Thermonuclear burn-up in deuterated methane CD$_4$.

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(Dated: August 10, 2010)

Abstract

The thermonuclear burn-up of highly compressed deuterated methane CD$_4$ is considered in the spherical geometry. The minimal required values of the burn-up parameter $x = \rho_0 \cdot r_f$ are determined for various temperatures $T$ and densities $\rho_0$. It is shown that thermonuclear burn-up in CD$_4$ becomes possible in practice if its initial density $\rho_0$ exceeds $\approx 5 \cdot 10^3$ g·cm$^{-3}$. Burn-up in CD$_2$T$_2$ methane requires significantly ($\approx 100$ times) lower compressions. The developed approach can be used in order to compute the critical burn-up parameters in an arbitrary deuterium containing fuel.

PACS: 52.35.Tc and 28.52.Cx

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In this communication the thermonuclear burn-up in highly compressed CD$_4$ (and CD$_2$T$_2$) methane is considered. Our goal is to determine the minimal required values for the burn-up parameter $x$ (where $x = \rho_0 \cdot r_f$), density $\rho_0$ and temperature $T$ at which thermonuclear burning starts locally and then propagates successfully to the rest of the highly compressed CD$_4$ methane. Let us assume that initially we have the infinite, homogeneous, immovable CD$_4$ (and CD$_2$T$_2$) methane (gas), which is assumed to be cold at time $t = 0$. The burn-up problem is considered in the following form. Inside homogeneous immovable CD$_4$ methane a quite large amount of energy $Q_0$ is instantaneously released in a very small (point) volume. The temperature in this volume increases very rapidly to extremely large values. As a result a high-temperature thermal wave forms, which begins to propagate from this very hot volume into the rest of the cold methane. After some time this wave transforms into the shock wave, which propagates and disappears, in the general case, in the cold methane.

However, if the three following conditions are obeyed: (1) the initial density of the highly compressed methane $\rho_0$ is quite large, (2) the temperature in the central, heated region exceeds the ignition temperature $T \geq T_c$ and (3) its volume is large enough $V \geq V_c$, then the thermonuclear burn wave may form and propagate out, igniting the rest of the cold CD$_4$. In general, the thermonuclear burn wave propagates initially as a thermal wave, and later as a detonation wave, i.e. the shock wave which supports itself [1] - [2].

In this study we want to determine the critical conditions, i.e. the minimal possible $\rho_0, T_c$ and $V_c$ values at which the combustion zone still expands to the infinity in the cold CD$_4$ or CD$_2$T$_2$ methane, respectively. In the general case, this problem is very complicated, but here we restricted ourselves to the spherical geometry only. This means that the initial hot spot has a spherical form, and later, the expanding thermal or detonation waves also remain spherically symmetric. In this case the arising gas flow is one-dimensional, and therefore, the appropriate equations can be also written in the one-dimensional form (see below). Finally, the initially complicated problem of $V_c$ determination is reduced to the one dimensional and relatively simple problem: to find a critical value for some linear $r_c$ parameter, which can be called by the critical radius of the initial hot spot. The consideration of the cylindrical and planar cases can be based on analogous assumptions, but our present study is concentrated mainly on the spherical symmetry, since this case is of specific interest for various applications in practice.

As is well known (see e.g. [1], [2]) high-temperature burn-up in thermonuclear fuel is
governed by the so-called burn-up equation. Let $r_f$ be the space radius of the hot (i.e. combustion) zone at the time $t$. The burn-up parameter $x = \rho_0 \cdot r_f$ (where $\rho_0$ is the initial density) plays a very important role below. The initial values of $r_f$ and $x$ are designated below as $r_0 = r_c$ and $x_0 = x_c$. This corresponds to the physical sense of $r_c$ as the minimal or critical radius of the hot zone for which the thermonuclear burn wave still propagates to infinity (in the cold $CD_4$ methane). The temperature $T$ behind the shock or thermal wave (i.e. in the hot or combustion zone) is assumed to be significantly higher than in the initially cold $CD_4$ methane, where $T_0 \approx 0$. Furthermore, without loss of generality we may assume that $T$ inside of the hot zone does not depend upon spatial coordinates. Likewise, the ion and electron temperatures are equal to each other and coincide with $T$. Such an approximation has a quite good accuracy for burn-up problems, unless the temperature $T$ exceeds $25$ keV \cite{1}. In this approximation the burn-up equation can be written (for detail see e.g. \cite{2}) in the form which contains only one variable, namely the burn-up parameter $x = \rho_0 \cdot r_f \ (g \cdot cm^{-2})$:  

$$
\frac{dT}{dx} = -\frac{\nu}{x} \cdot T + \frac{q(x, T)}{C \cdot V_{\text{max}}}
$$

(1)

where $q(x, T)$ is the so-called normalized energy release function, $\nu = 3, 2$ and 1 for the spherical, cylindrical and plane cases, respectively. $C$ is the specific heat per unit mass, which is assumed to be a constant on the spatial coordinates and time. $V_{\text{max}} = \frac{dr_f}{dt}$ is the maximal velocity of the hot (i.e. combustion) zone expansion. Since such a zone expands either by the detonation or thermal waves, the velocity $V_{\text{max}}$ is the larger of the two corresponding velocities, i.e. $V_{\text{max}} = \max\{\left(\frac{dr_f}{dt}\right)_d, \left(\frac{dr_f}{dt}\right)_t\}$. In general, it can be shown that the thermal wave may propagate the thermonuclear burning only at hardly attainable temperatures $T \geq 23 - 25$ keV and only in the dense equimolar $DT$–mixture. In all other cases, including $DT$–mixtures with low ($\leq 10\%$) tritium concentrations, highly compressed deuterium, $DT$–hydrides and deuterides of light elements, etc, the thermonuclear burning (at $T \leq 25$ keV) propagates by using only the relatively slow, high-temperature detonation expansion. This means that without loss of generality we can assume in the last equation $V_{\text{max}} = V_D$, where $V_D$ is the velocity of the detonation wave.

To determine the velocity $V_D$ we shall apply the strong explosion approximation, which has a good accuracy for the considered problems (see e.g. \cite{1}, \cite{2}). In this approximation
the expression for $V_D$ takes the form
\[
V_D = a(\gamma) \cdot \sqrt{C \cdot T} = \frac{3 \cdot \pi \cdot (\gamma + 1)^2 \cdot (\gamma - 1)}{8 \cdot (3 \gamma - 1)} \cdot \sqrt{C \cdot T} = 5.28443640 \cdot 10^{-3} \cdot \sqrt{C \cdot T} \tag{2}
\]
where $\gamma = \frac{5}{3}$ and $C$ is in $MJ \cdot g^{-1} \cdot keV^{-1}$ ($1MJ = 1 \cdot 10^6 J$), $T$ is in $keV$ and $V_D$ is in $cm \cdot nsec^{-1}$. The specific heat $C$ (in $MJ \cdot g^{-1} \cdot keV^{-1}$) per unit mass is of the form [3, 2]:
\[
C = \frac{144.7164346}{A} \cdot (1 + Z) \quad MJ \cdot g^{-1} \cdot keV^{-1} \tag{3}
\]
where the $A$ and $Z$ values are the mean atomic mass number and the mean atomic number, respectively. They must be expressed in units in which the proton charge and proton mass equal 1. Also we assume that the difference between the proton and neutron masses is negligible and also the mass defect for all considered nuclei equals zero exactly. Both these approximations are traditional in thermonuclear problems, and they have been widely used in earlier works (see e.g. [5] or [6]). Therefore, for the considered CD$_4$ methane $Z = 2.00$ and $A = 4.00$, while for the CD$_2$T$_2$ methane $Z = 2.00$ and $A = 4.40$. From here one easily finds: $C = 108.537260 \ MJ \cdot g^{-1} \cdot keV^{-1}$, $V_D = 5.50539183 \cdot 10^{-2} \cdot \sqrt{T} \ cm \cdot nsec^{-1}$ for CD$_4$ methane and $C = 98.6702963 \ MJ \cdot g^{-1} \cdot keV^{-1}$, $V_D = 5.24918515 \cdot 10^{-2} \cdot \sqrt{T} \ cm \cdot nsec^{-1}$ for CD$_2$T$_2$ methane ($T$ is in $keV$).

To discuss the energy release function $q(x, T)$ in Eq. (1) we introduce the useful pair notation $(\alpha, \beta)$ which designates the $(d, d)$—pair for CD$_4$ methane and $(d, t)$—pair for CD$_2$T$_2$ methane. By using this notation the corresponding energy release function $q(x, T) = q_{\alpha, \beta}(x, T)$ Eq. (1) can be written in the following universal form (for more detail see [2]):
\[
q_{\alpha, \beta}(x, T) = \mu_{\alpha, \beta}^2 \cdot s_{\alpha, \beta}(x, T) \cdot (1 + B_{\alpha, \beta} \cdot \frac{b_{\alpha, \beta} \cdot x}{1 + b_{\alpha, \beta} x}) - \frac{\kappa_{\alpha, \beta} \mu_{\alpha, \beta}^2 C_{\alpha, \beta} \sqrt{T}}{1 + c_{\alpha, \beta} \sqrt{\rho_0} \cdot x T^{-\frac{1}{2}}} \tag{4}
\]
where $\mu_{\alpha, \beta}$ is the ratio of the $(\alpha, \beta)$ fragment mass to the total atomic (quasi-molecular) mass and the factor $\kappa_{\alpha, \beta}$ takes the well known form [6]:
\[
\kappa_{\alpha, \beta} = \frac{1}{N_{\alpha, \beta}^2} \left( \frac{\sum_i n_i \cdot Z_i \cdot N_{\alpha, \beta}}{\sum_{\alpha} n_\alpha Z_\alpha \cdot N_{\alpha, \beta}} \right) = \frac{1}{N_{\alpha, \beta}^2} \left( \frac{\sum_i n_i \cdot Z_i}{\sum_{\alpha} n_\alpha Z_\alpha} \right) \frac{1}{2} \left( \frac{\sum_i n_i Z_i^2}{2} \right) \tag{5}
\]
where the summation is taken over all ions present. The factor $\frac{1}{N_{\alpha, \beta}^2} = \frac{1}{2} = \frac{1}{4}$ corresponds to the fact that CD$_4$ contains 4 deuterium atoms or two $d \cdot d$ fragments (i.e. $N_{dd} = 2$). Analogously, there are two $d \cdot t$ fragments in each molecule of CD$_2$T$_2$ methane (i.e. $N_{dt} = 2$). For instance, from the last equations for the CD$_2$T$_2$ methane one easily finds that
\( \mu_{\alpha,\beta} = \mu_{d,t} = \frac{2+2+3+3}{12+2+2+3+3} = \frac{10}{22} \) and \( \kappa_{\alpha,\beta} = \kappa_{d,t} = \frac{1}{4} \cdot \frac{6+1+1+1+1+1+1+1}{2} \cdot \frac{6^2+1^2+1^2+1^2+1^2}{2} = 25 \). Finally, \( \kappa_{d,t} \cdot \mu_{d,t}^2 = 5.165289256 \). The use of these two factors \( \mu_{\alpha,\beta} \) and \( \kappa_{\alpha,\beta} \) allows us to apply the same short (i.e. ionic) energy release functions \( s_{\alpha,\beta}(x,T) \) and bremsstrahlung constants the \( C_{\alpha,\beta} \) factor as for the appropriate pure deuterium or equimolar DT-mixture. At least in the first order such an approximation seems to be quite good [6]. In general, the parameters \( B_{\alpha,\beta}, b_{\alpha,\beta} \) and \( c_{\alpha,\beta} \) depend upon the ionic contents of the considered mixture. But for our present approximate purposes we shall assume that in the CD\(_4\) and CD\(_2\)T\(_2\) methane they are exactly the same as for pure deuterium and equimolar DT-mixture, respectively. Their numerical values can be found in [1], [2].

The short energy release functions \( s_{\alpha,\beta}(x,T) \) in Eq.(4) correspond to the energy gain produced only by positive high-energy ions arising in the thermonuclear reactions. They can be expressed also in the following general form:

\[
s_{\alpha,\beta} = Y \cdot (\sigma_{\alpha\beta} \cdot v_{\alpha\beta}) \quad \text{MJ} \cdot \text{cm}^3 \quad \frac{g^2 \cdot \text{nsec}}{2 \cdot \text{nsec}} \quad (6)
\]

where, e.g. \( \sigma_{dt} \) is the \((d,t)\)–reaction cross-section, \( v_{dt} \) is the relative velocity of the deuterium and tritium nuclei and \((\sigma_{dt} \cdot v_{dt})\) means the averaged (dimensionless) value over the whole range of relative velocities. The numerical factors \( Y \) are [2]: 8.17611 \cdot 10^{18} for the \((d,t)\)–reaction and 3.03891 \cdot 10^{19} for the \((d,d)\)–reaction when all reactions for \(^3\)He nuclei are ignored. However, if the \((d,d)\)–reaction is considered in the presence of high intensity neutron fluxes, then the factor \( Y \) equals 4.63191 \cdot 10^{19}. The last case represents the situation when all \(^3\)He nuclei react with neutrons, and the following \((d,t)\)–reactions are included in the consideration. Now, the remaining problem is to find an analytical or numerical expression for the \((\sigma_{dd} \cdot v_{dd})\) and \((\sigma_{dt} \cdot v_{dt})\) values which depend upon the temperature \( T \) only. Presently, they have been chosen from [2], but it should be mentioned that the analogous \((\sigma_{dt} \cdot v_{dt})\) function from [1] and \((\sigma_{dd} \cdot v_{dd})\) function from [7] produce quite close results.

Now, the solution of the burn-up equation Eq.(1) not contain any principle difficulties. Moreover, this solution can be written in the following integral form [2]:

\[
T(x) = T_0 \cdot \left( \frac{x_0}{x} \right) + \frac{1}{C \cdot x_0} \cdot \int_{x_0}^{x} dy \cdot \frac{g(y, T(y)) \cdot y^{\nu}}{V_D(y, T(y))} \quad (7)
\]

where \( x_0 = \rho_0 \cdot r_0 \) and and \( T(x_0) = T_0 \) are the initial value of the burn-up parameter and temperature, respectively. The general \( T(x) \) dependence has the following qualitative behaviour, when \( x \) grows: it starts at \( x = x_0 \) \((T(x_0) = T_0)\), then decreases, reaches a minimum and later
increases to infinity. The minimal temperature $T_{\text{min}}$ can not be negative and the appropriate condition $T_{\text{min}} = 0$ is used to determine the critical value $x_0 = x_c(= \rho_0 \cdot r_c)$ of the burn-up parameter.

The results of our numerical calculations are presented in Table I (for CD$_4$ methane) and Table II (for CD$_2$T$_2$ methane). These Tables contain the $x_c$ values for various temperatures $T$ ($4 \text{ keV} \leq T \leq 20 \text{ keV}$) and different initial densities $\rho_0$. In Table I the two subcases are considered for each temperature and density. The left subcolumn (a) in Table I represents the results when all reactions with $^3\text{He}$ nuclei are ignored. The right subcolumn (b) in this Table corresponds to the case, when all arising $^3\text{He}$ nuclei react with neutrons, i.e. the thermonuclear burn-up in the presence of high intense neutron fluxes. In the last case we assume that the $(n, ^{12}\text{C}; ^{13}\text{C}, \gamma)$ reaction has relatively small cross-sections. As it follows from Table I the propagation of the thermonuclear detonation wave in CD$_4$ methane simplifies significantly in the presence of high intense neutron fluxes. This effect is of paramount importance for relatively small (or realistic) temperatures $T \approx 4 - 6 \text{ keV}$. In these cases the critical burn-up parameter drops by approximately 3 times. The thermonuclear detonation in CD$_2$T$_2$ methane (see Table II) is not very sensitive to the presence (or absence) of any neutron fluxes.

In discussing the results from Tables I and II, we note that the thermonuclear burn-up in CD$_4$ methane requires significantly greater compression than in CD$_2$T$_2$ methane. For realistic temperatures $T \approx 4 - 6 \text{ keV}$ the deuterated CD$_4$ methane must be compressed $\approx 75 - 100$ times more strongly than CD$_2$T$_2$. In other words, the thermonuclear detonation in CD$_4$ methane may propagate successfully only if its initial densities $\rho_0 \approx 2.5 \cdot 10^3 - 5 \cdot 10^3 \text{ g} \cdot \text{cm}^{-3}$ or higher, which are quite close to the corresponding Fermi limits (see below). Analogous 'critical' densities for CD$_2$T$_2$ methane are $\approx 70 - 90 \text{ g} \cdot \text{cm}^{-3}$ and quite comparable with the densities required in order to produce thermonuclear burn-up in the dense deuterium or DT—mixtures with very low ($\leq 1\%$) tritium concentrations (see [2]). It should be mentioned also that the minimal energy $\mathcal{E}_c$, which is needed to produce thermonuclear ignition in the spherically symmetric case, can be easily evaluated in terms of the known $x_c$ parameter or critical radius $r_c$: $\mathcal{E}_c = \frac{4\pi}{3} \cdot x_c^3 \cdot \rho_0^{-2} \cdot C \cdot T = \frac{4\pi}{3} \cdot r_c^3 \cdot \rho_0 \cdot C \cdot T$, where $\rho_0$ is the initial density and $T$ is the burn-up temperature.

Note also, that the extremely violated energy balance in the deuterated CD$_4$ methane produces another complication which can not be found in CD$_2$T$_2$. The source of such a
complication is obvious. Indeed, the velocity of the detonation wave \( V_D \) increases with the temperature as \( V_D \approx a \cdot \sqrt{T} \), while the thermonuclear ignition in \( \text{CD}_4 \) takes obviously a significantly longer time than in \( \text{CD}_2\text{T}_2 \). Therefore, we would expect that at some critical temperature \( T_{cr} \) the shock wave breaks away from the burn wave and propagates into the cold fuel without producing any ignition. Since the energy losses related with the expansion of the hot zone are not compensated, the temperature behind the shock wave decreases rapidly \( T \approx r_f^{-3} \). Later, when the spatial radius of the hot zone increases to the \( r_{cr} \) value and its temperature drops to the \( T = T_{cr} \) value, then the thermonuclear burn-up begins, i.e. the initial, very fast shock wave slows down and transforms into the detonation wave with a lower temperature. In \( \text{CD}_4 \) methane such a critical temperature \( T_{cr} \approx 11.5 - 13.75 \text{keV} \) and its value depends on the initial density \( \rho_0 \). The existence of the critical temperature means also that in \( \text{CD}_4 \) methane the critical burn-up parameter \( x_c(T, \rho_0) \) has the finite, non-zero limit at \( T \to +\infty \): \( \lim_{T \to +\infty} \pi(T, \rho_0) = \pi(T_{cr}, \rho_0) \). In other words, for \( \text{CD}_4 \) methane the critical burn-up radius \( r_{cr}(\rho_0) = \pi(T_{cr}, \rho_0)/\rho_0 \) can be determined uniformly for each density, and moreover, its value: (1) does not vanish when temperature grows, and (2) does not depend on the temperature \( T \), if \( T \geq T_{cr} \). In fact, this means that the non-physical part of the burn-up curve (where \( dx_c/dT > 0 \)) is ignored and replaced by the straight line \( dx_c/dT = 0 \). For the \( \text{CD}_2\text{T}_2 \) methane \( dx_c/dT < 0 \) everywhere (at \( T \leq 20 \text{keV} \)) and the principal difference between the thermonuclear burn-up in \( \text{CD}_4 \) and \( \text{CD}_2\text{T}_2 \) can be easily understood from comparison of Tables I and II. The \( \pi \) and \( T_{cr} \) values are also presented (for each density) in Table I. Since \( T_{cr} \leq 13.75 \text{keV} \) in all cases, the higher temperatures (\( T \geq 14 \text{ keV} \)) are not considered in Table I.

Now, it is necessary to show that the obtained solution corresponds to the actual burn-up process. In general, successful burn-up means that the two following conditions are obeyed: firstly, the energy release in the hot zone behind the expanding thermonuclear burn wave must exceed all energy losses related with the hot zone expansion. This gives the burn-up equation Eq.\( \text{II} \) used above. Secondly, the thermonuclear burn wave must move faster than the following disassembly (or rarefaction) wave. The later is essentially the high-temperature sound wave. Its velocity \( C_s \) takes the following form:

\[
C_s(T) = \sqrt{\frac{\gamma_i \cdot P_i + \gamma_e \cdot P_e}{\rho_0}} = \sqrt{\frac{2}{3} \cdot \gamma \cdot C \cdot T}
\]

where \( T \) is in \( \text{keV} \), \( \gamma = \frac{5}{3} \) and \( C \) is determined above. \( P_i = n_i \cdot T \) and \( P_e = n_e \cdot T \) are
the ion and electron pressures, respectively. As follows from Eq.(8) in the strong explosion approximation we always have $V_D(T) > C_s(T)$ for arbitrary $T$. In other words, in this approximation the disassembly wave always moves behind the thermonuclear burn wave, i.e. in the combustion zone. The motion of the disassembly wave in the combustion area does not mean the termination of thermonuclear burning. However, if the disassembly wave passes the thermonuclear burn wave, then the fuel will be disassembled before the thermonuclear burning may start effectively. Such a situation can be found for both relatively small ($\approx 1 keV$) and very high ($> 100 keV$) temperatures.

In particular, at small temperatures and high compressions the electrons are very close to the Fermi limit of electron degeneracy. As a result, in this case, the velocity of the disassembly wave is significantly larger than it follows from Eq.(8). Indeed, in this limit the electron pressure is given by the same formula $P_e = n_e \cdot T_e$, where the effective electron temperature $T_e$ takes the form:

$$T_e = T_{ef} \cdot \left[ 1 + \frac{\pi^2}{15} \cdot \left( \frac{T_e}{T_{ef}} \right)^2 + \frac{7\pi^4}{150} \cdot \left( \frac{T_e}{T_{ef}} \right)^4 + \ldots \right]$$

(9)

where $T_{ef}$ is the equivalent Fermi temperature, given by (see e.g. [8]):

$$T_{ef}(keV) = \frac{2}{5} \cdot \left( \frac{3N_A}{8\pi} \right)^{\frac{1}{3}} \cdot \frac{h^2}{2m_e} \cdot \left( \frac{\rho_0 \cdot Z}{\bar{A}} \right)^{\frac{1}{3}} = 9.3019938991 \cdot 10^{-3} \cdot \left( \frac{Z}{\bar{A}} \right)^{\frac{1}{3}} \cdot (\rho_0)^{\frac{1}{3}}$$

(10)

where $N_A$, $h$ and $m_e$ are the Avogadro number, Planck constant and electron mass, respectively [9]. For instance, for $^{12}CD_4$ methane with $\rho_0 = 1 \cdot 10^6$ $g \cdot cm^{-3}$ one easily finds $T_{ef} = 58.50$ keV. Therefore, for $T_e = 1$ keV the effective electron temperature $T_e \approx T_{ef}$ and actual electron pressure $P_e$ exceeds its classical value in 58.60 times. In this case, the disassembly wave moves in $\approx 5$ times faster than it follows from Eq.(8). Briefly, we can say that the appropriate correction on the electronic degeneracy is obviously needed when the actual temperature $T (= T_e)$ is comparable with the equivalent Fermi temperature $T_{ef}$ [8].

In our present analysis the largest $T_{ef}$ value can be found in CD$_4$ methane, where $\rho_0 \leq 1 \cdot 10^4$ $g \cdot cm^{-3}$, i.e. $T_{ef} \leq 0.27199195$ keV. This indicates that the maximal $T_{ef}$ temperature is significantly smaller than temperatures used in the present study ($T \geq 4$ keV), i.e. the appropriate Fermi correction to the electron pressure $P_e$ is very small and can be ignored.

In conclusion, it should be mentioned that the presented approach predicts that the thermonuclear burn-up is quite likely also in the higher deuterated alkanes (C$_n$D$_{2n+2}$), alkenes (C$_n$D$_{2n}$), alkynes (C$_n$D$_{2n-2}$), etc. Indeed, the both factors $\kappa_{d,d}$ and $\mu_{d,d}$ have the finite limits
when $n \to \infty$ (these limits equal $\frac{1}{4}$ and $\frac{25}{16}$, respectively). The determination of the critical parameters for each of these polymers is straightforward. Moreover, the critical burn-up parameters for an arbitrary deuterium containing fuel can be computed by using the formulas presented above. In particular, for all deuterides of light elements with $Z \leq 9$ ($D^{19}F$ is included) such an analysis can be found in our paper [10].

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TABLE I: The critical values of the burn-up parameter $x_c$ (in $g \cdot cm^{-2}$) for various densities $\rho_0$ (in $g \cdot cm^{-3}$), temperatures $T$ (in keV) for CD$_4$ methane. The superscripts mean that: (a) all reactions with the $^3$He nuclei are ignored, and (b) the thermonuclear burn-up proceeds in very intense neutron fluxes.

| $\rho_0^{(a)}$ | $\rho_0^{(b)}$ | $\rho_0^{(a)}$ | $\rho_0^{(b)}$ | $\rho_0^{(a)}$ | $\rho_0^{(b)}$ |
|----------------|----------------|----------------|----------------|----------------|----------------|
| $T$            | $1 \cdot 10^3$ | $1 \cdot 10^3$ | $5 \cdot 10^3$ | $5 \cdot 10^3$ | $1 \cdot 10^4$ | $1 \cdot 10^4$ |
| 4              | 13125.467      | 4330.014       | 2834.009       | 986.851        | 1542.655       | 564.471        |
| 5              | 9340.429       | 3074.378       | 2019.777       | 703.628        | 1100.985       | 403.783        |
| 6              | 7783.949       | 2552.553       | 1679.783       | 583.248        | 913.755        | 334.134        |
| 7              | 7010.860       | 2288.540       | 1508.037       | 520.944        | 817.637        | 297.394        |
| 8              | 6585.202       | 2138.550       | 1411.503       | 484.666        | 762.567        | 275.570        |
| 9              | 6341.346       | 2047.877       | 1354.585       | 462.082        | 729.256        | 261.671        |
| 10             | 6206.550       | 1992.443       | 1321.531       | 447.699        | 709.104        | 252.551        |
| 11             | 6145.359       | 1960.466       | 1304.593       | 438.793        | 697.844        | 246.629        |
| 12             | 6135.744       | 1945.657       | 1299.618       | 433.884        | 693.165        | 243.037        |
| 13             | 6135.744       | 1943.435       | 1299.618       | 432.106        | 692.812        | 241.257        |

$\bar{\rho}$ | 6135.744 | 1943.435 | 1299.618 | 432.073 | 692.812 | 240.914|

$T_{cr}$ | 11.62 | 12.60 | 11.99 | 13.16 | 12.38 | 13.71|

TABLE II: The critical values of the burn-up parameter $x_c$ (in $g \cdot cm^{-2}$) for various densities $\rho_0$ (in $g \cdot cm^{-3}$), temperatures $T$ (in keV) for CD$_2$T$_2$ methane.

| $\rho_0$ | $\rho_0$ | $\rho_0$ | $\rho_0$ | $\rho_0$ | $\rho_0$ |
|----------|----------|----------|----------|----------|----------|
| $T$      | 50       | 100      | 200      | 50       | 100      | 200      |
| 4        | 96.6663  | 60.2729  | 39.7280  | 10 8.4305 | 7.2564  | 6.1622  |
| 5        | 50.6829  | 34.2076  | 24.0097  | 11 6.6828 | 5.9122  | 5.1495  |
| 6        | 31.1823  | 22.4731  | 16.5634  | 13 4.5962 | 4.2287  | 3.8281  |
| 7        | 20.9295  | 15.9492  | 12.2564  | 15 3.5037 | 3.3001  | 3.0624  |
| 8        | 14.8525  | 11.8714  | 9.4686   | 18 2.6749 | 2.5653  | 2.4291  |
| 9        | 10.9912  | 9.1474   | 7.5431   | 20 2.3703 | 2.2870  | 2.1811  |