A New Qualitative Prediction of the Parton Model for High Energy Hadron Collisions

Victor T. Kim†, Grigorii B. Pivovarov‡ and James P. Vary §

†: St.Petersburg Nuclear Physics Institute, 188350 Gatchina, Russia
‡: Institute for Nuclear Research, 117312 Moscow, Russia
§: International Institute of Theoretical and Applied Physics, Iowa State University, Ames, Iowa 50011-3022, USA

Abstract

Inclusive single jet production in hadron collisions is considered. It is shown that the QCD parton model predicts a nonmonotonic dependence of the inclusive cross section on the fraction of the energy deposited in the jet registered, if it is normalized on the same cross section measured at another collision energy. Specifically, if the cross section is normalized by the one measured at a higher collision energy, it possesses a minimum which depends on jet rapidity. This prediction can be tested at the Fermilab Tevatron, at the CERN LHC, and at the Very Large Hadron Collider under discussion.

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1 e-mail: kim@pnpi.spb.ru
2 e-mail: gbpivo@ms2.inr.ac.ru
3 e-mail: jvary@iastate.edu
The parton model, improved with the QCD running of the coupling and of the structure functions, has the well-deserved status of a paradigm. Its diverse justifications are of two kinds: the qualitative justifications of the early days of the parton model (like the prediction of the relation between structure functions $F_1$ and $F_2$ governed by the spin of the partons (Gross relation), for a review see, e.g., Ref. [1]) and the present-day abundant quantitative justifications (for a recent review, including a list of structure function parameterizations, see Ref. [2]).

The advent of the Fermilab Tevatron and the CERN LHC provides a new testing ground for the parton model—the kinematic conditions when the energies of the produced hadrons are large enough to be described by perturbation theory and, at the same time, are much smaller than the total energy of the collision (semi-hard kinematics). Because the parton model was originally invented and subsequently tested for the hard kinematics, the second condition makes it plausible that a substantial modification of the parton model will be needed to describe this semi-hard kinematic region. Note that for hard kinematics the transverse energy of the produced hadrons is on the order of the total collision energy. In particular, some fusion of Regge theory (see, e.g., Ref. [3]) and perturbative QCD may be required for the semi-hard kinematics. The Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [4] and, more generally, the experience with the resummation of the large energy logarithms (for a review see Ref. [5]) may be useful in constructing this new model.

The range of applicability of the QCD improved parton model is a subject of controversy at the moment. There are statements (see, e.g., [6, 7]) that the fitting capacity of the conventional parton model is sufficient to accommodate all the data on parton structure functions available at the semi-hard kinematics. On the other hand, the data from HERA on forward jet production at small-$x$ [8] may be interpreted as a manifestation of the BFKL Pomeron [4], which seems to be beyond the limits of the conventional parton model. The situation is further complicated by the observation that the range of applicability of the parton model may be different for different observables. In particular, the cross sections of processes with specific kinematics exhibit breakdown of the applicability of finite order perturbative QCD via the loss of insensitivity to the choice of the normalization scale. On the other hand, some dedicated combinations (ratios) of cross sections may be less sensitive to the inclusion of the higher order corrections. An example is the scaled cross section ratio [10, 11, 12, 13] since, as it follows from Ref. [14], it is relatively insensitive to the inclusion of the next-to-leading (NLO) correction.

Under these circumstances, it is crucial to have qualitative predictions from the conventional QCD-improved parton model (without resummation of the energy logarithms) for the new kinematic domain. If the predictions would turn out qualitatively incorrect, a substitute for the parton model would become indispensable.

In this paper, we present such a prediction. It is a prediction for the ratio of inclusive single jet production at a smaller energy $\sqrt{s_N}$ of the hadron collision to the one at a higher energy $\sqrt{s_D}$:

$$R(x, y) = \left( \frac{s_N d\sigma(s_N)}{dxdy} \right) / \left( \frac{s_D d\sigma(s_D)}{dxdy} \right).$$  

(1)

Here the cross section is made dimensionless by the rescaling with the corresponding total invariant energy of the collision squared $s_N(s_D)$. The ratio depends on the (pseudo)rapidity
\[ y = 1/2 \ln(k_+/k_-), \] where \( k_\pm = E \pm k_3 \) are the light-cone components of the momentum of the produced jet, and on the fraction of the energy \( x = (k_+ + k_-)/\sqrt{s_i}, \) \( i = N, D \) deposited in the jet produced (\( s_N \) is used for the definition of \( x \) in the numerator, \( s_D \) in the denominator, so \( x \) varies from zero to unity for both energies). Note that this scaling variable coincides in the center of mass system with \( x_R = E/E_{\text{max}} = 2E/\sqrt{s}, \) the radial Feynman variable.

The ratio \( R \) (taken at \( y \approx 0 \), i.e., for jets perpendicular to the collision axes) was used in Refs. [10, 11, 12] as a means to test QCD predictions for scaling violations. Note that without scaling violations the ratio \( R \) is exactly unity. The \( x \)-dependence of \( R \) comes from the presence of \( \Lambda_{QCD} \) in the running coupling and in the parton distribution functions.

Perturbative QCD calculations of Ref. [14] with hard kinematics (\( Q^2 \sim s \)) predict for \( R \) at \( y = 0 \) a steep increase around the value of 1.8-1.9 for \( x \) growing in the range above 0.1 (for the case \( \sqrt{s_N}/\sqrt{s_D} = 0.63 \text{ TeV}/1.8 \text{ TeV} \)). For moderate \( x \), the prediction is in reasonable agreement with CDF data [11, 12]. For \( x < 0.1 \) calculations are above the preliminary data of CDF [12]. This was among the reasons for the conclusion of Refs. [14, 15] that NLO perturbative QCD [16] with hard kinematics is insufficient for the description of absolute cross section of jets with transverse energy less than 50 GeV within accuracy 10\%. It was shown in Ref. [17] that resummation of the energy logarithms restores the agreement between theory and experiment.

In this paper we present the following result: the QCD-improved parton model predicts that \( R \) is not a monotonic function of its arguments, i.e. the single jet production cross section, if measured in the natural units of the same cross section taken at another (higher) energy of the collision, has extrema. Namely, it has minima (“dips”): there is a value of \( x \) for each \( y \) with the smallest ratio of jets produced. The reason this fact was overlooked is that for \( y = 0 \) (the only value for which the calculations were reported earlier) the minimum is at a value of \( x \) too small to be inside the acceptance of the existing detectors (\( x_{\text{dip}}(y = 0) < 0.01 \) at the Tevatron).

Fig. 1 presents the ratio for energies 0.63 TeV/1.8 TeV at the Fermilab Tevatron, Fig. 2 for energies 6 TeV/14 TeV at the CERN LHC, and Fig. 3 for energies 6 TeV/100 TeV of the Very Large Hadron Collider (VLHC) proposal [18]. Each curve on the plots presents the dependence of the ratio on \( x \) at different values of rapidity \( y \). Each curve ends at a lowest value of \( x \) where \( \alpha_S(Q^2) = 4\pi/[(11 - 2/3n_f)\ln(Q^2/\Lambda_{QCD}^2)] \) has the value of about 0.5 (it corresponds to \( Q = 0.7 \text{ GeV} \), and \( Q \) was taken to be half of the transverse energy of the jet produced). For lower values of \( x \) perturbative theory becomes unreliable because the coupling approaches unity.

There is another lower bound on the values of \( x \) at which our plots make sense, because there is a lowest energy for which the jet may be resolved. This energy is accepted now to be around 5 GeV [1] which corresponds to \( x > 0.016 \) for the Tevatron (Fig. 1), and to \( x > 0.0017 \) for the LHC (Fig. 2).

The cross sections for the plots of Figs. 1-3 were calculated with the formula

\[
\frac{sd\sigma}{dydx} = \frac{\pi [C_A\alpha_S(Q^2)]^2}{x} \left[ I(x, y, Q^2) + I(x, -y, Q^2) \right],
\]

where

\[
I(x, y, Q^2) = \frac{\pi [C_A\alpha_S(Q^2)]^2}{x} \left[ I(x, y, Q^2) + I(x, -y, Q^2) \right],
\]

\( ^4 \)At the HERA, lepton-hadron collider, jets are resolved from \( E_\perp = 3 \text{ GeV} \), and at the Tevatron, hadron-hadron collider, e.g., CDF Collaboration used to tagging jets from \( E_\perp = 8 \text{ GeV} \) [19].
where \( C_A = 3 \) for \( SU(3) \), and the two terms in the brackets of the right-hand-side correspond to the contributions with balancing jet of rapidity over \((I(x, y, Q^2))\) and below \((I(x, -y, Q^2))\) the one of the registered jet (the balancing jet is the unregistered jet whose transverse momentum balances the one of the registered jet; the presence of two terms corresponds to the symmetry of the cross section by which it is an even function of \( y \)); \( I(x, y, Q^2) \) is a convolution of the parton distribution functions; the normalization point for the coupling and for the parton distribution functions is chosen to be \( Q = 0.5E_\perp \), where \( E_\perp \) is the transverse energy of the jet produced [4].

In the region of small \( x \) (the only one we are interested in), the convolution \( I(x, y, Q^2) \) of the parton distribution functions can be expressed (if one neglects powers of small \( x \)) via the effective parton distribution functions \( F(x, Q^2) \) and the effective subprocess of Ref. [20]:

\[
I(x, y, Q^2) = \int_{z_{\text{min}}}^{z_{\text{max}}} dz F(\phi_+(x, y; z), Q^2) F(\phi_-(x, y; z), Q^2),
\]

where

\[
\phi_{\pm}(x, y; z) = \frac{x(1 \pm z^{\pm})}{1 + e^{\mp 2y}},
\]

and \( z_{\text{max}}, z_{\text{min}} \) are determined by \( \phi_+(x, y; z_{\text{max}}) = 1, \phi_-(x, y; z_{\text{min}}) = 1 \). In the calculation, \( F(x, Q^2) \) of the proton was used which coincides with that of the antiproton. Thus, the plots of Figs. 1-3 correspond to \( pp \) or \( p\bar{p} \) collision.

There is an important issue concerning the accuracy of present leading order (LO) calculation. The most important advantage of the scaled cross section ratio is that this is the ratio of two perturbative series with the same coefficients and with different scales in the
Figure 3: "Dips" at VLHC energies

running coupling. These scales are defined by the two initial collision energies at fixed scaling variable. One can show that the theoretical accuracy of the ratio in LO of perturbative QCD is at least not less than the accuracy of NLO calculations for absolute cross sections.

The minima in Figs. 1-3 originate from a competition between the running of the parton distribution functions and the running of the coupling constant. Namely, the ratio with frozen parton distribution functions is decreasing monotonously (this tendency is realized at small $x$), while the one with frozen coupling constant is growing monotonously (this latter tendency is realized for $x$ larger than the position of the minimum).

We suggest the following potential implications of the minima we have predicted with the parton model: (i) If one observes the minima experimentally, one employs the orthodox parton model and tries to account for observed positions and depths of the minima by taking into account higher order corrections, in particular, resummation of the energy logarithms. (ii) If one does not observe the minima experimentally, more radical changes are motivated such as an alternative model of the elementary constituents inside the hadrons for semi-hard asymptotics. One example might be the color dipole model [21].

Finally, we comment on the possibility for searching the minima at the Fermilab Tevatron, at the CERN LHC, and at the VLHC: Positions of the minima for the Tevatron energies (see Fig. 1) seem to be reached for both DØ and CDF detectors. The minima of the LHC plot (see Fig. 2) seem to be well inside the acceptance of, e.g., the FELIX [22], the ALICE [23] and the CMS [24] detectors.

We take the ratio of 6 TeV/14 TeV for the LHC, because, in addition to 14 TeV $pp$ collisions, lead-lead collisions at the LHC are planned with the collision energy of 6 TeV per nucleon-nucleon collision. Since nuclear collisions bring in nuclear effects which can distort our predicted curves, we also considered the ratio 6 TeV/100 TeV (Fig. 3). These latter predictions correspond to the VLHC energies [18].

Further consideration should be given for which pair of energies and value of rapidity
are most convenient for an experimental search of the minima. Also, more work is needed to make quantitative predictions for the locations and the shapes of the dips with the NLO corrections taken into account.

It is worth noting that in the case of nuclear collisions the effects of initial nuclear parton distributions (small-x EMC-effect \[25\]) and dynamical effects like quark-gluon plasma, jet quenching, etc. \[26\] will demand special consideration. We plan to consider these and related issues in future efforts.

Before making conclusions, we would like to note that many of the above ideas can be studied also for the case of heavy quarkonium production, where similar phenomena should be present \[27\].

To sum up, we find a new qualitative prediction of the QCD-improved parton model for hadron collisions and suggest its use to test the applicability of the parton model for certain regions of high energy hadron collisions.

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References

[1] B.L. Ioffe, V.A. Khoze and L.N. Lipatov, Hard Processes. Vol.1: Phenomenology, Quark Parton Model (North Holland, Amsterdam, 1984);
R.D. Field, Applications of Perturbative QCD (Addison-Wesley, Redwood City, 1989) Frontiers in Physics, Vol. 77

[2] CTEQ Collaboration, R. Brock et al., Rev. Mod. Phys. 67, 157 (1995);
H.L. Lai et al., Phys. Rev. D55, 1280 (1997)

[3] P.D.B. Collins, An Introduction to Regge Theory and High-Energy Physics (Cambridge, England, 1977)

[4] L.N. Lipatov, Yad. Fiz. 23, 642 (1976) [Sov. J. Nucl. Phys. 23, 338 (1976)];
E.A. Kuraev, L.N. Lipatov and V.S. Fadin, Zh. Eksp. Teor. Fiz. 71, 840 (1976) [Sov. JETP 44, 443 (1976)]; 72, 377 (1977) [45, 199 (1977)];
Ya.Ya. Balitskii and L.N. Lipatov, Yad. Fiz. 28, 1597 (1978) [Sov. J. Nucl. Phys. 28, 822 (1978)];
L.N. Lipatov, Zh. Eksp. Teor. Fiz. 90, 1536 (1986) [Sov. JETP 63, 904 (1986)]

[5] L.N. Lipatov, Phys. Rep. C286, 131 (1997)
[6] R.D. Ball and S. Forte, Phys. Lett. 405B, 317 (1997)

[7] F. Barreiro, C. Lópezm and F.J. Ynduráìn, Zeit. Phys. C72, 561 (1996);
F.J. Ynduráìn, FTUAM 96-19 (1996) Madrid, hep-ph/9605265

[8] ZEUS Collaboration, J. Breitweg et al., DESY-98-050 (1998), hep-ex/9805016
H1 Collaboration, C. Adloff et al., DESY-98-143 (1998), hep-ex/9809028, submitted to Nucl. Phys.

[9] J. Bartels, V. Del Duca, A. De Roeck, D. Graudenz, M. Wüsthoff, Phys. Lett. 384B, 300 (1996)

[10] UA2 Collaboration, J.A. Appel et al., Phys. Lett. 160B, 349 (1985);
UA1 Collaboration, G. Arnison et al., Phys. Lett. 172B, 461 (1986);
CDF Collaboration, F. Abe et al., Phys. Rev. Lett. 62, 613 (1989)

[11] CDF Collaboration, F. Abe et al., Phys. Rev. Lett. 70, 1376 (1993)

[12] CDF Collaboration, T. Devlin, Proc. of XXVIII ICHEP-96, July 25-31, 1996, Warsaw, Poland, edited by Z. Ajduk and A.K. Wroblewski (World Scientific, Singapore, 1997);
A.A. Bhatti, Proc. of the DPF96 Meeting, August 10-15, 1996, Minneapolis, MN (World Scientific, Singapore, 1997)

[13] J. Huston, Plenary talk at the XXIX Int. Conf. on High Energy Physics – ICHEP-98, Vancouver, Canada, July 23 - 29, 1998 (World Scientific, Singapore), hep-ph/9901352;
CDF Collaboration (preliminary), A. Akopian, PhD thesis, Rockefeller Univ. (1999),
http://www-cdf.fnal.gov/physics/new/qcd/QCD.html
D0 Collaboration (preliminary), J. Krane, PhD thesis, Nebraska Univ. (1999),
http://www-d0.fnal.gov/results/publications_talks/thesis/thesis.html

[14] S.D. Ellis, Proc. of the XXVIIIth Rencontres de Moriond, Les Arcs, France, March 20-27, 1993, edited by J. Tran Thanh Van (Editions Frontières, 1993) Vol.2, p.235

[15] W. Giele, talk presented at the CTEQ Symposium, Confronting QCD with Experiment: Puzzles and Challenges, Fermilab, IL, November 7-9, 1996

[16] S.D. Ellis, Z. Kunszt and D.E. Soper, Phys. Rev. Lett. 64, 2121 (1990);
F. Aversa, M. Greco, P. Chiappetta and J.Ph. Guillet, Phys. Rev. Lett. 65, 401 (1990);
W. Giele, E.W.N. Glover and D.A. Kosower, Phys. Rev. Lett. 73, 2019 (1994)

[17] V.T. Kim and G.B. Pivovarov, Phys. Rev. D57, R1341 (1998)

[18] C.W. Foster and E. Malamud, Low Cost Hadron Colliders at Fermilab: a Discussion Paper, FERMILAB-TM-1976 (1996);
Talks in Proc. of the DPF/DPB Summer Study on New Directions for High-Energy Physics, Snowmass ’96;
Talks at the Very Large Hadron Collider Physics and Detector Workshop, Fermilab, March 13-15, 1997
[19] CDF Collaboration, F. Abe et al., Phys. Rev. D57, 67 (1998)

[20] B.L. Combridge and C.J. Maxwell, Nucl. Phys. B239, 429 (1984)

[21] A.H. Mueller, Nucl. Phys. B415, 373 (1994);
    N.N. Nikolaev and B.G. Zakharov, Phys. Lett. 327B, 149 (1994)

[22] K. Eggert and C. Taylor, FELIX: a Full Acceptance Detector for the CERN LHC,
    CERN-PPE-96-136 (1996);
    FELIX Collaboration, E. Lippmaa et al., Letter of Intent, CERN/LHCC 97-45 (1997)

[23] ALICE Collaboration, Technical Proposal, CERN/LHCC 95-71 (1995)

[24] CMS Collaboration, Technical Proposal, CERN/LHCC 94-38 (1994)

[25] EMC Collaboration, J.J. Aubert et al., Nucl. Phys. B293, 740 (1987);
    M. Arneodo, Phys. Rept. C240, 301 (1994)

[26] X.N. Wang, Phys. Rept. C280, 287 (1997)

[27] V.T. Kim, G.B. Pivovarov, H.S. Song and J.P. Vary, in progress