A Preonic Model with Colour - Spin Symmetry

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We have constructed a preonic model starting from a coloured fermionic preon and by postulating a new symmetry, MUSY. This new symmetry is defined via the MU number involving colour, charge and spin properties of the preons. We show that all the known fields of the Standard Model (SM) can be constructed using the fermionic preon and 6 preonic scalars, its MUSY partners. As an example, we present well known β-decay process at MUSY level. MUSY also forbids some processes such as proton decay (preserving the baryon number) and hence, it is compatible with current experimental results. In this model the number of SM generations arises to be three naturally.

The MUSY generalization of the SUSY algebra is constructed and the MUSY invariant Lagrangian is also explicitly written. Similar to other preonic and supersymmetric models, a number of new particles are predicted. These particles do not interact with any of the SM fermions but only with the gauge bosons. These particles could be dark matter candidates.

I. INTRODUCTION

Although the Standard Model (SM) of particle physics is able to explain the experimental results obtained so far [1], its problems and open questions shed some doubt on its prospects as being the ultimate theory of fundamental particles and their interactions. The large number of free parameters in the model, the nature of the elementary particles, the reason for their observed mass hierarchy, the electroweak symmetry breaking mechanism are some examples of these open issues. A number of alternative theories have been proposed as the cure to these problems. For example, Grand Unified Models [2], Technicolour [3], Compositeness [4] and Supersymmetry [5] can be cited. The composite models are particularly interesting for the continued simplification they offer. Similar situations have been seen at least twice in the the past: The big and complex periodic table of elements has been understood in terms of three constituents and their combinations: the proton, the neutron and the electron. Similarly, to solve the hadron inflation of the 1960ies, the quark model had to be introduced. The hope of the authors is to propose a solution to some of the aforementioned problems with an effective model at the preonic level. Considering elementary particle family replication and especially the fermion mixings as hints for preonic structure of some of SM particles, a number of preonic models, with different levels of compositeness, have been studied by various groups. Below we list a few examples.

H. Harari [6] and A. M. Shupe [7] proposed two coloured fermionic preons (T, V) with which the first generation of SM quarks and leptons can be formulated as three preon bound states. In this model, the other fermion generations are thought to be excitations of the first generation and the SM bosons are assumed to be fundamental particles. Considering Harari-Shupe model, Buchmann and Schmid [8] later proposed some quark-like structures with hypercolours so that the model become compatible with ’t Hooft anomaly.

H. Fritzsch and G. Mandelbaum considered two fermionic and two bosonic preons (α, β, x, y) to construct all SM fundamental particles except the photon and gluons [9]. In this model, the first generation of SM quarks and leptons are composed of a fermionic and a bosonic preon, while W, Z and Higgs bosons are assumed to consist of a fermionic preon and its anti-particle. The other generations are constructed by adding one or more “gluons” of the preon binding force (haplodynamics) to the appropriate state. Therefore, this model is not able to give any constraint on the number of the generations in the SM.
J. Dugne, S. Fredriksson, J. Hansson and E. Predazzi, in their Preon Trinity Model \[10\], proposed three coloured fermionic preons \(\alpha, \beta, \delta\) with which they construct three scalar bound states \(x, y, z\). With the bound state of a fermionic and a scalar preon (or its anti-particle) they construct all SM fermions and more: one additional charged lepton, two neutral leptons and three additional quarks emerge naturally from the model. With the various combinations of the fermionic preons, they construct the electroweak gauge bosons and more: the model predicts two additional charged bosons and four additional neutral ones.

A. Celikel, M. Kantar and S. Sultansoy \[11\] proposed two scalar and two fermionic preons, all coloured. In this model leptons are bound states of one fermionic preon and one scalar antipreon whereas antiquarks consist of one fermionic and one scalar preon. As a result each SM lepton has its colour octet partner and each SM antiquark has its colour sextet partner.

S. Ishida and Sekiguchi \[12\] have discussed a left-right symmetric preon model assuming the quarks, leptons and weak bosons as composite states and the weak interaction is a secondary effective one without the need of breaking a fundamental symmetry. The fermions are formed by a fermionic and a bosonic preons where the fermionic preons are colour singlet but isospin doublet and bosonic preons are \(3 \oplus 1\) colour. The conservation of baryon and lepton number is included by hand.

The goal of this work is to use the key concepts from compositeness and supersymmetry ideas for constructing a new model with charge-colour and spin symmetry at preonic level. Using this preonic model, we attempt to reduce the number of free parameters, answer the questions on the number of elementary particle families, to propose a candidate for the Dark Matter (DM) and to solve the hierarchy problem. It will be shown that all three fermion generations of the SM can be obtained from a single chiral coloured preon using appropriate symmetries. Using current-current interactions, the known bosons of the SM naturally emerge in the right form, e.g. gluon masses vanishingly small. The baryon and lepton number conservations are emergent from the model itself, prohibiting the anomalies like the proton decay.

The rest of this manuscript is organized as follows: in the next section, the reader will be motivated for a colour and spin symmetric preonic model, MUSY model, which defines all known SM fundamental particles as preonic bound states. The section \[11\] will go into the dynamics of the MUSY model to define the MUSY algebra and to construct the effective MUSY Lagrangian. In Section \[16\] the preonic bound states forming SM particles are discussed. Section \[17\] contains a MUSY description of the well known processes of proton and \(\beta\)-decay followed by an assessment of the CKM and MNS mixings in MUSY model. In section \[18\] a number of testable MUSY predictions are considered: excited fermions and dark matter candidates. The last section contains some discussions on the phenomenology of the MUSY model.

### II. BUILDING THE MUSY MODEL

The construction of the model starts with a single fermionic chiral preon colour triplet of charge \(-e/6\). Firstly, we define a conserved “MU” \[21\] number for each preon as follows:

\[
MU = QC + S + H,
\]

where \(Q\) is electrical charge, \(C\) is number of colours (3 for colour, -3 for anticolour triplet, 1 for colour singlet), \(S\) is spin and \(H\) is helicity (\(+1/2\) for right, \(-1/2\) for left components). The conserved quantity \(MU\) will arise from a transformation which makes the action invariant, i.e., a MU symmetry or MUSY in short.

| \(Q\) | \(C\) | \(S\) | \(H\) | \(MU\) |
|---|---|---|---|---|
| \(\psi_a\) | \(-1/6\) | 3 | \(+1/2\) | \(+1/2\) |
| \(\bar{\psi}^a\) | \(-1/6\) | 3 | \(-1/2\) | \(-1/2\) |
| \(\phi_1\) | \(+1/2\) | 1 | 0 | \(+1/2\) |
| \(\bar{\phi}_1\) | \(-1/2\) | 1 | 0 | \(-1/2\) |

A symmetry transformation that changes spin-1/2 to spin-0, colour triplet to singlet (or vice versa) leads us to generate six scalar preons starting from the single initial fermionic preon with left and right states. Therefore, each helicity state of the fermionic preon will give three complex scalars since it is a colour triplet as well as a complex spinor field. According to these constraints, one may write the MU transformation mappings as follows using the notation of \[13\]:

Table I: Preons of the MUSY model: musions. The subscript \(i\) runs from 1 to 3 and \(a\) runs for three colour charges (Red, Green and Blue).
\[
\begin{align*}
\psi_{a,a} & \rightarrow \phi_i \\
\bar{\psi}^{\dot{a},a} & \rightarrow \bar{\phi}^\dagger_i \\
\phi_i & \rightarrow \psi_{a,a} \\
\bar{\phi}^\dagger_i & \rightarrow \bar{\psi}^{\dot{a},a}
\end{align*}
\] (2)

where $\psi_{a,a}$ and $\bar{\psi}^{\dot{a},a}$ are respectively left and Hermitian conjugate of the right handed fermionic preons with colour index $a = R, G, B$, and the fields $\phi_i$ and $\bar{\phi}^\dagger_i$ denote the scalar preons with family index $i = 1, 2, 3$. The quantum numbers of the MUSY preons are presented in Table I. One may notice that the transformation maps colours to families in addition to mapping fermions to bosons as in supersymmetry [13]. This property makes the model look like a “colour-family extension of supersymmetry”, that is, MUSY will reduce to a SUSY if one equates the colour indices to the fermion family indices, which means setting $a = i$ [14].

In supersymmetry, a fermionic parameter, denoted by $\epsilon$, was introduced to suppress the spinor indices in one side of the supersymmetric transformations [5]. In MU symmetry, as one may notice, there has to be a similar variable that would suppress both colour and family indices. Hence, a MU symmetric parameter, denoted by $\zeta_i^a$, is to be introduced. It will lead to CKM mixing of quarks and electroweak symmetry breaking as it will be discussed later in Section V. Notice that the reduction of MUSY to SUSY is then nothing but setting the parameter $\zeta_i^a$ proportional to Kronecker delta $\delta_{ia}$. The complete transformation will be discussed in Section IIIA.

III. DYNAMICS OF THE MUSY MODEL

Generally, most of the preonic models suffer from the lack of a convincing dynamical explanation for the problem of the bound states consisting of both fermion and scalar preons. The crucial problem in those models arises from the absence of an explicit interaction between preons. Some models consist of a Yukawa interaction in which the preons have a common gauge symmetry.

\[\begin{array}{c}
\psi
\end{array}\]

Figure 1: Vertex of fermionic and scalar preons (left). Bound state of a fermionic preon and a scalar preon forming a quark (right).

In MUSY model, there does not exist a Yukawa interaction between scalar preons and fermionic preons because of the chirality, i.e., a Yukawa term is not invariant under $SU(3)_L \otimes SU(3)_R$ or $SU(2)_L \otimes SU(2)_R$ global symmetry for which the scalar sector and the fermionic sector remains invariant, respectively. However, the MUSY model has an advantage of having Fermi-like interactions which lead one to have an approximate gauge symmetry as will be discussed in the next sections. These approximate gauge interaction terms, here denoted by $L_{int}$, will effectively generate supersymmetric Noether currents [15] of preons in low energies (i.e., SM scale) as follows:

\[
L_{\text{eff}} = -\frac{i}{2} \int d^4x T \{ L_{\text{int}}(x), L_{\text{int}}(x') \} = \frac{G}{\sqrt{2}} \bar{\zeta} J^\mu \zeta J_{N\mu} + \frac{G}{\sqrt{2}} \bar{\zeta} J^\mu \zeta J_{N\mu} + \text{Higher dimensional terms}
\]

where it is used the Wilson short distance method [16],

\[J^\mu_N = \gamma^\mu \gamma^\rho \Psi \partial_\rho \Phi^\dagger\]

is a fermionic current and have both colour and family indices, $\zeta$ is the MUSY parameter, $\gamma^\mu$ are Dirac matrices, $\Psi$ is the four-component fermionic preon, $\Phi$ is the scalar preon doublet. The $\Phi \partial^\rho \Phi^\dagger$ vertex occured in the effective
Lagrangian of the MUSY model is shown on the left of Figure 1. On the right of the figure, it is described the low energy transition of the bound state schematically.

In this Section it will be described the dynamics of the MUSY model in the following picture:

\[ \mathcal{L}_{\text{preon}}(\Psi, \Phi) \rightarrow \mathcal{L}_{\text{app.}}(\Psi, \Phi, G, W, B) \rightarrow \mathcal{L}_{\text{eff}}(q, \ell, G, W, B) + \mathcal{L}_{\text{DM}} \]  

(6)

where \( \Psi, \Phi \) are preons, \( q, \ell \) are quarks and leptons, \( G, W, B \) are vector bosons with approximate gauge symmetry and \( \mathcal{L}_{\text{DM}} \) is the Lagrangian of the composite fields other than SM fields.

A. MUSY transformation

MUSY transformation maps a colour-triplet Weyl spinor to three scalar fields, and vice versa, since bosonic number of degrees of freedom \( n_B \) and fermionic number of degrees of freedom \( n_F \) should be equal in order to handle the naturalness problem of scalar preons, that is, when \( n_F = n_B \), the huge mass correction to a scalar field vanishes. Indeed, a (complex) colour-triplet Weyl spinor has 12 degrees of freedom and each (complex) scalar have 2 degrees of freedom. The MU transformation also conserves the number of degrees of freedom between bosonic preons and fermionic preons as shown in Table II.

| Fields | fermions | bosons |
|--------|----------|--------|
| \( \psi \) | \( 3 \times 1 \times 2 \) | \( 3 \times 1 \times 2 \) |
| \( \phi_i \) | \( 1 \times 3 \times 1 \) | \( 1 \times 3 \times 1 \) |
| \( \phi^*_i \) | \( 1 \times 3 \times 1 \) | \( 1 \times 3 \times 1 \) |
| Total (on-shell) | 12 | 12 |

Naturally, the infinitesimal change in a scalar under a spin transformation will require its right-hand side to be a scalar such that the spinor indices are suppressed. Also, it can be seen immediately that MUSY transformation \( 2 \) should have a new parameter discussed previously to satisfy the family index in both sides of the equation. Therefore, the change in scalar takes the following form:

\[ \begin{align*}
\delta \phi_i &= \zeta_i^a \epsilon^a \psi_{ia} \\
\delta \hat{\phi}_i &= \zeta_{ia} \bar{\epsilon} \phi^*_{ia}
\end{align*} \]  

(7)

where \( \epsilon_{\alpha} \) is the so-called SUSY parameter, and \( \zeta_{ia} \) are complex constants in a global MUSY transformation, i.e. \( \partial_\mu \zeta_{ia} = 0 \) \[13\]. One can note that

\[ \begin{align*}
\delta \phi^* &= \delta \hat{\phi} \\
\delta \hat{\phi}^* &= \delta \phi
\end{align*} \]  

(8)

although \( \phi \) and \( \hat{\phi}^* \) themselves are not equal (as it should not), due to the condition of \( n_F = n_B \). Now, we may deduce the transformation of the fermionic preon for the free case by writing the MUSY Lagrangian density, which will be discussed in the next section, as follows:

\[ \mathcal{L}_{\text{free}} = -\partial^\mu \phi_i^* \partial_\mu \phi_i - \partial^\mu \hat{\phi}_i^* \partial_\mu \hat{\phi}_i + i \bar{\psi}_a \sigma_\mu \partial_\mu \psi_a \]  

(9)

where summation convention is used over repeated indices \( i, a \) and \( \mu \). Therefore, the change in fermion field, which makes the action invariant under MUSY transformations up to total derivatives, is written as follows:

\[ \delta \psi_{\alpha, a} = -i \zeta_{ia} \sigma_\mu \partial_\mu \left( \phi_i + \hat{\phi}_i^* \right) \]  

(10)

where \( \sigma_\mu \) are the Pauli matrices.
In summary, the MUSY transformations take the following form, immediately after taking the Hermitian conjugate of Eq. (10):

\[
\begin{align*}
\delta \phi_i &= \bar{\psi}_i \epsilon \psi_a \\
\delta \tilde{\phi}_i &= \bar{\psi}_a \epsilon \psi_i \\
\delta \psi_a &= -i \zeta_{a \sigma} \epsilon \partial \mu \left( \phi_i + \phi_i^\dagger \right) \\
\delta \tilde{\psi}_a &= i \bar{\zeta}_a \epsilon \sigma \partial \mu \left( \phi_i + \phi_i^\dagger \right) 
\end{align*}
\]  

(11)

where the spinor indices are suppressed. This transformation indeed looks like SUSY transformation of Wess-Zumino model, only with an additional colour transformation.

B. Effective MUSY Lagrangian

The indices of fermionic preons $\psi_a$, and scalars $\phi_i$ and $\tilde{\phi}_i$ do not indicate colour and family a priori. Instead, when the gauge bosons emerges with an approximate gauge symmetry from the current-current interactions as discussed in [19], preons will have colour or flavour post-priori. The simplest effective Lagrangian containing free fields and current-current interactions at preonic level can be written as follows (again suppressing the family and colour indices):

\[
\mathcal{L}_{\text{preon}} = -\partial^\mu \Phi \partial_\mu \Phi + i \bar{\psi} \sigma^\mu \partial_\mu \psi + i \bar{\tilde{\psi}} \sigma^\mu \partial_\mu \tilde{\psi} + f_0^2 j_0^\mu j_0^\mu + f_W^2 J_0^\mu J_0^\mu + f_0 j_0^\mu j_0^\mu 
\]  

(12)

where $\Phi$ is a doublet consist of the scalar preons, and $f_0$, $f_W$ and $f'$ are constants of dimension mass$^{-1}$, $j_0^\mu$, $j^\mu$, $J_0^\mu$ and $J^\mu$ are the currents of the preons which can be explicitly written as:

\[
\begin{align*}
\delta j_0^\mu &= \psi^\dagger \lambda_m \sigma^\mu \psi + \bar{\psi}^\dagger \lambda_m \sigma^\mu \bar{\psi}, \\
\delta j^\mu &= \psi^\dagger \lambda_m \sigma^\mu \psi + \bar{\psi}^\dagger \lambda_m \sigma^\mu \bar{\psi}, \\
\delta J_0^\mu &= \Phi^\dagger \tau_k \partial^\mu \Phi + (\partial^\mu \Phi^\dagger) \tau_k \Phi, \\
\delta J^\mu &= \Phi^\dagger \partial^\mu \Phi + (\partial^\mu \Phi^\dagger) \Phi. 
\end{align*}
\]  

(13)

Here $m = 1, \ldots, 8$ and $k = 1, 2, 3$ indicates the indices of the generators $\lambda_m$ and $\tau_k$ of groups $SU(3)$ and $SU(2)$, respectively.

One could easily see that Eq. (12) is a non-renormalizable Lagrangian including a current-current interaction and it is not invariant under MU transformations anymore. This non-invariance occurs due to the current-current interaction, and it could be cured by adding an auxiliary Lagrangian to $\mathcal{L}_{\text{preon}}$. The auxiliary Lagrangian could be written as:

\[
\mathcal{L}_{\text{aux}} = (m^\prime B^\mu - f^\prime J^\mu) (m^\prime B^\mu - f^\prime J^\mu) + (m_W W^\mu_k - f_W J_k^\mu) (m_W W_k^\mu - f_W J_k^\mu) + (m_G G^\mu_k - f_G J_k^\mu) (m_G G_k^\mu - f_G J_k^\mu) + (m_a G^\mu_a - f_a J_a^\mu) (m_a G_a^\mu - f_a J_a^\mu) 
\]  

(14)

where the new fields, $G^\mu_a$, $G^\mu_b$, $W^\mu_k$ and $B^\mu$ are the, so called, auxiliary fields. Then the two Lagrangians $\mathcal{L}_{\text{preon}}$ and $\mathcal{L} = \mathcal{L}_{\text{preon}} + \mathcal{L}_{\text{aux}}$ are equivalent since their partition functions are equal. A general approach to preonic models where currents act as approximate gauge fields, were investigated elsewhere [19].

When off-shell, the new fields give an extra degree of freedom and they should satisfy the MU invariance as these also transform under MU transformation. This requirement implies an additional term in the MU transformations of the preons as follows:

\[
\begin{align*}
\delta \phi_i &= \bar{\psi}_i \epsilon \psi_a \\
\delta \tilde{\phi}_i &= \bar{\psi}_a \epsilon \psi_i \\
\delta \psi_a &= -i \zeta_{a \sigma} \epsilon \partial \mu \left( \phi_i + \phi_i^\dagger \right) \\
\delta \tilde{\psi}_a &= i \bar{\zeta}_a \epsilon \sigma \partial \mu \left( \phi_i + \phi_i^\dagger \right) 
\end{align*}
\]  

(15)

where $D_\mu \Phi = \partial_\mu \Phi - ig Y' B_\mu \Phi - ig \Gamma_k W_k^\mu \Phi$ is the covariant derivatives are presented via the occurrence of the approximate gauge symmetry as motivated.
Since auxiliary fields do not have kinetic terms, the variation of the total Lagrangian will give equations of motion of these fields as follows:

\[
\begin{align*}
G_\mu^m &= (f_s/m_s) \left( \psi^\dagger \lambda_m \bar{\sigma}^\mu \psi + \bar{\psi} \lambda_m \sigma^\mu \bar{\psi} \right), \\
G_0^\mu &= (f_s/m_s) \left( \psi^\dagger \bar{\psi} \bar{\psi} + \bar{\psi} \psi \right) , \\
W_k^\mu &= (f_W/m_W) \left( \Phi^\dagger \tau_k \partial^\mu \Phi + (\partial^\mu \Phi^\dagger) \tau_k \Phi \right), \\
B^\mu &= (f'_W/m'_W) \left( \Phi^\dagger \partial^\mu \Phi + (\partial^\mu \Phi^\dagger) \Phi \right).
\end{align*}
\]

The coupling constants and masses of bosons obtained from loop diagrams will give rise to an approximate \( SU(3) \times SU(2) \times U(1) \) gauge symmetry, with an extra singlet, where all the gauge non-invariant parts are embedded into the mass term of the vector fields:

\[
\mathcal{L} = -\partial^\mu \Phi^\dagger \partial_\mu \Phi + i \bar{\psi} \bar{\sigma}^\mu \partial_\mu \psi + i \bar{\psi} \bar{\sigma}^\mu \partial_\mu \psi + m^2 B^\mu B_\mu + m_0^2 G_0^\mu G_0^\mu \\
+ m_W^2 W_k^\mu W_\mu k + m_s^2 G_m^\mu G_m^\mu + g_s j_m^\mu G_m^\mu + g_W j_k^\mu W_\mu k + g_0 j_0^\mu G_0^\mu
\]

where \( g_s, g_0, g' \) and \( g_W \) are dimensionless coupling constants satisfying

\[
\begin{align*}
g_s &= f_s m_s, \\
g_0 &= f_0 m_0, \\
g_W &= f_W m_W, \\
g' &= f' m'
\end{align*}
\]

The mass of the vector bosons \( G^\mu \) and \( G_0^\mu \) converges to zero as the corresponding constant \( f \) diverges and the masses of weak bosons arise from the spontaneous symmetry breaking as will be discussed in the next section. The procedure for obtaining mass to composite vector bosons from loop diagrams has already been investigated in detail \[19\]. The corresponding representations for the composite vector bosons are presented in Table III.

### Table III: Composite SM vector bosons

| Vector boson | Constituent Representation |
|--------------|----------------------------|
| \( G^\mu_m \) | \( \bar{\psi} \gamma \psi \) 8 |
| \( G_0^\mu \) | \( \bar{\psi} \gamma \psi \) 1 |
| \( W_k^\mu \) | \( \Phi^\dagger \tau_k \Phi \) 3 |
| \( B^\mu \) | \( \Phi^\dagger \Phi \) 1 |

### IV. STANDARD MODEL PARTICLES IN MUSY MODEL

In this section quarks, leptons and gauge bosons will be defined in the flavour basis. The corresponding states in the mass basis, resulting in CKM mixings, are discussed in the next section. The SM fermions are proposed as doublets made from fermionic and bosonic preons. Similarly, SM gauge bosons are assumed to be matrices acting on those doublets.

#### A. Standard Model fermions in MUSY model

The SM fermions are in the form:

\[
\begin{align*}
Q_i &= \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}, \\
\bar{u}_i &= \bar{u}, \bar{c}, \bar{t}, \\
\bar{d}_i &= \bar{d}, \bar{s}, \bar{b},
\end{align*}
\]
\[
L_i = \begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}, \quad
\bar{e}_i = \bar{e}, \quad \bar{\mu}, \quad \bar{\tau},
\bar{\nu}_i = \bar{\nu}_e, \quad \bar{\nu}_\mu, \quad \bar{\nu}_\tau,
\]

where \( Q_i \) and \( L_i \) are weak isospin doublets and the remaining ones are singlets. In order to construct SM fermions in terms of MUSY, one can write the SM particles as composite states of preons. A quark is then considered as a composite state made of a fermionic preon and a scalar preon. Thus, it has colour and fractional electric charge as it should. The preonic scalar is to be postulated as \( \phi_i \) for up-type and \( \tilde{\phi}_i \) for down-type quarks where \( i = 1, 2, 3 \) denotes the quark family index. Similarly, up and down type antiquarks are in the form \( \bar{\psi}\phi_i^* \) and \( \bar{\psi}\tilde{\phi}_i^* \), respectively. The full preon content of all left and right handed quark states can be found in Table IV. Note that we have written the MU numbers of composite particles by summing MU numbers of the preons.

| Quark (left) | Q | C | MU_L | Quark (right) | MU_R |
|-------------|---|---|------|-------------|------|
| u = \psi\phi_1 | 2/3 | 3 | 1 | \bar{u} = \bar{\psi}\phi_1^* | 0 |
| d = \psi\phi_1 | -\tilde{1}/3 | 3 | 0 | \bar{d} = \bar{\psi}\tilde{\phi}_1^* | -1 |
| c = \psi\phi_2 | 2/3 | 3 | 1 | \bar{c} = \bar{\psi}\phi_2^* | 0 |
| s = \psi\phi_2 | -\tilde{1}/3 | 3 | 0 | \bar{s} = \bar{\psi}\tilde{\phi}_2^* | -1 |
| t = \psi\phi_3 | 2/3 | 3 | 1 | \bar{t} = \bar{\psi}\phi_3^* | 0 |
| b = \psi\phi_3 | -\tilde{1}/3 | 3 | 0 | \bar{b} = \bar{\psi}\tilde{\phi}_3^* | -1 |

Leptons are thought to be composed of three fermionic preons and one bosonic preon as listed in Table V. To keep compatibility with the SM spin 1/2 particles, one fermionic preon will be opposite handed with respect to the other two. In terms of the colour structure, the state counting gives \( 3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10 \) states. The colour singlet state is taken to yield the SM leptons. The remaining states will be discussed later in Section V1B.

The masses of the SM fermions could arise from the quantum mass corrections from the four-point interaction of the fermionic preons, similar to the mass term in \( \phi_4 \)-model \[17\]. Fermionic component of the bound state resulting in SM particles could also help acquiring mass through (approximate) gauge interactions even if the fermionic preon does not interact directly with the scalar one \[13\]. In general, the correction to the squared mass of the scalar preon has two leading terms one from self-interaction and another one from an indirect interaction with the fermionic preon. However, since the self-interaction term does not exist due to the absence of the Yukawa coupling as MU symmetry requires, the correction to the squared mass of the scalar preon would be as follows:

\[
\Delta M^2 = -g_\phi^2 + g_\phi^2 \left[ \Lambda^2 - 24 M^2 \ln (\Lambda/M_\phi) + \cdots \right]
\]

where \( \Lambda \) is the cut-off scale and it gives contribution even if the bare mass \( M_\phi \) is set to zero. The mass terms will be finite since \( f_\phi \) and \( f_0 \) in Eq. \[13\] are expected to be in the same order of magnitude. At higher energies, the contribution from the fermionic preons is suppressed since there aren't any direct interactions between fermionic and scalar preons. However, the masses are still finite since \( f^2 \) and \( 1/\Lambda^2 \) terms are expected to be at the same order of magnitude. This can be understood by considering the preonic four-fermion interaction constant \( f^2 \) as an analogy to the Fermi constant \( G_F = g^2_W/16\pi^2 \) which is in the same order of magnitude with \( 1/\Lambda^2 \) where \( \Lambda_{\text{weak}} \approx M_W \) is the cut-off scale of the Fermi theory.

According to the discussion above, the masses of the composite states could be obtained without having an hierarchy problem and furthermore the composite states would acquire their masses with respect to the chiral symmetry breaking of the effective action similar to the baryons in chiral perturbation theory \[19\].

#### B. Standard Model gauge bosons in MUSY model

At the preonic scale, MUSY model does not have fundamental gauge interactions at all. This means that a preon does not decay into another preon. Since SM consists of the gauge group \( SU(3) \times SU(2) \times U(1) \), MUSY model should produce those gauge bosons, out of the preons in the model.

One way of forming the gauge bosons, similar to the construction of the scalar bosons in Technicolour models, is to form a fermionic bound state. However, one could easily see that fermionic composite gauge bosons would
have to be flavour singlets since the flavour is originated only from bosonic preons in MUSY model. Another way is current-current interactions becoming current-gauge interactions of SM fermions and thus all terms that are not invariant under global gauge transformations transferred into boson mass [19]. Fermionic (left and right) and bosonic current definitions are repeated here for the reader’s convenience:

\[
\begin{align*}
G^\mu_m &= g_s \left( \psi^\dagger \lambda_\mu \psi + \bar{\psi}^\dagger \lambda_\mu \bar{\psi} \right) \\
G^\mu_m &= g_s \left( \psi^\dagger \bar{\psi}_\mu \psi + \bar{\psi}^\dagger \lambda_\mu \bar{\psi} \right) \\
W^\mu_k &= g_W \left( \Phi^\dagger \tau_k \partial^\mu \Phi + (\partial^\mu \Phi^\dagger) \tau_k \Phi \right) \\
B^\mu &= g' \left( \Phi^\dagger \partial^\mu \Phi + (\partial^\mu \Phi^\dagger) \Phi \right)
\end{align*}
\]

where the fields \( \psi \), \( \bar{\psi} \) and \( \Phi \) are multiplets of the groups generated by \( \lambda_m \) and \( \tau_n \), respectively.

1. Gluons

The fermionic preon current gives a colour-changing vector gauge boson which is nothing but the gluon. Since the multiplets in the current are colour triplets, the gauge group is nothing but the SU(3)colour. Therefore, gluons are defined proportional to the colour-changing current which is unique in the model:

\[
G^\mu_m \sim \bar{\psi} \gamma^\mu \psi
\]

where \( \lambda_m \) are the Gell-Man matrices for \( m = 1, \cdots, 8 \) and \( \Psi \) refers to a four-component fermion taking \( \psi \) and \( \bar{\psi} \) as left and right handed colour triplet fermions, respectively. Table [VI] contains a summary of the preonic contents of the gluons according to the MUSY model. The mass of the gluons emerges as approximately zero since the four-point interaction of the fermionic preons is infinitely strong, i.e., \( m_s \rightarrow 0 \) as \( f_s \rightarrow \infty \) since \( g_s \) is known and finite from the experiments. It is crucial to have a sufficiently large \( f_s^2 \) because the preonic four-fermion interaction is expected to have a very large cut-off scale with respect to the scalar current-current interactions. This cut-off scale is the cut-off scale of the effective Lagrangian discussed in Section [III.B]. The same approach is applied to the field \( G^\mu_0 \) which is a colour singlet and interacts only with fermionic preons.

Table VI: SM Gluons according to the MUSY model. Here the indices of fermionic preons refers R for red, B for blue and G for green.
2. Electroweak bosons and weak mixing angle

The definition of the weak isotriplet and isosinglet fields emerge from the MUSY model as bosonic currents with $SU(2) \times U(1)$ generators since the only available preons are $\phi$ and $\hat{\phi}$, corresponding to up and down isospin states. The explicit form of the electroweak gauge bosons are shown in Table VII.

| Fields         | Contents                                                                 |
|----------------|---------------------------------------------------------------------------|
| $W_{1}^{\mu}$  | $\phi_{i}^{\dagger} \partial_{\mu} \phi_{i}$                           |
| $W_{2}^{\mu}$  | $i (\hat{\phi}_{j}^{\dagger} \partial_{\mu} \phi_{j} - \phi_{i}^{\dagger} \partial_{\mu} \hat{\phi}_{j})$ |
| $W_{3}^{\mu}$  | $\phi_{i}^{\dagger} \partial_{\mu} \phi_{i} - \hat{\phi}_{j}^{\dagger} \partial_{\mu} \hat{\phi}_{j}$ |
| $B^{\mu}$      | $\phi_{i}^{\dagger} \partial_{\mu} \phi_{i} + \hat{\phi}_{j}^{\dagger} \partial_{\mu} \hat{\phi}_{j}$ |

The explicit preon content of the weak and electromagnetic gauge bosons are given in Table VIII. The masses of the weak bosons are obtained via the mixing of the weak currents. With the well-known mixing mechanism as introduced in [9], it is possible to obtain a massless photon and masses of $W$ and $Z$ bosons compatible with the experimental results. Following the electroweak model, the physical states of charged gauge bosons ($W^{\pm}$ and $W^{-}$) can be defined as:

$$ W^{\pm \mu} = (W_{1}^{\mu} \mp i W_{2}^{\mu}) / \sqrt{2} $$

and the neutral bosons ($Z^{0}$ and photon) are defined through the weak mixing angle:

$$ Z^{\mu} = \cos \theta_{W} W_{3}^{\mu} - \sin \theta_{W} B^{\mu}, $$
$$ A^{\mu} = \sin \theta_{W} B^{\mu} + \cos \theta_{W} W_{3}^{\mu}. $$

Before the electroweak mixing, the masses of $W_{1}$, $W_{2}$ and $W_{3}$ are equal as discussed in Ref. [9]. However, when $W_{3}$ and $B$ are mixed to form the $Z$ boson and the physical photon as in Eq. (23), the contribution to the mass of $Z$ boson will be related to the decay constant $F_{3}$ to the constituents in the vector boson, which reads as follows:

$$ \langle 0 | J^{3\mu} | W^{3} \rangle = \langle 0 | \phi_{i}^{\dagger} \partial_{\mu} \phi_{i} - \hat{\phi}_{j}^{\dagger} \partial_{\mu} \hat{\phi}_{j} | W^{3} \rangle = i M_{W} F_{3} p^{\mu} $$

where $M_{W}$ is the mass of the $W^{3}$ boson (before the contribution), $p^{\mu}$ is the momentum. Therefore, the mixing parameter (as in [9]) becomes $\lambda = \sqrt{g/e} \sin \theta_{W} = g M_{W} / F_{3}$. Therefore the relation between the masses of the $Z$ and $W$ bosons can be written as:

$$ M_{Z}^{2} = M_{W}^{2} / (1 - \lambda^{2}) $$

This mechanism is an analogy of the mixing of the vector mesons of QCD and obtaining their masses.

| Fields         | Contents                                                                 |
|----------------|---------------------------------------------------------------------------|
| $W_{+}^{\mu}$  | $\phi_{i}^{\dagger} \partial_{\mu} \phi_{i}$                           |
| $W_{-}^{\mu}$  | $\phi_{i}^{\dagger} \partial_{\mu} \phi_{j}$                           |
| $Z^{\mu}$      | $\frac{1}{2}(g_{1}\phi_{i}^{\dagger} \partial_{\mu} \phi_{i} - g_{2}\hat{\phi}_{j}^{\dagger} \partial_{\mu} \hat{\phi}_{j})$ |
| $A^{\mu}$      | $\frac{1}{2}(g_{2}\phi_{i}^{\dagger} \partial_{\mu} \phi_{i} + g_{1}\hat{\phi}_{j}^{\dagger} \partial_{\mu} \hat{\phi}_{j})$ |
Figure 2: Beta Decay: on the left according to the SM and on the right according to MUSY model

V. SM PHENOMENOLOGY WITHIN MUSY

A. Baryon and Lepton number conservation

Let $\Delta_\psi$ indicate the difference between the number of fermionic preons and fermionic anti-preons, and similarly $\Delta_\phi$ the difference between the number of scalars and anti-scalars. Since a lepton includes three fermionic preons and one scalar preon, lepton number is defined as:

$$L = N_\ell - N_{\bar{\ell}} = \frac{\Delta_\psi - \Delta_\phi}{2}. \quad (27)$$

As for quarks, there is one fermionic preon for each scalar. Thus, one can define baryon number as follows:

$$B = \frac{N_q - N_{\bar{q}}}{3} = \frac{3\Delta_\phi - \Delta_\psi}{6}. \quad (28)$$

Since, there is no decay between fermionic preons and scalars in MUSY model, these quantum numbers are always conserved. Hence, baryon number violations such as proton decays is not allowed in MUSY model.

B. Beta decay

Remembering the MUSY content of the quarks as $u = \psi\phi_1$ and $d = \psi\hat{\phi}_1$ where the fermions are of the same colour, the decay can be written as

$$\psi\hat{\phi}_1 \to \psi\phi_1 + W^-.$$  \quad (29)

It is already introduced that the charged weak boson is originating from the current of scalar preons leading to an interaction of current and vector boson just as in SM. The decay would consist of four fermion and four scalar vertices at the preonic level as shown in the Figure.

C. CKM and MNS mixing

From some decay processes, it is known that quarks are observed in mixed flavour while leptons remain in flavour basis. One could suggest that this is because of the nature of quarks but not leptons. However, MUSY model gives a different mixing mechanism which is originated from the MUSY parameter $\zeta$ and the mixing is generated by the weak bosons. This can be seen immediately by writing the low-energy approximation of the effective MUSY Lagrangian since the operators obtained would include $\zeta_{ia}$ parameter as Noether current in Eq. \ begins{3} arises. In order to have the mixing correct which the values would be obtained from experiments, the leptonic vertex would have $\zeta^3$ where the deviation from the unitarity is in the order of MNS mixing.
VI. PREDICTIONS OF THE MUSY MODEL

A. Excited fermions

As previously discussed, SM fermions are constructed as bound states of preons: one scalar and one fermionic preon for quarks and one scalar and three fermionic preons for leptons. Therefore, the MUSY model contains a rich phenomenology due to the orbital or spin excitations of the SM fermions. For example, for each quark and lepton one expects to have a multitude of orbital excitations, denoted as $q^{(n)}$ and $\ell^{(n)}$. In the lepton sector, an additional possibility are the spin excitations of the from $\ell_{3/2} \equiv \psi\psi\phi$, giving spin $3/2$ particles and naturally their orbital excitations as well. So, each SM lepton has two colour-octet and one colour-decouplet partners.

B. New matter particles: The Oghuz

As discussed in the construction of the SM leptons, three fermionic preons yield a singlet, two octet and one decouplet in terms of group representations of colours. It has been stated that the singlet state corresponds to SM leptons, we therefore propose the remaining objects as dark matter candidates with an appropriate helicity state to give a spin $1/2$ particle because there occurs chargeless and colour-singlet fermions (like a spin-$3/2$ neutrino).

Table IX: BSM particle content of the MUSY model

| Left handed states | Q  | C  | S  |
|-------------------|----|----|----|
| $\bar{\psi}\psi\phi$ | $1/3$ | $3\oplus 6$ | 0  |
| $\bar{\psi}\psi\hat{\phi}$ | $-2/3$ | $3\oplus 6$ | 0  |
| $\psi\bar{\psi}\phi$ | $1/3$ | $3\oplus 6$ | 1  |
| $\psi\bar{\psi}\hat{\phi}$ | $-2/3$ | $3\oplus 6$ | 1  |
| $\psi\psi\psi\phi$ | 0   | $1 \oplus 8 \oplus 8 \oplus 10$ | $3/2$ |
| $\psi\psi\hat{\psi}\hat{\phi}$ | -1  | $1 \oplus 8 \oplus 8 \oplus 10$ | $3/2$ |

Possible bound states other than SM particles are in the form given in Table IX with their charge, colour and spin properties. These bound states are split into two subgroups which do not interact with each other: a group of bicoloured bosonic (vector or scalar) particles and another group of fermionic particles with $3/2$ spin. A suitable name these new particles could be “Oghuz” [22], and the two sub-groups could be called as “Bozoklar” and “Üçoklar” respectively. The Bozoklar interact within the group via four point interactions consist of either fermionic preons and bosonic anti-preons or fermionic anti-preons and bosonic preons. Therefore these “particles” can not interact with any of the SM bosons which consist of either all preons or all anti-preons and can be though as the Dark Matter (DM) candidates of the MUSY model. Additionally, these DM candidates form a group of particles with two fermionic preons and another group with either one or three fermionic preons.

VII. DISCUSSION AND CONCLUSION

A new symmetry, MU symmetry, including spin, colour and charge was introduced. It was shown that the MU symmetry is a generalization of the well known supersymmetry concept and in fact, reduces to SUSY in a specific scenario. Using MU symmetry, and starting from a single chiral colour triplet preon, the electroweak and QCD was built. The MUSY model proposes SM fermions and gauge bosons as the bound states of scalar and fermionic preons. It also predicts the number of families as three, relating it to the number of colours in strong interactions. Other naturally occurring features of the model are the inclusion of the lepton and baryon number conservation, the existence of the quark mixings and new fermions which do not interact with the SM fermions. These new fermions, named the Oghuz, could be the Dark Matter candidates. The higher excitations of the SM bound states, are also expected to be present in Nature and are to be sought at the present and future colliders.

With an effective Lagrangian the MUSY model proposes the gauge bosons of the SM as current-current interactions. It also contains vanishingly small gluon and photon masses due to infinitely strong four-point interaction of the fermionic preons. The electroweak sector remains very similar to the SM, including the weak mixing angle, arising from four-point scalar preon interactions. Although the effective Lagrangian approach does provide a working model, it would have been more elegant to embed all preonic states in a large enough gauge group similar to GUT models,
i.e., a gauge interaction between preons an extension of MUSY model. The applicability of this idea is yet to be investigated.

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[21] The word “mu” means “mystery, yet unknown, hidden” in Old Turkic (and in some Altaic languages). Today the word is living in Modern Turkish as interrogative.
[22] The name Oghuz is derived from the Turkish word “ok”, which means arrow or tribe. The reason for calling these particles as Oghuz is simple: in Turkish history, the Oghuz tribes were split into two fractions “Üçoklar” (Three Arrows) and “Bozoklar” (Grey Arrows). These two fractions would not interact one with another, just like the two sub-groups in the MUSY model.