Parity Violation and a Preferred Frame

Tsao Chang
Center for Space Plasma and Aeronomy Research
University of Alabama in Huntsville
Huntsville, AL 35899
Email: changt@cspar.uah.edu

Based on parity violation in the weak interaction and evidences from neutrino oscillation, a natural choice is that neutrinos may be spacelike particles with a tiny mass. To keep causality for spacelike particles, a kinematic time under a non-standard form of the Lorentz transformation is introduced, which is related to a preferred frame. A Dirac-type equation for spacelike neutrinos is further investigated and its solutions are discussed. This equation can be written in two spinor equations coupled together via nonzero mass while respecting maximum parity violation. As a consequence, parity violation implies that the principle of relativity is violated in the weak interaction.

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I. INTRODUCTION

Parity violation is a specific feature of the weak interaction. It was first argued by T.D. Lee and C.N. Yang in 1956 [1] and experimentally established by C.S. Wu in beta-transition of polarized Cobalt nuclei [2]. In the standard model, neutrinos are naturally massless. Three flavors of neutrinos are purely left-handed, but anti-neutrinos are right-handed. In recent years, many convincing evidences for neutrino oscillation come from the solar and atmospheric neutrino data have shown that neutrinos have tiny mass (about 1 eV) or mass difference [3-5].

If neutrino has a tiny rest mass, it would move slower than light. When taking a Lorentz boost with a speed faster than the neutrino, the helicity of the neutrino would change its sign in the new reference frame. In another word, parity would not be violated in the weak interactions. In order to solve this dilemma, The hypothesis that neutrinos may be spacelike particles is further investigated in this paper. To keep causality for spacelike particles, a kinematic time under a non-standard form of the Lorentz transformation is introduced, which is related to a preferred frame.

Besides neutrino oscillations, the cosmic ray spectrum at $E \approx 1 - 4$ PeV [6] has suggested that the electron neutrino is a tachyon. Using a model to fit data, it yields a value for $m^2(\nu_e) \approx -3 \text{ eV}^2$, which is consistent with the results from recent measurements in tritium beta decay experiments [7-9]. Moreover, the muon neutrino also exhibits a negative mass squared [10].

The negative value of the neutrino mass-square simply means:

$$E^2/c^2 - p^2 = m^2_{\nu}c^2 < 0 \quad (1)$$

The right-hand side in Eq.(1) can be rewritten as $(-m^2_{s}c^2)$, then $m_s$ has a positive value. The subscript $s$ means spacelike particle, i.e. tachyon. The negative value on the right hand side of Eq.(1) means that $p^2$ is greater than $(E/c)^2$. Based on special relativity and known as re-interpretation rule, tachyon as a hypothetical particle was proposed by Bilaniuk et al. in the 1960s [11-12]. Some reviews can be found in Ref.[13].

It is usually to introduce an imaginary mass to consider neutrinos as tachyons, but these efforts could not reach the point of constructing a consistent quantum theory. Some early investigations of Dirac-type equations for tachyons are listed in Ref.[14,15]. An alternative approach
was investigated by Chodos et al. [16]. A form of the lagrangian density for tachyonic neutrinos was proposed. Although they did not obtain a satisfactory quantum theory for tachyonic fermions, they suggested that more theoretical work would be needed to determine a physically acceptable theory.

II. GGT AND A PREFERRED FRAME

To keep causality for spacelike particles, a non-standard form of the Lorentz transformation has been studied, which is called the generalized Galilean transformation (GGT). The reason is as follows.

Within the context of the variation principle, it is well known that there is the freedom to choose different types of physical time [17]. In general, the invariant four-line element can be written in arbitrary coordinates

\[ ds^2 = g_{\mu\nu}dx^\mu dx^\nu \]  

(2)

where index \( \mu \) (or \( \nu \)) = 0,1,2,3. The 0th coordinate is the time coordinate, which is related to the measured time but may be not identical to it. The action integral for a free particle has a form:

\[ S = \int (mc)ds = \int L(x^\mu, \dot{x}^\mu)d\lambda \]  

(3)

where \( \dot{x}^\mu = dx^\mu/d\lambda \) and \( \lambda \) is called the evolution parameter, which also plays roll of time. \( L(x^\mu, \dot{x}^\mu) \) is the Lagrangian in four-dimensional space,

\[ L(x^\mu, \dot{x}^\mu) = (mc)ds/d\lambda = mc(g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu)^{1/2} \]  

(4)

When using the Euler-Lagrange equation in terms of the variation principle, we can obtain the geodesic equation. Notice that the above Lagrangian is valid for any choice of the evolution parameter \( \lambda \) in one reference frame. This method can be applied to the space-time theories in flat space though it is frequently used in general relativity for curved space. Furthermore, when we consider a specific evolution parameter as the physical time in all of the inertial frames, certain consistent rules must be imposed. Under this constraint, the definitions of physical time have only a few limited choices.

Besides the definition of relativistic time, another natural time, \( \lambda = \tilde{t} \), defined by the generalized Galilean transformation (GGT) has been studied [18-21]. Let us start with a simplified 2-D line element, \( ds^2 = \)
\[ c^2dT^2 - dX^2, \] in a preferred inertial frame \( \Sigma(X, T) \). Considering another inertial frame, \( S(x, t) \), which moves with a constant velocity \( v < c \) along the \( x \) axis with respect to \( \Sigma \), the 2-D form of GGT can be expressed as follows:

\[
x = \gamma(X - vT) \\
\tilde{t} = \gamma^{-1}T
\]

where \( \gamma = \left(1 - v^2/c^2\right)^{-1/2} \) is the factor of time dilation or length contraction. It is easily seen from Eq.(5) that the synchronization of distant events is absolute, independent of the choice of the reference frame. It has been shown that GGT is a non-standard form of the Lorentz transformation (LT)[18-21]. GGT and LT are equivalent if we describe particles with velocity less than (or equal to) light. On the other hand, when describing spacelike particles, GGT has its advantages since the GGT time, \( \tilde{t} \), always goes forward.

In terms of GGT, 4-D line element in Eq.(2) can be written as:

\[
ds^2 = \left(c\tilde{t} - \left(\frac{v}{c}\right) \cdot d\mathbf{r}\right)^2 - \left(d\mathbf{r}\right) \cdot d\left(\mathbf{r}\right) = \left(c\tilde{t}\right)^2 - \left(d\mathbf{r}\right) \cdot d\left(\mathbf{r}\right)
\]

It gives the relationship between GGT time \( \tilde{t} \) and the SR time \( t \). Therefore, GGT time is a supplement to SR time. For timelike particles, the 4-D momentum can be defined as

\[
P^\mu = m_0cdx^\mu/ds
\]

where the contravariant 4-D momentum \( P^\mu = (p, (ds/c\tilde{t})^{-1}m_0c) \). Using tensor calculus, the covariant 4-D momentum can be obtained as \( P_\mu = g_{\mu\nu}P^\nu \). It can be easily proven that the relation of energy and momentum under GGT is the same as in SR [18-21]. For spacelike particles, since \( ds^2 < 0 \), we need to introduce an invariant, \( d\tau = \sqrt{(-ds^2)}/c \), the contravariant 4-D momentum can be defined as

\[
P^\mu = m_sdx^\mu/d\tau = m_s\Gamma dx^\mu/d\tilde{t} = (m_s\Gamma\tilde{u}_s, m_s\Gamma c)
\]

where \( \Gamma = (d\tau/d\tilde{t})^{-1} \). In terms of Eq.(8), the relation of energy and momentum under GGT for spacelike particles is also the same as Eq.(1) [18-21]. As a natural choice, we assume that a tachyon has only positive energy in the preferred frame \( \Sigma \). However, the lowest limitation of
momentum and energy for tachyons are different in a non-preferred frame. It can be derived from Eq.(8) when velocity \( \tilde{u} \to \infty \):

\[
P_\infty = m_s c [1 - (n \cdot v/c)^2]^{-1/2} n
\]

\[
E_\infty = -m_s c^2 (u \cdot v/c^2) [1 - (n \cdot v/c)^2]^{-1/2}
\]

(9)

where the unit vector \( n = \tilde{u} / \tilde{u} \). In the preferred inertial frame, \( v = 0 \), then the low limit \( p_\infty = m_s c n \), and \( E_\infty = 0 \). In other inertial frames, the lowest energy is not equal to zero, which could have a limited negative value. In Eq.(9), when the unit vector \( n \) changes its direction from \( x \) to \(-x\), the sigh of \( (u \cdot v) \) is also changed. Because neutrinos are created in the weak interaction, the asymmetry for \( E_\infty \) space inversion may be a source of CP violation in the weak interaction. If we identify the preferred frame with the cosmic microwave background radiation (CMBR), the earth frame has a speed of \( (v/c \approx 10^{-3}) \) with respect to CMBR. Since the mass of e-neutrino is about \( 1 \text{eV} \), we obtain \( \Delta E_\infty \approx 10^{-3} \text{eV} \), which is a undetectable effect at present time.

### III. A SPACELIKE DIRAC-TYPE EQUATION

To follow Dirac's approach [22], the Hamiltonian form of spacelike Dirac-type equation for neutrinos can be written in:

\[
\hat{E} \Psi = c(\vec{\alpha} \cdot \hat{p}) \Psi + \beta_s m_s c^2 \Psi
\]

(10)

with \( \hat{E} = i\hbar \partial / \partial t \), \( \hat{p} = -i\hbar \nabla \). \( \vec{\alpha} = (\alpha_1, \alpha_2, \alpha_3) \) and \( \beta_s \) are \( 4 \times 4 \) matrix, which are defined as

\[
\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \quad \beta_s = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}
\]

(11)

where \( \sigma_i \) is \( 2 \times 2 \) Pauli matrix, \( I \) is \( 2 \times 2 \) unit matrix. It is easily to prove that there are commutation relations as follows:

\[
\alpha_i \alpha_j + \alpha_j \alpha_i = 2 \delta_{ij}
\]

\[
\alpha_i \beta_s + \beta_s \alpha_i = 0
\]

\[
\beta_s^2 = -1
\]

(12)
Furthermore, the relation between the matrix $\beta_s$ and the standard matrix $\beta$ is

$$\beta_s = \beta \gamma_5, \quad \text{where} \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \quad (13)$$

The spacelike Dirac-type equation (10) was briefly studied in [23]. It can be rewritten in covariant forms by multiplying matrices $\beta$ and $\gamma_5$. The covariant form has been discussed in Ref.[16] and [25-26] except the sign for the momentum operator can be negative. A more general form of Dirac equation with two mass parameters has also been studied in Ref.[27]. In addition, Eq.(10) was investigated in a different way [28].

We now study the spin-$\frac{1}{2}$ property of the neutrino (or antineutrino) as a tachyonic fermion. Denote the wave function $\Psi$ by two spinor functions $\varphi(\vec{x},t), \chi(\vec{x},t)$ the spacelike Dirac-type equation (10) can be rewritten as a pair of two-component equations:

$$i\hbar \frac{\partial \varphi}{\partial t} = -i\hbar \vec{\sigma} \cdot \nabla \chi + m_s c^2 \chi$$

$$i\hbar \frac{\partial \chi}{\partial t} = -i\hbar \vec{\sigma} \cdot \nabla \varphi - m_s c^2 \varphi \quad (14)$$

From Eq.(14), the conserved current can be derived:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0 \quad (15)$$

and we obtain

$$\rho = \Psi^\dagger \gamma_5 \Psi, \quad \vec{j} = c(\Psi^\dagger \gamma_5 \vec{\sigma} \Psi) \quad (16)$$

where $\rho$ and $\vec{j}$ are probability density and current; $\Psi^\dagger$ is the Hermitian adjoint of $\Psi$.

Considering a plane wave along the $z$ axis for a right-handed particle, $\nu$, the helicity $H = (\vec{\sigma} \cdot \vec{p})/|\vec{p}| = 1$, then Eq.(14) yields the following relation:

$$\chi = \frac{cp - m_s c^2}{E} \varphi \quad (17)$$

The plane wave can be represented by $\Psi(z,t) = \psi_\sigma \exp[i(\vec{\sigma} \cdot \vec{p} - Et)]$, where $\psi_\sigma$ is a four-component bispinor. Substituting this bispinor into
the wave equation (10) or (14), the explicit form of two bispinors with positive-energy states are listed as follows:

$$\psi_1 = \psi_{\uparrow(+)} = N \begin{pmatrix} 1 \\ 0 \\ A \\ 0 \end{pmatrix}, \quad \psi_2 = \psi_{\downarrow(+)} = N \begin{pmatrix} 0 \\ -A \\ 0 \\ 1 \end{pmatrix}$$ (18)

and other two bispinors with the negative-energy states are:

$$\psi_3 = \psi_{\uparrow(-)} = N \begin{pmatrix} 1 \\ 0 \\ -A \\ 0 \end{pmatrix}, \quad \psi_4 = \psi_{\downarrow(-)} = N \begin{pmatrix} 0 \\ A \\ 0 \\ 1 \end{pmatrix}$$ (19)

where the component $A$ and the normalization factor $N$ are

$$A = \frac{cp - m_s c^2}{|E|}, \quad N = \sqrt{\frac{p + m_s c}{2m_s c}}$$ (20)

For $\psi_1 = \psi_{\uparrow(+)}$, the conserved current in Eq.(15) becomes:

$$\rho = \Psi_1^\dagger \gamma_5 \Psi_1 = \frac{|E|}{m_s c^2}, \quad j = \frac{p}{m_s}$$ (21)

Clearly, the ratio $j/\rho$ represents the superluminal speed $u_s$. For $\psi_2 = \psi_{\downarrow(+)}$, the density $\rho$ is negative so that it should be discarded. If we consider the negative states as mathematics solutions in the preferred frame, then $\psi_1 = \psi_{\uparrow(+)}$ is the only solution with physical identity i.e. $\bar{\nu}_R$. It gives a natural choice that antineutrino is right-handed only. Therefore, maximum parity violation implies that the principle of relativity is violated in the weak interaction. If we identify the preferred frame with CMBR, the earth frame can be considered as the preferred frame approximately $(v/c \approx 10^{-3})$. Further study on the negative states and negative $\rho$ may be associated with complicated mathematics under GGT (see [24,25]), which will not be discussed here.

Let $\bar{\Psi} = \Psi^\dagger \beta$, we can obtain the following scalars:

$$\bar{\Psi}_1 \Psi_1 = \bar{\Psi}_3 \Psi_3 = 1$$ (22)
$$\bar{\Psi}_2 \Psi_2 = \bar{\Psi}_4 \Psi_4 = -1$$ (23)
In addition, the pseudo scalar for each spinor satisfies:

$$\bar{\Psi} \gamma_5 \Psi = 0 \quad (24)$$

**IV. PARITY VIOLATION FOR NEUTRINOS**

In order to compare the spacelike Dirac-type equation Eq.(10) with the two component Weyl equation in the massless limit, we now consider a unitary transformation of $\varphi$ and $\chi$:

$$\xi = \frac{1}{\sqrt{2}}(\varphi + \chi), \quad \eta = \frac{1}{\sqrt{2}}(\varphi - \chi) \quad (25)$$

where $\xi(\vec{x},t)$ and $\eta(\vec{x},t)$ are two-component spinor functions. In terms of $\xi$ and $\eta$, Eq.(16) becomes

$$\rho = \xi^\dagger \xi - \eta^\dagger \eta, \quad \vec{j} = c(\xi^\dagger \vec{\sigma} \xi + \eta^\dagger \vec{\sigma} \eta) \quad (26)$$

Moreover, Eq.(10) can be rewritten in Weyl representation:

$$i\hbar \frac{\partial \xi}{\partial t} = -ic\vec{\sigma} \cdot \nabla \xi - m_s c^2 \eta$$

$$i\hbar \frac{\partial \eta}{\partial t} = ic\vec{\sigma} \cdot \nabla \eta + m_s c^2 \xi \quad (27)$$

In the above equations, both $\xi$ and $\eta$ are coupled via the mass term $m_s$.

Comparing Eq.(27) with the well known Weyl equation, we take a limit, $m_s = 0$, then the first equation in Eq.(27) reduces to

$$\frac{\partial \xi_\varphi}{\partial t} = -c\vec{\sigma} \cdot \nabla \xi_\varphi \quad (28)$$

In addition, the second equation in Eq.(27) vanishes because $\varphi = \chi$ when $m_s = 0$.

Eq.(28) is the two-component Weyl equation for describing anti-neutrinos $\bar{\nu}$, which is related to the maximum parity violation discovered in 1956 by Lee, Yang and Wu [1,2]. They pointed out that no experiment had shown parity to be a good symmetry for weak interactions. Now we see that, in terms of Eq.(27), once if neutrino has some mass, no matter how small it is, two equations should be coupled together via the mass term while still respecting maximum parity violation.
Indeed, the Weyl equation (28) is only valid for the antineutrinos since the antineutrino always has right-handed spin, which is opposite to the neutrino. In order to describe the left-handed neutrino, we now take a minus sign for the momentum operator, then Eq.(2) becomes

\[ \hat{E} \Psi_\nu = -c(\vec{\alpha} \cdot \hat{\mathbf{p}})\Psi_\nu + \beta_s m_s c^2 \Psi_\nu \]  

(29)

Similar to the solutions for Eq.(10), Eq.(29) yields one physical solution for the neutrino: \( \psi_\nu = \psi_{\downarrow(+)} \). Therefore, only \( \bar{\nu}_R \) and \( \nu_L \) exist in nature. From Eq.(29), the two-component Weyl equation for massless neutrino becomes:

\[ \frac{\partial \xi_\nu}{\partial t} = c\vec{\sigma} \cdot \nabla \xi_\nu \]  

(30)

Some related discussions the CPT theorem can be found in Ref.[29].

V. REMARKS

Based on parity violation in the weak interaction and evidences from neutrino oscillation, a natural choice is that neutrinos may be spacelike particles with a tiny mass. A Dirac-type equation for spacelike neutrinos is further investigated and its solutions are discussed. This equation can be written in two spinor equations coupled together via nonzero mass while respecting maximum parity violation. As a consequence, parity violation implies that the principle of relativity is violated in the weak interaction.

Spacelike neutrinos have many peculiar features, which are very different from all other particles. Neutrino has left-handed spin in any reference frame, but anti-neutrino always has right-handed spin. This means that the speed of neutrinos must be equal to or greater than the speed of light. Otherwise, the spin direction of neutrino would be changed in some reference frames when taking a Lorentz boost. Moreover, as shown in this paper, the energy of a tachyonic neutrino (or anti-neutrino), \( E_\nu \), could be negative in non-preferred frames, which was studied in Ref.[30].

The electron neutrino and the muon neutrino may have slightly different proper masses. It provides a natural explanation why the numbers of e-lepton and \( \mu \)-lepton are conserved respectively in low energy experiments.

Comparing with the electron mass, the mass term of the e-neutrino in Eq.(10) is approximately close to zero. Moreover, from Eq.(24),
Ψγ₅Ψ = 0 for Spacelike neutrinos. It means that the mass term in Eq.(10) can be negligible in most experiments. In fact, the momentum of a neutrino is much greater than \((m_s c)\) in most measurements. For instance, let \(p_s = 10(m_s c) = 16 \text{ eV}/c\), the speed of the e-neutrino is about \(u_s = 1.005c\). It also yields the component of bispinor \(A \simeq 1\) in Eq.(18). Therefore, spacelike neutrinos behave just like the massless neutrinos. This similarity may also play role at the level of SU(2) gauge theory.

According to special relativity [31], if there is a spacelike particle, it might travel backward in time. Besides the re-interpretation rule introduced in the 1960s, another approach is to introduce a kinematic time under GGT [18-21]. Therefore, special relativity can be extended to the spacelike region without causality violation.

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