The optimum thickness of paraffin wax insulator filling a double glazed window

J M Jalil and S M Salih
Electromechanical Engineering Department, University of Technology, Iraq.

Email: jalalmjalil@gmail.com
Salih0499@gmail.com

Abstract. A numerical three-dimensional investigation is performed to estimate the optimum doping content (i.e. thickness) of the PCM filling a double glazed window. For the summer weather of Baghdad, Iraq. Finite Volume method combined with the enthalpy method is utilized to acknowledge the conduction related with phase change problems within the wax by using a constructed FORTRAN (F90) program. Based on the constructed numerical model, the transient heat transfer characteristics of the insulation were calculated and analyzed in detail. The obtained results indicate that optimum doping content of wax filling the window can be estimated. The required thickness of wax paraffin (melting point 40 °C) to keep temperature within the required range were as follows: May 2 cm, June 3 cm, July 3.5 cm, August 3.5 cm, September 3 cm and October 1.5 cm.

Key Words; thermal insulation, finite volume, wax paraffin, phase change

Nomenclature:
A: Area (m²)
a, b : Coefficient in the discretization equation
B: ‘Bottom' neighbor of grid P
b: Control-volume face between P and B
Cp: Specific heat (kJ/kg °C)
Cp_s: Specific heat of solid phase (kJ/kg °C)
Cp_l: Specific heat of liquid phase (kJ/kg °C)
H: Enthalpy (kJ/kg)
e: Control-volume face between P and E
E: ‘East' neighbor of grid P
L: Latent heat (kJ/kg)
i, j, k: Unit vector
k: Thermal conductivity (W/ m °C)
L: Liquid
N: ‘North’ neighbor of grid P
n: Control-volume face between P and N
P: Grid point
q: Heat generation (W/m³)
\( \bar{q} \): Average heat generation
S: ‘South’ neighbor of grid P
Introduction

In nowadays, the Phase Change Material (PCM) is a possible option for reducing the energy consumption and for increase the thermal comfort in buildings. The use of PCM in building applications provides the potential to increase the indoor thermal comfort for occupants due to the reduced indoor temperature fluctuations and lower global energy consumption. The possibility to integrate the PCM into the material of construction for cooling and heating the buildings gained the interest of researchers from all the world because the PCM have a high latent heat, meaning it is capable of storing and release high amounts of heat during its melting and solidifying process at a particular temperature [1].

Hasan et al. investigated the validation of installing layers of PCM as insulation Layers with different thickness located in walls and ceiling of a room located in Kut, Iraq. The acquired results showed a considerable amount of reduction in the indoor temperature and cooling load which led to saving in electricity consumption. Furthermore, Xie et al. investigated numerically the transient heat transfer characteristics of combined multilayer thermal insulation materials combined with two types of PCM. Goia et al. designed and built a test facility (glazing prototype) containing DGU_PCM (double glazed window with PCM) and DGU_GG (without PCM) and observed the performance of both of them within six months of the year focusing on the thermal comfort aspect and under real working condition. For the majority of the time, the DGU_PCM performance was much better however, the interior surface temperature of DGU_PCM was greater than DGU_GG which is a promising feature in winter but it must be avoided in summer. Ismail et al. studied numerically and experimentally the thermal efficiency of two types of window, traditional and composite (filled with PCM or air). The experimental investigation was carried out by means of spectrophotometry on both types. The reflectance test indicated that the reflectance of the single glass window was 12% to 13%, however, the reflectance of the window with PCM was 7%. Gowreesunker et al. analyzed numerically and experimentally the thermal and optical aspect of a double glazed unit with PCM compared to the usual double glazed unit. The obtained results illustrated that the transmittance during a fast phase change is unstable, however, throughout solid and liquid phases the visual transmittance rates are 40% and 90% respectively. Moreover, during the phase change process, the utilized PCM enhanced the thermal mass. Liu et al. demonstrated numerically the effect of semi-transparent property and zenith angle on the thermal performance of double glazed roof filled with PCM, the thickness of the PCM was also studied. The acquired results shown that the semi-transparent property, zenith angle and the thickness of the PCM have an enormous effect on the thermal performance of the double glazed roof. Li et al.
investigated numerically the thermal performance of the double glazed unit filled with PCM with different thermophysical parameters of PCM. The effects of specific heat capacity, density, thermal conductivity, latent heat and melting temperature of the PCM were studied. The obtained results indicated that all of the mentioned properties affect significantly the performance of the double glazed unit. Li et al. experimentally investigated the thermal behaviour of triple–pane window (TW) + PCM and compare it with (TW) and (DW + PCM). The obtained results indicate that the inside surface temperature of the TW + PCM was lower than the DW+PCM and TW by 2.7 and 5.5 °C, respectively. Additionally, minimized the heat entering the building by 16.6 % and 28% throughout the summer sunny day. Liu et al. developed a numerical model to determine the optical and thermal performance of glazed roof incorporate with PCM. In addition, the effect of air convection (h wind), PCM thickness and melting temperature were also studied, in the northeast China climates. Results obtained shows that regarding the internal surface temperature of the glazed roof the influence of the air convection should be considered, and the recommended PCM thickness and melting temperature were 12-20 mm and 16-18 °C, respectively.

The aim of this paper is to estimate the optimum thickness of the paraffin wax filling a double glazed window under the summer condition of Baghdad.

2. Numerical Formulations

Stefan problems were used to represent the heat conduction problems with phase change. This problem is addressed by the enthalpy transforming method; this method was offered to convert the energy equation into a non-linear equation with a single dependent variable enthalpy (H). The benefits of enthalpy method are that it can solve the problem when it's formulated in a fixed region, in addition to temperature, this method deals with the enthalpy as a dependent variable and separates the energy equation into a set of equations that include both temperature and enthalpy.

The analysis of the model in 3-D is regarding to Cao [11].

The energy equation is:

\[
\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \bar{q} = \rho \frac{\partial H}{\partial t}
\]

In addition, the state equation [11]

\[
\frac{dH}{dt} = C_p
\]

Where the specific heat case is constant for each phase and the change of phase happens at a single temperature [11],[12].

\[
T = \begin{cases} 
T_m + \frac{H}{C_{ps}} & H \leq 0 \quad \text{(Solid phase)} \\
T_m & 0 < H < L \quad \text{(Phase change)} \\
T_m + \frac{(H - L)}{C_{pl}} & H \geq L \quad \text{(Liquid phase)}
\end{cases}
\]

(3)

For the above relation, \( H = 0 \) was selected according to the phase change material (PCM) in their solid state to temperature \( T_m \). Also, the "Kirchhoff" temperature is presented as [13]:

\[
T^* = \int_{T_m}^{T} k(\eta) d\eta = \begin{cases} 
k_s(T - T_m), & T < T_m \\
0, & T = T_m \\
k_l(T - T_m), & T > T_m
\end{cases}
\]

(4)

Utilizing Equation (3) and the definition illustrated in equation (4) [11].
\[ T^* = \begin{cases} 
\frac{k_2 H}{C_{ps}}, & H \leq 0 \\
0, & 0 < H < L \\
\frac{k_1 (H - L)}{C_{pl}}, & H \geq L 
\end{cases} \]  

(5)

Now, enthalpy function is introduced as [11]:

\[ T^* = \lambda(H)H + S(H) \]  

(6)

In Kirchhoff temperature’s terms, adjusting equation (1) and replacing equation (6), Provides [14]:

\[ \rho \frac{\partial H}{\partial t} = \frac{\partial}{\partial x} \left( \frac{\partial (\lambda H)}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial (\lambda H)}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{\partial (\lambda H)}{\partial z} \right) + p + q \]  

(7)

With

\[ p = \frac{\partial}{\partial x} \left( \frac{\partial S}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial S}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{\partial S}{\partial z} \right) \]  

(8)

And for \( \lambda = \lambda(H) \) and \( S = S(H) \), the above equation exhibits the control-volume Finite-difference. Integrating the equation over the control volumes as [14]:

\[
\int \int \int \frac{\partial H}{\partial t} \Delta V = \int \int \int \frac{\partial}{\partial x} \left( \frac{\partial (\lambda H)}{\partial x} \right) \Delta V + \int \int \int \frac{\partial}{\partial y} \left( \frac{\partial (\lambda H)}{\partial y} \right) \Delta V + \int \int \int \frac{\partial}{\partial z} \left( \frac{\partial (\lambda H)}{\partial z} \right) \Delta V \\
+ \int \int \frac{\partial}{\partial x} \left( \frac{\partial S}{\partial x} \right) \Delta V + \int \int \frac{\partial}{\partial y} \left( \frac{\partial S}{\partial y} \right) \Delta V + \int \int \frac{\partial}{\partial z} \left( \frac{\partial S}{\partial z} \right) \Delta V + \int q \Delta V
\]  

(9)

By means of an explicit scheme, the time variation term becomes [11],

\[
\int \int \int \rho \frac{\partial H}{\partial t} \Delta V = \rho \Delta V \left( \frac{H_p - H_p^*}{\Delta t} \right)
\]  

(10)

Now, let the coefficient of \( H_p \) be \( a_N, a_s, a_E, a_W, a_T \) and \( a_B \), respectively. Writing equation (10) in the familiar standard form [14]:

\[ a_p H_p = a_N H_N + a_s H_S + a_E H_E + a_W H_W + a_T H_T + a_B H_B + b \]  

(11)

With, \( H_p^o \) representing the old value of \( H \) at P (grid point), the coefficients’ values are [14]:

\[ a_p = a_N + a_s + a_E + a_W + a_T + a_B \]

\[ a_E = \frac{\Delta t}{\rho \Delta V} \frac{\lambda_E A_x}{\delta x_e}, \quad a_W = \frac{\Delta t}{\rho \Delta V} \frac{\lambda_W A_x}{\delta x_w}, \quad a_N = \frac{\Delta t}{\rho \Delta V} \frac{\lambda_N A_y}{\delta y_n}, \quad a_S = \frac{\Delta t}{\rho \Delta V} \frac{\lambda_S A_y}{\delta y_s}, \quad a_T = \frac{\Delta t}{\rho \Delta V} \frac{\lambda_T A_z}{\delta z_t}, \quad a_B = \frac{\Delta t}{\rho \Delta V} \frac{\lambda_B A_z}{\delta z_b} \]  

(12)

\[ b = -[a_N + a_s + a_E + a_W + a_T + a_B - 1]H_p^o + b_N S_N + b_S S_S + b_E S_E + b_W S_W + b_T S_T + b_B S_B - b_p S_p + \frac{dt}{\rho} q \Delta \]  

(13)
Where, $b_p = b_E + b_w + b_x + b_N + b_T + b_B$

$$
\begin{align*}
E_B = \frac{\Delta t}{\rho \Delta V \delta x_e},& \quad b_w = \frac{\Delta t}{\rho \Delta V \delta x_w},& \quad b_N = \frac{\Delta t}{\rho \Delta V \delta y_n},& \quad b_s = \frac{\Delta t}{\rho \Delta V \delta y_s},
\end{align*}
$$

(14)

The last procedure is similar to liquid and solid regions.

3. Validation of the numerical model

A room with the dimensions of 1m*1m*1m was manufactured. The walls and ceiling were made of PVC sandwich panel (5 mm thickness) containing a window in the south side of it with the dimensions of 50*50 cm, in addition, the ground was made of wood with a thickness of 2 cm. the thickness of the PCM filling the glazed unit was 1.7 cm. The properties of the wax are given in table 1. The solar radiation of the sun was simulated by using 4 halogen lamps which placed 70 cm away from the window. Figure 1 illustrates the specifics details of the double glazed window doped (filled) with PCM, which showcase the incident solar radiation and the portion of it that was transmitted to the room. In addition, heat transfer processes (thermal convection and radiation) occur on the boundary of the interior and exterior surfaces, respectively.

The numerical model is validated by means of this experimental room, under the environmental condition of Baghdad (extremely hot summer) in 25/6/2018. Figure 2 shows the comparison between the experimental and numerical results regarding the interior surface temperature of the double glazed window (filled with PCM) and the indoor temperature of the room, respectively. Which displays a quite good agreement between them.

4. Results and discussion

The constructed FORTRAN computer program solved the energy equation with phase change in the layer of paraffin wax filled the double glazed window. At $x = 0$, the solar radiation of Baghdad weather was applied (monthly average radiation was taken for each month). As shown in figure 3 (a).

In order to evaluate the performance of the double glazed window (including the wax), two criteria were selected, temperature decrement factor ($\Delta t_{pcm}$) and internal surface temperature of the double glazed window. Where the performance of the glazed unit is accepted if the temperature decrement factor is low and internal surface temperature is within the accepted range (i.e. above the melting temperature of the wax by 4 to 5 °C or below it).

$$
\Delta t_{pcm} = \frac{T_{pcm,\text{max}} - T_{pcm,\text{min}}}{T_{a,\text{max}} - T_{a,\text{min}}}
$$

(15)

Where ($T_{pcm,\text{max}}$) and ($T_{pcm,\text{min}}$) are the maximum and minimum internal surface temperature of the double glazed unit respectively, on the other hand, ($T_{a,\text{max}}$) and ($T_{a,\text{min}}$) are the maximum and minimum ambient temperature [10]. Figure 3 (b) illustrate the ambient temperature levels for Baghdad in 2018. The solution was applied for summer months starting from May to October 2018. The results were plotted as follows: Figure 4 (a) for May, figure 4 (b) for June, figure 4 (c) for July, figure 4 (d) for August, figure 4 (e) for September and figure 4 (f) for October, respectively.

Figure 4 (a) shows the internal surface temperature of the unit with different PCM’s thicknesses in May. The tested thicknesses were 2.5 cm, 2 cm, and 1.5 cm. It can be seen that the 1.5 cm thickness could not keep the internal surface temperature of the window within the accepted range, however, thicknesses 2 and 2.5 cm would successfully maintain the temperature within the accepted range. Moreover, the temperature decrement factor was 0.73, 1.15 and 1.53, respectively. Figure 4 (b) shows the internal surface temperature of the unit in June. The required thickness to keep the temperature within the accepted range was 3 cm, in which temperature decrement factor was 1. Furthermore, figure 4 (c) shows the internal surface temperature of the unit in July. The required thickness was between 3.5 cm and 4 cm, and the temperature decrement factor was 0.95 and 0.76 respectively. Also, for August the required thickness was between 3 and 3.5 cm (temperature decrement factor was 1 and 0.74, respectively), as shown in figure 4 (d). Also, for September the required thickness was 3 cm, in
which temperature decrement factor was 0.92, as shown in figure 4 (e). However, figure 4 (f) illustrates October data, in which the required thickness was between 1.5 and 2 cm to achieve the accepted temperature and temperature decrement factor (i.e. 1.2 and 0.6, respectively). Consequently, figure 5 illustrate the variation of the required thickness of the wax for each month (summer period). Which demonstrate the required thickness of the PCM to ensure reliable performance from the window. And it’s start from 2 cm (in May) and reaches the maximum in July (3.5 cm) and returns back to 1.5 cm in October. Additionally, the recommended thickness for the whole summer period of Baghdad is 3 cm.

5. Conclusions
1. For the summer weather of Baghdad, the required thickness of the double glazed window filled with paraffin wax (melting point 40 °C) was as follows: May 2 cm, June 3 cm, July 3.5 cm, August 3.5 cm, September 3 cm, October 1.5 cm.
2. For the whole summer weather of Baghdad the required thickness is 3 cm.

References

[1] Socaciu L, Pleșa A, Ungureșan P and Giurgiu O 2014 Review on phase change materials for building applications. Leonardo Electronic Journal of Practices and Technologies 25 179-194.
[2] Hasan I, Hadi O, and Ahmed O 2018 Experimental investigation of phase change materials for insulation of residential buildings Sustainable cities and society 36 42-58.
[3] Xie T, He L and Tong X 2016 Analysis of insulation performance of multilayer thermal insulation doped with phase change material. International Journal of Heat and Mass Transfer 102 934-943.
[4] Goia F, Perino M and Serra V 2013 Improving thermal comfort conditions by means of PCM glazing systems. Energy and Buildings 60 442-452.
[5] Ismail K. and J. Henrıquez 2002 Parametric study on composite and PCM glass systems. Energy conversion and management 43.7 973-993.
[6] Gowreesunker L, Stankovic B, Tassou A and Kyriacou A 2013 Experimental and numerical investigations of the optical and thermal aspects of a PCM-glazed unit. Energy and Buildings 61 239-249.
[7] Liu C, Zhou Y, Li D, Meng F, Zheng Y and Liu X 2016 Numerical analysis on thermal performance of a PCM-filled double glazing roof. Energy and Buildings 125 267-275.
[8] Li D, Li Z, Zheng Y, Liu C, Hussein K and Liu X 2016 Thermal performance of a PCM-filled double-glazing unit with different thermophysical parameters of PCM. Solar Energy 133 207-220.
[9] Li S, Sun G, Zou, K and Zhang X 2016 Experimental research on the dynamic thermal performance of a novel triple-pane building window filled with PCM. Sustainable Cities and Society 27 15-22.
[10] Liu C, Wu Y, Bian J, Li D and Liu X 2018 Influence of PCM design parameters on thermal and optical performance of multi-layer glazed roof. Applied energy 212 151-161.
[11] Cao Y, Amir F and Won C 1989 A numerical analysis of Stefan problems for generalized multidimensional phase-change structures using the enthalpy transforming model. International journal of heat and mass transfer 32.7 1289-1298.
[12] Norton T, Delgado A, Hogan E, Grace P and Sun W 2009 Simulation of high pressure freezing processes by enthalpy method. Journal of food engineering 91 260-268.
[13] Cho H and Sunderland E 1969 Heat-conduction problems with melting or freezing. Journal of Heat Transfer 91(3) 421-426.
[14] Abdulmunem R and Jalal M 2018 Indoor investigation and numerical analysis of PV cells temperature regulation using coupled PCM/Fins. International Journal of Heat and Technology 36 121-1222.
Table 1. Thermal properties of materials

| Material       | Melting temp. (°C) | Density (Kg.m\(^{-3}\)) | Thermal cond. (W.m\(^{-1}\).K\(^{-1}\)) | Specific heat (J.kg\(^{-1}\).K\(^{-1}\)) | Latent heat (KJ.kg\(^{-1}\)) |
|----------------|--------------------|--------------------------|-----------------------------------------|------------------------------------------|-------------------------------|
| Glass          | --                 | 2500                     | 0.96                                    | 840                                      | --                            |
| Paraffin wax   | 40                 | 880                      | 0.2                                     | 1800                                     | 174.12                        |

Figure 1. Double glazed window filled with paraffin wax

(a) Temperature of the room with PCM experimentally (°C)
(b) Temperature of the inner glass of the window with PCM experimentally (°C)
Figure 2. Comparison between experimental and numerical results (a) Room temperature (b) Internal surface temperature of the double glazed window with 1.7 cm wax thickness

Figure 3. (a) The monthly average solar radiation in the summer of 2018 (b) Temperature levels for Baghdad in 2018.
Figure 4. Temperature of the inner surface of the double glazed window with different PCM thicknesses at (a) May (b) June (c) July (d) August (e) September (f) October
Figure 5. The variation of the required thickness of the paraffin wax filled double glazed window for the summer weather of Baghdad