Asymmetric resonance in selective reflection: Explanation via Fano-like mechanism

Denis V. Novitsky

B.I. Stepanov Institute of Physics, National Academy of Sciences of Belarus, Nezavisimosti Avenue 68, 220072 Minsk, Belarus.

Fano mechanism is the universal explanation of asymmetric resonance appearing in different systems. We report the evidence of Fano-like resonance in selective reflection from a resonant two-level medium. We draw an analogy with the asymmetric resonance previously obtained for the coupled oscillators. We also take into account the effects of dielectric background and local-field correction and connect our results with optical bistability.

PACS numbers:

From the first observation of asymmetric resonance in the beginning of the twentieth century, it was observed and studied in different physical systems. Many works of the last fifty years were inspired by the pioneering investigation of Ugo Fano [1] who proposed the universal mechanism of asymmetric resonances appearance. The essence of this mechanism is the process of destructive and constructive interference of two paths along which the system can evolve. One of these paths concerns the usual symmetric (Breit-Wigner) resonance, while the second one goes through the broad continuum. In other words, there is the superposition of discrete and continuum states. This superposition, or interference, can be obtained in different classical and quantum systems leading to the characteristic resonance with asymmetric profile known as Fano resonance.

Fano resonances were widely studied previously in terms of atomic, optical, and condensed-matter physics. The last decade was marked by enormous interest to Fano-like behavior of photonic, plasmonic, and nonos- sium systems. We report the evidence of Fano-like resonance in selective reflection from a resonant two-level medium. We draw an analogy with the asymmetric resonance previously obtained for the coupled oscillators. We also take into account the effects of dielectric background and local-field correction and connect our results with optical bistability.

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\[ \frac{\partial R}{\partial t} = -i \frac{\mu}{2\hbar}(EW + i(\Delta - \ell\varepsilon W)R - \gamma_2 R), \]
\[ \frac{\partial W}{\partial t} = -i \frac{\mu}{\hbar}(\ell^* E^\ast R - \ell E R^\ast) - 2i(\ell^* - \ell)\varepsilon|R|^2 - \gamma_1(W - W_{eq}), \]

where \( E \) is the amplitude of macroscopic electric field, \( \mu \) is the transition dipole moment, \( \Delta \) is the detuning of radiation from resonance, \( \gamma_1 \) and \( \gamma_2 \) are the population and polarization relaxation rates respectively, \( W_{eq} \) is the value of inversion at equilibrium. The parameter \( \varepsilon = 4\pi N\mu^2/3\hbar \) is responsible for Lorentz shift due to local-field correction, \( N \) is the density of two-level atoms per unit volume, \( \hbar \) is the Planck constant. Here \( \ell = (\varepsilon_d + 2)/3 \) is the local-field enhancement factor due to polarizability of host material with dielectric constant \( \varepsilon_d = n_d^2 \) (generally, complex, however we restrict ourselves to nonabsorbing dielectrics).

In stationary regime the dependence of inversion on light intensity is described by the cubic equation

\[ (W - W_{eq})[i(\delta - \ell bW) - 1]^2 + WI = 0. \]

Here \( I = \mu^2|\ell E|^2/\hbar^2\gamma_1\gamma_2 \), \( \delta = \Delta/\gamma_2 \) and \( b = \varepsilon/\gamma_2 \) are the normalized parameters.

Equation (3) has three solutions which take on real values in different ranges of parameters, which results in appearance of intrinsic optical bistability at certain conditions [11,12]. Therefore, one has to link these solutions to obtain the full spectrum. In order to simplify, we consider the case of absence of the local-field Lorentz shift at first. This corresponds to quite low values of \( \ell b \), below the threshold of bistable response. Further we will analyze the influence of Lorentz shift on the resonant reflection. So, neglecting the term \( \ell b W \) in Eq. (3), we
obtain the solutions of the Bloch equations as follows,
\[ W = \frac{W_{eq}(\delta^2 + 1)}{\delta^2 + 1 + I}, \]
\[ R = \frac{\mu P_{eq}(\delta - i)}{2\hbar \gamma_2 (\delta^2 + 1 + I)} E. \]

Now we can calculate the dielectric function of the complex medium containing two-level atoms inside a dielectric as
\[ \varepsilon = 1 + 4\pi P/E, \]
where the polarization is given by [10]
\[ P = \frac{\varepsilon_d - 1}{4\pi} E + \ell P_{res}. \]

Here \( P_{res} = 2N\mu R \) is the nonlinear polarization due to resonant atoms. Substituting the stationary solutions [11] and [10], the final expression is
\[ \varepsilon = \varepsilon_d + \frac{3\ell^2 b W_{eq}(\delta - i)}{\delta^2 + 1 + I}. \]

Let us consider a monochromatic electromagnetic wave incident normally to the interface between the outer medium with the refractive index \( n_0 \) and two-level medium with the dielectric permittivity [5]. Assuming the thickness of the latter medium much less than the light wavelength, we can avoid the necessity of taking into account the propagation effects. Then reflection of light by the interface can be described by the amplitude coefficient as follows,
\[ r(\delta) = \frac{\sqrt{\varepsilon - n_0}}{\sqrt{\varepsilon + n_0}}. \]

Using Eq. [8], one can easily obtain the explicit expression for the relation \( r(\delta) \). However, we will not write it out, since it is awkward and not illustrative. Further we just demonstrate it in graphical form.

Figure 1 shows spectral dependencies of refraction and absorption indices, as well as of modulus and argument of the reflection coefficient [9]. It is seen that the modulus of \( r \) follows the symmetrical curve in the case of two-level atoms in vacuum, while its phase demonstrates the phase jump by \( \pi \) near the resonance. The reflection from the dielectric itself is described by continuum-like curves. The combination of two-level atoms and dielectric results in fundamentally another behavior: Asymmetric line-shape for modulus, and the gain and the subsequent drop for the phase of the reflection coefficient. This behavior is similar to that of the excited oscillator coupled to another one [14]. We treat this behavior as the evidence of Fano resonance in reflection from two-level atoms embedded in a dielectric medium.

The role of asymmetry parameter plays the difference \( \Delta n = n_d - n_0 \) between the refractive indices of the host dielectric and the outer medium. This is seen from Fig. 2(a). At \( \Delta n = 0 \) we have a symmetric resonance, while at nonzero values of this difference the asymmetric curves are observed with minimal values of reflection on the left or on the right of \( \delta = 0 \) depending on the sign of \( \Delta n \). However, the situation is more complex: Comparison of Figs. 2(a) and (b) shows that curves at the same \( |\Delta n| \) and different signs are not exactly the same. It is also seen that the value of reflection at minimum is not zero and gets larger as the difference \( \Delta n \) is increasing. Continuing our analogy, one can say that, in the case of coupled oscillators, the curve cannot reach zero as well if only the
local field is not large at low values of coefficient

\begin{equation}
0.024 \quad 0.026 \quad 0.028 \quad 0.03 \quad 0.032
\end{equation}

and inversion is governed by the cubic equation \(3\). Comparison between this description and those by Eq. \(8\) is presented in Fig. 3. It is seen that influence of local field effect; dotted lines were calculated using Eq. \(8\).

Now let us consider the effect of local field on the asymmetrical resonance. The description of medium is then given by the same equations \(6\) and \(7\), where the microscopic polarization is

\begin{equation}
R = \frac{1}{2i} \frac{iW\Omega}{\delta - \Omega W} - 1.
\end{equation}

FIG. 3: Spectral curves for the absolute value of the reflection coefficient at different values of parameter \(b\): (a) \(b = 0.5\), (b) \(b = 1\), (c) \(b = 2\), (d) \(b = 5\). Solid lines correspond to the calculations taking into account of local field effect; dotted lines were calculated using Eq. \(8\).

The results considered above were obtained for the local interface between the outer medium and the two-level medium in the dielectric host. Further we try to verify them for more general and realistic case of the resonant medium of finite thickness. Our calculations were performed within the framework of the iteration matrix method discussed in detail previously \[16\]. This approach is sufficient to obtain stationary characteristics, though, in general, one needs to use the coupled Maxwell-Bloch equations to study light propagation \[9\]. Figure 4 shows that, for the layers thin in comparison with the radiation wavelength (for example, \(L = \lambda/10\)), we have qualitatively the same asymmetric resonance as in local case (Fig. \(1\)). However, if the the thickness of the layer is about or larger than \(\lambda\) some local minima appear in the spectral curves of the reflection coefficient. These additional minima are due to propagation effects, that is inhomogeneous distribution of the medium properties (refractive index) along layer’s thickness. At the same time, the dip in transmission is getting wider for thicker layers due to large absorption.

As demonstrates Fig. 4(c), the difference \(\Delta n\) can still be considered as the Fano asymmetry parameter though even for \(\Delta n = 0\) the curve of resonance (solid line) is distorted because of propagation effects. These distortions also grow up as the local-field parameter \(b\) increases [Fig. 4(d)].

FIG. 4: Spectral curves for the absolute value of (a, c, d) reflection coefficient, (b) transmission coefficient. The calculations were performed for the layers of different (a, b) thickness (in units of the light wavelength \(\lambda = 0.5\) \(\mu m\)); (c) outside refractive index \(n_0\) (at \(b = 1\), \(L = \lambda\)), (d) parameter \(b\) (at \(n_0 = 1\), \(L = \lambda\)). Note that local field correction is taken into account.
In conclusion, we considered a simple model of selective reflection from a resonant medium which consists of two-level atoms embedded in a dielectric host. The asymmetric curve of the reflection coefficient can be explained in terms of Fano resonance with the difference between refractive indices of outer and host media as the asymmetry parameter. This model can be considered as one more evidence of universality of Fano mechanism alongside with classical coupled resonators [14] and recently reported optically driven atomic force microscope cantilever [17].

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