THE CRITICAL ROTATION OF STRANGE STARS AND RAPIDLY ROTATING PULSARS
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ABSTRACT

We utilize the bulk viscosity of interacting strange quark matter to reevaluate the damping timescale. The presence of a medium effect of bulk viscosity leads to a stronger damping of \( r \)-modes, which can be over an order of magnitude for realistic parameters. We find that the \( r \)-mode instability window is narrowed because of the medium effect, and hence, when a pulsar reaches the instability window, it will slow down by gravitational wave emission to a period of only 1.78 ms instead of the 2.5 ms given by early estimates. As a theoretical upper rotation limit of pulsars, the period of 1.78 ms is very close to the two most rapidly spinning pulsars known, with periods of about 1.6 ms.

Subject headings: dense matter — gravitation — stars: oscillations — stars: rotation

1. INTRODUCTION

At high density, normal hadron matter has been predicted to have a deconfined transition, in which quark-gluon plasma is formed. This could have important consequences for compact stars, with central densities several times the nuclear saturation density. Since strange quark matter (SQM), a conglomerate of up, down, and strange quarks, has been suggested as a possible absolutely stable or metastable phase of nuclear matter (Witten 1984), it has been speculated that strange stars might exist in the universe (Alcock et al. 1986; Haensel et al. 1986; Colpi & Miller 1992). The studies including quark interactions within lowest order perturbative QCD in the MIT bag model also predicted that sufficiently heavy strangelets might be absolutely stable (Fahri & Jaffe 1984). If the SQM hypothesis is correct, then some (perhaps all) pulsars may be strange stars (Alcock et al. 1986; Haensel et al. 1986; Colpi & Miller 1992). Thus, it is important to probe possible observational evidences of the existence of strange stars through astrophysical investigations.

On the one hand, since self-bounds of SQM due to strong interaction are very important, the mass-radius relation of the assumed strange stars has been uncovered to be different from that of neutron stars. However, for the canonical mass of 1.4 \( M_\odot \), gravity dominates the strong interaction that leads to strange stars and neutrons being similar in size (Alcock et al. 1986). Therefore, the attempt to distinguish strange stars from neutron stars for given observed masses and radii of pulsars seems impossible. On the other hand, strange stars could indicate the distinguishable signal from neutron stars by their cooling properties, but the difference in the cooling behavior of strange stars and neutron stars will disappear if the direct Urca process is operating in the core of a neutron star, which is done either by the proton fraction rising above 10% (Lattimer, van Riper, & Prakash 1994) or by hyperons being present in the core of a neutron star (Page et al. 2000). Hence, this would also make it difficult to identify a strange star from cooling data.

Even since Andersson realized that the \( r \)-modes are unstable at all rates of rotation in perfect fluid stars (Andersson 1998), a series of papers have investigated the many implications for gravitational radiation detection and the evolution of pulsars (Friedman & Morsink 1998; Lindblom, Owen, & Morsink 1998; Owen et al. 1998; Kojima 1998; Madsen 1998, 2000; Andersson et al. 1999a, 1999b, 2002; Ho & Lai 2000; Rezzolla & Maartens 2000). In recent years, several crucial issues regarding the astrophysical relevance of the \( r \)-mode instability have been investigated. Key results concern the interaction between oscillations in core fluid and the crust (Bildsten 1998; Andersson et al. 2000; Lindblom, Owen, & Ushomirsky 2000), the role of the magnetic field (Spruit 2000; Rezzolla, Lamb, & Shapiro 2000; Mendell 2001), superfluidity (Lindblom & Mendell 2000; Andersson & Comer 2001), and the effect of exotic particles that are thought to exist in the deep neutron star core (Jones 2001; Lindblom, Toline, & Vallier 2001).

Meanwhile, Madsen (1998) pointed out that the \( r \)-mode instability may provide the means to distinguish strange stars from neutron stars. The main reason for this is that the viscosity coefficients are rather different in these two cases. While the shear viscosity of a strange star is comparable to that of a neutron star, the bulk viscosity would be many orders of magnitude stronger than its neutron star counterpart. This has interesting effects on the \( r \)-mode instability. Based on the characteristic \( r \)-mode instability window of strange stars, which is related to the gravitational wave emission and the viscosity, a few mechanisms have been discussed to explain the clustering of spin frequencies of low-mass X-ray binaries (Madsen 2000; Andersson et al. 2002).

However, the bulk viscosity coefficient in Madsen’s studies takes the one for the case of noninteracting quark gas in the MIT bag (Madsen 1992). In fact, if the interaction among quarks is considered, the bulk viscosity increases, which is calculated by Zheng et al. (2002). As we show in the following, the \( r \)-mode instability window for strange stars will be significantly modified when the improved bulk viscosity is adopted.

As demonstrated below, we find that the \( r \)-mode instability window is evidently narrowed because of the inclusion of medium effect (interactions among constituent particles). The medium effect makes the lowest critical spin frequency rise to 558 Hz from about 300–400 Hz (for the case of free quark gas). The limiting period of 1.78 ms corresponding to 558 Hz
In § 4, we give a summary.

In § 2, we recall the bulk viscosity damping of r-modes to bulk viscosity vanishes in the lowest order expression, the derived timescales must be based on the fully self-consistent second-order calculation of this coupling. The earlier estimates cannot gain this end (Lindblom et al. 1998; Andersson et al. 1999a; Kokkotas & Stergioulas 1999), but it has been completed by Lindblom, Mendell, & Owen (1999). We substitute our viscosity for the given low-T limit (T < 10^7 K) into the formula (eq. [6.2]) given by Lindblom et al. (1999) and find that the only difference compared to the early calculation is the change of the viscosity coefficient. Thus, we can immediately obtain the timescale by a simple comparison instead of repeating the complicated calculations

\[ \tau_b = \tau_b s (\pi G \bar{\rho}/\Omega^2) T_b^{-2} \]  

with

\[ \tau_b = 2.83 \times 10^3 \alpha^{-1} \bar{\rho} m_{100}^4, \]  

where \( T_b \) and \( m_{100} \) denote temperature in units of 10^7 K and the current mass of strange quark in units of 100 MeV, and \( \bar{\rho} \) is the mean density of the star.

Evidently, \( \tau_b \) will be determined with the chemical potential \( \mu_\eta \), and \( \mu_\delta \) can be obtained by solving the equations related to chemical equilibrium, electric charge neutrality, and conservation of the baryon number for given \( \bar{\rho} \), namely,

\[ \mu_\eta = \mu_{\delta}, \quad \mu_u = \mu - \mu_\eta, \]

\[ \frac{1}{2} n_u - \frac{1}{2} (n_d + n_u) - n_e = 0, \]

\[ n = \frac{1}{2} (n_u + n_d + n_e), \]

\[ \bar{\rho} = \left( \frac{E}{A} \right) n. \]

Here, \( n \) denotes the baryon number density and \( n_i = (1/6\pi^2) k_i^2 \) is the particle number density. In addition, \( E/A \) is the energy per baryon, and we take it as approximately the mass of a neutron in our calculation.

is further closer to the two most rapidly spinning pulsars known (with the periods of 1.56 and 1.61 ms) than the periods of 2.5 and 3 ms given by Madsen.

In addition, if it could be established that rapidly spinning pulsars are strange stars, the pulsar data would put certain constraints on the model parameter, namely, the current mass of strange quark mass \( m_q \). As we can see in Madsen’s article (2000), in the normal and especially in the two-flavor color superconducting phase (2SC) case, the theoretical results were not consistent with the pulsar data when \( m_q = 100 \) MeV. However, when we take the medium effects into account, the constraints to \( m_q \) from pulsar data appear to be deeply relaxed, and we show that \( m_q \) can be taken as 100 MeV in this Letter.

This Letter is organized as follows. In § 2, we recall the bulk viscosity of interacting quark matter and compute the damping timescale on r-modes. In § 3, we give the improved critical curve (spin frequency) in a spin frequency (\( \nu \))–temperature (\( T \)) plane. In § 4, we give a summary.

2. BULK VISCOSITY AND DISSIPATION ON r-MODES

Ever since Wang & Lu (1984) pointed out that SQM is characterized by a huge bulk viscosity relative to nuclear matter, some investigations have tried to calculate the relevant viscosity coefficient of SQM (Sawyer 1989; Madsen 1992; Goyal, Gupta, & Anad 1994). In the MIT bag model, it is thought that the exact solution had been obtained by Madsen (1992). However, the coupling among quarks in the bag was ignored, which had been considered in the study of the equation of state (EOS) of SQM (Schertler, Greiner, & Thoma 1997). Soon before, we found that the coupling’s effect on the bulk viscosity leads to an increase of the viscosity over an order of magnitude, although the medium modifications of the EOS of SQM were negligible (Zheng et al. 2002). To very good approximation, the relevant bulk viscosity coefficient still takes the form formulated by Madsen (1992),

\[ \zeta = \frac{\alpha T^2}{\omega^2 + \beta T^4}. \]  

but here \( \alpha \) and \( \beta \) were given by Zheng et al. (2002) and are extremely different from Madsen’s and strongly dependent on the coupling constant of strong interactions among quarks. We expressed them as

\[ \alpha = 9.39 \times 10^{22} \mu_\delta^3 \left( \frac{k_{\delta\, d}^2 - k_{\delta\, i}^2}{C_d - C_i} \right)^2 \quad (g \text{ cm}^{-1} \text{s}^{-1}), \]  

\[ \beta = 7.11 \times 10^{-4} \left[ \frac{\mu_\delta}{2} \left( \frac{1}{k_{\delta\, d} C_d} - \frac{1}{k_{\delta\, i} C_i} \right) \right]^2 \quad (\text{s}^{-2}), \]  

where \( k_{\delta\, i} = (\mu_\delta^2 - m_{0\delta}^2)^{1/2}, \quad C_i = \mu_i - m_i (\partial m_i/\partial \mu_i), \) and \( m_i^* \) was given in the references (Klimov 1982; Weldon 1982a, 1982b; Pisarski 1989; Blaizot & Ollitrault 1993; Vija & Thoma 1995).

The bulk viscosity is expected to be the dominated internal fluid mechanism in hot compact stars. The timescales for the bulk viscosity damping of r-modes need to be estimated. Since the coupling of the r-modes to bulk viscosity vanishes in the lowest order expression, the derived timescales must be based on the fully self-consistent second-order calculation of this coupling. The earlier estimates cannot gain this end (Lindblom et al. 1998; Andersson et al. 1999a; Kokkotas & Stergioulas 1999), but it has been completed by Lindblom, Mendell, & Owen (1999). We substitute our viscosity for the given low-T limit (T < 10^7 K) into the formula (eq. [6.2]) given by Lindblom et al. (1999) and find that the only difference compared to the early calculation is the change of the viscosity coefficient. Thus, we can immediately obtain the timescale by a simple comparison instead of repeating the complicated calculations

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Evidently, \( \tau_b \) will be determined with the chemical potential \( \mu_\eta \), and \( \mu_\delta \) can be obtained by solving the equations related to chemical equilibrium, electric charge neutrality, and conservation of the baryon number for given \( \bar{\rho} \), namely,

\[ \mu_\eta = \mu_{\delta}, \quad \mu_u = \mu - \mu_\eta, \]  

\[ \frac{1}{2} n_u - \frac{1}{2} (n_d + n_u) - n_e = 0, \]  

\[ n = \frac{1}{2} (n_u + n_d + n_e), \]  

\[ \bar{\rho} = \left( \frac{E}{A} \right) n. \]  

Here, \( n \) denotes the baryon number density and \( n_i = (1/6\pi^2) k_i^2 \) is the particle number density. In addition, \( E/A \) is the energy per baryon, and we take it as approximately the mass of a neutron in our calculation.
The coupling $g$ will extremely influence the solutions of the above equations. Following Schertler et al. (1997), $g$ is taken as a free parameter ranging from 0 to 5, and the equations (6), (7), and (8) are solved numerically. Figure 1 shows $\bar{\tau}_v$ as a function of $\bar{\rho}$ for different $g$ and $m_q = 200$ MeV. The timescales depend on $g$ remarkably, while they are nearly independent of $\bar{\rho}$. The timescale for $g = 5$ arrives at tens of times shorter than the case of $g = 0$. For a strange star with mass $M = 1.4 M_\odot$ and radius $R = 10$ km, we find that $\bar{\tau}_v$ ranges from $4.24 \times 10^{-2}$ s ($g = 0$) to $1.53 \times 10^{-3}$ s ($g = 5$). This is of great interest, because this will significantly increase the critical rotation angular velocity for the onset of $r$-mode instability.

3. CRITICAL SPIN FREQUENCY FOR THE ONSET OF THE $r$-MODE INSTABILITY

The $r$-mode unstable (or stable) regime of the relativistic stars—neutron stars as well as strange stars—depends on the competition between the gravitational radiation and various dissipation mechanisms. To plot the instability window of the $r$-mode or gain the critical rotation frequency for a given stellar model as a function of temperature, we need to acquire the characteristic timescales: damping and growing timescales of $r$-mode instability. Because of the emission of gravitational waves, the $r$-mode grows on a timescale

$$\tau_G = \bar{\tau}_G(\pi G\bar{\rho}\Omega^2)^{-1},$$

where $\bar{\tau}_G$ is $-3.26(1.57)$ s for $n = 1(0)$ polytropic EOS, which was respectively studied by Lindblom et al. (1999) and Kokkotas & Stergioulas (1999). For the viscous damping timescales, we also consider the shear viscosity besides bulk viscosity discussed in the last section. In strange stars, the timescale for the shear viscous damping is given by

$$\tau_S = \bar{\tau}_S(\alpha_s/0.1)^{5/3}T_\rho^{-5/3}.$$  

Here, $\bar{\tau}_S$ is $5.37(2.40) \times 10^8$ s corresponding to $n = 1(0)$, $\alpha_s$ is the strong coupling, and we take $\alpha_s = 0.1$ in the following calculations (Madsen 2000).

We can now evaluate the critical spin frequency as a function of temperature from the equation

$$\frac{1}{\tau_G} + \frac{1}{\tau_S} + \frac{1}{\tau_B} = 0. \quad (12)$$

Figure 2 shows the regions of $r$-mode (in)stability in a $r$-$T$ plane for a strange star with mass $M = 1.4 M_\odot$ and radius $R = 10$ km. The shading between the two curves displays the effect of the medium modification of quark masses on critical rotation frequencies. The medium effect narrows the $r$-mode instability window. The dotted curve corresponds to $g = 0$, reduced to the Madsen (2000) result, and the solid curve, corresponding to $g = 5$, is assumed to be the upper limit of the medium effect. The top curve has a minimum frequency denoted by $C$ as 558 Hz (the corresponding period is 1.78 ms).
which is very close to the two most rapidly spinning pulsars
known, with frequencies of 642 and 622 Hz (the periods are
1.56 and 1.61 ms), relative to the period of 2.5 ms for non-
interacting medium. This implies that a strange star would slow
down by gravitational wave emission when it reaches the in-
stability window and spin around in 1.78 ms instead of the
2.5–3 ms expected by Madsen. Figure 3 shows the results for
2SC stars. Similar to Figure 1, the medium effect also increases
the critical rotation frequency of 2SC stars, but it should be
stressed that the medium effect leaves the most rapidly stars
away from the instability window.

Figure 4 depicts the instability windows in which the current
mass of strange quark takes 100 MeV. If the rapidly rotation
pulsars could be regarded as strange stars, the medium effect
would relax the stringent constraint on the choice of QCD
parameters in contrast to the noninteracting medium case; for
example, the 2SC stars for smaller current mass $m_c$ can safely
exist because of the medium effect (Fig. 4, top dashed curve).

4. CONCLUSIONS AND DISCUSSIONS

We apply the bulk viscous coefficient including medium
effect to reevaluate and discuss the viscous damping time. The
time greatly depends on the strong coupling $g$. We find the
timescale $\tilde{\tau}_g$, rather than noninteracting perfect fluid in which
$\tilde{\tau}_g$ is constant (the fact is that $\tilde{\tau}_g$ vs. $\tilde{\rho}$ slowly increases for
nonvanishing $m_c$ as shown in Fig. 1), weakly decreases with
increasing mass density of stars for the given larger $g$. There-

fore, the medium effect due to strong interactions among quarks
adds the viscous dissipation of instability modes. For a star
with $M = 1.4 M_\odot$ and radius $R = 10$ km, we numerically
calculate the critical spin frequency as a function of temper-

ature. The medium effect leads to a significant lift of the critical
frequencies. We find that the maximum critical period in the
$r$-mode instability window is reduced to 1.78 from 2.5 ms,
appearing to be further in agreement with the data of the two
most rapidly spinning pulsars known. We also study the 2SC
stars and give our improved scenario. The current mass of
strange quark in the model has a broader range, taking the
medium effects into account.

Finally, it should be mentioned that we here consider only
the weak coupling of strong interactions among quarks in the
MIT bag. The long-range unperturbation interactions are con-
tained in the usual bag constant. Although the MIT bag constant
is able to contribute to the EOS of SQM, the long-range effects
on dynamical quantities, such as the viscous coefficient, etc.,
seem to be ignored because of the hiding of the microscopic
processes in the bag constant. If the processes can be taken
into account, we can conjecture that the limiting spin of strange
stars would shift farther upward. This is our future work.

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