Color defects in a gauge condensate

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The model of an approximate non-perturbative calculations in the SU(3) gauge theory is offered. This approach is based on the separation of initial degrees of freedom on ordered and disordered phases. The ordered phase is almost classical degrees of freedom, the disordered phase is completely quantum degrees of freedom. Using some approximations and simplifications for 2 and 4-points Green’s functions an effective Lagrangian describing both phases from the SU(3) Lagrangian is obtained. The calculations show that ordered phase is squeezed by disordered phase into defects. These defects are: an infinite flux tube filled with longitudinal color electric and magnetic fields; a color electric hedgehog; a defect having either two color electric dipoles + two color magnetic dipoles or two color electric dipoles or two color magnetic dipoles. It assumed that the color defects are quantum excitations in a gauge condensate. The equations for the disordered phase are an analog of Ginzburg-Landau equation.

I. INTRODUCTION

In QCD exists an open problem of the description of non-perturbative degrees of freedom. In the most known kind this problem is connected to the problem of quark confinement. In this work we offer an approximate model of non-perturbative quantization. We separate SU(3) degrees of freedom on two parts: the first one are almost classical degrees of freedom (ordered phase), the second one are entirely quantum degrees of freedom (disordered phase or gauge condensate). The ordered phase is described using classical field equations for a small gauge subgroup. A gauge condensate is presented by scalar fields with corresponding scalar field equations. In order to obtain the equations describing disordered phase we use several assumptions and simplifications for 2 and 4-points Green’s functions. Roughly speaking we suppose that 2 and 4-points Green’s functions can be simplified by the following way: $G_2 \approx \phi^2$ and $G_4 \approx G_2^2 + \alpha_1 G_2 + \alpha_2$ ($\alpha_{1,2}$ are constants).

In the modern language we derive field equations for the dimensions two and four condensates. As a consequence we have the possibility to describe the gauge condensate on the microscopical level as a field distribution. Using this idea we can investigate color defects in the gauge condensate. In our investigation the color defects are some defects which destroy the global spatial homogeneity of the disordered phase. It can be color flux tubes, color electric dipoles, color magnetic currents and so on.

The dimension two condensate $\langle A_\nu^2 \rangle$ has received a great deal of attention in the last few years, see for example [1]-[8]. The output of these investigations is that a non-vanishing condensate is favored as it lowers the vacuum energy. As a consequence of the existence of a non-vanishing condensate $\langle A_\nu^2 \rangle$, a dynamical mass parameter for the gluons can be generated in the gauge fixed Lagrangian, see [2], [9].

An interesting idea is implemented in the so-called “cut-off” model [12], [13] in which gluons having momenta smaller than a fixed value $K$ are bound inside non-perturbative structures. This idea is similar to the one we will use in this work: probably these gluons are in disordered phase which we investigate here.

In the other context the condensate notion arises in the dual superconductor model [14] where the vacuum of a non–Abelian gauge model may be regarded as a medium of condensed Abelian monopoles. The monopole condensate squeezes the chromoelectric flux (coming from the quarks) into a flux tube due to the dual Meissner effect. In this work we show that the gauge condensate squeezes the color electric and magnetic fields into defects considered here.

In Ref. [15] it is shown that it is possible to arrive at an effective dual Abelian-Higgs model, the dual and relativistic version of Ginzburg-Landau model for superconductor, from $SU(2)$ Yang-Mills theory based on the Faddeev-Niemi connection decomposition and the order-disorder assumptions for the gauge field.
II. HEISENBERG’S NON-PERTURBATIVE QUANTIZATION FOR QCD

In this subsection we will apply a version of Heisenberg’s non-perturbative quantization method \[ \text{to QCD.} \] The classical SU(3) Yang-Mills equations are

\[
\partial_\mu F^{\mu
u} = 0
\]

where \( F^{\mu
u} = \partial_\mu A^\nu - \partial_\nu A^\mu + g f^{BCD} A^B_\mu A^C_\nu \) is the field strength; \( B, C, D = 1, \ldots, 8 \) are the SU(3) color indices; \( g \) is the coupling constant; \( f^{BCD} \) are the structure constants for the SU(3) gauge group. In quantizing the system given in Eqs. (1) - via Heisenberg’s non-perturbative method one first replaces the classical fields by field operators \( A^B_\mu \rightarrow \hat{A}^B_\mu \). This yields the following differential equations for the operators

\[
\partial_\mu \hat{F}^{\mu
u} = 0.
\]

These nonlinear equations for the field operators of the nonlinear quantum fields can be used to determine expectation values for the field operators \( \hat{A}^B_\mu \), where \( \langle \cdots \rangle = \langle Q | \cdots | Q \rangle \) and \( |Q\rangle \) is some quantum state. One can also use these equations to determine the expectation values of operators that are built up from the fundamental operators \( \hat{A}^B_\mu \). For example, the “electric” field operator, \( \hat{E}^B_\mu = \partial_\mu \hat{A}^B_\mu - \partial_\nu \hat{A}^B_\nu + g f^{BCD} A^C_\nu \hat{A}^D_\mu \) giving the expectation \( \langle \hat{E}^B_\mu \rangle \). The simple gauge field expectation values, \( \langle A_\mu(x) \rangle \), are obtained by average Eq. (2) over some quantum state \( |Q\rangle \)

\[
\langle Q | \partial_\mu \hat{F}^{\mu
u} | Q \rangle = 0.
\]

One problem in using these equations to obtain expectation values like \( \langle A^B_\mu \rangle \), is that these equations involve not only powers or derivatives of \( \langle A^B_\mu \rangle \) (i.e. terms like \( \partial_\alpha \langle A^B_\mu \rangle \) or \( \partial_\alpha \partial_\beta \langle A^B_\mu \rangle \)) but also contain terms like \( \hat{G}^{BC} = \langle A^B_\mu A^C_\mu \rangle \). Starting with Eq. (3) one can generate an operator differential equation for the product \( \hat{A}_\mu^B \hat{A}_\nu^C \) thus allowing the determination of the Green’s function \( \hat{G}^{BC} \)

\[
\langle Q | \hat{A}_\mu^B(x) \partial_\mu \hat{F}^{\mu
u}(x) | Q \rangle = 0.
\]

However this equation will in turn contain other, higher order Green’s functions. Repeating these steps leads to an infinite set of equations connecting Green’s functions of ever increasing order. This construction, leading to an infinite set of coupled, differential equations, does not have an exact, analytical solution and so must be handled using some approximation.

Operators which are involved in equation (4) are only well determined if there is a Hilbert space of quantum states. Thus we need to look into the question of the definition of the quantum states \( |Q\rangle \) in the above construction. The resolution to this problem is as follows: There is an one-to-one correspondence between a given quantum state \( |Q\rangle \) and the infinite set of quantum expectation values for any product of field operators, \( \hat{G}^{\mu
u\cdots}(x_1, x_2 \ldots) = \langle Q | A^{\mu_1}_{\nu_1}(x_1) A^{\nu_2}_{\mu_2}(x_2) \cdots | Q \rangle \). If all the Green’s functions \( \hat{G}^{\mu
u\cdots}(x_1, x_2 \ldots) \) are known then the quantum states, \( |Q\rangle \) are known, i.e. the action of \( |Q\rangle \) on any product of field operators \( \hat{A}^B_\mu (x_1) \hat{A}^C_\nu (x_2) \) is known. The Green’s functions are determined from the above, infinite set of equations (following Heisenberg’s idea).

Another problem associated with products of field operators like \( \hat{A}^B_\mu (x) \hat{A}^C_\nu (x) \) which occur in Eq. (4) is that the two operators occur at the same point. For non-interacting field it is well known that such products have a singularity. In this paper we are considering interacting fields so it is not known if a singularity would arise for such products of operators evaluated at the same point. Heisenberg in his investigations of a quantized nonlinear spinor field repeatedly underscored that in a quantum field theory with strong interaction the singularities of propagators can be essentially smoothed out. Physically it is hypothesized that there are situations in interacting field theories where these singularities do not occur (e.g. for flux tubes in Abelian or non-Abelian theory quantities like the “electric” field inside the tube, \( \langle E^E_2(x) \rangle < \infty \), and energy density \( \varepsilon(x) = \langle (E^E_2(x))^2 \rangle < \infty \) are nonsingular). Here we take as an assumption that such singularities do not occur.

III. INFINITE COLOR FLUX TUBE DEFECT.

In this section we will present a cylindrically symmetric color defect in a gauge condensate – infinite flux tube between dyon and antidyon. We reiterate that we have not found an exact analytical method for solving the full equations for all the Green’s functions. Our basic approach in this case is to give some physically reasonable scheme
for cutting off the infinite set of equations for the Green’s functions. In addition our approximate calculations will involve the decomposition of the initial gauge group into a smaller gauge group: $SU(3) \rightarrow SU(2) \times U(1)$. Physically we will find that this reduction splits the initial degrees of freedom (the $SU(3)$ gauge potential components) into $SU(2) \times U(1)$ and coset components. The $SU(2) \times U(1)$ components will represent the ordered phase while the coset components, which will be represented via an effective scalar field, will represent the disordered phase. After these approximations one can perform analytical calculations which suggest the formation of a color electric flux tube. This is of interest since an important feature of the confinement phenomenon is the formation of electric flux tubes between the confined quarks.

We suppose that quantum $SU(3)$ gauge fields in quantum chromodynamics can be decomposed on a gauge condensate (ordered phase) and non-perturbative fluctuations (disordered phase). We suppose as well as that the flux tube is an axially symmetric defect in a gauge condensate.

### A. Reduction to a small subgroup

First we define the reduction of $SU(3)$ to $SU(2) \times U(1)$ following the conventions of Ref. [10]. Starting with the $SU(3)$ gauge group with generators $T^B$, we define the $SU(3)$ gauge fields, $A_\mu = A^B \partial^B$. $SU(2) \times U(1)$ is a subgroup of $SU(3)$ and $SU(3)/(SU(2) \times U(1))$ is a coset. Then the gauge field $A_\mu$ can be decomposed as

$$A_\mu = A^B_\mu \partial^B = A^a_\mu T^a + A^m_\mu T^m,$$

$$A^a_\mu \in SU(2), \quad A^m_\mu \in U(1) \quad \text{and} \quad A^m_\mu \in SU(3)/(SU(2) \times U(1))$$

where the indices $a, b, c, \ldots = 1, 2, 3$ belong to the subgroup $SU(2)$ and $m, n, \ldots = 4, 5, 6, 7$ to the coset $SU(3)/(SU(2) \times U(1)); B$ are $SU(3)$ indices. Based on this the field strength can be decomposed as

$$F^B_\mu \partial^B = F^a_\mu T^a + \tilde{F}^m_\mu T^m = F^a_\mu T^a + F^m_\mu T^m$$

where $f^{abc}$ are the structure constants of $SU(2)$. The $SU(3)$ Yang-Mills field equations can be decomposed as

$$d_\nu(F^{\mu\nu} + \Phi^{\mu\nu}) = -gf^{mn}A^m_\nu(F^{n\mu\nu} + G^{n\mu\nu}),$$

$$D_\nu(F^{\mu\nu} + G^{\mu\nu}) = -gf^{mn}B_\nu(A^m_\nu(h^{b\mu\nu} + \Phi^{b\mu\nu}) - a^b_\nu(F^{n\mu\nu} + G^{n\mu\nu})),$$

$$\partial_\nu(h^{\mu\nu} + g \tilde{F}^{m\nu} A^m_\nu A_\nu) = -g \tilde{F}^{m\nu} A^m_\nu (F^{m\mu\nu} + G^{m\mu\nu})$$

where $d_\nu[\cdots] = \partial_\nu[\cdots] + f^{abc}a^b_\nu[\cdots]c$ is the covariant derivative on the subgroup $SU(2)$ and $D_\nu[\cdots] = \partial_\nu[\cdots] + f^{mnA}a^m_\nu[\cdots]A$. The Heisenberg non-perturbative quantization procedure gives us

$$d_\nu(\tilde{h}^{\mu\nu} + \Phi^{\mu\nu}) = -g \tilde{F}^{m\nu} A^m_\nu (F^{m\mu\nu} + G^{m\mu\nu}),$$

$$D_\nu(F^{m\nu} + G^{m\nu}) = -gf^{mn}B_\nu(A^m_\nu(h^{b\mu\nu} + \Phi^{b\mu\nu}) - a^b_\nu(F^{n\mu\nu} + G^{n\mu\nu})) -$$

$$g \tilde{F}^{m\nu} A^m_\nu (F^{m\mu\nu} + G^{m\mu\nu}),$$

$$\partial_\nu(\tilde{h}^{\mu\nu} + g \tilde{F}^{m\nu} A^m_\nu A_\nu) = -g \tilde{F}^{m\nu} A^m_\nu (F^{m\mu\nu} + G^{m\mu\nu})$$
B. Basic assumptions

It is evident that an exact quantization is impossible for Eq’s. 19-21. Thus we look for some simplification in order to obtain equations which can be analyzed. Our basic aim is to cut off the set of infinite equations using some simplifying assumptions. For this purpose we will propose simple but physically reasonable ansätze for the 2 and 4-points Green’s functions – \( \langle A^\mu_\alpha(x)A^\nu_\beta(y) \rangle, \langle A^\mu_\alpha(x)A^\nu_\beta(z)A^\sigma_\gamma(u) \rangle, \langle A^\mu_\alpha(x)A^\nu_\beta(y)A^\sigma_\gamma(z)A^\tau_\delta(u) \rangle \) and \( \langle A^\mu_\alpha(x)A^\nu_\beta(y)A^\sigma_\gamma(z)A^\tau_\delta(u) \rangle \) – which are involved in averaged SU(3) Lagrangian. As was mentioned earlier we assume that there are two phases

1. The gauge field components \( A^\mu_\alpha (A^\mu_\alpha \in SU(2), A^\mu_8 \in U(1)) \) belonging to the small subgroup \( SU(2) \times U(1) \) are in an ordered phase. Mathematically this means that
\[
\langle a^\mu_\alpha(x) \rangle = (a^\mu_\alpha(x))_{cl}.
\]  
(22)

2. The gauge field components \( A^\mu_\alpha (m = 4, 5, ..., 7 \text{ and } A^\mu_\alpha \in SU(3)/(SU(2) \times U(1))) \) belonging to the coset \( SU(3)/(SU(2) \times U(1)) \) are in a disordered phase (i.e. they form a gauge condensate), but have non-zero energy. In mathematical terms this means that
\[
\langle A^\mu_\alpha(x) \rangle = 0.
\]  
(23)

According to lattice calculations the main contribution at calculations of any quantum quantities in QCD is brought with field configurations having Abelian monopoles. We use this fact to say that the contribution from the \( A^\mu_\alpha \) component of gauge potential is negligible in comparing with the contribution from the \( A^\mu_\alpha \) components \( \mu = 1, 2, 3 \)

\[
\langle A^\mu_\alpha(x)A^\nu_\beta(y) \rangle \ll \langle A^\mu_\alpha(x)A^\nu_\beta(y) \rangle
\]  
(24)

where \( \mu, \nu = 1, 2, 3 \) are the spatial indices. After this remark we define an approximate expression for the 2-point Green’s function as follows

\[
\langle A^\mu_\alpha(x)A^\nu_\beta(y) \rangle = -\eta_{\mu\nu}f_{\alpha}f_{\beta}(x)\phi^\alpha(x),
\]  
(25)

\[
\langle A^\mu_{\alpha}(x)A^\nu_\beta(y) \rangle \approx 0
\]  
(26)

here \( \phi^\alpha \) is a real SU(2) triplet scalar fields. Thus we have replaced the coset gauge fields in 2-points Green’s function by effective scalar fields, which will be the scalar field in our effective \( SU(2) \times U(1) \)-scalar system.

3. We suppose that the Green’s functions with odd numbers of \( A^\mu_{\alpha} \) are zero
\[
\langle A^\mu_\alpha(x) \cdots A^\nu_\beta(y) \rangle = 0
\]  
(27)

4. The correlations between the ordered (classical) and disordered (quantum) phases have the following forms

\[
\langle A^\mu_\alpha(x)A^\nu_\beta(y)b_\alpha(z) \rangle = k_1b_\alpha(z) \langle A^\mu_\alpha(x)A^\nu_\beta(y) \rangle,
\]  
(28)

\[
\langle A^\mu_\alpha(x)A^\nu_\beta(y)b_\alpha(z)b_\beta(u) \rangle = k_2b_\alpha(z)b_\beta(u) \langle A^\mu_\alpha(x)A^\nu_\beta(y) \rangle + \delta^{\alpha\beta}M_{1,\mu\nu}b_\alpha(z)b_\beta(u),
\]  
(29)

\[
\langle A^\mu_\alpha(x)A^\nu_\beta(y)A^\sigma_\gamma(z)A^\tau_\delta(u) \rangle = k_3k_4 \delta^{\alpha\gamma}A^\sigma_\gamma(z)A^\tau_\delta(u) \langle A^\mu_\alpha(x)A^\nu_\beta(y) \rangle + \delta^{\alpha\beta}M_{2,\mu\nu}A^\sigma_\gamma(z)A^\tau_\delta(u),
\]  
(30)

where \( k_{1,2} \) are constants; \( M_{1,2,\mu\nu} \) are matrices which will destroy the gauge invariance of a final Lagrangian.
5. The 4-points Green’s functions can be decomposed by the following way. For $A^m_\mu$ gauge potentials

$$\langle A^m_\mu(x) A^p_\nu(y) A^p_\alpha(z) A^q_\beta(u) \rangle = \lambda_1 \left( \langle A^m_\mu(x) A^p_\nu(y) \rangle \langle A^p_\alpha(z) A^q_\beta(u) \rangle + \right) + \frac{\mu_1^2}{4} \left( \delta^{mn} \eta_{\mu\nu} \langle A^p_\nu(z) A^q_\beta(u) \rangle + \delta^{pq} \eta_{\alpha\beta} \langle A^m_\mu(x) A^p_\nu(y) \rangle \right) + \frac{\mu_1^2}{4} \left( \delta^{mp} \eta_{\mu\alpha} \langle A^n_\nu(y) A^q_\beta(u) \rangle + \delta^{pq} \eta_{\beta\nu} \langle A^m_\mu(x) A^p_\nu(y) \rangle \right) + \frac{\mu_1^2}{4} \left( \delta^{mq} \eta_{\mu\beta} \langle A^p_\nu(y) A^q_\alpha(z) \rangle + \delta^{pq} \eta_{\alpha\nu} \langle A^m_\mu(x) A^p_\nu(y) \rangle \right) + \frac{\mu_1^4}{16} \left( \delta^{mn} \eta_{\mu\nu} \delta^{pq} \eta_{\alpha\beta} + \delta^{mp} \eta_{\mu\alpha} \delta^{nq} \eta_{\nu\beta} + \delta^{mq} \eta_{\mu\beta} \delta^{np} \eta_{\nu\alpha} \right)$$

where $\lambda_1, \mu_1$ are constants.

C. Derivation of an effective Lagrangian

For our quantization procedure we will take the expectation of the Lagrangian rather than for the equations of motions. Thus we will obtain an effective Lagrangian rather than approximate equations of motion. The Lagrangian we obtain from the original SU(3) pure gauge theory will turn out to be an effective SU(2) × U(1) Yang-Mills-Higgs system. The averaging SU(3) Lagrangian is

$$L_{eff} = -\frac{1}{4} \langle F_{\mu\nu}^A F^{A\mu\nu} \rangle - \frac{1}{2} \langle F_{\mu\nu}^A F^{A\mu\nu} \rangle - \frac{1}{4} \langle F_{\mu\nu}^A F^{A\mu\nu} \rangle$$

where $\mu, \nu = 0, 1, 2, 3$. According to item 2 of section 4.11.3

$$\langle F_{\mu\nu}^A F^{A\mu\nu} \rangle \ll \langle F_{\mu\nu}^A F^{A\mu\nu} \rangle$$

since $F_{\mu\nu}^A = -\partial_\mu A_\nu + g f^{ABC} A^B_\mu A^C_\nu$ (we consider the stationary fields only), $F_{\mu\nu}^A = \partial_\mu A_\nu - \partial_\nu A_\mu + g f^{ABC} A^B_\mu A^C_\nu$ and correspondingly $\langle F_{\mu\nu}^A F^{A\mu\nu} \rangle$ has terms like $\langle A^B_\mu A^B_\nu \rangle$ but $\langle F_{\mu\nu}^A F^{A\mu\nu} \rangle - \langle A^B_\mu A^B_\nu \rangle$. It means that the magnetic fields generating by the magnetic monopoles are more important by quantization in comparing with the electric fields. Therefore

$$L_{eff} \approx -\frac{1}{4} \langle F_{\mu\nu}^A F^{A\mu\nu} \rangle - \frac{1}{4} \langle \langle F_{\mu\nu}^A \Phi^{a\mu\nu} \rangle + \langle F_{\mu\nu}^A F_{\mu\nu} \rangle + \langle F_{\mu\nu}^A F_{\mu\nu} \rangle \rangle$$

In Eq. 5.1 $F^{a\mu\nu}$, $F^{s\mu\nu}$ and $F^{\mu\nu}$ are defined by equations 5.3 - 15.

1. Calculation of $\langle F_{\mu\nu}^A F^{a\mu\nu} \rangle$

We begin by calculating the first term on the r.h.s. of equation 5.4

$$\langle F_{\mu\nu}^a F^{a\mu\nu} \rangle = \langle F_{\mu\nu}^a F^{a\mu\nu} \rangle + 2 \langle F_{\mu\nu}^a \Phi^{a\mu\nu} \rangle + \langle \Phi^{a\mu\nu} \rangle.$$ (35)

Immediately we see that the first term on the r.h.s. of equation 5.5 is the SU(2) Lagrangian as we assume that $A^a_\mu$ and $F^a_{\mu\nu}$ are effectively classical quantities and consequently

$$\langle F_{\mu\nu}^a F^{a\mu\nu} \rangle = F_{\mu\nu}^a F^{a\mu\nu}.$$ (36)

The second term in equation 5.5 vanishes as $F_{\mu\nu}$ is the antisymmetrical tensor but $\langle \Phi^{a\mu\nu} \rangle$ is the symmetrical one (see Eq. 2021).

The last term which is quartic in the coset gauge fields can be calculated using 5.7

$$f^{a mn} f^{a pq} \langle A^m_\mu A^n_\nu A^p_\alpha A^q_\beta \rangle = \frac{9}{8} \lambda_1 g^2 \left( \phi^a \phi^a - \mu_1^2 \right)^2.$$ (37)

Up to this point the SU(2) part of the Lagrangian is

$$\langle F_{\mu\nu}^a F^{a\mu\nu} \rangle = \langle F_{\mu\nu}^a F^{a\mu\nu} \rangle + \frac{9}{8} \lambda_1 g^2 \left( \phi^a \phi^a - \mu_1^2 \right)^2.$$ (38)
2. Calculation of $\langle F_{\mu\nu}^m F^{\mu\nu} \rangle$

Next we work on the coset part of the Lagrangian

$$\langle F_{\mu\nu}^m F^{\mu\nu} \rangle = \langle F_{\mu\nu}^m F^{\mu\nu} \rangle + 2g f^{mna} \langle F_{\mu\nu}^m (A^{\nu} A^{\mu} - A^{\mu} A^{\nu}) \rangle +$$

$$2g f^{mans} \langle F_{\mu\nu}^m (b^\nu A^{\mu} - b^\mu A^{\nu}) \rangle + g^2 f^{mans} f^{mpsb} \langle (A^{n}_a A^{s}_a - A^{s}_a A^{n}_a) (A^{\nu} A^{\mu} - A^{\mu} A^{\nu}) \rangle +$$

$$g^2 f^{mna} f^{mpsb} \langle (b^\nu A^{\mu} - b^\mu A^{\nu}) (b^\nu A^{\mu} - b^\mu A^{\nu}) \rangle.$$  \hspace{1cm} (39)

After the calculations we have

$$\langle F_{\mu\nu}^m F^{\mu\nu} \rangle = -4 (\partial_{\mu} \phi^a) (\partial_{\nu} \phi^a),$$ \hspace{1cm} (40)

$$g^2 f^{mna} f^{mpsb} \langle (A^{n}_a A^{s}_a - A^{s}_a A^{n}_a) (A^{\nu} A^{\mu} - A^{\mu} A^{\nu}) \rangle = -g^2 k^2_2 (\phi^b \phi^b) A^{n}_a A^{s}_a + 2 (M^2)^{\mu\nu} A^{n}_a A^{s}_a,$$ \hspace{1cm} (41)

$$g^2 f^{mna} f^{mpsb} \langle (b^\nu A^{\mu} - b^\mu A^{\nu}) (b^\nu A^{\mu} - b^\mu A^{\nu}) \rangle = -3g^2 k^2_2 (\phi^b \phi^b) b^\mu b^{\nu} + 2 (m^2)^{\mu\nu} b^\mu b^{\nu},$$ \hspace{1cm} (42)

$$2g f^{mna} \langle F_{\mu\nu}^m (A^{\nu} A^{\mu} - A^{\mu} A^{\nu}) \rangle = -2g k^2 f^{abc} (\partial_{\mu} \phi^a) A^{b\nu} \phi^c,$$ \hspace{1cm} (43)

$$g^2 f^{mna} f^{mpsb} \langle (A^{n}_a A^{s}_a - A^{s}_a A^{n}_a) (b^\nu A^{\mu} - b^\mu A^{\nu}) \rangle = 0,$$ \hspace{1cm} (44)

$$g^2 f^{mna} f^{mpsb} \langle F_{\mu\nu}^m (b^\nu A^{\mu} - b^\mu A^{\nu}) \rangle = 0.$$ \hspace{1cm} (45)

where

$$(M^2)^{\mu\nu} = g^2 (M^2_{\alpha\nu} \eta^{\mu\nu} - M^2_{\mu\nu}),$$ \hspace{1cm} (46)

$$(m^2)^{\mu\nu} = 3g^2 (M^2_{\alpha\mu} \eta^{\nu\nu} - M^2_{\mu\nu}).$$ \hspace{1cm} (47)

In what follows we assign these matrices as diagonal

$$(M^2)^{\mu\nu} = M_{2\alpha\nu} \eta^{\mu\nu},$$ \hspace{1cm} (48)

$$(m^2)^{\mu\nu} = m_{2\mu\nu} \eta^{\nu\nu}. $$ \hspace{1cm} (49)

on $\mu$ there is no summing, $M^2_{\mu}$ and $m^2_{\mu}$ are constants. Finally

$$\langle F_{\mu\nu}^m F^{\mu\nu} \rangle = -4 (\partial_{\mu} \phi^a) (\partial_{\nu} \phi^a) - 2g k^2 f^{abc} (\partial_{\mu} \phi^a) A^{b\nu} \phi^c - g^2 k^2_2 (\phi^b \phi^b) A^{n}_a A^{s}_a - 3g^2 k^2_2 (\phi^b \phi^b) b^\mu b^{\nu} + 2 (m^2)^{\mu\nu} b^\mu b^{\nu} + 2 (M^2)^{\mu\nu} A^{n}_a A^{s}_a.$$ \hspace{1cm} (50)

3. Calculation of $\langle F_{\mu\nu}^m F^{\mu\nu} \rangle$

We end by calculating the second term on the r.h.s. of equation (34)

$$\langle F_{\mu\nu}^m F^{\mu\nu} \rangle = \langle h_{\mu\nu} h^{\mu\nu} \rangle + \langle A^{\mu} A^{\nu} A^{\nu} \rangle + g^2 f^{mna} f^{mpsb} \langle A^{n}_a A^{s}_a A^{\nu} \rangle.$$ \hspace{1cm} (51)

The first term on the r.h.s. of (34) has the classical field $h_{\mu\nu}$ consequently

$$\langle h_{\mu\nu} h^{\mu\nu} \rangle = h_{\mu\nu} h^{\mu\nu}.$$ \hspace{1cm} (52)

The second term in equation (34) vanishes as $h_{\mu\nu}$ is the antisymmetrical tensor but $\langle A^{\mu} A^{\nu} A^{\nu} \rangle$ is the symmetrical one (see Eq. (25)). The last term which is quartic in the coset gauge fields can be calculated using (34)

$$g^2 f^{mna} f^{mpsb} \langle A^{n}_a A^{s}_a A^{\nu} A^{\mu} \rangle = \frac{9}{8} \lambda^2 g^2 (\phi^a \phi^a - \mu^2)^2.$$ \hspace{1cm} (53)

Thus

$$\langle F_{\mu\nu}^m F^{\mu\nu} \rangle = h_{\mu\nu} h^{\mu\nu} + \frac{9}{8} \lambda^2 g^2 (\phi^a \phi^a - \mu^2)^2.$$ \hspace{1cm} (54)
\[ \langle F_{\mu
u}A^{\mu
u} \rangle = F_{\mu
u}F^{\mu\nu} + h_{\mu\nu}h^{\mu\nu} - 4 \left( \partial_{\mu}\phi^a \partial_{\nu}\phi^a + \frac{k_2}{2} g \epsilon^{abc} \partial_{\mu}\phi^a A^{\mu\nu} \phi^c + \frac{1}{4} g^2 k_2^2 A^a_{\mu} A^{\mu\nu} \phi^c \phi^c \right) + \frac{9}{4} \lambda_1 g^2 \left( \phi^a \phi^a - \mu_1^2 \right)^2 - 3k_2^2 g^2 \left( \phi^a \phi^a \right) b_{\mu} b_{\nu} + 2 \left( m^2 \right)^{\mu\nu} A^a_{\mu} A^a_{\nu} + \frac{1}{2} \left( m^2 \right)^{\mu\nu} b_{\mu} b_{\nu} \] 

(55)

After the redefinitions \( \phi^a \rightarrow \tilde{\phi}^a \), \( \mu_1 \rightarrow \sqrt{\frac{9 g^2}{4 k_2}} \rightarrow \lambda_1 \) we have the full averaged Lagrangian

\[ \mathcal{L}_{\text{eff}} = -\frac{1}{4} \langle F_{\mu
u}A^{\mu
u} \rangle = -\frac{1}{4} F_{\mu
u} F^{\mu\nu} - \frac{1}{4} h_{\mu\nu} h^{\mu\nu} - \frac{1}{2} \left( \tilde{D}_{\mu}\phi^a \right) \left( \tilde{D}^{\mu}\phi^a \right) - \frac{1}{8} g^2 k_2^2 A^a_{\mu} A^a_{\nu} \phi^c \phi^c b_{\mu} b_{\nu} - \frac{\lambda_1}{4} \left( \phi^a \phi^a - \mu_1^2 \right)^2 + \frac{3}{8} k_2^2 g^2 \left( \phi^a \phi^a \right) b_{\mu} b_{\nu} - \frac{1}{2} \left( m^2 \right)^{\mu\nu} A^a_{\mu} A^a_{\nu} - \frac{1}{2} \left( m^2 \right)^{\mu\nu} b_{\mu} b_{\nu} \] 

(56)

where \( \tilde{D}_{\mu}\phi^a = \partial_{\mu}\phi^a + \frac{3}{2} k_2 \epsilon^{abc} A^{\mu\nu} \phi^b \phi^c \). The original pure SU(3) gauge theory has been transformed into an SU(2) x U(1) gauge theory with broken gauge symmetry and coupled to an effective triplet scalar field. It is necessary to note that the coupling constant for an effective triplet scalar field is \( \frac{2}{3} k_2 \) not \( g \). This Lagrangian we will investigate numerically on the next sections.

Our final result given in \( \mathcal{L}_{\text{eff}} \) depends on several crucial assumptions. In the next section we will investigate a non-topological flux tube solution to this effective Lagrangian.

E. Initial equations

In this section we will investigate a non-topological flux tube solution to the effective Lagrangian \( \mathcal{L}_{\text{eff}} \), i.e. a cylindrically symmetric defect in a gauge condensate. We will see that the corresponding solution is an infinite flux tube filled with longitudinal and transversal electric and magnetic fields. It allows us to say that the obtained solution is the flux tube stretched between two color charge with chromoelectric and chromomagnetic fields. The separation between charge is infinite one.

We will start from the SU(2) x U(1) Yang - Mills - Higgs field equations of the Lagrangian \( \mathcal{L}_{\text{eff}} \) with broken gauge symmetry

\[ D_{\nu} F^{a\mu\nu} = g \epsilon^{abc} D^c_{\nu} \phi^b - \left( M^2 \right)^{\mu\nu} A^a_{\mu} - \frac{1}{4} k_2^2 g^2 A^{\mu\nu} \phi^c \phi^c, \] 

(57)

\[ \partial_{\nu} h^{\mu\nu} = \frac{3}{4} k_2^2 g^2 \left( \phi^a \phi^a \right) b_{\nu} - \left( m^2 \right)^{\mu\nu} b_{\nu}, \] 

(58)

\[ \tilde{D}_{\nu} \tilde{D}^a_{\mu} \phi^a = -\lambda_0 \left( \phi^a \phi^a - \mu_1^2 \right) + \frac{3}{4} g^2 k_2^2 b_{\mu} b_{\nu} \phi^a - \frac{1}{4} g^2 k_2^2 A^a_{\mu} A^{\nu\mu} \phi^b \phi^b \] 

(59)

here \( F_{\mu\nu} = \partial_{\mu} A^a_{\nu} - \partial_{\nu} A^a_{\mu} + g f^{abc} A^b_{\mu} A^c_{\nu} \) is the field tensor for the SU(2) gauge potential \( A^a_{\mu} \); \( a, b, c \in 1, 2, 3 \) are the color indices; \( D_{\nu} \phi = \partial_{\nu} \phi^a \) is the gauge derivative; \( \phi^a \) is the Higgs field; \( \lambda_1, g \) and \( \mu_1 \) are some constants; \( \left( M^2 \right)^{\mu\nu} \) and \( \left( m^2 \right)^{\mu\nu} \) are mass matrices which destroys the gauge invariance of the Yang - Mills - Higgs theory. In the present case we are postulating that the masses of the two gauge bosons are different. This was done in order to find electric flux tube solutions for the system \( \mathcal{L}_{\text{eff}} \). For equal masses electric flux tubes did not exist for the ansatz used. In numerically solving \( \mathcal{L}_{\text{eff}} \) the two mass come out close to one another, but are not equal.

We will use the following ansatz

\[ A^a_{\mu}(\rho) = \frac{f(\rho)}{g}; \quad A^a_{\nu}(\rho) = \frac{v(\rho)}{g}; \quad \phi^a(\rho) = \frac{\phi(\rho)}{g}; \] 

\[ b_{\mu} = \frac{1}{g} \{ b(\rho), 0, 0, w(\rho) \}; \quad \left( M^2 \right)^{\mu\nu} = \text{diag} \{ M_0, M_1, 0, 0 \}; \quad \left( m^2 \right)^{\mu\nu} = \text{diag} \left\{ m_0, 0, 0, \frac{m_3}{\rho^2} \right\} \] 

(60)

here \( z, \rho, \varphi \) are cylindrical coordinates. The color electric and magnetic fields are

\[ E^3_{\rho} = -\frac{1}{g} f_{\varphi}; \quad E^1_{\varphi} = -\frac{1}{g} f_{\rho}; \quad H^3_{\rho} = \frac{1}{m_3}; \quad H^3_{\varphi} = \frac{1}{g} \rho. \]

(61)
Substituting this into the Yang - Mills - Higgs equations \([\text{67} - \text{69}]\) gives us

\[
v'' + \frac{v'}{x} = v \left( \phi^2 - f^2 - M_0^2 \right),
\]

\[
f'' + \frac{f'}{x} = f \left( \phi^2 + v^2 - M_0^2 \right),
\]

\[
\phi'' + \frac{\phi'}{x} = \phi \left[ -f^2 + v^2 + \lambda_1 \left( \phi^2 - \mu_1^2 \right) + \frac{3}{4} k_1^2 \left( -2 + \frac{w^2}{\rho^2} \right) \right]
\]

\[
h'' + \frac{h'}{x} = h \left( \frac{3}{4} k_1^2 \phi^2 - m_0^2 \right),
\]

\[
w'' - \frac{w'}{x} = w \left( \frac{3}{4} k_1^2 \phi^2 - m_3^2 \right),
\]

here we redefined \(\phi/\phi_0 \rightarrow \phi, \ f/\phi_0 \rightarrow f, \ v/\phi_0 \rightarrow v, \ h/\phi_0 \rightarrow h, \ w/\phi_0 \rightarrow w, \ \mu_1/\phi_0 \rightarrow \mu_1, \ M_{0,1}/\phi_0 \rightarrow M_{0,1}, \ m_{0,3}/\phi_0 \rightarrow m_{0,3}, \ \rho \phi_0 \rightarrow x; \ \phi_0 = \phi(0)\).

\[\text{F. Numerical investigation}\]

For the numerical calculations we choose the following parameters values

\[
k_1 = \sqrt{\frac{4}{3}}, \quad \phi(0) = 1, \quad \lambda_1 = 0.1, \quad v(0) = 0.5,
\]

We apply the method of step by step approximation for finding of numerical solutions (the details of similar calculations can be found in Ref. \([11]\)).

**Step 1.** On the first step we solve Eq. \((\text{67})\) (having zero approximations \(f_0(x) = \frac{0.3}{\cosh^2(\frac{x}{4})}\), \(\phi_0(x) = 1.3 - \frac{0.3}{\cosh^2(\frac{x}{4})}\).

The regular solution exists for a special value \(M_{1,i}^*\) only. For \(M_1 < M_{1,i}^*\) the function \(v_1(x) \rightarrow +\infty\) and for \(M_1 > M_{1,i}^*\) the function \(v_1(x) \rightarrow -\infty\) (here the index \(i\) is the approximation number). One can say that in this case we solve a non-linear eigenvalue problem: \(v_1^*(x)\) is the eigenstate and \(M_{1,i}^*\) is the eigenvalue on this Step.

**Step 2.** On the second step we solve Eq. \((\text{64})\) using zero approximation \(\phi_0(x)\) for the function \(f(x)\) and the first approximation \(v_1^*(x)\) for the function \(v(x)\) from the Step 1. For \(M_0 < M_{0,1}^*\) the function \(f_1(x) \rightarrow +\infty\) and for \(M_0 > M_{0,1}^*\) the function \(f_1(x) \rightarrow -\infty\). Again we have a non-linear eigenvalue problem for the function \(f_1(x)\) and \(M_{0,1}^*\).

**Step 3.** On the third step we solve Eq. \((\text{65})\) using the first approximations \(f_1^*(x)\) and \(v_1^*(x)\) from the Steps 1, 2. For \(\mu_1 < \mu_{1,1}^*\) the function \(\phi_1(x) \rightarrow +\infty\) and for \(\mu_1 > \mu_{1,1}^*\) the function \(\phi_1(x)\) oscillates and tends to zero. Again we have a non-linear eigenvalue problem for the function \(\phi_1(x)\) and \(\mu_{1,1}^*\).

**Step 4.** On the fourth step we repeat the first three steps that to have the good convergent sequence \(v_1(x), f_1(x), \phi_1^*(x)\). Practically we have made three approximations.

**Step 5.** On the fifth step we solve Eq. \((\text{66})\) using the first approximations \(\phi_1^*(x)\) from the Step 3. This equation is exactly the Schrödinger equation for the function \(h(x)\) with the potential \(\phi^2(x)\) and eigenvalue \(m_{0,1}^*\).

**Step 6.** On the sixth step we solve Eq. \((\text{67})\) using the first approximations \(\phi_1^*(x)\) from the Step 3. For \(m_3 < m_{3,1}^*\) the function \(w_1(x) \rightarrow +\infty\) and for \(m_3 > m_{3,1}^*\) the function \(w_1(x) \rightarrow -\infty\). Again we have a non-linear eigenvalue problem for the function \(w_1(x)\) and \(m_{3,1}^*\).

**Step 7.** On this step we repeat Steps 1-6 necessary number of times that to have the necessary accuracy of definition of the functions \(v^*(x), f^*(x), \phi^*(x), h^*(x), w^*(x)\).

After Step 7 we have the solution presented in Fig.\(\text{'s}\) \([11 - 13]\). These numerical calculations give us the eigenvalues \(M_0^* \approx 1.2325025678, \ M_1^* \approx 1.18060857823, \ m_0^* \approx 1.18356622792, \ m_3^* \approx 1.29306895\) and eigenstates \(v^*(x), f^*(x), \phi^*(x), h^*(x), w^*(x)\).
It is easy to see that the asymptotical behavior of the regular solution of equations (63)-(67) is

\[
\begin{align*}
\phi(x) &\approx \mu_1 + \phi_\infty \exp \left\{ -x \sqrt{2\lambda_1 \mu_1^2} \right\}, \\
v(x) &\approx v_\infty \exp \left\{ -x \sqrt{\mu_1^2 - M_0^2} \right\}, \\
f(x) &\approx f_\infty \exp \left\{ -x \sqrt{\mu_1^2 - M_1^2} \right\}, \\
h(x) &\approx f_\infty \exp \left\{ -x \sqrt{\mu_1^2 - m_0^2} \right\}, \\
w(x) &\approx f_\infty \exp \left\{ -x \sqrt{\mu_1^2 - m_3^2} \right\},
\end{align*}
\]

(69) (70) (71) (72) (73)

where \( \phi_\infty, v_\infty, f_\infty, h_\infty, w_\infty \) are some constants.

Let us note the following specialities of the solution:

1. The flux of the chromoelectric field \( E_3^z(x) \) is nonzero

\[
\Phi_E = 2\pi \int_0^\infty E_3^z \rho d\rho = 2\pi \int_0^\infty f v \rho d\rho \neq 0.
\]

(74)
FIG. 3: The transversal chromoelectric $E_\rho^1(x)$ and chromomagnetic $H_\rho^2(x)$ fields.

2. The flux of the chromomagnetic field is zero

$$\Phi_H = 2\pi \int_0^\infty H_\rho^8 \rho d\rho = 2\pi \int_0^\infty \frac{dw}{d\rho} d\rho = 2\pi [w(\infty) - w(0)] = 0.$$ 

(75)

3. The longitudinal electric field

$$E_3^z = gA_1^1 A_2^2$$

(76)

is essentially non-Abelian field.

4. The longitudinal magnetic field

$$H_8^z = \frac{1}{\rho} \frac{dA_8^\phi}{d\rho}$$

(77)

is essentially Abelian.

5. Comparing the obtained flux tube with the Nielsen - Olesen flux tube we see that the scalar field $\phi^a$ is an analog of the Ginzburg - Landau wave function for Cooper pairs and Eq. (59) in QCD is the analog of the Ginzburg - Landau equation.

6. According to the previous remark the scalar field $\phi^a$ probably describes a monopole condensate.

IV. A DEFECT WITH TWO INFINITESIMALY CLOSED ELECTRIC AND MAGNETIC DIPOLES.

In the previous section we have obtained the color defect (infinite flux tube with the longitudinal electric field) where the color charges are on infinite distance from each other. In this section we want to investigate the case with infinitesimally closed color charges.

A. $SU(3) \rightarrow U(1) + coset$ decomposition

Now we will consider the case when $A_{\mu}^{a,m}$ ($a = 1, 2, 3, m = 4, 5, 6, 7$) degrees of freedom are quantized but $A_8^\mu$ remains in a classical phase. Again we start from the initial $SU(3)$ Lagrangian $\mathcal{L}_{SU(3)}$ where we have to average over $A_{\mu}^{a,m}$. In order to do this we have to add assumptions in addition to the section III.B

1. On this step the gauge field components $A_{\mu}^a$ ($A_{\mu}^a \in SU(2)$) are in a disordered phase (i.e. they form the gauge condensate), but have non-zero energy. In mathematical terms this means that

$$\langle A_{\mu}^a(x) \rangle = 0.$$ 

(78)
The first term is taking into account that thus we have to calculate the following effective Lagrangian.

The calculations are similar with the section III C 1. We begin by calculating the first term on the r.h.s. of equation 4. We suppose that the Green's functions with odd numbers of $A$ are in disordered phase but $A_{\mu}$ and $A_{\mu}$ quantum degrees of freedom are in disordered phase but $A_{\mu}$ remains in ordered phase.

2. The 4-points Green's function for $A_{\mu}$ gauge potential can be decomposed by the following way:

\[ \langle A_{\mu}^{a}(x)A_{\nu}^{b}(y)A_{\alpha}^{c}(z)A_{\beta}^{d}(u) \rangle = \lambda_2 \left[ \langle A_{\mu}^{a}(x)A_{\nu}^{b}(y) \rangle \langle A_{\alpha}^{c}(z)A_{\beta}^{d}(u) \rangle + \frac{\mu_2^2}{4} (\delta^{ab}\eta_{\mu\nu} \langle A_{\alpha}^{c}(z)A_{\beta}^{d}(u) \rangle + \delta^{cd}\eta_{\alpha\beta} \langle A_{\mu}^{a}(x)A_{\nu}^{b}(y) \rangle + \frac{\mu_2^2}{4} (\delta^{ac}\eta_{\mu\alpha} \langle A_{\nu}(y)A_{\beta}^{d}(u) \rangle + \delta^{bd}\eta_{\nu\beta} \langle A_{\mu}^{a}(x)A_{\alpha}^{c}(z) \rangle \rangle + \frac{\mu_2^2}{4} (\delta^{ad}\eta_{\alpha\beta} \langle A_{\beta}^{d}(u) \rangle \rangle + \delta^{bc}\eta_{\nu\alpha} \langle A_{\mu}^{a}(x)A_{\alpha}^{c}(z) \rangle + \delta^{ad}\eta_{\mu\beta} \delta^{bc}\eta_{\nu\alpha} \rangle \right] \]  

where $\lambda_2, \mu_2$ are constants.

3. The correlations between the ordered ($A_{\mu}^{b} = b_{\mu}$) and disordered ($A_{\mu}^{a,m}$) phases have the following forms:

\[ \langle A_{\mu}^{a,m}(x)A_{\nu}^{b,n}(x)b_{\alpha}(z) \rangle = k_1 b_{\alpha}(z) \langle A_{\mu}^{a,m}(x)A_{\nu}^{b,n}(x) \rangle, \]  

\[ \langle A_{\mu}^{a,m}(x)A_{\nu}^{b,n}(y)b_{\alpha}(z)b_{\beta}(u) \rangle = k_1^2 b_{\alpha}(z)b_{\beta}(u) \langle A_{\mu}^{a,m}(x)A_{\nu}^{b,n}(x) \rangle + \delta^{mn}M_{1,\mu\nu}b_{\alpha}(z)b_{\beta}(u), \]  

where $k_1$ are constant and $M_{1,\mu\nu}$ are matrices.

4. We suppose that the Green's functions with odd numbers of $A_{\mu}$ are zero:

\[ \langle A_{\mu}^{a}(x) \ldots A_{\nu}^{b}(y) \rangle = 0 \]  

5. The correlations between $A_{\mu}^{a}$ and $A_{\mu}^{m}$ quantum degrees of freedom are:

\[ \langle A_{\mu}^{a}(x)A_{\nu}^{b}(y)A_{\alpha}^{c}(z)A_{\beta}^{d}(u) \rangle = k_2^2 \langle A_{\mu}^{a}(x)A_{\nu}^{b}(x) \rangle \langle A_{\alpha}^{c}(z)A_{\beta}^{d}(u) \rangle. \]  

Thus we have to calculate the following effective Lagrangian:

\[ L_{eff} \approx -\frac{1}{4} \langle F_{\mu\nu}^{a}F_{\mu\nu}^{a} \rangle - \frac{1}{4} \langle F_{\mu\nu}^{a}F_{\mu\nu}^{m} \rangle - \langle F_{\mu\nu}^{m}F_{\mu\nu}^{m} \rangle \]  

Taking into account that $A_{\mu}^{a,m}$ are in disordered phase but $A_{\mu}^{8}$ remains in ordered phase.

**B. Calculation of $\langle F_{\mu\nu}^{a}F_{\mu\nu}^{m} \rangle$**

The calculations are similar with the section III C 1. We begin by calculating the first term on the r.h.s. of equation 85:

\[ \langle F_{\mu\nu}^{a}F_{\mu\nu}^{m} \rangle = \langle (\partial_{\mu}A_{\nu}^{a} - \partial_{\nu}A_{\mu}^{a})^2 \rangle + 2g f^{abc} \langle (\partial_{\mu}A_{\nu}^{a} - \partial_{\nu}A_{\mu}^{a}) A^{b\mu}A^{c\nu} \rangle + g^2 f^{abc} f^{ade} \langle A_{\mu}^{b}A_{\nu}^{c}A^{d\mu}A^{e\nu} \rangle + 2g f^{amn} \langle F_{\mu\nu}^{a}A^{m\mu}A^{n\nu} \rangle + g^2 f^{amn} f^{apq} \langle A_{\mu}^{m}A_{\nu}^{a}A^{p\mu}A^{q\nu} \rangle \]  

The first term is

\[ \langle (\partial_{\mu}A_{\nu}^{a} - \partial_{\nu}A_{\mu}^{a})^2 \rangle = -\frac{3}{2} \partial_{\mu}\phi^{m}\partial^{\mu}\phi^{m} \]
The first term on the r.h.s. of (51) has the classical field

\[ \langle \partial \mu A^\nu_a - \partial \nu A^\mu_a \rangle A^{b\mu} A^a_{\nu} \]

on r.h.s. of equation (51) vanishes as it has odd number of \( A^\mu_a \). The term \( \langle \mu F^\nu_{\mu \nu} A^\mu A^\nu \rangle \) is vanishes as \( F^\nu_{\mu \nu} \) is the antisymmetrical tensor but \( \langle A^{\mu \nu} A^{\mu \nu} \rangle \) is the symmetrical one (see Eq. (72)).

The two terms which are quartic in the coset gauge fields can be calculated using (83) and (84):

\[
g^2 f^{abc} f^{ade} \langle A^b_{\mu} A^d_{\nu} A^{\mu \nu} \rangle = \frac{9}{4} \lambda_2 g^2 \left( \phi^a \phi^m - \mu_2 \right)^2,
\]

\[
f^{mn} f^{pq} \langle A^m_{\mu} A^n_{\nu} A^{\mu \nu} \rangle = \frac{9}{8} \lambda_1 g^2 \left( \phi^a \phi^a - \mu_1 \right)^2.
\]

Up to this point the first term \( \langle F^a_{\mu \nu} F^{a\mu \nu} \rangle \) of the Lagrangian (88) is

\[
\langle F^a_{\mu \nu} F^{a\mu \nu} \rangle = - \frac{3}{2} \partial \mu \phi^m \partial \nu \phi^m + \frac{9}{8} \lambda_1 g^2 \left( \phi^a \phi^a - \mu_1 \right)^2 + \frac{9}{4} \lambda_2 g^2 \left( \phi^m \phi^m - \mu_2 \right)^2.
\]

### C. Calculation of \( \langle F^m_{\mu \nu} F^{m\mu \nu} \rangle \)

The next term is

\[
\langle F^m_{\mu \nu} F^{m\mu \nu} \rangle = \left( \langle \partial \mu A^m_{\nu} - \partial \nu A^m_{\mu} \rangle (\partial^\mu A^{m\nu} - \partial^\nu A^{m\mu}) \right) + 2g f^{mna} \langle \left( \partial \mu A^m_{\nu} - \partial \nu A^m_{\mu} \right) \langle A^{n\mu} A^{n \nu} - A^{a \mu} A^{a \nu} \rangle \rangle +
\]

\[
2g f^{mna} \langle \left( \partial \mu A^m_{\nu} - \partial \nu A^m_{\mu} \right) (b^\nu A^{m \nu} - b^\mu A^{m \nu}) \rangle +
\]

\[
g^2 f^{mna} f^{mpb} \langle \left( A^m_{\mu} A^n_{\nu} - A^n_{\mu} A^m_{\nu} \right) \langle A^{p \mu} A^{n \nu} - A^{b \mu} A^{n \nu} \rangle \rangle +
\]

\[
g^2 f^{mna} f^{mpb} \langle \left( b_a A^m_{\nu} - b_\mu A^n_{\nu} \right) \langle b^\nu A^{p \mu} - b^\mu A^{p \nu} \rangle \rangle +
\]

\[
g^2 f^{mna} f^{mpb} \langle \left( A^m_{\mu} A^n_{\nu} - A^n_{\mu} A^m_{\nu} \right) \langle b^\nu A^{p \mu} - b^\mu A^{p \nu} \rangle \rangle
\]

After the calculations we have

\[
\langle \partial \mu A^m_{\nu} - \partial \nu A^m_{\mu} \rangle (\partial^\mu A^{m\nu} - \partial^\nu A^{m\mu}) = -4 \left( \partial \mu \phi^a \right) \left( \partial^\mu \phi^a \right),
\]

\[
g^2 f^{mna} f^{mpb} \langle \left( A^m_{\mu} A^n_{\nu} - A^n_{\mu} A^m_{\nu} \right) \langle A^{p \mu} A^{n \nu} - A^{b \mu} A^{n \nu} \rangle \rangle = \frac{9}{4} g^2 k_2^2 \left( \phi^a \phi^a \right) \langle \phi^m \phi^m \rangle,
\]

\[
g^2 f^{mna} f^{mpb} \langle \left( b_a A^m_{\nu} - b_\mu A^n_{\nu} \right) \langle b^\nu A^{p \mu} - b^\mu A^{p \nu} \rangle \rangle = -3g^2 k_2^2 \langle \phi^a \phi^a \rangle b_\mu b^\mu + 2 \langle m^2 \rangle b_\mu b^\mu,
\]

\[
g^2 f^{mna} f^{mpb} \langle \left( A^m_{\mu} A^n_{\nu} - A^n_{\mu} A^m_{\nu} \right) \langle b^\nu A^{p \mu} - b^\mu A^{p \nu} \rangle \rangle = 0,
\]

\[
g^2 f^{mna} f^{mpb} \langle \left( b_a A^m_{\nu} - b_\mu A^n_{\nu} \right) \langle b^\nu A^{p \mu} - b^\mu A^{p \nu} \rangle \rangle = 0,
\]

\[
2g f^{mna} \langle \left( \partial \mu A^m_{\nu} - \partial \nu A^m_{\mu} \right) (b^\mu A^{p \nu} - b^\nu A^{p \mu}) \rangle = 0.
\]

Finally,

\[
\langle F^m_{\mu \nu} F^{m\mu \nu} \rangle = - 4 \left( \partial \mu \phi^a \right) \left( \partial^\mu \phi^a \right) + \frac{9}{4} g^2 k_2^2 \left( \phi^a \phi^a \right) \langle \phi^m \phi^m \rangle - 3g^2 k_2^2 \left( \phi^a \phi^a \right) b_\mu b^\mu + 2 \langle m^2 \rangle b_\mu b^\mu
\]

### D. Calculation of \( \langle F^8_{\mu \nu} F^{8\mu \nu} \rangle \)

We end by calculating the third term on the r.h.s. of equation (88)

\[
\langle F^8_{\mu \nu} F^{8\mu \nu} \rangle = \langle h_{\mu \nu} h^{\mu \nu} \rangle + 2g \langle h_{\mu \nu} A^{m \mu} A^{m \nu} \rangle + g^2 f^{8mn} f^{8pq} \langle A^m_{\mu} A^n_{\nu} A^{p \mu} A^{q \nu} \rangle.
\]

The first term on the r.h.s. of (88) has the classical field \( h_{\mu \nu} \) consequently

\[
\langle h_{\mu \nu} h^{\mu \nu} \rangle = h_{\mu \nu} h^{\mu \nu}.
\]

The second term in equation (88) vanishes as \( h_{\mu \nu} \) is the antisymmetrical tensor but \( \langle A^{m \mu} A^{m \nu} \rangle \) is the symmetrical one (see Eq. (72)). The last term which is quartic in the coset gauge fields can be calculated using (83)

\[
g^2 f^{8mn} f^{8pq} \langle A^m_{\mu} A^n_{\nu} A^{p \mu} A^{q \nu} \rangle = \frac{9}{8} \lambda_1 g^2 \left( \phi^a \phi^a - \mu_1 \right)^2.
\]

Thus

\[
\langle F^8_{\mu \nu} F^{8\mu \nu} \rangle = h_{\mu \nu} h^{\mu \nu} + \frac{9}{8} \lambda_1 g^2 \left( \phi^a \phi^a - \mu_1 \right)^2.
\]
E. An effective Lagrangian and field equations

Thus after the quantization $A_{\mu}^{a,m}$ degrees of freedom we have

$$-4 \langle L_{eff} \rangle = h_{\mu \nu} h^{\mu \nu} - \frac{3}{2} (\partial_{\mu} \phi^a)(\partial^{\nu} \phi^a) - 4 (\partial_{\mu} \phi^a)(\partial^{\mu} \phi^a) + \frac{9}{4} \lambda_1 (\phi^a \phi^a - \mu_1^a)^2 + \frac{9}{4} g^2 \lambda_2 (\phi^m \phi^m - \mu_2^m)^2 + \frac{9}{4} g^2 k_2^2 (\phi^a \phi^a)(\phi^m \phi^m) - 3 g^2 k_1^2 (\phi^a \phi^a) b_{\mu} b^{\mu} + 2 (m^2)^{\mu \nu} b_{\mu} b_{\nu}. \quad (103)$$

After the redefinition $\phi^m \rightarrow \frac{2}{\sqrt{3}} \phi^m$, $\phi^a \rightarrow \frac{\phi^a}{\sqrt{2}}$, $\mu_2 \rightarrow \frac{\mu_2}{\sqrt{2}}$, $\mu_1 \rightarrow \frac{\mu_1}{\sqrt{2}}$, $4 g^2 \lambda_2 \rightarrow \lambda_2$, $\frac{9}{16} g^2 \lambda_1 \rightarrow \lambda_1$ we have

$$L_{eff} = - \frac{1}{4} h_{\mu \nu} h^{\mu \nu} + \frac{1}{2} (\partial_{\mu} \phi^a)(\partial^{\nu} \phi^a) + \frac{1}{2} (\partial_{\mu} \phi^a)(\partial^{\mu} \phi^m) - \frac{\lambda_1}{4} (\phi^a \phi^a - \mu_1^a)^2 - \frac{\lambda_2}{4} (\phi^m \phi^m - \mu_2^m)^2 - \frac{3}{8} g^2 k_2^2 (\phi^a \phi^a)(\phi^m \phi^m) + \frac{3}{8} g^2 k_1^2 (\phi^a \phi^a) b_{\mu} b^{\mu} - \frac{1}{2} (m^2)^{\mu \nu} b_{\mu} b_{\nu}. \quad (104)$$

The field equations for the U(1)+(two scalar) system are

$$\partial^\mu \partial_\mu \phi^a = - \phi^a \left[ \frac{3 k_2^2}{4} g^2 \phi^m \phi^m + \lambda_1 (\phi^a \phi^a - \mu_1^a) - \frac{3}{4} g^2 k_1^2 b_{\mu} b^{\mu} \right], \quad (105)$$

$$\partial^\mu \partial_\mu \phi^m = - \phi^m \left[ \frac{3 k_2^2}{4} g^2 \phi^a \phi^a + \lambda_2 (\phi^m \phi^m - \mu_2^m) \right], \quad (106)$$

$$\partial_\nu h^{\mu \nu} = \frac{3}{4} g^2 k_1^2 (\phi^a \phi^a) b^{\mu} - (m^2)^{\mu \nu} b_{\nu}. \quad (107)$$

F. Numerical solution

We will use the following ansätz

$$b_{\mu} = \sqrt{\frac{4}{3 k_1^4}} \{ f(r, \theta), 0, 0, 0 \}, \quad \phi^a = \frac{2}{g} \frac{3 k_2}{3 k_2^2} \phi(r, \theta); \quad \phi^m = \frac{1}{g} \frac{1}{k_2 \sqrt{3}} \chi(r, \theta); \quad (108)$$

(here $r, \theta, \varphi$ are spherical coordinates. The corresponding color (but Abelian) electric and magnetic fields are

$$E_\theta^8 = \frac{1}{r} \frac{\partial b_\theta}{\partial \varphi}, \quad H_\theta^8 = \frac{1}{r \sin \theta} \frac{\partial b_\varphi}{\partial \theta}, \quad (109)$$

$$E_r^8 = - \frac{\partial b_r}{\partial \varphi}, \quad H_r^8 = \frac{1}{r \sin \theta} \frac{\partial b_\theta}{\partial \varphi}. \quad (110)$$

Substituting (108) into the U(1) - Higgs equations (105)-(107) gives us

$$\frac{1}{x^2} \frac{\partial}{\partial x} \left( x^2 \frac{\partial \phi}{\partial x} \right) + \frac{1}{x^2} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \phi}{\partial \theta} \right) = \phi \left[ \chi^2 + \lambda_1 (\phi^2 - \mu_1^2) - \left( f^2 - \frac{\nu^2}{x^2 \sin^2 \theta} \right) \right], \quad (111)$$

$$\frac{1}{x^2} \frac{\partial}{\partial x} \left( x^2 \frac{\partial \chi}{\partial x} \right) + \frac{1}{x^2} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \chi}{\partial \theta} \right) = \chi \left[ \phi^2 + \lambda_2 (\chi^2 - \mu_2^2) \right], \quad (112)$$

$$\frac{1}{x^2} \frac{\partial}{\partial x} \left( x^2 \frac{\partial f}{\partial x} \right) + \frac{1}{x^2} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial f}{\partial \theta} \right) = f \left[ \left( \frac{k_1}{k_2} \right)^2 \phi^2 - m_0^2 \right], \quad (113)$$

$$\frac{\partial^2 \nu}{\partial x^2} + \sin \theta \frac{\partial}{\partial \theta} \left( \frac{1}{\sin \theta} \frac{\partial \nu}{\partial \theta} \right) = \nu \left[ \left( \frac{k_1}{k_2} \right)^2 \phi^2 - m_3^2 \right]. \quad (114)$$

here we redefined $\phi/\phi_0 \rightarrow \phi$, $\chi/\phi_0 \rightarrow \chi$, $f/\phi_0 \rightarrow f$, $\nu/\phi_0 \rightarrow \nu$, $\frac{g k_2 \sqrt{7}}{2 \phi_0} \mu_1 \rightarrow \mu_1$, $\frac{g k_2}{\phi_0 \sqrt{12}} \mu_2 \rightarrow \mu_2$, $\frac{4}{3 g k_2} \lambda_1 \rightarrow \lambda_1$, $\frac{12}{g^2 k_2^2} \lambda_2 \rightarrow \lambda_2$, $r \phi_0 \rightarrow x; \phi_0 = \phi(0)$. The equations (111) - (114) are partial differential equations and it is not quite
clearly how they can be solved numerically to find eigenvalues \( \mu_1, \mu_2 \) and \( m_{0,3} \). In order to avoid this problem we assume that the effect from the U(1) Abelian field \( b_\mu \) is small and then

\[
\phi(x, \theta) \approx \phi(x), \quad \chi(x, \theta) \approx \chi(x),
\]

\[
f(x, \theta) = f(x) \left( \cos \theta - \frac{5}{3} \cos^3 \theta \right),
\]

\[
v(x, \theta) = v(x) \sin^2 \theta \cos \theta.
\]  

In addition we have to average Eq. (115) over the angle \( \theta \) using the following average

\[
\frac{1}{\pi} \int_0^\pi \phi^2(x, \theta) \sin \theta \, d\theta = \frac{8}{63} \pi f^2(x),
\]

\[
\frac{1}{\pi} \int_0^\pi \chi^2(x, \theta) \sin \theta \, d\theta = \frac{4}{15} \pi v^2(x).
\] 

Finally we have the following ordinary differential equations which can be numerically solved as an eigenvalue problem

\[
\phi'' + \frac{2}{x} \phi' = \phi \left[ \chi^2 + \lambda_1 (\phi^2 - \mu_1^2) - \left( \frac{8}{63} \pi f^2 - \frac{4}{15} \pi v^2 \right) \right],
\]

\[
\chi'' + \frac{2}{x} \chi' = \chi \left[ \phi^2 + \lambda_2 (\chi^2 - \mu_2^2) \right],
\]

\[
f'' + \frac{2}{x} f' - \frac{12}{x^2} f = f \left[ \left( \frac{k_1}{k_2} \right)^2 \phi^2 - m_0^2 \right],
\]

\[
v'' - \frac{6}{x^2} v = v \left[ \left( \frac{k_1}{k_2} \right)^2 \phi^2 - m_3^2 \right].
\] 

For this choice we should have

\[
E_\theta(x, \theta)|_{\theta=0,\pi} = 0, \quad H_\theta(x, \theta)|_{\theta=0,\pi} = 0,
\]

\[
E_r(x, \theta)|_{x=0} = 0, \quad H_r(x, \theta)|_{x=0} = 0.
\] 

The series expansions near \( x = 0 \) for the functions are

\[
\phi(x) = \phi_0 + \phi_3 \frac{x^2}{2} + \ldots,
\]

\[
\chi(x) = \chi_0 + \chi_3 \frac{x^2}{2} + \ldots,
\]

\[
f(x) = f_3 \frac{x^3}{6} + \ldots,
\]

\[
v(x) = v_3 \frac{x^3}{6} + \ldots
\] 

provide the constraints (124) (125). We will search for a regular solution with the following boundary conditions:

\[
\phi(0) = 1, \quad \phi(\infty) = \mu_1,
\]

\[
\chi(0) = \chi_0, \quad \chi(\infty) = 0,
\]

\[
f(0) = f(\infty) = 0,
\]

\[
v(0) = v(\infty) = 0.
\] 

The numerical calculations are carried out as well as in the section III with \( \frac{k_2}{k_1} = 6 \). The result is presented in Fig 4 and \( \mu_1 = 1.617525, \ldots, \mu_2 = 1.493004, \ldots, m_3 = 3.46539926281310275, \ldots \)
It is easy to see that the asymptotical behavior of the regular solution of equations (120)-(123) is

\[
\phi(x) \approx \mu_1 + \phi_\infty \exp \left\{ -x \sqrt{2 \lambda_1 \mu_1^2} \right\},
\]

(134)

\[
\chi(x) \approx \chi_\infty \exp \left\{ -x \sqrt{\mu_2^2 - \lambda_2 \mu_2^2} \right\},
\]

(135)

\[
f(x) \approx f_\infty \exp \left\{ -x \sqrt{\frac{k_1}{k_2} \mu_1^2 - m_0^2} \right\},
\]

(136)

\[
v(x) \approx v_\infty \exp \left\{ -x \sqrt{\frac{k_1}{k_2} \mu_1^2 - m_3^2} \right\},
\]

(137)

where \(\phi_\infty, \chi_\infty, f_\infty, v_\infty\) are some constants.

In Fig. (5) and (6) the distribution of color electric and magnetic fields are presented.

V. COLOR ELECTRIC HEDGEHOG

In the previous section the defect in the gauge condensate is considered which represents two infinitesimally closed color electric dipoles and two color magnetic currents. In this section we want to investigate an isolated electric
hedgehog in the gauge condensate. It is not pure electric charge as at the origin the radial electric field is zero. The origin of the electric hedgehog is the nonlinear interaction between ordered and disordered phases (nonlinear interaction between gauge potential components in the initial SU(3) Lagrangian).

The initial equations describing such defect are Eq’s (111)-(114) with \( v = 0 \) and we consider spherically symmetric case \( f = f(r), \phi = \phi(r), \chi = \chi(r) \)

\[
\phi'' + \frac{2}{x} \phi' = \phi \left[ \chi^2 + 2 \lambda_1 (\phi^2 - \mu_1^2) - f^2 \right],
\]

(138)

\[
\chi'' + \frac{2}{x} \chi' = \chi \left[ \phi^2 + 2 \lambda_2 (\chi^2 - \mu_2^2) \right],
\]

(139)

\[
f'' + \frac{2}{x} f = f \left[ \left( \frac{k_1}{k_2} \right)^2 \phi^2 - m_0^2 \right].
\]

(140)

The numerical calculations are the same as in the previous sections and the profiles of gauge components and radial electric field are presented in Fig’s 7, 8. In this case \( \mu_1 = 1.6098972\ldots, \mu_2 = 1.4854892\ldots, m_0 = 2.9355398046322565\ldots \)

![FIG. 7: The functions \( \phi^*(x), \chi^*(x), f^*(x) \)](image)

![FIG. 8: The radial electric field \( E_r(x) \)](image)

VI. TWO COLOR ELECTRIC DIPOLES

In this section we want to investigate Eq’s (111)-(114) with \( v = 0 \), i.e. without magnetic field

\[
\frac{1}{x^2} \frac{\partial}{\partial x} \left( x^2 \frac{\partial \phi}{\partial x} \right) + \frac{1}{x^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \phi}{\partial \theta} \right) = \phi \left[ \chi^2 + 2 \lambda_1 (\phi^2 - \mu_1^2) - f^2 \right],
\]

(141)

\[
\frac{1}{x^2} \frac{\partial}{\partial x} \left( x^2 \frac{\partial \chi}{\partial x} \right) + \frac{1}{x^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \chi}{\partial \theta} \right) = \chi \left[ \phi^2 + 2 \lambda_2 (\chi^2 - \mu_2^2) \right],
\]

(142)

\[
\frac{1}{x^2} \frac{\partial}{\partial x} \left( x^2 \frac{\partial f}{\partial x} \right) + \frac{1}{x^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial f}{\partial \theta} \right) = f \left[ \left( \frac{3k_1}{k_2} \right)^2 \phi^2 - m_0^2 \right]
\]

(143)

and

\[
f(x, \theta) = f(x) \left( \cos \theta - \frac{5}{3} \cos^3 \theta \right).
\]

(144)

Again we assume that the effect from the U(1) Abelian field \( b_\mu \) is small and then

\[
\phi(x, \theta) \approx \phi(x), \quad \chi(x, \theta) \approx \chi(x).
\]

(145)
Thus we will numerically investigate the following equations set

\[ \phi'' + \frac{2}{x} \phi' = \phi \left[ \chi^2 + \lambda_1 \left( \phi^2 - \mu_1^2 \right) - \frac{8}{63} f^2 \right], \quad (146) \]
\[ \chi'' + \frac{2}{x} \chi' = \chi \left[ \phi^2 + \lambda_2 \left( \chi^2 - \mu_2^2 \right) \right], \quad (147) \]
\[ f'' + \frac{2}{x} f' - \frac{12}{x^2} f = f \left[ \left( \frac{k_1}{k_2} \right)^2 \phi^2 - m_0^2 \right]. \quad (148) \]

The result is presented in Fig. 9 and \( \mu_1 = 1.616718 \ldots, \mu_2 = 1.4926212 \ldots, m_0 = 3.670495292 \ldots \) The field distribution of Abelian electric field \( E^k \) in this case is the same as in Fig. 5 and similar to the field distribution between two electric dipoles lying on the line.

\[ \text{FIG. 9: The functions } \phi^* (x), \chi^* (x), f^* (x) \text{ for two color electric dipoles.} \]

\section{VII. TWO COLOR MAGNETIC DIPOLES}

In this section we want to investigate Eq's (111)-(114) with \( f = 0 \)

\[ \frac{1}{x^2} \frac{\partial}{\partial x} \left( x^2 \frac{\partial \phi}{\partial x} \right) + \frac{1}{x^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \phi}{\partial \theta} \right) = \phi \left[ \chi^2 + \lambda_1 \left( \phi^2 - \mu_1^2 \right) + \frac{v^2}{x^2 \sin^2 \theta} \right], \quad (149) \]
\[ \frac{1}{x^2} \frac{\partial}{\partial x} \left( x^2 \frac{\partial \chi}{\partial x} \right) + \frac{1}{x^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \chi}{\partial \theta} \right) = \chi \left[ \phi^2 + \lambda_2 \left( \chi^2 - \mu_2^2 \right) \right], \quad (150) \]
\[ \frac{\partial^2 v}{\partial x^2} + \frac{\sin \theta}{x \sin \theta} \frac{\partial}{\partial \theta} \left( \frac{1}{\sin \theta} \frac{\partial v}{\partial \theta} \right) = v \left[ \left( \frac{3k_1}{k_2} \right)^2 \phi^2 - m_0^2 \right] \quad (151) \]

and

\[ v(x, \theta) = v(x) \sin^2 \theta \cos \theta. \quad (152) \]

Doing the same assumptions about smallness of the U(1) Abelian field \( b_{\mu} \) we have again the approximations (145) and the following equations set

\[ \phi'' + \frac{2}{x} \phi' = \phi \left[ \chi^2 + \lambda_1 \left( \phi^2 - \mu_1^2 \right) + \frac{4}{15 \pi} \frac{v^2}{x^2} \right], \quad (153) \]
\[ \chi'' + \frac{2}{x} \chi' = \chi \left[ \phi^2 + \lambda_2 \left( \chi^2 - \mu_2^2 \right) \right], \quad (154) \]
\[ v'' - \frac{6}{x^2} v = v \left[ \left( \frac{k_1}{k_2} \right)^2 \phi^2 - m_3^2 \right]. \quad (155) \]
For the simplicity we choose \( \psi \) classical state (ordered phase). The field equations for this system are

\[ A_{\mu} \]

Eq. (157) means that \( \vec{H} \) distribution of Abelian magnetic field \( \vec{H}^{A} \) in this case is the same as in Fig. 6 and similar to the field distribution between two magnetic dipoles.

\[
\mathcal{L}_{\text{eff}} = -\frac{1}{4} h_{\mu \nu} h^{\mu \nu} + \frac{1}{2} (\partial_{\mu} \phi^{a}) (\partial^{\mu} \phi^{a}) + \frac{1}{2} (\partial_{\mu} \phi^{a}) (\partial^{\mu} \phi^{a}) - \frac{1}{4} (\phi^{a} \phi^{a} - \mu_{1}^{2})^{2} - \frac{1}{4} (\phi^{m} \phi^{m} - \mu_{2}^{2})^{2} - \frac{3}{8} g^{2} k_{2} (\phi^{a} \phi^{a}) (\phi^{m} \phi^{m}) + \frac{3}{8} g^{2} k_{1}^{2} (\phi^{a} \phi^{a}) b_{\mu} b^{\mu} - \frac{1}{2} (m^{2})_{\mu \nu} b_{\mu} b_{\nu} + \bar{\psi} [\gamma^{\mu} (\hat{p}_{\mu} + gA_{\mu}^{B} \lambda^{B}) - m_{q}] \psi
\]

where \( \psi \) a quark field; \( m_{q} \) is the quark mass; \( \bar{\psi} \gamma^{\mu} A_{\mu}^{B} \lambda^{B} \psi = A_{\mu}^{B} \bar{\psi}_{\lambda a} \gamma_{\mu} \lambda_{\lambda} \psi_{,\lambda} \). In Eq. (156) we took into account that according to Section IV A

\[
\langle A_{\mu}^{B} \rangle = \begin{cases} 
0, & B = 1, \cdots, 7 \\
A_{\mu}^{B}, & B = 8 
\end{cases}
\]

Eq. (157) means that \( A_{\mu}^{B} \), \( B = 1, \cdots, 7 \) are quantized degrees of freedom (disordered phase) but \( A_{\mu}^{8} \) remains in classical state (ordered phase). The field equations for this system are

\[
\partial^{\mu} \partial_{\mu} \phi^{a} = -\phi^{a} \left[ \frac{3}{4} k_{2}^{2} g^{2} \phi^{m} \phi^{m} + \lambda_{1} (\phi^{m} \phi^{m} - \mu_{1}^{2}) - \frac{3}{4} g^{2} k_{1}^{2} b_{\mu} b^{\mu} \right], \\
\partial^{\mu} \partial_{\mu} \phi^{m} = -\phi^{m} \left[ \frac{3}{4} k_{2}^{2} g^{2} \phi^{a} \phi^{a} + \lambda_{2} (\phi^{m} \phi^{m} - \mu_{2}^{2}) \right], \\
\partial_{\mu} h^{\mu \nu} = \frac{3}{4} g^{2} k_{1}^{2} (\phi^{a} \phi^{a}) b^{\mu} - (m^{2})_{\mu \nu} b_{\nu} + gA_{\mu}^{8} \bar{\psi} \gamma^{\mu} \lambda^{8} \psi,
\]

\[
[\gamma^{\mu} (\hat{p}_{\mu} + gA_{\mu}^{8} \lambda^{8}) - m_{q}] \psi = 0.
\]

For the simplicity we choose \( \psi_{1a} = \psi_{2a} = \psi_{3a} \). For this choice the term \( \bar{\psi} \gamma^{\mu} \lambda^{8} \psi \) in Eq. (160) vanishes.

The solution of Eq’s (168) - (171) we search in the form

\[
b_{\mu} = \sqrt{\frac{4}{3k_{2}^{2}}} \{ f(r), 0, 0, 0 \}; \quad \phi^{a} = \frac{1}{g} k_{2} \phi(r); \quad \phi^{m} = \frac{1}{g} k_{2} \sqrt{3} \chi(r);
\]

\[
(m^{2})_{\mu \nu} = \text{diag} \{ m_{0}, 0, 0, 0 \}
\]

here \( r, \theta, \varphi \) are spherical coordinates. The corresponding color (but Abelian) radial electric field is

\[
E_{r} = -\frac{\partial b_{\varphi}}{\partial r}.
\]
FIG. 11: The radial functions $v(r), h(r)$.

The ansatz for the spinor field is in the standard form

$$
\psi = \begin{pmatrix} v(r) \Omega_{jlm} \end{pmatrix} (-1)^{l-l'-1} h(r) \Omega_{j'l'm}
$$

(164)

here $l = j + 1/2$, $l' = 2j - 1$. Thus we have the following equations set

$$
\phi'' + \frac{2}{x} \phi' = \phi \left[ \chi^2 + \lambda_1 (\phi^2 - \mu_1^2) - f^2 \right], \\
\chi'' + \frac{2}{x} \chi' = \chi \left[ \phi^2 + \lambda_2 (\chi^2 - \mu_2^2) \right],
$$

(165, 166)

$$
f'' + \frac{2}{x} f = f \left[ \left( \frac{k_1}{k_2} \right)^2 \phi^2 - m_0^2 \right],
$$

(167)

$$
\nu' + \frac{1 + \kappa}{x} \nu - (E + m - gf) h = 0,
$$

(168)

$$
h' + \frac{1 - \kappa}{x} h + (E - m - gf) f = 0
$$

(169)

here we have introduced the dimensionless coordinate $x = r \phi(0)$; redefined $v \rightarrow v/\phi_0$, $h \rightarrow h/\phi_0$ and

$$
\kappa = \begin{cases} 
- (j + \frac{1}{2}) = -(l + 1), & \text{if } j = l + \frac{1}{2} \\
+ (j + \frac{1}{2}) = l, & \text{if } j = l - \frac{1}{2}
\end{cases}
$$

(170)

Eq’s (165) - (167) have been solved in Section V The numerical calculations of Eq’s (168) (169) (with $\kappa = -1$) are presented in Fig. 11

IX. DISCUSSION AND CONCLUSIONS

In this work we have investigated color defects in a gauge condensate. The gauge condensate is defined as quantized non-perturbative degrees of freedom of the SU(3) gauge field (disordered phase). The color defects are residual degrees of freedom remaining in a classical state (ordered phase). Using some assumptions and approximations for 2 and 4-points Green’s functions the SU(3) Lagrangian is reduced to an effective Lagrangian. The 2-point Green’s function of quantum degrees of freedom is presented as a bilinear combination of scalar fields. The 4-point Green’s function is presented as a bilinear combination of 2-points Greens functions. The effective Lagrangian describes a gauge theory with broken gauge symmetry interacting with scalar fields. The gauge field describe the ordered phase and scalar fields – disordered phase.

In this model we have a few undefined parameters which are connected with a non-linear interaction between initial gauge fields. Some of them are defined as eigenvalues at which regular solutions of corresponding field equations exist. The obtained solutions describe: infinite flux tube filled with longitudinal color electric and magnetic fields; a color electric hedgehog; a defect having two color electric dipoles lying on a line and color magnetic dipoles; two color
electric dipoles; two color magnetic dipoles; parton-like defect which can be considered as a bag for the quark field. We assume that these defects are quantum excitations in a gauge condensate.

Finally we would like to list the main results of the paper:

- An approximate method for non-perturbative quantization of QCD is proposed.
- The proposed method is based on the decomposition of SU(3) degrees of freedom on ordered and disordered phases.
- The ordered phase is considered as classical gauge fields. This phase forms color defects inside of the disordered phase. For example, it can be: a flux tube filled with a longitudinal electric field, a hedgehog filled with a radial electric field and so on.
- The disordered phase is quantum degrees of freedom which squeezes the ordered phase into color defects.
- Probably the disordered phase is formed by Abelian monopoles.
- The ordered phase is described by Yang-Mills equations with broken gauge symmetry.
- The equations for the disordered phase are an analog of Ginzburg - Landau equation for the wave function of Cooper pairs.

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