From microphysics to dynamics of magnetars

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Abstract. MeV-scale magnetic fields in the interiors of magnetars suppress the pairing of neutrons and protons in the $S$-wave state. In the case of a neutron condensate the suppression is the consequence of the Pauli-paramagnetism of the neutron gas, i.e., the alignment of the neutron spins along the magnetic field. The proton $S$-wave pairing is suppressed because of the Landau diamagnetic currents of protons induced by the field. The Ginzburg-Landau and BCS theories of the critical magnetic fields for unpairing are reviewed. The macrophysical implications of the suppression (unpairing) of the condensates are discussed for the rotational crust-core coupling in magnetars and the neutrino-dominated cooling era of their thermal evolution.

1. Introduction

The powerful X-ray and soft $\gamma$-ray outburst activity observed in a number of astrophysical point sources has been attributed to the magnetic energy stored in compact objects. The \textit{magnetar} interpretation of these observational phenomena implies magnetic fields larger by a factor $10^3$ than the fields deduced for rotationally powered pulsars $B \sim 10^{12}$ G [1]. The interior fields of magnetars cannot be measured, but it has been frequently conjectured that they are stronger than their surface fields by several orders of magnitude; for recent reviews see [2, 3]. Large interior fields affect the equation of state of nucleonic and quark matter, potentially endangering the hydrostatic equilibrium of compact stars. In fact, observationally significant modification of gross parameters of compact stars (mass, radius, moment of inertia, etc.) requires extremely high fields $B \sim 10^{18} - 10^{19}$ G, for which the stars are close to loss of hydrostatic equilibrium as derived from the virial theorem [4, 5] and general relativistic numerical models [6, 7, 8].

Those physical phenomena in dense and strongly interacting matter which are controlled by the form of the quasiparticle spectrum in the vicinity of the Fermi surface are affected by much
lower fields of the order $B \sim 10^{16} - 10^{17}$ G. For such fields, electromagnetic interactions become of the order of the nuclear scales $\sim$ MeV characterizing low-energy fermionic excitation spectrum of nucleons (as opposed to the high-energy scale set by the Fermi energy). An example to be addressed below is pairing in nucleonic condensates, which is characterized by the MeV scale. Likewise, neutrino transport and radiation are dominated by this scale throughout the early evolution of compact stars as they cool via neutrino radiation from their interiors. Accordingly, we will also discuss the effects of magnetic fields on the neutrino radiation processes.

It is useful at this point to make the notion of MeV-scale magnetic fields more precise. The interaction energy of the magnetic field with the nucleon spin is $\mu_N B$, where $\mu_N = \frac{e\hbar}{2m_p}$ is the nuclear magneton. Substituting the values of constants we find that $\mu_N B \simeq \pi(B/10^{18}$ Gauss) MeV, i.e., fields of the order of $10^{17}$ G would substantially affect the MeV-scale pairing via the spin–$B$-field interaction. In the case of charged particles, the relevant magnetic-field scale follows from the Landau criterion for the critical velocity $v_\perp \sim \Delta/p_\perp$, where $\Delta$ is the pairing gap and $p_\perp$ is the characteristic momentum in the plain orthogonal to the field, restricted to $p_\perp \leq p_F$ by the Fermi momentum $p_F$. The corresponding energy scale is then given by $\nu_{pF} \sim \pi(v_\perp/c)(\xi/10$ fm$)(B/10^{16}$ Gauss) MeV, assuming that the Larmor radius $pFc/eB$ is of the order of the coherence length $\xi \simeq 10$ fm and $v_\perp/c \leq 0.3$. Thus, the range of characteristic fields relevant to the MeV-scale physics in the vicinity of the Fermi surface of nucleons is $10^{16} \leq B \leq 10^{17}$ G. As will be shown, this is indeed confirmed by explicit calculations.

The purpose of this review is to give a concise account of the progress achieved in understanding the effect of MeV-scale magnetic fields on nucleonic pairing [9, 10, 11] and its consequences for neutrino radiation [10] and crust-core coupling in magnetars [12]. The broad and actively pursued subject of the equation of state of various phases of dense matter in much stronger magnetic fields (comparable to the Fermi energy scale of fundamental fermions) is not discussed here.

This review is organized as follows. Section 2.1 summarizes the Ginzburg-Landau theory of charged (proton) and neutral (neutron) superfluid mixtures in the cores of magnetars and addresses the critical magnetic field for destruction of proton superconductivity. Section 2.2 discusses the suppression of pairing in the neutron $S$-wave condensate (relevant for crusts of magnetars). Section 3 focuses on implications the suppression of the proton and neutron pairing for dynamical coupling between the core and the crust of a magnetar. Finally, Sec. 4 considers the modifications of the neutrino emission induced by the magnetic fields. Section 5 contains some concluding remarks.

2. Unpairing of nucleonic condensates in magnetars

MeV-scale magnetic fields can destroy the superconducting coherence that is required for the formation of condensates in nucleonic matter. The interaction of the magnetic field with the neutron or proton spin induces an imbalance in the number of spin-up and spin-down particles, which implies that the Cooper pairing will be suppressed, because not all spin-up particles will find spin-down “partners” [11]. This so-called Pauli paramagnetic suppression acts for both proton and neutron condensates, but is the dominant suppression mechanism only in the case of neutrons. The proton condensate is already suppressed by a smaller magnetic field due to a different mechanism, associated with the Larmor motion of protons in the magnetic field, i.e., originating from the interaction of the charge of the proton with the $B$-field [9, 10]. The following Sections review these mechanisms in turn.

2.1. Critical unpairing of proton condensate

The BCS superconductors are characterized at least by three distinct length scales: (i) the London penetration depth $\lambda$, (ii) the coherence length $\xi$, and (iii) the interparticle distance $d$. It will be assumed henceforth that the last scale is much smaller than the other two in the
problem, i.e., the superconductor is in the weakly coupled regime. The ratio of the remaining two scales defines the type of the superconductivity via the Ginzburg-Landau (GL) parameter, \( \kappa = \lambda / \xi \) (see, e.g., [13]). In the range \( 1 / \sqrt{2} < \kappa < \infty \), the material is a type-II superconductor; otherwise it is type-I. In type-II superconductors the magnetic field is carried by electromagnetic vortices with quantum flux \( \phi_0 = \pi / e \) (here and below \( \hbar = c = 1 \)), while the field forms domain structures in a type-I superconductor. These two scales also define three distinct magnetic-field scales when combined with the flux quantum:

\[
H_{c1} \simeq \phi_0 \lambda^{-2}, \quad H_{cm} \simeq \phi_0 (\xi \lambda)^{-1}, \quad H_{c2} \simeq \phi_0 \xi^{-2}.
\]

In type-II superconductors the hierarchy of these fields is \( H_{c1} \leq H_{cm} \leq H_{c2} \) when \( \kappa \geq 1 \). At and above \( H_{c1} \) the creation of a single flux-tube (Abrikosov quantum vortex) is energetically favorable. The field \( H_{cm} \) is the thermodynamical magnetic field whose energy density is equal to the difference in the energy densities of superconducting and normal states. Finally, \( H_{c2} \) corresponds to the field at which superconductivity disappears; physically the density of flux-tubes in such a magnetic field is so high that the normal vortex cores overlap. Note that the thermodynamical magnetic field \( H_{cm} \) has no physical significance for type-II superconductors, which exist between the lower \( H_{c1} \) and upper \( H_{c2} \) critical fields.

The GL theory of neutron-proton superfluid mixtures is based on the functional [14, 15, 16]

\[
\mathcal{F}[\phi, \psi] = \mathcal{F}_n[\phi, \psi] + \mathcal{F}_p[\phi, \psi] + \frac{1}{4m_p} |D\psi|^2 + \frac{B^2}{8\pi},
\]

where \( \psi \) and \( \phi \) are the condensate wave functions for protons and neutrons, \( D = -i \nabla - 2eA \) is the gauge-invariant derivative, \( m_p \) is the proton mass and indices \( p \) and \( n \) label the quantities referring to the neutron and proton condensates. If we are interested only in the \( H_{c2} \) field, the wave function is small and the functional for the proton condensate can be written at temperature \( T \) and critical temperature \( T_{cp} \) of the superconducting phase transition as

\[
\mathcal{F}_p[\phi, \psi] = \alpha \tau |\psi|^2 + \frac{b}{2} |\psi|^4 + b' |\psi|^2|\phi|^2,
\]

where \( \tau = (T - T_{cp})/T_{cp} \). The quantities \( \alpha \) and \( b \) are the coefficients of the GL expansion, while \( b' \) describes the density-density coupling between the neutron and proton condensates. In carrying out the Ginzburg-Landau expansion (2) we have taken into account (assuming second order phase transition) that the pre-factor of the \( |\psi|^2 \) term changes its sign at \( T_c \), therefore it can be expanded in the small parameter \( \tau \) to leading order.

The current-current coupling, i.e., the entrainment of the proton condensate by the neutron condensate, can be absorbed in the effective gauge potential \( A \) [10, 14].

The variations \( \delta \mathcal{F}[\phi, \psi] / \delta \psi = 0 \), \( \delta \mathcal{F}[\phi, \psi] / \delta \phi = 0 \), and \( \delta \mathcal{F}[\phi, \psi] / \delta A = 0 \) constitute the coupled equations of motion describing the mixture of condensates. For simplicity, a homogeneous neutron background condensate is assumed. This is a good approximation given the fact that the neutron vortex size for typical neutron star rotation rates is of the order \( 10^{-4} \) cm, which by many orders of magnitude exceeds the proton vortex size \( \sim 100 \) fm.

Thus, assuming that the neutron condensate is static and homogeneous and that \( A \) is locally linear in the coordinates (constant \( B \)-field), the linearized GL equations provide the value

\[
H_{c2} = \frac{\phi_0}{2\pi \xi_p^2} \left[ 1 + \beta (b') \right],
\]

of the critical field \( H_{c2} \) [10], where \( m_p |\alpha \tau| = (2\xi_p)^{-2} \). The coupling of the proton condensate to the neutron condensate enhances the critical field by a factor \( \beta \simeq 0.2 \). The dependence of the
Figure 1. Conjectured layered structure of a magnetar with a constant magnetic field in the core (indicated by the dashed line). To the right of the intersection of this field with $H_{c2}$, the proton fluid is non-superconducting; to the left, i.e., at $B \leq H_{c2}$, it is superconducting. The crust contains a homogeneous $B$-field. The density is given in units of the nuclear saturation density $n_0$.

$H_{c2}$ field on density is illustrated in Fig. 1 according to [10]. The following features are notable. The maximum of $H_{c2}$ is attained close to the crust-core interface at a density $n_b = 0.5n_0$, where $n_0$ is the nuclear saturation density. Fig. 1 shows the conjectured layered structure of a magnetar: (a) an inner core void of superconductivity, (b) an outer core threaded by flux tubes, and (c) a crust containing a homogeneous field of magnitude $B$. If the field $B \geq \text{max}[H_{c2}]$ the intermediate flux-carrying region disappears, i.e., superconductivity in a magnetar is completely destroyed. The critical fields in a similar formalism for superfluid-superconducting mixture were studied recently in [17].

2.2. Critical unpairing of neutron $S$-wave condensate

The nature of the suppression of pairing in the neutron condensate differs from that in the proton condensate, because charge-neutral neutrons interact with the $B$-field via their spin magnetic moment. This has a destructive effect on $S$-wave neutron Cooper pairs, which involve spin-up and down partners. Clearly, a large enough magnetic field will quench pairing completely. In the following, this field will be referred to as $H_{c2}$, as no confusion should arise with the analogous quantity for protons.

$S$-wave pairing is relevant for the crusts of magnetars, i.e., the low-density regime below the saturation density of symmetrical nuclear matter. At higher densities the dominant pairing state in neutron matter shifts to the $^3P_2-^3F_2$ channel, which pairs neutrons in a total spin-1 state (see [18, 19] for a review). In this case, the spin-polarizing effect of the magnetic field on the internal structure of the spin-1 pairs is nondestructive [20, 21, 22].

In Fig. 2 we plot values of $H_{c2}$ for the neutron condensate as a function of density, determined based on a phase-shift-equivalent nucleon-nucleon interaction and numerical solutions of the BCS equations in the case of spin-polarized neutron matter [11]. The shape of the curve reflects the
Figure 2. Critical unpairing magnetic field $H_{c2}$ for the neutron condensate due to spin alignment in this field, for two temperatures indicated in the plot [11]. The density is given in units of the nuclear saturation density $n_0$. The bell-shaped structure of the critical field reflects the analogous shape of the gap function of $^1S_0$ paired neutrons, which attains a maximum at a certain density. The maximum in the gap value arises from the interplay between the increase in the density of states at the Fermi surface and decrease in the attraction in the pairing force as the density (and neutron Fermi energy) increases.

corresponding density dependence of the pairing gap, and its temperature dependence follows the BCS prediction: it is largest at $T = 0$ and decreases as the pairing gap decreases with increasing temperature. Thus, if the local field in a magnetar crust exceeds the value $H_{c2}$, the magnetic field will destroy the condensate. In the specific model of [11], the neutron fluid in magnetars will be non-superfluid (i.e., in a normal phase) for $B > 2.6 \times 10^{17}$ G. Note that at those densities where attractive $P$-wave interaction exists the suppression of the $S$-wave pairing may give rise to $P$-wave superfluid rather than normal spin-polarized fluid [11].

The non-superfluidity or partial superfluidity of magnetars will clearly have important implications for an array of microphysical quantities of neutron star crusts. These include their neutrino emissivities and transport properties, and consequently the thermal relaxation and dynamical coupling time scales that are important, most notably, for the damping of stellar oscillations and the interpretation of rotational anomalies such as glitches and anti-glitches. Finally, it is to be noted that the Pauli paramagnetic destruction mechanism discussed here for $S$-wave paired neutrons will apply as well to $S$-wave paired protons; however, the diamagnetic mechanism mentioned in the preceding section is more important for protons.

3. Crust-core coupling time scales in magnetars

We turn now to implications of the unpairing effect for the rotational coupling of a neutron superfluid in magnetar cores. In [12] it was argued that unpairing offers a new channel for coupling of the electron-proton plasma to the neutron vorticity in the core: if protons are unpaired then they are available for scattering off neutron quasiparticles in the vortex cores. This process is much more effective than the scattering of electrons off magnetized neutron vortices by electromagnetic forces, which is the dominant process in the type-I superconducting
case. In the more realistic case of type-II superconductivity, the coupling mechanism can be more complicated, because of the non-negligible interactions between the protonic flux-tubes and the neutron vortices.

The time scale of dynamical coupling of the superfluid to the normal plasma is important for the interpretation of rotational irregularities of magnetars, including glitches, anti-glitches, post-glitch relaxation, and non-axisymmetric motions such as precession. The influence of an interior fluid on precession has been discussed extensively in the literature for the case of ordinary neutron stars (see [23] and references therein). In the case of magnetars, the coupling time scale of the $P$-wave neutron superfluid when the protons form a normal fluid is an important ingredient of such considerations [12].

To illustrate this point, assume a constant field in the core of a magnetar and a fully unpaired proton fluid. The field will then couple to the electron fluid on plasma time scales, which are much shorter than the hydrodynamical time scales. Therefore, the unpaired core of a magnetar can be considered as a two-fluid system consisting of a superfluid neutron condensate component and a normal component formed by the proton and electron fluids.

The neutron superfluid rotates by forming an array of quantized vortices with areal number density

$$N_n = \frac{2\Omega}{\omega_0}, \quad \omega_0 = \frac{\pi}{m_n},$$

where $m_n$ is the bare neutron mass, $\Omega$ the rotation frequency of the star, and $\omega_0$ the quantum of neutron circulation. As seen from Eq. (4), any changes in the rotation frequency of a magnetar must be accompanied by changes in the number of neutron vortices. Because the vortices are created and destroyed at the interfaces, they need to move in the bulk of the superfluid to respond to variations in $\Omega$. The velocity of a vortex $v_L$ is determined by the equation of motion

$$\rho_n \omega_0 |(v_S - v_L) \times \nu| - \eta (v_L - v_N) = 0.$$  

This equation reflects the balance of forces acting on a vortex segment; the first term of the right is the Magnus force, and the second is the frictional force between the vortices and the normal liquid, with $\rho_n$ denoting the mass density of the superfluid component, $v_N$ is the velocity of the normal component and $\eta$ is the coordinate-dependent longitudinal friction coefficient and $\nu$ is the unit vector along the vortex circulation. The frictional force due to scattering of normal quasiparticles (electrons, muons, and unpaired protons) is given by

$$F = \frac{2}{\tau N_n} \int f(p,v_L) d^3p / (2\pi \hbar)^3 = -\eta v_L,$$

where $f(p,v_L)$ is the non-equilibrium distribution function in the frame where $v_N = 0$. Assuming a small perturbation, this function can be expanded about the equilibrium distribution Fermi function $f_0$ to obtain $f(p,v_L) = f_0(p) + (\partial f_0 / \partial \epsilon)(p \cdot v_L)$. For strongly degenerate systems, $\partial f_0 / \partial \epsilon \approx -\delta(\epsilon - \epsilon_{Fp})$ and $\eta = m_p^* n_p / \tau N_n$, where $n_p$ and $m_p^*$ are respectively the unpaired quasiparticle number density and effective mass. In our example, the dominant process determining the relaxation time $\tau$ is the scattering of unpaired protons off neutron vortex-core quasiparticles; its rate scales with the temperature as $\tau^{-1} \propto T \exp[-\epsilon_{1/2}^0 / T]$ and is proportional to the differential nuclear scattering cross-section $d\sigma / d\Omega$ between neutrons and protons. The exponential factor contains the energy scale $\epsilon_{1/2}^0 = \pi \Delta_n^2 / (4 \epsilon_{Fn})$, corresponding to the lowest energy state of a neutron quasiparticle confined in the vortex, with $\Delta_n$ denoting the neutron pairing gap and $\epsilon_{Fn}$ the neutron Fermi energy; (see [24] for further details.) The dynamical coupling time of the superfluid to the plasma is given by

$$\tau_D = \frac{1}{2\Omega} (\zeta + \zeta^{-1}),$$
Figure 3. Relaxation time, drag-to-lift ratio, and dynamical coupling time scale (7) as functions of baryon density in the stellar core for temperatures $T = 0.01$ and 0.05 MeV (or equivalently $T = 1.2 \times 10^8$ K and $T = 5.8 \times 10^8$ K). The computations are carried out with an angle-independent neutron-proton cross section $\sigma \simeq 60 \text{ fm}^2$ and for a rotation frequency $\Omega = 1 \text{ Hz}$. Note that these results apply only if the proton fluid is unpaired at a given density.

where the dimensionless drag-to-lift ratio $\zeta$ is related to the dimensioned friction $\eta$ via the relation $\zeta = \eta/\rho_n \omega_0$. Figure 3 summarizes the results of computations carried out in [12]. It is seen that for typical magnetar periods of about 10 sec, i.e., for spin rotations of about 1 Hz, the unpaired core couples to the plasma on dynamical time scales from several minutes at the crust-core boundary to a few seconds in the high-density core. Furthermore, the values of $\zeta$ obtained imply that the low-density outer core ($\zeta \simeq 0.2$) does not affect free precession, while the high-density inner core ($\zeta \simeq 0.4$) can cause significant damping of precession over a cycle. Thus, magnetar precession cannot be definitely excluded since the values of the drag-to-lift ratio are within the range of the crossover from undamped to damped precession. However, in the inner core, where $\zeta$ is large, the condensate vanishes for lower magnetic fields. We conclude that relatively low magnetic fields are sufficient to damp any free precession in magnetars. Of course, this argument applies only to free precession. Magnetic deformations of magnetars can be a continuous source of excitation of precession [25, 26]. The macroscopic arrangement of the magnetic field in the case of type-II superconducting stars and its implications for precession have been discussed elsewhere [27, 28, 29].

4. Neutrino radiation from magnetars
The suppression of pairing by MeV-scale magnetic fields will also have profound consequences on the thermal evolution of magnetars, because the dominant processes of neutrino radiation will not be suppressed by the Boltzmann factor containing the gap in the quasiparticle spectrum of baryons. At the same time, the processes that are intrinsic to condensates, such as the pair-breaking emission of neutrino-anti-neutrino pairs, will not operate by definition.

4.1. Direct Urca process: $n \rightarrow p + e + \bar{\nu}_e$
In strong $B$-fields the phase-space of nucleons is modified and the Urca process is allowed even below the threshold $x_p \simeq 11\%$, where $x_p$ is the proton fraction in nucleonic matter [30, 31]. The
Figure 4. Magnetic-field dependence of the emissivity $\epsilon$ of the Urca process (measured in units of the zero-field emissivity $\epsilon_0$), as calculated at temperatures $T = 0.01$ and $0.1$ MeV and density $n = n_0$ for the case $x = 0$, i.e., such that emission is forbidden kinematically in the zero-field limit. Three possibilities are considered: (i) normal neutron-star matter (dotted line), (ii) neutrons paired but protons normal (dashed line), and (iii) both neutrons and protons paired (dot-dashed lines). The parameter $x$ of Eq. (4) scales with the field $B$ according to $x \propto N^2/3 F_p\propto B^{-2/3}$.

kinematics of the Urca process in this case is conveniently characterized by the parameter [31]

$$x = \left[1 - (k_{Fe} + k_{Fp})^2/k_{Fn}^2\right] N_{Fp}^{2/3},$$

where $k_{Fi}, i = e, p, n,$ are the Fermi momenta of electrons ($e$), protons ($p$) and neutrons ($n$) and $N_{Fp} = k_{Fp}^2/2|e|B$ is the number of Landau levels populated by protons. Clearly, for $x > 0$ the Urca process is forbidden in the $B = 0$ case, but in a strong magnetic field some phase space opens up. Then, if the Urca process operates even at a fraction of its strength in the kinematically permitted region, it can become an important factor in the cooling the star's core, because other competing processes are weaker by orders of magnitude. In the case where $x < 0$, i.e., when the Urca process is kinematically permitted, magnetic fields induce de Haas-van Alven oscillations in the emissivity associated with the filling of the Landau levels.

As well known, proton and neutron pairing suppress the Urca process once nucleons make a transition to a superconducting or superfluid state. At asymptotically low temperatures, the emissivity is suppressed by a factor $\exp(-\Delta/T)$ for each participating nucleon, where $\Delta$ is the relevant pairing gap and $T$ is the temperature [32, 33, 34]. Clearly, termination of neutron and proton $S$-wave pairing by the $B$-field will mitigate this suppression. Because the neutron pairing gap in the $P$-wave channel is smaller than the proton pairing gap in the $S$-wave channel, destruction of proton superconductivity by a MeV-scale magnetic field will strongly modify the Urca emissivity. Numerical examples can be found in [10].

4.2. Field-assisted bremsstrahlung processes: $N \to N + \nu_f + \bar{\nu}_f$

The bremsstrahlung $N \to N + \nu_f + \bar{\nu}_f$ of neutrino pairs by a single nucleon (denoted $N$) is ordinarily prohibited by energy and momentum conservation. Accordingly, the leading charge-
neutral mechanism for neutrino-pair production is the so-called modified bremsstrahlung process \( N + N \rightarrow N + N + \nu_f + \bar{\nu}_f \). However, when the interaction energy of the \( B \)-field with the spin of a nucleon becomes of the order of the temperature, the single-nucleon bremsstrahlung process \( N \rightarrow N + \nu + \bar{\nu} \) becomes kinematically possible because of the paramagnetic splitting of the energies of nucleons with spin-up and spin-down in a strong \( B \) field [35]. Pairing will suppress this process exponentially at low temperatures, as discussed above in the case of the Urca process. However, if unpairing by a strong magnetic field takes place, this suppression will be nullified.

4.3. Pair-breaking processes: \([NN] \rightarrow N + N + \nu_f + \bar{\nu}_f\)

Nucleonic superfluidity results in a new class of neutrino bremsstrahlung processes that owe their existence to the condensate. Symbolically, these can be written as \([NN] \rightarrow N + N + \nu_f + \bar{\nu}_f\), where \([NN]\) stands for a Cooper pair. These processes are referred to as pair-breaking-formation (PBF) processes [36, 37, 38, 39, 40, 41]. The rates of neutrino emission via the PBF processes scale as \( \epsilon \propto \Delta^2 T^7 \), where \( \Delta \) is the pairing gap. Therefore, the unpairing of the \( S \)-wave condensates will have the plain effect of removing the PBF processes from the regions where the field locally exceeds the unpairing fields for protons and neutrons. Consequently, the net neutrino emission rate will be reduced asymptotically to the value which corresponds to the PBF emission by the \( P \)-wave condensate.

Assessment of the combined effect of unpairing on the cooling of neutron stars is difficult. While it is clear how the individual processes are affected by the strong magnetic field, their concerted effect needs to be studied in numerical simulations.

4.4. Specific heats

An additional ingredient that modifies the thermal evolution of magnetars is the specific heat of the interior matter, which quantifies the thermal inertia of the star. As well known, in the absence of superconductivity and superfluidity, the heat capacity of nucleonic fluids will scale linearly with temperature. This should be contrasted with the exponential suppression of the heat capacity in the superconducting \( S \)-wave state. In fully superconducting/superfluid neutron stars at low \( B \)-fields, electrons dominate the heat capacity. One can anticipate that for MeV-scale magnetic fields in which only protons are unpaired, the proton and electron specific heats will decrease linearly with temperature (as in normal Fermi liquids), whereas the heat capacity of the superfluid neutrons will be reduced exponentially in the regions of \( S \)-wave pairing and as a power-law in the regions of \( P \)-wave pairing. Increase of the specific heat of the interior matter will act to increase the cooling time scale of a magnetar.

In closing, it should be mentioned that other factors such as internal heating due to the Ohmic dissipation of the magnetic fields in the interior of the star will be an important factor in determining the temperature evolution of magnetars. The unpairing of protons will imply that the resistivity due to electron-proton scattering will be larger than in superconducting stars. Consequently, the time scales of the Ohmic dissipation will be shorter.

5. Final remarks

Magnetars pose new challenges at the microphysical level because the electromagnetic interactions (e.g. the magnetic field-nucleon spin coupling) become of the order of the nuclear MeV scale. As a consequence, we find an intimate interplay between the electromagnetic interactions and nuclear and weak processes that take place in the vicinity of the Fermi surfaces of nucleons. This contribution has focused on the recent progress in understanding the mechanisms of suppression of pairing in nucleonic matter by a magnetic field and their implications for macroscopic dynamics of magnetars, such as their rotational and thermal evolutions. These findings call for more detailed studies of the macroscopic dynamics of magnetars, which will entail the modification of the pairing properties of nucleonic fluids.
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