MSSM Higgs with dimension-six operators.

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Abstract

We investigate an extension of the MSSM Higgs sector by including the effects of all dimension-five and dimension-six effective operators and their associated supersymmetry breaking terms. The corrections to the masses of the neutral CP-even and CP-odd Higgs bosons due to the $d = 5$ and $d = 6$ operators are computed. When the $d = 5$ and $d = 6$ operators are generated by the same physics (i.e. when suppressed by powers of the same scale $M$), due to the relative tan $\beta$ enhancement of the latter, which compensates their extra scale suppression $(1/M)$, the mass corrections from $d = 6$ operators can be comparable to those of $d = 5$ operators, even for conservative values of the scale $M$.

We identify the effective operators with the largest individual corrections to the lightest Higgs mass and discuss whether at the microscopic level and in the simplest cases, these operators are generated by “new physics” with a sign consistent with an increase of $m_h$. Simple numerical estimates easily allow an increase of $m_h$ due to $d = 6$ operators alone in the region of $10^{-30}$ GeV, while for a much larger increase light new states beyond MSSM may be needed, in which case the effective description is unreliable. Special attention is paid to the treatment of the effective operators with higher derivatives. These can be removed by non-linear field redefinitions or by an “unfolding” technique, which effectively ensure that any ghost degrees of freedom (of mass $\lesssim M$) are integrated out and absent in the effective theory at scales much smaller than $M$. Considering general coefficients of the susy operators with a scale of new physics above the LHC reach, it is possible to increase the tree-level prediction for the Higgs mass to the LEPII bound, thus alleviating the MSSM fine-tuning.

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1 Introduction

The coming LHC experiments are a great opportunity to test directly the old idea of low-energy supersymmetry, as a possibility of new physics beyond the Standard Model. In the minimal supersymmetric version of this model (MSSM) one obtains definite predictions in particular for the Higgs sector. For an agreement with the LEPII constraints for the mass of the SM-like Higgs of $m_h > 114.4$ GeV [1], the MSSM requires that quantum corrections lift its tree-level bound $m_h \leq m_Z$. This is indeed possible and acceptable within the allowed parameter space, increasing the Higgs mass above the LEPII bound (for a recent MSSM fit see [2]). However, larger quantum corrections usually require larger soft terms, making it more difficult to satisfy the electroweak constraint $v^2 = -m_{soft}^2/\lambda$ with $\lambda$ the MSSM effective quartic coupling and $m_{soft}$ a linear combination of soft (masses)$^2$. With $\lambda$ fixed by the gauge sector and with $m_{soft} \sim TeV$ this condition is more difficult to respect, given the negative searches for supersymmetry so far and the mass bounds for sparticles. As a result, the MSSM appears fine-tuned [3, 4, 5, 6] although there is no universally agreed fine-tuning measure or
exact value. For further discussion in this direction see [7]. This situation could even be seen as undermining the original motivation for supersymmetry, prompting alternatives such as [8].

If one maintains the idea of TeV-scale supersymmetry, such a problem of the MSSM must be addressed. The most common idea to solve it is to assume that new physics beyond the MSSM is present somewhere in the region of a few TeV. To investigate this possibility, a general, model-independent approach can be considered, by parametrising this new physics using effective operators. This is possible by organising such operators in inverse powers of the scale $M$ of new physics which, when integrated out, generates these effective operators. One can later address the question of what “new physics” may generate these operators. In [9] operators of dimension $d = 5$ were considered in the Higgs sector, together with their microscopic origin and implications for the Higgs mass ($m_h$). Further analysis including all baryon and lepton number conserving $d = 5$ operators beyond MSSM was done in [10, 11], showing how generalised, spurion dependent field redefinitions reduce the number of effective operators to an irreducible, minimal set. As a result, the number of independent parameters is reduced with the benefit of improving the predictive power of the method. Further analysis of the MSSM with $(d = 5)$ effective operators studied the stability of the Higgs potential with these operators [12], the effects on the neutralino sector [13], baryogenesis [14], CP violation [15] or fine-tuning [16]. The presence of these operators of $d = 5$ can increase the effective quartic coupling $\lambda$ of the Higgs field and as a result the fine tuning [18] for the MSSM electroweak scale is reduced [16] (see also [17]). One obtains one-loop values $114 \leq m_h \leq 130$ GeV with a very acceptable fine-tuning $\Delta \leq 10$ at one-loop, for $M \sim 8$ to 10 TeV [16].

The purpose of this work is to extend these studies by considering, in a systematic way, operators of dimension $d = 6$ that can account for new physics beyond the MSSM Higgs sector. The motivation is that such operators can bring relevant contributions to $m_h$, even in the absence of $d = 5$ operators. This is indeed possible, since not all $d = 6$ operators are necessarily generated by the same new physics as the $d = 5$ ones. Even if both $d = 5$ and $d = 6$ operators are present, they could also be suppressed by a different high scale, if generated by different new physics; this possibility can, in principle, also be read from our results by keeping track of their coefficients. Finally, if all or some of the $d = 5, 6$ operators are generated by same new physics, while suppressed by an extra $1/M$ factor relative to the leading $d = 5$ operators, the $d = 6$ operators can nevertheless have an impact for the large $\tan \beta$ region of the parameter space. Indeed, some $d = 6$ operators acquire an enhancement factor $(\tan \beta)$ relative to the $d = 5$ operators, which compensates for their extra scale suppression. As a result, their effects on the Higgs mass can be comparable to those of $d = 5$ operators and
it is interesting to examine, in this setup, the new corrections to $m_h$ at classical level. The study is also relevant for examining the limits of the approximation of expanding in powers of $1/M$ by comparing leading and sub-leading terms of this expansion. Such corrections to $m_h$ can be as large as loop corrections to $m_h$. We identify individual $d = 6$ operators with the largest correction to $m_h$, and discuss their possible microscopic origin and the signs they are generated with in the simplest cases. Although we do not provide detailed examples of high-energy physics that give the desired signs, considering general coefficients of the susy operators of $d = 6$, our results show that one can increase the tree-level prediction for the Higgs mass to the LEPII bound (alleviating the fine-tuning of the MSSM as noticed earlier for $d = 5$ operators [9, 16]), even for a scale of new physics above the LHC reach.

While studying higher dimensional operators, one problem is associated with the presence in some of these of higher derivatives, i.e. the presence of ghosts degrees of freedom in the spectrum. In some cases one can use the equations of motion to set these operators “on-shell” [19, 20, 21], and remove the extra derivatives. We investigate this procedure and show that this ultimately means integrating out the ghost degrees of freedom. While this “on-shell” method is correct in the leading order (in $1/M$), it is not true beyond it. Appendix B provides detailed examples which investigate these issues, and supports this statement (see in particular Appendix B.2). A more general and correct procedure is to use instead non-linear field redefinitions to remove the derivative operators. A third and more interesting method is to re-write (“unfold”) the original theory with higher derivatives as a second-order theory (i.e. with at most two derivatives) with additional (ghost) superfields of mass of order $M$ [22]. After integrating classically these fields one obtains in the low energy action below $M$, an effective theory without higher derivatives and with (classically) renormalised interactions. The results obtained are identical to those obtained by using the non-linear field redefinitions mentioned earlier; in the leading order in $1/M$ the method of setting “on-shell” the operators with extra derivatives by equations of motion gives similar results.

The aforementioned presence of the ghost degrees of freedom near the scale $M$ simply warns us that beyond this scale the theory is unstable and UV incomplete. This is a generic situation in all effective theories, even in those obtained from renormalisable ones by integrating out a massive state and after truncating the effective action to a given order (in $1/M$). These problems are also present in our discussion with $d = 6$ operators. In the end, one eliminates the $d=5$ operators with extra derivatives via field redefinitions, to leave only polynomial (in superfields) $d=5$ operators, and $d=6$ operators in which it is possible to use the equations of motion (with a similar result with integrating out the ghost degrees of free-
In the presence of supersymmetry breaking additional effects are present, like: $\mu$-term renormalisation by susy-breaking terms, soft terms renormalisation, discussed in detail in [10].

The plan of the paper is as follows: Section 2 presents the list of operators and clarifies which of them have relevant contributions to the scalar potential. In Section 3 the scalar potential of the Higgs in MSSM with $d=5,6$ operators is computed. Section 4 shows the results for the masses of the CP even/odd higgses. We identify the operators with the largest contribution in Section 5. The Appendix provides technical details and shows how to replace higher derivative operators by non-derivative ones in an effective action with $d=5,6$ operators.

2 MSSM Higgs sector with $d=5$ and $d=6$ operators

The relevant part of the Lagrangian of our model contains a piece $L_0$ of the MSSM higgs sector, together with that due to relevant $d=5$ and $d=6$ operators. For $L_0$ we have

$$L_0 = \int d^4\theta \sum_{i=1,2} Z_i(S, S^\dagger) H_i^\dagger e^{V_i} H_i + \left\{ \int d^2\theta \mu_0 \left( 1 + B_0 m_0 \theta^2 \right) H_1.H_2 + h.c. \right\}$$

(1)

in a standard notation. Here $Z_i(S, S^\dagger) = 1 - c_i m_0^2 \theta^2 \theta^\dagger \theta$ with $i = 1, 2$ and $c_i = \mathcal{O}(1)$, $m_0$ is the supersymmetry breaking scale in the visible sector, $m_0 = \langle F_{\text{hidden}} \rangle / M_{\text{Planck}}$ with $F_{\text{hidden}}$ an auxiliary field in the hidden sector. As usual we assume this breaking is transmitted to the visible sector through gravitational interactions mediated by $M_{\text{Planck}}$.

We extend this Lagrangian by $d=5$ and $d=6$ operators. In the first class we have

$$L_1 = \frac{1}{M} \int d^2\theta \zeta(S) (H_2.H_1)^2 + h.c. = 2\zeta_{10} (h_2.h_1)(h_2.F_1 + F_2.h_1) + \zeta_{11} m_0 (h_2.h_1)^2 + h.c.$$  

$$L_2 = \frac{1}{M} \int d^4\theta \left\{ A(S, S^\dagger) D^\alpha \left[ B(S, S^\dagger) H_2 e^{-V_1} \right] D_\alpha \left[ C(S, S^\dagger) e^{V_1} H_1 \right] + h.c. \right\}$$

(2)

where

$$\frac{1}{M} \zeta(S) = \zeta_{10} + \zeta_{11} m_0 \theta^2, \quad \zeta_{10}, \zeta_{11} \sim 1/M,$$

(3)

with $S$ the spurion superfield, $S = \theta^2 m_0$. We assume that

$$m_0 \ll M$$

(4)

$^1$Other notations used: in [10] $\eta_2 = 2\zeta_{10}m_0^2$, $\eta_3 = -2m_0\zeta_{11}$; in [10] $\eta_2 \rightarrow \zeta_1$, $\eta_3 \rightarrow \zeta_2$; in [9] $\eta_2 \rightarrow 2\zeta_1$, $\eta_3 \rightarrow 2\zeta_2$. 

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so that the effective theory approach is reliable. If this condition is not respected and the “new physics” is represented by “light” states (like the MSSM states), then one should work in the model where these are not integrated out. \( A, B, C \) are general functions, which take into account supersymmetry breaking associated with these operators so, for example:

\[
A(S, S^\dagger) = a_0 + a_1 S + a_1^* S^\dagger + a_2 S S^\dagger, \quad \text{(similar for } B, C) \quad (5)
\]

They are general and account for effects of supersymmetry breaking in the presence of some massive states which when integrated out generate \( L_{1,2} \) with these susy breaking terms.

\( L_2 \) is eliminated by generalised, spurion-dependent field redefinitions as it was showed in detail in [10]. We assume this procedure was already implemented, therefore only \( L_1 \) is relevant for the discussion below. These redefinitions bring however a renormalisation of the usual MSSM soft terms and of the \( \mu \) term, and additional corrections of order \( 1/M^2 \). The latter are corrections to the \( d = 6 \) operators that are relevant for the Higgs sector, that we present shortly. Since we shall write down all \( d = 6 \) operators, these corrections are then ultimately accounted for by renormalisations (redefinitions) of the coefficients of the \( d = 6 \) terms. Since we take these coefficients arbitrary, without any restriction to generality we can assume these redefinitions are already implemented.

The list of \( d = 6 \) operators is [23]

\[
\begin{align*}
\mathcal{O}_j &= \frac{1}{M^2} \int d^4 \theta \ Z_j(S, S^\dagger) \left( H_j^\dagger e^{V_j} H_j \right)^2, \quad j = 1, 2, \\
\mathcal{O}_3 &= \frac{1}{M^2} \int d^4 \theta \ Z_3(S, S^\dagger) \left( H_1^\dagger V_1 H_1 \right) \left( H_2^\dagger V_2 H_2 \right), \\
\mathcal{O}_4 &= \frac{1}{M^2} \int d^4 \theta \ Z_4(S, S^\dagger) \left( H_2 H_1^\dagger \right), \\
\mathcal{O}_5 &= \frac{1}{M^2} \int d^4 \theta \ Z_5(S, S^\dagger) \left( H_1^\dagger V_1 H_1 \right) H_2 H_1 + h.c. \\
\mathcal{O}_6 &= \frac{1}{M^2} \int d^4 \theta \ Z_6(S, S^\dagger) \left( H_2^\dagger V_2 H_2 \right) H_2 H_1 + h.c. \\
\mathcal{O}_7 &= \frac{1}{M^2} \int d^2 \theta \ Z_7(S, 0) \frac{1}{16 g^2 \kappa} \text{Tr} W^\alpha W_\alpha \left( H_2 H_1 \right) + h.c. \\
\mathcal{O}_8 &= \frac{1}{M^2} \int d^4 \theta \left[ Z_8(S, S^\dagger) \left( H_2 H_1 \right)^2 + h.c. \right]
\end{align*}
\]

(6)

where \( W^\alpha = (-1/4) D^2 e^{-V} D^\alpha e^V \) is the chiral field strength of \( SU(2)_L \) or \( U(1)_Y \) vector superfields \( V_\alpha \) and \( V_Y \) respectively. Also \( V_{1,2} = V_a^\alpha (\sigma^\alpha/2) + (+1/2) V_Y \) with the upper (minus) sign for \( V_1 \). The expressions of these operators in component form, are given in Appendix A.
The remaining $d = 6$ operators are:

\[
\begin{align*}
\mathcal{O}_9 & = \frac{1}{M^2} \int d^4 \theta \, Z_9(S, S^\dagger) \ H_1^\dagger \nabla^2 e^{V_1} \nabla^2 H_1 \\
\mathcal{O}_{10} & = \frac{1}{M^2} \int d^4 \theta \, Z_{10}(S, S^\dagger) \ H_2^\dagger \nabla^2 e^{V_2} \nabla^2 H_2 \\
\mathcal{O}_{11} & = \frac{1}{M^2} \int d^4 \theta \, Z_{11}(S, S^\dagger) \ H_1^\dagger e^{V_1} \nabla^\alpha W^{(1)}_\alpha H_1 \\
\mathcal{O}_{12} & = \frac{1}{M^2} \int d^4 \theta \, Z_{12}(S, S^\dagger) \ H_2^\dagger e^{V_2} \nabla^\alpha W^{(2)}_\alpha H_2 \\
\mathcal{O}_{13} & = \frac{1}{M^2} \int d^4 \theta \, Z_{13}(S, S^\dagger) \ H_1^\dagger e^{V_1} W^{(1)}_\alpha \nabla^\alpha H_1 \\
\mathcal{O}_{14} & = \frac{1}{M^2} \int d^4 \theta \, Z_{14}(S, S^\dagger) \ H_2^\dagger e^{V_2} W^{(2)}_\alpha \nabla^\alpha H_2
\end{align*}
\]

Also $\nabla_\alpha H_i = e^{-V_i} D_\alpha e^{V_i} H_i$ and $W^\alpha_i$ is the field strength of $V_i$. To be even more general, in the above operators one should actually include spurion dependence under any $\nabla_\alpha$, of arbitrary coefficients to include supersymmetry breaking effects associated to them. Finally, the wavefunction coefficients introduced above have the structure

\[
\frac{1}{M^2} Z_i(S, S^\dagger) = \alpha_{i0} + \alpha_{i1} m_0 \theta \theta + \alpha_{i2} m_0^2 \theta \theta \theta, \quad \alpha_{ij} \sim 1/M^2.
\]

Regarding the origin of these operators: $\mathcal{O}_{1,2,3}$ can be generated in MSSM with an additional, massive $U(1)'$ gauge boson or $SU(2)$ triplets integrated out [9]. $\mathcal{O}_4$ can be generated by a massive gauge singlet or $SU(2)$ triplet, while $\mathcal{O}_{5,6}$ can be generated by a combination of $SU(2)$ doublets and massive gauge singlet. $\mathcal{O}_7$ is essentially a threshold correction to the gauge coupling, with a moduli field replaced by the Higgs. $\mathcal{O}_8$ exists only in non-susy case, but is generated when redefining away the $d = 5$ derivative operator [10], thus we keep it.

Let us consider for a moment the operators $\mathcal{O}_{9,...,14}$ in the exact supersymmetry case. Then, we can set “on-shell” some of these, by using the eqs of motion:\footnote{Superpotential convention: $\int d^2 \theta \mu_0 H_1 H_2 = \int d^2 \theta \mu_0 H_1^T (i \sigma_2) H_2 \equiv \int d^2 \theta \mu_0 e^{i \sigma_2} H_1^T H_2^\dagger$; $\epsilon^{12} = 1 = -\epsilon^{21}$.}

\[
\begin{align*}
-\frac{1}{4} \mathcal{D}^2 (H_2^\dagger e^{V_2}) + \mu_0 H_1^T (i \sigma_2) = 0, \\
\frac{1}{4} \mathcal{D}^2 (H_1^\dagger e^{V_1}) + \mu_0 H_2^T (i \sigma_2) = 0
\end{align*}
\]

With this we find that in the supersymmetric case:\footnote{Also using $(i \sigma_2) e^{-\Lambda} = e^{\Lambda^T} (i \sigma_2); \Lambda \equiv \Lambda^\tau T^\alpha; (i \sigma_2)^T = -(i \sigma_2); (i \sigma_2)^2 = -1_2$}

\[
\mathcal{O}_9 \sim \int d^4 \theta \ H_1^\dagger \nabla^2 e^{V_1} \nabla^2 H_1 = 16 \mu_0^2 \int d^4 \theta \ H_1^\dagger e^{V_1} H_1.
\]
and similar for $\mathcal{O}_{10}$. Regarding $\mathcal{O}_{11,12}$, in the supersymmetric case they vanish, following the definition of $\nabla^\alpha$ and an integration by parts. Further, $\mathcal{O}_{13,14}$ are similar to $\mathcal{O}_{9,10}$, which can be seen by using the definition of $W_\alpha^{(i)}$ and the relation between $\nabla^2$, $(\nabla')^2$ and $D^2$, $(D')^2$.

In conclusion, in the exact supersymmetric case, $\mathcal{O}_{9,...,14}$ give at most wavefunction renormalisations of operators already included. This was shown by using the equations of motion (“on-shell” method – we return to this issue shortly). Let us now consider supersymmetry breaking associated to these operators, due to their spurion dependence. Turning on supersymmetry breaking should not bring physical effects, as showed explicitly in [10] and could only give soft terms and $\mu$-term renormalisation by $\mathcal{O}(1/M^2)$ corrections. Since these terms are anyway renormalised by $\mathcal{O}_{1,...,8}$, where spurion dependence is included with arbitrary coefficients, then there is no loss of generality to ignore the supersymmetry breaking effects associated to $\mathcal{O}_{9,...,14}$ in the following discussion, which are anyway taken into account by $\mathcal{O}_{1,...,8}$. Following this discussion, one concludes that $\mathcal{O}_{9,...,14}$ are not relevant for the analysis of the Higgs potential performed below. Finally, there can be an additional operator of $d = 6$ from the gauge sector, $\mathcal{O}_{15} = (1/M^2) \int d^2\theta \, W^\alpha \Box W_\alpha$ which could affect the Higgs potential.\footnote{Its complete gauge invariant form is $\int d^4\theta \, Tr \, e^V e^{-V} D^2(e^V W_\alpha e^{-V})$.} Using the equations of motion for the gauge field it can be shown that $\mathcal{O}_{15}$ gives a renormalisation of $\mathcal{O}_{1,2,3}$, so its effects are ultimately included, since the coefficients $Z_{1,2,3}$ are arbitrary.

The careful reader may question the above use of the eqs of motion in some of the higher dimensional operators, in order to essentially remove those with more than two derivatives ($\mathcal{O}_{9,...,15}$). This “on-shell” procedure is justified by previous works [19, 20] and further detailed in [21]. A more general and correct approach is to use instead non-linear field redefinitions\footnote{These are actually employed to prove this “on-shell” method [21].} or an “unfolding” technique (see later). These two generally valid approaches are discussed in detail in Appendix B. We used these two approaches to check the validity of the above “on-shell” procedure, for the cases and approximation in which we applied it. This was also done to clarify, from a general perspective, what actually means to set “on-shell” the higher derivative operators.

To this purpose, consider for simplicity the case of operator $\mathcal{O}_9$ without gauge fields, when $\mathcal{O}_9 \sim (1/M^2) \int d^4\theta \, H_1^\dagger \Box H_1$. A Lagrangian with such a higher derivative operator contains additional poles corresponding to ghosts degrees of freedom. As shown in [22], see also Appendix [B.1] such theory can be reformulated and “unfolded” into a second order one (i.e with no more than two derivatives) with (one or two) additional ghost superfields of mass
of the order $M$. In such an effective theory, at energies well below the scale $M$, such ghost-like states can then be integrated out. The result is a wavefunction renormalisation only, which is in agreement with the result obtained by the “on-shell” method discussed above. Therefore, using the eqs of motion to set “on-shell” the higher derivative operator as done in (9), (10) corresponds to integrating out the massive ghost degrees of freedom associated with such operator. For details see Appendix B.1.

A result similar to the “unfolding” method is obtained by using non-linear field redefinitions. This was detailed in Appendix B.2 for the case of $d = 5$ operators. There it is shown that in the leading order in $1/M$ the “unfolding” method (integrating out the ghosts), the nonlinear field redefinition method and the “on-shell” method give similar results. Beyond this $1/M$ order however, the “on-shell” method should be appropriately modified to use the Euler-Lagrange equations for a higher derivative Lagrangian.

With these clarifications one can safely say that $O_{9,...,15}$ are not relevant for the following discussion of the Higgs potential. In conclusion the list of $d = 6$ operators that remain for our study of the Higgs sector beyond MSSM is that of eq.(6). Let us stress that not all the remaining operators $O_{1,...,8}$ of (6) are necessarily present or generated in a detailed model. Symmetries and details of the “new physics” beyond the MSSM that generated them, may forbid or favour the presence of some of them. Therefore, we regard these remaining operators as independent of each other, although in specific models correlations may exist among their coefficients $Z_i$. It is important to keep all these operators in the analysis, for the purpose of identifying which of them has the largest individual contribution to the Higgs mass, which is one of the main interests of this analysis. Finally, some of the $d = 6$ operators can in principle be present even in the absence of the $d = 5$ operators, if these classes of operators are generated by integrating different “new physics”. In specific models one simply sets to zero, in the results below, the coefficients of those operators of $d=5$ and/or $d=6$ not present/generated.

3 The scalar potential with $d=5$ and $d=6$ operators

Following the previous discussion, the overall Lagrangian of the model is

$$L_H = L_0 + L_1 + \sum_{i=1}^{8} O_i$$  \hspace{1cm} (11)

with the MSSM higgs Lagrangian $L_0$ of eq.(1), $L_1$ of eq.(2) and $O_{1,2,...,8}$ of eq.(6).
With the results in Appendix A we find the following contributions to the scalar potential:

\[
V_F = \frac{\partial^2 K}{\partial \rho_i \partial \rho_j^*} F_i F_j^* = |F_1|^2 + |F_2|^2 + \frac{\partial^2 K_6}{\partial \rho_i \partial \rho_j^*} F_i F_j^*
\]  

(12)

where \( K_6 \) is the contribution of \( \mathcal{O}(1/M^2) \) to the Kähler potential due to \( \mathcal{O}_{1,\ldots,8} \). The first two terms in the rhs give \( (h_i \text{ denote } SU(2)_L \text{ doublets, } |h_i|^2 \equiv h_i^+ h_i) \):

\[
V_{F,1} = |F_1|^2 + |F_2|^2 = |\mu_0 + 2\tilde{\xi}_{10} h_1 h_2|^2 \left( |h_1|^2 + |h_2|^2 \right) + \left[ \mu_0 \left( |h_1|^2 \rho_{21} + |h_2|^2 \rho_{11} + (h_1 h_2)^\dagger (\rho_{22} + \rho_{12}) \right) + h.c. \right]
\]

(13)

obtained using (A-11) and where \( \rho_{ij} \) are functions of \( h_{1,2} \):

\[
\rho_{11} = -2(\alpha_{10} \mu_0 + \alpha_{20} \mu_0 + \alpha_{51}^* m_0) |h_1|^2 - (\alpha_{30} \mu_0 + \alpha_{40} \mu_0 + \alpha_{61}^* m_0) |h_2|^2 - (\alpha_{50}^* \mu_0 + \alpha_{50}^* \mu_0) (h_2 h_1)^* + \left[ (\alpha_{50} + 2\alpha_{50}) \mu_0 + 2\alpha_{51}^* m_0 \right] (h_1 h_2)
\]

\[
\rho_{12} = (2\alpha_{51}^* m_0 + \alpha_{50}^* \mu_0) |h_1|^2 + (\alpha_{31}^* m_0 + \alpha_{50} \mu_0) |h_2|^2 - \left[ (2\alpha_{10} + \alpha_{30}) \mu_0 + \alpha_{51}^* m_0 \right] (h_1 h_2) + \alpha_{51}^* m_0 (h_2 h_1)^*
\]

(14)

\[
\rho_{21} = -2(\alpha_{20} \mu_0 + \alpha_{40} \mu_0 + \alpha_{61}^* m_0) |h_2|^2 - (\alpha_{30} \mu_0 + \alpha_{40} \mu_0 + \alpha_{51}^* m_0) |h_1|^2 - (\alpha_{51}^* m_0 + \alpha_{50}^* \mu_0) (h_2 h_1)^* + \left[ (\alpha_{50} + 2\alpha_{50}) \mu_0 + 2\alpha_{51}^* m_0 \right] (h_1 h_2)
\]

\[
\rho_{22} = (2\alpha_{21}^* m_0 + \alpha_{60}^* \mu_0) |h_2|^2 + (\alpha_{31}^* m_0 + \alpha_{60} \mu_0) |h_1|^2 - \left[ (2\alpha_{20} + \alpha_{30}) \mu_0 + \alpha_{61}^* m_0 \right] (h_1 h_2) + \alpha_{61}^* m_0 (h_2 h_1)^*
\]

(15)

The non-trivial field-dependent Kähler metric gives for the last term in \( V_F \) of eq. (12):

\[
V_{F,2} = |\mu_0|^2 \left[ 2 \left( \alpha_{10} + \alpha_{20} + \alpha_{40} \right) |h_1|^4 |h_2|^2 + (\alpha_{30} + \alpha_{40}) (|h_1|^4 + |h_2|^4) + 2 (\alpha_{10} + \alpha_{20} + \alpha_{30}) |h_1 h_2|^2 + (|h_1|^2 + 2 |h_2|^2) (\alpha_{50} h_2 h_1 + h.c.) + (2 |h_1|^2 + |h_2|^2) (\alpha_{60} h_2 h_1 + h.c.) \right]
\]

(16)

so that \( V_F = V_{F,1} + V_{F,2} \). Further, for the gauge contribution, we have:
\[ V_{gauge} = \frac{1}{2}(D^2_w + D^2_W) \left[ 1 + (\alpha_{70} h_2.h_1 + h.c.) \right] \]
\[ = \frac{g_1^2 + g_2^2}{8} \left( |h_1|^2 - |h_2|^2 \right) \left( (1 + f_1(h_{1,2}) |h_1|^2 - (1 + f_2(h_{1,2}) |h_2|^2 \right) \]
\[ + \frac{g_3^2}{2} (1 + f_3(h_{1,2}))|h_1 h_2|^2 \]  
\text{(17)}

obtained with (A-12) and where \( f_{1,2,3} \) are functions of \( h_{1,2} \):
\[ f_1(h_{1,2}) \equiv 4 \alpha_{10} |h_1|^2 + \left[ (2\alpha_{50} - \alpha_{70}) h_2.h_1 + h.c. \right] \]
\[ f_2(h_{1,2}) \equiv 4 \alpha_{20} |h_2|^2 + \left[ (2\alpha_{60} - \alpha_{70}) h_2.h_1 + h.c. \right] \]
\[ f_3(h_{1,2}) \equiv \tilde{\rho}_1 + \tilde{\rho}_2 + (\alpha_{70} h_2.h_1 + h.c.) \]  
\text{(18)}

with
\[ \tilde{\rho}_1(h_{1,2}) \equiv 2\alpha_{10} |h_1|^2 + \alpha_{30} |h_2|^2 + \left[ (\alpha_{50} - \alpha_{70}) h_2.h_1 + h.c. \right] \]
\[ \tilde{\rho}_2(h_{1,2}) \equiv 2\alpha_{20} |h_2|^2 + \alpha_{30} |h_1|^2 + \left[ (\alpha_{60} - \alpha_{70}) h_2.h_1 + h.c. \right] \]  
\text{(19)}

The scalar potential also has corrections \( V_{SSB} \) from supersymmetry breaking, due to spurious dependences in higher dimensional operators (of dimensions \( d = 5 \) and \( d = 6 \)); in addition we also have the usual soft breaking term from the MSSM. As a result
\[ V_{SSB} = -m_0^2 \left[ \alpha_{12} |h_1|^4 + \alpha_{22} |h_2|^4 + \alpha_{32} |h_1|^2 |h_2|^2 + \alpha_{42} |h_2.h_1|^2 \right] \]
\[ + (\alpha_{32} |h_1|^2 (h_2.h_1) + h.c.) + (\alpha_{62} |h_2|^2 (h_2.h_1) + h.c.) \]
\[ - \left[ m_0^2 \alpha_{82} (h_1 h_2)^2 + \zeta_{11} m_0 (h_2.h_1)^2 + \mu_0 B_0 m_0 (h_1 h_2) + h.c. \right] + m_0^2 (c_1 |h_1|^2 + c_2 |h_2|^2) \]  
\text{(20)}

Finally, in \( O_{1,..8} \) there are non-standard kinetic terms that can contribute to \( V \) when the scalar singlet components (denoted \( h_i^0 \)) of \( h_i \) acquire a vev. The relevant terms are:
\[ \mathcal{L}_H \supset (\delta_{ij}^* + g_{ij}^*) \partial_\mu h_i^0 \partial^\mu h_j^{0*}, \quad i, j = 1, 2. \]  
\text{(21)}

where the field dependent metric is:
\[ g_{11}^* = 4 \alpha_{10} |h_1^0|^2 + (\alpha_{30} + \alpha_{40}) |h_2^0|^2 - 2 (\alpha_{50} h_1^0 h_2^0 + h.c.) \]
\[ g_{12}^* = (\alpha_{30} + \alpha_{40}) h_1^0 h_2^0 - \alpha_{50} h_1^{0*} - \alpha_{60} h_2^{0*}, \quad g_{21}^* = g_{12}^* \]
\[ g_{22}^* = 4 \alpha_{20} |h_2^0|^2 + (\alpha_{30} + \alpha_{40}) |h_1^0|^2 - 2 (\alpha_{60} h_1^0 h_2^0 + h.c.) \]  
\text{(22)}
For simplicity we only included the $SU(2)$ higgs singlets contribution, that we actually need in the following, but the discussion can be extended to the general case. The metric $g_{ij}$ is expanded about a background value $(h_i^0) = v_i/\sqrt{2}$, then field re-definitions are performed to obtain canonical kinetic terms; these bring further corrections to the scalar potential. The field re-definitions are:

\[
\begin{align*}
 h_1^0 & \rightarrow h_1^0 \left(1 - \frac{\tilde{g}_{11}^*}{2}\right) - \frac{\tilde{g}_{21}^*}{2} h_2^0, \\
 h_2^0 & \rightarrow h_2^0 \left(1 - \frac{\tilde{g}_{22}^*}{2}\right) - \frac{\tilde{g}_{12}^*}{2} h_1^0, \quad \tilde{g}_{ij}^* \equiv g_{ij}\big|_{h_i^0 \rightarrow v_i/\sqrt{2}}
\end{align*}
\]  

(23)

Since the metric has corrections which are $O(1/M^2)$, after (23) only the MSSM soft breaking terms and the MSSM quartic terms are affected. The other terms in the scalar potential, already suppressed by one or more powers of the scale $M$ are affected only beyond the approximation $O(1/M^2)$ considered here. Following (23) the correction terms $O(1/M^2)$ induced by the MSSM quartic terms and by soft breaking terms in $V_{SSB}$ are:

\[
V_{k.t.} = m_1^2 (-\tilde{g}_{11}^*) |h_1^0|^2 + m_2^2 (-\tilde{g}_{22}^*) |h_2^0|^2 - \frac{1}{2} (\tilde{m}_1^2 + \tilde{m}_2^2) (\tilde{g}_{21}^* h_1^0 h_2^0 + h.c.)
\]

\[
+ \frac{1}{2} \left[ B_0 m_0 \mu_0 \left( (\tilde{g}_{11}^* + \tilde{g}_{22}^*) h_1^0 h_2^0 + \tilde{g}_{12}^* h_1^0 h_2^0 + \tilde{g}_{21}^* h_2^0 h_2^0 + h.c. \right) \right] - \frac{g^2}{8} \left( |h_1^0|^2 - |h_2^0|^2 \right) (\tilde{g}_{11}^* |h_1^0|^2 - \tilde{g}_{22}^* |h_2^0|^2 + h.c.)
\]

(24)

Using eqs. (11), (12), (13), (16), (17), (21), (24), we find the full scalar potential. With the notation $\tilde{m}_i^2 \equiv c_i m_0^2 + |\mu_0|^2$, $i = 1, 2$ ($c_{1,2}$, were introduced in $Z_i$ of eq.(1)) one has:

\[
V = V_{F,1} + V_{F,2} + V_G + V_{SSB} + V_{k.t.}
\]

(25)

\[
= V_{k.t.} + m_1^2 |h_1|^2 + m_2^2 |h_2|^2 - \left[ \mu_0 B_0 m_0 h_1 \cdot h_2 + h.c. \right] + \frac{\lambda_1}{2} |h_1|^4 + \frac{\lambda_2}{2} |h_2|^4 + \lambda_3 |h_1|^2 |h_2|^2 + \lambda_4 |h_1 \cdot h_2|^2
\]

\[
+ \left( \frac{\lambda_5}{2} (h_1 \cdot h_2)^2 + \lambda_6 |h_1|^2 (h_1 \cdot h_2) + \lambda_7 |h_2|^2 (h_1 \cdot h_2) + h.c. \right) + \frac{g^2}{8} (|h_1|^2 - |h_2|^2) (f_1(\bar{h}_{1,2}) |h_1|^2 - f_2(\bar{h}_{1,2}) |h_2|^2) + 4 |\zeta_{10}|^2 |h_1.h_2|^2 (|h_1|^2 + |h_2|^2)
\]

\[
+ \frac{g^2}{2} f_3(h_{1,2}) |h_1.h_2|^2
\]

where $g^2 = g_1^2 + g_2^2$, and $f_{1,2,3}(h_{1,2})$ are all quadratic in $h_i$, see eq. (18). Except $V_{k.t.}$, all other fields are in the $SU(2)$ doublets notation. The following notation for $\lambda_i$ was introduced:
\[ \lambda_1/2 = \lambda_1^0/2 - |\mu_0|^2 (\alpha_{30} + \alpha_{40}) - m_0^2 \alpha_{12} - 2m_0 \text{Re} [\alpha_{51} \mu_0] \]  
\[ \lambda_2/2 = \lambda_2^0/2 - |\mu_0|^2 (\alpha_{10} + \alpha_{40}) - m_0^2 \alpha_{22} - 2m_0 \text{Re} [\alpha_{61} \mu_0] \]  
\[ \lambda_3 = \lambda_3^0/2 - 2 |\mu_0|^2 (\alpha_{10} + \alpha_{20} + \alpha_{40}) - m_0^2 \alpha_{32} - 2m_0 \text{Re} [(\alpha_{51} + \alpha_{61}) \mu_0] \]  
\[ \lambda_4 = \lambda_4^0/2 - 2 |\mu_0|^2 (\alpha_{10} + \alpha_{20} + \alpha_{30}) - m_0^2 \alpha_{42} - 2m_0 \text{Re} [(\alpha_{51} + \alpha_{61}) \mu_0] \]  
\[ \lambda_5/2 = -m_0 \mu_0 (\alpha_{51} + \alpha_{61}) - m_0 \zeta_{11} - m_0^2 \alpha_{82} \]  
\[ \lambda_6 = |\mu_0|^2 (\alpha_{50} + 2 \alpha_{60}) + m_0^2 \alpha_{52} + m_0 \mu_0 (2 \alpha_{11} + \alpha_{31} + \alpha_{41}) + 2m_0 \mu_0^* \alpha^*_{31} + 2 \zeta_{10} \mu_0^* \]  
\[ \lambda_7 = |\mu_0|^2 (\alpha_{60} + 2 \alpha_{50}) + m_0^2 \alpha_{62} + m_0 \mu_0 (2 \alpha_{21} + \alpha_{31} + \alpha_{41}) + 2m_0 \mu_0^* \alpha^*_{31} + 2 \zeta_{10} \mu_0^* \]

Eq. (26) shows the effects of various higher dimensional operators on the scalar potential. As a reminder, note that all \( \alpha_{ik} \sim O(1/M^2) \), while \( \zeta_{11}, \zeta_{10} \sim O(1/M) \). The latter can dominate, but this depends on the value of \( \tan \beta \); when this is large, \( O(1/M^2) \) have comparable size. In specific models correlations exist among these coefficients. The above remarks apply to the case when the \( d = 5 \) and \( d = 6 \) operators considered are generated by the same “new physics” beyond the MSSM (i.e. are suppressed by the same scale). However, as mentioned earlier, this may not always be the case; in various models contributions from some \( d = 6 \) operators can be independent of those from \( d = 5 \) operators (and present even in the absence of the latter), if generated by different “new physics”. A case by case analysis is then needed for a thorough analysis of all possible scenarios for “new physics” beyond the MSSM higgs sector.

We also used the following notation for the corresponding MSSM contribution:

\[ \lambda_i^0/2 = \frac{1}{8} (g_2^2 + g_1^2), \quad \lambda_{i2}^0/2 = \frac{1}{8} (g_2^2 + g_1^2), \quad \lambda_3^0 = \frac{1}{4} (g_2^2 - g_1^2), \quad \lambda_4^0 = -\frac{1}{2} g_2^2, \]

One can include MSSM loop corrections by replacing \( \lambda_i^0 \) with radiatively corrected values [24].

The overall sign of the \( h^0 \) terms depends on the relative size of \( \alpha_{j0}, j = 1, 2, 5, 6, 7 \), and cannot be fixed even locally, in the absence of the exact values of these coefficients of the \( d = 6 \) operators. Effective operators of \( d = 5 \), \( (\zeta_{10}) \), also contribute to the overall sign, however these alone cannot fix it. At large fields values higher and higher dimensional operators become relevant and contribute to it. We therefore do not impose that \( V \) be bounded from below at large fields. For a discussion of stability with \( d = 5 \) operators only see [12].

Eqs. (25), (26) of \( V \) in the presence of \( d = 5, 6 \) effective operators are the main result of this section. For simplicity, one can take \( \tilde{g}_{12}, \tilde{g}_{21} \), real (similar for \( B_0 \mu_0 \)), possible if for example \( \alpha_{50}, \alpha_{60} \) are real and no vev for \( Im h_1 \); in the next section we shall assume that this is the case.
4 Corrections to the MSSM Higgs masses: analytical results

With the general expression for the scalar potential we compute the mass spectrum. From
the scalar potential, one evaluates the mass of CP-even Higgs fields \( h, H \):

\[
m_{h,H}^2 \equiv \left. \frac{1}{2} \frac{\partial^2 V}{\partial h_i \partial h_j} \right|_{(h) = v_i/\sqrt{2}, (\text{Im } h) = 0}
\]  

(28)

In the leading order \( \mathcal{O}(1/M) \) one has (upper signs for \( m_h \)):

\[
m_{h,H}^2 = \frac{m_Z^2}{2} + \frac{B_0 m_0 \mu_0 (u^2 + 1)}{2u} \pm \frac{\sqrt{w}}{2} + v^2 \left[ (2 \zeta_{10} \mu_0) q_1^\pm + (-2 m_0 \zeta_{11}) q_2^\pm \right] + \delta m_{h,H}^2
\]

(29)

with

\[
q_1^\pm = \frac{1}{4} \frac{u}{u^2 (1 + u^2) \sqrt{w}} \times \left[ - (1 - 6 u^2 + u^4) u \sqrt{w} \mp \left( m_Z^2 u (1 - 14 u^2 + u^4) - B_0 m_0 \mu_0 (1 + u^2) (1 + 10 u^2 + u^4) \right) \right]
\]

\[
q_2^\pm = \mp \frac{2u}{(1 + u^2)^2 \sqrt{w}} \left[ - B_0 m_0 \mu_0 (1 + u^2) - m_Z^2 u \right]
\]

(30)

where

\[
w \equiv m_Z^4 + \left[ - B_0 m_0 \mu_0 (1 + u^2)^3 + 2 m_Z^2 u (1 - 6 u^2 + u^4) \right] \frac{(-B_0 m_0 \mu_0)}{u^2 (1 + u^2)}, \quad u \equiv \tan \beta
\]  

(31)

In eq. (29)

\[
\delta m_{h,H}^2 = \mathcal{O}(1/M^2)
\]

(32)

and we also used that \( m_Z = g v/2 \). One also shows that the Goldstone mode has \( m_G = 0 \) and
the pseudoscalar \( A \) has a mass:

\[
m_A^2 = \frac{1 + u^2}{u} B_0 m_0 \mu_0 - \frac{1 + u^2}{u} \zeta_{10} \mu_0 v^2 + 2 m_0 \zeta_{11} v^2 + \delta m_A^2, \quad \delta m_A^2 = \mathcal{O}(1/M^2)
\]  

(33)

The corrections \( \mathcal{O}(1/M) \) of \( m_{h,H} \) and \( m_A \) showed in (29), (33), agree with earlier findings [9].

Ignoring for the moment the corrections \( \mathcal{O}(1/M^2) \), one eliminates \( B_0 \) between (29) and
(33) to obtain:
\[
m_{h,H}^2 = \frac{1}{2} \left[ m_A^2 + m_Z^2 \mp \sqrt{\tilde{w}} \right] \\
+ (2 \zeta_{10} \mu_0) v^2 \sin 2\beta \left[ 1 \pm \frac{m_A^2 + m_Z^2}{\sqrt{\tilde{w}}} \right] \\
+ \frac{(-2 \zeta_{11} m_0) v^2}{2} \left[ 1 \mp \frac{(m_A^2 - m_Z^2) \cos^2 2\beta}{\sqrt{\tilde{w}}} \right] \\
+ \delta' m_{h,H}^2, \quad \delta' m_{h,H}^2 = \mathcal{O}(1/M^2) \tag{34}
\]

where the upper (lower) signs correspond to \( h \) (\( H \)) respectively and

\[
\tilde{w} \equiv (m_A^2 + m_Z^2)^2 - 4m_A^2 m_Z^2 \cos^2 2\beta \tag{35}
\]

in agreement with [9]. This is important if one considers \( m_A \) as an input; it is also needed if one considers the limit of large \( \tan \beta \) at fixed \( m_A \) (see later).

Regarding the \( \mathcal{O}(1/M^2) \) corrections of \( \delta m_{h,H}^2, \delta m_A^2 \) and \( \delta' m_{h,H}^2 \) of eqs. (29), (33), (34) in the general case of including all operators and their associated supersymmetry breaking, they have a rather complicated form. For most purposes, an expansion in \( 1/\tan \beta \) of \( \delta m_{h,H}^2, \delta m_A^2, \delta' m_{h,H}^2 \) is accurate enough. The reason for this is that it is only at large \( \tan \beta \) that \( d = 6 \) operators bring corrections comparable to those of \( d = 5 \) operators. The relative \( \tan \beta \) enhancement of \( \mathcal{O}(1/M^2) \) operators compensates for the extra suppression factor \( 1/M \) that these operators have relative to \( \mathcal{O}(1/M) \) operators (which involve both \( h_1 \) and \( h_2 \) and thus are not enhanced in this limit). Note however that in some models only \( d = 6 \) operators may be present, depending on the details of the “new physics” generating the effective operators.

If we neglect the susy breaking effects of \( d = 6 \) operators (i.e. \( \alpha_{j1} = \alpha_{j2} = 0, \alpha_{j0} \neq 0, j = 1, \ldots, 8 \)) and with \( d = 5 \) operators contribution, one has\footnote{In the case of including the supersymmetry breaking effects from effective operators, associated with coefficients \( \alpha_{j1}, \alpha_{j2}, j = 1,2,\ldots,8 \), the exact formula is very long and is not included here.} for the correction \( \delta m_{h,H}^2 \) in eq. (29) (upper signs correspond to \( \delta m_{h}^2 \))

\[
\delta m_{h,H}^2 = \sum_{j=1}^{7} \gamma_{j}^\pm \alpha_{j0} + \gamma_{x}^\pm \zeta_{10} \zeta_{11} + \gamma_{z}^\pm \zeta_{10}^2 + \gamma_{y}^\pm \zeta_{11}^2 \tag{36}
\]

The expressions of the coefficients \( \gamma^\pm \) are provided in Appendix C and can be used for numerical studies. While these expressions are exact, they are complicated and not very transparent. It is then instructive to analyse an approximation of the \( \mathcal{O}(1/M^2) \) correction as an expansion in \( 1/\tan \beta \). We present in this limit the correction \( \delta m_{h,H}^2 \) of eq. (29), which also includes all supersymmetry breaking effects associated with all \( d = 5, 6 \) operators, (i.e. \( \alpha_{j1} \neq 0, \alpha_{j2} \neq 0, \zeta_{11} \neq 0, j = 1, \ldots, 8 \)) in addition to the MSSM soft terms. This has a simple expression:
\[
\delta m_h^2 = -2 v^2 \left[ \alpha_{22} m_0^2 + 2 \alpha_{61} m_0 \mu_0 + (\alpha_{30} + \alpha_{40}) \mu_0^2 - \alpha_{20} m_Z^2 \right] \\
+ \frac{v^2}{\tan \beta} \left[ 4 \alpha_{62} m_0^2 + 4 \mu_0 m_0 (2 \alpha_{21} + \alpha_{31} + \alpha_{41} + 2 \alpha_{81}) + 4 \mu_0^2 (2 \alpha_{50} + \alpha_{60}) \\
- m_Z^2 (2 \alpha_{60} - 3 \alpha_{70}) - \frac{v^2}{(B_0 m_0 \mu_0)} (2 \zeta_{10} \mu_0)^2 \right] + O(1/\tan^2 \beta) \quad (37)
\]

which is obtained with \((B_0 m_0 \mu_0)\) kept fixed. The result is dominated by the first line, including both susy and non-susy terms from the effective operators. This correction can be comparable to linear terms in \(\zeta_{10}, \zeta_{11}\) from \(d = 5\) operators for \((2 \zeta_{10} \mu_0) \approx 1/\tan \beta\) (see later). Not all \(O_{1,2,8}\) are necessarily present, so in some models some \(\alpha_{ij}, \zeta_{10}, \zeta_{11}\) could vanish. Also:

\[
\delta m_H^2 = -\frac{1}{4} (B_0 m_0 \mu_0) v^2 \alpha_{60} \tan^2 \beta + \frac{v^2 \tan \beta}{8} \left[ -8 B_0 m_0 \mu_0 \alpha_{20} - 4 \alpha_{62} m_0^2 \\
- 4 \mu_0 m_0 (2 \alpha_{21} + \alpha_{31} + \alpha_{41} + 2 \alpha_{81}) - 4 \mu_0^2 (2 \alpha_{50} + \alpha_{60}) + (2 \alpha_{60} - \alpha_{70}) m_Z^2 \right] \\
+ \frac{3}{4} B_0 m_0 \mu_0 v^2 (\alpha_{50} + \alpha_{60}) + \frac{v^2}{8 \tan \beta} \left[ -8 B_0 m_0 \mu_0 \alpha_{10} + (12 \alpha_{52} - 16 \alpha_{62}) m_0^2 \\
- 4 \mu_0 m_0 (-6 \alpha_{11} + 8 \alpha_{21} + \alpha_{31} + \alpha_{41} + 2 \alpha_{81}) - 4 \mu_0^2 (5 \alpha_{50} - 2 \alpha_{60}) \\
+ (6 \alpha_{60} + 20 \alpha_{60} - 13 \alpha_{70}) m_Z^2 + \frac{8 v^2}{B_0 m_0 \mu_0} (2 \zeta_{10} \mu_0)^2 \right] + O(1/\tan^2 \beta) \quad (38)
\]

which is obtained for \((B_0 m_0 \mu_0)\) fixed. Note the \(O(1/M^2)\) effects from \(d = 5\) operators \((\zeta_{10}^2)\).

Similar expressions exist for the neutral pseudoscalar \(A\). The results are simpler in this case and we present the exact expression of \(\delta m_A^2\) of (33) in the most general case, that includes all supersymmetry breaking effects from the operators of \(d = 5, 6\) and from the MSSM. One finds

\[
\delta m_A^2 = \frac{v^2}{8 \tan^2 \beta (1 + \tan^2 \beta)} \left[ -2 B_0 m_0 \mu_0 \alpha_{50} + \left[ - (4 \alpha_{31} + 4 \alpha_{41} + 8 \alpha_{81} + 8 \alpha_{11}) m_0 \mu_0 \\
- 4 \alpha_{52} m_0^2 - 8 B_0 m_0 \mu_0 \alpha_{10} - 4 (\alpha_{50} + 2 \alpha_{60}) \mu_0^2 + (2 \alpha_{50} - \alpha_{70}) m_Z^2 \right] \tan \beta \\
+ \left[ 2 B_0 m_0 \mu_0 (10 \alpha_{50} + 3 \alpha_{60}) + 16 \alpha_{52} m_0^2 + 16 (\alpha_{51} + \alpha_{61}) m_0 \mu_0 \right] \tan^2 \beta \\
+ 2 \left[ -4 B_0 m_0 \mu_0 (\alpha_{10} + \alpha_{20} + 2 \alpha_{30} + 2 \alpha_{40}) + 6 \left( \alpha_{50} + \alpha_{60} \right) \mu_0^2 - (\alpha_{50} + \alpha_{60} - \alpha_{70}) m_Z^2 \\
- 2 (\alpha_{62} + \alpha_{52}) m_0^2 - 4 (\alpha_{11} + \alpha_{21} + \alpha_{31} + \alpha_{41} + 2 \alpha_{81}) m_0 \mu_0 \right] \tan^3 \beta \\
+ \left[ 2 B_0 m_0 \mu_0 (3 \alpha_{50} + 10 \alpha_{60}) + 16 \alpha_{52} m_0^2 + 16 (\alpha_{51} + \alpha_{61}) m_0 \mu_0 \right] \tan^4 \beta \\
- \left[ 8 B_0 m_0 \mu_0 (\alpha_{20} + 4 \alpha_{50} + 2 \alpha_{60}) \mu_0^2 - (2 \alpha_{60} - \alpha_{70}) m_Z^2 + 4 \alpha_{62} m_0^2 \\
+ 4 \left( \alpha_{21} + \alpha_{31} + \alpha_{41} + 2 \alpha_{81} \right) m_0 \mu_0 \right] \tan^5 \beta - 2 B_0 m_0 \mu_0 \alpha_{60} \tan^6 \beta \right] \quad (39)
\]

15
While this is certainly an interesting case, because then appropriate value. The result is (assuming m_β

We emphasise that the large tan β case to consider at large tan β

A similar formula exists for the correction to

We also showed that \( \delta m_G = 0 \) so the Goldstone mode remains massless in \( O(1/M^2) \), which is a good consistency check. A result similar to that in eq. (37) is found from an expansion of (39) in the large tan β limit:

\[
\delta m_A^2 = -\frac{1}{4} (B_0 m_0 \mu_0) \alpha_{60} v^2 \tan^2 \beta + \frac{\tan \beta}{8} v^2 \left[ -8 B_0 m_0 \mu_0 \alpha_{20} - 4 \alpha_{62} m_0^2 \right] - (8 \alpha_{21} + 4 \alpha_{31} + 4 \alpha_{41} + 8 \alpha_{81}) m_0 \mu_0 - (8 \alpha_{50} + 4 \alpha_{60}) \mu_0^2 + 2 \alpha_{60} m_Z^2 - \alpha_{70} m_Z^2 \n
\]

\[
+ \frac{v^2}{4} \left[ B_0 m_0 \mu_0 (3 \alpha_{50} + 11 \alpha_{60}) + 8 m_0^2 \alpha_{82} + 8 m_0 \mu_0 (\alpha_{51} + \alpha_{61}) \right] \n
+ \frac{v^2}{8 \tan \beta} \left[ -8 B_0 m_0 \mu_0 (\alpha_{10} + 2 \alpha_{30} + 2 \alpha_{40}) - 4 (2 \alpha_{11} + \alpha_{31} + \alpha_{41} + 2 \alpha_{81}) m_0 \mu_0 \right] - 4 \alpha_{52} m_0^2 - (4 \alpha_{50} + 8 \alpha_{60}) \mu_0^2 - (2 \alpha_{50} + 4 \alpha_{60} - 3 \alpha_{70}) m_Z^2 \right] + O(1/\tan^2 \beta) \tag{40}
\]

We emphasise that the large tan β limits presented so far were done with \( (B_0 m_0 \mu_0) \) fixed. While this is certainly an interesting case, because then \( m_A \) becomes large\(^2\) a more physical case to consider at large tan β is that in which one keeps \( m_A \) fixed \( (B_0 m_0 \mu_0 \) arbitrary).

We present below the correction \( O(1/M^2) \) to \( m_{h,H}^2 \) for the case \( m_A \) is kept fixed to an appropriate value. The result is (assuming \( m_A > m_Z \), otherwise \( \delta m_h^2 \) and \( \delta m_H^2 \) are exchanged):

\[
\delta m_h^2 = -2 v^2 \left[ \alpha_{22} m_0^2 + (\alpha_{30} + \alpha_{40}) \mu_0^2 + 2 \alpha_{61} m_0 \mu_0 - \alpha_{20} m_Z^2 \right] - \frac{(2 \zeta_{10} \mu_0)^2 v^4}{m_A^2 - m_Z^2} 
\]

\[
+ \frac{v^2}{\tan \beta} \left[ \frac{1}{(m_A^2 - m_Z^2)} \left( 4 m_A^2 \left( (2 \alpha_{21} + \alpha_{31} + \alpha_{41} + 2 \alpha_{81}) m_0 \mu_0 + (2 \alpha_{50} + 4 \alpha_{60}) \mu_0^2 + \alpha_{62} m_0^2 \right) \n
- (2 \alpha_{60} - 3 \alpha_{70}) m_A^2 m_Z^2 - (2 \alpha_{60} + \alpha_{70}) m_Z^4 \right) \n
+ 8 \frac{(m_A^2 + m_Z^2) (\mu_0 m_0 \zeta_{10} \zeta_{11}) v^2}{(m_A^2 - m_Z^2)^2} \right] + O(1/\tan^2 \beta) \tag{41}
\]

A similar formula exists for the correction to \( m_H \):

\[
\delta m_H^2 = -2 \left( m_0 \mu_0 (\alpha_{51} + \alpha_{61}) + \alpha_{82} m_0^2 \right) v^2 + \frac{(2 \zeta_{10} \mu_0)^2 v^4}{m_A^2 - m_Z^2} \n
+ \frac{v^2}{\tan \beta} \left[ \frac{1}{m_A^2 - m_Z^2} \left( 2 m_A^2 \left( (2 \alpha_{11} - \alpha_{21}) m_0 \mu_0 + (\alpha_{60} - \alpha_{50}) \mu_0^2 + (\alpha_{52} - \alpha_{62}) m_0^2 - \alpha_{60} m_A^2 \right) \n
- 4 (\alpha_{11} + \alpha_{21} + \alpha_{31} + \alpha_{41} + 2 \alpha_{81}) m_0 \mu_0 + 6 (\alpha_{50} + \alpha_{60}) \mu_0^2 + 2 (\alpha_{52} + \alpha_{62}) m_0^2 \n
- (\alpha_{50} + 5 \alpha_{60} - 2 \alpha_{70}) m_A^2 m_Z^2 - (\alpha_{50} - \alpha_{60}) m_Z^4 \right) \n
- 8 \frac{(m_A^2 + m_Z^2) (\mu_0 m_0 \zeta_{10} \zeta_{11}) v^2}{(m_A^2 - m_Z^2)^2} \right] + O(1/\tan^2 \beta) \tag{42}
\]

\(^5\)and thus likely to re-introduce a little hierarchy to explain.
Corrections (41), (42) must be added to the rhs of eq. (34) to obtain the value of \( m_{h,H}^2 \) expressed in function of \( m_A \) fixed. The corrections in eqs. (36) to (42) extend the result in [9] to include all \( \mathcal{O}(1/M^2) \) terms and represent the main result of this section.

From eqs. (37), (41) we are able to identify the effective operators of \( d = 6 \) that give the leading contributions to \( m_{h,H}^2 \), which is important for model building. These are \( O_{2,3,4} \) in the absence of supersymmetry breaking and \( O_{2,6} \) when this is broken, see also eqs. (11). It is however preferable to increase \( m_{h,H}^2 \) by supersymmetric rather than supersymmetry-breaking effects of the effective operators, because the latter are less under control in the effective approach and one would favour a supersymmetric solution to the fine-tuning problem associated with increasing the MSSM Higgs mass above the LEPII bound. Therefore \( O_{2,3,4} \) are the leading operators, with the remark that \( O_2 \) has a smaller effect, of order \( (m_Z/\mu_0)^2 \) relative to \( O_{3,4} \) (for similar \( \alpha_{j0}, j = 2, 3, 4 \)). At smaller \( \tan \beta \), \( O_{5,6} \) can also give significant contributions, while \( O_7 \) has a relative suppression factor \( (m_Z/\mu_0)^2 \).

5 Analysis of the leading corrections and effective operators

In general one would expect that \( d = 6 \) operators give sub-leading contributions to the spectrum, compared to \( d = 5 \) operators, in the case that all these operators are present and originate from integrating the same massive "new physics" (i.e. are suppressed by powers of the same scale \( M \), which is not always the case). Even so, for large \( \tan \beta \) the latter acquire a relative suppression factor, and the two classes of operators can indeed give comparable corrections. At large \( \tan \beta \) with \( m_A \) fixed, by comparing \( \mathcal{O}(1/M) \) terms in eq. (34) against \( \mathcal{O}(1/M^2) \) terms in eqs. (37), (38), (39), one identifies the situation when these two classes of operators give comparable corrections:

\[
\frac{4m^2_A}{m_A^2 - m_Z^2} \frac{\zeta_{10} \mu_0}{\tan \beta} \approx \left| \alpha_{22} m_0^2 + (\alpha_{30} + \alpha_{40}) \mu_0^2 + 2\alpha_{61} m_0 \mu_0 - \alpha_{20} m_Z^2 + \frac{2(\zeta_{10} \mu_0)^2 v^2}{m_A^2 - m_Z^2} \right| \left| \zeta_{11} m_0 + \frac{4m^2_Z}{m_A^2 - m_Z^2} \frac{\zeta_{10} \mu_0}{\tan \beta} \right| \approx \left| (m_0 \mu_0 (\alpha_{51} + \alpha_{61}) + \alpha_{82} m_0^2) - \frac{2(\zeta_{10} \mu_0)^2 v^2}{m_A^2 - m_Z^2} \right| \quad (43)
\]

In this case \( \mathcal{O}(1/(M \tan \beta)) \) and \( \mathcal{O}(1/M^2) \) corrections are approximately equal (for \( M \approx m_0 \tan \beta \)). Similar relations can be obtained from comparing (29), (33), against \( \delta m_{h,H}^2 \) of (37), (38), (39). Note that if these relations are satisfied this does not necessarily mean a failure of the effective field theory expansion, since the "new physics" that generates these operators
may be different! Indeed, the corrections from \( d = 6 \) and \( d = 5 \) operators can be completely independent (uncorrelated). However, if all operators involved in (43) are generated by the same massive physics, one would expect that the lhs be smaller than the rhs. In this case one obtains conditions for the coefficients of the operators that should be considered in numerical analyses. The exact form of such conditions depends on which operators are present\(^8\). Note that operators with \( d > 6 \) could not acquire a \( \tan \beta \) enhancement relative to \( d = 6 \) operators to become comparable in size, and they will always have an extra suppression factor (\( \sim 1/M \)).

Let us now examine more closely the corrections to the Higgs masses due to \( d = 6 \) operators. The interest is to maximise the correction to the MSSM classical value of \( m_h \). From eq. (37) and (41) and their \( \alpha_{ij} \) dependence and ignoring susy breaking corrections (\( \alpha_{jk}, k \neq 0 \)), we saw that \( O_{3,4} \) bring the largest correction (at large \( \tan \beta \)), and to a lower extent also \( O_2 \).

At smaller \( \tan \beta \), \( O_{5,6,7} \) can have significant corrections. All this can be seen from the relative variation:

\[
\epsilon_{\text{rel}} \equiv \frac{m_h - m_Z}{m_Z} = \sqrt{\delta_{\text{rel}}} - 1,
\]

where

\[
\delta_{\text{rel}} \equiv 1 - \frac{4 m_A^2}{m_A^2 - m_Z^2} \tan^2 \beta \left[ 1 + \frac{v^2}{m_Z^2} \left\{ \frac{2 \zeta_{10} \mu_0}{\tan \beta} \frac{4 m_A^2}{m_A^2 - m_Z^2} + \frac{(-2 \zeta_{11} m_0)}{\tan^2 \beta} \frac{2 (m_A^4 + m_Z^4)}{(m_A^2 - m_Z^2)^2} \right\} \right. \\
- \left. \left[ \frac{2}{\mu_0} \left( \frac{\alpha_{22} m_0^2 + (\alpha_{30} + \alpha_{40}) m_0^2 + 2 \alpha_{61} m_0 \mu_0 - \alpha_{20} m_Z^2}{m_A^2 - m_Z^2} \right) + \frac{(2 \zeta_{10} \mu_0)^2 v^2}{m_A^2 - m_Z^2} \right] \right]
\]

\[
+ \frac{1}{\tan \beta} \frac{1}{m_A^2 - m_Z^2} \left[ 4 m_A^2 \mu_0 \left( 2 \alpha_{21} + \alpha_{31} + \alpha_{41} + 2 \alpha_{81} \right) m_0 + \left( 2 \alpha_{50} + \alpha_{60} \right) \mu_0 \right] \\
+ 4 \alpha_{62} m_0^2 m_Z^2 \left[ \frac{2 (2 \alpha_{60} - 3 \alpha_{70}) m_Z^2}{m_A^2 - m_Z^2} \right] + 8 \zeta_{10} \zeta_{11} m_0 \mu_0 v^2 \frac{m_A^2 + m_Z^2}{m_A^2 - m_Z^2}\right\} \\
+ O(1/\tan^4 \beta) + O(\tilde{m}/(M \tan^3 \beta)) + O(\tilde{m}^2/(M^2 \tan^2 \beta))
\]

(44)

where \( \tilde{m} \) is some generic mass scale of the theory such as \( \mu_0, m_Z, m_0, v \). The arguments of the functions \( O \) in the last line show explicitly the origin of these corrections (MSSM, \( d = 5 \) or \( d = 6 \) operators, respectively). Depending on the signs of coefficients \( \alpha_{jk}, \zeta_{10}, \zeta_{11} \) this relative variation can be positive and increase \( m_h \) above the MSSM classical upper bound \( \langle m_Z \rangle \). Eq. (44) gives the overall relative change of the classical value of \( m_h \) in the presence of all possible higher dimensional operators of \( d = 5 \) and \( d = 6 \) beyond the MSSM Higgs

\(^8\) As an example, assuming at least one \( d = 6 \) operator is generated by the same physics as the \( d = 5 \) one considered, and if we neglect the supersymmetry breaking associated effects, then from (43) \( d = 6 \) operators could give comparable corrections for \( |2 \zeta_{10} \mu_0| \approx g^2 / \tan \beta \approx 0.55 / \tan \beta \) or \( M \approx 2 \mu_0 \tan \beta / g^2 \approx 3.6 \times \mu_0 \tan \beta \).
sector, for large tan \( \beta \) with \( m_A \) fixed. The expansion is accurate enough to be used also at intermediate tan \( \beta \), but this also depends on the ratio \( \tilde{m}/M \); for small tan \( \beta \) the terms in the last line in (44) give an error estimate; alternatively one can use exact \( \delta m_{h,H}^2 \) in (36).

A similar result exists for the case the limit of large tan \( \beta \) is taken with \( B_0 m_0 \mu_0 \) fixed (instead of \( m_A \)). Then

\[
\delta_{\text{rel}} = 1 - \frac{4}{\tan^2 \beta} + \frac{v^2}{m_Z^2} \left( \frac{4 (2 \zeta_{10} \mu_0)}{\tan \beta} + \frac{2}{\tan^2 \beta} \left( \frac{-2 \zeta_{11} m_0}{B_0 m_0 \mu_0} + \frac{2 m_Z^2 (2 \zeta_{10} \mu_0)}{B_0 m_0 \mu_0} \right) \right)
- \frac{2}{\tan \beta} \left[ \alpha_{22} m_0^2 + 2 \alpha_{61} m_0 \mu_0 + (\alpha_{30} + \alpha_{40}) \mu_0^2 - \alpha_{20} m_Z^2 \right]
+ \frac{1}{\tan \beta} \left[ \frac{(2 \zeta_{10} \mu_0)^2 v^2}{-B_0 m_0 \mu_0} \right]
+ 4 (2 \alpha_{21} + \alpha_{31} + \alpha_{41} + 2 \alpha_{81}) m_0 \mu_0 + 4 (2 \alpha_{50} + \alpha_{60}) \mu_0^2 + 4 \alpha_{62} m_0^2 - (2 \alpha_{60} - 3 \alpha_{70}) m_Z^2 \right]
+ \mathcal{O}(1/\tan^4 \beta) + \mathcal{O}(\tilde{m}/(M \tan^3 \beta)) + \mathcal{O}(\tilde{m}^2/(M^2 \tan^2 \beta))
\]

(45)

which can be used in numerical applications even for smaller, intermediate values of tan \( \beta \).

In (44), (45), the \( d = 6 \) operators \( \alpha_{ij} \) dependence) give contributions which are dominated by tan \( \beta \)-independent terms. One particular limit to consider for \( \delta m_h^2 \) or \( \delta' m_h^2 \) is that in which the effective operators of \( d = 6 \) have coefficients such that these contributions or those in the first line in (37), (41) add up to maximise \( \delta_{\text{rel}} \). Since coefficients \( \alpha_{ij} \) are not known, as an example we can choose them equal in absolute value

\[
- \alpha_{22} = - \alpha_{61} = - \alpha_{30} = - \alpha_{40} = \alpha_{20} > 0
\]

(46)

In this case, at large tan \( \beta \):

\[
\delta m_h^2 \approx 2 v^2 \alpha_{20} \left[ m_0^2 + 2 m_0 \mu_0 + 2 \mu_0^2 + m_Z^2 \right]
\]

(47)

and similar for \( \delta' m_h^2 \). A simple numerical example is illustrative. For \( m_0 = 1 \) TeV, \( \mu_0 = 350 \) GeV, and with \( v \approx 246 \) GeV, one has \( \delta m_h^2 \approx 2.36 \times 10^{11} \) (GeV). Assuming \( M = 10 \) TeV and ignoring \( d = 5 \) operators, with \( \alpha_{20} \sim 1/M^2 \) and the MSSM value of \( m_h \) taken to be its upper classical limit \( m_Z \) (reached for large tan \( \beta \)), we obtain an increase of \( m_h \) from \( d = 6 \) operators alone of about \( \Delta m_h = 12.15 \) GeV to \( m_h \approx 103 \) GeV. An increase of \( \alpha_{20} \) by a factor of 2.5 to \( \alpha_{20} \sim 2.5/M^2 \) would give \( \Delta m_h \approx 28 \) GeV to \( m_h \approx 119.2 \) GeV, which is above the LEPII bound. Note that this increase is realised even for a scale \( M \) of “new physics” beyond the LHC reach.

Considering instead the larger, loop-corrected MSSM value of \( m_h \), to which we add the \( d = 6 \) operators effects, the relative increase of \( \Delta m_h \) due to \( d = 6 \) operators alone is mildly...
reduced. However, the effective operators of $d = 6$ could in this case reduce the amount of fine-tuning for the electroweak scale, since these operators can increase the effective quartic coupling of the Higgs and thus reduce the fine-tuning, even for a smaller increase of $m_h$.

This was indeed observed for the case of $d = 5$ operators in the presence of MSSM quantum corrections to $m_h$, when the overall mass of $m_h$ (see Appendix 1) can easily reach values of $130 \text{ GeV}$ with a reduced, acceptable fine-tuning (less than $10^{-18}$) of the electroweak scale (for a scale of effective operators close to $10 \text{ TeV}$). A similar result may be expected in the presence of $d = 6$ operators [25]. Finally, the above choice of $M = 10 \text{ TeV}$ was partly motivated by the fine-tuning results of [16] (valid for $d = 5$ operators) and also on convergence grounds: the expansion parameter of our effective analysis is $m_q/M$ where $m_q$ is any scale of the theory, in particular it can be $m_0$. For a susy breaking scale $m_0 \sim \mathcal{O}(1) \text{ TeV}$ (say $m_0 = 3 \text{ TeV}$) and $c_{1,2}$ of (1) (or $\alpha_{ij}$ of $Z_i(S, S^\dagger)$) of order unity (say $c_{1,2} = 2.5$) one finds for $M = 10 \text{ TeV}$ that $c_{1,2} m_0/M = 0.75$ which is already close to unity, i.e. at the limit of validity of the expansion in powers of $1/M$ of the effective approach considered.

From these considerations, one may see that effective operators of $d = 6$ can indeed bring a significant increase of $m_h$ to values compatible with the LEPII bound, however, the value of the increase depends on implicit assumptions, like the type and number of operators present and whether their overall sign as generated by the “new physics” is consistent with an increase of $m_h$. Let us briefly refer to this latter issue.

We therefore consider the case of the leading contribution to $m_h$ in the large tan $\beta$ case. One would prefer to generate, from a renormalisable model, the leading operators with supersymmetric coefficients satisfying

$$\alpha_{20} > 0, \, \alpha_{30} < 0, \, \alpha_{40} < 0 \quad (48)$$

in order to increase $m_h$. Let us recall that $\mathcal{O}_{1,2,3}$ can be easily generated by integrating out a massive gauge boson $U(1)'$ or $SU(2)$ triplets [9]. $\mathcal{O}_4$ can be generated by a massive gauge singlet or $SU(2)$ triplets. Let us discuss the signs of the operators when so generated:

(a): Integrating out a massive vector superfield $U(1)'$ under which Higgs fields have opposite charges (to avoid a Fayet-Iliopoulos term), one finds $\alpha_{20} < 0$ and $\alpha_{30} > 0$ (also $\alpha_{10} < 0$) [9], which is opposite to condition (48). This can however be changed, if for example there are additional pairs of massive Higgs doublets also charged under new $U(1)'$, and then $\mathcal{O}_3$ could be generated with $\alpha_{30} < 0$. (b): Integrating massive $SU(2)$ triplets that couple to the MSSM Higgs sector would bring $\alpha_{20} > 0, \, \alpha_{40} < 0, \, \alpha_{30} > 0$, so the first two of these satisfy (48). (c): Integrating a massive gauge singlet would bring $\alpha_{40} > 0$, which would actually
decrease $m_h$. Finally, at large $\tan \beta$, due to additional corrections that effective operators bring to the $\rho$ parameter \cite{26}, it turns out that $\alpha_{40}$ and $\alpha_{30}$ can have the largest correction to $m_h^2$, while avoiding $\rho$-parameter constraints. The case of a massive gauge singlet or additional $U(1)'$ vector superfield (giving $O_{3,4}$) would have the advantage of preserving gauge couplings unification at one-loop.

For smaller $\tan \beta$, operators $O_{5,6,7}$ could bring significant corrections to $m_h$; it is more difficult to generate these in a renormalisable set-up, when more additional states are needed. For example $O_{5,6}$ can be generated by integrating out a pair of massive Higgs doublets and a massive gauge singlet, but the overall sign of $\alpha_{50,60}$ would depend on the details of the model. This discussion shows that while effective operators can in principle increase $m_h$, deriving a detailed, renormalisable model where they are generated with appropriate signs for their (supersymmetric) coefficients is not a simple issue. These examples are however rather naive and other generating mechanisms for $O_i$ could be in place (in a renormalisable set-up\footnote{Further constraints exist from $\rho$-parameter: $\rho - 1 = -v^2/M^2 (\alpha_{10} \cos^4 \beta + \alpha_{20} \sin^4 \beta - \alpha_{30} \sin^2 \beta \cos^2 \beta) + O(v^4/M^4)$, see \cite{26}, which at large $\tan \beta$ is dominated by $\alpha_{20}$, while the effect of $\alpha_{30}$ is strongly suppressed; thus $\alpha_{30}$ is less constrained than $\alpha_{20}$ and a better choice for increasing $m_h$.}), with appropriate signs to increase $m_h$.

6 Conclusions

We investigated in detail the Higgs sector of the MSSM in the presence of all $d = 5$ and $d = 6$ effective operators that can be present in this sector. This was motivated by the attempt to better understand the MSSM Higgs sector and its consistency with the quantum stability of the electroweak scale, the associated amount of fine tuning, the LEPII bound on $m_h$ and the so far negative searches for TeV scale supersymmetry. New physics beyond the current MSSM Higgs sector, parametrised by these effective operators, could alleviate these problems while retaining at the same time the advantages of low-energy supersymmetry, which was a main motivation of this work. The effective operators description used here is little dependent on the exact details of the new physics which generates these operators.

Two classes of such effective operators were present and investigated: higher dimensional derivative and non-derivative operators. We showed in Appendix \text{A} that the former can be removed from the action through appropriate non-linear field redefinitions and this is essentially equivalent to integrating out the massive additional ghost degrees of freedom (of mass $\sim M$) that such operators bring. It was also clarified in Appendix \text{B} that the use...
of “on-shell” setting of these operators brings similar results and is appropriate only in the leading order in the suppressing scale. The remaining, non-derivative operators contribute to the Higgs sector and their effects on the scalar potential and on the CP even and odd Higgs masses were computed analytically.

Despite their suppression by an extra power of the high scale $M$ relative to the $d = 5$ operators, the relative $\tan \beta$ enhancement of the $d = 6$ operators compensates for this suppression, to bring corrections comparable to those of the $d = 5$ operators, in the case both classes of operators are generated from the same high energy physics. This may not always be the case and it is possible that some of the $d = 6$ operators be present even in the absence of the $d = 5$ operators, if these classes of operators are generated by different new physics beyond the MSSM Higgs sector. Since our analysis assumed independent coefficients for all operators (whether of $d = 5$ and $d = 6$), our results are general and can be applied even if only some of these operators are present, regardless of their origin.

We identified the effective operators which give the most significant contributions to $m_h$ in the limit of large $\tan \beta$ and these can be both supersymmetric and non-supersymmetric. The supersymmetric case is preferable and also more important since such contribution would essentially alleviate a problem of fine-tuning which is intrinsically susy-breaking related. Of these operators $O_{3,4}$ would have the advantage of avoiding further $\rho$-parameter constraints. At small $\tan \beta$ other operators ($O_5, O_6, O_7$) could bring relevant corrections to $m_h$. Numerically, the impact of $d = 6$ operators alone on the mass of the lightest Higgs can be in the region of $10 - 30$ GeV. In the presence of MSSM loop effects and eventually $d = 5$ operators (if also present), this effect can help keep a low electroweak scale fine tuning, while respecting the LEPII mass bound and the current bounds on superpartners masses. If a larger increase of $m_h$ is sought from “new physics” beyond MSSM, the effective approach may not be reliable, and one should instead consider other approaches, such as MSSM with additional light states which are not integrated out.

Simple possibilities were listed for the “new physics” that, upon being integrated out, could generate these operators, in a renormalisable set-up. The “new physics” could be associated with the presence of a massive gauge singlet ($O_4$), massive $U(1)'$ ($O_{1,2,3}$), massive $SU(2)$ triplets ($O_{1,2,3,4}$). Some of these cases can have difficulties, through their impact on unification, perturbativity up to the Planck scale, etc. Of these, a very interesting possibility is that of extra $U(1)'$ massive gauge boson or massive gauge singlet, which do not share these difficulties in the leading order. In the simplest mechanisms generating the corresponding, leading operators $O_{3,4}$, the overall coefficients of their supersymmetric part have however signs
opposite to those needed to maximise the classical correction to the lightest Higgs mass (at large $\tan \beta$). Nevertheless such operators could be generated in other ways, when correlations among the coefficients of the effective operators could also be present. The next step in this analysis would be to construct a renormalisable model that would generate in the effective action such operators with appropriate values for their coefficients.

**Note added:**
While this paper was being typewritten, a similar study appeared [31] which has a partial overlap with this work.

**Acknowledgements**

This work was partially supported by ANR (CNRS-USAR) contract 05-BLAN-007901, INTAS grant 03-51-6346, contract PITN-GA-2009-237920, MRTN-CT-2006-035863, CNRS PICS no. 3747 and 4172 and the ERC Advanced Grant - 226371 (“MassTeV”). E.D. thanks the GGI Institute in Florence and the Aspen Center for Physics for hospitality during the completion of this work. P.T. thanks the “Propondis” Foundation for the financial support during the last stages of this work. D.G. thanks the CERN Theory Group and École Polytechnique Paris for the financial support and S. Cassel, C. Grojean and G.G. Ross for interesting related discussions.
7 Appendix:

A Integrals of operators $O_{1, 8}$:

\[
O_1 = \frac{1}{M^2} \int d^4 \theta \ Z_1(S, S^\dagger) \ (H_1^\dagger e^{V_1} H_1)^2 \\
= 2 \alpha_{10} \left[ (h_1^\dagger h_1) \left[ (\mathcal{D}_\mu h_1)^\dagger (\mathcal{D}^\mu h_1) + h_1^\dagger \frac{D_1}{2} h_1 + F_1^\dagger F_1 \right] + |h_1^\dagger F_1|^2 + (h_1^\dagger \mathcal{D}^\mu h_1)(h_1^\dagger \mathcal{D}_\mu h_1) \right] \\
+ \left[ 2 \alpha_{11} m_0 (h_1^\dagger h_1)(F_1^\dagger h_1) + h.c. \right] + \alpha_{12} m_0^2 (h_1^\dagger h_1)^2 + \text{fermionic part} \quad (A-1)
\]

\[
O_2 = \frac{1}{M^2} \int d^4 \theta \ Z_2(S, S^\dagger) \ (H_2^\dagger e^{V_2} H_2)^2 \\
= 2 \alpha_{20} \left[ (h_2^\dagger h_2) \left[ (\mathcal{D}_\mu h_2)^\dagger (\mathcal{D}^\mu h_2) + h_2^\dagger \frac{D_2}{2} h_2 + F_2^\dagger F_2 \right] + |h_2^\dagger F_2|^2 + (h_2^\dagger \mathcal{D}^\mu h_2)(h_2^\dagger \mathcal{D}_\mu h_2) \right] \\
+ \left[ 2 \alpha_{21} m_0 (h_2^\dagger h_2)(F_2^\dagger h_2) + h.c. \right] + \alpha_{22} m_0^2 (h_2^\dagger h_2)^2 + \text{fermionic part} \quad (A-2)
\]

\[
O_3 = \frac{1}{M^2} \int d^4 \theta \ Z_3(S, S^\dagger) \ (H_1^\dagger e^{V_1} H_1) (H_2^\dagger e^{V_2} H_2), \\
= \alpha_{30} \left\{ (h_1^\dagger h_1) \left[ (\mathcal{D}_\mu h_1)^\dagger (\mathcal{D}^\mu h_2) + h_1^\dagger \frac{D_2}{2} h_2 + F_2^\dagger F_1 \right] + (h_1^\dagger F_1) (F_2^\dagger h_2) + (1 \leftrightarrow 2) \right\} \\
+ \alpha_{30} \left[ (h_1^\dagger \mathcal{D}_\mu h_1)(h_2^\dagger \mathcal{D}^\mu h_2) + h.c. \right] + \left\{ \alpha_{31} m_0 \left[ (h_1^\dagger h_1)(F_2^\dagger h_2) + (h_2^\dagger h_2)(F_1^\dagger h_1) \right] + h.c. \right\} \\
+ \alpha_{32} m_0^2 (h_1^\dagger h_1)(h_2^\dagger h_2) + \text{fermionic part} \quad (A-3)
\]

\[
O_4 = \frac{1}{M^2} \int d^4 \theta \ Z_4(S, S^\dagger) \ (H_2 \cdot H_1) (H_2 \cdot H_1)^\dagger, \\
= \alpha_{40} \partial_\mu (h_2 \cdot h_1) \partial^\mu (h_2 \cdot h_1)^\dagger + \left[ \alpha_{41} m_0 (h_2 \cdot h_1) (h_2^\dagger F_1 + F_2^\dagger h_1) + h.c. \right] \\
+ \alpha_{42} m_0^2 (h_2 \cdot h_1)(h_2 \cdot h_1)^\dagger + \alpha_{40} |h_2 \cdot F_1 + F_2 \cdot h_1|^2 + \text{fermionic part} \quad (A-4)
\]

\[
O_5 = \frac{1}{M^2} \int d^4 \theta \ Z_5(S, S^\dagger) \ (H_1^\dagger e^{V_1} H_1) H_2 \cdot H_1 + h.c. \\
= \alpha_{50} \left\{ \left[ (\mathcal{D}_\mu h_1)^\dagger (\mathcal{D}^\mu h_1) + h_1^\dagger \frac{D_1}{2} h_1 + F_1^\dagger F_1 \right] (h_2 \cdot h_1) + (h_1^\dagger \mathcal{D}_\mu h_1) \partial^\mu (h_2 \cdot h_1) \right\} \\
+ \left[ \alpha_{50} (F_1^\dagger h_1) + \alpha_{51} m_0 (h_1^\dagger h_1) \right] (h_2 \cdot F_1 + F_2 \cdot h_1) + m_0 \left[ \alpha_{51} (F_1^\dagger h_1) + \alpha_{51} (h_1^\dagger F_1) \right] (h_2 \cdot h_1) \\
+ \alpha_{52} m_0^2 (h_1^\dagger h_1)(h_2 \cdot h_1) + \text{h.c. of all + fermionic part} \quad (A-5)
\]

\[
O_6 = \frac{1}{M^2} \int d^4 \theta \ Z_6(S, S^\dagger) \ (H_2^\dagger e^{V_2} H_2) H_2 \cdot H_1 + h.c. \\
= \alpha_{60} \left\{ \left[ (\mathcal{D}_\mu h_2)^\dagger (\mathcal{D}^\mu h_2) + h_2^\dagger \frac{D_2}{2} h_2 + F_2^\dagger F_2 \right] (h_2 \cdot h_1) + (h_2^\dagger \mathcal{D}_\mu h_2) \partial^\mu (h_2 \cdot h_1) \right\} \\
+ \left[ \alpha_{60} (F_2^\dagger h_2) + \alpha_{61} m_0 (h_2^\dagger h_2) \right] (h_2 \cdot F_1 + F_2 \cdot h_1) + m_0 \left[ \alpha_{61} (F_2^\dagger h_2) + \alpha_{61} (h_2^\dagger F_2) \right] (h_2 \cdot h_1) \\
+ \alpha_{62} m_0^2 (h_2^\dagger h_2)(h_2 \cdot h_1) + \text{h.c. of all + fermionic part} \quad (A-6)
\]

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\[ O_7 = \frac{1}{M^2} \left[ \frac{1}{16g^2\kappa} \int d^2 \theta \ Z_7(S,0) \right] \left[ \alpha \right] + \text{fermionic part} \]
\[ = \frac{1}{2} (D^2_w + D^2_\gamma) \left[ \alpha_{70}(h_2 \cdot h_1) + \alpha_{70}^*(h_2 \cdot h_1)^* \right] + \text{fermionic part} \]  
(A-7)

\[ O_8 = \frac{1}{M^2} \int d^4 \theta \ \left[ Z_8(S,S^T) \right] \left[ (H_2 H_1)^2 + \text{h.c.} \right] \]
\[ = 2 \alpha_5 m_0 (h_2 \cdot h_1) (h_2 F_1 + F_2 h_1) + m_0^2 \alpha_5 \left( h_2 \cdot h_1 \right)^2 + \text{h.c.} + \text{fermionic part} \]  
(A-8)

\[ W^\alpha \]  

\[ (1/M^2) \ Z_i(S,S^T) = \alpha_{i0} + \alpha_{i1} m_0 \theta \theta + \alpha_{i1}^* m_0 \overline{\theta} \overline{\theta} + \alpha_{i2} m_0^2 \theta \theta \overline{\theta} \overline{\theta} \]  
(A-9)

and \[ \mathcal{D}^\mu h_i = \left( \partial^\mu + i/2 V_i^\mu \right) h_i, \quad h_i^T \mathcal{D}^\mu = (\mathcal{D}^\mu h_i)^\dagger. \]  

Further, \[ D_1 \equiv \tilde{D}_w \tilde{T} + (-1/2) D_Y \] and \[ D_2 \equiv \tilde{D}_w \tilde{T} + (1/2) D_Y, \]  

\[ T^n = \sigma^n/2. \]  

Finally, one rescales in all \[ O_i \] (i \[ \neq \] 7):  

\[ V_w \rightarrow 2 g_2 V_w, \quad V_y \rightarrow 2 g_1 V_y. \]  

Then \[ V_{1,2} = 2 g_2 \tilde{V}_w \tilde{T} + 2 g_1 (\mp 1/2) V_y \] with the upper sign (minus) for \[ V_1, \]  

where \[ V_{1,2} \] enter the definition of \[ O_{1,2}. \]  

Other notations used above: \[ H_1, H_2 = e^{ij} H_1^i H_2^j. \]  

Also \[ |h_1 \cdot h_2| = |h_1|^2 |h_2|^2 - |h_1^T h_2|^2; \ e^{ij} e^{kl} = \delta^{ik} \delta^{jl} - \delta^{il} \delta^{jk}, \]  

\[ \varepsilon^{ijkl} = 1, \]  

with

\[ h_1 = \begin{pmatrix} h_1^0 \\ h_1^- \end{pmatrix} = \begin{pmatrix} h_1^+ \\ h_2^- \end{pmatrix}, \quad Y_{h_1} = -1; \quad h_2 = \begin{pmatrix} h_2^+ \\ h_2^- \end{pmatrix} = \begin{pmatrix} h_2^0 \\ h_2^0 \end{pmatrix}, \quad Y_{h_2} = +1 \]  

(A-10)

Lagrangian (11) with the above \[ O_{1,8} \] leads to

\[ F_{1q}^q = - \left\{ e^{pq} h_1^p \left[ \mu_0 + 2 \zeta_{10}(h_1 \cdot h_2) + \rho_{11} \right] + h_1^q \rho_{12} \right\} \]
\[ F_{2q}^q = - \left\{ e^{pq} h_1^p \left[ \mu_0 + 2 \zeta_{10}(h_1 \cdot h_2) + \rho_{21} \right] + h_2^q \rho_{22} \right\} \]  
(A-11)

where \[ \rho_{ij} \] are functions of \[ h_{1,2}, \] given in eqs. (14), (15). Similarly

\[ D_{w}^a = - g_2 \left[ h_1^T h_1 \left( 1 + \tilde{\rho}_1 \right) + h_2^T h_2 \left( 1 + \tilde{\rho}_2 \right) \right], \quad T^n = \sigma^n/2 \]
\[ D_Y^a = - g_1 \left[ h_1^T \frac{1}{2} h_1 \left( 1 + \tilde{\rho}_1 \right) + h_2^T \frac{1}{2} h_2 \left( 1 + \tilde{\rho}_2 \right) \right] \]  

(A-12)

with notation (19). This gives

\[ D_w^a D_w^a = \frac{g_2^2}{4} \left[ (1 + \tilde{\rho}_1) |h_1|^2 - (1 + \tilde{\rho}_2) |h_2|^2 \right] \]
\[ = \frac{g_2^2}{4} \left[ (1 + \tilde{\rho}_1) |h_1|^2 - (1 + \tilde{\rho}_2) |h_2|^2 \right] \]  
(A-13)

used in the text, eq. (17).
B Integrating out the ghosts, field redefinitions, and “on-shell” operators.

Here it is shown that operators of $d = 5$ or $d = 6$ of type $O_{9,\ldots,15}$ encountered in (7) or similar, which contain higher derivatives, can be “removed” from the action: (1) by integrating out the ghost degrees of freedom, (2): using the eqs of motion to set “on-shell” the derivative operator, or (3) by using non-linear field re-definitions. Beyond the leading order method (2) is not always applicable, as showed later for $d = 5$ effective operators (Appendix B.2) and thus it should be used with care.

B.1 The case of $d = 6$ operators.

Let us consider first the case of $d = 6$ operators. We use here method (1) and (2). Similar results are found with method (3).

(1) Integrating out the (super)ghosts.

Take

$$L = \int d^4\theta \left[ \Phi^\dagger (1 + \Box/M^2) \Phi + S^\dagger S \right] + \left\{ \int d^2\theta W[\Phi, S] + h.c. \right\} + \mathcal{O}(1/M^3) \quad (B-1)$$

where $\Box \equiv -1/16 \nabla^2 D^2$ and $S$ denotes in this appendix some arbitrary superfield. The derivative operator is similar to $O_9$ in the absence of gauge interactions; here we show how to remove this operator. $W$ can contain non-renormalisable terms up to $\mathcal{O}(1/M^3)$. This $L$ can be re-written as a second order theory (for details see [22]) with a Lagrangian:

$$L = \int d^4\theta \left[ \Phi_1^\dagger \Phi_1 - \Phi_2^\dagger \Phi_2 - \Phi_3^\dagger \Phi_3 + S^\dagger S \right]$$
$$+ \int d^2\theta \left[ \mu_{13} \Phi_1 \Phi_3 + \mu_{23} \Phi_2 \Phi_3 + W[\Phi(\Phi_{1,2,3}); S] \right] + h.c. + \mathcal{O}(1/M^3) \quad (B-2)$$

where

$$\Phi(\Phi_{1,2,3}) = \eta^{-1/4} (\Phi_2 - \Phi_1), \quad \eta \equiv 1 + 4 m^2 / M^2 \quad (B-3)$$

and

$$\mu_{13} = \mu_{31} = \frac{1 - \sqrt{\eta}}{2 \eta^{1/4}} M = - \frac{m^2}{M} + \mathcal{O}(1/M^3)$$
$$\mu_{23} = \mu_{32} = \frac{1 + \sqrt{\eta}}{2 \eta^{1/4}} M = - M + \mathcal{O}(1/M^3) \quad (B-4)$$

\footnote{A very similar treatment follows if one considers in (B-1) an opposite sign in front of $\Box/M^2$ [22].}

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We can integrate out the massive super-ghosts by using their eqs of motion:

\[
\frac{1}{4} \bar{D}^2 \Phi_2 + \mu_{23} \Phi_3 + \eta^{-1/4} W' = 0 \\
\frac{1}{4} D^2 \Phi_3 + \mu_{13} \Phi_1 + \mu_{23} \Phi_2 = 0 \quad (B-5)
\]

giving

\[
\Phi_3 = \frac{1}{M} W'[-\Phi_1; S] + O(1/M^3) \\
\Phi_2 = \frac{1}{4M^2} \bar{D}^2 W'[-\Phi_1; S] - \frac{m^2}{M^2} \Phi_1 + O(1/M^3) \quad (B-6)
\]

where the derivatives are taken wrt the first argument. We have

\[
\mu_{23} \Phi_2 \Phi_3 = -\frac{1}{M^2} \left[ \frac{1}{4} \bar{D}^2 W'[-\Phi_1; S] - m^2 \Phi_1 \right] W'[-\Phi_1; S] + O(1/M^3) \\
\mu_{13} \Phi_1 \Phi_3 = -\frac{m^2}{M} \Phi_1 W'[-\Phi_1; S] + O(1/M^3) \\
W[\Phi(\Phi_{1,2,3}; S)] = W[-\Phi_1; S] + \frac{1}{4M^2} W'[-\Phi_1; S] \bar{D}^2 W'[-\Phi_1; S] + O(1/M^3) \quad (B-7)
\]

Using these one finds

\[
\mathcal{L} = \int d^4 \theta \left[ \Phi_1^\dagger \Phi_1 - \frac{1}{M^2} W'[-\Phi_1; S] W'[-\Phi_1; S] + S^\dagger S \right] \\
+ \left\{ \int d^2 \theta \ W[-\Phi_1; S] + h.c. \right\} + O(1/M^3) \quad (B-8)
\]

This result is valid at energy scales well below the mass of the ghost \( M \). A similar result is obtained in this leading order, by using the equations of motion to set on-shell the higher derivative term, (see below).

(2) Using eqs of motion to set “on-shell” the operators.

Consider again \( (B-1) \)

\[
\mathcal{L} = \int d^4 \theta \left[ \Phi^\dagger \Phi - \frac{1}{(16 M^2)} \bar{D}^2 \Phi^\dagger D^2 \Phi + S^\dagger S \right] + \int d^2 \theta \ W[\Phi, S] + h.c. + O(1/M^3) \quad (B-9)
\]

which can be re-written by using the eqs of motion for \( \Phi \).
\[
\mathcal{D}^2 \Phi^\dagger = 4 W'[\Phi; S] + \mathcal{O}(1/M^2) \tag{B-10}
\]

to find
\[
\mathcal{L} = \int d^4 \theta \left[ \Phi^\dagger \Phi - \frac{1}{M^2} W'^t [\Phi; S] \right. \\
+ \left. W' [\Phi; S] + S \dagger S \right] \\
+ \int d^2 \theta \ W'[\Phi; S] + h.c. + \mathcal{O}(1/M^3) \tag{B-11}
\]

where under the derivative of the superpotential one should include only the renormalisable terms of \( W \), which is correct under the approximation considered. This result is in agreement with that of (B-8), up to a trivial field redefinition.

**B.2 The case of \( d = 5 \) operators.**

We extend the previous discussion to the case of extra derivatives in the superpotential and we take the lowest order case \( (d = 5) \). Start with

\[
\mathcal{L} = \int d^4 \theta \left[ \Phi^\dagger \Phi + S \dagger S \right] \\
+ \left\{ \int d^2 \theta \left[ \frac{\sigma}{M} \Phi \Box \Phi + W[\Phi; S] \right] + h.c. \right\} \\
= \int d^4 \theta \left[ \Phi^\dagger \Phi + \frac{\sigma}{4M} (\Phi D^2 \Phi + h.c.) \right] \\
+ \left\{ \int d^2 \theta \ W[\Phi; S] + h.c. \right\} \tag{B-12}
\]

with \( \sigma = \pm 1 \) and where it is assumed that the superpotential part of the action can contain additional higher dimensional (non-derivative) operators which have mass dimensions \( d \leq 5 \). It is shown that one can remove these operators via field redefinitions or via integrating out the ghost degree of freedom. These methods are shown to be equivalent. In the leading order only setting “on-shell” the operator via the eqs of motion also gives a similar, correct result.

**(1). Integrating out the (super)ghosts:**

\[
\mathcal{L} = \int d^4 \theta \left[ \Phi^\dagger \Phi + S \dagger S \right] \\
+ \left\{ \int d^2 \theta \left[ \frac{1}{2} d_2 \Phi^2 + d_3 \Phi_1 \Phi_2 + \frac{1}{2} d_4 \Phi_1^2 + W[\Phi(\Phi_1, \Phi_2); S] \right] + h.c. \right\} \tag{B-13}
\]
with

\[ d_1 = \frac{(\sqrt{\eta'} - 1)^2}{8\sigma \sqrt{\eta'}} M = \mathcal{O}(1/M^3), \]
\[ d_2 = \frac{(\sqrt{\eta'} + 1)^2}{8\sigma \sqrt{\eta'}} M = \sigma M/2 + \mathcal{O}(1/M^3), \]
\[ d_3 = \frac{\eta' - 1}{8\sigma \sqrt{\eta'}} M = (k \sigma) m^2/M + \mathcal{O}(1/M^3) \]  

(B-14)

where \( k = 17/32 \) and

\[ \Phi \equiv \eta'^{-1/4}(\Phi_2 - \Phi_1), \quad \eta' \equiv 1 + (17/4) m^2/M^2, \]  

(B-15)

We can integrate out the massive ghost superfield \( M \gg m \) using

\[ \frac{1}{4} \mathcal{D}^2 \Phi_1^\dagger + \frac{\sigma}{2} \Phi_2 + W'[\Phi; S] \eta'^{-1/4} + \frac{\sigma k m^2}{M} \Phi_1 + \mathcal{O}(1/M^3) = 0 \]  

(B-16)

Denote in the following \( W' \equiv W'[-\Phi_1; S] \) where the derivative is wrt the first argument. Then

\[ \Phi_2 = -\frac{2\sigma}{M} W' + \frac{4}{M^2} W' W'' - \frac{2k m^2}{M^2} \Phi_1 + \frac{1}{M^2} \mathcal{D}^2 W' + \mathcal{O}(1/M^3) \]  

(B-17)

Taylor expand:

\[ \Phi = -\Phi_1 - \frac{2\sigma}{M} W' + \frac{4}{M^2} W' W'' + \frac{1}{M^2} \mathcal{D}^2 W' + \mathcal{O}(1/M^3) \]  

(B-18)

then Taylor expand \( W[\Phi; S] \) in function of \( W', W'' \) to \( \mathcal{O}(1/M^3) \); also

\[ \Phi_2^\dagger \Phi_2 = \frac{4}{M^2} W' W' + \mathcal{O}(1/M^3) \]
\[ d_3 \Phi_1 \Phi_2 = -\frac{2k m^2}{M^2} \Phi_1 W' + \mathcal{O}(1/M^3) \]
\[ \frac{1}{2} d_2 \Phi_2^2 = \frac{\sigma}{M} W^2 - \frac{4}{M^2} W^2 W'' + \frac{2k m^2}{M^2} \Phi_1 W' - \frac{1}{M^2} W' \mathcal{D}^2 W' + \mathcal{O}(1/M^3) \]
\[ W[\Phi; S] = W - \frac{2\sigma}{M} W^2 + \frac{6}{M^2} W^2 W'' + \frac{1}{M^2} W' \mathcal{D}^2 W' + \mathcal{O}(1/M^3) \]  

(B-19)

Add everything together

\[ \mathcal{L} = \int d^4 \theta \left[ \Phi_1^\dagger \Phi_1 - \frac{4}{M^2} W' W' + S^\dagger S \right] + \left\{ \int d^2 \theta \left[ W - \frac{\sigma}{M} W^2 + \frac{2}{M^2} W''^2 \right] + h.c. \right\} + \mathcal{O}(1/M^3) \]  

(B-20)
where the argument of $W, W', W''$ above is $[-\Phi_1; S]$ and derivatives are wrt the first argument. This is equivalent to the starting Lagrangian, with new (non-renormalisable) interactions but no derivative ones.

(2) Removing derivative operators using field redefinitions.

Let us show that a similar result is obtained if we use general, local field redefinitions. Start again with:

$$L = \int d^4\theta [\Phi^\dagger \Phi + S^\dagger S] + \left\{ \int d^2\theta \left[ \frac{\sigma}{M} \Phi \Box \Phi + W[\Phi; S] \right] + h.c. \right\}$$

First eliminate the $O(1/M)$ terms by redefinition:

$$\Phi \rightarrow \Phi - \frac{\sigma}{4M} \slashed{D}^2 \Phi^\dagger \quad (B-22)$$

The Lagrangian becomes:

$$L = \int d^4\theta \left[ \Phi^\dagger \Phi + S^\dagger S \right] + \frac{\sigma}{M} \left( W' \Phi^\dagger + h.c. \right) - \frac{3}{32M^2} \left( \Phi D^2 \slashed{D} \Phi^\dagger + h.c. \right)$$

Next

$$\Phi \rightarrow \Phi - \frac{\sigma}{M} W'[\Phi; S] \quad (B-24)$$

which gives

$$L = \int d^4\theta \left[ \Phi^\dagger \Phi + S^\dagger S - \frac{1}{M^2} \left( \Phi^\dagger W' W'' + h.c. \right) - \frac{1}{M^2} W' W'^\dagger - \frac{3}{32M^2} \left( \Phi D^2 \slashed{D} \Phi^\dagger + h.c. \right) \right.$$ 

$$\left. - \frac{1}{8} \left( W'' \Phi^\dagger \slashed{D}^2 \Phi^\dagger + h.c. \right) \right] + \left\{ \int d^2\theta \left[ W - \frac{\sigma}{M} W'' + \frac{1}{2M^2} W'^2 W'' \right] + h.c. \right\} \quad (B-25)$$

We eliminate now the $1/M^2$ terms by

$$\Phi \rightarrow \Phi + \frac{3}{32M^2} \slashed{D}^2 \Phi \quad (B-26)$$
giving
\[ L = \int d^4 \theta \left[ \Phi \Phi + S \right] - \frac{1}{M^2} \left( \Phi \Phi W W'' + h.c. \right) - \frac{1}{M^2} \left( \Phi \Phi W' W' + h.c. \right) - \frac{3}{8 M^2} (W' D^2 \Phi + h.c.) \]
\[ - \frac{1}{8 M^2} (W'' \Phi \Phi D^2 \Phi + h.c.) + \left\{ \int d^2 \theta \left[ W - \frac{\sigma}{M} W'^2 + \frac{1}{2 M^2} W' W'' \right] + h.c. \right\} \] (B-27)

Next consider
\[ \Phi \rightarrow \Phi + \frac{1}{8 M^2} W'' D^2 \Phi \] (B-28)
to find
\[ L = \int d^4 \theta \left[ \Phi \Phi + S \right] - \frac{3}{2 M^2} \left( \Phi \Phi W'' + h.c. \right) - \frac{1}{M^2} \left( \Phi \Phi W' W' + h.c. \right) - \frac{3}{8 M^2} (W' D^2 \Phi + h.c.) \]
\[ + \left\{ \int d^2 \theta \left[ W - \frac{\sigma}{M} W'^2 + \frac{1}{2 M^2} W' W'' \right] + h.c. \right\} \] (B-29)
then
\[ \Phi \rightarrow \Phi + \frac{3}{8 M^2} D^2 W' [\Phi; S] \] (B-30)
to obtain
\[ L = \int d^4 \theta \left[ \Phi \Phi + S \right] - \frac{3}{2 M^2} \left( \Phi \Phi W'' + h.c. \right) - \frac{1}{M^2} \left( \Phi \Phi W' W' + h.c. \right) \]
\[ + \left\{ \int d^2 \theta \left[ W - \frac{\sigma}{M} W'^2 + \frac{1}{2 M^2} W' W'' + \frac{3}{8 M^2} W' D^2 W' \right] + h.c. \right\} \] (B-31)
Finally
\[ \Phi \rightarrow \Phi + \frac{3}{2 M^2} W' [\Phi; S] W'' [\Phi; S] \] (B-32)
one finds
\[ L = \int d^4 \theta \left[ \Phi \Phi + S \right] - \frac{4}{M^2} W' W' \]
\[ + \left\{ \int d^2 \theta \left[ W - \frac{\sigma}{M} W'^2 + \frac{2}{M^2} W' W'' \right] + h.c. \right\} \] (B-33)
which agrees with the result in (B-20) up to and including \( O(1/M^2) \) terms.
(3) Setting “on-shell” the derivative operators by using the eqs of motion.

Let us discuss what happens if in action (B-12) we “removed” the higher derivative terms by using the equations of motion i.e. setting them “on-shell”. It will turn out that only in leading order \(1/M\) do we obtain a similar \(\mathcal{L}\) as in previous cases. The eq of motion is

\[
\mathcal{D}^2 \Phi^\dagger = 4 W' + \mathcal{O}(1/M) \tag{B-34}
\]

where \(W' \equiv \partial W[\Phi; S]/\partial \Phi\). This is used in (B-12), and after an additional shift to re-write higher dimensional D-terms as F-terms

\[
\Phi \rightarrow \Phi - (\sigma/M) W' \tag{B-35}
\]

we obtain:

\[
\mathcal{L} = \int d^4 \theta \left[ \Phi^\dagger \Phi + S^\dagger S\right] + \left\{ \int d^2 \theta \ W[\Phi - (\sigma/M) W'; S] + h.c. \right\}
\]

\[
= \int d^4 \theta \left[ \Phi^\dagger \Phi + S^\dagger S\right] + \left\{ \int d^2 \theta \left[ W[\Phi; S] - (\sigma/M) W'^2[\Phi; S]\right] + h.c. \right\} \tag{B-36}
\]

where we used a Taylor expansion in the last step. If we include the next order, after using

\[
\mathcal{D}^2 \Phi^\dagger = 4 W' - \frac{\sigma}{M} \mathcal{D}^2 W'^\dagger + \mathcal{O}(1/M^2) \tag{B-37}
\]

and after the following redefinitions

\[
\Phi \rightarrow \Phi - \frac{\sigma}{M} W' \tag{B-38}
\]

and

\[
\Phi \rightarrow \Phi + \frac{1}{M^2} W' W'' \tag{B-39}
\]

one finds

\[
\mathcal{L} = \int d^4 \theta \left[ \Phi^\dagger \Phi + S^\dagger S - \frac{3}{M^2} W' W'^\dagger\right] + \left\{ \int d^2 \theta \left[ W - \frac{\sigma}{M} W'^2 + \frac{3}{2M^2} W'^2 W''\right] + h.c.\right\} \tag{B-40}
\]

Although this agrees with (B-20) in \(\mathcal{O}(1/M)\), it disagrees with it in order \(\mathcal{O}(1/M^2)\). The reason for this is that the “on-shell” setting method of higher dimensional operators is derived using general field redefinitions only in the leading order \(\mathcal{O}(1/M)\). As a result this method should be used with care.
C Coefficients for the Higgs masses.

The coefficients in eq.(30) have the following expressions:

\[
\gamma_1^\pm = \pm \frac{v^2}{2u^2(1+u^2)^3 w^{1/2}} \\
\times \left[ (B_0 m_0 \mu_0)^2 (1 + u^2)^4 - 2m_Z^2 u^2 \left[ m_Z^2 (1 - u^2)^2 + (1 + u^2) (8\mu_0^2 u^2 \pm (u^2 - 1) w^{1/2}) \right] \right] \\
+ (B_0 m_0 \mu_0) u (1 + u^2)^2 \left[ m_Z^2 (1 + u^2) - (\pm w^{1/2} (1 + u^2) + 16\mu_0^2 u^2) \right] \tag{C-1}
\]

\[
\gamma_2^\pm = \pm \frac{v^2}{2(1+u^2)^3 w^{1/2}} \\
\times \left[ (B_0 m_0 \mu_0)^2 (1 + u^2)^4 - 2m_Z^2 u^2 \left[ 8\mu_0^2 (1 + u^2) + m_Z^2 (1 - u^2)^2 \pm w^{1/2} (1 - u^4) \right] \right] \\
- (B_0 m_0 \mu_0) u (1 + u^2)^2 \left[ 16\mu_0^2 - m_Z^2 (1 + u^2) \pm (1 + u^2) w^{1/2} \right] \tag{C-2}
\]

\[
\gamma_3^\pm = \gamma_4^\pm = \frac{\pm v^2}{u (1 + u^2)^2 w^{1/2}} \left\{ m_0^2 \left[ -B_0 m_0 \mu_0 (1 + u^2)^3 + m_Z^2 u (1 - 6u^2 + u^4) \mp u (1 + u^2)^2 w^{1/2} \right] \right. \\
+ \left. B_0 m_0 \mu_0 u^2 (1 + u^2) m_Z^2 + m_Z^2 u^3 \left[ m_Z^2 \mp w^{1/2} \right] \right\} \tag{C-3}
\]

\[
\gamma_5^\pm = \frac{\pm v^2}{8u^3 (1 + u^2)^3 w^{1/2}} \left[ (B_0 m_0 \mu_0)^2 (1 + u^2)^4 (-1 + 3u^2) - (B_0 m_0 \mu_0) u (1 + u^2)^2 \right. \\
\times \left[ - 2m_Z^2 (1 + 5u^2) + 2\mu_0^2 (1 + 8u^2 + 25u^4 + 2u^6) \mp (1 + u^2)(3u^2 - 1) w^{1/2} \right] \\
- u^2 m_Z^2 \left[ m_Z^2 (1 - 19u^2 - 4u^4 + 3u^6) - 2\mu_0^2 (1 + u^2)(1 - 16u^2 - 23u^4 + 2u^6) \right. \\
\pm \left. (1 + u^2)^2 (1 + 3u^2) w^{1/2} \right] + 2\mu_0^2 u^2 \left[ \mp (1 + u^2)^2 (1 - 9u^2 + 2u^4) w^{1/2} \right] \tag{C-4}
\]

\[
\gamma_6^\pm = \frac{\pm v^2}{8u^2 (1 + u^2)^3 w^{1/2}} \left[ (B_0 m_0 \mu_0)^2 u (1 + u^2)^4 (-3 + 3u^2) - (B_0 m_0 \mu_0) (1 + u^2)^2 \right. \\
\times \left[ 2m_Z^2 (5 + u^2) u^4 - 2\mu_0^2 (2 + 25u^2 + 8u^4 + u^6) \mp (1 + u^2) (u^2 - 3) u^2 w^{1/2} \right] \\
+ u m_Z^2 \left[ m_Z^2 (3 - u^2 - 19u^4 + u^6) u^2 - 2\mu_0^2 (1 + u^2)(2 - 23u^2 - 16u^4 + u^6) \right. \\
\pm \left. u^2 (1 + u^2)^2 (3 + u^2) w^{1/2} \right] - 2\mu_0^2 u \left[ \pm (1 + u^2)^2 (2 - 9u^2 + u^4) w^{1/2} \right] \tag{C-5}
\]

\[
\gamma_7^\pm = \frac{\pm v^2 m_Z^2}{16u^2 (1 + u^2)^3 w^{1/2}} \left[ -B_0 m_0 \mu_0 (1 + u^2)(1 + 40u^2 - 114u^4 + 40u^6 + u^8) \right. \\
+ m_Z^2 u (u + 30u^5 + u^9) \pm u (1 + u^2)^2 (1 - 10u^2 + 4u^4) w^{1/2} \right] \tag{C-6}
\]

\[
\gamma_x^\pm = \frac{\pm 8 \left( u^2 - 1 \right)^2 v^4}{u (1 + u^2)^3 w^{3/2}} \left[ m_Z^2 u - B_0 m_0 \mu_0 (1 + u^2)^2 \right] m_0 m_0 \mu_0 \tag{C-7}
\]

\[
\gamma_y^\pm = \frac{\pm (1 + u^2)^2 v^4}{(1 + u^2)^4 w^{3/2}} \left[ m_Z^2 u - B_0 m_0 \mu_0 (1 + u^2)^2 \right] (4 m_0^2) \tag{C-8}
\]
\[ \gamma^\pm_2 = \frac{\mp v^4}{\mu_0^2 u^2 (1 + u^2)^3 w^{3/2}} \times \left[ -2 (B_0 m_0 \mu_0)^3 u (1 + u^2)^4 + m_Z^2 u^2 (1 + u^2)(4 \mu_0^2 (-1 + u^2)^2 - u^2 (2 m_Z^2 \pm w^{1/2})) \right. \\
+ \left. 2 B_0 m_0 \mu_0 m_Z^2 u \left[ -2 \mu_0^2 (u^4 - 1)^2 + u^2 (m_Z^2 (1 - 14 u^2 + u^4) \pm (u^4 - 6 u^2 + 1) w^{1/2}) \right] \\
+ \left. (B_0 m_0 \mu_0)^2 (1 + u^2) \left[ \mu_0^2 (u^4 - 1)^2 + u^2 (2 m_Z^2 (1 - 14 u^2 + u^4) \mp (1 + u^2) w^{1/2}) \right] \right] (4 \mu_0^2) \tag{C-9} \]

### D One-loop \( m_h \) with \( d = 5 \) operators

For future reference, it is worth mentioning the value of \( m_h \) in the presence of one-loop corrections from top-stop and \( d = 5 \) operators \([16]\), mentioned in the text:

\[
m_h^2 = \frac{1}{2} \left[ m_A^2 + \tilde{m}_Z^2 - \sqrt{\tilde{w}} + \xi \right] \\
+ (2 \zeta_{10} \mu_0) v^2 \sin 2\beta \left[ 1 + \frac{m_A^2 + \tilde{m}_Z^2}{\sqrt{\tilde{w}}} \right] + \frac{(2 \zeta_{11} \mu_0)}{2} v^2 \left[ 1 - \frac{(m_A^2 - \tilde{m}_Z^2) \cos^2 2\beta}{\sqrt{\tilde{w}}} \right] \tag{D-1} \]

where

\[
\tilde{w} = [(m_A^2 - \tilde{m}_Z^2) \cos 2\beta + \xi]^2 + \sin^2 2\beta (m_A^2 + \tilde{m}_Z^2)^2 \\
m_A^2 = \tilde{m}_1^2 + \tilde{m}_2^2 + \xi/2 + (2 \zeta_{10} \mu_0) v^2 \sin 2\beta + \zeta_{11} \mu_0 v^2; \quad \xi = \delta m_Z^2 \sin^2 \beta \quad (D-2) \]

where \( \delta \) is the one-loop correction from top-stop Yukawa sector to \( \lambda_2^0 \) of \([27]\) which changes according to \( \lambda_2^0 \rightarrow \lambda_2^0 (1 + \delta) \) where \([7, 24]\)

\[
\delta = \frac{3 h_t^2}{g^2 \pi^2} \left[ \ln \frac{M_t^2}{m_t} + \frac{X_t}{4} + \frac{1}{32 \pi^2} (3 h_t^2 - 16 g_3^2) \right] \left[ X_t + 2 \ln \frac{M_t}{m_t} \right], \\
X_t = \frac{2 (A_t m_0 - \mu \cot \beta)^2}{M_t^2} \left[ 1 - \frac{(A_t m_0 - \mu \cot \beta)^2}{12 M_t^2} \right]. \tag{D-3} \]

with \( M_t^2 = m_{\tilde{t}_1} m_{\tilde{t}_2} \), and \( g_3 \) the QCD coupling. The combined effect of \( d = 5 \) operators and top Yukawa coupling \( h_t \) is that \( m_h \) can reach values of 130 GeV for \( \tan \beta \leq 7 \) with a small fine-tuning \( \Delta \leq 10 \) \([16]\) and with the supersymmetric coefficient \( \zeta_{10} \) giving a larger effect than the non-susy one, \( \zeta_{11} \). Even for a modest increase of \( m_h \) from \( d = 5 \) operators alone of order \( \mathcal{O}(10 \text{GeV}) \), their impact on the effective quartic coupling of the Higgs field is significant (due to the small value of the MSSM gauge couplings), and this explains the reduction of fine-tuning by the effective operators.
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