Drell-Yan double-spin asymmetry $A_{LT}$ in polarized $p\bar{p}$ collisions: Wandzura-Wilczek contribution

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Abstract

The longitudinal-transverse spin asymmetry $A_{LT}$ in the polarized Drell-Yan process depends on the twist-3 spin-dependent distributions of nucleon. In addition to the contributions expressed as matrix element of the twist-3 operators, these distributions contain the so-called Wandzura-Wilczek part, which is completely determined by a certain integral of the twist-2 spin-dependent parton distributions. We demonstrate that the recently obtained empirical information on the transversity distribution allows a realistic estimate of the Wandzura-Wilczek contribution to $A_{LT}$ for the case of polarized proton-antiproton collisions. In particular, our results in the Wandzura-Wilczek approximation indicate that rather large $A_{LT}$ can be observed in the proposed spin experiments at GSI, and its behavior as a function of dilepton mass obeys novel pattern, compared with the other double-spin asymmetries $A_{TT}$ and $A_{LL}$. Our results provide a guide for testing a signal of effects originating from the twist-3 operators associated with quark-gluon correlation.
The proposed polarization experiments with antiprotons at GSI [1] stimulate renewed interest in the polarized Drell-Yan processes to access the chiral-odd spin-dependent parton distributions of the nucleon. The double transverse-spin asymmetries $A_{TT}$ for lepton pair production in collisions of transversely polarized protons and antiprotons, $p^1 \bar{p}^1 \rightarrow l^+l^-X$, are estimated, and are found to be large enough to be measured at GSI [2], providing promising way to probe the chiral-odd twist-2 spin-dependent parton distribution, the transversity $h_1(x)$ [3, 4, 5, 6, 7]. In particular, the $p\bar{p}$ collisions at moderate energy in GSI experiments allow us to probe the relevant parton distributions in the “valence region”, in contrast to the complementary case of $pp$ collisions at, e.g., RHIC where the “sea-quark region” is mainly probed. The QCD corrections to $A_{TT}$ at GSI have been studied recently at next-to-leading order (NLO) [8] and at higher orders with the “threshold resummation” [9]. The resummation corrections relevant when the transverse-momentum of the produced lepton pair is small [10, 11] are also investigated [12]. It has been found that the behavior of these QCD corrections associated with the valence region is rather different from the corresponding effects involving the sea quarks for the $pp$-collision cases [13, 14, 11]. As a result, these QCD corrections are small at the kinematical regions corresponding to the GSI experiments, suggesting that the large LO $A_{TT}$ at GSI is rather robust. This fact also allows us to estimate the value of $A_{TT}$ at GSI using only the empirical information on the transversity distributions [12], which is recently extracted [15] through the LO global fit to the semi-inclusive deep inelastic scattering (SIDIS) data, in combination with the $e^+e^-$ data for the associated (Collins) fragmentation function.

The PAX Collaboration has proposed the Drell-Yan experiments in $p\bar{p}$ collisions at the CM energy $\sqrt{s}$ with $s = 30$ and 45 GeV$^2$ in the fixed-target mode, and those up to $s = 210$ GeV$^2$ in the collider mode [1]. Those GSI-PAX experiments will measure $A_{TT}$ for $0.2 \lesssim Q/\sqrt{s} \lesssim 0.7$ with $Q$ the mass of the produced dilepton, and indeed probe the transversity $h_1(x)$ in the valence region in a wide range of $x$. It should not be overlooked that the double-spin longitudinal-transverse asymmetry $A_{LT}$ is also readily accessible in those Drell-Yan experiments, in particular, in the fixed-target mode with the longitudinal polarization of the target: $A_{LT}$ plays a distinguished role in spin physics because it allows us to access the twist-3 spin-dependent parton distributions as leading effects [4], similarly as the longitudinal-transverse asymmetry associated with the structure function $g_2$ in the polarized DIS [16]. Thus the data of $A_{LT}$ will provide an experimental test whether the quark-gluon-quark correlations inside the nucleon is sizeable or not, in particular, in the chiral-odd spin structure that is not accessible by $g_2$ in DIS. These facts call for theoretical study of $A_{LT}$ to assess its potential at GSI experiments, which is the purpose of this Letter. Up to now $A_{LT}$ was estimated for the $pp$-collision cases [18], but the above mentioned situation for $A_{TT}$ suggests that the behavior of $A_{LT}$ will be different between the $pp$ and $p\bar{p}$ collisions. Also it is important to clarify the impact of the new empirical information [15, 19] of the transversity distribution $h_1(x)$ on the prediction of $A_{LT}$ at GSI, because $A_{LT}$ depends on $h_1(x)$. We will demonstrate that this empirical information for $h_1(x)$ indeed allows a useful estimate for $A_{LT}$ in $p\bar{p}$ collisions at GSI kinematics.

*For a test of chiral-odd quark-gluon-quark correlation using the SIDIS data, see [17].
We shall work at LO QCD, which provides a sufficient accuracy for our first estimate of $A_{LT}$ at GSI. We may anticipate that the mechanism associated with the valence region relevant to GSI kinematics could make the QCD corrections to $A_{LT}$ small, similarly to the case for $A_{TT}$ mentioned above. To calculate $A_{LT}$, we first recall the parton distributions of the nucleon. At LO, we need the spin-dependent quark distribution functions of twist-3 as well as of twist-2, which are defined as the nucleon matrix element of the chiral-odd and chiral-even quark bilocal operator with the light-like separation between the constituent fields [14, 20, 22].

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0)\sigma_{\mu\nu}i\gamma_5\psi(\lambda n)|PS\rangle = 2 \left[ h_1(x, \mu^2) (S_{\perp\mu} P_{\nu} - S_{\perp\nu} P_{\mu}) / M + h_L(x, \mu^2) M (P_{\mu} n_{\nu} - P_{\nu} n_{\mu}) (S \cdot n) \right], \quad (1)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0)\gamma_\mu\gamma_5\psi(\lambda n)|PS\rangle = 2 \left[ g_1(x, \mu^2) P_\mu (S \cdot n) + g_T(x, \mu^2) S_{\perp\mu} \right], \quad (2)$$

and we also need the unpolarized quark distribution defined as usually as

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0)\gamma_\mu\psi(\lambda n)|PS\rangle = 2 f_1(x, \mu^2) P_\mu, \quad (3)$$

where $|PS\rangle$ denotes the nucleon state with mass $M$, the four momentum $P_\mu = (P^+, P^-, 0_\perp)$, and the spin vector $S_\mu$ satisfying $P^2 = 2P^+ P^- = M^2$, $S^2 = -M^2$, and $P \cdot S = 0$, and a light-like vector $n_\mu = (0, n^-, 0_\perp)$ is introduced by the relation $P \cdot n = 1$. $S_\mu$ is decomposed as $S_\mu = (S \cdot n) P^\mu - M^2 (S \cdot n) n_\mu + S_{\perp\mu}$ with $P \cdot S_{\perp} = n \cdot S_{\perp} = 0$. In [14, 23], the gauge link operators which ensure gauge invariance are suppressed for simplicity. The distribution functions $h_{1,L}$, $g_{1,T}$ and $f_1$ depend on the factorization scale $\mu$, at which the bilocal operators in the LHS are renormalized, and those distribution functions are defined for each quark and anti-quark flavor $\psi = \psi^a (a = u, \bar{u}, d, \bar{d}, s, \bar{s}, \ldots)$ as $h^a_{1,L}$, etc. We remind that in the infinite momentum frame ($P^+ \to \infty$) the Lorentz structures associated with $h_1$, $g_1$ and $f_1$ are of $O(P^+)$ (twist-2), those for $h_L$ and $g_T$ are of $O(1)$ (twist-3), and those behaving as twist-4 ($O(1/P^+)$) are ignored in the RHS of (1)-(3). $g_1^u(x, \mu^2)$ is the familiar helicity distribution for $u$-quark carrying the momentum component $k^+ = x P^+$ inside the longitudinally polarized nucleon, and, similarly, $h_1^u(x, \mu^2)$ is the transversity distribution inside the transversely polarized nucleon [3, 4]. Note that the twist-3 distributions $h_L$ and $g_T$ are also associated with the longitudinal and transverse polarization of the nucleon, respectively.

The above mentioned classification of twist based on the power counting in the infinite momentum frame is directly related to the power of $1/Q$ with which the corresponding distributions appear in the physical cross sections, but does not exactly match the conventional and formal definition of twist as “dimension minus spin” associated with the relevant operator structure in [14 and 22]. As a result, the distributions $h_L$ and $g_T$ actually contain the piece that is expressed by matrix element of the twist-2 operators as [20, 14, 22] (see also...
Appendix in \[21\]

\[
h_L^a(x, \mu^2) = 2x \int_x^1 dy \frac{h_1^a(y, \mu^2)}{y} + \cdots, \\
g_T^a(x, \mu^2) = \int_x^1 dy \frac{g_1^a(y, \mu^2)}{y} + \cdots,
\]

where the ellipses stand for “genuine twist-3” contributions given as matrix element of the twist-3 operators; it is known that those twist-3 operators can be reexpressed as quark-gluon-quark three-body correlation operators on the lightcone, using the QCD equations of motion \([1, 22, 23, 24]\). In the following we call the twist-2 component, shown explicitly in (4) and (5), the Wandzura-Wilczek part. Because the operators with different geometric twist do not mix with each other under renormalization, the Wandzura-Wilczek part does not mix with the genuine twist-3 contributions under the QCD evolution with \(\mu^2\). Thus both \(x\)- and \(\mu^2\)-dependences of the Wandzura-Wilczek part are determined solely by those of the twist-2 distribution functions as \([1, 22, 23, 24]\). Taking into account only the Wandzura-Wilczek part in (4) and (5) yields the “Wandzura-Wilczek approximation” for \(h_L\) and \(g_T\).

With the above definitions for the parton distributions, we can write down the LO expression for the longitudinal-transverse spin asymmetry \(A_{LT}\) in the polarized \(p \bar{p}\) collisions. Before doing this, it is worthwhile to remind the LO formula of the other double-spin asymmetries \(A_{LL}\) and \(A_{TT}\) \([3, 4, 6]\). Using the quark distributions inside the proton,

\[
A_{LL} = \frac{\frac{\partial\sigma}{\partial Q^2} \frac{\partial\sigma}{\partial \Omega}}{\frac{\partial\sigma}{\partial Q^2} \frac{\partial\sigma}{\partial \Omega}} = \hat{a}_{LL} \frac{\sum_a e_a^2 g_1^a(x_1, Q^2) g_1^a(x_2, Q^2)}{\sum_a e_a^2 f_1^a(x_1, Q^2) f_1^a(x_2, Q^2)},
\]

\[
A_{TT} = \frac{\frac{\partial\sigma}{\partial Q^2} \frac{\partial\sigma}{\partial \Omega}}{\frac{\partial\sigma}{\partial Q^2} \frac{\partial\sigma}{\partial \Omega}} = \hat{a}_{TT} \frac{\sum_a e_a^2 h_1^a(x_1, Q^2) h_1^a(x_2, Q^2)}{\sum_a e_a^2 f_1^a(x_1, Q^2) f_1^a(x_2, Q^2)},
\]

for the production of the dilepton with the invariant mass \(Q\) and the longitudinal momentum component \(Q_z\) corresponding to the Feynman \(x_F\), where one of the leptons outgoes to the direction with the angle \(\Omega = (\theta, \phi)\). \(e_a\) represents the electric charge of the quark-flavor \(a\) and the summation is over all quark and anti-quark flavors, \(a = u, \bar{u}, d, \bar{d}, s, \bar{s}, \ldots\).

The scaling variables \(x_{1,2}\) represent the momentum fractions associated with the partons annihilating via the Drell-Yan mechanism, such that \(Q^2 = (x_1 P_1 + x_2 P_2)^2 = x_1 x_2 s\) and \(x_F = x_1 - x_2 (= 2Q_z/\sqrt{s} in the CM frame)\), where \(s = (P_1 + P_2)^2\) is the CM energy squared of the colliding proton and antiproton. This implies

\[
x_1 = \frac{1}{2} \left( x_F + \sqrt{x_F^2 + \frac{4Q^2}{s}} \right), \quad x_2 = \frac{1}{2} \left( -x_F + \sqrt{x_F^2 + \frac{4Q^2}{s}} \right).
\]

In (6) and (7), \(\hat{a}_{LL}\) and \(\hat{a}_{TT}\) represent the asymmetries in the parton level defined as

\[
\hat{a}_{LL} = 1, \quad \hat{a}_{TT} = \frac{\sin^2 \theta \cos 2\phi}{1 + \cos^2 \theta},
\]

Appendix in \[21\]
with the polar and azimuthal angles $\theta$ and $\phi$ in the dilepton rest frame with respect to the incoming beam and transverse-spin axes, respectively. The LO formula for $A_{LT}$ in $p^-\bar{p}^1 \to l^+l^-X$ or $p^1\bar{p}^- \to l^+l^-X$ can be expressed similarly as [4, 6]

$$A_{LT} = \frac{d\sigma}{dQ^2 dx dp d\Omega} = \hat{a}_{LT} \sum_a e_a^2 \left[ g_1^u(x_1, Q^2) x_2 g_2^d(x_2, Q^2) + x_1 h_L^u(x_1, Q^2) h_L^d(x_2, Q^2) \right] \sum_a e_a^2 f_1^a(x_1, Q^2) f_1^a(x_2, Q^2),$$

(10)

associating the variables $x_1$ and $x_2$ with the longitudinally and transversely polarized beams, respectively, with

$$\hat{a}_{LT} = \frac{M 2 \sin 2\theta \cos \phi}{Q \left( 1 + \cos^2 \theta \right)},$$

(11)

We note that $A_{LL}$ and $A_{TT}$ receive contribution only from the twist-2 distributions, while $A_{LT}$ is proportional to the twist-3 distributions and hence $\hat{a}_{LT}$ is suppressed by a factor $1/Q$ compared with (9).

To compute the above formulae (6), (7), and (10) with the GSI kinematics, we have to specify the LO parton distributions to be substituted. We use the LO GRV98 [25] and GRSV2000 (“standard scenario”) [26] distributions for the unpolarized and longitudinally-polarized quark distributions $f_1^a(x, Q^2)$ and $g_1^a(x, Q^2)$, respectively. For the LO transversity distribution $h_1^a(x, Q^2)$, we are guided by the recent information from the LO global fit [15, 19]: we find that a useful estimate can be obtained by assuming the relation

$$h_1^u(x, \mu^2) = g_1^u(x, \mu^2),$$

(12)

at a low scale $\mu$ ($\mu^2 = 0.26$ GeV$^2$ using the GRSV2000 $g_1^u(x, \mu^2)$); its QCD evolution from $\mu^2$ to $Q^2$ is controlled by the LO DGLAP kernel [27] for the transversity. It is worth noting that the above relation (12) at the low $\mu^2$, which is exact in the non-relativistic limit, is suggested also by the estimates from relativistic quark models for nucleon [3, 2, 28], matches the results by lattice QCD simulation [29, 30], and has been used in the previous estimates for $A_{TT}$ at GSI [2, 8, 12]. The obtained LO transversity distributions for $u$ and $d$ quarks, $xh_1^u(x, Q^2)$ and $xh_1^d(x, Q^2)$, are shown in Fig. 1(a) as a function of $x$ at $Q^2 = 2.4$ GeV$^2$. $xg_1^{1,d}(x, Q^2)$ is also shown in the same figure. For convenience, we have multiplied the factor $-1$ to the $d$-quark distributions. If we compare Fig. 1(a) with the results of the LO global fit [15, 19], we see that, for the valence region $0.2 \lesssim x \lesssim 0.7$ relevant for the GSI kinematics, our LO transversities lie slightly outside the error band of the fit, similarly as observed for the NLO case [12]. Therefore, our transversities will provide a realistic estimate of the upper bound of the relevant asymmetries, implied by the present empirical uncertainty in the transversities. (At present there are no data to constrain the transversity $h_1^q(x, Q^2)$ directly for $x > 0.4$, and in this region the uncertainty bands resulting from the LO global fit [15, 19] could be subject to the particular choice of the parameterization of $h_1^q(x, Q^2)$ assumed in the fitting procedure.) We see from Fig. 1(a) that $(h_1^u(x, Q^2))^2 \gg (h_1^d(x, Q^2))^2$ and $(g_1^u(x, Q^2))^2 \gg (g_1^d(x, Q^2))^2$ in the valence region, and likewise [25] for $f_1^a(x, Q^2)$. This also holds for higher $Q^2$, so that

$$\frac{A_{TT}}{\hat{a}_{TT}} \simeq \frac{h_1^u(x_1, Q^2) h_1^u(x_2, Q^2)}{f_1^u(x_1, Q^2) f_1^u(x_2, Q^2)},$$

(13)
Figure 1: (a) The transversity distributions \( x h_u^a(x, Q^2) \) and the helicity distributions \( x g_d^a(x, Q^2) \) at the scale \( Q^2 = 2.4 \text{ GeV}^2 \) for \( u \)- and \( d \)-quarks. (b) The twist-3 distributions \( x h_L^a(x, Q^2) \) and \( x g_T^a(x, Q^2) \) in the Wandzura-Wilczek approximation for \( u \)- and \( d \)-quarks. For convenience, we multiplied –1 for \( d \)-quark distributions in both figures.

...
Figure 2: (a) $\tilde{A}_{LT}$ and (b) $\tilde{A}_{LL}$ and $\tilde{A}_{TT}$ as a function of $x_F$ for $Q = 2.5$ and 4 GeV at $s = 45$ GeV$^2$. For $\tilde{A}_{LT}$, the chiral-even and -odd contributions are also shown separately.

The data on the transverse spin structure function $g_2$ from the polarized DIS experiments indicate that the genuine twist-3 contribution in (5) is small [16], and $g_T$ approximately follows the Wandzura-Wilczek result. The calculations of low moments of $g_T$ by lattice QCD simulation support this result [30, 32]. Also, estimates from nucleon models, combined with the QCD evolution for the relevant twist-3 operators [33, 24], suggest that the Wandzura-Wilczek part of (5) and (4) dominates $g_T$ and $h_L$ for $\mu^2 \gg 1$ GeV$^2$ [33, 35, 18]. For the present first estimate of $A_{LT}$ of (10) in $p\bar{p}$-collision, we employ the Wandzura-Wilczek approximation of Fig. 1(b) for $g_T$ and $h_L$.

In all the following numerical evaluation, we present the results for the “reduced asymmetries” $\tilde{A}_{YW} \equiv A_{YW}/\hat{a}_{YW} (Y, W = L, T)$\footnote{Note that $\hat{a}_{LT}$ of (11) is defined absorbing the suppression factor $M/Q$ specific to twist-3 cross section.}. We first consider the fixed-target mode, where $A_{LT}$ will be readily accessible. Figure 2(a) shows $\tilde{A}_{LT}$ as a function of $x_F$ for $Q = 2.5$ and 4 GeV at $s = 45$ GeV$^2$. Also shown are the separated contributions from the chiral-even and -odd distributions, corresponding to the first and second terms in the numerator in (10). The results may be compared with the behavior of $\tilde{A}_{LL}$ and $\tilde{A}_{TT}$ at the same kinematics, shown in Fig. 2(b). The curves for $\tilde{A}_{TT}$ reproduce the corresponding LO results in [8]. $\tilde{A}_{LL}$ and $\tilde{A}_{TT}$ are symmetric with respect to $x_F = 0$, while $\tilde{A}_{LT}$ is not symmetric (compare (6), (7) with (10)). $\tilde{A}_{LL}$ and $\tilde{A}_{TT}$ are almost flat as a function of $x_F$ for the GSI kinematics [8], in strong contrast to $\tilde{A}_{LT}$. These features of $\tilde{A}_{LT}$ come from the $x_F$ dependence of chiral-even and -odd contributions; in particular, the chiral-odd contribution shows the tendency to increase for decreasing $x_F$, while the chiral-even one shows opposite tendency. The values of $\tilde{A}_{LL}$ and $\tilde{A}_{TT}$ are more than 30% and are much larger than their typical values in the $pp$-collision cases [13, 18]. This is because of the fact that for the GSI kinematics the
valence contributions are dominant both in the numerator and the denominator of (6) and (7) and the small-$x$ rise of sea-distributions is absent in the denominator [2, 9, 8, 12]. We see in Fig. 2(a) that the similar mechanism leads to the significant value ($\gtrsim 10\%$) also for $\tilde{A}_{LT}$. In general, $\tilde{A}_{LT}$ is smaller than $\tilde{A}_{LL}$, $\tilde{A}_{TT}$ by the presence of the additional factor, $x_1$ or $x_2$, in (10) compared with (6), (7). Further suppression effect for $\tilde{A}_{LT}$ could be caused by the behavior of $q_T$ and $h_L$ observed in Fig. 1(b) in comparison with Fig. 1(a). When the sea-quark region is probed in $pp$ collisions, these effects, in particular the additional $x_1$, $x_2$ factor, lead to $\tilde{A}_{LT}$ much smaller than the corresponding $\tilde{A}_{LL}$, $\tilde{A}_{TT}$, as demonstrated in [18].

 Actually, the fixed-target mode discussed above mainly probes the region $x_{1,2} \gtrsim 0.4$ (see [8]), where the transversities involved in (13), (14) are poorly determined at present (see the discussion above [13]). In the collider mode we probe the smaller $x_{1,2}$: Figure 3 is same as Fig. 2 but for $Q = 2.5$, 4, 6 and 8 GeV and $s = 210$ GeV$^2$. We observe the similar pattern as in Fig. 2 except that in Fig. 3(a) each of chiral-even and -odd contributions changes its behavior between $Q = 2.5$ GeV and $Q = 6$ GeV. Also, all the asymmetries become somewhat smaller for higher energy, i.e., for smaller $Q/\sqrt{s}$. Actually, the mechanism relevant to this latter point leads to the behavior commonly observed in Figs. 2 and 3 i.e., the increasing $\tilde{A}_{LL}$ and $\tilde{A}_{TT}$ for increasing $Q$, and the corresponding moderate increase of $\tilde{A}_{LT}$. The corresponding behavior is also presented in Fig. 4 where the relevant asymmetries with $x_1 = x_2 = Q/\sqrt{s}$ ($x_F = 0$) are plotted as functions of $Q$ for $s = 30$, 45 and 210 GeV$^2$. As clarified in [12], the $Q$ dependence

\footnote{\tilde{A}_{LL} is slightly larger than $\tilde{A}_{TT}$, because $g_1^u$ is slightly larger than $h_1^u$ as in Fig. 1(a).}
Figure 4: (a) $\tilde{A}_{LT}$ at $x_F = 0$ as a function of $Q$ for $s = 30$, $45$ and $210$ GeV$^2$. (b) $\tilde{A}_{LL}$ and $\tilde{A}_{TT}$ at $x_F = 0$ as a function of $Q$ for $s = 30$, $45$ and $210$ GeV$^2$.

The behavior of $\tilde{A}_{LL}$ and $\tilde{A}_{TT}$ shown in Fig. 4(b) directly reflects the $x$ dependence of the corresponding distributions: \cite{13} implies that $A_{TT}$ is controlled by the ratio $h_1^u(x, Q^2)/f_1^u(x, Q^2)$. It is straightforward to see that the scale dependence of the $u$-quark distributions in this ratio almost cancels between the numerator and denominator in the valence region relevant at GSI, as $h_1^u(x, 1\text{GeV}^2)/f_1^u(x, 1\text{GeV}^2)$ (see Fig. 3 in \cite{12}). Thus the behavior of $h_1^u(x, 1\text{GeV}^2)/f_1^u(x, 1\text{GeV}^2)$ as a function of $x$ directly determines the $Q$-dependence of $A_{TT}$ with $x = Q/\sqrt{s}$. The same logic holds for $A_{LL}$. In the present case using GRV and GRSV parameterizations, the ratio $h_1^u(x, 1\text{GeV}^2)/f_1^u(x, 1\text{GeV}^2)$, as well as $g_1^u(x, 1\text{GeV}^2)/f_1^u(x, 1\text{GeV}^2)$, is actually an increasing function of $x$, leading to the $Q$-dependence in Fig. 4(b). Note, this mechanism characteristic for the GSI kinematics survives even when including the higher order QCD corrections \cite{12}. For $\tilde{A}_{LT}$, however, the cancellation of the scale dependence between the numerator and denominator in \cite{14} is less complete due to the additional $y$-integral for the Wandzura-Wilczek part, which, combined with the additional factor $x_1$ or $x_2$ ($= Q/\sqrt{s}$), results in the novel $Q$-dependence in Fig. 4(a). In particular, the suppression in the moderate $x$-region observed in Fig. 4(b) compared with Fig. 4(a) leads to the decreasing behavior of $\tilde{A}_{LT}$ for increasing $Q$ in the large $Q$ region, while the increasing behavior of $\tilde{A}_{LT}$ in the small $Q$ region is caused by that of the additional factor $x_{1,2} = Q/\sqrt{s}$.

To summarize, we have presented a first estimate of the longitudinal-transverse spin asymmetry $A_{LT}$ for the polarized Drell-Yan process in $p\bar{p}$ collisions at GSI kinematics. Guided by the new empirical information of the transversity, we performed the LO calculation of the Wandzura-Wilczek contribution to $A_{LT}$, which is directly related to the behavior of the transversity in the valence region. The results turned out to be significantly large,
and exhibited distinguished behaviors compared with the twist-2 asymmetries $A_{TT}$ and $A_{LL}$. These results serve as a useful guide for possible future $A_{LT}$ measurement at GSI.

In relation to the discussion on $A_{LT}$, we also emphasized that the large value of $A_{TT}$ at GSI kinematics is known to be quite stable when including the QCD corrections, and that the behavior of $A_{TT}$ as a function of dilepton mass is controlled by the $x$-dependence of the transversity. Thus, first of all, the measurements of $A_{TT}$ at GSI will provide the data that constrain the detailed shape of the transversity in the valence region, including the large $x$ regime where our knowledge on transversity is poor at present. The corresponding new information on the transversity will enable us to update our prediction of $A_{LT}$ in the Wandzura-Wilczek approximation. If the strong deviation from our updated results were observed in the GSI measurements of $A_{LT}$, this would provide an indication of large genuine twist-3 effect, associated with the chiral-odd distribution $h_L$. The QCD analysis of such data using the evolution equation for the corresponding twist-3 operators will reveal the quark-gluon-quark correlation inside the nucleon. One problem for this purpose is that the exact form of the evolution equation governing the genuine twist-3 contributions in $h_L$ is known to be quite sophisticated even at the LO level [23]. Fortunately, as in the case for the similar problem in the chiral-even distribution $g_T$ [33], it is proved [24] that, in the limit of large number of colors, $N_c \to \infty$, the corresponding evolution equation is simplified into the evolution of usual DGLAP-type, with the novel anomalous dimension known in analytic form. Since this simplification holds up to the corrections of $O(1/N_c^2) \sim 10\%$, the large-$N_c$ evolution for the genuine twist-3 contributions in $h_L$ provides a powerful and practical framework to solve the above problem.

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