Effects of temperature on thick branes and the fermion (quasi-)localization

Zhen-Hua Zhao, Yu-Xiao Liu, Yong-Qiang Wang, Hai-Tao Li

Institute of Theoretical Physics, Lanzhou University, Lanzhou 730000, People’s Republic of China
E-mail: zhaozhh09@lzu.edu.cn, liuyx@lzu.edu.cn, yqwang@lzu.edu.cn, liht07@lzu.edu.cn

Abstract: Following Campos’s work [Phys. Rev. Lett. 88, 141602 (2002)], we investigate the effects of temperature on flat, de Sitter (dS), and anti-de Sitter (AdS) thick branes in five-dimensional (5D) warped spacetime, and on the fermion (quasi-)localization. First, in the case of flat brane, when the critical temperature reaches , the solution of the background scalar field and the warp factor is not unique. So the thickness of the flat thick brane is uncertain at the critical value of the temperature parameter, which is found to be lower than the one in flat 5D spacetime. The mass spectra of the fermion Kaluza-Klein (KK) modes are continuous, and there is a series of fermion resonances. The number and lifetime of the resonances are finite and increase with the temperature parameter, but the mass of the resonances decreases with the temperature parameter. Second, in the case of dS brane, we do not find such a critical value of the temperature parameter. The mass spectra of the fermion KK modes are also continuous, and there is a series of fermion resonances. The effects of temperature on resonance number, lifetime, and mass are the same with the case of flat brane. Last, in the case of AdS brane, the critical value of the temperature parameter can less or greater than the one in the flat 5D spacetime. The spectra of fermion KK modes are discrete, and the mass of fermion KK modes does not decrease monotonically with increasing temperature parameter.

Keywords: Large Extra Dimensions, Field Theories in Higher Dimensions
1 Introduction

In the braneworld scenario our universe is a 3-brane embedded in a higher dimensional spacetime. The braneworld scenario has received a lot of attentions since it can provide us a novel approach to resolve the cosmological constant and the hierarchy problems [1–4], and reproduce the Newtonian law of gravity [5]. In the braneworld scenario all kinds of matter fields should be localized on the brane, and there is much literature on these issues [6–32]. The braneworld scenario also provides some new approaches to resolve the family and favor problems in particle physics [33–39].

Further, taking into account that our real universe is a system with the temperature. So it is interesting to investigate the effects of temperature in braneworld theories. Brevik et al. [40] investigated the quantum (in)stability of the AdS$_5$ braneworld universe at nonzero temperature. Campos discussed the critical phenomena of flat thick branes in warped spacetimes [41] with a five-dimensional (5D) complex background scalar field. Bazeia et al. [42] investigated the geometric transitions of thick braneworlds at high temperature limit. Different from the ideal thin braneworld models [4, 5], thick braneworld models usually need to introduce 5D background scalar fields to generate the branes [41, 43–69]. To introduce the temperature effects to thick braneworld theories, we can calculate the effective potential of the 5D background scalar fields at finite temperature. And the method is the same as the one proposed in [70–72] in four-dimensional flat spacetime. Ansari and Suresh [73] calculated such an effective potential of $\phi^4$ model in 5D flat spacetime with the one-loop correction. As far as we know, the effective potential of 5D scalar fields in thick braneworld theories has not been calculated. To do such a calculation is very meaningful but also difficult, not only because the spacetime is curved, but also because the background scalar field is coupled with the gravity. Although without an analysis function form of the effective potential with the temperature, one can still investigate the temperature effects
in braneworld theories [41, 42]. Within the $\phi^4$ model, only the mass parameter is corrected by the effects of temperature, so the variation of the mass parameter will reflect the effects of temperature on branes.

Our study is based on the work of Campos [41]. In this paper, the self-interaction potential of the background complex scalar field $\Phi$ has the form of

$$V(\Phi) = a|\Phi|^2 - b\phi_R(\phi_R^2 - 3\phi_I^2) + c|\Phi|^4 + C_5,$$

(1.1)

where $\phi_R$ and $\phi_I$ are the real and imaginary parts of the complex scalar field $\Phi$, respectively, and $C_5$ is a constant. If there is no background complex scalar field $\Phi$, $C_5$ will be the cosmological constant of 5D spacetime. Taking the effects of temperature into account, the mass parameter $a$ should be a function of the temperature $T$ [70–72]. But the parameters $b$ and $c$ will not vary with the temperature. This can be seen from $V(\Phi)$ after a shift of $\Phi$ around its vacuum expectation value $\Phi_0(\phi_{R0}, \phi_{I0})$: $\Phi \to \delta \Phi + \Phi_R \ [\phi_R \to \delta \phi_R + \phi_{R0}, \phi_I \to \delta \phi_I + \phi_{I0}]$. After that shift the coefficient before $|\Phi_0|^4$ is also c, so c will not vary with the temperature. The coefficients before $\phi_R^3$ and $\phi_R\phi_I^2$ are changed from $-b$ and $3b$ to $-b + 2c\delta \phi_R$ and $3b + 4c\delta \phi_R$, respectively. Because the vacuum expectation value of $\delta \phi$: $\langle 0 | \delta \phi_R | 0 \rangle = 0$, so the coefficients $b$ will also not vary with the temperature. Because we have not the exact function form of $a(T)$, we only take $a$ as the temperature parameter in this paper.

Campos found that the presence of gravity would lower the critical value of the temperature parameter of the phase transition comparing to 5D flat spacetime [41]. And the phase transition is characterized by the emergence of a double kink solution of the $\phi_I$. Below the critical value of the temperature parameter, the solution of the $\phi_I$ has a single kink form. Further, we find that, in the case of AdS brane, the critical value of $a$ can less or greater than the one in the 5D flat spacetime, and we do not find the double kink solution of the $\phi_I$ for the case of dS brane.

We also investigate the effects of temperature on fermion localization and quasi-localization on flat, dS, and AdS thick Branes. In order to localize fermions on thick branes, the coupling between fermions and the background scalar fields is needed. We consider the Yukawa coupling $\eta \bar{\Psi} \phi_I \Psi$. We find that, in the cases of flat and dS branes, the fermion zero mode can be localized on the branes, and there are quasi-localized massive fermion KK modes, namely, fermion resonances. The number and the lifetime of the fermion resonances increase with the temperature parameter $a$. But the resonance mass decreases with the increase of $a$. In the case of AdS brane, the spectrum of fermion KK modes is discrete, and the number of the discrete KK modes is infinite. The variation of masses of fermion KK modes does not decrease monotonically with the increase of $a$.

The paper is organized as follows. Firstly, the effects of temperature on flat, dS, and AdS thick branes are discussed in section 2. Secondly, the effects of temperature on the fermion localization and fermion resonances are investigated in section 3. For self-completeness and self-consistency, the Schrödinger equations and the corresponding potentials for the KK modes of left- and right-handed fermions are derived. Before performing numerical calculations, we analyze the asymptotic behaviors of the potentials at zero and infinity along the extra dimension. In subsection 3.1, we investigate the effects
of temperature on the number, lifetime, and mass of fermion resonances quasi-localized on flat and dS branes. We show the variation of masses of fermion KK modes localized on AdS brane with the temperature parameter $a$ in subsection 3.2. Finally, the conclusion and discussion are given in section 4.

2 Effects of temperature on thick branes

To investigate the effects of temperature on thick branes, we consider a 5D action with a complex scalar field $\Phi$ minimally coupled to gravity:

$$ S = \int d^4x dy \sqrt{-g} \left( \frac{1}{4} R - \frac{1}{2} g^{MN} \partial_M \Phi^* \partial_N \Phi - V(\Phi) \right), $$  

(2.1)

where $V(\Phi)$ is the same as the one in Eq. (1.1). To ensure the stability, the parameter $c$ must be positive. There are three degenerate global minima for the potential $V(\Phi)$ if $b > 0$ and $0 < a < 9a_c/8$, where $a_c = b^2/4c$ [41]. The three degenerate global minima are at $\Phi^1 = \phi_0$, $\Phi^2 = (-\frac{1}{2} + i\frac{\sqrt{3}}{2})\phi_0$, and $\Phi^3 = (-\frac{1}{2} - i\frac{\sqrt{3}}{2})\phi_0$, where $\phi_0 = \frac{3b}{8c}(1 + \sqrt{1 - 8\frac{a}{9a_c}})$ [41].

The line element in this model is assumed as

$$ ds^2 = e^{2A(z)}(\hat{g}_{\mu\nu}(x)dx^\mu dx^\nu + dz^2), $$  

(2.2)

where $\mu, \nu = 0, 1, 2, 3$, $\hat{g}_{\mu\nu}$ is the 4-dimensional metric on branes, and $e^{2A(z)}$ is the warp factor. The usual hypothesis is that $A$ and $\Phi$ are only the functions of the extra dimension coordinate $z$.

For the three types of maximally 4-symmetric branes (flat, dS and AdS), the equations of motion of $A(z)$, $\phi_I$, and $\phi_R$ have a unified form:

$$ \phi_I'' = -3A'\phi_I' + e^{2A} \frac{dV}{d\phi_I}, $$  

(2.3)

$$ \phi_R'' = -3A'\phi_R' + e^{2A} \frac{dV}{d\phi_R}, $$  

(2.4)

$$ A'' = A'^2 - \frac{1}{3}((\phi_I'^2 + \phi_R'^2) + \Lambda_4), $$  

(2.5)

$$ 6A'^2 = 2\Lambda_4 + \frac{1}{2}((\phi_I'^2 + \phi_R'^2) - 2e^{2A}V), $$  

(2.6)

where the prime stands for the derivative with respect to $z$. $\Lambda_4$ is the cosmological constant on branes, and $\Lambda_4 = 0, > 0$ and $< 0$ correspond to the cases of flat, dS and AdS branes, respectively. Equations (2.3)-(2.6) are a set of nonlinear ordinary differential equations, which can be solved numerically \(^1\) with the following boundary conditions:

$$ A(0) = A'(0) = \phi_R'(0) = \phi_I(0) = 0, $$  

(2.7)

$$ \phi_R(+\infty) = -\frac{1}{2}\phi_0, \text{ and } \phi_I(+\infty) = \sqrt{3}\phi_0. $$  

(2.8)

\(^1\)We can solve these equations with the FORTRAN code colsys and bvp solver, or the function bvp5c in Matlab.
Figure 1. The profiles of the background scalar fields $\phi_I$ and $\phi_R$ for flat brane. The three lines correspond to $a = 0.5, 0.8, 0.961$, respectively. The other parameters are set to $b = 2$ and $c = 1$.

Figure 2. The profiles of the warp factor $A$ for flat brane. The three lines correspond to $a = 0.5, 0.8, 0.961$, respectively. The other parameters are set to $b = 2$ and $c = 1$.

The solutions of $A$, $\phi_I$, and $\phi_R$ are varied with the temperature parameter $a$. For flat, dS, and AdS thick branes the variation of the solutions of $\phi_I$ and $\phi_R$ with $a$ is similar. So, we only show the solutions for the case of flat brane in Fig. 1 with $b = 2$, $c = 1$, and different values of $a$. Fig. 1 shows that the solutions of flat thick brane are dilated along the extra dimension with the increase of $a$, and the solution of $\phi_I$ tends to have a double kink form.

The solutions of $A(z)$ for flat and dS branes are similar, and the solutions for the case of flat brane are shown in Fig. 2.

For the case of AdS brane, the behavior of the warp factor is very different from the cases of flat and dS branes. This has been analyzed in Refs. [29, 74, 75]. In this case, with the metric (2.2) and the potential (1.1), the $z$ coordinate only runs from $-z_b$ to $z_b$, where $z_b$ is a finite value, and $A(z)$ is divergent at $\pm z_b$. This can be seen from the analysis of the solutions of Eqs. (2.3)-(2.6) at $z \to \pm z_b$. When $z \to \pm z_b$,

$$
\phi_R(\pm z_b) = -\frac{1}{2}\phi_0, \quad \phi_I(\pm z_b) = \pm \frac{\sqrt{3}}{2}\phi_0,
$$

(2.9)
Figure 3. The profiles of the warp factor $A(z)$ for AdS brane. The three lines correspond to $a = 1.07$, 0.9, and 0.5. The other parameters are set to $b = 2$, $c = 1$, and $\Lambda_4 = -0.1$.

and

$$\phi_R'(\pm z_b) = \phi_I'(\pm z_b) \to 0. \tag{2.10}$$

So Eqs. (2.5) and (2.6) turn into

$$A''(z \to \pm z_b) = A'^2(z \to \pm z_b) - \frac{1}{3} \Lambda_4, \tag{2.11}$$
$$6A'^2(z \to \pm z_b) = 2\Lambda_4 - e^{2A(z \to \pm z_b)} V_{z_b}, \tag{2.12}$$

where $V_{z_b} \equiv V(\phi_I(\pm z_b), \phi_R(\pm z_b))$ is a constant. The solution of the above equations is

$$A(z \to \pm z_b) \to \log \sqrt{\frac{2\Lambda_4}{V_{z_b}}} - \log \left[ \sin \left( \sqrt{-\frac{\Lambda_4}{3}} (z_b - |z|) \right) \right]. \tag{2.13}$$

So it is clear that the warp factor $A(z)$ is divergent at the boundaries $z = \pm z_b$. This is consistent with the numerical solution of $A(z)$ shown in Fig. 3, from which it can be seen that the value of $z_b$ decreases with increasing $a$.

The critical value of the bulk temperature parameter $a$ is defined such that the profile of $\phi_I$ has the form of a double kink \[41\], namely $\phi_I'(0) = 0$. For the case of five-dimensional flat spacetime ($A = 0$), we can easily determine the critical value of $a$, which is given by $a_c = b^2/(4c) \[41\]. But for five-dimensional warped spacetime with flat brane, the critical value of $a$ is not $a_c$ but a smaller effective critical value $a^*$ \[41\].

We find when $a = a^*$ the solutions of $\phi_I$, $\phi_R$, and $A$ are not unique \footnote{Because Eqs. (2.3-2.6) are non-linear, so in order to solve them numerically we need to offer initial guess solutions of $\phi_I$, $\phi_R$, and $A$. And different guess solutions follow different solutions of them.}. This means that at the critical temperature the width of flat thick brane is not fixed. The profiles of $\phi_I$ and $\phi_R$ with $a^* = 0.9617$ ($b = 2$, $c = 1$) are shown in Fig. 4 for flat brane. In the AdS brane case, we find that the effective critical value $a^*$ can greater than $a_c$. But for the dS brane, we do not find a double kink solution of $\phi_I$.

In the Appendix we have discussed the relation between $a^*$ and $a_c$ with the assumptions that as $a \to a^*$, $|(a_c - a^*)/a_c| \ll 1$ and $\phi_R(0)^2 \ll \phi_0^2$. And we show that, for the case of
Figure 4. The profiles of $\phi_{I}$ and $\phi_{R}$ with different solutions with $a_{s} = 0.9617$ for flat branes. The other parameters are set to $b = 2$ and $c = 1$.

Figure 5. The profiles of $\phi_{I}$ and $\phi_{R}$ with $a = 1.08$, $a = 1.09$, and $a = 1.10$ for AdS brane. The critical value $a_{s}$ is 1.084. The other parameters are set to $b = 2$, $c = 1$, and $\Lambda_{4} = -0.1$.

flat brane, $a_{s} < a_{c}$. But in the cases of dS and AdS branes, the relation between $a_{s}$ and $a_{c}$ is uncertain.

In the flat brane case, when the value of $a$ is greater than the critical value $a_{s}$, we do not find any solution of Eqs. (2.3)-(2.6). However, in AdS brane case, we do, and we show the profiles of $\phi_{I}$ in Fig. 5 for $a = 1.08$, $a = 1.09$, and $a = 1.10$ (the critical value $a_{s}$ is 1.084).

3 Effects of temperature on fermion localization and resonances

In order to localize a bulk fermion on these branes, we need to introduce the coupling of the fermion and the background scalar $\Phi$. A general form of the coupling is $\eta \bar{\Psi} F(\phi_{I}, \phi_{R}) \Psi$, where $\Psi$ is the fermion field and $F(\phi_{I}, \phi_{R})$ is a function of $\phi_{I}$ and $\phi_{R}$. The action of the fermion describing such coupling is

$$S_{\Psi} = \int d^{5}x \sqrt{-g} \left\{ \bar{\Psi} \Gamma^{M} (\partial_{M} + \omega_{M}) \Psi - \eta \bar{\Psi} F(\phi_{I}, \phi_{R}) \Psi \right\},$$

(3.1)
where $\Gamma^M = (e^{-A}\gamma^\mu, e^{-A}\gamma^5)$ are the curved space gamma matrices, $\eta$ is the coupling constant and $\eta > 0$. $\omega_M$ is the spin connection [76–78] and its nonvanishing components are [14, 79]

$$\omega_\mu = \frac{1}{2} A^\nu \gamma_\mu \gamma_5 + \hat{\omega}_\mu,$$

where $\hat{\omega}_\mu$ is the spin connection derived from the metric $\hat{g}_{\mu\nu}$. The resulting Dirac equation is

$$\left[ \gamma^\mu (\partial_\mu + \hat{\omega}_\mu) + \gamma^5 (\partial_z + 2A') - \eta e^A F(\phi_L, \phi_R) \right] \Psi = 0,$$

where $\gamma^\mu (\partial_\mu + \hat{\omega}_\mu)$ is the Dirac operator on branes. With the usual KK and chiral decomposition

$$\Psi(x, z) = e^{-2A} \sum_n \left( \psi_{Ln}(x) f_{Ln}(z) + \psi_{Rn}(x) f_{Rn}(z) \right),$$

where subscripts ‘L’ and ‘R’ denote the Left- and right-handed chiralities, respectively, and the four-dimensional massive Dirac equation

$$\gamma^\mu (\partial_\mu + \hat{\omega}_\mu) \psi_{Ln} = m_n \psi_{Ln}, \quad (3.4)$$

$$\gamma^\mu (\partial_\mu + \hat{\omega}_\mu) \psi_{Rn} = m_n \psi_{Rn}, \quad (3.5)$$

we can obtain the coupled equations of $f_{Ln}$ and $f_{Rn}$ [23]:

$$\left[ \partial_z + \eta e^A F(\phi_L, \phi_R) \right] f_{Ln}(z) = m_n f_{Ln}(z), \quad (3.6)$$

$$\left[ \partial_z - \eta e^A F(\phi_L, \phi_R) \right] f_{Rn}(z) = -m_n f_{Ln}(z), \quad (3.7)$$

where $f_{Ln}$ and $f_{Rn}$ satisfy the following orthonormality conditions:

$$\int_{-\infty}^{\infty} f_{Ln}^* f_{Rn} dz = \delta_{mn} \delta_{LR}. \quad (3.8)$$

Further, we obtain the Schrödinger equations [23]:

$$\left[ - \partial_z^2 + U_{IL}(z) \right] f_{Ln} = m_n^2 f_{Ln}, \quad (3.9a)$$

$$\left[ - \partial_z^2 + U_{IR}(z) \right] f_{Rn} = m_n^2 f_{Rn}, \quad (3.9b)$$

where the effective potentials are given by

$$U_{IL}(z) = \eta^2 e^{2A} F(\phi_L, \phi_R)^2 - \eta e^A F(\phi_L, \phi_R)' - \eta F(\phi_L, \phi_R)e^A A', \quad (3.10a)$$

$$U_{IR}(z) = U_{IL}(z)|_{\eta \to -\eta}. \quad (3.10b)$$

In order to determine the forms of $U_{IL}(z)$ and $U_{IR}(z)$, we should first construct the form of $F(\phi_L, \phi_R)$. There are two simplest forms of $F$: $F_1 = \phi_1$ and $F_2 = \phi_R$. With the solutions of $\phi_L$ and $\phi_R$, we find that $\phi_L$ is an odd function, and $\phi_R$ is an even function. So, in order to hold the $Z_2$ symmetry of $U_{IL}(z)$ and $U_{IR}(z)$, we choose the first form of $F$, i.e., $F_1 = \phi_1$. So

$$U_{IL}(z) = \eta^2 e^{2A} \phi_1^2 - \eta e^A \phi_1' - \eta \phi_1 e^A A', \quad (3.11a)$$

$$U_{IR}(z) = \eta^2 e^{2A} \phi_1^2 + \eta e^A \phi_1' + \eta \phi_1 e^A A', \quad (3.11b)$$
Figure 6. The profiles of the potentials of fermion KK modes for flat brane. The parameters are set to $\eta = 2$, $b = 2$, $c = 1$, and $a = 0.5, 0.8, 0.961$.

3.1 Flat and dS branes

Before solving Eq. (3.9) numerically, let us first analyze the behavior of $U_{fL}(z)$ and $U_{fR}(z)$ at $z = 0$ and $z \to \pm \infty$.

First, at $z = 0$, with the boundary conditions $A(0) = A'(0) = \phi I(0) = 0$, we get

$$U_{fL}(0) = -\eta \phi_I',$$
$$U_{fR}(0) = +\eta \phi_I'. \tag{3.12a}$$

And from Fig. 1, we see that $\phi_I'(0) > 0$, so $U_{fL}(0) > 0$ and $U_{fR}(0) < 0$ for positive coupling constant $\eta$.

Second, when $z \to \pm \infty$, $\phi_I'(z) = 0$. So we have

$$U_{fL}(z \to \pm \infty) = \eta^2 e^{2A} \phi_I^2 - \eta \phi_I e^A A'',$$
$$U_{fR}(z \to \pm \infty) = \eta^2 e^{2A} \phi_I^2 + \eta \phi_I e^A A'. \tag{3.13b}$$

And Eqs. (2.5) and (2.6) turn into

$$A'' = A'^2 - \frac{1}{3} \Lambda_4,$$
$$6A'^2 = 2 \Lambda_4 - e^{2A} V_\infty,$$ \tag{3.14}

where $V_\infty \equiv V(\phi_I(\pm \infty), \phi_R(\pm \infty))$ is a constant, and $\phi_R(\pm \infty) = -\frac{1}{2} \phi_0$, $\phi_I(\pm \infty) = \pm \frac{\sqrt{3}}{2} \phi_0$.

In the flat brane case, $\Lambda_4 = 0$, the solution of $A(z \to \pm \infty)$ is

$$A(z \to \pm \infty) = -\log \left( \sqrt{-V_\infty/6|z|} + c_1 \right), \tag{3.16}$$

where $c_1$ is an integration constant. So $e^{2A}|_{z \to \pm \infty} = 1/(\sqrt{-V_\infty/6|z|} + c_1)^2 |_{z \to \pm \infty} \to 0$ and $A'(z)|_{z \to \pm \infty} = -1/(\sqrt{-V_\infty/6|z|} + c_1)|_{z \to \pm \infty} \to 0$. Then we reach the conclusion that $U_{fR} \to 0$ and $U_{fL} \to 0$ when $z \to \pm \infty$.\～
Figure 7. The profiles of the zero mode of left-handed fermions for flat brane. The parameters are set to $\eta = 2$, $b = 2$, $c = 1$, and $a = 0.5, 0.8, 0.961$.

In the dS brane case, $\Lambda_4 > 0$, the solutions of $A(z)$, $e^{A(z)}$, and $A'(z)$ at $z \to \pm\infty$ are

\begin{align*}
A(z) &= \log \left[ \sqrt{\frac{2\Lambda_4}{V_\infty}} \text{sech} \left( \sqrt{\frac{\Lambda_4}{3}} (|z| + c_2) \right) \right], \quad (3.17) \\
e^{A(z)} &= \sqrt{\frac{2\Lambda_4}{V_\infty}} \text{sech} \left( \sqrt{\frac{\Lambda_4}{3}} (|z| + c_2) \right) \to 0, \quad (3.18) \\
A'(z) &= \mp \sqrt{\frac{\Lambda_4}{3}} \tanh \left( \sqrt{\frac{\Lambda_4}{3}} (|z| + c_2) \right) \to \mp \sqrt{\frac{\Lambda_4}{3}}. \quad (3.19)
\end{align*}

Then we come to the conclusion that $U_{fL}$ and $U_{fR}$ vanish at $z \to \pm\infty$.

With the numerical solutions of $A(z)$, $\phi_l$, and $\phi_R$, we plot the potentials $U_{fL}$ and $U_{fR}$ in Fig. 6 for flat brane. The profiles of the potentials for dS brane are similar with the case of flat brane, and we do not show them again. We can see that, for $\eta > 0$, the potential of left-handed KK modes $U_{fL}$ is a modified volcano-type potential, which has a well lower than zero, so only the left-handed fermions have zero mode. The Schrödinger Eqs. (3.9) can be solved using the Numerov method [80] with two initial conditions at $z = 0$, which can be set as

\begin{align*}
f_{L,R}(0) = d_1, f'_{L,R}(0) = 0 \quad (3.20)
\end{align*}

for the even parity KK modes and

\begin{align*}
f_{L,R}(0) = 0, f'_{L,R}(0) = d_2 \quad (3.21)
\end{align*}

for the odd parity KK modes, where $d_1$ and $d_2$ are arbitrary constants. Here we choose $d_1 = d_2 = 1$. The profiles of the zero mode for flat brane are shown in Fig. 7 with different values of the temperature parameter. The case for dS brane are similar.

The volcano-type potential implies that there will not exist discrete spectrum of fermion KK modes, but there may exist fermion resonances [79, 81–83]. The massive
KK modes cannot be normalized because their wave functions are oscillating when far away from the brane along the extra dimension. So Ref. [82] proposed a function

\[ P_{L,R}(m) = \frac{\int_{-z_c}^{z_c} |f_{L,R}(z)|^2 dz}{\int_{-z_{max}}^{z_{max}} |f_{L,R}(z)|^2 dz} \]  

as the relative probability for finding the resonances on a brane. Here \( 2z_c \) is about the width of the thick brane and \( z_{max} \) is set to \( z_{max} = 10z_c \). So for KK modes with \( m^2 \gg V_{L,R}^{max} \) (\( V_{L,R}^{max} \) is the maximum value of \( V_{L,R} \)), \( f_{L,R} \) can be approximately taken as plane waves, and the value of \( P_{L,R}(m) \) will trend to 0.

As an example, we plot the profiles of \( P_{L,R}(m) \) in Fig. 8 corresponding to \( a = 0.961 \) for flat brane. In this figure each peak corresponds to a resonant state. We can estimate the lifetime \( \tau \) of a resonance as \( \tau \sim \Gamma^{-1} \), where \( \Gamma = \delta m \) is the full width at half maximum of the peak [84]. It can be seen that the spectra of resonances are the same for both the left- and right-handed fermions.

In order to investigate the effects of temperature on resonances. We solve Eq. (3.9) with the following values of \( a \):

\[ a = \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.92, 0.94, 0.96, 0.961, 0.9612, 0.9614, 0.9616\} \]

Here we only take those resonances with \( m^2 \leq V_{L,R}^{max} \) into account.

Because the spectra of resonances or bound states for left- and right-handed fermions are the same, here and after we only discuss the spectrum of left-handed fermions.

In order to see the effect of temperature on the fermion potentials \( U_{IL} \) and \( U_{IR} \) more intuitively, we can take the volcano box potential approximation [45] for the potentials. For example in Fig. 9, the well’s depth, height, and width are \( V_1, V_2, \) and \( 2z_1 \), respectively. The width of barrier is \( |z_2 - z_1| \). Now back to see the potential \( U_{IL} \) in Fig. 6, we can find that with the increase of \( a \) the width of barrier and the width of the well are increasing, but the depth and height of the well are decreasing. Because the variation of height of the well is small and all the KK modes with \( m^2 \geq 0 \), from the knowledge of quantum mechanics, we can expect that with the increase of \( a \) the number of the resonances is increasing and their

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**Figure 8.** The profiles of \( P_{L,R} \) for left-handed fermions for flat brane with \( a = 0.961, b = 2, c = 1, \) and \( \eta = 2. \)**
Figure 9. The fermion potential $U_{\text{fL}}$ with $a = 0$ and its volcano box potential approximation. The other parameters are set to $b = 2$, and $c = 1$.

Figure 10. The variation of the resonance mass $m$ with $a$ for left-handed fermions for flat brane. The other parameters are set to $b = 2$, $c = 1$, and $\eta = 2$.

masses are decreasing. The numerical results support this expectation. The variation of the resonance mass $m$ with $a$ is shown in Fig. 10. We can see the resonance mass decreases with the increase of $a$. While the resonance lifetime $\tau$ increases with $a$, which is plotted in Fig. 11.

The variation of the number of resonances $n$ with the temperature parameter $a$ is shown in Fig. 12. It can be seen that the number of resonances $n$ is increasing with the temperature parameter $a$, and there is a threshold value for $a$ to produce more resonances. For example, for $a < 0.4$ there is no resonance, and for $0.4 \leq a < 0.9$ there is a resonance. This is very similar to the phenomenon of the particle generation in colliders, namely, if we want to find more particles we need to increase the center-mass energy and the generation of particles need the center-mass energy greater than a threshold value.

3.2 AdS brane

When $z \to 0$, the behaviors of the potentials of left- and right-handed fermions for AdS brane are similar to the ones in the flat braneworld scenario, so we will not repeat them here.
Figure 11. The variation of the resonance lifetime $\tau$ with $a$ for left-handed fermions for flat brane. The other parameters are set to $b = 2$, $c = 1$, and $\eta = 2$.

Figure 12. The variation of the number of resonances $n$ with $a$ for left-handed fermions for flat brane. The other parameters are set to $b = 2$, $c = 1$, and $\eta = 2$.

But when $z \to \pm z_b$, the behaviors of $A$ are very different from the ones in the cases of flat and dS branes, so we need to analyze the behaviors of the potentials of left- and right-handed fermions carefully. When $z \to \pm z_b$, the potentials turn into

$$U_{fL} = \eta e^A \phi_I (\eta e^A \phi_I - A'),$$

$$U_{fR} = \eta e^A \phi_I (\eta e^A \phi_I + A').$$

Inserting Eq. (2.13) into the above equations, we get

$$U_{fL} = -\frac{\Lambda_4}{3} \frac{\eta}{\eta_0} \frac{\eta/\eta_0 - \cos(z')}{\sin^2(z')},$$

$$U_{fR} = -\frac{\Lambda_4}{3} \frac{\eta}{\eta_0} \frac{\eta/\eta_0 + \cos(z')}{\sin^2(z')} ,$$

where $z' = \sqrt{-\Lambda_4/3(z_b - |z|)}$, $\eta_0 \equiv \frac{v_0}{\sqrt{3} v_0}$ with $v_0 \equiv V(\phi_I = \frac{\sqrt{3}}{2} \phi_0, \phi_R = -\frac{1}{2} \phi_0)$. Then we can see that, if $\eta > \eta_0$,

$$U_{fL}(z \to \pm z_b) \to +\infty,$$

$$U_{fR}(z \to \pm z_b) \to +\infty;$$
The plots of $U_{fL}$ and $U_{fR}$ for AdS brane. The other parameters are set to $\eta = 2$, $b = 2$, $c = 1$, and $\Lambda_4 = -0.1$.

if $\eta = \eta_0$,

\[ U_{fL}(z \to \pm z_b) \to -\frac{\Lambda_4}{6}, \quad (3.26a) \]
\[ U_{fR}(z \to \pm z_b) \to +\infty; \quad (3.26b) \]

if $0 < \eta < \eta_0$,

\[ U_{fL}(z \to \pm z_b) \to -\infty, \quad (3.27a) \]
\[ U_{fR}(z \to \pm z_b) \to +\infty. \quad (3.27b) \]

In the cases of $\eta = \eta_0$ and $0 < \eta < \eta_0$, the behaviors of left- and right-hand fermions potentials at $z \to \pm z_b$ are so different, and we will not expect the left- and right-handed fermions have the same KK modes spectra. So we only discuss the case of $\eta > \eta_0$. The plots of the potentials of left- and right-hand fermions are shown in Fig. 13.

From Fig. 13 and the above analysis, we know that, for $\eta > \eta_0$, all massive fermion KK modes are bound states and there is a discrete mass spectrum for them. We use the method proposed in Ref. [85] to calculate the fermion mass spectrum. And we calculate the fermion mass spectra with a set of values of $a$: $a = \{0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1, 1.08, 1.10\}$, and the other parameters are set to $\eta = 2$, $b = 2$, $c = 1$, and $\Lambda_4 = -0.1$. The masses of 39 KK modes are obtained.

From Figure 13, we can see that the profile of potentials $U_{fL}$ and $U_{fR}$ is similar to that of the infinite square-well potential. And we know that a particle in the infinite square-well potential will hold the the energy of $E_n \propto \frac{n^2}{d^2} \quad [n = 1, 2, 3, \ldots]$, where $d$ is the width of well, and $E_n$ decreases with $d$. Figure 13 shows that with the increase of $a$ the width of the potentials is decreasing, so we except that the masses of the KK modes increase with $a$. But this is not all consistent to our numerical results. For the 28th excited state and the ones above it, the masses of KK modes increase with $a$. For example, we show the
Figure 14. The $m-a$ curves of 28th-35th excited states for left-handed fermion with parameters $\eta = 2$, $b = 2$, $c = 1$, and $\Lambda_4 = -0.1$.

Figure 15. The $m-a$ curves of 1st-12th excited states for left-handed fermions for AdS brane with parameters $\eta = 2$, $b = 2$, $c = 1$, and $\Lambda_4 = -0.1$.

$m-a$ curves of 28th-35th excited states in Fig. 14. But the ones of below the 28th excited state masses of the KK modes do not vary monotonically with the increase of $a$ (see Fig. 15). This is because with the increase of $a$ that the values of the bottom of potentials are also not varying monotonically with the increase of $a$. For example the first excited state (left-handed fermion) mass $m_1(a = 0.2)$ greater than $m_1(a = 1.1)$ is because the value of the bottom of $U_{fL}$ at $a = 0.2$ is greater than the one of $U_{fL}$ at $a = 1.1$.

4 Conclusion and discussion

With the model used in this paper, we found the effects of temperature on flat and dS branes are similar, except the existence of the critical temperature. In the case of flat brane, there is a critical temperature. While it does not exist for dS brane. The thickness of flat brane is uncertain at the critical temperature. We think this is related to the potential given in (1.1). If we choose the $\phi^4$ model of a real scalar field, there will not be such a critical temperature even for flat brane.

In the case of AdS brane, we found that the critical temperature is greater than the one in 5D flat spacetime. This is just opposite to the case of flat brane, for which the
critical temperature is less than the one in 5D flat spacetime [41].

The variation of the temperature parameter will affect the spectrum of fermion KK modes. First, in the cases of flat and dS branes, the spectrum is continuous. We found that, for some parameters, there are fermion resonances on the brane. The fermion resonances can be thought as quasi-localized fermions with finite lifetime staying on the brane. The number of fermion resonances is finite and increases with the temperature parameter $a$, so does the resonance lifetime. This means that the brane with a bigger value of temperature parameter can trap more quasi-localized fermions on it. The masses of the quasi-localized fermions decrease with the increase of $a$. Second, in the case of AdS brane, if the Yukawa coupling is larger than some critical coupling constant, then there is a discrete spectrum of fermion KK modes, and all the KK modes (four-dimensional fermions) are localized on the brane. But the masses of fermion KK modes do not vary monotonically with the increase of the temperature parameter.

From the model discussed in this paper we can see: (a) In the case of AdS brane, the fermions will be bounded on our brane permanently and the number of fermions is infinite. But the model with an AdS brane is unreal. This is because we know that in our real universe the 4D cosmological constant is positive. (b) Corresponding to the AdS brane, the flat and dS branes are more similar to our real universe. In these cases the fermions are quasi-localized states, except the zero mode, with a finite lifetime staying on the branes. This means that the fermions can dissipate into the fifth dimension just like the quasi-localized gauge field [12] and the quasi-localized gravitons [84, 86–88]. From the point of view of 4D observer, one will have the chance to find energy non-conservation in the collider.

If we assume that the temperature parameter $a$ is a monotone increasing function of temperature $T$, then, for a collider experiment, the number of the produced new fermion resonances will increase with the center-of-mass energy of beams. This is consistent to results of the experiment. Further, our another result that the lifetime of fermions increase with the temperature, would be examined if the collider experiment find the signal of fermions dissipating into the fifth dimension.

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5 Appendix

Here we give the discussion about the relation between $a_*$ and $a_c$.

First at the point of $z = 0$, substituting $A(0) = A'(0) = \phi_I(0) = \phi_R'(0) = 0$ into Eq. (2.6) we get:

$$2\Lambda_4 + \frac{1}{2}\phi_I'(0) - V_0 = 0,$$

where

$$V_0 = V(\phi_I(0), \phi_R(0))
= a\phi_R^2(0) - b\phi_R^3(0) + c\phi_R^4(0) + C_5. \quad (5.2)$$

Because $\phi_R(z)$ is an even function of $z$, we can Taylor expand $\phi_R(z)$ around the point of $z = 0$:

$$\phi_R(z) = d_0 + d_2z^2 + d_4z^4 + \cdots. \quad (5.3)$$

So $\phi_R(0) = d_0$. Substituting this into Eq. (5.2) we get

$$V_0 = ad_0^2 - bd_0^3 + cd_0^4 + C_5. \quad (5.4)$$

And Eq. (5.1) is rewritten as

$$2\Lambda_4 + \frac{1}{2}\phi_I'(0) = ad_0^2 - bd_0^3 + cd_0^4 + C_5. \quad (5.5)$$

Further, when $z$ tends to its right boundary (for flat and dS branes it is $\infty$ but for AdS brane it is $z_b$, here we use symbol $\infty$ to stand for it).

$$V_\infty = a|\Phi(\infty)|^2 - b\phi_R(\infty)(\phi_R(\infty))^2 - 3\phi_I(\infty)^2 + c|\Phi(\infty)|^4 + C_5. \quad (5.6)$$

Substituting Eq. (2.8) into the above equation leads to

$$V_\infty = a\phi_0^2 - b\phi_0^3 + c\phi_0^4 + C_5, \quad (5.7)$$

where $\phi_0 = \frac{3\phi}{8a}(1 + \sqrt{(9 - 8a)/a_c}/3)$. So

$$C_5 = V_\infty - (a\phi_0^2 - b\phi_0^3 + c\phi_0^4). \quad (5.8)$$

Substituting the above expression for $C_5$ into Eq. (5.5) gives

$$a = (b\phi_0 - c\phi_0^2)\frac{\phi_0^2}{\phi_0^2 - d_0^2} - \frac{b - cd_0}{\phi_0^2 - d_0^2}d_0^2 - \frac{2\Lambda_4}{\phi_0^2 - d_0^2} + \frac{V_\infty}{\phi_0^2} - \frac{1}{2}\frac{\phi_I'(0)}{\phi_0^2 - d_0^2}. \quad (5.9)$$

When $\phi_I'(0) \to 0$, $a \to a_*$. We assume that $d_0^2 \ll \phi_0^2$ (this assumption is supported by the numerical results which can be seen from Figs. 1, 4, and 5). Neglecting the terms containing square and higher powers of $d_0$, Eq. (5.9) reads

$$a_* = (b\phi_0 - c\phi_0^2) - \frac{1}{\phi_0^2}(2\Lambda_4 - V_\infty). \quad (5.10)$$
Further we take \(a_s = a_c(1 - \delta)\) \([a_c = b^2/(4c) = 1]\) and assume that \(|\delta| \ll 1\). So \(\phi_0 = \frac{a_s}{a_c}(1 + \sqrt{1 + 2\psi_1})\). The Taylor expansions of Eq. (5.10) respect to \(\delta\) is

\[
\delta = a_c(1 - \delta^2 + 4\delta^3) - \frac{4c^2}{b^2}(2a_c - V_\infty)(1 - 2\delta + 7\delta^2 - 32\delta^3) + \mathcal{O}(\delta)^4. \tag{5.11}
\]

After neglecting the terms containing square and higher powers of \(\delta\), Eq. (5.11) reads

\[
a_s = a_c - \frac{4c^2}{b^2}(2a_c - V_\infty)(1 - 2\delta). \tag{5.12}
\]

So in the case of flat brane, we have \(\Lambda_4 = 0\) and \(V_\infty \leq 0\) (this can be found from Eq. (3.16)), which leads to \(a_s \leq a_c\).

But in the cases of dS and AdS branes we cannot confirm whether \(a_s > a_c\) or \(a_s \leq a_c\). That is because that, in the case of dS brane \(\Lambda_4 > 0\) and \(V_\infty > 0\) (this can be found from Eq. (3.17)), and in the case of AdS brane \(\Lambda_4 < 0\) and \(V_\infty < 0\) (this can be found from Eq. (2.13)), we are not able to determine whether \((2a_c - V_\infty) > 0\) or \(\leq 0\). Even in the case of dS brane, we cannot confirm the existence of the \(a_s\) using the numerical method.

In the above discussion we have made use of two assumptions: \(|\delta| \ll 1\) and \(d_0^2 \ll \phi_0^2\). This will weak the efficiency of our conclusion. So here we make use of the numerical method to check the consistency between the two assumptions and Eq. (5.12). From Eq. (5.12) we have

\[
\delta = 1 - \frac{b^2}{b^2 + 4c^2(2a_c - V_\infty)}. \tag{5.13}
\]

Using the numerical method, we have obtained a set of values of \(V_\infty, d_0\) and \(\delta\) corresponding to \(a = a_s\). All results are shown in Table 1 for the case of AdS brane (the corresponding solutions of \(\phi_1\) are shown in Fig. 16) and Table 2 for the case of flat brane (the corresponding solutions of \(\phi_1\) are shown in Fig. 17), which show that the value of \(a_c - a_s\) is almost the same with the \(\delta\) obtained from Eq. (5.12). So the consistency is satisfied.

**Table 1.** The numerical results of \(V_\infty, d_0\) and \(\delta\) corresponding to \(a = a_s\) for the case of AdS brane with \(b = 2, c = 1, \) and \(a_c = 1\).

| \(V_\infty\)  | \(d_0\)      | \(a_s\) | \(a_c - a_s\) | \(\delta\)          |
|-------------|-------------|--------|--------------|----------------------|
| -0.041001   | -0.000327   | 0.999  | 0.001        | 0.000999             |
| -0.052821   | -0.000350   | 1.028  | -0.028       | -0.0279383           |
| -0.071759   | -0.000366   | 1.051  | -0.051       | -0.0506862           |
| -0.095345   | -0.000339   | 1.070  | -0.070       | -0.0691242           |
| -0.123884   | -0.000382   | 1.084  | -0.084       | -0.082387            |

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Figure 16. The solutions of $\phi_I$ with different values of $\Lambda_4$ (given in Table 1) corresponding to $a = a_*$ for the case of AdS brane. The other parameters are set to $b = 2$ and $c = 1$.

Table 2. The numerical results of $V_\infty$, $d_0$ and $\delta$ corresponding to $a = a_*$ for the case of flat brane with $b = 2$, $c = 1$, and $a_c = 1$. The solutions I, II, and III of $\phi_I$ are showed in Fig. (17), and they correspond to the same value of $a_* = 0.9617$.

| $\phi_I$ | $V_\infty$ | $d_0$ | $a_*$ | $a_c - a_*$ | $\delta$ |
|----------|-------------|-------|-------|-------------|----------|
| solution I | -0.039713 | -0.000103 | 0.9617 | 0.0383 | 0.038196 |
| solution II | -0.039714 | -0.000064 | 0.9617 | 0.0383 | 0.038197 |
| solution III | -0.039714 | -0.000017 | 0.9617 | 0.0383 | 0.038197 |

Figure 17. Three solutions of $\phi_I$ with the same value of $a_* = 0.9617$ for the case of flat brane. The other parameters are set to $b = 2$ and $c = 1$.

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