The prevalent Landau damping theory for classical plasmas does not fully explain the Langmuir wave decay in dense plasmas. A dielectric function theory adapted from the condensed matter physics is extended in order to be applied to dense plasmas and warm dense matter. This theory, accounting for the Umklapp process, predicts much higher decay rates than the Landau damping theory, which is in better agreement with the existing experimental data obtained from the metals. It is demonstrated that this strong plasmon decay leads to the existence of a parameter regime where the backward Raman scattering is unstable while the forward Raman scattering is stable. By means of the backward Raman compression, intense x-ray pulses can be created in this regime.

I. INTRODUCTION

Understanding the warm dense matter and dense plasmas has recently become more important in the context of various practical problems, including the inertial confinement fusion [1, 2]. In these media, many interesting phenomena observed in classical plasmas would still occur, but they get modified due to the quantum diffraction and the degeneracy [3–6]. We consider one such phenomenon, the decay of the Langmuir wave, which plays an important role in various applications [7–12].

The Landau damping theory is often used to predict the plasmon decay in classical plasmas. On the other hand, the plasmon decay rate in metals is computed by complicated quantum theories accounting for the local-field correction or the density functional approach [13, 14]. However, neither of them is sufficient to predict the plasmon decay in warm dense matter or dense plasmas. In this paper, we introduce a theoretical framework to calculate the plasmon decay in dense plasmas by adapting the dielectric function approach used in the condensed matter physics. It is suggested that the modification of the free-electron wave packet due to the presence of ions is considerable, which in turn modifies the plasmon decay profiles. It is shown that the decay rate of the long wavelength plasmon is much higher than the prediction by the Landau damping theory, and that the rate is finite and non-negligible in the consideration of the Raman compression. It leads us to show that there exists a parameter regime where the backward Raman scattering (FRS) is stable, enabling the x-ray compression without a premature pump depletion.

The rest of the paper is organized as follows. The basic classical and quantum-mechanical Landau damping theory is summarized in Sec. II, and why the prevalent Landau damping theory is insufficient for metals and dense plasmas is argued in Sec. III. A dielectric function formalism, adapted from the condensed matter physics research is introduced and extended to be applicable to the dense plasmas and warm dense matter (Secs. IV and V). The plasmon damping rate is computed in this new framework and the existence of a regime in dense plasmas where the BRS can be unstable while the FRS is stable is demonstrated (Sec. VI), and then it is concluded in Sec. VII.

II. LANDAU DAMPING THEORY

Consider a wave of the form \( \phi(x,t) = \phi \exp(ik \cdot x - i\omega t) + \phi^* \exp(-ik \cdot x + i\omega t) \). The classical analysis of the Landau damping rate, \( \gamma_{cl} \), for small \( e\phi \) is [15]

\[
\frac{\gamma_{cl}}{\omega} = \frac{\pi \omega_{pe}^2}{2k^2} \frac{\partial f(\omega/k)}{\partial v},
\]

where \( k(\omega) \) is the wave vector (frequency), \( \omega_{pe} = \sqrt{4\pi n_e e^2/m_e} \) is the plasma frequency, \( n_e \) \( (m_e) \) is the electron density (mass), and \( f \) is the electron probability distribution function satisfying \( \int f d^3v = 1 \). The damping rate \( \gamma \) can be alternatively derived in the dielectric function formalism,

\[
\frac{\gamma}{\omega} = \frac{1}{2} \text{Im} [\varepsilon(k,\omega)],
\]

**References**

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where $\epsilon$ is the well-known dielectric function obtained through the random phase approximation:

$$\epsilon(k, \omega) = 1 + \frac{4\pi n_e e^2}{m_e k^2} \int \frac{k \cdot \nabla v_f}{\omega - k \cdot v}.$$  \hspace{1cm} (3)

In the regime where the x-ray compression would be feasible through the BRS, $n_e$ is so high that the electron quantum diffraction cannot be neglected. In such a case, the quantum version of the dielectric function, the so-called Lindhard function \[16\], needs to be used:

$$\epsilon(k, \omega) = 1 + \frac{\omega_p^2 m_e}{k^2} \int \frac{f(E_f) - f(E_i)}{\omega - (E_f - E_i)} d^3v,$$  \hspace{1cm} (4)

where $E_i = m_e v_i^2/2 \ (E_f = m_e v_f^2/2)$ is the electron kinetic energy of the momentum $m_e v_i = m_e v - \hbar k/2 \ (m_e v_f = m_e v + \hbar k/2)$. In order to derive Eq. (4), we first solve the time-dependent Schrödinger equation using a perturbation analysis for each free electron wave packet and arrive at

$$|q\rangle = |q_0\rangle + \frac{e^0 \exp(ik \cdot x - i\omega t)}{\hbar \omega - E(q + k) + E(q)} |q + k\rangle_0$$

$$- \frac{e^0 \exp(-ik \cdot x + i\omega t)}{\hbar \omega - E(q - k) + E(q)} |q - k\rangle_0,$$

where $E(p) = \hbar^2 |p|^2/2m_e$, $| \rangle_0$ denotes the free electron wave function, and $| \rangle$ denotes the perturbed wave function. The perturbed electron density is given as

$$\delta n_e(x, t) = \int d^3q \left[ \frac{g(q)}{(2\pi)^3} |\langle x|q\rangle|^2 \right],$$

where $g(q)$, satisfying $\int g(q) d^3q/(2\pi)^3 = n_e$, is the occupation number. $\delta n_e(k, \omega)$ reads

$$\delta n_e(k, \omega) = e^0 \int \frac{d^3q}{(2\pi)^3} g(q) \frac{f(q + k) - f(q)}{\hbar \omega - E(q + k) + E(q)}.$$

The dielectric function, to the first order in $e^0$, is

$$\epsilon(k, \omega) = 1 + \frac{4\pi e^2}{k^2} \alpha(k, \omega),$$

where $\alpha(k, \omega) = \delta n_e(k, \omega)/e^0$ is the susceptibility. The imaginary part of Eq. (4) is

$$\text{Im} \left[ \epsilon \right] = \frac{\omega_p^2 m_e}{k^2} \int \pi \delta(\omega - E_f - E_i) \left( f(E_f) - f(E_i) \right) d^3v.$$

Note that the integration is done in the velocity space which is simply related to the momentum space by $\hbar q/m_e = v$. In the limit $\hbar \ll 0$, the above equation is reduced to Eq. (3).

III. LANGMUIR WAVE DAMPING IN METALS AND BREAKDOWN OF THE LANDAU DAMPING THEORY

The Landau damping rate in the free-electron plasma can be computed by either the classical dielectric function (Eq. (1)) or the degenerate Lindhard dielectric function (Eq. (4)). The rate in rare dense plasmas is accurately predicted by these approaches. However, the rate in dense plasmas, warm dense matter, or metals is not. For an instance, the comparison between experimentally obtained damping rates in Al and theoretical computations exhibit large deviation (Fig. 1). The prediction by the Lindhard dielectric function \[16\] is essentially zero in this range of $k$, and is not visible in the figure. The damping rate from the DuBois’ theory \[14\], accounting for the dynamical correlations, predicts the damping to be proportional to $(k/k_F)^2$ in the long wavelength limit where $k$ goes to zero. However, the data from the electron stopping experiments in metals \[18\] suggest that the rate is finite in this limit, which is inconsistent with the prediction of the aforementioned theories \[20–23\]. The experimental decay rate for $k < 0.5k_F$ can be estimated as

$$\nu_3(k) = \eta(k) \omega_{pe} / \text{sec}.$$

where $\eta(k) = \eta_0 + d\eta/dk^2 (k/k_F)^2$. For a typical metal, $0.02 < \eta_0 < 0.2$ and $d\eta/dk \cong a \eta_0$, where $2 < a < 10$ \[18\]. It is tempting to explain that the finite plasmon decay rate for $k \cong 0$ arises from the electron-ion collisions. However, it is difficult to estimate the decay rate by this argument. Furthermore, the electron-ion collisions get diminished in degenerate plasmas by the electron diffraction and the degeneracy \[3 \ 4\], and it is not clear how the quantum effect can be incorporated into the plasmon damping.

The electron wave packets in metals are distorted from those of the free electrons through the strong interaction between the electrons and the ion lattices. Adler \[24\] computed the electronic dielectric function in the presence of the metal ion lattices, by expanding the wave packet in terms of the reciprocal lattice wave vector and using the quantum random phase approximation. Sturm \[20\] and Hasegawa \[25\] applied this dielectric function formalism to Eq. (2) and obtained the Langmuir wave decay rate, which is in good agreement with the experimental data \[18\]. In particular, in the long wavelength limit, the damping rate is predicted to be finite. The dominant mechanism of the decay in Alkali metals is shown to be the Umklapp process; an electron inside the Fermi shell gets excited by the Langmuir wave outside the shell, obtaining the momentum of $k + G$, where $k$ is the Langmuir wave vector and $G$ is the reciprocal vector.

In dense plasmas or warm dense matter, the electron density is so high that many physical properties are similar to those in metals, and so is the decay profile of the plasmon damping; the ion density is also high, and the distortion of the electron wave packet and the Umklapp excitation become significant.
perturbation theory of the first order is used.

The dielectric function, up to the first order in \( \phi \), is given to be

\[
e(k, \omega) = 1 + \frac{4\pi n_e e^2}{k^2} \sum_{\sigma_1, \sigma_2} e\phi(f(\sigma_1) - f(\sigma_2)) \times \frac{\langle \sigma_1 \exp(-i k \cdot x) \sigma_2 \rangle \langle \exp(i k \cdot x) \sigma_1 \rangle}{\hbar \omega - E_{\sigma_1} + E_{\sigma_2}}.
\]

With an appropriate choice for the eigenstate in a given condition, Eq. \((7)\) can be used in various situations. For example, Sturm \([20, 27]\) used the following eigenstate

\[
|\sigma\rangle = |q\rangle + \sum_{q_1 \neq q} |q_1\rangle V|q\rangle,
\]

where \(q\) and \(q_1\) are the wave vectors, and \(\langle q_1|V|q\rangle\) is the pseudo-potential which is experimentally measurable. It should be noted that, in case of metals, \(\langle q_1|V|q\rangle\) becomes finite only when \(q_1 - q\) is a reciprocal vector.

Now we apply the above formalism to dense plasmas or warm dense matter, where unlike metals, the ions can move as individual particles. Since the ions move much slower than the electrons and the Langmuir wave frequency is much faster than the ion relaxation time, we assume that the ions are spatially frozen at \(X_i\). We first consider the dynamical property of the electrons in the presence of the frozen ions, and then obtain the average dynamics by averaging over the probability distribution of \(X_i\), following the Born-Oppenheimer’s approximation. Without loss of generality, \(X_i\) can be assumed to have the following correlation average:

\[
\langle \sum_{i,j} \exp(i s \cdot (X_i - X_j)) \rangle_V = n_f(s),
\]

where \(V\) is the volume of the region under consideration, and \(n_f(s)\) is the static two-point correlation function of the ions. For independent ions, \(n_f(s) = n_i\). Only one species of ions of the charge \(Z\) is considered. When the ion positions are fixed, the electron’s free wave eigenfunction is modified to be

\[
|\sigma\rangle = |q\rangle + \int_i \frac{d^3q_1}{(2\pi)^3} \exp(-i q_1 \cdot X_i) U(|q - q_1|) |q_1\rangle,
\]

where \(U(q)\) is the Fourier transform of the ion-electron potential. In the absence of the electron screening, \(U(q) = 4\pi Ze^2/q^2\), and Eqs. \((7)\) and \((8)\) give the susceptibility

\[
\alpha(k, \omega) = \alpha_{\text{rpa}}(k, \omega) + \alpha_{\text{dense}}(k, \omega),
\]

where \(\alpha_{\text{rpa}}\), the well-known Lindhard susceptibility, is

\[
\alpha_{\text{rpa}} = \int \frac{d^3q}{(2\pi)^3} \frac{f(k + q) - f(q)}{\hbar \omega - E(k + q) + E(k)}.
\]
and the $\alpha_{\text{dense}}$, the dense plasma correction, is

$$
\alpha_{\text{dense}}(k, \omega) = \int \frac{d^3s}{(2\pi)^3} n_I(s) \int \frac{d^3q}{(2\pi)^3} \frac{U^2(s)}{A^2(q, k, s)} \times \frac{f(|q + k + s|) - f(q)}{\hbar c k \omega - E(|q + k + s|) + E(q)},
$$

(10)

where $A(q, k, s)$ is

$$
A^{-1}(q, k, s) = \frac{1}{E(q) - E(q + s)} - \frac{1}{E(q + k) - E(q + k + s)}.
$$

(11)

The imaginary part of $\alpha_{\text{dense}}$ is given as

$$
\text{Im}[\alpha_{\text{dense}}] = \int \frac{d^3s}{(2\pi)^3} n_I(s) \int \frac{d^3q}{(2\pi)^3} \frac{U^2(s)}{A^2(q, k, s)} \times \pi \delta(\hbar c k \omega - E(q + k + s) + E(q)) \times (f(q + k + s) - f(q)).
$$

(12)

Some comments are made on Eq. (10), which is the main result of this paper. First, the integration over $s$ corresponds to the summation of the contribution from the Umklapp process in metals. An electron with an initial momentum $hk$ absorbs the energy $\hbar c k \omega$ and the momentum $h(k + s)$. This absorption is accompanied by an exchange of the momenta between an electron and an ion. Second, in metals, the integration over $s$ is replaced by the summation over the reciprocal lattice vectors, which represents the summation of the inter-band transitions in the case of metals. Third, the ion correlation is incorporated into $n_I(s)$. The ions tend to be strongly correlated in the warm dense matter, which could be easily accounted for by this formalism. Fourth, from Eq. (2), the decay of the Langmuir wave is given as

$$
\frac{\gamma}{\omega} = \frac{1}{2} \text{Im}[\epsilon] = \frac{1}{2} \frac{4\pi e^2}{k^2} \text{Im}(\alpha_{\text{rpa}} + \alpha_{\text{dense}}).
$$

(13)

The first term in the right hand side accounts for the Landau damping, and the second term is the correction due to the presence of the ions. Lastly, $\alpha_{\text{dense}}$ does not arise purely from the quantum-mechanical effects, as this term does not vanish in the limit $h = 0$. In the limit $h = 0$ and $k = 0$, the decay rate becomes proportional to $n_I$ and $T^{-3/2}$, which is reminiscent of the electron-ion collision frequency. However, the exact coefficient, which is derived in Eq. (12), cannot be obtained from the ion-electron collision argument. When $h \neq 0$, the quantum degeneracy and the electron diffraction effects are self-consistently incorporated into Eq. (12).

V. DEGENERATE ELECTRON CASE

Consider the dense plasma where the electrons are completely degenerate. The Fermi-energy is given as $E_F = 36.4 \times (n/n_{\text{cr}})^{2/3} \text{eV}$, where $n_{\text{cr}} = 10^{24} \text{cm}^{-3}$. The computation in this Section is valid as long as $T_e < E_F$. In the partially degenerate and classical case, the electron screening and the quantum diffraction need to be treated carefully, which would be considered elsewhere [31].

The susceptibility $\alpha_{\text{rpa}}$ of the free electron plasma, computed by Lindhard [10], is given as

$$
\alpha_{\text{rpa}}(k, \omega) = \frac{3n_e}{m_e v_F^2} h(z, u),
$$

where $v_F = \sqrt{2E_F/m_e}$ is the Fermi velocity, $z = k/2k_F$, $u = \omega/kv_F$, and $h = h_r + i h_i$. The real part of $h$, $h_r$, is

$$
h_r = \frac{1}{2} + \frac{1}{8z} (1 - (z - u)^2) \log \left( \frac{|z - u + 1|}{|z - u + 1|} \right) - \frac{1}{8z} (1 - (z + u)^2) \log \left( \frac{|z + u + 1|}{|z + u + 1|} \right),
$$

and the imaginary part $h_i$ is

$$
h_i = \frac{\pi}{2} u, \quad \text{for } |z + u| < 1,
$$

$$
= \frac{\pi}{8z} (1 - (z - u)^2), \quad \text{for } |z - u| < 1 < |z + u|,
$$

$$
= 0, \quad \text{if } |z - u| > 1,
$$

In the limit $k = 0$, due to the delta function in Eq. (12), $A^{-1}(q, k, s)$ in Eq. (11) can be approximated as

$$
A^{-1}(q, k, s) = \frac{k \cdot s}{m_e \omega^2},
$$

using the screening potential $U(s) = 4\pi Z e^2/(s^2 + k_{tf}^2)$, where $k_{tf} = \sqrt{3\omega_F}/v_F$ is the Thomas-Fermi screening length. In this case, $A$ is independent of $q$, which can be taken out of the $q$-integration in Eq. (10). Using Eq. (9), Eq. (12) reads

$$
\text{Im}[\alpha_{\text{dense}}(k, \omega)] = \int \frac{d^3s}{(2\pi)^3} \frac{h^4(k \cdot s)^2 (4\pi Ze^2)^2}{m_e^2(\hbar c k)^4(s^2 + k_{tf}^2)^2} \times n_I(s) \text{Im}[\alpha_{\text{rpa}}(k + s, \omega)],
$$

which can be further simplified to

$$
\text{Im}[\alpha_{\text{dense}}(k, \omega)] = \frac{4E_F^2 Z(k_F e^2)^2 (4\pi)^2}{3\omega} \int \frac{d^3s}{k_{tf}^3 (2\pi)^3} \times \frac{(k \cdot s)^2}{k^2(s^2 + k_{tf}^2)^2} \text{Im}[\alpha_{\text{rpa}}(k + s, \omega)],
$$

(14)
where we assumed \( n_f(s) = n_f \). \( \eta(0) \) in Eq. (6) from the above equation is shown in Fig. 2.

For non-zero \( k \)'s, Eq. (14) cannot be used, as \( A \) is dependent on \( q \). A rather complicated expression for \( A \) is

\[
A^{-1}(\mathbf{q}, \mathbf{k}, s) = \frac{\hbar^2 \mathbf{k} \cdot \mathbf{s}}{m_e (\hbar \omega)^2} \times \left[ \left( 1 - \frac{\hbar \mathbf{k} \cdot (\mathbf{q} + s/2)}{m \omega} \right) \left( 1 - \frac{\hbar \mathbf{k} \cdot (\mathbf{q} - s/2)}{m \omega} \right) \right]^{-1}
\]

which should remain in the integrand for the \( q \)-integration in Eq. (10). For given \( s \) and \( \mathbf{k} \), we integrate over \( \mathbf{q} \) first and then over \( s \). For simplicity, we assume \( \mathbf{k} = (k_x, 0, 0) \) and \( s = (s \cos \theta, s \sin \theta, 0) \), and denote \( \mathbf{q} = q_1 \hat{x}_1 + q_2 \hat{x}_2 + q_3 \hat{x}_3 \), where \( \hat{x}_1 = (k + s \cos \theta, s \sin \theta, 0) / |\mathbf{k} + \mathbf{s}| \), \( \hat{x}_2 = (-s \sin \theta, 1 + \cos \theta, 0) / |\mathbf{k} + \mathbf{s}| \), and \( \hat{x}_3 = (0, 0, 1) \). In this coordinate system, \( A \) depends only on \( k \), \( s \), \( \theta \), and \( q_2 \):

\[
A^{-1}(\mathbf{k}, s, q_2) = \frac{\hbar^2 \mathbf{k} \cdot \mathbf{s}}{m_e (\hbar \omega)^2} \kappa^{-1} \kappa^+,
\]

where

\[
\frac{1}{\kappa^+} = 1 - \frac{k^2 + sk \cos \theta}{|\mathbf{k} + \mathbf{s}|^2} + \frac{\hbar}{m_e \omega} \left( \frac{sk \sin \theta q_2}{|\mathbf{k} + \mathbf{s}|} \pm \frac{sk \cos \theta}{2} \right).\]

The real part of \( \alpha \) remains still complicated, but the imaginary part given in Eq. (12) could be simplified as the delta function would eliminate \( q_1 \) integration and the integrand of \( dq \) is independent of \( q_3 \). We do not present the detailed steps here. After some tedious manipulation, the \( q \)-integration is reduced to an one-dimensional integration

\[
\int \frac{d^3 \mathbf{q}}{(2\pi)^3} A^2(\mathbf{q}, \mathbf{k}, s) \frac{\mathbf{F} e^{-i \mathbf{q} \cdot \mathbf{u}}}{\mathbf{F}} \text{Im} \left[ \frac{f(\mathbf{q} + \mathbf{k} + \mathbf{s}) - f(\mathbf{q})}{\hbar \omega - E(\mathbf{q} + \mathbf{k} + \mathbf{s}) + E(\mathbf{q})} \right]
\]

\[
= \left( \frac{\hbar^2 \mathbf{k} \cdot \mathbf{s}}{m_e (\hbar \omega)^4} \right) \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \frac{m_e}{4 \pi^{2} \hbar^{2} |\mathbf{s} + \mathbf{k}|} \kappa^+ \kappa^{-1} \left( R^-(k_F, z, u, q_2) - R^+(k_F, z, u, q_2) \right),
\]

which should remain in the integrand for the \( q \)-integration in Eq. (10). For given \( s \) and \( \mathbf{k} \), we integrate over \( \mathbf{q} \) first and then over \( s \). For simplicity, we assume \( \mathbf{k} = (k_x, 0, 0) \) and \( s = (s \cos \theta, s \sin \theta, 0) \), and denote \( \mathbf{q} = q_1 \hat{x}_1 + q_2 \hat{x}_2 + q_3 \hat{x}_3 \), where \( \hat{x}_1 = (k + s \cos \theta, s \sin \theta, 0) / |\mathbf{k} + \mathbf{s}| \), \( \hat{x}_2 = (-s \sin \theta, 1 + \cos \theta, 0) / |\mathbf{k} + \mathbf{s}| \), and \( \hat{x}_3 = (0, 0, 1) \). In this coordinate system, \( A \) depends only on \( k \), \( s \), \( \theta \), and \( q_2 \):

\[
A^{-1}(\mathbf{k}, s, q_2) = \frac{\hbar^2 \mathbf{k} \cdot \mathbf{s}}{m_e (\hbar \omega)^2} \kappa^{-1} \kappa^+,
\]

where

\[
\frac{1}{\kappa^+} = 1 - \frac{k^2 + sk \cos \theta}{|\mathbf{k} + \mathbf{s}|^2} + \frac{\hbar}{m_e \omega} \left( \frac{sk \sin \theta q_2}{|\mathbf{k} + \mathbf{s}|} \pm \frac{sk \cos \theta}{2} \right).\]

The real part of \( \alpha \) remains still complicated, but the imaginary part given in Eq. (12) could be simplified as the delta function would eliminate \( q_1 \) integration and the integrand of \( dq \) is independent of \( q_3 \). We do not present the detailed steps here. After some tedious manipulation, the \( q \)-integration is reduced to an one-dimensional integration

\[
\int \frac{d^3 \mathbf{q}}{(2\pi)^3} A^2(\mathbf{q}, \mathbf{k}, s) \frac{\mathbf{F} e^{-i \mathbf{q} \cdot \mathbf{u}}}{\mathbf{F}} \text{Im} \left[ \frac{f(\mathbf{q} + \mathbf{k} + \mathbf{s}) - f(\mathbf{q})}{\hbar \omega - E(\mathbf{q} + \mathbf{k} + \mathbf{s}) + E(\mathbf{q})} \right]
\]

\[
= \left( \frac{\hbar^2 \mathbf{k} \cdot \mathbf{s}}{m_e (\hbar \omega)^4} \right) \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \frac{m_e}{4 \pi^{2} \hbar^{2} |\mathbf{s} + \mathbf{k}|} \kappa^+ \kappa^{-1} \left( R^-(k_F, z, u, q_2) - R^+(k_F, z, u, q_2) \right),
\]

where \( R^\pm(k_F, z, u, q_2) = \sqrt{k_F^2 (1 - (z \pm u)^2) - q_2^2} \) if \( k_F^2 (1 - (z \pm u)^2) > 0 \), \( R^\pm(k_F, z, u, q_2) = 0 \) if \( k_F^2 (1 - (z \pm u)^2) \leq 0 \), \( z = |\mathbf{k} + \mathbf{s}|/2k_F \), and \( u = \omega/\kappa \). The right-hand side of the above equation is a function only of \( k \), \( s \), and \( \theta \). We do the \( s \)-integration in the spherical coordinate and arrive at

\[
\text{Im} [\alpha(k, \omega)] = \int \frac{2\pi^{2} d\mathbf{q} d\mathbf{s}}{(2\pi)^3} n_f(s) \left( \frac{\hbar^2 k s \mu^2}{m_e^2 (\hbar \omega)^4} \right)
\]

\[
\times \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \frac{m_e}{4 \pi^{2} \hbar^{2} |\mathbf{s} + \mathbf{k}|} \kappa^+ \kappa^{-1} \left( R^-(k_F, z, u, q_2) - R^+(k_F, z, u, q_2) \right),
\]

This is a three-dimensional integration that needs to be evaluated numerically. In Fig. (3), \( \eta \) in Eq. (10) as a function of \( k \) is shown for a hydrogen plasma of two different electron densities.

**VI. RAMAN SCATTERING**

In this Section, we discuss the existence of a regime where an x-ray pulse can be compressed by the BRS.
More detailed presentation can be found in Ref. 26 for the case of metals. Here we provide a similar computation in the case of degenerate dense hydrogen plasmas.

The BRS is a three-wave interaction where two light pulses and one Langmuir wave exchange the energy 3, 39–41. A one-dimensional interaction can be written as 33.4:

\[
\begin{align*}
&\left( \frac{\partial}{\partial t} + v_1 \frac{\partial}{\partial x} + \nu_1 \right) A_1 = -ic_1 A_2 A_3, \\
&\left( \frac{\partial}{\partial t} + v_2 \frac{\partial}{\partial x} + \nu_2 \right) A_2 = -ic_2 A_1 A^*_3, \\
&\left( \frac{\partial}{\partial t} + v_3 \frac{\partial}{\partial x} + \nu_3 \right) A_3 = -ic_3 A_1 A^*_2,
\end{align*}
\]

where \( A_i = eE_i/n_e\omega_i c \) is the ratio of the electron quiver velocity of the pump pulse \( (i = 1) \) and seed pulse \( (i = 2) \), relative to the velocity of the light c, and \( A_3 = n_e/n_e \) is the intensity of the inverse bremsstrahlung wave. \( \nu_1 (\nu_2) \) is the rate of the inverse bremsstrahlung of the pump (seed), \( \nu_3 \) is the plasmon decay rate, \( c_i = \omega_{pi}/2\omega_i \) for \( i = 1 \) and 2, \( c_3 = (\alpha q)^2/2\omega_3 \), and \( \omega_3 (\omega_2) \) is the frequency of the pump (seed). In the BRS (FRS), the energy conservation is given as \( \omega_1 = \omega_2 + \omega_{pe} \) (\( \omega_1 = \omega_2 + \omega_{pe} \)) and the momentum conservation is given as \( q = k_1 - k_2 \) (\( q = k_1 + k_2 \)).

The inverse bremsstrahlung \( (\nu_1 \text{ and } \nu_2) \) in metals is given as 33, 36.

\[
\nu(\omega, E) = 4\pi n_1 Z^2 \frac{e^4}{m^2 c^6} \frac{\omega_{pe}^2}{\omega^2} \frac{F(\alpha)}{\alpha^2/\kappa^2/2},
\]

where \( \kappa = (\hbar \omega/2m_e v_F^2) < 1 \), \( v_F \) is the Fermi energy and \( \alpha^2 = 2e^2 E^2/m_3 \hbar^3 \) with E being the electric field of the pulse, and the function \( F(\alpha) \) can be approximated as \( \alpha^2/6 \) when \( \alpha < 2 \) and \( (2/\alpha) \log(2\alpha) \) otherwise 33. In Eq. (16), it is assumed that \( \hbar \omega > E_F \) and \( \hbar \omega > T_e \) where \( T_e (E_F) \) is the electron temperature (Fermi energy). The above equation can be simplified, when \( \alpha < 2 \), as

\[
\nu = 2.9 \times 10^{15} Z K^{-3/2} \frac{\omega_{pe}^2}{\omega^2} \text{ sec}.
\]

If \( 1/\nu \) in Eq. (17) is shorter than the pulse duration, the pump pulse will be heavily damped before being compressed. Note that \( \nu_1 \) is proportional to \( \omega_{pe}^2/\omega^2 \).

From Eqs. (15) and (16), the BRS growth rate, assuming that the pump intensity is large enough to make the BRS unstable, is roughly estimated to be

\[
g_B = \frac{A_1^2}{\eta(q)} \frac{\omega_2}{\omega_{pe}} \frac{\omega_{pe}}{\omega} \text{ sec}.
\]

The larger \( g_B \) is, the stronger the BRS is.

Now, a possible operating regime for the Raman compression is obtained from Eqs. (16), (15), and (17). The stability condition for the FRS is given, from Eq. (15), as \( c_2 c_3 |A_1|^2 < v_2 \nu_2 \nu_3 \), where \( v_2 \) is obtained from Eq. (17) and \( \nu_3 \) is from Eq. (8), and \( c_3 = \omega_{pe}/2 \). This condition can be written as \( |A_1| > |A_{1F}| \) where

\[
|A_{1F}|^2 = \frac{4e^2(q_F)^2\nu_1}{\omega_{pe}^2} \frac{\omega_2}{\omega_{pe}}.
\]

and \( q_F = \omega_{pe}/c \). The same condition for the BRS can be estimated, using \( c_3 = (\omega_1 + \omega_2)^2/2\omega_{pe} \), as \( |A_1| < |A_{1B}| \), where

\[
|A_{1B}|^2 = \frac{v_3(q_B)}{\omega_{pe}^2} \frac{4\omega_{pe}^2 \omega_2}{(\omega_2 + \omega_1)^2}.
\]

and \( q_B = (\omega_1 + \omega_2)/c \). \( |A_{1B}|^2 \) in Eq. (20) is smaller than \( |A_{1F}|^2 \) in Eq. (19) by a factor of \( (\omega_1 + \omega_2)^2/\omega_{pe}^2(\eta(q_F)/\eta(q_B)) \), which is very large since \( \eta(q_F)/\eta(q_B) \sim 1 \). Therefore there exists a regime where the BRS is unstable but the FRS is stable. This is due to the strong inverse bremsstrahlung and a rather slowly varying damping of the Langmuir waves as a function of \( k \).

We provide an estimation for hydrogen plasmas with \( n_e = 10^{24} \text{ cm}^{-3} \) and \( 10^{26} \text{ cm}^{-3} \). In the former case, when \( \hbar \omega_{pe} = 37.11 \text{ eV} \) and \( \omega_1/\omega_{pe} = 10 \), it is estimated that \( \eta(k) \sim 0.06 \) from Fig. 8. The threshold for the FRS estimated from Eq. (19) is \( |A_{1F}|^2 = 3 \times 10^{-4} \), which corresponds to \( 3.4 \times 10^{19} \text{ W cm}^{-2} \). The threshold for the BRS estimated from Eq. (20) is \( |A_{1B}|^2 = 0.0075 \times 10^{-4} \) or \( 8.4 \times 10^{16} \text{ W cm}^{-2} \). For \( n_e = 10^{26} \text{ cm}^{-3} \), \( \hbar \omega_{pe} = 371.19 \text{ eV} \), \( \omega_1/\omega_{pe} = 10 \), \( \eta(0) \approx 0.18 \), and \( \eta(k = 2\omega/e) \approx 0.25 \) from Fig. 8. The threshold for the FRS estimated from Eq. (19) is \( |A_{1F}|^2 = 0.288 \times 10^{-4} \) which corresponds to \( 3.225 \times 10^{20} \text{ W cm}^{-2} \). The threshold for the BRS estimated from Eq. (20) is \( |A_{1B}|^2 = 0.000998 \times 10^{-3} \) or 1.116 \( \times 10^{18} \text{ W cm}^{-2} \).

\[ \text{VII. CONCLUSION} \]

The prevalent Landau damping theory predicts the plasmon decay rate to vanish in the long wavelength limit. On the other hand, the observations from the electron stopping experiments in metals suggest that it is finite in that limit. Using a well-known dielectric function formalism developed in the condense matter, we propose a theoretical framework predicting the plasmon decay rate in dense plasmas more accurately (Eq. (10)) than conventional theories. A few advantages of our approach are in order: First, the robustness of this framework is verified through the experiments with metals 20. Second, the quantum degeneracy and the diffraction of the electrons are self-consistently accounted for. Third, the inter-ion correlations could be incorporated for a wide range of physical parameter regimes through \( v_1(s) \), the static two-point correlation function among the ions. Lastly, the theory takes into account of the Umklapp process. The computation based on this dielectric function approach suggests that the plasmon decay rate in dense
plasmas in the long wavelength limit is much higher than the prediction by other theories.

This strong decay has implications for various processes in dense plasmas, warm dense matter and metals. For instance, it renders the FRS less harmful for the Raman compressor in metals and dense plasmas, compared to the FRS in rare dense plasmas. The plasmon decay in metals strongly depends on the direction relative to the lattice structure. As the electron temperature $T_e$ increases, the electron screening of the ions gets weaker, resulting in a stronger plasmon damping \[20, 27\]. As $T_e$ increases further, the ions would finally lose their lattice structure. A larger value of $\eta(k)$ might be preferred for a strong pump, as it would weaken the FRS; however, a small value of $\eta(k)$ might be preferred for a weak pump, as a strong BRS is necessary.

As a consequence, there exist a parameter regime in metals where the BRS compression is strong while the FRS is stable (Sec. VI). As an example, a pump pulse with a duration of hundreds of femto seconds could be compressed, via a plasmon, to a pulse of a few or sub-femto second without FRS pump depletion \[26\]. The computation of the decay rate for a wide range of parameter regimes, by extending our formalism given in Eq. (12), may lead to detailed engineering of the plasmon decay for the x-ray BRS compression.

The plasmon decay in dense plasmas is a complicated process. The Umklapp process dominates for low $k$ as presented here, and the Landau damping dominates for high $k$. In addition, the degeneracy and diffraction would suppress the damping \[6\]. Experiments measuring the damping rate in warm dense matter might be readily available by observing the loss of the electron energy in the thin heated foil experiment \[18\]. The effect of the phase transition on the plasmon decay is theoretically challenging, but is important for its practical application in the BRS x-ray compression and other processes.

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