How the growth of lake ice depends on the fluid dynamics underneath

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Convective flows coupled with solidification or melting in water bodies play a major role in shaping geophysical landscapes. Particularly in relation to the global climate warming scenario, it is essential to be able to accurately quantify how water-body environments dynamically interplay with ice formation or melting process. Previous studies have revealed the complex nature of icing process, but have often ignored one of the most remarkable particularity of water, its density anomaly, and the induced stratification layers, interacting and coupled in a complex way in presence of turbulence and phase change. By combining experiments, numerical simulations and theoretical model, we investigate solidification of fresh water, properly considering phase transition, water density anomaly, and real physical properties of ice and water phases, which we show to be essential for correctly predicting the different qualitative and quantitative behaviors. We identify, with increasing thermal driving, four distinct flow-dynamics regimes, where different levels of coupling among ice front, stably and unstably stratified water layers occur. Despite the complex interaction between the ice front and fluid motions, remarkably, the average ice thickness and growth rate can be well captured with the theoretical model. It is revealed that the thermal driving has major effects on the temporal evolution of the global icing process, which can vary from a few days to a few hours in the current parameter regime. Our model can be extended towards general situations where icing dynamics may also be influenced by factors like, e.g., undercooling conditions or water layer depth.

I. INTRODUCTION

Many geophysical patterns result from the interaction between fluid motions and the dynamical evolution of solid phase boundaries. Usually the dynamics of the solid boundaries are due to phase change or erosion. Examples range from sculpturing of glacier, ice shelf, iceberg and sea caves due to flows in the oceans, to congelation ice forming in ponds and lakes and many geological patterns [1], astrophysical landforms [2], as well as in our daily lives and many industrial processes [3, 4].

Generally, warm water is lighter and so it floats, whereas cold water is denser and therefore it sinks. However, this is not the case once water is around 4 degrees: water expands when it is colder than 4°C (the nonmonotonic relationship of density with temperature for water near 4°C is reported in SI Appendix, section C and Fig. S3). This is why ice layer first start forming on top of lakes, otherwise fishes and other aquatic creatures would not be able to survive the severe winter. This water density anomaly near the 4 degree results in the complex coupling between the ice layer, the gravitationally stably stratified layer of fluid (0 < T ≤ 4°C) and the unstably stratified layer (T > 4°C) [5, 6]. The stably stratified layer always exists in the ice-water system, but its may be enhanced or depleted under different levels of turbulence.

Connecting to the complex fluid dynamics in the water, the evolution of the ice-water interface and the phase change at the interface show very rich dynamics, which recently have received increasing attention. Rayleigh-Bénard (RB) convection, a fluid layer confined between a cold top plate and a hot bottom plate [7, 8], is an ideal model system to study the aforementioned coupled dynamics. Various studies have been performed on the flow in RB system with freezing or melting top boundary conditions. The focus has been on the studies on behaviors of global quantities such as the heat flux and the kinetic energy and the dynamics of the ice-water interface morphology with a melting boundary in a RB...
II. RESULTS AND DISCUSSION

A. Experiments and simulations

The experiments are performed in a Rayleigh-Bénard convection system of cuboid shape (aspect ratio $\Gamma = L_x/H = 1$, $L_x$, $H$ are the system width and height) heated up from the bottom and cooled down from the top. Water, as the working fluid, is deionized, ultrapured and degassed. The top plate undercooling temperature, $T_t$, and bottom plate temperature, $T_b$, are imposed by water-circulating bath, with $T_t < T_\phi$ and $T_t > T_\phi$ ($T_\phi$ is the water freezing point, $T_\phi = 0^\circ$C). In such a configuration, ice starts forming from the top plate and it grows till its saturation thickness. During the experimental process, there is a volume change induced by thermal expansion of water and water-ice phase change, so an open expansion vessel is connected to the experimental cell allowing to quantify the volume change, and therefore the pressure of the system remains atmospheric pressure. By monitoring the water volume change inside the expansion vessel, the evolution of the spatial average ice thickness can also be calculated (details are shown in SI Appendix, section A and B and Figs. S1 and Figs. S2). In addition to the experiments, two- and three-dimensional simulations are carried out by means of Lattice-Boltzmann method (LBM) numerical code [4, 7, 8]. In the simulations, we consider the source term from the latent heat at the ice-water interface [6] and the correction for the governing equations when the investigated domain consists of heterogeneous media, i.e., ice and water phases (SI Appendix, section E) [9]. Two- ($\Gamma = L_x/H = 1$) and three-dimensional simulations are conducted ($\Gamma = L_x/H = 1, L_y = 1/4H$, same as the experimental cell; $L_y$ being the system width). The boundary conditions are no-slip for the velocity, adiabatic at the sidewalls, and constant temperatures at the top and bottom plates. We assume thermophysical properties to be constant except for the density in the buoyancy term. The real water density property near to $4^\circ$C is well described with the equation $\rho = \rho_0(1 - \alpha^*|T - 4^\circ|)$, where $\alpha^*$ is not the usual thermal expansion coefficient but has units of $(K^{-1})$ with $q = 1.895$ and $\alpha^* = 9.30 \times 10^{-6}(K^{-1})$. This equation gives the maximum density of water $\rho_0 = 999.972$ kg/m$^3$ at $T = 4^\circ$C [11] (see also in SI Appendix, section C and Fig. S3).

One important control parameter of the system is the Rayleigh number, which is the dimensionless thermal forcing, and its definition formula is explained below (more details are shown in SI Appendix, section D). Another important control parameter is the Stefan number which relates the sensible heat to the latent heat, Ste = $C_{pi}(T_b - T_i)/L$ with $C_{pi}$ being the isobaric heat capacity of ice and $L$ the latent heat of solidification. In order to make sure that the fluid dynamics of the water region is the only influencing factor for the ice evolution, the top undercooling temperature, $T_t$ (correspondingly also the Stefan number), both in experiments and simulations is fixed at a typical value for winter, which we select as $T_t = -10^\circ$C and thus the Stefan number $\text{Ste} \sim 10^{-2}$. The bottom plate temperature, $T_b$ (connected to Rayleigh number to be explained below), is varied in a wide parameter regime, i.e., in experiments $3.8^\circ$C $\leq T_b \leq$ $8^\circ$C and in simulations $0.5^\circ$C $\leq T_b \leq$ $15^\circ$C (typical water temperature in winter).

An important response to the imposed Ra$_e$ and Ste is the overall heat flux transported vertically from bottom to top. The dimensionless heat flux is Nusselt number, Nu (more details are shown in SI Appendix, section D).

B. The final average ice position

We first compare the final average ice position, $h_0$, as a function of the bottom plate temperature from the experiments, the two dimensional (2D-) and two dimensional (3D-) simulations and from the theoretical model (the details of the model will be discussed later).

Figure 1A is a photo of the experimental domain at $T_b \approx 8^\circ$C when the system has reached the statistical equilibrium state. With the same operating conditions, the visualization from the 3D-simulation of the ice position and underneath temperature field in the fluid phase at the statistical equilibrium state is shown in Fig. 1B. As shown in Fig. 1A and B, the ice position is similar from the experiment and the numerical simulation at the same condition. Varying bottom plate temperature, $T_b$, in a large temperature range, the spatially average ice...
position at the equilibrium state as a function of $T_b$ is shown in Fig. 1C. Depending on $T_b$, the system may end up in a diffusive state (refer to the green shaded area of Fig. 1C) or in a convective state. There is a good agreement on the height of the spatially average ice-water interface among the experiments, the 2D- and 3D- simulations as well as the theoretical model with considering water density anomaly. However, it is noteworthy that when neglecting the water density anomaly the prediction of ice position from the model (see the violet line in Fig. 1C) deviates dramatically from the real value. The key reason is that the stably stratified layer (with temperature ranging from 0°C to 4°C), which results from the density anomaly of water, is crucial for the dynamics of the system. As the results from 2D- and 3D- simulations are similar, below we will explore the complex nature of the coupled dynamics mostly via 2D- simulations as these allow to more efficiently explore the phenomena in a wide range of parameters.

C. The coupled dynamics of the ice growth with the fluid motion

To investigate the physical mechanism, we highlight four distinct regimes based on the phenomenology of the equilibrium state as the bottom plate temperature increases from below to above 4°C (Fig. 1A–D). The four regimes that will be considered are as follows, where the first two letters of the acronyms specify the feature of stratification, which can be either the stably stratified (SS) or the unstably stratified (US), and the third letter of the acronyms specifies the mode of heat transport (and fluid motion) which can be either diffusion (D) or convection (C): 1) Regime-1: SSD with flat ice ($T_b \leq 4°C$); 2) Regime-2: SSD + USD with flat ice ($4°C < T_b \leq 5.1°C$); 3) Regime-3: SSD + USC with flat ice ($5.1°C < T_b \leq 6.9°C$); and 4) Regime-4: SSD + USD with deformed ice front ($T_b > 6.9°C$).

The boundaries between different regimes depend on the bottom plate temperature. Fig. 1A–D show typical cases from all four regimes from the simulations. Next, we discuss the details of the four regimes.

**Regime-1 ($T_b \leq 4°C$)** Figure 1A shows a typical case in this regime. The system is in a stably stratified state with purely diffusive heat transfer all the way from the beginning (see Fig. 1AI) till the end (see Fig. 1AIII), the corresponding sketch, which shows different layers at the statistical equilibrium state in the system, can be seen in Fig. 1AIII. The ice-water interface is always flat indicating that the instantaneous 0°C isotherm overlaps with the average position of the ice front, $h_0$. The temperature profiles are linearly dependent on the height both in the ice and water phases, with the different slopes corresponding to the different thermal conductivity in ice and water (Fig. 1AIV).

**Regime-2 ($4°C < T_b \leq 5.1°C$)** Raising the bottom plate temperature into this regime, the gravitationally unstably stratified layer (from the level of the bottom plate to the spatially average level of 4°C denoted as $h_4$, namely the horizontally average temperature is 4°C at $z = h_4$, with the temperature ranging from 4°C to
FIG. 2. The phenomenology of temporal dynamics and the feature at the statistical equilibrium states in the four regimes. Typical cases visualizations from Regimes-1–4: (A) $T_b = 3.8^\circ C$. (B) $T_b = 4.75^\circ C$ (see SI Movies1). (C) $T_b = 5.5^\circ C$ (see SI Movies2). (D) $T_b = 10^\circ C$ (see SI Movies3). Different time instances of temperature field and instantaneous $0^\circ C$ and $4^\circ C$ isotherms (thick black lines) for four typical regimes of the simulations (panels I and II in (A)–(D)). The sketches on panels III in (A)–(D) depict the coupled different layers of the system at the statistical equilibrium state in Regimes-1–4 respectively, in which the interface (horizontal lines) between neighboring layers (different color shaded areas) are space-average value. The dashed black line is for $h_0$; dotted blue line is for $h_4$; thick black curved lines are for instantaneous $0^\circ C$ and $4^\circ C$ isotherms respectively; dash-dotted lines are for the upper bound $h^*_4$ and lower bound $h^*_0$ of instantaneous $4^\circ C$ isotherms. Panels IV in (A)–(D): the temporal and space-average temperature profiles at the statistical equilibrium state corresponding to the four typical cases. In panels III&IV of (A)–(D), the blue-shaded, yellow-shaded and orange-shaded areas denote ice (ICE), stably stratified layer (SS) and unstably stratified layer (US) respectively. To make the flow structures more visible, two approaches are applied: 1) two colorbars for the temperature field corresponding to ice region($T_1(x,z)$) and water region ($T_W(x,z)$) are shown on the right of (D)I&II; 2) (A)I, (C)I&II, and (D)I&II show more isotherms (thin black lines) except for $0^\circ C$ & $4^\circ C$ isotherms (thick black lines), which are designed to make the hot and cold plumes more noticeable.
shrinks and $R_a$ consequently decreases. And thus the system ends up at a diffusive state in the entire water layer (SSD+USD) with effective Rayleigh number in US layer $R_a \sim 10$ smaller than $R_a$ (see Fig. 2[I]). This also simplifies why the 4°C isotherm becomes flat in the end (see Fig. 2[II]), and the corresponding sketch is shown in Fig. 2[III]. The entire system is in a diffusive state with a linear temperature profile (see Fig. 2[IV]) similar to that in Regime-1.

**Regime-3 (5.1°C < $T_b$ ≤ 6.9°C)** As $T_b$ is in Regime-3, with temperature range from 5.1°C to 6.9°C, there are rich fluid dynamics in the fluid layer below the ice. The system ends up in the convective state with $R_a \sim 10^5$ (see Fig. 2[II]). We can see hot plumes form from the bottom plate. During the lifetime of hot plumes, they detach from the bottom plate shortly after being generated; the plumes accumulate and become coherent plumes, which rise through the bulk region while experiencing heat exchange with the fluid around; if in a classical Rayleigh-Bénard system they would later on go through the cold boundary layer below the flat 4°C isotherm where they give out most of the energy and slow down to stop, however, in the stably and unstably stratified coupled system presented here, bunches of plumes can impact on and deform the 4°C isotherm because of turbulent bursts, which is similar to the penetration of turbulent bursts from unstable layer near the inner cylinder to the stable layer near the outer cylinder. The 4°C isotherm is no longer flat but develops some spatial variations (see the thick black line in Fig. 2[II]). The region from the spatially average height, $h_4$, of the instantaneous 4°C isotherm to its upper bound, $h_4^+$, belongs to the gravitationally stably stratified layer but there exists also some warmer patches of fluid with the temperature larger than 4°C from the unstably stratified layer. Due to mass conservation, the same amount of fluid, with a temperature smaller than 4°C coming from the gravitationally unstably stratified layer, goes downwards below the level of $h_4$ (see the downward cold plumes in the region from the lower bound of the instantaneous 4°C isotherm $h_b^-$ to $h_4$ in Fig. 2[III]). In other words, due to the nonmonotonical behavior of water with respect to the temperature, on average sense there is stably stratified layer with diffusive heat transfer (SSD, from the level of $h_4$ to that of $h_b$) and unstably stratified layer with convective heat transport (USC, from the level of the bottom plate to that of $h_b$), however instantaneously because of the penetration, there is strong fluid exchange between SSD and USC as indicated by the deformation of instantaneous 4°C isotherm. Because of the shield of SSD which still has a horizontal layer with fluid temperature purely smaller than 4°C ($h_b^-$ to $h_b$), the ice-water interface is still flat in Regime-3. In this regime, the temperature profile in the entire water layer is not a linear profile any more (see Fig. 2[IV]). In the entrainment layer (from the level of $h_4$ to the level of $h_b^+$) and underneath USC, the temperature profile reflects the turbulence-induced mixing. There is hot thermal boundary layer attached to the bottom plate, well-mixed bulk region of nearly uniform temperature and the cold thermal boundary layer, and all of these are similar to that in the classical Rayleigh-Bénard convection.

**Regime-4 (6.9°C < $T_b$) Upon further increasing $T_b$ to above 6.9°C, the level of upper bound of instantaneous 4°C $h_4^+$ is even higher than the spatially average level of ice position $h_b$, which indicates the hot plumes from USC can directly impact on the ice surface. On the impact spot, ice will melt and form a concave interface due to extra heat input. We can see that there is no horizontally stably stratified layer with fluid temperature purely smaller than 4°C which can shield the ice front from the turbulent convective motion. The 4°C isotherm line is not in a well-defined position, instead it displays intensive spatial fluctuations due to strong turbulent plumes, resulting in local melting or freezing of the ice front. The water layer consists a very wide range of USC at the equilibrium state (refer to Fig. 2[III]). The temperature profile is similar to that in Regime-3 but with a much thicker water layer thickness and a much thinner ice layer.

In summary, we can see that the heat transfer regimes of diffusion and convection can be even switched dynamically during the evolving process due to the fact that the USC thickness is changing, so the system may end up in a diffusive or convective state depending on the final effective Rayleigh number $R_a$ (namely $R_a$). The equilibrium state depends on the bottom plate temperature, $T_b$. Next, we assess the detailed ice dynamics in a more quantitative perspective.

The flow is highly dynamic in the Regime-3 and 4, and the intricate nature of the intensive interaction among the ice front, the entrainment layer, and the unstably-stratified layer leads to high fluctuations of 4°C and 0°C isotherm lines varying in a range (see the black-shaded area and the red-shaded area in Fig. 3). Nevertheless the global responses of the system, i.e., the spatial-average thicknesses of the ice-water interface, $h_b$ (where the horizontally average temperature is 0°C), and the, $h_4$ (where the horizontally average temperature is 4°C), match up well to the 1D-model for water (to be discussed below) except for some deviations in the Regimes-3 and 4. The ice-water interface and the 4°C isotherm line attach and adjust with each other, which results in a self-organizing large scale circulation, and the overall effects shape the ice front as shown in Fig. 2[II].

D. Theoretical model

The ice thickness can be properly predicted by taking into account the water density anomaly and the known scaling properties of turbulent thermal convection (namely the Nusselt number-Rayleigh number relation [19]). Next we introduce the theoretical model and we consider two situations: 1) for statistical equilibrium states, and 2) for the time-dependent transient states. Here, we assume one dimensional geometry and all the
the thicknesses as follows,
\[
\begin{align*}
H - h_0 &= \frac{-k_1 T_i}{k_W T_b - k_1 T_i} H, \\
H_0 - h_4 &= \frac{k_W T_b}{k_W T_b - k_1 T_i} H.
\end{align*}
\]

In the temperature range \( T_b > 4^\circ C \), we assume that the interfaces of the ice front and that between the stably stratified and the unstably stratified layers are both flat,
\[
\begin{align*}
&k_1 \frac{T_\phi - T_i}{H - h_0} = k_W \frac{4 - T_\phi}{h_0 - h_4}, \\
&T_\phi - T_i = Nu k_W \frac{T_b - 4}{h_4}.
\end{align*}
\]

The model for the heat flux in the unstably stratified layer is in the form of Nusselt number as a function of Rayleigh number (Nusselt number is the dimensionless heat flux defined as \( Nu = \frac{k_1 \partial T}{k_W (T_b - 4)/h_4} \)). The Rayleigh number dependence of Nu can be obtained from the simulations and is consistent with that of the classical Rayleigh-Bénard in the same parameter regime suggesting that the model we build is of a general form (more details are reported in SI Appendix, section D).

By this statistical equilibrium state model, the final ice position, as a function of \( T_b \) (see Fig. 1C and Fig. 3), and the \( 4^\circ C \) isotherm position, as a function of \( T_\phi \), can be calculated, and shows a good agreement with the results from simulations as well as experiments.

2) theoretical model for water: transient state

Following the analytical methods for the classical Stefan problem since the time-dependent evolving interface between ice and water (denoted as \( z = h_0(t) \)) where \( h_0(t) \) is the height at which \( T_W(h_0(t), t) = 0^\circ C \) is a priori unknown, a part of the solution will be to determine the boundary. As the phase transition occurs, there will be volume change due to the density difference between water and ice as well as the thermal expansion effect.

In order to simplify the problem, here we ignore this volume variation. Further, we consider the one-dimension heat transfer problem and assume that the physical properties are invariant with temperature while their values are different for the ice and water phase: the ice-water interface is fixed at phase change temperature \( T_\phi \) (recall \( T_\phi = 0^\circ C \)).

When \( T_b \leq 4^\circ C \), the basic control equations are
\[
\begin{align*}
\frac{\partial T_I(z, t)}{\partial t} &= \alpha_I \frac{\partial^2 T_I(z, t)}{\partial z^2}, \quad 0 < z < h_0(t), \\
\frac{\partial T_W(z, t)}{\partial t} &= \alpha_W \frac{\partial^2 T_W(z, t)}{\partial z^2}, \quad h_0(t) < z < H,
\end{align*}
\]
where \( \alpha \) is the thermal diffusivity, the subscripts “I” and “W” denote ice and water phase respectively. The boundary conditions read
\[
\begin{align*}
T_W(0, t) &= T_0, \\
\lim_{z \to h_0(t)^-} T_W(z, t) &= \lim_{z \to h_0(t)^+} T_I(z, t) = T_\phi, \\
T_I(H, t) &= T_i.
\end{align*}
\]
where the superscripts “+” and “−” indicate the direction when taking the limit, namely from smaller than \( h_0(t) \) towards \( h_0(t) \) and from larger than \( h_0(t) \) towards \( h_0(t) \), respectively. The nonlinear energy balance at the ice-water interface is

\[
L \rho_1 \frac{dh_0(t)}{dt} = k_1 \frac{\partial T_1(h_0(t)^+, t)}{\partial z} - k_W \frac{\partial T_W(z, t)}{\partial z} |_{z=h_0(t)^-} - k_W \frac{\partial T_W(z, t)}{\partial z} |_{z=h_0(t)^+},
\]

where \( L \) is latent heat for solidification of water, \( k \) the conductivity. From Eqsns. (21, 22, 23, 24), we obtain the solutions for temperature distributions in the ice and water,

\[
T_W(z, t) = T_b - \frac{T_b}{\text{erf}(\lambda_W)} \text{erf}\left(\frac{z - h_0(t)}{2\sqrt{\alpha_W}t}\right),
\]

\[
T_i(z, t) = \frac{T_i}{\text{erf}(\lambda_i)} \text{erf}\left(\frac{z - h_0(t)}{2\sqrt{\alpha_i}t}\right),
\]

where \( \text{erf} \) is the error function and

\[
\lambda_W = \frac{h_0(t)}{2\sqrt{\alpha_W}t}, \lambda_i = \frac{H - h_0(t)}{2\sqrt{\alpha_i}t}. \tag{10}
\]

When \( T_b > 4^\circ C \), the effective Rayleigh number can be calculated and the interface energy balance takes the form:

\[
L \rho_1 \frac{dh_0(t)}{dt} = k_1 \frac{\partial T_1(h_0(t)^+, t)}{\partial z} - \text{Nu} k_W \frac{T_b - T_\phi}{h_0(t)}. \tag{11}
\]

Based on Eqn. (21) and (11) with boundary conditions Eqn. (23), the position of the ice-water interface as a function of time can be solved, and therefore we can predict the temporal evolution of the global icing process. of the icing process (see Fig. 4B) (more details are reported in SI Appendix, section D).

**E. Growth dynamics of the ice layer**

The coupled interactions between the stably and the unstably stratified layers play a major role in determining the final saturation thickness of the ice layer and the time it takes to reach the saturation state (here we define the saturation time as 90% of the final statistical equilibrium state thickness). Remarkably, we find that the ice growth still follows a diffusive process \((1 - h_0)/H \propto t^{0.5}\) for all the cases as shown in Fig. 4A. The physical reason is that there is always a thermal diffusive layer attached to the ice front (ranging from \(0^\circ C\) to \(4^\circ C\)) no matter how thin this layer is. The saturation time \(t^*\) versus the bottom plate temperature is shown in Fig. 4B, which clearly shows a good agreement between simulations (black symbols) and experiments (orange symbols). We also compare the experimental and numerical results with that in the theoretical model. They show a good agreement except that there are some deviation in the Regime-2 and 3, which maybe due to the complex dynamics around the onset of convection. Further, the coexistence of stably and unstably stratified layers leads to the effective convective region smaller than the entire water depth, which may contribute to the discrepancy. Based on the investigated parameter regime, it is revealed that the temperature of the bottom surface has major effects on the icing time. To give the reader an impression of the physical saturation time scale, for example, when the bottom plate increases from \(T_b = 0.5^\circ C\) to \(T_b = 15^\circ C\), the saturation time can vary from a few days to a few hours.

Here, we use a fixed top undercooling temperature \((T_1 = -10^\circ C)\) as a typical example, nevertheless, it should be noted that in real natural situation, the icing dynamics may also be influenced by undercooling condition, whole water layer depth and other factors, to which our findings are still applicable and the model is easy to be extended to general situations.
The approach followed in this study, which is based on the matching of controlled laboratory scale experiments with fully resolved direct-numerical simulations sets a standard for future explorations on convection coupled to phase-change problems. We plan to continue by studying the effect of container aspect ratio, ice-water interface inclination, dissolved salt, overburden pressure, the topic of which are of great relevance for better modeling of geophysical and climatological large-scale processes.

IV. MATERIALS AND METHODS

Below, we provide basic information on the experiments, theoretical model and numerical simulations performed in this work. Further details and additional figures are provided in SI Appendix.

A. Experimental setup

The turbulent convection coupled with solidification of fresh water experiments were performed in a classical convection setup (SI Appendix, Fig. S1A). The experimental cell, of rectangular shape, consists of plexiglas sidewalls with height $H = 240\text{ mm}$ (length $L_x = 240$ mm and width $L_y = 60$ mm, i.e., aspect ratio $\Gamma = L_x/H = 1.0$). The working fluid is confined in between the copper top plate (cooled by circulating bath (PolyScience PP15R-40)) and the copper bottom plate (heated by by circulating bath (PolyScience PP15R-40)). In the experiments, ice forms on the top plate and grows in thickness until the system reaches a statistical equilibrium state. During the phase change process, there is a volume change. In order to release the pressure due to volume change induced by phase-change, an expansion vessel is connected to the experimental cell through a tube. The expansion vessel is open to the atmosphere so that the pressure of the experimental cell is kept constant. To avoid evaporation of the water in expansion vessel, we use a thin layer of silicon oil (immiscible with water) to seal the water surface. By monitoring the water level inside the expansion vessel, the spatial average ice position, $h_0$ (i.e., the ice thickness is $H - h_0$), can be calculated for each bottom plate temperature, $T_b$ (SI Appendix, section B). Six resistance thermistors (44000 series thermistor element, SI Appendix, Fig. S1C) are embedded into the top and bottom plates, respectively. To control the temperature the setup to avoid the heat exchange between the experimental cell and the environment, there are two kinds of techniques applied: 1) the experimental cell is wrapped in a sandwich structure: insulation foam, aluminium plate, and insulation foam; 2) a PID (Proportional-Integral-Derivative) controller (SI Appendix, Fig. S1B) is installed to the setup (more details are reported in SI Appendix, section A and section B). The working fluid is deionized and ultrapure water. Before the experiments, water is boiled twice to degas.
Since water density inverses at temperature 4°C, here we use the nonmonotonic relationship of density with temperature for water near 4°C and details are reported in SI Appendix, section C.

### B. Theoretical model

The theoretical model with considering density anomaly for the system are divided into at and we assume one dimensional geometry: 1) for statistical equilibrium states; and 2) for the time-dependent transient states. We also perform theoretical model without considering water density anomaly and prove that in this case the ice position can’t be predicted properly. More details about the theoretical model are reported in SI Appendix, section D.

### C. Numerical simulations

We use lattice-Boltzmann method (LBM) which is able to capture the turbulent convective dynamics in the water phase and also describe the phase change process at the ice-water interface. SI Appendix, section E provides more details about the relevant equations that govern phase change, fluid flow, and heat transfer solved by the LBM algorithm.

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SI: SUPPLEMENTARY INFORMATION

Section A: Experimental setup

Rayleigh-Bénard convection experiments coupled with solidification of fresh water are performed in a classical convection setup (see Fig. 5). Fig. 5A reports a sketch of the experimental cell, of rectangular shape, consisting of a plexiglas sidewall with height \( H = 240 \) mm (length \( L_y = 240 \) mm and width \( L_x = 60 \) mm, i.e., aspect ratio \( \Gamma = L_x/H = 1.0 \)). The working fluid is deionized and ultrapure water. Before conducting any experiments, water is boiled twice to degas. The working fluid is confined in between the copper top plate (cooled by a circulating bath, PolyScience PP15R-40) and the copper bottom plate (heated by a circulating bath, PolyScience PP15R-40). The top and bottom plates and the sidewalls are sealed using silicon O-ring. During the experiments, the top plate temperature, \( T_t \), and bottom plate temperature, \( T_b \), are kept constant, with \( T_t < 0^\circ \text{C} \) and \( T_b > 0^\circ \text{C} \). In order to focus on how the fluid dynamics of the water region influences the ice formation, the top plate temperature \( T_t \) (and therefore the Stefan number, \( \text{Ste} \)) is fixed in the experiments at a typical value in winter, which we select as \( T_t = -10^\circ \text{C} \) (i.e., \( \text{Ste} \sim 10^{-2} \)). The bottom plate temperature \( T_b \) (i.e., Rayleigh number, \( Ra \)) is varied in the temperature range of \( 3.8^\circ \text{C} \leq T_b \leq 8^\circ \text{C} \). In the experiments, ice forms on the top plate and grows in thickness until the system reaches a statistical equilibrium state. During the phase change process, there is a volume change. In order to release the pressure due to volume change induced by phase-change, an expansion vessel is connected to the experimental cell through a tube. The expansion vessel is open to the atmosphere so that the pressure of the experimental cell is kept constant. To avoid evaporation of the water in the expansion vessel, we use a thin layer of silicon oil (immiscible with water) to seal the water surface. By monitoring the water level inside the expansion vessel, the evolution of the spatial average ice thickness can be calculated. Six resistance thermistors (44000 series thermistor element, see Fig. 5C) are embedded into the top and bottom plates respectively (refer to the black shaded circles on the top and bottom plates in Fig. 5A for the positions of the thermistors). The experimental cell is wrapped in a sandwich structure: insulation foam, aluminium plate, and insulation foam. There is a PID (Proportional-Integral-Derivative) controller (see Fig. 5B) to control the temperature of the setup in order to avoid heat exchange between the experimental setup and the environment.

Section B: Calculation of ice thickness as a function of time in experiments

The evolution of the spatial average ice position, \( h_0 \) (the ice thickness is \( H - h_0 \)), can be calculated for each bottom plate temperature, \( T_b \), by monitoring the water level inside the expansion vessel.

There are three parts in the system: the Rayleigh-Bénard convection cell (RB cell, red dashed box), the expansion vessel (EV, green dashed box) and the tube (blue dashed box) which connects the RB cell and the EV (see Fig. 5A). During the experiments, the mass of water and ice in the system (RB cell + tube + EV) is conserved (the initial mass of the system \( m_0 \) is a priori known), the total volume (the ice volume and the water volume) will change due to isobaric thermal expansion of water and ice formation, both of which will induce a redistribution of the mass in the system.

The argument of mass conservation yields:
\[ m_0 = \rho_W(T_m) \cdot A_{RB} \cdot h_0(t) + \rho_I \cdot [A_{RB} \cdot (1 - h_0(t))] + m_{tube} + \rho_w(T_0) \cdot V_{EV}(t). \]  

where \( \rho_W \) is the water density as a function of the mean temperature of the water in the RB cell \( T_m = T_b/2 \), \( A_{RB} \) is the cross sectional area of the RB cell, \( \rho_I \) is the ice density evaluated at the mean temperature of the ice layer \( T_I/2 \), \( m_{tube} \) is the water mass in the connecting tube, and \( V_{EV} \) is the volume of water in the EV (green dashed box in Fig. 6). 

So the general form of the ice position, as a function of time \( h_0(t) \), is

\[ h_0(t) = \frac{m_0 - \rho_W(T_0) \cdot V_{EV}(t) - \rho_I A_{RB} - m_{tube}}{\rho_W(T_m) A_{RB} - \rho_I A_{RB}}. \]  

Next, we estimate the measurement error on the ice position, \( h_0 \). The expansion vessel is made of a burette on which there are scales, and therefore the volume of water in the expansion vessel can be read directly. The scale on the expansion vessel has the minimum value of 0.1ml which can lead to accuracy errors when calculating \( h_0 \). There is also another factor associated with water evaporation in the expansion vessel that may induce error. It has been mentioned in Subsection A that the expansion vessel is open to the atmosphere to keep the pressure constant, and we use an oil seal to decrease the evaporation of water from the expansion vessel. To evaluate the evaporation effect, we measure the evaporation rate of water in the expansion vessel on condition of oil seal, which is approximately 1ml decrease for three days. On top of these, the minimum and maximum variation in ice position are 0.5% to 7% variation of \( h_0 \).

**Section C: The nonmonotonic relationship of density with temperature for water near 4°C**

The working fluid in the experiments is deionized ultrapure water. Since water density inverses at the temperature of 4°C, here we use the nonmonotonic relationship of density with temperature for water near 4°C from Ref. [1],

\[ \rho_w = \rho_0 (1 - \alpha^* |T_b - 4|^q), \]  

with \( \rho_0 = 999.972 kg/m^3 \), \( \alpha^* = 9.30 \times 10^{-6} (K^{-q}) \), \( q = 1.895 \). The density of water, \( \rho_w \), as a function of temperature, \( T \), is shown in Fig. 7.

**Section D: Theoretical model**

*Theoretical model incorporating water density anomaly*

In this section, we introduce the theoretical model that accounts for the density anomaly of water. We consider two situations and we assume one dimensional geometry:

1) for statistical equilibrium states;
2) for the time-dependent transient states.

Next we discuss the details about the two situations.

**1) theoretical model for water: statistical equilibrium state**

When the system has reached the statistical equilibrium state, there is an energy balance between the heat flux through the ice and that through the water. When \( T_b > 4°C \), the water layer consists of a stably stratified layer (from 0°C to 4°C) and an unstably stratified layer (from 4°C to \( T_b \)). So there are three kinds of heat flux that balance at the statistical equilibrium state: 1) the diffusive heat flux in the ice layer; 2) the diffusive heat flux in the stably stratified layer; 3) the convective heat flux in the unstably stratified layer, from which we can calculate the average thicknesses of the ice layer \((H - h_0)\), stably stratified layer \((h_0 - h_4)\) and unstably stratified layer \((h_4, exists when \( T_b > 4°C \)) at the equilibrium state.

An important step is to model the convective heat flux in the unstably stratified layer, which is similar to the classical Rayleigh-Bénard convection that we will show later.

Here we define the effective Rayleigh number, \( Ra_e \), based on the unstably stratified layer, which induces the thermal buoyancy driving force when \( T_b > 4°C \),

\[ Ra_e = \frac{(\Delta \rho/\rho_0)g(h_4)^3}{\nu \kappa} = \frac{g \alpha^*(T_b - 4)^q(h_4)^3}{\nu \kappa}. \]
where \( g \) is the gravitational acceleration, \( \nu \) the kinematic viscosity, and \( \kappa \) the thermal diffusivity. Correspondingly, the effective Nusselt number is defined as the heat flux compensated by the diffusive heat flux based on the thickness of the unstably stratified layer \( h_4 \) and its temperature difference \((T_b - 4^\circ C)\),

\[
\text{Nu}_e = \frac{k^I}{k^W} \frac{\text{grad}(T)|_{z=H}}{(T_b - 4)/h_4}.
\] (16)

An empirical fit on the simulation data points similar to Ref. [2] is as follows,

\[
\text{Nu}_e = \begin{cases} 1, & \text{when } \xi \leq 0, \\ 1 + C_1 \xi, & \text{when } 1 < \xi \leq 1.23, \\ C_2 \xi^\beta, & \text{when } \xi > 1.23. \end{cases}
\] (17)

with \( \xi = (\text{Ra}_e - \text{Ra}_{cr})/\text{Ra}_{cr} \), \( C_1 = 0.88, C_2 = 0.27 \times \text{Ra}_{cr}^\beta \) with \( \beta = 0.27 \), and all these values are based on the simulation results. \( \text{Nu}_e \) as a function of \( \text{Ra}_e \) is shown in Fig. [8] where the simulations results are the red circles and the an empirical fit on the simulation data points similar to Ref. [2] given by Eqn. [17] is the black line.

Fig. [8] also reports the numerical results of classical Rayleigh-Bénard convection from Ref. [3]. We can see that, despite that our system has different conditions (ice layer, stably stratified layer and unstably stratified layer coexist and couple with one another), there is good agreement on the Nu-Ra relation between the current simulation results and the classical Rayleigh-Bénard convection, suggesting that the Nu-Ra relation is robust [4]. So we can use the principle in classical Rayleigh-Bénard convection to model our system.

In the temperature range \( T_b \leq 4^\circ C \). The system is in a diffusive state and independent of the water layer thickness, the total water layer is stably-stratified,

\[
k_I T_\phi - T_t \frac{H - h_0}{h_0 - h_4} = k_W \frac{T_b - T_\phi}{h_0 - h_4},
\] (18)

where \( T_\phi \) is the phase change temperature \((T_\phi = 0^\circ C)\), \( k_I \) and \( k_W \) are the thermal conductivity of ice and water, respectively. From which we obtain the results on the thicknesses as follows,

\[
\begin{align*}
H - h_0 &= \frac{-k_I T_t}{k_W T_b - k_I T_t} H, \\
h_0 - h_4 &= \frac{k_W T_b}{k_W T_b - k_I T_t} H.
\end{align*}
\] (19)

In the temperature range \( T_b > 4^\circ C \). We neglect the fact that the interfaces of the ice front and that between the stably and unstably stratified layers can be curved, at the statistical equilibrium state we have

\[
\begin{align*}
&k_I \frac{T_\phi - T_t}{H - h_0} = k_W \frac{4 - T_\phi}{h_0 - h_4}, \\
&k_I \frac{T_\phi - T_t}{H - h_0} = \text{Nu}_e k_W \frac{T_b - 4}{h_4}.
\end{align*}
\] (20)

2) theoretical model for water: transient state

Following the analytical methods of classical Stefan problem [5], since the time-dependent evolving interface between ice and water (denoted as \( z = h_0(t) \)), where \( h_0(t) \) is the height at which \( T_W(h_0(t), t) = 0^\circ C \) is a priori unknown, a part of the solution will be to determine the boundary. As the phase transition occurs, there is a volume change due to the density difference between water and ice as well as the thermal expansion effect. In order to simplify the problem, we ignore here this volume variation. Further, we consider the one-dimension heat transfer problem and assume that the physical properties are invariant with temperature while their values are different for the ice and water phases; the ice-water interface is fixed at the phase change temperature \( T_\phi \) (recall \( T_\phi = 0^\circ C \)).

When \( T_b \leq 4^\circ C \), the basic control equations are

\[
\frac{\partial T_I(z,t)}{\partial t} = \alpha_I \frac{\partial^2 T_I(z,t)}{\partial z^2}, \quad 0 < z < h_0(t),
\] (21)

\[
\frac{\partial T_W(z,t)}{\partial t} = \alpha_W \frac{\partial^2 T_W(z,t)}{\partial z^2}, \quad h_0(t) < z < H,
\] (22)
where $\alpha$ is the thermal diffusivity, the subscripts “I” and “W” denote ice and water phase respectively. The boundary conditions read

$$T_W(0, t) = T_b,$$

$$\lim_{z \to h_0(t)^-} T_W(z, t) = \lim_{z \to h_0(t)^+} T_I(z, t) = T_\phi,$$

$$T_I(H, t) = T_t,$$

(23)

where the superscripts “+” and “−” indicate the direction when taking the limit, namely from smaller than $h_0(t)$ towards $h_0(t)$ and from larger than $h_0(t)$ towards $h_0(t)$, respectively. The energy balance at the ice-water interface is

$$L \rho_I \frac{dh_0(t)}{dt} = k_I \frac{\partial T_I(z, t)}{\partial z} |_{z=h_0(t)^+} - k_W \frac{\partial T_W(z, t)}{\partial z} |_{z=h_0(t)^-},$$

(24)

where $L$ is the latent heat for solidification of water, $k$ the conductivity. From Eqns. (21, 22, 23, 24), we obtain the solutions for temperature distributions in the ice and water,

$$T_W(z, t) = T_b - \frac{T_b}{erf(\lambda_W)} erf\left(\frac{z}{2\sqrt{\alpha_W t}}\right),$$

$$T_I(z, t) = \frac{T_t}{erf(\lambda_I)} erf\left(\frac{z - h_0(t)}{2\sqrt{\alpha_I t}}\right),$$

(25)

where erf is the error function with

$$\lambda_W = \frac{h_0(t)}{2\sqrt{\alpha_W t}}, \quad \lambda_I = \frac{H - h_0(t)}{2\sqrt{\alpha_I t}}.$$

(26)

When $T_b > 4^\circ C$, the water layer consists of stably and unstably stratified layers and the interface between these two layers is $h_4(t)$, to simplify the problem of estimating the convective heat flux of the water layer, here we define the nominal Rayleigh number $Ra$ and Nusselt number $Nu$ based on the whole water layer from the bottom plate to $h_0(t)$ with temperature difference ($T_b - 0 ^\circ C$). The definitions for $Nu$ and $Ra$ are as follows:

$$Ra = \frac{g \alpha^* (T_b - 0)^3 (h_0(t))^3}{\nu \kappa},$$

$$Nu = \frac{k_I}{k_W} \frac{grad(T)|_{z=H}}{(T_b - 0)/h_0(t)}.$$

(27)

By comparing the definition of the nominal Rayleigh number, $Ra$, and Nusselt number, $Nu$, with the effective Rayleigh number, $Ra_e$, and effective Nusselt number, $Nu_e$, we can find the relations in between $Ra$ and $Ra_e$ as well as $Nu$ and $Nu_e$ as follows,

$$Ra = Ra_e \cdot \varphi_1^q \cdot \varphi_2^3,$$

$$Nu = Nu_e \cdot \varphi_1^{-1} \cdot \varphi_2,$$

(28)

with

$$\varphi_1 = \frac{T_b - 0}{T_b - 4},$$

$$\varphi_2 = \frac{h_0}{h_4}.$$

(29)

Using Eqns. (17, 28) we can have the model for $Nu$ as a function of $Ra$ so that the convective heat flux based on the whole water layer can be calculated. The energy balance at the ice-water interface takes the form:

$$L \rho_I \frac{dh_0(t)}{dt} = k_I \frac{\partial T_I(h_0(t)^+, t)}{\partial z} - Nu k_W \frac{T_b - T_\phi}{h_0(t)}.$$

(30)

Based on Eqn. (21) and (30) with boundary conditions Eqn. (23), the position of ice-water interface as a function of time can be solved, and therefore we can predict the temporal evolution of the global icing process.
For the theoretical model without considering the water density anomaly, there is no such thing as nominal or effective Rayleigh number, so here the Rayleigh number \(Ra^*\) is defined based on the whole water layer, i.e., the whole water layer thickness \(h_0\) and the corresponding temperature difference \((T_b - 0)\), which is shown as follows,

\[
Ra^* = \frac{g\alpha(T_b - 0)(h_0)^3}{\nu k},
\]

where \(\alpha\) is the thermal expansion coefficient of water evaluated at the mean temperature of the investigated range of \(T_b\) (\(\sim 7^\circ\text{C}\)).

At the equilibrium state, the energy balances between the diffusive heat flux in the ice layer and the heat flux in the whole water layer, which takes the form:

\[
k_I T_\phi - T_I = \frac{Nu_w k_I}{h_0} \frac{T_b - T_\phi}{h_0}.
\]

By using \(Ra^*\) to predict the Nusselt number in the Eqn. (30) we can solve the equation and get the ice position \(h_0\) for different \(T_b\) just shown in Fig. 1C of the main paper.

**Section E: Introduction to the numerical methods: governing equations and numerical simulations**

In this section we introduce the relevant equations that govern the phase change, the fluid flow, and the heat transfer. The governing equations in the water layer are,

\[
\nabla \cdot \bar{u}(x, y, z, t) = 0,
\]

\[
\frac{\partial \bar{u}}{\partial t} + \bar{u}(x, y, z, t) \cdot \nabla \bar{u}(x, y, z, t) = \frac{\nabla p}{\rho_0} + \nu \nabla^2 \bar{u}(x, y, z, t) + \alpha^* g |T(x, y, z, t) - 4| e_z,
\]

\[
\rho C_p \frac{\partial T(x, y, z, t)}{\partial t} + \nabla \cdot (\rho C_p \bar{u}(x, y, z, t) T(x, y, z, t)) = \nabla \cdot (k \nabla T(x, y, z, t)).
\]

where \(\bar{u}(x, y, z, t)\), \(p(x, y, z, t)\), \(T(x, y, z, t)\) are fluid velocity, pressure, and temperature fields, respectively; \(\nu, k, \rho, C_p, g\) are the kinematic viscosity of water, the thermal conductivity, the density, the specific heat, and the acceleration of gravity, respectively. When it is in water phase \(k = k_w\), \(\rho = \rho_w = \rho_0(1 - \alpha^*[T_b - 4])\), \(C_p = C_{pw}\), and when it is in ice phase \(k = k_I, \rho = \rho_I, C_p = C_{pi}\). All the physical properties of water and ice phase, except for \(\rho_w\) are evaluated at the mean temperatures in each phase which are \((T_b + 0)/2\) and \((T_i + 0)/2\), respectively.

The boundary conditions corresponding to the governing equations above are isothermal at the top and bottom plates, no-slip at the bottom plate and at the ice-water interface, adiabatic at the lateral boundaries, and no-slip and freezing (namely, Stefan condition [5]) at the phase-changing interface. The boundary conditions read:

\[
T(x, y, 0, t) = T_b,
\]

\[
T(x, y, H, t) = T_i,
\]

\[
u(x, y, 0, t) = 0,
\]

\[
u(x, y, h_0(x, y, t), t) = 0,
\]

\[
\frac{\partial T(0, y, z, t)}{\partial y} = 0,
\]

\[
\frac{\partial T(L_x, y, z, t)}{\partial y} = 0,
\]

\[
L \rho_0 \frac{dh_0(x, y, t)}{dt} = k_I \frac{\partial T_1(x, y, z, t)}{\partial z}|_{z=h_0(x, y, t)} - k_W \frac{\partial T_W(x, y, z, t)}{\partial z}|_{z=h_0(x, y, t)}.
\]

where \(L\) is the latent heat and \(h_0(x, y, t)\) is the position vector of a point belonging to the ice-water interface.

The boundary condition at the ice-water interface is often difficult to solve since it is time- and space-dependent. So an useful method is to separate the total enthalpy \(h\) into sensible heat and latent heat [6]:

\[
h = \begin{cases} 
L \phi_w + C_{pi} T, & \text{when } T < T_\phi, \\
L \phi_w + C_{pi} T_\phi, & \text{when } T = T_\phi, \\
L \phi_w + C_{pi} T_\phi + C_{pw}(T - T_\phi), & \text{when } T > T_\phi.
\end{cases}
\]
where $T_\phi$ is the phase change temperature ($T_\phi = 0$), and $\phi_l(x, y, z, t)$ is the liquid fraction in the system and the relation between $h_0(x, y, t)$ and $\phi_l(x, y, z, t)$ is $h_0(x, y, t) = \int_0^H \phi_l(x, y, z, t) \, dz$, where $H$ is the height of the investigated domain. In the ice phase, $\phi_w = 0$, and in the water phase, $\phi_w = 1$, which leads to an additional source term $S_1$ from the latent heat contribution at the ice-water interface in the energy conservation equation of Eqn. 33. On the other hand, we use the Lattice Boltzmann method (LBM) which is able to capture the turbulent convective dynamics in the water phase and also describe the phase change process at the ice-water interface. The basic principle and formulation has been extensively discussed e.g. in Refs. [4, 7, 8]. It is noteworthy that the key to accurately solve such problems is to recover the diffusion term in the energy conservation equation exactly and, similarly to [9], we implement the correction when the investigated domain consists of heterogeneous media which lead to another additional source term $S_2$ in the energy conservation equation of Eqn. 33. So the energy equation with consideration of two source terms $S_1$ and $S_2$ read

$$
\rho C_p \frac{\partial T(x, y, z, t)}{\partial t} + \nabla \cdot (\rho C_p \vec{u}(x, y, z, t)T(x, y, z, t)) = \nabla \cdot (k \nabla T(x, y, z, t)) + S_1 + S_2.
$$

where the first source term is $S_1 = -L \rho \frac{\partial \phi_w}{\partial t}$ and the second source term $S_2 = -\sigma k \nabla T(x, y, z, t) \nabla \frac{1}{\sigma} - \frac{\rho C_p}{(\rho C_p)_0} T(x, y, z, t) \vec{u}(x, y, z, t) \nabla \sigma$. Here $\sigma = \frac{\rho C_p}{(\rho C_p)_0}$ is the ratio of heat capacitance and $(\rho C_p)_0$ is reference heat capacitance [9].

**movie:** The temperature field overlapped with $0^\circ C$ and $4^\circ C$ isotherms for $T_b = 4.75^\circ C$ of Regime-2.

**Description:** The system starts from convection in the gravitationally unstably stratified layer, with the $4^\circ C$ isotherm deformed. As the ice grows, the effective height, $h_4$, shrinks and $Rae$ consequently decreases. And thus the system ends up at a diffusive state in the entire water layer. This also explains why the $4^\circ C$ isotherm becomes flat in the end.
FIG. 6. Sketch for the mass conserved region of the system. There are three parts: Rayleigh-Bénard convection cell (RB cell, red dashed box), the expansion vessel (EV, green dashed box) and the tube (blue dashed box) which connects the RB cell and EV.

FIG. 7. Water density anomaly: the nonmonotonic relationship of density with temperature for cold water near 4°C from Ref. [1].

_movie: The temperature field overlapped with 0°C and 4°C isotherms for $T_b = 5.5°C$ of Regime-3._
Description: The system begins with convection in the gravitationally unstably stratified layer, with the 4°C isotherm deformed. As the ice thickness grows, the convective intensity decreases nevertheless the system ends up in a convective state with a deformed 4°C isotherm. But at the final state, there is still a horizontally stably stratified layer, which protects the ice front from deforming.

_movie: The temperature field overlapped with 0°C and 4°C isotherms for $T_b = 10°C$ of Regime-4._
Description: The system begins with convection in the gravitationally unstably stratified layer, with the 4°C isotherm deformed. As the ice thickness grows, the convective intensity decreases a little bit and the system ends up in a
convective state. At the final state, the ice front is deformed with a pattern similar to the 4°C isotherm.

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