The purpose of this study was to quantify dynamic functional roles of upper body joint torques to the generation of racket head speed in consideration of the joint torque generating mode in the tennis serve motion. The upper body with a racket was modelled as a linked eleven-segment system consisting of the upper limbs, shoulder girdles, head, upper trunk and the racket. The contributions of the joint torque term, motion-dependent term, gravitational term and external force term to the generation of racket head speed were calculated from the equation of motion for the system. Furthermore, the joint torque was divided into two components, such as eccentric torque component, which shows negative sign of its torque power, and concentric torque component, which shows positive sign of its torque power. An algorithm which converts the motion dependent term into other terms was proposed. The results obtained in this study showed that 1) motion dependent term was the great contributor to the generation of racket head speed prior to the impact and 2) after converting motion dependent term into other terms, racket head speed was mainly obtained by eccentric torque component about internal rotational axis at racket side shoulder joint.
1. Introduction

Swing motions, such as tennis serve motion (Springings et al., 1994, Elliott et al., 1995; Bahamonde, 2000) and baseball pitching motion, are sports motions that accelerate a tip of hitting tool or distal end of body to high speed within a short time. Although previous studies indicate that motion dependent term (MDT) is a significant contributor to the generation of end-point speed of linked-segment systems in baseball pitching motion (Naito et al., 2008; Hirashima et al., 2008) and the soccer kicking and ball throwing motion (Putnam, 1993), dynamic contributions of joint torques to the generation of racket head speed during the tennis serve motion have not been investigated. From a viewpoint of torque generating mode, joint torques can be divided into eccentric and concentric torque components judged from the sign of joint torque powers. The purpose of this study was to quantify dynamic functional roles of upper body joint torques to the generation of racket head speeds with consideration of the torque generating mode in tennis serve motion.

2. Methods

2.1. Equation of motion for upper body and racket system

The upper body with a racket was modeled as a linked eleven-segment system consisting of the upper limbs, shoulder girdles, head, upper trunk, and the racket as shown in Figure 1.

The translational and rotational equations of motion for each segment of the upper body and racket system can be summed up in a matrix form as follows:

\[ \dot{M}V = PF + P_{ext}F_{ext} + Q_{act}T_{act} + H + G \]  

where \( M \) is the inertia matrix and \( V \) is the vector containing the translational and rotational velocity vectors of each segment's CG, \( P \) and \( P_{ext} \) are the coefficient matrix of vector \( F \) which contains all joint force vectors and the coefficient matrix of external force vector \( F_{ext} \), \( Q \) is the coefficient matrix of vector \( N \) which contains moment vectors at all joints, \( H \) is the vector containing jyro moment vectors of all segments, and \( G \) is the vector of the gravitational component.

The equation for constraint condition in which adjacent segments are connected by joint is expressed as follows:

\[ CV = 0 \]  

Fig.1 A schematic representation of upper body and racket model consisted of eleven rigid segments
The geometric equations for constraint axes of joints, such as, inversion/eversion axis of the elbow, and internal/external rotation of the wrist joint can be expressed in matrix form as follows:

$$AV = 0$$ \hspace{1cm} (3)

where \( A \) is the anatomical constraint coefficient matrix of the generalized velocity vector.

Substituting eqs.(2) and (3) after differentiating with respect to time into eq.(1), an analytical form of the equation of motion for the system can be obtained as follows:

$$\dot{V} = A_{Fe} F_{ext} + A_{Ta} T_{act} + \bar{A}_v V + A_G G$$ \hspace{1cm} (4)

where \( A_{Fe} \), \( A_{Ta} \), and \( A_G \) are the coefficient matrices of the external force vector \( F_{ext} \), active joint torque vector \( T_{act} \), and gravitational acceleration vector \( G \), respectively, and \( \bar{A}_v V \) denotes the motion dependent term.

2.2. Converting algorithm of motion dependent term into other terms

The equation of motion for the system, eq.(4), was discretized as follows:

$$\dot{V}(k) = A_v(k) + \bar{A}_v(k)V(k)$$ \hspace{1cm} (5)

$$A_v(k) = A_{Fe}(k)F_{ext}(k) + A_{Ta}(k)T_{act}(k) + A_G(k)G(k)$$ \hspace{1cm} (6)

The generalize acceleration vector was expressed by difference approximation shown as

$$\dot{V}(k) = \frac{V(k+1)-V(k)}{\Delta t}$$ \hspace{1cm} (7)

Combining eqs.(5) and (7) yields a recurrence formula for the generalized velocity vector \( V \) as follows:

$$V(k+1) = \Delta t A_v(k) + \Psi_v(k)V(k), \quad \Psi_v(k) = E + \Delta t \bar{A}_v(k)$$ \hspace{1cm} (8)

Eqs.(6) and (8) provide us the information about contribution of the input terms (e.g., individual axial torques at upper body joints, external force exerted on torso joint) to the generation of speed of racket face segment, which is the distal end segment of the articulated segment system.

2.3. Contribution to evaluation variable at the evaluating time

The contribution at every instant of each term to the generation of evaluating variables at evaluating time \( k_{eval} \) can be derived from eq.(8). For example, the generalized velocity vector at the time \( k_{eval} \) can be calculated from the input vector \( A_v \) as follows:

$$V(k_{eval}) = \Delta t \sum_{i=1}^{k_{eval}-1} \left[ \prod_{j=i}^{k_{eval}-1} \Psi_V(j) \right] A_v(h-1) \left[ \prod_{j=i}^{k_{eval}-1} \Psi_V(j) \right] V(0)$$ \hspace{1cm} (9)

The generalized velocity vector at the time \( k_{eval} \) can also be divided into the components caused by the individual terms as follows:

$$V(k_{eval}) = V_{Tact}(k_{eval}) + V_{Fext}(k_{eval}) + V_G(k_{eval}) + V_{V0}(k_{eval})$$ \hspace{1cm} (10)

where the four terms on the right side of eq.(10) show the generalized velocity vectors caused by active joint torques, external force vector exerted on the torso joint, gravity and the initial velocities of the segments, respectively.

The contribution of each term to a certain evaluating variable \( q_{eval} \), which is extracted from the generalized velocity vector at the time \( k_{eval} \), can be expressed as follows:
where the four terms on the right side of eq.(11) show the contributions of the active joint torques, the external force vector, gravity and the initial velocities, respectively.

The contribution of active joint torque at time $k$ can be furthermore divided into the contributions of individual joint torques about axes of the joints as follows:

$$\dot{C}_{T,act}(k) = \sum_{j, \text{Axis}} \dot{C}_{T,act,j,\text{Axis}}(k), \quad j, \text{Axis} = 1, 2, \cdots, n\text{Axis} \quad (12)$$

2.4. Consideration of joint torque generating mode

In order to consider the torque generating characteristics at joints, the torque generating mode was judged from the signs of joint torque power about each joint axis. When the sign of joint torque power is positive in which joint angular velocity and exerting torque are in same direction, the joint torque is regarded as concentric contraction mode. And when the sign is negative, the joint torque is regarded as eccentric contraction mode.

The equation for contribution of the concentric and eccentric torques at time $k$ can be expressed as follows:

$$\dot{C}_{T,act} = \dot{C}_{T,ecc} + \dot{C}_{T,con} \quad (13)$$

$$\dot{C}_{T,ecc}(k) = \sum_{i, \text{Axis}} \dot{C}_{T,\text{Axis},ecc}(k), \quad i, \text{Axis} = 1, 2, \cdots, n\text{Axis} \quad (14)$$

$$\dot{C}_{T,con}(k) = \sum_{i, \text{Axis}} \dot{C}_{T,\text{Axis},con}(k), \quad i, \text{Axis} = 1, 2, \cdots, n\text{Axis} \quad (15)$$

3. Experiment

A collegiate tennis player participated in the experiment as a subject. The subject performed the tennis first serve motion from deuce side position to the target area settled in the service area close to the center line. The motion was captured with 3-dimentional motion capture system (VICON-MX, 250Hz). The forward swing motion was analyzed from the time when the racket head speed was minimum value to the time of impact (Fig.2). The contributions were standardized with the duration of the forward swing motion.

![Fig.2 Stick picture of forward swing in the tennis serve motion](image)
4. Result and discussion

First, an example of the contribution of each term to the generation of racket head speed is shown in Figure 3(a). The contribution of the joint torque term increased gradually from 65 % to 90 % normalized time of the forward swing motion and decreased toward the impact. The contribution of the motion dependent term increased rapidly after 90 % normalized time toward the impact. At the impact, the motion dependent term was the largest contributor to the generation of the racket head speed.

Next, an example of the contribution of each term to the generation of racket head speed with conversion of the motion dependent term into the other terms is shown in Figure 3(b). Since the joint torque term became the primary contributor to the generation of the racket head speed over the forward swing period after converting the motion dependent term, the contribution of the motion dependent term was mainly generated by the joint torque term. Thus, the joint torque term plays significant important role to the generation of the racket head speed in the tennis serve motion.

Finally, according to eqs.(13)-(15), the total contribution of individual joint axial torques to the generation of the racket head speed at the impact with consideration of torque generating mode was shown in Figure 4. The bars in the figure express the speeds generated by the individual axial torques during forward swing motion, and the
The sum of those bars equals the total contribution of the joint torque term at the impact. The Contribution of the internal/external rotation (IER) axial torque at racket-side shoulder joint and the pronation/supination (PS) axial torque at racket-side elbow joint were large contributors. Almost part of the contribution of the IER torque at the shoulder joint is generated by the eccentric component of the torque. On the other hand, almost part of the contribution of the PS torque at the elbow joint is generated by the concentric component of the torque. The negative contributors may play other roles in the motion, such as angular velocity control of racket face and body control during the motion.

5. Conclusion

This study has successfully developed a method of quantifying dynamic functional roles of upper body joint torques on the head speed generating mechanism in consideration of torque generating mode during the tennis serve motion. The motion dependent term is a crucial factor in the mechanism during the motion. By using the proposed algorithm which converts the motion dependent term into the other terms, it has been clarified that 1) the largest dynamic contributor to the generation of the head speed was the eccentric torque about internal/external rotational axis at racket-side shoulder joint, and 2) the second contributor was the concentric torque about pronation/supination axis at racket-side elbow joint.

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