two-stage DEA model with partial impacts between inputs and outputs: application in refinery industries,” Annals of Operations Research, vol. 295, no. 1, pp. 285–312, 2020.

[10] X. Chen, Z. Liu, and Q. Zhu, “Performance evaluation of China’s high-tech innovation process: analysis based on the innovation value chain,” Technovation, vol. 74-75, pp. 42–53, 2018.

[11] S. Esfifdani, F. Hosseinizadeh Lotfi, Sh. Razavayan, and A. Ebrahimnejad, “Efficiency changes index in the network data envelopment analysis with non-radial model,” Asian-European Journal of Mathematics, vol. 13, no. 2, Article ID 2050031, 2020.

[12] K. Chen and J. Zhu, “Additive slacks-based measure: computational strategy and extension to network DEA,” Omega, vol. 91, Article ID 102022, 2020.

[13] R. H. Green, W. Cook, and J. Doyle, “A note on the additive data envelopment analysis model,” Journal of the Operational Research Society, vol. 48, no. 4, pp. 446–448, 1997.

[14] S. Esfifdani, F. Hosseinizadeh Lotfi, Sh. Razavayan, and A. Ebrahimnejad, “A slacks-based measure approach for efficiency decomposition in multi-period two-stage systems,” RAIRO—Operations Research, vol. 54, no. 6, pp. 1657–1671, 2020.

[15] N. Torabi Golsefid and M. Salahi, “Efficiency decomposition in a three-stage network structure: Cooperative DEA, Nash bargaining game models and conic relaxations,” RAIRO—Operations Research, vol. 55, no. 6, pp. 3677–3699, 2021.

[16] K. Tone, “A slacks-based measure of efficiency in data envelopment analysis,” European Journal of Operational Research, vol. 130, no. 3, pp. 498–509, 2001.

[17] H. Fukuyama and W. L. Weber, “A slacks-based inefficiency measure for a two-stage system with bad outputs,” Omega, vol. 38, no. 5, pp. 398–409, 2010.

[18] W. B. Liu, W. Meng, X. X. Li, and D. Q. Zhang, “DEA models with undesirable inputs and outputs,” Annals of Operations Research, vol. 173, no. 1, pp. 177–194, 2010.

[19] W. Liu, Z. Zhou, C. Ma, D. Liu, and W. Shen, “Two-stage DEA models with undesirable input-intermediate-outputs,” Omega, vol. 56, pp. 74–87, 2015.

[20] Q. Cui, Y. Li, and Y.-M. Wei, “Comparison analysis of airline energy efficiency under weak disposability and strong disposability using a virtual frontier slack-based measure model,” Transportation Journal, vol. 57, no. 1, pp. 112–135, 2018.

[21] S. Maleki, A. Ebrahimnejad, and R. Kazemi Matin, “Fareto–Koopmans efficiency in two-stage network data envelopment analysis in the presence of undesirable intermediate products and nondiscretionary factors,” Expert Systems, vol. 36, no. 2, Article ID e12393, 2019.

[22] E. Vaezi, S. Najafi, M. Haji Molana, F. Hosseinizadeh Lotfi, and M. Ahadzadeh Namin, “Measuring performance of a three-stage structure using data envelopment analysis and stackelberg game,” Journal of Industrial and Systems Engineering, vol. 12, no. 2, pp. 151–173, 2019.

[23] Z. Hu, S. Yan, X. Li, L. Yao, L. Yao, and Z. Luo, “Evaluating the oil production and wastewater treatment efficiency by an extended two-stage network structure model with feedback variables,” Journal of Environmental Management, vol. 251, Article ID 109578, 2019.

[24] Q. Cui, J.-I. Lin, and Z.-y. Jin, “Evaluating airline efficiency under “carbon neutral growth from 2020” strategy through a network interval slack-based measure,” Energy, vol. 193, Article ID 116734, 2020.

[25] F. Yang, D. Wang, L. Zhao, and F. Wei, “Efficiency evaluation for regional industrial water use and wastewater treatment systems in China: a dynamic interactive network slacks-based measure model,” Journal of Environmental Management, vol. 279, pp. 111–721, 2021.

[26] X. Shi, A. Emrouznejad, and W. Yu, “Overall efficiency of operational process with undesirable outputs containing both series and parallel processes: a SBM network DEA model,” Expert Systems with Applications, vol. 178, Article ID 115062, 2021.

[27] T. Kuosmanen, “Weak disposability in nonparametric production analysis with undesirable outputs,” American Journal of Agricultural Economics, vol. 87, pp. 1077–1082, Article ID 115062, 2005.

[28] Y. Zha and L. Liang, “Two-stage cooperation model with input freely distributed among the stages,” European Journal of Operational Research, vol. 205, no. 2, pp. 3327–3338, 2010.

[29] Y. Yu and Q. Shi, “Two-stage DEA model with additional input in the second stage and part of intermediate products as final output,” Expert Systems with Applications, vol. 41, no. 15, pp. 6570–6574, 2014.

[30] M. Tavassoli, G. R. Faramarzi, and R. Farzipoor Saen, “Efficiency and effectiveness in airline performance using a SBM-NDEA model in the presence of shared input,” Journal of Air Transport Management, vol. 34, pp. 146–153, 2014.

[31] S. Fathalikhani, “A two-stage DEA model considering shared inputs, free intermediate measures and undesirable outputs,” International Journal of Applied Operational Research, vol. 6, no. 2, pp. 33–45, 2016.
[32] J. Wu, Q. Zhu, X. Ji, J. Chu, and L. Liang, “Two-stage network processes with shared resources and resources recovered from undesirable outputs,” *European Journal of Operational Research*, vol. 251, no. 1, pp. 182–197, 2016.

[33] H. Wu, K. Lv, L. Liang, and H. Hu, “Measuring performance of sustainable manufacturing with recyclable wastes: a case from China’s iron and steel industry,” *Omega*, vol. 66, pp. 38–47, 2017.

[34] Z. Hu, S. Yan, L. Yao, and M. Moudi, “Efficiency evaluation with feedback for regional water use and wastewater treatment,” *Journal of Hydrology*, vol. 562, pp. 703–711, 2018.

[35] M. Nematizadeh, A. Amirteimoori, and S. Kordrostami, “Performance analysis of two-stage network processes with feedback flows and undesirable factors,” *Operations Research and Decisions*, vol. 29, no. 3, pp. 51–66, 2019.

[36] X. Zeng, Z. Zhou, Q. Liu, H. Xiao, and W. Liu, “Environmental efficiency and abatement potential analysis with a two-stage DEA model incorporating the material balance principle,” *Computers & Industrial Engineering*, vol. 148, Article ID 106647, 2020.

[37] D. Wang, L. Zhao, F. Yang, and K. Chen, “Performance evaluation of the Chinese high-tech industry: a two-stage DEA approach with feedback and shared resource,” *Journal of Industrial and Management Optimization*, 2021.

[38] R. Färe and S. Grosskopf, “Modeling undesirable factors in efficiency evaluation: comment,” *European Journal of Operational Research*, vol. 157, pp. 242–245, 2004.

[39] R. Fare and D. Primont, *Multi-output Production and Duality: Theory and Applications*, Kluwer Academic Publishers, Netherlands, 1995.

[40] M. Grant, S. Boyd, and Y. Ye, *Cvx: Matlab Software for Disciplined Convex Programming*, 2013.

[41] A. Hatami-Marbini, S. Saati, and S. M. Sajadi, “Efficiency analysis in two-stage structures using fuzzy data envelopment analysis,” *Central European Journal of Operations Research*, vol. 26, no. 4, pp. 909–932, 2018.

[42] M. Kachouei, A. Ebrahimnejad, and H. Bagherzadeh Valami, “Efficiency evaluation in fuzzy two-stage data envelopment analysis based on fuzzy arithmetic approach,” *Journal of Decisions and Operations Research*, vol. 7, no. 1, pp. 143–159, 2022.

[43] R. Shakouri, M. Salahi, and S. Kordrostami, “Stochastic p-robust approach to two-stage network DEA model,” *Quantitative Finance and Economics*, vol. 3, no. 2, pp. 315–346, 2019.

[44] M. Salahi, M. Toloo, and Z. Hesabirad, “Robust russell and enhanced measures in DEA,” *Journal of the Operational Research Society*, vol. 70, no. 8, pp. 1275–1283, 2019.

[45] M. Toloo and E. K. Mensah, “Robust optimization with nonnegative decision variables: a DEA approach,” *Computers & Industrial Engineering*, vol. 127, pp. 313–325, 2019.

[46] M. Toloo, E. K. Mensah, and M. Salahi, “Robust optimization and its duality in data envelopment analysis,” *Omega*, vol. 108, Article ID 102583, 2022.