Testing Cosmology with Double Source Lensing

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Double source lensing provides a dimensionless ratio of distance ratios, a “remote viewing” of cosmology through distances relative to the gravitational lens, beyond the observer. We use this to test the cosmological framework, particularly with respect to spatial curvature and the distance duality relation. We derive a consistency equation for constant spatial curvature, allowing not only the investigation of flat vs curved but of the Friedmann-Lemaître-Robertson-Walker framework itself. For distance duality, we demonstrate that the evolution of the lens mass profile slope must be controlled to \(\gtrsim 5\) times tighter fractional precision than a claimed distance duality violation. Using \textsc{LensPop} forecasts of double source lensing systems in Euclid and LSST surveys we also explore constraints on dark energy equation of state parameters and any evolution of the lens mass profile slope.

I. INTRODUCTION

The Friedmann-Lemaître-Robertson-Walker (FLRW) framework for describing the cosmology of our universe is highly successful. Considerable effort is dedicated toward determining the parameters of the cosmology, e.g. the energy densities of the components and their equations of state \cite{1}. Another path for cosmological investigation is to test the framework itself, e.g. testing whether gravity follows general relativity, whether spatial sections have constant curvature (flat or otherwise), and whether photons propagate along null geodesics. This can include a wide variety of techniques such as testing geometric quantities vs growth quantities, e.g. \cite{2–4}.

Here we focus on testing the broad framework, and in particular probing spatial curvature in a general manner as well as photon propagation in terms of the distance duality relation. A variety of probes can be used in such tests, with systematics needing to be stringently controlled for each probe. The more independent of cosmological and astrophysical parameters a probe is, the more general its conclusions may be. We therefore focus primarily on model independent constraints rather than parameter estimation (see e.g. \cite{5} for a review of parametric estimates of curvature), and geometric probes such as distance measures.

One distance measure stands out by being dimensionless and hence not dependent on an absolute scale to “anchor” it: double source lensing (DSL; \cite{6}). With DSL one can form a dimensionless quantity \(\beta\) from the observable image positions associated with each source strongly gravitationally lensed by a common foreground galaxy. Such a quantity is a ratio of distance ratios seen by the lens, i.e. a “remote viewing” of the universe, and furthermore is independent of the Hubble constant and fairly insensitive to the exact lens mass model (and even lens and source redshifts to some extent). As a geometric probe, it is well suited to testing the spacetime framework. Our goal is to supplement, not supplant, other probes, and enable a crosscheck in a more model independent manner.

In Section II we briefly review double source lensing. Section III forecasts expectations for the data set that upcoming wide-field surveys will deliver. Section IV explores the cosmological parameter leverage of DSL in constraining dark energy using the DSL forecasts. In Section V we apply DSL to probing spatial curvature, deriving a consistency test able to decide between a flat universe, one with constant curvature, and one that breaks the FLRW framework. Section VI turns to the question of photon propagation and the distance duality relation, in particular examining the systematics requirements for measuring a violation. We summarize and conclude in Section VII.

II. DOUBLE SOURCE LENSING

Lensed image positions, related to the Einstein radii of the circle of light imaged from a source aligned with the lens and observer, are observables. They depend on the mass of the lens and cosmological distances. Therefore, if one knows the mass of a lens, the Einstein radius is a direct geometrical probe of cosmology. However astrophysical lenses are galaxies or bigger and the masses of such objects are rarely known with sufficient precision to break the
mass-cosmology degeneracy inherent to the Einstein radius. Where multiple sources are present at different distances behind the same lensing mass this degeneracy can be lifted by taking the ratio of light deflection angles (Einstein radii in special cases) \[6\]. At high precision, the method also requires understanding the lens density profile and of perturbative lensing from the sources and other mass along the line of sight. For a two-source plane system with one primary lens, the lens equations for the sources are

\[ y = x - \beta_{12} \alpha_l(x) \]
\[ z = x - \alpha_0(x) - \alpha_{s1}(x - \beta_{12} \alpha_l(x)) \]

where \( x \) are positions on the image plane (as seen by the observer), \( y \) (\( z \)) is the unlensed position of the first (second) source, \( \alpha_l(x) \) is the reduced deflection at \( x \) caused by the primary lens acting on the final source plane, and \( \alpha_{s1} \) is the lensing deflection effect of mass of the first source. The primary quantity of cosmological interest, the ratio of distance ratios \( \beta \) will be used as the DSL probe, with

\[ \beta_{12} \equiv \beta(z_l, z_{s1}, z_{s2}) = \frac{r_{ls}(z_l, z_{s1})}{r_s(z_{s1})} \frac{r_{s}(z_{s2})}{r_{ls}(z_l, z_{s2})} \]

where \( r \) are the comoving distances. Note that \( \beta \) is the cosmological scaling factor relating the reduced deflection angles between two source planes (the physical deflection caused by the lens is independent of the source redshift). For a singular isothermal sphere lens mass profile this is the ratio of Einstein radii, but \( \beta \) is more fundamental than that as it is the term that enters into the multiplane lens equations above regardless of deflector mass profiles. It is measured during the fitting of a multiplane lens model to the imaging data. The reduced sensitivity of \( \beta \) to the mass profile(s) is key, as that is the dominant systematic in lensing. And as outlined in Sec. I and expanded upon in Secs. V and VI, its dimensionless and “remote viewing” nature are particularly valuable for model independent tests.

Figure 1 shows how \( \beta \) depends on the lensing system redshifts \( z_l, z_{s1}, z_{s2} \). For a broad range of parameter space \( \beta \) stays near 0.6–0.8. We also see that for given ratios \( z_{s1}/z_l \) and \( z_{s2}/z_{s1} \) the quantity \( \beta \) is quite insensitive to the lens redshift. DSL also has good complementarity with time delay distances, also from strong lensing, as seen in \[7\], is a valuable crosscheck on the use of time delay lensing in testing the cosmological framework \[8\], and has interesting leverage at high redshift \[9\].

III. FORECASTING THE FUTURE DOUBLE SOURCE PLANE POPULATION

Currently, only a small sample of galaxy scale DSL systems is known, and only the so-called Jackpot \[10–12\] has been used to precisely constrain cosmology. Collett & Auger \[13\] modeled the HST imaging, performing a pixelated reconstruction of both sources, and made a 1.1% measurement of \( \beta \). Converting this into constraints on dark energy, this single DSL, with a cosmic microwave background (CMB) data prior from \[14\], constrains a constant dark energy equation of state to \( \sigma(w) \approx 0.2 \). This number is from Collett & Auger \[13\] and actual data. While we do not focus here on parameter estimation, the important point is that future surveys are forecast to increase the known galaxy scale population by \( \sim 100 \times \) \[15\], with a similar increase in the compound lens population including DSL.

The Euclid satellite \[16\] should discover \( \sim 1700 \) galaxy-scale DSL systems that are suitable for cosmology. Rubin Observatory’s Legacy Survey of Space and Time (LSST) is forecast to discover a similar number of systems \[17\]. (Galaxy scale systems strongly lensing more than two sources are much rarer and we do not include them.) These forecasts are derived from the LENSPOP package \[15\] modified to include multiple background sources.

The LENSPOP approach assumes that all lens galaxies are singular isothermal ellipsoids, with velocity dispersions and ellipticities drawn from the population of elliptical galaxies observed by the Sloan Digital Sky Survey \[18\]. Potential lenses are assumed to be uniformly distributed in co-moving volume. Sources are elliptical exponential profiles, with number densities and colours drawn from the LSST galaxy simulations of \[19\].

Observations of these idealized lens systems are then simulated by mocking the LSST and Euclid point spread function and background noises. A lensed source is deemed to be detected if the signal-to-noise is greater than 20, the Einstein radius is greater than twice the seeing and the magnification is greater than 3. When simulating DSLs with LENSPOP, we neglect the mass effect of the first source \[20\]. We also use more stringent constraints than the LENSPOP defaults: we insist that both sources have one or more image arcs of length 0.3 arcseconds – this ensures a reasonable possibility that the density slope of the lens can be recovered from high-resolution imaging alone.

Since follow-up is needed for cosmological inference, we restrict ourselves to the smaller sample of 87 DSL forecasts, representing the number estimated to be discovered in the best-seeing single epoch imaging of LSST. These DSL were used in the LSST Science Requirements Document \[17\] and represent a conservative lower limit for the number of DSL that will have the high-resolution imaging and spectroscopic follow-up that is critical to do precision cosmology with LSST DSLs. (The James Webb Space Telescope and ground based extremely large telescopes could further
FIG. 1. Isocontours of $\beta$ are plotted vs $z_s/z_l$ and $z_{s2}/z_{s1}$ for three lens redshifts $z_l = 0.3, 0.75, \text{and } 1$. The contours vary very little with lens redshift $z_l$ for fixed ratios $z_{s1}/z_l$ and $z_{s2}/z_{s1}$. The DSL parameter $\beta$ shows a broad flat valley between 0.6–0.8 over a wide range of parameter space. The circles represent the mock data introduced in Secs. III and IV, leaving off those outside the plot bounds. The radius of the circles is proportional to the simulated $1/\sigma(\beta)$, so more precise data are more prominent, and the colors correspond to $z_l = [0.2, 0.3]$ (red), [0.3, 0.5] (blue), and $z_l > 0.5$ (cyan). 90% of the data have $\sigma(\beta) < 1.7\%$.

substantially improve the constraining power of each system.) We overplot the simulated systems in Fig. 1, for those within the axis range, with the radius of the circles proportional to $1/\sigma(\beta)$, so more visible data are more precise. The choice of 87 DSLs is likely to be extremely conservative by the late 2020s: Euclid is expected to find 1700 DSLs, and will provide high resolution imaging of each of them. The 4MOST Strong Lens Spectroscopic Legacy Survey will provide spectroscopic redshifts for tens of thousands of lenses, including compound lenses. Together these surveys will deliver a much larger sample of DSLs without the need for additional followup.

Cosmological forecasts also require us to know the precision with which $\beta$ will be constrained in each DSL. Simulating this inference is beyond the scope of this work, we instead adopt a simple approximation: we assume that $\beta$ in each system can be independently constrained with a fractional precision of the quadrature sum of the uncertainties $0.01(\text{arcsec}/\theta_E,1)$, $0.01(\text{arcsec}/\theta_E,2)$, and 0.01. The first two terms mimic the uncertainty with which Einstein radii are likely to be measured with Euclid whilst the extra 1% sets a floor from the uncertainty on the inferred density profile of the lenses. The results with HST [13] and simulations of Euclid time delay lenses [21] indicate that these uncertainties are not unreasonable. In fact, they may be somewhat conservative – compare the 1.1% precision obtained in 2014 [13].

IV. CONSTRAINING DARK ENERGY

As was pointed out in [6] and [7], DSL provide an independent and complementary cosmological probe from the standard ones. While they will be highly valuable in the consistency tests of the following sections, and as complementary data and as crosschecks, we briefly leave our model independent focus and explore their direct cosmological parameter leverage due to their unique combination of distance ratios. Figure 2 presents the results of Markov chain Monte Carlo constraints from 87 simulated DSL as described in Sec. III on the matter density and dark energy equation of state. For a spatially flat $\Lambda$CDM cosmology (with a constant dark energy equation of state, $w$, not fixed to $-1$) and assuming a uniform prior on $\Omega_M$ between 0 and 1, we find $w = -1.09^{+0.15}_{-0.29}$. Allowing the equation of
state to vary with the scale factor of the universe yields a constraint on its current value of $w_0 = -1.01^{+0.31}_{-0.32}$, whereas the derivative is poorly constrained: $w_a = 0.6^{+1.0}_{-2.0}$ (assuming a uniform prior between -2.5 and 2.5). Much of the posterior weight is in regions of the parameter space that are already strongly excluded by other cosmological data sets. This is particularly the case for the region with both $w_0 \gtrsim 1$ and $w_a \gtrsim -0.7$ which is ruled out by evidence for the existence of a matter dominated era in our Universe. For regions of the parameter space close to ΛCDM, an interesting behavior is observed: the $w_0$ constraint is almost insensitive to $w_a$. Assuming $|w_a| < 0.5$ yields very similar constraints on $w_0$ as setting $w_a \equiv 0$. For parameter fitting (as opposed to our main focus of model independent tests of the cosmological framework), much of the leverage of DSL will be in breaking degeneracies from other probes.

V. SPATIAL CURVATURE CONSISTENCY TEST

In addition to the standard cosmology parameter constraint estimation, we can also explore consistency tests of the FLRW framework. In this section and the following one, this is our main focus. Our universe appears to be consistent with being spatially flat, i.e. zero spatial curvature $k$ in the Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dx^2}{1- kx^2} + x^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right],$$

where $a(t)$ is the scale or expansion factor and $x$ is the coordinate distance while the first quantity in the spatial part of the metric gives the comoving distance element (squared) while the second quantity gives the usual spherically symmetric angular part. We can also write $k$ in terms of a spatial curvature quantity $\Omega_k = -k/(a_0^2 H_0^2)$, where $H_0$ is the Hubble scale today (and we are free to define $a_0 = 1$). Since $k$ has the dimensions of $1/(\text{distance})^2$ then one can write an effective curvature energy density $\Omega_{\text{curv}}(z) = \Omega_k a^{-2} = 1 - \Omega_{\text{total}}(z)$ scaling as $a^{-2} \sim (1+z)^2$, where $z$ is the redshift. Here $\Omega_{\text{total}}$ is the total energy density, the sum of the component energy densities.

Two important questions then are whether $k = 0 = \Omega_k$, hence $\Omega_{\text{total}}(z) = 1$ for all times, and whether the Robertson-Walker metric lying at the heart of the FLRW framework is correct in taking $k$, equivalently $\Omega_k$, as constant. One can certainly attempt to determine the value of the $\Omega_k$ parameter from observational data and there is a large and manifold effort to do so (e.g. [5, 23] and many others). Instead we focus on the framework itself: if we determine $\Omega_k$ from the data through a relation between observables (rather than parameter fitting), is it the same value at different
In the FLRW framework, independent of the specific energy density contents. This can be thought of as triangulating points on a surface to determine its curvature (see discussions by [8, 24]), and so we see that we need not simply distances from the observer, but distances between remote points to form a triangle. Such remote distances can be provided by gravitational lensing, where the distance between the lens and the source enters. Curvature consistency tests directly using observables have explored the use of time delay distance from strong lens systems, e.g., [8, 25–28]. This quantity does however have sensitivity to $H_0$ and the lens mass model.

The dimensionless ratio of distance ratios $\beta$ from double source lensing is an interesting alternative (it was briefly considered in [28] but used to fit the value of $\Omega_k$ rather than for a redshift dependent consistency test). Since $\beta$ involves four distances – the distance to each of the two sources from the observer, $r_{s1}$ and $r_{s2}$, and from the common lens, $r_{l1}$ and $r_{l2}$ – this method actually uses observations to form a concave quadrilateral rather than a triangle.

The remote comoving distances are related to the curvature in the FLRW framework by
\begin{equation}
r_{ls} = r_s \sqrt{1 + \Omega_k r_l^2} - r_l \sqrt{1 + \Omega_k r_s^2}.
\end{equation}

In terms of the observables $\beta$ and the various distances we can solve for the spatial curvature $\Omega_k$. While we obtain $\beta$ directly from the measurement of a DSL system, and can get distance measurements from standardized candles (Type Ia supernovae: SN) or rulers (baryon acoustic oscillations: BAO), those distances will not in general be at the precise redshifts of the lens and sources in the DSL system. We, therefore, need a nonparametric method of obtaining the desired distances; frequently Gaussian processes are employed for this (see, e.g., [29–32] and many others).

Given the observational data, the spatial curvature consistency relation for all redshifts is
\begin{equation}
\Omega_k = \frac{(1 - \beta)^4 r_l^{-4} - 2(1 - \beta)^2 r_l^{-2} \left(\beta^2 r_s^{-2} + r_s^{-2}\right) + \left(\beta^2 r_s^{-2} - r_s^{-2}\right)^2}{4\beta(1 - \beta) \left[\beta r_s^{-2} + (1 - \beta) r_l^{-2} - r_s^{-2}\right]}.
\end{equation}

Again we emphasize that this must hold in the FLRW framework, independent of the specific energy density components and parameter values. Any statistically significant deviation from constancy of $\Omega_k$ points to either a measurement systematic or a violation of the FLRW framework. A constant value of $\Omega_k \neq 0$ obtained over a wide range of redshifts would point to potentially more robust evidence against flatness than a standard parameter fit analysis.

From measurements of the distances and $\beta$, we can propagate their uncertainties to $\Omega_k$. Since they come from different measurements, plus the redshifts are well separated, it is a reasonable assumption to take the uncertainties to be independent. Thus we can add them in quadrature when propagating them to $\sigma(\Omega_k)$.

Figure 3 shows the result for 1% precision on each of the four observable quantities, with $\beta$ taken to be from a single DSL system having the $z_l$ shown. For simplicity we set $z_{s1} = 2z_l$ and $z_{s2} = 1.5z_{s1}$ as reasonable rules of thumb (this puts $\beta$ in the broad valley of Fig. 1, and see also [9]). Note that $\sigma(\Omega_k)$ improves for higher redshift systems, leveling off at $\sigma(\Omega_k) \approx 0.2$ for $z_l \gtrsim 0.9$. The uncertainty will reduce with multiple DSL, so (if all uncertainties can be reduced statistically) for example 16 DSL at $z_l = 0.9$ will deliver $\sigma(\Omega_k) \approx 0.05$ if the measurements are independent, and this can be carried out at multiple lens redshifts. The actual result will depend on where the systematics limits exist for the distances and the DSL distance ratio. High redshift lenses at $z_l \gtrsim 0.8$, such as should be readily found by Euclid, will be especially useful.

Following [8] we can define a redshift dependent curvature quantity $K$ that is zero for a flat universe. This “$K$ test” gives a somewhat different window on spatial curvature in that $K$ has specific redshift dependence predicted within the FLRW framework. Therefore one can use any measured deviations from this – as a function of redshift – as an indicator of issues with the framework, or observations. The $K$ test in terms of the observables is (compare Eq. 3)
\begin{equation}
K \equiv \beta - 1 - \frac{r_l}{r_{s1}} - \frac{r_l}{r_{s2}} \approx \frac{r_l r_{s2}}{2 r_{s1}} \left(\frac{r_{s1} - r_l}{r_{s2} - r_l}\right) (r_{s2} - r_{s1}) \Omega_k + \mathcal{O}(\Omega_k^2).
\end{equation}

Note that we always use the full form of $K$ to test curvature, with the first-order expansion just shown for intuition. In addition to the different redshift weighting mentioned above, the $K$ test also has a different weighting on data than Eq. (6) and so it has the potential to provide a crosscheck or different view of curvature.
FIG. 3. The uncertainty $\sigma(\Omega_k)$ from a single DSL with $\beta$ measured to 1% (plus comoving distance measurements to 1% from SN and BAO data) is plotted vs lens redshift $z_l$. The uncertainty will roughly square root down with the number of observations at each redshift, down to some systematics limit.

Figure 4 shows these two curvature quantities and their uncertainties as a function of $\Omega_k$ for a DSL system with $z_l = 0.75$, $z_{s1} = 1.5$, $z_{s2} = 2.25$. As before, for simplicity, we adopt a fractional $\beta$ precision of 1%, and lens and source distance precisions (from, e.g., supernova or BAO distances) of 1%. The intersection point between the curvature quantity ($\Omega_k$ or $K$) curve and its uncertainty curve determines the lower bound on the ability to measure the curvature parameter for a given, single DSL system. This is made more explicit in the right panel showing signal to noise like ratios. While a single DSL system has $S/N < 1$, multiple DSL can be used in these curvature consistency tests. Note that unlike the time delay distance consistency test of [8], the quantities $\Omega_k$ and $K$ do carry somewhat distinct information, with $K$ having more leverage at higher curvature amplitudes. One can certainly imagine applying one curvature consistency test with time delay distances and performing a crosscheck on any violation with DSL distance ratios.

While statistically the uncertainty estimation on curvature will never be as good as a model dependent parameter estimation (Planck [22] provides 0.016 uncertainty for $\Lambda$CDM, but this loosens by a factor two for $w$CDM, and even more so for $w_0w_a$CDM), these consistency tests do provide a more robust method will be highly valuable if evidence is presented from parameter estimation of either deviation from flatness or especially violation of the FLRW framework.

VI. DISTANCE DUALITY CONSISTENCY TEST

The DSL distances ratio involves angular diameter distances entering the light deflection. We can explore a consistency test between angular diameter distances and luminosity distances, such as from Type Ia supernovae, in a similar way to what we did in the previous section with spatial curvature. Converting both to comoving distances, we have

$$r_i^{SN} = d_L/(1 + z_i) \quad ; \quad r_i^\beta = d_A (1 + z_i) .$$

We expect consistency in terms of what is often called the distance duality relation $d_L = d_A (1 + z)^2$, attributable to Tolman, Ruse, Etherington [33–35] in various degrees of generality and with formal proofs by [24, 36]). This relation should hold when four very general conditions are valid (also see the pedagogical discussion in [37]): 1) metricity, 2) geodesic completeness, 3) photons propagate on null geodesics, and 4) adiabaticity. Violation of any one would have revolutionary consequences for cosmology.

Nevertheless, we can test this by writing

$$r_i^{SN} = r_i^\beta (1 + z)^\epsilon ,$$
and seeing if $\epsilon = 0$ is consistent with data. (See, e.g., [38–41] for a selection of other analyses using strong lensing.) That is, from supernova distances we predict what $\beta$ we should get, and compare it to the observed $\beta$. However, we emphasize that any discrepancy is not automatically interpretable as a violation of distance duality. We therefore consider and compare three sources of any such discrepancy: 1) violation of the distance duality, $\epsilon \neq 0$, 2) incorrect cosmological model assumed, specifically $\Omega_k \neq 0$, and 3) systematics.

Let’s begin with systematics. As an example we consider astrophysics in the form of an evolution in power law slope of the lens mass profile with redshift,

$$\gamma = 2 + \mu \left[ \frac{1 + z_l}{1.4} \right]^2 - 1. \quad (10)$$

Thus, the mass density profile near the Einstein radius, $\rho \sim r^{-\gamma}$, may have $\gamma = 2$ (the singular isothermal sphere, or SIS, value) at $z = 0.4$, but evolves from approximately $2 - \mu/2$ at $z = 0$ to $2 + 2\mu$ at $z = 1.5$. This is just a toy model to illustrate systematics, e.g. for $\mu = 0.05$ the slope would evolve from $\gamma \approx 1.975$ to 2.1. As the mass density profile affects the light deflection angle and hence Einstein radius, measuring $\beta$ as the ratio of Einstein radii would give

$$\beta(\mu) = [\beta(\mu = 0)]^{1/[\gamma(z_e^2) - 1]} \quad (11)$$

For considering the effect due to spatial curvature, to evaluate $\beta(\Omega_k)$ we simply use the appropriate distances for a universe with curvature parameter $\Omega_k$, as in the previous section.

Finally, in the case of violation of distance duality, the distances ratio predicted by supernova distances is (for a flat universe with $\gamma = 2$ lens mass profile – we vary each potential source of discrepancy one at a time)

$$\beta(\epsilon) = \frac{1 - \frac{r_l}{r_{s1}} \left( \frac{1+z_l}{1+z_{s1}} \right)^\epsilon}{1 - \frac{r_l}{r_{s2}} \left( \frac{1+z_l}{1+z_{s2}} \right)^\epsilon} \quad (12)$$

We can then compare the fractional deviation

$$\left( \frac{\delta \beta}{\beta} \right)_p = \frac{\beta(p) - \beta(0)}{\beta(0)} \quad (13)$$

for each of $p = \epsilon, \Omega_k, \mu$. Figure 5 shows the percent deviation in $\beta$ as a function of each offset, for the particular case of a DSL system with $z_l = 0.75$, $z_{s1} = 1.5$, $z_{s2} = 2.25$. The imparted deviations due to distance duality violation and
curvature are mostly comparable in magnitude, but are swamped by the necessity of tight systematics control. That is, before making any claim for violation of distance duality, one must ensure that the knowledge of the cosmological model (here specifically spatial curvature) and systematics is respectively at least as or much more rigorously known.

Figure 5 shows the dependence on \( z_l \) (keeping the same ratios \( z_{s1}/z_l = 2 \) and \( z_{s2}/z_{s1} = 1.5 \)) of the percent deviation, for the case of \( \epsilon = -0.05 \) or \( \Omega_k = -0.05 \) or \( \mu = +0.05 \). The impacts are roughly linear in \( z_l \). Again we find that equal amplitudes of offset in distance duality violation or curvature cause nearly the same deviation in \( \beta \) regardless of which parameter is offset; however, systematics must be particularly tightly controlled, by roughly five times better. Thus the cosmological model and especially astrophysical and experiment systematics have to be well-known before new physics such as a distance duality violation can be pursued.

One could constrain the lens mass profile slope, and hence \( \mu \), with follow-up observations of the lenses, e.g. through spectroscopy to measure lens kinematics. This is certainly the best approach, but we end by exploring the internal “self calibration” of the lens mass profile through measurements of \( \beta \) itself. For this, we restrict to a standard, flat cosmology and seek to fit \( \mu \), together with the cosmological parameters \( \Omega_m, w_0, \) and \( w_a \), using the mock sample of 87 LSST-like DSL systems. For a quick estimate we use the information matrix formalism, assume uniform 1% uncertainty on each \( \beta \), and study the effect of external priors from the CMB or from cosmic surveys in the form of an effective prior on \( \Omega_m \) of 0.01.

We find the lens mass profile is not strongly covariant with the cosmology parameters, with correlation coefficients \( \gamma_{\mu i} < 0.84 \) for each cosmology parameter \( i \), in all cases. Marginalized constraints are \( \sigma(\mu) \approx 0.022 \) for no or either prior, improving to 0.015 if both priors are applied. Note that \( \mu = 0.022 \) corresponds to the mass profile slope evolving over \( \gamma = [1.99, 2.05] \) from \( z = 0 \) to 1.5. From Fig. 5, however, we see that this would still swamp a comparable amplitude violation of distance duality. Restricting to \( \Lambda \text{CDM} \) we find \( \sigma(\mu) \approx 0.012 \) for no or all priors.

If we do want to carry out the distance duality test robustly, simultaneously fitting for the lens mass profile evolution and (flat) cosmology, we indeed find strong covariance of the distance duality parameter \( \epsilon \) with the mass profile evolution \( \mu \). Figure 7 shows the degeneracy between \( \mu \) and \( \epsilon \) using the 87 DSL forecasts. One can readily see that a vertical cut at \( \mu = 0 \) (i.e. fixing to no evolution) gives much tighter constraints on \( \epsilon \) than in the case marginalizing over \( \mu \). The 1\( \sigma \) marginalized uncertainties are \( \sigma(\epsilon) = 0.628, \sigma(\mu) = 0.029 \) for the case with an added \( \Omega_m \) prior of 0.01. Adding a CMB prior gives similar results, with \( \sigma(\epsilon) = 0.701, \sigma(\mu) = 0.029 \), while using both priors delivers \( \sigma(\epsilon) = 0.291, \sigma(\mu) = 0.029 \).
FIG. 6. Percent deviation in $\beta$ as a function of lens redshift (keeping $z_{s1} = 2z_l$, $z_{s2} = 3z_l$) is shown for offsets from zero of magnitude $-0.05$, $-0.05$, $+0.05$ respectively for distance duality violation $\epsilon$, spatial curvature $\Omega_k$, and mass density profile evolution $\mu$.

FIG. 7. 1$\sigma$ joint confidence contours on the distance duality violation parameter $\epsilon$ and the lens mass profile evolution parameter $\mu$ expected from the 87 DSL considered in this work, with various priors. Note the significant covariance between the parameters, i.e. evidence of new physics is only possible with stringent systematics control.

VII. CONCLUSIONS

Double source lensing offers a promising probe of cosmology, including the FLRW framework itself. It has the useful characteristics of “remote viewing” the universe with some distances independent of the observer, of being geometric, substantially independent of structure growth, and of being dimensionless, independent of the Hubble constant.

The number of DSL will increase some 1000 fold with the advent of wide-field surveys such as Euclid and LSST, though the number of the most robust systems will be limited by follow-up resources, particularly with the inevitable
demise of the Hubble Space Telescope, which has provided much of the high resolution imaging for strong lensing cosmology so far. Even so, we find that a data set of fewer than 100 DSL can have significant impact on not only cosmology parameter estimation (an independent constraint on the dark energy equation of state \( w = -1.09^{+0.15}_{-0.29} \), with this result essentially insensitive to the variation of the equation of state with cosmological scale factor) but also, the focus of this article, testing the fundamental cosmological framework. Furthermore, DSL has good complementarity with other probes [9], including time delay distances from other strong lensing systems [7].

The DSL property of measuring the distance between distant objects allows a form of triangulation (really quadrangulation) enabling tests of the homogeneous, isotropic FLRW framework. One can map spatial curvature quantities as a function of lens redshift – in a model-independent manner, especially valuable since parameter estimation approaches to \( \Omega_k \) tend to be sensitive to what other parameters are allowed to vary. Moreover we show consistency relations that investigate whether each system gives \( \Omega_k = 0 \) – testing flatness – and whether they are all constant – testing FLRW. While a small number of systems gives modest constraints, DSL provides an independent consistency test from other probes (such as time delay distances), and one can build up larger samples given follow-up resources. Moreover, DSL serves as an important crosscheck if a violation of FLRW is detected by another probe, essential for such a revolutionary result.

We show how to check the consistency of foundational light propagation properties, such as that light travels on null geodesics, through the distance duality test. However, we caution, and quantitatively demonstrate, that other cosmological and astrophysical effects can mimic the violation of the distance duality relation if not properly accounted for, such as spatial curvature and evolution in the lens mass profile slope. The latter dominates in general, and such a systematic would need to be controlled at the \( \sim 1\% \) level in order to cleanly observe a distance duality violation even as large as \( \sim 5\% \).

With upcoming surveys delivering new discoveries of DSL and other lens configurations, further probes may be developed as well. With DSL’s ratio of distance ratios, and reduced systematics from lens modeling, the field of strong gravitational lensing beyond the standard one source–one lens paradigm looks quite promising, and worthy of further attention.

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