The spatial–numerical association of response codes effect and math skills: why related?

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Evidence from multiple studies conducted in the past few decades converges on the conclusion that numerical properties can be associated with specific directions in space. Such spatial–numerical associations (SNAs), as a signature of elementary number processing, seem to be a likely correlate of math skills. Nevertheless, almost three decades of research on the spatial–numerical association of response codes (SNARC) effect, the hallmark of SNAs, has not provided conclusive results on whether there is a relation with math skills. Here, going beyond reviewing the existing literature on the topic, we try to answer a more fundamental question about why the SNARC effect should (and should not) be related to math skills. We propose a multiroute model framework for a SNARC–math skills relationship. We conclude that the relationship is not straightforward and that several other factors should be considered, which under certain circumstances or in certain groups can cause effects of opposite directions. The model can account for conflicting results, and thus may be helpful for deriving predictions in future studies.

Keywords: spatial–numerical associations; SNARC; spatialization; numerical cognition; math skills

Intuitions that space and numbers are somehow related were already expressed by Immanuel Kant in the 18th century.1 Furthermore, it has been argued that relating numbers to space improves mathematical skill and that spatial thinking plays a crucial role for math skills.2 In our view, there are two major issues with this argument. First, not all spatial–numerical associations (SNAs) are consistently related to math skills. Second, we know very little about why SNAs are related to math skills to begin with.

In this paper, we first outline the multitude of SNAs and then discuss their hallmark effect, the spatial–numerical association of response codes (SNARC) effect. For this effect, we propose a new model framework for how it is generated and modulated and why it does or does not correlate with math skills. The goal is to disentangle the link between the number–space relation and math skills.

A multitude of spatial–numerical associations

SNAs are a group of behavioral and neural phenomena showing that different aspects of numerical information are associated with different aspects of space.3–5 The following distinctions are of major importance:3

- Numerical information can be either approximate or exact.
- Numerical information might concern ordinality (i.e., a position in a sequence of numbers), cardinality (i.e., the numerosity of a set described by a given number), place-value structure (i.e., a structure of multidigit numbers in the Western Arabic system in which the relative position of numbers in a multi-symbol number determines its meaning), and functions (e.g., addition and subtraction).
The association itself can be implicit (i.e., its existence is measured indirectly, for instance, in reaction time patterns, response accuracies, or physiological signatures of neural processing) or explicit (i.e., it is observed in some overt behavior). To better understand the distinction between implicit and explicit SNAs, we can use an analogy to implicit and explicit associations in social psychology. One can look at explicit associations by asking participants to name their associations related to a specific category (e.g., atheists). They would produce several responses: some of them can be positive and some of them negative. Similarly, we can ask about explicit association of numbers in space, and see how the participants respond (e.g., by placing numbers of different values into space, counting spatially distributed objects and pointing to each of them while counting, or answering the question, “Where would you place number X?”). Apart from looking at explicit associations, one can also measure implicit associations, for example, with the implicit association test (IAT). In this test, participants respond to a target category (e.g., words related to atheism) with the same key that they are responding to either positive (e.g., “chocolate”) or negative (e.g., “mold”) objects. By looking at the reaction pattern (speed and accuracy), one can infer whether the target category is associated with positive or negative objects. Similarly, the reaction pattern (or spatial biases or biases in random number generation, for example) can implicitly measure associations between numbers and space so that participants are not directly asked. In such implicit measures (like the IAT or a SNA task), we can directly observe the behavior (e.g., the target category is often mistakenly responded to with the same key as positive objects, when it should be responded to with negative ones, or a small number is responded to with the left-hand key even though the task instruction requires it to be responded to with the right hand). However, such directly observable behavior provides different information than information gathered from explicit tasks we described above.

Numerical information can be associated with spatial extensions and specific directions in space (e.g., left-to-right, bottom-to-top, or front-to-back). Presently, we think that automaticity of processing might be an important aspect for our understanding of SNA, but it was not proposed in the original taxonomy. We include it now in the model and discuss its impact in the following sections. Several phenomena can be described within such an SNA taxonomy. The size congruity effect (i.e., if the physically larger number in a pair is also the numerically smaller one, then it takes longer to identify than if physical and numerical size correspond) is an approximate extension SNA. The number line estimation (i.e., a participant is to mark the location of a given number on a line knowing the value at the lower end, e.g., 0, and either the value at the upper end, e.g., 10 (bounded variant), or the length of a single unit (unbounded variant)) measures the exact extension SNA. Note that in the case of number line estimation, the number corresponds to an exact point on the line; therefore, we speak of exact rather than approximate SNA. The operational momentum (i.e., leftward/rightward spatial biases associated with subtraction/addition, respectively) is an instance of directional SNA with implicit coding of arithmetic functions. Biases in preferred direction of counting objects arranged in a row are examples of directional explicit associations of numerical cardinality.

Considering the multitude of existing SNAs described above, it is hard to provide an exhaustive

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This taxonomy is not morphological but hierarchical. In other words, it is like a tree with several branches and subbranches, rather than a multidimensional space within which we could describe every existing SNA. For this reason, some SNAs can be classified into two categories (i.e., approximate extension and exact extension). Some other SNAs can be classified into four categories (e.g., directional, implicit, cardinality, or directional explicit ordinality).

Even though exact numbers are presented in the task, the numerically larger number is associated with physically larger font. However, there is no exact mapping between specific numerical magnitudes and specific fonts sizes; thus the size congruity effect is considered approximate rather than an exact SNA.
SNA definition, which would be precise and at the same time characterize all possible SNAs. One solution to this problem is to refer to family resemblance. As originally suggested by Wittgenstein, if one tries to characterize the resemblance of family members posing together in a photo, one has the general impression that they are similar to each other. However, one cannot point to traits common to all of them. Rather, one can point to some features that are shared by several, but not all, family members, and these features overlap with each other. When one tries to think of SNAs in terms of family resemblance, it becomes obvious that more fine-grained analyses are necessary to understand why humans associate numbers with space. Furthermore, beyond the pure association of number and space, the role that different SNAs play for the development of math skills needs to be investigated in a more fine-grained manner.

The theoretical considerations and the model we propose are for the hallmark of implicit directional SNAs—the SNARC effect. In the SNA taxonomy, the SNARC effect is a directional implicit SNA of cardinality (i.e., number magnitudes are associated with directions in the left-to-right–oriented plane and the association is measured indirectly in the reaction time (RT) pattern). However, some instances of the SNARC effect are related to ordinality (i.e., ordinal positions of numbers) rather than cardinality. Going beyond the over-25-year-old discussion on whether the SNARC effect relates to math skills, we try to answer the question: why is the SNARC effect related, or not, to math skills?

Here, we restrict the discussion to the SNARC effect in order to provide a thorough model framework for this phenomenon. However, this framework may turn out to be useful for formulating and testing predictions regarding other SNA types in the future and, when a sufficient body of empirical evidence accumulates, be modified accordingly.

The SNARC effect

The SNARC effect denotes the phenomenon that in a bimanual parity judgment task, small magnitude numbers are responded to faster on the left-hand side and large magnitude numbers are responded to faster on the right-hand side. Even though the magnitude is response-irrelevant, it influences the RT pattern. The SNARC effect is very robust and has been replicated multiple times, including in a large-scale online setup. It has also been observed with several modifications to the original bimanual parity setup.

The SNARC effect is typically quantified at the individual level as a within-participant regression slope, in which dRTs (difference for RT; right-hand RT minus left-hand RT) are regressed on number magnitude. The negative slope indicates that the relative advantage of right-hand over left-hand RT increases with the increasing magnitude of the numbers the participant responds to. Thus, the more negative the slope, the stronger the effect. The slope can be expressed as an unstandardized (more common) or a standardized coefficient. Slope is a single measure of an individual's SNARC effect, which is then used in subsequent analyses considering individual differences. Most of the studies we refer to used either unstandardized slopes or standardized and both of them at the same time. Another method of quantifying the SNARC effect involves the variance explained by the regression predictor. This can be important because for some SNARC paradigms (parity judgment), a continuous predictor is the better predictor, while for other paradigms (e.g., magnitude classification), a categorical (contrast-coded small- and large-magnitude numbers) predictor is better suited to account for the actual data. Importantly, the conclusions of studies we discuss below did not differ depending on which specific method was used to quantify the SNARC effect. To summarize, while there are different ways to quantify the SNARC effect, for the most part these different computations lead to the same conclusions regarding links between the SNARC effect and math skills.

SNAs and math skills

The view that several aspects of elementary number processing underlie math skills is very compelling and intuitive. If one of these aspects of elementary number processing was the use of space as a building block to better represent numerical and mathematical knowledge and processes (which is behaviorally expressed by SNAs), one could use SNAs to build up and diagnose one of the underpinnings of numerical cognition. Possibly, one could even train math skills. Indeed, several studies have investigated the relationship between SNAs and math skills. As reviews show, there is no simple and
Table 1. Summary of currently available studies investigating the relationship between the SNARC effect and math skills in different age groups

| Groups tested                          | Children                                                                 | Adults                     |
|----------------------------------------|--------------------------------------------------------------------------|----------------------------|
| Special groups—low math skills level   | Bachot et al.\textsuperscript{33} +                                      | Hoffmann et al.\textsuperscript{36} – |
|                                        | Crollen et al.\textsuperscript{34} +                                   |                            |
|                                        | Crollen and Noel\textsuperscript{35}                                    |                            |
| Typical math skills level              | Schneider et al.\textsuperscript{37}                                    | Kramer et al.\textsuperscript{41} – |
|                                        | Hoffmann et al.\textsuperscript{38} +                                  | Dehaene et al.\textsuperscript{20} |
|                                        | Gibson and Maurea\textsuperscript{39}                                   | Fischer and Rottmann\textsuperscript{42} |
|                                        | Georges et al.\textsuperscript{40} +                                   | Bonato et al.\textsuperscript{43} |
|                                        |                                                                           | Bull et al.\textsuperscript{44} |
|                                        |                                                                         | Göbel et al.\textsuperscript{45} |
|                                        |                                                                         | Cipora et al.\textsuperscript{32} |
|                                        |                                                                         | Toomarian et al.\textsuperscript{46} |
| Special groups—high math skills level  | He et al.\textsuperscript{47}                                           | Cipora et al.\textsuperscript{48} – |

Note: Studies considering individuals with very low, typical, and very high math skills level are presented separately. Note that brain-lesioned patients or individuals with genetic disorders are not considered. Studies with significant effects are in bold font. Among these, studies in which a stronger SNARC effect was associated with better math skills are marked with +, and studies in which a stronger SNARC effect was related to poor math skills are marked with –.

An immediate answer for whether a relation exists, as different SNA types differentially relate to math skills. In the case of several SNAs, more pronounced or more precise spatial representation/better SNA task performance relates to better math skills. This is the case, for instance, for bounded number line estimation (a participant is shown a line, the endpoints of which are marked with numbers, for example, 0 and 100, and the task is to mark where a certain number (e.g., 67) would be located on this line).\textsuperscript{31} Individuals who perform the task more accurately are also better at math than those who perform poorly.\textsuperscript{29} However, the direction of causality has been questioned.\textsuperscript{30} The relations between other SNA types and math skills are more complex (i.e., might be modulated by cognitive control processes\textsuperscript{31}) and sometimes even the existence of these relations is under debate.\textsuperscript{3,28,32} Moreover, in several cases, such relationships have not been investigated at all.\textsuperscript{3} Such a complex and nuanced pattern of connections between SNAs and math skills is well exemplified in the case of the SNARC effect, which we discuss next.

The SNARC effect and math skills level—what we learned in the last quarter of the century

The SNARC effect, as a robust and easily replicable phenomenon, is a likely SNA correlate of math skills. The effect is implicit and taps into automatic, task-irrelevant number magnitude processing. It does not require participants to use any sort of declarative knowledge other than familiarity with number parity status. Therefore, finding a relationship between the SNARC effect and math skills (either positive or negative) would be theoretically sound. Owing to its implicit nature, one could not argue that the relationship between SNAs and math skills is, in fact, a relationship between one type of math skill and another. Such an argument is often raised in the case of bounded number line estimation.\textsuperscript{3} Up to now, several studies have investigated whether the SNARC effect is related to math skills. We summarize these studies in Table 1. Unfortunately, the pattern of results is not very straightforward. Three conclusions can be drawn from this overview of the literature: (1) the majority of studies did not find a significant relationship between math skills and the SNARC effect; (2) studies that have found a relationship have tested groups characterized either by very low or very high-level skills in math; and (3) most intriguingly, the direction of the observed relationship was inconsistent; in particular, it differed between children and adults.\textsuperscript{16} For children, a better level of math skills was related to a stronger SNARC effect, whereas for adults better math skills were related to a weaker SNARC effect.
Additionally, a more detailed look at the literature shows that math skills were operationalized in various ways: group membership, performance on (timed) arithmetic tests, or math grades. Some studies utilized more than one way of quantifying math skills. Consequently, depending on whether continuous or categorical variables were used to quantify math skills, different statistical approaches were used: group comparison and correlation/regression models. However, discrepancies in results cannot be accounted for by differences either in operationalization of math skills or in statistical models used to analyze the data. There is no systematic pattern that studies operationalizing math skills in a certain way are more likely to detect the relation between the SNARC effect and math skills. Moreover, in several studies using more than one method to quantify math skills, conclusions were consistent across all methods. It is worth noting that even if effects were observed, the corresponding effect sizes were small, and only one correlation in the literature exceeded 0.30.

In the following, we provide a tentative answer to the more fundamental question: why should the SNARC effect be related to math skills? This will help us understand if the appealing theory that posits SNAs as building blocks for more advanced math skills has a solid foundation. Our new framework also aims to explain the divergent patterns of results one finds in the literature.

The outline of the model

We propose a model for how math skills may be related to the SNARC effect. In our proposed model, we try to trace processes involved in the formation of the SNARC effect (as it is indexed by the behavioral observation of RT differences depending on the response side and number magnitude). We emphasize that multiple mechanisms may be involved, the influence of which may be in opposite

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**Figure 1.** Postulated mechanisms affecting the observed SNARC effect. Several may be either directly or indirectly related to math skills. The + and − signs indicate that a certain factor should be related to a stronger/weaker SNARC effect, respectively. The +/- sign, the dashed frame, and the dashed arrows indicate that the given influence of a particular mechanism and the direction of such an influence has not been fully determined. The − sign indicates that existing evidence shows that the more efficient processes should be related to a weaker SNARC effect, but more evidence is needed to support this claim. Please note that the strength of the observed SNARC effect is equivalent neither to the strength of automatic semantic processing of numbers (i.e., it must be associated with space) nor to the strength of the association between numbers and space, because factors related to handling interference are also at play.
directions (Fig. 1). First, for the SNARC to appear, the meaning (magnitude or ordinality) of the number needs to be processed automatically. Second, this information needs to be associated with space, and this process of associating is modulated by several factors. If and how this association is manifested in the RT pattern depends on inhibition and control of interference and information-processing efficiency. All of these mechanisms are discussed in the following sections.

Automatic semantic processing of numbers

The SNARC effect is typically considered to be a signature of semantic number processing. For the SNARC effect to appear, particularly in a parity task, it is necessary that magnitude (or, according to some authors, the number’s ordinality) information is processed automatically, as it is task-irrelevant. High automaticity of extracting semantic information about numbers may be a characteristic of individuals skilled in math. Moreover, differences between typically developing and dyscalculic children regarding automaticity of number processing can be observed even in very basic tasks, such as two-digit number comparison, unit-decade compatibility, or subitizing. For this reason, aside from being necessary for the SNARC effect to occur, automatic semantic processing of numbers may likely be a building block for more complex math skills. The semantic processing in this case would refer to processing of the most important information conveyed by numbers: their magnitude (i.e., cardinality) and/or position in an ordered sequence (i.e., ordinality). In a typical SNARC effect setup, these two types of information are usually highly correlated, and it is difficult to distinguish between them. However, which attribute—cardinality or ordinality—is (mainly) responsible for the SNARC effect in a given setting is currently under intense debate.

Consequently, if an individual lacks automatic number processing, he or she either has a weaker SNARC effect or does not exhibit it at all. However, this does not preclude that such an individual would be able to voluntarily recall the meaning of the number if he or she was required to. We can consider this phenomenon in analogy to the Stroop effect, which gets stronger with increasing proficiency in reading. For instance, if the names of colors are written in a language unknown to the participant (or the participant is illiterate) in a nonmatching colored font, there is no semantic processing and no interference with the font color. This is in contrast to the case of a native speaker (or literate participant) who experiences highly automated reading. Likewise, individuals skilled in math (who process magnitude more automatically) should be characterized by a stronger SNARC effect. However, one could hardly expect that an increase in automatic semantic number processing would imply a linear increase in the strength of the SNARC effect (factors discussed in the following sections). We rather postulate that a certain threshold for automatic processing needs to be reached. This should be the case for most educated adults in industrialized societies who are fluent in semantic processing of (single digit) numbers. Therefore, automated processing may have reached such a level that differentiation of the SNARC effect between individuals depending on math expertise is not possible even though they might still differ in respect to automatic semantic number processing itself. In the terms of our Stroop analogy, literature students and students of other disciplines display a similar Stroop effect because the reading proficiency of educated adults is at a similar level, even though it is possible to observe differences in reading speed or other measures of reading proficiency between these groups.

Importantly, this does not have to be the case for children, who might differ considerably in automatic semantic number processing, with some above or below the necessary threshold. Therefore, the relationship between the SNARC effect and math skills in children can be at least partly accounted for by differences in automatic access to number magnitude (ordinality) information. Children, who process numbers more automatically, might display a stronger SNARC effect.

Here, the Stroop–SNARC analogy ends, because the SNARC effect is not only about automatic semantic processing of numbers. The numerical information (magnitude/ordinality) needs to be further associated with space. What is more, several factors can influence this association even in individuals who have reached the necessary threshold of automatic semantic processing.

Given the importance of automatic semantic number processing, one may postulate that it accounts for some relationships between other types of SNAs and the level of math skills. This
Spatial association of magnitude information

The SNARC effect reflects the association of directions in space and number magnitude (ordinality), which goes beyond this semantic processing itself. Therefore, for the SNARC effect to appear, there must not only be automatic semantic number processing, but the magnitude (ordinality) information must also evoke a spatial association. There is some evidence showing that semantic number processing can be dissociated from spatial associations. The spatial association of number magnitude seems to be at the core of SNA and may likely be another interesting building block for the development of math skills. We identified several mechanisms, which were shown to influence spatial association: (1) selective attention, (2) abstractness and/or flexibility of the representation, (3) multiple codes underlying SNAs, (4) embodied influences, and (5) spatial skills. Some of them are related to math skills and some are inherent to task requirements. These factors might be relevant for other SNA types as well; we postulate that they have an influence on how numerical magnitude (ordinality) is associated with space, irrespective of the specific task being used to quantify SNA.

Selective attention. Numerical magnitudes can be semantically processed without the presence of spatial associations. It seems that selective attention oriented toward numbers allows/facilitates the process of associating them with space. This observation not only stems from typical SNARC bi-manual setups. A similar conclusion can also be drawn from multiple studies aimed at replicating the attentional SNARC effect. Specifically, these (failed) replications showed that perceiving numbers alone does not cause involuntary spatial shifts of attention. At the same time, when processing of numerical primes was enforced, the spatial effects (i.e., facilitation of detection times of left-/right-sided targets following small/large digit primes, respectively) were more likely to be observed. This mechanism seems to be more dependent on specific task requirements rather than on the math proficiency of the participants. In summary, selective attention is a necessary, or at least facilitating, factor for the emergence of the SNARC effect (and potentially other SNAs).

Abstractness and/or flexibility of representation. Higher levels of math proficiency are related to greater flexibility in the use of available knowledge structures. Mathematical concepts are highly abstract, and according to some authors, they are mostly based on the linguistic code; however, others propose a nonlinguistic nature. At the same time, language can enhance core mathematical representations. Moreover, expertise in every domain is characterized by higher abstraction of the available knowledge.

The role of a certain amount of flexibility becomes apparent even in the early acquisition of math skills: children need to learn that when counting objects, the order of counting is irrelevant (counting five tokens leads to the same result no matter where one starts counting, given that each token is counted once and only once). Flexibility is also important for high-level mathematics. Indeed, professional mathematicians employed more varied estimation strategies as compared with professional accountants (also skillful in arithmetic) and nonmathematician controls.

However, flexibility might not be the only skill that is stronger in individuals highly proficient in math. Such individuals are also characterized as having more abstract spatial associations. Importantly, mathematicians conduct mathematical proofs and typically handle very abstract symbols, variables, and constants. All these operations are far from simple (or even complex) arithmetic. Such intense training might facilitate abstractness and flexibility of number representations when using them within the requirements of a task. Such flexibility in the SNARC effect may build an SNA when needed, suppress it when not, or even build SNAs in different directions depending on task requirements.

As we know from the previous research, the SNARC effect (as well as other SNAs) is prone to situational influences. It is then possible that situational fluctuations are stronger in individuals whose representations are more flexible (mathematicians may be among them). These individuals may also be more flexible in adapting to different task demands or characteristics.
Crucially, in parity judgment, the spatial association is task irrelevant and efficient task performance requires its suppression. This holds true even if we consider the MARC effect (i.e., the markedness of response codes:73 faster left-/right-hand responses to odd and even numbers, respectively; see the section on multiple codes below). As long as we keep two blocks with reverse response-to-key assignment in the task, such an association is at best redundant for optimal performance in parity judgment. Therefore, individuals who are flexible and able to adapt to an ongoing task should not bother themselves with task-irrelevant spatial associations. Importantly, a similar mechanism may play a role for other SNAs (see the discussion below on interference effects).

To summarize, if professional mathematicians, because of their more flexible and more abstract processing, are indeed better at suppressing irrelevant SNAs, math proficiency could be related to a weaker or absent SNARC effect.16

Multiple codes. It has been postulated that the way in which numbers are associated with space can depend on (at least) five codes:74

1. The visuospatial simulation for numbers would be the mental number line, but simulations can be much more general, as other ordinal sequences can be also represented in a visuospatial manner.

2. Sequential working memory refers to the fact that the SNARC effect follows the sequential order in which discrete stimuli are stored in working memory.19 Although any sequence could be memorized75 for a given numerical stimulus set, it is assumed that the order in working memory follows the ascending number order in long-term memory if not instructed otherwise.

3. Linguistic markedness, a concept borrowed from linguistics, assumes that any opposing adjective pair can be categorized as either being a default base, unmarked form (e.g., “efficient”), or a derived, marked form (e.g., “inefficient”). If stimulus and response are congruent (i.e., both marked and unmarked), responses are faster than in an incongruent condition.73 In regard to SNAs, large and left are both marked. Consequently, linguistic markedness also predicts the SNARC effect.

4. If spatial–verbal instructions are inconsistent with the spatial response positions, (e.g., a right-hand key labeled as “left” and a left-hand key as “right”), the SNARC effect is reversed.76

5. Finally, the SNARC effect can also be influenced by other nonverbal processes.77 For instance, scanning and motor behavior, which proceed from one direction to another, can also trigger the SNARC effect.78,79

It remains unclear if and how strengthening/suppressing any of these codes may be related to math skills. This area requires further investigation to better understand the codes underlying the SNARC effect and the relative advantages of these codes related to math skills.

Embodied influences. Embodied processing refers to a general claim that bodily experiences, such as specific movements or sensations, are important part of mental, seemingly abstract processing.80 The association of numerical magnitudes with directions in space can be modulated by these embodied influences, such as finger-counting routines.22,81 Better finger gnosia is related to better math skills:82,83 children characterized by a greater ability to recognize and discriminate their fingers (on the basis of tactile information) perform better in math. On the other hand, the finger counting direction has been linked to the SNARC effect—individuals starting with the left hand have a stronger SNARC effect than those starting with the right hand.22,81 Importantly, no link between finger gnosia and the preferred finger counting direction has been reported. Nevertheless, the role of finger representation, and also the stability of finger counting routines,84 should be addressed in future studies on the links between the SNARC effect (and other SNAs) and math skills. We know that embodied training of number line estimation26,27 is beneficial for math skills, but it still remains to be examined whether the relation of embodied influences holds for implicit directional SNAs as well, that is, if left-to-right embodied training would really lead to greater improvements in math skills than right-to-left embodied training.
Spatial skills. Spatial skills are known to relate to math skills. For instance, spatial skills mediate the effect of superior performance in the number line estimation task by professional mathematicians and place-value integration seems to work better for subjects with better mental rotation skills. Spatial skills might also be related to the SNARC effect: a less pronounced SNARC effect is observed in people who reveal a weaker effect of mental rotation (i.e., their performance is less affected by the degree of rotation) of three-dimensional objects. Moreover, children with visuospatial disabilities were found to not show the SNARC effect. Please note that again the direction of the effect differs between adults and children. In this context, as was already postulated, geometry as a method of quantifying space in numerical terms and vice versa seems to be a likely correlate of the SNARC effect.

Spatial skills that facilitate associations of numerical magnitudes with space may apply to other SNA types, and might also explain the relationship between other types of SNAs with math skills (see the example of spatial skills mediating the relation between math skills and bounded number line estimation mentioned above).

Factors influencing the relation between the spatial mapping of the number representation and its behavioral signature

Inhibition and control of interference. Crucially, the construct of spatial association of numerical magnitude is not equivalent to its behavioral manifestation (the SNARC effect or any other SNA we measure). The SNARC effect, similar to some other SNA types, is an interference effect; the spatial location of response keys is task irrelevant. Therefore, irrespective of tendencies to spatially map number representations onto space (and the role of selective attention in this process, which we described above), domain-general cognitive processes should influence task performance via decreasing interference. This is also in line with our reading of the literature, which suggests that other domain-general cognitive processes beyond those mentioned above indeed attenuate the behavioral signatures of spatial association of numerical magnitude. These processes might play a role not only for the SNARC effect but also in other SNAs whose behavioral manifestation contains an interference component.

First, inhibition seems to be at play: it is important to remember that the spatial location of the response key is task irrelevant for the SNARC effect. If the association of number magnitude (ordinality) with response side were fully inhibited, no SNARC effect would appear. Accordingly, it was demonstrated that individuals characterized by a greater capacity to inhibit irrelevant semantic information have a weaker SNARC effect.

Second, cognitive control mechanisms, including the attentional control system, can be disturbed by anxiety; anxiety has been claimed to impair the attentional mechanism of attenuating distraction. This mechanism may explain why individuals characterized by higher math anxiety also have a stronger SNARC effect—anxiety makes them more prone to interference related to the task-irrelevant response side.

Information processing efficiency. Several studies show that more efficient information processing is related to a weaker SNARC effect. Specifically, participants who respond faster and whose RTs are less variable have a weaker SNARC effect. Information processing efficiency.
least part of the correlation can be explained by the fact that some subjects may be faster/slower at both tasks. Such a mechanism might also be relevant for other SNA instances.

The factors we discussed in this section may be relevant for the relation between the SNARC effect and math skills as they too are directly or indirectly related to math skills. However, these relationships might be even more complex and interactive; for instance, individuals characterized by high levels of math anxiety experience problems with attentional control (a domain-general cognitive process). At the same time, math anxiety is also related to the level of math skills. Therefore, just looking at the unique variance of single predictors may not be sufficient, as complex interactions between these predictors are possible, and indeed, based on the literature, even likely. To sum up, according to the model, the relationship between math skills and the SNARC effect (if it exists) does not need to be direct, as it may also be (partially) modulated by several domain-general processes.

**Causal relations**

First, we need to specify that we do not think the SNARC effect itself is a building block for math skills. What is more plausible is that several mechanisms of elementary number processing driving the SNARC effect, such as automatic magnitude processing or associating magnitude with space, may be building blocks for more complex math skills. Thus, the SNARC effect would be another implicit, involuntary, and maybe unbiased, outcome of these building blocks.

Coming back to the compelling theory of elementary number processing as building blocks for complex math skills, we wish to emphasize that its appeal remains unchanged and it may be true for some SNAs but not for the SNARC effect. The literature review and theoretical analysis we present here shows that the SNARC effect is not a good measure of such building blocks. This is because it is neither equivalent to the extent of automatic semantic number (magnitude/ordinality) processing nor to the spatial association of numerical magnitude. Rather, it is a combination of these, and other processes, which are responsible for resolving interference.

The most problematic aspect of the SNARC effect, as a result of being composed of automatic semantic processing, associating numbers with space, inhibition processes, and information processing efficiency, is that the direction of the postulated relationship between each of these processes and math skills differs. On the one hand, the strength of automatic semantic number (magnitude/ordinality) processing should positively correlate with math skills. On the other hand, high-level math skills should be related to factors attenuating the SNARC effect, such as higher flexibility or abstractness of representation, more efficient cognitive control processes, or better inhibition capabilities.

A crucial element of the building blocks theory relates to the direction of causal relations. The building blocks theory assumes that elementary number processing influences the development of math skills. So, elementary processes are the cause and math skills are the effect. Nevertheless, such a claim is also problematic. Automaticity of symbolic semantic number processing is not innate. First, an individual needs to learn the symbols and then automaticity comes with practice. It may be that some individuals are more receptive to this type of training, but it is true that the amount of training shapes automaticity. The same holds true for flexibility/abstractness of representation: there might be some individual differences, but long-term practice can also shape flexibility/abstractness. To sum up, when thinking of the relation between elementary number processing and math skills, one must not only consider the building blocks approach, but also feedback loops and the role of domain-general processes that make some individuals more receptive to training than others.

**What can we conclude from interference effects like the SNARC effect?**

As we discussed above, the SNARC effect is to some extent an interference effect. It is appealing to look at the effect as an implicit measure of information processing. However, one must keep in mind that interpreting the strong interference effect with reference to more efficient processing can be problematic. This is because two types of processes can lead to a weaker SNARC effect: (1) a somewhat weaker automatic activation of magnitude (ordinality) and its relation with space, and (2) a somewhat better interference control when the spatial–magnitude association in the task is incongruent with the mental association. Generally, it is
assumed that better automatic activation of magnitude (ordinality) and using space are beneficial to mathematical skills (as a domain-specific process), but better inhibition and interference control is also useful for mathematical skills (as a domain-general effect). Therefore, both a stronger and a weaker SNARC effect can, in principle, be related to better math skills, depending on which type of process is more salient for a given task in a given paradigm at a given age or within a given skill group. Importantly, this relationship may change with age and learning experience. During new skill acquisition (e.g., reading skills in a new language), interference caused by new knowledge (e.g., increased time needed to identify font colors of color names in this new language) would increase. However, there is also evidence that the interference does not increase linearly with increasing proficiency. Instead, the relationship has an inverted U-shape.\(^{100}\) From a certain point of new skill acquisition, increasing automaticity relates to an increase in the efficiency of cognitive control processes.\(^{101}\) Such an effect in the domain of numerical cognition was observed for the developmental trajectory of the size congruity effect we mentioned before.\(^{10}\) Developmentally, this interference effect first increases with increasing automaticity of number processing but then decreases with age and increasing efficiency of executive functions.

In general, interference effects tap suboptimal information processing. Some aspects of the stimuli are processed despite being irrelevant or even disruptive to the ongoing task. We would expect strong resistance to irrelevant information from a cognitive system that is performing optimally. Therefore, when we consider stronger interference as a measure of greater skill, we must keep in mind that greater skill is also expected from individuals who can fully focus on the ongoing task and not allow the processing of irrelevant information. For this reason, treating stronger interference as a potential correlate of better performance is at best problematic. This also holds true for other SNAs constituting an interference component.

The role of automatic number processing (as contrasted with the role of controlled processing) in formation of specific SNA types has not been directly considered in the SNA taxonomy. On the basis of the current review of literature, it seems that in case of spatial extensions, the automatic processing of magnitude is inherently spatially associated: it is hard if at all possible to mitigate, for example, a size congruity effect. At the same time (as we show in the model) for directional SNAs, the magnitude processing does not have to evoke associations with directions in space.

For now, it seems to us that for most of the implicit SNAs, we are tapping automatic processing, while the role of controlled processing seems rather limited. Moreover, automatic processing seems to play a crucial role in extension approximate SNAs (like the size congruity effect). On the other hand, for extension exact SNAs, both automatic and controlled processing play a role (i.e., it requires some controlled processing to mark a response in number line estimation). In case of explicit SNAs, the role of automatic and controlled processing is more unclear at the moment. Finally, we postulate that the larger the role of automatic processing, the larger the potential interference component in a given SNA. However, this aspect needs to be further elaborated in future studies.

**Conclusions**

In this article, we tried to go beyond the discussion about whether the SNARC effect correlates with math skills and tried to address the more basic question of why these two should be related. The answer to this question is not straightforward because different aspects of the relationship may imply correlations in different directions, the direction of causation is unclear, and the presence of potential suppressor variables is a possibility. Our model provides a tentative explanation for the puzzling results presented in the literature, such as numerous null results and conflicting data on the SNARC effect/math skills relationship in children and adults. When one considers the model, it is easy to propose that children may have different pathways (differences in automatic magnitude/ordinality processing) than adults (representation flexibility and inhibition of interference). As these factors might act independently and in opposite directions, they may cancel each other out, which would explain the null results both for children and adults. We wish to emphasize that the model is a theoretical proposal, and its goal is to stimulate and guide future investigations in regard to links between the SNARC effect (and other SNA types) and math skills. These investigations need to take into account that the SNARC effect is neither equivalent to automatic magnitude
(ordinality) processing nor to spatial associations of magnitude processing. It is also important to keep in mind that the SNARC effect is about directions in space, which is culture-sensitive, and not about spatial extensions, which are more closely related to math skills.

Another aspect that needs to be taken into account in future studies is the proposed theoretical model of a causal relation between the SNARC effect and math skills. It should consider elementary cognitive processes, both domain-general and domain-specific, which underlie both the SNARC effect and math skills. Finally, such investigations need to be a mixture of cognitive and differential psychology, because even such elementary processes as basic SNAs seem not only to be influenced by age, experience, culture, and more general cognitive abilities, but also by the learning of math and its practice.

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Author contributions

All authors developed the general idea, which was then refined and specified by K.C., who wrote the first draft of the manuscript. H.-C.N. and Y.H. commented on the manuscript, and all authors approved its final version.

Competing interests

The authors declare no competing interests.

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