Fractional charge in electron clusters: Mani and von Klitzing data of quantum Hall effect- Part II

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We have calculated the fractional charge of quasiparticles in a cluster of electrons. The 61 values have been calculated which are exactly the same as the measured values. In a previous eprint we have calculated 85 values which are the same as the measured values. Thus, we have calculated 146 fractional charges which are the same as the experimental values. In the case of 61 values, we are able to determine the spin of the cluster and hence the number of electrons in a cluster. The polarization in a magnetic field is determined. A new zero conductivity state is found which is the same as “superresistivity” previously reported in our book. Anderson and Brinkman, cond-mat/0302129, have noted that frequencies like $5/4\Omega_c$ do occur. We predict all fractional frequencies correctly and report a new state with a zero-frequency mode.

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1. Introduction

Recently, we[1] have calculated 85 fractional charges which are the same as the experimental values reported by Mani and von Klitzing[2]. In all of the 85 values, we have determined the number of electrons and their polarization. In some cases, it is clear that not all electrons are located at one site. The analysis of the data of figures 1-4 of Mani and von Klitzing is given in the previous eprint.

In the present paper, we note the data from figures 5 and 6 of Mani and von Klitzing and calculate these 61 values. All of the 61 calculated values are the same as the measured values. Such a perfect agreement between the calculated and the measured values has never been found in the physics problems. The earlier 85 values and the present 61 values make 146 values for which there is full agreement between the calculated and measured values. It was pointed out by Pan et al[3] that some of the measured values did not come out from their calculations suggesting the inadequacy of their model. Therefore, there is a clear need for alternative theoretical possibilities. In another eprint, we have shown[4] that the values of Pan et al are well predicted by our theory.

2. Description

From the central part of figure 5 of Mani and von Klitzing, we read the following experimental values,

$$6/17, 11/31, 9/25, 4/11, 7/19, 10/27, 11/29, 8/21, 5/13$$ (1)

$$2/5, 7/17, 8/19, 3/7, 4/9, 5/11$$ (2)

$$16/39, 12/29, 13/31, 10/23, 11/25.$$ (3)
The right-hand-side inset gives the following experimental values,

\[
\frac{16}{37}, \frac{29}{67}, \frac{23}{53}, \frac{17}{39}, \frac{24}{55}, \frac{25}{57}, \frac{18}{41}, \frac{11}{25}, \frac{4}{9} \tag{4}
\]

and the left-hand-side of figure 5 has the following,

\[
\frac{11}{27}, \frac{20}{49}, \frac{16}{39}, \frac{12}{29}, \frac{17}{41}, \frac{18}{43}, \frac{8}{19}, \frac{3}{7}. \tag{5}
\]

Table 1: The interpretation of fractional charges in terms of \( l \) and \( s \). The calculated values are the same as the experimental values.

| S.No. | \( l \) | \( s \) | \( \nu_- \) | \( n_e \) |
|-------|--------|--------|----------|--------|
| 1     | 8      | -5/2   | 6/17     | 5      |
| 2     | 15     | -9/2   | 11/31    | 9      |
| 3     | 12     | -7/2   | 9/25     | 7      |
| 4     | 5      | -3/2   | 4/11     | 3      |
| 5     | 9      | -5/2   | 7/19     | 5      |
| 6     | 13     | -7/2   | 10/27    | 7      |
| 7     | 14     | -7/2   | 11/29    | 7      |
| 8     | 10     | -5/2   | 8/21     | 5      |
| 9     | 6      | -3/2   | 5/13     | 3      |
| 10    | 2      | -1/2   | 2/5      | 1      |
| 11    | 8      | -3/2   | 7/17     | 3      |
| 12    | 9      | -3/2   | 8/19     | 3      |
| 13    | 3      | -1/2   | 3/7      | 1      |
| 14    | 4      | -1/2   | 4/9      | 1      |
| 15    | 5      | -1/2   | 5/11     | 1      |
| 16    | 19     | -7/2   | 16/39    | 7      |
| 17    | 14     | -5/2   | 12/29    | 5      |
| 18    | 13     | -5/2   | 13/31    | 5      |
| 19    | 11     | -3/2   | 10/23    | 3      |
| 20    | 12     | -3/2   | 11/25    | 3      |

While writing the book[5] we have found that the fractional effective charge is given by,

\[
e_{\text{eff}}/e = \frac{l + \frac{1}{2} \pm s}{2l + 1} \tag{6}
\]

When we substitute the values of \( l \) and \( s \), we obtain the calculated values of the fractional charge. Table 1 shows 20 calculated values for various values of \( l \) and \( s \). All of these 20 calculated values are the same as the measured values. Next, we calculate 9 values as given in Table 2. These calculated values are the same as the experimental values of eq.(4). We calculate another set of 8 values given in Table 3. These calculated values are the same as the experimental values of eq.(5). All the calculated values given in Tables 1-3 are thus the same as the experimental values. All of these use negative sign for the
spin. Hence, these are polarized in the magnetic field with all particles parallel to the field with no particle with opposite spin. From the spin, we can know the number of electrons. Thus there are electron clusters with number of electrons 1, 3, 5, 7 or 9, in a cluster. All these electron numbers per cluster are odd numbers. For even number of electrons, the denominator becomes even. For odd number of electrons, the denominator is odd.

**Table 2:** Some of the fractional charges in terms of $l$ and $s$. The calculated values are the same as the experimental values.

| S.No. | $l$ | $s$  | $\nu_-$ | $n_e$ |
|-------|-----|------|---------|-------|
| 1     | 18  | -5/2 | 16/37   | 5     |
| 2     | 33  | -9/2 | 29/67   | 9     |
| 3     | 26  | -7/2 | 23/53   | 7     |
| 4     | 19  | -5/2 | 17/39   | 5     |
| 5     | 27  | -7/2 | 24/55   | 7     |
| 6     | 28  | -7/2 | 25/57   | 7     |
| 7     | 20  | -5/2 | 18/41   | 5     |
| 8     | 12  | -3/2 | 11/25   | 3     |
| 9     | 4   | -1/2 | 4/9     | 1     |

*Table 3:* Some more fractional charges in which the calculated values are the same as the experimental values.

| S.No. | $l$ | $s$  | $\nu_-$ | $n_e$ |
|-------|-----|------|---------|-------|
| 1     | 13  | -5/2 | 11/27   | 5     |
| 2     | 24  | -9/2 | 20/49   | 9     |
| 3     | 19  | -7/2 | 16/39   | 7     |
| 4     | 14  | -5/2 | 12/29   | 5     |
| 5     | 20  | -7/2 | 17/41   | 7     |
| 6     | 21  | -7/2 | 18/43   | 7     |
| 7     | 9   | -3/2 | 8/19    | 3     |
| 8     | 3   | -1/2 | 3/7     | 1     |

Now we take the experimental values from figure 6 of Mani and von Klitzing. These measured values are given below:

\[
\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{4}{7}, \frac{3}{5}, \frac{2}{3}, 1, 2, 3, 4, \quad (7)
\]
\[
\frac{9}{25}, \frac{4}{11}, \frac{7}{19}, \frac{8}{21}, \frac{5}{13}, \frac{7}{17}, \frac{8}{19}, \frac{11}{25} \quad (8)
\]
\[
\frac{11}{19}, \frac{8}{13}, \frac{7}{11}, \frac{5}{7}, \frac{8}{11}, \frac{7}{9}, \frac{4}{5}, \frac{9}{11}, \quad (9)
\]
\[
\frac{9}{7}, \frac{4}{3}, \frac{7}{5}, \frac{8}{5}, \frac{5}{3}, \frac{7}{3}, \frac{8}{3}, \frac{11}{3}. \quad (10)
\]

The calculation of these values is given in Tables 4-7.
Table 4: For $s=\pm 1/2$ both the spin polarizations occur. The calculated values of the fractional charges in terms of $l$ and $s$ are the same as those measured. In the first line $\nu_-=0$ indicates zero-frequency mode which is characteristic of charge-density waves.

| S.No. | $l$ | $s$ | $\nu_+$ | $\nu_-$ |
|-------|-----|-----|---------|---------|
| 1     | 0   | $1/2$ | 1       | 0       |
| 2     | 1   | $1/2$ | $2/3$   | $1/3$   |
| 3     | 2   | $1/2$ | $3/5$   | $2/5$   |
| 4     | 3   | $1/2$ | $4/7$   | $3/7$   |
| 5     | 4   | $1/2$ | $5/9$   | $4/9$   |
| 6     | $\infty$ | $1/2$ | $1/2$   | $1/2$   |

Table 5: Several fractional charges calculated for $l$ and $s$. The calculated values are the same as the measured values.

| S. No. | $l$ | $s$ | $\nu_-$ |
|--------|-----|-----|---------|
| 1      | 12  | $-7/2$ | $9/25$  |
| 2      | 5   | $-3/2$ | $4/11$  |
| 3      | 9   | $-5/2$ | $7/19$  |
| 4      | 10  | $-5/2$ | $8/21$  |
| 5      | 6   | $-3/2$ | $5/13$  |
| 6      | 8   | $-3/2$ | $7/19$  |
| 7      | 9   | $-3/2$ | $8/19$  |
| 8      | 12  | $-3/2$ | $11/25$ |

Table 4 shows that as long as the spin is $\pm 1/2$, both spin polarizations occur except for the small difference due to Boltzmann factor. Table 5 shows that only negative spin occurs due to polarization in the magnetic field. Similarly, Tables 6 and 7 show only the positive spin. In some places large values of spin occur so that electrons must be distributed over several sites so we get the idea of sites and clusters.

We learn from this exercise that there are electron clusters with a small number of electrons per cluster. There are spin polarizations in the field so that in some region all spins are aligned in one direction only. We determine the number of electrons per cluster. [In ref.1 the fraction 29/49 was left out. It is well predicted by $l=24$ and $s=9/2$ with positive sign for the spin. Therefore 85 charges are predicted in this reference].

3. **Zero conductance, zero charge or superresistivity.**

When the charge of the quasiparticles is zero, the resistivity becomes infinite and hence the conductivity at this point is zero. This is the phenomenon of “superresistivity” as it was called in the book[5]. The quantized resistivity is given by,

\[
\rho_{xy} = \frac{h}{ie^2}
\]  

(11)
When \( i=0, \rho_{xy}=\infty \). This gives the divergence in the resistivity so that there is “super-resistivity”. In an approximate representation,

\[
\sigma_{xx} = \rho_{xx}/\rho_{xy}^2.
\]  \((12)\)

**Table 6:** More values of fractional charges in terms of \( l \) and \( s \). The calculated values are the same as the experimental values.

| S.No. | \( l \) | \( s \) | \( \nu_+ \) |
|-------|--------|--------|---------|
| 1     | 9      | 3/2    | 11/19   |
| 2     | 6      | 1/2    | 8/13    |
| 3     | 5      | 3/2    | 7/11    |
| 4     | 3      | 3/2    | 5/7     |
| 5     | 5      | 5/2    | 8/11    |
| 6     | 4      | 5/2    | 7/9     |
| 7     | 2      | 3/2    | 4/5     |
| 8     | 5      | 7/2    | 9/11    |

**Table 7:** More fractional charges. There are 61 fractional values in this paper. All of the calculated values are the same as the experimental values.

| S.No. | \( l \) | \( s \) | \( \nu_+ \) |
|-------|--------|--------|---------|
| 1     | 3      | 7/2    | 9/7     |
| 2     | 1      | 5/2    | 4/3     |
| 3     | 2      | 9/2    | 7/5     |
| 4     | 2      | 11/2   | 8/5     |
| 5     | 1      | 7/2    | 5/3     |
| 6     | 1      | 11/2   | 7/3     |
| 7     | 1      | 13/2   | 8/3     |
| 8     | 1      | 19/2   | 11/3    |

When \( \rho_{xy}=\infty, \sigma_{xx}=0 \). These are the “zero conductivity” points. The effective charge of the quasiparticles is,

\[
i = \frac{l + \frac{1}{2} \pm s}{2l+1}
\]  \((13)\)

which is zero for \( l + \frac{1}{2} \pm s=0 \). For \( l=0, s=1/2 \) for the negative sign, the charge becomes zero. This is because the charge acquires a vector nature due to \( l \) and \( s \). The effective charge is zero for \( l=0, s=-1/2; l=1, s=-3/2; l=2, s=-5/2; l=3, s=-7/2; \ldots \), etc. These are the points of infinite \( \rho_{xy} \) and zero conductivity.

Yang et al[6] have found the points of zero conductivity. In eq.(12) making \( \rho_{xy}=\infty, \sigma_{xx}=0 \) is consistent with the experimental observation of zero conductivity but now \( \rho_{xx} \) is not known. Let us search for points where \( \rho_{xx} \) may also be zero. If we use semiquantized
values, then by using classical Hall effect, we can write \( \sigma_{xx} = \rho_{xx} (neC/B)^2 \) which can give \( \rho_{xx} = 0 \) points as well as \( \sigma_{xx} = 0 \). Therefore, in the semiclassical model both the conductivity as well as the resistivity can show zeros. Recently, several authors have shown interest in this problem [7-10]. Anderson and Brinkman [7] have noted that the frequencies of \( 3/4 \Omega_c, 7/4 \Omega_c, 5/4 \Omega_c \) do occur. Our formula (6) works very well and predicts all fractions correctly.

4. Conclusions

There are single electrons which give the fractional charge as given in Table 4. There are clusters of 2, 3, 4, ..., 9 electrons. The electrons are not all on one site. They are spin polarized. The vector nature of charge is exhibited when it is described in terms of \( l \) and \( s \). There is a state of zero charge and “superresistivity”. Along with ref. 1, we have calculated 146 fractional charges in which all of the calculated values are the same as the experimentally measured values. The fractional charge 1/3 occurs in Table 4, in a natural way and when the field is varied, its Kramers conjugate with charge 2/3 is found. It is shown elsewhere [5] that the calculated temperature dependent spin polarization agrees with the experimental data. Similarly, there is a Bose-Einstein condensation at half filled Landau level.

Are there any charge-density waves? Yes, we do have a solution with zero frequency, \( l=0, s=-1/2 \). Are there any superconducting states? Yes, clusters of three electrons with all spin parallel do not have a Meissner effect but can superflow. At very large values of \( l \) it is possible to have singlet pairs. For \( l=0, s=1 \), there are two solutions, one has a charge of 3/2 and the other -1/2, the difference between the two being 2\( e \), can exhibits triplet type superconductivity but there are clusters so that uniform medium is not available.

5. References

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