MICROLENSING: PROSPECTS FOR THE FUTURE

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Abstract

Four ongoing microlensing experiments have produced important new results but also big puzzles, the major one being that the expected classes of lenses cannot account for the observed distribution of time scales. I discuss future experiments that could resolve these puzzles. By far the most important would be to launch a parallax satellite into solar orbit. I also discuss a number of non-dark-matter applications of microlensing, including searching for planets, measuring the rotation speeds of giant stars, and imaging a black hole at the center of a quasar.

1 Introduction

Within a few short years, microlensing has been transformed from an apparently hair-brained idea of a theorist into a practical experimental approach to search for dark matter. What are the most pressing questions posed by the current observations? Are there practical approaches for resolving these questions? How can we expect microlensing to develop as a technique in the future?

2 Major Results, Major Puzzles

The latest results from the MACHO collaboration’s observations toward the Large Magellanic Cloud (LMC) reported here by Sutherland seem to imply that of order half of the dark matter between the Sun and the LMC is composed of compact objects. However, the best estimate of the mean mass of these objects is $\sim 0.4 M_\odot$. If such objects were composed of hydrogen, they would be M or K dwarf stars and would certainly have been noticed (as I reported here earlier). This mass range is consistent with white dwarfs, but the production of white dwarfs would be accompanied by the generation of large amounts of carbon and other heavy elements, contrary to observations. In addition, we are now able to directly search for halo white dwarfs and do not find them, although the limits are not yet very strong. One possibility is that the objects being detected are not actually in the halo, but rather are in the disk of our Galaxy or in the LMC itself. While there are theoretical arguments against major contributions from both the disk and the LMC, it is
nonetheless striking that of the 8 events reported by MACHO, one (the binary event) is very likely in the LMC and another is quite possibly in the disk. The proper motion (angular speed) of the binary is apparently 20 km s$^{-1}$ $D_{\text{LMC}}^{-1}$, where $D_{\text{LMC}} \sim 50$ kpc is the distance to the LMC. This would be quite consistent with an LMC object, but halo objects should be moving 50 times faster. The unlensed light in MACHO event 5 has a color and flux consistent with a foreground disk M dwarf, but not consistent with any known population of LMC stars. It is natural to suppose that this unlensed light is in fact the lens.

It is also possible that the masses of the lenses are being overestimated. The mass estimates are based on the measured timescales of the events and on a model for the physical and velocity distributions of the lenses. Models are required because the timescale, $t_e$, is related to the underlying physical parameters of interest in a complicated way. The physical size of the Einstein ring, $r_e$, is given by

$$r_e = \sqrt{\frac{4GM_D D_o D_s}{c^2 D_{\text{ol}}}},$$

(1)

where $M$ is the lens mass and $D_{\text{ol}}, D_o, D_s$ are the distances between the observer, lens, and source. Then,

$$t_e = \frac{r_e}{|v|},$$

(2)

where $v$ is the transverse velocity of the lens relative to the line of sight from the observer to the source. If the lenses are in the halo, their space velocities must be of order $\sim 200$ km s$^{-1}$ by the virial theorem and from measurements of the Galactic potential. However, the exact structure of the potential is not known. Moreover, the flattening of the halo is not known and this could affect the timescale through the “$D_{\text{ol}}$” term in Eq. 1. Furthermore, if the halo were rotating, the lenses might have large absolute space motions but their projected velocities $v$ might nonetheless be small depending on the relative orientation of the lens and LMC velocities. In brief, the interpretation of the lensing observations toward the LMC is far from unique.

The observations toward the Galactic bulge are even more puzzling. Although many seem to believe that the current observations are well explained as lensing by “known” populations of stars, the required populations are not at all “known” by the people who study them. On the contrary, the events seem to require an enormous population of “unknown” brown dwarfs. What is known, is that half the dynamical mass of the bulge is accounted for by bulge stars with masses $M > 0.5 M_\odot$ that can be seen using the Hubble Space Telescope (HST) [11] (See also my other contribution to these proceedings.) These known stars contribute
only a tiny fraction of the observed microlensing events and almost none of the short ones. Even if the bulge mass function is extended to the brown dwarf limit using the locally measured disk mass function, very few of the short events are accounted for. Only if the remaining 1/3 of the bulge mass is put in brown dwarfs ($M \sim 0.08 M_\odot$), can the short events be explained. Of course, the problems of interpretation are similar to those for the LMC. While the kinematics of luminous stars can be directly measured, those of the unobserved lenses (the great majority) cannot. One can assume that the kinematics are similar and this assumption may be a reasonable one. However, if the inference is the existence of a huge population of otherwise unobservable objects, one must question whether their kinematics can really be assumed to be the same.

One would like to actually measure both the kinematics and the masses of the individual lenses toward both the LMC and the bulge, rather than relying on statistical arguments.

3 Parallax Satellite

By far the best idea for learning more about the lenses is to launch a parallax satellite into solar orbit. For objects in the relevant mass range, $0.1 M_\odot < M < 1 M_\odot$, the Einstein ring given by Eq. 1 is $r_e \sim 1–5$ astronomical units (AU). Hence, if one were to observe the event from $\sim 1$ AU away, the event would look very different. Specifically, the magnification $A$ is a function only of the projected separation of the lens and the source in units of the Einstein ring, $x$, which in turn changes with time according to the Pythagorean theorem:

$$A[x(t)] = \frac{x^2 + 2}{x \sqrt{x^2 + 4}}, \quad x(t) = \sqrt{\left(\frac{t - t_0}{t_e}\right)^2 + \beta^2}. \quad (3)$$

Here $t_0$ is the time of maximum magnification and $\beta$ is the impact parameter in units of the Einstein ring. Two observers separated by a distance $d_{\text{sat}}$ (projected onto the plane of the sky) will see events with parameters $(t_0, \beta)$ and $(t'_0, \beta')$ respectively. That is, they will be separated in the Einstein ring by $\Delta x = (\omega \Delta t, \Delta \beta)$ where $\omega \equiv t_e^{-1}$. The magnitude of $\Delta x$ is related to the physical parameters by simple geometry:

$$\tilde{r}_e = \frac{d_{\text{sat}}}{\Delta x}, \quad \tilde{r}_e = \frac{D_{\text{os}}}{D_{\text{ls}}} r_e. \quad (4)$$

where I have now introduced $\tilde{r}_e$, the Einstein radius projected onto the observer plane. With a parallax satellite, one therefore measures two parameters ($t_e$ and $\tilde{r}_e$) which are combinations of the three physical quantities ($M$, $v$, and $D_{\text{ls}}$). In addition, the direction of $\Delta x$ gives the
direction of $\mathbf{v}$ relative to the known direction of the satellite. (The careful reader will have noticed that while $\Delta t = t'_0 - t_0$ is completely determined, $\Delta \beta = \pm (\beta' \pm \beta)$ is determined only up to a four-fold ambiguity. It is actually possible to resolve this ambiguity using the slight difference in $t_e$ induced by the Earth-satellite relative motion. Detailed calculations show this to be feasible for both the LMC and the Galactic bulge.)

What is the value of a parallax satellite? First, it would give unambiguous confirmation of the microlensing nature of the events. Only two classes of events look significantly different when observed from 1 AU apart: microlensing and events within the solar system. Second, it would distinguish between events in the Galactic disk, halo, and LMC. The quantity $\tilde{v} \equiv \tilde{r}/t_e$ is very different for these three populations, being about 50, 300, and 3000 km/s$^{-1}$ respectively. Thus, the populations could be distinguished on an event by event basis. Third, $\tilde{r}$ is a function only of the mass and distance. Hence the individual (and statistical) mass estimates would be considerably less uncertain than currently where they are derived from $t_e$ which is a combination of three physical parameters. Finally, for some cases it is possible to measure an additional parameter and so determine the mass, distance, and velocity separately.

4 Proper Motions

If the lens transits the face of the source, the light curve deviates from the simple form given by Eq. 3 and this permits one to measure $x_*$, the value of the $x$ when the lens crosses the limb of the star. Since the angular size of the source, $\theta_*$, is usually known from its temperature and flux (and Stefan’s Law), one can therefore determine the angular Einstein radius, $\theta_e \equiv r_e/D_{\odot}$,

$$\theta_e = \frac{\theta_*}{x_*}. \quad (5)$$

This measurement is called a “proper motion” (astronomical jargon for angular speed) because it immediately yields this quantity, $\mu = \theta_e/t_e$. As mentioned above, combined parallax and proper motion measurements give a complete determination of the event parameters. For example, the mass is given by

$$M = \frac{c^2}{4G} \tilde{r}_e \theta_e. \quad (6)$$

Of course, such transits are rare, but using combined optical/infrared photometry, one can extend this technique to cases where the lens comes within two source radii of the source. This means that a fraction $\sim 2\theta_*/\theta_e$ of events can be measured. For some classes of events, such as low-mass objects seen toward the Galactic bulge, this fraction may be of order 10%, but for high-mass lenses or LMC events, such near-transits
are extremely rare. Hence, many other ideas have been developed to measure proper motions including interferometric resolution of the two lensed images and lunar occultations (both useful for high-mass bulge lenses) as well as spectroscopic measurements of spinning and binary source stars (both useful for LMC events). Taken together (and in concert with a parallax satellite) such techniques could give complete solutions for a significant subsample of events. This would greatly clarify our understanding of the lenses.

5 Pixel Lensing of M31

In order to observe microlensing, one must monitor stars, generally millions of them because the optical depth, \( \tau \) (the probability that a given star is being lensed at any given time) is usually \( \tau < 10^{-6} \). This requires dense star fields like the bulge or the LMC. However, if the fields get too dense, one cannot resolve the individual stars. Thus lensing searches toward galaxies more distant than the LMC would appear impossible since few if any individual stars are resolved. Two groups are nevertheless attempting lensing observations toward M31, the nearby giant spiral galaxy in Andromeda. It is always heartening to watch people attempt the impossible, more inspiring still when they succeed.

Before discussing why such observations are in fact possible, let me focus for a moment on why they are so important. First, M31 should have its own halo. It is highly inclined so our lines of sight to the far side of the disk pass through much more of the halo than those toward the near side. This permits us to study the structure of the halo along many lines of sight and to test the reality of microlensing: microlensing should be a strong function of position whereas astrophysical sources like variable stars should be symmetrically distributed. By contrast, we have only a few lines of sight along which to probe the halo of our own Galaxy. Second, M31 can be used to probe for exceptionally low mass lenses in our own Galaxy. For observations toward the LMC, one is fundamentally restricted to masses \( M > 10^{-7} M_\odot \). Smaller mass lenses have such small angular Einstein rings that they magnify only a fraction of the surface of a source star. By contrast, because it is 16 times further away, source stars in M31 are 256 times smaller, so one can probe to mass scales that are smaller by the same amount. Third, if dark matter in the form of lenses is confirmed for the Milky Way, we would like to begin studying the same stuff elsewhere in the universe, and M31 is a good place to start.

Now, why is this impossible project possible? If a resolved star is lensed, its flux changes from \( F = F_0 \) to \( F = F_0 A \). The difference is \( \Delta F = F_0 (A - 1) \). If the star is unresolved, its light \( F_0 \) falls on some pixel (or more precisely, resolution element) of the detector along with
the light $B$ from many other stars, for a total flux $F_0 + B$. During the lensing event, the total light grows to $F_0 A + B$, for a difference that is still $\Delta F = F_0 (A - 1)$. Thus, by measuring flux differences of pixels rather than fluxes of individual stars, one can effectively monitor dozens of stars in each pixel. Of course, there are drawbacks to this technique which come principally from the lower signal-to-noise ratio of the more distant (and so fainter) stars. Nevertheless, the main difficulty in carrying out these projects to date has been a lack of telescope time (due to the maddening – but quite common – conservatism of telescope time allocation committees) rather than any fundamental problem with the techniques. These projects promise important new results on microlensing within the next few years and, in addition, are a fantastic way to study previously unobservable variable stars in external galaxies.

6 Pixel Lensing of M87

Many of the most important astronomical objects (galaxies, globular clusters, star-forming regions) carry the designation ‘M’ for Messier, an 18th century astronomer. You might think that he was a far-sighted pioneer who catalogued objects that would not attract the interest of others for centuries to come. Not at all. Messier was interested in comets which today are very much an astronomical side show but were the cat’s pajamas in the 18th century. He often received alleged comet sightings which were then found to be bogus because the “comets” failed to move against the fixed stars. He therefore constructed a catalogue of this astronomical garbage so he would not be distracted from his all-important comet searches. Eighty-seventh on his list is now known to be a giant cannibal galaxy at the center of the Virgo Cluster, the nearest cluster of galaxies to the Milky Way.

M87 is an excellent candidate for pixel lensing. To see why, let’s think about how Massive Compact Halo Objects (machos) might have formed in our own Galaxy. According to the most recent lensing results, the total macho mass in the Milky Way is $M \sim 2 \times 10^{11} M_\odot$. This is of the same order as the total baryonic material in the visible components of the Galaxy, its disk and bulge. Hence, one might imagine that before the Milky Way had fully collapsed, half of its gas formed into the machos. The other half collapsed into a proto-disk and proto-bulge which then went on to form the stars that we see today. That would explain the equal amounts of machos and visible baryons. Now suppose that this same process went on in a Milky-Way-like galaxy forming on the outskirts of a cluster. Half the gas would still form machos, but before the other half could form stars, the galaxy would fall through the center of the cluster and be stripped of its gas by the hot intra-cluster gas. According to this scenario, there should then be equal masses of machos and intra-cluster
gas in clusters. The latter is measured in several clusters to be of order
20% of the dark matter. One should then expect an equal amount of
machos. How would one find them? By pixel lensing of M87, of course!
Since M87 is about 20 times farther away than M31, the experiment is
much more difficult to carry out. Nevertheless, it would be possible with
dedicated observations by HST, with a time commitment similar to that
used on the Hubble Deep Field which has yielded so many important
results.

7 Planet Searches

Microlensing began as a technique to search for dark matter, but gradu-
ally it is being realized that microlensing has many other potential
applications. The remainder of my contribution is aimed at giving you
some flavor of these possibilities.

If a planet is orbiting a star and the star is the lens in an ongoing
microlensing event, one could hope to detect the planet through its per-
turbations on the lensing light curve. The Einstein ring of the planet
is given by Eq. (6) except with the planet mass $m$ replacing the stellar
mass $M$. That is,

$$r_p = r_e \sqrt{\frac{m}{M}}.$$  \hspace{1cm} (7)

Thus, the probability that the planet will perturb the main event at
any given time is only $\sim (r_p/r_e)^2 = m/M$ which is too small to be
noticed in standard microlensing searches. However, the probability that
it will influence the event at sometime is $\sim (r_p/r_e) = \sqrt{m/M}$. For
Jupiter-mass planets, $m/M \sim 10^{-3}$, this is getting to be an interesting
number. Moreover, if the planet happens to lie near the Einstein ring
of the lensing star, the effect of the planet is to create a far reaching
"astigmatism" in the lens which can dramatically increase the probability
of a detection given intensive monitoring. If a solar-like system lay
half way toward the galactic center, Jupiter would be at $1.3 \, r_e$ and its
probability of detection would rise from $\sim 3\%$ to $17\%$. These effects make
microlensing a potentially powerful tool to search for planets. If a planet
were detected, one could measure the mass ratio $m/M$ and the projected
planet/star separation in units of the stellar Einstein ring. In addition,
if auxiliary information were available (such as from a parallax satellite)
additional constraints could be placed on the detected planetary system.

Two groups are already conducting microlensing searches for plan-
etes using microlensing "alert" events which are recognized in real time
by the MACHO and OGLE experiments. The EROS microlensing
experiment has recently begun monitoring the bulge and should begin
producing alerts soon.
Planets that are substantially smaller than Jupiter are harder to detect, in part because the probability of an event is lower and in part because the planet Einstein ring eventually becomes so small that it magnifies only part of the source star, thus reducing the size of the effect. Nevertheless, more ambitious future searches may be able to detect even Earth-mass planets.

8 Rotation Speed of Stars

One of the ideas that I briefly mentioned for measuring the proper motion of lensing events was to look for microlensing effects on spinning stars. The idea is that stellar lines are normally broadening by the rotation of stars because part of the stellar atmosphere is spinning toward us and is blue-shifted while the other part is spinning away and so is redshifted. During a microlensing event, there is a changing magnification gradient across the star, which means that either the blue-shifted or redshifted side is magnified more than the other. This causes a shift in the centroid of the line, i.e., an apparent shift in the star’s radial velocity. The shift is proportional to the rate of stellar rotation and to the angular size of the star in units of the Einstein ring. If the first quantity is measured (from the line broadening) the second can be determined. This method is especially useful for rapidly spinning stars (such as A type stars in the LMC) but it is useless for slow rotators (like K giant type stars in the bulge) in part because the effect is smaller, but more importantly because no one knows how fast K giant stars rotate!

The problem is that the rotation rate is probably an order of magnitude smaller than the atmospheric turbulence, so the broadening due to rotation cannot be disentangled from turbulent broadening. One would very much like to know how fast K giants rotate in order to learn about the evolution of angular momentum in stars. This evolution affects numerous other questions including rotational mixing and the survival of primeval lithium from the big bang.

How can the rotation speed of giants be measured? By microlensing! For lensing events where the star transits the source, one can measure the ratio of the source size to the Einstein ring (see § 4). As just stated, the line shift is proportional to this quantity and to the rotation rate. Therefore, if the line shift is also measured then the rotation rate is known. This is a difficult experiment but a feasible one because during a transit event the source star may be magnified by 10 or 20 times, making precision spectroscopy much easier. In fact, microlensing is probably the best way to study all aspects of stellar atmospheres, not just rotation. Because the lens acts as a giant, ever-changing magnifying glass, it in effect resolves the entire surface of the star.
9 Femtolens Imaging of a Quasar’s Black Holes

Space constraints prevent me from giving detailed descriptions of all applications of microlensing. Before presenting my final example, let me just mention that microlensing can be used to study the star-formation history of the universe, to measure the transverse velocities of distant galaxies, to determine the distribution of binary stars, and to search for low-mass, high-redshift compact objects such as primordial black holes or even axion mini-clusters.

What better way to confirm the hypothesis that quasars are powered by black holes than to actually image the black hole at the center of a quasar? To do this, one obviously needs a big telescope since the angular size of a $10^8 M_\odot$ black hole at a cosmological distance is only $\sim 10^{-9}$ arcsec. I propose to use a nearby dwarf star as the primary lens of such a telescope. By a simple calculation, there should be a dwarf star within about 20 pc of the Sun which lies within a few arcseconds of a quasar. Hence, if one were to travel a short distance of $\sim 40$ AU, the quasar and the dwarf star would be perfectly aligned. The main problem then is to determine what secondary optics must be placed at focus of the primary lens in order to actually image the black hole.

The Einstein ring of such a dwarf star is $r_e \sim 10^{-1}$ AU, corresponding to $\theta_e \sim 10^{-2}$ arcseconds. Now most dwarf stars have binary companions and if the first one you find does not, move on to one that does. Typically these companions are at separations $a \sim 10-100$ AU. As I mentioned above, companions induce an astigmatism on the lens. However, for $a \gg r_e$, the astigmatism is very slight and can be completely characterized by a shear $\gamma \sim (m/M)(r_e/a)^2 \sim 10^{-4}-10^{-6}$. The astigmatism is in the form of a caustic whose size is $\sim 2\gamma$, where here and afterwards I normalize all sizes to the Einstein ring. For sources lying inside the caustic, there are five images. One of these images lies near position of the companion and is of no interest here. The other three images lie near the Einstein ring. We maneuver our spacecraft so that the quasar lies just inside (at a distance $\xi$ from) one of the cusps of the caustic. Then one of the four images lies on the opposite side of the Einstein ring and is not highly magnified. The other three images lie close together on the same side of the Einstein ring and are highly magnified: they are stretched by $\sim \xi^{-1}$ in one direction (along the Einstein ring) and are contracted by a factor 1/2 in the other direction. How large a magnification is possible? The black hole must fit inside the cusp and this sets a lower limit $\xi > (4\gamma\rho^2)^{1/3}$ where $\rho \sim 10^{-9}$ arcsec is the size of the black hole. For typical parameters $\xi^{-1} \sim 10^6$. Unfortunately, because the images are stretched, this permits resolution of the quasar in only one dimension: all of the information in the other direction is compressed by a factor of 2 rather than being magnified. How can this other information be
Consider the three distinct images of a single point. Each of these images contains light not only from that point, but from an entire curve (in fact nearly a straight line) across the source. The three curves intersect at exactly one point, the point in question. In reality, the resolution elements are finite, so each resolution element contains light from an entire swath across the source. The three swaths intersect in a relatively small region. Now, let us consider bringing the light from two of the resolution elements together and then analyzing them with a spectrograph. Since most of the light in each resolution element comes from non-overlapping regions of the source, it will not interfere. However, the light from the intersecting region will appear in both resolution elements and so will interfere. By how much? This depends on the wavelength of the light and on the relative delay in arrival times of the light along the two paths represented by the two images. Rudy Schild discussed his measurement of the time delay of 1.1 years for the two images of the famous double quasar 0957+561. In the present instance, the time delays are \( \sim \mathcal{O}(10^{-15}) \) s. Hence the term femtolensing. This corresponds to the wavelength of optical light. By Fourier transformation of the interference spectrum, one can determine the amount of light at each position within the region. There is additional information because all three images can be interfered. In terms of physical size, the resolution element is substantially less than the size of the black hole, \( \sim 1 \) AU.

The requirements for this project are not trivial. A set of mirrors totaling at least 20 square meters must be accurately aligned in a one dimensional array extending over several hundred meters. The entire apparatus, must be sent \( \sim 40 \) AU from the Earth and then brought to a velocity exactly equal to that of the dwarf star (so that it remains aligned with the star-quasar line of sight). It will even be necessary to correct this velocity every few hours to compensate for the gravitational effects of the Sun and the accelerated motion due to the star’s companion. But microlensing is an incredibly powerful tool and we should be ambitious in deploying it.

Acknowledgments

This work was supported in part by NSF grant AST 9420746.

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