The Operator Manifold Formalism. II.
Physical Applications

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Abstract

Within the operator manifold approach developed in the previous part [1] we consider some key problems of particle physics, particularly, we derive the Gell-Mann-Nishijima relation and flavour group. This scheme enables to conclude that the leptons are particles with integer electric and leptonic charges and free of confinement, while quarks carry fractional electric and baryonic charges and imply the confinement. We consider the unified electroweak interactions with small number of free parameters. We exploit the background of the local expanded symmetry $SU(2) \otimes U(1)$ and P-violation. The Weinberg mixing angle is shown to have fixed value at 30°. Due to the Bose-condensation of relativistic fermion pairs the Higgs bosons arise on an analogy of the Cooper pairs in superconductivity. Within the present microscopic approach we predict the Kobayashi-Maskawa quark flavour mixing; the appearance of the CP-violation phase; derive the mass-spectrum of leptons and quarks, as well as other emerging particles, and also some useful relations between their masses.

1 Introduction

Based on the operator manifold formalism (Part I) [1] in the present paper we develop the microscopic approach to the problems of field theory. Although the complete picture is largely beyond the scope of it, nevertheless some key problems are already solved. The proliferation of lepton and quark flavours prompts us within this framework to consider the fields as composites. Certainly, it may seem foolhardy to set up such picture in the spacetime continuum. The difficulties here are well-known. The first problem is closely related to the fact that the expected mass differences of particles would be too large ($\geq 1T_eV$). Another problem concerns the transformations of particles. Our idea is to remove these difficulties by employing the multiworld geometry. The formalism of operator multimani fold yields the multiworld geometry decomposing into the spacetime continuum and internal worlds. Thus, all fields including the leptons and quarks, along with the spacetime component have nontrivial composite internal multiworld structure. While the various subquarks then are defined in the corresponding internal worlds. The microscopic structure of leptons, quarks and other particles are governed by the various possible conjunctions of subquarks implying concrete symmetries.

This article is organized as follows: In an enlarged framework of the operator multi-manifold we define and clarify the conceptual basis of subquarks and their characteristics stemming from the various symmetries of the internal worlds (sec.2). This scheme enables an insight to the key concepts of particle physics (sec.3-9), and to conclude that the leptons are particles with integer electric and leptonic charges and free of confinement, while the quarks carry fractional electric and baryonic charges and imply the confinement. We derive the Gell-Mann-Nishijima relation and the flavour group. The multiworld structure of
primary field (sec.10,11) is described by the gauge invariant Lagrangian involving nonlinear fermion interactions of the components somewhat similar to the theory by Heisenberg and his co-workers [10,11], but still it will be defined on the multiworld geometry. From this Lagrangian the whole complexity of the leptons, quarks and their interactions arises. The number of free parameters in this Lagrangian is reduced to primary coupling constant of the nonlinear interaction and gauge coupling, apart from some additional primary mass constants which are not essential for the conclusion. Based on it, we consider the unified electroweak interactions (sec.12-19). It follows that contemporary phenomenological standard model of electroweak interactions [4-8] is an approximation to the suggested microscopic approach. We exploit the background of the local symmetry $SU^{loc}(2) \otimes U^{loc}(1)$ (sec.12), the weak hypercharge and P (mirror symmetry)-violation (sec.13). The Weinberg mixing angle determining the symmetry reduction coefficient is shown to have a value fixed at 30°. We develop the microscopic approach to the isospinor Higgs boson with self-interaction and Yukawa couplings (sec.14,15). It involves Higgs boson as the collective excitations of bound quasi-particle pair. Tracing a resemblance with the Cooper pairs [17-19], within the framework of local gauge invariance of the theory incorporated with the phenomenon of P-violation in weak interactions we suggest a mechanism providing the Bose-condensation of relativistic fermion pairs. Unobserved effects produced by ready made Higgs bosons are suppressed. Finally we attempt to predict the mixing angles in the six-quark KM model (sec.17), the appearance of the CP-violation phase (sec.18), derive the mass-spectrum of leptons and quarks and other emerging particles, as well as some useful relations between their masses (sec.19).

2 Subquarks and Subcolour Confinement

Since our discussion within this section in many respects is similar to that of sec.3,4 in [1], here we will be brief. According to our strategy we admit that the $\eta$-type (fundamental) regular structure forms a stable system with infinite number of distorted $^i u$-type ordinary structures of different species ($i = 1, \ldots, N$). In this stable system the flat multimanifold $G_N$ is realized (eq.(4.3.6) in [1])

$$G_N = G_{\eta u_1} \oplus \cdots \oplus G_{u_N}.$$  

We assume that the distortion rotations ($G_{u_i} \theta \rightarrow G_{\tilde{u}_i}$) through the angles $^i \theta_{+k}$ and $^i \theta_{-k}$, $k = 1, 2, 3$ occur separately in the three dimensional internal spaces $R^3_{u_{i+}}$ and $R^3_{u_{i-}}$ composing six dimensional distorted submanifold $G_{\tilde{u}_i} = R^3_{u_{i+}} \oplus R^3_{u_{i-}}$. Furthermore, for the rotation angles we take the ansatz

$$^i \theta_{\pm k} = \delta_{0 m_i} ^i \theta_{\pm k}(\eta) + (1 - \delta_{0 m_i}) ^i \theta^c_{\pm k}, \quad (2.1)$$

Here $\delta$ is the Kronecker symbol, the angles $^i \theta_{\pm k}(\eta)$ and $^i \theta^c_{\pm k}$ are respectively local and global (specified by (c)), and

$$m_i^c \hat{\Psi}_u(0) \equiv \hat{\rho}_{u_i} ^i \hat{\Psi}_u(0), \quad (2.2)$$

where $\hat{\rho}_{u_i} = i \gamma^{(\lambda \alpha)} \partial_{u_i(\lambda \alpha)}$ eq.(4.2.5) [1], $^i \hat{\Psi}_u(0)$ is the plane wave function of state of regular ordinary structure. Then, the distortion rotations are local in the internal world if $m_i^c = 0,$
coefficients structures furnished by generalized operators of creation and annihilation as the expansion wave packets constructed by superposition of the link functions of distorted ordinary and global otherwise. As it is exemplified in sec.4 in [1], the laws apply in use the fields $i \Psi(\theta_\pm)$ and $i \Psi(\theta_-)$ are defined on the distorted internal spaces $R^3_{u_\pm}$ and $R^3_{u_-}$. The generalized expansion coefficients in eq.(2.3) imply

$$ < \chi_- | \{ i \tilde{\gamma}^{(a)}_{u_k} (p_{u_i}, s_i^1), i \tilde{\gamma}^{k'}_{u_+} (p'_{u'_i}, s'_{j}) \} | \chi_- > = - \delta_{ij} \delta_{kk'} \delta_{ss'} [G(p_{u_i} - p'_{u'_i})],$$

(2.4)

The condition eq.(4.3.5) in [1] of multiworld geometry realization reduces to

$$ \sum_{i=1}^{N} \omega_i \left[ \lim_{i \theta_+ \rightarrow i \theta_-} G^\theta_{u_i F} (i \theta_+ - i \theta_-) \right] = \lim_{\eta_f \rightarrow \eta'_f} G(\eta_f - \eta'_f),$$

(2.5)

provided $\omega_i = \frac{u^2_i}{u^2}$. Taking into account the expression of causal Green’s function for given (i)

$$ G^\theta_{u_i F} (i \theta_+ - i \theta_-) = - i \int \frac{d^3p_{u_i}}{(2\pi)^{3/2}} i \Psi_{u_+ + p} (i \theta_+ \pm i \theta_-) \Psi_{u_- + p} (i \theta_- \pm i \theta_+),$$

(2.6)

in the case

$$ \lim_{u_{i_1} \rightarrow u_{i_2}} \lim_{\eta_f \rightarrow \eta'_f} \left[ G(u_{i_1} - u_{i_2}) / G(\eta_f - \eta'_f) \right] = \lim_{u'_{i_1} \rightarrow u'_{i_2}} \lim_{\eta_f \rightarrow \eta'_f} \left[ G(u'_{i_1} - u'_{i_2}) / G(\eta_f - \eta'_f) \right] = \text{inv},$$

one gets

$$ \sum_k i \Psi_{u}(i \theta_+) i \Psi_{u}(i \theta_-) = \sum_k i \Psi'_{u}(i \theta'_{+}) i \Psi'_{u}(i \theta'_{-}) = \cdots = \text{inv}. $$

(2.7)

So, in the context of multiworld geometry it is legitimate to substitute a term of quark $(q_k)$ defined in sec.3 in [1] by subquark $(q'_k)$. Everything said will then remain valid, provided we make a simple change of quarks into subquarks, the colours into subcolours. Hence, we may think of the function $i \Psi_{u}(i \theta_+) -conjugated bispinor field of subquark $(q'_k)$ of species $(i)$ with subcolour $k$, and respectively $i \Psi'_{u}(i \theta_-) -conjugated bispinor field of antisubcolour $(k)$. The subquarks and antishubquarks may be local $(q'_k)$ or global $(q'_k)$. Then, the subquark $(q'_k)$ is the fermion with the half integer spin and subcolour degree of freedom. According to eq.(2.7), they obey a condition

$$ \sum_k i q_k i q'_k = \sum_k i q'_k i q_k = \cdots = \text{inv}.$$  

(2.8)
To trace a resemblance with sec.3 in [1], the internal symmetry group \(^iG = U(1), SU(2), SU(3)\) enables to introduce the gauge theory in internal world with the subcolour charges as exactly conserved quantities. Thereto the subcolour transformation are implemented on subquark fields right through local and global rotation matrices of group \(^iG\) in fundamental representation. Due to the Noether procedure the conservation of global charges ensued from the global gauge invariance of physical system, meanwhile reinforced requirement of local gauge invariance may be satisfied as well by introducing the gauge fields with the values in Lie algebra \(^i\hat{g}\) of group \(^iG\).

### 3 The Multiworld Structures: Leptons and Quarks

For our immediate purpose, however, we shall consider the collection of matter fields \(\Psi(\zeta)\) with nontrivial internal structure \(\Psi(\zeta) = \Psi(\eta)^{-1}\Psi(u_l)\cdots^N\Psi(\theta_N)\). We suppose that the component \(\Psi^i(\theta)\) is made of product of some subquarks and antishubquarks \(\Psi^i(\theta) = \Psi^i(\{i\eta\}, \{i\bar{\eta}\})\). The fields of subquarks \(i\eta\) and antishubquarks \(i\bar{\eta}\) form the multiplets transforming by fundamental \(iD(j)\) and contragradient \(i\bar{D}(j)\) irreducible representations of group \(^iG\). Hereinafter we admit that multiworld index \((i)\) will be running only through \(i = Q, W, B, s, c, b, t\) specifying the internal worlds formally taken to denote in following nomenclature: \(Q\)-world of electric charge; \(W\)-world of weak interactions; \(B\)-baryonic world of strong interactions; \(s, c, b, t\) are the worlds of strangeness, charm, bottom and top. Also we admit that \(m^i_l = 0\) for \(i = Q, W, B\) and \(m^i_l \neq 0\) for \(i = s, c, b, t\), where \(m^i_l\) is given by eq.(2.2). According to the ansatz eq.(2.1), the distortion rotations in the worlds \(Q, W\) and \(B\) are local \(i\theta_{\pm k}(\eta)\), while they are global in the worlds \(s, c, b, t\). Below we introduce the fields of leptons \((l)\) and quarks \((q_f)\) with different flavours \(f=u, d, s, c, b, t\). To develop some feeling for this problem and to avoid irrelevant complications, now we may temporarily skeletonize it by taking the leptons have following multiworld structure:

\[
l \equiv \Psi_l(\zeta) = \Psi(\eta)\Psi(u_Q)\Psi(u_W), \tag{3.1}
\]

while the quarks are in the form

\[
q_f \equiv \Psi_f(\zeta) = \Psi(\eta)\Psi(u_Q)\Psi(u_W)\Psi(u_B)q^c_f, \tag{3.2}
\]

where the superscript \((c)\) specified the worlds in which rotations are global

\[
q^c_u = q^c_s = 1, \quad q^c_s = \Psi^c(u_s), \quad q^c_c = \Psi^c(u_c), \quad q^c_b = \Psi^c(u_b), \quad q^c_t = \Psi^c(u_t). \tag{3.3}
\]

We can take this scheme as a starting point for our considerations. We assume that none of these components is accidentally zero and implies

\[
i\Psi^A(\cdots, \theta_{i_1}, \cdots \theta_{i_n}, \cdots) j\Psi^B(\cdots, \theta_{i_1}, \cdots \theta_{i_n}, \cdots) = \delta_{ij} \sum_{l=i_1, \ldots, i_n} f^A_{il} \Psi^l \Psi^l, \tag{3.4}
\]

namely, the contribution of each individual subquark \(i\bar{q}_l\) into the component of given world \((i)\) is determined by the partial formfactor; (invariant charge) \(f^A_{il}\). A fascinating opportunity has turned out to infer these formfactors if we assume that this contribution is
completely governed by the rules of nonlinear processes. Then we may utilize the equations of functional autoduality [12] adopted to describe a wide class of nonlinear processes with a different dynamical properties, including also the problems of renormalization in quantum field theory [13,14] and radiative transfer [15]. The significant feature of these equations is their universal character, namely they are independent from the details of concrete physical problems. Drawing the analogy with radiative transfer in the problem of diffuse reflection and transmission of reduced incident flux $f$, which penetrates to the depth $\tau$, we consider, in general, a plane-parallel medium of finite thickness $\tau + \tau_0$ and ask for the intensity diffusing reflected and the intensity diffusing transmitted $f_{AB} = \bar{f}$ below the surface $\tau_0$. Tracing the implications of the principles of invariance [16] in radiative transfer, the nonlinear functional equations for the scattering and transmission functions are derived in [15], which turned out to be identical to those of corresponding equations of renormalization group [17]. The basic functional equation governing the functions $\bar{f}$ may be written

$$\bar{f}(\tau, \tau_0, f) = \bar{f}(\tau - t, \tau_0 - t, \bar{f}(t, \tau_0, f)),$$

(3.5)

This equation is identical to functional equation of renormalization group for the massive invariant charge, where $\tau_0 \to \ln \frac{m^2}{\lambda^2}$ and $\tau = \ln x$

$$\bar{f}(x, y, f) = \bar{f}(x/\xi, y/\xi, \bar{f}(\xi, y, f)),$$

(3.6)

provided $\bar{f}(1, y, f) = f$. Reviewing the notation $\lambda$ is taken to denote the normalization momentum, $x = \frac{p^2}{\lambda^2}$ and $y = \frac{m^2}{\lambda^2}$ are the dimensionless momentum and mass parameters. The other case of massless invariant charge corresponds to the particular problem of diffuse reflection and transmission by semi-infinite medium. The basic differential equation can be obtained by differentiating functional equation with respect to $\xi$ and passing to the limit $\xi = 1$. In the aftermath one gets the well-known equation of Ovsyannikov-Källen-Symanzik, the solution of which can be written only if the function $\beta = \frac{\partial \bar{f}(\xi, y, f)}{\partial \xi} \bigg|_{\xi=1}$ is known.

The explicit forms of the functions $\Psi(u_i)$ determining microscopic structure of concrete fields will be important subject for discussion in the next sections. Here, we just merely point out that due to concrete symmetries of internal worlds, the multiworld structure of leptons eq.(3.1) and quarks eq.(3.2) will come into being if only following two conditions would be satisfied, namely, besides the multiworld geometry realization requirement eq.(2.5) a condition of multiworld connections must be held too, which will be discussed in section 5.

4 The Rotational Modes

We assign to each distortion rotation mode in the three dimensional spaces $\mathbb{R}^3_{u_1+}$ and $\mathbb{R}^3_{u_1-}$ a scale $1/3$, namely each of the subquarks associated with the rotations around the axes of given world carries the corresponding charge in the scale $1/3$; antisubquark carries respectively the $(-1/3)$ charge. In the case of the worlds $C=s,c,b,t$, where distortion
rotations are global and diagonal with respect to axes 1,2,3, the physical system of corresponding subquarks is invariant under the global transformations $f^{(3)}_C(\theta^c)$ of the global unitary group $SU^c_3$:

$$f^{(3)}_C = \begin{pmatrix} f_{11}^c & 0 & 0 \\ 0 & f_{22}^c & 0 \\ 0 & 0 & f_{33}^c \end{pmatrix} = \exp \left\{ -\frac{i}{3} \begin{pmatrix} \theta^c_1 & 0 & 0 \\ 0 & \theta^c_2 & 0 \\ 0 & 0 & \theta^c_3 \end{pmatrix} \right\},$$

where $f^{(3)}_C \left( f^{(3)}_C \right)^+ = 1$, $\|f^{(3)}_C\| = 1$. That is $\theta^c_1 + \theta^c_2 + \theta^c_3 = 0$. The simplest possibility is $\theta^c \equiv \theta^c_1 = \theta^c_2$, then one gets

$$f^{(3)}_C = \exp \left\{ -\frac{i}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \right\} \theta^c = \exp \left\{ -\frac{i}{3} \lambda^c \theta^c \right\} = e^{-iY^c\theta^c}, \quad (4.1)$$

provided with the operator of hypercharge $Y^c$ of diagonal group $SU^c_3$. If all worlds are involved, then $Y^c = s + c + b + t$. The case of Q-world, which makes a substantial contribution into the condition of multiverse connections will be discussed in detail in next section. We only notice that conservation of each rotation mode in Q- and B-worlds, where the distortion rotations are local, means that corresponding subquarks carry respectively the conserved charges $Q$ and $B$ in the scale 1,2,3, the physical system of corresponding subquarks under the transformations of these symmetries.

The incompatibility relations eq.(3.5.5) in [1] for global distortion rotations in the worlds C=s,c,b,t involved in the multiworld geometry realization condition eq.(2.5) (see eq.(5.2)).

5 Realization of Q-World and Gell-Mann-Nishijima Relation

The symmetry of Q-world of electric charge is assumed to be a local unitary symmetry $\text{diag} \left( SU^{loc}(3) \right)$ is diagonal with respect to axes 1,2,3. The unitary unimodular matrix $f^{(3)}_Q$ of local distortion rotations takes the form

$$f^{(3)}_Q = \begin{pmatrix} f_{11}^Q & 0 & 0 \\ 0 & f_{22}^Q & 0 \\ 0 & 0 & f_{33}^Q \end{pmatrix} = Q_1 e^{-i\eta_1} + Q_2 e^{-i\eta_2} + Q_3 e^{-i\eta_3} = e^{-i\bar{Q}\theta} = e^{-i\lambda_Q\theta_Q},$$
where \( f^{(3)}_Q (f^{(3)}_Q)^+ = 1, \quad \|f^{(3)}_Q\| = 1 \), provided

\[
Q_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad Q_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad Q_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},
\]

and \( \theta_1 + \theta_2 + \theta_3 = 0 \). Taking into account the scale of rotation mode, in the other than eq.(4.1) simple case \( \theta_2 = \theta_3 = -\frac{1}{3} \theta_Q \), it follows that \( \theta_1 = \frac{2}{3} \theta_Q \). Among the generators of the group \( SU(3) \) only the matrices \( \lambda_3 \) and \( \lambda_8 \) are diagonal. Then the matrix \( \lambda_Q \) may be written

\[
\lambda_Q = \frac{1}{2} \lambda_3 + \frac{1}{2\sqrt{3}} \lambda_8.
\]

Making use of the corresponding operators of the group \( SU(3) \) we arrive at Gell-Mann-Nishijima relation

\[
Q = T_3 + \frac{1}{2} Y,
\]

(5.1)

where \( Q = \lambda_Q \) is the generator of electric charge, \( T_3 = \frac{1}{2} \lambda_3 \) is the third component of isospin \( \vec{T} \), and \( Y = \frac{1}{\sqrt{3}} \lambda_8 \) is the hypercharge. The eigenvalues of these operators will be defined later on by considering the symmetries and microscopic structures of fundamental fields. We think of operators \( T_3 \) and \( Y \) as the multiworld connection charges and of relation eq.(5.1) as the condition of the realization of multiworld connections. Thus, during realization of multiworld structure the symmetries of corresponding internal worlds must be unified into more higher symmetry including also the \( \lambda_3 \) and \( \lambda_8 \). Meanwhile, the realization conditions of multiworld structure are embodied in eq.(2.5) and eq.(5.1), provided by the conservation law of each rotational mode eq.(4.1) in the corresponding internal worlds involved into condition eq.(2.5). For example, in the case of quarks eq.(3.2) and eq.(3.3), the eq.(2.5) reads

\[
\sum_{i=B,s,c,b,t} \omega_i G^\theta_{i F} (0) = G^\theta_{\eta F} (0),
\]

(5.2)

and according to eq.(4.2), the Gell-Mann-Nishijima relation is written down

\[
Q = T_3 + \frac{1}{2} (B + s + c + b + t).
\]

(5.3)

The other case of leptons eq.(3.1) closely related to the realization of W-world of weak interactions will be discussed in detail separately. The realization condition reduces to following:

\[
G^\theta_{Q F} (0) = G^\theta_{\eta F} (0), \quad u \equiv u_Q, \quad (i \equiv Q)
\]

(5.4)

and

\[
Q = T_3^w + \frac{1}{2} Y^w
\]

(5.5)

where \( T_3^w \) and \( Y^w \) are respectively the operators of third component of weak isospin \( \vec{T}^w \) and weak hypercharge (sec.6,12). The incompatibility relations eq.(3.5.5) in [1] for the local distortion transformations in Q-world lead to

\[
f^{Q}_{11} f^{Q}_{22} = \bar{f}^{Q}_{33}, \quad f^{Q}_{22} f^{Q}_{33} = \bar{f}^{Q}_{11}, \quad f^{Q}_{33} f^{Q}_{11} = \bar{f}^{Q}_{22},
\]
where \( f_{ii}^{Q} f_{ii}^{\bar{Q}} = 1 \), for \( i = 1, 2, 3 \). \( \| f_{Q}^{(3)} \| = f_{11}^{Q} f_{22}^{Q} f_{33}^{Q} = 1 \). This suggests two subcolour singlets \((q \bar{q})_{i}^{Q} = \text{inv}, \quad (q_{1} q_{2} q_{3})_{i}^{Q} = \text{inv} \). The corresponding electric charges are \( Q_{(q \bar{q})_{i}^{Q}} = 0 \), \( Q_{(q_{1} q_{2} q_{3})_{i}^{Q}} = 1 \). Using orthogonal unit vectors \( L_{i} \) the following subcolour singlets arise:

\[
(q \bar{q})_{i}^{Q} = L_{i} (q \bar{q})_{i}^{Q} \quad \text{and} \quad (q_{1} q_{2} q_{3})_{i}^{Q} = L_{i} (q_{1} q_{2} q_{3})_{i}^{Q},
\]

that

\[
(q_{1} q_{2} q_{3})_{1}^{Q} = L_{1} \left( f_{11}^{Q} f_{22}^{Q} f_{33}^{Q} \right) (q_{1} q_{2} q_{3})_{1}^{Q} = \left( f_{i1}^{Q} f_{11}^{Q} q_{1} \right) q_{2} q_{3} = (q_{1} q_{2} q_{3})_{1}^{Q} = \text{inv}, \quad (5.6)
\]

e tc, where \( f_{i1}^{Q} f_{11}^{Q} f_{11}^{Q} = L_{i} f_{11}^{Q} f_{22}^{Q} f_{33}^{Q} = L_{i} f_{11}^{Q} f_{11}^{Q} \) for given \( i \). According to it, in singlet combinations \((q_{1} q_{2} q_{3})_{i}^{Q} \) and \((q \bar{q})_{i}^{Q} \) only \( i \)-th subquark undergoes distortion transformations, which can be considered as the real dynamical field.

6 The Symmetries of the W- and B-Worlds

6.1 The W-World

It will be seen in sec. 12 that the symmetry of W-world of weak interactions is \( SU^{\text{loc}}(2) \otimes U^{\text{loc}}(1) \), invoking local group of weak hypercharge \( Y^{w} \) \((U^{\text{loc}}(1)) \). However, for the present it is worthwhile to restrict oneself by admitting that the symmetry of W-world is simply expressed by the group of weak isospin \( SU^{\text{loc}}(2) \). Namely, we start by considering a case of two dimensional distortion transformations through the angles \( \theta_{\pm} \) around two arbitrary axes in the W-world. In accordance with the results of sec.3 in [1], the fields of subquarks and antisubquarks will come in doublets, which form the basis for fundamental representation of weak isospin group \( SU^{\text{loc}}(2) \). The doublet states are complex linear combinations of up and down states of weak isotopic spin. Three possible doublets of six subquark states are \( \left( \begin{array}{c} q_{1} \\ q_{2} \\ q_{3} \end{array} \right)^{w} \), \( \left( \begin{array}{c} q_{2} \\ q_{3} \\ q_{1} \end{array} \right)^{w} \), \( \left( \begin{array}{c} q_{3} \\ q_{1} \\ q_{2} \end{array} \right)^{w} \).

6.2 The B-World

The B-world is responsible for strong interactions. The internal symmetry group is \( SU^{\text{loc}}_{c}(3) \) enabling to introduce gauge theory in subcolour space with subcolour charges as exactly conserved quantities (sec.3 in [1]). The local distortion transformations implemented on the subquarks \((q_{i})^{B} \), \( i = 1, 2, 3 \) through a \( SU^{\text{loc}}_{c}(3) \) rotation matrix \( U \) in the fundamental representation. Taking into account a conservation of rotation mode (sec.4), each subquark carries \((1/3)\) baryonic charge, while an antisubquark - \((-1/3)\) baryonic charge.

7 The Microscopic Structure of Leptons: Lepton Generations

After the quantitative discussion of the properties of symmetries of internal worlds, below we attempt to show how the known fermion fields of leptons and quarks fit into this scheme. In this section we start with the leptons. Taking into account the eq.(3.1), eq(5.4), we consider six possible lepton fields forming three doublets of lepton generations.
Here $e, \mu, \tau$ are electron, muon and tau meson, $\nu_e, \nu_\mu, \nu_\tau$ are corresponding neutrinos, $L_e \equiv L_1, L_\mu \equiv L_2, L_\tau \equiv L_3$, are leptonic charges. The leptons carry leptonic charges as follows: the $e$ and $\nu_e \rightarrow L_e = 1$; $\mu$ and $\nu_\mu \rightarrow L_\mu = 1$; $\tau$ and $\nu_\tau \rightarrow L_\tau = 1$. The leptonic charges are conserved in all interactions. The leptons carry also the weak isospins: $T^w_3 = \frac{1}{2}$ for $\nu_e, \nu_\mu, \nu_\tau$, and $T^w_3 = \frac{1}{2}$ for $e, \mu, \tau$. Corresponding electric charges are as follows: $Q_{\nu_e} = Q_{\nu_\mu} = Q_{\nu_\tau} = 0$, $Q_e = Q_\mu = Q_\tau = -1$. The Q-components $\Psi(u_Q)$ of lepton fields eq.(3.1) are made of singlet combinations of subquarks in Q-world. They imply subcolour confinement eq.(5.4). Then the multiworld geometry realization condition is already satisfied and leptons may emerge in free combinations without any constraint. Thus, in suggested theory there are three possible generations of six leptons with integer electric and leptonic charges and being free of confinement.

8 The Microscopic Structure of Quarks: Quark Generations

We assume that the microscopic structure of 18 possible multiworld quark fields is as follows:

$$
\begin{align*}
\Psi_{u_i} &= \Psi_{\eta_u} (\eta) (q_1 q_2)^Q (q_1)^w (q_i^B), \\
\Psi_{d_i} &= \Psi_{\eta_d} (\eta) (q_3 q_4)^Q (q_3)^w (q_i^B), \\
\Psi_{t_i} &= \Psi_{\eta_t} (\eta) (q_5 q_6)^Q (q_5)^w (q_i^B), \\
\Psi_{c_i} &= \Psi_{\eta_c} (\eta) (q_6 q_3)^Q (q_6)^w (q_i^B), \\
\Psi_{s_i} &= \Psi_{\eta_s} (\eta) (q_1 q_5)^Q (q_1)^w (q_i^B), \\
\Psi_{b_i} &= \Psi_{\eta_b} (\eta) (q_2 q_6)^Q (q_2)^w (q_i^B),
\end{align*}
$$

(8.1)

where the subcolour index $(i)$ runs through $i = 1, 2, 3$, the $(q_i^B)$ are given in eq.(3.3). Henceforth the subcolour index will be left implicit, but always a summation must be extended over all subcolours in B-world. These fields form three possible doublets of weak isospin in the W-world \( \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix} \). The quark flavour mixing and similar issues are left for treatment in sec.17. The corresponding electric charges of quarks read $Q_u = Q_c = Q_t = \frac{2}{3}$, $Q_d = Q_s = Q_b = -\frac{1}{3}$, in agreement with the rules governing
the multiworld connections eq.(5.5), where the electric charge difference of up and down quarks implies \( \Delta Q = \Delta T^3_w = 1 \). The explicit form of structure of \((q^+_f)\) will be discussed in next section. Here we only notice that all components of \((q^+_f)\) are made of singlet combinations of global subquarks in corresponding internal worlds. They obey a condition of subcolour confinement. According to eq.(5.2), the subcolour confinement condition for B-world still remains to be satisfied. Due to it the total quark fields obey confinement. Then quarks would not be free particles. Unwanted states (since not seen) like quarks or diquarks etc. are eliminated by construction at the very beginning. Thus, three quark generations of six possible quark Fermi fields exist. They carry fractional electric and baryonic charges and imply a confinement. Their other charges is left to be discussed below. Although within considered schemes the subquarks are defined in the internal worlds, however due to eq.(2.1) the resulting \(\eta\)-components, which we are going to deal with to describe the leptons and quarks defined in the spacetime continuum, are completely affected by them. Actually, as it is seen in subsec.3.3 and 3.4 of [1] the rotation through the angle \(\theta_{+k}\) yields a total subquark field

\[
q_k(\theta) = \Psi(\theta_{+k}) = \Psi^0(q) \Psi(\theta_{+k})
\]

where \(\Psi^0\) is a plane wave defined on \(G\). According to subsec.3.3 in [1] and eq.(2.1), one gets

\[
q_k(\theta(\eta)) = \Psi^0(q_u k) \Psi(\eta(k)) \Psi^0(q_u k) = q_u \Psi^0(q_u k) \Psi(\eta(k)) \equiv \psi_{+}(q_u k(\eta)) \Psi^0(q_u k)
\]

where \(\Psi^0\) is a plane wave defined on \(G\). The \(\psi_{+}(q_u k(\eta))\) is considered as the subquark field defined on flat manifold \(G\) with the same quantum numbers of \(q_u \Psi(\eta(k))\). Due to it, instead of the eq.(7.1) and eq.(8.1) we can consider on equal footing only the resulting \(\eta\)-components of leptons and quarks having the same structures, which enable to return to the Minkowski spacetime continuum \(G \rightarrow M^4\) (subsec.2.1 in [1]).

9 The Flavour Group \(SU_f(6)\)

We may think of the field component \((q^+_f)\) (f=u,d,s,c,b,t) eq.(3.3) associated with the global distortion rotations in the worlds s,c,b,t having following microscopic structure with corresponding global charges:

\[
q_u^c = q_d^c = 1, \quad q_s^c = (q_1^1 q_2^2 q_3^3)^s, \quad s = -1; \quad q_c^c = (q_1^c q_2^c q_3^c)^c, \quad c = 1; \quad q_b^c = (q_1^b q_2^b q_3^b)^b, \quad b = 1; \quad q_t^c = (q_1^t q_2^t q_3^t)^t, \quad t = 1.
\]  

During the realization of multiworld structure the global symmetries of internal worlds are unified into more higher symmetry including the generators \(\lambda_3\) and \(\lambda_8\) (sec.5). This global group is the flavour group \(SU_f(6)\) unifying all symmetries \(SU_i^c\) of the worlds Q,B,s,c,b,t: \(SU_f(6) \supset SU_f(2) \otimes SU^c_5 \otimes SU^c_5 \otimes SU^c_5 \otimes SU^c_5 \otimes SU^c_5\). Then the total symmetry reads \(G_{tot} \equiv G^{loc} \otimes G^{glob} = G^{loc} \otimes SU_f(6)\), provided \(G^{loc} \equiv SU^{loc}(3) \otimes G^{loc}_w\), where \(G^{loc}_w\) is the local symmetry of the electroweak interactions (sec.12). The other important aspects of standard model are left for investigation in the next sections. However, below we proceed with further exposition of our approach to consider a gauge invariant Lagrangian of primary field with multiworld structure and nonlinear fermion interactions of the components.
10 The Primary Field

All fields including the leptons eq.(7.1) and quarks eq.(8.1), along with the spacetime components have also multiworld components made of the various subquarks defined in the corresponding internal worlds, namely, the internal components are consisted of distorted ordinary structures

\[
\Psi(\theta) = \Psi(\eta) \Psi(\theta_Q) \Psi(\theta_W) \Psi(\theta_B) \Psi(\theta^c).
\]

(10.1)

The components \(\Psi(\theta_Q), \Psi(\theta_W), \Psi(\theta_B)\) are primary massless bare Fermi fields, but the component \(\Psi(\theta^c)\) has a mass \(m^c_i \neq 0\) eq.(2.2). We assume that this field arises from primary field in lowest state \((s_0)\) with the same field components consisted of regular ordinary structures. It is motivated by the argument that the regular ordinary structures directly could not take part in link exchange processes with the \(\eta\)-type regular structure [1]. Therefore the primary field defined on \(G_N\)

\[
\Psi(0) = \Psi(\eta) \Psi(0) \Psi(0) \Psi(0) \Psi(0)
\]

(10.2)

serves as the ready made frame into which the distorted ordinary structures of the same species should be involved. We now apply the Lagrangian of this field possessed local gauge invariance

\[
\tilde{L}_0(D) = \frac{i}{2} \{\bar{\Psi}_e(\zeta) i \gamma_i D_i \Psi_e(\zeta) - D_i \bar{\Psi}_e(\zeta) i \gamma_i \Psi_e(\zeta)\}.
\]

(10.3)

The latter is in the notation eq.(4.2.3) in [1], where \(D = \partial - igB(\zeta), B\) are gauge fields. Since the components \(\Psi\) and \(\bar{\Psi}\) will be of no consequence for a discussion, then we temporarily leave them implicit, namely \(i = Q, W\). The equation of primary field of multiworld structure with nonlinear fermion interactions of the components may be derived from an invariant action in terms of local gauge invariant Lagrangian, which looks like Heisenberg theory [2,3]

\[
\tilde{L}(D) = \tilde{L}_0(D) + \tilde{L}_I + \tilde{L}_B,
\]

(10.4)

provided by the Lagrangians of nonlinear fermion interactions of the components \(\tilde{L}_I = \sqrt{2} O_1 \otimes L_I,\) and gauge field \(\tilde{L}_B = \sqrt{2} O_1 \otimes L_B\). The binding interactions are in the form

\[
L_I = L_{Q_I} + L_{W_I}, \quad L_{Q_I} = \frac{\lambda}{4} Q_L Q_R, \quad L_{W_I} = \frac{\lambda}{2} S_W S^+_W, \\
L_B = -\frac{1}{2} Tr(B\bar{B}) = -\frac{1}{2} Tr \begin{pmatrix} B & B \\ \bar{B} & \bar{B} \end{pmatrix},
\]

(10.5)

where

\[
J_{Q,L,R} = V_T A, \quad V_{Q} = \bar{\Psi}_Q \gamma_{(\lambda\alpha)} \Psi_Q, \quad V^+_{Q} = V_{Q}^{(\lambda\alpha)} = \bar{\Psi}_Q \gamma_{(\lambda\alpha)} \Psi_Q, \\
A_{Q,(\lambda\alpha)} = \bar{\Psi}_Q \gamma_{(\lambda\alpha)} \gamma_5 \Psi_Q, \quad A^+_{Q,(\lambda\alpha)} = A_{Q,(\lambda\alpha)} = \bar{\Psi}_Q \gamma_{(\lambda\alpha)} \gamma_5 \Psi_Q, \quad S_W = \bar{\Psi}_W \Psi_W.
\]
γμ and γ5 = iγ0γ1γ2γ3 are Dirac matrices. According to Fiertz theorem the interaction Lagrangian \( L = \frac{\lambda}{2}(VV^+ - AA^+) \) may be written \( L = -\lambda(S_Q S_Q^+ - P_Q P_Q^+) \), provided \( S_Q = \bar{\Psi}_Q \Psi_Q \), \( P_Q = \bar{\Psi}_Q \gamma_5 \Psi_Q \). Hence

\[
\bar{L}(D) = \sqrt{2}\tilde{O}_1 \otimes L(D), \quad L(D) = L(D) - L(D) - L(D), \quad (10.6)
\]

where

\[
L(D) = \frac{1}{2}Tr(BB), \quad L(D) = L(Q) - L(Q) - L(Q), \quad (10.2)
\]

Here

\[
L'(0) = \frac{i}{2}\{\bar{\Psi}_\eta \gamma_D \Psi_\eta - \bar{\Psi}_\eta \gamma_D \bar{\Psi}_\eta\} = \psi_\eta + L(0) \psi_\eta,
\quad L'(0) = \frac{i}{2}\{\bar{\Psi}_\eta \gamma_D \Psi_\eta - \bar{\Psi}_\eta \gamma_D \bar{\Psi}_\eta\} = \psi_\eta + L(0) \psi_\eta,
\]

and

\[
L(0) = \frac{i}{2}\{\bar{\Psi}_\eta \gamma_D \Psi_\eta - \bar{\Psi}_\eta \gamma_D \bar{\Psi}_\eta\}, \quad L(0) = \frac{i}{2}\{\bar{\Psi}_\eta \gamma_D \Psi_\eta - \bar{\Psi}_\eta \gamma_D \bar{\Psi}_\eta\}.
\]

The Lagrangian eq.(10.6) has the global γ5 and local gauge symmetries. We consider only γ5 symmetry in Q-world, namely \( B_Q \equiv 0 \).

According to the operator multimanifold formalism (subsec.4 in [1]), it is the most important to fix the mass shell of the stable multiverse structure (eq.(4.2.1) in [1]). Thus, at first we must take the variation of the Lagrangian eq.(10.3) with respect to primary field eq.(10.2) (eq.(4.2.2) in [1]), then switch on nonlinear fermion interactions of the components. In other words we take the variation of the Lagrangian eq.(10.6) with respect to the components on the fixed mass shell. The equations of free field (\( B = 0 \)) of multiverse structure follow at once

\[
\hat{\partial}_\xi \Psi_\eta(\zeta) = i \gamma \hat{\partial} \Psi_\eta(\zeta) = i \gamma \hat{\partial} \Psi_\eta(\zeta) = 0, \quad \Psi_\eta \xi^\eta = -i\partial \Psi_\eta \gamma = 0, \quad (10.7)
\]

which lead to separate equations for the massless components \( \bar{\Psi}_\eta \Psi_\eta \) and \( \Psi_\eta \Psi_\eta \):

\[
\gamma(\lambda_0) \psi_{\eta(\lambda_0)} = i \gamma \hat{\partial} \psi_{\eta(\lambda_0)} = 0, \quad \gamma_{\lambda} \psi_{\eta} = i \gamma \hat{\partial} \psi_{\eta} = 0, \quad \gamma_{\lambda} \psi_{\lambda} = i \gamma \hat{\partial} \psi_{\lambda} = 0. \quad (10.8)
\]

The important feature is that the field equations (10.7) remain invariant under the substitution \( \psi_{\eta} \rightarrow \psi_{\eta} \), where \( \psi_{\eta} \) and \( \psi_{\eta} \) are respectively the massless and massive Q-component fields, to which merely the substitution \( \psi_{\eta} \rightarrow \psi_{\eta} \) is corresponded. Thus, in free state the massless field components \( \bar{\Psi}_\eta \Psi_\eta \) and \( \Psi_\eta \Psi_\eta \) are independent. Due to eq.(10.8), the Lagrangian

\[
L'(0) = \psi^+ \psi_{\eta} + \psi^+ \psi_{\eta} - \psi^+ \psi_{\eta} = \psi^+ \psi_{\eta} - \psi^+ \psi_{\eta} + L(0) + L(0) \psi_\eta. \quad (10.9)
\]
reduces to the following:

\[
L'_0(0) = L'_{\eta_0} - L'_{u_0} = L'_{Q_0} - L'_{\tilde{\Psi}_0}.
\]  
(10.10)

Thus, we implement our scheme as follows: starting with the reduced Lagrangian \( L'_0(0) \) of free field we shall switch on nonlinear fermion interactions of the components. After a generation of nonzero mass of \( \Psi_Q \) component in Q-world (next sec.) look for the corresponding corrections via the eq.(10.9) to the reduced Lagrangian eq.(10.10) of free field. These corrections mean the interaction between the components governed by the eq.(10.7) and eq.(10.9), and do not at all imply a mass acquiring process for the \( \eta \)-component (see eq.(11.6)).

11 A Generation of Mass of Fermions in Q-World

We apply the well known Nambu-Jona-Lasinio model [18] to generate a fermion mass in the Q-world and start from the chirality invariant total Lagrangian of field \( \Psi_q : L = L^{(0)}_Q - L_{\tilde{\Psi}_Q} \), where primary field \( \Psi_q \) is massless bare spinor implying \( \gamma_5 \) invariance, i.e. it is an eigenstate of chirality. However, due to interaction a rearrangement of vacuum state caused a generation of nonzero mass of fermion such like to appearance of energy gap in superconductor [9-11]. The energy gap in BCS-Bogoliubov theory of superconductivity is created by the electron-electron interaction of Cooper pairs. Pursuing this analogy in [18,19] it was assumed that the mass of Dirac quasi-particle excitation is due to some interaction between massless bare fermions, which may be considered as a self-consistent (Hartree-Fock) representation of it. This approach based on the main idea that due to a dynamical instability the field theory in general may admit also nontrivial solutions with less symmetry than the initial symmetry of Lagrangian, Namely, it is considered such possibility that the field equations may possess higher symmetry, while their solutions may reflect some asymmetries arisen due to fact that nonperturbative solutions to nonlinear equations do not in general possess the symmetry of the equations themselves. In [18,19] the solution of massive fermion is obtained which lack the initial \( \gamma_5 \) symmetry of the Lagrangian. On the analogy of Gor’kov’s theory [20,21] it is shown that if one takes into account only the qualitative dynamical effects connected with rearrangement of vacuum state, in addition to the trivial solution of equation of massless fermion a real Dirac quasi-particle will satisfy the equation with non-zero self-energy operator \( \Sigma(p, m, \tilde{\lambda}, \Lambda) \) depending on mass \( m \), coupling constant \( \tilde{\lambda} \) and cut-off \( \Lambda \). In the mean time \( \tilde{\lambda} = \lambda \Gamma(m, \tilde{\lambda}, \Lambda) \), where \( \lambda \) is a bare coupling, \( \Gamma \) is the vertex function. This theory leads to the expression of self-energy operator \( \Sigma_Q \) for the field \( \Psi_q \). In lowest order it is quadratically divergent, but with a cutoff can be made finite. Making use of passage \( G \rightarrow M^4 \) (subsec.2.1 in [1]), one shall proceed directly with the calculation. In momentum space one gets [18,19]

\[
\Sigma_Q = m_Q = -\frac{8\lambda i}{(2\pi)^4} \int \frac{m_Q}{p_Q^2 + m_Q^2 - i\varepsilon} F(p_Q, \Lambda)d^4p_Q,
\]  
(11.1)
where $F(p_Q, \Lambda)$ is a cutoff factor, $m_Q = |\Delta_Q|$, $\Delta_Q = 4\lambda < \Psi_{Q,R}^+ \Psi_{Q,L}^+ >$, $\Psi = \frac{1 + \gamma_5}{2} \Psi$, $< \cdots >$ specifies the physical vacuum averaging. Besides of trivial solution $m_Q = 0$, this equation has also nontrivial solution determining $m_Q$ in terms of $\lambda$ and $\Lambda$. Straightforward calculations with invariant cutoff yield the relation $\frac{2\pi^2}{\lambda\Lambda^2} = 1 - \frac{m^2_Q}{\Lambda^2} \ln \left( \frac{\Lambda^2}{m^2_Q} + 1 \right)$. The latter is valid only if $\frac{\lambda\Lambda^2}{2\pi^2} \approx 1$. After a vacuum rearrangement the total Lagrangian of initial massless bare field $\psi^L_{Q}^{(0)}$ gives rise to corresponding Lagrangian $L_{Q}^{(m)}$ of massive field $\psi^L_{Q}^{(m)}$: $L_{Q}^{(m)} = L_{Q}^{(0)} - L_{Q}^{(1)} = L_{Q}^{(m)}$ describing Dirac particle $(\gamma p_Q - \Sigma Q)\psi^L_{Q}^{(m)} = 0$. In lowest order $\Sigma Q = m_Q \ll \lambda^{-1/2}$. Within the refined theory of superconductivity, the collective excitations of quasi-particle pairs arise in addition to the individual quasi-particle excitations when a quasi-particle accelerated in the medium [11, 22-26]. This leads to the conclusion [18,19] that, in general, the Dirac quasi-particle is only an approximate particle excitation when a quasi-particle accelerated in the medium [11, 22-26]. This leads to the conclusion [18,19] that, in general, the Dirac quasi-particle is only an approximate description of an entire system with the collective excitations as the stable or unstable bound quasi-particle pairs. In a simple approximation there arise CP-odd excitations of zero mass as well as CP-even massive bound states of nucleon number zero and two. Along the same line we must substitute in eq.(10.7) the massless field $\psi^L_{Q}^{(0)}$ of zero mass as well as CP-even massive bound states of nucleon number zero and two. The

$$
\psi^L_{Q}^{(m)} \equiv \psi^L_{Q}^{(m)} \psi_{Q}^{(m)} \psi_{W}^{(m)} \psi_{W}^{(m)}.
$$

Then, we obtain

$$
\gamma p_Q \psi^L_{Q}^{(m)} = \Sigma Q \psi^L_{Q}^{(m)}, \quad \gamma p_W \psi^L_{W}^{(m)} = 0, \quad \gamma p_{\eta} \psi^L_{Q}^{(m)} = (\gamma p_Q + \gamma p_W) \psi^L_{Q}^{(m)} = \Sigma Q \psi^L_{Q}^{(m)}.
$$

This applies following corrections to eq.(10.9):

$$
L'_{Q}^{(m)} = \psi^L_{Q}^{(m)} \psi_{Q}^{(m)} \psi_{Q}^{(m)} (L_{Q}^{(0)} - \Sigma Q \psi^L_{Q}^{(m)} \psi_{Q}^{(m)} \psi_{W}^{(m)} \psi_{W}^{(m)} \rightarrow L_{Q}^{(0)} - \Sigma Q \psi^L_{Q}^{(m)} \psi_{Q}^{(m)} \psi_{W}^{(m)} \psi_{W}^{(m)};
$$

$$
L'_{Q}^{(m)} = (\psi^L_{Q}^{(m)} \psi_{Q}^{(m)} \psi_{Q}^{(m)} - \Sigma Q \psi^L_{Q}^{(m)} \psi_{Q}^{(m)} \psi_{W}^{(m)} \psi_{W}^{(m)} \rightarrow L_{Q}^{(0)} - \Sigma Q \psi^L_{Q}^{(m)} \psi_{Q}^{(m)} \psi_{W}^{(m)} \psi_{W}^{(m)}).
$$

where suffix $(m)$ in $\psi^L_{Q}^{(m)}$ is left implicit. Such redefinition $\psi^L_{Q}^{(0)} \rightarrow \psi^L_{Q}^{(m)}$ leaves the structure of that part of Lagrangian eq.(10.10) involving only the fields $\psi_{Q}^{(m)}$ and $\psi_{W}^{(m)}$ unchanged

$$
L_{Q}^{(m)} = L_{Q}^{(0)} - L_{W}^{(0)} = \left( L_{Q}^{(0)} - \Sigma Q \psi^L_{Q}^{(m)} \psi_{Q}^{(m)} \psi_{W}^{(m)} \psi_{W}^{(m)} - \left( L_{Q}^{(0)} - \Sigma Q \psi^L_{Q}^{(m)} \psi_{Q}^{(m)} \psi_{W}^{(m)} \psi_{W}^{(m)} = L_{Q}^{(m)} - L_{W}^{(m)},
$$

where the component $\psi_{Q}^{(m)}$ is left implicit. The gauge invariant Lagrangian eq.(10.6) takes the form

$$
L(D) = L(D) - L(D),
$$

where upon combining and rearranging relevant terms we separate the Lagrangians

$$
L(D) = \frac{i}{2} \left\{ \psi_{Q}^{(m)} \gamma_{Q} \psi_{Q}^{(m)} - \psi_{Q}^{(m)} \gamma_{Q} \psi_{Q}^{(m)} \right\} - f_{Q} \psi \psi_{Q}^{(m)} - \frac{1}{2} Tr(BB)
$$

$$
L(D) = \frac{i}{2} \left\{ \psi_{Q}^{(m)} \gamma_{Q} \psi_{Q}^{(m)} - \psi_{Q}^{(m)} \gamma_{Q} \psi_{Q}^{(m)} \right\} - \Sigma Q \psi \psi_{Q}^{(m)} - \frac{\lambda}{2} S_{W} S_{W}^{+} - \frac{1}{2} Tr(BB),
$$

(11.5)
provided \( f_{\eta} \equiv \Sigma_{\eta}(p_{\eta}, m_{\eta}, \lambda, \Lambda) \), \( \Psi = \Psi, \overline{\Psi} \). The eq.(11.5) is the Lagrangian that we shall be concerned within the following. The self-energy operator \( \Sigma_{\eta} \) takes into account the mass-spectrum of all expected collective excitations, which arise as the poles of the function \( \Sigma_{\eta} \). At \( \Sigma_{\eta} \approx m_{\eta} \ll \lambda \), in eq.(11.7) we may use the approximation \( \Sigma_{\eta}{\Psi} = \Sigma_{\eta}{\tilde{\Psi}}{\gamma}_{\eta} \left( \overline{\Psi} \overline{\Psi} \right) \tilde{\Psi} \approx \Sigma_{\eta}{\Psi} \). The Lagrangians eq.(11.6) and eq.(11.7) will be further evaluated.

12 The Electroweak Interactions: the P-Violation

The standard model of the unified \( SU(2) \otimes U(1) \) gauge theory of weak and electromagnetic interactions [4-8] and the colour \( SU(3) \) gauge theory of strong interactions [27-36] have become generally acceptable paradigm as the view on particles and interactions in the various models of the same class. Their phenomenological success, which suggests that the effective gauge group at ordinary energies is \( SU(3) \otimes SU(2) \otimes U(1) \) seemed to be proven for certain by many experiments. Below we extend our approach to the pertinent concepts and ideas of the unified electroweak interactions. We start with the idea that the local rotations in W-world are occurred at very beginning only around two arbitrary axes (sec.6), namely \( DimW \) in W-world always occur around all three axes. That is, instead of initial symmetry, we admit the spanning \( W_{loc}^{(2)} \to W_{loc}^{(3)} \) where \( DimW_{loc}^{(3)} = 3 \neq N_{w_{1}w_{2}}(q_{1}, q_{2}) = 2 \). Taking into account that at the very beginning all subquark fields in W-world are massless, we cannot rule out the possibility that they are transformed independently. On the other hand, when this situation prevails the spanning \( W_{loc}^{(2)} \to W_{loc}^{(3)} \) must be occurred compulsory in order to provide a necessary background for the condition eq.(5.5) to be satisfied. The most likely attitude here is that doing away this shortage the subquark fields \( q_{L_{1}}, q_{L_{2}}, q_{R_{1}}, \) and \( q_{R_{2}} \) tend to give rise to triplet. The three dimensional effective space \( W_{loc}^{(3)} \) will then arise

\[
W_{loc}^{(2)} \ni q_{(2)}^{w}(\bar{T}^{w} = \frac{1}{2}) \to q_{(3)}^{w} = \begin{pmatrix}
q_{R}(\bar{T}^{w} = 0) \\
q_{L}(\bar{T}^{w} = \frac{1}{2})
\end{pmatrix} = \begin{pmatrix}
q_{3}^{w} \\
q_{1}^{w} \\
q_{2}^{w}
\end{pmatrix} \equiv \begin{pmatrix}
q_{R_{2}} \\
q_{L_{1}} \\
q_{L_{2}}
\end{pmatrix} \in W_{loc}^{(3)}. \quad (12.1)
\]

The latter holds if violating initial P-symmetry the components \( q_{R_{1}}, q_{R_{2}} \) still remain in isosinglet states, namely the components \( q_{L} \) are forming isodoublet while \( q_{R} \) is a isosinglet: \( q_{L}(\bar{T}^{w} = \frac{1}{2}), \ q_{R}(\bar{T}^{w} = 0) \). So, the mirror symmetry is broken. Corresponding local transformations are implemented upon triplet \( q_{(3)}^{w} = f_{W_{loc}}^{(3)} q_{(3)}^{w} \), where the unitary matrix...
of three dimensional local rotations reads

\[
f^{(3)}_W = \begin{pmatrix}
  f_{33} & 0 & 0 \\
  0 & f_{11} & f_{12} \\
  0 & f_{21} & f_{22}
\end{pmatrix}^w.
\]

Making use of incompatibility relations eq.(3.5.5) in [1] one gets

\[
\|f^{(3)}_W\| = f_{33}(f_{11}f_{22} - f_{12}f_{21}) = f_{33}\varepsilon_{123}\varepsilon_{123}\|f^{(3)}_W\|f^{*}_{33},
\]

or \(f_{33}f^{*}_{33} = 1\). That is \(f_{33} = e^{-i\beta}\). While

\[
\|f^{(2)}_W\| = f_{11}f_{22} - f_{12}f_{21} = \|f^{(3)}_W\|f^{*}_{33} = \|f^{(3)}_W\|e^{i\beta},
\]

and due to condition \(\|f^{(3)}_W\| = 1\) it reads \(\|f^{(2)}_W\| = e^{i\beta} \neq 1\). Thus, the initial symmetry \(SU^{loc}(2)\) is broken. Restoring it the fields \(q_L\) must be undergone to additional transformations

\[f^{(2)}_W \rightarrow f^{(2)'}_W = \left(\begin{array}{c}
  f_{11}e^{-i\frac{\mathbf{T}_w}{2}}
  f_{12}e^{-i\frac{\mathbf{T}_w}{2}} \\
  f_{21}e^{-i\frac{\mathbf{T}_w}{2}}
  f_{22}e^{-i\frac{\mathbf{T}_w}{2}}
\end{array}\right)^w\]

in order to satisfy the unimodularity condition of matrix of the group \(SU^{loc}(2)\), namely \(\|f^{(2)'}_W\| = \|f^{(2)}_W\|e^{-i\beta} = 1\), \(f^{(2)'}_W \in SU^{loc}(2)\). Then, the expanded group of local rotations in W-world arises

\[
f^{(3)}_{exp} = \begin{pmatrix}
  e^{-i\beta} & 0 & 0 \\
  0 & f_{11}e^{-i\frac{\mathbf{A}_w}{2}} & f_{12}e^{-i\frac{\mathbf{A}_w}{2}} \\
  0 & f_{21}e^{-i\frac{\mathbf{A}_w}{2}} & f_{22}e^{-i\frac{\mathbf{A}_w}{2}}
\end{pmatrix} \in SU^{loc}(2) \otimes U^{loc}(1),
\]

where \(U = e^{-i\mathbf{T}_w\mathbf{\bar{\theta}}_w} \in SU^{loc}(2),\ U_1 = e^{-i\mathbf{Y}_w\theta_1} \in U^{loc}(1)\). Here \(U^{loc}(1)\) is the group of weak hypercharge \(Y_w\) taking the following values for left- and right-handed subquark fields: \(q_R : Y^w = 0, -2,\ q_L : Y^w = -1\). Whence \(q^{w}_{R(3)} = f^{(3)}_{exp}w^{(3)}\), namely

\[q^{'}_{L} = e^{-i\mathbf{T}_w\mathbf{\bar{\theta}}_w -i\mathbf{Y}_w\theta_1}q_L,\ q^{'}_{R} = e^{-i\mathbf{Y}_w\theta_1}q_R.\]

### 13 The Reduction Coefficient and the Weinberg Mixing Angle

The realization of weak interacting particles always has incorporated with the spanning eq.(12.1). This implies P-violation in W-world expressed in the reduction of initial symmetry group of local transformations of right-handed components \(q_R\), namely

\([SU(2)]_R \rightarrow [U(1)]_R\),

where subscript \((R)\) specified the transformations implemented upon right-handed components. The invariance of physical system of the fields \(q_R\) under initial group \([SU(2)]_R\) may be realized as well by introducing non-Abelian massless vector gauge fields \(A = \mathbf{T}_wA\) with the values in Lie algebra of the group \([SU(2)]_R\). Under a reduction eq.(13.1) the coupling constant \((g)\) changed into \((g')\) specifying the interaction strength between \(q_R\) and the Abelian gauge field \(B\) associated with the local group \([U(1)]_R\). Thereto
\[ g = g' \tan \theta_w, \text{ where } \theta_w \text{ is the Weinberg mixing angle, in terms of which the reduction coefficient reads } r_p = \frac{g - g'}{g + g'} = \frac{1 - \tan \theta_w}{1 + \tan \theta_w}. \] To define the \( r_p \) we consider the interaction vertices corresponding to the groups \([SU(2)]_R : gA\bar{q}_R\gamma_2 q_R\) and \([U(1)]_R : g'B\bar{q}_R\gamma_2 q_R\). To notice that the matrix \( \frac{\lambda_8}{2} \) is in the same normalization scale as each of the matrices \( \frac{\lambda_i}{2} \ (i = 1, 2, 3) : Tr \left( \frac{\lambda_i}{2} \right)^2 = Tr \left( \frac{\lambda_i}{2} \right)^2 \frac{1}{2} \) the vertex scale reads \((\text{Scale})_{SU(2)} = g \frac{\lambda_8}{2}, \) which is equivalent to \( g \frac{\lambda_8}{2} \). It is obvious that per generator scale could not be changed under the reduction eq.(13.1), i.e. \( (\text{Scale})_U(1) = (\text{Scale})_{SU(2)}, \) Stated somewhat differently, the normalized vertex for the group \([U(1)]_R \) reads \( \frac{1}{3}gB\bar{q}_R\gamma_2 q_R. \) In comparing the coefficients can then be equated \( \frac{g'}{g} = \tan \theta_w = \frac{1}{\sqrt{3}}, \) and \( r_p \approx 0.27. \) We may draw a statement that during the realization of multiverse structure the spanning eq.(12.1) compulsory occurred, which is the source of P-violation in W-world incorporated with the reduction eq.(13.1). The latter is characterized by the Weinberg mixing angle with the value fixed at 30°.

14 The Emergence of Composite Isospinor-Scalar Meson

The field \( q^w_\eta \) is the W-component of total field \( q_\eta = q_{\eta L} q_{\eta R} \equiv \Psi^{(2)} \eta \), where the field component \( q(\equiv \Psi) \) is left implicit. Instead of it, below we introduce the additional suffix \( (Q = 0, \pm) \) specifying electric charge of the field. At the beginning there is an absolute symmetry between the components \( q_1 = q_{\eta 1L} q_{\eta R} \) and \( q_2 = q_{\eta 2L} q_{\eta R} \). Hence, left- and right-handed components of fields may be written

\[
q_{1L} = q_{1L}^{(0)} q_{1L}^{(-)}, \quad q_{2L} = q_{2L}^{(-)} q_{2L}^{(0)}, \quad q_{1R} = q_{1R}^{(0)} q_{1R}^{(-)}, \quad q_{2R} = q_{2R}^{(-)} q_{2R}^{(0)}.
\]

On the example of one lepton generation \( e \) and \( \nu \) we shall exploit the properties of these fields. An implication of other fermion generations will be straightforward. Then

\[
q_{\eta L} = \begin{pmatrix} q_{1L}^{(0)} \\ q_{1L}^{(-)} \end{pmatrix}, \quad q_{\eta R} = \begin{pmatrix} q_{1R}^{(0)} \\ q_{1R}^{(-)} \end{pmatrix}, \quad q_{\eta 1L} = \begin{pmatrix} \nu_{1L} \\ e_{1L} \end{pmatrix}, \quad q_{\eta 2L} = \begin{pmatrix} q_{2L}^{(0)} \\ q_{2L}^{(-)} \end{pmatrix}, \quad q_{\eta R} = \begin{pmatrix} q_{2R}^{(0)} \\ q_{2R}^{(-)} \end{pmatrix}, \quad q_{\eta R} = \begin{pmatrix} q_{1R}^{(-)} \\ q_{1R}^{(0)} \end{pmatrix}.
\]
We evaluate the term $f_Q \bar{\Psi} \Psi$ in the Lagrangian eq.(11.6) as follows:

$$\bar{\Psi} \Psi = \Psi_L^+ \Psi_R + \Psi_R^+ \Psi_L \equiv q_L^+ q_R + q_R^+ q_L,$$

provided

$$q_L^+ q_R = q^+ q^+ q = q_{WL} W_{LR} \gamma^0 q_{WL} \gamma_R = \bar{L} \varphi R,$$

$$q_R^+ q_L = q^+ q^+ q = q_{WR} W_{LW} \gamma^0 q_{WR} \gamma_L = \bar{R} \varphi^+ L.$$

Hence, for appropriate values of the parameters, this term causes

$$\bar{\Psi} \Psi = \bar{q}(2) q(2) = \bar{L} \varphi R + \bar{R} \varphi^+ L,$$

where the isospinor-scalar meson field $\varphi$ reads

$$\varphi \equiv \gamma^0 q^+ q_{WL}, \quad \varphi^+ \equiv \gamma^0 q^+ q_{WR}.$$

A calculation gives

$$\varphi = (\varphi_1, \varphi_2), \quad \varphi_1 \equiv \left( q(-)^+ \right)^+ q_{WL}, \quad \varphi_2 \equiv \left( q(0)^+ \right)^+ q_{WL}, \quad \varphi^+ = (\varphi_1^+, \varphi_2^+).$$

Thereupon

$$\varphi_{1e} \equiv \left( q(-)^+ \right)^+ q_{WL} \equiv \varphi^{(+)}, \quad \varphi_{2e} \equiv \left( q(0)^+ \right)^+ q_{WL} \equiv \varphi^{(0)},$$

and

$$\varphi_{1\nu} \equiv \left( q(-)^+ \right)^+ q_{WL} \equiv c_\nu (\varphi^{(0)})^+, \quad \varphi_{2\nu} \equiv \left( q(0)^+ \right)^+ q_{WL} \equiv -c_\nu \varphi^{(-)},$$

where the charge conjugated field $\varphi^c$ is defined $(\varphi^c)_i = \varphi^* k \epsilon_{ik}, c_\nu$ is a constant. Then, the composite isospinor-scalar meson reads

$$\varphi^c \equiv \varphi = \left( \begin{array}{c} \varphi^{(+)} \\ \varphi^{(0)} \end{array} \right), \quad \varphi^c = c_\nu \varphi^c = c_\nu \left( \begin{array}{c} (\varphi^{(0)})^- \\ -\varphi^{(-)} \end{array} \right).$$

This field is a scalar as far as it is invariant under Lorentz transformations

$$q_R \to \exp \left[ i \frac{\sigma}{2} (\bar{\theta} - i \vec{\varphi}) \right] q_R, \quad q_L \to \exp \left[ i \frac{\sigma}{2} (\bar{\theta} + i \vec{\varphi}) \right] q_L,$$

where $\bar{\theta}$ is ordinary rotations, $\vec{\varphi}$ is the boost.

In accordance with eq.(5.5), the isospinor-scalar meson carries following weak hypercharge $\varphi : Y^w = 1$. Thus, the term $-f_Q \bar{\Psi} \Psi$ arisen in the total Lagrangian of fundamental fermion field eq.(11.5) accommodates the Yukawa couplings between the fermions and corresponding isospinor-scalar mesons in fairy conventional form

$$- f_Q \bar{\Psi} \Psi = - f_e \left( L \varphi e_R + e_R \varphi^+ R \right) - f_\nu \left( L \bar{\nu} c R + \bar{\nu} R \varphi^+_c L \right),$$

(14.1)

where $f_e \equiv f_Q = \Sigma Q$, $\nu_\nu \equiv c_\nu f_Q$. The rules regarding to this change apply the gauge invariant Lagrangian of isospinor-scalar $\varphi$-meson to be added to total Lagrangian eq.(11.5).

The $\varphi$-meson undergoes following gauge transformations: $\varphi'(\eta, u_W) = f_{exp}^{(3)}(\eta(\eta)) \varphi(u_W)$, where $\eta \in G$ and $u_W \in G$. It is due to fact that the $\varphi$-meson carries isospin and hypercharge. Making use of the local transformations implemented upon the components $q_L$ and $q_R$, one gets this transformation rule.
15 The Higgs Boson

A common feature of gauge theories is that introducing of ready made multiplets of zero mass scalar bosons as predicted by Goldstone theorem [37,38]. Such bosons tend to restore the primary symmetry [38,39]. In Higgs relativistic model [40,41] these unwanted massless Goldstone particles are eliminated by coupling in gauge fields. To break the gauge symmetry down and leading to masses of the fields, one needs in general, several kinds of spinless Higgs mesons, with conventional Yukawa couplings to fermion currents and transforming by an irreducible representation of gauge group. The Higgs theory like [37] involves these bosons as the ready made fundamental elementary fields, which entails various difficulties. Within our approach the self-interacting isospinor-scalar Higgs bosons arise as the collective modes of excitations of bound quasi-particle iso-pairs.

15.1 The Bose Condensate of Iso-Pairs

The ferromagnetism [42], Bose superfluid [43] and BCS-Bogoliubov model of superconductivity [9-11] are characterized by the condensation phenomenon leading to the symmetry-breaking ground state. It is particularly helpful to remember that in BCS-Bogoliubov theory the importance of this phenomenon resides in the possibility suggested by Cooper [44] that in the case of an arbitrary weak interaction the pair, composed of two mutually interacting electrons above the quiescent Fermi sea, remains in a bound state. The electrons filling the Fermi sea do not interact with the pair and in the same time they block the levels below the Fermi surface. The superconductive phase arises due to effective attraction between electrons occurred by exchange of virtual phonons [45]. In BCS microscopic theory of superconductivity instead of bound states, with inception by Cooper, one has a state with strongly correlated electron pairs or condensed state in which the pairs form the condensate. The energy of a system in the superconducting state is smaller than the energy in the normal state described by the Bloch individual-particle model. The energy gap arises is due to existence of the binding energy of a pair as a collective effect, the width of which is equal to twice the binding energy. According to Pauli exclusion principle, only the electrons situated in the spherical thin shell near the Fermi surface can form bound pairs in which they have opposite spin and momentum. The binding energy is maximum at absolute zero and decreases along the temperature increasing because of the disintegration of pairs. Pursuing the analogy with these ideas in outlined here approach a serious problem is to find out the eligible mechanism leading to the formation of pairs, somewhat like Cooper mechanism, but generalized for relativistic fermions, of course in absence of any lattice. We suggest this mechanism in the framework of gauge invariance incorporated with the P-violation phenomenon in W-world. To trace a maximum resemblance to the superconductivity theory, within this section it will be advantageous to describe our approach in terms of four dimensional Minkowski space $M^4$ corresponding to internal W-world: $G \rightarrow M^4 (\text{subsec.2.1 in [1]})$. Although we shall leave the suffix (W) implicit, but it goes without saying that all results obtained within this section refer to W-world. According to previous section, we consider the isospinor-scalar $\varphi$-meson arisen in W-world

$$\varphi(x) = \gamma^0 \Psi_L^\dagger(x) \Psi_R(x),$$
where \( x \in M^4 \) is a point of W-world. The following notational conventions will be employed throughout

\[
\Psi_L(x) \rightarrow \Psi_L(x), \quad M^4 \rightarrow M^4, \quad q_L \equiv \Psi_L(x) \rightarrow \Psi_L(x),
\]

where \( \Psi_R(x) = \gamma(1+\vec{\sigma} \vec{\beta})\Psi_L(x), \quad \vec{\beta} = \vec{v}, \quad \Psi_L(x) = \gamma(1-\vec{\sigma} \vec{\beta})\Psi_R(x), \quad \gamma = \frac{E}{m}, \)

provided by the spin \( \vec{\beta} \), energy \( E \) and velocity \( \vec{v} \) of particle. In terms of Fourier integrals

\[
\Psi_L(x) = \frac{1}{(2\pi)^4} \int \Psi_L(p_L)e^{iq_L \cdot x} d^4p_L, \quad \Psi_R(x) = \frac{1}{(2\pi)^4} \int \Psi_R(p_R)e^{iq_R \cdot x} d^4p_R,
\]

it is readily to get

\[
\varphi(k) = \int \varphi(x)e^{-ikx} d^4x = \gamma \int \frac{d^4p_L}{(2\pi)^4} \Psi_L^+(p_L)\Psi_L(p_L+k) = \gamma \int \frac{d^4p_R}{(2\pi)^4} \Psi_L^+(p_R-k)\Psi_R(p_R)
\]

provided by conservation law of fourmomentum \( k = p_R - p_L \), where \( k = k(\omega, \vec{k}), \quad p_{L,R} = p_{L,R}(E_{L,R}, \vec{p}_{L,R}) \). Our arguments on Bose-condensation are based on the local gauge invariance of the theory incorporated with the P-violation in weak interactions. The rationale for this approach is readily forthcoming from the consideration of gauge transformations of the fields eq.(15.1.2) under the P-violation in W-world

\[
\Psi'_L(x) = U_L(x)\Psi_L(x), \quad \Psi'_R(x) = U_R(x)\Psi_R(x),
\]

where the Fourier expansions carried out over corresponding gauge quanta with wave fourvectors \( q_L \) and \( q_R \)

\[
U_L(x) = \int \frac{d^4q_L}{(2\pi)^4} e^{iq_L \cdot x} U_L(q_L), \quad U_R(x) = \int \frac{d^4q_R}{(2\pi)^4} e^{iq_R \cdot x} U_L(q_R),
\]

and \( U_L(x) \neq U_R(x) \). They induce the gauge transformations implemented upon \( \varphi \)-field \( \varphi'(x) = U(x)\varphi(x) \). The matrix of induced gauge transformations may be written down in terms of induced gauge quanta

\[
U(x) \equiv U^+_L(x)U_R(x) = \int \frac{d^4q}{(2\pi)^4} e^{iq \cdot x} U(q),
\]

where \( q = -q_L + q_R \), \( q(q^0, \vec{q}) \). In momentum space one gets

\[
\varphi'(k') = \int \frac{d^4q}{(2\pi)^4} U(q)\varphi(k' - q) = \int \frac{d^4k}{(2\pi)^4} U(k' - k)\varphi(k).
\]

Conservation of fourmomentum requires that \( k' = k + q \). According to eq.(15.1.2) and eq.(15.1.5), we have

\[
-p_L' + p_R' = -p_L + p_R + q = -p_L'' + p_R = -p_L + p_R',
\]

where \( p_L' = p_L - q, \quad p_R' = p_R + q \). Whence the wave vectors of fermions imply the conservation law \( \vec{p}_L + \vec{p}_R = \vec{p}_L' + \vec{p}_R' \), characterizing the scattering process of two fermions with effective interaction caused by the mediating induced gauge quanta. We suggest
the mechanism for the effective attraction between the fermions in the following manner: Among all induced gauge transformations with miscellaneous gauge quanta we distinguish a special subset with the induced gauge quanta of the frequencies belonging to finite region characterized by the maximum frequency $\tilde{q}$: $\left( \tilde{q} = max\{q^0 \} \right)$ greater than the frequency of inducing oscillations fermion force $\bar{\varepsilon}_{L} - \bar{\varepsilon}_{L}^\prime < \frac{\tilde{q}}{\hbar}$. To the extent that this is a general phenomenon, we can expect under this condition the effective attraction (negative interaction) arisen between the fermions caused by exchange of virtual induced gauge quanta if only the forced oscillations of these quanta occur in the same phase with the oscillations of inducing force (the oscillations of fermion density). In view of this we may think of isospinor $\Psi_{L}$ and isoscalar $\Psi_{R}$ fields as the fermion fields composing the iso-pairs with the same conserving net momentum $\vec{p} = \vec{p}_{L} + \vec{p}_{R}$ and opposite spin, for which the maximum number of negative matrix elements of operators composed by corresponding creation and annihilation operators $a_{\vec{p}_{L}}^{+} a_{\vec{p}_{R}} a_{\vec{p}_{L}}^{+} a_{\vec{p}_{R}}$, (designated by the pair wave vector $\vec{p}$) may be obtained for coherent ground state with $\vec{p} = \vec{p}_{L} + \vec{p}_{R} = 0$. In the mean time the interaction potential reads

$$V = \sum_{\vec{p}^\prime_{L}, \vec{p}^\prime_{R}, \vec{p}_{R}, \vec{p}_{L}} \left( a_{\vec{p}^\prime_{L}}^{+} \right)^{T} a_{\vec{p}^\prime_{R}}^{+} \left( a_{\vec{p}_{L}}^{+} \right) a_{\vec{p}_{R}} = \sum_{\vec{p}^\prime_{L}, \vec{p}^\prime_{R}, \vec{p}_{R}, \vec{p}_{L}} a_{\vec{p}^\prime_{L}}^{+} a_{\vec{p}^\prime_{R}}^{+} a_{\vec{p}_{R}} a_{\vec{p}_{L}}, \quad (15.1.6)$$

implying the attraction between the fermions situated in the spherical thin shall near the Fermi surface

$$V_{\vec{p}\vec{p}^\prime} = \begin{cases} -V & \text{at} \quad |E_{\vec{p}^\prime} - E_{F}| \leq \tilde{q}, \quad |E_{\vec{p}^\prime}^\prime - E_{F}| \leq \tilde{q}, \\ 0 & \text{otherwise}. \end{cases} \quad (15.1.7)$$

The fermions filled up the Fermi sea block the levels below Fermi surface. Hence, the fermions are in superconductive state if the condition eq.(15.1.7) holds. Otherwise, they are in normal state described by Bloch individual particle model. Thus, the Bose-condensate arises in the W-world as the collective mode of excitations of bound quasi-particle iso-pairs described by the same wave function in the superconducting phase $\Psi = \langle \Psi_{L} \Psi_{R} \rangle$, where $\langle \cdots \rangle$ is taken to denote the vacuum averaging. The vacuum of the W-world filled up by such iso-pairs at absolute zero $T = 0$. We make a final observation that $\Psi_{R} \Psi_{R}^{+} = n_{R}$ is a scalar density number of right-handed particles. Then it readily follows:

$$(\Psi_{L} \Psi_{R})^{+} (\Psi_{L} \Psi_{R}) = \Psi_{L}^{+} \Psi_{L} \Psi_{R}^{+} \Psi_{R} = \frac{1}{n_{R}} \Psi_{R}^{+} \gamma^{0} \left( \gamma^{0} \Psi_{L}^{+} \Psi_{R} \right) \left( \Psi_{R}^{+} \Psi_{L} \gamma^{0} \right) \gamma^{0} \Psi_{R} = \varphi \varphi^{+}, \quad (15.1.8)$$

where $|\Psi|^{2} = \langle \varphi \varphi^{+} \rangle = |\langle \varphi \rangle|^{2}$. It is convenient to abbreviate the $\langle \varphi \rangle$ by the symbol $\varphi$. The eq.(15.1.8) shows that the $\varphi$-meson actually arises as the collective mode of excitations of bound quasi-particle iso-pairs.

### 15.2 The Non-Relativistic Approximation

In the approximation to non-relativistic limit ($\beta \ll 1, \quad \Psi_{L} \simeq \Psi_{R}, \quad \gamma^{0} \rightarrow 1$) by making use of Ginzburg-Landau’s phenomenological theory [46] it is straightforward to write down the free-energy functional for the order parameter in equilibrium superconducting phase in presence of magnetic field. The self-consistent coupled GL-equations are differential
equations like Schrödinger and Maxwell equations, which relate the spatial variation of
the order parameter $\Psi$ to the vector potential $\vec{A}$ and the current $\vec{j}$. In the papers [20,21],
by means of thermodynamic Green’s functions in well defined limit Gor’kov was able to
show that GL-equations are a consequence of the BCS-Bogoliubov microscopic theory of
superconductivity. The theoretical significance of these works resides in the microscopic
interpretation of all physical parameters of GL-theory. Subsequently these ideas were
extended to lower temperatures by others [47-49] using a requirement that the order
parameter and vector potential vary slowly over distances of the order of the coherence
length and that the electrodynamics be local (London limit). Namely, the validity of
derived GLG-equations is restricted to the temperature $T$, such $T_c - T \ll T_c$ and to the
local electrodynamics region $q\xi_0 \ll 1$, where $T_c$ is transition temperature, $\xi_0$ is coherent
length characterizing the spatial extent of the electron pair correlations, $q$ are the wave
numbers of magnetic field $\vec{A}$. Thereto the most important order parameter $\Psi$, mass $m_\Psi$
and coupling constant $\lambda_\Psi$ figuring in GLG-equations read

$$\Psi(\vec{r}) = \left(\frac{7\zeta(3)N}{4\pi k_B T_c}\right)^{1/2} \Delta(\vec{r}), \quad \Delta(T) \approx 3.1 k_B T_c \left(1 - \frac{T}{T_c}\right)^{1/2}, \quad \xi_0 \approx 0.18 \frac{h v_F}{k_B T_c},$$

$$m_\Psi^2 = 1.83 \frac{\hbar^2}{m \xi_0^2} \left(1 - \frac{T}{T_c}\right), \quad \lambda_\Psi^2 = \frac{1}{1.4 N(0)} \left(\frac{\hbar^2}{2m\xi_0^2}\right) \frac{1}{(k_B T_c)^2}. \quad (15.2.1)$$

Reviewing the notation $\Delta(\vec{r})$ is the energy gap, $e^* = 2e$ is the effective charge, $N(0)$
is the state density at Fermi surface, $N$ is the number of particles per unit volume in
normal mode, $v_F$ is the Fermi velocity, $m \equiv \Sigma_Q = f_Q$ is the mass of fermion field. The
transition temperature relates to gap at absolute zero $\Delta_0$ [9]. The estimate for the pair
size at $v_F \sim 10^8 \text{cm/s}$, $T_c \sim 1$ gives [50] $\xi_0 \approx 10^{-4} \text{cm}$. To set up the total Lagrangian
we must add a $SU(2)$ multiplet of spinless meson fields that couples to the gauge fields
in a gauge invariant way. According to eq.(15.2.1) these $\varphi$-meson fields acquire vacuum
expectation values, which subsequently gives the gauge fields a mass. This Lagrangian
of self-interacting iso-pairs spinless scalar $\varphi$-meson at degenerate vacuum of W-world is
given

$$L_\varphi(D) = L^0_\varphi(D) - V(|\ \varphi \ |),$$

provided by $V(|\ \varphi \ |) = \frac{1}{2} \lambda_\varphi^2 \left(\ | \varphi \ |^2 - \frac{\eta_\varphi^2}{2}\right)^2$, and $\eta_\varphi^2 = \frac{m_\varphi^2}{\lambda_\varphi^2}$,

$$m_\varphi \equiv m_\Psi, \quad \lambda_\varphi \equiv \lambda_\Psi.$$  

In accordance with a standard method one may introduce
the real-valued scalar field $\chi(x)$ describing the excitation in the neighbourhood of stable
vacuum $\eta_\varphi \frac{\sqrt{2}}{2}$: $\varphi'(x) = \frac{1}{\sqrt{2}} (\eta_\varphi + \chi(x))$, which reflects the spontaneous breakdown of
symmetry in W-world.

### 15.3 The Relativistic Treatment

We start with total Lagrangian eq.(11.7) of self-interacting fermion field in W-world,
which is arisen from the Lagrangian eq.(10.6) of primary fundamental field after the
rearrangement of the vacuum of Q-world

$$L_W(x) = \frac{i}{2} \left\{ \bar{\Psi}_W(x) \gamma^\mu \partial_\mu \Psi_W(x) - \bar{\Psi}_W(x) \gamma^\mu \partial_\mu \Psi_W(x) \right\} - m \bar{\Psi}_W(x) \Psi_W(x) - \frac{\lambda}{2} \bar{\Psi}_W(x) \left( \bar{\Psi}_W(x) \Psi_W(x) \right) \Psi_W(x). \quad (15.3.1)$$
Here, $m = \Sigma_Q$ is the self-energy operator of the fermion field component in Q-world, the suffix $(W)$ just was put forth in illustration of a point at issue. For the sake of simplicity, we also admit $B_W(x) = 0$, but of course one is free to restore the gauge field $B_W(x)$ whenever it should be needed. In lowest order the relation $m \equiv m_Q \ll \lambda^{-1/2}$ holds. The Lagrangian eq. (15.3.1) leads to the field equations

$$
(\gamma p - m)\Psi(x) - \lambda \left(\bar{\Psi}(x)\Psi(x)\right)\Psi(x) = 0,
$$

$$
\Psi(x)(\gamma p + m) = \lambda \Psi(x) \left(\bar{\Psi}(x)\Psi(x)\right) = 0,
$$

(15.3.2)

where the indices have been suppressed as usual. At non-relativistic limit the function $\Psi$ reads $\Psi \to e^{imc^2t}\Psi$, and Lagrangian eq. (15.3.1) leads to Hamiltonian used in [20].

Our discussion will be in close analogy with that of [20]. We make use of the Gor’kov’s technique and evaluate the equations (15.3.2) in following manner: The spirit of the calculation will be to treat interaction between the particles as being absent everywhere except the thin spherical shell $2\tilde{q}$ near the Fermi surface. The Bose condensate of bound particle iso-pairs occurred at zero momentum. The scattering processes between the particles are absent. We consider the matrix elements

$$
< T \left(\Psi_\alpha(x_1)\Psi_\beta(x_2)\bar{\Psi}_\gamma(x_3)\bar{\Psi}_\delta(x_4)\right) > = - < T \left(\bar{\Psi}_\alpha(x_1)\bar{\Psi}_\gamma(x_3)\right) > \cdot < T \left(\bar{\Psi}_\beta(x_2)\bar{\Psi}_\delta(x_4)\right) >
$$

$$
+ < T \left(\Psi_\alpha(x_1)\bar{\Psi}_\delta(x_4)\right) > \cdot < T \left(\Psi_\beta(x_2)\bar{\Psi}_\gamma(x_3)\right) > + < T \left(\Psi_\alpha(x_1)\Psi_\beta(x_2)\right) > \cdot < T \left(\bar{\Psi}_\gamma(x_3)\bar{\Psi}_\delta(x_4)\right) >,
$$

where

$$
< T \left(\bar{\Psi}_\gamma(x_3)\bar{\Psi}_\delta(x_4)\right) > \cdot < T \left(\bar{\Psi}_\delta(x_4)\bar{\Psi}_\gamma(x_3)\right) > =< T \left(\bar{\Psi}_\gamma(x_3)\bar{\Psi}_\delta(x_4)\right) > \cdot < T \left(\bar{\Psi}_\gamma(x_3)\bar{\Psi}_\delta(x_4)\right) >
$$

also introduce the functions

$$
< N | T \left(\gamma^0\Psi(x)\Psi(x')\right) | N + 2 > = e^{-2i\mu't}F(x-x'),
$$

$$
< N + 2 | T \left(\gamma^0\Psi(x)\gamma^0\Psi(x')\right) | N > = e^{2i\mu't}F^+(x-x'),
$$

(15.3.3)

and

$$
< N | T \left(\Psi_L(x)\Psi_R(x')\right) | N + 2 > = e^{-2i\mu't}F_{LR}(x-x'),
$$

$$
< N + 2 | T \left(\Psi_L(x)\gamma^0\Psi_R(x')\right) | N > = e^{2i\mu't}F_{LR}^+(x-x'),
$$

$$
< N | T \left(\Psi_R(x)\Psi_L(x')\right) | N + 2 > = e^{-2i\mu't}F_{RL}(x-x'),
$$

$$
< N + 2 | T \left(\Psi_R(x)\gamma^0\Psi_L(x')\right) | N > = e^{2i\mu't}F_{RL}^+(x-x').
$$

(15.3.4)

Thereupon

$$
F(x-x') = \begin{pmatrix} F_{LR}(x-x') \\ F_{RL}(x-x') \end{pmatrix}, \quad F^+(x-x') = \begin{pmatrix} F_{LR}^+(x-x') \\ F_{RL}^+(x-x') \end{pmatrix}.
$$

Here, $\mu' = \mu + m$, $\mu$ is a chemical potential. We omit a prime over $\mu$, but should understand under it $\mu + m$. Let us now make use of Fourier integrals

$$
G_{\alpha\beta}(x-x') = \int \frac{d\omega d\tilde{p}}{(2\pi)^4} G_{\alpha\beta}(p)e^{i\tilde{p}(x-x')-i\omega(t-t')}
$$

etc, which render the equation (15.3.2) easier to handle in momentum space

$$
(\gamma p - m)G(p) - i\lambda \gamma^0 F(0+)\tilde{F}(p) = 1,
$$

$$
\tilde{F}(p)(\gamma p + m - 2i\mu) + i\lambda \tilde{F}(0+)G(p) = 0,
$$

(15.3.5)
where \( F_{\alpha\beta}(0+) = e^{2i\mu t} < T (\gamma^0 \Psi_\alpha(x) \Psi_\beta(x)) = \lim_{x \to x'(t \to t')} F_{\alpha\beta}(x - x') \). Next we substitute

\[
\gamma p - m = (\omega' - \xi_p)\gamma^0, \quad (\gamma p + m - 2\mu \gamma^0) = \gamma^0 (\omega' + \xi_p^+),
\]

where

\[
\omega' = \omega - \mu' = \omega - m - \mu, \quad \xi_p = (\gamma p^0 + m)\gamma^0 - \mu' = (\gamma p^0 + n)\gamma^0 - m - \mu, \quad \xi_p^+ = \gamma^0 (\gamma p^0 + m) - m - \mu,
\]

and omit a prime over \( \omega' \) for the rest of this section. We employ

\[
F(0+) = -JI, \quad I = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad F^+(0+)F(0+) = -J^2 I^2 = J^2,
\]

and \( \tilde{\omega} + \tilde{\xi}_p = \gamma^0 (\omega + \xi_p) \). The gap function \( \Delta \) reads \( \Delta^2 = \lambda^2 J^2 \), where \( J = \int \frac{d\omega d\vec{k}}{(2\pi)^4} F^+(p) \).

Making use of standard rules [43], one may pass over the poles. This method allows oneself to extend the study up to limit of temperatures, such that \( T_c - T \ll T_c \), by making use of thermodynamic Green’s function. So

\[
F^+(p) = -i \lambda J (\omega - \xi_p + i\delta)^{-1} (\omega + \xi_p - i\delta)^{-1} - \frac{\pi \Delta}{\epsilon_p} n(\epsilon_p) \{ \delta(\omega - \epsilon_p) + \delta(\omega + \epsilon_p) \},
\]

\[
G(p) = \gamma^0 \{ u_p^2 (\omega - \xi_p + i\delta)^{1/2} + v_p^2 (\omega + \xi_p - i\delta)^{-1} + 2\pi i n(\epsilon_p) [u_p^2 \delta(\omega - \epsilon_p) - v_p^2 \delta(\omega + \epsilon_p)] \},
\]

where \( u_p^2 = \frac{1}{2} \left( 1 + \frac{\xi_p}{\epsilon_p} \right), \quad v_p^2 = \frac{1}{2} \left( 1 - \frac{\xi_p}{\epsilon_p} \right), \quad \epsilon_p = (\xi_p^2 + \Delta^2(T))^{1/2} \). and \( n(\epsilon_p) \) is the usual Fermi function \( n(\epsilon_p) = \left( \exp \frac{\epsilon_p}{T} + 1 \right)^{-1} \). Then

\[
1 = |\lambda| \frac{1}{2(2\pi)^3} \int d\vec{k} \frac{1 - 2n(\epsilon_k)}{\epsilon_k(T)} \left( |\xi_p| < \tilde{q} \right), \tag{15.3.7}
\]

determining the energy gap \( \Delta \) as a function of \( T \). According to eq.(15.3.7), the \( \Delta(T) \to 0 \) at \( T \to T_c \sim \Delta(0) \) [9].

### 15.4 Self-Interacting Potential of Bose-Condensate

To go any further in exploring the form and significance of obtained results it is entirely feasible to include the generalization of the equations (15.3.5) in presence of spatially varying magnetic field with vector potential \( \vec{A}(\vec{r}) \), which is straightforward \( (t \to \tau = it) \)

\[
\begin{aligned}
\left\{ -\gamma^0 \frac{\partial}{\partial \tau} - i\vec{\gamma} \left( \frac{\partial}{\partial \vec{r}} - ie\vec{A}(\vec{r}) \right) - m + \gamma^0 \mu \right\} G(x, x') + \gamma^0 \Delta(\vec{r}) \vec{F}(x, x') & = \delta(x - x'), \\
\vec{F}(x, x') \left\{ \gamma^0 \frac{\partial}{\partial \tau} + i\vec{\gamma} \left( \frac{\partial}{\partial \vec{r}} + ie\vec{A}(\vec{r}) \right) - m + \gamma^0 \mu \right\} - \Delta^*(\vec{r}) \gamma^0 G(x, x') & = 0,
\end{aligned}
\tag{15.4.1}
\]

where the thermodynamic Green’s function [51,52] is used and the energy gap function is in the form

\[
\Delta^*(\vec{r}) = \lambda F^+(\tau, \vec{r}; \tau, \vec{r}) = (\Delta^*_{LR}(\vec{r}), \Delta^*_{RL}(\vec{r})), \quad F(x, x') = \left( \begin{array}{c} F_{LR}(x, x') \\ F_{RL}(x, x') \end{array} \right),
\]
This function is logarithmically divergent, but with a cutoff of energy of interacting fermions at the spatial distances in order of $\frac{\hbar}{\tilde{q}}$ can be made finite, where $\tilde{q} \equiv \frac{q}{\hbar}$. If one uses the Fourier components of functions $\tilde{G}(x, x')$ and $F(x, x')$

$$G(\vec{r}, \vec{r}'; u) = T \sum_n e^{-iu\omega}G_\omega(\vec{r}, \vec{r}'), \quad G_\omega(\vec{r}, \vec{r}') = \frac{1}{2} \int_{-1/T}^{1/T} e^{iu\omega}G(\vec{r}, \vec{r}'; u) \, du,$$

where $u = \tau - \tau'$, $\omega$ is the discrete index $\omega = \pi T(2n + 1), \quad n = 0, \pm 1, \ldots$, then the eq. (15.4.1) reduces to

$$\left\{i\omega\gamma^0 - i\gamma^0 \left(\vec{\partial}_\tau - ie\vec{A}(\vec{r})\right) - m + \gamma^0 \mu \right\} G_\omega(\vec{r}, \vec{r'}) + \gamma^0 \Delta(\vec{r})\bar{F}_\omega(\vec{r}, \vec{r'}) = \delta(\vec{r} - \vec{r'}),$$

$$\bar{F}_\omega(\vec{r}, \vec{r'}) \left\{-i\omega\gamma^0 + i\gamma^0 \left(\vec{\partial}_\tau + ie\vec{A}(\vec{r})\right) - m + \gamma^0 \mu \right\} - \Delta^*(\vec{r})\gamma^0 G_\omega(\vec{r}, \vec{r'}) = 0,$$

(15.4.3)

where the gap function is defined by $\Delta^*(\vec{r}) \equiv \lambda T \sum_n F_n^+(\vec{r}, \vec{r'})$. The Bloch individual particle Green’s function $\bar{G}_\omega(\vec{r}, \vec{r'})$ for the fermion in normal mode is written

$$\left\{i\omega\gamma^0 - i\gamma^0 \left(\vec{\partial}_\tau - ie\vec{A}(\vec{r})\right) - m + \gamma^0 \mu \right\} \bar{G}_\omega(\vec{r}, \vec{r'}) = \delta(\vec{r} - \vec{r'}),$$

(15.4.4)

or the adjoint equation

$$\left\{i\omega\gamma^0 + i\gamma^0 \left(\vec{\partial}_\tau + ie\vec{A}(\vec{r})\right) - m + \gamma^0 \mu \right\} \bar{G}_\omega(\vec{r}, \vec{r'}) = \delta(\vec{r} - \vec{r'}).$$

(15.4.5)

By means of eq. (15.4.5) the eq. (15.4.3) gives rise to

$$G_\omega(\vec{r}, \vec{r'}) = \bar{G}_\omega(\vec{r}, \vec{r'}) - \int \bar{G}_\omega(\vec{s}, \vec{s'})\gamma^0 \Delta(\vec{s})\bar{F}_\omega(\vec{s}, \vec{s'}) \, d^3s,$$

(15.4.6)

and

$$\bar{F}_\omega(\vec{r}, \vec{r'}) = \int \bar{G}_\omega(\vec{s}, \vec{r'})\Delta^*(\vec{s})\gamma^0 \bar{G}_\omega(\vec{s}, \vec{r'}) \, d^3s.$$  

(15.4.7)

The gap function $\Delta(\vec{r})$ as well as $\bar{F}_\omega(\vec{r}, \vec{r'})$ are small ones at close neighbourhood of transition temperature $T_c$ and varied slowly over a coherence distance. This approximation, which went into the derivation of equations, meets our interest in eqs. (15.4.6), eq. (15.4.7). Using standard procedure one may readily express them in power series of $\Delta$ and $\Delta^*$ by keeping only the terms in $\bar{F}_\omega(\vec{r}, \vec{r'})$ up to the cubic and in $G_\omega(\vec{r}, \vec{r'})$ - quadratic order in $\Delta$. After averaging over the polarization of particles the following equation coupling $\Delta(\vec{r})$ and $\vec{A}(\vec{r})$ ensued:

$$\overline{\Delta^*(\vec{r})} = \lambda T \sum_n \int \bar{G}_\omega(\vec{r}, \vec{r'})G_{-\omega}(\vec{r}, \vec{r'})\Delta^*(\vec{r'}) \, d^3r' -$$

$$\lambda T \sum_n \int \int \tilde{G}_\omega(\vec{s}, \vec{r'})\bar{G}_{-\omega}(\vec{s}, \vec{l})\bar{G}_\omega(\vec{m}, \vec{l})\bar{G}_{-\omega}(\vec{m}, \vec{r})\Delta(\vec{s})\Delta^*(\vec{l})\Delta^*(\vec{m}) \, d^3s \, d^3l \, d^3m.$$

(15.4.8)

It is worthwhile to determine the function $\bar{G}_\omega(\vec{r}, \vec{r'})$. In the absence of applied magnetic field, i.e. $\vec{A}(\vec{r}) = 0$ the eq. (15.4.4) reduces to

$$\left\{(i\omega + \mu)\gamma^0 - \tilde{\gamma}\bar{p} - m \right\} \bar{G}_\omega^0(\vec{r}, \vec{r'}) = \delta(\vec{r} - \vec{r'}).$$

(15.4.9)
It is well to study at this point certain properties of the solution which we shall continually encounter

\[
\tilde{G}_0^0(\vec{r}, \vec{r}') = \frac{1}{2m} \left\{ (i\omega + \mu)\gamma^0 - \vec{\gamma}\vec{p} + m \right\} \tilde{G}_0^0(\vec{r} - \vec{r}'),
\]

(15.4.10)

where the function \( \tilde{G}_0^0(\vec{r} - \vec{r}') \) satisfies the equation

\[
\frac{1}{2m} \left( q^2 + \Delta \right) \tilde{G}_0^0(\vec{r} - \vec{r}') = \delta(\vec{r} - \vec{r}'),
\]

(15.4.11)

provided \( q^2 = (i\omega + \mu)^2 - m^2 = 2im\Omega + p_0^2 \) and \( \Omega = \omega\mu/m, \quad p_0^2 = \mu^2 - m^2 \). At \( \mu \gg |\omega| \) one has \( \frac{p_0^2}{2m} \gg |\Omega| \), and

\[
\frac{1}{2m} \left\{ 2im\Omega + p_0^2 + \Delta \right\} \tilde{G}_0^0(\vec{r} - \vec{r}') = \delta(\vec{r} - \vec{r}'),
\]

(15.4.12)

the solution of which reads

\[
\tilde{G}_0^0(\vec{r} - \vec{r}') = -\frac{m}{2\pi R} \exp(iqR),
\]

where

\[
q = \text{sgn} \Omega p_0 + i \frac{|\Omega|}{v}, \quad R = |\vec{r} - \vec{r}'|, \quad \text{sgn} \Omega = \frac{\Omega}{|\Omega|}.
\]

In approximation to non-relativistic limit \( \vec{p} \to 0 \) this Green’s function reduces to \( \tilde{G}_0^0(\vec{r} - \vec{r}') \) used in [20]. So, our discussion is consistent with well-known [20]. Making use of Fourier integrals we readily get

\[
\tilde{G}_0^0(\vec{p}) = \frac{\mu\gamma^0 + \vec{\gamma}\vec{p} + m}{q^2 - \vec{p}^2 + i0} = \tilde{G}_0^0(\vec{p}),
\]

where \( \tilde{\gamma} = \frac{1}{2m} (\mu\gamma^0 + \vec{\gamma}\vec{p} + m) \). One has

\[
\tilde{G}_0^0(\vec{p}) = \tilde{G}_0^0(\vec{p}) = \frac{1}{i\Omega - \xi} = \tilde{G}_0^0(-\vec{p}),
\]

where as usual \( \xi = \frac{\vec{p}}{2m} - \frac{\vec{p}_0}{2m} \). The Green’s function \( \tilde{G}_0^0(\vec{r}, \vec{r}') \) in presence of magnetic field differs from \( \tilde{G}_0^0(\vec{r} - \vec{r}') \) only by phase multiplier [20]

\[
\tilde{G}_0^0(\vec{r}, \vec{r}') = \exp \left\{ \frac{i\epsilon}{c} \left( \vec{A}(\vec{r}), \vec{r} - \vec{r}' \right) \right\} \tilde{G}_0^0(\vec{r} - \vec{r}').
\]

The technique now is to expand a second term in right-hand side of the eq.(15.4.8) up to the terms quadratic in \( (\vec{r} - \vec{r}') \). After calculations it transforms

\[
\left\{ \left( i\hbar \vec{\nabla} + \frac{e^*}{c} \vec{A} \right)^2 + \frac{2m}{\nu} \left[ \frac{2\pi^2}{\lambda m p_0} \left( \frac{\mu}{m} \right)^2 \left( \frac{\mu}{m} - 1 \right) + \left( \frac{\mu}{m} \right)^2 \left( \frac{T}{T_{cu}} - 1 \right) \right] \right\} \Psi(\vec{r}) = 0,
\]

(15.4.13)
where \( \nu = \frac{7\zeta(3)mv_F^2}{24(\pi k_B T_c)^2} \) and \( T_{c\mu} = \frac{m}{\mu} T_c \). Succinctly

\[
\left\{ \frac{p_A^2}{2} - \frac{1}{2} m^2 \Psi + \frac{1}{4} \lambda^2 \Psi | \Psi(\vec{r}) |^2 \right\} \Psi(\vec{r}) = 0, \tag{15.4.14}
\]

provided

\[
m^2_\Psi(\lambda, T, T_{c\mu}) = \frac{24}{7\zeta(3)} \left( \frac{\hbar}{\xi_0} \right)^2 \left( \frac{\mu}{m} \right)^2 \left[ 1 - \frac{T}{T_{c\mu}} - \left( \frac{\mu}{m} - 1 \right) \ln \frac{2\nu}{\Delta_0} \right],
\]

\[
\lambda^2_\Psi(\lambda, T_c) = \frac{96}{7\zeta(3)} \left( \frac{\hbar}{\xi_0} \right)^2 \frac{1}{N}, \quad \Psi(\vec{r}) = \Delta(\vec{r}) \frac{(7\zeta(3)N)^{1/2}}{4\pi k_B T_c}. \tag{15.4.15}
\]

According to the eq.(15.4.15), the magnitude of the relativistic effects, however, is found to be greater to account for the large contribution to the values \( m^2_\Psi \) and \( \lambda^2_\Psi \). Thereby the transition temperature decreases inversely by the relativistic factor \( \mu \). A spontaneous breakdown of symmetry of ground state occurs at \( \eta^2_\Psi(\lambda, T < T_{c\mu}) > 0 \), where

\[
\eta^2_\Psi(\lambda, T, T_{c\mu}) = \frac{m^2_\Psi}{\lambda^2_\Psi}.
\]

As far as \( \Psi = \left( \Psi_{LR} \Psi_{RL} \right) \), \( \Delta = \left( \Delta_{LR} \Delta_{RL} \right) \), and \( \Psi_{LR} = \Psi_{RL} \), \( \Delta_{LR} = \Delta_{RL} \), then the eq.(15.4.14) splits into the couple of equations for \( \Psi_{LR} \) and \( \Psi_{RL} \). Subsequently, a Lagrangian of the \( \phi \) will be arisen with the corresponding values of mass \( m^2_\Psi \equiv m^2_\varphi \) and coupling constant \( \lambda^2_\Psi \equiv \lambda^2_\varphi \).

### 15.5 The Four-Component Bose-Condensate in Magnetic Field

Now we are going to derive the equation of four-component bispinor field of Bose-condensate, where due to self-interaction the spin part of it is vanished. We start with the nonsymmetric state \( \Delta_{LR} \neq \Delta_{RL} \), where \( \Psi_{LR} \) and \( \Psi_{RL} \) are two eigenstates of chirality operator \( \gamma_5 \). In standard representation

\[
\Psi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \Psi_{LR} \\ \Psi_{RL} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \Psi_{LR} + \Psi_{RL} \\ \Psi_{LR} - \Psi_{RL} \end{pmatrix} = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix},
\]

\[
\Delta = \begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \Delta_{LR} + \Delta_{RL} \\ \Delta_{LR} - \Delta_{RL} \end{pmatrix}, \tag{15.5.1}
\]

\[
\gamma_5 \Psi_{LR} = \Psi_{LR}, \quad \gamma_5 \Psi_{RL} = -\Psi_{RL}, \quad \Delta_{LR} \neq \Delta_{RL}.
\]

The eq.(15.4.14) enables to postulate the equation of four-component Bose-condensate in magnetic field and equilibrium state

\[
i\hbar \frac{\partial \Psi}{\partial t} = \left\{ c\vec{\alpha} \left( \vec{p} + \frac{e^* A}{c} \right) + \beta mc^2 + M(F) + L(F) | \Psi |^2 \right\} \Psi = 0, \tag{15.5.2}
\]

or succinctly

\[
(\gamma p_A - m) \Psi = 0. \tag{15.5.3}
\]
This is in standard notation

\[
\gamma^0 = \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \bar{\alpha} = \gamma^0 \bar{\gamma} = \begin{pmatrix} 0 & \bar{\sigma} \\ \bar{\sigma} & 0 \end{pmatrix}, \quad F = F_{\mu\nu} \sigma^{\mu\nu} = \text{inv},
\]

\[
\sigma^{\mu\nu} = \frac{1}{2} [\gamma^\mu, \gamma^\nu], \quad \frac{i}{2} e^* F = - e^* \bar{\Sigma} \bar{H}, \quad \bar{H} = \text{rot} \bar{A}, \quad F_{\mu\nu} = (0, \bar{H}), \quad \bar{\Sigma} = \left( \begin{array}{cc} \bar{\sigma} & 0 \\ 0 & \bar{\sigma} \end{array} \right),
\]

\[
p_A = (p_{A0}, \bar{p}_A), \quad \bar{p}_A = i h \bar{\nabla} + e^* c \bar{A}, \quad p_{A0} = - \left( M(F) + L(F) | \Psi |^2 \right).
\]

The \( M(F) \) and \( L(F) \) are some functions depending upon the invariant \( F \), which will be determined under the requirement that the second-order equations ensued from the eq.(15.5.3) must match onto eq.(15.4.14). Defining the functions \( M(F) \) and \( L(F) \):

\[
M(F) = \left( M_0^2 + \frac{i}{2} e^* F \right)^{1/2}, \quad M_0 = \left( m^2 + \frac{1}{2} m_\varphi^2 \right)^{1/2}, \quad L(F) = - \frac{\lambda_\varphi^2}{8 M(F)},
\]

\[
L_0 = - \frac{\lambda_\varphi^2}{8 M_0}, \quad M(F) L(F) = M_0 L_0 = - \frac{1}{8} \lambda_\varphi^2,
\]

and taking into account an approximation fitting our interest that the gap function is small at close neighbourhood of transition temperature, one gets

\[
\left\{ p_{A0}^2 + m^2 - (M_0 + L_0 | \Psi |^2)^2 \right\} \Psi(\tilde{r}) \equiv \left\{ \bar{p}_A^2 - \frac{1}{2} m_\varphi^2 + \frac{1}{4} \lambda_\varphi^2 | \Psi |^2 \right\} \Psi(\tilde{r}) = 0. \quad (15.5.4)
\]

This has yet another important consequence. At \( \Delta_{LR} \neq 0 \) and imposed constraint \( (m + M(F) + L(F) | \Psi |^2)_{F \rightarrow 0} \rightarrow 0 \) we have

\[
\Delta_2 = \frac{1}{\sqrt{2}} (\Delta_{LR} - \Delta_{RL}) = 0, \quad \Psi_2 = 0. \quad (15.5.5)
\]

So, the \( | \Psi_0 | \) is the gap function symmetry-restoring value

\[
\Delta_2 \left( | \Psi_0 |^2 = \frac{m + M_0}{-L_0} \right) = 0, \quad \Delta_{LR} \left( | \Psi_0 |^2 \right) = \Delta_{RL} \left( | \Psi_0 |^2 \right),
\]

where, according to eq.(15.5.4), one has

\[
V \equiv \left[ m^2 - (M_0 + L_0 | \Psi |^2)^2 \right] \Psi^2 = \left[ - \frac{1}{2} m_\varphi^2 + \frac{1}{4} \lambda_\varphi^2 | \Psi |^2 \right] \Psi^2, \quad (15.5.6)
\]

and

\[
V \left( | \Psi_0 |^2 = \frac{m + M_0}{-L_0} \right) = \frac{1}{2} \eta_\varphi^2 (\lambda, T, T_{c\mu}) = 0. \quad (15.5.7)
\]

It leads us to the conclusion that the field of symmetry-breaking Higgs boson must be counted off from the \( \Delta_{LR} = \Delta_{RL} \) symmetry-restoring value of Bose-condensate \( | \Psi_0 | = \frac{1}{\sqrt{2}} \eta_\varphi (\lambda, T, T_{c\mu}) \) as the point of origin describing the excitation in the neighbourhood of stable vacuum eq.(15.5.7).

We may write down the Lagrangian corresponding to the eq.(15.5.3)

\[
L_\Psi = \frac{1}{2} \left\{ \bar{\Psi} \gamma p_A \Psi - \bar{\Psi} \gamma^* \bar{p}_A \Psi \right\} - m \bar{\Psi} \Psi.
\]
The gauge invariant Lagrangian eq.(11.7) takes the form
\[
L(D_W) = \frac{i}{2} \left\{ \Psi \gamma D_W \Psi - \Psi \gamma D_W \Psi \right\} - \Psi \left\{ m + \gamma^0 \left[ M(F) + L(F) \right] | \Psi|^2 \right\} \Psi.
\] (15.5.8)

At the symmetry-restoring point, this Lagrangian can be replaced by
\[
L(W_1 \gamma D_W) \rightarrow L(W_1) = \frac{1}{2} \left( D_W \varphi \right)^2 - V \left( | \varphi |^2 \right),
\]
provided
\[
V \left( | \varphi |^2 \right) = -\frac{1}{2} m_\varphi^2 \varphi^2 + \frac{1}{4} \lambda^2 \varphi^4.
\]

Taking into account the eq.(15.1.8), in which
\[
| \varphi |^2 = \left| \frac{\eta + \chi}{2} \right|^2,
\]
one gets
\[
L(W_\varphi \gamma D_W) = \frac{1}{2} \left( D_W \varphi \right)^2 - V \left( | \varphi |^2 \right),
\]
provided
\[
V \left( | \varphi |^2 \right) = -\frac{1}{2} m_\varphi^2 \varphi^2 + \frac{1}{4} \lambda^2 \varphi^4.
\] (15.5.9)

The average value of well-defined current source expressed in terms of spinless field \( \Psi \) eq.(15.5.3) is given by
\[
\vec{j}(\vec{r}) \big|_{\Sigma=0} = m^4 \left( m - M_0 - L_0 \right) \left( \gamma^0 + \frac{\mu}{m} \right) \left( \gamma^0 + \frac{\mu}{m} \right)^2 + 3 \beta^2 F \right\} \vec{j}(\vec{r}),
\] (15.5.11)
provided
\[
\vec{j}(\vec{r}) = \left( \frac{m}{\mu} \right)^3 \left\{ -ie^* h \frac{\partial \Psi^*}{\partial \vec{r}} - \Psi^* \frac{\partial \Psi}{\partial \vec{r}} \right\} - \frac{e^*^2}{mc} \vec{A} |\Psi|^2 \right\}.
\] (15.5.12)

Below we write the eq.(15.5.3) in the form on close analogy of the elementary excitations in superconductivity model described by coherent mixture of electrons and holes near the Fermi surface \([10, 11, 53]\). In chirial representation \( \Psi = \begin{pmatrix} \Psi_{LR} \\ \Psi_{RL} \end{pmatrix} \) one has
\[
p_{A0} \Psi_{LR} = \vec{p} \Psi_{LR} + m \Psi_{RL}, \quad p_{A0} \Psi_{RL} = -\vec{p} \Psi_{RL} + m \Psi_{LR},
\]
\[
p_{A0} = \pm \left( p^0_A + m^2 \right)^{1/2}.
\]

The two states of quasi-particle are separated in energy by \( 2 | p_{A0} | \). In the ground state all quasi-particles should be in lower (negative) energy states. It would take a finite energy \( 2 | p_{A0} | \geq 2m \) to excite a particle to the upper state (the case of Dirac particle). Thus, one may assume that the energy gap parameter \( m \) is also due to some interaction between massless bare fermions \([18,19]\).
15.6 Extension to Lower Temperatures

It is worth briefly recording the question of whether or not it is possible to extend the ideas of former approach to lower temperatures as it was investigated in the case of Gor’kov’s theory by others [47-49]. Here, as usual we admit that the order parameter and vector potential vary slowly over distances of the order of the coherence length. We restrict ourselves to the London limit and the derivation of equations will be proceeded by iterating to a low order giving only the leading terms. Taking into account the eq.(15.4.1), eq.(15.4.3), eq.(15.4.6) and eq.(15.4.7), it is straightforward to derive the separate integral equations for $G$ and $F^+$ in terms of $\Delta$, $\Delta^*$ and $\tilde{G}$. We introduce

$$K_{\Omega}(\vec{r}, \vec{s}) = \delta(\vec{r} - \vec{s}) \left\{ i\Omega + \frac{1}{2m} \left( \frac{\partial^2}{\partial \vec{s}^2} - i\frac{e}{c} \vec{A}(\vec{s}) \right)^2 + \mu_0 \right\} + \Delta(\vec{r}) \tilde{\gamma}_A(\vec{s}) \tilde{G}_{-\Omega}(\vec{s}, \vec{r}) \Delta^*(\vec{s}),$$

(15.6.1)

provided by $\mu_0 = \frac{p_0^2}{2m}$, and

$$F^+(\vec{s}, \vec{r}') = \tilde{\gamma}_A(\vec{s}) F^+_\Omega(\vec{s}, \vec{r}'), \quad \tilde{\gamma}_A(\vec{s}) = \frac{1}{2m} \{ \gamma^0(i\omega + \mu) - \tilde{\gamma}p_A(\vec{s}) + m \}.$$

We write down the coupled equations in the form

$$\int d^3s \ K_{\Omega}(\vec{r}, \vec{s}) G_\Omega(\vec{s}, \vec{r}') = \delta(\vec{r} - \vec{r}'), \quad (15.6.2)$$

and

$$\int d^3s \ F^+_\Omega(\vec{s}, \vec{r}') K_{-\Omega}(\vec{s}, \vec{r}) = \Delta^*(\vec{r}) \tilde{\gamma}_A(\vec{r}') \tilde{G}_{\Omega}(\vec{r}, \vec{r}'), \quad (15.6.3)$$

The mathematical structure of obtained equations are closely similar to that studied by [47,54,55] in somewhat different context. So, adopting their technique we introduce sum and difference coordinates, and Fourier transform with respect to the difference coordinates as follows:

$$K_{\Omega}(\vec{p}, \vec{R}) \equiv \int d^3r d^3s e^{-i\vec{p} \cdot (\vec{r} - \vec{s})} K_{\Omega}(\vec{r}, \vec{s})$$

(15.6.4)

with $\vec{R} = \frac{1}{2}(\vec{r} + \vec{s})$. We also involve similar expansions for all other functions. Then the eq.(15.6.2) and eq.(15.6.3) reduce to following:

$$\Theta \left[ K_{\Omega}(\vec{p}, \vec{R}) G_\Omega(\vec{p}', \vec{R}') \right] = 1,$$

$$\Theta \left[ F^+_\Omega(\vec{p}, \vec{R}) K_{-\Omega}(\vec{R}, \vec{R}') \right] = \Theta \left[ \Delta^*(\vec{R}) \tilde{\gamma}_A(\vec{p}', \vec{R}') \tilde{G}_{\Omega}(\vec{p}, \vec{R}') \right]$$

(15.6.5)

provided by the standard differential operator of finite order defined under the requirement that it produces the Fourier transform of the matrix product of two functions when it operates on the transforms of the individual functions [54,55]

$$\Theta \equiv \lim_{\vec{R} \to \vec{R}} \lim_{\vec{p} \to \vec{p}'} \exp \left[ \frac{i}{2} \left( \frac{\partial}{\partial \vec{R}} \frac{\partial}{\partial \vec{p}} - \frac{\partial}{\partial \vec{p}'} \frac{\partial}{\partial \vec{R}'} \right) \right].$$

(15.6.6)
One gets

\[ K_\Omega(\vec{p}, \vec{R}) = i\Omega - \epsilon(\vec{p}, \vec{R}) + \lim_{\vec{R}', \vec{R}'' \to \vec{R}} \exp \left[ i \frac{\partial}{\partial \vec{p}} \left( \frac{\partial}{\partial \vec{R}'} - \frac{\partial}{\partial \vec{R}''} \right) \right] \times \Delta^*(\vec{R}')\tilde{\gamma}_A(\vec{p}, \vec{R}') \Delta^*(\vec{R}''), \]  

where it is denoted \( \epsilon(\vec{p}, \vec{R}) \equiv \frac{1}{2m} \left( \vec{p} - \frac{e}{c} \vec{A}(\vec{R}) \right)^2 - \mu_0 \). To obtain resulting expressions we shall proceed with further calculations, but shall forbear to write them out as they are so standard. There is only one thing to be noticed about the integration. That is, due to the angular integration in momentum space, as mentioned above, the terms linear in the vector \( \vec{p} \) will be vanished, as well as the integration over the energies removes the linear terms in \( \epsilon(\vec{p}) \). So, we may expand the quantities in eq.(15.6.5) according to the degree of inhomogeneity somewhat like it we have done in equation (15.4.8) of gap function \( \Delta^*(\vec{r}) \), which in mixed representation transforms to the following:

\[ \Delta^*(\vec{R}) = T \sum_\omega \int \frac{d^3p}{(2\pi)^3} F^+_\omega(\vec{p}, \vec{R}) = T \sum_\omega \int \frac{d^3p}{(2\pi)^3} \tilde{\gamma}_A(\vec{p}, \vec{R}) F^+_\omega(\vec{p}, \vec{R}). \]  

The approximation was used to obtain the function \( F^+_\Omega \) must be of one order higher \( F^+_\Omega \simeq F^{(0)+}_\Omega + F^{(1)+}_\Omega + F^{(2)+}_\Omega \) than that for function \( \tilde{\gamma}_\Omega \simeq \tilde{\gamma}^{(0)}_\Omega + \tilde{\gamma}^{(1)}_\Omega \). Employing an iteration method of solution one replaces \( K \to \tilde{K} \), \( G \to \tilde{G} \) in eq.(15.6.5) and puts \( \Theta^{(0)} = 1 \), \( \tilde{K}^{(1)} = 0 \), \( \tilde{G}^{(1)} = 0 \). Hence \( \tilde{G} = \tilde{G}^{(0)} \).

The resulting equation for gap function is similar to those occurring in [47], although not identical. The sole difference is that in the resulting equation we use the expressions of \( \Omega \) and \( \xi \). With this replacement the equation reads

\[ \Delta^* = T \sum_\omega \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{\Delta^*}{\Omega^2 + \xi^2} + \left[ \frac{\partial}{\partial \vec{R}} + \frac{2ie}{c} \hat{A} \right]^2 \Delta^* + \Delta \left( \frac{\partial}{\partial \vec{R}} + \frac{2ie}{c} \hat{A} \right)^2 \frac{\partial}{\partial |\Delta|^2} \right\}, \]

where \( \xi(\vec{p}, \vec{R}) \equiv \left[ \epsilon^2(\vec{p}) + |\Delta(\vec{R})|^2 \right]^{1/2} \). The average value of the operator of current density at \( T \sim T_c \) follows at once

\[ \bar{j}_G(\vec{R}) = \left( \frac{\gamma_0 \gamma_3}{2} \right)_{\Sigma=0} \frac{2e}{m} \left\{ -\frac{i}{2} \left( \Delta^*(\vec{R}) \frac{\partial \Delta(\vec{R})}{\partial \vec{R}} - \frac{\partial \Delta^*(\vec{R})}{\partial \vec{R}} \Delta(\vec{R}) \right) - \frac{2e}{c} |\Delta(\vec{R})|^2 \hat{A}(\vec{R}) \right\} T \sum_\omega \int \frac{d^3p}{(2\pi)^3} \frac{p^2/6m^2}{\left( \Omega^2 + \xi^2(\vec{p}, \vec{R}) \right)^2}, \]  

where \( \left( \frac{\gamma_0 \gamma_3}{2} \right)_{\Sigma=0} = \frac{1}{8} \left( \gamma_0 + \frac{\mu}{m} \right) \left( \gamma_0 + \mu \right) \left( \gamma_0 + \mu \right)^2 + 3 \beta_F^2 \). At \( \Delta \ll \pi k_B T \) and \( \hat{A} \) is independent of position the eq.(15.6.9) and eq.(18.6.10) lead back to the equations (15.4.13) and (15.5.11). Actually, from such results it is then easy by ordinary manipulations to investigate the pertinent physical problem in several particular cases, but a separate calculation for each case would be needed.
The Lagrangian of Electroweak Interactions

The results obtained within the previous sections enable us to trace unambiguously rather general scheme of unified electroweak interactions. Below we remind some features allowing us to write down the final Lagrangian of electroweak interactions.

1. During the realization of multiworld connections of weak interacting fermions the P-violation compulsory occurred in W-world incorporated with the symmetry reduction eq.(13.1) characterized by the Weinberg mixing angle with the fixed value at 30°. This gives rise to the local symmetry $SU(2) \otimes U(1)$, under which the left-handed fermions transformed as six independent doublets, while the right-handed fermions transformed as twelve independent singlets.

2. Due to vacuum rearrangement in Q-world the Yukawa couplings arise between the fermion fields and corresponding isospinor-scalar $\varphi$-meson in conventional form.

3. In the framework of suggested mechanism providing the effective attraction between the relativistic fermions caused by the exchange of the mediating induced gauge quanta in W-world, the self-interacting isospinor-scalar Higgs bosons arise as Bose-condensate. Then we must add to the total Lagrangian a $SU(2)$ multiplet of spinless $\varphi$-meson fields coupled to the gauge fields in a gauge invariant way. We involve the Lagrangian of $\varphi$-meson with the degenerate vacuum of W-world, where the symmetry-breaking Higgs boson is counted off from the gap symmetry restoring value as the point of origin. In view of this a Lagrangian ensues from the eq.(11.5)- eq.(11.7), which is now invariant under local symmetry $SU(2) \otimes U(1)$, where a set of gauge fields are coupled to various multiplets of fields among which is also a multiplet of Higgs boson. Subsequently, we separate a part of Lagrangian containing only the fields defined on four dimensional Minkowski flat spacetime continuum $M^4$. The resulting Lagrangian reads

$$L = -\frac{1}{2} Tr G_{\mu\nu} G^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{L} D\bar{L} + i \bar{e}_R D e_R + i \bar{\nu}_R D\nu_R + |D_\mu \varphi|^2 - \frac{1}{2} \lambda^2_\varphi (|\varphi|^2 - \frac{1}{2} \eta^2_\varphi)^2 - f_e (L \varphi e_R + \bar{e}_R \varphi^+ L) - f_\nu (L \varphi \nu_R + \bar{\nu}_R \varphi^+ L) + \text{similar terms for other fermion generations.}$$

(16.1)

The $\varphi$ is taken to denote Higgs boson according to redefinition $\varphi(x) \equiv \varphi(x, x_w) = f_{\varphi}(x) \varphi(x_w)$, where the coordinates $x_w \in W$ are left implicit. The gauge fields $A_\mu(x)$ and $B_\mu(x)$ associate respectively with the groups $SU(2)$ and $U(1)$, where the gauge covariant curls are $F_{\mu\nu}, G_{\mu\nu}$. The corresponding gauge covariant derivatives are in standard form. One took into account corresponding values of the operators $T$ and $Y$ for left- and right-handed fields, and for isospinor $\varphi$-meson. The Yukawa coupling constants $f_e$ and $f_\nu$ are inserted in eq.(14.1), but in the case of quarks, according to eq.(2.2) and eq.(10.9), it respectively must be changed into $f_e \rightarrow f_q \equiv f_Q + m_\varphi^2$. In standard scenario a gauge invariance of the Lagrangian is broken when the $\varphi$-meson fields acquire a vacuum expectation value. Thereby the mass $m_\varphi$ and coupling constant $\lambda_\varphi$ are in the form eq.(15.4.15). The spontaneous breakdown of symmetry is vanished at $\eta^2_\varphi(\lambda, T > T_{c\mu}) < 0$. When this doublet obtains a vacuum expectation value, three of the gauge fields acquire masses. These fields will mediate the weak interactions. Consequently, a remaining massless gauge field will be identified as the photon field coupled to the electric current. The microscopic
structure of these fields reads
\[ W^+ \equiv \Psi^n (\eta) (q_1 q_2 q_3)^Q (q\bar{q})^w, \quad W^- \equiv \Psi^n (\eta) (\bar{q}_1 \bar{q}_2 q_3)^Q (\bar{q} q)^w, \]
\[ Z^0 \equiv \Psi^n (\eta) (q\bar{q})^Q (q\bar{q})^w, \quad A \equiv \Psi^n (\eta) (q\bar{q})^Q. \]

The expressions of the masses \(m_W\) and \(m_Z\) are changed if the Higgs sector is built up more compoundly. Due to Yukawa couplings the fermions acquire the masses after symmetry-breaking. The mass of electron reads \(m_e = \eta \phi \sqrt{2} f\) etc. One gets for the leptons \(f_e : f_\mu : f_\tau = m_e : m_\mu : m_\tau\). This mechanism does not disturb the renormalizability of the theory [56,57]. In approximation to lowest order \(f = \Sigma_Q \simeq m_Q \ll \lambda^{-1/2} \left( \lambda^{-1} = \frac{m_{p_0}}{2\pi^2 \ln \frac{2\omega}{\Delta_0}} \right)\), the Lagrangian eq.(16.1) leads to Lagrangian of phenomenological standard model. At \(f \sim 10^{-6}, \lambda \ll 10^{12}\). In this case the phenomenological standard model survives. The self-energy operator \(f = \Sigma_Q\) takes into account the mass-spectrum of expected various collective excitations of bound quasi-particle pairs produced by higher-order interactions, which arise as the poles of the function \(\Sigma_Q\) eq.(11.1).

17 Quark flavour Mixing and the Cabibbo Angle

An implication of quark generations into this scheme will be carried out in the same manner. Namely, under the group \(SU(2) \otimes U(1)\) the left-handed quarks transform as three doublets, while the right-handed quarks transform as independent singlets except of following differences:

1. The values of weak-hypercharge of quarks are changed is due to their fractional electric charges \(q_L : Y^w = \frac{1}{3}, \quad u_R : Y^w = \frac{4}{3}, \quad d_R : Y^w = -\frac{2}{3}\) etc.
2. All Yukawa coupling constants have nonzero values.
3. An appearance of quark mixing and Cabibbo angle, which is unknown in the scope of standard model.
4. An existence of CP-violating phase in unitary matrix of quark mixing. We shall discuss it in the next section.

Below, we attempt to give an explanation to quark mixing and Cabibbo angle. Here for simplicity, we consider the example of four quarks \(u, d, s, c\). The further implication of all quarks would complicate the problem only in algebraic sense. Let consider four left-handed quarks forming a \(SU(2)_L\) doublets mixed with Cabibbo angle \(\left( \begin{array}{c} u' \\ d \end{array} \right)_L\), and \(\left( \begin{array}{c} c' \\ s \end{array} \right)_L\), where \(u' = u \cos \theta + c \sin \theta, \quad c' = -u \sin \theta + c \cos \theta\). One must distinguish two kind of fermion states: an eigenstate of gauge interactions, i.e. the fields of \(u'\) and \(c'\); an eigenstate of mass-matrices, i.e the fields of \(u\) and \(c\). The qualitative properties of Cabibbo flavour mixing could be understood in terms of Yukawa couplings. Unlike the case of leptons, where the Yukawa couplings are characterized by two constants
Straightforward comparison of two states gives

\[
\frac{1}{\sqrt{2}} f_{u'}(\bar{u}'L u'_R + \bar{u}'_R u'_L)(\eta + \chi) = \frac{1}{\sqrt{2}} f_{c'}(\bar{c}'L c'_R + \bar{c}'_R c'_L)(\eta + \chi),
\]

\[
\frac{1}{\sqrt{2}} f_{u}(\bar{c}'L u'_R + \bar{c}'_R u'_L + \bar{u}'L c'_R + \bar{u}'_R c'_L)(\eta + \chi) = \frac{1}{\sqrt{2}} f_{c'}(\bar{c}'L u'_R + \bar{c}'_R u'_L)(\eta + \chi).
\]

Throughout a small part our discussion will be a standard [58], except one point: instead of mixing of fields \(d'\) and \(s'\) we consider a quite equivalent mixing of \(u'\) and \(c'\). The last expression may be diagonalized by means of rotation right through Cabibbo angle. In the sequel one gets \(m_u \bar{u}u + m_c \bar{c}c\), where \(m_u\) and \(m_c\) are masses of quarks \(u\) and \(c\).

Straightforward comparison of two states gives

\[
m_u = \frac{1}{\sqrt{2}} (f_u \cos^2 \theta + f_{c'} \sin^2 \theta - 2 f_{u'} \cos \theta \sin \theta) \eta, \]

\[
m_c = \frac{1}{\sqrt{2}} (f_u \sin^2 \theta + f_{c'} \cos^2 \theta + 2 f_{u'} \cos \theta \sin \theta) \eta, \]

\[
\tan 2\theta = \frac{2 f_{u'} \cos \theta \sin \theta}{f_c - f_u} \neq 0.
\]

Similar formulas can be worked out for the other mixing. Thus, the nonzero value of Cabibbo angle arises due to nonzero coupling constant \(f_{u'c'}\). Then the problem is to calculate all coupling constants \(f_{u'c'}, f_{c'c'},\) and \(f_{u'u'}\) generating three Cabibbo angles

\[
\tan 2\theta_2 = \frac{2 f_{u'} \cos \theta \sin \theta}{f_c - f_u}, \quad \tan 2\theta_3 = \frac{2 f_{c'} \cos \theta \sin \theta}{f_c - f_u}, \quad \tan 2\theta_1 = \frac{2 f_{u'} \cos \theta \sin \theta}{f_u - f_{u'}}.
\]

Taking into account the explicit form of Q-components of quark fields eq.(8.1)

\[
\Psi_{Q_u} = (q_1 q_2)^Q, \quad \Psi_{Q_{c'}} = (q_2 q_3)^Q, \quad \Psi_{Q_{c'}} = (q_3 q_1)^Q,
\]

also eq.(11.2) and eq.(14.1), we may write down

\[
f_{u'} \to \frac{1}{2} \left\{ \Psi_{u'_u} \hat{p}_u \Psi_{u'_u} - \left( \Psi_{u'_u} \hat{p}_u \right) \right\} \Psi_{u'_u}^\dagger = \left( \Sigma^1_Q + \Sigma^2_Q \right) \Psi_{u'_u} \Psi_{u'_u} \to \left( \Sigma^1_Q + \Sigma^2_Q \right), \quad (17.1)
\]

where for given \((i)\) one has \(\hat{p}_Q q_i^Q = \Sigma^i_Q q_i^Q\). In analogy, the \(f_{c'}\) and \(f_{u'c'}\) imply

\[
f_{c'} \to \frac{1}{2} \left\{ \Psi_{u'_u} \hat{p}_u \Psi_{u'_u} - \left( \Psi_{u'_c} \hat{p}_u \right) \right\} \Psi_{u'_u} \to \left( \Sigma^2_Q + \Sigma^3_Q + m_c \right), \quad (17.2)
\]

and

\[
f_{u'c'} \to \frac{1}{4} \left\{ \Psi_{Q'_w} \hat{p}_Q \Psi_{Q'_w} + \hat{p}_Q \Psi_{Q'_w} - \left( \Psi_{Q'_w} \hat{p}_Q \right) \Psi_{Q'_w} - \left( \Psi_{Q'_w} \hat{p}_Q \right) \Psi_{Q'_w} \right\} = \frac{1}{2} \left\{ \left( \Sigma^1_Q + \Sigma^2_Q \right) \Psi_{Q'_w} \Psi_{Q'_w} + \left( \Sigma^2_Q + \Sigma^3_Q \right) \Psi_{Q'_w} \Psi_{Q'_w} \right\}. \quad (17.3)
\]
In accordance with eq.(3.4), one has

\[ \Psi_{Q'Q''} \Psi_{Q','Q''} = (q_1 q_2)^Q (q_2 q_3)^Q = f_{Q'Q''} (q_2 q_2)^Q, \]

\[ \Psi_{Q'Q''} \Psi_{Q,Q'} = (q_2 q_3)^Q (q_1 q_2)^Q = f_{Q'Q''} (q_2 q_2)^Q, \]

where

\[ f_{Q'Q''} (q_2 q_2)^Q = \left( f_{Q'Q''} (q_2 q_2)^Q \right)^* = f_{Q'Q''} (q_2 q_2)^Q \equiv f_2. \]

The partial formfactors \( f \equiv f_{il}^{AB} \) imply the functional equation of renormalization group eq.(3.6). Hence \( f_{u',c'} = \frac{f_3}{2} (\Sigma_2^1 + \Sigma_2^3 + 2\Sigma_2^Q) \) and

\[ \tan 2\theta_2 = \frac{\tilde{f}_2 (\Sigma_2^1 + \Sigma_2^3 + 2\Sigma_2^Q)}{\Sigma_2^Q - \Sigma_2^1 + m_c}, \quad \tan 2\theta_3 = \frac{\tilde{f}_3 (\Sigma_2^3 + \Sigma_2^1 + 2\Sigma_2^Q)}{\Sigma_2^Q - \Sigma_2^2 + m_c - m_u}, \]

\[ \tan 2\theta_1 = \frac{\tilde{f}_1 (\Sigma_2^1 + \Sigma_2^3 + 2\Sigma_2^Q)}{\Sigma_2^Q - \Sigma_2^3 - m_u}, \]

where a rest of \( \tilde{f}_i \) reads \( \tilde{f}_3 = f_{Q'Q''}^{u,c'} = f_{Q'Q''}^{t,t'} \) and \( \tilde{f}_1 = f_{Q'Q''}^{t',t'} = f_{Q'Q''}^{u,c'} \). So, the Q-components of the quark fields \( u', c' \) and \( t' \) contain at least one identical subquark, due to which in eq.(3.4) the partial formfactors \( \tilde{f}_i \) have nonzero values causing a quark mixing with the Cabibbo angles eq.(17.4). Therefore, the unimodular orthogonal group of global rotations arises, and the quarks \( u', c' \) and \( t' \) come up in doublets \( (u',c'),(c',t'),(t',u') \). For the leptons these formfactors equal zero \( f_{lep}^o \equiv 0 \), because of eq.(7.1), namely the lepton mixing is absent. In conventional notation \( (u',d')_L, (c',s')_L, (t',b')_L \rightarrow (u,d')_L, (c,s')_L, (t,b')_L \), which gives rise to \( f_{u',c'} \rightarrow f_{d',s'}, f_{c',t'} \rightarrow f_{s',u'}, f_{t',u'} \rightarrow f_{b',d'}, f_{u'} \rightarrow f_{d'}, f_{c'} \rightarrow f_{s'}, f_{t'} \rightarrow f_{b'} \) and \( f_d \rightarrow f_u, f_s \rightarrow f_c, f_b \rightarrow f_t \).

18 The Appearance of the CP-Violating Phase

The required magnitude of the CP-(or time reversal T-related to CP by CPT theorem [63]) violating complex parameter \( \varepsilon \) [64] depends upon the specific choice of theoretical model for explaining the \( K^0 \rightarrow 2\pi \) decay. From the experimental data it is somewhere \( |\varepsilon| \simeq 2.3 \times 10^{-3} \). In the framework of Kobayashi-Maskawa (KM) parametrization of unitary matrix of quark mixing [65], this parameter may be expressed in terms of three Eulerian angles of global rotations in the three dimensional quark space and one phase parameter. Below we attempt to derive the KM-matrix with an explanation given to an appearance of the CP-violating phase. We recall that during the realization of multiworld structure the P-violation compulsory occurred in the W-world provided by the spanning eq.(12.1). The three dimensional effective space \( W_{v}^{loc}(3) \) arises as follows:

\[ W_{v}^{loc}(3) \ni q_v^{(3)} = \begin{pmatrix} \begin{pmatrix} q_R^w(T = 0) \\ q_L^w(T = \frac{1}{2}) \end{pmatrix} \end{pmatrix} \equiv \begin{pmatrix} (u_R,d_R) \\ (c_R,s_R) \\ (t_R,b_R) \end{pmatrix} = \begin{pmatrix} q_3^w \\ q_1^w \\ q_2^w \end{pmatrix}, \]

\[ = \begin{pmatrix} (u')_L \\ (c')_L \\ (t')_L \end{pmatrix} = \begin{pmatrix} (u')_L \\ (c')_L \\ (t')_L \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} q_3^w \\ q_1^w \\ q_2^w \end{pmatrix} \end{pmatrix}, \]

\[ (18.1) \]
where the subscript \((v)\) formally specifies a vertical direction of multiplet, the subquarks \(q^v_\alpha\) \((\alpha = 1, 2, 3)\) associate with the local rotations around corresponding axes of three dimensional effective space \(W^v_{loc}(3)\). The local gauge transformations \(f^v_{exp}\) are implemented upon the multiplet \(q^v_\alpha = f^v_{exp} q^v_{\alpha}\), where \(f^v_{exp} \in SU^v_{loc}(2) \otimes U^v_{loc}(1)\). If for the moment we leave it intact and make a closer examination of the composition of middle row in eq.\((18.1)\), then we distinguish the other symmetry arisen along the horizontal line \((h)\). The situation is exactly similar to that discussed in sec.12: due to the specific structure of \(W\)-world implying the condition of realization of multiworld connections eq.\((5.5)\) with \(\bar{T} \neq 0, \ Y^w \neq 0\), the subquarks \(q^w_\alpha\) tend to be compulsory involved into triplet. They form one “doublet” \(\bar{T} \neq 0\) and one singlet \(Y^w \neq 0\). Then the quarks \(u', c', t'\) form a \(SO^v(2)\) “doublet” and a \(U^v(1)\) singlet

\[
\begin{align*}
((u'^v, c'^v, t'^v) & \equiv ((q^w_1, q^w_2, q^w_3) \equiv q^h_3) \in W^h(3), \\
(u'^v, (c'^v, t'^v)) & \equiv ((q^w_1, q^w_2), t'^v) \equiv ((q^w_3, q^w_1, q^w_2)).
\end{align*}
\tag{18.2}
\]

Here \(W^h(3)\) is the three dimensional effective space in which the global rotations occur. They are implemented upon the triplets through the transformation matrix \(f^h_{exp} q^h_3\), which reads (eq.\((18.2)\))

\[
f^h_{exp} = \begin{pmatrix}
f_{33} & 0 & 0 \\
0 & c & s \\
0 & -s & c
\end{pmatrix}
\]

in the notation \(c = \cos \theta, \ s = \sin \theta\). This implies the incompatibility relation eq.\((3.5.5)\) [1], namely

\[
\|f^h_{exp}\| = f_{33}(f_{11}f_{22} - f_{12}f_{21}) = f_{33} \varepsilon_{123} \varepsilon_{123} \|f^h_{exp}\|f^h_{33}.
\tag{18.3}
\]

That is \(f_{33} f^h_{33} = 1\), or \(f_{33} = e^{i\delta}\) and \(\|f^h_{exp}\| = 1\). The general rotation in \(W^h(3)\) is described by Eulerian three angles \(\theta_1, \theta_2, \theta_3\). If we put the arisen phase only in the physical sector then a final KM-matrix of quark flavour mixing would result

\[
(u_L, c_L, t_L) V_{K-M} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \equiv (u'_L, c'_L, t'_L) \begin{pmatrix} d \\ s \\ b \end{pmatrix} \equiv (u_L, c_L, t_L) \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} =
\]

\[
(u_L, c_L, t_L) \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix},
\tag{18.4}
\]

where

\[
(u'_L, c'_L, t'_L) \equiv (u_L, c_L, t_L) V_{K-M}, \quad \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} \equiv V_{K-M} \begin{pmatrix} d \\ s \\ b \end{pmatrix}.
\]

The CP-violating parameter \(\varepsilon\) approximately is written [58, 62] \(\varepsilon \sim s_1 s_2 s_3 \sin \delta \neq 0\). While the spanning \(W^v_{loc}(2) \rightarrow W^v_{loc}(3)\) eq.\((18.1)\) underlies the P-violation and the expanded symmetry \(C^v_{loc}(3) = SU^v_{loc}(2) \otimes U^v_{loc}(1)\), the CP-violation stems from the similar spanning \(W^h(2) \rightarrow W^h(3)\) eq.\((18.2)\) with the expanded global symmetry group.
19 The Mass-Spectrum of Leptons and Quarks

The mass-spectrum of leptons and quarks stems from their internal multiworld structure eq.(7.1) and eq.(8.1) incorporated with the quark mixing eq.(17.4). We start a discussion with the leptons. It might be worthwhile to adopt a simple viewpoint on Higgs sector eq.(7.1) and eq.(8.1) incorporated with the quark mixing eq.(17.4). We start a discussion

whence the explicit expressions of the masses of leptons, in accordance with eq.(14.1), read

\[ m_i = \frac{\eta^i}{\sqrt{2}} f_i \equiv \frac{\eta^i}{\sqrt{2}} \Sigma Q^i, \]

where \( \Sigma Q^i : \Sigma^1_Q : \Sigma^2_Q : m_e : m_\mu : m_\tau, \) and

\[ m_{\nu_i} = \frac{\eta^i}{\sqrt{2}} f_{\nu_i} \equiv \frac{\eta^i}{\sqrt{2}} c_\nu \Sigma Q^i. \]

In the case of quarks, we admit in analogy that \( \eta^q \equiv \eta_u = \eta_d = \eta_s = \eta_c = \eta_b = \eta_t. \) Taking into account the eq.(9.1) and eq.(2.2), one may write down the expressions of the primary mass parameters of states \( q_i^Q(i = s, c, b, t) \)

\[ m_q^Q = m_{(q_i^Q)}^Q, \]

where in view of eq.(17.4) the Cabibbo angles imply

\[ \tan 2\theta_2 = \frac{\bar{f}_2 (m_e + m_\tau + 2m_\mu)}{m_\tau - m_e + \bar{m}_e}, \quad \tan 2\theta_3 = \frac{\bar{f}_3 (m_\mu + m_e + 2m_\tau)}{m_e - m_\mu + \bar{m}_t - m_c}, \]

and

\[ \tan 2\theta_1 = \frac{\bar{f}_1 (m_\tau + m_\mu + 2m_e)}{m_\mu - m_\tau - \bar{m}_t}. \]

(19.1)
This is legitimate for $\bar{m}_i = \frac{\eta'}{\sqrt{2}} m_i^c$, $i = s, c, b, t$. By making use of eq.(19.1), some useful relations between the masses of leptons and quarks are written

$$m_d = \frac{\eta'}{\eta} m_\tau, \quad m_s = \frac{m_d}{m_\tau} (m_e + \bar{m}_s), \quad m_b = \frac{m_d}{m_\tau} (m_\mu + \bar{m}_b),$$

$$m_u = \frac{m_d}{m_\tau} \left\{ (m_e + m_\mu) \cos^2 \theta_2 + (m_\mu + m_\tau + \bar{m}_e) \sin^2 \theta_2 - \frac{\bar{f}_2}{2} (m_e + m_\tau + \bar{m}_e) \right\},$$

$$2m_\mu \sin 2\theta_2 = \frac{m_d}{m_\tau} \left\{ (m_e + m_\mu) \cos^2 \theta_1 + (m_\tau + m_e + \bar{m}_\tau) \sin^2 \theta_1 + \frac{\bar{f}_1}{2} \right\}$$

$$m_c = \frac{m_d}{m_\tau} \left\{ (m_\mu + m_\tau + \bar{m}_e) \cos^2 \theta_2 + (m_e + m_\mu) \sin^2 \theta_2 + \frac{\bar{f}_2}{2} (m_e + m_\tau + \bar{m}_e) \right\},$$

$$2m_\mu \sin 2\theta_2 = \frac{m_d}{m_\tau} \left\{ (m_\mu + m_\tau + \bar{m}_e) \cos^2 \theta_3 + (m_\tau + m_e + \bar{m}_\tau) \sin^2 \theta_3 + \frac{\bar{f}_3}{2} \right\}$$

$$m_t = \frac{m_d}{m_\tau} \left\{ (m_\tau + m_e + \bar{m}_\tau) \cos^2 \theta_3 + (m_\mu + m_\tau + \bar{m}_e) \sin^2 \theta_3 + \frac{\bar{f}_3}{2} \right\}$$

$$2m_\tau \sin 2\theta_3 = \frac{m_d}{m_\tau} \left\{ (m_\tau + m_e + \bar{m}_\tau) \cos^2 \theta_1 + (m_e + m_\mu) \sin^2 \theta_1 - \frac{\bar{f}_1}{2} \right\}$$

with $\bar{f}_i \equiv \bar{f}(\tau_i, y, f_i)$ ($i = 1, 2, 3$). From the known values of the $\eta'$, angles $\theta_i$, and masses of quarks one may obtain the values of 12 quantities $\tau_i, y, f_i, \eta', m_s, \bar{m}_b, \bar{m}_t$, while the explicit form of the functions $\bar{f}(\tau_i, y, f_i)$ can be inferred from the equation of renormalization group. For instance, in approximation to the lowest order (sec.16) this equation reduced to Gell-Mann-Low’s equation, which in the scope of perturbation theory

$$\bar{f}(\tau, f) = f d(\tau, f) = \frac{\bar{f}_2 \tau}{3\pi} + \frac{\bar{f}_3}{9\pi^2} \left( \tau^2 + \frac{9}{4} \tau \right) + \cdots$$

This gives the leading terms as follows:

$$d(\tau, f)^{-1} = 1 - \frac{\tau}{3\pi} + \frac{3f}{4\pi} \ln \left( 1 - \frac{\tau}{3\pi} \right) + \cdots.$$ [17]

### 20 Concluding Remarks

Continuing the program developed in a previous paper [1] we attempt to suggest a microscopic approach to the properties of particles and interactions. Within this approach the fields have composite nontrivial internal structure. The condition of realization of multiworld connections is arisen due to the symmetry of Q-world of electric charge and embodied in the Gell-Mann-Nishijima relation. During a realization of multiworld structure the symmetries of corresponding internal worlds are unified into more higher symmetry including also the operators of isospin and hypercharge. Such approach enables to conclude that possible three lepton generations consist of six lepton fields with integer electric and leptonic charges and being free of confinement condition. Also three quark generations exist composed of six possible quark fields. They carry fractional electric and baryonic charges and obey confinement condition. The global group unifying all global symmetries of the internal worlds of quarks is the flavour group $SU_f(6)$. The whole complexity of leptons, quarks and other composite particles, and their interactions
arises from the primary field, which has nontrivial multiworld internal structure and involves nonlinear fermion self-interaction of the components. This Lagrangian contains only two free parameters, which are the coupling constants of nonlinear fermion and gauge interactions. Due to specific structure of W-world of weak interactions implying the condition of realization of multiworld connections, the spanning eq.(12.1) takes place, which underlies the P-violation in W-world. It is expressed in the reduction of initial symmetry of the right-handed subquarks. Such reduction is characterized by the Weinberg mixing angle with the value fixed at 30°. It gives rise to the expanded local symmetry $SU(2) \otimes U(1)$, under which the left-handed fermions transform as six independent $SU(2)$ doublets, while the right-handed fermions transform as twelve independent singlets. Due to vacuum rearrangement in Q-world the Yukawa couplings arise between the fermion fields and corresponding isospinor-scalar $\varphi$-meson in conventional form. We suggest the microscopic approach to Higgs bosons with self-interaction and Yukawa couplings. It involves the Higgs bosons as the collective excitations of bound quasi-particle iso-pairs. In the framework of local gauge invariance of the theory incorporated with the P-violation in weak interactions we propose a mechanism providing the Bose-condensation of iso-pairs, which is due to effective attraction between the relativistic fermions caused by the exchange of the mediating induced gauge quanta in W-world. The relativism qualitatively affects the value of mass of Higgs boson, coupling constant and transition temperature. The latter decreases inversely by relativistic factor. We consider the four-component Bose-condensate, where due to self-interaction its spin part is vanished. Based on it we show that the field of symmetry-breaking Higgs boson always must be counted off from the gap symmetry restoring value as the point of origin. Then the Higgs boson describes the excitations in the neighbourhood of stable vacuum of the W-world. The resulting Lagrangian of unified electroweak interactions of leptons and quarks ensues, which in lowest order approximation leads to the Lagrangian of phenomenological standard model. In general, the self-energy operator underlies the Yukawa coupling constant, which takes into account a mass-spectrum of all expected collective excitations of bound quasi-particle pairs arising as the poles. The implication of quarks into this scheme is carried out in the same manner except that of appearance of quark mixing with Cabibbo angle and the existence of CP-violating complex phase in unitary matrix of quark mixing. The Q-components of the quarks $u', c'$ and $t'$ contain at least one identical subquark, due to which the partial formfactors gain nonzero values. This underlies the quark mixing with Cabibbo angles. In lepton’s case these formfactors are vanished and lepton mixing is absent. The CP-violation stems from the spanning eq.(18.2) incorporated with the expanded group of global rotations. With a simple viewpoint on Higgs sector the masses of leptons and quarks are given in sec.19, which lead to some useful relations between the masses of leptons and quarks. From these relations one may define the values of 12 quantities $\tau_i, y, f_i, \eta_i, \tilde{m}_s, \tilde{m}_c, \tilde{m}_t$, where the explicit form of the functions $\tilde{f}(\tau_i, y, f_i)$ can be inferred from the equation of renormalization group. Although some key problems of particle physics are elucidated within outlined approach, nevertheless the numerous issues still remain to be solved. We hope that this approach will be an attractive basis for the future theory.

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