To the Shear Strength of Concrete

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Abstract. Interpretations of the "pure shear" phenomenon were presented. The incompatibility of "pure shear" for concrete as a special case of stress state and destruction form was indicated. Due to the variety of cases of concrete and reinforced concrete elements destruction by shearing, it is impossible to establish a concrete shear strength single characteristic, and all its empirical values are particular for individual cases. As an alternative to calculating shear strength, a sufficiently strength calculating general theory of concrete and reinforced concrete elements under complex inhomogeneous stress-strain states, developed on the basis of the ideal concrete plasticity premise on the destruction surface and variational method, was proposed. The developed methodology has been comprehensively tested. An example of using the theoretical results of concrete elements strength solving the problem of the with a two-way strip load application by the variational method in the plasticity theory to determine the concrete strength characteristics was given.

1. Introduction
The issues of concrete and reinforced concrete shear resistance occupy an important place in the general strength theory and have significant practical invention. The degree of their knowledge to a large extent determines the development level of such theory and the optimality of constructive solutions for an elements and joints number.

In connection with the need to solve a wide range practical problems of the strength of concrete and reinforced concrete elements under the action of shearing forces (key joints of slabs with girders and between themselves, girders with columns and columns with foundations in frame buildings; joints of wall panels between themselves and floor slabs in frameless buildings; contact joints of prefabricated-monolithic structures; beams and slabs in the shear forces zone; short elements, etc.), the issue of the concrete resistance to the so-called "pure shear" $f_{c,sh}$ is of particular importance. According to the researchers, its solution would make it possible to determine the strength of any structure that collapses by shearing, using the superposition principle – superimposing shear, compressive, and tensile stresses. Increased attention to this problem has been given since the second half of the XIX century, however, until now, the question of the concrete strength under the shear cannot be considered finally resolved [1-5].

Important enough is the definition of the concept of “pure shear”. In mechanics of a solid deformed body, “pure shear” (shear) is understood as “a kind of plane stress state in which only tangential stresses $\tau$ act on two mutually perpendicular areas oriented in a certain way” [6]. In this case, the
deformations are characterized by a change in the initially right angles of the rectangular element. The main stresses are equal in absolute value, opposite in sign ($\sigma_1 = -\sigma_3$) and directed at an angle of 45$^\circ$ to the edges of the rectangle, and $\sigma_2 = 0$. Thus, on the limit surface in the main stresses axes, the specified definition of “pure shear” corresponds to a point in the “tension-compression” zone, which lies on the angle bisector of the coordinate $\sigma_1$, $\sigma_3$.

However, in the educational and scientific literature concerning the study of concrete and reinforced concrete structures, certain features related to the nature of the elements destruction under the action of shearing forces are added to the definition of a “pure shear”, namely: in its pure form, a shear is a “division of an element into two parts over the cross section to which shearing forces are applied” [7] or “separation of material by shear along a plane on which only shear stresses act” [8].

In the above definitions of a “pure shear”, two sides of the phenomenon under consideration are distinguished or combined: force – a “pure shear” as a case of a plane stress state characterized by the action of only shear stresses and a kinematic – a “pure shear” as a form of fracture characterized by a mutual shear of the parts of an element, separated by shear plane.

In order to combine the above-mentioned sides of the phenomenon under consideration for concrete, for decades, the search for the “most suitable” sample was carried out to determine the concrete shear strength characteristics $f_{c,sh}$. Such a direction of searches seemed quite logical and corresponded to the known experimental data for plastic materials for which the indicated compatibility is possible. The variety of methods for the experimental study of the “pure shear” (more than 10 different samples were proposed) is actually divided into two directions, corresponding to one of the sides of the phenomenon under consideration [8-18]. However, to date, no sample has been found that allows combining the “pure shear” stress state with the fracture form by shearing.

2. Empirical dependences for determining shear resistance

In our opinion, these difficulties are impossible to overcome, because the desire to find the “most suitable” sample in the above sense does not take into account the real concrete physical and mechanical properties. For structurally inhomogeneous stone materials having different compressive and tensile strengths, the phenomena of a “pure shear” as a plane stress state case and destruction form are not compatible [19]. That is, for concrete it is necessary to distinguish between two interpretations of the concept of “pure shear”, which have independent significance. A “pure shear” as a special stress state case is important in the development of strength theories, and the destruction form is often found in practical problems.

Researchers have proposed many empirical dependences to determine the concrete shear resistance. They can be divided into three groups, in which $f_{c,sh}$ is defined as a function of the concrete strength: compression $f_{c,cube}$ ($f_{c,prizm}$) [8, 9, 13]; stretching $f_{cl}$ [8, 9]; depends on both concrete strength characteristics $f_c$ and $f_{cl}$ [14] (Figure 1). An analysis of these dependences shows a large discrepancy in the shear resistance numerical values obtained by various authors [8-17], and almost all of them are confirmed by specific series of experiments. This testifies to their particular nature, which is determined by the sample shape, the loading scheme, and experimental conditions. Thus, due to the variety of shear cases as a destruction form of concrete and reinforced concrete elements, it is impossible to establish a single characteristic of concrete strength on a shear, and only its particular values for individual cases are obtained.

Therefore, the need for developing a fairly general and accurate theory of calculating the strength of concrete and reinforced concrete elements under inhomogeneous stress-strain states is obvious. It should be based on the consideration of the elements destruction stage, explain the physical essence of known phenomena, predict new dependencies and properties, describe with the necessary accuracy the quantitative correspondences of the processes parameters under consideration, and take into account the stress-strain state specifics.
Figure 1. Dependence of concrete shear resistance $f_{c,sh}$ on its strength characteristics (stress in MPa).
Such a technique was developed at the Poltava National Technical Yuri Kondratyuk University [20, 21]. This is the theory of concrete ideal plasticity, supplemented by its applicability limitations and using the variational method for determining the ultimate load using the Balandin-Geniev potential [22]. The theory is sufficiently tested to solve the problems of strength of concrete and reinforced concrete elements during crushing, shearing, and punching [23-26].

3. Method for indirect determination of concrete compressive and tensile strength

In addition to solving the problems of strength testing, described in detail in [23-26], the variational method in the plasticity theory can be used to determine the concrete axial compression and tensile strength. As an example, we consider a concrete plate loaded from two sides during bearing-splitting. Its fracture surface consists of inclined shear planes bounding the compaction wedges whose vertices are joined by a vertical separation crack.

Figure 2 shows the kinematically possible diagram of the destruction of elements in a two-way strip load application.

![Figure 2. Kinematic scheme of the destruction of a concrete element during crushing-splitting.](image)

This problem unknowns are the ultimate load \( F_u \) or \( f_{c,loc} = F_u/A_{loc} \) (\( A_{loc} = l_{loc}b \), \( b \) the element thickness), the angle \( \gamma \) (the angle of destruction site inclination to the vertical) and the velocities ratio \( k = V_1/V_2 \) (when solving problems, we operate with relative velocities, and not their absolute values).

The function for determining the ultimate load is obtained by studying the functional of the variational method for a stationary state.

After determining unknown \( k \) and \( \gamma \) for the conditions \( f_{c,loc} / f_{c,prizm} \rightarrow \min \), we obtain a formula for the ultimate load, which in general can be written as

\[
f_{c,loc} = f \left(f_{c,prizm}, f_{ct}, \alpha \right),
\]

where \( f \) - unknown function, \( \alpha = h/l_{loc} \).

The obtained dependence (1) makes it possible to immediately determine both concrete strength characteristics \( f_{c,prizm} \) and \( f_{ct} \) from tests for bilateral central crushing-splitting of identical samples from the same concrete, but with two different sizes of the bearing area \( l_{loc1} \) and \( l_{loc2} \). This difference allows us to take into account the influence degree of \( f_{c,prizm} \) and \( f_{ct} \) on the samples strength. Obviously, dependences (2) constitute a equations system for determining \( f_{c,prizm} \) and \( f_{ct} \) from given \( \alpha_1 \) and \( \alpha_2 \), and measured in experiments \( f_{c,loc1} \) and \( f_{c,loc2} \).
\[ f_{c,loc1} = f \left( f_{c,prizm}, f_{ct}, \alpha_1 \right), \]
\[ f_{c,loc2} = f \left( f_{c,prizm}, f_{ct}, \alpha_2 \right), \]
where \( \alpha_1 = h/l_{oc1}, \alpha_2 = h/l_{oc2} \).

Solution analysis of the equations system (2) made it possible to obtain graphs convenient for the practical application of the proposed method. As a result, the following algorithm was developed: 1) in relation to the experimental crushing strengths \( \varphi = f_{c,loc1}/f_{c,loc2} \), corresponding to two length values of the test samples bearing area, using the left ordinate and curve A (Figure 3), the desired value \( \chi = f_{ct}/f_{c,prizm} \) is located on the abscissa axis; 2) from the found value \( \chi \), using the curve B and the right ordinate axis, we find the correlation \( f_{c,loc1}/f_{c,prizm} \), which allows us to determine \( f_{c,prizm} \); 3) value \( f_{ct} = \chi f_{c,prizm} \) is determined.

Figure 3. Graphs for determining the concrete strength characteristics \( f_{c,prizm} \) and \( f_{ct} \) from tests of standard cubes for crushing-splitting.

The experiments main goal was to verify the accuracy of the proposed method by comparing it with direct determination of \( f_{c,prizm} \) and \( f_{ct} \) from prism tests for axial compression and tension in the concrete strength range \( f_{c,cube} = 10 \text{ – } 30 \text{ MPa} \), which is practically important for application. As a result of the experimental data analysis, the optimum bearing pads length was \( l_{oc1} = 15 \text{ mm} \) and \( l_{oc2} = 50 \text{ mm} \).

At the second stage, 5 series of samples were tested, differing in composition and strength.

The results of data statistical processing of the tested samples showed the average ratio \( f'_{c,prizm} / f_{c,prizm} \) of compressive strength obtained by the proposed method, \( f'_{c,prizm} \), to \( f_{c,prizm} \) obtained from direct tests, 0.97 with a variation coefficient of 5%, accordingly, for the ratio \( f'_{ct} / f_{ct} \) – 1.07 with a coefficient of variation of 10%.

4. Conclusion
For structurally inhomogeneous materials having different compressive and tensile strengths, the phenomena of a "pure shear" as a plane stress state case and destruction form are not compatible.

Due to the variety of shear cases as a destruction form of concrete and reinforced concrete elements, it is impossible to establish a single characteristic of concrete shear strength, but it is necessary to solve individual problems for specific cases of destruction by shear.

As a tool for solving problems, a variational method is proposed in the plasticity theory of concrete.
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