Model dependence of elliptic flow differences

M D Cozma
IFIN-HH, Reactorului 30, 077125 Măgurele-Bucharest, Romania
E-mail: cozma@niham.nipne.ro

Abstract. An isospin dependent version of the QMD transport model is used to study the influence of the isovector part of the equation of state of nuclear matter on observables that can be measured in heavy-ion collisions at intermediate energy. The model dependence of neutron-proton elliptic flow difference is studied for AuAu collisions at an incident energy of 400 MeV per nucleon. It is found that the sensitivity to microscopical nucleon-nucleon cross-sections, momentum dependence of the optical potential, compressibility modulus of nuclear matter and width of nucleon wave function are moderate compared to the dependence on the stiffness of the isospin asymmetric part of the equation of state. It is concluded that neutron-proton elliptic flow difference is a suitable observable for setting constraints on the supra-saturation density dependence of symmetry energy.

1. Introduction

One of the remaining open questions in nuclear physics is the equation of state (EoS) of isospin asymmetric nuclear matter (asy-EoS), i.e. the density dependence of the symmetry energy (SE). Its precise knowledge is mandatory for a proper understanding of nuclear structure of rare isotopes, dynamics and products of heavy-ion collisions, and most importantly for astrophysical processes such as neutron star cooling and supernovae explosions [1, 2]. Intermediate energy nuclear reactions involving radioactive beams have allowed, by studying the thickness of neutron skins, deformation, binding energies and isospin diffusion to constrain the density dependence of SE at densities below saturation ($\rho_0$) [3, 4]. Existing theoretical models describing its density dependence generally agree with each other in this density regime, but their predictions start to diverge well before regions with densities $\rho \geq 2\rho_0$ are reached [2].

Nuclear matter at supra-saturation densities is created in the laboratory in the process of collision of heavy nuclei. Several observables that can be measured in such reactions have been determined to bear information on the behavior of the SE above $\rho_0$: the neutron/proton ratio of squeezed out nucleons [5], light cluster emission [6], $\pi^-/\pi^+$ multiplicity ratio in central collisions [7, 8], double neutron to proton ratios of nucleon emission from isospin-asymmetric but mass-symmetric reactions [9], elliptic flow ratios [10] and others. The FOPI experimental data for the $\pi^-/\pi^+$ ratio [11] have been used to set constraints on the supersaturation density behavior of SE by various authors with contradicting results: Xiao et al. [7] made use of the IBUU transport model supplemented by the isovector momentum dependent Gogny inspired parametrization of SE [12] to point toward a soft asy-EoS, while the study of Feng et al. [8], which employed IQMD and a power-law parametrization of the symmetry energy $S(\rho) = S_0 (\rho/\rho_0)\gamma$, favors a stiff SE. The study of elliptic flow ratios (npEFR) of Ref. [10] favors, by making use of the power-law parametrization of SE, an asy-EoS dependence on density above saturation point close to a
linear one: \( \gamma = 0.9 \pm 0.4 \).

Elliptic flows of protons and neutrons cannot be used separately to constrain the isovector part of the equation of state above saturation point due to their sizable dependence on the values of transport model parameters, that are either inaccurately determined or do not represent measurable quantities, like in-medium nucleon-nucleon (NN) cross-sections, compressibility modulus of nuclear matter and width of nucleon wave function [13]. A precise knowledge of the dependence on these parameters would allow the elimination of the most uncertain ones leaving one with a set of observables that bear no or almost no model dependence. In practice, one is forced to make assumptions that are verified or disproved by employing a definite transport model. For example, neutron proton elliptic flow ratios (npEFR), studied in Ref. [10], require a scenario in which elliptic flows scale linearly with model parameters. In this study we consider the scenario in which the derivatives of elliptic flows of neutrons and protons with respect to model parameters are assumed to be equal, i.e. we study the model dependence of neutron-proton elliptic flow difference (npEFD). Differences in the constraints on the density dependence of SE extracted from npEFR and npEFD can be viewed as due to residual model dependence and can be used to estimate the accuracy of such an approach.

2. The model

The quantum molecular dynamics (QMD) model [20, 21] is employed to simulate heavy-ion collisions. It has been updated by adding explicit density dependence of the microscopical nucleon-nucleon cross-sections and building in an isospin dependent EoS [13]. The same model has been previously used to study dilepton emission in heavy-ion collisions [14, 15, 16], stiffness of the equation of state of symmetric nuclear matter [17] and various in-medium effects relevant for the dynamics of heavy-ion collisions [18, 19]. The ingredients of the model most relevant for the current study are presented in the following.

One of the key ingredients of the transport models are the microscopic elastic and inelastic cross-sections. For the case of elastic collisions theoretically [22, 23] and experimentally driven [24, 25] studies hint for important in-medium modifications. For the case of total cross-sections the following density dependent parametrizations are being used [22, 23]

\[
\begin{align*}
\sigma_{np}(E_{lab}, \rho) &= [31.5 + 0.092 \times (20.2 - E_{lab}^{0.53} )^{2.9}] \times \frac{1.0 + 0.0034 E_{lab}^{1.51} \rho^2}{1.0 + 21.55 \rho^{1.34}} \\
\sigma_{pp}(E_{lab}, \rho) &= [23.5 + 0.256 \times (18.501 - E_{lab}^{0.52} )^{3.1}] \times \frac{1.0 + 0.1667 E_{lab}^{1.05} \rho^3}{1.0 + 9.704 \rho^{1.2}}
\end{align*}
\]

for energies below the pion production threshold; here \( E_{lab} \) is the incident kinetic energy in MeV, while the density \( \rho \) is expressed in fm\(^{-3}\). Values of the elementary cross-sections of reactions involving at least one excited baryon (either \( \Delta \) or \( N^* \)) are supposed to remain unmodified in a dense nuclear medium. Alternatively the Cugnon parametrization of vacuum NN cross-sections [26] can be used. The only sizable difference between the two mentioned parametrizations occurs in the neutron-proton channel at incident energies below 100 MeV.

The dependence of in-medium nucleon-nucleon cross-sections on isospin asymmetry is introduced by making use of the assumption that NN matrix elements retain their vacuum analytical expressions in the medium, leading to a isospin asymmetry dependence indirectly through the expressions of the effective nucleon masses as functions of \( \beta \) [27]

\[
\sigma_{N_1N_2}(\rho, \beta) = \sigma_{N_1N_2}(\rho, \beta = 0) \frac{m_{N_1}(\rho, \beta)m_{N_2}(\rho, \beta)}{m_{N_1}(\rho, \beta = 0)m_{N_2}(\rho, \beta = 0)}.
\]

The above expression is used with minimal changes above the pion production threshold to describe both the density and isospin asymmetry dependence of NN cross-sections by considering
Figure 1. (Left) Sensitivity of npEFD to changes in the compressibility modulus. For the isoscalar part of EoS the Skyrme type parametrization has been used. The width of the bands corresponds to a variation of $K$ from $K=210$ MeV (soft EoS) to $K=380$ MeV (stiff EoS). (Right) Variations in the values of the npEFD due to different parametrizations of the momentum dependent part of the isoscalar EoS. Different bands correspond, in both panels, to various stiffnesses for asy-EoS.

3. Elliptic flow difference

The azimuthal distribution of protons (or neutrons) resulted in heavy-ion collisions can be approximately described by $dN/d\phi = (N/(2\pi) [1 + v_1 \cos \phi + 2v_2 \cos 2\phi]$, $v_1$ and $v_2$ being called the sidewards and elliptic flow (EF) parameters respectively. The elliptic flow can be extracted
from simulated or experimental data by computing the following average over the respective particle species in the final state \( v_2 = \frac{1}{N} \sum_{i=1,N} (p_{xi}^2 - p_{yi}^2)/p_T^2 \). The neutron-proton elliptic flow difference can readily be obtained: \( v_2^n - v_2^p = v_2^n - v_2^p \). The results presented in this Section are restricted to Au+Au collisions at an incident energy of 400 MeV per nucleon.

The most reliable extraction of the compressibility modulus \( K \) of nuclear matter has been made possible by studying the multiplicity ratio of \( K^+ \) production in heavy (Au+Au) over light (C+C) nuclei at incident energies close to 1 AGeV pointing towards a soft EoS. At lower incident energies the situation is not as clear: an even softer EoS is favored while the study of sidewards flow by the FOPI collaboration favors a soft or hard EoS of state depending on system size. In view of this we have investigated the sensitivity of \( K \) to changes from a soft (\( K=210 \text{ MeV} \)) to a hard (\( K=380 \text{ MeV} \)) EoS. Results are shown in left panel of Fig. 1 for different choices of the parameter \( x \) determining the stiffness of asy-EoS. A clear separation of results corresponding to different values of the asy-EoS stiffness is observed. The kinematic constraints adopted here are compatible with those of the FOPI collaboration [28].

The importance and impact of momentum dependent interactions on heavy-ion observables is well known [29], yet the momentum dependence of the nucleon optical potential is not fully agreed upon. We study its impact on npEFD by employing two popular parametrizations: the Gogny inspired one (Eq. 3) and \( U_{ndi} = -0.054 + 0.00158 \cdot \ln [500 (\bar{p}_1 - \bar{p}_2)^2 + 1] \) [30]. They differ mainly in the high momentum region where the Aichelin parametrization [30] becomes repulsive while the former stays attractive at all energies. In the right panel of Fig. 1 changes in the predicted values for npEFD are studied when switching between the two parametrizations of the optical potential, while preserving the saturation properties of nuclear matter. It is observed that while for a soft asy-EoS scenario the impact of different parametrizations of the optical potential is negligible the situation changes considerably once an asy-stiff scenario is used. This is in agreement with the results of Ref. [31].

An additional indetermination of theoretical estimates is brought in by in-medium values of the nucleon-nucleon cross-sections and the value of the nucleon wave-packet width \( L \) [13]. For the former we employ two different vacuum parametrizations: Cugnon and Li-Machleidt and several scenarios for the density and asymmetry dependence of second (details in Ref. [13]). The width of the nucleon wave function is usually set in literature to \( 2L^2 = 4 \text{ fm}^2 \) for light systems, while for heavy systems an increase to the value \( 2L^2 = 8 \text{ fm}^2 \) is found necessary in order to generate nuclei with stable static properties (e.g. rms). The sensitivity of npEFR to variations in these two model parameters are plotted in Figure 2, in the left and right plots for NN cross-sections and nucleon wave-function width respectively. The variations amount to about 20% of the splitting of npEFR between a super-stiff and super-soft asy-EoS in each case.

To conclude, we have studied the sensitivity of npEFR to different model parameters like microscopic NN cross-sections, compressibility modulus of nuclear matter, optical potential and width of nucleon functions as compared to the sensitivity to the stiffness of the asy-EoS. We have found that summed together (in quadrature) the indetermination of npEFR due to inaccurately known model parameters amounts to about 40% of the splitting of the same observable between the super-stiff and super-soft asy-EoS scenarios. This would allow, by comparing with the experimental npEFD data of the FOPI-LAND [32] or the ASY-EOS [33] Collaborations, to set constraints on the density dependence of symmetry energy above saturation point with a very limited model bias.

4. Acknowledgments
M.D.C. would like to thank the organizers of the summer school “Dynamics of Open Nuclear Systems” 2012 for the invitation and generous financial support and W. Trautmann and for extensive discussions on the topic. M.D.C. acknowledges financial support from PN09370103 grant of the Romanian Ministry of Education and Research.
Figure 2. Sensitivity of the npEFD due to changes in the parametrization for the microscopic NN cross-sections (left) and the width of the nucleon waves function (right).

References

[1] Baran V, Colonna M, Greco V and Di Toro M 2005 Phys. Rep. 410 335
[2] Li B-A, Chen K W and Ko C M 2008 Phys. Rep. 464 113
[3] Li B-A, Ko C M and Bauer W 1998 Int. J. Mod. Phys. E 7 147
[4] Chen L-W, Ko C M and Li B-A 2005 Phys. Rev. Lett. 94 032701
[5] Yong G-C, Li B-A and Chen L-W 2007 Phys. Lett. B 650 344
[6] Chen L-W, Ko C M and Li B-A 2003 Phys. Rev. C 68 017601
[7] Xiao Z, Li B-A, Chen L-W, Yong G-C and Zhang M 2009 Phys. Rev. Lett. 102 062502
[8] Feng Z-Q and Jin G-M 2010 Phys. Lett. B 683 140
[9] Li Q, Li Z and Stöcker H 2006 Phys. Rev. C 73 051602
[10] Russotto P, Wu P Z, Zoric M, Chartier M, Leifels Y, Lemmon R C, Li Q et al. 2011 Phys. Lett. B 697 471
[11] Reisdorf W et al. [FOPI Collaboration] 2007 Nucl. Phys. A 781 459
[12] Das C B, Das Gupta S, Gale C and Li B-A 2003 Phys. Rev. C 67 034611
[13] Cozma M D 2011 Phys. Lett. B 700 139
[14] Shekhter K, Fuchs C, Faessler A, Krivoruchenko M and Martemyanov B 2003 Phys. Rev. C 68 014904
[15] Cozma M D, Fuchs C, Santini E and Fassler A 2006 Phys. Lett. B 640 170
[16] Santini E, Cozma M D, Faessler A, Fuchs C et al. 2008 Phys. Rev. C 78 034910
[17] Fuchs C, Faessler A, Zabrodin E and Zheng Y-M 2001 Phys. Rev. Lett. 86 1974
[18] Fuchs C, Essler P, Gaı̂tanos T and Wolter H H 1997 Nucl. Phys. A 626 987
[19] Uma Maheswari V S, Fuchs C, Faessler A, Sehn L, Kosov D S and Wang Z 1998 Nucl. Phys. A 628 669
[20] Khoa D T et al. 1992 Nucl. Phys. A 548 102
[21] Uma Maheswari W S et al. 1998 Nucl. Phys. A 628 669
[22] Li G Q and Machleidt R 1993 Phys. Rev. C 48 1702
[23] Li G Q and Machleidt R 1994 Phys. Rev. C 49 566
[24] Li Q, Li Z, Soff S, Bleicher M and Stöcker H 2006 J. Phys. G 32 407
[25] Gaitanos T, Fuchs C and Wolter H H 2005 Phys. Lett. B 609 241
[26] Cugnon J, Mizutani T and Vandermuelen J 1981 Nucl. Phys. A 352 505
[27] Li B-A and Chen L-W 2005 Phys. Rev. C 72 064611
[28] Andronic A et al. 2001 Nucl. Phys. A 679 765
[29] Aichelin J, Rosenhauer A, Pei ́ert G, Stoecker H and Greiner W 1987 Phys. Rev. Lett. 58 1926
[30] Aichelin J 1991 Phys. Rep. 202 233
[31] Zhang L, Gao Y, Du Y, Zuo G H and Yong G C 2012 Eur. Phys. Jour. A 48 30
[32] Leifels Y et al. [FOPI Collaboration] 1993 Phys. Rev. Lett. 71 963
Lambrecht D et al. [FOPI-LAND Collaboration] 1994 Z. Phys. A 350 115
[33] Russotto P et al. [ASY-EOS Collaboration] arXiv:1209.5961 [nucl-ex].