Jeans instability and antiscreening in the system of matter–antimatter with antigravitation

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Abstract. The hypothesis of antigravitational interaction of elementary particles and antiparticles is applied to the simple two-component hydrodynamic model Λ-CDM (Lambda cold–dark matter) with gravitational repulsion and attraction. An increase in the Jeans instability rate, the presence of antiscreening, and the dominant role of the gravitational repulsion as a possible mechanism of spatial separation of matter and antimatter in the Universe are shown, as well as the observable acceleration of far galaxies. The sound wave is found for the two-component gravitational–antigravitational system. The suggested approach permits to reestablish the idea about baryon symmetry of the Universe, causing its steady large-scale flatness and accelerated Universe expansion.

1. Introduction
The Λ-CDM standard model of Big Bang cosmology looks formally very successful, but it includes two vastly dominating and unknown—directly unobserved components—cold–dark matter (CDM) and dark energy (DE), filling more than 95% of the Universe. Moreover, this model needs the hypothesis of initial hyperinflation to provide a successful parametrization of the present cosmological data, including the observed perfect flatness (also its earlier stages directly after the Big Bang) and recently discovered accelerating expansion of the Universe on its large scale. The low value of the Einstein’s cosmological constant Λ in the Λ-CDM model is hypothetically associated with an intrinsic energy density of the vacuum, acting as repulsive DE and having the unknown, controversial physical nature. Taking into account this unsatisfactory situation with the repulsive vacuum DE and attractive DM, some authors arrive at unconventional matter–antimatter symmetric cosmology. In this model matter and antimatter are assumed to be always equally presented after Big Bang, because they gravitationally repulse each other and survive in spite of annihilation processes. A gravitationally symmetric rest exists in the form of mixed—gravitationally neutral—matter–antimatter Universe, providing zero gravity mass density on the large scale. Thus, antimatter (carrying positive inertial mass) is supposed to have a negative gravitational mass (or negative gravitational “charge”, like opposite electrostatic charges in plasma) for all particles–antiparticles, e.g., neutron–antineutron, electron–positron, proton–antiproton etc. as well as for atoms and anti-atoms. The first discussion, related with such hypothesis for electrons and positrons has been done by L Schiff [1] (see also references therein). Later on, different aspects of this idea have been discussed (see, e.g., [2, 3]). In particular, the criticism, concerning the hypothesis of anti-gravitational
interaction of matter and antimatter was considered in detail and in many points rejected in [3, 4]. This concerns, e.g., the violation of the Equivalence Principle (EP) of the Einsteinian General Relativity. Some essential features of the symmetric cosmology (like the Dirac–Milne one), including the antigravity instead the cosmological constant, have been considered recently (see [5–7] and references therein). The authors [6, 7] show how the EP (historically formulated for attractive matter–matter gravity) can be naturally emerged and non-contradictorily expanded for the resulting model of the repulsive matter–antimatter antigravity. The authors [5] underline a serious motivation for studying in detail the Dirac–Milne-like Universe cosmology, because it “does not have any horizon problem...and solves two major problems in standard cosmology without requiring additional ingredients such as dark energy or inflation”. Initial separation of the gravitationally repulsive matter and antimatter in the Dirac–Milne Universe could start at the very early cosmological epoch of quark-gluon plasma. According to the Dirac–Milne Universe model, the transition of quark-gluon plasma to the electron-baryon plasmas corresponds approximately to an age of $6 \times 10^5$ s ([5]), in contrast with the more short time interval of order $\simeq 1$ s of the quark-gluon plasma existence in the $\Lambda$-CDM-model. Huge average distances between antigravitationally separated matter and antimatter clusters, which prevent annihilation in the current cosmological time, correspond roughly to the minimal value $L \simeq 7$ kpc or is even much more for certain estimations. These distances completely exclude annihilation between matter and antimatter clusters. In contrast, it has been well demonstrated that in the same—totally symmetric—matter–antimatter Universe, but without the assumed repulsive antigravity, the matter/antimatter survival is impossible, because the average distance between matter and antimatter clusters must have a size of the observable Universe. Otherwise, annihilations at the frontiers of matter and antimatter clusters should generate a diffuse gamma ray emission that would contradict observational data [8].

Recently, fine astronomical observations discovered the accelerated Universe expansion [9, 10]. This discovery gives a strong support to the hypothesis of matter–antimatter antigravity. Some futures of the repulsive gravitational behavior were found also for an astronomically short distance of about 2–5 Mpc (see [11] and further discussion in [12]).

The hypothesis of gravitational repulsion between matter and antimatter leads to their gravitational decoupling, but the exact mechanism still needs to be established. The solution of this complex problem cannot be considered within one paper. However, generalization of the Jeans instability and antiscreening, considered below are the effective mechanisms support to the hypothesis about gravitational repulsion of the particle-antiparticle interaction and the baryon-antibaryon symmetry of Universe. In the present paper, in contrast to the Dirac–Milne Universe model [5] based on Einstein’s equations of General Relativity, we investigate the purely classical—weak-gravity approximation—Newtonian “cosmology”, retaining the matter–antimatter antigravity. Due to the observable global Universe flatness, this approach applicable to large scales in the gravitational-antigravitatalional Universe model and leads to essential simplifications and new findings.

In this paper based on the simple model of two-component gravitational–antigravitational (TGA) model of homogeneous gravitationally neutral system of matter-clusters (MC) and antimatter-clusters (AMC), we consider the generalization of the Jeans instability and compare it with (a) the instability in homogenous gravitating matter “dust”, proposed by Jeans [2] and (b) the properties of electrically neutral plasma. We demonstrate that, in contrast to the well-known neutral system of electric charges, the neutral system of gravitational charges (gravitationally neutral matter–antimatter system) has an “anti-screening” property preventing matter–antimatter “charge” recombination and further annihilation. The remarkable features of gravitationally neutral matter–antimatter system lead to new opportunities to solve the problem of baryon asymmetry of the Universe and to disclose the deeply interconnected $DE/DM$ nature. The background of the supposed matter–antimatter symmetry, coupled with the gravity-
antigravity symmetry is the inevitable spatial separation of the masses of opposite gravitational charges during the early stages of the Universe evolution.

2. The features of the Newtonian gravity-antigravity model

Let us summarize the features of the Newtonian model investigated below:

(a) We assume the gravitationally neutral Universe (GNU) originated via a local Big Bang inside infinite and flat Euclidean 3D-space existed before the Big Bang. The GNU is steadily flat and Hubble-like expanding via the gravitational neutrality on a large scale:

(b) Cosmological constant $\Lambda = 0$; therefore, the vacuum energy density is negligible.

(c) The GNU consists of the cold and gravitationally neutral (weightless) homogeneous mixture of matter + DM clusters and antimatter + dark antimatter clusters filling a 3D-sphere with limited radius $R_U(t)$. It has zero gravity mass density on a large scale, where flat Euclidean large-scale space geometry inside $R_U(t)$ is always applicable.

The homogeneously distributed matter and antimatter clusters are “cold”, having low velocities and weak gravity on a large scale. These circumstances allow us to use the non-relativistic Newtonian approximation and classical Poisson equation. The Newtonian weak-gravity approximation is applicable to the matter–antimatter symmetric Universe along its long evolution, because it always consists of the gravitationally neutral and matter–antimatter mixture after the Big Bang.

In what follows, it is shown that the DE concept becomes physically transparent and self-consistent in the gravitationally neutral GNU-model, but the GNU-neutrality needs also correct inclusion the cold DM in the model under consideration. Indeed, the average DM density in the Universe is of order 5 times higher than Ordinary Matter (OM) average density. The DM is also gravitationally Newton-like attractive to OM. There are some cosmological observations and DM theories [13] which suggest that DM consists of hypothetical (yet unknown) stable dark “elementary particles” similarly to OM. Therefore, in the GNU-model with symmetrically coexisting OM / Ordinary Antimatter (OAM), we should also take into account the symmetrically coexisting DM / Dark Antimatter (DAM) counter-components. These last counter-components equally dominate in the GNU -Universe.

Thus, for simplicity, we assume that the globally gravitationally neutral Universe is the gravitationally neutral “dust-like” composition of $+m_{gr}$ of (DM + OM) clusters and $-m_{gr}$ of (DAM + OAM) clusters. Zero value of the average gravitational “charge” density necessarily provides the observed Universe flatness on a large scale. The existence of the DM, as is known, is conditioned by the necessity of explaining the observable relative stability of galactic clusters, galaxies, and their formation rate [14, 15]. The characteristic sizes of the DM clouds in the galaxy area can be of the order of or significantly larger than the characteristic size of the visible galaxy itself. According to the suggested GNU scenario, the ordinary antimatter (OAM) clusters should be also visible, as the ordinary matter (OM) clusters. The negative gravitational clusters possess (similarly to DM) the invisible DAM, which provides the relative stability of the formation rate of the visible antigalaxies. Based on the simple model of two-component gravitational–antigravitational (TGA) model with electrically neutral “particles”, e.g., gravitationally neutral system of $+m_{gr}$ matter-clusters (MC) and $-m_{gr}$ antimatter-clusters (AMC), in sections 3 and 4, we consider the generalization of the classical Jeans instability. We also demonstrate that, in contrast to the electrostatically neutral plasma of opposite electric charges, the gravitationally neutral system of opposite gravitational “charges” has the specific “anti-screening” property (section 5). This “gravitational plasma” can never recombine: opposite gravitational charges are repulsive. In section 6, the effect of the repulsive matter–antimatter potential on the hydrodynamic force is considered for the simple configurations of the gravitational system. Correlation energy of this system is formally considered in section 7 using the plasma-like approach. In the GNU, the concept of the ordinary baryon symmetry (together with the assumed
DM-DAM “particle-antiparticle” symmetry) can be reformulated as the general concept of global particle-antiparticle symmetry of the Universe for all evolution stages. The background of this symmetry is the spatial separation of the masses of opposite gravitational charges during the early Universe evolution stages. The way of these evolution and separation, especially during early epochs when annihilation processes of quark-gluon and hadron plasmas play an essential role, should be clarified in the future development of the considered model.

3. Jeans instability of matter
Isaac Newton (in his letter to Richard Bentley, 1692) first suggested that self-gravity in the infinite Universe would lead to the observed mass distribution [16]. The first quantitative description of matter fragmentation due to self-gravity has been done by Jeans [17] in 1902. According to [17], self-gravitating infinite uniform gas at rest should be unstable against small perturbations proportional to \( \exp\left[i(kr - \omega t)\right] \). Linearization of equations of ideal hydrodynamics and Poisson equation for the gravitational potential results in the dispersion equation

\[
\omega^2 = c^2 k^2 - \Omega^2,
\]

(1)

where \( \Omega = (4\pi G \rho)^{1/2} \) is the Jeans gravitational frequency, \( \rho \) is the density, \( c = (\gamma T/m)^{1/2} \) is the adiabatic sound velocity, \( \gamma = 5/3 \) is the ratio of specific heats, \( T \) is the gas temperature in energy units, \( m \) is the particle mass, and \( G \) is the gravitational constant.

As seen from equation (1), \( \omega^2 \) becomes negative, and the instability arises when the perturbation wavelength, \( \lambda = 2\pi/k \) exceeds the critical value:

\[
\lambda > \lambda_c = c \sqrt{\frac{\pi}{G\rho}}.
\]

(2)

The pressure gradient tends to quench the instability. This force dominates over the gravity force resulting in stabilization with \( \lambda \leq \lambda_c \). Thus, originally uniform gas, due to the instability, should break into clots with characteristic sizes of the order of \( \lambda_c \). It is noteworthy that \( k_c = \lambda_c / 2\pi = c / (4\pi G \rho)^{1/2} \) is only the characteristic linear scale, which may be constructed of the parameters inherent for the problem under consideration.

Kinetic theory of the Jeans instability was given in [18–21] using methods of plasma physics. It is noteworthy also that more realistic models, like expanding Newtonian world-model [22], lead to the same Jeans instability criterion (2). In fact, very similar results have been obtained in [23] using Friedmann solution for the expanding Universe.

4. Modes and instability for two component gravitational hydrodynamics with annihilation
Let us start from hydrodynamic equations for two types of electrically neutral particles (e.g., neutrons and antineutrons), assuming that their “gravitational charges” are opposite and the gravitational interaction of the particles and antiparticles is repulsive

\[
\frac{\partial n_a}{\partial t} + \text{div}(n_a \mathbf{V}_a) = -\nu_a \dot{a} n_a n_{\bar{a}},
\]

(3)

\[
m n_a \left[ \frac{\partial \mathbf{V}_a}{\partial t} + (\mathbf{V}_a \times \nabla) \mathbf{V}_a \right] = -\frac{\partial p_a}{\partial r} + n_a \mathbf{F}_a, \quad \mathbf{F}_a = -m_a \nabla \Phi.
\]

(4)

Here \( n_a \) is the density of particles of type \( a \), \( p_a = n_a(r,t)T \) is the respective pressure, index \( a = \{ m, -m \} \) and \( \bar{a} = -a \). The gravitational masses have opposite signs (for certainty, \( m > 0 \)). However, the inertial mass is always positive and is \(| m_a | = m \). We suppose that temperature \( T \)
The quantity $\nu_{a,\bar{a}} = \nu_{\bar{a},a} = \nu$ is the characteristic rate for the particle-antiparticle annihilation process.

The Poisson equation reads

$$\Delta \Phi = 4\pi G m (n_m - n_{-m}).$$

(5)

After linearization of Eqs. (3)-(5) on small deviations from the homogeneous state $n^0_m = n^0_{-m}$, $V^0_m = V^0_{-m} = 0$ and $\Phi = 0$ we arrive to the dispersion relation

$$\omega^4 + i2\omega^3 n^0 \nu + 2\omega^2 (\Omega^2 - c^2 k^2) + i\omega 2 n^0 \nu (2\Omega^2 - c^2 k^2) + c^4 k^4 - 2c^2 \Omega^2 k^2 = 0,$$

(6)

where the following notations are used $c^2 = T/m$, $\Omega^2 = 4\pi G m n^0$. For $k = 0$ equation (6) reads

$$\omega^4 + 2i\omega^3 n^0 \nu + 2\omega^2 \Omega^2 + 4i\omega n^0 \nu \Omega^2 = 0.$$

(7)

The explicit nontrivial solutions of equation (7) are

$$\omega_{1,2} = \pm i\sqrt{2} \Omega, \quad \omega_3 = -2in\nu.$$

(8)

The unstable solution $\omega_{2,3} = i\sqrt{2} \Omega$ is similar to the Jeans instability in the system of gravitational masses. However, the rate of this “balling” instability is $\sqrt{2}$ times higher than the Jeans one due to the presence of the subsystem of masses with the opposite gravitational charge. Comparison of the rates of “balling”, and Jeans instabilities is shown on figure 1.

In the recent work, Chris Collins [24] considered the so-called “Planck-cluster problem” and estimated the number of massive clusters in the Universe. According to the estimation, this number is about a factor of two below the prediction based on the Planck CBR analysis. In our opinion this estimation can be connected with the factor $\sqrt{2}$ in the increment of the above instability.

The obtained instability rate confirms the assumption [7] that gravitational repulsion of the matter–antimatter system accelerates consolidation of matter and antimatter clusters.

These clusters (“balls”) of the opposite gravitational charge continue to repulse one from another, providing the accelerated expansion of the whole system. The third solution describes the purely damping annihilation mode.

For the case of the model with $\nu = 0$ the solutions equation (6) for finite $k$ reads

$$\omega_1^2 = -2\Omega^2 + c^2 k^2, \quad \omega_2^2 = c^2 k^2.$$

(9)

The peculiarity of the “balling” instability is stabilization on the smaller characteristic size than the Jeans instability. The second essential peculiarity of the system with antigravitation is the appearance of the sound (linear on $k$) mode. It is easy to see that the sound mode for a small, but finite rate of annihilation has the damping spectrum

$$\omega_3 = ck - in^0 \nu.$$

(10)

Therefore, the “sound wave” exists only for $k \gg k_0 = n^0 \nu / c$. For the opposite inequality $k \ll k_0 = n^0 \nu / c$ this mode is absent, according to (8).
5. Two-component gravitational system: anti-screening of an immobile gravitational charge

For the case of the point gravitational mass $\mu_0$ (providing the spherical symmetry) in the two-component infinite gravitational system with immobile ($V_e = V_i = 0$) masses $m_+ = -m_0 = m$, the stationary distribution at $T = \text{const}$ is described by equations

$$\triangle \Phi(r) = 4\pi G[nn_m - mn_{m,-}] + 4\pi G\mu_0 \delta(r)$$

$$= 4\pi G[nn_{m,0} \exp(-m\Phi/T) - mn_{m,0} \exp(m\Phi/T)] + 4\pi G\mu_0 \delta(r).$$

(12)

Let us suppose (as in the case of an electrically charged particles in quasineutral weakly non-ideal plasma) that linearization is possible. For the case $n_{m,0} = n_{-m,0} = n_0$, introducing the value of gravitational radius $R_G^2 = T/4\pi Gm^2 n_0$ we arrive at the equation

$$\triangle \Phi(r) + 2\Phi/R_G^2 = 4\pi G\mu_0 \delta(r).$$

(13)

For the Fourier-component $\Phi(k) = \int dr \exp(-ik \cdot r)\Phi(r)$

$$-k^2 \Phi(k) + \frac{2}{R_G^2} \Phi(k) = 4\pi G\mu_0 \delta(k),$$

(14)

$$\left[-k^2 + \frac{1}{R_G^2}\right] \Phi(k) = 4\pi G\mu_0,$$

(15)
where \(2/R_G^2 \equiv 1/R_G'^2\). Therefore, in \(r\)-space

\[
\Phi(r) = -\frac{\mu_0 G}{r} \cos\left(\frac{r}{R_G'}\right),
\]

\[n_m = n_0 \exp(-m\Phi/T) \rightarrow n_m = n_0 \exp\left[\frac{m\mu_0 G}{T r} \cos\left(\frac{r}{R_G'}\right)\right] \simeq n_0[1 + \frac{m\mu_0 G}{T r} \cos\left(\frac{r}{R_G'}\right)], \]

\[n_{-m} = n_0 \exp(m\Phi/T) \rightarrow n_{-m} = n_0 \exp\left[-\frac{m\mu_0 G}{T r} \cos\left(\frac{r}{R_G'}\right)\right] \simeq n_0[1 - \frac{m\mu_0 G}{T r} \cos\left(\frac{r}{R_G'}\right)]. \]

Figure 2. Comparison of electrostatic screening of the charge \(+Q\) in the quasineutral plasma (a) and the anti-screening effect for the gravitational “charge” \(+M\) (b). The arrows show direction of the gravitational charges motion (b).

These distributions are crucially different from the two-component plasma of electric charges. If the signs of the gravitational masses \(m\) and \(\mu_0\) are the same the density of masses \(m\) increases around \(\mu_0\), whereas the density of masses \(m_–\) decreases around \(\mu_0\). Comparison of the density distributions for quasineutral plasma and for two-component gravitational–antigravitational system is shown on figure 2. Both densities tends to \(n_0\) where \(r \to \infty\). Since the density \(n_m(r \to 0) \to \infty\) linearization is not valid for the problem in general. However, due to linearization, it is easy to understand that the physical picture in the gravitational–antigravitational two-component system (which can be determined as anti-screening) is crucially different from the plasma case. The potential \(\Phi\) has the oscillatory behavior.

The attraction of the masses of the same gravitational charge leads to the creation of massive clouds of the same gravitational mass (gravitational clusters of particles and antiparticles), whereas the clouds of opposite gravitational masses tend to break up. This trend can play a fundamental role for the global astrophysical processes.

6. Two-component gravitational system: force

In the previous sections we considered the processes in the initially homogeneous infinite \(TGA\) system. Let us consider now forces in the finite initially inhomogeneous \(TGA\) system.
After integration of the Poisson equation for the case of spherical symmetry $\Delta \Phi = 4\pi Gm(n_m(r) - n_m(r'))$, where $\Delta = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 \frac{\partial \Phi(r)}{\partial r})$. The force acting on one particle equals $F_m = -m \nabla \Phi$ and $F_{-m} = m \nabla \Phi$

$$F_{m,r}(r) = -m \frac{\partial \Phi(r)}{\partial r} = -\frac{4\pi Gm^2}{r^2} \int_0^r dr' r'^2 (n_m(r') - n_{-m}(r')) = 0$$

$$F_{m,r}(r) = -m \frac{\partial \Phi(r)}{\partial r} = -\frac{4\pi Gm^2}{r^2} \int_0^r dr' r'^2 (n_m(r') - n_{-m}(r')). \quad (19)$$

For homogeneous distributions of both components inside a sphere of radius $R_0$, the $r$-component of the force acting on one $m$ particle placed on the surface of the sphere is given by

$$F_{m,r}(R_0) = -\frac{4\pi Gm^2}{R_0^2} \int_0^{R_0} dr' r'^2 (n_m(r') - n_{-m}(r')) = -\frac{4\pi Gm^2((N_m - 1) - N_{-m})}{R_0^2}. \quad (20)$$

For $N_m = N_{-m}$ the force is positive $F_{m,r}(R_0) = \frac{Gm^2}{R_0^2} > 0$, hence, is always directed from the center.

For the particles of the type $m_{-}$

$$F_{-m,r}(r) = m \frac{\partial \Phi(r)}{\partial r} = \frac{4\pi Gm^2}{r^2} \int_0^r dr' r'^2 (n_m(r') - n_{-m}(r'))$$

$$F_{-m,r}(R_0) = \frac{4\pi Gm^2}{R_0^2} \int_0^{R_0} dr' r'^2 (n_m(r') - n_{-m}(r')) = \frac{4\pi Gm^2(m(N_e - (N_i - 1))}{R_0^2}. \quad (21)$$

Since $N_m = N_{-m}$, the force is also the same and positive $F_{-m,r}(R_0) = \frac{Gm^2}{R_0^2} > 0$. Therefore, for homogeneous spherically symmetric distribution, the force acting on the particle of arbitrary sign of mass, placed on the surface of the sphere, is always directed from the center.

However, this calculation seems too primitive since the distortion of density around the probe gravitational charge placed on the sphere $R_0$ is not taken into account. Let us estimate the force more accurately assuming that the potential created by this mass $\mu_0$ on the boarder is of the order of (16). Since the potential of this charge in vacuum is $\Phi_0(r) = -\mu_0 G/r$, the potential created by surrounding particles is $\delta \Phi(r) = \Phi(r) - \Phi_0(r)$ (in this approximation, we neglect the spherical asymmetry violation)

$$\delta \Phi(r) = \frac{\mu_0 G}{r} [1 - \cos(\frac{r}{R_G})]. \quad (22)$$

Therefore, the force $F_{\mu_0,r}(r \to 0)$ affected on the probe charge equals

$$F_{\mu_0,r}(r \to 0) = \lim_{r \to 0} \frac{\partial \delta \Phi}{\partial r} \approx \frac{3\mu_0^2}{4R_G^2}. \quad (23)$$

This force is directed outside the spherical volume (along the unit vector $r/r$) independently on the sign of gravitational charge and provides the accelerating expansion of the system under consideration.

Let us now calculate the force for two spherical homogeneous distributions with different radii $R_m$ and $R_{-m}$. For certainty take $R_{-m} > R_m$. The “global mass neutrality” requires $n_m R_m^3 = n_{-m} R_{-m}^3 = N_m = N_{-m}$. Therefore, $n_{m,0} > n_{-m,0}$. The force $F_{m,r}$ inside $R_m$:

$$F_{m,r}(r < R_m) = -\frac{4\pi Gm^2}{r^2} \int_0^{R_m} dr' r'^2 (n_{m,0}(r') - n_{-m,0}(r')) \to$$

$$= -\frac{4\pi Gm^2(r_{m,0} - r_{-m,0})}{3} < 0. \quad (24)$$
This force is directed to the center. This means the sort of particles $m$ will constrict to the center.

The force $F_{-m,r}$ inside $R_m$

$$F_{-m,r}(r) = \frac{4\pi G m^2 r (n_{m,0} - n_{-m,0})}{3} > 0. \quad (25)$$

This force is directed outward the center and the type of particles $m_-$ will be pushed outside the sphere $R_m$.

The forces $F_{m,r}$ (however, the $m$ particles are absent for $r > R_m$, but we can determine the force acted on an added probe $m$ particle), as well as the force $F_{-m,r}$ outside $R_m$, but inside $R_m > R_m$:

$$F_{m,r}(R_m < r < R_{-m}) = -\frac{4\pi G m^2}{r^2} \int_0^{R_m < r < R_{-m}} dr' r'^2 (n_{m,0}(r') - n_{-m,0}(r')) \rightarrow \frac{-4\pi G m^2 (n_{m,0} R_m^3 - n_{-m,0} r^3)}{3r^2} < 0. \quad (26)$$

This force directed to the center, since $(n_{m,0} R_m^3 - n_{-m,0} r^3) > 0$ for $R_m < r < R_{-m}$.

$$F_{-m,r}(R_m < r < R_{-m}) = \frac{4\pi G m^2 (n_{m,0} R_m^3 - n_{-m,0} r^3)}{3r^2} > 0. \quad (27)$$

This force directed outward the center, since $n_{m,0} R_m^3 - n_{-m,0} r^3$ since $R_m < r < R_{-m}$. The particles of type $m_-$ will be pushed out from the initial sphere.

Finally, for $r > R_i$

$$F_{m,r}(r > R_i) = -\frac{4\pi G m^2 (n_{m,0} R_i^3 - n_{-m,0} R_i^3)}{3r^2} = 0, \quad (28)$$

$$F_{-m,r}(r > R_i) = \frac{4\pi G m^2 (n_{m,0} R_i^3 - n_{-m,0} R_i^3)}{3r^2} = 0. \quad (29)$$

The gravitational forces acting outside the larger radius are zero due to the “global mass neutrality” requirement $n_{m,0} R_m^3 = n_{-m,0} R_{-m}^3 = N_m = N_{-m}$. The force connected with pressure can be easily included. The pressure prevents the compression by gravitational force directed to the center and contributes to the gravitational force directed outward the sphere.

The same way can be used to consider the different (in particular smooth) spherically-symmetrical profiles and more complicated non-symmetrical profiles. We see the dominative role of gravitational repulsive force between the initially separated “clouds” of particles with the opposite gravitational charges.

7. Correlation energy calculation

Although the homogeneous $TGA$ system is unstable, it is interesting to calculate the energy of $TGA$ gas and to compare the result with the energy of weakly interacted plasma. The correlation energy can be written in the form [25]

$$E_{corr.} = \frac{1}{2V^2} \sum_{a,b} N_a N_b \int \int u_{a,b} w_{a,b} dV_a dV_b, \quad (30)$$
where the potentials are \( u_{a,b} = -m_a m_b G/r \) (repulsion interaction of the particles with opposite gravitational masses) and \( w_{a,b} \) is the pair correlation function. Taking into account these notations and following the standard procedure applied to Coulomb systems, we arrive at the equation for \( w_{a,b} \)

\[
\Delta w_{a,b}(r) = -\frac{4\pi m_a m_b G}{T} \delta(r) - \frac{4\pi m_b G}{TV} \sum_c N_c m_c w_{a,c}(r). \tag{31}
\]

The solution to this system of equations can be found in the form \( w_{a,b}(r) = m_a m_b \omega(r) \), where the function \( \omega(r) \) obeys the equation

\[
\Delta \omega(r) + \frac{1}{R_G^2} \omega(r) = -\frac{4\pi G}{T} \delta(r), \tag{32}
\]

where \( 2R_G^2 = T/4\pi n_0 m^2 G \) and the equalities \( m_+ = -m_- = m > 0 \), \( N_m = N_{-m} \), \( n_{m,0} = n_{-m,0} = n_0 = N_m/V \) are taken into account. The solution for \( \omega(r) \) reads

\[
\omega(r) = \frac{G}{rT} \cos(r/R_G). \tag{33}
\]

For \( E_{\text{corr.}} \), we obtain

\[
E_{\text{corr.}} = -V \frac{T}{32\pi R_G^2} \int_0^\infty \cos \zeta d\zeta = -VT\frac{(8\pi n_0 m^2 G)^{3/2}}{32\pi T^{3/2}} \int_0^\infty \cos \zeta d\zeta, \tag{34}
\]

Regularization of the integral in (34) leads to the equality \( E_{\text{corr.}} = 0 \) for the used approximation of probability \( w_{a,b} \).

8. Conclusions

The hypothesis about the antigravitational interaction between elementary particles and antiparticles leads to the possibility of creating massive gravitationally charged clusters of the opposite gravitational charge during the Universe evolution. The observable flatness of the Universe on a large scale provides the Newtonian equations applicability for consideration of the model under consideration.

On this basis, assuming the condition of the initial homogeneity of particles and antiparticles and global gravitational neutrality (connected with the unbroken fundamental particle-antiparticle symmetry), we found the change of the Jeans instability rate. The obtained instability rate is higher (by a factor of 2) than the rate based on Jeans instability increment. In describing the clustering process, this factor serves as an argument in favor of the considered clustering in the matter–antimatter model, comparable to the clustering of the gravitating matter model, discovered by Jeans [2]. The found rate difference solves the so-called “Planck-cluster problem”, where the estimated number of massive clusters in the Universe is about two times lower than the prediction based on the Planck CBR analysis [24]. We suppose that the relation between this change and the estimated number of massive clusters in the gravitationally neutral—“weightless” Universe belongs to definite cosmological confirmations of the GNU-concept. The existence of “antiscreening” in the model under consideration is also shown. Both effects lead to the conclusion that not the space itself is now and was enormously expanded via the Big Bang hyperinflation, but assumedly gravitationally repulsive matter and antimatter have the unstoppable tendency to spatial separation, causing the observed quasi-linear Hubble-like Universe expansion. The force calculation supports the hypothesis that the mechanism of the observable acceleration of the far located galactic clusters in the Universe
can be explained based on the considered antigravitational interaction between matter and antimatter clusters. Therefore, this interaction can play the role of the interconnected DE and DM in the globally gravitationally neutral Universe. The essential advantage of the hypothesis is the opportunity to reestablish the fundamental unbroken baryon symmetry of the Universe, where the matter quantity is always equal to the antimatter quantity. However, due to the gravitational–antigravitational massive matter–clouds and antimatter–clouds repulsion, they are separated and escaped annihilation during the gravitational Universe evolution. Creation of these massive clouds is conditioned by the gravitational attraction of particles (as well as antiparticles) of the same type. The structure of the observable Universe at huge distances can be similar to soap suds or huge bubbles, whose surfaces consist of gravitational and antigravitational “dust” (e.g., galactic and antigalactic clusters), where an average surface gravity mass density on the large scale remains zero. The problem of the non-linear time evolution for the models with the antigravitational particle-antiparticle interaction has to be considered analytically and numerically, taking into account the transformation of elementary particles and antiparticles, electromagnetic radiation, and creation of elements at the later stage of the gravitationally neutral Universe evolution. The large-scale waves can be considered in spirit of [26]. The similarity approach (see, e.g. [27]) can be used for development of the thermodynamic models of TGA.

The gravitational interaction of matter with antimatter has not been conclusively observed by physicists at ground-based laboratories. The first sufficiently sensitive and decisive experimental tests of the antihydrogen gravitational properties will be conducted at the end of 2015 in ALPHA [28], AEgIS [29] and GBAR [30] experiments at CERN.

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