NEUTRINO PROPAGATION THROUGH FLUCTUATING MEDIA

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The description of neutrino propagation through a fluctuating medium is summarized. Fluctuation-Dissipation-Theorem arguments relate microscopic fluctuations to thermodynamic quantities, allowing these to be very generally studied in astrophysical environments. Fluctuation-induced modifications to neutrino oscillations have been studied in detail within the Sun and within the envelope of supernovae, and although surprisingly large effects have been found none appear to be observable in the near future.

1 Introduction

The same properties which make neutrinos elusive make them interact uniquely within astrophysical and cosmological environments. The incredible feebleness of their interactions permit neutrinos to escape from deep within stars and supernovae, making them superb probes with which to potentially ‘observe’ the interior workings of these objects.

On the other hand, neutrino interactions are not negligible within these environments since the extreme temperatures and densities which arise there preclude their neglect. Despite the potential importance of these neutrino-matter interactions, some of the approximations inherent in these analyses have only recently been systematized. It is the purpose of this talk to briefly summarize recent studies of how neutrinos interact with dense, hot environments.

2 The Nature of the Problem

A great deal is known about how photons interact with matter, and much (but not all) of the intuition developed for photons can be carried over to the study of neutrinos. We therefore pause here to review the description of photons propagating through a medium such as a dielectric.
2.1 Photons

At a microscopic level, photons couple to dielectric materials by interacting with the electrons within its constituent atoms. The underlying electron-photon interaction is expressed in terms of the electromagnetic current, \( j^\mu \), and the electromagnetic potential, \( A_\mu \), by the interaction lagrangian density:

\[
\mathcal{L} = e A_\mu j^\mu. \tag{1}
\]

Although the smallness of \( e \) ensures interactions with any particular atom are weak, the interaction probability becomes unity once sufficiently many atoms are encountered. For everyday dielectrics sufficiently many atoms are encountered over a characteristic distance, \( \xi \), which is usually much smaller than the macroscopic distances over which the propagation of light is to be understood.

What is generically a complicated, strong-coupling problem considerably simplifies if the medium’s fluctuations are correlated over distances, \( \ell \), which are sufficiently small in comparison with \( \xi \). In this case, although fluctuations can strongly influence the evolution of electromagnetic waves, they do not build up correlations over significant distance scales.

The condition \( \ell \ll \xi \) permits simplification for two reasons. First, the absence of correlations over distances larger than \( \ell \) permits photon evolution to be described by a Markov process. That is, it ensures that the evolution may be described by a differential equation which is local in time and space. Second, the smallness of \( \ell \) compared to \( \xi \) permits the resulting differential equation to be evaluated perturbatively in the microscopic couplings.

When these conditions are satisfied, the interactions of photons with the medium are well described by Maxwell’s equations, supplemented by a dielectric function which describes the influence of the medium. Furthermore, the dielectric function itself may be computed perturbatively in powers of \( e \), with the fluctuations contributing to the electromagnetic polarizability:

\[
\Pi^{\mu\nu}(x) = e^2 \langle j^\mu(x) j^\nu(0) \rangle. \tag{2}
\]

Notice that the leading term arises at second order in \( e \) because, for dielectrics, the lower-order contribution vanishes because \( \langle j^\mu(x) \rangle = 0 \).

A virtue of this kind of formulation is that it separates issues. The correlation function, eq. (2), is seen to control photon propagation regardless of the microscopic details of the fluctuations themselves. The nature of a particular fluctuation only enters once \( \Pi^{\mu\nu} \) is evaluated in order to obtain the dielectric function explicitly for the system of interest.
2.2 Neutrinos

Similar considerations apply to neutrino interactions with fluctuating media. The microscopic interactions can be taken to be

$$\mathcal{L}_F = \frac{i G_F}{\sqrt{2}} \sum_k (\overline{\nu_k} \gamma_\mu \nu_k) \ J^K_\mu (x),$$

where the sum on $k$ runs over the three neutrino species and $J^K_\mu$ describes the contribution of other particles to the weak interactions.

As before, a perturbative calculations are possible for fluctuations having sufficiently small correlation lengths. An important difference in the neutrino case is that the leading influence of the medium is described by neutrino coupling to the mean, $\langle J^K_\mu \rangle$. For many astrophysical applications the medium of interest is a parity invariant mix of protons, electrons and neutrons, for which

$$\langle J^K_\mu (x) \ J^K_\nu (0) \rangle = 2 \delta_{kf} j^K_e (x) - j^K_n (x) + (1 - 4s^2_w) [j^K_p (x) - j^K_e (x)],$$

where $j^K_f$ (for $f = e, p$ and $n$) are the currents which follow the flow of electron, proton and neutron densities. This neutrino interaction lagrangian describes the usual resonant oscillations in matter.

Corrections to this approximation describe neutrino scattering from the fluctuations about the mean. These are proportional to the correlation function $\langle J^K (x) J^K (0) \rangle$, whose size must be evaluated for various fluctuations.

3 Types of Fluctuations

Two major kinds of fluctuations are usually examined when considering particle propagation through a medium: (i) equilibrium fluctuations and (ii) position-dependent fluctuations, each of which are now considered in turn.

3.1 Spatial or Temporal Fluctuations

The influence of spatial variations in $\langle J^K_\mu (x) \rangle$ (or in the solar magnetic field, $B(x)$) on neutrino oscillations has recently received considerable study in the literature. Most of these model the variations as delta-correlated gaussian noise in either the density of the sun, $\rho(x)$, or a supernova envelope, or in solar magnetic fields. Solar neutrino evolution through more realistic fluctuations with long correlation lengths, such as helioseismic waves, has also been investigated.

Although the neutrino-oscillation effects can be surprisingly large, none of detectable size has yet been discovered. The difficulty in producing detectable
effects can be seen if the following expression is used, which generalizes to
the presence of fluctuations the Parke formula for resonant neutrino survival
probability:

\[ P_e(t, t') = \frac{1}{2} + \left( \frac{1}{2} - P_J \right) \lambda \cos 2\theta_m(t') \cos 2\theta_m(t). \]  

(5)

Here \( P_J \) is the usual ‘jump’ probability and \( \theta_m(t) \) is the matter mixing angle
evaluated at the position occupied by the neutrino at time \( t \).

The contributions of fluctuations are summarized by the coefficient \( \lambda \),
which is given by

\[ \lambda \equiv \exp \left[ -2G_F^2 \int_{t'}^{t} d\tau A(\tau) \sin^2 2\theta_m(\tau) \right]. \]  

(6)

Here \( A(t) \equiv \int_{t'}^{t} d\tau \left( \delta n_e(t) \delta n_e(\tau) \right) \) is the relevant measure of the strength
of fluctuations. An important consequence of this equation is its implication
that the probability of \( \nu_e \) survival depends almost exclusively on fluctuation
properties at the position of the MSW resonance, since \( \sin^2 2\theta_m \) is typically
sharply peaked at this point. It is the difficulty of obtaining large fluctuations
deep within the radiation zone, where the neutrino resonance takes place, which
decisively suppresses the influence of fluctuations in solar neutrino experiments.

### 3.2 Equilibrium Fluctuations

Equilibrium fluctuations are those within the statistical ensemble whose average
gives the local thermodynamic properties. For small correlation lengths,
and for nonrelativistic systems, the correlation function of interest boils down
to a measure of local density fluctuations within the medium.

Such fluctuations are quite generally related by the fluctuation-dissipation
theorem to the equilibrium thermodynamic susceptibilities of the system. For instance, in a grand-canonical ensemble local density fluctuations are related
to the system’s compressibility:

\[ \langle [n(x) - \bar{n}(x)][n(0) - \bar{n}(0)] \rangle = \left( \frac{\partial n}{\partial p} \right)_T n k_B T \delta^3(x). \]  

(7)

Notice that the generality of this result implies it works equally well for media which are composed of weakly- or strongly-interacting systems. So long
as they are in equilibrium, these systems can differ only through their equations
of state. For a dilute, weakly-coupled system, the ideal-gas law \( p = n k_B T \)}
applies, and the right-hand-side of eq. (7) reduces to $n \delta^3(x)$. The resulting neutrino scattering cross section gives the usual result proportional to $n$.

More complicated equations of states can change the scattering rate appreciably. A study of the implications of this expression for neutrino propagation within supernova cores is presently underway.

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