Simulation of Senise landslide in clayey silty soil using a strain softening soil model and Updated Lagrangian H-Adaptive approach

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ABSTRACT

Senise landslide in the southern Italy occurred in July 1986 due to the existence of a thin layer of clayey silt, interbedded with a slightly cemented sand formation. Excavation at the toe of the slope has lead to the occurrence of landslide. This is accompanied by progressive failure, mainly due to the softening behaviour of the thin clayey silt layer. The landslide is simulated using the developed finite element code that combines the Updated Lagrangian formulation and a strain softening model. This code is capable of simulating the development of the progressive failure. Simulation of slope failure in highly strain softening soils results in the distortion of finite element mesh; therefore, a proper numerical discretisation is prepared by an H-Adaptive remeshing technique to ensure that the analysis and leads to reliable results and solution convergence. The results of this study show the location and extent of failure zones. The failure is shown to be triggered by excavation at the toe of the slope and extends upward along the thin clayey silt layer. Compared with the observation, the numerical analysis could simulate the location of failure surface and the sliding soil mass reasonably well.

Keyword: sliding mass, large deformation analysis, progressive failure, finite element, updated lagrangian

1 INTRODUCTION

Failure of slopes and embankments are commonly caused by changes made to effective stress, material properties or the geometry of slopes and is accompanied with movement of large mass of soil. The magnitude of deformations after failure has a key role in the design of many slopes and embankments. A realistic approach to estimate post-failure deformations is to take into account the continuous movement of the slopes, from the initiation of failure to stabilisation of geo-material, employing large deformation analysis using finite element method (Mohammadi & Taiebat, 2013). This paper aims to show the capabilities of a large deformation finite element analysis in simulating the post failure behaviour of the Senise slope after landslide. Senise landslide occurred due to the existence of a thin layer of clayey silt, interbedded with a slightly cemented sand formation. The softening behaviour of the thin clayey silt layer and excavation at the toe of slope resulted in the occurrence of landslide. The finite element program developed for this study is based on the Updated Lagrangian (UL) formulation and the finite strain theory. Application of the UL formulation is crucial for proper modelling of the slide event. An extended Mohr-Coulomb constitutive model represents the strain softening behaviour of the slope material (Potts, Dounias, & Vaughan, 1990). The H-adaptive remeshing technique is also used that allows for simulation of slopes with very large post-failure deformations. The accuracy and capabilities of the numerical tool are demonstrated by simulation of the Senise landslide. The results of analysis of the landslide are also presented and discussed. The numerical study clearly shows that a progressive failure occurred in the subsoil, which was triggered by deep excavations performed at the toe of the slope. Also the location of failure surface and the sliding soil mass has been predicted reasonably well.

2 THE SENISE SLOPE

The Senise landslide occurred at Timpone on 26 July 1986 in southern Italy which affected a large area with great dimensions. It is categorised as a translational slide, where the landslide occurs along a weak planar surface approximately parallel to the slope with little rotation or backward tilting.
2.1 Geometry

The dimensions of the landslide were about 150 m in width, and about 230 m in length. The depth of soil mass involved in landslide, varied between 10 to 15 m except at the toe which was about 5 m (Guerricchio & Melidoro, 1988). A plan of the landslide is shown in Fig 1. The buildings in the landslide area were constructed in 1980s (Del Prete & Hutchinson, 1988) and involved deep excavations, as deep as 10 m.

2.2 Soil Condition

A schematic geotechnical section of Senise hill is shown in Fig 2. The subsoil is essentially consists of yellow sand interbedded by clayey silt levels, which overtops blue grey clay (Troncone, 2005). The location of water table was about 23 m below the slope toe (Del Prete & Hutchinson, 1988), also it should be noted that the landslide occurred after a period of very scarce rain (Guerricchio & Melidoro, 1988).

2.3 Triggering Mechanism

The progressive failure occurred in the sub-soil was triggered by an excavation at the toe of slope. Although the cuts were protected by 9-10 m concrete retaining walls, the existence of a thin clayey silt layer interbedded with a slightly cemented sand formation contribute to the slide. The landslide resulted in movement of a large mass of soil by about 30 m along the A–A direction (Fig 1 and Fig 2).

3 PROGRESSIVE FAILURE

3.1 Silty - Clayey Softening Clay

The soil involved in the movement were characterised by a pronounced strain softening behaviour. The FoS of the slope, considering a slip surface similar to that actually occurred in the field, was calculated for both peak and residual strength parameters as $\text{FoS}_{\text{peak}}=1.73$ and $\text{FoS}_{\text{res}}=0.60$ (Troncone, 2005). This should imply that progressive failure has occurred, with mobilised strength varying between the peak and the residual values along the sliding surface, depends on the magnitude of strain developed due to the excavation at the toe. The three-dimensional (3D) back-analysis of this landslide also confirmed that a progressive failure occurred at Senise (Conte, Donato, & Troncone, 2013).

3.2 Mechanism of Progressive Failure

Materials with strain softening behaviour, such as rocks, dense sands or over-consolidated clays, exhibit strength loss under large deformations. The strength-strain relationship of these materials shows a peak strength followed by a gradual reduction of shear strength to a residual value at a relatively large strain. This behaviour may lead to the development of progressive failure in problems involving large strains as recognised first by Terzaghi and Peck (1948). Progressive failure is initiated by the failure of a zone of large shear strain, localised along a potential failure surface. The reduction in the post-peak strength of this zone and the redistribution of stresses result in propagation of the shear zone and a further increase in shear strains, which in turn triggers failure of a larger section of the soil mass. This may eventually lead to the formation of a continuous failure surface and collapse of the slope. Excavation and fill of the existing slopes changes the effective stress and initiate progressive failure of slopes.

4 PROCEDURE

4.1 Strain Softening Material Model

In this paper, the strain softening behaviour is taken into account using the framework of the Mohr-Coulomb failure criteria. In this model the shear strength parameters, such as cohesion and internal friction angle, are defined as functions of the shear strain, or more generally, the deviatoric strain.
invariant, \( \varepsilon_D \), which for plain strain problems can be defined as:

\[
\varepsilon_D = \frac{1}{\sqrt{6}} \sqrt{(\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_x)^2 + (\varepsilon_y)^2 + (\gamma_{xy})^2}
\]

where \( \varepsilon_x \) and \( \varepsilon_y \) are the normal strains in x and y directions, and \( \gamma_{xy} \) is the engineering shear strain.

The process of strain softening begins when \( \varepsilon_D \) exceeds the strain corresponding to the peak shear strength, \( \varepsilon_{D\text{peak}} \), and then follows as the strength parameters, \( \Omega, (\text{cohesion, } c, \text{ or friction angle, } \tan(\phi)) \) gradually decrease from a peak value, \( \Omega_{\text{peak}} \), to its residual value, \( \Omega_{\text{res}} \), at a residual strain level, \( \varepsilon_{D\text{res}} \). The relationship between strength and strain is schematically shown in Fig 3. The material strength can be defined in terms of a piecewise tri-linear function of the deviatoric strain invariant:

\[
\Omega(\varepsilon_D) = \begin{cases} 
\Omega_{\text{peak}} & \varepsilon_D \leq \varepsilon_{D\text{peak}} \\
\Omega_{\text{peak}} - \frac{(\varepsilon_{D\text{peak}} - \varepsilon_{D\text{res}})(\varepsilon_D - \varepsilon_{D\text{peak}})}{\varepsilon_{D\text{peak}} - \varepsilon_{D\text{res}}} & \varepsilon_{D\text{peak}} \leq \varepsilon_D < \varepsilon_{D\text{res}} \\
\Omega_{\text{res}} & \varepsilon_D \geq \varepsilon_{D\text{res}}
\end{cases}
\]

The simple strain softening model employed in this study can be regarded as an extension of the standard elasto-plastic Mohr-Coulomb model, provided that the extra model parameters \( \Omega_{\text{peak}}, \Omega_{\text{res}}, \varepsilon_{D\text{peak}}, \varepsilon_{D\text{res}} \) are defined. These parameters can be obtained from laboratory tests such as triaxial, direct shear and ring shear tests on material samples.

![Simplified relationship](image)

Fig 3. Variation of a shear strength parameter, \( \Omega \), with shear strain, \( \varepsilon_D \).

4.2 Large deformation Analysis of Slopes

Slope failure is often accompanied by large strains and large changes in slope geometry, therefore application of large deformation analysis to slope failure studies is appropriate. Large deformation analysis using Updated Lagrangian formulation can simulate the changing geometry of slopes during failure. To analyse the Senise landslide, the finite strain theory together with the UL formulation has been implemented in a finite element code.

In a small deformation finite element analysis the constitutive relation is commonly expressed in terms of the Cauchy stress and strain rates. The stresses at any increment, \( \sigma_{ij}^{\text{f+\Delta t}} \), can be related to the incremental strains, \( \Delta \varepsilon_{kl} \), and the stresses developed in the previous increments, \( \sigma_{ij}^\text{f} \), by a standard integration procedure as:

\[
\sigma_{ij}^{\text{f+\Delta t}} = \sigma_{ij}^\text{f} + \int_0^{\Delta \varepsilon_{kl}} C_{ijkl}(\sigma, \kappa) d\varepsilon_{kl}
\]

where \( C \) is the stress-strain matrix, \( \sigma \) is the true (Cauchy) stress tensor, \( \varepsilon \) denotes the strain tensor, and \( \kappa \) represents hardening/softening parameters.

In a UL large deformation analysis the configuration of the body changes continuously and may result in a substantial rigid body rotation of elements, therefore the Cauchy stress may not be integrated directly. The rigid body rotation should not cause any strain in the material to ensure the objectivity of the constitutive equations. Hence, a stress transformation should be included in the stress integration scheme to correct stresses and to account for stress objectivity (Belytschko, Liu, & Moran, 2000). In this study, the Jaumann (1911) stress transformation is incorporated into the stress strain relationship to account for the objectivity of stresses. The Jaumann rate of the Cauchy stress is given by:

\[
\sigma_{ij}^\text{\Delta t} = \frac{d\sigma_{ij}}{dt} = \sigma_{ij} - \sigma_{ik}w^k_w^l w_{ik} \sigma_{lj}
\]

where \( w \) is the spin tensor which is defined in terms of the velocity vector, \( v_i \) as:

\[
w_{ij} = 1/2 (\partial v_i / \partial x_j - \partial v_j / \partial x_i)
\]

The stress increment in the finite strain formulation considered here can be calculated by:

\[
\sigma_{ij}^{\text{f+\Delta t}} = \sigma_{ij}^\text{f} + \int_0^{\Delta \varepsilon_{kl}} \sigma_{ijkl} \omega_{kl} d\varepsilon_{ij} =
\]

\[
\sigma_{ij}^\text{f} + \int_0^{\Delta \varepsilon_{kl}} \sigma_{ijkl} \omega_{kl} + \sigma_{ik} \omega_{lj} + \int_0^{\Delta \varepsilon_{kl}} C_{ijkl}(\sigma, \kappa) d\varepsilon_{kl}
\]

where \( \omega \) is the rigid body rotation term that can be calculated based on the displacement vector, \( u \), as:

\[
\omega_{ij} = 1/2 (\partial u_i / \partial x_j - \partial u_j / \partial x_i).
\]
5 NUMERICAL ANALYSIS OF THE SENISE LANDSLIDE

5.1 Analysis Process

An analytical tool has been developed by implementing the large deformation formulations and the strain softening model into the finite element program, AFENA (Carter & Balaam, 1995). The Senise landslide is simulated in a two-dimensional finite element analysis under plane strain condition. The soil is assumed to obey the Mohr Coulomb failure criterion, modified to include the strain softening behaviour. The element type used in the spatial discretisation of the slope is the isoparametric 6 noded triangles. The initial finite element meshes used in the analysis, before and after the excavation of the toe, are shown in Fig 4. The slope is under self weight gravitational loading. The initial stresses due to the soil gravity are distributed in the continuum by simulating staged construction of the slopes, i.e., switching on the gravity in layers of elements sequentially. The strength parameters during the construction of the slope are set to peak values to prevent failure during the staged construction. These parameters reduce during the analysis due to the strains generated by excavation near the toe, and as a consequence the movement of the soil mass commences. Water table is not considered in the analysis as its effect was assumed to be negligible.

The solution procedure adopted in the analyses is based on the modified Newton-Raphson method, using the elastic stiffness of the material.

5.2 Model assumptions

The soil parameters used in the analyses are reported in table 1. The values of Young’s modulus E have been deduced from the results of the triaxial tests performed by Del Prete and Hutchinson (1988), considering confining pressures corresponding approximately to those existed, on average, in the slope. No specific experimental value for Poisson’s ratio, \( \nu \), is available, therefore it is assumed to be 0.25. It should be noted that the peak, \( c'_p \) and \( \phi'_p \), and residual, \( c'_r \) and \( \phi'_r \), strength parameters considered for the yellow sand and clayey silt fall within the range of values deduced from the direct shear tests performed by Viggiani & Di Maio (1991). The soil dilation angle, \( \psi \), has been assumed to be zero, implying a non-associated flow rule with zero volume change during yielding of the soil material.

![Fig 4. Finite element mesh Finite element mesh before and after excavation](image)

Previous studies on the effects of the rate of strength reduction \( \varepsilon_{Dres} \) and stiffness parameter of soil (E) on the initiation of progressive failure in a typical slope showed that the mobilized shear strength, and consequently the post-failure deformation of slopes, depends on the value of E. Slope materials with larger Young’s moduli generate smaller shear strains and require a smaller residual strain parameter, \( \varepsilon_{Dres} \), to exhibit progressive failure (Mohammadi & Taiebat, 2013). The Young’s modulus of the soil significantly affects the critical residual strain parameter, a value below which progressive failure occurs accompanied by a large deformation. Therefore, the residual strain parameter and the modulus of elasticity of the slope material should be adopted with care in finite element analysis of slopes with strain softening materials.

5.3 Results

The numerical analysis of problems with progressive failure is often affected by lack of convergence and mesh dependency of the solution.
Previous Elasto-plastic analysis of the landslide performed by Troncone (2005) did not yield convergence when excavation was modelled to its maximum height, \( H_{\text{max}} \). This analysis was based on infinitesimal strain formulation which does not have any capability to estimate the post-failure deformations.

In the current analysis, the adoption of the UL formulation enables the assessment of deformations occurring during and after failure, however large changes of geometry results in mesh distortion as illustrated in Fig 5; therefore, an H-Adaptive mesh refinement technique has been used to minimise the effects of mesh distortions on the results.

The excavation at the toe results in development of large strains and softening of the soil which triggers progressive failure. The large deformation of the slope after failure causes distortion of the finite element mesh and results in inaccuracy of the solution. To maintain accuracy of the results the mesh has been updated a few times during the analysis, using h-adaptive remeshing technique, as shown in Fig 6. The incremental displacement field is shown in Fig 7, which confirms most of the movements are above the weak clayey silt layer.

The analysis shows the accurate location of the failure surface. The maximum movement of the soil, occurs before stabilisation of the slope, is predicted to be 11 m, as compared to the approximately 30 m observed in the field. It should be noted that the analysis is limited to simulation of slow failures, and the inertia effects are not considered in the analysis. Therefore, the soil movement of the soil mass predicted by the analysis is expected to be less than that observed in the field. The movement of soil mass and localisation of strains result in reduction of soil shear strengths and development of progressive failure in the slope.

Application of a strain softening model in a FEM is often accompanied by numerical instabilities and lack of convergence (Conte, Silvestri, & Troncone, 2010; Lo & Lee, 1973; Potts et al., 1990; Potts, Kovacevic, & Vaughan, 1997; Troncone, 2005). In the analyses performed in this study, the implementation of the UL formulation in modelling the post-failure behaviour proved to be useful in stabilising the solutions. A modified Newton-Raphson method was employed initially where the elastic stiffness matrix was formed only once at the beginning of the analysis. However, oscillation in the residual forces was observed in some cases which led to unstable solutions and unreliable results. The solution procedure was modified by updating the elastic stiffness matrix with the updated geometry of the problem at some intermediate stages during the analysis. This method was extremely effective in eliminating the problem of oscillation of the residual forces and produced stable results. The usual procedure of checking the norm of the load vector was found to be adequate as the convergence criterion with a 10^{-8} tolerance.

**6 CONCLUSIONS**

A large deformation finite element formulation together with a simple strain softening material behaviour were incorporated in a finite element analysis of the Senise landslide. The deformation of the sliding mass was captured with a reasonable accuracy by the finite element analysis. In addition the final orientation and geometry of the slope was predicted. The remeshing technique activated in intermediate steps during analysis has facilitated the simulation of failure by maintaining the accuracy of
the solution and avoid mesh distortion as a result of large deformations. The numerical problem associated with lack of convergence that normally occurs in problems involving shear strain localisation has been successfully tackled. The rate of strain softening and stiffness of slope material should be adopted with care in the analysis for correct initiation of progressive failure in the slopes. The failure is shown to be triggered by the excavation at the toe of the slope and spreads upward along the thin clayey silt layer. Compared with the observation, the numerical tool could simulate the location of failure surface and the sliding soil mass reasonably well.

Fig 7. Incremental displacement field at iterations 18000, 42000

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