Exact Learning of Primitive Formal Systems Defining Labeled Ordered Tree Languages via Queries

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SUMMARY A formal graph system (FGS) is a logic programming system that directly manipulates graphs by dealing with graph patterns instead of terms of first-order predicate logic. In this paper, based on an FGS, we introduce a primitive formal ordered tree system (pFOTS) as a formal system defining labeled ordered tree languages. A pFOTS program is a finite set of graph rewriting rules. A logic program is well-known to be suitable to represent background knowledge. The query learning model is an established mathematical model of learning via queries in computational learning theory. In this learning model, we show the exact learnability of a pFOTS program consisting of one graph rewriting rule and background knowledge defined by a pFOTS program using a polynomial number of queries.

1. Introduction

Graph grammar (see [1]) has been applied to a wide range of fields including pattern recognition and image analysis. Uchida et al. [2] introduced a framework called a formal graph system (FGS) as a graph grammar system. An FGS is a logic programming system that deals with term graph patterns instead of terms of first-order predicate logic.

Large amounts of graph-structured data are now accessible on the Internet. Graph-structured data having tree structures, such as HTML/XML files, glycan data and parsing structures of natural languages, are called tree-structured data and can be represented by ordered trees. A tree language is a set of ordered trees. In order to represent structural features from tree-structured data, we propose an ordered term tree pattern [3], [4], which is a tree pattern that has an ordered tree structure and some internal structured variables. A variable of an ordered term tree pattern has a variable label and can be replaced with an arbitrary ordered tree by hyperedge replacement (see [1]) according to the variable label. Computational learning theory is studied in the field of foundations of computer science. In computational learning theory, the learnability of tree languages has been studied intensively [5]. We consider the problem of identifying an unknown tree language from a specified class of tree languages. In this paper, we use an ordered term tree pattern as an expression of a target tree language, such that each tree language defined using it.

We showed in [4] that the class of tree languages defined by ordered term tree patterns is polynomial time inductively inferable from positive data. The query learning model of Angluin [6] is an established mathematical model of learning via queries in computational learning theory. In this learning model, a learning algorithm accesses oracles, which answer specific types of queries, and collects information about a target.

First of all, we introduce a primitive graph rewriting rule of the form \( p_0(t_0) \leftarrow p_1(t_1), p_2(t_2), \ldots, p_n(t_n) \) satisfying the following conditions. (1) If \( n \geq 1 \), then \( t_0 \) is an ordered term tree pattern of whose variables are distinct. Otherwise, \( t_0 \) is a tree consisting of two vertices and one edge between them. (2) For each \( i (1 \leq i \leq n) \), \( t_i \) is an ordered term tree pattern consisting of two vertices and one variable between them. (3) For any \( i (1 \leq i \leq n) \), the variable label in \( t_i \) appears in \( t_0 \). Here, for \( i (0 \leq i \leq n) \), the symbols \( p_i \) and \( p_i(t_i) \) are called a predicate symbol and a literal, respectively. A primitive graph rewriting rule of the form \( p(t_0) \leftarrow \) is called a fact.

Secondly, as a special type of FGS defining labeled ordered tree languages obtained by replacing variables of ordered term tree patterns with specified ordered trees, we introduce a primitive formal ordered tree system (pFOTS) program, which is a finite set of primitive graph rewriting rules. Moreover, for a pFOTS program \( \Gamma \), we give a condition, called the predicate symbol identifiable condition (PSI condition), on two different edge labels of two facts in order to identify one predicate symbol. According to the manner of logic programming system, the tree language defined using a pFOTS program \( \Gamma \) and its predicate symbol \( r \), denoted by \( L(\Gamma, r) \), is defined as the set of all trees \( T \) such that there exists a derivation starting from the unit goal \( '\leftarrow r(T)' \) and ending with the empty goal \( 'r' \) derived using \( \Gamma \) (see [2]). In Fig. 1, as examples we give a pFOTS program \( \Gamma_{en} \), a new graph rewriting rule \( \alpha \) and the tree language \( L(\Gamma_{en} \cup \{\alpha\}, r) \) defined using the pFOTS program \( \Gamma_{en} \), the graph rewriting rule \( \alpha \) and the predicate symbol \( r \). Moreover, we give a derivation \( D \) starting from the unit goal \( '\leftarrow r(T)' \) and ending with the empty goal \( 'r' \) derived using \( \Gamma_{en} \), where \( T \) is the leftmost tree displayed in \( L(\Gamma_{en} \cup \{\alpha\}, r) \) in Fig. 1. The language \( L(\Gamma_{en} \cup \{\alpha\}, r) \) represents a set of parse trees of a natural language.
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\[ \Gamma_{en} = \begin{cases} 
  e_{n1} = 'q^S(\text{Yes} \rightarrow \text{No})', & e_{n2} = 'q^S(\text{No} \rightarrow \text{Yes})', & e_{n3} = 'q^S(\text{He} \rightarrow \text{No})', \\
  e_{n4} = 'q^S(\text{She} \rightarrow \text{No})', & e_{n5} = 'q^S(\text{works} \rightarrow \text{No})', & e_{n6} = 'q^S(\text{runs} \rightarrow \text{No})', \\
  e_{n7} = 'q^S(\text{in a park} \rightarrow \text{No})', & e_{n8} = 'q^S(\text{in a shop} \rightarrow \text{No})'. 
\end{cases} \]

\[ \alpha = 'r(\text{She} \rightarrow \text{works})', \quad q^S(\text{She} \rightarrow \text{works}), \quad q^S(\text{Yes} \rightarrow \text{No}), \quad q^S(\text{She} \rightarrow \text{works}), \quad q^S(\text{Yes} \rightarrow \text{No}), \quad q^S(\text{She} \rightarrow \text{works}), \quad q^S(\text{Yes} \rightarrow \text{No}). \]

\[ \mathcal{L}(\mathcal{G}_{en} \cup \{\alpha\}, r) = \begin{cases} 
  \mathcal{D}(\text{She} \rightarrow \text{works}), \quad \mathcal{D}(\text{Yes} \rightarrow \text{No}), \quad \mathcal{D}(\text{She} \rightarrow \text{works}), \quad \mathcal{D}(\text{Yes} \rightarrow \text{No}). 
\end{cases} \]

An elementary formal system (EFS) [7] is a logic programming system over strings and is regarded as a special type of pFOTS, since any string is expressed by an ordered tree having only one leaf. We assume that our considered class does not contain the empty tree language. Under this assumption, from the insufficiency result [8] for a
special type of EFS, we can see that there exists no learning algorithm identifying a target tree language defined by a pFOTS program using a polynomial number of equivalence, membership and subset queries. This insufficiency result is due to the fact that a positive example is not necessarily obtained by using equivalence and subset queries. A logic program is well-known to be suitable to represent background knowledge. We call an FGS program or a pFOTS program given as background knowledge a background FGS program or a background pFOTS program, respectively. Let Γ be a pFOTS program as background knowledge satisfying the PSI condition. Thirdly, we present learning algorithms of a target tree language defined using a pFOTS program consisting of one graph rewriting rule and a background pFOTS program Γ satisfying the PSI condition. Finally, by giving examples of FGS programs, we discuss the learnability of a target tree language defined using an FGS program consisting of one graph rewriting rule and a background FGS program that does not satisfy at least one of the conditions of a primitive graph rewriting rule and the PSI condition, by using one positive example and a polynomial number of membership queries.

For the learning of ordered tree languages, Suzuki et al. [4] showed that the class of languages defined by ordered term tree patterns is polynomial time inductively inferable from positive data. Matsumoto et al. [9] showed that finite unions of ordered term tree pattern languages are exactly learnable with a polynomial number of queries. For the learning of graph grammars, Okada et al. [10] showed that some classes of graph pattern languages defined by FGS programs are exactly learnable with a polynomial number of equivalence and restricted subset queries. Shoudai et al. [11] showed that the regular FGS languages of bounded degree with the 1-finite context property and bounded treewidth property are learnable from positive data and membership queries with current distributional learning techniques [12].

This paper is organized as follows. In Sect. 2, we introduce an ordered term tree pattern [3], [4], a primitive formal ordered tree system and the PSI condition on a pFOTS program, and explain the query learning model [6] briefly. In Sect. 3, we propose a learning algorithm that exactly identifies a target tree language defined using one graph rewriting rule and a background pFOTS program having only one predicate symbol. In Sect. 4, we present a learning algorithm that identifies a target tree language defined using one graph rewriting rule and a background pFOTS program having two or more predicate symbols. In Sect. 5, under the assumption that a background FGS program Ξ is not a pFOTS program, we discuss whether or not a target tree language defined using one graph rewriting rule and Ξ is exactly identified by using one positive example and a polynomial number of membership queries. This paper is the full version of the paper [13], with proofs, extended results and complete definitions.

2. Preliminaries

2.1 Term Tree Patterns

Let Σ and Λ be finite alphabets, and X an infinite alphabet. We assume (Σ ∪ Λ) ∩ X = ∅. Let T be a rooted ordered tree (a tree, for short). We denote by V(T) and E(T) the vertex set and the edge set of T, respectively. We denote by (u₀, u₁) the edge between u₀ and u₁ such that u₁ is a child of u₀. For a set or list S, the number of elements in S is denoted by |S|.

Definition 1: Let T be a tree and H₇ a subset of E(T). A term tree pattern t obtained from T and H₇ is a 3-tuple (V, E, H) where V = V(T), E = E(T) \ H₇ and H = H₇. An element (u₀, u₁) in H₇ is called a variable of t and denoted by [u₀, u₁] instead of (u₀, u₁). Every vertex and edge of T is labeled with a symbol in Σ and Λ, respectively. Every variable h = [u₀, u₁] is labeled with a symbol x in X. Such a symbol x is called a variable label of h. We call u₀ the parent port of h and u₁ the child port of h.

For a term tree pattern t obtained from T and H₇, if T and H₇ are clear from the context, we omit them, i.e., we write a term tree pattern t simply. For a term tree pattern t, we denote by V(t), E(t), and H(t) the vertex set, the edge set, and the variable set of t, respectively. Moreover we denote the vertex label of v ∈ V(t) by φ(v) ∈ Σ, the edge label of e ∈ E(T) by ψ(e) ∈ Λ, and the variable label of h ∈ H(T) by λₗ(h) ∈ X. For two vertices u, u′ ∈ V(T), we say that u is the parent of u′ in t if u is the parent of u′ in T. Similarly we say that u′ is a child of u in t if u′ is a child of u in T. For a vertex u ∈ V(T) with no child, we call u a leaf of t. We define the order of the children of each internal vertex u in t as the order of the children of u in T.

Definition 2: We say that two term tree patterns t and t′ are isomorphic, denoted by t ≡ t′, if there is a bijection f from V(t) to V(t′) such that for any u, u′, v, v′ ∈ V(t) the following conditions 1–7 hold:

1. the root of t is mapped to the root of t’ by f,
2. u is the next sibling of u’ if and only if f(u) is the next sibling of f(u’),
3. (u, u’) ∈ E(t) if and only if (f(u), f(u’)) ∈ E(t’),
4. [u, u’] ∈ H(t) if and only if [f(u), f(u’)] ∈ H(t’),
5. φ(u) = φ(f(u)),
6. ψₗ((u, u’)) = ψₗ(f((u, u’))), and
7. λₗ([u, u’]) = λₗ([f(u), f(u’)]) if and only if λₗ([f(u), f(u’)]) = λₗ([f(v), f(v’)]).

A term tree pattern t is linear if all variables in t have mutually distinct variable labels in X. The set of all linear term tree patterns is denoted by LOT’T. A term tree pattern is said to be primitive if it consists of two vertices and one variable between them. We regard a term tree pattern t with
Let $2.2$ Primitive Formal Ordered Tree System

$\Sigma$ of two distinct vertex labels in following way. For each variable $h, t$ tree pattern having at least one variable. Let $x$ correspond to the vertices $u, v$ and by identifying the vertices $t$ denote by $L$ the set of all trees. We denote by $\theta$ as a tree.

For each variable $h = [u_0, u_1]$ a list of two distinct vertices in $T$ where $u_0$ is the root of $T$ and $u_1$ is a leaf of $T$. Then, the form $x := (T, \theta)$ is called a binding for $x$. A new term tree pattern $t'$ is obtained by applying the binding $x := (T, \theta)$ to $t$ in the following way. For each variable $h = [v_0, v_1]$ labeled with $x$ in $t$, we attach a copy $T'$ of $T$ to $t$ by removing $h$ from $H(t)$ and by identifying the vertices $v_0$ and $v_1$ with the vertices $u'_0$ and $u'_1$ of $T'$, respectively, where the vertices $u'_0$ and $u'_1$ of $T'$ correspond to the vertices $u_0$ and $u_1$ of $T$, respectively. We define a new sibling ordering on every vertex $v$ in $t'$ in a natural way [4], [14]. A substitution $\theta$ is a finite set of bindings for distinct variable labels. The term tree pattern obtained from $t$ by applying all bindings in $\theta$ to $t$ simultaneously is denoted by $t\theta$. For a term tree pattern $t$, the term tree pattern language of $t$, denoted by $L(t)$, is the set of all trees obtained from $t$ by replacing all variables in $r$ with arbitrary trees.

2.2 Primitive Formal Ordered Tree System

Let $\Pi$ be a set of unary predicate symbols. We assign a list of two distinct vertex labels in $\Sigma$ to every predicate symbol $p$ in $\Pi$. The list is called the pointer of $p$ and denoted by $\text{pointer}(p)$. An atom is an expression of the form $p(t)$. We use $\text{pointer}(p)$ to specify how to bind a tree to a variable of $t$ in an atom $p(t)$. The first label of $\text{pointer}(p)$ specifies the root of $t$ and the second specifies one of the leaves of $t$. In Sect. 2.1, we gave the definition of a binding $x := (T, \sigma)$. In any binding, we need to give the list $\sigma$ of vertices. Roughly speaking, if $T$ is a tree that is generated by a predicate symbol $p$, the first (resp. second) vertex of $\sigma$ is the vertex of $T$ that has the first (resp. second) label of pointer($p$). In Figs. 1 and 2, all predicate symbols $q_1, q_2, \ldots$ have the same pointer $(a, b)$.

Let $A, B_1, \ldots, B_n$ be atoms $(n \geq 0)$. A graph rewriting

rule (rule, for short) is a clause of the form $A \leftarrow B_1, \ldots, B_n$. The atom $A$ is called the head and the part $B_1, \ldots, B_n$ is called the body of the rule. A rule is called a fact if $n = 0$. We denote by $o(t, x)$ the number of variables in $t$ labeled with $x$.

Definition 3: A rule $p(t) \leftarrow q_1(t_1), \ldots, q_n(t_n)$ is said to be primitive if the following conditions 1–3 hold.

1. If $n = 0$, i.e., the rule is a fact, then $t$ is a tree consisting of two vertices and one edge between them.

2. If $n \geq 1$, then the following conditions are satisfied.

   a. $t$ is a linear term tree pattern such that $|V(t)| > 2$ holds.

   b. Every $t_i$ $(1 \leq i \leq n)$ is a primitive term tree pattern.

3. For every variable $x \in X$, $o(t, x) = o(t_1, x) + \cdots + o(t_n, x) \leq 1$.

For example, all rules in $\Gamma_{\text{en}}$ and the rule $\alpha$ in Fig.1 are primitive. Moreover, all rules in $\Gamma_{\text{OR}}$ and a rule $\beta$ in Fig.2 are also primitive.

Definition 4: A finite set $\Gamma$ of primitive rules is said to be a primitive Formal Ordered Tree System program (pFOTS program, for short) if for each predicate symbol $p$ appearing in $\Gamma$, there is at least one fact whose predicate symbol is $p$.

For a rule $\alpha \leftarrow p(t) \leftarrow q_1(t_1), \ldots, q_n(t_n)$, we denote by $\Pi^p(\alpha)$ and $\Pi^p(\alpha)$ the sets of all predicate symbols in...
the head and body of \( \alpha \), respectively, that is, \( \Pi^p(\alpha) = \{ p \} \) and \( \Pi^b(\alpha) = \{ q_1, \ldots, q_n \} \). For a pFOTS program \( \Gamma \), we denote by \( \Pi(\Gamma) \) the set of all predicate symbols in \( \Gamma \), that is, \( \Pi(\Gamma) = \bigcup_{\text{pred}} \Pi^p(\alpha) \cup \Pi^b(\alpha) \). For any predicate symbol \( p \in \Pi(\Gamma) \), we denote by \( \Lambda(\Gamma, p) \) the set of all edge labels in the facts or rules in \( \Gamma \) whose predicate symbols are \( p \). Figure 2 shows a pFOTS program \( \Gamma_{\text{OR}} \), where \( \Pi(\Gamma_{\text{OR}}) = \{ p \} \) and \( \Lambda(\Gamma_{\text{OR}}, p) = \{ a, b \} \).

A pFOTS program \( \Gamma \) satisfies the predicate symbol identifiable condition (PSI condition, for short) if for each predicate symbol \( p \in \Pi(\Gamma) \), there exist two edge labels \( a_1, a_2 \in \Lambda \) (\( a_1 \neq a_2 \)) such that \( P(a_1) \cap P(a_2) = \{ p \} \), where \( P(a) = \{ q \in \Pi(\Gamma) \mid \text{edge label } q(T) \leftrightarrow \text{edge label } a \} \).\( \Gamma_{mp} = \{(\overrightarrow{a-a-b}), p(\overrightarrow{a-b-b}), q(\overrightarrow{a-b-b}), q(\overrightarrow{a-c-b})\} \)

\[
\begin{align*}
\text{Fig. 3 pFOTS program } \Gamma_{mp}
\end{align*}
\]

Let \( \Gamma \) be a pFOTS program and \( \beta \) a goal. A refutation derived using \( \Gamma \) is a finite derivation from \( \beta_0 \) derived using \( \Gamma \) ending with the empty goal. Let \( F = ( (\beta_i, \theta_i, \alpha_i) )_{i \in S} \) be a refutation from a unit goal \( \beta \) derived using \( \Gamma \). A refutation tree \( T_{i+1} \) of \( F \) is a tree satisfying the following conditions:

1. Every vertex is labeled with a unit goal or the empty goal.
2. The label of the root is the unit goal \( \beta_0 = \beta \).
3. Every leaf is labeled with the empty goal.
4. \( T_0 \) is a tree consisting of only one vertex labeled with \( \beta \). For any \( i = 0, \ldots, t \), \( T_{i+1} \) is a tree obtained from \( T_i \) by applying \( (\beta_i, \theta_i, \alpha_i) \) to \( T_i \) as follows: let \( \beta_i \) be a goal \( \leftarrow A_1, \ldots, A_k \). We assume that \( T_i \) has a leaf labeled with \( \leftarrow A_j \) (\( 1 \leq j \leq n_i \)). Let \( \alpha_i \) be a rule \( A' \leftarrow B_1', \ldots, B_q', (q_i \geq 0) \) and \( \theta_i \) a unifier of \( A \) and \( A' \). If \( q_i = 0 \), that is, \( \alpha_i \) is a fact \( A' \leftarrow \), then \( T_{i+1} \) is obtained by adding a new vertex labeled with empty goal, as the child of the vertex that is labeled with \( \leftarrow A_j \). Otherwise, \( T_{i+1} \) is obtained by adding new \( q_i \) vertices labeled with \( \leftarrow B_1', \ldots, B_{q_i}' \), respectively, as the children of the vertex that is labeled with \( \leftarrow A_j \).

Figure 4 shows an example of a refutation from a unit goal derived using the pFOTS program \( \Gamma_{OR} \) in Fig. 2 and its refutation tree.

For a pFOTS program \( \Gamma \) and a predicate symbol \( p \) in \( \Pi(\Gamma) \), we denote by \( L(\Gamma, p) \) the set of all trees \( T \in OT \) such that there exists a refutation from the unit goal \( \leftarrow p(T) \) derived using \( \Gamma \). We say that a subset \( L \subseteq OT \) is a pFOTS lan-
Fig. 4 Tree in $\mathcal{LOTT}$, refutation $F$ from the unit goal $\leftarrow p(T)$ derived using $\Gamma_{OT}$ and refutation tree $T$ of $F$. For each $i$ ($1 \leq i \leq 5$), $ot'_i$ is a variant of a rule $oti$ in Fig. 2. The square box with thick line denotes the empty goal.
get \( L_\epsilon \in L \) if \( A \) outputs a representation \( \pi \in R \) satisfying \( L(\pi) = L(\pi_\epsilon) \). In the next section, we will present a learning algorithm that exactly identifies a target \( L_\epsilon \in \mathcal{P}\mathcal{G}R\mathcal{R}(\Gamma, r) \) using one positive example and a polynomial number of membership queries, that is, the algorithm outputs a primitive rule \( \alpha \in \mathcal{P}\mathcal{G}R\mathcal{R}(\Gamma, r) \) satisfying \( L(\Gamma \cup \{\alpha\}, r) = L_\epsilon \).

3. Exact Learning of Tree Languages with Background pFOTS Programs Having Only One Predicate Symbol via Queries

In this paper, for a background pFOTS program \( \Gamma \) and the set \( \mathcal{P}\mathcal{G}R\mathcal{R}(\Gamma, r) \) of representations with \( r \in \Pi \setminus \Pi(\Gamma) \), we consider the learnability of the class \( \mathcal{P}\mathcal{G}R\mathcal{R}(\Gamma, r) \) using one positive example and a polynomial number of membership queries. From Theorem 16 in [4], if a background pFOTS program \( \Gamma \) satisfies the PSI condition, a target tree language \( L_\epsilon \), in \( \mathcal{P}\mathcal{G}R\mathcal{R}(\Gamma, r) \) is identified by identifying a rule \( \alpha \), in \( \mathcal{P}\mathcal{G}R\mathcal{R}(\Gamma, r) \) such that \( L_\epsilon = L(\Gamma \cup \{\alpha\}, r) \) holds. Hence, by using one positive example and a polynomial number of membership queries, we construct a primitive rule \( \alpha \in \mathcal{P}\mathcal{G}R\mathcal{R}(\Gamma, r) \) that satisfies \( L_\epsilon = L(\Gamma \cup \{\alpha\}, r) \) on the basis of the following strategy.

(1) Construct contraction operators from \( \Gamma \).
(2) Construct a minimal tree \( T_{\min} \) with respect to the number of edges in \( L_\epsilon \), by recursively applying the constructed contraction operators to a given positive example in \( L_\epsilon \).
(3) Construct a target primitive rule \( \alpha \), from \( T_{\min} \) such that \( L_\epsilon = L(\Gamma \cup \{\alpha\}, r) \) holds as follows.
   (a) Identify a term tree pattern in the head of \( \alpha \), by replacing some edges in \( T_{\min} \) with appropriate variables.
   (b) Identify all predicate symbols in the body of \( \alpha \).

For a background pFOTS program \( \Gamma \) satisfying the PSI condition and the set \( \mathcal{P}\mathcal{G}R\mathcal{R}(\Gamma, r) \) of representations with \( r \in \Pi \setminus \Pi(\Gamma) \), in Sect. 3.2 and Sect. 4, we give two query learning algorithms based on the above strategy for two cases of \( |\Pi(\Gamma)| = 1 \) and \( |\Pi(\Gamma)| \geq 2 \), respectively. In the case of \( |\Pi(\Gamma)| = 1 \), it is not necessary to identify predicate symbols in the body of a target primitive rule \( \alpha \), that is, process (3)-(b) of the strategy is unnecessary. On the other hand, in the case of \( |\Pi(\Gamma)| \geq 2 \), we have to identify predicate symbols in the body of a target primitive rule \( \alpha \), since, \( L(\Gamma, p) \neq L(\Gamma, q) \) holds for two distinct predicate symbols \( p, q \in \Pi(\Gamma) \). To make our results easier to understand, we explain (1) and (2) of the above strategy in detail in Sect. 3.1. Moreover, in Sect. 3.2, we show results for the case of \( |\Pi(\Gamma)| = 1 \). Finally, in Sect. 4, we show results for the case of \( |\Pi(\Gamma)| \geq 2 \).

3.1 Algorithm for Finding Minimum Positive Example by Contraction Operators

In this subsection, we explain our ideas for (1) and (2) in the above strategy.

Let \( \Gamma \) be a pFOTS program. For convenience, i.e., in order to describe our learning algorithm simply, we use two kinds of special symbols that are not in \( \Sigma \cup \Delta \). For any \( p \in \Pi(\Gamma) \), we use the symbol \( \epsilon \) to specify two vertex labels in \( \text{pointer}(p) \), and the symbols "?\( \_p \)" to specify the edge labels in \( \Lambda(\Gamma, p) \). Let \( p \) be a predicate symbol of the head of a rule in \( \Gamma \). Let \( \alpha = \langle p(\epsilon_i) \leftarrow p_1(\epsilon_i), \ldots, p_n(\epsilon_i) \rangle \) (\( n \geq 1 \)) be a primitive rule in \( \Gamma \) and \( \beta = \langle p(\bar{\epsilon}) \leftarrow \epsilon \rangle \) a fact in \( \Gamma \). Let \( H(\epsilon_i) = \{h_1, \ldots, h_n\} \). Without loss of generality, we assume that for any \( i (1 \leq i \leq n) \), the variable label of \( h_i \) is the same as that of the unique variable of \( \epsilon_i \). Let \( P_{\alpha} \) be the tree obtained from \( \alpha \) in the following way: each \( h_i (1 \leq i \leq n) \) is replaced with an edge whose label is the special symbol "?\( _p \)", and each vertex label in \( \text{pointer}(p) \) is changed to the special symbol \( \epsilon \). Let \( Q_\beta \) be the tree obtained from \( \beta \) by replacing the labels of two vertices of \( \beta \) with the special symbol \( \epsilon \). The pair \( (P_\alpha, Q_\beta) \) is called the contraction pair of \( \alpha \) and \( \beta \). For a pFOTS program \( \Gamma \) and a predicate symbol \( p \in \Pi(\Gamma) \), the contraction pair set of \( \Gamma \) and \( p \), denoted by \( \mathcal{C}(\Gamma, p) \), is the set of all contraction pairs of \( \alpha \) and \( \beta \) that satisfy the following conditions 1–3:

1. \( \alpha \) is a primitive rule in \( \Gamma \) with \( \Pi^h(\alpha) \neq \emptyset \).
2. \( \beta \) is a fact in \( \Gamma \) such that the edge label appearing in the head of \( \beta \) is the lexicographically first symbol in \( \Lambda(\Gamma, p) \).
3. \( \Pi^h(\alpha) = \Pi^h(\beta) = \{p\} \).

The contraction pair set of \( \Gamma \), denoted by \( \mathcal{C}(\Gamma) \), is the set \( \bigcup_{p \in \Pi(\Gamma)} \mathcal{C}(\Gamma, p) \).

Example 1: For the pFOTS program \( \Gamma_{en} \) in Fig. 1, we present the contraction pair set \( \mathcal{C}(\Gamma_{en}) = \{\langle Q_{en}^1, P_{en}^1 \rangle, \langle P_{en}^2, Q_{en}^2 \rangle, \langle P_{en}^3, Q_{en}^3 \rangle\} \) of \( \Gamma_{en} \), where \( P_{en}^1, P_{en}^2, P_{en}^3 \) and \( Q_{en}^1, Q_{en}^2, Q_{en}^3 \) are shown in Fig. 5.

Example 2: For the pFOTS program \( \Gamma_{OT} \) in Fig. 2, we present the contraction pair set \( \mathcal{C}(\Gamma_{OT}) = \{\langle Q_{OT}^1, P_{OT}^1 \rangle, \langle P_{OT}^2, Q_{OT}^2 \rangle, \langle P_{OT}^3, Q_{OT}^3 \rangle\} \) of \( \Gamma_{OT} \), where \( P_{OT}^1, P_{OT}^2, P_{OT}^3 \) and \( Q_{OT}^1, Q_{OT}^2, Q_{OT}^3 \) are shown in Fig. 6.

Let \( T \) be a tree in \( OT \). A subgraph \( S \) of \( T \) is said to be a subtree of \( T \) if \( S \) is a tree. A vertex \( u \) in \( V(S) \) is called a boundary vertex of \( S \) if \( u \) is adjacent to a vertex in \( V(T) \setminus V(S) \). A subtree \( S \) of \( T \) is said to be a 2-port subtree of \( T \) if the root of \( S \) is either the root of \( T \) or a boundary vertex of \( S \), and all leaves of \( S \) except at most one boundary vertex are leaves of \( T \). Let \( r_S \) be the root of \( S \) and \( \ell_S \) the boundary vertex that is a leaf of \( S \) if it exists.

For a 2-port subtree \( S \) of \( T \) and a contraction pair \( (P, Q) \) of \( \Gamma \), we say that \( S \) matches \( P, \) denoted by \( S \approx P, \) if there is a bijection \( f : V(S) \rightarrow V(P) \) that satisfies the following conditions 1–5:

1. the root of \( S \) is mapped to the root of \( P \) by \( f \),
2. for any \( u \) and \( u' \) in \( V(S) \), \( u \) is the next sibling of \( u' \) if and only if \( f(u) \) is the next sibling of \( f(u') \),
3. for any \( u \) and \( u' \) in \( V(S) \), \( (u, u') \in E(S) \) if and only if \( (f(u), f(u')) \in E(P) \),
4. for any $u \in V(S)$, if $u$ is neither $r_S$ nor $\ell_S$, then $\varphi_S(u) = \varphi_P(f(u))$ holds, and
5. for any $(u, u') \in E(S)$, if $\varphi_P(f(u), f(u')) =$ “$\rightarrow$” for some $p \in \Pi(\Gamma)$, then $\psi_S((u, u')) \in \Lambda(\Gamma, p)$ holds, otherwise $\psi_S((u, u')) = \varphi_P(f(u), f(u'))$.

Note that if $\ell_S$ exists, then $\varphi_P(f(\ell_S)) = \epsilon$ holds.

Let $(P, Q)$ be a contraction pair in CPS$(\Gamma)$. A contraction operator, denoted by $\triangleright_{(P,Q)}(T, S)$, is a function that returns a new tree from a tree $T$ and a 2-port subtree $S$ of $T$ as output. If $S \neq P$ holds, we define the tree $\triangleright_{(P,Q)}(T, S)$ as $T$. We assume that $S \cong P$ holds. Let $f$ be a bijection from $V(S)$ to $V(P)$ realizing $S \cong P$. Let $\ell_S'$ be the leaf of $S$ such that $\varphi_P(f(\ell_S')) = \epsilon$ holds. If $\ell_S$ exists, then $\ell_S' = \ell_S$ holds. The tree $\triangleright_{(P,Q)}(T, S)$ is obtained from $T$ by removing all of the edges in $E(S)$, and by identifying $r_S$ of $S$ with the root of $Q$ and the children of $\ell_S'$ in $S$ with the leaf of $Q$.

The following lemma on pFOTS plays an important role in our algorithms.

**Lemma 1:** Let $\Gamma$ be a pFOTS program, $r$ a predicate symbol in $\Pi \setminus \Pi(\Gamma)$, $\alpha$ a primitive rule in $pGR(\Gamma, r)$, and $t_o$ a linear term tree pattern appearing in the head of $\alpha$. For any tree $T \in L(\Gamma \cup \{\alpha\}, r)$, if $|V(T)| > |V(t_o)|$ holds, then there are a 2-port subtree $S$ of $T$ and a contraction pair $(P, Q) \in CPS(\Gamma)$ such that $\triangleright_{(P,Q)}(T, S) \in L(\Gamma \cup \{\alpha\}, r)$ holds.

**Proof.** Since $T \in L(\Gamma \cup \{\alpha\}, r)$, there is a refutation tree $T'$ from the unit goal $\leftarrow r(T)$. Let $T'$ be the root of $T'$. Since $|V(T')| > |V(t_o)|$, there is a proper descendant $\chi$ of $v_T$ all of whose children in $T'$ are labeled with facts in $\Gamma$. Let $\leftarrow p(S')$ be the label of $\chi$ for a tree $S' \in OT$ and a predicate symbol $p \in \Pi(\Gamma)$. Then there are a rule $p(t_0) \leftarrow q_1(t_1), \ldots, q_n(t_n)$ and a substitution $\theta$ such that $S' \cong t_0\theta$ holds and all $t_i (1 \leq i \leq n)$ are trees of only one edge. From the definition of the contraction pair set $\Pi(\Gamma)$ and $p$, there is a contraction pair $(P, Q)$ in $CPS(\Gamma, p)$ such that $S' \cong P$ holds. From the definition of a refutation tree, there is a 2-port subtree $S$ of $T$ such that $S$ is isomorphic to $S'$ except the two vertex labels of $S'$ specified by $pointer(p)$. Therefore $S \cong P$ holds. The refutation tree of the tree $\triangleright_{(P,Q)}(T, S)$ is obtained from $T'$ by removing all of the children and grandchildren of $\chi$, and by replacing $\chi$ with the vertex $\chi'$ having the unit goal $\leftarrow p(Q)$ as its label and its child having the empty goal as its label. Therefore the tree $\triangleright_{(P,Q)}(T, S) \in L(\Gamma \cup \{\alpha\}, r)$ holds.

**Procedure 1 Contraction**

**Input:** A tree $T$ in the target $L$, and the contraction pair set $CPS(\Gamma)$ of $\Gamma$.

**Output:** A tree $T_{\text{min}} \in L(t_o)$ s.t. $|V(T_{\text{min}})| \leq |V(T')|$ holds for all $T' \in L_{\text{r}}$.

1: $T_{\text{min}} := T$;
2: repeat
3: for all $(P, Q) \in CPS(\Gamma)$ do
4: for all subtrees $S$ of $T_{\text{min}}$ satisfying $S \cong P$ do
5: $T_{\text{tmp}} := \triangleright_{(P,Q)}(T_{\text{min}}, S)$;
6: if $|V(T_{\text{tmp}})| < |V(T_{\text{min}})|$ then
7: $T_{\text{min}} := T_{\text{tmp}}$;
8: end if
9: end for
10: end for
11: until $T_{\text{min}}$ does not change;
12: output $T_{\text{min}}$;

**Lemma 2:** Let $\Gamma$ be a pFOTS program, $r$ a predicate symbol in $\Pi \setminus \Pi(\Gamma)$, $\alpha$ a primitive rule in $pGR(\Gamma, r)$, and $t_o$ a linear term tree pattern appearing in the head of $\alpha$. For any tree $T \in L(\Gamma \cup \{\alpha\}, r)$ and the contraction pair set $CPS(\Gamma)$ of $\Gamma$, Procedure 1 (Contraction) finds a tree $T_{\text{min}} \in L(\Gamma \cup \{\alpha\}, r)$ with $|V(T_{\text{min}})| = |V(t_o)|$ using $O(n^2)$ membership queries where $n = |V(T)|$.

**Proof.** We will prove that, given any tree $T \in L(\Gamma \cup \{\alpha\}, r)$ and the contraction pair set $CPS(\Gamma)$ of $\Gamma$ as inputs, Procedure 1 (Contraction) outputs a tree $T_{\text{min}}$ obtained from $T$ such that $|V(T_{\text{min}})| = |V(t_o)|$ holds. We assume that $|V(T_{\text{min}})| > |V(t_o)|$ holds. From Lemma 1, there are a 2-port subtree $S$ of $T_{\text{min}}$ and a contraction pair $(P, Q) \in CPS(\Gamma)$ such that $\triangleright_{(P,Q)}(T_{\text{min}}, S) \in L(\Gamma \cup \{\alpha\}, r)$ and $|V(\triangleright_{(P,Q)}(T_{\text{min}}, S))| < |V(T_{\text{min}})|$ holds. Therefore if $|V(T_{\text{min}})| > |V(t_o)|$, the repeat-loop of Procedure 1 does not finish. Hence $|V(T_{\text{min}})| = |V(t_o)|$ holds. The number of vertices of $T_{\text{min}}$ decreases by at least one every iteration at lines 2–13 of Procedure 1. The for-loop at lines 4–11 of Procedure 1 enumerates $O(n)$ subtrees, since the number of subtrees of $T$ that are nearly isomorphic to the first element of one pair of CPS$(\Gamma)$ is $O(|CPS(\Gamma)| \cdot n)$. Note that $|CPS(\Gamma)| \leq |\Gamma|$ holds. Then, since
Algorithm 2 \textsc{Learning}(\Gamma, T)

\textbf{Input:} A tree $T \in OT$ that is a positive example of a target tree language $L$, i.e., $T \in L_
u$.

\textbf{Output:} A primitive rule $\alpha \in p\mathcal{GRR}(\Gamma, r)$ satisfying $L(\Gamma \cup \{\alpha\}, r) = L_
u$.

1: // Preprocessing Step
2: Make the contraction pair set CPS(\Gamma) of \Gamma;
3: // Contraction Step
4: $T_{\min} := \text{Contraction}(T, CPS(\Gamma));$ (Procedure 1)
5: // Variable Replacement Step
6: $\gamma := \text{Variable Replacement}(T_{\min});$ (Procedure 3)
7: // Postprocessing Step
8: $\alpha := \text{Make Rule}(\gamma);$ (Procedure 4)
9: output $\alpha$;

Procedure 3 \textsc{Variable Replacement}

\textbf{Input:} A tree $T_{\min}$ in the target $L_
u$.

\textbf{Output:} A term tree pattern $t \in LOT^\Gamma$.

1: $t := T_{\min}$;
2: for all $e \in E(T_{\min})$ do
3: \hspace{1em} $\psi_{\text{tmp}}(e) := \alpha$, where $\alpha \in \Lambda(\Gamma, p) \setminus \{\psi_{\text{tmp}}(e)\}$;
4: \hspace{1em} if $\text{MQ}(\text{tmp}) = \text{yes}$ then
5: \hspace{2em} $h_e := [u, u']$, where $e = (u, u')$;
6: \hspace{2em} $\text{tmp} := (V(t), E(t) \setminus \{e\}, H(t) \cup \{h_e\})$;
7: \hspace{2em} $\Delta_{\text{tmp}}(h_e) := x$, where $x \in X \setminus \{\lambda(h) \mid h \in H(t)\}$;
8: end if
end for
12: output $t$;

the number of primitive rules in $\Gamma$ is constant, Procedure 1 uses $O(n^3)$ membership queries.

3.2 Learning Algorithm Given Background pFOTS Program Having Only One Predicate Symbol

Let $\Gamma$ be a background pFOTS program such that $\Pi(\Gamma) = \{p\}$ holds and $r$ a predicate symbol in $\Pi(\Gamma)$, $\Pi(\Gamma)$, for a target tree language $L_\nu \subseteq \mathcal{GRR}(\Gamma, r)$ with $r \in \Pi(\Gamma)$, Algorithm 2 (\textsc{Learning}_{\Gamma,r}) finds a primitive rule $\alpha \in \mathcal{GRR}(\Gamma, r)$ satisfying $L_\nu = L(\Gamma \cup \{\alpha\}, r)$ by using one positive example and membership queries in the following way. First of all, Algorithm 2 makes the contraction pair set $CPS(\Gamma)$ of $\Gamma$ from the pFOTS program $\Gamma$. Secondly, in Procedure 1 (\textsc{Contraction}), Algorithm 2 constructs the minimal tree $T_{\min}$ in the target language $L_\nu$ by recursively applying contraction operators defined using $CPS(\Gamma)$ to the tree given as one positive example. Thirdly, in Procedure 3 (\textsc{Variable Replacement}), Algorithm 2 constructs a term tree pattern $t$ obtained from $T_{\min}$ by recursively replacing edges of $r$ with variables using membership queries. Finally, in Procedure 4 (\textsc{Make Rule}), Algorithm 2 makes the primitive rule $\alpha \in \mathcal{GRR}(\Gamma, r)$ from $t$.

We will prove the correctness and the time complexity of Algorithm 2 (\textsc{Learning}_{\Gamma,r}) when for a background pFOTS program $\Gamma$, $|\Pi(\Gamma)| = 1$ holds. From Lemma 1, we have the following theorem.

\textbf{Theorem 1:} Let $\Gamma$ be a fixed pFOTS program such that $\Gamma$ has at least two facts and $|\Pi(\Gamma)| = 1$ holds, $r$ a predicate symbol in $\Pi \setminus \Pi(\Gamma)$, and $L_\nu$ a target tree language in $\mathcal{GRR}(\Gamma, r)$. Then, a primitive rule $\alpha \in \mathcal{GRR}(\Gamma, r)$ satisfying $L_\nu = L(\Gamma \cup \{\alpha\}, r)$ is exactly identified using $O(n^3)$ membership queries and one positive example $t \in L_\nu$, where $n = |V(T)|$.

\textbf{Proof.} Algorithm 2 (\textsc{Learning}_{\Gamma,r}) consists of four steps. In \textsc{Preprocessing Step}, the contraction pair set $CPS(\Gamma)$ of $\Gamma$ is created. In \textsc{Contraction Step}, Algorithm 2 decreases the size of $T_{\min}$ using the contraction operators constructed from $CPS(\Gamma)$ repeatedly while $\text{MQ}(\text{tmp})$ returns yes. After that, in \textsc{Variable Replacement Step}, Algorithm 2 makes a linear tree term pattern $t$ by replacing edges of $T_{\min}$ with new variables if $\text{MQ}(\text{tmp})$ returns yes. Then, in \textsc{Postprocessing Step}, Algorithm 2 makes a primitive rule $\alpha$ from $t$ such that the predicate symbol of the head of $\alpha$ is $r$ and $L_\nu = L(\Gamma \cup \{\alpha\}, r)$ holds.

Let $\alpha_r$ be a primitive rule in $\mathcal{GRR}(\Gamma, r)$ such that $L_\nu = L(\Gamma \cup \{\alpha_r\}, r)$ holds, and $t_r$ the linear term tree pattern appearing in the head of $\alpha_r$. From Lemma 2, in \textsc{Contraction Step}, for a tree $T \in L_\nu$ and the contraction pair set $CPS(\Gamma)$ of $\Gamma$, Algorithm 2 finds a tree $T_{\min} \in L_\nu$ with $|V(T_{\min})| = |V(T)|$ using $O(n^3)$ membership queries. Here, we will prove that after \textsc{Variable Replacement Step}, $t_r \equiv t$ holds. We see that after \textsc{Contraction Step}, there is a bijection $f : V(T_{\min}) \rightarrow V(t)$ such that for any $u, u' \in V(T_{\min})$, $(u, u') \in E(T_{\min})$ if and only if $(f(u), f(u')) \in E(t)$ or $([f(u), f(u')] \subseteq H(t)$. In particular, if $(f(u), f(u')) \in E(t)$, then $\psi_{\text{tmp}}(u, u') = \psi_{\text{tmp}}((f(u), f(u')))$. The for-loop at lines 2–11 of Procedure 3, for an edge $e = (u, u') \in E(T_{\min})$, if $(f(u), f(u')) \in E(t)$, $\text{MQ}(\text{tmp})$ does not return yes. That is, if $\text{MQ}(\text{tmp})$ returns yes, then $(f(u), f(u')) \in H(t)$ holds. Conversely, if $(f(u), f(u')) \in H(t)$, whatever the edge label of $e = (u, u')$ is, $\text{MQ}(\text{tmp})$ returns yes. Therefore, we have $t \equiv t_r$. Trivially, in \textsc{Variable Replacement Step}, Procedure 3 uses at most $n$ membership queries. Then Algorithm 2 (\textsc{Learning}_{\Gamma,r}) uses $O(n^3)$ membership queries. Hence, the theorem holds.

From Theorem 1, when the pFOTS program $\Gamma_{OT}$, which defines the set of all ordered trees, is given as background knowledge, we can see that the following corollary holds.

\textbf{Corollary 1:} Let $\Gamma_{OT}$ be the pFOTS program in Fig. 2 and $r$ a predicate symbol in $\Pi(\Gamma_{OT})$. Then, Algorithm 2 (\textsc{Learning}_{\Gamma_{OT},r}) exactly identifies a target tree language in
Algorithm 5 LearningPluralPred(T, r)
Input: A tree $T \in OT$ that is a positive example of a target tree language $L_*$, i.e., $T \in L_*$.
Output: A primitive rule $\alpha \in pGRR(\Gamma, r)$ satisfying $L(\Gamma) \cup \{\alpha, r\} = L_*$. 
1: // Preprocessing Step
2: Make the contraction pair set CPS($\Gamma$) of $\Gamma$;
3: // Contraction Step
4: $T_{\text{min}} := \text{Contraction}(T, CPS(\Gamma))$; (Procedure 1)
5: // Variable Replacement Step
6: $(t, X) := \text{VariableReplacement}(T_{\text{min}})$; (Procedure 6)
7: // Postprocessing Step
8: $\alpha := \text{MakeRule}(t, X)$; (Procedure 7)
9: output $\alpha$.

$\mathcal{L}(\text{LOTT})$ using one positive example $T$ and $O(n^2)$ membership queries, where $n = |V(T)|$.

4. Exact Learning of Tree Languages with Background pFOTS Programs Having Two or More Predicate Symbols with Queries

In this section, for a background pFOTS program $\Gamma$ satisfying the PSI condition and the set $pGRR(\Gamma, r)$ of representations with $r \in \Pi \setminus \Pi(\Gamma)$, we consider the learnability of the class $pGRR_P(\Gamma, r)$ in the query learning model under the assumption of $|\Pi(\Gamma)| \geq 2$. If a background pFOTS program $\Gamma$ has two or more predicate symbols such as $\Gamma_e$ with $|\Pi(\Gamma_e)| = 4$ in Fig. 1, it is necessary to determine the predicate symbols in the body of a target rule. We identify those predicate symbols in Variable Replacement Step, while identifying the variables of the term tree pattern in the head of the target rule.

Let $\Gamma$ be a pFOTS program such that $\Gamma$ satisfies the PSI condition and $|\Pi(\Gamma)| \geq 2$ holds. We can present a learning algorithm, denoted by Algorithm 5 (LearningPluralPred(T, r)), by replacing Procedures 3 and 4 of Algorithm 2 with Procedures 6 and 7, respectively. By using Algorithm 5, given a pFOTS program having two or more predicate symbols and satisfying the PSI condition as background knowledge, we have the following theorem.

Theorem 2: Let $\Gamma$ be a background pFOTS program such that $\Gamma$ satisfies the PSI condition and $|\Pi(\Gamma)| \geq 2$, $r$ a predicate symbol in $\Pi \setminus \Pi(\Gamma)$, and $L_*$ a target tree language in $pGRR_P(\Gamma, r)$. Then, a primitive rule $\alpha \in pGRR(\Gamma, r)$ satisfying $L_* = L(\Gamma) \cup \{\alpha, r\}$ is exactly identified using one positive example $T \in L_*$ and $O(n^2)$ membership queries with $n = |V(T)|$.

Proof: In a similar way to Theorem 1, the correctness of Algorithm 5 (LearningPluralPred(T, r)) can be shown. Here we will estimate the time complexity of Algorithm 5. Since $\Gamma$ is a pFOTS program satisfying the PSI condition, Variable Replacement Step is recursively applied to a tree $T_{\text{min}}$ obtained after Contraction Step at most $2 \cdot V(T_{\text{min}})$ times. After that, we obtain a primitive rule $\alpha$ by using Procedure 7 such that $L(\Gamma) \cup \{\alpha, r\} = L_*$ holds. Hence, since $|V(T_{\text{min}})| \leq n$, $\alpha$ is exactly identified using $O(|\Gamma| \cdot n^3)$ membership queries and

Procedure 6 VariableReplacement
Input: A tree $T_{\text{min}}$ in the target $L_*$.
 Output: A term tree pattern $t \in \mathcal{L}(\text{LOTT})$ and a collection of sets of variable labels $\{X_p\}_{p \in \Pi(\Gamma)}$.
1: $t := T_{\text{min}}$;
2: for all predicate symbols $p \in \Pi(\Gamma)$ do
3: Let $(a_p, b_p)$ be a predicate identifier of $p$;
4: $X_p := \emptyset$;
5: end for
6: for all $e \in E(T_{\text{min}})$ do
7: $T_{\text{imp}}^1 := T_{\text{min}}$;
8: $T_{\text{imp}}^2 := T_{\text{min}}$;
9: for all predicate symbols $p \in \Pi(\Gamma)$ do
10: if $MQ(T_{\text{imp}}^1) = \text{yes}$ and $MQ(T_{\text{imp}}^2) = \text{yes}$ then
11: $h_e := [a, u']$, where $e = (u, u')$;
12: $t_{\text{imp}} := (V(t), E(t) \setminus \{e\}, H(t) \cup \{h_e\})$;
13: $\lambda_{t_{\text{imp}}}(h_e) := x$, where $x \in X \setminus \{\lambda_{t_{\text{imp}}}(h) | h \in H(t)\}$;
14: $X_p := X_p \cup \{x\}$;
15: end if
16: $t := t_{\text{imp}}$;
17: break;
18: end for
19: end for
20: end for
21: output $t$ and $\{X_p\}_{p \in \Pi(\Gamma)}$.

Procedure 7 MakeRule
Input: A linear term tree pattern $t \in \mathcal{L}(\text{LOTT})$ and a collection $\mathcal{X}$ of sets of variable labels $\{X_p\}_{p \in \Pi(\Gamma)}$.
Output: A primitive rule $\alpha \in pGRR(\Gamma, r)$.
1: Let $H(t) := \{h_1, \ldots, h_t\}$;
2: for $i := 1$ to $t$ do
3: Let $p_i$ be a predicate symbol in $\Pi(\Gamma)$ s.t. $\lambda_{t}(h_i) \in X_{p_i}$;
4: Let $u_i$ and $v_i$ be distinct new vertices;
5: $t_i := ((u_i, v_i), \emptyset, \{(u_i, v_i)\})$;
6: $\phi_i(u_i) := \text{pointer}(p_i)$;
7: $\lambda_{t_i}(u_i, v_i) := \lambda_{t}(h_i)$;
8: end for
9: output $\alpha = \langle r(t) \leftrightarrow p_1(t_1), \ldots, p_t(t_t) \rangle$.

one positive example. Since $|\Gamma|$ is constant, the statement holds.

We can extend pFOTS programs to deal with the class of ordered tree languages in case that $|\Lambda|$ is infinite. by using the special atom determining whether or not the edge label is in $\Lambda$. Therefore, it is easy to see that Theorems 1 and 2 hold if $|\Lambda|$ is infinite.

5. Discussion

In this section, under some relaxed conditions of pFOTS programs, that is, the assumption that background knowledge $\Gamma$ has rules that do not satisfy at least one of the conditions 1 and 2 in Definition 3, we discuss whether or not a target tree language $L_* = L(\Gamma) \cup \{r\}$ with background knowledge $\Gamma$ and $r \in \Pi \setminus \Pi(\Gamma)$, we need to construct a linear term tree pattern $t$, in the head of a target
primitive rule $\alpha_*$ such that $L_\alpha = L(\Gamma \cup \{\alpha_*\}, r)$ holds. If $\Gamma$ is a pFOTS program, we can see that $|E(t_\alpha)| + |H(t_\alpha)| = |E(T_{min})|$ holds, where $T_{min}$ is a minimal tree such that there exists a refutation from the unit goal $\leftarrow r(T_{min})$ derived using $\Gamma \cup \{\alpha_*\}$. Therefore, we can construct a target term tree pattern $t_\alpha$ obtained from $E(T_{min})$ by replacing edges with variables recursively while membership queries return “yes”. We consider the case that $\Gamma$ has a fact $p(t) \leftarrow$ such that $t$ consists of two or more edges. Let $T_{min}$ be a minimal tree such that there exists a refutation $F$ from the unit goal $\leftarrow r(T_{min})$ derived using $\Gamma \cup \{\alpha_*\}$ satisfying $(\beta, \theta, \gamma)$ holds, since $\Gamma \cup \{\alpha_*\}$ does not appear in a target graph rewriting rule $\beta$ and any rule in $\Lambda(\Gamma, p)$ does not appear in a target graph rewriting rule $\beta$ and any rule in $\Lambda(\Gamma, p)$. However, $L(\Gamma \cup \{\beta\}, r) \neq L(\Gamma \cup \{\alpha_*\}, r)$ holds, since $T_{bbb} \notin L(\Gamma \cup \{\beta\}, r)$ but $T_{bbb} \notin L(\Gamma \cup \{\alpha_*\}, r)$. This example shows that identification of the target language may fail even if we recursively apply the variable replacements to $T_{min}$ while membership queries return “yes”. For a similar reason, we can see that identification of a target rule language may fail for any background knowledge containing rules having variables consisting of 3 ports or more fail.

Moreover, let a pFOTS program $\Gamma$ have at most one fact for each predicate symbol in $\Pi(\Gamma)$. We can easily construct a target primitive rule $\alpha_*$ without any membership query, if $\Gamma$ satisfies the following condition. For each predicate symbol $p$ in $\Pi(\Gamma)$, the edge label in $\Lambda(\Gamma, p)$ does not appear in a target graph rewriting rule $\alpha_*$. For any rule in $\Gamma$ except a fact whose predicate symbol is $p$. Otherwise, since we can use membership queries only, we need to identify edges replaced with variables by using subtrees instead of edge labels. If $\Gamma$ is a pFOTS program such that $L(\Gamma, p)$ equals the set of all ordered trees having only one edge label, we conjecture that, using a similar strategy to that of the proof of Theorem 22 in [4], a target tree language defined

$$
\begin{align*}
\Gamma_{CE} = \{ & p\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow, \ p\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow, \ p\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow, \ p\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow \\
& p\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow p\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow, \ p\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow, \ p\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow \\
& p\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow p\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow, \ p\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow, \ p\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow \\
& \alpha_* = r\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow p\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow, \ p\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow, \ p\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow \}
\end{align*}

\begin{align*}
\beta = r\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow p\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow, \ p\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow, \ p\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow, \ p\left(\begin{array}{c}
\alpha \\
\beta \\
\gamma
\end{array}\right) \leftarrow \}
\end{align*}

Fig. 7  FGS program $\Gamma_{CE}$, which is not a pFOTS program, target primitive rule $\alpha_*$ and language $L(\Gamma_{CE} \cup \{\alpha_*\}, r)$
using a pFOTS program such as $\Gamma$ and a predicate symbol $r \in \Pi \setminus \Pi(\Gamma)$ can be identified by using one positive example and a polynomial number of membership queries with respect to the number of edges of the positive example. Otherwise, that is, if $\Gamma$ is an arbitrary pFOTS program, to identify a target tree language $L_r$ by using one positive example and a polynomial number of membership queries, we need a new strategy for constructing an appropriate primitive rule $\alpha$ such that $L(\Gamma \cup \{\alpha\}, r) = L_r$, holds.

To identify a target tree language defined using a background FGS program $\Xi$, such as $\Gamma_{CE}$ and one graph rewriting rule in $pGRR(\Xi, r)$, by using one positive example and a polynomial number of membership queries, Procedures 3 of $\text{Learning}(\Xi, r)$ or 6 of $\text{LearningPluralPred}(\Xi, r)$ is necessary to be modified.

6. Conclusions

We have introduced a primitive formal ordered tree system (pFOTS) as a formal system defining ordered tree languages based on FGS [2]. For a pFOTS program $\Gamma$ as background knowledge, we have shown the exact learnability of the class $pGRR\rho(\Gamma, r)$ of tree languages defined using pFOTS programs each of which consists of one primitive graph rewriting rule and $\Gamma$ in cases of $|\Pi(\Gamma)| = 1$ and $|\Pi(\Gamma)| \geq 2$, by using one positive example and a polynomial number of membership queries. Moreover, by giving an FGS program $\Gamma_{CE}$ whose facts do not satisfy the condition 1 of Definition 3 as background knowledge, we have shown that a target tree language defined using $\Gamma_{CE}$ and a primitive rule cannot be identified, by our learning algorithm using one positive example and a polynomial number of membership queries.

As future work, we will consider the exact learnability of tree languages defined using pFOTS programs as background knowledge in case that the number of facts for each predicate symbol is only one, by using one positive example and membership queries. Let a pFOTS program $\Gamma$ have at most one fact for each predicate symbol in $\Pi(\Gamma)$. Kato et al. [8] showed that any target language of all strings defined using a special type of EFS [7] is identified, by using a polynomial number of membership and superset queries. Since any string is expressed by an ordered tree having only one leaf, we will consider the learnability of tree languages defined using pFOTS programs without background knowledge by expanding Kato’s results to pFOTS programs. Moreover we will expand our results to pFOTS programs without background knowledge using more powerful queries such as equivalence queries, subset queries, superset queries and predicate membership queries (see [6], [16]).

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