High-current arc discharge in air

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Abstract. Computational model of high-current pulsed arc discharge in air is proposed. This is, in general, a two-dimensional model with taking into account gas dynamics of the discharge channel, real air thermodynamics in a wide range of pressure and temperature, electrodynamics of the discharge including pinch effect, and radiation. The developed model was applied to simulate the electric discharge in air for the currents of 1 - 250 kA and characteristic rise times in 13 - 25 µs, and results of calculations were compared with experimental ones. It was concluded that most of characteristics of the discharge are predicted well. Namely, arc column radius and shock wave position agree well with experimental data for all current amplitudes and rise times considered. Radial distributions of temperature and electron density also satisfactorily agree with experimental data. It was found that pinch effect should be considered for currents higher than 100 kA.

1. Introduction
Studies of high-current pulsed arc discharge in air are of significant interest both in themselves and for the modeling of lightning discharges in the atmosphere. Lightning is considered as a large-scale arc discharge with a strong current flowing through the channel and intense electromagnetic radiation. The study of electrical and radiation characteristics of high-current laboratory arc and lightning discharges has always been an important research subject in the field of lightning protection [1-5].

The lightning channel heated by strong discharge current can reach a peak temperature of about 30,000 K and electron density higher than 10¹⁸ cm⁻³ [1-3]. Analysis on lightning spectrum is an effective way to study the physical parameters of the plasma channel. It should be mentioned the early works of Uman et al. [6] and Orville [7,8], who performed optical emission spectroscopy (OES) measurements on lightning discharge. More recent experimental studies of natural lightning strikes [9,10] by OES technique have also estimated the channel temperature about 29 000 K and electron density higher than 10¹⁸ cm⁻³. It should also be noted recent theoretical studies on lightning channel structure with high current peak levels such as the works of Aleksandrov et al [11] and Alanakyan et al [12], Ripoll et al [13-14]. As already noted, theoretical studies of processes in high-current lightning discharges are of considerable interest in the field of lightning protection [15].

Currently, detailed experimental analysis of lab-scale arc discharges is available in the literature for the wide range of discharge currents. In [16], pulsed spark discharge at I = 1 – 2 kA has been studied. Recently, Sousa Martins et al [17-19] performed a study of arc discharges in air for the current amplitude of I = 10 – 100 kA [17,18], and I = 100 – 250 kA [19]. Representative set of time-dependent radial distributions of main parameters of the discharge has been reported. As well, time evolution of the...
plasma channel radius and the shock wave position has been recorded. The results obtained for high-current pulsed arc discharges and presented in [16-19] are very useful for developing models and validation computational tools.

One of the aims of the paper is to develop a physical and computational model of pulsed high-current discharge capable of simulating arc discharge in a wide range of current, \( I = 1 - 300 \) kA. The second goal is a verification of the model by comparing numerical results with the experimental ones. Namely, time-dependent radial profiles of temperature and electron density will be compared with those obtained in experiments [17-18]. As well experimental and numerical time-dependent channel radius and position of the shock wave will be compared. The influence of pinch effect on the arc characteristics at high currents will be discussed.

2. Computational model of the discharge

The computational model proposed in the paper is based on coupled solution of conservation laws for mass, momentum, energy, and radiation energy. In general, the model is those developed in papers [20-21]. To simulate high current discharge the following features are supplied. 1) Local thermodynamic equilibrium (LTE) is assumed to be valid at every space point. 2) Radiation losses are estimated from radiation energy transfer equation assuming spectrum-averaged total radiation energy transfer is suitable. 3) Electrodynamic part of the total set of equations takes into account pinch effect, i.e. compression of the current channel due to interaction of the electric current with magnetic field induced by this current. 4) At this stage, we shall assume (following papers [16-19]) that all processes evolve only in radial direction.

According to these papers [16-19], the current flows in \( z \)-direction, so the arc expands in radial direction, \( r \)-direction. Original arc is initiated by explosion of carbon wire which connects two tungsten electrodes. As has been registered an [16-10], central part of the arc can be considered as axially symmetric discharge for several tens of microseconds. On later stages, the axial symmetry is loosed.

2.1 Governing equations

With these assumptions, the computational model originating from the model developed in [20-21] reads as.

\[
\frac{\partial \rho}{\partial t} + \text{div}(\rho \mathbf{V}) = 0
\]

\[
\frac{\partial \rho \mathbf{V}}{\partial t} + \text{div}(\rho \mathbf{V} \otimes \mathbf{V} + \mathbf{s}) = -\text{grad}(P) + \mathbf{J} \times \mathbf{B}
\]

\[
\frac{\partial \rho e^0}{\partial t} + \text{div}(\rho \mathbf{V} h^0 + \mathbf{V} \mathbf{s} + \mathbf{q}) = \mathbf{JE} + Q_r
\]

\[
h^0 = e^0 + \frac{P}{\rho}, e^0 = e + \frac{V^2}{2}
\]

Here, \( t \) is time, \( \rho \) is density, \( \mathbf{V}=(0, V, 0) \) is velocity, \( e \) is specific energy, \( e^0 \) is total specific energy, \( h^0 \) is total specific enthalpy, \( P \) is thermodynamic pressure, \( \mathbf{J} \) is electric current density, \( \mathbf{E} \) is electric field strength, \( Q_r \) is radiation power losses, \( \mathbf{s} \) is viscous stress tensor, \( \mathbf{q} \) is heat flux density. Here, all quantities are functions of time \( t \) and radial coordinate, \( r \). It is assumed that \( z \)-coordinate directs along the arc.

One of the key features of the model is simulation of the equations of state. In the paper, we shall apply approximations for air plasma properties proposed in paper [22] for air in local thermodynamic equilibrium (LTE). All quantities of interest (for example, specific energy, molecular mass and other) are considered as functions of pressure \( P \) and temperature \( T \). Also, estimates for transport coefficients \( \mu \) and \( \lambda \) are taken from [22]; \( \mu \) is viscosity in the stress tensor \( \mathbf{s} \), and \( \lambda \) is thermal conductivity in the heat flux vector expression, \( \mathbf{q} = -\lambda \nabla T \).

Analysis of experimental data [17-19] shows that characteristic magnetic field (azimuthal) induced by the discharge current \( I > 100 \) kA can be of order of 1-2 T. Magnetic pressure corresponding to such
values of magnetic induction are of order of 5-10 atm. Therefore, second term in right-hand side of
Equation (2) was written. To calculate magnetic field, we use quasi-steady Maxwell’s equations.
Consider vectors $E$, $J$ and $B$ in Equations (1) – (3) as:
$$E = (E_z, 0, 0), \quad J = (J_z, 0, 0), \quad B = (0, 0, B_\theta).$$
$E_z$, $J_z$, and $B_\theta$ are considered as functions of $t$ and $r$. Then, Faraday’s law reads as
$$\frac{\partial B_\theta}{\partial t} - \frac{\partial E_z}{\partial r} = 0 \quad (5)$$
Ampere’s law is written as
$$\frac{1}{r} \frac{\partial (r B_\theta)}{\partial r} = \mu_0 j_z \quad (6)$$
Finally, we assume that Ohm’s law is valid,
$$j_z = \sigma \cdot (E_z + V \cdot B_\theta) \quad (7)$$
In (7) $\sigma$ is electric conductivity, which is also computed from LTE approximations of [22]. Substituting
(6) and (7) to (5) results in 2nd order partial differential equation for magnetic field.

To estimate the energy losses due to radiation from the discharge we use the transfer equation for
total radiation energy, namely
$$\nabla \left( - \frac{1}{3k} \nabla U \right) = \kappa \left( \frac{4\pi}{c} B(T) - U \right) = S_r \quad (8)$$
In (8) $U = \frac{1}{c} \int I d\Omega$ is the density of total radiation energy, $I$ is radiation intensity, $\kappa$ is absorption
coefficient, $c$ is speed of light, and $B(T)$ is Plank function. In accordance with the model, we apply mean
Plank absorption coefficient $k(P,T)$ for spectrum-averaged radiation energy. Then, right-hand-side (8)
reads as: $S_r = \frac{c}{4} (4\sigma_{SB} T^4 - cU)$, where $\sigma_{SB}$ is Stefan-Boltzman constant, $Q_r = cS$, in (3).

For mean Plank absorption coefficient $k(P,T)$ we use approximations [23] for $T < 20$ kK, and approximations [24] are used for 20 kK < $T < 300$ kK.

2.2. Initial and boundary conditions
The set of Equations (1) – (8) is solved with the following initial and boundary conditions. Initial
temperature is specified as $T(r, t = 0) = T_0 \cdot \exp(-r^2 / R^2)$. Here, $T_0$ varied between 5000 and
18000 K. $R$ is characteristic spatial scale of initial disturbance varied between 0.5 and 2 mm. The density
was specified as those corresponding to normal pressure 1 atm and room temperature 300 K. The
pressure was computed from LTE conditions. Preliminary test runs have shown that variation of $T_0$ and
$R$ has no difference at times more than 3-5 $\mu$s. All results below were obtained with $T_0 = 9000$ K and $R = 2$ mm.

One of the most important characteristics of the discharge is a calculation of total current versus time.
Basing on figure 1 from paper [19] and figure 3 from paper [18], we shall approximate the total current
as follows:
$$I(t) = I_0 (t/\tau)^n \exp((1 - t/\tau) \cdot n) \quad (9)$$
Here, $I_0$ is the current amplitude achieved at time moment $\tau$, $n$ is number of order of 1.

The computational domain is taken large enough in order the boundary conditions have no effect on
the solution. At the symmetry axis, $r = 0$, zero derivatives are set for all variables except one: $V = 0$ for
radial velocity. The boundary conditions for magnetic field $B_\theta$ are using Biot-Savart law, namely $B_\theta(r=0) = 0$, and $B_\theta(L, t) = \mu_0 l(t)/2\pi L$. Here, $L$ is the size of computational domain, $\mu_0$ is magnetic
permeability of vacuum. For radiation energy $\frac{\partial U}{\partial r} (r = 0) = 0$ is set at the symmetry axis, and $\frac{2}{3k} \frac{\partial U}{\partial r} + U = 0$ is set at $r = L$. The set of Equations (1) – (9) is solved using methods described in [20,21]. In
addition to gas dynamics equations (1) – (4), the magnetic field equations (5) – (7) are solved at every
time-step. As well, radiation energy equation (8) is solved at every time step.
3. Simulation of the discharge at $I = 10 - 100$ kA

The computational model presented above was applied to analyze a high current discharge investigated in papers [17,18]. We consider the following discharge parameters: $\tau = 13$ µs for $I_0 = 100$ kA, $\tau = 15$µs for $I_0 = 25$ kA and $I_0 = 50$ kA, and $\tau = 21$µs for $I_0 = 10$ kA. The degree $n = 1.2$ was used in (9) to specify the current $I(t)$.

Some important details of these discharge can be seen from figure 1 and figure 2. Almost all “computational” curves in these figures agree well with the experimental those. We conclude that such a good agreement is due to three factors.

![Figure 1](image1.png)  
**Figure 1.** Temperature profiles (a) and electron number density profiles (b) at $t = 6$ µs for the currents $I = 10-100$ kA. Symbol’s lines were taken from figure 10 of paper [18], solid lines – current paper.

![Figure 2](image2.png)  
**Figure 2.** Temperature profiles (a) and electron number density profiles (b) at $t = 9$ µs for the currents $I = 10-100$ kA. Symbol’s lines were taken from figure 10 of paper [18], solid lines – current paper.

First, approximation of thermodynamic properties presented in paper [22] works well. Second, it seems that mean-Plank absorption coefficient approach works well to simulate high current air discharges. These two factors are of primary importance in balancing heating from the current and radiative losses from the hot channel. Third important factor is approximation of transport properties, among them the electric conductivity coefficient is of primary importance.
Comparison of electric conductivity profiles for 100 kA discharge is presented in figure 3. It is seen from figures 1 - 3 that agreement of calculated and experimental data is within experimental uncertainty. So, all key components of the model, thermodynamic, radiative, transport properties of air under considered conditions are proved to be suitable to simulate the high current air discharges.

Finally, figure 4 presents the dependencies of arc channel radius versus current amplitude. It is seen that evolution of “computational discharge” agrees well with the experimental one. It should be noticed that variation of initial arc radius may change the arc radius within 10-20% for current amplitude less than 50 kA. Note that pinch effect is insignificant for the current amplitudes below 50 kA.

We incorporated one-dimensional pinch effect into the computational model to describe the high current discharge. Typically, it is not necessary for the currents lower than ~ 100 kA. However, it follows from estimations made above that magnetic compression should influence on the discharge characteristics at current amplitudes higher than ~ 100 kA. Direct comparison of the characteristics of \( I_0 = 100 \text{kA}, \tau = 13 \mu\text{s} \) discharge is given in figure 5.

It is seen that evolution of temperature is similar for both models, i.e., with pinch off and pinch on. The pinch effect is well exposed in pressure and electron density curves, especially at earlier times. This is because the pressure drop in radial direction is proportional to the square of total current and inversely proportional to the square of characteristic radius. The influence of pinch effect should increase as the current increases.

4. Summary
The 10 – 100 kA arc discharges in air were numerically investigated in the paper by means of the computational model developed to study lightning return strike. The numerical results were compared with experimental ones presented in papers [17,18].

Comparison of calculated and numerical temperatures and electron densities at the current level of 1 – 100 kA was found to be well. As well, the evolution of arc channel radius agrees well with the experimental one. This allows us to say that the model developed is suitable for simulating air discharges at the current level 1 - 100 kA, which is typical for the lightning return strike stage. It should be noticed that local equilibrium (LTE) approach in conjunction with the total radiation power transport approach appeared to capture all significant features of high current discharges. We demonstrated agreement of calculated and experimental time-dependent profiles in temperature and electron density for the whole range of discharge currents of 1 - 100 kA.
As far as compression of arc column with own magnetic field (pinch effect) is concerned, we think this should be taken into account for the current amplitudes higher than ~ 100 kA. Arc column radii are closer to experimental values when the pinch effect is considered. The pressure and electron density profiles look more correct, and their shape is closer to the experimental one when the pinch is considered.

![Figure 5](image)

**Figure 5.** Temperature (a), pressure (b), and electron density (c) for 100 kA/13µs discharge. Solid lines – pinch effect is on, dotted lines – pinch effect is off.

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