A new salinity equation for sound speed instruments

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Abstract

A new equation is presented for the calculation of salinity from triplets of temperature, pressure, and sound speed in marine and estuarine waters. The sixty nine term, sixth order, equation is valid over an environmental range of 0–40°C, 0–6000 dbar, and 0–40 psu. The equation can reproduce a UNESCO speed of sound equation derived reference parameter space to an RMS error of ± 0.0014 psu. The limitation for practical use is the ± 0.05 m s⁻¹ accuracy of the best of the current equations for calculating sound speed. A new reference dataset, either laboratory or in-situ, is required to match the capabilities of new sound speed measuring instruments.

Sound speed measurements, alongside temperature and pressure, have previously been used to derive seawater salinity and density over limited ranges where conductivity measurements have not been available or have been unreliable (Lovett 1968). Such analyses involved the fitting of a multivariate polynomial equation for salinity, and/or density, given in terms of pressure, temperature and sound speed, generally using very limited historical reference datasets. This was a perfectly acceptable approach when sound speed could be measured accurately to 0.1–0.5 ms⁻¹ and salinity was only required to be derived to 0.1–0.5 ‰. However, more recently, there have been significant advances in marine sound speed measurement, clear examples being the commercially available instruments by Valeport Ltd (UK), achieving accuracies better than 0.02 ms⁻¹ at sampling rates up to 200 Hz. Such advances in production instruments promises the potential for routine sound speed derivation of salinity and density to around 0.01 psu or kg m⁻³ respectively. Furthermore accuracies far in excess of this, possibly to 0.003 ms⁻¹ seem to be an expected target within the next 5–10 yr.

It has been conventional for several decades to derive the salinity of seawater from measurements of conductivity, or perhaps more accurately, conductivity ratio to a known standard. Conductivity measurements (or inductance measurements as is the case with a number of sensors) are only affected by the dissolved ionic, electrolytic, solutes in the seawater. The definition of salinity, however, is the total mass fraction of all dissolved matter in seawater. Therefore the measurement of conductivity has always been an imperfect proxy; however, while continuous measurements of other properties of seawater dependent on salinity remained elusive or too inaccurate, conductivity measurement would remain the preferred method to determine salinity. That was fine when we still considered a constant ratio of dissolved material globally in seawater, but the advent of the new TEOS-10 Gibbs equation of state for seawater (IOC, SCOR and IAPSO 2010) acknowledges that the oceanographic community has known for some time the ratio of dissolved components in seawater is not a constant, and will affect the real salinity measurement to the 10⁻² level and more in places around the globe.

With the advent of the TEOS-10 Gibbs function equation of state (EOS) for seawater, adopted internationally in 2010, a new historical measurement based geographical reference table, referenced to available global biogenic silicate concentration measurements, has been created to attempt to relate the practical salinity, S_p, derived through conductivity measurement to an absolute in situ salinity, or density salinity, S_d. However, in the acknowledgement that TEOS-10 has made to the heterogeneity of dissolved constituents in seawater, it is surprising that there is still no acknowledgement of the advances made in alternative measurements for the less imperfect, more inclusive, determination of salinity, perhaps even the promise of determining S_d directly.

In the next section, we will discuss the background and objectives to the work presented in this paper, including the requirement for a more complex equation for the derivation of sound speed. In the third section, we describe the derivation of our new sixth order equation following laboratory and theoretical testing of earlier 3rd and 4th order equations like those of previous authors. Finally, in the fourth section, we discuss the limitations of the current study, mainly associated with the lack of reference datasets, before finally discussing future requirements and challenges in the post TEOS-10 oceanographic era (IOC, SCOR and IAPSO 2010).
Background and objectives

A new equation for salinity calculation from sound speed, temperature and pressure was recently developed by Applied Microsystems (AML) (Dakin 1999), using 1631 data points created from Chen and Millero’s seawater sound speed equation (Chen and Millero 1977). The accuracy of the AML equation with respect to Chen and Millero’s equation is 0.035 m s⁻¹. AML’s purpose for deriving this equation was to enable accurate sound velocity data to be collected for the support of acoustic devices, while still provide salinity and density data without having to use a CTD in addition to an SVP.

The sound speed equation of Chen and Millero (1977) was adopted by UNESCO as the international standard (Fofonoff and Millard 1983) and was modified by Wong and Zhu (1995) who recalculated the coefficients to adopt the International Temperature Scale of 1990, ITS-90. Brewer et al. (2015) presented the latest equation to derive sound speed from salinity, temperature and pressure, calculating a fit to a UNESCO derived quadruple dataset. Using a potentially more elegant approach, the Brewer et al. (2015) equation estimates sound speed to a similar uncertainty as earlier equations, including the UNESCO equation itself, but with fewer terms. In addition, the Brewer et al. (2015) equation can be easily arithmetically inverted for salinity as sound speed varies linearly with salinity. However, their equation is only valid for a limited salinity range (>15).

This paper arises from an industry request to develop an equation to derive salinity from measured temperature, sound speed and pressure, covering a fully realistic global environment range of temperature and salinity. Frequently the request arises as a result of customers whose primary interest is in measuring in-situ sound speed, but occasionally want a reference value for salinity. Frequently such customers, described in more detail later, are interested in the more extreme regions of temperature and salinity parameter space, i.e., brackish or low salinity riverine or estuarial locations including seasonally low temperatures, or extreme evaporation high salinity high temperature regions. Existing equations, providing standard deviation errors at the ±10⁻² m s⁻¹, tend to have maximum and minimum errors at the ±10⁻¹ m s⁻¹ level; and these extreme errors are typically in the extreme regions of global temperature and salinity parameter space. Our objective therefore, was to develop a sound speed equation fit to a derived quadruple dataset that could match the accuracy of that dataset (Order 10⁻² m s⁻¹) at the extremes of global temperature and salinity parameter space; i.e., an equation that would minimize the addition of further error.

As a rule of thumb, well published, online and in print, and not attributable to any one particular author, sound speed increases by ~1.4 m s⁻¹ for every 1 psu increase in salinity, ~4 m s⁻¹ for every 1°C increase in temperature and by ~34 m s⁻¹ for every 2000 dbar increase in pressure, within typical ranges of salinity in the ocean (e.g., 31–38). Conductivity, on the other hand, increases by ~1.0 mS cm⁻¹ for every 1 psu increase in salinity, 1°C increase in temperature or ~2000 dbar in pressure. Therefore current routine accuracy of both temperature and pressure sensors are sufficient to offer sound speed as a slightly more sensitive measurement for changes in salinity or density than conductivity, provided we are only concerned with absolute accuracies of ~0.01 psu or kg m⁻³. The limiting factor currently is the measurement of temperature; most commercially available marine temperature sensors are quoted to 0.001°C which limits the measurement of sound speed to ~0.004 m s⁻¹.

Sound velocity sensors, measuring sound speed by the time of flight of an emitted acoustic signal, are often considered more robust than conductivity sensors by the commercial/operational user. Sound speed sensors are considered largely self-cleaning, and less prone to fouling or proximity effects than resistive or inductive conductivity sensors respectively. Of course, both a conductivity cell and a sound speed sensor would be badly affected by the attachment of something large like a goose-barnacle in the wrong place. However, conductivity cells, as a result of their size and geometry, are frequently bio-fouled by small and soft-bodied plankton such as salps. This kind of bio-fouling, by generally passive suspended particles, is much more common in data from short period lowered or towed measurement platforms; which is one of the reasons why some manufacturers and operators have favored inductive style conductivity sensors. Similarly the larger size and more open geometry of sound speed sensors renders them significantly more resistant to this kind of bio-fouling.

Although the promise of sound speed measurements to derive salinity does not currently reach WOCE (World Ocean Circulation Experiment) standards of deep mode water salinity values (0.001–0.003 psu), the marketplace for WOCE standard accuracy is rather small, limited to a handful of research organizations in each of those nations with a well-established marine scientific research infrastructure. The major market place, the regulatory environmental monitoring and the oil and gas industry, is only interested in accuracies at least one order of magnitude lower than this; however, they need this as a maximum error across their range of water parameters, which often include salt waters near one or more extremes of parameter space, rather than a standard deviation error across all parameter space. Critically this larger marketplace is not interested in the delicate measurement of conductivity; the development of a more robust means of achieving their requirements, like that through sound speed measurement would be welcome.

Deriving 3rd and 4th order equations (not presented in this paper) like that of AML, demonstrated that they are robust in calculating salinity from temperature, pressure, and sound velocity. This was evidenced in a stable response as temperature, sound velocity, and pressure changed during laboratory measurement series over narrow, but significant, ranges in increasing and decreasing gradients. However, while the standard deviation error over a UNESCO sound speed equation derived dataset (described in the next section) was better
than \( \pm 0.02 \) psu, peak errors rose to an order of magnitude greater than this particularly at the extreme fresh and cold range of environmental validity. Such cold estuarine like environmental conditions could be of significant importance to likely customers for the types of instruments our equation needs to support. Interestingly, both a 3rd and a 4th order equation can be derived that do considerably better than Lovett’s (1968) equation which is technically then out of range and typically greater than 0.5 psu off.

As always we are near the limit of what we can achieve bearing in mind the very different measurement of conductivity (the basis for PSU) and sound speed, and the knowledge that saltwater composition does vary, upon which the new TEOS10 equation of state is based. Any equation capable of explaining a UNESCO sound speed equation derived dataset, is likely to be limited by the accuracy of the UNESCO equation itself; \( \pm 0.05 \text{ m s}^{-1} \) is all that is claimed. Nonetheless it was noted that Bilaniuk and Wong (1993) require a fifth order equation in temperature for their freshwater determination of sound speed and the UNESCO equation (Chen and Millero 1977) for determination of sound speed has a fifth order temperature term and a combined 6th order term involving pressure and temperature. Thus it seems consistent that we should derive a full 6th order equation and simplify this as far as sensitivity tests will allow, to attempt to achieve reliable accuracies of better than 0.01 relative to the UNESCO equation.

### Deriving a full 6th order equation

In the absence of up to date high quality datasets of simultaneously measured temperature, conductivity, sound speed and pressure, we have used a manufactured dataset over a parameter volume from temperature (0–40°C, in 0.5°C steps), salinity in psu (0–40 in 0.5 psu steps) and pressure (0–6000 dbar in 100 dbar steps). A sound speed was calculated for each STP combination using the Wong and Zhu (1995), ITS90, version of the UNESCO sound velocity equation determined by Chen and Millero (1977). Since some of the values in the parameter space are significantly outside naturally occurring combined ranges we restrict the data volume in a manner similar to that adopted by Jackett et al. (2006) that can be characterized as follows:

\[

\Delta S_{\text{TPC}} = \left( c-c_w \right) \left( e_1 T + e_2 P + e_3 T^2 + e_4 P^2 + e_6 T^3 + e_7 P^3 + e_8 T^4 + e_9 P^4 + e_{10} T^5 + e_{11} P T \right) \\
+ \left( c-c_w \right)^2 \left( f_1 T + f_2 P + f_3 T^2 + f_4 P^2 + f_5 T^3 + f_6 P^3 + f_7 T^4 + f_8 P^4 + f_9 T^5 + f_{10} P T \right) \\
\left( e_{12} P T^2 + e_{13} P^2 T + e_{14} P^2 T^2 + e_{15} P^3 T + e_{16} P^4 T + e_{17} T^3 P^2 + e_{18} T^4 P \right) \\
\left( a_1 T + a_2 T^2 + a_3 T^3 + a_4 T^4 + a_5 T^5 + a_6 T^6 \right)
\]

\[

\Delta S_{\text{P}} = \left( c-c_w \right) \left( g_1 T + g_2 P + g_3 T^2 + g_4 P^2 + g_5 P^3 + g_6 P^4 + g_7 T^3 + g_8 T^4 + g_9 T^5 + g_{10} T^6 \right)
\]

\[

\Delta S_{\text{C}} = \left( c-c_w \right) \left( h_1 T + h_2 P + h_3 T^2 + h_4 P^2 + h_5 T^3 + h_6 P^3 + h_7 T^4 + h_8 P^4 + h_{10} T^5 \right)
\]

\[

\Delta S_{\text{T}} = \left( c-c_w \right) \left( i_1 T + i_2 T^2 + i_3 T^3 + i_4 T^4 + i_5 T^5 + i_6 P T \right)
\]

\[

\Delta S_{\text{PC}} = \left( c-c_w \right) \left( j_1 T + j_2 T^2 + j_3 T^3 + j_4 T^4 + j_5 T^5 + j_6 P T \right)
\]

\[

\Delta S_{\text{TPC}} = \left( c-c_w \right) \left( k_1 T + k_2 T^2 + k_3 T^3 + k_4 T^4 \right)
\]

\[

\Delta S_{\text{TPC}} = \left( c-c_w \right) \left( l_1 T + l_2 T^2 + l_3 T^3 + l_4 T^4 \right)
\]

\[

\Delta S_{\text{TPC}} = \left( c-c_w \right) \left( m_1 T + m_2 T^2 + m_3 T^3 \right)
\]

### a. For pressures 0–5500 dbar,

- Maximum temperature: \( 40 - \left( (25/5500) \times \text{pressure} \right) \)
- Minimum salinity: \( 0 + \left( (30/5500) \times \text{pressure} \right) \)
- Minimum salinity: \( 0 + \left( (30/5500) \times \text{pressure} \right) \)

### b. For pressures 5500–6000 dbar,

- Max temp = 15
- Min salinity = 30

This is a slightly more restricted depth range than that often quoted for Sound Speed equations (6000 rather than 10,000 dbar), for three reasons. There are very few measurements worldwide at over 6000 dbar. Commercially available instruments are generally depth rated to less than or around 6000 dbar. Lovett (1968) using Wilson’s dataset, did not appear to have any data with which the 0–10,000 dbar claim could be supported.

Finally we replaced the surface fresh water values from the UNESCO sound velocity equation with those derived from the 148 point Bilaniuk and Wong (1993) equation. This gave us a manufactured reference dataset of 174837 quadruplets of temperature, salinity, sound speed, and pressure.

Defining Practical Salinity as:

\[

S = S_0 + \Delta S_T + \Delta S_P + \Delta S_C + \Delta S_{\text{TPC}},
\]

with \( S_0 = 0.000 \). Then the full 81 term sixth order equation can be written in many ways, but is of the following component form:

\[

\Delta S_T = a_1 T + a_2 T^2 + a_3 T^3 + a_4 T^4 + a_5 T^5 + a_6 T^6
\]

\[

\Delta S_P = b_1 P + b_2 P^2 + b_3 P^3 + b_4 P^4 + b_5 P^5 + b_6 P^6
\]

\[

\Delta S_C = d_1 (c-c_w) + d_2 (c-c_w)^2 + d_3 (c-c_w)^3 + d_4 (c-c_w)^4 + d_5 (c-c_w)^5 + d_6 (c-c_w)^6
\]

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where, \(c_w = 1402.388 \text{ m s}^{-1}\) is the sound speed of fresh water at surface pressure and 0°C.

The 81 coefficients, \(a_1-n_1\), were derived by minimizing the least squares error,

\[
E = \sum_{i=1}^{N} \left( S_i - (\Delta S_{T_i} + \Delta S_{P_i} + \Delta S_{c_i} + \Delta T_{P,c_i}) \right)^2,
\]

(2)

to the, \(N\), points in our UNESCO equation derived data volume. Minimizing this error with respect to each of the parameters, \(a_1-n_1\), produces 81 simultaneous equations of the form,

\[
0 = \frac{\partial E}{\partial a_1} = 2 \sum_{i=1}^{N} \left( S_i - (\Delta S_{T_i} + \Delta S_{P_i} + \Delta S_{c_i} + \Delta T_{P,c_i}) \right) T_i
\]

\[
0 = \frac{\partial E}{\partial a_2} = 2 \sum_{i=1}^{N} \left( S_i - (\Delta S_{T_i} + \Delta S_{P_i} + \Delta S_{c_i} + \Delta T_{P,c_i}) \right) T_i^2
\]

etc

e tc

\[
0 = \frac{\partial E}{\partial f_{14}} = 2 \sum_{i=1}^{N} \left( S_i - (\Delta S_{T_i} + \Delta S_{P_i} + \Delta S_{c_i} + \Delta T_{P,c_i}) \right) (c_i - c_w)^2 P_{14} T_i^2
\]

e tc

e tc

\[
\Delta S_{TPC} = (c-c_w) \left( e_1 T + e_2 P + e_3 T^2 + e_4 P^2 + e_5 T^3 + e_6 P^3 + e_7 T^4 + e_8 P^4 + e_9 T^5 + e_{10} P^5 + e_{11} PT \right)
\]

\[+ e_{12} PT^2 + e_{13} P^2 T + e_{14} P^2 T^2 + e_{15} PT^3 + e_{16} P^2 T^3 \]

\[+ e_{17} P^3 T^2 + e_{18} P^3 T^3 + (c-c_w)^2 \left( f_1 P + f_2 P + f_3 P^2 + f_4 P^3 + f_5 T + f_6 P T + f_7 T^2 \right) \]

\[+ f_{18} P^2 T + f_{19} P^2 T^2 + f_{20} P^2 T^3 + \left( g_1 T + g_2 P + g_3 T^2 + g_4 P^2 + g_5 T^3 + g_6 P^3 + g_7 P T + g_8 P^2 T + g_9 P^2 T^2 \right) \]

\[+ (c-c_w)^3 \left( h_1 T + h_2 P + h_3 T^2 + h_4 P^2 \right) \]

\[+ (c-c_w)^4 \left( i_1 T + i_2 P + i_3 T^2 + i_4 P^2 \right) \]

\[+ (c-c_w)^5 \left( j_1 T + j_2 P \right) \]

\[+ P(j_1 T + j_2 T^2 + j_3 T^3 + j_4 T^4 + j_5 T^5) \]

\[+ P^2(k_1 T + k_2 T^2 + k_3 T^3 + k_4 T^4) \]

\[+ P^3(l_1 T + l_2 T^2 + l_3 T^3) \]

\[+ P^4(m_1 T + m_2 T^2) \]

Following a least squares fit to the UNESCO parameter volume as before, this produces the 76 coefficients given in Table 2. The 76 term equation maps the UNESCO ocean parameter volume to a residual error of ±0.00134 psu; with maximum errors of −0.0289 and 0.0147: showing no reduction in accuracy from the 81 term full equation.

The coefficient solutions are given in Table 1, each with a significance expressed as a t-value. This equation maps the UNESCO ocean parameter volume to a residual error of ±0.00134 psu; with maximum errors of −0.0289 and 0.0148. These values are considerably smaller than the quoted accuracy of the UNESCO sound speed equation itself as discussed earlier. The t-value gives us a measure of significance for the terms in the equation; the higher the modulus of the value, the higher the significance. By sequentially removing the less significant terms the equation was simplified in stages.

**Simplification to 76 and to 69 terms**

A 76 term equation resulted from the omission of the \(a_5, b_3, c_15, b_5, \) and \(n_1\) terms, reducing the equation components to,

\[\Delta S_T = a_1 T + a_2 T^2 + a_3 T^3 + a_4 T^4 + a_6 T^6 \]

\[\Delta S_P = b_1 P + b_2 P^2 + b_3 P^3 + b_5 P^5 + b_5 P^6 \]

\[\Delta S_c = d_1 (c-c_w)^2 + d_2 (c-c_w)^3 + d_3 (c-c_w)^4 \]

\[+ d_4 (c-c_w)^5 + d_5 (c-c_w)^6 \]

\[
0 = \frac{\partial E}{\partial m_2} = 2 \sum_{i=1}^{N} \left( S_i - (\Delta S_{T_i} + \Delta S_{P_i} + \Delta S_{c_i} + \Delta T_{P,c_i}) \right) P^4 T_i^2
\]

\[
0 = \frac{\partial E}{\partial n_1} = 2 \sum_{i=1}^{N} \left( S_i - (\Delta S_{T_i} + \Delta S_{P_i} + \Delta S_{c_i} + \Delta T_{P,c_i}) \right) P^3 T_i
\]

The equation can then be simplified further, until the t-values are virtually the same for all terms, by removal of the \(b_5, b_6, d_6, c_10, f_7, l_1, \) and \(m_1\) terms. The resulting 69 term equation has the reduced components,

\[\Delta S_T = a_1 T + a_2 T^2 + a_3 T^3 + a_4 T^4 + a_6 T^6 \]
Table 1. Coefficient estimates and t-values for the 81 term 6th order equation.

| Coefficient | Estimate | t-value | Coefficient | Estimate | t-value |
|-------------|----------|---------|-------------|----------|---------|
| $a_1$       | -3.694463 E +00 | -11.864 | $\beta$     | -5.161707 E -14 | -11.474 |
| $a_2$       | 3.794849 E -02  | 11.83   | $\gamma$    | 2.093267 E -06  | 11.487  |
| $a_3$       | 3.250509 E -03  | 11.806  | $\eta$      | 1.037336 E -07  | 11.39   |
| $a_4$       | 1.034984 E -04  | 11.746  | $\eta_1$    | 1.151616 E -09  | 8.6     |
| $a_5$       | -3.898871 E -08 | -3.761  | $\eta_2$    | 4.278277 E -11  | 11.181  |
| $a_6$       | -7.214226 E -09 | -11.761 | $\eta_3$    | -3.881513 E -13 | -5.549  |
|             |           |         | $\eta_4$    | 4.840743 E -10  | 11.103  |
| $b_1$       | -1.102838 E -01 | -11.864 |             |           |         |
| $b_2$       | 1.945909 E -05  | 11.585  | $g_1$       | -6.931994 E -06 | -11.641 |
| $b_3$       | -3.566639 E -09 | -2.931  | $g_2$       | -1.013249 E -07 | -10.971 |
| $b_4$       | -1.164933 E -10 | -10.317 | $g_3$       | -3.464199 E -07 | -11.525 |
| $b_5$       | -2.803511 E -13 | -10.728 | $g_4$       | 1.236342 E -10  | 10.126  |
| $b_6$       | -1.534459 E -16 | -9.534  | $g_5$       | -1.145887 E -09 | -9.697  |
|             |           |         | $g_6$       | 3.713794 E -13  | 11.297  |
| $c_1$       | 7.323328 E -01  | 11.864  | $g_7$       | -1.350005 E -08 | -11.011 |
| $c_2$       | 1.151352 E -03  | 11.777  | $g_8$       | -1.509950 E -10 | -10.365 |
| $c_3$       | -2.902402 E -05 | -11.684 | $g_9$       | 3.291511 E -12  | 7.341   |
| $c_4$       | 3.150110 E -07  | 11.473  |             |           |         |
| $c_5$       | -1.297414 E -09 | -10.82  | $h_1$       | 3.988934 E -08  | 11.378  |
| $c_6$       | -5.220544 E -13 | -5.154  | $h_2$       | 2.333364 E -10  | 7.353   |
|             |           |         | $h_3$       | 4.388962 E -10  | 10.434  |
| $e_1$       | -4.673162 E -03 | -11.683 | $h_4$       | -1.473092 E -12 | -11.59  |
| $e_2$       | -4.519012 E -04 | -11.764 | $h_5$       | -2.766152 E -13 | -0.278  |
| $e_3$       | -2.242701 E -03 | -11.786 |             |           |         |
| $e_4$       | -7.083059 E -07 | -10.851 | $i_1$       | -3.576964 E -11 | -9.279  |
| $e_5$       | -1.232879 E -04 | -11.739 | $i_2$       | 2.522568 E -12  | 11.93   |
| $e_6$       | 1.386976 E -09  | 9.868   |             |           |         |
| $e_7$       | -1.238906 E -06 | -11.384 | $j_1$       | 8.007027 E -04  | 11.636  |
| $e_8$       | 5.779118 E -12  | 10.652  | $j_2$       | 2.565795 E -04  | 11.735  |
| $e_9$       | 9.937542 E -09  | 11.973  | $j_3$       | 1.328650 E -05  | 11.678  |
| $e_{10}$    | 4.152925 E -15  | 11.442  | $j_4$       | 1.484785 E -07  | 11.319  |
| $e_{11}$    | -1.094978 E -04 | -11.677 | $j_5$       | -8.290561 E -10 | -11.256 |
| $e_{12}$    | -1.025450 E -05 | -11.63  |             |           |         |
| $e_{13}$    | -1.710897 E -07 | -10.957 | $k_1$       | 5.907303 E -06  | 11.522  |
| $e_{14}$    | -1.009825 E -08 | -10.949 | $k_2$       | 5.025304 E -07  | 11.427  |
| $e_{15}$    | 3.943536 E -11  | 2.863   | $k_3$       | 1.485834 E -08  | 11.302  |
| $e_{16}$    | -2.419770 E -07 | -11.46  | $k_4$       | 1.448682 E -10  | 11.957  |
| $e_{17}$    | -8.061875 E -12 | -11.892 |             |           |         |
| $e_{18}$    | -1.689671 E -10 | -11.973 | $l_1$       | 1.945342 E -09  | 5.1     |
|             |           |         | $l_2$       | 3.304822 E -10  | 8.971   |
| $f_1$       | 4.640807 E -04 | 11.75   | $\beta$     | 1.787428 E -11  | 11.913  |
| $f_2$       | 9.697120 E -06 | 11.534  |             |           |         |
| $f_3$       | 4.793062 E -05 | 11.711  | $m_1$       | -7.205355 E -12 | -7.141  |
| $f_4$       | 3.657485 E -09 | 5.233   | $m_2$       | 5.129721 E -13  | 11.403  |
| $f_5$       | 1.123870 E -06 | 11.541  |             |           |         |
| $f_6$       | -4.560148 E -11 | -10.749 | $n_1$       | 5.668886 E -16  | 1.091   |
| $f_7$       | -1.917525 E -09 | -11.092 |             |           |         |
**Table 2.** Coefficient estimates and t-values for the 76 term 6th order equation.

| Coefficient | Estimate      | t-value | Coefficient | Estimate      | t-value |
|-------------|---------------|---------|-------------|---------------|---------|
| a1          | -3.694347 E +00 | -11.98  | f7          | -1.941518 E -09 | -11.57  |
| a2          | 3.805745 E -02  | 11.98   | f8          | -5.310686 E -14 | -11.8   |
| a3          | 3.272441 E -03  | 11.99   | f9          | 2.155816 E -06  | 12      |
| a4          | 1.050758 E -04  | 11.99   | f10         | 1.090649 E -07  | 12.02   |
| a6          | -7.564520 E -09 | -12     | f11         | 1.425401 E -09  | 12.02   |
|             |                |         | f12         | 4.591421 E -11  | 11.98   |
| b1          | -1.102820 E -01 | -11.98  | f13         | -3.216855 E -13 | -11.1   |
| b2          | 1.969670 E -05  | 11.97   | f14         | 5.086486 E -10  | 12.01   |
| b4          | -1.024956 E -10 | -11.81  | g1          | -7.010323 E -06  | -11.99  |
| b5          | -2.678577 E -13 | -11.7   | g2          | -1.047023 E -07  | -11.94  |
| b6          | -1.714186 E -16 | -11.37  | g3          | -3.538030 E -07  | -11.99  |
| d1          | 7.332820 E -01  | 11.98   | g4          | 1.079837 E -10  | 11.86   |
| d2          | 1.153107 E -03  | 11.98   | g5          | -1.166947 E -09  | -11.69  |
| d3          | -2.913498 E -05 | -11.97  | g6          | 3.761111 E -13  | 11.94   |
| d4          | 3.171860 E -07  | 11.96   | g7          | -1.425392 E -08  | -12     |
| d5          | -1.302492 E -09 | -11.88  | g8          | -1.589531 E -10  | -12.02  |
| d6          | -6.711967 E -13 | -10.21  | g9          | 2.942745 E -12  | 11.42   |
| e1          | -4.710512 E -03 | -11.97  | h1          | 4.056006 E -08  | 11.98   |
| e2          | -4.533073 E -04 | -11.98  | h2          | 2.573752 E -10  | 11.42   |
| e3          | -2.257785 E -03 | -11.99  | h3          | 4.421836 E -10  | 11.88   |
| e4          | -7.469049 E -07 | -11.96  | h4          | -1.496395 E -12  | -11.99  |
| e5          | -1.249587 E -04 | -11.99  |             |                |         |
| e6          | 1.137143 E -09  | 11.89   | i1          | -3.499613 E -11  | -11.73  |
| e7          | -1.295940 E -06 | -11.99  | i2          | 2.620637 E -12  | 12.01   |
| e8          | 5.391921 E -12  | 11.87   |             |                |         |
| e9          | 1.019852 E -08  | 12.01   | i3          | 8.113682 E -04  | 11.98   |
| e10         | 4.428288 E -15  | 11.59   | i4          | 2.603147 E -04  | 12      |
| e11         | -1.110718 E -04 | -11.99  | i5          | 1.366624 E -05  | 12.01   |
| e12         | -1.054051 E -05 | -12.01  | i6          | 1.609560 E -07  | 12.02   |
| e13         | -1.863629 E -07 | -12.01  | i7          | -8.832268 E -10 | -11.99  |
| e14         | -1.128502 E -08 | -12.03  |             |                |         |
| e15         | -2.559667 E -07 | -12.02  | k1          | 6.110480 E -06  | 12      |
| e16         | -8.424215 E -12 | -11.93  | k2          | 5.349917 E -07  | 12.02   |
| e17         | -1.746508 E -10 | -11.99  | k3          | 1.635674 E -08  | 12.02   |
|             |                |         | k4          | 1.447396 E -10  | 11.91   |
| f1          | 4.669553 E -04  | 11.98   | l1          | 3.097202 E -09  | 11.66   |
| f2          | 9.822959 E -06  | 11.97   | l2          | 4.122212 E -10  | 11.95   |
| f3          | 4.852802 E -05  | 11.99   | l3          | 1.815324 E -11  | 11.93   |
| f4          | 5.141493 E -09  | 11.58   | l4          |                |         |
| f5          | 1.154808 E -06  | 11.99   |             |                |         |
| f6          | -4.176367 E -11 | -11.95  | m1          | -5.301923 E -12 | -11.53  |
|             |                |         | m2          | 5.185494 E -13  | 11.81   |
Table 3. Coefficient estimates and t-values for the 69 term 6th order equation.

| Coefficient | Estimate     | t-value | Coefficient | Estimate     | t-value |
|-------------|--------------|---------|-------------|--------------|---------|
| a1          | -3.695851E+00| -9.248  | f9          | 1.809726E-06 | 9.251   |
| a2          | 3.68474E-02  | 9.25    | f10         | 8.820074E-08 | 9.252   |
| a3          | 3.081724E-03 | 9.251   | f11         | 9.048890E-10 | 9.246   |
| a4          | 9.616883E-05 | 9.251   | f12         | 1.898597E-11 | 9.238   |
| a6          | -4.259169E-09| -9.242  | f13         | -4.226753E-13| -9.241  |
| b1          | -1.102869E-01| -9.248  | f14         | 4.067971E-10 | 9.247   |
| b2          | 1.947376E-05 | 9.247   | g1          | -6.381374E-06| -9.252  |
| b4          | -6.045637E-11| -9.155  | g2          | -9.590636E-08| -9.248  |
| d1          | 7.324290E-01 | 9.248   | g3          | -3.167521E-07| -9.252  |
| d2          | 1.132294E-03 | 9.25    | g4          | 5.007374E-11 | 9.174   |
| d3          | -2.804244E-05| -9.251  | g5          | -1.522413E-09| -9.25   |
| d4          | 2.984912E-07 | 9.251   | g6          | 1.638435E-13 | 9.241   |
| d5          | -1.253726E-09| -9.251  | g7          | -1.146659E-08| -9.25   |
| e1          | -4.291753E-03| -9.248  | g8          | -1.214922E-10| -9.249  |
| e2          | -4.479172E-04| -9.25   | g9          | 3.566895E-12 | 9.247   |
| e3          | -2.118181E-03| -9.251  | h1          | 3.636617E-08 | 9.251   |
| e4          | -6.969598E-07| -9.245  | h2          | 2.880166E-10 | 9.226   |
| e5          | -1.140338E-04| -9.251  | h3          | 4.668968E-10 | 9.251   |
| e6          | 7.945546E-10 | 9.211   | h4          | -9.142480E-13| -9.247  |
| e7          | -1.179964E-06| -9.251  | h5          | 1.715735E-12 | 9.249   |
| e8          | 1.145059E-12 | 9.177   | h6          | 1.715735E-12 | 9.249   |
| e9          | 5.920094E-09 | 9.248   | h7          | -3.850991E-11| -9.25   |
| e11         | -1.003958E-04| -9.252  | h8          | 7.299315E-04 | 9.248   |
| e12         | -8.380591E-06| -9.252  | h9          | 2.319620E-04 | 9.252   |
| e13         | -1.262917E-07| -9.249  | h10         | 1.150563E-05 | 9.251   |
| e14         | -7.905725E-09| -9.25   | h11         | 1.386883E-07 | 9.249   |
| e16         | -2.121439E-07| -9.251  | h12         | -7.606725E-10| -9.237  |
| e17         | -2.788394E-12| -9.241  | h13         | 5.085449E-06 | 9.251   |
| e18         | -8.583556E-11| -9.248  | h14         | 3.739732E-07 | 9.251   |
| f1          | 4.392833E-04 | 9.251   | k1          | 3.850449E-06 | 9.251   |
| f2          | 9.359168E-06 | 9.25    | k2          | 3.739732E-07 | 9.251   |
| f3          | 4.414043E-05 | 9.252   | k3          | 1.274588E-08 | 9.248   |
| f4          | 4.790080E-09 | 9.17    | k4          | 5.138444E-11 | 9.21    |
| f5          | 1.041642E-06 | 9.251   | k5          | 6.902247E-12 | 9.235   |
| f6          | -1.724117E-11| -9.223  | k6          | 5.085449E-06 | 9.251   |
| f8          | -9.946432E-15| -9.226  | k7          | 1.153594E-13 | 9.192   |
Following a least squares fit to the UNESCO parameter volume as before, this produces the 69 coefficients given in Table 3. The 69 term equation maps the UNESCO ocean parameter volume to a residual error of ±0.00135 psu; with maximum errors of −0.0291 and 0.0151: therefore significantly simplifying the equation at statistically little cost. We see that at this point the magnitude of the \( t \)-value test statistic varies between only 9.155 and 9.252 making it very difficult to justify removing further terms.
In Fig. 1, the salinity residuals, the parameter volume salinity minus our 69 term equation derived salinity, are plotted for temperature ranges in a 1565 point sub-set of our UNESCO equation derived parameter volume. The biggest positive residuals are associated with zero salinity, evidenced by the two peaks in the left half of the figure. This is perhaps to be expected as the UNESCO sound speed equation is dependent on the sensitive measurement of conductivity at approaching zero salinity. Conversely the biggest negative residuals are associated with a combination of high temperature, greater than 38°C, and salinity, 33–40 psu, as evidenced in the lower part of the left and center of the figure. Pressure appears to be well handled by the new equation throughout the entire range 0–6000 dbar.

In Fig. 2, we present profiles from an Arctic dataset (84°29′N, 90°22.5′E) of the quadruple measurement of temperature, salinity, pressure, and sound speed. The difference between measured salinity and that derived through our 69 term equation is presented in the right hand panel.

**Discussion**

The errors inherent in the permissible realistic ocean data volume are those of the UNESCO sound velocity equation...
quoted as $\sim 0.050$ ms$^{-1}$ (Dushaw et al. 1993). This is better than the 0.067 ms$^{-1}$ achieved with the 48 point density equation of the new TEOS-10 Gibbs function equation of state (IOC, SCOR and IAPSO TEOS10 manual 2010); although the full density equation derivation is quoted at 0.035 ms$^{-1}$, but over a reduced salinity range. The classical Del Grosso (1974) equation and the most recent Brewer et al. (2015) equation can be considered good to 0.025–0.045 ms$^{-1}$ however, again over a significantly reduced salinity range. These inherent errors translate to a similar error of approximately $\pm 0.050$ psu in salinity for the uncertainty of the parameter volume for fitting a reverse equation for salinity in terms of pressure, sound speed and temperature (Emery 1977). Therefore our new 81, 76, and 69 term equations add little significant error to that inherent in the estimated parameter space itself.

We should remember that salinity is defined as the total mass fraction of dissolved material in water; and that Practical Salinity (psu) is an uncomfortable approximation to salinity, albeit one that enabled the mass estimation and mapping of salinity globally through the advent of electronic instruments that could accurately and rapidly measure the conductivity of a sample of water in-situ. Now that instruments, such as those by Valeport Ltd., can rapidly measure sound speed to an accuracy that promises to rival that of the measurement of conductivity in the near future, formulations in terms of absolute salinity may fare better given that sound speed is sensitive to the full range of dissolved material not just the ionic varieties. The promise of sound speed as an alternative to conductivity is particularly important in productive surface waters or in waters of low or intermediate practical salinity, where non-ionic solutes form an increasingly significant fraction of total dissolved material.

The recent sound speed equation of Brewer et al. (2015), suggesting that a linear relationship can be assumed between salinity and sound speed, is supported by a further inspection of our UNESCO equation based dataset (Fig. 3). We are therefore somewhat surprised that our statistical significance testing did not result in the removal of more sound speed terms than just the 6th order term. We note, however, that previous equations have also required a more complex dependency on salinity/sound speed as discussed earlier. We would encourage a further examination of the van’t Hoff (1884) approach of Brewer et al. (2015), to cover a wider range of salinities and reduced minimum and maximum residuals.

The latest generation sound speed sensors can make measurements at a rate of up to 200 Hz, opening up a potential new approach to the determination of microstructure. Thus our new equation, in conjunction with these technologies, will open up a pathway for the cheap and reliable investigation of turbulent scour processes and mixing associated with the installation of offshore structures among other applications. It is unfortunate that the current accuracy of the sound speed equation for the TEOS-10 Absolute Salinity based equation of state is not quoted to be as good as the UNESCO equation. The problem is that we have a circular argument, where none of the equations or comparators are truly independent; this will ultimately remain so until we have a new dataset of simultaneously measured high resolution sound speed, temperature, pressure and true salinity, or at least the salinity components that the oceanographic community is going to approximate to absolute salinity under TEOS-10 and it’s future evolutions. The integration of understanding of the variation in compressibility, viscosity and chemical relaxation (Leander 1989; Denisov et al. 2004; Millero and Huang 2013) as a function of solute composition would be a profitable line of future research to speed up the transition to sound speed measurement; and indeed the application of such research will support future evolution of the TEOS-10 equation of state geographical correction.

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Conflict of Interest
None declared.