Analysis of bending vibration characteristics of rotating composite boring bar

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Abstract. In this study, the dynamic characteristics of composite boring bar are theoretically investigated. The structural dynamic model of composite boring bar having clamped-free supports is developed by employing Bernoulli-Euler beam theory and considering dissipative characteristics of viscoelastic composite material. The Galerkin method is used to solve the equations of bending vibration. The complex characteristic equation for bending vibration is derived. Numerical examples present the variation of the natural frequency, damping and dynamic stiffness with ply angle, and the variation of the natural frequency and damping with rotating speed are also obtained. The study results show that ply angle, stacking sequences and ratio of length over outer radius have significant influence on the dynamic characteristics of composite boring bar. Conclusively, lamination design can significantly improve dynamic performance and enhance chatter stability in machining.

1. Introduction

Boring is a precision manufacturing technology that is widely applied in hole processing with complicated structures. Since the diameter of boring bar is affected by hole diameter and depth, the boring bar with cutting cool can only adopt slender cantilever structure, which can result in poor stiffness of the boring bar, thereby easily leading to deformation and chatter. In order to effectively reduce chatter, cutting depth and feed rate should be reduced, which can greatly affect the cutting efficiency. Chatter is a kind of self-excited vibration, which can be classified into two types during machining process, namely, mode coupled chatter and regenerative chatter. The latter, induced by the change of cutting thickness because of the phase difference between chatter mark produced in the last cutting and the vibration displacement in this cutting, can impose the greatest damages in cutting process. The regenerative chatter starts quickly and the vibration amplitude exhibits exponential increases, accompanied by both cutting depth and cutting forces. Great dynamic cutting force may produce poor-quality processing surface, leading to the fracture of boring bar or the damages on spindle bearings.

Previous researches demonstrate that limit cutting depth is proportion to the boring bar’s dynamic stiffness [1]. Therefore, in order to suppress the chatter of boring bar and enhancing cutting stability, scholars mainly adopted passive control methods based on various dynamic vibration absorbers and impact dampers [2]. These chatter suppression methods are proved to be effective and need the increase of additional systems. Moreover, the parameters of vibration absorbers are strongly related to the machine tool’s technological state and easily restricted by the installation conditions of the cutting system. In fact, changing the boring bar’s structural dynamic characteristics from the selection of...
boring bar structure is an effective path that is worth exploring. It is well known that composite materials are featured by high static stiffness, damping and specific stiffness. Based on finite-element analysis, Nagano et al. developed the analysis model of composite boring bars composed of steel core embedded pith-based carbon fiber reinforced plastic and examined the shape of steel core on stiffness and natural frequency. Results show that the length-to-diameter ratio of the composite boring bar with no chatter is approximately twice of that of metal boring bar [3]. Lee et al. employed vibrating experiment for examining the dynamic characteristics of the composite boring bar consisting of damping core, carbon fiber epoxy composite main shaft and steel cover, and found that the dynamic stiffness and the cutting capability of the prepared composite boring bar approximately exceed traditional boring bar made up of tungsten carbide by approximately 30% and 33%, respectively [4].

For a cutting system, stability analysis includes structural dynamic modeling, the modelling of cutting force as well as their relationship. Currently, scholars mainly adopted one degree of freedom (DOF) or multi-DOF lumped mass model [5-8], finite element model [3-4, 9-10] and distributed parameter model [11-12] for structural modeling of boring cutting system. However, above distributed parameter models are only applicable to isotropic metal boring bars. The structural dynamic characteristics of the distributed parameters of composite boring bars have been poorly investigated to date. Ren Y S et al. established the stability analysis model of the rotor system with rotating composites, in which the composite shaft follows simply-supported boundary conditions and the effects of rotary inertia, gyroscopic effect and the damping characteristics of composites were taken into account [13]. This model can be used for analyzing vibration and stability of simply-supported shaft rotor system with rotating damping.

By applying the model as described in Ref. [13] to structural dynamic modeling and dynamic characteristics of composite boring bar, this study derives motion equations based on Hamilton principle. The bending vibration equation is then solved using Galerkin method. Next, through numerical calculation, the dynamic stiffness characteristics of composite boring bar are examined, while the effects of rotating speed on natural frequency and damping ratio and the effects of fiber ply angle, lamination pattern and length-to-diameter ratio on dynamic stiffness, critical speed and instability threshold are analyzed. Finally, model effectiveness is validated by contrast with the results reported in previous literatures.

2. Motion equation and solution

![Figure 1. Schematic of the rotating composite boring bars.](image)

Based on Hamilton principle, the vibration differential equation of the rotating composite boring bar as shown in Figure 1 was formulated as:

$$\int_0^T (\delta U + \delta W - \delta T) dt = 0.$$  \hspace{1cm} (1)

where $\delta T$ and $\delta U$ denote the variation of kinetic energy and potential energy, respectively, and $\delta W$ denotes the dissipative strain energy.
The functional of the composite boring bar can be written as:
\[ T = \int_0^L \left[ m \left( \dot{u}_x^2 + \dot{u}_y^2 + \dot{u}_z^2 \right) + I \left( \dot{\gamma}_x + \dot{\gamma}_y \right) - 4I \Omega \dot{\gamma}_y + 2I \left( \dot{\phi} + \Omega \right)^2 \right] dx \]  \tag{2}
where \( m \) and \( I \) denote the mass and the inertial moment per unit length. \( m \) and \( I \) can be written as:
\[ m = \pi \sum_{k=1}^N \rho_k \left( r_{k+1}^2 - r_k^2 \right) \]
\[ I = \frac{\pi}{4} \sum_{k=1}^N \rho_k \left( r_{k+1}^4 - r_k^4 \right) \]  \tag{3}
where \( \cdot \) denote the differentiate with respect to time (\( t \)), and \( \rho_k \) denotes the density of single-layer composite material.

The variation of the potential energy of the composite boring bar can be written as:
\[ \delta U = \frac{1}{2} \int_V \left( \sigma_x \delta \varepsilon_x + \tau_{xa} \delta \gamma_{xa} \right) dV \]  \tag{4}

Stress and strain should satisfy the following relation:
\[ \sigma_x = \overline{Q}_{11} \varepsilon_x + \overline{Q}_{16} \gamma_{xa}, \]
\[ \tau_{xa} = \overline{Q}_{16} \varepsilon_x + \overline{Q}_{66} \gamma_{xa} \]  \tag{5}
where \( \sigma_x \) and \( \tau_{xa} \) denote normal stress and shear stress on the boring bar; \( \varepsilon_x \) and \( \gamma_{xa} \) denote normal strain and shear strain of the boring bar; \( x, r, \alpha \) are cylindrical coordinates; \( \overline{Q}_{ij} (i, j = 1, 6) \) denotes the off-axis stiffness coefficient of single-layer composite material.

The dissipative strain energy of the composite boring bar can be written as:
\[ \delta W = \int_V \left( \sigma_x^d \delta \varepsilon_x + \tau_{xa}^d \delta \gamma_{xa} \right) dV \]  \tag{6}

Where \( \sigma_x^d \) and \( \sigma_x^d \) denote dissipative stresses. Both satisfy the following relation:
\[ \sigma_x^d = \overline{Q}_{11}^d \varepsilon_x + \overline{Q}_{16}^d \gamma_{xa}, \]
\[ \tau_{xa}^d = \overline{Q}_{16}^d \varepsilon_x + \overline{Q}_{66}^d \gamma_{xa} \]  \tag{7}

Based on Bernoulli-Euler beam theory, strain-displacement relation can be written as:
\[ \varepsilon_x = \frac{\partial u_x}{\partial x} - k_w r \cos \alpha + k_w' r \sin \alpha, \]
\[ \gamma_{xa} = \frac{\partial \phi}{\partial x} \]  \tag{8}
where \( u_x \) denotes the displacement of neutral axis along x-axis direction, \( k_w \) and \( k_w' \) denote the curvatures neutral axis along z-axis and y-axis direction, and \( \phi \) denotes the torsion angle of the cross-section along x-axis direction.

By substituting equation (5) and equation (8) into equation (4), equation (7) and equation (8) into equation (6), and the variation of equation (2) into equation (1), the bending-bending coupling vibration differential equation of rotating composite boring bar can be derived as:
\[ m \ddot{u}_x - I \frac{\partial^2 \dot{u}_x}{\partial x^2} - 2I \Omega \frac{\partial^2 \dot{u}_y}{\partial x^2} + D_{11} \frac{\partial^4 u_x}{\partial x^4} + D_{11}^d \frac{\partial^4 u_x}{\partial x^4} + D_{11} \Omega \frac{\partial^4 u_x}{\partial x^4} = 0, \]
\[ m \ddot{u}_y - I \frac{\partial^2 \dot{u}_y}{\partial x^2} + 2I \Omega \frac{\partial^2 \dot{u}_y}{\partial x^2} + D_{11} \frac{\partial^4 u_y}{\partial x^4} + D_{11}^d \frac{\partial^4 u_y}{\partial x^4} - D_{11} \Omega \frac{\partial^4 u_y}{\partial x^4} = 0, \]  \tag{9}
where \( D_{11} \) and \( D_{11}^d \) are bending stiffness coefficient and damping coefficient, respectively. As regard to the definitions of \( D_{11} \) and \( D_{11}^d \), the details can be seen in Ref. [14], which will not be repeated in this study.
Let $\bar{u}_y = \frac{u_y}{L}, \bar{u}_z = \frac{u_z}{L}, \bar{x} = \frac{x}{L}, \bar{y} = \int \frac{D_{11} t}{mL^4} = \gamma t, \bar{\Omega} = \frac{\Omega}{\gamma}, \bar{I} = \frac{I}{mL^2}, \bar{\varphi} = \frac{D_{11}^{eff}}{D_{11}}$, by introducing the complex variable $U = \bar{u}_y + i\bar{u}_z$ (in which $i = \sqrt{-1}$), the vibration differential equation in plural form can be described as:

$$\frac{\partial^2 U}{\partial t^2} - \bar{I} \frac{\partial^4 U}{\partial x^4} + 2i\bar{\Omega} \frac{\partial^2 U}{\partial x^2} + \bar{\varphi} \frac{\partial^5 U}{\partial x^5} - \frac{\Omega}{\gamma} \frac{\partial^4 U}{\partial x^4} = 0.$$  \hspace{1cm} (10)

It is assumed that equation (10) satisfies the solution of clamped-free support conditions, i.e., one end is fixed and one end is free, and can be written as:

$$U = \exp(i\lambda x) \Phi_n(\bar{x})$$  \hspace{1cm} (11)

in which the mode shape function $\Phi_n(\bar{x})$ can be written as:

$$\Phi_n(\bar{x}) = \cos z_n \bar{x} - \cos h z_n \bar{x} - \frac{\cos z_n + \cos h z_n}{\sin z_n + \sin h z_n} (\sin z_n \bar{x} - \sin h z_n \bar{x})$$

$$\cos z_n ch z_n = -1$$  \hspace{1cm} (12)

$$n = 1, 2, 3, \ldots$$

In equation (11), $\lambda_n = \omega_n + id_n (n=1, 2, \ldots)$ is a complex characteristic value, whose real part ($\omega_n$) and imaginary part ($d_n$) represent natural circular frequency and modal damping, respectively. Obviously, if $d_n > 0$, the rotor system is stable; otherwise, the system is unstable.

By substituting equation (11) into equation (10) and solving equation (10) using Galerkin method, the following complex characteristic equation can be acquired:

$$(1 + n^2 \pi^2 \bar{I}) \lambda_n^2 - n^2 \pi^2 (2i\bar{\Omega} + i n^2 \pi^2 \bar{\varphi}) \lambda_n - n^4 \pi^4 (1 - i \bar{\Omega} \bar{\varphi}) = 0.$$  \hspace{1cm} (13)

### 3. Dynamic stiffness

The dynamic performance of the cutting system of the composite boring bar mainly depends on the first natural frequency and damping ratio. Previous research demonstrates that the maximum cutting depth corresponding to no occurrence of cutting in boring process is proportional to the system’s dynamic stiffness [15].

The dynamic stiffness of the composite boring bar can be written as:

$$D = \frac{3\zeta (EI)_{equiv}}{l^2}$$  \hspace{1cm} (14)

where $\zeta$, $(EI)_{equiv}$, and $l$ denote damping ratio, equivalent bending stiffness and length, respectively.

The first natural frequency of the composite boring bar, denoted as $\omega_1$, can be written as [16]:

$$\omega_1 = \frac{3.516}{l^2} \sqrt{(EI)_{equiv}/(\rho S)_{equiv}}$$  \hspace{1cm} (15)

where $(\rho S)_{equiv}$ denotes the equivalent mass per unit length.

By solving $(EI)_{equiv}$ from equation (15) and then substituting $(EI)_{equiv}$ into equation (14), the following expression can be acquired:

$$D = 0.243l \omega_1^2 \zeta (\rho S)_{equiv}$$  \hspace{1cm} (16)

### 4. Numerical analysis and results

#### 4.1. Model validation

Using the established model and calculation method, the numerical results of natural frequency and modal damping of the circular hollow composite cantilever beam can be solved, as listed in Table 1.
By comparison with the results as described in Ref. [17] via finite-element model analysis, it can be concluded that the calculated natural frequencies fit well with the results in Ref. [18] but the calculated modal damping values differ from previous results to certain degree. This may be due to the fact that no tension-bending coupling stiffness was not involved in the present bending vibration model.

**Table 1.** Calculated results of natural and mode damping of hollow circular-section composite cantilever beam.

| Lamination pattern | Mode            | Natural frequency (Hz) | Modal damping (%) |
|--------------------|-----------------|------------------------|-------------------|
|                    |                 | Ref. [17] | This study | Ref. [17] | This study |
| [0°]ₙ₁           | First flapping  | 3.2       | 3.33     | 0.66     | 1.08     |
|                   | First sweeping  | 3.2       | 3.33     | 0.66     | 1.08     |
| [90°]ₙ₁           | First flapping  | 1.9       | 1.92     | 2.35     | 2.28     |
|                   | First sweeping  | 1.9       | 1.92     | 2.35     | 2.28     |
| [0°/90°/45°/-45°]ₙ₁ | First flapping | 2.4       | 2.55     | 1.44     | 1.68     |
|                   | First sweeping  | 2.4       | 2.55     | 1.44     | 1.68     |
| [45°/-45°]ₙ₁     | First flapping  | 2.0       | 2.39     | 2.45     | 1.91     |
|                   | First sweeping  | 2.0       | 2.39     | 2.45     | 1.91     |

Table 2 lists the predicted results of critical rotating speed of rotating composite shaft using the established model as well as the calculated results in other literatures. The composite shaft is made up of boron fiber/epoxy laminated materials in the lamination pattern of (layer-up) [90° /45°/-45° /0° /90°], with a length of 2.47 m, a mean diameter of 12.69 cm and a mean thickness of 1.321 mm [19]. Results show that the present results are close to the calculated results based on equivalent modulus beam theory (EMBT) by taking into account tension-bending coupling effects.

**Table 2.** The critical speed of composite shaft rotor.

| Method                                      | Critical rotating speed (rpm) |
|---------------------------------------------|-------------------------------|
| Simplified homogenized beam theory (SHBT)   | 5767                          |
| Equivalent modulus beam theory (EMBT)       | 5780                          |
| Vibrating experiment                        | 5500                          |
| Donnell’s shell finite element              | 4942                          |
| Timoshenko beam finite element              | 5762                          |
| Layerwise Timoshenko beam theory (LBT)      | 5620                          |
| Equivalent modulus beam theory (EMBT)       | 5747                          |
| Present                                     | 5710                          |

4.2. Natural frequency, damping ratio and dynamic stiffness of fixed boring bar

Figure 2 displays the comparisons of natural frequency, damping ratio and dynamic stiffness between composite boring bar and metal boring bar, respectively, in which the composite boring bar has a mean radius of 0.176 m and a thickness of 0.01016 m and adopts the lamination pattern of [±θ]ₙ₁. Table 3 lists the mechanical parameters of the composites. The density, elastic modulus, Poisson’s ratio and damping ratio of steel are 7850 kg/m³, 210 GPa, 0.3 and 0.05, respectively. It should be noted that above fundamental parameters are used in this study unless special illustration. The length-to-diameter ratio of the boring bar is 40 and \( \bar{\Omega} = 0 \), i.e., the effect of the rotating speed is not taken into account.
Table 3. Mechanical properties of carbon/epoxy composite materials [18].

| \( \rho \) (kg/m\(^3\)) | \( E_{11} \) (GPa) | \( E_{22} \) (GPa) | \( G_{12} \) (GPa) | \( G_{23} \) (GPa) | \( \nu_{12} \) | \( \psi_1 \) (%) | \( \psi_2 \) (%) | \( \psi_4 \) (%) | \( \psi_5 = \psi_6 \) (%) |
|---|---|---|---|---|---|---|---|---|---|
| 1446.2 | 172.7 | 7.2 | 3.76 | 3.76 | 0.3 | 0.45 | 4.22 | 7.05 | 7.05 |

As shown in Figure 2(a), although the modulus of the composites in principal direction \( E_{11} \) is slightly lower than the elastic modulus of steel, the natural frequency of the composite boring bar exceeds that of steel boring bar at a ply angle of below 50° since the density of the composites is smaller than that of steel. It can also be observed that natural frequency increases with the decrease of ply angle. This is due to the fact that \( E_{11} \) is significantly larger than transverse elastic modulus \( E_{22} \) (as shown in Table 3). Therefore, at a smaller ply angle, the bar’s bending stiffness \( D_{11} \) is greater, thereby leading to greater natural frequency.

At a ply angle of 0°~90°, the damping ratio of the composite boring bar is larger than that of steel boring bar, as shown in Figure 2(b). It can also be observed from Figure 2(b) that the damping ratio increases with the increase of ply angle. Since the damping capacity along the fiber’s transverse direction far exceeds that along the vertical direction (see Table 3), a greater ply angle suggests that more fibers are laminated near the transverse direction and the damping along this direction is larger.

At a ply angle of 0°~90°, the dynamic stiffness of the composite boring bar is significantly larger than that of steel boring bar. In addition, the dynamic stiffness of the composite boring bar drops with the increasing ply angle, which is consistent with the variation tendency of the natural frequency with the ply angle.

![Figure 2](image-url)  
**Figure 2.** Comparison between composite boring bar and steel boring bar in dynamic characteristics: (a) Natural frequency; (b) Damping ratio; (c) Dynamic stiffness.

Figure 3 displays the comparison of dynamic performance among three composite boring bars with different length-to-diameter ratios, from which we can observe that the composite boring bar with greater length-to-diameter ratio as smaller natural frequency and damping ratio, thereby resulting in smaller dynamic stiffness. Therefore, the composite boring bar with greater length-to-diameter ratio exhibits poorer chatter stability and easily produces cutting chatter.

Tables 4~6 display the effects of the stacking sequence (SS) on the composite boring bar’s natural frequency (NF), damping ratio (DR), equivalent bending stiffness (EBS) and dynamic stiffness (DS). Results show that, if the number of layers with ply angle near the axial direction is greater, the composite boring bar exhibits larger bending stiffness and higher natural frequency, and if the number of layers with ply angle near the transverse direction is larger, the composite boring bar’s damping is larger. However, dynamic stiffness seems to depend on natural frequency. It can also be observed from Tables 4~6 that length-to-diameter ratio imposes similar effect on the dynamic properties of the composite boring bar, as shown in Figure 3.
The damping at a certain rotating speed is also referred to rotating speed, while these two factors drop with the increase of rotating speed in the lower branch corresponding to backward whirl. The critical rotating speed of the rotating boring bar is the horizontal coordinate of the intersection point of line $\omega = \Omega$ and the upper branch of the natural vs. rotating speed. The instability threshold of the rotating boring bar is the rotating speed in the lower branch of the damping vs. rotating speed when the damping ratio equals to 0. It should be noted that the damping at a certain rotating speed is also referred to rotating speed.

![Figure 3. Effect of length aspect ratio on the dynamic characteristics of composite boring bar: (a) natural frequency; (b) damping ratio; (c) dynamic stiffness.](image)

**Table 4.** Effect of stacking sequences on the dynamic characteristics of composite material boring bar (L/d=40).

| SS     | $[\pm 15^\circ]_{b3}$ | $[\pm 30^\circ]_{b3}$ | $[\pm 45^\circ]_{b3}$ | $[90^\circ / 0^\circ / 45^\circ / -45^\circ]_{b3}$ |
|--------|-----------------------|-----------------------|-----------------------|-----------------------------------------------|
| NF (Hz)| 3.3480                | 2.7350                | 1.9172                | 0.7272                                         |
| DR     | 0.8308                | 0.9124                | 1.0821                | 1.5490                                         |
| EBS ($10^3$N/m) | 26498                | 17693                | 8705.9                | 1258.7                                         |
| DS ($10^3$N/m) | 0.0425                | 0.0312                | 0.0182                | 0.0037                                         |

**Table 5.** Effect of stacking sequences on the dynamic characteristics of composite material boring bar (L/d=50).

| SS     | $[\pm 15^\circ]_{b3}$ | $[\pm 30^\circ]_{b3}$ | $[\pm 45^\circ]_{b3}$ | $[90^\circ / 0^\circ / 45^\circ / -45^\circ]_{b3}$ |
|--------|-----------------------|-----------------------|-----------------------|-----------------------------------------------|
| NF (Hz)| 2.1444                | 1.7521                | 1.2287                | 0.4667                                         |
| DR     | 0.5313                | 0.5834                | 0.6916                | 0.9886                                         |
| EBS ($10^3$N/m) | 26498                | 17693                | 8705.9                | 1258.7                                         |
| DS ($10^3$N/m) | 0.0112                | 0.0082                | 0.0048                | 9.8302e-04                                     |

**Table 6.** Effect of stacking sequences on the dynamic characteristics of composite material boring bar (L/d=60).

| SS     | $[\pm 15^\circ]_{b3}$ | $[\pm 30^\circ]_{b3}$ | $[\pm 45^\circ]_{b3}$ | $[90^\circ / 0^\circ / 45^\circ / -45^\circ]_{b3}$ |
|--------|-----------------------|-----------------------|-----------------------|-----------------------------------------------|
| NF (Hz)| 1.4896                | 1.2171                | 0.8537                | 0.3244                                         |
| DR     | 0.3688                | 0.4050                | 0.4800                | 0.6858                                         |
| EBS ($10^3$N/m) | 26498                | 17693                | 8705.9                | 1258.7                                         |
| DS ($10^3$N/m) | 0.0037                | 0.0027                | 0.0016                | 3.2953e-04                                     |

4.3. Effects of the rotating speed on natural frequency and damping ratio

Figure 4(a) and Figure 4(b) display the variations of the first natural frequency and damping ratio of steel and composite boring bars with a length-to-diameter ratio of 40 and a same stacking sequence of $[0^\circ]_{16}$. Results show that, if the boring bar rotates, the variation curves of natural frequency and damping ratio with the rotating speed exhibit bifurcation because of the existence of gyroscopic effect. To be specific, in the upper branches corresponding to forward whirl both natural frequency and damping ratio increase with the increase of rotating speed, while these two factors drop with the increase of rotating speed in the lower branch corresponding to backward whirl. The critical rotating speed of the rotating boring bar is the horizontal coordinate of the intersection point of line $\omega = \Omega$ and the upper branch of the natural vs. rotating speed. The instability threshold of the rotating boring bar is the rotating speed in the lower branch of the damping vs. rotating speed when the damping ratio equals to 0. It should be noted that the damping at a certain rotating speed is also referred to rotating speed.
damping or internal damping of the rotating system [18]. When non-rotating damping i.e., the damping ratio, the critical rotating speed is identical with the instability threshold. For example, as shown in Figure 4(a) and (b), the critical rotating speeds/instability thresholds of steel and composite boring bars with identical stacking sequence of $[0^\circ]_{16}$ are 109 rpm and 216 rpm, respectively.

Figure 4. Comparison between composite boring bar and steel boring bar in natural frequency vs. rotating speed and damping vs. rotating speed: (a) natural frequency; (b) damping ratio.

Figure 5 displays the variations of the first natural frequency and damping ratio with the rotating speed of the composite boring bars at different ply angles. As shown in Figure 5(a), at a rotating speed of 0, the natural frequency increases with the decreasing ply angle, as a result the critical rotating speed increases with the decreasing ply angle. This is because that $E_{11}$ is significantly larger than $E_{22}$ (as listed in Table 3). Therefore, as the ply angle drops, the boring bar’s bending stiffness increases, accompanied by the increase of natural frequency and critical rotating speed.

As shown in Figure 5(b), the instability threshold drops with the increase of ply angle. This is due to the fact that the instability threshold is inversely proportional to the internal damping of the composite. Since the fiber’s transverse damping capability far exceeds vertical damping capability (as listed in Table 3), the ply angle is greater (i.e., more fibers are laminated near the axis’s transverse direction), the internal damping is greater along this direction, and the instability threshold is smaller.

Figure 5. Natural frequency and damping versus rotating speed for different ply angle: (a) natural frequency; (b) damping ratio.

Figure 6(a) and Figure 6(b) display the effects of the length-to-diameter ratio of the composite boring bar with a stacking sequence of $[\pm 30^\circ]_6$ on the first natural frequency versus rotating speed and
first damping ratio versus rotating speed, respectively. Apparently, the critical rotating speed and the instability threshold of the composite boring bar drops with the increase of the length-to-diameter ratio, which is consistent with the conclusions in Ref. [18].

Figure 6. Natural frequency and damping versus rotating speed for different length aspect ratio: (a) natural frequency; (b) damping ratio.

5. Conclusions
Based on Hamilton principle and Bernoulli-Euler beam theory, a dynamic theoretical analysis model of the composite boring bar are is proposed, in which the composites damping is also taken into account. The proposed model can be used for revealing the composite boring bar’s natural frequency, damping ratio and dynamic stiffness as well as their variation rules with ply angle and length-to-diameter ratio. Moreover, the critical rotating speed and instability threshold of the rotating composite boring bar are calculated. According to the calculation results, the boring bar made up of composites exhibits more favorable dynamic performances than metal boring bar, which can thus greatly enhance the stability in cutting process. Main conclusions are described below.

(1) The ply angle, stacking sequence and the length-to-diameter ratio all can affect the natural frequency and the damping ratio of the composite boring bar, thereby affecting dynamic stiffness and cutting stability. Results demonstrate that, as the ply angle increases, the natural frequency of the composite boring bar drops, while the damping ratio and the dynamic stiffness increase.

(2) On account of gyroscopic effect, both the variation curves of the natural frequency and the damping ratio with the rotation speed for the rotating composite boring bar exhibit bifurcation, and both critical rotating speed and instability threshold drop with the increase of ply angle or length-to-diameter ratio.

(3) The effects of the rotating speed and the internal damping on the cutting stability of the composite boring bar are not discussed in this study. However, these problems should be further considered in investigating the chatter stability of composite boring bar, which will be the research focus in future studies.

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