We study experimentally and theoretically discrete solitons in crystalline structures consisting of several tens of laser-cooled ions confined in a radiofrequency trap. Resonantly exciting localized, spectrally gapped vibrational modes of the soliton, a nonlinear mechanism leads to a nonequilibrium steady state of the continuously cooled crystal. We find that the propagation and the escape of the soliton out of its quasi-one-dimensional channel can be described as a thermal activation mechanism. We control the effective temperature of the soliton's collective coordinate by the amplitude of the external excitation. Furthermore, the global trapping potential permits controlling the soliton dynamics and realizing directed transport depending on its topological charge.

Transport is one of the most basic phenomena studied in physics. In particular, molecular scale directed transport of matter and energy is of considerable interest [1–3]. Biological 'molecular motors' are submicron machines that consume nondirectional energy to enable directed transport, typically restricted to a one-dimensional (1D) 'track'. At these scales, the fundamental question arises how the self-propelled motion permits directed transport while competing against stochastic forces. Similarly, membrane channels, nanopores and nanotubes can be modeled as 1D or quasi-1D systems with a cross section comparable to the size of the transported ion or molecule. Here, the entire channel forms the machine controlling the rate and direction of matter transport between two regions against a gradient (e.g. electrochemical), acting in addition to the noise.

A prominent model for such dynamics is the Brownian ratchet, or Brownian motor [4]. The basic assumption for the ratchet effect is that all mean forces in the system vanish, and for the Brownian ratchet - the presence of significant noise. To allow the emergence of nonvanishing (mean) currents, the breaking of a symmetry is required - either spatial, temporal, stochastic or spontaneous.

A natural generalization of the single-particle ratchet to a many-body, nonlinear setting can be achieved with solitons, non-perturbative solutions that manifest a collective particle-like behavior [5,6] (fig. 1). However, solitons are not point particles and have some extension in space, in addition to carrying internal degrees of freedom, e.g. oscillatory localized modes. When mobilized, these topologically protected excitations permit the transport of mass, energy, charge, spin and other conserved quantities, in a broad range of optical, atomic, soft matter and solid-state systems [7–28]. Starting with the first theory studies of soliton ratchets, it became clear that the internal modes play a crucial role in the dynamics [29–36]. In particular the modes in general can couple to external periodic excitations, and due to nonlinearities, induce the motion of the soliton. Furthermore, in lattice systems, an effective pinning potential for discrete solitons appears, the so-called Peierls-Nabarro (PN) potential. An ab initio theory of Brownian discrete soliton ratchets is nontrivial and currently limited to 1D systems with a continuum limit [37]. Experimentally accessible are continuum soliton ratchets in Josephson junction devices [38–42], while proposals exist for optical [43], atomic [44] and solid state systems [45].

In this Letter, we demonstrate experimental spectroscopy of internal vibrational modes of a micrometer-
scale discrete soliton. We find that the discrete soliton is capable of rectifying a simple harmonic drive of high frequency, that has a negligible effect in the absence of the soliton. Energy from the transverse drive is converted by a nonlinear mechanism to heat, feeding the low frequency localized mode and propelling the quasi-1D soliton along the crystal axis. At the presence of damping and fluctuations, we show that the discrete soliton can be directed towards one end of its channel at a rate conditional on its topological charge and controllable by global external potentials. Therefore the presented mechanism could serve as a model for soliton-based transport of mass, electric charge or other conserved quantities.

Trapped ions are well suited for studying fundamental concepts down to the quantum level, featuring unique control in preparation, manipulation, and detection of electronic and motional degrees of freedom. Isolated in ultrahigh vacuum, they can be laser cooled to micro-Kelvin temperatures and localized to nanometer scale. The effective potential for an ion near the center of a radiofrequency (RF) Paul trap is approximately harmonic in 3D, with characteristic trapping frequencies $\omega_{(x,y,z)}$. Considering multiple ions, the potential has to be supplemented by the mutual Coulomb interaction and an ordered, self-assembled crystal is formed that can be scaled to a mesoscopic size of interest to investigate many-body physics. Figure 1 shows schematically how, for appropriate trapping frequencies $\omega_x < \omega_y < \omega_z$, the doubly degenerate ground state of such a crystal takes a planar, inhomogeneous zigzag configuration (we denote the mirrored configuration by zigzag). To realize both configurations in one crystal, a localized interface, a domain wall (e.g. the ‘kink’ and ‘kink’ shown in fig. 1(b)], must form, with a higher energy and the properties of a topological soliton.

Such discrete solitons have been recently characterized theoretically and manipulated experimentally and are predicted in circular and helical configurations. In particular, they are proposed to permit quantum coherent manipulation of their internal modes using the rich toolbox of quantum optics developed for trapped ions.

To describe the dynamics of discrete solitons, we start by considering the $3N$ normal modes of $N$ trapped ions, assuming small oscillations around their equilibrium positions. We consider one representative realization with $N = 34$ and experimentally determined trap frequencies $\omega_{(x,y,z)} = 2\pi \times (38.2, 232.3, 293.0) \pm 0.1$ kHz. For the zigzag configuration we find mode frequencies $\omega_{\text{zigzag}}^{(1,\ldots,102)}$ in the range $2\pi \times 38.2$ kHz to $2\pi \times 328$ kHz, while with a kink, the additional nonlinearity broadens the range of $\omega_{\text{kink}}^{(1,\ldots,102)}$ to $2\pi \times 23.2$ kHz to $2\pi \times 345$ kHz. The zigzag and kink have identical modes. A distinct set of internal modes can be attributed to the kink, with the eigenvectors localized at 8 to 10 ions defining the kink.

In the experiment, we first probe the internal modes of the discrete soliton, using a spectroscopic protocol consisting of four steps: (1) Inducing a phase transition from a gas of trapped ions to a Coulomb crystal by laser cooling, (2) in-situ imaging of the crystal to reveal the potential presence of the kink, (3) excitation of the kink using a weak periodic drive, and (4) detecting the configuration, analyzing the kink’s response to the excitation.

In step (1), a kink is formed and stabilized near with near 0.5 probability. (2) The crystallized ions scatter laser photons that are collected in a charge-coupled device (CCD) camera, resolving the ion-ion separation with sub-micrometer accuracy, allowing to differentiate the crystal configurations. (3) During an excitation of duration $t_d$, we modulate the peak voltage ($U_{RF}$) on the

![FIG. 2. Spectroscopy and directed escape of a discrete soliton.](image)
Experimental survival probability of the soliton as function of \( t_d \).

**FIG. 3.** Resonant drive of the radial kink mode leads to a thermally activated escape out of the PN-potential. (a) Experimental survival probability of the soliton as function of \( t_d \) for \( \omega_d/(2\pi) = 327 \text{ kHz} \) and \( \epsilon = 10^{-3} \times \{1.15 \text{ (green triangles)}, 1.30 \text{ (black inverted triangles)}, 1.45 \text{ (blue squares)}, 1.74 \text{ (red discs)} \} \). Solid lines represent exponential fits yielding the corresponding lifetimes of the kink, \( \tau(\epsilon) = \{544 \pm 35 \text{ ms}, 248 \pm 16 \text{ ms}, (71 \pm 5) \text{ ms}, (23 \pm 2) \text{ ms}\} \) respectively. Errorbars represent the 1\( \sigma \) confidence interval. The residual kink loss rate for \( \epsilon = 0 \) is subtracted based on a calibration measurements. (b) Lifetime of the kink in dependence on \( \epsilon \) derived from panel (a), fitted with an overdamped Kramers’ model \(^{69}\) for symmetric confinement (see text for details), indicated by the gray solid line. The experiment control parameter \( \epsilon \) is numerically found to be linearly related to the mean kinetic energy of the ions, leading to an effective temperature. The gray shaded region represents 1\( \sigma \) uncertainty of the fit. We extract a related barrier height of \((26.5 \pm 1.0) k_B T_x\).

Trap’s RF electrodes applying a voltage \( U_d \sin(\omega_d t) \) calibrated by the experimentally determined transfer function of the RF circuit. Defining the relative excitation depth \( \epsilon = U_d/U_{RF} \), the force acting on each ion is derivable from the potential

\[
V_d \propto \epsilon \sin(\omega_d t) \left[ y^2 - z^2 \right].
\]

The constant of proportionality is set by the experimental setup, whereas \( \epsilon \) remains fully controllable. The excitation acts uniformly on the ions’ radial coordinates \( y \) and \( z \), while they remain continuously Doppler cooled by a beam tilted by \(<5^\circ\) from the axial direction \( x \).

We run this sequence for crystals of \(^{24}\text{Mg}\) ions, at least 100 times for each datapoint. Figure 2(a) shows two main resonances where the kink [fig. 2(b)(I)] escapes from the crystal, when scanning \( \omega_d \) up to the highest mode frequencies for \( t_d = 85 \text{ ms} \). These two resonances are close (within their width) to the frequencies of two internal modes of the kink, \( \omega_{kink}^{100} \) and \( \omega_{kink}^{101} \), derived for \( \epsilon \to 0 \). The potential of eq. (1) excites the normal modes, and the reduced peak amplitude for \( \omega_{kink}^{101} \) results from a smaller projection of its eigenvector components for each ion on the radial direction. To shed light on the mechanism leading to the disappearance of the kink at resonance, we image the kink during the driving time \( t_d \). With small \( \epsilon \) and \( \omega_d \) chosen at a resonance, we find an axially blurred trace [fig. 2(b)(II)]. We identify this as an induced excitation of the lowest-frequency kink mode, \( \omega_{kink}^{1} \), a localized shear mode of the two opposing ion chains oscillating \( \pi \)-out of phase, capable of axially translating the soliton. Increasing \( \epsilon \) leads to the dynamics imaged in fig. 2(b)(III), demonstrating that due to the radial drive the kink reaches the axial edge of the crystal and escapes, while the crystal as a whole remains intact.

Detailed Molecular Dynamics (MD) simulations confirm these conclusions. In addition it reveals that in the limit of vanishing damping and kinetic energy, the individual localized mode resonant with \( \omega_d \) is parametrically excited, and, on a slower timescale comparable with \( 1/\omega_{kink}^{1} \approx 50 \mu s \), the energy leaks via nonlinear coupling to the rest of the modes. Additionally considering laser cooling along the \( x \)-axis, the dynamics of a single ion at the low-temperature limit of Doppler cooling can be modeled as a Brownian harmonic oscillator \(^{66–69}\). The damping coefficient \( \gamma_x \) and diffusion coefficient \( D_x \) are determined by the experiment parameters and obey a fluctuation-dissipation relation \( D_x = \gamma_x k_B T_x/m \). The temperature \( T_x \) is of the order of the Doppler-cooling limit \( T_D \approx 1 \text{ mK} \) and \( \gamma_x/m \approx 2\pi \times 0.3 \text{ kHz} \). Similar equations hold for the radial coordinates, with \( \gamma_y, \gamma_z \ll \gamma_x \). Adding these Langevin dynamics to the MD simulations including the trap, drive, and Coulomb interactions of all ions, reveals that a nonequilibrium steady state is reached on a millisecond timescale. The characterization of this state is nontrivial \(^{70}\), however the mixing of spatial directions by the quasi-2D crystal modes leads to a very effective radial damping, and a mean kinetic energy in the crystal at the steady state, \( E_k \), can be defined and is linearly related to \( \epsilon \). Furthermore, using the measured experiment parameters, the shape and position of the resonances in fig. 2 are reproduced quantitatively \(^{70}\).

To further investigate the dynamics of the escape, we experimentally determine the survival probability of the kink as function of \( t_d \), for different values of \( \epsilon \), with \( \omega_d \) resonant at \( \omega_{kink}^{100} \). The survival probabilities shown in fig. 3(a) can be fitted by an exponential decay yielding a mean lifetime \( \tau(\epsilon) \) that decreases with \( \epsilon \) [fig. 3(b) \(^{71}\)]. This evidences a thermal activation mechanism for the kink’s escape across a barrier. As it is known from numerical simulations that the PN potential in a trap becomes effectively harmonic \(^{52}\), the value of the PN potential at the edges of the crystal defines the height of the barrier \( W \). To model the dynamics of the escape, we use the numerically obtained positions of all ions to define an in-
stansstantaneous collective kink coordinate \[ \{kink, kink\} \], adapted from \[ \{67\} \], for the kink’s axial position along the quasi-2D crystal featuring its 1D track. Following its time evolution we find that it is overdamped, by using its time derivative to define the velocity and the velocity autocorrelation function. We fit the effective damping rate in the experimentally relevant temperature regime \[ \{70\} \] originating from phonon scattering, finding \[ g(E_k) \propto E_k^{1/2} \]. Then, assuming that the effective kink coordinate is subject to thermal noise at an effective temperature \[ \{74\} \] defined by \[ k_B T/2 = E_k/(3N) \], we apply Kramers’ model in the overdamped limit \[ \{65, 75\} \] to describe the motion of the soliton \[ \{70\} \{77\} \]. With a barrier height \[ W \], the predicted mean lifetime of the Brownian particle is

\[
\tau \propto g(T) e^{W/(k_B T)}, \tag{2}
\]

with the proportionality constant depending on details of the potential well, independent of \( T \). This approach yields the lifetime averaged over the kink and kink and both directions, and gives \( W = (26.5 \pm 1.0) \ k_B T_D \) based on the experimentally determined \( \tau(\epsilon) \) \[ \{fig. 3(b)\} \].

Finally, we experimentally find a substantial directionality of the soliton transport dependent on its topological charge. We define the transport directionality (TD) as the difference of probabilities to escape to the right and to the left, normalized by their sum. The TD of the kink remains close to zero for all \( \epsilon \) \[ \{fig. 4(a)\} \], while for the kink we find a substantial bias to the right. The existence of a mean current requires a broken symmetry. We extend the description of the harmonic trap potential, accurate at the center of the trap, by nonlinear terms \[ \{78\} \] of third order along the \( x \)-axis \( (L_x) \) and \( y \)-axis \( (L_y) \) and also fourth order and mixed terms. The charge density in the trap is sensitive to these terms and we exploit the positions of the ions as a sensor, by minimizing the weighted least-mean-square shift of the imaged ion positions and the measured frequencies \( \omega_x \) and \( \omega_y \), from their numerically obtained values as a function of the nonlinear coefficients. In particular, we find \( L_x > 0 \), which leads to a shift of the whole crystal towards \( x < 0 \), increasing the left-side PN barrier and decreasing it on the right. Furthermore, \( L_y < 0 \) shifts the crystal to \( y > 0 \), and due to the different radial densities of the kink and kink, results in different PN barriers. The mean PN barrier numerically obtained is \( W = 25.3 \ k_B T_D \), coinciding within errorbars with the experimental value. An intricate interplay of the various global nonlinearity parameters explains the directionality measured in \( \{fig. 4\} \) and we obtain an asymmetric shift of about \( 2 k_B T_D \) for \( W \) on the left and on the right. This differential shift is comparable to the increase of \( T \) with \( \epsilon \), which reduces the soliton’s sensitivity to the differences in the height of these barriers, as evidenced in \( \{fig. 4\} \). Thus, the directionality can be controlled via the nonlinear terms of the global trapping potential and the amplitude of the drive.

To conclude, the external radial drive can be tuned to couple selectively to a kink mode, and energy is drawn and converted to heat by the soliton. This establishes a power transfer, leading to a nonequilibrium steady state given the laser damping. However, despite being microscopically out-of-equilibrium, we find that the axial motion of the soliton can be described by integrating out all degrees of freedom leaving one effective coordinate. The directed transport mechanism (that is the manifestation of the underlying nonequilibrium conditions) arises as a consequence of different barrier heights, conditioned on the topological charge, entering Kramers’ model, with an effective temperature. Typically, realizing a ratchet mechanism requires asymmetric gradients at the single particle scale where the combination of nonlinearity, noise, and nonequilibrium drive raise challenges for an efficient control of the transport. In this work we show how a large scale potential permits the robust control of the soliton transport, its direction and its rate.

These physical processes connect to a broad range of recent work: among those are investigations of nonequilibrium states \[ \{79, 81\} \], the physics of single-particle ratchets in granular chains \[ \{82\} \], in quantum systems \[ \{83, 84\} \], with power law interactions \[ \{85\} \], and recent experiments and theory studies with trapped ions concerning thermal activation \[ \{86\} \], escape dynamics \[ \{87\} \], and steady state heat current formation \[ \{88\} \{91\} \]. The unique controllability of trapped ions further permits investigations of the dynamics of transport. Controlling the global potential allows concatenating crystals along a linear axis with a broken spatial symmetry, or studying ring-configurations \[ \{48, 99\} \], providing periodic boundary conditions \[ \{92\} \{93\} \]. Ground state cooling of internal modes enables access-
ing the quantum regime in mesoscopic, quasi-2D crystals [14, 15]. In addition, trapping of several discrete solitons has been realized [50, 57]. Kink mode excitation has been demonstrated via intensity modulation of a laser beam focused on an individual ion of the crystal [10]. This enables the study of energy transport between kink-lattices, mediated via phonons.

Related work on local kink mode spectroscopy in Coulomb crystals was performed in the context of friction and Aubry transitions [19].

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[1] Frank Jülicher, Armand Ajdari, and Jacques Prost. Modeling molecular motors. Rev. Mod. Phys., 69(4):1269, 1997.
[2] Peter Hänggi and Fabio Marchesoni. Artificial brownian motors: Controlling transport on the nanoscale. Rev. Mod. Phys., 81(1):387, 2009.
[3] R Dean Astumian, Shyantani Mukherjee, and Arieh Warshel. The physics and physical chemistry of molecular machines. ChemPhysChem, 17(12):1719–1741, 2016.
[4] Sergey Denisov, Sergej Flach, and Peter Hänggi. Tunable transport with broken space–time symmetries. Phys. Rep., 538(3):77–120, 2014.
[5] R. Rajaraman. Solitons and Instantons: An Introduction to Solitons and Instantons in Quantum Field Theory. North-Holland Personal Library. North-Holland, 1987.
[6] T. Dauxois and M. Peyrard. Physics of Solitons. Cambridge University Press, 2006.
[7] O.M. Braun and Y.S. Kivshar. The Frenkel-Kontorova Model: Concepts, Methods, and Applications. Texts and Monographs in Physics. Springer, 2004.
[8] Sergej Flach and Andrey V. Gorbach. Discrete breathers - advances in theory and applications. Physics Reports, 467(1-3):1 – 116, 2008.
[9] Michael Öster, Magnus Johansson, and Anders Erksson. Enhanced mobility of strongly localized modes in waveguide arrays by inversion of stability. Phys. Rev. E, 67:056606, 2003.
[10] Ljup ko Hadžievski, Aleksandra Maluckov, Milutin Stepić, and Detlef Kip. Power controlled soliton stability and steering in lattices with saturable nonlinearity. Phys. Rev. Lett., 93:033901, 2004.
[11] T. R. O. Melvin, A. R. Champneys, P. G. Kevrekidis, and J. Cuevas. Radiationless traveling waves in saturable nonlinear schrödinger lattices. Phys. Rev. Lett., 97:124101, 2006.
[12] Rodrigo A. Vicencio and Magnus Johansson. Discrete soliton mobility in two-dimensional waveguide arrays with saturable nonlinearity. Phys. Rev. E, 73:046602, 2006.
[13] O. F. Oxtoby and I. V. Barashenkov. Moving solitons in the discrete nonlinear schrödinger equation. Phys. Rev. E, 76:036603, 2007.
[14] H. Susanto, P. G. Kevrekidis, R. Carretero-González, B. A. Malomed, and D. J. Frantzeskakis. Mobility of discrete solitons in quadratically nonlinear media. Phys. Rev. Lett., 99:214103, 2007.
[15] Uta Naether, Rodrigo A. Vicencio, and Magnus Johansson. Peierls-nabarro energy surfaces and directional mobility of discrete solitons in two-dimensional saturable nonlinear schrödinger lattices. Phys. Rev. E, 83:036601, 2011.
[16] V. Ahufinger, A. Sanpera, P. Pedri, L. Santos, and M. Lewenstein. Creation and mobility of discrete solitons in bose-einstein condensates. Phys. Rev. A, 69:053604, 2004.
[17] Boris A Malomed. Soliton management in periodic systems. Springer New York, 2006.
[18] Jasur Abdullaev, Dario Poletti, Elena A. Ostrovskaya, and Yuri S. Kivshar. Controlled transport of matter waves in two-dimensional optical lattices. Phys. Rev. Lett., 105:090401, 2010.
[19] Thomas Fogarty, C Cormick, H Landa, Vladimir M Stojanović, E Demler, and Giovanna Morigi. Nanofriction in cavity quantum electrodynamics. Phys. Rev. Lett., 115(23):233602, 2015.
[20] Thomas Fogarty, Haggai Landa, Cecilia Cormick, and Giovanna Morigi. Optomechanical many-body cooling to the ground state using frustration. Phys. Rev. A, 94(2):023844, 2016.
[21] Tim Sanchez, Daniel TN Chen, Stephen J DeCamp, Michael Heymann, and Zvonimir Dogic. Spontaneous motion in hierarchically assembled active matter. Nature, 491(7424):431–434, 2012.
[22] Andrew Ward, Fedoor Hilitski, Walter Schwenger, David Welch, AWC Lau, Vincenzo Vitelli, L Mahadevan, and Zvonimir Dogic. Solid friction between soft filaments. Nat Mater, 14(6):583–588, 2015.
[23] Thomas Bohlein, Jules Mikhael, and Clemens Bechinger. Observation of kinks and antikinks in colloidal monolayers driven across ordered surfaces. Nat Mater, 11(2):126–130, 2012.
[24] Alexei Bylinskii, Dorian Gangloff, Ian Counts, and Vladan Vuletić. Observation of Aubry-type transition in finite atom chains via friction. Nat Mater, 2016.
[25] Tae-Hwan Kim and Han Woong Yeom. Topological solitons versus nonsolitonic phase defects in a quasi-one-dimensional charge-density wave. Phys. Rev. Lett., 109(24):246802, 2012.
[26] Sergio Brazovskii, Christophe Brun, Zhao-Zhong Wang, and Pierre Monneau. Scanning-tunneling microscope imaging of single-electron solitons in a material with incommensurate charge-density waves. Phys. Rev. Lett., 108(9):096801, 2012.
[27] Siegmar Roth and David Carroll. Conducting polymers: Solitons and polarons. One-Dimensional Metals: Conjugated Polymers, Organic Crystals, Carbon Nanotubes, Second Edition, pages 85–112, 2015.
[28] P Karpov and S Brazovskii. Phase transitions in ensembles of solitons induced by an optical pumping or a strong electric field. Phys. Rev. B, 94(12):125108, 2016.
[29] Niurka R Quintero, Angel Sánchez, and Franz G Mertens. Anomalous resonance phenomena of solitary waves with internal modes. Phys. Rev. Lett., 84(5):871, 2000.
[30] Niurka R Quintero, Angel Sánchez, and Franz G Mertens.
Anomalies of ac driven solitary waves with internal modes: Nonparametric resonances induced by parametric forces. *Phys. Rev. E*, 64(4):046601, 2001.

[31] Mario Salerno and Niurka R Quintero. Soliton ratchets. *Phys. Rev. E*, 65(2):025602, 2002.

[32] S Flach, Yaroslav Zolotaryuk, AE Miroshnichenko, and MV Fistul. Broken symmetries and directed collective energy transport in spatially extended systems. *Phys. Rev. Lett.*, 88(18):184101, 2002.

[33] Luis Morales-Molina, Niurka R Quintero, Franz G Mertens, and Angel Sánchez. Internal mode mechanism for collective energy transport in extended systems. *Phys. Rev. Lett.*, 91(23):234102, 2003.

[34] CR Willis and M Farzaneh. Soliton ratchets induced by excitation of internal modes. *Phys. Rev. E*, 69(5):056612, 2004.

[35] Pedro J Martínez and Ricardo Chacón. Disorder induced control of discrete soliton ratchets. *Phys. Rev. Lett.*, 100(14):144101, 2008.

[36] J Cuevas, Bernardo Sánchez-Rey, and Mario Salerno. Regular and chaotic transport of discrete solitons in asymmetric potentials. *Phys. Rev. E*, 82(1):016604, 2010.

[37] Bernardo Sánchez-Rey, Niurka R Quintero, Jesús Cuevas-Maraver, and Miguel A Alejo. Collective coordinates theory for discrete soliton ratchets in the sine-gordon model. *Phys. Rev. E*, 90(4):042922, 2014.

[38] E. Trías, J. J. Mazo, F. Falco, and T. P. Orlando. Depinning of kinks in a josephson-junction ratchet array. *Phys. Rev. E*, 61:2257–2266, 2000.

[39] G. Carapella and G. Costabile. Ratchet effect: Demonstration of a relativistic fluxon diode. *Phys. Rev. Lett.*, 87:077002, 2001.

[40] A. V. Ustinov, C. Coqui, A. Kemp, Y. Zolotaryuk, and M. Salerno. Ratchetlike dynamics of fluxuoms in annular josephson junctions driven by biharmonic microwave fields. *Phys. Rev. Lett.*, 93:087001, 2004.

[41] D. E. Shalóm and H. Pastoriza. Vortex motion rectification in josephson junction arrays with a ratchet potential. *Phys. Rev. Lett.*, 94:177001, 2005.

[42] M. Beck, E. Goldobin, M. Neuhaus, M. Siegel, R. Kleiner, and D. Koelle. High-efficiency deterministic josephson vortex ratchet. *Phys. Rev. Lett.*, 95:090603, Aug 2005.

[43] Andrey V Gorbach, Sergey Denisov, and Sergej Flach. Optical ratchets with discrete cavity solitons. *Optics letters*, 31(11):1702–1704, 2006.

[44] Dario Poletti, Tristram J. Alexander, Elena A. Ostrovskaya, Baowen Li, and Yuri S. Kivshar. Dynamics of matter-wave solitons in a ratchet potential. *Phys. Rev. Lett.*, 101:150403, 2008.

[45] Y. Zolotaryuk and M. M. Osmanov. Directed motion of domain walls in biaxial ferromagnets under the influence of periodic external magnetic fields. *The European Physical Journal B*, 79(3):257–262, 2011.

[46] D. J. Wineland and Wayne M. Itano. Laser cooling of atoms. *Phys. Rev. A*, 20(4):1521–1540, 1979.

[47] R. Blümel, J. M. Chen, E. Peik, W. Quint, W. Schleich, Y. R. Shen, and H. Walther. Phase transitions of stored laser-cooled ions. *Nature*, 334:309, 1988.

[48] H. Landa, S. Marcovitch, A. Retzker, M. B. Plenio, and B. Reznik. Quantum Coherence of Discrete Kink Solitons in Ion Traps. *Phys. Rev. Lett.*, 104:043004, 2010.

[49] A. del Campo, G. De Chiara, G. Morigi, M. B. Plenio, and A. Retzker. Structural Defects in Ion Chains by Quenching the External Potential: The Inhomogeneous Kibble-Zurek Mechanism. *Phys. Rev. Lett.*, 105:075701, 2010.

[50] G. De Chiara, A. del Campo, G. Morigi, M. B. Plenio, and A. Retzker. Spontaneous nucleation of structural defects in inhomogeneous ion chains. *New J. Phys.*, 12(11):115003, 2010.

[51] Ch. Schneider, D. Porras, and T. Schaeetz. Experimental quantum simulations of many-body physics with trapped ions. *Rep. Prog. Phys.*, 75(2):024401, 2012.

[52] M. Mielenz, H. Landa, J. Brox, S. Kahra, G. Leschhorn, M. Albert, B. Reznik, and T. Schaeetz. Trapping of topological-structural defects in coulomb crystals. *Phys. Rev. Lett.*, 110(133004), 2013.

[53] K. Pyka, J. Keller, H. L. Partner, R. Nigmatullin, T. Burgermeister, D. M. Meier, K. Kuhlmann, A Retzker, M. B. Plenio, W. H. Zurek, A. del Campo, and T. E. Mehlstubler. Topological defect formation and spontaneous symmetry breaking in ion coulomb crystals. *Nat Comm*., 4:2291, 2013.

[54] S. Ulm, J. Roßnagel, G. Jacob, C. Degünther, S. T. Dawkins, U. G. Poschinger, R. Nigmatullin, A. Retzker, M. B. Plenio, F. Schmidt-Kaler, and K. Singer. Observation of the kibble–zurek scaling law for defect formation in ion crystals. *Nat Comm*., 4, 08 2013.

[55] S. Ejtemae and P. C. Haljan. Spontaneous nucleation and dynamics of kink defects in zigzag arrays of trapped ions. *Phys. Rev. A*, 87:051401, 2013.

[56] H Landa, B Reznik, J Brox, M Miedenz, and T Schaeetz. Structure, dynamics and bifurcations of discrete solitons in trapped ion crystals. *New J. Phys.*, 15(9):093003, 2013.

[57] H L Partner, R Nigmatullin, T Burgermeister, K Pyka, J Keller, A Retzker, M B Plenio, and T E Mehlstubler. Dynamics of topological defects in ion Coulomb crystals. *New J. Phys.*, 15(10):103013, 2013.

[58] Florian Cartarius, Cecilia Cormick, and Giovanna Morig. Stability and dynamics of ion rings in linear multipole traps. *Phys. Rev. A*, 87:013425, 2013.

[59] H. Landa, A. Retzker, T. Schaeetz, and B. Reznik. Entanglement Generation Using Discrete Solitons in Coulomb Crystals. *Phys. Rev. Lett.*, 113:053001, 2014.

[60] Ramil Nigmatullin, Adolfo del Campo, Gabriele De Chiara, Giovanna Morigi, Martin B Plenio, and Alex Retzker. Formation of helical ion chains. *Phys. Rev. B*, 93(1):014106, 2016.

[61] Alexandra V Zampetaki, J Stockhofe, and P Schmelcher. Dynamics of nonlinear excitations of helically confined charges. *Phys. Rev. E*, 92(4):042905, 2015.

[62] S. Marcovitch and B. Reznik. Entanglement of solitons in the fренkel-kontorova model. *Phys. Rev. A*, 78:052303, 2008.

[63] D. Wineland. Superposition, Entanglement, and Raising Schroedinger’s Cat. *Nobel Lecture*, 2012.

[64] We have verified that the dynamic nature of the Paul trap leads in our setup to only small frequency shifts.

[65] H.A. Kramers. Brownian motion in a field of force and the diffusion model of chemical reactions. *Physica*, 7(4):284 – 304, 1940.

[66] D. Leibfried, R. Blatt, C. Monroe, and D. Wineland. Quantum dynamics of single trapped ions. *Rev. Mod. Phys.*, 75(1):281–324, 2003.

[67] Mathieu Marcianete, Caroline Champenois, Annette Calisti, Jofre Pedregosa-Gutierrez, and Martina Knoop. Ion dynamics in a linear radio-frequency trap with a single cooling laser. *Phys. Rev. A*, 82(3):033406, 2010.
[68] J. Javanainen. Light-pressure cooling of trapped ions in three dimensions. *Applied Physics*, 23(2):175–182, 10 1980.

[69] J. Javanainen and S. Stenholm. Laser cooling of trapped particles i: The heavy particle limit. *Applied Physics*, 21(3):283–291, 03 1980.

[70] J. Brox, P. Kiefer, M. Bujak, T. Schaezt, and H. Landa. *In preparation.*

[71] Heather L. Partner, Ramil Nigmatullin, Tobias Burgermeister, Jonas Keller, Karsten Pyka, Martin B. Plenio, Alex Retzker, Wojciech H. Zurek, Adolfo del Campo, and Tanja E. Mehlstäuber. Structural phase transitions and topological defects in ion Coulomb crystals. *Physica B: Condensed Matter*, 460:114 – 118, 2015. Special Issue On Electronic Crystals (ECRYS-2014).

[72] R. Boesch, P. Stancioff, and C. R. Willis. Hamiltonian equations for multiple-collective-variable theories of non-linear klein-gordon equations: A projection-operator approach. *Phys. Rev. B*, 38:6713–6735, 1988.

[73] A. H. Castro Neto and A. O. Caldeira. Transport properties of solitons. *Phys. Rev. E*, 48:4037–4043, 1993.

[74] More elaborate models could be considered in a second step. [91][92].

[75] Peter Hänggi, Peter Talkner, and Michal Borkovec. Reaction-rate theory: fifty years after kramers. *Rev. Mod. Phys.*, 62(2):251, 1990.

[76] P Grigolini, H-L Wu, and VM Kenkre. Brownian motion and finite-temperature effects in the discrete nonlinear schrödinger equation: Analytic results for the nonadiabatic dimer. *Phys. Rev. B*, 40(10):7045, 1989.

[77] PS Lomdahl and WC Kerr. Do Davydov solitons exist at 300K? *Phys. Rev. Lett.*, 55(11):1235, 1985.

[78] N. Akerman, S. Kotler, Y. Glickman, Y. Dallal, A. Keister, and R. Ozeri. Single-ion nonlinear mechanical oscillator. *Phys. Rev. A*, 82:061402, 2010.

[79] E Dieterich, J Camunas-Soler, M Ribezzi-Crivellari, U Seifert, and F Ritort. Single-molecule measurement of the effective temperature in non-equilibrium steady states. *Nat. Phys.*, 11(11):971–977, 2015.

[80] AY Grosberg and J-F Joanny. Nonequilibrium statistical mechanics of mixtures of particles in contact with different thermostats. *Phys. Rev. E*, 92(3):032118, 2015.

[81] Étienne Fodor, Cesare Nardini, Michael E Cates, Julien Tailleur, Paolo Visco, and Frédéric van Wijland. How far from equilibrium is active matter? *Phys. Rev. Lett.*, 117(3):038103, 2016.

[82] V Berardi, J Lydon, PG Kevrekidis, C Daraio, and R Carretero-González. Directed ratchet transport in granular chains. *Phys. Rev. E*, 88(5):052202, 2013.

[83] Leonardo Ermann and Gabriel G Carlo. Quantum parameter space of dissipative directed transport. *Phys. Rev. E*, 91(1):010903, 2015.

[84] Christopher Grossert, Martin Leder, Sergey Denisov, Peter Hänggi, and Martin Weitz. Experimental control of transport resonances in a coherent quantum rocking ratchet. *Nat Commun.*, 7, 2016.

[85] Benno Liebchen and Peter Schmelcher. Interaction induced directed transport in ac-driven periodic potentials. *New J. Phys.*, 17(8):083011, 2015.

[86] J. Liang and P. C. Haljan. Hopping of an impurity defect in ion crystals in linear traps. *Phys. Rev. A*, 83:063401, 2011.

[87] Christoph Petri, Stefan Meyer, Florian Lenz, and Peter Schmelcher. Correlations and pair emission in the escape dynamics of ions from one-dimensional traps. *New J. Phys.*, 13(2):023006, 2011.

[88] Guin-Dar Lin and LM Duan. Equilibration and temperature distribution in a driven ion chain. *New Journal of Physics*, 13(7):075015, 2011.

[89] Alejandro Bermúdez, M Bruderer, and MB Plenio. Controlling and measuring quantum transport of heat in trapped-ion crystals. *Phys. Rev. Lett.*, 111(4):040601, 2013.

[90] Michael Ramm, Thaned Pruttivarasin, and Hartmut Häffner. Energy transport in trapped ion chains. *New J. Phys.*, 16(6):063062, 2014.

[91] Nahuel Freitas, Esteban A Martinez, and Juan Pablo Paz. Heat transport through ion crystals. *Physica Scripta*, 91(1):013007, 2015.

[92] U. Schramm, T. Schätz, and D. Habs. Bunched crystalline ion beams. *Phys. Rev. Lett.*, 87:184801, Oct 2001.

[93] U. Schramm, T. Schätz, and D. Habs. Three-dimensional crystalline ion beams. *Phys. Rev. E*, 66:036501, Sep 2002.

[94] J. Kiethe, R. Nigmatullin, D. Kalincev, T. Schmirander, U. Schramm, T. Schätz, and D. Habs. Three-dimensional crystalline ion beams. *Phys. Rev. E*, 85:036501, Oct 2012.

[95] Jannis Schuecker, Markus Diesmann, and Moritz Helias. How far from equilibrium is active matter? *Phys. Rev. Lett.*, 111(1):010601, 2013.

[96] Alexander Geiseler, Peter Hänggi, and Gerhard Schmid. Kramers escape of a self-propelled particle. *The European Physical Journal B*, 89(8):175, 2016.