Cosmological Origin of Small-Scale Clumps and DM Annihilation Signal

Veniamin BEREZINSKY
Laboratori Nazionali del Gran Sasso, Istituto Nazionale di Fisica Nucleare
I-67010 Assergi (AQ), Italy
Vyacheslav DOKUCHAEV
Institute for Nuclear Research of the Russian Academy of Sciences,
60th Anniversary of October Prospect 7a, Moscow 117312, Russia
Yury EROSHENKO
Institute for Nuclear Research of the Russian Academy of Sciences,
60th Anniversary of October Prospect 7a, Moscow 117312, Russia

Abstract

We study the cosmological origin of small-scale DM clumps in the hierarchical scenario with the most conservative assumption of adiabatic Gaussian fluctuations. The mass spectrum of small-scale clumps with $M \leq 10^3 M_\odot$ is calculated with tidal destruction of the clumps taken into account within the hierarchical model of clump structure. Only $0.1 - 0.5\%$ of small clumps survive the stage of tidal destruction in each logarithmic mass interval $\Delta \ln M \sim 1$. The mass distribution of clumps has a cutoff at $M_{\text{min}}$ due to diffusion of DM particles out of a fluctuation and free streaming at later stage. $M_{\text{min}}$ is a model dependent quantity. In the case the neutralino DM, considered as a pure bino, $M_{\text{min}} \sim 10^{-8} M_\odot$. The evolution of density profile in a DM clump does not result in the singularity because of formation of the core under influence of tidal interaction. The radius of the core is $R_c \sim 0.1 R$, where $R$ is radius of the clump. The applications for annihilation of DM particles in the Galactic halo are studied. The number density of clumps as a function of their mass, radius and distance to the Galactic center is presented. The enhancement of annihilation signal due to clumpiness, valid for arbitrary DM particles, is calculated. In spite of small survival probability, the global annihilation signal in most cases is dominated by clumps, with major contribution given by small clumps. The enhancement due to large clumps with $M \geq 10^6 M_\odot$ is very small.
1. Introduction

The gravitationally bound structures in the universe are developed from primordial density fluctuations $\delta(x,t) = \delta \rho / \rho$. They are produced at inflation from quantum fluctuations. The predicted power spectrum of these fluctuations has a nearly universal form $P(k) \propto k^{n_p}$, with $n_p \approx 1$. At radiation-dominated epoch the fluctuations grow logarithmically slowly. After transition at $t = t_{eq}$ to the matter-dominated epoch, the fluctuations grow as $\delta \propto t^{2/3}$. The gravitationally bound objects are formed and detached from cosmological expansion when fluctuations enter the non-linear stage $\delta \geq 1$. The non-linear stage of fluctuation growth has been studied both by analytic calculations [1] and [2] and in numerical simulations [3, 4, 6] for Large Scale Structure (LSS). The density profile in the inner part of these objects is given by $\rho(r) \propto r^{-\beta}$, with $\beta \approx 1.7 - 1.9$ in analytic calculations [2], $\beta = 1$ in simulations of NFW [3] and $\beta = 1.5$ in simulations of Moore et al. [4] and Jing and Suto [6]. In this work we apply this approach to the smallest DM objects in the universe, which we shall call clumps. The clumps, being the smallest structures, are produced first in the universe, and it makes difference of our consideration from LSS formation. The theoretical observation of this work is the importance of tidal interaction in the process of DM clump formation: the central nonsingular core is formed in the clumps and large fraction of clumps are tidally disrupted.

We use in the calculations the hierarchical model, in which due to merging of objects a small clump is hosted by the bigger one, the latter is submerged to more bigger etc. We use the standard cosmology with WMAP parameters. The primordial spectrum index is $n_p = 0.99 \pm 0.04$ (WMAP) or $n_p = 0.93 \pm 0.03$ (WMAP+2dF+Ly$\alpha$).

2. Tidal Destruction of Clumps in Hierarchical Model

The destruction of clumps by the tidal interaction occurs at the epoch of their hierarchical formation, long before the formation of galaxies. This interaction arises when two clumps pass near each other and when a clump moves in the external gravitational field of the bigger host to which this clump belongs. In both cases a clump is exited by the external gravitational field, i. e. its constituent particles obtain additional velocities in the c. m. system. The clump is destroyed if its internal energy increase $\Delta E$ exceeds the corresponding total energy $|E| \sim GM^2/2R$. In [7] we have calculated the rate of excitation energy production by both aforementioned processes. The dominating process is given by tidal interaction in the gravitational field of the host clumps, with the main
contribution from the smallest host clump. We use the Press-Schechter formalism [8, 9] for hierarchical clustering. A small-scale clump during its life can be a constituent part of many host clumps of successively larger masses. After tidal disruption of the lightest host, a small clump becomes a constituent part of the larger host etc. Transition of a small clump from one host to another occurs nearly continuously in time up to the formation of a big enough host, where tidal destruction becomes inefficient.

The fraction of mass in the form of clumps which escape the tidal destruction in each logarithmic mass interval $\Delta \ln M \sim 1$ is found as

$$\xi_{\text{int}} \simeq 0.01(n + 3).$$

In other words the mass function of clumps is $\xi_{\text{int}}dM/M$. Since $n$ is close to $-3$, only a small fraction of clumps about $0.1 - 0.5\%$ survive the stage of tidal destruction. However, this fraction is enough to dominate the total annihilation rate in the Galactic halo.

3. Core of Dark Matter Clump

We use the following parameterization of the density profile in a clump:

$$\rho_{\text{int}}(r) = \begin{cases} 
\rho_c, & r < R_c; \\
\rho_c \left(\frac{r}{R_c}\right)^{-\beta}, & R_c < r < R; \\
0, & r > R.
\end{cases}$$

In [2] the relative core radius of the clump is estimated as $x_c = R_c/R \simeq \delta_{\text{eq}}^3 \ll 1$ from consideration of the perturbation of the velocity field due to damped mode of the cosmological density perturbations. Here $\delta_{\text{eq}}$ is an initial density fluctuation value at the end of radiation dominated epoch. In [10] the core is considered to be produced for spherically symmetric clump by inverse flow caused by annihilation of DM particles. We show [7] that these phenomena are not the main effects and that much stronger disturbance of the velocity field in the central part of clumps is produced by tidal forces. The tidal forces influence the nearly radial motion of DM particles at the time of clump formation. As a result these particles obtain some angular momentum which prevents the formation of singularity. Once the core is produced it is not destroyed in the evolution followed. The core formation proceeds mainly near the time of the clump maximal expansion $t_s$. At this moment the clump decouples from an expansion of the universe and contracts in the non-linear regime. Soon after this period a clump enters the hierarchical stage of evolution, when the tidal forces can destroy it, but surviving clumps retain their cores.
The calculations proceed in the following way (see [7] for details). The background gravitational field (including that of the host clumps) is expanded in series in respect to the distance from the point with maximum density in a fluctuation. The motion of a DM particle in this field is studied. The spherically symmetric term of the expansion causes the radial motion of a particle in the oscillation regime. Spherically non-symmetric term describes the tidal interaction. It results in deflection of a particle trajectory from a center (point with maximum density). The average (over statistical ensemble) deflection gives the radius of the core $R_c$. After statistical averaging, $R_c$ is expressed through the amplitude of the fluctuation $\delta_{eq}$ and the variance $\sigma_{eq}$ (or $\nu = \delta_{eq}/\sigma_{eq}$) as

$$x_c = R_c/R \approx 0.3\nu^{-2}f^2(\delta_{eq}),$$

where the function $f(\delta_{eq}) \sim 1$ is given in Ref [7]. The fluctuations with $\nu \sim 0.5 - 0.6$ have $x_c \sim 1$, i.e. they are practically destroyed by tidal interactions. Most of galactic clumps are formed from $\nu \sim 1$ peaks, but the main contribution to the annihilation signal is given by the clumps with $\nu \approx 2.5$ for which $x_c \approx 0.05$.

4. Clumps of Minimal Mass

The mass spectrum of clumps has a low-mass cutoff at $M = M_{\text{min}}$, which value is determined by a leakage of DM particles from the overdense fluctuations in the early universe. CDM particles at high temperature $T > T_f \sim 0.05m_\chi$ are in the thermodynamical (chemical) equilibrium with cosmic plasma. After freezing at $t > t_f$ and $T < T_f$, the DM particles remain for some time in kinetic equilibrium with plasma, when the temperature of CDM particles $T_\chi$ is equal to temperature of plasma $T$. At this stage the CDM particles are not perfectly coupled to the cosmic plasma. Collisions between a CDM particle and fast particles of ambient plasma result in exchange of momenta and a CDM particle diffuses in the space. Due to diffusion the DM particles leak from the small-scale fluctuations and thus their distribution obtain a cutoff at the minimal mass $M_D$.

The DM particles get out of the kinetic equilibrium when the energy relaxation time for DM particles $\tau_{\text{rel}}$ becomes larger than the Hubble time $H^{-1}(t)$. This conditions determines the time of kinetic decoupling $t_d$. At $t \geq t_d$ the CDM matter particles are moving in the free streaming regime and all fluctuations on the scale of free-streaming length $\lambda_{fs}$ and smaller are washed away. The corresponding minimal mass $M_{fs} = (4\pi/3)\rho_\chi(t_0)\lambda_{fs}^3$, is much larger than $M_D$ and therefore $M_{\text{min}} = M_{fs}$. In [7] we have performed the calculations using two methods: the transparent physical method, based on the description of diffusion and free streaming, and more formal method based on solution of kinetic equation.
for DM particles starting from the period of chemical equilibrium. Both methods agree perfectly. The minimal mass in the DM mass distribution is determined by the process of free-streaming. For the case of neutralino (bino) as DM particle this minimal mass equals to $M_{\text{min}} = 1.5 \times 10^{-8} M_\odot$ for neutralino mass $m_\chi = 100$ GeV and the mass of selectron and sneutrino $M = 1$ TeV. Our calculations agree reasonably well with that of [11], while $M_{\text{min}}$ from [12] coincides with our value for $M_D$.

5. Annihilation Signal Due to Small Clumps

There is distribution of clumps in the Galactic halo over three parameters, mass $M$, radius $R$, and distance from the Galactic Center $l$: $n_{\text{cl}}(M, R, l)$. This distribution also depends on the parameters which describe the internal structure of the clumps, $\beta$ and $x_c = x_c(M, R)$, from Eq. [2]. With the number density of clumps in the halo written as $dN_{\text{cl}} = n_{\text{cl}}(l, M, R)d^3ld^MldR$, the observed signal at the position of the Earth from DM particle annihilation in the clumps is given by quantity

$$I_{\text{cl}} = \frac{1}{4\pi} \int_0^\pi d\zeta \sin \zeta \left( \int_0^{r_{\text{max}}(\zeta)} \frac{2\pi r^2dr}{r^2} \int_{M_{\text{min}}}^{M_{\text{max}}} dM \int_{R_{\text{min}}}^{R_{\text{max}}} dR \right) \times n_{\text{cl}}(l(\zeta, r), M, R)\mathcal{N}_{\text{cl}}(M, R),$$

where $r$ is distance from the Sun (Earth) to a clump and $\zeta$ is angle between the line of observation and the direction to the Galactic center, $r_{\text{max}}$ is the distance from the Sun to the halo’s outer border and $\mathcal{N}_{\text{cl}}(M, R)$ is annihilation rate in the single clump of mass $M$ and radius $R$.

Additional annihilation signal is given by unclumpy DM in the halo with homogeneous (i.e. smoothly spread) density $\rho_{\text{DM}}(l)$, where $l$ is a distance to the Galactic Center.

$$I_{\text{hom}} = \langle \sigma_{\text{ann}}v \rangle \frac{\pi}{2} \int_0^\pi d\zeta \sin \zeta \left( \int_0^{r_{\text{max}}(\zeta)} dr^2 \rho_{\text{DM}}^2(l(\zeta, r))/m_\chi^2 \right).$$

The enhancement $\eta$ of the signal due to a presence of clumps is given by

$$\eta = \frac{I_{\text{cl}} + I_{\text{hom}}}{I_{\text{hom}}}.$$
Fig. 1. The global enhancement $\eta$ of the annihilation signal from the Galactic halo as a function of the minimal clump mass $M_{\text{min}}$, for clump density profile with index $\beta = 1.5$ and for different indices $n_p$ of primeval perturbation spectrum. The curves are marked by the values of $n_p$.

that space density of clumps in the halo, $n_{\text{cl}}(l)$ is proportional to the unclumpy DM density, $\rho_{\text{DM}}(l)$: $n_{\text{cl}}(l) = \xi \rho_{\text{DM}}(l)/M$ with $\xi \ll 1$. This assumption holds with a good accuracy for the small-scale clumps. The signal from small clumps is determined mainly by clumps of the minimal mass. In calculations [7] we used different density profiles in the clumps, the distribution of DM clumps over their masses $M$ and radii $R$, and the distribution of clumps in the galactic halo. The enhancement depends on the nature of DM particle only through $M_{\text{min}}$. For $\beta = 1.5$ enhancement is given in Fig. 1. Details of calculations and the plots for other values of $\beta$ can be found in [7]. The enhancements $\eta$ for $n_p = 1$ or less is not large: typically it is 2-5 for $M_{\text{min}} \sim 10^{-8} M_\odot$. For example, $\eta = 5$ for $n_p = 1.0$ and $M_{\text{min}} = 2 \cdot 10^{-8} M_\odot$. It strongly increases for smaller $M_{\text{min}}$ and larger $n_p$. For example, for $n_p = 1.1$ and $M_{\text{min}} = 2 \cdot 10^{-8} M_\odot$, enhancement is $\eta = 130$. Our approach is based on the hierarchical clustering model in which smaller mass objects are formed earlier than the larger ones, i.e. $\sigma_{\text{eq}}(M)$ diminishes with the growing of $M$. This condition is satisfied for objects with mass $M > M_{\text{min}} \simeq 2 \cdot 10^{-8} M_\odot$ only if the primordial power spectrum has the power.
index $n_p > 0.84$. The enhancement of the annihilation signal is absent, e.g. $\eta \simeq 1$, for $n_p < 0.9$.

6. Annihilation Signal Due to Big Clumps

Numerical simulations reveal in the galactic halos the big clumps with masses $10^8 - 10^{10} M_\odot$. At $t \sim t_{eq}$ these clumps are characterized by an effective power spectrum $P(k) \propto k^n$ with $n \approx -2$ (in contrast to $n \approx -3$ for small clumps), and thus the survival probability given by Eq. (1) is larger for the big clumps. Indeed, the effective power spectrum index

$$n = -3 \left[ 1 + 2 \frac{\partial \ln \sigma_{eq}(M)}{\partial \ln M} \right]$$

(7)

tends to $n = n_p - 4 = -3$ for small $M$ and to $n \approx -2$ for $M$ in the interval $10^6 - 10^9 M_\odot$. On the other hand the mean internal number density of DM particles in the big clumps is much smaller in comparison with that in the small clumps, and it compensates the first effect.

The number density $n(M)$ of the big clumps with $M \geq 10^8 M_\odot$ in the numerical simulations found [4] as $n(M) dM \propto dM/M^{\gamma}$ with $\gamma \approx 1.9 - 2.0$ and with a mass fraction of clumps $\varepsilon \sim 0.1 - 0.2$. Observations of halo lensing [5] give smaller values $\varepsilon \sim 0.06 - 0.07$. It is interesting to note that the mass function of clumps, obtained from Eq. (1), is close (including the normalization coefficient) to that obtained in the numerical simulations for big clumps with mass $M \geq 10^8 M_\odot$. Strictly speaking our calculations are not valid for big clumps, because of their destruction in the halo up to the present epoch $t_0$ and accretion of new clumps into the halo. Nevertheless, for the small interval of masses, where the power-law spectrum can be used as a rather good approximation, our approach appears to be roughly valid.

Calculations of the enhancement factor $\eta$ from simplified Eqs. (4) and (5) are performed by using $\varepsilon = 0.1$, the number density distribution of clumps in the Galaxy $n_{cl}(l) \propto \rho_{DM}(l)$, and internal density distribution of the DM particles in clumps

$$\rho(r) = \rho_c (r/a)^{-\beta} (1 + r/a)^{-\kappa}$$

(8)

valid down to the core radius $R_c$. The core is defined as $\rho(r) = \rho_c = \text{const}$ at $r \leq R_c$. The NFW profile has $\beta = 1$ and $\kappa = 2$, while the Moore et al. profile has $\beta = \kappa = 1.5$.

The other important parameter is clump radius $R$, which determines the average density $\bar{\rho}$ at the epoch of DM virialization in the clump. This quantity is calculated from the value of overdensity $\delta$ at the linear stage of clump formation.
The values of $\delta$ have Gaussian distribution and the normalization of fluctuation spectrum was performed as usual to the value of r.m.s. fluctuation at the 8 Mpc scale $\sigma_8 \simeq 1$. This approach corresponds to the picture that the big clumps in the Galactic halo are similar to the small protogalaxies (galactic building blocks), which escape the tidal destruction when capturing by the Galaxy. The tidal stripping of the outer parts of the clumps change their structure. Nevertheless this process is not important for clumps with mass $M \ll 10^{10} M_\odot$. For minimal and maximal masses of the big clump we use $M_{\text{min}} = 10^8 M_\odot$ and $M_{\text{max}} = 10^{10} M_\odot$.

As a result of calculations, identical to those in Section 5, we have found the enhancement factor $\eta = 1.02$ for the NFW profile \[3\] with $\gamma = 1.9$, $\varepsilon = 0.1$, $R_c = a$, $R/a = 5$ (the case of existence of the core), and $\eta = 1.14$ for the case of absence of the core $R_c = 0$.

For the Moore et al. profile \[4\], $\eta = 1.06$ for $\varepsilon = 0.1$, $R_c/a = 0.5$, $R/a = 5$ and $\eta = 1.16$ for a smaller core $R_c/a = 0.2$.

Diminishing of $M_{\text{min}}$ increases the enhancement weakly, approximately as $\eta(M_{\text{min}}) \propto M_{\text{min}}^{-0.35}$. We conclude that enhancement of the annihilation signal due to the big clumps is small.

We are grateful to Ben Moore for providing us with the data which allow us to normalize the density distribution of DM in the clumps.

References

[1] Y. B. Zeldovich, Astrofizika 6 (1970) 319.
[2] A. V. Gurevich and K. P. Zybin, Sov. Phys. Usp. 165 (1995) 723.
[3] J. F. Navarro, C. S. Frenk, and S. D. M. White, Astrophys. J. 462 (1996) 563.
[4] B. Moore et al., Astrophys. J. 524 (1999) L19.
[5] N. Dalal and C. S. Kochanek, [ArXiv:astro-ph/0111456](http://arxiv.org/abs/astro-ph/0111456)
[6] Y. P. Jing and Y. Suto, Astrophys. J. 529 (2000) L69.
[7] V. S. Berezinsky, V. I. Dokuchaev, and Yu. N. Eroshenko, Phys. Rev. D68, (2003) 103003.
[8] W. H. Press and P. Schechter, Astrophys.J. 187 (1974) 425.
[9] C. Lacey and S. Cole, Mon. Not. R. Astron. Soc. 262 (1993) 627.
[10] V. Berezinsky, A. Bottino, and G. Mignola, Phys. Lett. B391 (1997) 355.
[11] D. J. Schwarz, S. Hofmann, and H. Stocker, Phys. Rev. D64 (2001) 083507.
[12] K. P. Zybin, M. I. Vysotsky, and A. V. Gurevich, Phys. Lett. A260 (1999) 262.
[13] J. M. Bardeen et al., Astrophys. J. 304 (1986) 15.