Structure and Dynamics of Charmonium

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(Dated: March 30, 2022)

Abstract

We apply a confining q̄q potential to charmonium and open charm states in order to model wave functions and to begin studying structure. Results (in momentum space) provide form-factor input to a four-flavor effective chiral Lagrangian which models dynamics of charmonium in hot hadronic matter. Estimates are made for \( J/\psi \) dissociation cross sections and rates within a fireball. Our study attempts to improve on previous comover suppression models since it includes gauge-invariant form-factor formalism constrained by quark-model phenomenology.
I. INTRODUCTION

Signals of momentary quark and gluon deconfinement in high-energy heavy-ion reactions are studied today more aggressively than ever before owing to current experimental activities at the Relativistic Heavy-Ion Collider (RHIC)[1]. Hard probes represent a particular piece to the overall puzzle whose goal upon assembly is to fully understand the strongly interacting many-body dynamics of ultra-relativistic heavy-ion collisions, complete with definite evidence for quark-gluon plasma (QGP) formation and propagation. The hard probes provide complementary information to such softer probes as photon and dilepton production. Very briefly for charm, the idea of Matsui and Satz[2] proposes that in events where QGP is formed, screening effects tend to break apart the $c\bar{c}$ states leaving a “gap” between observed charmonium and expected. It is very convenient to look for evidence of charmonium breakup by studying the mass distribution of muon pairs and trimming away the background of non-$J/\psi$ contributions. There have already been suggestions that such a comparison might suggest glimpses of QGP[3].

Meanwhile, several authors have begun to systematically assess comover absorption using a variety of different approaches in order to improve understanding of the background due to such effects as light meson plus charmonium interactions leading to breakup[4, 5, 6, 7, 8, 9, 10, 11]. Breakup of this type could possibly be misinterpreted as subhadronic effects when indeed, it is ordinary hadronic many-body physics. The aim of this study is to further refine estimates of hadronic cross sections for breakup of $J/\psi$ by constraining form factors with quark-model phenomenology and confining potentials, and then to use the form factors in a four-flavor chiral Lagrangian.

Our article is organized in the following way. We discuss in Sect. II the confining potential and resulting meson wave functions. They provide information on the hadronic form factors to be later used in charmonium dynamics. Sect. III includes a brief summary of the four-flavor chiral Lagrangian used to model the dynamics of the hadronic matter constituents. It also discusses gauge-invariant implementation of finite hadron size effects, namely form factors. Results are presented and discussed in Sect. IV and finally, Sect. V summarizes and concludes.
II. CONFINING POTENTIAL

Accomplishments of quark-model studies include rather detailed comparisons of calculated hadron spectra versus observed for a long list of light mesons. The models are constrained as firmly as possible by confronting such spectroscopic details as masses and decay widths. We take a recent result from the literature[4] and extend it beyond the light mesons to include $D$’s $\eta_c$, $J/\psi$ and $\psi'$ (no isospin dependence is included, so we do not mention $\chi_c$ states explicitly). The potential is

$$V_{ij} = -\frac{\kappa}{r} + \lambda r^p + \Lambda + \frac{2\pi \kappa'}{3m_i m_j} \frac{\exp(-r^2/r_0^2)}{\pi^{3/2} r_0^3} \vec{\sigma}_i \cdot \vec{\sigma}_j,$$

(1)

where the range $r_0$ of the hyperfine term is taken to be mass dependent

$$r_0(m_i, m_j) = A \left( \frac{2m_i m_j}{m_i + m_j} \right)^B.$$

(2)

A set of parameters is chosen to give the usual Coulomb plus linear form for the central part

$m_u = m_d = 0.315$ GeV; $m_s = 0.577$ GeV; $m_c = 1.836$ GeV

$\kappa = 0.5069; \quad \kappa' = 1.8609; \quad \lambda = 0.1653 \text{ GeV}^2; \quad p = 1$

$\Lambda = -0.8321$ GeV; $B = 0.2204; \quad A = 1.6553 \text{ GeV}^{B-1}.$

(3)

By numerically solving the Schrödinger equation bound state wave functions are obtained, from which meson masses and rms radii are readily computed. Results for a selected set of mesons are listed in table I. Much of the motivation to do subnuclear structure in this way is to use the information as form-factor input to an effective Lagrangian description of the strongly-interacting hadrons. Typically, one views the spatial density as the Fourier transform of a (momentum space) distribution—the form factor. As is usually assumed in field theoretic modeling of this type, and is consistent with quark counting rules, a monopole structure is used. We extrapolate from massless to massive probes, and use a monopole-charge-form-factor-inspired expression (only meant as an indicator rather than a consistent model calculation) for the cutoff or off-shellness parameter

$$\Lambda = \sqrt{m^2 + \frac{6}{\langle r^2 \rangle}}.$$

(4)

The monopole form factor to be discussed later uses $\Lambda$ in attempts to describe three-point vertices where a meson of mass $m$ and rms radius $\sqrt{\langle r^2 \rangle}$ is forced to go off shell. In rough terms, the size of the interaction vertex is inversely proportional to the cutoff parameter. And again, it is the off shell particle in a three point vertex which governs the physics.
III. FOUR-FLAVOR CHIRAL LAGRANGIAN

In the absence of firm experimental constraints on a full set of mesonic interactions involving strangeness and charm, effective theories, with a specified chirally symmetric interaction, are quite useful. Indeed, there has been a renewed interest in these approaches since full understanding and control of the nonperturbative effects confinement necessitates is still beyond grasp. Chiral symmetry and current conservation represent minimum requirements for any effective hadronic field theory. We take such an approach here by extending the usual two-flavor chiral Lagrangian to not three, but a four-flavor set of fields. The strange quark mass being greater than up and down quark masses probably already brings about some limitations for the Lagrangian’s usefulness, and the charmed quark mass certainly breaks the symmetry to a deeper extent. And yet, it is not unreasonable to relegate this breaking to the mass terms, and insist that the interaction remain fully symmetric.

The full details, starting from the nonlinear sigma model, introducing vector and axial vector fields into gauge covariant derivatives and then subsequently gauging away all of the axial degrees of freedom, have been published elsewhere[7]. We therefore include here the

| Meson | Mass (MeV) [Obs.] | Mass (MeV) [Calc.] | $\sqrt{\langle r^2 \rangle}$ (fm) | $\Lambda$ (GeV) |
|-------|------------------|--------------------|----------------------------------|----------------|
| $\pi$ | 138              | 138                | 0.59                             | 0.80           |
| $K$   | 496              | 490                | 0.59                             | 0.96           |
| $\rho$| 769              | 770                | 0.92                             | 0.93           |
| $K^*$ | 894              | 904                | 0.81                             | 1.07           |
| $\phi$| 1019             | 1021               | 0.70                             | 1.23           |
| $a_1$ | 1230             | 1208               | 1.24                             | 1.29           |
| $D$   | 1867             | 1862               | 0.61                             | 2.03           |
| $D^*$ | 2008             | 2016               | 0.70                             | 2.12           |
| $\eta_c$ | 2980          | 3005               | 0.37                             | 3.25           |
| $J/\psi$ | 3097           | 3101               | 0.40                             | 3.32           |
| $\psi'$ | 3686           | 3641               | 0.79                             | 3.73           |
interaction terms alone. They are written very compactly as

\[ \mathcal{L}_{\text{int}}^I = i g \text{Tr} (\rho_\mu [\partial^\mu \phi, \phi]) - \frac{g^2}{2} \text{Tr}([\phi, \rho^\mu]^2) + \frac{g}{4} \text{Tr}([\rho^\mu, \rho^\nu]) + \frac{g^2}{4} \text{Tr}([\rho^\mu, \rho^\nu]^2) \]  (5)

where \( \phi \) and \( \rho_\mu \) are \( 4 \times 4 \) matrices with entries containing pseudoscalar and vector fields, respectively.

Since a strict chiral symmetry is respected, there remains just one (chiral) coupling constant to fix. We do so by making certain the rho meson is correctly described. The choice \( g = 4.3 \) gives \( \Gamma_\rho = 151 \) MeV at the pole mass of 770 MeV, and gives decay widths for strangeness and charm excitations listed in Table II.

| particle       | Chiral Lagrangian | Experiment       |
|----------------|-------------------|------------------|
| K(892)\(^0\)   | 44.5 MeV          | 50.5 ± 0.6 MeV   |
| K(892)\(\pm\)  | 44.5 MeV          | 49.8 ± 0.8 MeV   |
| D(2007)\(^0\)  | 10.1 keV          | < 2.1 MeV, 90% CL|
| D(2010)\(^\pm\) | 21.1 keV          | 96 ± 4 ± 22 keV  |

The very recent charged \( D^* \) decay measurement coming in at 96 keV allows for a \( D^*D\pi \) coupling constant to be fully twice as large as the chiral symmetry proposes. This would increase the dissociation cross section be precisely a factor of two. For now, however, we adhere to the chiral symmetry prediction.

The interactions identified in Eq. (5) do not include anomalous processes of type vector-vector-pseudoscalar. We therefore extend the set of interactions by introducing

\[ \mathcal{L}_{\text{int}}^{II} = g_{V\phi} \text{Tr} \left( \epsilon_{\mu\nu\alpha\beta} \partial^\mu V^\nu \partial^\alpha V^\beta \phi \right) , \]  (6)

with coupling constants constrained individually via vector meson dominance. One of the channels now open with \( \mathcal{L}^{II} \) is \( J/\psi + \pi \rightarrow \eta_c + \rho \), an important contributor.

### A. Form Factors

Effective theories attempt to model composite objects which necessarily have finite extent, and are therefore responsible for finite-sized interaction vertices. Three-point functions,
representing full details of the interactions including loop effects to arbitrary order, are notoriously difficult to handle consistently. One-boson-exchange models, which involve three- and possibly four-point vertices, approximate the full calculation. Since in such models at most one particle per vertex goes off shell, a Lorentz invariant form factor accounts for dressing the vertex. So, rather than attempt to calculate higher-order contributions, we assume a specific monopole form for the momentum dependent vertex coupling “constants”, and use the off-shell or cutoff parameters from Table I. All three-point vertices are therefore given the following monopole

\[ h(t) = \frac{\Lambda^2}{t - m^2} + \frac{|t - m^2|}{\Lambda^2}, \]

where \( t \) here is the squared four-momentum of the off-shell particle. Notice that when \( t \rightarrow m^2 \), then \( h \rightarrow 1 \), which is indeed how the pole coupling constants are all defined.

Four-point functions are modified from their typical \( g^{\mu\nu} \) form to the most general linear combination of all possible lowest-order Lorentz invariant structures constructible out of the external momenta. Specifically, in the reaction \( J/\psi + \pi \rightarrow D + D^* \), the four-point vertex becomes

\[ \Gamma^{\mu\nu} = A g^{\mu\nu} + B \left( p'^\mu_D p'^\nu_D + p'^\mu_\pi p'^\nu_\pi \right) + C \left( p'^\mu_D p'_\pi + p'^\mu_\pi p'_D \right) + D \left( p'^\mu_D p'_\pi + p'^\mu_\pi p'_D \right), \]

and then the expansion coefficients \( A - E \) are chosen so that the overall scattering amplitude is fully gauge invariant, \( \partial_\mu M^\mu = 0 \). The choice is not unique; but on the other hand, gauge invariance alone is not enough to uniquely fix an amplitude. It does however represent an absolute minimum requirement of any reliable model.

**IV. RESULTS**

With all the interactions identified in the model, all the vertices constrained as much as possible, a list of reactions involving light meson plus \( J/\psi \) can be enumerated and calculated. We begin looking at \( \pi, K, \eta, \rho, \omega, K^* \), and so on.
A. Cross Sections

The required dynamical quantity for estimates of $J/\psi$ production and possible suppression are the breakup cross sections for the individual reactions. There are too many specific channels studied here to completely discuss them all. Instead, we show the two leading contributors in Fig. [1]. The pion-induced reactions involve $D + \bar{D}^*$, $D + D^*$, and $\eta_c + \rho$ final states and the rho-induced reactions involve $D + \bar{D}$, $D^* + \bar{D}^*$ and $\eta_c + \pi$ final states. In a rough summary, the cross sections range from a few tenths to a few millibarns.

B. Dissociation Rates

Consider the reaction $J/\psi + b \to 1 + 2$. The rate within a fireball for this to occur is the following.

$$d\Gamma_{J/\psi} = \frac{1}{2E_{J/\psi}} d_b \frac{d^3p_b}{(2\pi)^3 2E_b} f_b \left|\mathcal{M}\right|^2 \tilde{f}_1 \tilde{f}_2 (2\pi)^4 \delta^4(p_{J/\psi} + p_b - p_1 - p_2)$$

$$\times \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2},$$

where $d_b$ is the degeneracy of species $b$, $f_b$ is the Bose-Einstein distribution, and $\tilde{f} \equiv 1 + f$ to account for the medium. We show in Fig. [3] the total dissociation rates at two fixed temperatures as functions of the $J/\psi$ momentum.

C. Flow

It seems fairly clear by looking at the experimental results from RHIC that significant flow is present in the reaction zone [13]. Comover suppression is not expected to be significantly affected if the heavy charm is comoving. We suppose here, that it is not. We look at the possibility that $J/\psi$ breakup rates could depend on the radial flow velocity. There is no reason to expect the $J/\psi$'s to have thermal momentum distributions since the elastic cross sections are too small to allow complete thermalization [7]. We therefore estimate the new dissociation rates assuming rapid radial expansion, but with $J/\psi$ given a fixed three-momentum in the rest frame of the fireball. We then average over all solid angles with equal
FIG. 1: Cross sections for pion- and rho-induced dissociation. For complete details on all final states see Ref. [7].
FIG. 2: Dissociation rates in a fireball at fixed temperatures, 150 and 200 MeV as a function of $J/\psi$ (nonthermal) momentum. Rates included $\pi$, $K$, $\rho$ and $K^*$-induced breakup.

weight. The expression we use "with flow" is therefore

$$d\Gamma^\text{wf}_{J/\psi} = \frac{d\Omega_{p_{J/\psi}}}{4\pi} d\Gamma_{J/\psi},$$

(10)
where $d\Gamma_{J/\psi}$ from Eq. (9) is used but the equilibrium distribution for particle $b$ is now $f_{eq}(p_b \cdot U)$, where $U = \gamma (1, v)$ is the four-velocity. We use $|v| = 0.6$. Rates in the presence of flow are reported in Fig. 3.

V. CONCLUSIONS

We have estimated charmonium structure using a confining quark potential calibrated with a long list of hadronic states. The momentum space wave functions provide form-factor input to an effective four-flavor chiral Lagrangian describing charmonium dynamics in a strongly-interacting many-particle system. Flow was introduced, albeit in a rather simplistic way, and was shown to affect the results.

$J/\psi$ breakup cross sections of several tenths to possibly a few millibarns were found. Kinetic theory was used to benchmark the dissociation rates in a fireball. At high $J/\psi$ momentum (5 GeV/$c$) and high system temperature (200 MeV), a dissociation rate of 10 MeV was found. With flow present in the system, the dissociation rate increased by a factor of roughly 3. Future studies will include a folding of a more realistic $p_T$ distribution for $J/\psi$.

Acknowledgement(s)

I thank Prof. Charles Gale for useful exchanges regarding this project. This work was supported in part by the National Science Foundation under grant numbers PHY-9814247 and 0098760.

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[13] See, for example, Declan Keane’s contribution to this meeting.
FIG. 3: Dissociation rates in the presence of rapid radial flow $|v| = 0.6$ assuming the $J/\psi$ has fixed three-momentum randomly distributed.