Digital control of structurally unstable power facilities using a polynomial approach

V I Zakhvatov, S L Podvalny and A V Mikhailusov

Voronezh State Technical University, 14 Moscow Avenue, Voronezh, 394026, Russia

Abstract. The problem of improving the quality of control of structurally unstable power facilities is viewed based on a polynomial controller with a dynamic change in settings using the inverse pendulum on a carriage as an example. The mathematical model of the control object was built; a polynomial controller was synthesized using the method of symbolic computation of systems of differential equations with an additional input for specifying the geometric mean root from the outside. Reverse pendulum model on a carriage and a model of regulators were created using the MATLAB Simulink visual modeling environment to analyze the quality and parameters of the created control system. As a result of modeling, direct dependence of the position setting time of pendulum on the value of the geometric mean root was obtained which confirmed the possibility of external control of the speed of the control system. The possibility to control the control actions level to prevent saturation of executive devices was investigated and confirmed. In fact, a system of external parametric control of the processes dynamics was created and the described regulator does not require adaptation and training.

1. Introduction
In connection with the constant complexity growth of the designed systems, the control means for the dynamic processes are getting more and more complex. And a particular difficulty is the management of structurally unstable power objects that are often of a non-minimal-phase nature. This type of objects is also called complex, since its control necessitates the creation of regulators capable of operating in different modes to ensure the maximum area of attraction. Examples of such objects are balancing mechanisms, various types of motors, the movement of cranes with a suspended load, etc.

Improving the quality of complex objects control often requires the control of the dynamics of processes, and the control of control actions themselves to prevent saturation of executive devices [1, 2].

A classic example of a complex object is a reverse pendulum on a carriage. Different positions of the pendulum require different control modes, which makes it impossible to use a static regulator in case of need in the maximum area of attraction.

2. Problem formulation
In this work, it was required to carry out the process of synthesizing a polynomial controller with its settings changing dynamically thus making possible an adjustment to the control object parameters changes without the re-synthesis of the control system. External parametric control was supposed to allow the parametric control of system parameters.
To test the control quality it was required to build a model of a reverse pendulum on a carriage using the MATLAB Simulink, standard solutions in the form of PID controllers were used in addition, to compare the results of these solutions with the resulting control system.

3. Initial conditions
To build a model of the system it was required to enter the initial parameters. First of all, a carriage with a reverse pendulum, shown in figure 1 is controlled by the force F. The model shows the acting forces and, it should be noted that the direction of the pendulum angle, relative to the vertical axis, is counterclockwise. And the growth of the horizontal coordinate of the carriage movement is directed from left to right (a similar system is discussed in [3]).

![Figure 1. Direction of forces.](image1)

![Figure 2. Carriage forces.](image2)

![Figure 3. Pendulum forces.](image3)

4. Building the mathematical model of pendulum
Figures 2 and 3 show the forces acting on the pendulum and on the carriage.

The described physical system has two degrees of freedom both of them are represented by the coordinate of the carriage position and the pendulum deflection angle. Here, guided by Newton's laws, the equations (1, 2) were written for both degrees of freedom.
\[
\frac{d^2 x}{dt^2} = \frac{1}{M} \sum_{\text{cart}} F_x = \frac{1}{M} \left( F - N b \frac{dx}{dt} \right)
\]

(1)

\[
\frac{d^2 \theta}{dt^2} = \frac{1}{l} \sum_{\text{pend}} \tau = \frac{1}{l} \left( NL \cos(\theta) + PL \sin(\theta) \right)
\]

(2)

5. System linearization
Since the stabilization occurs in the vicinity to pendulum’s zero deflection angle, several substitutions could be made:

After substitutions, new equations of motion (3) and (4) were obtained.

\[(l + ml^2) \ddot{\phi} - mgl \dot{\phi} = ml \ddot{x} \quad (3)\]

\[(M + m) \ddot{x} + b \dot{x} - ml \ddot{\phi} = u \quad (4)\]

For further linearization, we apply the Laplace transforms for the equations (5) and (6).

\[(l + ml^2) \Phi(s)s^2 - mgl \Phi(s) = mlX(s)s^2 \quad (5)\]

\[(M + m)X(s)s^2 + bX(s)s - ml\Phi(s)s^2 = U(s) \quad (6)\]

6. Getting the transfer function
As soon as linearized equations were obtained, it became possible to compose the transfer function of the system of the reciprocal pendulum on the carriage which is necessary for the future for the synthesis of the controller.

Since the input of the system is \(U(s)\) (applied force), \(X(s)\) is the carriage position, and the system output is \(\Phi(s)\) (pendulum deflection angle). The coordinate equation (7) was compiled.

\[X(s) = \left[ \frac{l(m + l^2)}{ml} - \frac{g}{s^2} \right] \Phi(s) \quad (7)\]

For more convenient recording, a replacement (8) was made

\[q = [(M + m)(l + ml^2) - (ml)^2] \quad (8)\]

Transfer function after substitution (9).

\[
\frac{\Phi(s)}{U(s)} = \frac{ml}{q} \frac{s^2}{s^4 + \frac{b(l + ml^2)}{q} \frac{s^3}{q} \frac{(M + m)mg}{q} \frac{s^2}{q} - \frac{bmgl}{q}} \quad (9)
\]

After abbreviation the complete form of the pendulum transfer function was obtained (10).

\[
\frac{\Phi(s)}{U(s)} = \frac{ml}{q} \frac{s^2}{s^4 + \frac{b(l + ml^2)}{q} \frac{s^3}{q} \frac{(M + m)mg}{q} \frac{s^2}{q} - \frac{bmgl}{q}} \quad (10)
\]

And the equation (11) shows the carriage transfer function:

\[
\frac{X(s)}{U(s)} = \frac{\frac{(l + ml^2)}{s} \frac{mg}{q}}{s^4 + \frac{b(l + ml^2)}{q} \frac{s^3}{q} \frac{(M + m)mg}{q} \frac{s^2}{q} - \frac{bmgl}{q}} \quad (11)
\]

A simple analysis of equations (10) and (11) shows that we are dealing with objects of non-minimal-phase type whereas an extensive number of publications are devoted to its control with the detailed simulation using the Matlab [4], wherein, as a rule, the systems of adaptive control with an observer or a reference model are usually built [5, 6]. The identification of such models is particularly difficult in these systems, which complicates significantly the final version of the closed-loop control.
Due to these circumstances, the search for simpler control algorithms is permanently in progress. It is the search for simplified control options that our further discussion is devoted to.

7. Controller synthesis

As an integral part of this task a polynomial controller synthesis was required that would satisfy all necessary conditions and have a minimum order [7, 8].

To reduce the system order the authors neglected the carriage friction which helped to obtain the transfer function of the second-order pendulum (12).

\[
\frac{\Phi(s)}{U(s)} = \frac{ml}{s^2 - \frac{(M+m)mg}{q}}
\]

Transfer function equation after substitution (12).

\[
W_0 = \frac{50}{11s^2 - 343}
\]

Since the final transfer function is of the second order, we could synthesize a first-order polynomial controller using the symbolic methods for solving systems of differential equations according to the method presented in [9].

Using the polynomial controller synthesis method presented in [8], the transfer function of the controller Wp was obtained.

In the process of synthesis, an additional parameter was introduced, which would be an additional input parameter of the final controller. This parameter is the geometric mean root of the characteristic equation (Ω), which in turn will set the system bandwidth.

The final transfer function of the controller Wp with an additional parameter Ω is shown in (14).

\[
W = \frac{(0.66\Omega^2 + 6.86)s + (0.22\Omega^3 + 20.58\Omega)}{s + 3\Omega}
\]

8. System performance analysis

Setting time was considered as the time during which the pendulum is set to a position with a deflection angle of <0.01 radian.

In fig. 4 an oscillogram is shown by means of which the pendulum installation time was estimated. In this case, the starting angle is 0.5 radian and the geometric mean root is 36.

Analyzing Figure 4, one can see that the setup time is 2 seconds. This is due to the large value of the initial deviation and the small value of the geometric mean.

The dependence of the installation time on the geometric mean root was considered. To do so, we will conduct a series of experiments with an initial deflection angle of 0.3 radians. The resulting dependence graph is shown in Figure 6.

Experimental results confirmed the dependence of the installation time on the value of the geometric mean root.

Within the framework of the developed system, the control action level can be controlled in turn by changing the geometric mean root. In fig. 6 the results of experiments are shown with a change in Ω and at a constant starting angle in the form of a dependence of the maximum level of on the value of Ω.
A series of experiments confirmed the possibility of controlling the maximum impact level by means of changes in the geometric mean root in the characteristic equation.

A number of experiments were carried out to outline the attraction area which showed that using a carriage regulator with its significant effect on a carriage position, the attraction area is limited by 1 radian deflection angle.
9. Conclusions
In this work, a control system for structurally unstable power objects (a reverse pendulum on a carriage) was created basing on a polynomial controller with the ability to change the settings dynamically.

During the simulation of the synthesized control system and subsequent experiments, direct dependence of the installation speed on the geometric mean root was established.

The possibility to control the control actions level was also confirmed and made it possible to control the saturation of executive devices.

The comparison of the polynomial controller with the PID controller showed that at small deflection angles the controller works with comparable quality but with an increase in the initial pendulum angle, the polynomial controller works better.

With the analysis of the parameters of the considered regulator, several promising areas for further research were identified:
- improving the quality of prevention of the executive devices saturation;
- bandwidth control based on spectral characteristics of measurement noise;
- system speed is controlled without the readjustment of the controller;
- dynamic control of the area of attraction of systems;
- development of multi-alternative systems.

In fact, the possibility of external parametric control of the processes dynamics was confirmed and the described regulator does not require adaptation and training (the settings are changing instantly).

All the tasks have been completed and the final parameters of the control system meet the specified conditions.

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