Research Article

Distributed Adaptive Neural Consensus Tracking Control for Multiple Euler-Lagrange Systems with Unknown Control Directions

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This paper investigates the distributed adaptive neural consensus tracking control for multiple Euler-Lagrange systems with parameter uncertainties and unknown control directions. Motivated by the Nussbaum-type function and command-filtered backstepping technique, the error compensations and neural network approximation-based adaptive laws are established, which can not only overcome the computation complexity problem of backstepping but also make the consensus tracking errors reach to the desired region although the control directions and system nonlinear dynamics are both unknown. Numerical example is given to show the proposed algorithm is effective at last.

1. Introduction

In recent years, the Euler-Lagrange system (ELS) has been widely used in various fields, since the mathematical models of many mechanical systems are represented by the ELS, such as the satellite, manipulator, and sensor networks [1–5]. Due to the low-power consumption, high reliability, and low communication burden, the distributed control of multiple ELSs has become an important research direction [6, 7]. For distributed control of multiple ELSs, the consensus is concerned by many scholars, which is necessary to design the coordinated protocol such that all ELSs’ states reach to an agreement on certain interest through local information exchange [8–10]. Nowadays, many consensus algorithms have been given for multiple ELSs, which can be divided into leaderless consensus and leader-following consensus. All states of the closed-loop system converge to a common value under the leaderless consensus [8], and all states converge to the leader’s state under the leader-following consensus [10].

Backstepping is one of the most commonly used approaches for consensus control of multiple ELSs [11–14], such as [12] considered the backstepping-based attitude consensus control for a group of flexible spacecraft; [13] proposed the backstepping-based adaptive sliding mode control approach for multiple manipulators; [14] studied the backstepping-based synchronization of uncertain networked Lagrangian systems. However, for the traditional distributed backstepping, the state is used as the virtual input for each subsystem, and the derivatives of virtual input are used in the next step. Therefore, a computationally complicated problem may arise since each step needs to duplicate the differentiation of virtual signals [15]. To solve the explosion of complexity (EOC) problem, the dynamic surface control (DSC) was proposed in [16, 17], which can eliminate the EOC by applying the first-order filters. Recently, [18] studied adaptive control of multiple flexible manipulators using neural network (NN) and DSC. However, since the filtering errors are not compensated in [18], the DSC can not further improve the control quality [19]. Then, the command-filtered backstepping is established, in which the virtual control differentiation is approximated by the output of the filter and the error caused by the command filter is compensated by the error compensation system [20, 21]. Zhao et al. [22] proposed the adaptive control for multiple spacecraft systems by using command-filtered backstepping, but the system models are assumed to be completely known.
Since the ELS usually works in complex working condition, the system uncertainties cannot be avoided. How to extend the command filter and error compensation-based backstepping for multiple ELSs with uncertainties to achieve the leader-following consensus is remained unsolved, which should be further studied.

On the contrary, in the design of a controller, the selection of control direction is extremely meaningful. However, in some physical models, it is difficult to choose the directions of control [23–25]. In recent years, some control methods combined with adaptive fuzzy and NN-based backstepping control to deal with the nonlinear systems with unknown control directions (UCDs). For example, the problem of adaptive fuzzy control was studied for stochastic nonlinear systems with UCDs in [26]. In [27], the adaptive control design for nonlinear systems with UCDs was given. For uncertain MIMO nonlinear systems with UCDs, the NN-based adaptive control was investigated in [28]. Chen et al. [29, 30] considered the Nussbaum function will be extended to facilitate the problem of unknown nonlinear dynamics by using NN approximation technique. In addition, the transmission matrix $K_i = \text{diag}[k_{i,1}, \ldots, k_{i,m}] \in \mathbb{R}_+^{m\times n}$ with $k_{i,j} > 0$, $j = 1, \ldots, n$ is assumed to be unknown, and the sign of $k_{i,j}$ is also assumed to be unknown, but $\text{sgn}(k_{i,j}) = \text{sgn}(k_{i,p})$ for $1 \leq i \neq j \leq N, d, p = 1, \ldots, n$.

Remark 1. As pointed out in [31], the control directions of all ELSs are assumed to be the same is reasonable. For example, the consensus can only be achieved for those ELSs that have the same control directions. Furthermore, for the practical model as two-link planar elbow manipulators [32] or revolute manipulators [33], it always has similar joints, thus we assume that each joint has the same control direction.

Define the output of the leader as $y_d \in \mathbb{R}^n$, $y_{\dot{d}}$, and $\dot{y}_d$ are continuous bounded and smooth signals.

Property 1. For any $x \in \mathbb{R}^n, x^T(M_i(q_i) - 2C_i(q_i, \dot{q}_i))x = 0$.

Property 2. For $\lambda_{\min}, \lambda_{\max} \in \mathbb{R}$,

$$\lambda_{\max}I_n \leq M_i(q_i) \leq \lambda_{\max}I_n.$$  

Assumption 1. $\|d_i\| \leq \bar{d}_i$, and $\bar{d}_i > 0$ is an unknown constant.

Assumption 2. $\bar{G}$ contains a directed spanning tree with the leader being the root.

Define new variables as

$$s_{i1} = q_i,$$

$$s_{i2} = \dot{q}_i.$$  

\section*{2. System Descriptions}

Denote $G = (\mathcal{V}, \epsilon)$ as a weighted directed graph, which can show the communications among agents in a network. For a network with $N$ agents, $\mathcal{V} = \{1, 2, \ldots, N\}$ is nodes’ set, and $\epsilon \subseteq \mathcal{V} \times \mathcal{V}$ is the edges’ set. $(i, j) \in \epsilon$ means that there is an edge from $j$ to $i$, and the neighbors’ set of $i$ is denoted as $\mathcal{N}_i = \{j | (j, i) \in \epsilon\}$. The weighted matrix is denoted by $A = [a_{ij}] \in \mathbb{R}^{N \times N}$ with $a_{ij} > 0$ for $(j, i) \in \epsilon$, $a_{ij} = 0$ for $(j, i) \notin \epsilon$, and $a_{ii} = 0$ for $\forall i$. Then, the Laplacian matrix can be given as $L = D - A$, where $D = \text{diag}[d_1, d_2, \ldots, d_N]$ and $d_i = \sum_{j=1}^N a_{ij}$. $G$ exists a direct path from $i$ to $j$ means that there has a sequence of successive edges with the form of $(i, k), (k, l), \ldots, (m, j)$. Furthermore, $G$ exists a spanning tree means that there is a node such that this node can arrive every other node by a directed path.

$\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is an extension graph to describe the networked system with $N$ followers and one leader. $B = [b_{11}, \ldots, b_{1N}]$ is defined as the adjacency matrix of the leader, where $b_{i} > 0$ means that there is an edge from the leader to $i$, otherwise $b_{i} = 0$.

Now, we consider the networked ELSs under $N$ followers and one leader, a digraph $\mathcal{G}$ is used to describe the communications among them. The dynamic modeled for the $i$th following ELS is given by

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i + d_i,$$

$$\tau_i = K_i u_i,$$

where $q_i \in \mathbb{R}^n$ is the joint vector, $M_i(q_i) \in \mathbb{R}_+^{n\times n}$ is symmetric inertia matrix, $C_i(q_i, \dot{q}_i) \in \mathbb{R}_+^{n\times n}$ is the matrix of centripetal moments and Coriolis moments, $G_i \in \mathbb{R}^n$ is the gravity term, $\tau_i \in \mathbb{R}^n$ is the torque input vector, $u_i \in \mathbb{R}^n$ is the voltage input vector to amplifier for the $i$th ELS, and $d_i \in \mathbb{R}^n$ is the disturbance torque, respectively. The transmission matrix $K_i = \text{diag}[k_{i,1}, \ldots, k_{i,m}] \in \mathbb{R}_+^{m\times n}$ with $k_{i,j} > 0$, $j = 1, \ldots, n$ is assumed to be unknown, and the sign of $k_{i,j}$ is also assumed to be unknown, but $\text{sgn}(k_{i,j}) = \text{sgn}(k_{i,p})$ for $1 \leq i \neq j \leq N, d, p = 1, \ldots, n$. 

\section*{Complexity}

The main contributions are stated as follows.

(1) Compared with the traditional backstepping and DSC-based controller design methods in [14, 18, 31], the proposed command-filtered backstepping-based controller design method cannot only avoid the traditional explosion terms but also eliminate the error caused by the used command filter by using the compensation system.

(2) Compared with the command-filtered backstepping-based controller design method in [22] with the complete known system model, the proposed adaptive neural control design method can solve the problem of unknown nonlinear dynamics by using NN approximation technique. In addition, the Nussbaum function will be extended to facilitate control design to deal with the UCD problem that is not considered in [14, 18, 20–22].
Then,
\[
\begin{align*}
\dot{s}_{i,1} &= s_{i,2}, \\
M_i(s_{i,1})\dot{s}_{i,2} &= -C_i(s_{i,1}, s_{i,2})s_{i,2} - G_i(s_{i,1}) + K_im_i + d_i, \\
y_i &= s_{i,1}.
\end{align*}
\] (4)

**Lemma 1.** If Assumption 2 is satisfied, then the eigenvalues of \(H = L + B\) have positive real parts.

**Lemma 2** (see [34]). If the continuous function \(N(s) : \mathbb{R} \rightarrow \mathbb{R}\) satisfies
\[
\begin{align*}
\lim_{s \rightarrow -\infty} \sup_{r \in [0, t_f]} \int_{0}^{s} N(\zeta)d\zeta &= +\infty, \\
\lim_{s \rightarrow +\infty} \inf_{r \in [0, t_f]} \int_{0}^{s} N(\zeta)d\zeta &= -\infty,
\end{align*}
\] (5)

\(N(s)\) is a Hamburger-type function. \(\zeta^2 \cos(\zeta), e^{\zeta^2} \cos(\pi/2\zeta)\) are commonly Hamburger-type functions.

**Lemma 3** (see [34]). Let \(V(t), \zeta(t)\) be smooth functions defined on \([0, t_f]\) with \(V(t) \geq 0, \forall t \in [0, t_f]\), and \(N(\zeta)\) be an even smooth Hamburger-type function. The following inclusions hold:
\[
V(t) \leq c_0 + e^{-c_1t} \int_{0}^{t} [g(t)N(\zeta) + 1]\zeta e^{-c_1r}dr,
\] (6)

where \(c_0\) represents some suitable constant, \(c_1\) is positive constant, and \(g(t)\) is a time-varying parameter which takes values in the unknown closed intervals \(I = [l_-, l_+]\) with \(0 \notin I\), and then \(V(t), \int_{0}^{t} [g(t)N(\zeta) + 1]\zeta e^{-c_1r}dr\) must be bounded on \([0, t_f]\).

**3. Main Results**

For the ith follower, the local neighborhood tracking errors are given as
\[
\bar{s}_{i,1} = \sum_{j=1}^{N} a_{ij}(y_i - y_j) + b_i(y_i - y_d), \quad \bar{s}_{i,2} = s_{i,2} - \pi_i,
\] (7)

where \(\pi_i(t) = \varphi_i(t) = [\varphi_{i,1}(t), \ldots, \varphi_{i,n}(t)]^T, \varphi_{i,1}(t), \varphi_{i,2}(t), z = 1, \ldots, n\) is given by using the finite-time command filter
\[
\begin{align*}
\varphi_{i,1} &= l_{i,1}, \\
\dot{l}_{i,1} &= -r_{i,1}\left[\varphi_{i,1} - \alpha_{i,1}\right]^{1/2}\text{sgn}(\varphi_{i,1} - \alpha_{i,1}) + \varphi_{i,2}, \\
\dot{\varphi}_{i,2} &= -r_{i,2}\text{sgn}(\varphi_{i,2} - l_{i,1}), \quad z = 1, \ldots, n,
\end{align*}
\] (8)

and the input of (8) is the virtual control signal \(\alpha_{i,1} = [\alpha_{i,1,1}(t), \ldots, \alpha_{i,1,n}(t)]^T\).

**Lemma 4** (see [35]). If \(\alpha_{i,1,2}\) is not disturbed by noise, that is, \(\alpha_{i,1,2} = \alpha_{i,1,0,2}\), then \(r_{i,1}\) and \(r_{i,2}\) can be properly chosen such that
\[
\varphi_{i,1,2} = \alpha_{i,1,0,2}, \quad l_{i,1,2} = \dot{l}_{i,1,2},
\] (9)

are achieved in finite time. If the noise of input satisfies \(|\alpha_{i,1,2} - \alpha_{i,1,0,2}| \leq k_{i,2}\), then
\[
\begin{align*}
&|\varphi_{i,1,2} - \alpha_{i,1,0,2}| \leq \mu_{i,1,2}k_{i,2} = \Pi_{i,1,2}, \\
&|l_{i,1,2} - \dot{l}_{i,1,2}| \leq \lambda_{i,1,2}k_{i,2}^{1/2} = \Pi_{i,2,2},
\end{align*}
\] (10)

are achieved in finite time, where \(\mu_{i,1,2} > 0\) and \(\lambda_{i,1,2} > 0\).

To eliminate the affection of the filtering error \(\pi_i - \alpha_{i,1}\), the following error compensation system is established:
\[
\dot{\pi}_{i,1} = -k_i\pi_{i,1} + (b_i + d_i)(\pi_i - \alpha_{i,1}),
\] (11)

where \(k_{i,1} > 0, \pi_{i,1}(0) = 0\). Furthermore, define
\[
\eta_{i,1} = \bar{s}_{i,1} - \pi_{i,1},
\] (12)

and choose the virtual control signal as
\[
\alpha_{i,1} = \frac{1}{(b_i + d_i)}(-k_i\bar{s}_{i,1} + \sum_{j=1}^{N} a_{ij}\bar{s}_{i,2} + b_i\bar{y}_d),
\] (13)

and the control torque is chosen as
\[
\begin{align*}
u_i &= N(\zeta)\begin{pmatrix} 
\frac{1}{2\kappa_{i,1}}\bar{s}_{i,1} & \bar{\theta}_i S_{i,1}^T \bar{s}_{i,1} & 
\ldots & 
\frac{1}{2\kappa_{i,n}}\bar{s}_{i,1} \bar{\theta}_i S_{i,1}^T \bar{s}_{i,n} 
\end{pmatrix} 
\end{align*}
\] (14)

where \(k_{i,2} > 0\) is a constant, is the Nussbaun gain matrix, and is defined as \(N(\zeta) = \text{diag}_{i}[N(\zeta_{i,1}), \ldots, N(\zeta_{i,n})]\), with \(\zeta_i = [\zeta_{i,1}, \ldots, \zeta_{i,n}]^T \in \mathbb{R}^n\), \(\zeta_i\) is obtained by using the following dynamic process:
\[
\begin{align*}
\zeta_i &= \text{diag}[(\zeta_{i,2,1}, \ldots, \zeta_{i,2,n})](k_{i,2} + 1)\bar{s}_{i,2} + \\
&\quad \begin{pmatrix} 
\frac{1}{2\kappa_{i,2}}\bar{s}_{i,2} \bar{\theta}_i S_{i,2}^T \bar{s}_{i,1} & 
\ldots & 
\frac{1}{2\kappa_{i,n}}\bar{s}_{i,2} \bar{\theta}_i S_{i,2}^T \bar{s}_{i,n} 
\end{pmatrix} 
\end{align*}
\] (15)

Let \(\theta_i = \max\{\|W_{i,2}\|^2\}\), then the adaptive law to estimate \(\theta_i\) is designed by
\[
\dot{\hat{\eta}}_i = -r_i \eta_i \hat{\eta} + \sum_{z=1}^{n} \frac{1}{2 \eta_i z} \hat{\eta}_T \tilde{s}_{i, z} \tilde{S}_{i, z},
\]  

(16)

where \( r_i > 0 \) and \( \eta_i > 0 \) are constants.

**Theorem 1.** For the ELS (1) with Assumptions 1-2, then the \( \xi_{i, 1} \) in (11), \( a_{i, 1} \) in (13), \( u_i \) in (14), and \( \hat{\eta}_i \) in (16) can be chosen such that \( y_i - y_d \) converges to a sufficiently small neighborhood of the origin, and all the signals in the closed-loop system are bounded.

**Proof**

**Step 1.** Consider the Lyapunov function as

\[
V_{i, 1} = \frac{1}{2} \hat{\eta}_T \eta_i.
\]

(17)

Then,

\[
\dot{V}_{i, 1} = \eta_i \eta_i \hat{\eta}_T \left( \tilde{s}_{i, 1} - \dot{\xi}_{i, 1} \right) \\
= \eta_i \eta_i \left( \sum_{j=1}^{N} a_{ij} (\dot{\xi}_{j, 1} - \dot{\xi}_{i, 1}) + b_i (\dot{\xi}_{i, 1} - \dot{y}_d) - \dot{\xi}_{i, 1} \right) \\
= \eta_i \eta_i \left( -\sum_{j=1}^{N} a_{ij} \dot{s}_{j, 2} + (b_i + d_i) \tilde{s}_{i, 2} - b_i \dot{y}_d - \tilde{\xi}_{i, 1} \right) \\
= \eta_i \eta_i \left( (b_i + d_i) a_{i, 1} + (b_i + d_i) (\eta_i - \alpha_i) + \tilde{\xi}_{i, 1} - \sum_{j=1}^{N} a_{ij} \dot{s}_{j, 2} \right).
\]

(18)

Substituting (11) and (13) into (18), one has

\[
\dot{V}_{i, 1} = \eta_i \eta_i \left( -k_{i, 1} \tilde{s}_{i, 1} + k_{i, 1} \tilde{\xi}_{i, 1} + (b_i + d_i) \tilde{s}_{i, 2} \right) \\
= -k_{i, 1} \eta_i \eta_i + \eta_i \eta_i \left( b_i + d_i \right) \tilde{s}_{i, 2}.
\]

(19)

**Step 2.** Consider another

\[
V_{i, 2} = V_{i, 1} + \frac{1}{2} \tilde{s}_{i, 2} \tilde{S}_{i, 2}.
\]

(20)

Then, one can write

\[
\dot{V}_{i, 2} = \dot{V}_{i, 1} + \frac{1}{2} \tilde{s}_{i, 2} \tilde{S}_{i, 2}.
\]

(21)

From Property 1, we have \( \tilde{s}_{i, 2} (-C_i \tilde{s}_{i, 2} + (1/2) \tilde{s}_{i, 2} \tilde{S}_{i, 2} = (1/2) \tilde{s}_{i, 2} (M_i - 2C_i) \tilde{s}_{i, 2} = 0) \). Denoting \( \Phi_i = -C_i \eta_i - M_i \eta_i + (b_i + d_i) \eta_i, \) one has

\[
\dot{V}_{i, 2} = -k_{i, 1} \eta_i \eta_i + \tilde{s}_{i, 2} \left( K_i \eta_i + d_i + \Phi_i \right).
\]

(22)

Substituting \( u_i \) into (22) yields

\[
\dot{V}_{i, 2} = -k_{i, 1} \eta_i \eta_i + \tilde{s}_{i, 2} \left( K_i \eta_i + d_i + \Phi_i \right)
\]

\[
= \eta_i \eta_i \left( k_{i, 1} \tilde{s}_{i, 2} + \tilde{s}_{i, 2} \left( K_i \eta_i + d_i + \Phi_i \right) \right)
\]

\[
= \eta_i \eta_i \left( \frac{1}{2} \tilde{s}_{i, 2} \tilde{S}_{i, 2} \right) + d_i + \Phi_i.
\]

(23)
Adding and subtracting $\sum_{z=1}^{n} \dot{\xi}_{i,z}$ on the right-hand side of (23) yields

$$\dot{V}_{i,2} = -k_{i,1} \eta_{i,1} \eta_{i,1} + \bar{z}_{i,2}^T K_{i,2} \dot{S}_{i,2} + \frac{1}{2t_{i,1}} \bar{z}_{i,2,1} \Phi_{i,1} \dot{S}_{i,2} + d_{i} + \Phi_{i} + \sum_{z=1}^{n} \dot{\xi}_{i,z} - \sum_{z=1}^{n} \dot{\xi}_{i,z}$$

where $W_{i,z} \in \mathbb{R}^l$ is the ideal weight matrix, $S_{i,z} \in \mathbb{R}^l$ is the basis vector, $l > 1$ is the number of NN nodes, and $|\xi_{i,z}| \leq \varepsilon_{i,z}, \varepsilon_{i,z} > 0$ is the approximation error. According to Young’s inequality, it yields

$$\bar{z}_{i,2}^T \Phi_{i} = \sum_{z=1}^{n} \bar{z}_{i,2,z} \Phi_{i,z} \leq \sum_{z=1}^{n} \left( \frac{1}{2t_{i,z}} \bar{z}_{i,2,z}^2 \|W_{i,z}\|^2 + \frac{1}{2} \bar{z}_{i,2,z}^2 + \frac{1}{2} \bar{z}_{i,2,z}^2 + \frac{1}{2} \bar{z}_{i,2,z}^2 \right).$$

where $t_{i,z} > 0$ is a constant. Furthermore,

$$\bar{z}_{i,2,2d_{i}} \leq \frac{\bar{z}_{i,2,2}^T \bar{z}_{i,2,2}}{2} + \frac{\bar{z}_{i,2,2}^2}{2} + \frac{\bar{z}_{i,2,2}^2}{2}$$

Substituting (26) and (27) into (24) yields

$$\dot{V}_{i,2} \leq -k_{i,1} \eta_{i,1} \eta_{i,1} - k_{i,2} \bar{z}_{i,2} \bar{z}_{i,2} + \frac{1}{2t_{i,1}} \bar{z}_{i,2,1} \Phi_{i,1} \bar{z}_{i,2} + d_{i} + \Phi_{i} + \sum_{z=1}^{n} (\varepsilon_{i,z}) K_{i,z} + 1) \dot{\xi}_{i,z}.$$
which yields $-k_{i,2}\bar{s}_{i} + 2\bar{s}_{i}\bar{s}_{i,2} \leq -(k_{i,2}/\lambda_{\max})\bar{M}_{i,2}\bar{s}_{i,2}$, and $M_{i,2}\bar{s}_{i,2} = -(2k_{i,2}/\lambda_{\max})\bar{M}_{i,2}\bar{s}_{i,2}/2$. Furthermore, substituting $d_{i}^{T}\bar{s}_{i} \leq -\left((1/2)d_{i}^{T} + (1/2)d_{i}^{T}\right)$ into (30) yields

$$V \leq -\sum_{i=1}^{N} a_{i}^{T}\pi_{i}^{T} + \sum_{i=1}^{N} \left(-2a_{i}^{T}M_{i,2}\bar{s}_{i,2} + \sum_{i=1}^{N} \bar{s}_{i,2}^{2}\right) + \sum_{i=1}^{N} \bar{s}_{i,2}^{2}$$

Then, by using Young’s inequality,

$$\dot{V} \leq \sum_{i=1}^{N} \left(-k_{i,1}\bar{s}_{i,1}^{T}\xi_{i,1} + \frac{\bar{s}_{i,1}^{T}\xi_{i,1}^{2}}{2} + \bar{a}_{i}^{T}\xi_{i,1}^{2}/2\right)$$

$$\leq -2\bar{k}_{0}V + \bar{a}_{i}^{T}$$

(38)

where $\bar{k}_{0} = 2\min\{k_{i,1} - 1/2\}$, $\bar{a}_{i} = \sum_{i=1}^{N} (\bar{s}_{i,1}^{T}/2)$. Thus, we have $\|\bar{s}_{i,1}\| \leq \sqrt{(2\bar{k}_{0}/a) + \sqrt{(2\bar{a}_{i}/k_{0})}}$. Define $X_{1} = [x_{1}^{T}, \ldots, x_{N}^{T}]$, $\Theta_{1} = [y_{1}^{T} - y_{d}^{T}, \ldots, y_{N}^{T} - y_{d}^{T}]^{T}$, then $\Theta_{1} = (H \otimes I_{N})^{-1}X_{1}$ can be obtained, so $\|y_{i} - y_{d}\| \leq \sqrt{(\sqrt{2\bar{k}_{0}/a) + \sqrt{(2\bar{a}_{i}/k_{0})}} / \sigma_{\min}(H)}$.

Remark 2. Note that before the command filter achieves stability, the filtering error $\pi_{i} - \bar{\xi}_{i,1}$ is always existed, this may influence the control qualities. The error compensation signal in (11) can be used at the first step of backstepping to remove the affection of the error $\pi_{i} - \bar{\xi}_{i,1}$, which can guarantee better transient performance than the DSC in [16–18].

Remark 3. Larger $k_{i,1}, k_{i,2}$ can guarantee the smaller convergence neighborhood for the consensus tracking error. Furthermore, the principle of choosing the parameters $r_{i,1}$ and $r_{i,2}$ is $r_{i,1}$ and $r_{i,2}$ should be sufficiently large, and $r_{i,2}$ is chosen first [35].

4. Numerical Results

The multiple manipulator system with six followers and one leader is used to verify the given algorithm. The information communications among the followers and leader are shown in Figure 1. Each follower is assumed to be the two-link robot manipulator, in which $M_{i}(q_{i}) \in \mathbb{R}^{2\times2}$ and $C_{i}(q_{i}, \dot{q}_{i}) \in \mathbb{R}^{2\times2}$ are chosen as

$$M_{i11} = v_{i,1} + 2v_{i,2}\cos(q_{i,2}),$$

$$M_{i21} = M_{i22} = v_{i,3} + v_{i,2}\cos(q_{i,2}),$$

$$C_{i11} = -v_{i,2}\sin(q_{i,2})\dot{q}_{i,1},$$

$$C_{i12} = -v_{i,2}\sin(q_{i,2})(\dot{q}_{i,1} + \dot{q}_{i,2}),$$

$$C_{i21} = v_{i,2}\sin(q_{i,2})\dot{q}_{i,1},$$

$$C_{i22} = 0,$$

with $v_{i,1} = I_{i,1} + m_{i,2}l_{i,2}^{2} + m_{i,1}l_{i,2}^{2} + m_{i,2}l_{i,2}^{2} + m_{i,2}l_{i,2}^{2}$, $v_{i,2} = m_{i,2}l_{i,2}^{2}$, $l_{i,2} = 1$, $l_{i,2}$ are the links’ masses, $I_{i,1}$ and $I_{i,2}$ are the inertia’s moments, $l_{i,1}$ and $l_{i,2}$ are the links’ lengths, and $l_{i,1}$ and $l_{i,2}$ are the links’ mass centers. We further assume that $G_{i}(q_{i}) = 0$ for simplicity. The system parameters are given by
Figure 1: The interaction topology among ELSs.

\[ I_{1,1} = 0.48 \text{kgm}^2, \]
\[ I_{1,2} = 0.36 \text{kgm}^2, \]
\[ m_{1,1} = 1.4 \text{kg}, \]
\[ m_{1,2} = 1.6 \text{kg}, \]
\[ l_{1,1} = 1.8 \text{m}, \]
\[ l_{1,c1} = 0.9 \text{m}, \]
\[ l_{1,c2} = 1.1 \text{m}, \]
\[ l_{1,3} = 0.49 \text{kgm}^2, \]
\[ l_{1,4} = 0.37 \text{kgm}^2, \]
\[ m_{2,1} = 1.5 \text{kg}, \]
\[ m_{2,2} = 1.8 \text{kg}, \]
\[ l_{2,1} = 1.6 \text{m}, \]
\[ l_{2,c1} = 1 \text{m}, \]
\[ l_{2,c2} = 1.3 \text{m}, \]
\[ l_{2,3} = 0.46 \text{kgm}^2, \]
\[ l_{2,4} = 0.35 \text{kgm}^2, \]
\[ m_{3,1} = 1.3 \text{kg}, \]
\[ m_{3,2} = 1.5 \text{kg}, \]
\[ l_{3,1} = 1.7 \text{m}, \]
\[ l_{3,c1} = 0.8 \text{m}, \]
\[ l_{3,c2} = 1 \text{m}, \]
\[ l_{3,4} = 0.43 \text{kgm}^2, \]
\[ l_{4,1} = 0.41 \text{kgm}^2, \]
\[ m_{4,1} = 1.6 \text{kg}, \]
\[ m_{4,2} = 1.7 \text{kg}, \]
\[ l_{4,1} = 1.6 \text{m}, \]
\[ l_{4,c1} = 1.1 \text{m}, \]
\[ l_{4,c2} = 1.3 \text{m}, \]
\[ l_{4,5} = 0.42 \text{kgm}^2, \]
\[ l_{5,2} = 0.38 \text{kgm}^2, \]
\[ m_{5,1} = 1.4 \text{kg}, \]
\[ m_{5,2} = 1.7 \text{kg}, \]
\[ l_{5,1} = 1.5 \text{m}, \]
\[ l_{5,c1} = 1 \text{m}, \]
\[ l_{5,c2} = 1.2 \text{m}, \]
\[ l_{6,1} = 0.51 \text{kgm}^2, \]
\[ l_{6,2} = 0.44 \text{kgm}^2, \]
\[ m_{6,1} = 1.6 \text{kg}, \]
\[ m_{6,2} = 1.9 \text{kg}, \]
\[ l_{6,1} = 1.8 \text{m}, \]
\[ l_{6,c1} = 1.2 \text{m}, \]
\[ l_{6,c2} = 1.4 \text{m}. \]

The transmission matrix \( K \) is chosen as \( K = \text{diag}(-1, -1) \). The initial conditions are

\[ q_{1,1}(0) = 0.3\pi \text{ rad}, \]
\[ q_{1,2}(0) = -0.35\pi \text{ rad}, \]
\[ q_{2,1}(0) = -0.25\pi \text{ rad}, \]
\[ q_{2,2}(0) = 0.3\pi \text{ rad}, \]
\[ q_{3,1}(0) = 0.25\pi \text{ rad}, \]
\[ q_{3,2}(0) = -0.33\pi \text{ rad}, \]
\[ q_{4,1}(0) = -0.3\pi \text{ rad}, \]
\[ q_{4,2}(0) = 0.2\pi \text{ rad}, \]
\[ q_{5,1}(0) = 0.3\pi \text{ rad}, \]
\[ q_{5,2}(0) = -0.2\pi \text{ rad}, \]
\[ q_{6,1}(0) = -0.3\pi \text{ rad}, \]
\[ q_{6,2}(0) = 0.3\pi \text{ rad}, \]
\[ \dot{q}_{1,1}(0) = 0 \text{ rad}, \]
\[ \dot{q}_{1,2}(0) = 0 \text{ rad}, \]
\[ \dot{q}_{2,1}(0) = 0 \text{ rad/s}, \]
\[ \dot{q}_{2,2}(0) = 0 \text{ rad/s}, \]
\[ \dot{q}_{3,1}(0) = 0 \text{ rad/s}, \]
\[ \dot{q}_{3,2}(0) = 0 \text{ rad/s}, \]
\[ \dot{q}_{4,1}(0) = 0 \text{ rad/s}, \]
\[ \dot{q}_{4,2}(0) = 0 \text{ rad/s}, \]
\[ \dot{q}_{5,1}(0) = 0 \text{ rad/s}, \]
\[ \dot{q}_{5,2}(0) = 0 \text{ rad/s}, \]
\[ \dot{q}_{6,1}(0) = 0 \text{ rad/s}, \]
\[ \dot{q}_{6,2}(0) = 0 \text{ rad/s}, \]
\[ \ddot{q}_i(0) = 0, \quad i = 1, 2, 3. \]

The disturbance torque is given by \( d_i = [0.1 \sin(it), 0.1 \cos(it)]^T, \quad i = 1, \ldots, 6, \) and the joint position of leader is chosen as \( y_d(t) = (2\sin(t), 2\cos(t))^T. \)
Figure 2: Response curves of $q_i$ and $y_{d,i}, i = 1, \ldots, 6$.

Figure 3: Response curves of $\pi_i$ and $\alpha_{i,1,j}, i = 1, 2, 3$. 

Figure 4: Continued.
Figure 4: Response curves of $\pi_i$ and $\alpha_{i,1}$, $i = 4, 5, 6$.

Figure 5: Response curves of $\bar{\theta}_i$, $i = 1, 2, 3$.

Figure 6: Response curves of $\bar{\theta}_i$, $i = 4, 5, 6$. 

Complexity
The gains are given by $k_{i,1} = k_{i,2} = 50$, $r_{i,1,z} = r_{i,2,z} = 60$, $\rho_i = r_i = 1 (i = 1, \ldots, 6, z = 1, 2)$. The used Nussbaum-type function is $\zeta^2 \cos(\zeta)$. The responses of $q_i (i = 1, \ldots, 6)$ and $y_d$ are shown in Figure 2, which show that the joint positions of follower manipulators ultimately converge to the joint position of the leader manipulator with sufficient accuracy although the UCDs are existed. The responses of $\alpha_i, 1$ and $\pi_i (i = 1, \ldots, 6)$ are given in Figures 3 and 4, and the responses of $\tilde{\theta}_i (i = 1, \ldots, 6)$ are shown in Figures 5 and 6, respectively. We see that the virtual control can be fast filtered. It can also be seen that $\tilde{\theta}_i (i = 1, \ldots, 6)$ converge to zero. The reason for this phenomenon is that the $\sigma$-modification technique is used for the adaptive law in (16) to prevent the parameter drift. Furthermore, (34) can guarantee $\| \tilde{\beta}_{i,z} \| \leq \sqrt{\frac{2b}{a}}$, so as in Remark 2, we know that larger $\sigma_{i,1}, \sigma_{i,2}$ can guarantee the smaller $\sqrt{\frac{2b}{a}}$, which means that the term $\sum_{z=1}^{n} (1/2r_{i,z}^2) r_i \tilde{\beta}_{i,z} S_{i,z}^T S_{i,z}$ in (16) is sufficiently close to zero. When the term $\sum_{z=1}^{n} (1/2r_{i,z}^2) r_i \tilde{\beta}_{i,z} S_{i,z}^T S_{i,z}$ is sufficiently small, $\tilde{\theta}_i (i = 1, \ldots, 6)$ can sufficiently be close to zero.

Then, we consider the joint positions are disturbed by measurement noises, and the noises for joint positions $q_{i,1}$ and $q_{i,2}, i = 1, \ldots, 6$ are chosen as in Figures 7 and 8, respectively. The responses of $q_i (i = 1, \ldots, 6)$ and $y_d$ under measurement noises are shown in Figure 9, which show that the proposed algorithm can effectively attenuate the affection of measurement noises.

To further show the algorithm effectiveness, the proposed distributed command-filtered backstepping is compared with the command-filtered backstepping without considering error compensation. If the error compensation is not considered, it actually reduces to the distributed DSC in [18], and the same control parameters are chosen. We apply the overall tracking error $OTE = \| y_1 - y_d, \ldots, y_6 - y_d \|^2$ to evaluate the tracking performances. The OTE under the two algorithms are shown in Figure 10, and we can see...
the proposed distributed command-filtered backstepping achieves better transient performance and better tracking performance.

5. Conclusions

The command-filtering backstepping and adaptive neural control are used to study the consensus tracking problem for multiple ELSs with UCDs. In the distributed control design, the finite-time command filter is constructed to solve the EOC in the classical backstepping, and the error compensation mechanism is applied to overcome the problem of filtering error between command filtering and virtual signal. Meanwhile, the NN is used to approximate unknown nonlinear dynamics, and the Nussbaum-type function is applied to attenuate the affection of UCDs. It is shown that the consensus tracking errors converge to any desired neighborhood of the origin under the proposed control scheme, and the multiple two-link robot manipulators are used in simulation to show the effectiveness.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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