Trapping Abelian anyons in fractional quantum Hall droplets

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We study the trapping of Abelian anyons (quasiholes and quasiparticles) by a local potential (e.g., induced by an AFM tip) in a microscopic model of fractional quantum Hall liquids with long-range Coulomb interaction and edge confining potential. We find, in particular, at Laughlin filling fraction $\nu = 1/3$, both quasihole and quasiparticle states can emerge as the ground state of the system in the presence of the trapping potential. As expected, we find the presence of an Abelian quasihole has no effect on the edge spectrum of the quantum liquid, unlike in the non-Abelian case [Phys. Rev. Lett. 97, 256804 (2006)]. Although quasiholes and quasiparticles can emerge generically in the system, their stability depends on the strength of the confining potential, the strength and the range of the trapping potential. We discuss the relevance of the calculation to the high-accuracy generation and control of individual anyons in potential experiments, in particular, in the context of topological quantum computing.

I. INTRODUCTION

Shortly after the discovery of the fractional quantum Hall effect, Laughlin realized that electrons in such a system form an incompressible quantum liquid with excitations of fractional charge. These exotic particle excitations are dubbed (Abelian) anyons. To interchange two anyons, one obtains a phase factor $e^{i\theta}$ for the wave function, where $\theta$ is neither an integral multiple of $2\pi$ as required by bosons, nor an odd multiple of $\pi$ as required by fermions. The presence of these particles with fractional statistics is an indication of topological phases. So far, experiments have confirmed the fractional charge but the direct observation of the fractional statistics remain questionable. Recent experiments demonstrated the so-called super-periods in the conductance oscillations in a fractional quantum Hall quasiparticle interferometer, which appear to be consistent with fractional statistics. However, some theoretical works raised subtleties in the interpretations.

A second family of anyons is believed to exist in the fractional quantum Hall state at $\nu = 5/2$. The even-denominator state is believed to be a $p$-wave paired state, known as the Moore-Read state or the Pfaffian state, which supports half-flux quantum vortex excitations. Such particle excitations carry $e/4$ charge each and, when interchanged, not just add a phase factor to the wave function, but evolve unitarily in its degenerate (or quasi-degenerate for finite systems) ground state manifold. They are dubbed non-Abelian anyons, which are also speculated to exist at $\nu = 12/5$. The existence of the non-Abelian anyons, although not confirmed by experiments yet, is of vital importance to topological quantum computing.

In theory, the wave functions of quasihole excitations can be written explicitly in analytic functions for both the Laughlin case and the Moore-Read case. They are also exact eigenstates of some special Hamiltonians with short-range two-body and three-body interactions, respectively. Exact diagonalization of finite systems has fruitfully revealed some of these quasihole/quasiparticle states. In systems with Coulomb interaction, such ground state descriptions appear to be sufficient even for electrons on a Corbino disk geometry in Abelian cases, as well as for electrons on a disk geometry in the non-Abelian case at $\nu = 5/2$. In the latter case, the change of the edge spectrum in the presence of an odd number of non-Abelian anyons at the origin implies the non-Abelian statistics of such excitations. In addition, up to four non-Abelian quasiholes have been induced and oriented tetrahedrally on a sphere, which results in two nearly degenerate states with very similar charge density profile (presumably a topologically protected qubit).

To achieve fault-tolerant quantum computing in the topological fashion, one needs to be able to create individual, paired, or a small cluster of anyons. One of the simplest experimental approaches is probably to use a biased AFM (atomic force microscopy) tip to create and trap anyons. One may then easily move the anyons localized at the tip to realize braiding to fulfill computation. However, the feasibility of creating anyons at an AFM tip has not yet been systematically studied even on the numerical level. In a earlier work by one of the authors and collaborators, it is demonstrated that a short-range repulsive local potential (as produced by a sharp AFM tip) can induce both $+e/4$ and $+e/2$ quasiholes, depending on the potential strength, in a $\nu = 5/2$ system. However, a mixture of long-range Coulomb interaction and short-range three-body interaction is used, and it is not clear whether negatively charged quasiparticles can be created in a similar fashion.

In this work, we study the excitation and trapping of both quasiholes and quasiparticles with a local potential in a microscopic model of fractional quantum Hall droplets with both long-range Coulomb interaction and...
realistic edge confining potential. We focus on the Laughlin 
primary filling fraction $\nu = 1/3$, although the ap-
proach can be applied to other filling fractions, including 
the intriguing $\nu = 5/2$ case\textsuperscript{24,26} to obtain similar results. 
We find that both positively charged quasiholes and neg-
atively charged quasiparticles can be excited generically 
by a finite-range tip potential with appropriate sign and 
strength. We confirm that edge spectrum of the system 
is not affected by the presence of a single quasihole, char-
acteristic of its Abelian nature. Our results suggest it 
is possible to trap individual anyons, as needed in topolog-
ical quantum computer proposals. We also discuss the 
stability of anyons when the strength of the confining 
potential varies.

The rest of the paper is organized as follows. In Sec. 
II, we consider the short-range hard-core potential, which 
generates the Laughlin state and the single-quasihole 
state as exact zero-energy ground states. We consider 
long-range Coulomb interaction in Sec. III, where we 
apply tip potentials of $\delta$-function, Gaussian, and expo-
ential forms. We summarize our results and discuss the 
relevance to experiments in the context of topological 
quantum computing in Sec. IV.

II. HARD CORE INTERACTION

In this section, we study the two-dimensional electron 
system on a disk at filling fraction $\nu = 1/3$ with 
short-range hard-core interaction between electrons. In 
Haldane’s pseudopotential language, $V_m = \delta_{1,m}$. The 
Laughlin state\textsuperscript{2} at the primary filling factor $\nu = 1/3$

$$\Psi_{1/3}(z_1 \cdots z_N) = \prod_{i>j}^{N} (z_i - z_j)^3 \exp \left\{ -\frac{1}{3} \sum_{i=1}^{N} |z_i|^2 \right\}$$  \hspace{1cm} (1)

is the exact ground state with zero energy in the sub-
space with total angular momentum $M_{\text{tot}} = M_L = 
3N(N-1)/2$ for $N$ electrons in at least $N_{\text{orb}} = 3N-2$ or-
bitals. In fact, it is the zero-energy ground state with the 
smallest allowed angular momentum; other zero-energy 
states (for larger $N_{\text{orb}}$) are known as edge states. We plot 
the density profile of the Laughlin state for 10 electrons in 
28 orbitals using palette-mapped 3D plot in Fig. II(a) and, 
for comparison, along the radial direction in Fig. II(b).

As Laughlin pointed out, the state with a single quasi-
hole at $\xi$ can be written as

$$\Psi_{1/3}^{qh}(\xi, z_1 \cdots z_N) = \prod_{i=1}^{N} (z_i - \xi) \Psi_{1/3}(z_1 \cdots z_n).$$  \hspace{1cm} (2)

In the disk geometry, $\xi$ can be placed at the origin to 
preserve rotational symmetry. Obviously, this is a zero-
energy ground state in the $M_{\text{qph}} = 3N(N-1)/2 + N$ 
momentum subspace for $N_{\text{orb}} > 3N-2$. In general, 
there can be additional zero-energy states in the same 
momentum subspace, with the wave function being the 
Laughlin state multiplied by a symmetric polynomial of 
order $N$. Such degeneracy can be lifted either by limiting 
$N_{\text{orb}} = 3N-1$, or by the addition of an impurity potential 
$H_W = W e_0^\dagger c_0$ at the $m = 0$ orbital. In Fig. II(c) and 
Fig. II(d), we plot the density profile of the quasihole 
wave function. A density deficiency around the origin 
is clearly visible, indicating the presence of a quasihole 
roughly the size of one magnetic length $l_B$.

On the other hand, the quasiparticle state of the cor-
responding Laughlin state is of some ambiguity. There 
is no zero-energy state obtained in the exact diagonaliza-
tion at $M_{\text{tot}} = M_{\text{qph}} = 3N(N-1)/2 - N$, because 
the excitation gap of the Laughlin liquid is finite. Here, 
it is hard to compare it with the variational quasiparticle 
wave function

$$\Psi_{1/3}^{qp}(\xi, z_1 \cdots z_N) = \prod_{i=1}^{N} e^{-|z_i|^2/4} \left[ 2 \frac{\partial}{\partial z_i} - \xi^2 \right] e^{|z_i|^2/4} \Psi_{1/3}(z_1 \cdots z_N)$$

proposed by Laughlin, which is not known as the exact so-
lution of any simple Hamiltonian. We assume that, like 
the quasihole state, the quasiparticle state is the ground 
state of the Hamiltonian of interested at the appropri-
ate angular momentum $M_{\text{qph}}$. Here we plot the density 
profile as such a candidate for a quasiparticle state in 
Fig. II(e) and Fig. II(f).

FIG. 1: (color online). The density profiles of the 2D elec-
trons with hard-core interaction. The Fig. (a) and (b) are the 
density of Laughlin state. Fig. (c) and (d) describe the quasi-
hole state while we add an external potential $W_0 c_0^\dagger c_0$ at $m=0$ or-
bit($W_0 = 0.1$). Fig. (e) and (f) are for the quasiparticle 
candidate state, which looks in the same shape as in ref.\textsuperscript{22}. 
Hamiltonian in the second quantization language reads

\[ H_C = \frac{1}{2} \sum_{m\ell n l} V_{m\ell n l}^c c_{m\ell n l}^c c_{n l m} + \sum_m U_m c_{m 0}^c c_m \]  

(5)

where \( c_m^c \) (\( c_m \)) creates (annihilates) an electron at the \( m \)-th orbital. \( V_{m\ell n l}^c \) are Coulomb matrix elements

\[ V_{m\ell n l}^c = \int d^2r_1 \int d^2r_2 \phi_{m\ell n l}(\vec{r}_1) \phi_{n l m}(\vec{r}_2) \frac{e^2}{\epsilon r_{12}} \phi_{n l m}(\vec{r}_1), \]

and \( U_m \) the background confining potential

\[ U_m = \frac{N e^2}{\pi R^2 \epsilon} \int d^2r \int_{\rho<R} d^2\rho \frac{\mid \phi_m(\vec{r}) \mid^2}{\sqrt{|\vec{r}^2 - \rho^2| + d^2}} \]  

(7)

In order to study the quasiparticle and quasihole excitations, we include an external local potential \( H_W \), created by an AFM tip, for example. So the complete Hamiltonian is

\[ H = H_C + H_W. \]  

(8)

In the following, we will consider three different forms of \( H_W \): (i) a short-range potential at the origin of the disk \( H_W = W_0 c_0^c c_0 \); (ii) a Gaussian potential \( H_W = W_2 \sum_m \exp(-m^2/2\xi^2)c_{m 0}^c c_m \); and (iii) an exponential potential \( H_W = W e \sum_m \exp(-m/\xi)c_{m 0}^c c_m \).

### A. Short-range potential at origin

A short-range potential can be produced by a very sharp AFM tip. By sharp we mean the range of the tip potential on the 2DEG is smaller than one magnetic length, the size of a single-particle wave function in the lowest Landau level. In this case, we can model the potential by \( H_W = W_0 c_0^c c_0 \), located at the origin in the disk geometry. A previous study has applied the short-range potential to create a single +e/4 quasihole and two +e/4 quasiholes (or a +e/2 quasihole) in a model of the fractional quantum Hall liquid at \( \nu = 5/2 \) with Coulomb interaction and an edge confining potential.

To begin with, we apply the same short-range potential \( H_W \) to the electron liquid at \( \nu = 1/3 \). We present the results of a system of \( N = 8 \) electrons in 26 orbitals (large enough so that edge excitations have low enough energies). The background charge is still confined to a disk of \( R = \sqrt{2N/\nu} = \sqrt{48} \) corresponding to the lowest 24 orbitals, at a distance \( d = 0.5l_B \) above the electron layer. We expect the ground state of Laughlin nature has a total angular momentum of \( M_L = 3(N - 1)/2 = 84 \), which is found to be right for zero and small \( W_0 \). When we increase \( W_0 \) above \( 0.26 \pm 0.01 \), the total angular momentum of the global ground state jumps from 84 to 92, indicating the excitation of a +e/3 quasihole. The density profile of the quasihole state is similar to that found for the hard-core potential (Fig. 1c and d), in which the electron density approaches zero at the origin.
In Figure 3, we compare the low-energy excitations of the system with and without the quasihole excitation. We identify the edge excitations, labeled by solid red bars, following the approach developed by one of the authors and his collaborators.\(^27\) We observe that in energy relative to the ground state, the edge spectrum looks almost identical with and without the quasihole, implying the Abelian nature of the quasihole. This contrast to the case of a Moore-Read state, where the presence of a +e/4 quasihole changes the fermionic edge excitations. The number of the edge states (including the ground state) are 1, 1, 2, 3, and 5 for Δ\(M\) = 0-4, as expected by the chiral boson edge theory.\(^{28,29}\)

The short-range potential is useful to generate a single +e/3 quasihole if the edge confinement is not too strong. However, since it only affect the local potential at a single orbital, a second quasihole cannot be induced, since one cannot deplete more than (on average) 1/3 charge in a single orbital in the Laughlin case. For the same reason, the short-range potential does not support a single quasiparticle (charged −e/3), as one can see from Fig. 2, a quasiparticle occupies several orbitals, unlike a quasihole. Therefore, we proceed to study local potentials after exciting a quasihole at the origin by a short range potential \(W_0\epsilon_0\) with \(W_0 = 0.3\).

![Figure 3](image1)

**FIG. 3**: (color online). (a) The low-lying energy states (edge states are marked by solid red bars) for 8 electrons in 26 orbitals with Coulomb interaction. (b) The energy spectrum after exciting a quasihole at the origin by a short range potential \(W_0\epsilon_0\) with \(W_0 = 0.3\).

![Figure 4](image2)

**FIG. 4**: (color online). The threshold strength \(W_\epsilon\) for the excitation of a quasihole (red dots) or a quasiparticle (green squares) in \(\nu = \frac{1}{3}\) systems with 6-10 electrons with Coulomb interaction, using a Gaussian potential \(H_W = W_\epsilon \sum_m \exp(-m^2/2s^2)\epsilon_m^\dagger \epsilon_m\) with width \(s = 2\), with a longer range.

### B. Gaussian-shaped potential

We now considered the Gaussian-shaped potential \(H_W = W_\epsilon \sum_m \exp(-m^2/2s^2)\epsilon_m^\dagger \epsilon_m\), or the potential has a value \(W_\epsilon \exp(-m^2/2s^2)\) on the \(m\)th orbital. The width of the potential is \(s\), while the strength of the potential \(W_\epsilon\). In the limit of \(s \to 0\), the Gaussian potential evolves into the short-range \(\delta\)-potential discussed in the previous subsection.

For fixed \(s = 2\) and \(d = 0.5l_B\), we vary \(W_\epsilon\) to study the change of the total angular momentum of the global ground state. For example, in a system of \(N = 10\) electrons in 30 orbitals, we find the total angular momentum \(M_{tot}\) jumps from \(M_L = 3N(N-1)/2 = 135\) to \(M_{qh} = 3N(N-1)/2 - N = 145\) at \(W_\epsilon = 0.16 \pm 0.01\), indicating the presence of one quasihole at the origin. For an attractive potential, \(M_{tot}\) drops from \(M_L = 135\) to \(M_{qh} = 3N(N-1)/2 - N = 125\) at \(W_\epsilon = -0.13 \pm 0.01\), indicating the emergence of a quasiparticle at the origin. Figure 4 shows the value of \(W_\epsilon\) at which \(M_{tot}\) changes for systems with 6-10 electrons. We find that the threshold values for the generation of one quasihole or one quasiparticle approaches a constant value of roughly \(\pm 0.15\) as the system size increases. It is not surprising that the threshold value is of the same energy scale as the bulk energy gap, but it also depends on the detail of the potential.

Figure 4 shows the electron density profiles for (a) the Laughlin state, (b) a one-quasihole state for \(W_\epsilon = 0.2\), and (c) a one-quasiparticle state for \(W_\epsilon = -0.2\). The density accumulation or depletion at the origin indicates the presence of a quasiparticle or quasihole. Compared to the case of the hard-core potential, the quasihole is slightly larger, while the quasiparticle is smaller with a...
the quasiparticle state has a large value around interaction. In Fig. 5(c), we find the electron density, which arises commonly in systems with Coulombian potential in systems with Coulomb interaction can- a quasiparticle-quasihole symmetry. A Gaussian potential with Coulomb interaction. A Gaussian potential

$$H = W_\ell \sum_m \exp(-m^2/2s^2)c_m^\dagger c_m$$

with $s = 2$ is applied.

The main difference between $d = 0.5l_B$ and $d = 1.0l_B$ occurs along the boundaries of quasiholes, not along the quasiparticle boundaries. This, we believe, is due to the fact that for a fixed number of electrons the quasihole states (not the quasiholes themselves) have larger size than the quasiparticle states, thus more susceptible to the edge confinement. The difference is more evident at smaller $s$ (sharper tips). In particular, a $\delta$-tip can excite

well-defined peak at the origin. Therefore, in the more realistic case with Coulomb interaction (and not too narrow a tip), the quasihole state and the quasiparticle state have roughly the same perturbation to the Laughlin ground state except for the opposite signs, suggesting a quasiparticle-quasihole symmetry.

Since the quasiparticle state we obtain for the Gaussian potential in systems with Coulomb interaction cannot be easily compared with the variational wave function [Eq. (3)], we want to make sure it is not a stripe phase, which arises commonly in systems with Coulomb interaction. In Fig. 5(c), we find the electron density of the quasiparticle state has a large value around the origin and edge. A stripe phase of $N = 10$ electrons in 30 orbitals with a somewhat similar density distribution and the same total angular momen-
a quasihole in the case of \( d = 1.0l_B \), but not in the case of \( d = 0.5l_B \) for not too large \( W_g \). This, as illustrated in Fig. 4, suggests that a finite width \( s \approx 2l_B \) may be more robust for the excitation of quasiholes and quasiparticles.

C. Exponential-shaped potential

In this section we discuss the exponential-shaped potential \( H_W = W_e \sum_m \exp(-m/\xi) \phi_m \phi_m \). In real space, this corresponds to a Gaussian potential \( V(z) = W_g e^{-|z|^2/2\sigma^2} \), which may not be too difficult to prepare in experiments. Explicitly, by projecting the real-space potential into the lowest Landau level, we obtain the matrix elements

\[
\langle \phi_m | V | \phi_m \rangle = \frac{W^e_g}{2\pi 2^m m!} \int_0^\infty \exp \left( -\frac{z^2}{2\sigma^2} - \frac{|z|^2}{l_B} \right) \frac{z^{2m} d^2 z}{l_B^{m+2}}
\]

\[
= W_g \left( \frac{\sigma^2}{l_B + \sigma^2} \right)^{m+1}
\]

where the decay length is \( \xi = 1/\ln(1 + l_B^2/\sigma^2) \) and the effective strength \( W_e = W_g \sigma^2/(\sigma^2 + l_B^2) \). Again, \( |\phi_m\rangle \) is the lowest Landau Level wave function with angular momentum \( m \).

After applying the exponential potential \( H_W = W_e \sum_m \exp(-m/\xi) \phi_m \phi_m \) with \( \xi = 1/\ln 2 \) (or \( \sigma = l_B \) in real space), we are also able to trap a single quasihole or a quasiparticle. Again, we consider a system of 10 electrons in 30 orbitals, with neutralizing confining charge located at a distance of \( d = 0.5l_B \) above the electron layer. The electron density profiles for the Laughlin state, the one-quasihole state, and the one-quasiparticle state are plotted in Fig. 5. The density profiles look very similar to those for the Gaussian potential discussed in the previous subsection (Fig. 3). Like the Gaussian case, the quasiparticle state and the quasihole state have roughly the same density perturbation (but with opposite signs) to the Laughlin ground state.
FIG. 8: (color online). The electron density profiles for (a) the one-quasiparticle state ($W_e = -0.2$) and (b) the one-quasihole state ($W_e = 0.28$) for the exponential potential $H_W = W_e \sum_c \exp(-m/\xi)c_m^\dagger c_m$ with $\xi = 1/\ln 2$ (or $\sigma = l_B$ in real space). The system has 10 electrons in 30 orbitals with Coulomb interaction. The neutralizing confining charge is located at a distance of $d = 0.5l_B$ above the electron layer.

IV. CONCLUSION AND DISCUSSION

To summarize, we study the trapping of quasiholes and quasiparticles by a local potential (e.g. induced by an AFM tip) in a microscopic model of fractional quantum Hall liquids with short-range hard-core interaction or long-range Coulomb interaction with an edge confining potential due to neutralizing charge. We find, in particular, at the Laughlin filling fraction $\nu = 1/3$, both quasihole and quasiparticle states can be energetically favorable for the ground state of the Coulomb system for tip potentials of various shape and strengths. The presence of the Abelian quasihole has no effect on the edge spectrum of the quantum liquid, unlike in the non-Abelian case when fermionic excitations are present.

Although quasiholes and quasiparticles can emerge generically in the system, its stability depends on the strength of the confining potential, the strength and the range of the tip potential. Experimentally the quantum Hall plateau at $\nu = 1/3$ was found in a high magnetic field ($\sim 15$ T)\(^1\). In this case the magnetic length $l_B \approx 70$ Å. Based on our microscopic calculation, we estimate a optimal range of the tip potential to be 140 Å. The size falls in the right range of AFM tip size under current technology. The Laughlin state in the context of topological quantum computing is of less interest due to its Abelian nature, although it can be used for topological quantum memory. Nevertheless, it is much easier to model in numerical studies than the non-Abelian Moore-Read state\(^2\) and the even more complicated Read-Rezayi (parafermion) states\(^3\). We expect the results found here can be of help for the excitation and trapping of quasiholes or quasiparticles in the Moore-Read case in future experiments. In the Moore-Read case at filling fraction of 5/2, a smaller magnetic field $\sim 5$ T is usually applied. Thus with a longer magnetic length we can have even wider tips, which should not be a technical challenge.

With the well-known difficulties of the exact diagonalization method in highly entangled systems such as the fractional quantum Hall liquids, the search for the ground states with a few parameters is a time-consuming job. The Moore-Read case is even more complicated, since the even-denominator state has a smaller excitation gap and is competing with stripe phases\(^4\). One might wish to develop more efficient numerical methods to approach the ground state properties. One development in recent years is the application of density-matrix renormalization group (DMRG) method\(^5\) to the fractional quantum Hall systems. We implement the method in the disk geometry with results in excellent agreement with exact diagonalization in small systems\(^6\). However, we find the time to reach convergence (especially near the origin) in larger systems is impractically long for the extensive search for the ground states discussed in the current paper.

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