Vortex Mass in BCS Systems: Kopnin and Baym-Chandler Contributions.

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PACS numbers: 67.57.Dd, 74.60.Gs

It is well known that in the BCS superfluids and superconductors the most important contribution to the vortex mass originates from the vortex core. The core mass in these systems is proportional to the area of the core $\alpha^2$, where $\alpha$ is the coherence length (see [1] for the vortex mass in superconductors and [2] for the vortex mass in superfluid $^3$He-B). This core mass is essentially larger than the logarithmically divergent contribution, which does not depend on the core pressibility. In the logarithmically divergent regime, the latter contains the speed of sound in the dense medium and is smaller by a factor $a=\frac{\alpha}{\ln \alpha}$, where $a$ is the interatomic distance. The core pressibility mass of the vortex core is in the superfluids, where the core size is small, $a$. A vortex, according to Kopnin, theory the core mass comes from the fermion trapped in the vortex core [3,4]. Recently the problem of another mass of the hydrodynamic origin was raised in Ref. [5]. It is the so-called back ow mass discussed by Baym and Chandler [3], which also can be proportional to the core area. Here we compare these two contributions in the superclean regime and at low $T_\text{c}$ using the model of the continuous core.

The continuous-core vortex is one of the best models which helps to solve many problems in the vortex core physics. Instead of consideration of the singular core, one can smoothen the $1/r$-singularity of the superfluid velocity by introducing the point gap nodes in the core region. As a result the superfluid/superconducting state in the vortex core of any system acquires the properties of the A-phase of superfluid $^3$He with its continuous vorticity and point gap nodes [6,7]. Using the continuous-core model one can show for example that the Kopnin (spectral core) force comes directly from the Adler-Bell-Jackiw chiral anomaly equation [8], and this shows the real origin of this anomalous force. In this model one can easily separate different contributions to the vortex mass. A actually this is not only the model: The spontaneous smoothing of the velocity singularity occurs in the core of both types of vortices observed in $^3$He-B [9] in heavy fermionic and high-$T_\text{c}$ superconductors such smoothening can occur due to admixture of di erent pairing states in the vortex core. It appears that both the Kopnin mass and the Baym-Chandler mass are related to the norm al componen. In general the norm al componen of the super uid liquid com es from two sources: (i) the local contribution, which comes from the system of quasiparticles, and (ii) the associated mass related to the back ow. The latter is dominated in porous materials, where some part of super uid com ponent is hydrodynamically trapped by the pores and thus is removed from the overall superuid motion. The normal com ponent, which gives rise to the vortex mass contains precisely the same two contributions. (i) The local contribution com es from the quasiparticles locali ed in the vortex core and thus moving with the core. This is the origin of the Kopnin mass according to Ref. [9]. (ii) The associated mass contribution arises because the pro le of the local density of the norm al com ponent in the vicinity of the vortex core disturbs the super ow around the vortex, when the vortex mass moves. This creates the back ow and thus some part of the super uid com ponent is trapped by the moving vortex, resulting in the Baym-Chandler mass of the vortex.

Let us consider this on the example of the simplest continuous-core vortex [8]. It has the following distribution of the unit vector $\hat{l}(r)$ which shows the direction of the point gap nodes in the smooth core

$$\hat{l}(r) = \cos \psi \hat{r} + \sin \psi \hat{z} \tag{1}$$

where $z; r; \psi$ are cylindrical coordinates. For super uid $^3$He the $3\text{-vector}$ in the smooth core changes from $\hat{l}(0) = \hat{z}$ to $\hat{l}(\infty) = \hat{r}$, which represents the doubly quantized continuous vortex. For the smoothly...
which gives for the Koppin mass an estimation:

$$M_{\text{Koppin}} = R_k \left( \frac{k_2}{r} = 4 \right) \left( \frac{k_1}{r} = 0 \right)$$

For the soft-core vortex the interlevel spacing is \( \Delta E = \frac{\hbar^2}{m R} \) which gives the Koppin mass \( M_{\text{Koppin}} \) where \( m \) is the mass density of the liquid.

For our purposes it is instructive to consider the normal component associated with the vortex as the local quantity, determined at each point in the vortex core. Such consideration is valid for the smooth core with the radius \( R \), where the local classical description of the Fermiionic spectrum can be applied. The main contribution comes from the point gap nodes, where the classical spectrum has the form:

$$E_0 = \frac{\hbar^2}{2m} \left( k^2 + \frac{\Delta^2}{4} \right)$$

and \( \Delta \) is the gap amplitude. In the presence of the gradient of \( \Delta \)-field, which acts on the quasiparticles as an effective magnetic field, this gapless spectrum leads to the nonzero local DOS and finally to the following local density of the normal component at \( T = 0 \) (see Eq. 5.24) in the review [13]):

$$\rho_{\text{local}}(r) = \frac{1}{2} \int d^2 r \sin^2 \left( \frac{\theta}{2} \right) R :$$

The integral of this normal density tensor over the cross section of the soft core gives the same Koppin mass of the vortex but in the local density representation [3]:

$$M_{\text{Koppin}} = \frac{1}{2} \int d^2 r \rho_{\text{local}}(r) \sin^2 \left( \frac{\theta}{2} \right) R :$$

Note that area law for the vortex mass is valid only for vortices with \( R \), but in general one has the linear law:

$$M_{\text{Koppin}} = R^2 :$$

which is to be added to the actual mass of the cylinder to obtain the total inertial mass of the body. In superfluids this part of superfluid component moves with external body and thus can be associated with the normal component. The similar mass is responsible for the normal component in porous materials and in aerogel, where some part of superfluid is hydrodynamically trapped by the pores. It is removed from the overall superfluid motion and thus becomes the part of the normal fluid.

In the case when the vortex is trapped by the wire, the Eq. (3) gives the vortex mass due to the back of the moving core. This is the simplest realization of the back of the vortex discussed.
by Baym and Chandler [6]. For such vortex with the wire core the Baym-Chandler mass is the dominating mass of the vortex. The Kopnin mass which can result from the non-al excitations trapped near the surface of the wire is essentially less.

Let us now consider the Baym-Chandler mass for the free vortex at \( T = 0 \) using again the continuous-core model. In the wire-core vortex this mass arises due to the back ow caused by the inhomogeneity of \( s : s (r > R) = 0 \) and \( s (r < R) \). Similar but less severe inhomogeneity of \( s = \frac{n}{R} \) occurs in the continuous-core vortex due to the nonzero local normal density in Eq. (6). Due to the profile of the local super uid density the external ow is disturbed near the core according to the continuity equation

\[
\dot{r} (v_s) = 0 ;
\]

If the smooth core is large, \( R \), the deviation of the super uid component in the smooth core from its asymptotic value outside the core is small: \( s = \frac{n}{R} \) and can be considered as perturbation. Thus if the asymptotic value of the velocity of the super uid component with respect to the core is \( v_{s0} = v_s \), the disturbance \( v_s = r \) of the super ow in the smooth core is given by:

\[
r^2 = v_s^2 = \frac{1}{v_{s0}^2} \cdot \frac{1}{n};
\]

The kinetic energy of the back ow gives the Baym-Chandler mass of the vortex

\[
M_{BC} = \frac{2}{v_{s0}^2} \cdot \frac{1}{n} ;
\]

In the simple approximation, when the normal component in Eq. (6) is considered as isotropic, one obtains

\[
M_{BC} = \frac{1}{2} \cdot \frac{2}{n} \cdot \frac{1}{v_{s0}^2} ;
\]

The Baym-Chandler mass does not depend on the core radius \( R \), since the large area \( R^2 \) of integration in Eq. (6) is compensated by small value of the normal component in the rared core, \( n = \frac{1}{R} \). That is why if the smooth core is large, \( R \), this mass is parametrically smaller than the Kopnin mass in Eq. (6).

In conclusion, both contributions to the mass of the vortex result from the mass of the normal component trapped by the vortex. The difference between Kopnin mass and Baym-Chandler back ow mass is only in the origin of the normal component trapped by the vortex. The relative importance of two masses depends on the vortex core structure: (1) For the free continuous vortex with the large core size \( R \), the Kopnin mass dominates: \( M_{BC} \approx R \), while the Baym-Chandler mass is proportional to the core area, \( M_{BC} \approx R^2 \), and

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