How to use the cosmological Schwinger principle for 
energy flux, entropy, and “atoms of space–time” to 
create a thermodynamic space–time and multiverse

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Abstract. We make explicit an idea by Padmanabhan in DICE 2010 [1], as to finding “atoms 
of space–time” permitting a thermodynamic treatment of emergent structure similar to Gibbs 
treatment of statistical physics. That is, an ensemble of gravitons is used to give an “atom” of 
space–time congruent with relic GW. The idea is to reduce the number of independent variables 
to get a simple emergent space–time structure of entropy. An electric field, based upon the 
cosmological Schwinger principle, is linked to relic heat flux, with entropy production tied in 
with candidates as to inflaton potentials. The effective electric field links with the Schwinger 
1951s result of an E field leading to pairs of $e^+e^-$ charges nucleated in space–time volume $V \cdot t$. 
Note that in most inflationary models, the assumption is for a magnetic field, not an electric 
field. An electric field permits a kink–anti-kink construction of an emergent structure, which 
includes Glinka’s recent pioneering approach to a Multiverse. Also an E field allows for an 
emergent relic particle frequency range between one and 100 GHz. The novel contribution is a 
relic E field, instead of a B field, in relic space–time “atom” formation and vacuum nucleation 
of the same.

1. Introduction. What is to be analyzed?
T. Padmanabhan at DICE 2010 introduced the theme of this document [1]. That is, to 
reverse engineer GR emergent structure into initial component space–time “atoms,” to permit a 
“Gibbs” style treatment of the thermodynamics of space–time physics [2, 3, 4]. To make a link 
with graviton physics, we emphasize entropy generation of initial structure via a relic electric 
field. Note that Padmanabhan [5] has, for weak gravitational fields emphasized generation of 
electric field, but the treatment of the author as given is for relic E fields. This, if one uses 
a Lenz’s law transformation has been discussed in relic space–time conditions for the early 
formation of electromagnetic fields in between early galaxies [6]. The present document, while 
assuming an eventual magnetic field via a Lenz’s law transformation of a relic E field brings 
up electromagnetics in very early universe conditions via conditions similar to a cosmological 
Swinger effect [7]. The formation of a relic electric field would be to initiate formation of relic 
particles from the initial inflaton structure.

This paper answers the question as to what would be optimal conditions for initial entropy 
production. To begin this inquiry, we start by examining candidates for the initial configuration 
of the normalized energy density. The normalized energy density of gravitational waves, as given
by Maggiore [8] is

\[ \Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} \equiv \int_{\nu=0}^{\nu=\infty} d(\log \nu) \cdot \Omega_{gw}(\nu) \Rightarrow h_0^2 \Omega_{gw}(\nu) \approx 3.6 \cdot \left[ \frac{n_\nu}{10^{37}} \right] \cdot \left( \frac{\nu}{1 \text{ kHz}} \right)^4. \tag{1} \]

\( n_\nu \) is a frequency-based count of gravitons per unit cell of phase space. Eq. (1) leads to, as given in Figure 1, candidates as to early universe models to be investigated experimentally. The author, Beckwith, wishes to determine inputs into \( n_\nu \) above, in terms of frequency, and also initial temperature. What is in the brackets of the exponential is a way of counting the number of space–time \( e^+e^- \) charges nucleated in a space–time volume \( V \cdot t \). Beckwith used a very similar constructions with Density wave physics [9] and also has extended this idea to use in graviton physics, in a kink–anti-kink construction [9]. The idea, after one knows how to obtain a counting algorithm, with additional refinements will be to use what can be understood by the above analogy, assuming a minimal mass, \( m \approx m_{\text{eff}} \), as Beckwith brought up [10]. This will be discussed as inputs into the models represented by Figure 1.

**Figure 1.** From the LIGO Scientific Collaboration [11] shows the relation between \( \Omega_g \) and frequency.

Beckwith has derived a way to use two Friedman equations for estimating Entropy [12]. Also, by applying a Gaussian bifurcation mapping, Beckwith defined the initial degrees of space–time freedom up to over 1000 per unit of phase space during inflation [12]. We define entropy in terms of slow roll parameters and an effective electric field. The effective E field is a way to join vacuum nucleation with inflaton physics, and also nucleation of counted “pairs” of vacuum nucleated structures [9], allowing the use of Ng’s “infinite quantum statistics” [13] for inflaton/inflationary physics. Finally, the electric field is a way to present creation of relic particles up to the point of chaotic dynamics becoming dominant, in the permitted frequency range. The route to chaos has a well-defined sequence of evolution, and formulation of the emergent structure electric field permits identification of key bandwidth frequency ranges for relic particles, before the creation of a stochastic background of signal-to-noise chaos, permitting identification of key frequency patches to identify as high-frequency signals of the relic particles created at the start of inflation. What Beckwith intends to do is to use the emergent structure set by the cosmological
Schwinger method, as outlined by Martin [7] to obtain the number of emergent particles in initial phase-space counting, and tie in the phase space numerical counting and entropy with different candidates for the inflaton potential. Beckwith, via dimensional analysis [10] made the following identifications. \( \text{Force} = qE = [T\Delta S/dist] = \hbar [\omega_{\text{final}} - \omega_{\text{initial}}]/\text{dist} \) which in terms of inflaton physics leads to \((\text{if} V' = dV/d\phi, V'' = d^2V/d\phi^2 \text{ where} V \text{ is an inflaton potential, and } \text{dist} = \text{distance of Planck length, or more})\)

\[
\left[ \frac{T\Delta S}{\text{dist}} \right] = \left[ \frac{\hbar}{\text{dist}} \right] \left[ 2k^2 - \frac{1}{\eta^2} \left[ M_p^2 \cdot \left[ \frac{6}{16\pi} - \frac{3}{4\pi} \cdot \frac{[V''']^2}{[V']} - \frac{3}{4\pi} \cdot M_p^2 \cdot \frac{[V'']}{V} \right] \right] \right]^{\frac{1}{2}}. \tag{2}
\]

Eq. (2) divided by a charge, \(q\), gives a relic electric field. While the existence of a charge, \(q\), as an independent entity, at the onset of inflation is open to question, what the author, Beckwith [10], is paying attention to is using inputs into free energy, as can be specified by the following: If one identified the evolution of temperature, with energy, and made the following identification, \(T\) for time, and \(\Omega_0\) for a special frequency range, as inputs into [10],

\[
E_{\text{thermal}} = \frac{5}{2} k_B T_{\text{temp}} \times [\Omega_0 T] \sim \tilde{\beta}
\]

Here, the thermal energy, as given by temperature ranging as \(T_{\text{temp}} \in (0, 10^{19} \text{ GeV})\), up to the Planck interval of time \(t_P \sim 10^{-44} \text{ sec}\), so that one is looking at \(\tilde{\beta} \approx \left| F \right| \equiv \frac{5}{2} k_B T \cdot \mathbf{N}\), as a free energy. For this parameter, if \(\mathbf{N}\), as an initial entropy, arrow of time configuration, were fixed, then the change in temperature would lead to change in “free energy,” so that work, is here change in energy, and \(dE = T dS - p dV\). In basic physics, this would lead to force being work (change in energy) divided by distance. Then \(\Delta \beta \equiv (5/2)k_B T_{\text{temp}} \cdot \mathbf{N} \sim \text{Force} \cdot \text{dist}\). We assume that there would be an initial fixed entropy arising, with \(\mathbf{N}\) a nucleated structure arising in a short time interval as a temperature \(T_{\text{temp}} \in (0, 10^{19} \text{ GeV})\) arrives. So [10] leads to a force value

\[
\frac{\Delta \tilde{\beta}}{\text{dist}} = \frac{5}{2} k_B \Delta T_{\text{temp}} \cdot \frac{\mathbf{N}}{\text{dist}} \sim qE_{\text{net}} \text{ electric field } \sim [T\Delta S/dist]. \tag{4}
\]

The parameter, as given by \(\Delta \tilde{\beta}\) will be one of the parameters used to define chaotic Gaussian mappings, whereas the right-hand side of Eq. (4) is Eq. (2), which is force, which is linked to inflaton physics in Eq. (1). Next will be the identification of inflation physics, as dimensionally argued in both Eq. (1) and Eq. (3), to choices in the inflaton potential. To see that, consider the following, as given by Eq. (6) below [10]. Candidates as to the inflaton potential would be in powers of the inflaton—i.e., in terms of \(V \sim \phi^N\), with perhaps \(N = 2\) an admissible candidate (chaotic inflation). For \(N = 2\), one gets the result [10]

\[
[\Delta S] = \left[ \frac{\hbar}{T} \right] \left[ 2k^2 - \frac{1}{\eta^2} \left[ M_p^2 \cdot \left[ \frac{6}{4\pi} - \frac{12}{4\pi} \cdot \left[ \frac{1}{\phi^2} \right]^2 - \frac{6}{4\pi} \cdot \frac{1}{\phi^2} \right] \right] \right]^{1/2} \sim n_{\text{particle count}}, \tag{5}
\]

which in full dimensional correctness takes finally the form

\[
[\Delta S] = \frac{\hbar c}{T} \left[ 2k^2 + 8 \left( \sqrt{\frac{3}{8\pi}} \cdot \frac{M_p c}{\hbar} \cdot \frac{1}{\eta \phi} \right)^2 \right]^{\frac{1}{2}}. \tag{6}
\]

Making a comparison with a weighted average of \(\Delta S \sim 10^5\) and varying values of a scalar field of \(0 < \phi < 2\pi\), when one has \(\eta \in (10^{-44} \text{ sec}, 0)\), and \(0 < T \leq T_P \sim 10^{19} \text{ GeV}\) leads to a rich
phenomenology, where one could see variations as of a time parameter, and how the wave vector value, \( k = 2\pi/\lambda \) where \( \lambda \) is wavelength, evolved, especially if \( \Delta S \sim 10^5 \) remains constant. That is, why did the value of the wave vector value, \( k \), vary so much, in a short period of time—less than Planck time? As mentioned before in [10], this question asks how the initial wave vector, \( k \), forms and to what degree variation in the inflaton \( 0 < \phi < 2\pi \) occurs. That is, it gives a way to vary the inflaton, and understand relic entropy generation.

2. First principle evaluation of initial bits of information, as opposed to numerical counting, and entropy

A consequence of Verlinde’s [14] generalization of entropy as also discussed by Beckwith [10] and the number of “bits” yields the following consideration, which will be put here for startling effect. Namely, if a net acceleration is such that \( a_{\text{accel}} = 2\pi k_B c T/\hbar \) as mentioned by Verlinde [15] and Beckwith [12] as the Unruh result, and that the number of “bits” is

\[
    n_{\text{Bit}} = \frac{\Delta S}{\Delta x} \cdot \frac{c^2}{\pi \cdot k_B^2 T} \approx 3 \cdot \frac{(1.66)^2 g^*}{\Delta x \cong \ell_P} \cdot \frac{c^2 \cdot T^2}{\pi \cdot k_B^2}, \quad (7)
\]

Eq. (7) has a \( T^2 \) temperature dependence for information bits, as opposed to [10]

\[
    S \sim 3 \cdot \left[ 1.66 \cdot \sqrt{g_*} \right]^2 T^3 \sim n_f. \quad (8)
\]

Should the \( \Delta x \cong \ell_P \) order of magnitude minimum grid size hold, then when \( T \sim 10^{19} \text{ GeV} \) [10],

\[
    n_{\text{Bit}} \approx \frac{c^2}{\pi \cdot k_B^2 T} \cong \frac{3 \cdot (1.66)^2 g^*}{\Delta x \cong \ell_P} \cdot \frac{c^2 \cdot T^2}{\pi \cdot k_B^2} \sim 3 \cdot \left[ 1.66 \cdot \sqrt{g_*} \right]^2 T^3. \quad (9)
\]

The situation for which one has [10, 14] \( \Delta x \cong \ell^1/\ell_P^{2/3} \) with \( \ell \sim \ell_P \) corresponds to \( n_{\text{Bit}} \propto T^3 \) whereas \( n_{\text{Bit}} \propto T^2 \) if \( \Delta x \cong \ell^1/\ell_P^{2/3} \gg \ell_P \). This issue of either \( n_{\text{Bit}} \propto T^3 \) or \( n_{\text{Bit}} \propto T^2 \) will be analyzed in future publications. If the bits of information can be related to a numerical count, the next step will be to make a linkage between thermal heat flux, due to the initial start of inflation, with degrees of freedom rising from a point, almost zero to over 1000 in a Planck time interval.

2.1. How to set up a bifurcation diagram for creation of \( N(T) \sim 10^9 \) degrees of freedom at the start of inflation

In a word, the way to introduce the expansion of the degrees of freedom from nearly zero, at the maximum point of contraction to having \( N(T) \sim 10^9 \) is to first of all define the classical and quantum regimes of gravity in such a way as to minimize the point of the bifurcation diagram affected by quantum processes [10]. That is, classical physics, with smoothness of space–time structure down to a grid size of \( \ell_P \sim 10^{-33} \) centimeters at the start of inflationary expansion. When doing this construction, what would be needed would be to look at the maximum point of contraction, set at \( \ell_P \sim 10^{-33} \) centimeters as the quantum “dot,” as a de facto measure zero set, as the bounce point, with classical physics behavior before and after the bounce “through” the quantum dot.

Dynamic-systems modeling could be directly employed right “after” evolution through the “quantum dot” regime, with a transfer of crunched in energy to Helmholtz free energy, as the driver “force” for a Gauss map type chaotic diagram right after the transition to the quantum “dot” point of maximum contraction. The diagram would look like an application of the Gauss mapping of [10, 15]:

\[
    X_{i+1} = \exp\{-\vec{a} \cdot x_i^2\} + \tilde{\beta}. \quad (10)
\]
In dynamic-systems type parlance, one would achieve a diagram, with tree structure looking like what was given by Binous [16], using material written up by Lynch [15, 10]—by looking at his bifurcation diagram for the Gauss map. Binous’s demonstration plots the bifurcation diagram for user-set values of the parameter \( x_{i+1} \). For the author’s purposes, the parameter \( x_{i+1} \) and \( \tilde{x}_i^2 \) as put in Eq. (10) would represent the evolution of a number of degrees of freedom, with ironically, the near zero behavior, plus a Helmholtz degree of freedom parameter set fed into \( \tilde{\beta} \). The quantum “dot” contribution would be a measure set zero glitch in the mapping given by Eq. (10), with the understanding that where the parameter \( \tilde{\beta} \) “turns on” would be right AFTER the “bounce” through the infinitesimally small quantum “dot” regime. Far from being trivial, there would be a specific interactive chaotic behavior initiated by the turning on of parameter \( \tilde{\beta} \), corresponding, as brought up by Dickau [17], with a connection between octo-octonionic space and the degrees of freedom available at the beginning of inflation. That is, turning on the parameter \( \tilde{\beta} \) would be a way to have Lisi’s E8 structure [18] be nucleated at the beginning of space–time. As the author sees it, \( \tilde{\beta} \) would be proportional to the Helmholtz free energy, \( F \), where as Mandl [19, p. 272] relates, the usual definition of \( F = E - TS \) becomes, instead, here, using partition function, \( Z \), with \( \overline{N} \) a “numerical count factor,” so that [10, 19]

\[
F = -k_B T \cdot \ln Z(T, V, \overline{N}).
\]  

Note that Ng [13] sets a modification of \( Z_N \sim \frac{1}{N!} \left( \frac{V}{N} \right)^N \) as in the use of his infinite quantum statistics, with the outcome that one will be setting a temperature dependent free energy [5]

\[ F = -k_B T (\ln(V/\lambda^3) + 5/2) \]  

with \( V \sim \ell_p^3 \), and the Entropy obeying [10, 13]

\[ S \approx N \cdot (\ln \left[ \frac{V}{N\lambda^3} \right] + 5/2) \frac{\text{Ng infinite Quantum Statistics}}{N} \cdot (\ln \left[ \frac{V}{\lambda^3} \right] + 5/2) \approx N. \]  

\[ (12) \]

2.2. Linking driving “frequency” as a result of change in temperature to relic particles

After presenting this initial argument, we will bring up similar, more refined attempts by Donnelly and Jacobson [20], and also Rosenblum, Pikovsky, and Kurths [21] via their phased synchronization of chaotic oscillators to suggest necessary refinements ss to the linkage of the chaotic Gaussian mapping with structure formation. The first part of the following discussion is meant to be motivational, with references from the other papers being refinements that will be worked on in additional publications. The main idea is that \( \beta \) increasing up to a maximum temperature \( T \) would enable the evolution and spontaneous construction of the Lisi E8 structure as given by [18]. As Beckwith wrote up [10], including in additional energy due to an increase of \( \beta \) due to increasing temperature \( T \), would have striking similarities to the following. Observe the following argument as given by Mukhanov and Winitzki [22, 10], as to additional particles being “created” due to what is an infusion of energy in an oscillator, obeying two equations of motion:

\[
\ddot{q}(t) + \omega_0^2 q(t) = 0, \quad \text{for} \quad t < 0 \text{ and } t > \tilde{T}; \\
\ddot{q}(t) - \Omega_0^2 q(t) = 0, \quad \text{for} \quad 0 < t < \tilde{T}.
\]  

\[ (13) \]

Given \( \Omega_0 \tilde{T} \gg 1 \) with a starting solution of \( q(t) = q_1 \sin \omega_0 t \) if \( t < 0 \), Mukhanov and Winitzki state that for \( t > T \)

\[
q_2 \approx \frac{1}{2} \sqrt{1 + \frac{\omega_0^2}{\Omega_0^2}} \exp \left\{ \Omega_0 \tilde{T} \right\}.
\]  

\[ (14) \]

The Mukhanov and Winitzki argument leads to an exercise which Mukhanov and Winitzki [22] claim is a solution. The exercise yields an increase in number count, as can be given by first
setting the oscillator in the ground state with \( q_1 = \omega_0^{-1/2} \), with the number of particles linked to amplitude by \( \tilde{n} = (1/2)(q_0^2/\omega_0 - 1) \), leading to

\[
\tilde{n} = (1/2) \left( 1 + \frac{\omega_0^2}{\Omega_0^2} \right) \cdot \sinh^2 \left\{ \Omega_0 \tilde{T} \right\}
\]  

That is, for nonzero \( \Omega_0 \tilde{T} \), Eq. (15) leads to exponential expansion of the numerical state. For sufficiently large \( \Omega_0 \tilde{T} \), Eq. (13) and Eq. (14) are equivalent to placing of energy into a system, leading to vacuum nucleation. A further step in this direction is given on p. 82 of Mukhanov and Winitzki, leading to a Bogoliubov particle number density becoming exponentially large:

\[ n \sim \sinh^2[m_0 \eta_1]. \]  

Interestingly, Glinka et al [23] using quantum field theory methods showed that number of universes generated from stabilized vacuum is

\[ n = \sinh^2 r(\phi), \]  

where \( r(\phi) \) is one of the Bogoliubov angles, and \( \phi = \phi_0 a(x^0) \) is the cosmological scale factor with \( \phi_0 = \sqrt{\frac{2}{3\pi}} M_\text{P}. \)

Eq. (15) and (16) are, for sufficiently large \( \Omega_0 \tilde{T} \), a way to quantify what happens if initial thermal energy is placed in a harmonic system, leading to vacuum particle "creation." Eq. (16) is the formal Bogoliubov coefficient limit of particle creation. Note that \( \tilde{q}(t) - \tilde{\omega}_0 \tilde{q}(t) = 0 \) for \( 0 < t < \tilde{T} \) corresponds to a thermal flux of energy into a time interval \( 0 < t < \tilde{T} \).

If \( \tilde{T} \approx t_\text{P} \approx 10^{-44} \text{ sec} \) or some multiple of \( t_\text{P} \) and if \( \Omega_0 \propto 10^{10} \text{ Hz} \), then Eq. (13), and Eq. (15) plus its generalization as given in Eq. (16) may be a way to imply either vacuum nucleation, or transport of gravitons from a prior to the present universe. To generalize, what is done from Eq. (13) to Eq. (16), as brought up by Rosenblum, Pikovsky, and Kurths [21], would be to seek an explicit coupling of the coupled oscillations used to set up Eq. (10) with Eq. (13) to Eq. (16) explicitly. In order to take such a coupling, of chaotic oscillators, one would need to move beyond the idea given in this document’s section E to look at \( \text{Force} = qE = [T\Delta S/\text{dist}] = h[\omega_\text{final} - \omega_\text{initial}]/\text{dist} \), possibly setting the initial \( \omega_\text{initial} = 0 \). That is, look at candidates for \( \omega_\text{final} \) from the perspective of frequencies about and around \( \Omega_0 \propto 10^{10} \text{ Hz} \). The author’s initial results seem to indicate that in doing so, there is a range of permitted \( \Omega_0 \) values from 1 GHz up to 100 GHz, and this range of permitted \( \Omega_0 \) values due to the choices in grid size from a minimum value \( t_\text{P} \propto 10^{-33} \text{ centimeters} \), to those several thousand times larger, for reasons brought up earlier. Secondly, Donnelly and Jacobson’s linearized perturbations [20] and dispersions relations sections, in terms of an aether background have in its Eq. (28) and Eq. (30) potentially useful limits as to additional constraints for the frequency \( [T\Delta S/\text{dist}] = h[\omega_\text{final} - \omega_\text{initial}]/\text{dist} \) to obey, in the limits that \( k \to 0 \) as in Eq. (3) above. If, as an example, Donnelly and Jacobson’s dispersion relationship [20] as of their Eq. (30) can be reconciled with the limits of \( k \to 0 \) as in our Eq. (3), and \( [T\Delta S/\text{dist}] = h[\omega_\text{final} - \omega_\text{initial}]/\text{dist} \), this may entail a rethinking concerning if Donnelly and Jacobson’s damped harmonic oscillator for the inflaton, with a driving term—such as their Eq. (85)—is applicable, i.e., his [21],

\[
\ddot{\phi} + \theta \dot{\phi} + m^2 \phi + \mu N \phi = 0.
\]  

This Eq. (18) is a driven damped harmonic oscillator, with a “source” term. That is, the idea is that with an aether, or some similar background, possibly with the aether having the same function as DE, up to a point, with damping put in, as seen above, as a further modification
as to inflaton potentials which inputs as to our Eq. (3) to Eq. (7) should be considered. All this will be tried in future extensions of this project. The author’s own work, at least in low red shift regimes up to a billion years ago [9], emphasizes the role of massive gravitons having, in higher than four dimensions qualitative overlap in dynamic behavior as to the speed of cosmological expansion a billion years ago as attributed to DE. Beckwith’s own work [9], as also Alves et al. [24], involves what is known as the deceleration parameter to show a resumption of a speed up of cosmological acceleration one billion years ago. Making use of Eq. (18) would be a way, to perhaps link between massive gravitons, as given by Beckwith [9] as well as Alves et al. [24] and the implications of Donnelly and Jacobson’s damped driven harmonic oscillator [20] as represented by Eq. (18) above, which would make our investigation more comprehensive.

The remainder of this document will be in setting the “atoms” of space–time brought up in [1], via a comparison of energy for GW, for relic gravitons, with an eye toward thermal inputs into the inflation field geometry.

3. Effective “electric field” as proportional to temperature, to the first power. Its interpretation.

\[ n_f = \frac{1}{4} \cdot \left[ \sqrt{\frac{\nu(a_{\text{initial}})}{\nu(a)}} - \sqrt{\frac{\nu(a)}{\nu(a_{\text{final}})}} \right] \]  

Eq. (19) above, factually derived and used by Glinka [25, 26] in his pioneering approach to Multiverse, could be investigated as being part of the bridge between phenomenology of what inflaton potentials should be used. That is, the inputs into Eq. (4) lead to a number of permissible inputs into the inflaton potential which should be looked at: the values of the inflaton field which are acceptable, for \( \phi \in (0, 2\pi) \). What remains to be seen would be if the Schwinger numerical counting formula [27] as given in the argument of the exponential of Eq. (2) leads to a way to represent a numerical counting procedure giving reality to Ng’s infinite quantum statistics [13], via \( S \sim N \) with \( N \) representing a numerical counting of some assembly of effective mass of gravitons [9, 13]. The goal would be to make a linkage between the instanton–anti-instanton construction as Beckwith used in [10, 28] below, for electric fields, and an emergent graviton, along the lines of the false vacuum nucleation procedure seen below [29].

In order to connect with GR, one needs to have a higher dimensional analog of this pop up, as discussed in [9]. With comparisons between Eq. (4) and Figure 2, we hope to find appropriate inflaton potentials and their scaling behavior so Eq. (4) and the space–time nucleation, with an effective electric field, are clearly understood by an extension of the Schwinger result given in Eq. (2) and its Ng. counterpart \( S \sim N \). Also one of the most exciting parts of this inquiry would be to find if there is a way to make a linkage with Eq. (19) and DE with the work done by the author with massive gravitons, to mimic DE one billion years ago. The author’s motivation was in his work as reported in Dark Side of the Universe 2010 [29], to obtain a working DM/DE joint model, in a way better than current work done with Chaplygin Gas models [30, 31]. Having a linkage given between Eq. (18) of a damped driven inflaton field, with an “aether” in space–time, possibly linked to DE would be a way to show to what degree inflaton physics can be linked to a synthesis of DM/DE models [9]. If such a linkage can be made, with the author’s view of space–time inflation being driven by an E field inflaton field, as a source of entropy, it would give insight as to what role the formation of relic particles plays in entropy production. Last, but not least. A model using a Gaussian bifurcation map to generate chaotic dynamics as to creation of many degrees of freedom—up to 1000—in place of usual maximum between 100 and 120, with a turn on of Helmholtz free energy right after the start of inflation [10]. The author is convinced that the generation of Helmholtz free energy, as portrayed, is a classical phenomenon, but that the quantum gravity congruent grid size, possibly
Figure 2. The pop-up effects of an instanton–anti-instanton in Euclidian space from references [9, 28].

as small as a Planck length is very important. A great deal of analytical work has been done to isolate the regime where Classical and Quantum Gravity intersect. It is the author’s firm conviction that this debate can be settled by ascribing the regime of space–time as of the order of, or slightly larger than the length of a Planck size grid size, $\ell_P \sim 10^{-33}$ centimeters (at best it would be about two to three orders of magnitude larger) as where gravity is a purely quantum phenomenon, whereas when the Helmholtz free energy term [10] is turned on, right AFTER the growth of space–time past $\ell_P \sim 10^{-33}$ centimeters as where chaotic, classical dynamics plays a dominant role. Proving this latter conjecture would allow for, among other things helping to isolate the frequency ranges for production of relic particles, whose faint traces show up in an otherwise almost perfect “white noise” of relic GW/relic graviton frequencies from the big bang. Numerous authors, including Buonanno [32] have stated that this white noise is not analyzable. If one takes into account that the approach to chaos has well defined steps up to its initiation, physically, the author asserts that this last assumption may be falsifiable experimentally.

4. Conclusion: Analyzing the problem of graviton “counting,” atoms of space–time, GW, and Khrennikov’s signal theory treatment of QM with regards to the representation of gravitons and inflaton fields

According to Khrennikov [33], the classical and quantum probabilities can be delineated, in CM, by

$$\langle f \rangle_\mu = \int_M f(\mu) d\mu(\phi)$$

(20)

Here, $M$ is the state space, and $f$ is a functional in classical probability to be estimated, while $\mu$ is the measure. Eq. (20) should be contrasted with a QM presentation of the probability to be estimated with

$$\langle \hat{A} \rangle_\rho = \text{Tr}\rho \hat{A}$$

(21)

Khrennikov’s main claim is that randomness is the same in classical mechanics as in QM, and furthermore delineates a way to make a linkage between Eq. (20) and Eq. (21) via use of, if
t is the time scale of fluctuation, and T is the time of measurement that one can write, to order \( \vartheta(t/T) \)

\[
\int_M f_A(\mu) d\mu(\phi) = \langle \hat{A} \rangle_{\rho} = \text{Tr} \rho \hat{A} + \vartheta(t/T)
\]

(22)

Note that our supposition, via its cartoon sketch in Figure 2 above which we delineate as a vacuum nucleation of a graviton as a kink–anti-kink super position in space–time, with the overlap of order \( \vartheta(t/T) \) being the source for the tiny four-dimensional graviton rest mass given by Padmanabhan [4, 5]. That is, this is our way of stating that QM has an embedding, via Eq. (21) above in a more complex nonlinear theory to be built up. How that is built up will enable a fuller creation of “space–time atoms” as stated by Padmanabhan [1, 4] for a thermodynamic treatment of the evolution of the inflaton, in terms of gravitons and GW physics as mentioned by the author [10]. We seek to do this by, among other things using Elze’s negative probability [34] as a way to delineate the dynamics of evolution of the pop up represented in Figure 2 above in future work. The hope is to establish the details of the embedding of QM as part of a larger nonlinear theory and also to answer L. Lusanna’s [35] statement of the problem as he sees it. That is, a Swinger space–time kink–anti-kink pair would have a different embedding group structure for its representation than that of the Poincaré group commonly used in GR. The group structure is different, so reconciling the different structures would be part and parcel of resolving issues as of Eq. (22) above.

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