The Calculation of Firing Safety Zone Based on Ballistic Curve Model

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Abstract. This paper discusses the determination of the safety zone of artillery fire drill. For the convenience of research, the safety zone is divided into absolute safety zone and relative safety zone. The ballistic curve model starts from the analysis of the trajectory of the projectile, through the analysis of the forces on the flying projectile, using Newton's second law and the differential equations, the ballistic curve model describing the trajectory of the projectile is established. After that, the trajectory curve parameters of given launch angle are calculated by using MATLAB software, and the farthest range of projectile is obtained. According to the relationship between the range and the angle of the projectile in the ballistic curve model, the angle of the projectile can be obtained, and then combined with the upper limit of the angle of the projectile launching error, the range of the projectile falling point can be obtained.

1. Introduction
The problem of establishing a safe area must be considered when artillery is shooting. Taking the projectile's launching speed as an example: \( v_0 = 200 \text{m/s} \), the launching angle is not limited, and it is required to obtain the safe area of the shooting exercise by analyzing the trajectory curve.

For the determination of the safety zone, it is obvious that we should start from the killing range of the shell. The killing range of shell is determined by the range, killing radius, firing error and other parameters. Among them, the killing radius and firing error of the shell are determined by the performance of the weapon itself, so the determination of the range of the shell is the key to solve this problem. It is clear that beyond the maximum range is the absolute safety zone. But in the actual shooting performance, it will not use the farthest range of the shell to hit the target, because the shooting accuracy will be very poor. Therefore, the relative safety zone should also be considered when the weapon aims at a certain target within the range.

Based on Newton's second law, the differential equation describing the ballistic curve of projectile can be established. Then, the function relation between the range and the angle of the projectile is derived by using the equation. Thus, the firing angle corresponding to the farthest range and any range can be obtained. Combined with the upper limit of firing error angle, the absolute safety zone and relative safety zone of artillery firing exercise can be determined.

2. The Establishment of Model

2.1. The Establishment of Ballistic Curve Model
It is assumed that the flying projectile is an ideal particle, and the air resistance is only related to the velocity and not affected by the wind. Analyze the force of its operation in the air, as shown in the following figure.
In the figure, \( f \) is the angle between the shell speed and the horizontal direction, \( \theta \) is the radius of curvature of the shell trajectory, and \( \rho \) is the radius of curvature of the shell trajectory. By analyzing the above figure, considering the acceleration in tangent direction and centripetal acceleration in normal direction of projectile trajectory, the ballistic differential equation model can be obtained:

\[
\begin{align*}
\frac{dV}{dt} &= -f(V) - mg \sin \theta \\
\frac{V^2}{\rho} &= mg \cos \theta
\end{align*}
\]

(1)

Considering the calculation method of air resistance, reference [1], when the velocity of projectile is not more than 200m/s, it is generally considered that the air resistance is directly proportional to the square of velocity. Therefore:

\[ f(V) = kV^2 \]

(2)

It can be solved by combining (1) and (2):

\[ V^2 = \cos^2 \theta \left[ C - \frac{1}{2} K \ln \left( \frac{1+\sin \theta}{1-\sin \theta} \right) \right] - K \sin \theta \]

(3)

\[ \frac{dx}{d\theta} = \frac{V^2}{g} \cos \theta \left( C - \frac{1}{2} K \ln \left( \frac{1+\sin \theta}{1-\sin \theta} \right) \right) \]

\[ \frac{dy}{d\theta} = \frac{V^2}{y} \tan \theta \left( C - \frac{1}{2} K \ln \left( \frac{1+\sin \theta}{1-\sin \theta} \right) \right) \]

\[ \frac{dt}{d\theta} = \frac{V}{g} \left( C - \frac{1}{2} K \ln \left( \frac{1+\sin \theta}{1-\sin \theta} \right) \right) \]

(4)

\[ x = \int_{\alpha_1}^{\alpha_2} \frac{1}{g} \left[ \cos^2 \theta \left( C - \frac{1}{2} K \ln \left( \frac{1+\sin \theta}{1-\sin \theta} \right) \right) \right] d\theta \]

\[ y = \int_{\alpha_1}^{\alpha_2} \tan \theta \left[ \cos^2 \theta \left( C - \frac{1}{2} K \ln \left( \frac{1+\sin \theta}{1-\sin \theta} \right) \right) \right] d\theta \]

\[ t = \int_{\alpha_1}^{\alpha_2} \sin \theta \left[ \cos^2 \theta \left( C - \frac{1}{2} K \ln \left( \frac{1+\sin \theta}{1-\sin \theta} \right) \right) \right] d\theta \]

(5)

(6)

(7)
\( \theta_i \) is the angle between the instantaneous velocity direction of the shell and the horizontal line, and \( \theta_0 \) is the launching angle.

2.2. The Establishment of Security Zone Model

Considering the problem of safety area in shooting exercise, a plane rectangular coordinate system is established, which takes the position of gun as the origin and the line between gun and target as the x-axis.

Obviously, the area beyond the maximum damage range of the shell is an absolutely safe area. During the actual shooting exercise, the shell aims at a certain target within the range. The artillery determines the firing angle according to the target point, and the firing angle determined by the artillery must have error, which will cause the shell to fall in an area around the target point. This area, together with the shell damage extension area, is an unsafe area. As shown in the figure below:

![Diagram](image)

A: Relative safety zone  
B: Absolute safety zone  
C: Hazardous area

**Figure 2.** The schematic diagram of each area

The absolute safe area is outside the circle with the origin as the center and \( L + r \) as the radius. \( L \) is the maximum range, which can be obtained from the ballistic curve model, and \( r \) is the killing radius of the shell.

In the coordinate system, the absolute safe area can be expressed as:

\[
x^2 + y^2 > (L + r)^2
\]  

(8)

For the target point at \( x_0 \) meter from the gun aiming line, substitute the data into the ballistic curve model (1) and (2), and the theoretically corresponding launching angle \( \theta \) can be obtained. Considering the aiming error \( \delta \), the maximum offset target distance can be obtained by using the ballistic curve model:

\[
l = \max\{X(\theta + \delta) - x_0, X(\theta - \delta) - x_0\}
\]  

(9)

\( X(\theta) \) is the range when the launching angle is \( \theta \), which can be calculated by formula (4).
According to formula (9), the circle with $x_0$ as the center and $l + r$ as the radius is the dangerous area:

$$(x - x_0)^2 + y^2 < (l + r)^2$$  \hspace{1cm} (10)

Removing this area and absolute safety area is relative safety area. Its expression is:

$$\begin{cases} (x - x_0)^2 + y^2 > (l + r)^2 \\ x^2 + y^2 < (L + r)^2 \end{cases}$$  \hspace{1cm} (11)

In conclusion, formula (9) and (11) are the safe area models.

3. The Solution of Software Simulation

It is difficult to solve the differential equation in the trajectory curve model, and it can not get the explicit solution of the trajectory equation. In this paper, MATLAB software is used to program the numerical solution of the differential equation (refer to the data in [2], take the air resistance coefficient). Some special solutions are listed in chart form.

**Table 1.** The special solution of ballistic differential equation

| Initial velocity direction (degrees) | Range (m)   | Height (m) | Flight time (seconds) |
|-------------------------------------|-------------|------------|-----------------------|
| 10                                  | 1407.697    | 54.951     | 8.081                 |
| 20                                  | 2180.484    | 206.135    | 14.115                |
| 30                                  | 2680.128    | 416.314    | 19.91                 |
| 40                                  | 2864.622    | 671.952    | 24.933                |
| 50                                  | 2779.426    | 930.843    | 29.434                |
| 60                                  | 2368.117    | 1172.503   | 32.162                |
| 70                                  | 1730.695    | 1368.242   | 33.875                |
| 80                                  | 925.291     | 1501.343   | 35.058                |
| 90                                  | 0.000       | 1542.751   | 36.017                |
Figure 3. The trajectory of projectile with different angles

Table 2. The maximum range parameter

| Parameters for maximum range |
|-----------------------------|
| Initial velocity direction (degrees) | Range (m) | Height (m) | Flight time (seconds) |
| 35.359 | 2974.579 | 549.544 | 24.395 |

In reference [2], the maximum killing radius of the shell \( r = 17.8 \) meter, and the error \( \delta = 0.5 \) degrees. Taking \( x_0 = 2680 \) as an example, combined with the ballistic curve model, the relevant parameters of the solution are as follows:

Table 3. The related parameters of safety zone model

| \( L \) | \( l \) | \( l + r \) | \( L + r \) |
|-------|-------|-------|-------|
| 2974.579 | 8.14 | 15.94 | 2992.4 |

It can be seen that the absolute safety area is:

\[
x^2 + y^2 < 8954500
\]

And the relatively safe areas are:

\[
\begin{cases}
(x - 2680)^2 + y^2 > 254.0836 \\
x^2 + y^2 < 2992.4 
\end{cases}
\]

4. Evaluation of the Model

It is an important tactical calculation in artillery battle command system to determine the firing safety zone. In this paper, the research on the safe area of artillery acting can be solved by computer, which can meet the requirement of automatic calculation, and solve the problem of accurately determining...
the safe area in theory, which provides an effective calculation method for the development of new artillery battle command system.

5. References
[1] Lirong Li, Using numerical method to solve the problem of ballistic curve, *Inner Mongolia Agricultural University, March 2003, vol 24*

[2] Liangyu Zhao, Shuxing Yang, Ballistic curve fitting method based on MATLAB and iSIGHT, *Tactical missile technology, September 2006, p 87-89.*