We present results from a recently completed project to calculate next-to-leading logarithmic resummed distributions for a variety of event shapes in the 1+1-jet limit of DIS. This allows fits for the strong coupling and for non-perturbative effects using the large amount of data on these observables from HERA. Spin-offs include the discovery of a new class of logs for certain final state observables (non-global observables); a program that allows a speed-up by an order of magnitude of certain fixed-order calculations in DIS with DISENT or DISASTER++; and the development of state-of-the-art PDF evolution code.

1 Introduction

Event-shapes are observables sensitive to the flow of energy and momentum in hadronic final states. They have been extensively studied in $e^+e^-$ collisions, for example for the measurement of the strong coupling, tests of QCD through fits for the colour factors and the study of novel approaches to hadronization. Typically the most discriminatory studies make use of event-shape distributions, which are compared to next-to-leading perturbative predictions that are resummed in the 2-jet limit.

Recently the HERA experiments have also started considering event shapes, defined in the current hemisphere of the Breit frame. In particular, distributions have been measured by H1 and while only mean values have been so far studied by ZEUS it is our understanding that they intend to extend their studies to distributions.

Resummed predictions for $e^+e^-$ event shapes have existed in the literature since the early nineties but until recently no such calculations were available for DIS observables (an exception is jet rates). Because of the strong similarity between a hemisphere of an $e^+e^-$ event and the current hemisphere of the DIS Breit frame, it is natural to assume that the extension from...
$e^+e^-$ to DIS will be fairly straightforward. It turns out not to be so, for both conceptual and technical reasons. In what follows we outline some of the issues that arise, and then present some preliminary comparisons to data.

2 Resummation issues in DIS

The most obvious new issue to arise in DIS compared to $e^+e^-$ is that of collinear factorization. Despite the fact that the observables are all defined in the current-hemisphere ($\mathcal{H}_C$) of the Breit frame, owing to details of the kinematics those defined with respect to the photon axis are sensitive to emissions in the remnant hemisphere ($\mathcal{H}_R$) through recoil effects. Requiring the event-shape to have a value close to that of the $1+1$-jet limit, one therefore forbids emissions in the whole of the phase space ($\mathcal{H}_C$ and $\mathcal{H}_R$). However, collinear factorization at the scale $Q^2$ is conditional on there being no restrictions on emissions in the remnant hemisphere. Requiring the event shape to have a value less than some $V$, which translates to a limit on the largest possible transverse momentum of collinear emissions in $\mathcal{H}_R$, $k_t^2 \lesssim V^n Q^2$ ($n$ is observable-dependent), has the consequence that collinear factorization can only be recovered if parton distributions are evaluated at a scale of the order of $V^n Q^2$. This is actually a familiar result from calculations of the $p_t$ distribution of Drell-Yan pairs and has also been observed in more complicated multi-jet event shapes.

A second issue is that of non-global logarithms, which arise in observables sensitive only to emissions in a restricted portion of phase space (e.g. $\mathcal{H}_C$) such as the jet mass, and additionally in observables whose sensitivity to emissions is discontinuous across one or more boundaries in phase space (an example is the thrust $\tau_{ZE}$). An erroneous assumption that has widely been made in the literature is that (to single-logarithmic accuracy) in order to suppress radiation into $\mathcal{H}_C$ (say), it suffices to suppress primary radiation from the various hard 'legs' into that hemisphere.

While at leading order in $\alpha_s \ln V$ this is correct, starting from second order, configurations such as those in fig. become relevant. The crosses indicate emissions which must be forbidden in order for the observable to have a small value. The grey emissions are those that do not directly affect the value of the observable. The left-hand picture represents the configuration relevant at second order: a soft emission (1) in $\mathcal{H}_R$, which does not contribute to the observable, radiates an even softer emission (2) into $\mathcal{H}_C$, which does contribute to the observable. The strong ordering in energies $Q \gg E_1 \gg E_2$ leading to one power of $\ln V$ for each power of $\alpha_s$. While this term is calculable analytically, at all orders one needs to forbid coherent radiation into $\mathcal{H}_C$ from arbitrarily complicated ensembles of large-angle energy-ordered gluons in $\mathcal{H}_R$ (right-hand picture). This is complicated both from the point of view of the colour structure and of the geometry. The former can be dealt with approximately in the large $N_C$ approximation, while the latter can so far only be treated numerically. Some insight into the dynamics associated with these non-global logs was obtained in the context of a more general study of energy flow distributions where one finds that in the limit of large $\ln 1/V$ not only is radiation into $\mathcal{H}_C$
forbidden, but radiation at intermediate energy scales is also forbidden in a neighbouring ‘buffer’ region of $\mathcal{H}_R$. The size (in rapidity) of this buffer region increases with $\ln V$ and the overall suppression factor coming from non-global logs seems, at least in part, to be associated with the suppression of primary radiation into the buffer.

3 Technical issues

When implementing the resummations as computer programs to allow comparisons to data a number of technical issues arose. When this project started there existed only two subtraction-based programs for NLO calculations in DIS, namely DISENT\textsuperscript{15} and DISASTER++\textsuperscript{16}, which were known to disagree for certain observables.\textsuperscript{17} Comparisons with the expansion of the resummed results made it possible to identify DISASTER++ as the one giving correct predictions.\textsuperscript{c}

Unfortunately, of the two programs, DISASTER++ is an order of magnitude slower, and we would have needed over a year’s computing time to obtain the fixed-order predictions (needed to describe the observable in the region outside the $1 + 1$ jet limit) to sufficient accuracy. However, the traditional approach to such calculations involves considerable duplication of effort: one needs distributions at several $x$ and $Q^2$ values and typically one uses separate events for each $x$ and $Q^2$ value. But the matrix elements (modulo their $y_{Bj}$-dependence) and the calculation of the event-shape are both independent of $x$ and $Q^2$, so each event in an NLO Monte-Carlo program can be ‘reapplied’ to several $x$ and $Q^2$ values. We have written a program DISPATCH,\textsuperscript{d} which acts as a wrapper to DISENT and DISASTER++ so as to automate such a procedure.

Another spin-off from this project is the development of a high-precision PDF evolution code,\textsuperscript{19} which has been used in collaboration with Vogt\textsuperscript{20} to produce reference NNLL evolutions to an accuracy of 1 part in $10^5$.

4 Comparison to data

The left-hand plot of fig. 2 shows a comparison between our matched resummed distributions\textsuperscript{6,9,19} and the H1 data\textsuperscript{2} for the $C$-parameter, $C_E$. Non-perturbative contributions have

\textsuperscript{c}The recently released NLOJET program\textsuperscript{18} is also in agreement with DISASTER++.

\textsuperscript{d}Available from http://www.lpthe.jussieu.fr/~salam/dispatch/
been included using $1/Q$ corrections, whose size have been hypothesized to be governed by a universal parameter $\alpha_0$. We have fitted for both $\alpha_s$ and $\alpha_0$, using only the red points of the distributions. The results (1-$\sigma$ contours) for $C_E$ and a number of other DIS observables are shown in the right-hand plot (green curves) and compared to $e^+e^-$ results for mean values (red curves), with the jet masses measured in so-called massless schemes. The agreement both within DIS and across experiments is strong confirmation of the universality hypothesis for $\alpha_0$.

There remain some observables in DIS where the agreement is less good, notably those measured with respect to the photon axis ($\tau_{\gamma E}, B_{\gamma E}$), and the situation worsens if one includes lower $Q$ (also lower $x$) data. The detailed origin of the problem remains to be understood, though it may well be associated with higher-order corrections being relatively larger at lower $Q$ values. In this context, higher precision data (and higher resolution data as well), especially at larger $Q$ values, will be interesting as it will make it possible to pin down any systematic $Q$ dependence over and above that expected from the theoretical predictions used so far, perhaps for example from shape functions.

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