STUDY ON $\nu_\mu$, $\nu_\tau$ DECAYS AND CP-NON INVARINANCE
AT EARLY EPOCH OF THE UNIVERSE$^*$

Wu-Sheng Dai$^1$, Song Gao$^2$ and Xue-Qian Li$^1$

1. Department of Physics, Nankai University,
Tianjin, 300071, China.

2. Institute of High Energy Physics, Academia Sinica,
P. O. Box 918 (4), Beijing 100039, China.

Abstract

We study the $\nu_\mu$ and $\nu_\tau$ decays in the early epoch of the universe. If $m_{\nu_\tau} > 2m_e$, there would be a CP asymmetry between $\nu_\tau \to e^+ + e^- + \nu_e$ and $\bar{\nu}_\tau \to e^+ + e^- + \bar{\nu}_e$. The resultant CP non-invariance is a function of temperature and density and can reach $10^{-7}$ for a reasonable temperature range, but it is noticed that if $m_{\nu_\tau} > 2m_\mu \sim 200$ MeV, the CP non-invariance can be much larger.

PACS Numbers: 14.60G, 98.80C

Typeset using REV\TeX

$^*$This work is partly supported by the National Science Foundation of China (NSFC)
I. Introduction.

As a possible candidate of the hot dark matter, the characteristics of neutrinos need to obey many constraints from both cosmology and experiments on the Earth. The recent research indicates that the upper limit of the dark matter neutrinos must be less than 10 eV [1]. The earth experiments give an upper bound on $\nu_e$ mass as $m_{\nu_e} < 7.2$ eV [2], the measurements by a PSI group [3] has given an upper limit of $\nu_\mu$ as $m_{\nu_\mu} < 160$ keV. For $\nu_\tau$, there are several groups which have obtained various upper bounds as [4] in Table I.

| Exp.char. | CLEO  | ARGUS | OPAL  | ALEPH |
|----------|-------|-------|-------|-------|
| Produced $\tau^+\tau^-$ | 1.77M | 325K  | 36K   | 76K   |
| $3\pi^\pm\pi^0/5\pi^\pm$ | $5\pi^\pm$ | $5\pi^\pm(\pi^0)$ | $5\pi^\pm/5\pi^\pm\pi^0$ |
| $2^{nd}$ $\tau$ | 53/60 | 20    | 5     | 23/2  |
| Method   | 1–D   | 1–D   | 2–D   | 2–D   |
| $m_{\nu_e}$ (<95% CL) | 32.6  | 31    | 74    | 23.8  |

Table I. The mass is in unit of MeV.

The stringent constraints demand that either the neutrino as the hot dark matter component is lighter than 10 eV, or can decay fast enough, so that its relic contents are not substantial in the present universe. Of course, there is possibility for existence of heavy unstable neutrinos to which the cosmology sets lower bounds [5]. But a crucial problem is to investigate the decays of the lighter neutrinos $\nu_\mu$ and $\nu_\tau$.

The cosmology demands that corresponding to a few MeV, the minimum lifetime is $\tau \leq 2.5 \times 10^8$ sec. for Dirac neutrinos and $\tau \leq 4.3 \times 10^8$ sec. for Majorana neutrinos [6].

As long as $\nu_\tau$ is heavier than $2m_e \sim 1.0$ MeV, it can decay via $\nu_\tau \rightarrow \nu_e + e^+ + e^-$ and the decay rate at tree level can be written as
\[ \Gamma = \frac{G_f^2 m_{\nu_\tau}^5}{192\pi^3} |U_{\nu_\tau e}^* U_{\nu_\tau e}|^2, \]  

(1)

where the electroweak (EW) corrections and the mass effects of \(m_e\) are omitted, \(U_{\nu_\tau e}\), \(U_{\nu_\tau e}\) are the Cabibbo-Kobayashi-Maskawa matrix elements. Provided \(m_{\nu_\tau} \sim 31\) MeV and \(U_{\nu_\tau e} \sim 0.01\), \(U_{\nu_\tau e} \sim 1\), the lifetime \(\tau\) can be about 11 sec. Therefore if \(m_{\nu_\tau}\) is sufficiently heavy, it can satisfy the cosmology constraint. However, if \(m_{\nu_\tau}\) were 30 times lighter (say, 1~2 MeV), its lifetime would be close to the cosmology bound.

A more serious problem exists for \(\nu_\mu\), if its mass is about 160 KeV. Due to the phase space restriction, \(\nu_\mu\) cannot decay via any reactions at tree level and the most probable decay mode is \(\nu_\mu \rightarrow \nu_e \gamma\) which can only occur via the penguin diagram. Because of the loop suppression and the GIM mechanism, the decay rate would be very small, if the temperature effects are not taken into account. In fact, the lifetime can be as long as \(10^{22}\) sec. i.e. \(3 \times 10^{14}\) yrs.

Nieves et al. systematically studied the properties of neutrinos in medium with finite temperature and density [7], especially, they investigated \(\nu_\tau \rightarrow \nu_e + \gamma\) and found a tremendous change, because then the GIM suppression is dismissed, so that the decay rate can be nine orders larger than that at zero temperature [8]. Following the trend, we will employ their results to analyze the \(\nu_\mu\) lifetime, provided \(\nu_\mu \sim 160\) KeV. We further study the temperature and density effects on the decay \(\nu_\tau \rightarrow \nu_e + e^+ + e^-\). Obviously, the temperature and density effects occur at the propagators which only exist in the penguin loop. At zero temperature, the loop correction is small with a factor \(\alpha/\pi \sim 2 \times 10^{-3}\) with respect to the tree level. For higher temperature and density which exist at the early epoch of the universe, there are two more energy scales besides the mass of \(\nu_\tau\), e.g. the temperature \(T\) and the electron chemical potential \(\mu\) which corresponds to the density. Therefore once \(T, \mu > m_{\nu_\tau}\), there is a chance to elevate the amplitude by orders and this enhancement may compensate the loop suppression. So we investigate the temperature and density effects via loops, concretely the penguin diagrams.

The penguin diagram at zero-temperature has been carefully studied by many
authors [3]. Nieves et al. studied the penguin contributions to $\nu_\tau \to \nu_e + \gamma$, where $\gamma$ is a real photon satisfying on-shell condition $q^2 = 0$. On contrast, in the process $\nu_\tau \to \nu_e + e^+ + e^-$, the propagating photon is a virtual intermediate boson and $q^2 \neq 0$, therefore the results would be expected to deviate from that in $\nu_\tau \to \nu_e + \gamma$. Moreover, it is well known that as $q^2 \neq 0$, the penguin loop may possess an imaginary part which induces a CP asymmetry for $\nu_\tau \to \nu_e + e^+ + e^-$ and $\bar{\nu}_\tau \to \bar{\nu}_e + e^+ + e^-$ [10]. The temperature and density effects could modify the CP non-invariant phenomenon. In fact, any CP non-invariant source in the early universe is interesting and worth careful studies.

Our paper is organized as following. After the introduction, we give the necessary formulations, in Sec.III, we present our numerical results, while the last section is devoted to a brief discussion and conclusion.

II Formulations

(i) For $\nu_\mu \to \nu_e + \gamma$.

Because the final phase space forbids the reaction $\nu_\mu \to \nu_e + e^+ + e^-$ as long as $m_{\nu_\mu} < 1$ MeV, so the most reasonable decay mode for $\nu_\mu$ is the radiative decay $\nu_\mu \to \nu_e + \gamma$.

Nieves et al. [7] [8] assumed that only electrons (positrons) in the atmosphere, for the case of a nonrelativistic (NR) electron background,

$$\frac{\Gamma^{(NR)}}{\Gamma} = 1.39 \times 10^9 r F(V) \left[ \frac{n_e}{10^{24} \text{cm}^{-3}} \right]^2 \left( \frac{1 \text{eV}}{m} \right)^4, \quad (2)$$

whereas for an extreme relativistic (ER) background,

$$\frac{\Gamma^{(ER)}}{\Gamma} = 1.5 \times 10^9 r F(V) \left( \frac{T}{m} \right)^4 \quad (3)$$

where $r = |U_{e\nu_e}^* U_{\nu_e\nu_{\mu}}|/|U_{\tau\nu_e}^* U_{\tau\nu_{\mu}}|$ which is a large number about $10^4 \sim 10^5$ unlike that in $\nu_\tau \to \nu_e \gamma$ it is close to unity, $\Gamma$ is the width at zero temperature.

$$F(V) = (1 - V^2)^{1/2} \left[ \frac{2}{V} \ln \left( \frac{1 + V}{1 - V} \right) - 3 \right]$$
where $\mathcal{V}$ is the three-velocity of the decaying neutrino and $F(\mathcal{V})$ can take values between 1 and 1.55. The superscripts in eqs. (2, 3) denote the non-relativistic and extra-relativistic cases respectively and $n_e$ is the electron density in the surrounding, $m$ is the mass of $\nu_\mu$.

The zero temperature $\Gamma$ is expressed as

$$
\Gamma \approx \frac{1}{2} G_F^2 \left( \frac{3}{32\pi^2} \right)^2 m^5 \sum_{l=e,\mu,\tau} \frac{m_l^2}{M_W^2} U_{l\nu_\mu}^* U_{l\nu_e}^2.
$$

Obviously due to the GIM mechanism, only $\tau$ contributes substantially at zero temperature.

(ii) For $\nu_\tau \rightarrow \nu_e + e^+ + e^-$. 

The dispersive part of the amplitude comes from both the tree level and the penguin diagrams, while the absorptive part only comes from the penguin. It is noted that besides the absorptive phase determined by a ratio of the absorptive part and the dispersive part, there are the CKM phases and they would be dealt with separately.

The propagator of fermions at finite temperature and density can be written as [11]

$$
S_F(p) = (p + m) \left[ \frac{1}{p^2 - m^2 + i\epsilon} + 2\pi\delta(p^2 - m^2)\eta(p \cdot v) \right],
$$

where

$$
\eta(x) = \frac{\theta(x)}{e^{\beta(x-\mu)} + 1} + \frac{\theta(-x)}{e^{-\beta(x-\mu)} + 1},
$$

and $\beta = 1/kT$, $\mu$ is the chemical potential and $\nu_\alpha$ is the four-velocity of the medium, in the Lab frame $\nu_\alpha = (1, \vec{0})$.

The Feynman diagrams are shown in Fig.1 where (a) corresponds to the tree diagram, obviously at this energy scale, (c) gives rise of negligible contributions only. In the real photon emission case where $q^2 = 0$, $\delta(p^2 - m_e^2)$ and $\delta((p-q)^2 - m_e^2)$ cannot be non-zero simultaneously as long as $m_e^2 \neq 0$, therefore, in the reaction $\nu_\tau \rightarrow \nu_e + \gamma$ the penguin diagram does not possess an absorptive part. However, as $q^2 \neq 0$, ...
product of the two $\delta$–functions can be non-zero, therefore an absorptive part of the penguin diagram emerges. For the energy scale of $m_{\nu\tau} \sim 31\text{ MeV}$, only electron-loop contributes an absorptive part, but not $\mu$ and $\tau$, because $m_{\nu\tau} < 2m_\mu$. It is a well-known fact in the zero-temperature field theory, but for the finite temperature and density situation, their effects on the absorptive part appear via $\eta(p \cdot v)\eta((p-q) \cdot v)$.

The calculations for the absorptive part concern an integration

$$\int d^4p \delta(p^2 - m_{\nu}^2)\delta((p-q)^2 - m_{\tau}^2)f(p).$$

which was investigated by Kobes and Semenoff [11].

Thus, the total temperature and density dependent amplitude would be

$$M = -i\left(\frac{g}{2\sqrt{2}}\right)^2 \bar{u}_{\nu e} \gamma_\alpha (1 - \gamma_5)u_{\nu \tau} \bar{e} \gamma^\alpha (1 - \gamma_5)e \frac{1}{k^2 - M_W^2} U_{\nu e}^* U_{\nu \tau}$$

$$+ (-i)\bar{u}_{\nu e} \Gamma^{(\text{eff})}_\alpha \frac{1}{q^2} \bar{e} \gamma^\alpha e,$$

(8)

where $\Gamma^{(\text{eff})}_\alpha$ stands for the loop contributions of $e$, $\mu$ and $\tau$ as

$$\Gamma^{(\text{eff})}_\alpha = \sum_{l=e,\mu,\tau} \left(\Gamma^{D(\text{eff})}_\alpha + i\Gamma^{A(\text{eff})}_\alpha\right) U_{l\nu e}^* U_{l\nu \tau},$$

(9)

and the superscripts D and A represent the dispersive and absorptive parts of the penguin diagram respectively.

In the derivations, we follow Nieves et al. [7] to use their conventions and decomposition as

$$T_{\alpha\beta} = M_T R_{\alpha\beta} + M_L Q_{\alpha\beta} + M_P P_{\alpha\beta},$$

(10)

where

$$R_{\alpha\beta} \equiv \tilde{g}_{\alpha\beta} - Q_{\alpha\beta},$$

(11)

$$Q_{\alpha\beta} \equiv \frac{\tilde{v}_\alpha \tilde{v}_\beta}{\tilde{v}^2},$$

(12)

$$P_{\alpha\beta} \equiv \frac{i}{Q^2} \epsilon_{\alpha\beta\lambda\mu} q^\mu v^\nu,$$

(13)

and $\tilde{g}_{\alpha\beta} \equiv g_{\alpha\beta} - \frac{q_{\alpha} q_{\beta}}{q^2}$, $\tilde{v}_\alpha \equiv \tilde{g}_{\alpha\beta} v^\beta$. 6
Then the following calculations are straightforward and standard, even though tedious, we ignore the details in the work to save space.

Thus the amplitude squared can be easily written down in a standard framework. The decay width is

$$\Gamma_{\text{total}} = \int \frac{d^3p_{\nu_\tau}}{(2\pi)^3 f_{\nu_\tau}} \frac{1}{4E_{\nu_\tau}} \int \frac{d^3p_{e^+}}{(2\pi)^3 2E_{e^+}} \frac{1}{(2\pi)^3 2E_{e^+}} \frac{d^3p_{e^-}}{(2\pi)^3 2E_{e^-}} \frac{d^3p_{\nu_e}}{(2\pi)^3 2E_{\nu_e}} \cdot |M|^2 \delta^4(p_{\nu_\tau} - p_{e^+} - p_{e^-} - p_{\nu_e})(1 - f_{\nu_\tau})(1 - f_{e^+})(1 - f_{e^-}),$$

where \( f_i \)'s are the distribution functions of fermions, \((1 - f_i)\) denotes the Pauli blocking of the produced \(e^+, e^-\) and \(\nu_e\). It is suggested that in the early universe, one may use \(\frac{d^3p}{2E}\sqrt{-g}\) where \(g\) is the determinant of the metric matrix, instead of \(\frac{d^3p}{2E}\), but here we assume that at this concerned time period of the universe evolution, the universe is flat enough and \(\sqrt{-g}\) is close to unity.

(iii) The CP asymmetry.

For \(\nu_\tau \to e^+ + e^- + \nu_e\), the amplitude can be recast into another form which is more appropriate for the CP asymmetry discussions as

$$M = A'(0)e^{i\theta_e} + A'(e)e^{i\theta_e} + A'(\mu)e^{i\theta_\mu} + A'(\tau)e^{i\theta_\tau},$$

where

$$A'(0) = |A(0)^*U^\tau_{\nu_\tau} U_{\nu_e}|,$$

$$A'(e) = \sqrt{A(e)^2 + A(e)^2_{D}}U_{\nu_e}^* U_{\nu_\tau},$$

$$A'(\mu) = |A(\mu)^*U^\mu_{\nu_\tau} U_{\nu_e}|,$$

$$A'(\tau) = |A(\tau)^*U^\tau_{\nu_\tau} U_{\nu_e}|,$$

and \(A(0)\) is the tree level quantity, while \(A(l)\)'s correspond to the loop as \(l = e, \mu, \tau\) respectively.

The absorptive phase \(\theta_e\) which is equivalent to the strong phase in \(K \to 2\pi\) process, comes from the absorptive part of the penguin loop as

$$\tan^{-1} \theta_e = \frac{A(A)}{A(D)}, \quad \theta_\mu = \theta_\tau = 0.$$
Since $m_{\nu_{\tau}} < 2m_{\mu}$, only the electron loop contributes an absorptive part according to the Cutkosky cutting rule [12] and the weak phases $\delta_{e}$, $\delta_{\mu}$, $\delta_{\tau}$ originate from the CKM phase. Under a CP transformation, the weak phases change signs, while $\theta_{e}$ does not. The interferences give rise to a CP asymmetry.

The CP asymmetry is proportional to

$$a \equiv \frac{\int \prod_k (dLIPS_k) (|M|^2 - |M'|^2)}{\int \prod_k (dLIPS_k) (|M|^2 + |M'|^2)} = \frac{\int \prod_k (dLIPS_k) (\sum_{i,j} 2|A'_i||A'_j| \sin(\delta_i - \delta_j) \sin(\theta_i - \theta_j))}{\int \prod_k (LIPS_k) (\sum_{i,j} [|A'_i|^2 + 2|A'_j||A'_i| \cos(\delta_i - \delta_j) \cos(\theta_i - \theta_j)])},$$  (21)

where $dLIPS_k$ denotes the Lorentz Invariant Phase Space of final particle $k$.

The numerator comes from interferences among all the terms (tree level and $e$, $\mu$, $\tau$ loops), obviously, it is zero unless both the weak and strong phases of the two terms are different. Therefore in our case, the numerator is proportional to

$$2|A'_{(e)}||A'_{(\mu)}| \sin(\delta_e - \delta_{\mu}) + |A'_{(\tau)}| \sin(\delta_e - \delta_{\tau})\sin \theta_e.$$  (22)

It is observed that there is no contribution to the CP asymmetry from the interference between the tree amplitude and $A'_i$ because they either have the same weak phase (with $A'_{(e)}$) or the same absorptive phase (with $A'_{(\mu)}$ and $A'_{(\tau)}$). As some authors suggested that in the surrounding there were no $\mu$ and $\tau$ leptons, then $A'_{(\mu)}$ and $A'_{(\tau)}$ are the zero temperature amplitudes which are somewhat smaller than that with higher temperature and density.

We can argue that since $m_\tau \sim 1.7$ GeV $\gg m_\mu$, one can expect it to decay very fast, so that there is no $\tau$-lepton in the atmosphere, but probably a certain amount of muons. In our later calculations we assume that both $\mu$ and $\tau$ exist when the reaction takes place.

III. The numerical results.

In the calculations, we study the high temperature and density effects for the early universe. The studies on the evolution of the universe suggested that the
reasonable temperature and density limits may be $4 \times 10^{12}$ K and $\mu = 600$ MeV, so we take them as the extreme bounds \textsuperscript{[3]}. In the calculations we adopt the CKM matrix elements as given in the data book \textsuperscript{[14]} and ref. \textsuperscript{[15]}.

(i) For the $\nu_\mu$ decay, the zero temperature calculations predict a rather long lifetime as $\tau_{\nu_\mu} \sim 3.5 \times 10^{12}$ yrs which is longer than the lifetime of our universe.

The calculations show that at $T = 10^{12}$ K, the lifetime of $\nu_\mu$ decreases to $2 \times 10^3$ sec. But since the temperature of the universe cannot hold so high for more than a few seconds, thus one can expect that an average lifetime of $\nu_\mu$ would be $10^8$ sec, which roughly coincides with the cosmology requirements. We will discuss this consequence in the last section.

(ii) For $\nu_\tau$ decays.

There are two important decay modes $\nu_\tau \rightarrow e^+ + e^- + \nu_e$ and $\nu_\tau \rightarrow \nu_e + \gamma$. At zero temperature, because the former reaction can be realized via a tree diagram, so its contribution dominates over the radiative decay mode which can only occur via loops.

The finite temperature and density effects change the characteristics of the decay modes. The radiative decay was studied by Nieves et al. in detail, while we investigate the temperature and density effects on $\nu_\tau \rightarrow e^+ e^- \nu_e$ in this work. Because at high temperature and density, the new energy scale $T$ and the chemical potential $\mu$ would replace or partially replace the mass term, so can result in a larger decay rate. Our numerical results indicate that as $T$ and $\mu$ get large enough, the loop contribution can be one times larger than the tree contributions. (see Fig.2).

In fact we have also found the dependence of the decay rate on the density as $\Gamma$ increases fast with the density increment. It is the main aim of this work. The dependence on the chemical potentials are shown in Fig.2, the three curves correspond to various temperatures. Its significance would be discussed in next section.

It is noted that the penguin contributions increase fast with increments of tem-
perature either, for $\mu = 0$, at about $T = 6 \times 10^{11}$ K, the tree amplitude and the penguin have the same magnitude, then the penguin contribution takes over and dominates.

(iii) The CP asymmetry.

One could expect an absorptive part emerging at zero temperature as an absorptive angle is resulted \[17\]

$$\tan \theta_e = \frac{G_F \frac{2\alpha}{\sqrt{2}} \sqrt{\alpha}}{G_F \left( \frac{-2\alpha}{3\pi} \right) \ln \frac{m_e^2}{M_W^2}}$$

which is about -0.4, but as the temperature is not zero, this ratio becomes very small that $\theta_e \sim 10^{-6}$. Concrete values are shown in Table II.

| $\mu$ | 0  | 50 | 100 | 200 | 300 | 400 | 500 | 600 |
|-------|----|----|-----|-----|-----|-----|-----|-----|
| $\tan \theta_e \times 10^{-6}$ | 4.1 | 4.05 | 3.9 | 3.43 | 2.79 | 2.12 | 1.53 | 1.03 |
| $a \times 10^{-7}$ | 4.1 | 4.1 | 4.08 | 3.98 | 3.2 | 2.6 | 2.1 | 1.5 |

Table II. in the table, $T = 4 \times 10^{12}$ K, $\mu$ is in MeV.

In fact, in the interaction

$$\int d^4 p \delta(p^2) \delta((p - q)^2) \frac{1}{e^{\beta(E - \mu)}} + \frac{1}{1} (2\pi)^2 F(p),$$

where we ignore the small electron mass and $F(p)$ is a function of $p$, the distributions are generally smaller than unity, on contrast, at the zero temperature, it is

$$\int d^4 \delta(p^2) \delta((p - q)^2)(\pi)^2 F(p),$$

so it can be expected that the finite temperature and density effects cannot make the integration values to be much larger than the zero-temperature case. Especially, as the chemical potential $\mu$ gets larger, the second distribution in (24) would decrease very steeply, thus the integration turns smaller accordingly and so does $\tan \theta_e$. Therefore, we should think that the absorptive part remains at the same order as
the zero temperature value. Our careful numerical computation confirms it. Thus a rapid increase of the dispersive part of the penguin loop with temperature and density causes $\tan \theta_e$ to decrease fast and the effects disfavor the CP asymmetry. We will discuss it in next section.

IV. Discussion and conclusion.

We have evaluated the temperature and density effects on the $\nu_{\mu}$ and $\nu_\tau$ lifetimes and estimated a possible CP asymmetry $a$ for various $T$ and $\mu$.

(i) For the $\nu_{\mu}$ lifetime, it can only decay via the radiative mode, we estimate its lifetime as about $10^3$ sec. as $m_{\nu_{\mu}} = 160$ KeV for very high temperature. However, obviously, such high temperature cannot hold for so long and would decrease very fast, and so does the $\nu_{\mu}$ decay rate since it is proportional to $T^4$. Considering this, $\nu_{\mu}$ lifetime is close to $10^8$ sec. which is the bound set by cosmology. It is possible that as $\nu_{\mu} \leq 160$ KeV, its lifetime bound would be even larger, then one may conclude that to be consistent with the cosmology, either the $m_{\nu_{\mu}}$ is much lower than the upper bound to be close to 10 eV, or there is some new physics which speeds the decay, for example axion exists etc.

(ii) The temperature and density effects on the $\nu_\tau$ lifetime. The process $\nu_\tau \rightarrow \nu_e + \gamma$ was carefully studied by Nieves et al. We study the effects on $\nu_\tau \rightarrow e^+e^-\nu_e$ for which there is a tree diagram. For $\nu_\tau \rightarrow e^+e^-\nu_e$, the temperature and density effects occur via the penguin loop which is $10^{-3} - 10^{-4}$ orders smaller than the tree amplitude at zero temperature. The numerical results indicate that the dispersive loop contribution is enhanced to the same order as the tree amplitude. Moreover, at $T = 10^{12}$ K, the two decay modes are of the same orders.

We have found that as temperature increases, the rates of both modes increase, below a certain temperature (about $T = 10^{12}$ K) the rate of $\nu_\tau \rightarrow e^+e^-\nu_e$ is larger than that of $\nu_\tau \rightarrow \nu_e + \gamma$, at the temperature, they are at the same order, but while the temperature further increases the radiative decay has a larger rate. The
concrete temperature also depends on the density, but the trend is obvious. As Babu et al. pointed out that the window for MeV neutrino might be closed \[16\], if so as aforementioned, \(m_{\nu_{\tau}}\) gets to keV order, its lifetime would become as troublesome as for \(\nu_{\mu}\) of 160 keV.

(iii) The CP asymmetry.

At zero temperature, it is known that the penguin diagram induces a CP violation which can be observed at B- and D-decays. As estimated, the absorptive angle which is equivalent to the strong scattering phase, is relatively large. It is natural to ask if it can result in a CP non-invariance at finite temperature and density which exist in the early universe.

Since \(\nu_{\tau} \rightarrow \nu_{e} + \gamma\) does not possess an absorptive phase because \(q^2 = 0\), it cannot induce a CP violation. Whereas, for \(\nu_{\tau} \rightarrow e^+ e^- \nu_e\), there indeed exists an absorptive angle, but there is the tree amplitude which is overwhelming at zero temperature. Therefore, even though the absorptive angle is large, the CP violation is very small because the tree contribution exists at the denominator in expression (21). When the temperature is very high, the penguin contributions are much enhanced and can reach the same order as the tree contribution, however, it is observed that only the dispersive part increases, while the absorptive part remains at the same order. Thus the absorptive angle \(\theta_e\) turns out to be rather small and the total CP violation at high temperature and density keeps the same order as at zero temperature and remains to be small. Therefore, even though the decay can serve as one of the CP non-invariance sources, but definitely is not the main one to explain the matter dominance over anti-matter.

Acknowledgment We would like to thank prof. R.K. Su for helpful discussions.

Figure Captions
Fig.1 The Feynman diagrams for $\nu_\tau \to e^+ + e^- + \nu_e$, where (a) is the tree diagram.

Fig.2 The dependence of the decay rate on the chemical potential, the three curves correspond to $T = 1 \times 10^{10} \text{K}$ (solid), $T = 5 \times 10^{11} \text{K}$ (dashed), and $T = 4 \times 10^{12} \text{K}$ (dotted), respectively.
REFERENCES

[1] D. Morrison, CERN-PPE/95-47; P. Close and G. Ellis, Nature 370 (1994) 609.

[2] C. Weinheimer et al., Phys.Lett. B300 (1993) 210.

[3] k. Assamagan et al., Phys.Lett. B335 (1994) 21.

[4] S. Gentile and M. Pohl, CERN-PPE/95-147.

[5] B.W. Lee and S. Weinberg, Phys.Rev.Lett. 39 (1977) 165; J. Ellis et al., CERN-TH-5853/90.

[6] E. Masso and A. Pomarol, Phys.Rev.D40 (1989) 2519.

[7] J. Nieves, Phys.Rev. D40 (1989) 866; J. D’Olivo, J. Nieves and P. Pal, Phys.Rev. D40 (1989) 3679; J. Nieves and P. Pal, Phys.Rev. D40 (1989) 1693.

[8] J. D’Olivo, J. Nieves and P. Pal, Phys.Rev.Lett. 64 (1990) 1088.

[9] A. Vainstein, V. Zakharov and M. Shifman, JETP Lett. 22 (1975) 55; S. Chia, Phys.Lett. B130 (1983) 315.

[10] D. Du and Z. Xing, Phys.Lett. B280 (1992) 292; Phys.Rev. D48 (1993) 4155; Phys.Lett. B349 (1995) 215.

[11] R. Kobes and G. Semenoff, Nucl.Phys. B260 (1985) 714.

[12] see, for example, H. Politzer, Phys.Rept. C14 (1978) 129.

[13] E. Kolb and M. Turner, *The Early Universe*, Addison-Wesley Pub. Co. (1990).

[14] The Data group, Phys.Rev. D50 (1994) 1173.

[15] O. Peres, V. Pleitez and R. Funchal, Phys.Rev. D50 (1994) 513.

[16] K. Babu, T. Gould and I. Rothstein, Phys.Lett. B321 (1994) 140.

[17] D. Wu, X. Li and P. Wang, Commun.Theor.Phys. 13 (1990) 355.
Fig. 2

\[ \frac{\Gamma}{\Gamma_{\text{tree}}} \]