A Note on AdS/SYM Correspondence on the Coulomb Branch

Yi-Yen Wu

Department of Physics and Astronomy
Johns Hopkins University
Baltimore, Maryland 21218, USA

Abstract

We study Maldacena’s conjecture and the AdS/SYM correspondence on the Coulomb branch. Several interesting aspects of this conjectured AdS/SYM correspondence on the Coulomb branch are pointed out and clarified.
1 Introduction

One of the remarkable AdS/CFT dualities (Anti-de Sitter space/Conformal Field Theory) conjectured by Maldacena is the equivalence between \( \mathcal{N}=4 \) four-dimensional supersymmetric Yang-Mills theory (SYM) and Type IIB string theory on \( \text{AdS}_5 \times \text{S}^5 \). Prescriptions for testing this correspondence have been given in \([2, 3]\), and this conjecture has been tested in many ways. (See \([4]\) for a recent review.) Recently, there is also some evidence for this conjecture at finite \( N \) \([5]\). AdS/SYM correspondence is natural from the point of view of holography \([3, 6]\), and another interesting aspect of this correspondence is the study of Type IIB branes in \( \text{AdS}_5 \times \text{S}^5 \) and its SYM correspondence \([7, 8, 9, 10]\).

In \([1]\), the AdS/SYM correspondence on the Coulomb branch has also been suggested. Recently, Douglas and Taylor \([14]\) proposed that D3-branes in the AdS\(_5\) bulk are equivalent to an \( \mathcal{N}=4 \) four-dimensional SYM on the Coulomb branch where the adjoint Higgs scalars have non-vanishing vacuum expectation values (vevs). In this letter, we will look more closely at this proposal. In Section 2, we study the AdS/SYM correspondence on the Coulomb branch following Maldacena’s original argument in \([1]\). The relevance of Witten’s study of D3-branes in the AdS\(_5\) bulk \([8]\) is emphasized. An interesting duality between D3-brane configurations in the AdS\(_5\) bulk suggested by this conjectured AdS/SYM correspondence on the Coulomb branch is also pointed out. In Section 3, we analyze an interesting aspect of this AdS/SYM correspondence on the Coulomb branch. Closely related to Witten’s study of D3-branes as domain walls \([8]\), a consistency check at finite \( N \) for the conjectured AdS/SYM correspondence on the Coulomb branch is also given by studying D3-branes in the AdS\(_5\) bulk. (We note that Witten’s argument for D3-branes in the AdS\(_5\) bulk as domain walls of Type IIB string theory on \( \text{AdS}_5 \times \text{S}^5 \) in \([8]\) also applies to M2-branes (M5-branes) in the AdS\(_4\) (AdS\(_7\)) bulk, and they are therefore domain walls of M-theory on \( \text{AdS}_4 \times \text{S}^7 \) (AdS\(_7\)\times\text{S}^4). As we will see, this suggests that many of the arguments in this letter can be extended to the study of M2-branes (M5-branes) in the AdS\(_4\) (AdS\(_7\)) bulk for M-theory on \( \text{AdS}_4 \times \text{S}^7 \) (AdS\(_7\)\times\text{S}^4).)

2 AdS/SYM Correspondence on the Coulomb Branch

\(^1\)Recently there is also a study of Type IIB branes in \( \text{AdS}_5 \times \text{X}_5 \) and its \( \mathcal{N}=1 \) SYM correspondence \([12]\), where \( \text{X}_5 \) is a five-dimensional Einstein manifold \([13]\).

\(^2\)Some recent studies related to \([14]\) can be found in \([14, 16]\).
2.1 Argument à la Maldacena and the Conjecture

The D3-brane in AdS$_5 \times$S$^5$ considered throughout this letter is a D3-brane unwrapped on S$^5$ and parallel to the AdS$_5$ boundary. That is, it sits at a point of S$^5$ and its world volume fills the four non-radial directions of AdS$_5$. According to Witten [8], $\Sigma(5)$ changes by ±1 unit as we cross this D3-brane, where $\Sigma(5)$ is the flux of R-R 5-form field strength $F(5)$ over S$^5$. $\Sigma(5) \rightarrow \Sigma(5) \pm 1$ corresponds to two kinds of D3-branes, where the $F(5)$’s supported on them are opposite in sign. A naive question is whether and how these D3-branes are stable in AdS$_5$? In the study of mesons and baryons as strings and wrapped 5-branes in AdS$_5$ [7, 8, 9, 17], the AdS gravitational force acting on the string and wrapped 5-brane is balanced by the tension of the string(s) ending on the AdS$_5$ boundary, and therefore these configurations are stable. Unlike other branes, D3-branes carry R-R five-form field strength $F(5)$. This $F(5)$ interacts with the background $F(5)$ of AdS$_5 \times$S$^5$, which results in a R-R force acting on the D3-brane. These D3-branes are of two kinds: their $F(5)$’s are opposite in sign. It is natural to expect that, for one kind of the D3-branes, the R-R force cancels the AdS gravitational force, and therefore it is stable. For the other kind of the D3-branes whose $F(5)$ is opposite in sign to that of the former, the R-R force adds to the AdS gravitational force; it is unstable and accelerates toward the AdS$_5$ horizon. The former is exactly the analogy of cancellation of NS-NS and R-R forces between two parallel D3-branes in flat ten-dimensional spacetime, and the latter is analogous to the instability of parallel anti-D3-brane and D3-branes.

For the AdS/SYM correspondence on the Coulomb branch, naturally the main concern will be stable D3-branes in the AdS$_5$ bulk.

In the following, we try to study the AdS/SYM correspondence on the Coulomb branch by “deriving” this correspondence following Maldacena’s argument [1]. To avoid unnecessary overlap with [1], the notations and conventions of [1] are always assumed whenever it is possible. We begin with $N$ D3-branes in flat ten-dimensional spacetime. Consider the case where we have two groups of parallel D3-branes, $(N-M)$ D3-branes and $M$ D3-branes, separated by $\vec{r}$. If we start with Type IIB string theory and take the decoupling limit $\alpha' \rightarrow 0$ while holding $\vec{W} = \vec{r}/\alpha'$ fixed, we obtain an $\mathcal{N}=4$ four-dimensional $U(N)$ SYM with gauge group $U(N)$ spontaneously broken to $U(N-M) \times U(M)$ by Higgs scalar vev’s in the adjoint representation of $U(N)$, where the $N$ eigenvalues of Higgs scalar vev’s parametrize the positions of $N$ D3-branes.

---

3In this letter, “stable” means stable against the AdS gravitational force.

4It is natural to refer to the above stable D3-brane as “D3-brane in AdS$_5$”, and the above unstable D3-brane as “anti-D3-brane in AdS$_5$”. However, here we simply call them stable and unstable D3-branes in AdS$_5$. 
Next, consider the supergravity (SUGRA) solution of the above D3-brane configuration and take the same decoupling limit. The resulting metric is

\[
\frac{ds^2}{\alpha'} = \frac{U^2}{\sqrt{4\pi g \left(N - M + \frac{MU^4}{|U-W|^4}\right)}} dx_\parallel^2 + \sqrt{4\pi g \left(N - M + \frac{MU^4}{|U-W|^4}\right)} dU^2
\]

\[+ \sqrt{4\pi g \left(N - M + \frac{MU^4}{|U-W|^4}\right)} d\Omega_5^2, \tag{1}\]

where \(U = |\vec{U}|\), \(\vec{U}/U\) is a point on \(S^5\), and \(d\Omega_5\) is the volume element of \(S^5\). According to [1], the resulting theory in the SUGRA approximation is Type IIB SUGRA on the background (1). Furthermore, we should ask what the underlying theory is beyond the SUGRA approximation. As we will argue, the background (1) represents the SUGRA solution describing \(M\) stable D3-branes of Type IIB string theory on \(AdS_5 \times S^5\). Therefore, the underlying theory should be Type IIB string theory on \(AdS_5 \times S^5\) with \(M\) stable D3-branes in the AdS\(_5\) bulk.

Now we argue that the static background (1) is the SUGRA solution describing \(M\) stable D3-branes of Type IIB string theory on \(AdS_5 \times S^5\). Firstly, (1) solves Type IIB SUGRA equations of motion trivially. Secondly, as we move from \(U = \infty\) to \(U = 0\), the geometry of (1) changes from \((AdS_5 \times S^5)_N\) to \((AdS_5 \times S^5)_{N-M}\). This corresponds to Type IIB string theory on \(AdS_5 \times S^5\), where \(\Sigma_{(5)} = (N-M)\) at \(U=0\) and \(\Sigma_{(5)} = N\) at \(U=\infty\), and therefore \(\Sigma_{(5)}\) changes by \(M\) units across the AdS\(_5\) bulk. According to Witten [8], \(\Sigma_{(5)} \to \Sigma_{(5)} + 1\) when we cross a D3-brane along \(U = \infty \to 0\), and \(\Sigma_{(5)} \to \Sigma_{(5)} - 1\) when crossing a D3-brane whose \(F_{(5)}\) is opposite in sign to that of the former. As argued in the beginning of this section, only one of them is stable; the other always accelerates toward \(U = 0\). We conclude that there must be \(M\) stable D3-branes in the AdS\(_5\) bulk. Thirdly, \(\vec{W}\) is naturally identified as the position of \(M\) D3-branes, where (1) is singular at \(\vec{U} = \vec{W}\). Furthermore, note that the metric of (1) does not depend on \(x_\parallel\), and therefore it does describe D3-branes unwrapped on \(S^5\) and parallel to the AdS\(_5\) boundary. Therefore we argue that the background (1) is indeed the Type IIB SUGRA solution describing \(M\) stable D3-branes of Type IIB string theory on \(AdS_5 \times S^5\), where \(\Sigma_{(5)} = (N-M)\) for \(U < |\vec{W}|\) and \(\Sigma_{(5)} = N\) for \(U > |\vec{W}|\). More precisely, according to

\footnote{Consider in the decoupling limit [1] the SUGRA solution for two groups of parallel M2-branes, \((N-M)\) M2-branes and \(M\) M2-branes, separated by \(\vec{r}\). Similarly we can argue that this SUGRA solution in the decoupling limit \((\vec{r}/\ell_p^{3/2})\) is fixed as \(\ell_p \to 0\) describes \(M\) stable M2-branes in the AdS\(_4\) bulk for M-theory on \(AdS_4 \times S^7\). The observation that M2-branes in the AdS\(_4\) bulk are domain walls of...}
Witten \[8\] this is Type IIB string theory residing in the \((\text{AdS}_5 \times \mathbb{S}^5)_{N-M}\) string vacuum on one side of \(M\) D3-branes, and residing in the \((\text{AdS}_5 \times \mathbb{S}^5)_N\) vacuum on the other side. The \(M\) D3-branes are domain wall separating the two AdS\(_5 \times \mathbb{S}^5\) vacua. We emphasize that, throughout this letter, this AdS string picture of Witten \[8\] is always assumed whenever we talk about D3-branes of Type IIB string theory on AdS\(_5 \times \mathbb{S}^5\) with definite \(\Sigma(5)'s\).

Following Maldacena’s argument \[1\], the above argument leads to a conjectured AdS/SYM correspondence between field theory and string theory. The field theory is an \(\mathcal{N}=4\) SYM with gauge group \(U(N)\) spontaneously broken to \(U(N-M) \times U(M)\) by Higgs scalar \(vev's\) in the adjoint representation of \(U(N)\).\[8\] The string theory is Type IIB string theory on AdS\(_5 \times \mathbb{S}^5\), with \(M\) stable coincident D3-branes in the AdS\(_5\) bulk, where Type IIB string theory resides in the \((\text{AdS}_5 \times \mathbb{S}^5)_N\) vacuum on the \(U=\infty\) side of D3-branes and resides in the \((\text{AdS}_5 \times \mathbb{S}^5)_{N-M}\) vacuum on the \(U=0\) side. Note that the presence of stable D3-branes in the AdS\(_5\) bulk breaks the \(SO(1,1)\) part of AdS\(_5\) isometry group \(SO(4,2)\) because of the \(U\)-position(s) of D3-branes. This corresponds to the fact that conformal symmetry of an \(\mathcal{N}=4\) SYM is broken because of the energy scale(s) introduced by non-vanishing Higgs scalar \(vev's\). This “position/scale correspondence” is a general feature of AdS/CFT(SYM) correspondence \[18\], and has been observed in many examples \[1, 19, 20, 21, 22\].

It is straightforward to generalize the argument of this section to obtain the AdS/SYM correspondence for Type IIB string theory on AdS\(_5 \times \mathbb{S}^5\) with \(M\) stable D3-branes in the AdS\(_5\) bulk. These \(M\) D3-branes consist of \(K\) groups of coincident stable D3-branes. \((M = \sum_{i=1}^{K} M_i, \ M_i \text{ is the number of D3-branes in the } i \text{th group.})\) The relevant SUGRA solution is obtained by replacing

\[
\left( N - M + \frac{MU^4}{|\vec{U} - \vec{W}|^4} \right) \Rightarrow \left( N - M + \sum_{i=1}^{K} \frac{M_i U^4}{|\vec{U} - \vec{W}_i|^4} \right)
\]

in (1), where \(W_1 \gg W_2 \gg \cdots \gg W_K\). The corresponding field theory is an \(\mathcal{N}=4\) SYM with gauge group \(U(N)\) spontaneously broken to \(U(N-M) \times U(M_1) \times \cdots \times U(M_K)\) by Higgs scalar \(vev's\) in the adjoint representation of \(U(N)\).

M-theory on AdS\(_4 \times \mathbb{S}^7\) (see the last remark in Section 1; more precisely, \((\text{AdS}_4 \times \mathbb{S}^7)_2 \rightarrow (\text{AdS}_4 \times \mathbb{S}^7)_{Z \pm 1}\) across an M2-brane in the AdS\(_4\) bulk) is again essential to this argument. The same consideration for M5-branes then leads to the SUGRA solution describing stable M5-branes in the AdS\(_7\) bulk for M-theory on AdS\(_7 \times \mathbb{S}^4\).

\[6\] It has been argued that the SYM gauge group appropriate for AdS/SYM correspondence should be \(SU(N)\) rather than \(U(N)\), where the \(U(1)\) part of \(U(N)\) decouple \[3, 4\]. Throughout this letter we adopt a similar point of view that the \(U(1)\) part of the boundary \(U(N)\) SYM is “frozen” or “non-dynamical” \[21\]. Only the remaining \(SU(N)\) part of the \(U(N)\) SYM is dynamical.
2.2 A Duality for Stable D3-Branes in AdS$_5$

We discuss in this section an interesting implication of the conjectured AdS/SYM correspondence on the Coulomb branch. The argument à la Maldacena in Section 2.1 leads to the SUGRA solution (1) and an $\mathcal{N}=4$ SYM with gauge group $U(N)$ spontaneously broken to $U(N-M) \times U(M)$ by Higgs scalar vev’s in the adjoint representation of $U(N)$. The SUGRA solution and the Higgs scalar vev’s of SYM are written explicitly as follows:

\[
\frac{ds^2}{\alpha'} = \sqrt{4\pi g \left( \frac{N-M}{|U|^4} + \frac{M}{|U-W|^4} \right)} dx^2 + \sqrt{4\pi g \left( \frac{N-M}{|\bar{U}|^4} + \frac{M}{|\bar{U}-\bar{W}|^4} \right)} d\bar{U}^2. \quad (3)
\]

\[
\langle \bar{X} \rangle = \begin{pmatrix}
0_{N-M,N-M} & 0_{N-M,M} \\
0_{M,N-M} & \bar{W}_{M,M}
\end{pmatrix}. \quad (4)
\]

$\bar{X}$ denotes the six Higgs scalars in the adjoint of $U(N)$. $\bar{W}_{M,M}$ is $\bar{W}$ times $M \times M$ identity matrix. As argued in Section 2.1, an $\mathcal{N}=4$ $U(N)$ SYM with Higgs scalar vev’s specified by (4) corresponds to Type IIB string theory on AdS$_5 \times S^5$ ($\Sigma(5)=N$ at $U=\infty$)\(^8\) with $M$ stable D3-branes at $\bar{U} = \bar{W}$ in the AdS$_5$ bulk.

The same argument can be trivially repeated by another simple choice of coordinates, i.e., fixing the origin of coordinates $\bar{U}$ on the $M$ D3-branes instead of the $(N-M)$ D3-branes. This amounts to replacing $\bar{U}$ by $\bar{U} + \bar{W}$ in (3). The SUGRA solution and the Higgs scalar vev’s of SYM are:

\[
\frac{ds^2}{\alpha'} = \sqrt{4\pi g \left( \frac{N-M}{|\bar{U}+\bar{W}|^4} + \frac{M}{|\bar{U}|^4} \right)} dx^2 + \sqrt{4\pi g \left( \frac{N-M}{|\bar{U}+\bar{W}|^4} + \frac{M}{|\bar{U}|^4} \right)} d\bar{U}^2. \quad (5)
\]

\[
\langle \bar{X} \rangle = \begin{pmatrix}
-\bar{W}_{N-M,N-M} & 0_{N-M,M} \\
0_{M,N-M} & 0_{M,M}
\end{pmatrix}. \quad (6)
\]

Note that the SUGRA solutions (3) and (5) are related by a coordinate shift $\bar{U} \rightarrow \bar{U} + \bar{W}$. And the two $U(N)$ SYM’s are related by a constant $U(1)$ shift, $\bar{X} \rightarrow \bar{X} + \bar{W}_{N,N}$, in their Higgs scalar vev’s (4) and (6). $\bar{W}_{N,N}$ is $\bar{W}$ times $N \times N$ identity matrix. As noted

---

\(^7\)For convenience, here we use $\bar{U}$ instead of $U$ and $\Omega_5$.

\(^8\)That is, near the AdS$_5$ boundary at $U=\infty$, Type IIB string theory resides in the $(\text{AdS}_5 \times S^5)_N$ vacuum. In the AdS$_5$ bulk, Type IIB string theory may reside in different AdS$_5 \times S^5$ vacua if there are stable D3-branes present \[^8\]. Throughout this letter, this is what we mean by “Type IIB string theory on AdS$_5 \times S^5$ ($\Sigma(5)=N$ at $U=\infty$)”. 

5
in [21], the $U(1)$ part of the boundary $U(N)$ SYM is “frozen”. For Higgs scalars in the adjoint of $U(N)$, this “frozen” $U(1)$ is related to the fixing of coordinates in the SUGRA description. It is then clear that a coordinate shift $\vec{U} \rightarrow \vec{U} + \vec{W}$ in SUGRA corresponds to a constant $U(1)$ shift $\vec{X} \rightarrow \vec{X} + \vec{W}_{N,N}$ in the Higgs scalars of boundary SYM. As argued in Section 2.1, an $\mathcal{N}=4\ U(N)$ SYM with Higgs scalar vev’s specified by (6) corresponds to Type IIB string theory on AdS$_5 \times S^5$ ($\Sigma_{(5)}=N$ at $U=\infty$) with $(N-M)$ stable D3-branes at $\vec{U} = -\vec{W}$ in the AdS$_5$ bulk.

The above arguments suggest an interesting duality between two D3-brane configurations: Type IIB string theory on AdS$_5 \times S^5$ ($\Sigma_{(5)}=N$ at $U=\infty$) with $M$ stable D3-branes at $\vec{U} = \vec{W}$, and Type IIB string theory on AdS$_5 \times S^5$ ($\Sigma_{(5)}=N$ at $U=\infty$) with $(N-M)$ stable D3-branes at $\vec{U} = -\vec{W}$. According to the above arguments or directly from the conjectured AdS/SYM correspondence on the Coulomb branch, the two $U(N)$ SYM’s describing these two D3-brane configurations are simply related by a constant $U(1)$ shift, $\vec{X} \rightarrow \vec{X} \pm \vec{W}_{N,N}$, in the Higgs scalars. Therefore, these two D3-brane configurations are “dual” to each other in the sense that they are related to each other simply by a coordinate shift $\vec{U} \rightarrow \vec{U} \pm \vec{W}$.

This duality seems surprising from the point of view of AdS$_5 \times S^5$ string theory. On the other hand, it can be regarded as a consequence of the conjectured AdS/SYM correspondence on the Coulomb branch. The above arguments suggest how this duality arises. Whether or how this duality makes sense for AdS string theory remains to be seen.

### 3 D3-Branes in AdS$_5$ and Coulomb Branch of SYM$_4$

Together with the conjectured AdS/SYM correspondence on the Coulomb branch, the original AdS/SYM correspondence conjectured by Maldacena can be understood in a more general context: an $\mathcal{N}=4$ four-dimensional $U(N)$ SYM is dual to Type IIB string theory on AdS$_5 \times S^5$, where the R-R 5-form flux over $S^5$ is $\Sigma_{(5)}=N$ at $U=\infty$. Whether $U(N)$ is spontaneously broken by adjoint Higgs scalar vev’s corresponds to whether there are stable D3-branes in the AdS$_5$ bulk. There is an interesting aspect of this AdS/SYM correspondence from the point of view of SYM. Consider the $\mathcal{N}=4\ U(N)$ SYM for AdS/SYM correspondence. A Higgs scalar in the adjoint representation of

---

9Therefore, it is a duality which formally takes $M$ to $(N-M)$, and $\vec{U} = \vec{W}$ to $\vec{U} = -\vec{W}$.

10It is also straightforward to extend this duality to D3-brane configurations which contain several groups of coincident stable D3-branes in the AdS$_5$ bulk.

11See Footnote 8.
$U(N)$ can have at most $N$ independent eigenvalues. In terms of the AdS/SYM correspondence in Section 2.1, this simple fact of SYM means that an $\mathcal{N}=4$ $U(N)$ SYM corresponds to Type IIB string theory on $\text{AdS}_5 \times S^5$ ($\Sigma_{(5)}=N$ at $U=\infty$) with $M$ stable D3-branes in the AdS$_5$ bulk, and $M$ can only be $0, 1, \cdots, (N-1)$, depending on whether and how the $U(N)$ is broken by Higgs scalar vev's. Therefore for Type IIB string theory ($\Sigma_{(5)}=N$ at $U=\infty$) with $M$ ($M=N$ or $M>N$) stable D3-branes, it seems that this AdS/SYM correspondence breaks down and there is no dual SYM description. The other possibility is that these string theory configurations with $M \geq N$ are ill defined. This seeming obstruction to $M \geq N$ is derived from SYM consideration alone. If the same obstruction can be understood by purely string/SUGRA consideration, it can be regarded as a consistency check at finite $N$ for the conjectured AdS/SYM correspondence on the Coulomb branch. Such a string/SUGRA analysis is possible by studying D3-branes in the AdS$_5$ bulk as follows.

There are two kinds of D3-branes in the AdS$_5$ bulk: $\Sigma_{(5)} \to \Sigma_{(5)} \pm 1$ when we cross a D3-brane along $U=\infty \to 0$. As argued in Section 2.1, only one of them is stable against AdS gravity. Which one is stable can be determined, for example, by a string theory calculation. However, for our purpose a simple SUGRA argument as follows will be sufficient. It has been argued that the static SUGRA solution ([1]) describes D3-branes of Type IIB string theory on $\text{AdS}_5 \times S^5$, where $\Sigma_{(5)}$ always decreases when we cross a D3-brane along $U=\infty \to 0$. Because only stable D3-branes have static SUGRA solution, the existence of ([1]) determines that $\Sigma_{(5)} \to \Sigma_{(5)} - 1$ when we cross a stable D3-brane along $U=\infty \to 0$. This observation is essential to the following discussion.

Next, consider Type IIB string theory on $\text{AdS}_5 \times S^5$ ($\Sigma_{(5)}=N$ at $U=\infty$) with $M$ stable D3-branes in the AdS$_5$ bulk, where these $M$ D3-branes consist of $K$ separate groups of coincident stable D3-branes. ($M=\sum_{i=1}^{K} M_i$. $M_i$ is the number of D3-branes in the $i$th group ($i=1, 2, \cdots, K$) indexed along $U=\infty \to 0$.) Firstly, consider $M<N$. According to Witten [3] and the above observation, the Type IIB string vacuum jumps from one to another, $(\text{AdS}_5 \times S^5)_{(N)} \to (\text{AdS}_5 \times S^5)_{(N-M_1)} \to \cdots \to (\text{AdS}_5 \times S^5)_{(N-M_1-M_2-M_3-\cdots-M_K)} \to (\text{AdS}_5 \times S^5)_{(N-M)}$, as we cross each group of coincident D3-branes along $U=\infty \to 0$. The $i$th group of D3-branes is a domain wall interpolating between the $(\text{AdS}_5 \times S^5)_{(N-M_1-M_2-M_3-\cdots-M_i)}$ and $(\text{AdS}_5 \times S^5)_{(N-M_1-M_2-M_3-\cdots-M_{i-1})}$ string vacua. For $M<N$, the above spacetime is always of the anti-de Sitter type. According to the AdS holographic principle proposed in [3], a boundary field theory description therefore should exist for $M<N$. This is consistent with the expectation of the conjectured

\[12\] And $\Sigma_{(5)} \to \Sigma_{(5)} + 1$ when we cross an unstable D3-brane along $U=\infty \to 0$. 

7
AdS/SYM correspondence on the Coulomb branch.

Secondly, consider $M=N$. As we cross the last ($K$th) group of stable D3-branes along $U = \infty \to 0$, naively we expect that $\Sigma(3)=M_K \to \Sigma(3)=0$. Note that $\Sigma(3)=0$ indicates that the spacetime is no longer of the anti-de Sitter type. Because the AdS holographic principle [3] has no natural generalization to non-AdS spacetime [23], this suggests that the case of $M=N$ does not have a boundary field theory description in the sense of [3]. On the other hand, since the spacetime across the $K$th group of D3-branes is not of AdS type by a naive analysis, it may be not even appropriate to talk about Type IIB string theory on $AdS_5 \times S^5$ ($\Sigma(5)=N$ at $U=\infty$) with $M=N$ stable D3-branes in the $AdS_5$ bulk in the beginning. Thirdly, for Type IIB string theory on $AdS_5 \times S^5$ ($\Sigma(5)=N$ at $U=\infty$) with $M$ ($M>N$) stable D3-branes in the $AdS_5$ bulk, the above consideration for $M=N$ also applies because $M$ ($M>N$) D3-branes always contain $N$ D3-branes as a subset.

In conclusion, the string/SUGRA analysis shows that only the cases of $M = 0, 1, 2, \cdots, (N-1)$ stable D3-branes can have dual SYM descriptions. This obstruction to $M \geq N$ is exactly what is expected from purely SYM consideration. In this sense this string/SUGRA analysis can be regarded as a consistency check for the conjectured AdS/SYM correspondence on the Coulomb branch.

**Acknowledgement**

I thank Chong-Sun Chu and Pei-Ming Ho for many useful discussions. This work was supported in part by NSF grant No. PHY-9404057.

**References**

[1] J. Maldacena, “The Large $N$ Limit of Superconformal Field Theories and Supergravity”, [hep-th/9711200](http://arxiv.org/abs/hep-th/9711200).

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, “Gauge Theory Correlators from Non-Critical String Theory”, Phys. Lett. B428 (1998) 105, [hep-th/9802109](http://arxiv.org/abs/hep-th/9802109).

[3] E. Witten, “Anti de Sitter Space and Holography”, [hep-th/9802150](http://arxiv.org/abs/hep-th/9802150).
[4] J. Maldacena, “The Large \( N \) Limit of Field Theories and Gravity”,
Talk presented at Strings'98 Conference,
http://www.itp.ucsb.edu/online/strings98/maldacena.

[5] O. Aharony and E. Witten, “Anti-de Sitter Space and the Center of the Gauge
Group”, hep-th/9807205.

[6] G. ’t Hooft, “Dimensional Reduction in Quantum Gravity”, gr-qc/9310006;
L. Susskind, “The World as a Hologram”, J. Math. Phys. 36 (1995) 6377,
hep-th/9409089;
L. Susskind, “The Holographic Principle”, Talk presented at Strings'98 Conference,
http://www.itp.ucsb.edu/online/strings98/susskind.

[7] S.-J. Rey and J. Yee, “Macroscopic Strings as Heavy Quarks in Large \( N \) Gauge
Theory and Anti-de Sitter Supergravity”, hep-th/9803001;
J. Maldacena, “Wilson Loops in Large \( N \) Field Theories”, Phys. Rev. Lett. 80
(1998) 4859, hep-th/9803002;
J.A. Minahan, “Quark-Monopole Potentials in Large \( N \) Super Yang-Mills”,
hep-th/9803111.

[8] E. Witten, “Baryons and Branes in Anti-de Sitter Space”, hep-th/9805112.

[9] D.J. Gross and H. Ooguri, “Aspects of Large \( N \) Gauge Theory Dynamics as seen
by String Theory”, hep-th/9805129.

[10] T. Banks and M.B. Green, “Non-Perturbative Effects in AdS\(_5\)×S\(^5\) String Theory
and D=4 SUSY Yang-Mills”, JHEP 05 (1998) 002, hep-th/9804170.

[11] C.-S. Chu, P.-M. Ho and Y.-Y. Wu, “D-Instanton in AdS\(_5\) and Instanton in SYM\(_4\)”,
hep-th/9806103;
I.I. Kogan and G. Luzón, “D-Instantons on the Boundary”, hep-th/9806197;
M. Bianchi, M.B. Green, S. Kovacs and G. Rossi, “Instantons in Supersymmetric
Yang-Mills and D-Instantons in IIB Superstring Theory”, hep-th/9807033.
[12] S.S. Gubser and I.R. Klebanov, “Baryons and Domain Walls in an \( \mathcal{N}=1 \) Superconformal Gauge Theory”, [hep-th/9808075](http://arxiv.org/abs/hep-th/9808075).

[13] I.R. Klebanov and E. Witten, “Superconformal Field Theory on Threebranes at a Calabi-Yau Singularity”, [hep-th/9807080](http://arxiv.org/abs/hep-th/9807080); S.S. Gubser, “Einstein Manifolds and Conformal Field Theories”, [hep-th/9807164](http://arxiv.org/abs/hep-th/9807164).

[14] M.R. Douglas and W. Taylor IV, “Branes in the Bulk of Anti-de Sitter Space”, [hep-th/9807225](http://arxiv.org/abs/hep-th/9807225).

[15] J.A. Minahan and N.P. Warner, “Quark Potentials in the Higgs Phase of Large \( N \) Supersymmetric Yang-Mills Theories”, JHEP **06** (1998) 005, [hep-th/9805104](http://arxiv.org/abs/hep-th/9805104).

[16] A.A. Tseytlin and S. Yankielowicz, “Free Energy of \( \mathcal{N}=4 \) Super Yang-Mills in Higgs Phase and Non-Extremal D3-Brane Interactions”, [hep-th/9809032](http://arxiv.org/abs/hep-th/9809032).

[17] A. Brandhuber, N. Itzhaki, J. Sonnenschein and S. Yankielowicz, “Baryons from Supergravity”, [hep-th/9806158](http://arxiv.org/abs/hep-th/9806158).

[18] L. Susskind and E. Witten, “The Holographic Bound in Anti-de Sitter Space”, [hep-th/9805114](http://arxiv.org/abs/hep-th/9805114).

[19] Miao Li and Tamiaki Yoneya, “Short Distance Space-Time Structure and Black Holes in String Theory: A Short Review of the Present Status”, [hep-th/9806240](http://arxiv.org/abs/hep-th/9806240).

[20] T. Banks, M.R. Douglas, G.T. Horowitz and E. Martinec, “AdS Dynamics from Conformal Field Theory”, [hep-th/9808016](http://arxiv.org/abs/hep-th/9808016).

[21] V. Balasubramanian, P. Kraus, A. Lawrence and S.P. Trivedi, “Holographic Probes of Anti-de Sitter Spacetimes”, [hep-th/9808017](http://arxiv.org/abs/hep-th/9808017).

[22] A.W. Peet and J. Polchinski, “UV/IR Relations in AdS Dynamics”, [hep-th/9809022](http://arxiv.org/abs/hep-th/9809022).

[23] E. Witten, Talk presented at Strings’98 Conference, [http://www.itp.ucsb.edu/online/strings98/witten](http://www.itp.ucsb.edu/online/strings98/witten).