Emergence of the NMSGUT

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Abstract. We trace the emergence of the “New Minimal” supersymmetric SO(10) GUT (NMSGUT) out of the debris created by our demonstration that the MSGUT is falsified by the data. The NMSGUT is based on $210 \oplus 10 \oplus 120 \oplus 126 \oplus 126$ Higgs system. It has only spontaneous CP violation and Type I seesaw. With only 24 real superpotential parameters it is the simplest model capable of accommodating the known 18 parameter fermion mass data set and yet has enough freedom to accommodate the still unknown Leptonic CP violation and neutrino mass scale parameters. Our focus is on the two most salient features uncovered by our analysis: the domination of the $126$ Yukawa couplings by those of the $10$, $120$ (required for evasion of the no-go that trapped the MSGUT) and the inescapable raising of the Baryon violation scales (and thus suppression of proton decay) decreed by a proper inclusion of the threshold effects associated with the calculated superheavy spectra. These two structural features are shown to be complementary.

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INTRODUCTION

Following the discovery of neutrino mass Supersymmetric SO(10) theories, particularly ones based on the $210 \oplus 126 \oplus 126$ Higgs system, have received a great deal of attention[1, 2, 3]. SO(10) unification has multiple virtues that seem to pre-destine it for canonicity: a SM family together with a conjugate neutrino fits in one chiral 16-plet of SO(10). Using the $126$ Higgs both the Type I and Type II seesaw mechanisms find a natural implementation. The $210$-plet Higgs together with the $126 \oplus 126$ breaks[4] SO(10) to the MSSM but preserves R-parity as a part of its gauge symmetry, so that the theory presents its stable LSP as dark matter. The simple superpotential allows[13] an explicit and complete solution of the spontaneous symmetry breaking in terms of a single complex control parameter ($x$) and complete calculation of the superheavy spectrum. The parameter $x$ satisfies a simple cubic equation dependent linearly on a parameter ratio ($\xi$): so the map $x \rightarrow \xi$ is $1 - t \rightarrow 1$. The ‘SU(5) conspiracy’[11] enforced by RG flows calculated from the computed spectra -as is required in the Susy case[4][12]- restricts one to the single step breaking case. From the spectra detailed information on the parameter space of the theory can be deduced at an unprecedented level of control and refinement thus pinning down the theory so that it is rendered falsifiable. Unexpectedly this yields an explanation for the suppression of proton decay[10].

The other aspect concerns the generic features of fermion mass spectra in SO(10).

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Fermion masses and mixings (22 parameters in all) extrapolated to the superheavy scale using a grand desert assumption are the primary data that any Susy GUT must accommodate. An early attempt to implement a complete fermion data fit using only the $10 + 126$ representations\cite{14} set the rules for much of the later work. After initial difficulties the increasingly precise observation of neutrino masses stimulated and facilitated the successful ‘generic’ fits of all fermion data using both Type I and Type II seesaw mechanisms\cite{2, 3}. Thus, till 2005, the Babu-Mohapatra program seemed successful. However it was just assumed that the overall scale and relative strength of Type I versus Type II seesaw masses were realizable in Susy GUTs. Our MSGUT survey\cite{7} revealed serious difficulties in obtaining Type II over Type I dominance as well as in obtaining large enough Type I neutrino masses. Using a convenient parametrization in terms of the single “fast” parameter ($x$) which controls MSGUT ssb we gave\cite{9} a complete proof of the failure of the Seesaw mechanism in the context of the MSGUT. The nature of the obstruction uncovered by us led us to suggest\cite{9} a natural scenario that dealt with the problem by extending the MSGUT with a $120$ FM Higgs representation. In this scenario\cite{9, 16} the $120$ -plet and the $10$ -plet fit the dominant charged fermion masses. Small $126$ -plet couplings give appreciable contributions only to light charged fermion masses and enhance Type I seesaw masses to viable values since Type one seesaw masses are inversely proportional to these couplings. In what follows we give details of a new and simpler Minimal Supersymmetric GUT candidate that has emerged from the implementation of our suggestion. We will then summarize the challenges that this so called New MSGUT(NMSGUT) faces in its attempt to reach the status of a falsifiable and predictive theory.

**SEESAW FAILURE IN THE MSGUT**

The Type I and Type II neutrino masses in the MSGUT are\cite{9}:

\[
M^I_V = (1.70 \times 10^{-3} \text{eV}) F_I \hat{n} \sin \beta \\
M^I_{\nu} = (1.70 \times 10^{-3} \text{eV}) F_{II} \hat{f} \sin \beta \\
\hat{n} = (\hat{h} - 3 \hat{f})^{\dagger} (\hat{h} - 3 \hat{f})^{-1}
\]

where $\hat{h}, \hat{f}$ are proportional to the $10, 126$ Yukawa couplings to fermions, $\beta$ is the MSSM Higgs vev ratio parameter, and $F_I(x), F_{II}(x)$ are specified in the MSGUT, for explicit forms see\cite{9}.

In typical BM-Type II fits\cite{3} the maximum eigenvalue $\hat{f}_{\text{max}} \sim 10^{-2}$ while $\hat{h}_{\text{max}} \sim 10^{2}$. Thus $R = F_I/F_{II} \leq 10^{-3}$ in order that the pure BM-Type II not be overwhelmed by the BM-Type I values. Such $R$ values are un-achievable\cite{7, 9} in the MSGUT parameter space (while preserving baryon stability, perturbativity etc). Furthermore in the BM-Type I fits, typically $\hat{n}_{\text{max}} \sim 5 \hat{f}_{\text{max}} \sim .5$ for the maximal eigenvalues of $\hat{n}$. Thus values of $F_I \sim 100$ are required in order to reach realistic values of seesaw masses for the heaviest neutrino. We demonstrated\cite{7, 9} that such values are not even nearly achievable over the complex $x$ plane except where they also violate some aspect of successful unification. The same argument also applies to mixed Type I and Type II\cite{2}. Later another group independently confirmed our results\cite{15}. 
THE NEW $10 - 120 - \mathbf{126}$ SCENARIO

To resolve the difficulty with the overall neutrino mass scale we proposed\[9, 16\] that $\mathbf{126}$ yukawa couplings be reduced (well below the level where they are important for 2-3 generation masses) and introduced a $\mathbf{120}$-plet to do the work of charged fermion mass fitting previously accomplished by $\mathbf{126}$ couplings. The small $\mathbf{126}$ yukawa coupling would boost the value of the Type I seesaw masses by increasing $\hat{h}$ (and suppress Type II seesaw masses). Note that an extension of the MSGUT by $\mathbf{120}$-plet, with only spontaneous CP violation but with Type II dominant seesaw mechanism had been considered previously \[19\]. However the assumption of the viability of the Type II generic fits used there has already been explained to be invalid. Thus our scenario with Type I dominant and a characteristic and distinct pattern of yukawa couplings (which has proven successful and viable) must be considered as a qualitatively different theory. As we shall see this also applies to the mechanisms of lowering proton decay rates advocated in the two scenarios.

The Dirac masses in such GUTs are then generically given by\[9, 10\]

$$
m^d = v(h + \hat{f} + \hat{g})
$$

$$
m_\nu = v(h - 3\hat{f} + r_5'\hat{g})
$$

$$
m^l = v(r_1h + r_2\hat{f} + r_6\hat{g})
$$

$$
m^l = v(r_1h - 3r_2\hat{f} + r_7\hat{g})
$$

$$
\hat{g} = 2ig\sqrt{2/3}(\alpha_6 + i\sqrt{3}\alpha_5)\sin\beta ; \quad \hat{h} = 2\sqrt{2}h\alpha_1\sin\beta ; \quad \hat{f} = -4\sqrt{2/3}if\alpha_2\sin\beta
$$

The right handed neutrino mass is $M_\nu = \hat{f}\hat{\sigma}$ and the Type I seesaw formula is

$$
M_\nu' = vr_4\hat{h} ; \quad \hat{h} = (h - 3\hat{f} - r_5'\hat{g})\hat{f}^{-1}(\hat{h} - 3\hat{f} + r_5'\hat{g})
$$

where $\hat{\sigma} = i\sigma\sqrt{3}/\alpha_2\sin\beta$, $\sigma$ is the GUT scale vev of the $\mathbf{126}$ while $\alpha_i, \bar{\alpha}_i$ refer to fractions of the MSSM doublets contributed by various doublets present in the GUT Higgs representations \[13, 18, 6, 7, 9\]. See \[10\] for the form of the coefficients $r_i$ in terms of the $\alpha_i, \bar{\alpha}_i$ and the expression for the $\alpha_i, \bar{\alpha}_i$ in terms of the parameters of the NMSGUT.

As regards spontaneous CP violation one finds that it requires the parameters in the superpotential to be real and the six independent phases\[20, 21\] in the generic fermion mass matrices must thus arise from the various doublet vevs or, in our language, where $< h_i > \rightarrow \alpha_i v_{d}, < \bar{h}_i > \rightarrow \bar{\alpha}_i v_{d}$, from the complexity of of the coefficients $\alpha_i, \bar{\alpha}_i$. This can be shown\[10\] to require that the fast control parameter ‘$x$’ of the MSGUT and NMSGUT superheavy spectra is itself complex while the superpotential parameter ratio $\xi$ that it determines (via the complex solution branches of the fundamental cubic equation satisfied by $x$) is real. The values of the other (slow) superpotential parameters are restricted to a narrow range $\sim 1$ by unification constraints. Thus one can actually scan\[6, 7, 9, 10\] the complete behaviour of the theory as function of a single relevant real parameter $\xi$. Subsequent semi-analytic work by us\[16, 17\] but more effectively the purely numerical analysis in\[20, 21\] lent decisive support to our fitting scenario.
In [20] CP violation is spontaneous but free parameters are reduced to 21 from 25 by fixing $f_{12} = f_{23} = g_{13} = 0$ by imposing a $Z_2$ symmetry. The “downhill-simplex” nonlinear fitting method is used to minimize a $\chi^2$ fitting function constructed from the values of 18 observables (9 charged fermion masses, 4 CKM parameters, 2 neutrino oscillation mass splittings, 3 PMNS angles) and the experimental uncertainties thereof. The eigenvalues found are

$$
\hat{h} : 0.58 ; -0.0213 ; 0.000375 \\
\hat{f} : 0.105 ; 0.021 ; 0.00033 \\
\hat{r}_{5\hat{g}}' : \pm 0.65 ; 0 \\
\hat{n} : 31.19 ; 5.996 ; 1.045
$$

Similarly in [21] the same authors achieved even better fits with fewer(18) parameters by fixing the 6 phases(6 parameters less) so that CP was violated but without imposing any $Z_2$ symmetry(3 parameters more) on the Yukawas. They then obtained a fit to the 18 known fermion mass parameters with the eigenvalues

$$
\hat{h} : 0.473 ; 0.00037 ; 8.15 \times 10^{-5} \\
\hat{f} : 2.45 \times 10^{-3} ; 1.49 \times 10^{-3} ; 5.8 \times 10^{-5} \\
\hat{r}_{5\hat{g}}' : \pm 0.108 ; 0 \\
\hat{n} : 220.035 ; 39.867 ; 7.974
$$

Another similar fit but with inverted neutrino mass heirarchy was also found. Clearly both fits illustrate how $\hat{f}$ is reduced and $\hat{n}$ enhanced sufficiently to allow accommodation in the NMSGUT. In addition the fits fix the 1 Dirac and 2 Majorana phases of the leptonic mixing matrix and the overall neutrino mass scale. Since, however, these are not known at present it seems that the parameter freedom fixed arbitrarily by these authors for numerical and illustrative purposes may finally be called upon to shoulder the task of also fitting the experimental values of these phases. In the NMSGUT[10] there are 24 real superpotential parameters which are reduced to 23 by the fine tuning condition that keeps the MSSM doublets light but raised by two to count the electroweak vev and the susy parameter $\tan \beta$ which are not fixed within the Susy GUT. The fermion mass data(12 masses + CKM phase + 3 CKM angles + 3 PMNS angles + 3 PMNS phases) consists of 22 parameters at most. Thus somewhat disappointingly, even after reserving 4 parameters for the remaining mass data there is still a surplus of 3 parameters left out of the total of 25 parameters. The excess of three parameters most likely implies that additional data such as that from proton decay will be required before the NMSGUT becomes falsifiable or its parameters determined. The situation without a CP preserving Lagrangian is of course much(15 real parameters more) worse.
PROTON DECAY SCALES IN THE NMSGUT

In [6, 7, 9, 10] we developed the analysis of RG constraints on the NMSGUT following the approach of [22] in which $M_X$ is taken to be the mass of the lightest gauge multiplet which mediates proton decay (and not the point where the 3 MSSM gauge couplings cross). In this section we summarize information on the $x$ values allowed by imposing plausible ‘realistic’ constraints on the magnitudes of the threshold corrections to the gauge couplings. We pay particular attention to the scenario [23] where $M_X$ and with it all dangerous $d = 5$ proton decay mediating Higgs triplet (of which there are three distinct types $t[3, 1, \pm \frac{2}{3}], P[3, 3, \pm \frac{2}{3}]$ and $K[3, 1, \pm \frac{8}{3}]$ in this theory [10]) masses are pushed above $10^{16} GeV$. Quite reasonably, we demand

$$|\Delta G| \equiv |\Delta (\alpha_G^{-1}(M_X))| \leq 10$$

$$2 \geq \Delta_X \equiv \Delta (\log_{10} M_X) \geq -1$$

$$|\Delta W| \equiv |\Delta (\sin^2 \theta_W(M_S))| < .02$$

(5)

to implement perturbative gauge dominance, 10% uncertainty in $\sin^2 \theta_W(M_S)$ and $d = 6$ proton decay mediation suppression.

We find threshold corrections [6, 7, 9, 10]

$$\Delta^{(th)}(\log_{10} M_X) = .0217 + .0167 \sum_{M'} (5\bar{b}'_1 + 3\bar{b}'_2 - 8\bar{b}'_3) \log_{10} \frac{M'}{M_X}$$

$$\Delta^{(th)}(\sin^2 \theta_W(M_S)) = .00004 - .00024 \sum_{M'} (4\bar{b}'_1 - 9.6\bar{b}'_2 + 5.6\bar{b}'_3) \log_{10} \frac{M'}{M_X}$$

$$\Delta^{(th)}(\alpha_G^{-1}(M_X)) = .1565 + .01832 \sum_{M'} (5\bar{b}'_1 + 3\bar{b}'_2 + 12\bar{b}'_3) \log_{10} \frac{M'}{M_X}$$

(6)

Where $\bar{b}'_i = 16\pi^2 b_i'$ are 1-loop $\beta$ function coefficients ($\beta_i = b_i g_i^3$) for multiplets with mass $M'$. These corrections, together with the two loop gauge corrections, modify the one loop values corresponding to the successful gauge unification of the MSSM, see [6, 7, 9] for details.

The parameter $\xi = \lambda M / \eta m$ is the only numerical parameter that enters into the cubic eqn. that determines the parameter $x$ in terms of which all the superheavy vevs are given. It is thus the most crucial determinant of the mass spectrum.

The rest of the coupling parameters divide into “diagonal”($\lambda, \eta, \rho$) and “non-diagonal” ($\gamma, \tilde{\gamma}, \xi, \tilde{\xi}, k$) couplings with the latter exerting a very minor influence on the unification parameters. We have therefore fixed the non diagonal parameters at representative values $\sim 1$ throughout.

A crucial point [7] is that the threshold corrections depend only on ratios of masses and are independent of the overall scale parameter which we choose to be the mass parameter $m$ of the 210-plet. Since $M_X = 10^{16.25 + \Delta X} GeV$ it follows that $m \sim 10^{16.25 + \Delta X}$. It is thus clear that this factor will enter every superheavy mass so that they must all rise or fall in tandem with $M_X$ i.e exponentially with $\Delta X$. 
FIGURE 1. NMSGUT critical behaviour: \( \rho = \rho_c = .15 \). Regions of the \( x \)-plane compatible with the unification constraints \([5]\) are shaded. The darkest regions have \( 2 \geq \Delta_X > 1 \) (corresponding to \( M_X > 10^{17.25} \text{GeV} \)), the next darkest \( 1 \geq \Delta_X > 0 \) the lightest shade \( 0 \geq \Delta_X > -1 \) and the white regions are disallowed.

As an example of the regions of parameter space allowed by the unification constraints we present Figs. 1 and Fig. 2 from \([10]\). It is clear that the bulk of the allowed parameter space has the mass of the lightest baryon violating gauge bosons, as well as all other superheavy masses including Higgs and Higgsino triplets that mediate \( d = 5 \) proton decay, raised by a factor of 10 or more. This can be seen even more clearly in the case of the CP preserving superpotential which requires that we use the complex solution for \( x \) and a real value for the parameter ratio \( \xi \). The corresponding plot of \( \Delta_X \) versus \( \xi \) is given in Fig. 3 for the region where there is a sensible variation of \( \xi \). The constraint on \( \Delta_W \) restricts one to \( |\xi| < 11 \) while that on \( \Delta_G \) excludes a narrow region around \( \xi = -5 \). As a result the values of all superheavy mass scales are raised well above the one loop unification scale of \( 10^{16.25} \text{GeV} \). We present a representative set of allowed values in Table 1. It is clear that the MSSM running in the grand desert will be practically unaffected and it is the modification of the relation between the one loop unification mass and the mass of the lightest baryon number violating gauge boson (and thereby all other superheavy masses) that is responsible for the elevation of the dangerous superheavy masses. The upshot of the mass scales raised in tandem is that both \( d = 6 \) and \( d = 5 \) baryon violation will be strongly suppressed over most of the viable NMSGUT parameter space. Thus we see that the NMSGUT deals with the difficulty regarding \( d = 5 \) mediated baryon decay in a very natural and unforced way. This may be contrasted with the mechanisms for suppression of proton decay in \([19]\). No fine tuning of Yukawas or artificially arranged cancellation or any introduction of a plethora of uncontrollable non-renormalizable terms is required at all. The suppression is generic in the viable regions of the parameter space and is practically inescapable on complex branches of the ssb solution.

As a corollary to the raising of the scale of baryon violation, however, the \( \overline{126} \) vev responsible for the right handed neutrino mass is also raised significantly so that
FIGURE 2. NMSGUT critical behaviour: $\rho = .51$ i.e larger than $\rho_c = .15$ : Regions of the x-plane compatible with the unification constraints (5) are shaded. The darkest regions have $2 \geq \Delta_X > 1$ (corresponding to $M_X > 10^{17.25}\text{GeV}$), the next darkest $1 \geq \Delta_X > 0$ the lightest shade $0 \geq \Delta_X > -1$ and the white regions are disallowed.

FIGURE 3. Plot of $\Delta_X$ against $\xi$ on the CP violating solution branch $x_+ (\xi)$ at representative allowed values of the diagonal parameters.

compatibility with Leptogenesis constraints also actually favours the small $\overline{126}$ Yukawas that were already indicated by our fermion fitting scenario. This is another instance of the interplay between neutrino masses and baryon violation in SO(10). Another difficulty alleviated by the raised mass scales is that the Landau pole in the SO(10) gauge coupling that occurs once the superheavy thresholds have intervened in the gauge evolution is pushed closer to the Planck scale. This ties in well with our scenario[24] that envisions a (calculable) UV condensation of coset gauginos in the supersymmetric GUT which drives the breaking of the GUT symmetry[24]. Moreover since the cutoff
of the perturbative theory is about an order of magnitude less than the scale of UV condensation and the Planck scale - which coincide, it is natural to surmise that the gauge strong coupling dynamics induces gravity characterized by $M_P \sim \Lambda_X$. It is worth remarking that such a scenario naturally overcomes all the fundamental objections\[25\] that led to an abandonment of the induced gravity scenarios of the 1980s.

To conclude: minimal SO(10) supersymmetric GUTs enjoyed a falsification which led to a simpler version compatible with all data. Minimality inspired investigations of Susy SO(10) GUTs have thus entered a kind of limbo. The vision\[14\] of predictive SO(10) Susy GUTs seems to have been an un-realizable dream. To break out the difficult issues of fine tuned scale separation and of supersymmetry breaking must be faced:

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**TABLE 1.** Example of Masses and couplings favouring high $M_X$, $\xi = -8.5$ on the $x = x_+ (\xi)$ branch for real couplings

| Field [SU(3),SU(2),Y] | Masses ( Units of $10^{18}$ Gev) |
|------------------------|---------------------------------|
| A[1,1,4]               | 1.19                            |
| B[6,2,5/3]             | 0.49                            |
| C [8,2,1]              | 1.13, 0.91, 0.27                 |
| D[3,2,7/3]             | 2.03, 1.74, 0.47                 |
| E[3,2,1/3]             | 2.12, 1.56, 1.01, 1.01, 0.78, 0.55 |
| F[1,1,2]               | 2.16, 0.41, 0.41, 0.14           |
| G[1,1,0]               | 1.19, 0.85, 0.39, 0.33, 0.33, 0.06 |
| h[1,2,1]               | 3.35, 2.46, 2.06, 1.34, 0.36     |
| I[3,1,10/3]            | 1.46                            |
| J[3,1,4/3]             | 1.99, 1.22, 0.61, 0.61, 0.22     |
| K[3,1, 8/3]            | 1.23, 0.49                       |
| L[6,1,2/3]             | 1.45, 0.12                       |
| M[6,1, 8/3]            | 1.44                            |
| N[6,1,4/3]             | 1.52                            |
| O[1,3, 2]              | 3.86                            |
| P[3,3, 2/3]            | 3.0, 0.21                        |
| Q[8,3,0]               | 1.29                            |
| R[8,1, 0]              | 2.39, 0.59                       |
| S[1,3,0]               | 2.64                            |
| t[3,1,2/3]             | 3.47, 2.26, 1.44, 1.03, 0.73, 0.18, 0.12 |
| U[3,3,4/3]             | 2.18                            |
| V[1,2,3]               | 1.69                            |
| W[6,3,2/3]             | 2.05                            |
| X[3,2,5/3]             | 0.98, 0.98, 0.6                  |
| Y[6,2, 1/3]            | 0.7                             |
| Z[8,1,2]               | 2.37                            |

Parameter Values

| $\lambda = 1.0$, $\eta = .95$, $\gamma = .7$, $\bar{\gamma} = .6$, $\bar{\xi} = .7$ |
| $\bar{\xi} = .85$, $k = .35$, $m_0 = .8$, $p = .95$, $m/\lambda = 5.28 \times 10^{16}$ Gev |

Min [Mass] (Units of $10^{18}$ Gev)

| 0.06(G[1,1,0]), 0.12(L[6,1,2/3], t[3,1,2/3]) |

Max [Mass] (Units of $10^{18}$ Gev)

| 3.86(O[1,3,2]) |

Threshold Corrections

| $\Delta_X = 1.739$, $\Delta_W = -0.0149$, $\Delta_G = 0.4994$ |

Gauge Masses (Units of $10^{18}$ Gev)

| $m_{\lambda e} = 1.01$, $m_{\lambda x} = 0.41$, $m_{\lambda d} = 0.67$, $m_{\lambda d} = 0.61$, $m_{\lambda x} = 0.97$ |
perhaps by tackling the fascinating issue of dynamical symmetry breaking due to the inevitable asymptotic strength of the NMSGUT[24].

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