Anomaly Mediated Supersymmetry Breaking, D-terms and R-symmetry

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We explore two distinct resolutions to the tachyonic slepton puzzle in the Anomaly Mediated Supersymmetry Breaking scenario. Both are based on extending the MSSM by an anomaly free $U_1$ symmetry, and in both cases exact RG invariance is preserved.

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1 Introduction

Recently there has been interest in a specific and predictive framework for the origin of soft supersymmetry breaking within the MSSM, known as Anomaly Mediated Supersymmetry Breaking (AMSB). The basic AMSB solution is given by [1]:

\[
\begin{align*}
M &= M_0 \frac{\beta_g}{g} \\
h^{ijk} &= -M_0 \beta_{Y^{ijk}} \\
(m^2)^i_j &= \frac{1}{2} |M_0|^2 \mu \frac{d\gamma_i}{d\mu}.
\end{align*}
\]

where \(M\) is the gaugino mass, \(h^{ijk}\) the \(\phi^3\) coupling \(Y^{ijk}\) the superpotential Yukawa coupling and \((m^2)^i_j\) the \(\phi\phi^*-\)mass. They are all given in terms of the gravitino mass, \(M_0\), and the RG functions \(\beta_g\) and \(\gamma^i_j\) of the unbroken theory. It is interesting that two of these relations were first developed in an attempt to construct RG trajectories [2]; the results for \(M, h, m^2\) satisfy exactly the formulae for the corresponding \(\beta\)-functions given, for example in [2].

Direct application of the AMSB solution to the MSSM leads, unfortunately, to negative (mass)\(^2\) sleptons: in other words, to a theory without a vacuum preserving the \(U_1\) of electromagnetism. We explore two distinct resolutions [3][4] of this dilemma, both based on generalising the AMSB solution, while retaining the crucial property of RG invariance, which makes the low energy theory insensitive to the nature of new physics at high scales. (For some other approaches see Refs. [5]-[14]). Both our ideas are based on extending the MSSM with an extra \(U_1\); in the second case this \(U_1\) being associated with an \(R\)-symmetry. In both cases it transpires that requiring RG invariance of the generalised AMSB solution means that the \(U_1\) must have no mixed anomalies with the MSSM gauge group. Also in both cases, a distinguishing feature of the results is the existence of sum rules for the sparticle masses.

2 Fayet-Iliopoulos D-terms

A modification of the AMSB solution which has been studied in some detail is the simple replacement \(m^2 \rightarrow \hat{m}^2\) where

\[
(h^{\hat{m}^2})^i_j = (m^2)^i_j + m_0^2 \delta^i_j.
\]  

Here \(m^2\) is the basic AMSB solution form Eq. (1) and \(m_0^2\) is constant. This is not RG invariant (for constant \(m_0^2\)), but if instead we have

\[
(h^{\hat{m}^2})^i_j = (m^2)^i_j + m_0^2 \sum_{a=1}^{N} k_a(Y_a)^i_j
\]
then $\hat{m}^2$ is RG invariant, as long as each $Y_a$ corresponds to a $U_1$ invariance of the superpotential $W$ and also has vanishing mixed anomaly with each MSSM gauge group factor. This apparent miracle occurs because in fact the modification to $m^2$ proposed in Eq. (3) is precisely that introduced by a set of Fayet-Iliopoulos (FI) $D$-terms.

In the MSSM, there is a non-zero FI-term, but this cannot alone solve the slepton problem because its $(mass)^2$ contributions to the LH and RH sleptons have opposite signs, being dictated by the hypercharge of the relevant field. The minimal solution we proposed was the introduction of a single extra $U_1$, which we denote $U'_1$. The MSSM does not admit such a generation independent) anomaly-free $U_1$ so we need to introduce some new fields to cancel the associated anomalies. However cancellation of the mixed anomalies can be achieved within the MSSM itself; the hypercharges of the quark and Higgs multiplets are determined in terms of the lepton hypercharges, so that the MSSM admits two independent mixed-anomaly-free $U_1$ groups, the existing $U'_Y$ and another (which could be chosen to be $U_1^{B-L}$). If we also require absence of $(U')^3$ and $U'_1$-gravitational anomalies this can be achieved by introducing a set of MSSM singlets with $U'_1$ charges $s_i$ and imposing the constraints

$$\sum_{i=1}^N s_i = -3(2Y'_L + Y'_\tau), \quad \text{and}$$

$$\sum_{i=1}^N s_i^3 = -3(2Y'_L + Y'_\tau)^3,$$

(4)

where the existing MSSM $U_1$ corresponds of course to $s_i = 2Y'_L + Y'_\tau = 0$.

The classification of rational solutions to Eq. (4) is an interesting Diophantine problem; but as explained in [3], all we require for the RG invariance of Eq. (3) is the existence of the sets of charges $Y_a$; there need be no relic of the associated gauge symmetry (or the singlets $S_i$) in the low energy theory. This is the point of view we will take from now on.

In Table (1) we give a possible set of $U'_1$ charges with the $U'_Y$ ones for comparison. This set of $Y'$ charges correspond to requiring $\text{Tr}(YY') = 0$; of course the result is a linear combination of $U'_Y$ and $U_1^{B-L}$. The outcome is that the squark and slepton masses are given by

$$m_Q^2 = m_Q^2 + \frac{1}{6}\zeta_1 + \zeta_2 Y'_Q, \quad m_{\bar{Q}}^2 = m_{\bar{Q}}^2 - \frac{2}{3}\zeta_1 + \zeta_2 Y'_\tau,$$

$$m_{\tilde{e}}^2 = m_{\tilde{e}}^2 + \frac{1}{3}\zeta_1 + \zeta_2 Y'_\tau, \quad m_{\tilde{\tau}}^2 = m_{\tilde{\tau}}^2 - \frac{1}{2}\zeta_1 + \zeta_2 Y'_L,$$

$$m_{\tilde{\nu}}^2 = m_{\tilde{\nu}}^2 + \zeta_1 + \zeta_2 Y'_\nu,$$

(5)

where dependence on the FI coefficients, the Higgs vevs and the singlet sector is subsumed into the parameters $\zeta_{1,2}$. There is a substantial region of the $\zeta_{1,2}$-plane...
Table 1: Table of $U_1$ and $U'_1$ hypercharges.

|         | $Q$ | $L$ | $t^c$ | $b^c$ | $\tau^c$ | $H_1$ | $H_2$ | $S_i$ |
|---------|-----|-----|-------|-------|-----------|-------|-------|-------|
| $Y$     | $\frac{1}{6}$ | $-\frac{1}{2}$ | $-\frac{2}{3}$ | $\frac{1}{3}$ | 1 | $-\frac{1}{2}$ | $\frac{1}{2}$ | 0 |
| $Y'$    | $\frac{7}{3}$ | $-7$ | $\frac{5}{3}$ | $-\frac{19}{3}$ | 3 | 4 | $-4$ | $s_i$ |
\[(\overline{q}^i + \overline{q}^j + \overline{q}^k)Y_{ijk} = 0\]
\[2 \text{Tr} [\overline{q}C(R)] + Q = 0, \tag{9}\]

where \(Q\) is the one loop \(\beta_g\) coefficient and \(C(R)\) is the quadratic matter Casimir. It is easy to show that Eq. \((9)\) corresponds precisely to requiring that the theory have a non-anomalous \(\mathcal{R}\)-symmetry (which we denote \(\mathcal{R}\), to avoid confusion with our notation \(R\) for group representations). If we set
\[\overline{q}^i = 1 - \frac{3}{2}r^i, \tag{10}\]
then we see that Eq. \((9)\) corresponds to \((r^i + r^j + r^k)Y_{ijk} = 2Y_{ijk}\), which is the conventional \(\mathcal{R}\)-charge normalisation.

Turning to the MSSM we find that, as in the previous section, our solution will retain the crucial RG invariance as long as all the mixed anomalies of the \(\mathcal{R}\)-symmetry with the MSSM gauge group vanish. The MSSM does not admit such a generation independent \(\mathcal{R}\)-symmetry; however it does admit one that permits only 3rd generation Yukawa couplings and has identical \(\mathcal{R}\)-charges for the first two generations. We find that this can be achieved for for arbitrary values of the leptonic charges with the quark and Higgs charges determined as follows (we work with the fermionic charges, related to the \(\mathcal{R}\)-charges by \(q_f = r - 1\)):

\[
q_3 = \frac{4}{9} - \frac{1}{3}l_3 - \frac{1}{9} \kappa
\]
\[u_3 = -\frac{22}{9} - \frac{2}{3}l_3 - e_3 + \frac{1}{9} \kappa
\]
\[d_3 = -\frac{4}{9} + \frac{4}{3}l_3 + e_3 + \frac{1}{9} \kappa
\]
\[q_1 = -\frac{101}{90} - \frac{1}{3} \kappa + \frac{1}{15}l_3 + \frac{1}{5} e_3 + \frac{1}{30} \kappa + \frac{1}{18} \kappa
\]
\[u_1 = -\frac{79}{90} - \frac{2}{3} \kappa - \frac{16}{15} l_3 - \frac{6}{5} e_3 - \frac{1}{30} \kappa - \frac{1}{18} \kappa
\]
\[d_1 = \frac{101}{90} + \frac{4}{3} \kappa + \frac{4}{15} l_3 + \frac{4}{5} e_3 - \frac{1}{30} \kappa - \frac{1}{18} \kappa
\]
\[h_2 = -h_1 = l_3 + e_3 + 1, \tag{11}\]

where \(\kappa = l_1 - l_3 + e_1 - e_3 - 3\), and \(\overline{\kappa} = -12l_3 - 16e_3 + 10e_1 - 23\). As explained above, we have imposed \(q_1 = q_2\) etc.

Thus for any set of rational values for the leptonic charges there exist rational values for all the charges. it is clear therefore that we can potentially resolve the
Table 2: The fermionic \( R \)-charges for the case \( \Delta_d = \Delta_L \)

|   | \( q_3 \) | \( l_3 \) | \( u_3 \) | \( d_3 \) | \( e_3 \) | \( q_1 \) |
|---|---|---|---|---|---|---|
|   | \( -\frac{2}{3} \) | \( -\frac{2}{3} \) | \( -\frac{2}{3} \) | \( -\frac{2}{3} \) | \( \frac{1}{3} \) | \( e \) | \( e -\frac{43}{72} \) |

|   | \( l_1 \) | \( u_1 \) | \( d_1 \) | \( e_1 \) | \( H_1 \) | \( H_2 \) |
|---|---|---|---|---|---|---|
|   | \( -\frac{2}{2} + \frac{5}{21} \) | \( -\frac{2}{3} + \frac{19}{72} \) | \( \frac{5}{3} - \frac{77}{72} \) | \( e + \frac{9}{8} \) | \( -\frac{2}{6} - \frac{5}{6} \) | \( \frac{2}{6} + \frac{5}{6} \) |

The tachyonic slepton problem, since we can choose the lepton \( R \)-charges so that all the \( \overline{\theta} \) contributions to Eq. (5) are positive for the sleptons. Of course we will need to check that the corresponding contributions for the squarks and Higgses do not cause problems.

We can get a different handle on the \( R \)-charge assignments by relating them to a possible origin of the light quark and lepton masses. Suppose there are higher-dimension terms in the effective field theory of the form (for the up-type quarks)

\[
H_2 Q_i u_j^c \left( \frac{\theta}{M_U} \right)^{a_{ij}} \text{ or } H_2 Q_i u_j^c \left( \frac{\overline{\theta}}{M_U} \right)^{a_{ij}},
\]

where \( \theta, \overline{\theta} \) is a pair of MSSM singlet fields with \( R \)-charges \( \pm r_\theta \) that get equal vacuum expectation values, and \( M_U \) represents some high energy new physics scale (with similar terms for the light down quarks and leptons). Evidently the \( R \)-charge assignments will then dictate the texture of the Yukawa couplings, via the relation \( h_2 + q_1 + u_1 + a_{11} r_\theta = -1 \) and similar identities.

If we suppose identical textures for the down quarks and leptons then we find

\[
\kappa = \frac{-3}{2}, \quad \bar{\kappa} = \frac{-21}{2} - \frac{9}{4} \lambda,
\]

where \( \lambda = 2 l_3 + e_3 \). The only value of \( \lambda \) we have found which leads to nice textures with only one pair of \( \theta, \overline{\theta} \) fields is \( \lambda = -\frac{1}{3} \), which leads to the set of fermionic \( R \)-charges shown in Table (2).

With this charge assignment we find, (for arbitrary \( e \)) but setting \( r_\theta = \frac{3}{8} \), Yukawa textures of the form

\[
\Delta_u = \begin{pmatrix} e^4 & e^4 & e \\ e^4 & e^4 & e \\ e^5 & e^5 & 1 \end{pmatrix}, \quad \Delta_d = \Delta_L = \begin{pmatrix} e^4 & e^4 & e \\ e^4 & e^4 & e \\ e^3 & e^3 & 1 \end{pmatrix}
\]
The quark and lepton mass hierarchies and the CKM matrix can be produced with matrices of these generic structures, see \[4\]. The phenomenology of Flavour Changing Neutral Currents (in both hadronic and leptonic sectors) and CP-violation effects clearly deserve a detailed investigation.

It is easy to show that as long as \(-\frac{1}{3} < \epsilon < \frac{1}{3}\) and \(m_0^2 < 0\), the contribution to each slepton mass term due to the \(q\) term in Eq. (8) will be positive, and we may expect to achieve a viable spectrum; however, it turns out that it is still non-trivial to obtain an acceptable minimum because, for example, if \(\epsilon = 0\) and \(m_0^2 < 0\), the \(m_0^2 q\) contributions to Eq. (8) from \(u_3\), \(q_1\) and \(d_1\) are negative. We find in fact that we need to have \(\epsilon < 0\).

A variety of mass spectra for \(m_0 = 40\text{TeV}\) (corresponding to a gluino mass of around 1TeV), but with different values of \(\tan \beta\), \(\epsilon\) and \(m_0^2\), is presented in table 3; we were unable to find any values of \(\epsilon\) and \(m_0^2\) corresponding to an acceptable spectrum for \(\tan \beta\) significantly larger than 10. The heaviest sparticle masses scale with \(m_0\) and are given roughly by \(M_{\text{SUSY}} = \frac{1}{4\epsilon} m_0\). A characteristic feature of AMSB models is the near-degenerate light charged and neutral winos; this prediction, as in the FI case, is preserved in the scenario presented here. The main distinction from the FI case is the large splitting between the third generation and the other two, caused by the generation-dependent \(\mathcal{R}\)-charge assignments. Moreover, unusual is the possibility (exemplified in the first three columns of table 3) that the \(\tilde{\nu}_\tau\) is the LSP. As is well known, radiative corrections give a sizeable upward contribution to the mass of the light CP-even Higgs, and so we have included the one-loop calculation.

As in the FI case, however, a salient feature of the model is the existence of sum rules for the sparticle masses. These sum rules follow from Eq. (11); and thus for the particular solution exhibited in table 3 they are independent of \(\epsilon\). We find for example the following relation:

\[
m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2 + m_{\tilde{b}_1}^2 + m_{\tilde{b}_2}^2 - 2(m_t^2 + m_b^2) - 2.75m_G^2 = 0.92m_0^2 \text{TeV}^2,
\]

where we have again taken \(\tan \beta = 5\) and \(m_0 = 40\text{TeV}\).

Note the similarity with the corresponding one in the FI scenario described in the previous section, Eq. (7); the distinction lies in the non-zero RHS in Eq. (14), which can be traced back to the dependence on \(\gamma\) in Eq. (8).

4 Conclusions

We have shown that by extending the MSSM with a \(U_1\) or a \(U_1^R\) (which may or may not be associated with a physical vector boson), it is possible to construct solutions to the running equations for \(m^2\), \(M\) and \(h\) that are completely RG invariant, and leads to a phenomenologically acceptable theories, resulting in a distinctive spectrum with sum
Table 3: The sparticle masses (given in GeV) for the $U^R_1$ case

| $\tan \beta$ (sign $\mu_s$) | 2(+) | 2(−) | 5(+) | 5(+) | 10(+) |
|-----------------------------|------|------|------|------|-------|
| $e$                         | −1/9 | −1/9 | −1/9 | −2/9 | −2/9  |
| $\tilde{m}_0$ (TeV$^2$)     | −0.1 | −0.1 | −0.1 | −0.25| −0.2  |
| $\tilde{t}_{1,2}$           | 652,882 | 615,908 | 567,876 | 302,879 | 404,875 |
| $\tilde{b}_{1,2}$           | 865,977 | 865,977 | 843,974 | 853,1009 | 843,987 |
| $\tilde{\tau}_{1,2}$       | 94,110 | 87,116 | 75,127 | 136,289 | 86,251 |
| $\tilde{u}_{L,R}$           | 918,997 | 918,997 | 917,997 | 880,1084 | 892,1057 |
| $\tilde{d}_{L,R}$           | 920,887 | 920,887 | 921,887 | 884,776 | 896,814 |
| $\tilde{e}_{L,R}$           | 260,423 | 260,423 | 261,423 | 473,664 | 418,590 |
| $\tilde{\nu}_\tau$         | 83    | 83    | 73    | 277   | 234   |
| $\tilde{\nu}_e$             | 251   | 251   | 249   | 467   | 410   |
| $h$                         | 96    | 105   | 119   | 114   | 124   |
| $H$                         | 598   | 598   | 585   | 121   | 308   |
| $A$                         | 593   | 593   | 584   | 110   | 307   |
| $H^\pm$                     | 599   | 599   | 590   | 137   | 318   |
| $\tilde{\chi}_1^{\pm}$     | 98    | 116   | 104   | 101   | 106   |
| $\tilde{\chi}_2^{\pm}$     | 628   | 625   | 663   | 449   | 530   |
| $\tilde{\chi}_1$           | 98    | 115   | 103   | 99    | 103   |
| $\tilde{\chi}_2$           | 364   | 372   | 367   | 357   | 365   |
| $\tilde{\chi}_3$           | 619   | 620   | 662   | 446   | 532   |
| $\tilde{\chi}_4$           | 637   | 628   | 672   | 470   | 544   |
| $\tilde{g}$                 | 1008  | 1008  | 1008  | 1008  | 1008  |

rules for the sparticle masses. In both cases the additional source of supersymmetry-breaking may be provided by the vacuum expectation value of a $D$-term.

In a recent paper[14], it was shown how our first scenario can be compatible with currently fashionable braneworld scenarios, with breaking of both supersymmetry and the extra $U_1$ occurring on a hidden brane; the incorporation of massive neutrinos
was also considered. It would be interesting to perform a similar construction for the \( \mathcal{R} \)-symmetry case.

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