A multi-agent-based approach to tax morale

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Abstract

We embed the behaviour of tax evasion into the standard two-dimensional Ising model. In the presence of an external magnetic field, the Ising model is able to generate the empirically observed effect of tax morale, i.e. the phenomenon that in some countries tax evasion is either rather high or low. The external magnetic field captures the agents’ trust in governmental institutions. We also find that tax authorities may curb tax evasion via appropriate enforcement mechanisms. Our results are robust for the Barabási-Albert and Voronoi-Delaunay networks.

[Keywords: Opinion dynamics, Sociophysics, Ising model, Tax morale.]

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1 Introduction

Tax evasion can vary widely across nations, reaching extremely high values in some developing countries. Wintrobe and Gérxhani (2004) explain the observed higher levels of tax evasion in generally less developed countries with a lesser amount of trust that people accord to governmental institutions. Many empirical studies confirm that tax payers are more willing to comply, the more they feel that the government represents their preferences, i.e. the more content they are with how their government uses tax revenue (see, e.g.: Schnellenbach, 2002; Torgler, 2004; Hyun, 2005; Cummings et al., 2007). We consider the concept of tax morale important because it seems for the most part to explain why compliance differs across countries even if the level of enforcement is roughly the same. Our aim is to add to the experimental and empirical studies conducted on tax morale by applying a multi-agent-based approach. In economics, the problem of tax evasion from a multi-agent-based perspective has received little attention to date (see Bloomquist (2006) for a recent overview).

In two previous studies (Zaklan et al., 2008; Westerhoff et al., 2008) we modify the two-dimensional Ising model to analyse how enforcement affects tax evasion dynamics. The key idea in these papers is that agents may either behave honest (tax payers) or dishonest (tax evaders), a decision which is subject to group influence. We further augment the Ising model by adding an enforcement mechanism, which consists of two elements: a probability of an audit ($p$) and a period of honesty ($k$) in which detected evaders remain honest. We find evidence that enforcement has a direct and indirect effect on aggregate tax evasion. First, enforcement obviously has a direct effect on aggregate tax evasion since any tax evader who is audited becomes honest for a certain period of time. Second, there is an important indirect effect of enforcement on aggregate tax evasion since agents who live in an environment with more honest agents are also likely to be honest, due to group influence. We also find that even low levels of enforcement may help to prevent substantial fluctuations in tax evasion.

The aim of the current work is to develop our previous studies by showing that
our tax evasion model can also replicate the effect of tax morale, if an external magnetic field is incorporated. By modifying the probabilities of evading or not evading tax duties, the external magnetic field captures agents’ trust in governmental institutions. By implementing a positive and a negative external magnetic field and contrasting the resulting equilibrium levels of tax evasion with the baseline setting, where the external magnetic field is absent, we show that our model can explain both high and low levels of tax morale. To demonstrate that this remains valid regardless of the level of enforcement, we depict the equilibrium values of tax evasion corresponding to each considered external magnetic field for different levels of enforcement.

The remainder of our manuscript is organised as follows: in section 2 we briefly present our tax evasion model, which is based on the standard two-dimensional Ising model on a square lattice. In section 3, we discuss how the external magnetic field influences tax evasion. By varying the compliance level systematically and observing the resulting equilibrium levels of tax evasion for different external magnetic fields, we illustrate that the external magnetic field may be an appropriate tool which enables different degrees of tax morale to be modeled, for any level of compliance. In section 4, we also embed our model into the Barabási-Albert network and the Voronoi-Delaunay random lattice. We find that the effect of the external magnetic field also remains valid in these networks. The last section presents a number of conclusions.

2 The model

We use the so-called single spin-flip heat-bath algorithm to simulate the Ising model on a 1000 × 1000 square lattice with periodic boundary conditions. In this algorithm, irrespective of the current state of the chosen spin, its new state is chosen with probabilities $P_\pm = e^{-\beta E_\pm}/(e^{-\beta E_+} + e^{-\beta E_-})$, where $P_\pm$ are the probabilities of choosing the up (+) and down (−) states, respectively (Chowdhury and Stauffer, 2002), and $\beta$ is defined as $1/(k_B \cdot T)$. The total energy ($E$) is given by the Hamiltonian $H = -\sum_{<i,j>} J_{ij} S_i S_j - B \sum_i S_i$ (we use $J_{ij} = J = 1$). As an external magnetic
field we use $h = 2B/k_B T$. A negative external magnetic field acts to augment the probability of becoming dishonest, whereas a positive field lowers this probability. In our previous models we set $B = 0$, and hence neglect the influence of the external magnetic field. We interpret a negative external magnetic field ($h < 0$) as agents’ low confidence in governmental institutions, meaning that tax evasion is consequently high. On the other hand, a positive external magnetic field ($h > 0$) implies above-average confidence in the tax authorities: since individuals feel well represented by the government, they feel more comfortable about paying their tax duty, and hence evade paying tax less frequently.

We interpret this setup in the following way: in every time period each lattice site is occupied by a tax payer who can either be honest ($S_i = +1$) or a tax evader ($S_i = −1$). We assume that everybody is honest, initially. In consecutive time periods agents have the opportunity to change their agent-type, enabling cheating agents to become honest and honest citizens to become tax evaders. Each agent’s social network is made up of four nearest neighbours, and they may either prefer tax evasion, reject it or be indifferent.

Tax evaders have the greatest influence to turn honest citizens into tax evaders if they constitute a majority in the given neighbourhood. If the majority evades paying tax, one is likely to also evade. On the other hand, if most people in the vicinity are honest, the respective individual is likely to become a decent citizen if she was a tax evader before. The strength of neighbourhood influence can be controlled by adjusting the temperature, $T$.

For very low temperatures, individuals mainly base their decision regarding tax evasion on what the neighbourhood does. A rising temperature has the opposite effect. Individuals then decide more autonomously. We only use temperatures below $T_c$ ($\approx 2.269$) for our simulations, since we are interested in modeling the effect of group influence.

As an enforcement measure, we introduce a probability of an efficient audit ($p$). If tax evasion is detected, the individual remains honest for a certain number of periods. We denote the period of time for which detected tax evaders remain honest by the variable $k$. We assume that $k$ is a random number between either 0 and 10
or 0 and 50 periods. Both of these intervals, which we will use, express that agents are ashamed and feel guilty about their behaviour if cheating is detected, but that the shame is of temporary nature, which is typical for the normal type of shame (e.g. Potter-Efron and Efron (1993)). Obviously, the second interval allows detected agents to be more ashamed on average, having evaded taxes: agents have a longer memory of their wrongdoing.

One time unit is one sweep through the entire lattice. Audits are stochastically independent from other agents and from the history any agent has.

3 Dynamics of the model

In Figure 1, the probability of an audit is set to 1% and the temperature to a level 25% below $T_c$. The upper left panel in Figure 1 ($k \in [0, 10]$ and $h = -0.25$) illustrates that tax evasion may spread throughout the population within only 1000 time steps, given that the interval for the parameter $k$ is chosen to be sufficiently small. The level of tax evasion comes to rest at about 90 percent. The three panels below, also in the first column of Figure 1, visualise the distribution of agent types after 50, 500 and 1000 time steps, respectively. It can be seen that both honest people (white) and evaders (black) increasingly group together in clusters as time passes. After 50 time steps, when the aggregate tax evasion is at about 4 percent, the clusters are still relatively small. After 500 time steps (about 35% of the population then evades taxes), the clusters grow to such an extent, that individuals with different opinions regarding tax evasion clearly separate. After 1000 time steps, tax evasion eventually prevails, as the diagram at the bottom left clearly illustrates.

The column on the right of Figure 1 shows that higher values of $k$ ($k \in [0, 50]$) prevent this spread of tax evasion, keeping it at about 4% over time. The other three diagrams in the right-hand column also illustrate the distribution of agent types after 50, 500 and 1000 time steps, respectively, and show that honest individuals remain dominant over time.
The left column of Figure 2 also implies a maximum duration of honesty of 10 periods, whereas the right column shows a maximum duration of honesty of 50 periods, after having been subject to an audit. We analyse for different strengths of group influence (by systematically lowering the temperature further below $T_c$, we accord more importance to group influence) how the equilibrium levels of tax evasion evolve under different influences of the external magnetic field for different enforcement levels: we implement these by gradually augmenting the probability of an audit from 0 to 5% (in steps of 0.1 percent).

Each equilibrium value in tax evasion is calculated by first allowing a transient phase of 5000 time steps and by then forming the simple average of the tax evasion levels over the next 1000 time steps. In Figure 2 the lines are marked identically to denote the same influence of the external magnetic field: diamonds imply $h = -0.5$, squares denote the absence of the external field (i.e. $h = 0$) and pluses represent the case where the magnetic field is positive ($h = 0.5$).

Figure 2 illustrates that a positive external field ($h = 0.5$) works to lower tax evasion, compared to the baseline case, where an external field is absent. On the contrary, a negative external field works to augment the problem of tax evasion: the curve where $h = -0.5$ (diamonds) is above and the curve where $h = 0.5$ (pluses) is below the line where $h = 0$ (squares), for all considered probabilities of an audit. Recall that a positive value of $h$ stands for the case in which agents have great trust in governmental institutions and that a negative value of $h$ stands for the case in which agents distrust governmental institutions. In this sense our model is able to replicate the phenomenon of tax morale.

Note that, if the external magnetic field is negative, an increase in the audit rate has the effect of reducing non-compliance substantially. Given that individuals are sufficiently affected in the case of detection (right-hand column of Figure 2) and group influence is sufficiently strong (panel on the bottom right), non-compliance may be reduced from about 100% to below 10% by increasing the audit rate to 5%.
4 Modifications

We now consider our model on other network structures to obtain further support for our results. In particular, we use the Voronoi-Delaunay random lattice and the Barabási-Albert network model. The Voronoi-Delaunay lattice (i.e. tessellation of the plane for a given set of points) is constructed as follows (Lima et al., 2000). First, a given number of points are randomly distributed in a plane of a given size. For each point, the polygonal cell, consisting of the region of space nearer to that point than to any other point, must be determined. Whenever two such cells share an edge, they are considered to be neighbours. Using the Voronoi tessellation, the dual lattice can be obtained as follows: if two cells are neighbours, a link is placed between the two points located in the cells. The triangulation of space is obtained from the links. The network constructed in this way, which we use for simulation, is called the Voronoi-Delaunay lattice.

The Barabási-Albert network (Barabási and Albert, 1999) is established such that the probability of a new site being connected to one of the already existing sites is proportional to the number of connections the existing site has already accumulated over time: the chance for an agent to obtain a new connection is greater if he is well connected already.

In these variations of our simple square lattice model, we also choose 1,000,000 agents and calculate the equilibrium levels of tax evasion. The results are displayed in Figures 3 and 4. As can be seen in both networks, the Voronoi-Delaunay lattice (Figure 3) and the Barabási-Albert network (Figure 4), higher values of $k$ (i.e. $k \in [0, 50]$ versus $k \in [0, 10]$) also lead to lower equilibrium values in tax evasion in any considered parameter setting, as in the square lattice network: identically marked curves in panels on the right are lower than the corresponding curves, which are in the same row, but on the left.

— Figures 3 and 4 go about here —

In the Voronoi-Delaunay network (Figure 3), the curves, where the external magnetic field is absent ($h = 0$) and where it is positive ($h = 0.5$) resemble much the
corresponding lines in the square lattice network. Yet enforcement seems to work
better. Especially when looking at the case where the external magnetic field is
negative \((h = -0.5)\), one can clearly see that non-compliance may be reduced from
very high to quite low levels. For example, if group influence is strong \((T = 0.8 \cdot T_c)\)
and the agent’s memory typically high \((k \in [0,50])\), the influence of the negative
external magnetic field, which favors non-compliance, may quickly be reduced by
augmenting the probability of an audit to such low levels as 0.5%.

Tax evasion is quite different under the Barabási-Albert network (Figure 4).
If a negative external field is applied \((h = -0.5)\), tax evasion is lower for small
audit rates than in the square lattice or in the Voronoi-Delaunay network. While
tax evasion increases to nearly 100% in the other two networks if no audits are
conducted, in the Barabási-Albert network, on the other hand, non-compliance only
rises to about 70% (for \(p = 0\)). While tax evasion is lower for small audit rates
in the Barabási-Albert network, if a negative external field is applied, in the case
of no external field \((h = 0)\) or a positive one \((h = 0.5)\), equilibrium tax evasion
is much higher than in the other two networks. Also audit rates have a different
impact on tax evasion, compared to the other networks. While in the square lattice
and the Voronoi-Delaunay lattice tax evasion under a zero external and a positive
external magnetic field are only marginally reduced, in the Barabási-Albert network
tax evasion can be reduced substantially, by increasing the audit rate to 5% (here
we mean the amount of more compliance that can be established by implementing
higher levels of enforcement). If a negative external field is applied, it can be seen,
as in the other two networks, that tax evasion decreases with an increasing audit
rate and that for any level of enforcement (i.e. audit rate) non-compliance is higher
than for a zero or a positive external magnetic field.

For sufficiently small enforcement levels, we find for the applied negative external
magnetic field \((h = -0.5)\) that the equilibrium values in tax evasion are higher, the
further the temperature is decreased. This is most obvious for the square lattice

\footnote{However, if the temperature is below a certain threshold (i.e. about \(T = 0.55 \cdot T_c\) for the square lattice with \(k \in [0,10]\)), the negative external magnetic field we apply is too small, to trigger
non-compliance to spread. For sufficiently low temperatures the state of compliance hence prevails
for the considered negative external magnetic field, even for low audit rates.}
with $k \in [0, 10]$: low enforcement levels (i.e. small values of $p$) are less efficient to reduce tax evasion, the more individuals focus on what their neighbours do (i.e. for lower temperatures). In the presence of the negative external magnetic field, the number of honest agents is small for low enforcement levels. At lower temperatures honest agents (some agents become honest due to enforcement) have less influence on cheating agents to become honest, so that, compared to higher temperatures, tax evasion is higher for low enforcement levels. If enforcement levels increase, the tax evasion of more people is detected and these agents then usually become compliant for a while. If the number of agents is sufficiently large (due to a high level of enforcement), their influence on the non-compliant individuals grows. The lower the temperature is, the greater is their influence: non-compliance hence is smaller at lower temperatures, if enforcement is sufficiently high. This can be observed also in the Voronoi-Delaunay network for $k \in [0, 10]$ and in the Barabási-Albert network for lower values of $k_{\text{max}}$, e.g. $k_{\text{max}} = 5$.

Finally we briefly discuss why the non-compliance curve, which captures the influence of the negative external magnetic field ($h = -0.5$), declines more rapidly in the Barabási-Albert and in the Voronoi-Delaunay network, compared to the square lattice, when the audit rate is increased. We conjecture that this can be explained by the greater mean size of the neighbourhood in these two networks (on average, the neighbourhood comprises 6 agents in the Barabási-Albert network and 8 agents in the Voronoi-Delaunay lattice), compared to the square lattice (each agent has only 4 nearest neighbours). Setting the audit rate to a positive value forces some agents in the neighbourhood to become honest for $k$ periods. The greater the size of the neighbourhood is, the more honest agents will be included in the considered agent’s social network. These honest agents then exert influence on the respective individual to become honest, too. Hence, in the Voronoi-Delaunay lattice, which on average includes most agents in a neighbourhood, increasing the audit rate slightly has the effect of reducing non-compliance the most. In the Barabási-Albert network, whose neighbourhood is intermediate in size, the same increase of the audit rate consequently works less effectively to establish more compliance, compared to the Voronoi-Delaunay lattice, although it is still better than in the square lattice.
5 Conclusion

By incorporating the possibility of tax evasion into the Ising model we are able to replicate, within the context of a multi-agent-based model, the empirical fact regarding the existence of tax morale by isolating the effect of the external magnetic field on tax evasion. While in this study our focus is on the influence of the external magnetic field on tax evasion, other extensions to our simple model appear interesting and economically important. Allowing agents to differ in their period of honesty after detection seems to be just the first step in creating a more realistic tax evasion model. Economic variables, which are important for the decision regarding tax evasion, such as personal income, may need to be incorporated into the model. Also, an intuitive extension to our hitherto very simple model may be to allow for a third type of agent, who is neither entirely compliant nor entirely non-compliant, but somewhere inbetween.

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Figure 1: Illustration of how a negative external magnetic field (here we use $h = -0.25$) may cause most of society to become non-compliant. (A more detailed description is contained in section 3.)
Figure 2: The panels on the left imply $k \in [0, 10]$ and those on the right $k \in [0, 50]$. They illustrate the equilibrium levels of tax evasion for different probabilities of an audit (the same applies to Figs. 3 and 4) in the square lattice network (cp. section 3).
Figure 3: The same simulation setting as in Figure 2 applies to the Voronoi-Delaunay lattice, which has a critical temperature of $T_c = 3.802$. (A more detailed description is given in section 4.)
Figure 4: The same simulation setting as in Figure 2 applies to the Barabási-Albert network, which has a critical temperature of $T_c = m \cdot \log(N_{SITES})/2$, where $m = 4$ and $N_{SITES} = 1,000,000$. (A more detailed description is contained in section 4.)