Was The Electromagnetic Spectrum A Blackbody Spectrum In The Early Universe?

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Abstract

It is assumed, in general, that the electromagnetic spectrum in the Primordial Universe was a blackbody spectrum in vacuum. We derive the electromagnetic spectrum, based on the Fluctuation-Dissipation Theorem that describes the electromagnetic fluctuations in a plasma. Our description includes thermal and collisional effects in a plasma. The electromagnetic spectrum obtained differs from the blackbody spectrum in vacuum at low frequencies. In particular, concentrating on the primordial nucleosynthesis era, it has more energy for frequencies less than 3 to 6$\omega_{pe}$, where $\omega_{pe}$ is the electron plasma frequency.

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I. INTRODUCTION

It is usually assumed in Cosmology that the primordial plasma was a homogeneous plasma and that the electromagnetic field was a blackbody spectrum in vacuum. These assumptions are used, for example, in standard Big Bang Nucleosynthesis calculations. Deviations from a blackbody spectrum in vacuum and a homogeneous plasma can affect Primordial Nucleosynthesis and in this letter we concentrate on this epoch.

In the epoch of Big Bang Nucleosynthesis, the universe was a thermal bath of photons, electrons, positrons, baryons and neutrinos. The usual treatment considers the universe as a homogeneous plasma in thermal equilibrium and the electromagnetic field as a blackbody spectrum in vacuum. For example, in the energy density calculation, the energy density of the photons is given by the energy density of the blackbody spectrum in vacuum.

In this manner, the Primordial Universe is treated as an ideal gas: collective effects are assumed to be negligible. However, a plasma differs from an ideal gas due to correlations, for example, two-particle correlations. In an ideal gas, the distribution function for two-particles is: $F_2(x_1, x_2) = F_1(x_1)F_2(x_2)$. In a plasma we have an additional term $F_2(x_1, x_2) = [1 + P_{12}(x_1, x_2)]F_1(x_1)F_2(x_2)$, where $P_{12}$ is the Debye-Huckel screening. ($P_{12} \propto \exp(- |x_2 - x_1|/\lambda_D)/|x_2 - x_1|$, where $\lambda_D$ is the Debye length.)

A plasma, even in thermal equilibrium, has fluctuations, that is, the physical variables such as temperature, density and electromagnetic fields, fluctuate. Even for a non-magnetized plasma, where the average magnetic field is zero, $\langle B \rangle = 0$, the squared average is not zero: $\langle B^2 \rangle \neq 0$.

The study of the electromagnetic fluctuations in a plasma has been made in numerous studies, including those of Dawson, Rostoker et al., Sitenko and Gurin and Akhiezer et al. Most of the results are compiled in Sitenko and Akhiezer et al. The electromagnetic fluctuations are described by the Fluctuation- Dissipation Theorem. The intensity of such fluctuations is highly dependent on how the plasma is described, for example, on the dissipation mechanisms present in it. It is necessary to describe the
plasma in the most complete way.

Cable and Tajima [3] (see also [4,5]) studied the magnetic field fluctuations, for several cases. Two of there descriptions concern the primordial plasma, which is an isotropic, non-magnetized and non-degenerate plasma. In particular, they studied: a) a cold, gaseous plasma and b) a warm, gaseous plasma described by kinetic theory.

In their study, Cable and Tajima [3] in case (a) used the cold plasma description with a constant collision frequency. In case (b) they analyzed the spectrum only for low frequencies, with the warm plasma description for phase velocity $\omega/k$ less or equal to the thermal velocity of the electrons, $v_e$, and the ions, $v_i$, in a collisionless description.

Through the study of the electromagnetic fluctuations, that is, the magnetic and electric field transverse fluctuations, given for example by the Fluctuation-Dissipation Theorem, a reliable electromagnetic spectrum can be obtained. In this letter, we present a model that extends the work of Cable and Tajima [3]. It includes in the same description collisional and thermal effects. By using the Fluctuation-Dissipation Theorem relations, we derive the electromagnetic spectrum in a plasma. We concentrated, in particular, in the primordial nucleosynthesis era, in the case of the electron-positron plasma at high temperatures and the electron-proton plasma at low temperatures. The first case is the plasma at the beginning of the primordial nucleosynthesis, when the number of electrons and positrons was comparable to the number of photons. The second case is the plasma at low temperatures, after the annihilation of the electrons and positrons where the number of electrons and protons was $\eta \sim 10^{-10}$ smaller than the number of photons.

In Section II we present the expression of the electromagnetic fluctuations, in Section III our model, and in Section IV a discussion of our results and conclusions.

II. ELECTROMAGNETIC FLUCTUATIONS

The spectrae of fluctuations of the magnetic and electric transverse fields, for an isotropic plasma, given by the Fluctuation-Dissipation Theorem (for the deduction see Sitenko and
Akhiezer et al. [2]), are
\[
\langle B^2 \rangle_{k\omega} = \frac{2 \hbar}{e^{\hbar\omega/T} - 1} \left( \frac{k c}{\omega} \right)^2 \frac{\text{Im} \varepsilon_T}{\varepsilon_T - \left( \frac{k c}{\omega} \right)^2} ; \quad \langle E_{T^2} \rangle_{k\omega} = \frac{2 \hbar}{e^{\hbar\omega/T} - 1} \left( \frac{k c}{\omega} \right)^2 \frac{\text{Im} \varepsilon_T}{\varepsilon_T - \left( \frac{k c}{\omega} \right)^2}, \tag{1}
\]
where \( \varepsilon_T \) is the transverse dielectric permittivity of the plasma.

An intuitive way to understand the above expressions, is that the Fluctuation-Dissipation Theorem takes into account the emission and absorption processes in a plasma, and knowing that in equilibrium they are equal, the fluctuation level is obtained.

To determine the fluctuations of the magnetic and electric transverse fields in a plasma in equilibrium or quasi-equilibrium, it is sufficient to know the transverse dielectric permittivity of the plasma, in particular, the dissipation mechanisms present for each frequency and wave number (\( \text{Im} \varepsilon_T \)). This depends on the treatment used to describe the plasma.

Another important feature of the electromagnetic spectral distributions can be seen from Eq. (1). The equation \( \varepsilon_T(\omega, k) - \left( \frac{k c}{\omega} \right)^2 = 0 \) determines the transverse eigenfrequencies of the plasma. In the transparency region (\( \text{Im} \varepsilon_T \ll \text{Re} \varepsilon_T \)), the magnetic and the electric field transverse spectrae have \( \delta \)-function-like maximae near the eigenfrequencies (i.e., the frequency spectrum of the fluctuations contains only the transverse eigenfrequencies in the plasma). Knowing that photons have the dispersion relation in the plasma \( \omega^2 = \omega_p^2 + k^2 c^2 \), we note that for frequencies \( \omega \gg \omega_p \) the eigenfrequencies dominate and the electromagnetic transverse field spectrum behaves like a blackbody spectrum.

### III. BGK COLLISION TERM

To describe completely the plasma, we present a model that includes thermal effects as well as collisional effects. For this we need a kinetic description that takes into account collisions. We use the Vlasov equation in first order [3] (in the plasma parameter \( g = 1/n \lambda_D^3 \), where \( \lambda_D \) is the Debye length and \( n \) is the particle density). The collisions are described by the collision term in this equation.

Charged particles simultaneously interact with all the particles in the Debye sphere,
which is a large number and the Fokker-Planck collision term is most appropriate for a fully
ionized plasma. It describes the effect of the microscopic fields produced by all the particles
in the plasma. However, the kinetic equations with the Fokker-Planck collision term are very
hard to solve. Therefore, we used the BGK collision term as a rough guide to the inclusion
of collisions in the plasma. The BGK collision term describes the binary collisions as used
by Cable and Tajima \cite{3} and is given by, (for a derivation see Clemnow and Dougherty or
Alexandrov et al. \cite{6})

\[
\left( \frac{\partial f}{\partial t} \right)_C = -\eta (f - f_{\text{max}}),
\]

where \( \eta \) is the collision frequency (considered constant and equal to the Coulomb collision
frequency). \( f_{\text{max}} \) is given by \( f_{\text{max}}(x, v, t) = N(x, t)f_0(v)/N_0 \), where the number density is
\( N = N_0 + N_1 \) and \( f = f_0 + f_1 \), \( f_0 \) being the unperturbed Maxwellian distribution. (This
collision term conserves the number of particles.) Substituting this collision term in the
equation of Vlasov in first order, it is straightforward to obtain, for an isotropic plasma, the
transverse dielectric permittivity \( \varepsilon_T \), (generalized for several species):

\[
\varepsilon_T(\omega, k) = 1 + \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega^2} \left( \frac{\omega}{\sqrt{2}kv_{\alpha}} \right) Z \left( \frac{\omega + i\eta_{\alpha}}{\sqrt{2}kv_{\alpha}} \right),
\]

where \( \alpha \) is the label for each species of the plasma, \( v_{\alpha} \) the thermal velocity for each species
and \( Z(z) \) the Fried & Conte function \cite{7},

\[
Z(z) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dt \ e^{t^2}. \quad (4)
\]

(If relativistic temperature effects are included, the substitution \( \omega_{p\alpha} \rightarrow \omega_{p\alpha}/\sqrt{\gamma} \) is made.)

It is interesting to comment and emphasize that both the cold plasma description and the
warm plasma collisionless description, used by Cable and Tajima \cite{3}, are particular solutions
of this model. For \( |z|^2 \gg 1 \), where \( z = (\omega + i\eta)/\sqrt{2}kv_e \), we obtain the cold plasma dielectric
permittivity and for \( \eta \rightarrow 0 \) the warm plasma collisionless dielectric permittivity.

We substitute the transverse dielectric permittivity in Eq. (1), obtaining the magnetic
and the electric transverse field spectrum, \( \langle B^2 \rangle_{k\omega} \) and \( \langle E_T^2 \rangle_{k\omega} \), respectively. Integrating
over wave number (and dividing by $(2\pi)^3$) gives us the magnetic and electric field transverse spectrae, $\langle B^2 \rangle_\omega$ and $\langle E_T^2 \rangle_\omega$, respectively.

Our model uses a kinetic theory description with a collision term that describes the binary collisions in the plasma and a cut-off has to be taken, since for very small distances the energy of the Coulomb interactions of the particles exceeds their kinetic energy which violates the applicability of the condition of the perturbation expansion (in the plasma parameter $g \ll 1$). This occurs approximately for distances $r_{\text{min}} \sim e^2/T$, or more exactly, the distance of closest approach between a test particle and an electron in a plasma, $r_{\text{min}} = (k_{\text{max}})^{-1} \approx [Mv^2/(m + M) | eq | ]^{-1}$, where $M$, $v$ and $q$ are respectively, the mass, velocity and charge of the test particle [8].

Treating properly the effects of distant encounters, the cut-off procedure can be removed. This was shown by Thompson and Hubbard, and Hubbard, in several works [9], analyzing the Fokker-Planck equation and its coefficients. They showed that the cut-off procedure is unnecessary, when higher order terms in the Fokker-Planck equations are maintained. The kinetic equations with the Fokker-Planck collision term are very hard to solve, however. We used the BGK collision term where a cut-off is necessary. We took $k_{\text{max}}$ consistent with this collision term.

The electromagnetic spectrum is obtained by summing the magnetic and electric field transverse spectrae:

$$S(\omega) = \frac{\langle B^2 \rangle_\omega}{8\pi} + \frac{\langle E_T^2 \rangle_\omega}{8\pi}.$$  \hspace{1cm} (5)

In Figure 1a, we plot the electromagnetic spectrum $S(\omega)$ given by Eq. 5 (divided by the normalization $S_0 = \omega_{pe}^2 k_B T / (c^3)$) vs $\omega / \omega_{pe}$. We study an electron-positron plasma at $T = 10^{10}$ K and $n_e = 1.4 \times 10^{31}$ $cm^{-3}$. This is the plasma at the beginning of the Primordial Nucleosynthesis era, when neutrinos decoupled from the plasma and the neutron to proton ratio become frozen-in. (This ratio essentially determines the primordial $^4He$ abundance.) The dotted curve is $S(\omega)$ (our model) compared to the blackbody spectrum in vacuum (the solid curve). In Figure 1b, we plot the same curves as in Figure 1a, but
extended to high frequencies. In Figure 1c and 1d, we did the same as Figure 1a and 1b, but for an electron-proton plasma with \( T = 10^9 \) K and \( n_e = 5.4 \times 10^{26} \). In the epoch of Primordial Nucleosynthesis, at lower temperatures, the electrons and positrons annihilate and the plasma is reduced to a plasma of protons and electrons.

It can be seen that we obtain the blackbody spectrum naturally for high frequencies, and in the case of the high-temperature plasma \( (T = 10^{10} \) K), for frequencies \( \omega \leq 3 \omega_{pe} \), the spectrum has more energy than the blackbody spectrum. In the case of the low-temperature plasma \( (T = 10^9 \) K) it has more energy for frequencies \( \omega \leq 6 \omega_{pe} \).

**IV. DISCUSSION AND CONCLUSIONS**

The unique manner to obtain the electromagnetic transverse spectrum is analyzing the magnetic and electric field transverse fluctuations. This is the only manner to obtain information, not only about modes that propagate, like photons, but also modes that do not propagate. These modes appear, not only at low frequencies but also at high frequencies, resulting from the correlations in the plasma. Only at very high frequencies, the photons contribute uniquely to the magnetic and electric field transverse spectrae.

We present a model that incorporates, in the same description, the thermal and collisional effects and used the *Fluctuation-Dissipation Theorem* that describes the electromagnetic fluctuations. We use the Vlasov equation with the BGK collision term.

The final electromagnetic spectrum for the primordial plasma at the epoch of Big Bang Nucleosynthesis behaves like a blackbody spectrum in vacuum for high frequencies. However, for low frequencies, it is distorted. It has more energy than the blackbody spectrum in vacuum. In the case of the high-temperature plasma \( (T = 10^{10} \) K), this range is for frequencies \( \omega \leq 3 \omega_{pe} \) and for the low-temperature plasma \( (T = 10^9 \) K) the range is for \( \omega \leq 6 \omega_{pe} \), where \( \omega_{pe} \) is the electron plasma frequency.

This additional energy is due to the collective modes of the plasma. The reason why the collective modes of the plasma can have more energy for \( \omega \leq \omega_{pe} \) than the photons in
vacuum, can be understood as follows. Photons are massless bosons with the dispersion relation $\omega^2 = k^2 c^2$. For the energy interval $0 \leq \omega \leq \omega_{pe}$, the wave number interval is $k = 0$ to $k = \omega_{pe}/c$. A relatively small amount of phase space is involved. For the collective motions of the plasma, in general, we have a larger amount of phase space. For example, for plasmons with energy $\omega \sim \omega_{pe}$, the amount of phase space extends to a maximum $k$ of $k_D \simeq \omega_{pe}/v_T$, where $v_T$ is the thermal electron velocity, which is greater than $\omega_{pe}/c$ for the photons. In general, for a given frequency for $\omega < \omega_{pe}$, the greater phase space available to the collective modes of the plasma (than that of the photons) implies more energy, or a higher spectrum.

This result (the additional energy that appears in the electromagnetic spectrum compared to the blackbody energy usually assumed) can affect several fields in Cosmology, in particular, Big Bang Nucleosynthesis. This extra energy, that has not been previously taken into account, causes the Universe to expand more rapidly at a given temperature. In particular, it causes the neutrinos to decouple earlier (at a higher temperature) and the neutron to proton ratio to freeze-in at a higher value. (In order to estimate the total additional energy, we have to add also, the longitudinal energy due to the longitudinal electric field spectrum. For $T = 0.8\ MeV$, we obtain for the additional energy, $\Delta \rho \simeq 1\% \rho_\gamma$, where $\rho_\gamma = \rho_{BB}$, the energy density of the blackbody photon spectrum in vacuum. A complete study, estimating the additional energy for diverse temperatures and densities, is in preparation [10].

Another interesting aspect is how the additional energy affects the spectrum of the microwave background. The additional energy at $z_{DEC}$ (the redshift when the spectrum was formed) occurred at frequencies $\sim 10^{-9} \ \omega_{peak}$ ($\omega_{peak} = 2.8k_B T/h$). However, non-linear effects in plasma can bring the additional energy to higher frequencies. This is very interesting and should be investigated in the future.

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FIGURES

FIG. 1. The electromagnetic spectrum \( \ln[\mathcal{S}(\omega)/\mathcal{S}_0] \) vs \( \omega/\omega_{pe} \) (where \( \mathcal{S}(\omega) = \langle B^2 \rangle/8\pi + \langle E_T^2 \rangle/8\pi \) and \( \mathcal{S}_0 = \omega_{pe}^2 k_B T/c^3 \) is the normalization) for: (a) The electron-positron plasma at \( T = 10^{10} \) K and \( n_e = 1.4 \times 10^{31} \text{cm}^{-3} \) (the dotted curve is our model and the solid curve is the blackbody spectrum in vacuum); (b) The same as case (a), extended to high frequencies; (c) The same as case (a) but for an electron-proton plasma at \( T = 10^9 \) K and \( n_e = 5.4 \times 10^{26} \); and (d) The same as case (c), extended to high frequencies.
\[ \ln \left( \frac{S(\omega)}{S_0} \right) \]

(a) $\omega/\omega_{pe}$

(b) $\omega/\omega_{pe}$

(c) $\omega/\omega_{pe}$

(d) $\omega/\omega_{pe}$