Can LSND be included in a 3-Neutrino framework?

O. Haug 1, Amand Faessler1 and J. D. Vergados1,2

1 Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany

2 Theoretical Physics Division, Ioannina University, Ioannina, Greece

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We study the special features emerging from a three lepton generation analysis of the available neutrino oscillation data (solar, atmospheric and LSND). First we find that it is possible to explain all three sets of data in terms of the standard left handed neutrinos without the need of sterile neutrinos. Second we find a significant difference in the mass matrix extracted from the data, depending on the analysis (without or with LSND), if the mass of the lightest neutrino, which cannot be determined from the neutrino oscillation data alone, is relatively small, i.e. \( \leq 0.1 \text{ eV} \). To compare with other processes we used the R-parity violating Minimal Supersymmetric Standard Model (\( R_p \)-MSSM) for the theoretical description of the neutrino masses. Using the oscillation data we were able to constrain the parameters of the model. In particular we were able to obtain values for the coupling constants of the \( R_p \)-MSSM.

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Searches for neutrino masses have intensively been performed during the last decades. But till now no evidence for massive neutrinos was found in experiments aiming to measure the mass directly, but only upper limits were set as follows \[ \text{(1)} \]

\[
m_{\nu_e} < 3.5 \text{eV}, m_{\nu_\mu} < 160 \text{KeV}, m_{\nu_\tau} < 23 \text{MeV}.
\]

On the other hand some hints for massive neutrinos have been seen more than 30 years ago in experiments measuring the electron neutrino flux coming from the sun \[ \text{(2)} \]. In these experiments the flux of \( \nu_e \) coming from the sun to the earth was found to be much smaller
than the expectations. This observed difference of the expected vs the measured flux could be explained in terms of neutrino oscillations. The $\nu_e$ mixing, due to oscillations to other neutrino flavors, leads to a smaller number of $\nu_e$'s at the detector. These oscillations between different states can only occur if the weak eigenstates are different from the mass eigenstates and if the mass eigenstates are not all degenerate. This means that some neutrinos must be massive. The probability for a $\nu$, which was produced in the flavor state $|\nu_\alpha\rangle$ with energy $E$, to be detected in the flavor state $|\nu_\beta\rangle$ is given in a three family mixing scheme by

$$P(\alpha \rightarrow \beta) = \delta_{\alpha,\beta} - 4 \sum_{i<j=1}^{3} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2 \left[ \frac{\Delta m_{ij}^2 L}{4E} \right],$$

(2)

where $L$ is the source detector distance. Here $\Delta m_{ij}^2 \equiv |m_i^2 - m_j^2|$ is the difference of the squared masses of the mass eigenstates $i$ and $j$, $U_{\alpha i}$ are the elements $\alpha i$ of the mixing matrix $U$, assuming it is real (negligible CP violation), which connects the flavor eigenstates $|\nu_\alpha\rangle$ and the mass eigenstates $|\nu_i\rangle$ by $|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle$. If CP-violation is negligible the mixing matrix $U$ can be parameterized in analogy to the CKM matrix by three angles,

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix},$$

(3)

where $s_{ij}$ and $c_{ij}$ stand for $\sin(\theta_{ij})$ and $\cos(\theta_{ij})$, respectively.

The search for neutrino oscillations has been the subject of many recent experiments [3–10]. The experiments which found evidence for the existence of $\nu$ oscillations can be characterized by the sources of neutrinos which they use as solar [4,5], atmospheric [6–8] and accelerator neutrino experiments [9,10]. For the atmospheric and solar neutrino experiments several groups find evidence for neutrino oscillations. In the field of accelerator neutrino experiments only one group, the LSND collaboration, has claimed evidence for neutrino oscillations, but this result has not yet been confirmed by any other experiment. Furthermore it is commonly believed that all the three experiments (solar, atmospheric and LSND) cannot be accommodated with just two independent $\Delta m^2$. Thus the LSND results were put in
doubt. We will see that this outcome is due to the essentially two flavor scenario, which has hitherto been employed in most analyses. We will see that in a rigorous three generation analysis all the available data can be accommodated and can be used to constrain the neutrino mass matrix. Since the emerging solution is, unfortunately, not unique, we propose a way to test in the future this solution and thus, indirectly, to test the LSND results.

As we have already mentioned, the analysis of neutrino oscillation experiments is often done independently for each experiment within a two family scenario. The oscillation probability with the same notation as in eqn. (2) in a two generation mixing scheme is

\[ P(\alpha \rightarrow \beta) = \sin^2 2\theta \sin^2 \left(1.27 \Delta m^2 \frac{L}{E}\right), \]  

where \( \Delta m^2 \) is in Units eV^2, \( L \) in m and \( E \) in MeV. The result of analyses of the solar neutrino experiments for the mass squared difference \( \Delta m^2 \) by including matter effects (Mikheyev-Smirnov-Wolfenstein \[11,12\]) is

\[ 4 \times 10^{-6} eV^2 \leq \Delta m^2_{sun} \leq 1.2 \times 10^{-5} eV^2. \]  

For the atmospheric \( \nu \) experiments one finds disappearance oscillations of \( \nu_\mu \) probably into \( \nu_\tau \) and obtains for \( \Delta m^2 \)

\[ 4 \times 10^{-4} eV^2 < \Delta m^2 < 6 \times 10^{-3} eV^2. \]  

Fitting now the results from the LSND collaboration for oscillations of \( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \) coming form \( \mu^+ \) decay at rest and \( \nu_\mu \rightarrow \nu_e \) which originate form the decay of \( \pi^+ \) also in a two family analysis yields a mass splitting of

\[ 0.1 eV^2 < \Delta m^2 < 1.0 eV^2. \]  

The three different splittings \( \Delta m^2 \) of eqns. (5) to (7) seem to be not compatible with a mixing of only three \( \nu \) families and led to the conclusion that either the LSND result is wrong or that there should be a fourth sterile neutrino which mixes with the others. But before one makes such severe claims one should carefully scrutinize the methods which have
been used to obtain these results. By looking at eqns. (2) and (4) one realizes that the squared mass differences obtained by fits using a two family scenario and by using a three family scenario need not to be the same. Indeed the squared mass difference extracted from eqn. (4) are a convolution of the squared mass differences in the three family scheme. Therefore the need for a sterile neutrino does not necessarily follow from the above argument. Thus one should analyze the experimental data not in the oversimplified two family but in a three family mixing scheme. This has been done by several authors [13–19]. The results of these analyses are displayed in table I and II. In table I we show the results of such analyses which excluded the LSND results while in table II the results of analyses including the LSND results are given. At first we want to examine the influence of the inclusion of the LSND result on the oscillation parameters in the different analyses. By comparing the two tables we see that the influence of the inclusion of the LSND experiment on the mixing angles \( \theta_{13} \) and \( \theta_{23} \) is negligible and is still relatively small for \( \theta_{12} \). The mass splitting \( \Delta m_{12}^2 \) of the lower lying masses is found to be larger by including the LSND result in most analyses. Also for the mass splitting \( \Delta m_{23}^2 \) the inclusion of the LSND result leads to a larger splitting. In average, the inclusion of the LSND result in global fits to all data leads only to a larger mass splitting \( \Delta m_{ij}^2 \), which is due to the small oscillation length (\( L \simeq 20m \)) in LSND. To see now the consequences of this difference we calculated the entries of the mass matrix in the weak basis for the different analyses. Oscillations fix the difference of the masses squared but not the absolute scale of the masses. For values of the smallest mass \( m_1 \) larger than 1 eV we find no significant difference between the two types of analyses. For three cases of the smallest mass, \( m_1 = 0.0, 0.01 \) and 0.1 eV, we show the average value for the neutrino mass matrix for the two types of analyses in table III. By comparing the results one finds that there exists a significant difference between the two types of solutions for these small masses. Because of the non observation of the neutrinoless double beta decay \((0\nu\beta\beta)\) one expects such small masses for Majorana neutrinos [20,21]. As it is well known the mass extracted from the \( 0\nu \) double beta decay experiments is the absolute value of the quantity:
\[ \langle m_{\nu} \rangle = \sum_{j=1}^{3} U_{\alpha j}^{2} \lambda_{j}^{CP} m_{j} \]  

(8)

where \( \lambda_{j}^{CP} \) are the CP eigenvalues of the neutrino mass eigenstate \( \nu_{j} \) with mass \( m_{j} \), which, of course, is positive. From the data of TABLE II we find:

\[ \langle m_{\nu} \rangle = 0.006, 0.013, 0.101, \text{ respectively (without LSND)} \]  

(9)

\[ \langle m_{\nu} \rangle = 0.026, 0.032, 0.115, \text{ respectively (with LSND)} \]  

(10)

The small values can differentiate between the two types of analysis, but, unfortunately, they are far below the present experimental limits. They are, however, within the goals of the planned experiments \([20,21]\). The experimentally most interesting values obtained for large \( m_{1} \) do not seem to depend on the type of the analysis (with or without LSND).

To decide now which of the two kind of analyses is correct one has to check the consistency of the results with other physical observables. To compare with other processes we have to consider now a model which is able to accommodate massive neutrinos. This means that we have to use a model which is an extension of the Standard Model (SM). One natural way to extend the SM is supersymmetry (SUSY) \([22]\). A popular SUSY extension is the R-parity violating supersymmetric SM (\( R_{p}\)-MSSM). Within this model we have for each SM particle a supersymmetric partner, called sparticles. For details about the \( R_{p}\)-MSSM see e.g. \([23]\). Within this model all neutrinos acquire masses by mixing with the the SUSY partners of the photon, the \( Z^{0} \) and the two neutral SUSY Higgs fields, see fig. \([1]\) and by loop corrections, see fig. \([2]\). For details how neutrinos acquire masses within this model see \([24–26]\). Using the values for the entries of the neutrino mass matrix given in table \([1]\) we extracted the values of the coupling constants for single term dominance. They are given in table \([4]\). We see that for the couplings \( \lambda_{133}^{(i)} \) exists a significant difference only in the cases of \( m_{1} = 0.00 \) and \( m_{1} = 0.01 \) eV. These small masses are found in a predictive model which use an underling symmetry to describe quark and lepton masses \([24]\).

To conclude, we discussed the analyses of neutrino oscillation data and showed that it is necessary to use a three family mixing scheme to do such an analysis. We presented results
of different three family analyses including and excluding the LSND result and found that
the inclusion mainly influences the mass splitting $\Delta m_{23}^2$. We found that to distinguish the
two different approaches and test by this the LSND result will only be possible if the smallest
mass is smaller than 0.01 eV.

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| $\Delta m^2_{12}[eV^2]$ | $\Delta m^2_{23}[eV^2]$ | $\theta_{12}[Deg.]$ | $\theta_{23}[Deg.]$ | $\theta_{13}[Deg.]$ | ref. |
|--------------------------|--------------------------|---------------------|---------------------|---------------------|-----|
| $3 \times 10^{-6} - 7 \times 10^{-5}$ | 0.01 | 53-62 | 28-37 | <13 | 14 |
| $4 \times 10^{-6} - 7 \times 10^{-5}$ | 1.0 | 51-72 | 27-32 | <4 | 13 |
| $4 \times 10^{-6} - 7 \times 10^{-5}$ | 0.1 | 51-72 | 28-33 | <3 | 13 |
| $10^{-4}$ | $8 \times 10^{-4}$ | 39.23 | 45 | 26.6 | 18 |

**TABLE I.** Results of neutrino oscillation analyses in a three family mixing scheme excluding the LSND results.
| $\Delta m_{12}^2 [eV^2]$ | $\Delta m_{23}^2 [eV^2]$ | $\theta_{12} [Deg.]$ | $\theta_{23} [Deg.]$ | $\theta_{13} [Deg.]$ | ref. |
|-----------------------------|-----------------------------|------------------------|------------------------|------------------------|-----|
| $10^{-4} - 10^{-3}$         | 0.3                         | 35.5                   | 27.3                   | 13.1                   | [15]|
| $10^{-4} - 10^{-3}$         | 0.3                         | 54.5                   | 27.3                   | 13.1                   | [15]|
| $2.87 \times 10^{-4}$       | 1.11                        | 45                     | 28.9                   | 4.2                    | [17]|
| $10^{-4} - 10^{-3}$         | 0.4                         | 37.6                   | 26.5                   | 10.3                   | [16]|
| $4 \times 10^{-6} - 7 \times 10^{-5}$ | 1                           | 51-72                  | 27-32                  | 3-4                    | [13]|

**TABLE II.** Results of neutrino oscillation analyses in three family mixing scheme including the LSND results.

| $m_1 = 0.00 \text{ eV}$ | $m_1 = 0.01 \text{ eV}$ | $m_1 = 0.10 \text{ eV}$ |
|-------------------------|-------------------------|-------------------------|
| without LSND             |                         |                         |
| $0.006 \ 0.012 \ 0.007$ | $0.013 \ 0.010 \ 0.008$ | $0.101 \ 0.005 \ 0.005$ |
| $0.012 \ 0.109 \ 0.099$ | $0.010 \ 0.114 \ 0.095$ | $0.005 \ 0.181 \ 0.076$ |
| $0.007 \ 0.099 \ 0.111$ | $0.008 \ 0.095 \ 0.117$ | $0.005 \ 0.076 \ 0.186$ |
| LSND included            |                         |                         |
| $.026 \ .064 \ .072$    | $.032 \ .061 \ .073$   | $.115 \ .050 \ .067$   |
| $.064 \ .292 \ .339$    | $.061 \ .296 \ .336$   | $.050 \ .354 \ .301$   |
| $.072 \ .339 \ .457$    | $.073 \ .336 \ .459$   | $.067 \ .301 \ .496$   |

**TABLE III.** Results for the neutrino mass matrix in the weak basis. Given are the averaged values for matrix elements using the analyses discussed in tables[1] and [14] for $\lambda_j^{CP}=1$ which we found to give the largest differences. With LSND one obtains larger off-diagonal elements.
TABLE IV. Values for trilinear couplings for single term dominance using neutrino oscillation analyses and different mass schemes for $m_1$. For the calculation of these values of the trilinear coupling constants we assumed as usual that masses and soft parameters are of the same order and are parameterized by $M_{SUSY}$ which is expected to lie in the region from 100 to 1000 GeV.

| $m_1$ [eV] | without LSND | LSND included |
|------------|--------------|---------------|
|            | 0.00 0.01 0.10 | 0.00 0.01 0.10 |
| $\lambda_{133} \times 10^4$ | 0.4 0.6 1.6 | 0.8 0.9 1.7 |
| $\lambda_{233} \times 10^4$ | 1.6 1.7 2.1 | 2.7 2.7 2.9 |
| $\lambda_{333} \times 10^4$ | 1.6 1.7 2.1 | 3.3 3.3 3.5 |
| $\lambda_{133} \times 10^4$ | 1.7 2.6 7.1 | 3.6 4.0 7.6 |
| $\lambda_{233} \times 10^4$ | 7.4 7.5 9.5 | 12.1 12.2 13.3 |
FIG. 1. Mixing of the neutrinos and neutralinos.

Fig. 1

\[ \tilde{\nu}^*, \tilde{B}^* \]

\[ \nu_\alpha \]

\[ \langle \tilde{\nu} \rangle \]

\[ \nu_\alpha \]

\[ \mu_\alpha \]

\[ \mu \]

\[ \tilde{H}_2^0 \]

\[ \tilde{H}_1^0 \]

FIG. 2. Radiative contributions to the neutrino mass in the $R_p$-MSSM.

Fig. 2

\[ (A^d + \mu \tan \beta)(m_d)_{\alpha\beta} \]

\[ \hat{d}_{\alpha L}^\dagger \times \hat{d}_{\beta R}^\dagger \]

\[ \nu_i \]

\[ \nu_j \]

\[ \lambda_{i \alpha k} \]

\[ \lambda_{j \beta k} \]

\[ (A^e + \mu \tan \beta)(m_e)_{\alpha\beta} \]

\[ \hat{e}_{\alpha L}^\dagger \times \hat{e}_{\beta R}^\dagger \]

\[ \nu_i \]

\[ \nu_j \]

\[ \lambda_{i \alpha k} \]

\[ \lambda_{j \beta k} \]