Ultra-high energy neutrino-nucleon cross section and radiative corrections

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A cubic kilometer scale experiment has been proposed to detect cosmic neutrinos of energy from
tens of GeV up to the highest energies observed for cosmic rays, \( \sim 10^{20} \text{eV} \), or possibly even beyond. Detection efficiencies depend crucially on the neutrino-nucleon cross section at these energies at which radiative corrections beyond the lowest order approximation could become non-negligible. The differential cross sections can be modified by more than 50\% in some regions of phase space. Estimates of corrections to the quantities most relevant for neutrino detection at these energies give, however, less dramatic effects: The average inelasticity in the outgoing lepton is increased from \( \gtrsim 0.19 \) to \( \gtrsim 0.24 \). The inclusive cross section is reduced by roughly half a percent. The dominant uncertainty of the standard model ultra-high energy neutrino-nucleon cross section therefore still comes from uncertainties of the parton distributions in the nucleon at very low momentum fractions.

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1. INTRODUCTION

Several proposals have recently been put forward for the search for cosmic neutrinos above tens of GeV up to the high energy end of the cosmic ray spectrum, and possibly beyond. The most well developed technique is to detect Cherenkov light from the muon produced in a charged-current reaction of an ultra-high energy (UHE) muon neutrino with a nucleon in either ice or water. Several prototype detectors based on this technique have been constructed, namely DUMAND at a depth of nearly 5 km in the ocean near Hawaii (now out of commission), the neutrino telescope at Lake Baikal about 1 km deep, NESTOR at about 3.5-km depth in the Mediterranean near Greece, and AMANDA up to 2 km deep in South Pole ice [4]. Other techniques have been proposed such as detection of horizontal air showers [5], or detection of acoustic [6] or radio waves [7] associated with the neutrino induced cascades.

For a given neutrino flux, the efficiency of all these methods depends predominantly on the neutrino-nucleon interaction cross section. To lowest order in the electroweak (EW) coupling, this cross section has been discussed in detail in the literature, see, e.g., Refs. [8] for the most recent work. Due to uncertainties in the extrapolation of quark distribution functions in the nucleon to very small fractional momentum transfers, \( x \lesssim 10^{-4} \), and large (negative) squared four-momentum transfers, \( Q^2 \gtrsim 10^5 \text{GeV}^2 \), best estimates for energies around \( 10^{20} \text{eV} \) (in the laboratory frame) vary by factors of a few.

On the other hand it is well known that higher order processes can become important or even dominant for electromagnetic (EM) interactions at UHE. For example, energy exchange between two leptons \( l_1 \) and \( l_2 \) is dominated by EM bremsstrahlung, \( l_1 + l_2 \to l_1 + l_2 + \gamma \), for center-of-mass energies \( s \) exceeding the square of the electron mass, rather than by ordinary Mott scattering, \( l_1 + l_2 \to l_1 + l_2 \). The relevant cross section rises with the logarithm of \( s \).

EW radiative corrections to deep inelastic neutrino-nucleon scattering have been calculated before in the literature: Ref. [8] contains a discussion of the leading log approximation for which only corrections involving photons are relevant. These corrections contain a factor \( \ln(s/m_{l}^2) \) where \( m_{l} \) is the charged lepton mass and also depend on the behavior of the parton distribution functions. Single and double differential cross sections have been evaluated in Ref. [8] for neutrino energies \( E_{\nu} \) up to a few hundred GeV, using rough estimates of the parton distributions available back then.

To compare with HERA measurements [9] at \( s \approx 10^5 \text{GeV}^2 \), corresponding to \( E_{\nu} \approx 50 \text{TeV} \), more recently complete analytical expressions have been presented in Refs. [10,11]. At these energies, corrections have been shown to be up to 50\% in certain areas of phase space. Consequently, at energies approaching \( 10^{20} \text{eV} \), radiative corrections could be larger still and may play an important role. We therefore found it worthwhile to extend estimates of radiative corrections to UHE, using updated parton distribution functions and with a special emphasize on the quantities relevant for UHE neutrino detection.

The rest of this paper is organized as follows: In Sec. II we introduce our estimates of radiative corrections at UHE. In Sec. III we present numerical results. We fi-
nally summarize our findings and resulting consequences in Sec. IV.

II. ESTIMATES OF RADIATIVE CORRECTIONS AT ULTRA-HIGH ENERGIES

In principle, the full EW radiative corrections complete to order \( g^2 \) where \( g \) is the EW coupling constant can be computed from the expressions given in Ref. [10] and references therein. These, however, involve hundreds of terms and are somewhat hard to reproduce. In contrast, the leading logarithmic approximation is comparatively simple and is expected to be very good at the UHEs we are interested in. Furthermore, to be concrete we will restrict ourselves to charged-current reactions which are more relevant for most of the detection methods relying on Cherenkov radiation from muons. We use the usual kinematic variables, \( Q^2 = 2M E_{\nu} x y \), with \( M \) the nucleon mass, \( y = (E_{\nu} - E\ell)/E_{\nu} \), and \( E\ell \) the energy of the outgoing charged lepton in the laboratory frame. The contribution from the lepton leg to the double differential cross section in leading log approximation can then be written as

\[
\frac{d^2\sigma}{dx dy} = \frac{d^2\sigma_0}{dx dy} + \frac{\alpha}{2\pi} \ln \left( \frac{2M E_{\nu}(1 - y + xy)}{m_l^2} \right) \times \int_0^1 dz \frac{1 + z}{1 - z} \left[ y \Theta(z - z_{\text{min}}) \frac{d^2\sigma_0}{dx dy}(\tilde{x}, \tilde{y}) - d^2\sigma_0(x, y) \right],
\]

where \( \alpha \approx 1/137 \) is the EM fine structure constant,

\[
\tilde{x} = \frac{xy}{z + y - 1}, \quad \tilde{y} = \frac{y + z - 1}{z}, \quad z_{\text{min}} = 1 - y + xy,
\]

and \( d^2\sigma_0/dx dy \) is the lowest order cross section. Eq. (1) is expected to be a good approximation for \( \ln(2M E_{\nu} u/m_l^2) \gg \ln(1 - y) \). Assuming isoscalar nucleons (i.e., averaging over protons and neutrons), the average of the lowest order cross section over neutrinos and antineutrinos is given by

\[
\frac{d^2\sigma_0}{dx dy} = \frac{2G_F^2 M E_{\nu} x}{\pi} \left( \frac{m_W^2}{Q^2 + m_W^2} \right)^2 (2 - 2y + y^2) \times \left[ \frac{1}{2} q_\nu(Q^2, x) + q_s(Q^2, x) \right].
\]

Here, \( G_F \) is Fermi’s constant, \( m_W \) is the mass of the W boson, and \( q_\nu(Q^2, x) \) and \( q_s(Q^2, x) \) are the valence and sea quark distributions of the nucleon.

The first term under the integral in Eq. (1) comes from bremsstrahlung of the lepton. Its \( z \to 1 \) infrared divergence is cancelled by the second term under the integral which describes lepton propagator corrections due to virtual photons such that the total result is finite. It was argued in Ref. [8] that the lepton contribution to the QED correction is dominant and that contributions from the quarks do not have to be included because they are accounted for in the quark distribution functions.

For comparison we also calculated the hard photon bremsstrahlung contribution \( \sigma_{\text{hb}}(E_{\nu}) \) to the inclusive cross section. In the following we denote the modulus of the 3-momenta of the incoming neutrino and quark by \( k \) and \( q' \), respectively, and we also use \( q_0^2 = ((q')^2 + M^2)^{1/2} \). Furthermore, let \( p \) be the modulus of the 3-momentum of the incoming neutrino and \( q_{kp}, \mu_{kp} \), and \( \mu_{pq} \) the cosine of the angle between the 3-momenta indicated. After integrating out the \( \delta \)-functions for 4-momentum conservation we have

\[
\sigma_{\text{hb}}(E_{\nu}) = \frac{1}{4} \frac{M m_l}{E_{\nu}} \int dk dq' d\mu_{kp} d\mu_{k'q'} dx k \times \left[ (1 - \mu_{kp}^2)(1 - \mu_{k'q'}^2) - (\mu_{pq}^2 - \mu_{kp}\mu_{k'q'})^2 \right]^{1/2} \times \frac{q_{s}(Q^2, x) + q_{s}(Q^2, x)}{x^2},
\]

where \( \mu_{pq} \) is given by

\[
\mu_{pq} = \frac{q_0^2}{q'} \left( 1 + \frac{M}{x E_{\nu}} \right) + \frac{k}{x E_{\nu}} \left( \mu_{kp} - \frac{q_0^2}{q'} \right) + \frac{k}{q'} (1 - \mu_{kp}) - \frac{M}{q'} + \frac{M(k - M) + m_l^2/2}{q' x E_{\nu}},
\]

and the integration in Eq. (4) is performed over all areas where \(-1 \leq \mu_{pq} \leq 1 \) and where the angle dependent factor in the integrand is real. The squared matrix element \( \overline{M^2} \) in Eq. (3) is averaged and summed over polarizations of incoming and outgoing particles, respectively, as well as over the electric charges of the quarks in the relevant distributions. In computing this matrix element we used the software REDUCE. The Feynman diagrams for bremsstrahlung photons attached to the incoming and outgoing quark, the outgoing lepton and to the intermediate W boson as well as all interference terms were included. We note that as opposed to Eq. (1) this causes some double counting of radiative quark corrections, when using the same quark distribution functions as for Eq. (1). This is, however, expected to cause only a small error.

As expected, Eq. (4) diverges logarithmically for \( k \to 0 \). One can, however, still get a rough estimate of the radiative corrections without having to compute the virtual corrections. To this end one observes that a natural cutoff scale for \( k \) is given by the quark and lepton masses, and we will therefore restrict the range of integration to \( k \geq \min(M, m_l) \). This will not provide a precise value for the EM radiative corrections but at least a rough estimate.

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For the quark distribution functions we use the CTEQ3 distributions in the deep-inelastic scattering factorization scheme parametrization at \( Q^2 = (1.6 \text{ GeV})^2 \) \cite{12}. Assuming a scaling of the sea quark distribution with \( \ln Q^2 \), the resulting values for the average inelasticity \( \langle y \rangle \) are 0.190 and 0.307 for the lowest order cross section, and 0.240 and 0.334 for the corrected one, respectively. We note in passing that corresponding values in an accelerator experiment such as HERA, the proposed methods are not very sensitive to the phase space distribution of produced hadrons and charged leptons but rather act as calorimetric detection techniques. The only exception may be the quantity \( y \) which is the fraction of the neutrino energy initially going into the hadronic channel of the cascade ensuing, and equals 1 minus the fraction transferred to the outgoing charged lepton whose Cherenkov radiation is observed in the optical techniques. We therefore restrict ourselves to the quantities \( \sigma(E_\nu) \) and \( y(d\sigma/dy)(E_\nu, y)/\sigma(E_\nu) \).

Fig. 2 presents the distribution of \( y \), comparing the lowest order result from Eq. \( (3) \) and the result including radiative corrections in leading log approximation from Eq. \( (4) \) for a neutrino energy \( E_\nu = 10^{20} \text{ eV} \) and \( E_\nu = 10^6 \text{ eV} \). The resulting values for the average inelasticity \( \langle y \rangle \) are 0.190 and 0.307 for the lowest order cross section, and 0.240 and 0.334 for the corrected one, respectively. We note in passing that corresponding values for the neutral-current neutrino-nucleon scattering cross sections are similar. The up to 3-dimensional integrals involved in these calculations have been evaluated using standard numerical techniques.

Fig. 3 compares the fractional corrections \( \delta(E_\nu) \equiv \sigma(E_\nu)/\sigma_0(E_\nu) - 1 \) to the inclusive lowest order cross section resulting from Eqs. \( (3) \) and \( (4) \) as functions of \( E_\nu \). To perform the 3- and 5-dimensional integrations, respectively, we used the adaptive Monte Carlo routine VEGAS developed by Peter Lepage \cite{13} in the version from Ref. \cite{14}. In order to ensure a reasonably smooth integrand and sufficiently accurate results, we resorted to

III. NUMERICAL RESULTS

We now present results of numerical evaluations of Eqs. \( (3) \) and \( (4) \). We first note that for neutrino detection at UHE the probably most relevant quantity is the inclusive charged-current cross section \( \sigma(E_\nu) \) because, unlike

![Fig. 1. The lowest order inclusive charged-current neutrino-nucleon cross section as given by Eqs. \( (3) \) and \( (4) \) as a function of neutrino energy in the nucleon rest frame.](image1)

![Fig. 2. The logarithmic distribution of \( y = 1 - E_l/E_\nu \) from the lowest order calculation, Eq. \( (3) \) (dashed lines) and including radiative corrections calculated in leading logarithmic approximation, Eq. \( (4) \) (solid lines) for \( E_\nu = 10^{20} \text{ eV} \) (thick lines) and for \( E_\nu = 10^6 \text{ eV} \) (thin lines).](image2)
FIG. 3. Absolute values of fractional radiative corrections $\delta(E_{\nu}) = \sigma(E_{\nu})/\sigma_0(E_{\nu}) - 1$ to the inclusive lowest order cross section as functions of $E_{\nu}$. The solid line shows the negative of the total correction in leading log approximation [Eq. (3)] and the dashed line is the hard bremsstrahlung contribution [Eq. (4)]. Wiggles in the curves are due to the finite accuracy of the Monte Carlo integration which is at the few percent level.

integration variables that are power laws in $x$ and, typically, logarithms in the other variables. Note that the leading log approximation for the total correction is negative and roughly an order of magnitude smaller than the hard bremsstrahlung contribution which tends to be cancelled by virtual corrections.

IV. DISCUSSION AND CONCLUSIONS

As can be seen from Fig. 2 at energies around $10^{20}$ eV the radiative corrections to the single differential charged-current neutrino-nucleon cross section $(d\sigma/dy)(E_{\nu}, y)$ are negative for $y \lesssim 0.02$ and positive otherwise. For $y \lesssim 10^{-3}$ they grow larger than 50%, whereas for $y \gtrsim 0.2$ they are of the order of 30%. The average inelasticity $\langle y \rangle$ is increased from $\simeq 0.19$ to $\simeq 0.24$ whose potential influence on the development of the neutrino induced cascade is probably the strongest effect of radiative corrections. The corrections to the total cross section are negative and roughly constant at about half a percent (see Fig. 3). Apart from physics beyond the standard model (see, e.g., Refs [5][6]), uncertainties of the UHE neutrino-nucleon cross section to date therefore are by an ample margin dominated by the uncertainties in the parton distributions in the nucleon at very low momentum fractions.

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[1] For a review see, e.g., T. K. Gaisser, F. Halzen, and T. Stanev, Phys. Rep. 258, 173 (1995).
[2] S. Yoshida, H. Dai, C. H. Jui, and P. Sommers, Astrophys. J. 479, 547 (1997).
[3] J. J. Blanco-Pillado, R. A. Vazquez, and E. Zas, Phys. Rev. Lett. 78, 3614 (1997).
[4] See, e.g., J. G. Learned, Phys. Rev. D 19, 3293 (1979).
[5] see, e.g., G. A. Askar’yan, Soviet Physics JETP 14, 441 (1962); *ibid* 48, 988 (1965); M. A. Markov and I. M. Zheleznykh, Nucl. Inst. Methods, A 248, 242 (1986); F. Halzen, E. Zas, and T. Stanev, Phys. Lett. B 257, 432 (1991); E. Zas, F. Halzen, and T. Stanev, Phys. Rev. D 45, 362 (1992).
[6] G. M. Frichter, D. W. McKay, and J. P. Ralston, Phys. Rev. Lett. 74, 1508 (1995).
[7] R. Gandhi, C. Quigg, M. Hall Reno, and I. Sarcevic, Astropart. Phys. 5, 81 (1996).
[8] A. De Rújula, R. Petronzio, and A. Savoy-Navarro, Nucl. Phys. B 154, 394 (1979).
[9] S. Aid et al. (H1 collaboration), Z. Phys. C 67, 565 (1995).
[10] D. Yu. Bardin, Č. Burdík, P. Ch. Christova, and T. Riemann, Z. Phys. C 44, 149 (1989).
[11] H. Spiesberger, Nucl. Phys. B 349, 109 (1991).
[12] H. Lai et al. (CTEQ collaboration), Phys. Rev. D 51, 4763 (1995).
[13] G. P. Lepage, J. Comp. Phys. 27, 192 (1978).
[14] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes in Fortran*, 2nd ed. (Cambridge University Press, Cambridge, 1992).
[15] A. V. Butkevich, A. B. Kaidalov, P. I. Krastev, A. V. Leonov-Vendrovski, and I. M. Zheleznykh, Z. Phys. C 39, 241 (1988).
[16] J. Bordes et al., e-print hep-ph/9705463; e-print astro-ph/970703.