Strength and deformability of segmental vaults with embedded steel beams

**Popov Anton**¹[0000-0002-9008-6228] and **Yagofarov Dinar**¹[0000-0002-7298-9837]

¹ Kazan State University of Architecture and Engineering, Kazan 420043, Russia
E-mail: d.yagofarov@list.ru

**Abstract.** The use of curved arches and vaulted structures in mass construction dates back to ancient Rome. Rack-and-beam systems are changed by rack-and-arch systems, the latter have a greater assurance factor and cover a much longer span. For many centuries, architects have mastered and improved the design of arched floors, constantly improving their details. In the middle of the XVIII century the appearance of segmental vaults with embedded steel beams made it possible to abandon the massive brick vaults. Such slabs are widely adopted not only in religious buildings, but also in public buildings that are now recognized as architectural monuments. Often these constructions form the face of the city, and, therefore, the question of preservation, as well as their adaptation for modern use is urgent question. The latter implies the changing of functional load, and sometimes the design model of the building. Therefore, in most cases, the priority emergency measures are required. In this article, the analytical solution of the strength issue and stability of segmental vaults with embedded steel beams on the basis of engineering research and design solutions are offered [14, 20].

**Keywords:** segmental vaults, damaged structures, reconstruction.

**1 Introduction**

Before the first half of the XIX century slabs in buildings were arranged in the form of cylindrical brick vaults or entirely from wood. From the end of the XIX century they began to run from segmental vaults with embedded steel beams, also known as Prussian vaults. Brick, concrete, and reinforced concrete were used as aggregates (figure 1). Slabs with reinforced concrete rolling on rolled steel beams became known as Monier vaults. The appearance of such structures in the construction practice of the XVIII century is associated with the previous Industrial Revolution. In due time, this was an innovative solution that allowed to abandon the bulky height brick vaults but at the same time not losing the size of the span, as well as the load-bearing capacity [1].

**Figure 1.** The typology of segmental vaults with embedded steel beams:

a. segmental vaults with embedded iron and steel rails;

b. segmental vaults with a thickness of ½ brick, with embedded iron and steel I-beams;

c. segmental vaults with a thickness of ¼ brick, with embedded iron and steel I-beams;

d. segmental vaults of rubble concrete, concrete and reinforced concrete ("Monier"), with embedded steel I-beams
From the point of view of fire resistance, durability and load-bearing capacity, the filling between steel beams in the form of stone arches is the most rational solution compared to wood, especially for sub-basement slabs. The disadvantages of such rolling can be attributed to heavy sole weight, the complexity of construction and the lack of correct calculation methods. For these reasons, from the beginning of the XX century constructors started to use such less commonly, and with the spread of reinforced concrete structures, they completely lost their popularity. Nevertheless, a considerable number of buildings have been preserved with the use of this structural solution of slab in large cities, where many historical buildings are dated back to the XIX-XX centuries [5, 6, 7].

2 Materials and methods
The segmental vaults with embedded steel beams have sprung arch designs, in general determining the scheme of their destruction. Numerous works on the auscultation, strengthening and restoration of historical buildings were carried out by the authors [14,20]. The data provided in fundamental works edited by N. K. Lakhtin [12] and V. R. Bernhard [3] allowed systematizing the damage of such a structure (Figure 2).

Figure 2. The most typical damage of segmental vaults with embedded steel beams: a. the surface corrosion of the metal beams; b. the deep corrosion of the metal beams up to rust-through damage of the wall; c. the longitudinal cracks along the metal beams formed by expansion of corrosion products; d. the longitudinal cracks on the surface of the vault; e. the destruction of stone masonry.

Often these damages occur together, within a single beam span (Figure 3). Therefore, complex solutions are required to restore the load-bearing capacity of the segmental vaults with embedded steel beams.

Figure 3. Representative photo of the defects and the damages of brick segmental vaults with embedded steel beams [20]
Considering the nature of the destruction and damage of segmental vaults, as well as the causal relationship of their formation, we can conclude that the vaults, being sprung arch design, have a significant margin of viability. This is due to the effect of self-healing of cracks on their surface. Crack opening and cracks formed by shifting individual blocks along the mortar joints are most often formed on the surface of vaults [17]. The mechanism of self-healing for the first and second cases is different, but in both cases it leads to a slight, in general, geometric change in the forming surface of the vault. The mechanism of self-healing of the crack opening is due to the fact that, regardless of the zone of crack formation (in the lower or upper part). A significant concentration of stresses is formed on the zone opposite from the crack, at the same time when the strength limit of the brickwork is reached, it is crushed, and the crack closes up at this moment. In this case, the vault as a whole passes from one stationary geometrically unchangeable state to another also geometrically unchangeable, however, with a different form of the generator than the initial one. Such geometric modifications of the segmental vaults generator are possible until the rising height of f is close to zero (figure 4), and in some cases have negative value, as it is shown in figure 5 [9,25,26].

**Figure 4.** Deformed vault with rising height =0

**Figure 5.** Deformed vault with a negative value of the rising height.

The solution of the strength and stability issues of the vaults are given in the works of G. Milani, J. McInerney, M. Rossi, N. K. Lakhtin, V. G. Zalesky, V. R. Bernhard, G. B. Bessonov and many others, but they are limited to a direct solution of the design issue, namely, by the selection of the geometric parameters of vaults, as well as the strength characteristics of materials. In modern conditions, when
the design of vaulted slabs in newly erected buildings is rare, the main task is to check the strength and deformability of existing vaulted slabs, taking into account defects and damage.

In modern construction practice, the numeral methods of solving strength and stability issues of vaults are widely adopted.

For numeral research segmental vaults with embedded steel beams were modeled in the PC by ANSYS 18.1.

As a result of the model calculation, the pictures of strain development in the concrete vault and steel profile under loading are obtained. The picture of deformations clearly shows the sequence of deformations in the steel profile and concrete vault; the strain development from a more stressed zone to a less stressed one, from the middle to the ends of the slab and from the extreme fibers deep into the section [2]. In a steel beam, the strain development with increasing load is more intense than in a concrete vault. It can be explained by a smaller section of the stretched belt than the compressed concrete section. The computer picture of strains confirms the dynamics of the distribution of defects and damages detected during the field observation in cross sections, both in height and length of these slabs.

![Figure 6. The computer graphics of the stress distribution in the lower surface in Nz (compression/tension).](image)

The analysis of the stress distribution in the body of the vault shows that the main stresses are concentrated in the diagonal directions from the center of the vault to the supporting zones of the steel beams (see Figure 6). This is due to the fact that as loading in the longitudinal direction segmental vaults are included in the work.

In the early works of the authors it was shown [22] that the most suitable model for using numeral methods is the Heck-Braun model, which is in contrast to models with a surface flow according to the Mohr-Coulomb theory and the others allows including damage criteria. Taking into account the wide range of stress levels that can occur in masonry, the dependence obtained using the Mohr-Coulomb model does not fully describe the existing situation. In this regard, the most suitable for masonry is the Heck-Braun fracture criterion, which has a nonlinear approximation of the strength of materials. It includes shear resistance and tensile strength, in the form of continuous functions. This criterion, together with Hooke’s law of isotropic linear elasticity, defines the Heck-Braun model.

On the basis of the above hypotheses and analyzing the results of numeral methods, the solution of the segmental vaults is offered by using the analytical method. Using the well-known solution of G. B. Bessonov and considering a two-hinged arch as a statically indeterminate hinge-supported curved rod with fixed supports.

Based on the equilibrium condition the moment and normal force are defined by the expression:

\[ M \leq M_p^0 + M_{r}X_1 \]  \hspace{1cm} (1)

\[ N \leq N_p^0 + N_{r}X_1 \]  \hspace{1cm} (2)
where
\( M^0_p \) - the moment from the load on the main system;
\( N^0_p \) –normal force from the load on the main system;
\( M_1 \) - the moment from the conditional unit load \( X_1, X_2, X_3 \) applied to the main system;
\( N_1 \) –normal force from a conditional unit load \( X_1, X_2, X_3 \) applied to the main system;
\( X_1, X_2, X_3 \) - so-called "extra" unknowns, applied to the main system instead of eliminated unnecessary bonds and determined by the solution of a system of canonical equations, which is transformed depending on the number of unknowns.

In our case, for two-hinged arches with one unknown, the equation has the form:
\[ X_i \delta_{i1} + \Delta_{i1} = 0 \] (3)

The coefficients of the equations represent movements on the main system from the action of unit forces applied instead of "extra" unknown ones. The mentioned coefficients are determined by Maxwell-Mohr Eq. (4):

\[ \delta_{i1} = \sum \int \frac{M_1 M_1}{EJ} ds + \sum \int \frac{N_1 N_1}{EF} ds \] (4)
\[ \Delta_{i1} = \sum \int \frac{M_1 M_0 p}{EJ} ds + \sum \int \frac{N_1 N_0 p}{EF} ds \] (5)

For calculating the vault there is used a method of statically indeterminate force system, i.e., it's necessary to determine the laws of change of bending moments, shear and longitudinal force on the main system for cargo and auxiliary states, \( M_1 \) and \( M_p \) respectively, by the Eq. (4) and (5) the coefficients of the canonical Eq. (3) of force method, the solution of which is unknown force \( X_i \) in one of the supports.

The integrals included in Eq. (4) and (5) that determine the coefficients \( \delta_{i1} \), \( \Delta_{i1} \) can be calculated in various ways. In the case of an arch of constant stiffness and a residual simple load (i.e., with a simple form of the moment function), analytical calculation of integrals can be used.

When calculating the displacements \( \delta_{i1} \), \( \Delta_{i1} \) integrals of these coefficients can be calculated with sufficient accuracy by multiplying the diagrams \( M_1, N_1 \) etc. To do this, the arch with diagrams \( M^0_p, N^0_p \) from a given load, is divided into separate sections. The curved axis of the arch is replaced by a polygonal axis inscribed in it, each link of which serves as the basis for the corresponding diagram area. Multiplication of diagram areas is performed according to the Vereshchagin rule. The accuracy of the calculation depends on the degree of approximation of the polyline length to the curved axis, that is, on the number of summed sections. The recommended number of them is \( 6-8 \) per half-frame with symmetrical load.

In a two-sided (once statically indeterminate) arch, the extra unknown \( X_1 \) is usually taken to be the n-spacer, calculated by the Eq. (6):

\[ H = \frac{\Delta_{i1}^p}{\delta_{i1}} \] (6)

Substituting the values of cargo and unit movements, we get:
\[ H = \frac{\sum \mathcal{M}_1 M^0_E ds + \sum \mathcal{N}_1 N^0_F ds}{\sum \mathcal{M}_1 \mathcal{M}_1 ds + \sum \mathcal{N}_1 \mathcal{N}_1 ds} \]  
\tag{7}

It should be noted that the influence of longitudinal forces is very noticeable for sloping arches of large thickness \( \delta > 0.1l \), as well as for arches that are close to non-stationary. In cases when \( f > \frac{1}{5} \) and \( \delta < 0.1l \), can be taken:

\[ H = \frac{\sum \mathcal{M}_1 M^0_E ds}{\sum \mathcal{M}_1 \mathcal{M}_1 ds} \]  
\tag{8}

For arches of constant cross-section at \( \text{EJ} = \text{const} \), \( \text{EF} = \text{const} \), according to the rules of integration, the multiplier-constant can be taken as the sign of the integral, in our case the constant is stiffness, then the Eq. (8) will have the form:

\[ H = \frac{\sum \mathcal{M}_1 M^0_E ds}{\sum \mathcal{M}_1 \mathcal{M}_1 ds} \]  
\tag{9}

The calculation results are in good agreement with the test results [14].

3 Results
1. There was presented a model for the calculation of the segmental vaults.
2. It is assumed that the algorithm provides the opportunity to obtain relatively high accuracy while minimizing operations.
3. The use of the Heck-brown model for solving stability and deformability issues of segmental vaults is established.

References
[1] Alexakis H and Makris N 2016 Chapter 12: Validation of the discrete element method for the limit stability analysis of masonry arches. Computational modeling of masonry structures using the discrete element method (Ed) Paulo B. Lourenço (eds) V Sarhosis, K Bagi, J V Lemos and G Milani (Hershey, PA, US: Engineering Science Reference) pp 292–325
[2] Baratta A, Corbi O 2015 Computers & Structures 147 pp 244-249
[3] Berngard V R 1901.Arki i svody. Rukovodstvo k ustranstvu i raschetu arochnykh i svodchatykh perekrytij [Arches and vaults. Guide to the design and calculation of arched and vaulted floors] (Saint-Petersburg) p 128(rus)
[4] Bespalov V V, Ucer D, Salmanov I D, Kurbanov I N and Kupavykh S V 2018 Deformation compatibility of masonry and composite materials Magazine of Civil Engineering 2 pp 136–150 DOI: 10.18720/MCE.78.11
[5] Block P and Lachauer L 2014 Three-dimensional equilibrium analysis of Gothic masonry vaults International Journal of Architectural Heritage 8(3) pp 312–35 DOI:10.1080/15583058.2013.826301
[6] Capone M, Campi M and Catuogno R 2015 Gothic Churches in Paris St Gervais et St Protais Image Matching 3d Reconstruction to Understand the Vaults System Geometry The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences, eds. D Gonzalez-Aguilera, F Remondino, J Boehm, T Kersten, and T Fuse (Spain: Avila) pp 423–30
[7] Carozzi F G, Poggi C, Bertolesi E and Milani G 2018 Ancient masonry arches and vaults strengthened with TRM, SRG and FRP composites: experimental evaluation Composite Structures 187 pp 466–80 DOI: 10.1016/j.compstruct.2017.12.075
[8] Costa C, A Arêde, M Morais and A Aníbal 2015 Detailed FE and DE modelling of stone masonry arch bridges for the assessment of load-carrying capacity Procedia Engineering 114 pp 854–61 DOI:10.1016/j.proeng.2015.08.039

[9] Hamdy G, Kamal O, Al-Hariri O and El-Salakawy T 2018 Plane and vaulted masonry elements strengthened by different techniques: testing, numerical modeling and nonlinear analysis Journal of Building Engineering 15 pp 203–217 DOI: 10.1016/j.jobe.2017.11.009

[10] Gilbert M 2007 Limit analysis applied to masonry arch bridges: State-of-the-art and recent developments ARCH’07 – Proc. of the 5th Int. Conf. on Arch Bridges (Guimarães) pp 13-28

[11] Klyuev S V, Klyuev A V, Shorstova E S and Gafarova N G 2017 The effect of particulate reinforcement on strength and deformation characteristics of fine–grained concrete Magazine of Civil Engineering 75(7) pp 66–75 DOI: 10.18720/MCE.75.6

[12] Lakhtin N K 1911 Raschet arok i svodov [The calculation of arches and vaults] (Saint-Petersburg) p 493(rus)

[13] Makris N, and H Alexakris 2013 The effect of stereotomy on the shape of the thrust-line and the minimum thickness of semicircular masonry arches Archive of Applied Mechanics 83(10) pp 1511–33 DOI:10.1007/s00419-013-0763-4

[14] Matveev I Ju et. al Report on the results of natural testing of the building "Barracks for 800 seats, a dining room for 1200 seats" (Military Space Academy named after AF Mozhaisky at the address: Moscow, St. Petersburg, Pionierskaya St., 28.

[15] Matveev I Ju, Popov A O, Birjuleva D K et. al. 2012-2015 Arhivnye materialy po provedeniju avtorskogo nadzora LLC CTSS "JekspertProektStroj" (Kazan)

[16] McInerney J and M DeLong 2014 Discrete element modelling of groin vault displacement capacity International Journal of Architectural Heritage 9(8) pp 1037–49 DOI:10.1080/15583058.2014.923953

[17] Milani G, M Simoni and A Tralli 2014 Advanced numerical models for the analysis of masonry cross vaults: A case-study in Italy Engineering Structures 76 pp 339–58. DOI:10.1016/j.engstruct.2014.07.018

[18] O'Dwyer D 1999 Funicular analysis of masonry vaults Computers & Structures 73(1–5) pp 187–97 DOI:10.1016/S0045-7949(98)00279-X

[19] Oliveira D V, Lourenço P B and Lemos C 2010 Geometric issues and ultimate load capacity of masonry arch bridges from the northwest Iberian Peninsula Engineering Structures 32(12) pp 3955–65 DOI: 10.1016/j.engstruct.2010.09.006

[20] Popov A O, Matveev I Ju, Birjuleva D K et. al. 2012 Otchety ob obsledovanii tehnichestkogo sostojanija stroitel'nyh konstrukcij istoricheskikh zdanij [Reports on the inspection of the technical condition of building structures of historic buildings] LLC CTSS "JekspertProektStroj" Kazan, 2012-2015 years (rus)

[21] Popov A O, Matveev I Ju and Kuramshin K V 2015 The strength of normal sections and deformability of brick cylindrical vaults reinforced by carbon canvases Izvestiya KGASU 2 pp 168 – 175

[22] Popov A O, Mavljuto A M and Jagofarov D Sh 2016 Ispol'zovanie modeli Heka-Brauna pri provedenii poverochnyh raschetov kamennyh svodov [Use of Huck-Brown models for verification calculations of stone arches] Uspehi sovremennoj nauki i obrazovanija 12 pp 65-70 (rus)

[23] Quadjflieig T, Stolyarov O and Gries T 2017 Carbon fibres as sensor for buildings [Carbonfaserbewehrung als Sensor für Bauwerke] Beton und Stahlbetonbau 112(8) pp 541–544

[24] Roca P, Cervera M and Gariup G 2010 Structural analysis of masonry historical constructions. Classical and advanced approaches Archives of Computational Methods in Engineering 17 pp 299–325 DOI:10.1007/s11831-010-9046-1
[25] Rossi M, Calderini C, Lagomarsino S and Milani G 2014 Seismic response of masonry vaulted structures: experimental and numerical modelling. 9th Int. Mason, Conf. (Guimarães) ed. P B Lourenço, B A Haseltine and G Vasconcelos (Portugal: Universidade do Minho)

[26] Riveiro B, Caamaño J C, Arias P and Sanz E 2011 Photogrammetric 3D modelling and mechanical analysis of masonry arches: An approach based on a discontinuous model of voussoirs Automation in Construction 20(4) pp 380–88 DOI:10.1016/j.autcon.2010.11.008

[27] Stolyarov O, Quadflieg T and Gries T 2017 Characterization of shear behavior of warp–knitted fabrics applied to composite reinforcement Journal of the Textile Institute 108(1) pp 89–94

[28] Ucer D, Ulybin A, Zubkov S and Elias–Ozkan SmT 2018 Analysis on the mechanical properties of historical brick masonry after machinery demolition Construction and Building Materials. 161 pp 186–195 DOI: 10.1016/j.conbuildmat.2017.11.090

[29] Zakrevskaya L V, Lubin P A, Avdeev S N, Gandelsman I A and Filippov S V 2017 Dome houses made of soil–concrete based on local raw materials Magazine of Civil Engineering 75(7) pp 123–128 DOI: 10.18720/MCE.75.12