EXPERIMENTAL RESEARCH ON THE VIBRATION CHARACTERISTICS OF BRIDGE’S HORIZONTAL ROTATION SYSTEM

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ABSTRACT

As a new construction method, the bridge horizontal rotation construction method can reduce the impact of traffic under the bridge. During the horizontal rotation of the bridge, the overall structure will inevitably lead to a vibration response due to the construction error of the contact surface of the spherical hinge. Due to the large weight of the structure and the longer cantilever of the superstructure, the vibration at the spherical hinge will be amplified at the girder end, which will adversely affect the stability of the structure. Taking a 10,000-ton rotating bridge as a reference, a scaled model was made to test the vibration of the girder during the rotating process of the horizontal rotating system. And by analyzing the frequency domain curve of girder vibration and the results of simulation calculation, it is found that the vertical vibration displacement response is related to the first three modes of longitudinal bending of the girder structure, but has nothing to do with the higher modes or other modes. Applying the harmonic response analysis module in ANSYS software method, it is proposed that the structural vibration effect will reach the smallest by controlling the rotating speed in order to control the excitation frequency within the first-order mode frequency of girder.

KEYWORDS
Bridge rotation, Horizontal rotation system, Structural vibration test, Finite element simulation, Vibration control

INTRODUCTION

The rotating construction method of the bridge is an important construction method. The bridge is constructed along the parallel direction of the highway and the railway without affecting the traffic. After the bridge is completed, it is rotated 60°-120° along the central axis of rotation under the action of horizontal traction, and finally consolidated the middle and side piers. This is mainly used for the traffic node that crosses some busy roads and railways. The stability of the bridge’s horizontal rotation system during the rotation process is the most important factor that will affect construction safety; indeed, the instability of the structure will cause serious accidents.

Presently, the research on structural stability mainly includes static factors and dynamic factors. In terms of static load: Wang Lifeng, Yuan Chongwei and others analyzed the sensitivity of design parameters in the monitoring of rotating construction. In their research, self-weight and concrete elastic modulus are selected. Also, they conduct a force analysis of the rotating structure on static load conditions such as cable tension, concrete shrinkage and creep, and temperature gradient. The results of the study show that the weight, tension and creep of the cable have a greater impact on the stability of the rotation process [1]. In the study of the control and stability of the rotating construction of a large-tonnage continuous rigid frame bridge, Lu Jinhua used the rate of change of deflection θ and the rate of stress change ω as the judgment index of the bridge stability [2]. Based on the analysis of concrete bulk density, the construction process, elastic modulus, overweight on
one side of the girder, the concrete shrinkage as well as the creep on the rotating bridge and the static wind load on the stability of the transverse bridge, they conclude that concrete bulk density, the change of tension control stress exerts a significant impact on the construction accuracy. Also, it is concluded that the influence of parameters such as the extension of the closing time on the bridge is within a controllable range. In the sensitivity analysis of the construction parameters of the large-span turning. When the static wind load acts alone, the center of rotation (the position of the spherical hinge) will not overturn [3-4]. Ma Shufen proposed that pre-stress effect and bulk density are the sensitive factors among the material factors, and the modulus of elasticity is the secondary factor. The temperature effect is the main sensitive factor, while the shrinkage and creep of concrete is the secondary sensitive factor [5].

In terms of dynamic factors, Xu Chao, Wang Changfeng and others used theoretical mechanics and material mechanics to derive and analyze the torque during the acceleration of the bridge rotation during the start-up phase of the construction of the bridge, and simplified the torque to the shear stress of the temporary support. In their research, they analyzed the influence of the bearing capacity of the temporary support with and without the stirrups, which provides a reference for the check calculation of the temporary support during the construction process [6]. Huang Weiwen used ANSYS software to establish a finite element model [7], and applied it to comparing and analyzing the dynamic characteristics of the rotating bridge so as to study the impact of wind loads on the main girder structure. The results show that the natural frequency of the maximum cantilever state is much lower than that of the bridge state and that the stability during rotation is poor because of its lower stiffness; the static wind response calculated according to the specification is the most unfavorable working condition. Gao Ri et al. studied the influence of train-induced vibration on the stability of rotating construction by using acceleration sensors to test the vibration of the bridge foundation when different trains passed. The results uncover that the train-induced vibration exerts little effect on the stability of the structure, and the vibration displacement and acceleration of the girder are less than the seismic fortification intensity value of the region [8]. Wang Changjie and He Wei analyzed the seismic response of a T-shaped rigid frame rotating bridge in the completed state, and calculated the bending moment envelope diagram of the main girder and piers of the superstructure under earthquake action by the response spectrum method. It is also found in the study that the structure is always in a state of elastic stress, and that the load-bearing capacity of the structure meets the design requirements [9].

During the production process of concrete and steel structure spherical hinges, certain flatness errors will inevitably occur. The bridge will cause self-excited vibration during the rotation process. When the frequency of the vibration reaches or approaches the natural frequency of the structure, it will cause violent vibration. Hence, research on the aforementioned factors should be paid enough attention.

MODEL TEST OF BRIDGE HORIZONTAL ROTATION SYSTEM

Model design

The bridge horizontal rotation system is generally composed of five parts: the superstructure, the pier, the cap and the hinge structure. The upper structure of a continuous rigid frame bridge is generally about 100 m, and the length of a single cantilever is 50 m. The scale of the model is 1/50: the girder adopts two 2-meter-long HW150x150 steel, the bridge pier adopting a reinforced concrete structure with a length, width and height of 400,200,500 mm separately, and 12 vertical compression steel bars of 14 mm, 6 of 8 mm stirrups respectively. The length, width, and height of the platform are 400, 400, and 150 mm respectively. The concrete spherical hinge structure between the caps, the diameter of the spherical hinge is 10 cm, the thickness is 5 cm, and the radius of the convex and concave surfaces of the spherical hinge is 0.225 m. In order to test the compressive stress distribution during the horizontal rotation of the spherical hinge, 17 stress measuring holes are uniformly arranged along the radial direction in the steel structure spherical hinge, located at 1/4R, 1/2R, 3/4R, and R respectively. The rotating model is shown in Figure 2-1. The vibration of the structure is collected by the 941-B vibration pickup, as shown in Figure 2-2.
In the research, the continuous rigid frame bridge swivel model is placed on the rotating table, and two 941-B seismic vibration pick-up on both sides of the girder end are used to test the vertical vibration of the girder during the rotation, as shown in Figure 2-3.

**Vibration test during horizontal rotation**

![Figure 2-1 Swivel model](image1.png)

(a) Model design  (b) Model production

![Figure 2-2 Vibration test of girder](image2.png)

(a) Sensors at the girder end  (b) Sensor debugging

**Fig. 2-2 Vibration test of girder**
Model making

The swivel model is poured into the girder field, and the piers and caps are made according to the design dimensions, and the cement, coarse aggregate, fine aggregate and other materials required for the C30 concrete are prepared. Based on the design, the steel frame in the cap and the pier body are tied up; then install the measuring point stress sensor on the stirrups and main bars in the measuring point. Then the concrete is poured; the standard maintenance lasts for 28 days. After the pier body is formed, bolts are implanted on the top of the pier, and two I-girders are installed on the top. Finally, the girder and pier strain gauge are pasted. The prefabrication process of the swivel model is shown in Figure 2-4. Before pasting the strain gauges, it is necessary to polish the steel bars, I-girder and concrete surfaces to remove the rusty parts and floating soil on the surface to ensure accurate stress testing. After pasting, the strain gauge should be insulated to prevent short-circuit and failure. Finally, waterproof the strain gauge at the measuring point, and lead the wire out of the model. After the test pier is prefabricated in the girder field, it is necessary to check the structure size, the position of the steel bar and the strength of the pier body. The maximum error between the actual size and design of the swivel model obtained by measurement is 3%; the position and quantity of the steel bars are checked by the steel bar locator, and the deviation range of the steel bars is around 3% to 5%; The concrete was tested, and the measured strengths were 37.1MPa and 38.2MPa respectively. The size and strength of the rotating model meets the design requirements. The supporting radius \( R_1 \) of the spherical hinge is 5 cm, the thickness 5 cm, and the radius \( R \) of the spherical hinge 22.5 cm. The structure of the spherical hinge is shown in Figure 2-4e. The supporting radius \( R_1 \) of the spherical hinge is 5 cm, the thickness 5 cm, and the radius \( R \) of the spherical hinge is 22.5 cm. The structure of the spherical hinge is shown in Figure 2-4e.

![Prefabrication process of test model](image)
In order to test the radial compressive stress, sensors are installed at the rotation center and also at the positions of 1/4R, 1/2R, 3/4R, and R. The concrete spherical hinge is poured through a steel formwork, and positioning steel bars are set inside the spherical hinge, and strain gauges are embedded inside, as shown in Figure 2-5.

ESTABLISHMENT OF SIMULATION MODEL OF BRIDGE HORIZONTAL ROTATION SYSTEM

**Unit selection**

This study applies ANSYS software to establish the solid element model of the swivel pier. The girder188 unit is used to establish the girder element model when the flatness error of the concrete spherical hinge was studied on the structural vibration response, and the solid65 element is used to establish the solid element model when the mechanical behavior of the spherical hinge was studied. The force of the spherical hinge during the rotation is actually a contact problem. The analysis of the contact problem is an important issue in elastic mechanics. In order to accurately simulate the relative sliding between the upper and lower spherical hinges, this research uses Contact170 and Contact174. Establish the spherical hinge contact unit [10].

Solid65 is a type of unit commonly used to simulate concrete materials. The characteristic of the unit is that the compressive strength is much greater than the tensile strength (usually about 10 times). It adds concrete strength parameter indexes on the basis of Solid45 unit, which can be used to simulate the state of cracking and crushing of concrete. The Solid65 unit diagram is shown in Figure 3-1. The Solid65 element has eight nodes and is a spatial hexahedron. When the four nodes of K, P, O, and L overlap, the unit turns a spatial tetrahedron. These parameters can be set when
the unit is divided into nodes. When the unit is simulating cracking and crushing, attention should be paid to the loading rate. If the speed is too high, the fracture surface will be inaccurate. The spherical hinge model and unit division are shown in Figure 3-2a and b. Girder188 is a spatial three-dimensional finite strain element with 2 nodes [10]. Each node owning 6 degrees of freedom (3 degrees of freedom in translation and 3 degrees of freedom in rotation). The connection line between the two nodes of the girder unit is the X axis of the local coordinate system, and the unit possesses the ability to withstand various deformations; thus, this unit is often used in the girder of the bridge superstructure and can handle linear and nonlinear problems. The model and element division are shown in Figure 3-2c.

In order to ensure the accuracy of the calculation of the compressive stress of the spherical hinge, the mesh simulation of the upper and lower spherical hinge contact surface area is encrypted, as shown in Figure 3-2b. And to simulate the counterweight of the upper structure of the main girder, mass21 mass units are set on the girder nodes. The mass unit has a total of six parameters, respectively, the mass $M_x$, $M_y$, and $M_z$ and the moments of inertia $I_{xx}$, $I_{yy}$, and $I_{zz}$ in the three
directions. The connecting position of the pier and the girder uses the cp command to transfer the bending moment and shear force of the girder to the pier, as shown in Figure 3-2c. In order to accurately simulate the constraints of the turntable, horizontally, the girder unit model constrains the translational displacements $Ux, Uy$ and $Uz$ of the nodes at the pier bottom position; vertically, it constrains the rotation $Rotx$ and $Rotz$, but relaxes the longitudinal rotation constraint $Roty$. The vertical translational displacement $Uz$ of the node is constrained at a position that is 5cm from the center of rotation (the boundary position of the spherical hinge), as shown in Figure 3-2d.

**Simulation of Boundary Conditions of Spherical Hinge**

The connection between spherical hinges is a contact problem: contact analysis is generally divided into the contact between a rigid body and a flexible body and the contact between a flexible body and a flexible body. In the former study, it is assumed that one or more contact surfaces are rigid bodies, which have greater structural rigidity than flexible bodies. Generally, this method is used to analyze the contact between a soft material and a hard material, mainly used for analysis between steel structure and concrete structure, or concrete and soil structure, etc. The contact analysis between rigid bodies mainly involves their similar or equal structural rigidity. The structure would turn deformed during the whole process of contact. Based on this, this research adopts the flexible body-flexible body contact analysis. The contact methods are divided into three types, point-to-point contact, point-to-surface contact and face-to-face contact, each contact method corresponding to its own set of contact units. Among them, the point-to-point contact situation is like the boundary condition of a simply supported structure. This type of contact requires accurate positioning of the contact point. There could be a small sliding situation between the contact surfaces. At this time, the Contact178, Contact12 and Contact52 units in ANSYS will be used. The point-to-surface contact situation is used for the anchor bolt or orange connection of the steel structure under which condition it is not necessary to accurately locate the specific position of the point and the surface. When larger sliding and deformation are generated between them, the three units of Contact48, Contact49 and Contac26 in ANSYS will be applied. The surface-to-surface direct contact unit is suitable for the face-to-face contact unit of rigid body-flexible body and also flexible body-flexible body. The rigid surface is set as the target unit surface, the flexible surface as the contact unit surface. The target surface uses Targe169 and Targe170 to simulate 2D and 3D surfaces, and the contact surface uses Contact171, Contact172 and Contact173 to simulate 2D and 3D contact surfaces. In this research, the analysis methods of Contact170 and Contact174 elements are used in the finite element contact analysis.

When analyzing the force between spherical hinges, both the compressive stress on the contact surface and the relative sliding friction force should be taken into consideration. They would act together on the contact surface, which conforms to the relevant theories on the Coulomb friction model. In other words, the cross section remains stationary when the upper and lower surfaces of the spherical hinge are in contact. When a rotational force is generated, the static friction of the cross section prevents this rotational force, and the structure remains stationary. The friction at this time is called the bonding state. When the rotation force reaches a certain limit, the structure begins to have a relative sliding tendency. At this time, the rotation force is greater than the frictional resistance. This state is called the sliding state. In the study, this limit was achieved by defining the coefficient of friction. ANSYS proposes a maximum equivalent shear stress value, which has nothing to do with contact pressure, but is manually specified. When the rotational force is greater than the equivalent shear force, the structure will slide. The TAUMAX value in ANSYS is then set to determine the shear stress value, which is used to solve the problem that the friction calculated in the Coulomb model is greater than the material limit [11-12]. The friction model is shown in Figure 3-3.

**RESULTS AND DISCUSSION**

**Results of structural modal**

This research uses ANSYS to calculate the first 6-order mode of the rotating model; the results are shown in Table 4-1 and Figure 4-1.
Tab. 4-1: Six modes in front of swivel pier

| Mode Number | Frequency Hz | Vibration form                                           |
|-------------|--------------|----------------------------------------------------------|
| 1           | 14.213       | the girder and the bridge pier bent longitudinally, the 1st mode |
| 2           | 16.465       | The girder twisted transversely, the 1st mode            |
| 3           | 22.168       | The girder bent transversely, the 1st mode               |
| 4           | 22.744       | the girder transversely, the 2nd mode                    |
| 5           | 35.448       | the girder bent longitudinally, the 2nd mode             |
| 6           | 49.925       | the girder bent longitudinally, the 3rd mode             |

Fig.4-1 - Three modes before longitudinal bending of girder

Vibration test results of girder during horizontal rotation

The girder vibration form is forced vibration, which has nothing to do with the external load during construction. Therefore, the structure itself needs optimizing. The influence of such vibration can be effectively reduced by controlling the rotation speed and changing the boundary conditions of the spherical hinge. In the experiment, the concrete spherical hinge had certain unevenness during the prefabrication process. A displacement vibration picker was installed at the cantilever end of the girder to test the vertical displacement response of the girder during the horizontal rotation. A total of 42.5s was collected during the rotation, and the sampling frequency was 200 Hz. The measured displacement time history curves are shown in Figures 4-2a, 4-3a, and 4-3c, which are converted in the frequency domain, as shown in Figures 4-2b, 4-3b, 4-3d.
Fig. 4-2 Displacement time history and frequency domain distribution of girder
Figure 4-2a shows that the maximum vertical displacement value of the cantilever girder end during rotation is only 1/20000 of the span of the cantilever girder. According to the frequency distribution curve, there appear three peak displacements when the frequency reaches 13.1 Hz, 32 Hz, and 48 Hz. In order to obtain the vibration conditions in different periods, the displacement time history curves of the 7-17 s vibration displacement amplitude period are small and the 32-42 s vibration displacement amplitude period are larger, as shown in Figure 4-3a and 4-3c. The frequency curve is obtained via Fourier transform. Based on Figure 4-3b and 4-3d, it can be seen that the vertical vibration at the end of the cantilever in the frequency domain space still produces three peak displacements when the frequency reaches 13.1 Hz, 32 Hz, and 48 Hz. The three frequency is then tested and turns out to be irrelevant to the time period.

Simulation analysis results

The results of experiments and modal analysis show that during the horizontal rotation of the rotating model, the vibration response of the vertical displacement of the girder end is relatively large when the external excitation frequency reaches the second-order mode frequency of the longitudinal bending of the girder; when it reaches the first and third-order mode frequencies, the pier produces longitudinal unbalanced moments. In order to study the influence of the flatness of the concrete spherical hinge on the vibration of the girder, the flatness of the contact surface was simulated by changing the frequency and position of the dynamic load.

The vibration response of the load of different frequencies to the girder can be analyzed by the harmonic response in ANSYS [10]. In order to simulate the vibration response of the spherical hinge unevenness to the structure, a vertical excitation load is set at the edge of the spherical hinge, the magnitude of which is equal to the supporting reaction force, and the results of the Uz displacement response of the girder end under 0~100 Hz are analyzed, shown in Figure 4-4.
The calculation results of finite element and the test results uncover that the vertical displacement of the girder ends all produce peaks at the first three modes of longitudinal bending of the girder, which has nothing to do with the magnitude of the excitation but the frequency of the excitation. Due to the boundary conditions and the accuracy of the structural stiffness simulation, the natural frequency of the girder calculated by the finite element method is slightly larger compared with the simulation analysis results. The displacement response of the girder at the first and third frequencies is relatively small, the displacement response at the second frequency is relatively large, but the displacement-frequency curve is consistent with the experimental curve. The bending moment-frequency curve shown in Figure 4-5 is based on the change of the size and position of the excitation, and the analysis of the bending moment at the bottom of the pier and the bending moment at the root of the girder cantilever.

Fig.4-4 - Comparison of displacement response at 0 ~ 100 Hz

Fig.4-5 Time history curve of Bending moment-frequency curve
In the figure, F=2.5 kN means the spherical hinge reaction force, and the excitation load is 0.5, 1, 2 times the support reaction force for analysis. From Figure 4-5a, it can be seen that the pier bottom will have a larger bending moment under the first-order and third-order anti-symmetric modes, and the pier bottom bending moment is 0 under the second-order mode. The bending moment of the pier bottom changes linearly with the magnitude of the excitation. The bending moment values generated under the first and third mode frequencies are 464~1882 Nm and 4375~17735 Nm; the bending moments of the third mode frequency is ten times that of the first order. However, the third-order frequency value is relatively large, and it is difficult to induce the third-order mode of the girder during the rotation process, which is generally controlled by the first-order mode frequency. According to the test results of the anti-overturning moment of the spherical joint, the friction moment of the concrete spherical joint is 328.5 Nm, which is smaller than the bending moment caused by structural vibration, and the structure will lose stability along the longitudinal direction. The position of the maximum bending moment at the bottom of the pier is related to the frequency and position of the excitation. The curve in green below is gained is based on the increase of the diameter of the concrete spherical hinge. This kind of increase changes the position and constraint conditions of the excitation, and further changes the natural frequency of the structure, which cause the bending moment-frequency curve of the bottom of the pier to move backward.

Based on Figure 4-5b, it can be seen that the second-order mode frequency is the controlling factor of the girder amplitude, and its bending moment value has nothing to do with the magnitude of the excitation load and the location of the excitation load. It is also detected that the influence of the third-order mode on the bending moment of the girder is less than that of the second-order mode, and the first-order mode has basically no influence on the bending moment. Besides, the changing of the excitation position will change the frequency of the peak bending moment at the cantilever root of the girder. Therefore, in order to reduce the influence of the flatness error of the spherical hinge contact surface on the vibration of the girder structure, the excitation frequency should be controlled below one by reducing the excitation frequency to a value that is within the range of the first-order natural frequency. At this time, the vibration response of the girder during rotation is relatively small, and it will not cause dynamic instability.

When the spherical hinge rotates on a fixed axis of a rigid body around the center of rotation, the structure will be displaced along the vertical direction as the two areas on the contact surface are in contact with each other. At this time, the corresponding position of the lower spherical hinge will produce a vertical reaction force. According to the Load code for the design of building structures, the structural dynamic coefficient is generally taken as 1.1~1.3. When doing time history analysis, the magnitude of the excitation is 1.2 times the value of the vertical reaction force of the spherical hinge. The position where the excitation produces the greatest effect is at the R position of the outermost edge of the spherical hinge. At this time, the frequency of the excitation load is \(\omega/2\pi\). When there are a total of \(n\) uneven areas on the contact surface, the excitation frequency is increased to \(n\omega/2\pi\). Under normal circumstances, there are certain requirements for the flatness of the cross-section during the manufacture of the spherical hinge. Assuming that the maximum number of uneven areas of the spherical hinge is 10, the frequency range of the external excitation is \(\omega/2\pi\sim5\omega/\pi\) at this time. The study of the relationship between the rotation angular velocity and the axial tensile stress of the girder proposes the calculation method of the limit rotation angular velocity. It is gained that the maximum rotation angular velocity of the rotating model is 3.77 rad/s, and the excitation frequency range is from 0.6 to 6 Hz. The vibration response is not significant when the excitation frequency is less than the first-order natural frequency. When \(\omega\) reaches 8.9 rad/s, the excitation frequency is from 1.4 to 14.2 Hz, and the structure reaches the first-order natural frequency; when \(\omega\) reaches 22.3 rad/s, the excitation frequency is around 3.6~35.5 Hz, and the structure reaches the second-order natural frequency; when \(\omega\) reaches 31.4 rad/s, the excitation frequency is 5 to 50 Hz, and the structure reaches the third-order natural frequency. Therefore, the angular velocity of rotation should be less than 8.9 rad/s, and the limitation of angular velocity of rotation finally reaches 3.77 rad/s.

As the weight of the rotating model is relatively small and the natural frequency of the structure is relatively high, the calculated rotational angular velocity limit is then relatively high. Based on the
literature, the first-order frequency of the girder of the 10,000-ton continuous rigid frame swivel bridge is generally between 0.4 and 1.5Hz [13], so the angular velocity of rotation should be controlled within 0.25 rad/s so as to avoid structural vibration caused by the unevenness of the spherical hinge [15]. Presently, the rotation angular velocity used in the actual domestic rotating engineering is generally between 0.01 and 0.02 rad/min, and the rotation time is between 90 and 120 min. The research results of this research then conclude that the rotational angular velocity can be increased to more than 0.05 rad/min, and the turning time will be shortened to 20-60 min, greatly improving the construction efficiency and reducing the impact of the construction on the railway traffic.

Theoretical derivation of girder end vertical vibration velocity and acceleration

In order to monitor the stability of the bridge horizontal rotation system in the process of turning in real-time, the relationship between the vertical speed and acceleration of the girder end and the corresponding first three-order natural frequency should be established, and the stability of the structure should be judged by monitoring the speed and acceleration indicators.

In order to obtain the relationship between the frequency of the first three modes of the girder longitudinal bending and the vertical velocity Vz and acceleration az of the girder end, the vertical velocity and acceleration values of the cantilever girder end are monitored to determine the state of the structure during the turning process, and the girder is derived an expression between Vz, az and ω [14]. The expression of the vertical displacement y on the axis x of the girder is as follows:

\[ y = Y(x)\sin(\theta + \omega t) \]  

In the above formula, \( Y(x) \) is the mode shape displacement function, \( \theta \) the phase angle and \( \omega \) is the mode shape frequency. Suppose \( \lambda^4 = \omega^2m/EI \), the expression of \( Y(x) \) solution can be:

\[ Y(x) = C_1\cosh \lambda x + C_2\sinh \lambda x + C_3\cos \lambda x + C_4\sin \lambda x \]  

For a bar with one end constrained and one end free, the node numbers on both sides are i and j respectively, and the translational displacement of constrained i is \( y_i \). At this time, the relationship between force and displacement is as follows:

\[
\begin{bmatrix}
[ M_i ] \\
[ F_j ] \\
[ M_j ]
\end{bmatrix} = 
\begin{bmatrix}
4EI & -6EI & 2EI \\
L & -L^2 & L \\
-6EI & 12EI & -6EI \\
2EI & -6EI & 4EI \\
L & -L^2 & L
\end{bmatrix}
\begin{bmatrix}
\theta_i \\
y_j \\
\theta_j
\end{bmatrix}
\]  

The vibration at any point of the girder can be expressed by three parameters: vertical displacement \( y_1 \), longitudinal displacement \( x_1 \) and pier bottom rotation angle \( \theta_{2i} \), the vertical vibration displacement expression can be derived as follows:

\[
y_1 = Y_1(x_1) \times \sin(\theta + \omega t) \]  

\[
x_1 = C_9\sin(\theta + \omega t) \]  

\[
\theta_{2i} = C_{10}\sin(\theta + \omega t) \]  

In order to determine the coefficients \( C_5 \sim C_{10} \), six equations need to be solved. On the other hand, six balance equations can be established according to cantilever girder boundary conditions and internal force balance conditions,

1. When the bending moment at the end of the cantilever is 0, the formula can be like this:

\[ C_5\cosh \lambda L_1 + C_6\sinh \lambda L_1 - C_7\cos \lambda L_1 - C_8\sin \lambda L_1 = 0 \]  

2. When the shear force at the end of the cantilever reaches 0, the formula is as follows:

\[ C_5\sinh \lambda L_1 + C_6\cosh \lambda L_1 + C_7\sin \lambda L_1 - C_8\cos \lambda L_1 = 0 \]  

3. When the displacement of the cantilever root of the girder is 0, the formula can be derived like this:

\[ C_6 + C_7 = 0 \]

4. If taking the pier shear balance condition into consideration, the formula can be like this:
In the above formula, \( E_2 \) and \( l_2 \) represent the elastic modulus of the pier and the bending moment of inertia. The formulas of the pier angle and displacement are as follows:

\[
\theta_{2j} = (C_6 + C_9) \lambda \sin(\theta + \omega t) \quad (4-11)
\]

\[
y_{2j} = C_6 \sin(\theta + \omega t) - (C_6 + C_9) h_k \times \lambda \sin(\theta + \omega t) \quad (4-12)
\]

Based on Newton’s second law, the following formula is derived: \( F = ma = F_{2j} \),

\[
m = \frac{1}{2} m_1 L_1 + 2m_2 L_2 \quad (4-13)
\]

\[
a = \frac{d^2x_1}{dt^2} = -\omega^2 \times C_9 \sin(\theta + \omega t) \quad (4-14)
\]

In the formula, \( m_1 \) and \( L_1 \) are the quality and span of the main girder; \( m_2 \) and \( L_2 \) are the quality and height of the pier. Based on the equations from (4-10) to (4-14), a simplified equation can be obtained as follows:

\[
6E_2 l_2 \times (l_2 + 2h_k) \lambda C_6 + 6E_2 l_2 \times (l_2 + 2h_k) \lambda C_8 + 6E_2 l_2 L_2 C_{10} + \left( \frac{1}{2} m_2 L_2^4 \omega^2 + 2m_1 L_1^3 L_1 \omega^2 - 12E_2 l_2 \right) C_9 = 0 \quad (4-15)
\]

(5) Similarly, according to the balance conditions of the bending moments of the top and bottom sections of the pier, the following formula is derived:

\[
2E_1 l_1^2 l_2^2 C_5 - E_2 l_1 \times \left( 4L_2 + 6h_k \right) \lambda C_6 - 2E_1 l_1^2 l_2^2 C_7 - E_2 l_1 \times \left( 4L_2 + 6h_k \right) \lambda C_8 - 2E_2 l_2 L_2 C_{10} + \left( \frac{1}{2} m_2 L_2^4 h_k \omega^2 + 2m_1 L_1^3 L_2 h_k \omega^2 + 6E_2 l_2 \right) C_9 = 0 \quad (4-16)
\]

\[
2E_2 l_2 \times (l_2 + 3h_k) \lambda C_6 + 2E_2 l_2 \times (l_2 + 3h_k) \lambda C_8 - 6E_2 l_2 C_9 + \left( 4E_2 l_2 L_2 + kL_2^2 \right) C_{10} = 0 \quad (4-17)
\]

Combine the equations from (4-7) to (4-9), and (4-15) to (4-17), with the frequency \( \omega \) and \( C_5 \sim C_{10} \) as unknowns. Since \( C_5 \sim C_{10} \) is not all zeros, while the result is zero, the determinant of the coefficient matrix composed of frequencies then should be zero. Therefore, the different frequencies and modes of the cantilever girder structure can be obtained, which can be used to obtain the coefficients.

During the rotation of the continuous rigid frame bridge, it is difficult to measure the speed and acceleration of the spherical hinge in order to test whether the cantilever girder structure reaches the first three-order natural frequency. However, it is relatively easy to test the vertical velocity and acceleration value of the end of the cantilever during rotating construction. Therefore, based on the formula (4-4), the vertical velocity \( Vz \) at the end of the cantilever girder, and the expression of acceleration \( a_z \) the following equation can be derived.

\[
Vz = C_5 \omega \cos \lambda L_1 + C_6 \omega \sin \lambda L_1 + C_7 \omega \sin \lambda L_1 + C_8 \omega \sin \lambda L_1 \quad (4-18)
\]

\[
a_z = C_5 \omega^2 \cos \lambda L_1 + C_6 \omega^2 \sin \lambda L_1 + C_7 \omega^2 \sin \lambda L_1 + C_8 \omega^2 \sin \lambda L_1 \quad (4-19)
\]

Substitute the natural frequency of the first three modes of the girder to obtain the coefficient value, and finally obtain the maximum vertical vibration speed limit \( Vz \) and acceleration value \( a_z \) at the end of the cantilever from equations (4-18) and (4-19). The speed and acceleration of the girder end are monitored to determine the stability of the structure during horizontal rotation. The measured vertical velocity and acceleration of the girder end should be smaller than \( Vz \) and \( a_z \) gained at the first three-order mode frequency of the girder; otherwise, only when the cause is found should the rotating construction be continued.

**CONCLUSIONS**

To conclude, this research starts with the model test, and draws the following conclusions after the test and simulation analysis.

(1) First, concrete spherical hinges will inevitably produce certain flatness errors during the manufacturing process, which will cause vibrations during rotation. Through the test of the vibration displacement of the girder, it is found that the vertical vibration displacement is related to the...
The frequency of the first three modes of the girder longitudinal bending, and has nothing to do with the higher modes of the girder longitudinal bending and other modes.

(2) Second, this research establishes finite element simulation models of the rotating structure and uses the harmonic response analysis method to simulate the influence of the flatness error of the spherical hinge contact surface on the structural vibration. It is then proposed that the control of the rotating speed and the excitation frequency within the first-order mode frequency range of the girder can make the structural vibration response at a minimal scale.

(3) Third, this research provides the expression of the relationship between the vertical vibration velocity and acceleration of the girder end and the vibration frequency of the main girder during the rotation of the bridge horizontal rotation system. It is finally proposed that the structure stability can be predicted by monitoring the vertical velocity and acceleration of the cantilever girder end during the horizontal rotation.

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