Four-Dimensional Effective Supergravity and Soft Terms in 

$M$–Theory

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Abstract

We provide a simple macroscopic analysis of the four-dimensional effective supergravity of the Hořava-Witten $M$-theory which is expanded in powers of $\kappa^2/\rho V^{1/3}$ and $\kappa^2 \rho/V^{2/3}$ where $\kappa^2$, $V$ and $\rho$ denote the eleven-dimensional gravitational coupling, the Calabi-Yau volume and the eleventh length respectively. Possible higher order terms in the Kähler potential are identified and matched with the heterotic string corrections. In the context of this $M$-theory expansion, we analyze the soft supersymmetry-breaking terms under the assumption that supersymmetry is spontaneously broken by the auxiliary components of the bulk moduli superfields. It is examined how the pattern of soft terms changes when one moves from the weakly coupled heterotic string limit to the $M$-theory limit.
Recently Hořava and Witten proposed that the strong coupling limit of $E_8 \times E_8$ heterotic string theory can be described by 11-dimensional supergravity (SUGRA) on a manifold with boundary where the two $E_8$ gauge multiplets are restricted to the two 10-dimensional boundaries respectively ($M$-theory). The effective action of this limit has been systematically analyzed in an expansion in powers of $\kappa^{2/3}$ where $\kappa^2$ denotes the 11-dimensional gravitational coupling. At zeroth order, the effective action is simply that of the 11-dimensional supergravity which includes only the bulk fields whose kinetic terms are of order $\kappa^{-2}$. However at first order in the expansion, there appear a variety of additional terms including the 10-dimensional boundary action of the $E_8 \times E_8$ gauge multiplets which is of order $\kappa^{-4/3}$. It was also noted that a four-gaugino term appears at higher order which would lead to an interesting phenomenological consequence when supersymmetry (SUSY) is broken by the gaugino condensation on the hidden boundary.

Some phenomenological implications of the strong-coupling limit of $E_8 \times E_8$ heterotic string theory has been studied by compactifying the 11-dimensional $M$-theory on a Calabi-Yau manifold times the eleventh segment. The resulting 4-dimensional effective theory can reconcile the observed Planck scale $M_P = 1/\sqrt{8\pi G_N} \approx 2.4 \times 10^{18}$ GeV with the phenomenologically favored GUT scale $M_{GUT} \approx 3 \times 10^{16}$ GeV in a natural manner, providing an attractive framework for the unification of couplings. In this framework, $M_{GUT}$ corresponds to $1/V^{1/6}$ where $V$ is the Calabi-Yau volume, while $M_P^2 = 2\pi \rho V/\kappa^2$ for $\pi \rho$ denoting the length of the eleventh segment. Then by choosing $\pi \rho \approx 4V^{1/6}$ together with $M_{GUT} = 1/V^{1/6} \approx 3 \times 10^{16}$ GeV, one could get the correct values of $\alpha_{GUT}$ and $M_P$, which was not allowed in the weakly coupled heterotic string theory. An additional phenomenological virtue of the $M$-theory limit is that there can be a QCD axion whose high energy axion potential is suppressed enough so that the strong CP problem can be solved by the axion mechanism. These phenomenological virtues have motivated many of the recent studies of the 4-dimensional effective theory of $M$-theory.

As is well known, the 4-dimensional effective action of the weakly coupled heterotic string theory can be expanded in powers of the two dimensionless variables: the string coupling
\( \epsilon_\alpha = \frac{e^{2\beta}/(2\pi)^5 }{ } \) and the worldsheet sigma model coupling \( \epsilon_\sigma = 4\pi\alpha'/V^{1/3} \) [14]. The effective action of \( M \)-theory can be similarly analyzed by expanding it in powers of the two dimensionless variables: \( \epsilon_1 = \kappa^{2/3}\pi\rho/V^{2/3} \) and \( \epsilon_2 = \kappa^{2/3}/\pi\rho V^{1/3} \). To compute the 4-dimensional effective action, one first expands the 11-dimensional action in powers of \( \kappa^{2/3} \) to find the compactification solution which is expanded in powers of \( \epsilon_1 \) and \( \epsilon_2 \). The subsequent Kaluza-Klein reduction of the 11-dimensional action for this compactification solution will lead to the desired 4-dimensional effective action. At the leading order in this expansion, the Kähler potential, superpotential and gauge kinetic functions have been computed in [14-17]. It is rather easy to determine the order \( \epsilon_1 \) correction to the leading order gauge kinetic functions [14-17], while it is much more nontrivial to compute the order \( \epsilon_1 \) correction to the leading order Kähler potential, which was recently done by Lukas, Ovrut and Waldram [15]. As we will argue later, the holomorphy and Peccei–Quinn symmetries guarantee that there is no further correction to the gauge kinetic functions and the superpotential at any finite order in the \( M \)-theory expansion [18]. However generically the Kähler potential is expected to receive corrections which are higher order in \( \epsilon_1 \) or \( \epsilon_2 \). An explicit computation of these higher order corrections will be highly nontrivial since first of all the 11-dimensional action is known only up to the terms of order \( \kappa^{2/3} \) relative to the zeroth order action (except for the order \( \kappa^{4/3} \) four-gaugino term) and secondly the higher order computation of the compactification solution and its Kaluza-Klein reduction are much more complicated.

In this paper, we wish to provide a simple macroscopic analysis of the 4-dimensional effective SUGRA action by expanding it in powers of \( \epsilon_1 \) and \( \epsilon_2 \), and apply the results for the computation of soft SUSY–breaking terms. As we will see, this analysis allows us to extract the form of possible higher order corrections to the Kähler potential and also to estimate their size for the physically interesting values of moduli. An interesting feature of the 4-dimensional effective SUGRA is that the forms of the gauge kinetic function and superpotential are not changed (up to nonperturbative corrections) when one moves from the weakly coupled heterotic string domain to the \( M \)-theory domain in the moduli space.
The same is true for the leading order Kähler potential, i.e. the Kähler potential of the weakly coupled heterotic string computed at the leading order in the string loop and sigma model perturbation theory is the same as the $M$-theory Kähler potential computed at the leading order in the $M$-theory expansion. In fact, one can smoothly move from the $M$-theory domain with $\epsilon_s \gg 1$ to the heterotic string domain with $\epsilon_s \ll 1$ while keeping $\epsilon_1$ and $\epsilon_2$ to be small enough. This means that the $M$-theory Kähler potential expanded in $\epsilon_1$ and $\epsilon_2$ is valid even on the heterotic string domain up to (nonperturbative) corrections which can not be taken into account by the $M$-theory expansion. Then as in the case of the gauge kinetic functions, one can determine the expansion coefficients in the $M$-theory Kähler potential by matching the heterotic string Kähler potential which can be computed in the string loop and sigma model perturbation theory.

About the issue of SUSY breaking, the possibility of SUSY breaking by the gaugino condensation on the hidden boundary has been studied \cite{2,11,12} and also some interesting features of the resulting soft SUSY–breaking terms were discussed in \cite{11}. Here we analyze the soft SUSY–breaking terms under the more general assumption that SUSY is spontaneously broken by the auxiliary components of the bulk moduli super fields in the model. We examine in particular how the soft terms vary when one moves from the weakly coupled heterotic string limit to the $M$-theory limit in order to see whether these two limits can be distinguished by the soft term physics. Our analysis implies that there can be a sizable difference between the heterotic string limit and the $M$-theory limit even in the overall pattern of soft terms.

Let us first discuss possible perturbative expansions of the 4-dimensional effective SUGRA action of $M$-theory. As in the case of weakly coupled heterotic string theory, the effective SUGRA of compactified $M$-theory contains two model–independent moduli superfields $S$ and $T$ whose scalar components can be identified as

\begin{align}
\text{Re}(S) &= \frac{1}{2\pi} (4\pi\kappa^2)^{-2/3} V, \\
\text{Re}(T) &= \frac{6^{1/3}}{8\pi} (4\pi\kappa^2)^{-1/3} \rho V^{1/3}, \quad (1)
\end{align}
where these normalizations of $S$ and $T$ have been chosen to keep the conventional form of the
gauge kinetic functions in the effective SUGRA. (See (7) for our form of the gauge kinetic
functions. Our $S$ and $T$ correspond to $\frac{1}{4\pi} S$ and $\frac{1}{8\pi} T$ of [3] respectively.) The pseudoscalar
components $\text{Im}(S)$ and $\text{Im}(T)$ are the well known model–independent axion and the Kähler
axion whose couplings are constrained by the non–linear Peccei–Quinn symmetries:

$$U(1)_S : S \rightarrow S + i\beta_S, \quad U(1)_T : T \rightarrow T + i\beta_T,$$  \hspace{1cm} (2)

where $\beta_S$ and $\beta_T$ are continuous real numbers. These Peccei-Quinn symmetries appear as
exact symmetries at any finite order in the $M$-theory expansion, but they are explicitly
broken by nonperturbative effects that will not be taken into account in this paper.

The moduli $S$ and $T$ can be used to define various kind of expansions which may be
applied for the low–energy effective action. For instance, in the weakly coupled heterotic
string limit, we have

$$\begin{align*}
\text{Re}(S) &= e^{-2\phi} \frac{V}{(2\alpha')^3}, \\
\text{Re}(T) &= \frac{6^{1/3} V^{1/3}}{32\pi^3 \alpha'},
\end{align*}$$  \hspace{1cm} (3)

where $\phi$ and $\sqrt{2\alpha'}$ denote the heterotic string dilaton and length scale respectively. One
may then expand the effective action of the heterotic string theory in powers of the string
loop expansion parameter $\epsilon_s$ and the world–sheet sigma model expansion parameter $\epsilon_\sigma$:

$$\begin{align*}
\epsilon_s &= \frac{e^{2\phi}}{(2\pi)^5} \approx 0.3 \frac{[4\pi^2 \text{Re}(T)]^3}{\text{Re}(S)}, \\
\epsilon_\sigma &= \frac{4\pi\alpha'}{V^{1/3}} \approx 0.5 \frac{1}{4\pi^2 \text{Re}(T)}.
\end{align*}$$  \hspace{1cm} (4)

Here we are interested in the possible expansion in the $M$-theory limit of the strong
heterotic string coupling $\epsilon_s \gg 1$ for which $\pi \rho \lesssim \kappa^{2/9}$ and $V \lesssim \kappa^{4/3}$ and so the physics can
be described by 11-dimensional supergravity. Since we have two independent length scales,
$\rho$ and $V^{1/6}$, there can be two dimensionless expansion parameters in the $M$-theory limit
also. The analysis of the 11-dimensional theory suggests that the expansion parameters
should scale as $\kappa^{2/3}$ which may be identified as the inverse of the membrane tension. One obvious candidate for the expansion parameter in the $M$-theory limit is the straightforward generalization of the string world-sheet coupling $\sim \alpha'/V^{1/3}$ to the membrane world-volume coupling $\sim \kappa^{2/3}/\rho V^{1/3}$. Note that in the $M$-theory limit, heterotic string corresponds to a membrane stretched along the eleventh dimension. Information for the other expansion parameter comes from the Witten's strong coupling expansion of the compactification solution [3], implying that one needs an expansion parameter which is linear in $\rho$. The analysis of [3] shows that a $\rho$-independent modification of the bulk physics at higher order in $\kappa^{2/3}$ can lead to a modification of the boundary physics which is proportional to $\rho$. As was noticed in [4, 15], this leads to an expansion parameter which scales as $\kappa^{2/3}\rho/V^{2/3}$. The above observations lead us to expand the 4-dimensional effective SUGRA action of the Hořava-Witten $M$-theory in powers of

$$\epsilon_1 = \kappa^{2/3} \pi \rho / V^{2/3} \approx \frac{\text{Re}(T)}{\text{Re}(S)},$$

$$\epsilon_2 = \kappa^{2/3} / \pi \rho V^{1/3} \approx \frac{1}{4\pi^2 \text{Re}(T)},$$

(5)

where (1) has been used to arrive at this expression of $\epsilon_1$ and $\epsilon_2$. Note that $\epsilon_1 \epsilon_2 \approx 1/[4\pi^2 \text{Re}(S)] \approx \alpha_{\text{GUT}}/\pi$ which is essentially the 4-dimensional field theory expansion parameter. Thus if one goes to the limit in which one expansion works better while keeping the realistic value of $\alpha_{\text{GUT}}$, the other expansion becomes worse. Here we will simply assume that both $\epsilon_1$ and $\epsilon_2$ are small enough so that the double expansion in $\epsilon_1$ and $\epsilon_2$ provides a good perturbative scheme for the effective action of $M$-theory. As we will argue later, this expansion works well even when $\epsilon_1$ becomes of order one, which is in fact the case when $M_{\text{GUT}} \approx 3 \times 10^{16}$ GeV.

Possible guidelines for the $M$-theory expansion of the 4–dimensional effective action would be (i) microscopically, the expansion parameter scales as $\kappa^{2/3}(\pi \rho)^{-n}V^{(n-3)/6}$ and it has a sensible limiting behavior when the Calabi-Yau volume or the 11-th segment becomes large, (ii) macroscopically, the expansion parameter scales as integral powers of $\text{Re}(S)$ and $\text{Re}(T)$. In $M$-theory, $V$ can be arbitrarily large independently of the value of $\pi \rho$. However
as was noted in [3], for a given value of the averaged Calabi-Yau volume \( V \), in order to avoid that one of the boundary Calabi-Yau volume shrinks to zero, \( \pi \rho \) is restricted as \( \pi \rho \leq \kappa^{-2/3}V^{2/3} \). Then one can demand that the expansion parameter \( \kappa^{2/3}(\pi \rho)^{-n}V^{(n-3)/6} \) should not blow up in the limit that \( V \to \infty \) for a fixed value of \( \pi \rho \) and also in another limit that \( \pi \rho \to \infty \) and \( V \to \infty \) while keeping \( V^{1/6} \ll \pi \rho \leq \kappa^{-2/3}V^{2/3} \). Obviously this condition is satisfied only for \(-1 \leq n \leq 3\). There are then only three possible expansion parameters which meet the guidelines (i) and (ii), \( \epsilon_1 \) and \( \epsilon_2 \) in (7) and finally \( \epsilon_3 = \kappa^{2/3}/(\pi \rho)^3 \approx 10^{-3}\text{Re}(S)/[\text{Re}(T)]^3 \) which scales as the inverse of the string loop expansion parameter \( \epsilon_s \).

As will be discussed later, the 4-dimensional effective action computed in the heterotic string limit can be smoothly extrapolated to the \( M \)-theory limit, suggesting that there can be a complete matching between the heterotic string effective action (expanded in \( \epsilon_s \) and \( \epsilon_\sigma \)) and the \( M \)-theory effective action (expanded in \( \epsilon_1 \) and \( \epsilon_2 \)). In view of this matching, it is not likely that there is an additional correction which would require to introduce the third expansion parameter \( \epsilon_3 \), although we can not rule out this possibility at this moment. Even when there is such correction, \( \epsilon_3 \) is smaller than \( \epsilon_2 \) and \( \epsilon_1 \) by one or two orders of magnitude for the phenomenologically interesting case that both \( \text{Re}(S) \) and \( \text{Re}(T) \) are essentially of order one, which is necessary to have \( M_{\text{GUT}} \approx 3 \times 10^{16} \) GeV together with the correct value of \( \alpha_{\text{GUT}} \) (see the discussion below (17)). This would justify our scheme of including only the expansions in \( \epsilon_1 \) and \( \epsilon_2 \).

To be explicit, let us consider a simple compactification on a Calabi-Yau manifold with the Hodge-Betti number \( h_{1,1} = 1 \) and \( h_{1,2} = 0 \). In this model, the low-energy degrees of freedom include first the gravity multiplet and \( S \) and \( T \) which are the massless modes of the 11-dimensional bulk fields. If the spin connection is embedded into the gauge connection in the observable sector boundary, we also have the \( E_6 \) gauge multiplet and the charged matter multiplet \( C \) together with the hidden \( E_8 \) gauge multiplet as the massless modes of the 10-dimensional boundary fields. It is then easy to compute the Kähler potential \( K \), the observable and hidden sector gauge kinetic functions \( f_{E_6} \) and \( f_{E_8} \), and the superpotential \( W \) at the leading order in the \( M \)-theory expansion. Obviously the leading contribution to
the moduli Kähler metric is from the 11-dimensional bulk field action which is of order $\kappa^{-2}$, while the charged matter Kähler metric, the gauge kinetic functions, and the charged matter superpotential receive the leading contributions from the 10-dimensional boundary action which is of order $\kappa^{-4/3}$. Including only the non-vanishing leading contributions, one finds

$$K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T}) + \frac{3|C|^2}{(T + T)},$$

$$f_{E_6} = f_{E_8} = S,$$

$$W = d_{pqr}C^pC^qC^r,$$  \hspace{1cm} (6)

where $d_{pqr}$ is a constant $E_6$-tensor coefficient.

The holomorphy and the Peccei-Quinn symmetries of (2) imply that there is no correction to the superpotential at any finite order in the $S$ and $T$-dependent expansion parameters $\epsilon_1$ and $\epsilon_2$. However the gauge kinetic functions can receive a correction at order $\epsilon_1$ in a way consistent with the holomorphy and the Peccei-Quinn symmetries. An explicit computation leads to

$$f_{E_6} = S + \alpha T, \quad f_{E_8} = S - \alpha T,$$  \hspace{1cm} (7)

where the integer coefficient $\alpha = \frac{1}{8\pi^2} \int \omega \wedge [\text{tr}(F \wedge F) - \frac{1}{2} \text{tr}(R \wedge R)]$ for the Kähler form $\omega$ normalized as the generator of the integer $(1,1)$ cohomology. As was discussed in [4,5], the coefficient $\alpha$ in the $M$-theory gauge kinetic function can be determined either by a direct $M$-theory computation [3] or by matching the string loop threshold correction to the gauge kinetic function [19,20]. At any rate, the holomorphy and the Peccei-Quinn symmetries imply that there is no further correction to the gauge kinetic functions at any finite order in the $M$-theory expansion.

1 Usually $\alpha$ is considered to be an arbitrary real number. For $T$ normalized as (1), it is required to be an integer [4].
Let us now consider the possible higher order corrections to the Kähler potential. With the Peccei-Quinn symmetries, the Kähler potential can be written as

\[ K = \hat{K}(S + \bar{S}, T + \bar{T}) + Z(S + \bar{S}, T + \bar{T})|C|^2, \]

(8)

with

\[ \hat{K} = \hat{K}_0 + \delta \hat{K}, \quad Z = Z_0 + \delta Z, \]

(9)

where \( \hat{K}_0 = -\ln(S + \bar{S}) - 3 \ln(T + \bar{T}) \) and \( Z_0 = 3/(T + \bar{T}) \) denote the leading order results in (8), while \( \delta \hat{K} \) and \( \delta Z \) are the higher order corrections. Before going to the M-theory expansion of \( \delta \hat{K} \) and \( \delta Z \), it is useful to note that the bulk physics become blind to the existence of boundaries in the limit \( \rho \to \infty \). (This is true up to the trivial scaling of the 4-dimensional Planck scale \( M_P^2 \sim \rho \).) However some of the boundary physics, e.g. the boundary Calabi-Yau volume, can be affected by the integral of the bulk variables over the 11-th dimension and then they can include a piece linear in \( \rho \). This implies that \( \delta \hat{K}/\hat{K}_0 \), being the correction to the pure bulk dynamics, contains only a non-negative power of \( 1/\rho \) in the M-theory expansion, while \( \delta Z/Z_0 \) which concerns the couplings between the bulk and boundary fields can include a piece linear in \( \rho \). Since \( \epsilon_1^m \epsilon_2^n \sim \rho^{n-m} \), one needs \( m \geq n \) for the expansion of \( \delta \hat{K}/\hat{K}_0 \) and \( m \geq n - 1 \) for the expansion of \( \delta Z/Z_0 \). Taking account of these, the M-theory expansion of the Kähler potential is given by

\[
\delta \hat{K} = \sum_{(n+m \geq 1, m \geq n)} A_{nm} \epsilon_1^n \epsilon_2^m \\
= \sum_{m \geq 1} \frac{A_{0m}}{4\pi^2 \text{Re}(T)} + \frac{A_{11}}{4\pi^2 \text{Re}(S)} \left[ 1 + \mathcal{O}(\frac{1}{4\pi^2 \text{Re}(S)}, \frac{1}{4\pi^2 \text{Re}(T)}) \right],
\]

\[
\delta Z = \frac{3}{(T + \bar{T})} \sum_{(n+m \geq 1, m \geq n-1)} B_{nm} \epsilon_1^n \epsilon_2^m \\
= \frac{3}{(T + \bar{T})} \sum_{m \geq 1} \frac{B_{0m}}{4\pi^2 \text{Re}(T)} + \frac{3B_{10}}{2\text{Re}(S)} \left[ 1 + \mathcal{O}(\frac{1}{4\pi^2 \text{Re}(S)}, \frac{1}{4\pi^2 \text{Re}(T)}) \right],
\]

(10)

where the \( n = 0 \) terms are separated from the other terms with \( n \geq 1 \). Here the coefficients \( A_{nm} \) and \( B_{nm} \) are presumed to be of order one, and then they can have a logarithmic
dependence on Re(S) or Re(T), while all the power-law dependence on Re(S) and Re(T) appear through the expansion parameters $\epsilon_1$ and $\epsilon_2$ which are presumed to be small enough.

The above expansion would work well in the $M$-theory limit:

$$[4\pi^2 \text{Re}(T)]^3 \gg \text{Re}(S) \gg \text{Re}(T) \gg \frac{1}{4\pi^2},$$  \hspace{1cm} (11) $$

while the heterotic string loop and sigma model expansions work well in the heterotic string limit:

$$\text{Re}(S) \gg [4\pi^2 \text{Re}(T)]^3, \quad \text{Re}(T) \gg \frac{1}{4\pi^2}. \hspace{1cm} (12)$$

By varying Re(S) while keeping Re(T) fixed, one can smoothly move from the $M$-theory limit to the heterotic string limit (or vice versa) while keeping $\epsilon_1 \approx \text{Re}(T)/\text{Re}(S)$ and $\epsilon_2 \approx 1/[4\pi^2\text{Re}(T)]$ small enough. Obviously then the $M$-theory Kähler potential expanded in $\epsilon_1$ and $\epsilon_2$ remains to be valid over this procedure, and thus is a valid expression of the Kähler potential even in the heterotic string limit. This means that, like the case of the gauge kinetic functions, one can determine the expansion coefficients in (10) by matching the heterotic string Kähler potential which can be computed in the string loop and sigma model perturbation theory. Since $\epsilon_1^n \epsilon_2^m \sim \epsilon_s^n \epsilon_{\sigma}^{m+2n}$, $(n, m)$-th order in the $M$-theory expansion corresponds to $(n, m + 2n)$-th order in the string loop and sigma-model perturbation theory. Thus all the terms in the $M$-theory expansion have their counterparts in the heterotic string expansion. It appears that the converse is not true in general, for instance the term $\epsilon_s^q \epsilon_{\sigma}^q$ with $q < 2p$ in the heterotic string expansion does not have its counterpart in the $M$-theory expansion. However all string one-loop corrections which have been computed so far lead to corrections which scale (relative to the leading terms) as $\epsilon_s^2 \epsilon_{\sigma}^2$ or $\epsilon_s^3 \epsilon_{\sigma}^3$, and thus have $M$-theory counterparts. This leads us to suspect that all the terms that actually appear in the heterotic string expansion have $q \geq 2p$ and thus have their counterparts in the $M$-theory expansion. Then there will be a complete matching (up to nonperturbative corrections) of the Kähler potential between the $M$-theory limit and the heterotic string limit, like the case of the gauge kinetic function and superpotential.
Let us now collect available informations on the coefficients in (10), either from the heterotic string analysis or from the direct M-theory analysis. Clearly in the heterotic string limit, \( n \) corresponds to the number of involved string loops. It has been argued that for (2,2) Calabi-Yau compactifications, a non-vanishing correction to the Kähler potential starts to occur at 3-rd order in sigma-model perturbation theory\(^2\) and thus

\[
A_{01} = A_{02} = B_{01} = B_{02} = 0 ,
\]

while \( A_{03} \) and \( B_{03} \) are generically nonvanishing\(^2\). The coefficients \( A_{03} \) and \( B_{03} \) were explicitly computed for some (2,2) Calabi-Yau compactifications, yielding for instance\(^2\)–\(^7\)

\[
A_{03} = \frac{3}{11} B_{03} = -15\zeta(3)/8 , \quad -51\zeta(3)/16 , \quad -111\zeta(3)/16 , \quad -27\zeta(3)/2 ,
\]

where \( \zeta(3) \approx 1.2 \). Also the orbifold computation of the string one-loop correction to \( \hat{K} \)\(^2\) suggests that \( A_{11} \) is generically nonvanishing with a possible dependence on \( \ln(T + \bar{T}) \), more explicitly \( A_{11} \approx \frac{1}{4} \delta_{GS} \ln(T + \bar{T}) \) for the orbifold case where the Green–Schwarz coefficient \( \delta_{GS} \) is generically of order one. The coefficient \( B_{10} \) has been recently computed in the context of the M-theory expansion\(^1\), yielding

\[
B_{10} = \frac{2}{3} \alpha .
\]

Putting these informations altogether, we finally obtain the following higher order corrections to the leading order Kähler potential in (6):

\[
\begin{align*}
\delta \hat{K} &= \frac{A_{03}}{[4\pi^2 \text{Re}(T)]^3} \left[ 1 + O\left( \frac{1}{4\pi^2 \text{Re}(T)} \right) \right] \\
&\quad + \frac{A_{11}}{4\pi^2 \text{Re}(S)} \left[ 1 + O\left( \frac{1}{4\pi^2 \text{Re}(S)} \right), \frac{1}{4\pi^2 \text{Re}(T)} \right], \\
\delta Z &= \frac{3}{(T + \bar{T})} \frac{B_{03}}{[4\pi^2 \text{Re}(T)]^3} \left[ 1 + O\left( \frac{1}{4\pi^2 \text{Re}(T)} \right) \right] \\
&\quad + \frac{\alpha}{\text{Re}(S)} \left[ 1 + O\left( \frac{1}{4\pi^2 \text{Re}(S)}, \frac{1}{4\pi^2 \text{Re}(T)} \right) \right].
\end{align*}
\]

\(^2\) For (2,0) compactifications, there may be a correction at lower order, and then our subsequent discussion should be accordingly modified.
As a phenomenological application of the \( M \)-theory expansion discussed so far, in the following we are going to analyze the soft SUSY-breaking terms under the assumption that SUSY is spontaneously broken by the auxiliary components \( F^S \) and \( F^T \) of the moduli superfields \( S \) and \( T \). To simplify the analysis, we will concentrate on the moduli values leading to \( M_{GUT} \approx 3 \times 10^{16} \) GeV. Using (1) and the \( M \)-theory expression of the 4-dimensional Planck scale, \( M_P^2 = 2\pi\rho V/\kappa^2 \), one easily finds

\[
M_{GUT} = 1/V^{1/6} \approx 3.6 \times 10^{16} \left( \frac{2}{\text{Re}(S)} \right)^{1/2} \left( \frac{1}{\text{Re}(T)} \right)^{1/2} \text{GeV} .
\]

(17)

Since from (7) one obtains \( \text{Re}(S) + \alpha \text{Re}(T) = \text{Re}(f_{E_6}) = 1/g_{GUT}^2 \approx 2 \), the above relation implies that \( \text{Re}(T) \) is essentially of order one when \( M_{GUT} \approx 3 \times 10^{16} \) GeV. Clearly, if \( \text{Re}(T) \) is of order one, we are in the \( M \)-theory domain with \( \epsilon_s \gg 1 \). (See (4)). One may worry that the \( M \)-theory expansion (10) would not work in this case since \( \epsilon_1 = \text{Re}(T)/\text{Re}(S) \) is of order one also. However as we have noticed, any correction which is \( n \)-th order in \( \epsilon_1 \) accompanies at least \( (n-1) \)-powers of \( \epsilon_2 \) and thus is suppressed by \( (\epsilon_1\epsilon_2)^{n-1} \approx (\alpha_{GUT}/\pi)^{n-1} \) compared to the order \( \epsilon_1 \) correction. This allows the \( M \)-theory expansion (10) to be valid even when \( \epsilon_1 \) becomes of order one. Obviously if \( \text{Re}(T) \) is of order one, only the order \( \epsilon_1 \) correction to \( Z \), i.e. \( \delta Z = \alpha/\text{Re}(S) \), can be sizable. The other corrections are suppressed by either \( \epsilon_1\epsilon_2 \approx 1/4\pi^2\text{Re}(S) \) or \( \epsilon_2^3 \approx 1/[4\pi^2\text{Re}(T)]^3 \) and thus smaller than the leading order results at least by \( \mathcal{O}((\epsilon_2/\pi)) \). Thus we will include only \( \delta Z = \alpha/\text{Re}(S) \) in the later analysis of soft terms, while ignoring the other corrections to the Kähler potential\(^3\).

Summarizing the above discussion, our starting point of the soft term analysis is the effective SUGRA model given by

\[
K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T}) + \left( \frac{3}{T + \bar{T}} + \frac{\alpha}{S + \bar{S}} \right) |C|^2 ,
\]

\(^3\) In fact, in the weakly coupled heterotic string limit, it is quite possible that \( 1/4\pi^2\text{Re}(T) \) is not so small, and then the sigma model corrections of order \( 1/[4\pi^2\text{Re}(T)]^3 \) can significantly affect the soft terms [24,27]. However in the \( M \)-theory limit, \( 1/4\pi^2\text{Re}(T) \) is quite small and thus the effects of these sigma model corrections are negligible compared to those of \( \delta Z = \alpha/\text{Re}(S) \).
\[ f_{E_6} = S + \alpha T , \quad f_{E_8} = S - \alpha T , \]
\[ W = d_{pq} C^p C^q C^r . \quad (18) \]

Here the superpotential and gauge kinetic functions are exact up to nonperturbative corrections, while there can be small additional perturbative corrections to the Kähler potential which are of order \( 1/4\pi^2 \text{Re}(S) \) or \( 1/[4\pi^2 \text{Re}(T)]^3 \). Since the above SUGRA model includes only a single \( T \)-modulus, the resulting soft terms would describe the case that only one combination of the \( T \)-moduli, if there are more than one, participates in the SUSY breaking. Later, we will also discuss the multimoduli case.

Applying the standard (tree–level) soft term formulae \([28,29]\) for the above SUGRA model \((18)\), we can compute the soft terms straightforwardly. Since the bilinear parameter, \( B \), depends on the specific mechanism which could generate the associated \( \mu \) term, let us concentrate on gaugino masses, \( M \), scalar masses, \( m \), and trilinear parameters, \( A \). After normalizing the observable fields, these are given by
\[ M = \frac{\sqrt{3} C m_{3/2}}{(S + \bar{S}) + \alpha(T + \bar{T})} \left( (S + \bar{S}) \sin \theta e^{-i\gamma_S} + \frac{\alpha(T + \bar{T})}{\sqrt{3}} \cos \theta e^{-i\gamma_T} \right) , \]
\[ m^2 = V_0 + m_{3/2}^2 - \frac{3m_{3/2}^2 C^2}{3(S + \bar{S}) + \alpha(T + \bar{T})} \times \left\{ \alpha(T + \bar{T}) \left( 2 - \frac{\alpha(T + \bar{T})}{3(S + \bar{S}) + \alpha(T + \bar{T})} \right) \sin^2 \theta 
+ (S + \bar{S}) \left( 2 - \frac{3(S + \bar{S})}{3(S + \bar{S}) + \alpha(T + \bar{T})} \right) \cos^2 \theta 
- \frac{2\sqrt{3} \alpha(T + \bar{T})(S + \bar{S})}{3(S + S) + \alpha(T + \bar{T})} \sin \theta \cos \theta \cos(\gamma_S - \gamma_T) \right\} , \]
\[ A = \sqrt{3} C m_{3/2} \left\{ \left( -1 + \frac{3\alpha(T + \bar{T})}{3(S + \bar{S}) + \alpha(T + \bar{T})} \right) \sin \theta e^{-i\gamma_S} 
+ \sqrt{3} \left( -1 + \frac{3(S + \bar{S})}{3(S + \bar{S}) + \alpha(T + \bar{T})} \right) \cos \theta e^{-i\gamma_T} \right\} , \quad (19) \]

where we are using the parametrization introduced in \([30]\) in order to know what fields, either \( S \) or \( T \), play the predominant role in the process of SUSY breaking
\[ F^S = \sqrt{3} m_{3/2} C(S + \bar{S}) \sin \theta e^{-i\gamma_S} , \]
\[ F^T = m_{3/2} C(T + \bar{T}) \cos \theta e^{-i\gamma_T} , \quad (20) \]
with $m_{3/2}$ for the gravitino mass, $C^2 = 1 + V_0/3m_{3/2}^2$ and $V_0$ for the tree-level vacuum energy density. In what follows, given the current experimental limits, we will assume $V_0 = 0$ and $\gamma_S = \gamma_T = 0 \text{ (mod } \pi)$. More specifically, we will set $\gamma_S$ and $\gamma_T$ to zero and allow $\theta$ to vary in a range $[0, 2\pi)$.

Notice that the structure of these soft terms is qualitatively different from those of the weakly coupled heterotic string case [30] which can be recovered from (19) by taking the limit $\alpha(T + \bar{T}) \ll (S + \bar{S})$:

$$M = -A = \sqrt{3}m_{3/2}\sin \theta, \quad m^2 = m_{3/2}^2 \left(1 - \cos^2 \theta\right).$$

(21)

Whereas there are only two free parameters in (21), viz $m_{3/2}$ and $\theta$, the $M$–theory result (19) is more involved due to the additional dependence on $(S + \bar{S})$ and $\alpha(T + \bar{T})$ even when we set $C = 1$ and $\gamma_S = \gamma_T = 0$. As a consequence, even in the dilaton–dominated SUSY–breaking scenario with $|\sin \theta| = 1$, it is no longer true that simple results $M = -A = \pm \sqrt{3}m$ and $m = m_{3/2}$ hold. (We will discuss in more detail this scenario below.) Nevertheless we can simplify the analysis by taking into account, as already mentioned above, that the real part of the gauge kinetic functions in (18) are the inverse squared gauge coupling constants and thus

$$(S + \bar{S}) + \alpha(T + \bar{T}) \approx 4$$

(22)

to produce the known values of $\alpha_{GUT}$. We then have only one more parameter, say $\alpha(T + \bar{T})$, than the result (21) in the heterotic string limit. In fact, this parameter is severely constrained by $\text{Re}(f_{Es}) > 0$, leading to $\alpha(T + \bar{T}) < (S + \bar{S})$. Using (22), we then obtain the following bound

$$0 < \alpha(T + \bar{T}) \lesssim 2.$$  

(23)

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4 Of course, in this limit the corrections of order $1/(4\pi^2\text{Re}(T))^3$ in (18) which are ignored in (19) can be important, however we will ignore this point for simplicity.
Given these results, and recalling the analysis of the GUT scale (17) where we obtained that \( \text{Re}(T) \) should be of order one, we show in Fig. 1 the dependence on \( \theta \) of the soft terms \( M, m, \) and \( A \) in units of the gravitino mass for different values of \( \alpha(T + \bar{T}) \). Several comments are in order. First of all, some ranges of \( \theta \) are forbidden by having a negative scalar mass-squared. The figures clearly show that the smaller the value of \( \alpha(T + \bar{T}) \), the smaller the forbidden regions become. In the weakly coupled heterotic string case shown in Fig. 2, the forbidden region vanishes since the squared scalar masses are always positive (see (21)). Notice however that even in the extreme case \( \alpha(T + \bar{T}) = 2 \), shown in Fig. 1(a), the allowed regions correspond to values of \( \theta \) such that \( |\sin \theta| < 0.9 \) and therefore most of the dilaton/modulus SUSY-breaking scenarios are possible. About the possible range of soft terms, the smaller the value of \( \alpha(T + \bar{T}) \), the larger the range becomes. For example, for \( \alpha(T + \bar{T}) = 2 \) (Fig. 1(a)), those ranges are \( 0.5 < |M|/m_{3/2} < 1, 0 < m/m_{3/2} < 0.5 \) and \( 0.71 < |A|/m_{3/2} < 0.87 \), whereas for \( \alpha(T + \bar{T}) = 1 \) (Fig. 1(c)), they are \( 0.25 < |M|/m_{3/2} < 1.32, 0 < m/m_{3/2} < 0.7 \) and \( 0.3 < |A|/m_{3/2} < 1.25 \).

In order to discuss the SUSY spectra further, it is worth noticing that scalar masses are always smaller than gaugino masses. This is shown in Fig. 3 where the ratio \( m/|M| \) versus \( \theta \) is plotted for different values of \( \alpha(T + \bar{T}) \). Notice that in the heterotic string limit which corresponds to the straight line \( m/|M| = 1/\sqrt{3} \), the limit \( \sin \theta \to 0 \) is not well defined since all \( M, A, m \) vanish in that limit. One then has to include the string one–loop corrections (or the sigma–model corrections) to the Kähler potential and gauge kinetic functions which would modify the boundary conditions (21). This problem is not present in the \( M \)–theory limit since gaugino masses are always different from zero.

Finally, given the numerical results and also Fig. 1 and Fig. 2, it is straightforward to compare the \( M \)–theory limit with the weakly coupled heterotic string limit in the dilaton–dominated case with \( |\sin \theta| = 1 \). For example, whereas \( m^2 = m_{3/2}^2 \) in the heterotic string limit, now in the \( M \)–theory limit, \( m^2 \) can be much smaller and even negative. See Fig. 1(a) with respect to the possibility of a negative \( m^2 \) in the dilaton–dominated scenario, where \( |\sin \theta| = 1 \) is excluded. For a further comparison, let us consider the case when several
moduli $T_i$ and the associated matter $C_i$ are present \cite{27}. The soft scalar masses in this case are given by

$$m_{ij}^2 = m_{3/2}^2 Z_{ij} - F^m \left( \partial_m \partial_n Z_{ij} - Z^{k\ell} \partial_m Z_{il} \partial_n Z_{kj} \right) \bar{F}^n,$$  \hspace{1cm} (24)

where $F^m = F^S, F^T_i$, and $Z_{ij}$ and $Z^{ij}$ denote the Kähler metric and its inverse of the matter fields $C_i$. After normalizing the fields to get canonical kinetic terms, although the first piece in (24) will lead to universal diagonal soft masses, the second piece will generically induce non–universal contributions due to the presence of off–diagonal Kähler metric:

$$Z_{ij} = (\partial^2 \hat{K}^T / \partial T_i \partial \bar{T}_j) e^{-\hat{K}^T / 3} + \delta Z_{ij} (S + \bar{S}, T_i + \bar{T}_i),$$  \hspace{1cm} (25)

where

$$\hat{K}^T = - \ln k_{ijk}(T_i + \bar{T}_i)(T_j + \bar{T}_j)(T_k + \bar{T}_k)$$  \hspace{1cm} (26)

and $\delta Z_{ij}$ corresponds to the $S$-dependent correction in the $M$-theory expansion (or the string-loop correction). If one ignores $\delta Z_{ij}$, the matter Kähler metric is $S$-independent, and as a consequence in the dilaton–dominated scenario with $F^T_i = 0$, the normalized soft scalar masses are universal as $m_i = m_{3/2}$. However including the $S$-dependent $\delta Z_{ij}$, one generically loses the scalar mass universality even in the dilaton–dominated case. In fact, this was noted in \cite{31} for the string–loop induced $\delta Z_{ij}$ which is small in the weakly coupled heterotic string limit. Our main point here is that in the $M$-theory limit $\delta Z_{ij}$ can be as large as the leading order Kähler metric, and then there can be a large violation of the scalar mass universality even in the dilaton–dominated scenario.

It is worth noticing here that the above comments about the dilaton–dominated scenario may also be applied to orbifold compactifications assuming that similar $M$–theory effects are present. For example, the Kähler metric of untwisted matter field $C_i$ may be given by

$$Z_i = \frac{1}{T_i + \bar{T}_i} + \frac{\alpha_i}{S + \bar{S}},$$  \hspace{1cm} (27)

where the second piece corresponds to the high order correction in the $M$-theory expansion. Then the soft scalar mass of $C_i$ in the dilaton–dominated scenario can be obtained from (19).
with the substitution $\alpha(T + \bar{T}) \rightarrow 3\alpha_i(T_i + \bar{T}_i)$ and $|\sin \theta| = 1$, which shows clearly that the scalar mass universality is lost. This is also true for $\alpha_i = \alpha$ since still the VEVs of the $T_i$’s will be different in general.

Let us now discuss the predictions for the low–energy ($\approx M_W$) sparticle spectra in this $M$–theory scenario. As is well known there are several particles whose masses are rather independent of the details of $SU(2)_L \times U(1)_Y$ radiative breaking and are mostly given by the boundary conditions and the renormalization group running [29]. In particular, that is the case of the gluino $\tilde{g}$, all the sleptons $\tilde{l}$ and first and second generation squarks $\tilde{q}$. Since, as discussed above, always $m < |M|$ at the GUT scale, the qualitative mass relations at the electroweak scale in the $M$-theory scenario turn out to be

$$M_{\tilde{g}} \approx m_{\tilde{q}} > m_{\tilde{f}},$$

where gluinos are slightly heavier than squarks. We recall that slepton masses are smaller than squark masses because they do not feel the important gluino contribution in the renormalization. The precise values in (28) depend on the ratio $r \equiv m/|M|:

$$M_{\tilde{g}} : m_{\tilde{Q}_L} : m_{\tilde{d}_R} : m_{\tilde{L}_L} : m_{\tilde{e}_R} \approx 1 : \frac{1}{3} \sqrt{7.6 + r^2} : \frac{1}{3} \sqrt{7.17 + r^2} : \frac{1}{3} \sqrt{7.14 + r^2} : \frac{1}{3} \sqrt{0.53 + r^2} : \frac{1}{3} \sqrt{0.15 + r^2}.$$  

(29)

For example for $r = 1/\sqrt{3}$, which is always the case of the weakly coupled heterotic string limit, one obtains $1 : 0.94 : 0.92 : 0.91 : 0.3 : 0.23$, whereas for the extreme case of $r = 0$ the result is $1 : 0.92 : 0.89 : 0.88 : 0.24 : 0.13$. Clearly, this type of analysis would allow us to distinguish the $M$–theory limit from the weakly coupled heterotic string limit. If the observed SUSY spectrum is inconsistent with the above results for $r = 1/\sqrt{3}$, the $M$–theory limit may be the answer. On the other hand, we see in Fig. 3 that $r = 1/\sqrt{3}$ can be obtained for particular values of $\theta$ in the $M$–theory limit also. Thus if the SUSY spectrum turns out to be consistent with $r = 1/\sqrt{3}$, one should analyze the rest of the SUSY mass spectra, taking into account the details of the electroweak radiative breaking. This more detailed analysis would allow us to distinguish clearly between both limits. To this respect, we note that even
for $r$ which is close to $1/\sqrt{3}$, the pattern of soft terms in the $M$–theory limit significantly differs from that in the heterotic string limit (see Figs. 1, 2). Although most of our analysis has been made for the simple case that only $S$ and one of the possible $T$-moduli participate in SUSY breaking, this kind of analysis can be easily generalized to a more general case.

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Figure 1: Soft parameters in unit of $m_{3/2}$ versus $\theta$ for different values of $\alpha(T + \bar{T})$ in the $M$–theory limit. Here $M$, $m$ and $A$ are the gaugino mass, the scalar mass and the trilinear parameter respectively.
Figure 2: Soft parameters in unit of $m_{3/2}$ versus $\theta$ in the weakly coupled heterotic string limit.

Figure 3: $m/|M|$ versus $\theta$ for different values of $\alpha(T + \bar{T})$. The straight line corresponds to the weakly coupled heterotic string limit.