A Relationship Function Between Loadability Margin and OMIB Simulation Results for Heavy Loading Power System

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Abstract. A novel voltage stability relation function based on an one-machine-infinite-bus (OMIB) equivalent was proposed, which shows detailed relationship between voltage stability and transient stability. Since the OMIB hybrid simulation has the ability to take into consideration any kind of system detailed dynamic model, the proposed voltage stability relation function embodies all the system’s nonlinearity, which is stricter than traditional static voltage stability indexes. The feasibility and validity of the proposed index were demonstrated on 9-bus, 14-bus, 39-bus and 692-bus systems.

Introduction

Proximity of voltage instability is usually measured by the distance between the operating point on the PV curve and the limit of the same curve [1]-[3], i.e. system loadability margin. However, loadability margin analysis does not include the system detailed dynamic model, and may draw a false (un)stable conclusion when system operates near a critical stable operating point.

While one-machine-infinite-bus (OMIB) simulation can take into consideration any kind of system detailed dynamic model, i.e. system’s detailed nonlinearity. A relation function between loadability margin and OMIB simulation results is proposed in this paper, which shows detailed relationship between voltage stability and transient stability. Since OMIB hybrid simulation can use system detailed dynamic model, the proposed relation function embodies all the system’s nonlinearity, which is stricter than traditional static voltage stability indexes. The feasibility and validity of the proposed function are demonstrated on 9-bus, 14-bus, 39-bus and 692-bus systems.

OMIB Hybrid Simulation Method

Based on the extended equal area criterion [4]-[5], OMIB hybrid simulation methods based on time domain simulations are proposed in [6]-[8].

First, a time-domain simulation of a contingency for the multi-machine system is run, until any two of the system machines reach an angular deviation of 360°, which is defined as here as the point in time \( t_{\text{max}} \). At \( t_{\text{max}} \), the system machines are sorted in descending order with respect to their rotor angles, and the largest angular deviation (“maximum gap”) between any two adjacent machines is used to identify all machines above the maximum gap. These machines are aggregated into one equivalent machine, and the remaining machines are aggregated into another equivalent machine [9]. The two equivalent machines are then used to form an OMIB system, as described in detail in [6]-[8].

According to OMIB equivalent, faults can be classified for the first swing into stable, generally unstable, very unstable, and critically (un)stable faults. The stable fault trajectory is depicted Figure 1 in the OMIB P-\( \delta \) plane, with its stability criterion being:
\[
\begin{align*}
\left\{ P_a(t) &= P_m(t) - P_e(t) < 0 \\
\dot{\omega}(t) &= 0 \\
\dot{\omega}(t) &> 0 \ \forall \ t_o < t < t_r 
\end{align*}
\]  

(1)

where \( P_a \) is the equivalent OMIB accelerating power, \( P_m \) is mechanical power, \( P_e \) is the electrical power, \( \dot{\omega} \) is the equivalent rotational speed deviation, \( t_o \) represents the initial time at which the system is in steady state conditions (i.e. right before the fault), and \( t_r \) is the time at which the maximum oscillation equivalent angle \( \hat{\delta}_r \) is reached.

The “generally” unstable criterion is given by:

\[
\begin{align*}
\left\{ P_a(t_u) &= P_m(t_u) - P_e(t_u) = 0 \\
\frac{dP_a}{dt}_{t=t_u} &> 0 \\
\dot{\omega}(t) &> 0 \ \forall \ t > t_o 
\end{align*}
\]  

(2)

This is depicted in Figure 2, where the OMIB trajectory reaches the unstable equilibrium \( \hat{\delta}_u \) point at time \( t_u \) after the fault clearance. The critically (un)stable trajectory evaluation criterion is given by:

\[
\begin{align*}
\left\{ P_a(t_u) &= P_m(t_u) - P_e(t_u) = 0 \\
\dot{\omega}(t_u) &= 0 
\end{align*}
\]  

(3)

which are equivalent to \( \hat{\delta}_r = \hat{\delta}_u \), at time \( t_u \). Finally, a “very” unstable trajectory is depicted in Figure 3, which corresponds to:
\[ P_a(t) > 0 \quad \forall \quad t_0 < t < t_{\text{end}} \]  
(4)

where \( t_{\text{end}} \) is the total simulation time.

**PV Curves of Four Test Systems**

Proximity of voltage instability is usually measured by the distance between the operating point on the PV curve and the limit of the same curve, i.e. system loadability margin. So first of all, PV curves of four test systems, i.e. IEEE 3-machine 9-bus system, IEEE 4-machine 14-bus system, 10-machine 39-bus New England benchmark system, and a real provincial 138-machine 692-bus system, are computed by using CPF in PSAT [10] which can be seen in Figure 4-7. The continuation power flow (CPF) computations are performed considering voltage limits and reactive power limits for all the systems except 14-bus system in which only voltage limits are considered.

For 9-bus system and 14-bus system, the base case powers are used as load directions, with equations as following:

\[
\begin{align*}
\begin{cases}
P_G &= (\lambda + \gamma k_G) P_G^0 \\
P_L &= \lambda P_L^0 \\
Q_L &= \lambda Q_L^0
\end{cases}
\end{align*}
\]  
(5)

where \( P_G^0 \), \( P_L^0 \) and \( Q_L^0 \) are the base case generator and load power; \( \lambda \) is the loading parameter; \( k_G \) is the distributed slack bus variable and \( \gamma \) is the generator participation coefficients.

For 39-bus system and 692-bus system, the generator and load power directions, \( P_S^0 \), \( P_D^0 \) and \( Q_D^0 \), are defined, with equations as follow:

\[
\begin{align*}
\begin{cases}
P_G &= P_G^0 + (\lambda + \gamma k_G) P_S^0 \\
P_L &= P_L^0 + \lambda P_D^0 \\
Q_L &= Q_L^0 + \lambda Q_D^0
\end{cases}
\end{align*}
\]  
(6)

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Figure 5. P-V Figure for 14-Bus System after Disturbance.

Figure 6. P-V Figure for 39-Bus System after Disturbance.
A three-phase to ground fault takes place at some line in each system and is cleared 50ms later by line tripping. PV curves are compared before and after the contingency occurs, which are depicted in Figure 4-7. In Figure 4, the maximum loadability for IEEE 3-machine 9-bus system is 381MW after contingency, but it is not sure whether the system is stable or not when system load is less than 381MW. Since the PV curves are obtained from system steady state, transient voltage stability can not be concluded from the PV curves, especially when system operates near the maximum loading point.

Relation Function Using OMIB Simulation

OMIB simulation can take into consideration any kind of system detailed dynamic model, i.e. system’s detailed nonlinearity. So it is significantly meaningful to explore the relationship between loadability margin and OMIB simulation results.

There are two obstacles in establishing a novel relation function based on OMIB hybrid simulation. First, does the unstable mode change along all the load levels? Second, what kind of relation function can be defined using OMIB simulation?

OMIB hybrid simulations are performed on 9-bus, 14-bus, 39-bus and 692-bus systems, respectively using the same three-phase to ground fault in Section III followed by line tripping 50ms later, under different loading level. A two-axis four-order dynamic machine model with IEEE type-DC1 exciter [11] and constant mechanical input is used to model all generators for the 9-bus and 39-bus system; sixth order model with type-DC1 exciter and constant mechanical input for 14-bus and 692-bus system. Exponential load-recovery model are adopted for all the loads in the four systems [12].

From computational results of the four test system, system experience stable, critical stable, “generally” unstable and “very” unstable operating condition with the load increasing; the unstable mode is consistent for the first three systems; but as to the 692-bus system, the unstable mode is only consistent for those loads which are not less than the critical stable load, or are little lower than the critical stable load level; while when the load decreases far from critical load level, the two equivalent machines used to form an OMIB system change their leading/lagging order.

Although the unstable mode may be not consistent during the whole loading levels, the system unstable modes under critical loading level and a little lower loading level are consistent, which is most concerned since power systems are operating close to their security limits.
It can be seen from the power-angle characteristics (seen in Figure 8-10) of OMIB equivalent system that if the active electrical power maximum point in the first half swing becomes higher the decreasing area becomes larger and system will be more stable. So \( \rho_1 \) (seen in equation (7)) can be tried as the voltage stability relation function. \( P_{emax} \) is the maximum electrical power during the first half swing and \( P_{mmax} \) is the corresponding mechanical power, seen in Figure 8~10. If the active electrical power \( P_e \) reaches a local maximum point (seen in Figure 8 and Figure 10) during the first half swing, the electrical power of this point is then taken as the value of \( P_{emax} \). This will produce convenience to compare voltage stability relation function between “generally” unstable condition and stable condition. If active electrical power monotonously increases during the first half swing (seen in Figure 9), the electrical power at the first swing return point is taken as the value of \( P_{emax} \).

\[
\rho_1 = \frac{P_{emax} - P_{mmax}}{P_{mmax}}
\]  

(7)

The first-swing angle of OMIB equivalent system returns more and more quickly when load level is decreasing, and the maximum electrical power is decreasing relatively at the same time, resulting in the voltage index \( \rho_1 \) decreases with the load decreasing. So the angle \( \theta \) in the Figure 8~10 can be used to depict the OMIB rotor angle return speed, and the first voltage stability relation function \( \rho_1 \) can be properly revised as \( \rho_2 \) (seen in equation (8)).

\[
\rho_2 = \begin{cases} 
\frac{P_{emax} - P_{mmax}}{P_{mmax}} \tan(\theta) & \text{"generally" unstable and stable} \\
\frac{P_{emax} - P_{mmax}}{P_{mmax}} & \text{"very" unstable}
\end{cases}
\]

(8)

Furthermore the voltage stability relation function needs to be further modified to identify the stable loading boundary as depicted in equation (9), where \( \rho_{v\text{r}} \) is voltage stability relation function \( \rho_2 \) for critically stable condition for the same contingency. Figure 11-14 depict the relation curve between the proposed voltage stability relation function \( \rho_3 \) and total load level for the four test systems, i.e. IEEE 3-machine 9-bus system, IEEE 4-machine 14-bus system, 10-machine 39-bus New England benchmark system, and a real provincial 138-machine 692-bus system respectively. From the computational results, \( \rho_3 \) decreases, monotonously and fast, with load increasing for all the systems and can well represent the voltage stability within a largest load varying range. So \( \rho_3 \) is highly recommended as the voltage stability relation function, which takes all the detailed model and dynamics of the system into consideration; and moreover the computation burden is extremely light.

\[
\rho_3 = \frac{\rho_2 - \rho_{v\text{r}}}{{\rho_{v\text{r}}}}
\]

(9)

To obtain the voltage stability relation function \( \rho_3 \), there are only two steps needed. Firstly system critical stable loading level should be searched in order to determine the unstable mode; this search is only taken once for any specific generator and load power direction. Then OMIB hybrid simulation is performed only for the rotor angle first-swing period, i.e. 2-6 seconds for most power system.
In Figure 11, the maximum load beyond which the IEEE 3-machine 9-bus system becomes unstable for the same contingency depicted in Section III is 337MW, which is smaller than both the maximum loadability 381MW after contingency and the maximum loadability 477MW before contingency according to P-V curves in Figure 4. So other three systems do.

System loadability margin produced by P-V curve is static data, determined by power system network structure and parameters, and system operating point, also closely connected with generator reactive power limitation, however system stable condition is influenced by other more detailed factors: fault severity, e.g. the fault clearing time, dynamic characteristics of synchronous machine and load, etc. Voltage stability relation function $\rho^{(3)}_V$ can express system loadability better than traditional static voltage stability indexes under contingency.

**Summary**

A novel voltage stability relation function based on an one-machine-infinite-bus (OMIB) equivalent is proposed in this paper. This index is stricter than traditional static voltage stability indexes and valid in a relatively large load variation range near the critical stable loading level, which will help to deal with load perturbation problems in optimal control[13-15], e.g. in solving transient stability constrained optimal power flow and transient stability excitation control optimization.

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