Multiplicity distributions in the low x regime in a simple model

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In this work we introduce small changes in the model proposed by E. Levin and D. Kharzeev for multiplicity distributions of particles produced in proton-proton collisions. We compare the predictions of the model with the available experimental data from the LHC. We also consider the most recent version of the model proposed by E. Gotsman and E. Levin. These two versions of the model give a good description of the measured multiplicity distributions in the central pseudo-rapidity region ($|\eta| < 0.5$). The agreement with data is not so good when we consider the particles measured over a wider pseudo-rapidity interval ($|\eta| < 2.4$). Interestingly, the agreement becomes better when one selects events with higher transverse momentum ($p_T > 500$ MeV).

I. INTRODUCTION

The most recent review paper on multiplicity distributions was written ten years ago [1]. Its last section presented a number of predictions for the future collisions that would be performed at the LHC.

After ten years of operation, the LHC delivered an impressive amount of data on multiplicity distributions [2–8]. The collision energies range from 0.9 to 13 TeV. These data allow us to study the energy evolution of multiplicity distributions and to look for qualitative changes in the reaction dynamics. One of the expected changes was a transition from the soft (non-perturbative) to the semi-hard (perturbative) QCD regime [9]. Previous studies had concluded that this transition was already happening in the pre-LHC era. It was conjectured that semi-hard events would have larger multiplicities and this would induce a “shoulder” in the large $n$ region of the multiplicity distribution $P(n)$. Up to $\sqrt{s} = 1$ TeV, $P(n)$ was well described by a negative binomial distribution (NBD). At the Tevatron at $\sqrt{s} = 1.8$ TeV the appearance of the “shoulder” was confirmed and this led to the two-component model proposed in [9] in which the measured data were described by a combination of two NBDs, one representing the soft and one the semi-hard component. This “shoulder” was also responsible for the violation of the Koba-Nielsen-Olsen (KNO) scaling [1]. More recently, the data taken at $\sqrt{s} = 7, 8$ and 13 TeV show that the best fit is obtained with the inclusion of a third negative binomial distribution [10, 11].

At the LHC another transition was expected to be seen: the transition to the low x regime and to the BFKL dynamics with the possible manifestation of gluon saturation effects [12, 13]. Compared to the previous accelerators, the LHC proton-proton collisions occur at a center of mass energy which is one order of magnitude higher. Increasing the energy we have access to the low momentum component of the proton wave function, i.e., we “see” more partons with smaller momentum fraction $x$ of the proton. At the same time, we may have collisions between partons which are more off-shell, i.e., which have a higher virtuality $Q^2$. The changes in the parton distribution functions $f(x, Q^2)$ when they enter in the low $x$ and high $Q^2$ regimes are described by the BFKL and DGLAP QCD evolution equations respectively. The solutions of these equations show that the number of partons increases with $1/x$ and with $Q^2$. We can picture this process thinking that when we boost a proton its partons go through a branching process, with a copious production of new partons. This has direct implications for the final particle multiplicity. At lower collision energies particle production is dominated by string fragmentation, which describes hadron formation and which is a non-perturbative QCD process. At higher energies, and especially at the LHC, particle production is dominated by parton branching, which is studied with perturbative QCD, with string fragmentation playing a secondary role. The measured multiplicity distributions at the LHC are thus an excellent testing ground for the predictions of the evolution equations.

The best theory of strong interactions is QCD. The part of QCD which describes dense partonic systems formed in high energy collisions is the Color Glass Condensate (CGC) theory [12, 13]. In practical applications of the CGC it is necessary to make approximations and/or additional assumptions to obtain results which can be compared to data. In several works it has been shown that with the CGC based models it is possible to obtain a reasonable description of the global features of multiparticle production in high energy collisions. In the case of multiplicity distributions there

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is a lot of work to be done. In [14] the authors used the IP-Glasma model and calculated the multiplicity distribution of particles produced with $|\eta| < 0.5$ in pp collisions at 7 TeV. They found discrepancies between the model predictions and data at large multiplicities. In [15, 16] the authors used CGC models to study particle production and $P(n)$ was assumed to be a negative binomial distribution with the parameters $k$ and $n$ given by the CGC models. The data analysis was restricted to the lower LHC energies and to central pseudo-rapidity intervals. In the present analysis we will include the most recent data from higher energies and consider other rapidity and $p_T$ windows. Moreover we will test a $P(n)$ which is related to BFKL dynamics.

In [17] it was shown that the number of particles produced from the glasma follows a negative binomial distribution, which depends on the parameter $k$, which turns out to be proportional to the saturation scale $k \propto Q_s^2$. We know that the saturation scale grows with the collision energy and hence so does $k$. As $k \to \infty$ the negative binomial distribution goes to a Poisson distribution, which is a very narrow distribution. Therefore the CGC-Glasma prediction is that at increasing energies the multiplicity distribution should “shrink”. So far, the existing data have shown the opposite trend. The observed broadening of the multiplicity distributions can be quantified through the moments $C_n$ which are increasing functions of the energy, implying broader multiplicity distributions. The non-observation of the shrinkage of $P(n)$ is an indication that some ingredient is still missing in the theory and it must be completed.

In theoretical studies of MD’s there are top-down and bottom-up approaches. In the first group we find works [17–21], firmly rooted in QCD, which discuss in detail the parton branching process using different formalisms, different techniques and different approximations. Special attention is given to gluon saturation, which takes place at very high gluon densities, when gluon-gluon fusion becomes important and reduces the growth of gluon number, having a direct impact on the final hadron multiplicity. In a non-perturbative approach, it was suggested in [22] that the presence of instantons could affect the observed multiplicity distributions.

In the second group we find works [10, 11, 23–28] that try to extract the maximum amount of information from data, using sophisticated statistical tools, paying attention to the subtleties of the measured distributions and building up models, with assumptions that may motivate an a posteriori theoretical study.

In the top-down approach most of the works did not reach the point where they address the data and try to reproduce them. In the bottom-up approach most the works did not reach the point where they establish a clear connection to QCD.

In a third group, we find works in which the authors try to build the bridge between QCD and data. In this category we include the studies performed with more phenomenological models [29, 30] and with event generators [31], which are built to reproduce all the observable quantities in a high energy collision. The accumulated experimental data were studied with bottom-up models and with phenomenological models. The global features of the MD’s are captured by the models but they fail to reproduce the data in one or another kinematical region. In the compilation of the theoretical results presented in [31], the event generators either underestimate or overestimate the large $n$ tail of the distribution $P(n)$. Inspite of the recent improvements, such as, for example, in [31], the discrepancies persist. There is not yet a complete and satisfactory theoretical description of multiplicity distributions.

In this work we try to follow the approach of the third group: we choose a simple model of the QCD branching process and perform a comprehensive comparison with the existing data. This model was first proposed some time ago [32], it was used recently [33] to study entanglement entropy in particle production and it was improved very recently in [34, 35], where a preliminary discussion of the LHC data was presented. In the next section we briefly describe the model and in the last section we present our results.

II. THE MODEL

A. The Kharzeev - Levin model

In Ref. [32, 33], the authors have developed a model for multiplicity distributions based on the BFKL equation, which we will call KL model. They propose the following evolution equation for the parton multiplicity distribution $P_n$:

$$\frac{dP_n(Y)}{dY} = -\Delta n P_n + (n-1) \Delta P_{n-1}(Y)$$

which has the simple solution:

$$P_n(Y) = P_{KL}(n) = e^{-\Delta Y} \left(1 - e^{-\Delta Y}\right)^{n-1}$$
where $Y = \ln(1/x)$ and $\Delta$ is the BFKL Pomeron intercept, $\Delta = 4 \ln 2 \, \bar{\alpha}_s$ with $\bar{\alpha}_s = \alpha_s N_c / \pi$. From the above expression we obtain the mean multiplicity:

$$ \langle n \rangle = \sum_n n P(n) = e^{\Delta Y} = \left( \frac{1}{x} \right)^\Delta $$

The variable $x$ is defined here as in [36, 37]:

$$ x = \frac{q_0^2}{s} $$

where $q_0$ is a constant. Inserting (4) into (3) we obtain:

$$ \langle n \rangle = \left( \frac{s}{q_0^2} \right)^\Delta $$

The energy scale $q_0$ can be a mass or the average transverse momentum and hence it might be different for different data sets, but it should not depend on the collision energy $\sqrt{s}$.

The above expression is a prediction of the model. Interestingly, it can be derived from the CGC formalism. In fact, the first estimate of the mean multiplicity of produced particles in the CGC framework was done in [38] and in [39]. According to [38] (see also [29]) in the saturation regime the number of produced partons is given by

$$ \langle n \rangle = C Q_s^2 R^2 \alpha_s(Q_s^2) $$

where $C$ is a constant, $R$ is the transverse size of the projectile (proton in our case) and $\alpha_s(Q_s^2)$ is the strong coupling evaluated at $Q_s^2$, which is the saturation scale given by

$$ Q_s^2 = Q_0^2 \left( \frac{x_0}{x} \right)^\lambda $$

where $\lambda = 0.2 - 0.3$, $Q_0^2$ and $x_0$ are constants. From the above equations we conclude that:

$$ \langle n \rangle = C Q_0^2 \left( \frac{x_0}{q_0^2} \right)^\lambda \frac{R^2(s)}{\alpha_s(s)} s^\lambda $$

The radius of the proton, $R$, is related to confinement and can not be estimated from CGC physics. It has some weak energy dependence which comes from nonperturbative effects. In [40] the proton radius was parametrized as

$$ R^2(s) = R_0^2 \left( \frac{s}{s_p} \right)^{0.05} $$

Assuming that $\alpha_s(s) \simeq const$ and substituting (9) into (8) we find

$$ \langle n \rangle = \left( \frac{s}{s_0} \right)^{\lambda_{eff}} $$

which is equivalent to (5) and where all the constants were packed in the parameter $s_0$ and $\lambda_{eff} = \lambda + 0.05$.

At lower energies, $\sqrt{s} = 10 - 50$ GeV, particle production is dominated by soft, non-perturbative dynamics, and the calculated quantities have typically a weak (log) energy dependence. In the $\sqrt{s} \approx 1$ TeV region it was argued [9] that the mean multiplicity has a soft ($n_s$) and a semi-hard ($n_h$) component which depend on the collision energy $\sqrt{s}$ as $\langle n_s \rangle \propto \ln s$ and $\langle n_h \rangle \propto \ln^n s$. At the LHC energies it was shown in [3] and [39] that power law expressions such as (5) and (10) are able to reproduce the data. However these data are also compatible with other parametrization forms, such as $\langle n_{ch} \rangle \propto \ln^3 s$, as was shown in [11].

In the next section we will compare (2) with the available experimental data from LHC. Eq. (2) depends on two parameters, $\Delta$ and $q_0$, which will be adjusted to fit the data. Before doing this we will describe the improvement of the KL model proposed in [34] and [35].
B. The Gotsman - Levin model

The formalism developed in [33] was further explored in [34]. In [34] it was pointed out that (2) yields multiplicity distributions which are too broad. This behavior was attributed to the fact that (2) describes dilute systems. According to the authors, a more accurate description of central rapidity data, requires the study of the collision between dense systems. In [35], Gotsman and Levin derived the following formula for the multiplicity distribution:

\[ P_n(Y) = P_{GL}(n) = \frac{2 \ e^{-z}}{\sqrt{\pi z} N}, \]  \hspace{1cm} (11)

where \( z = \frac{n}{N} \) and

\[ N = e^{\Delta Y} = e^{\Delta \ln(\frac{\eta}{q})} = e^{\ln(\frac{\eta}{q})} = \left( \frac{n}{q_1} \right)^\Delta \]  \hspace{1cm} (12)

It is useful to write the simple relation between \( P_{KL}(n) \) and \( P_{GL}(n) \) [35]:

\[ R = \frac{P_{GL}(n)}{P_{KL}(n)} = \sqrt{\frac{1}{\pi z}} = \sqrt{\frac{N}{\pi n}} \]  \hspace{1cm} (13)

This more singular behavior of \( P_{GL}(n) \) at small \( n \) will be visible in all the figures shown below.

III. RESULTS AND DISCUSSION

In this section we compare the results of the model with the experimental data from LHC. Three data sets were used, each one with several center of mass energies (\( \sqrt{s} \)) and each set corresponding to a different cut in the minimum transverse momentum (\( p_T \)) of the measured particles and a different pseudo-rapidity (\( \eta \)) window. They are:

- Set I: \( p_T > 100 \text{ MeV}, |\eta| < 0.5 \) and energies \( \sqrt{s} = 900, 2360 \) and 7000 GeV. These data are from [3].
- Set II: \( p_T > 100 \text{ MeV}, |\eta| < 2.4 \), and energies \( \sqrt{s} = 900, 7000 \) and 8000 GeV [7] and 8000 [4] and 13000 GeV [5].
- Set III: \( p_T > 500 \text{ MeV}, |\eta| < 2.4 \) and energies \( \sqrt{s} = 900, 2360 \) and 7000 GeV [2], 8000 GeV (\( |\eta| < 2.5 \)) [4] and 13000 GeV [8] and (\( |\eta| < 2.5 \)) [9].

In Figs. 1, 2 and 3, we show the the fits of (2) (solid lines) and (11) (dashed lines) to the data on multiplicity distributions from the LHC. For each figure we adjusted three numbers \( \Delta, q_0 \) and \( q_1 \). They are listed in Table I. As can be seen both models overshoot the data at large \( n \) in all data sets. The model GL, as expected, overshoots the data at small \( n \). In Figs. 1 and 2, the theoretical curves have essentially the correct shape. In contrast, in Fig. 2 in some cases, the data show a curvature which is absent in the theoretical curves. A careful comparison between set I and set II was done in [3]. The conclusion then was that set I is compatible with KNO scaling while it is violated in set II. There seemed to be something different happening in set II. Now set II has been enlarged with new data points from higher energies and the agreement with models KL and GL is better. From the perspective of low \( x \) physics, set III is the most interesting data sample because the cut \( p_T > 500 \text{ MeV} \) removes a (significant ?) part of the non-perturbative events. These events are not related to parton branching or to the evolution equations. As can be seen in Table I, the value of \( \Delta \) is larger, indicating a stronger dependence on the energy \( \sqrt{s} \), typical of perturbative physics. Finally, the overall agreement between theory and data is better.

In [34] and [35] \( P_{GL}(n) \) and \( P_{KL}(n) \) were compared with some experimental data but here we perform a systematic comparison to all relevant available data. In those works the final goal was to compute the entanglement entropy and the multiplicity distributions were a crucial ingredient. To this end it is important to have an accurate description of data.

The model presented here can be improved in many ways. The multiplicity distributions are affected by fluctuations in several quantities. The most obvious one is the impact parameter \( b \) [19, 23]. Another one is the fraction (called inelasticity \( \chi \)) of the c.m.s energy which is converted into secondary particles. It is typically 0.5, but it changes from event to event and it depends on the energy according to a distribution \( \chi |K(b, s)| \) [23, 24]. The inclusion of these fluctuations will certainly modify the shape of \( P(n) \). Another aspect to be considered is the number of sources which emit particles. When we use \( P_{GL}(n) \) or \( P_{KL}(n) \) above it is understood that there is only one emitting system. However, in high energy proton-proton collisions there is a separation between particle production at low rapidities...
TABLE I: Fit parameters.

| Data Set | Δq0 (GeV) | q1 (GeV) |
|----------|-----------|---------|
| I        | 0.13      | 4.46    |
| II       | 0.13      | 0.02    |
| III      | 0.16      | 2.85    |

FIG. 1: Multiplicity distributions compared with data set I. Data are from CMS (2011) [3]. Left panel: $P(n)$ for different energies. Solid lines: $P_{KL}(n)$. Dashed lines: $P_{GL}(n)$. Sets of points were multiplied by powers of ten for clarity. Right panel: Ratio Theory ($P_{KL}(n)$)/Data.

To summarize: multiplicity distributions deserve more attention from theorists. In Refs. [33–35] simple distributions were derived from the QCD evolution equations. Here we have performed a comprehensive comparison between these multiplicity distributions and LHC data. The results are encouraging but there is still a lot to be done.

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[1] J. F. Grosse-Oetringhaus and K. Reygers, J. Phys. G 37, 083001 (2010).
[2] G. Aad et al. [ATLAS], New J. Phys. 13, 053033 (2011).
[3] V. Khachatryan et al. [CMS], JHEP 1101, 079 (2011).
[4] G. Aad et al. [ATLAS Collaboration], Eur. Phys. J. C 76, 403 (2016).
[5] G. Aad et al. [ATLAS Collaboration], Eur. Phys. J. C 76, 502 (2016).
[6] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 758, 67 (2016).
[7] S. Acharya et al. [ALICE Collaboration], Eur. Phys. J. C 78, 816 (2018).
[8] A. M. Sirunyan et al. [CMS Collaboration], Eur. Phys. J. C 78, 697 (2018).
[9] A. Giovannini and R. Ugoccioni, Phys. Rev. D 59, 094020 (1999); Erratum: [Phys. Rev. D 69, 059903 (2004)].
[10] I. Zhorovskyy, Eur. Phys. J. C 78, 816 (2018).
[11] M. Biyajima and T. Mizoguchi, Int. J. Mod. Phys. A 34, 1950203 (2019).
FIG. 2: Multiplicity distributions compared with data set II. Data for $\sqrt{s} = 900, 7000$ and 8000 GeV from ALICE (2017) [7]. Data from ATLAS (2016) for 8000 [4] and 13000 GeV [5]. Solid lines: $P_{KL}(n)$. Dashed lines: $P_{GL}(n)$. Left panel: $P(n)$ for different energies. Sets of points were multiplied by powers of ten for clarity. Right panel: Ratio Theory ($P_{KL}(n)/$Data).

FIG. 3: Multiplicity distributions compared with data set III. Data for energies $\sqrt{s} = 900, 2360$ and 7000 GeV from ATLAS (2011) [2], for 8000 GeV from ATLAS (2016) [4]. Data for 13000 GeV from CMS (2018) [8] and from ATLAS (2016) [6]. Solid lines: $P_{KL}(n)$. Dashed lines: $P_{GL}(n)$. Left panel: $P(n)$ for different energies. Sets of points were multiplied by powers of ten for clarity. Right panel: Ratio Theory ($P_{KL}(n)/$Data).

[12] Yuri V. Kovchegov and Eugene Levin, “Quantum Chromodynamics at High Energies”, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, Cambridge University Press, 2012.
[13] F. Gelis, E. Iancu, J. Jalilian-Marian and R. Venugopalan, Ann. Rev. Nucl. Part. Sci. 60, 463 (2010).
[14] B. Schenke, P. Tribedy and R. Venugopalan, Phys. Rev. C 89, 024901 (2014).
[15] A. Dumitru, D. E. Kharzeev, E. M. Levin and Y. Nara, Phys. Rev. C 85, 044920 (2012).
[16] P. Tribedy and R. Venugopalan, Nucl. Phys. A 850, 136 (2011).Erratum: [Nucl. Phys. A 859, 185 (2011)].
[17] F. Gelis, T. Lappi and L. McLerran, Nucl. Phys. A 828, 149 (2009).
[18] T. Liou, A. H. Mueller and S. Munier, Phys. Rev. D 95, 014001 (2017).
[19] L. Domíngé, G. Giacalone, C. Lorcé, S. Munier and S. Pekar, Phys. Rev. D 98, 114032 (2018).
[20] A. Dumitru and V. Skokov, Phys. Rev. D 96, 056029 (2017).
[21] G. H. Arakelyan, Y. M. Shabelski and A. G. Shuvaev, Eur. Phys. J. C 80, 592 (2020).
[22] V. V. Khoze, F. Krauss and M. Schott, JHEP 2004, 201 (2020); V. V. Khoze, D. L. Milne and M. Spannowsky,
[23] P. C. Beggio and F. R. Coriolano, Eur. Phys. J. C 80, 437 (2020).
[24] R. Aggarwal and M. Kaur, Adv. High Energy Phys. 2020, 5464682 (2020).
[25] S. Sharma and M. Kaur, Phys. Rev. D 99, 096016 (2019).
[26] M. Rybczyński, G. Wilk and Z. Włodarczyk, Phys. Rev. D 99, 094045 (2019).
[27] M. Rybczyński, G. Wilk and Z. Włodarczyk, Ukr. J. Phys. 64, 738 (2019).
[28] H. W. Ang, A. H. Chan, M. Ghalfar, M. Rybczyński, G. Wilk and Z. Włodarczyk, Eur. Phys. J. A 56, 117 (2020).
[29] A. Dumitru and E. Petreska, arXiv:1209.4105
[30] A. Dumitru and Y. Nara, Phys. Rev. C 85, 034907 (2012).
[31] C. Bierlich, S. Chakraborty, G. Gustafson and L. Lönblad, arXiv:2010.07505 [hep-ph].
[32] E. Levin and M. Lublinsky, Nucl. Phys. A 730, 191 (2004).
[33] D. E. Kharzeev and E. M. Levin, Phys. Rev. D 95, 114008 (2017).
[34] E. Gotsman and E. Levin, Phys. Rev. D 102, 074008 (2020).
[35] E. Gotsman and E. Levin, arXiv:2008.10911 [hep-ph].
[36] J. Bartels, E. Gotsman, E. Levin, M. Lublinsky and U. Maor, Phys. Lett. B 556, 114 (2003).
[37] F. Carvalho, F. O. Duraes, V. P. Goncalves and F. S. Navarra, Mod. Phys. Lett. A 23, 2847 (2008).
[38] D. Kharzeev and M. Nardi, Phys. Lett. B 507, 121 (2001).
[39] D. Kharzeev, E. Levin and M. Nardi, Nucl. Phys. A 747, 609 (2005).
[40] H. G. Dosch, E. Ferreira and A. Kramer, Phys. Rev. D 50, 1992 (1994).
[41] R. A. Lacey, P. Liu, N. Magdy, M. Csanád, B. Schweid, N. N. Ajitanand, J. Alexander and R. Pak, Universe 4, 22 (2018).
[42] F. S. Navarra, O. V. Utyuzh, G. Wilk and Z. Włodarczyk, Phys. Rev. D 67, 114002 (2003); G. N. Fowler et al., Phys. Rev. C 40, 1219 (1989).