Edge Adaptive Hybrid Regularization Model For Image Deblurring

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Abstract

A spatially fixed parameter of regularization item for whole images doesn’t perform well both at edges and smooth areas. A large parameter of regularization item reduces noise better in smooth area but blurs edges, while a small parameter sharpens edges but causes residual noise. In this paper, an automated spatially dependent regularization parameter hybrid regularization model is proposed for reconstruction of noisy and blurred images which combines the harmonic and TV models. The algorithm detects image edges and spatially adjusts the parameters of Tikhonov and TV regularization terms for each pixel according to edge information. In addition, the edge information matrix will be carefully designed regularization techniques based on prior information on images.

Numerical simulation results demonstrate that the proposed model effectively protects the image edge while eliminating noise and blurs and outperforms the state-of-the-art algorithms in terms of PSNR, SSIM and visual quality.

1. Introduction

Images are frequently degraded by noise and blurring during the image acquisition and transmission procedures [22, 35]. As a result, image denoising and deblurring are fundamental preprocessing steps for various image processing tasks, such as image segmentation, edge detection, and pattern recognition. The mathematical model for image degradation [1, 3, 20, 24] can be formulated as follows. Let \( u \in \mathbb{R}^{M \times N} \) be the clear image of size \( M \times N \), and \( f \in \mathbb{R}^{M \times N} \) be the observed image with Gaussian white noise. Then,

\[
f = A \otimes u + \varepsilon.
\]  

where \( A \) is the blurring operator, \( A \otimes u \) is the convolution of \( A \) with \( u \) as

\[
[A \otimes u](i,j) = \sum_{(s,t) \in \Omega} A[(i,j) - (s,t)] \times u(s,t).
\]

and \( \varepsilon \in \mathbb{R}^{M \times N} \) represents the Gaussian white noise with mean 0 and standard deviation \( \sigma \). Let \( \mathcal{F} \) denote the vector space of 2D images defined on \( \Omega = \{1, 2, \cdots, M\} \times \{1, 2, \cdots, N\} \). For each 2D image \( u \), the total number of pixels is \( M \times N \), and \( u(i,j) \) denotes the image value at pixel \( (i,j) \in \Omega \). Restoring the unknown image \( u \) from the degraded image \( f \) with noise \( \varepsilon \) is a typical ill-posed problem. Hence, effective image deblurring methods usually rely on carefully designed regularization techniques based on prior information on \( u \).

To preserve significant edges, Rudin, Osher, and Fatemi[37] proposed the celebrated total variation (TV) model for image restoration. By this approach, the recovery of the image \( u \) is based on solving the following minimization problem

\[
\min_u \frac{\mu}{2} \|A \otimes u - f\|^2 + \|\nabla u\|_1,
\]

where \( \nabla u = [\nabla_x u, \nabla_y u] \in \mathbb{R}^{2 \times MN} \) is the gradient in the discrete setting with \( \nabla u(i,j) = (\nabla_x u(i,j), \nabla_y u(i,j)) \), and \( \nabla_x u \) and \( \nabla_y u \) denote the horizontal and vertical first-order difference with Dirichlet boundary condition respectively, which are defined as follows

\[
\nabla_x u(i,j) = \begin{cases} u(i+1,j) - u(i,j), & \text{if } i < M; \\ 0, & \text{if } i = M; \end{cases}
\]

\[
\nabla_y u(i,j) = \begin{cases} u(i,j+1) - u(i,j), & \text{if } j < N; \\ 0, & \text{if } j = N; \end{cases}
\]

and \( \|\nabla u\|_1 \) is \( l_1 \)-norm of \( \nabla u \)

\[
\|\nabla u\|_1 = \sum_{(i,j) \in \Omega} |\nabla u(i,j)|,
\]

\[
|\nabla u(i,j)| = \sqrt{(\nabla_x u)^2(i,j) + (\nabla_y u)^2(i,j)}.
\]
As the total variation term is powerful for preserving sharp edges, the TV model is widely used for image processing. Over the years, various research efforts have been devoted to studying, solving, and extending the TV model. In particular, a two-phase approach for deblurring images [6, 7] was proposed to handle the impulse noise. More related models and algorithms have been reported in [9, 10, 11, 12, 15, 19, 23, 32, 34, 36, 33, 39, 42, 43, 44, 45].

However, image deblurring with TV model often leads to some undesirable staircase effect, namely, the transformation of smooth regions into piece wise constant ones. There are at least two possible ways to handle this issue. One is to use the tight-frame approaches [9, 10]. The other is to combine the TV regularization term with some more advanced models. Note that the harmonic model [38] with Tikhonov regularization and fourth-order PDE (LLT) filter [31] can effectively reduce noise. Therefore, some hybrid regularization models combining the TV model and LLT models [25, 28, 31, 36] or harmonic model [18, 29] are proposed to preserve edges while restraining staircase effects at the same time. Furthermore, Liu et al. [27] combine image sharpening operator and framelet regularization [8] for image deblurring, and the model can be expressed as,

\[
\min_u \frac{\mu}{2} \|A \otimes u - f\|^2_2 + \|Wu\|_1 + \alpha \|u - B(f)\|_1, \tag{5}
\]

where \(W\) is the framelet transformation, \(B(f) \in \mathbb{R}^{M \times N}\) is a sharpened image, and \(\mu, \alpha\) are positive parameters. In the optimization problem (2), the positive parameter \(\mu\) controls the trade-off between a good fit of \(f\) and a smoothness requirement due to the total variation regularization. In general, images are comprised of multiple objects at different scales. This suggests that different values of \(\mu\) localizing at image features of different scales are desirable to obtain better results. Generally speaking, for the texture region, we can use larger \(\mu\) as it leads to less smoothing and better detail preservation. On the other hand, for the flat region, smaller \(\mu\) is desired in order to get a better smoothing and noise-canceling result. For this reason, in [2, 4] the multi-scale total variation (MTV) models were proposed, with spatially varying parameters based on (2). The corresponding multi-scale versions of model (2) is represented as

\[
\min_u \frac{1}{2} \|A \otimes u - f\|^2_{2,\mu} + \|\nabla u\|_1, \tag{6}
\]

where \(\mu \in \mathbb{R}^{M \times N}\) is spatially varying non-negative parameter matrix and the norm

\[
\|A \otimes u - f\|^2_{2,\mu} = \langle A \otimes u - f, \mu \ast (A \otimes u - f) \rangle.
\]

As in Matlab, \(\mu \ast (A \otimes u - f)\) denotes point-wise product between the elements \(\mu\) and \(A \otimes u - f\) as follows,

\[
[\mu \ast (A \otimes u - f)](i, j) = \mu(i, j) \times [A \otimes u - f](i, j). \tag{7}
\]

Borrowing the idea of (6), the multi-scale technique can be applied to the TV term, which yields

\[
\min_u \frac{\mu}{2} \|A \otimes u - f\|^2_2 + \|\nabla u\|_1,\alpha, \tag{8}
\]

where \(\alpha \in \mathbb{R}^{M \times N}\) is a non-negative spatially varying parameter matrix, \(\|\nabla u\|_1,\alpha\) is given by

\[
\|\nabla u\|_1,\alpha = \sum_{(i,j) \in \Omega} \alpha(i, j) |\nabla u(i, j)|. \tag{9}
\]

Introducing a local parameter gives more flexibility to the model in exchange for the robustness. For this reason, Dong et al. [16] proposed a method to decide whether to use a spatially varying parameter, relying on a confidence interval technique depending on the expected maximal local variance estimate. In this paper, we study the properties of the image edges and propose an edge adaptive hybrid regularization model by introducing an edge detection operation to a combination of the TV and harmonic models. The edge detection operation is used to decide, in a robust way, where the edges locate in the noisy image \(f\) and generate an edge information matrix. The decision on the acceptance or rejection of a scaling parameter is based on the edge information matrix. Our edge information matrix can be obtained fully automatically and updated in each iteration for preserving edges. Furthermore, the proposed model is convex once the local parameters are fixed, and hence it can be solved efficiently by the semi-proximal alternating direction method of multipliers (sPADMM) [17, 26, 21] which has linear convergence rate under mild condition. The contributions of this work are summarized as follows:

1. We propose an algorithm which can automatically detect the image edges and adjust the parameters for the Tikhonov and TV regularization terms for each pixel according to the edge information. This enables the algorithm to effectively remove noise on the smooth area and sharpen the edges during deblurring.

2. We build a convex optimization model which is easily solved by sPADMM with a linear-rate convergence.

3. We conduct extensive experiments to display that the proposed method outperforms the state-of-the-art methods for reconstruction from noisy and blurred images.

The remainder of the paper is organized as follows. In section 2, we present our model, describing how to adjust the parameters of TV and Tikhonov regularization terms according to edge information. The simulation results are exhibited and analyzed in section 3, followed by conclusions in section 4.
2. Edge adaptive hybrid regularization model for image deblurring

In this section, we will explain that the proposed model can smartly balance the relationship between the edge and smooth areas and achieve the goal of retaining the edge and simultaneously eliminating the staircase effect. The sPADMM is used to efficiently solve the proposed model.

2.1. Model Establishment

As mentioned above, the harmonic model is effective in suppressing noise in smooth areas of the image. However, it blurs edges concomitantly. The total variation based methods protect the image edges efficiently while having a staircase effect in the smooth region. In this paper, we combine these two models to make up for their respective shortcomings. Note that the simple combination alone is not distinctive enough for image restoration. The fixed parameter between the edge and smooth areas well and achieves the goal of retaining the edge and simultaneously eliminating the staircase effect. Note that the TV model sometimes erroneously treats the noise in the smooth area as image edge, while the proposed model eliminates those misjudged edges by edge detection.

2.2. Algorithm

As mentioned, our proposed model can be efficiently solved by sPADMM. For this, we first introduce an auxiliary variable k to take the place of the \( \nabla u \) in the both term of \( \| \nabla u \|_{1, \alpha} \) and \( \| \nabla u \|_{2, \alpha} \), then the model (10) can be reformulated as the following constrained optimization problem

\[
\min_u J(u) = \min_u \left\{ \| k \|_{1, \alpha_1} + \frac{1}{2} \| k \|_{2, \alpha_2}^2 + \frac{\mu}{2} \| A \otimes u - f \|_2^2 \right\},
\]

(14)

where \( \alpha_1 \) and \( \alpha_2 \in \mathbb{R}^{M \times N} \) are scaling parameters, \( \| \nabla u \|_{1, \alpha_1} \) is given by (9) with \( \alpha_1 \) instead of \( \alpha \), \( \| \nabla u \|_{2, \alpha_2} \) is given by

\[
\| \nabla u \|_{2, \alpha_2} = \sqrt{\sum_{(i,j) \in \Omega} \alpha_2(i,j)|\nabla u(i,j)|^2}.
\]

(11)

The augmented Lagrangian function of the problem (14) is defined as

\[
L(u, k; \lambda) = \| k \|_{1, \alpha_1} + \frac{1}{2} \| k \|_{2, \alpha_2}^2 + \frac{\mu}{2} \| A \otimes u - f \|_2^2 + \langle \lambda, k - \nabla u \rangle + \frac{\beta}{2} \| k - \nabla u \|_2^2,
\]

(15)

where \( \lambda = [\lambda_1, \lambda_2] \in \mathbb{R}^{2M \times N} \) is the Lagrange multiplier parameter matrix with \( \lambda(i,j) = (\lambda_1(i,j), \lambda_2(i,j)) \), \( \beta > 0 \) is the penalty parameter. In each iteration of the sPADMM, we minimize \( L \) with respect to \( u \) for fixed \( k \) and then minimize \( L \) with respect to \( k \) for fixed \( u \). After

\[
[G \otimes \nabla u](i,j) = \sum_{(s,t) \in \Omega} G[(i,j) - (s,t)] \times \nabla u(s,t).
\]
these, we update the multiplier $\lambda$. Hence, we introduce the sPADMM to solve (15) by updating $u$, $k$ and $\lambda$ as follows:

$$
\begin{align*}
    u^n &= \arg \min_u L(u, k^{n-1}; \lambda^{n-1}) + \frac{1}{2} \|u - u^{n-1}\|_2^2, \\
    k^n &= \arg \min_k L(u^n, k; \lambda^{n-1}) + \frac{1}{2} \|k - k^{n-1}\|_2^2, \\
    \lambda^n &= \lambda^{n-1} + \eta \beta (k^n - \nabla u^n).
\end{align*}
$$

(16) (17) (18)

Here, for any $x \in \mathbb{R}^{M \times N}$, $S \in \mathbb{R}^{M \times N}$ is the self-adjoint positive semidefinite matrix, $Sx$ is the matrix multiplication of $S$ and $x$, $\|x\|_s = \sqrt{\langle x, Sx \rangle}$ denotes the matrix norm. If we take $S_1 = 0$ and $S_2 = 0$, the sPADMM will be the alternating direction method of multipliers (ADMM) [5]. In our algorithm, $S_1$ and $S_2$ are positive matrices. The sPADMM for our EAHR model is given by Algorithm 1.

**Algorithm 1:** sPADMM for the EAHR model

**Input:** Noisy & Blurry image $f$, standard deviation $\sigma$, blurring function $A$, Gaussian kernel $G$

**Initialize:** 1) $S_1$, $S_2$, $\alpha_1$, $\alpha_2$, $\beta_1$, $\theta_2$, $\tau$, $\mu$, $\beta$, $\epsilon$,

2) BM3D [14, 13] preprocessed blurred image $f$

3)$u^0 = 0; k^0 = 0; \lambda^0 = 0$

for $n = 1 : MaxIter$

| Update |

- $u^n$ by Eq: (20)
- $k^n$ by section 2.2.1
- $\lambda^n$ by Eq: (18)
- $\alpha_1$ and $\alpha_2$ by Eqs: (12) and (13)

if $\min \left\{ \frac{\|u^n - u^{n-1}\|_2}{\|u^n\|_2}, \frac{\|k^n - \nabla u^n\|_2}{\|\nabla u^n\|_2} \right\} \leq \epsilon$ then

| break |

end

**Output:** Restored image $u^n$

In each iteration of sPADMM, the subproblems (16) and (17) need to be solved. The $u$-subproblem is written as

$$
\begin{align*}
    \min_u &\left\{ \frac{\mu}{2} \|A \otimes u - f\|_2^2 + \langle \lambda^{n-1}, k^{n-1} - \nabla u \rangle \\
    &+ \frac{\beta}{2} \|k^{n-1} - \nabla u\|_2^2 + \frac{1}{2} \|u - u^{n-1}\|_2^2 \right\}.
\end{align*}
$$

(19)

Note that the subproblem (19) is a quadratic optimization problem subject to the optimality condition

$$
\begin{align*}
    \mu A^T \otimes (A \otimes u - f) - \nabla^T \lambda^{n-1} - \beta \nabla^T k^{n-1} \\
    + \beta \Delta u + S_1 (u - u^{n-1}) = 0,
\end{align*}
$$

which is solved by Fourier transform [39] as follows

$$
\begin{align*}
    u &= \frac{\mu A^T \otimes f + \nabla^T (\lambda^{n-1} + \beta k^{n-1}) + S_1 (u^{n-1})}{\mu A^T \otimes A + \beta \Delta + S_1}
\end{align*}
$$

(20)

Next, we analyze the nonsmooth subproblem (17) in detail to find its global minimizer.

### 2.2.1 Solving subproblem (17)

In order to solve subproblem (17), we first introduce a proposition as follows.

**Proposition 4.1.** For any $\alpha, \beta > 0$, and $s, t \in \mathbb{R}^2$, the minimizer of

$$
\min_{s \in \mathbb{R}^2} \left\{ f(s) = \alpha |s| + \frac{\beta}{2} |s - t|^2 \right\}
$$

(21)

is given by

$$
\begin{align*}
    s^* &= \arg \min_s f(s) = \frac{|s^*|}{|t|} \cdot t,
\end{align*}
$$

(22)

where $\frac{\alpha}{\beta} \cdot 0 = 0$ and $0 = (0, 0)$.

Proof: If $s \neq 0$, we have

$$
\frac{\partial f}{\partial s} = \alpha \frac{s}{|s|} + \beta (s - t).
$$

Let $\frac{\partial f}{\partial s} = 0$, we get

$$
\begin{align*}
    t &= s + \frac{\alpha}{\beta} \frac{s}{|s|}.
\end{align*}
$$

(23)

From the equation above, we have

$$
|t|^2 = (|s| + \frac{\alpha}{\beta})^2, \text{ or } |t| = |s| + \frac{\alpha}{\beta}.
$$

(24)

It follows from (23) and (24) that

$$
|s| |t| = |s| |s| + \frac{\alpha}{\beta} \cdot s
$$

and

$$
|s| |t| = |s| |s| + \frac{\alpha}{\beta} \cdot s.
$$

Then the equation (22) holds when $s \neq 0$. The equation (22) also holds when $s = 0$. Therefore, Proposition 4.1 is proved.

The subproblem (17) is written as

$$
\begin{align*}
    \min_k &\left\{ \|k\|_{1, \alpha_1} + \frac{1}{2} \|k\|_{2, \alpha_2}^2 + \frac{\mu}{2} \|A \otimes u^n - f\|_2^2 \\
    &+ \langle \lambda^{n-1}, k - \nabla u^n \rangle + \frac{\beta}{2} \|k - \nabla u^n\|_2^2 \right\}
\end{align*}
$$

(25)
The problem above is equivalently expressed as
\[
\min_{k(i,j)} \left\{ \alpha_1(i,j) |k(i,j)| + \alpha_2(i,j) |k(i,j)|^2 + \langle \lambda, z - q \rangle + \frac{\beta}{2} |z - q|^2 \right. \\
\left. + \frac{1}{2} k(i,j) - k(i,j)^{-1} \|\lambda\|^2 \right\}.
\]

For convenience, let \( z = k(i,j), q = \nabla u(i,j), p_1 = \alpha_1(i,j), p_2 = \alpha_2(i,j) \) and \( \lambda = \lambda(i,j) \), then we rewrite the problem above as follows
\[
\min_{z} \left\{ p_1 |z| + p_2 |z|^2 + \langle \lambda, z - q \rangle + \frac{\beta}{2} |z - q|^2 \right. \\
\left. + \frac{1}{2} z - z^{-1} \|\lambda\|^2 \right\}.
\]

(26)

where \( |z| = \sqrt{z^2 + z^2} \). Let
\[
m = \frac{\beta}{\beta + 2p_2 + S_2} (q - \lambda) + \frac{S_2 z^{-1}}{\beta},
\]
the problem (26) is equivalent to
\[
\min_{z} \left\{ p_1 |z| + \frac{\beta + 2p_2 + S_2}{2} |z - m|^2 \right\}.
\]

(27)

From Proposition 4.1, the optimal solution of this problem is
\[
z^* = \frac{z^*}{m} m.
\]

(28)

3. Numerical Simulation

In this section, we present simulation results with twelve testing images, including 7 gray images and 5 color images, as is shown in Figure 1. We consider three types of blurring functions, Gaussian blur (GB), Motion blur (MB) and Average blur (AB). For instance, the notation GB(9, 5)/\( \sigma = 5 \) represents Gaussian kernel with the free parameter 5 and size 9 \( \times \) 9, and additive Gaussian noise with standard deviation \( \sigma = 5 \). To illustrate the effectiveness of the proposed model, we compare our model with the classic TV [37], DCA with \( L_1 = 0.5L_2 \) [30], TRL2 [41] and SOCF [27]. All the experiments are performed under Windows 10 and MATLAB R2018a running on a desktop (Intel(R) Core(TM) i5-8250 CPU @ 1.60 GHz). The termination criterion for all experiments is defined as follows
\[
\min \left\{ \frac{\|u^n - u^{n-1}\|^2}{\|u^n\|^2}, \frac{\|k^n - \nabla u^n\|^2}{\|\nabla f\|^2} \right\} \leq \epsilon,
\]

where \( n \) is the number of iterations, the tolerance value is set as \( \epsilon = 5 \epsilon - 5 \). Quantitatively, we evaluate the quality of image restoration by the peak signal to noise ratio (PSNR) and structural similarity index (SSIM). The SSIM is defined by Wang et al. [40] and PSNR is given by
\[
\text{PSNR} = 10 \log_{10} \frac{255^2}{\text{MSE}}
\]

where
\[
\text{MSE} = \frac{1}{MN} \sum_{(i,j)\in\Omega} (u(i,j) - \hat{u}(i,j))^2,
\]
\( M \times N \) is the image size, \( u \) is the original image and \( \hat{u} \) is the restored image.

From (13), \( \theta_1, \theta_2 \in (0, 1) \) are scaling parameters, and \( \tau > 0 \) is a threshold which is used to adjust the value of \( \alpha_1 \) and \( \alpha_2 \). There are three parameters \( (\mu, \beta, \tau) \) in our algorithm 1 that need to be adjusted. We choose the self-adjoint positive definite linear operators \( S_1 = S_2 = \Omega \) with \( t = 0.1 \) and the maximum iterative number (denoted by \( MaxIter \)) is set as 500. Through a multitude of experiments, we choose better parameters to improve the quality of restoring images damaged by blur and noise. For Gaussian blur GB(9, 5)/\( \sigma = 5 \), the images corrupted by blur and noise restoration effect is better while choosing \( \mu = 2100, \beta = 0.25 \) and \( \tau = 0.95 \) for all images. Similarly, we set \( \mu = 3500, \beta = 0.2, \tau = 0.9 \) for Motion blur MB(20, 60)/\( \sigma = 5 \) and \( \mu = 6000, \beta = 0.2, \tau = 0.9 \) for Average blur AB(20, 60)/\( \sigma = 3 \).

![Set of 12 testing images](http://www.cs.tut.fi/~foi/GCF-BD3D/) Plate from [27]. Duck, Building, Car from the Berkeley segmentation database Dataset(https://www.cs.albany.edu/~xypan/research/snr/Kodak.html)

In order to verify the performance of our proposed model, we have tested the recovery results of gray images and color images under different blur kernels and different Gaussian noise levels. Tables 1, 2 and 3 show the values of PSNR, SSIM of the classic TV [37], DCA with \( L_1 = 0.5L_2 \) [30], TRL2 [41] and SOCF [27] on Gaussian blur GB(9, 5)/\( \sigma = 5 \) and Motion blur MB(20, 60)/\( \sigma = 5 \) and Average blur AB(20, 60)/\( \sigma = 3 \). The number in bold means the best result. Tables 1, 2 and 3 confirm that our
method achieves the best results in PSNR and SSIM in most cases. In addition, Table 4 lists the computation time on a desktop (Intel(R) Core(TM) i5-8250 CPU @ 1.60 GHz), which reveals that our method is faster than DCA [30] and SOCF [27]. But is about three to five times slower than TV [37]. Acceleration will be left in the future work.

In order to further demonstrate the superiority of our algorithm on image restoration more intuitively, we present visual results of image deblurring. In particular, we consider Gaussian blur $GB(9, 5)/\sigma = 5$, Motion blur $MB(20, 60)/\sigma = 5$ and Average blur $AB(9, 9)/\sigma = 3$ for Shepp-Logan, Cameraman and Duck. Figures 2, 3 and 4 show that our method yields the best image quality in terms of removing noise, preserving edges, and maintaining image sharpness. Looking carefully at the details in Figure 2 with $GB(9, 5)/\sigma = 5$, it's easy to find that TV [37] over-smoothes images, TRL2 [41] introduces staircase effect, DCA [30] sharpens edges but loses some detail information, there exits the ringing phenomenon by SOCF [27] and ours achieve the best visual quality. Figures 3 and 4 show that our algorithm gets even greater advantages in the cases of $MB(20, 60)/\sigma = 5$ and $AB(9, 9)/\sigma = 3$. Methods TV [37], DCA [30] and SOCF [27] have the same problem with $GB(9, 5)/\sigma = 5$, and TRL2 [41] lefts excessive residual noise, while the proposed method always maintains good visual quality.

In brief, our algorithm (EAHR) has competitive performance in sharpening the edges and removing the noise, which outperforms other mainstream methods both in PSNR/SSIM values and visual quality.
Table 3: The value of PSNR and SSIM of the test images recovered by different models for AB(9, 9)/σ = 3.

| Image       | PSNR/SSIM Degraded | TV [37] | DCA [30] | TRL2 [41] | SOCF [27] | Ours       |
|-------------|---------------------|---------|----------|-----------|-----------|------------|
| Shepp-Logan | 18.65/0.708         | 24.39/0.947 | 25.82/0.926 | 24.94/0.853 | 24.73/0.938 | 27.35/0.951 |
| Shape150    | 18.16/0.612         | 26.25/0.900 | 28.48/0.933 | 27.97/0.746 | 27.46/0.941 | 28.97/0.918 |
| House       | 23.96/0.618         | 29.09/0.807 | 29.44/0.823 | 29.61/0.768 | 30.45/0.825 | 31.60/0.837 |
| Boat        | 23.23/0.521         | 25.82/0.662 | 26.59/0.703 | 27.12/0.708 | 27.23/0.721 | 28.14/0.751 |
| Pepper      | 21.25/0.613         | 24.21/0.763 | 25.45/0.781 | 25.78/0.777 | 27.02/0.823 | 27.89/0.820 |
| Cameraman   | 20.70/0.561         | 23.87/0.743 | 24.47/0.766 | 24.78/0.642 | 24.66/0.704 | 25.98/0.792 |
| Hill        | 24.85/0.528         | 27.46/0.661 | 28.03/0.695 | 28.39/0.709 | 28.43/0.708 | 28.96/0.736 |
| Plate       | 17.08/0.766         | 22.98/0.930 | 23.86/0.941 | 23.54/0.935 | 25.04/0.950 | 26.80/0.963 |
| Duck        | 24.12/0.709         | 28.43/0.821 | 28.59/0.837 | 29.21/0.845 | 29.34/0.850 | 30.62/0.863 |
| Building    | 19.47/0.650         | 20.70/0.734 | 20.87/0.750 | 21.02/0.749 | 21.48/0.778 | 22.00/0.794 |
| Hats        | 27.38/0.897         | 29.83/0.945 | 29.86/0.945 | 30.02/0.935 | 31.63/0.952 | 31.53/0.955 |
| Car         | 23.93/0.785         | 26.40/0.847 | 26.43/0.850 | 26.49/0.843 | 27.16/0.859 | 27.67/0.863 |
| Average     | 21.90/0.664         | 25.79/0.813 | 26.49/0.829 | 26.57/0.793 | 27.05/0.837 | 28.13/0.854 |

Table 4: The computation time(s) for all compared methods on a desktop (Intel(R) Core(TM) i5-8250 CPU @ 1.60 GHz).

| Image size | TV [37]      | DCA [30]   | TRL2 [41]  | SOCF [27]  | Ours  |
|------------|--------------|------------|------------|------------|-------|
| GB(9, 5)/σ = 5 | 512 × 512 gray | 2.60       | 136.99     | 6.71       | 15.02 | 8.16 |
|            | 768 × 512 RGB | 12.93      | 720.79     | 26.79      | 60.66 | 59.15 |
| MB(20, 60)/σ = 5 | 512 × 512 gray | 3.61       | 159.15     | 11.38      | 23.96 | 9.08 |
|            | 768 × 512 RGB | 20.44      | 653.47     | 50.99      | 112.01 | 70.42 |
| AB(9, 9)/σ = 3 | 512 × 512 gray | 1.89       | 53.53      | 6.28       | 6.32  | 7.21 |
|            | 768 × 512 RGB | 11.51      | 352.9      | 44.39      | 55.75 | 56.23 |

Figure 2: Restoration results of the case GB(9, 5)/σ = 5 for images Shepp-Logan, Cameraman and Duck respectively.
Figure 3: Restoration results of the case MB(20, 60)/σ = 5 for images Shepp-Logan, Cameraman and Duck respectively.

Figure 4: Restoration results of the case AB(9, 9)/σ = 3 for images Shepp-Logan, Cameraman and Duck respectively.

4. Conclusion

For an entire image, the spatially fixed regularization parameters don’t perform well both at edges and smooth areas. The larger parameters are favorable to reduce the noise in the smooth area, but they also blur the edges. The small parameters enable regularization based algorithms to sharpen the edges, but the denoising is not sufficient. In this paper, we have presented an automated spatially dependent regularization parameter selection framework for restoring a image from a noisy and blur image. An edge detector with high noise robustness is used to detect the edges of the image then generates an edge information matrix. According to this matrix, the automated spatially dependent binarization regularization parameters are given. With the parameters, the regularization algorithm perform outstandingly both at edges and smooth area. Once fixed the automated spatially dependent parameters, the newly-established model is convex, then it can be solved by sPADMM with a linear-rate convergence. Extensive experiments on different types of blurring kernels and different levels of Gaussian noise have been conducted to show that
our approach is robust and outperforms other state-of-the-art deblurring methods. In addition, our proposed model not only effectively overcome the false edges and staircase effect but also make up for the same insufficiency of the harmonic model’s diffusion ability in all directions, and better protect the edge to restore the internal smooth area. Due to the limited space, we only display the experimental results in three cases: Gaussian blur $GB(9, 5)/\sigma = 5$, Motion blur $MB(20, 60)/\sigma = 5$ and Average blur $AB(9, 9)/\sigma = 3$. The future work will includes:

1. We will accelerate our algorithm to shorten the recovery time.
2. For remaining more image details, we will examine an edge detector with higher accuracy and robustness and automated spatially dependent parameters according to image texture.
3. Our model is now only suitable for non-blind deblurring, and we will extend it to blind deblurring.

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References

[1] R. Acar and C. Vogel. Analysis of bounded variation penalty methods for ill-posed problems. Inverse Problems, 10:1217–1229, 1994.

[2] A. Almansa, C. Ballester, V. Caselles, and G. Haro. A tv based restoration model with local constraints. Journal of Scientific Computing, 34(3):209–236, 2008.

[3] F. Benvenuto, A La Camera, C. Thys, A. Ferrari, H Lantéri, and M. Bertero. The study of an iterative method for the reconstruction of images corrupted by poisson and gaussian noise. Inverse Problems, 24(3):35016–35020, 2008.

[4] M. Bertalmio, V. Caselles, B. Rougé, and A. Solé. Tv based image restoration with local constraints. Journal of Scientific Computing, 19:95–122, 2003.

[5] Stephen Boyd, Neal Parikh, Eric Chu, Borja Peleato, and Jonathan Eckstein. Distributed optimization and statistical learning via the alternating direction method of multipliers. Foundations & Trends in Machine Learning, 3(1):1–122, 2010.

[6] Jian Feng Cai, Raymond H. Chan, and Mila Nikolova. Two-phase approach for deblurring images corrupted by impulse plus gaussian noise. Inverse Problems and Imaging, 2(2):187–204, 2008.

[7] Jian Feng Cai, Raymond H. Chan, and Mila Nikolova. Fast two-phase image deblurring under impulse noise. Journal of Mathematical Imaging and Vision, 36(1):46–53, 2010.

[8] Jian Feng Cai, Bin Dong, Stanley Osher, and Zuowei Shen. Image restorations: Total variation, wavelet frames, and beyond. Journal of the American Mathematical Society, 25(2):1033–1089, 2012.

[9] Jian Feng Cai, Stanley Osher, and Zuowei Shen. Linearized bregman iterations for frame-based image deblurring. SIAM Journal on Imaging Sciences, 2(1):226–252, 2009.

[10] Jian Feng Cai, Stanley Osher, and Zuowei Shen. Split bregman methods and frame based image restoration. SIAM Journal on Multiscale Modeling & Simulation, 8(2):337–369, 2009.

[11] Antonin Chambolle. An algorithm for total variation minimization and applications. Journal of Mathematical Imaging and Vision, 20:89–97, 2004.

[12] Dali Chen, Shenshen Sun, Congrong Zhang, Yang Quan Chen, and Dingyu Xue. Fractional-order tv-l2 models for image denoising. Central European Journal of Physics, 11(10):1414–1422, 2013.

[13] Kostadin Dabov, Alessandro Foi, Vladimir Katkovnik, and Karen Egiazarian. Color image denoising via sparse 3d collaborative filtering with grouping constraint in luminance-chrominance space. In 2007 IEEE International Conference on Image Processing, volume 1, pages I–313. IEEE, 2007.

[14] Kostadin Dabov, Alessandro Foi, Vladimir Katkovnik, and Karen Egiazarian. Image denoising by sparse 3-d transform-domain collaborative filtering. IEEE Transactions on image processing, 16(8):2080–2095, 2007.

[15] Yiqiu Dong, Torsten Görtner, and Stefan Kunis. An algorithm for total variation regularized photoacoustic imaging. Advances in Computational Mathematics, 41:423–438, 2015.

[16] Yiqiu Dong, Michael Hintermüller, and M. Monserrat Rincon-Camacho. Automated regularization parameter selection in multi-scale total variation models for image restoration. Journal of Mathematical Imaging & Vision, 40(1):82–104, 2011.

[17] Maryam Fazel, Ting Kei Pong, Defeng Sun, and Paul Tseng. Hankel matrix rank minimization with applications to system identification and realization. SIAM Journal on Matrix Analysis and Applications, 34(3):946–977, 2013.

[18] Ali Gholami and S. Mohammad Hosseini. A balanced combination of tikhonov and total variation regularizations for reconstruction of piecewise-smooth signals. Signal Processing, 93(7):1945–1960, 2013.

[19] Tom Goldstein and Stanley Osher. The split bregman method for l1 regularized problems. SIAM Journal on Imaging Sciences, 2(2):323–343, 2009.

[20] Zheng Gong, Zuowei Shen, and Kim-Chuan Toh. Image restoration with mixed or unknown noises. SIAM Journal on Multiscale Modeling and Simulation, 12(2):458–487, 2014.

[21] Deren Han, Defeng Sun, and Liwei Zhang. Linear rate convergence of the alternating direction method of multipliers for convex composite programming. Mathematics of Operations Research, 43(2):622–637, 2018.

[22] Jie Huang, Marco Donatelli, and Raymond Chan. Nonstationary iterated thresholding algorithms for image deblur-
[23] Yumei Huang, Michael K. Ng, and You-Wei Wen. A fast total variation minimization method for image restoration. *SIAM Journal on Multiscale Modeling and Simulation*, 7(2):774–795, 2008. 2

[24] Tongtong Jia, Yuying Shi, Yonggui Zhu, and Lei Wang. An image restoration model combining mixed 1/12 fidelity terms. *Journal of Visual Communication and Image Representation*, 38:461–473, 2016. 1

[25] Fang Li, Chaomin Shen, Jingsong Fan, and Chunli Shen. Image restoration combining a total variational filter and a fourth-order filter. *Journal of Visual Communication and Image Representation*, 18(4):322–330, 2007. 2

[26] Min Li, Defeng Sun, and Kim-Chuan Toh. A majorized admm with indefinite proximal terms for linearly constrained convex composite optimization. *SIAM Journal on Optimization*, 26(2):922–950, 2016. 2

[27] Jingjing Liu, Yifei Lou, Guoxi Ni, and Tieyong Zeng. An image sharpening operator combined with framelet for image deblurring. *Inverse Problems*, 36(4), 2020. 2, 5, 6, 7, 8

[28] Kui Liu, Jieqing Tan, and Liefu Ai. Hybrid regularizers-based adaptive anisotropic diffusion for image denoising. *SpringerPlus*, 5(1):1–24, 2016. 2

[29] Jitendra M. Malik and Pietro Perona. Scale-space and edge detection using anisotropic diffusion. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 12(7):629–639, 1990. 2

[30] Jin Jin Mei, Yiqiu Dong, and Ting Zhu Huang. Simultaneous image fusion and denoising by using fractional-order gradient information. *Journal of Computational and Applied Mathematics*, 351:212–227, 2018. 2

[31] Marius Lysaker, Arvid Lundervold, and Xue-cheng Tai. Noise removal using fourth-order partial differential equation with applications to medical magnetic resonance images in space and time. *IEEE Transactions on Image Processing*, 12(12):1579–1590, 2003. 2

[32] Liyan Ma, Li Xu, and Tieyong Zeng. Low rank prior and total variation regularization for image deblurring. *Journal of Scientific Computing*, 70:1336–1357, 2017. 2

[33] Jitendra M. Malik and Pietro Perona. Scale-space and edge detection using anisotropic diffusion. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 12(7):629–639, 1990. 2

[34] Zhou Wang, Alan Conrad Bovik, Hamid Rahim Sheikh, and Eero P. Simoncelli. Image quality assessment: from error visibility to structural similarity. *IEEE Transactions on Image Processing*, 13(4):600–612, 2004. 5

[35] Chunlin Wu, Zhifang Liu, and Shuang Wen. A general truncated regularization framework for contrast-preserving variational signal and image restoration: Motivation and implementation. *Science China Mathematics*, 61(9):1711–1732, 2018. 5, 6, 7, 8

[36] Chunlin Wu and Tai Xue-Cheng. Augmented lagrangian method, dual methods, and split bregman iteration for rof, vectorial tv, and high order models. *SIAM Journal on Imaging Sciences*, 3(3):300–339, 2010. 2

[37] Xiongjun Zhang, Minru Bai, and Michael K. Ng. Nonconvex-tv based image restoration with impulse noise removal. *SIAM Journal on Imaging Sciences*, 10(3):1627–1667, 2017. 2

[38] Shixiu Zheng, Zhenkuan Pan, Guodong Wang, and Xu Yan. A variational model of image restoration based on first and second order derivatives and its split bregman algorithm. In *International Conference on Audio, Language and Image Processing (ICALIP)*, pages 860–865, 2012. 2