State-dependent driving: A new route to non-equilibrium steady states

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While studying systems driven out of equilibrium, one usually employs a drive that is not directly coupled to the degrees of freedom of the system. In contrast to such a case, we here unveil a hitherto unexplored situation of state-dependent driving, whereby a direct coupling exists between the two. We demonstrate the ubiquity of such a driving, and establish that it leads to a nontrivial steady-state that is qualitatively opposite to what is observed in other driven systems. Further, we show how state-dependent driving in a many-body system can be effectively captured in terms of a single-particle model. The origin of this description may ultimately be traced to the fact that state-dependent driving results in a force that undergoes repeated resetting in time.

INTRODUCTION

A common parlor trick to demonstrate inertia is to place a coin on a post card, and then quickly pull the card. If the pull is quick enough, the coin remains almost in its place while the card gets pulled out. While the trick and its many variations (pulling a book out of a stack of books, or, the game Jenga) are themselves very instructive, they nevertheless leave a few pertinent questions related to the steady-state dynamics unanswered. The coin is attached to the card by forces of friction. Common wisdom tells us that if the acceleration of the card is larger than a critical value, the coin will begin to slip with respect to the card. However, as soon as the coin starts to slip, the frictional coupling between the card and the coin weakens. Since there is dissipation, the coin loses momentum and eventually sticks to the substrate transiently and then goes on to repeat the entire cycle of stick-slip motion. In such a scenario, does the coin move with uniform velocity or does the constant energy injection cause the coin to accelerate? In this paper, we will show that this particular simple experiment and some other variants of it represent a largely unexplored dynamical scenario in which the rate of energy injection into a system depends on the dynamical state of the system.

In a more general setting, the aforementioned scenario would be an example of an interacting system evolving in presence of drive and dissipation. This otherwise well studied problem has in the present case an added complexity arising from the fact that the individual entities have access to different internal states. Thus, there can be two levels of description of the system, one in terms of its degrees of freedom \( x_i \) and the other in terms of their internal states \( y_{\alpha} \). If the system has \( D \) degrees of freedom and \( M \) internal states, then we have \( i = 1, 2, \ldots, D \), and \( \alpha = 1, 2, \ldots, M \). In the above example of a coin on a paper, one may associate \( x_i \) with the velocity of the center of mass of the coin, while the internal states may be identified with the qualitatively different frictional states of slipping and sticking. If the coin is replaced by a sphere, then the internal states may be identified with the more easily identifiable rolling (low friction state) and sliding (high friction state) motion that the object undergoes on the surface.

For the aforementioned class of systems, input of energy through the external drive at the level of \( x_i \)'s affects the evolution of the \( y_{\alpha} \)'s and may cause in time random transitions between the different \( y_{\alpha} \)'s. These transitions in turn would act back on the time evolution of the \( x_i \)'s, resulting in an intricate, (and intriguing, in many-body complex systems) interplay between the dynamics of the degrees of freedom and the internal states \([1,2]\). It is then evidently of interest to ask: what is the long-time behavior of the system? Does the system achieve a stationary state with a time-independent distribution of the degrees of freedom? We may in general anticipate that a balance between drive and dissipation does in fact lead the system to a stationary state. Presence of an external drive precludes the possibility for the stationary state to be an equilibrium one. Consequently, any stationary state the system relaxes to at long times would be a generic non equilibrium stationary state (NESS) \([3]\).

In this work, we highlight the subtleties that come into play when there is interplay between the degrees of freedom and internal states by presenting three experiments - (i) a single coin on a platform oscillating along one axis, (ii) a sphere and (iii) collection of spheres placed on a orbiting platform \([4]\). The first experiment is one dimensional in nature and from its detailed study, we establish that at a phenomenological level, the coupling between the plate and the coin can be modeled by a viscous force-like velocity dependent term. In the second experiment, the center of mass of the sphere covers a two-dimensional space, and it does so under the influ-

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ence of two orthogonal forces. From this experiment, we establish that the stochasticity in the trajectory of the sphere comes from the randomness in the amplitudes of the two orthogonal forces. In the third experiment, we introduce interactions between the particles and establish that the resulting velocity distribution attains a unique velocity distribution with the tail, which is exponential in nature, gaining weight with increasing density. This observation is surprising, since collision-induced cooling is at the heart of many observable NESS states in granular systems. Following the experimental results, we present a minimalistic one-particle theoretical model that captures the essence of the experimental findings and provides us with a framework to understand such scenarios.

The idea of state-dependent driving is routed in various biological systems where the functionality of a state is maintained by an organism by constantly regulating the activity of its various internal processes. In the context of active systems, this would translate to the idea of prescribing a local density-dependent mobility to the particles. It is known that such quorum-sensing interactions can give rise to motility induced phase transitions [5]. Another example is that of enzymes locally enhancing diffusion by self-regulating the phoretic [6] and hydrodynamic forces generated by themselves. This can lead to such functional behavior as antichemotaxis [7].

FIG. 1. (a) First experimental setup: a coin on a plate that is being vibrated sinusoidally in time. Images are captured at 180 fps from a camera fixed in the laboratory frame. (b) Second experimental setup: Stainless steel balls of diameter $2a = 800 \mu m$ confined in an acrylic cylindrical container of diameter $D = 150 mm$ and height $1 mm$, with a glass plate used as a lid for the container to enforce a two-dimensional geometry for the problem. The container is placed on a square platform, and the entire assembly along with a camera (used for imaging) is made to perform an orientation-preserving horizontal circular motion. The net motion of the assembly may be imagined as a combination of two circular rigid-body motions of the platform: (i) a rotation about shaft 1 with angular frequency $\omega$, and (ii) an opposing rotation with angular frequency $-\omega$ about shaft 2 that passes through the center of the platform. (c) Typical temporal variation of the dichotomous noise $\xi(t)$ used in the phenomenological dynamics (1 and 3) to model both the experiments.

FIG. 2. A coin on an oscillating plate: The top panels show results from experiments. The trace of the velocity of center of mass of the coin (black) and a point on the plate (red) at driving frequency $f = 6$ Hz are both shown in panel (a). The distribution of the magnitude of the velocity of the centre of mass of the coin in the laboratory frame and obtained in the long-time limit is plotted in panel (b) at the same value of $f$. The measured time-averaged amplitude of the displacement of the centre of mass of the coin, obtained in the long-time limit, and the same for the plate are both shown in panel (c) as a function of the driving frequency $f$. Here $f_c$ is the critical frequency below which the coin is completely stuck to the plate. The solid line indicates slope of -1. The bottom panels show results from numerical simulation of the phenomenological model (1) with $m = 1$, $\gamma = 5$, and $\beta = 100f$; the driving frequency $f$ has the same value as in the top panels. The red trace in panel (d) is that of the velocity of the plate, $v_p(t) = -2\pi f x_{p0} \sin(2\pi ft)$. It is evident from a comparison of results in the top and bottom panels that the phenomenological model (1) is able to capture qualitatively and rather remarkably the experimentally-observed features.

RESULTS

A coin on an oscillating plate

A coin is placed on a plate that is being vibrated in the $x$ direction, see Fig. 1(a). The motion of both the coin and the plate is measured simultaneously. The plate moves sinusoidally, $x_p(t) = x_{p0} \cos(2\pi ft)$, where $x_{p0}$ is the position of a point on the plate, $x_{p0}$ is the amplitude of vibration, and $f$ is the driving frequency. For small values of $f(\leq f_c = 4.4$ Hz), the coin remains stuck to the plate, while for larger values of $f$, it begins to slip with respect to the plate. Typical trajectories of the velocity of the center of mass of the coin and a point on the plate for $f = 6$ Hz are shown in Fig. 2(a). In the phase in which the coin slips with respect to the plate, its center of mass drifts slowly in time. However, for small time intervals, its response $x_c(t)$, namely, the location of its center of mass, is approximately sinusoidal, and has the same frequency of variation as that of the plate. The probability distribution of $|v|$, the magnitude of the velocity of the center of mass of the coin, is plotted in Fig. 2(b); the distribution is obtained by sampling the trajectories in the long-time limit over a time window. The variation of
The most evident way to describe the system would be to use the Coulomb friction model. If the driving frequency is such that the amplitude of the sinusoidal forcing exerted by the plate is larger than the maximum static friction force, i.e., \( |(2\pi f)^2 x_p| > \mu mg \), with \( \mu \) being the coefficient of friction between the coin and the plate and \( m \) the mass of the coin, the coin will undergo both stick and slip motion periodically. In the stick state, the equation of motion of the coin can be given as:

\[
m\partial_t v = -\mu mg,
\]

where \( \mu \) is the coefficient of friction between the coin and the plate. However, as soon as the driving force exceeds \( \mu mg \), the coin will start to slip with respect to the plate and it will then begin to decelerate due to the friction force acting in the opposite direction. Thus in a single period, there would be two instances of both stick and slip motion. Hence the velocity of the coin will be partly sinusoidal and partly linear in a single period. This model gives rise to a velocity which is a strongly distorted sine wave. This is in stark contrast to the approximate sinusoidal nature of the velocity observed in the experiments.

For the velocity of the coin to be approximately sinusoidal, it has to undergo many transitions between the stick and slip state in a single period for the reasons mentioned above. As there is a priori no reason for these transitions to occur periodically, it can be assumed that they occur randomly in time. We thus propose a stochastic variant of the Coulomb friction model for the motion of the coin on the vibrated plate which captures the essential qualitative features of its dynamics observed in the experiments described above. To be consistent with the experiment, we choose to describe the motion of the center of mass of the coin in an inertial frame, i.e., in the laboratory frame. With respect to such a frame, the plate on which the coin is placed is being vibrated sinusoidally in time, so that its velocity at time \( t \) reads \( v_p(t) = v_0 \sin(2\pi ft) \). When the coin is in the stick state, the equation of motion of its center of mass is the usual Newton’s equation of motion:

\[
m\partial_t v = F(t); \quad F(t) = m_p v_p(t)/\tau_p,
\]

where \( v \) is the velocity of the coin, while \( F(t) \) is the force experienced by the coin due to transfer of momentum from the plate to the coin. Here, the quantity \( m_p \) is the mass of the plate and \( \tau_p \) is the timescale over which the momentum of the vibrating plate is transferred to the coin. The timescale over which the velocity of the coin changes significantly is much larger than the scale \( \tau_p \), and hence, on the former timescale, the coin appears to be in the stick state. On the other hand, when the coin is in the slip state, it is detached from the plate, and consequently, it moves in presence of an effective damping force that dissipates in time the initial momentum of the coin at the instant of detachment from the plate. Hence, in the slip state, the equation of motion of the coin is given by:

\[
m\partial_t v = -\gamma v,
\]

where \( \gamma > 0 \) is a phenomenological dissipation constant. The coin toggles randomly in time between the stick and the slip state. Introducing a random variable \( \xi \) taking on values 1 or 0 corresponding to the stick and the slip state, respectively, one may on the basis of the foregoing write down the equation of motion of the coin as a stochastic differential equation, a Langevin-like equation, of the form:

\[
m\partial_t v = -\gamma v + \xi(t)(F(t) + \gamma v).
\]

Equation (1) has to be supplemented by another equation describing the time evolution of the instantaneous \( \xi \), namely, \( \xi(t) \). While derivation of an exact form of the latter would invariably involve a detailed modeling of the friction force, we here offer a phenomenological description for the evolution of \( \xi \) in terms of a stochastic Markov process, namely, between times \( t \) and \( t + dt \), the variable \( \xi(t) \) is updated to read \( \xi(t + dt) = 1 \) with probability \( \beta dt \), while \( \xi(t + dt) = 0 \) with the complementary probability \( 1 - \beta dt \). Here, \( \beta > 0 \) is a dynamical parameter. It then follows that the random time \( \tau \) between two successive occurrences of the value unity for \( \xi \) is distributed as an exponential:

\[
\rho(\tau) = \beta e^{-\beta \tau}; \quad \tau \in [0, \infty),
\]

and that the average \( \tau \) is given by \( \langle \tau \rangle = 1/\beta \). As a function of time, \( \xi(t) \) appears as a set of impulses distributed randomly in time, as shown in Fig. 2(c).

It is interesting and pertinent that we draw a parallel between Eq. (1) and the usual form of the Langevin equation that one encounters in describing say the paradigmatic Brownian motion. Besides the nature of the stochastic noise, which in the latter is a Gaussian, white noise and in Eq. (1) is a dichotomous noise, one very important difference is the following. In the case of the Brownian motion, the strength of the noise term is a constant that does not depend on the value of the dynamical variable in question (more precisely, the constant is by virtue of the fluctuation-dissipation theorem related to the equilibrium temperature of the ambient medium). By contrast, in Eq. (1), the strength of the noise term is explicitly dependent on the value of the dynamical variable \( v \) being studied. Another point worth mentioning is that the noise in the case of Brownian motion is considered stationary, while in our case, the noise is non-stationary.

Note that in the dynamics described by Eq. (1), the force experienced by the coin is being continually reset in time between the pure sinusoidal drive and the entirely dissipative form as the random variable \( \xi \) toggles in time between its two-possible values. Equation (1) may be considered representative of a class of stochastic dynamical systems in which the force resets stochastically in time between a set of possible values, the two-state process considered here being the simplest one may conceive.
The dynamics described by Eq. (1) involves two time scales \(1/\beta\), setting the average time between two impulses, and \(1/f\). We work in the regime in which \(1/\beta \ll 1/f\). Consequently, in the long-time limit, we expect the velocity \(v\) to vary sinusoidally in time with occasional jump in values as an effect of toggling acting on a time scale that is faster compared to the sinusoidal variation. As far as the parameter \(\gamma\) is concerned, its magnitude would set the cut-off scale of the velocity in the slip state.

Figure 2 panels (d)-(f) depict results obtained from numerical simulation of the dynamics (1). In panel (d), we see that consistent with our expectations mentioned above, the velocity does represent an almost sinusoidal variation in time and correspondingly, the velocity distribution in (e) exhibits a peak at a value around the amplitude value of the sinusoidal variation in (d). The toggling phenomenon manifests itself as deviations from a pure sinusoidal variation in (d) and contributes to a width around the peak in the distribution in (e). It is remarkable that our phenomenological model (1) is able to capture qualitatively in a rather striking manner the experimental features of both the coin dynamics in (a) and the velocity distribution in (b). Moreover, the distribution has a tail that is exponential, as seen in panel (e). In simulations, this is a result of dissipation set by \(\gamma\) in the dynamics (1). Not only, one may obtain for the model (1) and as a function of the frequency \(f\) the time-averaged amplitude of the displacement \(x_\text{c}(t)\) in the long-time limit, based on the following heuristic arguments. In the extreme limit in which \(\beta \to \infty\) at a fixed \(f\) so that \(1/\beta \ll 1/f\), the impulses in Fig. (1c) are infinitesimally close together over a finite observation time window \(T\) at long times. Moreover, from the nature of the dichotomous noise, it is evident that the different impulses are uncorrelated with one another. Consequently, over time \(T\), when the number of impulses is \(T/\beta\), the standard deviation of the total force experienced by the coin is \(\sigma(F) \approx T/\beta \sigma(v_\text{p})/\tau_\text{p}\), where the factor \(m\sigma(v_\text{p})/\tau_\text{p}\) is the standard deviation of the force that the coin experiences due to a single impulse. It then follows that the standard deviation of the displacement of \(x_\text{c}\) would scale as \(\sigma(x_\text{c}) \propto T/\beta \sigma(v_\text{p})/f^2\), for we have \(dv/dt = \cos(2\pi ft)\) implying on the basis of dimensional considerations that \(x_\text{c} \propto 1/f^2 \cos(2\pi ft)\). Since in our case, we have \(\beta \propto f\), we get \(\sigma(x_\text{c}) \sim T\sigma(v_\text{p})/f\), so that the standard deviation of the amplitude of the displacement when averaged over time \(T\) in the long-time limit would scale as \(\sigma(v_\text{p})/f\).

The numerical simulation results in panel (f) indeed show such a \(1/f\) variation of the averaged amplitude as a function of \(f\), and again, quite remarkably, a similar variation is presented by the experimental data for the coin depicted in panel (c). As for the coin-trick problem that was proposed in the introduction, we expect the coin to move with a constant velocity. The magnitude of the velocity would depend on the coefficient of friction and the acceleration of the card being pulled out.

FIG. 3. Panel (a) shows the experimental trajectory of a single spherical particle on an orbital shaker driven at frequency \(f = 2.1\) Hz, in a reference frame co-moving with the orbital shaker (camera frame). Panel (b) shows the probability distribution of the magnitude of the \(x\)-component \(v_x\) of the velocity of the center of mass of the ball. Simulated trajectory is shown in inset.

A sphere on an orbital shaker

In the above experiment, the stochasticity in the forcing term arises from the toggling between the stuck and the slip state of the coin. In the second experiment that we describe now, we place a steel ball (a sphere of mass \(m\), radius \(a\), and moment of inertia \(I\)) on a plate connected to an orbital shaker. The schematic setup of the experimental setup is shown in Fig. (1b).

Here, the system can toggle between the rolling (without slipping) and the slip state. With an orbital shaker, the plate moves in an orientation-preserving manner such that each point on the plate moves on a circle of the same radius \(r\): a given point \(Q(x_p, y_p)\) on the plate moves as \(x_p = x_{po} + r \cos(2\pi ft)\), \(y_p = y_{po} + r \sin(2\pi ft)\), where \((x_{po}, y_{po})\) is the center of the circle about which the chosen point \(Q\) moves. Each point on the plate has a center associated with it. The motion of the sphere comes from the force that is exerted at the point of contact between the sphere and the moving plate. This frictional force affecting the velocity \(v_b\) of the center of mass of the ball as \(m\partial_t v_b = -\mu mg\hat{v}_p\), with \(\mu\) the coefficient of friction, \(g\) the acceleration due to gravity, and \(v_p\) being the velocity of a point on the plate, exerts a torque \(\Gamma\) about an axis parallel to the platform and passing through the center of mass of the sphere, i.e.,

\[
I \partial_t \omega_b = -\hat{k} \times m\partial_t v_b = a\mu mg(\hat{k} \times \hat{v}_p), \tag{2}
\]

where \(\omega_b\) is the angular velocity of the sphere. An isolated sphere on the plate performs a swirling motion \(\hat{k}\). An example of such a motion for \(f = 2.1\) Hz in the reference frame of the moving plate (camera frame) is shown in Fig. (3a). In experiments, the long-time trajectory of both the position and the velocity of the sphere is not deterministic.

Effectively, this is a two-dimensional version of the previous problem, wherein the particle-substrate interaction generates the randomness in the motion. Individually, both the \(x\)- and the \(y\)-components of the motion are ap-
proximately sinusoidal in nature. The abrupt alteration in the trajectory marked as $P_1$ in Fig. 4(b) is an example of such scattering. Occasional collisions with the boundary leads to additional randomness in the trajectories. An instance of this kind of scattering is shown as $P_2$ in Fig. 4(b). These scattering events introduce a random phase difference between the two components which results in the randomization of the trajectories. The probability distribution of the magnitude of the $x$-component of the velocity, $|v_x|$, of a rolling sphere for representative value of $f$ is plotted in Fig. 4(b). A similar distribution is seen for $|v_y|$ also. In the tails, the velocity distribution varies as $P(|v|) \sim \exp(-|v|^\alpha)$, with $\alpha \approx 1$. This is very different from the well-known Maxwell distribution, in which case $\alpha = 2$.

Similar to previous experiment, we postulate that the noise in the trajectory arises from occasional slipping of the ball with respect to the plate. These partial slips have a concomitant toggling between the sliding and the rolling state of the system, and hence, similar to the previous example, the resulting noise can also be considered dichotomous. Thus, the Langevin-like equation of motion of the velocity of the center of mass of the ball has the form

$$m\partial_t v_\sigma = -\gamma v_\sigma + \xi(t)(F_\sigma(t) + \gamma v_\sigma),$$

(3)

with $\sigma = \{x,y\}$. Here, $\xi(t) = 0$ and 1 correspond to sliding and rolling states respectively.

However, when there are more than one particle in the system, the inter-particle collisions also contribute to the overall noise in the trajectories. In the following subsection, we consider the case of many spheres on an orbital shaker.

**Collection of spheres on an orbital shaker**

For the purpose of tracking, particles are imaged at 110 fps for a duration of 10 minutes. Particle tracking is done as per the algorithm mentioned in [11]. Care has been taken to ensure that the driven plate is kept as much horizontal as possible which can be seen from the inset in Fig. 5(b). It shows the averaged intensity of a stack of 600 images taken at an interval of 1 s. The homogeneous distribution of particle density clearly shows that particles are mostly concentrated at the center of the plate. This also indicates that particle-wall collisions occur rarely as compared to inter particle collisions (wall is indicated by the dotted circular red line).

The trajectories shown in Fig. 4(b) and (c) are obtained by averaging a sequence of images, each taken with a time interval of 100 ms. The micrographs in Fig. 4(c) to (d) highlight the influence exerted by one particle on the other. When two spheres collide tangentially, two events can occur - (i) the spheres alter their direction and continue swirling, or, (ii) they form a bound dyad-like pair; This pair moves together mostly in a reciprocating manner, rolling back and forth in the direction that is perpendicular to the line joining the centers of the two balls. In the direction that is along the line joining the two centers, the pair does a sliding motion. Figure 4(c) shows two trajectories $\pi_1$ and $\pi_2$ that correspond to a collision and formation of a transiently stable dyad structure. In its dyad form, the two trajectories move parallel to each other. This structure is short lived and often spontaneously break apart. When it does so, the trajectories $\pi_1$ and $\pi_2$ depart from each other. The destabilisation of the dyads can be brought about either by a particle-substrate or a particle-particle scattering event.
In the case of the trajectory shown in Fig. 4(c), it is the particle-substrate scattering that destabilises the dyad.

Occasionally, a third ball bumps into the two-particle pair. This can either destabilise the dyad or can form a compact three-particle triad. For spheres in contact, rolling in the same direction causes shearing of the contact region \[1\]. Thus, a triad cannot roll and, hence, it either becomes static or does small sliding motion. Figure 4(d) shows a sequence of micrographs that captures the event corresponding to the formation of the triad. These micrographs are obtained by using exposure time of 10 ms. This long exposure produces a motion-induced dilation effect of the objects. Thus, moving objects in these micrographs will appear extended, and its shape tells us about the nature of motion, e.g., a roller’s motion will appear as a curved line, and dyads will appear as parallel straight lines. For static objects, motion-induced elongation is absent, and the objects appear as dots. The leftmost frame shows a dyad and a nearby rolling isolated sphere. The other two particles in the frame are present to provide a reference. After collision, the three spheres form a static triad. Absence of substantial motion for this triad can be inferred from the lack of motion- induced dilation (see the second frame of the micrograph in Fig. 4(d) and first frame in Fig. 4(e)). For any higher-order structure involving more than 3 spheres that can form, the only available mode of movement is sliding. So the transitions in these structures are limited to stuck to sliding states. For the values of \( f \) reported in this paper, the clusters (dyads, triads and higher order structures) are only transiently stable. Structures involving more than 3 particles are destabilised by particle collisions. An instance of this destabilisation can be seen in Fig. 4(e).

The very fact that two moving particles can come to a halt after a collision points us to instances where the momentum is not conserved. Moreover, the transitions between the rolling, sliding and the stuck states are hysteretic in nature, i.e., the frequency at which a moving particle makes a transition to the stuck state is lower than the corresponding transition from a stuck to a moving state. Figure. 4 (a) shows this hysteresis between stuck and rolling states for a single sphere.

With each hysteretic transition, a certain amount of kinetic energy is lost. It would thus be natural to expect higher particle density to result in higher frictional dissipation and hence lower velocities. However, contrary to this, we observe in Fig. 5 (a), (b) and (c) the tail of the velocity distribution \( P(|v_x|) \sim \exp(-|v_x|^\alpha) \), with \( \alpha \approx 1 \), to increase with the density. There is a large spread in the value of \( \alpha \) for driven granular gas obtained in literature \[12, 13\], and there exists multiple models to explain this deviation, e.g., presence of clustering \[13\], non-uniformity of granular temperature \[14, 15\], nature of noise injected by the drive \[16\]. In most experiments, the strength of the noise is assumed to be a parameter controlled externally, e.g., energising a shaker more injects more noise to a granular gas, and it is assumed to be independent of the density of the particles. In the present situation, noise is injected when the particle toggles its state. Thus, with increasing density, the toggling increases in ways that are described in Fig. 4 and, hence, the strength of the noise increases. In our model dynamics Eq. (3), which we now invoke as an effective single-particle dynamics to describe qualitatively the features observed in our experiment, the parameter \( \beta \) parameterizes the frequency of this toggling. Indeed, the tail of the velocity distribution obtained by numerically integrating Eq. (3) increases as a function of \( \beta \). This variation is shown in Fig. 5(d). The nature of the driving ensures strong correlation in the motion of the spheres. These correlations decrease with increasing density of the spheres.

**DISCUSSION**

Systems with multiple components maintain themselves in a stable state by constantly regulating energy flow and dissipation, e.g., homeostasis in a biological setting is maintained by constant regulation of chemical processes, a governor in a combustion engine uses the inertial forces acting on it to limit the fuel injection. In physical terms, one can think of the system to be consisting of two parts, the body and the environment. The body is the place where the energy is dissipated and to maintain the processes in the body energy has to flow from the environment to the body. The rate of dissipation is a function of the inward energy flux and by controlling this...
flux, the body maintains a desired steady state. Though such self-sustained steady states are common to biological, chemical and system science settings, there seems to be a gap in realizing this regulation process in a more prosaic physical setting, particularly in situations where the energy injection and dissipation processes are clearly identified. In this paper, we have shown that for multiple experimental settings that span from a single particle to multiple particles, a state-dependent energy injection process can lead to a nontrivial stationary state in driven frictional systems. This stationary state is maintained by continued toggling between the different frictional states of the system. The energy injected to any of these states is a function of the state itself. We also provide a simple single parameter theoretical description of the various experimental realizations. Our work provides a new paradigm for finding routes to achieve non-equilibrium steady states. It is only natural that future work in this direction would be to understand the conditions under which the steady-state is maintained and the processes by which a given steady state becomes unstable and a new steady state is arrived at. This could possibly open new avenues to understand modes of failure leading to destabilisation in complex systems.

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