The formation of nonlinear sound fields in the boundary region of the bubble medium

I A Ogorodnikov

Kutateladze Institute of Thermophysics, 1 Lavrentyev Ave., 630090, Novosibirsk, Russia

*E-mail: igoro47@yandex.ru

Abstract. The formation of sound fields in the boundary region of a bubble medium under the influence of a powerful sound pulse for different volume concentrations of bubbles is studied. Two characteristic ranges of the volume concentration of bubbles, to the limits of which different types of wave fields are formed, are determined. At low values of volume concentrations in the boundary region, a long-lived space-limited standing sound wave is formed. A standing sound wave contains an order of magnitude more energy than other areas of the bubble medium. At large values of the volume concentration of bubbles in the boundary region of the bubble medium, an excitation zone forms. The structure of the zone is a periodic wave propagating deep into the bubble medium. The wavelength in the excitation zone increases with increasing distance from the boundary.

1. Introduction

The study of sound propagation in inhomogeneous bubble media is of scientific and applied interest [1, 2], in particular for improving methods of sonar in the ocean. Physical interest is that a bubble is one of two examples of natural monopoles. The second example of monopoles is stars. The traditional approach, which served to study many interesting properties of weakly nonlinear waves in a liquid with bubbles, uses equations of Korteweg de Vries, Burgers, Khokhlov-Zabolotskaya, Klein-Godon [3,4,5,6]. However, this approach has significant limitations. The equations are obtained under the assumption of finite, but small values of pressure in the wave and, accordingly, nonlinear, but small deviations of the radius of the bubbles from the initial value of the radius. The second limitation is the assumption of slow changes in the parameters of the bubble medium, which provides the introduction of averaged parameters. Models of this type do not allow studying problems with sharp changes in the volume concentration of bubbles in the medium at scales shorter than the length of the studied waves and large amplitudes of the excitation waves, when the volume of the bubbles changes several times. In addition, due to the application of averaging procedures, these models prevent from analyzing the behavior of individual bubbles in a bubble medium and their influence on the formation of wave fields.

In [7], on the basis of the microscopic approach, a nonlinear wave model was obtained to simulate sharp changes in the volume concentration of bubbles and an arbitrary distribution of bubble sizes in a bubble medium.

The aim of the work is to analyze the characteristics of emerging sound fields and the dynamic behavior of bubbles in the boundary region of a homogeneous bubble medium and to determine the effect of the volume concentration of bubbles at large amplitude of the exciting wave.
2. Physical statement of the problem
In an unlimited fluid, the part is uniformly filled with the same air bubbles. A plane sound wave in the form of a short pulse of large amplitude is incident from a pure liquid onto a region with bubbles. The subject of study is the formation of wave structures and the dynamic behavior of bubbles in the boundary region of the bubble medium.

3. Nonlinear wave system of equations
The radiation characteristics of the bubble layer were studied in a one-dimensional formulation using the wave model [7]. The system of equations has the form:

\[
\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = -\frac{1}{c^2} \frac{\partial}{\partial t} \left( p \frac{\partial}{\partial t} \ln(1 - \alpha) \right) \tag{1}
\]

\[
RP \frac{d^2 R_k}{dt^2} + 2 \left( \frac{dR_k}{dt} \right)^2 + \frac{4 \mu}{\rho_0 R_k} \frac{dR_k}{dt} + \frac{2\sigma}{\rho_0} = \frac{1}{\rho_0} \left[ P_0 + \frac{2\sigma}{R_{k0}} \right] - \frac{P_0}{\rho_0} \frac{\partial}{\partial t} \frac{p(x_k, t)}{\rho_0} \tag{2}
\]

\[
\alpha(x, t) = \sum_k V_k(t) \cdot \delta(x - x_k(t)) \tag{3}
\]

\[
V_k(t) = \frac{4}{3} \pi R_k^3(t). \tag{4}
\]

\(k = 1, ..., N\) defines the ordinal number of the bubble in the study area, \((x-x_k)\) characterizes the position of the \(k\)th bubble in space, \(P(x, t)\) is the pressure in wave, \(P_0\) is the initial pressure in the medium, \(c\) is the speed of sound in water, \(\alpha\) is the volume concentration of bubbles, \(R_0\) is the radius of the \(k\)th bubble, \(V_k\) is the volume of \(k\)th bubble, \(\rho\) is the density of liquid, \(\sigma\) is the surface tension of liquid, \(\mu\) is the viscosity of liquid, \(t\) is the time, and \(x\) is the spatial coordinate. Gas in bubbles follows the adiabatic law, and pressure and density of water are related by expression \(P(x, t) = (x, t)c^2\).

For numerical solutions of system (1) - (4), dimensionless variables are introduced:

\[
\delta R_k = R_k / R_{k0}, \quad \delta P = P / P_{w0}, \quad \delta t = t / (R_0 \sqrt{(\rho_0 / P_{w0})}), \quad \delta x = x / (cR_0 \sqrt{(\rho_0 / P_{w0})}). \tag{5}
\]

The method of solution and verification of the system of equations (1) - (4) are described in [7]. All calculations were performed for the amplitude of the sound excitation wave \(P_{w0}=1\ \text{MPa}\) and wavelength \(\tau=30 \cdot 10^{-6}\ \text{s}\).

The choice of the duration of the sound wave exciting the bubble medium is due to the fact that at these values a single resonant soliton is formed in the medium [7].

Solutions of the system of equations (1) - (4) were obtained for the values of the volumetric concentration of bubbles \(\alpha \cdot 10^{-6}, 10^{-3}, 10^{-2}\). The values of the remaining parameters used in the study are as follows: \(R_0=0.25 \cdot 10^{-3}\ \text{m}, \gamma=1.4, \rho=103\ \text{kg/m3}, s=1500\ \text{m/s}\).

4. Discussion of the results
Figure 1 shows a graph of dependence of the equilibrium speed of sound Mallock cm on the volume concentration of bubbles in a liquid with bubbles. The equilibrium velocity is determined by the formula \(c_m=\sqrt{(\gamma P V/(\rho_0 \alpha (1-\alpha)))}\), which is derived under the condition of equilibrium of gas pressure in bubbles and pressure in the wave [8]. Two areas are highlighted on the graph. In the first region, the volume concentration of bubbles is \(\alpha < 5 \cdot 10^{-4}\) and the equilibrium speed of sound is close to the speed of sound in a pure liquid. In the second region, the equilibrium speed of sound is significantly lower than the speed of sound in a pure liquid.
Figure 2 presents the spatial profile of the wave field $\delta P (x)$ and the profile of deviations of the radii of the bubbles $\delta R (x)$ from equilibrium values at a fixed point in time in the boundary region of the bubble medium.

The calculation results are presented in a dimensionless form. Numerical solutions are obtained for $\alpha = 10^{-6}$. A sound impulse, moving through a bubble medium, does not change its original shape. The speed of a sound pulse in a bubble medium is 1482.89 m/s. The reflection coefficient $r$, expressed as the ratio of energy of the reflected wave to the energy of the incident wave $E_{ref}/E=0.006$ at an energy in the incident wave $E=5.76$ J.

For these conditions of the problem, a sound pulse travels through the medium, causing strong nonlinear pulsations of the bubbles.

At maximum compression, the bubble radius assumes the values of $0.428 R_0$, while the local value of the volume concentration of bubbles decreases to $7.8 \cdot 10^{-8}$, which is 12.8 less than the initial value.

The volume concentration of bubbles in the region of maximum expansion of the bubbles increases 3.4 times with respect to the initial value.

The spatial profile of the rarefaction and compression is shown behind the main pulse in figure 2. The trace of the sound pulse moves along the bubble medium with the speed of the pulse. Traditional models for studying nonlinear waves in a bubble medium [3,4,5,6] do not allow studying wave processes at given parameters.

At long times, which exceed the duration of the exciting sound pulse by an order of magnitude, a standing wave is formed in the boundary region of the bubble medium. Figure 3 presents a series of instantaneous profiles of the pressure field, and figure 4 shows a series of instantaneous profiles of the spatial distribution of the radii of the bubbles. The graphs determine the zone of the standing wave and allow you to determine its parameters. At these values, a standing wave was formed in the form of a half-wave $\lambda/2$, where $\lambda$ is the wavelength with two external nodes.
Pressure fluctuations and pulsations of bubbles are in-phase at all points of the bubble medium. Counter waves are absent. The spatial size of the incident sound pulse for given parameters is $4.4 \cdot 10^{-2}$ m, and the half-wave length is $\lambda = 16.5 \cdot 10^{-2}$ m, which is 3.72 times more than the spatial size of the exciting pulse.

The amplitude in the standing wave is $2 \cdot 10^{9}$ Pa, and the energy concentrated in the region of the standing wave $E_{w} = 3.6 \cdot 10^{4}$ J is an order of magnitude higher than the average energy in the bubble medium. Due to this, the region of the standing wave is an emitter into a pure liquid and into a bubble medium. A characteristic feature of this region is that the bulk of the energy is concentrated in the bubbles and is equal to $3.57 \cdot 10^{-3}$ J.

In the wave field in this region, the energy is two orders of magnitude less and it is $2.94 \cdot 10^{-5}$ J. The amplitude of a sound wave in water is $1.14 \cdot 10^{3}$ Pa, and in a bubble medium $4.14 \cdot 10^{3}$ Pa. Sound wave in pure water at wavelength contains $1.36 \cdot 10^{4}$ J of energy. In bubble medium at a wavelength it contains $7.89 \cdot 10^{4}$ J of energy. The wave component of the field contains energies $E_{w} = 6.0 \cdot 10^{4}$ J. The bubbles contain 3 times less energy equal to $E_{b} = 1.88 \cdot 10^{4}$ J. The greater amount of energy in the wave in the bubble medium is due to the fact that the wave radiated into the bubble medium propagates through the excited medium and this radiation is synchronized with vibrations in the bubble medium.

The formation of a standing wave in the border zone depends on the duration of the exciting sound wave. A decrease in the duration of the sound wave and, as a consequence, the expansion of the spectrum lead to the expansion of the region, in which the standing wave is formed, and the appearance of internal nodes.

As an example, figure 5 shows a series of instantaneous pressure profiles in the boundary region, when the bubble medium is excited by a sound pulse of the same amplitude with lesser duration $\tau = 24 \cdot 10^{-6}$ s.

In this case, the standing wave consists of three half-waves with two internal nodes and has a half-wave length of $\lambda/2 = 9.45 \cdot 10^{-2}$ m.

![Figure 5. A series of instantaneous pressure field profiles. Three half-waves.](image-url)

In the region where the equilibrium speed of sound is significantly lower than the speed of sound in a pure liquid, another field structure is formed in the boundary zone of the bubble medium. Figure 6 presents spatial profiles of the pressure field $\delta P(x)$ and the corresponding distribution of the radii of the bubbles $\delta R(x)$ at a fixed point in time at $\alpha = 10^{3}$.

The reflection coefficient $r = 0.168$, so over 80% of the energy of the exciting sound wave enters the bubble medium. A bubble medium at a given volume concentration of bubbles has sufficient energy intensity so that under the strong influence of an excitation wave, a resonant soliton, a sound precursor, and an excited zone are formed inside the bubble medium [9].

The excited zone is formed in the form of a traveling wave. Fluctuations in the pressure field and the radii of the bubbles in the excitation zone occur in antiphase.
Pressure fluctuations and pulsations of bubbles are in-phase at all points of the bubble medium. Figure 8 shows the solution when $\alpha=10^{-2}$.

Counter waves are absent. In this case, an almost complete reflection of the sound wave from the region of the liquid with bubbles occurs. Reflection coefficient $r=0.849$. Fluctuations in wave pressure and pulsation of bubbles occur in anti-phase. The rate of expansion of the excited zone is greater than the equilibrium speed of sound at the corresponding values of the volume concentration of bubbles. This is explained by the fact that, despite the large reflection coefficients, sound fields with amplitude of $10^4 \text{ Pa}$ are formed inside the bubble medium. The propagation velocity of waves of this intensity is greater than the equilibrium velocity of sound [7].

**Conclusions**

Studies have shown that near the interface between a pure liquid and a bubble medium, two types of wave structures are formed inside a liquid with bubbles under the influence of a powerful sound pulse. At small values of the volume concentration of bubbles in the boundary region, a standing wave localized in space is formed. The number of half-waves in a standing wave depends on the duration of the exciting sound pulse. The energy density in the region of the standing wave is much higher than that in the rest of the bubble medium. Pressure fluctuations in the field of a standing wave are in phase with pulsations of the bubbles. In the region of large values of the volume concentration of bubbles in the border zone, regular traveling waves are formed in which the wavelength decreases with distance from the border. Fluctuations in the pressure field and pulsations of the bubbles are in anti-phase.

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**Figure 6.** Profiles of pressure fields and bubble radii.

**Figure 7.** Synchronization of pressure field profiles and bubble radii.

**Figure 8.** Spatial profiles of the pressure field and the radii of the bubbles near the border.
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