Planewave diffraction at the periodically corrugated boundary of vacuum and a negative phase–velocity material

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Abstract

Considering the diffraction of a plane wave by a periodically corrugated half–space, we show that the transformation of the refracting medium from positive/negative phase–velocity to negative/positive phase–velocity type has an influence on the diffraction efficiencies. This effect increases with increasing corrugation depth, owing to the presence of evanescent waves in the troughs of the corrugated interface.

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I. INTRODUCTION

The rediscovery [1] of isotropic dielectric–magnetic materials exhibiting phase velocity vector opposed in direction to the time–averaged Poynting vector has prompted a flurry of publications during the last four years [2, 3]. Many interesting effects have been predicted, with some experimental backing as well [4, 5, 6].

Though several names have been proposed for this class of materials, we think that the most descriptive is: negative phase–velocity (NPV) materials [7]. In contrast, the phase velocity and the time–averaged Poynting vectors are co–parallel in positive phase–velocity (PPV) materials. PPV materials are, of course, commonplace and require no introduction.

That the intrinsic difference between NPV and PPV materials has recognizable consequences is easily gauged from a simple problem: reflection and refraction of a plane wave due to a homogeneously filled half–space. Let vacuum be the medium of incidence, while \( \epsilon_2 \) and \( \mu_2 \) denote the relative permittivity and relative permeability of the medium of refraction. Let a linearly plane wave be incident on the planar interface of the two mediums at an angle \( \theta_0 \), \( |\theta_0| < \pi/2 \), from the normal to the interface, and \( \rho(\theta_0) \) be the reflection coefficient. If the transformation \( \{ \epsilon_2 \rightarrow -\epsilon_2^*, \mu_2 \rightarrow -\mu_2^* \} \) is implemented, then \( \rho(\theta_0) \rightarrow \rho^*(\theta_0) \), where the asterisk denotes the complex conjugate [8]. Thus, the replacement of a NPV/PPV half–space by an analogous PPV/NPV half–space changes the phase of the reflection coefficient but not its magnitude.

What would happen if the interface were to be corrugated [9]? Surface–relief gratings are periodically corrugated surfaces that are commonly used in electromagnetics, and many theoretical techniques are available to compute their diffraction efficiencies [10]. Therefore, we decided to compute and compare the diffraction efficiencies of PPV and NPV surface–relief gratings. In this report, we present our chief results here. Section II contains a sketch of the theoretical method we chose, while Section III is a discussion of the numerical results obtained. An \( \exp(-i\omega t) \) time–dependence is implicit.

II. THEORY

In a rectangular coordinate system \( (x, y, z) \), we consider the periodically corrugated boundary \( y = g(x) = g(x + d) \) between vacuum and a homogeneous, isotropic, linear
material, with $d$ being the corrugation period. The region $y > g(x)$ is vacuous, whereas the medium occupying the region $y < g(x)$ is characterized by complex-valued scalars 

$$
\varepsilon_2 = \varepsilon_2R + i\varepsilon_2I \quad \text{and} \quad \mu_2 = \mu_2R + i\mu_2I.
$$

If this medium is of the NPV type, then

$$
\varepsilon_2R|\mu_2| + \mu_2R|\varepsilon_2| < 0; \quad (1)
$$

otherwise

$$
\varepsilon_2R|\mu_2| + \mu_2R|\varepsilon_2| > 0. \quad (2)
$$

A linearly polarized electromagnetic plane wave is incident on this boundary from the region $y > g(x)$ at an angle $\theta_0$, $|\theta_0| < \pi/2$, with respect to the $y$ axis.

Let the function $f(x, y)$ represent the $z$–directed component of the total electric field for the $s$–polarization case, and the $z$–directed component of the total magnetic field for the $p$–polarization case [12]. Outside the corrugations, $f(x, y)$ is rigorously represented by the following Rayleigh expansions [9]:

$$
f(x, y) = \exp \left[ i (\alpha_0 x - \beta_0^{(1)} y) \right] + \sum_{n=-\infty}^{+\infty} \rho_n \exp \left[ i (\alpha_n x + \beta_n^{(1)} y) \right], \quad y > \max g(x), \quad (3)
$$

$$
f(x, y) = \sum_{n=-\infty}^{+\infty} \tau_n \exp \left[ i (\alpha_n x - \beta_n^{(2)} y) \right], \quad y < \min g(x). \quad (4)
$$

Here, $\{\rho_n\}_{n=-\infty}^{+\infty}$ and $\{\tau_n\}_{n=-\infty}^{+\infty}$ are scalar coefficients to be determined; and

$$
\begin{align*}
\alpha_0 &= \frac{\omega}{c} \sin \theta_0 \\
\alpha_n &= \alpha_0 + 2n\pi/d \\
\beta_n^{(1)} &= \sqrt{\frac{\omega^2}{c^2} - \alpha_n^2} \\
\beta_n^{(2)} &= \sqrt{\frac{\omega^2}{c^2} \varepsilon_2 \mu_2 - \alpha_n^2}
\end{align*}
$$

where $c$ is the speed of light in vacuum. Note that $\beta_n^{(1)}$ is either purely real or purely imaginary; and the conditions

$$
\begin{align*}
\text{Re} \left[ \beta_n^{(1)} \right] &\geq 0 \\
\text{Im} \left[ \beta_n^{(1)} \right] &\geq 0
\end{align*}
$$

\forall n \quad (6)$

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are appropriate for plane waves in the vacuous half-space $y > g(x)$. The refracting half-space $y < g(x)$ being filled by a material medium, $\varepsilon_{2t} > 0$ and $\mu_{2t} > 0$ by virtue of causality. The refracted plane waves must attenuate as $y \to -\infty$, which requirement leads to the condition

$$\text{Im} \left[ \beta_n^{(2)} \right] > 0.$$  \hspace{1cm} (7)

Fulfilment of this condition automatically fixes the sign of $\text{Re} \left[ \beta_n^{(2)} \right]$, regardless of the signs of $\varepsilon_{2R}$ and $\mu_{2R}$. We must note here that the transformation $\{\varepsilon_{2R} \to -\varepsilon_{2R}, \mu_{2R} \to -\mu_{2R}\}$ alters the signs of the real parts of all $\beta_n^{(2)}$.

Boundary conditions at $y = g(x)$ require the continuity of the tangential components of the total electric field and the total magnetic field. Hence,

$$\begin{aligned}
f(x, g(x)+) &= f(x, g(x)-) \\
\hat{n} \cdot \nabla f(x, g(x)+) &= \sigma^{-1} \hat{n} \cdot \nabla f(x, g(x)-)
\end{aligned}$$  \hspace{1cm} (8)

where $\sigma = \mu_2$ for the $s$-polarization case and $\sigma = \varepsilon_2$ for the $p$-polarization case, while $\hat{n}$ is a unit vector normal to the boundary.

At this stage we invoke the Rayleigh hypothesis [9] — that is, we assume that expansions (3) and (4), which are strictly valid outside the corrugated region, can be used in the boundary conditions (8). Doing so, and then projecting into the Rayleigh basis $\{\exp(i \alpha_n x)\}_{m=-\infty}^{+\infty}$, we obtain a system of linear equations for $\{\rho_n\}_{n=-\infty}^{+\infty}$ and $\{\tau_n\}_{n=-\infty}^{+\infty}$. Following Maradudin [13, p. 427], we write down the system in matrix form as

$$\begin{bmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{bmatrix} \begin{bmatrix} \mathcal{R} \\ \mathcal{T} \end{bmatrix} = \begin{bmatrix} \mathcal{U} \\ \mathcal{V} \end{bmatrix}.$$  \hspace{1cm} (9)

The $(m, n)$-th elements of the four matrixes on the right side of (9) are

$$\begin{aligned}
\mathcal{M}_{11} &\bigg|_{mn} = -D_{mn}(\beta_n^{(1)}) \\
\mathcal{M}_{12} &\bigg|_{mn} = D_{mn}(-\beta_n^{(2)}) \\
\mathcal{M}_{21} &\bigg|_{mn} = \beta_n^{(1)} D_{mn}(\beta_n^{(1)}) - \alpha_n E_{mn}(\beta_n^{(1)}) \\
\mathcal{M}_{22} &\bigg|_{mn} = \frac{1}{\sigma} \left[ \beta_n^{(2)} D_{mn}(-\beta_n^{(2)}) + \alpha_n E_{mn}(-\beta_n^{(2)}) \right]
\end{aligned}$$  \hspace{1cm} (10)
while the $m$–th elements of the four column vectors in the same equation are

$$\mathcal{R}\bigg|_m = \rho_m$$

$$\mathcal{T}\bigg|_m = \tau_m$$

$$\mathcal{U}\bigg|_m = D_{m0}(-\beta_0^{(1)})$$

$$\mathcal{V}\bigg|_m = \beta_0^{(1)} D_{m0}(-\beta_0^{(1)}) + \alpha_0 E_{m0}(-\beta_0^{(1)})$$

The integrals appearing in the foregoing equations are defined as

$$D_{mn}(u) = \frac{1}{d} \int_0^d \exp \left[ -i \frac{2\pi}{d} (m - n) x + iug(x) \right] dx$$

and

$$E_{mn}(u) = \frac{1}{d} \int_0^d g'(x) \exp \left[ -i \frac{2\pi}{d} (m - n) x + iug(x) \right] dx,$$

with the prime denoting differentiation with respect to argument.

Equation (9) has to be appropriately truncated and solved to determine the reflection coefficients $\rho_n$ and refraction coefficients $\tau_n$. Diffraction efficiencies

$$e_r^n = \frac{\text{Re} \left[ \beta_n^{(1)} \right]}{\beta_0^{(1)}} |\rho_n|^2,$$

are defined for the reflected orders. The normalized power absorbed across one period of the corrugated interface is given by

$$P_a = \text{Re} \left[ \frac{1}{\beta_0^{(1)}} \sum_{n, m} \left\{ \alpha_n E_{mn} \left( (\beta_m^{(2)})^* - \beta_n^{(2)} \right) + \beta_n^{(2)} D_{mn} \left( (\beta_m^{(2)})^* - \beta_n^{(2)} \right) \tau_n \tau_m^* \right\} \right].$$

The principle of conservation of energy requires that

$$\sum_n e_r^n + P_a = 1.$$

When we implemented the procedure presented, we checked that the condition (16) was satisfied to an error of 10 ppm. This was usually achieved by retaining 15 terms (i.e., $-7 \leq n \leq 7$) in the Rayleigh expansions (3) and (4) of the fields.
III. NUMERICAL RESULTS AND DISCUSSION

We chose the corrugations to be sinusoidal: \( g(x) = 0.5h \cos(2\pi x/d) \). For this type of boundary between vacuum and a penetrable dielectric medium, good results have been obtained for \( h/d < 0.3 \) [14, 15]. We calculated diffraction efficiencies for refracting mediums of both the PPV \((\epsilon_2 = 5 + i0.01, \mu_2 = 1 + i0.01)\) and the NPV \((\epsilon_2 = -5 + i0.01, \mu_2 = -1 + i0.01)\) types. Calculations were made for both the \( s \)- and the \( p \)-polarization cases. Fixing the ratio \( \omega d/c = 2\pi/1.1 \), we plotted the diffraction efficiencies \( e^r_0 \) and \( e^r_{-1} \) as well as the absorption \( P_a \) as functions of \( \theta_0 \in [0, \pi/2) \) for \( h/d = 0.07 \) (Figure 1), \( h/d = 0.14 \) (Figure 2) and \( h/d = 0.21 \) (Figure 3).

When \( h/d = 0 \) — i.e., when the interface is planar — it has been shown [8] that the transformation \( \{\epsilon_{2R} \rightarrow -\epsilon_{2R}, \mu_{2R} \rightarrow -\mu_{2R}\} \) does not change \( e^r_0 \). No wonder, the same transformation does not seem to be very effective in affecting \( e^r_0 \) when \( h/d = 0.07 \). As the corrugations grow deeper (i.e., as \( h/d \) increases in value), the presented data shows that the transformation of the refracting medium from NPV/PPV to PPV/NPV increasingly affects \( e^r_0 \) and \( P_a \).

Why should this be so? Now, for a planar interface, the transformation \( \{\epsilon_{2R} \rightarrow -\epsilon_{2R}, \mu_{2R} \rightarrow -\mu_{2R}\} \) leaves the magnitude of the reflection coefficient only unchanged for non–evanescent incident plane waves; but that is not a true statement for incident evanescent plane waves [16]. In the troughs of the corrugated interface, the total field that exists has both specular \((n = 0)\) and nonspecular \((n \neq 0)\) components. Most of the nonspecular components are like evanescent plane waves because they are characterized by \( \text{Re} \left[ \beta_n^{(1)} \right] = 0 \). Their presence ensures that the diffraction efficiencies are affected by the transformation of the refracting medium from NPV/PPV to PPV/NPV.

Before concluding, let us point out that the numerical results presented here for NPV surface–relief gratings agree with the results of a perturbational approach, thereby validating the limited use of the Rayleigh hypothesis for NPV gratings in the same way as for PPV gratings [17]. Also, the emergence of homogeneous NPV materials promises new types of gratings which could be significantly different from their PPV counterparts.
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FIG. 1: Diffraction efficiencies $e'_0$ and $e'_{-1}$ as well as the normalized absorbed power $P_a$ as functions of the incidence angle $\theta_0$, for a sinusoidally corrugated interface between vacuum and a linear homogeneous medium. The interface function $g(x) = 0.5 \, h \cos(2\pi x/d)$, where $h/d = 0.07$ and $\omega d/c = 2\pi/1.1$. The refracting medium is of either the PPV ($\epsilon_2 = 5 + i0.01, \mu_2 = 1 + i0.01$) or the NPV ($\epsilon_2 = -5 + i0.01, \mu_2 = -1 + i0.01$) type. Calculations were made for both the $s$– and the $p$–polarization cases.
FIG. 2: Same as Figure 1 but for $h/d = 0.14$. 
FIG. 3: Same as Figure [11] but for $h/d = 0.21$. 

(a) $e_0^f$

(b) $e_{-1}^f$

(c) $P_a$