Light tetraquark state candidates

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Abstract

In this article, we study the axialvector-diquark-axialvector-antidiquark type scalar, axialvector, tensor and vector $ss\bar{s}\bar{s}$ tetraquark states with the QCD sum rules. The predicted mass $m_X = 2.08 \pm 0.12 \text{GeV}$ for the axialvector tetraquark state is in excellent agreement with the experimental value $2.052 \pm 0.042 \text{MeV}$ from the BESIII collaboration and supports assigning the new $X$ state to be a $ss\bar{s}\bar{s}$ tetraquark state with $J^{PC} = 1^{+-}$. The predicted mass $m_X = 3.08 \pm 0.11 \text{GeV}$ disfavors assigning the $\phi(2170)$ or $Y(2175)$ to be the vector partner of the new $X$ state. As a byproduct, we obtain the masses of the corresponding $qq\bar{q}\bar{q}$ tetraquark states. The light tetraquark states lie in the region about $2 \text{GeV}$ rather than $1 \text{GeV}$.

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1 Introduction

Recently, the BESIII collaboration studied the process $J/\psi \to \phi \eta \eta'$ and observed a structure $X$ in the $\phi \eta'$ mass spectrum [1]. The fitted mass and width are $m_X = 2.002.1 \pm 27.5.15 \text{MeV}$ and $\Gamma_X = (129 \pm 17 \pm 7) \text{MeV}$ respectively with assumption of the spin-parity $J^P = 1^-$, the corresponding significance is $5.3\sigma$; while the fitted mass and width are $m_X = (2062.8 \pm 13.1 \pm 4.2) \text{MeV}$ and $\Gamma_X = (177 \pm 36 \pm 20) \text{MeV}$ respectively with assumption of the spin-parity $J^P = 1^+$, the corresponding significance is $4.9\sigma$. The $X$ state was observed in the $\phi \eta'$ decay model rather than in the $\eta \eta$ decay model, they maybe contain a large $ss\bar{s}\bar{s}$ component, in other words, it maybe have a large tetraquark component. In Ref.[2], Wang, Luo and Liu assign the $X$ state to be the second radial excitation of the $h_1(1380)$. In Ref.[3], Cui et al assign the $X$ to be the partner of the tetraquark state $Y(2175)$ with the $J^{PC} = 1^{++}$.

We usually assign the lowest scalar nonet mesons $\{f_0(500), a_0(980), k_0(800), f_0(980)\}$ to be tetraquark states, and assign the higher scalar nonet mesons $\{f_0(1370), a_0(1450), K^*_0(1430), f_0(1500)\}$ to be the conventional $3P_0$ quark-antiquark states [4, 5, 6]. In Ref.[7], we take the nonet scalar mesons below $1 \text{GeV}$ as the two-quark-tetraquark mixed states and study their masses and pole residues with the QCD sum rules in details, and observe that the dominant components of the nonet scalar mesons below $1 \text{GeV}$ are conventional two-quark states. The light tetraquark states maybe lie in the region about $2 \text{GeV}$ rather than lie in the region about $1 \text{GeV}$.

In this article, we take the axialvector diquark operators as the basic constituents to construct the tetraquark current operators to study the scalar ($S$), axialvector ($A$), tensor ($T$) and vector ($V$) tetraquark states with the QCD sum rules, explore the possible assignments of the new $X$ state. We take the axialvector diquark operators as the basic constituents because the favored configurations from the QCD sum rules are the scalar and axialvector diquark states [8, 9], the current operators or quark structures chosen in the present work differ from that in Ref.[3] completely.

The article is arranged as follows: we derive the QCD sum rules for the masses and pole residues of the $ss\bar{s}\bar{s}$ tetraquark states in section 2; in section 3, we present the numerical results and discussions; section 4 is reserved for our conclusion.

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2 QCD sum rules for the $ss\bar{s}s$ tetraquark states

We write down the two-point correlation functions $\Pi_{\mu\nu\alpha\beta}(p)$ and $\Pi(p)$ firstly,

$$\Pi_{\mu\nu\alpha\beta}(p) = i \int d^4 x e^{ipx} \langle 0 | T \{ J_{\mu\nu}(x) J^\dagger_{\alpha\beta}(0) \} | 0 \rangle,$$

$$\Pi(p) = i \int d^4 x e^{ipx} \langle 0 | T \{ J_0(x) J_0^\dagger(0) \} | 0 \rangle,$$

where $J_{\mu\nu}(x) = J_{2,\mu\nu}(x)$, $J_{1,\mu\nu}(x)$,

$$J_{2,\mu\nu}(x) = \frac{\epsilon^{ijk} \epsilon^{imn}}{\sqrt{2}} \left\{ s^{Tj}(x) C_{\gamma\mu} s^k(x) \hat{s}^m(x) \gamma_\nu C s^{Tn}(x) + s^{Tj}(x) C_{\gamma\mu} s^k(x) \hat{s}^m(x) \gamma_\mu C s^{Tn}(x) \right\},$$

$$J_{1,\mu\nu}(x) = \frac{\epsilon^{ijk} \epsilon^{imn}}{\sqrt{2}} \left\{ s^{Tj}(x) C_{\gamma\mu} s^k(x) \hat{s}^m(x) \gamma_\nu C s^{Tn}(x) - s^{Tj}(x) C_{\gamma\mu} s^k(x) \hat{s}^m(x) \gamma_\mu C s^{Tn}(x) \right\},$$

$$J_0(x) = \frac{\epsilon^{ijk} \epsilon^{imn}}{\sqrt{2}} s^{Tj}(x) C_{\gamma\mu} s^k(x) \hat{s}^m(x) \gamma_\nu C s^{Tn}(x),$$

where the $i, j, k, m, n$ are color indexes, the $C$ is the charge conjugation matrix. Under charge conjugation transform $\hat{C}$, the currents $J_{\mu\nu}(x)$ and $J_0(x)$ have the properties,

$$\hat{C} J_{2,\mu\nu}(x) \hat{C}^{-1} = + J_{2,\mu\nu}(x),$$

$$\hat{C} J_{1,\mu\nu}(x) \hat{C}^{-1} = - J_{1,\mu\nu}(x),$$

$$\hat{C} J_0(x) \hat{C}^{-1} = + J_0(x).$$

At the hadronic side, we can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J_{\mu\nu}(x)$ and $J_0(x)$ into the correlation functions $\Pi_{\mu\nu\alpha\beta}(p)$ and $\Pi(p)$ to obtain the hadronic representation $[10][11]$. After isolating the ground state contributions of the scalar, axialvector, vector and tensor tetraquark states, we get the results,

$$\Pi_{2,\mu\nu\alpha\beta}(p) = \frac{\lambda^2_X}{m^2_{X_T} - p^2} \left( \frac{\tilde{g}_{\mu\nu} \tilde{g}_{\alpha\beta} \tilde{g}_{\mu\nu} \tilde{g}_{\alpha\beta}}{2} - \frac{\tilde{g}_{\mu\nu} \tilde{g}_{\alpha\beta}}{3} \right) + \cdots,$$

$$\Pi_{1,\mu\nu\alpha\beta}(p) = \frac{\tilde{\lambda}^2_{X_A}}{m^2_{X_A} - p^2} (p^2 g_{\mu\nu} g_{\alpha\beta} - p^2 g_{\mu\alpha} g_{\nu\beta} - g_{\mu\alpha} p_{\nu\beta} - g_{\nu\beta} p_{\mu\alpha} + g_{\nu\beta} p_{\mu\alpha} + g_{\mu\alpha} p_{\nu\beta})$$

$$+ \frac{\tilde{\lambda}^2_{X_V}}{m^2_{X_V} - p^2} (-g_{\mu\alpha} p_{\nu\beta} - g_{\nu\beta} p_{\mu\alpha} + g_{\nu\beta} p_{\mu\alpha} + g_{\nu\beta} p_{\mu\alpha} + g_{\nu\beta} p_{\mu\alpha} + g_{\nu\beta} p_{\mu\alpha}) + \cdots$$

$$\Pi_1(p^2) = \Pi_2(p^2) = \frac{\lambda^2_{X_T}}{m^2_{X_T} - p^2} + \cdots,$$

$$\Pi_0(p^2) = \Pi_0^+(p^2) = \frac{\lambda^2_{X_T}}{m^2_{X_T} - p^2} + \cdots,$$

where $\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{p_{\mu} p_{\nu}}{p^2}$, the subscripts $2^+$, $1^+$, $1^-$ and $0^+$ denote the spin-parity $J^P$ of the corresponding tetraquark states. The pole residues $\lambda_X$ and $\tilde{\lambda}_X$ are defined by

$$\langle 0 | J_{2,\mu\nu}(0) | X_T(p) \rangle = \lambda_{X_T} \epsilon_{\mu\nu},$$

$$\langle 0 | J_{1,\mu\nu}(0) | X_A(p) \rangle = \tilde{\lambda}_{X_A} \epsilon_{\mu\nu\alpha\beta} \epsilon^\alpha p^\beta,$$

$$\langle 0 | J_{1,\mu\nu}(0) | X_V(p) \rangle = \tilde{\lambda}_{X_V} (\epsilon_{\mu p^\nu} - \epsilon_{\nu p^\mu}),$$

$$\langle 0 | J_0(0) | X_S(p) \rangle = \lambda_{X_S},$$

(8)
where the $\varepsilon_{\mu\nu}$ and $\varepsilon_\mu$ are the polarization vectors of the tetraquark states.

Now we contract the $s$ quarks in the correlation functions with Wick theorem, there are four $s$-quark propagators, if two $s$-quark lines emit a gluon by itself and the other two $s$-quark lines contribute a quark pair by itself, we obtain a operator $GG\bar{s}s\bar{s}$, which is of order $O(\alpha_s^k)$ with $k = 1$ and of dimension 10. In this article, we take into account the vacuum condensates up to dimension 10 and $k \leq 1$ in a consistent way. For the technical details, one can consult Refs.\[12]. Once the analytical expressions of the QCD spectral densities are obtained, we take the quark-hadron duality below the continuum thresholds $s_0$ and perform Borel transform with respect to the variable $P^2 = -p^2$ to obtain the QCD sum rules:

$$\lambda_X^2 \exp\left(-\frac{m_X^2}{T^2}\right) = \int_{0}^{s_0} ds \rho(s) \exp\left(-\frac{s}{T^2}\right),$$ (9)

where $\rho(s) = \rho_S(s), \rho_A(s), \rho_V(s)$ and $\rho_T(s)$,

$$\rho_S(s) = \frac{s^4}{384\pi^6} - \frac{13s m_s \langle \bar{s}g_s\sigma Gs \rangle}{384\pi^4} + \frac{2s \langle \bar{s}s \rangle^2}{3\pi^2} - \frac{17\langle \bar{s}s \rangle \langle \bar{s}\bar{s}\sigma Gs \rangle}{48\pi^2} + \frac{s^2 \langle \alpha_s GG \rangle}{192\pi^4} \langle \bar{s}s \rangle \delta(s) + \frac{19m_s \langle \bar{s}s \rangle}{96\pi^2} \langle \alpha_s GG \rangle \delta(s) + \frac{16m_s \langle \bar{s}s \rangle^3}{3} \langle \bar{s}s \rangle \delta(s) - \frac{(\bar{s}s)^2 \langle \alpha_s GG \rangle}{24} \delta(s),$$ (10)

$$\rho_A(s) = \frac{s^4}{1152\pi^6} - \frac{s^2 m_s \langle \bar{s}s \rangle}{12\pi^4} + \frac{s m_s \langle \bar{s}g_s\sigma Gs \rangle}{9\pi^4} + \frac{4s \langle \bar{s}s \rangle^2}{9\pi^2} - \frac{5\langle \bar{s}s \rangle \langle \bar{s}\bar{s}\sigma Gs \rangle}{18\pi^2} - \frac{s^2 \langle \alpha_s GG \rangle}{2304\pi^4} \langle \bar{s}s \rangle \delta(s) + \frac{3m_s \langle \bar{s}s \rangle}{64\pi^2} \langle \alpha_s GG \rangle \delta(s) - \frac{16m_s \langle \bar{s}s \rangle^3}{9} \langle \bar{s}s \rangle \delta(s) - \frac{2\langle \bar{s}s \rangle^2 \langle \alpha_s GG \rangle}{27} \delta(s),$$ (11)

$$\rho_V(s) = \frac{s^4}{1152\pi^6} + \frac{s^2 m_s \langle \bar{s}s \rangle}{12\pi^4} - \frac{7s m_s \langle \bar{s}g_s\sigma Gs \rangle}{72\pi^4} - \frac{2s \langle \bar{s}s \rangle^2}{9\pi^2} + \frac{5\langle \bar{s}s \rangle \langle \bar{s}\bar{s}\sigma Gs \rangle}{18\pi^2} + \frac{s^2 \langle \alpha_s GG \rangle}{768\pi^4} - \frac{79m_s \langle \bar{s}s \rangle}{1728\pi^2} \langle \alpha_s GG \rangle + \frac{16m_s \langle \bar{s}s \rangle^3}{9} \delta(s) - \frac{2\langle \bar{s}s \rangle^2 \langle \alpha_s GG \rangle}{81} \langle \bar{s}s \rangle \delta(s) - \frac{\langle \bar{s}s \rangle \langle \bar{s}s \rangle^2}{18\pi^2} \delta(s),$$ (12)

$$\rho_T(s) = \frac{s^4}{576\pi^6} - \frac{3s^2 m_s \langle \bar{s}s \rangle}{20\pi^4} + \frac{29s m_s \langle \bar{s}g_s\sigma Gs \rangle}{96\pi^4} - \frac{8s \langle \bar{s}s \rangle^2}{9\pi^2} - \frac{37\langle \bar{s}s \rangle \langle \bar{s}\bar{s}\sigma Gs \rangle}{48\pi^2} - \frac{11s^2 \langle \alpha_s GG \rangle}{1920\pi^4} + \frac{43m_s \langle \bar{s}s \rangle}{864\pi^2} \langle \alpha_s GG \rangle - \frac{64m_s \langle \bar{s}s \rangle^3}{9} \langle \bar{s}s \rangle \delta(s) - \frac{4\langle \bar{s}s \rangle^2 \langle \alpha_s GG \rangle}{27} \langle \bar{s}s \rangle \delta(s),$$ (13)

and $\lambda_{X_{A/V}} = m_{X_{A/V}}\lambda_{X_{A/V}}$.

We derive Eq.(9) with respect to $\tau = \frac{1}{T}$, then obtain the QCD sum rules for the masses of the tetraquark states through a fraction,

$$m_X^2 = -\int_{0}^{s_0} ds \frac{d}{ds} \rho(s) \exp(-\tau s) \int_{s_0}^{s} d\tau \rho(s) \exp(-\tau s).$$ (14)

3 Numerical results and discussions

We take the standard values of the vacuum condensates $\langle q\bar{q} \rangle = -(0.24 \pm 0.01 \text{ GeV})^3$, $\langle \bar{q}g_s\sigma Gq \rangle = m_0^2 \langle \bar{q}q \rangle$, $m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2$, $\langle \bar{s}s \rangle = (0.8 \pm 0.1) \langle \bar{q}q \rangle$, $\langle \bar{s}g_s\sigma Gs \rangle = m_0^2 \langle \bar{s}s \rangle$, $\langle \frac{\bar{s}s}{\pi} \rangle = (0.33 \text{ GeV})^4$.
Table 1: The Borel parameters, continuum threshold parameters, pole contributions, contributions of the vacuum condensates of dimension 10, masses and pole residues of the tetraquark states, where the subscripts S, A, T and V denote the scalar, axialvector, tensor and vector tetraquark states, respectively.

|       | $T^2$(GeV$^2$) | $\sqrt{s_0}$(GeV) | pole | $|D(10)|$ | $m_X$(GeV) | $\lambda_X$(10$^{-2}$GeV$^3$) |
|-------|---------------|-----------------|------|---------|-----------|-----------------
| sssS  | 1.4 - 1.8     | 2.65 ± 0.10     | (40 - 73)%| ≪ 1%  | 2.08 ± 0.13 | 2.73 ± 0.55    |
| sssA  | 1.5 - 1.9     | 2.65 ± 0.10     | (41 - 72)%| < 1%   | 2.08 ± 0.12 | 1.87 ± 0.34    |
| sssT  | 1.5 - 1.9     | 2.75 ± 0.10     | (41 - 72)%| < 1%   | 2.22 ± 0.11 | 3.02 ± 0.53    |
| sssV  | 2.1 - 2.7     | 3.60 ± 0.10     | (42 - 73)%| ≤ 1%   | 3.08 ± 0.11 | 6.47 ± 1.07    |
| qqqS  | 1.2 - 1.6     | 2.40 ± 0.10     | (40 - 76)%| ≪ 1%  | 1.86 ± 0.11 | 1.95 ± 0.37    |
| qqqA  | 1.3 - 1.7     | 2.40 ± 0.10     | (40 - 73)%| ≤ 1%   | 1.87 ± 0.10 | 1.30 ± 0.22    |
| qqqT  | 1.4 - 1.8     | 2.65 ± 0.10     | (42 - 74)%| ≤ 1%   | 2.13 ± 0.10 | 2.58 ± 0.41    |
| qqqV  | 1.9 - 2.5     | 3.40 ± 0.10     | (41 - 74)%| ≤ 2%   | 2.86 ± 0.11 | 4.94 ± 0.92    |

at the energy scale $\mu = 1$ GeV [10, 11, 13], and choose the $\overline{MS}$ mass $m_s(\mu = 2$ GeV) = 0.095 ± 0.005 GeV from the Particle Data Group [14], and evolve the $s$-quark mass to the energy scale $\mu = 1$ GeV with the renormalization group equation, furthermore, we neglect the small $u$ and $d$ quark masses.

We choose suitable Borel parameters and continuum threshold parameters to warrant the pole contributions (PC) are larger than 40%, i.e.

$$ PC = \int_{0}^{s_0} ds \rho(s) \exp\left(-\frac{s}{\lambda_X}\right) \geq 40\% , $$ (15)

and convergence of the operator product expansion. The contributions of the vacuum condensates $D(n)$ in the operator product expansion are defined by,

$$ D(n) = \int_{0}^{s_0} ds \rho_n(s) \exp\left(-\frac{s}{\lambda_X}\right) , $$ (16)

where the subscript $n$ in the QCD spectral density $\rho_n(s)$ denotes the dimension of the vacuum condensates. We choose the values $|D(10)| \sim 1%$ to warrant the convergence of the operator product expansion. In Table 1, we present the ideal Borel parameters, continuum threshold parameters, pole contributions and contributions of the vacuum condensates of dimension 10. From the Table, we can see that the pole dominance is well satisfied and the operator product expansion is well convergent, we expect to make reliable predictions.

We take into account all uncertainties of the input parameters, and obtain the values of the masses and pole residues of the $ss\bar{s}s$ tetraquark states, which are shown explicitly in Fig.1 and Table 1. In this article, we have assumed that the energy gaps between the ground state and the first radial state is about 0.6 GeV [15]. In Fig.1, we plot the masses of the scalar, axialvector, tensor and vector $ss\bar{s}s$ tetraquark states with variations of the Borel parameters at larger regions than the Borel windows shown in Table 1. From the figure, we can see that there appear platforms in the Borel windows.

The predicted mass $m_X = 2.08 \pm 0.12$ GeV for the axialvector tetraquark state is in excellent agreement with the experimental value (2062.8 ± 13.1 ± 4.2) MeV from the BESIII collaboration [1], which supports assigning the new $X$ state to be an axialvector-diquark-axialvector-antidiquark type $ss\bar{s}s$ tetraquark state. The predicted mass $m_X = 3.08 \pm 0.11$ GeV for the vector tetraquark state lies above the experimental value of the mass of the $\phi(2170)$ or $Y(2175)$, $m_\phi = 2188 \pm 10$ MeV, from the Particle Data Group, and disfavors assigning the $\phi(2170)$ or $Y(2175)$ to be vector partner of the new $X$ state. If the $\phi(2170)$ have tetraquark component, it maybe have color octet-octet
Figure 1: The masses with variations of the Borel parameters $T^2$, where the $A$, $B$, $C$ and $D$ denote the scalar, axialvector, tensor and vector tetraquark states, respectively.
component \[10\]. As a byproduct, we obtain the masses and pole residues of the corresponding $qq\bar{q}\bar{q}$
tetraquark states, which are shown in Table 1. The present predictions can be confronted to the
experimental data in the future.

Now we perform Fierz rearrangement to the currents both in the color and Dirac-spinor spaces,

\[
J_0 = 2\bar{s}s + 2\bar{s}i\gamma_5s\bar{s}\gamma_5s + \bar{s}\gamma_\alpha s\bar{s}\gamma^\alpha s - \bar{s}\gamma_\alpha\gamma_5s\bar{s}\gamma^\alpha\gamma_5s,
\]

\[
J_{1,\mu\nu} = \sqrt{2}\left\{ i\bar{s}s\bar{s}_\mu\gamma_5s - \bar{s}_\mu\gamma_5s\bar{s}_\nu + ig_{\mu\nu\alpha\beta}\bar{s}\gamma_\alpha\gamma_5s\bar{s}\gamma^\beta s \right\},
\]

\[
J_{2,\mu\nu} = \frac{1}{\sqrt{2}}\left\{ 2\bar{s}\gamma_5s\bar{s}_\mu\gamma_5s - 2\bar{s}_\mu\gamma_5s\bar{s}_\nu + 2g^{\alpha\beta}\bar{s}_\mu\gamma_\alpha s\bar{s}_\nu + g_{\mu\nu}(\bar{s}s\bar{s}s - \frac{1}{2}\bar{s}\gamma_5s\bar{s} - \bar{s}_{\alpha\beta}\bar{s}_{\alpha\beta}s) \right\}.
\]

The diquark-antidiquark type currents can be re-arranged into currents as special superpositions of
color singlet-singlet type currents, which couple potentially to the meson-meson pairs or molecular
states, the diquark-antidiquark type tetraquark states can be taken as special superpositions of
meson-meson pairs, and embodies the net effects. The decays to their components are Okubo-
Zweig-Iizuka supper-allowed, we can search for those tetraquark states in the decays,

\[
X_S \rightarrow \eta'/\eta', \ f_0(980)f_0(980), \ \phi(1020)\phi(1020),
\]

\[
X_{A/V} \rightarrow f_0(980)h_1(1380), \ \phi(1020)\eta', \ \phi(1020)\phi(1020),
\]

\[
X_T \rightarrow \eta'/\eta', \ f_0(980)f_0(980), \ \phi(1020)\phi(1020).
\]

4 Conclusion

In this article, we construct the axialvector-diquark-axialvector-antidiquark type currents to inter-
polate the scalar, axialvector, tensor and vector $ss\bar{s}\bar{s}$ tetraquark states, then calculate the con-
tributions of the vacuum condensates up to dimension-10 in the operator product expansion, and
obtain the QCD sum rules for the masses and pole residues of those tetraquark states. The pre-
dicted mass $m_X = 2.08 \pm 0.12$ GeV for the axialvector tetraquark state is in excellent agreement
with the experimental value, $m_X = (2062.8 \pm 13.1 \pm 4.2)$ MeV, from the BESIII collaboration and
supports assigning the new X state to be an axialvector-diquark-axialvector-antidiquark type $ss\bar{s}\bar{s}$
tetraquark state. The predicted mass $m_X = 3.08 \pm 0.11$ GeV for the vector tetraquark state lies
above the experimental value of the mass of the $\phi(2170)$, $m_\phi = 2188 \pm 10$ MeV, from the Particle
Data Group, and disfavors assigning the $\phi(2170)$ to be the vector partner of the new X state. As
a byproduct, we also obtain the masses and pole residues of the corresponding $qq\bar{q}\bar{q}$
tetraquark states. The present predictions can be confronted to the experimental data in the future.

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