Evolution of collectivity as a signal of quark gluon plasma formation in heavy ion collisions

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(Dated: August 11, 2011)

A measurement for studying the mass dependence of the dilepton interferometry in relativistic heavy-ion collision experiments as a tool to characterize the quark-gluon phase is proposed. In calculations involving dileptons, we show that the mass dependence of radii extracted from the virtual photon (dilepton) interferometry provide access to the development of collective flow with time. It is argued that the non-monotonic variation of HBT radii with invariant mass of the lepton pairs signals the formation of quark gluon plasma in heavy ion collisions. Our proposal of experimentally measuring the ratio, $R_{\text{out}}/R_{\text{side}}$, for dileptons can be used to estimate the average life times of the partonic as well as the hadronic phases.

PACS numbers: 25.75.+r,25.75.-q,12.38.Mh

I. INTRODUCTION

The primary goal of the nuclear collision experiments at ultra-relativistic energies is to create and study a new state of matter called the Quark Gluon Plasma (QGP) [1]. Most of the experimental observables for QGP, however, contain contributions from both the partonic as well as hadronic phases. As a consequence the disentanglement of the signals for the partonic phase remains a challenge and has been successful to a limited extend [1]. Some of the probes, which are produced early in the interactions like jets [2] and heavy quarks [3] or real photons [4–6] and dileptons [7–10] are considered to be particularly useful.

The transverse momentum ($p_T$) distribution of photons reflect the temperature of the source as their productions from a thermal system depend on the temperature ($T$) of the bath through the thermal phase space factors of the participants of the reaction that produces the photon. However, the thermal phase space factor may be changed by several factors - e.g. the transverse kick due to flow received by low $p_T$ photons from the low temperature hadronic phase will mingle with the high $p_T$ photons from the partonic phase, making the task of detecting and characterizing QGP more difficult. For dilepton the situation is, however, different because in this case we have two kinematic variables - out of these two, the $p_T$ spectra of lepton pairs is affected but the $p_T$ integrated invariant mass ($M$) spectra is unaltered by the flow. Moreover, it is expected that the large $M$ pairs originate from the early time and the low $M$ pairs predominantly produced in the late time. Therefore, the $M$ distribution can act as a chronometer of the heavy ion collisions. This suggests that a simultaneous measurement of $p_T$ and $M$ spectra along with a judicious choice of $p_T$ and $M$ windows will be very useful to characterize the QGP and the hadronic phases. Precise measurements of lepton pairs in pp collisions at a given collision energy is of paramount importance for detecting the thermal spectra in heavy ion collisions at the same energy.

Experimental measurements of two-particle intensity interferometry has been established as a useful tool to characterize the space-time evolution of the heavy-ion collision [11]. For the case of dileptons, such an interferometry needs to be carried out over dilepton pairs, theoretically representing a study of the correlations between two virtual photons. Although, the dilepton production rate is down by a factor of $\alpha$ compared to real photon, the analysis involving lepton pairs has been successfully used to get direct photon yields at RHIC [12]. In contrast to hadrons, two-particle intensity interferometry by using lepton pairs, like photons, have almost no interaction with the surrounding hadronic medium hence can provide information on the history of the evolution of the hot matter very efficiently. From the experimental point of view, dilepton interferometry encounters considerable difficulties compared to hadron interferometry due to small yield of the dileptons from the early hot and dense region of the matter and the associated large background primarily from the electromagnetic decay processes of hadrons at freeze-out. However, recent work demonstrate that it is still possible to carry out experimentally such interferometry studies [13]. With a high statistics data already collected at RHIC in the year 2010 by both STAR and PHENIX collaborations having dedicated detectors (Time-Of-Flight [14] and Hadron Blind Detector [15]) with good acceptance for dilepton measurements, also augurs well for the dilepton interferometry analysis.

In this work we present this new proposal for carrying out an experimental measurement of dilepton interferometry at RHIC. We establish through a hydrodynamical model based space-time evolution for central 0-5% Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV...
the promise of such a dilepton interferometry analysis will hold out to understand the properties of the partonic phase.

II. DILEPTON INTERFEROMETRY IN HEAVY ION COLLISION

As interferometry of the dilepton pairs actually reflect correlations between two virtual photons, the analysis then concentrates on computing the Bose-Einstein correlation (BEC) function for two identical particles defined as,

\[ C_2(\vec{k}_1, \vec{k}_2) = \frac{P_2(\vec{k}_1, \vec{k}_2)}{P_1(\vec{k}_1)P_1(\vec{k}_2)} \]  

where \( \vec{k}_i \) is the three momentum of the particle \( i \) and \( P_1(\vec{k}_i) \) and \( P_2(\vec{k}_1, \vec{k}_2) \) represent the one- and two- particle inclusive lepton pair spectra respectively. The correlation function, \( C_2 \) has been evaluated with the following source function:

\[ P_1(\vec{k}) = \int d^4x \omega(x, k) \]  

and

\[ P_2(\vec{k}_1, \vec{k}_2) = P_1(\vec{k}_1)P_1(\vec{k}_2) + \int d^4x_1d^4x_2 \omega(x_1, K)\omega(x_2, K) \cos(\Delta x^\mu \Delta k^\mu) \]  

where \( K = (k_1 + k_2)/2 \), \( \Delta k^\mu = k_1^\mu - k_2^\mu = q^\mu \), \( x \) and \( k \) are the four co-ordinates for position and momentum variables respectively and \( \omega(x, k) \) is the source function related to the thermal emission rate of virtual photons per unit four volume, \( R \) as follows:

\[ \omega(x, k) = \int_{M_1^2}^{M_2^2} dM^2 \left[ \frac{dR}{dM^2d^2k_Tdy} \right] \]  

The inclusion of the spin of the virtual photon will reduce the value of \( C_2 - 1 \) by \( 1/3 \). The correlation functions can be evaluated for different average mass windows, \( \langle M \rangle (\equiv M_{\pi^+\pi^-}) = (M_1 + M_2)/2 \). The leading order process through which lepton pairs are produced in QGP is \( q\bar{q} \rightarrow t^+t^- \). For the low \( M \) dilepton production from the hadronic phase the decays of the light vector mesons \( \rho, \omega \) and \( \phi \) have been considered including the continuum \[ 3, 8, 17 \]. Since the continuum part of the vector meson spectral functions are included in the current work the processes like four pions annihilation \[ 18 \] are excluded to avoid double counting. For the space time evolution of the system relativistic hydrodynamical model with cylindrical symmetry \[ 19 \] and boost invariance along the longitudinal direction \[ 20 \] has been used. The initial temperature \( (T) \) and proper thermalization time \( (\tau_i) \) of the system is constrained by hadronic multiplicity \( (dN/dy) \) as \( dN/dy \sim T^3\tau_i \). The equation of state (EoS) which controls the rate of expansion/cooling has been taken from the lattice QCD calculations \[ 21 \]. The chemical \( (T_{ch}) \) and kinetic \( (T_{fo}) \) freeze-out temperatures are fixed by the particle ratios and the slope of the \( p_T \) spectra of hadron \[ 22 \]. The values of these parameters are displayed in Table I.

With all these ingredients we evaluate the correlation function \( C_2 \) for different invariant mass windows as a function of \( q_{side} \) and \( q_{out} \) which are related to transverse momentum of individual pair as follows: \( q_{out} = (k_{1T}^2 - k_{2T}^2)/f(k_{1T}, k_{2T}) \) and \( q_{side} = \)
is closely related to the transverse size of the system. The presence of non-thermal sources. A representative fit of the correlation functions are shown in Fig. 1 (solid line) corresponding to low and high mass which are expected to be dominated by radiations (see Fig. 2) from QGP (⟨M⟩ ≈ 0.3, 1.6 GeV) and hadronic phase (⟨M⟩ ≈ 0.77 GeV) respectively. A clear difference is seen in C2 for different ⟨M⟩ windows when plotted as a function of q_{side}. The differences are, however, small when BEC is studied as a function of q_{out}. The source dimensions can be obtained by parameterizing the calculated correlation function of the (time like) virtual photon with the empirical (Gaussian) form:

\[ C_2 = 1 + \lambda \exp(-R_i^2 q_i^2). \]  

where the subscript \( i \) stand for out and side and \( \lambda (= 1/3 \) here) represents the degree of chaoticity of the source. The deviation of \( \lambda \) from 1/3 will indicate the presence of non-thermal sources. A representative fit to the correlation functions are shown in Fig. 1 (solid lines).

While the radius (R_{side}) corresponding to q_{side} is closely related to the transverse size of the system and considerably affected by the collectivity, the radius (R_{out}) corresponding to q_{out} measures both the transverse size and duration of particle emission. The extracted R_{side} and R_{out} for different ⟨M⟩ are shown in Fig. 3. The R_{side} shows non-monotonic dependence on ⟨M⟩. The R_{side} reduces with ⟨M⟩, reaches its minimum value at ⟨M⟩ ≈ m_0 and then increases again at high ⟨M⟩ approaching values close to the corresponding R_{side} for the QGP phase. It can be shown that R_{side} ≈ 1/(1 + E_{collective}/E_{thermal}). In the absence of radial

![FIG. 1: (color online) Correlation function for dilepton pairs as a function of q_{side} [(a), for k_{1T} = k_{2T} = 2 GeV and ψ = 0] and q_{out} [(b), for ψ = ψ_2 = 0 and k_{2T} = 2 GeV] for three values of ⟨M⟩. The solid lines shows the parameterization of C2 using Eq. 5](image)

![FIG. 2: (color online) Invariant mass distribution of lepton pairs from quark matter and hadronic matter.](image)

![FIG. 3: (color online) R_{side} and R_{out} as a function of ⟨M⟩. The dashed, dotted and the solid line (with asterisk) indicate the HBT radii for the QGP, hadronic and total dilepton contributions from all the phases respectively. The open circles are obtained by switching off the contributions from ρ and ω.](image)
flow, $R_{side}$ is independent of $M$. With the radial expansion of the system a rarefaction wave moves toward the center of the cylindrical geometry as a consequence the radial size of the emission zone decreases with time. Therefore, the size of the emission zone is larger at early times and smaller at late time. The high $⟨M⟩$ regions are dominated by the early partonic phase where the collective flow has not been developed fully i.e. the ratio of collective to thermal energy is small hence show larger $R_{side}$ for the source. In contrast, the lepton pairs with $M \sim m_\rho$ are emitted from the late hadronic phase where the size of the emission zone and hence the $R_{side}$ is smaller due to larger collective flow. The ratio of collective to thermal energy for such cases is quite large, which is reflected as a dip in the variation of $R_{side}$ with $⟨M⟩$ around the $ρ$-mass region (Fig. 3 upper panel). Thus the variation of $R_{side}$ with $M$ can be used as an efficient tool to measure the collectivity in various phases of the matter. The dip in $R_{side}$ at $⟨M⟩ \sim m_\rho$ is due to the contribution dominantly from the hadronic phase. The dip, in fact vanishes if the contributions from $ρ$ and $ω$ are switched off (circle in Fig. 3). We observe that by keeping the $ρ$ and $ω$ contributions and setting radial velocity, $v_r = 0$, the dip in $R_{side}$ vanishes, confirming the fact that the dip is caused by the radial flow of the hadronic matter. Therefore, the value of $R_{side}$ at $⟨M⟩ \sim m_\rho$ may be used to estimate the average $v_r$ in the hadronic phase.

The $R_{out}$ probes both the transverse dimension and the duration of emission. Therefore, unlike $R_{side}$, it does not remain constant even in the absence of radial flow. The large $M$ regions are populated by lepton pairs from early partonic phase where the effect of flow is small and the duration of emission is also small - resulting in smaller values of $R_{out}$. For lepton pair from $M \sim m_\rho$ the flow is large which could have resulted in a dip as in $R_{side}$ in this $M$ region. However, $R_{out}$ probes the duration of emission too which is large for hadronic phase. The larger duration compensates the reduction of $R_{out}$ due to flow in the hadronic phase resulting in a bump in $R_{out}$ in this region of $M$ (Fig. 3 lower panel). Both $R_{side}$ and $R_{out}$ approach QGP values for $⟨M⟩ \sim 2.5$ GeV implying dominant contributions from the partonic phase in this region of $M$.

It is clear from the results displayed in Fig. 2 that the high $M(> m_\rho)$ and low $M(< 0.5$ GeV) regions are dominated by the radiation from early QGP phase when collective motion is not fully developed - as a result the size of the emission zones for these $M$ regions are large. However, lepton pairs for $M$ around $ρ$ mass dominantly originate from the late hadronic phase when the collectivity is large and consequently the size of the homogeneous emission domain is small. This results in non-monotonic variation of $R_{side}$ with $M$ as discussed above. Therefore, such a non-monotonic behaviour of the HBT radii will signal the presence two different phases during the evolution - indicating the creation of QGP which will inevitably reverts to hadrons because of the cooling due to expansion.

The quantities $R_{out}$ and $R_{side}$ are proportional to the average size of the system $24$. However, in the ratio, $R_{out}/R_{side}$ some of the uncertainties associated with the space time evolution get canceled out. The quantity, $R_{out}/R_{side}$ gives the duration of particle emission $25–27$ for various domains of $M$.

Figure 3 shows the variation of the ratio, $R_{out}/R_{side}$ and the difference $\sqrt{R_{out}^2 − R_{side}^2}$ as a function of $⟨M⟩$ for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Both show a non-monotonic dependence on $⟨M⟩$. The smaller values of both the quantities, particularly at high mass region, reflect the contributions from the early partonic phase of the system. The peak around $ρ$-meson mass reflects dominance of the contribution from the late hadronic phase as discussed before.

Now we discuss below two experimental challenges in such studies. We quote some numbers from the PHENIX measurements, keeping in mind that the situation will further improved by increasing the luminosity as well as collision energy. The number of events can be computed from the luminosity ($L$), the $pp$ inelastic cross-section ($σ$) and the run time ($T$) of the machine as,

$$N_{event} = L \times σ \times T$$  \hspace{1cm} (6)
For RHIC, the luminosity, $\mathcal{L}$ is of the order of $50 \times 10^{22}$ cm$^{-2}$ s$^{-1}$ and $\sigma = 40$ mb. If RHIC runs for 12 weeks then the number of events, $N_{\text{event}} = 1.45 \times 10^{10}$. For the $M$, $810 \leq M(\text{MeV}) \leq 990$, the differential number ($dN/2\pi p_T dp_T dy$) measured by PHENIX collaboration in Au+Au collisions [10] at $\sqrt{s_{NN}} = 200$ GeV is given by:

$$\frac{dN}{2\pi p_T dp_T dy} \bigg| y = 0 = \frac{N_{\text{part}}}{2} \times 1.29 \times 10^{-7}$$

in the $p_T$ bin 1-1.8 GeV. Therefore, the (differential) number of pairs in the above range of $p_T$ and $M$ is $\sim 1.45 \times 10^{10} \times \frac{N_{\text{part}}}{2} \times 1.29 \times 10^{-7} \sim \frac{N_{\text{part}}}{2} \times 1870$. Similarly for $M$, $500 \leq M(\text{MeV}) \leq 750$, the measured value of the above quantity is:

$$\frac{dN}{2\pi p_T dp_T dy} \bigg| y = 0 = \frac{N_{\text{part}}}{2} \times 2.235 \times 10^{-7}$$

for the $p_T$ bin 1.4-1.8 GeV. This indicates that the (differential) number of pairs in this kinematic domain is $\sim 1.45 \times 10^{10} \times \frac{N_{\text{part}}}{2} \times 2.235 \times 10^{-7} \sim \frac{N_{\text{part}}}{2} \times 3240$. For 0–10% centrality the number of participants for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV is about 330. The number of lepton pairs in the $p_T$ range 1.4-1.5 GeV is $\sim 5.3 \times 10^4$ for the $M$ window 0.5-0.75 GeV. For 12 weeks of run time the number of events estimated with the current RHIC luminosity is $\sim 1.45 \times 10^{10}$. Then the number of pairs produced per event is $\sim 3 \times 10^{-5}$ in the kinematic range mentioned above. The probability to have 2 pairs of dileptons is $\sim 10^{-9}$. Therefore, roughly $10^9$ events are required to make the HBT interferometry with lepton pairs possible.

It is expected that further increase in luminosity at RHIC by a factor 2 beyond the year 2012 to about $10^{20}$ cm$^{-2}$ s$^{-1}$ may be a motivating factor for such measurements. The increase in lepton pair production at higher collision energy at Large Hadron Collider (LHC) may also provide a reason to pursue such measurements.

The possibility of dilution of signal due to addition of random pairs, which one may encounter in the analysis of experimental data is discussed below. We have added some “mixture” to the dilepton source with exponential energy distribution, i.e., we have replaced $\omega$ by $\omega + \delta \omega$ where $\delta \omega$ has exponential energy (of the pair) dependence and weight factor is as large as that of $\omega$ itself. Then we find that the resulting change in the HBT radii is negligibly small. This can be understood from the expression for $C_2$ (Eq. 7):

$$C_2 = 1 + \frac{\int d^4 x_1 \omega(x_1, K) \cos(\alpha_1) \int d^4 x_2 \omega(x_2, K) \cos(\alpha_2)}{\int d^4 x \omega(x, k_1) \int d^4 x \omega(x, k_2)} + \frac{\int d^4 x_1 \omega(x_1, K) \sin(\alpha_1) \int d^4 x_2 \omega(x_2, K) \sin(\alpha_2)}{\int d^4 x \omega(x, k_1) \int d^4 x \omega(x, k_2)}$$

where

$$\alpha_1 = \tau_1 M_{1T} \cosh(y_1 - \eta_1) - r_1 k_{1T} \cos(\theta_1 - \psi_1) - \tau_1 M_{2T} \cosh(y_2 - \eta_1) + r_1 k_{2T} \cos(\theta_1 - \psi_2)$$

$$\alpha_2 = \tau_2 M_{2T} \cosh(y_2 - \eta_2) - r_2 k_{2T} \cos(\theta_2 - \psi_2) - \tau_2 M_{1T} \cosh(y_1 - \eta_2) + r_2 k_{1T} \cos(\theta_2 - \psi_1)$$

$M$ domain in In+In collisions at 158 A GeV beam energy measured by NA60 collaboration [2] indicates substantial broadening of the $\rho$ spectral function. In the present work we artificially modify the spectral function of the $\rho$ by increasing its width by a factor of two and keeping the pole mass at its vacuum value and then evaluate $C_2$ with the modified spectral function to explicitly check the change in the magnitudes of the HBT radii. We observe that an increase in the width of $\rho$ by a factor of two leads to the reduction of $R_{side}$ by about 10% for invariant mass windows below $\rho$-peak. This reduction is due to the contributions from the decays of broader $\rho$ which undergoes larger flow compared to the QGP phase. The nature of the variation of $R_{side}$ with $\langle M \rangle$ remains unaltered. However, a huge broadening of $\rho$ for which the contribution from the hadronic phase
becomes overwhelmingly large compare to the QGP phase may alter the nature of the variation of $R_{side}$ in the low $M$ domain.

IV. SUMMARY

In summary, the dilepton pair correlation functions has been evaluated for Au+Au collisions at RHIC energy. The additional kinematic variable, $M$ for dilepton pairs make it a more useful tool for characterizing the different phases of the matter formed in heavy ion collisions compared to the HBT interterometry with direct photons. The HBT radii extracted from the dilepton correlation functions show non-monotonic dependence on the invariant mass, reflecting the evolution of collective flow in the system which can be considered as a signal for the QGP formation in heavy ion collisions. The $M$ dependence of the $R_{out}/R_{side}$ and $\sqrt{R_{out}^2 - R_{side}^2}$ which can be experimentally measured could be used to characterize the source properties at various stages of the evolution.

Acknowledgment: We are grateful to Tetsufumi Hirano for providing us the hadronic chemical potentials. We thank Nu Xu for very useful discussions. J A and P M supported by DAE-BRNS project Sanction No. 2005/21/5-BRNS/2455.

[1] I. Arsene et al., Nucl. Phys. A 757, 1 (2005); B.B. Back et al., Nucl. Phys. A 757, 28 (2005); J. Adams et al., Nucl. Phys. A 757, 102 (2005); K. Adcox et al., Nucl. Phys. A 757, 184 (2005).
[2] J. Adams et al., Phys. Rev. Lett. 91, 072304 (2003); Phys. Rev. Lett. 91, 172302 (2003); S.S. Adler et al., Phys. Rev. Lett. 91, 072301 (2003).
[3] A. Adare et al., Phys. Rev. Lett. 98, 172301 (2007).
[4] L. D. McLerran and T. Toimela, Phys. Rev. D 31, 545 (1985).
[5] J. Alam, S. Raha and B. Sinha, Phys. Rep. 273, 243 (1996).
[6] J. Alam, S. Sarkar, P. Roy, T. Hatsuda and B. Sinha, Ann. Phys. 286, 159 (2001).
[7] C. Gale and J.I. Kapusta, Nucl. Phys. B 357, 65 (1991).
[8] R. Rapp and J. Wambach, Adv. Nucl. Phys. 25, 1 (2000).
[9] R. Arnaldi et al. for NA60 Collaboration, Phys. Rev. Lett. 96, 162302 (2006); Phys. Rev. Lett. 100, 022302 (2008).
[10] A. Adare et al., Phys. Rev. C 81, 034911 (2010).
[11] U. Heinz and B. V. Jacak, Ann. Rev. Nucl. Part. Sci. 49, 529 (1999); T. Csörgő and B. Lörstad, Phys. Rev. C 54, 1390 (1996); B. R. Schlei and N. Xu, Phys. Rev. C 54, 2155 (1996); D. H. Rischke and M. Gyulassy, Nucl. Phys. A 608, 479 (1996).
[12] A. Adare et al., Phys. Rev. Lett. 104, 132301 (2010).
[13] D. Peressounko, Phys. Rev. C 67, 014905 (2003); J. Alam et al., Phys. Rev. C 67, 054902 (2003); S. A. Bass et al., Phys. Rev. Lett. 93, 162301 (2004).
[14] B. Bonner et al., Nucl. Inst. and Meth. in Physics Research A 508, 181 (2003).
[15] A. Kozlov et al., Nucl. Inst. and Meth. in Physics Research A 523, 345 (2004).
[16] J. Cleymans, J. Finger and K. Redlich, Phys. Rev. D 35, 2153 (1987).
[17] E. V. Shuryak, Rev. Mod. Phys. 65, 1 (1993).
[18] P. Lichard and J. Juran, Phys. Rev. D 76, 094030 (2007).
[19] H. von Gersdorff, M. Kataja, L. McLerran and P. V. Ruuskanen, Phys. Rev. D34, 794 (1986).
[20] J. D. Bjorken, Phys. Rev. D 27, 140 (1983).
[21] C. Bernard et al., Phys. Rev. D 75, 094505 (2007).
[22] T. Hirano and K. Tsuda, Phys. Rev. C 66, 054905 (2002).
[23] U. A. Weidemann and U. Heinz, Phys. Rep. 319, 145 (1999).
[24] D. H. Rischke and M. Gyulassy, Nucl. Phys. A 608, 479 (1996).
[25] M. Herrmann and G. F. Bertsch, Phys. Rev. C 51, 328 (1995).
[26] S. Chappman, P. Scotto and U. Heinz, Phys. Rev. Lett. 74, 4400 (1995).
[27] S. Pratt, Phys. Rev. D 33, 1314 (1986).
[28] G. E. Brown and M. Rho, Phys. Rep. 269, 333 (1996).