GUT’s have the ability to defy sphalerons

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Abstract

We show that the baryon asymmetry produced by the out of equilibrium decay of heavy GUT scalars can be the baryon asymmetry that is observed today. No restrictions need be imposed on the initial values of $B$, $L$ and $B - L$, nor on the neutrino masses; no new symmetries need be gauged nor new fermions introduced. We find two mechanisms that can bring this about for any GUT gauge group. Two illustrative models are discussed that are robust, and need just two (at most four) more GUT scalar fields than the minimal model. The additional scalar fields can also help in generating an adequately large value of the CP violation parameter and in efficiently annihilating the monopoles. Our work should firmly re-establish heavy GUT scalars as probable progenitors of today’s baryon asymmetry.
The baryon (lepton) asymmetry $B(L)$, with $B - L = 0$, generated well above the electroweak phase transition temperature ($T_{EW}$) is known to be erased by the electroweak instanton (sphaleron) interactions [1]. A non-zero value of $B - L$ invariably comes hand in hand with massive neutrinos and to prevent the lepton number violating interactions, such as decay of heavy right handed Majorana neutrinos, from being in thermodynamic equilibrium at the same time as the sphaleron interactions, a combination that can potentially wipe out baryon asymmetry with any value of $B - L$, special restrictions have to be imposed on the neutrino masses [2]. This state of affairs was largely responsible for a waning of interest, over the years, in the grand unified theory ($GUT$) based baryogenesis schemes [3] and fostered the emergence of low temperature baryogenesis scenarios, most notably electroweak baryogenesis [4].

Electroweak baryogenesis schemes, basically, attempt to harness the baryon number violating sphaleron interactions to produce a baryon asymmetry during the course of electroweak phase transition (EWPT). If the phase transition is sufficiently strongly first order then the expanding bubble walls provide the site for the departure from thermal equilibrium. $C$ and $CP$ violation are largely model dependent. Attractive though the idea is, a large body of concerted effort has not been able to establish its viability beyond reasonable doubt. An abundance of models that claim to produce a baryon asymmetry in the range allowed by nucleosynthesis constraints exists, and yet the debate on whether the mechanism is fundamentally correct and well understood rages on [5]. While waiting for the dust to clear on the issue of electroweak baryogenesis, some workers have begun devising strategies to protect the baryon asymmetry generated at high temperatures ($>> T_{EW}$) from sphaleron depredations.

Lew and Riotto [6] introduce mirror fermions to render baryon and lepton number currents anomaly free and then gauge the baryon and lepton number symmetries. When $U(1)_B$ and $U(1)_L$ are broken, cosmic string loops arise that ultimately collapse producing fermions. Since the baryon number is anomaly free, the sphalerons leave it unaffected. Campbell et al. [7] have an interesting idea. They observe that if there is no mixing of lepton generations and lepton number violating interactions are in thermal equilibrium for just one or two (but not all three) generations simultaneously with the sphaleron interactions, then a non-zero value of $B$ can survive even if initially $B - L = 0$;
but $1/3B - L_i$ or $2/3B - (L_i + L_j)$ must be non-zero where $i$ (and $j$) is (are) the generation(s) for which the lepton number violating interactions are not in thermal equilibrium. Dreiner and Ross [8] have shown that, for a second or weakly first order EWPT, inclusion of the particle masses while analyzing the chemical equilibrium equations for $T < T_{EW}$ gives $B \sim 10^{-7} \Delta L$ for $(B - L)_{\text{initial}} = 0$, where $\Delta L$ is the initial lepton generations’ asymmetry $\Delta L = \sum_{i>j} (L_i - L_j)$ and it is expected to be of the same order of magnitude as the initial lepton number. While Davidson et al. [9] show that the inclusion of thermal mass effects for $T \gtrsim T_{EW}$ also gives $B \sim 10^{-7} \Delta L$ at $T \sim T_{EW}$. Thus, irrespective of the order (first, second or weakly first) of EWPT a non-zero, though severely diluted, value of $B$ should survive today if initially, at $T >> T_{EW}$, $B - L$ had been zero.¹

1. We find that a non-zero $B$ can survive the sphaleron interactions if the electromagnetic charge $Q$ carried by the particles in chemical equilibrium is not zero. To maintain the electrical charge neutrality of the universe, a charge $-Q$ can be carried by the particles that are not in chemical equilibrium.² The *value of $(B - L)$ is immaterial*, only

$$ (B - L) \neq \frac{NQ}{(4N + 2m)} $$

as

$$ B = \frac{(8N + 4m)(B - L) - 2NQ}{(22N + 13m)} $$

and

$$ L = \frac{-(14N + 9m)(B - L) - 2NQ}{(22N + 13m)} . $$

(1)

$N(m)$ is the number of standard model type fermion generations (Higgs doublets).

If lepton number violating interactions are in thermal equilibrium for all the generations, $(B - L)$ is not conserved but
\[ B = \frac{2NQ}{(10N + 3m)} \]

\[ L = \frac{-6NQ}{(10N + 3m)} \]  \hspace{1cm} (2)

And if only \( n \) (but not all \( N \)) generations have lepton number violations in thermal equilibrium then, \( (B - L)_{N-n} \equiv \frac{(N-n)B}{N} - \sum_i (N-n)_{Li} \) is conserved and

\[ B = \frac{4N[(4N + 2m)(B - L)_{N-n} - (N - 4n)Q]}{(4N + 2m)(13N - 4n) - 8N(N - 4n)} \]

\[ L = -\frac{9B}{4} - \frac{N(Q - 2B)}{(4N + 2m)} \]  \hspace{1cm} (3)

These observations\(^3\) follow from a straightforward exercise in solving the chemical equilibrium equations, \( a \ la \) Harvey and Turner [13]. The particles we have considered are \( N \) standard model type fermion generations, \( m \) standard model type Higgs doublets and the standard model gauge bosons. Right handed neutrinos may also be present, but in the presence of Majorana mass terms their chemical potential is zero. The particles that stay out of equilibrium will usually be of the heavy GUT scalar type.

The main point of this paper is to show that the baryon asymmetry produced by the decay of heavy GUT scalars(X) can be the baryon asymmetry observed today. And the models that make this possible are simply a modified version of the model [14] that gives rise to a large value of the CP violation parameter (\( \epsilon \)) or the model [15] for efficient annihilation of magnetic monopoles: a feature that makes our main point particularly alluring. No extra symmetries need be gauged, no new fermions added, no special restrictions imposed on neutrino masses and the baryon asymmetry at \( T > T_{EW} \) need not be large.

The mechanisms that bring about this happy turn of events are two:

(i) heavy GUT scalars(X) may decay during a phase of temporarily broken electromagnetic gauge invariance, \( U(1)_{em} \), producing not just a non-zero \( B \)
but also a non-zero $Q$, and

(ii) an asymmetry may be produced in the numbers of charged heavy $GUT$ scalars($X$) which may decay at different times producing, again, a non-zero $B$ and a non-zero $Q$.

We, first, cursorily deal with an implementation of mechanism (i) and then give details of a model that executes mechanism (ii).

2.a. In [15] we had presented a model for efficient annihilation of magnetic monopoles, which is achieved by temporarily breaking $U(1)_{em}$ for $10^7 GeV \lesssim T \lesssim 10^8 GeV$. Below $10^7 GeV$ the monopoles begin dominating the energy density of the universe, and in a matter dominated universe the monopoles almost cease to annihilate [16]. The monopole annihilation is expected to yield $GUT$ gauge bosons and $GUT$ scalars which can rapidly decay into fermions.

In our model, $U(1)_{em}$ is broken by the non-zero thermal expectation value of a charged but $SU(3)_c \times SU(2)_L$ - singlet scalar which may couple to leptons [17]. This gives rise to lepton number and charge violating mass terms during the broken $U(1)_{em}$ epoch, thereby facilitating emergence of a non-zero $Q$ and $B - L$ when the heavy $GUT$ scalar bosons, produced by monopole annihilations, rapidly decay out of equilibrium.

Upon restoration of $U(1)_{em}$ at $T \lesssim 10^7 GeV$ a charge $-Q$ emerges out of the vacuum, if the universe is finite sized [18], in the form of charged scalar particles (mass $\sim 10^6 GeV$) responsible for breaking $U(1)_{em}$ and the universe regains charge neutrality. These scalar particles may eventually decay into leptons at $T < T_{EW}$, making $B - L = 0$ but leaving $B$ unaffected.

In [19] the significance of a non-zero $Q$ had not yet dawned on us, and we were quite satisfied with being able to obtain $B - L \neq 0$ for $T \gtrsim T_{EW}$ in cases, such as $SU(5)$, where it would otherwise be zero. #4

It should now be clear that the $GUT$ gauge group can be other than $SU(5)$ and our model will still help preserve $B$ generated in the course of monopole annihilations and subsequent decay of heavy $GUT$ scalar bosons, even if lepton number violating interactions are present.

If the reheat temperature ($T_{RH}$) after inflation is less than $T_{GUT}$, then the monopoles are simply inflated away. But we can still arrange a temporarily
broken $U(1)_{em}$ phase around $T_{RH}$ enabling the GUT scalar bosons produced by inflaton decay to decay into fermions with $Q \neq 0$ thus allowing a non-zero $B$ to persist at $T < T_{EW}$.

2.b. Finally, a model that is known to give rise to a large value of the $CP$ violation parameter ($\epsilon$) in the course of decay of the heavy GUT scalars is shown to be capable of protecting the baryon asymmetry, generated by the scalar decays, from sphaleron interactions. For the ease of presentation we shall stick to $SU(5)$, but we stress that analogous models can be constructed for any other GUT gauge group.

The Weinberg (Three-Higgs) model \[14\] consists of, eponymous, three scalar fields $\phi_i$ in the fundamental 5 representation of $SU(5)$, with interactions

$$V(\phi) = \mu_r^2 \phi_r^\dagger \phi_r + a_{rs}(\phi_r^\dagger \phi_r)(\phi_s^\dagger \phi_s) + b_{rs}(\phi_r^\dagger \phi_s)(\phi_s^\dagger \phi_r) + c_{rs}(\phi_r^\dagger \phi_r)(\phi_r^\dagger \phi_r),$$

(4)

where $r,s$ are summed from 1 to 3. Hermiticity requires that $a_{rs}$ and $b_{rs}$ be real and symmetric, and $c_{rs}$ be Hermitian. $\phi_3$ is chosen not to couple to fermions, but $\phi_1$ and $\phi_2$ have Yukawa couplings such as

$$L_{Yuk} = \bar{\psi}_m,\alpha,\beta \chi_n^\alpha([f^1_m]^n_\alpha \phi_1^\dagger + (g^1_2)^mn_\alpha \phi_2^\dagger] + \epsilon_{\alpha\beta\mu\nu} \lambda \psi_n^{\alpha\beta} C \psi_n^{\mu\nu} [(g^1_1)^mn_\lambda \phi_1^\lambda + (f^2_2)^mn_\lambda \phi_2^\lambda] + \text{h.c.}$$

(5)

where $m,n$ are summed over fermion generations, $C$ is the charge-conjugation matrix; and fermions are put in a right-handed five-dimensional representation $\chi^\alpha$ and a left-handed, ten-dimensional representation $\psi^{\alpha\beta}$; $\alpha, \beta, \mu, \nu, \lambda$ are $SU(5)$ indices running from 1 to 5.

When $SU(5)$ is broken by the vacuum expectation value ($\sim 10^{15} GeV$) of a scalar field in the adjoint 24 representation, the color triplets $\phi^a_i$ acquire superheavy masses which we choose to satisfy $M_3 > M_1 > M_2$; $a$ is the color index. The $SU(2)_L$ doublets $\phi^h_3$ and $\phi^h_2$ are also allowed to acquire heavy masses ($m_3 > m_2 < M_i$). $\phi^h_1$ becomes the standard Higgs doublet whose vacuum expectation value ($\sim 10^2 GeV$) gives mass to the fermions.

The superheavy color triplet $\phi^a_3$ has two decay channels: $\phi^a_1$ plus $\phi^h_3$ and $\phi^h_1$ or $\phi^h_2$ plus $\phi^h_3$ and $\phi^h_3$. The decays of $\phi^a_3$ will violate CP invariance because of the complex $c_{rs}$. The interference between tree and one-loop diagrams gives, for $b_{rs} << c_{rs}$,
\[ r_{3 \rightarrow 1} - \bar{r}_{3 \rightarrow 1} \equiv \epsilon = \frac{\text{Im}(c_{12}c_{23}c_{31})}{(4\pi)|c_{13}|^2 + |c_{23}|^2} \quad (6) \]

where \( r(\bar{r})_{3 \rightarrow 1} \) is the partial decay rate of \( \phi^a_3(\bar{\phi}^a_3) \) into \( \phi^a_1(\bar{\phi}^a_1) \) plus relatively lighter Higgs. In fact \( \epsilon \) is the number of \( \phi^a_1 \) produced in the decay of a \( \phi^a_3 - \phi^a_3 \) pair. CPT ensures that \( r_{3 \rightarrow 2} - \bar{r}_{3 \rightarrow 2} \equiv -\epsilon \), hence the number of \( \bar{\phi}^a_2 \)'s produced in the decay of a \( \phi^a_3 - \phi^a_3 \) pair is also \( \epsilon \).

Since some of the Yukawa couplings \((g_1)_{mn}\) may be as large as \( O(1) \), \( \phi^a_1 \) readily decays into fermions producing

\[ B - L = 3(-2/3) \quad , \quad (7) \]

where we have summed the contributions of the three \( \phi^a_i \)’s. The largest of the Yukawa couplings, \((f_2)_{mn}\) and \((g_2)_{mn}\), may be such that \( \bar{\phi}^a_2 \) does not decay until well after \( T_{EW} \) when we expect the sphaleron interactions to have dropped out of thermal equilibrium.

If initially \( T > T_{GUT} \), \( \phi^a_3(\bar{\phi}^a_3) \) begin decaying when the temperature \( T_3 \) has fallen below \( M_3 \) if, (for \( c_{31} \sim c_{32} \))\#5.

\[ M_3 \gtrsim (2 \times 10^{14})|c_{31}|^2 \text{GeV} \quad . \quad (8) \]

Then \( \phi^a_1 \) decays into fermions at (say) \( T_1 \lesssim M_1 < M_3 \), and if \( T_3 < M_1 \) then \( \phi^a_1 \) and \( \bar{\phi}^a_2 \) produced in \( \phi^a_3 \) decay are never in chemical equilibrium. \( \bar{\phi}^b_2, (\phi^b_3, \bar{\phi}^b_3) \) will be in chemical equilibrium with fermions if \( m_2(m_3) < T_1 \) due to scalar mixing through \( c_{12} \). Now the charge carried by the particles in chemical equilibrium is\#6

\[ Q = -1 \quad (9) \]

and it is balanced by the charge carried by \( \bar{\phi}^b_2 \), which is \( 3(+1/3) \). At \( T \sim m_3 > m_2, \phi^b_3 \) and \( \bar{\phi}^b_3 \) annihilate. For \( m_2 < T < m_3 \) the particles in chemical equilibrium are \( N(=3) \) fermion generations, \( m(=2) \) Higgs doublets \( \phi^b_1 \) and \( \bar{\phi}^b_2 \), and the standard model gauge bosons. At \( T \sim m_2 \)

\[ B = -29/46 \quad L = 63/46 \]

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\[ \mu_0 = \frac{3}{184} \quad Q = -1 \]  
\[(10)\]

where \( \mu_0 \) is the chemical potential of the Higgs doublets.

At \( T < m_2 \), \( \phi_2^h \) drops out of chemical equilibrium and we are left with just one Higgs doublet in chemical equilibrium. For \( T_{EW} \leq T < m_2 \)

\[ B = -\frac{4609}{7268}, \quad L = \frac{9927}{7268}, \quad Q = -\frac{181}{184} \quad . \]  
\[(11)\]

Sometime after the sphalerons have dropped out of thermal equilibrium, \( \bar{\phi}_2^a \) and \( \bar{\phi}_2^h \) decay into fermions giving

\[ B_2 = 1 - \frac{r}{2}, \quad L_2 = -1 - \frac{r}{2}, \quad Q_2 = \frac{181}{184} \]  
\[(12)\]

where \( r \equiv \frac{4g_2^2}{(4g_2^2 + 3f_2^2)} \); \( m, n \) have been summed over fermion generations. Now the net values are

\[ B = \frac{2659}{7268} - \frac{r}{2}, \quad L = \frac{2659}{7268} - \frac{r}{2}, \quad Q = 0 \]  
\[(13)\]

If \( \phi_3^a(\phi_3^h) \) had been produced at \( T < m_2 \) by the collapse or annihilation of a topological defect or decay of the inflaton then \( \phi_2^h \) is never in chemical equilibrium and \( \phi_1^a \) readily decays with

\[ B - L = -2, \quad Q = 0 \],

\[ B = -\frac{56}{79}, \quad L = \frac{102}{79} \]  
\[(14)\]

And when \( \bar{\phi}_2^a \) and \( \bar{\phi}_2^h \) decay at \( T < T_{EW} \), the net values are

\[ B = \frac{23}{79} - \frac{r}{2}, \quad L = \frac{23}{79} - \frac{r}{2}, \quad Q = 0 \]  
\[(15)\]

\((f_2, g_2)_{mn}, r \) can always be such that \( B \neq 0 \).

A few words on particle masses and decays.

(i) \( M_1 > 10^{14} GeV \) to prevent too rapid a proton decay [20]. For \( \phi_1^a \) to decay out of equilibrium \( M_1 > 5 \times 10^{15} GeV \). But in the Weinberg model out of equilibrium decay of \( \phi_1^a \) is possible for smaller values of \( M_1 \) if \( M_1 > T_3 \).
(ii) The values of $B(L)$ in all the equations are per $\phi_1^a(\bar{\phi}_2^a)$ particle decay; and since $B \sim 10^{-1}$ the number density of $\phi_1^a(\bar{\phi}_2^a)$ should be $(n_3/s)\epsilon \sim 10^{-10}$ to allow $(n_B/s)$ to lie in the range $(4-6) \times 10^{-11}$ allowed by nucleosynthesis. $n_3$ is the number density of $\phi_2^a(\bar{\phi}_3^a)$ that decay.

(iii) To prevent too large an entropy production when $\bar{\phi}_2^a$ and $\bar{\phi}_2^b$ decay at $1 MeV < T < T_{EW}$, $M_2 < 10^{12} GeV$. (Recall $m_2 < M_2$). $M_2$ can be larger but then $(n_3/s)\epsilon$ would have to be correspondingly large to accommodate the increase in entropy. And, roughly, $(m_2, M_2) > 10^2 GeV$ or else $\phi_2$ should have been, more or less, seen by the present accelerators.

(iv) For $M_2(m_2)$ in the range allowed by (iii), $(f_2, g_2)_{mn}$ can easily be chosen to avoid too large a rate of proton decay and flavour changing neutral current processes. Roughly, for $(M_2, m_2) > 10^6 GeV$ and $(f_2, g_2)_{mn} < 10^{-10}$ there are no problems.

Just by weakly coupling the scalar field $\phi_2$ to fermions, the Weinberg (Three-Higgs) model, for generating a large value of CP violation parameter $\epsilon$, has been empowered with the ability to protect baryon asymmetry, generated well above $T_{EW}$, from sphaleron interactions. It should be noted that even if $B = 0$ at $T \gtrsim T_{EW}$ (say $\bar{\phi}_2^b$ is never in chemical equilibrium but rapid lepton number violating interactions are present while sphalerons are in thermal equilibrium) an adequately large baryon asymmetry can still be produced by the out of equilibrium decays of $\bar{\phi}_2^a$ at $T < T_{EW}$ (and if such is the case then today $B - L = 2$). A very robust model for baryogenesis we have indeed.

Yukawa couplings as small as $10^{-10}$ should not seem very unusual, for in the Standard Model the electron Yukawa coupling is $2 \times 10^{-6}$ and should neutrinos turn out to have Dirac masses in the eV range Yukawa couplings of order $10^{-10} - 10^{-11}$ will be needed.

3. A simple solution to a vexing problem has been found. It appears that today’s observed baryon asymmetry is as likely to have been produced by the decay of heavy GUT scalars as by any other viable mechanism.

The traditional GUT’s appear to have an inherent ability to fight off the sphalerons. All that the minimal models need are two (at the most four) additional GUT scalar fields to be adequately empowered: a very
economical bargain. And the additional scalar fields do not just shield the baryon asymmetry from sphalerons but also eliminate monopoles or generate and adequately large value of the CP violation parameter: a two for one kind of offer that makes an already economical bargain even more attractive.

After a long, and now seen to be undeserved, exile to the margins, baryogenesis via decay of heavy GUT scalars seems set to regain its position on the main stage.
FOOTNOTES

[1] Cline et al. [10] had proposed that an asymmetry in the number of right-handed electrons $e_R$ could protect the baryon asymmetry. But detailed calculations [11] belied this hope by revealing that $e_R$ enter chemical equilibrium, at $T \lesssim 10 TeV$, well before the sphalerons have dropped out of thermal equilibrium. Antaramian et al. [12] also discuss survival of baryon asymmetry but without any models.

[2] A particle species is said to be in chemical equilibrium if the rate ($\Gamma$) of the reactions that alter its number is large enough to keep it in thermal equilibrium, $\Gamma >> H$.

[3] Eqs. (1) – (3) pertain to $T >> T_{EW}$. If EWPT is second or weakly first order, sphalerons may remain in thermal equilibrium up to $T \sim m_W < T_{EW}$ and the numerical coefficients in eqs.(1) – (3) will be different, but qualitatively our results remain unaffected.

[4] The ability of non-zero $Q$ to protect $B$ from being decimated by the sphalerons becomes fully manifest only in the presence of rapid lepton-number violating interactions for all the generations.

[5] The $\phi_3^a - \bar{\phi}_3^a$ annihilations can be ignored (at $T_3 \lesssim M_3$) if $\Gamma_{ann} < H$ which holds for $T > 3 \times 10^{14} GeV$.

[6] Total (charge, baryon number, lepton number) is $\left( \frac{n_3}{s} \right) \epsilon(Q, B, L)$, where $n_3$ is the number density of $\phi_3^a (\bar{\phi}_3^a)$ that decay.
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