Detecting Dual Superconductivity in the Ground State of Gauge Theories - II.

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A monopole creation operator is constructed: its vacuum expectation value is an order parameter for dual superconductivity in that, if different from zero it signals spontaneous breaking of the U(1) symmetry corresponding to monopole charge conservation. The operator is tested on compact U(1) gauge theory on lattice. For SU(2) gauge theory it clearly demonstrates that confinement is produced by dual superconductivity.

I. INTRODUCTION

A possible mechanism of colour confinement in Quantum Chromo-Dynamics (QCD) is dual superconductivity of the vacuum \[1\rightarrow4\]. According to this scenario, the chromoelectric field is channelled into Abrikosov flux tubes \[5\], in the same way as the ordinary magnetic field is in superconductors: the word dual indicates the interchange of roles between electric and magnetic fields and charges. The chromoelectric field mediating the force between coloured particles is squeezed by Meissner effect into flux tubes of constant energy per unit length, giving rise to the confining linear potential. These flux tubes behave as strings \[3\rightarrow4\]. The existence of strings in hadronic physics is supported by phenomenology \[7\rightarrow8\]. They have also been visualized by numerical simulations of QCD on the lattice \[9\rightarrow10\]. Some evidence in favour of dual superconductivity of the vacuum has been produced by Montecarlo simulations \[11\].

A clear cut test of the mechanism would be the detection of monopole condensation in the ground state, analogous to the condensation of Cooper pairs in the ground state of an ordinary superconductor. Condensation implies that the vacuum is a superposition of states with different charge, which, in turn, is nothing but a spontaneous breaking of the U(1) symmetry related to charge conservation \[12\]. Such a breaking is signaled by a non-
vanishing vacuum expectation value ($vev$) of any operator carrying non-trivial charge, as, e.g. the scalar field of the Landau-Ginzburg model of superconductivity. That $vev$ is called a disorder parameter in the language of statistical mechanics.

In QCD the monopoles which are expected to condense and generate superconductivity are Dirac monopoles of a residual $U(1)$ symmetry, which survives after a suitable gauge fixing, known as abelian projection \[1\]. An abelian projection is defined as the gauge transformation which diagonalizes any operator transforming in the adjoint representation of the gauge group. The points where two eigenvalues of that operator coincide correspond to singularities of the gauge transformation and have the topology of world-lines of point monopoles \[1,15\]. Such monopoles have been observed on the lattice \[16\]. Of course the location and the number of monopoles do depend on the choice of the operator used to define the abelian projection: a possibility \[1\] is that physics is independent of it. The relevant abelian degrees of freedom can also be fixed by a somewhat different procedure, known as maximal abelian projection \[15\]. Our operator allows to investigate unambiguously what abelian projection, if any, defines the monopoles relevant to confinement.

In this paper, we will present the construction of the operator, which is rather general and can be used for any kind of solitons. We will analyze how the operator works in compact $U(1)$ gauge theory \[17\].

For that theory, there exists a construction of a disorder variable describing monopole condensation \[18\], which is rigorous but based on a particular form of the action (the Villain action). Our operator coincides with the above one in the Villain case, but can be used with different forms of the action. This is particularly important in order to use it for non-abelian theories, where the effective $U(1)$ action after abelian projection is not known.

For $U(1)$ we get a spectacular signal of monopole condensation \[17\]. We then apply the same construction to $SU(2)$ gauge theory: we consider the monopoles corresponding to the abelian projection in which the Polyakov loop is diagonal, and we investigate the condensation of monopoles across the deconfining phase transition. Here again we find a spectacular signal of monopole condensation. The monopoles corresponding to that abelian projection do condense in the confined phase: Confinement is produced by dual superconductivity.

The next steps of our investigation, which is in progress, will be to study the behaviour of monopoles defined by different abelian projections, to test ‘tHooft ideas, and, in particular, the properties of the maximal abelian projection.

In sect. 2 the construction of the monopole creation operator is presented. Sect.3 contains the results for compact $U(1)$. Sect.4 the results for monopole defined by abelian projection diagonalising the Polyakov loop in $SU(2)$ gauge theory. Sect.5 contains a few concluding remarks.

II. THE MONOPOLE CREATION OPERATOR.

For the sake of definiteness, we shall consider here only the case of a $U(1)$ monopole. Our procedure can easily be generalized to all kind of solitons.

Let $B_\mu(x,y) = (0, b(x,y))$ be the classical field produced in the location $x$ by a Dirac monopole at rest in $y$. We can make any choice for the gauge, e.g. by putting the string along the positive $z$-axis. Then, defining $r = x - y$ and $r = ||r||$, we have:
\[ b_i(x, y) = g \varepsilon_{3ij} \frac{r_j}{r(r - r_3)} \]  

where \( g \) is the charge of the monopole, satisfying the Dirac quantization condition \( e g = \frac{n}{2} \). If \( \Pi_i(x, t) \) is the conjugate momentum to \( A_i(x, t) \), then the operator

\[ \mu(y, t) = \exp\{i \int d^3x \, b_i(x, y) \Pi_i(x, t)\} \]  

creates a monopole at the location \( y \) and time \( t \). This can be immediately seen in the Schrödinger representation of the fields, where we have:

\[ \mu(y, 0)|A(x, 0)\rangle = |A(x, 0) + b(x, y)\rangle \]  

Equation (3) is a trivial consequence of the canonical commutation relations between the fields and their conjugate momenta, and is nothing but the field-theoretic equivalent of the familiar statement that \( e^{i \varphi a} \) translates the coordinate \( q \) by \( a \). \( \mu(y, t) \), applied to any field configuration, adds a monopole to it. This can be restated in terms of commutation relations as:

\[ [A_i(x, t), \mu(y, t)] = b_i(x, y)\mu(y, t) \]  

\[ [\Pi_i(x, t), \mu(y, t)] = 0 \]

which also show that the electric field of the configuration is left unchanged. It is worthwhile to notice that the specific choice of the gauge for \( b \) is irrelevant: what really matters here is topology, which is independent of it. Our operator \( \mu \) is similar to operators introduced in the literature by different constructions, in various contexts [19].

Now, if the ground state of the theory has a definite monopole number \( N \), then, under a magnetic \( U(1) \) rotation, we have: \( U|0\rangle = e^{i \varphi N}|0\rangle \). Since \( U\mu U^\dagger = e^{i \varphi} \mu \), then \( \langle 0|\mu|0\rangle = \langle 0|\mu|0\rangle e^{i \varphi} \) which implies \( \langle 0|\mu|0\rangle = 0 \). Therefore if \( \langle 0|\mu|0\rangle \neq 0 \) then \( |0\rangle \) is not \( U(1) \) invariant and there is spontaneous symmetry breaking of \( U(1) \). A translation by a static field \( g(x) \) such that \( \text{curl } g = 0 \) in this language corresponds to a pure gauge transformation:

\[ \gamma(t) = \exp\left\{i \int d^3x \, g_i(x) \Pi_i(x, t)\right\} \]

By gauge-invariance \( \langle 0|\gamma(t)|0\rangle = 1 \). Performing the Wick rotation to Euclidean space, we obtain:

\[ \mu_E(y, y_4) = \exp\left\{-g \int d^3x \, b_i(x, y) \Pi_i(x, y_4)\right\} \]  

From now on, we will be interested only in the Euclidean quantity and drop for simplicity the \( E \) subscript. We will first compute \( \langle \mu \rangle \equiv \langle 0|\mu_E(y, y_4)|0\rangle \) for free photons. Rescaling the fields by a factor \( 1/\sqrt{\beta} \) with \( \beta = 1/e^2 \), we have

\[ \langle \mu \rangle = \frac{1}{Z} \int DA \left\{ \exp\left[-\frac{\beta}{4} \int d^4xF_{\mu\nu}F_{\mu\nu}\right]\times \exp\left[-\beta \int d^3xF_{0i}(x, y_4)b_i(x, y)\right]\right\} \]
The integral is Gaussian and can be directly computed giving:

\[
\langle \mu \rangle = \exp \left\{ \frac{\beta}{2} \int \frac{d^4k}{(2\pi)^4} (F_{0i}(k)F_{0j}(-k))b_i(-k) b_j(k) \right\} 
\]

\[
(F_{0i}(k)F_{0j}(-k)) = \delta_{ij} - \frac{k^2\delta_{ij} - k_i k_j}{k_0^2 + k^2} 
\]  \hspace{1cm} (9)

If \( b \) is such that \( k_i b_i(k) = 0 \), we have, performing the integral over \( k_0 \) in the second term,

\[
\langle \mu \rangle = \exp \left\{ \frac{\beta}{2} \int \frac{d^4k}{(2\pi)^4} |b(k)|^2 - \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} |k| |b(k)|^2 \right\} 
\]  \hspace{1cm} (10)

In the analogous calculation for a gauge transformation \( \gamma \), \( g_i(k) \propto k_i \), the second term of eq.(9) does not contribute, and

\[
\langle \gamma \rangle = \exp \left\{ \frac{\beta}{2} \int \frac{d^4k}{(2\pi)^4} |g(k)|^2 \right\} 
\]  \hspace{1cm} (11)

We realize that the first term in the exponent of eq.(10) is a normalization which can be subtracted by taking instead of \( \langle \mu \rangle \) the ratio \( \langle \bar{\mu} \rangle = \langle \mu \rangle / \langle \gamma \rangle \) where \( \langle \gamma \rangle \) is defined by means of any gauge transformation \( g \), such that:

\[
\int \frac{d^4k}{(2\pi)^4} |g(k)|^2 = \int \frac{d^4k}{(2\pi)^4} |b(k)|^2 
\]  \hspace{1cm} (12)

Then for free photons, which, in the compactified version of the theory, correspond to the deconfined phase, \( \beta \gg \beta_c \)

\[
\langle \bar{\mu} \rangle = \exp \left\{ -\frac{\beta}{4} \int \frac{d^3k}{(2\pi)^3} |k| |b(k)|^2 \right\} 
\]  \hspace{1cm} (13)

The integral in the exponent, once regularized at small distances, tends to \(+\infty\) as \( V \to \infty \). In the infinite volume limit \( \langle \bar{\mu} \rangle = 0 \) as it should be, since the perturbative vacuum has zero magnetic charge. The same situation appears when computing the overlap of the Fock vacuum to the Bogolubov rotated vacuum in a superconductor [20].

An alternative way of looking at the normalization factor in eq.(10) is to go back to the very definition of Feynman path integral: when computing the vacuum expectation value of an operator like \( e^{ip(t)a} \), after the usual discretization of time is performed in intervals of size \( \delta \), the operator appears in a matrix element of the form

\[
\langle x_{n+1} | e^{-iH\delta} e^{ipx} | x_n \rangle = \int \frac{dp}{(2\pi)^3} |x_{n+1} | e^{-iH\delta} e^{ipx} | p \rangle \langle p | x_n \rangle 
\]

\[
= \int \frac{dp}{(2\pi)^3} e^{-i\left( \frac{p^2}{2m} + V(x_n) \right)} e^{ipx} e^{ip(x_{n+1} - x_n)} 
\]

The integral over \( p \) can be performed giving

\[
e^{-i(x_{n+1} - x_n)^2 \frac{m}{\beta} + i\alpha (x_{n+1} - x_n)m + V(x_n)} e^{-i\alpha^2} 
\]

The first factor is the lagrangian definition, the second factor corresponds to the subtraction operated in going from \( \langle \mu \rangle \) to \( \langle \bar{\mu} \rangle \) in eq.(13).
III. LATTICE FORMULATION. \textit{U(1) GAUGE THEORY.}

A lattice version of the operator $\mu$ is obtained by replacing $eF_{0i}$ by the plaquette $P_{0i}$, or better, by its imaginary part: $eF_{0i} \rightarrow \text{Im } P_{0i}$ and discretizing the field $b_i$. Then the disorder parameter becomes:

$$\langle \mu \rangle = \frac{1}{Z} \int DU \exp \{-\beta [S + \sum_{n_0=y_4} b_i(n, y) \text{Im } P_{0i}(n)]\}$$

or, if we want to cancel the unwanted normalization, we can divide by:

$$\langle \gamma \rangle = \frac{1}{Z} \int DU \exp \{-\beta [S + \sum_{n_0=y_4} g_i(n) \text{Im } P_{0i}(n)]\}$$

obtaining

$$\langle \bar{\mu} \rangle = \frac{\int DU \exp \left\{-\beta [S + \sum_{n=y_4} b_i(n, y) \text{Im } P_{0i}(n)]\right\}}{\int DU \exp \left\{-\beta [S + \sum_{n=y_4} g_i(n) \text{Im } P_{0i}(n)]\right\}} \tag{14}$$

We stress once again that any gauge function $g_i$ is acceptable, provided the normalization condition (12) is satisfied. In the following we shall use Wilson’s action $S = \frac{1}{2} \sum_{a,\mu,\nu} (1-P_{\mu\nu})$.

If we blindly compute $\langle \mu \rangle$ or $\langle \bar{\mu} \rangle$ by numerical simulations, a first technical difficulty arises. We are faced with the usual problems encountered in computing quantities like a partition function, which are exponentials of extensive quantities, proportional to the number of degrees of freedom. The distribution of the values is not Gaussian and the error does not decrease by increasing statistics (see, e.g. [21], where the same problem appears in a different context). To avoid that, we will compute the quantity:

$$\rho = \frac{d}{d\beta} \log \langle \bar{\mu} \rangle = \frac{d}{d\beta} \log \langle \mu \rangle - \frac{d}{d\beta} \log \langle \gamma \rangle \tag{15}$$

At $\beta = 0$, $\langle \mu \rangle = \langle \gamma \rangle = 1$, and therefore:

$$\langle \bar{\mu} \rangle = \exp \left[ \int_0^\beta d\beta' \rho(\beta') \right] \tag{16}$$

Putting $S_b = \sum b_i(n) \text{Im } P_{0i}(n) |_{n_0=0}$ and $S_g = \sum g_i(n) \text{Im } P_{0i} |_{n_0=0}$ we get:

$$\rho = \langle S + S_g \rangle_{S+S_g} - \langle S + S_b \rangle_{S+S_b} \tag{17}$$

which can be evaluated by numerical simulations. The subscript on the average indicates the action defining the Feynman integral. The two quantities on the rhs have the same strong coupling expansion. Thus, the use of $\langle \bar{\mu} \rangle$ instead of $\langle \mu \rangle$, besides producing a cancellation of the spurious normalization of the Feynman path integral, can help in eliminating the lattice artefacts produced by the discretization which can spoil the continuum limit. This brings us to a second, more physical difficulty, which is the continuum limit. In fact, while for $QCD$,
which is asymptotically free, we expect that, at sufficiently high $\beta$, lattice artefacts should cancel, for a model like $U(1)$ this point is not so clear in principle. In Ref. [18] a proof is given of monopole condensation in the confined phase of $U(1)$, defining a disorder variable for the Villain action. We have checked that our $\langle \mu \rangle$ operator exactly coincides with the one of Ref. [18], when we use the Villain action: we expect that for Wilson action the same will hold. We have then computed numerically $\rho$.

For a good order parameter, we would expect $\rho$ to be zero, or $\langle \bar{\mu} \rangle = 1$ below the critical value of the coupling $\beta_c$ and then to show a large negative peak around $\beta_c$, corresponding to a drop to zero of $\langle \bar{\mu} \rangle$. At larger values of $\beta$, we have free photons and Eq. (12) should hold.

Figure 1 shows the behaviour of $\rho$ for a 12$^4$ lattice, for a monopole in the center of the space lattice. In order to be able to identify the signal as a genuine physical result (i.e. not due to lattice artefacts), we have performed a number of checks:

1. We have changed the form of $b_i$ by a gauge transformation to get the Wu-Yang expression of the monopole potential. The result does not change qualitatively.

2. $\langle \gamma \rangle$ shows practically no signal at $\beta_c$ within the errors, and does not change appreciably by changes of $g_i$. To test that we have computed $\langle \gamma \rangle$ for two different choices of $g_i(x)$, both satisfying the condition Eq. (12): $\langle \gamma_1 \rangle$ for $g_1(x) = \text{const.}$ and $\langle \gamma_2 \rangle$ for $g_2(x) \propto x/|x|^2$. In Fig. 2 we display $\rho_{gauge} = \frac{d}{d\beta} \ln \frac{C(d)}{\langle \gamma \rangle^2}$ and $\rho = \frac{d}{d\beta} \ln \langle \mu \rangle$ for a 6$^4$ lattice. $\rho_{gauge}$ shows no relevant signal at $\beta_c$.

3. We have measured the correlation function between a monopole antimonopole pair at large time distance, in the same position in space. To do that we define $C(d) = \langle \mu(0,0) \mu(0,d) \rangle$ and

$$S_{bb}(d) = \sum_n b_i(n,0) [\text{Im} P_{0i}(n,0) - \text{Im} P_{0i}(n,d)]$$

We measure $\frac{d}{d\beta} \ln \frac{C(d)}{\langle \gamma \rangle^2} = \rho_{(b,b)}(d)$ or

$$\rho_{(b,b)}(d) = 2\langle S + S_g \rangle_{S+S_g} - \langle S + S_{bb}(d) \rangle_{S+S_{bb}} - \langle S \rangle_S$$

Since $\frac{C(d)}{\langle \gamma \rangle^2} \bigg|_{\beta=0} = 1$ we have

$$\frac{C(d)}{\langle \gamma \rangle^2} = \exp \left[ \int_0^\beta \rho_{(b,b)}(d) d\beta \right] \quad (18)$$

By the cluster property we should have at large $d$ that $C(d) \rightarrow \langle \mu(0,0) \rangle^2$, or $\rho_{(b,b)}(d) \rightarrow 2\rho$. Fig. 3 shows that this expectation is indeed verified. We notice that the height of the negative peak of $\rho$ at $\beta_c$ increases with volume [see Fig.1 and Fig.2]. The value of $\beta_c$ as defined by the position of our peak is $\beta_c = 1.01(1)$ for a 6$^4$ lattice and $\beta_c = 1.009(1)$ for a 12$^4$ lattice.

We have taken monopole charge $n = 4$ to get a good signal with a relatively low statistics (typically $10^4$ configurations per value of $\beta$): smaller charges ($n = 1, 2$) give similar results but the signals are smaller and more noisy. Our statistical errors are shown in the figures, when they are larger than the symbols used.
IV. SU(2) GAUGE THEORY. MONOPOLE CONденSATION AND
CONFINEMENT

The monopoles which should condense in QCD vacuum in the confined phase are Dirac monopoles of the $U(1)$ field defined by the abelian projection [1]. The density of current of such monopoles is

$$j_\mu = \partial_\mu F_{\mu\nu}$$

where, in the notation of ref. [2],

$$F_{\mu\nu} = \frac{1}{|Q|} Q_a G_{\mu\nu}^a - \frac{1}{g|Q|^3} \varepsilon_{abc} Q^a (D_\mu Q^b)(D_\nu Q^c)$$

$$a, b, c = 1, 2, 3$$

$Q^a$ is the Higgs field in the Georgi - Glashow model. Abelian projection is a gauge transformation which brings $Q$ in the third direction, $Q = (0, 0, Q^3)$. In QCD the role of $Q^a$ should be played by some operator transforming in the adjoint representation [1]: what this operator is, if any, is the problem under investigation.

After abelian projection the second term of eq.(19) gives zero and $F_{\mu\nu} = G^3_{\mu\nu}$.

As a first candidate for $Q^a$ we have chosen $p^a = \frac{1}{2} \text{Tr} (-i \ln P \sigma^a)$ where $P$ is the Polyakov line [1], $\exp(\int A_0 dx^0)$. This choice corresponds to take $A_0$ as the Higgs field. For that choice $D_0 Q = 0$ and

$$F_{0i} = \frac{p^a}{|p|} G_{0i}^a$$

Since the expression (19) is gauge invariant, we do not need to perform any gauge transformation to make $\ln P$ diagonal. We simply define the gauge invariant quantity

$$F_{0i} = \frac{1}{2} \text{Tr} (-i \ln P G_{0i})$$

The operator creating the relevant monopole is

$$\mu(y, t) = \exp\{- \int d^3 x b_i(x, y) F_{0i}(x, t)\}$$

Exactly as for $U(1)$ we can define $\langle \bar{\mu} \rangle = \langle \mu \rangle / \langle \gamma \rangle$ where

$$\langle \gamma(t) \rangle = \exp\{- \int d^3 x g_i(x) F_{0i}(x, t)\}$$

We shall take for $g_i(x)$ a constant field, such that eq.(12) is satisfied.

The lattice version of $F_{0i}$ will be

$$F_{0i} = \frac{1}{2} \text{Tr} \{ -i \ln P \Pi_{0i} \}$$

and

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\[
\langle \bar{\mu} \rangle = \frac{\int \mathcal{D}U \exp \left\{ -\beta [S + \sum_{n=y_4} b_i(n, y) F_{0i}(n)] \right\}}{\int \mathcal{D}U \exp \left\{ -\beta [S + \sum_{n=y_4} g_i(n) F_{0i}(n)] \right\}}
\]

We will define \( \rho \) as in eq.(13), and therefore again:

\[
\langle \bar{\mu} \rangle = \exp \left[ \beta_0 d \rho(\beta') \right]
\]

As in \( U(1) \) we measure the behaviour of \( \rho(\beta) \). across the deconfining phase transition The simulations are performed on asymmetric lattices, with temporal size \( N_T \) smaller than spacial size \( N_S \).

Fig. 4,5,6,7 show the behaviour of \( \rho \) for \( 8^3 \times 4, 12^3 \times 4, 12^3 \times 6 \) and \( 16^3 \times 6 \) lattices. The behaviour of \( \rho \) is similar to the \( U(1) \) case: \( \rho \) is compatible with zero below the deconfining transition, has a sharp negative peak at the deconfining phase transition, and a negativetail at high \( \beta \), corresponding to a phase of free “photons” (see eq.(13). Qualitatively the shape of \( \langle \bar{\mu} \rangle \) is

\[
\langle \bar{\mu} \rangle = \theta(\beta_c - \beta)
\]

The negative peak increases with space volume at fixed \( N_T \) (compare \( 8^3 \times 4 \) with \( 12^3 \times 4 \), \( 12^3 \times 6 \) with \( 16^3 \times 6 \)) which means that as \( V \rightarrow \infty \), for \( \beta > \beta_c \), \( \langle \bar{\mu} \rangle \rightarrow 0 \).

The peak is placed exactly at the official location of the phase transition for \( N_T = 4 \) and moves as expected from renormalization group from \( N_T = 4 \) to \( N_T = 6 \).

The usual order parameter \( |\langle P \rangle| \) is plotted for comparison in the figures.

The depth of the peak does not change appreciably from \( 8^3 \times 4 \) to \( 12^3 \times 6 \), showing that the signal mainly depends on the physical volume of the lattice, i.e. on its size in \( \text{fm}^3 \), and not on the geometrical volume of it.

We have also measured the correlation function

\[
\langle \bar{\mu}(10, 0, 0, 0), \bar{\mu}(0, 0, 0, 0) \rangle
\]

on a \( 8^2 \times 20 \times 4 \) lattice to check the cluster property, and tested that it is equal to the square of the disorder parameter on the same lattice (Fig. 8).

We can finally conclude that the monopoles defined by abelian projection identified by the Polyakov loop are charges of an \( U(1) \) symmetry which is spontaneously broken in the confined phase, and restored above \( \beta_c \), or that confinement is produced by dual superconductivity.

**V. CONCLUDING REMARKS**

We have demonstrated that confinement in \( SU(2) \) gauge theory is due to dual superconductivity of the vacuum. We have identified the monopole charges which condense and produce it. Our method consists in a direct detection of spontaneously breaking of the \( U(1) \) symmetry corresponding to monopole charge.

The method has been successfully checked on \( U(1) \) compact gauge theory.

A number of open questions are currently under investigation
i) Are there different abelian projections defining monopoles which condense in QCD vacuum? Is t’Hooft’s idea about the physical equivalence of different abelian projections correct? In particular we are investigating the so called maximal abelian gauge, defined by the gauge transformation which maximises the quantity

\[ \sum_{n,\mu} \text{Tr} \left\{ U_{\mu}(n)\sigma_3 U_{\mu}^\dagger(n)\sigma_3 \right\} \]

On this gauge there is a lot of evidence collected in favour of dual superconductivity [15,16].

ii) The extension to SU(3) of the above construction.

iii) The application of our method to other systems, like e.g. the 3d x-y model, in which a phase transition is driven by the condensation of solitons.
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FIGURE CAPTIONS

Fig.1 $\rho$ versus $\beta$ on a $12^4$ lattice.

Fig.2 $\rho$ and $\rho_{gauge}$ versus $\beta$ on a $6^4$ lattice.

Fig.3 $\rho b\bar{b}(d)$ versus $\beta$ at $d = 4, 7, 9$, compared to $2\rho$.

Fig.4 $\rho$ (circles) and $\langle |P| \rangle$ (squares), $8^3 \times 4$ $SU(2)$ gauge theory.

Fig.5 $\rho$ (diamonds) and $\langle |P| \rangle$ (squares), $12^3 \times 4$ $SU(2)$ gauge theory.

Fig.6 $\rho$ (triangles) and $\langle |P| \rangle$ (squares), $12^3 \times 6$ $SU(2)$ gauge theory.

Fig.7 $\rho$ (open squares) and $\langle |P| \rangle$ (squares), $16^3 \times 6$ $SU(2)$ gauge theory.

Fig.8 $\rho b\bar{b}(d)$ at $d = 10$ (circles) compared to $2\rho$ (triangles) and their difference (squares). $SU(2)$ gauge theory.