LORENTZ VIOLATION AND EXTENDED SUPERSYMMETRY

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We construct a collection of Lorentz violating Yang-Mills theories exhibiting supersymmetry.

1. Introduction and background
Symmetry has played a fundamental role in the construction of field theories purporting to describe fundamental physics. Nowhere is this more true than in the construction of the Minimal Supersymmetric Standard Model where symmetries mixing bosonic and fermionic states lead to tightly constrained theories often exhibiting remarkable properties. This is particularly true for $\mathcal{N} = 4$ extended supersymmetric Yang-Mills theories,\(^1\) which are known to be finite.

In this note we construct field theories which exhibit $\mathcal{N} = 4$ extended supersymmetry and Lorentz violation. To do so we combine ideas of Berger and Kostelecký\(^2\) involving supersymmetric scalar theories exhibiting Lorentz violation, and well-known constructions of extended supersymmetric theories involving dimensional reduction (nicely described in the work of Brink, Schwartz and Scherk\(^1\)). We begin by establishing notation in the context of the standard construction.

Consider a gauge theory involving a single fermion $\lambda$ and lagrangian
\[
\mathcal{L} = -\frac{1}{4} F^{2} + \frac{i}{2} \bar{\lambda} \gamma^{\mu} \partial_{\mu} \lambda,
\]
where $F$ is the field strength, $F^{\mu\nu} = [D^{\mu}, D^{\nu}]/ig$ and $D$ is the covariant derivative,
\[
D^{\mu} = \partial^{\mu} + igA^{\mu}.
\]
To simplify notation, we will first consider the abelian case. To implement supersymmetry we introduce a supercharge, $Q$, satisfying

$$[P_\mu, Q] = 0, \quad \{Q, \bar{Q}\} = 2\gamma^\mu P_\mu,$$

(3)

where $\gamma^\mu$ are the standard Dirac matrices

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu},$$

(4)

and the energy momentum 4-vector $P_\mu$ generates spacetime translations.

The construction of a supercharge is elegantly carried out in the context of superspace. More precisely, introduce four independent anticommuting variables, $\theta$, and consider the general vector superfield

$$V(x, \theta) = C(x) + i\bar{\theta}\gamma^5 w(x) - \frac{i}{2}\bar{\theta}\gamma^5 \theta M(x) - \frac{1}{2}\bar{\theta} N(x) + \frac{1}{2}\bar{\theta}\gamma^5 \gamma^\mu \theta A_\mu$$

$$-i\bar{\theta}\gamma^5 \theta [\lambda + \frac{i}{2}\bar{\theta} w(x)] + \frac{1}{2}(\theta\bar{\theta})^2(D(x) - \frac{1}{2}\bar{\theta}_\mu \partial^\mu C(x))$$

(5)

and the $Q$ operator

$$Q = -i\partial_\theta - \gamma^\mu \partial_\mu.$$}

(6)

Fixing an arbitrary spinor $\alpha$, standard analysis\(^3\) of the operator $\delta_Q V = -i\bar{\alpha} Q V$ leads to the supersymmetry transformations defining an $\mathcal{N} = 1$ supersymmetric theory.

2. $\mathcal{N} = 1$ supersymmetry

Following Berger and Kostelecký\(^2\), we introduce Lorentz violation by defining a twisted derivative:

$$\tilde{\partial}^\mu = \partial^\mu + k^\mu{}_{\nu}\partial_\nu,$$

(7)

where $k^\mu{}_{\nu}$ is a symmetric, traceless, dimensionless tensor parametrizing Lorentz violation. To obtain a gauge invariant theory, we also twist the underlying connection:

$$\tilde{A}^\mu = A^\mu + k^\mu{}_{\nu}A_\nu.$$}

(8)

These perturbations lead to a general vector superfield

$$\tilde{V}(x, \theta) = C(x) + i\bar{\theta}\gamma^5 w(x) - \frac{i}{2}\bar{\theta}\gamma^5 \theta M(x) - \frac{1}{2}\bar{\theta} N(x) + \frac{1}{2}\bar{\theta}\gamma^5 \gamma^\mu \theta \tilde{A}_\mu$$

$$-i\bar{\theta}\gamma^5 \theta [\lambda + \frac{i}{2}\bar{\theta} w(x)] + \frac{1}{2}(\theta\bar{\theta})^2(D(x) - \frac{1}{2}\bar{\theta}_\mu \partial^\mu \tilde{C}(x))$$

(9)

and perturbed $Q$ operator

$$\tilde{Q} = -i\partial_\theta - \gamma^\mu \partial_\mu.$$}

(10)
Using Wess-Zumino gauge we obtain a lagrangian
\[ \tilde{\mathcal{L}} = \frac{1}{4} \tilde{F}^2 + \frac{i}{2} \tilde{\partial} \lambda + \frac{1}{2} D^2, \tag{11} \]
where $D$ is an auxiliary chiral field and $\tilde{F}$ is the twisted field strength, $\tilde{F}^{\mu\nu} = \tilde{\partial}^{\mu} \tilde{A}^{\nu} - \tilde{\partial}^{\nu} \tilde{A}^{\mu}$. The twisted field strength can be written in terms of the standard SME parameters: $\tilde{F}^{\mu\nu} = F^{\mu\nu} + k_{\mu\nu\alpha\beta} F^{\alpha\beta}$, where
\[ k_{\mu\nu\alpha\beta}^{\mu\nu\alpha\beta} = 2(2k^{\alpha\mu} + (k^2)_{\alpha}^{\mu})g^{\beta\nu} + 4(k^{\mu\alpha} + (k^2)^{\mu\alpha})k_{\beta\nu} + (k^2)^{\alpha\mu}(k^2)^{\beta\nu}. \tag{12} \]
Direct calculation confirms that the action is invariant under the supersymmetry transformations
\[ \delta \tilde{A}^{\mu} = -i \tilde{\alpha}^{\gamma} \gamma^{\mu} \lambda, \]
\[ \delta \lambda = \frac{i}{2} \sigma^{\mu\nu} \tilde{F}_{\mu\nu}^{\alpha} - \gamma^{5} D\alpha, \]
\[ \delta D = \tilde{\alpha}^{\gamma} \beta^{\gamma} \lambda. \tag{13} \]
This defines an $\mathcal{N} = 1$ supersymmetric theory with Lorentz violation.

3. $\mathcal{N} = 4$ supersymmetry

To build an $\mathcal{N} = 4$ supersymmetric theory we work in 4 + 6-dimensional spacetime. We represent the $32 \times 32$ gamma matrices via $\Gamma^{\mu} = \gamma^{\mu} \otimes I_8$ where $I_8$ is the $8 \times 8$ identity matrix and $\mu = 0, 1, 2, 3$, and
\[ \Gamma^4 = \Gamma^{14} + \Gamma^{23}, \quad \Gamma^6 = \Gamma^{34} + \Gamma^{12}, \quad i\Gamma^8 = \Gamma^{24} + \Gamma^{13}, \]
\[ \Gamma^5 = \Gamma^{24} - \Gamma^{13}, \quad i\Gamma^7 = \Gamma^{14} - \Gamma^{23}, \quad i\Gamma^9 = \Gamma^{34} - \Gamma^{12}, \tag{14} \]
where
\[ \Gamma^{ij} = \gamma_5 \otimes \begin{pmatrix} 0 & \rho^{ij} \\ \rho_{ij} & 0 \end{pmatrix} \tag{15} \]
and the $4 \times 4$ matrices $\rho$ are defined by
\[ (\rho^{ij})_{kl} = \delta_{ik} \delta_{jl} - \delta_{jk} \delta_{il}, \]
\[ (\rho_{ij})_{kl} = \frac{1}{2} \epsilon_{ijklm} (\rho^{mn})_{kl} = \epsilon_{ijkl}. \tag{16} \]
We consider the lagrangian
\[ \mathcal{L} = -\frac{1}{4} \tilde{F}^2 + \frac{i}{2} \tilde{\partial} \lambda, \tag{17} \]
where $\tilde{\partial} = \Gamma^{\mu}(\partial_\mu + k_{\mu\nu} \partial^{\nu})$ is the twisted derivative with $k_{\mu\nu}$ parametrizing $SO(1, 9)$ violation and $\tilde{F}$ is the corresponding perturbed field strength.
Imposing both the Weyl and the Majorana condition and compactifying, the fermion $\lambda$ satisfies

$$\lambda = \begin{pmatrix} L\chi \\ R\tilde{\chi} \end{pmatrix},$$

(18)

where $L$ and $R$ denote left and right projection operators, respectively, the spinor $\chi$ transforms as a 4 of $SU(4)$ and the (independent) spinor $\tilde{\chi}$ transforms as a $\bar{4}$ of $SU(4)$.

Choosing the Lorentz violating parameters with care leads to Lorentz violating extended supersymmetric theories which are easy to describe. For example, taking the $k_{\mu\nu}$ to vanish in the compactified directions leads to a lagrangian of the form

$$L = -\frac{1}{4} F^2 + i \bar{\chi} \partial L \chi + \frac{1}{4} \partial_{\mu} \phi_{ij} \partial^{\mu} \phi^{ij},$$

(19)

where the complex scalar fields $\phi_{ij}$ transform as a 6 of $SU(4)$ and are given by

$$\phi_{i4} = \frac{1}{\sqrt{2}} (A_{i+3} + i A_{i+6}),$$

$$\phi^{jk} = \frac{1}{2} \epsilon^{ijkl} \phi_{lm} = (\phi_{jk})^*. \tag{20}$$

The associated action is invariant under the supersymmetry transformations

$$\delta \tilde{A}^\mu = -i (\tilde{\alpha}^i \gamma_{\mu} L^i + \tilde{\chi}_i \gamma_{\mu} L^i),$$

$$\delta \phi_{ij} = -i \sqrt{2} (\tilde{\alpha}_j R \tilde{\chi}_i - \tilde{\alpha}_i R \tilde{\chi}_j + \epsilon_{ijkl} \partial^k L^l),$$

$$\delta L^i = \frac{i}{2} \sigma^{\mu\nu} \tilde{F}^\mu_{\nu} L^i - \sqrt{2} \gamma^i \partial_{\mu} \phi^{ij} R \tilde{\alpha}_j,$$

$$\delta R \tilde{\chi}_i = \frac{i}{2} \sigma^{\mu\nu} \tilde{F}^\mu_{\nu} R \tilde{\alpha}_i + \sqrt{2} \gamma^i \partial_{\mu} \phi^{ij} L \alpha^j. \tag{21}$$

Similarly, choosing the $k_{\mu\nu}$ parameters to vanish in the spacetime directions $\mu = 0, 1, 2, 3$, we obtain a Lagrangian of the form

$$L = -\frac{1}{4} F^2 + i \tilde{\chi} \partial L \chi + \frac{1}{4} \partial_{\mu} \tilde{\phi}_{ij} \partial^{\mu} \tilde{\phi}^{ij},$$

(22)

where $\tilde{\phi}_{ij} = \phi_{ij} + \Lambda_{ijkl} \phi_{kl}$ with the matrix $\Lambda_{ijkl}$ containing the effect of Lorentz violation in the compactified directions. The associated action is invariant under the supersymmetry transformations.
\[ \delta A^\mu = -i(\bar{\alpha}_i \gamma^\mu L\chi^i - \bar{\chi}^i \gamma^\mu L\alpha^i), \]
\[ \delta \tilde{\phi}_{ij} = -i\sqrt{2}(\bar{\alpha}_j R\tilde{\chi}^i - \bar{\alpha}_i R\tilde{\chi}^j + \epsilon_{ijkl} \tilde{\alpha}^k L\chi^l), \]
\[ \delta L\chi^i = \frac{i}{2}\sigma^{\mu\nu} F_{\mu\nu} L\alpha^i - \sqrt{2} \gamma^\mu \partial_\mu \tilde{\phi}^{ij} R\tilde{\alpha}_j, \]
\[ \delta R\tilde{\chi}^i = \frac{i}{2}\sigma^{\mu\nu} F_{\mu\nu} R\tilde{\alpha}_i + \sqrt{2}\gamma^\mu \partial_\mu \tilde{\phi}^{ij} L\alpha^j. \] (23)

Note that if the scalars \( \tilde{\phi}_{ij} \) are identified with physical scalars \( \phi_{ij} \) we restore \( SU(4) \) symmetry and remove any Lorentz violating effects. If, however, the \( \phi_{ij} \) couple to other sectors, Lorentz effects may reappear in these sectors.

4. Extensions and clarifications

The above results warrant a number of additional comments:

- These constructions can be carried out in the nonabelian case where they yield supersymmetric theories which exhibit Lorentz violation.\(^4\)
- The same techniques can be applied to obtain an \( \mathcal{N} = 2 \) supersymmetric theory with Lorentz violation. The construction proceeds by working in \( 4+2 \)-dimensional spacetime and using dimensional reduction.\(^4\)
- Because these constructions involve changing the structure of the underlying superalgebra,\(^4,5\) the no-go results of Nibbelink and Pospelov\(^6\) do not apply.

References

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