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Theoretical expectations for the muon’s electric dipole moment

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Abstract

We examine the muon’s electric dipole moment $d_\mu$ from a variety of theoretical perspectives. We point out that the reported deviation in the muon’s $g - 2$ can be due partially or even entirely to a new physics contribution to the muon’s electric dipole moment. In fact, the recent $g - 2$ measurement provides the most stringent bound on $d_\mu$ to date. This ambiguity could be definitively resolved by the dedicated search for $d_\mu$ recently proposed. We then consider both model-independent and supersymmetric frameworks. Under the assumptions of scalar degeneracy, proportionality, and flavor conservation, the theoretical expectations for $d_\mu$ in supersymmetry fall just below the proposed sensitivity. However, nondegeneracy can give an order of magnitude enhancement, and lepton flavor violation can lead to $d_\mu \sim 10^{-22}$ e cm, two orders of magnitude above the sensitivity of the $d_\mu$ experiment. We present compact expressions for leptonic dipole moments and lepton flavor violating amplitudes. We also derive new limits on the amount of flavor violation allowed and demonstrate that approximations previously used to obtain such limits are highly inaccurate in much of parameter space. © 2001 Published by Elsevier Science B.V.

1. Introduction

Electric dipole moments (EDMs) of elementary particles are predicted to be far below foreseeable experimental sensitivity in the standard model. In extensions of the standard model, however, much larger EDMs are possible. Current EDM bounds are already some of the most stringent constraints on new physics, and they are highly complementary to many other low energy constraints, since they require CP violation, but not flavor violation.

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The field of precision muon physics will be transformed in the next few years [1]. The EDM of the muon is, therefore, of special interest. A new experiment [2] has been proposed to measure the muon’s EDM at the level of

\[ d_\mu \sim 10^{-24} \text{e cm}, \]  

more than five orders of magnitude below the current bound [3]

\[ d_\mu = (3.7 \pm 3.4) \times 10^{-19} \text{e cm}. \]  

The interest in the muon’s EDM is further heightened by the recent measurement of the muon’s anomalous magnetic dipole moment (MDM) \( a_\mu \) = \((g_\mu - 2)/2\), where \( g_\mu \) is the muon’s gyromagnetic ratio. The current measurement \( a_\mu^{\text{exp}} = 11659202(14)(6) \times 10^{-10} \) [4] from the muon \((g - 2)\) experiment at Brookhaven differs from the standard model prediction \( a_\mu^{\text{SM}} [5,6] \) by 2.6\( \sigma \):

\[ \Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (43 \pm 16) \times 10^{-10}. \]  

The muon’s EDM and \( a_\mu \) arise from similar operators, and this tentative evidence for a nonstandard model contribution to \( a_\mu \) also motivates the search for the muon’s EDM.

In this study, we examine the prospects for detecting a nonvanishing muon EDM from a variety of theoretical perspectives. We first note that the reported deviation in the muon’s \( g - 2 \) can be due partially or even entirely to a new physics contribution to the muon’s electric dipole moment. In fact, at present the result from the muon \((g - 2)\) experiment provides the most stringent bound on \( d_\mu \). We derive this bound and comment on the conclusions that may be drawn about \( d_\mu \) from the \( a_\mu \) measurement alone.

We then move to more concrete frameworks, where additional correlations constrain our expectations. In particular, we consider supersymmetry and examine quantitatively the implications of the electron EDM and lepton flavor violating processes [7,8]. Our aim is to impose as few theoretical prejudices as possible and draw correspondingly general conclusions. For studies of the muon EDM in specific supersymmetric models, see, e.g., Refs. [9,10].

Finally, although we use exact expressions for all flavor-conserving amplitudes in this study, we also provide compact expressions in the mass insertion approximation for branching ratios of radiative lepton decays and for leptonic EDMs and MDMs both with and without lepton flavor violation. These include all leading supersymmetric effects and are well-suited to numerical calculations.

2. Model-independent bounds from the muon \((g - 2)\) experiment

Modern measurements of the muon’s MDM exploit the equivalence of cyclotron and spin precession frequencies for \( g = 2 \) fermions circulating in a perpendicular and uniform magnetic field. Measurements of the anomalous spin precession frequency are, therefore, interpreted as measurements of \( a_\mu \).

The spin precession frequency also receives contributions from the muon’s EDM, however. For a muon traveling with velocity \( \beta \) perpendicular to both a magnetic field \( B \)
Fig. 1. Regions in the \((a_{\mu}^{NP}, d_{\mu}^{NP})\) plane that are consistent with the observed \(|\omega_a|\) at the 1\(\sigma\) and 2\(\sigma\) levels. The current 1\(\sigma\) and 2\(\sigma\) bounds on \(d_{\mu}^{NP}\) [3] are also shown.

and an electric field \(E\), the anomalous spin precession vector is

\[
\omega_a = -a_\mu \frac{e}{m_\mu} B - d_\mu \frac{2c}{\hbar} \beta \times \mathbf{B} - \frac{e}{m_\mu c} \left( \frac{1}{\gamma^2 - 1} - a_\mu \right) \beta \times \mathbf{E} - d_\mu \frac{2e}{\hbar} \mathbf{E}.
\]

In recent experiments, the third term of Eq. (4) is removed by running at the 'magic' \(\gamma \approx 29.3\), and the last term is negligible. For highly relativistic muons with \(|\beta| \approx 1\), then, the anomalous precession frequency is

\[
|\omega_a| \approx |B| \left[ \left( \frac{e}{m_\mu} \right)^2 \left( a_\mu^{SM} - 2a_\mu^{SM} a_{\mu}^{NP} \right) + \left( \frac{2c}{\hbar} \right)^2 d_{\mu}^{NP^2} \right]^{1/2}, \tag{5}
\]

where NP denotes new physics contributions, and we have assumed \(a_{\mu}^{NP} \ll a_{\mu}^{SM}\) and \(d_{\mu}^{NP} \gg d_{\mu}^{SM}\).

The observed deviation from the standard model prediction for \(|\omega_a|\) has been assumed to arise entirely from a MDM and has been attributed to a new physics contribution of size \(\Delta a_\mu\). However, from Eq. (5), we see that, more generally, it may be due to some combination of magnetic and electric dipole moments from new physics. More quantitatively, the effect can also be due to an EDM contribution

\[
|d_{\mu}^{NP}| \approx \frac{he}{m_\mu} \sqrt{\frac{1}{2} a_{\mu}^{SM^2} \left( \Delta a_\mu - a_{\mu}^{NP} \right)} \approx 3.0 \times 10^{-19} e \text{ cm} \sqrt{1 - \frac{a_{\mu}^{NP}}{43 \times 10^{-10}}}, \tag{6}
\]

where \(a_{\mu}^{NP}\) has been normalized to the current central value given in Eq. (3). In Fig. 1 we show the regions in the \((a_{\mu}^{NP}, d_{\mu}^{NP})\) plane that are consistent with the observed deviation in \(|\omega_a|\).

In fact, the observed anomaly may, in principle, be due entirely to the muon’s EDM! This is evident from Eqs. (2) and (6), or from Fig. 1, where the current 1\(\sigma\) and 2\(\sigma\) upper bounds on \(d_{\mu}^{NP}\) [3] are also shown. Alternatively, in the absence of fine-tuned cancellations
between $a_{\mu}^{NP}$ and $d_{\mu}^{NP}$, the results of the muon ($g - 2$) experiment also provide the most stringent bound on $d_{\mu}$ to date, with 1$\sigma$ and 2$\sigma$ upper limits

$$\Delta a_{\mu} < 59.75 \times 10^{-10} \implies |d_{\mu}^{NP}| < 3.5(3.9) \times 10^{-19} \text{ cm}. \quad (7)$$

This discussion is completely model-independent. In specific models, however, it may be difficult to achieve values of $d_{\mu}$ large enough to saturate the bound of Eq. (7). For example, in supersymmetry, assuming flavor conservation and taking extreme values of superparticle masses ($\sim 100$ GeV) and $\tan \beta$ ($\tan \beta \sim 60$) to maximize the effect, the largest possible value of $a_{\mu}$ is $a_{\mu}^{\text{max}} \sim 10^{-7}$ \cite{11}. Very roughly, one therefore expects a maximal muon EDM of order $(e/2m_{\mu}c)d_{\mu}^{\text{max}} \sim 10^{-20} \text{ cm}$ in supersymmetry.

Of course, the effects of $d_{\mu}$ and $a_{\mu}$ are physically distinguishable: while $a_{\mu}$ causes precession around the magnetic field’s axis, $d_{\mu}$ leads to oscillation of the muon’s spin above and below the plane of motion. This oscillation is detectable in the distribution of positrons from muon decay, and further analysis of the recent $a_{\mu}$ data should tighten the current bounds on $d_{\mu}$. Such analysis is currently in progress \cite{12}. The proposed dedicated $d_{\mu}$ search will provide a definitive answer, however, by either measuring a nonzero $d_{\mu}$ or constraining the contribution of $d_{\mu}$ to $|\omega_a|$ to be insignificant.

3. Theoretical expectations from the muon’s MDM

The muon’s EDM and anomalous MDM are defined through\footnote{Here and below we set $\bar{h} = c = 1$.}

$$L_{\text{EDM}} = -i d_{\mu}^{NP} \bar{\mu} \sigma^{mn} \gamma_5 \mu F_{mn}, \quad (8)$$

$$L_{\text{MDM}} = a_{\mu}^{NP} \frac{e}{4m_{\mu}} \bar{\mu} \sigma^{mn} \mu F_{mn}, \quad (9)$$

where $\sigma^{mn} = \frac{i}{2} [\gamma^m, \gamma^n]$ and $F$ is the electromagnetic field strength. These operators are closely related. In the absence of all other considerations, one might expect their coefficients to be of the same order. Parameterizing them as $d_{\mu}^{NP} / 2 = 1 \text{m}A$ and $a_{\mu}^{NP} e / (4m_{\mu}) = \text{Re} A$ with $A \equiv |A| e^{\phi_{\text{CP}}}$, one finds

$$d_{\mu}^{NP} = 4.0 \times 10^{-22} \text{ cm} \frac{a_{\mu}^{NP}}{43 \times 10^{-10} \tan \phi_{\text{CP}}}. \quad (10)$$

The measured discrepancy in $|\omega_a|$ then constrains $\phi_{\text{CP}}$ and $d_{\mu}^{NP}$. The preferred regions of the $(\phi_{\text{CP}}, a_{\mu}^{NP})$ plane are shown in Fig. 2. For ‘natural’ values of $\phi_{\text{CP}} \sim 1$, $d_{\mu}^{NP}$ is of order $10^{-22} \text{ cm}$. With the proposed $d_{\mu}^{NP}$ sensitivity of Eq. (1), all of the 2$\sigma$ allowed region with $\phi_{\text{CP}} > 10^{-2}$ yields an observable signal.

At the same time, while this model-independent analysis indicates that natural values of $\phi_{\text{CP}}$ prefer $d_{\mu}^{NP}$ well within reach of the proposed muon EDM experiment, very large values of $a_{\mu}^{NP}$ also require highly fine-tuned $\phi_{\text{CP}}$. For example, the contributions of $d_{\mu}^{NP}$ and $a_{\mu}^{NP}$ to the observed discrepancy in $a_{\mu}$ are roughly equal only if $|\pi / 2 - \phi_{\text{CP}}| \sim 10^{-3}$. This is
Fig. 2. Regions of the \((\phi_{CP}, d_{\mu}^{NP})\) plane allowed by the measured central value of \(|\omega_{a}|\) (solid) and its 1\(\sigma\) and 2\(\sigma\) preferred values (shaded). The horizontal line marks the proposed experimental sensitivity to \(d_{\mu}^{NP}\). A consequence of the fact that EDMs are \(CP\)-odd and \(d_{\mu}^{SM} \approx 0\), and so \(d_{\mu}^{NP}\) appears only quadratically in \(|\omega_{a}|\). Without a strong motivation for \(\phi_{CP} \approx \pi/2\), it is, therefore, natural to expect the EDM contribution to \(|\omega_{a}|\) to be negligible, and we assume in the following that the \(|\omega_{a}|\) measurement is indeed a measurement of \(a_{\mu}\).

4. The electron EDM and naive scaling

The EDM operator of Eq. (8) couples left- and right-handed muons, and so requires a mass insertion to flip the chirality. The natural choice for this mass is the lepton mass. On dimensional grounds, one, therefore, expects

\[
d_{\mu}^{NP} \propto \frac{m_{\mu}}{\tilde{m}^2},
\]

where \(\tilde{m}\) is the mass scale of the new physics. If the new physics is flavor blind, \(d_{f} \propto m_{f}\) for all fermions \(f\), which we refer to as ‘naive scaling’. In particular,

\[
d_{\mu} \approx \frac{m_{\mu}}{m_{e}} d_{e}.
\]

The current bound on the electron EDM is \(d_{e} = 1.8 (1.2) (1.0) \times 10^{-27} e\, cm\) [13]. Combining the statistical and systematic errors in quadrature, this bound and Eq. (12) imply

\[
d_{\mu} \lesssim 9.1 \times 10^{-25} e\, cm,
\]

at the 90\% CL, which is barely below the sensitivity of Eq. (1). Naive scaling must be violated if a nonvanishing \(d_{\mu}\) is to be observable at the proposed experiment. On the other hand, the proximity of the limit of Eq. (13) to the projected experimental sensitivity
of Eq. (1) implies that even relatively small departures from naive scaling may yield an observable signal.

5. Violations of naive scaling in supersymmetry

Is naive scaling violation well-motivated, and can the violation be large enough to produce an observable EDM for the muon? To investigate these questions quantitatively, we consider supersymmetry. (For violations of naive scaling in other models, see, for example, Ref. [14].) Many additional mass parameters are introduced in supersymmetric extensions of the standard model. These are in general complex and so are new sources of $CP$ violation. These parameters may be correlated by a fundamental theory of supersymmetry breaking that includes a specific mechanism for mediating the breaking. In fact, all viable mechanisms of mediating supersymmetry breaking are designed to suppress flavor violation, and so $CP$-violating observables that also involve flavor violation, such as $\epsilon_K$, are also suppressed. However, EDMs do not require flavor violation, and constraints on quark and electron EDMs are some of the main challenges for supersymmetric models. For a recent discussion of the supersymmetric $CP$ problem in various supersymmetry breaking schemes, see Ref. [15].

In full generality, the relevant dimensionful supersymmetry parameters for leptonic EDMs are the slepton mass matrices $m^2$, trilinear scalar couplings $A$, gaugino masses $M_1$ and $M_2$, the Higgsino mass $\mu$, and the dimension two Higgs scalar coupling $B$. Schematically, these enter the Lagrangian through the terms

$$L \supset m^2_{LL} \tilde{L}_i \tilde{L}_j, \quad m^2_{LR} \tilde{\tilde{L}}_i \tilde{\tilde{L}}_j, \quad A_{ij} \tilde{H}_d \tilde{L}_i \tilde{E}_j,$$

$$M_1 \tilde{B} \tilde{B}, \quad M_2 \tilde{W} \tilde{W}, \quad \mu \tilde{H}_u \tilde{H}_d, \quad B \tilde{H}_u \tilde{H}_d, \quad \tag{14}$$

where $i, j$ are generational indices, $L$ and $E$ denote SU(2) doublet and singlet leptons, respectively, $\tilde{H}_u$ and $\tilde{H}_d$ are the up- and down-type Higgs multiplets, and $\tilde{B}$ and $\tilde{W}$ are gauginos, the U(1) Bino and SU(2) Winos. The parameter $\tan \beta = \langle H^0_u \rangle / \langle H^0_d \rangle$, the ratio of Higgs boson vacuum expectation values, will also enter below.

The U(1)$_R$ and U(1)$_{PQ}$ symmetries allow us to remove two phases — we choose $M_2$ and $B$ real. Throughout this study, we assume that the gaugino masses have a common phase, as is true in many well-motivated theories where the gaugino masses are either unified at some scale or otherwise have a common origin. We also begin by assuming supersymmetric flavor conservation, that is, that the sfermion masses and trilinear couplings are diagonal in the fermion mass basis. (We will consider flavor violation in Section 5.3.) With these assumptions, only the $\mu$ and $A_\ell$ parameters are complex. We define $\phi_\mu \equiv \text{Arg}(\mu)$ and $\phi_{A_\ell} \equiv \text{Arg}(A_\ell)$.

Leptonic electromagnetic dipole moments arise at one-loop from chargino–neutralino and neutralino–charged slepton diagrams. In all figures and results presented here, we evaluate flavor-conserving amplitudes exactly. However, for purposes of exposition, it is convenient to consider the fermion mass basis and to adopt the mass insertion approximation for sleptons, neutralinos, and charginos. (We have checked that the mass
insertion approximation is accurate to about 5% in almost all of parameter space, justifying the intuition derived from this simplification.) In the mass insertion approximation, for large and moderate $\tan \beta$ and neglecting subdominant terms, there are five contributions with the following Feynman diagrams and amplitudes [16]:

$$A_{\ell}^a = -g'^2 M_1 \left[ A_\ell \langle H_d^0 \rangle - m_\ell \mu \tan \beta \right] K_N \left( m_{\tilde{\ell}_L}^2, |\mu|^2, M_1 \right),$$

$$A_{\ell}^b = -g'^2 M_1 m_\ell \mu \tan \beta K_N \left( m_{\tilde{\ell}_R}^2, |\mu|^2, M_1 \right),$$

$$A_{\ell}^c = \frac{1}{2} g'^2 M_1 m_\ell \mu \tan \beta K_N \left( m_{\tilde{\ell}_L}^2, |\mu|^2, M_1 \right),$$

$$A_{\ell}^d = -\frac{1}{2} g'^2 M_2 m_\ell \mu \tan \beta K_N \left( m_{\tilde{\ell}_L}^2, |\mu|^2, M_2 \right),$$

$$A_{\ell}^e = g'^2 M_2 m_\ell \mu \tan \beta K_C \left( m_{\tilde{\nu}_\ell}^2, |\mu|^2, M_2 \right).$$

In these diagrams, an external photon connected to any charged internal line is implicit, and $\delta_{LR}^{\ell\ell} = (A_\ell \langle H_d^0 \rangle - m_\ell \mu \tan \beta) / m_{\tilde{\ell}_R}^2$. The functions $K_N$ and $K_C$ are mass dimension $-4$ functions entering the neutralino and chargino diagrams, respectively, and are given in Appendix A.

Defining

$$A_{\ell}^{\text{tot}} = A_{\ell}^a + A_{\ell}^b + A_{\ell}^c + A_{\ell}^d + A_{\ell}^e,$$

the EDM and anomalous MDM of a lepton $\ell$ are simply

$$d_\ell = \frac{1}{2} e \text{Im} A_{\ell}^{\text{tot}}, \quad a_\ell = m_\ell \text{Re} A_{\ell}^{\text{tot}}.$$

From the amplitudes of Eq. (15), naive scaling is seen to require

- Degeneracy: Generation-independent $m_{\tilde{\ell}_R}$, $m_{\tilde{\ell}_L}$, and $m_{\tilde{\nu}_\ell}$.
- Proportionality: The $A$ terms must satisfy $\text{Im}(A_\ell) \propto m_\ell$.
- Flavor conservation: Vanishing off-diagonal elements of $m_{\tilde{\ell}_L}^2$, $m_{\tilde{\ell}_R}^2$, and $A$.

The last requirement, flavor conservation, has been assumed in all of our discussion so far. As we will see, relaxing this assumption also leads to naive scaling violation. We now consider violations of each of these properties in turn.
5.1. Nondegeneracy

Scalar degeneracy is the most obvious way to reduce flavor changing effects to allowable levels. Therefore, many schemes for mediating supersymmetry breaking try to achieve degeneracy. However, in many of these, with the exception of simple gauge mediation models, there may be non-negligible contributions to scalar masses that are generation-dependent. Furthermore, there are classes of models that do not require scalar degeneracy at all. For example, scalar nondegeneracy is typical in alignment models [17], where flavor-changing effects are suppressed by the alignment of scalar and fermion mass matrices rather than by scalar degeneracy. Scalar nondegeneracy is also typical in models with anomalous U(1) contributions to the sfermion masses. In fact, in models where the anomalous U(1) symmetry determines both sfermion and fermion masses, the sfermion hierarchy is often inverted relative to the fermion mass hierarchy [18–20], and so smuons are lighter than selectrons, as required for an observable $d_\mu$. In summary, there is a wide variety of models in which deviations from scalar degeneracy exist.

We now consider a simple model-independent parameterization to explore the impact of nondegenerate selectron and smuon masses. We set $m_{\tilde{e}_R} = m_{\tilde{e}_L} = m_{\tilde{e}}$ and $m_{\tilde{\mu}_R} = m_{\tilde{\mu}_L} = m_{\tilde{\mu}}$ and assume vanishing $A$ parameters. For fixed values of $M_1$, $M_2$, $|\mu|$, and $\tan \beta$, then, to a good approximation both $d_e$ and $d_\mu$ are proportional to $\sin \phi_\mu \tan \beta$, and we assume that $\sin \phi_\mu \tan \beta$ saturates the $d_e$ bound.

Contours of $d_\mu$ are given in Fig. 3. The contributions to $d_\mu$ have been evaluated exactly (without the mass insertion approximation). Observable values of $d_\mu$ are possible even for small violations of nondegeneracy; for example, for $m_{\tilde{\mu}} / m_{\tilde{e}} \lesssim 0.9$, muon EDMs greater than $4.4 \times 10^{-27}$ e cm for $\tan \beta = 50$. The shaded regions are preferred by $a_\mu$ at 1σ and 2σ for $\tan \beta = 50$.

![Fig. 3. Contours of $d_\mu$ in units of $10^{-24}$ e cm for varying $m_{\tilde{e}_R} = m_{\tilde{e}_L} = m_{\tilde{e}}$ and $m_{\tilde{\mu}_R} = m_{\tilde{\mu}_L} = m_{\tilde{\mu}}$ for vanishing $A$ terms, fixed $|\mu| = 500$ GeV and $M_2 = 300$ GeV, and $M_1 = (g_1^2 / g_2^2) M_2$ determined from gaugino mass unification. The parameter combination $\sin \phi_\mu \tan \beta$ is assumed to saturate the bound $d_e < 4.4 \times 10^{-27}$ e cm. The shaded regions are preferred by $a_\mu$ at 1σ and 2σ for $\tan \beta = 50$.](image-url)
than $10^{-24}$ \(e\text{cm}\) are possible. The current value of \(a_\mu\) also favors light smuons and large EDMs. The smuon mass regions preferred by the current \(a_\mu\) anomaly are given in Fig. 3 for \(\tan \beta = 50\). Within the 1\(\sigma\) preferred region, \(d_\mu\) may be as large as \((10) \times 10^{-24} e\text{cm}\) for \(m_\tilde{e} < 2 \text{TeV}\). Our assumed value of \(\tan \beta\) is conservative; for smaller \(\tan \beta\), the preferred smuon masses are lower and the possible \(d_\mu\) values larger.

5.2. Nonproportionality

Naive scaling is also broken if the \(\text{Im} A_\ell\) are not proportional to Yukawa couplings \(y_\ell\). Just as in the case of nondegeneracy, deviations from proportionality are found in many models. Even in models constructed to give proportionality, there are often corrections to the \(A\) terms, so that

\[
A_{ij} = y_{ij} A_0 + a_{ij},
\]

where the second term is smaller than the first term, but violates proportionality. In flavor models, the \(A\) terms do not obey proportionality at all. Rather they are of the form

\[
A_{ij} = c_{ij} y_{ij} A_0,
\]

where \(c_{ij}\) are order one coefficients. Clearly, violations of proportionality may affect \(d_\mu\) not only by changing the magnitude of the \(A\) terms, but also through the possible appearance of new phases, either in \(a_{ij}\) or in \(c_{ij}\). Note that these possibilities also lead to flavor violation, the subject of the following section.

We will not study the possibility of nonproportionality in detail. For large \(\tan \beta\), the \(A\) term contribution to the EDM is suppressed relative to the typically dominant chargino contribution by roughly a factor of \((g_2^2 M_2 / g'^2 M_1) (y_\mu \text{ Im} \tan \beta / \text{ Im} A_\mu)\), where we have used the amplitudes of Eq. (15). However, there are many possibilities that may yield large effects. In Ref. [8], for example, it was noted that \(A_e\) may be such that the chargino and neutralino contributions to \(d_e\) cancel, while, since \(A_e \neq A_\mu\), there is no cancellation in \(d_\mu\), and observable values are possible.

5.3. Flavor violation

In all of the discussion so far, we have neglected the possibility of supersymmetric lepton flavor violation. However, such flavor violation is present, at least at subleading order, in most models of high-scale supersymmetry breaking [15]. Moreover, large smuon–stau mixing, of particular importance here, is well-motivated by the evidence for large \(\nu_\mu - \nu_\tau\) mixing observed in atmospheric neutrinos [21]. Explicit examples of this connection in models with left–right gauge symmetry are given in Refs. [9,10]. The relation between neutrino and slepton mixing is also a general feature of Abelian flavor models [22]. In the simplest of these models, highly mixed states have similar masses, contradicting the most straightforward interpretation of the neutrino data. This difficulty may be circumvented in less minimal models by generating hierarchical neutrino masses from the neutrino mass matrix and large mixing by arranging for the gauge and mass eigenstates of the SU(2)
lepton doublets to be related by large rotations. However, because lepton doublets contain charged leptons in addition to neutrinos, these rotations also generate large misalignments between the charged leptons and sleptons, producing large slepton flavor mixing.

Smuon–stau mixing leads to a potentially significant enhancement in \(d_{\mu}\), because it breaks naive scaling by introducing contributions with a tau mass insertion so that \(d_{\mu} \propto m_{\tau}/\tilde{m}_{23}^2\). However, to evaluate the significance of this enhancement, we must first determine how large the flavor violation may be. This effect may be isolated by assuming that all charged sleptons are roughly degenerate with characteristic mass \(m_\tilde{\ell}\). In the basis with lepton mass eigenstates and flavor-diagonal gauge interactions, slepton flavor violation enters through off-diagonal masses in the slepton mass matrix. As usual, we parameterize the chirality-preserving off-diagonal masses by 

\[
\delta_{LL}^{23} \equiv \frac{m_{23}^{\mu}}{m_{23}^{\tilde{\mu}}}, \quad \delta_{RR}^{23} \equiv \frac{m_{23}^{E}}{m_{23}^{\tilde{E}}}, \quad \delta_{LR}^{23}, \quad \delta_{RL}^{23}.
\]

There may also be flavor violation in the left–right couplings; we parameterize these by \(\delta_{LR}^{23}\) and \(\delta_{RL}^{23}\). We begin by assuming real \(\delta's\); however, very large effects are possible for imaginary \(\delta's\), and we consider this possibility at the end of this section.

The off-diagonal masses induce \(\tau \rightarrow \mu \gamma\) transitions. Eight contributions are parametrically enhanced by \(m_{\tau}/m_{\mu}\) (retaining the possibility that \(\delta_{LR}^{23}, \delta_{RL}^{23} \propto m_{\tau}\)). Their Feynman diagrams and amplitudes are

\[
B(\tau \rightarrow \mu \gamma) = \frac{12 \pi^3 \alpha}{G_F m_{\tau}^2} \left[ |\mathcal{M}_L|^2 + |\mathcal{M}_R|^2 \right] B(\tau \rightarrow e \bar{\nu}_e \nu_\tau),
\]

where the amplitudes \(A'_i\) are given in Eq. (15). The branching ratio may then be written as

\[
\mathcal{M}_L = \frac{m_{\tau}}{m_{\mu}} \left[ \frac{\partial A_{\mu}^{tot}}{\partial \ln m_{\tilde{\mu}}^2} + \frac{\partial A_{\mu}^{tot}}{\partial \ln m_{\tilde{\nu}_\mu}^2} \right] \delta_{LL}^{23} + A_{\mu}^{a} \frac{1}{\delta_{22}^{LR} \delta_{23}^{LR}},
\]

where

\[
\mathcal{M}_L = \frac{m_{\tau}}{m_{\mu}} \left[ \frac{\partial A_{\mu}^{tot}}{\partial \ln m_{\tilde{\mu}}^2} + \frac{\partial A_{\mu}^{tot}}{\partial \ln m_{\tilde{\nu}_\mu}^2} \right] \delta_{LL}^{23} + A_{\mu}^{a} \frac{1}{\delta_{22}^{LR} \delta_{23}^{LR}},
\]
Fig. 4. The ratios $\delta_{LL}^{23 \text{only}}/\delta_{LL}^{\text{max}}$ (left) and $\delta_{RR}^{23 \text{only}}/\delta_{RR}^{\text{max}}$ (right). The values $\delta_{LL}^{\text{max}}$ and $\delta_{RR}^{\text{max}}$ are the upper bounds allowed by $\tau \to \mu \gamma$ determined with all leading contributions and with only the Bino-mediated contribution, respectively. We fix $m_\tilde{\ell} \approx 300$ GeV and $\tan \beta = 50$. The shaded regions are excluded by LEP, the requirement of a neutral LSP, and the condition that the vacuum not be charge-breaking.

\begin{equation}
\mathcal{M}_R = \frac{m_\tau}{m_\mu} \frac{\partial A^\text{tot}_\mu}{\partial \ln m^2_\mu} \delta_{RR}^{23} + A^0_\mu \frac{1}{\delta_{LL}^{22}} \delta_{RL}^{23}.
\end{equation}

Eqs. (15), (16), (21) and (22) provide a compact form for the branching ratio for radiative lepton decays in the mass insertion approximation and are well-suited to numerical evaluation. (Note that terms subleading in $m_\tau/m_\mu$ and linear in the $\delta$s are also easily computed as $\mathcal{M}_L = (\partial A^\text{tot}_\mu/\partial \ln m^2_\mu) \delta_{LR}^{23} \delta_{RL}^{23}$ and $\mathcal{M}_R = (\partial A^\text{tot}_\mu/\partial \ln m^2_\mu) \delta_{LL}^{23}$, but we neglect them in the analysis to follow.)

The flavor-violating mass insertions are bounded by the current constraint $B(\tau \to \mu \gamma) < 1.1 \times 10^{-9}$ [23]. It is important to note that the Higgsino-mediated decays give the dominant contribution unless $\mu \gg M_2, M_1$ [24]. The often-used bounds of Gabbiani, Gabrielli, Masiero and Silvestrini [25] assume a photino neutralino, and so effectively include only the Bino-mediated contribution. In Fig. 4 we show contours of the ratio of the upper bound on $\delta_{LL,RR}^{23}$ determined with only the Bino-mediated contribution included to the upper bound determined with all of the leading diagrams included. We see that the Bino-only bounds are reasonably accurate only for $|\mu| \gtrsim 1$ TeV, a region that is forbidden by the requirement that electromagnetic charge be an unbroken symmetry. (We have assumed that the diagonal entries of the stau and smuon mass matrices are equal; in the forbidden region, $m^2_{\tilde{\tau}_1} < 0$.) Analyses based solely on the Bino contribution are highly misleading in most regions of parameter space, especially for moderate and large $\tan \beta$. In particular, for $\delta_{LL}^{23}$, the constraint from $\tau \to \mu \gamma$ is always far more stringent than one would conclude from a Bino-only analysis. Similar conclusions apply to constraints from $\tau \to e \gamma$, $\mu \to e \gamma$, and $\mu - e$ conversion and will be presented elsewhere [26].
We now determine what effect flavor violation may have on $d_\mu$. Flavor violation induces four $m_\tau/m_\mu$ enhanced contributions to the muon’s EDM:

\[
\frac{d_\mu}{d_\mu} = \frac{m_\tau}{m_\mu} \frac{\partial^2 A_\mu}{\partial \ln m_\mu^2 \partial \ln m_\mu^2} \left[ \delta_{LL}^{23} \delta_{RR}^{23} \delta_{LL}^{23} \delta_{RR}^{23} \right].
\]

The additional flavor-violating contribution to the muon’s EDM is then simply $d_\mu^{FV} = \frac{1}{2} \epsilon \text{ Im } A_\mu^{FV}$, where

\[
A_\mu^{FV} = \frac{m_\mu}{\mu} \frac{\partial^2 A_\mu}{\partial \ln m_\mu^2 \partial \ln m_\mu^2} \left[ \delta_{LL}^{23} \delta_{RR}^{23} + \delta_{LL}^{23} \delta_{RR}^{23} \right] + \frac{\delta_{LL}^{23} \partial A_\mu}{\delta_{LL}^{23} \partial \ln m_\mu^2} \delta_{LL}^{23} + \frac{\delta_{RR}^{23} \partial A_\mu}{\delta_{RR}^{23} \partial \ln m_\mu^2} \delta_{RR}^{23}.
\]

Contours of the maximal possible EDM $d_\mu^{max}$ in the presence of $LL$ and $RR$ flavor violation are given in Fig. 5. To obtain $d_\mu^{max}$, $\phi_\mu$ is taken to saturate the constraint from $d_\epsilon$, and the sign of the flavor-violating contribution is chosen to add constructively to the flavor-conserving piece. The parameters $\delta_{LL}, \delta_{RR}$ are also taken to maximize $d_\mu$ given the constraint from $\tau \to \mu \gamma$; we make use of the fact that, subject to the constraint $ax^2 + by^2 \leq c$ with $a, b, c > 0$, the product $xy$ is maximized for $ax^2 = by^2 = c/2$. We also require $\delta_{LL,RR}^{23} \leq 1/2$ so that the mass insertion approximation is valid. We find that flavor violation may enhance $d_\mu$. While the enhancement is not enormous, it does bring the maximal possible value of $d_\mu$ into the range of the proposed experimental sensitivity in parts of the parameter space.

We have performed a similar analysis for chirality-violating flavor violation. In the case of nonvanishing $\delta_{LR,RL}^{23}$, enhancements above the proposed sensitivity are not found.

Note, however, that, to investigate various effects independently, we have assumed real off-diagonal masses. In fact, however, this division of new physics effects is rather artificial, as off-diagonal masses need not be real and generically have $O(1)$ phases.

For concreteness, we consider two cases: in the first, we take $\text{ Arg } (\delta_{LL}^{23} \delta_{RR}^{23}) = \phi_\delta$ and $\delta_{LL}^{23} = \delta_{RR}^{23} = 0$, while in the second, we let $\delta_{LL}^{23} = \delta_{RR}^{23} = 0$ and $\text{ Arg } (\delta_{LL}^{23} \delta_{RR}^{23}) = \phi_\delta$. In
Fig. 5. Contours of $d_{\mu}^{\text{max}}$ in units of $10^{-24}$ e cm in the presence of flavor violation. Regions consistent with the observed $\Delta a_\mu$ at the 1$\sigma$ and 2$\sigma$ levels are also shown. On the left, $m_\tilde{e} \approx 300$ GeV and $\tan \beta = 50$, and on the right, $M_2 = 300$ GeV and $|\mu| = 500$ GeV. $M_1$ is fixed by gaugino mass unification. The excluded regions are as in Fig. 4.

Fig. 6. Contours of $d_{\mu}$ (solid) and $a_\mu$ (dashed) as functions of $\phi_3$, the $C P$-violating phase present in flavor-violating slepton mass terms. In each pair of contours, the upper is for $\text{Arg}(\delta^{LL}_{23} \delta^{RR}_{23}) = \phi_3$ and $\delta^{LR}_{23} = \delta^{RL}_{23} = 0$, while the lower is for $\delta^{LL}_{23} = \delta^{RR}_{23} = 0$ and $\text{Arg}(\delta^{LR}_{23} \delta^{RL}_{23}) = \phi_3$. We fix $m_\tilde{e} = 300$ GeV, $\tan \beta = 30$, $|\mu| = 500$ GeV, and $M_2 = 300$ GeV, and $M_1$ is fixed by gaugino mass unification.

In both cases, the phase $\phi_3$ is irrelevant for $B(\tau \to \mu \gamma)$, as only the magnitudes of the $\delta$s enter in Eq. (21). This phase is also not constrained by $d_\epsilon$, as it has no direct couplings to the first generation. However, it contributes directly to $d_\mu$, as is clear in Eq. (23).

In Fig. 6 we show the dependence of $d_\mu$ on the phase $\phi_3$ for both cases. We see that in the $LL/RR$ case, it is easy to achieve values of $d_\mu \sim 10^{-22}$ e cm, two orders of magnitude above the proposed sensitivity. For the $LR/RL$ case, $d_\mu$ well above the proposed sensitivity is also possible. Such values are consistent with all present constraints.
In particular, note that we have also given values of $a_\mu$ including both the flavor-conserving contribution and the flavor-violating amplitude of Eq. (23). As may be seen in Fig. 6, the values of $\phi_3$ are also perfectly consistent with the currently preferred $a_\mu$.

6. Conclusions

The proposal to measure the muon EDM at the level of $10^{-24} \, e \, cm$ potentially improves existing sensitivities by five orders of magnitude. Such a leap in sensitivity is rare in studies of basic properties of fundamental particles and merits attention.

In this study we have considered the muon EDM from a number of theoretical perspectives. We noted that the recent results from the muon $(g - 2)$ experiment, although widely interpreted as evidence for a nonstandard model contribution to $a_\mu$, may alternatively be ascribed entirely to a nonstandard model contribution to $d_\mu$. In fact, these results provide the most stringent constraints on the muon EDM at present. Theoretical prejudices aside, this ambiguity will be definitively resolved only by improved bounds on (or measurements of) $d_\mu$, such as will be possible in the proposed $d_\mu$ experiment.

Considering only the indications from a nonstandard model contribution to $a_\mu$, ‘naturalness’ implies muon EDMs far above the proposed sensitivity. In more concrete scenarios, however, additional constraints, notably from the electron’s EDM and lepton flavor violation, impose important restrictions. Nevertheless, we have noted a number of well-motivated possibilities in supersymmetry: nondegeneracy, nonproportionality, and slepton flavor violation. Each of these may produce a muon EDM above the proposed sensitivity.

For simplicity, we have focused for the most part on one effect at a time. From a model-building point of view, however, this is rather unnatural. For example, nondegeneracy of the diagonal elements of the scalar mass matrices is typically accompanied by flavor violation. At the very least, if the soft masses are diagonal but nondegenerate in a particular interaction basis, off-diagonal elements will be generated upon rotating to the fermion mass basis. As noted above, nonproportionality will also typically be accompanied by flavor violation. In both of these cases, then, the lepton flavor violation leads to new contributions to $d_\mu$, as well as to new constraints from lepton flavor violating observables. The amount of flavor violation present is highly model-dependent, and so we have not considered this in detail. In Section 5.3, however, we have noted that $d_\mu$ may be greatly enhanced by two or more simultaneous effects, leading to values of $d_\mu \sim 10^{-22} \, e \, cm$, far above the proposed sensitivity.

If a nonvanishing $d_\mu$ is discovered, it will be unambiguous evidence for physics beyond the standard model. At the currently envisioned sensitivity, it will also imply naive scaling violation, with important implications for many new physics models. In addition, the measurement of nonstandard model contributions to both $d_\mu$ and $a_\mu$ will provide a measurement of new $CP$-violating phases, with little dependence on the overall scale of the new physics. Such information is difficult to obtain otherwise. Low energy precision experiments may not only uncover evidence for new physics before high energy collider
experiments, but may also provide information about the new physics that will be highly complementary to the information ultimately provided by colliders.

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Appendix A

The functions $K_N$ and $K_C$ of Eq. (15) are

\begin{align}
K_N(x^2, y^2, z^2) &= J_5(x^2, x^2, y^2, y^2, z^2) + J_5(x^2, x^2, y^2, y^2, z^2), \\
K_C(x^2, y^2, z^2) &= 2I_4(x^2, y^2, z^2) + 2I_4(x^2, y^2, z^2) - K_N(x^2, y^2, z^2),
\end{align}

where the loop functions $J_n$ and $I_n$ are defined iteratively [16] through

\begin{align}
J_n(x_1^2, \ldots, x_n^2) &= \frac{1}{x_1^2 - x_n^2} \left[ I_{n-1}(x_1^2, \ldots, x_{n-1}^2) - I_{n-1}(x_2^2, \ldots, x_n^2) \right], \\
I_n(x_1^2, \ldots, x_n^2) &= I_{n-1}(x_1^2, \ldots, x_{n-1}^2) + x_n^2 I_n(x_1^2, \ldots, x_n^2),
\end{align}

with

\begin{align}
I_2(x_1^2, x_2^2) &= - \frac{1}{16\pi^2} \left\{ \frac{x_1^2}{x_1^2 - x_2^2} \ln \frac{x_1^2}{A^2} + \frac{x_2^2}{x_2^2 - x_1^2} \ln \frac{x_2^2}{A^2} \right\}.
\end{align}

Their arguments are the gaugino masses $M_1$ and $M_2$, the Higgsino mass parameter $\mu$, and the scalar soft supersymmetry breaking masses

\begin{align}
m_{\ell L}^2 &= (m_{LL}^2)_{\ell \ell} + \left( -\frac{1}{2} + \sin^2 \theta_W \right) m_Z^2 \cos 2\beta, \\
m_{\ell R}^2 &= (m_{RR}^2)_{\ell \ell} - \sin^2 \theta_W m_Z^2 \cos 2\beta, \\
m_{\ell i}^2 &= (m_{LL}^2)_{\ell \ell} + \frac{1}{2} m_Z^2 \cos 2\beta.
\end{align}

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