Observation of antichiral edge states in a circuit lattice

YuTing Yang1,2,3, DeJun Zhu2, ZhiHong Hang2,4*, and YiDong Chong3,5*

1 School of Materials Science and Physics, China University of Mining and Technology, Xuzhou 221116, China; 2 School of Physical Science and Technology & Collaborative Innovation Center of Suzhou Nano Science and Technology, Soochow University, Suzhou 215006, China; 3 Division of Physics and Applied Physics, School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore 637371, Singapore; 4 Institute for Advanced Study, Soochow University, Suzhou 215006, China; 5 Centre for Disruptive Photonic Technologies, Nanyang Technological University, Singapore 637371, Singapore

Received January 24, 2021; accepted January 29, 2021; published online March 24, 2021

We construct an electrical circuit to realize a modified Haldane lattice exhibiting the phenomenon of antichiral edge states. The circuit consists of a network of inductors and capacitors with interconnections reproducing the effects of a magnetic vector potential. The next nearest neighbor hoppings are configured differently from the standard Haldane model, and as predicted by earlier theoretical studies, this gives rise to antichiral edge states that propagate in the same direction on opposite edges and co-exist with bulk states at the same frequency. Using pickup coils to measure voltage distributions in the circuit, we experimentally verify the key features of the antichiral edge states, including their group velocities and ability to propagate consistently in a Möbius strip configuration.

antichiral edge state, modified Haldane model, topological circuit

PACS number(s): 73.20.At, 07.50.Ek, 03.65.Vf

Citation: Y. T. Yang, D. J. Zhu, Z. H. Hang, and Y. D. Chong, Observation of antichiral edge states in a circuit lattice, Sci. China-Phys. Mech. Astron. 64, 257011 (2021), https://doi.org/10.1007/s11433-021-1675-0

1 Introduction

The Haldane model [1] is a simple but rich theoretical model that exemplifies the physics of two-dimensional topological insulators [2-5]. When magnetic fluxes are appropriately threaded through a honeycomb lattice, with zero net flux in each unit cell, the band structure hosts a band gap spanned by chiral edge states; on a rectangular strip, the edge states localized on opposite edges will propagate in opposite directions. The edge states are protected by topological band invariants (Chern numbers) of the bulk bands. The Haldane model has been realized in condensed matter systems [6], and very similar models have been used to create classical wave analogues of topological insulators based on electric circuits [7-11], photonics [12-14], and acoustics [15-17]. The Haldane model has also inspired the development of more complex topological insulators, such as the time-reversal (T) invariant Kane-Mele model [18,19], which consists of spin-up and spin-down sectors that can be regarded as two copies of the Haldane model.

Recently, Colomès and Franz [20] discovered that a subtle modification to the Haldane model leads to a strikingly different behavior. With a different configuration of magnetic fluxes, equivalent to reversing the next-nearest-neighbor (NNN) hoppings in one sublattice, the lattice exhibits “an-
tichiral edge states” that propagate in the same direction on opposite sides of a rectangular strip. Moreover, the bulk spectrum is ungapped, so on a finite rectangular strip the transmission in one direction is edge-dominated whereas transmission in the opposite direction must occur via the bulk [20]. To our knowledge, there is thus far no experimental demonstration of this effect, despite proposals to realize it using strained materials [21], ferromagnetic materials with Dzyaloshinskii-Moriya interactions [22], exciton polaritons [23], gyromagnetic photonic crystals [24], graphene [25,26], and other systems [27].

Here, we use electric circuits to experimentally realize antichiral edge states and study their properties. Circuit metamaterials have been the subject of recent theoretical and experimental interest [28-40] due to the ease with which they can be designed and fabricated to realize different topological phases, as well as unusual lattice configurations that are hard to achieve on other platforms. Circuits have been used to demonstrate nonlinear topological boundary states [33,34], topological corner modes [35-38], and four-dimensional topological insulators [39,40]. Most notably, Jia et al. [7] have shown how a Haldane-type Chern insulator phase can be accessed using a lattice of capacitors (C) and inductors (L) with braided interconnections. Although LC circuits are time-reversal symmetric, the braiding decomposes the spectrum into two degenerate decoupled sectors that are individually T-broken [28-43], with the physical T symmetry mapping each sector to the other. Utilizing this idea, we design and fabricate a braided LC circuit lattice that realizes the modified Haldane model. Using different samples with electrical connections simulating periodic or closed boundaries, we probe the bulk and edge excitation spectra as well as the antichiral propagation characteristics of the edge states, which agree well with theoretical predictions. Moreover, we show experimentally that the antichiral states are able to propagate consistently (i.e., without switching sectors) along the edge of a Möbius strip edge—a property that the chiral edge states of a Chern insulator does not have [7]. This work points the way toward using circuit metamaterials for future studies of more complicated T-broken materials, including higher dimensional lattices and unusual sample geometries.

2 Results and discussion

The schematic of the modified Haldane model [20] is shown in Figure 1(a). Each unit cell contains two sites, A and B. The NNN hoppings between A sites and between B sites have π/2 phase shifts in the directions indicated by the arrows. The NNN hopping phases are same between A and B sites in modified Haldane model, which is inverted for Haldane model. The nearest-neighbor (NN) hoppings have zero phase. Figure 1(b) shows schematically how such hoppings can be realized using interconnected capacitor and inductor elements. On each lattice site there are two inductors X and Y, whose ends are labeled as X± and Y±; the voltages across the inductors are $U_{X} = V_{X} - V_{X'}$ and $U_{Y} = V_{Y} - V_{Y'}$, respectively. All the inductors have the same inductance $L$, and inductors at different sites are connected via capacitors. For NN (zero phase) hoppings, we connect each end $X_±$ to $Y_±$ with capacitors of capacitance $C_1$. For NNN (π/2 phase) hoppings, we use capacitors of capacitance $C_2$, and the connections are braided so that $U_{X'±} \rightarrow -U_{Y'±}$ and $U_{Y'±} \rightarrow U_{X'±}$. Defining $U_{i'±} = U_{i±} \pm iU_{q}$, we find that the NNN hoppings correspond to $U_{i} \rightarrow iU_{i}$, and $U_{i'} \rightarrow -iU_{i'}$, as desired [7,43]. Henceforth, we will focus on one of the two “spin” sectors, specifically the spin-up.

For steady-state solutions of angular frequency $\omega$, we can show using Kirchhoff’s laws (see Supplementary Information) that

$$
\begin{align*}
E = \begin{pmatrix}
U_{k±}^d & \left( P_k(\phi) \right) & T_k \\
U_{k±}^u & T_k^* & P_k(-\phi) \\
\end{pmatrix}
\begin{pmatrix}
U_{k±}^d \\
U_{k±}^u \\
\end{pmatrix}
\end{align*}
$$

where $E = 3t_1 + 6t_2 - 2\omega_0^2 \omega_0 = 1/\sqrt{LC}$, and C is a reference capacitance such that $C_1 = t_1 C$ and $C_2 = t_2 C$. This has the form of the modified Haldane model, with the caveat that the eigenvalue $E$ is not equal to the eigenfrequency. The Hamiltonian matrix elements are defined by $T_k = t_1(e^{i\phi} + e^{-i\phi})$ and $P_k = 2t_2[\cos(kv_1 + \phi) + \cos(kv_2 + \phi) + \cos(kv_3 + \phi)]$, where $\phi = \pi/2$ is the NNN hopping phase. As indicated in the right panel of Figure 1(a), $e_1, e_2, e_3$ are the NN bond vectors, and $v_j (j=1, 2, 3)$ are the NNN bond vectors.

We choose the circuit parameters to be $L = 3.3$ mH, $C_1 = 330$ pF and $C_2 = 33$ pF, so that the eigenfrequency is related to $E$ by $f = \frac{1}{2\pi \sqrt{LC}} \sqrt{\frac{2}{3.6-E^2}}$, where $C = 330$ pF (i.e., $t_1 = 1$ and $t_2 = 0.1$). Figure 1(c) shows the resulting band diagram, with the physical eigenfrequency $f$ as the vertical axis, for a strip that is infinite along x and 20 unit cells wide along y, with zigzag boundaries. In agreement with the prior findings of Colomés and Franz [20], the Dirac points are shifted in opposite directions, and joined by a two-fold degenerate arc.

Figure 1(d) shows the intensity distributions for four of the eigenstates at wavenumber $k = \pi/a$. The middle two panels, labelled $u_1$ and $u_3$, correspond to the two degenerate eigenmodes at frequency 113.63 kHz (red dashes in Figure 1(c)); they are localized to opposite edges of the strip, despite having the same group velocity as shown in Figure 1(c). The other eigenstates are bulk, as exemplified by the eigenstates labelled $u_2$ and $u_3$, which occur at frequencies 105.08 and 124.28 kHz (blue dashes in Figure 1(c)). The results shown here are for the spin-up states. For the spin-
down states, the antichiral edge states have the opposite group velocity (Figure S1).

We implement the lattice using a FR4 printed circuit board (PCB), as shown in Figure 2(a). The black cylinders in the photograph are unshielded wire-wound inductors with 13 Ω series resistance, and the yellow components are coupling ceramic capacitors. The circuit parameters are as stated in the previous paragraph. The PCB contains additional traces that can be used to connect additional inductors. The PCB is 2 sites wide in the $x$ direction and 10 sites (5 unit cells) wide in the $y$ direction; the site numbers are explicitly labeled (1 to 20) in the photograph.

We first connect the left and right boundaries using capacitive connections in order to realize periodic boundary conditions along $x$. Since the circuit is two unit cells wide along $x$, this is equivalent to probing $k = 0$ and $k = \pi/a$ in the band diagram of Figure 1(a), with the latter allowing the antichiral edge states to be accessed. Additional shorted-out capacitors are added to the top and bottom $y$ boundaries to achieve clean zigzag boundaries, compensating for the change in on-site potential caused by missing couplings at the lattice terminations. The input signal is produced by a function generator (Tektronix AFG3022C) connected to 9-turn, 8-mm-diameter air-core driving coils. The output signal is obtained with a pickup coil of the same dimensions, connected to a lock-in amplifier (Zurich Instrument MFLI). We place driving coils on the $X$ inductors at sites 2 and 12, and use the pickup coil to measure the voltage amplitude on the $Y$ inductor at site 1. The results are shown in the red curve in Figure 2(b). The response is peaked at 111.1 kHz, close to the predicted frequency of the antichiral edge states shown in Figure 1(c). Although the antichiral edge states coexist with bulk states, they are preferentially excited due to the strong spatial overlap with the driving source. Note that this driving scheme excites both spin-up and spin-down states, but the antichiral edge states in both spin sectors are degenerate at $k = \pi/a$. These experimental findings agree well with the results of circuit simulations, shown by the red curve in Figure 2(c). Circuit AC (steady state) analysis is performed using the LTspice circuit simulator, in which a 1 V sine wave is used as the source. The voltage at each inductor node is probed, and the amplitude and phase are used to derive the complex signal. In the simulations, the response peaks at 113.64 kHz. This small frequency shift can be attributed to fabrication errors, such as the approximately 5% tolerance in the capacitances and inductances of the various circuit components.

Next, we study the bulk lattice by connecting the top and bottom inductors of the strip and removing the additional shorted-out capacitors, which is equivalent to applying periodic boundary conditions to opposite edges of the strip. To probe the spatially averaged density of states, we excite the lattice using one driving source at site 1 and another at sites 4, 6, or 8, on the $X$ inductors, with the pickup coil located at
the X inductor on site 17, and take the averages of the three data sets. The results, plotted as the black curve in Figure 2(b), show no significant dip in the frequency range of interest. This agrees with the theoretical expectation that this bulk bandstructure, unlike that of the standard Haldane model, lacks a band gap. The circuit simulation results, shown as the black curves in Figure 2(c), exhibit qualitatively similar behavior.

The localized nature of the edge states can be observed by taking voltage amplitude measurements at different sites. Figure 2(d) shows the experimentally measured voltage amplitudes on the Y inductors at different sites, for the previously-discussed strip geometry (i.e., open boundary conditions along the edges of the strip, with driving coils on the X inductors at sites 2 and 12, corresponding to the red curve in Figure 2(b)). Here, the red curves show the response at the peak frequency of 111.1 kHz, which is strongly localized on the top edge (the edge states on the bottom edge are not excited since the sources are located on the top edge). For comparison, the blue curves show the response at 117.6 kHz, away from the eigenfrequency of the edge states, for which the response is not localized on the edge.

To further characterize the antichiral edge states, we prepare a circuit corresponding to a finite lattice of 128 sites in a rectangular geometry (Figure 3(a)). Owing to fabrication limitations, the sample consists of two PCBs connected by cable assemblies. We apply two driving coils to the X and Y inductors at a corner site (marked by a black star in Figure 3(b)), with a 90° relative phase shift in order to selectively excite spin-up states. In this configuration, the driving coils should excite antichiral edge states that propagate leftward along the upper edge. Figure 3(b) shows the experimentally obtained spin-up voltage amplitude distribution at 112.7 kHz (the frequency matching the antichiral edge states at $k = \pi/a$, as seen in Figure 2(b)). A strong voltage response is observed at both sample edges, a result that agrees well with the steady-state voltage distributions obtained in circuit simulations (Figure S3(a)). These results are consistent with the interpretation that antichiral edge states are initially excited on the top edge, undergo reflection at the left boundary into the bulk states, and reflect off the right boundary into left-moving antichiral edge states on both edges [20]. This behavior is further confirmed by time-domain circuit simulations (Figure S4).

We then determine the group velocities of the edge states by measuring the dwell time $d\phi/d\omega$, where $\phi$ is the phase of the complex spin-up voltage measured by the pickup coils at each site, and $\omega$ is the angular frequency. The rate of change of dwell time with distance along the edge is the group velocity [44]. Figure 3(c) shows the experimental results for the dwell times on the top and bottom edges; each data point is estimated from spin-up voltage measurements at 5 equally-spaced frequencies between 112.5 and 112.9 kHz (Figure S3(b)). From a linear least-squares fit of these results, we estimate group velocities of $-0.045$ sites/μs (top edge) and $-0.042$ sites/μs (bottom edge). The group velocities on both edges are negative, consistent with theoretical predictions. The corresponding circuit simulations (see
Figure 3(d)) predict group velocities of $-0.047$ sites/μs (top edge) and $-0.045$ sites/μs (bottom edge). From the band diagram of a strip of the same width and infinite length (similar to Figure 1(c) but with reduced width), the group velocities of the antichiral edge states is estimated to be $-0.059$ sites/μs. These experimental results unambiguously verify the antichiral nature of the edge states.

One of the most interesting features of electrical circuits is the ability to set up lattice geometries that are difficult or impossible to realize on other platforms. Jia et al. [7], for instance, showed that a rectangular sample can be converted into a Möbius strip by placing appropriate electrical connections between sites on two opposite boundaries. However, an unpaired Chern insulator cannot be placed in a Möbius strip geometry—an edge state, upon crossing one boundary, passes through to the opposite edge of the strip moving in the same direction, which is inconsistent with chiral propagation. In an actual circuit, the edge states switch to the opposite spin upon crossing the boundary [7]; in other words, the Möbius strip geometry necessarily couples the two spin sectors. The modified Haldane lattice, however, can be self-consistently implement on a Möbius strip since its edge states are antichiral. To test this notion, we implement a circuit with a Möbius strip configuration via twisted connections between sites on opposite boundaries, as shown in Figure 4(a). As shown in Figure 4(b) and (c), the antichiral edge states are able to traverse the entire Möbius strip edge, traveling in the same direction along the top and bottom edges. These results are consistent with circuit simulations (Figure S5).

3 Conclusions

We have experimentally verified the key features of the modified Haldane lattice proposed by Colomés and Franz [20], including the existence of antichiral edge states that have the same group velocity on opposite edges, the lack of a bulk gap, the transfer of energy between edge and bulk states during successive reflections within a finite sample, and the ability to exist self-consistently on a Möbius strip. These results demonstrate the flexibility of electrical circuits, as an experimental platform, for realizing topological phases and other related lattice phenomena [7,28]. In particular, we have used the “braiding” trick, originally used by Jia et al. [7] to realize a Chern insulator, to implement a new type of effective vector potential (complex inter-site hoppings). In the future, this approach might be used to implement circuit lattices with even more complicated vector potentials. Bulk bandstructure features, such as the Berry curvature, can also
be probed using previously-established tomographic methods \cite{41,45}. It would be also interesting to further investigate the use of circuit lattices to implement nontrivial vector potentials in other non-traditional geometries such as Klein bottles.

We thank You Wang, Qiang Wang and Udvas Chattopadhyay from Nanyang Technological University for helpful discussions. This work was supported by the National Natural Science Foundation of China (Grant Nos. 11874274, and 12004425), the Natural Science Foundation of Jiangsu Province (Grant Nos. BK20170058, and BK20200630), and a Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD). YiDong Chong was supported by the Singapore MOE Academic Research Fund Tier 3 (Grant No. MOE2016-T3-1-006).

Supporting Information
The supporting information is available online at http://phys.scichina.com and https://link.springer.com/journal/11433/. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

1. F. D. M. Haldane, Phys. Rev. Lett. 61, 2015 (1988).
2. M. Z. Hasan, and C. L. Kane, Rev. Mod. Phys. 82, 3045 (2010), arXiv: 1002.3895.
3. X. L. Qi, and S. C. Zhang, Rev. Mod. Phys. 83, 1057 (2011), arXiv: 1008.2026.
4. L. Lu, J. D. Joannopoulos, and M. Soljačić, Nat. Photon. 8, 821 (2014), arXiv: 1408.6730.
5. A. B. Khanikaev, and G. Shvets, Nat. Photon. 11, 763 (2017).
6. G. Jotzu, M. Messer, R. Desbuquois, M. Lebrat, T. Uehlinger, D. Greif, and T. Esslinger, Nature 515, 237 (2014), arXiv: 1406.7874.
7. N. Jia, C. Owens, A. Sommer, D. Schuster, and J. Simon, Phys. Rev. X 5, 021031 (2015).
8. V. V. Albert, L. I. Glazman, and L. Jiang, Phys. Rev. Lett. 114, 173902 (2015), arXiv: 1410.1243.
9. T. Hofmann, T. Helbig, C. H. Lee, M. Greiter, and R. Thomale, Phys. Rev. Lett. 122, 247702 (2019).
10. Z. Q. Zhang, B. L. Wu, J. Song, and H. Jiang, Phys. Rev. B 100, 184202 (2019), arXiv: 1906.04064.
11. M. Ezawa, Phys. Rev. B 100, 081401 (2019), arXiv: 1904.03823.
12. S. Raghu, and F. D. M. Haldane, Phys. Rev. A 78, 033834 (2008), arXiv: cond-mat/0602501.
13. Z. Wang, Y. D. Chong, J. D. Joannopoulos, and M. Soljačić, Nature 461, 772 (2009).
14. Y. Poo, R. X. Wu, Z. Lin, Y. Yang, and C. T. Chan, Phys. Rev. Lett. 106, 093903 (2011).
15. R. Fleury, D. L. Sounas, C. F. Sieck, M. R. Haberman, and A. Alù, Science 343, 516 (2014).
16. Z. Yang, F. Gao, X. Shi, X. Lin, Z. Gao, Y. Chong, and B. Zhang, Phys. Rev. Lett. 114, 114301 (2015), arXiv: 1411.7100.
17. M. Xiao, W. J. Chen, W. Y. He, and C. T. Chan, Nat. Phys. 11, 920 (2015).

Figure 4 (Color online) Propagation of antichiral edge states in a Möbius strip. (a) Schematic of the Möbius strip circuit. Twisted electrical connections are applied to the left and right boundaries of the physical sample, so that the upper edge of the strip continues to the lower edge and vice versa. (b) Experimental results showing the voltage amplitude distribution at 116.4 kHz with spin up excitation. Red arrows indicate the propagation directions of the edge and bulk states. (c) Experimentally measured relative dwell times along the top edge (red) and bottom edge (blue). The slopes of the linear least-squares fits correspond to group velocities of −0.039 sites/μs (top edge) and −0.044 sites/μs (bottom edge).
