Semileptonic Transition of $\Lambda_b$ Baryon

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Abstract Semileptonic transition of $\Lambda_b$ baryon is studied using the Hypercentral constituent quark model. The six-dimensional hyperradial Schrödinger equation is solved in the variational approach to get masses and wavefunctions of heavy baryons. The matrix elements of weak decay are written in terms of overlap integrals of the baryon wave function. The Isgur-Wise function is determined to calculate exclusive semileptonic decay $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}$. The calculated decay rate and branching ratio of $\Lambda_b$ baryon are consistent with other theoretical predictions and with the available experimental observations.

Keywords Bottom Baryons, Semileptonic Decays, Nonrelativistic quark model

1 Introduction

Inclusive and exclusive semileptonic decays of heavy flavour hadrons play an important role in the calculation of fundamental parameters of the electroweak standard model and towards a deeper understanding of QCD. Semileptonic decays of heavy hadrons are also a unique tool for determining the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, for studying the internal structure of hadrons.

The semileptonic decays of heavy mesons have been studied extensively as mentioned in Refs. 1, 2, 3, 4, 13 and references there in, but less attempts have been made to study the semileptonic decays of heavy baryons compare to that of heavy mesons. The chosen semileptonic $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}$ transition is one of the prominent decay channels out of the manifold available channels of the $\Lambda_b$ baryon reported by PDG 7. This particular semileptonic transition has been investigated using different theoretical approaches such as Covariant Confined Quark Model 8, QCD Sum Rules 9, 10, 11, quark Model 12, Bethe-Salpeter Equation 13, Lattice QCD 14, 15, Zero recoil sum rules 16, Relativistic Quark Model 17, 18, Light Front Approach 19 etc. Also the experimental group like DELPHI collaboration 20 and LHCb collaboration 21 reported their measurement on the slope parameter $\rho^2$ in the Isgur-Wise function and the branching ratio of the semileptonic process of $\Lambda_b$ baryon.

All this experimental measurements and theoretical calculations make the study of semileptonic decay of $\Lambda_b$ interesting. A precise calculation of form factors involve in the process of weak decay has been unrevealed for many years due to perturbative nature of QCD. The Heavy-Quark Effective Theory (HQET) provides the framework to include non-perturbative corrections to study of hadrons containing heavy quarks. In the limit of infinite heavy-quark mass, all the form factors describing the semileptonic decay of a heavy baryons are proportional to the universal function only which is known as the Isgur-Wise (IW) function.

In this paper, we extend the study of our earlier work 22, 23, 24, 25, 26 on the mass spectra of heavy baryons to the study of exclusive semileptonic decay of $\Lambda_b$ baryon. This paper is organized as follows: The hypercentral Constituent Quark Model (HCQM) is applied to get masses and wave function of heavy baryons presented in section 2. We furnished detail calculation of Isgur-Wise function and decay rate of semileptonic transition of $\Lambda_b$ baryon in section 2. In section 2 we presented results and also drawn an important conclusion. Finally we summarized our present study on semileptonic transition of $\Lambda_b$ baryon in Section 3.
2 Hypercentral Constituent Quark Model (HCQM) for Baryons

The exact solution of the QCD equations is very complex, so one has to rely upon conventional quark models. The assumptions in various conventional quark models are different, but they have a simple general structure in common including some basic features like confinement and asymptotic freedom and for the rest built up by means of suitable assumptions. In this article, we have adopted HCQM to study masses of heavy baryons (\(A_c, A_b\)) and semileptonic transition of \(A_b\) baryon. For detail informations on Hypercentral Constituent Quark Model (HCQM), see references \[27\][28][29].

The relevant degrees of freedom for the relative motion of the three constituent quarks are provided by the relative Jacobi coordinates \(\mathbf{\rho}\) and \(\lambda\) which are given by \[22\][21] as

\[
\rho = \frac{1}{\sqrt{2}}(r_1 - r_2) \quad (1)
\]

\[
\lambda = \frac{m_1 r_1 + m_2 r_2 - (m_1 + m_2) r_3}{\sqrt{m_1^2 + m_2^2 + (m_1 + m_2)^2}} \quad (2)
\]

The respective reduced masses are given by

\[
m_\rho = \frac{2m_1 m_2}{m_1 + m_2} \quad (3)
\]

\[
m_\lambda = \frac{2m_3 (m_1^2 + m_2^2 + m_1 m_2)}{(m_1 + m_2)(m_1 + m_2 + m_3)} \quad (4)
\]

Here, \(m_1, m_2\) and \(m_3\) are the constituent quark masses.

The angle of the Hyperspherical coordinates are given by \(\Omega_\rho = (\theta_\rho, \phi_\rho)\) and \(\Omega_\lambda = (\theta_\lambda, \phi_\lambda)\). We define hyper radius \(x\) and hyper angle \(\xi\) by

\[
x = \sqrt{\rho^2 + \lambda^2} \quad \text{and} \quad \xi = \arctan \left( \frac{\rho}{\lambda} \right) \quad (5)
\]

In the center of mass frame \((R_{c.m.} = 0)\), the kinetic energy operator can be written as

\[
P_\rho^2 = -\frac{\hbar^2}{2m} (\Delta_\rho + \Delta_\lambda)
\]

\[
= -\frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{5}{x} \frac{\partial}{\partial x} + \frac{L^2(\Omega)}{x^2} \right) \quad (6)
\]

where \(m_{\rho, \lambda} = m_1 + m_2 + m_3\) is the reduced mass and \(L^2(\Omega) = L^2(\Omega_\rho, \Omega_\lambda, \xi)\) is the quadratic Casimir operator of the six-dimensional rotational group \(O(6)\) and its eigenfunctions are the hyperspherical harmonics, \(Y_{\Lambda_\gamma}(\Omega_\rho, \Omega_\lambda, \xi)\) satisfying the eigenvalue relation, \(L^2 Y_{\Lambda_\gamma}(\Omega_\rho, \Omega_\lambda, \xi) = (\gamma(\gamma + 4)) \frac{Y_{\Lambda_\gamma}(\Omega_\rho, \Omega_\lambda, \xi)}{\gamma(\gamma + 4)}\). Here, \(l_\rho\) and \(l_\lambda\) are the angular momenta associated with the \(\mathbf{\rho}\) and \(\lambda\) variables respectively and \(\gamma\) is the hyper angular momentum quantum number.

The confining three-body potential is chosen within a string-like picture, where the quarks are connected by gluonic strings and the potential increases linearly with a collective radius \(x\) as mentioned in \[30\]. In the hypercentral approximation, the potential is expressed in terms of the hyper radius \(x\) as

\[
\sum_{i<j} V(r_{ij}) = V(x) + \ldots \quad (7)
\]

In this case the potential \(V(x)\) not only contains two-body interactions but it contains three-body effects also. The three-body effects are desirable in the study of hadrons since the non-Abelian nature of QCD leads to gluon-gluon couplings which produce three-body forces.

The model Hamiltonian for baryons in the HCQM is then expressed as

\[
H = \frac{P_\rho^2}{2m} + V(x) \quad (8)
\]

The six-dimensional hyperradial \(\text{Schrodinger}\) equation corresponding to the above Hamiltonian can be written as

\[
\left[ \frac{d^2}{dx^2} + \frac{5}{x} \frac{d}{dx} - \frac{\gamma(\gamma + 4)}{x^2} \right] \psi_{\nu\gamma}(x) = -2m [E - V(x)] \psi_{\nu\gamma}(x) \quad (9)
\]

where \(\psi_{\nu\gamma}(x)\) is the hyper-radial wave function. For the present study, we consider the hypercentral potential \(V(x)\) as the hyper-Coulomb plus linear potential which is given as

\[
V(x) = \frac{\tau}{x} + \beta x + V_0 \quad (10)
\]

Here, the hyper-Coulomb strength \(\tau = -\frac{2}{3} \alpha_s\), \(\frac{2}{3}\) is the color factor for the baryon. \(\beta\) corresponds to the string tension of the confinement. We fix the model parameter \(\beta\) and \(V_0\) to get the experimental ground state mass of \(A_b\) baryon. The parameter \(\alpha_s\) corresponds to the strong running coupling constant, which is written as

\[
\alpha_s = \frac{\alpha_s(\mu_0)}{1 + \left( \frac{33 - 2n_f}{12\pi} \right) \alpha_s(\mu_0) \ln \left( \frac{m_1 + m_2 + m_3}{m_0} \right) \quad (11)
\]

In above equation, the value of \(\alpha_s\) at \(\mu_0 = 1\) GeV is considered 0.6 as shown in Table \[1\]. The six-dimensional hyperradial Schrodinger equation described by equation \[9\] has been solved in the variational scheme with the hyper-Coulomb trial radial wave function given by \[31\][32].

\[
\psi_{\nu\gamma} = \left[ \frac{(\nu - \gamma)! (2g)^6}{(2\nu + 5)(\nu + \gamma + 4)!} \right]^{\frac{1}{2}} (2gx)^{\gamma} \quad \times e^{-gx} L_{\nu-\gamma}^{2\gamma+4}(2gx) \quad (12)
\]

The wave function parameter \(g\) and hence the energy eigenvalue are obtained by applying virial theorem. The
baryon masses are determined by the sum of the model quark masses plus kinetic energy and potential energy as
\[ M_B = \sum_i m_i + \langle H \rangle \quad (13) \]

3 Semileptonic Transition of \( \Lambda_b \to \Lambda_c \ell \bar{\nu} \)

In the approximation of infinite heavy quark masses \( m_{b,c} \to \infty \), the masses of heavy quarks \( b \) and \( c \) are much larger than the strong interaction scale \( \Lambda_{QCD} \). The spin of the heavy quark decouples from light quark and gluon degrees of freedom. This flavour and spin symmetry provides several model independent relations for the heavy to heavy baryonic form factors. In the heavy quark limit, the six form factors for the heavy to heavy baryonic form factors. In the

\[ F_q \left( q^2 \right) = G_1 (q^2) = \xi (\omega), \quad F_2 = F_3 = G_2 = G_3 = 0 \quad (14) \]

where \( \omega \) is the scalar invariant \( \omega \equiv v_{\Lambda_b} \cdot v_{\Lambda_c} \) which is related to the squared four-momentum transfer between the heavy baryons, \( q^2 \), by an equation
\[ \omega = \frac{m_{\Lambda_b}^2 + m_{\Lambda_c}^2 - q^2}{2m_{\Lambda_b}m_{\Lambda_c}} \quad (15) \]

In the literature, various approaches exist to calculate Isgur-Wise function in absence of any standard formulation. Here, the Isgur-Wise function can be calculated using Taylor’s series expansion at the zero recoil point \( \xi (\omega) \big|_{\omega=1} = 1 \) as
\[ \xi (\omega) = 1 - \rho^2 (\omega - 1) + c (\omega - 1)^2 + \ldots \quad (16) \]

where \( \rho^2 \) is the magnitude of the slope and \( c \) is the curvature (convexity parameter) of Isgur-Wise function \( \xi (\omega) \) at \( \omega = 1 \). \( \rho^2 \) and \( c \) can be written as
\[ \rho^2 = - \frac{d\xi (\omega)}{d\omega} \big|_{\omega=1} \quad ; \quad c = \frac{d^2\xi (\omega)}{d\omega^2} \big|_{\omega=1} \quad (17) \]

The Isgur-Wise function for the weak decay of heavy baryons transition in the HCQM can be written as overlap integrals of the baryon wave functions and has the form
\[ \xi (\omega) = 16\pi^2 \int_0^\infty |\psi_{\nu\gamma} (x)|^2 \cos (px) x^3 \, dx \quad (18) \]

Generally, overlap integral involving final and initial wavefunction is used to calculate transition matrix elements. In above equation, only \( |\psi (x)|^2 \) comes into the picture instead of overlap integral of final and initial state. This is because, we have investigated the Isgur-Wise function near the zero recoil point \( \omega = 1 \), where the four velocities of the baryons before and after transitions are identical. Now, we are expanding \( \cos (px) \) as
\[ \cos (px) = 1 - \frac{p^2 x^2}{2!} + \frac{p^4 x^4}{4!} + \ldots \quad (19) \]

and considering \( p^2 = 2m^2 (\omega - 1) \), where \( p^2 \) is the square of virtual momentum transfer. After substituting Eqn. (19) into Eqn. (18) and then comparing Eqn. (18) with Eqn. (16), the slope and curvature of Isgur-Wise function in HCQM can be derived as
\[ \rho^2 = 16\pi^2 m^2 \int_0^\infty |\psi_{\nu\gamma} (x)|^2 x^7 \, dx \quad (20) \]
\[ c = \frac{8}{3} \pi^4 m^4 \int_0^\infty |\psi_{\nu\gamma} (x)|^2 x^9 \, dx \quad (21) \]

It is evident from Eqn. (18) that in this HCQM at the zero recoil point \( \omega = 1 \), \( \xi (\omega) = 1 \). Once the Isgur-Wise function is obtained, one can predict semileptonic transition of heavy baryon. The differential decay width for semileptonic transition of heavy baryon can be written as
\[ d\Gamma \bigg|_{\omega=1} = \frac{2}{3} m_{\Lambda_b} m_{\Lambda_c} A \, \xi^2 (\omega) \sqrt{\omega^2 - 1} \times [3\omega (\eta + \eta^{-1}) - 2 - 4\omega^2] \quad (22) \]

Here, \( \eta = m_{\Lambda_b} / m_{\Lambda_c} \) and \( A = \frac{G_F^2}{24\pi} |V_{cb}|^2 \) \( Br (\Lambda_c \to ab) \). \( G_F \) is the Fermi coupling constant and \( |V_{cb}| \) is the Kobayashi-Maskawa matrix element. \( Br (\Lambda_c \to ab) \) is the branching ratio through which \( \Lambda_c \) is observed.

To calculate total decay width, we integrate above Eqn. (22) over the solid angle as
\[ \Gamma = \int_{\omega_{max}} d\omega \frac{d\Gamma}{d\omega} \, d\omega \quad (23) \]

where the upper bound of the integration \( \omega_{max} \) is the maximal recoil \( q^2 = 0 \) and it can be written as
\[ \omega_{max} = \frac{m_{\Lambda_b}^2 + m_{\Lambda_c}^2}{2m_{\Lambda_b}m_{\Lambda_c}} \quad (24) \]

where \( m_{\Lambda_b} \) and \( m_{\Lambda_c} \) are the masses of \( \Lambda_b \) and \( \Lambda_c \) baryons.

Table 1 Quark mass parameters (in GeV) and constants used in the calculations.

| \( m_u \) | \( m_d \) | \( m_s \) | \( m_{b,c} \) | \( m_t \) | \( \alpha_s (\mu_0=1 \text{ GeV}) \) |
|-----|-----|-----|-----|-----|-----|
| 0.330 | 0.350 | 1.55 | 4.95 | 4 | 0.6 |
4 Result and Discussions

We have chosen the quark mass parameter as $m_u = 0.33$ GeV, $m_d = 0.35$ GeV, $m_c = 1.55$ GeV and $m_b = 4.95$ GeV (See Table 1) to calculate the masses of $Λ_c$ and $Λ_b$ baryons in the Hypercentral Constituent Quark Model (HQCQM). The computed masses of $Λ_c$ and $Λ_b$ baryons are mentioned in Table 2. The calculated mass of $Λ_c$ baryon is 2.232 GeV and mass of $Λ_b$ baryon is 5.619 which are in good agreement with experimental results and other model predictions.

The behaviour of the variation of Isgur-Wise function with respect to $ω$ is shown in Fig. 1. The slope ($ρ^2$) at zero recoil of the baryonic Isgur-Wise function $ξ(ω)$ is computed and the result along with the other theoretical predictions are listed in Table 3. Our calculated value for the slope at zero recoil of the baryonic Isgur-Wise function is 1.58 which fairly agrees with other theoretical predictions within the theoretical errors. The result obtained from relativistic quark model [13] for slope of the Isgur-Wise function is 1.51 which indicates good agreement with our prediction. Our predicted value for $ρ^2$ of the Isgur-Wise function is in accordance with the experimental value 1.63±0.07±0.08 recently reported by LHCb collaboration [21]. The overall range of the slope predicted by all the theoretical predictions varies from 1.2 to 1.61. The Spectator quark model [43] has predicted relation between slope of baryonic Isgur-Wise function and slope of mesonic Isgur-Wise function through $ρ^2_ρ = 2ρ^2_M - 1/2$. After submitting the slope of mesonic Isgur-Wise function $ρ^2_M ≈ 1$, which is reported by the Heavy Flavor Averaging Group [53], the value obtained for $ρ^2_ρ = 1.5$. So our computed result is also consistent with spectator quark model [43]. By comparing the slopes of the Isgur-Wise function at the zero recoil point for the heavy baryon and for the heavy meson, we predict that the Isgur-Wise function for the baryons should be much steeper function of $ω$ than the corresponding function for mesons.

Our computed value of convexity parameter $c$ is 0.42. The other theoretical model (reference [33]) has predicted the value of convexity parameter $c = 0.56$ which is comparatively higher than our prediction.

We are able to calculate the decay width and branching ratio of $Λ_b → Λ_c \ell \bar{ν}$ semileptonic decay from the obtained Isgur-Wise function. The plot for differential decay width is shown in Fig. 2. The experimental value of $m_{Λ_b} = 5.619$ GeV and $m_{Λ_c} = 2.286$ GeV (PDG [7]) are used to calculate $Λ_b → Λ_c \ell \bar{ν}$ semileptonic decay.

Table 3 provides comparison of theoretical predictions for the $Λ_b → Λ_c \ell \bar{ν}$ semileptonic decay parameters with available experimental data. While comparing our results for decay width with other theoretical predictions, we have converted the GeV unit to s$^{-1}$ in some predictions. Our calculated result for semileptonic decay width of $Λ_b$ baryon is $4.11 \times 10^{10}$ s$^{-1}$. From Table 3, we see that the decay widths from different theoretical predictions vary from $2.15 \times 10^{10}$ s$^{-1}$ to $6.09 \times 10^{10}$ s$^{-1}$. The relativistic quark model [17] has predicted the value of semileptonic decay width $Γ = 4.42 \times 10^{10}$ s$^{-1}$ which
The transition properties for $A_b \to A_c \ell \bar{\nu}$ semileptonic decay are studied within the framework of a Hypercentral Constituent Quark Model. After fixing the model parameters using the ground state mass of $A_b$ baryon, the slope at zero recoil of the baryonic Isgur-Wise function is computed. With the help of Isgur-Wise function, exclusive semileptonic decay width and branching ratio of $A_b$ baryon are calculated. The computed results for $A_b \to A_c \ell \bar{\nu}$ semileptonic decay and its branching ratio are in agreement with available experimental observations and with other model predictions. The HCQM gives plausible predictions for the Isgur-Wise function, decay width and branching ratio corresponding to the $A_b \to A_c \ell \bar{\nu}$ semileptonic decay.

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Table 4 Comparison of theoretical predictions for the $A_b \to A_c \ell \bar{\nu}$ semileptonic decay parameters with available experimental data.

| Decay Width $\Gamma$ (in $10^{13}$ s$^{-1}$) | Reference | Branching Ratio $Br(\%)$ | Reference |
|------------------------------------------|-----------|--------------------------|-----------|
| 4.11                                    | This Work | 6.04                     | This Work |
| 3.52                                    | [9]       | 6.04±1.70                | [9]       |
| 5.02                                    | [18]      | 6.9                      | [18]      |
| 4.42                                    | [17]      | 6.48                     | [17]      |
| 4.86                                    | [8]       | 6.9                      | [8]       |
| 2.15±0.08±0.11                          | [14]      | 4.83                     | [14]      |
| 5.39                                    | [45]      | 6.2±1.3                  | Expt [17] |
| 3.52±2.2                                | [15]      | 5.6±1.1±1.6              | Expt [17] |
| 5.9                                     | [46]      |                          |           |
| 4.92                                    | [35]      |                          |           |
| 4.2–5.7                                 | [13]      |                          |           |
| 5.14                                    | [15]      |                          |           |
| 5.1                                     | [49]      |                          |           |
| 6.09                                    | [50]      |                          |           |
| 5.08±1.3                                | [51]      |                          |           |
| 5.82                                    | [92]      |                          |           |
| 5.39                                    | [12]      |                          |           |

is in good agreement with our computed result. Our calculated branching ratio for the $A_b \to A_c \ell \bar{\nu}$ semileptonic decay is 6.04%. We have used mean life time $\tau_{A_b} = 1.47 \times 10^{-12}$ s and value of $|V_{cb}| = 0.041$ as given in PDG [7] to calculate the branching ratio. The present calculated result for branching ratio is nicely agreed with average experimental value $6.2^{+1.4}_{-1.3}\%$ within experimental error reported by PDG [7].

### 5 Conclusions

The transition properties for $A_b \to A_c \ell \bar{\nu}$ semileptonic decay are studied within the framework of a Hypercentral Constituent Quark Model. After fixing the model parameters using the ground state mass of $A_b$ baryon, the slope at zero recoil of the baryonic Isgur-Wise function is computed. With the help of Isgur-Wise function, exclusive semileptonic decay width and branching ratio of $A_b$ baryon are calculated. The computed results for $A_b \to A_c \ell \bar{\nu}$ semileptonic decay and its branching ratio are in agreement with available experimental observations and with other model predictions. The HCQM gives plausible predictions for the Isgur-Wise function, decay width and branching ratio corresponding to the $A_b \to A_c \ell \bar{\nu}$ semileptonic decay.

# Fig. 2

The variation of differential decay rate for the $A_b \to A_c \ell \bar{\nu}$ semileptonic decay.
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