Quasiparticle agglomerates in the Read–Rezayi and anti-Read–Rezayi states

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Received 27 June 2012
Accepted for publication 20 July 2012
Published 30 November 2012
Online at stacks.iop.org/PhysScr/T151/014052

Abstract

We calculate the dominant excitations for the k-level (k ∈ ℕ) Read–Rezayi (RR) states and their particle–hole conjugates, the anti-Read–Rezayi (RR̄), proposed for quantum Hall states. These states are supposed to be built over the second Landau level with total filling factor ν = 2 + ν∗ with ν∗ = k/(k + 2) for RR and ν∗ = 2/(k + 2) for RR̄. In the k-level RR states, based on ℤk parafermions, the dominant excitations are the fundamental quasiparticles (qps) with fractional charge e∗ k = e/(k + 2), with e the electron charge, if k = 2, 3. For k = 4 the single-qp and the 2-agglomerate, with charge 2e∗ k, have the same scaling and both dominate, while for k > 4 the 2-agglomerates are dominant. Anyway the dominance of the 2-agglomerates can be affected by the presence of environmental renormalizations. For all the k-level RR states, the single-qp and the 2-agglomerate have the same scaling and both dominate. In this case, only the presence of environmental renormalizations can make one dominant over the other. We determine the conditions in which the environmental renormalizations of the charged and neutral modes make the Abelian 2-agglomerates dominate over the non-Abelian single-qps in the two models and for any value of k. We conclude by observing that, according to these predictions, the dominance of 2-agglomerates, at very low energies for the ν = 5/2, can be an interesting indication supporting the validity of the anti-Pfaffian model in comparison with the Pfaffian.

PACS numbers: 73.43.–f, 71.10.Pm

1. Introduction

Fractional quantum Hall systems are a unique platform in condensed matter physics to study the peculiar properties of low-dimensional electron systems. The two-dimensional nature of the electron gas opens the possibility to explore a richer class of electron liquids with exotic excitations and intriguing statistical properties. Quasiparticles (qps) with fractional charge, and consequently fractional statistics (neither bosonic nor fermionic), were found [1–3]. The fractional statistics can be not only Abelian but also non-Abelian [4, 5], such as for the Pfaffian or anti-Pfaffian models developed for ν = 5/2 [6–8].

Low-energy effective theories for the edge states have been demonstrated successfully to derive transport and noise properties in the simplest testing device: the quantum point contacts (QPCs) [9]. In the Laughlin sequence ν = 1/(2n + 1) with n ∈ ℤ, the gapless modes at the edges can be described in terms of chiral Luttinger liquid (χLL) with minimal excitations of charge e∗ = e/(2n + 1), where e is the electron charge [10]. The effective edge description for the Jain sequence ν = p/(2np + 1), with p ∈ ℤ, was obtained within the hierarchical models where the minimal charge is e∗ = e/(2np + 1) [11, 12]. In such cases for |p| > 1 the hierarchy predicts the presence of |p| channel (one charged and |p| − 1 neutral) with a hidden SU(|p|) symmetry. The experimental observations of shot noise in the QPC at extremely weak backscattering confirmed the value of the fundamental charges supporting the validity of previous models [2, 3, 13, 14].

More recently, at the lowest possible temperatures, an unexpected increasing of the carrier charges was reported. For Jain’s series with |p| > 1 (such as, for example, ν = 2/5, 2/3) the carrier charge grew up to |νe| [15, 16]. For ν = 2/5 (p = 2), at low enough energy, the dominant excitation
RR, which are based on a non-Abelian [17–19]. This could explain the evolution of the effective charge with a crossover between the two more dominant excitations: the single-qp and the $|p|$-agglomerate. This mechanism, also taking into account the non-universal value of the $\chi$LL exponents (environmental renormalizations) [20], seems enough to explain the experiments for both co-propagating and counter-propagating edge state models [21, 22].

Lately, a similar observation was reported also for $v = 5/2$ where, on further lowering the temperature, the effective carrier charge $e^* = e/(2p+1)$ [23]. We have shown that, also for the case of a non-Abelian model, such as the anti-Pfaffian, in the presence of a renormalization mechanism for the bosonic modes, the agglomerates $e/2$ could dominate at low enough energies [20, 24]. We reported elsewhere very good agreement with the observations using this approach [25].

The $v = 5/2$ Pfaffian and anti-Pfaffian non-Abelian models correspond, respectively, to the $k$-level Read–Rezayi (RR) theory [5] and its particle–hole conjugate, the anti-Read Rezayi (RR) [7, 8], with $k = 2$. Consequently, it is legitimate to ask if similar crossovers could be, in principle, also observed in the generic models based on $k$-level RR theories. Here, we will discuss this issue in detail finding that agglomerates may be dominant at low energies for both RR and RR models with some relevant exceptions [5, 26].

In particular, we found that for $k > 4$ the 2-agglomerate dominates the transport in the $k$-level RR model but, when the charged modes are renormalized by environmental effects, the single-qp could become dominant again. We also found, for RR, an unexpected result for $k = 2, 3$ where the single-qps are always dominant even in the presence of renormalization effects.

For $k$-level RR models the situation is even more complex. Without any renormalization the single-qp and the 2-agglomerate are equally dominant because they have exactly the same scaling dimensions for any $k$. We will see that charged mode renormalization, induced by the external environment, favors the dominance of the single-qp, while the neutral one helps the 2-agglomerates. In this paper, we precisely determine the conditions on the renormalization strengths where one excitation will dominate over the other.

The rest of the paper is organized as follows. In section 2, we present the edge state models we will investigate: the RR and RR. The peculiar algebraic properties of parafermions and the excitation structure of the RR states are described in section 2.1. The peculiar forms of the excitations in terms of the composition of a parafermionic neutral sector and the charge bosonic sector are also discussed. In section 2.2, we investigate instead the excitation structure of the RR models in the disorder-dominated limit [26]. In section 3, we will discuss which are the dominant excitations of the two models by investigating the operator scaling-dimension of the excitations. We will also take into account the possible effects of renormalization induced by the external environment. Finally, in section 4, we will conclude the discussion pointing out some consequences for the non-Abelian models of $v = 5/2$.

2. Edge state models

For the filling factor in the lowest Landau level (LL), the Laughlin and the hierarchical construction were quite successful. Unfortunately, for the fractional values in the second LL, such as $v = 2 + 1/2, 2 + 2/5, 2 + 3/5, 2 + 2/3, \ldots$, many theories are not able to correctly describe the system just because the ‘vacuum’ is now constituted by two filled LLs. Hereafter we consider two of the most successful proposals, discussed in the literature: the RR states and their particle–hole conjugate $\overline{RR}$, which are based on a non-Abelian extension of the fractional statistics [27]. The interest in these models is partly determined by their intrinsic non-Abelian nature that is potentially relevant for topological quantum computation [28].

2.1. RR models

A serious step to go beyond the discussed pitfalls was proposed by Read and Rezayi [5] that introduced a completely new class of wavefunctions, a sort of generalization of the concept of the Pfaffian state [29]. In particular, it was shown, using conformal field theory arguments for $k \geq 2$, that the eigenstate of $k + 1$-body $\delta$-function interaction can be written in terms of a generalization of the Majorana fermions: the $Z_k$ parafermions. These parafermions correspond to an SU(2)/U(1) coset where the central charge is given by $c = (2k - 2)/(k + 2)$ [30]. The $k$-level RR state describes filling factor $v^* = k/(k + 2)$ in terms of a charged and a neutral sector.

The imaginary time action for the edge states in these models is [5, 26]

$$S = \int d\tau dx \left[ \frac{1}{4\pi v_p} \left[ \partial^\tau \varphi_\rho (i\partial_x + v_p \partial_x) \varphi_\rho + L_k \right] \right],$$

where $\varphi_\rho$ is the bosonic charged mode propagating at the velocity $v_p$ with $v_p = k/(k + 2)$ and satisfying the commutation relation

$$[\varphi_\rho(x), \varphi_\rho(x')] = i\pi v_p \text{sgn}(x - x').$$

The particle density operator on the edge can be written as $\rho(x) = \partial_x \varphi_\rho/(2\pi)$. The neutral sector $L_k$ coincides with the SU(2) Weiss–Zumino–Witten model, where the $U(1)$ has been gauged [26]. For example, in the case of $v = 5/2 = 2 + 1/2$, when $k = 2$, the neutral mode is the Majorana fermion $\Psi$ of the Pfaffian state $L_2 = i\Psi(i\partial_x + v_p \partial_x)\Psi$ propagating at velocity $v_p$.

In general, in the parafermionic neutral sector, there are primary fields $\Phi_{j,m}$ with the integer or half-integer number $j$ satisfying $0 \leq j \leq k/2$ and $m \in \{-j, -j + 1, \ldots, j\}$. It is easy to see that $j$ and $m$ are both half-integer or integer (such as in the usual spin algebra where $j$ is the total spin and $m$ is a projection along one axis). The primary fields satisfy additional identities such as $\Phi_{j,m} = \Phi_{j,-m}$ and $\Phi_{j,m} = \Phi_{k/2-j,m+k/2}$ that reduce the number of them to $k(k + 1)/2$. For example, for $k = 2$ we have three primary fields: the identity $\Phi_{0,0} = 1$, the twist field $\Phi_{1/2,1/2} = \sigma$ and the Majorana fermion $\Phi_{1,0} = \Psi$. 


These operators $\Phi_{j,m}$ have the $k$-dependent conformal dimensions [26, 27, 30–33]

$$\delta_{j,m,k} = \frac{j(j+1)}{k+2} - \frac{m^2}{k}$$

(3)

and satisfy the fusion algebra

$$\Phi_{j,m} \times \Phi_{j',m'} \rightarrow \sum_{j''=\min(j+j',k-j')}^{\min(j+j',k-j')} \Phi_{j'',m+m'}$$

(4)

derived from the operator product expansion.

The fundamental charge for this class of states is $e^* = e/(k+2)$. Allowed qps excitations can be written in terms of the product of the neutral field operators $\Phi_{j,m}$ and a standard vertex operator for the charge sector $e^{\omega \psi}$ with the prescription

$$\Psi \propto \Phi_{j,m} e^{\omega \psi}$$

(5)

with the coefficient $\alpha$, which determines the charge of the excitation, given by

$$\{\rho(x), e^{\omega \psi}\} = -\alpha \psi \delta(x-x') e^{\omega \psi}.$$  

(6)

The coefficients $\alpha$, for all the possible excitations, can be determined by requiring the monodromy condition [12, 34–36]. The unit charge operator, with fermionic statistics, obtained with this approach is the electron operator

$$\Psi^{(e)} \propto \Phi_{k/2, 1-k/2} e^{\omega \psi(k+2)/k}.$$  

(7)

The most general excitation is labeled by three numbers $(n, j, m)$ and is written as

$$\Psi_{n,m} \propto \Phi_{j,m} e^{\omega \psi} | k \rangle,$$

(8)

where the integer $n = (m k + 2m)$, with $u \in \mathbb{Z}$. The excitation charge $q_{n,m} = en/(k+2)$ is an integer multiple of the minimal charge $e^*_k = e/(k+2)$. For $n = 1$ we have the single-qp (minimal charge) and for $n \geq 2$ the $n$-agglomerate of qps. For an even $k$ and half-integer (integer) $j$, the $n$-agglomerate charge must be an odd (even) multiple of the minimal charge $e^*_k$.

The single-qp with minimal charge $e^*_k$ is represented by the superpositions of operators according to

$$\Psi^{(qp)} \propto \sum_{m=-1/2}^{1/2} \gamma_m \Phi_{1/2,m} e^{\omega \psi}/k,$$

(9)

where $\gamma_m$ are arbitrary coefficients [27, 31].

For example, for $n = 5/2$, the qp operator is $\Psi^{(qp)} \propto \Phi_{1/2, \pm 1/2} e^{\omega \psi}/2 \equiv \sigma^{e^*_k/2}$ with charge $e^* = e/4$. Recently, experimental observations confirmed the presence of this quarter of an electron charge giving strong support to models based on RR states for this fraction [37–40].

2.2. The particle–hole conjugate RR models

In full analogy with the anti-Pfaffian state, for $v = 5/2$, Bishara et al [26] introduced the $k$-level particle–hole conjugate Read–Rezayi (RR) model. The edges, in these models, are composed by one filled LL$^3$ and a $k$-level RR state of holes superimposed on it. Then the bulk filling factor is $\nu = 2 + \nu^*$ with $\nu^* = 1 - k/(k+2) = 2/(k+2)$.

Hereafter we present only the theory for RR at a fixed point of the disorder-dominated phase because, only for such a limit, an appropriate value of quantized conductance is properly obtained [7, 8, 18, 26]. At this fixed point, one can write the Lagrangian in terms of a bosonic charged mode $\psi_\rho$, with interaction parameter $v_\rho = 2/(k+2)$ and propagating velocity $v_\rho$ and a counter-propagating neutral sector. This sector is composed of two modes with the same velocity $v_\rho$: one bosonic mode $\phi_\rho$, with interaction parameter $v_\phi = 2/k$, and a $k$-level parafermion with the Lagrangian $L_\ell$. The low-energy effective Lagrangian for the edge is [26]

$$\mathcal{L} = \frac{1}{4 \pi v_\rho} \partial_\tau \psi_\rho \left( i \partial_\tau + v_\rho \partial_x \right) \psi_\rho + \frac{1}{4 \pi v_\phi} \partial_\tau \phi_\phi \left( - i \partial_\tau + v_\phi \partial_x \right) \phi_\phi + L_\ell,$$

(10)

where the commutation relations of the fields are

$$[\psi_j(x), \phi_{j'}(x')] = i \pi \delta_{j,j'} \xi_j v_j \text{sgn}(x-x')$$

(11)

with $j = \rho, \sigma$ and where $\xi_j = 1$ ($\xi_\sigma = -1$) indicates the downstream (upstream) mode propagation. The electron operator is given by the $m$-multiplet superposition of the operators $\Phi_{k/2,m}$ according to

$$\Psi^{(e)} \propto \sum_{m=-k/2}^{k/2} \gamma_m' \Phi_{k/2,m} e^{\omega \psi}/(k+2)$$

(12)

with $\gamma_m'$ arbitrary coefficients. All the admissible excitations of the theory are calculated by applying the monodromy condition over these electron operators. Finally, the generic allowed qp excitation is labeled by three numbers $(n, j, m)$

$$\Psi_{n,j,m} \propto \Phi_{j,m} e^{\omega \psi}/n e^*_k,$$

(13)

where $n$ assumes even (odd) values when $j$ is an integer (half-integer). The charge of the generic excitations of equation (13) is $q_{n,j,m} = n e^*_k/(k+2) = n e^*_k$, an integer multiple of the minimal charge $e^*_k = e/(k+2)$. The independence of the charge from $m$ assumes that the operator of an excitation with fixed charge $n e^*_k$, and angular momentum $j$, is given by a $m$-multiplet superposition

$$\Psi_{n,j} \propto \sum_{m=-j \pm 1} \gamma_m'' \Phi_{j,m} e^{\omega \psi}/n e^*_k$$

(14)

with $\gamma_m''$ arbitrary coefficients.

5 The two lowest LLs are again the ‘vacuum’ of the theory and will be neglected.

6 The bar notation over the operators, such as $\bar{\Psi}$, is used to distinguish RR operators from the RR case discussed before.
3. Single-qp versus agglomerate dominance

In the following, we will discuss which are, at low energies, the dominant excitations in the RR and RR models. This can be done by looking at the long-time behavior at $T = 0$ of the imaginary time two-point Green function $(T, \Psi(t)\Psi(t')^\dagger) \propto |t|^{-2\Delta}$ [41] for the general qp operators. Let us start considering the RR states.

3.1. The RR states

The total scaling dimension for the RR states of a generic $n$-agglomerate operator $\Psi_{n,i,m}$ of equation (8) is given by

$$\Delta_{n,i,m}^{(RR)} = \frac{g_\rho}{2} \left( \frac{n^2}{k(k+2)} \right) + \frac{j(j+1)}{k+2} - \frac{m^2}{k}, \quad (15)$$

where the first term is the charge contribution and the other terms come from the neutral parafermionic sector (cf conformal dimension of equation (3)).

In the previous formula, we assumed that, in general, the charge sector could also be ‘renormalized’ by the presence of interactions with the external environments with a factor $g_\sigma \geq 1$. Many mechanisms can cause such renormalization effects such as the coupling with phonons [42], dissipation induced by the electromagnetic environment [43, 44] or the combined effect of out of equilibrium $1/f$ noise and dissipation [20, 24]. We do not know of any mechanisms acting on the parafermions and, consequently, we do not assume any renormalization for the scaling dimension in this sector.

In general, from the previous formula, and from the structure of operators in the RR theory given in equation (8), one can see which is the most dominant excitation of the theory (i.e. the excitation operator with minimal scaling dimension). We found that the $n = 1$ single-qp $\Psi_{1,1/2,\pm 1/2}$ (with charge $e\sigma$) is the dominant excitation for odd $n$, while the 2-agglomerate $\Psi_{2,0,0}$ (with charge $2e\sigma$) dominates for even $n$. The single-qp, based on the primary fields $\Phi_{1,1/2,\pm 1/2}$, has no trivial fusion rules in the parafermionic sector and presents a non-Abelian statistics. The 2-agglomerate, based on the identity operator $\Phi_{0,0} \equiv I$, is instead an Abelian excitation. Between these two excitations one has to find which is the most relevant by comparing directly their scaling dimensions $\Delta_{1,1/2,\pm 1/2}^{(RR)}$ and $\Delta_{2,0,0}^{(RR)}$.

In particular, the single-qp $\Psi_{1,1/2,1/2}$ always dominates for $k = 2, 3$. For $k = 4$ the 2-agglomerate has the same scaling of the single-qp and only for $k > 4$ the 2-agglomerate becomes more relevant.

If we take into account an environmental renormalization $g_\rho \geq 1$, which acts only on the charged modes, the single-qp is typically favored and, in general, always dominates when the renormalization is strong enough, $g_\rho > (k-1)/3$.\footnote{Note that for $k = 3$ one could also imagine the presence of the single-qp $\Psi_{1,1\pm 1}$ but due to the parafermionic identification property $\Phi_{1,1+1} = \Phi_{1,1-1} = \Phi_{1,1+1}$, one demonstrates that it coincides with the single-qp $\Psi_{1,1\pm 1}$.}

3.2. The $\bar{R}R$ states

Let us now consider the $\bar{R}R$ states. The $n$-agglomerate operator, given in equation (13), has the scaling dimension similar to the previous case but now we have an additional contribution from a bosonic neutral mode $\varphi_\sigma$. The scaling dimension becomes

$$\Delta_{n,i,m}^{(\bar{R}R)} = \frac{g_\rho}{4} n^2 + \frac{j(j+1)}{k+2} + \frac{m^2}{k} (g_\sigma - 1), \quad (16)$$

where we took into account the renormalization factors $g_\rho$ and $g_\sigma$ of the charged and neutral bosonic modes, respectively. We discussed in [20] the mechanisms that could determine the renormalization of the neutral modes with the restriction $g_\rho \geq 1$, similar to the charge modes\footnote{These inequalities, with the additional condition for renormalization $g_\rho \geq 1$, show that for $k = 2, 3$ the single-qp is indeed always dominating.}. It is important to note that neutral mode renormalizations can also be stronger than the charged one when $g_\rho > g_\rho$ [20]. In conclusion, hereafter we treat $g_\rho, g_\sigma \geq 1$ as completely independent parameters.

The contribution to the scaling dimension of the bosonic neutral mode component $g_\rho \omega_\nu$, alone is $g_\rho m^2/k$, and it is added to the parafermionic sector contribution of equation (15) giving the result reported in equation (16). Obviously, one has to also take into account that the charge sector has a different $\nu_\rho$, with respect to the RR model. All these differences contribute to creating the peculiar behavior of the RR model described hereafter.

Indeed, we firstly observe that, in the absence of renormalizations ($g_\rho = g_\sigma = 1$), the term proportional to $m^2$ vanishes. The most dominant excitations are of two classes: the non-Abelian single-qp $\Psi_{1,1/2,\pm 1/2}$, with minimal charge $e\sigma = e/(k+2)$, and the Abelian 2-agglomerate $\Psi_{2,0,0}$, with charge $2e\sigma$. These excitations have equal scaling $\Delta_{1,1/2,\pm 1/2}^{(\bar{R}R)}, \Delta_{2,0,0}^{(\bar{R}R)} = 1/(k+2)$.

If, instead, we assume a renormalization of the bosonic modes ($g_\rho, g_\sigma > 1$) depending on the precise values of the parameters, one type of excitation will dominate over the other.

Here we simply calculate the condition to have dominance of the 2-agglomerate over the single-qp $\Delta_{2,0,0}^{(\bar{R}R)} < \Delta_{1,1/2,\pm 1/2}^{(\bar{R}R)}$. This leads to the general relation between the renormalization parameters

$$g_\rho < \left[ 1 + \frac{(k+2)(g_\sigma - 1)}{3k} \right], \quad (17)$$

for which the 2-agglomerate is dominant over the single-qp.

For example, if charged modes are not renormalized ($g_\rho = 1$) and instead the neutral modes are ($g_\sigma > 1$), dominance of the 2-agglomerates is guaranteed. The opposite happens when $g_\sigma = 1$ and $g_\rho > 1$ where indeed the single-qp is dominant.

In general, charged renormalization favors the dominance of the single-qp (in force of their smaller charge) while the neutral mode renormalization leads to dominance of the 2-agglomerate (because it has no neutral contribution $j = m = 0$).\footnote{In [20] it was also demonstrated that the renormalization mechanism, based on the interplay between noise and dissipation, is valid also in the disorder-dominated phase as required for RR models.}
In conclusion, to determine which excitation will be dominant we need precise knowledge of non-universal renormalization parameters \( g_\rho \) and \( g_\sigma \). These parameters can be, in principle, deduced by fitting the transport properties and cross-checking \( a \) posteriori if the excitation observed to be dominant coincides with these theoretical predictions.

For example, we recently considered the anti-Pfaffian case of 5/2 where a comparison with experimental observation \([23, 25]\) is possible. In such a case, the renormalization parameters can be extracted by looking at the scaling of the transport properties because the power laws are also directly affected by the renormalization parameters. We found that the dominance of the 2-agglomerate at low temperatures is predicted to be in full agreement with the backscattering conductance and QPC noise properties. The same experiment consistently indicates that at higher temperatures (higher energies) the dominant charge evolves from the 2-agglomerate to the single-qp. This behavior is observed because the single-qp has higher scaling dimension with respect to the 2-agglomerate. So, with increasing temperatures, single-qp could naturally overcome the agglomerate contribution. This example shows how the comparison of the transport and noise properties in the QPC setup in the weak backscattering could validate some of the results described here.

4. Conclusions

In conclusion, we have demonstrated that, in the \( k \)-level RR states, the single-qp is always the dominant excitation for \( k = 2, 3 \). In the presence of sufficient strong renormalization of the charge sector, \( g_\rho > (k - 1)/3 \), the single-qp dominance is guaranteed for all \( k \)-level RR models.

For the RR states, instead, the single-qp and the 2-agglomerate are equally relevant. In the presence of renormalization effects, the single-qp dominance is favored by renormalizations of the charge modes, while the 2-agglomerate dominance is favored by neutral renormalizations.

Finally, we note that the observation of the dominance of a 2-agglomerate or, even, the crossover between this excitation and the single-qp, in the backscattering conductance and noise transport in QPCs, could provide an indication of the applicability of the \( k \)-level RR or RR models to some specific Hall states in the second LL.

In particular, our analysis may be relevant for \( v = 5/2 \) where the agglomerates were recently seen \([23]\). According to our results, this would indicate that the Pfaffian (2-level RR state) mode is probably excluded because it predicts the dominance of the single-qp independently of any renormalization effect\(^{10}\). Furthermore, the good agreement of the anti-Pfaffian with the intriguing observation of a neutral counter-propagating mode \([16]\) and various transport properties \([23, 25, 37]\) supports the appropriateness of the second model. Anyway the discussion is still highly debated \([45]\).

\(^{10}\) This conclusion is closely associated with the reasonable assumption that the conformal dimension of the parafermionic sector cannot be modified by an external environment. If a similar mechanism is found, this conclusion has to be reassessed.

Certainly, the observation of dominance of agglomerates at low temperatures would probably be possible for many models, including the two most important non-Abelian RR and RR sequences. Conversely, the same observation of dominance of an agglomerate could return interesting information on the model nature.

Acknowledgments

We thank M Sassetti, G Viola and M Carrega for valuable discussions and acknowledge support from the CNR STM 2010 program, the EU-FP7 via ITN-2008-234970 NANOCTM and CNR-SPIN via Seed Project PGESE001.

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