Double Hierarchy Hesitant Fuzzy Linguistic LINMAP Method for Multiple Attribute Group Decision Making

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Keywords: Double hierarchy hesitant fuzzy linguistic term set, LINMAP, Multiple attribute group decision making.

Abstract. Compared with hesitant fuzzy linguistic term set (HFLTS), double hierarchy hesitant fuzzy linguistic term set (DHHFLTS) has a second hierarchy linguistic term set, which is a detailed supplementary of each linguistic term included in the first hierarchy linguistic term set. Thus, DHHFLTS can express more complex and sufficient evaluation information than HFLTS. To solve multiple attribute group decision making (MAGDM) problems in which the assessment values of alternatives and the preference relations between alternatives are denoted by double hierarchy hesitant fuzzy linguistic elements, this paper extends the traditional LINMAP method under double hierarchy hesitant fuzzy linguistic environment. A practical example is provided to demonstrate the applicability of the proposed method.

Introduction

Multiple attribute group decision making (MAGDM), an important research field of decision science, is to rank alternatives based on the evaluation information of alternatives associated with multiple attributes. Due to the cost concern, the limited knowledge of decision makers, the nature of considered objects and the unpredictability of events, it is sometimes impossible for decision makers to obtain the quantitative values but only the qualitative ones over the decision variables, and thus Zadeh [1] proposed the fuzzy linguistic approach in which all decision variables are words or sentences in a natural or artificial language but not in the numerical values. However, in fuzzy linguistic approach, only single and simple linguistic terms can be used to express the qualitative opinions, Rodrigues [2] proposed the hesitant fuzzy linguistic term set (HFLTS) in which the membership degree of an element to a given set is denoted by a set of consecutive linguistic terms. Furthermore, Gou [3] added a second hierarchy linguistic term set to be a detailed supplementary of each linguistic term included in the first hierarchy hesitant fuzzy linguistic term set, and proposed the concept of double hierarchy hesitant fuzzy linguistic term set (DHHFLTS).

The Linear Programming Technique for Multidimensional Analysis of Preference (LINMAP) is one of well-known and useful methods for solving multiple attribute decision making problems [4]. The basic idea of LINMAP is to define the consistency and inconsistency indices based on the pairwise comparisons over the alternatives. Based on the consistency and inconsistency indices, a linear programming model is constructed to derive the ideal solutions and attribute weights. Due to the advantage of LINMAP, it has been extended to different environment, such as the intuitionistic fuzzy LINMAP [5], the hesitant fuzzy LINMAP [6], the hybrid LINMAP [7], and the hesitant fuzzy linguistic LINMAP [8]. To the best of our knowledge, there is no LINMAP method proposed for solving double hierarchy hesitant fuzzy linguistic information. Therefore, this paper proposes a double hierarchy hesitant fuzzy linguistic LINMAP method for MAGDM problems in which the assessment values of alternatives and the preference relations between alternatives are denoted by double hierarchy hesitant fuzzy linguistic elements.
To do so, the rest of this paper is organized as follows: Section 2 introduces some definitions related to DHHFLTS; In Section 3, an extended LINMAP method is proposed under double hierarchy hesitant fuzzy linguistic environment; In Section 4, a practical example is provided to demonstrate the applicability of the proposed method; The paper is concluded in Section 5.

Preliminaries

To facilitate the following discussion, some basic definitions related to DHHFLTS are introduced in this section.

Definition 1 [3]. Let \( s = \{ s_t | t = -\tau, ..., -1,0,1, ..., \tau \} \) and \( o = \{ o_k | k = -\zeta, ..., -1,0,1, ..., \zeta \} \) be the first and second hierarchy linguistic term sets, respectively, and they are fully independent, then a double hierarchy linguistic term set (DHLTS), \( S_o \), is in a mathematical form of

\[
S_o = \{ s_{-\tau < o_k >} | t = -\tau, ..., -1,0,1, ..., \tau; k = -\zeta, ..., -1,0,1, ..., \zeta \}.
\]

we call \( s_{-\tau < o_k >} \) a double hierarchy linguistic term (DHLT), where \( o_k \) expresses the second hierarchy linguistic term when the first hierarchy linguistic term is \( s_t \).

Definition 2 [3]. Let \( X = \{ x_1, x_2, ..., x_n \} \) be a finite universe of discourse, and \( S_o = \{ s_{-\tau < o_k >} | t = -\tau, ..., -1,0,1, ..., \tau; k = -\zeta, ..., -1,0,1, ..., \zeta \} \) be a DHLTS, then a double hierarchy hesitant fuzzy linguistic term set (DHHFLTS) on \( X, H_{S_o} \), is in a mathematical form of

\[
H_{S_o} = \{ h_{s_{-\tau < o_k >}}(x_i) > x_i \in X \},
\]

where \( h_{s_{-\tau < o_k >}}(x_i) \), a set of some values in \( S_o \), expresses the possible membership degree of the element \( x_i \) to a given set \( H_{S_o} \), which is denoted as

\[
h_{s_{-\tau < o_k >}}(x_i) = \{ (s_{-\tau < o_k >}, x_i) | s_{-\tau < o_k >} \in S_o; l = 1,2, ..., L; \phi_l = -\tau, ..., -1,0,1, ..., \tau; \varphi_l = -\zeta, ..., -1,0,1, ..., \zeta \},
\]

with \( L \) being the number of double hierarchy linguistic terms (DHLTs) in \( h_{s_{-\tau < o_k >}}(x_i) \) and \( s_{-\tau < o_k >} \) in each \( h_{s_{-\tau < o_k >}}(x_i) \) being the continuous terms in \( S_o \). For convenience, we call \( h_{s_{-\tau < o_k >}}(x_i) \) a double hierarchy hesitant fuzzy linguistic element (DHHFLE).

Definition 3 [3]. Let \( s_{-\tau < o_k >} \) be a DHLT and \( \gamma \in [0,1] \) be a real number, then the transformation functions between them are shown as follows:

\[
f : [-\tau, \tau] \times [-\zeta, \zeta] \rightarrow [0,1],
\]

\[
f(\phi_l, \varphi_l) = \begin{cases} 
\frac{1}{\tau} \times \phi_l + \zeta + \frac{\tau + \phi_l - 1}{2\tau} = \frac{\phi_l + (\tau + \phi_l)}{2\zeta \tau} = \gamma_l, & \tau + 1 \leq \phi_l \leq \tau - 1 \\
\frac{1}{2\tau} \times \phi_l + \frac{\zeta + \frac{\tau + \phi_l - 1}{2\tau}}{\zeta} = \frac{\phi_l + (\tau + \phi_l)}{2\zeta \tau} = \gamma_l, & \phi_l = \tau \\
\frac{1}{2\tau} \times \frac{\varphi_l}{\zeta} = \frac{\varphi_l}{2\zeta \tau} = \gamma_l, & \phi_l = -\tau 
\end{cases}
\]
\[ f^{-1} : [0, 1] \rightarrow [-\tau, \tau] \times [-\varsigma, \varsigma], \]

\[ f^{-1}(\gamma_i) = \begin{cases} [2\tau_i - \tau] & < O_{(2\tau_i - \tau)} > \\ [2\tau_i - \tau] + 1 & < O_{(2\tau_i - \tau - 1)} > \\ \tau - 1 & < O_{(2\tau_i - \tau - 1 - 1)} > \end{cases} \]

Based on the definition given above, the transformation functions between DHHFLE \( h_{S_0} \) and hesitant fuzzy element (HFE) \( h_{\gamma} = \{\gamma_i | \gamma_i \in [0, 1]; l = 1, ..., L\} \) are shown as follows:

\[
F : \Phi \times \Psi \rightarrow \Theta, \\
F(h_{S_0}) = F \left( s_{\theta < o_{\phi}} \in S_{\phi}; l = 1, 2, ..., L; \phi_i \in [-\tau, \tau]; \varphi_i \in [-\varsigma, \varsigma] \right) \\
= \{\gamma_i | \gamma_i = f(\phi_i, \varphi_i)\} = h_{\gamma};
\]

\[
F^{-1} : \Theta \rightarrow \Phi \times \Psi, \\
F^{-1}(h_{\gamma}) = F^{-1} \left( [\gamma_i | \gamma_i \in [0, 1]; l = 1, 2, ..., L\right) \\
= \{s_{\theta < o_{\phi}} > | \phi_i < o_{\phi} \rightarrow f^{-1}(\gamma_i)\} = h_{S_0}.\]

**Definition 4 [3].** Let \( h_{S_0}, h_{S_{\theta}}, \) and \( h_{S_{o_2}} \) be any three DHHFLEs, and \( \lambda \) be a real number, then the operational laws between DHHFLEs are shown as follows:

1. \( h_{S_0} \oplus h_{S_{o_2}} = F^{-1} \left( \bigcup_{\eta_i \in F(h_{S_0})} \left\{ \eta_i + \eta_j - \eta_k \right\} \right); \) (8)

2. \( h_{S_0} \odot h_{S_{o_2}} = F^{-1} \left( \bigcup_{\eta_i \in F(h_{S_0})} \left\{ \eta_i \eta_j \right\} \right); \) (9)

3. \( \lambda h_{S_0} = F^{-1} \left( \bigcup_{\eta_i \in F(h_{S_0})} \left\{ 1 - (1 - \eta)^{\lambda} \right\} \right); \) (10)

4. \( (h_{S_0})^* = F^{-1} \left( \bigcup_{\eta_i \in F(h_{S_0})} \left\{ \eta_i^* \right\} \right); \) (11)

5. \( (h_{S_0})^o = F^{-1} \left( \bigcup_{\eta_i \in F(h_{S_0})} \left\{ -\eta_i \right\} \right); \) (12)

**Definition 5.** Let \( h_{S_{\theta}} = \left\{ s_{\theta < o_{\phi}} ; s_{\theta < o_{\phi}} \in S_{\phi}; l = 1, 2, ..., \# L_\phi; \phi_i \in [-\tau, \tau]; \varphi_i \in [-\varsigma, \varsigma] \right\} \) and

\[ h_{S_{o_2}} = \left\{ s_{\theta < o_{\phi}}^2 ; \right\]
\[ s^2_{\eta \eta} \in S_\eta; l = 1, 2, \ldots, \# L_\eta; \phi_l \in [\tau, \tau]; \varphi_l \in [-\zeta, \zeta] \] be any two DHHFLEs. In general, \#L_1 is not equivalent to \#L_2. For convenience, the short DHHFLE can be extended by the mean value of its upper and lower bounds. Then the normalized Euclidean distance between \(h_{S_{i0}}\) and \(h_{S_{02}}\) is defined as
\[ D\left(h_{S_{i0}}, h_{S_{02}}\right) = \sqrt{\frac{1}{\#L} \sum_{l=1}^{\#L} (\eta'_l - \eta'_2)^2}, \] (13)
where \#L = \max(\#L_1, \#L_2); \(\eta'_l\) and \(\eta'_2\) are the l-th smallest element of \(F\left(h_{S_{i0}}\right)\) and \(F\left(h_{S_{02}}\right)\), respectively.

**Double Hierarchy Hesitant Fuzzy Linguistic LINMAP Method**

Let \(A = \{A_1, A_2, \ldots, A_m\}\) be the alternative set, \(C = \{C_1, C_2, \ldots, C_n\}\) be the attribute set and \(E = \{E_1, E_2, \ldots, E_q\}\) be the decision maker set. Based on DHLTS \(S_\eta\), the decision maker (DM) \(E_p(p=1, 2, \ldots, q)\) uses DHHFLEs \(h_{S_{i0}}^p = \{h_{S_{i0}}^p, b = 1, 2, \ldots, l_{ij}^p\}\) to give the ratings of alternatives \(A_i(i=1, 2, \ldots, m)\) under attributes \(C_j(j=1, 2, \ldots, n)\), where \(l_{ij}^p\) is the number of DHLTs in \(h_{S_{i0}}^p\) and \(h_{S_{00}}^p\) is the b-th smallest DHLT in \(h_{S_{i0}}^p\). Thus, the evaluation information given by \(E_p(p=1, 2, \ldots, q)\) can be expressed by a double hierarchy hesitant fuzzy linguistic decision matrix \(H_{S_{i0}}^p = (h_{S_{i0}}^p)_{\text{mean}}\). Suppose that \(E_p(p=1, 2, \ldots, q)\) provides the preference relations between alternatives by an ordered pair set \(\hat{\Omega}_p = \{(k, i), \hat{t}_p(k, i) | a_{k \geq i} \geq a_i\}\) with the truth degrees \(\hat{t}_p(k, i)\) denoted by DHHFLEs. Let \(\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T\) be the weight vector of attributes, where \(\omega_j\) is the weight of attribute \(C_j\) \((j=1, 2, \ldots, n)\), \(\omega_j \geq 0\) \((j=1, 2, \ldots, n)\) and \(\sum_{j=1}^{n} \omega_j = 1\). Due to the lack of information or time, decision makers usually give partial preference relations of attribute weights, denoted by \(\Lambda\). In what follows, a double hierarchy hesitant fuzzy linguistic LINMAP method is proposed to solve MAGDM problems described above, which are shown as follows:

**Step 1.** Transform the double hierarchy hesitant fuzzy linguistic decision matrices \(H_{S_{i0}}^p = (h_{S_{i0}}^p)_{\text{mean}}\) into the normalized double hierarchy hesitant fuzzy linguistic decision matrices \(H_{S_{i0}}^p = (h_{S_{i0}}^p)_{\text{mean}}\).

\[ h_{S_{i0}}^p = \begin{cases} h_{S_{i0}}^p, & j \in B, \\ \left(h_{S_{i0}}^p\right)^\top, & j \in C, \end{cases} \] (14)

where \(B\) and \(C\) represent the benefit attribute set and the cost attribute set, respectively.

**Step 2.** Construct the group consistency and inconsistency indices. Denote positive ideal solution (PIS) by \(r^+ = (r_{ij}^+, r_{ij}^+, \ldots, r_{ij}^+)\) and negative ideal solution (NIS) by \(r^- = (r_{ij}^-, r_{ij}^-, \ldots, r_{ij}^-)\), which are unknown a priori and need to be determined. \(r_{ij}^+ = h_{S_{i0}}^p = \{h_{S_{i0}}^p, b = 1, 2, \ldots, l_{ij}\}\) and \(r_{ij}^- = h_{S_{i0}}^p = \{h_{S_{i0}}^p, b = 1, 2, \ldots, l_{ij}\}\) are DHHFLEs, in which \(h_{S_{i0}}^p\) and \(h_{S_{i0}}^-\) are the b-th smallest DHLTs in \(h_{S_{i0}}^p\) and \(h_{S_{i0}}^-\), respectively.

According to Eq.(13), the square of distance between \(h_{ij}^p\) and \(r^+ (r^-)\) can be obtained as follows:

\[ S_{ij}^{\text{p}} = \sum_{j=1}^{\#L} \sum_{l=1}^{l_{ij}} \left|F\left(h_{S_{i0}}^p\right) - F\left(h_{S_{i0}}^p\right)\right|^2, \] (15)
\[
S_{ij}^p = \sum_{j=1}^{\Omega_p} \sum_{i=1}^{\Omega_p} \left[ F\left( h_{ijp}^p \right) - F\left( h_{ijp}^q \right) \right] \quad p = 1, 2, \ldots, q; i = 1, 2, \ldots, m.
\]

(16)

Then, the relative closeness of \( h_j^p \) can be defined as

\[
D_j^p = \frac{S_{ij}^p}{S_{ij}^p + S_{ji}^p}, \quad p = 1, 2, \ldots, q; i = 1, 2, \ldots, m.
\]

(17)

For each \((k, i) \in \Omega_p\), if \( D_k^p \geq D_i^p \), the objective ranking order is \( a_k \geq a_i \), which is consistent with the preference relation given by \( E_p \). Conversely, if \( D_k^p < D_i^p \), the objective ranking order is \( a_k \leq a_i \), which is inconsistent with the preference relation given by \( E_p \). A double hierarchy hesitant fuzzy inconsistency index can be defined to measure the inconsistency degree between the objective ranking order of alternatives \( a_k \) and \( a_i \) and the preference relation given by \( E_p \) as follows:

\[
\tilde{B}_{ip} = \begin{cases} \tilde{r}_p(k, i)(D_i^p - D_k^p), & D_i^p < D_k^p \\ 0, & D_k^p \geq D_i^p \end{cases}, \quad p = 1, 2, \ldots, q; k, i = 1, 2, \ldots, m.
\]

(18)

Eq. (18) can be rewritten as \( \tilde{B}_{ip} = \tilde{r}_p(k, i)\max\left\{0, D_i^p - D_k^p\right\} \), and thus the group inconsistency index is defined as follows:

\[
\tilde{B} = \sum_{p=1}^{q} \sum_{(i,j) \in \Omega_p} \tilde{r}_p(k, i)\max\left\{0, D_i^p - D_k^p\right\}.
\]

(19)

Similarly, the group consistency index can be obtained as

\[
\tilde{G} = \sum_{p=1}^{q} \sum_{(i,j) \in \Omega_p} \tilde{r}_p(k, i)\max\left\{0, D_k^p - D_i^p\right\}.
\]

(20)

**Step 3.** Establish a bi-objective mathematic programming model by minimizing the group inconsistency index as well as maximizing the group consistency index to determine the attribute weight vector \( \omega \), PIS \( r^+ \) and NIS \( r^- \).

\[
\begin{align*}
\min & \quad \tilde{B} = \sum_{p=1}^{q} \sum_{(i,j) \in \Omega_p} \tilde{r}_p(k, i)\max\left\{0, D_i^p - D_k^p\right\} \\
\max & \quad \tilde{G} = \sum_{p=1}^{q} \sum_{(i,j) \in \Omega_p} \tilde{r}_p(k, i)\max\left\{0, D_k^p - D_i^p\right\}
\end{align*}
\]

s.t. \( \omega \in \Lambda \)

**Step 4.** Calculate the relative closeness \( D_i^p \) of the alternative \( A_i \) \((i = 1, 2, \ldots, m)\) provided by DM \( E_p \) \((p = 1, 2, \ldots, q)\) by Eq.(17), and then rank alternatives according to the descending order of \( D_i^p \).

**Step 5.** Generate the individual ranking matrix \( X^p = (x_{ij}^p)_{m \times m} \), where

\[
x_{ij}^p = \begin{cases} 1, & \text{if DM } E_p \text{ ranks alternative } a_i \text{ in the } i \text{-th position} \\ 0, & \text{otherwise} \end{cases}
\]

(22)

**Step 6.** Derive the collective ranking matrix \( X = (x_{ij})_{m \times m} \), where
\[ x_y = \begin{cases} 1, & \text{if the decision group ranks alternative } a_i \text{ in the } i - \text{th position} \\ 0, & \text{otherwise} \end{cases} \] (23)

By minimizing the deviation between the individual order of alternatives for each DM and the collective order, a single-objective assignment model is constructed as follows:

\[
\min \left\{ \sum_{i=1}^{n} \sum_{j=1}^{m} \left| x^p_{ij} - x_y \right| \right\} \\
\sum_{i=1}^{n} x_{ij} = 1 \quad (j \in M) \\
s.t. \sum_{j=1}^{m} x_{ij} = 1 \quad (i \in M) \\
x_y = 0 \text{ or } 1 \quad (i, j \in M) 
\] (24)

By solving Eq.(24), we can obtain the collective ranking matrix and select the best alternative.

**Numeric Example**

There is an investment company which wants to invest money into one of the five companies (adapted from [8]). A1 is a car company, A2 is a food company, A3 is a computer company, A4 is a weapon company, A5 is a TV company. The investment company needs taking a decision according to the following four criteria: C1 is the risk analysis, C2 is the growth analysis, C3 is the social-political impact analysis and C4 is is the environmental impact analysis. Let \( S = \{ s_{-3} : extremely \ bad, s_{-2} : very \ bad, s_{-1} : bad, s_0 :medium, s_1 : good, s_2:very \ good, s_3:perfectly \ good \} \) and \( O = \{ o_{-3} : far \ from, o_{-2} : only \ a \ little, o_{-1} : a \ little, o_0 : just \ right, o_1 : much, o_2 :very \ much, o_3: entirely \} \) be the first and second hierarchy LTS, respectively. Based on \( S \) and \( O \), the company invites three decision makers \( E_p (p=1,2,3) \) to provide the ratings of alternatives \( A_i (i=1,2,3,4,5) \) under attributes \( C_j (j=1,2,3,4) \). The double hierarchy hesitant fuzzy linguistic decision matrices \( H_{x_{ij}}^p = (h_{x_{ij}}^p)_{5\times4} \) are listed in Tables 1-3.

![Table 1](image1)

![Table 2](image2)

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According to the judgement and preference of decision makers, they provide the double hierarchy hesitant fuzzy linguistic preference relations as follows:

\[ \Omega_1 = \{(1,2), \tilde{r}_1(1,2) >, (2,4), \tilde{r}_1(2,4) >, (3,5), \tilde{r}_1(3,5) >\}, \]

\[ \Omega_2 = \{(3,1), \tilde{r}_2(3,1) >, (2,3), \tilde{r}_2(2,3) >, (4,3), \tilde{r}_2(4,3) >, (2,5), \tilde{r}_2(2,5) >\}, \]

\[ \Omega_3 = \{(5,2), \tilde{r}_3(5,2) >, (4,1), \tilde{r}_3(4,1) >, (3,4), \tilde{r}_3(3,4) >\}, \]

where

\[ \tilde{r}_1(1,2) = \langle s_{101}, s_{200} \rangle, \quad \tilde{r}_1(2,4) = \langle s_{101}, s_{200} \rangle, \quad \tilde{r}_1(3,5) = \langle s_{101}, s_{200} \rangle, \quad \tilde{r}_2(2,3) = \langle s_{101}, s_{200} \rangle, \quad \tilde{r}_2(3,1) = \langle s_{101}, s_{200} \rangle, \]

\[ \tilde{r}_2(4,3) = \langle s_{101}, s_{200} \rangle, \quad \tilde{r}_2(2,5) = \langle s_{101}, s_{200} \rangle, \quad \tilde{r}_3(5,2) = \langle s_{101}, s_{200} \rangle, \quad \tilde{r}_3(4,1) = \langle s_{101}, s_{200} \rangle, \quad \tilde{r}_3(3,4) = \langle s_{101}, s_{200} \rangle. \]

The weight vector of decision makers is \( w_p = (0.4, 0.4, 0.2) \), and the partial preference information of attribute weights is provided as \( \lambda = \{\omega_1 \geq 0.2, \omega_4 - \omega_3 \leq \omega_1 - \omega_2, \omega_3 \geq 0.05, \omega_4 \geq 0.05\} \). In what follows, the proposed method is used to select the most desirable investment alternative.

Firstly, based on normalizing the double hierarchy hesitant fuzzy linguistic decision matrices \( H_{S_0}^p = (h_{S_0}^p)_{5 \times 4} \), a bi-objective mathematical programming model is constructed by Eq.(21) to determine the PIS, NIS and attribute weight vector, which are shown as follows:

\[ r^+ = \{(0.127, 0.525), (0.734, 0.891), (0.216, 0.348), (0.269, 0.639)\}, \]

\[ r^- = \{(0.230, 0.321), (0.841, 0.891), (0.042, 0.151), (0.453, 0.544)\}, \]

\[ \omega = (0.2, 0.05, 0.7, 0.05)^T. \]

Then, according to Eq.(17), the closeness degree \( D_i^p (p=1,2,3,4,5) \) can be obtained, and the individual ranking matrices \( X_i^p (p=1,2,3) \) are constructed according to the descending order of \( D_i^p \). By minimizing the deviation between the individual order of alternatives for each DM and the collective order, the collective ranking matrix \( X \) is obtained as follows:

\[ X = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}. \]

\[ X^1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad X^2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad X^3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \]

Finally, according to the collective ranking matrix \( X \), the collective ranking order of alternatives can be obtained as \( A_1 > A_2 > A_3 > A_5 > A_4 \), and thus the best investment alternative is \( A_1 \).

**Conclusions**

This paper develops an extended LINMAP method to solve MAGDM problems under double hierarchy hesitant fuzzy linguistic environment. In this method, a bi-objective double hierarchy inconsistency index as well as maximizing the group consistency index to determine the positive ideal solution, negative ideal solution and attribute weight vector. The proposed method has the following
advantages: (1) the assessment values of alternatives and the preference relations between alternatives given by decision makers are denoted by DHHFLs; (2) both the group inconsistency index and group consistency index are considered in this method, which makes the results more precise. In the future, a consistency analysis of double hierarchy hesitant fuzzy linguistic preference relations can be conducted and other methods can be applied to measure the deviations between the evaluation information provided by decision makers and the ideal solutions.

References

[1] L.A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning, J. Information Sciences. 8 (1975) 199-249.

[2] R.M. Rodriguez, L. Martínez, F. Herrera, Hesitant Fuzzy Linguistic Term Sets for Decision Making, J. IEEE Transactions on Fuzzy Systems. 20(1) (2012) 109-119.

[3] X. Gou, H. Liao, Z. Xu, F. Herrera, Double Hierarchy Hesitant Fuzzy Linguistic Term Set and Multimoora Method: A Case of Study to Evaluate the Implementation Status of Haze Controlling Measures, J. Information Fusion. 38(C) (2017) 22-34.

[4] V. Srinivasan, A.D. Shocker, Linear programming techniques for multidimensional analysis of preferences, J. Psychometrika. 38(3) (1973) 337-369.

[5] D.F. Li, G.H. Chen, Z.G. Huang, Linear programming method for multi-attribute group decision making using IF sets, J. Information Sciences. 180(9) (2010) 1591-1609.

[6] X. Zhang, Z. Xu, Interval programming method for hesitant fuzzy multi-attribute group decision making with incomplete preference over alternatives, J. Computers & Industrial Engineering. 75(1) (2014) 217-229.

[7] S.P. Wan, Y.L. Qin, J.Y. Dong, A hesitant fuzzy mathematical programming method for hybrid multi-criteria group decision making with hesitant fuzzy truth degrees, J. Knowledge-Based Systems. (2017) 138.

[8] Y. Xu, A. Xu, H. Wang, Hesitant fuzzy linguistic linear programming technique for multidimensional analysis of preference for multi-attribute group decision making, J. International Journal of Machine Learning & Cybernetics. 7(5) (2016) 845-855.

[9] I. Beg, T. Rashid, TOPSIS for Hesitant Fuzzy Linguistic Term Sets, J. International Journal of Intelligent Systems. 28(12) (2013) 162–1171.