Registration-based Image Segmentation using Lattice Boltzmann Method

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Abstract. Image segmentation is the first and important step leading to image analysis and interpretation. Using image segmentation we can divide the image into different regions. In this study, a Lattice Boltzmann model is proposed for registration-based CTA segmentation. The energy model is established and can be minimized by variational method. The gradient descent flow was solved by the Lattice Boltzmann method. In the experiments, a reference image was selected from CTA image series and artificially segmented by experts. By aligning the images to the reference image, the bias of the voxels are obtained. At the same time the contours of the desired object are estimated at the low contrast region.

1. Introduction
Image segmentation is a hot research field of image processing. That is because image segmentation is the first and important step leading to image analysis and interpretation[1]. Using image segmentation we can divide the image into different regions. From the segmented image, the desired objects are identified from the background for the quantitative measurement and analysis. Up to now, variant image segmentation algorithms have been proposed, for example active contour models[2][3][4], Fuzzy C means clustering [5][6], Gaussian-Mixture Model (FGMM) [7][8] and Lattice Boltzmann model(LBM)[9].

As one of the major applications of image processing, Medical image segmentation is relatively difficult because of the low spatial resolution, low contrast between the tissues, weak object boundary, strong noise jamming, and etc. cause extra demands on segmentation. For example, in the Computed Tomography Angiography (CTA) images, we need to segment not only the focal lumen which appears as a focal object, but also the thrombus. However, thrombus is a diffuse object connected to lumen, and always has low contrast to the surrounding tissues. Therefore, it comes to be particularly difficult to segment both lumen and thrombus. To deal with this problem, we proposed a Lattice Boltzmann model for registration-based CTA segmentation. The energy model is established and can be minimized by variational method. The gradient descent flow was solved by the Lattice Boltzmann method. In the experiments, a reference image was selected from CTA image series and artificially segmented by experts. By aligning the images to the reference image, the bias of the voxels are obtained. At the same time the contours of the desired object are estimated at the low contrast region.

2. Lattice Boltzmann Model for Registration-based Segmentation Model
Suppose a reference image \( R \) and a template image \( T \) are given, the main aim of image registration is to search a locale or global optimal transformation \( F \) to align \( T \) into the reference \( R \). In other words,
we should find the displacement vector \((ux, uy)\) such that: \(T(x - ux, y - uy) = R(x, y)\). This problem can be expressed by the sum of squared differences (SSD) [10]:

\[
E = \frac{1}{2} \int_\Omega \left[ T(x - ux, y - uy) - R(x, y) \right]^2 \, dx \, dy.
\] (1)

Thus the problem changes to search the optimal \((ux, uy)\) to minimize energy \(E\). According to the object region and \((ux, uy)\) in Reference, we can segment the objection in Template. To compensate the errors in the registration, we need to add another energy term:

\[
E = E_1 + E_2,
\] (2)

and

\[
\begin{cases}
E_1 = \frac{1}{2} \int_\Omega \left[ T(x - ux, y - uy) - R(x, y) \right]^2 \, dx \, dy
\end{cases}
\]

\[
\begin{cases}
E_2 = \int_\Omega g(x - ux, y - uy) \delta(x - ux, y - uy) \left| \nabla \phi(x - ux, y - uy) \right| \, dx \, dy
\end{cases}
\] (3)

where \(E_2\) can attract the object contour to the edges[11]. \(\phi\) is the distance function in which the object contour is embedded. \(g(x, y)\) is defined as \(\frac{1}{1 + \left| \nabla R(x, y) \right|^2}\). Minimize the energy function \(E\) by variational method, and add regular term \(\text{div}(\nabla ux)\) and \(\text{div}(\nabla uy)\). we obtain:

\[
\begin{align*}
\frac{\partial ux}{\partial t} & = \text{div}(\nabla ux) + \left[ T(x - ux, y - uy) - R(x, y) \right] \frac{\partial T}{\partial x} + g \frac{F_x}{|\nabla H|} + g_x \left| \nabla H \right| \\
\frac{\partial uy}{\partial t} & = \text{div}(\nabla uy) + \left[ T(x - ux, y - uy) - R(x, y) \right] \frac{\partial T}{\partial y} + g \frac{F_y}{|\nabla H|} + g_y \left| \nabla H \right|
\end{align*}
\]

(4)

To solve equation (4), the Lattice Boltzmann method is applied. Similar to document[9], we choose a D2Q9 LB model. Without loss of generality, the evolution equation of LB model can be written as:

\[
f_i (\vec{r} + \sigma_s \vec{e}_i, t + \sigma_t) - f_i (\vec{r}, t) = -\frac{1}{\tau} \left( f_i (\vec{r}, t) - f_i^{\text{eq}}(\vec{r}, t) \right) + \sigma_i F
\] (5)

where

\[
\begin{cases}
\sum_{i=0}^{8} f_i = \sum_{i=0}^{8} f_i^{\text{eq}} = \rho \\
f_i^{\text{eq}} = \rho / 9
\end{cases}
\]

(6)

\(f_i (\vec{r} + \sigma_s \vec{e}_i, t + \sigma_t)\) is the particle density distribution function at node \(\vec{r} + \sigma_s \vec{e}_i\), and time \(t + \sigma_t\) with lattice spacing \(\sigma_s\). \(\sigma_t\) denotes the time step, and \(\vec{e}_i\) the direction. Referencing to document [9], we can get the macroscopic equation as:

\[
\frac{\partial \rho}{\partial t} + 2\sigma_t \text{Div} \left[ \left( \tau - \frac{1}{2} \right) \nabla \rho \right] + 9 \sigma_t \rho F.
\] (7)

Therefore the LB model is defined as:
\[
\begin{align*}
fx_i (\vec{r}, t) - \frac{1}{\tau} (fx_i (\vec{r}, t) - fxeq_i (\vec{r}, t)) + \frac{\alpha}{9} Mx \\
fy_i (\vec{r}, t) - \frac{1}{\tau} (fy_i (\vec{r}, t) - fyeq_j (\vec{r}, t)) + \frac{\alpha}{9} My
\end{align*}
\]

The macroscopic equation is as follows:

\[
\begin{align*}
\frac{\partial u_x}{\partial t} &= \frac{2\sigma_x}{3c} \text{Div} \left( \frac{V_{ux}}{V_{ux}} \right) + \alpha F_x \\
\frac{\partial u_y}{\partial t} &= \frac{2\sigma_y}{3c} \text{Div} \left( \frac{V_{uy}}{V_{uy}} \right) + \alpha F_y \\
F_x &= \left[ T(x - u_x, y - u_y) - R(x, y) \right] \frac{\partial T}{\partial x} + g \frac{F_x}{\nabla H} + g_x \nabla H \\
F_y &= \left[ T(x - u_x, y - u_y) - R(x, y) \right] \frac{\partial T}{\partial y} + g \frac{F_y}{\nabla H} + g_y \nabla H
\end{align*}
\]

\(F_x\) and \(F_y\) can be estimated by standard method, e.g. up-wind scheme.

3. Experiment and discussion

The algorithm is listed as following:

1) Initialize the distance function \(\phi\) and the values of \(u_x, u_y\) as zero;
2) implement the evolution equation (8);
3) Update \(u_x, u_y\) according to formula (6);
4) Update equilibrium distribution functions \(f_i^{eq}\) according to formula (6);
5) Update \(F_x, F_y\); find the contour \(\phi = 0\);
6) Go to step 2).

**Figure 1.** Experiment 1: The test images (first row) and segmentation (second row).
Figure 2. Experiment 2: The test images (first row) and segmentation (second row).

Figure 3. Reference images

Figure 1 shows the segmentation on a case that the object has complete boundary. The test images (the first row of figure 1) are the slices of #99, #101, #103, #105. The reference image are shown in figure 3(a) in which the red region are segment artificially. The segmentation of each slices are shown in the second row of figure 1.

Figure 2 gives the experiment results on the CTA images (#300, #303, #306, #309, #312) which has low contrast and lost boundaries. Figure 3(b) is the reference images. The experiment indicates that our model can estimate the boundaries which are embedded in the noise effectively.

4. Conclusion
In this paper, we proposed a Lattice Boltzmann model for registration-based CTA segmentation. The energy model is established and can be minimized by variational method. The gradient descent flow was solved by the Lattice Boltzmann method. In the experiments, two kinds of cases of CTA image are tested. The segmentation results shows our model can segment the object and estimate the contours of the desired object are estimated at the low contrast region. Moreover, Our method has natural parallelism, it can be implemented on almost all of the massive parallel computing platforms including embedded FPGAS, DSPs, GPUs and etc.

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