Determination of the Strong Coupling Constant and Multijet Cross Section Ratio Measurements

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Abstract. Concepts and results of determinations of the strong coupling in hadron collisions are discussed. A recent $\alpha_s$ result from the inclusive jet cross section in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV is presented which is based on perturbative QCD calculations beyond next-to-leading order. Emphasis is put on the consistency of the conceptual approach. Conceptual limitations in the approach of extracting $\alpha_s$ from cross section data are discussed and how these can be avoided by using observables that are defined as ratios of cross sections. For one such observable, the multijet cross section ratio $R_{3/2}$, preliminary results are presented.

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INTRODUCTION

The strong coupling constant, $\alpha_s$, is one of the fundamental parameters of the Standard Model of Particle Physics. The energy dependence of $\alpha_s$ is predicted by the renormalization group equation (RGE). The value of $\alpha_s$ has been determined in many different processes, including a large number of results from hadronic jet production, in either $e^+e^-$ annihilation or in deep-inelastic $ep$ scattering (DIS) up to energies of 209 GeV [1]. Prior to the analysis presented in this article, however, only a single result had been obtained from jet production in hadron-hadron collisions. This $\alpha_s$ result is $\alpha_s(M_Z) = 0.1178^{+0.0081\text{(exp.)}+0.0071\text{(scale)}}_{-0.0095\text{(scale)}}^{+0.0047\text{(scale)}} \pm 0.0059\text{(PDF)}$, extracted by the CDF collaboration from the inclusive jet cross section in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV [2]. All individual uncertainty contributions for this result are larger than those from comparable results from $e^+e^-$ annihilation or DIS [1].

The first part of this article presents a recent $\alpha_s$ determination from the DØ collaboration from the inclusive jet cross section with significantly improved precision. Conceptual limitations of the approach are discussed, and how those are addressed in the DØ analysis. The second part introduces a new observable to which these conceptual limitations do not apply, and which will be valuable for future $\alpha_s$ determinations in new energy regimes accessible at the Tevatron and at the LHC.

DETERMINATION OF THE STRONG COUPLING CONSTANT

A new DØ analysis [3] extracts the value of $\alpha_s$ from inclusive jet cross section data in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV. It is based on a recent DØ measurement of the inclusive jet cross section [4] with unprecedented precision at a hadron collider. The perturbative
QCD (pQCD) prediction for the inclusive jet cross section is given by

\[ \sigma_{\text{pert}}(\alpha_s) = \left( \sum_n \alpha_s^n c_n \right) \otimes f_1(\alpha_s) \otimes f_2(\alpha_s), \]

where the \( c_n \) are the perturbative coefficients, the \( f_{1,2} \) are the parton distribution functions (PDFs) of the initial state hadrons, and the “\( \otimes \)” sign denotes the convolution over the momentum fractions \( x_1, x_2 \) of the hadrons. The sum runs over all powers \( n \) of \( \alpha_s \) which contribute to the calculation. The DØ result is based on NLO pQCD (\( n = 2, 3 \)) plus 2-loop contributions from threshold corrections [5] (\( n = 4 \)). The latter reduce the scale dependence of the calculations, leading to a significant reduction of the corresponding uncertainties. While the \( f_{1,2} \) have no explicit \( \alpha_s \) dependence, our knowledge of \( f_{1,2} \) depends on \( \alpha_s \) (due to \( \alpha_s \) assumptions in the PDF analyses). Since the RGE uniquely relates the value of \( \alpha_s(\mu_r) \) at any scale \( \mu_r \) to the value of \( \alpha_s(M_Z) \), all equations can be expressed in terms of \( \alpha_s(M_Z) \). The total theory prediction for inclusive jet production is given by the pQCD result in (1), multiplied by a correction factor for non-perturbative effects

\[ \sigma_{\text{theory}}(\alpha_s(M_Z)) = \sigma_{\text{pert}}(\alpha_s(M_Z)) \cdot c_{\text{non-pert}}. \]

The pQCD results are computed in FASTNLO [6] which is based on NLOJET++ [7, 8] and the calculations from Ref. [5]. To determine \( \alpha_s(M_Z) \), recent PDF results are used and \( \alpha_s(M_Z) \) is varied in \( \sigma_{\text{pert}}(\alpha_s(M_Z)) \) (i.e. simultaneously in the matrix elements and in the PDFs) until \( \sigma_{\text{theory}}(\alpha_s(M_Z)) \) agrees with the data. There are, however, two conceptual issues when extracting \( \alpha_s \) from cross section data.

1. When performing the DGLAP evolution of the PDFs, all PDF analyses are assuming the validity of the RGE which has so far been tested only for energies up to 209 GeV. Since extracting \( \alpha_s \) at higher energies means testing (and therefore questioning) the RGE, using these PDFs as input would be inconsistent.

2. DØ jet data have been used in all recent global PDF analyses. The PDF uncertainties are therefore correlated with the experimental uncertainties in those kinematic regions in which the DØ jet data had strong impact on the PDF results. As shown in Figs. 51–53 in Ref. [10], this is the case for the proton’s gluon density at \( x > 0.2 – 0.3 \). Since the correlations between PDF uncertainties and experimental uncertainties are not documented, the \( \alpha_s \) extraction should avoid using those data points which already had significant impact on the PDF results.

In light of the second issue, the DØ \( \alpha_s \) extraction uses only data points which are insensitive to \( x > 0.2 – 0.3 \). Since all of these data points have \( p_T \) below 145 GeV, the first issue does not become relevant here. This leaves 22 (out of 110) inclusive jet data points.

The \( \alpha_s \) extraction uses PDFs from the MSTW2008 analysis [9] which were obtained at NNLO (consistent with the precision of the theory calculation used here). These PDFs have been determined for 21 \( \alpha_s(M_Z) \) values between 0.107 and 0.127 [10]. The continuous \( \alpha_s(M_Z) \) dependence of the pQCD cross sections is obtained by interpolating the cross section results for the PDF sets for different \( \alpha_s(M_Z) \) values. PDF uncertainties are computed using the twenty uncertainty eigenvectors (corresponding to 68% C.L.).
uncertainties in the pQCD calculation due to uncalculated higher-order contributions are estimated from the $\mu_{r,f}$ dependence of the calculations when varying the renormalization and factorization scales in the range $0.5 \leq (\mu_{r,f}/p_T) \leq 2$. In a first step, data points with same $p_T$ are combined to determine nine values of $\alpha_s(p_T)$ for $50 < p_T < 145$ GeV. These results are shown in Fig. 1 and compared to results obtained in DIS. A combined determination from all 22 data points yields a result of

$$\alpha_s(M_Z) = 0.1161^{+0.0034}_{-0.0033}^{(exp.)} +0.0010_{-0.0016}^{(non-pert.)} +0.0011_{-0.0012}^{(PDFs)} +0.0025_{-0.0029}^{(scale)}.$$  

This is currently the most precise result from a hadron collider, with similar precision as recent results from jet production in DIS.

**MEASUREMENT OF MULTIJET CROSS SECTION RATIOS**

The conceptual issues discussed above, which limit the $x$ range, and therefore also the $p_T$ range of the data points used in the $\alpha_s$ determination, are both related to the fact that the observable is sensitive to the proton PDFs which are required as external input in the $\alpha_s$ determination. These limitations can be avoided by studying observables which are largely independent of the PDFs, but still sensitive to $\alpha_s$. One class of such observables are ratios of multijet cross sections. The variable $R_{3/2}$ represents the conditional probability that a given inclusive dijet event also has a third jet. It is defined as the ratio of the inclusive 3-jet and dijet cross sections and investigated as a function of $p_{T_{\text{max}}}$, the transverse momentum of the leading jet in an event, which is a
common scale for the 3-jet and the dijet production processes. Therefore $R_{3/2}(p_{T_{\text{max}}})$ is directly sensitive to $\alpha_s$ at the scale $\mu_r = p_{T_{\text{max}}}$ while the PDFs cancel to a large extent in the cross section ratio. Technically, the $n$-jet cross section (for $n = 2, 3$) is defined by all events with $n$ or more jets with $p_T$ above $p_{T_{\text{min}}}$, in a given rapidity region (here: $|y| < 2.4$ for the $n$ leading jets). The preliminary results of a recent DØ measurement of $R_{3/2}$ [11], obtained for different values of $p_{T_{\text{min}}} = 50, 70, 90\text{GeV}$, are corrected to particle level and presented in Fig. 2 as a function of $p_{T_{\text{max}}}$. The data are well described by theory calculations based on NLO pQCD plus non-perturbative corrections, for different PDFs [10, 12, 13, 14] using in all cases $\alpha_s(M_Z) = 0.118$ (in the matrix elements and in the PDFs). In the future, measurements of $R_{3/2}$ and related observables will provide a solid basis for determinations of $\alpha_s$ over the whole $p_T$ region, accessible at the Tevatron and the LHC, and without conceptual issues related to the proton PDFs. Such results will allow testing the RGE in a novel energy regime.

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