Topological Lifshitz transition and one-dimensional Weyl mode in HfTe₅

Landau band crossings typically stem from the intra-band evolution of electronic states in magnetic fields and enhance the interaction effect in their vicinity. Here in the extreme quantum limit of topological insulator HfTe₅, we report the observation of a topological Lifshitz transition from inter-band Landau level crossings using magneto-infrared spectroscopy. By tracking the Landau level transitions, we demonstrate that band inversion drives the zeroth Landau bands to cross with each other after 4.5 T and forms a one-dimensional Weyl mode with the fundamental gap persistently closed. The unusual reduction of the zeroth Landau level transition activity suggests a topological Lifshitz transition at 21 T, which shifts the Weyl mode close to the Fermi level. As a result, a broad and asymmetric absorption feature emerges due to the Pauli blocking effect in one dimension, along with a distinctive negative magneto-resistivity. Our results provide a strategy for realizing one-dimensional Weyl quasiparticles in bulk crystals.

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In magnetic fields, electrons in crystals undergo cyclotron motion and transform the energy bands into discrete Landau levels. The formation of Landau levels gives rise to various phenomena such as Shubnikov–de Haas oscillations and integer and fractional quantum Hall effects, as well as composite fermions. In two-dimensional (2D) systems like graphene and quantum wells, these Landau levels are non-dispersive. When additional energy such as spin or valley splitting exceeds the cyclotron energy, Landau levels meet with each other at critical magnetic fields, and electron interaction is enhanced as a result. This type of intra-band Landau level crossing, which comes from the overlap of energy levels without dispersion, has been widely investigated and serves as an essential tool to modulate and analyse the Landau level spectrum. By contrast, Landau levels in three-dimensional (3D) systems change into Landau bands due to the dispersion along the magnetic field direction. As the field varies, the evolution of the cyclotron energy and band splitting potentially lead to the Landau band crossings existing only at discrete momentums. If one is considering an inverted-band system such as a weak topological insulator (TI), the spin splitting drives the lowest (zeroth) Landau bands of the conduction and valence bands moving towards each other and finally crossing above a critical magnetic field at discrete momentums. Hereafter, the ‘inter-band Landau level crossing’ denotes the crossing between conduction and valence Landau levels in momentum space. Notably, such inter-band Landau level crossings persistently close the fundamental bandgap rather than the Landau gap alone. The further increased magnetic field only shifts the crossing momentums but keeps the bandgap closed.

On the other hand, band crossing in momentum space generates quasiparticles, which has drawn enormous research interest. The most

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After reaching the quantum limit at $E_F$, the Fermi level (purple) crosses only the zeroth Landau band. Characteristic band inversion leads to the crossing of the zeroth Landau bands after the critical field of $B_c$. With the Fermi level staying high, the system still behaves as a trivial semimetal until the Lifshitz transition at $B_c$. Fermi surfaces experience a splitting, which is accompanied by a topological transition where the zeroth Landau bands form an effective 1D and spin-polarized band crossing near the Fermi energy ($E_F$).

Field-driven topological phase transitions and the 1D Weyl mode

Considering a weak 3D TI with an in-plane inverted gap, a series of magnetic-field-driven phase transitions are proposed at critical fields of $B_{c0}, B_1, B_2$, as shown in Fig. 1. Applying a magnetic field ($B$) first leads to the formation of 1D Landau bands, which are parabolically dispersed along the field direction especially around the band edge at zero momentum. Once reaching the quantum limit at $B_{c0}$, the Fermi level crosses only with the zeroth Landau band. By increasing the magnetic fields, band inversion drives the two zeroth Landau bands, moving towards and eventually forming inter-band crossings at $B_c$. At first, the Fermi level remains higher than the Lifshitz transition energy, so the system behaves as a trivial semimetal. Above $B_c$, a Lifshitz transition takes place accompanied by a topological phase transition with the Fermi surface divided into two parts with opposite spin textures. Such topological Lifshitz transitions originate from the dispersive Landau bands, and are different from those in the Bloch bands. The non-zero topological number of each phase is denoted below the Fermi surface in Fig. 1. The overall phase transitions and main physics can be modelled by the ideal low-energy Hamiltonian

$$H(k) = \hbar v_F k_x \sigma_z - \hbar v_F k_y \sigma_x + \hbar v_F k_z \sigma_0 + [\Delta + M (k_x^2 + k_y^2)] \tau_2 \sigma_0,$$

where $k$ is the momentum, and $\tau$ and $\sigma$ are the Pauli matrices acting on the orbital and spin degrees of freedom, respectively. $\Delta$ and $\sigma_0$ are reduced Planck’s constant and the unit matrix, respectively. The subscripts $x, y, z$ correspond to the $a, c, b$ crystal axis of the HTe. Band parameters include the energy gap $\Delta$, Fermi velocity $v_F$, and, in-plane and out-of-plane band inversion parameters $M$ and $\sigma_0$. The Fermi velocity and band inversion parameters also act as linear and parabolic contributions of energy dispersion. Influence from additional perturbation terms such as spin–orbit coupling and inversion symmetry breaking are further discussed in Supplementary Section III.

Material realization and anisotropic band structure

Among various TI candidates, we find that HTe meets the material criteria to realize the above proposal. Similar to ZrTe$_5$, HTe is at the boundary between weak TI and strong TI, with the electronic structure and band topology sensitive to the $b$-axis lattice constant. The access to the extreme quantum limit has led to various intriguing phenomena in ZrTe$_5$ and HTe (refs. 34–36). Photoemission and optical experiments reveal a temperature-dependent Fermi level and a TI phase at low temperature (Fig. 2a–d). Figure 2a–d exhibits the quantum oscillations from our transport measurement. A large positive magneto-resistivity is observed with a small Fermi vector $k_{Fac} = 5.9 \times 10^{-3}$ Å, in agreement with

Fig. 1 | Schematic plot of proposed magnetic-field-driven phase transitions.

A weak 3D TI at zero field with $\Delta > 0, M < 0$ and $\sigma_0 > 0$. The Landau quantization of the TI features both band inversion and full spin-polarization of the zeroth Landau bands, denoted by red and blue lines, while the black and grey lines denote the original energy bands and high-index Landau bands, respectively.

After reaching the quantum limit at $B_{c0}$, the Fermi level (purple) crosses only the zeroth Landau band. Characteristic band inversion leads to the crossing of the zeroth Landau bands after the critical field of $B_c$. With the Fermi level staying high, the system still behaves as a trivial semimetal until the Lifshitz transition at $B_c$. Fermi surfaces experience a splitting, which is accompanied by a topological transition where the zeroth Landau bands form an effective 1D and spin-polarized band crossing near the Fermi energy ($E_F$).
previous reports\textsuperscript{19, 20}. $\Delta R$ and $R_0$ denote the resistivity variation and original resistivity without magnetic field. The fitted quantum limit in our sample is around $B_0 = 1.5 \pm 0.1$ T. Assuming an ellipsoid Fermi surface\textsuperscript{20}, we can obtain the cyclotron mass from the temperature-dependent oscillation amplitude, as $m_e = 0.016 m_e$, $m_s = 1.2 m_e$, and $m_0 = 0.028 m_e$, where $m_e$ is the free electron mass (details are in Supplementary Sections I and II). Small $m_e$ and $m_s$ values suggest quasi-linear in-plane dispersion, while $m_0$ indicates a parabolic dispersion along $k_z$ (Fig. 2d). Hence, we set the $\nu_{\parallel}$ term to be zero while $M_\parallel$ remains finite for HfTe$_5$ (ref. \textsuperscript{19}). With positive $\Delta$, the in-plane band inversion of the TI requires $M < 0$. As shown later, the sign of $M_\parallel$ determines the presence of inter-band Landau level crossings and a Lifshitz transition. The $a-c$ plane is treated as isotropic because the cyclotron motion averages the in-plane response. The low carrier concentration along with the special anisotropic dispersion and band inversion serve as the prerequisites to observe the topological Lifshitz transition within a magnetic field of 35 T.

**High-field magneto-infrared spectrum**

The evolution of Landau bands is detected by magneto-infrared spectroscopy with optical transmittance $T_y/T_0$ (Fig. 2e) measured at the $a-c$ plane of HfTe$_5$ under magnetic fields applied along the $a$ axis (Faraday geometry). Here $T_y$ and $T_0$ are the transmittance measured in magnetic field $B$ and zero field, respectively. A series of absorption peaks develops and evolves with the magnetic fields as shown in the relative magneto-absorbance $A_y = -\ln(T_y/T_0)$ (Fig. 2f). For those optical transitions labelled as $T_1, T_2, T_3, \ldots$, the transition energy approximately follows $\omega \propto (\sqrt{n + 1} + \sqrt{n}) \sqrt{B}$ with $n = 1, 2, 3, \ldots$, which indicates a Dirac-type band in HfTe$_5$. Since the joint DOS diverges at the band edge, we focus on the $k_\parallel = 0$ case at this stage and start from the optical transitions with the non-zeroth Landau band described by Eq. (2). The Landau bands are labelled as $L_n$ with $n = \pm 1, \pm 2, \ldots$ and $s = \pm 1$ denoting the Landau index and spin index, respectively. The zeroth ($|n| = 0$) Landau bands are fully spin-polarized, distinct from all others. Band inversion leads to a field-dependent energy shift, which is most prominent for the zeroth Landau bands. Figure 2g presents a schematic plot of the Landau band edge energy $E(k_\parallel = 0)$ versus $B$. The red and blue lines in Figs. 2g and 3c denote spin-polarized Landau bands. The optical transitions between Landau bands are restricted by a selection rule of $\Delta|n| = \pm 1$ for the ideal model. The photon energy for the $T_n$ transition ($L_{n+1} \rightarrow L_{n+1}$ and $L_{(n+1)} \rightarrow L_n$) follows Eq. (5) in the Methods, by which the transition index is assigned accordingly. The extracted Fermi velocity with $v_F = 4.58 \times 10^5$ m s$^{-1}$ is consistent with transport measurement. The observed inter-Landau-level transitions give the same intercept at zero field, corresponding to a small bandgap of $\sim 5$ meV.

For optical transitions with energy lower than $T_1$ in Fig. 2f, they are readily seen to arise from the zeroth Landau bands. Here we name these three transitions $T_0, T_1, T_2$ with increasing energy and refer to them together as $T_0$. As the Fermi level drops below the band edge of $L_1$ at $B_0 = 1.5$ T, only the zeroth Landau bands are occupied so that the strength of the $T_0$ transition is much higher than $T_1$, as shown in Fig. 3a(i). When further increasing the magnetic field, $L_{-1}$ and $L_{-2}$ come closer due to band inversion and eventually touch at the critical field.
$B_i$. This touching results in the splitting of $T_a$ and $T_p$ as shown by Fig. 3a(ii),(iv) and the black arrow in Fig. 2f. The presence of small spin–orbit coupling mixes the spin of the Landau band, which enables the spin-flipped $T_a$ and $T_p$ transition at $B > B_i$ and also explains the absence of $T_a$ splitting at $B < B_i$ as summarized in Table 1. To extract the $B_i$ value, we perform a multi-peak Lorentz fitting on the magneto-absorbance spectrum and extract the peak position of $T_a$ and $T_p$ as shown in Fig. 3a(iv). The splitting feature is confirmed and agrees with the model given by the solid lines. The intersecting magnetic field gives $B_i = 4.5$ T. Details of $T_0$ transitions including the intensity distribution, field-dependent spin mixing and multi-peak Lorentz fitting are given in Supplementary Sections III and XI. The energy of the intra-band transition $L_{\pm 0} \rightarrow L_{\mp 1}$ is close to that of the inter-band transition $L_{\pm 0} \rightarrow L_{\pm 1}$ after the system reaches the quantum limit, and therefore they merge in $T_o$. Previous studies have discussed a picture of Landau levels with band edge touching in ZrTe$_5$ using a strong TI model or and its first derivative in $v$ prove the reduction of $T_0$ and give $B_i = 4.5$ T. Details of $T_0$ transitions including the intensity distribution, field-dependent spin mixing and multi-peak Lorentz fitting are given in Supplementary Sections III and XI. The energy of the intra-band transition $L_{\pm 0} \rightarrow L_{\mp 1}$ is close to that of the inter-band transition $L_{\pm 0} \rightarrow L_{\pm 1}$ after the system reaches the quantum limit, and therefore they merge in $T_o$. Previous studies have discussed a picture of Landau levels with band edge touching in ZrTe$_5$ using a strong TI model or

Fig. 3 | Optical activity of Landau level transitions and field-driven phase transitions. a. Relative magneto-absorbance spectra at different magnetic field regimes: after reaching the deep quantum limit at $B_0 < B < B_1$, showing $T_0$ as the most prominent transition with only the zeroth Landau band occupied (i); after the touching of the zeroth Landau band edge at $B_1 < B < B_2$, showing the split $T_a$ and $T_p$ indicating the crossing of the zeroth Landau bands (ii); and after the Lifshitz transition at $B > B_2$, showing the intensity reduction of $T_0$, in contrast with the continuous intensity boost of $T_p$ and $T_s$, suggesting the emergence of a 1D Weyl mode near the Fermi level (iii). The fitted peak position in iv confirms the splitting behaviour, which agrees with the model (grey solid lines) and gives $B_i = 4.5$ T (red dashed line). Grey dashed line denotes that $T_a$ is not active below $B_i$. The normalized Pauli-blocking-induced spectral weight $SW_{PB}$ of $T_0$ and also explains the unusual reduction of $T_0$ and the 1D Weyl mode. c. Landau band extrema energy versus magnetic field, focusing on the $T_0$ transition. The activity of the optical transition is plotted by arrows with and without red crosses. $T_s$ in yellow experiences activity reduction through $B_i$ while $T_0$ experiences the opposite. d. Model prediction of the real part of the magneto-optical conductivity spectrum based on the fitting parameters, which reproduce the experimental features of inter-Landau-level transitions.
assuming a fixed Fermi energy. Their model gives a distinct \( k_z \) dispersion variation and Fermi level variation, as discussed below.

We note that the position of the Fermi level is vital to determine the optical activity of the transitions beyond the quantum limit for HfTe\(_5\). In 2D systems, the Landau levels are non-dispersive so that the Fermi level ultimately stays exactly at the lowest level of occupied bands. However, for 3D systems, the dispersion along the field direction of the Landau bands may shift the Fermi level away from the Landau band energy at \( k_z = 0 \). In a 3D trivial insulator, the Fermi level always stays higher than \( L_{00} \), but gradually converges to its band extrema due to the increasing DOS with magnetic fields (Fig. 3b, left panel). For a strong TI, the Fermi level first decreases with magnetic field followed by an upturn (Fig. 3b, middle panel), since the system becomes fully gapped again after the band edge touching. As for a weak 3D TI, the Fermi level continues to drop after the band edge touching due to the persistent gap closure (Fig. 3b, right panel). The dispersion of zeroth Landau bands along \( k_z \) can be directly obtained from the proposed Hamiltonian \( E_0(k_z) = -\Delta + M_r^2 k_z^2 \) (More details in the Methods). The effect of the Fermi level shifting with fields has been mostly overlooked in previous studies of the ZrTe\(_5\) or HfTe\(_5\) Landau level spectrum. Below, we will show that Landau level transitions in HfTe\(_5\) can be explained well by this picture.

Generally speaking, the optical transition activity of \( T_0 \) persists beyond the quantum limit, and the intensity grows with fields, since the Fermi level no longer crosses any Landau band edge. However, as indicated by the bottom orange arrow in Fig. 2f, \( T_0 \) becomes weakened and gradually disappears above \( B_2 = 21 \) T, while all other \( T_0 \) transitions get enhanced. Figure 3a(ii),(iii),(iv) also clearly indicates the reduction of \( T_0 \). This striking phenomenon contrasts the traditional argument and suggests that the Fermi level further crosses \( L_{00} \) around \( B_0 \). Above \( B_0 \), the emptying of \( L_{00} \) forbids all transitions initializing from \( L_{00} \) (Fig. 3c), and hence \( T_0 \) vanishes accordingly. By contrast, \( T_0 \) and \( T_2 \) persist above \( B_0 \), Figure 3c shows the assigned transitions for \( T_0 \) and \( T_2 \). The subscript numbers '1' and '2' denote the optical transition with opposite circular polarization. \( T_{1a} \) and \( T_{2a} \) are intra-band transitions after \( B_2 \), while \( T_{1g}^* \) and \( T_{2g}^* \) are inter-band transitions. While \( T_{1g} \) (solid dark green) maintains the optical activity through \( B_2 \), \( T_{2g} \) (dashed dark green) becomes active, resulting in the increased overall intensity of \( T_0 \) above \( B_2 \). By comparison, \( T_{1g}^* \) (solid light green) experiences similar variation as \( T_{1a} \), but is at the same energy as \( T_0 \). Considering the spin conservation nature and the overlapping of \( T_{1a} \) and \( T_{1g} \), \( T_0 \) shows the highest intensity among the \( T_0 \) transitions. The remaining \( T_2 \) transition naturally comes with the highest energy of the transition \( L_{00} - L_{2g}^* \)

## Optical and transport response of 1D Weyl mode

The field-induced 1D Weyl mode also features a high DOS near the Fermi level. We first compare this mode with the conventional 3D Weyl node formed by Bloch band crossing. Under high magnetic fields, the low-energy excitation resembles effective 1D Weyl quasiparticles because the Landau bands disperse only along \( k_z \). Moreover, the Fermi velocity of the obtained 1D Weyl mode can be tuned from \( 5 \times 10^4 \) m s\(^{-1} \) at \( B_0 = 21 \) T to \( 7 \times 10^4 \) m s\(^{-1} \) at the highest experimental field of 35 T. The theoretically allowed Fermi velocity is expected to reach \( 2 \times 10^5 \) m s\(^{-1} \) before annihilating the Weyl points at the boundary of the Brillouin zone around ~300 T (estimation details are given in Supplementary Sections VI and VIII).
quantitative verification) from the inversion symmetry breaking at low temperatures. The complete selection rules including $\Delta |m| = 0$ and $\Delta s = \pm 2$, and the quantitative derivation are given in Supplementary Table I and Supplementary Section III. In the experiment, we observe a newly formed set of peaks $T_w$ with a prominent high-energy tail above $B_2$, as shown in Fig. 2f and denoted by the black arrow in Fig. 4d, whose peak height increases with the fields. The presence of $T_w$ near $B_2$ suggests the relation with the induced 1D Weyl mode (hence it is named $T_w$), but the low frequency distinguishes $T_w$ from the $T_s$ transition. The typical features of $T_w$ fit well with the 1D Pauli blocking behaviours discussed above.

For a 1D Weyl fermion, the chiral anomaly takes a simple form of $d(n^{1D}_{RL}/dT) = \pm eE/h$ where $n^{1D}_{RL}$ denotes the number of right-handed and left-handed 1D Weyl fermions (Fig. 4c). Here $E, h, e$ are the external electric field, Planck’s constant and elementary charge, respectively. The chiral anomaly essentially describes a 1D conductivity channel. To verify the influence of the topological Lifshitz transition on the electrical properties of HfTe$_5$, we measure the electrical resistivity along the $a$ and $b$ axes at pulsed magnetic fields. With magnetic field applied along the $a$ axis, longitudinal voltage $V_{||}$ and $V_{\perp}$ are measured with current $I$ along $a$ and $b$ axes, respectively. As shown in Fig. 4e, longitudinal resistivity along $a$ axis $R_{xx}$ exhibits prominent linear behaviour before saturating at high fields, which may result from the impurity scattering characterized by the screened Coulomb potential (Supplementary Section X contains more discussion). By contrast, longitudinal resistivity along $b$ axis $R_{yy}$ experiences a clear drop around 20 T, which is reminiscent of the chiral-anomaly-induced negative magneto-resistivity in 3D Weyl semimetals. The critical field for negative $R_{yy}$ is close to the field of the topological Lifshitz transition ($B_2 \approx 21$ T). Above this field, the 1D chiral anomaly gives rise to the $b$-axis conductive channel with chiral current proportional to $E$. Recall that the magnetic field increases the Landau band DOS near the Fermi level (Fig. 4a(ii)), resulting in a chiral current proportional to the magnetic fields. Consequently, field-induced 1D quasiparticles in HfTe$_5$ present a chiral anomaly with an $\mathbf{E} \cdot \mathbf{B}$-like characteristic similar to the 3D counterpart, which explains the anomalous decrease of $R_{yy}$ above $B_2$ compared with $R_{aa}$.

In contrast with symmetry-protected topological phases such as 3D Dirac and Weyl semimetals, the 1D Weyl mode in HfTe$_5$ is not symmetry protected. Based on our magneto-infrared spectroscopy results, the gap size should be extremely small, such as a few millielectronvolts, with negligible influence in our experimental regime. The formation of the 1D Weyl mode is essentially much more difficult in a strong TI ($M > 0, M_T < 0$). Without spin–orbit coupling, applying quantum limit magnetic fields in a strong TI leads to the cross of the zeroth Landau bands before $B_2$ and a gap opening after that. Therefore, dropping the Fermi level through $L_{\perp}$ before $B_2$ and realizing the 1D Weyl mode near the Fermi level are not accessible, as shown in Fig. 3b. HfTe$_5$, in our case, features in-plane band inversion ($M < 0$) but an out-of-plane trivial gap ($M_T > 0$) as a weak TI. The phase transition at $B_2$ changes the in-plane gap...
to be trivial but, most importantly, turns the out-of-plane direction into a band inversion state, which guarantees a Weyl mode after $B_z$. The observed Landau band Lifshitz transition can be further detected by other spectroscopic techniques such as scanning tunnelling microscopy under high magnetic field, which could further verify the 1D Weyl mode and explore the electromagnetic response in real space.

**Conclusion**

In summary, we report the magneto-infrared spectroscopic evidence of a field-induced topological Lifshitz transition and 1D Weyl mode in HfTe$_5$. In magnetic fields, band inversion results in the inter-band Landau level crossings accompanied by the persistent closure of the bandgap. With the Fermi level further dropping with fields, the reduction in optical transitions indicates the presence of a topological Lifshitz transition and a 1D Weyl mode near the Fermi level. The observed 1D Pauli blocking behaviour and negative magneto-resistivity agree well with the electromagnetic response of the 1D Weyl fermions. The overall magneto-infrared features and field-driven phase transitions can be quantitatively explained by the $\mathbf{1k \cdot p}$ model. The realization of a 1D Weyl mode from inter-band Landau levels establishes a unified strategy of topological phase engineering.

**Online content**

Any methods, additional references, Nature Research reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at [https://doi.org/10.1038/s41563-022-01364-5](https://doi.org/10.1038/s41563-022-01364-5).

**References**

1. Shoenberg, D. *Magnetic Oscillations in Metals* (Cambridge Univ. Press, 2009).
2. Klitzing, K. V., Dorda, G. & Pepper, M. New method for high-accuracy determination of the fine-structure constant based on quantized Hall resistance. *Phys. Rev. Lett.* 45, 494–497 (1980).
3. Tsui, D. C., Stormer, H. L. & Gossard, A. C. Two-dimensional magnetotransport in the extreme quantum limit. *Phys. Rev. Lett.* 48, 1559–1562 (1982).
4. L uptatini, M. et al. Spin reversal of a quantum Hall ferromagnet at a Landau level crossing. *Phys. Rev. Lett.* 125, 067404 (2020).
5. Jungwirth, T. & MacDonald, A. H. Pseudospin anisotropy classification of quantum Hall ferromagnets. *Phys. Rev. B* 63, 035305 (2000).
6. Ferreira, G. J., Freire, H. J. P. & Egues, J. C. Many-body effects on the $\rho_{xx}$ ringlike structures in two-subband wells. *Phys. Rev. Lett.* 104, 066803 (2010).
7. Taychatanapat, T., Watanabe, K., Taniguchi, T. & Jarillo-Herrero, P. Quantum Hall effect and Landau-level crossing of Dirac fermions in trilayer graphene. *Nat. Phys.* 7, 621–625 (2011).
8. Das, I. et al. Symmetry-broken Chern insulators and Rashba-like Landau-level crossings in magic-angle bilayer graphene. *Nat. Phys.* 17, 710–714 (2021).
9. Weyl, H. Gravitation and the electron. *Proc. Natl Acad. Sci. USA* 15, 323–334 (1929).
10. Wan, X., Turner, A. M., Vishwanath, A. & Savrasov, S. Y. Topological semimetal and Fermi-arc surface states in the electronic structure of pyrochlore iridates. *Phys. Rev. B* 83, 205101 (2011).
11. Weng, H., Fang, C., Fang, Z., Bernevig, B. A. & Dai, X. Weyl semimetal phase in noncentrosymmetric transition-metal monophosphides. *Phys. Rev. X* 5, 011029 (2015).
12. Lv, B. Q. et al. Experimental discovery of Weyl semimetal TaAs. *Phys. Rev. X* 5, 031013 (2015).
13. Xu, S. Y. et al. Discovery of a Weyl fermion semimetal and topological Fermi arcs. *Science* 349, 613–617 (2015).
14. Morali, N. et al. Fermi-arc diversity on surface terminations of the magnetic Weyl semimetal Co$_3$Sn$_2$S$_2$. *Science* 365, 1286–1291 (2019).
15. Huang, X. et al. Observation of the chiral-anomaly-induced negative magnetoresistance in 3D Weyl semimetal TaAs. *Phys. Rev. X* 5, 031023 (2015).
16. Zhang, C. et al. Room-temperature chiral charge pumping in Dirac semimetals. *Nat. Commun.* 8, 13741 (2017).
17. Suzuki, T. et al. Large anomalous Hall effect in a half-Heusler antiferromagnet. *Nat. Phys.* 12, 1119–1123 (2016).
18. Zhang, C. et al. Quantum Hall effect based on Weyl orbits in Cd$_3$As$_2$. *Nature* 565, 331–336 (2019).
19. Yuan, X. et al. The discovery of dynamic chiral anomaly in a Weyl semimetal NbAs. *Nat. Commun.* 11, 1259 (2020).
20. Chen, R. Y. et al. Magneto-infrared spectroscopy of Landau levels and Zeeman splitting of three-dimensional massless Dirac fermions in ZrTe$_5$. *Phys. Rev. Lett.* 115, 176404 (2015).
21. Ma, Q. et al. Direct optical detection of Weyl fermion chirality in a topological semimetall. *Nat. Phys.* 13, 842–847 (2017).
22. Yuan, X. et al. Chiral Landau levels in Weyl semimetal NbAs with multiple topological carriers. *Nat. Commun.* 9, 1854 (2018).
23. Wu, L. et al. Giant anisotropic nonlinear optical response in transition metal monopnictoide Weyl semimetals. *Nat. Phys.* 13, 350–355 (2017).
24. Polatkan, S. et al. Magneto-optics of a Weyl semimetal beyond the conical band approximation: case study of TaP. *Phys. Rev. Lett.* 124, 176402 (2020).
25. Volovik, G. E. Topological Lifshitz transitions. *Low. Temp. Phys.* 43, 47–55 (2017).
26. Bernevig, B. A., Hughes, T. L. & Zhang, S.-C. Quantum spin Hall effect and topological phase transition in HgTe quantum wells. *Science* 314, 1757–1761 (2006).
27. Jiang, Y. et al. Unraveling the topological phase of ZrTe$_5$ via magneto-infrared spectroscopy. *Phys. Rev. Lett.* 125, 046403 (2020).
28. Jiang, Y. et al. Landau-level spectroscopy of massive Dirac fermions in single-crystalline ZrTe$_5$ thin flakes. *Phys. Rev. B* 96, 041101 (2017).
29. You, L., Zhang, Z.-Y. & Wang, Y.-X. Magneto-optic signatures in the gapped Dirac semimetal with mixed linear and parabolic dispersions of ZrTe$_5$ New J. Phys. 23, 123033 (2021).
30. Wang, Y. et al. Gigantic magnetoehiral anisotropy in the topological semimetal ZrTe$_5$. *Phys. Rev. Lett.* 128, 176602 (2022).
31. Weng, H., Dai, X. & Fang, Z. Transition-metal pentatelluride ZrTe$_5$ and HfTe$_5$: a paradigm for large-gap quantum spin Hall insulators. *Phys. Rev. X* 4, 011002 (2014).
32. Monserrat, B. & Narayan, A. Unraveling the topology of ZrTe$_5$ by changing temperature. *Phys. Rev. Res.* 1, 033181 (2019).
33. Morice, C., Lettl, E., Kopp, T. & Kampf, A. P. Optical conductivity and resistivity in a four-band model for ZrTe$_5$ from ab initio calculations. *Phys. Rev. B* 102, 155138 (2020).
34. Wang, H. et al. Discovery of log-periodic oscillations in ultraquantum topological materials. *Sci. Adv.* 4, eaau5096 (2018).
35. Chen, Z.-G. et al. Spectroscopic evidence for bulk-band inversion and three-dimensional massive Dirac fermions in ZrTe$_5$. *Nat. Phys. Sci. Adv.* 116, 861–821 (2017).
36. Tang, F. et al. Three-dimensional quantum Hall effect and metal-insulator transition in ZrTe$_5$. *Nature* 569, 537–541 (2019).
37. Martino, E. et al. Two-dimensional conical dispersion in ZrTe$_5$ evidenced by optical spectroscopy. *Phys. Rev. Lett.* 122, 217402 (2019).
38. Qin, F. et al. Theory for the charge-density-wave mechanism of 3D quantum Hall effect. *Phys. Rev. Lett.* 125, 206601 (2020).
39. Zhang, C. et al. Magnetic-field-induced nonlinear transport in HfTe$_5$. *Natl Sci. Rev.* [https://doi.org/10.1093/nsr/nwab208](https://doi.org/10.1093/nsr/nwab208) (2021).
40. Zhang, Y. et al. Electronic evidence of temperature-induced Lifshitz transition and topological nature in ZrTe$_5$. Nat. Commun. 8, 15512 (2017).
41. Xu, B. et al. Temperature-driven topological phase transition and intermediate Dirac semimetal phase in ZrTe$_5$. Phys. Rev. Lett. 121, 187401 (2018).
42. Kamm, G., Gillespie, D., Ehrlich, A., Peebles, D. & Levy, F. Fermi surface, effective masses, and energy bands of HfTe$_5$ as derived from the Shubnikov–de Haas effect. Phys. Rev. B 35, 1223–1229 (1987).
43. Zhang, J. L. et al. Anomalous thermoelectric effects of ZrTe$_5$ in and beyond the quantum limit. Phys. Rev. Lett. 123, 196602 (2019).
44. Cai, W. & Ting, C. S. Screening effect on the Landau-level broadening for electrons in GaAs-Ga$_{1-x}$Al$_x$As heterojunctions. Phys. Rev. B 33, 3967–3972 (1986).
45. Jiang, Z. et al. Infrared spectroscopy of Landau levels of graphene. Phys. Rev. Lett. 98, 197403 (2007).
46. Han, X., Salehi, M., Oh, S. & Wu, L. A new type of cyclotron resonance from charge-impurity scattering in the bulk-insulating Bi$_2$Se$_3$ thin films. J. Phys. Appl. Phys. 55, 364004 (2022).
47. Akrap, A. et al. Magneto-optical signature of massless Kane electrons in Cd$_3$As$_2$. Phys. Rev. Lett. 117, 136401 (2016).
48. Mak, K. F. et al. Measurement of the optical conductivity of graphene. Phys. Rev. Lett. 101, 196405 (2008).
49. Abrikosov, A. A. Quantum magnetoresistance. Phys. Rev. B 58, 2788–2794 (1998).
50. Rodionov, Y. I., Kugel, K. I., Aronzon, B. A. & Nori, F. Effect of disorder on the transverse magnetoresistance of Weyl semimetals. Phys. Rev. B 102, 205105 (2020).

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Methods

Material choice and crystal preparation

HfTe₅ is chosen to achieve the proposed 1D Weyl mode due to its low Fermi energy, accessible quantum limit and location at the strong/weak TI phase boundary. The HfTe₅ single crystals were prepared by the standard chemical vapour transport method. Stoichiometric mixtures of Hf and Te powder were sealed in an evacuated quartz tube. Iodine was added as the transport agent. The tube was placed in a two-zone furnace with a hot end temperature setting at 770 K and a temperature gradient of 100 K. After reaching the designed temperature, the condition was held for two weeks. Needle-like single crystals were obtained after cooling to room temperature.

Magneto-infrared measurement

Magneto-infrared spectroscopy was performed using a Fourier-transform infrared (FTIR) spectrometer (Bruker IFS-66) with a 35 T resistive magnet at the National High Magnetic Field Laboratory, Tallahassee. The collimated infrared beam from the spectrometer was propagating inside an evacuated beamline and focused at the top of the probe with a diamond window. Then the infrared beam was guided through a brass light pipe to the sample space, which was cooled to liquid helium temperature by a small amount of helium exchange gas. The thin HfTe₅ flakes with a thickness of tens of micrometres were mechanically exfoliated from the bulk HfTe₅ single crystals. After mounting them on the transmission sample holder, a d.c. magnetic field was applied along the crystallographic b axis in the Faraday geometry. The infrared beam went through the sample and then was detected by a 4.2 K composite silicon bolometer located just a short distance below. The single transmission spectra were collected with an acquisition time of about 3 min.

Magneto-transport measurement

Low-field magneto-transport measurements were carried out using a superconducting magnet with a standard lock-in technique. High-field magneto-transport measurements were performed in a pulsed magnet of up to 50 T.

The k·p model of HfTe₅

The ideal Hamiltonian of HfTe₅ is given by Eq. (1); when applying a magnetic field B along the z axis, the energy of the Landau bands in the TI are as follows:

\[ E_n(k_z = 0) = -\frac{M}{2} \sqrt{\left( \Delta + 2 \frac{v_F}{\hbar} |n| \right)^2 + 2 \frac{\hbar v_F}{\epsilon_F} |n|}, n = \pm 1, \pm 2, \pm 3 \ldots, \]

where \( \epsilon_F = \sqrt{\hbar v_F |n|} \) and \( n, \alpha = \pm 1 \) and \( s = \pm 1 \) denote the Landau index, carrier type index and spin index, respectively; \( l_B = \sqrt{\hbar e B} \) is the magnetic length. The optical transitions between Landau levels in the ideal case generally follow selection rules of \( \Delta|n| = \pm 1 \). Therefore, the optical transitions occur from both \( L_n \rightarrow L_{n+1} \) and \( L_{n+1} \rightarrow L_n \), where \( L_n \) denotes the nth Landau level. The optical transitions at \( k_z = 0 \) require a photon energy of

\[ \omega(n, B) = \sqrt{\left( \Delta + 2 \frac{v_F}{\hbar} |n| + 1 \right)^2 + 2 \frac{\hbar v_F}{\epsilon_F} |n| + 1} \]

Detailed discussions including the Zeeman effect and other perturbation terms are provided in Supplementary Sections III and X.

Data availability

Source data are provided with this paper. All other supporting data are available from the corresponding authors upon reasonable request.

References

1. Chen, R. et al. Optical spectroscopy study of the three-dimensional Dirac semimetal ZrTe₅. Phys. Rev. B 92, 075107 (2015).
2. Liu, Y. et al. Zeeman splitting and dynamical mass generation in Dirac semimetal ZrTe₅. Nat. Commun. 7, 12516 (2016).
3. Galecki, S. et al. Unconventional Hall response in the quantum limit of HfTe₅. Nat. Commun. 11, 5926 (2020).
4. Zhao, P.-L., Lu, H.-Z. & Xie, X. C. Theory for magnetic-field-driven 3D metal-insulator transitions in the quantum limit. Phys. Rev. Lett. 127, 046602 (2021).
5. Fu, B., Wang, H.-W. & Shen, S.-Q. Dirac polarons and resistivity anomaly in ZrTe₅ and HfTe₅. Phys. Rev. Lett. 125, 256601 (2020).
6. Fu, B., Wang, H.-W. & Shen, S.-Q. Dirac polarons and resistivity anomaly in ZrTe₅ and HfTe₅. Phys. Rev. Lett. 125, 256601 (2020).
7. Wang, C. Thermodynamically induced transport anomaly in dilute metals ZrTe₅ and HfTe₅. Phys. Rev. Lett. 126, 126601 (2021).
8. Zheng, G. et al. Field-induced topological phase transition from a three-dimensional Weyl semimetal to a two-dimensional massive Dirac metal in ZrTe₅. Phys. Rev. B 96, 121401 (2017).
9. Liang, T. et al. Anomalous Hall effect in ZrTe₅. Nat. Phys. 14, 451–455 (2018).
10. Xiong, H. et al. Three-dimensional nature of the band structure of ZrTe₅ measured by high-momentum-resolution photoemission spectroscopy. Phys. Rev. B 95, 195119 (2017).
11. Zhang, Y. et al. Temperature-induced Lifshitz transition in topological insulator candidate HfTe₅. Sci. Bull. 62, 950–956 (2017).
12. Li, Q. et al. Chiral magnetic effect in ZrTe₅. Nat. Phys. 12, 550–554 (2016).
13. Böttner, B. et al. Single valley Dirac fermions in zero-gap HgTe quantum wells. Nat. Phys. 7, 418–422 (2011).
14. Klier, J., Gornyi, I. V. & Mirlin, A. D. Transversal magnetoresistance in Weyl semimetals. Phys. Rev. B 92, 205113 (2015).
15. Pesin, D. A., Mishchenko, E. G. & Levchenko, A. Density of states and magnetotransport in Weyl semimetals with long-range disorder. Phys. Rev. B 92, 174202 (2015).
16. Xiao, X., Law, K. T. & Lee, P. A. Magnetoeconductivity in Weyl semimetals: effect of chemical potential and temperature. Phys. Rev. B 96, 165101 (2017).
17. Könye, V. & Ogata, M. Magnetoresistance of a three-dimensional Dirac gas. Phys. Rev. B 98, 195420 (2018).

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**Author contributions**
X.Y. conceived the idea and supervised the overall research. X.M., B.L. and Y.M. carried out the growth of the HfTe, single crystals. M.O., W.W., Z.S. and Y.D. performed the magneto-infrared experiments. C.Z. and Y.W. conducted the magneto-transport experiments. W.W., X.Y., Z.Y., F.Q. and H.-Z.L. performed the theoretical analyses based on the $\mathbf{k} \cdot \mathbf{p}$ model. X.Y., W.W., C.Z., Z.S. and J.C. wrote the paper with the help of all the coauthors.

**Competing interests**
The authors declare no competing interests.

**Additional information**

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