Modeling of a well-type composite detector

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Abstract
A probabilistic approach has been used for understanding the operation of a well-type composite detector. This detector comprises of identical cubical shaped fourteen detectors. Assuming the isotropic absorption and scattering of gamma-rays, we have estimated performance parameters like the addback factor, peak-to-total ratio and peak-to-background ratio. Our model assume nearest neighbour interactions and we have performed calculations up to fourth order interactions of gamma-rays inside the detector.

Keywords
Modeling, spectrometer, gamma-ray, phenomenology, radiation, spectroscopy.

AMS Subject Classification
60G99, 97M10, 00A71, 97K50.

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1. Introduction

For more than two decades, the composite germanium detectors like the Clover, and Cluster detectors have played a huge role in performing high resolution gamma-ray spectroscopic studies of the atomic nuclei [2]. Using these sophisticated detectors, which comprise of smaller identical high purity germanium (HPGe) detectors arranged in a compact way, we could obtain high detection efficiency without sacrificing the energy resolution [2]. The composite detector has two modes of operation - single detector and addback modes. During the addback mode of operation [2], there is increased full energy peak efficiency and lower background.

Recently, investigations are being carried out with even more sophisticated detectors which are basically complex versions of the series and parallel combination of smaller detectors [3, 4]. Even though a lot of experimental and simulation studies has been carried out to understand the operation and exploit the features of these sophisticated detectors, it would be very interesting to explore the possibilities for developing a mathematical model for them by taking into account the scattering of gamma-rays inside these detectors.

When an incident gamma-ray interacts with one of the detector modules of a composite detector, there can be partial or complete absorption of its energy. Scattered out gamma-ray(s) (corresponding to partial absorption event) can interact with neighbouring module or escape the composite detector completely and is thereby lost. It is possible to estimate the performance parameters, like peak-to-total ratio, peak-to-background ratio and addback factor if one could associate probability with each of the absorption and scattering processes inside the composite detector. Using this philosophy, a simplified formalism has been developed for understanding the operation of various types of composite gamma-ray detectors [5–7]. Recently, using a single probability, we modelled composite detectors – like the pyramidal shaped detectors [8], and the stacked detectors [9]. Recently, we have performed a detailed investigation of a three level well-type composite detector consisting of nineteen detector modules [10]. In the present paper, we will apply the same approach for the modeling of a two level well-type composite detector having fourteen detector modules. The schematic diagrams of the three dimensional and lateral view of this detector are shown in figures 1(a) and 1(b), respectively.
Figure 1. The two level well-type composite detector along with its lateral view are schematically shown in figures (a), and (b), respectively. The direction of the incident gamma-ray is shown by red arrow.

## 2. Modeling of composite detector

Let us assume equal absorption probability for successive gamma scatterings inside the composite detector consisting of identical cubical shaped detector modules. Let the probability of full energy peak absorption in a single detector module be \( x \), then the scattering out probability is \( (1 - x) \).

The fourteen modules of the composite detector have been named: A – N. We will now consider the scenario where \( N \) monoenergetic gamma rays initially interact with module H, such that at a time only one gamma-ray interacts with a module. Figure 2 shows the various possible gamma-ray interactions during the first, second, third and fourth interactions. In these four sub-figures, we have also mentioned the total absorbed and scattered out counts during each interaction, where \( \eta = Nx, \delta = \frac{1}{6}(1 - x) \).

We will now introduce nomenclature for simplification in performing analytical calculations. For a module namely \( Z \), let

- the number of \( \gamma \)-rays absorbed in module \( Z = A_Z \),
- the number of \( \gamma \)-rays scattered from module \( Z = S_Z \),
- the number of \( \gamma \)-rays scattered to module \( Z = S_Z \)

The calculations are quite similar to those of our recent works [8–10]. We have shown below the detailed calculations only for first and second interactions, the rest could be carried our similarly with the help of figure 2.

For first interaction:
\[
\begin{align*}
A_H &= Nx \\
S_H &= N(1-x) \\
S_D &= S_G = S_J = \frac{1}{6}N(1-x)
\end{align*}
\]

For second interaction:
\[
\begin{align*}
A_D &= A_G = A_J = \frac{1}{6}N(1-x)x
\end{align*}
\]

Figure 2. Schematic block diagrams of the gamma-ray interactions with the module H for first, second, third and fourth interactions are shown in figures (a), (b), (c), and (d), respectively. The incident gamma-ray (indicated by thick black arrow) first interacts with module H. The various possible scatterings of gamma-rays from a module to neighbouring module and outside the detector are shown by red and cyan arrows, respectively. The brown arrow represents the scattering of six gamma-rays.
\[ S_D = S_G = S_I = \frac{1}{6} N (1-x)(1-x) \]
\[ S_E = S_A = S_K = S_N = \frac{1}{16} N (1-x)(1-x) \]
\[ S_C = S_J = 2 \times \frac{1}{6} N (1-x)(1-x) \]
\[ S_H = 3 \times \frac{1}{16} N (1-x)(1-x) \]

In case of the interaction of \( N \) mono-energetic incident gamma-rays with module \( G \), the possible gamma-ray interactions are briefly shown in figure 3.

### 3. Discussion

Considering up to the fourth order interactions of gamma-rays, we could obtain expressions for the total absorbed counts, and the total counts contributing to the background. Using these results, we could estimate the addback factor, peak-to-total and peak-to-background ratios. Using figures 2 and 3, we get

**Total absorbed counts**

\[
\begin{align*}
\text{Total absorbed counts} &= 2\eta + 6\eta \delta + 20\eta \delta^2 + 58\eta \delta^3 \\
&= 2N(x + \alpha) \\
\text{where } \alpha &= x\delta[3 + 10\delta + 29\delta^2] \\
\text{Peak-to-total ratio} &= \frac{2N(x+\alpha)}{2N} = x + \alpha
\end{align*}
\]

The addback factor \( f \), defined as the ratio of FEP efficiency in addback mode to that in single detector mode, given by

\[
\begin{align*}
\text{f} &= \frac{2N(x+\alpha)}{2N} \\
&= 1 + \frac{x\delta[3 + 10\delta + 29\delta^2]}{x} \\
&= 1 + 3\delta + 10\delta^2 + 29\delta^3
\end{align*}
\]

Similarly, **Total scattered counts**

\[
\begin{align*}
\text{Total scattered counts} &= 6N\delta + 16N\delta^2 + 62N\delta^3 + 180N\delta^4 \\
&= 2N \beta \\
\text{where } \beta &= \delta \psi, \text{ and } \psi = 3 + 8\delta + 31\delta^2 + 90\delta^3
\end{align*}
\]

**Peak-to-background ratio**

\[
\begin{align*}
\text{Peak-to-background ratio} &= \frac{2N(x+\alpha)}{2N \beta} = \frac{x + \alpha}{\beta}
\end{align*}
\]

Note that \( \alpha \) is the contribution to FEP due to multiple hit event, while \( \beta \) is the contribution to background. The expression for addback factor shows that the minimum value of \( f \) is 1.0 at \( x = 1.0 \), while the upper limit is 1.91, obtained for \( x = 0 \), which corresponds to the maximum possible energy in our model where the isotropic scattering will be valid. Using the above results, it is possible to calculate the relative contribution of absorbed counts as well as scattered counts during the first-fourth interactions. As an example, the relative contribution of absorbed counts for third interaction to total absorbed counts is \( \frac{10\delta^2}{f} \), while the relative contribution of scattered counts for third interaction to total scattered counts is \( \frac{31\delta^2}{\psi} \).

**Figure 3.** Schematic block diagrams of the gamma-ray interactions with the module \( G \) for first, second, third and fourth interactions are shown in figures (a), (b), (c), and (d), respectively.
4. Summary and Conclusion

A simple model based on probabilistic understanding of the gamma interaction process in a medium, has been presented for understanding the operation of a fourteen element well-type composite gamma detector in addback mode. Considering isotropic scattering of gamma-rays and their partial as well as complete absorption, expressions are obtained for peak-to-total, peak-to-background ratios and addback factor, in terms of the absorption probability in a single detector module.

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