Interferometric control of the photon-number distribution

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We demonstrate deterministic control over the photon-number distribution by interfering two coherent beams within a disordered photonic lattice. By sweeping a relative phase between two equal-amplitude coherent fields with Poissonian statistics that excite adjacent sites in a lattice endowed with disorder-immune chiral symmetry, we measure an output photon-number distribution that changes periodically between super-thermal and sub-thermal photon statistics upon ensemble averaging. Thus, the photon-bunching level is controlled interferometrically at a fixed mean photon-number by gradually activating the excitation symmetry of the chiral-mode pairs with structured coherent illumination and without modifying the disorder level of the random system itself.

Optical interferometers implement deterministic field transformations that trace an interferogram by sweeping a phase – which enables applications across all of optics and photonics.1 The detected intensity is modulated, but the underlying photon number distribution $P_n$ does not change: the Poisson statistics associated with coherent light remain Poissonian, and the Bose-Einstein statistics that are the hallmark of thermal light remain Bose-Einstein. Introducing dynamical randomness in the interferometer can change the photon statistics. An early example in neutron interferometry implemented a chopper in one arm of a two-path interferometer2–5, while an optical ‘stochastic interferometer’ devised by De Martini et al. changes the photon statistics by inserting a randomly varying optical element6–10. Along a different vein, coherent light traversing a random medium may gradually acquire chaotic behavior and under certain conditions may even attain Bose-Einstein statistics. This phenomenon has been observed in several systems, including weakly transmitting barriers embedded in a waveguide9, quasi one-dimensional disordered samples9, and optically dense slabs containing multiple scatterers10–15. In these approaches, changing the photon statistics is predicated on varying the system disorder level – with the amount of fluctuations (photon bunching) typically proportional to the disorder level.

We present here a different strategy for tuning photon statistics in a random structure that does not require modifying its disorder level. Instead, the ensemble-averaged $P_n$ is varied deterministically by an external element in the form of a relative phase between two input channels of a random network – analogously to a traditional two-path interferometer. Reconstructing $P_n$ reveals deterministic interferometric tuning of photon-bunching across the super-thermal and sub-thermal regimes with fixed mean photon-number $\bar{n}$. This effect has its origin in the smooth and deterministic breaking of the excitation symmetry in certain random lattices, which can be achieved by sculpting the lattice excitation. Our experiment makes use of a one-dimensional (1D) disordered photonic lattice of evanescently coupled waveguides with off-diagonal disorder.16 Additionally, we measure two statistical quantities: the normalized second-order photon correlation $g^{(2)}$, which does not depend on $\bar{n}$, and Mandel’s $Q$-parameter, which does.17 In previous work18, we measured the statistical parameter $g^{(2)}$. Although $g^{(2)}$ can be computed from the photon-number distribution, the converse is not true. The counting distribution has features that are not captured by a single parameter, its normalized second-order moment $\tilde{g}^{(2)}$. The novelty in our work lies in revealing the tunability of the shape of the distribution itself, by demonstrating the transition between a Poisson distribution, to a Bose-Einstein-like distribution, and the emergence of other intermediate photon counting distributions.

The concept of interferometric control over $P_n$ is depicted in Fig. 1. We start by considering coherent light with a Poissonian distribution $P_n=\bar{n}^n e^{-\bar{n}}/n!$, $n=0,1,2,\ldots$, that is fed into a single channel of a disordered lattice consisting of an array of coupled optical elements (waveguides19–21, resonators22–24, or fiber loops25,26). The output $P_n$ can be changed by varying the lattice disorder level $\Delta$ (to be defined below). Surprisingly, high-order coherences do not decline while increasing $\Delta$ [Ref.27]. If the lattice is endowed with disorder-immune ‘chiral symmetry’,28–30, a photonic thermalization gap31 emerges upon ensemble averaging: the regime of sub-thermal photon statistics is forbidden at any disorder level, while super-thermal statistics are inaccessible to lattices lacking chiral symmetry.31 We make use of the normalized second-order photon correlation $g^{(2)}=((n(n-1))/\langle n^2 \rangle)$ (\langle \cdot \rangle denotes ensemble averaging over both the quantum state and the lattice disorder) as a scalar measure of randomness to delineate the sub-thermal $1<g^{(2)}<2$ and super-thermal $g^{(2)}>2$ regimes.32 Instead of a monotonic trend towards ‘thermalization’ with increasing $\Delta$ in lattices characterized by chiral symmetry (such as those with off-diagonal disorder16,33), $P_n$ exhibits super-thermal statistics with a gradual decline towards thermal statistics upon increasing $\Delta$ [Fig. 1(a)]. Such a lattice can thus tune the photon statistics in the super-thermal regime. Alternatively, in lattices lacking chiral symmetry (such as those with diagonal disorder34,35), $P_n$ exhibits sub-thermal statistics with a gradual decline towards Poisson statistics upon increasing $\Delta$ [Fig. 1(b)]. This lattice can thus tune the photon statistics in the sub-thermal regime.

To tune the photon statistics without altering $\Delta$, we sculpt the excitation over multiple lattice sites; e.g., by varying the relative phase between two sites. In traditional interferometry, a relative phase $\theta$ is introduced between two fields before they are superposed to trace an intensity interferogram1. In the interferometric scheme introduced here, a relative phase $\theta$ is introduced between adjacent channels of a random network with chiral symmetry fed with coherent light. The two fields superpose within the network and $P_\theta(\theta)$ measured in a single output channel reveals a deterministic tuning of $P_\theta(\theta)$ between super-thermal and sub-thermal regimes while sweeping $\theta$ [Fig. 1(c)].

The above-described effect stems from the properties of the eigenmodes and eigenvalues of a lattice endowed with chiral symmetry. To appreciate the underlying physics, consider a generic tight-binding lattice model with nearest-neighbor-only coupling. The complex field amplitude $E_x(z)$ at site $x$ after traveling along $z$ is described by a set of coupled differential equations,

$$\begin{align*}
-i \frac{dE_x}{dz} &= \beta_x E_x + C_{x+1,x} E_{x+1} + C_{x-1,x} E_{x-1},
\end{align*}$$

where $\beta_x$ is the wave number for site $x$ and $C_{x+1,x}$ the coupling coefficient between site $x$ and site $x+1$ – all of which may be

\[\text{Ref.}\]
random variables when the lattice is disordered. We define a Hermitian coupling matrix or Hamiltonian $H$ for the lattice where $\{b_x\}$ correspond to the diagonal elements and $\{C_{x,x\pm1}\}$ occupy the next two diagonals. The eigenvalues $\{b_j\}$ and eigenfunctions $\{\psi^{(j)}_x\}$ of $H$ are defined through $H\psi^{(j)}_x = (\vec{\beta} - b_j)\psi^{(j)}_x$, where $\vec{\beta}$ is the average wave number. Since $H$ is real and symmetric, $\{\psi^{(j)}_x\}$ are all real. If $H$ can be recast into a block-diagonal form after setting $\vec{\beta} = 0$ in the interaction picture, this indicates that the lattice is endowed with chiral symmetry. Lattices with off-diagonal disorder ($b_x = \vec{\beta}$ and $C_{x,x\pm1}$ are random) satisfy chiral symmetry, whereas lattices featuring diagonal disorder ($C_{x,x\pm1} = C$ and $b_x$ are random) do not. Henceforth, we focus our attention on lattices with off-diagonal disorder. A consequence of chiral symmetry is that $b_j = -b_j$ and $\psi^{(j)}_x = (-1)^j\psi^{(-j)}_x$. However, for the impact of this skew-symmetry to be manifested, the members of the skew-symmetric paired modes with indices $\pm j$ must be excited with equal weights, which we refer to as ‘activating’ the chiral symmetry. To visualize the activation of chiral symmetry, we associate a phasor with each mode at each lattice site. Then, in the rotating frame which we refer to as ‘activating’ the chiral symmetry with equal weights (symmetric excitation), the phasor sum (depicted in gray) takes on either real or imaginary values depending on the lattice site. This symmetry is absent when the chiral symmetry is not activated (asymmetric excitation) or absent (diagonal disorder). Thus, the phasor sum is always complex (not only real or imaginary).

Figure 1 | The concept of interferometric control over $P_n$.
(a,b) Coherent light with Poissonian statistics is fed into a channel of a random network while varying the disorder level $\Delta$. The emerging light (a) spans the regime of super-thermal statistics when the lattice is endowed with chiral symmetry, or (b) spans the sub-thermal regime when the network lacks chiral symmetry. (c) Coherent light is split into two paths and a relative phase $\theta$ is introduced before being launched into a random network with chiral symmetry at fixed $\Delta$. Modulating $\theta$ breaks the mode-excitation symmetry and enables spanning the sub- and super-thermal regimes. (d) When the chiral symmetric eigenmode pairs in a lattice with off-diagonal disorder (corresponding phasors in a single lattice site depicted in blue and red) are activated with equal weights (symmetric excitation), the phasor sum (depicted in gray) takes on either real or imaginary values depending on the lattice site. This symmetry is absent when the chiral symmetry is not activated (asymmetric excitation) or absent (diagonal disorder). Thus, the phasor sum is always complex (not only real or imaginary).

An input optical field $E_x(z=0)$ can be analyzed in a basis of lattice eigenmodes, $E_x(0) = \sum_j c_j \psi^{(j)}_x$, where $c_j = \sum_i \psi^{(j)}_z E_z(0)$ is the amplitude of the $j$th mode $\psi^{(j)}_z$. The field subsequently evolves after a distance $z$ into $E_x(z) = \sum_j c_j e^{jz}$. In the special case of a single-site excitation at the input $E_x(0) = \delta_{x,0}$, then $|c_j| = |c_{-j}| = |\psi^{(j)}_0|$, for all $j$, so that chiral symmetry is activated. This is not necessarily the case for more general field excitations. For example, when two adjacent sites are excited equally $E_x(0) = \delta_{x,0} + \delta_{x,1}$, the modal coefficients are $c_{\pm j} = \psi^{(j)}_0 \pm \psi^{(j)}_1$, and chiral symmetry is activated $|c_j| = |c_{-j}|$ only when the relative phase is $\theta = \pm \pi/2$. Gradually varying the phase $\theta$ for fixed relative amplitudes ($|A_j| = |A_{-j}|$ in our experiment) tunes the chiral-symmetry breaking: maximal symmetry breaking at $\theta = 0$ or $\pi$, and symmetry activation at $\theta = \pm \pi/2$.

The photonic lattice we utilized consists of an array of 101 identical 35-mm-long waveguides with nearest-neighbor-only evanescent coupling. The average separation between the waveguides is 17 μm, resulting in an average coupling coefficient $C \approx 1.71 \text{ cm}^{-1}$ at a wavelength of 633 nm. The coupling coefficients are selected independently from a uniform probability distribution function with mean $C$ and half-width $\Lambda$ in units of $C$; our sample has $\Lambda \approx 0.6$.

The optical arrangement used in demonstrating deterministic interferometric control over $P_n$ is illustrated in Fig. 2. A single-mode coherent beam from a He-Ne laser is attenuated by a neutral density filter and split into two paths via a beam splitter. A relative phase shift $\theta$ is introduced via a delay in one path varied in 20-nm steps. The two beams are then brought together by a second beam splitter in parallel but closely spaced paths, which are imaged to two neighboring waveguides. The output facet of the array is imaged with a magnification of 8× via a lens (focal length $f=3.5$ cm) to a plane in which we scan a multimode fiber whose core optimally couples light from an individual waveguide. The multimode fiber delivers the collected light to a single-photon-counting module, and the output photon-number distribution $P_n$ is reconstructed using three photon-detection time windows: 20, 40, and 60 μs. The input intensity level is reduced to low levels
while maintaining \( \bar{n} \) while examining a quantity extracted from \( P_n \) while minimizing the accidental arrivals of multiple photons. We measure single realizations of \( P_n(\theta) \) (averaged over \( 10^4 \) shots of the detection window) at the central lattice site \( x=0 \), and then average \( P_n(\theta) \) over an ensemble of 15 disorder realization for each value of \( \theta \) by shifting the input excitation site.

We present in Fig. 3 our measurements confirming the deterministic interferometric tuning of \( P_n(\theta) \) in the excitation waveguide \( (x=0) \) while varying \( \theta \). As \( \theta \) is swept, \( P_n(\theta) \) varies periodically (period \( \pi \)) between sub-thermal to super-thermal statistics [Fig. 3(a)]. The measured mean photon-number \( \bar{n}(\theta) \) does not vary with \( \theta \) (\( \bar{n}=\approx 5.5 \) with a standard deviation \( =0.5 \)). Although \( \bar{n} \) remains constant, the photon-number distribution itself varies with \( \theta \), achieving maximal bunching when the chiral symmetry is fully activated \( \theta=\pi/2,3\pi/2 \), and minimal bunching at \( \theta=0,\pi \) when chiral symmetry is dormant. To better examine the salient changes in the photon statistics with \( \theta \), we plot in Fig. 3(b,c) \( P_n(\theta) \) for the extrema at \( \theta=0 \) and \( \pi/2 \). The increase in the probability of higher photon numbers at the distribution tail when \( \theta=\pi/2 \) as compared to that when \( \theta=0 \) is a clear signature of photon bunching. In comparison, we also plot the Bose-Einstein distribution for thermal light with the experimentally determined \( \bar{n}=\approx 5.5 \). At high \( n \), the symmetric-excitation statistics exhibit higher probabilities than the Bose-Einstein distribution, whereas the broken-symmetric excitation has lower. Our experimental data [Fig. 3(c)] is in excellent agreement with the simulations [Fig. 3(b)].

Further analysis of the measured distributions \( P_n(\theta) \) helps bring about the changes that take place in the photon statistics. First, we examine a quantity extracted from \( P_n(\theta) \) that does not depend on \( \bar{n} \) for the field considered here: the normalized second-order photon correlation function \( g^{(2)}(\theta) \) [Ref. 17]. We plot in Fig. 4(a) \( g^{(2)}(\theta) \) and note clearly that it varies sinusoidally with \( \theta \), between the sub-thermal and super-thermal regimes. The statistics are tuned from super-thermal \( (g^{(2)}>2) \) to sub-thermal \( (g^{(2)}<2) \), all while maintaining \( \bar{n} \) fixed with \( \theta \) [Fig. 4(a), inset]. Although \( \bar{n} \) changes according to the photon-counting window \( (\bar{n}=\approx 5.5,11,16.5 \) for counting windows of 20,40,60 ps, respectively), the three interferometric traces of \( g^{(2)}(\theta) \) are indistinguishable.

Next we consider a quantity that does indeed depend on \( \bar{n} \): Mandel’s Q-parameter, \( Q=\text{Var}(n)/(\langle n\rangle^2)-1=\bar{n}g^{(2)}-1 \), which is thus linear in \( \bar{n} \) for the field considered here. Varying \( \theta \) at a fixed \( \bar{n} \) modulates \( Q(\bar{n},\theta) \) between two limits identified by the dashed inclined lines in Fig. 4(b); we identify in Fig. 4(b) only the values corresponding to \( \theta=0 \) and \( \pi/2 \), which are the extrema of this oscillation between the super-thermal and sub-thermal regimes. Increasing \( \bar{n} \) (longer counting windows) leads to a linear growth in the two limits of \( Q \) modulation.

In conclusion, we have demonstrated that the photon-number distribution \( P_n \) and hence any photon statistic such as \( g^{(2)} \) or Mandel’s Q-parameter – can be tuned deterministically by varying a relative phase between two equal-amplitude beams launched into adjacent sites of a disordered photonic lattice. Such interferometric control over \( P_n \) is possible by judicious excitation-
photonic lattices

(a) Mandel’s parameter. The latter approach has the advantage of producing the lattice while alternating the relative phases between adjacent sites. The latter approach has the advantage of producing the lattice while alternating the relative phases between adjacent sites.

(b) Mandel’s parameter. The latter approach has the advantage of producing the lattice while alternating the relative phases between adjacent sites. The latter approach has the advantage of producing the lattice while alternating the relative phases between adjacent sites.

Finally, we note that multiple experiments have been reported in which non-classical light is coupled into ordered or disordered photonic lattices \(^{(41-44)}\). In these reports, only small photon numbers are involved (usually two) in either Fock or entangled photon states, and while the problem of coupling light into ordered or disordered photonic lattices \(^{(41-44)}\) is not a new one, there have been no studies in the nature of the underlying photon statistics. Our work extends these studies into photon distributions with higher mean photon-number and demonstrates unambiguously interferometric control over \(P_n\) itself.

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