Hot-electron Effect in A Cylindrical Nanoshell

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A general expression for calculating the thermal power transferring from 3-dimensional electron to any D-dimensional phonon subsystem is derived in this paper. Electron-phonon coupling in a free suspended cylindrical nanoshell is studied, in which acoustic phonons are confined to quasi-one dimension but electrons behave three-dimensionally. The temperature dependence of the thermal power is obtained analytically, and the low-temperature crossover from the $T^3$ to $T^3/(1 - \nu^2) + 9\gamma T^4/[T^3(1 - \nu^3/2)]$ dependence is also observed. The corresponding quantities are estimated for the material parameters from a metallic nanotube.

PACS numbers: 63.22.+m, 63.20.Kr, 85.85.+j

I. INTRODUCTION

The development of modern micro-manufacturing and extremely low temperature technology enables people to explore the physical properties of condensed matter in nanoscales, where the phonon thermal conductance at low-temperature is one of the most active area that attracts much interest of researchers. In addition to its significance for investigating fundamental phonon physics and macroscopic quantum phenomena, it may find applications in nanoscale bolometers and calorimeters with unprecedented sensitivity and even in quantum information processing. It is an established fact that the conduction electrons in a metal at low temperatures are only weakly coupled to the lattice phonons. Therefore, the electron-electron scattering will cause nonequilibrium electrons to equilibrate on a timescale that is typically much shorter than the electron-phonon relaxation time, resulting in an electron distribution that is thermal, but at a temperature higher than that of the lattice. Understanding this hot-electron effect is crucial in studies of transport in semiconductors and metals at low temperatures.

The electron-phonon scattering rate in conductors has been studied both theoretically and experimentally with good agreement between theory and experiment. The theoretical work has focused on the rate of energy transfer between electrons and acoustic phonons in an infinite bulk metal. The weak coupling between electrons and phonons allows separate temperatures $T_{el}$ and $T_{ph}$ to be defined for the electron and phonon subsystems, and at low temperatures significant differences in these temperatures are easily achieved. In the deformation-potential coupling, the primary theoretical result states that the rate of energy exchange $P$ from the electrons to the phonons is given by

$$ P = \Sigma V (T_{el} - T_{ph}^n), $$

(1)

where $n = 5$, $V$ is the volume of the system, and $\Sigma$ is a material parameter given by

$$ \Sigma = \frac{8 \zeta(5) k_B^5 \varepsilon_F^2 N_{el}(\varepsilon_F)}{3\pi^3 \hbar^4 \rho v_F v_l^4} $$

(2)

Here $\zeta(m)$ is the Riemann zeta function, $\varepsilon_F$ is the Fermi energy, $N_{el}$ is the electronic density of states (DOS) per unit volume, $\rho$ is the mass density, $v_l$ is the bulk longitudinal sound speed, and $v_F$ is the Fermi velocity.

All of the experiments to date, however, have been performed on thin metal films deposited on semiconducting substrates, whereas the theory has assumed bulk metals. The metal films typically have a thickness $d$ of the order of ten to several hundred nanometers, with lateral dimensions of a few microns. Although the electrons in the films are three dimensional, the Fermi wavelength being much smaller than any of the physical dimensions, the phonons at low temperatures will have wavelengths larger than $d$. Therefore, one expects similar temperature dependence of the thermal power described by Eq. (1), but with different pre-factors $\Sigma$ and exponents $n$ ranging from $n = 3$ to $n = 6, 10, 11, 17, 18$.

In addition, there is a growing interest in the low-temperature properties of electrons and phonons in mechanically suspended nanostructures, where the phonon spectrum is strongly modified from that in a bulk metal, and thus the similar behavior as in thin films was observed. Actually, experimental results dictated that the reduction of phonon dimensionality leads to weak temperature dependent of the heat flux between electrons and phonons. In many cases, the power transferred from electrons into phonons follows the $P \sim T^{(D+2)}$ temperature dependence, where $D$ is the dimension of space seen by the phonons.

The $T^3$ dependence of the heat flux obtained for suspended one-dimensional metallic nanowires in Ref. 25 also supports the above conjecture. It stimulates us to investigate the hot-electron effect in a geometry of cylindrical nanoshell in which electrons behave three-dimensionally but phonons are confined in quasi-one di-
mension. This geometry can approximate a single-walled metallic buckytube and also resemble the microtubules found in many parts of the human body.\textsuperscript{27} Single-walled carbon nanotubes are remarkable quasi-dimensional materials because the nanotubes can be either metallic or semiconducting depending on their helicity.\textsuperscript{28} Therefore, applications of single-walled carbon nanotubes as nanoscale electronic devices have long been expected. In practice, metallic carbon nanotubes are very good conductors and thus can be used to realize interconnection of integrated circuits. Simple circuits based on semiconducting carbon nanotube field-effect transistors have already been demonstrated. The investigation of the electron-phonon interaction in nanotubes plays an important role in understanding their properties and thus has attracted much interest of researchers.\textsuperscript{29–31} Hot carrier energy relaxation is a promising candidate of probing the electron-phonon coupling in carbon nanotubes because it is directly related to inelastic scattering.\textsuperscript{30,31} In the current work, we will derive an analytical expression (12) is obtained for the system consisting of a three-dimensional electron gas and a three-dimensional solid. The Hamiltonian is

\[
\mathcal{H} = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} \hat{c}_{\mathbf{k}}^\dagger \hat{c}_{\mathbf{k}} + \sum_{n} \hbar \omega_n \hat{a}_{\mathbf{n}}^\dagger \hat{a}_{\mathbf{n}} + \delta \mathcal{H},
\]

where \( \hat{c}_{\mathbf{k}}^\dagger \) and \( \hat{c}_{\mathbf{k}} \) are electron creation and annihilation operators, with \( \mathbf{k} \) the momentum, and \( \hat{a}_{\mathbf{n}}^\dagger \) and \( \hat{a}_{\mathbf{n}} \) are bosonic phonon creation and annihilation operators. The vibrational modes, labeled by an index \( n \), are eigenfunctions of the continuum elasticity equation

\[
v_{\text{el}}^2 \nabla \times \nabla \times \mathbf{u} - v_{\text{el}}^3 \nabla \cdot \mathbf{u} = \omega^2 \mathbf{u}
\]

for linear isotropic media, along with accompanying boundary conditions. \( v_{\text{el}} \) and \( v_{\text{el}} \) are the bulk transverse and longitudinal sound velocities. The electron-phonon interaction \( \delta \mathcal{H} \) is described by the deformation potential

\[
\delta \mathcal{H} \equiv \frac{2}{3} \varepsilon_{\text{F}} \int_{V_{\text{el}}} d^3r \psi^\dagger \psi \cdot \mathbf{u},
\]

with

\[
\mathbf{u}(\mathbf{r}) = \sum_{n} (2\hbar \omega_n)^{-\frac{1}{2}} [\hat{f}_{\mathbf{n}}(\mathbf{r}) a_n + \hat{f}_{\mathbf{n}}^\dagger(\mathbf{r}) a_{\mathbf{n}}^\dagger]
\]

and the quantized displacement field. The vibrational eigenfunctions \( \hat{f}_{\mathbf{n}}(\mathbf{r}) \) are defined to be solutions of the elasticity field equations, normalized over the phonon volume \( V_{\text{ph}} \) according to \( \int_{V_{\text{ph}}} d^3r \mathbf{f}_{\mathbf{n}}^\dagger \cdot \mathbf{f}_{\mathbf{n}'} = \delta_{\mathbf{n} \mathbf{n}'} \). It will be convenient to rewrite the electron-phonon interaction as

\[
\delta \mathcal{H} = \sum_{\mathbf{k}q} [g_{\mathbf{n}} \hat{c}_{\mathbf{k}+\mathbf{q}}^\dagger \hat{c}_{\mathbf{k}} a_n + g_{\mathbf{n}}^* \hat{c}_{\mathbf{k}-\mathbf{q}}^\dagger \hat{c}_{\mathbf{k}}^\dagger a_{\mathbf{n}}],
\]

with coupling constant

\[
g_{\mathbf{n}} = \frac{2}{3} \varepsilon_{\text{F}} (2\hbar \omega_n)^{-\frac{1}{2}} \int_{V_{\text{el}}} d^3r \nabla \cdot \mathbf{f}_n e^{-i\mathbf{q} \cdot \mathbf{r}}.
\]

The quantity we calculate is the thermal energy per unit time transferred from the electrons to the phonons,

\[
P \equiv \sum_{\mathbf{k}qn} \hbar \omega_n \left[ \Gamma_{\mathbf{n}}^{\text{em}}(\mathbf{k} \rightarrow \mathbf{k} - \mathbf{q}) - \Gamma_{\mathbf{n}}^{\text{abs}}(\mathbf{k} \rightarrow \mathbf{k} + \mathbf{q}) \right],
\]

where

\[
\Gamma_{\mathbf{n}}^{\text{em}}(\mathbf{k} \rightarrow \mathbf{k} - \mathbf{q}) = 2\pi \left| g_{\mathbf{n}} \right|^2 \left[ n_{\text{B}}(\omega_n) + 1 / n_{\text{F}}(\varepsilon_{\mathbf{k}}) \right] \times \left[ 1 - n_{\text{F}}(\varepsilon_{\mathbf{k}+\mathbf{q}}) \right] \delta(\varepsilon_{\mathbf{k}-\mathbf{q}} - \varepsilon_{\mathbf{k}} + \hbar \omega_n),
\]

is the golden-rule rate for an electron of momentum \( \mathbf{k} \) to scatter to \( \mathbf{k} - \mathbf{q} \) while emitting a phonon \( n \), and

\[
\Gamma_{\mathbf{n}}^{\text{abs}}(\mathbf{k} \rightarrow \mathbf{k} + \mathbf{q}) = 2\pi \left| g_{\mathbf{n}} \right|^2 n_{\text{B}}(\omega_n) n_{\text{F}}(\varepsilon_{\mathbf{k}}) \times \left[ 1 - n_{\text{F}}(\varepsilon_{\mathbf{k}+\mathbf{q}}) \right] \delta(\varepsilon_{\mathbf{k}+\mathbf{q}} - \varepsilon_{\mathbf{k}} - \hbar \omega_n)
\]

is the corresponding phonon absorption rate. \( n_{\text{B}} \) is the Bose distribution function with temperature \( T_{\text{ph}} \), and \( n_{\text{F}} \) is the Fermi distribution with temperature \( T_{\text{el}} \). The factor of 2 in \( \langle 12 \rangle \) accounts for spin degeneracy. It is possible to obtain an exact expression for \( P \); the result (suppressing factors of \( h \) and \( k_{\text{B}} \)) is

\[
P \equiv \frac{m^2 \varepsilon_{\text{F}}^2}{8\pi^4} \sum_{\mathbf{k}qn} \int_{0}^{\infty} d\omega \delta(\omega - \omega_n) \left[ \frac{1 + \exp\left(\frac{\omega}{T_{\text{el}} - 1}\right)}{1 + \exp\left(\frac{\omega}{T_{\text{el}} - 1}\right)} \right] \times \int d^3k \frac{|g_{\mathbf{n}}|^2}{|\mathbf{q}|} \left[ \omega + T_{\text{el}} \ln \left( \frac{1 + \exp\left(\frac{\omega + \omega_{\text{D}}}{T_{\text{el}} - 1}\right)}{1 + \exp\left(\frac{\omega + \omega_{\text{D}}}{T_{\text{el}} - 1}\right)} \right) \right].
\]

The logarithmic term in \( P \) can be shown to be negligible in the temperature regime of interest and will be dropped.

Although Eq. \( \langle 12 \rangle \) is obtained for the system consisting of 3-dimensional phonon and 3-dimensional electron subsystems, we can simple extend it into those geometries that are three-dimensional seen by electrons but arbitrary D-dimensional by phonons. The DOS weighted form of \( P \) (suppressing factors of \( h \) and \( k_{\text{B}} \)) reads

\[
P \equiv \frac{2m^2 \varepsilon_{\text{F}}^2 \Omega_{\text{ph}}}{9\pi D} \int_{0}^{\omega_{\text{D}}} d\omega F(\omega) \left( \frac{\omega}{e^{\omega/T_{\text{el}} - 1}} - \frac{\omega}{e^{\omega/T_{\text{ph}} - 1}} \right),
\]

where \( \omega_{\text{D}} \) is the Debye frequency, \( \Omega_{\text{ph}} \) is the generalized volume of the D-dimensional phonon subsystem, and \( F(\omega) \) is a strain-weighted vibrational DOS, defined by

\[
F(\omega) \equiv \sum_{n} U_n \delta(\omega - \omega_n),
\]
with
\[ U_n = \frac{1}{V_{el}} \int_{V_{el}} d^3r \int d^3r' \nabla \cdot \mathbf{f}_n(r) \nabla' \cdot \mathbf{f}_n^*(r') \times \int \frac{d^3k}{(2\pi)^3} k^{-1} e^{-ik \cdot (r-r')} . \tag{14} \]

Here, \( U_n \) can be interpreted as an energy associated with mass-density fluctuations interacting via an potential determined by the last integration in Eq. (14). To this end we have reduced the calculation of \( P \) to the calculation of \( F(\omega) \). Allen\textsuperscript{24} has derived a related weighted-DOS formalism.

For the conventional hot-electron effect in bulk materials, one deals with 3-dimensional phonon subsystem and \( \Omega_{ph} = V_{el} = V \). Caring out the last integration in Eq. (14) gives
\[ U_n = \frac{1}{2\pi^2 V} \int d^3r d^3r' \frac{\mathbf{\nabla} \cdot \mathbf{f}_n(r) \mathbf{\nabla}' \cdot \mathbf{f}_n^*(r')}{|r-r'|^2} , \tag{15} \]

where the longitudinal acoustic phonon eigenfunction with momentum \( \mathbf{k} \) is
\[ \mathbf{f}_k = e^{i\mathbf{q} \cdot \mathbf{r}} \frac{1}{\sqrt{V}} \mathbf{e}_k . \tag{16} \]

Substituting this eigenfunction into Eq. (15) and carrying out the integration, we obtain the weighted DOS
\[ F(\omega) = \frac{1}{2\pi^2 V} \omega^3 \tag{17} \]
The conventional expression for the net rate of energy relaxation in Eq. (1) and the corresponding coefficient in Eq. (2) can be easily reduced by this DOS.

### III. HOT-ELECTRONS IN A CYLINDRICAL NANOSHELL

In this section we will derive the rate of the thermal energy relaxation in a solid in which the electrons move three-dimensionally but the phonons are confined in a cylindrical nanoshell in which the external pressure on the inner surfaces is equal to that on the outer surfaces, and the transverse dimensions are far smaller than the length in the \( z \)-direction, and the thickness \( h \) and the radius \( R \) satisfy condition \( h \ll R \). There has been extensively studies on their properties of the acoustic phonons in cylindrical shells.\textsuperscript{27} In this work, we employ the result of Ref.\textsuperscript{27}, in which the eigenfunction reads,
\[ \mathbf{f}(r) = \begin{pmatrix} -iC_r \\ C_\varphi \\ C_z \end{pmatrix} \exp(i\varphi + iq_z z) \tag{18} \]

where \( \varphi \) is the azimuth angular, \( q_z \equiv q/R \) is the wavevector in the \( z \)-direction. \( C_r, C_\varphi \) and \( C_z \) are respectively the normalization constants in three orthogonal directions. The dimensionless eigenmodes \( \Omega = R\omega/c_l \) are given by
\[ \begin{align*}
\Omega_{I}^{m}(q) & \equiv \sqrt{q^2 + m^2 + 1}, \\
\Omega_{II}^{m}(q) & \equiv \sqrt{\nu_-(q^2 + m^2)}, \\
\Omega_{III}^{m=0}(q) & \equiv \sqrt{1 - \nu^2} \frac{q}{1 + q}, \\
\Omega_{m}^{III}(q) & \equiv \sqrt{1 - \nu^2} \frac{q^2}{m \nu^2 + 1 + q^2} .
\end{align*} \tag{19} \]

Here \( q \) is the dimensionless wavevector, \( m \) is the branch index, \( \nu_- = (1 - \nu)/2 \), \( \nu \) is the Poisson ratio, and \( c_l = \sqrt{E/\rho(1-\nu^2)} \) is the longitudinal sound velocity ( \( E \) is Young’s modulus). In the axisymmetric case with \( m = 0 \) the I, II and III modes correspond to pure radial, torsional, and longitudinal modes respectively. They correspond to pure longitudinal modes, torsional, and radial respectively in the large \( q \) limitation.

Simply calculation via Eq. (8) reveals that only the acoustic modes with \( \Omega_{0}^I \) and \( \Omega_{0}^II \) contribute to the electron-phonon interaction. The corresponding dispersion relations are
\[ \begin{align*}
\omega_1(q) &= \frac{c_l}{R} \Omega_{0}^I = \omega_c \sqrt{q^2 + 1} , \\
\omega_2(q) &= \frac{c_l}{R} \Omega_{0}^{III} = \omega_c \frac{q \sqrt{1 - \nu^2}}{q + 1} .
\end{align*} \tag{20} \]

with \( \omega_c \equiv c_l/R \), the cutoff frequency for the first mode. Here one may find that the minimum of \( \omega_1 \) is always bigger than the maximum of \( \omega_2 \). The normalization constants for the first mode are respectively \( C_r = i\nu A_1, C_\varphi = 0 \) and \( C_z = q A_1 \) with \( A_1 \equiv [V_{el}(\nu^2 + q^2)]^{-1/2} \). Those for the second mode are \( C_r = -i\nu q A_2, C_\varphi = 0 \) and \( C_z = A_2 (1 + 2q + q^2)/2(1 + q^2) \) with \( A_2 \equiv (1 + q^2)(V_{el}[\nu^2 q^2(1 + q)^4 + (1 + 2q + \nu^2 q^2)^2])^{-1/2} \).

Obviously, phonons in this subsystem move quasi-one dimensionally and thus we have \( \Omega_{ph} = L \) with \( L \) the length of the cylindrical nanoshell. Employing identity \( \int_{-\infty}^{\infty} dk k^{-1} e^{-ik(z-z')} = i\pi \text{sign}(z' - z) \), Eq. (14) gives
\[ U_{q_z}^{m} = \frac{i}{2V_{el}} \int_{V_{el}} d^3r d^3r' \mathbf{\nabla} \cdot \mathbf{f}_n(r) \mathbf{\nabla}' \cdot \mathbf{f}_n^*(r') \text{sign}(z' - z) , \]

where the superscript \( m = 1 \) or \( 2 \) distinguishes two vibrational modes associated with the two dispersion relations defined in Eq. (20) and (21), respectively. The strain-weighted vibration DOS is
\[ F(\omega) = F_I(\omega) + F_2(\omega) \tag{25} \]

with \( F_m(\omega) \) defined by
\[ \begin{align*}
F_m(\omega) & \equiv \sum_{q_z} U_{q_z}^{m} \delta(\omega - \omega_1(q_z)) \\
& = \frac{i L}{4\pi RV_{el}} \int_{-\infty}^{\infty} dq \delta(\omega - \omega_m(q)) \\
& \times \int_{V_{el}} d^3r d^3r' \mathbf{\nabla} \cdot \mathbf{f}_n(r) \mathbf{\nabla}' \cdot \mathbf{f}_n^*(r') \text{sign}(z' - z) . \tag{26}
\end{align*} \]
Here, \( F_1(\omega) \) is valid in the range confined by \( \omega \geq \omega_c \). Carrying out the integrations over \( \mathbf{r}, \mathbf{r}', \) and \( q \), we obtain

\[
F_1(\omega) = \frac{\omega}{2\pi c_l^2} \frac{1 - \omega^2/\omega_c^2}{(1 - \nu^2) - \omega^2/\omega_c^2} \tag{27}
\]

and

\[
F_2(\omega) = \frac{\omega}{2\pi c_l^2} \frac{\sqrt{1 - \nu^2}}{\sqrt{1 - \nu^2 - \omega/\omega_c}} \frac{\nu^2 \omega^2/\omega_c^2 + (1 - \omega^2/\omega_c^2)^2 (\sqrt{1 - \nu^2 - \omega/\omega_c})^2}{(1 - \omega^2/\omega_c^2)^2}. \tag{28}
\]

Then the thermal power can be calculated by the following formula,

\[
P = \frac{2m^2 c_l^2 L}{9\pi \rho} \left[ \int_{\omega_c}^{\omega_D} d\omega F_1(\omega) \left( \frac{\omega}{e^{\omega/\gamma T_{el}} - 1} - \frac{\omega}{e^{\omega/\gamma T_{ph}} - 1} \right) + \int_0^{\omega_D} d\omega F_2(\omega) \left( \frac{\omega}{e^{\omega/\gamma T_{el}} - 1} - \frac{\omega}{e^{\omega/\gamma T_{ph}} - 1} \right) \right]. \tag{29}
\]

The strain-weighted DOS has distinct high- and low-frequency character crossing over at \( \omega = \omega_c \) because \( F_1(\omega) \) has a cut off at this frequency. Clearly, \( \omega_c \) defines a nature crossover temperature

\[
T^* = \frac{\hbar \omega_c}{k_B} \tag{30}
\]

distinguishing the high and low temperature behaviors.

At high temperature, \( F_1(\omega) \) dominates the contribution, and the large frequency asymptotic behavior of strain-weighted DOS reads

\[
F(\omega) \approx F_1(\omega) = \frac{\omega}{2\pi c_l^2},
\]

leading to the high-temperature expression of thermal power

\[
P = \Sigma_{1D} L [\Phi_3(\omega_D/T_{el}) T_{el}^3 \Phi_3(\omega_D/T_{ph}) T_{ph}^3], \tag{31}
\]

here \( \Sigma_{1D} \) is the material parameter, defined by

\[
\Sigma_{1D} = \frac{\zeta(3)k_B^4}{18\rho \pi^2 c_l^4}, \tag{32}
\]

with \( \Phi_m(y) \equiv [(m-1)! \zeta(m)]^{-1} \int_0^y dx x^{m-1}/(e^x - 1) \).

It can be shown that the \( \Phi_3 \) factors are equal to unity at temperatures above \( T^* \) but sufficiently smaller than the Debye temperature. Thus, expression (31) reduces to

\[
P = \Sigma_{1D} L (T_{el}^3 - T_{ph}^3). \tag{33}
\]

The \( T^3 \) law for the thermal power for cylindrical nanoshells revealed by this relation is the same to that in one dimensional phonon systems where electron cooling devices are manipulated. The crossover from the \( T^5 \) law in three-dimensional phonon systems to the \( T^3 \) law in their one-dimensional counterparts shows that the electron-phonon interaction changes very significantly when dimensionality of the phonon subsystem reduces, which agrees with the effect of phonon dimensionality on the electron-phonon coupling observed by other researchers.

Now we apply our theory to a metallic carbon nanotube with the Poisson ratio \( \nu = 0.186 \), the Young’s modulus \( E = 1050 \) GPa, the mass density \( \rho = 2.26 \) g/cm\(^3\), and the Fermi energy \( \epsilon_F = 1.0 \) eV. The calculated longitudinal sound velocity is \( c_l = 21.9 \times 10^3 \) m/s and the the material parameter \( \Sigma_{1D} = 1.08 \times 10^{-10} \) W m\(^{-1} \) K\(^{-3} \).

At low-temperature when \( T < T^* \), the dominant contributions to the thermal power are due to the small frequency asymptotical behavior of \( F(\omega) \), which are

\[
F_1(\omega) \approx \frac{\omega}{2\pi c_l^2}, \quad \omega \geq \omega_c,
\]

\[
F_2(\omega) \approx \frac{\omega}{2\pi c_l^2} \left[ \frac{1}{1 - \nu^2} + \frac{3\omega}{\omega_c (1 - \nu^2)^{3/2}} \right].
\]

Combining these two expressions with Eq. (29), we get the low-temperature expression of the thermal power, i.e.,

\[
P = \Sigma_{1D} L [\Lambda(T_{el}) T_{el}^3 \Lambda(T_{ph}) T_{ph}^3] + \Sigma_{1D} L [\Pi(T_{el}) T_{el}^4 - \Pi(T_{ph}) T_{ph}^4] \tag{34}
\]

where

\[
\Lambda(T) = \frac{2\Phi_3(\omega_D/T) - \Phi_3(\omega_c/T)}{1 - \nu^2},
\]

\[
\Pi(T) = \frac{9\gamma}{T^*(1 - \nu^2)^{3/2}} \Phi_4(\omega_D/T),
\]

\( \gamma \equiv \zeta(4)/\zeta(3) \).

At extremely low temperature, i.e. \( T \ll T^* \), \( \omega_c/T \gg 1 \), and thus \( \Phi_3 \) and \( \Phi_4 \) factors tend to unity. Therefore, we have \( \Lambda(T) = 1/(1 - \nu^2) \) and \( \Pi(T) = 9\gamma / T^*(1 - \nu^2)^{3/2} \), and hence expression (34) reduces to

\[
P = \Sigma_{1D} L \left[ \frac{T_{el}^3 - T_{ph}^3}{1 - \nu^2} + \frac{9\gamma}{T^*(1 - \nu^2)^{3/2}} \right], \tag{35}
\]

which reveals a crossover of the \( T^3 \) law for the temperature dependence of the thermal power at low temperature. The crossover temperature for the metallic carbon nanotube is \( T^* = 24.6 \) K, which is in the temperature zone that the current technology can achieve. Hence we expect the experimental observation of this crossover phenomenon.

**IV. CONCLUSION**

We have advanced a general expression for calculating the thermal power transferring from 3-dimensional electron to any \( D \)-dimensional acoustic phonon subsystem.
It transfers the calculation of thermal power to the computation of the strain-weighted DOS for phonon subsystems of any dimensionality. Employing this method, we have investigated the hot-electron effect in free suspended cylindrical nanoshells, in which electrons behave three-dimensionally but phonons are confined to quasi-one dimension. The temperature dependence of the thermal power is obtained analytically, and the low-temperature crossover from the $T^3$ to $T^3/(1 - \nu^2) + 9\gamma T^4/[T^4(1 - \nu^2)^{3/2}]$ dependence is also deduced. The result shows that reduction of phonon dimensionality leads to weak temperature dependence of the heat flux between electrons and acoustic phonons. Even though we only estimate the corresponding quantities for the material parameters from a metallic carbon nanotube, the formula developed in this work is also applicable to any kind of the cylindrical shells in nanoscale, where the coupling between electrons and acoustic phonons is important.

ACKNOWLEDGMENTS

This work was supported by the Key Project of Chinese Ministry of Education.(No.108118)