Intelligent vehicle lane change trajectory control algorithm based on weight coefficient adaptive adjustment

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Abstract
In order to improve the trajectory smoothness and the accuracy of lane change control, an adaptive control algorithm based on weight coefficient was proposed. According to lane change trajectory constraint conditions, the sixth-order polynomial lane change trajectory applied to intelligent vehicles was constructed. Based on the vehicle model and the model predictive control theory, the time-varying linear variable path vehicle predictive model was derived by combining soft constraint of the side slip angle. Combined with fuzzy control algorithm, the weight coefficient of the deviation of the lateral displacement was dynamically adjusted. Finally, the FMPC (model predictive controller based on fuzzy control) and MPC controller were compared and analyzed by co-simulation of CarSim and Simulink under different speeds. The simulation results show that the designed FMPC controller can track the lane change trajectory better, and the controller has better robustness when the vehicle changes lanes at different speeds.

Keywords
Intelligent vehicles, vehicle model, lane change trajectory, predictive model, fuzzy control, weight coefficient

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Introduction
Intelligent vehicle is a new generation integrated system integrating Internet, communication, intelligent control, environment awareness and path planning.¹–⁴ The application of intelligent driving technology greatly improves the active safety, ride comfort and handling stability of automobiles.⁵ At present, it has become the focus of research in various countries and an important part of intelligent transportation system. Active vehicle lane change is one of the common behaviors in daily driving. A safe and effective lane change trajectory and a smooth and comfortable lane change process have become an important indicator for evaluating the ability of intelligent driving vehicles to actively change lanes.⁶

At present, path planning algorithms can be divided into dynamic search algorithms and geometric model algorithms.⁷–¹¹ Path planning based on dynamic search originated from robotics. Through the analysis of the surrounding environment and the constraints of conditions, the optimal target path was programmed
The classical dynamic search algorithm includes Dijkstra algorithm, A* algorithm, RRT algorithm, V-graph algorithm and so on. Although these algorithms have good planning and solving speed, they are easy to fall into the local optimal solution, and the path planning effect need to be improved. Neural network algorithm, genetic algorithm, ant colony algorithm, etc., need a lot of iterative operations, and the current vehicle hardware conditions are difficult to meet the requirements. Path planning based on geometric model includes cosine trajectory, circular trajectory, isokinetic trajectory, trapezoidal acceleration trajectory, polynomial trajectory. These trajectory models are intuitive, accurate, and have a small amount of calculation. Therefore, they are widely used. Among them, Nelson proposed a fifth-order polynomial lane change trajectory model to ensure the real-time performance and smoothness of lane change trajectory through multiple constraints. However, the curvature change rate of the trajectory model at the end of lane change may not be zero, which may lead to lateral instability. On the basis of Nelson, Taehyun Shim added a sixth-order variable in consideration of the minimum travel distance and collision avoidance conditions, but the process of determining the highest-order coefficients of its lane change trajectory model is more complicated and the amount of calculation is large. Therefore, in order to improve the calculation efficiency and the handling stability of the vehicle during lane change, a sixth-order polynomial lane change trajectory is proposed on the basis of considering the constraints of the curvature change rate at the end of the lane change.

The purpose of trajectory tracking is to allow the vehicle to follow the planned path. The control method is to obtain the corresponding control parameters through the constraints of vehicle kinematics and dynamics. Commonly used control algorithms include PID control algorithm, fuzzy control algorithm, sliding mode control algorithm, LQR control algorithm. These algorithms respond slowly to changes in the vehicle's driving environment, so the accuracy of trajectory tracking is affected to some extent. The model predictive control algorithm can predict the vehicle’s motion state within a certain period of time according to the vehicle’s current motion state. Moreover, multiple targets can be constrained at the same time, so that the optimal control parameters can be solved within a limited time period. The weight coefficient of the objective function of the traditional model predictive control algorithm is usually taken as a fixed value, but in actual use, the control system needs to adapt to different working conditions to improve the accuracy of control tracking. Therefore, in this paper, based on the linearized time-varying predictive controller, the FMPC controller is designed, and the fuzzy control algorithm is used to dynamically adjust the weight coefficient of the lateral displacement deviation to improve the accuracy and robustness of the control system. Finally, the FMPC and MPC controller were compared and analyzed by co-simulation of CarSim and Simulink under different speeds.

The main contributions of this paper are as follows:

1. A sixth-order polynomial lane change trajectory is designed and compared with the fifth-order polynomial lane change trajectory.
2. A FMPC (model predictive controller based on fuzzy control) controller is designed, and the FMPC and MPC controller were compared and analyzed by co-simulation of CarSim and Simulink under different speeds.

### Vehicle lane change trajectory planning

When the vehicle is in danger of collision with the vehicle in front, if the side lane change conditions permit, the collision can be avoided by active lane change. As shown in Figure 1, vehicle A is driving at a higher speed $v_0$, and vehicle B is driving at a lower speed $v_1$. At this point, vehicle A detects that there is a risk of collision with vehicle B, and the condition of lane change to the adjacent lane is allowed. Vehicle A actively changes lanes, effectively avoiding collision accidents.

When designing a lane change trajectory, both the comfort of the lane change process and the handling stability must be considered. In the planning of commonly used fifth-order polynomial lane change trajectory, constraints on the position and velocity of starting and ending points, as well as constraints on the curvature during lane change have been taken into account, but the influence of the rate of curvature change at the end of lane change has not been taken into account. Therefore, on the basis of the fifth-order polynomial lane change trajectory, the new trajectory is constrained as shown in Table 1.

![Figure 1. A scenario of vehicles actively changing lanes to avoid collisions.](image-url)
Table 1. Constraints of lane change trajectory.

| Number | Constraints                                                                                                                                                                                                 |
|--------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1      | From the beginning to the end of the lane change, the lane change curve is smooth and continuous without abrupt changes, and the longitudinal distance is as short as possible. |
| 2      | When $X = 0$, $Y(X) = 0$, $Y'(X) = 0$, $K(X) = 0$                                                                                                                                                    |
| 3      | When $X = X_c$, $Y(X) = d_c$, $Y'(X) = 0$, $K(X) = 0$, $K'(X) = 0$                                                                                                                                        |

Where $X$ is the longitudinal displacement of the vehicle, and $Y$ is the lateral displacement of the vehicle, and $K$ is the curvature of the lane change trajectory, and $X_c$ is the terminal longitudinal displacement of the lane change trajectory, and $Y_c$ is the terminal lateral displacement of the lane change trajectory, and $d_c$ is the lateral displacement of the terminal of the lane change trajectory, and $Y_c$ is the lateral speed of the vehicle, and $K$ is the rate of change of the curvature of the lane change trajectory.

Under the constraints of Table 1, a sixth-order lane change trajectory proposed in this paper is shown in equation (1).

$$Y(X) = a_0 + a_1X + a_2X^2 + a_3X^3$$

$$+ a_4X^4 + a_5X^5 + a_6X^6$$

Equation (1)

The curvature $K$ is shown in equation (2).

$$K = \frac{\frac{d^2Y}{dX^2}}{\left(1 + \left(\frac{dY}{dX}\right)^2\right)^{\frac{3}{2}}}$$

Equation (2)

Since the vehicle changes $d_c \ll X_c$ during the actual lane change, so $\frac{dY}{dX} \approx 1$. The curvature $K$ can be simplified as shown in equation (3).

$$K \approx \frac{d^2Y}{dX^2}$$

Equation (3)

When the constraint conditions 1, 2, and 3 are satisfied, the lane change trajectory is shown in equation (4).

$$Y(X) = d_c\left(20 \frac{X^3}{X_c^3} - 45 \frac{X^4}{X_c^4} + 36 \frac{X^5}{X_c^5} - 10 \frac{X^6}{X_c^6}\right)$$

Equation (4)

If the curvature reaches its maximum at $X_c$, where $K(X_c) = \ddot{Y}(X_c) = 0$, the equation (5) can be found.

$$\frac{X_c}{X} = 4 - \sqrt{6}$$

Equation (5)

Assuming that $X \approx Vt$ during the lane change, the maximum lateral acceleration reached at $X_c$ must satisfy equation (6).

$$a_{\text{max}} \geq \frac{d_cV^2}{X_c^2}\left(120 \frac{X_c}{X_c} - 540 \frac{X_c^2}{X_c^2} + 720 \frac{X_c^3}{X_c^3} - 300 \frac{X_c^4}{X_c^4}\right)$$

Equation (6)

In order to meet the condition 1, the longitudinal distance of the lane change trajectory is as short as possible, so $X_c$ is taken as equation (7).

$$X_c = V \sqrt{\frac{d_c}{a_{\text{max}}} \left(120 \frac{X_c}{X_c} - 540 \frac{X_c^2}{X_c^2} + 720 \frac{X_c^3}{X_c^3} - 300 \frac{X_c^4}{X_c^4}\right)}$$

Equation (7)

At the speeds of 54, 72, and 108 km/h, the traditional fifth-order polynomial lane-changing trajectory and the improved sixth-order polynomial lane-changing trajectory are drawn, as shown in Figure 2.

In Figure 2, the dotted line represents the vehicle’s fifth-order polynomial lane change trajectory at different vehicle speeds, and the solid line represents the vehicle’s sixth-order polynomial lane change trajectory at different vehicle speeds. The $a_{\text{max}}$ of both trajectories is $2.5 \text{ m/s}^2$. It can be seen from Figure 2 that, compared with the fifth-order polynomial lane-changing trajectory, the longitudinal displacement of the lane change track of the sixth-order polynomial lane-changing trajectory with the curvature change rate constraint at the terminal moment increases significantly with the increase of vehicle speed, and the curvature of the lane change trajectory decreases significantly. At the position close to the lane change terminal, the lane change trajectory decreases significantly.
trajectory of the sixth-order polynomial is smoother, which can effectively reduce the vehicle instability during high-speed driving, thus improving the handling stability and passenger’s comfort.

Vehicle dynamics modeling

Vehicle dynamics model is the basis of trajectory tracking and model prediction, but complex dynamics model will increase the difficulty of calculation and reduce the convergence speed. Therefore, the following assumptions are made when establishing vehicle dynamics model:28

1. The vehicle runs on a smooth road without obvious road roughness, ignoring the influence of vertical force.
2. The suspension and body parts are a rigid body without relative motion.
3. The transmission ratio of the steering system is fixed, and the angle input can act on the wheel directly.
4. The effects of aerodynamics are ignored, and load transfer is not considered.

Under the above assumptions, a three-degree-of-freedom vehicle model as shown in Figure 3 is established.

In Figure 3, XOY is the earth coordinate system, and xoy is the car body coordinate system, and \( F_y, F_{cy} \) are the longitudinal force and lateral force on the front wheels, and \( F_{cf}, F_{cy} \) are the forces of the front wheels in the x and y directions, and \( F_{cr}, F_{cy} \) are the longitudinal force and lateral force on the rear wheels, and \( F_{cr}, F_{cy} \) are the forces of the rear wheels in the x and y directions. Assuming that the front wheels are steering wheels, \( \delta_f \) is the front wheel angle, and \( \alpha_f, \alpha_r \) are side angles of front and rear wheels, and \( \beta \) is side slip angle, and \( v_x \) is longitudinal speed of the vehicle, and \( v_y \) is lateral speed of the vehicle, and \( \phi \) is yaw angle, and \( a, b \) are the distance between the front axle and the center of mass and the distance between the rear axle and the center of mass.

Considering longitudinal, transverse and yaw motions, the vehicle dynamics equation is established:

\[
m(v_x - v_y \dot{\phi}) = 2F_{sf} + 2F_{cx} \tag{8}
\]

\[
m(v_x + v_y \dot{\phi}) = 2F_{cr} + 2F_{cy} \tag{9}
\]

\[
I_c \ddot{\phi} = 2aF_{sf} - 2bF_{cr} \tag{10}
\]

Further expansion, we can get the following:

\[
m\dot{v}_x = m v_y \dot{\phi} + 2(F_y \cos \delta_f - F_{cf} \sin \delta_f + F_{cr}) \tag{11}
\]

\[
m\dot{v}_y = -m v_x \dot{\phi} + 2(F_y \sin \delta_f + F_{cf} \cos \delta_f + F_{cr}) \tag{12}
\]

\[
I_c \ddot{\phi} = 2(aF_y \sin \delta_f + aF_{cf} \cos \delta_f - bF_{cr}) \tag{13}
\]

where \( I_c \) is the moment of inertia.

In the case of small side slip angle and slip ratio, the longitudinal force and lateral force of front and rear tire can be described by linear relationship.35 Therefore, it can be obtained that:

\[
F_y = k_y \cdot s_f \tag{14}
\]

\[
F_{cf} = k_{cf} \cdot \alpha_f \tag{15}
\]

\[
F_{cr} = k_{cr} \cdot \alpha_r \tag{16}
\]

where, \( k_y, k_{cf}, k_{cr} \) are the longitudinal stiffness of the front and rear wheels, and \( k_{cf}, k_{cr} \) are the cornering stiffness of front and rear wheels, and \( s_f, s_r \) are longitudinal slip rates of front and rear wheels, and \( \alpha_f, \alpha_r \) are side angles of front and rear wheels.

Under the condition of small side slip angles, the side slip angles of front and rear wheels can be approximately expressed as follows:

\[
\alpha_f = \frac{v_x + a \dot{\phi}}{v_x} - \delta_f \tag{18}
\]

\[
\alpha_r = \frac{v_x - b \dot{\phi}}{v_x} \tag{19}
\]

The longitudinal slip rates of front and rear wheels are follows:

\[
s_f = \frac{v_x - r_f \omega_f}{v_x} \times 100\% \tag{20}
\]

\[
s_r = \frac{v_x - r_r \omega_r}{v_x} \times 100\% \tag{21}
\]

where \( r_f, r_r \) are the rolling radius of the front and rear wheels, \( \omega_f, \omega_r \) are the angular velocities of the front and rear wheels.

Substituting equations (14) to (19) into equations (8) to (10), ignoring the influence of front-wheel driving
force on the vehicle’s yaw motion \( F_{yy} \sin \delta_y \approx 0 \), a vehicle dynamics model considering the tire model at a small angle can be obtained \( \sin \delta_y \approx \delta_y, \cos \delta_y \approx 1 \):

\[
mv_y = mv_y \dot{\phi} + 2 \left( k_{yf} \cdot s_f - k_{s} \cdot \left( \frac{v_y + a \dot{\phi}}{v_x} - \delta_y \right) \right) \cdot \delta_f + k_{s} \cdot s_v
\]

\[
mv_y = -mv_y \dot{\phi} + 2 \left( k_{yf} \cdot \left( \frac{v_y + a \dot{\phi}}{v_x} - \delta_y \right) + k_{s} \cdot \frac{v_y - b \cdot \dot{\phi}}{v_x} \right)
\]

\[
\dot{\phi} = 2 \left( a \cdot k_{s} \cdot \left( \frac{v_y + a \dot{\phi}}{v_x} - \delta_y \right) - b \cdot k_{s} \cdot \left( \frac{v_y - b \cdot \dot{\phi}}{v_x} \right) \right)
\]

\[
X = v_x \cos \phi - v_y \sin \phi
\]

\[
\dot{Y} = v_y \sin \phi + v_x \cos \phi
\]

Assuming that the state vector of the system at the current moment is \( \xi_0 \) and the input vector is \( \mu_0 \), the Taylor equation is used to expand the state equation at the current moment and ignore the higher-order terms. The equation of state can be rewritten as follows:

\[
\dot{\xi} = f(\xi_0, \mu_0) + A_t(\xi - \xi_0) + B_t(\mu - \mu_0)
\]

\[
\eta = C_t \xi
\]

where, \( A_t = \left[ \frac{\partial f}{\partial \xi} \right]_{\xi_0, \mu_0}, B_t = \left[ \frac{\partial f}{\partial \mu} \right]_{\xi_0, \mu_0} \).

Within the predictive horizon, the system can obtain the reference state vector \( \hat{\xi}_0(k) = \xi_0 \) by applying the constant control vector \( \hat{\mu}_0(k) = \mu_0 \). After discretization, the state equation can be obtained as follows:

\[
\xi(k + 1) = A_{k,t} \xi(k) + B_{k,t} \mu(k) + d_{k,t}
\]

\[
\eta(k) = C_{k,t} \xi(k)
\]

where, \( A_{k,t} = I + TA_t, B_{k,t} = TB_t, T \) is sampling time, \( d_{k,t} = \hat{\xi}(k + 1) - \hat{A}_{k,t} \hat{\xi}(k) - \hat{B}_{k,t} \hat{\mu}(k) \),

The following equation of state can be obtained from equations (22) to (26) of vehicle dynamics equation:

\[
\dot{\xi} = f(\xi, \mu)
\]

where, \( \xi = [v_x, v_y, \phi, \dot{\phi}, Y, X]^T \) is state vector, and \( \mu = \delta_y \) is control vector.

**Design of linear time-varying model predictive controller**

**Derivation of linear time-varying model predictive controller**

When tracking the lane change trajectory, under the premise of ensuring the stability of the vehicle, the front wheel angle is controlled. In order to improve the prediction accuracy and solution speed, the nonlinear model needs to be linearized and discretized.

\[
\hat{A}_{k,t} = \left[ \begin{array}{c}
1 + 2T(k_{yf} + k_0) \\
\phi T \\
2T(k_{yf} + k_0) \\
2T(k_{yf} + k_0) \\
-\phi T \\
\end{array} \right] \left[ \begin{array}{c}
-\phi T \\
1 + 2T(k_{yf} + k_0) \\
2T(k_{yf} + k_0) \\
T \cos \phi_k \\
T \sin \phi_k
\end{array} \right]
\]

\[
B_{k,t} = \left[ \begin{array}{c}
-2T_{k_{yf}} \\
-2T_{k_{yf}} \\
2T_{k_{yf}} \\
0 \\
0
\end{array} \right] \left[ \begin{array}{c}
-\phi T \\
1 + 2T(k_{yf} + k_0) \\
2T(k_{yf} + k_0) \\
T \cos \phi_k \\
T \sin \phi_k
\end{array} \right]
\]

If the control vector \( \mu(k) \) in the discrete equation of state is transformed into \( \Delta \mu(k) \), the equation of state can be rewritten as:

\[
\hat{\xi}(k + 1 | r) = \hat{A}_{k,t} \hat{\xi}(k | r) + \hat{B}_{k,t} \Delta \mu(k | r) + \hat{d}_{k,t}
\]

\[
\eta(k | r) = \hat{C}_{k,t} \hat{\xi}(k | r)
\]

where, \( \hat{\xi}(k | r) = \left( \frac{\xi(k | r)}{\mu(k | r)} \right), \Delta \hat{\mu}(k | r) = \mu(k | r) - \mu(k - 1 | r), \hat{A}_{k,t} = \left( \begin{array}{c}
A_{k,t} \setminus B_{k,t} \setminus I_1
\end{array} \right), \hat{B}_{k,t} = \left( \begin{array}{c}
B_{k,t} \setminus 0
\end{array} \right), \hat{d}_{k,t} = \left( \begin{array}{c}
d_{k,t} \setminus 0
\end{array} \right), \hat{C}_{k,t} = C_{k,t}
\]

The output of equations (32) and (33) in the predictive horizon \( N_p \) and control horizon \( N_c \) is shown in equation (34):

\[
\hat{Y}(t) = \psi \hat{\xi}(t | r) + \Theta \Delta U(t) + \Gamma \phi(t)
\]
where, \( \tilde{Y}(t) = \begin{bmatrix} \eta(t+1|t) \\ \eta(t+2|t) \\ \vdots \\ \eta(t+N_p|t) \end{bmatrix} \), \( \psi_t = \begin{bmatrix} \tilde{C}_{t+1,i}^{\dagger} \tilde{A}_{t+1,i} \\ \tilde{C}_{t+2,i}^{\dagger} \tilde{A}_{t+2,i} \\ \vdots \\ \tilde{C}_{t+N_p,i}^{\dagger} \prod_{i=1}^{t+N_p-1} \tilde{A}_{i,i} \end{bmatrix} \).

\[
\Delta U(t) = \begin{bmatrix} \Delta \mu(t) \\ \vdots \\ \Delta \mu(t+N_p|t) \end{bmatrix}, \quad \phi(t) = \begin{bmatrix} \tilde{d}(t) \\ \vdots \\ \tilde{d}(t+|N_p|) \end{bmatrix}, \quad \tilde{\Gamma}_t = \begin{bmatrix} \tilde{C}_t+1,i & \tilde{A}_t+1,i \\ \tilde{C}_t+2,i & \tilde{A}_t+2,i \\ \vdots \\ \tilde{C}_t+N_p,i & \prod_{i=1}^{t+N_p-1} \tilde{A}_{i,i} \end{bmatrix}. 
\]

\[
\Theta_t = \begin{bmatrix} \tilde{C}_t+1,i & \tilde{B}_t+1,i \\ \tilde{C}_t+2,i & \tilde{B}_t+2,i \\ \vdots \\ \tilde{C}_t+N_p,i & \prod_{i=1}^{t+N_p-1} \tilde{B}_{i,i} \end{bmatrix}. 
\]

\[
\alpha(k|t) = \tilde{\alpha}_k \tilde{\xi}(k|t) + \tilde{D}_a,k_i \Delta \mu(k|t) + e_{k,i} \quad (35)
\]

where, \( \tilde{\alpha}_k \tilde{\xi}(k|t) = \frac{\partial}{\partial \xi(t)} \tilde{\xi}(k|t) \), \( \tilde{\alpha}_k \tilde{\xi}(k|t) = \frac{\partial}{\partial \xi(t)} \tilde{\xi}(k|t) \), \( e_{k,i} \), \( \tilde{\alpha}_k \), \( \tilde{\xi}_k, \tilde{\mu}_k \), can be obtained from \( \tilde{\xi}_k, \tilde{\mu}_k \), that is

\[
\tilde{\alpha}_k = f_a(\tilde{\xi}_k, \tilde{\mu}_k). 
\]

If the control vector \( \mu(k) \) in the discrete equation of state is transformed into \( \Delta \mu(k) \), the equation (35) can be rewritten as:

\[
\alpha(k|t) = \tilde{\alpha}_k \tilde{\xi}(k|t) + \tilde{D}_a,k \Delta \mu(k|t) + e_{k,i} \quad (36)
\]

where, \( \tilde{\alpha}_k \tilde{\xi}(k|t) = (\tilde{C}_a,k_i \tilde{A}_k,i) \), \( \tilde{D}_a,k \Delta \mu(k|t) = (\tilde{D}_a,k \Delta \mu(k|t)) \).

The output of equations (36) in the predictive horizon \( N_p \) and control horizon \( N_c \) is shown in equation (37):

\[
\tilde{Y}_{sc}(t) = \psi_{sc,i} \tilde{\xi}(t) + \Theta_{sc,i} \Delta U(t) + \Gamma_{sc,i} \psi(t) + \Lambda(t) \quad (37)
\]
Equations (34) and (37) are the prediction models of the lane change trajectory of the vehicle. According to the above model and relaxation factor, the objective function of rolling optimization can be set as follows:

\[
J(\xi(t), \mu(t - 1), \Delta U(t), \varepsilon) = \min_{\Delta U(t)} \left[ \sum_{j=1}^{N_e} \left| \eta(t + i|t) - \eta_{ref}(t + i|t) \right|^2_Q + \sum_{i=0}^{N_e-1} \left| \Delta \mu(t + i|t) \right|^2_R + \sum_{i=0}^{N_e-1} \left| \mu(t + i|t) \right|^2_S + \rho \varepsilon^2 \right]
\] (38)

Where, the first term reflects the requirement for the tracking ability of the reference trajectory, and the second term reflects the requirement for the smoothness of the lane change process, and the third term reflects the requirement for the control vector. The fourth term is the relaxation factor, which is used to prevent the objective function from no solution in the control horizon. The constraints of the objective function are as follows:

\[
\begin{align*}
\text{s.t.} & \quad \Delta U_{\text{min}} \leq \Delta U(t) \leq \Delta U_{\text{max}} \\
& \quad U_{\text{min}} \leq U(t) \leq U_{\text{max}} \\
& \quad \tilde{Y}_{\text{min}} \leq \tilde{Y}(t) \leq \tilde{Y}_{\text{max}} \\
& \quad \tilde{Y}_{\text{min}} \leq \tilde{Y}_{\text{sc}}(t) \leq \tilde{Y}_{\text{sc, max}} + \varepsilon \\
& \quad 0 \leq \varepsilon \leq \varepsilon_{\text{max}}
\end{align*}
\] (39)

**Dynamic adjustment of weight coefficient**

In the theory of model predictive control, the weight coefficient of the objective function of the traditional algorithm is usually taken as a constant value. But in practice, the fixed weight coefficient may reduce the accuracy and robustness of the control system due to different working conditions. Therefore, in order to improve the accuracy of tracking the lane change trajectory, it is necessary to dynamically adjust the weight coefficient of the deviation of the lateral displacement. When the average deviation \(\tilde{Y}\) between the actual vehicle lateral displacement and the expected lateral displacement in the predictive horizon is large and the change rate of the average deviation \(\Delta \tilde{Y}\) is large, it indicates that the tracking ability of the vehicle controller is poor, and the weight coefficient of the lateral displacement deviation should be increased to ensure the accuracy of lane change trajectory tracking. Therefore, \(\tilde{Y}\) and \(\Delta \tilde{Y}\) are taken as the input indexes of the fuzzy controller to dynamically adjust the weight coefficient \(Q_T\) of lateral displacement deviation. The domain of discourse of \(\tilde{Y}\) is [0, 0.2], the domain of discourse of \(\Delta \tilde{Y}\) is [−2, 18], and according to Zhang et al., the domain of discourse of \(Q_T\) is [200, 800]. The degree of membership of \(\tilde{Y}, \Delta \tilde{Y}\), and \(Q_T\) is shown in Figure 4(a)–(c).

As shown in Figure 5, when \(\tilde{Y}\) and \(\Delta \tilde{Y}\) are smaller, it indicates that the trajectory tracking effect is better, so the value of \(Q_T\) is smaller. When \(\tilde{Y}\) is smaller and \(\Delta \tilde{Y}\) is increasing, it indicates that trajectory tracking has the possibility of mutation, so the value of \(Q_T\) should be appropriately increased. When \(\tilde{Y}\) increases, it indicates that the error of trajectory tracking becomes larger, and the value of \(Q_T\) needs to be increased in time to improve the tracking error.

The fuzzy control regulation rule of the weight coefficient \(Q_T\) of lateral displacement deviation is shown in Table 2:

When \(\tilde{Y}\) is the value of S and \(\Delta \tilde{Y}\) is the value of N, \(Q_T\) is the value of S. When \(\tilde{Y}\) is the value of S and \(\Delta \tilde{Y}\) is the value of PS, \(Q_T\) is the value of M. When \(\Delta \tilde{Y}\) is the value of S and \(\Delta \tilde{Y}\) is the value of PS, \(Q_T\) is the value of PL. When \(\tilde{Y}\) is the value of M and \(\Delta \tilde{Y}\) is the value of N, \(Q_T\) is the value of M. When \(\tilde{Y}\) is the value of M and \(\Delta \tilde{Y}\) is the value of PS, \(Q_T\) is the value of M. When \(\Delta \tilde{Y}\) is the value of M and \(\Delta \tilde{Y}\) is the value of PL, \(Q_T\) is the value of L. When \(\Delta \tilde{Y}\) is the value of L, no matter how the value of \(\Delta \tilde{Y}\) changes, \(Q_T\) is the value of L.

**Simulation design and verification**

**The block diagram of the control system**

CarSim and Simulink are used to establish the experimental simulation platform to realize the design and verification of the model predictive controller. The block diagram of the control system is shown in Figure 6.
In this paper, the model predictive controller based on fuzzy control is called FMPC.

Vehicle parameters
The main parameters of the vehicle model are shown in Table 3.

The main parameters of the controller
Sampling period: \( T_0 = 0.02 \) s.

Predictive horizon and control horizon: \( N_p = 10 \), \( N_c = 5 \). Weight coefficient matrix: 
\[
Q = \begin{bmatrix}
200 & 0 \\
0 & Q_Y
\end{bmatrix},
\]
\( R = 400 \), \( S = 100 \), \( \rho = 1000 \).
Constraints: $-9.8^\circ \leq \delta_r \leq 9.8^\circ$, $-0.8^\circ \leq \Delta \delta_r \leq 0.8^\circ$, $-17.2^\circ \leq \varphi \leq 12^\circ$, $-2.8^\circ \leq \alpha \leq 2.8^\circ$, $-3 \leq Y \leq 5$.

Reference trajectory:

\[
\begin{align*}
Y_{ref}(X) &= d_x \left( 20 \frac{b}{\alpha X} - 45 \frac{b}{\alpha X^2} + 36 \frac{b}{\alpha X^3} - 10 \frac{b}{\alpha X^4} \right) \\
\varphi_{ref}(X) &= \arctan \left( d_y \left( 60 \frac{c}{\alpha X} - 180 \frac{c}{\alpha X^2} + 180 \frac{c}{\alpha X^3} - 60 \frac{c}{\alpha X^4} \right) \right)
\end{align*}
\]

\[ (40) \]

**Simulation analysis**

In order to verify the effectiveness and robustness of the control algorithm, the vehicle controller based on FMPC and MPC is compared and analyzed. The forward travel speeds of 54, 72, and 108 km/h are taken for simulation verification under the condition of road adhesion coefficient of 0.85.

At different speeds, the tracking reference curve and actual lateral displacement curve, the deviation of tracking reference curve and actual lateral displacement curve, the maximum deviation of tracking reference and actual lateral displacement curve are shown in Figures 7(a)–(c) and 8(a)–(c), and Table 4, respectively.

From Figures 7(a)–(c) and 8(a)–(c), and Table 4, it can be seen that when the vehicle changes lanes at a speed of 54 km/h, the peak value of the lateral displacement tracking deviation based on FMPC controller is 0.046 m, and the peak value of the lateral displacement tracking deviation based on MPC controller is 0.171 m. The lateral displacement tracking deviation based on FMPC controller is significantly smaller than that based on MPC controller, and its error accuracy is improved by 271.7%. It is shown that the FMPC controller has better trajectory tracking performance than MPC controller under the condition of medium or low speed. With the increase of vehicle speed and the decrease of track curvature, the tracking errors of lateral displacement based on these two controllers are further reduced. When the vehicle changes lanes at a speed of 108 km/h, the peak value of the lateral displacement tracking deviation based on FMPC controller is 0.025 m, and the peak value of the lateral displacement tracking deviation based on MPC controller is 0.092 m. The error accuracy is improved by 268%. It shows that the FMPC controller also has better trajectory tracking performance than MPC controller at high speed, and the lateral displacement tracking error based on FMPC controller does not exceed 0.05 m at different vehicle speeds. Therefore, the deviation from the tracking reference curve and the actual lateral displacement curve further indicates that, at different vehicle speeds, the designed FMPC controller has better tracking characteristics and robustness to the lateral displacement of the vehicle.
Figure 7. The tracking reference curve and actual lateral displacement curve.

Figure 8. The deviation of tracking reference and actual lateral displacement curve.
At different speeds, the reference curve and the actual curve of the front wheel angle, the deviation between reference angle and actual angle of the front wheel are shown in Figures 9(a)–(c) and 10(a)–(c).

From Figures 9(a)–(c) and 10(a)–(c), it can be seen that, at different speed, the jitter of the front wheel angle based on the MPC controller is relatively large, the deviation between reference angle and actual angle of the front wheel is large, and the chatter of the deviation is relatively severe. However, the front wheel angle curve and the deviation curve based on FMPC controller are relatively smooth at different speeds. When the vehicle speed changes lane at 54 and 72 km/h, the peak deviation of front wheel angle based on FMPC controller is less than 0.1°. When the vehicle changes lanes at 108 km/h, although the angular deviation at the starting point of lane change has a short-term surge, it is still far less than the constraint boundary and converges smoothly in the process of lane change. The control effect based on FMPC controller is still better than that based on MPC controller. Therefore, at different vehicle speeds, the designed FMPC controller has better tracking characteristics and robustness to vehicle front wheel angle.

At different speeds, the reference yaw angle curve and the actual yaw angle curve, the deviation between the reference yaw angle and the actual yaw angle, the peak deviation between the reference yaw angle and the actual yaw angle are shown in Figures 11(a)–(c) and 12(a)–(c), and Table 5, respectively.

From Figures 11(a)–(c) and 12(a)–(c), Table 5 combined with Figures 2 and 9(a)–(c), it can be seen that, at low speed, the longitudinal displacement of vehicle lane change is shorter, and the front wheel angle changes greatly, so the vehicle movement is more violent. Therefore, the deviation between the reference yaw angle and the yaw angle based on the two controllers is large, but it is still within the acceptable range. However, with the increase of vehicle speed, the longitudinal displacement of lane change becomes larger, the lane change time is longer, the change of front wheel angle is relatively small, and the vehicle movement is comparatively gentle. At this time, the deviation between the reference yaw angle and the yaw angle...
Figure 10. The deviation between reference angle and actual angle of the front wheel.

Figure 11. The reference yaw angle curve and the actual yaw angle curve.
based on the two controllers is relatively small. When the vehicle changes lane at 54 km/h, the peak deviation of yaw angle based on FMPC is 1.111° and that based on MPC is 1.426°, and the error accuracy is improved by 28.4%. When the vehicle changes lanes at 72 km/h, the peak deviation of yaw angle based on FMPC is significantly smaller than that based on MPC, and the error accuracy is improved by 59.8%. When the vehicle changes lane at 108 km/h, the control effect of both controllers is better, but the accuracy of the peak deviation of yawing Angle based on FMPC controller can still be improved by 21.3%. Therefore, at different vehicle speeds, the designed FMPC controller has better tracking characteristics for vehicle yaw angle, and has better robustness without losing control.

At different vehicle speeds, the sideslip angle of mass center based on FMPC and MPC controller is shown in Figure 13(a)–(c).

From Figures 13(a)–(c) and 9(a)–(c), it can be seen that, when the vehicle changes lanes at medium or low speeds, the vibration of the side slip angle of mass center based on MPC controller is more severe due to the large change range of the front wheel angle. While the $Q_F$ of the FMPC controller can be dynamically adjusted according to the lateral displacement deviation and the rate of change of the lateral displacement deviation. Therefore, the change of the sideslip angle of mass center based on FMPC is relatively gentle. When the vehicle changes lanes at high speed, due to the decrease of the front wheel angle, the change of the side slip angle of mass center based on the two controllers is relatively smooth without severe shaking. Although the variation range of the side slip angle of the mass center increases, it is still within the acceptable range. Therefore, at different vehicle speeds, the designed FMPC controller has better tracking performance and robustness to the sideslip angle of mass center.

### Table 5. The peak deviation between the reference yaw angle and the actual yaw angle.

| Speed (km/h) | FMPC (°) | MPC (°) | Improved accuracy (%) |
|-------------|----------|---------|-----------------------|
| 54          | 1.111    | 1.426   | 28.4                  |
| 72          | 0.366    | 0.585   | 59.8                  |
| 108         | 0.225    | 0.273   | 21.3                  |

![Figure 12. The deviation between the reference yaw angle and the actual yaw angle.](image-url)
Conclusions

In order to improve the trajectory smoothness and the accuracy of lane change control, in this paper, on the basis of the constraints of the curvature change rate at the end of the lane change, a sixth-order polynomial lane change trajectory was proposed, and compared with the traditional fifth-order polynomial lane-changing trajectory. The simulation results show that at the position close to the lane change terminal, the lane change trajectory of the sixth-order polynomial is smoother, which can effectively reduce the vehicle instability during high-speed driving, thus improving the handling stability and passenger’s comfort.

On the basis of the traditional model predictive control algorithm, combined with the soft constraint of the side slip angle, the fuzzy control algorithm was used to dynamically adjust the weight coefficient of the lateral displacement deviation. The designed FMPC controller and MPC controller are simulated and compared under different speed conditions. The simulation results show that, at different vehicle speeds, compared with MPC controller, the designed FMPC controller can better control the vehicle state, and track the planned lane change trajectory more accurately by controlling the front wheel angles. The lane changing process is smoother and more stable. Therefore, the designed FMPC controller has better tracking characteristics and robustness.

In this paper, the design of lane change trajectory and controller was only carried out under the condition of the vehicle changing lanes at a constant speed, and the simulation verification was carried out under fine road adhesion conditions. In the next work, we will further improve the application range of lane change trajectory and controller, and further verify the robustness of the controller under different road adhesion conditions.

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**References**

1. Yan Y, Li CS and Tang FM. Lane-changing trajectory planning of the autonomous vehicle based on the quintic polynomial model. *J Mach Des* 2019; 36: 42-47.
2. Zhang JX, Shi ZT, Yang X, et al. Robust adaptive control for continuous wheel slip rate tracking of vehicle with state observer. *Meas Control* 2020; 53: 1331–1341.
3. Ziegler J, Bender P, Schreiber M, et al. Making berthia drive—an autonomous journey on a historic route. *IEEE Intel Transp Syst Mag* 2014; 6: 8–20.
4. Jiang H, Tian H and Hua Y. Model predictive driver model considering the steering characteristics of the skilled drivers. *Adv Mech Eng* 2019; 11: 1–14.
5. Lu S, Lian M, Cao Z, et al. Active rectifying control of vehicle with tire blowout based on adaptive fuzzy proportional–integral–derivative–derivative control. *Adv Mech Eng* 2019; 11: 1–13.
6. Ji J, Tang ZR, Wu YM, et al. Path planning and tracking for lane changing based on model predictive control. *China J Highway Transp* 2018; 31: 172–179.
7. Zhang JX and Zhou SY. Robust adaptive anti-windup wheel slip tracking control for intelligent vehicle with fast terminal sliding mode observer. *Proc Inst Mech Eng D J Automob Eng* 2020; 231: 3373–3384.
8. Song B, Wang Z and Sheng L. A new genetic algorithm approach to smooth path planning for mobile robots. *Assembly Autom* 2016; 36: 138–145.
9. Cai Z and Peng Z. Cooperative coevolutionary adaptive genetic algorithm in path planning of cooperative multi-mobile robot systems. *J Intell Robot Syst* 2002; 33: 61–71.
10. Bhattacharya P and Gavriloa ML. Roadmap-based path planning-using the voronoi diagram for a clearance-based shortest path. *IEEE Robot Autom Mag* 2008; 15: 58–66.
11. Guo H, Cao D, Chen H, et al. Model predictive path following control for autonomous cars considering a measurable disturbance: implementation, testing, and verification. *Mech Syst Signal Process* 2019; 118: 41–60.
12. Kumar S, Parhi DR, Muni MK, et al. Optimal path search and control of mobile robot using hybridized sine-cosine algorithm and ant colony optimization technique. *J Intell Robot Syst* 2020; 47: 535–545.
13. Maneev VV and Syryamkin MV. Optimizing the A* search algorithm for mobile robotic devices. *IOP Conf Ser Mater Sci Eng* 2019; 516: 012054.
14. Zheng YC, Wang J, Guo D, et al. Study of multi-objective path planning method for vehicles. *Environ Sci Pollut Res* 2020; 27: 3257–3270.
15. Thi TM, Cosmin C, Duc TT, et al. A hierarchical global path planning approach for mobile robots based on multi-objective particle swarm optimization. *Appl Soft Comput* 2017; 59: 68–76.
16. Cai Z, Cui X, Su X, et al. A novel vector based dynamic path planning method in urban road network. *IEEE Access* 2019; 8: 9046–9060.
17. Xu MH, Liu YQ, Huang QL, et al. An improved Dijkstra’s shortest path algorithm for sparse network. *Appl Math Comput* 2007; 185: 247–254.
18. Wang YJ, Liu ZX, Zuo QZ, et al. Local path planning of autonomous vehicles based on A* algorithm with equal-step sampling. In: The 37th Chinese control conference (CCC), Wuhan, China, 25–27 July 2018, pp.7828–7833.
19. New York: IEEE.
20. Li YJ, Wu W, Yong G, et al. PQ-RRT*: an improved path planning algorithm for mobile robots. *Expert Syst Appl* 2020; 152: 1–11.
21. Zhu DD and Sun JQ. A new algorithm based on dijkstra for vehicle path planning considering intersection attribute. *IEEE Access* 2021; 9: 19761–19775.
22. Pashkevich A, Kazhevik M and Ruano A. Neural network approach to collision free path-planning for robotic manipulators. *Int J Syst Sci* 2006; 37: 555–564.
23. Feng P. The research of dynamic path planning based on improving fuzzy genetic algorithm in the vehicle navigation. *Adv Mater Res* 2012; 424-425: 73–76.
24. Kim D, Chung W and Park S. Practical motion planning for car-parking control in narrow environment. *IET Control Theory A* 2010; 4: 129–139.
25. Wang JH, Xiao RH and Ma YL. Research on welding robot path planning using ant colony optimization. *Adv Mater Res* 2011; 201–203: 1926–1929.
26. Laumond JP and Jacobs PE. A motion planner for non-holonomic mobile robots. *IEEE Trans Robot Autom* 1994; 10: 577–593.
27. Choi JW, Curry RE and Elkaim GH. Continuous curvature path generation based on Bezier curves for autonomous vehicles. *Int J Appl Math* 2010; 40: 179–185.
28. Nelson WL. Continuous-curvature paths for autonomous vehicles. In: 1989 IEEE international conference on robotics & automation, Scottsdale, AZ, 14–19 May 1989, pp.1260–1264. New York: IEEE.
29. Shim T, Adireddy G and Yuan H. Autonomous vehicle collision avoidance system using path planning and model-predictive-control-based active front steering and wheel torque control. *Proc Inst Mech Eng D J Automob Eng* 2012; 226: 767–778.
30. Deng HP, Ma B, Zhao HG, et al. Path planning and tracking control of autonomous vehicle for obstacle avoidance. *Acta Armam* 2020; 41: 585–594.
31. Ren Y, Zheng L, Zhang W, et al. A study on active collision avoidance control of autonomous vehicles based on model predictive control. *Autom Eng* 2019; 41: 404–410.
32. Yaghoub P, Mehdi M and Majid O. Design of an optimal active stabilizer mechanism for enhancing vehicle rolling resistance. *J Cent South Univ* 2016; 23: 1142–1151.
33. Wen S, Chen MZQ, Zeng Z, et al. Fuzzy control for uncertain vehicle active suspension systems via dynamic sliding-mode approach. *IEEE Trans Syst Man Cybern Syst* 2016; 47: 24–32.
33. Robert R, Frank S, Juliane H, et al. Nonlinear model predictive path-following control for highly automated driving. *IFAC PapersOnLine* 2019; 52: 350–355.

34. Bruschetta M, Maran F and Beghi A. A fast implementation of MPC-based motion cueing algorithms for mid-size road vehicle motion simulators. *Vehicle Syst Dyn* 2017; 55: 802–826.

35. Lei QS, Peng YY, Wang GS, et al. Study of optimal model and optimizing method for the electronically controlled air suspension system structure. *Acta Armam* 2018; 39: 1259–1267.

36. Zhang LX, Wu GQ and Guo XX. Path tracking using linear time-varying model predictive control for autonomous vehicle. *J Tongji Univ* 2016; 44: 1595–1603.