Experimental Study of Turbulence Beginning of Supersonic Boundary Layer on Swept Wing at Mach Numbers 2 - 4

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Abstract. The paper is devoted to an experimental study of natural disturbances evolution and laminar-turbulent transition in a three-dimensional supersonic boundary layer on swept wing at different Mach numbers. Characteristic zones of disturbances evolution are determined. It is found that for Mach numbers 2 and 2.5 measurements were performed in the region of linear stage of disturbances evolution, and the experimental data can be compared with the results of the calculations on linear stability theory. Oscillograms, amplitude-frequency spectra, pulsation profiles and statistical diagrams of natural fluctuations are obtained. Secondary instability of supersonic boundary layer on swept wing is under consideration

1. Introduction

The problem of transition to turbulence in 3 D boundary layers is very important and very complicated. In a 3 D case exist along with the well-known Tollmien Schlichting waves, which development results to the turbulent transition in the 2 D boundary layers, stationary vortexes with axes directed along the outer streamlines and some traveling waves (not T S waves). Development of all instability disturbances and their relative role in transition strongly depend on the environmental conditions. Most theoretical and experimental results on stability of three-dimensional boundary layer are obtained for subsonic flow. Some recent studies in this field are discussed in reviews (Arnal et.al. (1990); Saric et.al. (2003); Lekoudis (1979); Mack (1982)) and other papers. However very few theoretical and experimental investigations of supersonic 3-D boundary layer stability have been fulfilled up to date.

Malik et.al. (1996) studied secondary instability on stationary crossflow disturbances in swept cylinder boundary layer at Mach number M=3.5. The secondary analysis yields three unstable modes with the peak growth rate at frequencies about 100 kHz, 1.05 kHz, and 970 kHz. The most unstable travelling crossflow disturbance has a peak frequency of about 50 kHz; therefore, the unstable frequency for secondary instability is an order of magnitude higher than that of the travelling crossflow disturbance. Mielke & Kleiser (2000) studied laminar-turbulent transition in a 3-D supersonic boundary layer by mean DNS using the temporal model. Linear stability analysis shows the dominance of crossflow instability. The secondary instability analysis reveals a broad band of secondary unstable modes travelling in streamwise direction. Cattafesta et. al. (1995) experimentally and theoretically studied transition on a swept wing model at M=3.5. Using the envelope e^N eN method for linear stability calculation
obtained the N factor and compared results with the observed transition locations. Travelling disturbances with N=13 provide a good correlation with the transition data over a range of unit Reynolds numbers and angles of attack. Disturbances with frequencies 40-60 kHz have the largest N factors, and it is assumed that the transition is more likely caused by them. Attempt of transition prediction with accounts for all major stages was made theoretically by Choudhary et. al. (2003). Numerical studies of secondary instability, transition prediction and control for swept wing supersonic boundary layers was made by Li & Choudhary (2010), were experimental configuration of Saric & Reed (2002) was modeled. Experiments Saric & Reed (2002) on passive flow control by distributed roughness was made at Mach number M=2.4 on 73-degree swept wing with thickness-to-chord ratio of 4%. Li & Choudhary examined evolution of travelling disturbances and the destabilization of high-frequency secondary instabilities in the presence of finite amplitude stationary crossflow vortices of a specified spanwise wavelength and varying initial amplitudes. Excitation of several modes of secondary instability with frequencies about 1 MHz was detected. The 3-D boundary layer stability to the stationary disturbances in a linear formulation was investigated in Asai et. al. (2000). It was obtained that the boundary layer became unsteady to the stationary mode, when the crossflow reached a sufficiently small value, which is less than 1% of an external flow.

Stability of supersonic boundary layer on swept wing was studied experimentally only in ITAM Semionov et. al. (2003); Ermolaev et. al. (1995); Semionov et. al. (2008). Experiments Semionov et. al. (2003); Ermolaev et. al. (1995); Semionov et. al. (2008) were made at Mach number M=2 on 40-degree swept wing with thickness-to-chord ratio of 7.8. Evolution of natural fluctuations in the boundary layer on a swept wing was studied by Ermolaev et. al. (1995); Semionov et. al. (2003). It was shown that the character of distribution of the mean and fluctuating characteristics of the boundary layer is similar to the case of subsonic velocities. It was obtained at M=2, that the disturbances growth in three-dimensional boundary layer occurs much faster, than in the flat plate case. The results of an experimental study of evolution of controlled disturbances on a swept-wing model for Mach number M = 2 are presented by Semionov et. al. (2003); Semionov et. al. (2008). The wave characteristics of traveling waves are obtained. The evolution of disturbances at frequencies of 10, 20, and 30 kHz is similar to the development of traveling waves for subsonic velocities. The angle of inclination of the wave vector for energy-carrying disturbances is directed across the flow, and the group-velocity vector is aligned with the steady cross-flow disturbance.

Linear stage of cross-flow instability in relation to stationary and unsteady disturbances was investigated theoretically by Gaponov & Smorodsky (2008). Direct quantitative comparison of theory with our experiments Semionov et. al. (2003) was presented. A good agreement of the theory with measurements performed in T-325 has been obtained only for spanwise scales of cross-flow vortices. However computed growth rates differ significantly from measurements. Principal cause of such discrepancy of theoretical and experimental data was nonlinearity. In this case the distance from a leading edge up to a point of transition becomes more in some times in comparison with the previous experiments. It has allowed investigating in detail disturbances evolution in supersonic boundary layer on swept wing, especially at an initial linear stage. New experiments Semionov et.al. (2010); Yermolaev et. al. (2011) were made on symmetrical wing with a 45 sweep angle, a 3-percent-thick circular-arc airfoil at M=2. The detailed data of disturbances development up to transition location are obtained for the first time. Characteristic zones of disturbances evolution are determined. A position of instability region of secondary flow is experimentally defined. Some features of disturbances evolution, characteristic only for a supersonic boundary layer are revealed.

This work is a continuation of previous studies Semionov et.al. (2010); Yermolaev et. al. (2011). Some new results of experimental study of stability of supersonic boundary layer on swept wing at different Mach number are presented in the paper. And secondary instability of
supersonic boundary layer on swept wing is under consideration.

2. Experimental Set-up

The experiments were conducted at the Institute of Theoretical and Applied Mechanics of the Siberian Division of the Russian Academy of Sciences in the T-325 supersonic wind tunnel with test-section dimensions 0.2 × 0.2 × 0.6 m at Mach numbers M = 2 - 4 at low unit Reynolds numbers. Mach number was changed with the step 0.5. The model length was 0.4 m, its width was 0.2 m, and the maximum thickness was 12 mm. The model was mounted at zero incidences in the central section of the test section of the wind tunnel. Photo of the model in test section with hot-wire probe is presented in fig. 1.

The oscillations were measured by a constant-temperature hot-wire anemometer. Single-wire tungsten probes of diameter 5 μm (or 10 μm) and length 1.2 mm was used. The overheat ratio of the wire was 0.8, and the measured disturbances corresponded to mass-flow fluctuations. The fluctuating and mean characteristics of the flow were measured by an automated data acquisition system. The fluctuation signal from the hot-wire anemometer was measured by a 12-bit A/D converter with a digitization on time 1.33 s, and a mean voltage was fixed by a voltmeter. The length of each realization was 65536 points. With the help of the discrete Fourier transform (DFT) on time t the amplitude-frequency spectra were determined:

\[ e'_f(x', z', y) = \sum_k e'_f(x', z', y, t_k) \exp[i\omega t_k] \Delta t_k \]

where T — length of digital time trace, \( \Delta t_k = t_{k-1} - t_k \) and \( e'(x', z', y, t_k) \) — digital oscillogram of a pulsation signal from a hot–wire anemometer. Values of the mass flow fluctuations \( \rho U \) were determined by the method described in Kosinov et. al. (1999). To measure a transition position hot-wire sensor were used.

To determine the nonlinear interaction of perturbations used well-known fact, that the Gaussian signal indicate a linear process (linear independence of the harmonic components of the signal), and any significant deviation from the normal distribution is the nonlinearity of the process Kosinov & Semisynov (2003); Kendall & Kimmel (1991); Chokani et. al. (2005); Nikias & Raghuveer (1987). Evaluation for normality of probability density distribution was carried out according to Lvovskiy et.al. (1982). The estimates of ”skewness” and ”kurtosis” of the measured pulsation signals were made:

\[ m_k = \frac{1}{N} \sum_{i=1}^{N} (x(i) - x)^k \]
\[ g_1 = \frac{m_3}{m_2^{3/2}} \]  
\[ g_2 = \frac{m_3}{m_2^2} - 3 \]

where \( m_k \) - central moment of k-order, \( g_1 \) - parameter of skewness, \( g_2 \) - parameter of kurtosis.

3. Results and Analysis

Evolution of natural disturbances in supersonic boundary layer of swept wing was investigated in detail for the first time. Oscillograms, amplitude-frequency spectra, mean velocity profiles, pulsation profiles and statistical diagrams of natural fluctuations were obtained. At the initial stage of the study have been obtained curves of the growth of natural disturbances at different Mach numbers. Measurements were performed in the critical layer, where the pulsations have a maximum value. When you move the hot-wire sensor along the longitudinal coordinate \( x \), the mean voltage \( E \) in the diagonal of the bridge was kept constant by moving along the normal coordinate \( y \). Dependencies of mass flux versus Reynolds number \( Re = Re_1 \times x \) (where \( x \) - longitudinal coordinate) are shown in fig. 2. Presented in fig. 2 data are obtained for the cases \( M = 2 \) and \( 2.5 \), \( Re_1 = 5 \times 10^6 \text{m}^{-1} \); \( M = 3 \), \( Re_1 = 6.6 \times 10^6 \text{m}^{-1} \); \( M = 3.5 \), \( Re_1 = 7.3 \times 10^6 \text{m}^{-1} \); \( M = 4 \), \( Re_1 = 10.2 \times 10^6 \text{m}^{-1} \).

Maxima in the distributions correspond to the position of laminar-turbulent transition. The minimum value of transition Reynolds number \( Re_{tr} \) obtained for \( M = 3.5 \), which agrees qualitatively with the experimental transition results on a flat plate at the same facility. Note that at each point measurements were obtained the amplitude-frequency spectra and statistical distributions too. With the help of statistical analysis it was found that for Mach numbers 2 and 2.5 measurements can be performed in the region of linear stage of disturbances evolution, and the experimental data can be compared with the results of the calculations on linear stability theory. Obtained experimental data at \( M = 2 \) are described in detail in Semionov et.al. (2010); Yermolaev et. al. (2011). A position of instability region of secondary flow at \( M = 2 \) was experimentally defined, and the growth of traveling disturbances observed at \( Re_x \approx 0.35 \times 106 \). Obtained, that in a supersonic swept wing boundary layer in region \((0.35 \times 10^6 < Re_x < 0.7 \times 10^6)\) at \( M = 2 \) and in region \((0.25 \times 10^6 < Re_x < 0.6 \times 10^6)\) at \( M = 2.5 \) linear disturbances evolution was observed. At Mach 3, 3.5 and 4 non-linear processes were observed from the beginning of region of measurements (50 mm from the leading edge of the swept wing).
Measurements across boundary layer were made for all mentioned above Mach numbers. Obtained distributions of mass flux pulsation \( <m'> \) in dependence of normal coordinate \( y \) at \( M=2.5 \) for initial stage of disturbances evolution are shown in fig. 3, and for latest stage - in fig. 4. Results of measurements of profiles of pulsations at \( M=3, 3.5 \) and 4 are presented in fig. 5 - 7 respectively. Dependencies of \( <m'> (y) \) at \( M=2 \) have two maxima, first corresponds to critical layer, second - subsonic layer Semionov et.al. (2010); Yermolaev et. al. (2011). Two maxima in dependencies were observed in the case of flat plate too. But in supersonic boundary layer on flat plate second maximum was lesser than maximum in critical layer. Fast growth of disturbances corresponding to the second maximum was observed in the case of swept wing in nonlinear region of disturbances evolution at \( M=2 \). Additional mode of perturbation was observed above the critical layer (see left plot in fig. 3). These perturbations are rapidly escalating, and in the last sections leads to a complex form of the main peak (see right plot in fig. 3). Note that the presence of this mode of disturbances can be noticed in the profiles measured at Mach 2 Semionov et.al. (2010); Yermolaev et. al. (2011). As shown in the graphs presented in Figures 3-5, the role of disturbances above the critical the layer in process of the laminar-turbulent transition increases with the growth of Mach number. At the same time at high Mach numbers the maximum fluctuation in the profiles near the surface of the model is fixed only in the initial sections.

![Figure 3](image1.png) \quad ![Figure 4](image2.png)

**Figure 3.** Distributions of mass flux pulsation \( <m'> \) versus normal coordinate \( y \) at \( M=2.5 \).

**Figure 4.** Distributions of mass flux pulsation \( <m'> \) versus normal coordinate \( y \) at \( M=2.5 \).

A statistical analysis was made for all experimental data obtained at \( M=2 \) (for measurements in the critical layer of streamwise disturbances evolution and for profile measurements). Figure 8-11 shows examples of the results of statistical analysis performed for the profiles of the pulsations. Deviations of "skewness", "kurtosis" are close to zero and are the same as in free stream for the profiles measured at \( Re_x = 0.35 \times 10^6 \) (\( x=70 \) mm) and \( Re_x = 0.6 \times 10^6 \) (\( x=120 \) mm), so we can say that processes are linear. The first significant deviation of the histograms obtained from a normal distribution were observed at \( Re_x = 0.7 \times 10^6 \) (\( x=140 \) mm) above and below of the critical layer. Note that when \( Re_x = 0.7 \times 10^6 \) in the vicinity of the second maximum (near the surface of the model) the processes are linear, but when \( Re_x = 0.9 \times 10^6 \) (\( x=180 \) mm) deviation from the normal distribution have grown considerably. With increasing nonlinearity, disturbances begin to grow rapidly, especially in the vicinity of the second peak. And as noted earlier Semionov et.al. (2010); Yermolaev et. al. (2011), the perturbations in the vicinity of the maximum near the surface of model on the amplitude becomes comparable to the disturbances.
in the critical layer.

Amplitude-frequency spectra of disturbances in dependence Reynolds number \( Re \) at \( M=2 \) are presented in fig.12. The measurements were conducted in the layer of maximum fluctuations. At \( M=2 \) with increasing of Reynolds number there was an intensive excitation and growth of pulsations in a range of frequencies from 10 up to 35 kHz on an initial stage of disturbances development. This experimental data are in a good agreement with calculations based on the linear stability theory Gaponov & Smorodsky (2008). The growth of pulsations in a range of frequencies from 10 up to 80 kHz was observed near to transition location. Amplitude-frequency spectra of disturbances in dependence Reynolds number \( Re \) at another Mach numbers are presented in fig.13 - 15. As was mentioned above, in a supersonic swept wing boundary layer at \( M = 2 \) and 2.5 linear disturbances evolution was observed. At Mach 3, 3.5 and 4 non-linear processes were observed from the beginning of region of measurements. At \( M=2.5 \) growth of pulsations in a range of frequencies \( 10 \div 45 \) kHz on an initial stage of disturbances development till \( 10 \div 200 \) kHz near to transition location were obtained. The same frequency ranges at \( M=2.5 \) are wider. When comparing these experimental data with calculations Li & Choudhary (2010), a large discrepancy in the values of the frequency of increasing disturbances. In Li & Choudhary
(2010) obtained that frequency of primary disturbances is about 120 kHz, significantly greater than obtained in the experiment at similar Mach numbers. A possible reason for this discrepancy may be that the calculations were made for the case of a subsonic leading edge (73-degree swept wing geometry), and the experiments were performed for the case of a supersonic leading edge (χ = 45°). Further expanding of the frequencies range was observed with increasing Mach number.

Was performed comparing the results of statistical analysis and analysis of amplitude-frequency spectra. Fluctuations grow at the frequency range from 8 to 35 kHz in the region of linear evolution of disturbances at M=2. It is known that at subsonic speeds on a swept wing mechanism of secondary instability leads to excitation and fast growth of high-frequency disturbances with a frequency one order higher than the fundamental disturbance, and this mechanism takes place in a region of nonlinear disturbances evolution Li & Choudhary (2010); Kohama et.al. (1991). It can be assumed that the growth of high-frequency part of the spectra (f > 35 kHz) is caused by the mechanism of secondary instability at supersonic speeds for M =
2. Note, that in Li & Choudhary (2010) obtained that frequencies of secondary disturbances is about 1 MHz, that is significantly higher frequency features of hot wire anemometry. Similar analysis, but no so detailed was performed for the experimental data obtained at $M = 2.5$. Fluctuations grow at the frequency range from 8 to 45 kHz in the region of linear evolution of disturbances at $M = 2.5$, and high-frequency part of the spectra ($f > 45$ kHz) increase in nonlinear region of disturbances evolution. Similar results in nonlinear region of disturbances evolution are obtained for other Mach numbers.

![Figure 12](image1.png) **Figure 12.** Evolution of amplitude-frequency spectra of disturbances at $M = 2$.

![Figure 13](image2.png) **Figure 13.** Evolution of amplitude-frequency spectra of disturbances at $M = 2.6$.

![Figure 14](image3.png) **Figure 14.** Evolution of amplitude-frequency spectra of disturbances at $M = 3$.

![Figure 15](image4.png) **Figure 15.** Evolution of amplitude-frequency spectra of disturbances at $M = 4$. 
4. Conclusions

Evolution of natural disturbances in supersonic boundary layer of swept wing is investigated in detail. Oscillograms, amplitude-frequency spectra, pulsation profiles and statistical diagrams of natural fluctuations are obtained. The characteristic zones of disturbances development are defined. It is found that for Mach numbers 2 and 2.5 measurements can be performed in the region of linear stage of disturbances evolution, and the experimental data can be compared with the results of the calculations on linear stability theory. At Mach 3, 3.5 and 4 non-linear processes are observed from the beginning of measurements (50mm from the leading edge of the wing). Profiles of pulsation have two maxima, first corresponds to critical layer, second - subsonic layer. Fast growth of disturbances corresponding to the second maximum was observed in the case of swept wing in nonlinear region of disturbances evolution. Experimentally is defined a position of instability region of secondary flow at M=2 and 2.5. With the help of statistical analysis and analysis of amplitude-frequency spectra confirmed the existence of the mechanism of secondary instability and its features are revealed in supersonic flow of swept wing. Fluctuations grow at the frequency range from 8 to 35 kHz in the linear region. Nonlinear processes lead to increasing of high-frequency disturbances. Assume that at M = 2 the growth of high-frequency part of the spectrum \( f \div 35 \) kHz is caused by secondary instability. Similar results are obtained for other Mach numbers.

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