On Fluctuations of Conserved Charges: Lattice Results Versus Hadron Resonance Gas

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Abstract. We compare recent lattice results on fluctuation and correlations of strangeness, baryon number and electric charge obtained with p4 improved staggered action with the prediction of hadron resonance gas model. We show that hadron resonance gas can describe these fluctuations reasonably well if the hadron properties are as calculated on the lattice.

1. Introduction
In recent years a lot of progress has been made in understanding the QCD transition at non-zero temperature through lattice calculations (see [1, 2] for reviews). In particular, fluctuations of conserved charges have been studied on the lattice and provided insight on how the relevant degrees of freedom change from hadronic to partonic [3, 4, 5, 6, 7, 8, 9, 10]. At low temperatures it is expected that hadrons are good degrees of freedom and thermodynamic quantities can be well described by hadron resonance gas.

Hadron resonance gas (HRG) turned out to be very successful in describing particle abundances produced in heavy ion collisions [11]. It was also used to estimate QCD transport coefficients [12] as well as chemical equilibration rates [13] close to the transition temperature. Thermodynamic quantities calculated in lattice QCD with rather large quark mass agree well with the HRG model if the masses of the hadrons in the model are tuned appropriately to match the large quark mass used in lattice calculations [14]. Furthermore, the ratios of certain fluctuations calculated on the lattice are also in reasonably good agreement with HRG model predictions at low temperatures [15, 16, 10].

Fluctuations of conserved charges are the most suitable quantities to test the validity of HRG model. They can be defined as derivatives of the pressure with respect to chemical potentials and as such appear in the calculations of thermodynamic quantities at finite baryon density via Taylor expansion. The recent lattice calculations performed with p4 and asqtad actions at or very close to the physical quark mass [6, 7, 10] gave fluctuations that were quite different from the results obtained in HRG model. It was pointed out that lattice discretization effects on the hadron spectrum are responsible for this discrepancy [17]. When taking into account the lattice spacing dependence of the hadron masses in the hadron resonance gas calculations a good agreement between lattice and HRG calculations has been found. Strangeness fluctuations calculated with and stout [8, 9] and HISQ [18] improved staggered fermion actions agree better with the HRG result (see also the discussion in Ref. [19]). This is due to largely reduced discretization effects in the hadron spectrum for these actions. In Ref. [17] we studied baryon

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number and strangeness fluctuations in HRG, where the hadron masses have been modified to include discretization effects present in lattice calculations. The comparison was done using lattice data obtained with asqtad action on $N_r = 6$ and 8 lattices. On the other hand, in the most detailed lattice study of baryon number, strangeness and electric charge fluctuations and correlations up to sixth order was performed with p4 action using lattices with temporal extent $N_r = 4$ and $N_r = 6$ [10]. In this contribution we are going to extend our calculations in modified HRG model to study different fluctuations and compare them with lattice results obtained with p4 action on $N_r = 4$ and 6 lattices. As mentioned above this is also important for constructing realistic equation of state at finite baryon density for hydrodynamic models similarly as this was done for zero baryon density in Ref. [17].

2. Fluctuations of strangeness, baryon number and electric charge

Derivatives of the pressure with respect to chemical potentials of conserved charges, e.g. baryon number ($B$), electric charge $Q$ and strangeness $S$ at zero chemical potentials can be well calculated in lattice QCD

$$
\chi_X^Y = T \frac{\partial^2 p(T, \mu_B, \mu_Q, \mu_S)}{\partial \mu_X^{\mu_Y}} \bigg|_{\mu_B = \mu_Q = \mu_S = 0}, \quad X = B, Q, S. \tag{1}
$$

These are related to quadratic and higher order fluctuations of conserved charges $\chi_2^X = \langle X^2 \rangle / (VT^3)$, $\chi_3^X = \langle (N_X^2) - 3(N_X^2)^2 \rangle / (VT^3)$ etc.\(^1\) Mixed derivatives of the pressure give correlations of conserved charges

$$
\chi_{11}^{XY} = T \frac{\partial^2 p(T, \mu_B, \mu_Q, \mu_S)}{\partial \mu_X \partial \mu_Y} \bigg|_{\mu_B = \mu_Q = \mu_S = 0} = \langle XY \rangle / (VT^3), \quad X, Y = B, Q, S. \tag{2}
$$

We have calculated these quantities in HRG with hadron spectrum modified to take into account the lattice artifacts. Unfortunately, there is no detailed calculations of the hadron spectrum with p4 action. Therefore the lattice spacing and quark mass dependence of the hadron masses was evaluated using the formulas derived in Ref. [17] based on asqtad calculations. We expect that cutoff dependence of the hadron masses is similar for p4 and asqtad actions, except in the pseudo-scalar meson sector. It is known that the quadratic splitting of non-Goldstone pseudo-scalar mesons is about two times larger for p4 action than for asqtad action [20]. Therefore when evaluating the contributions of kaons and pions to different quantities we simply doubled the quadratic splittings used in Ref. [17]. To get agreement between lattice results and HRG it turned out to be necessary to modify the masses of excited baryons states. Since the cutoff dependence of the excited baryon masses is not known, it was assumed that masses of all excited states up to certain threshold $m_{cut}^B$ are modified the same way as the ground state baryon masses [17]. Values $m_{cut}^B = (1.7 - 2.5)\text{GeV}$ have been considered in the previous analysis.

In this paper we consider all resonances with mass up to 2.5GeV and use $m_{cut}^B = 1.9\text{GeV}$ in all calculations. The quadratic fluctuations of strangeness and electric charge calculated in HRG and compared to the p4 lattice results are shown in Fig. 1. The lattice results are well below the HRG curve obtained with physical hadron masses. This is a general feature for all fluctuations. However, taking into account the discretization effects in the hadron spectrum we find a reasonable agreement between HRG and lattice results. In the strange sector discretization effects are slightly overestimated in our approach. We also studied quadratic baryon number fluctuations and baryon number - strangeness correlations and the corresponding results are shown in Fig. 2. The baryon number fluctuations are well described by modified HRG, although the in the low temperature region cutoff effects are under-predicted for $N_r = 6$. For

\(^1\) Here we consider the case of zero chemical potential, so $\langle N_X \rangle = 0$. 

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baryon number-strangeness fluctuations the agreement between lattice and modified HRG is not very good. This may imply that the cutoff effects in strange baryon sector are smaller than in non-strange baryon sector and are overestimated in our approach. Interestingly, the new calculations with HISQ action also suggest smaller cutoff effects in baryon number-strangeness fluctuations compared to baryon number fluctuations [19]. Finally we have considered fourth order fluctuations of the electric charge and baryon number. The numerical results are shown in Fig. 3. The HRG calculations with the modified hadron masses agree reasonably well with the p4 lattice data. Note, however, that here the deviations from the resonance gas show up at smaller temperatures.

3. Conclusions
We have studied quadratic and fourth order fluctuations of baryon number, electric charge and strangeness fluctuations in HRG model, where hadron masses have been adjusted to take into
account the discretization errors in the hadron spectrum present in the lattice calculations. We have found reasonably good agreement with the lattice calculations of the fluctuations performed using p4 action on $N_{\tau} = 4$ and 6 lattices. Our calculations explain why all the lattice results fall below the physical HRG result. Our approach does not give a good description of baryon number - strangeness fluctuations. To resolved this issue a more refined treatment of the cutoff effects in the baryon sector is needed.

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[1] C. E. DeTar, PoS LATTICE2008, 001 (2008)
[2] P. Petreczky, Nucl. Phys. A 830, 11C (2009); P. Petreczky, Nucl. Phys. Proc. Suppl. 140, 78 (2005)
[3] R. V. Gavai and S. Gupta, Phys. Rev. D 64, 074506 (2001).
[4] R. V. Gavai and S. Gupta, Phys. Rev. D 73, 014004 (2006)
[5] C. Bernard et al. [MILC Collaboration], Phys. Rev. D 71, 034504 (2005)
[6] A. Bazavov et al., Phys. Rev. D 80, 014504 (2009)
[7] M. Cheng et al., Phys. Rev. D 81, 054504 (2010)
[8] Y. Aoki, Z. Fodor, S. D. Katz and K. K. Szabo, Phys. Lett. B 643, 46 (2006)
[9] S. Ejiri, F. Karsch and K. Redlich, Phys. Lett. B 633, 275 (2006)