Interplay of intrinsic and extrinsic mechanisms to the spin Hall effect in a two-dimensional electron gas

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Abstract

In order to describe correctly the interplay of extrinsic and intrinsic spin-orbit mechanisms to the spin Hall effect, it is necessary to consider different sources of spin relaxation. We take into account the spin relaxation time $\tau_{DP}$ due to the Dyakonov-Perel mechanism as well as the Elliot-Yafet spin-relaxation time $\tau_s$ due to the spin-orbit scattering from impurities. The total spin Hall conductivity depends crucially on the ratio $\tau_s/\tau_{DP}$.

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1. Introduction

In a recent paper[1], we have provided a general framework to describe the electric-field control of the electron spin in a two-dimensional electron gas (2DEG) with diffusive electron spin dynamics when both intrinsic and extrinsic spin-orbit interactions are present. The extrinsic mechanism for the spin Hall effect arises to first order in the spin-orbit coupling strength, $\lambda_0^2$ (see below) [2, 3]. However, when also the intrinsic mechanism is present [4, 5, 6], a first-order calculation is no longer sufficient, since it is important to consider the Elliot-Yafet spin relaxation due to the spin-orbit scattering from impurities, which arises in the order $\lambda_0^4$. Here, by using a Keldysh Green function approach [7], we provide a microscopic basis of the equation of motion for the spin density that governs the interplay of intrinsic and extrinsic mechanisms. The layout of the paper is as follows. In the next section we introduce the model Hamiltonian and the method. In section 3 we derive an expression for the spin current and an associated continuity equation. In section 4, we consider the specific case of the spin Hall effect and derive a formula for the spin Hall conductivity. Finally, we state our conclusions in section 5.

2. The model and the method

In the presence of both extrinsic and intrinsic spin-orbit interaction as well as normal potential scattering from impurities $V(x)$, the Hamiltonian for the 2DEG can be written in terms of a spin-dependent [SU(2)] vector potential $\mathbf{A}$

$$H = \frac{p^2}{2m} - \mathbf{A} \cdot \mathbf{p} + V(x).$$

The vector potential is the sum of intrinsic (Rashba type) and extrinsic contributions

$$\mathbf{A} = m \sigma \times \left( \alpha \mathbf{e}_z + \frac{\lambda_0^2}{4} \partial_x V(x) \right) \equiv \frac{1}{2} \hat{\mathbf{A}} \sigma \sigma^\mu,$$  

where $\sigma$ is the vector of Pauli matrices and $\alpha = x, y, z$. The coupling constants $\alpha$ and $\lambda_0^2$ characterize the strength of the intrinsic and extrinsic spin-orbit interaction, respectively. The advantage of introducing the SU(2) vector potential is that one can immediately derive a continuity equation for the spin density

$$\partial_t s^\mu + \partial_x j^\mu + \kappa_{abc} A^b \cdot j^c = 0,$$  

where the spin density is defined in terms of the Heisenberg field operators $s^\mu = (1/2)(\phi^\dagger(x,t)\sigma^\mu\phi(x,t))$ and the spin current has the expression $j^\mu = (1/4)(\partial_t \phi^\dagger(x,t)\sigma^\mu\phi(x,t))$, with the velocity operator given by

$$v = \frac{p - \mathbf{A}}{m}.$$  

Although Eq. (3) is formally exact, it cannot directly convey information for the disorder averaged spin current, since the disorder potential appears also in the vector potential $\mathbf{A}$. In order to carry out the average over the disorder, it is convenient to use the Green function approach as developed in Ref. [7]. It is also useful to explicitly separate the spin-orbit interaction due to the intrinsic mechanism from that due to the extrinsic mechanism. To this end we define a space-independent vector potential

$$\mathbf{A} = \alpha m \sigma \times \mathbf{e}_z$$  

and a spin-dependent disorder potential

$$U(x) = V(x) - \frac{\lambda_0^2}{4} \sigma \cdot \partial_x V(x) \cdot \mathbf{p}.$$
The disorder averaged spin density is given by

\[ s^\sigma(x, t) = -i \int \frac{d\epsilon}{2\pi} \int \frac{d^2 p}{(2\pi)^2} \text{Tr}[\sigma^\sigma G^{\text{<}}(\epsilon, p, x, t)], \]  

(7)

where \( G^{\text{<}}(\epsilon, p, x, t) \) is the lesser component of the disorder averaged Green function, here given in Wigner coordinates. The current density is written as a sum of two terms, \( j^i = j^i_0 + j^i_0 \), associated to the normal, \( v_0 = (p - A)/m \), and to the anomalous disorder-dependent components of the velocity, respectively. Explicitly the normal current is

\[ j^i_0(x, t) = -\frac{i}{4} \int \frac{d\epsilon}{2\pi} \int \frac{d^2 p}{(2\pi)^2} \text{Tr}[\sigma^\sigma \{v_0, G^{\text{<}}(\epsilon, p, x, t)\}]. \]  

(8)

where the symbol \( \{, \} \) denotes the anticommutator. Since the anomalous velocity contribution to the current contains explicitly the disorder potential, its expression can be obtained only after specifying the disorder model and the approximations used. Later on, we will identify the expression for the anomalous current from the self-energy terms contributing to the continuity equation.

In Wigner coordinates and after a gradient expansion, the equation of motion for the Green function \( \tilde{G}(\epsilon, p, x, t) \) reads

\[ i\hbar \partial_t \tilde{G} + \partial_\epsilon \cdot \{p \frac{A}{2m}, \tilde{G}\} + \frac{A \cdot p}{m} \tilde{G} = \tilde{\Sigma}, \]  

(9)

The Green function has a matrix structure in both the Keldysh and spin space, and the symbol \( \{, \} \) indicates a commutator. An external electric field \( E \) is included via the substitution \( \partial_\epsilon \rightarrow \partial_\epsilon - eE_\sigma \partial_{\sigma} \). The self-energy \( \tilde{\Sigma} \) entering the RHS of the Eq. (9) contains all the effect of the disorder potential defined in Eq. (6).

We assume the standard model of uncorrelated impurities with

\[ V(x_1)V(x_2) = n_v v_0^2 \delta(x_1 - x_2), \]  

(10)

where \( n_v \) is the impurity concentration and \( v_0 \) the scattering amplitude. In the Born approximation it is sufficient to define the moments of the disorder distribution up to the second order as done in Eq. (10). At the level of the Born approximation, the self-energy is given by the diagram of Fig. 1(a) and reads

\[ \tilde{\Sigma}(x_1, x_2) = \overline{U(x_1)\overline{G}(x_1, x_2)U(x_2)}. \]  

(11)

To zero order in \( \lambda^2_0 \), Eq. (11) yields

\[ \tilde{\Sigma}^0(p, x) = n_v v_0^2 \sum_p \tilde{G}(p', x). \]  

(12)

This contribution leads to the standard elastic scattering time

\[ \tau_{\text{e}}^{-1} = 2\pi N_0 n_v v_0^2, \]

where \( N_0 \) is the single-particle density of states of the Fermi gas. To first order in \( \lambda^2_0 \), Eq. (11) yields the three terms

\[ \tilde{\Sigma}^1_0(p, x) = -\frac{\lambda^2_0}{4} n_v v_0^2 \sum_p \{p, \sigma_b p_\sigma' G(p', x)\}, \]  

(13)

\[ \tilde{\Sigma}^1_1(p, x) = \frac{\lambda^2_0}{8} n_v v_0^2 \sum_p \{\epsilon p_\sigma' p_\sigma', \partial_\sigma G(p', x)\}, \]  

(14)

\[ \tilde{\Sigma}^1_2(p, x) = \frac{\lambda^2_0}{8} n_v v_0^2 \sum_p \{p, \sigma_b, \partial_\sigma G(p', x)\}; \]  

(15)

again the external electric field is included in the space derivative \( \partial_\sigma \equiv \partial_{\sigma_x} - eE_\sigma \partial_{\sigma_x} \). The self-energy \( \tilde{\Sigma}^1 \) is related to the mechanism for "swapping of spin currents". \( \tilde{\Sigma}^1_0 \) and \( \tilde{\Sigma}^1_1 \) are side-jump contributions. Notice that to lowest order in the gradient expansion, only the "swapping term" is different from zero. The side-jump contributions arise when considering the next-to-leading order in the gradient expansion. Finally, to second order in \( \lambda^2_0 \), Eq. (11) yields

\[ \tilde{\Sigma}^2 = \frac{\lambda^2_0}{16} n_v v_0^2 \sum_k \epsilon^\sigma \tilde{G}(k, x) \epsilon^\sigma (p \cdot k)^2, \]  

(16)

which corresponds to the Elliot-Yafet spin relaxation mechanism.

Skek-scattering arises beyond the Born approximation, starting from the third order diagram shown in Fig. 1(b). By using a disorder model with third moments different from zero and defined by \( \mathcal{B} \),

\[ V(x_1)V(x_2)V(x_3) = n_v v_0^2 \delta(x_1 - x_2) \delta(x_2 - x_3), \]  

(17)

the diagram of Fig. 1(b) yields to first order in \( \lambda^2_0 \), the terms

\[ \tilde{\Sigma}^3_{SS, o} = \frac{i \lambda^2_0}{4} n_v v_0^2 \sum_{kk'} \tilde{G}(k, x) \tilde{G}(k', x) \epsilon \cdot \mathbf{K} \]  

(18)

\[ \tilde{\Sigma}^3_{SS, s} = \frac{i \lambda^2_0}{4} n_v v_0^2 \sum_{kk'} \mathbf{k} \times \mathbf{p} \cdot \sigma \tilde{G}(k, x) \tilde{G}(k', x). \]  

(19)

To make contact with the diagrammatic language of the Kubo formula, we notice that the two self-energies correspond to the diagrams of Fig. 2 in Ref. [3].

3. The continuity equation and the spin current

The insertion of Eqs. (12–16) and (18–19) into the equation-of-motion (7) allows to derive a continuity equation for the spin density polarized along the \( a \)-axis. After integrating over \( (\epsilon, p) \) and taking the trace of Eq. (9), one obtains

\[ \partial_t s^a + \partial_\epsilon \cdot j^a_0 = -\partial_\epsilon \cdot j^a_0 + \frac{1}{\tau_s} s^a. \]  

(20)

The gradient together with the commutator on the LHS of Eq. (9) are the origin of the covariant derivative of the spin current

\[ \partial_\epsilon \cdot j^a = \partial_\epsilon \cdot j^a_0 + \epsilon_{abc} A^b \cdot j^c_0. \]  

(21)
Most of the contributions to the self-energy disappear in the integrated equation. The only terms surviving the integration are those originating from $\Sigma_{\alpha}^1$ and $\Sigma^2$. The term containing $\Sigma_{\alpha}^1$ of Eq. (14) yields a contribution which can be written as a divergence and hence, apart from a minus sign, defines the anomalous contribution to the current $j_{\alpha}^s$. The spin relaxation time, which is obtained from Eq. (16), only applies to the in-plane spin components (cf. the two $\sigma$ matrices before and after the Green function) and reads $\tau_{\alpha}^{-1} = \tau^{-1}(\alpha D_{\alpha} F \tau)^{1}$.

Clearly, also the explicit expression for the spin current can be obtained starting from Eq. (9). To do this, we followed the procedure of Ref. [7], namely we integrated Eq. (9) over $\xi = p^2/2m - \mu$ in order to obtain a Boltzmann-like equation for the quasiclassical Green function,

$$\hat{g}(\mathbf{p}, \mathbf{x}) = \frac{i}{\pi} \int d\xi \mathcal{G}^{R/A}(\mathbf{p}, \mathbf{x}).$$

(22)

Technically the equations simplify considerably by using that the retarded and advanced quasiclassical Green functions are given by a constant, $\hat{g}_{AA}^R \approx \pm 1$.

After some more steps we found, in the diffusive limit, when the spin splitting due to the intrinsic spin-orbit interaction, $\sigma_{D\alpha} F$, is smaller than the disorder broadening, $\tau^{-1}$ the spin current as

$$j_{\alpha}^s = -D \partial_t s^\alpha + \sigma_{\alpha}^{ext} e_{ab} E_b + \gamma_{\alpha}^s E_j.$$

(23)

The contribution from the extrinsic spin-orbit mechanism has the form first predicted by Dyakonov and Perel[11] and is due to $\sigma_{\alpha}^{ext} = \sigma_{ss}^{H} + \sigma_{sj}^{H}$ with the standard expressions for the side-jump (sj) and skew-scattering (ss) contributions [3]

$$\sigma_{sj}^{H} = \sigma_D \frac{\xi}{4 e \tau}, \quad \sigma_{ss}^{H} = \frac{1}{4}(p_F \lambda)(2\pi \eta_0)\sigma_{sj}^{H},$$

(24)

$$\sigma_D = 2e^2 \eta_0 D$$

being the Drude conductivity and $D = v_F^2 \tau/2$ the diffusion coefficient. It is worthwhile pointing out that the anomalous disorder-dependent contribution to the current is one half of the side-jump contribution, $j_{\alpha j}^{ext} = (1/2)\gamma_{\alpha}^{sj} e_{ab} E_b$.

The intrinsic spin-orbit coupling yields a contribution to the spin current due to the $SU(2)$ magnetic field $B^\alpha = \Delta_k \times \mathbf{A}^\alpha$,

$$\gamma_{\alpha}^s E_j = \frac{\tau_D \sigma_D}{8ne} (B^\alpha \times \mathbf{E})_j.$$

(25)

For the Rashba model the only non-zero component of $B^\alpha$ is $B_x^2 = 2(2ma)^2$ leading to $\gamma_{\alpha}^s E_j = e_{ab} \sigma_{int}^{H} E_b$ with $\sigma_{int}^{H} = \frac{\lambda}{2}(p_F \tau)^2$. The extra terms of Ref. [10], associated to $\Sigma_{\alpha}^2$, are not relevant in the present context and therefore have been ignored in Eq. (25).

4. The spin Hall conductivity

After these formal aspects on how to derive Eqs. (26) and (27) we now apply the equations to a specific problem, namely the spin Hall effect[11][12][13][14][15]. More precisely we study the transverse spin current that is generated by a uniform electric field applied along the $x$-axis. According to Eq. (23) the spin current flowing along the $y$-axis reads

$$j_y = 2maD \gamma^x + (\sigma_{ext}^{H} + \sigma_{int}^{H}) E_x,$$

(26)

where the first term originates from the covariant derivative defined in Eq. (21). The presence of the in-plane spin density, $s^\alpha$, requires that Eq. (25) must be solved together with the $y$-component of the continuity equation (27)

$$\partial_t s^y + 2ma \left( j_y - \frac{1}{2} \sigma_{sj}^{H} E \right) + \frac{1}{\tau} s^y = 0.$$

In the absence of the extrinsic mechanism ($\lambda_0 = 0$), as it has been noticed previously [16][17][13], the above equation implies the vanishing of the spin current and spin Hall conductivity [19][20][21][22]. As a result the in-plane spin polarization acquires the electric-field dependent value predicted by Edelstein[23].

In the presence of both the extrinsic and intrinsic mechanisms, by solving Eqs. (26) and (27) together, one gets $j_y = \sigma_{sj}^{H} E$ with

$$\sigma_{sj}^{H} = \frac{1}{\tau_x + 1/\tau_D} \left( \sigma_{int}^{H} + \sigma_{ss}^{H} + \frac{1}{2} \sigma_{sj}^{H} \right) + \frac{1}{2} \sigma_{sj}^{H},$$

(28)

where we have introduced the Dyakonov-Perel relaxation time $\tau_{\alpha}^{-1} = (2ma)^2 D$. To first order in $\lambda_0^2, \tau_{\alpha} = 0$ and one obtains $\sigma_{sj}^{H} = (1/2)\sigma_{ss}^{H}$ for any $\alpha \neq 0$, in agreement with [3]. Clearly, the ratio $\tau_D/\tau_{ss}^{H}$ acts as a control knob of the spin Hall conductivity. This suggests that by adjusting the constant $\alpha$ by a suitably applied gate voltage, one can vary experimentally the magnitude of the spin Hall current. For illustrative purposes in Fig. 2 we plot the spin Hall conductivity for GaAs as a function of the dimensionless parameter $2\alpha D \tau_F$.

5. Conclusions

In summary we have presented a microscopic derivation of the equation of motion for the Keldysh Green function in the presence of both extrinsic and intrinsic spin-orbit interaction as well as scattering from impurities. In particular we have given explicit expressions for the disorder averaged self energy at the
level of Born approximation and to the first order beyond the Born approximation. This has allowed us to derive an expression for the spin current and the associated continuity equation. It has been shown that the ratio of the Elliot-Yafet and Dyakonov-Perel spin relaxation times is the important parameter controlling the interplay of extrinsic and intrinsic mechanisms to the spin Hall effect in a 2DEG.

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