HIGGS MASS PREDICTION

W. HOLLIK
Institut für Theoretische Physik, Universität Karlsruhe,
D-76128 Karlsruhe, Germany

In this talk the Higgs boson effects in electroweak precision observables are reviewed and the possibility of indirect information on the Higgs mass from electroweak radiative corrections and precision data is discussed.

1 Introduction

By the present high precision experiments stringent tests on the standard model of electroweak and strong interactions are imposed. Impressive achievements have been made in the determination of the $Z$ boson parameters, the $W$ mass, and the confirmation of the top quark at the Tevatron with mass $m_t = 175 \pm 6$ GeV, but direct experimental evidence for the Higgs boson is still lacking.

Also a sizeable amount of theoretical work has contributed over the last few years to a steadily rising improvement of the standard model predictions (for a review see ref.\cite{ref5}). The availability of both highly accurate measurements and theoretical predictions provides tests of the quantum structure of the standard model thereby probing the empirically yet unknown Higgs particle via its contribution to the electroweak radiative corrections.

2 Theoretical basis

2.1 Radiative corrections

The possibility of performing precision tests is based on the formulation of the standard model as a renormalizable quantum field theory preserving its predictive power beyond tree level calculations. With the experimental accuracy being sensitive to the loop induced quantum effects, also the Higgs sector of the standard model is probed. The higher order terms induce the sensitivity of electroweak observables to the top and Higgs mass $m_t, M_H$ and to the strong coupling constant $\alpha_s$, which are not present at the tree level.

Before one can make predictions from the theory, a set of independent parameters has to be taken from experiment. For practical calculations the physical input quantities $\alpha, G_{\mu}, M_Z, m_f, M_H; \alpha_s$ are commonly used for fixing the free parameters of the standard model. Differences between various schemes are formally of higher order than the one under consideration. The
study of the scheme dependence of the perturbative results, after improvement by resumming the leading terms, allows us to estimate the missing higher order contributions.

Two fermion induced large loop effects in electroweak observables deserve a special discussion:

- The light fermionic content of the subtracted photon vacuum polarization corresponds to a QED induced shift in the electromagnetic fine structure constant. The recent update of the evaluation of the light quark content\cite{6,7} yield the result

\[ (\Delta \alpha)_{\text{had}} = 0.0280 \pm 0.0007. \]  

Other determinations\cite{8} agree within one standard deviation. Together with the leptonic content, $\Delta \alpha$ can be resummed resulting in an effective fine structure constant at the $Z$ mass scale:

\[ \alpha(M_Z^2) = \frac{\alpha}{1 - \Delta \alpha} = \frac{1}{128.89 \pm 0.09}. \]  

- The electroweak mixing angle is related to the vector boson masses by

\[ \sin^2 \theta = 1 - \frac{M_W^2}{M_Z^2} \Delta \rho + \cdots \equiv s_{\text{W}}^2 + c_{\text{W}}^2 \Delta \rho + \cdots \]  

where the main contribution to the higher order quantity $\Delta \rho$ is from the $(t, b)$ doublet\cite{9} in 1-loop and neglecting $m_b$ given by:

\[ \Delta \rho^{(1)} = 3x_t, \quad x_t = \frac{G_\mu m_t^2}{8\pi^2\sqrt{2}}. \]  

Higher order irreducible contributions have become available, modifying $\Delta \rho$ according to

\[ \Delta \rho = 3x_t \cdot [1 + x_t \rho^{(2)} + \delta \rho_{QCD}] \]  

The electroweak 2-loop part\cite{10} is described by the function $\rho^{(2)}(M_H/m_t)$ derived in\cite{11} for general Higgs masses. $\delta \rho_{QCD}$ is the QCD correction to the leading $G_\mu m_t^2$ term\cite{12}:

\[ \delta \rho_{QCD} = -2.86a_s - 14.6a_s^2, \quad a_s = \frac{\alpha_s (m_t)}{\pi}. \]  

The Higgs contribution to $\rho$ is only logarithmic for large Higgs masses.
2.2 The vector boson masses

The correlation between the masses $M_W$, $M_Z$ of the vector bosons in terms of the Fermi constant $G_\mu$, in 1-loop order is given by 14:

$$\frac{G_\mu}{\sqrt{2}} = \frac{\pi \alpha}{2 s_W^2 M_W^2} \left( 1 + \Delta r(\alpha, M_W, M_Z, M_H, m_t) \right).$$  (7)

The decomposition

$$\Delta r = \Delta \alpha - \frac{c_W^2}{s_W^2} \Delta \rho + (\Delta r)_{\text{remainder}},$$  (8)

separates the leading fermionic contributions $\Delta \alpha$ and $\Delta \rho$. All other terms are collected in the $(\Delta r)_{\text{remainder}}$, the typical size of which is of the order $\sim 0.01$.

The presence of large terms in $\Delta r$ requires the consideration of higher than 1-loop effects. The modification of Eq. (7) according to

$$1 + \Delta r \to \frac{1}{(1 - \Delta \alpha) \cdot (1 + \frac{c_W^2}{s_W^2} \Delta \rho) - (\Delta r)_{\text{remainder}}} \equiv \frac{1}{1 - \Delta r}$$  (9)

accommodates the following higher order terms ($\Delta r$ in the denominator is an effective correction including higher orders):

- The leading log resummation of $\Delta \alpha$: $1 + \Delta \alpha \to (1 - \Delta \alpha)^{-1}$
- The resummation of the leading $m_t^2$ contribution in terms of $\Delta \rho$ in Eq. (5). Beyond the $G_\mu m_t^2 \alpha_s$ contribution in Eq. (5), the complete $O(\alpha \alpha_s)$ corrections to the self energies are available from perturbative calculations and by means of dispersion relations.
- With the quantity $(\Delta r)_{\text{remainder}}$ in the denominator non-leading higher order terms containing mass singularities of the type $\alpha^2 \log(M_Z/m_f)$ from light fermions are also incorporated.

2.3 $Z$ boson observables

With $M_Z$ as a precise input parameter, the predictions for the partial widths as well as for the asymmetries can conveniently be calculated in terms of effective neutral current coupling constants for the various fermions. The effective
couplings follow from the set of 1-loop diagrams without virtual photons, the non-QED or weak corrections. These weak corrections can be written in terms of fermion-dependent overall normalizations $\rho_f$ and effective mixing angles $s_f^2$ in the NC vertices:

$$J^{NC}_\nu = \left(\sqrt{2}G_\mu M_Z^2\right)^{1/2} \left(g_V^\nu \gamma_\nu - g_A^\nu \gamma_\nu \gamma_5\right)$$

$$= \left(\sqrt{2}G_\mu M_Z^2 \rho_f\right)^{1/2} \left((I_3^f - 2Q_f s_f^2)\gamma_\nu - I_3^f \gamma_\nu \gamma_5\right).$$

$\rho_f$ and $s_f^2$ contain universal parts (i.e. independent of the fermion species) and non-universal parts which explicitly depend on the type of the external fermions. The universal parts arise from the self-energies and contain the Higgs mass dependence. The Higgs contributions to the non-universal vertex corrections are suppressed by the small Yukawa couplings.

**Asymmetries and mixing angles:** The effective mixing angles are of particular interest since they determine the on-resonance asymmetries via the combinations

$$A_f = \frac{2g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2}.$$  \hspace{1cm} (11)

Measurements of the asymmetries hence are measurements of the ratios

$$g_V^f / g_A^f = 1 - 2Q_f s_f^2$$

or the effective mixing angles, respectively.

**Z width and partial widths:** The total $Z$ width $\Gamma_Z$ can be calculated essentially as the sum over the fermionic partial decay widths

$$\Gamma_Z = \sum_f \Gamma_f + \cdots, \quad \Gamma_f = \Gamma(Z \to f \bar{f})$$  \hspace{1cm} (13)

The dots indicate other decay channels which, however, are not significant. The fermionic partial widths, when expressed in terms of the effective coupling constants read up to 2nd order in the fermion masses:

$$\Gamma_f = \Gamma_0 \left((g_V^f)^2 + (g_A^f)^2(1 - \frac{6m_f^2}{M_Z^2})\right) \cdot (1 + Q_f^2 \frac{3\alpha}{4\pi}) + \Delta \Gamma_{QCD}$$

with

$$\Gamma_0 = N_C^f \frac{\sqrt{2}G_\mu M_Z^3}{12\pi}, \quad N_C^f = 1 \text{ (leptons),} \quad = 3 \text{ (quarks).}$$

and the QCD corrections $\Delta \Gamma_{QCD}$ for quark final states.
2.4 Accuracy of the standard model predictions

For a discussion of the theoretical reliability of the standard model predictions one has to consider the various sources contributing to their uncertainties:

The experimental error of the hadronic contribution to $\alpha(M_Z^2)$, Eq. (2), leads to $\delta M_W = 13$ MeV in the $W$ mass prediction, and $\delta \sin^2 \theta = 0.00023$ common to all of the mixing angles, which matches with the experimental precision.

The uncertainties from the QCD contributions, besides the 3 MeV in the hadronic $Z$ width, can essentially be traced back to those in the top quark loops for the $\rho$-parameter. They can be combined into the following errors:

$$\delta(\Delta \rho) \simeq 1.5 \cdot 10^{-4}, \quad \delta s^2_\ell \simeq 0.0001$$

for $m_t = 174$ GeV.

The size of unknown higher order contributions can be estimated by different treatments of non-leading terms of higher order in the implementation of radiative corrections in electroweak observables (‘options’) and by investigations of the scheme dependence. Explicit comparisons between the results of 5 different computer codes based on on-shell and \text{MS} calculations for the $Z$ resonance observables are documented in the “Electroweak Working Group Report” in ref. Table 1 shows the uncertainty in a selected set of precision observables. Quite recently (not included in Table 1) the non-leading 2-loop corrections $\sim G^2_\mu m_t^2 M_Z^2$ have been calculated for $\Delta \rho$ and $s^2_\ell$. They reduce the uncertainty in $M_W$ and $s^2_\ell$ considerably, by about a factor 0.2.

3 Precision data and virtual Higgs bosons

In table 2 the standard model predictions for $Z$ pole observables and the $W$ mass are put together for a light and a heavy Higgs particle with $m_t = 175$ GeV. The last column is the variation of the prediction according to $\Delta m_t = \pm 6$ GeV. The input value $\alpha_s = 0.123$ is the one from QCD observables at the $Z$ peak. Not included are the uncertainties from $\delta \alpha_s = 0.006$, which amount to 3 MeV for the hadronic $Z$ width. The experimental results on the $Z$ observables are from combined LEP and SLD data. $\rho$ and $s^2_\ell$ are the leptonic neutral current couplings in eq. (10), derived from partial widths and asymmetries under the assumption of lepton universality. The table illustrates the sensitivity of the various quantities to the Higgs mass. The effective mixing angle turns out to be the most sensitive observable, where both the experimental error and the uncertainty from $m_t$ are small compared to the variation with $M_H$. Since a light Higgs boson goes along with a low value of $s^2_\ell$, the strongest upper
Table 1: Largest half-differences among central values (Δc) and among maximal and minimal predictions (Δg) for m_t = 175 GeV, 60 GeV < M_H < 1 TeV, α_s(M^2_Z) = 0.125 (from ref. 22)

| Observable O | ΔcO | ΔgO |
|--------------|-----|-----|
| M_W (GeV)    | 4.5 × 10^{-3} | 1.6 × 10^{-2} |
| Γ_e (MeV)    | 1.3 × 10^{-2} | 3.1 × 10^{-2} |
| Γ_Z (MeV)    | 0.2 | 1.4 |
| s_W^2        | 5.5 × 10^{-5} | 1.4 × 10^{-4} |
| s_b^2        | 5.0 × 10^{-5} | 1.5 × 10^{-4} |
| R_{had}      | 4.0 × 10^{-3} | 9.0 × 10^{-3} |
| R_b          | 6.5 × 10^{-5} | 1.7 × 10^{-4} |
| R_c          | 2.0 × 10^{-5} | 4.5 × 10^{-5} |
| σ_0^had (nb) | 7.0 × 10^{-3} | 8.5 × 10^{-3} |
| A_{FB}^1     | 9.3 × 10^{-5} | 2.2 × 10^{-4} |
| A_{FB}^0     | 3.0 × 10^{-4} | 7.4 × 10^{-4} |
| A_{FB}^2     | 2.3 × 10^{-4} | 5.7 × 10^{-4} |
| A_{LR}       | 4.2 × 10^{-4} | 8.7 × 10^{-4} |

bound on M_H is from A_{LR} at the SLC 25, whereas LEP data alone allow to accommodate also a relatively heavy Higgs (see figure 1). Further constraints on M_H are to be expected in the future from more precise M_W measurements at LEP 2.

Besides the direct measurement of the W mass, the quantity s_W^2 resp. the ratio M_W/M_Z is indirectly measured in deep-inelastic neutrino scattering, in particular in the NC/CC neutrino cross section ratio for isoscalar targets. The world average from CCFR, CDHS and CHARM, including the new CCFR result 26

\[ s_W^2 = 1 - M_W^2/M_Z^2 = 0.2244 \pm 0.0044 \]

is fully consistent with the direct vector boson mass measurements.

**Standard model fits and Higgs mass range:** Assuming the validity of the standard model a global fit to all electroweak results from LEP, SLD, M_W, νN and m_t, allows to derive information on the allowed range for the Higgs mass. Although the Higgs mass dependence of the electroweak parameters is only logarithmic, the already quite accurate value for m_t leads to some sensitivity to M_H. The Higgs mass dependence of the χ^2 of an overall fit is shown in figure 27. As one can see, the impact of R_b, which is on the way to the standard...
Table 2: Precision observables: experimental results and standard model predictions.

| observable      | exp. (1996) | $M_H = 65\text{ GeV}$ | $M_H = 1\text{ TeV}$ | $\Delta m_t$ |
|-----------------|-------------|------------------------|-----------------------|--------------|
| $M_Z$ (GeV)     | 91.1863 ± 0.0020 | input                  | input                |              |
| $\Gamma_Z$ (GeV)| 2.4946 ± 0.0027   | 2.5015                 | 2.4923 ± 0.0015       |              |
| $\sigma_0^{\text{had}}$ (nb) | 41.508 ± 0.056       | 41.441                | 41.448 ± 0.003        |              |
| $\Gamma_{\text{had}}/\Gamma_Z$ | 20.778 ± 0.029       | 20.798                | 20.770 ± 0.002        |              |
| $\Gamma_b/\Gamma_{\text{had}}$ | 0.2178 ± 0.0011       | 0.2156                | 0.2157 ± 0.0002       |              |
| $\Gamma_c/\Gamma_{\text{had}}$ | 0.1715 ± 0.0056       | 0.1724                | 0.1723 ± 0.0001       |              |
| $\rho_t$        | 1.0043 ± 0.0014     | 1.0056                | 1.0036 ± 0.0006       |              |
| $s^2_f$         | 0.23165 ± 0.00024    | 0.23115               | 0.23265 ± 0.0002      |              |
| $M_W$ (GeV)     | 80.356 ± 0.125      | 80.414                | 80.216 ± 0.038        |              |

The model value, is only marginal whereas $A_{LR}$ is decisive for a restrictive upper bound for $M_H$ (this is different from the results based on the data from the last year):

- including $A_{LR}$:
  $$ M_H = 146^{+112}_{-68}\text{GeV}, \quad M_H < 364\text{GeV}(95\% \text{C.L.}) $$  \hspace{1cm} (14)

- without $A_{LR}$:
  $$ M_H = 250^{+187}_{-112}\text{GeV}, \quad M_H < 622\text{GeV}(95\% \text{C.L.}) $$  \hspace{1cm} (15)

Similar results have been obtained by Passarino. The fit results by the LEP-EWWG are slightly higher (see also):

- all data:
  $$ M_H = 149^{+148}_{-82}\text{GeV}, \quad M_H < 450\text{GeV}(95\% \text{C.L.}) $$  \hspace{1cm} (16)

These numbers do not yet include the theoretical uncertainties of the standard model predictions. The LEP-EWWG has performed a study of the influence of the various 'options' discussed in section 2.4 on the bounds for the Higgs mass with the result that the 95\% C.L. upper bound is shifted by +100 GeV to higher values. It has to be kept in mind, however, that this error estimate is based on the uncertainties as given in table 1. Since the recent improvement in the theoretical prediction is going to reduce the theoretical uncertainty especially in the effective mixing angle one may expect also a significant smaller
Figure 1: Dependence of the leptonic mixing angle on the Higgs mass. The theoretical predictions correspond to $m_t = 175 \pm 6$ GeV. The SLD (0.23061 \pm 0.00047) and LEP (0.23200 \pm 0.00027) measurements are separately shown. The star is the result of a combined fit to LEP and SLD data, the squares are for separate fits (from ref. [27], updated version).

4 Conclusions

The quantum structure of the electroweak standard model allows in principle to probe the mass of the as yet experimentally unknown Higgs boson through its contribution to the radiative corrections for electroweak precision observables. Although the dependence on $M_H$ is only logarithmic, the experimental precision in the $Z$ boson parameters and the top quark mass have meanwhile reached a level where a sensitivity to the Higgs mass becomes visible, with preference to a light Higgs. The present upper bound on $M_H$ is dominated by the result on $A_{L,R}$. The instability of the Higgs mass range obtained from global fits with or without $A_{L,R}$ recommends to consider the present mass bound with some caution. The only safe conclusion is that we are well below the critical range where the standard Higgs becomes non-perturbative. For the future, the theoretical error on the Higgs mass bounds once the 2-loop terms $\sim G^2_p m_t^2 M_Z^2$ are implemented in the codes used for the fits. At the present stage the codes are without the new terms.
reduction of the theoretical uncertainties and more precise experimental values for $M_W$ and $m_t$ will be the important ingredients in improving the indirect Higgs search.

**Acknowledgements:** I want to thank W. de Boer, P. Gambino, M. Grünewald, G. Passarino, U. Schwickerath and G. Weiglein for helpful discussions and valuable informations.

**References**

1. A. Blondel, ICHEP96 (plenary talk), Warsaw, July 1996
2. M. Rijssenbeck, ICHEP96 (talk), Warsaw, July 1996
3. CDF Collaboration, F. Abe et al., *Phys. Rev. Lett.* 74 (1995) 2626; D0 Collaboration, S. Abachi et al., *Phys. Rev. Lett.* 74 (1995) 2632;
4. P. Tipton, ICHEP96 (plenary talk); P. Grannis, ICHEP96 (talk), Warsaw, July 1996
5. Reports of the Working Group on *Precision Calculations for the Z Resonance*, CERN 95-03 (1995), Eds. D. Bardin, W. Hollik, G. Passarino
6. S. Eidelman, F. Jegerlehner, *Z. Phys.* C 67 (1995) 585
7. H. Burkhardt, B. Pietrzyk, Phys. Lett. B 356 (1995) 398
8. M.L. Swartz, Phys. Rev. D 53 (1996) 5268; A.D. Martin, D. Zeppenfeld, Phys. Lett. B 345 (1995) 558; K. Adel, F.J. Yndurain, hep-ph/9509378
   D.H. Brown, W.A. Worstell, hep-ph/9607315
9. M. Veltman, Nucl. Phys. B 123 (1977) 89; M.S. Chanowitz, M.A. Furman, I. Hinchcliffe, Phys. Lett. B 78 (1978) 285
10. J.J. van der Bij, F. Hoogeveen, Nucl. Phys. B 283 (1987) 477
11. R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci, A. Vicere, Phys. Lett. B 288 (1992) 95; Nucl. Phys. B 409 (1993) 105; J. Fleischer, F. Jegerlehner, O.V. Tarasov, Phys. Lett. B 319 (1993) 249
12. A. Djouadi, C. Verzegnassi, Phys. Lett. B 195 (1987) 265
13. L. Avdeev, J. Fleischer, S. Mikhailov, O. Tarasov, Phys. Lett. B 336 (1994) 560; E: Phys. Lett. B 349 (1995) 597; K.G. Chetyrkin, J.H. Kühn, M. Steinhauser, Phys. Lett. B 351 (1995) 331
14. A. Sirlin, Phys. Rev. 22 (1980) 971; W.J. Marciano, A. Sirlin, Phys. Rev. 22 (1980) 2695
15. W.J. Marciano, Phys. Rev. D 20 (1979) 274
16. M. Consoli, W. Hollik, F. Jegerlehner, Phys. Lett. B 227 (1989) 167
17. A. Djouadi, Nuovo Cim. A 100 (1988) 357; D. Yu. Bardin, A.V. Chizhov, Dubna preprint E2-89-525 (1989); B.A. Kniehl, Nucl. Phys. B 347 (1990) 86; F. Halzen, B.A. Kniehl, Nucl. Phys. B 353 (1991) 567; A. Djouadi, P. Gambino, Phys. Rev. D 49 (1994) 3499
18. B.A. Kniehl, J.H. Kühn, R.G. Stuart, Phys. Lett. B 214 (1988) 621; B.A. Kniehl, A. Sirlin, Nucl. Phys. B 371 (1992) 141; Phys. Rev. D 47 (1993) 883; S. Fanchiotti, B.A. Kniehl, A. Sirlin, Phys. Rev. D 48 (1993) 307
19. A. Sirlin, Phys. Rev. D 29 (1984) 89
20. For a review see: K.G. Chetyrkin, J.H. Kühn, A. Kwiatkowski, in [5], p. 175
21. B.A. Kniehl, in [5], p. 299
22. D. Bardin et al., in [5], p. 7
23. G. Degrassi, P. Gambino, A. Vicini, hep-ph/9603374, P. Gambino, private communication
24. S. Bethke, in: Proceedings of the Tennessee International Symposium on Radiative Corrections, Gatlinburg 1994, Ed. B.F.L. Ward, World Scientific 1995
25. E. Torrence (SLD Coll.), ICHEP96 (talk), Warsaw, July 1996
26. K. McFarlane (CCFR Coll.), ICHEP96 (talk), Warsaw, July 1996
27. W. de Boer, A. Dabelstein, W. Hollik, W. Möslle, U. Schwickerath, hep-ph/9607286, updated version based on the data given at ICHEP96,
Warsaw, July 1996

28. P. Chankowski, S. Pokorski, hep-ph/9509207; J. Ellis, G.L. Fogli, E. Lisi, *Z. Phys. C* **69** (1996) 627; S. Dittmaier, D. Schildknecht, G. Weiglein, hep-ph/9602436; G. Passarino, hep-ph/9604344

29. G. Passarino, talk at CRAD96, Cracow, August 1996 (updated from hep-ph/9604344)

30. M. Grünewald, ICHEP96 (talk), Warsaw, July 1996

31. J. Ellis, G.L. Fogli, E. Lisi, hep-ph/9608329