Anomalous acoustic reflection on a sliding interface or a shear band

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We study the reflection of an acoustic plane wave from a steadily sliding planar interface with velocity strengthening friction or a shear band in a confined granular medium. The corresponding acoustic impedance is utterly different from that of the static interface. In particular, the system being open, the energy of an in-plane polarized wave is no longer conserved, the work of the external pulling force being partitioned between frictional dissipation and gain (of either sign) of coherent acoustic energy. Large values of the friction coefficient favor energy gain, while velocity strengthening tends to suppress it. An interface with infinite elastic contrast (one rigid medium) and V-independent (Coulomb) friction exhibits spontaneous acoustic emission, as already shown by M. Nosonovsky and G.G. Adams (Int. J. Ing. Sci., 39, 1257 (2001)). But this pathology is cured by any finite elastic contrast, or by a moderately large V-strengthening of friction.

We show that (i) positive gain should be observable for rough-on-flat multicontact interfaces (ii) a sliding shear band in a granular medium should give rise to sizeable reflection, which opens a promising possibility for the detection of shear localization.

45.70.-n, 43.40.+s, 46.55.+d

I. INTRODUCTION

The question of the origin and nature of shear localization in disordered systems, such as soft glasses or confined granular media, which are jammed at equilibrium, but flow when sheared beyond a threshold stress, is a long standing one. Due in particular to recent progress in theoretical descriptions, it is presently the subject of renewed interest. Hence the need for identifying appropriate, non invasive methods of experimental investigation which could complement the optical and NMR imaging ones, recently put to use by Pignon et al [2], and by Coussot et al [3]. We intend to show in this article that the propagation of sound pulses appears as a promising possibility. Namely, we will show that the presence of a shear band of thickness small compared with the acoustic wavelength should give rise to strong anomalous reflection of a transverse acoustic signal for well defined incidence ranges. Such a method could therefore provide a relatively handy fingerprint of shear localization in confined granular media.

An extreme case of localized shear flow is that of frictional sliding of the interface between two cohesive macroscopic solids. In such a configuration, the very structure of the system prelocalizes shear to the nanometer-thick layer which forms the molecular adhesive contact(s). The role of the above-mentioned threshold stress is played here by the so-called static friction force. Below this threshold, the interface is elastically pinned (jammed), and responds elastically to a shearing force. So, the corresponding mechanical boundary condition is simply that the displacement fields in the two media must be fully continuous across the interface. But, beyond this shear level, sliding sets in and, along the direction of relative motion, the boundary condition is now provided by the dynamic friction law, which states that the shear and normal stresses are proportional. Obviously, such a discontinuity of the boundary conditions must result in a discontinuous change of the acoustic reflection and transmission coefficients when the system is set into sliding.

This was pointed out already long ago by Chez et al [4], who studied the reflection of a sound wave propagating in a plane orthogonal to the sliding direction, and polarized in the plane of incidence on an interface with the Coulomb frictional behavior (constant friction coefficient). However, due to the choice of this particular geometry, they overlooked an interesting effect. Indeed, the sliding system is an open one: energy is being pumped in from an external source – the external driving machine which imposes the sliding velocity. So, as soon as the acoustic displacement field has a non-zero component, in the interfacial plane, along the direction of the sliding motion, additional mechanical work (of a priori either sign) is extracted from the external source, and interfacial acoustic scattering is no longer energy-conserving. This opens in principle the possibility of acoustic gain at reflection, i.e. conversion of incoherent into coherent mechanical energy – quite an exciting prospect indeed!

Now, from the point of view of the propagation of an acoustic signal of wavelength λ, a shear band of thickness d \( \ll \lambda \) in a confined granular medium appears as the equivalent of a frictional interface between two identical solids. Indeed, the band can then be considered as a surface of mechanical discontinuity between the non sliding adjacent regions, which behave as (disordered) elastic solids. Experimental studies by rock mechanicians [5] [6] of systems constituted of two bulk rock pieces separated by an interposed layer of granular material (called "gouge")
have established that in such systems (i) sliding occurs in a narrow band within the gouge (ii) the dynamics is ruled by a standard solid friction law, the associated friction coefficients having a magnitude comparable with those for solids in direct contact.

Reflection and transmission of waves with a polarization component along the sliding direction of an interface with constant friction coefficient between dissimilar media have been recently studied by Nosonovsky and Adams [7], though in a different perspective. Namely, they focussed primarily on the possible generation of slip pulses - a dynamical feature that seems to be specific of pure Coulombic friction. In the course of this article we will rederive some of their results, which will be extended to the more realistic case of velocity-dependent friction and to the shear band problem.

This article is organized as follows.

In Section 2, we first write down the equations for the most general case of a monochromatic acoustic wave incident upon a planar frictional interface between two semi-infinite solids with different elastic moduli. We then specialize in Section 3 to the case where the two media is very large (e.g. a gel sliding on top of a glass plate). We show that, if the stiffer medium is assumed strictly non deformable and friction taken to be Coulomb-like, the reflection coefficient of a wave with the sliding direction and polarization in the plane of incidence is highly pathological: not only is a huge gain at reflection possible for some particular incidence range, but spontaneous acoustic emission from the surface is predicted - a result already obtained recently by Nosonovsky and Adams [8].

These singularities are to be related with those already found by Adams [9], and Ranjith and Rice [10], in their studies of interfacial waves in the same system. They result, as is well known in mechanics, from the specific singular character which the Coulomb model, which takes the friction coefficient to be a mere constant, shares with the Hill model of plasticity. Indeed, we show that (i) a very small finite relative compliance of the stiffer medium is sufficient to destroy the acoustic emission singularity (ii) improving upon the Coulomb description by taking into account a velocity-strengthening dependence of the dynamic friction coefficient of the order of what is measured for real systems also cures this singularity. Moreover, the possibility of energy gain at reflection is found to be strongly dependent upon the strength of the velocity dependence, and hence on the type of system: while it should be negligible for a gel-on-glass system, it might be observable with a rough-on-flat multicontact interface in well defined incidence ranges.

Section 4 is devoted to the symmetric case of two mechanically identical solids relevant to the shear band problem. In this situation, and when the velocity dependence of the friction coefficient is taken into account, we predict that the sliding shear band should give rise to a clear acoustic signature - namely a reflection coefficient with magnitude of order typically of order unity for incident signals in well defined ranges of incidence angles.

II. GENERAL FORMULATION

Consider two elastically isotropic semi-infinite media (M; M’) with shear moduli and Poisson ratios (μ, ν; μ’, ν’) and densities (ρ, ρ’), occupying respectively (Figure 1) the upper (x_2 > 0) and lower (x_2 < 0) half spaces, and in continuous contact with each other along the x_2 = 0 plane. Medium (M’) is assumed to be in stationary sliding motion with respect to (M) at velocity v along x_1 and towards x_1 < 0. In this reference state, the (homogeneous) normal and shear stresses τ^∗_{22}, τ^∗_{12}, are related by the dynamic friction law:

\[ τ^∗_{12} = -f(v)τ^∗_{22} \]  

with f(v) the friction coefficient.

An emitter linked to (M) is sending from infinity towards the interface a plane acoustic wave of frequency ω, propagating in the x_1, x_2 plane at incidence angle θ (Figure 1) and polarized in the plane of incidence. That is, the associated displacement has a finite component along the sliding direction. In order to fix ideas, and for the sake of simplicity, we restrict in most of what follows the algebraic formulation to the case of a transverse (shear) incident wave - the case of an incident longitudinal (dilatational) signal follows straightforwardly. The elastic displacement field (u_1, u_2) in (M) obeys the Lamé equations:

\[ \ddot{u}_1 = c_L^2 \frac{∂^2 u_1}{∂x_1^2} + (c_L^2 - c_T^2) \frac{∂^2 u_2}{∂x_1∂x_2} + c_T^2 \frac{∂^2 u_1}{∂x_2^2} \]  

\[ \ddot{u}_2 = c_L^2 \frac{∂^2 u_2}{∂x_2^2} + (c_L^2 - c_T^2) \frac{∂^2 u_1}{∂x_1∂x_2} + c_T^2 \frac{∂^2 u_2}{∂x_1^2} \]  

c_L, c_T are the longitudinal and transverse sound velocities in (M), and
The solution of equations (2), (3) reads:

\[ u_1 = e^{i(kx_1 - \omega t)} \left[ e^{-iqT x_2} + \alpha e^{iqT x_2} + \beta e^{iqL x_2} \right] \]

\[ u_2 = e^{i(kx_1 - \omega t)} \left[ \frac{k}{qT} e^{-iqT x_2} - \alpha \frac{k}{qT} e^{iqT x_2} + \frac{qL}{k} e^{iqL x_2} \right] \]

where \( \epsilon \) specifies the amplitude of the incident wave, \( k \) is the \( x_1 \) component of its wavevector, and

\[ q_{T,L}^2 = \frac{\omega^2}{c_{T,L}^2} - k^2 \]

with \( q_T \) real positive and \( \text{Re} q_L \geq 0 \) and \( \text{Im} q_L \geq 0 \).

In the lower medium (M'), which moves at velocity \((-v)\) in our reference frame, analogously:

\[ u'_1 = e^{i(kx_1 - \omega t)} \left[ \alpha' e^{-iq_T' x_2} + \beta' e^{-iq_L' x_2} \right] \]

\[ u'_2 = e^{i(kx_1 - \omega t)} \left[ \alpha' \frac{k}{q_T'} e^{-iq_T' x_2} - \beta' \frac{q_L'}{k} e^{-iq_L' x_2} \right] \]

where

\[ \omega' = \omega + vk \quad q_{T,L}'^2 = \frac{\omega'^2}{c_{T,L}'^2} - k^2 \]

In order to determine the amplitude reflection \((\alpha, \beta)\) and transmission \((\alpha', \beta')\) coefficients into the transverse and longitudinal channels we must now specify the four boundary conditions (BC) to be satisfied along the deformed M/M' interface.

Since the acoustic stresses we consider are small, \( O(\epsilon) \), we expect the contact to persist everywhere, so that

(i) normal displacements on both sides of the interface are equal: \( u \cdot \hat{n} = u' \cdot \hat{n} \) and mechanical equilibrium imposes continuity of normal and shear stresses:

(ii) \( \tau_{nn} = \tau^{'}_{nn} \)

(iii) \( \tau_{nt} = \tau^{'}_{nt} \)

The sliding velocity perturbation \((v_t - v)\) being also \( O(\epsilon) \), \( v_t \) remains positive (sliding persists) everywhere, and we assume that the dynamic friction condition now holds locally, namely that, at each interfacial point:

(iv) \( \tau_{nt} + f(v_t) \tau_{nn} = 0 \)

These boundary conditions, which have been taken for granted in existing studies of interfacial waves and shear fractures, actually imply important physical assumptions, which should be made explicit.

On the one hand, the (macroscopic) contact is supposed to be continuous and homogeneous - hence the statement of homogeneous stresses in the reference state. Now, it is well known that such is not the prevalent case. Most interfaces, being formed between rough solids, are actually constituted of a large, but rather sparse, set of microcontacts [11] [12], in between which the surfaces are mechanically free. Then, it is clear that conditions (i)-(iii) are valid only in a coarse-grained sense, i.e. provided that the length scale \( 2\pi/k \) of variation of the acoustic fields along \( x_1 \) is much larger than the average distance \( d \) between microcontacts, that is for \( kd << 1 \).

Intercontact distances lie commonly in the range of hundreds of micrometers. Only in the case where one at least of the solids is extremely compliant are macroscopic contacts truly continuous. This is for example the case for elastomers or gels.

On the other hand, the mere existence of a finite static threshold proves that frictional dissipation results from the triggering by the applied shear of structural instabilities. It is now documented that the corresponding structural rearrangement events take place in the nanometer-thick adhesive interfacial layers, and affect volumes of, typically, nanometric scale [12] [13] - comparable with the "shear transformation zones" invoked by Falk and Langer [14] to model the plasticity of amorphous solids. Let us call \( b \) the size of such a zone. A friction law represents the result of a statistical average over a large number of such dissipative events. So, it can only make sense on a scale much larger than \( b \), that is for \( kb << 1 \).
All this means that boundary conditions (i)-(iv) above must be understood as valid only on scales larger than some cut-off length \( L = \max(b, d) \) on the order of, typically, (i) a fraction of micrometer for conforming solid contacts (ii) a fraction of millimeter for the more common, multicontact interfaces.

Finally, since \( v_I \) now becomes a time-dependent quantity, condition (iv) implies that we assume the friction relation to hold instantaneously on the scale \( \omega^{-1} \). It is known that this does not apply to the case where the steady state \( f(v) \) is velocity weakening (\( df/dv < 0 \)). Indeed, such behavior necessarily results from the action of some underlying structural dynamics leading to aging when sticking and rejuvenation upon sliding, such as that associated with the slow creep growth of the real area of contact relevant to multicontact interfaces [15] [16]. Then, the equations describing non steady friction must involve explicitly at least one more dynamical "state" variable, and condition (iv) becomes insufficient. Note also that it is in this regime that steady sliding may be unstable with respect to stick-slip.

So, we restrict ourselves in what follows to the velocity-strengthening case \( df/dv > 0 \). This can be expected to hold, for rough-on-rough interfaces, only at sliding velocities in the \( \text{mm.sec}^{-1} \) range, large enough for rejuvenation effects to be saturated [17]. However, it has been shown [18] to prevail down to the \( \mu\text{m.sec}^{-1} \) range when working with rough-on-flat interfaces where contacts keep their identity when sliding, which makes contact area saturation easily realizable.

The position of our deformed interface is given by

\[
x_{2I} (x_1, t) = u_2 (x_1 - u_1 (x_1, 0, t))
\]  

We assume from now on that acoustic deformations are small enough for us to work in the linearized approximation. Then, the normal and tangent unit vectors \( \hat{n}, \hat{t} \) are simply:

\[
\hat{n}(x_1, t) = \left(-\frac{\partial u_2}{\partial x_1} x_1, 0, t, 1 \right) \\
\hat{t}(x_1, t) = \left(\frac{\partial u_2}{\partial x_1} x_1, 0, t, 1 \right)
\]  

Let the stress field in, say, M, be denoted \( \tau_{ij}^* + \delta \tau_{ij} \), \((i, j) = 1, 2\), with

\[
\delta \tau_{ij} = \mu \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{2\nu}{1-2\nu} \delta_{ij} u_{kk} \right]
\]  

Then, to first order, at the interface in M

\[
\tau_{nn} = n_i \tau_{ij} n_j = \tau_{22}^* + \delta \tau_{22} - 2\tau_{12}^* \left( \frac{\partial u_2}{\partial x_1} \right)
\]  

\[
\tau_{nt} = \tau_{12}^* + \delta \tau_{12} + (\tau_{22}^* - \tau_{11}^*) \left( \frac{\partial u_2}{\partial x_1} \right)
\]  

and the local sliding velocity

\[
v_I = v + (\dot{u}_1 - \dot{u}'_1)
\]  

Conditions (i)-(iv) then become:

\[
[u_2 - u'_2]_{x_1, 0, t} = 0
\]  

\[
\left[ \delta \tau_{22} - \delta \tau'_{22} - 2\tau_{12}^* \left( \frac{\partial u_2}{\partial x_1} - \frac{\partial u'_2}{\partial x_1} \right) \right]_{x_1, 0, t} = 0
\]  

\[
\left[ \delta \tau_{12} - \delta \tau'_{12} + (\tau_{22}^* - \tau_{11}^*) \left( \frac{\partial u_2}{\partial x_1} - \frac{\partial u'_2}{\partial x_1} \right) \right]_{x_1, 0, t} = 0
\]  

\[
\left[ \delta \tau_{12} + f(v) \delta \tau_{22} - \tau_{22}^* \frac{\partial u_2}{\partial x_1} + f'(v) \tau_{22}^* (\dot{u}_1 - \dot{u}'_1) \right]_{x_1, 0, t} = 0
\]
where \( f'(v) = df/dv \), and we have set

\[ \tau^* = - (\tau^*_{22} - \tau^*_{11} - 2f(v)\tau^*_{12}) \]  

\( \tau^*_{22} < 0 \) (compressive normal stress), \( \tau^*_{12} = -\tau^*_{22} > 0 \), and we can reasonably assume sliding to occur under zero lateral stress, i.e. \( \tau^*_{11} = 0 \), so that, in general, \( \tau^* > 0 \).

Note that the \( \tau^* \)-dependent terms in eqs (17)-(20), which account for the fact that the BC’s must be enforced along the deformed interface, have usually been overlooked in previous works on interfacial waves. As will appear below, they give rise to corrections on the order of \( (\tau^*/\mu) \). In the case of hard materials, externally applied stresses are in practice always considerably smaller than elastic moduli, and these corrections can safely be neglected. However, such may not be the case when dealing with very compliant materials such as gels or elastomers. For example, for the case of gel/glass interfaces [19] sliding stress levels may be a sizeable fraction of \( \mu \), and the full expressions should be retained.

Then, using eqs (5), (6), (8), (9) and (13), the BC’s yield the following set of four non homogeneous linear equations for \( \alpha, \beta, \alpha', \beta' \):

\[ -\frac{k}{q_T} \alpha + \frac{q_L}{k} \beta - \frac{k}{q_T'} \alpha' + \frac{q_L'}{k} \beta' = -\frac{k}{q_T} \]  

\[ -2\mu \alpha + \mu \frac{q^2_T - k^2}{q_T} \beta + 2\mu' \alpha' - \mu' \frac{q^2_T - k^2}{q_T} \beta' = 2\mu \]  

\[ \mu \frac{q^2_T - k^2}{q_T} \alpha + 2\mu q_L \beta + \mu \frac{q^2_T - k^2}{q_T'} \alpha' + 2\mu' q_L' \beta' = \mu \frac{q^2_T - k^2}{q_T} \]  

\[ \left[ \frac{q^2_T - k^2}{q_T} - 2fk + \frac{\tau^*}{\mu} \right] \alpha + \left[ 2q_L + f \frac{q^2_T - k^2}{k} - \frac{\tau^*}{\mu} q_L \right] \beta + \frac{Af \omega}{c_T} (\alpha + \beta - \alpha' - \beta') = \left[ \frac{q^2_T - k^2}{q_T} + 2fk + \frac{\tau^*}{\mu} k^2 \right] - \frac{Af \omega}{c_T} \]  

where we have defined:

\[ A = \frac{f'(v)c_T}{f(v)} \left| \frac{\tau^*_{22}}{\mu} \right| \]  

which measures the dimensionless strength of the velocity-strengthening effect.

Equations (22)-(25) above are relevant to the case of a transverse incident wave. We will also display some results for a longitudinal one. In this latter case, the only modification concerns the first terms in the r.h.s. of equations (5), (6) which give the expression of the total acoustic displacement field. This results in leaving the l.h.s.’s of the final equations (22)-(25) unchanged, only the r.h.s.’s being modified.

Since the general solution of our problem depends on a large number of parameters (four elastic moduli, two densities, the angle of incidence, the friction coefficient and its derivative) it will be more illuminating to focus on a few simple cases, namely that of very large elastic contrast (quasi non deformable \( M' \)), and the symmetric case of two identical materials, relevant to the problem of shear band detection.

### III. LARGE ELASTIC CONTRAST CASE

We consider here the case of maximum asymmetry, where \( M' \) is considerably stiffer than \( M \). In order to disentangle the respective effects of the elastic contrast ratio \( R = \mu'/\mu \) and of the velocity dependence of friction, we first treat the extra simple limit of a non deformable \( M' \) and of pure Coulomb friction, then examine how the results thus obtained are modified (i) when \( R \) is large but finite (ii) when \( f'(v) \neq 0 \).
A. Non deformable $M'$ and Coulomb friction

The boundary conditions reduce to imposing that $u_2 = 0$ and $\tau_{nt} + f \tau_{nn} = 0$ along the $x_2 = 0$ plane. Equations (22)-(25) become, in this $R^{-1} = f' = 0$ limit, where $\alpha' = \beta' = 0$:

$$\frac{k}{qT} \alpha - \frac{qL}{k} \beta = \frac{k}{qT}$$

(27)

$$\left[\frac{k^2 - q_L^2}{qT} + 2fk\right] \alpha + \left[-2qL + f \frac{k^2 - q_L^2}{k}\right] \beta = \frac{k^2 - q_L^2}{qT} - 2fk$$

(28)

which yield for the $T \to T$ and $T \to L$ reflection coefficients:

$$\alpha = 1 + \frac{4fqL}{\Delta} \quad \beta = \frac{4fk^2}{qT \Delta}$$

(29)

where:

$$\Delta = qL \left(\frac{k}{qT} + \frac{qT}{k}\right) + f \left(q_L - 2qL - \frac{k^2}{qT}\right)$$

(30)

Note that, for $f \to 0+$, and for incident waves propagating towards $x_1 > 0$ ($k > 0$), $\Delta > 0$. That is, at least in the limit of weak friction and in this incidence range, clearly $\alpha > 1, \beta > 0$, and the reflected energy flux is larger than the incident one. In order to try and clear up this a priori surprising prediction, let us look in more detail into the question of energy balance in our system. Consider the volume $(V)$ of a slice of unit depth along $x_3$ of medium $(M)$, limited by its area $(S)$ ($-L < x_1 < L$) along the interface, and the semicylinder $(C_\infty)$ of radius $L(L \to \infty)$. In the situation we are considering, of an incident wave of constant amplitude, the average over an acoustic period of the elastic energy stored within $(V)$ is time independent. This means that energy conservation simply imposes that the net energy flux at infinity $\dot{W}_\infty$, flowing out of $C_\infty$, associated with the acoustic waves, must equal the increment associated with the acoustic perturbation of the work injected per unit time into the system via the work of the stresses acting on the moving interface, $\dot{W}_{ext}$, which is pumped from the driving machine. This is proved in detail in the Appendix, where we show that, in the present case on a non deformable $M'$, per unit area of $(S)$:

$$\langle \dot{W}_{ext} \rangle = (-\dot{u}_1 \tau_{12})$$

(31)

where $\langle \ldots \rangle$ stands for the average over the acoustic period. We then easily prove (Appendix A) that this is exactly equal to the net acoustic energy flux:

$$\dot{W}_\infty = \langle \dot{W}_{refl} \rangle - \langle \dot{W}_{inc} \rangle$$

(32)

In other words, a sliding frictional interface is in principle able to transform external incoherent mechanical work into coherent acoustic radiation - i.e. to act as an "acoustic laser". In order to make this more precise, we plot on Figure 2a the relative powers reflected into the T and L channels, given by (see Appendix):

$$w_{TT} = |\alpha|^2$$

(33)

$$w_{TL} = \frac{q_Lk^2}{k^2} |\beta|^2$$

if $q_L$ real

$$= 0$$

if $q_L$ imaginary

(34)

against the incidence angle $\theta = \sin^{-1}(c_Tk/\omega)$ and for $f = 0.2$, $c_T/c_L = 0.5$. It is seen that, for all positive incidences, a transverse incident wave is predicted to be notably amplified upon reflection, while the power in the L channel remains quite small, even close to $\theta_{om} = \sin^{-1}(c_T/c_L)$ beyond which the L wave becomes evanescent, where it is maximum. However a much more surprising result, already derived by Nosonovsky and Adams [8] is that $w_{TT}$ and $w_{TL}$ are found to diverge at a negative incidence $-\theta_{cr}$, where the denominator $\Delta$ (Eq.(30)) of the expressions for $\alpha$ and $\beta$ (Eq.(29) vanishes. Using equation (7), the condition $\Delta = 0$ reads:

$$\sqrt{\frac{c_T^2}{c_L^2}} - \sin^2 \theta = -\frac{f \cos 2\theta \sin \theta}{1 - f \sin 2\theta}$$

(35)
which is easily checked graphically to have a single negative solution \(-\theta_{cr}\) with \(\theta_{cr} < \theta_{lim}\), whatever the values of \(f\) and of \(c_T/c_L\). The smaller \(f\), the closer \(\theta_{cr}\) approaches \(\theta_{lim}\). In the case shown on Figure 2a, \(\theta_{lim} - \theta_{cr} \approx 7^\circ\). In the case (Figure 2b) of L incidence, the corresponding singularity occurs for \(\theta\) close above \(-90^\circ\) In other words, in this admittedly oversimplified limit (rigid \(M'\), pure Coulomb friction), homogeneous sliding at constant velocity is impossible: indeed, any infinitesimal perturbation is able to trigger the emission, all along the interface, of a coupled set of transverse and longitudinal acoustic waves, of a priori undetermined amplitude, travelling respectively at angles \(\theta_{cr}\) and \(\sin^{-1}\left(\frac{c_T^2}{c_L^2 \sin^2 \theta_{cr} - 1}\right)\) in the back direction. On the one hand this entails that an infinite external energy would be pumped in. On the other hand, as soon as the local interfacial velocity will vanish, the interface will stop (stick). So this pathologic behavior signals that homogeneous sliding is here absolutely unstable - a signature to look for. In the case shown on Figure 2b, the corresponding singularity occurs for \(\theta_{cr}\) close above \(-90^\circ\) when \(\sin \theta \approx 1\) and \(\cos \theta < 0\), the zero of \(\Delta\) always occurs for \(\theta_{cr}\). As seen above, the zero of \(\Delta\) always occurs for \(\theta_{cr} < \theta_{lim}\) where \(q_L\), and hence \(\Delta\), is real. On the other hand it appears that, for \(q_T^2 < 0\), pure imaginary, \(\Gamma\) is a complex quantity. So, no zero of \(D\) which would evolve continuously from \(-\theta_{cr}\) exists, and one finds that, for finite \(R^-1\), \(\alpha\) and \(\beta\) no longer diverge, but exhibit, in the vicinity of \(-\theta_{cr}\), a maximum of order \(R\): a finite compliance of \(M'\), however small it is, is sufficient to kill the pathology found in the totally rigid limit. This we have confirmed by computing the relative reflected powers \(w_{IJ}\) \((I, J = T, L)\) for various values of \(R\). The results for \(w_{TT}\) are shown on Figure 3: the smaller \(R\), the lower the maximum of \(\epsilon\), which reduces to a few units for \(R \lesssim 40\). Of course, in practice, for \(R >> 1\), the very large values of the reflection coefficients at maximum mean that a very small incident amplitude \(\epsilon\) would suffice to induce a finite response, such that \(\nu + \dot{u}_1\) would vanish, leading to interfacial stick - i.e. homogeneous slip, though no longer absolutely unstable from a strict mathematical point of view, would be very weakly steady, as it could not resist perturbations of finite but very small amplitude.

### B. Finite elastic contrast and Coulomb friction

Let us now consider the case of a large but finite elastic contrast ratio \(R = \mu'/\mu >> 1\). We assume for the sake of simplicity equality of the Poisson ratios \(\nu = \nu'\). In this case, except for quasi-normal incidence angles, \(q_T^2 < 0\): there is total reflection at the interface, with only evanescent transmission into \(M'\). \(\alpha\) and \(\beta\) must now be obtained by solving the full equations (22) - (25) for constant \(f\). Except in the vicinity of \(\theta = -\theta_{cr}\), a finite \(R^-1\) simply acts as a regular perturbation, leading to a correction \(O(R^-1)\). For \(\theta \approx -\theta_{cr}\), however, one must consider in detail the behavior of the determinant \(D\) of the system (22) - (25). To first order in \(R^-1\):

\[
D \propto \Delta(q_T, q_L) + R^{-1} \Gamma(q_T, q_L, q_T', q_L') \tag{36}
\]

As seen above, the zero of \(\Delta\) always occurs for \(\theta_{cr} < \theta_{lim}\) where \(q_L\), and hence \(\Delta\), is real. On the other hand it appears that, for \(q_T^2 < 0\), pure imaginary, \(\Gamma\) is a complex quantity. So, no zero of \(D\) which would evolve continuously from \(-\theta_{cr}\) exists, and one finds that, for finite \(R^-1\), \(\alpha\) and \(\beta\) no longer diverge, but exhibit, in the vicinity of \(-\theta_{cr}\), a maximum of order \(R\): a finite compliance of \(M'\), however small it is, is sufficient to kill the pathology found in the totally rigid limit. This we have confirmed by computing the relative reflected powers \(w_{IJ}\) \((I, J = T, L)\) for various values of \(R\). The results for \(w_{TT}\) are shown on Figure 3: the smaller \(R\), the lower the maximum of \(\epsilon\), which reduces to a few units for \(R \lesssim 40\). Of course, in practice, for \(R >> 1\), the very large values of the reflection coefficients at maximum mean that a very small incident amplitude \(\epsilon\) would suffice to induce a finite response, such that \(\nu + \dot{u}_1\) would vanish, leading to interfacial stick - i.e. homogeneous slip, though no longer absolutely unstable from a strict mathematical point of view, would be very weakly steady, as it could not resist perturbations of finite but very small amplitude.

### C. Infinite elastic contrast and \(v\)-strengthening friction

The problem is now specified by eqs. (22) and (24) in which \(\alpha' = \beta' = 0\), and the position of the singularity of \(\alpha, \beta\), if any, is therefore given by the zero of:

\[
\Delta_A = q_L \left(\frac{k}{q_T} + \frac{q_T}{k}\right) + f \left(q_T - 2q_L - \frac{k^2}{q_T}\right) + A \frac{\omega f(v)}{c_T} \left(\frac{q_L}{k} + \frac{k}{q_T}\right) \tag{37}
\]

where the dimensionless parameter measuring the velocity dependence effect is defined by equation (26):

\[
A = \frac{f(v)c_T |\tau_{22}|}{\mu}
\]

When solving numerically for \(\Delta_A = 0\), we find that the singularity disappears for \(A \geq A_{cr}\). More precisely, we find that, as \(A\) increases, the zero of \(\Delta_A\) approaches \(-\theta_{lim}\), which it reaches for \(A = A_{cr}\), beyond which it disappears,
due to the fact that no zero can occur in the regime where the L wave is evanescent. The threshold value $A_{cr}$ is independent of the value of $f(v)$, but it depends upon the sound velocity ratio. For a reasonable choice of this parameter ($c_T/c_L < 0.7$), $A_{cr}$ is at most a few units. For example, for the presently used value $c_T/c_L = 0.5$, we obtain $A_{cr} = 1$. This behavior is illustrated on Figure 4, where we plot the relative reflected power $w_{TT}$ against incidence angle for various values of $A$. It is also seen that the larger $A$, the smaller the gain at reflection. It appears that at another characteristic $A$ value ($A = 2$ for our parameter values), the gain becomes negative ($w_{TT} < 1$) for all incidences. That is, a strengthening $v$-dependence of $f$ is very efficient to kill the amplification effect.

It is therefore important to evaluate an order of magnitude of $A$ for real interfaces with a large elastic contrast, a good example of which is that of gel/glass couples. Frictional sliding at the interface between glass and a 5% gelatin aqueous gel has recently been studied in detail by Bamberger et al [19]. Using their data, we find that, for $v$ in the mm.sec$^{-1}$ range, $f(v)/f(v) \approx 2.10^{3}$ sec.m$^{-1}$, while $(c_T | \tau_{22}^* | /\mu) \approx 2$ m.sec$^{-1}$, so that $A \approx 4.10^{3}$ is a very large number.

In this $A \gg 1$ limit, the rigid M' version of equation (24) reduces, to lowest order in $A^{-1}$, to :

$$\alpha + \beta + 1 = 0 \quad (38)$$

that is, due to the high relative cost of increasing its instantaneous velocity, the interface remains locked to the homogeneous sliding state and $u_1 \equiv 0$.

So, for gel/glass systems, acoustic reflection should not in practice be able to distinguish between the static and the sliding interfaces.

Another realization of the high elastic contrast situation is provided by the multicontact interface between rough glassy PMMA and atomically flat silanized glass, recently studied by Bureau et al [18]. For this couple, $\mu'/\mu \simeq 20$.

Thanks to the flatness of the glass surface, it is possible to saturate the slow growth of the real area of contact, which is responsible, in the case of rough-on-rough systems, for the velocity weakening behavior of the dynamic friction coefficient and of the associated stick-slip dynamics. Then, for $v \gtrsim 1$µm.sec$^{-1}$, $f(v)$ is velocity strengthening and of the form :

$$f(v) = f_0 \left[1 + \zeta \log \left(\frac{v}{v_0}\right)\right]$$

where $f_0 \approx 0.2$, and $\zeta \approx 2 - 4.10^{-2}$. With $\mu_{PMMA} = 1$ GPa, $c_T \approx 10^3$ m.sec$^{-1}$, and under normal stresses on the order of 5 kPa, one gets :

$$A \approx \frac{100 - 200}{v_{\mu m.}\text{sec}^{-1}} \quad (40)$$

That is, for sliding velocities in the $100$µm.sec$^{-1}$ range, $A$ is typically on the order of $1 - 2$.

Finally, note that, since in this configuration the average distance between the micrometric regions which form the real area of contact is on the order of a fraction of millimeter, such an experiment would ask for acoustic signals in the range of of a few hundred kHz at most, in order for our continuum description to be valid.

The relative powers reflected into the two channels for T and L incident waves under these conditions are plotted on Figures 5 and 6, for $A = 0, 1, 2$. (Since in this case typical values of $\tau^*/\mu$ and $v/c_T$ are $< 10^{-5}$, calculations have been performed for $\tau^* = 0$, $\omega' = \omega$.) Inspection of these results and comparison with the case of the non moving interface (Figure 7) lead us to the following conclusions.

- The more favorable channels for observing gain at reflection are the diagonal (LL and TT) ones.
- In both cases, a very narrow peak of $w$, reminiscent of the singularity obtained in the limit of Coulomb friction on a rigid substrate, is predicted for $\theta \simeq -\theta_{nm}$. However, due to its narrowness, observing it would be certainly be excessively demanding in terms of directional accuracy.
- It thus appears more feasible to investigate one of the two following configurations : LL at large negative incidence angles, in the range of $-45^\circ$, and (TT) at large positive incidence $\theta \approx 60^\circ$.
- Gain decreases very rapidly with increasing $A$, i.e. with decreasing $v$ (see eq.(40)) : while, for $A = 1$ it reaches up to 20% in (LL) and 35% in (TT), already for $A = 2$ it has practically collapsed to zero. So, according to equation (40), the best situation corresponds to the largest possible driving velocities, in practice in the range of a fraction of mm/sec.

With these conditions in mind, it seems quite possible to obtain substantial acoustic gain at reflection on a sliding rough-on-flat multicontact interface.
IV. SYMMETRIC CASE

We now turn to the opposite limit where M and M’ have the same elastic properties. It is a priori relevant to the case of multicontact interfaces between two pieces of the same material. However, the above mentioned rough-on-flat configuration is inappropriate in this case, since the asperities on the rough surface would give rise on the flat one to unavoidable and poorly qualified plastic damage and wear effects. On the other hand, as already mentioned, in the rough-on-rough configuration where such problems are irrelevant, geometric aging interrupted by motion results in a velocity-weakening behavior of \( f(v) \) for velocities up to at least the \( \text{mm.sec}^{-1} \) range, not very easy to access in a stationary sliding laboratory experiment. So, such symmetric solid on solid interfaces turn out not to be well adapted in practice to test our predictions.

The most interesting case, as already mentioned in Section I, is that of an established shear band, in particular in a highly confined granular medium. In such highly disordered systems, internal stress inhomogeneities (the so-called stress-chain phenomenon) of course give rise to scattering of acoustic waves. However, this is all the smaller that the effective acoustic wavelength in the medium is larger with respect to the correlation length of stress fluctuations, known to be on the order of a few grain diameters \( D \) only. In this \( kD \ll 1 \) limit, as shown by experiments on the propagation of acoustic pulses [20], where one can separate out unambiguously the signal corresponding to the propagation of a "coherent pulse", a description in terms of an effective acoustic medium is legitimate. Note that, as shear band thickness is also expected to be, for low shearing rates, on the order of a few grain diameters \( D \)

\[ \delta \rho/\rho \approx (\delta/\rho)^2(d\omega/cT)^2 \] for velocities up to at least the \( \text{mm.sec}^{-1} \) range, not very easy to access in a stationary sliding laboratory experiment. So, such symmetric solid on solid interfaces turn out not to be well adapted in practice to test our predictions.

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9 that, although, as expected, \( w_{TT} \) decreases as \( A \) grows, at normal incidence it remains non negligible up to the sizeable value \( A = 2 \), where it still is on the order of 25%.

V. CONCLUSION

In summary, from the above results, we conclude to the interest of performing experimental studies of the reflection of acoustic pulses on sliding solid interfaces and shear bands, the potential interest of which is different in each case.

As discussed in section III.C, experiments with a rough-on-flat multicontact interface open the possibility of obtaining acoustic gain, through the conversion by the frictional system into incoherent mechanical energy pumped from the driving system into coherent acoustic vibrations. However, such experiments are certainly delicate to realize, since they ask for working at non normal incidences, and, due to the effect of velocity strengthening of dynamic friction coefficients, at not very small velocities, in the range of typically a few hundred \( \mu \text{m/sec} \).

On the other hand, acoustic pulse reflection appears as a promising method for detecting shear localization in a confined granular medium. The best configuration - a transverse pulse at normal incidence - is more easily realizable since it may be implemented with a single transducer acting as both emitter and receiver. Moreover, the expected signature (a strongly reduced transit time before return of the reflected pulse) should be easy to identify - its presence confirming at the same time the expected quasi-independence of the sliding stress on the sliding velocity.

We therefore strongly hope that this work will motivate such experiments in the near future.

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APPENDIX A: ENERGY BALANCE AT A FRICTIONAL INTERFACE

We restrict ourselves to the case of a rigid substrate with an infinite elastic contrast \( R = \mu' / \mu \to \infty \) considered in Sec. III A. This situation is sketched in Fig. 10, which concerns specifically the case of a transverse wave incident below the critical angle \( \theta_{\text{lim}} \), so that also the longitudinal reflected wave is propagating. In our 2D geometry, the incident beam irradiates a strip \( S_0 \) of the interface perpendicular to the sliding direction \( x_1 \) and extending from \( -L_0 \) to \( L_0 \) along this direction. It has an area \( S_0 \) per unit length along \( x_3 \). The reflected beams stem from the irradiated area. The beams are wide enough to make the fringe effects negligible and easy to exclude as additive constants to the principal quantities proportional to \( S_0 \). The \( S_0 \) strip is now overlapped by a wider region \( S \) stretching from \( -L \) to \( L \) such that a semicylindrical dome \( C_{\infty} \) raised over it encloses the regions of beam overlap and interference (as well as the fields of attenuated waves, should they arise). The bottom of the dome is infinitesimally above the interface, so that the equations of motion (2), (3) are valid both inside and on the surface of the region \( V \) enclosed by the dome and the bottom plane. Then, the total elastic energy \( W \) inside the dome can be studied using the differential energy conservation law [24], [25] (see also [4])

\[
\text{div} \mathbf{j} + \dot{e} = 0 \quad \text{i.e.,} \quad j_{\ell} \dot{\ell} + \dot{e} = 0,
\]

where \( e \) is the local energy density, whose form will not be needed, and the energy current density \( \mathbf{j} \) is given by

\[
\mathbf{j} = -\dot{u} \mathbf{\tau} \quad \text{i.e.,} \quad j_{\ell} = -\dot{u}_\ell \tau_{\ell}
\]

We consider elastic fields having a steady homogeneous component and a single frequency acoustic component. As usual, in bilinear expressions like Eq. (A2), we turn to real parts of all quantities involved, which is consistent in a linear theory. Thus, we have

\[
\dot{u}(r, t) = \text{Re}(-i\omega u^0(r)e^{-i\omega t})
\]

\[
\mathbf{\tau}(r, t) = \mathbf{\tau}^* + \text{Re}(\delta \mathbf{\tau}^0(r)e^{-i\omega t})
\]

Introducing this into (A2), we may single out the steady flow component of \( \mathbf{j} \) by averaging over the period \( 2\pi / \omega \):

\[
\langle j \rangle = \frac{1}{2}(-i\omega u^0 \delta \mathbf{\tau}^0 + i \omega \mathbf{\bar{u}}^0 \delta \mathbf{\tau}^0)
\]
where overbar denotes complex conjugation. The time average \( \langle W \rangle \) of the total elastic energy per unit length of the \( V \) region for a stationary irradiation is zero. Integration of the time average of Eq. (A1) then yields a balance equation for the steady time averaged energy flows across the surface of \( V \):

\[
\int_{C_\infty} dS \langle \hat{n} \rangle \cdot \hat{n} = \int_S dS (\langle -\hat{j} \rangle) \cdot \hat{n}
\]  

(A6)

with \( \hat{n} \) the outer normal of \( V \). With the sign conventions of Figure 10, we obtain the following relation for the gain \( \dot{W}_\infty \), defined as the imbalance between the incident and reflected energy flows:

\[
\dot{W}_\infty = \langle \dot{W}_T \rangle + \langle \dot{W}_L \rangle - \langle \dot{W}_{inc} \rangle = \langle \dot{W}_{ext} \rangle
\]  

(A7)

The energy flows associated with the plane wave beams crossing the dome \( C_\infty \) are easily obtained, if we use the well known steady current density for a plane wave [24]

\[
|\langle j \rangle_{L,T}| = \frac{1}{2} \rho \omega^2 c_{L,T} \cdot |U|^2
\]

valid for both polarizations; the quantity \( U \) is a possibly complex amplitude. The plane waves appearing in (A7) are defined in Eqs. (5), (6). The coefficients \( \alpha, \beta \) were obtained in Sec. III A. The three total energy flows are the products of current densities and beam cross sections. Thus,

\[
\langle \dot{W}_{inc} \rangle = \frac{1}{2} \rho \omega^2 c_T \cdot \varepsilon^2 \left( \frac{\omega}{c_T q_T} \right)^2 \cdot S_0 \frac{c_T q_T}{\omega}
\]

\[
\langle \dot{W}_T \rangle = \frac{1}{2} \rho \omega^2 c_T \cdot \varepsilon^2 |\alpha|^2 \left( \frac{\omega}{c_T q_T} \right)^2 \cdot S_0 \frac{c_T q_T}{\omega}
\]

\[
\langle \dot{W}_L \rangle = \frac{1}{2} \rho \omega^2 c_L \cdot \varepsilon^2 |\beta|^2 \left( \frac{\omega}{c_L q_L} \right)^2 \cdot S_0 \frac{c_L q_L}{\omega}
\]  

(A8)

The ratios of the flows \( w_{T,L} = \langle \dot{W}_J \rangle / \langle \dot{W}_{inc} \rangle \) \((J = T, L)\) then have the form given in Eqs. (33),(34) in the main text.

It remains to interpret the right hand side \( \langle \dot{W}_{ext} \rangle \) of (A7). First, using explicit expressions (A2), (A5) for \( j \), we obtain (cf. (31))

\[
\langle \dot{W}_{ext} \rangle = \int_{S_0} dS \langle \dot{u}_1 \tau_{12} \rangle
\]  

(A9)

\[
\langle \dot{W}_{ext} \rangle = \int_{S_0} dS \frac{1}{2} (-i \omega u_{10} \delta \tau_{12}^{(0)} + i \omega u_{10} \delta \tau_{12}^{(0)})
\]  

(A10)

Eq. (A10) may serve to check equation (A7) explicitly. Two points appear explicitly: (i) Only those irradiated parts of the interface where \( \dot{u}_1 \neq 0 \) do contribute: the effect is connected with the free sliding of the interface points along the sliding direction. (ii) All three waves superimposed enter \( \langle \dot{W}_{ext} \rangle \). This is particularly remarkable in the case of an evanescent L wave, in which there is no longitudinal flow at infinity, yet the energy gain at the interface cannot be obtained correctly without including the L wave contribution right at the interface.

At each point of the interface, the external force acting on medium \( M \) is

\[
F = \tau \hat{n} = \tau^* \hat{n} + \text{Re}(\delta \tau) \hat{n} \equiv F^* + \delta F
\]

The time average is

\[
\langle F \rangle = F^*
\]

The averaged macroscopic force acting on \( S \) is thus \( F_S = SF^* \). Medium \( M' \) is pulled at the overall velocity \( \mathbf{v} \), thus the power spent by this force is \( F_S \cdot \mathbf{v} \), independently of the presence of oscillatory acoustic fields. It is now easy to see that
The first term is the power dissipated against the friction forces: \( \mathbf{v} - \dot{\mathbf{u}} \) is the local relative interfacial velocity, while \( \mathbf{F} \) has only a frictional component along the interface. Equation (A11) thus expresses the partitioning of the total work done per unit time by the external force into the (irreversibly) dissipated power and the net acoustic gain.
FIG. 1. Schematic representation of the system: a transverse incident wave impinges at incidence $\theta$ onto the sliding interface, giving rise to two reflected and two transmitted waves.

FIG. 2. Relative powers $w_{IJ}(\theta)$ ($I, J = T, L$) reflected from the interface between a compliant medium and a rigid one versus incidence angle $\theta$. Friction is velocity independent. $R = \infty$, $f = 0.2$, $c_T/c_L = 0.5$. Incident wave (a) transverse, $w_{TT}$ (—), $w_{TL}$ (— - -) (b) longitudinal, $w_{LL}$ (—), $w_{LT}$ (— - -).

FIG. 3. Evolution of the relative reflected power $w_{TT}(\theta)$ with elastic contrast ratio $R$ for Coulombic friction. $f = 0.2$, $c_T/c_L = 0.5$, $R = \infty$ (· · ·), 40 (· - ·), 20 (· - · -), 10 (—). (a) Full incidence range; (b) Blow-up for $\theta$ close to $-\theta_{lim} = -30^\circ$.

FIG. 4. Evolution of the relative reflected power $w_{TT}(\theta)$ with strength $A$ of the velocity dependence of the friction coefficient for rigid $M'$. $f = 0.2$, $c_T/c_L = 0.5$, $R = \infty$. $A = 0$ (—), 1 (· - ·), 2 (· - · -). (a) Full incidence range; (b) Blow-up for $\theta$ close to $-\theta_{lim} = -30^\circ$. Two more curves, for $A = 0.5$ (· · ·), 0.9 (thin full line) show the shift and narrowing of the singularity as $A = A_{cr} = 1$ is approached. For $A \geq A_{cr}$, the peak amplitude becomes finite.

FIG. 5. Evolution of relative reflected powers with increasing $A$. Incident $T$ wave, $R = 20$, $f = 0.2$, $c_T/c_L = 0.5$. $A = 0$ (—), 1 (· - ·), 2 (· - · -). (a) $w_{TT}(\theta)$, (b) $w_{TL}(\theta)$. The height of the peak for $\theta = -\theta_{lim}$ is maximum, but finite, for $A = 1$.

FIG. 6. Same plot as in Fig. 5, but for an incident $L$ wave. (a) $w_{LL}(\theta)$, (b) $w_{LT}(\theta)$.

FIG. 7. Static interface: relative powers reflected from incident wave (a) transverse, $w_{TT}$ (—), $w_{TL}$ (— - -) (b) longitudinal, $w_{LL}$ (—), $w_{LT}$ (— - -). $R = 20$, $c_T/c_L = 0.5$.

FIG. 8. Lastically symmetric case ($R = 1$), and Coulombic friction ($A = 0$): relative powers reflected from incident wave (a) transverse, $w_{TT}$ (—), $w_{TL}$ (— - -) (b) longitudinal, $w_{LL}$ (—), $w_{LT}$ (— - -). $f = 0.2$, $c_T/c_L = 0.5$.

FIG. 9. Evolution of the relative reflected power $w_{TT}(\theta)$ with strength $A$ of the velocity dependence of the friction coefficient for symmetric case $R = 1$. $f = 0.2$, $c_T/c_L = 0.5$. $A = 0$ (—), 1 (· - ·), 2 (· - ·).

FIG. 10. Energy flows for rigid $M'$. 

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Figure a shows the behavior of $W_{TT}$ as a function of the angle of incidence. The graph displays two distinct peaks, indicating significant variations in $W_{TT}$ with changes in the angle of incidence.

Figure b illustrates the variation of $W_{LL}$ and $W_{LT}$ with the angle of incidence. The curves exhibit a rapid decrease in both $W_{LL}$ and $W_{LT}$ as the angle of incidence increases, highlighting the sensitivity of these quantities to the angle of incidence.

The axes represent the angle of incidence, with values ranging from $-90$ to $90$ degrees, and the y-axes denote the corresponding $W$ values, ranging from 0 to 3.
$W_{TT}$ vs. angle of incidence
