Dependence of hadron spectra on decoupling temperature and resonance contributions

K.J. Eskola\textsuperscript{a,b}, H. Niemi\textsuperscript{a}, P.V. Ruuskanen\textsuperscript{a,b}, S.S. Räsänen\textsuperscript{a,b}

\textsuperscript{a}Department of Physics, P.O.Box 35, FIN-40014 University of Jyväskylä, Finland
\textsuperscript{b}Helsinki Institute of Physics, P.O.Box 64, FIN-00014 University of Helsinki, Finland

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Using equilibrium hydrodynamics with initial conditions for the energy and net baryon number densities from the perturbative QCD + saturation model, a good simultaneous description of the measured pion, kaon and (anti)proton spectra in central Au+Au collisions at $\sqrt{s} = 130$ AGeV is found with a single decoupling temperature $T_{\text{dec}} = 150 \ldots 160$ MeV. The interplay between the resonance content of the EoS and the development of the transverse flow leads to inverse slopes and $\langle p_T \rangle$ of hadrons which increase with decreasing $T_{\text{dec}}$. The origin of this result is discussed.

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Introduction. The amount of initially produced matter in ultrarelativistic heavy ion collisions at collider energies has been suggested to be controlled by gluon saturation\cite{1,2,3}. For heavy nuclei, $A \sim 200$, the saturation scale which determines the dominant transverse momentum scale is expected to be $1 \ldots 2$ GeV, which can lead to quite large values for the initial transverse energy $\frac{dE}{dy}$\cite{1,2,3}. For instance, the initial $dE_T/dy$ at $y \sim 0$ obtained from the perturbative QCD (pQCD) + saturation model\cite{2} exceeds the value observed in central Au+Au collisions at Brookhaven RHIC by a factor 2.5 ... 3\cite{2,3}.

A mechanism to transfer transverse energy into the longitudinal motion is provided by the asymmetry of the initial collective motion of produced matter. The observed rapidity distributions of final particles suggest that the matter is produced in a state of rapid longitudinal expansion\cite{4}. In the collective expansion energy is transferred through the work by pressure, $pdV$. Hence the strong initial expansion in longitudinal direction leads to a large transfer of energy from the transverse into the longitudinal motion. The space–time evolution of a dense matter with collective effects and with a QCD phase transition is describable in terms of relativistic hydrodynamics\cite{4,5,6,7}.

Hadron spectra are often argued to be insensitive to the early stages of the collision. This is not quite true for events with large multiplicities since secondary collisions will lead to transverse collective motion and correlations in the properties of particle spectra which are difficult to understand otherwise. A clear correlation resulting from radial flow is the mass dependence of slopes of transverse spectra. For non-central collisions hydrodynamics predicts the experimentally observed elliptic flow\cite{4}.

As energy is transferred into longitudinal direction during the expansion, it is usually expected that the transverse spectra of final pions become steeper at lower decoupling temperatures. However, since the number of hadrons and resonance states increases rapidly with mass, a considerable fraction of the energy of thermal matter can be in the form of heavy resonances. When the total transverse energy of all hadrons decreases with the decreasing decoupling temperature, the energy released from the reduction of the number of resonances and heavy particles can lead to an increase of the slope of the spectra of the remaining final stable hadrons. The energy release from the latent heat in a phase transition has a similar but smaller effect. In this work we study the details of these effects and the interplay between them by comparing the expansion of a massive pion gas with that of a hadron resonance gas.

Slopes of hadron spectra from heavy ion experiments at Brookhaven AGS and CERN SPS, can be reproduced with a kinetic freeze-out temperature $\sim 120$ MeV\cite{2}. To reproduce the strange particle abundances in thermal models, chemical freeze-out temperatures higher than 120 MeV are needed\cite{3}. Also at RHIC, a higher decoupling temperature is supported by the heavy particle yields\cite{1}. In this note we argue that in central Au+Au collisions at $\sqrt{s} = 130$ AGeV also the kinetic freeze-out takes place effectively at higher temperatures: solving the transverse expansion within boost invariant hydrodynamics with initial conditions for the energy and net baryon number densities from the pQCD + saturation model\cite{2}, we describe simultaneously the measured $\pi^\pm$, $K^\pm$ and $p(\bar{p})$ spectra with a single decoupling temperature $T_{\text{dec}} = 150 \ldots 160$ MeV. Similar observation has been made in Ref.\cite{4} using a fireball parametrization.

We describe briefly our framework to calculate the hadron spectra and then discuss how the hadron and resonance content in the Equation of State (EoS), and the phase transition affect the dependence of the slope of the spectra on the decoupling temperature. Finally, the pion, kaon and (anti)proton spectra are compared with the experimental results from PHENIX\cite{15,16,17}.

The theoretical framework. The calculational frame which we use has been discussed in detail in\cite{4}. The initial particle production is calculated from the pQCD + saturation model based on lowest order pQCD interactions and a cut-off scale $p_{\text{cut}}$ determined from a saturation condition for the final state minijets\cite{2}. The $K$-factor 2.3
has been fixed on the basis of the next-to-leading order calculation of minijet transverse energy [8].

The transverse energy weighted minijet cross section in a central rapidity unit, \( \sigma(E_T)/(p_{sat}, \sqrt{S}, A_i, |y| \leq 0.5) \), is the key quantity for determining the initial energy density at the time of formation of the matter. As a new ingredient, we include the corresponding quantity for the initial net baryon number, \( \sigma(N_B) = (\sigma(N_q) - \sigma(N\bar{q}))/3 \), where the computation of \( \sigma(N_{q(\bar{q})}) \) is based on the flavour decomposition of the minijet cross sections in [8].

Below, we shall consider Au+Au collisions at \( \sqrt{S} = 130 \text{ AGeV} \) with a 5 % centrality cut. As explained in [4], this corresponds to central collisions of effective nuclei with \( A_{eff} = 181 \). The number of participants which we get for such collisions is 346, consistent with the PHENIX result 348 ± 10 [9]. For the computation of the initial densities, we obtain \( \sigma(E_T) = 67.0 \text{ mbGeV} \), \( \sigma(N_B) = 0.64 \text{ mb} \) and \( p_{sat} = 1.06 \text{ GeV} \). Thus, the initial \( E_T = 1750 \text{ GeV} \) and \( N_B = 16.7 \) in the interval \( |y| \leq 0.5 \) [9].

The matter is assumed to be (approximately) thermalized at the time \( \tau_0 = 1/p_{sat} = 0.19 \text{ fm/c} \). For central collisions, this approach has predicted successfully [2, 8] the particle multiplicities at RHIC [20] and also, when amended with a hydrodynamical description of the transverse expansion of produced matter, the transverse energies of final particles, \( dE_T/dy \), within 10...20 % [9].

The high temperature phase of our EoS is QGP with a bag constant \( B \) and the low temperature phase a hadron resonance gas (HRG) with hadrons and hadron resonances up to \( M = 2 \text{ GeV} \) and a repulsive mean field among the hadrons [7]. The values of the bag constant \( B \) and the mean field strength \( K \) are so chosen that the transition temperature is \( T_c = 165 \text{ MeV} \).

The thermal spectra of final stable hadrons are obtained by folding the thermal motion with the fluid motion using the Cooper and Frye prescription [8, 21]. The dependence of the spectra on the transverse flow velocity \( v_r = \tanh y_r \) and on the decoupling temperature \( T_{dec} \) enters through the factors \( (p_T/T_{dec}) \cosh y_r \) and \( (m_T/T_{dec}) \sinh y_r \) as the arguments of modified Bessel functions [21]. E.g., for pions, the contributions from regions where \( v_r \geq 0.5 \) behave approximately as \( \sim \exp(-p_T/e^{y_r}T_{dec}) = \exp(-p_T/T_{eff}) \) with \( T_{eff} = e^{y_r}T_{dec} \). This shows explicitly how the inverse slope, \( T_{eff} \), depends at large \( p_T \) on the decoupling temperature and on the transverse flow rapidity \( y_r \). Due to the exponential dependence on \( y_r \), a small increase in the flow can compensate the decrease in \( T_{dec} \). The net effect on \( T_{eff} \), as the decoupling temperature and the transverse flow change, depends on the EoS and, in particular, on the assumed particle content of the hadron phase.

To obtain the spectra of final stable hadrons, we first calculate the spectra of all hadrons and hadron resonances which are included in the EoS. Then the decay contributions from all resonance states are added to the spectra of stable hadrons. All strong and electromagnetic two and three body decay modes with known branching ratios are included [22].

![FIG. 1: Left: Transverse spectra of thermal \( \pi^+ \) for two decoupling temperatures \( T_{dec} \) in the four cases studied; decay pions are not included for (b) and (d). The spectra for (c) and (d) have been scaled. Right: The change of the transverse flow rapidity \( y_r \), as a function of temperature along a flow line starting at \( r = 5 \text{ fm} \) at \( \tau_0 \). The flow rapidities \( y_r, 1 \) at \( T_1 = 160 \text{ MeV} \) are indicated. On the dashed-dotted line the effective temperature \( T_{eff} = e^{y_r}T_{dec} \) would remain constant.](image)

The effect of resonances and phase transition on \( T_{dec} \) dependence of spectra. Before comparing our results with data, we try to clarify the \( T_{dec} \) dependence of final hadron spectra by comparing the spectra of thermal pions at \( T_{dec} = 160 \text{ and } 100 \text{ MeV} \) in four different cases: (a) massive pion gas (PG) without phase transition (PhT), (b) hadron resonance gas (HRG) without PhT and (c) PG with PhT to QGP, (d) HRG with PhT to QGP. To make the differences in the effects on flow more transparent, we first consider only thermal pions also in cases (b) and (d). The spectra of positive pions are shown in the left panel of Fig. 1 for each case. Except in the case (a) the slopes in the interval 1.5 \( \leq p_T \leq 4 \text{ GeV} \) change considerably. Pions in this part of the spectrum come mainly from matter with large transverse velocity where \( T_{eff} = e^{y_r}T_{dec} \) holds.

In the expansion the temperature of the matter drops and the transverse velocity increases. These changes have opposite effects on the slope. They nearly cancel if the growth of the flow in terms of an increase in \( y_r \) satisfies \( \Delta y_r = y_r - y_{r,1} = \log(T_1/T) \) when the temperature drops from \( T_1 \) to \( T \). In the right panel of Fig. 1 we plot \( \Delta y_r \) as a function of \( \log(T_1/T) \) between \( T_1 = 160 \text{ MeV} \) and \( T = 100 \text{ MeV} \) for a fluid element moving along a flow line which initially starts at \( r = 5 \text{ fm} \). This fluid element belongs to the region which dominates the tail of the spectra. We see that for PG without PhT, the case (a) with the small change of slope (cf. Fig. 1, left), the curve is close to the dashed-dotted line for \( e^{y_r}T_{dec} = \text{const} \). For the other cases the increase in flow with the decrease
in temperature is stronger and the changes in $\Delta y$ are comparable. This indicates a similar relative increase in $T_{\text{eff}}$, the inverse slope of the spectra, in the cases (b)-(d).

The development of transverse flow is fastest for PG with large pressure gradients and slower for the other cases with softer EoS. In the cases (b) and (d) with HRG, a considerable amount of energy is stored in the heavy hadron and resonance states. As the matter expands, the release of energy from these states slows down the decrease in temperature leading to a stronger growth of the flow, as a function of decreasing temperature, than in the PG of the case (a). In the case (c), the latent heat released at the strong first order PhT leads to a similar behavior of the $y_r(T)$ as in the case (b) and (d). The reason why the change of slopes between (b) and (d) is smaller than between (a) and (c) is the weaker phase transition in the case (d). The strength of the PhT is controlled by $s_Q/s_H$, the ratio of entropy densities of the QGP and hadron phase at $T_c$ which is $\approx 15$ for the case (c) and $\approx 3$ for (d). If the ratio in the case (c) is artificially forced to that in (d) by decreasing the number of degrees of freedom in the QGP, the changes in the pion spectra from (a) to (c) and (b) to (d) become approximately the same. We conclude, that for the realistic EoS with HRG and QGP, the change in the slope of pions, is caused mainly by heavy hadrons and resonances in the EoS and to much lesser degree by the phase transition.

The values of the average transverse momentum $\langle p_T \rangle$ and multiplicities of positive thermal pions are collected to the Table I. In addition, the column (e) shows the same quantities for HRG+QGP when also the pions from the resonance decays (RD) are included. The results show that $\langle p_T \rangle$ of thermal pions increase $\approx 10$ % when $T_{\text{dec}}$ drops from 160 to 100 MeV in all the cases (a)-(d). However, the change is $\approx 21$ % in (e), where 66 % of the pions come from the decays at $T_{\text{dec}} = 160$ MeV (cf. Table I). In this case the low-$p_T$ part of the pion spectrum, that gives the dominant contribution to multiplicity and $\langle p_T \rangle$, is filled by decay pions from slowly moving resonances. As $T_{\text{dec}}$ decreases the fraction of resonances drops increasing the growth of the average transverse momentum. The same behaviour is seen in the case (b), if decay pions are included. The last three columns of Table I show the total multiplicity $dN/dy$ of all particles and their total transverse energies $dE_T/dy$ and $dE_T/d\eta$, where $E_T = E \sin \theta$.

| $T_{\text{dec}}$ | $dN/\eta$ | $\langle p_T \rangle$ | $dN/\eta$ | $\langle p_T \rangle$ | $dN/\eta$ | $\langle p_T \rangle$ | $dN/\eta$ | $\langle p_T \rangle$ |
|-----------------|-----------|-------------------|-----------|-------------------|-----------|-------------------|-----------|-------------------|
| 120             | 204       | 0.96              | 192       | 1.02              | 192       | 1.02              | 192       | 1.02              |
| 150             | 204       | 0.96              | 192       | 1.02              | 192       | 1.02              | 192       | 1.02              |

Comparison with data. Next we demonstrate that (i) using the initial conditions from the pQCD + saturation model and (ii) the EoS HRG+QGP with large number of hadron states in the low temperature phase, it is possible to find a single decoupling temperature which reproduces both the measured abundances of pions, kaons and (anti)protons and the shapes of their spectra.

Transverse spectra of pions, kaons and (anti)protons, calculated at $T_{\text{dec}} = 120$, 150 and 160 MeV, are shown in Fig. 2 with the data from PHENIX. Spectra of negative (positive) particles are in the left (right) panel. We find the overall agreement between the data and the calculations at $T_{\text{dec}} = 150 \ldots 160$ MeV very satisfactory, considering that we have not adjusted parameters other than the decoupling temperature. A $\approx 30$ % feed-down from hyperons is included in the $p(\overline{p})$ data but not in the calculation. Especially, we would like to draw attention to the $T_{\text{dec}}$ dependence of the slopes: at $T_{\text{dec}} \sim 150 \ldots 160$ MeV the slopes agree with the slopes.
of the data but at 120 MeV all spectra are too shallow.

In Table II we list the multiplicities and the thermal fractions $F$ for positive pions, kaons and protons, also at $T_{\text{dec}} = 160, 150$ and 120 MeV. Our results on the multiplicities agree within the experimental errors with the measured values [15]. For pions the thermal fraction $F$ increases strongly because the number of heavier resonances drops fast. This stronger suppression of heavier states at lower temperature is the sole source for the relative change between the multiplicities of pions and heavier particles; it does not depend on the flow at all.

The widening of spectra with decreasing $T_{\text{dec}}$ is also seen in Table II as an increase in the average transverse momenta. For pions and kaons our results at $T_{\text{dec}} = 150 \ldots 160$ MeV are $\sim 10$% larger than the measured values and slightly outside the experimental error bars [13]. The $dE_T/d\eta$ which we obtain with $T_{\text{dec}} = 150 \ldots 160$ MeV agrees within 8% with the latest PHENIX result [16], when the difference in the definitions of $E_T$ is taken into account.

PHENIX has also reported $p$ ($\bar{p}$) yields corrected for hyperon feed-down, $dN/dy = 19.3 \pm 0.6$ (13.7 $\pm$ 0.7) [17]. Our results in Table II for $T_{\text{dec}} = 150$ MeV agree with the measurement. With $T_{\text{dec}} = 150$ (160) MeV we also obtain $\Lambda/\Lambda = 0.70$ (0.74), again consistent with the PHENIX result 0.75 $\pm$ 0.09 [17]. Note that the deviation of $\bar{p}$/$p$ and $\Lambda/\Lambda$ from unity is due to the net baryon number content of the initial matter at saturation.

We have not performed a $\chi^2$ fit for finding the best value for $T_{\text{dec}}$ since our aim is to see how well we can describe the main features of the data while keeping our framework as simple as possible: pQCD + saturation to calculate the initial state, EoS to govern the expansion, and $T_{\text{dec}}$ to specify the freeze-out. Also all the input parameters with the exception of $T_{\text{dec}}$ are based on our previous studies [2, 4, 19] and we have not tried to tune them here.

Summary. Using a hydrodynamic framework, we have studied the effect of the EoS and the decoupling temperature on the particle spectra and multiplicities. We have demonstrated an interesting and significant interdependence between the hadron content of the EoS, the decoupling temperature and the strength of the flow. Taking the initial conditions from the pQCD + saturation model [2, 17], we have calculated the transverse spectra and the multiplicities of pions, kaons and (anti)protons at the RHIC energy $\sqrt{s}$ = 130 AGeV with a 5% centrality cut in gold–gold collisions. The agreement between our results with $T_{\text{dec}} = 150 \ldots 160$ MeV and the data measured by the PHENIX Collaboration [13, 16] is very good both for the shape and the normalization of the spectra.

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* Electronic address: kari.eskola, harri.niemi, vesa.ruuskanen, sami.rasanen@phys.jyu.fi

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