HBT search for new states of matter in A+A collisions

S.V. Akkelin¹, Yu.M. Sinyukov¹

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Abstract

A method allowing studies of the hadronic matter at the early evolution stage in A+A collisions is developed. It is based on an interferometry analysis of approximately conserved values such as the averaged phase-space density (APSD) and the specific entropy of thermal pions. The plateau found in the APSD behavior vs collision energy at SPS is associated, apparently, with the deconfinement phase transition at low SPS energies; a saturation of this quantity at the RHIC energies indicates the limiting Hagedorn temperature for hadronic matter. It is shown that if the cubic power of effective temperature of pion transverse spectra grows with energy similarly to the rapidity density (that is roughly consistent with experimental data), then the interferometry volume is inverse proportional to the pion APSD that is about a constant because of limiting Hagedorn temperature. This sheds light on the HBT puzzle.

¹ Bogolyubov Institute for Theoretical Physics, Kiev 03143, Metrologichna 14b, Ukraine.

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1 Introduction

The main goal of experiments with ultra-relativistic heavy ion collisions is to study the new forms of strongly interacting matter which can be created under the extreme conditions. High densities and temperatures that arise in quasi-macroscopic systems formed during collision processes can result in the phase transitions from hadronic gas (HG) to Quark-Gluon Plasma (QGP) or sQGP [1]; the initial very dense pre-thermal stage of the collisions is, apparently, associated at RHIC energies with a specific form of matter - Color Glass Condensate (CGC) [2]. The bulk of hadronic observables ("soft physics") are related, however, only to the very last period of the matter evolution, so called thermal or kinetic freeze-out - the end of the collective expansion of hadronic gas when the system decays. The evolution of wave function of a single particle, e.g. pion which can be created, annihilated and scattered many times in a dense stochastic (incoherent) surrounding, can be hardly considered. In fact, the single particle spectra bring information about the state of matter, e.g., its temperature and flows, at the very end of the hadron gas evolution, and the HBT interferometry [3] is related directly to the structure of emission function, i.e. the space-time density of last hadronic collisions. Thus, the correlation measurements itself cannot be related directly to the preceding hot and dense stages of the matter evolution in A+A collisions where a formation of new forms of matter is expected. Our basic idea is to use the "conserved observables" which are specific functionals of spectra and correlation functions - integrals of motion. Then such an observables, being conserved during the matter evolution (or some period of the evolution) can be related to the state of matter at the early evolution stage.
A structure of the integrals of motion, besides the trivial ones associated with the energy, momentum and charges, depends strongly on a scenario of the matter evolution. The actual numbers of partonic degrees of freedom released in ultra-relativistic nuclear-nucleus collisions are fairly big, up to tens of thousands. As it is commonly supposed such a quasi-macroscopic system becomes thermal at the early stage of the collision process: the time of thermalization for the RHIC energies is estimated to be from 0.6 fm/c (the phenomenology analysis [4]) to 1 fm/c (the "black hole thermalization" [5]) and even more, 3 fm/c (the pure pQCD result [6]). Then the system expands nearly isoentropically - the latter is standard point in the hydrodynamic approach to A+A collisions and is advocated theoretically for the RHIC energies [1, 7]. As for the SPS energies, the approximation of perfect hydrodynamics successfully describes the spectra and correlations while overestimates the elliptic flows [8]. The latter can be connected with some viscosity effects at this energies. Keeping this in mind, one can, nevertheless, consider, at least approximately, the entropy produced in A+A collisions as integral of motion that carries out the information as for very initial thermal stage of these processes. The another "conserved observable" is found recently [9]: it is the pion phase-space density $\langle f \rangle$ averaged over momentum (totally or at fixed rapidity) and configuration space. It is also about a constant during the stage of chemically frozen expansion. So, measurements of the entropy and average phase-space density (APSD) in thermal hadronic systems at the final, freeze-out stage of A+A collisions makes it possible to look into the previous stages such as the partonic thermalization and the hadronization (or chemical freeze-out).

In the paper we express the entropy and APSD of thermal pions through the observed spectra and interferometry radii irrespective of unknown form of freeze-out (isothermal) hypersurface and transverse flows developed based on the approach proposed in the Ref. [9]. Our aim is to study the properties of the matter around hadronization stage at different energies of A+A collisions, from AGS to RHIC, provide the general analysis of the results and to make conclusions as for possible new forms of matter formed in these processes.

## 2 The entropy and APSD as observables in A+A collisions

To clarify the problem let us start from the entropy of thermal pions. As well known the expression for entropy of a gas of bosons/fermions has the following covariant form

$$S = (2J + 1) \int \frac{d\sigma^\mu p_\mu d^3 p}{(2\pi)^3 p^0} \left[-(2\pi)^3 f \ln((2\pi)^3 f) \pm (1 \pm (2\pi)^3 f) \ln(1 \pm (2\pi)^3 f)\right],$$

where $\pm$ sign corresponds to bosons/fermions with total spin $J$ and $\sigma$ is some hypersurface in Minkowski space. The value depends on the distribution function, or the phase-space density $f(x,p)$, that should be known to make the corresponding estimates. It is easy to show, however, that the phase-space density, e.g. of $\pi^-$, cannot be extracted from the two- and many- particle spectra even if the system at kinetic freeze-out is characterized by the locally equilibrated distribution function. To make it clear let us write the Wigner function (an analogy of the phase-space density for the quantum systems) for weakly interacting particles in the mass-shell approximation [12]:

$$f(x,p) = (2\pi)^{-3} \int d^4 q \delta(q \cdot p) e^{-iqx} \langle a^+(p - (1/2)q)a(p + (1/2)q) \rangle.$$  \hspace{1cm} (2)

1Unlike many authors (see, e.g., [10, 11]) we utilize almost totally averaged value of the APSD, $\langle f \rangle$, instead of momentum-dependent one ($f(p)$) because the extracted value of the later quantity is affected essentially by assumed form of freeze-out hypersurface as well as a assumed profile and intensity of flow on it, see Ref. [11] and correspondent discussion in Ref. [9].
Here the brackets $\langle \ldots \rangle$ mean the averaging of the product of creation and annihilation operators with a density matrix referred to the space-like hypersurface where particles become or are already nearly free. If the freeze-out is sudden, one uses usually a thermal density matrix at the freeze-out hypersurface. The invariant single- and double- (identical) particle spectra have the forms:

$$n(p) = \langle a^+(p)a(p) \rangle, \quad n(p_1,p_2) = n(p_1)n(p_2) + |\langle a^+(p_1)a(p_2) \rangle|^2,$$

and the correlation function is defined as the following

$$C(p,q) = n(p_1,p_2)/n(p_1)n(p_2).$$

It is easy to see from Eq. (3) that the complex phase of the two- operator average $\langle a^+(p_1)a(p_2) \rangle$ cannot be extracted from the single- and double- particle spectra. It is possible to show that the same takes place even when many-particle spectra are included into an analysis. Therefore one cannot reconstruct the distribution function $f$, and over momentum at fixed particle rapidity, $y = 0$,

$$\langle f(\sigma,y) \rangle_{y=0} = \frac{\int (f(x,p))^2 p^d\sigma_\mu d^2p_T}{\int f(x,p) p^d\sigma_\mu d^2p_T} = \frac{(2\pi)^{-3} \int p_0^{-1} n^2(p)(C(p,q) - 1)d^3q d^2p_T}{dN/dy},$$

directly from the experimental data in full accordance with the pioneer Bertsch idea \cite{13}. Using the standard Bertsch-Pratt parametrization the last equality in (5) can be re-written through the interferometry radii, as it is presented in Eq. (7).

On the face of it, the extracted value of the APSD (5) does not help to determine the entropy (1), and so, some phenomenological functions that can reproduce the approximate Gaussian behavior of the correlation function are usually suggested \cite{14}. However, since we cannot extract the phase of the two- operator average, there is the infinite set of distribution functions compatible with the observables and, therefore, the entropy calculated will depend on the class of the functions we choose which. Here we propose the method to estimate the entropy using just the APSD without assuming any concrete expression for the phase-space density.

The method is similar to what was proposed in Ref. \cite{9} for analysis of the overpopulation of the phase-space. The idea is based on the standard approach for spectra formation \cite{15} that supposes the thermal freeze-out in expanding (with 4-velocity field $u^\nu$) locally equilibrated system happens at some space-time hypersurface with uniform temperature $T$ and particle number density $n$ (or chemical potential $\mu$). Then, within this approximation which is probably appropriate in some "boost-invariant" mid-rapidity interval, $\Delta y \lesssim 1$, the phase-space density of pions,

$$f = f_{l,eq}(x,p) = (2\pi)^{-3}(\exp\left(\frac{u_\mu(x)p^\mu - \mu_\mu}{T}\right) - 1)^{-1},$$

totally averaged over the hypersurface of thermal freeze-out, $\sigma = \sigma_{th}$, and momentum except the longitudinal one (rapidity is fixed, e.g., $y = 0$) will be the same as the totally averaged phase-space
density in the static homogeneous Bose gas [9]:

\[ (2\pi)^3 \langle f(\sigma, y) \rangle_{y=0} = \frac{\int d^3p f_{eq}^2}{\int d^3p f_{eq}} = \frac{2\pi^{5/2}}{\int d^3p f_{eq}} \int \left( \frac{1}{R_O R_S} \frac{d^2N}{d\eta dy} \right)^2 \frac{d\eta}{dN/dy}, \]

where

\[ \bar{T}_{eq} \equiv (\exp(\beta(p_0 - \mu) - 1) - 1)^{-1} \]

and \( \beta \) and \( \mu \) coincide with the inverse of the temperature and chemical potential at the freeze-out hypersurface.\(^2\) The \( \kappa = 1 \) if one ignores the resonance decays. Here we neglect interferometry cross-terms since they are usually rather small in the mid-rapidity region for symmetric heavy ion central collisions at high energies.

The last expression in Eq. (7) just corresponds to the last term in (5) calculated in the Bertsch-Pratt parametrization. The important result presented by the first equality in (7) is based on the properties of relativistic invariance of the distribution function (6) and its local isotropy in momentum in the rest frames of each fluid element. Then, using the "boost-invariance" within homogeneity length, \( \Delta y \approx 1 \), the integrals over \( d^2p \) of diverse functions \( F_i \) of the locally-equilibrium distribution, \( F_i(f_{eq}) \), contain the common factor, "effective volume" \( V_{eff} \) (\( \eta \) is rapidity of fluid), that completely absorbs the flows \( u^\mu(x) \) and form of hypersurface \( \sigma(x) \) in mid-rapidity. For instance, if \( F_i(f_{eq}) = f_{eq} \), then

\[ \frac{dN}{dy} = \int d^3p d\sigma d^2p T f_{eq} = V_{eff} \int d^3p \frac{\bar{T}_{eq}}{(2\pi)^3} = n_{th} V_{eff} \]

where \( n_{th} \) is thermal density of equilibrium ideal Bose gas, and similar takes place for \( F_i(f_{eq}) = f_{eq}^2 \) in (5). Thus, the effective volume \( V_{eff} \) is cancelled in the corresponding ratios. This factorization property has been found first for Eq. (9) in Refs. [16, 17], multiple used for an analysis of particle number ratios (see, e.g., Ref. [18]) and recently generalized for a study of the APSD in Ref. [9].

In this work we apply the approach to an analysis of the thermal pion entropy per particle, or specific entropy of thermal pions. Using the same approximation of the uniform freeze-out temperature and density and Eq. (11) with some local equilibrium distribution we get the following expression for specific entropy in mid-rapidity:

\[ \frac{dS}{dy} = \frac{\int d^3p [-(\bar{T}_{eq} \ln \bar{T}_{eq} + (1 + \bar{T}_{eq}) \ln(1 + \bar{T}_{eq})] - \bar{T}_{eq} \ln \bar{T}_{eq} + (1 + \bar{T}_{eq}) \ln(1 + \bar{T}_{eq})]}{\int d^3p f_{eq}} \]

In the above ratio due to the factorization property the effective volume is cancelled and the final expression depends only on the two parameters: the temperature and chemical potential at freeze-out. The temperature can be obtained from the fit of the transverse spectra for different particle species and we will use the value \( T = 120 \) MeV as a typical "average value" for SPS and RHIC experiments.

\(^2\)Note here that \( (2\pi)^3f \) is unitless not only in the natural units where \( \hbar = c = 1 \) but also in conventional system of units (e.g. SI) where \( (2\pi)^3f \) is expressed via the Plank constant: \( (2\pi)^3f = h^3f \). Therefore the above value can be interpreted as distribution of the population numbers in elementary phase-space cells unlike phase-space density \( f \) that carries units \( h^{-3} \).
Another parameter, the chemical potential, cannot be extracted from the spectra, its value could be fairly high even for the thermal pions because of chemical freeze-out and this parameter is crucial for an estimate of the entropy. We extract the chemical potential from an analysis of the APSD following to (7). The factor $\kappa$ is accounting for a contribution of the short-lived resonances to the spectra and interferometry radii and absorbs also the effect of suppression of the correlation function due to the long-lived resonances [9]. Because of the chemical freeze-out a big part of pions, about a half, are produced by the short-lived resonances after thermal freeze-out. It leads to an increase of the APSD despite the maximal particle and entropy densities of pions at post-hydrodynamic stage of the evolution is reached at the end of the hydrodynamic expansion - at thermal freeze-out as discussed in detail in Ref. [9]. To estimate the thermal characteristics and "conserved observables" at the final stage of hydrodynamic evolution by means of Eqs. (7), (10) one needs to eliminate non-thermal contributions to the pion spectra and correlation functions from resonance decays at post-freeze-out stage. To do this we use the results of Ref. [9] where a study of the corresponding contributions within hydrodynamic approach gives the values of parameter $\kappa$ to be $\kappa = 0.65$ for SPS and $\kappa = 0.7$ for RHIC, if half of pions is produced by the resonances at post-freeze-out stage. Then, from Eqs. (7), (8) one can extract the pion chemical potential at thermal freeze-out. This makes it possible to estimate the average phase-space density, the specific entropy of thermal pions and other thermal parameters of the system at the end of the hydrodynamic expansion as explained above.

3 The analysis of experimental data and the results

To evaluate the APSD of negative pions by means of Eq. (7) we utilize the yields, transverse momentum spectra and interferometry radii of $\pi^-$ at mid-rapidity measured in central heavy ions collisions by the E895 and E802 Collaborations for AGS energies [19, 20], NA49 Collaboration for SPS CERN energies [21, 22, 23], STAR and PHENIX Collaborations for RHIC BNL energies [24, 25, 26].

To calculate the APSD we have to integrate over the whole $p_T$ region. Always, if possible, we use the pion yields and values of radii in each measured $p_T$ bin instead of analytical parameterizations for the transverse spectra and interferometry radii. To interpolate between successive data points we use an polynomial functions, the degree of the polynomial curves is chosen to be 2 or 3. Outside the whole measured $p_T$ region we make analytical extrapolations with parameters that catch the main tendencies of the data points in the measured $p_T$ region. We utilize the analytical parameterizations that are typically used in fitting procedures and are motivated by hydro model calculations. Namely, for the transverse spectra at high $p_T$ we assume the exponential parameterizations with slopes that correspond to average ones in the measured $p_T$ region (that is, actually, the average slope in the measured $p_T \lesssim 1$ GeV points). For small unmeasured $p_T$ our exponential extrapolations are taken with the slopes which are defined from a requirement of coincidence of the resulting total $\pi^-$ yields at mid-rapidity with the values presented by experimental Collaborations. To extrapolate interferometry radii behind the measured $p_T$ bins, we utilize the widely used simple analytical parameterizations for interferometry radii, $a/(b + c \cdot m_T^d)$, with numerical parameters which are taken, if possible, from the fit given by the experimental Collaboration, or we determine them ourselves if they are not presented. Also, we use the approximation $R_O = R_S$ for the outward and sideward interferometry radii below the minimal measured $p_T$ momentum, because this equality should take place for $p_T = 0$ due to evident (and well known) symmetry reasons. We neglect
interferometry cross-terms because they are rather small at mid-rapidity (see, e.g., [23])), and, for example, at SPS energies \( \sqrt{R_0^2 R_L^2 - R_{O,L}^2} \) is equal to \( R_O R_L \) with a few percents accuracy. Since the interferometry radii are measured by PHENIX Collaboration for 0 – 30 % centrality events at \( \sqrt{s_{NN}} = 200 \) GeV [26], we increase the interferometry volume measured by PHENIX Collaboration at this c.m. energy by a factor of 1.215 to get the interferometry volume corresponding to the most central 0 – 5 % centrality bin in accordance with \( N_{\text{part}} \) dependence of the Bertsch-Pratt radius parameters found in Ref. [26].

The results for the APSD at mid-rapidity for all negative pions at the AGS, SPS, RHIC energies in logarithmic c.m. energy scale are presented in Fig. 1. Note that for SPS energy domain we use the results of NA49 Collaboration since it presents interferometry data as well as the data on transverse momentum spectra which are necessary to get the APSD values, while the CERES Collaboration demonstrates the interferometry radii only.

The non-monotonic structure of the APSD behavior seen in Fig. 1 in the AGS-SPS energy domain could be an indicator of new physical phenomena as it is discussed in next Section, so a statistical significance of this structure is important and we demonstrate it in Fig. 2 at AGS, SPS energies in \( \sqrt{s_{NN}} \) scale. The comment is the following. Because of fast (exponential) decrease in of transverse momentum spectra with \( m_T \) the uncertainties of the spectra and HBT radii in the region of high \( p_T \) has only a little influence on the calculated APSD. The latter are mostly affected by the values of the HBT radii at low \( p_T \). The most important are systematic (from a choice of analytic \( p_T \) parameterizations) uncertainties in this region behind the lower measured \( p_T \) bin, and experimental errors in the HBT data. Taking into account that the experimental errors in the HBT radii do not exceed typically 0.5 fm (see, e.g., [23]), and estimating the resulting systematic uncertainties from our choice of \( p_T \) parameterizations as 0.5 fm, we get the uncertainty of 1 fm in each HBT radius. For a rough estimate of relative errors in the APSD values let us assume that average value of HBT radius in the region of interest is approximately 5 fm. Then, supposing that all uncertainties are independent and accidental, we can estimate that those errors are approximately 30 percents. As one can see from Fig. 2, the APSD in AGS energy domain can be approximated with quite good accuracy by linear in \( \sqrt{s_{NN}} \) function, \( a \sqrt{s_{NN}} + b \), where \( a = 0.03768 \) GeV\(^{-1/2} \) and \( b = 0.01345 \). Then the extrapolation of this tendency to SPS energy domain results in rather high values for APSD, for example, at \( \sqrt{s_{NN}} = 12.3 \) GeV we get \( (2\pi)^3 \langle f \rangle = 0.477 \) that is far from the estimated error bars, \( (2\pi)^3 \langle f \rangle = 0.203 \pm 0.06 \). While the final conclusion about statistical significance of the observed tendencies needs in more detail studies both theoretical and experimental, our estimate from the above analysis is that it is very likely that experimental data really indicate non-monotonic behavior of the APSD as a function of collision energy in the AGS-SPS energy domain.

In Fig. 3 the APSD of thermal negative pions at the SPS and RHIC energies are presented. Here and below we demonstrate the values for the thermal negative pions at the RHIC energies which are mean values of STAR and PHENIX data. Note, that in Fig. 3 we present also the APSD of negative thermal pions at chemical freeze-out in assumption of chemical equilibrium, \( (\mu_\pi = 0) \). We use the temperatures of chemical freeze-out for different colliding energies from Refs. [27], where they were found from analysis of particle number ratios. We do not calculate the APSD of thermal pions at the AGS energies because estimates of resonance contributions to the pion spectra have been done in [9] for the SPS and RHIC energies only.

The APSD of negative thermal pions are used then to extract the chemical potentials \( \mu \) of them at thermal freeze-out with \( T_{lh} = 120 \) MeV at different SPS and RHIC energies, and after that to calculate the specific entropies, \( s = \frac{dS}{dN/dy} \) [10], the entropies, \( \frac{dS}{dy} = s * \frac{dN}{dy} \), and the densities
n_{th}$, see Eq. (9), of negative thermal pions. The values founded are presented in Figs. 4, 6, 7. Also we demonstrate in Fig. 5 the interferometry volume $V_{int} = (2\pi)^{3/2}R_O R_S R_L$ calculated at small $p_T \simeq 0.06 - 0.07$ GeV as the function of the rapidity densities, $dN^{-}/dy$, of all negative pions at mid-rapidity in central nucleus-nucleus collisions. For the RHIC energies we use in Fig. 5 the interferometry radii measured by STAR Collaboration because the Collaboration presents the interferometry measurements in lower $p_T$ bins as compare to PHENIX Collaboration. If there are no experimental data in selected $p_T$ bin, $p_T \simeq 0.06 - 0.07$ GeV, we calculated the correspondent values using the analytical parameterizations of the interferometry radii as explained above. The obtained values, $V_{int}$, are used then to evaluate the ratios $(dN^{-}/dy)/V_{int}$ that are demonstrated and compared with the thermal densities $n_{th}$ in Fig. 6.

In Fig. 8 we present, in addition to Fig. 7 where the entropies $dS/dy$ of negative thermal pions are demonstrated, the rapidity densities, $dN^{-}/dy$, of all negative pions at mid-rapidity in central nucleus-nucleus collisions for the AGS, SPS and RHIC energies. The experimental values for the pion rapidity densities are taken from Refs. [19, 20, 21, 22, 24, 25]. We used the rapidity densities of pions instead of those for all charged particles, since pions are the new produced particles which are not contained initially in colliding nuclei, and, therefore, more directly represent a mechanism of the particle production in A+A collisions. It is especially important for collision processes with relatively low multiplicities at the AGS energies, where a large fraction of registered charged particles are protons which were not produced in collision processes being initially in colliding nuclei. The lines in Fig. 8 represent the logarithmic law of energy dependence for negative pion multiplicities: $a \log_{10}(\sqrt{s_{NN}}/b)$, where $a = 160(230)$, $b = 1.91$ GeV (3 GeV) for solid (dashed) lines respectively. Note, that we use the STAR point $dN^{-}/dy = 249$ for $\sqrt{s_{NN}} = 130$ GeV because this value - the result of a use of the Bose-Einstein distribution as a fit function, is closer then another STAR value based on a Boltzmann-like fit (see detail in [24]) to the value $dN^{-}/dy = 270 \pm 3.5$ reported by PHENIX Collaboration and is also closest to the result $dN^{-}/dy = 287 \pm 20$ that is deduced by STAR Collaboration from their measurements of negative hadrons, antiprotons and negative kaons spectra in Ref. [28]. Note that the latter value (it is not presented in Fig. 8) is even closer to the fitting line, which is showed in Fig. 8 as the solid line, than demonstrated experimental value.

4 Discussion and interpretation of the results

Let us start from an analysis of the $\sqrt{s_{NN}}$ dependance of the averaged phase-space density per unit of rapidity (APSD). Figure 1 is based on Eq. (5) or the last term in Eq. (7) with $\kappa = 1$ and demonstrates a behavior of the “raw” APSDs (which are, actually, the “asymptotic” APSDs related to the times when pions are detected) accounting for all negative pions, $\pi^-$, thermal and from resonance decays at post thermal stage. One can see that the APSD grows significantly with energy at the AGS energies, then has the plateau starting from the lowest SPS energy, 20 AGeV, till 80 AGeV and then begins to grow again, apparently very slowly at RHIC as one can conclude from the non quite compatible experimental data of the STAR and PHENIX Collaborations.

Unlike a fast decrease of particle $n(x)$ and phase-space $f(x,p)$ local densities, the totally averaged phase-space densities of thermal pions are conserved during a chemically frozen evolution. Roughly, it is proportional to the total (“raw”) APSD at the SPS and RHIC energies, see Fig. 3. If the same properties take place at the AGS energies too, then one can easily interpret the behavior of the averaged phase-space density in Fig. 1. When the energy of collisions at AGS grows, the
initial hadronic density and phase-space density increase; since the pion APSD is conserved, its observed value also grows. Its rise stops at low SPS energies, this means that the initial density of pions also stops increasing. The simplest explanation is: an excess of the initial energy begins to be transformed into new, non pionic (hadronic) degrees of freedom, possible to quarks and gluons. The pure hadronic stage appears later when the densities became smaller than the initial ones, therefore the initial APSD of pions depends then not on the initial energy density but on the density determined by the hadronization temperature $T_c$. Figure 3 demonstrates that the APSD of the thermal pions grows indeed very slowly starting from high SPS energies that can reflect the fact of saturation of the temperature of the phase transition at the RHIC energies. The APSD at the thermal freeze-out is slightly higher than at chemical one. It is because the conservation of the APSD should take place at perfectly chemically frozen hadronic evolution; there is, however, a residual effect of increase of pion number because of an excess of resonance decays into the expanding gas over back processes of the recombination. This difference does not contradict to typical estimates that roughly 2/3 of pions at hadronization stage are "hidden" in resonances and about half of pions are already thermal to the end of the hydro evolution. The pion APSDs at the chemical freeze-out are calculated using the thermal parameters of that stage that were found from an analysis of particle number ratios at SPS and RHIC in Refs. A dependance of the chemical potential $\mu$ of thermal pions at thermal freeze-out on $\sqrt{s_{NN}}$ is demonstrated in Fig. 4. One can see that the chemical potential of thermal pions at freeze-out is saturated somewhere between 50 and 60 MeV.

Figures 5 and 6 are related to the behavior of the interferometry volume $V_{\text{int}}$ on multiplicities, $dN^{\pi^-}/dy$, in central collisions at different energies of AGS, SPS, and RHIC. The interferometry volume has a tendency to grow over a broad energy range, as one can see from Fig. 5. However for central Pb+Pb (Au+Au) collisions at different energies, the corresponding increase is much slow than the proportionality law between $V_{\text{int}}(\sqrt{s})$ and $dN^{\pi^-}/dy(\sqrt{s})$. Also, there is the statistically reliable violation of a monotonic behavior of $V_{\text{int}}$ manifested in the evident decrease of $V_{\text{int}}$ in AGS energy interval. The phenomenon has been associated with the supposed constancy of the (kinetic) freeze-out value of pion mean free path while the transition from the nucleon to pion dominated matter happens within that energy range. Note that the relatively steep rise of the pion APSDs (see Figs. 1 and 2) at a moderate increase of pionic multiplicities at the AGS energies (see Fig. 8) is caused by the discussed decrease of $V_{\text{int}}$ with beam energy. Of course, the conservation of the APSD in absence of the deconfinement phase transition leads in any case to some rise of its value with initial energy density. As follows from Fig. 6, the $dN^{\pi^-}/dy$ grows with energy significantly faster than $V_{\text{int}}$. This fact is the main component of the HBT puzzle. To understand it qualitatively, let us very roughly estimate the APSD supposing that the transverse spectra have mainly exponential behavior vs transverse mass $m_T$, $\propto \exp (-m_T/T_{\text{eff}})$, where effective temperature $T_{\text{eff}}$ depends on the thermodynamic temperature at the hypersurface of thermal freeze-out $\sigma$ and flows at $\sigma$. Then, assuming that integral $I$ over dimensionless variable $m_T/T_{\text{eff}}$ depends on energies of collisions fairly smoothly, one can write

$$V_{\text{int}}(\sqrt{s}) \approx I \frac{dN/dy}{\langle f \rangle T_{\text{eff}}^3}$$  \hspace{1cm} (11)$$

where the interferometry volume is taken here at the smallest $m_T$. Thus, a proportionality between $V_{\text{int}}$ and the particle numbers $dN/dy$ is destroyed by a factor $\langle f \rangle T_{\text{eff}}^3$. So, if the APSD and $V_{\text{int}}$ only slightly grow with energy, mostly an increase of $T_{\text{eff}}^3$ could compensate a growth of $dN/dy$ in Eq. (11). One can see that it is the case: for example, the ratio of cube of effective temperatures of
negative pions at $\sqrt{s_{NN}} = 200$ GeV (RHIC) to one at 40 AGeV (CERN SPS) gives approximately 2, while the ratio of corresponding mid-rapidity densities is approximately equal to 3 \cite{21, 24, 25}. It could be only in the case of an increase of the pion flows in A+A collisions with energy. If the intensity of flows grows, it leads to a reduction of homogeneity lengths and the corresponding interferometry radii \cite{33}. This effect almost "compensates" a growth of final geometrical sizes of the system with energy in observed interferometry volumes. Indeed, as one can see from Fig. 6 the freeze-out densities for pions, $n_{th}$, become noticeably smaller than formally defined HBT densities, $\frac{dN/dy}{V_{int}}$, starting from the high SPS energies. In other words, the interferometry volume at those energies becomes to be significantly smaller than the effective one occupied by the system and defined by Eq. (9).

The result (11) brings about some more details. If at any fixed energy $\sqrt{s_{NN}}$ we look at the evolution in time of $V_{int}$, we found that it can be nearly constant since the values $dN/dy$, APSD $\langle f \rangle$ and effective temperature $T_{eff}$ in r.h.s. of Eq. (11) are approximately conserved for the thermal pions during the chemically frozen hydro-evolution \cite{9}. As the result, the "HBT microscope" at diverse energies "measures" the radii that are similar to the sizes of colliding nuclei. It provides an explanation to the phenomenological observations \cite{23, 35} that in central Pb+Pb and Au+Au collisions the interferometry volumes grow rather slowly with energy, and only due to the longitudinal interferometry radius grows (transverse sizes are equal), while, at the same energy, the $V_{int}$ depends strongly on the sizes of colliding nuclei and on the impact parameters in non-central collisions.

Our other observations are based on one more conserved value, the entropy. In principle, the entropy $S$ of the pion component alone can be changed even in the perfect fluid as well as the pion numbers $N$ (see discussion above). One can expect, however, that such deviations will be small for the ratios $s = \frac{dS/dy}{dN/dy}$ or for the specific entropy. Then using the result (11) for the latter value at the thermal freeze-out and the chemical potential extracted from an APSD analysis, one can determine the specific entropy of pions. The corresponding estimates give the values of $s$ to be approximately equal to 4 at SPS except for the top SPS energy, where $s \approx 3.79$. At RHIC energies for $\sqrt{s_{NN}} = 130$ GeV, the corresponding averaged value is equal to 3.66 (one can conclude from Fig. 1 that such a low value is, probably, artefact and result of relatively high discrepancy between the STAR and PHENIX data), and for $\sqrt{s_{NN}} = 200$ GeV the specific entropy of negative pions is equal to 3.82.

The total entropy of thermal negative pions per unit of rapidity, that is $s$ multiplied by the rapidity densities of thermal pions, $(dN_{\pi^-}/dy)/2$, is presented in Fig. 7. One can see that the entropy starts to grow faster at the SPS energies but at the RHIC energies this tendency is canceled. To understand better the situation let us look at the "raw" data in Fig. 8 representing a behavior of the negative pion rapidity density in the AGS, SPS and RHIC experiments. From this picture, that makes the tendencies which are seen in Fig. 7 more evident, one can conclude that at the SPS energies there is, indeed, an anomalously large slope of an increase of the pion entropy (and the number of pions) with energy. The observed multiplicities of negative pions at SPS at 80 AGeV (158 AGeV) are by factors of 1.09 (1.143) larger than ones extrapolated in accordance with the tendency (dashed line) observed at the AGS energies and, apparently, at the RHIC energies where relatively low pion multiplicities could be remnants of CGC formation \cite{2}. Taking into account that chemically equilibrated thermal model correctly describes particle number ratios, including pions, at the AGS and RHIC energies \cite{30} and that at the top SPS energy there is a problem of a "pion
we can suppose that the observed "excess" of pions is caused by a mechanism shifting the pion production at the SPS energies from the equilibrium. This mechanism could be also in some degree responsible for the reduction of K/π ratios as compare to ones in chemical equilibrium model [37] - the effect was observed by NA49 Collaboration ("horn" puzzle) [22]. Some increase of the pion APSD at SPS 158 AGeV, see Figs. 1, 2, 3 could be also caused by the "extra pion" production at high SPS energies.

What could be the reason of the "extra pion" production out of equilibrium at SPS energies? The intriguing possibility is that such an effect is the manifestation of the QCD critical end point (CEP) which is the terminating point of the first order phase transition line (about physics of the CEP and its location at QCD phase diagram see, e.g., [38]). Indeed, since the CEP acts as an attractor of the isentropic trajectories of the thermodynamic system evolution [39], the critical domain can influence the particle spectra for some range of collision energies, e.g., this could be responsible for the "step" behavior of the kaon effective temperature, discovered by NA49 Collaboration at the SPS energies [22]. Then nonequilibrium features, which accompany the phase transition in the expanding systems, e.g., a rise of the bulk viscosity in the mixed phase due to a finite relaxation time and variation of sound velocity in the transition region [40], could lead to a dissipation of kinetic energy and to entropy production. Another effect, that can result in the "extra pion" production in A+A collisions near the CEP, is a significant reduction (that is maximal in the vicinity of the CEP at the QCD phase diagram) of σ meson mass from its vacuum value, and mass shift of other resonances (ρ, etc.) due to sigma exchanges [41, 42]. As a result these species are rather numerous around the CEP. As m_σ < 2m_π in a vicinity of the CEP [41, 42], such sigma mesons cannot decay into ππ in this state and can do it only after σ meson masses are increased that happens when the density in A+A collisions are reduced by the system expansion. One can speculate that when it happens the rate of inelastic collisions can be not high enough to push the new produced pions into chemical equilibrium, while the elastic collisions can still thermalize them. That is why there could be no peculiarities in the pion transverse momentum spectra in low p_T region, as predicted in Ref. [41], while an enhancement of pion yields can be considered as a possible manifestation of the CEP in A+A collisions. Since the "pion excess" is maximal at the highest SPS energy 158 AGeV, it could mean, unlike present expectations [38], that the CEP is situated in the QCD phase diagram closer to the chemical freeze-out point at highest SPS energy than at the lowest SPS one. This possibility is argued for recently in Refs. [44] and [45]. It is noteworthy that because of an inhomogeneity of the baryonic chemical potential and temperature in rapidity at the chemical freeze-out hypersurface (see, e.g., an analysis that has been done in Ref. [46]), the condition for the thermodynamic trajectory of system evolution to pass around the CEP could be realized just in mid-rapidity region. Then the enhancement of pion production can be observed from diverse particle number ratios in the unit central rapidity interval rather than from 4π abundances.

At the RHIC energies both mechanisms, bulk viscosity and sigma mass reduction resulting in intensive entropy and pion multiplicities rise, can be inefficient since at small net baryon density, which are typical for that energies, the crossover far from critical domain around CEP might, apparently, happen (see, e.g., [47]).

A few remarks are in order here. Presently, one of the interpretation of a larger rate of an
increase of pion production in A+A collisions at the SPS energies as compared to AGS energies and properly normalized \( p + p \) (or \( p + p \)) collisions is based on the statistical model of the early stage (SMES) [48] (see also [49]). In this model the kink-like change (and horn-like structure of \( K^+/\pi^+ \) ratios) is a direct consequence of an onset of the deconfinement and liberation of massless quark-gluon degrees of freedom in the Lorentz-contracted fireball at the initial stage of A+A collisions. However it seems that both basic assumptions, namely, massless quark-gluon plasma and the Lorentz-contracted fireball, are not supported by further studies. The effective masses of quarks and gluons in the QGP are temperature-dependent and grow with temperature (see, e.g., Ref. [50] where the validity of *quasiparticle* picture of the quark-gluon plasma at high temperatures is advocated) that turns down the possibility to treat the QGP as weak coupling *massless* quark-gluon system even for \( T \gtrsim 3T_c \). It is also clearly seen from the lattice QCD results [51] where the pressure and energy density are both below the Stefan-Boltzmann limit even at very high temperatures which will be hardly reached in heavy ion collisions. Another crucial assumption in the SMES is a formation of the longitudinally Lorentz-contracted fireball in rest, that is actually the Landau-type initial conditions for hydrodynamic expansion. It was demonstrated [52], however, that the Landau-type initial conditions are unable to reproduce effective temperatures together with other data (multiplicities and rapidity distributions) at the SPS energies, and that these quantities can be described altogether only when one uses large initial volume with an appropriate velocity distribution (see also [53]).

5 Conclusions

The method allowing analysis of an early stage of the hadronic matter evolution in A+A collisions is developed. It is based on studies of approximate integrals of motions for the evolution of hadronic systems, such as the totally averaged phase-space density (APSD) and the specific entropy of thermal pions. We express these quantities through experimental data on the spectra and Bose-Einstein correlations in a way that does not depend on the freeze-out hypersurface and collective flows developed. Our estimates of the APSD at hadronization stage are close to the corresponding ones that we calculate based on the results of analysis of particle number ratios. A behavior of the pion APSD vs collision energy has a plateau at low SPS energies that indicates, apparently, the transformation of initial energy to non-hadronic forms of matter at SPS; a saturation of that quantity at the RHIC energies can be treated as an existence of the limiting Hagedorn temperature of hadronic matter, or maximal temperature of deconfinement \( T_c \). It is noteworthy that observation as for compatible values of the momentum-dependent APSD \( \langle f(p) \rangle \) for collisions of different nuclei at top SPS and top AGS energies has been interpreted in Ref. [10] as the universal properties of kinetic freeze-out in heavy ion collisions. Our study shows, however, that the pion APSD is approximately conserved value and so has no direct link to the freeze-out criterion and final thermodynamic parameters, being connected rather to the initial phase-space density of hadronic matter in A+A collisions [9].

A behavior of the entropy of thermal pions and measured pion multiplicities in central rapidity region vs energy demonstrates an anomalously high slope of an increase of the pion entropy/multiplicities at SPS energies compared to what takes place at the AGS and RHIC energies. This additional growth could be, probably, a manifestation of the QCD critical end point. The observed phenomenon can be caused by the dissipative effects that usually accompany phase transitions, such as an increase of the bulk viscosity, and also by peculiarities of pionic decays of \( \sigma \) mesons.
and other resonances with masses that are reduced, as compare to its vacuum values, in vicinity of the QCD CEP. At the RHIC energies there is no anomalous rise of pion entropy/multiplicities, apparently, because the crossover transition takes place far from the CEP and no additional degrees of freedom appear at that scale of energies: quarks and gluons were liberated at previous energy scale.

We also analyze a behavior of the interferometry radii with energy in a context of the HBT puzzle. We show that if cubic power of the effective temperature of pion spectra grows with energy similar to the rapidity density then the interferometry volume is inversely proportional to the pion APSD. The behavior of the latter with collision energy is nearly constant starting from the high SPS energies because of the limiting Hagedorn temperature, $T_c$, for hadronic matter. Roughly, the similarity between rise of effective temperature cubed and pion rapidity density takes place within the wide interval: from the lowest SPS energy to the highest RHIC energy. Therefore interferometry volume in Pb+Pb and Au+Au central collisions is nearly constant, more precisely, it grows much slower (mostly due to an increase of longitudinal radius associated with total lifetime of the system) than rapidity density. At the same time, at each fixed energy the pion APSD, rapidity density and effective temperature of pion spectra are approximately conserved during the evolution, therefore the interferometry volume, that is the function of above values, is nearly the same as at the initial moment of hadronic evolution, if it were measured. It explains the experimental observations that the interferometry volumes are changed only a little with energy for central collisions of the same nuclei and, at the same energy, they are proportional to the initial system extension in non-central collisions and central collisions of nuclei with different atomic numbers. The further experimental analysis of a such type of the correlations between the interferometry volumes, initial system sizes, multiplicities and slopes of the transverse spectra are still needed to clarify the picture.

Summarizing this work, we point out that the available interferometry data not only do not contradict possible dramatic transformations of the matter in A+A collisions, as it is usually concluded, but being analyzed properly give deep insight into the physics of the phase transitions. The precise localization of the collision energy region where the transition to QGP at finite net baryonic densities happens is very important for understanding of the physics of deconfinement transition. At the moment no single experiment has collected data that by themselves show non-monotonic behavior of physical observables as a function of collision energy in the AGS-SPS energy domain. This can throw doubt upon an experimental significance of the observed APSD behavior which leads to the treatment of that as the result of a phase transition and critical end point. In view of this the Compressed Baryonic Matter (CBM) project at the future Facility for Antiproton and Ion Research (FAIR) in Darmstadt is particularly important because it makes possible the systematic studies of heavy ion collisions at beam energy range between 10 and 40 AGeV. Our results as for the localization of the QCD CEP indicate the importance of the experiments with relatively light nuclei at the top SPS energy and future RHIC energy scans in which the corresponding part of the $(T - \mu_B)$ plane can be reached. Also, a testing of our prediction of the APSD saturation at the top RHIC and LHC energies because of the limiting Hagedorn temperature, associated with the deconfinement temperature at zero net baryonic densities, is of a great importance.
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Figure 1: The average phase-space density of all negative pions at mid-rapidity, \((2\pi)^3 \langle f(y) \rangle\), (circles, squares, stars and triangles) as function of c.m. energy per nucleon in heavy ion central collisions.
Figure 2: The average phase-space density of all negative pions at mid-rapidity, \((2\pi)^3 \langle f(y) \rangle\), as function of c.m. energy per nucleon at the AGS-SPS energy domain. The error bars take into account uncertainties of the calculated APSD, see the text for details.
Figure 3: The average phase-space density of thermal ("direct") negative pions, \((2\pi)^3 \langle f(y) \rangle^{th}\) (rhombus), and the average phase-space density of negative pions at the stage of chemical freeze-out, \((2\pi)^3 \langle f(y) \rangle^{ch}\) (crosses), as functions of c.m. energy per nucleon in heavy ion central collisions.
Figure 4: The chemical potential of thermal ("direct") negative pions, $\mu_{th}$, (rhombus) as function of c.m. energy per nucleon in heavy ion central collisions.
Figure 5: The interferometry volumes, \( V_{\text{int}} = (2\pi)^{3/2}R_O R_S R_L \), (circles, squares, and stars) of negative pions at \( p_T \simeq 0.06 - 0.07 \) GeV vs rapidity densities of the negative pions, \( dN^{\pi^-}/dy \), at mid-rapidity in heavy ion central collisions at different energies.
Figure 6: The ratio of rapidity densities of all negative pions to the corresponding interferometry volumes, \( \frac{dN^{\pi^-}/dy}{V_{int}} \) (circles, squares and stars) and the ratio of rapidity densities of negative thermal pions to their effective volumes, that is thermal densities \( n_{th} \) (crosses) vs c.m. energies per nucleon in heavy ion central collisions.
Figure 7: The rapidity density of entropy for negative thermal pions, $dS_{\pi^-}^{th}/dy$, (squares and stars) as function of c.m. energy per nucleon in heavy ion central collisions.
Figure 8: The rapidity density of negative pions, $dN^{\pi^-}/dy$, as function of c.m. energy per nucleon in heavy ion central collisions.