Joint Resummation for Higgs Production

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Abstract

We study the application of the joint resummation formalism to Higgs production via gluon-gluon fusion at the LHC, defining inverse transforms by analytic continuation. We work at next-to-leading logarithmic accuracy. We find that at low $Q_T$ the resummed Higgs $Q_T$ distributions are comparable in the joint and pure-$Q_T$ formalisms, with relatively small influence from threshold enhancement in this range. We find a modest (about ten percent) decrease in the inclusive cross section, relative to pure threshold resummation.
1. Introduction

The gluon-gluon fusion process $gg \rightarrow hX$ is expected to be the dominant mechanism for the production of rather light Higgs bosons at the LHC [1]. Next-to-leading order (NLO) QCD corrections to the total production cross section for this process are known to be large, of the order of 70% [2]. This made the computation of NNLO corrections to the process an important task, which was completed very recently [3]. The NNLO corrections are found to be substantial as well, even at LHC energies, where they increase the total cross section by an additional 30% [3] over NLO. The dominant part of these contributions is directly related to soft and collinear gluon emission, as demonstrated in Refs. [4, 5].

Soft gluon emission is manifested through the presence of logarithmic corrections in expressions for partonic subprocesses. These corrections arise from cancellations between real and virtual contributions to the differential cross section at each order in perturbation theory. They can be large when a cross section is sensitive to their behavior near the boundary of phase space. This is the case for threshold corrections where terms of the form $\alpha_s^n \ln^{2n-1}(1-z)/(1-z)$ grow large in the limit $z = Q^2/\hat{s} \rightarrow 1$, that is, when the partonic center-of-mass energy $\hat{s}$ approaches the invariant mass $Q$ of the Higgs particle. Similarly, recoil corrections appearing in transverse momentum distributions, $\alpha_s^n \ln^{2n-1}(Q^2/Q_T^2)/Q_T^2$, become large when the Higgs boson is produced with a small transverse momentum $Q_T \ll Q$. Because both the total production rate and the production characteristics, including the transverse momentum distribution, will influence the search and analysis strategies for the Higgs, a close study of higher-order corrections in the soft and/or collinear limit is desirable.

Resummation techniques for Higgs production have been established separately in the threshold [6] and in the recoil [7, 8, 9] cases, based on earlier work [10, 11, 12] on the related case of Drell-Yan dimuon production. A new analysis for threshold resummation, including the recently calculated [4] next-to-next-to-leading logarithmic (NNLL) coefficients $C^{(2)}$ and $D^{(2)}$, can be found in [13, 14]. The resummed Higgs-$Q_T$ distribution has also recently been reinvestigated in [15, 16, 17], using the NNLL $B^{(2)}$ coefficient calculated in [18, 19], and in [17] matched to the Higgs cross section at fixed $Q_T$ at NLO [20]. A complete understanding of soft gluon effects in differential distributions also requires a study of the relation between the two sets of corrections. A joint treatment of these corrections was proposed in [21, 22]. It relies on a refactorization of short-distance and long-distance physics at fixed transverse momentum and energy [22]. In this formalism, resummation of logarithmic corrections takes place in the impact parameter $b$ space [12], Fourier conjugate to transverse momentum $Q_T$ space, and at the same time in Mellin-$N$ space [10, 11], conjugate to $z$ space. This guarantees simultaneous conservation of energy and transverse momentum of the soft radiation. At present, the joint resummation formalism has been developed to next-to-leading logarithmic (NLL) accuracy. A full phenomenological study of the joint resummation formalism, as applied to electroweak Z production, was undertaken in [23].

It may seem surprising that threshold resummation is relevant in Higgs production at LHC energies, given that partonic threshold for the production of a Higgs at rest involves partonic fractions of the order of $10^{-2}$, even smaller than for the Z boson at the Tevatron, where the numerical effect of threshold resummation is rather small [23]. Nevertheless, the results of [3, 4, 5]
and [6, 13, 14] show that this is the case. Much of the difference between the Higgs and the Z boson can be attributed to the difference between the color charge $C_A$ for the gluons that initiate the former and $C_F$ for the quarks that initiate the latter. This difference reflects itself in an increased sensitivity to Sudakov logarithms associated with partonic threshold for gluon-induced processes. Gluon-induced processes, however, are also potentially sensitive to radiation at low $x$. Indeed, the steep $x$-dependence of the gluon distribution functions at low $x$ enhances their sensitivity to threshold singularities. By examining the Higgs cross section at measured $Q_T$, we will find an intriguing interplay between these two regions, which should have important influence of the shape of the resummed distribution.

In any implementation of a resummation formula one has to address the issue that the running strong coupling in the resummed expression is probed at virtualities reaching down to $\Lambda_{QCD}$ and below. A number of methods have been proposed to deal with this singularity, while leaving intact the full resummed cross section [24, 25], relying on the analytic structure of the resummed cross section. In our previous study of Z boson production, [23], we made use of the minimal prescription [25] for the Mellin-inverse, and the prescription of [26] for the inverse Fourier transform, diverting it into the complex $b$-plane according to the phase structure of the integrand (see below). In this way, we obtained a well-defined resummed cross section, valid for all nonzero $Q_T$, which described the $Q_T$ distribution of Z bosons produced at the Tevatron. A complete fit requires a fitted non-perturbative parameter in the form of a Gaussian in $b$ space [9]. In general, however, the effect of the nonperturbative parameter is quite modest, as also found in Ref. [27]. Toward large $Q_T$, a matching procedure between the fixed-order and the resummed results led to good agreement with the data there as well. The treatment of the inverse $b$-space transform can be carried out as well in pure $Q_T$ resummation as in joint resummation. We can therefore compare the two, to better understand the relationship between threshold and transverse momentum resummation.

In this paper we study Higgs production via gluon-gluon fusion for both $Q_T$ and joint resummation. The leading order cross section is of order $\alpha_s^2$ and is mediated via a heavy quark loop. Because the Higgs couples to fermions proportionally to their masses, we make the by-now standard simplification of keeping only top quark loop contributions to the process. We also treat the mass of the Higgs boson as much lighter than twice the mass of the top quark. Under these conditions, the Higgs-gluon interaction can be described in the language of an effective Lagrangian [2], replacing the top quark loop by a simple effective $ggh$ vertex. It has been shown that the heavy top quark approximation is accurate within a few percent in the case of NLO calculations [6] for actually a much wider range of Higgs masses. Owing to similarities between the Drell-Yan production mechanism and the gluon fusion mechanism for Higgs production (under the above assumptions), the joint formalism developed in [23] can be fairly easily translated to the latter case. It turns out, however, that there are additional effects in the Higgs resummation case that require a more detailed analysis, as we will show. As suggested above, they are related to the gluon initial state for the reaction, resulting in amplified resummation effects through the basic replacement $C_F = 4/3 \rightarrow C_A = 3$ in the step from Drell-Yan to Higgs, and to the important role of $1/x$ terms in the splitting functions.

The joint resummation formalism for Higgs production in $N$ and $b$ space is described in
section 2, along with its relations to threshold and \( Q_T \) resummations. In section 3, the method for inverting transforms is reviewed, along with the relation to more standard \( Q_T \) resummations. Section 4 presents our numerical results for the joint and pure \( Q_T \) cases. In section 5 we discuss the total (i.e., \( Q_T \)-integrated) cross section and the possible role of terms not brought under control by joint resummation. Finally, we draw conclusions.

2. Joint resummation for Higgs production

2.1. The jointly-resummed cross section

We start from the general formula for joint resummation [22, 23], applied to Higgs production:

\[
\frac{d\sigma_{AB}^{\text{res}}}{dQ^2 d^2\vec{Q}_T} = \frac{\pi \tau_0^h \delta(Q^2 - m_h^2)}{2\pi^2} \int \frac{dN}{\tau^{-N}} \int \frac{d^2b}{(2\pi)^2} e^{i\vec{Q}_T \cdot \vec{b}} \times C_{g/A}(Q, b, N, \mu, \mu_F) \exp\left[ E_{gg}^{\text{PT}}(N, b, Q, \mu) \right] C_{g/B}(Q, b, N, \mu, \mu_F).
\]

This is the same general form as for vector boson production, but adapted to the Born cross section for Higgs production in the heavy top mass limit, \( m_h \ll 2m_t \). The Higgs mass is \( m_h \), and \( \tau = Q^2/S \), where \( Q \), which is set equal to \( m_h \) in the approximation associated with threshold and \( Q_T \) resummation, sets the scale of the hard scattering. As is characteristic of joint resummation, the cross section is a double inverse transform, in impact parameter \( b \) and Mellin moment \( N \). It resums logarithms in \( b \) and \( N \) to the level of NLL. Let us first summarize the content of the factorization implicit in this expression.

Each of the factors in Eq. (1) is associated with a specific range of perturbative dynamics involved in the annihilation process. These ranges are set by the physical scales that enter the problem. From soft to hard these are, \( \Lambda_{QCD} \) at the scale of hadronic binding, followed by scales introduced by the transforms: \( Q/N \) and \( 1/b \), followed by \( m_h = Q, m_t \) and, finally \( s \). We introduce a function \( \chi(bQ, N) \), to be specified in section 2.3., that interpolates between \( N \) in the threshold limit, \( N \gg bQ \) and \( bQ \) in the limit of large impact parameter, \( bQ \gg N \), and is chosen in a way that makes it possible to generate logarithms in \( 1/b \) and \( Q/N \) at the level of next-to-leading logarithm in each. At scales lower than \( Q/\chi \), radiation is fully allowed, while beyond these scales, the transforms in \( N \) and \( b \) constrain final-state radiation. The resummations appropriate to these scales are specified in terms of finite-order coefficients at each logarithmic order.

2.2. Coefficients and anomalous dimensions

The shortest distances, with an off-shellness of order \( m_t \), are represented by the perturbative function \( H(\alpha_s) \). It collects the effects of hard virtual corrections and reads to first order [2, 18]:

\[
H(\alpha_s) = 1 + \frac{\alpha_s}{2\pi} \left(2\pi^2 + 11\right).
\]
This is multiplied by the overall normalization constant
\[ \sigma_0^h = \frac{\sqrt{2}G_F \alpha_s^2(m_h)}{576\pi} \] (3)
corresponding to the the heavy top mass limit, \( m_h \ll 2m_t \), with \( G_F \) the Fermi constant.

At the longest scales, the functions \( C \) represent evolution between \( \Lambda_{\text{QCD}} \) and \( Q/\chi(bQ,N) \). They are given by
\[ C_{g/H}(Q,b,N,\mu,\mu_F) = \sum_{j,k} C_{g/j}(N,\alpha_s(Q/\chi)) \mathcal{E}_{jk}(N,Q/\chi,\mu_F) f_{k/H}(N,\mu_F) . \] (4)

Here, the \( f_{j/H}(N,\mu_F) \) are the parton distribution functions for hadron \( H \) at any convenient factorization scale \( \mu_F \). The \( C_{g/j} \) may be determined from the coefficients of the \( \delta^2(\tilde{Q}_T) \) contributions in the order \( \alpha_s \) correction to the Higgs cross section \([12]\), given our choice of \( H^{(1)} \) in Eq. (2). They read
\[ C_{g/g}(N,\alpha_s) = 1 + \frac{\alpha_s}{4\pi} \pi^2, \] (5)
\[ C_{g/q}(N,\alpha_s) = \frac{\alpha_s}{2\pi} C_F \frac{1}{N+1} = C_{g/\bar{q}}(N,\alpha_s) . \] (6)

The matrix \( \mathcal{E}(N,Q/\chi,\mu_F) \) in Eq. (4) represents the evolution of the parton densities from scale \( \mu_F \) to scale \( Q/\chi \) to NLL accuracy in \( \ln \chi \) \([23]\). In effect, the combinations of parton distributions, evolution factors and matching coefficients follow the evolution of the initial-state hadrons from the QCD scale up to the scale at which the transforms suppress radiation.

As indicated in Eq. (4), we adopt the renormalization scale \( Q/\chi \) in the \( C \)-coefficients. This is guided by standard \( Q_T \)-resummation, where it was argued \([12]\) that the scale should be proportional to \( 1/b \). The choice of the scale in the \( C \)-coefficients actually only affects NNLL terms in the resummation.

We note that in the inclusive limit \( b = 0 \) the coefficients \( C_{g/j} \) and \( H \) do not quite reduce to the corresponding factor in threshold resummation. First, there is no analog of the nondiagonal \( C_{g/q} \) and \( C_{g/\bar{q}} \) terms in the pure threshold case, since they are not associated with logarithmic behavior at large \( N \). Second, the scale of \( \alpha_s \) in \( C_{g/j} \) reduces to \( Q/\bar{N} \) rather than \( Q \) for \( b = 0 \). Finally, for the diagonal coefficient in our formulas above the combined effect at \( \mathcal{O}(\alpha_s(Q^2)) \) is \( H \times C_{g/g} \times C_{g/g} \approx 1 + \frac{\alpha_s}{4\pi} (11 + 3\pi^2) \), whereas the corresponding coefficient in the threshold case is \([14]\) \( 1 + \frac{\alpha_s}{2\pi} (11 + 4\pi^2) \). Such differences are present as well in the case of \( Z \) boson production through quark annihilation \([23]\). A development of the joint formalism to NNLL in both the threshold and small-\( Q_T \) limits would account for them.

We recall that organizing the \( C \) functions in the form of Eq. (4) in Mellin-\( N \) moment space has a great advantage: it enables us to explicitly separate the evolution of the parton densities between the scales \( \mu_F \) and \( Q/\chi \), embodied by the matrix \( \mathcal{E} \) \([23]\). In this way, we can avoid the problem normally faced in \( Q_T \) resummation that one needs to call the parton densities at scales far below their range of validity, so that some sort of “freezing” (or related prescription) for handling them is required. Our matrix \( \mathcal{E} \), on the other hand, may be expanded to LL and NLL
accuracy, consistent with all our approximations, as shown in ref. [23]. We then only need the parton distribution functions at the “large” scale $\mu_F \sim Q$, whereas normally in $Q_T$ resummation the product $\sum_k E_{jk}(N, 1/b, \mu_F) f_{k/H}(N, \mu_F)$ is identified with $f_{j/H}(N, 1/b)$. We also recall that the moment variable $N$ and, as will be discussed below, also the impact parameter $b$ are in general complex-valued in our approach, so that it is even more desirable to separate the complex scale $Q/\chi$ from that in the parton densities. In practical applications, one usually has available only parton distributions in $x$-space. This obstacle can be overcome in various ways. We have described one practical approach in Appendix B of [23]. A very simple other method is to fit the parton distributions at scale $\mu_F$ in the $x$-region of relevance by a simple functional form that allows to take Mellin-moments analytically [28]. We have used both methods and found excellent agreement between the corresponding numerical results.

The exponential $E_{gg}^{PT}(N, b, Q, \mu)$ in Eq. (1) is the Sudakov factor for Higgs production in joint resummation. It summarizes QCD dynamics in the transverse momentum range, $Q/\chi \leq k_T \leq Q$, where real radiation is kinematically constrained and virtual corrections produce an exponentiating double-logarithmic suppression. We may think of it as interpolating between the scale of the partonic coefficients $\mathcal{C}$ and the short-distance annihilation process. $E_{gg}^{PT}(N, b, Q, \mu)$ is equivalent, with trivial changes in color factors, to the corresponding exponent in $Z$ production [23], and is given by

$$E_{gg}^{PT}(N, b, Q, \mu) = -\int_{Q^2/\chi^2}^{Q^2} \frac{dk_T^2}{k_T^2} \left[ A_g(\alpha_s(k_T)) \ln \left( \frac{Q^2}{k_T^2} \right) + B_g(\alpha_s(k_T)) \right],$$

accurate to NLL level. Dependence on the renormalization scale $\mu$ is implicit in Eq. (7) through the expansion of $\alpha_s(k_T)$ in powers of $\alpha_s(\mu)$ and logarithms of $k_T/\mu$.

The anomalous dimensions in Eq. (7) are perturbative series in $\alpha_s$, $A_g(\alpha_s) = \sum_{i=0}^{\infty} \left( \frac{\alpha_s}{\pi} \right)^i A_g^{(i)}$, and similarly for $B_g$. By examining the fixed-order cross section at NLO [2] the following familiar values necessary for NLL resummation can be derived [18, 29],

$$A_g^{(1)} = C_A, \quad B_g^{(1)} = \frac{1}{6} \left( 11 C_A - 4 T_R N_F \right),$$

$$A_g^{(2)} = \frac{C_A}{2} \left[ C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{10}{9} T_R N_F \right],$$

where $C_A = 3$, $C_F = 4/3$, $T_R = 1/2$, and $N_F$ is the number of flavors. NLL accuracy requires using $A_g^{(1)}$, $B_g^{(1)}$ and $A_g^{(2)}$ in Eq. (7).

From an NNLO [3] calculation one can also determine a second-order coefficient for the function $B(\alpha_s)$. This coefficient contributes to the cross section at the level of NNLL. Given our choices for $H^{(1)}$ and hence for $C^{(1)}$, and assuming the NLL expression for the Sudakov exponent (7), this coefficient is uniquely determined to be

$$B_g^{(2)} = C_A^2 \left( -\frac{4}{3} + \frac{11}{36} \pi^2 - \frac{3}{2} \zeta_3 \right) + \frac{1}{2} C_F T_R N_F + C_A N_F T_R \left( \frac{2}{3} - \frac{\pi^2}{9} \right).$$
Even though the contributions associated with the coefficients $H^{(1)}$, $C^{(1)}$ and $B_g^{(2)}$ are all only of NNLL order, we will include them in our study. It was found in previous studies on $Q_T$ resummation for Higgs production that the combined contribution of these coefficients is actually numerically rather significant, partly due to the size of $C_A$ relative to $C_F$.

As was shown in ref. [19], there is actually some freedom in how one apportions contributions to $H^{(1)}$, $C^{(1)}$ and $B_g^{(2)}$ individually. A choice for one coefficient uniquely determines the others, but it is possible to shift terms among them, resulting in different but formally equivalent schemes for the resummation. We believe that our choice in Eq. (2) is well motivated physically, since with this choice the function $H$ contains all genuinely hard short-distance effects at scales above $Q$. The function $B_g^{(2)}$, on the other hand, enters with dependence on scales much smaller than $Q$, as can be seen from Eq. (7), so excluding hard contributions from $B_g^{(2)}$ seems natural. We have nevertheless also investigated an alternative choice, for which all terms in $H$ are absorbed into the $C$ functions, so that $H = 1$, with $B_g^{(2)}$ adapted accordingly. This scheme has been more customary in previous studies. We found that our numerical results presented below do not change significantly if this “resummation scheme” [19] is used. We note that for our choice of $H^{(1)}$, unlike for the $H = 1$ scheme, the corresponding numerical value for $B_g^{(2)}$ is very small. A more detailed discussion of the interrelations between $H^{(1)}$, $C^{(1)}$ and $B_g^{(2)}$ may be found in [19].

There are other NNLL contributions in the exponent, besides those associated with $H^{(1)}$, $C^{(1)}$ and $B_g^{(2)}$. The third-order coefficient $A_g^{(3)}$ also contributes at NNLL. Its value is so far known numerically [30], with its contributions proportional to $N_F^2$ and $N_F$ determined analytically [31, 32, 33]. We have checked that when we include $A_g^{(3)}$ at the level of its currently known approximation the impact on our numerical results is insignificant.

We also note that in the context of joint resummation the form of Eq. (7) for the Sudakov exponent is itself only valid to NLL. The original jointly resummed Sudakov exponent given in [22] is, in fact, far more general than NNL and will correctly reproduce logarithms of any order for threshold and $Q_T$ resummation in the limits of large $N$ and $b$, respectively. For large $b$ and fixed $N$, the structure in (7) is correct to all logarithmic order, as is known from standard $Q_T$ resummation, where $\chi \sim b$. In the opposite (threshold) limit of $N$ large and $b$ finite, we should in general introduce an additional function that reflects coherent soft radiation at wide angles, called $g_3$ in [10] and $D$ in [33, 34]. For the purposes of this study, we follow standard $Q_T$ resummation as closely as possible and therefore use the form (7) also at NNLL. We emphasize at this point that a full investigation of NNLL effects in the joint resummation formalism would be very desirable. Our taking into account of $H^{(1)}$, $C^{(1)}$ and $B_g^{(2)}$ can therefore be only an approximation for the full NNLL effects. We defer a full NNLL analysis to a future publication.

In the next subsection we discuss the Sudakov exponent in more detail.
2.3. The Sudakov exponent and the choice of $\chi$

The exponent $E_{gg}^{PT}(N, b, Q, \mu)$ in Eq. (7) isolates the Sudakov suppression in the logarithm of the full NLL eikonal cross section in $N$- and $b$-space [22],

$$E_{gg}^{(eik)}(N, b, Q, \mu_F) = 2 \int_0^{Q^2} \frac{dk_T^2}{k_T^2} \left\{ A_g(\alpha_s(k_T)) \left[ J_0(bk_T) K_0\left(\frac{2Nk_T}{Q}\right) + \ln\left(\frac{\bar{N}k_T}{Q}\right) \right] \right\}$$

$$-2 \ln \bar{N} \int_0^{Q^2} \frac{dk_T^2}{k_T^2} A_g(\alpha_s(k_T)),$$

where here and below, we denote

$$\bar{N} = Ne^{\gamma_E},$$

with $\gamma_E$ the Euler constant. As in Ref. [23], we rewrite Eq. (10) in a form that is accurate to next-to-leading logarithm in both $b$ and $N$,

$$E_{gg}^{(eik)}(N, b, Q, \mu, \mu_F) = -\int_{Q^2/\chi^2}^{Q^2} \frac{dk_T^2}{k_T^2} \left[ A_g(\alpha_s(k_T)) \ln\left(\frac{Q^2}{k_T^2}\right) + B_g(\alpha_s(k_T)) \right]$$

$$+ \int_{\mu_F^2}^{Q^2/\chi^2} \frac{dk_T^2}{k_T^2} \left[ -2A_g(\alpha_s(k_T)) \ln\left(\bar{N}\right) - B_g(\alpha_s(k_T)) \right].$$

Here we have added and subtracted a term with $B_g$, the term constant in $N$ in the DGLAP splitting functions. To NLL, $B_g$ coincides with the function defined in the previous section. We have also neglected logarithms of the factorization scale divided by $Q$. The Sudakov exponent is the first term in this expression. We will reinterpret the second term as the large-$N$ limit of the evolution of parton distributions in the next section.

The function $\chi(bQ, N)$ in Eq. (10) organizes logarithms of $N$ and $b$ in joint resummation [23] and is given by

$$\chi(bQ, N) = \bar{b} + \frac{\bar{N}}{1 + \eta \frac{b}{N}},$$

where we define

$$\bar{b} \equiv bQe^{\gamma_E}/2.$$

This form of $\chi(N, b)$ is chosen to reproduce threshold logarithms at NLL for large $N$ at fixed $b$, and impact parameter logarithms at large $b$ with $N$ fixed. The functional form ensures that corrections at low $Q_T$ (hence, at large $b$) avoid unphysical singularities suppressed by $1/Q_T$ compared to leading terms: as discussed in [23], any nonzero value for the parameter $\eta$ in Eq. (13) eliminates corrections of order $b^{-1}$ in the large-$b$ limit, relative to leading behavior. It therefore does not introduce spurious logarithmic singularities in $Q_T d\sigma/dQ^2 dQ_T^2$ as $Q_T \to 0$. To NLL, the cross section is independent of $\eta$, as is the $Q_T$-integrated cross section, found by setting $b$ to zero. The shape of the cross section depends to some extent on the precise value of $\eta$ that
we choose, through subleading terms. For very large values of \( \eta \), \( \chi \) approaches its value in “pure recoil” \( b \)-space resummation, \( \chi = \bar{b} \), although the integrated cross section is then ill-defined.

In the following, we choose the value of \( \eta = 1 \). We motivate this choice by comparing the expansions of the logarithm of \( \chi \) in terms of \( \ln \bar{b} \) at fixed \( \bar{N} \), and of \( \ln \bar{N} \) at fixed \( \bar{b} \). For \( \eta = 1 \) these expansions give rise to “power-like” corrections of the form \( (\bar{N}/\bar{b})^2 \) in the former case, and \( (\bar{b}/\bar{N})^2 \) in the latter, with no linear \( \bar{b}/\bar{N} \) corrections. We readily verify that the full NLL exponent (10) also has no corrections linear in \( \bar{b}/\bar{N} \) for \( \bar{b}/\bar{N} \to 0 \). The choice \( \eta = 1 \) treats the remaining two sets of corrections, \( (\bar{N}/\bar{b})^2 \) and \( (\bar{b}/\bar{N})^2 \), symmetrically. To estimate the sensitivity to \( \eta \), we will also exhibit results for \( \eta = 1/2 \) and \( \eta = 2 \) below. In Ref. [23], on the other hand, where we used the framework of joint resummation to analyse \( Z \) production at the Tevatron, a choice of \( \eta = 1/4 \) was made. It led to a relatively simple structure of singularities in the complex \( b \) plane. We checked that the \( \eta \) dependence of the \( Q_T \) distribution for \( Z \) bosons was insignificant [23]. This is, however, not necessarily the case for Higgs production at the LHC. The treatment of the inverse transforms (i.e. contours in the complex \( N \) and \( b \) spaces) for differing values of \( \eta \) is discussed in section 3.

The expansion of \( E_{gg}^{\text{PT}} \) into leading, next-to-leading, and next-to-next-to-leading logarithms gives the form [22, 23]

\[
E_{gg,\text{NNLL}}(N, b, Q, \mu) = \frac{2}{\alpha_s(\mu)} h_g^{(0)}(\beta) + 2 h_g^{(1)}(\beta, Q, \mu) + 2 \alpha_s(\mu) h_g^{(2)}(\beta, Q, \mu) ,
\]

where each function \( h_g^{(i)} \) is a power series in

\[
\beta = b_0 \alpha_s(\mu) \ln (\chi) ,
\]

with

\[
b_0 = \frac{11C_A - 4T_R N_F}{12\pi} .
\]

The leading- and next-to-leading logarithmic functions \( h_g^{(0)} \) and \( h_g^{(1)} \) have already been presented in [22, 23]. For convenience, we provide them again in Appendix A. As stated above, we do not attempt a full NNLL study of the resummed cross section, but we do wish to include those NNLL terms that are known to produce numerically significant effects. At the level of the Sudakov exponent, this term is given by the contribution involving \( B_g^{(2)} \) and gives rise to the function \( h_g^{(2)} \):

\[
h_g^{(2)}(\beta, Q, \mu) = -\frac{B_g^{(2)}}{\pi^2 b_0} \frac{\beta}{1 - 2\beta} + \ldots ,
\]

where the ellipses denote further, neglected, terms proportional to \( A_g^{(1)}, B_g^{(1)}, A_g^{(2)} \), as well as to the third-order coefficient \( A_g^{(3)} \). As mentioned above, we have checked that those terms among the neglected terms that are fully known lead to numerically insignificant effects.
2.4. Relation to threshold resummation

In the resummed cross section, Eq. (1), the partonic functions $C_{g/H}(Q, b, N, \mu, \mu_F)$, Eq. (4), incorporate the parton distribution functions $f_{j/H}(N, \mu_F)$, the evolution matrices $E_{jk}(N, Q/\chi, \mu_F)$, and the standard $C$ coefficients coming from matching the fixed order to the resummed result.

Within a strict threshold approximation, only parton-diagonal contributions to evolution are taken into account, which for Higgs production means the sole use of the large-$N$ limit of the $gg$ anomalous dimension. The matrix $E(N, Q/\chi, \mu_F)$ then only has an entry in the $gg$ position and reads

$$E(N, Q/\chi, \mu_F) = \exp \left[ -\frac{2A_g^{(1)} \ln \bar{N} - B_g^{(1)}}{2\pi b_0} s(\beta) \right] \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},$$

where \[23\]

$$s(\beta) \equiv \ln \left( \frac{\alpha_s(\mu_F)}{\alpha_s(Q/\chi)} \right) = \ln(1 - 2\beta) + \text{NLL}.$$  \hspace{1cm} (20)

In Eq. (19) we have for the sake of brevity only written down the LO/LL part of the evolution; extension to NLO/NLL is straightforward and included in our analysis. The $E$ in Eq. (19) may then be combined with the Sudakov exponent in (15) at $\beta = b_0 \alpha_s \ln \bar{N}$ (i.e., $b = 0$) to reproduce the familiar exponent of Higgs threshold resummation [6, 13, 14].

However, as we discussed in [23], one can improve the above near-threshold approximation and use the full NLO singlet evolution matrix in $E$, with full $N$-dependence for the anomalous dimensions. This amounts to the replacement

$$-2A_g(\alpha_s) \ln \left( \bar{N} \right) - B_g(\alpha_s) \longrightarrow \gamma_N(\alpha_s),$$

or equivalently, in terms of parton distribution functions,

$$\delta_{gg} \exp \left[ -\frac{2A_g^{(1)} \ln \bar{N} - B_g^{(1)}}{2\pi b_0} s(\beta) \right] f_{g/H}(N, \mu_F) \longrightarrow E_{jk}(N, Q/\chi, \mu_F) f_{k/H}(N, \mu_F).$$

In this way, $E$, and hence the functions $C$ in Eq. (1), obtains a full matrix structure and becomes an ordered exponential. On the other hand, the standard threshold resummation formula is recovered from Eq. (1) by integrating over $\vec{Q}_T$ to set $b = 0$, and then undoing the replacements of Eqs. (21) and (22).

One of the assets of the joint resummation of Eq. (1), incorporating (22), is to resum collinear, but non-soft, terms of the form $\alpha_s^n \ln^{2n-1}(\chi)/N$ to all orders [23]. Such terms are obviously subleading with respect to the threshold approximation, but may have some relevance for phenomenology. They were discussed in [6, 13, 14] in the context of threshold resummation, where they occur as $\alpha_s^n \ln^{2n-1}(\bar{N})/N$. In this formalism they are brought under control automatically. In our analysis of Higgs production, we actually find that other formally subleading terms can also be rather important. We will discuss this issue in section 5 below.
3. Inverse transforms and matching

We have already mentioned in the introduction the importance of suitably defining the inverse Mellin and Fourier transforms. We follow here closely our previous phenomenological study for electroweak annihilation [23], where all details may be found. Let us briefly recall the main ingredients.

The contour for the inverse Mellin transform is chosen according to the “minimal prescription” of [25]. It is parameterized as

\[ N = C + z e^{\pm i \phi}, \tag{23} \]

where the upper (lower) sign applies to the upper (lower) branch of the contour, with \( 0 \leq z \leq \infty \) (\( \infty \geq z \geq 0 \)). The constant \( C \) is required to lie to the right of the rightmost singularity of the parton distribution functions. At any finite order in perturbation theory, all values of \( C > 0 \), and all angles \( \pi > \phi > \pi/2 \) are equivalent. In the resummed cross section, however, the singularity at \( \chi = \rho_L = \exp[1/2b_0 \alpha_s(\mu)] \) introduces an ambiguity in the transform, which may be resolved by choosing \( C < \rho_L \) [25].

The inverse Fourier integral is also defined as a contour integral [23, 26]. It is performed using the identity

\[ \int d^2b \ e^{i \vec{q} \cdot \vec{b}} f(b) = 2\pi \int_0^\infty db \ J_0(bq) f(b) = \pi \int_0^\infty db \ [h_1(bq,v) + h_2(bq,v)] f(b), \tag{24} \]

and employing Cauchy’s theorem to deform the integration over real \( b \) into a contour in the complex \( b \) plane [26, 23], as shown in Fig. 1. Here the auxiliary functions \( h_{1,2} \) [26] are related to Hankel functions. They distinguish between the positive and negative phases in Eq. (24).

The \( b \) integral can thus be written as a sum of two contour integrals, of the integrand with \( h_1 \) (\( h_2 \)) along a contour in the upper (lower) half of the \( b \) plane. The precise form of the contours becomes unimportant as long as the contours do not run into the Landau pole or singularities associated with the particular form (13) of the function \( \chi \). One also needs to make sure that the lines of singularities do not cross, which would make it impossible to draw a fixed contour without running into a line. This can be ensured by choosing the opening angle of the \( N \) contour to fulfill the condition \( \pi/2 < \phi < \pi - \tan^{-1} \left( \sqrt{|4\eta - 1|} \right) \).

This choice of contours in complex transform space is completely equivalent to the original form, Eq. (24) when the exponent is evaluated to finite order in perturbation theory. In the presence of the Landau pole arising in the resummed formula, it is a natural extension of the \( N \)-space contour redefinition above [25], using a generalized “minimal” exponent (15). We emphasize that joint resummation with its contour integration method provides an alternative to the standard \( b \) space resummation. Joint resummation has built-in a perturbative treatment of large \( b \) values, eliminating the need for a \( b_* \) [12] prescription for the exponent, or for a freezing of the scale of parton distributions at large \( b \) or low \( Q_T \). In this way, we can derive entirely perturbative cross sections for Higgs production. We note that this definition of the \( b \)-space contour was also adopted in the recent study [17], while in [27] an extrapolation of the perturbative exponent to large \( b \) is used to define the inverse transform.
At large values $Q_T \sim m_h$, we need to match the resummed cross section to fixed-order perturbation theory. This is achieved in the following simple way [17, 23]:

$$\frac{d\sigma}{dQ^2dQ_T^2} = \frac{d\sigma^{\text{res}}}{dQ^2dQ_T^2} \frac{d\sigma^{\text{exp}(k)}}{dQ^2dQ_T^2} + \frac{d\sigma^{\text{fixed}(k)}}{dQ^2dQ_T^2},$$

where $d\sigma^{\text{res}}/dQ^2dQ_T^2$ is given in Eq. (1) and $d\sigma^{\text{exp}(k)}/dQ^2dQ_T^2$ denotes the terms resulting from the expansion of the resummed expression in powers of $\alpha_s(\mu)$ up to the order $k$ at which the fixed-order cross section $d\sigma^{\text{fixed}(k)}/dQ^2dQ_T^2$ is taken. The last two terms on the right-hand-side of Eq. (25) should cancel each other at small $Q_T$, since order by order the terms generated by the resummed formula must reproduce the singular behavior of the cross section at small $Q_T$. In our study, we do the matching using $k = 1$ (see [7]) in Eq. (25).

4. Predictions for the resummed Higgs cross section

For our calculations of the Higgs transverse momentum distribution at the LHC, obtained in the framework of joint resummation, we choose $m_h = 125$ GeV and the factorization and renormalization scales $\mu = \mu_F = Q = m_h$. We use the CTEQ5M [35] parton distribution function parameterizations. The contour parameters are $\phi = 17/32\pi$ and $\phi_b = \phi + \tan^{-1}\sqrt{|4\eta - 1|}$, with $\eta = 1$.

Fig. 2 shows our result for $d\sigma/dQ_T$ as a function of $Q_T$. The solid curve shows the result obtained for the formulas given in Sec. 2. For comparison, we also display the result for “pure-$Q_T$” resummation, i.e., for the case when we do not take into account threshold resummation effects, $\chi = \bar{b}$ (dashed line). We find that the corresponding result is somewhat higher and broader than our jointly resummed curve. We can compare this curve to previous analyses in
the literature [9, 15, 16]. It turns out that our “pure-$Q_T$” resummed result is about 30% higher (in the peak of the distribution) than the one of the RESBOS [15] code, and about 20% higher than the result of [16]. On the other hand, our distribution is lower by about 15% than the NLL result of [17], matched with the NLO predictions. These differences appear to be due in part to different treatments of the large-$b$ region in the various formalisms, and to different organizations of the coefficients.

![Transverse momentum distribution for Higgs production at the LHC in the framework of joint resummation and of “pure-$Q_T$” resummation.](image)

Figure 2: Transverse momentum distribution for Higgs production at the LHC in the framework of joint resummation and of “pure-$Q_T$” resummation.

The jointly resummed distribution in Fig. 2 was obtained after adding a small non-perturbative term of the form $-g b^2$ to the exponent. As discussed in [23], it is an advantage of joint resummation that no non-perturbative contribution is required in order to obtain results for any nonzero $Q_T$. The reason for introducing a non-perturbative component here is purely technical. Our choice of $\eta = 1$ necessitates small values of angle $\phi$, making it time consuming to achieve numerical stability at low $Q_T$. The value of the parameter $g$ was adopted from the study [36], where an analysis of Upsilon hadroproduction was performed as a means of studying the non-perturbative contributions in processes with two gluons in the initial state. We use the value $g = 1.67$ GeV$^2$ corresponding to the lower end of a range of values extracted from data. In any case, on the basis of previous studies of the dependence of $b$-space resummation for Higgs production on the non-perturbative contribution [16, 36, 37], we expect the dependence on the parameter $g$ to be weak for values of $Q_T$ around or above the peak of the cross section. In effect, a nonzero value
of $g$ simply enables us to extrapolate the cross section to $Q_T = 0$.

In section 2 we introduced the parameter $\eta$ in the definition of the function $\chi$, Eq. (13), and as discussed above chose $\eta = 1$. To investigate the dependence of the results on the value of this parameter, we computed the $Q_T$ distributions for $\eta = 1/2$ and $\eta = 2$. As can be seen from Fig. 3 the numerical sensitivity of the results to the value of $\eta$ is rather small in this range.

Figure 3: Dependence of the jointly resummed transverse momentum distribution for Higgs production at the LHC on the value of the parameter $\eta$.

5. Joint resummation and the role of non-threshold effects

In a sense, Fig. 2 completes our study, since it predicts a cross section that remains to be measured. Nevertheless, a number of cross checks in joint resummation suggest that Fig. 2 is not the final word, and that to get a full picture of the Higgs cross section in $Q_T$ it may be desirable to extend the formalism to include the contributions that give rise to dominant small-$x$ behavior.

In this section, we compare the jointly resummed cross section in more detail to existing fixed-order and threshold-resummed calculations.
5.1. **NLO expansion in $Q_T$ space**

Fig. 4 shows the difference $d\sigma_{\text{fixed}(1)}/dQ_T - d\sigma_{\text{exp}(1)}/dQ_T$ and the fractional deviation

$$\Delta \equiv \left[ \frac{d\sigma_{\text{fixed}(1)}}{dQ_T} - \frac{d\sigma_{\text{exp}(1)}}{dQ_T} \right] / \frac{d\sigma_{\text{fixed}(1)}}{dQ_T},$$

(26)

using $m_h = 125$ GeV, scales $\mu = \mu_F = Q = m_h$, and the CTEQ5M [35] parton distribution functions. We find very good agreement in the region $Q_T < 10$ GeV where resummation is necessary. Beyond 10 GeV, the agreement is naturally less exact but still reasonable. This is the region where matching to finite order is appropriate, as expressed by Eq. (25). From the figure we see that the ‘recoil’ approximation, $\chi \rightarrow \bar{b}$, is slightly more accurate at low $Q_T$, while the joint form remains stable at larger values of transverse momentum. The somewhat lower value of the expanded joint cross section is due in part to the lower scale at which the gluon distribution is evaluated compared to pure $Q_T$-resummation ($Q/\chi$ as opposed to $1/b$). At larger values, the stability of the jointly resummed cross section makes it easier to match to fixed order (a property shared with the $Q_T$-resummed treatment in Ref. [17]).

![Fractional deviation $\Delta$](image)

**Figure 4**: Fractional deviation $\Delta$ (as defined in Eq. (26)) between the “exact” $\mathcal{O}(\alpha_s)$ result and the $\mathcal{O}(\alpha_s)$ expansion of the jointly and the pure-$Q_T$ resummed cross sections.

5.2. **Total cross section and subleading terms in $N$ space**

When integrated over $Q_T$, the jointly resummed distribution formally reduces to the threshold resummed cross section in the large-$N$ limit. When we numerically compare the two formalisms,
however, we find that the total cross section in joint resummation is around ten percent lower than pure threshold resummation at NLL: after matching to NLO we find the NLL threshold-resummed cross section to be 39.4 pb (consistent with results shown in [14]), whereas our jointly resummed cross section integrates to 35.0 pb. This slight disagreement implies that terms that are subleading with regard to the threshold approximation play a fairly important role. Since our formalism has been designed to automatically reproduce all behavior singular at $Q_T \to 0$, we are led to associate the “missing area” with contributions to the Higgs cross section at rather high $Q_T$.

It is instructive to investigate these points in more detail. To this end, we expand the jointly resummed, and $Q_T$-integrated, cross section to fixed order $\alpha_s$ (not counting the overall $\alpha_s^2$ of the cross section), and compare it to the “exact” expressions for the Higgs production cross section at this order. For simplicity, we set the renormalization and factorization scales to $m_h$. Expansion of Eq. (1) to $\mathcal{O}(\alpha_s)$ gives for the partonic cross sections in the $gg$ and $gq$ scattering channels:

$$
\hat{\sigma}^{gg} = \sigma_g^{(0)} \frac{\alpha_s}{2\pi} \left\{ -4C_A \ln^2 \bar{N} + 8\pi b_0 \ln \bar{N} + 11 + 3\pi^2 \right. \\
-2 \ln \bar{N} \left[ \frac{4C_A}{N(N-1)} + \frac{4C_A}{(N+1)(N+2)} - 4C_A S_1(N) + 4\pi b_0 \right] \right\} ,
$$
$$
\hat{\sigma}^{gq} = \sigma_g^{(0)} \frac{\alpha_s}{2\pi} C_F \left\{ -2 \ln \bar{N} \frac{N^2 + N + 2}{N(N+1)(N-1)} + \frac{1}{N+1} \right\} ,
$$

(27)

where $S_1(N) = \sum_{j=1}^{N} j^{-1} = \psi(N+1) + \gamma_E$, with $\psi$ the digamma function. Since we are interested in the near-threshold region, we can expand this further to the large-$N$ limit. Using $\psi(N+1) = \ln N + 1/(2N) + \mathcal{O}(1/N^2)$, we find:

$$
\hat{\sigma}^{gg} = \sigma_g^{(0)} \frac{\alpha_s}{2\pi} \left\{ 4C_A \ln^2 \bar{N} + 4C_A \frac{\ln \bar{N}}{N} + 11 + 3\pi^2 \right\} + \mathcal{O} \left( \frac{\ln \bar{N}}{N^2} \right) ,
$$
$$
\hat{\sigma}^{gq} = \sigma_g^{(0)} \frac{\alpha_s}{2\pi} C_F \frac{1}{N} \left[ -2 \ln \bar{N} + 1 \right] + \mathcal{O} \left( \frac{\ln \bar{N}}{N^2} \right) .
$$

(28)

The full (“exact”) NLO cross total section has been calculated in refs. [2]. When taking Mellin-$N$ moments of these results, we obtain

$$
\hat{\sigma}^{gg}_{\text{exact}} = \sigma_g^{(0)} \frac{\alpha_s}{2\pi} \left\{ 4C_A S_1^2(N) - 8C_A \left[ \frac{1}{N(N-1)} + \frac{1}{(N+1)(N+2)} \right] S_1(N) + 11 + 4\pi^2 \\
+ \frac{3C_A}{N(N+1)} + \frac{13C_A}{(N-1)(N+2)} + 4C_A \left[ \frac{1}{(N-1)^2} - \frac{1}{N^2} - \frac{1}{(N+1)^2} + \frac{1}{(N+2)^2} \right] \right\} ,
$$
$$
\hat{\sigma}^{gq}_{\text{exact}} = \sigma_g^{(0)} \frac{\alpha_s}{2\pi} C_F \left\{ -2S_1(N) \frac{N^2 + N + 2}{N(N+1)(N-1)} + \frac{2}{(N-1)^2} - \frac{2}{N^2} - \frac{1}{(N+1)^2} \\
+ \frac{5}{2(N-1)} - \frac{1}{N} - \frac{1}{2(N+1)} \right\} .
$$

(29)
At large $N$, this gives

$$\hat{g}^g = \frac{\sigma_g^{(0)} }{2\pi} \frac{\alpha_s}{\hat{g}^g} \left\{ 4C_A \ln^2 \hat{N} + 4C_A \frac{\ln \hat{N}}{\hat{N}} + 11 + 4\pi^2 \right\} + \mathcal{O} \left( \frac{\ln \hat{N}}{\hat{N}^2} \right),$$

$$\hat{g}^q = \frac{\sigma_g^{(0)} }{2\pi} \frac{\alpha_s}{\hat{g}^q} \frac{C_F}{\hat{N}} \left[ -2 \ln \hat{N} + 1 \right] + \mathcal{O} \left( \frac{\ln \hat{N}}{\hat{N}^2} \right).$$

(30)

Comparing Eqs. (28) and (30), we see that they agree even to the subleading order $\mathcal{O}(1/N)$, except for the contribution proportional to $\pi^2$ discussed in section 2.2. As noted there, had we fully developed the joint formalism to NNLL for both the threshold and the small-$Q_T$ limits, it would automatically account for this difference in the $C$ coefficients. At the level of our rudimentary inclusion of NNLL effects at large $b$, we need to accept the mismatch in the $\pi^2$ term.

In addition, there are differences between the $\mathcal{O}(\alpha_s)$ expansion of the resummed cross section, Eq. (27), and the exact $\mathcal{O}(\alpha_s)$, Eq. (29), that are formally even more suppressed than the above $\pi^2$ term, or even the $\mathcal{O}(\ln \hat{N}/\hat{N})$ terms mentioned earlier, but that still have a fairly large impact on the results. These correspond to the terms $\propto 1/(N - 1)^k$ ($k = 1, 2$) in these equations. For instance, there is a term $-8C_A \ln \hat{N}/N(N - 1)$ in Eq. (27) that arises from our treatment of evolution. In Eq. (29), on the other hand, we find the combination $-8C_A S_1(N)/N(N - 1) + 13C_A/(N - 1)(N + 2) + 4C_A/(N - 1)^2$. Numerically, the former term leads to a sizeable negative contribution in Eq. (27), while the latter terms cancel each other to a large extent in Eq. (29). This is the main reason why our jointly resummed cross section integrates to a smaller area than one obtains for the standard threshold-resummed cross section on the basis of Eq. (30), where of course no $\propto 1/(N - 1)^k$ contributions are present since they are suppressed from the threshold point of view.

The relevance of the $1/(N - 1)^k$ contributions results from the fact that these terms actually constitute the most singular behavior in the limit opposite to partonic threshold, $z = Q^2/\hat{s} \to 0$. For Higgs masses of a few hundred GeV and LHC’s center-of-mass energy one is actually quite far from threshold. We note that at higher orders in $\alpha_s$ the leading terms singular at $N = 1$ generalize to $\alpha^k_s/(N - 1)^{2k}$, corresponding to $\alpha^k_s \ln^{2k-1}(z)/z$ in $z$ space. These terms, too, can be resummed to all orders in $\alpha_s$, and the resummation was achieved in Ref. [38]. To what extent it would be possible to incorporate this small-$z$ resummation into our joint formalism is an interesting question that, however, is beyond the scope of this paper. That the leading and subleading terms $\propto 1/(N - 1)^k$ ($k = 1, 2$) almost entirely cancel numerically in the full NLO inclusive cross section, Eq. (29), is rather remarkable, albeit probably not accidental. It is only because of this feature that the near-threshold approximation for the NLO cross section, Eq. (30), becomes a good approximation.

Assuming that threshold resummation indeed does yield a reliable answer for the integrated total cross section, it becomes then an interesting question what the implications for the $Q_T$-dependence of the Higgs cross section will be, if it is derived in such a way that it the area underneath the distribution integrates to the threshold-resummed cross section. A possible, but clearly not unique, way of making up for the “missing” area is to redefine the $C$ coefficients of
Eq. (5) in the following way:

\[
C_{g/g} (N, \alpha_s) = 1 + \frac{\alpha_s}{4\pi} \left\{ \frac{\pi^2}{2} + \frac{1}{1 + b^2} \left[ \frac{13C_A}{(N - 1)(N + 2)} + \frac{4C_A}{(N - 1)^2} \right] \right\},
\]

\[
C_{g/q} (N, \alpha_s) = \frac{\alpha_s}{2\pi} C_F \left\{ \frac{1}{N + 1} + \frac{1}{1 + b^2} \frac{2}{(N - 1)^2} \right\}. \tag{31}
\]

Adding terms of this form to the \( C \) coefficients is completely consistent with resummation for both the threshold and \( Q_T \) resummation cases. The term \( 1/(1+b^2) \) is designed so that it becomes unity in the \( Q_T \)-integrated cross section and does not produce behavior singular in \( Q_T \). We do not claim that the coefficients in (31) are necessarily more realistic or better motivated than the standard ones of Eq. (5); we rather regard them as one model for the effects associated with the \( \propto 1/(N - 1)^k \) terms on the Higgs \( Q_T \) distribution, for which the \( Q_T \)-integrated cross section becomes numerically equivalent to a NLL threshold-resummed cross section.

In Fig. 5 we show the effect of the modified \( C \) coefficients on the Higgs \( Q_T \)-distribution. As expected, there is no difference between the \( Q_T \) distributions with the \( C \) coefficients in the forms (5) and (31) in the region where the distribution peaks; however, the new coefficients cause the \( Q_T \) distribution to be higher at intermediate / large values of \( Q_T \). The value of the total cross section is now about 41 pb.

6. Conclusions

In this paper we have presented a study of the transverse momentum distribution of Higgs bosons in the context of the joint resummation formalism. Guided by the literature on the more standard \( Q_T \) resummation, we have further developed the joint resummation formalism to make it suitable for application to the Higgs production cross section. This included the implementation of some NNLL terms that have a significant numerical effect on the Higgs \( Q_T \) distribution.

We have found in our study that effects associated with threshold resummation have a modest importance for the Higgs cross section at small to moderate \( Q_T \). For this region, where the distribution peaks, our results further confirm the applicability of standard \( Q_T \) resummation.

On the other hand, threshold resummation is very relevant for the \( Q_T \)-integrated cross section, as is known from previous studies in the literature. For joint resummation, the integrated cross section is formally identical to the threshold resummed one in the large-\( N \) limit, but contains non-leading contributions that go beyond standard threshold resummation and which, as we found, decrease the cross section. They are associated with small-\( x \) terms from parton evolution, in a sense of opposite origin to the threshold logarithms. We have argued that their presence indicates that the Higgs \( Q_T \) distribution at rather large \( Q_T \) receives sizeable contributions also from sources not related to Sudakov logarithms. Our study also suggests that it will be useful to extend joint resummation to NNLL. Such an extension will help clarify technical issues, such as the choice of scales in coefficients and parton distributions, which have a modest but significant effect on the magnitude of the cross section even near the peak.
Figure 5: Effect of introducing extra terms in the $C$ coefficients, as shown in Eq. (31) and discussed in the text, on the transverse momentum distribution for Higgs production at the LHC in the framework of joint resummation.

Acknowledgments

We are grateful to D. de Florian and F. Hautmann for valuable discussions. The work of G.S. was supported in part by the National Science Foundation, grants PHY9722101 and PHY0098527. W.V. is grateful to RIKEN, Brookhaven National Laboratory and the U.S. Department of Energy (contract number DE-AC02-98CH10886) for providing the facilities essential for the completion of this work. A.K. thanks the U.S. Department of Energy (contract number DE-AC02-98CH10886) for financial support.

A Leading- and next-to-leading logarithmic expansion

Here we give the functions $h_g^{(0)}$ and $h_g^{(1)}$ that determine the jointly resummed exponent (15) to leading and next-to-leading logarithmic accuracy [22, 23]:

$$h_g^{(0)}(\beta) = \frac{A_g^{(1)}}{2\pi b_0^2} [2\beta + \ln(1-2\beta)] ,$$

(32)
$$h_g^{(1)}(\beta, Q, \mu) = \frac{A_g^{(1)}b_1}{2\pi b_0} \left[ \frac{1}{2} \ln^2(1 - 2\beta) + \frac{2\beta + \ln(1 - 2\beta)}{1 - 2\beta} \right] + \frac{B_g^{(1)}}{2\pi b_0} \ln(1 - 2\beta) + \frac{1}{2\pi b_0} \left[ A_g^{(1)} \ln \left( \frac{Q^2}{\mu^2} \right) - A_g^{(2)} \right] \left[ \frac{2\beta}{1 - 2\beta} + \ln(1 - 2\beta) \right],$$

(33)

with the coefficients $A_g^{(i)}$, $B_g^{(i)}$ from Eq. (8), $\beta$ from Eq. (16), and $b_0$ as in Eq. (17). In addition,

$$b_1 = \frac{17C_A^2 - 10C_AT_RN_F - 6C_FT_RN_F}{24\pi^2}. \quad (34)$$

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