This note presents LHCb results on CP violation searches performed on charm decays that have been released after FPCP2012.

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\[1\] On behalf of the LHCb collaboration.
1 Introduction

In the Standard Model (SM) CP violation (CPV) in the charm sector is restricted to Cabibbo-suppressed decays and is predicted to be very small. Due to the smallness of the SM expectations, CPV in charm has been seen as a portal to new physics (NP). Indeed, until recently it was often assumed that the observation of CP asymmetries in D meson decays at the $\mathcal{O}(10^{-2})$ level would be an indication of new sources of CPV. SM predictions, however, suffer from the large uncertainty on the magnitude of penguin amplitudes. According to recent estimates, it is conceivable that CP asymmetries at this level could be accommodated within the SM [1, 2, 3].

From the experimental side, the sensitivity of CPV searches has increased dramatically in the past few years, especially with the advent of the large LHCb data sets. CP asymmetries at the $\mathcal{O}(10^{-2})$ level in D meson decays seem now to be excluded. With errors reaching the level of a few per mille, the current CPV searches probe a regime where CP asymmetries would be consistent with the SM expectations. The interpretation of an eventual observation of CP asymmetries in charm would not be straightforward.

This note collects recent LHCb results on CPV searches, all based on the full 2011 data set (1.0 fb$^{-1}$ at $\sqrt{s} = 7$ TeV). An update of the measurement of the difference in time-integrated CP asymmetry between $D^0 \to K^-K^+$ and $D^0 \to \pi^-\pi^+$, \[ \Delta A_{CP} \equiv A_{CP}(D^0 \to K^-K^+) - A_{CP}(D^0 \to \pi^-\pi^+) \] [5], is presented, along with results from an independent measurement of the same observable using $D^0$ from semileptonic $b$-hadron decays. Results from CPV searches using charged D mesons are also reviewed.

2 Update on $\Delta A_{CP}$ from prompt $D^{*+}$

2.1 Analysis strategy and sample selection

As in the previous measurement [4], the flavor of the initial state is determined by the charge of the soft pion ($\pi^+_s$) in the decays $D^{*+} \to D^0\pi^+_s$ and $D^{*-} \to \bar{D}^0\pi^-_s$.

The decay-time-dependent CP asymmetry $A_{CP}(f; t)$ for the decays $D^0 \to f$ is defined as

\[ A_{CP}(f; t) = \frac{\Gamma(D^0(t) \to f) - \Gamma(\bar{D}^0(t) \to f)}{\Gamma(D^0(t) \to f) + \Gamma(\bar{D}^0(t) \to f)}, \] (1)

where $f$ is a CP eigenstate, $f = K^-K^+, \pi^-\pi^+$. One can express $A_{CP}(f; t)$ as a sum of two terms: a direct, time-independent component, associated with CPV in the decay amplitudes and dependent on the final state; and an indirect, time-dependent component, universal to a good approximation, associated with CPV in mixing or in
the interference between mixing and decay. The time-integrated asymmetry $A_{CP}(f)$ may be written to first order as

$$A_{CP}(f) = a_{CP}^{\text{dir}} + \frac{\langle t \rangle}{\tau} a_{CP}^{\text{ind}},$$

(2)

where $\langle t \rangle$ is the average decay time in the reconstructed sample and $\tau$ is the average $D^0$ lifetime.

The difference between $CP$ asymmetries for the $D^0 \to K^- K^+$ and $D^0 \to \pi^- \pi^+$ is thus

$$\Delta A_{CP} = a_{CP}^{\text{dir}}(K^- K^+) - a_{CP}^{\text{dir}}(\pi^- \pi^+) + \frac{\Delta(t)}{\tau} a_{CP}^{\text{ind}}.$$  

(3)

In this analysis we measure $\Delta(t)/\tau = [11.19 \pm 0.13 \pm 0.17] \%$. Indirect $CP$ violation is a small contribution to $\Delta A_{CP}$.

The raw asymmetry for tagged $D^0$ decays to a final state $f$ is

$$A_{\text{raw}}(f) = \frac{N(D^{*+} \to D^0(f)\pi_s^+)}{N(D^{*+} \to D^0(t)\pi_s^+)} - \frac{N(D^{*-} \to \bar{D}^0(f)\pi_s^-)}{N(D^{*-} \to \bar{D}^0(t)\pi_s^-)},$$

(4)

where $N$ is the number of reconstructed events after background subtraction.

The raw charge asymmetry has a component due to detector, $A_D$, and production, $A_P$, effects. If these effects are small, the raw asymmetry is, to first order,

$$A_{\text{raw}}(f) = A_{CP}(f) + A_D(f) + A_D(\pi_s^+) + A_P(D^{*+}).$$

(5)

Since the two modes are self-conjugate final states, there is no detection asymmetry: $A_D(K^- K^+) = A_D(\pi^- \pi^+) = 0$. $A_D(\pi_s^+)$ and $A_P(D^{*+})$ are independent of the final state. These terms cancel, to first order, in the difference $A_{\text{raw}}(K^- K^+) - A_{\text{raw}}(\pi^- \pi^+)$:

$$\Delta A_{CP} = A_{\text{raw}}(K^- K^+) - A_{\text{raw}}(\pi^- \pi^+).$$

(6)

The following presents a terse summary of the sample selection, which is fully described in Ref. [5]. The sample is selected imposing requirements on the kinematic properties and on the decay time of the $D^0$ candidate. The $D^0$ decay products are required to form a displaced vertex with good fit quality, pointing back to the primary vertex. It is also required that the $D^0$ daughter tracks have good fit quality, transverse momentum above a minimum value and momentum directions that do not point to the primary vertex. The RICH system is used to distinguish between kaon and pions. Fiducial requirements are imposed to exclude kinematic regions with large charge asymmetry in the soft pion detection efficiency. The candidates must pass the specific high level software trigger. The $D^{*+}$ yields are $2.24 \times 10^6$ for $D^0 \to K^- K^+$ decays, and $0.69 \times 10^6$ for $D^0 \to \pi^- \pi^+$ decays.
2.2 Differences from previous analysis

There are some changes to the reconstruction and analysis procedure with respect to the previous measurement:

- full 2011 data set (1.0 fb$^{-1}$ at 7 TeV);
- improved reconstruction, with a more accurate alignment and calibration;
- weighting procedure to account for small remaining differences in the kinematics of the $D^{*+} \rightarrow D^0(K^-K^+)\pi^+_s$ and $D^{*+} \rightarrow D^0(\pi^--\pi^+)\pi^+_s$ decays;
- better mass resolutions due to a constrained fit that forces the soft pion to come from the primary vertex.

2.3 Fit model and results

For each $D^0$ final state the data are divided into four disjoint samples according to magnet polarity and hardware trigger decision. The raw asymmetry is determined, for each final state, by a simultaneous fit of the $\delta m \equiv m(h^-h^+\pi^+) - m(h^-h^+) - m(\pi^+)$ $(h = K, \pi)$ distributions from the four sub-samples. The $\delta m$ spectrum is obtained accepting $D^0$ candidates in the mass interval $1.844 < m(h^-h^+) < 1.884$ MeV/$c^2$

The signal is described by a sum of three Gaussian functions with a common mean convolved with an asymmetric function to improve the description of the tails. The background is described by the empirical function $B(\delta m) \propto |\delta m - m_0|^a e^{-b(\delta m - m_0)}$. The parameters $a$, $b$ and $m_0$ describe the shape of the function.

For each subsample — magnet polarity and hardware trigger category — $\Delta A_{CP}$ is computed. The combined value is computed as a weighted average across the sub-samples. Systematic uncertainties are assigned by loosening the fiducial requirement on the soft pion; by estimating the impact of a potential background from mis-reconstructed $D^{*+}$; by evaluating the asymmetry using sideband subtraction instead of a fit; by comparing with the result obtained with no kinematic weighting; and by excluding events in which the soft pion has a large impact parameter with respect to the primary vertex. The latter is the dominant source of systematic uncertainty.

The final result is

$$\Delta A_{CP} = (-0.34 \pm 0.15({\text{stat.}}) \pm 0.10({\text{syst.}}))\% \quad (7)$$
Figure 1: Fits to the $\delta m$ spectra from $D^{*+} \rightarrow D^{0}(K^-K^+)\pi^+$. Candidates are divided into four independent sub-samples according to magnet polarity and hardware trigger decision. The normalized residuals are shown below the fits. The fit procedure is described in the text.

3 $\Delta A_{CP}$ from semileptonic $b$-hadron decays to $D^0\mu^-X$

3.1 Analysis strategy

An independent measurement of $\Delta A_{CP}$ was performed using $D^0$ from semileptonic $b$-hadron decays to $D^0\mu^-X$ [7]. The initial flavor of the $D^0$ is tagged by the charge of the muon: a positive muon is associated with a $D^0$, and a negative muon with a $D^0$ meson. The $X$ denotes any other particle(s) that are not reconstructed.

The raw charge asymmetry is written in terms of the $D^0$ decay rate $\Gamma$, the muon detection efficiency $\varepsilon$, and the $D^0$ production rate in semileptonic $b$-hadron decays, $P$,

$$A_{\text{raw}} = \frac{\Gamma(D^0)\varepsilon(\mu^-)P(D^0) - \Gamma(\bar{D}^0)\varepsilon(\mu^+)P(\bar{D}^0)}{\Gamma(D^0)\varepsilon(\mu^-)P(D^0) + \Gamma(\bar{D}^0)\varepsilon(\mu^+)P(\bar{D}^0)}. \quad (8)$$

The raw asymmetry can be written to first order as
Figure 2: Fits to the $\delta m$ spectra from $D^{*+} \rightarrow D^0(\pi^-\pi^+)\pi_s^+$ with normalized residuals. Candidates are divided into four independent sub-samples according to magnet polarity and hardware trigger decision. The fit procedure is described in the text.

As in the analysis with prompt $D^{*+}$, the difference between the raw asymmetries measured in $D^0 \rightarrow \pi^-\pi^+$ and $D^0 \rightarrow K^-K^+$ decays cancels the detection and production asymmetries,

$$
\Delta A_{CP} = A_{raw}(K^-K^+) - A_{raw}(\pi^-\pi^+) \simeq A_{CP}(K^-K^+) - A_{CP}(\pi^-\pi^+). \quad (10)
$$

The cancellation requires the kinematic distributions of the muon and the $b$—hadron to be the same for both $D^0$ final states. This is ensured by a weighting procedure which equalises the kinematic distributions.
In this analysis the ratio between the difference of the $D^0$ and $\bar{D}^0$ mean decay time and the world average $D^0$ lifetime is

$$\frac{\Delta(t)}{\tau} = (1.8 \pm 0.2 \pm 0.7)\%,$$

making the contribution of indirect $CP$ violation in eq. 3 marginal.

### 3.2 Data set and selection

As for the prompt $D^{*+}$ sample, candidates are required to be accepted by a specific trigger decision. In the hardware stage about 87% of candidates in the final selection are accepted by the hardware trigger based only in the muon system, 3% are accepted by the hadronic calorimeter only, and the remaining 10% by both systems. In the software stage candidates are selected by either a single muon trigger or by a topological trigger \[6\]. The offline selection imposes requirements on track and vertex fit quality, on impact parameter for each track with respect to the primary vertex and on transverse momentum of the muon and the $D^0$ daughters. Requirements on the $D^0$ decay topology are minimal in order to have similar decay-time acceptance for both $D^0$ final states. Particle identification is required for all tracks. A detailed description of the sample selection is found in \[7\].

The invariant mass distributions of muon tagged $D^0$ candidates are shown in Fig. 3. The signal yields are determined by a binned maximum likelihood fit. The signal is modelled by a sum of two Gaussian functions with common means and different widths. The combinatorial background is represented by an exponential function. A Gaussian function is used to represent the tail of the $D^0 \rightarrow K^-\pi^+$ background. The total number of signal events is $(558.9 \pm 0.9) \times 10^3$ for $D^0 \rightarrow K^-\pi^+$ and $(221.6 \pm 0.8) \times 10^3$ for $D^0 \rightarrow \pi^-\pi^+$.

### 3.3 Determination of the asymmetries

The raw asymmetries are determined with simultaneous fits to the $D^0$ mass distributions for positive and negative muon tags. A weighting procedure is used in order to eliminate small differences in kinematic distributions between the two final states, that are result of the particle identification, the phase space differences and the correlation between the muons and the $D^0$. The event weighting improves the cancellation in eq. \[10\].

In some cases the $D^0$ flavor is not correctly tagged by the muon charge. The mistag probability is estimated using the $D^0 \rightarrow K^-\pi^+$ decay. The wrongly tagged decays include a small fraction of the doubly-Cabibbo-suppressed decay $D^0 \rightarrow K^+\pi^-$, $(0.393\pm0.007)\%$ \[8\]. After correcting for this fraction the difference between mistag probabilities for $D^0$ and $\bar{D}^0$ is $(0.006\pm0.021)\%$ and is neglected.
Figure 3: Invariant mass distributions for $D^0 \rightarrow K^- K^+$ (a,c) and $D^0 \rightarrow \pi^- \pi^+$ (b,d) muon tagged candidates for the two magnet polarities. The fit result is superimposed, with the contributions from signal, combinatorial background and the $D^0 \rightarrow K^- \pi^+$ reflection. Underneath each plot the pull in each mass bin is shown.

Systematic uncertainties are due to:

- possible difference in the relative contribution from $B^0$ and $B^+$ decays between the $D^0 \rightarrow K^- K^+$ and $D^0 \rightarrow \pi^+ \pi^-$ modes;
- difference in $B$ decay time acceptance for the two $D^0$ modes;
- differences in mistag rate for the two $D^0$ modes;
- the $D^0$ fit model;
- $\Lambda_c^+$ background in $D^0 \rightarrow K^- K^+$;
- low lifetime background in $D^0 \rightarrow \pi^+ \pi^-$. 

The small lifetime background in $D^0 \rightarrow \pi^+ \pi^-$ is the dominant source of systematic uncertainty. Many cross checks have been performed to verify the stability of the result, including the use of fiducial cuts on the muon, a comparison between different
trigger decisions and different particle identification requirements. The stability of
the raw asymmetries is also investigated as a function of quantities such as the $D^0$
decay time, the $b$-hadron flight distance, the reconstructed $D^{0}-\mu$ invariant mass, the
transverse momentum and pseudorapidity of the muon and the $D^0$ meson.

The difference in $CP$ asymmetries between $D^0 \to K^- K^+$ and $D^0 \to \pi^+ \pi^-$ modes,
measured using $D^0$ mesons produced in semileptonic $B$ decays is found to be

$$\Delta A_{CP} = (0.49 \pm 0.33 \text{(stat.)} \pm 0.14 \text{(syst.)})\%$$

4 Search for CPV in charged $D$ decays

4.1 Analysis strategy

Searches for $CP$ violation in charm have been mostly performed using decays of the $D^0$
meson. A comprehensive programme, however, has to include $CP$ violation searches
using charged $D$ decays as well. Since there is no mixing in charged $D$, a non-zero
$CP$ asymmetry would be an indication of direct CPV.

The large branching fraction of $D^0 \to K^- K^+$ decay compared to $D^0 \to \pi^+ \pi^-$,
and of the $D^+ \to K^- K^+ \pi^+$ decay compared to $D^+ \to \pi^- \pi^+ \pi^+$, suggests a significant
contribution from penguin amplitudes.

An investigation of CPV in the $\phi \pi^+$ region of the Dalitz plot of the $D^+ \to K^- K^+ \pi^+$ decay — hereafter referred to as $D^+ \to \phi \pi^+$ — is performed using the full
2011 LHCb data set (1.0 fb$^{-1}$). The $\phi \pi^+$ region is defined by $1.00 < m_{K^- K^+} < 1.04$
GeV/$c^2$.

The decay $D^+ \to K^0_s \pi^+$ is used as a control channel (CPV in this channel is
possible via interference between Cabibbo-favoured and doubly Cabibbo-suppressed
amplitudes, but it is assumed to be negligible).

The $CP$ asymmetry in the $D^+ \to \phi \pi^+$ decay is, to first order, given by

$$A_{CP}(D^+ \to \phi \pi^+) = A_{raw}(D^+ \to \phi \pi^+) - A_{raw}(D^+ \to K^0_s \pi^+) + A_{CP}(K^0/K^0), \quad (11)$$

where $A_{CP}(K^0/K^0)$ is the correction for CPV in the neutral kaon system and $A_{raw}$
is defined as

$$A_{raw} = \frac{N_{D^+} - N_{D^-}}{N_{D^+} + N_{D^-}}.$$

A concurrent measurement of $CP$ asymmetry is performed with the $D^+_s \to K^0_s \pi^+$
decay, using the $D^+_s \to \phi \pi^+$ decay as a control channel. To first order one has

$$A_{CP}(D^+_s \to K^0_s \pi^+) = A_{raw}(D^+_s \to K^0_s \pi^+) - A_{raw}(D^+_s \to \phi \pi^+) + A_{CP}(K^0/K^0). \quad (12)$$
In both measurements it is assumed that asymmetries induced by the production and detection of the $D^+$ and the $D^-$ cancel in the difference between signal and control channel raw asymmetries.

Finally, a third measurement is performed. It is possible that a constant $CP$-violating asymmetry is modulated by the rapidly varying strong phase across the $\phi \pi^+$ region. In this case there could be a cancellation of the asymmetry when different parts of the $\phi \pi^+$ region are added to compute $A_{CP}$. The $\phi \pi^+$ region is divided into four rectangular regions, A-D, shown in Fig. [4] A complementary observable is defined as

$$A_{CP|S} = \frac{1}{2} (A^A_{raw} + A^C_{raw} - A^B_{raw} + A^D_{raw}).$$  \hspace{1cm} (13)

This observable is robust against systematic biases from the detector and is not affected by the $D^+$ production asymmetry.

![Figure 4: Observed density of events in the Dalitz plot of the $D^+ \rightarrow K^- K^+ \pi^+$ decay.](image)

4.2 Data set and selection

The $D^+_{(s)} \rightarrow \phi \pi^+$ candidates are reconstructed by combining two oppositely charged particles identified as kaons by the RICH detector. Pairs of oppositely charged pions are combined to form $K^0_s$ from $D^+_{(s)} \rightarrow K^0_s \pi^+$. Both $K^0_s$ and $D^+_{(s)}$ are required to have a vertex with good fit quality. Further requirements are applied in order to reduce the background from random track combinations, from partially reconstructed charm decays and to avoid $D$ candidates coming from $B$ decays. A detailed description of the selection criteria can be found in [9].

The invariant mass distributions of selected candidates are shown in Fig. [5]
Figure 5: Invariant mass distributions for (a) $D^+ \to \phi \pi^+$, (b) $D^- \to \phi \pi^-$, (c) $D^+ \to K^0_s \pi^+$ and (d) $D^- \to K^0_s \pi^-$ candidates. The data are represented by symbols with error bars. The red dashed lines indicate the signal lineshapes. The green solid line represents the combinatorial background whereas the dotted lines represent the background from mis-reconstructed $D^+_s \to \phi \pi^+ \pi^0$ or $D^-_s \to K^0_s K^+$.  

4.3 Determination of yields and asymmetries

For the measurement of $A_{CP}$, the data was divided into 12 bins of $D$ transverse momentum $p_T$ and pseudorapidity $\eta$. The signal yields and the corresponding $A_{CP}$ are measured in each bin. A weighted average over the bins is performed to obtain the final result. This procedure is adopted to reduce biases due to small differences in kinematics of the $D^+ \to \phi \pi^+$ and $D^+ \to K^0_s \pi^+$ decays.

The shapes of the $D^+_s \to K^0_s \pi^+$ signals are described by single Cruijff functions [10]. In the $\phi \pi^+$ final state Crystal Ball functions [11] are added to the Cruijff functions to account for the tails of the mass peaks. Monte Carlo simulations are used to test the signal lineshapes. The background is fitted with a straight line plus a Gaussian component to account for partially reconstructed $D^+_s \to K^0_s \pi^+ \pi^0$ and $D^-_s \to \phi \pi^+ \pi^0$ decays. In the $K^0_s \pi^+$ case there is also a cross-feed component from
Decay | yield ($\times 10^3$)
---|---
$D^+ \rightarrow \phi \pi^+$ | 1576.9 ± 1.5
$D_s^+ \rightarrow \phi \pi^+$ | 3010.2 ± 2.2
$D^+ \rightarrow K^0_s \pi^+$ | 1057.8 ± 1.2
$D_s^+ \rightarrow K^0_s \pi^+$ | 25.6 ± 0.2

Table 1: $D^+$ and $D_s^+$ yields from fits to the $\phi \pi^+$ and $K^0_s \pi^+$ spectra.

the $D_s^+ \rightarrow K^0_s K^+$. In each bin the data is further divided into four sets, according to the charge of the $D$ and magnet polarity. The yields are determined by simultaneous fits over the four sub-samples. The overall yields are shown in Table 1.

The main systematic uncertainties in $A_{CP}$ result from differences in the kinematics of $\phi \pi^+$ and $K^0_s \pi^+$ final states, causing imperfect cancellation of detector induced asymmetries, and from the response of the hardware trigger, which is known to be charge asymmetric. Systematic effects from binning, fit model, kaon detection asymmetry, $D$ mesons from $B$ decays, and from kaon $CP$ violation are also considered.

The results are

\[
A_{CP}(D^+ \phi \pi^+) = (-0.04 \pm 0.14 \pm 0.14)\% \\
A_{CP|S}(D^+ \phi \pi^+) = (-0.18 \pm 0.17 \pm 0.18)\% \\
A_{CP}(D_s^+ \rightarrow K^0_s \pi^+) = (+0.61 \pm 0.83 \pm 0.14)\% ,
\]

which are consistent with no $CP$ violation.

5 Concluding remarks

$CPV$ in charm is a phenomenon predicted by the SM that was not yet observed. Finding $CPV$ in charm is obviously a very important goal, but understanding its nature will require different types of inputs.

The LHCb experiment has an extensive programme of $CPV$ searches in the charm sector, including two-, three- and four-body decays of both neutral and charged $D$ mesons. With the 2011 data set (in 2012 LHCb recorded twice as much data at higher energy: $2 fb^{-1}$ at 8 TeV), errors on the $CP$ asymmetry are smaller than 0.2%. This means LHCb is now probing the regime of the SM expectations.

A naive combination of the two $\Delta A_{CP}$ measurements in two-body $D^0$ decays, assuming negligible indirect $CPV$, yields

\[
\Delta A_{CP} = (-0.15 \pm 0.16)\% 
\]
These and the results with charged $D$ mesons are the most sensitive searches and show a consistent picture: at this time we have no evidence of CPV in the charm sector.

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