Test of some fundamental principles in physics via quantum interference with neutrons and photons

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Abstract

The limitations and possibilities that the concept of quantum interference offers as a tool for testing fundamental physics are explored here. The use of neutron interference as an instrument to compare measurement readouts with some of the principles behind metric theories of gravity will be analyzed, as will some discrepancies between theory and experiment. The main restrictions that this model embodies for the study of some of the features of the structure of space–time will be explicitly pointed out. For instance, the conditions imposed by the necessary use of the semiclassical approximation. Additionally, the role that photon interference could play as an element in this context is also considered. In this realm we explore the differences between first-order and second-order coherence experiments, and underline the fact that the Hanbury–Brown–Twiss effect could open up some interesting experimental possibilities in the analysis of the structure of space–time. The void, in connection with the description of wave phenomena, implicit in the principles of metric theories is analyzed. The conceptual difficulties that this void entails are commented upon.
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1. Introduction

1.1. What is fundamental physics?

The work that you are about to read bears the title *Test of some fundamental principles in physics via quantum interference with neutrons and photons*, so it would be a good idea if we from the very beginning had a precise idea of the meaning of the words appearing in the title, just for the sake of completeness. Already from its etymology the word physics is related to the concept of experiment. Indeed, the origin of this word is Greek, and its meaning is *nature* [1]. In other words, it is a science which attempts to provide a description and explanation to natural phenomena. Nowadays this remark may be considered superfluous, but there was a time in which the explanation of the phenomena in nature did not entail the necessity of confronting the corresponding models against any experiment. Many times the disobedience of this unspoken rule was accompanied with personal problems. In this context we only need remember that Galileo was forbidden to hold Copernican views [2], or the tragic death of Miguel Servet [3]. Fortunately now the possibility of questioning any scientific model does not always imply a perilous situation.

Now that we have mentioned the fact that physics is an experimental science, we face a new question, what is the meaning of the phrase fundamental principles in physics? We could argue that fundamental principles in physics denotes those assumptions that are considered the bedrock of our description of nature. Though our answer is a correct one it also seems not to be very illustrative. Let us provide a more profound definition resorting to an example. Consider the Newtonian description of dynamics [4]. The core part of this theory is contained in the famous three laws of motion, and no problem in the realm of classical mechanics can be solved without resorting, in one way or another, to these laws. In other words, the Newtonian laws of motion, or any of their equivalent formulations, Poisson brackets, Hamilton formulation, etc, [5] constitute an example of fundamental principles in physics, since they provide the possibility of making predictions about the behavior of nature. Clearly, these predictions can be confronted against measurement readouts. Fortunately the story does not finish here. Indeed, the Newtonian view of the universe also assumes some other concepts, which are taken for granted as a conceptual background. For instance, in this perspective of the universe the ideas of time, space and simultaneity are absolute, i.e. they do not depend on the observer [4]. The possibility of testing these premises remained for several centuries outside the possibilities of mankind. The technology that allowed the option of checking them appeared at the end of the nineteenth century. Among the experiments in this context, we may mention the Michelson and Morley proposal [6], which is in essence an interference experiment and the Trouton–Noble experiment [7–9]. In this last case a suspended parallel-plate capacitor is held by a fine torsion fiber and is charged. If the aether theory were correct, the change in Maxwell’s equations due to the Earth’s motion through the aether would lead to a torque causing the plates to align perpendicularly to the motion.

The options that light interferometry offers us do not end with Michelson–Morley, some of the most important proposals to test gravity involve laser interferometry [10].

We may at this point answer our original question stating that the concept of fundamental principles in physics will embody, not only some fundamental equations, as could be the Newtonian motion laws, or the Schrödinger equation, but also basic assumptions concerning the structure of time and space. Additionally, as will be shown below, some physical theories, such as quantum mechanics, have more than one interpretation. Therefore, another issue to be addressed in this work comprises the use of interference devices in order to discard some models concerning quantum theory, for instance the hidden variables theory [11, 12].
Now that we have clearly stated the goals in this work let us answer a second question that at this point can be posed: what can be obtained from the application of interference techniques in connection with fundamental principles in physics? The answer to this question can be contemplated from two different points of view: (i) the unification between general relativity and quantum theory is currently one of the most challenging theoretical tasks in modern physics. The situation in this issue now requires the help of phenomenology as an important element which shall mitigate some of the deficiencies that the models show. Indeed, phenomenology can help to fix bounds upon some parameters that appear in connection with these theories [13], and which cannot be obtained from the corresponding approaches [14, 15]; (ii) looking at the experimental side of this issue we notice that the requirements involved in these kinds of experiments have spurred the development of new technology, as the case of the detection of gravitational waves clearly shows [16]. In other words, theoretical work and the development of technology live in a state that could be denoted a symbiosis, since each of them benefits from the other one.

1.2. What is classical interference?

The idea of interference takes us to the notion of wave motion, a concept that at this point allows us to remain completely within the realm of classical physics. One of the most important physical phenomena involving the concept of interference is the propagation of light. Robert Boyle and Robert Hooke made the first observation of interference, though the wave behavior of light was not recognized until Fresnel confirmed Young’s experiments. The delay in this aspect is explained by the fact that this hypothesis disagreed with the corpuscular model introduced by Newton [17]. By the way, this last remark also shows that sometimes the weight of a scientific figure becomes more important than experimental evidence.

The existence of interference (either of light, of quantum matter, etc) requires the fulfillment of several conditions. The first one is the superposition principle, that exists in connection with motion equations which are linear differential equations. We may define this principle stating that the sum of solutions to the corresponding motion equation is also a solution of the equation [6]. If this principle does not hold, then the phenomenon of interference, in general, disappears.

Let us illustrate this last assertion with an example. Consider a string, for instance, it can be the string of a violin, when the performer plays a staccato note (a bowed string behaves in a very different way [18]), such that any pulse generated by the performer travels with speed \( v \). Let us denote the amplitude of this pulse by \( y \). It can be shown that the dynamics of any pulse, in a very idealized scheme, is given by [17]

\[
\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}.
\]

(1)

Notice that if \( f \) and \( g \) are twice differentiable, then \( f(vt - x) \) and \( g(vt + x) \) both satisfy (1). Define \( h(\pm)(x, t) = f(vt - x) \pm g(vt + x) \). The linearity of the motion equation implies that \( h(\pm)(x, t) \) satisfies (1).

Let us now analyze the consequences of a non-linear term in the equation, and, in order to do this, consider a generalization of (1) in the following form

\[
\frac{\partial^2 y}{\partial x^2} + \frac{\partial (y)^2}{\partial x} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}.
\]

(2)

Assume that \( f(vt - x) \) and \( g(vt + x) \) satisfy it, and introduce \( h(x, t) \) into (2)

\[
\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 g}{\partial x^2} + \frac{\partial (f)^2}{\partial x} + \frac{\partial (g)^2}{\partial x} + 2 \frac{\partial (fg)}{\partial x} = \frac{1}{v^2} \left( \frac{\partial^2 f}{\partial t^2} + \frac{\partial^2 g}{\partial t^2} \right).
\]

(3)
It is readily seen that \( h_{\text{xy}}(x, t) \) is not a solution to our motion equation. In other words, the sum of two solutions of the motion equation is not anymore a solution. This remark has several consequences. For instance, as will be shown below, the effect called interference is related to the feature of summing solutions and ending up with another solution. Therefore, phenomena ruled by non-linear motion equations will not, in general, have interference as one of their characteristics.

Now that we fathom better the implications of a non-linear motion equation let us analyze the possibilities that its counterpart offers us. Harking back to (1), assume now that at a certain point \( \tilde{x} \) and at a certain time \( \tilde{t} \) we have that \( f(\tilde{v}t - \tilde{x}) = -a \) and \( g(\tilde{v}t + \tilde{x}) = +a \), where \( a > 0 \). The physical meaning of \( a \) can be easily understood with the example of a violin. We may suppose that positive values of \( y \) denote displacements along the positive direction of the \( y \)-axis. This last remark implies that the right propagating pulse \( f(\tilde{v}t - \tilde{x}) \) has, at the aforementioned position and time, a negative displacement along the \( y \)-axis, whereas \( g(\tilde{v}t - \tilde{x}) \) has a positive displacement. Clearly, we have that \( h_{\text{xy}}(\tilde{x}, \tilde{t}) = 0 \). The displacement of the string is neither \( a \) nor \( -a \); as a matter of fact, there is a vanishing displacement of the string. This is a simple example of destructive interference.

This case of a staccato note on a violin string is an example of the superposition principle, and clearly it does not exhaust the possibilities in this direction. Indeed, physical situations satisfying the superposition principle abound in physics. For instance, in an adiabatic process the propagation of sound can be contemplated as a first-order linear differential equation for pressure as a function of the density of the air [19]. The propagation of light, a case analyzed below, is also related to the superposition principle. Additionally, we may mention the motion of water, under certain circumstances. These simple examples show that every day we are in contact with the phenomenon of interference, though we may not notice it.

At this point we may give an answer to the question: what is interference? It is a direct consequence of the superposition principle, and it is the fact that at any point in which two perturbations are superposed the resulting perturbation is the algebraic sum of each one of them.

Preparing the material for the topics that we will discuss here let us mention that a very important phenomenon associated with a linear differential equation of motion is the propagation of light. Indeed, the behavior of the electromagnetic field is governed by Maxwell’s equations [20]. The debate between the corpuscular behavior of light and the possibility of a wave description went on for many years. One of the reasons behind this longevity can be tracked down to the fact that the observation of interference requires very restrictive conditions [17]. In connection with light, interference appears as a set of dark and bright bands denoted by fringes. The bright regions appear when a number of waves add together to produce an intensity maximum of the resultant wave, a case called constructive interference. On the other hand, destructive interference happens if the involved waves add together to produce an intensity minimum of the resultant wave. In our oil-dominated world we may find these fringes on oil films on a wet roadway, a situation that, unfortunately, is quite common nowadays.

But, what are the conditions that allow the detection of interference? As a part of the answer we may say that interference always involves small dimensions. For instance, two slits separated by only a couple of millimetres, or in the case of the aforementioned oil films, have a thickness of very few millimetres. These dimensions are not imposed by the wavelength of light, but by a fundamental property called coherence [21]. This last statement could sound puzzling, so let us explain the concept of coherence a little bit better. This idea will be handled in section 3, though here we provide a brief introduction to the main concepts.

Consider two electric fields. They are said to be coherent if the interference term

\[
\langle \vec{E}_1 \cdot \vec{E}_2 \rangle = \sqrt{T_1 T_2} \cos (\delta)
\]

(4)
is non-zero within the region occupied by both electric fields, here \( I_i = \langle \vec{E}_i \cdot \vec{E}_i \rangle \), and \( \delta \) is the phase between these two fields. We may rephrase this last remark saying that two waves are coherent if the associated electric fields have a constant phase relation \( \delta \). In other words, the property of coherence is related to the comparison of the relative phase between these fields. This comparison can be done in two different ways, temporal and spatial [21].

We may even assert that the simplest manifestations of correlations in light fields are interference effects that arise when two light beams are superposed. A good comprehension of this property is due, since any experiment intending to prove some physical model, and embodied in the context of interference, must take into account the fundamental features defining this property.

From the last paragraph a question assails us, namely, in the case of light, what are the conditions that entail the possibility of observing interference? At this point it is noteworthy to comment that in optics the corresponding *elongations* to be specified are the strengths of the electric and magnetic fields. Obviously, in this process we must take into consideration the phases of the involved waves. The situation is simplest when the phases do not vary noticeably in time. Of course, this is a very rough idealization, and, as shown below, we only require them to be constant over time intervals of the length of the observation time. When this last condition is fulfilled an interference pattern will emerge. Let us now make this last statement more precise. For the sake of simplicity take the case of two plane waves with equal frequencies and linear polarization. In addition, the amplitudes will be considered constant and real, \( E_1(\vec{r}, t) \) and \( E_2(\vec{r}, t) \), where \( \vec{k}(j) \), \( \omega(j) \) and \( \theta(j) \) denote the wave vector, the frequency and the phase, respectively.

\[
E_{(j)}(\vec{r}, t) = E_{(j)} \exp\left\{i\left((\vec{k}(j) \cdot \vec{r} - \omega(j)t - \theta(j))\right)\right\}, \quad j = 1, 2. \tag{5}
\]

The usual definition of intensity is one-half of the square of the electric field strength averaged over several oscillation periods [6]. The superposition principle states that the total electric field strength is the sum of \( E_1(\vec{r}, t) \) and \( E_2(\vec{r}, t) \). Then we obtain for the intensity.

\[
I(\vec{r}, t) = E_1^2(\vec{r}, t) + E_2^2(\vec{r}, t) + 2E_1(\vec{r}, t)E_2(\vec{r}, t) \cos\left\{\left((\vec{k}(2) - \vec{k}(1)) \cdot \vec{r} - (\omega(2) - \omega(1))t - (\theta(2) - \theta(1))\right)\right\}. \tag{6}
\]

The definition of a monochromatic wave also implies that \( \theta(j) \) must be constants. If the frequencies are the same, then we have a standing interference pattern, that hinges critically upon the difference \( \theta(2) - \theta(1) \). On the other hand, if the waves differ in frequency, then we obtain a sinusoidal dependence on time of the interference pattern. This analysis is quite simple, nevertheless, the experimental requirement of monochromaticity was within reach only after the appearance of lasers [22].

The problem with light emitted by conventional sources is that it exhibits fast fluctuations, in both amplitude and phase of the electric field. This entails that two independent beams, i.e. two beams emitted from different sources or from two different parts of the same source, cannot produce a detectable interference pattern. Indeed, a fleeting glance at (6) shows that the pattern formed at a certain time and point will be displaced by a random fraction of the fringe spacing each time that the phases, \( \theta(2) \) and \( \theta(1) \), change their value.

Up to this point everything seems to be in its place. Nevertheless, from the last arguments we cannot understand how the Michelson–Morley experiment provided, by means of an interference experiment, the data that led, at least partially, Einstein to the formulation of the Special Theory of Relativity. The answer is very simple. The interference term appearing on the right-hand side of (6) depends only upon the parameter \( \theta(2) - \theta(1) \), in other words, the only condition that we need is \( \theta(2) - \theta(1) = \text{const} \). The phases may fluctuate, but not
independently, a definite correlation between them must exist. Experimentally, this can be achieved by making the interfering beams replicas of a primary beam. The use of a beam splitter suffices. This is the core of the idea in the Michelson–Morley experiment [20].

How far can we walk testing fundamental principles in physics if we resort only to classical interference experiments? We should not belittle the possibilities that classical experiments could provide us. Indeed, remember that one of the most important sets of data that Einstein had at his disposal (in the formulation of the Special Theory of Relativity [23]) was the null experiment, no existence of aether [24], that the Michelson–Morley experiment implied. This experiment contains the empirical evidence behind one of the two premises of special relativity, namely, the speed of light is a constant, i.e. independent from the motion of the observer [25].

Moreover, the possibilities of light interferometry also embrace the test of the validity of Lorentz symmetry [26, 27], at least if we consider the introduction of some kind of deformed dispersion relation, a fact related to some quantum-gravity theories [28, 29]. This point will be explained in section 2.

Up to now everything remains in the realm of classical physics. If the reader has been patient enough and followed our arguments up to this point she/he could have the impression that classical and quantum interference processes are clearly distinctive events. This is not the case. For instance, in the detection process of light the corpuscular behavior becomes dominant. Indeed, an interference pattern will be formed only if there is a large number of photons impinging upon the detecting screen [22].

1.3. What is quantum interference?

Let us now address the issue of quantum interference. The advent of quantum theory opened up an ocean of possibilities in connection with interference experiments. Indeed, the use of particles in relation to interference devices is completely excluded in the context of classical mechanics [5], since in classical mechanics particles are not endowed with wave-like behavior. This may sound redundant, but as mentioned below, particles in quantum mechanics may show wave-like features. Quantum interference can be considered as the interference of the de Broglie wave associated with an ensemble of particles with itself. Ascribing these wave-like and particle-like features to one physical entity, such as light, or neutrons, is a very common characteristic in modern physics [30]. Additionally, it is inexorably linked to the concept of measurement in quantum theory [31]. This dual behavior has been the reason for many debates, and has spurred several modifications to the interpretation of theory, for instance, the statistical interpretation [32], in which the quantum description of any system is from the very beginning assumed as incomplete. In other words, quantum mechanics is considered not a fundamental theory, but a phenomenological model.

The emergence of quantum theory cannot be ascribed to the labor of one researcher, rather it is the confluence of the ideas of many people, beginning with Planck and his work on black body radiation, and continuing with Schrödinger, Dirac, Pauli, etc. In the non-relativistic limit, and for particles with no spin, the fundamental motion equation is [33]

\[ i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi. \tag{7} \]

It is a linear equation, and, according to the Copenhagen interpretation [30], the variable \(|\psi(\vec{r}, t)|^2\) is related to the probability of detecting the corresponding system in a neighborhood of point \(\vec{r}\) at time \(t\). Additionally, \(\psi\) is easily expressed as a vector of a Hilbert space. Since the description of a quantum system is given by elements of a Hilbert space, then the sum of two of them is an allowed physical state. This brings into quantum theory the phenomenon
of interference. This has been, since the very inception of the theory, a controversial and controverted point, and will be one of the core points in this present work.

Let us explain this fact a little bit better. Consider a cat whose state will be described by quantum mechanics; here we will make use of the argument of Schrödinger’s cat [30]. Our cat always remains inside a closed box containing a radioactive substance, that is also a quantum system. The state of the radioactive substance can be expressed as the superposition of two orthogonal states, a non-decayed state and a decayed one. In consequence, the state of the cat will be expressed in terms of two orthogonal states, \( \psi_{(d)} \) and \( \psi_{(a)} \), the cat is dead and the cat is alive, respectively. The physical state of the cat, \( \psi_{(c)} \), then reads \( \psi_{(c)} = \alpha_{(d)} \psi_{(d)} + \alpha_{(a)} \psi_{(a)} \), where \( \alpha_{(a)} \) and \( \alpha_{(d)} \) are complex numbers whose modulus is strictly larger than 0 and smaller than 1.

The first conceptual problem that we encounter is the meaning of \( \psi_{(c)} = \alpha_{(d)} \psi_{(d)} + \alpha_{(a)} \psi_{(a)} \). According to our classical understanding of the world the cat is dead or alive, and no place for a superposition of these two mutually excluding cases can be considered within the classical perspective of the universe. The superposition principle related to (7) entails a new possibility, which was absent in the classical version. Clearly, a second problem appears in connection with (7). Indeed, we may pose the following question: how do we recover a classical world from a quantum theory? This is a very complicated problem, one that has several approaches, some of them even deny the existence of the measurement problem in quantum mechanics [34]. On the other hand, we may find approaches in which the solution is obtained as a consequence of the interaction of the corresponding system with its environment [35], an idea named the decoherence model [36]. At this point we may ask ourselves: is there any relation between the last paragraph and the task of the present work? This is a good question, and luckily the answer is affirmative. Let us fathom better this last statement. The decoherence model predicts the existence of non-classical states, which, due to the interaction with the environment, have a very short life [37]. These non-classical states emerge as a direct consequence of the superposition principle, and therefore, in principle, they define a way in which the decoherence model can be experimentally tested. In other words, quantum interference could allow us to discard the decoherence model, or to take it as a fundamental element of the theory.

Now that we know that interference experiments can shed some light upon some thorny points of the foundations and interpretation of quantum theory we may address some additional issues. For instance, can quantum interference experiments tell us something about the relation between the structure of space–time and quantum mechanics? Once again, we receive a gleaming yes. Indeed, we may find a series of experiments that exhibit how gravity appears in the realm of quantum theory. This experiment, first performed in 1974 [38], contains an interference pattern of thermal neutrons induced by gravity. A brief description of this experiment is as follows. A nearly monoenergetic beam of neutrons is split into two parts by silicon crystals, they follow paths with different gravitational potential and afterwards they are brought together. The difference in the gravitational potential implies the emergence of a phase shift between the two beams. This difference in phase goes like (here \( m \) denotes the neutron mass)

\[
\Delta \phi \sim \left( \frac{m}{\hbar} \right)^2. \tag{8}
\]

This is a purely quantum mechanical process, since the limit \( \hbar \to 0 \) implies that the interference pattern gets washed out; this experiment will be explained in a more complete way below. The importance of this experiment cannot be reduced to one aspect. For instance, the dependence of (8) on the mass of the involved particles has originated a hot debate about the possibility of a non-geometric element of gravity in the quantum realm [39, 40]. In other words, gravity-induced quantum interference can be used to discuss the possible breakdown,
at the quantum level, of the equivalence principle [41]. This statement enhances the relevance of quantum interference as a tool for testing the structure of space–time.

Finally, as another point in which fundamental physics does have a fundamental role in the development of technology let us mention that quantum interference plays an important role in the implementation of entangling quantum gates between atomic qubits [42], a fact that lies right in the center of the present efforts of quantum computation [43].

Summing up, interference is a fundamental element of quantum theory and can be used to test the validity of the equivalence principle, in the construction of quantum computation, etc.

1.4. Geometric phases in quantum theory

The concept of geometric phases could be an important tool in connection with proposals that try to test some aspect of fundamental physics, and here we explain its meaning. As a matter of fact, examples of geometric phases, classical and quantum, abound in physics, and many ordinary situations that we do not usually associate with geometric phases can be rephrased in terms of them. As an illustrative example of this let us consider the precession of a Foucalt pendulum. The usual analysis of its movement is done in terms of Coriolis force [5], nevertheless, we may contemplate this situation from a different perspective. Indeed, suppose that we have a point mass with its corresponding gravitational field, and that our Foucalt pendulum is transported along a closed curve, say \( A \), in the aforementioned gravitational field.

At this point we introduce two conditions upon the period and amplitude of the motion of the pendulum, i.e. they are smaller than the typical time and distance of the transport motion, respectively. If additionally the curve lies on the surface of a sphere, then, when the pendulum returns to its initial position, its invariant plane will have rotated by some non-vanishing angle. In the case in which the pendulum has been transported along constant latitude, say \( \alpha \), the rotation angle reads \( 2\pi \cos(\alpha) \) [5]. This is a simple example of a classical phase. In general, we may state that if an integrable classical Hamiltonian describes a bound motion which depends on parameters that suffer a very slow change, then the adiabatic theorem [5] states that the action variables of the motion are conserved. For angle variables the change does not merely contain the time integral of the instantaneous frequency, it also shows an extra angle which depends only upon the circuit in the parameter space [44].

All the foregoing arguments have concerned classical systems. Let us now address the issue of quantum phases. The first work in which a quantum phase was explicitly derived and analyzed was done by Berry [45]. The first derivation of this phase involved the adiabatic theorem, which states that if a certain Hamiltonian \( H \) changes gradually from some initial form \( H^{(i)} \) to a final one \( H^{(f)} \), and a particle was initially in the \( n \)th eigenstate of \( H^{(i)} \), then it will be carried (according to the Schrödinger equation) into the \( n \)th eigenstate of \( H^{(f)} \) [46].

Consider a Hamiltonian \( H(t) \) which shows a non-trivial time dependence; then the eigenfunctions and, eigenvalues themselves are also time dependent,

\[
H(t)\psi_n(x, t) = E_n(t)\psi_n(x, t).
\]

At this point the adiabatic theorem is introduced, and, in consequence, if \( H(t) \) changes in a gradual way, then our particle picks up at most a time-dependent phase factor. This means that the wavefunction, written in terms of its eigenfunction, reads

\[
\Psi_n(x, t) = \psi_n(x, t) \exp\left\{-\frac{i}{\hbar} \int_0^t E_n(\tau) \, d\tau\right\} \exp\left\{i\gamma_n(t)\right\}.
\]

This new phase \( \gamma_n(t) \) is called the geometric phase.

Introducing (10) into the time-dependent Schrödinger equation, the expression for the geometric phase can be deduced. For instance, let us assume that the corresponding
Hamiltonian embodies more than one parameter changing with time (for the case of only one parameter the geometric phase is trivial), say $R_1(t), \ldots, R_N(t)$. Under these circumstances it can be shown [45] that if the Hamiltonian returns to its original form after a time $T$, then the geometric phase can be expressed as a line integral around a closed loop $\tilde{C}$ in the corresponding parameter space

$$\gamma_n(T) = i \oint_{\tilde{C}} <\psi_n|\nabla_R \psi_n> \cdot d\tilde{R}. \quad (11)$$

The acceptance of the argument that the phase of a wave function is arbitrary, i.e. it contains no physical information, is deeply rooted in quantum mechanics. However, phase variations and the so-called geometric phase are gauge invariant quantities, i.e. they may embody relevant physical information. The observability of this effect is related to variations of the corresponding phase shifts, i.e. auxiliary interference patterns are required [48].

The Aharonov–Bohm [47] effect can be thought of as an example of Berry’s phase, and its first experimental verification was done resorting to electron holography [49]. Its existence has been verified in other domains, for instance, using an optical fiber [50]. Since the experimental verification of geometrical phases usually involves some kind of interference experiment, then we may use them to test some possible effects related to metric theories.

The Aharonov–Bohm effect can be considered one of the first examples of a geometric phase, and deserves an analysis of its own, since it involves some very interesting features. Consider a particle with electric charge $q$, and an ideal solenoid (this condition is imposed just for the sake of clarity) with cylindrical symmetry, such that inside the solenoid the magnetic field is constant, and along the axis of symmetry, and zero outside, see figure 1.

A source point, $S$, emits particles, and they can follow two different trajectories, to the left or to the right of the solenoid, as shown by the arrows in figure 1. Finally, at point $D$ they are brought together. There is a striking feature that emerges in the interference pattern. Though the particles never enter the region in which the magnetic field is non-vanishing, a non-null phase difference between the two beams emerges

$$\Delta = \frac{q}{\hbar c} \oint \vec{A} \cdot d\vec{l}. \quad (12)$$
This last expression could deceive us, i.e., we could state that, since in electrodynamics the vector potential $\vec{A}$ is not unique [51], this expression is gauge-dependent. This is not the case, and can be easily proved resorting to Stokes’ theorem, i.e., this phase becomes

$$\Delta \phi = \frac{q}{\hbar c} \int_S \nabla \times \vec{A} \cdot d\vec{s}. \quad (13)$$

But $\nabla \times \vec{A} = \vec{B}$, and therefore $\Delta$ is gauge-invariant. The situation in the context of the Aharonov–Bohm effect is very rich. For instance, as a quite surprising case let us mention that, under certain circumstances, neutral particles with magnetic moment can exhibit the Aharonov–Bohm effect [52]. A quantum effect of the Aharonov–Bohm type for particles with an electric dipole appears, as a consequence of the relation between the topological properties of the phase shift and the linear and angular momentum of the electromagnetic field [53]. This topic is spiced with a hot debate about the force that a neutron experiences in connection with the Aharonov–Casher effect. Indeed, some authors state that there is an electric force on a classical model for a neutron [54], whereas others deny the existence of this force [55].

At this point we must have a more careful treatment of the experimental situation associated with the gauge-invariance of the effects of the Aharonov–Bohm type. Let us start mentioning that these kinds of experiments require the observation of the relative displacement of the interference patterns, i.e., a sole interference pattern does not suffice. The experimental detection of the phase shift variation, $\Delta \phi$, needs one value of $\vec{A}$, and a second one, say $\vec{A}_0$, which plays the role of a reference parameter, see, for instance, the reference beam mentioned in [49]. Hence the fringes related to $\phi(\vec{A})$ are compared against those emerging from $\phi(\vec{A}_0)$, and, in consequence, $\phi(\vec{A}) - \phi(\vec{A}_0)$ can be obtained. Clearly, a gauge transformation leaves this last quantity unaltered.

Finally, we may also mention that there are several types of Aharonov–Bohm effects. Indeed, for instance, the so-called molecular Aharonov–Bohm effect [56] does not share some of the properties to which we are used in the standard Aharonov–Bohm effect, i.e., the molecular version is neither non-local nor topological. Additionally, in the context of gravity there is an analogue of this effect when particles are constrained to move in a region where the Riemann curvature tensor does not vanish [57].

2. Neutron interference

2.1. Metric theories of gravity and quantum interference

2.1.1. The weak equivalence principle. Modern physics has its foundations in two theories, namely, quantum mechanics [30] and general relativity [58]. Any experimental test of general relativity must clearly bear in mind the postulates of the theory. In order to have a clear idea of the bounds involved with quantum interference experiments we now establish the different premises of this theory. The starting point is the weak equivalence principle (WEP), which states: if an uncharged test body is placed at an initial event of space-time and given an initial velocity, then its subsequent trajectory will be independent of its internal structure and composition [41].

This principle has its experimental foundation in the universality of free fall, an experiment first performed by Galileo [2]. It entails a relation between two different concepts of mass. Let us address this point. Consider the case of a classical particle freely falling in a homogeneous gravitational field. The motion equation is given by [5]

$$m(\dot{\vec{r}}) = -m(p)g\hat{z}. \quad (14)$$
In this last expression \( m(\mathrm{i}) \), \( m(\mathrm{p}) \) and \( \hat{z} \) denote the inertial mass the passive gravitational mass, and the unit vector along the \( z \)-axis (we assume that the gravitational field is along this last axis and has a direction contrary to the positive direction of \( z \)). The inertial mass already appears in Newton’s second law of motion [5], whereas the passive gravitational mass is the response of the particle to the presence of a gravitational field, in this case the field of the Earth. There is an additional concept of mass, active gravitational mass [41], though we will not consider it here. The equivalence principle states that \( m(\mathrm{i}) = m(\mathrm{p}) \). In other words, the mass term drops out from the motion equation, and this simple fact is the reason that allows us to state that gravity, in classical mechanics, is a purely geometric theory. This point had already been distinguished by Newton since he pointed out that the universality of free fall implies that \( m(\mathrm{i})/m(\mathrm{p}) \) is a constant for all bodies. This premise has been subjected to many experimental tests, from Galileo [2] to the sophisticated experiments of our time [59–62]. In general relativity the universality of free fall is a fundamental element in the formulation of the theory in purely geometrical terms, it appears in WEP [63].

2.1.2. Einstein equivalence principle. Nevertheless, WEP does not suffice to define general relativity. The axiom that divides the theories of gravity into metric theories (those that satisfy it) and non-metric theories (those that do not) is the Einstein equivalence principle (EEP): (i) WEP is valid; (ii) the outcome of any local non-gravitational test experiment is independent of the velocity of the freely falling measuring device (a condition usually known as local Lorentz invariance) and (iii) the outcome of any local non-gravitational test experiment is independent of the where and when in the universe it is performed (denoted as local position invariance) [41].

Since physics is an experimental science, we must state quite clearly the status of general relativity in this aspect. The experimental confirmation of general relativity ranges from the classical tests (deflection of light, in which a light beam suffers an optical bending induced by the gravitational field of a body [64]) to, for instance, experiments that are designed to detect a time variation of the Newtonian gravitational constant [65]. Any test of the predictions of general relativity is, in some way, an indirect test of its axioms. Of course, we must be quite careful, since, for instance, any experimental confirmation of the gravitational red-shift prediction would not be a proof of the validity of general relativity. It would rather be a proof of the validity of all metric theories of gravity which also contain this effect, such as Brans–Dicke theory [66], or some theories with prior geometry [67].

2.1.3. Gravitomagnetism. We have explained that general relativity is a generalization of the Newtonian gravity theory [58], and some experiments in which this difference appears have been mentioned and described. There is one effect present in most metric theories, including general relativity, which has no Newtonian counterpart, namely, the gravitomagnetic effect, also known as Lense–Thirring effect [68]. The name of this effect is justified, since in post-Newtonian approximation it differs from Newtonian gravity as a magnetic force differs from an electric one. This effect emerges as a consequence of mass-energy currents. Let us explain this last statement a little bit better, and in order to do this we will resort to an analogy with electrodynamics. In electrodynamics, in a frame in which an electrically charge is at rest, we have an electric field. If the sphere starts to rotate, then a magnetic field appears, and the strength of this field hinges upon the angular velocity. In a similar manner, in general relativity a non-rotating massive sphere produces a Schwarzschild field [58]. As soon as the sphere begins to rotate the gravitomagnetic effect emerges as an additional element that modifies the structure of space–time. In contrast to this situation in Newtonian theory the only
source of gravity is mass; if the mass of a sphere rotates or not it is completely irrelevant for the calculation of the gravitational field. Though this field has already been detected [69], it has to be clearly stated that this experiment was performed employing classical systems. Nevertheless, the possible consequences for quantum systems, particularly on the coupling spin-gravitomagnetic field, require a much deeper analysis, i.e. it is almost always assumed that the coupling orbital angular momentum–gravitomagnetism can be extended to explain the coupling spin-gravitomagnetic field [70]. Nevertheless, this assumption must be subject to experimental scrutiny [39].

2.1.4. Gravity-induced interference. The behavior of the mass parameter is quite different in classical mechanics and in quantum theory. How does the situation look in quantum mechanics? Is the extrapolation from the classical realm to the quantum domain straightforward? The answer is no. The situation is rather different in the Schrödinger equation (once again we assume a homogeneous gravitational field) as can be seen from the structure of the corresponding motion equation [71]

\[-\frac{\hbar^2}{2m} \nabla^2 \psi + mgz\psi = i\hbar \frac{\partial \psi}{\partial t}.\]  

(15)

It is readily seen that now the mass does not cancel. Also note that (15) entails that mass always appears in the combination \(m/\hbar\). In other words, the detection of quantum effects of gravity will inexorably imply the emergence also of mass, and in consequence, we may wonder if at the quantum level the equivalence principle remains valid or if it has to be restricted to the classical world? The first direct evidence of the presence of a gravitational field as a non-trivial quantum effect was obtained in 1974 by Colella, Overhauser, and Werner [38], a proposal known as COW. There is an additional result that could also be considered a test of gravity in the quantum domain, the red-shift experiment performed by Pound and Rebka, in which the effects of gravity upon the frequency of a photon are experimentally confirmed [72]. Nevertheless, a careful analysis of this case shows that \(\hbar\) does not appear explicitly, and hence the interpretation of this experiment as a test of gravity in the quantum domain seems to be somewhat feeble. The COW proposal has been repeated in a series of experiments which showed an outstanding sophistication of the involved technology [73–75].

The phenomenon dealt with in this series of experiments can be denoted a gravity-induced quantum interference. The idea is to use an almost monoenergetic beam of thermal neutrons. This last statement means a kinetic energy of about 20 MeV, which is tantamount to a speed of 2000 \(\text{ms}^{-1}\). The primary beam is split into two parts, such that each of the new beams travels along different paths. These paths define a parallelogram of sides \(l_1\) and \(l_2\). Let us denote the vertices of this parallelogram by A, B, C and D, see figure 2.

The primary beam is split at vertex A, one of the secondary beams follows the side defined by the line passing through vertices A and C, this side has a length of \(l_1\), upon arrival at vertex C the beam is deflected by a silicon crystal and moves along the side defined by the line passing through vertices C and D, whose size reads \(l_2\). In a similar way the remaining secondary beam travels along \(A - B - D\). At D we find the detecting screen. If these two paths, \(A - B - D\) and \(A - C - D\), lie in a horizontal plane, then there is vanishing relative phase shift between the two beams. If we now rotate the plane, say an angle \(\theta\) around the side \(A - C\) of the parallelogram, then a non-vanishing relative phase shift appears, due to the fact that the paths these secondary beams follow are located at heights associated with different gravitational potential. It can be shown that the phase difference between the two secondary beams, \(\Delta \phi\), has the following form

\[\Delta \phi = \frac{m^2 g l_1 l_2 \lambda \sin(\theta)}{\hbar^2}.\]  

(16)
In this last expression \( m \) and \( \lambda \) denote the mass of the neutrons and the de Broglie wavelength of the neutrons, respectively. The interference pattern has a purely quantum mechanical origin, i.e. in the limit \( \hbar \to 0 \) the interference pattern gets washed out. Notice also that (16) tells us that mass does indeed appear in the form of a function of the combination \( m/\hbar \), as expected from the analysis of the corresponding Schrödinger equation (15). The possibility of resorting to heavier species for these kinds of experiments has also been considered [76], though the situation with heavier samples introduces the excitation of the internal states as a new variable to be considered, a fact that experimentally can be considered a shortcoming [77].

The arguments proving that (16) is gauge-invariant lie along the same line of reasoning as those used in connection with the Aharonov–Bohm effect; see second paragraph on page 11.

2.1.5. Postulates of metric theories and gravity-induced experiments. What are the consequences of this experiment? Here we arrive at a controversial and controverted issue. Indeed, the appearance of the mass parameter in the interference pattern has led some people to accept the idea that, in the quantum domain, gravity is not purely geometric [33,39,40]. Of course, as we have mentioned before, general relativity is weaved with several hypotheses, one of them, the equivalence principle. This axiom is closely related to a geometrical interpretation of gravity [58]. In other words, this interpretation implies the breakdown, in the quantum world, of the equivalence principle. The phrase **equivalence principle** here means WEP. This principle states that the motion of a particle can be reduced to purely geometrical parameters and nothing else. The appearance of \( m \) in this phase difference, and that is the claim [33,39,40], could mean the breakdown of WEP in the quantum domain. On the other hand, this same experiment has been the main ingredient to state that the COW experiments prove the validity of WEP. The argument reads: The experiment proves that the Newtonian potential \( mg \cdot r \) has to be taken into account in Schrödinger’s equation, and that this potential impinges upon the interference pattern, as any other potential [73]. Clearly, a careful analysis of this issue has to be done. For instance, a fleeting glance at the first experimental verification of this gravity-induced interference has been considered as a sound experimental verification of the equivalence principle in the quantum limit [73].

Nevertheless, as pointed out [78] the COW experiment does not suffice to prove the validity in the quantum level of the aforementioned principle. An experiment that validates this statement, up to an accuracy of 4%, [78] resorts to neutron interferometry on an accelerated inertial coordinate system. This experiment does allow us to consider the possible validity in the quantum world of the equivalence between a gravitational field and an accelerated
system. Additionally, it also provides an indirect test of the weak equivalence principle. Let us explain this last assertion a little bit better. Notice that behind the equivalence between a gravitational field and an accelerated coordinate system we may find an additional requirement, namely, inertial mass has to be equal to the gravitational passive mass, see the argument after equation (4) in [78]. In other words, the measurement readouts of [78] provide an indirect test of the weak equivalence principle.

We may also comment that COW may be formulated in terms that do not include the particle’s mass [83]. This could be considered as a proof of the fact that quantum theory fulfills the weak equivalence principle, though we must also add that this point has also some thorny aspects. For instance, any reformulation of the phase shift in terms of the wavelength of the particle masks the mass dependence since the de Broglie wavelength establishes a relation between mass and wavelength. Further experiments and theoretical predictions can be found in the literature [79, 80].

There is an additional result which at this point has a particular relevance in connection with our discussion of the validity of the equivalence principle in the quantum realm. Indeed, the classical origin of the gravitational modification of the phase of a neutron beam has already been shown [81]. A careful analysis of this last reference allows us to have a clear picture of the assumptions of a COW experiment. We may notice that from square one [81] considers the semiclassical approximation for the motion of neutrons and the eikonal limit for massless particles. The crucial question is the following one: does [81] prove that the weak equivalence principle is fulfilled in the quantum realm, or does it only show that in the semiclassical region of quantum mechanics this principle holds? This question requires a thorough analysis since the fulfillment of a certain property in a very particular limit (the semiclassical one) does not guarantee the validity of the involved property in the most general scheme.

The generalization of the weak equivalence principle has already been put forward [82, 83], though also some possible conceptual difficulties have been pointed out [84]. At this point it is noteworthy to stress the fact that this debate is still alive, and that no conclusive evidence, in one direction or in the other, exists. This means that more work is needed in this direction [85].

The possibility of resorting to these gravity-induced experiments and using them as a tool for testing the foundations of general relativity requires some modifications in the experimental proposal. For instance, to test EEP requires, if we wish to prove (or disprove) local Lorentz invariance or local position invariance (see above), to perform the experiment in different freely falling frames. A corroboration of the validity, in the quantum domain, of these two invariance properties would be obtained if the corresponding readouts are always the same, independent of the freely falling reference frame and of where and when the experiment has been carried out. Additionally, the validity of EEP implies that gravitation has to be described by a metric theory. This condition implies that: In any locally freely falling frame the non-gravitational laws of physics are those of special relativity [86]. Therefore, if the results coincide with the outcome (when in Schrödinger’s equation we impose the condition $g = 0$), then we could state that for a quantum system locally the effects of gravity can be transformed (gauged) away. In other words, this experiment would be a confirmation that gravitation can be formulated within the context of a metric theory. These kinds of experiments have, up to now, not been performed [86].

The phrase gauged away demands a deeper explanation. The free fall experiment in the form of the Einstein elevator [58] implies that for an observer in free fall there is a, sufficiently small, neighborhood in space–time in which the gravitational field can be considered null. In other words, in this neighborhood everything takes place as if the observer were in an inertial coordinate frame. This implies that, at least locally, gravity can be gauged away. Mathematically this can also be easily understood. Indeed, consider a point $P$ in our manifold;
the flatness theorem [58] tells us that there is a neighborhood around $P$ in which the metric is given by the Minkowskian one. Since (this point will be explained further below) the gravitational potential is encoded in the metric, the gravitational force is contained in the derivatives of the metric. But the derivatives of the Minkowskian metric all vanish, and in consequence, in this neighborhood of $P$ there is no gravitational field. This is the meaning of the phrase \textit{gauged away}, for any point in our manifold we may find a neighborhood in which the gravitational field vanishes.

Nevertheless, there is an additional interpretation for the phrase ‘gauge-dependent’ in the context of gravitation. A gauge transformation can be introduced in the case of the linearized version of Einstein equations. Indeed, in this situation the metric can be written in the following form

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \quad (17)$$

In this last expression $\eta_{\mu\nu}$ denotes the Minkowskian metric and $|h_{\mu\nu}| \ll 1$. A gauge transformation is defined by $|\xi^\nu_\mu| \ll 1$)

$$x'^\nu = x^\nu + \xi^\nu. \quad (18)$$

It can be shown (see p 438, exercise 18.1 in [58]) that a gauge transformation does not modify the components of the Riemann tensor. Of course, the contractions of the Riemann tensor are also invariant under these kinds of transformations. In this sense, (16) is also gauge-invariant.

For the sake of completeness let us mention a couple of quantum experiments (though not involving interference) testing local Lorentz invariance, which are known as Hughes–Drever experiments [87,88]. In these experiments the idea is to find a bound to any possible anisotropy in the inertial mass of quantum systems looking at the spacing among the energy levels of a particular energy state. Finally, the effects of rotation upon the interference pattern of thermal neutrons have also been considered [89, 90].

The description of wave propagation, within the principles of any metric theory, has conceptual difficulties. Indeed, the principles of these models are based upon the motion of point-like objects. This fact implies that for wave phenomena, for instance, quantum theory, the semiclassical approach has to be, from square one, imposed, otherwise we encounter a question, which in the context of metric theories, has no answer [91]. The same problem emerges if the description of light is attempted outside the eikonal limit. This last statement does not mean that wave propagation cannot occur in metric theories. It only implies that the propagation of wave phenomena faces, in the case of non-vanishing wavelength, conceptual difficulties in the context of the postulates of these theories.

2.1.6. Geometric phases and gravitomagnetism. One of the problems in the detection of gravitomagnetism comprises the fact that it involves tiny perturbations in the orbit of the used satellites [92]. At this point we may pose the following question: could this field be detected without having to measure very small changes, either in the trajectory or in other physical observables?

Additionally, the detection of this effect has always been contemplated in the realm of classical systems, and this last remark takes us to another question directly related to the present work: could this field be detected resorting to a quantum interference experiment? Additionally, could this interference experiment be used to test some fundamental property of physics?

The joint answer to these three questions will be provided by the following proposal. Consider a 1/2-spin particle immersed in the gravitomagnetic field of a rotating sphere (this
field will be described in the PPN formalism for any metric theory of gravity [58]). Additionally, we assume that its rotation axis also spins. It will be shown that the interaction between spin and gravitomagnetism predicts a geometric phase for the wave function, which does not depend upon the strength of the interaction. This last comment gives an affirmative answer to our first question. We may wonder at what stage the strength of the gravitomagnetic field appears in our proposal. This work will show that the strength of the gravitomagnetic field defines the adiabatic regime [46].

Let us now proceed to explain the proposed experiment of this part of the work. A beam of 1/2-spin particles (all in the same initial state) is split into two. One of the beams will not be allowed to interact with $\vec{J}$ (the definition of this parameter appears below), whereas the second one will have its spin state always pointing in the direction of $\vec{J}$. Clearly, this last condition is obtained, as a consequence of the adiabatic theorem, when $\vec{J}$ spins sufficiently slowly around a certain axis. After this angular momentum vector completes one cycle we proceed to recombine these two beams. We will show that the final probability involves a geometric phase factor, which will be non-vanishing for the case of a non-trivial coupling between spin and gravitomagnetism. This last explanation answers our second question; yes, there is a interference proposal which involves the possible detection of the gravitomagnetic field in the quantum domain. This last statement additionally provides an answer to the third question. As shown below, the emergence of the aforementioned geometric phase will appear only if the coupling between spin and gravitomagnetism given in the literature [70], but never subject to experimental confirmation, is correct.

Of course, we need a source of gravitomagnetism, therefore let us consider a rotating uncharged, idealized spherical body with mass $M$ and angular momentum $\vec{J}$. In the formalism of the weak field and slow motion limit the gravitomagnetic field may be written, using the PPN parameters $\Delta_1$ and $\Delta_2$ [58], as

$$\vec{B} = \left(\frac{7\Delta_1 + \Delta_2}{4}\right) \frac{G}{c^2} \frac{\vec{J} - 3(\vec{J} \cdot \hat{x})\hat{x}}{|\vec{x}|^3}.$$  \hspace{1cm} (19)

In this last expression our parameters allow us to recover different metric theories. For instance, $\frac{7\Delta_1 + \Delta_2}{4} = 2$ implies general relativity, while Brans–Dicke [66] appears if $\frac{7\Delta_1 + \Delta_2}{4} = \frac{128\omega}{8+4\omega}$. An interesting point emerges in Ni’s theory [94], where $\frac{7\Delta_1 + \Delta_2}{4} = 0$, i.e. there is no gravitomagnetic field. This last example teaches us that a metric theory does not necessarily embody a non-trivial gravitomagnetic effect.

As an additional condition we will assume that $\vec{J}$ rotates around a certain axis, $\vec{e}_3$, with angular velocity $\omega$, and that the direction of this axis and that of the angular momentum defines an angle $\theta$. This implies that in our coordinate system

$$\vec{J} = J \left[ \cos(\omega t) \sin(\theta) \hat{e}_1 + \sin(\omega t) \sin(\theta) \hat{e}_2 + \cos(\theta) \hat{e}_3 \right].$$  \hspace{1cm} (20)

As mentioned before, there is a spin 1/2-system immersed in the gravitomagnetic field of $M$, and located on $\vec{e}_3$ at a distance $r$ from the center of our rotating sphere. Usually the coupling between gravitomagnetism and orbital angular momentum in a copy of the behavior of orbital angular momentum under the presence of a magnetic field [68]. This has a profound physical explanation, since in the weak-field limit Einstein equations resemble the motion equations of electrodynamics. Here we assume that the expression describing the precession of orbital angular momentum can also be used for the description of the dynamics in the case of intrinsic spin, i.e. the behavior of orbital angular momentum is copied into the behavior of spin, which is a physical quantity without a classical analogue. This seems to be a reasonable assumption, though we must underline the fact that up to now there is no experimental evidence supporting
The Hamiltonian becomes (here $\vec{S}$ denotes the corresponding spin-operator)
\[
\hat{H} = -\vec{S} \cdot \vec{B}.
\]
(21)

We now introduce the following definitions
\[
\omega_1 = \frac{7\Delta_1 + \Delta_2 \, G J}{2c^2 r^3},
\]
(22)

we may rewrite (21) as
\[
\hat{H} = -\frac{\hbar \omega_1}{2} \begin{pmatrix} -2 \cos(\theta), & e^{-i\omega t} \sin(\theta) \\ e^{i\omega t} \sin(\theta), & 2 \cos(\theta) \end{pmatrix}.
\]
(23)

The associated energy values are
\[
E(\pm) = \pm \frac{\hbar \omega_1}{2} \sqrt{1 + 3 \cos^2(\theta)}.
\]
(24)

The eigenvector related to $E(+)\rangle$ reads
\[
\psi(+)\rangle(t) = \frac{\sin(\theta)}{\sqrt{2 + 6 \cos^2(\theta) - 4 \cos(\theta)\sqrt{1 + 3 \cos^2(\theta)}}} \begin{pmatrix} 1 \\ 2 \cos(\theta) - \sqrt{1 + 3 \cos^2(\theta)} \sin(\theta) \end{pmatrix} e^{i\omega t}.
\]
(25)

According to Berry [45], if $\omega_1 \gg \omega$, with the initial spin state $\psi(+)\rangle(t = 0)$, then the spin state is provided by
\[
\psi(+)\rangle(t) = e^{iE(+)\langle t/\hbar} e^{\gamma(+)\langle t} \psi(+)\rangle(t),
\]
(26)

where $\gamma(+)\langle t$ is Berry’s phase, a geometric term given by [45]
\[
\gamma(+)\langle t = \int_0^t \left< \psi(+)\rangle(t') \right| \frac{\partial \psi(+)\rangle(t')}{\partial t'} > dt'.
\]
(27)

We may now write
\[
\gamma(+)\langle t = -\omega t.
\]
(28)

At this point it is noteworthy to comment that the geometric phase is independent of the magnitude of the gravitomagnetic field. A glance at the previous work in this context [68, 69] shows that the quantity to be observed always depends upon the magnitude of the gravitomagnetic field, a fact that represents a drawback, in the experimental realm. In the present proposal, this factor does not emerge.

If $t = 2\pi/\omega$, this condition means that $\vec{J}$ has completed one rotation around $\vec{e}_3$, then the geometric phase turns out to be
\[
\gamma(+)\langle t = -2\pi.
\]
(29)

The condition defining the adiabatic regime is given by $\omega_1 \gg \omega$, and we may rephrase this condition, resorting to (22), as
\[
\frac{7\Delta_1 + \Delta_2 \, G J}{2c^2 r^3} \gg \omega.
\]
(30)

The magnitude of the gravitomagnetic field appears in this last expression, i.e. it defines the adiabatic regime. In this sense, this present proposal is quite different from the usual experimental ideas, which must detect tiny changes in the corresponding physical parameter [68, 69].

Moreover, it was mentioned before that the extant experimental confirmations of the gravitomagnetic effect involve macroscopic objects, satellites as a matter of fact [68, 69]. A careful analysis of the theoretical background behind this experiment shows that it embodies
the coupling between gravitomagnetism and orbital angular momentum, a physical quantity of classical origin. The behavior of spin under the presence of gravitomagnetism is always assumed to be a copy of the case of orbital angular momentum. Although this is a reasonable assumption, it has no experimental evidence supporting it. The present proposal gives the possibility of testing an aspect of fundamental physics, namely, the manner in which spin couples to gravitomagnetism. At this point we may wonder if the use of geometric phases could tell us something about the validity, at quantum level, of the postulates of metric theories, for instance, WEP. It is noteworthy to comment that the so-called Hyper project attempts to detect the Lense–Thirring effect resorting to atomic interferometers. In addition it plans to measure the fine structure constant, as well as the quantum-gravity induced foam structure of space [93].

The existence of gravitomagnetism is linked to the concept of metric theories [41]. Then the experimental corroboration of the existence of this effect is an indirect test of the postulates of metric theories, but it is not a direct test, for instance, of WEP. Indeed, the quantum properties appear in connection with the spin space, and not with the behavior of the system in the configuration space. This can be seen from the fact that the Hamiltonian (21) does not contain the mass of the particle. In other words, any postulate of metric theories related to the mass parameter cannot be tested directly with these kinds of proposals.

Let us now fathom better the conditions under which the adiabatic regime appears. We assume, for the sake of simplicity, that our sphere is a homogeneous one (which implies \( J = 2M^2R^2\Omega /5 \), here \( \Omega \) is the angular velocity of \( M \)). Then the validity of the adiabatic regime is guaranteed if

\[
\frac{MR^2\Omega}{\omega r^3} \gg \frac{5c^2}{G(\Delta_1 + \Delta_2)}.
\]

This last expression defines the region for the experimental parameters (\( M, R \), etc) in which the proposal could be used, assuming a certain metric theory given by \( \Delta_1 \) and \( \Delta_2 \).

2.2. Non-metric theories of gravity and quantum interference

2.2.1. Torsion and gravity. The current experimental data do not contradict general relativity [41,86], and, in consequence, we could take for granted its validity, in the classical realm and in the quantum domain. Another possibility in this context is to analyze generalizations of general relativity and understand the bounds that the extant experiments [41,86] impose upon the extra parameters that these generalizations contain. In this part of the present work we will take this point of view, and analyze the options that COW experiments offer in the context of one of the generalizations of general relativity. The question now is: what generalization of general relativity? The answer to this question will be given in terms of the voids that the current experiments have. A fleeting glance at the available experimental information on general relativity clearly shows that there are more classical than quantum tests of this theory. This is the starting criterion for our generalization of general relativity which gives a promising possibility for testing its principles in the quantum domain. Now that we have defined our criterion, we must once again face our question: what generalization? The answer is given, partially by the Casimir invariants of the Poincaré group, i.e. mass and spin. Mass is connected with the translational part of this group, whereas spin with the rotational one [95]. The Poincaré group lies very deep in the principles of special relativity, but if we look at general relativity we notice at once that mass is taken into account as a source of the gravitational field, but spin is not [58,41]. This situation contains an asymmetry which will provide us with our generalization. Indeed, a possible generalization of general relativity is the Einstein–Cartan
theory [96], which introduces into its formalism the concept of torsion, and connects it with spin. Mass is a source for the gravitational field in general relativity, and in the same spirit, the other Casimir invariant of the Poincaré group, spin, will in Einstein–Cartan theory be a source of a gravitational field. In other words, spin will modify the geometry of space–time. In other metric theories of gravity there is a coupling between the energy–momentum tensor [58] and the metric of the corresponding manifold. In a similar way, spin will be coupled to a geometric element of the manifold, i.e. the so-called contorsion tensor, which is related to rotational degrees of freedom of space–time [96]. The introduction of this new element into the structure of the corresponding space–time is done at the level of the so-called affine connection, in which a non-symmetric part is included. This new element behaves as a tensor, in contrast to the symmetric part [96].

\[ S_{ij}^k = \frac{1}{2} \left( \Gamma_{ij}^k - \Gamma_{ji}^k \right). \] (32)

This is denoted as Cartan’s torsion tensor, the \( \Gamma_{ij}^k \) are called the affine connections. Geometrically Cartan’s tensor entails that if we build infinitesimal parallelograms in our space–time, then they, in general, do not close. This tensor is used to define the contorsion tensor, here denoted by \( K_{ij}^k \) (\( \Gamma_{ij}^k \) are the Christoffel symbols computed as usual, with the corresponding metric [58]).

\[ \Gamma_{ij}^k = \Gamma_{ij}^k - K_{ij}^k. \] (33)

The corresponding Einstein tensor has the usual form

\[ G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R. \] (34)

Though the structure is the same, there is a great difference with general relativity [96], namely, the Ricci tensor, \( R_{ij} \), is asymmetric. Additionally, there is no torsion outside the spinning matter distribution. Torsion is bound to matter and cannot propagate through the vacuum. The influence of spin outside the matter distribution appears through its influence on the metric tensor. Torsion cannot influence macroscopic bodies [97, 98], its effects appear only in the evolution of spin. Clearly, the interaction between spin and torsion modifies the trajectory of a microscopic particle, i.e. Einstein–Cartan theory predicts a motion that will differ from that emerging in the context of a metric theory of gravity, as general relativity [96].

2.2.2. Non-Newtonian gravity. We have mentioned several experiments that do not contradict the theoretical predictions of general relativity. Just for the sake of completeness, let us mention some additional results that are also along this line. With this brief recount of the experimental information we will try to understand better those regions in which general relativity has not been tested very profoundly. This aspect is important since it allows us to justify soundly a series of interference experiments, which takes into account a generalization of gravity. This conceptual broadening of gravity becomes important in a region in which general relativity has not been subjected to a very stringent analysis.

Among these proofs we may find, for instance, the gravitational time dilation measurement [99, 100], gravitational deflection of electromagnetic waves [92], time delay of electromagnetic waves in the field of the sun [101], or the geodetic effect [102]. A fleeting glance at these observational results confirms the fact that they are tests of weak field corrections to the Galileo–Newton mechanics, i.e. they do not involve any time dependence of the gravitational field.

High-precision timing observations of pulsars have also been used in the context of diverse topics. For instance, timing observation over many years provided tight upper limits on the energy density associated with gravitational waves [103]. The results between theory and
experiment agree at a level of $10^{-3}$. The possibilities that binary pulsars offer do not finish here, as they can also be used as laboratories for testing strong-field gravity [104]. Concerning binary pulsars at this point, it is noteworthy to mention that they are a confirmation of general relativity at the classical level. Here we mean that the observations and predictions comprise the orbital dynamics of a binary pulsar, for instance, orbital period, eccentricity [105], etc.

After more than a decade of experiments [106], there is no compelling evidence for any kind of deviations from the predictions of Newtonian gravity. At this point it is noteworthy to comment on an argument put forward in this context. Gibbons and Whiting’s (GW) phenomenological analysis of gravity data [107] has proved that the very precise agreement between the predictions of Newtonian gravity and observation for planetary motion does not preclude the existence of large non-Newtonian effects over smaller distance scales, i.e. precise experiments over one scale do not necessarily constrain gravity over another scale.

This has an important consequence since GW results conclude that the current experimental constraints over possible deviations did not severely test Newtonian gravity over the 10–1000 m distance scale, usually called the ‘geophysical window’.

New constraints on the possible ranges of a Yukawa term have been given in an experiment carried out in 2000 [108]. This new experiment improves the current limit for ranges between 10 and 1000 km. Nevertheless, in the short range it can say nothing about distances smaller than 1 cm. This experiment is performed on a classical system, namely, a 3 ton $^{238}$U attractor rotates around a torsion balance, which contains Cu and Pb macroscopical test bodies. In this experiment the differential acceleration of the test bodies toward the attractor was measured.

The displacement induced by an oscillating mass acting as a source of gravitational field on a micromechanical resonator must be mentioned [109], since it could provide evidence about scalar interactions in the short range below 1 mm.

Among the models that in the direction of non-inverse-square forces currently exist we have Fujii’s proposal [110]. In this idea a ‘fifth force’, coexisting simultaneously with gravity, comprises a modified Newtonian potential. The corresponding Yukawa term is given by

$$V(r) = -G_\infty (mM/r)(1 + \alpha e^{-r/\lambda}),$$

where $G_\infty$ describes the interaction between $m$ and $M$ in the limit case $r \to \infty$, i.e. $G = G_\infty(1 + \alpha)$, where $G$ is the Newtonian gravitational constant. This kind of deviation terms arises from the exchange of a single new quantum of mass $m_5$, the Compton wavelength of the exchanged field is

$$\lambda = \frac{\hbar}{m_5c}.$$  

This field is usually called dilaton.

If we take a look at the experimental efforts that have been done in order to test the inverse-square law we will find that they can be separated into two large classes: (i) those experiments which involve the direct measurement of the magnitude $G(r)$, they compare preexisting laboratory Cavendish measurements of $G$ [112]; and (ii) the direct measurement of $G(r)$ with $r$ [113]. A relevant characteristic of these efforts has to be mentioned, i.e. they always remain at the classical level, the action of the Yukawa term is always on classical systems, namely, classical test masses (Cavendish case), or in the case of mine and borehole experiments, once again, classical test particles are employed. One of the exceptions around this topic is the use of the Casimir effect [115, 114], where the Planck constant, $\hbar$, appears as a parameter in the experiment. Another quantum analysis may be found in [116].

The information contained in all these works makes us wonder why we need to analyze some possible deviation of the Newtonian inverse-square force law. The answer to this question is two-fold.

The first point to be mentioned here stems from the fact that the agreement between general relativity and experiment might be compatible with the existence of a scalar contribution to gravity, such as a dilaton field [117]. This dilaton field emerges in several theoretical attempts that try to formulate a unified theory of elementary particle physics. As one of their
consequences they predict the existence of new forces (which are usually referred to as ‘fifth force’), whose effects extend over macroscopic distances [118]. In some way, these new forces simulate the effects of gravity, but a crucial point is that they are not described by an inverse-square law, and, moreover, they generally violate the weak equivalence principle (WEP) [118]. In other words, the presence of these kinds of forces, coexisting with gravity, could be detected by apparent deviations from the inverse-square law, or from the violation of WEP. These last arguments give us a partial answer to the last question, namely, a strong theoretical motivation for analyzing possible deviations from Newtonian gravity is to probe for new fundamental forces in nature.

Additionally, as mentioned at the beginning of this section, most of the tests to which general relativity has been subjected lie in the classical domain. The number of quantum tests is smaller [41, 86]. Our proposal, see below, resorts to quantum theory, and in this sense it tends to fill this gap.

2.2.3. Torsion-induced interference. The importance of the COW experiments of gravity-induced interference, and the corresponding improvement in the precision that they have reached, is not constrained to the aforementioned debate concerning the validity at the quantum realm of the postulates around metric theories of gravity. There is an additional fact that we now face. Indeed, the precision associated with them tells us that there is a discrepancy of one percent between theory and experiment [119], which requires an explanation. Several models can be proposed in this direction, and among them we may find some which take into account variables usually neglected. For instance, this discrepancy could be a direct consequence of the way in which dynamical diffraction interacts with bending and strains in the interferometer [75].

In this part of our work we analyze the possible role that torsion could play as an element explaining part of this discrepancy between experiment and theory. There is some previous work concerning the detection of torsion as a fundamental geometrical element of space–time; nevertheless, this analysis [120] cannot explain the discrepancy just mentioned, since it is not a neutron or atomic interferometric experiment, it is a Hughes–Drever type proposal and the presence of torsion is tested through shifts in Zeeman lines.

At this point we pose the following question: could the coupling between the spin of the neutrons and the torsion of space–time be held responsible for part of the discrepancy?

The case of a 1/2-spin particle immersed in a Riemann–Cartan space–time [96] is now considered, and now we pose the following question: what does the contribution to the interference pattern, stemming from the coupling spin–torsion, look like? More precisely, let us suppose that the spin part of the neutron beam’s wave function is the coherent linear superposition of two contributions, one with the z-component of the spin 1/2, and the other one with −1/2. It can be proved that the presence of torsion could be detected, in principle, heeding the changes that appear as a function of the way in which the superposition is done [121]. At this point it has to be mentioned that the possible effects of torsion upon a neutron interference experiment have already an old story [122]. The presence of torsion can be held responsible for the appearance of some peculiarities. For instance, the particle’s orbit is non-geodesic [123]. At this point it is noteworthy to comment that in these last works the analysis of the aforementioned discrepancy was not done.

The results obtained from our analysis are quite interesting, since they share some traits that already emerged in the gravity-induced interference pattern. For instance, the quantum mechanical trait of this effect depends on powers of \( m/\hbar \), and hence has a striking similarity with the conclusions of [38].
Let us consider a neutron interferometer, as in the COW experiment, and assume that there is a coupling between the torsion of space–time and the spin of our neutrons. The quantum mechanical description of the neutrons now requires a Hilbert space which has to be the tensor product of two contributions, to wit, spin state space, $E_s$, and the orbital state space, $E_r$ [124].

The dynamics of the state vector related to the neutron beam is given in the non-relativistic limit of the Dirac equation, in a Newtonian approximation of Riemann–Cartan space–time, by the Pauli equation [122].

$$i\hbar \frac{\partial |\psi\rangle}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 |\psi\rangle - i\frac{\hbar^2}{m} k_{(0)} \sigma^I \partial_I |\psi\rangle - mV|\psi\rangle - \hbar c \kappa^I \sigma^I |\psi\rangle.$$  \hspace{1cm} (35)

In this last expression the following terms have been introduced, $c$ is the speed of light, $V$ the Newtonian gravitational potential, $\sigma^I$ Pauli matrices and $\kappa_{\mu}$ the axial part of the space–time torsion. Additionally, in (35) we assume that $k_{(0)} = 0$. This simplification will allow us to fathom, in a clear manner, the consequences, upon the interference pattern, of the space part of the axial part of the torsion. The motion equation now becomes

$$i\hbar \frac{\partial |\psi\rangle}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 |\psi\rangle - mV|\psi\rangle - \hbar c \kappa^I \sigma^I |\psi\rangle.$$  \hspace{1cm} (36)

If $\phi$ describes the spin state vector, i.e. $\phi \in E_s$, then its dynamics is governed by the following equation

$$i\hbar \frac{\partial \phi}{\partial t} = -\hbar c \kappa_\alpha \sigma^\alpha \phi.$$  \hspace{1cm} (37)

Obviously a solution reads

$$\phi(t) = \exp\left\{ic \int_0^t \kappa_\alpha \sigma^\alpha \, dt\right\} \phi(t = 0).$$  \hspace{1cm} (38)

Let us now consider an experiment similar to COW, figure 3. In other words, two particles, starting at point $(O)$, move along two different trajectories, say $C$ and $\tilde{C}$. Afterwards these beams are detected at a certain point $S$. Additionally, we take for granted the validity of the semiclassical approach.

Let us now specify our trajectories explicitly. $C$ is made up of two contributions, namely, $(O)$–$(A)$ which is horizontal, whose length reads $l$, and $(A)$–$(S)$, vertical, and with length equal to $L$. $\tilde{C}$ also comprises two parts, $(O)$–$(B)$ vertical, with length $L$, and $(B)$–$(S)$ horizontal, and size $l$. The horizontal axis is $x$, and $y$ points upwards, such that the Newtonian potential reads $V = gy$. 

---

**Figure 3.** Experimental device to detect torsion-induced phase shifts.
Then,
\[ \kappa_n(A) = \kappa_n(0) + \frac{\partial \kappa_n}{\partial x}(0) l, \]
\[ \kappa_n(B) = \kappa_n(0) + \frac{\partial \kappa_n}{\partial y}(0) L. \]

This entails that at the screen \( S \) (for the spin wave function that passes through \( A \), \( \phi_A(S) \), and for that passing through \( B \), \( \phi_B(S) \)) we have
\[ \phi_A(S) = \exp \left\{ i c \sigma_n \left[ \alpha_A \kappa_n(0) + \beta_A \frac{\partial \kappa_n}{\partial x}(0) + \gamma_A \frac{\partial \kappa_n}{\partial y}(A) \right] \right\} \phi(t = 0), \]
\[ \phi_B(S) = \exp \left\{ i c \sigma_n \left[ \alpha_B \kappa_n(0) + \beta_B \frac{\partial \kappa_n}{\partial x}(B) + \gamma_B \frac{\partial \kappa_n}{\partial y}(0) \right] \right\} \phi(t = 0). \]

In these last two expressions we have (approximately)
\[ \alpha_A = \frac{m\tilde{\lambda}}{\hbar} \left\{ l + \frac{L}{2} - \left( \frac{m\tilde{\lambda}}{\hbar} \right)^2 gL^2/8 \right\}, \]
\[ \beta_A = \frac{m\tilde{\lambda}}{\hbar} \left\{ (l + L)/2 - \left( \frac{m\tilde{\lambda}}{\hbar} \right)^2 gL^2/8 \right\}, \]
\[ \gamma_A = \frac{m\tilde{\lambda}}{\hbar} \left\{ L^2/2 \left[ \frac{1}{4} - \left( \frac{m\tilde{\lambda}}{\hbar} \right)^2 gL/4 \right] + L \left[ \frac{1}{2} - \left( \frac{m\tilde{\lambda}}{\hbar} \right)^2 g(2L + 3l)/4 \right] \right\}, \]
\[ \alpha_B = \frac{m\tilde{\lambda}}{\hbar} \left\{ l + \frac{L}{2} + \left( \frac{m\tilde{\lambda}}{\hbar} \right)^2 gL \left[ l - \frac{L}{8} \right] \right\}, \]
\[ \beta_B = \frac{3L^2m\tilde{\lambda}}{\hbar} \left\{ \frac{1}{4} - \left( \frac{m\tilde{\lambda}}{\hbar} \right)^2 gL/8 \right\}, \]
\[ \gamma_B = \frac{m\tilde{\lambda}}{\hbar} \left\{ \frac{L^2}{2} \left[ \frac{3}{4} - \left( \frac{m\tilde{\lambda}}{\hbar} \right)^2 13gL/48 \right] \right\}. \]

We must add that in our equations \( \tilde{\lambda} = \lambda/(2\pi) \), and \( \lambda \) denotes the initial wavelength of the neutron beam.

These wave functions may be cast in terms of a rotation of the initial state
\[ \phi_n(S) = \exp \left\{ -i \frac{1}{2} \vec{\theta} \cdot \vec{\tau} \right\} \phi(t = 0). \]

Here \( v = A, B \). The definition of the components of the unit vectors and the rotation angles are given by
\[ \tau_n^{(A)} = \left\{ \alpha_A \kappa_n(0) + \beta_A \frac{\partial \kappa_n}{\partial x}(0) l + \gamma_A \frac{\partial \kappa_n}{\partial y}(A) \right\}, \]
\[ (\vec{\tau})_n = \frac{1}{\sqrt{\left( \tau_x^{(A)} \right)^2 + \left( \tau_y^{(A)} \right)^2 + \left( \tau_z^{(A)} \right)^2}}, \]
\[ \theta_A = -2c \sqrt{\left( \tau_x^{(A)} \right)^2 + \left( \tau_y^{(A)} \right)^2 + \left( \tau_z^{(A)} \right)^2}. \]

In a similar manner for \( B \).
These results allow us to distinguish two different situations:

(i) If \( |\theta_{\phi}|, |\theta_{\sigma}| \ll |\phi_n| \). Therefore \( \vec{n}_A = \vec{n}_B \), the axis of rotation of the beams is the same, and, in consequence, they differ only in the angle of rotation, \( \theta_A \neq \theta_B \).

(ii) If the foregoing condition does not hold, then not only \( \theta_A \neq \theta_B \), also additionally, \( \vec{n}_A \neq \vec{n}_B \).

Let us now assume that \( \phi(t = 0) \) is the linear coherent superposition of states \( \chi_{(+)} \) and \( \chi_{(-)} \), where \( \sigma_{rX(s)} = \pm \chi_{(s)} \), namely

\[
\phi(t = 0) = c_{(+)} \chi_{(+)} + c_{(-)} \chi_{(-)}.
\]

The interference pattern at \( S \) is a function of the complete state vector, i.e. \( |\psi\rangle \), whose dynamics evolves according to (35). This last argument may be rephrased stating

\[
I = ||\psi_{(A)}||^2 + ||\psi_{(B)}||^2.
\]

It embodies two different contributions, one stemming from \( \mathcal{E}_r \) and the second one from \( \mathcal{E}_r \). In other words, we find that

\[
I = 2 + 2 \cos \left( \frac{m}{\hbar} g l \hat{L} \right) \left[ \phi^{+}_A(S) \phi_B(S) + \phi_B(S) \phi_A(S) \right].
\]

Therefore

\[
I = 2 + 2 \cos \left( \frac{m}{\hbar} g l \hat{L} \right) \left[ \cos \left( \frac{\theta_A}{2} \right) \cos \left( \frac{\theta_B}{2} \right) + [\vec{n}_A \cdot \vec{n}_B] \sin \left( \frac{\theta_A}{2} \right) \sin \left( \frac{\theta_B}{2} \right) \right]
- 2 \sin \left( \frac{m}{\hbar} g l \hat{L} \right) \left[ \sin \left( \frac{\theta_A}{2} \right) \sin \left( \frac{\theta_B}{2} \right) \left[ \vec{n}_A \times \vec{n}_B \right] \sin \left( \frac{\theta_A}{2} \right) \cos \left( \frac{\theta_B}{2} \right) \vec{n}_A
- \sin \left( \frac{\theta_B}{2} \right) \cos \left( \frac{\theta_A}{2} \right) \vec{n}_B \right] \cdot [2 \Re(c_{(+)}^* c_{(-)}) \vec{e}_x - 2 \Im(c_{(+)}^* c_{(-)}) \vec{e}_y + (c_{(+)}^2 - |c_{(-)}|^2)^2 \vec{e}_z].
\]

It is readily seen that \( \cos \left( \frac{m}{\hbar} g l \hat{L} \right) \) corresponds to the interference term in COW [38]. In other words, if we discard torsion, then the usual COW result is recovered. Additionally, \( \vec{e}_n \) denotes the unit vector along the \( n \)-axis.

Under these conditions we have that

\[
I = 2 + 2 \cos \left( \frac{m}{\hbar} g l \hat{L} \right) \left[ \cos \left( \frac{\theta_A}{2} \right) \cos \left( \frac{\theta_B}{2} \right) + [\vec{n}_A \cdot \vec{n}_B] \sin \left( \frac{\theta_A}{2} \right) \sin \left( \frac{\theta_B}{2} \right) \right].
\]

(56)

Let us now analyze some cases, for instance, \( c_{(+)} \neq c_{(-)} \).

Here we consider \( c_{(+)} \neq c_{(-)} \).

\[
I = 2 + 2 \cos \left( \frac{m}{\hbar} g l \hat{L} \right) \left[ \cos \left( \frac{\theta_A}{2} \right) \cos \left( \frac{\theta_B}{2} \right) + [\vec{n}_A \cdot \vec{n}_B] \sin \left( \frac{\theta_A}{2} \right) \sin \left( \frac{\theta_B}{2} \right) \right]
- 2 \sin \left( \frac{m}{\hbar} g l \hat{L} \right) \left[ \sin \left( \frac{\theta_A}{2} \right) \sin \left( \frac{\theta_B}{2} \right) \left[ \vec{n}_A \times \vec{n}_B \right] \sin \left( \frac{\theta_A}{2} \right) \cos \left( \frac{\theta_B}{2} \right) \vec{n}_A
- \sin \left( \frac{\theta_B}{2} \right) \cos \left( \frac{\theta_A}{2} \right) \vec{n}_B \right] \cdot [|c_{(+)}|^2 - |c_{(-)}|^2] \vec{e}_z.
\]

(57)

Neglecting all derivatives of the axial part of the torsion, a condition tantamount to \( \vec{n}_A = \vec{n}_B \), we obtain

\[
I = 2 + 2 \cos \left( \frac{m}{\hbar} g l \hat{L} \right) \cos \left( \frac{m \tilde{\lambda}}{\hbar} \right) \frac{3}{3} g c l^2 \frac{K}{K} = 2 \kappa(0) \frac{3}{3} g c l^2 \frac{K}{K} \frac{|c_{(+)}|^2 - |c_{(-)}|^2}{K} \times \sin \left( \frac{m}{\hbar} g l \hat{L} \right) \sin \left( \frac{m \tilde{\lambda}}{\hbar} \right) \frac{3}{3} g c l^2 \frac{K}{K}.
\]

(58)
In the foregoing expression the following definition has been introduced $K = \sqrt{\kappa^2(0)_z + \kappa^2(0)_y + \kappa^2(0)_x}$. 

Expression (55) provides enough leeway to consider the possibility of detecting the consequences of torsion, upon the interference pattern. This can be done modifying the values of $c(\pm)$ and $c(-)$. Indeed, choosing $c(\pm) = 1$ and $c(-) = 0$, 

$$I = 2 + 2 \cos \left( \frac{m}{\hbar} g l \hat{\lambda} \right) \cos \left( \frac{m \hat{\lambda}}{\hbar} g c l^2 K \right) 
- 2\kappa(0)_z / K \sin \left( \frac{m}{\hbar} g l \hat{\lambda} \right) \sin \left( \frac{m \hat{\lambda}}{\hbar} g c l^2 K \right).$$

(59)

If now $c(\pm) = 0$ and $c(-) = 1$, then

$$I = 2 + 2 \cos \left( \frac{m}{\hbar} g l \hat{\lambda} \right) \cos \left( \frac{m \hat{\lambda}}{\hbar} g c l^2 K \right) + 2\kappa(0)_z / K \sin \left( \frac{m}{\hbar} g l \hat{\lambda} \right) \sin \left( \frac{m \hat{\lambda}}{\hbar} g c l^2 K \right).$$

(60)

If the parameters are switched from $\{c(\pm) = 1, c(-) = 0\}$ to $\{c(\pm) = 0, c(-) = 1\}$ a sign change in the second term of the right-hand side appears. This effect vanishes if the torsion is null. In other words, this sign change is a direct consequence of torsion, and appears only if we modify the linear superposition of the initial spin state vector. As a matter of fact, considering a series of experiments in which we begin with $\{c(\pm) = 1, c(-) = 0\}$ and gradually change these two values (the first parameter diminishes, whereas the second one increases), then the role that the absolute value of the second term plays, peters out. This happens when $c(-) = 1/\sqrt{2}$. Afterwards, it starts to appear once again.

At this point we proceed to estimate the order of magnitude of the torsion contributions, and then relate it to the current experimental discrepancy. We have already mentioned that the theoretical prediction possesses a discrepancy on the order of one percent in the phase shift [75]. Additionally, a sufficiently (for our purposes) stringent experimental bound reads $K \sim 10^{-15} m^{-2} [122]$ and, hence (employing the typical experimental values [38, 75] and denoting by $\Gamma_1$ the contribution to this discrepancy), we deduce

$$\Gamma_1 \sim 10^{-16}.$$ 

(61)

Let us now analyze this last expression. It is easily seen that the involved experimental discrepancy cannot be understood exclusively by torsion effects. The reason stems from the fact that the value of $\Gamma_1$ is too tiny to provide the necessary explanation. The introduction of only one new element, Cartan’s torsion tensor, cannot give an explanation to this fact. Additional work is needed in this direction.

At this point let us remember that the COW experiment has an additional feature, which has spurred a hot debate about the validity in the quantum domain of WEP. Indeed, the appearance of the mass term in the interference expression $[\hat{\lambda} m]^2$ has been understood by some authors as a possible manifestation of non-geometricity [39, 40] in the gravitational field. Taking a look at (58), it is readily seen that under the aegis of torsion this trait, not only does not vanish, but on it an additional term is bestowed, i.e. $[\hat{\lambda} m]^3$. The only difference between the present model and the usual COW case has been the introduction of torsion in the motion equation for the neutrons, see (35). This implies that the emergence of this extra term in the interference pattern is a consequence of the coupling between the torsion tensor and the spin of our neutrons. A fleeting glance at this term shows us that it involves the mass of the particle, and we may
wonder why the mass appears in the interference pattern, since the new term involves torsion. Looking at (34), we find the answer, the Ricci tensor and also its contraction are a mixture between the metric and the contorsion tensor, since the metric is coupled to mass–energy and the contorsion to the spin. It should not surprise us that mass appears in the term related to the influence of torsion in the interference pattern.

We should be careful in the interpretation of this fact since it could be conjectured that the term \[ \frac{m}{\hbar} \] confirms that the emergence of the term \[ \frac{m}{\hbar} \] in the usual COW experiment is related to a violation of WEP. Indeed, we could claim if torsion entails the violation of WEP, and it appears in the interference pattern as a parameter involving mass, that \[ \frac{m}{\hbar} \] is a manifestation of a violation of WEP in the quantum domain. Of course, this last argument has to be supported by further experimental evidence.

2.2.4. Non-Newtonian gravity-induced interference. Now we analyze the theoretical predictions, at the quantum level, that a Yukawa term could have. But before doing this let us first comment on some interesting proposals designed to measure the Newtonian gravitational constant in the realm of atom interferometry. The pertinency at this point of these proposals stems from the fact that a good knowledge of the value of \( G \) is a prerequisite in any proposal attempting to determine any non-Newtonian term associated with gravity, as the expressions below will clearly show.

These ideas may be divided into two different models. Let us first mention the so-called MAGIA project [125, 126] in which free-falling laser cooled atoms will be used as probes for the gravitational potential. The acceleration will be detected resorting to the Raman atom interferometry techniques, and a value for \( G \) will then be deduced. The second proposal [127] is based upon a gradiometer which measures the differential acceleration of two samples of laser-cooled Cs atoms. Clearly, these two ideas employ independent methods in the measuring process of \( G \).

The idea now in the context of non-Newtonian gravity is to include in the theoretical background of the gravitational field a Yukawa term and calculate the corresponding effects in an experimental proposal which is very similar to the COW construction [38]. The goal is to understand the way in which the parameters, appearing in Fujii’s model, impinge upon the interference pattern of a COW experiment. The parameter \( \lambda \) is related to the range of this new force, whereas \( \alpha \) has no dimensions and is connected with the intensity of the interaction [110]. We must mention that in this context the analysis of the effects that a fifth force could have in a COW experiment has not been done [111].

Consider a Yukawa modification to the Newtonian gravitational potential [110]

\[
V(r) = -G_\infty \frac{mM}{r} \left(1 + \alpha e^{-\frac{r}{\lambda}}\right). \tag{62}
\]

The Lagrangian of a particle with mass \( m \), moving in this field, is

\[
L = \frac{m}{2} \dot{r}^2 + G_\infty \frac{mM}{r} \left(1 + \alpha e^{-\frac{r}{\lambda}}\right). \tag{63}
\]

Since we are considering a terrestrial experiment we introduce a definition in order to have a parameter related to the height above the surface of the Earth at which the experiment is carried out. Define \( r = R + l \), where \( R \) is the Earth’s radius and \( l \) the height over the Earth’s surface. Keeping terms up to second order in \( l \)

\[
L = \frac{m}{2} \dot{r}^2 + G_\infty \frac{mM}{R} \left(1 + \alpha + \frac{\alpha R}{\lambda} \left(1 - \frac{R}{2\lambda} - 1\right) - \left[1 + \alpha + \frac{\alpha R}{2\lambda} - 1\right] \right) + \frac{1}{R^2} l^2. \tag{64}
\]
Two particles, starting at point $O$, move along two different trajectories, $C$ and $\tilde{C}$, and afterwards they are detected at a certain point $S$, see figure 3. Of course, we require the semiclassical approximation, i.e. the size of the wavelengths of the packets is much smaller than the size in which the field changes considerably.

The wave function reads

$$\psi(\vec{r}, t) \sim \frac{1}{[E - V(\vec{r})]^2} \exp \left\{ \pm \frac{i}{\hbar} \int_0^{(S)} \sqrt{2m[E - V(\vec{r})]} \, d\tilde{L} - \frac{i}{\hbar} E t \right\},$$

where

$$V(\vec{r}) = -G \frac{mM}{R} \left[ 1 + \alpha + \frac{\alpha R}{\lambda} \left( \frac{R}{2\lambda} - 1 \right) \right] - \left[ 1 + \alpha - \frac{\alpha R}{2\lambda^2} \right] \left[ l + \frac{1 + \alpha}{R^2} l^2 \right].$$

The line integral in (65) has to be calculated along $C$ and $\tilde{C}$, which are two different trajectories.

The interference term at the detection point is given by

$$I = \cos \left\{ -\frac{g m^2 L S}{\hbar^2} \left[ 1 - \frac{\alpha R^2}{2\lambda^2(1 + \alpha)} - \frac{l_S}{R} \right] \right\}.$$ 

As a matter of fact, the result of COW does not contain the term that is quadratic in $l_S$. It appears in our result since we have introduced a less stringent approximation, to derive the results of COW we need only a homogeneous Newtonian gravitational field, and expression (64) includes the case of an inhomogeneous gravitational field, i.e. the term $\frac{(1 + \alpha)}{R^2}$.

The difference between the Newtonian and non-Newtonian cases divided by the Newtonian value is (approximately)

$$\Delta = \frac{\alpha R^2}{2\lambda^2(1 + \alpha)} \left( 1 + \frac{l_S}{R} \right).$$

The Compton wavelength of our new quantum particle (with mass $m_S$) reads $\lambda = \hbar / cm_S$. Introducing it into (67)

$$\Delta = \frac{\alpha (R cm_S)^2}{2\hbar^2(1 + \alpha)} \left( 1 + \frac{l_S}{R} \right).$$

We may now cast the interference term in the following form

$$I = I_N \cos \left\{ \frac{g m^2 L S \lambda \alpha R^2}{2\hbar^2 \lambda^2(1 + \alpha)} \right\} \pm \sqrt{1 - I_N^2} \sin \left\{ \frac{g m^2 L S \lambda \alpha R^2}{2\hbar^2 \lambda^2(1 + \alpha)} \right\}.$$ 

This last expression contains a deviation from the usual inverse-square law. If the usual experimental parameters related to COW [38] are $\alpha \sim 10^{-3}$ and $\lambda \sim 10^4$ cm [128], then

$$\frac{g m^2 L S \lambda \alpha R^2}{2\hbar^2 \lambda^2(1 + \alpha)} \sim 10^7 (cm)^{-1} l_S.$$ 

The interference pattern now becomes a simple function of $l_S$

$$I = I_N \cos \left\{ 10^7 (cm)^{-1} l_S \right\} \pm \sqrt{1 - I_N^2} \sin \left\{ 10^7 (cm)^{-1} l_S \right\}.$$ 

The possibility of testing non-Newtonian gravity appears in connection with the dependence of \((72)\) on \(l_S\). The COW experiment can, in principle, be used to detect a Yukawa type modification to the inverse-square law of gravity. A word of caution has to be added in this context. From the very beginning it was clearly stated that we require the validity of the semiclassical approach. In this model we have the Newtonian contribution plus a Yukawa term; this implies that the semiclassical approximation must take into account these two fields. The semiclassical limit requires the wavelength of the particle to be smaller than the distance in which the Newtonian potential and the Yukawa term have a noticeable change.

2.3. Non-demolition variables and gravity-induced interference

2.3.1. Non-demolition variables and restricted path integral. Nowadays one of the fundamental problems in modern physics comprises the so-called quantum measurement problem [30]. Though there are several attempts to solve this old conundrum (some of them are equivalent [129]), here we will explain the restricted path integral formalism (RPIF) [130]. This particular choice has behind it a sound reason. Indeed, RPIF allows us to test not only the possibility of a Yukawa term addition to the gravitational interaction [110], but also one of the solutions to the quantum measurement problem. This last point is quite relevant since we may find some postures which claim either that there is no measurement problem in quantum mechanics [32] or that quantum mechanics is an incomplete theory [11]. RPIF provides us with the possibility of testing the existence of a fifth force and one of the solutions to a fundamental problem (if we accept that it is a problem) in quantum theory.

Let us now briefly mention the main ideas behind RPIF. This model explains a continuous quantum measurement with the introduction of a restriction on the integration domain of the corresponding path integral [130]. This last condition can also be reformulated in terms of a weight functional that has to be considered in the path integral. Clearly, this weight functional contains all the information about the interaction between the measuring device and the measured system.

A measurement in the context of quantum mechanics possesses some traits which are absent in the classical version. For instance, in the latter there is no impediment for the simultaneous measuring of two variables with arbitrary precision. This is, in some cases, forbidden in the quantum realm [30, 131]. The issue of quantum theory of measurement was considered for some decades as an almost esoteric topic. The main reason behind this attitude was the irrelevance of the quantum theory of measurement in the technological aspect. Nevertheless, this started to change in the 1980s, when technology began to catch up with theory.

The concept of non-demolition variables stems from a technological requirement. The development of gravitational wave detectors required methods for measuring macroscopic variables at levels of precision approaching and even exceeding the standard quantum limit imposed by Heisenberg’s uncertainty relation. The theoretical analysis of the traditional schemes associated with measurement proved that the corresponding precisions can never go beyond the standard quantum limit. This fact spurred the development of non-traditional measurement procedures, which are known as quantum non-demolition measurements [131].

The basic idea around the concept of quantum non-demolition (QND) measurements is to carry out a sequence of measurements of an observable in such a way that the measuring process does not diminish the predictability of the results of subsequent measurements of the same observable. The main idea of a QND measurement is to deduce a variable such that the unavoidable disturbance of the conjugate observable does not disturb the evolution of the chosen variable [132].
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The dynamical evolution of a system, subject to a measuring process, limits the class of observables that may be measured repeatedly with arbitrary precision. Let us explain this a little bit better and assume that in our case we continuously monitor the observable \( A(t) \). Then \( A(t) \) is a QND variable if the commutation relation \( [B(t'), B(t'')] = 0 \) is fulfilled, for any \( t' \) and \( t'' \), where \( B(t) = \exp\{i \frac{\hat{F} t}{\hbar}\} A(t) \exp\{-i \frac{\hat{F} t}{\hbar}\} \) is the corresponding operator in the Heisenberg picture [130].

Let us now suppose that in our case \( A(t) = \rho \rho + \sigma_z \), where \( \rho \) and \( \sigma \) are functions of time. In this particular case, the condition that determines when \( A(t) \) is a QND variable may be written as a differential equation [131]

\[
\frac{df}{dt} = \frac{f^2}{m} - m\Omega^2,
\]

where \( f(t) = \sigma/\rho \).

It is readily seen that a solution to the differential equation is

\[
f(t) = -m\Omega \tanh(\Omega t).
\]

Choosing \( \rho(t) = 1 \), we find that in our case a possible QND variable is provided by

\[
A(t) = p - m\Omega z \tanh(\Omega t).
\]

2.3.2. Non-demolition variables and non-Newtonian gravity. The idea in this part is two-fold: firstly, the effects of a Yukawa term upon a quantum system (that is continuously monitored) will be calculated and compared against the corresponding results containing only the usual Newtonian gravitational potential (in a proposal that has as a new element the fact that it is not a COW type experiment); secondly, new theoretical predictions for one of the models in the context of quantum measurement theory will be found. The first goal offers us the possibility of testing a fifth force in the form of a Yukawa term [110], whereas the second one provides us with a series of theoretical predictions that embody the premises of one of the models that claim to be a solution to the quantum measurement problem, i.e. RPIF.

These two goals will be achieved obtaining a non-demolition variable for the case of a particle subject to a gravitational field which contains a Yukawa term such that \( \lambda \) (this parameter is related to the range of this new force) has the same order of magnitude as the radius of the Earth. This proposal, as will become clear below, does not involve an interference experiment in the sense of COW. From this remark we may pose the following question: could this idea be considered an interference experiment? The answer is no, in the sense of a COW interference device, but yes in the context of the meaning of the Feynman path integral version of quantum mechanics, in which the interference among all the possible trajectories between two points of the configuration space are fundamental to obtaining the properties of the corresponding quantum system. In this sense it is an interference experiment and the aspect of the principles of quantum theory that this idea could allow us to test is the response of a quantum system to series of continuous measurements, i.e. the understanding of a quantum system under a continuous measurement process will be improved.

We have already mentioned that we will assume a certain value for one of the parameters involved in a fifth force. Let us now explain the reasons behind this choice. The extant experiments [111] set constraints for \( \lambda \) for ranges between 10 and 1000 km [108], but the case in which \( \lambda \sim \) Earth’s radius remains rather unexplored [111]. Afterwards, we will consider, along the ideas of the restricted path integral formalism (RPIF) [130], the continuous monitoring of a non-demolition parameter, and then calculate not only the corresponding propagators but also the probabilities associated with the different measurement outputs. These probabilities will
provide an interesting way not only to prove the existence of a Yukawa term as an additional gravitational element but also to prove RPIF. The comparison with the purely Newtonian case will be effects, and the effects of the Yukawa term upon our quantum system will be obtained. In order to do this we will consider a particular measurement output, and then the corresponding probabilities, in the Newtonian and non-Newtonian situations, will be evaluated, and finally the ratio between them will be obtained.

Suppose that we have a spherical body with mass $M$ and radius $R$. Let us now consider the case of a Yukawa term \[110\], hence the potential and the Lagrangian are given by expressions (62) and (63) respectively.

We now define $r = R + z$, where $R$ is the body’s radius, and $z$ the height over its surface. If $R/\lambda \sim 1$ (which means that the range of this Yukawa term has the same order of magnitude as the radius of our spherical body), and if $z \ll R$, then we may approximate the potential as follows

$$V(r) = -G_{\infty} \frac{M}{R} \left(1 + \alpha \right) - \left[\frac{1 + \alpha}{R} + \frac{\alpha}{2\lambda}\right] z + \left[\frac{1 + \alpha}{2R^2} + \frac{\alpha}{2R\lambda} + \frac{\alpha}{2\lambda^2}\right] z^2.$$  \hspace{1cm} (76)

The Lagrangian of our particle of mass $m$ becomes now

$$L = \frac{p^2}{2m} + G_{\infty} \frac{mM}{R} \left[1 + \alpha \right] - \left[\frac{1 + \alpha}{R} + \frac{\alpha}{2\lambda}\right] z + \left[\frac{1 + \alpha}{2R^2} + \frac{\alpha}{2R\lambda} + \frac{\alpha}{2\lambda^2}\right] z^2.$$  \hspace{1cm} (77)

If our particle moves from point $N$ to point $W$, then the propagator becomes

$$U(W, \tau''; N, \tau') = \left(\frac{m}{2\pi\hbar T}\right)^{\frac{1}{2}} \exp \left\{ \frac{-im}{2\hbar T} \left[(xW - xN)^2 + (yW - yN)^2\right] \right\} \times \int d[z(t)] \exp \left\{ \frac{i}{\hbar} \int_{\tau'}^{\tau''} \left[\frac{m}{2} \dot{z}^2 + G_{\infty} \frac{mM}{R} \left(1 + \alpha\right) \right] z + \left[\frac{1 + \alpha}{2R^2} + \frac{\alpha}{2R\lambda} + \frac{\alpha}{2\lambda^2}\right] z^2 \right\} dt,$$  \hspace{1cm} (78)

where $T = \tau'' - \tau'$.

Now consider the following two definitions

$$F = -G_{\infty} \frac{mM}{R} \left[1 + \alpha + \frac{\alpha}{2\lambda}\right],$$  \hspace{1cm} (79)

$$\alpha^2 = -2 \frac{G_{\infty} M}{R} \left[\frac{1 + \alpha}{2R^2} + \frac{\alpha}{2R\lambda} + \frac{\alpha}{2\lambda^2}\right],$$  \hspace{1cm} (80)

where $G = G_{\infty}[1 + \alpha]$, and $G$ is the Newtonian gravitational constant \[118\]. In our case, \[\frac{1}{2R^2} + \frac{\alpha}{2R\lambda} + \frac{\alpha}{2\lambda^2}\] > 0, hence, $\omega = i\Omega$, where $\Omega \in \mathbb{R}$.

The propagator now becomes

$$U(W, \tau''; N, \tau') = \left(\frac{m}{2\pi\hbar T}\right)^{\frac{1}{2}} \exp \left\{ \frac{-im}{2\hbar T} \left[(xW - xN)^2 + (yW - yN)^2\right] \right\} \int_{z_N}^{z_W} d[p] d[z(t)] \times \exp \left\{ \frac{i}{\hbar} \int_{\tau'}^{\tau''} \left[\frac{p^2}{2m} + \left(1 + \alpha\right) \frac{G_{\infty} mM}{R} + Fz + \frac{m}{2} \Omega^2 z^2 \right] \right\} dt.$$  \hspace{1cm} (81)

According to RPIF, if the variable $A(t)$ is continuously monitored, then we must introduce a particular expression for our weight functional, i.e. for $w_{[a(t)]}[A(t)]$. As was mentioned before, the weight functional $w_{[a(t)]}[A(t)]$ contains the information concerning the measuring process. Here $a(t)$ denotes the measurement readout. At this point it is noteworthy to comment that the more probable the ‘trajectory’ $A(t)$ is, according to the output $a$, the bigger $\omega_{[a(t)]}[A(t)]$ becomes \[130\]. In other words, the value of $\omega_{[a(t)]}[A(t)]$ is approximately one
for all ‘trajectories’ \(A(t)\) that agree with the measurement output \(a(t)\), and it is almost 0 for those that do not match with the result of the experiment. Clearly, an issue remains to be addressed, namely, the choice of our weight functional. In order to solve this difficulty, let us mention that the results coming from a Heaviside weight functional \([133]\) and those coming from a gaussian one \([130]\) coincide up to the order of magnitude. Therefore we may consider a gaussian weight functional as an approximation to the correct expression.

We choose as our weight functional the following expression

\[
\omega_{a(t)}(A(t)) = \exp \left\{ -\frac{2}{T \Delta a^2} \int_{\tau'}^{\tau''} [A(t) - a(t)]^2 dt \right\}. \tag{82}
\]

\(\Delta a\) denotes the resolution of the measuring device.

With our weight functional choice \((82)\) the new propagator may be written as follows:

\[
U_{a(t)}(W, \tau''; N, \tau') = \left( \frac{m}{2\pi \hbar T} \right)^{3/2} \exp \left\{ \frac{im}{2\hbar T} [(x_w - x_N)^2 + (y_w - y_N)^2] \right\} \int_{\mathcal{C}} \! d[p] d*z(t) \bigg] \int_{\tau'}^{\tau''} \left[ \frac{p^2}{2m} + \frac{G_{\infty} m M}{R} + F z + \frac{m}{2} \Omega^2 z^2 \right] dt \bigg] \int_{\tau'}^{\tau''} [A - a]^2 dt \bigg] \]. \tag{83}
\]

This propagator involves two gaussian integrals, and it can be calculated \([134]\)

\[
U_{a(t)}(W, \tau''; N, \tau') = \left( \frac{m}{2\pi \hbar T} \right)^{3/2} \exp \left\{ \frac{im}{2\hbar T} [(x_w - x_N)^2 + (y_w - y_N)^2] \right\} \int_{\mathcal{C}} \! d[p] d*z(t) \bigg] \int_{\tau'}^{\tau''} \left[ \frac{p^2}{2m} + \frac{G_{\infty} m M}{R} + F z + \frac{m}{2} \Omega^2 z^2 \right] dt \bigg] \int_{\tau'}^{\tau''} [A - a]^2 dt \bigg] \]. \tag{84}
\]

The probability, \(P_{a(t)}\), of obtaining as measurement output \(a(t)\) is given by the expression

\[
P_{a(t)} = |U_{a(t)}|^2 \tag{130}.\]

Hence, in this case

\[
P_{a(t)} = \exp \left\{ \frac{-2T \Delta a^2}{4m^2 \hbar^2 + T^2 \Delta a^4} \int_{\tau'}^{\tau''} a^2(t) dt \right\} \times \exp \left\{ \frac{\hbar}{m \Omega^2} \int_{\tau'}^{\tau''} \left[ 2I_1 I_2 I_3 + I_4 (I_2^2 - I_1^2) \right] dt \right\}. \tag{85}
\]

In this last expression we have that

\[
I_1 = \frac{F}{\hbar} + \frac{4m^2 \hbar \Omega a}{4m^2 \hbar^2 + T^2 \Delta a^4} \tanh(\Omega t), \tag{86}
\]

\[
I_2 = \frac{2ma \Omega T \Delta a^2}{4m^2 \hbar^2 + T^2 \Delta a^4} \tanh(\Omega t), \tag{87}
\]

\[
I_3 = \frac{4m^2 \hbar^2 [1 + \tanh^2(\Omega t)] + T^2 \Delta a^4}{4m^2 \hbar^2 [1 + \tanh^2(\Omega t)]^2 + T^2 \Delta a^4}, \tag{88}
\]

\[
I_4 = \frac{2m \hbar T \Delta a^2 \tanh^2(\Omega t)}{4m^2 \hbar^2 [1 + \tanh^2(\Omega t)]^2 + T^2 \Delta a^4}. \tag{89}
\]
As mentioned before, one of the ideas in this proposal involves the possibility of detecting a Yukawa term in the range of $\lambda \sim R$. If the probabilities in the non-Newtonian and Newtonian cases are compared, then we find a manner to evaluate the feasibility of the proposal. The use of systems with internal structure complicates the analysis of interference experiments of the COW type [76,77] (here we mean that it requires the introduction of additional approximations, for instance, the so-called rotating wave approximation in the description of the S-matrices associated with the beam splitters [77]). In consequence, we assume the motion of thermal neutrons, i.e. $m \sim 10^{-27}$ kg, and a Compton wavelength of $\lambda \sim 10^{-10}$ m. The experiment lasts $T \sim 10^{-4}$ s, which is the flight time of the thermal neutrons when the traveled distance is $l \sim 10^{-2}$ m.

Additionally, we require an estimation of the resolution, $\Delta a$, of a measuring device which could detect $A(t)$. The current technology cannot detect $A(t)$ [132]. At this point we must add that one of the research areas that could, in the near future, allow the continuous monitoring of the position of a quantum particle is the use of Paul traps [135]. Let us now address the fact that one of the research areas that could, in the near future, allow the continuous monitoring of some fundamental principles in physics via quantum interference with neutrons and photons 1969. In consequence, we assume the motion of thermal neutrons, i.e. $m \sim 10^{-27}$ kg, and a Compton wavelength of $\lambda \sim 10^{-10}$ m. The experiment lasts $T \sim 10^{-4}$ s, which is the flight time of the thermal neutrons when the traveled distance is $l \sim 10^{-2}$ m.

The measurement output, $a(t)$, is assumed to be a constant, i.e. it equals the initial momentum of the thermal neutrons. In other words, we suppose that the involved measurement output fulfills the condition $a(t) = a^* \sim 10^{-24}$ kg m s$^{-1}$.

The current experimental status imposes, already, some restrictions upon the possible values of $\alpha$, as a function of $\lambda$. In our case, if $\lambda \sim 10^6$ m, then $|\alpha| \leq 10^{-2}$ (see p 62 of [118]). Additionally, it is noteworthy to comment that the LAGEOS satellite, and more generally the so-called satellite laser ranging technique, provides very accurate measurements for the case in which $\lambda \approx 1$ AU [137], and also in the range of planetary distances, i.e. $10^5$ m < $\lambda$ < $10^7$ m [138].

Let us now denote the prediction in the case in which the Yukawa term is absent by $P_{[\alpha(t)]=0}^N$, then we have, approximately, that

$$P_{[\alpha(t)]}^\omega/P_{[\alpha(t)]}^N = \exp \left( \frac{a^*^2 \Delta a^2}{2m^2\hbar^2} \sqrt{\frac{Gm^2M}{R}} \right). \quad (90)$$

Hence

$$P_{[\alpha(t)]}^\omega/P_{[\alpha(t)]}^N = 10^{-2}. \quad (91)$$

Considering our rough example, the possibility of detecting this Yukawa term would depend upon the fact that the involved measuring apparatus could detect probabilities with a precision better than one per cent.

Let us explain this point and assume that we have performed this experiment several times, say $s \gg 1$, such that the results read $a_1, a_2, \ldots, a_s$. Define now a partition of the set of results [139], namely, $a_k \in cl\{a_i\}$, if and only if, $P_{[a_k]} = P_{[a_i]}$. Clearly, we may define the cardinality of these sets, $C[cl\{a_i\}]$. If $s$ is sufficiently large, then we have that $C[cl\{a_i\}]/C[cl\{a_i\}]$ will be a good approximation to the ratio of the corresponding probabilities. Our expressions entail that this ratio does depend, in a non-trivial way, upon the parameters appearing in the Yukawa term. In other words, with $m, a_1, \ldots, a_s$ (85) can be evaluated, and then confronted against $P_{[\alpha(t)]}^\omega/P_{[\alpha(t)]}^\omega$, and afterwards, compared with $C[cl\{a_i\}]/C[cl\{a_i\}]$.

The appearance of the mass term in the COW experiment leads us to ask about the role that this parameter plays in this new scheme. A fleeting glance at (85) shows us that two particles with different mass, say $m$ and $\bar{m}$ have different probabilities. In other words, the difference in mass does impinge on a physically detectable function. Nevertheless, now the dependence
upon $m$ is not restricted to functions of $m/\hbar$, i.e. the inclusion of a measuring process renders a complication in this function. This means that the way in which the mass parameter emerges in the COW experiment does not have a general character in the quantum domain.

Concerning (85) there is a second point that draws our attention. Indeed, notice that the probabilities associated with the different measurement outputs of $a(t)$ do depend upon the precision of the measuring device. This feature is a characteristic of RPIF. This model is equivalent to other formulations of the quantum measurement process [129], and therefore this feature is a property of several approaches in this context. In other words, those models which deny the objectivity of this problem cannot predict the dependence upon the precision of the experimental device embodied in (85), i.e. we have a theoretical prediction that allows us to test the validity of RPIF, or of any of its equivalent formulations. As mentioned before, this comprises our second goal in this part of the work.

3. Photon interference

3.1. Deformed dispersion relations and coherence properties of light

Optical interferometry has played a fundamental role in some experimental aspects of gravitational physics [140]. For instance, we may mention that there are gravity-waves detectors which are built following the Michelson interferometer [141], or that the Sagnac ring interferometer [20] constitutes the bedrock for the ring laser gyroscopic device, which could be used to test the different metric theories of gravity in the weak-field and slow motion limit [58].

The use of light as a metric theory probe lies mainly in the possibility of generating interference patterns, which have to be related to the properties of the corresponding model. This simple comment leads us to state, briefly, the conditions that light has to satisfy in order to show interference. In this short trip we will encounter the concepts of coherence, order of coherence, etc. Almost at the beginning of the present work the relation between interference and the corresponding motion equations was mentioned. Since the motion of light is governed by a set of equations which allow the superposition principle, we may ask, when does light present interference? Everyday experience suggests that interference must be subject to very special conditions. These conditions are closely related to the concept of coherence [6, 21]. Let us now address briefly the concept of coherence with an example. Consider two waves whose electric fields read $\vec{E}_1$ and $\vec{E}_2$. They are said to be coherent if the following interference term does not vanish in the region occupied by these electric fields, i.e.

$$\langle \vec{E}_1 \cdot \vec{E}_2 \rangle = \sqrt{I_1 I_2} \cos(\delta).$$

In this last expression $\langle \vec{E}_i^2 \rangle = I_i$, $i = 1, 2$, $\langle \rangle$ denotes time average, and $\delta$ the phase difference between the two waves. We may rephrase this stating that two waves are coherent if the corresponding fields possess a constant phase. This entails that coherence is associated with the comparison of the relative phase of two light beams. Of course, this comparison can be done resorting to the dependence of the waves in terms of temporal or spatial parameters. The mathematical description of a comparison between two quantities is done by the so-called correlation operation [6]. The temporal correlation of an electromagnetic wave with itself can be determined by a Michelson–Morley interferometer, whereas the spatial correlation is detected by Young’s two-slit experiment [20].

Young’s experiment has been mentioned as an example of first-order coherence. Let us delve deeper into this concept. Consider monochromatic light (whose source is depicted in figure 4 by S). This beam is split using an opaque screen with two pinholes. A detecting
Figure 4. First-order coherence experiment.

The intensity can be cast in terms of the first-order correlation function \( G^{(1)} \) in the following form \[20\]
\[
\langle I(\vec{r}, t) \rangle = |A|^2 G^{(1)}(\vec{r}_1, \vec{r}_1; t - t_1, t - t_1) + |B|^2 G^{(1)}(\vec{r}_2, \vec{r}_2; t - t_2, t - t_2) + 2 \text{Re}[ABG^{(1)}(\vec{r}_1, \vec{r}_2; t - t_1, t - t_2)].
\]

The first two terms are associated with the average intensities at the photo-detector stemming from each one of the pinholes, and the last one is responsible for the interference. In this last expression \( A \) and \( B \) are complex factors depending upon the particular geometry of the pinholes, whose coordinate vectors are \( \vec{r}_1 \) and \( \vec{r}_2 \), respectively, whereas \( t_1 \) and \( t_2 \) are the traveling times from these pinholes to the photo-detector.

The essence of the Hanbury–Brown–Twiss effect involves the measurement of intensity from two photo-detectors spaced a distance \( l \) apart. The main assumption is that the fluctuations in the outputs of the photo-detectors must be correlated if the amplitudes of the two waves are correlated. The difference between an intensity interferometer and a Michelson–Morley device lies in the fact that the former measures the square of the modulus of the complex degree of coherence, whereas the latter also detects the phase \[6\].

At this point we may wonder, why should the Hanbury–Brown–Twiss effect be considered in the realm of gravitational physics? The answer comes from the present status of this area. Indeed, the quantization of the gravitational field defines a long-standing puzzle in modern physics. Unavoidably, some of the current attempts in this direction are accompanied by the breakdown of Lorentz symmetry \[29, 13\]. At this point it is noteworthy to mention that up to now there is no experimental evidence purporting a possible violation of this symmetry. Obviously, we have to be more careful with our language because the phrase violation of Lorentz symmetry could mean many things, since this feature embodies several characteristics. For instance, local Lorentz invariance could be violated, i.e., the results of a local non-gravitational experiment would not be independent of the velocity of the involved frame \[41\]. Additionally, local position invariance \[41\] could break down. In other words, there would be a local non-gravitational experiment in which the corresponding results do depend upon the spacetime location of the frame. A violation of local Lorentz invariance does not entail, inexorably, also a violation of local position invariance, and vice versa, of course.

In the present manuscript the meaning of the phrase Lorentz violation will take a very precise expression; it will embody a modification to the so-called dispersion relation,

\[
E^2 = p^2c^2\left[1 - \alpha\left(E\sqrt{G/(c^2\hbar)}\right)^\mu\right].
\]
In this last expression \( \alpha \) is a coefficient, usually of order 1, whose precise value depends upon the considered quantum-gravity model, and \( n \), the lowest power in Planck’s length leading to a non-vanishing contribution, is also model-dependent. This kind of symmetry breakdown is related to non-critical string theory, non-commutative geometry and canonical gravity [28]. The relevance of these kinds of violations can be easily understood remembering that they involve a region in which one of the fundamental symmetries of modern physics becomes only an approximation. In consequence those experimental proposals that could test this violation acquire relevance. As will be shown below, a consequence of (94) is connected to a modification of the speed of light, i.e. in this scheme it is not constant [28].

\[
v = c \left[ 1 - \alpha \left( E \sqrt{G/(c^5 \hbar)} \right)^n \right]^{3/2} \left[ 1 + \alpha (n/2 - 1) \left( E \sqrt{G/(c^5 \hbar)} \right)^n \right]^{-1}.
\] (95)

This last remark opens up the possibility of searching for these kinds of effects in the context of photon interferometry [28].

3.2. First-order coherence experiments and non-Newtonian gravity

In this part of the work we analyze the possibility of detecting a Yukawa contribution to the gravitational field through the interference process associated with the modifications upon the first-order coherence properties in the process of light emission of a particular quantum system [142]. In order to do this consider two identical atoms (located at \( P \) and \( P' \)), such that each one of them has two levels, and a single photon, where only one of these two atoms can be excited. The initial state of our system reads

\[
a |0, 1\rangle + b |0, 0\rangle |\phi\rangle.
\] (96)

In this last expression some parameters require an explanation. Here \(|0\rangle, |1\rangle, |0'\rangle, |1'\rangle\) denote the ground and excited states of the two atoms, while \(|0\rangle\) is the vacuum of the electromagnetic field, and finally, \(|\phi\rangle\) designates the photon. The complex numbers \(a\) and \(b\) are normalization constants. It is already known that after a time larger than the mean decay time, \(t_m\), the system decays to

\[
|\alpha\rangle = \frac{1}{\sqrt{2}} |0, 0'\rangle \left[ |\gamma\rangle + |\gamma'\rangle \right].
\] (97)

In this last expression \(|\gamma\rangle\) and \(|\gamma'\rangle\) denote the photon states emitted from sites \( P \) and \( P' \), respectively.

At this point we introduce a Yukawa-type term [110]

\[
V(r) = - \frac{G_{\infty} M M}{r} \left[ 1 + \alpha \exp \left( -r/\lambda \right) \right].
\] (98)

Once again, we have introduced the following parameters: \(G_{\infty}\) describes the interaction between \( M \) and \( m \) in the limit case \( r \to \infty \), i.e. \( G_N = G_{\infty}(1 + \alpha)\), where \( G_N \) is the Newtonian constant.

This implies that the gravitational potential generated by \( M \) reads

\[
U(r) = - \frac{G_{\infty} M}{r} \left[ 1 + \alpha \exp \left( -r/\lambda \right) \right].
\] (99)

This part of the paper analyzes the topic of quantum interference experiments in the context of tests of fundamental physics, and at this point we connect with our main goal. Our interference experiment will detect at point \( S \) the light that results from the decay of the system. There is a red-shift in the frequency which appears as a consequence of the fact that the electromagnetic field \( \text{climbs} \) in a region where a non-vanishing gravitational field is
present [58]. We may now wonder what kind of interference experiment is being considered, i.e. does it measure temporal or spatial coherence properties of light? The answer is spatial coherence, it is a Young’s experiment and this can be seen resorting to the two-slit analogy in which the present proposal can be reformulated [142].

At the emission point the frequency is $\nu$. The difference in gravitational potential between the emission point and detection point is $\Delta U$, and, in consequence, the frequency at the detection point is [58]

$$\tilde{\nu} = \frac{\nu}{1 + \Delta U/c^2}. \quad (100)$$

The electromagnetic field operator can be separated into two parts, namely, with positive and negative frequency parts [20]. Nevertheless, in the case of an experiment which employs absorptive detectors the measurements are destructive, and, in consequence, only that part of the field operator containing annihilation operators, $E^{(+)}(r, t)$, has to be considered. In order to simplify the model we will assume that the field is linearly polarized, and that the radiation emitted from $P$ (or $P'$) is monochromatic. This approach to our situation embodies, tacitly, the fact that we have a quantum interference process, i.e. the electromagnetic field is described by operators.

The possibility of performing these kinds of experiments near the Earth’s surface will be introduced at this point, i.e. we have the condition $R \gg |z|$ ($R$ is the Earth’s radius, whereas $z$ is the height with respect to the Earth’s surface).

$$r = R + z. \quad (101)$$

The field operator containing the annihilation operator reads, approximately [143]

$$E^{(+)}(r, t) = \Xi \hat{a} \exp \left\{ -i \nu \left[ - \frac{g_0}{c^2} \frac{1 + \alpha e^{-R/\lambda}}{1 + \alpha} \left( t - \hat{k} \cdot r \right) \right] \right\}. \quad (102)$$

The parameter $\hat{g}$ is related to the climbed distance, $\hat{k}$ denotes the unitary vector in the direction of propagation, $\Xi$ is a constant with dimensions of electric field, $\hat{a}$ is the corresponding annihilation operator and $g_0 = g_\infty (1 + \alpha)$ is the effective acceleration of gravity at laboratory distances.

The first-order correlation function is given by [20] (remembering that $h$ and $h'$ are the climbed distances coming from $P$ and $P'$)

$$G^{(1)}(r, r; t, t) = |\Xi|^2 \left[ 1 + \cos \left( \left[ \hat{g} h \hat{k} - h' \hat{k}' \right] \cdot r + \hat{g} \Delta h \right) \right]. \quad (103)$$

In (103) we have introduced the following definition:

$$\hat{g} = \frac{g_0}{c^2} \frac{1 + \alpha e^{-R/\lambda}}{1 + \alpha}. \quad (104)$$

Clearly, the condition $g_0 = 0$ allows us to recover the usual Young’s interference pattern [6], i.e. the pattern without gravitational field. Additionally, $\Delta h = h' - h$. The time dependence of the interference pattern disappears if $\Delta h = 0$, see (103), and under this restriction the first-order correlation function becomes

$$G^{(1)}(r, r; t, t) = |\Xi|^2 \left[ 1 + \cos \left( A \left[ 1 + \hat{g} (h + h') \right] \right) \right]. \quad (105)$$

The behavior of the interference pattern in the absence of gravitation is encoded in $A$, which depends upon the geometry of the interferometer and also on the wavelength of the emitted radiation [20].

The detection of the effects of a fifth force inside the so-called ‘geophysical window’ [107] imposes restrictions upon the parameters involved. Here we may consider the following values $\alpha \in [10^{-3}, 10^{-2}]$ and $\lambda = 10$ m [144].
The expression for the correlation function, together with $g_0 = G_0 M / R^2$, allows us to obtain a condition on $R$ as a function of $h$ and $h'$. For instance, if $\tilde{g}[h + h'] \sim 10^{-8}$, then

$$(h + h') / R^2 \sim 10^{-4} \text{ m}.$$  \hfill (106)

From the correlation function [20] we may see that there are certain time values, $t_n (n \in \mathbb{N})$, such that

$$\tilde{g} v t_n = 2 \pi n.$$  \hfill (107)

Hence the interval between $t_{n+1}$ and $t_n$ is

$$\Delta t_n \equiv t_{n+1} - t_n = \frac{2 \pi}{\tilde{g} v \Delta h}.$$  \hfill (108)

The purely Newtonian case implies

$$\Delta t_n^{(N)} = \frac{2 \pi c^2}{g_0 v \Delta h}.$$  \hfill (109)

The detection of a non-Newtonian contribution implies the comparison between the usual case and the new one. This last remark implies that the non-Newtonian times are provided by

$$\Delta t_n^{(NN)} = \frac{2 \pi c^2}{g_0 v \Delta h 1 + \alpha e^{-R/\lambda}}.$$  \hfill (110)

The ratio between these last two parameters entails

$$\frac{\Delta t_n^{(NN)}}{\Delta t_n^{(N)}} = \frac{1 + \frac{1}{1 + \alpha e^{-R/\lambda}}}.$$  \hfill (111)

In an approximate way we have

$$\Delta t_n^{(NN)} / \Delta t_n^{(N)} \sim 1 + 10^{-3}.$$  \hfill (112)

The question about the measurability of this term depends not only on the order of magnitude of (112), but also on the resolution of the detecting device. Clearly, the possibility of detecting a fifth force depends on the condition that the difference between the non-Newtonian and Newtonian cases has to be larger than the resolution of the experimental apparatus, i.e. $|\Delta t_n^{(NN)} - \Delta t_n^{(N)}| > \Delta T$, where $\Delta T$ denotes the time resolution of the measuring device.

Now that we know the condition that the corresponding measuring device has to satisfy, in order to provide information about the existence of a Yukawa term, let us now address the conditions that the use of the optical spectrum entails in this proposal. In other words, consider, for the emitted field, $\lambda^{(r)} \sim 400 \text{ nm}$. Finally, we must have an estimation of the order of magnitude of $\Delta t_n^{(NN)}$. Nevertheless, this variable contains, implicitly, a condition on $\lambda$ and $\alpha$, a set of data that we do not know. To solve this impasse we will contemplate this point from a different perspective, namely, from the experimental point of view. This phrase means that we will ascribe to $\Delta t_n^{(NN)}$ an order of magnitude not far from the current resolutions. The possibility of detecting time intervals similar to 50 fs, based on the interference of two-photon probability amplitudes [145], means that time differences $0.1 \sim \mu \text{s}$ could be within the technological margin. Hence $\Delta t_n^{(NN)} \sim 0.01 \mu \text{s}$ leads us to a constraint upon $\Delta h$, as a function of $R$

$$\Delta h / R^2 \sim 10^{-8} \text{ m}^{-1}.$$  \hfill (113)

If the experiment were performed near the Earth’s surface ($R \sim 10^6 \text{ m}$), then $\Delta h \sim 10^4 \text{ m}$. The difference in climbing distance has the order of magnitude of tens of kilometres, a result that implies that this proposal is not very feasible. Let us now recall the case of an experiment performed some years ago, in which a test of relativistic gravitation was carried out comparing
the frequencies emitted from hydrogen masers located in a spacecraft and also at an Earth station [99, 100]. The connection with expression (113) stems from the fact that in the experiment mentioned in [99, 100] one of the experimental devices was located at an altitude of 10,000 km. If we resort to a value of $R = 16,000\text{ km}$, then (113) renders a decrease in $\Delta h$ of one order of magnitude, $\Delta h \sim 10^3 \text{ m}$, a decrease not large enough to provide a feasible situation.

3.3. First-order coherence experiments and deformed dispersion relations

Several quantum-gravity theories are endowed with a modified dispersion relation [29, 13], which can be characterized, from a phenomenological point of view, by corrections hinging upon Planck’s length, i.e. $l_p$

$$E^2 = p^2 c^2 \left[1 - \alpha \left( E \sqrt{G/(c^5 \hbar)} \right)^n \right].$$  \hspace{1cm} (114)

From these comments it becomes evident that these theories are not finished. For instance, in loop quantum gravity $\alpha$ and $n$ depend upon the way in which the semiclassical states are calculated [146]. This last argument stresses the importance of any test that could lead to the detection of these kinds of parameters, or at least to set bounds upon them.

The relation between momentum and wave number, $p = \hbar k$, leads us to conclude that

$$k = \frac{E}{c \hbar} \left[1 - \alpha \left( E \sqrt{G/(c^5 \hbar)} \right)^n \right]^{1/2}. \hspace{1cm} (115)$$

Since, experimentally, these modifications are quite small (otherwise they would have already been detected) the following expansion is justified

$$k = \frac{E}{c \hbar} \left[1 + \frac{\alpha}{2} \left( E \sqrt{G/(c^5 \hbar)} \right)^n + \frac{3}{8} \alpha^2 \left( E \sqrt{G/(c^5 \hbar)} \right)^{2n} + \ldots \right]. \hspace{1cm} (116)$$

A deformed dispersion relation has important physical implications, among which we may find a non-trivial dependence of the speed of light upon the energy [13]. In our case, the speed of light has already been given; see (95).

In this part we resort to a Sagnac interferometer [6], which is a first-order coherence experiment, and has already been used in the context of tests of the gravitomagnetic effect [147]. The radius and angular velocity of this rotating interferometer are $b$ and $\Omega$, respectively. Since the idea is to analyze the possibility of carrying out these kinds of experiments near the Earth’s surface, then we must consider the presence of a gravitational potential

$$U(z) = gz. \hspace{1cm} (117)$$

Let us now define our experimental proposal. At the highest point of the interferometer a light beam, with frequency $\tilde{\nu}$, enters the device, and it is split up into two parts, one rotating in the same direction as the interferometer, i.e. clockwise, whereas the second beam travels in the opposite direction. The choice for the zero of this potential will be at the lowest point of the interferometer. The plane where this apparatus is located has its normal vector perpendicular to the $z$-axis. The presence of a gravitational potential implies that each beam suffers a shift in the frequency [58]

$$\tilde{\nu} = \frac{\tilde{\nu}}{1 + \Delta U/v^2}. \hspace{1cm} (118)$$

We have emphasized the fact that this kind of breakdown of Lorentz symmetry entails that the speed of light is energy-dependent, i.e. frequency dependent. This last point will be the core
factor in this idea. The speed of each one of the beams will be changing as they move along the interferometer, and, therefore, an interference pattern must emerge, which shall contain information about the parameters of the deformed dispersion relation [148].

We may approximate the speed of the beams as follows:

$$v^3 + c\left[\alpha \left(\frac{n}{2} + 1\right)\left(\hbar \nu / E_p\right)^n - 1\right]v^2 + n\Delta U v - cn\Delta U = 0. \tag{119}$$

A fleeting glance at (119) shows that if we set $\alpha = 0$, then $v = c$ is, indeed, a solution. In other words, we recover the usual situation.

Harking back to our more general case, we have that a solution is

$$v = c\left[1 - n\Delta U / c^2 - \frac{\alpha}{3} \left(\frac{n}{2} + 1\right)\left(\hbar \nu / E_p\right)^n\right]. \tag{120}$$

Define now the angle $\theta$, which measures the rotation of the beams. Since we divided the original beam, there will be two parameters, $\theta_1$ and $\theta_2$. Here $\theta_i = 0$, with $i = 1, 2$, coincides with the point at which the beam enters into the interferometer. If $\beta$ is the angle described by the interferometer, then any point at the edge of the interferometer has the following z-coordinate

$$z = b\left[1 + \cos(\beta)\right]. \tag{121}$$

The beam enters the interferometer at the point

$$z_0 = 2b, \tag{122}$$

then

$$\Delta U = gb\left[\cos(\beta) - 1\right]. \tag{123}$$

Additionally, we have

$$v = \frac{b}{c} \frac{d \theta}{d \tau}. \tag{124}$$

The traveled time required by the beams, as a function of the angle is deduced joining expressions (120) and (124)

$$t - t_0 = \frac{b}{c} \int_0^\theta \left[1 - \frac{\alpha}{3} \left(\frac{n}{2} + 1\right)\left(\hbar \nu / E_p\right)^n - \frac{2gb}{c^2} \left[\cos(\beta) - 1\right]\right]^{-1} d \tilde{\theta}. \tag{125}$$

Therefore,

$$\frac{ct}{2b} \left[1 - \frac{\alpha}{3} \left(\frac{n}{2} + 1\right)\left(\hbar \nu / E_p\right)^n + \frac{2gb}{c^2}\nu^2\right] - \left(\frac{2gb}{c^2}\right)^{1/2} = \theta / 2. \tag{126}$$

The beam moving in the opposite direction, with respect to the displacement of the interferometer, meets the detection device after it has moved an angle equal to

$$\beta_1 = t_1 \Omega. \tag{127}$$

The angle described by the beam then reads

$$\theta_1 = 2\pi - \beta_1. \tag{128}$$

As for the remaining beam, it meets the interferometer after the measuring device has rotated an angle

$$\beta_2 = t_2 \Omega. \tag{129}$$

The angle described by the second beam is given by

$$\theta_2 = 2\pi + \beta_2. \tag{130}$$
Introducing these two times, \( t_1 \) and \( t_2 \), into (126) renders

\[
\begin{align*}
t_1 &= \frac{2\pi b}{c} \left[ \gamma + \frac{b\Omega}{c} \right]^{-1}, \\
t_2 &= \frac{2\pi b}{c} \left[ \gamma - \frac{b\Omega}{c} \right]^{-1},
\end{align*}
\]

(131)

\[
\gamma = \left\{ \left[ 1 - \frac{a}{3} \left( \frac{n}{2} + 1 \right) \left( \hbar \tilde{\nu} / E_p \right)^n + \frac{2gb}{c^2} \right] - \left( \frac{2gb}{c^2} \right)^2 \right\}^{1/2}.
\]

(132)

(133)

These two last expressions entail (\( \Delta t = t_2 - t_1 \))

\[
\Delta t = \frac{4\pi \hbar^2 \Omega}{c^2} \left[ \left[ 1 - \frac{a}{3} \left( \frac{n}{2} + 1 \right) \left( \hbar \tilde{\nu} / E_p \right)^n + \frac{2gb}{c^2} \right] - \left( \frac{2gb}{c^2} \right)^2 - \frac{b^2\Omega^2}{c^2} \right]^{-1}.
\]

(134)

Setting \( a = 0 \) we recover the usual value, \( \Delta \theta^{(0)} \), for the phase shift [6, 20]. Note that the presence of a violation of Lorentz symmetry in the form of a deformed dispersion relation does impinge upon the difference in time of flight, and in consequence on the interference pattern.

Since the feasibility of the proposal shall also be addressed, we will consider the case \( n = 1 \). This means that the time difference becomes

\[
\Delta t = \frac{4\pi \hbar^2 \Omega}{c^2} \left[ 1 + \alpha \left( \hbar \tilde{\nu} / E_p \right) - \frac{4gb}{c^2} + \frac{4gb}{c^2} \left( \hbar \tilde{\nu} / E_p \right)^2 - \frac{4gb}{c^2} \left( \hbar \tilde{\nu} / E_p \right)^2 + \frac{b^2\Omega^2}{c^2} \right].
\]

(135)

In the analysis of the interference pattern we require the optical path and phase differences associated with (135). In this case they read, respectively,

\[
\Delta L = \frac{\Delta t}{c},
\]

(136)

\[
\Delta \theta = \frac{\Delta L}{\lambda}.
\]

(137)

In consequence

\[
\Delta \theta = \frac{4\pi \hbar^2 \Omega}{c \lambda} \left[ 1 + \alpha \left( \hbar \tilde{\nu} / E_p \right) - \frac{4gb}{c^2} + \frac{4gb}{c^2} \left( \hbar \tilde{\nu} / E_p \right)^2 - \frac{4gb}{c^2} \left( \hbar \tilde{\nu} / E_p \right)^2 + \frac{b^2\Omega^2}{c^2} \right].
\]

(138)

The feasibility of the model hinges upon the difference \( \Delta \theta - \Delta \theta^{(0)} \). Clearly, if \( \delta \theta \) denotes the corresponding experimental resolution, then the requirement to be fulfilled reads

\[
\delta \theta < | \Delta \theta - \Delta \theta^{(0)} |.
\]

(139)

This may be cast in the following form

\[
\delta \theta < \frac{4\pi \hbar^2 \Omega}{c \lambda} \alpha \left( \hbar \tilde{\nu} / E_p \right) \left[ 1 + \frac{2gb}{c^2} \right].
\]

(140)

For any terrestrial proposal of this experiment we have that \( g \) is approximately 9.8 m s\(^{-2}\). We need two experimental parameters in order to analyze the feasibility of the proposal. One of them is connected with the experimental resolution of the device, which will be taken as \( \delta \theta \sim 10^{-6} \) [6]. The remaining one is associated with the value of \( \hbar \tilde{\nu} \). For this parameter we will choose an optical transition, i.e. \( \hbar \tilde{\nu} \sim 10^{-3} \) J. At our disposal we have two variables, namely, the radius of the interferometer, \( b \), and its angular velocity, \( \Omega \). Consider the case in which \( b = 10^{-1} \) m, a value within the current technology. Then

\[
\Omega > 10^{25} \text{ s}^{-1}.
\]

(141)

Obviously, this condition lies outside the current technology. Demanding a more realistic value for the angular velocity entails values for the radius that are impossible to achieve in
an experiment. Finally, let us discuss another possibility. For instance, consider a smoother experimental condition
\[ b^2 \Omega \sim 10 \text{ m}^2 \text{s}^{-1}. \]  

(142)

Then we obtain a condition on the energy of the beam being used in our experiment which does not lie anymore in the realm of optical transitions
\[ \hbar \tilde{\nu} > 10^9 \text{ J}. \]  

(143)

These last two examples show us that Sagnac’s device has an additional parameter, which is absent in the case of a Michelson–Morley experiment. In Sagnac’s idea we have three experimental parameters at our disposal, \( b, \Omega, \) and \( \hbar \tilde{\nu} \), whereas in a Michelson–Morley experiment, we have only the frequency (or energy) of the beam and the difference in optical length. In this sense we may say that Sagnac is richer, though as this simple model shows the existence of this additional variable does not imply that the experiment falls within current technological possibilities.

3.4. Second-order coherence experiments and non-Newtonian gravity

Up to now our experimental proposals in the optic realm can be embodied in the context of first-order coherence [6, 21, 20]. It has been shown, in connection with Sagnac interferometry, which is a first-order coherence experiment, that the detection of a deformed dispersion relation seems to require either very large angular velocities and radii or very large energies for the light beam. This represents a technical problem, and we may wonder if there is another manner in which this kind of effect could be detected, without leaving the domain of optics. The answer stems from the possibility of resorting to higher-order coherence experiments, for instance, the so-called Hanbury–Brown–Twiss effect (HBTE), which falls within the group of second-order coherence experiments [149].

As mentioned before, HBTE is a technique which involves intensity correlation between signals collected at two points in space. Note that in this last phrase, we tacitly introduce a new experimental parameter which is absent in the first-order coherence models. Indeed, HBTE needs an additional distance variable related to the fact that in this idea, in contrast to the situation of any first-order coherence device, two collecting points are required. The intention in this part of the work is to use this extra parameter and see if it could lead us to an experiment closer to the current technology.

The experiment is defined as follows [142]. We have two atoms, located at points \( P \) and \( P' \), but now there are two detection points, \( S_1 \) and \( S_2 \). Initially the atoms are excited, but there is no electromagnetic field, hence the initial state vector reads
\[ |\alpha(t = 0)\rangle = |1, 1\rangle |\tilde{0}\rangle. \]  

(144)

The system decays, after a sufficiently larger interval, \( t_m \).
\[ |\alpha(t >> t_m)\rangle = |0, 0\rangle |\gamma, \gamma\rangle. \]  

(145)

Here \( |0, 0\rangle, |1, 1\rangle \) denote the ground and excited states of the two atoms, while \( |\tilde{0}\rangle \) is the vacuum of the electromagnetic field, and, finally, \( |\gamma\rangle \) and \( |\gamma\rangle \) are the photon states emitted from sites \( P \) and \( P' \), respectively.

Assuming a plane wave approximation for the emitted radiation, the definition of second-order correlation function [149, 150] tells us that the interference pattern is provided by the following term
\[
\cos \left\{ \left[ (k - k') \cdot (r_2 - r_1) + \tilde{g} [h_2 k' - h_2 k] \cdot r_2 - \tilde{g} [h_1 k' - h_1 k] \cdot r_1 + \tilde{g} v t \left[ \Delta h - \Delta h' \right] \right] \right\}.
\]  

(146)
In this last expression the following parameters have been introduced; \( h_1 \) and \( h_2 \) are the climbed distances for the radiation emitted by \( P \) and detected at \( S_1 \) and \( S_2 \), respectively (we have the same argument if the light is emitted in \( P' \)). Additionally, \( \Delta h = h_2 - h_1 \), \( \Delta h' = h_2' - h_1' \).

With these results we may now compare HBTE against the first-order coherence situation. Indeed, a fleeting glance at (103) and (146) allows us to understand that the order of magnitude of some of the experimental parameters, for instance, \( R \) or \( t \), will be the same as in the analysis of a Young’s experiment. In this sense the extra distance parameter related to HBTE offers no improvement, i.e. the technical problems are not significantly smoothed.

The time independent terms appearing in (146) share the structure of the corresponding terms of (103). This last statement means that if in the HBTE case we impose the following condition \( \hat{g}[\hat{h}_1 \hat{k} - \hat{h}_2 \hat{k}] \cdot \hat{r}_2 \sim 10^{-8} \), then

\[
|\Delta h'|/R^2 \sim 10^{-4} \text{ m}^{-1}.
\]  

(147)

In (147) \(|\Delta h'| \) appears, and not \( h + h' \), as in the case of the first-order coherence case; see expression (105).

\[
\Delta t_n = \frac{2\pi}{\hat{g}v|\Delta h - \Delta h'|}.
\]  

(148)

In other words, if we go from Young’s experiment to HBTE, then \( \Delta h \) is replaced by \( |\Delta h - \Delta h'| \). The additional distance factor that HBTE involves emerges in \( |\Delta h - \Delta h'| \), which is a parameter that requires four distance parameters, in lieu of the two that appear in \( h + h' \).

### 3.5. Second-order coherence and deformed dispersion relations

Now we consider a deformed dispersion relation and analyze its effects in a second order coherence device. The conclusions elicited from the proposal related to the detection of non-Newtonian gravity with a second-order coherence device do imply that there is no significant improvement in the feasibility of the corresponding experiment, compared with the case of a first-order coherence device. Therefore, we may wonder if this scheme could provide an improvement for the case of a deformed dispersion relation. Fortunately, here, as will be shown below, the situation is more optimistic, and the extra parameter associated with HBTE pays off [26]. As before, we take a deformed dispersion relation; see (94).

Since we expect very tiny corrections, expression (116) may be used as a good approximation to the wave number.

The idea now is to consider two photons propagating along the axis defined by the unit vector \( \hat{e} \), though with different energies.

\[
k = \hat{k} \hat{e},
\]  

(149)

\[
k' = k' \hat{e}.
\]  

(150)

Let us now consider the detection of these photons resorting to HBTE [149,150]. In other words, we have two photo-detectors located at points \( A_1 \) and \( A_2 \), with position vectors, \( \vec{r}_1 \) and \( \vec{r}_2 \), respectively.

The second-order correlation function reads [20]

\[
G^{(2)}(\vec{r}_1, \vec{r}_2; t, t) = \mathcal{E} \left( 1 + \cos \left( (\vec{k} - \vec{k'}) \cdot (\vec{r}_2 - \vec{r}_1) \right) \right).
\]  

(151)

\( \mathcal{E} \) denotes a constant factor with dimensions of the electric field. If \( \Delta \theta^{(n)} \) is the phase difference for \( n \), then the interference pattern (to second-order in \( \Delta E \)) is

\[
\Delta \theta^{(n)} = \frac{l \Delta E}{\hbar c} \left[ \left( 1 + \frac{n + 1}{2} \alpha [E \sqrt{G/(c^5 \hbar)]^n \right) + \frac{3}{8} \alpha^2 (2n + 1)(E \sqrt{G/(c^5 \hbar)]^{2n}} \right) + \frac{\Delta E}{E} \left( \frac{n(n + 1)}{4} \alpha^2 [E \sqrt{G/(c^5 \hbar)]^{2n}} \right) \right].
\]  

(152)
Here $E = E' + \Delta E$, and in addition, $l = \hat{e} \cdot (\vec{r}_2 - \vec{r}_1)$. The feasibility of the detection of this kind of correction depends upon the value of $n$, at least in the context of a first-order correlation function. In consequence, we will divide our situation in the same manner, namely, to first order in $\Delta E$ we have (approximately) that

$$
\Delta \theta^{(1)} = \frac{l \Delta E}{c \hbar} \left[ 1 + \alpha [E \sqrt{G/(c^5 \hbar)}] \times \left( 1 + \frac{9}{8} \alpha [E \sqrt{G/(c^5 \hbar})] \right) \right],
$$

(153)

$$
\Delta \theta^{(2)} = \frac{l \Delta E}{c \hbar} \left[ 1 + \frac{3}{2} \alpha [E \sqrt{G/(c^5 \hbar)}]^2 \times \left( 1 + \frac{5}{4} \alpha [E \sqrt{G/(c^5 \hbar})]^2 \right) \right].
$$

(154)

The corrections could be detectable if $|\Delta \theta^{(n)} - \Delta \theta^{(LS)}| > \Delta \theta^{(exp)}$. In this last expression $\Delta \theta^{(LS)}$ and $\Delta \theta^{(exp)}$ denote the phase difference in the case in which $\alpha = 0$, and the experimental resolution, respectively. Additionally, $\Delta E = (E/\gamma)$, with $\gamma > 1$.

The last expressions show that the phase difference is a function of the value of $n$. This, in addition, entails that if we impose a condition upon these phase differences, namely, $\Delta \theta^{(n)} = \Delta \theta^{(LS)}$, then $l$ becomes a function of $n$, see expression (152). This dependence will be denoted by $l^{(n)}$, and now we proceed to find this dependence for some particular cases.

At this point we must mention that this parameter is a direct consequence of the use of HBTE. Indeed, it is a function of the distance between the two photo-detectors. To first order in $E \sqrt{G/(c^5 \hbar)}$

$$
l^{(1)} \geq \frac{c \hbar \gamma}{a E^2} \sqrt{c^5 \hbar/G} \Delta \theta^{(exp)},
$$

(155)

$$
l^{(2)} \geq \frac{2 c^6 \hbar^2 \gamma}{3 \alpha G E^3} \Delta \theta^{(exp)}.
$$

(156)

If we assume the following values for our parameters, $\Delta \theta^{(exp)} \sim 10^{-4}$ [17], $\alpha \sim 1$, $\gamma \sim 10^2$ and $E \sim 10^{-6}$ J [28, 29], then

$$
l^{(1)} \geq 10^{-5} \text{ m},
$$

(157)

$$
l^{(2)} \geq 10^5 \text{ m}.
$$

(158)

The energy that has been considered has the order of magnitude of the highest energy that nowadays can be produced in a laboratory. These two last expressions mean that if the corrections to the dispersion relation entail $n = 1$, then a HBTE type-like experiment with a distance between the photo-detectors greater than $10^{-5}$ m could detect the extra term. In the remaining case, $n = 2$, the required distance implies the impossibility of detecting the correction, with the assumption introduced for the involved energy.

In the context of first-order correlation functions [148] the possibility of detecting the case $n = 2$ is, currently, a totally impossible task. Nevertheless, our approach introduces an additional parameter, and therefore, if we consider the case of an energy of $E \sim 10$ J (which is tantamount to the energy that could be involved in the observation of gamma-ray bursts), then

$$
l^{(2)} \geq 10^5 \text{ m}.
$$

(159)

In the present model the presence of our extra parameter ($l$) allows us to get closer to its possible detection. Let us now analyze the feasibility in the context of $n = 2$, which is a tougher situation, experimentally, to handle than $n = 1$. The experimental parameter that should be measured is the normalized correlation coefficient of the fluctuations in the photoelectric current outputs [6], $C(l)$. The connection with the difference in time of arrival stems from HBTE, namely, the squared modulus of the degree of coherence function, $\gamma$, is proportional to the normalized correlation function of the photocurrent fluctuations

$$
C(l) = \delta |\gamma(l)|^2.
$$

(160)
The parameter $\delta$ is the average number of photoelectric counts due to light of one polarization registered by the detector in the corresponding correlation time. Experimentally, for thermal sources of temperature below $10^5$ K, $\delta$ is always smaller than 1 [6]. In order to enhance the effect, i.e. to have a larger value of $\delta$, we may consider the fact that the number of average photons, as a function of the involved frequency, $\nu$, reads [6]

$$\delta = 2\xi(3) \frac{\nu^3}{c^2\pi^2}.$$  \hspace{1cm} (161)

Here $\xi$ is the so-called Riemann zeta function. Clearly, higher energy implies a larger mean number of photons. Hence, for an energy of $E \sim 10^1$ J we expect a value of $\delta$ not as small as in the case of $10^5$ K. In other words, the higher the energy of the light beam, the larger the constant between $C(l)$ and $\gamma$ becomes. Of course, this last fact cannot be considered a shortcoming of the proposal.

In terms of the photocurrents fluctuations at the two photo-detectors

$$C(l) = \frac{\langle \Delta I_1(t) \Delta I_2(t) \rangle}{\sqrt{\langle [\Delta I_1(t)]^2 \rangle \langle [\Delta I_2(t)]^2 \rangle}}.$$ \hspace{1cm} (162)

The feasibility of detecting a deformed dispersion relation in this context depends upon the aforementioned fluctuations. We may find already in the literature some models that explain the pulse width of a gamma-ray burst (GRB) in terms of the involved energy [151], as a power law expression, at least in the case in which the sources are observed as fireballs. In other words, we may find non-vanishing sources for $C(l)$. In this case we expect to have a better experimental situation. In consequence, we may assert that this proposal is a feasible one.

### 3.6. Degree of coherence function and deformed dispersion relations

The Hanbury–Brown–Twiss effect was a watershed in optics since when it was announced a possible contradiction with quantum theory was put forward. The situation was understood with the work of Glauber [152], who gave a firm theoretical background to the quantum theory of coherence. One of the fundamental concepts in this context is provided by the so-called degree of coherence function [6,20]. This function is endowed with a deep physical meaning. We may state that the interference properties depend strongly upon the degree of coherence function. For instance, in the case of a first-order coherence function it can be proved [20] that the visibility of the fringes is a function of the first-order degree of coherence function. In this part of the work the functional dependence of the degree of coherence function, when Lorentz symmetry is broken down in the form of a deformed dispersion relation, will be analyzed (94). This function has a quantum origin, and therefore the idea proposed here involves a quantum interference experiment.

In order to deduce the dependence of the degree of coherence function let us consider two beams with energies $E_1$ and $E_2$, respectively, such that $E_2 = E_1 + \Delta E$. The experimental device is a Michelson–Morley apparatus. Each frequency produces an interference pattern, and at this point it will be supposed that the corresponding beat frequency is too high to be detected [20], i.e. the output intensity is obtained adding the intensities associated with each frequency contained in the input. Under these conditions the measured intensity reads [27]

$$I = I_1 \left[ 1 + \cos(\omega_1 \tau_1) \right] + I_2 \left[ 1 + \cos(\omega_2 \tau_2) \right].$$ \hspace{1cm} (163)

In this last expression $I_1$ and $I_2$ denote the intensities of the two beams, $\omega_1$, $\omega_2$ the corresponding frequencies, and

$$\tau_1 = \frac{2d}{c_1}, \hspace{1cm} \tau_2 = \frac{2d}{c_2}.$$ \hspace{1cm} (164)
The difference in length in the two interferometer arms is denoted by $d$, and $c_1$, $c_2$, the corresponding velocities. As mentioned before, the velocity has a non-trivial energy dependence [13], i.e. $c_1 \neq c_2$.

From now on we will assume that $I_1 = I_2$ (this is no restriction at all), such that $I_0 = I_1 + I_2$. The detected intensity can be cast in the following form

$$I = I_0 \left[ 1 + \gamma(d) \right].$$  \hspace{1cm} (166)

In this last equation the so-called degree of coherence function has been introduced, the one for our situation reads ($k_1$ and $k_2$ are the corresponding wave numbers)

$$\gamma(d) = \cos \left( \frac{(k_1 + k_2)d}{2} \right) \cos \left( \frac{(k_1 - k_2)d}{2} \right).$$  \hspace{1cm} (167)

A fleeting glance at (115) clearly shows that (167) does depend upon $\alpha$ and $n$, and, in consequence, the roots of the degree of coherence function will be modified by the presence of a deformed dispersion relation.

The expression providing us the roots of the degree of coherence function is

$$\frac{(k_1 - k_2)d}{2} = \frac{\pi}{2}.$$  \hspace{1cm} (168)

We may cast (168) as

$$d = \frac{\hbar \pi}{2} \left( \frac{\Delta E}{E_1} \right)^n \times \left[ (n + 1) \frac{\Delta E}{E_1} + \frac{n(n + 1)}{2} \left( \frac{\Delta E}{E_1} \right)^2 + \cdots \right]^{-1}. \hspace{1cm} (169)$$

Now we introduce the following definition $\beta = \Delta E/E_1$, a real number smaller than 1. In the present proposal we will consider two possible values for $n$, namely, $n = 1, 2$.

For the case $n = 1$ we have that the roots of the degree of coherence function become, approximately

$$d = \frac{\hbar \pi}{2} \left[ \beta - \frac{\alpha}{2} \left( E_1 \sqrt{G/(\epsilon^2 \hbar)} \right) \left( 2 + \beta \right) \right].$$  \hspace{1cm} (170)

Assume that $\alpha \sim 1$. The possibility of detecting this deformed dispersion relation will hinge upon the fulfillment of the condition

$$|D - d| > \Delta d.$$  \hspace{1cm} (171)

In this last equation $D$ denotes the usual value in the difference of the interferometer arms at which the degree of coherence function vanishes (that is when $\alpha = 0$), whereas $\Delta d$ is the corresponding experimental resolution. Then

$$\frac{\Delta E}{E_1} > \frac{2\Delta d}{\pi \lambda_p} - 1. \hspace{1cm} (172)$$

Since it was assumed that our device cannot detect the beat frequencies, i.e. if $T$ denotes the time resolution of the measuring device, then

$$|\omega_2 - \omega_1|T/2 \gg 1.$$  \hspace{1cm} (173)

This last condition may be rewritten as

$$T \Delta E > h.$$  \hspace{1cm} (174)

In other words, (172) and (174) are the two conditions to be fulfilled, if the case $n = 1$ and $\alpha \sim 1$ is to be detected.

Similarly, for $n = 2$ and $\alpha \sim 1$ the roots of the degree of coherence function read, approximately

$$d = \frac{\hbar \pi}{E_1} \left[ \beta - \frac{\alpha}{2} \left( E_1 \sqrt{G/(\epsilon^2 \hbar)} \right)^2 \left[ 3 + 3\beta + \beta^2 \right] \right].$$  \hspace{1cm} (175)
The expression tantamount to (172) is
\[
\left\{ 3 + 3 \left( \frac{\Delta E}{E_1} \right)^2 + \left( \frac{E_1}{\Delta E} \right)^2 \right\} E_1 > \frac{\hbar}{\pi t_p^2}.
\] (176)

The impossibility of detecting beat frequencies translates, once again, as
\[ T \Delta E > \hbar. \] (177)

The experimental feasibility of this idea is related to (172), which implies that (together with \( \Delta E/E_1 < 1 \)) the resolution of the measuring device has to be close to Planck’s length, i.e. \( \Delta d \sim l_p \), a condition far away from the current technology.

4. Conclusions

4.1. Neutron interference: limitations and possibilities

4.1.1. Metric theories and semiclassical approximation. The use of neutron interference to test the structure of space–time has been a fundamental tool and provided a deeper insight into the connection between gravity and quantum theory. From the very beginning it has been mentioned that there are several experimental devices explicitly designed for the detection of the phenomenon of interference [6].

At this point we may wonder what kind of coherence, either spatial, or temporal, is involved in a COW experiment? Note that the experimental proposal divides a neutron beam several times [38], and, afterwards, the interference pattern is measured. Clearly this situation falls within the case of temporal coherence.

We have been considering the concept of coherence from diverse perspectives, i.e. temporal coherence, spatial coherence, etc. Let us at this point delve a little bit deeper into these ideas.

We may state that optical coherence addresses the statistical description of fluctuations and that the phenomena of optical coherence may be regarded as manifestations of the correlations between them [6, 21]. We may also ask, what is the relation between coherence and interference? The answer to this question leads us to accept that interference is intimately associated with optical coherence and that it may be considered the simplest example revealing correlation between light beams. These last remarks could be enlightening but do not provide a physical insight into the origin of coherence.

Searching for a more profound understanding of the concept of coherence let us consider a point-like source of light, and, for the sake of clarity, we assume that the emitted beam is quasimonochromatic. The first question that we pose concerns the conditions of our source that give rise to the concept of temporal coherence. At any point subjected to the influence of our light beam a very swiftly oscillating field exists. For a very small time interval, about 10 ns, this field behaves as a sinusoid, and then its phase shows a random jump, and once again, behaves as a sinusoid, and so on. Two aspects must be mentioned about this last remark. Firstly, this random jump emerges as a consequence of the emitting process of the source, i.e. it is a direct consequence of the laws that govern emitting processes. Secondly, the time interval between two successive random jumps of the phase is known as temporal coherence. If we denote by \( \Delta \nu \) the bandwidth in frequency of our quasimonochromatic source, then the time during which the relation \( \Delta t = 1/\Delta \nu \) is satisfied is called temporal coherence, \( \Delta t_c \). The concept of coherence length is defined in terms of coherence time, namely, \( \Delta l_c = c \Delta t_c \). We may endow this last parameter with a physical meaning, namely, it is the distance in which the wave has a sinusoidal behavior.

We may say that temporal coherence is a manifestation of spectral purity, i.e. if the source were ideally monochromatic, then the beam would be a perfect sinusoid, but, of course, this is
an idealization, and the concept of temporal coherence is a parameter which determines how far the situation is from the ideal case.

What about the concept of spatial coherence? In order to address this issue we must recognize that in the previous example we, tacitly, accepted an additional assumption, the emitting source is point-like. Spatial coherence is related to this supposition. Indeed, let us now consider an extended object, which emits electromagnetic radiation. Now consider two points on this object, say \( A \) and \( B \). Concerning this particular situation we may wonder if the phase of the light emitted by \( A \) is correlated with the phase of the light emitted by \( B \). The concept of spatial coherence measures the correlation shown by two different points on the emitting source.

Now that we have a deeper comprehension of the concepts of temporal and spatial coherence let us proceed to explain their relation to the different experimental proposals that have been here considered. Concerning the wave front division interferometers, we may state that the prototype of them is Young’s proposal. Clearly, a single beam is diffracted by two holes which can be considered as two sources emitting the same frequency [153]. This last comment leads us to conclude that this device measures spatial coherence, i.e. it compares the light emitted from two different points of an extended body. A Michelson–Morely apparatus belongs to the family of amplitude splitting interferometers. Just for the sake of completeness let us mention that Mach–Zehnder and Sagnac devices are also elements of this family. In these kinds of interferometers if the difference in optical path is larger than the coherence length, then the interference pattern will not be detectable [153]. In other words, amplitude splitting devices allow us to measure coherence length and, in consequence, temporal coherence.

Harking back to the COW experiment, we have stressed that it requires the use of the semiclassical approach. This restriction is imposed by two facts with a very different origin. On the one hand this origin has an experimental core. Indeed, the breakdown of the semiclassical approximation, for a COW experiment performed on the surface of the Earth, would require a spreading of the wave packet larger than the radius of the Earth, as shown below. On the other hand, even if this experimental condition were somehow solved a conceptual difficulty would emerge.

We now proceed to explain this conceptual problem. We may understand the semiclassical approximation as a situation in which the wave packet becomes a point. This statement needs an explanation. The wave packet becomes a point means that the wavelength is much smaller (almost a point) than any other physically significant length parameter of the situation, i.e. the distance in which the potential has a noticeable change [33]. Here is where the problem with the possible generalizations of COW experiments lies. In metric theories the motion of point particles and null rays are endowed with a special status. In contrast, wave properties do not pertain to intrinsic geometric properties of space–time [70]. Obviously, if we loosen the restrictions inherent to the semiclassical approximation, then the motion of the neutrons will share all the conceptual difficulties that wave motion has in general relativity, and, in consequence, the interpretation of the measurement readouts will face difficulties.

These last remarks do impinge upon those COW-like experiments that attempt to test fundamental physics in the context of gravitation. The theoretical framework embodied in the postulates of metric theories cannot analyze, consistently, those proposals in which the semiclassical approximation breaks down. We may fathom this last statement noting that these cases would involve the description of wave phenomena, which represents a conceptual problem in metric theories. This fact sets a limit to the information about gravitational theories that can be elicited from COW experiments. In connection with this remark there is an additional aspect of this experiment to be discussed. Indeed, we know that the Newtonian
gravitational potential (here denoted by $\phi$) enters as an element of the components of the metric tensor, i.e. it is proportional to the deviations of the metric from the Minkowskian situation, $\phi \sim h_{\mu\nu}$, $(g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu})$. Here we assume a weak-field limit of the theory [58], and, therefore, the gravitational acceleration, $g$, is proportional to the Christoffel symbols, $g \sim \Gamma^\rho_{\mu\nu}$. Since the postulates of metric theories of gravity imply that locally gravitational effects can be gauged away, then the COW experiment involves degrees of freedom that do not have an invariant meaning under coordinate transformations (if we believe in metric theories). In this last phrase we bear in mind an experiment in which the whole experimental device falls freely. Clearly, this is not the case in the usual experiment. Indeed, the detecting screen is at rest with respect to the Earth, a fact that implies that the screen is not falling freely.

To consider effects in this kind of experiment involving second derivatives of the metric tensor (which cannot in general be gauged away [58]) the spreading of the wave packet has to be larger than the region in which physics behaves according to special relativity. This means that the spreading of the wave packet $\Delta x$ must fulfill the condition $\Delta x \geq (1/|\phi_{\mu\nu}|)$, i.e. larger than the inverse of the absolute value of the second-order derivatives of the gravitational potential. What does this last condition mean? For the case of the Earth, whose radius and mass will be denoted by $R$ and $M$, respectively, the aforementioned restriction entails $\Delta x \geq R\sqrt{R/M}$ (here we use geometrized units, in which length and mass have the same units [58]). In the weak-field limit $(M/R) < 1$, and in consequence $\Delta x \geq R$. The introduction of this condition would imply the breakdown of the semiclassical approximation, which is always present as a condition in COW [38,75]. In other words, the use of COW experiments to detect elements of the gravitational field with an invariant geometrical meaning requires two initial premises. (i) The validity of the semiclassical approximation, which means that the size of the wave packet has to be smaller than the region in which the gravitational potential has a noticeable change [33]. (ii) A wave packet larger than the region in which the flatness theorem is valid [58], which means that the wave packet has to be larger than the region in which the gravitational field has a considerable change. Clearly, these two conditions cannot be fulfilled simultaneously, i.e. the lesson to be elicited here is that we cannot resort to COW experiments and test those degrees of freedom of the gravitational field which are coordinate invariant. This is, perhaps, the most important limitation of COW experiments in connection with the principles of metric theories.

To finish this part let us address the measuring process associated with the Aharonov–Bohm effect. This issue is an interesting one, since the experimental verification of this effect triggered a hot debate. Indeed, for some time it was claimed that no effect of inaccessible electromagnetic fields could be present in quantum mechanics, namely, continuity conditions for the vector potential would be responsible for the disappearance of the Aharonov–Bohm effect [154]. The detection of this effect was done in the context of a spatial coherence device [155], i.e. a Young type experiment. This fact becomes clear if we notice that two electron beams were used as part of the experimental idea [155]. Of course, this last remark does not imply a spatial coherence experiment. This conclusion appears after noting that these two electron beams were not obtained dividing a primeval electron beam. In other words, COW experiments resort to temporal coherence, whereas the experimental verification of the Aharonov–Bohm effect is related to spatial coherence.

4.1.2. Torsion, fifth force, and semiclassical approximation. Finally, the discrepancy between theory and experiment [75, 119] that the COW presents nowadays cannot be solved by the inclusion of torsion, as we have shown. Other effects, for instance, the consequence of the rotation of the Earth upon the interference pattern [156, 90], the influence of gravity on the beam splitter [157] or the possible dependence of the dynamical diffraction on the bending
and strains in the interferometer, do not provide a complete answer to this discrepancy [75]. This is an issue that up to now remains an open problem.

As shown in the corresponding section, COW experiments can also be used to test the possibility of a fifth force, though once again, the semiclassical approximation has to be used. Since in this case an additional interaction appears, the semiclassical limit will involve a second condition, i.e. the wavelength of the thermal neutrons has to be smaller than $\lambda$, the Compton wavelength of the field, which means smaller than the range of the force. In other words, since thermal neutrons imply a wavelength of about $10^{-8}$ m, this model will work if $\lambda > 10^{-8}$ m. When this condition is not fulfilled, the measurement readouts will not match the theoretical background.

Now that we better understand the limitation of COW experiments, let us address the issue that the case of non-demolition variables offers us. In general, mass not only does not disappear from the interference pattern, but it acquires a more complicated dependence upon mass than in COW experiments. In the context of the principles of quantum theory it predicts a very particular dependence of the probabilities upon the precision of the measuring device, and, therefore, it provides a way to confront the RPIF model [130], or any of its equivalent formulations [129], against measurement readouts. It has to be underlined that the current technology is far from being capable of carrying out the required experiments.

Usually, the experiments in the quantum domain are divided into two groups [158, 116, 159], namely, experiments which involve either the evolution of free particles (neutron interferometry falls within this group) or spectroscopy of bound states, the Hughes–Drever experiment is a typical case of this situation [87, 88]. The aforementioned limitations concerning neutron interferometry, due to the need of the semiclassical approximation, demands a new type of experiments in the quantum realm [160, 161]. One possibility in this direction could be provided by experiments joining gravitation and quantum measurement theory, since they could lead to a conceptual development in gravitation [82].

Summing up, the void that wave phenomena have in the principles of metric theories is the main hindrance to test the principles of metric theories in the quantum domain, at least by means of interference experiments. Clearly this implies that, for instance, if we are looking for the existence of a fifth force, then the wavelength of the particle has to be smaller than the range of the sought force. If the fifth force had a Compton wavelength smaller than the wavelength associated with that of thermal neutrons then this idea cannot be employed. This comment clearly defines a stringent limitation for this technique.

4.2. Photon interference: limitations and possibilities

4.2.1. First coherence experiments and gravity. Stress has been laid on the fact that the description of wave phenomena has conceptual difficulties in the context, not only of general relativity but also of metric theories. The limitations that we have mentioned in the domain of neutron interference will be shared by the technique of photon interference. In other words, we may test the principles of metric theories as long as the eikonal limit is a good approximation to the motion of light. If this limit is abandoned, then the results obtained by the corresponding experiment will imply premises falling outside the assumptions behind metric theories. The consequences of this conceptual drawback have to be contemplated in the realm of the present proposal, which involves, unavoidably, a wave phenomenon. The analysis of the results of an experiment of this sort shall be done in the domain in which the aforementioned shortcoming can be circumvented, at least partially. Otherwise the corresponding results would be weaved with effects that are not contemplated by our metric theory. For instance, since visible light has a wavelength between 400 and 700 nm, then a fifth force with a Compton length
smaller than 400 nm cannot be detected with an optical experiment. This clearly represents a shortcoming.

Let us now make a brief summary of the applications of first-order effects as tools to test gravitational physics. The Michelson–Morley idea has been an important experimental device in gravitational physics, but this is not the only optical possibility. Sagnac’s proposal has an additional parameter, which is absent in the case of a Michelson–Morley experiment. A Sagnac interferometer involves $b$, $\Omega$ and $\hbar\nu$, the radius, the angular velocity of the interferometer and the energy of the beam, respectively. In this sense we may say that Sagnac is richer, though as shown above, the detection of a violation of Lorentz symmetry, in the form of a deformed dispersion relation, lies outside the technological possibilities.

Another experiment which falls within the group of the first-order coherence type is Young’s, which has been considered in this work as an idea for testing non-Newtonian contributions to the gravitational force. Unfortunately, the proposal requires traveling distances, for the involved light beams, that avert (if the experiment is to be carried out near the Earth’s surface) the feasibility of the proposal. The detection of a fifth force by means of first-order experiments seems to be impossible.

4.2.2. Second coherence experiments and gravity. The just mentioned drawbacks concerning first-order experiments, in the context of test of fundamental physics, lead us to seek additional options. The realm of the Hanbury–Brown–Twiss effect, which is a second-order coherence effect, has been partially explored. For non-Newtonian contributions it has been proved that it offers no significant improvement with respect to Young’s situation, i.e. very large traveling distances have to be considered.

Nevertheless, the breakdown of Lorentz symmetry can be tested by resorting to HBTE. The additional distance parameter, which this model introduces, pays off in the quest of these violations. It has an additional advantage. Up to now the use of interference, either neutron, or optical, has always required the fulfillment of the semiclassical limit (for quantum systems), or of the eikonal limit (for light). Nevertheless, the measurement output in HBTE is the normalized correlation coefficient of the fluctuations in the photoelectric current obtained with the photo-detectors located at the two detection points [6, 20]. Obviously, it depends upon the distance, $l$, between the photo-detectors. This correlation function also hinges upon the properties of the gravitational field, but $l$ is not the distance that a wave travels. It can be much smaller, or larger, than the wavelength of the corresponding system and no conceptual difficulty would emerge. In this sense HBTE circumvents the aforementioned restriction.

We may sum up the limitations and the possibilities of photon interference to test the principles of metric theories stating that in the context of first-order coherence experiments the possibility of setting bounds to some gravitational effects, either fifth force, or deformed dispersion relations, is practically null. The quest for more optimistic scenarios leads us to consider higher-order effects, such as HBTE, which, for some possible gravitational features could provide interesting results [162].

4.3. Neutron and photon interference: coincidences

In this work the topic of interference has been divided into two realms, i.e. neutron and photon interference. This could lead us to conclude that, though sharing a set of common properties (stemming from a linear motion equation), they do not have too much in common. The truth is that there is a profound physical relation between these two phenomena. Indeed, the analogy between the case of light propagating in a moving dielectric and the motion of a charged particle (described by quantum mechanics), under the presence of a magnetic field has already been
underlined [163]. This result implies, among other things, that: (i) interference is a phenomenon which does not, necessarily, depend upon the nature of the wave; rather, it hinges on the propagation of wave motion in a medium; (ii) some of the phenomena detected using neutrons may also be measured resorting to photons. Of course, the detection in neutronic experiments of effects obtained with photons in also possible. For instance, an optical Aharonov–Bohm effect has already been put forward [164]. Furthermore, there is an interesting analogy between the propagation of light in a moving dielectric and the motion of light in a gravitational field, where the metric of the moving medium is defined by its dielectric properties [165]. This last remark poses interesting questions. For instance, the motion of light in a medium (under the condition that polarization does not impinge on the motion of light) resembles the motion of light in general relativity [165]. If we now take into account the effects of polarization of light, what is the corresponding analogy in the context of a gravitational theory? A metric theory or a more general model?

4.4. Additional alternatives

Of course, the principles of metric theories can be tested by resorting to experiments based on astrophysical sources, an already known possibility, as the analysis of the motion of Mercury [58] proves. In our case we may mention, for instance, the Auger project [166], a proposal that possesses undeniable advantages, but that has a drawback, i.e. in some of its aspects it is not controlled by the experimentalist, like all proposals hinging upon astrophysical sources. Another possibility is the analysis of the effects that, for instance, a violation of Lorentz symmetry has on white dwarfs [167], though these kinds of tests fall outside the realm of interference models.

Finally, the use of atom interferometry opens up an interesting spectrum of possibilities. Indeed, it has already been mentioned that atom interferometry provides several ways in which the Newtonian gravitational constant can be measured [125–127]. Fortunately, the options that this technique offers do not end at this topic. For instance, it could be feasible to obtain a very good accuracy, up to 1 part in $10^{15}$, in tests of general relativity resorting to atom interferometry [168]. The detection of rotation or gravity may also be contemplated from the perspective of atom interferometry [169].

These cases illustrate quite vividly the possibilities that atom interferometry could offer in the context of gravitational physics, namely, tests of some of the postulates associated with metric theories, etc.

4.5. Mathematical notation

In this part a short list containing the main mathematical symbols is provided.

1. Gravitational symbols.
   (a) Newton’s gravitational constant, $G$
   (b) Einstein tensor, $G_{\mu\nu}$
   (c) Ricci tensor, $R_{\mu\nu}$
   (d) Ricci scalar of curvature, $R$
   (e) Christoffel symbols, $\Gamma^k_{ij}$
   (f) Contorsion tensor, $K^k_{ij}$
   (g) Planck’s energy, $E_p$
   (h) Metric tensor, $g_{\mu\nu}$
   (i) Acceleration of gravity, $g$
   (j) Affine connection, $\Gamma^k_{ij}$
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(k) Torsion tensor, $S_{ij}^k$
(l) Gravitomagnetism-related PPN parameters, $\Delta_2, \Delta_1$

2. Quantal symbols.
(a) Pauli matrices, $\sigma^l$
(b) Planck’s constant, $\hbar$
(c) Berry’s phase, $\gamma$

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