Multi-state discrimination below the quantum noise limit at the single-photon level

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Measurements approaching the ultimate quantum limits of sensitivity are central in quantum information processing, quantum metrology, and communication. Quantum measurements to discriminate multiple states at the single-photon level are essential for optimizing information transfer in low-power optical communications and quantum communications, and can enhance the capabilities of many quantum information protocols. Here, we theoretically investigate and experimentally demonstrate the discrimination of multiple coherent states of light with sensitivities surpassing the quantum noise limit (QNL) at the single-photon level under realistic conditions of loss and noise based on strategies implementing globally-optimized adaptive measurements with single photon counting and displacement operations. These discrimination strategies can provide realistic advantages to enhance information transfer at low powers, and are compatible with photon number resolving detection, which provides robustness at high powers, thus allowing for surpassing the QNL at arbitrary input power levels under realistic conditions.

INTRODUCTION

Assessing the information about the state of a quantum system is fundamentally limited by quantum noise. This task requires performing measurements discriminating among different states, and when these states are nonorthogonal, discrimination cannot be performed perfectly due to their intrinsic overlap. Measurements for the discrimination of nonorthogonal states, such as coherent states, can enable secure communication, and assist quantum information protocols for quantum repeaters, quantum computing, entanglement generation with high fidelity, quantum signatures, and quantum fingerprinting. Moreover, efficient discrimination measurements of multiple coherent states can enhance information transfer in communication and, by performing collective measurements over long sequences of states, allow for achieving the ultimate limits for classical information transfer (classical capacity) in communication channels with loss and noise.

Quantum mechanics allows for measurements that in principle achieve the ultimate sensitivity limits for the discrimination of nonorthogonal states. These measurements can optimize many quantum information and communication protocols. The lowest error probability allowed by quantum mechanics for discriminating nonorthogonal states, such as coherent states with different phases, is referred to as the Helstrom bound. This bound is much lower than the limit of ideal conventional measurements, referred to as the quantum noise limit (QNL), which can be reached by the homodyne (heterodyne) measurement for two (multiple) phase states. Strategies for the discrimination of two coherent states below the QNL have been investigated theoretically and demonstrated experimentally, including the optimal strategy saturating the Helstrom bound based on photon counting and real-time feedback. The discrimination of multiple coherent states below the QNL was first investigated by Bondurant, generalizing the optimal two-state strategy to four states. Further theoretical and experimental work demonstrated that the discrimination of multiple states below the QNL can be achieved with current technology, and that some realizable measurements show the same exponential scaling as the Helstrom bound in the limit of high powers. Recent work showed that photon-number resolution (PNR) provides robustness against realistic noise for discrimination of multiple states. And an experimental realization with detectors with finite PNR demonstrated multi-state discrimination below the QNL at high input-power levels, which provides advantages for coded communication over heterodyne measurement.

Discrimination of multiple states surpassing the QNL in the single-photon regime has a large potential for applications in quantum information and enhancing information transfer in low-power communication. Moreover, optimized measurements for nonorthogonal states at the single-photon level can potentially be used to enhance the rate of secure communications in quantum key distribution (QKD) with coherent states, which require states at low powers to ensure security. However, even though theoretical advancements have shown that this discrimination task is possible, the discrimination of multiple states surpassing the QNL at the single-photon level under realistic conditions with noise and loss has not been experimentally demonstrated.

Here, we theoretically investigate and experimentally demonstrate the discrimination of multiple nonorthogonal states at the single-photon level below the QNL based on adaptive measurements, globally optimized displacement operations, and photon counting. These strategies are compatible with PNR detection to extend discrimination to high powers, and thus enable discrimination strategies of multiple states below the QNL at arbitrary input-power levels under realistic conditions. These optimized discrimination measurements more closely approach the quantum limits of detection for multiple states at the single-photon level, and can be used to optimize quantum information protocols assisted by coherent states, and enhance information transfer in optical communication. We show that these...
optimized measurements can provide advantages for increasing information transfer with coherent states at low powers beyond what can be achieved with the heterodyne measurement at the QNL. We expect that these measurements can be used in joint-detection schemes over sequences of coherent states for approaching the quantum limits for information transfer at the single-photon level.

RESULTS

Optimized discrimination measurements

Figure 1(a) shows the measurement scheme for the discrimination of multiple nonorthogonal states \(|\alpha_k\rangle\) below the QNL at the single-photon level implementing \(N\) adaptive measurements based on displacement operations \(\hat{D}(\beta)\) optimized to minimize the probability of error \(P_e\). This optimization gives rise to discrimination strategies surpassing the QNL at arbitrary input power levels under realistic noise and loss requiring just a few adaptive measurements, while being compatible with PNR, which increases robustness at high powers.

Compared to previous strategies described in Ref.\(^{23}\) that require measurements with infinite feedback bandwidth, the optimized strategies discussed here are based on a finite number of adaptive measurements, which is practical and allows for the demonstration of multi-state discrimination beyond the QNL at the single-photon level with current technologies.

The optimized strategies implement globally optimized displacements \(\hat{D}(\beta)\) of the input state \(|\alpha_k\rangle\) from a set of states with \(|\alpha_k\rangle\in \{|e^{i\phi_k}|\alpha_k\rangle\}\) \((\phi_k = \frac{2\pi k}{M}\) \((k = 1, 2, \ldots, M)\)) to the vacuum state \(|0\rangle\), single-photon counting, and recursive Bayesian inference. In a given adaptive measurement \(j\) \((j = 1, 2, \ldots, N)\), the displacement of the input state \(|\alpha_k\rangle\) results in state \(|\psi\rangle = \hat{D}(\beta)|\alpha_k\rangle\), which is followed by photon counting with probability of \(n\) photon detection given by \(P(n|\alpha_k, \beta) = |\langle n|\hat{D}(\beta)|\alpha_k\rangle|^2 = \frac{\alpha^2}{n!}e^{-\bar{n}}\). Here \(\bar{n} = |\alpha|^2 + |\beta|^2 - 2V|\alpha|\beta\cos[\arg(\alpha_k) - \arg(\beta)]\) is the mean photon number of the state \(|\psi\rangle\) and \(V\) is the visibility of the displacement operation.\(^{23,15}\) After an adaptive-measurement period \(j\), the strategy estimates the posterior probabilities of the possible input states \(|\alpha_k\rangle\) in \(j\) based on the photon detection result \(d_j\in\{0, 1, \ldots\}\) and the displacement field \(\beta\) as:

\[
P_{post}(\{\alpha_k\}|\beta, d_j) = \frac{P(d_j|\{\alpha_k\}, \beta) P_{prior}(\{\alpha_k\})}{\sum_{\{\alpha_k\}} P(d_j|\{\alpha_k\}, \beta) P_{prior}(\{\alpha_k\})},
\]

where \(P_{post}(\{\alpha_k\}|\beta, d_j)\) and \(P_{prior}(\{\alpha_k\})\) are the posterior and prior probabilities in \(j\), respectively, for all possible input states \(|\alpha_k\rangle\), and \(\sum_{\{\alpha_k\}}\) indicates the sum over all states \(|\alpha_k\rangle\). The discrimination strategy uses recursive Bayesian inference to update prior probabilities \(P_{prior}(\{\alpha_k\})\) in subsequent adaptive measurements, so that the posterior probability estimated in adaptive measurement \(j\), \(P_{post}(\{\alpha_k\}|\beta, d_j)\), is defined as the prior probability \(P^j_{prior}(\{\alpha_k\})\) for the next adaptive measurement, \(j + 1\). The phase of the displacement field \(\beta\) in \(j + 1\) is chosen to test the most likely state based on the maximum-posterior probability criterion: the phase of \(\beta\) is set equal to the phase of the state for which \(P_{post}\) is maximum. This criterion for choosing the phases of \(\beta\) is applied to all the optimized strategies described below.

The overall probability of error \(P_e\) after the last adaptive measurement \(N\) can be expressed as:

\[
P_e = 1 - \sum_{D_H} P_{D_H} \max_{\{\alpha_k\}} \left[ P_{N, prior}(\{\alpha_k\}) P(d_N|\{\alpha_k\}, [\beta]) \right]
\]

where \([\beta]\) is the set of all the displacement fields used in all the adaptive measurements \(j = 1, \ldots, N\), which can be optimized to minimize the final probability of error. \(\max_{\{\alpha\}}\) takes the maximum over input states.
\{(α_k)\} of \(P^N_{\text{prior}}(\{α_k\})P(\{d_N|α\},[β^{opt}])\), which is proportional to the posterior probability in \(N\), and the sum \(\sum_{D_{NH}}\) is realized over all detection histories \(D_H = \{d_1, d_2, ..., d_N\}\) with occurrence probabilities \(P_{DH}\). Note that \(P_{\text{prior}}(\{α_k\})\) in Eq. (2) depends on the detection histories \(D_H\) and the displacement fields \([β]\) in all the previous adaptive measurements. The freedom to choose \([β]\) allows for finding global optimizations of the set of displacement field amplitudes \([β]\) that minimize the overall probability of error, and gives rise to different discrimination strategies with different complexities and degrees of sensitivity. These optimized strategies for which \(|β| ≠ |α|\) enable measurements surpassing the QNL in the low-power regime, a task that measurements without optimized amplitudes (non-optimized strategies) for which \(|β| = |α|\) cannot achieve.

The simplest optimized discrimination strategy finds a constant value of the amplitude of the displacement field \([β]\) over all the \(N\) adaptive measurements to minimize the probability of error, so that

\[
\frac{δP_e}{δβ} = 0.
\]  

Figure 1(b) shows the probability of error for the discrimination of four nonorthogonal coherent states \(\{|α\},|ia\rangle,|−α\rangle,|−ia\rangle\) achieved by this strategy for \(N=10\). We observe that this strategy is sufficient to surpass the QNL for multiple states at the single-photon level. This strategy, which we call “flat optimization,” is the discrete version of that described in (34) and only requires a finite \(N\), which translates to measurements with finite bandwidth and provides advantages for realistic implementations.

Performing more sensitive measurements requires implementing global optimization over the magnitudes of the displacement fields \([β]\) in individual adaptive measurements. A simple strategy that we refer to as “sequential optimization” implements \(N\) optimized displacement amplitudes \([β] = \{β_1, β_2, ..., β_N\}\) one in each adaptive measurement, but optimized simultaneously so that

\[
\frac{δP_e}{δ\{β_1,...,N\}} = 0.
\]  

Figure 1(b) shows the improved performance of this strategy over the “flat optimization” strategy. This “sequential optimization” strategy is similar to the optimal discrimination strategy for two states described by Dolinar (35), but for multiple states. We observe that the displacement amplitude ratio \(|β|/|α|\) starts with a large value and decreases as a function of time, as seen in Fig. 1(c). However, in contrast to the optimal measurement for two states (35), the displacement magnitude ratio is not a monotonically decreasing function of time, and may not be optimal as \(N → ∞\).

Higher sensitivities for discrimination of multiple states can be achieved by strategies based on globally optimized displacement operations that are conditional on the detection histories. These strategies have as many optimal displacement amplitudes as the number of possible detection histories \(D_H\) minus one. For detectors without PNR capabilities, i.e., with only two possible detection outcomes, the number of detection histories is \(2^N\), and the displacement amplitudes \([β] = \{β_1, β_2, ..., β_2^N − 1\}\) are optimized to minimize the final probability of error

\[
\frac{δP_e}{δ\{β_1,...,2^N−1\}} = 0.
\]  

These strategies, which we refer to as “historical optimization,” generalize strategies for discrimination of multiple states based on coherent displacements, photon counting and finite number of adaptive measurements with maximum posterior probability criteria.

Figure 2(a) shows the improved performance of the historical optimization strategy over the “flat optimization” and the “sequential optimization.” The historical optimization is closer to the Helstrom limit for a small number of adaptive measurements. Fig. 2(d) shows the highly complex evolution of the optimized displacement amplitudes for this strategy for 100 possible detection histories \(D_H\) for \(N = 10\). It is unknown if the historical optimization strategy would reach the Helstrom limit for \(N → ∞\). Since the number of parameters to globally optimize grows as \(2^N\), finding the optimal \([β]\) for large \(N\) numerically becomes computationally intensive. However, this strategy as well as the flat and sequential optimization strategies can be realized experimentally with current technologies. Once optimal displacements for different strategies are determined, they can be coded in a high-bandwidth electronic controller. The controller updates the values of phase and amplitudes of the optimal displacements conditioned on detection results in each adaptive measurement using fast feedback. In this way, the overhead in computation time for estimating the optimal \([β]\) is done offline, and the complexity in the implementation of different strategies becomes comparable in the experiment. This method allows us to experimentally demonstrate discrimination of multiple nonorthogonal states below the QNL at the single-photon level.

**Experimental Demonstration**

Figure 2(a) shows a schematic of the experimental demonstration of optimized strategies for the discrimination of four coherent states \(\{|α_k\}\) below the QNL. A 633 nm laser with an acousto-optic modulator (AOM) prepares flat top 27 μs long pulses defining the temporal extent of the input state \(|α_k\rangle\). Phase modulator (PM1) prepares the phase of \(|α_k\rangle\) with a given mean photon number \(\langle n\rangle\) calibrated with a transfer-standard calibrated detector. Phase (PM2) and amplitude (AM) modulators prepare the phase and the amplitude of the optimal displacement fields \([β]\) for different optimized discrimination strategies. The displacement operations are performed by interference in a 99/1 beam splitter (BS2), achieving an average visib-
FIG. 2: Experimental implementation of multi-state discrimination with optimized displacements. (a) Experimental configuration of strategies with optimized discrimination of four nonorthogonal coherent states \{ |α⟩, |iα⟩, |−α⟩, |−iα⟩ \} implementing \( N = 10 \) adaptive measurements. A phase modulator PM1 prepares the input state, and phase PM2 and amplitude AM modulators prepare the optimized displacements \[ β \]. The displacement operation is implemented by interference in a 99/1 beam splitter (BS2), and a field-programmable gate array (FPGA) processes the detection result from the single-photon detector (SPD) to prepare the optimized displacements \[ β \] for subsequent adaptive measurements. The phase of \[ β \] is controlled with a 2-bit fast multiplexer switch and PM2, and its amplitude with a 1-byte depth digital-to-analog converter (DAC) and the AM. SMF, single-mode fiber; DM, dichroic mirror; PZT, piezo; AOM, acousto-optic modulator. (b, c) Example of the intensity \( |β|^2 \) of the optimized displacement field \[ β \] for the sequential global optimization strategy for \( N = 10 \), \( DE = 70\% \), \( \nu = 99.6\% \) and \( (n) = 0.5 \) for (b) target and (c) observed.

FIG. 3: Experimental results. Error probability for the discrimination of four nonorthogonal states \{ |α⟩, |iα⟩, |−α⟩, |−iα⟩ \} with three optimized strategies, flat (light-brown dots) sequential (green dots) and historical (blue dots) optimization, and the non-optimized strategy (magenta dots). The Helstrom bound (black line) and the QNL (red line) are shown for reference together with the QNL with the same experimental conditions of detection efficiency \( DE_{ExP} = 70\% \) (gray line). While discrimination strategies without optimized displacements do not surpass the QNL at the single-photon level, globally optimized strategies can reach error rates below the QNL without any correction for detection efficiency as shown for \( (n) = 1 \). Furthermore, for a system with the same \( DE = 70\% \), optimized strategies surpass the QNL at arbitrary small powers in the presence of noise and imperfections. Error bars represent 1 statistical standard deviation from 4 runs of \( 1 \times 10^5 \) independent experiments per data point. The theoretical predictions (dashed lines) are based on numerical simulations with the experimentally determined detection efficiency \( DE_{ExP} = 70\% \), visibility of the displacement \( \nu_{ExP} = 99.6\% \), and dark counts of 0.1% per pulse.

Figure 3 shows the experimental results of the discrimination of four nonorthogonal coherent states below the QNL at the single-photon level based on three optimized strategies: flat optimization, sequential optimization, and historical optimization. Included in the figure are the non-optimized strategy, the Helstrom bound, the ideal QNL, and the QNL scaled to the same system detection efficiency (70%) as in our experiment. The theoretical predictions are shown in dashed lines and are based on numerical simulations using Eq. (2) with the optimal displacements \[ β \] for the flat, sequential, and historical optimization strategies satisfying Eq. (3), Eq. (4), and Eq. (6), respectively, with the experimental parameters...
DISCUSSION
We investigated the potential advantages of optimized strategies for multiple states for increasing information transfer in communication. Recent theoretical work showed that nonconventional receivers for coherent states based on photon counting can provide higher communication rates than what is possible with conventional ones. The optimized discrimination strategies for multiple states presented here can achieve higher levels of information transfer than both non-optimized strategies and what could be achieved with coherent states and heterodyne detection at the single photon level. Fig. 4 shows the mutual information achieved by the “flat optimization” discrimination measurement for four states \{\ket{i\alpha}, \ket{i\alpha}, \ket{-i\alpha}, \ket{-i\alpha}\} with DE=1 and N = 10. This optimized measurement surpasses the capacity of the heterodyne measurement using an arbitrary number of coherent states in the single-photon regime at \langle n \rangle = 1, which is not possible with non-optimized strategies. Other optimized measurements are expected to provide greater advantages over the heterodyne measurement in the single-photon regime. However, we note that flat optimization may be easier to implement than sequential and historical optimizations, and may be more practical in communication, albeit with a slightly lower attainable sensitivity. This example shows the potential of optimized measurements for low-power communication with multiple states to increase capacity based on single-state decoding, which can be further optimized with larger alphabets and optimal distribution of the input state. Moreover, optimized discrimination strategies can assist joint quantum measurements over sequences of coherent states with multiple phases in approaching the ultimate limits in capacity at the single-photon level. While in the present work we used long pulses and feedback with a SPD with deadtime of about 35 ns, the optimized strategies investigated here can be implemented using feed forward, instead of feedback, by splitting the input state in space and using optical delay lines, as described in Refs., allowing for higher repetition rates with much shorter pulses. Together with fast electronic controllers and gated SPD reaching GHz speeds, these optimized strategies could allow for high sensitivity measurements at higher communication rates.

It has been shown that optimized measurements of two nonorthogonal coherent states based on post-selection can lead to higher secret key rates in QKD than what can be achieved with homodyne measurements and post selection. The optimization methods described here for measurements with definite outcomes may be useful for increasing the rate in QKD protocols with measurements without post-selection. These measurements may lead to higher rates than the heterodyne detection, and could be experimentally investigated with our current setup.

Conclusion
Optimized measurements for the discrimination of multiple nonorthogonal states at the single-photon level can increase the information transfer in communications.
and enhance security in quantum communication. We demonstrate discrimination of multiple nonorthogonal states below the QNL at the single-photon level under realistic loss, noise, and system imperfections based on globally optimized strategies requiring only a few adaptive measurements. These optimized discrimination measurements can enable low-power communication with multi-level encoding surpassing the QNL; yield higher information transfer at low powers; and could have applications in QKD with multiple coherent states. Our work makes measurements achieving sensitivities beyond conventional Gaussian measurements a more realistic alternative to enhance information transfer beyond what can be achieved with conventional technologies. We expect that this work will motivate further research in finding optimal technologies. We expect that this work will motivate further work in finding optimal technologies.

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