Lorentz Invariant CPT Violating Effects for a Class of Gauge-invariant Nonlocal Thirring Models

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Abstract

CPT violation and Lorentz invariance can coexist in the framework of non-local field theory. Local gauge-invariance may not hold for the few non-local interaction terms. However, the gauge-invariance for the non-local interaction term can be formulated by the inclusion of Swinger non-integrable phase factor. In this article we have proposed a class of CPT violating Lorentz invariant Nonlocal Gauge-invariant models which can be termed as non-local gauge-invariant Thirring models. The inclusion of non-locality will modify the current conservation laws. Also, the possible particle antiparticle mass-splitting in this respect is discussed.

Keywords: CPT-violation, Non-local interaction, Thirring Model, Particle anti-particle mass-splitting

1. Introduction

Recently the work of Chaichain et. all [1, 2, 3, 4] had open a new possibility of theoretical understanding of the phenomenon of particle antiparticle mass splitting which was recently speculated by the data analysis of recent experiments[5, 6, 7]. Some phenomenological description are in this aspect also available in the literature[8].

According to the discussion available till date, strong support for the effect of non-local interaction present in the process is identified. As, it is well known fact that within the Lorentz invariant framework every local field theory will obey the well established CPT theorem which immediately followed by the equality of mass of particle and corresponding antiparticle, it is natural to give the first effort by using the non-local theory to deal with the particle-antiparticle mass splitting phenomenon. The crucial debate for the sensible CPT violating non-local theory is that whether such theory can have sensible S-matrix. The S-matrix structure for the non-local field theory have been studied by several author [9, 10, 11, 12]. First claim of Yukawa [9] was criticized by Yennie [10] by showing that for general case of non-local field theory the convergence of S-matrix was ambiguous. Probably, the most concrete mathematical formalism for the S-matrix for non-local field theory was constructed by Ruijsenaars [11]. In that article they have constructed S-matrix for general non-local integrable field theory and also they explicitly had studied the non-local Thirring model for different physical and non-physical sectors. Later in the year 1988 the covariant, unitary and causal S-matrix was constructed by Kuryshkin and Zorin [12]. Therefore, it will not be a over ambitious claim if one say that “local and non-local fields may share the same S-matrix”[13]. In this article we shall not distract ourselves by those controversy issues which are still on a shaky ground. Rather, we shall try to observe the results after inclusion of non-local CPT violating interaction for the Thirring model.

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in Gauge invariance manner. Also, another controversy issue is that the Unitarity will be broken in general for the inclusion of non-locality. In our scheme, we treat the non-local coupling sufficiently small such that the effects of violation of unitarity is expected to be minimal.

The concrete mathematical formulation and as well as physical implications of non-local field theory were studied in great detail in the literature of theoretical physics by several authors [14, 15, 18, 16, 17]. Recently, Chaichian et. all [1] has constructed a class of nonlocal Lorentz invariant field theories which do not respect the CPT invariance principle. In [2, 3, 4] they had also proved the possibility of mass-splitting for the non-local interaction term and they had argued that the equality of masses of particles and antiparticles is due to Lorentz invariance rather than to CPT.

The main idea behind the process is to introduce an infrared divergent form factor [1, 2] in the interaction term. One can use the form factor $F((x-y)^2)\theta(x^0-y^0)\delta((x-y)^2)$; where $\theta(x)$ is the step function ($\theta(x) = 1$ for $x > 0$ and $\frac{1}{2}$ for $x = 0$ and vanishes otherwise). The weight function $F((x-y)^2)$ may be taken as for example Gaussian type $F((x-y)^2) = \exp \left(-\frac{(x-y)^2}{l^2}\right)$; $l$ being the non-locality length in the considered theory [1] (it should be noted that in the limit $l \to 0$ the weight function becomes a delta function which made the form factor infrared divergent).

In the similar manner we have inserted the infrared diverging form factor in the nonlocal interaction term and have studied the Thirring model coupled with electromagnetic field in the aid of the current conservation and as well as mass splitting of particle and corresponding antiparticle. To keep track with the Gauge-invariance, one can easily understand that we have to introduce nonintegrable phase factor (this can be replaced by a first quantized very massive particle propagation; an analogue of the Chan-Paton factor in string theory [4]) in the nonlocal interaction part.

One point we like to mention is that, it was first (with the best of our knowledge) successful attempt by Chaichian and his collaborators [4] to incorporate the gauge-invariant scenario in Lorentz invariant CPT-violating non-local cases.

We have choose the Thirring model coupled with Electro-magnetic field to study, because, it is well known that for specific values of the parameter of Sine Gordon system it becomes equivalent to the Thirring model [19]. The various aspects as well as solutions of Thirring model for 1+1 dimension have been studied by several authors [20, 21, 22, 23]. The most important (perhaps) observation for the Thirring model is though the Sine-Gordon equation is the theory of massless scalar field, the Sine- Gordon soliton [19] can be identified with the fundamental fermion of the Thirring model. In a recent article [24] the present authors had discussed the situation of particle anti-particle mass-splitting for a class of non-local Thirring models without invoking the local gauge-invariant scenario. The result of that article indicates that the equality of masses holds for gauge-invariant case; while mass splittings were observed for the cases which were not gauge-invariant.

In this article we have studied the aspects of modification of current conservation laws and possible particle anti-particle mass-splitting for three different possible cases of non-local local U(1) gauge-invariant interaction terms.

2. Gauge invariant Nonlocal Thirring models

We consider the action

$$S = \int d^4x \{\bar{\psi}(x)i\not\!D\psi(x) - m\bar{\psi}(x)\psi(x)$$

$$-\lambda j^\mu(x)j_\mu(x) - i\mu \int d^4y[\theta(x^0-y^0)$$

$$-\theta(y^0-x^0)]\delta((x-y)^2 - l^2)J(x,y)\}$$

$$-\frac{1}{4} \int d^4x F^{\mu\nu}(x)F_{\mu\nu}(x)$$

(1)

Here, $\psi$ are the Dirac fields, $\not\!D = \gamma^\mu D_\mu$, $\gamma^\mu$ are the usual Dirac matrices, $D_\mu = \partial_\mu - iA_\mu$ is the covariant derivative, $j^\mu(x) = \bar{\psi}(x)\gamma^\mu\psi(x)$, $\lambda$ and $\mu$ are the coupling constants, $\theta(x)$ stands for the usual Heviside step function, $\delta(x)$ is the usual Dirac delta function, $F^{\mu\nu}(x) = \partial_\mu A_\nu - \partial_\nu A_\mu$. 
J(x, y) in the nonlocal term will be different for different cases under consideration.

Because of the frequent appearance of the combination \[\theta(x^0 - y^0) - \theta(y^0 - x^0)\] \[\delta((x - y)^2 - l^2)\] we define for neatness

\[\theta(x^0 - y^0) - \theta(y^0 - x^0)\] \[\delta((x - y)^2 - l^2) = \Theta(x, y)\] (2)

Now, to keep track with the local gauge-invariance we will consider only following three choices of \(J(x, y) \in \{J_1(x, y), J_2(x, y), J_3(x, y)\}\).

\[J_1(x, y) = j^e(x)j_1(x, y)\] (3)
\[J_2(x, y) = j^e(x)j_1(x, y)\] (4)
\[J_3(x, y) = j^e(x)j_3(y)\] (5)

For the third case above, the interaction term is automatically gauge invariant. For the rest of the two cases the correct choice of \(j^e(x, y)\) should be (we have to introduce Schwinger non-integrable phase factor):

\[j^e(x, y) = \bar{\psi}(x)\gamma^\mu e^{ie^\gamma_5 A_\mu(z)dz^\mu} \psi(y)\] (6)

Lorentz invariance is manifested by the \(\theta\) function. CPT property can be verified by straightforward manner. In the local field theory models, the purely imaginary coupling (\(i\mu\), with \(\mu\) real) never appears. Because of the anti-linear property of time reversal operator (T), the appearance of the non-integrable phase factor is replaced by a real factor. In the local field theory models, Lorentz invariance is manifested by the \(\theta\) function.

3. Current conservation

As the fermion pair creation can be examined through the lowest order interaction, we expand the non-integrable phase factor to the lowest order in O(\(e\)) as follows

\[e^{ie^\gamma_5 A_\mu(z)dz^\mu} = 1 + ie^\gamma_5 A_\mu(z)dz^\mu\] (9)

Now we shall study each of the three cases separately.

3.1. Case-1

In that case the action is given by

\[S_1 = \int d^4x \{\bar{\psi}(x)i\gamma^\mu D_\mu \psi(x) - m\bar{\psi}(x)\psi(x)\]
\[-\lambda j^\mu(x)j_\mu(x) - i\mu \int d^4y \Theta_\delta(x, y)\]
\[J_1(x, y)\} - \frac{1}{4} \int d^4xF_{\mu\nu}(x)F_{\mu\nu}(x)\] (10)

In this case the relevant part with \(A_\mu\) contribution is given by

\[S_1^I = e \int d^4x \bar{\psi}(x)A(x)\psi(x) + e\mu \int d^4y \Theta_\delta(x, y)j^\mu(x)\bar{\psi}(x)\]
\[\gamma^\mu \int_{\tau_y}^{\tau_x} [\delta^4(z(\tau) - w) \frac{dz^\mu}{d\tau}] d\tau \psi(y)\] (11)

And the corresponding electromagnetic current is

\[J^\mu(w) = \frac{\delta}{\delta A_\mu(w)} S_1^I = e\bar{\psi}(w)\gamma^\mu \psi(w)\]
\[+ e\mu \int d^4x d^4y \Theta_\delta(x, y)j^\mu(x)\bar{\psi}(x)\]
\[\gamma^\mu \int_{\tau_y}^{\tau_x} [\delta^4(z(\tau) - w) \frac{dz^\mu}{d\tau}] d\tau \psi(y)\] (12)

Where \(z^\mu\) stands for the coordinate of the massive particle. In true sense the inclusion of path integral for the very massive particle as in equation (8) will alter the \(\delta^4(z(\tau) - w) \frac{dz^\mu}{d\tau}\) term as

\[\frac{1}{Z} \int d^4z' \delta^4(z'(\tau) - w)\langle x, \tau_x | z', \tau \rangle \frac{dz'^\mu}{d\tau} d\tau\]
\[\langle z', y, \tau_y \rangle = \frac{1}{Z} \langle x, \tau_x | \delta^4(z(\tau) - w)\]
\[\frac{dz^\mu(\tau)}{d\tau} | y, \tau_y \rangle\] (13)
where \( Z \) is the normalization factor of the path integral partition function.

Now the current \( J^\mu_t(w) \) is indeed conserve \((\partial_\mu J^\mu_t(w) = 0)\) by virtue of the equation of motion
\[
i\partial_\mu \psi(x) - m\psi(x) - \lambda(\gamma^\mu \psi(x)j_\mu(x) \\
+ j^\mu(x)\gamma_\mu \psi(x)) - i\mu \int d^4y \Theta_\delta(x,y) [\gamma^\mu \\
\psi(x)j_\mu(x,y) + j^\mu(x)\gamma_\mu(1 + \\
i\epsilon \int_y A_\mu(z)dz^\mu)\psi(y)] = 0 \tag{14}
\]
and its conjugate equation. One have to just note the following identity
\[
\frac{\partial}{\partial \mu^\nu} \delta^4(z(\tau) - w) \frac{dz^\mu}{d\tau} = - \frac{d}{d\tau} \delta^4(z(\tau) - w) \tag{15}
\]
and the following relation
\[
\delta^4(z(\tau) - w)|y, \tau_y = \delta^4(y - w)|y, \tau_y \tag{16}
\]
In the similar manner just mentioned, we can study the other two cases.

### 3.2. Case- 2

The relevant part for the lowest order pair creation in that case is given by
\[
S^I_2 = \int d^4x \{ e\bar{\psi}(x)A(x)\psi(x) \\
+ e\mu \int d^4y \Theta_\delta(x,y)\bar{\psi}(x)\gamma^\mu(\int_y A_\alpha(z) \\
dz^\alpha)\psi(y)\bar{\psi}(x)\gamma_\mu(\int_y A_\alpha(z)dz^\alpha)\psi(y)\} \tag{17}
\]
And the current,
\[
J^\mu_t(w) = e\bar{\psi}(w)\gamma^\mu \psi(w) \\
+ e\mu \{ \int d^4x d^4y \Theta_\delta(x,y)\bar{\psi}(x)\gamma^\mu \\
\int_{\tau_y}^{\tau_x} [\delta^4(z(\tau) - w) \frac{dz^\mu}{d\tau}] d\tau \psi(y)\bar{\psi}(x) \\
\gamma_\mu \psi(y) + \bar{\psi}(x)\gamma^\mu \psi(y)\bar{\psi}(x) \gamma_\mu \\
\int_{\tau_y}^{\tau_x} [\delta^4(z(\tau) - w) \frac{dz^\mu}{d\tau}] d\tau \psi(y)\} \tag{18}
\]
which is conserve in the same way as stated for the case-1 above.

### 3.3. Case- 3

The relevant contribution,
\[
S^I_3 = \int d^4x \{ e\bar{\psi}(x)A(x)\psi(x)\} \\
\therefore J^\mu_t(w) = \bar{\psi}(w)\gamma^\mu \psi(w) \tag{19}
\]
One should note that in the case-3 the fermion lines will not be discontinuous contrary to the previous two cases where the fermion lines were discontinuous. This is expected because the inclusion of non-integrable phase factor was handled by the replacement with first quantized very massive (indefinite) particle propagation which have no role in the case-3 (as this is automatically gauge-invariant).

Intuitively one may argue that the inclusion of the intermediate very massive (indefinite) particle propagation may break the equality of masses of particle and corresponding anti-particle and one will observe the possible mass-splitting between particle and antiparticle. Indeed, this is the case and in the next section possible mass-splitting is illustrated.

### 4. Particle Anti-particle Mass-Splitting

One may argue that the asymptotic single particle scattering state probably is not the feature of the non-local theory. But as we take the non-local coupling \((\mu)\) small such that the violation of unitarity is minimal. So, we are assuming the ansatz
\[
\psi(x) = U(p)e^{-ipx} \tag{21}
\]
for the equation of motion.

#### 4.1. Case- 1

The use of this ansatz (21) in eq.(14) will give the following dispersion relation
\[
\not{p} + e\not{A} = m + (\gamma^\mu U(p)\not{\bar{U}}(p)\gamma_\mu + \\
\bar{U}(p)\gamma^\mu U(p)\gamma_\mu)(\lambda + i\mu \int d^4y \Theta_\delta(x,y) \\
(1 + i\epsilon \int_y A_\mu(z)dz^\mu)e^{-ip(y-x)}) \tag{22}
\]
If we choose a light-like frame \((\vec{p} = 0)\) and use the so called ”Gordon identity” \((\bar{U}(p)\gamma^\mu U(p)\gamma_\mu =\)

4
The appearance of nonzero path dependent term $\beta$ is natural due to the inclusion of non-integrable phase factor. After the transformation $p \to -p$ and sandwiching the equation with $\gamma^5$, we get

$$\gamma^0 p_0 + e \gamma^\mu A_\mu = m + \lambda (4 \gamma^0 p_0 - 2m)$$

where,

$$f_\pm (p) = \int dz \theta (z^0) \delta (z^2 - l^2) e^{\pm ipz}$$

The situation is quite different. Because, the above mentioned procedures will give the relation

$$\gamma^0 p_0 = m + (\gamma^0 p_0 - 2m)$$

which remains unchanged if we change $p \to -p$ and sandwich the equation with $\gamma^5$ i.e, this do not flips chirally. So, this type of interaction will not give the particle anti-particle mass-splitting contrary to the previous two cases.

5. Conclusions

Within the minimal violation of unitarity, we can tell that non-local interactions may be one of the prominent candidate for the description of particle anti-particle mass-splitting which has been recently speculated for neutrino anti-neutrino case. At the starting point one may think that within the Lorentz-invariant CPT violating formulation, the inclusion of infrared divergent form factor is sufficient to formulate the mass-splitting of particle anti-particles. But, the case-3 of this article shows that local gauge-invariant principal is an essential ingredient for the equality of masses of particle and corresponding antiparticles. As the non-local interactions were failed to be local gauge-invariant in the rest of the two cases of this article, we had included by hand a Swinger-nonintegrable phase factor which was handled with the replacement of very massive (indefinite) particle propagation (similar to Chan-Paton factor in String-theory) which essentially breaks the particle anti-particle mass equality. In few articles the mass-splitting scenario was tried to be explained with the help of inclusion of infrared divergent form factor without invoking the gauge-invariant scenario. Actually, in those articles the Lorentz-invariant CPT violating non-local interactions were not local gauge-invariant. Therefore, an important question arises - which symmetry is the main controller for the equality of particle anti-particle mass? After all the
local gauge-invariance is nothing but the manifestation of the fact that to deal with the algebraic operation between fields and their derivatives at two different points in meaningful way so that the difference in phase transformation is compensated, one consider the comparator function in unitary way and as a result connections (Gauge field) arises. Therefore, it may be helpful to study the scenario by various aspects of the comparator to deal with the situation (just a speculation); this may be an interesting study in future. Also, it may be an interesting study to apply the same technique of inclusion of non-local interaction term for other types of non-linear systems.

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