Pierson-Moscowitz spectrum simulation based on the rough sea surface

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Abstract. By analyzing a variety of traditional wave spectrum models, combining the actual wavelength and wave surface displacement to compare the electromagnetic field propagation process, the P-M (Pierson-Moscowitz) spectrum is used to theoretically derive the electrical characteristics of the rough sea surface and its influence on the propagation process of high-frequency electromagnetic waves. Based on the bilinear superposition algorithm and Matlab numerical calculation software, the electrical characteristics of rough seas and the theoretical results of electromagnetic wave propagation in rough seas are simulated and verified. The results show that the modeling of rough sea surface based on P-M spectrum has the advantages of high reducibility and stable frequency spectrum.

1. Introduction

With the continuous development of communication systems such as radar and naval vessels, the propagation of electromagnetic waves on rough seas has become a major research focus. In order to more accurately describe the process of electromagnetic wave propagation when the wavelength and wave surface displacement are comparable, how to build a wave model that is more suitable for this situation has become a difficult problem that needs to be overcome. With the increase of observational data and the continuous development of wave spectrum, people have gradually summarized many empirical spectra. Among them, wave spectra such as Neumann, P-M and JONSWAP are the most commonly used. In order to better reflect the influence of the rough sea on electromagnetic wave propagation, the selected wave spectrum should have the following characteristics: 1. In a fully grown state, the spectrum energy is more concentrated and the spectrum is more stable; 2. In order to clarify the influence of sea waves on electromagnetic wave propagation, above the sea surface The same wind speed at the same height has a greater impact on the sea surface; 3. It is suitable for a wide range of sea depths.

When studying rough seas, it is mainly divided into linear and non-linear models, and the relatively mature one is the linear model. The bilinear superposition model adopted in this paper is one of them. By superimposing the amplitude and direction of the wave spectrum in detail, it shows the statistical and random characteristics of the wave spectrum, and finally realizes the simulation of the wave spectrum.
2. Wave spectrum model

2.1. Neumann Spectrum
This is a semi-empirical method that reflects the internal structure of a wave spectrum through the external characteristics of the wave. [2] It first appeared in the 1950s. Although this wave spectrum is relatively rough in observational data, it still provides a theoretical basis for a series of wave spectra that appeared later. The basic formula of the spectrum in the frequency domain is [3]

$$S(\omega) = C \frac{\pi}{4} \frac{1}{\omega^4} \exp\left(- \frac{2g^2}{U_{7.5}^2 \omega^2}\right)$$  \hspace{1cm} (1)

Where $U_{7.5}$ is the wind speed of 7.5m above the sea surface, the constant $C=3.05\text{m/s}^2$, and $g$ is the acceleration of gravity. It can be seen that the frequency spectrum is divided into two parts, $C \frac{\pi}{4} \frac{1}{\omega^4}$ and $\exp\left(- \frac{2g^2}{U_{7.5}^2 \omega^2}\right)$, the first part determines the high frequency part, and the latter part determines the low frequency part. At the same time, the frequency spectrum takes into account the influence of wind speed on ocean waves.

2.2. P-M Spectrum
The P-M spectrum is a fully grown wave spectrum. It was fitted by Pearson and Moskevich in 1964 based on a large number of data measured at fixed points in the North Atlantic. It also conforms to the basic form of the Neumann spectrum. Equation (2) gives the spectral function of the P-M spectrum [4][5]

$$S(\omega) = \alpha \frac{g^2}{\omega^5} \exp\left(- \beta \left(\frac{g}{U_{19.5} \omega}\right)^4\right)$$ \hspace{1cm} (2)

Where $\alpha = 8.1 \times 10^{-3}$, $\beta = 0.74$, $U_{19.5}$ is the average wind speed at 19.5m above the sea surface.

Figure 1 shows the spectral waveforms of the Neumann spectrum and P-M spectrum in the situation. Since $U_{19.5}$ in the P-M spectrum is the wind speed at 19.5m on the sea surface, $U_{7.5}$ in the Neumann spectrum is the wind speed at 7.5m on the sea surface. The wind speed $U_{10}$ at 10m above the sea surface is usually used in the calculation, and the wind speed at different heights can be converted by the following method.
Where $U_z$ is the wind speed at any height $z$, the Kappa constant $k = 0.4$, and the drag coefficient corresponding to $U_{10}$ can be obtained according to the empirical formula $C_{10} = 0.5U_{10}^{0.5} \times 10^{-3}$.

It can be seen that, corresponding to the wind speed $U_{19.5}=16.09\text{m/s}$ at 19.5m on the sea surface of the P-M spectrum, and $U_{7.5}=14.53\text{m/s}$ at 7.5m on the sea surface of the Neumann spectrum, the two spectra correspond to the frequencies corresponding to the extreme values. It is very close, but compared to the Neumann spectrum, the P-M spectrum has relatively more concentrated energy, and the convergence rate is relatively faster. As for the fully grown wave spectrum with wind speed as the parameter, its characteristics are stable amplitude and stable wavelength, so the energy will be concentrated in a certain frequency domain. In contrast, under the condition of 10m above the sea surface, the P-M spectrum is more severely affected by wind speed, which is convenient to reflect the electromagnetic wave propagation process under the conditions of comparable wavelength and wavefront displacement. In summary, the P-M spectrum is more suitable for this background than the Nuamann spectrum. Simulate the sea surface conditions.

3. Bilinear superposition method to establish a wave model

The displacement of the wave surface of ocean waves can actually be regarded as a three-dimensional model formed by superimposing many cosine waves with different parameters such as frequency, amplitude and direction on the coordinate system.\[^6\] The bilinear superposition is mainly to superimpose the amplitude and direction in a small amount under the premise of traversing the frequency spectrum. The three-dimensional form can be expressed as\[^7\]

$$\zeta(x, y, t) = \sum_{n=1}^{\infty} a_n \cos(k_n \cos \theta_n + k_n \sin \theta_n - \omega_n t + \varepsilon_n)$$

where $\zeta$ represents the wave surface displacement of the sea surface at any $(x, y)$ in the constructed coordinate system, which is a smooth and uniform normal process. Where $a_n$ represents the amplitude of each single wave, $k_n$ represents the wave number, $\omega_n$ represents the frequency, and $\varepsilon_n$ represents the random phase on $0 - 2\pi$. According to the definition of spectrum energy, both sides integrate $\omega$ and $\theta$ at the same time:

$$\int_0^{2\pi} \int_0^{\omega} S(\omega) d\omega d\theta = \sum_{n=1}^{\infty} \sum_{\theta=0}^{2\pi} \frac{1}{2} a_n^2$$

(5)

At this time, the formula (5) becomes

$$\int_0^{\omega+\Delta\omega+\Delta\theta} \int_0^{\theta+\Delta\theta} S(\omega) d\omega d\theta = \int_0^{\omega} \sum_{\theta=0}^{2\pi} \frac{1}{2} a_n^2$$

(6)

When the frequency is between $\omega$ and $\omega + d\omega$, and the direction is between $\theta$ and $\theta + d\theta$ for summing and integration, when $d\omega$ and $d\theta$ is close to infinity, the left side of equation (6) $S(\omega)$ can be regarded as a constant, and the right side $a_n$ can also be regarded as a constant. At this time, Becomes

$$S(\omega, \theta) \delta\omega \delta\theta = \frac{1}{2} a_n^2$$

(7)

Which is $a_n = \sqrt{2S(\omega, \theta) \delta\omega \delta\theta}$.
Figure 2 shows the relative value of wave surface displacement in the same normal direction under different sea states. It can be seen that with the continuous increase of wave levels, the wave surface displacement and sea level fluctuations increase relatively, and the fluctuations also become severe. At the same time, it matches the wave height range defined by the international standard sea state classification.[3]

4. P-M spectrum modeling algorithm and simulation results

In the process of establishing the wave spectrum model of the P-M spectrum, not only the correspondence between the wind level and the sea states must be considered, but also the refinement of the meshing distance in each direction must be considered. This distance must be much smaller than the subsequent electromagnetic wave experiment wavelength. At the same time, in order to consider the authenticity of the wave spectrum after simulation, it is necessary to add color processing to the algorithm: 1. Add a light source and use the surfl() algorithm in matlab to enhance the visibility of the simulation model; 2. According to the wave surface of the wave Displacement, the corresponding color gradient processing of the waves can visualize the height of different waves and improve the resolution. In this algorithm, the x and y directions are within 500m, and the step distance is 1m; the wavefront displacement and wavefront phase generated at each point are superimposed by not less than 100 points, so the calculation involved in this algorithm It is very huge, the specific algorithm flow is as follows:

![Figure 3. Flow chart of P-M spectrum simulation of bilinear superposition algorithm](image)

The sea surface simulation under sea state 5 is shown in Figure 4:
Figure 4. P-M spectrum simulation diagram under sea state 5

It can be seen that under the sea state 5 based on the P-M wave spectrum, the wave surface displacement is controlled within a height of 2.5-4.0m, which meets the standard sea state classification. At this time, if the electromagnetic wave propagation in the short-wave frequency band is used, the wavelength and wave surface displacement are comparable; the simulated P-M spectrum is the most severely affected by the wind speed parameter in the common spectrum; at the same time, the simulation process of the wave model has nothing to do with the sea area, and the water depth can be ignored. The impact on sea conditions. This shows that the wave model based on P-M spectrum simulation meets the required application background and can be used for subsequent simulation of electromagnetic wave propagation.

5. Conclusion

In this paper, through the comparative analysis of various wave spectrum models, the P-M spectrum is used to theoretically derive the electrical characteristics of the rough sea surface and its influence on the propagation of electromagnetic waves. Theoretical analysis results show that modeling rough ocean waves based on P-M spectrum has the advantages of high reducibility and stable frequency spectrum. The bilinear superposition algorithm and Matlab numerical calculation software are used to simulate and verify the electrical characteristics of rough seas and the theoretical results of high-frequency electromagnetic wave propagation in rough seas. The simulation results are consistent with the theoretical analysis results. Through theoretical analysis and simulation verification, this paper established a rough sea surface with a high degree of reduction, which laid a solid foundation for analyzing the propagation characteristics of electromagnetic waves in complex sea conditions. At the same time, the propagation characteristics of electromagnetic waves on rough seas were carried out in the shortwave frequency band. The analysis provides a theoretical basis for shortwave sea surface communication.

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