The cosmological constant as an eigenvalue of the Hamiltonian constraint in Horáva-Lifshits theory.

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In the framework of Horáva-Lifshits theory, we study the eigenvalues associated to the Wheeler-DeWitt equation with the meaning of vacuum states, i.e. cosmological constants. The explicit calculation is performed with the help of a variational procedure with trial wave functionals of the Gaussian type. We analyze both the case of the detailed balanced condition and the case without it. We find the existence of an eigenvalue depending on the coupling constant and on the physical scale.

I. INTRODUCTION

The cosmological constant represents one of the challenges of this century\cite{1}. If one is tempted to compute it from the zero point energy of some physical field, one discovers an enormous discrepancy with the observation data. This discrepancy amounts to be of $10^{120}$ order of magnitude: this is known as the cosmological constant problem. Many attempts to explain such a discrepancy have been done, but they appear to be far from being satisfactory. Recently, Hořava proposed a modification of Einstein gravity motivated by the Lifshitz theory in solid state physics\cite{2}\cite{3}. Such modification allows the theory to be power-counting ultraviolet (UV)-renormalizable and should recover general relativity in the infrared (IR) limit. Nevertheless Hořava-Lifshitz (HL) theory is non-covariant. Indeed, in this approach space and time exhibit Lifshitz scale invariance of the form

$$t \rightarrow \ell^z t \quad \text{and} \quad x^i \rightarrow \ell x^i \quad (1)$$

with $z \geq 1$. $z$ is called the dynamical critical exponent and in the present case it is fixed to $z = 3$. The breaking of the 4D diffeomorphism invariance allows a different treatment of the kinetic and potential terms for the metric: from one side the kinetic term is quadratic in time derivatives of the metric, form the other side the potential has high-order space derivatives. In particular the UV behavior is dominated by the square of the Cotton tensor of the 3D geometry by means of a $k^6$ contribution to the propagator leading to a renormalizable power-counting theory. The original HL theory is based on two assumptions – detailed balance and projectability\cite{4}\cite{11}. The projectability condition is a weak version of the invariance with respect to time reparametrization and therefore to the Wheeler-DeWitt equation\cite{17}. Motivated by these interesting features, we ask ourselves if the problem of the cosmological constant in HL theory has better chances to be solved than in ordinary Einstein gravity\cite{2}. Indeed, the modification of the gravitational field at short distances could produce new contributions that allow what in ordinary Einstein gravity is forbidden. In ordinary Einstein gravity, there exists a well accepted model connected with the cosmological constant: this is the Friedmann-Robertson-Walker metric, whose line element is

$$ds^2 = -N^2 dt^2 + a^2(t) d\Omega_3^2. \quad (2)$$

$d\Omega_3^2$ is the usual line element on the three sphere and $N$ is the lapse function. In this background, we have simply

$$R_{ij} = \frac{2}{a^2(t)} \gamma_{ij} \quad \text{and} \quad R = \frac{6}{a^2(t)}. \quad (3)$$

For simplicity we consider the case in which $k = 1$. The generalization to $k = 0, -1$ is straightforward. The Wheeler-DeWitt equation for such a metric is

$$H \Psi(a) = \left[ -\frac{\partial^2}{\partial a^2} + \frac{9 \pi^2}{4 G^2} \left( a^2 - \frac{\Lambda}{3} a^4 \right) \right] \Psi(a) = 0. \quad (4)$$

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1 Different aspects of HL theory are discussed in Refs.\cite{5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16}.

2 See also Ref.\cite{18}, for a different approach on the cosmological constant problem in HL theory.
It represents the quantum version of the invariance with respect to time reparametrization. If we define the following reference length $a_0 = \sqrt{\frac{3}{\Lambda}}$, the Eq. (4) assumes the familiar form of a one-dimensional Schrödinger equation for a particle moving in the potential

$$U(a) = \frac{9\pi^2 a_0^2}{4G^2} \left[ \left( \frac{a}{a_0} \right)^2 - \left( \frac{a}{a_0} \right)^4 \right]$$

(5)

with total zero energy and without a factor ordering. Eq. (4) can be cast into the following form

$$\int D\Psi\Psi^* (a) \left[ -\frac{\partial^2}{\partial a^2} + \frac{9\pi^2 a_0^2}{4G^2} a^2 \right] \Psi (a) = 3\Lambda \pi^2 \frac{4G^2}{a_0^4},$$

(6)

which appears to be an expectation value. In particular, it can be interpreted as an eigenvalue equation with a weight factor on the normalization. The application of a variational procedure in ordinary gravity with a trial wave function of the form

$$\Psi = \exp \left(-\beta a^2\right)$$

(7)

shows that there is no real solution compatible with the procedure. The purpose of this paper is to obtain an eigenvalue equation for the cosmological constant like the one in Eq. (6) but in HL theory with the help of the Wheeler-DeWitt equation. Nevertheless, as pointed out by Mukohyama[19], there are four versions of the theory: with/without the detailed balance condition; and with/without the projectability condition. In this paper, we will consider the problem with and without detailed balanced condition.

II. HL GRAVITY WITH DETAILED BALANCED CONDITION

With the assumption of the detailed balanced condition, the action for the Hořava theory assumes the form

$$S = \int_{\Sigma \times I} dt d^3x \left( \mathcal{L}_K - \mathcal{L}_P \right),$$

(8)

where

$$\mathcal{L}_K = N \sqrt{\bar{g}} \frac{2}{\kappa^2} \left( K^{ij} K_{ij} - \lambda K^2 \right)$$

(9)

is the Lagrangian kinetic term. The extrinsic curvature is defined as

$$K_{ij} = -\frac{1}{2N} \delta_{ij} + \nabla_i N_j + \nabla_j N_i$$

(10)

with $K = K^{ij} g_{ij}$ is its trace and $N_i$ is the shift function. For the FRW metric, there is no shift function. The Lagrangian potential term $\mathcal{L}_P$ is

$$\mathcal{L}_P = N \sqrt{\bar{g}} \left[ \frac{\kappa^2}{2w^2} C^{ij} C_{ij} - \frac{\kappa^2 \mu}{2w^2} \epsilon^{ijkl} R_{il} \nabla_j R_k + \frac{\kappa^2 \mu^2}{8} R^{ijkl} R_{ij} - \frac{\kappa^2 \mu^2}{8} \left( \frac{1 - 4\lambda}{4} R^2 + \Lambda W R - 3\Lambda_W^2 \right) \right],$$

(11)

where

$$C^{ij} = \epsilon^{ijkl} \nabla_k \left( R_l^j - \frac{1}{4} R \delta_l^j \right)$$

(12)

is the Cotton tensor. Plugging the FRW metric (2) into $\mathcal{L}_P$, we obtain

$$\mathcal{L}_P = N \sqrt{\bar{g}} \frac{\kappa^2 \mu^2}{8} \left[ \frac{3}{(1 - 3\lambda) a^4} - \frac{6\Lambda_W}{(1 - 3\lambda) a^2} + \frac{3\Lambda_W^2}{(1 - 3\lambda)} \right]$$

(13)

and the action for the potential part reduces to

$$S_P = -\int_{\Sigma \times I} dt d^3x \mathcal{L}_P = -\frac{\kappa^2 \mu^2}{8} \int_I dt N 2\pi^2 a^3 \left[ \frac{3}{(1 - 3\lambda) a^4} - \frac{6\Lambda_W}{(1 - 3\lambda) a^2} + \frac{3\Lambda_W^2}{(1 - 3\lambda)} \right].$$

(14)
Concerning the kinetic part one gets
\[ \mathcal{L}_K = N \sqrt{g} \frac{2}{\kappa^2} (K^{ij} K_{ij} - \lambda K^2) = a^3 \sin^2 \chi \sin \theta \frac{3}{N} \left( \frac{\dot{a}}{a} \right)^2 (1 - 3\lambda) \] (15)
and the corresponding action is
\[ S_K = \int_{\Sigma \times I} dt d^3x \mathcal{L}_K = \int_I dt 2\pi^2 a^3 \frac{3}{N} \left( \frac{\dot{a}}{a} \right)^2 (1 - 3\lambda). \] (16)
Now, we can compute the canonical momentum and we find
\[ \pi_a = \frac{\delta S_K}{\delta \dot{a}} = \frac{12\pi^2}{\kappa^2} a \dot{a} (1 - 3\lambda), \] (17)
where we have set \( N = 1 \). The resulting Hamiltonian is computed by means of the usual Legendre transformation leading to
\[ H = \int_{\Sigma} d^3 x \mathcal{H} = \int_{\Sigma} d^3 x [\pi_a \dot{a} - \mathcal{L}] \]
\[ = \frac{\kappa^2 \pi_a^2}{12\pi^2 a (1 - 3\lambda)} + \frac{2\pi^2 \kappa^2 \mu^2}{8} \left[ \frac{3}{a (1 - 3\lambda)} - \frac{6\Lambda_W a}{(1 - 3\lambda)} + \frac{3\Lambda^2_W a^3}{(1 - 3\lambda)} \right]. \] (18)
The classical constraint can be read off quite straightforwardly. We obtain
\[ \pi_a^2 + 9\mu^2 \pi^4 \left[ 1 - 2\Lambda_W a^2 + \Lambda^2_W a^4 \right] = 0. \] (19)
This is the classical constraint, which has been analyzed in Ref. [10]. However, we are interested in the Wheeler-DeWitt equation associated to Eq. (19). This is simply given by
\[ -\frac{\partial^2 \Psi}{\partial a^2} + 9\mu^2 \pi^4 \left[ 1 - 2\Lambda_W a^2 + \Lambda^2_W a^4 \right] \Psi = 0, \] (20)
where we have promoted the canonical momentum \( \pi_a \) to an operator \( \pi_a = -i \partial_a \). Even if the WDW equation can be solved, \( \Lambda_W \) cannot be treated as an eigenvalue, because \( \Lambda_W \) is entangled with the scale factor. Things do not change if we make an analytic continuation of parameter \( \mu \rightarrow i\mu \). To this purpose we need to slightly modify the action satisfying the detailed balanced condition [4] by adding an appropriate IR term of the form
\[ \mathcal{L}_P^{\text{small}} = N \sqrt{g} \frac{\kappa^2}{8(1 - 3\lambda)} \left[ \mu^4 R - 2\Lambda^6 \right], \] (21)
where \( \Lambda \) is a dimensionless parameter. Then Eq. (18) modifies into
\[ H = \frac{\kappa^2 \pi_a^2}{12\pi^2 a (1 - 3\lambda)} + \frac{\pi^2 \kappa^2 \mu^2}{4} \left[ \frac{3}{a (1 - 3\lambda)} + \frac{6 (\mu^2 - \Lambda_W a) a}{(1 - 3\lambda)} + \frac{3\Lambda^2_W a^3}{(1 - 3\lambda)} - \frac{2\Lambda^4 a^3}{(1 - 3\lambda)} \right]. \] (22)
Simplifying somewhat, we obtain for the WDW equation
\[ H \Psi = -\frac{\partial^2 \Psi}{\partial a^2} + 3\pi^4 \mu^2 \left[ 3 + 6 (\mu^2 - \Lambda_W) a^2 + 3\Lambda^2_W a^4 - 2\Lambda^4 a^4 \right] \Psi = 0, \] (23)
which looks like Eq. (4). Eq. (23) can be recast into the form
\[ \int \mathcal{D} a \Psi^* (a) \left[ -\frac{\partial^2}{\partial a^2} + 3\pi^4 \mu^2 \left( 3 + 6 (\mu^2 - \Lambda_W) a^2 + 3\Lambda^2_W a^4 \right) \right] \Psi (a) \]
\[ \int \mathcal{D} a \Psi^* (a) [a^4] \Psi (a) = 6\pi^4 \Lambda^6 \] (24)
We compute the value of \( \Lambda \), by adopting a variational technique with a trial wave functional of the form
\[ \Psi = \exp (-\beta a^2). \] (25)
We find
\[ \int_0^\infty da \Psi^* \Psi = \sqrt{\frac{\pi}{8\beta}}, \]  
(26)
which represents the normalization of the wave function. The other terms become
\[ -\int_0^\infty da \Psi^* \frac{\partial^2 \Psi}{\partial a^2} = \int_0^\infty da \Psi^* \left[ 2\beta - 4\beta^2 a^2 \right] \Psi = 2\beta \sqrt{\frac{\pi}{8\beta}} - 4\beta^2 \sqrt{\frac{\pi}{8}} \left( 1 + \frac{\beta^2}{2} \right) = \beta \sqrt{\frac{\pi}{8\beta}}. \]  
(27)
\[ 18\pi^4 \mu^2 \left( \mu^2 - \Lambda_W \right) \int_0^\infty da \Psi^* a^2 \Psi = 18\pi^4 \mu^2 \left( \mu^2 - \Lambda_W \right) \sqrt{\frac{\pi}{8}} \left( 1 + \frac{\beta^2}{2} \right) \]  
(28)
and
\[ \int_0^\infty da \Psi^* a^4 \Psi = \sqrt{\frac{\pi}{8}} \left( \frac{3}{16} \beta^{-\frac{3}{2}} \right). \]  
(29)
Plugging Eqs. (26) . . . (29) into Eq. (21), one gets
\[ 6\pi^4 \Lambda^6 = \beta \sqrt{\frac{\pi}{83}} + 9\pi^4 \mu^2 \sqrt{\frac{\pi}{83}} + 18\pi^4 \mu^2 \left( \mu^2 - \Lambda_W \right) \sqrt{\frac{\pi}{8}} \left( 1 + \frac{\beta^2}{2} \right) + 9\pi^4 \mu^2 \Lambda_W^3 \sqrt{\frac{\pi}{8}} \left( \frac{3}{16} \beta^{-\frac{3}{2}} \right) \]  
(30)

Simplifying somewhat, we obtain
\[ \frac{9}{8} \pi^4 \Lambda^6 = \beta^3 + 9\pi^4 \mu^2 \beta^2 + \frac{9}{2} \pi^4 \mu^2 \left( \mu^2 - \Lambda_W \right) \beta + \frac{27}{16} \Lambda_W^3 \pi^4 \mu^2. \]  
(31)
We now compute the extreme of \( \Lambda \equiv \Lambda (\beta) \) to obtain
\[ \frac{\partial \Lambda}{\partial \beta} = 0 \implies 3\beta^2 + 18\pi^4 \mu^2 \beta + \frac{9}{2} \pi^4 \mu^2 \left( \mu^2 - \Lambda_W \right) = 0, \]  
(32)
whose solutions are
\[ \beta_{1,2} = 3\pi^4 \mu^2 \left[ -1 \pm \sqrt{1 - \frac{1}{6\pi^4} (1 - x)} \right]; \quad x := \frac{\Lambda_W}{\mu^2}. \]  
(33)
We can recognize three cases: a) \( x = 1 \), b) \( x \gg 1 \) and c) \( 0 < x \ll 1 \).

a) \( x = 1 \). This implies that
\[ \beta_{1,2} = -3\pi^4 \mu^2 \left( 1 \pm 1 \right). \]  
(34)
Both solutions lead to a non-normalizable wave-function and therefore will be discarded.

b) \( x \gg 1 \). For this choice, one solution must be discarded for normalization conditions and we remain with
\[ \beta_1 \simeq -3\pi^4 \mu^2 \left( 1 - \sqrt{1 + \frac{x}{6\pi^4}} \right) \simeq \pi^2 \mu^2 \sqrt{\frac{3x}{2}}. \]  
(35)

Finally, we have the last case
c) \( 0 < x \ll 1 \). The two solutions can be approximated with
\[ \beta_{1,2} \simeq -3\pi^4 \mu^2 \pm 3\mu^2 \pi^4 \sqrt{1 - \frac{1}{6\pi^4}} \simeq 3\pi^4 \mu^2 (-1 \pm 0.999) \]  
(36)
and this is close to the case a) and therefore will be discarded.

Therefore, only case b) can give potential solutions to the problem. Plugging solution (35) into Eq. (31), one approximately obtains
\[ \Lambda = \left[ \frac{3}{2} x^2 - \pi^2 \frac{8x}{3} \sqrt{\frac{3x}{2}} + \pi^4 12x + 4\pi^2 \sqrt{\frac{3x}{2}} \right] \simeq \frac{3}{2} x^2. \]  
(37)
Here the dominant contribution has the correct sign. Therefore we conclude that in this range there exists a family of solutions to the cosmological constant depending on the variable \( x \). In the next section, we abandon the detailed balanced condition to see if we can obtain other different solutions.
III. HL GRAVITY WITHOUT DETAILED BALANCED CONDITION

In Refs. [21, 22], a more general potential form which avoids the detailed balanced condition has been proposed. Its expression is

\[ \hat{\mathcal{L}}_P = N \sqrt{g} \left[ g_0 \zeta^6 + g_1 \zeta^4 R + g_2 \zeta^2 R^2 + g_3 \zeta^2 R^{ij} R_{ij} + g_4 R^4 \right] + g_5 R \left( R^{ij} R_{ij} + g_6 R^{ij} R^{jk} + g_7 R \nabla^2 R + g_8 \nabla_i R_{jk} \nabla^i R^{jk} \right). \]  

(38)

Couplings \( g_a (a = 0 \ldots 8) \) are all dimensionless and powers of \( \zeta \) are necessary to maintain such a property of \( g_a \).

Plugging the FRW background into \( \hat{\mathcal{L}}_P \), one gets

\[ \hat{\mathcal{L}}_P = N \sqrt{g} \left[ g_0 \zeta^6 + g_1 \zeta^4 \frac{6}{a^2 (t)} + \frac{6 \zeta^2}{a^4 (t)} (6g_2 + g_3) + \frac{6}{a^6 (t)} (36g_4 + 6g_5 + g_6) \right] \]

(39)

The term \( g_0 \zeta^6 \) plays the role of a cosmological constant. In order to make contact with the ordinary Einstein-Hilbert action in 3 + 1 dimensions, we set without loss of generality \( g_0 \zeta^6 = 2 \Lambda \) and \( g_1 = -1 \). In case one desires to study the negative cosmological constant, the identification trivially will be \( g_0 \zeta^6 = -2 \Lambda \). After having set \( N = 1 \), the Legendre transformation leads to

\[ \mathcal{H} = \pi_a \dot{a} - \mathcal{L}_P \]

(40)

and the Hamiltonian becomes

\[ H = \int_{-\infty}^{\infty} dt \mathcal{H} = - \frac{\kappa^2 \pi_a^2}{12 \pi^2 a (3 \lambda - 1)} + 2 \pi^2 a^3 (t) \left[ 2 \Lambda - \frac{6 \zeta^4}{a^2 (t)} - \frac{6 \zeta^2 b}{a^4 (t)} - \frac{6 c}{a^6 (t)} \right], \]

(41)

where

\[ \begin{cases} 6g_2 + g_3 = b \\ 36g_4 + 6g_5 + g_6 = c \end{cases}. \]

(42)

The WDW equation can be easily extracted to give

\[ \pi_a^2 \Psi + \frac{(3 \lambda - 1)}{\kappa^2} 24 \pi^4 a^4 (t) \left[ -2 \Lambda - \frac{6 \zeta^4}{a^2 (t)} - \frac{6 \zeta^2 b}{a^4 (t)} - \frac{6 c}{a^6 (t)} \right] \Psi = 0 \]

(43)

and adopting the same procedure of Eq. (23), we can write

\[ \int_0^{\infty} da \Psi^* \left\{ \pi_a^2 \Psi + \frac{(3 \lambda - 1)}{\kappa^2} 24 \pi^4 a^4 (t) \left[ -2 \Lambda - \frac{6 \zeta^4}{a^2 (t)} - \frac{6 \zeta^2 b}{a^4 (t)} - \frac{6 c}{a^6 (t)} \right] \right\} \Psi = 0. \]

(44)

We can rearrange the previous expression to obtain

\[ \int_0^{\infty} da \Psi^* \left\{ \pi_a^2 \Psi + \frac{(3 \lambda - 1)}{\kappa^2} 144 \pi^4 \left[ \zeta^4 a^2 (t) - \zeta^2 b - c a^{-2} (t) \right] \right\} \Psi = \left( \frac{3 \lambda - 1}{\kappa^2} \right) 48 \pi^4 \Lambda. \]

(45)

It is now clear the role of \( \Lambda \). As a trial wave functional, we adopt the same form of Eq. (25) and using the results of Eqs. (26) ... (29) we can write

\[ \beta \sqrt{\frac{3}{8 \beta}} + \frac{(3 \lambda - 1)}{\kappa^2} 144 \pi^4 \left[ \zeta^4 \sqrt{\frac{a}{8}} \left( \frac{1}{2} \beta^{-\frac{1}{2}} \right) - \zeta^2 b \sqrt{\frac{a}{8}} + c \sqrt{2 \pi} \beta \right] \sqrt{\frac{3}{16}} \beta^{-\frac{1}{2}} = \left( \frac{3 \lambda - 1}{\kappa^2} \right) 48 \pi^4 \Lambda, \]

(46)

where we have used the following relation to compute

\[ -c \int_0^{\infty} da \Psi^* a^{-2} \Psi = -c \int_0^{\infty} \frac{dx}{2 \sqrt{2} x^3} \exp (-2 \beta x) = -c \Gamma \left( -\frac{1}{2} \right) \sqrt{2 \beta} = c \sqrt{2 \pi} \beta. \]

(47)
By simplifying somewhat, one gets
\[(1 + 4\tilde{c}c)\beta^3 + \tilde{c} \left[ \frac{\zeta^4}{4} \beta - \beta^2 \zeta^2 b \right] = 9\pi^4 \frac{(3\lambda - 1)}{\kappa^2} \Lambda, \tag{48}\]
where we have defined
\[\tilde{c} = \frac{(3\lambda - 1)}{\kappa^2} 144\pi^4. \tag{49}\]
By applying the variational procedure, one obtains
\[\frac{d\Lambda}{d\beta} = 0 = 3 (1 + 4\tilde{c}c) \beta^2 - 2\tilde{c}b\zeta^2\beta + \zeta^4 \tilde{c} = \Rightarrow \beta_{1,2} = \frac{\tilde{c}b\zeta^2 \pm \zeta^2 \sqrt{(\tilde{c}b)^2 - 3 (1 + 4\tilde{c}c)}}{3 (1 + 4\tilde{c}c)}. \tag{50}\]
In order to have two real solutions, we impose that
\[\tilde{c}b^2 \geq (1 + 4\tilde{c}c) \frac{3}{4}. \tag{51}\]
Since the coefficients \(b\) and \(c\) are a linear combination of the coupling constants \(g_i\), for simplicity, we investigate two cases: a) the case, when \(\tilde{c}b^2 \gg (1 + 4\tilde{c}c) \frac{3}{4}\); b) the case when \(\tilde{c}b^2 = (1 + 4\tilde{c}c) \frac{3}{4}\). For the case a), we get
\[\beta_1 = \frac{2\tilde{c}b\zeta^2}{3 (1 + 4\tilde{c}c)}. \tag{52}\]
Plugging \(\beta_1\) into Eq. \(48\), we obtain
\[\zeta^6 \frac{\tilde{c}^2 b}{6 (1 + 4\tilde{c}c)} \left[ 1 - \frac{8\tilde{c}b^2}{9 (1 + 4\tilde{c}c)} \right] = 9\pi^4 \frac{(3\lambda - 1)}{\kappa^2} \Lambda. \tag{53}\]
If we approximate \(1 + 4\tilde{c}c \simeq 4\tilde{c}c\), we get
\[\zeta^6 \frac{2b}{3c} \left[ 1 - \frac{2b^2}{9c} \right] = \Lambda < 0, \tag{54}\]
because of the approximation chosen. Therefore this range of solutions will be discarded. For the case b), we obtain
\[\beta_{1,2} = \beta = \frac{\zeta^2}{4b}. \tag{55}\]
Eq. \(48\) becomes
\[\zeta^6 \left[ \frac{\tilde{c}}{12b} \right] = \zeta^6 \frac{4}{3b} = \Lambda = \frac{g_0 \zeta^6}{2}. \tag{56}\]
This implies
\[\frac{8}{3b} = g_0. \tag{57}\]

IV. CONCLUSIONS

In this letter, we have considered the recent proposal of Horava to compute the cosmological constant in a modified non-covariant gravity theory at the Lifshitz point \(z = 3\). We have used the Wheeler-DeWitt equation as a backbone of all calculations with the cosmological constant regarded as an eigenvalue\(^{23}\). We have analyzed the situation with detailed balanced condition and without the detailed balanced condition. A variational approach with Gaussian wave functions has been used to do calculation in practice. The background geometry is a Friedmann-Robertson-Walker metric with \(k = 1\). Concerning the detailed balanced condition, we have found an approximate eigenvalue not in the original configuration but with a slight modification in the IR region. The main reason is related to the fact that the original parameter \(\Lambda_W\) cannot be considered as an eigenvalue in this formulation. What we have found is that we have only results in agreement with a big cosmological constant, therefore with Planck era estimations. This means that the cosmological constant problem in this approach is not solved. It is interesting to note that only the dimensionless ratio \(\Lambda_W/\mu^2\) comes into play. As regards the case without detailed balanced condition, we obtain two cases:
a) \( g_0 \) is large. For example, it could be fine tuned to Planck era values. This implies that \( b = 6g_2 + g_3 \ll 1 \). This can be achieved if \( g_2, g_3 \ll 1 \) or \( 6g_2 \simeq -g_3 \). When \( g_2, g_3 \ll 1 \) we fall into a perturbative regime, while when \( 6g_2 \simeq -g_3 \) this could not be the case. In this respect, this version of HL theory and the version with detailed balanced condition behave in the same way, except eventually for the smallness of the coupling constants \( g_2 \) and \( g_3 \).

b) \( g_0 \) is small and fine tuned to \( g_0 \sim 10^{-120} \) as from observation. This implies that \( b = 6g_2 + g_3 \gg 1 \), that it means that at least either \( g_2 \) or \( g_3 \) must be very large preventing a perturbative approach.

Note that in the version of HL theory without balanced condition, there is no trace of the other coupling constants: only \( g_2 \) and \( g_3 \) come into play. We can further observe that another big difference between HL theory with and without balanced condition is that the latter admits a possible transition from big values to small values of the cosmological constant. However, it is likely that an improvement could be obtained by introducing a renormalization group equation for \( g_2 \) and \( g_3 \) and a running \( \Lambda \). Another improvement for having bounds on parameters can be extracted from the equation of state \( p = \omega \rho \) with \( \omega = -1 \). Using the Gaussian wave function's value of the exponent \( \beta \) obtained by the variational procedure into the equation of state, we can extract other informations on \( g_2 \) and \( g_3 \). Needless to say that the analysis has been done without any matter field and also this addition could improve the final result.
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