Quark Confinement and Surface Critical Phenomena

K.J. Juge, J. Kuti and C.J. Morningstar

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

October 1999

Published Proceedings of the XVII International Symposium on Lattice Field Theory,
Pisa, Italy, June 29-July 3, 1999

Operated by Universities Research Association Inc. under Contract No. DE-AC02-76CH03000 with the United States Department of Energy
Disclaimer

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, expressed or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Distribution

Approved for public release; further dissemination unlimited.

Copyright Notification

This manuscript has been authored by Universities Research Association, Inc. under contract No. DE-AC02-76CH03000 with the U.S. Department of Energy. The United States Government and the publisher, by accepting the article for publication, acknowledges that the United States Government retains a nonexclusive, paid-up, irrevocable, worldwide license to publish or reproduce the published form of this manuscript, or allow others to do so, for United States Government Purposes.
Quark Confinement and Surface Critical Phenomena *

K.J. Jugea, J. Kutib and C.J. Morningstarb

aFermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510
bDept. of Physics, University of California at San Diego, La Jolla, California 92093-0319

Surface critical phenomena and the related onset of Goldstone modes probe the fundamental properties of the confining flux in Quantum Chromodynamics. New ideas on surface roughening and their implications for lattice studies of quark confinement are presented. Problems with the oversimplified string description of the Wilson flux sheet are discussed.

1. The spectrum of the confining flux

A rather comprehensive determination of the rich energy spectrum of the confined chromoelectric flux between static sources in the fundamental representation of SU(3), was reported earlier [1,2] for separations r ranging from 0.1 fm to 4 fm. The full spectrum is summarized in Fig. 1 with different characteristic behavior on two scales separated approximately at r=2 fm.

No string spectrum for r \leq 2 fm

The \Sigma^+_g ground state is the familiar static quark-antiquark potential which is dominated by the rather dramatic linearly-rising behavior once r exceeds about 0.5 fm. Although the empirical function \( E_{\Sigma^+_g}(r) = -c/r + \sigma r \) approximates the ground state energy very well for r \geq 0.1 fm, the fitted constant c = 0.3 has no relation to the running Coulomb law whose loop expansion breaks down before r = 0.1 fm separation is reached [4]. Early indoctrination on the popular string interpretation of the confined flux for r \leq 2 fm was mostly based on the observed shape of the \( \Sigma^+_g \) ground state energy: the linear shape of the ground state potential for r \geq 0.5 fm and the approximate agreement of the curvature shape for r \leq 0.5 fm with the ground state Casimir energy \(-\pi/(12r)\) of a long confined flux [5]. The excitation spectrum clearly contradicts earlier claims [3] on the simple string interpretation of the linearly rising confining potential. The gluon excitation energies lie well below the string predictions and the level orderings and degeneracies are in violent disagreement with expectations from a fluctuating string.

*Talk presented by J. Kuti.
Goldstone modes for $r > 2$ fm

A feature of any low-energy description of a fluctuating flux sheet in euclidean space is the presence of Goldstone excitations associated with the spontaneously-broken transverse translational symmetry. These transverse modes have energy separations above the ground state given by multiples of $\pi/r$ (for fixed ends). The level orderings and approximate degeneracies of the gluon energies at large $r$ match, without exception, those expected of the Goldstone modes. However, the precise $\pi/r$ gap behaviour is not observed. The spectrum is consistent with massive capillary waves on the surface of the flux sheet, with a cutoff dependent mass gap. The most likely explanation for this gap is Peierls-Nabarro lattice pinning of the confining flux sheet at small correlation lengths. Our new results [S] on the same spectrum in SU(2) for $D=3,4$, and a detailed test of the strong coupling spectrum in SU(2) for $D=3$ lend further support to the above summary of the earlier findings.

2. Z(2) model at $D=3$

The purpose of the Z(2) project is to understand flux formation and the string excitation spectrum from high statistics simulations and the loop expansion on the analytic side. The WKB approximation of flux formation in the sine-Gordon field representation of the monopole plasma was discussed earlier [7]. The Z(2) model maps into the Ising model by duality transformation which was exploited before in the study of large Wilson loops [8]. By invoking universality, the critical region of the Z(2) model is mapped into $D=3$ $\Phi^4$ scalar field theory in the study of flux formation. The confining flux sheet of the Wilson loop corresponds to a twisted surface in the Ising representation which is described by a classical soliton solution of the $\Phi^4$ field equations. Excitations of the flux are given by the spectrum of the fluctuation operator $M = -\nabla^2 + U''(\Phi_{\text{soliton}})$ where $U(\Phi)$ is the field potential energy of the $\Phi^4$ field. The spectrum of the fluctuation operator $M$ of the finite surface is determined from a two-dimensional Schrödinger equation with a potential of finite extent [7]. In the limit of asymptotically large surfaces, the equation becomes separable in the longitudinal and transverse coordinates. The transverse part of the spectrum is in close analogy with the quantization of the one-dimensional classical $\Phi^4$ soliton. There is always a discrete zero mode in the spectrum which is enforced by translational invariance in the transverse direction. Figs. 2,3,4,5 illustrate some of the results which are consistent with our findings in QCD.

Acknowledgements

One of us (J. K.) would like to acknowledge valuable discussions with S. Renn, P. Hasenfratz, and F. Niedermayer on surface Goldstone modes. This work was supported by the U.S. DOE, Grant No. DE-FG03-97ER40546.
Figure 3. Wavefunctions of the excited $Z(2)$ flux at spatial Wilson loop size $r=20$ for $\xi \approx 2$. The first two Goldstone modes and a massive intrinsic excitation are shown.

REFERENCES

1. K.J. Juge, J. Kuti, and C. Morningstar, Nucl. Phys. B (Proc. Suppl.) 63, 326 (1998).
2. K.J. Juge, J. Kuti, and C. Morningstar, Nucl. Phys. B (Proc. Suppl.) 73, 590 (1999).
3. S. Perantonis and C. Michael, Nucl. Phys. B 347, 854 (1990).
4. M. Peter, Nucl. Phys. B 501, 471 (1997).
5. M. Lüscher, Nucl. Phys., B180 (1981) 317.
6. K.J. Juge, J. Kuti, and C. Morningstar, to be published.
7. J. Kuti Nucl. Phys. B (Proc. Suppl.) 73, 72 (1999) and to be published.
8. M. Caselle, R. Fiore, F. Gliozzi, Nucl. Phys., B486 (1997) 245.

Figure 4. Effective mass plot of the string spectrum for a very large Wilson loop at $\xi \approx 1/2$ before the Peierls-Nabarro gap opens up.

Figure 5. The Peierls-Nabarro gap (negligible at large correlation lengths, or inverse glueball mass) implies pinning and mass generation for the Goldstone modes.