The Dilated Chiral Quark Model

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ABSTRACT

It is argued that constituent quarks live in an effective theory that possesses an approximate conformal invariance. An effective lagrangian is constructed which in the large-$N_c$ approximation incorporates Regge asymptotic constraints. The resulting picture explains why linear-sigma models provide successful constituent quark descriptions, both at zero and finite temperature. Our analysis suggests an interesting relation between non-linearly realized conformal invariance and the completion of chiral multiplets in the broken symmetry phase.

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Particle physicists have long puzzled over why the non-relativistic quark model works so well for hadrons made up of light quarks. Although the emergence of QCD has not resolved this conundrum, a partial explanation can be found in the chiral quark model of Manohar and Georgi \[1\]. If one assumes that the chiral symmetry breaking energy scale is larger than the confinement scale, then the effective low-energy field theory between these scales consists of the interactions among constituent quarks, gluons, and *elementary* Goldstone bosons of chiral symmetry. Because they are now massive and interact via weak gluon exchange (which accounts for spin-dependent corrections), the constituent quarks can be treated approximately as non-relativistic particles. It remains unexplained, however, why constituent quarks behave as bare Dirac particles, with axial vector coupling \( g_A \) close to (but smaller than) one and no anomalous magnetic moment, \( \mu_q \). Some time ago Weinberg \[2\] suggested that this could be understood by imposing on the chiral quark model the requirement of Regge asymptotic behavior. He argued that in the large-\( N_c \) limit \( g_A = 1 \) and \( \mu_q = 0 \). Various authors then showed that leading \( 1/N_c \) corrections push \( g_A \) down but leave \( \mu_q \) unchanged \[3,5\]. In this letter we concentrate on the asymptotic behaviour of the \( \pi-\pi \) interaction in the chiral quark model. (The importance of this process in establishing the leading \( 1/N_c \) corrections to \( g_A \) has been emphasized in [Ref. 4.]) We pursue the following approach: we take the chiral quark model seriously as an effective theory of QCD, and consider the constraints that follow from imposing Regge asymptotic behaviour.

Our starting point is an old “theorem” of Weinberg \[7\] which states that the content of quantum field theory is given entirely by general physical principles like unitarity and cluster decomposition, together with the assumed internal symmetries. QCD is characterized by \( SU(3)_c \) gauge invariance and (in the chiral limit to which we will restrict ourselves in this letter) a global \( SU(2)_L \otimes SU(2)_R \) symmetry, which is however not obviously manifest in the spectrum. Its usual representation employs current quark fields that realize chiral symmetry linearly, but have non-trivial bilinear vacuum expectation values. Weinberg’s “theorem” suggests that an “equivalent” field theory exists, which employs constituent quark fields that realize chiral symmetry non-linearly and therefore includes explicitly also Goldstone boson fields. Since these additional degrees of freedom destroy the good asymptotic behaviour that one would expect of QCD, this effective description is useful only at low-energies; the introduction of additional degrees of freedom is necessary to ensure true QCD behavior. Furthermore, it is not unreasonable to suppose that these additional degrees of freedom have, as do pions, an origin in the symmetries of QCD. We will show that conformal invariance emerges as an important symmetry in this respect.

Since the chiral quark model is a non-renormalizable field theory, quanta not present in the low-energy theory should appear at higher energies in order to effect the necessary cancellations among graphs. This expectation can be made practical by combining low-energy theorems of chiral symmetry with dispersion sum rules (particularly in the \( \pi-\pi \) sector where the amplitude at threshold is completely determined by the geometry of the coset space) that find their justification in Regge asymptotic behaviour. In the large-\( N_c \) approximation, one would expect that sum rules for Goldstone boson scattering off a quark or meson target are saturated by single particle states in the narrow width

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1 This conclusion has been, however, challenged recently in [Ref. 3](#) (See also [Ref. 4](#)).
approximation [3]. The sum rules can then be shown to possess algebraic content [3]. We will briefly review this formalism.

Chiral symmetry severely constrains low-energy scattering processes with Goldstone bosons on external lines, and allows an expansion of amplitudes in powers of momenta. What is less well-known is that one can also obtain interesting constraints by expanding chiral tree amplitudes in inverse powers of momenta. In this case, the parameters associated with the heavy particle content contribute to the power of energy which is protected by chiral symmetry, and for which there is therefore no invariant counterterm. The algebraic consequences of chiral symmetry arise from the need for cancellations among these heavy-particle contributions. (For higher powers of energy there exist chiral invariant contact interactions with arbitrary coefficients, and therefore no constraints are obtained.) The complete set of Adler-Weisberger sum rules can be put into the form

\[ [X_i, X_j] = i\epsilon_{ijk} T_k, \]  
where \( T_i \) denotes the isospin generator,

\[ [T_i, T_j] = i\epsilon_{ijk} T_k, \]  
and \( X_i \) is the isovector, reduced matrix element for emission of a pion with isospin \( i \) between one particle states. Hence,

\[ [T_i, X_j] = i\epsilon_{ijk} X_k. \]  
Eqs. (1) through (3) imply that particles appearing in chiral tree-graphs furnish representations of \( SU(2) \otimes SU(2) \). In general, strongly interacting particles would be expected to fall into complicated reducible representations. One can learn more by considering sum rules of superconvergence type. The absence of Regge trajectories with “exotic” quantum numbers leads to sum rules which can be expressed in algebraic form as

\[ [X_j, [M^2, X_i]] = -[M^2_i]\delta_{ij}. \]  
These sum rules reveal that the diagonal-mass-squared matrix can be written as \( M^2=M^2_0+M^2_4 \), where \( M^2_0 \) is a chiral singlet, and \( M^2_4 \), the symmetry breaking part of the mass matrix, transforms as the fourth component of a chiral four-vector. An important consequence of this sum rule is that the masses of any two particles in an irreducible representation are equal.

In Ref. 2 Weinberg considered the representation involving the constituent quark; Eqs. (1) through (4) then constrain \( g_A \). Sum rules for pions scattering off baryon targets are readily constructed, yielding commutation relations for baryon states similar to the Gervais-Sakita-Dashen-Manohar large-\( N_c \) consistency conditions [14]: for a baryon \( T_i^N = \sum_{a=1}^N T_i(a) = O(1) \), and \( X_i^N = \sum_{a=1}^N X_i(a) = N(X_i^{N(0)} + 1/N X_i^{N(1)} + O(1/N^2)) \) with \( X_i^{N(0,1)} = O(1) \), so Eq. (1) gives \( [X_i^{N(0)}, X_j^{N(0)}] = 0 \), \( [X_i^{N(0)}, X_j^{N(1)}] + [X_i^{N(1)}, X_j^{N(0)}] = 0 \), and so on. Consistency conditions for the baryon mass matrix follow in a similar fashion from Eq. (4). (The reason behind the breaking of the chiral symmetric structure for baryons in the \( N_c \to \infty \) limit is unclear to us.)
We now turn to the π-π sector in the chiral quark model. Since confinement is a large-separation effect, we will assume that \( q\bar{q} \) states have no place in the effective theory\(^2\). This assumption eliminates several chiral quark model paradoxes. If \( q\bar{q} \) states are a part of the spectrum, then there is a pseudoscalar \( q\bar{q} \) bound state as well as an elementary pion \([1]\). Furthermore, there is a duality between skyrmion stability and the excitation(s) that unitarize the π-π interaction. That is, vector mesons like \( \rho \), which couples strongly to the pion, are responsible for the stability of skyrmions. In the chiral quark model, the existence of stable skyrmions would lead to a second double-counting problem. Therefore, consistency itself suggests that mesonic states are generated by the running of \( \alpha_s \) at the confinement scale. We will see that this simple point has far reaching consequences.

If there are no color-singlet excitations in the effective theory with non-trivial transformation property under the unbroken isospin subgroup, then π must be joined by at least one isoscalar in a representation of the form \( 4 \oplus 1 \oplus ... \oplus 1 \). We can go further by considering the representation involving the pion in the color-singlet sector. There a quartet consisting of π, \( \epsilon \), and the longitudinal components of \( \rho \) and \( a_1 \) can be shown to fill out the reducible representation given by \( 4 \oplus 6 \) of chiral \( SU(2) \otimes SU(2) \) \([2]\). Consistency suggests that above the confinement scale, the pion falls into the simplest irreducible representation, the 4, in which a single scalar acts as the fourth component of a chiral four vector. According to Eq. (4), there can be no mass splitting within the multiplet, and hence the scalar should be more or less degenerate with the pion.

Since all quanta in the effective theory are accounted for, the interactions among the constituent quarks, pions, and the scalar should be weak enough that no bound states appear in the spectrum. Hence one might expect the existence of an additional symmetry that keeps the interactions involving the scalar under control. One might also suspect that such a symmetry is spontaneously broken, since Goldstone bosons are the only “natural” massless scalars. Fortunately, such a symmetry exists. The QCD lagrangian, in the absence of large quantum effects (for example, at a fixed point of the \( \beta \)-function), possesses an approximate conformal invariance, which if relevant below the chiral symmetry breaking scale is necessarily spontaneously broken. The resulting elementary dilaton field is the natural chiral partner of the elementary pion. This approach has the virtue of allowing a systematic effective lagrangian analysis, including symmetry breaking, based on the operator scaling properties of QCD. Furthermore, there is the appealing feature that all the quanta in the effective theory, besides the QCD degrees of freedom, find their origin in symmetry.

With this in mind, consider the coset space,

\[
\frac{G}{H} = \frac{SU(2)_L \otimes SU(2)_R \otimes C}{SU(2) \otimes P},
\]

\[\mathcal{C}\] denotes the conformal group, and \( \mathcal{P} \) the Poincaré group. (We also assume invariance under the discrete symmetries of QCD.) The Goldstone triplet \( \pi \) is contained in a matrix field \( U = \xi \xi, \xi = exp(\mathrm{i}\pi T_3) \) which transforms under \( SU(2)_L \otimes SU(2)_R \) as \( \xi \rightarrow L \xi v^\dagger = v^\dagger R = v^\dagger \),

where \( v \), a non-linear function of \( L \), \( R \), and \( \pi \), is implicitly defined. The condition that

\[\text{Ref. 11}\]
a chiral transformation be conformally invariant requires that the Goldstone boson fields have scale dimension, $d_\psi=0$. Out of $\xi$ one can construct $V_\mu = \frac{1}{2}((\xi\gamma^\mu\xi + \xi\partial_\mu\xi)\gamma^\mu); V_\mu \rightarrow vV_\mu v^\dagger + v\partial_\mu v^\dagger$, and $A_\mu = \frac{1}{2}i(\xi\gamma^\mu\xi - \xi\partial_\mu\xi)\gamma^\mu); A_\mu \rightarrow vA_\mu v^\dagger$. We introduce a constituent quark doublet, which transforms as $\psi \rightarrow v\psi$ under $SU(2)_L \otimes SU(2)_R$, and as $\psi \rightarrow e^{d_\psi\lambda}\psi$ under dilatations. The scale dimension $d_\psi$ has no invariant significance when dilatation symmetry is spontaneously broken \[12\], and so choice of $d_\psi$ corresponds to the definition of the constituent quark field. It is convenient to introduce the dilaton field $\sigma$, which scales inhomogeneously, by way of a scalar field $\chi=\exp(\frac{\sigma}{f_d})$ with canonical scale dimension, $d_\chi=1$.

The most general dilatation invariant chiral quark lagrangian\(^3\) at leading order is then given by

$$
\mathcal{L} = \bar{\psi}(\mathcal{D} + V)\psi \chi^{2d_\psi+3} + g_A\bar{\psi}\gamma_5\psi - m\bar{\psi}\chi^{2d_\psi+4} + \frac{1}{4}f_d^2\partial(U\partial U^\dagger)\chi^2 + \frac{1}{2}\partial_\mu\partial_\mu\chi - \frac{1}{2}\partial(U\partial U^\dagger)G_{\mu\nu}G^{\mu\nu} + ig_s(\bar{\psi}\gamma_\mu\psi)(\partial_\mu\chi)\chi^{2d_\psi+2} - V(\chi) + \ldots,
$$

(6)

where $m$ is the constituent quark mass and the covariant derivative is given by $D_\mu = \partial_\mu + igG_{\mu\nu}$, with $G_{\mu\nu}$ the gluon field of field strength $G_{\mu\nu}$. Although the lagrangian, Eq. (6), is manifestly scale invariant, the derivative interactions are not necessarily conformally invariant. It is not difficult to show that conformal invariance determines $g_s = -(d_\psi - \frac{3}{2}) \[12\]$. Clearly the canonical choice $d_\psi = \frac{3}{2}$ is most convenient. We then have, defining $\chi \equiv f_d\chi$ and $\kappa \equiv \frac{f_d^2}{f_d}$,

$$
\mathcal{L} = \bar{\psi}(\mathcal{D} + V)\psi + g_A\bar{\psi}\gamma_5\psi - \sqrt{\kappa}\frac{m}{f_d}\bar{\psi}\psi\chi + \frac{1}{4}\kappa tr(\partial U\partial U^\dagger)\chi^2 + \frac{1}{2}\partial_\mu\partial_\mu\chi - \frac{1}{2}tr(G_{\mu\nu}G^{\mu\nu}) - V(\chi) + \ldots
$$

(7)

In the absence of explicit dilatation breaking, the potential $V(\chi)$ is a flat function, and therefore the dynamical mechanism responsible for fixing the vacuum expectation value of $\chi$ is absent\(^4\). Thus, conformal symmetry is spontaneously broken only if it is explicitly broken \[13\]. Hence we must include minimal symmetry breaking. In QCD, breaking of conformal invariance is governed by the trace of the energy-momentum tensor: the divergence of the dilatation current $J_\mu$ is

$$
\partial^\mu J_\mu = \Theta^\mu = \frac{\beta(g)}{2g}G_{\mu\nu}G^{\mu\nu}
$$

(8)

in the chiral limit. If there are low-energy effective theories of QCD that possess an approximate conformal invariance, then the symmetry breaking terms in the effective theory should transform like $\lambda\partial^\mu J_\mu$, where $\partial^\mu J_\mu$ is the most general dimension-four, gauge invariant Lorentz scalar constructed out of the effective fields. Stability of the vacuum implies

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\(^3\) Meaning of course a lagrangian that yields a dilatation invariant action.

\(^4\) Note that although $\chi^4$ looks conformally invariant, stability of the vacuum requires subtracting $4f_d^3\sigma$, which is clearly not invariant.
\[ V(\chi) = -\frac{k m_\sigma^2}{8 f_\pi^2} \left[ \frac{1}{2} \chi^4 - \chi^4 \log \left( \frac{\kappa \chi^2}{f_\pi^2} \right) \right], \tag{9} \]

where \( m_\sigma \) is the dilaton mass. There is of course an infinite tower of additional symmetry breaking terms involving the interactions of the dilaton with itself and with the other degrees of freedom in the effective theory, but these are expected to be suppressed by powers of the dilaton mass divided by the chiral symmetry breaking scale.

How do we put the pions and the dilaton in a “Goldstone quartet”? We can transform our effective theory to a linear basis, \( \Sigma = U \chi \sqrt{\kappa} \) where \( \Sigma \equiv \sigma' + i \vec{\tau} \cdot \vec{\pi}' \), and \( Q = \frac{1}{2} ((\xi + \xi^\dagger) + \gamma_5 (\xi - \xi^\dagger)) \psi \). In terms of the new fields the lagrangian, Eq. (7), is given by

\[ \mathcal{L} = i \tilde{Q} D Q - \frac{1}{2} tr(\tilde{G}_{\mu\nu} \tilde{G}^{\mu\nu}) + \frac{1}{4} tr(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger) - \frac{m}{2 f_\pi} \tilde{Q} [\Sigma + \Sigma^\dagger - \gamma_5 (\Sigma - \Sigma^\dagger)] Q + \left( \frac{1}{16} - \frac{\kappa}{16} \right) tr(\Sigma \Sigma^\dagger)^{-1} tr(\partial_\mu (\Sigma \Sigma^\dagger)) tr(\partial^\mu (\Sigma \Sigma^\dagger)) \]

\[ - \frac{i}{4} (1 - g_A) tr(\Sigma \Sigma^\dagger)^{-2} \tilde{Q} [tr(\phi(\Sigma \Sigma^\dagger)) \{\Sigma, \Sigma^\dagger\} - 2 tr(\Sigma \Sigma^\dagger) (\Sigma \phi \Sigma^\dagger + \Sigma^\dagger \phi \Sigma)] Q \]

\[ - \frac{i}{2} (1 - g_A) tr(\Sigma \Sigma^\dagger)^{-1} \tilde{Q} [\gamma_5 (\Sigma \phi \Sigma^\dagger - \Sigma^\dagger \phi \Sigma)] Q + \frac{m_\sigma^2}{64 f_\pi^2} [tr(\Sigma \Sigma^\dagger)]^2 - \frac{m_\sigma^2}{32 f_\pi^2} [tr(\Sigma \Sigma^\dagger)]^2 \log[tr(\Sigma \Sigma^\dagger)]/2 f_\pi^2 + ... \tag{10} \]

As the conformal anomaly is turned off the interactions with singularities in field space in the symmetric phase should vanish, requiring \( \kappa, g_A \to 1 \). On the broken symmetry side of the phase transition, the large-\( N_c \) algebraic sum rules, Eq. (1) and Eq. (4), are satisfied with precisely these values in what we call the “dilaton limit”, defined by \( m_\sigma \to 0 \), and \( \kappa = g_A = 1 \). In the dilaton limit (dropping the primes on the \( \pi \) and \( \sigma \) fields) the lagrangian simplifies to

\[ \mathcal{L} = i \tilde{Q} D Q - \frac{1}{2} tr(\tilde{G}_{\mu\nu} \tilde{G}^{\mu\nu}) + \frac{1}{2} \partial_\mu \bar{\pi} \partial^\mu \pi + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{m}{f_\pi} \tilde{Q} [\sigma - i \gamma_5 \bar{\pi} \cdot \vec{\tau}] Q + \frac{m_\sigma^2}{16 f_\pi^2} (\sigma^2 + \bar{\pi}^2)^2 - \frac{m_\sigma^2}{8 f_\pi^2} (\sigma^2 + \bar{\pi}^2)^2 \log[(\sigma^2 + \bar{\pi}^2)/f_\pi^2] + ... \tag{11} \]

This is our main result. The effective theory of constituent quarks is renormalizable with chiral symmetry breaking induced by a generalized Coleman-Weinberg potential \[14\]. For our purposes, renormalizability means that all quanta necessary to ensure good asymptotic behaviour of scattering amplitudes are already present in a well defined lagrangian\(^5\). It is amusing that the above lagrangian follows directly from Weinberg’s “theorem” as the only known field theory that is “equivalent” (in the manner described above) to QCD. The emergence of a low-lying scalar in the large-\( N_c \) limit is not at odds with the quasiparticle

\(^5\) By well defined we mean that the masses of all quanta in the effective theory are substantially less than the chiral symmetry breaking scale.
philosophy inherent to the chiral quark picture. Moreover, since the scalar is a Goldstone boson, one can systematically include corrections to the dilaton limit. The scenario advocated here is much in the spirit of the “vector limit” of Ref. 15.

Perhaps the most convincing support for the point of view advocated in this letter lies in the study of chiral symmetry restoration at finite temperature. In the dilaton limit, the zero temperature effective theory of constituent quarks is very near the chiral symmetry breaking phase transition. As temperature approaches its critical value, the $X_i$ become true symmetry generators$^6$, and the important modes for discussing chiral symmetry restoration are the quarks and the “Goldstone quartet”. In fact, it has been shown $^7,8$ that simple constituent quark models with elementary $\pi$ and $\sigma$ modes reproduce the pattern of chiral symmetry restoration observed in finite temperature lattice Monte Carlo calculations. (Extension of our model to finite temperature is in progress.)

What are the phenomenological implications of our effective lagrangian? Presumably away from the large-$N_c$ limit the dilaton mass increases; of course a substantial dilaton mass is already achieved by inclusion of explicit chiral symmetry breaking effects. As mentioned earlier, the philosophy of the chiral quark model is that, except for long-range effects that can be simulated by a confining potential or a bag, the quark-gluon coupling is small and allows for a perturbative treatment (the first term of which splits the delta from the nucleon). Because the lagrangian Eq. (11) results from a derivative expansion, leading order S-matrix elements consist of the sum of tree graphs involving quarks and the Goldstone bosons only. If we further assume that confinement effects can be neglected for most nucleon structure purposes, the system of coupled differential equations that follows is known to possess non-topological soliton solutions. It is found $^9$ that these baryonic solutions are essentially determined by the coupling of the quark to the Goldstone quartet, being insensitive to details of the effective potential; in particular, a rather low sigma mass can be tolerated while still producing a realistic soliton mass. Generically, the main drawback of such models is seen from the crudest non-relativistic approximation: $g_A$ has to be smaller than one in order to obtain the correct nucleon axial coupling. Of course $g_A$ is shifted away from unity by loop effects; it is encouraging that for large values of the dilaton mass $g_A$ indeed decreases $^3$. The “dilated” chiral quark model also provides at least a partial explanation of the successes of constituent quark models of nuclear forces $^20$, where the exchange of a low-lying scalar excitation is the most successful way of generating the necessary intermediate-range attraction.

It has long been known that there is a cryptic connection between renormalizability and conformal invariance $^{21}$. We find it particularly interesting that conformal invariance and chirality conspire to yield good asymptotic behaviour in an effective theory in which both symmetries are spontaneously broken$^7$. In the context of chiral quarks, we have made use of large-$N_C$ algebraic sum rules to argue that the absence of confining effects implies the relevance of conformal invariance, with the elementary dilaton field completing the chiral multiplet and rendering the effective theory renormalizable. It is tempting to speculate

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$^6$ Algebraic realizations and the chiral symmetry breaking phase transition are discussed in Ref. 16.

$^7$ This would appear to be related to the need for a particle with the properties of the Higgs boson in gauge field theories with spontaneous symmetry breakdown.
that there exists a deep connection between conformal invariance and chiral symmetry breaking.

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