Analyzing the stress state of a thin-walled cylindrical tank, depending on the design features

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Abstract. The problem of the rational hole location on the surface of a thin-walled cylindrical gravity tank was studied in order to minimize the stresses around the concentrator. The studies were carried out for three cylindrical tanks with internal diameters of 250, 400 and 500 mm, respectively, with a height of 500 mm and a wall thickness of 20 mm. The diameter of the hole in the tank wall was varied as follows: 6, 10, 14, 18, 20, 23, 25, 30, 32, 36, 40, 45, and 50 mm. The holes were located on the lateral cylindrical surface of the tank, with the following variation of the location heights in relation to the total length of the cylinder element: 4/8, 3/8, 2/8, 1/8, 1/16, 1/32. As a result, it was determined that the increase of the internal diameter of a cylindrical tank increases the equivalent stress around the concentrator. Within some ranges, a change in the diameters of the hole does not actually affect the stress pattern. Using the holes of certain diameters can reduce the stresses around the concentrator.

1. Introduction

Cylindrical pressure tanks are widely used in all industrial areas. Tanks were widely used as energy accumulators in pneumatic and hydraulic actuators in technological equipment, facilitating energy saving [1-5]. Their destruction often leads to serious accidents and threatens not only the equipment, but also the life of the personnel [6]. The design of high-pressure vessels suggests the presence of a large number of inlet/outlet volutes, technological holes, also being stress concentrators, which complicates the strength calculation of the shell. The presented design approach allows a preliminary assessment of the stress-strain structure state, assessment of the safety coefficients, the correctness of the selected materials. In order to successfully design a high-pressure tank, it is necessary to correctly determine the most probable destruction areas, to evaluate the stress-strain pattern both in the area of concentrators and in the whole structure. Also, it is necessary to determine the magnitude of the stresses arising in the tank due to high temperature and pressure and to select tank material. The existing calculation methods use simplifications that do not allow obtaining an exact result [7].

Research objective: Determination of the rational location of the stress concentrator as a hole on the lateral surface of the cylindrical tank according to minimization of the stresses in the structure.

Strength analysis was carried out for thin-walled cylindrical tanks with three internal diameters of 250, 400 and 500 mm. The pressure in the tank was 5 MPa. The thickness of the tank walls was 20 mm. For these tanks, the diameter and the rational position of the technological hole on the cylindrical tank wall were determined, which corresponded to the lowest stress value of both the concentrator and the structure as a whole.
2. Materials and methods

2.1. Studying the stresses around a hole in a high pressure cylindrical tank

Let us assume that a cylindrical tank is a flat plate with a width $W$ and with a hole in the center of a diameter $d$, which is equal to the circumference of the pressure tank, and its height $h$ is equal to the height of the cylindrical pressure tank as shown in Figure 1 (a).

The stresses around the hole (Figure 1 (b)), are described by the following equations for the conditions $\theta = 0, r = a$:

$$\sigma_r = \frac{a}{2} \left(1 - \frac{a^2}{r^2}\right) + \frac{a}{2} \left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right) \cos(2\theta)$$  \hspace{1cm} (1)

$$\sigma_t = \frac{a}{2} \left(1 + \frac{a^2}{r^2}\right) - \frac{a}{2} \left(1 + \frac{3a^4}{r^4}\right) \cos(2\theta)$$  \hspace{1cm} (2)

$$\tau_{rt} = -\frac{a}{2} \left(1 - \frac{3a^4}{r^4} + \frac{4a^2}{r^2}\right) \sin(2\theta)$$  \hspace{1cm} (3)

For the circumference of a hole with $r=a$ radius: ellipsoidal stress $\sigma_r=0$; tangential stress $\sigma_t=\sigma(1-2\cos(2\theta))$; shear stress $\tau_{rt}=0$. The maximum tangential stress is at the points $\theta=90^\circ$ and $\theta=270^\circ$, along the perimeter of the hole and the vertical axis in the direction of tension $\sigma_t = 3\sigma$, but for $r=a$. When $\theta = 90^\circ$ and $\theta = 270^\circ$, then $\sigma_r=-\sigma$. This shows that a tensile stress acts on a hole and the smaller the hole radius, the greater the concentrating stresses around it ($\sigma_{\text{max}}$) as compared with the stresses acting at a considerable distance from the hole ($\sigma_0$).

The stress concentration is expressed by the stress ratio ($K_t$) and is determined by the following well-known formula (4) [8,9]:

$$K_t = \frac{\sigma_{\text{max}}}{\sigma_0}$$  \hspace{1cm} (4)

Taking into account axial stretching, the stress ratio near the plate hole is determined by the formula (5) for the $\frac{d}{W} > 0.65$:

$$K_t = 3.0039 - 3.753 \frac{d}{W} + 7.9735 \left(\frac{d}{W}\right)^2 - 9.2659 \left(\frac{d}{W}\right)^3 + 1.8145 \left(\frac{d}{W}\right)^4 + 2.9684 \left(\frac{d}{W}\right)^5$$  \hspace{1cm} (5)

$$K_t = 2 + \left(1 - \frac{d}{W}\right)^3 \quad \text{for} \quad \frac{d}{W} > 0.65$$

Taking into account the $\frac{d}{W}$ ratio, the stress concentration factor can be taken from the diagram in Figure 1 (a).

Next step was calculating the stresses in the tank structure using CAE Ansys [10–13], for the following diameters ($d$) of the holes: 6; 10; 14; 18; 20; 23; 25; 30; 32; 36; 40; 45; 50 mm, located in the center of the plate with a width of $W=50$ mm.
3. Results

The results of the tank structure strength calculation using the maximum tangential stress method in CAE Ansys and the analytical calculation are presented in Table 1. In the cases considered, the dangerous stresses are tangential, the remaining stresses are neglected due to their insignificance.

Table 1. The stress change in the holes area, located in the center of the generator cylinder of three tanks.

| d, mm | von stresses Mises, Cylinder Ø 250 mm, MPa | von stresses Mises, Cylinder Ø 400 mm, MPa | von stresses Mises, Cylinder Ø 500 mm, MPa |
|-------|------------------------------------------|------------------------------------------|------------------------------------------|
|       | Ansys | analytical | Ansys | analytical | Ansys | analytical |
| 6     | 88.2  | 97.6       | 133.4 | 155.6       | 163.2 | 192.4       |
| 10    | 87.9  | 96.7       | 134.2 | 154.2       | 170.2 | 190.5       |
| 14    | 87.9  | 95.8       | 131.7 | 152.8       | 167.4 | 188.8       |
| 18    | 91.5  | 95.8       | 129.3 | 151.4       | 169.8 | 187.1       |
| 20    | 91.8  | 95        | 135.1 | 150.7       | 168.7 | 186.3       |
| 23    | 94.4  | 94.6       | 137.6 | 149.7       | 161.5 | 185         |
| 25    | 94.7  | 93.6       | 135.6 | 149.1       | 164.6 | 184.3       |
| 30    | 99.8  | 92.6       | 140.2 | 147.6       | 172.6 | 182.3       |
| 32    | 101   | 92.2       | 142.5 | 147         | 175.7 | 181.6       |
| 36    | 105.1 | 91.5       | 149.4 | 145.8       | 179.3 | 180.2       |
| 40    | 109.1 | 90.8       | 149.7 | 144.7       | 182.2 | 178.8       |
| 45    | 112.4 | 90        | 158   | 143.3       | 181.8 | 177.1       |
| 50    | 122.4 | 89.2       | 155.6 | 142         | 183.9 | 175.5       |

The values of tangential, normal and equivalent stresses obtained using CAE Ansys for tanks with diameters of 250, 400, 500 mm, and technological holes in them with a diameter of 25 mm, located in the middle of the generator cylinder are presented in Figures 2,3,4, respectively.
a) 

b) 

c)
Figure 2. Stresses in a cylindrical tank with a diameter of 250 mm and a hole with a diameter of 25 mm, located in the middle of the generator cylinder: a) - tangential; b) - normal; c) - diagonal; d) - equivalent.
Figure 3. Stress change in a cylindrical tank with a diameter of 400 mm and a hole with a diameter of 25 mm, located in the middle of the generator cylinder: a) - tangential; b) - normal; c) - diagonal; d) - equivalent.
Figure 4. Stress change in a cylindrical tank with a diameter of 500 mm and a hole with a diameter of 25 mm, located in the middle of the generator cylinder: a) - tangential; b) - normal; c) - diagonal; d) - equivalent.

4. Discussion
Stress analysis showed their increase with increasing diameter of the tank. For a tank with a diameter of 250 mm, the maximum stresses were as follows: tangential – 93 MPa (Figure 2 (a)); normal – 19 MPa (Figure 2 (b)); equivalent – 94 MPa (Figure 2 (d)). For a tank with a diameter of 400 mm, the maximum stresses were as follows: tangential – 121.9 MPa (Figure 3 (a)); normal – 30 MPa (Figure 3
(b)); equivalent – 122 MPa (Figure. 3 (d)). For a tank with a diameter of 500 mm, the maximum stresses were as follows: tangential – 166 MPa (Figure. 4 (a)); normal – 41 MPa (Figure.4 (b)); equivalent – 164 MPa (Figure.4 (d)).

Figure 5 shows the correlation between the equivalent stress and the diameter of the technological holes located in the middle of the generator cylindrical tank, located at a pressure of 5 MPa. Analytical calculation of stresses showed a constant decrease of the working stresses as the technological hole diameter increases. This means that the maximum equivalent stresses at the edges of the concentrator (hole) are affected not only by the stresses and stress concentration ratio, but also by the sweep area of the tank cylinder.

The results obtained with the help of Ansys differ only slightly from the analytical calculation results. An analysis of the graphical dependencies depicted in Figure 5 showed the following:

- for a cylindrical tank with a diameter of 250 mm, increasing the diameter of the technological hole to 14 mm inclusively does not affect the working stresses. The further increase in the diameter of the hole increases the stresses in the concentrator;
- for a cylindrical tank with a diameter of 400 mm, increasing the diameter of the technological hole to 18 mm inclusively does not affect the working stresses. The further increase in the diameter of the hole increases the stresses in the concentrator;
- for a cylindrical tank with a diameter of 500 mm, increasing the diameter of the technological hole up to 23 mm inclusively does not affect the working stresses. The further increase in the diameter of the hole increases the stresses in the concentrator;

![Figure 5](image)

**Figure 5.** The equivalent stress variation in the area of the holes with various diameters placed in the middle of the tank cylinders heights, according to the analytical calculations and calculations obtained with Ansys.

In the latter case, there is a decrease in stresses in the concentrator when using holes with a diameter of 23 mm. In all cases, the analytically obtained dependences show a decrease in equivalent stresses, depending on the diameter of the technological hole. The calculation in CAE Ansys showed characteristic convergence with analytical data only to the specified diameter values of the holes. After that the stresses grow, not decrease. Such a divergence is associated with the assumptions made in the analytical calculation. Calculations are performed for a plate of limited area containing a hole
subjected to tensile force, and Ansys calculates the structure as a whole. Therefore, the bending that is present in a cylindrical tank is not taken into account in the plate, which is the cause of the discrepancies in the results of the analytical calculation and the calculation of the FEM in Ansys. In accordance with the minimal working stresses in the concentrator, the technological hole with a diameter of 14 mm is rational for a cylindrical tank with a diameter of 250 mm, hole with a diameter of 18 mm is rational for a cylindrical tank with a diameter of 400 mm, and the 23 mm hole is rational for the 500 mm cylindrical tank. The values of stresses in the area of these holes and tanks are minimal and are 87.3, 129.3 and 161.5 MPa, respectively.

The correlation between the tangential, normal and diagonal stresses for three tanks, depending on the diameter of the technological hole is shown in Figure 6, 7, 8. It should be noted that normal and diagonal stresses are relatively small compared with tangential ones, and their dependencies are characteristic close.

Figure 6. The correlation between the tangential stresses and the diameter of the technological holes located in the middle of the height of the cylindrical tank surface.
Figure 7. Correlation between normal stresses and the diameter of technological holes located in the middle of the height of the cylindrical tank surface.

Figure 8. Correlation between diagonal stresses and the diameter of technological holes located in the middle of the height of the cylindrical tank surface.

In order to determine the rational location of the hole along the height of the generator cylindrical surface of the tank, stress-strain states of tanks with a diameter of 250, 400 and 500 mm with a 25 mm hole were analyzed for the following heights: 1/32, 1/16, 1/8, 2/8 3/8, 4/8.
Figure 9 shows the dependence of the equivalent von Mises stress for a 25 mm hole on the heights of the technological hole with a diameter of 25 mm. A rational location of 25 mm hole for all three tanks is 1/16 height.

![Figure 9](image)

**Figure. 9.** The dependence of the equivalent von Mises stress on the location height of a hole with a diameter of 25 mm for three tanks.

The calculation results of the equivalent von Mises stress [14] for three cylindrical tanks, depending on the height of the hole with a diameter of 25mm, are presented in Table 2.

| Place of hole | Stresses von Mises, MPa |
|---------------|------------------------|
|               | Cylinder 250 mm, MPa   | Cylinder 400 mm, MPa | Cylinder 500 mm, MPa |
| 1/32          | 85.2                  | 127.2                 | 154                   |
| 1/16          | 80.8                  | 120.3                 | 147.1                 |
| 1/8           | 89.6                  | 133.3                 | 152.2                 |
| 1/4           | 90.1                  | 135.9                 | 170.2                 |
| 3/8           | 90.9                  | 136.3                 | 171.3                 |
| 1/2           | 91.8                  | 137.6                 | 169.6                 |

It should be noted that the maximum equivalent stresses for the investigated tanks with a diameter of 250 and 400 mm do not depend on the variation in the location height of the technological hole in the range from 1/2 to 1/8. For a tank with a diameter of 500 mm, this range is smaller and is from 1/2 to 1/4. Further displacement of the hole from 1/8 to 1/16 on the cylindrical tank surface leads to a drop in the equivalent stress with a minimum at the 1/16 height. When varying heights from 1/16 to 1/32, the equivalent stresses increase and for the 250 mm tank they take almost initial values.

### 5. Conclusions

- Analytical calculations give results that are close to practical, but the structure calculation using FEM CAE Ansys allows a substantial rationalization of the structure for the minimum stresses in it;
the hole height in the cylindrical tank wall and the hole diameter affect the stresses in the concentrator;
- normal and diagonal stresses can be neglected in the analytical calculation of the structure;
- the carried out researches allowed defining the rational location height of a technological hole for tanks regardless of their diameters. This height is 1/16 of the full tank height;
- the most rational diameters of technological holes of 18 and 23 mm for tanks with diameters of 400 and 500 mm, respectively, were found based on the lowest values of equivalent stresses.

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