Quantum critical nematic fluctuations and spin excitation anisotropy in iron pnictides

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Quantum criticality in iron pnictides involves both the nematic and antiferromagnetic degrees of freedom, but the relationship between the two types of fluctuations has yet to be clarified. Here we study this problem within an effective Ginzburg-Landau theory for both channels in the presence of a small external uniaxial potential, which breaks the \( C_4 \)-symmetry in the \( B_{1g} \) sector. We establish an identity that connects the spin excitation anisotropy, which is the difference of the dynamical spin susceptibilities at \( \vec{Q}_1 = (\pi, 0) \) and \( \vec{Q}_2 = (0, \pi) \), with the dynamical magnetic susceptibility and static nematic susceptibility. Using this identity, we introduce a scaling procedure to determine the dynamical nematic susceptibility in the quantum critical regime, and illustrate the procedure for the case of the optimally Ni-doped \( \text{BaFe}_2\text{As}_2 \) [Y. Song et al., Phys. Rev. B \textbf{92}, 180504 (2015)]. The implications of our results for the overall physics of the iron-based superconductors are discussed.

Iron-based superconductors have presented many intriguing and often puzzling properties [1–6]. Among these is the onset of the tetragonal-to orthorhombic structural phase transition at a temperature just above or at the antiferromagnetic (AF) phase transition [7]. When they are split, the region between the two transitions is called a nematic phase, where the \( C_4 \) tetragonal symmetry is broken while the \( O(3) \) spin rotational symmetry is preserved. It has been well established that the nematic transition is driven by electron correlations, with \( B_{1g} \) anisotropies in electronic, orbital and magnetic properties [8–10]. Several channels are entwined in the nematic correlations, including spin [11–14], electronic [15, 16] and orbital [17, 18] degrees of freedom.

One way to make progress is to consider the quantum critical regime, where the criticality singularities can be isolated from regular contributions. This facilitates the study of the relationship between the responses in the nematic and other channels. Our strategy in this paper is to assume that the magnetic fluctuations drive nematic correlations, establish a precise relationship between the singular responses in the magnetic and nematic sectors, and use this relationship to assess the underlying assumption and determine the critical properties. The parent ground state of the iron-pnictide superconductor is an AF state with the ordering wave vector \( \vec{Q}_1 = (\pi, 0) \) or \( \vec{Q}_2 = (0, \pi) \). Their spatial patterns are shown in Fig. 1(a) and 1(b). The AF state breaks not only the usual \( O(3) \) spin rotational symmetry, but also a \( Z_2 \) symmetry between the \( \vec{Q}_1 \) and \( \vec{Q}_2 \) magnetic state. In iron pnictides, the bad-metal behavior [19, 20] motivated a theoretical proposal for the electronic excitations into coherent and incoherent parts. The tuning of the coherent electron weight was proposed to give rise to concurrent quantum criticality in both the \( (\pi, 0) \) AF and Ising-nematic channels [11, 21]. The existence of quantum criticality has been most extensively evidenced by experiments in \( \text{BaFe}_2\text{As}_2 \) with P-for-As doping to the regime of optimal superconductivity [25, 29].

A defining characteristic of quantum criticality is the inherent mixing of statics and dynamics. Singular magnetic responses in the quantum critical regime have been observed through dynamical measurements at both the optimally P-for-As- and Ni-for-Fe-doped \( \text{BaFe}_2\text{As}_2 \) [30, 31]. Singular nematic responses in the quantum critical regime have also been observed over a variety of optimally doped iron pnictides [32], albeit in DC measurements. The comparison already demonstrates the concurrent nature of the quantum criticality in the magnetic and nematic channels [33]. However, to elucidate the relationship between the singular nematic and magnetic responses, it would be desirable to determine the dynamical nematic susceptibility in the quantum critical regime. In general, the low-energy dynamical nematic susceptibility in the quantum critical regime is not readily accessible experimentally.

In this Letter, we elucidate the relation between the singular dynamical magnetic and nematic responses by exploiting the relationship between the dynamical nematic susceptibility and spin excitation anisotropy. The latter, defined as the difference of the dynamical spin susceptibilities at \( \vec{Q}_1 = (\pi, 0) \) and \( \vec{Q}_2 = (0, \pi) \), under a uniaxial strain that breaks the \( C_4 \) symmetry in \( B_{1g} \) channel, has been measured by inelastic neutron scattering experiments in the optimally doped iron pnictides [10, 31, 35]. We analyze the singular part of the dynamical responses in both the \( O(3) \) AF and \( Z_2 \) Ising-nematic sectors. Within an effective Ginzburg-Landau theory involving both the AF and nematic degrees of freedom, we establish a general identity [Eq. (6)] among the spin excitation anisotropy, the dynamical magnetic susceptibility, and the nematic susceptibility. Based on a scaling analysis, we further show how this identity can be used to explore the properties of a quantum critical point (QCP), where both the magnetic and nematic channels are concurrently critical. Through the scaling procedure, we extract the dynamical nematic susceptibility from the spin excitation anisotropy, and also determine the dynamic exponent \( z \) and the scaling dimension of the nematic order parameter \( d_\Delta \). The procedure is illustrated in the context of the inelastic neutron scattering results for the optimally Ni-doped \( \text{BaFe}_2\text{Si}_2 \) under an external stress [31], which are summarized in Figs. 1(c) and 1(d).

Effective model: We start from an effective field theory for
the magnetism of iron pnictides [11, 36]

\[ S_0 = S_2 + S_4 \]

\[ S_2 = \sum_{q = \vec{Q}, \vec{\pi}, \omega_n} \left\{ G_0^{-1}(q) \left( |\vec{m}_A(q)|^2 + |\vec{m}_B(q)|^2 \right) \right\} \]

\[ S_4 = \int_0^\beta d\tau \int d^2x \left[ u_1 (\vec{m}_A^2 + \vec{m}_B^2) - u_1 (\vec{m}_A \cdot \vec{m}_B)^2 \right] \]

where \( \vec{m}_A \) and \( \vec{m}_B \) are the Neel order parameter fields on the sublattices A and B, respectively, and \( G_0^{-1}(\vec{q}, i\omega_n) = r + \omega_n^2 + c\vec{q}^2 + \gamma|\omega_n| \) with the mass term \( r \) and Landau damping term \( \gamma|\omega_n| \) resulting from the coherent electronic excitations [11]. For convenience, we neglect the interaction \(-u_2 (\vec{m}_A^2 - \vec{m}_B^2)^2 \) and the spatial anisotropy term \( v (\vec{q}_2^2 - \vec{q}_1^2) \vec{m}_A(q) \cdot \vec{m}_B(-q) \), which do not affect our scaling analysis.

We consider the problem in the presence of an external uniaxial stress, and focus on the effect of the induced strain in the \( B_{1g} \) channel, which couples linearly to the quantity \( \vec{m}_A \cdot \vec{m}_B \). Absorbing this coupling into a Hubbard-Stratonovich transformation, it leads to a coupling \( S_{\lambda, \Delta} = \lambda \int_0^\beta d\tau \int d^2x \Delta \) after integrating the strain degree of freedom, where \( \lambda \) is the strength of the external stress. Leaving details to the Supplementary Material (SM) (where additional terms that do not affect our scaling analysis are also described), we arrive at the following effective action:

\[ S = \tilde{S}_2 + S_3 + \ldots + S_{\lambda, \Delta} \]

\[ \tilde{S}_2 = \sum_{q = \vec{Q}, i\omega_n} \left\{ \chi_0^{-1}(q) \left( |\vec{m}_A(q)|^2 + |\vec{m}_B(q)|^2 \right) \right\} + \Delta(q) \chi_0^{-1}(q) \Delta(-q) \]

\[ S_3 = \int_0^\beta d\tau \int d^2x 2\Delta \vec{m}_A \cdot \vec{m}_B \]

\[ S_{\lambda, \Delta} = \lambda \int_0^\beta d\tau \int d^2x \Delta \]

where we have absorbed the mass term into the field conjugate to \( \vec{m}_A^2 + \vec{m}_B^2 \) (see SM) so that now \( \chi_0^{-1}(\vec{q}, i\omega_n) = G_0^{-1}(\vec{q}, i\omega_n) - r = \omega_n^2 + c\vec{q}^2 + \gamma|\omega_n| \), the ellipsis denotes the terms that are not essential to our scaling analysis (see SM), and \( \chi_0^{-1}(\vec{q}, i\omega_n) \) is the bare magnetic and nematic propagator, respectively.

The spin excitation anisotropy \( \chi_d(\omega) \) and the dynamical magnetic susceptibility \( \chi_s(\omega) \) are defined as the difference and summation of the dynamical spin susceptibility \( \chi(\vec{q}, \omega) \) between the two ordering wave vector \( \vec{Q}_1 = (\pi, 0) \) and \( \vec{Q}_2 = (0, \pi) \), respectively:

\[ \chi_s(\omega) \equiv \chi(\vec{Q}_1, \omega) + \chi(\vec{Q}_2, \omega) \]

\[ \chi_d(\omega) \equiv \chi(\vec{Q}_1, \omega) - \chi(\vec{Q}_2, \omega) \]

Connecting the spin excitation anisotropy to nematic susceptibility: On symmetry grounds, the spin excitation anisotropy \( \chi_d(\omega) \) should be related to the Ising-nematic fluctuations, since it measures the degree of the asymmetry of the magnetic fluctuations between the two wave vectors \( \vec{Q}_1 \) and \( \vec{Q}_2 \). However, the precise relation has not been considered

![Diagram](image341x227to539x332)

![Diagram](image453x483to555x658)

**FIG. 1.** (a) The spin configurations of the ground state of the parent iron-based superconductors with ordering wave vector \( \vec{Q}_1 = (\pi, 0) \) or (b) \( \vec{Q}_2 = (0, \pi) \). The blue and red arrows denote the spins forming the staggered magnetizations on the sublattices A and B, respectively. Also shown are the energy dependences of (c) the imaginary part of the spin excitation anisotropy \( \chi_d''(\vec{Q}_1) - \chi_d''(\vec{Q}_2) \) vs. energy and (d) the dynamical magnetic susceptibility \( \chi''(\vec{Q}_1) + \chi''(\vec{Q}_2) \) in BaFe\(_{2-x}\)Ni\(_x\)As\(_2\) measured by inelastic neutron scattering experiments at \( T = 5K < T_N \) near the optimal doping \( x = x_c \approx 0.1 \): the former(latter) is fit in the power law form \( E^{-\alpha}(E^{-\beta}) \) with the exponent \( \alpha \approx 1.0 (\beta \approx 0.5) \).
before. A detailed analysis, given in SM, shows that:

\[
\chi_s (\omega) \equiv \chi \left( \bar{Q}_1, \omega \right) + \chi \left( \bar{Q}_2, \omega \right) = \tilde{\chi}_m \left( 0, \omega \right) + O \left( \lambda^2 \right)
\]

and

\[
\chi_d (\omega) \equiv \chi \left( \bar{Q}_1, \omega \right) - \chi \left( \bar{Q}_2, \omega \right) = \lambda \nabla \left( 0, \omega \right) \tilde{\chi}_m^2 \left( 0, \omega \right) \tilde{\chi}_m \left( 0, 0 \right) + O \left( \lambda^2 \right)
\]

where \( \tilde{\chi}_m \left( q, \omega \right) \equiv \tilde{\chi}_{mA} \left( q, \omega \right) = \tilde{\chi}_{mA} \left( q, \omega \right) \) is the magnetic propagator, \( \tilde{\chi}_\Delta \left( q, \omega \right) \) is the nematic propagator, and \( \nabla \) is the irreducible vertex function involving two external magnetic order parameter fields \( \tilde{m}_A \) and \( \tilde{m}_B \) and one nematic order parameter field \( \Delta \). For both of the Eq. (5) and Eq. (6), we use the symmetry \( \tilde{m}_A \leftrightarrow \tilde{m}_B \) respected by the action (2), as discussed in the SM.

The identity, Eq. (6), will play a central role in the following analysis. The diagrammatic representation of this identity is shown in Fig. 2.

\[
\begin{align*}
\lambda & \delta & \Delta \\
\tilde{m}_A & \tilde{\chi}_m & \tilde{m}_B \\
\nabla & \tilde{\chi}_m & \tilde{\chi}_m
\end{align*}
\]

**FIG. 2.** The diagrammatic representation of the identity Eq. (6). The double black line and double cyan dashed line denote the renormalized magnetic propagator \( \tilde{\chi}_m \) and nematic propagator \( \tilde{\chi}_\Delta \), respectively. The blue circle is the vertex function \( \nabla \), and the red cross small circle is the external \( C_4 \) symmetry breaking potential.

**Scaling analysis:** We now apply the identity, Eq. (6), to extract the nematic susceptibility from the spin excitation anisotropy. Our focus is on the singular parts of these quantities in the quantum critical regime.

Due to the scale invariance in the quantum critical regime, the irreducible two-point correlation function \( \tilde{\chi}_m \left( 0, \omega \right) \) and the irreducible vertex function \( \nabla \left( 0, \omega \right) \) should obey a power law form with specific exponents. Therefore, we expect the spin excitation anisotropy \( \chi_d \left( \omega \right) \) to also obey the power law form with a specific exponent.

To derive these exponents, we carry out a scaling analysis of the irreducible vertex functions, using the generating functional \( \Gamma \left( m, \Delta \right) \)

\[
\Gamma \left( m, \Delta \right) = \sum \frac{1}{n_m n_\Delta} \int \langle q \rangle \langle p \rangle \int \langle \nu \rangle \int \langle \nu \rangle \prod_{i=1}^{n_m} \prod_{j=1}^{n_\Delta} \Gamma \left( m, \Delta \right) \left( \{ q \}, \{ p \}, \{ \omega \}, \{ \nu \} \right)
\]

where we have defined the abbreviated notations \( \int \langle q \rangle \langle p \rangle \equiv \int \prod_{i=1}^{n_m} d^D q_i \prod_{j=1}^{n_\Delta} d^D p_j \delta \left( \sum_{i=1}^{n_m} q_i + \sum_{j=1}^{n_\Delta} p_j \right) \) and

\[
\int \langle \nu \rangle \langle \nu \rangle = \int \prod_{i=1}^{n_m} d \omega_i \prod_{j=1}^{n_\Delta} d \nu_j \delta \left( \sum_{i=1}^{n_m} \omega_i + \sum_{j=1}^{n_\Delta} \nu_j \right),
\]

with \( D \) being the spatial dimensionality. In addition, we have introduced \( \Gamma \left( m, \Delta \right) \) to represent an irreducible vertex function with \( n_m \) and \( n_\Delta \) external magnetic and nematic order parameter fields, respectively.

In the quantum critical regime, under the rescaling \( q \rightarrow e^{-q}, \omega \rightarrow e^{-z \omega} \), the magnetic and nematic order parameter fields transform according to \( m \rightarrow e^{-d_m \eta} m, \Delta \rightarrow e^{-d_\Delta \eta} \Delta \), where \( d_m \) and \( d_\Delta \) are their respective scaling dimensions. Since \( \Gamma \left( m, \Delta \right) \) is a dimensionless quantity, the irreducible vertex function \( \Gamma \left( m, \Delta \right) \) must satisfy [39–41]:

\[
\Gamma \left( m, \Delta \right) \left( q, \omega \right) = e^{-d_m \eta} \Gamma \left( m, \Delta \right) \left( q e^{-l}, \omega e^{-z l} \right)
\]

where \( d_\nu = n_m \left( d_m + D + z \right) + n_\Delta \left( d_\Delta + D + z \right) - (D + z) \), and \( z \) is the dynamic exponent [42].

From the action, Eq. (2), it is straightforward to specify the scaling dimension [43] of the magnetic order parameter \( \tilde{m} \):

\[
d_m = - D + z + 2 - \eta.
\]

Because \( \eta \), the anomalous dimension, is typically small, we will carry out our analysis assuming \( \eta \approx 0 \); what happens when \( \eta \neq 0 \) is shown in the SM. It then follows from Eq. (8) that:

\[
\Gamma \left( m, \Delta \right) \left( q, \omega \right) = e^{\left[ n_m - n_\Delta (d_\Delta + D + z) - (D + z) \left( \frac{n_m}{n_\Delta} - 1 \right) \right]} \Gamma \left( m, \Delta \right) \left( q e^{-l}, \omega e^{-z l} \right)
\]

The magnetic propagator is determined by a two-point irreducible vertex function:

\[
\tilde{\chi}_m^{-1} \left( q, \omega \right) \equiv \Gamma \left( 2, \Delta = 0 \right) \left( q, \omega \right) = e^{2(0) \tilde{\chi}_m^{-1}} \left( q e^{-l}, \omega e^{-z l} \right)
\]

where in the last step we choose \( l \) such that \( q e^{-l} = 1 \). This, in turn, implies:

\[
\tilde{\chi}_m \left( 0, \omega \right) = \chi_s \left( \omega \right) \sim \omega^{-\frac{\eta}{2}}
\]

The function \( \nabla \) appearing in Eq. (9) and Fig. 2 is a three-point irreducible vertex function. The scaling procedure leads
\[ \nabla (q, \omega) \equiv \Gamma_{n=0, n=1}^{-1} (q, \omega) \]
\[ = e^{-[2-z-D(z)]} \chi (q e^{-1}, \omega e^{-1}) 
\]
\[ = q^{-[2-z-D(z)]} \chi (1, \omega q^{-z}) \]  

In turn, this gives rise to the following frequency dependence:
\[ \nabla (0, \omega) \sim \omega^{-2-z-D(z)} \]  

Collecting all these, we can now determine from Eq. (6) the scaling form for the spin excitation anisotropy:
\[ \chi_d (\omega) \sim \omega^{-2-z-D(z)} \]  

Conversely, by measuring the singular parts in the energy dependence of the spin excitation anisotropy \( \chi_d (\omega) \) and dynamical magnetic susceptibility \( \chi_s (\omega) \) in the quantum critical regime, we can determine the dynamical exponent \( z \) and the scaling dimension of the nematic order parameter \( d_\Delta \) through the following relations:
\[ \frac{-2}{z} = \frac{\partial \ln \chi_s (\omega)}{\partial \ln \omega} \]
\[ \frac{-d_\Delta - (D+z) - 2}{z} = \frac{\partial \ln \chi_d (\omega)}{\partial \ln \omega} \]  

The above determine \( d_\Delta \) and \( z \) for a given spatial dimensionality \( D \). In turn, we can determine the singular dynamical properties of the nematic degree of freedom, which we now turn to.

**Dynamical nematic susceptibility:** We now turn to the analysis of the dynamical nematic susceptibility, \( \chi_d (0, \omega) \), a task that seems impossible given that the identity Eq. (6) involves only the static nematic susceptibility \( \chi_d (0, 0) \). The key point is that the irreducible vertex function \( \nabla (0, \omega) \) couples the nematic and magnetic order parameter fields, and captures the critical singularity in the dynamical nematic correlations.

To make this point clear, we note that, according to the scaling analysis, the critical part of the dynamical nematic susceptibility \( \chi_d (q, \omega) \) obeys the following form:
\[ \chi_d^{-1} (q, \omega) \equiv \Gamma_{n=0, n=1}^{-1} (q, \omega) \]
\[ = e^{-[2-z-D(z)]} \chi_d^{-1} (q e^{-1}, \omega e^{-1}) \]
\[ = q^{-[2-z-D(z)]} \chi_d^{-1} (1, \omega q^{-z}) \]  

This, in turn, implies the following result for the dynamical nematic susceptibility \( \chi_d (0, \omega) \):
\[ \chi_d (0, \omega) \sim \omega^{-2+z-D(z)} \]  

**The case of quantum criticality in \( \text{BaFe}_2\text{As}_2 \) with optimal Ni-doping:** In the Ni-doped \( \text{BaFe}_2\text{As}_2 \), the singular energy dependences of the spin excitation anisotropy and magnetic susceptibility were observed near the optimal doping \( x = x_c \approx 0.1 \) by inelastic neutron scattering experiments \[31\] as shown in the Figs. [1(c) and 1(d)] (where \( T = 5K < T_S \) only affects the results at the lowest measured frequencies). The experimental data suggest that the spin excitation anisotropy \( \chi_d (\omega) \) and dynamical magnetic susceptibility \( \chi_s (\omega) \) are best-fitted in power laws with different exponents \( \alpha \) and \( \beta \), respectively:
\[ \chi_d (\omega) \sim \omega^{-\alpha} \]
\[ \chi_s (\omega) = \chi_m (0, \omega) \sim \omega^{-\beta} \]  

with \( \alpha \approx 1.0 \) and \( \beta \approx 0.50 \).

While there can be different physical reasons that could alter the values of the exponents \( \alpha \) and \( \beta \), it is intriguing that
\[ \alpha \approx 2\beta \]  

We demonstrate that the identity Eq. (6) we established earlier serves as a natural way to explore the physics behind the relation (20).

By the identity Eq. (6), we see that the relation (20) implies:
\[ \nabla (0, \omega) \sim \frac{\chi_d (\omega)}{\chi_m (0, \omega)} \sim \omega^0 . \]  

Following the scaling analysis of the irreducible vertex function, Eq. (14), we conclude that these power law forms are associated with an underlying QCP at which the scaling dimension of the nematic order parameter \( d_\Delta \) is:
\[ d_\Delta = 2 - D - z \]  

In other words, the relation (20) implies a specific link between the scaling dimension of the nematic order parameter \( d_\Delta \) and dynamic exponent \( z \).

This relation lontains non-trivial information about the nematic degree of freedom. To see this further, consider a general form of the nematic propagator suitable for the quantum critical regime:
\[ \chi_d^{-1} (q, \omega) = b_1 q^n + b_2 \frac{[\omega]}{q^n} \]  

For this propagator, we must have:
\[ z = n + a \]  

and
\[ d_\Delta = -\frac{D + z + n}{2} = -\frac{D + 2n + a}{2} \]  

On the other hand, according to Eqs. (25) and (26):
\[ d_\Delta = 2 - D - z = 2 - D - n - a \]  

Compare Eqs. (25) and (26), we can derive:
\[ a = 4 - D = 2 \]
when the spatial dimensionality $D = 2$.

The above result follows from $\alpha \approx 2\beta$ (Eq. 20). Further using $\alpha \approx 1.0$ (and $\beta \approx 0.50$), we find that $n = 2$. Thus, the quantum-critical nematic susceptibility is found to be:

$$\chi^{-1}_{\Delta}(q, \omega) = b_1 q^2 + b_2 \frac{\left|\omega\right|}{q^2}$$

(28)

To reiterate, the relation (20) suggests the presence of a non-trivial critical dynamical term $\left|\omega\right|/q^2$ in the nematic propagator. The origin of such dynamical term in the nematic sector and its relation with the microscopic physics of the iron pnictides will be investigated in a separate work.

Discussion and Conclusion: To summarize, we have shown how the singular component of the spin excitation anisotropy connects with its counterparts in both the nematic and dynamical magnetic susceptibilities. This identity has allowed us to extract the critical properties from the experiments in an optimally doped iron pnictides under a uniaxial strain, including several critical exponents and a singular nematic susceptibility as a function of both frequency and wavevector. Our results demonstrate the success of the spin-driven nematicity for understanding the measured responses in the magnetic and nematic channels. Our approach allows us to determine the dynamical nematic susceptibility, which is difficult to directly measure experimentally. The singular fluctuations in both the nematic and magnetic channels appear in the regime of optimized superconductivity within the iron-pnictide phase diagram. Thus, both are expected to influence the development of the superconductivity.

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Supplemental Material

Ginzburg-Landau Theory

We start from the Ginzburg-Landau description for the magnetism in iron pnictides, Eq. (1) of the main text. After performing the Hubbard-Stratonovich transformation to the quartic term \( S_4 \) in the action of Eq. (1), the resulting action is:

\[
S_0 = S_{2,\bar{m},\Delta,\eta} + S_{3,\bar{m},\Delta,\eta}
\]

\[
S_{2,\bar{m},\Delta,\eta} = \sum \left\{ \chi_{\bar{m}}^{-1}(q) \left| \bar{m}_A(q) \right|^2 + \left| \bar{m}_B(q) \right|^2 \right\}
\]

\[
+ \Delta \left( q \right) \chi_{\Delta}^{-1}(q) \Delta(q) + \frac{1}{4u_1} \left[ \left( \eta(q) - \tau \right) \left( \eta(q) - \tau \right) \right]
\]

\[
S_{3,\bar{m},\Delta,\eta} = \int_0^\beta d\tau \int d^2x \left[ 2\Delta \bar{m}_A \cdot \bar{m}_B + 2i\eta \left( \bar{m}_A^2 + \bar{m}_B^2 \right) \right]
\]

The field \( \Delta \) is the Ising-nematic order parameter field because it is the conjugate to the bilinear \( \bar{m}_A \cdot \bar{m}_B \), which changes sign under the transformation \( \bar{m}_A \to \pm \bar{m}_A, \bar{m}_B \to \mp \bar{m}_B \). The field \( \eta \) is the conjugate of \( \bar{m}_A^2 + \bar{m}_B^2 \). The nematic propagator \( \chi_{\Delta} \) will have the form

\[
\chi_{\Delta}^{-1}(q, i\omega_n) = \frac{1}{u_1} - \Pi_\Delta (q, i\omega_n).
\]

The constant term \( \frac{1}{u_1} \) comes from the Hubbard-Stratonovich transformation of the quartic term in Eq. (1), while \( \Pi(\vec{q}, i\omega_n) \) will come from integrating out high energy \( \bar{m}_A \) and \( \bar{m}_B \) fields as well as background coherent-fermion fields.

Nematicity and the Spin Excitation Anisotropy

The spin excitation anisotropy \( \chi_d(\omega) \) and the dynamical magnetic susceptibility \( \chi_s(\omega) \) is defined as:

\[
\chi_s(\omega) = \chi(\vec{Q}_1, \omega) + \chi(\vec{Q}_2, \omega)
\]

\[
\chi_d(\omega) = \chi(\vec{Q}_1, \omega) - \chi(\vec{Q}_2, \omega)
\]

where \( \vec{Q}_1 = (\pi, 0) \) and \( \vec{Q}_2 = (0, \pi) \) are ordering wave vectors, and the dynamical spin susceptibility \( \chi(\vec{q}, \omega) \) is:

\[
\chi(\vec{q}, \omega) = \frac{1}{u_1} \Pi_\Delta (\vec{q}, i\omega_n)
\]

Our next step is to express the dynamical magnetic susceptibility \( \chi_s(\omega) \) and the spin excitation anisotropy \( \chi_d(\omega) \) in terms of the magnetic order parameter fields \( \bar{m}_A, \bar{m}_B \), and the nematic order parameter field \( \Delta \). To do so, we just need to recognize that in the iron pnictides, the local spin operator field \( S(\vec{r}, \tau) \) can be represented as \( S(\vec{r}, \tau) = \bar{m}_1(\vec{r}, \tau) e^{i\vec{q}_1 \cdot \vec{r}} + \bar{m}_2(\vec{r}, \tau) e^{i\vec{q}_2 \cdot \vec{r}} \), where \( \bar{m}_1(\vec{r}, \tau) = (\bar{m}_A(\vec{r}, \tau) - \bar{m}_B(\vec{r}, \tau))/2 \) and \( \bar{m}_2(\vec{r}, \tau) = (\bar{m}_A(\vec{r}, \tau) + \bar{m}_B(\vec{r}, \tau))/2 \), is the magnetic order parameter associated with ordering wave vector \( \vec{Q}_1 \) and \( \vec{Q}_2 \), respectively. We can then express the spin excitation anisotropy \( \chi_d(\omega) \) and dynamical magnetic susceptibility \( \chi_s(\omega) \) in terms of the magnetic order parameter fields \( \bar{m}_A \) and \( \bar{m}_B \):

\[
\chi_d(\omega) \equiv \chi(\vec{Q}_1, \omega) - \chi(\vec{Q}_2, \omega)
\]

\[
\chi_s(\omega) \equiv \chi(\vec{Q}_1, \omega) + \chi(\vec{Q}_2, \omega)
\]

Here, the last equality for both Eq. (34) and Eq. (35) hold since the the model (2) respect the symmetry \( \bar{m}_A \leftrightarrow \bar{m}_B \), and there is no condensation of the \( \langle \bar{m}_A^2 - \bar{m}_B^2 \rangle \). We also neglected the terms such as \( \langle \bar{m}_A \vec{Q}_2 - \vec{Q}_1, \omega \rangle \bar{m}_B \). Because \( |\vec{Q}_1 - \vec{Q}_2| \approx \Lambda \), where \( \Lambda \) is the momentum cutoff of the theory, term can only be generated by very severe spatial fluctuations which had already been coarse-grained in the construction of the starting Ginzburg-Landau description (0).

The expectation values in Eqs. (34) and (35) are calculated under the action \( S = S_0 + S_{\lambda,\Delta} \). Now we treat the uniaxial strain \( S_{\lambda,\Delta} \) as a perturbation and expand the action with respect to \( S_0 \):

\[
\langle \mathcal{T}_\tau \bar{m}_A(0, \tau) \bar{m}_B(0, \tau) \rangle_{S_0 + S_{\lambda,\Delta}} = \langle \mathcal{T}_\tau \bar{m}_A(0, \tau) \bar{m}_B(0, \tau) \rangle_{S_0} + O(\lambda^2)
\]

\[
= \langle \mathcal{T}_\tau \bar{m}_A(0, \tau) \bar{m}_B(0, \tau) \rangle_{S_0} + O(\lambda^2)
\]

\[
= \int d\omega e^{i\omega \tau} \bar{\chi}_{m_A}(0, \omega') + O(\lambda^2).
\]

\[
= \int d\omega e^{i\omega \tau} \bar{\chi}_{m_A}(0, \omega') + O(\lambda^2).
\]
We have used \( \langle T_\tau \vec{m}_A (0, \tau) \vec{m}_A (0, 0) \Delta (0, \tau') \rangle_{S_0} = 0 \), and the fact that \( \chi_m \left( \vec{q}, \omega \right) \) is the magnetic propagator in the momentum space. Following Eq. (35) and Eq. (36), we have:

\[
\chi_s (\omega) = \chi_{m, AB} (0, \omega) = \chi_{m} (0, \omega) \tag{37}
\]

where \( \chi_{m, AB} (0, \omega) = \chi_{m, B} (0, \omega) \) because of the symmetry \( \vec{m}_A \leftrightarrow \vec{m}_B \) respected by the action (2). We have defined \( \chi_{m} (0, \omega) = \chi_{m, A} (0, \omega) = \chi_{m, B} (0, \omega) \).

We see that \( \chi_s (\omega) \) is just the dynamical magnetic propagator \( \chi_{m} (0, \omega) \). Similarly, for the spin excitation anisotropy \( \chi_d (\omega) \):

\[
\langle T_\tau \vec{m}_A (0, \tau) \cdot \vec{m}_B (0, 0) \rangle_{S_0, S_{\Delta, \lambda}} = \langle T_\tau \vec{m}_A (0, \tau) \cdot \vec{m}_B (0, 0) \rangle_{S_0} - \lambda \int d\tau' \langle \vec{m}_A (\tau, \tau') \cdot \vec{m}_B (0, 0) \Delta (0, \tau') \rangle_{S_0} + O (\lambda^2)
\]

\[
= -\lambda \int d\tau' \langle T_\tau \vec{m}_A (0, \tau) \cdot \vec{m}_B (0, 0) \Delta (0, \tau') \rangle_{S_0} + O (\lambda^2) \tag{38}
\]

where \( \langle T_\tau \vec{m}_A (0, \tau) \cdot \vec{m}_B (0, 0) \rangle_{S_0} \propto \langle \Delta \rangle_{S_0} \delta_{\tau, 0} \) vanishes when temperature \( T > T_S \), the nematic phase transition temperature, or \( T < T_S \) but near the critical point at which \( \langle \Delta \rangle_{S_0} \rightarrow 0 \). Since in this work, we focus on the quantum critical region, we can safely discard this term.

On the other hand, since any three-point correlation function can always be factorized as the product of suitable irreducible two-point correlation function and irreducible vertex function, we have:

\[
\int d\omega' e^{i\omega'\tau} \chi (\omega') = \chi_0 (\omega') - \chi (\vec{q}_2, \omega')
\]

\[
\chi_d (\omega) \equiv \chi \left( \vec{q}_1, \omega \right) - \chi \left( \vec{q}_2, \omega \right)
\]

\[
= \lambda \vec{V} \chi_m (0, \omega) \chi_\Delta (0, 0) \chi_m (0, -\omega) + O (\lambda^2)
\]

\[
= \lambda \vec{V} \chi_m (0, \omega) \chi_\Delta (0, 0) + O (\lambda^2) \tag{39}
\]

where the \( \chi_\Delta \) is the nematic propagator, and \( \vec{V} \) is the vertex function involving two external magnetic parameter fields \( \vec{m}_A \) and \( \vec{m}_B \), and one nematic order parameter field \( \Delta \). Again, we use \( \chi_{m, A} (0, \omega) = \chi_{m, B} (0, \omega) = \chi_{m} (0, \omega) \). Because of the time reversal symmetry, we also have \( \chi_{m} (0, \omega) = \chi_{m} (0, -\omega) \).

Note that when we derive the identities (37) and (40), we only expand the uniaxial strain term \( S_{\lambda, \Delta} \) perturbatively. Therefore, both the identities (37) and (40) are valid non-perturbatively as far as \( S_0 \) is concerned.

**SCALING ANALYSIS OF SPIN EXCITATION ANISOTROPY WHEN \( \eta \neq 0 \)**

In this section, we carry through the scaling analysis of the spin excitation anisotropy \( \chi_d (\omega) \) and dynamical magnetic susceptibility \( \chi_s (\omega) \) with a non-zero anomalous dimension \( \eta \) for the magnetic order parameter field \( \vec{m} \). The scaling dimension of the magnetic order parameter field is:

\[
d_m = -\frac{D + z + 2 - \eta}{2} \tag{41}
\]

with \( \eta \neq 0 \).

Following a procedure similar to that presented in the main text, in the quantum critical regime the frequency dependence of the magnetic propagator is:

\[
\chi_m (0, \omega) \sim \omega^{-\frac{2-a}{z}} \tag{42}
\]

and the frequency dependence of the vertex function is:

\[
\vec{V} (0, \omega) \sim \omega^{-\frac{(2-\mu-2\Delta)-(D+z)}{2z}} \tag{43}
\]

Consequently, by the identity Eq. (3), the spin excitation anisotropy is:

\[
\chi_d (\omega) \sim \omega^{-\frac{(2-\mu-2\Delta)-(D+z)}{2z}} \tag{44}
\]

Therefore, with the knowledge of the anomalous dimension \( \eta \), again we can detect the dynamical exponent \( z \) and the scaling dimension of the nematic order parameter \( d_\Delta \) by measuring the dynamical nematic susceptibility \( \chi_s (\omega) \) and the spin excitation anisotropy \( \chi_d (\omega) \) according to:

\[
\frac{- (2-\eta)}{z} = \frac{\partial \ln \chi_s (\omega)}{\partial \ln \omega}
\]

\[
\frac{-d_\Delta - (D+z) - (2-\eta)}{z} = \frac{\partial \ln \chi_d (\omega)}{\partial \ln \omega} \tag{45}
\]

For the singular energy dependence of the spin excitation anisotropy and the dynamical magnetic susceptibility observed in the Ni-doped BaFe2As2 near optimal doping, if anomalous dimension \( \eta \neq 0 \), again the relation (20) implies that:

\[
\vec{V} (0, \omega) \sim \frac{\chi_d (\omega)}{\chi_m (0, \omega)} \sim \omega^0 \tag{46}
\]

and thus following Eq. (43):

\[
d_\Delta = 2 - \eta - D - z \tag{47}
\]

Again, to see what this implies for the nematic degree of freedom, let’s consider the nematic propagator to take a general form in the quantum critical regime:

\[
\chi_\Delta^{-1} (q, \omega) = b_1 q^a + b_2 \frac{\omega}{q^a} \tag{48}
\]

by which we then know:

\[
z = n + a \tag{49}
\]

and

\[
d_\Delta = -\frac{D + z + n}{2} = -\frac{D + 2n + a}{2} \tag{50}
\]

Compare it with Eq. (47), it is straightforward to show that:

\[
a = 4 - D - 2n = 2 - 2\eta \tag{51}
\]