Hybrid fuzzy kinetic model of phosphorite pellets drying process

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Abstract. The chemical-technological process of phosphate pellets drying is investigated in the paper. The hybrid fuzzy kinetic model of phosphorite pellets drying process is proposed. The features of the proposed model are as follows: firstly, differential equations with fuzzy thermal-physical characteristics of pellets are used to estimate the thermal conductivity of pellets; secondly, a fuzzy rule-based model is used to determine the most difficult parameters to calculate. The method based on the developed hybrid model is proposed for analyzing the drying process of phosphorite pellets. The proposed method uses an original approach that allows solving the problem of increasing uncertainty of results in iterative calculations based on the modal interaction of fuzzy thermal characteristics represented by fuzzy numbers.

1. Introduction

One of the ways to ensure competitive advantages of the chemical and metallurgical industry is an integrated approach to the production of enriched pelletized raw materials (pellets). To do this, it is necessary to ensure the optimal granulometric composition of pellets, the absence of moisture in them, as well as the minimum content of carbonate inclusions, volatile gases and impurities [1-3].

One of the most significant problems in the pellets production is to ensure the required conditions for chemical-technological processes [4, 5]. For such conditions primarily include: a large number of parameters; the complexity and nonlinearity of these interdependent parameters; the uncertainty of the dynamics of processes; the uniqueness of the equipment for pellet production; the significant influence of environmental factors [6, 7]. The implementation of these conditions, in turn, requires the development of methods for analysis and management of chemical-technological processes based on the methodology of fuzzy modeling.

Studies in which fuzzy kinetic models of various dynamic processes are proposed are known. In [8], the fuzzy model based on fractional calculus for the analysis of dynamic processes with fuzzy parameters is proposed. To solve these problems under uncertainty, the fuzzy numerical method based on generalized Legendre fractional polynomials has developed. In [9], the fuzzy fractional kinetic model based on the fuzzy operational matrix of the generalized Laguerre polynomials. In [10], fuzzy models for analyzing biochemical reactions with fuzzy kinetic parameters are considered.
However, the analysis of complex dynamic processes of pellet drying requires the hybridization of fuzzy models based on the principle of “functional substitution”, when one of the models is picked out as a dominating model, and its separate components are replaced with other models [11]. Thus, the limitations inherent in each of the models separately are compensated.

The paper proposes the hybrid fuzzy kinetic model of phosphorite pellets drying process. The originality of the proposed model is as follows:

- firstly, differential equations with fuzzy thermal-physical characteristics of pellets are used to estimate the thermal conductivity of pellets;
- secondly, a fuzzy rule-based model is used to determine the most difficult parameters to calculate (for the temperature dependence of a wet thermometer on the temperature of gases and humidity).

2. Features of the pellets drying process

A large amount of thermal energy is spent on chemical and power-technological processes for drying pellets [12]. The physical-mechanical bound moisture is situated in the large capillaries and on the outer surface of the pellets. It is retained by capillary pressure. The physical-chemical bound moisture is retained on the inner surface of the pores in the pellets by adsorption forces. Chemically bound moisture is retained most firmly and is not removed when heated to 100-120°C [13].

The removing of the moisture requires energy costs, approximately equal to the heat of vaporization [14].

In convective drying, the wet pellet receives heat from the heat carrier gas (air and fuel combustion products). The drying agent – the heat-carrier gas – absorbs moisture that is removed from the raw pellets and thereby increases its moisture content [15].

The dynamics of external moisture exchange in the process of pellets drying is determined by the change in moisture vapor concentration across the boundary layer and the change in temperature of the drying agent near the surface of the wet material of the pellet. The difference in concentration creates a vapor flow from the surface, and the temperature difference between the main mass of the drying agent and the surface of the pellet provides heat to the wet body.

Near the wet surface of the granule, the following boundary layers are formed: hydrodynamic, thermal, and layer of concentration. Experimental data on the intensity of heat and mass transfer of the surface of a wet pellet with a stream of drying agent are usually presented as a relationship between the similarity criteria [16]. The relationship between the similarity criteria depends on the interaction conditions of the gas flow and the surface of the pellet. For the evaporation intensity of water from the surface of pellets into a drying agent filtered through their layer, the dependence $\text{Nu}_m = 2.2 \text{Re}^{0.5} \text{Pr}_m^{0.33}$ is proposed, where the diameter of the pellet is the determining size and the velocity of the heat carrier gas depends on the cross section of the apparatus [17].

Wet pellet is a capillary-porous body. The process of moisture and heat transfer inside the pellet occur due to various physical effects. Analysis of the moisture and heat transport processes inside capillary-porous materials based on account for the elementary types of transfer for real process of pellets drying is not possible [18]. Experiments have shown that the kinetic characteristics vary significantly for the entire range of changes in moisture content and temperature [19]. With intensive heating of the wet pellet, vaporization process takes place inside its porous structure. The overpressure does not have time to instantly relax through the porous structure of the pellet, and this effect leads to the movement of moisture.

The pellets drying on a conveyor indurating machine requires special attention in the initial period of heat treatment, due to the “thermal shock” of pellets. In practice, pellets drying takes place under non-isothermal conditions with a non-constant temperature and pressure gradient [20]. Therefore, high temperature of the heat carrier gas in the drying zone of the conveyor firing machine leads to a pellets overwetting (up to 25%) in medium and upper horizons of the pellets layers, as a result, the loss of mechanical strength and deterioration of the dense layer structure [21].
Thus, it is necessary that the kinetic model of the pellet drying process should take into account changes in the moisture content along the pellet radius, as well as temperature gradients and moisture flow through the surface of the evaporation front [22].

3. Hybrid model of pellets drying process
The model is developed under the assumption that moisture moves relatively freely inside the porous structure of the pellet, and as it dries, the localized front of moisture evaporation deepens. Heat is supplied to the evaporation front due to the thermal conductivity of the dried outer layer of the pellet (figure 1), where heat is consumed to convert the liquid into vapor.

Figure 1. Formation of a localized evaporation front in the pellet.

As a result of evaporation, an overpressure is created inside the pellet. The formed vapors are filtered from the evaporation front to the outer surface of the pellet [23]. The rate of moisture removal from the pellet depends on two successive resistances – thermal and filtration. The vapor pressure and the temperature at the evaporation front are established during the drying and are related to each other as saturated steam parameters.

The thermal conductivity of the pellet is described by a partial differential equation for a sphere with fuzzy thermal characteristics: \( \tilde{\rho}_m \) – density; \( \tilde{c}_m \) – heat capacity; \( \tilde{\lambda} \) – thermal conductivity; \( \tilde{T}_m \) – temperature of the pellet:

\[
\tilde{\rho} = \frac{d\tilde{\gamma}}{dt} = \frac{3\tilde{\lambda} \left( \tilde{T}_m \mid_{x=0} - \tilde{T}_m \right) \sqrt{\gamma}}{\tilde{\rho}_m \tilde{c}_m Q R^2 \left( \sqrt{\gamma} - 1 \right)}
\]

\[
\tilde{\rho}_m \tilde{c}_m \frac{\partial \tilde{T}_m}{\partial \tau} = \frac{1}{\tilde{x}^2} \frac{\partial}{\partial \tilde{x}} \left( \tilde{\lambda} \tilde{x} \frac{\partial \tilde{T}_m}{\partial \tilde{x}} \right) - Q \tilde{\rho}_m.
\]  

(1)

Figures 2a and 2b show examples of fuzzy thermal conductivity and fuzzy heat capacity in the form of triangular fuzzy numbers, respectively [24].

Figure 2. Fuzzy parameters of thermal conductivity (a) and heat capacity (b) in the form of fuzzy triangular numbers.
The drying rate is determined by the kinetic equation ([12]) with the fuzzy thermal characteristics: \( \tilde{R} \) – radius; \( \tilde{T}_m \) – temperature of the wet thermometer; \( \tilde{T}_g \) – the heat carrier gas temperature:

\[
\dot{\tilde{\theta}} = \frac{d\tilde{\gamma}}{d\tau} = \frac{3\lambda(\tilde{T}_m|_{\tau = \tilde{R}} - \tilde{T}_m)}{\rho_0\tilde{\gamma}_0 Q\tilde{R}^2(\sqrt{\gamma}_\infty - 1)}
\]

with initial and boundary (including fuzzy) conditions:

\[
\tau = 0, \tilde{T}_m = \tilde{T}_{m0}, u = \tilde{u}_0, \tilde{\gamma} = \tilde{I}
\]

\[
x = 0, \frac{\partial \tilde{T}_m}{\partial x} = \tilde{0},
\]

\[
x = \tilde{R}, -\tilde{\lambda} \left( \frac{\partial \tilde{T}_m}{\partial x} \right) = \alpha_f (\tilde{T}_g - \tilde{T}_m)_{x=\tilde{R}},
\]

where \( Q \) – specific heat of evaporation, J/m³; \( \tilde{\gamma} = (\tilde{G}/\tilde{R})^3 \) – relative degree of pellet drying.

In this equation, a difficulty is the calculation of a fuzzy value of the wet thermometer temperature \( \tilde{T}_m \) with known fuzzy values of the temperature of gases \( \tilde{T}_g \) and moisture content \( \tilde{x}_w \). The availability of experimental data allows us to develop the fuzzy rule-based model (the structure of model is presented in table 1) for setting this dependence \( \tilde{T}_m = f(\tilde{T}_g, \tilde{x}_w) \) in the drying process of phosphate pellets in the fuzzy kinetic model (2).

Table 1. The structure of the fuzzy rule-based model \( \tilde{T}_m = f(\tilde{T}_g, \tilde{x}_w) \).

| Rule number | Fuzzy input variables | Fuzzy output variable |
|-------------|-----------------------|----------------------|
| \( R_1 \)  | \( L \) \( L \)     | \( L \)              |
| \( R_2 \)  | \( L \) \( M \)     | \( L \)              |
| \( R_3 \)  | \( L \) \( H \)     | \( M \)              |
| \( R_4 \)  | \( M \) \( L \)     | \( L \)              |
| \( R_5 \)  | \( M \) \( M \)     | \( M \)              |
| \( R_6 \)  | \( M \) \( H \)     | \( M \)              |
| \( R_7 \)  | \( H \) \( L \)     | \( M \)              |
| \( R_8 \)  | \( H \) \( M \)     | \( H \)              |
| \( R_9 \)  | \( H \) \( H \)     | \( H \)              |

For the description, the same terms of all variables are given \( \{L \text{ – low, } M \text{ – middle, } H \text{ – high}\} \). The fragment of the rule base is presented below:

\( R_1: \) IF \( \tilde{T}_g \) is \( L \) AND \( \tilde{x}_w \) is \( L \), THEN \( \tilde{T}_m \) is \( L \);

\( R_5: \) IF \( \tilde{T}_g \) is \( M \) AND \( \tilde{x}_w \) is \( M \), THEN \( \tilde{T}_m \) is \( M \);

\( R_9: \) IF \( \tilde{T}_g \) is \( H \) AND \( \tilde{x}_w \) is \( H \), THEN \( \tilde{T}_m \) is \( H \).

One of the known fuzzy inference algorithms is used to calculate the output fuzzy variable \( \tilde{T}_m \) [25].

Figure 3 illustrates the dependence \( \tilde{T}_m = f(\tilde{T}_g, \tilde{x}_w) \).
4. Analysis of the phosphorite pellets drying process based on a hybrid fuzzy kinetic model

The method based on the developed hybrid model is proposed for analyzing the drying process of phosphorite pellets. In addition, the proposed method uses an original approach that allows solving the problem of increasing uncertainty of results in iterative calculations based on the modal interaction of fuzzy thermal characteristics represented by fuzzy numbers \[24\].

The essence of this approach is a finite-difference representation of equations (1) and (2) on a uniform grid with \(N\) partitions along the radius of pellets and by time \(K\):

\[
\frac{\rho \tilde{c}_i^k}{\Delta \tau} \left( \frac{T_{m,i}^k - T_{m,i+1}^{k-1}}{\Delta \tau} \right) = \frac{1}{\Delta x^2} \left( x_{i+1/2}^2 \tilde{x}_{i+1/2}^k \left( T_{m,i+1}^k - T_{m,i}^k \right) - x_{i-1/2}^2 \tilde{x}_{i-1/2}^k \left( T_{m,i}^k - T_{m,i-1}^k \right) \right) + \sum q_i^k,
\]

where,

\[
\tilde{x}_{i+1/2} = \frac{\tilde{x}_i + \tilde{x}_{i+1}}{2}, \quad \tilde{x}_{i-1/2} = \frac{\tilde{x}_i + \tilde{x}_{i-1}}{2}, \quad q_S = \sum q_i.
\]

The equation (3) is written in an implicit form, therefore, the sweep method is used to solve it:

\[
\tilde{A_i} C_{i+1} - \tilde{C_i} T_i + \tilde{B_i} T_{i+1} = \tilde{D_i}, \quad i = 2, ..., N - 1,
\]

where,

\[
\tilde{A_i} = \frac{\Delta \tau}{\Delta x^2} x_{i+1/2}^2 \tilde{x}_{i+1/2}^k, \quad \tilde{C_i} = \frac{\Delta \tau}{\Delta x^2} \left( x_{i+1/2}^2 \tilde{x}_{i+1/2}^k + x_{i-1/2}^2 \tilde{x}_{i-1/2}^k \right) + x_i^2 \rho \tilde{c}_i,
\]

\[
\tilde{B_i} = \frac{\Delta \tau}{\Delta x^2} x_{i+1/2}^2 \tilde{x}_{i+1/2}^k, \quad \tilde{D_i} = x_i^2 \left( \rho \tilde{c}_i T_i + \Delta \tau q_S^k \right).
\]

Notification. For clarity, at each time layer \(\Delta \tau\), the \(k\)-th indices are not indicated. The solution is looked up by a recursive expression sweep method:

\[
\tilde{T_i} = \tilde{\beta}_{i+1,i} \tilde{T}_{i+1} + \tilde{\gamma}_{i+1}, \quad i = 2, ..., N.
\]

For the coefficients \(\tilde{\beta}_{i+1,i}\) and \(\tilde{\gamma}_{i+1}\) the valid relations are:
\[ \tilde{\beta}_{i+1} = \frac{\tilde{B}}{\tilde{C}_{i} - \tilde{A}_{i}}, \quad \tilde{\gamma}_{i+1} = \frac{\tilde{A}_{i} \tilde{B}_{i} + \tilde{D}_{i}}{\tilde{C}_{i} - \tilde{A}_{i}}, \quad i = 2, \ldots, N - 1. \]

Taking into account the boundary conditions, fuzzy values of the coefficients \( \tilde{\beta}_{i}, \tilde{\gamma}_{i} \) are calculated, as well as the temperature \( T_{\gamma+i} \) at the right border (surface) of the pellet is determined.

Let us consider as an example the execution of some operation \( \ast \) on fuzzy values \( \tilde{A} \) and \( \tilde{B} \), given in the form of fuzzy triangular numbers \( \tilde{A}(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3) \) and \( \tilde{B}(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) \), with the resultant fuzzy number \( \tilde{C}(\tilde{c}_1, \tilde{M}(\tilde{C}), \tilde{c}_3) \). Here \( \tilde{a}_1, \tilde{b}_1, \tilde{c}_1 \) are the left bounds, \( \tilde{M}(\tilde{A}), \tilde{M}(\tilde{B}), \tilde{M}(\tilde{C}) \) – the modal values and \( \tilde{a}_3, \tilde{b}_3, \tilde{c}_3 \) – the right bounds of the fuzzy triangular numbers \( \tilde{A}, \tilde{B}, \tilde{C} \), respectively.

The algorithm for performing operations \( \ast \) on fuzzy numbers \( \tilde{A} \) and \( \tilde{B} \) is as follows.

**Step 1.** Decomposition \( \tilde{A} \) and \( \tilde{B} \) into \( \alpha \)-levels \( \tilde{A}_{\alpha} = [\tilde{a}_{1(\alpha)}, \tilde{a}_{2(\alpha)}] \) and \( \tilde{B}_{\alpha} = [\tilde{b}_{1(\alpha)}, \tilde{b}_{2(\alpha)}] \).

**Step 2.** Perform the operation for each \( \alpha \)-level: \( \tilde{C}_{\alpha} = \tilde{A}_{\alpha} \ast \tilde{B}_{\alpha}, \tilde{C}_{\alpha} = [\tilde{c}_{1(\alpha)}, \tilde{c}_{2(\alpha)}] \).

**Step 3.** Composing the results of operations on \( \alpha \)-levels: \( \tilde{C} = \bigcup_{\alpha \in [0,1]} \tilde{C}_{\alpha} \).

When performing iterative calculations on fuzzy numbers, the problem of increasing (accumulating) the fuzziness of the result is solved by the method, which is based on the so-called modal interaction of fuzzy numbers. This method consists of using the following modified operations on the intervals of \( \alpha \)-levels of fuzzy numbers \( \tilde{A} \) and \( \tilde{B} \):

\[
\begin{align*}
C_{\alpha} &= A_{\alpha} + B_{\alpha} = [c_{1(\alpha)}, c_{2(\alpha)}] = \left[ \min \left( a_{1(\alpha)} + M(\tilde{B}), b_{1(\alpha)} + M(\tilde{A}) \right), \max \left( a_{3(\alpha)} + M(\tilde{B}), b_{3(\alpha)} + M(\tilde{A}) \right) \right], \\
C_{\alpha} &= A_{\alpha} - B_{\alpha} = [c_{1(\alpha)}, c_{2(\alpha)}] = \left[ \min \left( a_{1(\alpha)} - M(\tilde{B}), -b_{1(\alpha)} + M(\tilde{A}) \right), \max \left( a_{3(\alpha)} - M(\tilde{B}), -b_{3(\alpha)} + M(\tilde{A}) \right) \right], \\
C_{\alpha} &= A_{\alpha} \cdot B_{\alpha} = [c_{1(\alpha)}, c_{2(\alpha)}] = \left[ \min \left( a_{1(\alpha)} \cdot M(\tilde{B}), a_{3(\alpha)} \cdot M(\tilde{B}), M(\tilde{A}) \cdot b_{1(\alpha)}, M(\tilde{A}) \cdot b_{3(\alpha)} \right), \max \left( a_{1(\alpha)} \cdot M(\tilde{B}), a_{3(\alpha)} \cdot M(\tilde{B}), M(\tilde{A}) \cdot b_{1(\alpha)}, M(\tilde{A}) \cdot b_{3(\alpha)} \right) \right], \\
C_{\alpha} &= \frac{A_{\alpha}}{B_{\alpha}} = [c_{1(\alpha)}, c_{2(\alpha)}] = \left[ \min \left( \frac{a_{1(\alpha)}}{M(\tilde{B})}, \frac{a_{3(\alpha)}}{M(\tilde{B})}, \frac{M(\tilde{A})}{b_{1(\alpha)}}, \frac{M(\tilde{A})}{b_{3(\alpha)}} \right), \max \left( \frac{a_{1(\alpha)}}{M(\tilde{B})}, \frac{a_{3(\alpha)}}{M(\tilde{B})}, \frac{M(\tilde{A})}{b_{1(\alpha)}}, \frac{M(\tilde{A})}{b_{3(\alpha)}} \right) \right].
\end{align*}
\]

**5. Experimental results and the simulation veracity using the hybrid fuzzy kinetic model**

Experiments were performed assuming the initial moisture content of the pellet \( u_{0} = 10 \text{ kg/kg} \) (± 0.5%). This corresponds to the moisture content of a raw pellet with a diameter of \( d = 2 \text{ cm} \) (± 0.5 cm), descending from a disc pelletizer with a speed of 1.2 m/s at various temperatures of the heat carrier gas.

The simulation results (figures 4, 5) correspond to the experimental data. This confirms the simulation veracity using a hybrid fuzzy kinetic model.

At 100°C the drying process does not start until the temperature of the pellet reaches the temperature of the wet thermometer. The higher the temperature of the heat carrier gas, the more intensive the drying process is. The drying process is active in the first minutes, and then it slows down. This is due to the fact that at the beginning moisture is removed from the surface layer of the pellet. Then the evaporation front moves inward, and heat is supplied through the dried material layer.

Moreover, the temperature of the wet core of the pellet, having reached the temperature of the wet thermometer, remains constant, while the temperature of the dried outer layers increases and approaches the temperature of the drying agent (figure 4). Thus, the temperature gradient is greatest inside the granule along the perimeter of the evaporation front. This gradient is higher, the higher the temperature of the heat carrier gas. Therefore, the drying in forced model leads to thermogradient destruction of the
pellet. In addition, it is necessary to take into account the amount of moisture flux \( I \) (kg/(m\(^2\)·s)). Critical parameters of the heat carrier gas for pellets of various sizes are determined. For example, for pellets of dimensions 1–2 cm and temperature range 150–600°C, value \( I \) varies from 4 to 16 \( \times \) 10\(^{-3} \) kg/(m\(^2\)·s).

![Image of temperature dependence](image)

**Figure 4.** Temperature dependence at the border and in the center of the pellet on time, at different temperatures of the heat carrier gas: 1 – 500°C, 2 – 300°C, 3 – 100°C. The experimental data are presented: in the center (○) and on the border (■) of the pellet.

The figure 5 shows an example of dependence of fuzzy values \( \tilde{T}_m \) as a function of time at the heat carrier gas temperature of 300°C, performed using the developed model. Figure 5 shows that the simulation results, using the proposed hybrid fuzzy kinetic model, are veracity. Thus, all crisp values \( T_m \), which obtained from experiments, correspond to the calculated values of the membership functions of the fuzzy variable \( \tilde{T}_m \) at a level not lower than 0.5.

![Image of fuzzy dependence](image)

**Figure 5.** An example of a fuzzy dependence of fuzzy values \( \tilde{T}_m \) as a function of time at the heat carrier gas temperature of 300°C (● – the experimental points).
Conclusion
The paper proposes the hybrid fuzzy kinetic model of phosphorite pellets drying process. The originality of the proposed model is as follows: firstly, differential equations with fuzzy thermal-physical characteristics of pellets are used to estimate the thermal conductivity of pellets; secondly, a fuzzy rule-based model is used to determine the most difficult parameters to calculate (for the temperature dependence of a wet thermometer on the temperature of gases and humidity).

The method based on the developed hybrid model is proposed for analyzing the drying process of phosphorite pellets. The proposed method uses an original approach that allows solving the problem of increasing uncertainty of results in iterative calculations based on the modal interaction of fuzzy thermal characteristics represented by fuzzy numbers.

Comparative evaluation of experimental data with simulation results allows us to draw conclusions, firstly, about the adequacy of the proposed hybrid fuzzy kinetic model, and secondly, about the reliability of the results of modeling using this model.

The obtained results will be used in the future to exclude the softening and destruction of the pellets due to “thermal shock” and the formation of an overwatered horizon in a multi-layer mass moving on a conveyor indurating machine.

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