Neutrino Magnetic Moment

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Abstract. Current experimental and observational limits on the neutrino magnetic moment are reviewed. Implications of the recent results from the solar and reactor neutrino experiments for the value of the neutrino magnetic moment are discussed. It is shown that spin-flavor precession in the Sun is suppressed.

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INTRODUCTION

A minimal extension of the Standard Model (with non-zero neutrino masses) yields a neutrino magnetic moment of \[ \mu_\nu = \frac{3 e G_F m_\nu}{8 \pi^2 \sqrt{2}} = \frac{3 G_F m_e m_\nu}{4 \pi^2 \sqrt{2}} \mu_B \]

(1)

where \( \mu_B = e/2m_e \) is the Bohr magneton. Note that the neutrino magnetic moment is proportional to the neutrino mass as required by the symmetry principles. Since the recent solar, atmospheric and reactor neutrino experiments indicate the existence of non-zero neutrino masses, we also know that neutrino has a magnetic moment. Using the neutrino parameters deduced from analyses of those experiments \[ \mu_\nu \geq (4 \times 10^{-20}) \mu_B \]. Larger values of magnetic moments are possible in extensions of the Standard Model, as indicated by the inequality sign in this value. If the magnetic moment is generated by physics at scale \( \Lambda \) we can write

\[ \mu_\nu \sim \frac{e \mathcal{G}}{\Lambda} \]

(2)

where \( \mathcal{G} \) represents the combination of the coupling constants and appropriate \( 2\pi \) factors. If we remove the external photon from the diagrams leading to Eq. (2) we get a contribution to the mass of the order

\[ \delta m_\nu \sim \mathcal{G} \Lambda. \]

(3)

These equations imply that

\[ \delta m_\nu \sim \frac{\Lambda^2}{m_e} \left( \frac{\mu_\nu}{\mu_B} \right). \]

(4)

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If one assumes that the scale $\Lambda$ is not significantly higher than the electroweak scale, current neutrino mass limits imply $|\mu| \leq 10^{-14} \mu_B$ for Dirac neutrinos [3]. It is however possible to introduce models where the magnetic moment and mass do not come from the same number of loops, and relax this bound (see e.g. Ref. [4]).

It is well-established that the neutrinos mix and the discussion above illustrates that magnetic moment is properly defined in the mass basis [5]. In this basis Dirac neutrinos can have both diagonal and off-diagonal moments, whereas Majorana neutrinos can only have transition moments. More specifically, if the magnetic moment operator is designated by $\mu$, then $\mu = \mu^\dagger$ for Dirac neutrinos, and $\mu^T = -\mu$ for Majorana neutrinos.

**LIMITS ON THE NEUTRINO MAGNETIC MOMENT**

There are a number of possible physical processes involving a neutrino with a magnetic moment. Among these are the $\nu - e$ scattering, spin-flavor precession in an external magnetic field, plasmon decay, and the neutrino decay. For the first process, using the magnetic moment operator $\mu$, the total cross section at an experiment where the final neutrino is not observed can be written in the Born approximation as

$$\sigma \sim \sum_i |\langle \nu_i | \mu | \nu_e \rangle|^2,$$

(5)

where $|\nu_i\rangle, i = 1, 2, 3$ represent the mass eigenstates and we assumed that electron neutrinos are used. Since the neutrino mixing matrix in $|\nu_e\rangle = \sum_i U_{ei} |\nu_i\rangle$ is unitary, Eq. (5) takes the form

$$\sigma \sim \langle \nu_e | \mu^\dagger \mu | \nu_e \rangle.$$

(6)

Detecting a neutrino magnetic moment then implies detecting them in mass eigenstates. Consequently the measured magnetic moment of the neutrino, in principle, depends on the distance from its source [5]:

$$\mu_e^2 = \sum_i U_{ej} U_{ij} \mu_{ij} \exp(-iE_jL) \right|_2.$$

(7)

The differential scattering cross section for electron neutrinos or antineutrinos on electrons is given by [6]

$$\frac{d\sigma}{dT} = \frac{G_F^2 m_e}{2\pi} \left[ (g_V + g_A)^2 + (g_V - g_A)^2 \left( 1 - \frac{T}{E_\nu} \right)^2 + \left( \frac{g_A^2 - g_V^2}{E_\nu^2} \right) \frac{m_e T}{E_\nu^2} \right]$$

$$+ \frac{\alpha^2 \mu^2}{m_e^2} \left[ \frac{1}{T} - \frac{1}{E_\nu} \right],$$

(8)

where $T$ is the electron recoil kinetic energy, $g_V = 2 \sin^2 \theta_W + 1/2$, $g_A = +1/2(-1/2)$ for electron neutrinos (antineutrinos), and the neutrino magnetic moment is expressed in units of $\mu_B$. The first line in Eq. (8) is the standard electroweak contribution and the second line represents the contribution of the neutrino magnetic moment. Clearly the
magnetic moment contribution is dominant at low recoil energies. The magnetic moment cross section will exceed the standard electroweak cross-section for recoil energies

$$\frac{T}{m_e} < \frac{\pi^2 \alpha^2}{(G_F m_e^2)^2} \mu^2 \nu,$$

(9)
i.e. the lower the smallest measurable recoil energy is, the smaller values of the magnetic moment can be probed. To perform such an experiment either solar or reactor neutrinos have been used. SuperKamiokande collaboration looked for distortions in the energy spectrum of solar neutrinos scattered off the electrons in their detector. No clear signal was observed. Combined with the other solar neutrino and KamLAND experiments a limit of $\mu \lesssim 1.1 \times 10^{-10} \mu_B$ at 90% C.L. was obtained [7]. The MUNU collaboration, using reactor neutrinos, recently obtained a slightly better limit of $\mu \lesssim 9 \times 10^{-11} \mu_B$ at 90% C.L. [8]. Another possibility for doing such experiments is to utilize low-energy beta beams [9]. A detailed study of neutrino-electron scattering using low-energy beta-beams in general is given in Ref. [10] and limits on the neutrino magnetic moment in particular are given in Ref. [11]. The latter work finds that a tritium source may yield a better bound.

Neutrinos change helicity in magnetic moment scattering. This fact has been used to put limits from astrophysics and cosmology on neutrino magnetic moment. If $\mu$ is sufficiently large, then the proto-neutron star formed in a core-collapse supernova can cool faster since the right-handed components are sterile. It was found that $\mu \gtrsim 10^{-12} \mu_B$ would be inconsistent with the observed cooling time of SN1987a [12]. In the Early Universe the existence of right-handed Dirac neutrinos that may be produced in magnetic scattering increase the number of effective degrees of freedom altering neutrino counting through the big-bang nucleosynthesis yields [13]. (Similar limits do not apply to Majorana neutrinos since antineutrino states are already counted).

The tightest astrophysical bound on neutrino magnetic moment comes from the red-giant stars. A large enough magnetic moment implies enhanced plasmon decay rate, $\gamma^* \to \nu \nu$, inside the star. Since the neutrinos freely escape the stellar environment this process in turn cools a red giant star faster, delaying helium ignition. Existing observations of globular cluster stars lack any evidence of this effect, yielding a limit of $\mu \lesssim 3 \times 10^{-12} \mu_B$ [14].

The discussion above shows the neutrino magnetic moment is presently known to be in the range

$$(9 \times 10^{-11}) \mu_B \geq \mu \geq (4 \times 10^{-20}) \mu_B.$$  
(10)
The large width of this range represents possible physics beyond the standard model which can be explored using the neutrino magnetic moment measurements.

**IMPLICATIONS OF NEUTRINO SPIN-FLAVOR PRECESSION**

If neutrinos have magnetic moments, large magnetic fields that exist in astrophysical environments may give rise to an additional spin-flavor precession coupled to the usual matter-enhanced neutrino oscillations [13, 16]. Spin-flavor precession changes the helicity of the neutrinos, and if the neutrinos are of Majorana type, this yields a solar
antineutrino flux \[^{17, 18, 19}\]. Since the electron antineutrino yields a very distinctive two-neutron signal on charged-current deuteron break-up, Sudbury Neutrino Observatory (SNO) measurements were able to put a limit of \(\Phi_{\bar{\nu}_e} \leq 3.4 \times 10^4 \text{cm}^{-2}\text{s}^{-1}\) at 90% C.L. \[^{20}\]. This corresponds to less than 0.8% of the standard solar model \(^8\)B flux. The KamLAND, experiment, being directly sensitive to antineutrino scattering in their scintillator liquid, provides a slightly better bound of \(\Phi_{\bar{\nu}_e} \leq 3.7 \times 10^2 \text{cm}^{-2}\text{s}^{-1}\) at 90% C.L. \[^{21}\]. This is less than \(2.8 \times 10^{-4}\) of the Standard Solar Model \(^8\)B \(\nu_e\) flux.

A complete analysis of the spin-flavor precession scenario in the Sun requires detailed knowledge of the solar magnetic fields. Unfortunately information about solar magnetic fields is rather incomplete. If the magnetic field is greater than \(10^8\) G, magnetic pressure becomes the same order of magnitude as the matter pressure obviating the Standard Solar Model. For the neutrino masses and the mixing angles deduced from the solar and reactor neutrino experiments, both the spin-flavor and the MSW resonances are very close together in the inner radiative zone. It was shown that magnetic fields greater than \(\sim 10^7\) G, localized at about 0.2 \(R_\odot\), would cause the sound speed profile to be at variance with the helioseismic observations \[^{22}\].

The MSW resonance takes place in the Sun where the condition

\[
\sqrt{2} G_F N_e = \frac{\delta m^2}{2E_\nu} \cos 2\theta
\]

is satisfied. The spin-flavor precession resonance takes place before the solar neutrinos reach the MSW resonance point. It is where

\[
\frac{G_F}{\sqrt{2}} (2N_e - N_n) = \frac{\delta m^2}{2E_\nu} \cos 2\theta
\]

for the Dirac neutrinos and where

\[
\sqrt{2} G_F (N_e - N_n) = \frac{\delta m^2}{2E_\nu} \cos 2\theta
\]

for the Majorana neutrinos. In these equations \(N_e\) and \(N_n\) are the electron and neutron densities, respectively. For the observed neutrino parameters these resonances significantly overlap, hence previous approaches treating them as isolated resonances \[^{19}\] are not applicable \[^{23, 24}\]. However, there is a limit which may provide an analytical insight into the problem of overlapping resonances. In the limit \(N_n = 0\), clearly the resonances of Eqs. (11), (12), and (13) are at the same location. The electron neutrino survival probability is \[^{25}\]

\[
P(\nu_e \rightarrow \nu_e) = \frac{1}{2} - \frac{1}{2} \cos 2\theta \left(1 - 2P_{\text{hop}}\right),
\]

where \(P_{\text{hop}}\) is the hopping probability between matter eigenstates. It can be shown that \[^{23}\], in the limit \(N_n = 0\), the hopping probability is given by

\[
\exp \left\{ \frac{i}{\pi} \int_{r_0}^{r_0'} dr r \frac{\delta m^2}{2E} \left[ \frac{(\mu B)^2}{\sqrt{\zeta^2(r) - 2\zeta(r) \cos 2\theta + 1}} \right] \right\},
\]

(15)
where we used the semiclassical treatment of the matter-enhanced neutrino oscillations to calculate the hopping probability \[26\]. In Eq. (15), \(\zeta(r) = \frac{2\sqrt{2} G_F N_e(r)}{\delta m^2/E} \), and \(r_0^*\) and \(r_0\) are the turning points (zeros) of the integrand. Matter-enhanced neutrino oscillations in the Sun are primarily adiabatic \[2\], hence the hopping probability is very small to begin with. Eq. (15) implies that the reduction factor of the hopping probability,

\[
\exp \left[ -\pi \frac{(\mu B)^2 2E}{\delta m^2} \right],
\]

is also very small. For a 10\(^5\) G magnetic field, a magnetic moment of 10\(^{-12}\) \(\mu_B\), and \(E \sim 10\) MeV, this hopping reduction factor is \(\sim 10^{-3}\). An exact numerical calculation, using the neutrino mixing parameters \(\delta m^2 = 8 \times 10^{-5} \text{ eV}^2\), \(\tan^2 \theta = 0.4\), and relatively large values of \(\mu_\nu = 10^{-11} \mu_B\) and \(B = 10^5\) G, finds that the electron neutrino survival probabilities calculated with the MSW resonance only \((B = 0)\) and calculated with both resonances \((B \neq 0)\) differ by less than \(10^{-5}\) \[23\]. For reasonable values of the solar magnetic fields the effect of the neutrino magnetic moment on the solar neutrino flux is minuscule. It should be noted however that large fluctuations of the magnetic fields could impact spin-flavor precession \[27\].

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