Establishing Base Elements of Perspective in Order to Reconstruct Architectural Buildings from Photographs

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Abstract. The use of perspective images, especially historical photographs for retrieving information about presented architectural environment is a fast developing field recently. The photography image is a perspective image with secure geometrical connection with reality, therefore it is possible to reverse this process. The aim of the hereby study is establishing requirements which a photographic perspective representation should meet for a reconstruction purpose, as well as determination of base elements of perspective such as a horizon line and a circle of depth, which is a key issue in any reconstruction. The starting point in the reconstruction process is geometrical analysis of the photograph, especially determination of the kind of perspective projection applied, which is defined by the building location towards a projection plane. Next, proper constructions can be used. The paper addresses the problem of establishing base elements of perspective on the basis of the photograph image in the case when camera calibration is impossible to establish. It presents different geometric construction methods selected dependently on the starting assumptions. Therefore, the methods described in the paper seem to be universal. Moreover, they can be used even in the case of poor quality photographs with poor perspective geometry. Such constructions can be realized with computer aid when the photographs are in digital form as it is presented in the paper. The accuracy of the applied methods depends on the photography image accuracy, as well as drawing accuracy, however, it is sufficient for further reconstruction. Establishing base elements of perspective presented in the paper is especially useful in difficult cases of reconstruction, when one lacks information about reconstructed architectural form and it is necessary to lean on solid geometry.

1. Introduction

Picturing reality by means of photographs can give very varied effects dependently on the way the object is framed, as well as photographic technique used. The photography image is usually a perspective image with secure geometrical connection with reality. This connection is provided by development of geometric rules of linear perspective. Due to the fact that perspective is the representation of an object from a specific viewing position as well as the image and the object correspond with each other it is possible to reverse this process. That is reconstruct shape and position of the object on the basis of its perspective image. Reconstruction as a restitution process was pioneered by J. H. Lambert in eighteen century and it has reminded an interesting topic of investigation since that time. These days the dimensional reconstruction from perspective images, especially photographs are getting more and more popular [1-4]. Several methods of reconstruction of 3D objects from their perspective images have been proposed [5-9]. However, in every reconstruction process it is necessary to establish base elements of perspective such as a principal point and a circle of death. They are easy to establish when the camera
calibration is known. There are also some approaches studied, which obtain camera geometry and its internal parameters to provide dimensional analysis of the reconstructed model [10-11]. However, the paper addresses the problem of establishing base elements of perspective on the basis of the photographic perspective image in the case when camera calibration is impossible to establish.

We begin in section 2 giving a historical background of the perspective construction and reconstruction process. Section 3 describes our approach to determination of base elements of perspective and section 4 discusses the applied methods. Section 5 concludes.

2. Historical background of geometrical construction and reconstruction of perspective
There are evidences that perspective as the method of representing the world from the given position was known by Greek painters and geometers during classical antiquity. However, this technique was apparently not formalized until the beginning of the Renaissance. The first artist to develop geometrical system of perspective was Filippo Brunelleschi. However, only Battista Alberti in the first treatise Della Pittura described the system of drawing perspective as the intersection of projection rays with a projection surface. This approach to perspective as to the method of representation is valid until this times. A lot of artists with Leonardo da Vinci among others contributed in exploration of both theoretical and practical implications of perspective. However, only creation of descriptive geometry as a branch of mathematics enabled development of perspective as the method of projection within the frame of projective geometry. The comprehension of the concept of perspective as a transformation method has been discussed in different studies [12]. Perspective transformation is a central projection which gives a natural appearance of an object as seen by the eye. However, such a projection usually does not preserve shape of the presented object. That is due to the fact that in projective geometry parallelism and the length of segments of lines is not preserved. The perspective image is treated as an intersection figure of the bundle of projection rays with a projection surface. The points of the projected object and their images are said to be in perspective position when taking a photograph. Next, this position is broken, however, point groups of the object and corresponding groups of points on the photograph remain in a very important geometrical projective relation. If the point group is included in the plane parallel to the projection surface it means that corresponding distances between them and between their images are proportional. But, if the plane of point groups is not parallel to the projection surface, this relationship is no longer valid. However, the so-called cross ratio of four points is preserved. Due to this fact, as well as thanks to the mutual correspondence of the object with its photographic image, it is possible to reverse perspective projection. It enables establishing foundation upon which it is possible to restitute a projected object. One of the first works which treat about perspective projection theory and its restitution is Lambert’s Free Perspective, where the author discusses how to derive a view point and the orthogonal plan of the building on the basis of its perspective image. Due to this work Lambert is regarded as a creator of theoretical photogrammetry.

3. Descriptive methods of estimating base elements of perspective in order to reconstruction process
Before any reconstruction attempt, it is necessary to evaluate photograph’s quality that is: sharpness of contour lines and contrasts, geometry content as well as accuracy.

The restitution process is possible only if the photograph fulfils the restitution requirements. These requirements can be different dependent on the purpose of reconstruction. For special purposes, such as technical documentation for instance one should base only on accurate and reliable data as well as clear geometry.

In the case of photographs taken by the camera which calibration is unknown, it is only possible to reconstruct the object from the photograph according to projective geometry rules.
The first step for establishing base elements of perspective on the basis of the photographic image is determination of the kind of perspective projection applied on the photograph. It should be clear if the object is displayed in one-point, two-point or three-point perspective. The method of perspective projection applied depends on the object’s location towards the projection plane. In general, the base elements of perspective can be establishing on the basis of geometrical figures composing architectural forms or being included in their ornamentation. Sometimes they can be other elements, even not connected with architectural forms, but elements whose geometrical structure is evident or known.

3.1. Establishing base elements of two-point perspective on the basis of the square
Most of the constructions for retrieving base elements of perspective base on a square included in the horizontal or vertical plane. Figure 1 shows such an example when a building is displayed in two-point perspective. Here, the starting point is a square element of a lattice attached to the building, figure 2. The intersection points $Z_1$ and $Z_2$ of two groups of horizontal and parallel lines which are mutually perpendicular, determine a horizon line $h$. Due to the fact that $Z_1$ and $Z_2$ are vanishing points of the mutually perpendicular lines, the eye after rotation lies on a circle $o_1$ between $Z_1$ and $Z_2$. The line $z$ perpendicular to $h$ and going through $Z_2$ is a vanishing line of the vertical lattice plane. Therefore, vanishing points $V_1$ and $V_2$ of the diagonals $d_1$ and $d_2$ lie on the line $z$. Then, the eye after rotation (here the point $O_o$) is an intersection point of circles $o_1$ and $o_2$. On the basis of the location of $O_o$, the principal point $O_\tau$ can be distinguished on a horizon line $h$ and then, the radius of a depth circle is equal the distance of $O_\tau$ from $O_o$, figure 1.

Figure 1. Determination of base elements of perspective on the base of the square: a - geometric construction, b - view of the lattice
3.2. Establishing base elements of two-point perspective on the basis of the ellipses being projections of circles

It is not easy to find representation of a square element on a photograph, if the presented object cannot be seen in reality. Very often it is easier to find ellipses being a perspective projections of circles, due to the fact that circle arches are common forms in old architecture and its ornamentation. The key in such cases is determination of a square in which the circle is inscribed. In the figure 3, it is presented the reconstruction of base elements of perspective on the basis of the projection of the circularly shaped lawn. We draw a trapeze in which the ellipse is inscribed. The trapeze $ABCD$ is a perspective view of a square having edges parallel and perpendicular to the projection plane. Due to this fact its diagonals are inclined relative to the projection plane at 45 degrees. The intersection point of two edge lines $AD$ and $BC$ is a principle point $O\tau$, whereas the intersection point of the diagonal line $d$ with a horizon line $h$ determines the radius of a depth circle $o_d$.

Another presented example shows establishing base elements of perspective on the basis of a façade ornament, which is a semicircle, figure 4b. The building is displayed in two-point perspective. The
Semicircle ornament is included in a vertical façade plane not parallel to the projection plane. Due to this fact, the trapeze in which the whole circle is inscribed has two edges perpendicular to the horizon line $h$, figure 3b. Establishing a proper trapeze enables further reconstruction shown in figure 4a. This reconstruction drawing is similar to the construction described for the case 1 and presented in the figure 1a.

Figure 4. Establishing base elements of perspective on the basis of the circle projection included in a vertical plane: a - geometric construction, b - establishing a key trapeze

3.3. Establishing base elements of three-point perspective

In this case we start from establishing vanishing points $Z_1$, $Z_2$, $Z_3$ of parallel lines of three mutually perpendicular directions, figure 5. Furthermore, these points define three vanishing lines of the three mutually perpendicular planes. We draw perspective views of two additional projecting planes $\alpha$ and $\beta$ being perpendicular to two of our distinguished planes. The intersection point of $\alpha'$ and $\beta'$ is a principle point $O\tau$. Next, we establish the eye after rotation around $Z'Z^\beta$. It is the point $O' = o^1 \cap o^2$. Finally, we establish the eye after the rotation around $\alpha'$, which is the point $O'' = o^1 \cap k$. The distance of $O''$ from $O\tau$ gives a radius of the depth circle $o_d$. 
Figure 5. Establishing base elements of tree-point perspective

4. Results and discussions
Retrieving base elements of perspective is a starting point for every reconstruction process. It can be done automatically or manually. The first method is preferable when the photograph has high quality which means high resolution and definition of vanishing line, as well as the image geometry is strong enough. The manual extraction of the base elements of perspective can be applied even in the case of poor quality photographs with poor perspective geometry. Manual retrieving base elements of perspective is universal as it can base on different geometric constructions selected appropriately according to the starting assumptions. Such constructions can be realized with computer aid when the photographs are in digital form as it was shown in the presented examples. The accuracy of the presented methods depends on the photographic image accuracy, as well as the accuracy of drawing construction. However, if we draw with computer aid zooming proper fragments of the photograph, the accuracy is sufficient for further reconstruction.
5. Conclusions
The presented methods of retrieving base elements of perspective can be applied in reconstructing architecture from photographs when camera calibration is totally unknown. For that reason, some knowledge of geometric reconstruction can be useful, especially in the most difficult cases of reconstruction when lack information about a reconstructed architectural form. In such cases it is necessary to lean on solid geometry.

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