An Analysis Model for Water Cone Subsidence in Bottom Water Drive Reservoirs

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Abstract. Water coning in bottom water drive reservoirs, which will result in earlier water breakthrough, rapid increase in water cut and low recovery level, has drawn tremendous attention in petroleum engineering field. As one simple and effective method to inhibit bottom water coning, shut-in coning control is usually preferred in oilfield to control the water cone and furthermore to enhance economic performance. However, most of the water coning researchers just have been done on investigation of the coning behavior as it grows up, the reported studies for water cone subsidence are very scarce.

The goal of this work is to present an analytical model for water cone subsidence to analyze the subsidence of water cone when the well shut in. Based on Dupuit critical oil production rate formula, an analytical model is developed to estimate the initial water cone shape at the point of critical drawdown. Then, with the initial water cone shape equation, we propose an analysis model for water cone subsidence in bottom water reservoir reservoirs. Model analysis and several sensitivity studies are conducted.

This work presents accurate and fast analytical model to perform the water cone subsidence in bottom water drive reservoirs. To consider the recent interests in development of bottom drive reservoirs, our approach provides a promising technique for better understanding the subsidence of water cone.

1. Introduction

Water coning in bottom water drive reservoirs, which will result in earlier water breakthrough, rapid increase in water cut and low recovery level, has drawn tremendous attention in petroleum engineering field (Liu et al., 2011; Xu et al., 2010). As stated in the literatures, most of the researches about water cone focus on studying the water coning behavior as it grows up (Jiang et al., 2007; Cai et al., 2008; Li et al., 2010; Duan et al., 2011). In order to control the water cone and furthermore to enhance economic performance, different kinds of measures have been taken in the oilfields, especially the shut-in coning control method. With the shut-in coning control method (i.e. production is stopped), no external pressure works on the reservoir fluids, and a flow of water from the location of higher potential to lower potential. However, until now the reported studies for water cone subsidence are very scarce (Nie et al., 2012; Lee...
et al., 1995). Therefore, it is of great significance to present an analytical model for analyzing the subsidence of water cone in bottom drive reservoirs.

2. Water cone subsidence in bottom water drive reservoirs

As stated in the literature, water cone shape above the point of critical drawdown is important to study cone subsidence (Lee et al., 1995). In this paper, the initial water cone shape, as the cone subsidence begins, is defined here at the point of critical drawdown (seen in Fig. 1).

![Fig. 1 Schematic plot of the initial water cone shape](image)

According to Dupuit critical oil production rate formula (Hagoort J. 1988)

\[
q = \frac{\pi k \Delta \rho g (d^2 - h_p^2)}{\mu \ln (r_e/r_w)}
\]

(1)

This expression is only valid within the range of \( h_p \) as

\[
\frac{2d}{3} \leq h_p < d
\]

(2)

According to Dupuit critical oil production rate formula, the height of the water cone corresponding to radial distance from the well bore can be written as

\[
h = (d - h_p) \left( \frac{d}{d - h_p} - \sqrt{\frac{d}{d - h_p}} \right) - \frac{q \mu}{\pi k \Delta \rho g (d - h_p)^2} \ln \left( \frac{r_e}{r} \right)
\]

(3)

Eq. (3) can be rewritten as

\[
h_D = \frac{h}{d - h_p} = \left( \frac{1}{\varepsilon} - \frac{1}{\varepsilon^2 + q_D \ln r_D} \right)
\]

(4)

with

\[
\varepsilon = \frac{d - h_D}{d}; \quad r_D = \frac{r}{r_e}; \quad q_D = \frac{q \mu}{\pi k \Delta \rho g (d - h_p)^2}
\]

(5)

where \( \varepsilon \) is the ratio of unperforated interval height and thickness of oil.

With the initial water cone condition Eq. (4), the unsteady state flow model for water cone subsidence is proposed in this paper. The detailed derivation of the solution is shown in the Appendix. Considering that initial time \( t=0 \), when subsidence of water cone is started, the instantaneous cone height is found to be
The height of cone apex in dimension less ratio with the initial apex height, as a function of time is given by the following equation:

\[
h = \sum_{m=1}^{\infty} \frac{2h_c J_0(\lambda_m r)}{r_c^2 J_1^2(\lambda_m r_c)} \left( \int_0^r \frac{1}{\varepsilon} \sqrt{1 + q_0 \ln \frac{r}{r_c}} J_0(\lambda_m r) dr \right)
\]  

(6)

The height of cone apex in dimension less ratio with the initial apex height, as a function of time is given by the following equation:

\[
H_t = \frac{H}{H_c} = \sum_{m=1}^{\infty} \frac{2h_c}{r_c^2 J_1^2(\lambda_m r_c)} \left( \int_0^r \frac{1}{\varepsilon} \sqrt{1 + q_0 \ln \frac{r}{r_c}} J_0(\lambda_m r) dr \right)
\]  

(7)

3. Results and discussions

In this section, we will study the water cone subsidence in detail. The presented model Eq. (4) demonstrates that \( \varepsilon \) and \( q_0 \) are the basic parameters, which can describe the initial water cone shape in dimensionless form. Fig.2 and Fig.3 illustrate initial water cone shape curves with different \( \varepsilon \) and \( q_0 \), respectively. As shown in Fig.2 and Fig.3, the dimensionless initial water cone height \( h_0 \) decreases with the increased dimensionless radial distance, which is expected. Fig.2 illustrates that the larger \( \varepsilon \) will lead to greater dimensionless water cone height \( h_0 \). The main reason for this is the larger \( \varepsilon \) value reflects the larger unperforated interval height. Fig.3 shows that the dimensionless water cone height increases with the dimensionless production rate \( q_0 \), which is consistent with the former studies (Guo and Lee, 1992; Guo et al., 1992; Lee et al., 1995; Menouar and Hakim, 1995; Nie et al., 2012).

**Fig. 2** Effect of \( \varepsilon \) on the initial water cone shape  
**Fig. 3** Effect of \( q_0 \) on the initial water cone shape

Fig.4 illustrates the variation of dimensionless water cone height \( H/H_c \) with time at different \( \varepsilon \). As shown in Fig.4, the dimensionless water cone height \( H/H_c \) decreases with time, which is expected. Fig.4 also demonstrates that the larger \( \varepsilon \) will lead to greater dimensionless water cone height. Fig.5 studies the lateral subsidence of the water cone. As shown in Fig.5, the dimensionless water cone height \( h/H_c \) decreases with dimensionless radial distance \( r_0 \), which shows a variation trend similar to that of the available experimental data (Nie et al., 2012). Fig.5 also illustrates that the dimensionless water cone height \( h/H_c \) decreases with time.
4. Conclusions

Based on Dupuit critical oil production rate formula, a theoretical model is developed to estimate the initial water cone shape at the point of critical drawdown. Then with the initial water cone shape equation, an analysis model for water cone subsidence in bottom water drive reservoirs is presented. With the theoretical study of the model in this paper, following conclusion can be drawn:

1. A set of curves to study the water cone subsidence is presented, which can make the approach easier to be used in practice.

2. The dimensionless initial water cone height $\frac{h_D}{H_C}$ decreases with the increased dimensionless radial distance $\frac{r_D}{r}$ and increases with the dimensionless production rate $\frac{q_D}{H_C}$. The larger $\varepsilon$ will lead to greater dimensionless initial water cone height $h_D$.

3. The dimensionless water cone height $\frac{H}{H_C}$ decreases with time, and the larger $\varepsilon$ will lead to greater dimensionless water cone height. The dimensionless water cone height $\frac{h}{H_C}$ decreases with dimensionless radial distance $\frac{r_D}{r}$ (or the increased time).

4. The analytical model presented in this paper is useful for water cone subsidence in bottom water reservoirs. However, flow in the vertical direction is neglected in the model, which may arise about some limitations for the model. A more accurate solution can be obtained by considering the vertical flow, and this work is in processing.

Appendix. Derivation of the model

The mathematical model for water cone subsidence must satisfy the following assumptions:

1. The homogeneous reservoir is a large circular formation which is isolated from any other external sources, and transient flow condition prevails in the reservoir;

2. Transient flow of compressible fluids through homogeneous porous medium is governed by the diffusivity equation;

3. During the flow, capillary pressure is negligible and abrupt two-phase interface exists;

4. After the well is shut in (i.e. the production is stopped), without the presence of any external effect, water flow phenomena along the original oil-water interface level is a radial transient shallow confined flow.

Neglecting the flow in a vertical direction, for a vertical well, the equation in cylindrical coordinates can be written in simplified form as

$$ \frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi}{\partial r} = \frac{\mu \rho_w (c_w + \phi c_v)}{k \Delta \rho} \frac{\partial \varphi}{\partial t} $$  \hspace{1cm} (A-1)

where $\varphi$ is the potential function which is defined as

$$ \varphi = \frac{k}{\mu_w} \Delta \rho g h $$  \hspace{1cm} (A-2)

Since, $h$ is the only variable in the above expression Eq. A-2, Eq. A-1 can be rearranged as
The initial and boundary conditions for Eq. A-3 are as follows

\( h(r, t = 0) = h_0(r) = H_c \left( \frac{1}{\epsilon} - \frac{1}{\epsilon^2} + q_0 \ln \frac{r}{r_e} \right) \)  
\( h(r = r_e, t) = 0 \)  
\( \frac{\partial h(r = 0, t)}{\partial r} = 0 \)  

Assuming \( h(r, t) = R(r)T(t) \), Eq. A-3 can be rewritten as

\[ \frac{1}{R} \frac{\partial^2 R}{\partial r^2} + \frac{1}{R} \frac{\partial R}{\partial r} + \frac{\mu \rho_w (c_w + \phi c_r)}{k \Delta \rho} \frac{1}{T} \frac{\partial T}{\partial t} = -\lambda^2 \]

where \( \lambda \) is an arbitrary constant. Solving individually for \( R \) and \( T \), the general solution for \( h \) is

\[ h = e^{-k \Delta \rho \frac{1}{\mu \rho_w (c_w + \phi c_r)} \lambda^2 t} \left[ A J_0 (\lambda r) + B Y_0 (\lambda r) \right] \]

Applying the condition Eq. A-4c to Eq. A-5, the coefficient \( B = 0 \) can be obtained. From the condition Eq. A-4b, we can get series of \( \lambda_m \), which satisfies the following equation

\[ J_0 (\lambda_m r_e) = 0 \quad (m = 1, 2, 3 \cdots) \]

Based on Eq. A-5 and Eq. A-6, \( h(r, t) \) can be written as

\[ h(r, t) = \sum_{m=1}^{\infty} A_m J_0 (\lambda_m r) e^{-k \Delta \rho \frac{1}{\mu \rho_w (c_w + \phi c_r)} \lambda_m^2 t} \]

Now applying the condition Eq. A-4a, we have

\[ h(r, t = 0) = h_0(r) = \sum_{m=1}^{\infty} A_m J_0 (\lambda_m r) \]

Thus the solution for instantaneous water cone height is

\[ h = \sum_{m=1}^{\infty} 2H_c J_0 (\lambda_m r) e^{-k \Delta \rho \frac{1}{\mu \rho_w (c_w + \phi c_r)} \lambda_m^2 t} \left( \int_{r_e}^{r} r \left( \frac{1}{\epsilon} - \frac{1}{\epsilon^2} + q_0 \ln \frac{r}{r_e} \right) J_0 (\lambda_m r) \, dr \right) \]

The transient cone height at its apex can be found by placing \( r = 0 \) in the above expression

\[ H = \sum_{m=1}^{\infty} 2H_c e^{-k \Delta \rho \frac{1}{\mu \rho_w (c_w + \phi c_r)} \lambda_m^2 t} \left( \int_{r_e}^{r} r \left( \frac{1}{\epsilon} - \frac{1}{\epsilon^2} + q_0 \ln \frac{r}{r_e} \right) J_0 (\lambda_m r) \, dr \right) \]

The dimensionless water cone height can be determined as

\[ H = \frac{H}{H_c} = \sum_{m=1}^{\infty} 2e^{-k \Delta \rho \frac{1}{\mu \rho_w (c_w + \phi c_r)} \lambda_m^2 t} \left( \int_{r_e}^{r} r \left( \frac{1}{\epsilon} - \frac{1}{\epsilon^2} + q_0 \ln \frac{r}{r_e} \right) J_0 (\lambda_m r) \, dr \right) \]

**Nomenclature**

\( A \) Constant of solution for governing differential equation

\( B \) Constant of solution for governing differential equation

\( c_w \) Volume compressibility coefficient of water

\( c_r \) Volume compressibility coefficient of rock
Thickness of oil zone
Gravity constant
Instantaneous height of water cone at any radial distance
Perforated interval height of well
Instantaneous height of water cone at \( r=0 \)
Time dependent dimensionless cone apex height
Critical cone height at \( r=0 \) and \( t=0 \)
Permeability of porous media
Integer numbers
Dupuit critical oil production rate
Dimensionless production rate
Radial distance
Dimensionless radial distance
Drainage radius
Well radius
The time
Velocity potential function
The porosity
Arbitrary constant
Water viscosity
Density of water
Difference between densities of water phase and oil phase
The ratio of unperforated interval height and thickness of oil

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