A Scalable Architecture for Coherence-Preserving Qubits

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We propose scalable architectures for the coherence-preserving qubits introduced by Bacon, Brown, and Whaley [Phys. Rev. Lett. 87, 247902 (2001)]. These architectures employ extra qubits providing additional degrees of freedom to the system. We show that these extra degrees of freedom can be used to counter errors in coupling strength within the coherence-preserving qubit and to combat interactions with environmental qubits. The presented architectures incorporate experimentally viable methods for inter-logical-qubit coupling and can implement a controlled phase gate via three simultaneous Heisenberg exchange operations. The extra qubits also provide flexibility in the arrangement of the physical qubits. Specifically, all physical qubits of a coherent-preserving qubit lattice can be placed in two spatial dimensions. Such an arrangement allows for universal cluster state computation.

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The encoding of logical qubits (LQ) into subspaces of multiple physical qubits is a powerful means of protecting quantum information from decoherence while allowing for universal quantum computation\cite{PhysRevLett.87.247902, Bacon2001}. Experimental examples of these decoherence free subspaces have been realized on nuclear magnetic resonance\cite{PhysRevLett.88.130501}, ion trap\cite{PhysRevLett.93.150501}, and optical systems\cite{Pirandola2007}, have been suggested for superconducting qubits\cite{Chuang1995}, and have been used to implement encoded quantum algorithms\cite{Hill2011}. Logical qubits of this type also allow performance of quantum logic maximizing the use of readily available operations while partially or completely avoiding operations that may add complexity to the computing hardware or a significant amount of time to the computation. Specifically, this type of subspace has been suggested to perform universal quantum computation with only the Heisenberg exchange interaction for a traditional circuit based quantum computation\cite{PhysRevLett.87.247902, Bacon2001, Bacon2004, Bacon2005} and cluster state computation\cite{Gross2005a}.

The best protected LQs introduced to date are the coherence-preserving, or supercoherent, qubits (SQ) of Ref.\cite{PhysRevLett.87.247902}. Supercoherent qubits, comprised of four physical qubits with equal coupling between all pairs, minimize decoherence by establishing an energy gap between the logical qubit subspace and the other eigenstates of the system. This forces all local interactions with the environment to supply energy to the system. In addition, SQs allow for universal quantum computation using only the Heisenberg exchange coupling\cite{PhysRevLett.87.247902}. This increases the speed of the computation for quantum dot implementations and removes the strenuous quantum hardware demands of local magnetic fields\cite{Bacon2001} or $g$-factor engineering\cite{Bacon2005} which would be required for single physical qubit rotations. However, a number of fundamental issues were left open in the original work on the SQ architecture. Chief among them are a scalable method to couple SQs, and the physical arrangement of the qubits within the SQ, such that there is equal coupling between all pairs.

In this paper, we introduce a scalable architecture for SQs with a practical two-dimensional arrangement of the qubits. This flexible arrangement incorporates additional degrees of freedom which can be used to correct errors in SQ construction and unwanted interactions from environmental qubits. We demonstrate that these scalable SQs have nearly the same robustness as the SQs of Ref.\cite{PhysRevLett.87.247902} against the dominant form of decoherence in III-V quantum dots: hyperfine coupling to the nuclear spins.

The inter-logical-qubit coupling in the original SQs created the most severe obstacle to scalability. To insure the system stays in the SQ subspace, Ref.\cite{PhysRevLett.87.247902} suggests using equal couplings between all pairs of the eight physical qubits comprising the pair of SQs to be coupled. This would be difficult in practice even for two SQs and the challenge would grow even more acute as the number of SQs is scaled up.

A more practical solution was suggested in Ref.\cite{Gross2005a}. If couplings between SQs are performed adiabatically\cite{Gross2005a}, the system will return to the logical SQ subspace after the interaction. The adiabaticity requirement is not difficult to achieve as adiabatic evolution is required for \textit{all} approaches using spins in quantum dots\cite{Gross2005a}.

The two-SQ interaction plus equal logical $z$ rotations on each of the two SQs performs a conditional phase gate which, together with the single qubit rotations of Ref.\cite{Gross2005a}, form a universal set of gates. In practice, the operations to perform the conditional phase gate may be done simultaneously, thus the gate requires only one time interval.
FIG. 1: (Color online) Proposed layouts for supercoherent qubits (SQ) with extra qubits to mediate couplings. The arrangement shown on the left has four physical qubits arranged such that the distance (and hence the coupling) between 2 and 4 is equal to the distance between the qubits on the edges of the rhombus. The extra qubits 5 and 6 mediate the coupling between 1 and 3. The right figure shows a proposed layout with four extra qubits mediating the coupling between 1 and 3. Any even length mediating chain can be used to create a supercoherent qubit.

The above shows that inter-SQ interactions can be implemented via control of the couplings between physical qubits of different SQs. The concern raised in [11], that coupling between SQs will cause the state of the system to leave the logical SQ subspace, is solved by the adiabaticity of the interactions.

Another stringent restriction in the actual construction of SQs is the need for equal couplings between all pairs within an SQ. Coupling between quantum dots is exponentially dependent on the distance between the electrons within the dots and can be controlled by electrostatic gates. A two dimensional square layout of the four physical qubits, as implied in [11] and suggested in [13], is impractical because the qubits diagonally across from each other are a further distance apart then neighboring qubits around the perimeter of the square. Rather, for practical SQ realization, the four physical qubits should be arranged such that there is equal distance between them. An obvious possibility is to arrange the four qubits in a tetrahedron, thus ensuring equal distance between the qubits. However, a tetrahedron architecture requires that the physical qubits be placed in three dimensions and leaves no room for flexibility in the physical qubit arrangement. An ideal SQ architecture would allow for the qubits to be arranged in only two dimensions and include additional degrees of freedom that would provide flexibility in the physical placement of the qubits and the couplings between the qubits.

All of these issues can be solved by adding more physical qubits to the SQ. Additional qubits give the needed flexibility in the arrangement of the quantum dots while maintaining the energy gap between the logical subspace and the other states of the system. These modified SQs are immune to global decoherence, but are no longer immune to decoherence from single environmental qubits. Nevertheless, the system is exceedingly robust against such errors, especially when compared with previously suggested encodings. In addition, decoherence from a single environmental qubit can be combatted by modifying the strengths of couplings within the SQ. More importantly, the modified SQs are nearly as robust as the original SQs against the most important form of decoherence: decoherence affecting each physical qubit in an uncorrelated manner.

Our primary proposed architecture is a six-qubit SQ with four qubits arranged in a rhombus, such that the distance between qubits along an edge is equal to the distance along the shorter of the diagonals. Two extra qubits are used to mediate the coupling across the longer diagonal in the two dimensional arrangement shown in Fig. 1b (Any even-length chain may be used to mediate the coupling across the longer diagonal: In Fig. 1b, four extra qubits are used). We model the system with the Heisenberg Hamiltonian $H = J_{ij} S^i \cdot S^j$. For ease of calculation we assume the couplings within the rhombus are equal to one. There is a continuum of possible values for the couplings to the mediating qubits. Assuming $J_{16} = J_{15}$, the coupling $J_{56}$ yielding the degenerate singlet ground state can be shown to be

$$J_{56} = (J_{16} \sqrt{J_{16}^4 + 8J_{16}^2 + 12J_{16}^2 + 8J_{16} + 4} \right) / (4J_{16} + 4).$$

(1)

The most convenient value of the couplings can be chosen based on external considerations such as ease of physical layout. Importantly, we note that $J_{16}$ can be set greater than, less than, or equal to 1 (with a minimum of $\approx .85$) allowing for many possible arrangements of the physical qubits. Examples of SQ coupling strengths are given in Table I.

The flexibility of the extra couplings allows for corrections of imperfection in values of other couplings. For example, using the coupling constants of the first line in Table I let us assume that misplacement of qubits causes the value of $J_{24}$ to be 1.1 instead of 1. Due to the extra degrees of freedom afforded by the six-qubit SQ, this can be corrected by reducing the value of $J_{56}$ to approx-

| $J_{16}$ | $J_{56}$ | Energy Gap |
|---------|---------|------------|
| 1       | $\frac{1}{2}(-1 + \sqrt{33})$ | 0.5931 ... 1.931 ... |
| 1.17672 | $\frac{1}{2}$ | .3855 ... |
| 2       | $\frac{1}{4}(4 + \sqrt{37})$ | 3.3609 ... .8519 ... |
or a rotation about the axis in the $z$ component between the eigenvalues of two coupled qubits are shown in Fig. 3. Similar two-logical-qubit gate operations. The low-lying states, and the degenerate state, $\langle \lambda_{11} | \langle \lambda_{10} |$ for the 6-dot SQ, the degenerate ground state is obtained for hopping parameters $t_{ij} = \sqrt{U}J_{ij}/4$ only in the infinite $U$ limit. For finite $U$, the hopping parameters need to be adjusted slightly. The dominant source of decoherence in III-V quantum dots is the local random magnetic field generated by the nuclei in each dot. The effect on a SQ from this type of decoherence is a splitting of the degenerate ground state into four states spanned by the logical computational basis, $\lambda_{00}$, $\lambda_{11}$, the eigenvalues for the two-SQ logical $|00\rangle$ and $|11\rangle$ states, and the degenerate state, $\lambda_{01}$, $\lambda_{10}$. The $J_{6,13} = J_{5,15}$ coupling splits the degenerate ground state into four states spanned by the logical computational basis. The above plots use coupling strengths from the second line of Table I. In both cases, the gap between the logical subspace and the rest of the system remains large even when $J_{5,7}$ or $J_{6,13}$ are greater than one.

Other inter-six-qubit-SQ coupling methods are possible. However, the ones discussed above are the best we know of for satisfying the requirements of a diagonal inter-SQ coupling and ease of arrangement of dots. We note that fine tuning of the couplings is necessary in order to account for additional interactions arising from multi-electron terms. When using a Hubbard Hamiltonian $H = t_{ij}c_{i\sigma}^\dagger c_{j\sigma} + Un_{i\uparrow}n_{i\downarrow}$ for the 6-dot SQ, the degenerate ground state is obtained for hopping parameters $t_{ij} = \sqrt{U}J_{ij}/4$ only in the infinite $U$ limit. For finite $U$, the hopping parameters need to be adjusted slightly. The dominant source of decoherence in III-V quantum dots is the local random magnetic field generated by the nuclei in each dot. The effect on a SQ from this type of decoherence is a splitting of the degenerate ground state (i.e., the logical qubit subspace), by $\Delta E$, leading to a precession, or decoherence, time given by

\[
\Delta E = \frac{\lambda_0^2 - \lambda_1^2}{2\gamma}
\]

approximately .5443. As another example, errors due to stray couplings between qubits 4-5 and 4-6 can be corrected by modifying the $J_{5,6}$ coupling. This may be necessary due to the relatively close proximity of qubit 4 to qubits 5 and 6.

For the six-qubit SQ, single SQ (logical) rotations are performed by changing one Heisenberg coupling strength between appropriate pairs of qubits. Depending on the choice of coupling, this performs a logical $z$ rotation or a rotation about the axis in the $x-z$ plane $120^\circ$ from the $z$ axis. Combinations of these operations are sufficient to perform any $SU(2)$ rotation.

While a logical qubit chain is sufficient for the implementation of universal circuit-based quantum computations, most algorithms can be implemented more efficiently on a higher dimensional lattice. Additionally a two dimensional lattice of logical qubits is necessary to perform universal cluster-based quantum computation. To this end we identify two different coupling schemes to perform logical operations between pairs of SQs. These are shown in Fig. 2. Both coupling schemes result in diagonal two-logical-qubit gate operations. The low-lying eigenvalues of two coupled qubits are shown in Fig. 3. For SQs coupled horizontally in Fig. 2, two of the eigenvalues in the computational space are equal $\lambda_{01} = \lambda_{10}$. For vertically coupled SQs, all four eigenvalues are different. In both cases, the two-SQ gate operations can be combined with single logical-qubit $z$-rotations to perform a controlled phase gate. Since these operations commute, they may be performed simultaneously, and the controlled phase gate can be implemented with a single pulse of the exchange interactions.
effective magnetic field, mental data suggests a ratio between the strength of the logical qubits of Ref. [12]. We note that current experiments are the precession times of a single qubit and the 3-dot case was used). Also shown is the ratio given in the first line of Table I. The figure shows the dramatic improvement in decoherence time gained by properly encoding quantum information. The SQs can be connected within a given logical qubit (for the 6-qubit case the ratio increases linearly with \( J \)). Immediately noticeable is the orders of magnitude improvement in decoherence time for the 4- and 6-qubit SQs. In addition, upon increasing \( J \), the decoherence time for both of these increases linearly and the six-qubit case loses little in robustness against this type of decoherence when compared with the four-qubit SQ.

In conclusion, we have introduced flexible architectures for supercoherent qubits with the goal of reducing the severe constraints required for equal inter-qubit couplings. These constraints include placing qubits at exact distances from each other which forces the placing of qubits in three dimensions. The schemes introduced here increase flexibility while keeping the energy gap necessary to protect the SQ from sources of decoherence. The additional degrees of freedom can be used to correct mismatches in intra-SQ couplings and to reduce coupling from environmental qubits. The SQs can be connected in both one and two-dimensional arrangements, and their natural implementation of diagonal logical operations makes them particularly suitable for cluster-state quantum computation. Most importantly, the supercoherent qubits show a dramatic increase in robustness against decoherence due to nuclei, the primary source of decoherence in III-V quantum dots.

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\[
\Delta T = \frac{\hbar}{2\pi \Delta E}
\]

The precession times for the SQ containing 4 and 6 physical qubits are shown in Fig. 4 as a function of \( J \), the coupling between two physical qubits within a given logical qubit (for the 6-qubit case the ratio given in the first line of Table I was used). Also shown are the precession times of a single qubit and the 3-dot logical qubits of Ref. [12]. We note that current experimental data suggests a ratio between the strength of the effective magnetic field, \( H_b \), and \( J \) of \( 10^{-6} \lesssim H_b/J \lesssim 0.01 \). Immediately noticeable is the orders of magnitude increase in decoherence time for the 4- and 6-qubit SQs. In addition, upon increasing \( J \), the decoherence time for both of these increases linearly and the six-qubit case loses little in robustness against this type of decoherence when compared with the four-qubit SQ.

FIG. 4: (Color online) Precession time of logical qubits due to the random magnetic field of nuclei in the quantum dot as a function of \( J \), the intra-logical-qubit coupling. When the logical qubit is just a single qubit or consists of three physical qubits, as in Ref. [12], the precession time is independent of \( J \). For the four-qubit and six-qubit SQs the precession time increases linearly with \( J \). The figure shows the dramatic improvement in decoherence time gained by properly encoding qubits. The magnetic field strength used is 0.06 µeV. This and other quantities are based on the experimental work in [12].

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