Invisible decay of orthopositronium vs extra dimensions

S.N. Gninenko\textsuperscript{a},\textsuperscript{1} N.V. Krasnikov\textsuperscript{a},\textsuperscript{2} A. Rubbia\textsuperscript{b}\textsuperscript{3}

\textsuperscript{a}Institute for Nuclear Research of the Russian Academy of Sciences, Moscow 117312
\textsuperscript{b}Institut für Teilchenphysik, ETHZ, CH-8093 Zürich, Switzerland

Abstract

In models of Randall-Sundrum type orthopositronium ($o - Ps$) can disappear due to tunnelling into additional dimension(s). The experimental signature of this effect is the invisible decay of orthopositronium. We point out that this process may occur at a rate within two or three orders of magnitude of the present experimental upper limit. We discuss this in details and stress that the existence of invisible decay of orthopositronium in vacuum could explain the $o - Ps$ decay rate puzzle. Thus, our result enhances the existing motivation and justifies efforts for a more sensitive search for $o - Ps \to \text{invisible}$ decay in a near future experiment. Possible manifestations of new physics in other rare (exotic) decays of orthopositronium are also discussed.

1 Introduction

Positronium ($Ps$), the positron-electron bound state, is the lightest known atom, which is bounded and self-annihilates through the same, electromagnetic interaction. At the current levels of experimental and theoretical precision this is the only interaction present in this system, see e.g. \cite{1}. This feature has made positronium an ideal system for testing of the QED calculations accuracy for bound states, in particular for the triplet ($1^3S_1$) state of $Ps$, orthopositronium ($o - Ps$). Due to the odd-parity under C-transformation $o - Ps$ decays predominantly into three photons. As compared with singlet ($1^1S_0$) state (parapositronium), the ”slowness” of $o - Ps$ decay rate, due to the phase-space and additional $\alpha$ suppression factors, gives an enhancement factor $\simeq 10^3$, making it more sensitive to an admixture of new interactions which are not accommodated in the Standard Model.

The study of the $o - Ps$ system has a long history \cite{1}. However, in spite of the substantial efforts devoted to the theoretical and experimental determination

\textsuperscript{1} Sergei.Gninenko@cern.ch
\textsuperscript{2} Nikolai.Krasnikov@cern.ch
\textsuperscript{3} Andre.Rubbia@cern.ch
of the $o-Ps$ properties, there is a long-standing puzzle: the $o-Ps$ decay rate in vacuum measured by Ann Arbor group, $\Gamma^{\text{exp}} = 7.0482 \pm 0.0016 \mu s^{-1}$ [4], has a $\simeq 5\sigma$ discrepancy with the predicted value $\Gamma = 7.03830 \pm 0.00007 \mu s^{-1}$ [5] (see also [6]). This discrepancy has been recently confirmed by more precise calculations of Adkins et al. [7], including corrections of the order $\alpha^2$.

The results of the recent Tokyo measurements of $o-Ps$ decay rate in low density SiO$_2$ powder corrected for matter effects [8] agree, however within the errors with the result of [7], thus making the situation confusing. So, it is difficult to disagree with the Adkins et al. [7] statement that: ...no conclusions can be drawn until the experimental situation is clarified.

Originally it was thought that an exotic $o-Ps$ decay, not taken into account in the calculation of $\Gamma$ and given a relative contribution to the $o-Ps$ decay rate at the level of $(\Gamma^{\text{exp}} - \Gamma)/\Gamma \simeq 10^{-3}$ would solve the discrepancy. At present, it is believed that practically all visible exotic decays of $o-Ps$ (i.e. decays accompanied by at least one photon in the final state) are excluded experimentally (for review, see e.g. [2,3]). However, there is still an intriguing explanation both of the discrepancy and Tokyo results. The idea was first discussed by S. Glashow [9] and is based on $o-Ps \rightarrow$ mirror $o$-Ps oscillations resulting in invisible decay of $o-Ps$ in vacuum, see Section 4 and ref. [10,11].

In this paper we point out that in models with additional infinite dimension(s) of Randall-Sundrum type positronium can disappear into additional dimension(s). The experimental manifestation of this effect is the invisible decay of positronium. Thus, this enhances motivation for further experimental search for this decay mode and makes it very interesting and exciting.

The paper is organised as follows. In Section 2 we remind the results on $o$-Ps decay rate calculation in the Standard Model. In Section 3 we briefly review the experimental results and phenomenological models on exotic visible decays of $o$-Ps and improve some bounds. In section 4 we discuss phenomenological models for $o-Ps \rightarrow$ invisible decays. We propose a model leading to $o-Ps \rightarrow$ invisible decays which could be at the experimentally interesting level. We also point out that models with infinite additional dimension(s) of Randall-Sundrum type predict the disappearance of orthopositronium as a result of tunnelling into additional dimension(s). The estimate of corresponding transition rate is presented. Section 5 contains concluding remarks.
2 Orthopositronium decays in Standard Model

The three photon decay width of the orthopositronium is [5]-[7],

$$\Gamma(o - Ps \rightarrow \gamma\gamma\gamma) = \frac{2(\pi^2 - 9)\alpha^6 m_\varepsilon^2}{9\pi}[1 - 10.28661(1)\frac{\alpha}{\pi} - \frac{\alpha^3}{3}\ln\left(\frac{1}{\alpha}\right)$$

$$+ B_0\left(\frac{\alpha}{\pi}\right)^2 - \frac{3\alpha^3}{2\pi}\ln\frac{1}{\alpha} + O(\alpha^3\ln(\alpha))] (1)$$

The coefficient $B_0$ has been recently calculated to be $B_0 = 44.52(26)$ [7], resulting in

$$\Gamma(o - Ps \rightarrow \gamma\gamma\gamma) = 7.039934 \pm 0.000010 \mu s^{-1} (2)$$

The five photon $o - Ps \rightarrow 5\gamma$ decay branching ratio is of order $\alpha^2$ [12]

$$Br(o - Ps \rightarrow 5\gamma) = 0.19(1)\left(\frac{\alpha}{\pi}\right)^2 \approx 1.0 \times 10^{-6} (3)$$

and therefore does not significantly influence the total decay width. Experimental result on $o$-Ps $\rightarrow 5\gamma$ decay gives [13]

$$Br(o - Ps \rightarrow 5\gamma) = [2.2^{+2.6}_{-1.6}(stat) \pm 0.5(syst.)] \times 10^{-6} (4)$$

in agreement with the theoretical prediction of Eq.(3).

Within the Standard Model orthopositronium can also decay weakly into neutrino-antineutrino pair. The $o - Ps \rightarrow \nu_\varepsilon \bar{\nu}_\varepsilon$ decay occurs through $W$ exchange in $t$ channel and $e^+e^-$ annihilation via $Z$. The decay width is [14]

$$\Gamma(o - Ps \rightarrow \nu_\varepsilon \bar{\nu}_\varepsilon) = \frac{G_F^2\alpha^3 m_\varepsilon^2}{24\pi^2}(1 + 4\sin^2\theta_W)^2 \approx 6.2 \times 10^{-18}\Gamma(o - Ps) (5)$$

For other neutrino flavours only $Z$-diagram contributes. For $l \neq e$ the decay width is [14]

$$\Gamma(o - Ps \rightarrow \nu_l \bar{\nu}_l) = \frac{G_F^2\alpha^3 m_\varepsilon^2}{24\pi^2}(1 - 4\sin^2\theta_W)^2 \approx 9.5 \times 10^{-21}\Gamma(o - Ps) (6)$$

Thus, in the Standard Model the $o - Ps \rightarrow \nu\bar{\nu}$ decay width is too small and its contribution to the total decay width can also be neglected.

3 Visible exotic decays of orthopositronium

Visible exotic decays of $o - Ps$ can be classified into following categories: i) $o - Ps \rightarrow \gamma X$ ii) $o - Ps \rightarrow \gamma\gamma X$ iii) $o - Ps \rightarrow N\gamma$, where $X$ is a new light particle(s) and $N = 2, 4, ..$ (for review, see e.g. [2,3]).
For the decay $o - Ps \rightarrow \gamma + X$, where $X$ is a long-lived particle the experimental bound is [15]

$$Br(o - Ps \rightarrow \gamma + X) < 1.1 \times 10^{-6}, (m_X < 800 \text{ keV}) \quad (7)$$

If $X$ decays within detector, e.g. into $2\gamma$, the experimental bounds depend on the $X$-particle mass [16]-[19]

$$Br(o - Ps \rightarrow \gamma + X \rightarrow 3\gamma) < 2 \times 10^{-4}, (300 \text{ KeV} < m_X < 900 \text{ keV}) \quad (8)$$

$$Br(o - Ps \rightarrow \gamma + X \rightarrow 3\gamma) < 2.8 \times 10^{-5}, (m_X < 30 \text{ keV}) \quad (9)$$

$$Br(o - Ps \rightarrow \gamma + X \rightarrow 3\gamma) < 2 \times 10^{-5}, (900 < m_X < 1013 \text{ keV}) \quad (10)$$

Note that all above experimental limits are based on a search for a peak in the energy spectrum of photons arising from the 2-body decay of $o - Ps$. In the case of an exotic 3-body $o - Ps \rightarrow \gamma + X_1 + X_2$ decay (which still might be a solution of the $o - Ps$ decay puzzle [2,20]), the signal cannot manifest itself through the peak in the $\gamma$ energy spectrum, thus making the limits for this decay mode weaker. The current estimate for $Br(o - Ps \rightarrow \gamma X_1 X_2)$ from the results of the indirect experiment of ref. [30] is around $10^{-4}$ [2].

### 3.1 Exotic $o - Ps \rightarrow \gamma X$ decay

Consider the model with light pseudoscalar [21] (e.g. axion) which predict the existence of the exotic decay $o - Ps \rightarrow \gamma + X$ decay. The interaction Lagrangian [21] is

$$L_X = g_X \bar{\psi} \gamma_5 \psi X \quad (11)$$

The $o - Ps \rightarrow X\gamma$ decay width is determined by the formula [21]

$$\Gamma(o - Ps \rightarrow \gamma X) = \frac{8}{3} g_X^2 \cdot \frac{\alpha^3 m_e^2}{8\pi} (1 - \frac{m_X^2}{m_{o-Ps}^2}) = g_X^2 (1 - \frac{m_X^2}{m_{o-Ps}^2}) \times 5.84 \cdot 10^4 \mu s^{-1} \quad (12)$$

Strong bound on $\alpha_{Xe} \equiv g_X^2/4\pi$ arises from the results on anomalous magnetic moment of electron. The measurements of $a_e = (g_e - 2)/2$ [22] give

$$a_e^{exp} = 0.0011596521884(43), \quad (13)$$

$$a_e^{exp} = 0.0011596521879(43) \quad (14)$$
whereas the prediction is

\[ a_e^{SM} = \frac{\alpha}{2\pi} - 0.32847844400\left(\frac{\alpha}{\pi}\right)^2 + 1.181234017\left(\frac{\alpha}{\pi}\right)^3 - 1.5098(384)\left(\frac{\alpha}{\pi}\right)^4 + 1.66(3) \times 10^{-12} \text{(hadronic and electroweak contributions)} \] (15)

The measurements (13,14) provide the best determination of the fine structure constant [23]

\[ \alpha^{-1}(a_e) = 137.03599958(52) \] (16)

Another determination of \( \alpha \), from the quantum Hall effect (QHE)[24]

\[ \alpha^{-1}(QHE) = 137.03600300(270) \] (17)

has larger error. If the new light particle \( X \) exists, it would give contribution to the anomalous magnetic moment of the electron, but would not influence the determination of the fine structure constant from the QHE. The value of the fine structure constant extracted from the QHE leads to

\[ a_e = .0011596522172(229) \] (18)

The estimate of a possible additional contribution to the electron anomalous magnetic moment results in

\[ |a_e^{exp} - a(QHE)| \lesssim 4.6 \cdot 10^{-11} \quad (90\% C.L.) \] (19)

For \( m_X^2 \ll m_e^2 \) the \( X \)-boson contribution to anomalous magnetic moment of electron is

\[ (g - 2)X \approx -\frac{1}{8\pi^2}g_X^2 \] (20)

From the experimental bound (19) we find that \( g_X^2/4\pi \lesssim 6 \cdot 10^{-10} \) and hence the bound is \( Br(o - Ps \rightarrow \gamma X) \lesssim 6 \times 10^{-5} \), that is weaker than the current experimental bound (7).

### 3.2 Exotic \( o - Ps \rightarrow \gamma \gamma X \) decay

Consider the case of \( o - Ps \rightarrow \gamma \gamma X \) decay mode. The negative search for direct \( e^+e^- \) annihilation \( e^+e^- \rightarrow \gamma + X \) leads for the case of long-lived \( X \) particle to the bounds [25]

\[ Br(o - Ps \rightarrow \gamma \gamma X) < 4.2 \times 10^{-6}, (m_X < 200 \text{ keV}) \] (21)

\[ Br(o - Ps \rightarrow \gamma \gamma X) < 2.1 \times 10^{-5}, (m_X < 2m_e) \] (22)
In ref. [26] a more stringent bound

\[ Br(o - Ps \to \gamma\gamma X) < 4 \times 10^{-6} \]  

has been obtained. Consider the model with vector $X$-boson and with the interaction Lagrangian

\[ L_X = g_X \bar{\psi}\gamma_\mu\psi X^\mu \]  

One can find that for $m_X \ll m_e$

\[ Br(o - Ps \to \gamma\gamma X) = \frac{3\alpha_X}{\alpha}, \]  

where $\alpha_X = g_X^2/4\pi$. From an experimental bound (23) one can find that

\[ \alpha_X < 10^{-8} \]  

Light $X$-boson leads to additional contribution $\alpha_X/2\pi$ to the anomalous magnetic moment of electron. From the bound (19) we find that

\[ \alpha_X \lesssim 3 \times 10^{-10}, \]  

\[ Br(X \to \gamma\gamma X) \lesssim 1.2 \cdot 10^{-7} \]  

The bound (28) is approximately 30 times smaller than the experimental bound (23). Note that if $X$-boson interacts only with the right-handed or left-handed electrons such interaction does not give contribution at one loop level to $a_e$ and the contribution to the anomalous magnetic moment of the electron is of the order $O(\alpha\alpha_X/\pi^2)$. As a consequence we find that the anomalous magnetic moment of electron gives much weaker bound $\alpha_X \lesssim O(10^{-7})$.

### 3.3 Exotic $o - Ps \to 2\gamma$ and $o - Ps \to 4\gamma$ decays

The decay $o - Ps \to 4\gamma$ is forbidden in QED by C-invariance. The corresponding experimental bound is [27]

\[ Br(o - Ps \to 4\gamma) < 2.6 \times 10^{-6} \]  

The $o - Ps$ has spin one and cannot decay into two photons due to conservation of angular momentum which follows from the isotropy of space. Hence, searching for decays $o - Ps \to 2\gamma$ tests spatial isotropy. The best current result is [28]

\[ Br(o - Ps \to 2\gamma) < 3.5 \times 10^{-6} \]  

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4 **Invisible decays of orthopositronium**

The experimental signature of the \( o - Ps \rightarrow \text{invisible} \) decay is the absence of an energy deposition of \( \sim 1 \text{ MeV} \), which is expected from the ordinary \( o - Ps \) annihilation, in a \( 4\pi \) hermetic calorimeter surrounding the \( o - Ps \) formation region. The first experiment on \( o - Ps \rightarrow \text{invisible} \) decay was performed a long time ago [29], and then was repeated with a higher sensitivity by Mitsui et al. [30]. The best current experimental bound is [30]

\[
Br(o - Ps \rightarrow \text{invisible}) < 2.8 \times 10^{-6}
\]

(31)

Consider several motivations for further experimental searching for this decay mode.

4.1 **Millicharged particles**

The first one is related to the fundamental problem of charge quantization. In 1986, Holdom [31] showed that, by adding a second, unobserved, photon (the “shadow” photon) one could construct in a natural way grand unified models, which contain particles with an electric charge very small compared to the electron charge. This work has stimulated new theoretical and experimental tests (for a recent review see [32] and references therein). If such milli-charged particles exist with a small mass, the \( o - Ps \) could decay apparently invisibly, since the particles would mostly penetrate any type of calorimeter without interaction. The corresponding decay width is [33]

\[
\Gamma(o - Ps \rightarrow X\bar{X}) = \frac{\alpha^5 Q_X^2 m_e}{6} \cdot k \cdot F\left(\frac{m_X^2}{m_e^2}\right),
\]

(32)

where \( Q_X \) is an electric charge of the \( X \)-particle \((Q_e \equiv 1)\), \( k = 1 \), \( F(x) = (1 - \frac{1}{2}x)(1 - x)^{\frac{3}{2}} \) for spin \( \frac{1}{2} \) and \( k = \frac{1}{4} \), \( F(x) = (1 - x)^{\frac{3}{2}} \) for spin-less \( X \)-particle. For spin \( \frac{1}{2} \) millicharged \( X \)-particle and for \( m_X \ll m_e \) one can find from experimental bound (31) that \( Q_X \lesssim 8.6 \cdot 10^{-5} [30] \). Search for the \( o - Ps \rightarrow \text{invisible} \) decay with the sensitivity \( Br(o - Ps \rightarrow \text{invisible}) \simeq 10^{-8} \) would touch the parameter space not excluded by the results of the recent experiment at SLAC [34].
4.2 Mirror World

In refs. [9] - [11] an explanation of orthopositronium decay rate puzzle within the model with the Mirror Universe [35] has been discussed. The existence of the effective mixing $\epsilon F^{\mu\nu}F'^{\mu\nu}$ between our photon and mirror photon would mix ordinary and mirror orthopositronium, see e.g. [38], resulting in $o - Ps$ - mirror $o - Ps$ oscillations [9]. This leads to an apparent increase in the $o - Ps$ decay rate. Since the mirror decays are not detected, the experimental signature of this effect is $o - Ps \rightarrow invisible$ decay with the branching ratio

$$Br(o - Ps \rightarrow invisible) = \frac{2(2\pi\epsilon f)^2}{\Gamma^2 + 4(2\pi\epsilon f)^2},$$

(33)

where $\epsilon$, $\Gamma$, and $f$ are, respectively the mixing strength between the photon and mirror photon, the decay rate of $o - Ps$ into three photons and the contribution to the ortho-para splitting from the one-photon annihilation diagram involving $o - Ps$ ($f = 8.7 \times 10^4 MHz$) [9]. The limit on photon-mirror photon mixing strength extracted from the bound (31) taking into account the suppression collision factor, is $\epsilon < 10^{-6}$ [10] and it is not strong enough to exclude a possible mirror contribution to the $o - Ps$ decay rate. A vacuum experiment on $o - Ps \rightarrow invisible$ decay with the sensitivity to the mixing strength $\epsilon \simeq 10^{-7}$ is necessary to confirm or rule out the mirror world effect [11].

4.3 New light X-boson

Consider now the model with a light vector $X$-boson and with the interaction of Eq.(24) which also leads to invisible decay of $o - Ps$.[5] Suppose in addition $X$-boson interacts with other particles (fermions) or (as a consequence of the Higgs mechanism) with itself and scalar field. The contribution of the $X$-boson to the electron anomalous magnetic moment is given by the well known formula

$$\delta a_e = \frac{\alpha_X}{\pi} \int_0^1 \frac{x^2(1 - x)}{x^2 + (1 - x)\frac{m_X^2}{m_e^2}},$$

(34)

For $m_X \ll m_e$ from the bound (19) we find that $\alpha_X < 3 \times 10^{-10}$. For the opposite case of heavy $X$-boson ($m_X \gg m_e$) the bound on the anomalous

\[ \text{For many interesting ideas on mirror matter see e.g. ref. [36] and recent book [37] and references therein.} \]

\[ \text{For the recent phenomenological bounds in models with light vector } X\text{-boson related to the muon } (g - 2) \text{ and, so-called NuTeV anomalies see, respectively [39] and [40], and also [41].} \]
electron magnetic moment leads to the bound
\[ \alpha_X \frac{m_e^2}{m_X^2} < 4.5 \cdot 10^{-10} \]  \hspace{1cm} (35)

For the reaction \( o - Ps \rightarrow X^* \rightarrow X_1 \bar{X}_1 \) (here \( X^* \) is virtual \( X \)-boson and \( X_1 \) is a fermion (sterile neutrino) or a scalar particle) one can find that

\[ Br(o - Ps \rightarrow X^* \rightarrow X_1 \bar{X}_1) = \]
\[ \frac{3\pi}{4(\pi^2 - 9)} \cdot k \cdot F\left(\frac{m_X^2}{m_e^2}\right) \left(1 - \frac{m_X^2}{m_{\bar{5} - Ps}^2}\right)^{-2} \frac{\alpha_X \alpha_{XX_1}}{\alpha^3}, \]

where \( F(x) \) has been defined before and \( \alpha_{XX_1} = g_{XX_1}^2 / 4\pi \). From the bound (19) we find for \( m_X \ll m_e \) that

\[ Br(o - Ps \rightarrow X^* \rightarrow X_1 \bar{X}_1) \lesssim k \times 2 \cdot 10^{-3} \cdot \alpha_{XX_1} \]  \hspace{1cm} (37)

To have experimentally interesting branching of the order \( O(10^{-6}) \), setting \( \alpha_X = 3 \times 10^{-10} \) we must have \( \alpha_{XX_1} \sim 5 \cdot 10^{-4} \).

In the opposite limit \( m_X \gg m_e \) the corresponding bound reads

\[ Br(o - Ps \rightarrow X^* \rightarrow X_1 \bar{X}_1) \leq k \times 3 \cdot 10^{-3} \cdot \alpha_{XX_1} \cdot \frac{m_e^2}{m_X^2} \]  \hspace{1cm} (38)

For \( X \)-boson mass close to the orthopositronium mass, we have the enhancement factor \( (1 - \frac{m_X^2}{m_{\bar{5} - Ps}^2})^{-2} \) in formula (36) for the branching ratio and hence the coupling constant \( \alpha_X \alpha_{XX_1} \) could be smaller.

The discussed above explanation [9] of the \( o \)-Ps lifetime discrepancy based on the model with mirror Universe [9] in fact uses the enhancement factor \( (1 - \frac{m_X^2}{m_{\bar{5} - Ps}^2})^{-2} \). According to our terminology we can treat \( X \)-boson as a mirror orthopositronium which due to mixing of our photon with mirror photon has direct coupling with electrons and decays into 3 invisible mirror photons. Vector boson \( X \) acquires a mass via Higgs mechanism and for both light \( X \)-boson and the Higgs scalar \( \phi \) the reaction (“Higgs-Strahlung” process)

\[ o - Ps \rightarrow X^* \rightarrow X\phi \]  \hspace{1cm} (39)

also leads to invisible \( o - Ps \) decay mode. The corresponding formula for the \( o - Ps \) branching in the limit \( m_x \ll m_e, m_\phi \ll m_e \) coincides with formula (36) for \( k = \frac{1}{4} \) (scalar case) and leads to similar bound on the product \( \alpha_X \alpha_{X\phi} \).
4.4 Extra dimensions

Recently the models with infinite additional dimensions of Randall-Sundrum type (brane-world models) [42],[43] have become very popular. There is a hope that models with a big compactification radius [42]-[48] will provide the natural solution to the gauge hierarchy problem. For instance, as it has been shown [42] in the five dimensional model, there exists a thin-brane solution to the 5-dimensional Einstein equations which has flat 4-dimensional hypersurfaces,

\[ ds^2 = a^2(z)\eta_{\mu\nu}dx^\mu dx^\nu - dz^2. \] (40)

Here

\[ a(z) = \exp(-k(z - z_c)) \] (41)

and the parameter \( k > 0 \) is determined by the 5-dimensional Planck mass and bulk cosmological constant. For the model with metric (40) the effective four-dimensional gravitational constant is

\[ G_{(4)} = G_{(5)}k\frac{1}{\exp(2kz_c) - 1} \] (42)

One can solve the gauge hierarchy problem in this model if \( k \sim M_{EW} = 1 \text{TeV} \), \( G_{(5)} \sim k^{-3} \). As it follows from the expression (42) the Planck scale in this model is

\[ M_{PL} \sim \exp(kz_c)M_{EW} \] (43)

that means the existence of exponential hierarchy between Planck and electroweak scales. For \( z_c \approx 37 \cdot k^{-1} \) we have correct quantitative relation among Planck and electroweak scales. Note that in this model the mass of the first gravitational Kaluza-Klein state is \( m_{grav} \sim k \). As it has been shown in ref.[49] the massive matter becomes unstable due to tunnelling effect and disappears into additional 5-th dimension. For massive scalar particle \( \Phi \) with the mass \( m \) the transition rate into additional dimension is given by the formula [49]

\[ \Gamma(\Phi \rightarrow \text{additional dimension}) = \frac{\pi m}{16} \left(\frac{m}{k}\right)^2 \] (44)

For massive vector particles the expression for its transition rate into additional dimension(s) is not explicitly known. To make quantitative estimate we shall use the formula (44) for the decay width of spin 1 particle.

We stress that \( o - Ps \) is a good candidate for the searching for effect of disappearance into additional dimension(s) since it has specific quantum numbers similar to those of vacuum and is a system which allows its constituents a
rather long interaction time. For the orthopositronium invisible decay into additional dimension(s)

\[ o - Ps \rightarrow \gamma^* \rightarrow \text{additional dimension(s)} \]  

(45)

the corresponding branching ratio is

\[
Br(o - Ps \rightarrow \gamma^* \rightarrow \text{additional dimension(s)}) = \frac{9\pi}{4(\pi^2 - 9)} \cdot \frac{1}{\alpha^2} \cdot \frac{\pi}{16} \left(\frac{m_o - Ps}{k}\right)^2 \approx 3 \cdot 10^{4} \left(\frac{m_o - Ps}{k}\right)^2
\]

(46)

Important bound on the parameter \( k \) arises from data on \( Z \rightarrow \text{invisible} \) decay. LEP1 bound on the number of neutrinos \( n_\nu = 3.00 \pm 0.06 \) [50] extracted from direct measurements of the \( \Gamma(Z \rightarrow \text{invisible}) \) decay rate results at the 2\( \sigma \)-level in the bound \( \Gamma(Z \rightarrow \text{additional dimension(s)}) \lesssim 0.02 \text{ GeV} \) and leads to \( k \gtrsim 2.7 \text{ TeV} \). Using this we find

\[
Br(o - Ps \rightarrow \text{additional dimension(s)}) \lesssim 4 \cdot 10^{-9}
\]

(47)

To solve the gauge hierarchy problem models with additional infinite dimension(s) must have the \( k \lesssim O(10) \text{ TeV} \). It means that

\[
Br(o - Ps \rightarrow \text{additional dimension(s)}) \gtrsim O(10^{-10})
\]

(48)

Since these estimates give only an order of magnitude for the lower and upper limits on corresponding branching ratio, we believe that the region of \( Br(o - Ps \rightarrow \text{invisible}) \simeq 10^{-9} - 10^{-8} \) is of great interest for observation of effect of extra dimensions. However, more accurate calculations of the tunnelling effect might provide more stringent bounds on transition rate of \( o - Ps \) into extra dimension(s).

5 Conclusion

Due to its specific properties, orthopositronium is an important probe of QED and also physics beyond the Standard Model. We have reviewed phenomenological models of rare \( o - Ps \) decay modes and updated some existing bounds. We have considered a model with infinite additional dimension(s) in which orthopositronium may disappear as a result of tunnelling into additional infinite dimension(s). The experimental signature of this effect is the invisible decay of orthopositronium which may occur at a rate within several (two or three) orders of magnitude of the present experimental upper limit. This result strengthens current motivations and justify efforts for a more sensitive search of the \( o - Ps \rightarrow \text{invisible} \) decay in a near future experiment [51].
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