Mechanisms of the $f_0(980)$ production in the reaction

$$\pi^- p \rightarrow \pi^0\pi^0 n$$

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Abstract

The model of the pure one-pion exchange mechanism, which gives a good description of the GAMS results on the alteration of the $S$-wave $\pi^0\pi^0$ mass spectrum in the $f_0(980)$ region in the reaction $\pi^- p \rightarrow \pi^0\pi^0 n$ with increasing $-t$, is compared with the recent detailed data on the $m$ and $t$ distributions of the $\pi^- p \rightarrow \pi^0\pi^0 n$ events obtained by the BNL-E852 Collaboration. It is shown that the predictions of this model are not confirmed by the BNL data. Therefore the observed phenomenon should be explained by the different exchange mechanism. It is most likely to be the $a_1$ exchange mechanism.

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I. INTRODUCTION

In two recent experiments on the reaction \( \pi^- p \rightarrow \pi^0 \pi^0 n \) at high energies performed by the GAMS Collaboration at Institute of High Energy Physics [1,2] and the BNL-E852 Collaboration at Brookhaven National Laboratory (BNL) [3,4] it has been found a very interesting phenomenon consisting in the alteration of the \( S \)-wave \( \pi^0 \pi^0 \) mass spectrum in the vicinity of the \( f_0(980) \) resonance with increasing of \(-t\), where \( t \) is the square of the four-momentum transferred from the incoming \( \pi^- \) to the outgoing \( \pi^0 \pi^0 \) system. If, for small values of \(-t\), where the reaction \( \pi^- p \rightarrow \pi^0 \pi^0 n \) is dominated by the one-pion exchange mechanism, the \( f_0(980) \) resonance manifests itself in the \( S \)-wave \( \pi^0 \pi^0 \) mass spectrum as a dip due to its strong destructive interference with the large and smooth background, then, for large values of \(-t\), it appears as a peak [1-4].

The GAMS and BNL-E852 results are based on high statistics and impose severe demands on the phenomenological models constructed for their explanation.

Historically, the first description of the GAMS results on the \( f_0(980) \) resonance [1] has been performed in Ref. [5] on the basis of the pure one-pion exchange (POPE) model. To explain the observed dip and peak behavior of the \( f_0(980) \) in this model the authors of Ref. [5] had to provide the individual contributions to the full \( S \)-wave \( \pi^\ast+ (t) \pi^- \rightarrow \pi^0 \pi^0 \) amplitude (where \( \pi^\ast(t) \) is a Reggeized pion) with rather exotic \( t \) dependencies. With minor modifications such a treatment of the GAMS data [1] has been also reproduced in a series of the subsequent publications [6,7]. Then, we suggested a crucially new explanation of the GAMS results on the \( f_0(980) \) production in which a main role was assigned to the \( \pi^- p \rightarrow f_0(980)n \) reaction amplitude with the quantum numbers of the \( a_1 \) Regge pole in the \( t \) channel [8]. In brief, our scenario came to the following. At small \(-t\), the reaction \( \pi^- p \rightarrow (\pi^0 \pi^0)_S n \) [hereafter \((\pi \pi)_S\) denotes a \( \pi \pi \) system with the orbital angular momentum \( L = 0 \)] is dominated by the one-pion exchange mechanism, and the \( f_0(980) \) resonance appears as a minimum in the \((\pi^0 \pi^0)_S \) mass spectrum. However, the one-pion exchange contribution decreases very rapidly with \(-t\) and the \( a_1 \) exchange mechanism becomes the dominant one in the reaction \( \pi^- p \rightarrow (\pi^0 \pi^0)_S n \) at large \(-t\) [8]. The \( f_0(980) \) resonance produced via the \( a_1 \) exchange shows itself as a peak in the \((\pi^0 \pi^0)_S \) mass spectrum because precisely such a manifestation of the \( f_0(980) \) has been observed in all known reactions in which the \( f_0(980) \) production channel differs from that of the elastic \( \pi \pi \) interaction [8].

In spite of the quite satisfactory descriptions of the GAMS data in Refs. [5] and [8], both above mentioned models are certainly in need of further experimental tests [8]. Note that the scenario considered in Ref. [8] can be rejected only by the measurements of the reaction \( \pi^- p \rightarrow (\pi^0 \pi^0)_S n \) on polarized targets because only they will make it possible an explicit separation of the \( \pi \) and \( a_1 \) exchange mechanisms. As to the test of the POPE model [5], it can be easily fulfilled experimentally owing to the specific predictions of this model, for example, for the \( t \) distributions of the \( \pi^- p \rightarrow (\pi^0 \pi^0)_S n \) events in the region \( 0 < -t < 0.2 \text{ GeV}^2 \) for \( m < 1 \text{ GeV} \) [where \( m \) is the invariant mass of the \((\pi^0 \pi^0)_S \) system]. In part, we have already drawn attention to this circumstance in Ref. [8]. It is necessary to note that the comparison with the available GAMS data does not allow to reveal all predictions hidden in the POPE model [5]. Fortunately, the recent data on the \( m \) and \( t \) distributions of the \( \pi^- p \rightarrow (\pi^0 \pi^0)_S n \) events presented by the BNL-E852 Collaboration [4] give a unique possibility to carry out the detailed comparison of the POPE model with experiment. Such a comparison is the main goal of this work.
In Sec. II, we briefly recall the initial POPE model constructed in Ref. [5] for the description of the dip and peak behavior of the \((\pi^0\pi^0)_s\) mass spectrum in the \(f_0(980)\) region. All subsequent versions [6,7,9,10] of this model are also briefly discussed. We emphasize that the POPE model [5] leads to a full violation of the \(t\) dependence factorization hypothesis for the \(S\)-wave \(\pi^*(t)\pi \rightarrow \pi\pi\) amplitude. In contrast, this hypothesis, as it is well known, has been widely used previously as a simple and reliable working tool for obtaining the data on the lower \(\pi\pi\) scattering partial waves (see, for example, Refs. [8,11-16]). Here we also discuss possibilities of the unambiguous experimental verification of the POPE model predictions associated with the above violation. In Sec. III, we perform a detailed comparison of the POPE model [5] with the BNL data [4]. Our conclusions are briefly summarized in Sec. IV.

II. MODEL OF THE ONE-PION EXCHANGE AMPLITUDE FOR THE REACTION \(\pi^-p \rightarrow (\pi^0\pi^0)_s n\)

The double differential distribution in \(m\) and \(t\) of the \(\pi^-p \rightarrow (\pi^0\pi^0)_s n\) reaction events at fixed incident pion momentum is defined by the authors of the POPE model [5] as follows

\[
\frac{d^2N}{dmdt} = C \left| \frac{\sqrt{-t}}{m^2_{\pi} - t} F(t) \ a_{\pi\pi}(m, t) \right|^2, \tag{1}
\]

where \(C\) is the normalization constant, \(F(t)\) is the form factor pertaining to the \(\pi^*(t)NN\) vertex and \(a_{\pi\pi}(m, t)\) is the \(S\)-wave \(\pi^*(t)\pi \rightarrow \pi\pi\) amplitude with isospin \(I = 0\). To construct the amplitude \(a_{\pi\pi}(m, t)\) the \(K\) matrix method was used in Ref. [5], and to describe the data in the region \(0.7 < m < 1.2\) GeV contributions of two resonances coupled to the \(\pi\pi\) and \(K\bar{K}\) channels and some background terms were taken into account in the \(K\) matrix. From the general formula for the amplitude \(A = \hat{K}(t)(I - i\hat{\rho}\hat{K})^{-1}\), where \(\hat{A}\) and \(\hat{K}\) are \(2 \times 2\) matrices describing the transitions in the \(\pi\pi\) and \(K\bar{K}\) channels, and \(\hat{\rho}\) is a diagonal matrix of the phase volumes, it follows that

\[
a_{\pi\pi}(m, t) = \frac{K_{\pi\pi}(t) + i\rho_K[K_{\pi\pi}(t)K_{\pi\pi} - K_{\pi\pi}(t)K_{\pi\pi}]}{1 - i\rho_KK_{\pi\pi} - i\rho_KK_{\pi\pi} + \rho_K\rho_K[K_{\pi\pi}K_{\pi\pi} - K_{\pi\pi}K_{\pi\pi}]}, \tag{2}
\]

where, according to Ref. [5], \(\rho_{\pi} = (1 - 4m^2_{\pi}/m^2)^{1/2}\), \(\rho_K = (1 - 4m^2_K/m^2)^{1/2}\) \((\rho_K \rightarrow i|\rho_K|\) for \(0 < m < 2m_K\), \(K_{ab} = K_{ab}(t = m^2_{\pi}) = K_{\pi\pi} = K_{\pi\pi}\),

\[
K_{ab}(t) = \left[ \frac{g_a(t)g_b}{M^2_t - m^2} + \frac{G_a(t)G_b}{M^2_t - m^2} + f_{ab}(t) \right] \left( 1 - \frac{m^2_{\pi}}{2m^2} \right), \tag{3}
\]

\(f_{K\bar{K}}(t) = 0\) \((a = \pi, K; b = \pi, K; g_{K\bar{K}} = g_K\) and \(G_{K\bar{K}} = G_K\).  

In order to simplify the discussion of the expression (2), it is convenient, for the moment, to neglect in Eq. (3) all background terms \(f_{ab}(t)\) and the quantity \(m^2_{\pi}/2m^2\) which is negligible for \(m \approx 1\) GeV. With these simplifications in mind, Eq. (2) can be rewritten in the following more transparent form:

\[
a_{\pi\pi}(m, t) = \frac{g_{\pi}(t) [D_2(m)g_{\pi} + \Pi_{12}(m)G_{\pi}] + G_{\pi}(t) [D_1(m)G_{\pi} + \Pi_{12}(m)g_{\pi}]}{D_1(m)D_2(m) - \Pi^2_{12}(m)}, \tag{4}
\]

\(^1\)Using the representation (2), it is easy to show that the authors of Ref. [5] missed in Eq. (1) the \(m\) dependent factor \(m\rho_{\pi}\) which is approximately equal to 1 only in the vicinity of \(m = 1\) GeV.
where \( D_1(m) = M_1^2 - m^2 - i g_{\pi\pi}^2 \rho_{\pi} - i g_{\pi K}^2 \rho_K \) and \( D_2(m) = M_2^2 - m^2 - i G_{\pi\pi}^2 \rho_{\pi} - i G_{\pi K}^2 \rho_K \) are the inverse propagators for the initial bare resonances, and \( \Pi_{12}(m) = i g_{\pi K} G_{\pi K} \rho_K + i g_{\pi K} G_{\pi K} \rho_K \) is the amplitude describing the transitions between these resonances through the real \( \pi \pi \) and \( K\bar{K} \) intermediate states. In Eq. (4), it is easily recognized the amplitude of the process \( \pi^*(t)\pi \rightarrow \pi\pi \) with \( L = I = 0 \) due to the contributions of two mixed resonances coupled to the \( \pi\pi \) and \( K\bar{K} \) channels.

It is now well understood that the observed alteration of the \( (\pi^0\pi^0)_S \) mass spectrum can be described with the considered model only if the destructive interference between two resonances at \( m \approx 1 \) GeV, which occurs in the low \(-t\) region, is replaced by the constructive one with increasing \(-t\). According to Eq. (4), this means a change of the interference type between the terms proportional to \( g_\pi(t) \) and \( G_\pi(t) \). In its turn, this is possible only if, as \(-t\) increases, one of the residues, for example, \( g_\pi(t) \), decreases in absolute value, vanishes at a certain value \( t = t_0 \), and then changes its sign. According to the fit to the GAMS data presented in Ref. [5], this has to occur for \(-t < 0.2 \) GeV\(^2\). Hence, due to such an approach, the \( t \) dependence of the amplitude \( a_{\pi\pi}(m, t) \) must not factorize at \( m \approx 1 \) GeV even in the low \(-t\) region. Here, in addition to the remark mentioned in the Introduction about the \( t \) dependence factorization hypothesis, we note that the results on the \( \pi\pi \) scattering obtained by using this hypothesis were always in close agreement with those of the more general Chew-Low extrapolation method [11-16]. In its simplest and most frequently used form [8,11-16], the factorization hypothesis implies in this case that, at least for small values of \(-t\), i.e. in the region \( 0 < -t < (0.15 - 0.20) \) GeV\(^2\), the amplitude \( a_{\pi\pi}(m, t) \) is proportional to the on-mass-shell amplitude \( a_{\pi\pi}(m, t = m_0^2) \). In doing so, the factor of proportionality is generally taken in the form \( \exp[i (t - m_0^2)] \). On the other hand, if one explains the GAMS data in the framework of the POPE model [5], then the factorization hypothesis must be rejected from the outset.

In Ref. [5], the following parametrization for the residues \( g_\pi(t), G_\pi(t), f_{\pi\pi}(t), \) and \( f_{\pi K}(t) \) was postulated:

\[
\begin{align*}
g_\pi(t) &= g_\pi + (1 - t/m_\pi^2) t g_\pi/m_\pi^2, \\
G_\pi(t) &= G_\pi + (1 - t/m_\pi^2) t G_\pi/m_\pi^2, \\
f_{\pi\pi}(t) &= (1 - t/m_\pi^2) t f_{\pi\pi}/m_\pi^2, \\
f_{\pi K}(t) &= f_{\pi K} + (1 - t/m_\pi^2) t f_{\pi K}/m_\pi^2.
\end{align*}
\]

In the best of the three fit variants given in Ref. [5], \( M_1 = 0.773 \) GeV, \( M_2 = 1.163 \) GeV, \( g_\pi = 0.848 \) GeV, \( g_\pi' = 0.0479 \), \( G_\pi = 0.848 \) GeV, \( G_\pi' = -0.0259 \) GeV, \( f_{\pi\pi} = 0.0963 \), \( f_{\pi\pi} = 0.687 \), and \( f_{\pi K} = 0.3918 \). It follows from Eq. (5) that \( g_\pi(t) \) vanishes at \( t \approx -0.0728 \) GeV\(^2\). Hence, with increasing \(-t\), a dip in the \( (\pi^0\pi^0)_S \) mass spectrum in the \( f_0(980) \) region gradually disappears and eventually turns into a resonancelike enhancement [5]. Here it is worth noting that the amplitude (2) on the mass shall \( [a_{\pi\pi}(m, t = m_0^2)] \) vanishes at \( m = m_0 \approx 0.986 \) GeV, i.e. just below the \( K\bar{K} \) threshold, due to the destructive

\[2\]Furthermore, as \(-t\) varies from 0 to 1 GeV\(^2\), the functions \( g_\pi^2(t) \) and \( G_\pi^2(t) \) increase, respectively, by approximately factors of 22000 and 6000. The appearing enormous rise with \(-t\) of the amplitude \( a_{\pi\pi}(m, t) \) in Eq. (1) is compensated by the very rapidly dropped form factor \( F(t) = [(\Lambda - m_\pi^2)/(\Lambda - t)]^4 \) with \( \Lambda = 0.1607 \) GeV\(^2\) which the authors of Ref. [5] ascribed to the nucleon vertex (see also Refs. [6,7,10]). The critical discussion of such an ascription leading to unsolvable difficulties in different reactions has been given in Ref. [8]. For example, the above form factor would yield an abnormally sharp drop of the one-pion exchange (OPE) contribution to the differential cross section of the charge exchange reaction \( pn \rightarrow np \). Since \( d\sigma^{\text{(OPE)}}(np \rightarrow np)/dt \sim |F(t)|^4 \), then, in the \(-t\) region from 0 to 0.2 GeV\(^2\) this cross section drops like \( \exp\{56t\} \), which is comparable only to the fall of the cross sections of diffractive processes on complex nuclei.
interference between the various contributions, and that the phase shift of $a_{\pi\pi}(m,t=m_\pi^2)$ goes through $180^\circ$ at this point in close agreement with the experimental data [12,13].

Analyzing the model of Ref. [5] we revealed that, as $-t$ increases, the amplitude (2) also vanishes but for different values of $m < 2m_K$. The zero “trajectory” of the amplitude (2) in the plane of the $m$ and $t$ variables is shown in Fig. 1. It is seen that with increasing $-t$ the amplitude zero shifts, gradually speeding up, from the region of $m \approx 2m_K$ to the lower mass region. For example, as $-t$ increases from 0.09 GeV$^2$ only by 0.026 GeV$^2$, it crosses the wide region of $m$ from 0.91 to 0.60 GeV.

Thus, we discover at once two striking predictions of the POPE model [5]. First, for each fixed $(\pi^0\pi^0)_S$ invariant mass value $m < 2m_K$ (or more precisely, for each small $m$ bin) the presence of a dip in the $t$ distribution, $dN/dt$, is predicted in the low $-t$ region. For example, in any interval of $m$ from the region $0.6 < m < 0.91$ GeV, a dip in $dN/dt$ must be located near $-t \approx 0.1$ GeV$^2$, and, as $m$ increases from 0.91 to 0.986 GeV, it must move to $t = 0$. Second, the model predicts that the $m$ distribution of the $\pi^- p \rightarrow (\pi^0\pi^0)_S n$ reaction events, $dN/dm$, for $0.6 < m < 0.9$ GeV must be suppressed in the vicinity of $-t \approx 0.1$ GeV$^2$ because in this region of the variables the one-pion exchange amplitude is close to zero, but, for $m > 0.9$ GeV it must sharply increase. Thus, the model of Ref. [5] describing the GAMS data [1] on the alteration of the $(\pi^0\pi^0)_S$ mass spectrum in the $f_0(980)$ resonance region for $-t > 0.3$ GeV$^2$ can be unambiguously checked owing to its predictions for the $dN/dt$ and $dN/dm$ distributions for $0 < -t < (0.2 - 0.25)$ GeV$^2$ and $0.6$ GeV $< m < 2m_K$. Certainly, to do this much more detailed data are required than those presented by the GAMS Collaboration. Let us recall that the GAMS data [1] on the reaction $\pi^- p \rightarrow (\pi^0\pi^0)_S n$ include single $dN/dm$ distribution in the region $0.8 < m < 1.2$ GeV for $0 < -t < 0.2$ GeV$^2$ (i.e. for the low $-t$ region as a whole) and, in addition, the $dN/dm$ distributions in the region $0.6 < m < 1.4$ GeV for five overlapping intervals of $-t$ covering the region $0.3 < -t < 1$ GeV$^2$.

In the subsequent versions [6,7,9,10] of the POPE model [5], the $K$ matrix analysis of the $I^{PC} = 00^{++}$ waves has been extended to the more wide regions of $m$ and a larger number of the coupled channels. In Ref. [6], four resonances coupled to $\pi\pi$, $K\bar{K}$, $\eta\eta$, and $4\pi$ channels have been included in the $K$ matrix and the region up to 1.55 GeV has been analyzed. Five resonances coupled to five channels have been taken into account in Refs. [7,9,10] and the region of the data description has been extended up to 1.9 GeV. Certainly the further resonances with masses in the range 1.2 – 1.9 GeV [6,7,9,10] exert some influence on the mass region below 1 GeV. However, with the exception of some details, all essential predictions of the two-resonance model [5] for $m < 1$ GeV remain valid. For example, the most essential feature of the one-pion exchange amplitude parametrization proposed in Ref. [5], namely, the passage through zero of the residue of the lowest-mass resonance with increasing $-t$, takes place in all subsequent variants. The mass of the lightest resonance varies with the $K$ matrix parametrization way from 0.65 to 0.86 GeV [5-7,9,10]. According to the best fit of Ref. [5] the residue of the resonance vanishes at $-t = 0.0728$ GeV$^2$ (this fact has been already mentioned above), according to Ref. [6] (solution I) at $-t = 0.117$ GeV$^2$, according to Ref. [7] (solution I) at $-t = 0.0683$ GeV$^2$, and according to Ref. [9] at $-t = 0.038$ GeV$^2$. Unfortunately, in Ref. [10], the parameter values needed for the determination of the zero location are absent.

Note that after the publication of our work [8] involving new explanation of the GAMS results and criticism of the POPE model [5] the $a_1$ exchange contribution also appeared in Ref. [9]. However, this contribution was taken into account in Ref. [9] by a “purely
cosmetic way” since, in doing so, the parametrization of the one-pion exchange amplitude and its dominant role in the description of the observed alteration phenomenon actually left unchanged. As the $a_1$ exchange contribution is really small in the low $-t$ region, it is reasonable that the predictions of the model [9] for small $-t$ and $m < 1$ GeV as a whole turned out to be close to those of the POPE model [5] which were qualitatively described above. In fact, this claim can be done immediately on inspection of Figs. 3 and 5 of Ref. [9] showing the predicted $m$ and $t$ distributions of the $\pi^- p \rightarrow (\pi^0\pi^0)_S n$ events. It is revealing that in the last publication [10] the authors again do not take into account the $a_1$ exchange mechanism as previously in Refs. [5-7].

III. COMPARISON WITH THE BNL DATA

The BNL-E852 Collaboration presented the high-statistics $m$ distributions of the $\pi^- p \rightarrow (\pi^0\pi^0)_S n$ reaction events in the region $2m_{\pi} < m < 2.2$ GeV with the 0.04 GeV-wide step in $m$ for nine sequential fine bins in $t$ covering the region $0 < -t < 0.4$ GeV$^2$ and for a single wide $-t$ interval from 0.4 to 1.5 GeV$^2$ [4]. The BNL data, which we use to check the predictions of the POPE model [5], are shown in Figs. 2 and 3. Let us stress that we are not concerned with the fitting of these data in the framework of the POPE model [5]. We just use the model with those values of its parameters which ensure the best fit to the GAMS data [1] and compare its predictions with the BNL data [4] both on the $m$ distributions pertaining to the six fine $t$ bins covering the region $0 < -t < 0.2$ GeV$^2$ and on the $t$ distributions for six 0.04 GeV-wide intervals in $m$ which we selected as an example from the region $0.6 < m < 1.12$ GeV. Similar detailed distributions have not been presented by the GAMS Collaboration [1,2]. The only parameter the value of which is needed to be determined once again is the overall normalization constant $C$ in Eq. (1). We found this parameter by normalizing the theoretical distribution to the total number of events in the interval $0.6 < m < 1.2$ GeV for $0.01 < -t < 0.03$ GeV$^2$. The data on the distribution $dN/dm$ for this region of the $m$ and $t$ variables are shown in Fig. 2b. Note that among all the isometric intervals of $t$ the interval $0.01 < -t < 0.03$ GeV$^2$ contains the maximal number of the $\pi^- p \rightarrow (\pi^0\pi^0)_S n$ events in the region $0.6 < m < 1.2$ GeV. We consider such a choice of overall normalization to be quite applicable to give a descriptive comparison between the experimental and theoretical distributions in $m$ and $t$.

Figure 2 shows that there is a satisfactory qualitative agreement of the experimental and theoretical distributions $dN/dm$ in the intervals $0 < -t < 0.01$ GeV$^2$ and $0.01 < -t < 0.03$ GeV$^2$. However, with increasing $-t$, the shape of the theoretical distributions in $m$ sharply changes. Note that this fact is in line with the expectations given in Sec. II. In addition, it can be seen from Fig. 2 that, according to the POPE model [5], the transformation of a dip in the $f_0(980)$ region to a resonancelike bump occurs in the $-t$ range from 0.1 to 0.2 GeV$^2$, i.e. too rapidly. As is also clearly seen from Fig. 2, the experimental distributions $dN/dm$ leave, in fact, similar to each other throughout the low $-t$ region from 0 to 0.2 GeV$^2$ and all of them have a dip on the place of the $f_0(980)$ resonance. Let us emphasize again that unlike the detailed information presented by the BNL-E852 Collaboration for $0 < -t < 0.2$ GeV$^2$, the GAMS Collaboration has presented for this $t$ region, containing some 90% of all $\pi^- p \rightarrow (\pi^0\pi^0)_S n$ events, a single “global” distribution $dN/dm$, and it is precisely this rough one that has been fitted successfully by using the POPE model [5].
The BNL data [4] on the $t$ distributions and the corresponding theoretical predictions are shown in Fig. 3. It is seen that the POPE model [5] predicts the presence of a dip in these distributions in the low $-t$ region if $m < 2m_K$. In direct contradiction, no such dip is observed in reality.

It is evident that the character of the POPE model predictions cannot be altered if the finite experimental resolutions in $m$ and $t$ are taken into account in the construction of the theoretical curves. In any case, the agreement of the model with the BNL data cannot be improved essentially.

IV. CONCLUSION

The question whether the observed alteration of the $(\pi^0\pi^0)_S$ mass spectrum in the reaction $\pi^-p \rightarrow (\pi^0\pi^0)_S n$ with increasing $-t$ can be described exclusively in terms of the amplitude with quantum numbers of the $\pi$ Regge pole in the $t$ channel is absolutely valid and deserves to be thoroughly considered. Therefore the first attempt to solve this question undertaken in Ref. [5] was of great importance. In our opinion, the merit of this work is the formulation of the particular one-pion exchange model containing some clear predictions which can be easily tested by experiment. The above analysis shows that these predictions are in rough contradiction with the detailed BNL data on the $m$ and $t$ distributions of the $\pi^-p \rightarrow (\pi^0\pi^0)_S n$ events. However, from our point of view, it is valuable that the way outlined in Ref. [5] has been completed conclusively. Note that the GAMS Collaboration selected the highest statistics on the reaction $\pi^-p \rightarrow \pi^0\pi^0 n$ [1,2], that is why the publication of their $m$ and $t$ distributions of the $(\pi^0\pi^0)_S$ production events for fine $t$ and $m$ bins for $0 < -t < 0.2$ GeV$^2$ and $m < 1$ GeV is highly desirable.

In accordance with the aforesaid discussion, it is pertinent also to note that those consequences that were extracted in Refs. [9] and [17] from the analyses of the experimental data based on the models of Refs. [5-7,9,10] are not justified.

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FIGURE CAPTIONS

Fig. 1. The zero “trajectory” of the amplitude a_{\pi\pi}(m,t) in the model of Ref. [5] in the plane of the m and −t variables.

Fig. 2. The (\pi^0\pi^0)_S mass spectra, dN/dm, in the reaction \pi^- p \to \pi^0\pi^0 n for six sequential intervals of −t shown just in the plots. The data are from the BNL-E852 Collaboration [4]. The curves are constructed by using Eqs. (1)–(3), (5), and (6). The used values of the parameters are mentioned in the text.

Fig. 3. The t distributions, dN/dt, of the \pi^- p \to (\pi^0\pi^0)_S n reaction events for six intervals of the invariant mass of the (\pi^0\pi^0)_S system, m, shown just in the plots. The data are from the BNL-E852 Collaboration [4]. Here, as well as in Fig. 14 of Ref. [4], the data for the intervals 0 < −t < 0.01 GeV² and 0.01 < −t < 0.03 GeV² are combined. The curves are constructed by using Eqs. (1)–(3), (5), and (6). The used values of the parameters are mentioned in the text.
Fig. 1.
Fig. 2.
Fig. 3.