Non-Hermitian Edge Burst

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We unveil an unexpected non-Hermitian phenomenon, dubbed edge burst, in non-Hermitian quantum dynamics. Specifically, in a class of non-Hermitian quantum walk in periodic lattices with open boundary condition, an exceptionally large portion of loss occurs at the system boundary. The physical origin of this edge burst is found to be an interplay between two unique non-Hermitian phenomena: non-Hermitian skin effect and imaginary gap closing. Furthermore, we establish a universal bulk-edge scaling relation underlying the non-Hermitian edge burst. Our predictions are experimentally accessible in various non-Hermitian systems including quantum-optical and cold-atom platforms.

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Standard quantum mechanics postulates Hermiticity of Hamiltonian, yet non-Hermitian Hamiltonians are useful in many branches of physics. For example, open systems with gain and loss naturally exhibit non-Hermitian physics [1]. Recently, there has been growing interest in non-Hermitian topological physics. In particular, the bulk-boundary correspondence principle is drastically reshaped by the non-Hermitian skin effect (NHSE), namely the boundary localization of bulk-band eigenstates [2–11]. It indicates that the boundary plays an even more profound role in non-Hermitian systems compared to their Hermitian counterparts.

In this Letter, we unveil a boundary-induced dynamical phenomenon, dubbed “edge burst,” in a class of non-Hermitian systems. For concreteness, we consider quantum-mechanical time evolution of particles (called “quantum walkers”) in a lossy lattice. Intuitively, a walker starting from a certain site far from the edges is expected to escape predominantly from nearby sites. However, a prominent peak in the loss probability is found at the edge. More unexpectedly, the relative height of this peak grows with the distance from the initial site to the edge. Furthermore, we find that this edge burst exhibits a unique scaling behavior, originating from a universal bulk-edge scaling relation. This provides an underlying theory that not only tells the precise conditions for edge burst, but also has implications beyond.

We note that the appearance of an edge peak has been reported in a very recent work, though it was incorrectly attributed to topological edge states [12]. Our work demonstrates that the edge burst stems entirely from the non-Bloch bulk bands, highlighting it as a robust phenomenon insensitive to edge perturbations.

Non-Hermitian edge burst.—For concreteness, we consider a one-dimensional lossy lattice shown in Fig. 1(a). During the quantum walk, the walker can escape from B sites. The Schrödinger equation \( i\frac{d\psi_x}{dt} = H\psi_x \) reads

\[
i \frac{d\psi_x}{dt} = t_1\psi_x + t_2\psi_{x-1} - \gamma \psi_x = -\gamma \psi_x \tag{1}
\]

with loss rate \( \gamma > 0 \). The corresponding Bloch Hamiltonian is

\[
H(k) = (t_1 + t_2 \cos k)\sigma_x + \left( t_2 \sin k + i\frac{\gamma}{2} \right)\sigma_z - i\frac{\gamma}{2}I. \tag{2}
\]

where \( \sigma_{x,z} \) are the Pauli matrices, with \( \sigma_z = 1 \) (−1) corresponding to A(B) sublattice, and \( I \) is the identity matrix. This model is similar to that of Ref. [13], except that it is purely lossy. Notably, it features the NHSE, which distinguishes it from earlier quantum-walk models [14]. Intuitively, the \( -\pi/2 \) fluxes in the triangles generate rotational motions, such that the A and B chains favor opposite directions of motion; the loss then generates a net chiral motion along the A chain by suppressing the backflow on the B chain. Alternatively, the NHSE can be seen via the equivalence of the model, under a basis change, to the non-Hermitian Su-Schrieffer-Heeger model with left-right asymmetric hopping [2].

The wave function norm decreases as \( \langle d/dt | \psi(t) \rangle | \psi(t) \rangle = i\langle \psi(t) | H^\dagger - H | \psi(t) \rangle = -\sum x 2\gamma |\psi_x(t)|^2 \), and the probability that the walker escapes from location \( x \) is

\[
P_x = 2\gamma \int_0^\infty dt |\psi_x(t)|^2. \tag{3}
\]

Note that \( \sum_x P_x = 1 \) is satisfied under the initial-state normalization \( \langle \psi(0) | \psi(0) \rangle = 1 \). Let us consider a walker starting from \( x = x_0 \), with \( \psi_x^A = \delta_{x,x_0} \) and \( \psi_x^B = 0 \). It appears natural to expect that \( P_x \) would decay away from \( x_0 \), which is confirmed by numerical simulations [Fig. 1(b)]. We also notice that the \( P_x \) distribution is left-right asymmetric.
The preference of walking left can be attributed to the NHSE, all eigenstates being localized at the left edge [2].

The most intriguing feature is the exceptionally high peak at the left edge, namely the edge burst, which stands out from the almost invisible decaying tail [Fig. 1(e)]. Such a peak was numerically seen in Ref. [12]. However, it was unclear when and why the peak occurs. It was attributed to topological edge states, which turns out to be incorrect. In fact, both (b) and (c) in Fig. 1 are within the topologically nontrivial regimes (i.e., there are topological edge modes) [2,4], yet the edge burst occurs only in (c), which looks puzzling.

To quantify the edge burst, we calculate the relative height, defined as $P_{\text{edge}}/P_{\text{min}}$, where $P_{\text{edge}} = P_1$, and $P_{\text{min}} = \min\{P_1, P_2, \ldots, P_x\}$ is the minimum of $P$ between the starting point and the edge. The existence and absence of edge burst manifests in $P_{\text{edge}}/P_{\text{min}} \gg 1$ and $P_{\text{edge}}/P_{\text{min}} \sim 1$, respectively. We see in Fig. 1(d) that the relative height increases with $x_0$ for $t_1 \in (0, t_2]$ (approximately), and rapidly decreases to order of unity otherwise, with $t_2 = 0.5$ fixed. In Fig. 1(e), we plot the relative height for $t_1 = 0.40$ and 0.63, which grows with $x_0$ in the former case. The numerical fitting $P_{\text{edge}}/P_{\text{min}} \sim (x_0)^{1.03}$ is close to being linear. We note that NHSE is present for all $t_1 \neq 0$, and therefore Figs. 1(d) and 1(e) tell us that NHSE by itself does not guarantee edge burst.

To unveil the origin of edge burst, we plot both $P_{\text{edge}}$ and bulk $P_x$ in Fig. 2. Figures 2(a) and 2(b) indicate that $P_{\text{edge}}$ follows a power law for $|t_1| \leq |t_2|$, 

$$P_{\text{edge}} \sim |x_0|^{-\gamma},$$

and an exponential law $P_{\text{edge}} \sim |\lambda_x|^x_0$ for $|t_1| > |t_2|$. Figures 2(c) and 2(d) indicate similar behaviors in the bulk.

FIG. 1. (a) The model. Each unit cell, labeled by spatial coordinate $x$, contains two sites A and B. (b),(c) The spatially resolved loss probability $P_x$ for a walker initiated at $x_0 = 50$. $t_1 = 0.63$ for (b) and $t_1 = 0.4$ for (c). The chain length $L = 60$. (d) The relative height $P_{\text{edge}}/P_{\text{min}}$ with varying $t_1$, for $x_0 = 50$ and 25. Here, $P_{\text{edge}} = P_{x=1}$ and $P_{\text{min}} = \min\{P_1, P_2, \ldots, P_x\}$. (e) Relative height with $x_0$ varying from 40 to 140, for $t_1 = 0.63$ (black square) and $t_1 = 0.40$ (blue triangle) [marked in (d)]. $L = 150$. Throughout (b)–(e), $t_2 = 0.5, \gamma = 0.8$ are fixed.

FIG. 2. (a),(b) The height of edge peak in double logarithmic (a) and logarithmic (b) plot. (c),(d) The bulk distribution of $P_x$ in double logarithmic (c) and logarithmic (d) plot. $L = 200$ for (a)–(d), and $x_0 = 150$ for (c),(d). (e) Energy spectra under periodic boundary condition (PBC). The green, red, and blue spectrums close the imaginary gap (dissipative gap), i.e., touch the real axis, while the black spectrum exhibits a nonzero imaginary gap. (f) Generalized Brillouin zone (GBZ). Throughout (a)–(f), $t_2 = 0.5, \gamma = 0.5, t_1 = 0$ (green), 0.3 (red), 0.5 (blue), and 0.6 (black).
\[ P_x \sim |x - x_0|^{-\alpha_b}, \]  

for \(|t_1| \leq |t_2|\), and exponential law \( P_x \sim (\lambda_b)_{t_0-x} \) (\( \lambda_b < 1 \)) for \(|t_1| > |t_2|\). Note that Eq. (5) is valid only for \( x \) in the bulk, i.e., not too close to the edge; also note that \( \alpha_b \neq \alpha_e \). The algebraic (i.e., power-law) behavior of bulk \( P_x \) reflects the algebraic decay of wave function norm in the time domain, which originates from the Bloch energy spectrum touching the real axis, i.e., closing the imaginary gap [Fig. 2(e)]. In other words, algebraic decay corresponds to max[Im\( E(k) \)] = 0, with \( E \) denoting the eigenspectrums of \( H \). It can be readily checked that the imaginary gap closes for \(|t_1| \leq |t_2|\) [15].

In the language of open quantum system, the algebraic behavior means that the dissipative gap (or Liouvillian gap) closes [16]. In fact, our non-Hermitian \( H \) in Eq. (1) can be reformulated in terms of the quantum master equation, 
\[ \frac{d\rho}{dt} = -i[H, \rho] + \sum_{ij} (L_i \rho L_j^\dagger - \frac{1}{2} \{ L_i^\dagger L_j, \rho \}), \]
where \( H = \sum_{ij} h_{ij} c_i^\dagger c_j \), with \( h \) denoting the Hermitian part of \( H \) in Fig. 1(a), namely, \( h_{ij} = H_{ij}(\gamma = 0) \), and the dissipator \( L_x = \sqrt{2} \gamma c_x^\dagger \). Note that \( c_i \) can be either bosonic or fermionic, which does not affect the single-particle dynamics. The effective non-Hermitian Hamiltonian \( H_{eff} = \hat{\mathcal{H}} - \sum_{i,j} \frac{1}{2} L_i^\dagger L_i = \hat{\mathcal{H}} - i\gamma \sum_{i,j} c_i^\dagger c_j = \sum_{i,j} \Delta_i c_i^\dagger c_j \). In this context, \( \max[\text{Im}\ E(k)] = 0 \) corresponds to closing the dissipative (imaginary) gap.

Given the imaginary gap closing, namely \(|t_1| \leq |t_2|\), we always see the edge burst except at \( t_1 = 0 \). The \( t_1 = 0 \) point is special in two aspects. First, NHSE is absent at this parameter value. Second, the periodic-boundary-condition (PBC) energy spectrum encloses zero area in the complex plane [green triangle in Fig. 2(e)]. These two features are concurrent. In fact, a precise correspondence has been established between the existence (absence) of NHSE and the complex energy enclosing nonzero (zero) area [17,18]. The zero and nonzero enclosed area is also known as having trivial and nontrivial point-gap topology, respectively [19–21].

Summarizing the above numerical findings, we infer that the edge burst stems from the interplay between two prominent non-Hermitian phenomena, NHSE and imaginary gap closing. The latter is a non-Hermitian counterpart of being gapless in Hermitian systems. This imaginary gaplessness and NHSE jointly induce the edge burst. 

**Bulk-edge scaling relation.**—The exponent \( \alpha_e \) in Eq. (4) and \( \alpha_b \) in Eq. (5) characterize the edge and bulk dynamics, respectively. One of our central results is the scaling relation
\[ \alpha_e = \alpha_b - 1 \]  
in the presence of NHSE and imaginary gap closing. For our specific model, it holds true when \(|t_1| \leq |t_2|\) (such that imaginary gap closes) and \( t_1 \neq 0 \) (such that NHSE is present). At the NHSE-free point \( t_1 = 0 \), we have \( \alpha_e = \alpha_b \) instead. Remarkably, although both \( \alpha_b \) and \( \alpha_e \) are model and parameter dependent, the relation Eq. (6) remains universal. Numerical fitting in Figs. 2(a) and 2(c) yields \( \alpha_e - \alpha_b = 0.99, 1.03 \) for \( t_1 = 0.3, 0.5 \), respectively, which is in reasonable agreement with Eq. (6). For \( t_1 = 0 \), the fitting yields \( \alpha_b - \alpha_e = 0.09 \), being close to the theoretical value 0 for the NHSE-free cases.

Before calculating \( \alpha_e, \alpha_b \), and proving Eq. (6), we observe that this equation implies edge burst. In fact, Eq. (5) implies that \( P_x \) takes the minimum near (but not too close to) the edge, and \( P_{\text{min}} \sim x_0^{-\alpha_e} \). Therefore, it follows from Eq. (6) that
\[ P_{\text{edge}}/P_{\text{min}} \sim x_0^{\alpha_e-\alpha_b} \sim x_0. \]  
Thus, as the starting point \( x_0 \) moves away from the edge, the relative height of edge peak increases. This is precisely the origin of the edge burst.

Now we calculate \( \alpha_b, \alpha_e \) and derive Eq. (6) using Green’s function, which has been a useful tool in non-Hermitian systems [22–27]. The integrand in Eq. (3) can be expressed as \( \langle x, B(G(t_\gamma(t = 0)) \rangle x_0, A \rangle \), where \( G(t) = -i\Theta(t)e^{-itH} \), with \( \Theta(t) \) standing for the Heaviside step function. It is convenient to work in the frequency (energy) domain using \( G(t) = (1/2\pi\int_{-\infty}^{\infty} do G(o)e^{-i0t}) \), in which the Green’s function reads \( G(o) = [1/(o + i0^+ - H)] \). Now we can recast Eq. (3) into
\[ P_x = \frac{1}{\pi} \int_{-\infty}^{\infty} do \langle x, B(G(o)) | x_0, A \rangle^2, \]
where the initial state \( \langle \psi(t = 0) \rangle = x_0, A \rangle \) has been inserted. To calculate \( \alpha_b \), it is more convenient to consider an infinite chain. The relevant Green’s function reads
\[ \langle x, B(G(o)) | x_0, A \rangle = \int_{\beta_1 = \beta_{\text{min}}}^{2\pi} \frac{d\beta}{2\pi e^{\beta/2}} \frac{1}{o + i0^+ - H(\beta)} \]
\[ \simeq \frac{e^{\beta/2}}{2\pi e^{\beta/2}} \frac{1}{o + i0^+ - H(\beta)} \]
where \( H(\beta) \) is the analytic continuation of \( H(k) \) in Eq. (2). \( H(\beta) \equiv H(k)|_{\beta = \beta_{\text{min}}} \). For our specific model, \( \{1/(o + i0^+ - H(\beta))\}_{BA} = \{t_1 + t_2((\beta + 1/2)/2)\} / \det(o + i0^+ - H(\beta)) \). This integration can be done by the residue theorem, and the asymptotic behavior at \( x \to x_0 \to \infty \) is determined by the roots of \( \det(o + i0^+ - H(\beta)) = 0 \) [23]. As a quadratic equation, it has two roots that we order as \( \beta_{L}(\omega) \geq \beta_{R}(\omega) \). Following Ref. [23], we have \( \langle x, B(G(o)) | x_0, A \rangle \sim f_L f_L^{x_0} \) for \( x < x_0 \), and \( \langle x, B(G(o)) | x_0, A \rangle \sim f_R f_R^{x_0} \) for \( x > x_0 \) \((\beta_L(\omega) \geq 1 \geq \beta_R(\omega)) \) is satisfied for real-valued \( o \), where \( f_{L/R} \) are \( x \) independent and their precise values do not concern us [15]. Accordingly, \( P_{\infty} \), in which the superscript \( \infty \) stands for the infinite chain, is given by
where the subscripts \( L \) and \( R \) correspond to \( x < x_0 \) and \( x > x_0 \), respectively. For \(|x - x_0|\) large, the integral of Eq. (10) is dominated by \(|\beta| \) closest to 1. In fact, the existence (absence) of a real \( \omega \) satisfying \( |\beta(\omega)| = 1 \) determines the algebraic (exponential) behavior of \( P_x \). To satisfy \( |\beta(\omega)| = 1 \) for real-valued \( \omega \) is to close the imaginary gap of Bloch Hamiltonian, because the gap-closing point \( \omega_0 \) satisfies \( \delta(\omega_0 - H(\beta)) = 0 \) with \( |\beta| = 1 \). These \( \omega_0 \) values are marked as \( A_1, A_2, \) and \( B \) in Fig. 2(e), and the corresponding \( \beta \) values in Fig. 2(f).

As we focus on \( x < x_0 \), the relevant root is \( \beta_L(\omega) \). Let us write \( \omega = \omega_0 + \delta \omega \), and then expand \( \beta_L(\omega), f_L(\omega) \) to the lowest order of \( \delta \omega \), so that \( |\beta_L(\omega)| \approx 1 + K \delta \omega p \approx \exp(K \delta \omega p) \), and \( |f(\omega)|^2 \approx \delta \omega m \). Now \( P_x \approx \int d(\delta \omega) \delta \omega m \exp(-2K \delta \omega p |x - x_0|) \sim |x - x_0|^{-m+1} / n \), and therefore \( \alpha_b = (m + 1) / n \). In contrast, when the imaginary gap opens, we have \( |\beta_L(\omega)| > 1 \) and exponential decay \( P_x \sim \{\min[|\beta_L(\omega)|]\}^{-2|x-x_0|} \). For our model, the imaginary gap closing regime is \(|t_1| \leq |t_2|\), in which the bulk \( P_x \) indeed exhibits algebraic behavior. Furthermore, taking \( t_2 > 0 \), we have \((n, m) = (1/2, 0), (2, 2), (4, 4)\) for \( t_1 = 0, t_1 \in (0, t_2), \) and \( t_1 = t_2 \), respectively. This leads to the bulk exponent [15]

\[
\alpha_b = \frac{m + 1}{n} = \frac{2}{5}, \frac{3}{2}, \frac{5}{4},
\]

(11)

for these three cases, which is in reasonable agreement with the numerical values \( \alpha_b = 2.13, 1.51, 1.39 \) obtained from Fig. 2(c).

Now let us consider a chain with open boundary condition (OBC) at \( x = 1 \) and \( L \) [Fig. 1(b)]. The NHSE of our model localizes all eigenstates exponentially to the edge. This effect can be precisely characterized by the generalized Brillouin zone (GBZ), which is the trajectory of \( \beta \) associated with OBC eigenstates [2,28–32]. In our model, the GBZ is a circle with radius |\( \beta | = \sqrt{|(t_1 - y/2)/(t_1 + y/2)|} < 1 \) for \( t_1 > 0 \), indicating NHSE with skin modes localized at the left edge [2]. The NHSE induces leftward walking, and the walker becomes trapped at the left edge once it arrives there. We compare the \( P_x \) of the (effectively) infinite chain and finite chain [Fig. 3(a)], which indicates that \( P_x^\infty \) is almost the same as OBC \( P_x \) for \( x \) not too close to the edge. In view of the probability sum \( \sum_x P_x = 1 \) in both cases, we conclude that the missing part, namely the edge accumulation in the OBC case and \( \sum_{x=\infty} P_x^\infty \) in the infinite-chain case, must be equal. This observation leads to the estimation

\[
P_{\text{edge}} \approx \int_0^{x_0} \int_{\infty}^{x_0} |x - x_0|^{-\alpha_b} dx \]

\[
\approx \int_{x_0}^{\infty} x^{-\alpha_b} dx = (x_0)^{-\alpha_b+1}.
\]

(12)

Therefore, we see that \( \alpha_b \) in Eq. (4) equals \( \alpha_b - 1 \). As explained by Eq. (7), this “\(-1\)” in exponent means a dramatic enhancement of \( P_{\text{edge}} \) compared to the decay tail of \( P_x \), generating the edge burst. In contrast, when the imaginary gap is nonzero, we have \( P_{\text{edge}} \sim \int_0^{x_0} (\lambda b)^{\alpha_b - 1} dx \sim \int_{x_0}^{\infty} (\lambda b)^{\alpha_b - 1} dx \sim (\lambda b)_0^{\alpha_b - 1} \), which is of the same order as the decay tail (taking \( x = 0 \) in \( P_x \sim \lambda b_0^{\alpha_b - 1} \)), and therefore no edge burst exists. Moreover, it implies \( \lambda = \lambda_b \). Numerical fitting in Figs. 2(b) and 2(d) yields \( \lambda_b \approx 0.916 \) and \( \lambda \approx 0.917 \), being close to each other.

Our calculations above demonstrate the respective role of imaginary gap closing and NHSE in creating the edge burst. The former causes the algebraic decay of \( P_x \) in the bulk, while the latter drives chiral motion and contributes the crucial “\(-1\)” to the right-hand side of Eq. (6).
Since $\alpha_n$ is a bulk-band quantity, Eqs. (12) and (6) unambiguously tell that the edge burst is a bulk-band phenomenon independent of edge details. The bulk-band nature can also be seen in the longtime behavior of wave function. In fact, we can write $H = \sum_n E_n |n_R\rangle \langle n_L|$ in terms of the right and left eigenstates $|n_R\rangle$ and $|n_L\rangle$, then $\langle x, B | \psi(t) \rangle = \sum_n e^{-iE_n t} \langle x, B | n_R \rangle \langle n_L | x_0, A \rangle$. It follows that $\max \{ \text{Im}(E_n) \}$ dominates the longtime behavior, and $|\langle x, B | \psi(t) \rangle | \sim e^{\max \{ \text{Im}(E_n) \}}$ for $t \to \infty$. Under OBC, the bulk band consists of skin modes localized at the edge, and $E_n$ should be calculated from GBZ [2,28]. According to Longhi [29], $\max \{ \text{Im}(E) \}$ of bulk band occurs at a saddle point $\beta_s$ on GBZ, satisfying $\langle \partial E / \partial \beta \rangle_{\beta_s=0} = 0$. We numerically calculate the time dependence of edge-site wave function, which indeed follows an exponential law with exponent close to $\max \{ \text{Im}(E) \}$ [insets of Fig. 3(a)], confirming the bulk-band nature of edge burst. To further back up our results, we change the sign of $t_1$ so that the skin modes and edge burst are seen at the right edge; the results again support our picture [Fig. 3(b)]. Results from other models, including those with bipolar NHSE [33], also confirmed our theory [15].

Conclusions.—We unveil a boundary-induced non-Hermitian dynamical phenomenon, dubbed the edge burst, which is an unexpected interplay between imaginary (dissipative) gap and NHSE. Its origin is identified as a universal bulk-edge scaling relation [Eq. (6)]. Our theory can be readily confirmed in various non-Hermitian platforms including, for example, the photon quantum walk in which NHSE has been realized and the dissipative gap can be conveniently tuned [8,34]. Dissipative cold atom systems with NHSE are also a promising platform [35,36].

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