Negative Binomial Regression in Underdispersion (Case Study: Neonatal Mortality in Jambi City)

Corry Sormin¹, Gusmanely Z.¹

¹Department of Mathematics, Faculty of Science and Technology, Universitas Jambi

Abstract. Neonatalis is birth before 28 days of a baby. Factors that are considered to affect neonatal mortality include the number of visits in the 1st and 3rd trimester, the number of pregnant women receiving Tetanus Dipheria Immunization, the estimated number of neonatal infants with complications, the number of infants receiving Hepatitis B Immunization for less than 24 hours, the number of infants receiving BCG Immunization and number of 1 and 3 neonatal visits. Neonatal mortality is still very rare so that the right analysis is used, namely Negative Binomial Regression. This research aim to investigate negative binomial regression in underdispersion on neonatal mortality at Jambi City. These two regression methods are specifically used for Poisson distributed data because they are rare. The stages of the research that will be carried out are the Poisson distribution test and the equidispersion assumption, parameter estimation, model feasibility test, and selection of the best model. The results obtained that the best model without the variable number of 3rd-trimester visits or without the variable number of infants who received BCG immunization with AIC was 36.3.

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Corresponding Author:
Corry Sormin
Department of Mathematics, Faculty of Science and Technology, Universitas Jambi
Email: corry.sormin@unja.ac.id

1. Introduction
The National Population and Family Planning Agency (BKKBN) noted that the neonatal mortality rate based on the results of the Indonesian Demographic and Health Survey (SKDI) decreased, from 32 per 1,000 live births in 2012 to 15 per 1,000 live births in 2017[1,2,3,4,5]. The endogenous infant
mortality rate or Neonatal mortality is the number of deaths occurring in the first month expressed in per thousand live births after the baby is born [6,7,8,9,10,11,12]. One of the causes of neonatal death is the factor brought by the child from birth obtained from the parents when fertilization occurs or during pregnancy[13,14]. The existence of endogenous factors associated with pregnancy, causing neonatal death [15,16,17,18,19,20].

To assess the extent of the factors that cause neonatal death, an analysis is needed using negative binomial regression [21,21,23,24,25,26,27]. Often the data obtained has a variance from the response variable that is greater than mean (overdispersion) so that the assumption of equidispersion (mean and variance of the response variable is the same) in Poisson regression is not fulfilled. This shows that to overcome the problem of Overdispersion, the Negative Binomial Regression method is used.

2. Review Literature
2.1 Negative Binomial Regression

The Negative Binomial has the following probability density form \[ f(y) = \frac{\Gamma(y+1)}{\Gamma\left(\frac{1}{\alpha}\right)\Gamma(y+1)} \left(\frac{\frac{1}{\alpha}}{\mu + \frac{1}{\alpha}}\right)^\frac{1}{\alpha} \left(\frac{\mu}{\mu + \frac{1}{\alpha}}\right)^y \]

with \( \mu > 0, \frac{1}{\alpha} > 0, \) and \( y = 0,1,2, \ldots \). In general, the model of the Binomial Regression model is defined as follows:

\[ Y_i = \exp(\beta_0 + \beta_1 X_{1i} + \cdots + \beta_p X_{pi}) + \epsilon_i \]

Estimation in this regression uses Maximum Likelihood Estimation with likelihood function:

\[ L\left(\frac{1}{\alpha}, \mu\right) = \prod_{i=1}^{n} \frac{\Gamma\left(y_i + \frac{1}{\alpha}\right)}{\Gamma\left(\frac{1}{\alpha}\right)\Gamma(y_i + 1)} \left(\frac{\frac{1}{\alpha}}{\mu + \frac{1}{\alpha}}\right)^\frac{1}{\alpha} \left(\frac{\mu}{\mu + \frac{1}{\alpha}}\right)^{y_i} \]

In the Negative Binomial Distribution, the connecting function commonly used is the log link of the form \( g(\mu_i) = \log(\mu_i) = X_i^T \beta \) or in other words \( \mu_i = \exp(X_i^T \beta) \). As a result, obtainde:

\[ L(r, \beta) = \prod_{i=1}^{n} \frac{\Gamma\left(y_i + \frac{1}{\alpha}\right)}{\Gamma\left(\frac{1}{\alpha}\right)\Gamma(y_i + 1)} \left(\frac{\frac{1}{\alpha}}{\mu_i + \frac{1}{\alpha}}\right)^\frac{1}{\alpha} \left(\frac{\mu_i}{\mu_i + \frac{1}{\alpha}}\right)^{y_i} \]

After obtaining the likelihood function, the the logarithm of the function will be searched as below:

\[ \log L(r, \beta) = \sum_{i=1}^{n} \log \left(\frac{\Gamma\left(y_i + \frac{1}{\alpha}\right)}{\Gamma\left(\frac{1}{\alpha}\right)\Gamma(y_i + 1)}\right) + \frac{1}{\alpha} \log \left(\frac{1}{\exp(X_i^T \beta) + \frac{1}{\alpha}}\right) + y_i \log \left(\frac{\exp(X_i^T \beta)}{\exp(X_i^T \beta) + \frac{1}{\alpha}}\right) \]

The estimation of the parameters \( \alpha \) dan \( \beta \) will be quite complicated so that other solutions with iterative numerical methods will be needed to solve these non linear equations.
3. Results and Discussion

3.1. Poisson Distribution Testing

Before starting the model formation, it will first be tested whether the response variable in this case is the number of neonatal mortality not following the Poisson distribution or not. The hypothesis used is the number of neonatal mortality not following the Poisson distribution. Obtained $\alpha = 1.000$ means data follows the Poisson distribution.

3.2 Assumption of Equidispersion

*Binomial Negative Regression* analysis has different assumptions from the *Poisson Regression* which must meet the equidispersion assumption that is the mean is equal to the variance. The analysis requires that the response variable does not have the same mean value as the variance value. Obtained *Mean* = 0.25 and *Variance* = 0.197, this means that the equidispersion assumption is not met. As a result, the *Binomial Negative Regression* analysis can be continued.

3.3 *Binomial Negative Regression* Analysis

Based on the estimation results, the estimated regression equation for the number of neonatal mortality is as follows:

$$ \hat{Y} = \exp(-3.79851 + 0.005323X_1 + 0.001796X_2 + 0.002733X_3 - 1.635959X_4 + 0.007466X_5 - 0.000363X_6 + 0.229378X_7 + 0.004048X_8) $$

The result of the likelihood ratio (G) of 18,33262 can be concluded that the *Negative Binomial Regression* model can be used.

Next, the **Remove** method will be used in the regression analysis to find a better model in the *Binomial Negative Regression* analysis. First, the variable that gives the largest $z$ value to the model will be removed. The selection of the best model will be seen from the smallest AIC value which is summarized in the Table 1.

**Table 1.** The selection of the best model will be seen from the smallest AIC value which is summarized

| Model            | AIC  |
|------------------|------|
| All Variables    | 38.3 |
| Without Variable $X_2$ | 36.3 |
| Without Variable $X_4$ | 36.3 |

The final result of the predictive regression equation for the number of neonatal mortality:

$$ \hat{Y} = \exp(-3.779736 + 0.005897X_1 + 0.002751X_3 - 1.581273X_4 + 0.007014X_5 - 0.000377X_6 + 0.223X_7 + 0.003954X_8) \quad (1) $$
or the following equation:
\[ \hat{Y} = \exp(-3.81735 + 0.00509X_1 + 0.0018X_2 + 0.00275X_3 - 1.63411X_4 + 0.007555X_5 + 0.2289X_7 + 0.00403X_8) \] (2)

The interpretation of the model in equation (1), \( \beta_0 \) is -3.779736 which means that the probability of neonatal mortality in jambi city is 0.0228. Parameter \( \beta_6 \) of 0.000377 means the number of infants who received BCG immunization as much as one person will reduce the chance of 0.9996 neonatal mortality in Jambi City if other variables are considered constant. The interpretation of the model in equation (2), \( \beta_0 \) of -3.81735 means that the probability of neonatal mortality per public health center in Jambi City of 0.022.

4. Conclusion
The regression model for the number of neonatal mortality per public health center is as follows
\[ \hat{Y} = \exp(-3.779736 + 0.005897X_1 + 0.002751X_3 - 1.581273X_4 + 0.007014X_5 - 0.000377X_6 + 0.223X_7 + 0.003954X_8) \]

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