Spinor Field Realizations of $W_{2,6}$ String and $W_6$ String

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Abstract

In this paper the spinor field BRST charges of the W2,6 string and W6 string are constructed, where the BRST charges are graded.

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I Introduction

As is well known, much work on the $W$ algebra and $W$ string has received a considerable attention since 1990's[1-6]. The BRST method is the simplest way to build a $W$ string by far. In the work[7], we mentioned that the explicit forms of the BRST charge for the $W_4$ and $W_5$ algebra[8,9] are so complex and it is difficult to be generalized to a general $W_N$ string. And we point out that the reason why are in two parts: because the method is not graded and it just only belongs to the usual scalar field realizations. At the same time, we found the methods to construct the spinor field realizations of $W_2,s$ strings and $W_N$ strings. We assume the BRST charges of the $W_2,s$ strings or $W_N$ strings are graded, this Ansatz makes their BRST charges become very easy. And the exact constructions of $W_{2,5}$ string and $W_5$ string have been obtained[7].

Naturally the computational complexity rises rapidly when we construct the spinor field realization of $W_2,s$ and $W_N$ strings with higher ‘s’ or ‘N’. It is necessary to give out a general program since the calculation by hand becomes more difficult. In fact, we have written the program which can give out the precise solutions in order to check our Ansatz and compute any solutions of the spinor realizations of $W_2,s$ and $W_N$ strings. The spinor field BRST charges of $W_{2,6}$ string and $W_6$ string have been constructed firstly by using it. Especially we improved the results of $W_{2,4}$ string in the work[10] and obtained a more general result. Of course, we also checked the results of $W_{2,3}$ and $W_{2,5}$ string[7,11], and we only spend several minutes on the calculation of $W_{2,3}$ string in particular. So the Ansatz has been showed again and all the spinor realizations of $W_2,s$ and $W_N$ strings can be obtained theoretically by using our program.

This paper is organized as follows. First we give a review of the grading BRST method to construct spinor field realizations of the $W_2,s$ strings and $W_N$ strings. Then we introduce the thought of our program which can construct the BRST charge of $W_2,s$ strings. Subsequently, we mainly give out the spinor field BRST charges of $W_{2,6}$ string and $W_6$ string. And the new result of $W_{2,4}$ string is also included. Finally, a brief conclusion is given.

II Review of BRST Method about Spinor $W_{2,S}$ Strings and $W_N$ Strings

Following Ref. [11], the BRST charge for the spinor field realization of $W_{2,s}$ strings takes the form:

\[
Q_B = Q_0 + Q_1,
\]

\[
Q_0 = \oint dz \ c( T^{eff} + T_\psi + K T_{bc} + y T_{\beta \gamma} ),
\]

\[
Q_1 = \oint dz \ \gamma F(\psi, \beta, \gamma ),
\]

\[
T_\psi = -\frac{1}{2} \partial \bar{\psi} \psi,
\]

\[
T_{\beta \gamma} = s \beta \partial \gamma + (s - 1) \partial \beta \gamma,
\]

\[
T_{bc} = 2 \beta \partial c + \partial bc,
\]

\[
T^{eff} = -\frac{1}{2} \eta_{\mu \nu} \partial Y^\mu Y^\nu,
\]
where $K, y$ are pending constants and $Y^\mu$ is a multi-spinor. It is easy to verify that the condition $Q_0^2 = 0$ is satisfied for any 's', so the nilpotency condition $Q_B^2 = 0$ can be translated into $Q_1^2 = \{Q_0, Q_1\} = 0$. Using these conditions, the precise form of $F(\psi, \beta, \gamma)$ and exact $y$ can be determined.

We can assume the BRST charge of a general $W_N$ string is also graded [10], and it can be given as follows:

$$Q_B = Q_0 + Q_1 + \cdots + Q_{N-2} = Q_0 + \sum_{i=1}^{N-2} Q_i.$$  \hfill (8)

Then the nilpotency condition $Q_B^2 = 0$ translates into

$$Q_0^2 = Q_i^2 = \{Q_0, Q_i\} = 0,$$

$$\{Q_i, Q_j\}_{i<j} = 0.$$  \hfill (10)

Observing these conditions, we can consider the equations $Q_0^2 = Q_i^2 = \{Q_0, Q_i\} = 0$ correspond to the case of $W_{2,(i+2)}$ strings for each 'i', so $Q_i$ should take the case of $W_{2,(i+2)}$ strings directly. And the correct solution can be selected out by using the equations $\{Q_i, Q_j\}_{i<j} = 0$.

The BRST charge of a spinor $W_N$ string can be written as follows:

$$Q_B = Q_0 + Q_1 + \cdots + Q_{N-2},$$  \hfill (11)

$$Q_0 = \oint dz \, c(T_{eff} + T_\psi + KT_{bc} + \sum_{i=1}^{N-2} y_i T_{\beta_i \gamma_i}),$$  \hfill (12)

$$Q_i = \oint dz \, \gamma_i F_i(\psi, \beta_i, \gamma_i),$$  \hfill (13)

$$T_\psi = -\frac{1}{2} \partial \psi \psi,$$

$$T_{bc} = 2b \partial c + \partial bc,$$

$$T_{\beta_i \gamma_i} = (i + 2)\beta_i \partial \gamma_i + (i + 1) \partial \beta_i \gamma_i,$$

$$T_{eff} = \frac{1}{2} \eta_{\mu \nu} \partial Y^\mu Y^\nu,$$  \hfill (17)

where the ghost fields $b, c, \beta_i, \gamma_i$ are all bosonic and communicating whilst the spinor field $\psi$ has spin $\frac{1}{2}$ and is anti-communicating. They satisfy OPEs:

$$b(z)c(\omega) \sim \frac{1}{z - \omega}, \quad \beta_i(z)\gamma_i(\omega) \sim \frac{1}{z - \omega}, \quad \psi(z)\psi(\omega) \sim -\frac{1}{z - \omega},$$  \hfill (18)

in the other case the OPEs vanish.

III Thought of the Corresponding Program

According to above theory, we should obtain the spinor field realization of $W_{2,(i+2)}$ strings firstly. So producing a general program for $W_{2,s}$ strings becomes very important. Constructing the form of $F(\psi, \beta, \gamma)$ and getting the value of 'y' are our main work. The thought of our program for determining $F(\psi, \beta, \gamma)$ and 'y' can be described as follows. Firstly, write out all
the possible terms of $F$ with $\psi, \beta, \gamma$ considering the spin of each term ‘s’ and ghost number zero. Then leave out all the total differential terms in $\gamma F(\psi, \beta, \gamma)$ since their contribution to $Q_1$ is zero. By using the nilpotency conditions $Q_2 = \{Q_0, Q_1\} = 0$, we can obtain corresponding equations subsequently. Finally, the value of ‘y’ and all the coefficients in $F$ would be obtained by solving these equations. Thus the form of BRST charge is constructed.

Here it is not necessary to list the program, if someone need it, we would like to offer it at any time.

IV Exact Spinor Field Constructions of the $W_{2,6}$ String

Apply the grading BRST method and our program, we obtained the results of the $W_{2,6}$ string as follows.

The general form of $F$ is:

$$F_6(\psi, \beta, \gamma) = f_6[1] \beta^6 \gamma^6 + f_6[2] \beta^5 \partial \gamma^4 + f_6[3] \beta^4 (\partial \gamma)^2 \gamma^2 + f_6[4] \beta^3 (\partial \gamma)^3 + f_6[5] \beta^4 \gamma^4 \partial \psi \psi + f_6[6] \beta^3 \partial \gamma^2 \partial \psi \psi + f_6[7] \beta^2 (\partial \gamma)^2 \partial \psi \psi + f_6[8] \beta^3 \gamma^3 \partial^2 \beta + f_6[9] \beta^2 \gamma^2 \partial^2 \beta \partial \gamma$$

$$+ f_6[10] \beta^2 \gamma \partial^2 \beta \partial \psi \psi + f_6[11] (\partial^2 \beta)^2 \gamma^2 + f_6[12] \beta^2 \gamma \partial^2 \beta \gamma + f_6[13] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[14] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[15] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[16] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[17] \beta \gamma \partial \gamma \partial^2 \beta \gamma$$

$$+ f_6[18] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[19] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[20] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[21] \beta \gamma \partial \gamma \partial^2 \beta \gamma$$

$$+ f_6[22] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[23] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[24] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[25] \beta \gamma \partial \gamma \partial^2 \beta \gamma$$

$$+ f_6[26] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[27] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[28] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[29] \beta \gamma \partial \gamma \partial^2 \beta \gamma$$

$$+ f_6[30] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[31] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[32] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[33] \beta \gamma \partial \gamma \partial^2 \beta \gamma$$

$$+ f_6[34] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[35] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[36] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[37] \beta \gamma \partial \gamma \partial^2 \beta \gamma$$

$$+ f_6[38] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[39] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[40] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[41] \beta \gamma \partial \gamma \partial^2 \beta \gamma$$

$$+ f_6[42] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[43] \beta \gamma \partial \gamma \partial^2 \beta \gamma + f_6[44] \beta \gamma \partial \gamma \partial^2 \beta \gamma.$$

Three constructions have been worked out as follows:

(1) $y = 0$ and

$$f_6[5] = f_6[6] = f_6[7] = f_6[10] = f_6[15] = f_6[19] = f_6[20] = f_6[21] = f_6[33] = f_6[36]$$

$$= f_6[41] = f_6[44] = 0, \quad f_6[16] = f_6[22] = f_6[24] = f_6[34] = C, \quad f_6[23] = -\frac{1}{2}C,$$

$$f_6[27] = -\frac{1}{6}C, \quad f_6[31] = f_6[35] = \frac{2}{3}C, \quad f_6[40] = \frac{1}{3}C,$$

where $C$ and other coefficients are arbitrary constants but do not vanish at the same time.

(2) $y = 1$

The forms of $f_6[i]$ ($i = 1, 2, ..., 44$) are complex comparatively. We do not list this result here.

(3) $y$ is an arbitrary constant and

$$f_6[5] = f_6[6] = f_6[7] = f_6[10] = f_6[15] = f_6[19] = f_6[20] = f_6[21] = f_6[33] = f_6[36]$$

$$= f_6[41] = f_6[44] = 0, \quad f_6[1] = 16(-302879179L_2 + 102793785L_3)/281288882105,$$

$$f_6[2] = \frac{2341}{5}f_6[1], \quad f_6[3] = 2(-379632019L_2 + 102044487L_3)/66420043,$$

$$f_6[4] = 242(-5815338L_2 + 10140597L_3)/332100215,$$

$$f_6[8] = 2(2065520459L_2 - 5774235L_3)/3653102365,$$

$$f_6[9] = 9(168697384L_2 - 384949 L_3)/66420043,$$

$$f_6[11] = (476150536L_2 + 406049487L_3 - 531360344L_5)/265680172,$$

$$f_6[12] = 3(-730884597L_2 + 350022565L_3)/1461240946,$$

$$f_6[13] = -302879179L_2 + 102793785L_3)/281288882105,$$

$$f_6[14] = 9(168697384L_2 - 384949 L_3)/66420043.$$
Thus these solutions, the values of \( \beta \) at string and give out the new constructions of
\[(2)\]
where other coefficients are arbitrary constants but do not vanish at the same time.

\[(1)\]
\[V.1\] The solutions of \( W_{2,3} \) string
\[F_{3} = f_{3}[1] \beta^{3} \gamma^{3} + f_{3}[2] \beta \gamma^{2} \partial \beta + f_{3}[3] \partial \beta \partial \gamma + f_{3}[4] \beta \gamma \psi \partial \psi + f_{3}[5] \beta \partial^{2} \gamma + f_{3}[6] \psi \partial^{2} \psi.\]

(1) \( y = 0 \) and
\[f_{3}[4] = f_{3}[6] = 0,\]
where other coefficients are arbitrary constants but do not vanish at the same time.

(2) \( y = 1 \) and
\[f_{3}[1] = \frac{1}{160}(-7N_{1} + 3N_{2}), \quad f_{3}[2] = \frac{1}{16}(7N_{1} - 3N_{2}), \quad f_{3}[3] = N_{1},\]
\[f_{3}[4] = -\frac{1}{2}(11N_{1} - 39N_{2}), \quad f_{3}[5] = N_{2}, \quad f_{3}[6] = 11N_{1} - 39N_{2},\]
where \( N_{1} \) and \( N_{2} \) are arbitrary constants but do not vanish at the same time.

(3) \( y \) is an arbitrary constant and
\[f_{3}[4] = f_{3}[6] = 0, \quad f_{3}[1] = -8P, \quad f_{3}[2] = 80P, \quad f_{3}[3] = 195P, \quad f_{3}[5] = P,\]
where \( P \) is an arbitrary nonzero constant.

\[V.2\] The new solutions of \( W_{2,4} \) string
\[F_{4} = f_{4}[1] \beta^{2} \gamma^{4} + f_{4}[2] (\partial \beta) \gamma^{2} \partial \gamma + f_{4}[3] \beta^{2} \gamma^{2} \partial \gamma + f_{4}[4] \beta \gamma \psi \partial \psi + f_{4}[5] \beta^{2} \gamma \psi \partial \psi \]
\[+ f_{4}[6] \partial \beta \gamma \psi \partial \psi + f_{4}[7] \beta \partial \gamma \psi \partial \psi + f_{4}[8] \beta \partial^{2} \gamma \partial \gamma + f_{4}[9] \partial^{2} \beta \partial \gamma + f_{4}[10] \partial \beta \partial^{2} \gamma.\]
\[ + f 4[11] \partial \phi \partial^2 \psi + f 4[12] \beta \partial^3 \gamma + f 4[13] \psi \partial^3 \psi. \]

(1) \( y = 0 \) and
\[ f 4[5] = f 4[6] = f 4[7] = f 4[11] = f 4[13] = 0, \]
where other coefficients are arbitrary constants but do not vanish at the same time.

(2) \( y = 1 \) and
\[ f 4[1] = \frac{1}{43869}, (-58Z_1 - 114Z_2 + 18207Z_3), \quad f 4[2] = \frac{1}{18317}(-116Z_1 - 63Z_2 + 15426Z_3), \]
\[ f 4[3] = \frac{79}{3} f 4[1], \quad f 4[4] = \frac{1}{232}(-5Z_2 + 114Z_3), \quad f 4[5] = \frac{1}{9}(-4Z_1 + 3Z_2), \]
\[ f 4[6] = Z_1, \quad f 4[7] = Z_2, \quad f 4[8] = Z_3, \quad f 4[9] = \frac{1}{96}(-7Z_2 + 1134Z_3 + 232Z_4), \]
\[ f 4[10] = Z_4, \quad f 4[11] = \frac{1}{7}(33Z_1 + Z_2), \quad f 4[12] = \frac{53}{1592}(Z_2 - 798Z_3 + 928Z_4), \]
\[ f 4[13] = \frac{1}{111}(-11Z_1 + 16Z_2), \]
where \( Z_1, Z_2, Z_3 \) and \( Z_4 \) are arbitrary constants but do not vanish at the same time.

(3) \( y \) is an arbitrary constant and
\[ f 4[5] = f 4[6] = f 4[7] = f 4[11] = f 4[13] = 0, \quad f 4[1] = \frac{867}{22330} V_1, \quad f 4[2] = \frac{2571}{2552} V_1, \]
\[ f 4[3] = \frac{289}{315} V_1, \quad f 4[4] = \frac{57}{110} V_1, \quad f 4[8] = V_1, \quad f 4[9] = \frac{189}{110} V_1 + \frac{1}{5} V_2, \]
\[ f 4[10] = V_2, \quad f 4[12] = -\frac{133}{202} V_1 + \frac{1}{3} V_2, \]
where \( V_1 \) and \( V_2 \) are arbitrary constants but do not vanish at the same time.

Comparing these new results with that in Ref.[10], we found they are more general.

**V.3 Exact spinor field constructions of \( W_6 \) string**

In this case the \( BRST \) charge can be written as \( Q_B = Q_0 + Q_1 + Q_2 + Q_3 + Q_4 \), where \( Q_i (i = 1, 2, 3, 4) \) must be one of the spinor field realization of \( W_2, s = (s = 3, 4, 5, 6) \) string respectively. The selection rule equation(10) namely \( \{ Q_1, Q_2 \} = \{ Q_1, Q_3 \} = \{ Q_1, Q_4 \} = \{ Q_2, Q_3 \} = \{ Q_2, Q_4 \} = \{ Q_3, Q_4 \} = 0 \). For these selection rules, we list two tables in which the correct combinations are denoted as sign ‘√’ whilst the incorrect combinations are denoted as sign ‘×’. Then the exact constructions of the spinor field grading \( BRST \) charge for the \( W_6 \) string are obtained. Especially we give out two simple correct combinations here.

**V.3.1 The simplest combination of \( Q_0, Q_1^{(1)}, Q_2^{(1)}, Q_3^{(1)} \) and \( Q_4^{(1)} \)**
\[
Q_B = \int d z \left[ c \left( -\frac{1}{2} \eta_{\mu\nu} \partial Y^{\mu} Y^{\nu} - \frac{1}{2} \partial \psi \partial \phi + 2Kb \partial c + K \partial bc \right) + f_1 \beta_1^3 \gamma_1^3 + f_2 \beta_2^4 \gamma_2^4 + f_3 \beta_3^5 \gamma_3^5 + f_4 \beta_4^6 \gamma_4^6 \right]
\]
where \( f_1 (i = 1, 2, 3, 4) \) are arbitrary nonzero constants.
This is one of the simplest formulae of \( Q_B \).

**V.3.2 Another simple combination of \( Q_0, Q_1^{(3)}, Q_2^{(3)}, Q_3^{(3)} \) and \( Q_4^{(3)} \)**
\[
Q_B = \int d z \left[ c \left( -\frac{1}{2} \eta_{\mu\nu} \partial Y^{\mu} Y^{\nu} - \frac{1}{2} \partial \psi \partial \phi + 2Kb \partial c + K \partial bc + 3\beta_1 \partial \gamma_1 + 2\partial \beta_1 \gamma_1 \right.ight.
\]
\[ + 4\beta_2 \partial \gamma_2 + 3\partial \beta_2 \gamma_2 + 5\beta_3 \partial \gamma_3 + 4\partial \beta_3 \gamma_3 + 6\beta_4 \partial \gamma_4 + 5\partial \beta_4 \gamma_4 \n\]
\[ + g_1 (-8\beta_1^3 \gamma_1^3 + 80\beta_1^2 \gamma_1^2 \partial \beta_1 + 195\beta_3 \beta_1 \partial \gamma_1 + g_1 \partial \beta_1 \partial \gamma_1) \n\]
\[ + g_2 (2\partial \beta_2 \partial \gamma_2 + 3\partial \beta_2 \partial \gamma_2 + 2\partial \beta_2 \partial \gamma_2) \n\]
\[ + g_3 (3\partial \beta_3 \partial \gamma_3 + \partial \gamma_3 \partial \beta_3 + 2\partial \beta_3 \partial \gamma_3 \gamma_3) \n\]
\[ + g_4 (6 \partial^3 \beta_4 \partial^2 \gamma_4 + 4 \partial^2 \beta_4 \partial^3 \gamma_4 - \partial^5 \beta_4 \gamma_4) \]

where \( g_i (i = 1, 2, 3, 4) \) are any arbitrary nonzero constants.

\textbf{V.3.3 The complete combinations of} \( Q_i (i = 1, 2, 3, 4) \)

The exact combinations could be found in Table 1 and Table 2.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
\textbf{combinations} & \textbf{\( Q_2^{(1)} \)} & \textbf{\( Q_2^{(2)} \)} & \textbf{\( Q_2^{(3)} \)} & \textbf{\( Q_3^{(1)} \)} & \textbf{\( Q_3^{(2)} \)} & \textbf{\( Q_3^{(3)} \)} & \textbf{\( Q_4^{(1)} \)} & \textbf{\( Q_4^{(2)} \)} & \textbf{\( Q_4^{(3)} \)} \\
\hline
\( Q_1^{(1)} \) & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\
\( Q_1^{(2)} \) & \checkmark & \times 1 & \checkmark & \checkmark & \times 2 & \times * & \times 4 & \times * & \checkmark \\
\( Q_1^{(3)} \) & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\
\hline
\( Q_2^{(1)} \) & \_ & \_ & \_ & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\
\( Q_2^{(2)} \) & \_ & \_ & \_ & \checkmark & \times 3 & \times * & \times 5 & \times * & \checkmark \\
\( Q_2^{(3)} \) & \_ & \_ & \_ & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\
\hline
\( Q_3^{(1)} \) & \_ & \_ & \_ & \_ & \_ & \checkmark & \checkmark & \checkmark & \checkmark \\
\( Q_3^{(2)} \) & \_ & \_ & \_ & \_ & \_ & \times * & \times 6 & \times * & \checkmark \\
\( Q_3^{(3)} \) & \_ & \_ & \_ & \_ & \_ & \checkmark & \checkmark & \checkmark & \checkmark \\
\hline
\end{tabular}
\caption{The precise combinations of \( Q_1, Q_2, Q_3 \) and \( Q_4 \)}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
\textbf{combinations} & \textbf{\( Q_1^{(1)} \)} & \textbf{\( Q_1^{(2)} \)} & \textbf{\( Q_1^{(3)} \)} \\
\hline
\( Q_1^{(1)} \) & \checkmark & \checkmark & \checkmark \\
\( Q_1^{(2)} \) & \checkmark & \times & \checkmark \\
\( Q_1^{(3)} \) & \checkmark & \checkmark & \checkmark \\
\hline
\end{tabular}
\caption{The special combinations of \( Q_i \) and \( Q_j \) when \( f6[31] = 0 \)}
\end{table}

1 If we take \( f4[6] = f4[7] = 0 \) (we name it condition 1), this combination will be correct.
2 If we take \( f5[8] = f5[9] = f5[14] = 0 \) (we name it condition 2), this combination will be correct.
3 If we take condition 1 or 2, this combination will be correct.
4 If we take \( f6[6] = f6[9] = f6[10] = f6[15] = f6[20] = f6[25] = f6[29] = f6[31] = 0 \) (we name it condition 3), this combination will be correct.
5 If we take condition 1 or 3, this combination will be correct.
6 If we take condition 2 or 3, this combination will be correct.

* If we take \( f6[31] = 0 \), all the '\( \times * \)' in Table 1 will become '\( \checkmark \)'. Then each part of Table 1 changes to Table 2.

\textbf{VI Conclusion}

In conclusion, the spinor field grading \textit{BRST} charges of \( W_{2,6} \) string and \( W_6 \) string have
been constructed. And a special program was obtained to construct spinor field $BRST$ charges of a general $W_{2,s}$ strings by using the $BRST$ method. With this procedure, the results are more accurate, and the process is accelerated rapidly. We have checked the solutions of $W_{2,3}$ string as well as $W_{2,5}$ string and given out a more general result of $W_{2,4}$ string in spinor field. Observing these solutions, we found they are standard. Of course the more realizations of $W_{2,s}$ strings can be calculated by using our program. Using these results, we will discuss their physical states in our next work.

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