High-multiplicity final states and transverse-momentum dependent parton showering at hadron colliders

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If large-angle multigluon radiation gives significant contributions to parton showers in the LHC high-energy region, appropriate generalizations of parton branching methods are required for Monte Carlo simulations of exclusive high-multiplicity final states. We discuss the use in this context of transverse-momentum dependent kernels which factorize in the region of high energies. We give examples based on $ep$ and $p\bar{p}$ multi-jet data, and point to possible developments for distributions associated with massive final states at the LHC.

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1 Introduction

Complex final states with high particle multiplicity are central to many aspects of the LHC physics program. Theoretical predictions for these processes require advanced QCD calculational tools, which rely both on perturbative results (at present, mostly next-to-leading-order) \cite{1} and on parton shower event generators for realistic collider simulations \cite{2}.

This article discusses aspects of spacelike parton showers that depend on the structure of QCD multiparton matrix elements in the multiple-scale region of large center-of-mass energy $\sqrt{s}$ and fixed transferred momenta, and are likely to affect the form of the final states at high multiplicity.

Let us recall that the physical picture underlying the most commonly used branching Monte Carlo generators \cite{2, 3} is based on collinear evolution of jets developing, both “forwards” and “backwards”, from the hard event \cite{4}, supplemented (in the case of certain generators) by suitable constraints for angularly-ordered phase space \cite{5}. The angular constraints are designed to take account of coherence effects from multiple soft-gluon emission \cite{5, 6, 7}.

The main new effect one observes when trying to push this picture to higher and higher energies is that soft-gluon insertion rules \cite{6, 7} based on eikonal emission currents \cite{8, 9} are modified in the high-energy, multi-scale region by terms that depend on the total transverse momentum transmitted down the initial-state parton decay chain \cite{10, 11, 12}. As a result, the physically relevant distribution to describe initial-state showers becomes the analogue not so much of an ordinary parton density but rather of an “unintegrated” parton density, dependent on both longitudinal and transverse momenta.\footnote{Theoretical aspects of unintegrated pdfs from the point of view of QCD high-energy factorization are discussed in \cite{13}. Associated phenomenological aspects are discussed in \cite{14, 15}, and references therein (see also \cite{16, 17} for recent new work). See works in \cite{18} for first discussions of a more general, nonlocal operator formulation of u-pdfs applied to parton showers beyond leading order.}

The next observation concerns the structure of virtual corrections. Besides Sudakov form-factor effects included in standard shower algorithms \cite{2, 3}, one needs in general virtual-graph...
terms to be incorporated in transverse-momentum dependent (but universal) splitting functions \[10, 18, 19, 20, 21\] in order to take account of gluon coherence not only for collinear-ordered emissions but also in the non-ordered region that opens up at high $\sqrt{s}/p_\perp$.

These finite-$k_\perp$ corrections to parton branching have important implications for multiplicity distributions and the structure of angular correlations in final states with high multiplicity. In the next section we discuss examples of such effects in the case of multi-jet production in $ep$ and $p\bar{p}$ collisions. In Sec. 3 we go on to possible developments involving the hadroproduction of massive states. In particular, we point to studies beginning to investigate the role of showering corrections versus multiparton interaction corrections in Monte Carlo event generators \[22, 23, 24\]. We give final remarks in Sec. 4.

2 Jet-jet correlations

In a multi-jet event the correlation in the azimuthal angle $\Delta \phi$ between the two hardest jets provides a useful measurement, sensitive to how well QCD multiple-radiation effects are described. In leading order one expects two back-to-back jets; higher-order radiative contributions cause the $\Delta \phi$ distribution to spread out. Near $\Delta \phi \sim \pi$ the measurement is mostly sensitive to infrared effects from soft-gluon emission; the behavior as $\Delta \phi$ decreases is driven by hard parton radiation. At the LHC such measurements may become accessible relatively early and be used to test the description of complex hadronic final states by Monte Carlo generators \[25\].

Experimental data on $\Delta \phi$ correlations are available from the Tevatron \[26\] and from Hera \[27, 28\]. These analyses indicate that the comparison of data with Monte Carlos and perturbative results are very different in the two cases. Observe in Fig. 1 that the Tevatron $\Delta \phi$ distribution drops by about two orders of magnitude over a fairly narrow range, essentially still close to the two-jet region. The measurement is dominated by leading-order processes, with small sub-leading corrections. Correspondingly, data are reasonably well described both by collinear showers (HERWIG and new PYTHIA tuning) and by fixed-order NLO calculations \[25, 26\].

The Hera $\Delta \phi$ measurements, on the other hand, are much more sensitive to higher orders, Fig. 2 \[27\]. NLO results for di-jet azimuthal distributions are affected by large corrections in the small-$\Delta \phi$ and small-$x$ region, and begin to fall below the data for three-jet distributions in the smallest $\Delta \phi$ bins \[27\]. These measurements are likely relevant for extrapolation of initial-state showering effects to the LHC, given the large phase space available for jet production, and relatively small ratio of jet transverse energy to the center-of-mass energy.

Refs. \[29, 30\] analyze the effects of finite-$k_\perp$ corrections to initial-state showers, using data \[27\] on jet angular and momentum correlations, and factorization at fixed transverse momentum \[11\] valid for high energies. Fig. 3 shows results from the collinear HERWIG Monte Carlo \[31\] and from the $k_\perp$-shower CASCADE Monte Carlo \[15\] for the distributions in $\Delta \phi$ and $\Delta \rho_t$ \[27, 29\], measuring the transverse momentum imbalance between the leading jets. The largest differences between the two Monte Carlos are at small $\Delta \phi$ and small $\Delta \rho_t$, where the two highest $E_T$ jets are away from the back to back region and one has effectively three hard, well-separated jets. By examining the angular distribution of the third jet, Ref. \[29\] finds significant contributions from regions where the transverse momenta in the initial state shower are not ordered. The description of the measurement by the $k_\perp$-shower is good, whereas the collinear-based HERWIG shower is not sufficient to describe the observed shape.

The physical picture underlying the $k_\perp$-shower method involves both transverse-momentum dependent pdfs and matrix elements. Fig. 4 \[30\] illustrates the relative contribution of these
different components to the result, showing different approximations to the azimuthal dijet distribution normalized to the back-to-back cross section. The solid red curve is the full result \cite{29}. The dashed blue curve is obtained from the same unintegrated pdf’s but by taking the collinear approximation in the hard matrix element. The dashed curve drops much faster than the full result as $\Delta \phi$ decreases, indicating that the high-$k_T$ component in the ME \cite{20} is necessary to describe jet correlations for small $\Delta \phi$. The dotted (violet) curve is the result obtained from the unintegrated pdf without any resolved branching. This represents the contribution of the intrinsic distribution only, corresponding to nonperturbative, predominantly low-$k_T$ modes. That is, in the dotted (violet) curve one retains an intrinsic $k_T \neq 0$ but no effects of coherence. We see that the resulting jet correlations in this case are down by an order of magnitude. The

![Figure 1: Dijet azimuthal correlations measured by D0 along with the HERWIG and PYTHIA results \cite{26}.

Figure 2: (left) Azimuth dependence and (right) Bjorken-$x$ dependence of ep di-jet distributions \cite{27}, compared with NLO results.

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Figure 3: (left) Angular correlations and (right) momentum correlations \[29\] in three-jet final states measured by \[27\], compared with $k_T$-shower (CASCADE) and collinear-shower (HERWIG) Monte Carlo results.

Inclusion of the perturbatively computed high-$k_T$ correction distinguishes the calculation \[29\] from other shower approaches (see e.g. \[32\]) that include transverse momentum dependence in the pdfs but not in the matrix elements.

Figure 4: The dijet azimuthal distribution \[30\] normalized to the back-to-back cross section: (solid red) full result (u-pdf $\oplus$ ME); (dashed blue) no finite-$k_T$ correction in ME (u-pdf $\oplus$ \[ME_{collin.}\]); (dotted violet) u-pdf with no resolved branching.

The above observations underline the role of accurate multi-jet measurements in events associated with proton scattering off virtual photons \[16\]. Further phenomenological analyses of available jet correlation data (and multiplicity distributions \[29\]) will be helpful. In this respect note that the effects of coherence emphasized above dominate for sufficiently small $\Delta \phi$ and small $x$. But the unintegrated formulation of parton showers is potentially more general \[18, 33, 34\]. Such analyses can be of use in attempts to relate \[35\] shower effects in DIS.
3 Massive final states

Corrections to collinear-ordered showers affect heavy mass production, including the structure of the final states associated with heavy flavor and heavy boson production. We next point to examples that depend on the physics of unintegrated gluon distributions.

Measurements of angular correlations for bottom quark jets have recently been performed at the Tevatron [38, 39, 40]. See [16, 41, 42, 43] for reviews of related phenomenology. Results for b-jet distributions in invariant mass and azimuthal angle are shown in Fig. 5 [39] and Fig. 6 [40]. Monte Carlo simulations based on PYTHIA, HERWIG and MC@NLO do not appear to give satisfactory descriptions of the observations [2, 39] especially at small ∆φ. The measurement of b-jet correlations has considerable interest, given their potential sensitivity to soft underlying events [39, 44] and possibly models for multiple-parton interactions [22, 23]. In this context it is worth noting the possible role of showering corrections, at the level of single-parton interactions, due to transverse-momentum dependent parton branching.

![Figure 5: Invariant-mass distribution and azimuthal-angle distribution for production of b-jets at the Tevatron.](image)

To this end let us recall that heavy flavor hadroproduction is dominated for sufficiently high energies by gluon splitting into heavy-quark pairs \( g \rightarrow QQ \) where \( g \) is produced from the spacelike jet. The high-energy asymptotic behavior is controlled by a triple-pole singularity [13] in the complex plane of the Mellin moment conjugate to the transferred \( k_1 \).

The coefficient functions associated to this singularity enhance regions that are not ordered in transverse momentum in the initial state shower. In fact, such contributions are already found to be significant at the level of the NLO correction [42, 43].

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2This is unlike the DIS jet correlation data discussed in the previous section, where multiparton contributions are believed to be much suppressed [14].

3It is possible that terms of this kind at orders higher than NLO are responsible for the rather large theoretical uncertainties found [44, 45] in the NLO predictions when going from the Tevatron to the LHC.
A typical contribution to \( g \rightarrow Q\bar{Q} \) is pictured in Fig. 7a. Note that for small \( \Delta \phi \) this graph gives effectively a contribution of leading order. Corrections of the next order from additional jet emission are shown in Fig. 7b. In the notation of Fig. 7, the triple-pole behavior is produced from regions in which \( m_Q^2 \ll (k_T + k'_T)^2 \ll k_T^2 \approx k'_T^2 \), where \( m_Q \) is the heavy quark mass, and \( k_T, k'_T \) are transverse momentum vectors. Collinear shower calculations, even if supplemented by NLO matrix elements as in MC@NLO, are not designed to take account of this behavior. This is likely to reduce the numerical stability of predictions as one goes to higher and higher energies. It may cause a non-negligible contribution from showering to be missed in the \( b \)-jet \( \Delta \phi \) distribution at small \( \Delta \phi \). On the other hand, such corrections can be obtained by methods based on transverse-momentum dependent parton branching, as those discussed in the previous section. It is of interest to analyze these contributions in comparison with those, e.g. in Fig. 6, from multiple interactions.

| Data/MC |
|---------|
| 0       |
| 1       |
| 2       |
| 3       |

\[ \Delta \phi \text{ (rad)} \]

Figure 6: Comparison of data and Monte Carlos for \( b \)-jet azimuthal correlations at the Tevatron [41].

\[ \frac{1.96 \text{ TeV}, L = 260 \text{ pb}^{-1}}{\text{CDF Run II Preliminary}} \]

\[ \Delta \phi > 32 \text{ GeV}, E_T > 35 \text{ GeV} \]

\[ \eta > 0.4, |\text{cone} - 1.2| < 1.2 \]

| Data/MC |
|---------|
| 0       |
| 1       |
| 2       |
| 3       |

\[ \Delta \phi \text{ (rad)} \]

| Data/MC |
|---------|
| 0       |
| 1       |
| 2       |
| 3       |

\[ \Delta \phi \text{ (rad)} \]

Figure 7: (a) Heavy quark hadroproduction from gluon showering at high energy; (b) next correction from extra jet emission.

Such an analysis would pay off as one goes from the Tevatron to the LHC. Parton showers

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at the LHC will be more influenced by the asymptotic high-energy pole. Let us observe that effects of a similar physical origin will affect the structure of final states associated to production processes predominantly coupled to gluons, e.g. central scalar boson production \cite{45}. See studies of showering effects in this case \cite{24}. These can affect the description of soft underlying events and mini-jets \cite{44, 22} as well as the use of exclusive production channels \cite{16}.

4 Concluding remarks

Final states with high particle multiplicity acquire qualitatively new features at the LHC compared to previous collider experiments due to the large phase space opening up for events characterized by multiple hard scales, possibly widely disparate from each other. This brings in potentially large perturbative corrections to the hard-scattering event and potentially new effects in the parton-shower components of the process.

If large-angle multigluon radiation gives significant contributions to parton showers at the LHC, appropriate generalizations of parton branching methods are required. We have discussed applications of transverse-momentum dependent kernels for parton showering that follow from factorization properties of QCD multiparton matrix elements in the high energy region, which are valid not only for collinear emission but also at finite angles.

While we have focused on observables that are sensitive primarily to the physics of initial-state gluonic showers, expressible in terms of “unintegrated” gluon densities, treatments of quark contributions to showers at unintegrated level are being worked on (see e.g. \cite{16, 17}). In this respect, theoretical results for splitting kernels \cite{20} already applied to inclusive phenomenology could also be of use in calculations for exclusive final states.

We have considered jet and heavy-mass production processes in the central rapidity region. Techniques are being developed \cite{47} to allow one to also address multi-particle hard processes at forward rapidities.

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