A modified active disturbance rejection control for fast steering mirror in aerospace application

Xintao Zheng*, Hai Huang and Weipeng Li

School of Astronautics, Beihang University, Beijing 100083, China

*Corresponding author; E-mail: zhengxintao@buaa.edu.cn

Abstract. Active disturbance rejection control (ADRC) has proved to be an effective tool in dealing with engineering problems related to the plant along with dynamic uncertainties, disturbances, nonlinearities. A modified active disturbance rejection control (MADRC) method is proposed to design the Fast Steering Mirror (FSM) controller. The modified extended state observer (ESO) incorporate with identified plant information is employed to reject the disturbances, nonlinear factors and modelling errors which meant to constraint the plant to a nominal form. Therewith, an well-designed analytical method is introduced to calculate the parameters of a feedback loop to force the plant response like a pre-designed model. Besides, a Lead-Leg Controller (LLC) consisting of two Tracking-Differentiator (TD) is designed to improve the performance of the controller. LLC generates a filtered output based on the expected plant dynamics to increase the FSM bandwidth, and LLC can also help to improve the stability of the system by restricting the tracking error in a limited range when there is an abrupt command input. The MADRC strategy is proven an effective solution for FSM. Simulation results show that the MADRC method can achieve a excellent performance without tedious tuning work. The experiment results proved the validation in practical application.

1. Introduction
FSM provides two-axis, high-bandwidth rotation with sub-micro radian resolution with an unique structural design and sensing technology. FSM is a key component in active optical control systems [1-7] such as image stabilization [2], astronomy, laser beam stabilization, laser pointing [5], free space communication [3], etc. The role of FSM is to correct displacement and angular misalignment of the beam caused by vibration, turbulence, as well as the thermal effects, therefore, the bandwidth and accuracy performance are very important features of FSMs.

FSM are mainly composed of base, mirror holder, supporting structure, actuator component, angle detecting device and the control and drive system. With the development of technology, the system configuration and structural designs of FSM have been nearly standardized and reached a bottleneck, the FSM design has not much room for improvement, and what affects the performance most is the control strategy, which leaves a great requirement to the algorithm to improving the performance.

The study on FSM control strategy focuses on three fields. One is the classic control theory represent by PID controller. As a traditional controller, PID cannot achieve the maximum performance when utilized in FSM. It usually has limited bandwidth and control accuracy, but still valued for the easy-deployment and reliable features, there are still more than 90% of the control strategy utilized in industrial are PID or derivatives of PID. The second is based on modern control theory, which performs excellent in some application where a precise mathematical model of the plant can be obtained, unfortunately, obtaining such an precise mathematical model is unpractical at most scenes. The last is
about the intelligent algorithm include the neural networks, adaptive algorithm, fuzzy algorithm and so on. These algorithm have the advantages of excellent performance, but usually difficult to solve the robustness and stability problem.

Considering the various restrictions in using the existing method, an MADRC method is employed. ADRC [8] is an emerging technology proposed by Prof. Han in 1998. The central idea is to treat the internal uncertainties and external disturbances as a lumped disturbance. ADRC estimate the lumped disturbance in real time through an ESO, then compensate the disturbance by a feedback controller. The ADRC algorithm has a general expression to simplify the deployment and has a flexible usage to assisted obtaining an expected performance. MADRC is an evolution form of the original ADRC, an modified ESO incorporate with identified plant information is employed to reject the disturbances, nonlinear factors and modelling errors which meant to obtain a precise and disturbance-free plant. Besides, a LLC consisting of two Tracking-Differentiators is employed to improve the performance of the controller.

The rest of this paper is organized as follows. The mathematical description of the FSM is obtained in section 2. The design of the original ADRC and MADRC is presented in Section 3. The simulation results are included in Section 4. The experiments results are shown in Section 5. Section 6 provides the conclusions.

2. Modelling
A sketch of the FSM structure is depicted in Figure 1. A pair of voice coil actuators utilized in push-pull mode provides the torque to the mirror. A special joint is designed to support the mirror-related component. The angle sensors mounted on the base of FSM provide position feedback of the mirror.

![Figure 1](https://example.com/figure1.png)

**Figure 1.** Configuration of the FSM

The mechanical part of FSM plant can be simplified to a classic second order spring-mass-damper plant as Eq. (1)

\[
M = J \cdot \dot{\theta} + c \cdot \dot{\theta} + K \cdot \theta
\]  

(1)

Where \( M \) is the output torque of actuators, \( J \) is the moment of inertia along the rotary axis, \( \theta \) is the angle of the mirror, \( c \) is the damping ratio and \( K \) is the rotational stiffness.

Similarly, the electronic part can expressed as Eq. (2) approximately

\[
U = B \cdot l \cdot \dot{x} + L \cdot \frac{di}{dt} + i \cdot R
\]  

(2)

where \( U \) is the input voltage of motor driver, \( B \) is the magnetic field intensity, \( l \) is the length of coil wire, \( x \) is the displacement of motor, \( L \) is total inductance of the circuit, \( i \) is current of motor and \( R \) is resistance of motor.

Consider the relation between moment \( M \) and current \( i \), we get

\[
M = i \cdot l \cdot B \cdot D
\]  

(3)

Where \( D \) represent the distance of the motor pairs.

The relation of \( x \) and \( \theta \) can simplified to \( x = D\theta / 2 \) for the small angle \( \theta \).

\[
x_1 = \theta, x_2 = \dot{\theta}, x_3 = i
\]  

(4)

Consider the above equations, the state-space equation is derived,
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{K}{J} \cdot x_1 - \frac{c}{J} \cdot x_2 + \frac{BID}{J} \cdot x_3 \\
\dot{x}_3 &= -\frac{BID}{2L} \cdot x_2 - \frac{R}{L} \cdot x_3 + \frac{1}{L} \cdot u \\
y &= x_1
\end{align*}
\]

(5)

3. Control design

3.1. Conventional LADRC

The structure of original ADRC is a nonlinear form which is very complex and has too much parameters to tune, that makes it difficult to apply in practice. To overcome the difficulty, the linear form of ADRC is introduced in [10-11] where linear ESO and linear state feedback are used. The number of the parameters in the LADRC is reduced to two, the controller bandwidth \(c\omega_c\) and the observer bandwidth \(o\omega_o\). These two parameters are closely related to the performance of the closed-loop system, LADRC is readily applicable in industrial control.

The structure of ADRC is depicted in Figure 2. ADRC include three parts, the tracking differentiator (TD), extended state observer (ESO) and a feedback control law.

![Figure 2. Structure of ADRC](image)

Take advantage of the model-free feature, the original LADRC assumes the plant be a cascaded integral model for simplicity which is easy to implement. Rewrite Eq.(5) to a state-space form,

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= f_i(x_1, x_2, x_3) + bu \\
y &= x_1
\end{align*}
\]

(6)

Where \(y\) and \(u\) are output and input respectively, \(f_i\) represent the plant dynamics and assumed unknown. Consider the external disturbance \(w\), and regard the internal dynamics and the external disturbance as a lumped disturbance \(f\), rewrite Eq.(6) as

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= f(x_1, x_2, x_3, w, t) + b_0 u \\
y &= x_1
\end{align*}
\]

(7)

Where \(f = (b - b_0)u + f_i(x_1, x_2, x_3) + w\), \(b_0\) is the estimation of \(b\), the basic idea is to obtain \(\hat{f}\), an estimate of \(f\), and use it in the control law, \(u = \left(\hat{f} + u_0\right) / b_0\), to reduce the plant to a cascaded integrator control problem. To construct the ESO, the plant in Eq.(7) is written as
Then the state observer based on Eq. (8) is expressed as
\[
\dot{x} = Ax + Bu + Eh
\]
\[
y = Cx
\]
(9)
Where
\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
\end{bmatrix},
B = \begin{bmatrix}
0 \\
0 \\
b_0 \\
0 \\
\end{bmatrix},
C = [1 \ 0 \ 0 \ 0],
E = \begin{bmatrix}
0 \\
0 \\
1 \\
\end{bmatrix}
\]
(10)
A full-order Luenberger state observer can be constructed as
\[
z = Az + Bu + L(y - \hat{y})
\]
\[
\hat{y} = Cz
\]
(11)
And \( L \) is the observer gain vector, which can be obtained by the pole placement method,
\[
L = [\beta_1 \ \beta_2 \ \beta_3 \ \beta_4]^T
\]
(12)
The controller is chosen as
\[
u = \frac{-z_4 + u_0}{b_0}
\]
(13)
Then plant is reduced to a 3rd-order integral system,
\[
\ddot{y} = (f - z_4) + u_0 \approx u_0
\]
(14)
Which can be effectively controlled with a simple feedback controller
\[
u_0 = k_1 (r - z_1) - k_2 (r - z_2) - k_3 z_3
\]
(15)
Where \( r \) is the command input.

3.2. Modified ADRC
The original ADRC has a very simple structure. The classic ADRC minimizes the amount of modeling information required to controller design. It assumes that the system parameters in Eq. (5) are totally unknown. However, if we can get some information of the plant and incorporate into the ESO design, it will improve the accuracy of the ESO. Ref [13] originally use the partially available modeling information to design an alternative ADRC. In this paper, all of the information of the plant are adopted to design the MADRC, and a new feedback control law and a well-designed LLC method is proposed to improve the performance of FSM. The algorithm is depicted in Figure 3.
3.2.1. Modified extended state observer (MESO)
For the FSM, the modeling information can be used to improve the performance of the system. Consider Eq.(6), \( f \) and \( b \) can be identified based on the input and output data, and we can use this information in the controller design. The corresponding state-space realization of Eq. (5) is expressed as

\[
A = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
-a_0 & -a_1 & -a_2 \\
\end{bmatrix}, \quad \quad B = \begin{bmatrix} 0 \\
\end{bmatrix}
\]

\[
C = \begin{bmatrix} b_0 & b_1 & b_2 \end{bmatrix}
\]

An extended state observer is used to estimate the unknown lumped disturbance. Define the extended plant as

\[
\begin{align*}
\dot{z} &= A_x z + B_x u + E_x h \\
y &= C_x z 
\end{align*}
\]

Where

\[
A_x = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
-a_0 & -a_1 & -a_2 & 1 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}, \quad B_x = \begin{bmatrix} 0 \\
\end{bmatrix}
\]

\[
C_x = \begin{bmatrix} b_0 & b_1 & b_2 & 0 \end{bmatrix}, \quad E_x = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T
\]

Consider the plant information and the following state-space form

\[
\begin{align*}
\dot{z} &= A_x z + B_x u + E_x (y - C_x z) \\
&= (A_x - L_x C_x) z + B_x u + L_x y
\end{align*}
\]

Take the bandwidth principle as suggested in [12] calculate the observer coefficients for ESO, the characteristic equation of \( A_x - L_x C_x \) is

\[
sI - (A_x - L_x C_x) = s^4 + l_1 s^3 + l_2 s^2 + l_3 s + l_4 = 0
\]

Where \( l_1, l_2, l_3, l_4 \) are the coefficients related to the plant coefficients \( a_0, a_1, a_2, b_0 \). Assume that all observer poles are placed at \( \omega_o \), the characteristic Eq.(22) is rewritten as \( (s + \omega_o)^4 = 0 \), where \( \omega_o \) is the desired bandwidth. Based on Ackerman formula, the observer gain vector is calculated as:
Where $z_1, z_2, z_3$ is the estimation of system states and $z_4$ is the estimation of the lumped disturbance. Then a nominal plant consistent with the identified model is obtained if we set $u_0 = u - z_4$, so that we can design a feedback control law with the work of MESO.

### 3.2.2. Linear feedback control law

A mathematical method is proposed to assist designing the feedback control law. Rewrite Eq.(5) to transfer function form, and determine unknown coefficients with an identification method or deriving from the design values, the original plant is expressed as

$$P_i(s) = \frac{b_{i0}}{(s + a_{i2})(s^2 + a_{i1}s + a_{i0})}$$  \label{eq:24}

Remark: the coefficients involved in Eq.(24) allow estimation errors, because the estimation errors can be compensated by MESO as well as the external disturbance. Then we get a nominal plant which is very similar to Eq.(24)

$$P_n(s) = \frac{b_{n0}}{(s + a_{n2})(s^2 + a_{n1}s + a_{n0})}$$  \label{eq:25}

As we have constrained the system to a nominal model with the help of MESO, assume the closed-loop transfer function we desired is

$$P_c(s) = \frac{b_{n0}}{(s + \omega_{i1})(s + \omega_{i2})^2}$$  \label{eq:26}

Remark: the coefficient $b_{n0}$ in Eq.(26) is unchanged compared to the coefficient in Eq.(25). Because $b_{n0}$ is related to the inertia of the moving component, keeping $b_{n0}$ unchanged can improve the efficiency of ESO. Then the feedback control law can design as

$$u = s_1z_1 + s_2z_2 + s_3z_3 + s_4z_4$$  \label{eq:27}

Where $s_1, s_2, s_3, s_4$ is calculated based on the difference between $P_n(s)$ and $P_c(s)$.

### 3.3. Lead-Leg Controller

TD is an excellent tool in generating smooth signals and limiting the maximum output based on the motor’s capability to avoid impact and prevent overflow of the output. Usually, under the limitation of sensor noise and sampling rate, the closed loop cannot set a very high feedback gain to achieve a required bandwidth. Therefore, a lead-leg controller is designed to improve the FSM bandwidth. In this paper, two TD is adopt to designed the Lead-Leg Controller as described in equation(28).

$$\begin{align*}
    f_{h1} &= f_{han}(x_{i1}(k) - v(k), x_{i2}(k), r_{i0}, h_{i0}) \\
    x_{i1}(k+1) &= x_{i1}(k) + h_{i2}(k) \\
    x_{i1}(k+1) &= x_{i1}(k) + h_{i1} \\
    f_{h2} &= f_{han}(x_{i2}(k) - (x_{i1}(k+1) + cx_{i2}(k+1)), x_{i2}(k) - x_{i2}(k+1), r_{i0}, h_{i0}) \\
    x_{i2}(k+1) &= x_{i2}(k) + h_{i2}(k) \\
    x_{i2}(k+1) &= x_{i2}(k) + h_{i2} \\
    y &= m_{h2} + nx_{21} + px_{22}
\end{align*}$$  \label{eq:28}
where \( v(k) \) is the command input, \( \text{fhan} \) is a nonlinear function referenced in [8] aimed to find a time-optimal solution that guarantees the fastest convergence without any overshoot. \( c, m, n \) and \( p \) are coefficients determined by the nominal model, \( r \) and \( h \) are parameters to adjust the \( \text{fhan} \) function to get a desired response speed and smoothness transient profile.

4. Simulation results
Once we constraint the original plant to a nominal plant, we can get a disturbance-free and ideal linear plant, and the rest of the work will be easy. So the performance of ESO is the thing that we should to test. To explore the performance of the ESO, a co-simulation environment is established with Simulink and LMS, the dynamic model is assembled in LMS and control blocks is built in Simulink which depicted in Figure 4.

![Figure 4. FSM control algorithm with Simulink and LMS](image)

The model information used in ESO is identified from LMS.

\[
P_s(s) = \frac{3.1057 \times 10^9}{(s + 5316)(s^2 + 41.39s + 5.262 \times 10^3)}
\]  

(29)

Let \( \omega_1 = \omega_2 = 4398(700Hz) \) and we get

\[
L = [3.94 \times 10^6 \ 1.62 \times 10^2 \ 2.17 \times 10^1 \ 1.20 \times 10^5]^T
\]  

(30)

Let \( \omega_0 = \omega_1 / 3 = 1466 \), the coefficients of the feedback law is

\[
s = [0 \ -9.60 \times 10^7 \ 6.18 \times 10^6 \ 2.87 \times 10^9]
\]  

(31)

The closed-loop response of the plant is plotted in Figure 5 along with the nominal plant.
Figure 5. Bode plot of the plant response, the expected closed-loop response, the closed-loop response under ADRC control and the closed-loop response under MADRC control.

The simulation results show that both the original and modified ESO can achieve a good closed-loop performance, and the closed-loop response under the MADRC control strategy performs better than the original ADRC. By setting the parameters in Eq. (26), we can have arbitrary bandwidth without overshoot as long as the sensor noisy and sampling rate can satisfy the desired level.

5. Experiments setup and results
Simulation cannot cover all the uncertainties in practice application, so an experimental setup is shown in Figure 6 to verify the performance of the proposed MADRC. The experiment devices include a FSM, a Controller and a collimator. The collimator is used to track and record the angle of the mirror. The MADRC algorithm is loaded in the Controller.

Figure 6. Experiment setup

The performance of ESO has confirmed effective in last section. In order to verify the performance of LLC, different amplitude of sweep signals is sent into the controller, and use the collimator to detect the angle of the FSM, the results are shown in Figure 7.
The changes in magnitude and phase at different input signal. The experiment results show that the control strategy of LLC has a stable performance under different input signal. The ability of different response under different input benefit from the nonlinear TD in LLC, which can always achieve a matched bandwidth under any amplitude of inputs.

6. Conclusions
In this paper, a modified ADRC strategy of FSM was investigated. It was shown that the ESO with plant information can improve the stabilities significantly. The new feedback control law can simplify the parameter tuning process, and the lead-correction loop consisting of Tracking-Differentiator can improve the bandwidth without causing a stability problem. Simulation results show that the MADRC can achieve an excellent performance without tedious work, and the experiment results proved the validation in practical application.

7. References
[1] R. W. Cochran and R. H. Vassar, 1990. “Fast steering mirrors in optical control systems,” in Advances in Optical Structure Systems, Proc. SPIE 1303, p245–251.
[2] J. Zhang, Q. Yang, K. Saito, K. Nozato, D. R. Williams and E. A. Rossi, 2015. "An adaptive optics imaging system designed for clinical use," Biomedical Optics Express 6(6), p2120-2137.
[3] W. Liu, K. N. Yao, D. N. Huang, X. D. Lin, L. Wang and Y. W. Lv, 2016. "Performance evaluation of coherent free space optical communications with a double-stage fast-steering mirror adaptive optics system depending on the Greenwood frequency," Optics Express 24(12).
[4] J. M. Hilkert, 2009. “Development of mirror stabilization line-of-sight rate equations for an unconventional sensor-to-gimbal orientation,” Proc. SPIE 7338, p733803.
[5] A. A. Portillo, G. G. Ortiz, and C. Racho, 2001. “Fine pointing control for optical communications,” in Proc. IEEE Aerospace Conf., Big Sky, MT, p 1541-1550.
[6] J. Tian, W. S. Yang, Z. M. Peng, T. Tang and Z. J. Li, 2016. "Application of MEMS Accelerometers and Gyroscopes in Fast Steering Mirror Control Systems," Sensors 16(4).
[7] MA Jia-guang, 1989. The basic technologies of the acquisition, tracking and pointing system, Opto Electronic Engineering, 16, pp. 1-42.
[8] J. Han, 2009. "From PID to active disturbance rejection control," IEEE transactions on Industrial Electronics 56(3), p900-906.
[9] L. Dong and J. Edwards, 2011. "Active disturbance rejection control for an electro-statically actuated MEMS device," International Journal of Intelligel Control and Systems 16(3), p160-169.
[10] Z. Gao, 2006. "Scaling and bandwidth-parameterization based controller tuning." Proceedings of the American control conference. Vol. 6.
[11] Z. Gao, 2006. "Active disturbance rejection control: a paradigm shift in feedback control system design," 2006 American control conference. IEEE,p 2399-2405.
[12] Z. Gao, 2003. "Scaling and bandwidth-parameterization based controller tuning. Proceedings of..."
2003 American control conference, ” Denver, CO, p 4989- 96.

[13] L. Dong, J. Edwards, 2011. "Active disturbance rejection control for an electro-statically actuated mems device. " International Journal of Intelligent Control & Systems, 16(3), p160-169.