The σ-Meson Production in Excited \( \Upsilon \) Decay Processes

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We analyze the ππ production amplitudes in the excited \( \Upsilon \) decay processes, \( \Upsilon(2S) \rightarrow \Upsilon(1S)\pi^+\pi^- \), \( \Upsilon(3S) \rightarrow \Upsilon(1S)\pi^+\pi^- \) and \( \Upsilon(3S) \rightarrow \Upsilon(2S)\pi^+\pi^- \), and the ππ and KK production amplitudes in the charmonium decay processes, \( \psi(2S) \rightarrow J/\psi\pi^+\pi^- \) and \( J/\psi \rightarrow \phi\pi^+\pi^- \) including the possible effect of light σ production. The amplitudes are parametrized by the sum of Breit-Wigner amplitudes for the σ and the other relevant particles and of the direct 2π-production amplitude, following the VMW method. All the ππ (and KK) mass spectra are reproduced well with the obtained values of σ-parameters, \( m_\sigma = 526^{+48}_{-37} \text{MeV} \) and \( \Gamma_\sigma = 301^{+145}_{-100} \text{MeV} \), which is almost consistent with the values in our previous phase shift analyses.

§1. Introduction

Whether the light σ-meson really exists or does not is an important problem in hadron physics. For many years, its existence had been neglected phenomenologically mainly due to the negative result of the analyses of \( I = 0 \) S-wave ππ scattering phase shift.

In many ππ-production experiments, a large event concentration or a bump structure in the spectra of ππ invariant mass \( m_{\pi\pi} \) around 500 MeV had been observed, however, conventionally it was not regarded as σ-resonance, but as a mere ππ-background, under influence of the so called “universality argument.” In this argument, it is stated that because of the unitarity of S-matrix and of the analyticity of the amplitudes, the ππ production amplitude \( \mathcal{F} \) takes the form \( \mathcal{F} = \alpha(s)\mathcal{T} \) (\( \mathcal{T} \) being the ππ scattering amplitude), with a slowly varying real function \( \alpha(s) \). The pole position of S-matrix is determined solely through the analysis of \( \mathcal{T} \), which was believed to have no light σ-pole at that time.

Recently the data of ππ-scattering phase shift had been reanalyzed by many groups including ours and the existence of light σ(450 ~ 600) was strongly suggested. The result of no σ-existence in the conventional analyses was pointed out to be due to the lack of consideration on the cancellation mechanism guaranteed by chiral symmetry, and shown to be not correct. Furthermore, we have pointed out that the “universality argument” should be revised, taking into account the quark physical picture of hadrons: The essential point is that the strong interaction is a residual interaction of QCD among all color-neutral bound states of quarks, anti-
quarks and gluons, and accordingly the requirement of unitarity and analyticity must be made on the $S$-matrix elements with the right “quark physical bases,” including not only the stable $\pi$ meson but also the “stable” $\sigma$ meson (which is also considered to be a $q\bar{q}$ bound state). The universality argument (and the conventional application of final state interaction (FSI) theorem) with the $S$-matrix bases including only stable particles, is not correct: Since this $\sigma$-meson has, in principle, an independent production coupling generally with a strong phase from the $2\pi$ system.

Thus, the production amplitude $F$ has “independent” properties from the scattering amplitude $T$, except that the pole position of resonant particles should be universal through both amplitudes. Accordingly, the data of many $\pi\pi$-production experiments, which had been analyzed following the universality argument, must be reanalyzed independently from $T$ by including the effect of $\sigma$-production. We parametrize $F$ as a sum of Breit-Wigner amplitudes for the relevant resonances (including $\sigma$-resonance) and of the direct $2\pi$ production amplitude, following the VMW method. In the VMW method the above mentioned universal pole singularity is explicitly taken into account, and this method is consistent with the unitarity of the total $S$-matrix (and also with the FSI theorem, applied correctly) with the above mentioned right bases. The VMW method had already been applied to the processes of $J/\psi \to \omega \pi\pi$ decay, $pp \to 3\pi^0$ annihilation by us $^6$, and essentially similar method to the process $D^- \to \pi^-\pi^+\pi^+$ by E791 $^8$. The large event concentration in low $m_{\pi\pi}$ region in $J/\psi \to \omega \pi\pi$ and $D^- \to \pi^-\pi^+\pi^+$ was satisfactorily explained by $\sigma$-production, while in $pp \to 3\pi^0$ clear evidences for $\sigma$-meson production were shown. In this paper we apply this method to the analyses of the hadronic decays of excited $\Upsilon$, $\Upsilon(2S,3S)$, and $\psi(1S,2S)$.

§2. Method of the analyses

We analyze $^9$ the $m_{\pi\pi}$ spectra of the processes, $\Upsilon(2S) \to \Upsilon(1S)\pi^+\pi^-$, $\Upsilon(3S) \to \Upsilon(1S)\pi^+\pi^-$, $\Upsilon(3S) \to \Upsilon(2S)\pi^+\pi^-$ and $\psi(2S) \to J/\psi\pi^+\pi^-$ following the VMW method in one-channel form. The $F$ is given by a coherent sum of $\sigma$-Breit-Wigner amplitude and of direct $2\pi$-production amplitude as

\[ F = \frac{e^{i\theta_\sigma}r_\sigma}{m_\sigma^2 - s - i\sqrt{s}\Lambda_\sigma(s)} + r_{2\pi}e^{i\theta_{2\pi}}, \]

\[ \Lambda_\sigma(s) = \frac{g_\sigma^2p_1(s)}{8\pi s}, \quad p_1(s) = \sqrt{\frac{s}{4} - m_\pi^2}, \]

where $r_\sigma$ ($r_{2\pi}$) is the $\sigma(2\pi)$ production coupling and $\theta_\sigma$ ($\theta_{2\pi}$) corresponds to the production phase. These parameters are process-dependent. The $J/\psi \to \phi\pi^+\pi^-$, $\phi K^+K^-$ process is analyzed by VMW method in two-channel form, where $f_0(980)$, $f_0(1370)$, $f_0(1500)$, as well as $\sigma$, are included. All the relevant spectra are fitted by using common values of the parameters, the mass of $m_\sigma$ and the $\sigma\pi\pi$ coupling $g_{\sigma\pi\pi}$.
The results of our analysis are shown in Fig. 1. The spectra of six different processes are reproduced well by the interference between the $\sigma$-Breit-Wigner amplitude with $m_\sigma = 526^{+48}_{-37}$MeV, $\Gamma_\sigma = 301^{+145}_{-100}$MeV, and the direct $2\pi$ production amplitude. The corresponding pole position is $\sqrt{s}_{\text{pole}} = (535^{+48}_{-36}) - i(155^{+76}_{-53})$MeV. The total $\chi^2$ is $\chi^2/(N_{\text{data}} - N_{\text{param}}) = 86.5/(150 - 37) = 0.77$. The contribution of $\sigma$ and of direct $2\pi$ production amplitudes to the spectra are given, respectively, by dot and dot-dashed lines in this figure. It is notable that the destructive interference between $\sigma$ amplitude and $2\pi$ amplitudes explains the suppression of the spectra in the threshold region of $\Upsilon(2S \rightarrow 1S)$ and $\psi(2S \rightarrow 1S)$ decays, while in $\Upsilon(3S \rightarrow 1S)$ decay these two amplitudes interfere constructively, and the steep increase from the threshold is reproduced. These threshold behaviors of the production amplitudes are shown to be consistent with the restriction from chiral symmetry. It is especially interesting that the double peak structure, with the bottom around the $\sigma$-peak position, of the spectra in $\Upsilon(3S \rightarrow 1S)$ decays is also reproduced well by the above interference between the direct $2\pi$ production amplitude with zero phase and the $\sigma$-production amplitude with a moving phase. That is, we are observing the very phase motion of the $\sigma$-Breit-Wigner formula through the variation of the spectra in this process.

The obtained values of masses and widths of $\sigma$-meson given above are almost consistent with the results obtained in our previous $\pi\pi$ phase shift analysis, $m_\sigma=535\sim675$MeV and $\Gamma_\sigma=385\pm70$MeV. These results give strong evidence for existence of the light $\sigma(450–600)$.

The consistency of our result of analyses with the general constraints from chiral symmetry is discussed in the separate talk.

In this conference, there was raised a doubt on the $\sigma$-existence along the line of the universality argument:

i) on $J/\psi \rightarrow \omega\pi\pi$ process. The experimental cos$\theta$-distributions show the parabolic shape around cos$\theta \approx 0$ in the range of $m_{\pi\pi}$, 300 through 700 MeV, coming from the interference between the $S$ and $D$ waves. However, the reverse of the direction of the parabolic shape, which is expected from the relative phase motion, to the background $D$-wave phase, of the $\sigma$-Breit-Wigner amplitude, was not observed. This criticism will become invalid, if we take into account the possible strong phase. For example, $\theta_{\sigma}=-90^\circ$.

ii) on $pp$-central collision, $pp \rightarrow pp\pi\pi$. The result of the partial wave analysis by WA102 shows that the non-resonant $\pi\pi$-production from one pion exchange is enough to reproduce the $S$- and $D$-wave components in low $m_{\pi\pi}$ region. We consider that it is necessary, in addition to this, to consider the effect of the $\sigma$-Breit-Wigner amplitude, since the relative phase between $S$- and $D$-wave amplitudes shows a structure around $m_{\pi\pi} \sim 500$MeV region, implying the interference between the non-resonant $r_{2\pi}e^{i\theta_{2\pi}}$ and $r_{\sigma}e^{i\theta_{\sigma}}$ in our formula Eq. (2.1). By the way, the experimental relative phase is completely different from that in the $\pi\pi$-scattering. This fact clearly shows the
universality-argument (where $\alpha = F/T$ is supposed to be real) is not correct.

iii) on $\pi\pi$-scattering. No rapid phase variation due to $\sigma(600)$ has been observed. We had mentioned in many occasions\(^{10}\) the reason why the $\sigma$-Breit-Wigner phase motion is not directly observed in $\pi\pi$-scattering: Because of the constraints from chiral symmetry the $\sigma$-amplitude must be strongly cancelled out by the non-resonant repulsive $\pi\pi$-amplitude in $\pi\pi$-scattering.

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Fig. 1. The result of the fit to the \( \pi\pi \) (or \( KK \)) mass spectra of (a) \( \Upsilon(2S) \rightarrow \Upsilon(1S)\pi\pi \), (b) \( \Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi \), (c) \( \Upsilon(3S) \rightarrow \Upsilon(2S)\pi\pi \), (d) \( \psi(2S) \rightarrow J/\psi\pi\pi \), (e) \( J/\psi \rightarrow \phi\pi\pi \), and (f) \( J/\psi \rightarrow \phi KK \).

The bold line represents the fit, while the dotted (dot-dashed) does the contribution of \( \sigma \)(direct \( 2\pi \))-production. By multiplying appropriate proportional factors, the different data sets are adjusted to the one with the largest number of events. For experimental references, see \(^9\).