STOCHASTIC BACKGROUND FROM COALESCENCES OF NEUTRON STAR–NEUTRON STAR BINARIES

T. REGIMBAU
Unité Mixte de Recherche 6162 Artemis, CNRS, Observatoire de la Côte d’Azur, BP 4229, 06304 Nice Cedex 4, France; regimbau@obs-nice.fr

AND

J. A. DE FREITAS PACHECO
Unité Mixte de Recherche 6202 Cassiopeîe, CNRS, Observatoire de la Côte d’Azur, BP 4229, F-06304 Nice Cedex 4, France; pacheco@obs-nice.fr

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ABSTRACT

In this work, numerical simulations were used to investigate the gravitational stochastic background produced by coalescences of double neutron star systems occurring up to $z \sim 5$. The cosmic coalescence rate was derived from Monte Carlo methods using the probability distributions for massive binaries to form and for a coalescence to occur in a given redshift. A truly continuous background is produced by events located only beyond the critical redshift $z_c = 0.23$. Events occurring in the redshift interval $0.027 < z < 0.23$ give rise to a “popcorn” noise, while those closer than $z = 0.027$ produce a shot noise. The gravitational density parameter $\Omega_{gw}$ for the continuous background reaches a maximum around 670 Hz with an amplitude of $1.1 \times 10^{-9}$, while the popcorn noise has an amplitude about 1 order of magnitude higher and the maximum occurs around a frequency of 1.2 kHz. The signal is below the sensitivity of the first generation of detectors but could be detectable by the next generation of ground-based interferometers. Correlating two coincident advanced LIGO detectors or two EGO interferometers, the expected S/N are respectively 0.5 and 10.

Subject headings: binaries: close — gravitational waves — stars: neutron

1. INTRODUCTION

The merger of two neutron stars, two black holes, or a black hole and a neutron star are among the most important sources of gravitational waves (GW), due to the huge energy released in the process. In particular, the coalescence of double neutron stars (DNSs) may radiate about $10^{53}$ ergs in the last seconds of their inspiral trajectory, at frequencies up to 1.4–1.6 kHz, a range covered by most of the ground-based laser interferometers, such as VIRGO (Bradaschia et al. 1990), LIGO (Abramovici et al. 1992), GEO (Hough 1992), or TAMA (Kuroda et al. 1997). In addition to the amount of energy involved in these events, the rate at which they occur in the local universe is another parameter characterizing whether these mergers are potential interesting sources of GW. In spite of the large amount of work performed in the past few years, uncertainties persist in estimates of the DNS coalescence rate. In a previous investigation, we have revisited this question (de Freitas Pacheco et al. 2006; Regimbau et al. 2005), taking into account the galactic star formation history derived directly from observations and including the contribution of elliptical galaxies when estimating the mean merging rate in the local universe. Based on these results, we have predicted an initial detection rate of one event every 125 and 148 yr by LIGO and VIRGO, respectively, and up to 6 detections per year in their advanced configurations.

In addition to the emission produced by the coalescence of the nearest DNSs, the superposition of a large number of unresolved sources at high redshifts will produce a stochastic GW background. In past years, different astrophysical processes capable of generating a stochastic background have been investigated. Distorted black holes (Ferrari et al. 1999; de Araujo et al. 2000) and bar-mode emission from young neutron stars (Regimbau 2001) are examples of sources able to generate a shot noise (time interval between events large in comparison with duration of a single event), while supernovae or hypernovae (Blair & Ju 1996; Coward et al. 2001, 2002; Buonanno et al. 2005) are expected to produce an intermediate “popcorn” noise. On the other hand, the contribution of triaxial rotating neutron stars (Regimbau & de Freitas Pacheco 2001), including magnetars (Regimbau & de Freitas Pacheco 2006), constitutes a truly continuous background.

Populations of compact binaries such as cataclysmic variables are responsible for the existence of a Galactic background of GW in the mHz domain, which could represent an important source of confusion noise for space detectors such as LISA (Evans et al. 1987; Hils et al. 1990; Bender & Hils 1997; Postnov & Prokhorov 1998; Neleman et al. 2001; Timpano et al. 2005). These investigations have been extended recently to the extragalactic contribution. Schneider et al. (2001), Kim et al. (2003), and Cooray (2004) considered cosmological populations of double and mixed systems involving black holes, neutron stars, and white dwarfs, while close binaries originating from low- and intermediate-mass stars were discussed by Farmer & Phinney (2002).

In this work, using the DNS merging rate estimated in our previous study, we have estimated the gravitational wave background spectrum produced by these coalescences. Numerical simulations based on Monte Carlo methods were performed in order to determine the critical redshift $z_c$ beyond which the duty cycle condition required to have a continuous background ($D > 1$) is satisfied. Unlike previous studies, which focus their attention on the early low-frequency inspiral phase covered by LISA (Schneider et al. 2001; Farmer & Phinney 2002; Cooray 2004), here we are mainly interested in the few thousand seconds before the last stable orbit is reached, when more than 96% of the
gravitational wave energy is released. The signal frequency is in the range 10–1500 Hz, which is covered by ground-based interferometers. The paper is organized as follows. In § 2 the simulations are described; in § 3 the contribution of DNS coalescences to the stochastic background is calculated; in § 4 the detection possibility with laser beam interferometers is discussed, and finally, in § 5 the main conclusions are summarized.

2. THE SIMULATIONS

In order to simulate by Monte Carlo methods the occurrence of merging events, we have adopted the following procedure. The first step is to estimate the probability for a given pair of massive stars, supposed to be the progenitors of DNSs, to be formed at a given redshift. This probability distribution is essentially given by the cosmic star formation rate (Coward et al. 2002) normalized in the redshift interval $0 \leq z \leq 5$, e.g.,

$$P_f(z) = \frac{dR_f(z)/dz}{N_p}.$$  

(1)

The normalization factor in the denominator is essentially the rate at which massive binaries are formed in the considered redshift interval, e.g.,

$$N_p = \int_0^5 [dR_f(z)/dz] dz,$$  

(2)

which depends on the adopted cosmic star formation rate, as we shall see later.

The formation rate of massive binaries per redshift interval is

$$R_p(z_f) = \frac{dR_f(z_f)/dz_f}{1+z_f} = \lambda_p R_c(z_f) dV(z_f) dz_f.$$  

(3)

In the equation above, $R_c(z)$ is the cosmic star formation rate (SFR) expressed in $M_\odot$ Mpc$^{-3}$ yr$^{-1}$ and $\lambda_p$ is the mass fraction converted into DNS progenitors. Hereafter, rates per comoving volume will always be indicated by the superscript “**” and rates with indexes ”$z_f$” or ”$z$,” refer to differential rates per redshift interval, including all cosmological factors. The $(1+z)$ term in the denominator of equation (3) corrects the star formation rate by time dilatation due to the cosmic expansion. In the present work we assume that the parameter $\lambda_p$ does not change significantly with the redshift, and thus it will be considered as a constant. In fact, this term is the product of three other parameters, namely,

$$\lambda_p = \beta_{NS} f_b \lambda_{NS},$$  

(4)

where $\beta_{NS}$ is the fraction of binaries that remain bound after the second supernova event, $f_b$ is the fraction of massive binaries formed among all stars, and $\lambda_{NS}$ is the mass fraction of neutron star progenitors.

According to the results by de Freitas Pacheco et al. (2006) and Regimbau et al. (2005), $\beta_{NS} = 0.024$ and $f_b = 0.136$, values that will be adopted in our calculations. Assuming that progenitors with initial masses above 40 $M_\odot$ will produce black holes and considering an initial mass function (IMF) of the form $\xi(m) = Am^{-\gamma}$, with $\gamma = 2.35$ (Salpeter’s law), normalized within the mass interval 0.1–80 $M_\odot$, such as $\int m \xi(m) dm = 1$, it results finally in $\beta_{NS} = \int \xi(m) dm = 5.72 \times 10^{-3}$ $M_\odot$ and $\lambda_p = 1.85 \times 10^{-5} M_\odot$. The evaluation of the parameters $\beta_{NS}$ and $f_b$ depends on different assumptions, which explains why estimates of the coalescence rate of DNSs found in the literature may vary by 1 or even 2 orders of magnitude. The evolutionary scenario of massive binaries considered in our calculations (see de Freitas Pacheco et al. 2006 for details) is similar to that developed by Belczynski et al. (2002), in which none of the stars ever had the chance of being recycled by accretion. Besides the evolutionary path, the resulting fraction $\beta_{NS}$ of bound NS-NS binaries depends on the adopted velocity distribution of the natal kick. The imparted kick may unbind binaries that otherwise might have remained bound or, less likely, conserve bound systems that would have been disrupted without the kick. The adopted value for $\beta_{NS}$ corresponds to a one-dimensional velocity dispersion of about 80 km s$^{-1}$. This value is smaller than those usually assumed for single pulsars but consistent with recent analyses of the spin period-eccentricity relation for NS-NS binaries (Dobbie et al. 2005). Had we adopted a higher velocity dispersion in our simulations (230 instead of 80 km s$^{-1}$), the resulting fraction of bound systems is reduced by 1 order of magnitude, e.g., $\beta_{NS} = 0.0029$. If, on the other hand, the fraction of bound NS-NS systems after the second supernova event depends on the previous evolutionary history of the progenitors and on the kick velocity distribution, then on the other hand, estimates of the fraction $f_b$ of massive binaries formed among all stars depends on the ratio between single and double NS systems in the Galaxy and on the value of $\beta_{NS}$ itself (de Freitas Pacheco et al. 2006). We have estimated relative uncertainties of about $\sigma_{f_b}/f_b \approx 0.5$ and $\sigma_{\beta_{NS}}/\beta_{NS} \approx 0.75$, leading to a relative uncertainty in the parameter $\lambda_p$ of about $\lambda_p/f_b \approx 0.9$. However, we emphasize that these are only formal uncertainties resulting from our simulations, which depend on the adopted evolutionary scenario for the progenitors. A comparison with other estimates can be found in de Freitas Pacheco et al. (2006).

The element of comoving volume is given by

$$dV(z) = 4\pi r(z)^2 \frac{c}{H_0} E(\Omega_m, z) dz,$$  

(5)

with

$$E(\Omega_m, z) = [\Omega_m (1+z)^3 + \Omega_\Lambda]^{1/2},$$  

(6)

where $\Omega_m$ and $\Omega_\Lambda$ are the present values of the density parameters due to matter (baryonic and nonbaryonic) and vacuum, respectively, corresponding to a nonzero cosmological constant. A “flat” cosmological model ($\Omega_m + \Omega_\Lambda = 1$) was assumed. In our calculations, we have taken $\Omega_m = 0.30$ and $\Omega_\Lambda = 0.70$, corresponding to the so-called “concordance” model derived from observations of distant Type Ia supernovae (Schmidt et al. 1998) and the power spectra of the cosmic microwave background fluctuations (Spergel et al. 2003). The Hubble parameter $H_0$ was taken to be 65 km s$^{-1}$ Mpc$^{-1}$.

Porciani & Madau (2001) provide three models for the cosmic SFR history up to redshifts $z \sim 5$. Differences among these models are mainly due to various corrections applied, in particular those due to extinction by the cosmic dust. In our computations, we have considered the second model, labeled SFR2 (Madau et al. 1998), but numerical results using SFR1 (Steidel et al. 1999) will also be given for comparison. Both rates increase rapidly between $z \sim 0–1$ and peak at $z \sim 1–2$, but SFR1 decreases gently after $z \sim 2$ while SFR2 remains more or less constant (Fig. 1).

The next step consists of estimating the redshift $z_b$, at which the progenitors have already evolved so that the system consists of
two neutron stars. This moment also fixes the beginning of the inspiral phase. If \( \tau_b \) \((\approx 10^8 \text{ yr})\) is the mean lifetime of the progenitors (average weighted by the IMF in the interval 9–40 \( M_\odot \)), then

\[
z_b = z_f - H_0 \tau_b (1 + z_f) E(z_f). \tag{7}
\]

Once the beginning of the inspiral phase is established, the redshift at which the coalescence occurs is estimated by the following procedure. The duration of the inspiral phase depends on the orbital parameters just after the second supernova and on the neutron star masses. The probability for a given DNS system to coalesce in a timescale \( \tau \) was initially derived by de Freitas Pacheco (1997), confirmed by subsequent simulations (Vincent 2002; de Freitas Pacheco et al. 2006) and is given by

\[
P_\tau(\tau) = B/\tau. \tag{8}
\]

Simulations indicate a minimum coalescence timescale of \( \tau_0 = 2 \times 10^5 \text{ yr} \), but a considerable number of systems have a coalescence timescale higher than the Hubble time. The normalized probability in the range \( 2 \times 10^5 \text{ yr} \) up to 20 Gyr implies \( B = 0.087 \). Therefore, the redshift \( z_\tau \) at which the coalescence occurs after a timescale \( \tau \) is derived from the equation

\[
H_0 \tau = \int_{z_\tau}^{z_b} \frac{dz}{(1 + z) E(z)}, \tag{9}
\]

which was solved in our code by an iterative method. The resulting distribution of the number of coalescences as a function of \( z_\tau \) is shown in Figure 2 for both star formation rates, SFR1 and SFR2, while the corresponding coalescence rate per redshift interval, \( R_{zc}(z) \), is shown in Figure 1. In the same figure, for comparison, we have plotted the formation rate \( R_{zf}(z) \) (eq. [3]). Note that the maximum of \( R_{z}(z) \) is shifted toward lower redshifts with respect to the maximum of \( R_{zf}(z) \), reflecting the time delay between the formation of the progenitors and the coalescence event. The coalescence rate \( R_{z}(z) \) does not fall to zero at \( z = 0 \), because a non-negligible fraction of coalescences (~3% for SFR2 and ~5% for SFR1) occur later than \( z = 0 \).

3. THE GRAVITATIONAL WAVE BACKGROUND

The nature of the background is determined by the duty cycle, defined as the ratio of the typical duration of a single burst \( \bar{\tau} \) to the average time interval between successive events, e.g.,

\[
D(z_c) = \int_0^{z_c} \bar{\tau}(1 + z') R_{zc}(z') \, dz'. \tag{10}
\]

The critical redshift \( z_c \) at which the background becomes continuous is fixed by the condition \( D(z_c) > 1 \). Since we are interested in the last instants of the inspiral, when the signal is within the frequency band of ground-based interferometers, we took \( \bar{\tau} = 1000 \text{ s} \), a duration that includes about 96% of the total energy released (see Table 1). From our numerical experiments and imposing \( D = 1 \), one obtains \( z_c = 0.23 \) when the SFR2 is used and \( z_c = 0.27 \) for SFR1. About 96% (94% in the case of SFR1) of coalescences occur above such a redshift, contributing to the production of a continuous background. Sources in the redshift interval \( 0.27 < z < 0.23 \) (SFR2) or \( 0.032 < z < 0.27 \) (SFR1) correspond to a duty cycle \( D = 0.1 \), and they are responsible for a cosmic “popcorn” noise.

The gravitational fluence (given here in ergs cm\(^{-2} \text{ Hz}^{-1} \)) in the observer frame produced by a given DNS coalescence is

\[
f_{gw} = \frac{1}{4\pi d_L^2} \frac{dE_{gw}}{d\nu}(1 + z_c), \tag{11}
\]

where \( d_L = (1 + z_c) r \) is the distance luminosity, \( r \) is the proper distance (which depends on the adopted cosmology), \( dE_{gw}/d\nu \) is the gravitational spectral energy, and \( \nu = (1 + z_c) \nu_c \) is the frequency in the source frame. In the quadrupolar approximation and for a binary system with masses \( m_1 \) and \( m_2 \) in a circular orbit,

\[
dE_{gw}/d\nu = K \nu^{-1/3}. \tag{12}
\]

| \( \nu_{\text{min}} \) (Hz) | \( \bar{\tau} \) | \( \Delta E/E_I \) (%) |
|-------------------------|----------------|------------------|
| 100                     | 2 s            | 84               |
| 10                      | 2000 s         | 96               |
| 1                      | 5.26 days      | 99               |

TABLE 1 FOR VALUES OF EMISSION FREQUENCY, TIME LEFT TO LAST STABLE ORBIT AND PERCENTAGE OF ENERGY RELEASED
where the fact that the gravitational wave frequency is twice the orbital frequency was taken into account. Then

\[ K = \frac{(Gm_1m_2)^{2/3}}{3} \left( \frac{m_1m_2}{m_1 + m_2} \right)^{1/3}. \]  

(13)

Assuming \( m_1 = m_2 = 1.4 \), one obtains \( K = 5.2 \times 10^{50} \text{ ergs Hz}^{-2/3} \).

The spectral properties of the stochastic background are characterized by the dimensionless parameter (Ferrari et al. 1999)

\[ \Omega_{gw}(\nu_o) = \frac{1}{c^3 \rho_c} \nu_o F_{\nu_o}, \]  

(14)

where \( \nu_o \) is the wave frequency in the observer frame, \( \rho_c \) is the critical mass density needed to close the universe, related to the Hubble parameter \( H_0 \) by

\[ \rho_c = \frac{3H_0^2}{8\pi G}, \]  

(15)

and \( F_{\nu_o} \) is the gravitational wave flux (given here in ergs \( \text{cm}^{-2} \text{ Hz}^{-1} \text{ s}^{-1} \)) at the observer frequency \( \nu_o \), integrated over all sources at redshifts \( z_c > z_o \), namely,

\[ F_{\nu_o} = \int_{z_c}^{z_{max}} f_{\nu_o} dR_c(z). \]  

(16)

Instead of analytically solving the equation above by introducing, for instance, an adequate fit of the cosmic coalescence rate, we have calculated the integrated gravitational flux by summing individual fluences (eq. [11]), scaled by the ratio between the expected number of events per unit of time and the number of simulated coalescences, or in other words, the ratio between the total formation rate of progenitors (eq. [2]) and the number of simulated massive binaries, e.g.,

\[ F_{\nu_o} = \frac{N_p}{N_{sim}} \sum_{i=1}^{N_{max}} f_{\nu_o}^i. \]  

(17)

The number of runs (or \( N_{sim} \)) in our simulations was equal to \( 10^7 \), representing an uncertainty of \( \leq 0.1\% \) in the density parameter \( \Omega_{gw} \). Using the SFR2, the derived formation rate of progenitors is \( N_p = 0.031 \text{ s}^{-1} \), whereas for the SFR1, one obtains \( N_p = 0.024 \text{ s}^{-1} \). For each run, the probability distribution \( P_f(z) \) defines, via Monte Carlo, the redshift at which the massive binary is formed. The beginning of the inspiral phase at \( z_{b} \) is fixed by equation (7). Then, in the next step, the probability distribution of the coalescence timescale and equation (9) define the redshift \( z_c \) at which the merging occurs. Thefluence produced by this event is calculated by equation (11), stored in different frequency bins in the observer frame, and added according to the equation above.

Figure 3 shows the density parameter \( \Omega_{gw} \) as a function of the observed frequency derived from our simulations. The density parameter \( \Omega_{gw} \) increases as \( \nu_o^{2/3} \) at low frequencies and reaches a maximum amplitude of about \( 1.1 \times 10^{-5} \text{ around 670 Hz} \) in the case of SFR2 and a maximum of \( 8.4 \times 10^{-10} \text{ around 630 Hz} \), in the case of SFR1. A high-frequency cutoff at \( \sim 1200 \text{ Hz} \) \( \sim 1170 \text{ Hz} \) for SFR1 is observed, approximately corresponding to the frequency of the last stable orbit at the critical redshift \( z_s = 0.23 \text{ (} z_s = 0.27 \text{ for SFR1)} \). Calculations performed by Schneider et al. (2001), in spite of the similar local merging rates, indicate that the maximum occurs at lower frequencies \( \sim 100 \text{ Hz} \) with an amplitude (scaled to the Hubble parameter adopted in this work) lower by a factor of 7. However, as those authors have stressed, their calculations are expected to be accurate in the frequency range \( 10 \mu\text{Hz} \) to \( 1 \text{ Hz} \), since they have set the value of the maximum frequency \( \nu_{max} \) to about that expected at a separation of 3 times the “last stable orbit” (LSO), e.g., \( \nu_{max} \approx 0.19\nu_{LSO} \). Thus, a direct comparison with our results is probably meaningless.

A more conservative estimate can be obtained if one adopts a higher duty cycle value, namely, \( D > 10 \), corresponding to sources located beyond \( z \sim 1.05 \text{ (} z \sim 1.075 \text{ for SFR1)} \). Our results are plotted in Figure 4, which includes, for comparison, the popcorn noise contribution arising from sources between \( 0.027 < z < 0.23 \text{ (} 0.032 < z < 0.27 \text{ for SFR1)} \), corresponding to the intermediate zone between a shot noise \( (D < 0.1) \), and a continuous background \( (D > 1) \). When increasing the critical redshift (or removing the nearest sources), the amplitude of \( \Omega_{gw} \)
decreases and the spectrum is shifted toward lower frequencies. The amplitude of the popcorn background is about 1 order of magnitude higher than the continuous background, with a maximum of about $\Omega_{gw} = 1.3 \times 10^{-8} (8.8 \times 10^{-9}$ for SFR1) around a frequency of 1.2 kHz.

Since some authors use, instead of $\Omega_{gw}$, the gravitational strain $S_h^{1,2}$, defined by Allen & Romano (1999) as

$$ S_h(\nu_o) = \frac{3H_0^2}{10\pi^2} \nu_o^{-1} \Omega_{gw}(\nu_o), \quad (18) $$

we show this quantity in Figure 5.

4. DETECTION

Because the background obeys a Gaussian statistic and can be confounded with the instrumental noise background of a single detector, the optimal detection strategy is to cross-correlate the output of two (or more) detectors, assumed to have independent spectral noises. The cross correlation product is given by (Allen & Romano 1999) as

$$ Y = \int_{-\infty}^{\infty} \tilde{s}_1(f) \tilde{Q}(f) \tilde{s}_2(f) df, \quad (19) $$

where

$$ \tilde{Q}(f) \propto \frac{\Gamma(f) \Omega_{gw}(f)}{\sqrt{P_1(f) P_2(f)}} \quad (20) $$

is a filter that maximizes the signal-to-noise ratio (S/N). In the above equations, $P_1(f)$ and $P_2(f)$ are the power spectral noise densities of the two detectors, and $\Gamma$ is the non-normalized overlap reduction function, characterizing the loss of sensitivity due to the separation and the relative orientation of the detectors. The optimized S/N for an integration time $T$ is given by (Allen 1997) as

$$ (S/N)^2 = \frac{9H_0^4}{8\pi^4} T \int_0^{\infty} df \frac{\Gamma^2(f) \Omega_{gw}^2(f)}{f^2 P_1(f) P_2(f)}. \quad (21) $$

Table 2

| Configuration | LHO-LHO | LHO-LLO | LLO-VIRGO | VIRGO-GEO |
|---------------|---------|---------|-----------|-----------|
| Initial       | $4 \times 10^{-7}$ | $4 \times 10^{-6}$ | $8 \times 10^{-6}$ | $8 \times 10^{-6}$ |
| Advanced      | $6 \times 10^{-9}$ | $1 \times 10^{-9}$ | $\ldots$ | $\ldots$ |

Notes:—Integration time $T = 1$ yr, detection rate $\alpha = 90\%$, and false-alarm rate $\gamma = 10\%$. LHO and LLO stand for LIGO Hanford Observatory and LIGO Livingston Observatory.

In the literature, the sensitivity of detector pairs is usually given in terms of the minimum detectable amplitude for a flat spectrum ($\Omega_{gw} = \text{const.}$; Allen & Romano 1999), e.g.,

$$ \Omega_{\min} = \frac{4\pi^2}{3H_0^2 \sqrt{T}} \times \left[ \text{erfc}^{-1}(2\alpha) - \text{erfc}^{-1}(2\gamma) \right] \left[ \int_0^{\infty} df f^6 P_1(f) P_2(f) \right]^{-1/2}. \quad (22) $$

The expected minimum detectable amplitude for the most sensitive pair of detectors in the world, after 1 yr of integration, are given in Table 2 for detection rate $\alpha = 90\%$ and false-alarm rate $\gamma = 10\%$. The power spectral density expressions used for the current calculation can be found in Damour et al. (2001). The $\Omega_{\min}$ is of the order of $10^{-6}$–$10^{-5}$ for the first generation of interferometers combined as LIGO/LIGO and LIGO/VIRGO. Their advanced counterparts will permit an increase of 2 or even 3 orders of magnitude in sensitivity ($\Omega_{\min} \sim 10^{-9}$–$10^{-8}$). The pair formed by the co-located and co-aligned LIGO Hanford detectors, for which the overlap reduction function is equal to 1, is potentially 1 order of magnitude more sensitive than the Hanford/Livingston pair, provided that instrumental and environmental noises could be removed.

However, because the spectrum of DNS coalescences is not flat and the maximum occurs out of the optimal frequency band of ground-based interferometers, which is typically around 50–300 Hz (as shown in Fig. 6), the S/N is slightly reduced. Considering the co-located and co-aligned LIGO interferometer pair,
we find a signal-to-noise ratio of $S/N \sim 0.002$ ($S/N \sim 0.5$) for the initial (advanced) configuration. Unless the coalescence rate is substantially higher than the present expectations, our results indicate that their contribution to the gravitational background is out of reach of the first and second generations of interferometers. On the other hand, the sensitivity of the future third generation of detectors, currently in discussion, could be high enough to gain 1 order of magnitude in the expected $S/N$. One such detector is the Large Scale Cryogenic Gravitational Wave Telescope (LCGT), sponsored by the University of Tokyo and the European antenna EGO (B. S. Sathyaprakash 2005, private communication). EGO will incorporate signal recycling, diffractive optics on silicon mirrors, cryo-techniques, and kV-class lasers, among other technological improvements. A possible sensitivity for this detector is shown in Figure 5, compared to the expected sensitivity of advanced LIGO. Around 650 Hz, the planned strain noise [${S_\ell}(\nu)$] is about $8 \times 10^{-24}$ Hz$^{-1/2}$ for the advanced LIGO configuration, while at this frequency, the planned strain noise for EGO is $2 \times 10^{-24}$ Hz$^{-1/2}$, which represents a gain by a factor of \sim 4. Considering two interferometers located at the same place, we find $S/N \sim 10$.

On the other hand, the popcorn noise contribution could also be detected by new data analysis techniques currently under investigation, such as the search for anisotropies (Allen & Ottewill 1997) that can be used to create a map of the GW background (Cornish 2001), the maximum likelihood statistic (Drasco & Flanagan 2003), or methods based on the probability “event horizon” (PEH) concept (Coward & Burman 2005), which describes the evolution, as a function of the observation time, of the cumulated signal throughout the universe. The PEH of the GW signal evolves quickly from contributions of high-redshift populations, forming a real continuous stochastic background, to low redshift and less probable sources that can be resolved individually, while the PEH of the instrumental noise is expected to evolve more slowly. Consequently, the GW signature could be distinguished from the instrumental noise background.

5. CONCLUSIONS

In this work, we have performed numerical simulations using Monte Carlo techniques to estimate the occurrence of double neutron star coalescences and the gravitational stochastic background produced in these events. Since the coalescence timescale obeys a well-defined probability distribution $[P(\tau) \propto 1/\tau]$ derived from simulations of the evolution of massive binaries (de Freitas Pacheco et al. 2006), the cosmic coalescence rate does not follow the cosmic star formation rate and presents a necessary time lag. In the case in which the sources are supernovae or black holes, the gravitational burst is produced in a very short timescale after the formation of the progenitors. Therefore, the time lag is negligible, and the comoving volume where the progenitors are formed is practically the same as that where the gravitational wave emission occurs, introducing a considerable simplification in the calculations. This is not the case when NS-NS coalescences are considered, since timescales comparable to or even higher than the Hubble timescale have non-negligible probabilities. The maximum probability of forming a massive binary occurs at $z \sim 1.7$, depending slightly on the adopted cosmic star formation rate, whereas the maximum probability at which a coalescence will occur is around $z \sim 1.4$.

We have found that a truly continuous background is formed only when sources are located beyond $z > 0.23$ ($z > 0.27$ for the SFR1 case), including 96% (94% for SFR1) of all events, and the critical redshift corresponds to the condition $D > 1$. Sources in the redshift interval 0.027 < $z < 0.23$ (0.032 < $z < 0.27$ for SFR1) produce a “popcorn” noise. Our computations indicate that the density parameter $\Omega_{gw}$ has a maximum around 670 Hz (630 Hz for SFR1), attaining an amplitude of about $1.1 \times 10^{-9}$ ($8.3 \times 10^{-9}$ for SFR1). The low-frequency cutoff around 1.2 kHz essentially corresponds to the gravitational redshifted wave frequency associated with the last stable orbit of sources located near the maximum of the coalescence rate.

The computed signal is below the sensitivity of the first and the second generation of detectors. However, using the planned sensitivity of third-generation interferometers, we found that after 1 yr of integration, the cross-correlation of two EGO-like coincident antennas gives the optimized signal-to-noise ratio of $S/N \sim 10$.

The popcorn contribution is 1 order of magnitude higher, with a maximum of $\Omega_{gw} \sim 1.3 \times 10^{-8}$ ($8.8 \times 10^{-9}$ for SFR1) at $\sim 1.2$ kHz. This signal, which is characterized by the spatial and temporal evolution of the events as well as by its signature, can be distinguished from the instrumental noise background, and adequate data analysis strategies for its detection are currently under investigation (Allen & Ottewill 1997; Cornish 2001; Drasco & Flanagan 2003; Coward & Burman 2005).

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