Nonlinear Effects in the Amplitude of Cosmological Density Fluctuations

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ABSTRACT

The amplitude of cosmological density fluctuations, $\sigma_8$, has been studied and estimated by analysing many cosmological observations. The values of the estimates vary considerably between the various probes. However, different estimators probe the value of $\sigma_8$ in different cosmological scales and do not take into account the nonlinear evolution of the parameter at late times. We show that estimates of the amplitude of cosmological density fluctuations derived from cosmic flows are systematically higher than those inferred at early epochs because of nonlinear evolution at later times. Here we derive corrections to the value of $\sigma_8$ and compare amplitudes after accounting for this effect.

Subject headings: Cosmology, Large-Scale Structure, Flows

1. Introduction

In our current understanding (Bardeen et al. 1986; Eisenstein & Hu 1998), large-scale cosmological structure arises from small Gaussian initial fluctuations imprinted in the very early universe that then evolve and are amplified by gravitational instability. One manifestation of gravity is large-scale motions, of particular interest because they respond to the otherwise invisible dark matter. Relative motions of pairs of galaxies (Feldman et al. 2003) as well as bulk flows (Pike & Hudson 2005; Sarkar et al. 2007) and velocity shear (Watkins & Feldman 2007; Feldman & Watkins 2008) measurements have been used to estimate $\Omega_m$ and $\sigma_8$, respectively the matter density and the amplitude of matter density perturbations on the scale of $8\,h^{-1}\text{Mpc}$. In particular, from pair-
wise velocities we found in Feldman et al. (2003) $\sigma_8 = 1.13^{+0.22}_{-0.23}$. From microwave background temperature fluctuations at the epoch of decoupling redshift, at a redshift $z \approx 10^3$, the WMAP collaboration (Dunkley et al. 2008) find $\sigma_8 = 0.80 \pm 0.04$. These differ only slightly, at the level of 1.5-$\sigma$, and it is a success of the model that inferences by such different methods applied at two greatly different epochs are in good agreement.

Recently, more and better observations of the peculiar velocity field have been undertaken. The new surveys are deeper, denser, and more reliable (Masters et al. 2006; Springob et al. 1996). With denser surveys and with better sky coverage, we have improved our understanding of the distance indicators needed to extract the peculiar velocities. Malmquist biases and other systematic errors are being handled better, and thus we are able to extract more and better information from surveys and get consistent and robust results from independent surveys (Feldman et al. 2003; Radburn-Smith et al. 2004; Pike & Hudson 2005; Sarkar et al. 2007; Watkins & Feldman 2007; Feldman & Watkins 2008). Using the newest surveys and analyses techniques, bulk flow measurements put a constraint $\sigma_8 > 1.11$ (0.88) at a 95% (99%) significance level (Watkins et al. 2008). Peculiar velocity observations will continue to improve, reducing statistical errors by making more measurements and by using better distance indicators.

With further improvements, it will become increasingly important to recognize that the early- and late-time observations are in fact measuring different quantities, the first extrapolated to the present using the linear perturbation growth factor, $D(t)$ (Peebles 1980), while the latter measured in the nonlinearly evolved present-day Universe. In a $\Lambda$CDM universe eventually dominated by a cosmological constant, the linear growth factor will saturate at a maximum value, as gravitational clustering is balanced by the effective force of accelerated expansion. The limiting value is given by

$$\lim_{t \to \infty} D(t) = \frac{2 \Gamma(2/3) \Gamma(11/6)}{\sqrt{\pi}} \left( \frac{\Omega_m}{1 - \Omega_m} \right)^{1/3} \approx 1.01 \text{ for } \Omega_m = 0.26.$$  

(1)

Note that approximate expressions for the growth factor given in Lahav et al. (1991) only apply to the past; the growth factor equation (Heath 1977) must be integrated anew for this case.

The scale $8 h^{-1}$ Mpc is chosen to be at the threshold of nonlinearity, but even on this scale there are small but significant systematic differences between the two that can be accounted for by applying models describing fully the dynamics of gravitational clustering used in the data reduction process. Implications of nonlinear evolution were confirmed by measurements of the galaxy bispectrum in redshift surveys (Scoccimarro et al. 2001; Feldman et al. 2001; Verde et al. 2002). In this paper we reconcile the two estimates of $\sigma_8$, taking the nonlinear dynamics into account. We find that the linear/nonlinear correction is modestly significant, given the improving accuracy in cosmology: the linear value of $\sigma_8$ can be smaller by $\sim 10\%$ than the nonlinear value, a systematic difference that rivals current statistical uncertainties. In the following, we first describe the procedure for recovering $\sigma_L$ from $\sigma$. We then compare our revised parameters to other estimates from the CMB, weak lensing, the galaxy power spectrum, etc.

The mean square density contrast at redshift $z$ in a spherical volume $V$ with a comoving radius $R$ is given by the expression

$$\sigma^2(R, z) = \frac{1}{V^2} \int_V d^3r d^3s \xi(|r - s|, z),$$  

(2)

where $\xi(r)$ is the two-point correlation function, and we take

$$\sigma_8 = \sigma(8 h^{-1} \text{ Mpc}, 0).$$  

(3)

In Feldman et al. (2003) we obtained the true present-day value of the $\sigma_8$ parameter, derived self-consistently from the empirical correlation function, as estimated from the PSCz survey (Hamilton & Tegmark 2002). This is to be compared with the $\sigma_8$ parameter reported by WMAP, derived from observations at an early time corresponding to redshift $z \approx 1000$ when the evolution of the density contrast is in the linear regime, and then propagated to the present time using the perturbation theory linear growth factor (Peebles 1980),

$$\sigma_8 = \sigma(8 h^{-1} \text{ Mpc}, t) \frac{D(t_0)}{D(t)}. $$  

(4)

The WMAP collaboration describe the $\sigma_8$ parameter in their Table 1 as the "linear theory am-
plitude” (Dunkley et al. 2008). The value of $\sigma_8$ derived from shear is “slightly higher than the WMAP value, although not inconsistent.”

2. Linear and nonlinear amplitudes

We use two different methods to estimate the nonlinear corrections, one based in perturbation theory, which allows us to express the correction as a simple expression, and one using a phenomenological mapping based on conservation of pair counts and calibrated using numerical simulations, which allows us to explore the effects of changing many parameters individually. One-loop perturbative corrections to the leading order variance $\sigma_L(r)$ for power-law power spectra are given by Lokas et al. (1996); Scoccimarro & Frieman (1996a)

$$\sigma^2 = \sigma^2_L + \beta \sigma^4_L,$$  \hspace{1cm} (5)

where the factor $\beta$ is related to the logarithmic slope of the two-point correlation function $\gamma(r) = -d \ln \xi / dr$ as Scoccimarro & Frieman (1996b)

$$\beta = 1.843 - 1.168 \gamma.$$ \hspace{1cm} (6)

For non-power-law spectra, $\gamma$ is a slowly varying function of scale. A convenient representation of the correlation function over scales of interest has two power laws (Hamilton & Tegmark 2002),

$$\xi(r) = q^2 (x_1^{-\gamma_1} + x_2^{-\gamma_2}),$$ \hspace{1cm} (7)

where $x_1 = r/r_1$, $r_1 = 2.33 h^{-1}$ Mpc, $r_2 = 3.51 h^{-1}$ Mpc, $\gamma_1 = 1.72$, $\gamma_2 = 1.28$, and $q = \sigma(8 h^{-1}$ Mpc)/0.888. The effective slope of the correlation function is then

$$\gamma(r) = -d\ln \xi(r)/dr = \gamma_1 x_1^{-\gamma_1} + \gamma_2 x_2^{-\gamma_2}/x_1^{-\gamma_1} + x_2^{-\gamma_2},$$ \hspace{1cm} (8)

independent of the amplitude $q$. At $r = 8 h^{-1}$ Mpc, the effective slope is $\gamma = 1.393$, for which $\beta = 0.216$. To map from $\sigma^2$ to $\sigma^2_L$, we have to invert equation (5):

$$\sigma^2_L(r) = \sqrt{1 + 4 \beta \sigma^2(r) - 1}/2\beta.$$  \hspace{1cm} (9)

Note that for $\beta \to 0$, the above expression gives $\sigma^2_L = \sigma^2$, as it should. In Feldman et al. (2003) we obtained $\sigma = 1.13$. Using this value with $\beta = 0.216$ in equation (9), we obtain for the central value

$$\sigma_L = 1.02,$$ \hspace{1cm} (10)

a modest but significant decrease.

Our other method of relating linear and nonlinear variance uses the mapping of scale proposed in Hamilton et al. (1991), for which conservation of mass or pair counts relates a scale in the linear regime to a nonlinear “collapsed” scale

$$r^2_L = \int_0^R d(r^3) [1 + \xi(r)] = R^3 [1 + \sigma^2(R)].$$ \hspace{1cm} (11)

The variance $\sigma^2$ is then a (nearly) universal function $\sigma^2(R) = g[\sigma^2_L(r_L)]$. The relation is verified and the function $g$ identified in numerical simulations (Hamilton et al. 1991; Peacock & Dodds 1996).

In Figure 1 we show nonlinear $\sigma$ plotted against the inferred linear $\sigma$ (bundle of curves); for comparison the isolated solid line shows $\sigma_L$. The perturbation theory result of equation (9) is shown by the long-dashed line. To verify that perturbative result still makes sense even when $\sigma_L \approx 1$, we also plot the phenomenological mapping results for a variety of parameters. In this mapping we must identify why $\sigma_L$ has changed, which may be from a change of fluctuation amplitude, evolution epoch, or scale; each has a slightly different effect. In general, the perturbative curve agrees well with the phenomenological results, except when Hubble constant or scale changes. This occurs because a change in scale substantially changes the value of $\gamma$ and so of $\beta$. This is yet another confirmation of the reliability of perturbative calculations. It also shows that the PSCz $\xi(r)$, assumed in the perturbative calculation, agrees well with the $\Lambda$CDM $\xi(r)$, assumed in the phenomenological mapping.

In the phenomenological relation, the nonlinear signal range

$$\sigma = 1.13^{+0.22}_{-0.22},$$ \hspace{1cm} (12)

maps to

$$\sigma_L = 1.02^{+0.16}_{-0.18}.$$ \hspace{1cm} (13)

Note that since $\sigma$ is steeper than $\sigma_L$, the range in $\sigma_L$ is a bit narrower than the one in $\sigma$. These values and the resulting limits are also show in 1.
Fig. 1.— Fully evolved variance $\sigma$ plotted vs. linear variance $\sigma_L$. For comparison, the isolated solid line shows $\sigma = \sigma_L$. Different curves show different mappings produced from the variation of several different parameters: the short-dashed line differs by a change of amplitude; the solid line differs in redshift or amount of evolution (and hence saturates at $\sigma_L \approx 0.95$ as discussed in Eq. 1); the dotted line differs in Hubble constant ($h = 0.47 - 1.7$) or scale ($R = 5 - 19$ Mpc) which changes $n_{\text{eff}}$ and thus $\gamma$ and so tracks a little differently from the others. Second order perturbation theory as in Eq. (5) gives the long-dashed line. We also show the WMAP (Dunkley et al. 2008) result ($\sigma_L = 0.80$, which maps to $\sigma = 0.85$), and that from measurements of cosmological flows ($\sigma = 1.13$ corresponding to $\sigma_L = 1.02$, with the vertical and horizontal intervals showing the range in Eqs. (12) and (13)).

3. Discussion

Like many cosmological experiments, measurements of cosmic flows are sensitive to a windowed integral of the matter power spectrum. In general, such an observable can be characterized as

$$x_i = \int_0^\infty dk \ W(k) P_p(k) T^2_i(k)$$

where $P_p(k)$ gives the primordial power spectrum (typically but not necessarily parameterized as $P_p(k) \propto k^{n_s}$, where $n_s$ is the spectral index), $T^2_i(k)$ gives the transfer function which contains the physics of the evolution of the particular observable from the primordial spectrum, and $W(k)$ which describes the experimental setup (e.g., sky coverage and depth). This formalism describes straightforward measurements of the galaxy power spectrum, in which case $x_i = P_{\text{gal}}(k_i)$, the CMB spectrum for which $x_i = C_{\ell_i}$, and, in this work, the amplitude of the cosmological velocity field. In fact, the details of the latter depend on which measurement is performed. It is crucial to note that the transfer function depends implicitly upon the other cosmological parameters and hence the lack of knowledge thereof will (or at least should) translate to increased uncertainty upon the amplitude.

Bulk flow and shear measure the velocity-velocity power spectrum (or covariance), whereas pairwise velocities measure the density-velocity cross-spectrum. Under linear evolution, the density contrast is proportional to the divergence of the peculiar velocity in real space, or $v \propto k \rho$ in Fourier space, so these power spectra differ by powers of wavenumber $k$ from the density power spectrum, which can be absorbed into the appropriate transfer function.

An amplitude parameter such as $\sigma_8$ is in essence a spectral observable as well. We define $\sigma_8$ as the root-mean-square amplitude of fluctuations, averaged in spherical balls of radius $8h^{-1}$ Mpc. Using a slightly more general notation, we define the window function $W_R(r, r') = W_R(r - r')$, normalized so that $\int d^3r \ W_R(r) = 1$. For our spherical top hat, $W_R(r) = 1/(4\pi R^3)$ when $|r' - r| \leq R$, 0 otherwise, so the windowed density contrast around a particular point $r$ is simply

$$\delta_R(r) = \int d^3s W_R(r - s) \delta(s)$$

(15)
Because the density contrast has a vanishing ensemble average (equivalent to spatial average under the assumption of ergodicity), the windowed density contrast is also zero-mean. However, its mean square satisfies

$$\sigma_R^2 \equiv \langle (\delta_R(r))^2 \rangle = \int_0^\infty \frac{dk}{k} |\tilde{W}_R(k)|^2 k^3 P(k)/(2\pi^2)$$

(16)

where $\tilde{W}_R(k)$ is the Fourier transform of $W_R(r)$ and $P(k)$ is the power spectrum. The mean square fluctuation that we would measure from the actual density contrast depends upon the actual nonlinear power spectrum, but we can analogously define the linear variance by replacing the power spectrum in the above expression with the linear spectrum, constrained to have the same amplitude for $k \rightarrow 0$ (i.e., on large scales where nonlinear evolution is irrelevant). As we have seen above, the difference between the linear and nonlinear variance can be 10-15%, and depends upon the value of the cosmological parameters.

Rather then directly measure the variance in $\delta_R(r)$, we usually indirectly measure $\sigma_s$ through the power spectrum itself; knowledge of the cosmological parameters then allows us to determine the linear $\sigma_s$ parameter via integration. Absent a direct measurement of the power spectrum itself over all relevant scales, a measurement of an amplitude parameter such as $\sigma_s$ thus requires the deconvolution of the actions of the transfer function and the window function, as well as a careful consideration of exactly the definition of the quantity (i.e., linear vs. nonlinear evolution and therefore possible deconvolution of this effect as well).

It is useful to recall the original motivation for the use of $\sigma_s$ to define the clustering amplitude: the variance in the number density of optical galaxies was observed to be unity when measured in $8h^{-1}$ Mpc balls (Davis & Peebles 1983; Strauss & Willick 1995). Hence, a scale-independent linear bias (Kaiser 1988), defined as the square root of the ratio of the number density variance to the mass variance has a simple expression in terms of $\sigma_s$ ($b^2 = \sigma_s^2/\sigma_{\text{gal}}^2 = 1/\sigma_{\text{mass}}^2$). Since this original work, of course, the model for bias has become more complicated, depending on the details of the galaxy population (e.g. morphology, prevailing neighborhood density, magnitude limit, etc.), scale, and redshift.

Here we review various recent measurements of the linear $\sigma_s$ in this context. They are summarized in Table 1.

**Deep and distant surveys**

**CMB:** Because the CMB itself measures a complicated (albeit linear) functional of the power spectrum, and because of degeneracies in the determination of the CMB power spectrum from the cosmological parameters, the value of $\sigma_s$ from the CMB depends on details of the data-analysis procedure, in particular on the assumed Bayesian priors on the cosmological parameters considered. The “recommended” value for a flat $\Lambda$CDM model (http://lambda.gsfc.nasa.gov) is $\sigma_s = 0.80 \pm 0.04$, but considering different models and priors on those parameters can give variations over $0.7 \lesssim \sigma_s \lesssim 0.9$.

**Ly$\alpha$:** Any measurements of cosmological parameters using the Ly$\alpha$ forest at $z > 1$ depends crucially on detailed comparison between observations and numerical models as well as the determination of the amount of absorption, known as the flux decrement. However, there are wide disagreements about the value of the decrement (Croft et al. 2002; Zaldarriaga et al. 2003; Seljak et al. 2006). Once the bias parameters from calculations are found, the normalization for the mass power spectrum that is a larger scale and earlier epoch analog of $\sigma_s$ can be estimated. The value of $\sigma_s$ found without Ly$\alpha$ data is typically smaller than $\sigma_s$ found incorporating Ly$\alpha$ data, e.g. Tytler et al. (2004) found $\sigma_s = 0.85 \pm 0.02$ with and $\sigma_s = 0.80 \pm 0.03$ without Ly$\alpha$ data, results that agree with the trend indicated by the thesis presented in this paper.

**Clusters:** The number density of rich clusters of galaxies depends upon the nonlinear evolution of the matter power spectrum. In practice, the number density of collapsed halos can be predicted fairly well using the Press & Schechter (1974) formalism, although matching these predictions to the measured X-Ray flux or optical richness of clusters can be difficult and require considerable expertise in modeling. Recent measurements give $\sigma_n = 0.75 \pm 0.01$ (Vikhlinin et al. 2008).
Intermediate-scale surveys

Cosmic Shear: Weak lensing measurements of cosmic shear are sensitive to a complicated line-of-sight integral of the matter density, which translates into a complicated effective window function. Moreover, lensing measurements are complicated by considerable systematic uncertainty due to instrumental effects and intrinsic galaxy properties. Current medium-scale (100 square degrees) weak lensing measurements (Benjamin et al. 2007) give $\sigma_8 = (\Omega_m/0.24)^{0.59} = 0.84 \pm 0.05$.

Sunyaev-Zeldovich (ACBAR): On smaller angular scales, the CMB is sensitive to fluctuations due to the Sunyaev-Zeldovich effect, the scattering of CMB photons by hot cluster gas. The amplitude of this effect is due to the small-scale, highly-nonlinear power spectrum that scales as $\sigma_8^4$ (Reichardt et al. 2008). Current measurements probe the epoch of cluster formation and provide an independent measure of $\sigma_8$. Reichardt et al. (2008) find $\sigma_8 = 0.94^{+0.03}_{-0.04}$ from secondary cluster SZ anisotropies in the CMB. In the future we hope to have a direct measurement of the number density of SZ clusters using experiments such as Planck, AMI and AMIBA.

Galaxies: When estimating the power spectrum (PS) from galaxy surveys (Cole et al. 2005; Tegmark et al. 2004; Eisenstein et al. 2005) the value of $\sigma_8$ usually denotes the normalization whereas the densities of the various matter constituents determine the shape of the PS. However, the normalization has some interesting degeneracies with the shape parameters and it should be noted that its meaning is not straightforward and depends on the depth of the survey. In general, the amplitude is determined at some mean redshift ($z > 0$) whereas the fitted shape parameters are estimated strictly at $z = 0$. The PS normalization is estimated using the rms density contrast averaged over spheres of 8 $h^{-1}$Mpc radius. Thus the normalization of the estimated PS corresponds to an estimated value $\sigma_8^\text{gal}$ which should not be confused with an estimate for the true value of $\sigma_8$. A typical value for the parameter is given by Szalay et al. (2003) to be $\sigma_8 = 0.92 \pm 0.06$.

Local surveys

Flows: Bulk flows, shear and pairwise velocity studies incorporate direct measurements of the peculiar velocities of galaxies to estimate cosmological parameters and specifically $\sigma_8$. From the SFI+++ survey (Masters et al. 2006) an estimate of $\sigma_8 \Omega^{0.6} = 0.52 \pm 0.06$, Zaroubi et al. (2002) using density-density and velocity-velocity comparisons found $\sigma_8 \Omega^{0.6} = 0.45 \pm 0.05$ both of which translate to $\sigma_8 \gtrsim 1$. At scales of $100 h^{-1}$ Mpc, Lavaux et al. (2008) found $\sigma_8 = 1.31 \pm 0.80$ from 2MASS, although they only directly measure the velocity field within $30 h^{-1}$ Mpc, on which scales they estimate a somewhat lower fluctuation amplitude. As mentioned above, pairwise velocity analysis (Feldman et al. 2003) estimates give $\sigma_8 = 1.13^{+0.22}_{-0.23}$.

Recent bulk flow measurements using the best available peculiar velocity surveys (Watkins et al. 2008) require $\sigma_8 > 1.11 (0.88)$ at a 95% (99%) confidence level.

4. Conclusion

We presented a formalism to calculate the amplitude of cosmological density fluctuations, $\sigma_8$, that incorporates the nonlinear evolution of the parameter. Estimates of $\sigma_8$ depend directly on the epoch and scale of the surveys used. When using deep, high-redshift surveys (CMB, Lyα, clusters, although it remains to be seen whether the latter truly belong to this category despite the low value of $\sigma_8$), the results are low values of $\sigma_8$. When analyzing shallow, local data (peculiar velocities) we get high $\sigma_8$. Galaxy redshift surveys give intermediate results for $\sigma_8$. We showed that nonlinear effects tend to systematically suppress $\sigma_8$ estimates when nonlinear effects are not taken into account. When estimates from deep surveys are being corrected for this effect, most estimates from various independent surveys agree quite well with each other.

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Table 1: $\sigma_8$ from various estimators

| Method                              | Parameter value |
|-------------------------------------|-----------------|
| CMB (Dunkley et al. 2008)           | $\sigma_L = 0.80 \pm 0.04$ |
| LY-α (Tytler et al. 2004)           | $\sigma_L = 0.85 \pm 0.02$ |
| Cosmic Shear (Benjamin et al. 2007) | $\sigma_L = 0.84 \pm 0.05$ |
| Clusters (Vikhlinin et al. 2008)    | $\sigma_L = 0.75 \pm 0.01$ |
| SZ (ACBAR) (Reichardt et al. 2008) | $\sigma_8 = 0.94^{+0.03}_{-0.04}$ |
| Galaxies (Cole et al. 2005; Tegmark et al. 2004; Eisenstein et al. 2005) | $\sigma^\text{gal}_8 = 0.92 \pm 0.06$ |
| Flows                               |                 |
| Pairwise velocities (Feldman et al. 2003) | $\sigma_8 = 1.13^{+0.22}_{-0.23}$ |
| Bulk flow (Watkins et al. 2008)     | $\sigma_8 > 1.11 (0.88) \text{ at } 95\% (99\%)$ |

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