Implementation of genetic algorithms for supersonic airfoil optimization

A Michelotti, A Cavini, R Giacopino, F Misino and L Piottoli
Department of Mechanical and Aerospace Engineering, Politecnico di Torino, Corso Duca degli Abruzzi 24, Turin, TO 10129, Italy
PoliTOrbital, Politecnico di Torino, Corso Duca degli Abruzzi 24, Turin, TO 10129, Italy
Email: politorbital.polito@gmail.com

Abstract. This paper describes the exploitation of Genetic Algorithms for the selection and optimization of supersonic airfoils. The main objective of the optimization is to ensure the possibility to reach the maximum efficiency at a given lift coefficient. The surrogate model presented in this paper implements the shock-expansion theory, and the optimization problem is constrained with respect to three design variables, since Bézier curves are used for the parameterization of the Diamond-Shaped and the 40-elements Double Circular Arc airfoil geometries. The airfoils were optimized for three different Mach numbers and two different lift coefficients. A thickness-constrained optimization has been run to evaluate the possibility to obtain a specific lift coefficient at a given Mach number and to understand how this parameter influences the optimized airfoil shape performance in terms of efficiency and the range of feasible angles-of-attack. An off-design evaluation is also presented, allowing for a comparison of the optimized geometries in terms of versatility. High fidelity models were validated with RAE 2822 airfoil using STAR CCM+, and they were compared to the surrogated model to ensure higher quality in the results. In conclusion, the Genetics Algorithms optimization coupled with the shock-expansion theory model results to be a fast and valuable solution to select and optimize airfoil solutions. The application of this approach has shown that a 20% reduction in airfoil thickness leads to a 25% efficiency improvement, while widening the range of viable angles of attack.

1. Introduction
As supersonic flight regime is computationally expensive, the aim of this paper is to investigate a methodology to study and optimize supersonic airfoil shapes using Genetic Algorithms with low computational demand.

1.1. Supersonic Airfoil profiles
Two kinds of suitable airfoil shapes for supersonic flight, the Diamond and the Double Circular Arc ones, were compared.

1.1.1. Double circular arc airfoil. This profile is characterized by a smoother evolution of the flow around the surface [1]. This should guarantee a better boundary layer behaviour in viscous flow. For convenience, the original ones will be denoted as symmetrical throughout the paper: when not, the optimized ones are considered.
2. Methodology

2.1. Shock-expansion theory
The shock-expansion theory is characterized by the study of sequences of shockwaves and expansion fans around a geometry made up of straight-line segments.
It leads to the exact calculation of the forces applied on a two-dimensional supersonic airfoil in inviscid flow [2].

2.2. Optimization algorithm
Using MATLAB, an algorithm which to implement the shock-expansion theory along the profiles has been written.
This methodology provided a well-suited surrogated model for all the further investigations.
After that, a Genetic Algorithm has been used to optimize the geometry to reduce the Drag coefficient: the problem was constrained with a target Lift coefficient value.
In the end, the results obtained from the two study cases were compared.

2.2.1. Problem formulation. The aerodynamics design problem was a Lift-constrained and Drag minimization one. The generic expression for this kind of problems might be:

$$\mathbf{x}^* = \arg\min \mathbb{H}(\mathbf{f}(\mathbf{x})) \mid \mathbf{h}(\mathbf{x}) = 0, \mathbf{l} \leq \mathbf{x} \leq \mathbf{u}$$

in which $$\mathbf{x}^*$$ is a vector containing optimized design variables, $$\mathbf{x}$$ represents the design variables, $$\mathbf{f}(\mathbf{x})$$ is the fitness function which must be minimized, $$\mathbf{h}(\mathbf{x})$$ is the vector of equality constraints and $$\mathbf{l}$$ and $$\mathbf{u}$$ are the lower and upper boundaries of the design variables.

2.2.2. Airfoil parameterization. To modify the geometries involved, Bézier curves were defined as

$$s(t) = \sum_{i=0}^{n} v_i B_i^n(t) \quad t \in [0,1]$$

in which Bernstein polynomials were used

$$B_i^k(t) = \binom{k}{i} (1 - t)^{(k-i)} t^i$$

with $$i = 0, ..., k$$ and $$k$$ equal to curves grade. The choice of a quadratic curve led to the use of three control points: two of them were coincident with the leading and the trailing edge and the last one was free, so the shape of the airfoil could be defined.

2.2.3. Design variables. Three variables were chosen:
- the $$x$$ and $$y$$ coordinates of the control point $$P(x, y)$$
- the angle of attack $$\alpha$$.

The last one was primarily used to reach the specific Lift coefficient value used as a constraint.
The resulting design variable vector was:

$$\mathbf{x} = [x_{ctrl}, y_{ctrl}, \alpha]$$

2.2.4. Boundary conditions. The boundary conditions applied to the design variables are expressed in table 1.
Table 1. Design variables bound conditions.

| Parameter | Lower Bound | Upper Bound |
|-----------|-------------|-------------|
| $x_{ctrl}$ | 0           | 1           |
| $y_{ctrl}$ | 0.1         | 2           |
| $\alpha$  | -20         | 0           |

In figure 1 the bound geometries are compared to the original ones.

Figure 1. Diamond-shaped (a) and Double Circular arc (b) bound geometries compared to the original ones.

2.3. Genetic Algorithm
The ga function combined with the auxiliary patternsearch function was used for this purpose. Both functions are part of the Optimization Toolbox.

2.3.1. Optimization parameters. The parameter used in the GA process are defined in table 2.

Table 2. Optimization parameters.

| Parameter          | Value                   |
|--------------------|-------------------------|
| Number of intervals| [2, 40]                 |
| Chord length [m]   | 1                       |
| Mach number        | [2.5, 3, 3.5]           |
| Lift coefficient value | [0, 0.5]           |
| Population size    | 50                      |
| Selection process  | Tournament              |
| Hybrid function    | patternsearch           |

In Tournament selection, $k$ individuals are randomly chosen, and their fitness values are compared: the best one is then selected, and it is placed into the mating pool. The patternsearch hybrid function uses the pattern search optimization method which does not require the evaluation of the gradient of the objective function.

2.4. CFD validation
Well-known case studies were considered first: two-dimensional inviscid and turbulent transonic flow around the RAE-2822 Airfoil with wind tunnel experimental data provided by NASA [4] was compared with obtained STAR-CCM+ results.
2.4.1. Physical Model. The parameters used for defining the physical model are given in table 3.

| Parameter                        | Value                                      |
|----------------------------------|--------------------------------------------|
| Mesher                           | Polygonal                                  |
| Grid dimensions [m]              | C-type – 200 x 200                         |
| Minimum / Maximum cell size [m]  | 2E-3 / 10                                  |
| Gas model                        | Ideal, Compressible, Viscous               |
| Turbulence model                 | k-ω SST                                    |
| Solver                           | Energy Coupled                             |
| Time                             | Steady                                     |
| Mach number                      | 0.729                                      |
| Angle of Attack [deg]            | 2.31                                       |
| Reynolds number                  | 6.5E6                                      |
| Prism layer near wall thickness [m] | 4.152E-6                                 |
| Total boundary layer height [m]  | 0.016                                      |
| Number of layers                 | 25                                         |

2.4.2. Adaptive mesh refinement. Two field functions were used to refine the initial mesh in proximity of the expected shockwave location and the most complex flow field zones. The first function in equation (1) represents the sampling of the reference quantity expressed in logarithmic scale and allowed to evaluate the velocity variations.

\[
f(V) = \log(|\nabla(|\nabla(V)|)|) \tag{1}
\]

Subsequently, the domain zones with higher gradients, such as in the presence of an oblique impact, were detailed using equation 2.

\[
f(V) = \log(|\nabla(|\nabla(V)|)|) \tag{2}
\]

The results can be seen in figure 2.

![Figure 2. Mesh adaptation and field values using the refinement function.](image)

2.4.3. Data comparison. The relative errors between the different simulations and the experimental data are shown in table 4 as function of the mesh cell number.
Table 4. Comparison between experimental data and simulation results

| Cell Number | Lift Coefficient | Experimental Data | Relative Error | Drag Coefficient | Experimental Data | Relative Error |
|-------------|------------------|-------------------|---------------|------------------|------------------|---------------|
| 186k        | 6.993E-1         | 0.675             | 0.036         | 1.145E-2         | 0.0101           | 0.13          |
| 327k        | 6.987E-1         | 0.675             | 0.034         | 1.129E-2         | 0.0101           | 0.12          |
| 614k        | 6.976E-1         | 0.675             | 0.033         | 1.106E-2         | 0.0101           | 0.09          |

In figure 3 the different pressure coefficient distributions around the airfoil are compared [5].

2.5. CFD validation of the optimized geometries

The methodology developed previously was used to simulate both shapes optimized with the hybrid-function at \( Mach = 3.5 \) and \( C_L = 0.5 \).

In figure 4 the higher-order results are compared with the shock-expansion theory described before.

The curves are very close, but the lift coefficient values are less than the analytical ones: this might be due to numerical approximation in CFD simulations.

Another important aspect is that, unlike shock-expansion theory, the CFD simulations highlight a wider range of angles of attack for the Double Circular Arc shape.

Also, in the second plot, the original shape data is well fitted, and its angle of attack range is higher than the analytical one.
2.6. **Thickness-constrained optimization**

To better understand the optimization process, the maximum airfoil thickness has been used as a constraint. Having neglected the effects of the real gas model, a constraint over the local maximum Mach number due to the expansion of the flow around the airfoil has been set. Because real gas effects begin when air dissociation occurs, the local maximum Mach number has been set as 5 and 6. In particular, the lower the value, the more stringent the requirement. The goal was to achieve the highest desired lift coefficient value for a specific airfoil thickness for every Mach number used in the optimization process. Figure 5 shows the conducted test cases.

![Figure 5](image_url)

**Figure 5.** Comparison of obtained lift coefficient values at different Mach numbers.

The green block means that the optimization was successful; a red block means otherwise. As can be seen, considering a local maximum Mach number of 6, a 0.5 lift coefficient cannot be obtained for any Mach number other than 2.5; this is because the flow energy is too high for higher Mach number values. For thickness values up to 0.1, a lift coefficient of 0.4 can be obtained even for a Mach number of 3.5. Lowering the local maximum Mach number to 5, the highest reachable lift coefficient for a Mach number of 3.5 is 0.3.

2.7. **Effects of thickness reduction**

As can be seen from figure 6 and figure 7, as thickness reduces from 0.1 to 0.08, the angle of attack range becomes wider, the efficiency has a 25% improvement, and its maximum value is higher for lower angles of attack. The symmetrical double circular arc airfoil has higher efficiency but a 0.4 lift coefficient cannot be reached.

![Figure 6](image_url)

**Figure 6.** Comparison between optimized airfoils for $C_L = 0$ with (a) thickness=0.1 and (b) thickness=0.08 at Mach=3.5. The thinner the airfoil, the better the overall performance.
Figure 7. Comparison between optimized airfoils for $C_L = 0.4$ with (a) thickness=0.1 and (b) thickness=0.08 at Mach=3.5. The thinner the airfoil, the better the overall performance.

2.8. Comparison between $C_L = 0$ and $C_L = 0.4$ at Mach=3.5 and thickness=0.08

For a 0.08 thickness, the airfoil optimized for a zero lift coefficient condition is more versatile and has higher efficiency than both the one optimized for a 0.4 lift coefficient scenario and the symmetrical one, but $C_L = 0.4$ is not reachable in on-design conditions.

Figure 8. Geometry comparison between airfoils optimized at $C_L = 0$ and $C_L = 0.4$.

Figure 9. Off-design comparison between profiles showed in figure 8 at Mach=2.5. The optimized profile for Mach=3.5 and $C_L = 0.4$ cannot reach that $C_L$ at Mach=2.5 due to the detached shockwave.
Figure 10. Off-design comparison between profiles showed in figure 8 at Mach=3.5. Neither the airfoil optimized for $C_L = 0$ at Mach=3.5 and the symmetrical one can reach $C_L = 0.4$ at Mach = 3.5. The one optimized for $C_L = 0.4$ and Mach=3.5 is the only one that can reach that value in that condition.

3. Conclusions

The Genetic Algorithm optimization is fast and valid, especially when coupled with an additional optimization function using the shock-expansion theory as a surrogate model in high Mach number and even detached shockwave scenarios.

For a local maximum Mach number up to 6, a 0.5 lift coefficient cannot be obtained using symmetrical airfoils with thickness values up to 0.08 because the flow has too much energy when the freestream Mach number is 3.5. In this case, $C_L = 0.4$ can be reached with the same thickness constraint. For a local maximum Mach number of 5, the maximum $C_L$ in the same conditions is reduced to 0.3. Lower thickness values increase the overall efficiency, reducing drag and consequently the amount of fuel needed.

For a given lift coefficient, higher values of efficiency can be reached with an optimized airfoil, but the constraining local maximum Mach number is reached for a lower maximum angle of attack. A non-zero lift coefficient condition leads to lower efficiency compared to a symmetrical Double Circular Arc airfoil, but also this kind of profile can barely reach the imposed $C_L$ value. The higher the thickness, the smaller the performance difference between the symmetrical and the optimized airfoil.

Symmetrical Double Circular Arc airfoils with a thickness value lower than 0.1 are very versatile as first-try solutions.

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