Dynamical foundations of the brane-induced gravity

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Abstract

We present a comprehensive formalism to derive precise expressions for the induced gravity of the braneworld, assuming the dynamics of the Dirac–Nambu–Goto type. The quantum fluctuations of the brane at short distances give rise to divergences, which should be cut off at the scale of the inverse thickness of the brane. It turns out that the induced-metric formula is converted into an Einstein-like equation via quantum effects. We determine the coefficients of the induced cosmological and gravity terms, as well as those of the terms including the extrinsic curvature and the normal connection gauge field. The latter is the characteristic of the brane-induced gravity theory, distinguished from ordinary non-brane-induced gravity.

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1. Introduction

General relativity is based on the premises that the spacetime is curved affected by matter according to the Einstein equation and that the objects move along the spacetime geodesics. The gravitations are apparent phenomena of the inertial motions in the curved spacetime. This successfully explains why the motion in the ‘gravitational’ field is universal, i.e. blind to the object properties (mass, charge, etc), and why the ‘gravitational’ forces are subject to Newton’s law of gravitation (within the ordinary precision). It is supported by further precise observations. It raises, however, another fundamental question why the spacetime is curved so as the Einstein equation indicates. A possible answer was as follows: suppose our spacetime is a (3+1)-dimensional embedded object like domain walls or vortices (braneworld [1–41]) in higher dimensional spacetime, and the gravity is induced through quantum fluctuations (brane-induced gravity [4, 5, 10, 14–18, 27, 28, 34, 35]). The cosmological constant should be finely tuned. Then, the brane-gravity field emerges as a composite, which obeys an Einstein-like equation at least at low curvatures, just like in the case of the (non-brane)-induced gravity theory [42–46].

The ideas of the braneworld and the brane-induced gravity have been studied extensively over the last three decades. Physical models were constructed with topological defects in higher dimensional spacetime [5–18], and they were realized as ‘$D$-branes’ in the superstring
theory [19–22]. They were applied to the hierarchy problem with large extra dimensions, or with warped extra dimensions [20, 23–25]. It was argued that the brane-induced gravity would imply infrared modifications of the gravity theory [26–28]. The ideas have been studied in wide areas including basic formalism [29–33], brane-induced gravity [34, 35], particle physics phenomenology [36–38] and cosmology [39–41] with many interesting consequences.

In this paper, we establish a precise formalism to derive the expressions for the quantum-induced effects on the brane. For definiteness, we follow the simplest model with the Dirac–Nambu–Goto-type dynamics [47]. Such a model was first considered in [4] with scalar fields which we now interpret as the position coordinate of the brane in higher dimensions. Such an interpretation motivated us to consider the model of braneworld as a topological object moving in higher dimensions in [5]. The idea of the brane-induced gravity has been considered repeatedly in the literature. They were, however, more or less naive both in model setting and derivation. Here, we present a comprehensive formalism from its foundation to the precise outcomes. Among the quantum fluctuations, the only meaningful ones are those transverse to the brane. The quantum loop effects are divergent, which should be cut off at the scale of the inverse thickness of the brane. We adopt the regularization scheme used in the original work [4], and calculate them at the one-loop level. We determine the coefficients of the induced cosmological and gravity terms, as well as those of the terms including the extrinsic curvature and the normal connection gauge field. The induction of the latter terms is the characteristic of the brane-induced gravity theory, distinguished from the ordinary (non-brane)-induced gravity. It turns out that the induced-metric formula is converted into an Einstein-like equation via the quantum short-distance effects.

The plan of this paper is as follows. First, we define the model (section 2), and then we derive the quantum effects (sections 3–7). We define the brane fluctuations (section 3), formulate the quantum effects (section 4), specify the method to regularize the divergences (section 5), classify the possible induced terms according to symmetries (section 6) and calculate them via the Feynman diagram method (section 7). Then, we interpret the quantum-induced terms and show why the induced gravity can avoid the problem (sections 8 and 9). The cosmological terms are fine-tuned (section 8), and the Einstein like gravity and other terms are induced (section 9). The final section (section 10) is devoted to discussions.

2. The model

We consider a quantum theoretical braneworld described by the Dirac–Nambu–Goto-type Lagrangian. We will see that the quantum effects of the brane fluctuations give rise to effective braneworld gravity. Let \( X^I(x^\mu) \) \((I = 0, 1, \ldots, D - 1)\) be the position of our three-brane in the \( D \)-dimensional spacetime (bulk), parameterized by the brane coordinate \( x^\mu \) \((\mu = 0, 1, 2, 3)\), where \( I = 0 \) and \( \mu = 0 \) indicate the time components. Let \( G^{IJ}(X^K) \) be the bulk metric tensor at the bulk point \( X^K \). This is taken to obey some bulk gravity theory. Then, we consider a braneworld with dynamics given by the Dirac–Nambu–Goto-type Lagrangian (density) [47]:

\[
\mathcal{L}_{\text{br}} = -\lambda \sqrt{-\text{det} \left( \frac{\partial X^I}{\partial x_\mu} \frac{\partial X^J}{\partial x_\nu} G_{IJ}(X^K) \right)},
\]

where \( \lambda \) is a constant. Or we write it as

\[
\mathcal{L}_{\text{br}} = -\lambda \sqrt{-g_{\mu\nu}} g^{\mu\nu}
\]

We would like to thank Professor G Gibbons for having suggested us the existence of these extrinsic effects in the present model.
with abbreviations \( g^{[X]} = \det g^{[X]} \), and
\[
g^{[X]}_{\mu\nu} = X^{I}_{\mu} X^{J}_{\nu} G_{IJ}(X^K),
\]
where (and hereafter) indices following a comma (,) indicate differentiation with respect to the corresponding coordinate component, and \([X]\) is attached to remind that they are abbreviations for expressions written in terms of \(X^I\). Note that \(g^{[X]}\) is the induced metric on the brane with (3).

We assume that \(X^I\) appears nowhere other than in \(L_{\text{br}}\) in the total Lagrangian \(L_{\text{tot}}\) including the bulk Lagrangian. The system is invariant under the general-coordinate transformation of the bulk and the brane separately. The Lagrangian (1) is the simplest among those with this symmetry.

Note that we do not a priori have the kinetic term of the metric in the basic Lagrangian (1). It will be induced through quantum effects. The metric emerges as a composite field, and the gravitation is an induced phenomenon but not an elementary one. If this is successful, we achieve a great conceptual advantage, since it means that the familiar and important phenomenon of the gravitation is explained by the more fundamental ingredients. Thus, it is worthwhile and urgent to examine what is true and what is not in this idea. This is the basic spirit of the induced gravity theory [42–46]. As for the dynamics of the bulk metric, the situation is more vague and ambiguous. We do not know what is the dynamics, i.e. what kinetic term we have, and it is not clear even whether the kinetic term exists or not, i.e. whether it is dynamical or not. Every case well deserves intensive investigations. It is too naive to take that the bulk Einstein equation provides the brane Einstein equation at the brane, because large discrepancies take place due to the extrinsic curvature terms. The brane moves according to its equation of motion and is deformed in manners different from what the naive gravity equation indicates. One may elaborate the elementary brane gravity from the elementary bulk gravity under specific conditions by hand in specific models [25, 39]. On the other hand, many people relied on the mechanism of brane-induced gravity to realize gravitation on the brane [5, 10, 14–18, 27, 28, 34, 35]. In the spirit of the brane-induced gravity, we do not a priori assume elementary gravity, and we take it to be induced from more fundamental ingredients.

Now, the equation of motion from (1) is given by
\[
\frac{\partial}{\partial X^{I}_{\mu\nu}} g^{[X]} = 0,
\]
where \(X^{I}_{\mu\nu}\) is the double covariant derivative with respect to both of the general-coordinate transformations on the brane and to those in the bulk:
\[
X^{I}_{\mu\nu} = X^{I}_{\mu} - X^{I}_{\lambda} \gamma^{[X]}_{\mu \lambda} + X^{I}_{\nu} \gamma^{[X]}_{\nu \lambda} - \Gamma^{I}_{JK} X^{J}_{\mu} X^{K}_{\nu}
\]
with the affine connections on the brane and bulk
\[
\gamma^{[X]}_{\mu \lambda} = \frac{1}{2} g^{[X]} \left( g^{[X]}_{\mu \nu} + g^{[X]}_{\nu \lambda} - g^{[X]}_{\lambda \nu} \right),
\]
\[
\Gamma^{I}_{JK} = \frac{1}{2} G^{IJ} \left( G_{IJK} + G_{IKJ} - G_{JKI} \right),
\]
respectively. We expect that this gives a good approximation at low curvature limit in many dynamical models of the braneworld (e.g. topological defects, spacetime singularities, \(D\)-branes, etc). It is remarkable that, as we shall see below, this simple model exhibits brane gravity and gauge theory like structure through the quantum effects.

For convenience of quantum treatments, we consider the following equivalent Lagrangian to (1):
\[
\mathcal{L}_{\text{be}}' = -\frac{\lambda}{2} \sqrt{-g} \left[ g^{\mu\nu} X^{I}_{\mu} X^{J}_{\nu} G_{IJ}(X^K) - 2 \right],
\]
where \( g_{\mu
u} \) is an auxiliary field, \( g = \det g_{\mu\nu} \) and \( g^{\mu\nu} \) is the inverse matrix of \( g_{\mu\nu} \). Note that \( g_{\mu
u} \), unlike \( g^{(X)}_{\mu\nu} \) above, is treated as a field independent of \( X^I \). Then the Euler Lagrange equations with respect to \( X^I \) and \( g_{\mu\nu} \) are given by

\[
g^{\mu\nu} X^I_{;\mu\nu} = 0, \tag{9}
g_{\mu\nu} = X^I_{;\mu\nu} G_{IJ}(X^K), \tag{10}
\]
respectively, where the covariant derivative

\[
X^I_{;\mu\nu} = X^I_{,\mu\nu} - X^I_{,\lambda\gamma\lambda\gamma} + X^J_{,\mu} X^K_{,\nu} G_{JK} \tag{11}
\]
is written in terms of the brane affine connection

\[
\gamma_{\mu\nu}^I = \frac{1}{2} g^{\rho\sigma} (g_{\rho\mu,\nu} + g_{\rho\nu,\mu} - g_{\mu\nu,\rho}) \tag{12}
\]
with respect to the auxiliary field \( g_{\mu\nu} \). Now \( g_{\mu\nu} \) in (9) is independent of \( X^I \), and, instead, we have an extra equation (10), which guarantees that \( g_{\mu\nu} \) is the induced metric. If we substitute (10) into (9), we obtain the same equation as (4). Thus, the systems with the Lagrangians \( \mathcal{L}_{\text{br}} \) and \( \mathcal{L}'_{\text{br}} \) coincide. Furthermore, the argument that their Dirac bracket algebra coincide [44] indicates their quantum theoretical equivalence. We proceed hereafter based on the Lagrangian \( \mathcal{L}'_{\text{br}} \) instead of \( \mathcal{L}_{\text{br}} \).

3. Brane fluctuations

In order to extract the quantum effects of \( \mathcal{L}'_{\text{br}} \), we deploy a semi-classical method, where we consider those due to small fluctuations of the brane around some classical solution (say \( Y^I(x^\mu) \)) for \( X^I(x^\mu) \) of the equation of motion (4) [15]. Namely, the solution \( Y^I(x^\mu) \) obeys the classical equation

\[
g^{\mu\nu} Y^I_{;\mu\nu} = 0, \tag{13}
g_{\mu\nu} = Y^I_{;\mu\nu} G_{IJ}(Y^K). \tag{14}
\]
In quantum treatment, \( X^I \) itself in the Lagrangian \( \mathcal{L}'_{\text{br}} \) does not necessarily obey the equation of motion (4), and may fluctuate from \( Y^I(x^\mu) \). Among the fluctuations, only those transverse to the brane are physically meaningful, because those along the brane remain within the brane and cause no real fluctuations of the brane. They are absorbed by general-coordinate transformations. In order to describe them, we choose \( D - 4 \) independent normal vectors \( n_m^I(x^\mu) \) (\( m = 4, \ldots, D - 1 \)) at each point on the brane with the normality condition

\[
n_m^I Y^J_{,\mu} G_{IJ}(Y^K) = 0. \tag{15}
\]
Then, we express the fluctuations as

\[
X^I(x^\mu) = Y^I(x^\mu) + \phi^m(x^\mu)n_m^I(x^\mu), \tag{16}
\]
where \( \phi^m(x^\mu) \) is the transverse fluctuation along \( n_m^I(x^\mu) \) (\( m = 4, \ldots, D - 1 \)). The arbitrariness of \( n_m^I \) in choice under the condition (15) gives rise to gauge symmetry under the group GL\((D - 4)\) of the general linear transformations of the normal space at each point on the brane. If we define \( g_{mn}^I = n_m^I n_n^J G_{IJ}(Y^K) \) and its inverse \( g^{mn} \), we have the completeness condition,

\[
Y^I_{,\mu} Y^J_{,\nu} g^{\mu\nu} + n_m^I n_n^J g^{mn} = G^{IJ}(Y^K). \tag{17}
\]
Bulk coordinate indices \( I, J, \ldots (= 0, \ldots, D - 1) \) are raised and lowered by the metric tensors \( G_{IJ} \) and \( G^{IJ} \). We can read off from (8) and (10) that the auxiliary field \( g_{\mu\nu} \) plays the role the metric tensor on the brane. Hereafter, we raise and lower the brane coordinate indices.
μ, ν, . . . (0, . . ., 3) by \( g_{\mu\nu} \) and \( g^{\mu\nu} \) (but not by \( g^{(X)}_{\mu\nu} \) and \( g^{(X)\mu\nu} \)). We raise and lower the normal space indices \( m, n, . . . (4, . . ., D - 1) \) by \( g_{mn} \) and \( g^{mn} \).

A problem of the definition (16) of \( \phi^m \) is that \( \phi^m \) lacks the bulk general-coordinate invariance. In fact, it is transformed in a complex way under the general-coordinate transformations of the bulk. In contrast, the invariant definition of \( \phi^m \) requires a complex formula instead of (16). For quantum treatments, however, it is desirable to have a relation linear in \( \phi^m \) like (16). It requires further careful considerations. Therefore, in this paper, we restrict ourselves to the case where the bulk is flat. Namely, there exists the Cartesian frame with

\[
G_{IJ} = \eta_{IJ} = \text{diag}\{1, -1, -1, \ldots, -1\},
\]

where we have \( \Gamma^I_{jk} = 0 \). In this case, we have \( R^{IJKL} = 0 \) in any frame, and \( \phi^m \) restores the bulk general-coordinate invariance. The general case of curved bulk will be considered in a separate forthcoming paper.

Now we substitute (16) into the Lagrangian (8) and obtain

\[
L'_br = L'_0 + L'_\phi \quad \text{with (19)}
\]

\[
L'_0 = L'_{br\phi=0} = -\frac{\lambda}{2} \sqrt{-g} \left[ g^{\mu\nu} \partial_\mu \gamma^I_{\mu\nu} G_{IJ} - 2 \right],
\]

\[
L'_\phi = -\frac{\lambda}{2} \sqrt{-g} \left[ g^{\mu\nu} g^{\rho\sigma} \phi^m A_{\mu\nu} B_{\rho\sigma} + g^{\mu\nu} g_{mn}(\phi_{\mu\nu} + g^{\mu\nu} A_{\mu\nu}) \right],
\]

where \( A_{\mu\nu} \) and \( B_{\mu\nu} \) are the normal connection and the extrinsic curvature, respectively. They are given by

\[
A_{\mu\nu} = n^I_{\mu\nu} G_{IJ},
\]

\[
B_{\mu\nu} = n^I_{\mu\nu} \gamma^J_{\mu\nu} G_{IJ},
\]

where \( n^I_{\mu\nu} \) is the covariant derivative:

\[
n^I_{\mu\nu} = n^I_{\mu\nu} + n^I_{\mu\nu} \gamma^{MK}_{\mu\nu} G_{IJ},
\]

which coincides with the ordinary derivative \( n^I_{\mu\nu} \) under the assumption \( G_{IJ} = \eta_{IJ} \) for this paper. We can see that (21) is the Lagrangian for the quantum scalar fields \( \phi^m \) on the curved brane interacting with the given external fields \( A_{\mu\nu} \) and \( B_{\mu\nu} \). For later use, it is convenient to rewrite (21) into

\[
L'_\phi = -\frac{\lambda}{2} \sqrt{-g} \left[ -\partial_\mu \sqrt{-g} A_{\mu\nu} + \partial_\mu \sqrt{-g} A_{\mu\nu} - \sqrt{-g} A_{\mu\nu} \square \phi^m - \sqrt{-g} \left( A_{\mu\nu} \phi^m + B_{\mu\nu} \phi^m \right) \right],
\]

where total derivatives are neglected.

4. Quantum effects

The quantum effects of the field \( \phi^m \) are described by the effective Lagrangian \( L^{\text{eff}} \)

\[
\int L^{\text{eff}} \, d^4x = -i \ln \int [d\phi^m] \exp \left[ i \int L'_{\phi} \, d^4x \right].
\]

where \([d\phi^m]\) is the path integration over \( \phi^m \). To perform it, we rewrite (25) into the form

\[
L'_{\phi} = \frac{\lambda}{2} \phi^m \left( -\delta^m_n \square + V^m_n \right) \phi^n,
\]
with $\Box = \eta^{\mu\nu} \partial_{\mu} \partial_{\nu}$ and
\begin{equation}
\gamma^m_n = \delta^n_m \partial_{\mu} A_n^{\mu} + \partial_{\mu} A_m^{\mu} + \mathcal{Z}^m_n
\end{equation}
\begin{equation}
\mathcal{H}^{\mu\nu} = \eta^{\mu\nu} - \sqrt{-g} g^{\mu\nu}
\end{equation}
\begin{equation}
A_m^{\mu} = -\sqrt{-g} A_m^{\mu}\n\end{equation}
\begin{equation}
\mathcal{Z}^m_n = -\sqrt{-g} (A^{m\mu} A_n^{\mu} - B^{m\mu\nu} B_n^{\mu\nu}),
\end{equation}
where the differential operator $\partial_{\mu} \equiv \partial/\partial x^\mu$ is taken to operate on the whole expression in its right side in (27). The path integration in (26) is performed to give
\begin{equation}
\int \mathcal{L}^{\text{eff}} \, d^4x = \sum_{l=0}^{\infty} \frac{1}{2l} \text{Tr} \left( \frac{1}{\Box} \gamma^m_n \right)^l,
\end{equation}
up to additional constants, where $\text{Tr}$ indicates the trace over the brane coordinate variable $x^\mu$ and extra dimension index $m$. The terms in (32) can be calculated with Feynman-diagram method. In terms of the Fourier transforms
\begin{equation}
\hat{H}^{\mu\nu}(q_l) = \int d^4x H^{\mu\nu}(x) e^{iq_l x},
\end{equation}
\begin{equation}
\hat{A}_m^{\mu}(q_l) = \int d^4x A_m^{\mu}(x) e^{iq_l x},
\end{equation}
\begin{equation}
\hat{Z}_m^n(q_l) = \int d^4x Z_m^n(x) e^{iq_l x},
\end{equation}
the effective Lagrangian $\mathcal{L}^{\text{eff}}$ is written as
\begin{equation}
\mathcal{L}^{\text{eff}} = \sum_{l=0}^{\infty} \frac{1}{2l} \prod_{i=1}^l \int \frac{d^4q_l}{(2\pi)^4} e^{-iq_l q_i} G_l,
\end{equation}
\begin{equation}
G_l = \int \frac{d^4p}{i(2\pi)^4} \prod_{i=1}^l \frac{1}{-p_{i+1}^2} \tilde{\gamma}_m^{m_{i-1},i}(p_i, q_i),
\end{equation}
\begin{equation}
\tilde{\gamma}_m^{m_{i-1},i}(p_i, q_i) = -\delta_0^{m_i}(p_i)_{\mu}(p_{i-1})_{\mu} \hat{H}^{\mu\nu}(q_i) - i(p_i + p_{i-1})_{\mu} \hat{A}_m^{\mu}(q_i) + \hat{Z}_m^n(q_i),
\end{equation}
where $p_i = p + q_1 + \cdots + q_i$ and $m_0 = m_l$. The function $G_l$ is nothing but the Feynman amplitude for the one-loop diagram with $l$ internal lines of $\phi^m$ and $l$ vertices of $\mathcal{H}^{\mu\nu}$ (figure 1). Unfortunately, the $p$-dependence of the integrand in (36) with (37) indicates that the integration over $p$ diverges at most quartically. The divergences will be regulated in the following section. Then, we can perform the integration over $p$ to obtain the function $G_l$. The $q_l$’s are replaced by differentiation $i\partial_0$ of the $k$th vertex function according to the inverse Fourier transformation in (36). Collecting all the contributions, which are functions of the fields $g_{\mu\nu}, A_m^{\mu\nu}$ and $B_m^{\mu\nu}$ and their derivatives, we can obtain the expression for the effective Lagrangian $\mathcal{L}^{\text{eff}}$.

5. Divergences and regularization

The $p$-dependence of the integrand in (36) with (37) indicates that the integration over $p$ diverges at most quartically. We expect, however, that fluctuations with smaller wave length than the brane thickness are suppressed. Then, the momenta higher than the inverse of the thickness are cutoff. In order to model the cutoff without violating full symmetry of $\mathcal{L}^{\text{eff}}$, we introduce three Pauli–Villers regulators $\Phi_j^m$ with very large mass $M_j$ ($j = 1, 2, 3$), which are
Figure 1. The Feynman diagrams. The dashed lines indicate the \( \phi^m \)-propagators, and the wavy lines indicate external fields of \( A_{\mu mn} \), \( B_{\mu \nu mn} \), or \( h_{\mu \nu} \). The dots indicate an infinite series of diagrams with a dashed-line loop and with more external wavy lines than two. By virtue of the symmetries of the system, we have only to calculate the three diagrams explicitly drawn here.

taken equal finally, \( M_j \rightarrow \Lambda \), following the original paper [4]. Precisely, it amounts to consider the regularized effective Lagrangian

\[
\mathcal{L}^{\text{reg}} = \mathcal{L}^{\text{eff}} + \sum_{j=1}^{3} C_j \mathcal{L}^{\text{eff}}_{M_j}
\]

where \( C_j \) are the coefficients defined by

\[
\sum_{j=1}^{3} C_j (M_j)^{2k} = -\delta^{(0)}(k = 0, 1, 2),
\]

and \( \mathcal{L}^{\text{eff}}_{M_j} \) is the effective Lagrangian for the quantum effects from \( \mathcal{L}_{\phi_j}^{\prime} \) which is the same as \( \mathcal{L}_{\phi_j}^{\prime} \) except that \( \phi^m \) is replaced by the regulator field \( \Phi^m_j \) with mass \( M_j \).

\[
\int \mathcal{L}^{\text{eff}}_{M_j} \, d^4x = -i \ln \left[ \int [d\Phi^m_j] \exp \left[ i \int \mathcal{L}_{\phi_j}^{\prime} \, d^4x \right] \right].
\]

\[
\mathcal{L}_{\phi_j}^{\prime} = \mathcal{L}_{\phi_j}^{\prime} |_{\phi^m = \Phi^m_j} + \frac{1}{2} \lambda M_j^2 \sqrt{-g} \Phi^m_j \Phi^m_n \eta_{mn}.
\]

Note that the added mass term also preserves the full symmetry of \( \mathcal{L}_{\phi_j}^{\prime} \). Performing the path integration over \( \Phi^m_j \), we have

\[
\int \mathcal{L}^{\text{eff}}_{M_j} \, d^4x = \sum_{l=0}^{\infty} \frac{1}{(2\pi)^4} \frac{1}{\mathcal{L}} \text{Tr} \left( \frac{1}{\Box + M_j^2} \mathcal{V}^{m}_{M_j} \right),
\]

\[
\mathcal{V}^{m}_{M_j} = \mathcal{V}^{m} + \mathcal{J} \delta^{m}_{n},
\]

\[
\mathcal{J} = 1 - \sqrt{-g}.
\]

with \( \mathcal{V}^{m}_{n} \) in (28). In terms of the Fourier transform

\[
\tilde{F}(q) = \int d^4x \mathcal{F}(x) e^{iqx}.
\]

we have

\[
\mathcal{L}^{\text{eff}}_{M_j} = \sum_{l=0}^{\infty} \frac{1}{2l} \prod_{i=1}^{l} \int \frac{d^4q_i}{(2\pi)^4} e^{-\frac{\mathcal{L}}{2} G^{\prime}_{M_j}},
\]

with
Note that $B_m$ and regular at infinity vanish according to (40). The function

$$G_{Mj} = \int \frac{d^4p}{i(2\pi)^4} \prod_{\ell=1}^{j} \frac{1}{-p_\ell^2 + M_{Mj}^\ell} \tilde{V}_{Mj}^{\mu} \eta_{\mu\nu} (p_\ell, q\ell),$$

with $\tilde{V}_{Mj}^{\mu}(p\ell, q\ell)$ in (38). In dimensional regularization, the divergent parts of the Feynman amplitude $G_{Mj}^\ell$ behaves like

$$G_{Mj}^\ell \sim \epsilon^{-1} (G_1M_j^\ell + G_2M_j^\ell + G_0),$$

where this is evaluated at the spacetime dimension $4 - 2\epsilon$, and $G_{2\epsilon}$ are the appropriate coefficient functions. The singularities at $\epsilon = 0$ reflect the divergences in the $p$-integration. We can see that, when they are summed with the coefficients $C_j$ over $j$ in (39), they cancel out according to (40). Therefore, the $p$-integrations in $L_{\text{reg}}$ converge. Any positive power contributions of $M_j$ regular at infinity vanish according to (40). The function $G_{Mj}^\ell$ involves logarithmic singularities in $M_j$, which tend, in the equal mass limit $M_j \rightarrow \Lambda$,

$$\sum_{j=1}^{3} C_j M_j^\ell \ln M_j^2 \rightarrow -\Lambda^4/2,$$

$$\sum_{j=1}^{3} C_j M_j^\ell \ln M_j^2 \rightarrow \Lambda^2/2,$$

$$\sum_{j=1}^{3} C_j \ln M_j^2 \rightarrow -\ln \Lambda^2.$$

### 6. Classification of the terms

Thus, the divergent part $L_{\text{div}}$ of the regularized effective Lagrangian $L_{\text{reg}}$ consists of the terms which are proportional to $\Lambda^4$, $\Lambda^2$, or $\ln \Lambda^2$, and are monomials of $H^{\mu\nu}$, $F^\mu$, $A^{\mu\nu}_n$, $Z^{\mu\nu}_n$ and their derivatives. The expressions $H^{\mu\nu}$, $F^\mu$, $A^{\mu\nu}_n$ and $Z^{\mu\nu}_n$ are written in terms of the fields $g_{\mu\nu}$, $A^{\mu\nu}_n$, and $B^{\mu\nu}_n$ according to (29), (45), (30) and (31). Introducing the notation $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$, we rewrite $g^{\mu\nu}$ and $\sqrt{-g}$ in $H^{\mu\nu}$, $F^\mu$, $A^{\mu\nu}_n$ and $Z^{\mu\nu}_n$ according to

$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} + h_{(2)}^{\mu\nu} + h_{(3)}^{\mu\nu} + \cdots,$$

$$\sqrt{-g} = 1 + h/2 - h_{(2)}/4 + h^2/8 + \cdots,$$

with

$$h_{(n)}^{\mu\nu} = h_{(n)}^{\mu}, h_{(n)}^{\mu} = h_{(n)}^{\mu\nu} h^{\nu\nu},$$

$$h = h^{\mu\nu}.$$  

Then, $L_{\text{div}}$ becomes an infinite sum of monomials of $h_{\mu\nu}$, $A^{\mu\nu}_n$, $B^{\mu\nu}_n$ and their derivatives. Let us denote the numbers of $h_{\mu\nu}$, $A^{\mu\nu}_n$, $B^{\mu\nu}_n$ and the differential operators in the monomial by $N_h$, $N_A$, $N_B$ and $N_\delta$, respectively. The Lagrangian $L_{\text{reg}}$ should have mass dimension 4, while $h_{\mu\nu}$, $A^{\mu\nu}_n$, $B^{\mu\nu}_n$, and the differential operator has mass dimension 0, 1, 1 and 1, respectively. Therefore, the numbers $N_h$, $N_A$, $N_B$ and $N_\delta$ are restricted by

$$N_h + N_A + N_B + N_\delta \leq 4 - 2k_{\text{div}},$$

Note that $h_{\mu\nu}$ and its derivatives are not brane tensors, and their superscripts are not defined by multiplying $g^{\mu\nu}$. For convenience of calculations, we define here that their subscripts are raised by $\eta^{\mu\nu}$.  

8
Table 1. Invariant forms.

| $k_{\text{div}}$ | $N_h$ | $N_g$ | $N_\partial$ | Invariant forms |
|------------------|------|------|------------|----------------|
| 2                | 0    | 0    | 0          | $R^{(2)}$, $B_mB^\mu$ |
| 1                | 0    | 0    | 2          | $R^2$, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ |
| 0                | 2    | 0    | 2          | $R_{\mu\nu}B^\mu$, $R_{\mu\nu}B^\mu B_{\mu\nu}$, $R_{\mu}\nabla^{\mu}$, $\nabla_{\mu}B^{\mu}$, $B_{\mu\nu}B^{\mu\nu}$, $B^{\mu\nu}B_{\mu\nu}$, $B_{\mu\nu}B^{\mu\nu}B^{\rho\sigma}$, $B_{\mu\nu}B^{\mu\nu}B^{\rho\sigma}$, $B_{\mu\nu}B^{\mu\nu}B^{\rho\sigma}B^{\rho\sigma}$ |
| 0,1,2            | 0    | 0    | 4          | $B_{\mu\nu\lambda\kappa}B_{\mu\nu\lambda\kappa}$, $B_{\mu\nu\lambda\kappa}B_{\mu\nu\lambda\kappa}$, $B_{\mu\lambda\rho\kappa}B_{\mu\lambda\rho\kappa}$, $B_{\mu\lambda\rho\kappa}B_{\mu\lambda\rho\kappa}$ |
| 1,2              | 0    | 0    | 2          | $A_{\mu\nu\lambda\kappa}B_{\mu\lambda\rho\kappa}$, $A_{\mu\nu\lambda\kappa}B_{\mu\lambda\rho\kappa}$ |
| 2,3,4            | 0    | 0    | 4          | $A_{\mu\nu\lambda\kappa}A_{\mu\nu\lambda\kappa}$ |

where $k_{\text{div}} = 2, 1, 0$ for $\Lambda_4$, $\Lambda_2$, and $\Lambda_1$ terms, respectively. On the other hand, the number $N_h$ of $h_{\mu\nu}$ is not restricted. Relation (58) allows only finite numbers of values of $N_h$, $N_g$, and $N_\partial$, according to which we can classify the terms of $L_{\text{div}}$. Each class involves infinitely many terms for arbitrary values of $N_h$.

They are, however, not all independent, because they are related by high symmetry of the system under the general-coordinate transformations on the brane and GL(4) gauge transformations of the normal space rotation. Although the original system is invariant under the general-coordinate transformations in the bulk also, it is not available here, because we restrict ourselves to the flat bulk in this paper. The general case will be discussed in the forthcoming paper. Owing to the symmetry of the system, only a finite number of terms are allowed. The general-coordinate transformation symmetry requires that the effective Lagrangian density is proportional to $\sqrt{-g}$ times a sum of invariant forms. We list the allowed invariant forms in Table 1. In the table and thereafter, we use the following abbreviations:

\[
B^{(2)} = B_{\mu\nu}B_{\mu\nu}, \quad B_m = B_{m\nu}.
\]

\[
B^{(4)} = B_{\mu\nu\rho\kappa}B_{\mu\nu\rho\kappa}.
\]

\[
B_{\mu\nu\lambda\kappa} = B_{\mu\nu\lambda\kappa} + A_{\mu\nu\lambda\kappa}B_{\mu\nu\lambda\kappa}.
\]

\[
A_{\mu\nu\lambda\kappa} = \partial_{\mu}A_{\nu\lambda\kappa} - \partial_{\nu}A_{\mu\lambda\kappa} + A_{\mu\lambda\kappa}A_{\nu\mu\lambda\kappa} - A_{\mu\nu\lambda\kappa}A_{\nu\mu\lambda\kappa},
\]

where (61) and (62) are the covariant derivatives of the full symmetry, and (63) is the field strength of the gauge field $A_{\mu\nu\lambda\kappa}$. In the table, $R^2$, $R_{\mu\nu}R^{\mu\nu}$ and $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ are not all independent, but related by Gauss–Bonnet relation, and some other combinations are related due to the Gauss–Codazzi–Ricci formulae.

7. Calculation

Thus, we can calculate the coefficients of the term $\sqrt{-g}$ times the invariant forms by calculating the lowest order contributions in $h_{\mu\nu}$. The lowest contributions to the term with
$N_1 = N_2 = N_3 = 0$ are $O(h_{\mu\nu})$, while those to $N_4 = N_5 = 0$ and $N_6 \neq 0$ are $O((h_{\mu\nu})^2)$, because the $O(h_{\mu\nu})$ terms are total derivatives. Therefore, their lowest terms are in the one- and two-point functions $G^1$ and $G^2$. We can see from (28)–(31) that $B_{\mu\nu}^1\mu\nu$ always appears in the combination $B_{\mu\nu}^1\mu\nu B_{\mu\nu}^1\mu\nu$. Therefore, the only possible forms including $B_{\mu\nu}^1\mu\nu$ are $B^{(2)}_1$, $BB^{(3)}_1$, $(B^{(4)}_1)^2$ and $B^{(4)}_1$, among many forms listed in table 1. Their lowest terms are those with $O((h_{\mu\nu})^0)$ and are also in $G^1$ and $G^2$. The only possible form including $A_{\mu\mu\nu}$ only is $A_{\mu\mu\nu}A_{\mu\mu\nu}$ and its lowest term is of $O((h_{\mu\nu})^0)$, and it is in $G^2$. Thus, it suffices to calculate $G^1$ and $G^2$ in order to determine full contributions to $L^{\mu\nu}$.

From (48), (49) and (38), they are given by

\begin{equation}
G_{M_j}^1 = -N_{ex}\tilde{G}_{\mu\nu}^1 J_{\mu\nu} + N_{ex}M_j^2\tilde{F}_1 + \tilde{Z}_{m}m J, \tag{64}
\end{equation}

\begin{equation}
G_{M_j}^2 = N_{ex}\tilde{G}_{\mu\nu}^1 J_{\mu\nu} + 2i\tilde{G}_{\mu\nu}^1 \tilde{A}_{\mu}^n J_{\mu\nu} - (N_{ex}M_j^2\tilde{F} + \tilde{Z}_{n}m)\tilde{H}_{\mu\nu}(J_{\mu\nu} - q_\mu q_\nu J)/4
-\tilde{A}_{m}^n J_{\mu\nu} = 2i(M_j^2\tilde{F}_m^n + \tilde{Z}_{m}^n)\tilde{A}_{\mu}^n J_{\mu}
+(N_{ex}M_j^2\tilde{F}_1 + M_j^2\tilde{F}_m^n + \tilde{Z}_{m}^n)J, \tag{65}
\end{equation}

where $N_{ex} = D - 4$ is the number of the extra dimensions, $q_\mu$ is the momentum flowing in and out through the vertices, and

\begin{equation}
I_{\mu\nu} = \int \frac{d^4p}{i(2\pi)^4} \frac{p_\mu p_\nu}{[-p^2 + M_j^2]}, \tag{66}
\end{equation}

\begin{equation}
I = \int \frac{d^4p}{i(2\pi)^4} \frac{1}{[-p^2 + M_j^2]}, \tag{67}
\end{equation}

\begin{equation}
J_{\mu\nu\rho\sigma} = \int \frac{d^4p}{i(2\pi)^4} \frac{(p + q)_\mu p_\rho (p + q)_\rho}{[-(p + q)^2 + M_j^2][-p^2 + M_j^2]}, \tag{68}
\end{equation}

\begin{equation}
J_{\mu\nu\rho} = \int \frac{d^4p}{i(2\pi)^4} \frac{(p + q)_\mu p_\nu (2p + q)_\nu}{[-(p + q)^2 + M_j^2][-p^2 + M_j^2]}, \tag{69}
\end{equation}

\begin{equation}
J_{\mu\nu} = \int \frac{d^4p}{i(2\pi)^4} \frac{(2p + q)_\mu (2p + q)_\nu}{[-(p + q)^2 + M_j^2][-p^2 + M_j^2]}, \tag{70}
\end{equation}

\begin{equation}
J_\mu = \int \frac{d^4p}{i(2\pi)^4} \frac{(2p + q)_\mu}{[-(p + q)^2 + M_j^2][-p^2 + M_j^2]}, \tag{71}
\end{equation}

\begin{equation}
J = \int \frac{d^4p}{i(2\pi)^4} \frac{1}{[-(p + q)^2 + M_j^2][-p^2 + M_j^2]}, \tag{72}
\end{equation}

In the dimensional regularization, for large $M_j^2$, they are calculated to be

\begin{equation}
I_{\mu\nu} = -I_{\mu\nu} M_j^2 \eta_{\mu\nu}, \quad I = I_{\mu\nu} M_j^2, \tag{73}
\end{equation}

\begin{equation}
J_{\mu\nu\rho\sigma} = I_{\mu\nu\rho\sigma} \left[ \left( \frac{M_j^4}{8} - \frac{M_j^2 q^2}{24} + \frac{q^4}{240} \right) S_{\mu\nu\rho\sigma} - \left( \frac{M_j^2}{12} - \frac{q^2}{60} \right) T_{\mu\nu\rho\sigma} \right. 
+ \left( \frac{M_j^2}{6} - \frac{q^2}{40} \right) T_{\mu\nu\rho\sigma} + \frac{1}{30} q_\mu q_\nu q_\rho q_\sigma, \tag{74}
\end{equation}

\begin{equation}
J_{\mu\nu} = -I_{\mu\nu} [2M_j^2 \eta_{\mu\nu} + (q_\mu q_\nu - q^2 \eta_{\mu\nu})]/3, \tag{75}
\end{equation}
\( J_{\mu\nu\rho} = 0, \quad J_\mu = 0, \quad J = I_J \) (76)

with \( I_J = M_J^{-2\varepsilon} / (4\pi)^2 \varepsilon \) and

\[
S_{\mu\nu\lambda\rho} = \eta_{\mu\nu} \eta_{\lambda\rho} + \eta_{\mu\lambda} \eta_{\nu\rho} + \eta_{\mu\rho} \eta_{\nu\lambda},
\]

(77)

\[
T_{\mu\nu\lambda\rho} = \eta_{\mu\nu} q_{\lambda\rho} + \eta_{\mu\lambda} q_{\nu\rho} + \eta_{\mu\rho} q_{\nu\lambda} + \eta_{\nu\lambda} q_{\mu\rho} + \eta_{\nu\rho} q_{\mu\lambda} + \eta_{\lambda\rho} q_{\mu\nu},
\]

(78)

\[
T_{\mu\nu}^\prime = \eta_{\mu\nu} q_{\lambda\rho} + \eta_{\nu\lambda} q_{\mu\rho}.
\]

(79)

We substitute (73)–(74) into (64) and (65), and substitute them into (36) to obtain \( \mathcal{L}^{\mu\nu\lambda\rho} \), and rearrange the terms into a sum of monomials of \( h_{\mu\nu} \), \( A^\mu_{\nu\lambda\rho} \), \( B_{\mu\nu\lambda\rho} \) and their derivatives. Each term is proportional to \( I_J M^{2\varepsilon}_J \) \((k = 0, 1, 2)\), which, when regularized via (39), behave as

\[
\sum_j C_j I_j \rightarrow \ln \Lambda^2 / (4\pi)^2,
\]

(80)

for large \( \Lambda \) (the equal mass limit of \( M_J \)). The terms are classified as follows.

(i) The terms with \( N_A = N_B = 0 \) are given by (see footnote 4)

\[
N_A \equiv N_B = 0 \quad \frac{\Lambda^4}{32(4\pi)^2} \left[ \frac{\Lambda^2}{2} (4h - 2h_{(2)} + h^2) + \frac{\Lambda^2}{3} (h^{\mu\nu,\lambda} h_{\mu\nu,\lambda} - 2h^{\mu\nu,\lambda} h_{\mu,\lambda} + 2h^{\mu\nu,\lambda} h_{\nu,\lambda} - h^{\mu\nu} h_{\mu,\lambda}) \right.

+ \frac{\ln \Lambda^2}{15} \left. \left\{ h^{\mu\nu,\lambda\rho} h_{\mu\nu,\lambda\rho} - 2h^{\mu\nu,\lambda\rho} h_{\mu,\lambda\rho} - 4h^{\mu\nu,\mu\nu} h^{\lambda\lambda} + 3(h^{\mu\nu} h^{\mu\nu})^2 \right\} \right]
\]

(81)

up to total derivatives. Because the full expression should have the symmetry, they should be the lower order expression of \( \sqrt{-g} \) times the invariant forms in table 1. The terms in (81) are to be compared with the lower contributions for \( \sqrt{-g} \) in (55) and

\[
\sqrt{-g} R = -\frac{1}{2} (h^{\mu\nu,\lambda} h_{\mu\nu,\lambda} - 2h^{\mu\nu,\lambda} h_{\mu,\lambda} + 2h^{\mu\nu,\lambda} h_{\nu,\lambda} - h^{\mu\nu} h_{\mu,\lambda}),
\]

(82)

\[
\sqrt{-g} R^2 = \left( h^{\mu\nu,\mu\nu} \right)^2 - 2h^{\mu\nu,\mu\nu} h^{\lambda\lambda} + (h^{\mu\nu} h^{\mu\nu})^2,
\]

(83)

\[
\sqrt{-g} R_{\mu\nu} R^{\mu\nu} = -\frac{1}{2} \left( h^{\mu\nu,\lambda\rho} h_{\mu\nu,\lambda\rho} - 2h^{\mu\nu,\lambda\rho} h_{\mu,\lambda\rho} + 2(h^{\mu\nu,\mu\nu} h^{\lambda\lambda})^2 \right.

- 2h^{\mu\nu,\mu\nu} h^{\lambda\lambda} + (h^{\mu\nu} h^{\mu\nu})^2,
\]

(84)

where total derivatives are neglected.

(ii) The lowest contributions to \( \mathcal{L}^{\text{reg}} \) with \( N_B \neq 0 \) and \( N_I = 0 \) are

\[
\frac{1}{4(4\pi)^2} \left( \Lambda^2 B^{(2)} + \ln \Lambda^2 B^{(4)} \right),
\]

(85)

which are taken as the lowest parts of the forms \( \sqrt{-g} B^{(2)} \) and \( \sqrt{-g} B^{(4)} \).

(iii) The lowest contribution with \( N_B \neq 0 \) and \( N_I = 2 \) is

\[
\frac{\ln \Lambda^2}{12(4\pi)^2} (h^{\mu\nu,\mu\nu} + h^{\mu\mu}) B^{(2)}
\]

(86)

which is the lowest part of the form \( \sqrt{-g} RB^{(2)} \).
(iv) The lowest contribution with $N_A \neq 0$ is
\[
\frac{1}{24(4\pi)^2}(A_{\mu\nu,\nu} - A_{\mu\nu,\nu})(A^{\mu\nu,\nu} - A^{\mu\nu,\nu}),
\]
which is the lowest part of the form $\sqrt{-g}A_{\mu\nu}A^{\mu\nu}$ with $N_A = 2$. Note that it suffices to determine the coefficient of the form in $L^{\text{reg}}$.

Collecting the results of (i)–(iv), we finally obtain the expression for the divergent part $L^{\text{div}}$ of $L^{\text{reg}}$:
\[
L^{\text{div}} = \sqrt{-g}/(4\pi)^2\left[ N_{\text{ex}} \left\{ \frac{\Lambda^4}{8} - \frac{\Lambda^2}{24}R + \frac{\ln \Lambda^2}{240}(R^2 + 2R_{\mu\nu}R^{\mu\nu}) \right\} 
+ \frac{\Lambda^2}{4}B^{(2)} + \frac{\ln \Lambda^2}{4}B^{(4)} - \frac{\Lambda^2}{12}RB^{(2)} - \frac{\ln \Lambda^2}{24}A_{\mu\nu}A^{\mu\nu} \right],
\]
where $B^{(2)}$, $B^{(4)}$ and $A_{\mu\nu}$ are defined in (59), (60) and (63), respectively, and $N_{\text{ex}}$ is the number of the extra dimensions. The divergences cannot be renormalized because the original action does not have these terms. They give rise to genuine quantum-induced effects.

8. Cosmological constant

Thus, we have derived the quantum effects of the brane fluctuations. Among them, the $\Lambda^4$ term in (88) gives huge a contribution to the cosmological term. To this term, the starting Lagrangian $L'_{\text{br}}$ in (8) by itself also has a contribution. From phenomenological points of view, it should be very tiny. Therefore, the large contributions should cancel out each other to give the tiny cosmological term. The condition for the cancellation is
\[
\lambda = -N_{\text{ex}}\Lambda^4/128\pi.
\]
This is, however, an extremely unnatural fine tuning. It is a serious problem common to the quantum theories including gravity in general. The present formulation has no solution to this longstanding problem.

Furthermore, it may give rise to another contribution which may mimic the cosmological term in the effective equation of motion for $g_{\mu\nu}$. The energy–momentum tensor in the equation has the term
\[
\lambda Y_{\mu}^I Y_{\nu}^J G_{IJ}/2,
\]
which may look like the cosmological term if the embedding is almost flat. In such cases, we can adjust the cosmological term to the phenomenological tiny value by, for example, adopting the conformally flat embedding
\[
Y^\mu = [1 + N_{\text{ex}}\Lambda^4/128\pi\lambda]^{1/2}x^\mu, \quad Y^m = 0
\]
instead of the condition (89). This is also an extremely unnatural fine tuning. Thus, the present model is not satisfactory in natural understanding of the cosmological constant. It is, however, not a problem for the model alone, but a serious puzzle for general quantum theoretical models with gravity. It is an open problem, and we wish that it will be solved in the future. The problem will be partly addressed in our forthcoming paper. Here, we phenomenologically adjust the tiny cosmological term via fine tuning of (89) or (91).
9. Induced gravity

If the cosmological term is suppressed, the main contribution in the quantum effects (88) comes from the $R$ term. It is nothing but the Einstein–Hilbert action, which supply the kinetic term for the auxiliary field $g_{\mu\nu}$. The sign of the term is right one to give ordinary attractive gravity in accordance with the observation, and its magnitude indicates that the cutoff $\Lambda$ is order of the Planck scale. The term with $(R^2 + 2R_{\mu\nu}R^{\mu\nu})$ gives small corrections of $O(\log \Lambda^2/\Lambda^2)$ as far as the brane curvature is small. The terms with $B^{(2)}$, $RB^{(2)}$ and $B^{(4)}$ are the mass and interaction terms of the field $B_{\mu\nu}$. Note that no kinetic term for $B_{\mu\nu}$ appears. This is because the $B_{\mu\nu}$ interacts with $\phi^m$ only in the combination $B_{\mu\nu}B^{\mu\nu}$, but not in single. The term with $A_{\mu\nu}$ squared gives the kinetic and the interaction terms of the field $A_{\mu\nu}$ as the gauge field. The fields $A_{\mu\nu}$ and $B_{\mu\nu}$ appear as fields on the brane. We should, however, be careful because they are not independent and are defined by (22) and (23) in terms of $Y^I$ and $n^m$.

The quantum-induced terms in (88) modify the equations of motion. The equation (9) for $Y^I$ is modified through the $B^{(2)}$, $RB^{(2)}$, $B^{(4)}$ and $(A_{\mu\nu})^2$ terms in (88). The classical solution $Y^I$ is deformed according to it. The correction terms are suppressed by at least a factor of $O(\Lambda^{-2})$ for small curvatures. The equation (10) for $g_{\mu\nu}$, the induced-metric formula, is converted into the Einstein equation with the $O(R^2)$ correction terms and the energy–momentum tensor for the fields $\phi^m$, $A_{\mu\nu}$, $B_{\mu\nu}$ and $Y^I$. The equation (10) holds as operator relation. In classical realizations, however, it suffers from a large quantum corrections. Then, we no longer have the induced-metric formula.

10. Discussions

The metric $g_{\mu\nu}$ emerges in the channel of intermediate quantum states composed of $\phi^m$'s, despite its absence in the original setup of the system (1). Hence, it is interpreted as a composite of the brane fluctuation fields $\phi^m$. Then the natural question is what is the further quantum effects of the composite metric. Within the semi-classical treatments, it suffices to calculate only the one-loop diagrams. The system does not include multi-loop diagrams. This is the virtue of the linear definition (16) of the brane fluctuation. Beyond the semi-classical approximation, however, we should take into account the higher order diagrams with the internal lines of composite metric fields.

The quantum induction mechanism of composite fields is common phenomena to various composite field theories [48]. A class of non-renormalizable theories with this mechanism becomes equivalent to some renormalizable models (with finite momentum cutoff) under the ‘compositeness condition’ that the wave-function renormalization constant vanishes [49, 50]. This renders us clues to formulate unambiguously the non-renormalizable theories at higher orders [51]. For example, the Nambu–Jona-Lasinio model is equivalent to the Yukawa model with the vanishing renormalization constants of the scalar and the pseudoscalar fields, and the latter renders an unambiguous higher-order descriptions of the former.

In the present case, however, the induced composite field theory is the modified Einstein gravity and is not renormalizable. We have no definite way to calculate the quantum effects due to the metric itself at higher orders. It shares the problems with the general quantum gravity theories. So we cannot apply all the achievements of the composite field theories with the compositeness condition. They are, however, very suggestive in considering properties of the quantum fluctuations. In composite theories, it is plausible that the quantum effects due to the composite would require different treatments. For example, if the cutoff for the composites is much smaller than that for the constituents, the effects can be suppressed [52].
expansion would be useful, as is in the various composite field theories [50, 51, 53]. We need further ideas and investigations for the more complete treatments.

We can see in (88) that the quantum effects give rise to the terms including the extrinsic curvature $B_{\mu \nu}$ and the normal connection $A_{\mu ij}$, in addition to the induced-gravity terms (see footnote 3). The induction of these terms is characteristic of the brane-induced gravity theory, distinguished from the ordinary (non-brane)-induced gravity. The fact was recognized in [14, 16] in the general braneworld scheme, and they were actually calculated in [18, 34] for the domain-wall-type braneworld. The forms of induced terms depend on the brane dynamics. The simplest case of the Nambu–Goto action was considered in [4] within the four dimensional field theory. In the model, however, only the gravity is induced, but no other terms. This is because the spacetime spanned by the scalar fields is not the real one, and hence it assumes no symmetry of the whole spacetime involving the brane. Therefore, we cannot define the normal to the brane. In contrast, the present model (1) possesses general-coordinate invariance of the bulk, as well as that of the brane. Therefore, the fluctuations along the brane is meaningless, and the only physical ones are those transverse to the brane, as are defined by (15)–(17). This is the origin how it includes the $A_{\mu ij}$ and $B_{\mu \nu}$ dependent terms. They should be determined according to the brane dynamics, as is done here. It would be an interesting and urgent subject to derive the induced terms in various brane dynamics and seek for the models suited for applications.

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References

[1] Fronsdal C 1959 Nuovo Cimento 13 988
[2] Joseph D W 1962 Phys. Rev. 126 319
[3] Regge T and Teitelboim C 1975 Proc. Marcel Grossman Meeting on Relativity (Oxford, 1977) pp 77–87
[4] Akama K 1978 Prog. Theor. Phys. 60 1900
[5] Akama K 1983 Lect. Notes Phys. 176 267
[6] Rubakov V A and Shaposhnikov M E 1983 Phys. Lett. B 125 136
[7] Maia M D 1984 J. Math. Phys. 25 2090
[8] Nicolai H and Wetterich C 1985 Phys. Lett. B 150 347
[9] Visser M 1985 Phys. Lett. B 159 22
[10] Pavšič M 1985 Class. Quantum Grav. 2 869
[11] Callan C G Jr and Harvey J A 1985 Nucl. Phys. B 250 427
[12] Hughes J, Liu J and Polchinski J 1986 Phys. Lett. B 180 570
[13] Gibbons G W and Wiltshire D L 1987 Nucl. Phys. B 287 717
[14] Akama K 1987 Prog. Theor. Phys. 78 184
[15] Watanabe I 1987 Bulletin of the College of Liberal Arts and Sciences vol 17 (Tokyo: Tokyo Medical and Dental University) pp 1–17
[37] Korutlu B 2008 arXiv:0801.3579
Landsberg G 2008 Proc. 13th Lomonosov Conf. on Elementary Particle Physics (Moscow, 23–29 August) (Moscow State U.) pp 99–109
Clark T E, Love S T, Nitta M, Veldhuis T ter and Xiong C 2008 Phys. Rev. D 78 115004
Clark T E, Love S T, Nitta M, Veldhuis T ter and Xiong C 2009 Nucl. Phys. B 810 97–114

[38] Sarrazin M and Petit F 2011 Phys. Rev. D 83 035009
Sarrazin M and Petit F 2010 Phys. Rev. D 81 035014
Sarrazin M, Pignol G, Petit F and Nussizhvesky V V 2012 Phys. Lett. B 712 213
Garrido N and Hernandez H H 2013 Prog. Theor. Exp. Phys. 2013 021B01
Fujimoto Y, Nagasawa T, Nishiwaki K and Sakamoto M 2012 arXiv:1209.5150

[39] Kanti P, Kogan I I, Olive K A and Pospelov M 1999 Phys. Lett. B 468 31
Binetruy P, Deffayet C and Langlois D 2000 Nucl. Phys. B 565 269
Shiromizu T, Maeda K I and Sasaki M 2000 Phys. Rev. D 62 024012
Maartens R, Wands D, Bassett B A and Heard I P C 2000 Phys. Rev. D 62 041301
Garriga J and Tanaka T 2000 Phys. Rev. Lett. 84 2778
Koyama K and Soda J 2000 Phys. Rev. D 62 123502
Copeland E J, Liddle A R and Lidsey J E 2001 Phys. Rev. D 64 023509
Maartens R 2004 Living Rev. Rel. 7 7

[40] Koyama K 2008 Gen. Rel. Grav. 40 421
Heydari-Fard M and Sepangi H R 2007 Phys. Rev. D 76 104009
Saridakis E N 2009 Nucl. Phys. B 808 224
Adam C, Grandi N, Sanchez-Guillen J and Wereszczynski A 2008 J. Phys. A: Math. Theor. 41 212004
Adam C, Grandi N, Sanchez-Guillen J and Wereszczynski A 2009 J. Phys. A: Math. Theor. 42 159801 (erratum)
Atazadeh K, Farhoudi M and Sepangi H R 2008 Phys. Lett. B 660 275

[41] Jardim I C, Landim R R, Alencar G and Costa Filho R N 2011 Phys. Rev. D 84 064019
Maity D 2012 Phys. Rev. D 86 084056
Sousa L J S, Silva C A S and Almeida C A S 2012 Phys. Lett. B 718 579–83

[42] Sakharov A D 1967 Dokl. Akad. Nauk SSSR 177 70
Sakharov A D 1968 Sov. Phys.—Dokl. 12 1040

[43] Akama K, Chikashige Y and Matsuki T 1978 Prog. Theor. Phys. 59 653
Akama K, Chikashige Y, Matsuki T and Terazawa H 1978 Prog. Theor. Phys. 60 868
Zee A 1979 Phys. Rev. Lett. 42 417
Adler S L 1980 Phys. Rev. Lett. 44 1567
Akama K 1981 Phys. Rev. D 24 3073

[44] Akama K 1979 Prog. Theor. Phys. 61 687

[45] Barcelo C, Liberati S and Visser M 2001 Class. Quantum Grav. 18 3595
Barcelo C, Visser M and Liberati S 2001 Int. J. Mod. Phys. D 10 799
Barcelo C, Liberati S and Visser M 2005 Living Rev. Rel. 8 12
Barcelo C, Liberati S and Visser M 2011 Living Rev. Rel. 4 3

[46] Broda B and Szanecki M 2009 Phys. Lett. B 674 64
Wetterich C 2011 Phys. Lett. B 704 612
Wetterich C 2012 Phys. Rev. D 85 104017
Wetterich C 2013 Lect. Notes Phys. 863 67–92
Wetterich C 2012 Ann. Phys. 327 2184
Sexty D and Wetterich C 2013 Nucl. Phys. B 867 290–329

[47] Dirac P A M 1962 Proc. Roy. Soc. Lond. A 268 57
Nambu Y 1995 Duality and hadrodynamics (Copenhagen High-Energy Summer Symp. 1970) Broken Symmetry: Selected Papers of Y Nambu ed T Eguchi and K Nishijima (Singapore: World Scientific) pp 280–301
Goto T 1971 Prog. Theor. Phys. 46 1560
Nambu Y 1974 Phys. Rev. D 10 4262

[48] Nambu Y and Jona-Lasinio G 1961 Phys. Rev. 122 345
Bjorken J D 1963 Ann. Phys. 24 174
Terazawa H, Chikashige Y and Akama K 1977 Phys. Rev. D 15 480

[49] Jouvet B 1956 Nuovo Cimento 5 113
Vaughn M T, Aaron R and Amado R D 1961 Phys. Rev. D 124 1258
Salam A 1962 Nuovo Cimento 25 224
Weinberg S 1963 Phys. Rev. 130 776

[50] Shizuya K I 1980 Phys. Rev. D 21 2327
[51] Akama K 1996 Phys. Rev. Lett. 76 184
    Akama K 1998 Nucl. Phys. A 629 37C
    Akama K 2004 Phys. Lett. B 583 207
    Akama K and Hattori T 1997 Phys. Lett. B 392 383
    Akama K and Hattori T 1998 Phys. Lett. B 445 106
    Akama K and Hattori T 2004 Phys. Lett. B Phys. Rev. Lett. 93 211602
    Akabane A and Akama K 2004 Prog. Theor. Phys. 112 757
[52] Akama K 1980 Prog. Theor. Phys. 64 1494
    Akama K 1990 Prog. Theor. Phys. 84 1212
[53] Tomboulis E 1977 Phys. Lett. B 70 361
    Tomboulis E 1980 Phys. Lett. B 97 77