Time-domain Representation of Passband Scattering Parameters

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Abstract—This paper presents a simple and accurate method for the inclusion of linear, time-invariant (LTI) networks, described by RF frequency-domain data, within equivalent baseband time-domain simulations. The time-domain representation is formulated as an equivalent baseband discrete-time impulse response, which may be convolved with the equivalent baseband form of the input signal, to obtain the corresponding equivalent baseband output. This allows networks which are most accurately described in the frequency domain, such as frequency-dispersive transmission lines, to be efficiently included as part of a transient time-domain simulation.

Index Terms—Transient simulation, convolution, S-parameters, behavioural modelling.

I. INTRODUCTION

Simulation of nonlinear high-frequency circuits has traditionally focused on both pure transient and harmonic balance (HB) simulation techniques. While HB admits simple inclusion of frequency-domain descriptions, it is limited to steady-state solutions involving only quasi-periodic signals. Transient simulation, on the other hand, is well-suited to dealing with highly nonlinear systems subject to complex digitally modulated excitations, with SPICE the archetype example of the power of this technique. For RF systems, however, it is not immediately clear how frequency-domain data (e.g. S-parameter descriptions) can be accommodated within such time-domain simulations.

Several techniques have been proposed to address this problem by carefully transforming baseband frequency-domain descriptions into a form that may be included within a transient simulation [1], [2]. In addition, previous work has shown that it is possible to extend [2] to include RF passband descriptions within equivalent baseband behavioural simulation frameworks [3].

This paper presents an extension of [4], allowing tabulated passband frequency-domain data to be transformed to a complex-valued equivalent baseband, discrete-time, impulse response which may be included within a transient solver.

II. FREQUENCY-DOMAIN REPRESENTATION

Consider a passband scattering parameter description of an LTI one-port system $F(\omega)$, with bandwidth $2\omega_m$. The equivalent baseband form of this signal is defined as $\tilde{F}(\omega)$, and we note that it does not possess conjugate symmetry in general. At this point we may approximate $\tilde{F}(\omega)$ as the following frequency-domain complex Fourier series

$$\tilde{F}(\omega) = \sum_{k=-\infty}^{\infty} s_k e^{-jk\Omega \omega},$$

where the $s_k$ are the Fourier coefficients, the frequency repetition ‘period’ is $2\omega_m$ and hence $\Omega = \pi/\omega_m$, while the lower summation limit is zero, to ensure time-domain causality. The frequency-domain equivalent baseband scattered response $\tilde{B}(\omega)$ to an incident pseudowave $\tilde{A}(\omega)$, is given as

$$\tilde{B}(\omega) = \tilde{S}(\omega)\tilde{A}(\omega).$$

The time-domain impulse response may now be obtained through an inverse Fourier transform

$$\tilde{h}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{S}(\omega) \cdot 1 \cdot e^{j\omega t} \, d\omega$$

$$= \sum_{k=0}^{\infty} s_k \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega(t-\tilde{k})} \, d\omega$$

$$= \sum_{k=0}^{\infty} s_k \tilde{h}(t - \tilde{k}),$$

where $\tilde{k} = k\Omega/\omega_m$.

The expression for $\tilde{h}(t)$ above may be interpreted as a discrete-time impulse response $\tilde{s}[k] = \tilde{h}(k\pi/\omega_m)$, with uniform time-step of $\pi/\omega_m$. Note that the impulse response weights $s_k$ are complex, since this represents the equivalent baseband form of the original passband network. In general for a multiport network, $\tilde{s}[k]$ could be a $P \times P$ matrix of discrete-time impulse response sequences. It is noted that, in practice, the Fourier series expansion in (1) must be limited to some finite upper harmonic index $N$.

The impulse response $\tilde{s}[k]$ may now be included as part of a transient simulation, via a discrete-time complex convolution operation

$$\tilde{b}[n] = (\tilde{s} \ast \tilde{a})[n] = \tilde{s}[0]\tilde{a}[n] + \sum_{k=1}^{n} \tilde{s}[k]\tilde{a}[n - k].$$

The terms within the summation symbol above represent the past convolution history, and thus contain only known data, while the first term contains the known impulse response $\tilde{s}[0]$, along with the unknown incident pseudowave $\tilde{a}[n]$, which must be solved for at each new time-step.

Fig. 1. One-port network which is to be represented in the frequency domain (see Fig. 3 for frequency response). Note how the delay of the second transmission line has been chosen such that the total response is not periodic.
III. EXTRATION OF FOURIER COEFFICIENTS

Consider tabulated equivalent baseband S-parameter data \( \hat{F}(\omega_i) \) with compact support in the range \([-\omega_m, \omega_m]\), written as the column vector,

\[
F = \begin{bmatrix} 
\hat{F}(\omega_m) \\
\vdots \\
\hat{F}(\omega_i) \\
\vdots \\
\hat{F}(\omega_m) 
\end{bmatrix},
\]

and let us also define the following coefficient matrix

\[
M = \begin{bmatrix} 
1 & \exp(jk\omega_m) & \cdots & \exp(jN\omega_m) \\
\vdots & \ddots & \ddots & \vdots \\
1 & \exp(jk\omega_i) & \cdots & \exp(jN\omega_i) \\
\vdots & \ddots & \ddots & \vdots \\
1 & \exp(-jk\omega_m) & \cdots & \exp(-jN\omega_m) 
\end{bmatrix}.
\]

Equation (1) may now be expressed as

\[
Ms + E = F,
\]

where \( s \) is the column vector of impulse response coefficients, \( s_k \), and \( E \) is the Fourier approximation error. This error is minimised in a least squares sense by taking

\[
s = (M^H \cdot M)^{-1} \cdot M^H \cdot F,
\]

i.e. the weights \( s_k \) can be extracted through a simple matrix operation. Note that \( M^H \) represents the conjugate transpose of \( M \). In the next section we validate the proposed method through a simple practical example.

IV. PRACTICAL EXAMPLE

Consider the non-commensurate transmission line network shown in Fig. 1. The \( S \)-parameters of this one-port have been measured over a frequency range from 9.5 GHz to 10.5 GHz, and a complex Fourier expansion in the form (1) has been extracted via (5) to yield a complex valued discrete-time impulse response \( \tilde{s}[k] \), which is shown in Fig. 2. The frequency-domain approximation this admits is shown in Fig. 3.

The transmission line network of Fig. 1 is now excited with a multisin input (four equal amplitude tones, equally spaced between 9.8 GHz and 10.2 GHz). The current \( i_s \) into the line is now solved for in the transient regime using a commercial simulation environment, and an in-house transient solver implementing the equivalent baseband convolution method of this paper. Note that the commercial solver operates at passband frequencies, and requires a built-in model for the transmission line. The baseband time-domain output of both solvers is shown in Fig. 4 with excellent agreement visible.

V. CONCLUSION

A method for the inclusion of passband frequency-domain data within transient simulators is presented. In general, the time-domain description is very compact (Fig. 2). The time domain solution shows excellent agreement compared to the results from a commercial solver operating at much faster time-step (Fig. 4).

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