Mathematical model of an electric vehicle with a non-flat battery of photovoltaic converters

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Abstract. One of the major environmental problems is emissions of CO2 and other harmful substances from motor vehicles (MV). According to the tests, the most environmentally friendly MV (measurement of emissions on MVs) are electric vehicles (EV). But a low emission level can only be ensured by the use of environmentally friendly sources of electricity used to charge EV: hydroelectric power plants, wind power plants, tidal, geothermal, wave, solar power plants, etc. However, the transmission of electric energy is connected with losses. One of the ways to use them is the application of batteries of photovoltaic converters (PVC) within the EV structure as the most suitable in terms of weight and size characteristics. The use of PVC in MV is not a new idea, but due to the increase in the efficiency of PVC, their relevance becomes more obvious.

Purpose of work: development of mathematical models, scientific substantiation and rationale of the calculation method of EV energy efficiency increase by enlarging the cruising range when using PVC batteries. Research methods: methods of ground MV movement theory, system design, theory of electric circuits, mathematical modeling, etc. Results: development and scientific substantiation of the calculation method for a non-flat PVC battery within the EV structure and its effect on run or mileage change. Summary: 1) the mathematical model of EV rectilinear motion on flat non-deformable horizontal ground, suitable for the study of the processes of energy exchange between the EV and PVC batteries and the effect of these processes on the mileage, has been developed; 2) the application of the calculation method for non-flat PVC batteries to be used within the EV structure has been scientifically substantiated.

1. Introduction

To solve environmental problems associated with emissions of harmful substances by motor vehicles (MV), it is necessary to develop environmentally friendly MV.

It is known that electric vehicles (EV) are environmentally friendly transport. But the environmental friendliness of an EV depends on the way it receives electric energy from the electric grid. For example, coal-fired power plants emit 1001 g of CO2 into the atmosphere (not counting more harmful substances) when generating 1 kW·h of electricity, fuel oil-burning power plants – 840 g, gas-burning power plants – 469 g. The average emissions of CO2 per 1 kW·h by countries are as follows: China – 745 g, Russia – 597 g, the UK – 225 g [1].

Low emissions of CO2 and other harmful substances can only be ensured by the application of environmentally friendly renewable energy sources used for EV charging [2]. In addition, in EV and vehicles with combined power units, braking in most cases occurs not by mechanical brakes, but by electric drives or actuators in the recuperation mode, which significantly reduces the release of...
microparticles of dust from brake pads and discs, including cancerogenic ones [[3]]. Also widely known is the problem of an insufficient EV cruising range due to the small amount of electric energy stored on board of EV, compared to the chemical energy contained in gasoline used to refuel the tank of the vehicle [[4]].

Therefore, the task of increasing the cruising range of EV, including application of batteries of PVC, is relevant. Among the MV manufacturers that use PVC batteries in EV are Volkswagen, Toyota, Sanyo, Ford, etc. [[5]].

There are also works of Russian researchers, which are devoted to the application of PVC batteries in EV [[6]-[8], [1], [9], [10]]. But no one has yet worked particularly on the specifics of non-flat PVC batteries and connection between the MV body geometry and electric parameters of PVC batteries and cruising range.

For mathematical modeling of different variants of a battery of PVC within the MV structure, it is necessary to use, among other things, a mathematical model (hereinafter "model") of the EV itself consisting of other models, for example, those of wheels, transmission, electric machine (EM), battery, etc.

2. Vehicle movement model

The rolling resistance force according to Coulomb’s law [[11]]

\[ F_f = m \cdot g \cdot \cos \alpha \cdot f, \ N, \]

where:

- \( m \) – the weight of the EV, kg;
- \( g \) – free fall acceleration on the Earth's surface, 9.81 m/s²;
- \( \alpha \) – angle of inclination of the support surface (road), rad;
- \( f \) – coefficient of rolling resistance of the tire from the translational speed.

In the absence of data on the terrain, the \( \alpha \) angle of inclination of the road is assumed to be 0.

Aerodynamic drag (resistance) force of the EV [[9]]

\[ F_w = \frac{C_x S_M \rho_{air} V^2}{2}, \ N, \]

where:

- \( C_x \) – coefficient of aerodynamic resistance of the vehicle (according to the vehicle manufacturer);
- \( S_M \) – midsection area, m² (according to the vehicle manufacturer);
- \( \rho_{air} \) – density of air (under normal conditions 1.204 kg/m³);
- \( V \) – speed of the rectilinear movement of the vehicle, m/s.

Force of inertia of the EV

\[ F_a = m \cdot a, \ N, \]

where:

- \( a \) – acceleration of the EV,
- \( a = \frac{dv}{dt}, \ m/s^2. \)

The total resistance force acting on an EV and consisting of the three forces listed above

\[ F_x = F_f + F_w + F_a, \ N. \]

3. Wheel model

The coefficient of rolling resistance of the tire from the translational speed [[11], [12]]:
\[ f = f_0 + k_f \cdot V^2, \text{N}, \]

where:
- \( f_0 \) – coefficient of rolling resistance of a wheel at a speed close to 0;
- \( k_f \) – coefficient of increase of rolling resistance of a wheel from speed.

Torque on the wheel axis taking into account the moment of inertia of the wheel

\[ M_w = F_\text{L}r + a \cdot \frac{I_w N_w}{r}, \text{Nm}, \]

where:
- \( r \) – average radius of EV wheels, m;
- \( I_w \) – EV wheel moment of inertia, kg\( \cdot \)m\(^2\);
- \( N_w \) – number of EV wheels.

**4. Transmission model**

For the transmission model, the rectilinear movement of the vehicle is assumed without taking into account the wheels slip. In this case, all the wheels rotate at the same speed, and the angular velocity of the EM is determined as

\[ \omega_{EM} = \frac{v_{utr}}{r}, \text{rad/s}, \]

where:
- \( u_{utr} \) – transmission gear ratio.

The torque on the shaft of the EM when transmitted through the transmission from the wheel is calculated as

\[ M_{EM} = \frac{M_w}{u_{tr} \eta_{tr}} \frac{\text{sgn}(M_w)}{s}, \text{Nm}, \]

where:
- \( M_w \) – torque at the wheels, N\( \cdot \)m;
- \( \eta_{tr} \) – transmission efficiency;
- \( \text{sgn}(M_w) \) – wheel torque sign function: mechanical efficiency is determined by the value of friction forces, so when accelerating it increases the load moment on the EM (sign -1), and when recuperating-reduces it (sign +1).

The moment of inertia of the rotor of the EM, given to the torque on the axis of the EM

\[ M_f = I_{EM} \frac{d\omega_{EM}}{dt} = I_{EM} \frac{dv_{utr}}{dt} = I_{EM} \frac{u_{tr}}{r}, \text{N} \cdot \text{m}, \]

where:
- \( I_{EM} \) – moment of inertia of the rotor of the EM, kg\( \cdot \)m\(^2\).

**5. Electric machine model**

The torque on the axis of an EM is the sum of the torques \( M_{EM} \) and \( M_f \). The efficiency of the EM is given by a table-valued function from the angular velocity and the load moment:

\[ \eta_{EM} = f(\omega_{EM}, M_{EM} + M_f). \]

Power consumed by an EM taking into account its efficiency

\[ P_{EM} = (M_{EM} + M_f) \cdot \omega_{EM} \cdot \eta_{EM}^{\text{sgn}(M_{EM}+M_f)}, \text{W}, \]
where:
\( \eta_{EM} \) – EM efficiency.

6. Inverter model
The current strength at the input of the traction inverter taking into account its efficiency

\[
i_{dc} = \frac{P_{EM} \eta_{sgn}(P_{EM})}{u_{b,hv}}, \text{ A},
\]

where:
\( \eta_{dc} \) – traction inverter efficiency.
\( u_{b,hv} \) – GB1 high-voltage (traction) battery voltage, V.

The total power losses in the power leads (wires) of the electric drive and at the equivalent resistance of the battery

\[
P_{\text{lost}} = i_{dc}^2 R_{bat} + \frac{i_{dc}^2 l_{\text{wire}} \rho_{\text{wire}}}{q_{\text{wire}}}, \text{ W}.
\]

where:
\( R_{bat} \) – equivalent active resistance of the battery, Ohm;
\( l_{\text{wire}} \) – wire length, m;
\( q_{\text{wire}} \) – wire cross section, mm²;
\( \rho_{\text{wire}} \) – wires material resistivity, for copper at 20 °C it would be around 0.01724-0.0180 Ohm·mm²/m.

7. Model of onboard consumers
The current strength taken by the load of onboard consumers with power \( P_{\text{add}} \) is

\[
i_{\text{add}} = \frac{P_{\text{add}}}{u_{b,lv}}, \text{ A},
\]

where:
\( P_{\text{add}} \) – power of onboard consumers, W;
\( u_{b,lv} \) – GB2 low-voltage (12 V) battery voltage, V.

At the same time, random oscillations with an amplitude of ±25% from the average value must be added to the current strength value of onboard consumers (A1-An) to simulate random deviations.

8. Traction battery model
In the pulse mode of the DC/DC converter, the charge and discharge processes of the low-voltage buffer battery are taken into account, so additional consumers are powered by the low-voltage battery, and the charge/discharge power of the traction battery

\[
P_b = P_{Ch} \cdot \eta_{dcdc} - P_{dc} - P_{\text{lost}}, \text{ W},
\]

where:
\( P_{Ch} \) – electric power consumed by the DC/DC converter, W;
\( \eta_{dcdc} \) – DC/DC converter efficiency;
\( P_{dc} \) – electric power consumed by the traction inverter, W;
\( P_{\text{lost}} \) – the total electrical power losses, W.

9. Buffer battery model
The differential equation of energy exchange between the onboard consumers, traction battery, buffer battery and PVC battery
\[
\frac{dE_{GB2}}{dt} = \frac{P_{b.PVC} \cdot \eta_{MPPT} - P_{Ch} - P_{add} - i_{GB2}^2 \cdot R_{GB2} - P_{lost.cond}}{W},
\]

where:
- \(P_{b.PVC}\) – electrical power generated by the PVC battery, W;
- \(\eta_{MPPT}\) – MPPT converter efficiency;
- \(P_{Ch}\) – electric power consumed by the DC/DC converter, W;
- \(P_{add}\) – power of additional (onboard) consumers, W;
- \(P_{lost.cond}\) – power losses in the power wires between GB2 buffer battery and MPPT converter, W;
- \(i_{GB2}\) – charge/discharge current of the GB2 buffer battery, A;
- \(R_{GB2}\) – equivalent active resistance of the GB2 battery, Ohm;
- \(E_{GB2}\) – GB2 traction battery energy, J.

The power losses in the power wires of the charge/discharge circuits of the GB2 buffer battery may be neglected due to their small length and active resistance.

High-voltage traction battery voltage \(u_{b,lv}\) in the first approximation can be taken as a constant.

10. DCDC converter model

The power consumed by DC/DC converter \([9]\)

\[
P_{Ch} = u_{b,lv} \cdot i_{dc,lv} \cdot \eta_{dc}^{sgn(i_{dc,lv})}, W,
\]

where:
- \(i_{dc,lv}\) – current on the low-voltage side, for maximum efficiency \(i_{dc,lv} = 125\) A.
- \(\eta_{dc}^{sgn(i_{dc,lv})}\) – converter efficiency taking into account the direction of current.

From the above equations by means of simple transformations it is possible to deduce the equation of rectilinear motion of the EV:

\[
\frac{dV}{dt} = \frac{\left(\frac{m_{EM} \cdot u_{tr} \cdot \eta_{tr}^{\text{sp}}(\text{MW})}{J_t} - F - F_{\text{fr}}\right)}{m + \left(u_{tr} \cdot \eta_{tr}^{\text{sp}}(\text{MW})\right)},
\]

which, taking into account the absence of the moment of inertia of the transmission and the change of the sign of the torque of the wheel, is identical to the differential equation of rectilinear motion of the EV \([9]\).

11. MPPT converter model

The MPPT converter efficiency is a function of the power and voltage of the PVC battery \([9]\):

\[
\eta_{mppt} = f(P_{PVC}, u_{PVC}),
\]

where:
- \(P_{PVC}\) – power generated by PVC, W,
- \(u_{PVC}\) – voltage of the PVC battery, V.

The current output of the MPPT converter

\[
i_{mppt,\text{out}} = \frac{i_{PVC} \cdot u_{PVC} \cdot \eta_{mppt}}{u_{b,lv}}, A,
\]

where \(i_{PVC}\) – current at the output of the PVC battery, A.

12. Model of the PVC battery

The power generated by each individual PVC is directly proportional to the flow of solar radiation taken by it, if other conditions are equal \([13]\):
where:
\( P_{pvc,\text{nom}} \) – PVC nominal power at nominal solar radiation density, W;
\( E \) – current density of the solar radiation, W/m²;
\( E_{\text{nom}} \) – nominal density of the solar radiation, W/m²;
\( I_{pvc} \) – current generated by PVC, A;
\( U_{pvc} \) – voltage generated by PVC, V.

Approximation of PVC CVC (Current-Voltage Characteristic) can be performed by various methods [10, 13, 14]. An alternative function can be proposed based on the curves of the tire adhesion to the road (figure). The tire model called "Magic Formula" provides a good approximation of such characteristics and is used to approximate PVC current depending on the voltage [15]

\[
i_{pvc} = D \cdot \sin \left( C \cdot \tan^{-1} \left( B \cdot u - E \left( B \cdot u - \tan^{-1} (B \cdot u) \right) \right) \right), A,
\]

where:
\( u \) – argument in the following form: \( u = u_{piv} - u_{pv}, \text{V} \);
\( u_{piv} \) – PVC idle voltage, V;
\( u_{pv} \) – PVC voltage, V;
\( B, C, E \) – dimensionless empirical coefficients;
\( D \) – maximum current of PVC depending on the intensity of solar radiation, A.

In case of series connection of PVCs into the battery in accordance with Kirchhoff's laws, the current through all the elements is the same, so the CVCs of the individual elements are added by voltage. In case of parallel connection of PVC in the battery, the voltage applied to parallel circuits is the same, therefore CVCs of separate circuits are added by current. The geometric interpretation of the calculation algorithms is shown respectively in figures.
13. MPPT algorithm

The control system (CS) of the PVC battery is an MPPT and implements one of the algorithms for finding the maximum power point. In the simulation, the task of PVC CS (MPPT) comes down to the selection of the point at which the power generated by the PVC battery is maximum:

\[ p_{pv,nom} = \max_{i=0}^{m} (U_i \cdot I_i), \, W, \]

where:
- \( i \) – number of the instant count of the simulated CVC;
- \( m \) – maximum number of the count of the simulated CVC;
- \( U_i \) – voltage value in each of the counts of the simulated CVC;
- \( I_i \) – current in each of the counts of the simulated CVC.

14. Solar illumination model

There are several methods for calculating the density of solar radiation: Atwater’s, Perrin’s, Yang’s, Bird’s, Gueymard’s, REST, Winter’s, Iqbal’s methods (table 1). Of these, the Perrin’s, Gueymard’s, REST and Iqbal’s methods do not take into account diffuse solar radiation, which unacceptably reduces...
the accuracy of the calculation of the total solar radiation flux. Of the remaining ones, the Bird's method gives the minimum root mean square error of calculating the total solar radiation among them [[16]].

Table 1. Root mean square error of calculations of solar radiation energy density according to the tested methods.

| No. | Method  | Direct SR | Full SR | Diffuse SR |
|-----|---------|-----------|---------|------------|
| 1   | Atwater | 16        | 15      | 51         |
| 2   | Bird    | 15        | 14      | 43         |
| 3   | Perrin  | 21        | ---     | ---        |
| 4   | Yang    | 18        | 19      | 48         |
| 5   | Gueymard| 49        | ---     | ---        |
| 6   | REST    | 22        | ---     | ---        |
| 7   | Winter  | 26        | 15      | 49         |
| 8   | Iqbal   | 28        | ---     | ---        |

15. Test cycles
The results of calculations performed in accordance with the models depend on the operation modes of the vehicle and PVC battery. Operation modes are determined by test cycles. Standard cycles are selected to make it possible to compare both the initial parameters and the obtained results of the calculations.

The standards for certification of vehicles and EV are defined by UNECE Regulation No. 101, according to which the experimental assessment of the characteristics of the vehicle being developed is conducted. UNECE Regulation No. 101 currently uses the New European Driving Cycle (NEDC) combined conditions (cycle), which is selected as the first basic cycle for EV testing.

The New European Driving Cycle (NEDC) combined conditions (cycle) data are taken from UNECE Regulation No. 101 [[17]].

16. Conclusion
– The mathematical model has been developed that integrates not only mathematical models of rectilinear motion of an EV, the Bird’s sky model and the PVC element model, but also the PVC battery model taking into account the geometry of the MV body.
– This mathematical model makes it possible not only to calculate more accurately the amount of electricity generated by a non-flat PVC battery, but also to develop recommendations for a more rational use of PVC for recharging traction batteries of electric transport.

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