Optimal Strategies of Product Price, Quality, and Corporate Environmental Responsibility

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Abstract: With the awakening of environmental consciousness, more and more firms desire to go “green” by shifting their focus of corporate social responsibility (CSR) from charitable contributions to environmental actions called corporate environmental responsibility (CER). We develop a monopoly differential game to depict optimal corporate strategies of product price, quality, and CER. Using the Hamilton–Jacobi–Bellman (HJB) equation, we analyze optimal feedback equilibrium strategies for pricing and investing in both quality and CER with/without government subsidies. Numerical simulations show that government subsidy can improve CER and profit.

Keywords: corporate social responsibility (CSR); corporate environmental responsibility (CER); government subsidy; social welfare; feedback equilibrium

1. Introduction

As part and parcel of corporate social responsibility (CSR), corporate environmental responsibility (CER) complies with the rise of today’s environmental consciousness in environmental evolutions such as climate change. CER encompasses all the practices put in place by firms to reduce emissions, increase efficiency, and integrate sustainability into their daily operations. Employees, consumers, and stakeholders are placing a premium on working for, spending their money on, and standing by brands or companies that prioritize CER. Laudable green CER strategies can improve managerial altruism, consumer loyalty, corporation recommendations, brand sentiment, and cost-cutting efficiency. For that reason, more and more companies desire to go “green”. Therefore, it is meaningful for us to consider CER when we analyze firms’ decisions.

In recent years, more and more researchers also have paid much attention to CSR and CER [1–6]. Qin et al. [7] constructed a CER conceptual framework for researchers and proposed a conceptual model for policymakers. Suganthi [8] examined a general research framework considering CSR, green practice performance, and employees’ pro-environmental behavior. CSR can not only put enterprises into competitive disadvantage due to investment in CSR [9–11] but also help firms gain competitive advantage, because environmentally responsible behavior can obtain support from stakeholders (e.g., governments, suppliers, consumers, employees, and local communities) [12], expand their market share [9], reduce operational risk, and obtain long-term growth [13]. Some other researchers [14–16] have also indicated that CSR has a positive effect on corporate profits from different perspectives. Generally speaking, CSR helps corporations to gain better corporate goodwill [17–19], and better corporate goodwill helps corporations acquire more resources, and earn optimal profits. Examining the role of CER in CSR, Liu et al. [20] found that CER is positively associated with CSR to a significant degree. Dang et al. [21] insisted that CER is also a double-edged sword under different mediation
effects, such as strategic similarity and organizational slack. Han, Yu, and Kim [3] uncovered that CER is a significant contributor to improving corporate goodwill and loyalty intentions. In the following, we will extend the Nerlove–Arrow model [22] to construct a monopoly differential game model by incorporating the effect of product quality, price, and CER on corporate goodwill to explore optimal corporate strategies.

The remainder of this paper is organized as follows. We review the relevant literature in Section 2. We propose a differential monopoly game model in Section 3. We analyze the equilibria without government subsidies in Section 4. We study the equilibria with government subsidies in Section 5. We validate the results by numerical simulations in Section 6. We discuss the results in Section 7. Finally, the paper concludes in Section 8.

2. Literature Review

Though there is no widely accepted definition of CER [7,8,23–30], for the sake of convenience, we support that CER is one of three facets of CSR, and focuses on pollution prevention and cleaner production. Furthermore, we regard the following terms as equivalent to CER: CSR in the environment, environmental CSR, environmental corporate responsibility. Like CSR, CER can impact the performance of micro-, small- and medium-sized enterprises from financial and innovative standpoints. As a kind of CSR, corporate contributions to charity may also have a long-term effect on a firm’s image and profits [31,32]. CER can facilitate firms to achieve support from external stakeholders, gain competitive advantages [33], reduce equity financing costs [34], affect investment efficiency for the long-term [35] and in green IT capital [36].

There is some literature about relationships between price, quality, and corporate responsibility (CR), as shown in Table 1. In this study, we analyze the relationship between price, quality, and CER by using the infinite-time differential game. Since the differential game will be used to analyze optimal corporate strategies of price, quality, and CER, some applications of the differential game are reviewed, as shown in Table 2. In this study, we investigate the feedback equilibria by setting corporate goodwill and CER knowledge accumulation as state variables, and setting pricing, investing in quality, and CER as control variables.

| Study                                | Main Relationship                      | Method                             | Reference |
|--------------------------------------|----------------------------------------|------------------------------------|-----------|
| De Giovanni and Zaccour (2019)       | Quality and product price              | Two-stage model                    | [37]      |
| Li et al. (2019)                     | Price and quality strategies           | Tobit regression and ordinary least square models | [38]      |
| Voros (2019)                         | Price and quality                      | Finite-time differential game       | [39]      |
| Hosseini-Motlagh et al. (2019)       | Price, sustainability level, and CSR   | Stackelberg game                   | [40]      |
| Khosroshahi et al. (2019)            | Price, transparency, and CSR           | Stackelberg game                   | [41]      |
| Jeong and Yoon (2014)                | Quality and CSR image                  | Empirical and causal approaches    | [42]      |
| Gatti, Caruana, and Snehota (2012)   | CSR and perceived quality              | Structural equation model           | [43]      |
| Nie, Wang, and Meng (2019)           | CER and profit                         | Static game                        | [44]      |
| Wong et al. (2018)                   | CER and income                         | Content analysis approach           | [45]      |
| Jiang, Xue and Xue (2018)            | CER and performance                    | Multi-variables regression          | [46]      |
Table 2. Differential games with more state or control variables.

| Study                        | State Variables                      | Control Variables                                           | Solution Type                  | Reference |
|------------------------------|--------------------------------------|-------------------------------------------------------------|--------------------------------|-----------|
| Lin and Wang (2019)          | Accumulation of sharing knowledge    | Effort level of knowledge sharing, degree of incentive      | Feedback                       | [48]      |
| Jiang et al. (2019)          | Pollutant stock                      | Emission capacity, pollution governance investment, eco-compensation ratio | Feedback                       | [49]      |
| Xin and Sun (2018)           | Product prices, water right prices   | Production planning, water saving                           | Open-loop, closed-loop, feedback | [50]      |
| Yang and Xu (2019)           | Carbon stock, inventory level        | Production output, product flow, product transaction, resource investment, carbon emission, carbon permit | Numerical                       | [51]      |
| Lu, Zhang, and Tang (2019)   | Corporation goodwill                | Advertising effort, retail margin, wholesale price, profit rate of cost | Feedback                       | [52]      |
| Wu (2018, 2019)              | Network effect, innovation level     | Price                                                       | Feedback                       | [53,54]  |
| Xin, Peng, and Sun (2019)    | Pollutant stock level                | Production output, abatement effort                         | Feedback                       | [55]      |
| Esahabani (2019)             | Product price                        | Production output                                           | Open-loop, closed-loop         | [56]      |
| Lu et al. (2019)             | Product price                        | Order quantity, advertising effort, wholesale price         | Feedback                       | [57]      |
| Kicsiny and Varga (2019)     | Water resource volume, payoff        | Consumption flow rate                                       | Numerical                       | [58]      |
| Chan, Zhou, and Wong (2019)  | Cumulative profit                    | New production output                                       | Numerical                       | [59]      |

3. Model Formulation and Notation

As explained in Sections 1 and 2, we consider an optimal dynamic problem over infinite time, in which a monopolist produces a single product and implements CER to promote corporate goodwill. General speaking, consumers are inclined to associate high quality and CER with high prices, where higher prices and CERs improve the corporate goodwill. Corporate goodwill directly affects sales. CER knowledge accumulation and investment in CER and quality all affect the cost. Besides, the classical supply–demand theory shows that (i) price is adversely related to sales, and (ii) the cost negatively affects the profits. We depict these relationships in the following block diagram, as shown in Figure 1.

![Figure 1. Block diagram of the proposed model.](image)

Tables 3 and 4 list the main notations used throughout the paper.
Table 3. Variables and descriptions.

| Variables | Description |
|-----------|-------------|
| $p(t)$   | The product price at time $t$ |
| $u(t)$   | Investment in CER at time $t$ |
| $x(t)$   | The corporate goodwill at time $t$ |
| $s(t)$   | CER knowledge accumulation from time 0 to $t$ |
| $z(t)$   | Investment in product quality at time $t$ |
| $C_p(z(t))$ | The marginal cost of production |
| $C_{CER}(u(t), s(t))$ | The cost of CER at time $t$ |
| $D(t)$   | The demand function at time $t$ |
| $G_S(u(t))$ | The marginal government subsidy function at time $t$ |
| $\pi_i(t)$ | The net profit rate with at time $t$, $i = 1, 2$ denotes without and with government subsidy, respectively. |

Table 4. Parameters and descriptions.

| Parameters | Description |
|-----------|-------------|
| $\bar{p}$ | The expected price for the brand with current corporate goodwill, $\bar{p} > 0$ |
| $x_0$     | The initial level of corporate goodwill, $x_0 \geq 0$ |
| $\pi_0$   | The initial CER knowledge accumulations, $\pi_0 > 0$ |
| $k_1$     | The price effect on the corporate goodwill, $k_1 > 0$ |
| $k_2$     | The effect of CER investment on the corporate goodwill, $k_2 > 0$ |
| $k_3$     | The effect of quality investment on the corporate goodwill, $k_3 > 0$ |
| $\delta$  | The depreciation rate of the corporate goodwill, $\delta > 0$ |
| $b_0$     | The rate of government subsidy, $b_0 \geq 0$ |
| $b_1$     | The effect of CER investment on CER cost, $b_1 > 0$ |
| $b_2$     | The learning rate of CER, $b_2 > 0$ |
| $a$       | The demand intercept, $a > 0$ |
| $a_1$     | The effect of quality investment on demand, $a_1 > 0$ |
| $a_2$     | The price effect on demand, $a_2 > 0$ |
| $a_3$     | The corporate goodwill effect on demand, $a_3 > 0$ |
| $\sigma$  | The effect of CER investment on the knowledge accumulations, $\sigma > 0$ |
| $r$       | The discount rate, $r > 0$ |
| $\lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22}$ | Dynamic adjoint variables |
| $\eta$    | The effect of quality investment on the margin production cost, $\eta > 0$ |
| $C_1, C_2, C_3, C_4$ | Constants |

Incorporating the effects of price $p(t)$, investment in quality $z(t)$ and CSRI $u(t)$ on corporate goodwill $x(t)$, we extend the well-known Nerlove–Arrow model [22] to the following dynamic equation describing the time evolution of the corporate goodwill:

$$\dot{x}(t) = k_1(p(t) - \bar{p}) + k_2z(t) + k_3u(t) - \delta x(t), \quad x(0) = x_0. \quad (1)$$

To formulate the demand problem in the monopolistic market, we extend the inverse demand function to the following demand function $D(t)$, which depends jointly on the investment in quality $z(t)$, price $p(t)$, and corporate goodwill $x(t)$:

$$D(t) = a + a_1z(t) - a_2p(t) + a_3x(t). \quad (2)$$

According to [60, 61], we employ the following linear marginal cost function of unitary production:

$$C_p(z(t)) = \eta z(t). \quad (3)$$

Borrowing from the thought of [62], we employ the following equation to measure the CSR knowledge accumulations:

$$s(t) = s_0 + \sigma \int_0^t u(h)dh, \quad (4)$$
which can be differentiated w.r.t. time $t$ and gives
\[ \dot{s}(t) = \sigma u(t). \] (5)

Inspired by [62,63], we consider the monopolist’s cost function of CSR as follows:
\[ C_{CER}(u(t), s(t)) = b_1 u^2(t) - b_2(s(t) - s_0). \] (6)

In this paper, we assume that all the demand can be satisfied, and there is no stock. We regard the demand function (2) as the product quantity under this circumstance. Then we can obtain the following monopolist’s instantaneous profits without government subsidies:
\[ \pi_1(t) = \left[p(t) - C_p(z(t))\right] D(t) - C_{CER}(u(t), s(t)) \]
\[ = (p(t) - \eta z(t))(a + a_1 z(t) - a_2 p(t) + a_3 x(t)) - b_1 u^2(t) + b_2(s(t) - s_0). \] (7)

In the real world, a government tends to provide subsidies for firms that undertake CSR. In the following, we will explore the difference of monopolist’s optimal strategies between the case with and without government subsidies. For the sake of simplicity, we employ the following linear marginal subsidy function:
\[ G_S(u(t)) = b_0 u(t). \] (8)

Referring to Equation (7), we write the following instantaneous profits with government subsidies:
\[ \pi_2(t) = \left[p(t) - C_p(z(t))\right] D(t) - C_{CER}(u(t), s(t)) + G_S(u(t)) \]
\[ = (p(t) - \eta z(t))(a + a_1 z(t) - a_2 p(t) + a_3 x(t)) + b_0 u(t) - b_1 u^2(t) + b_2(s(t) - s_0). \] (9)

To get the optimal combination of the product price, product quality, and CSRI to maximize its discounted infinite-horizon profit stream with/without government subsidies under the evolution of the corporate goodwill and CSR knowledge accumulations, we can depict it as the following differential game model:
\[ \max_{p(t), z(t), u(t)} \Pi = \int_0^\infty e^{-rt} \pi_i(t) dt, \quad i = 1, 2, \] (10)
\[ \text{s.t.} \left\{ \begin{array}{l}
\dot{s}(t) = \sigma u(t), \\
\dot{x}(t) = k_1(p(t) - \bar{p}) + k_2 z(t) + k_3 u(t) - \delta x(t).
\end{array} \right. \] (11)

where $p(t), z(t),$ and $u(t)$ are control variables; $s(t)$ and $x(t)$ are state variables.

4. The Case without Government Subsidy

In this section, we will perform the open-loop, closed-loop, feedback equilibrium analysis for the case without government subsidies. Moreover, the subscript $F$ indicates the feedback equilibrium of variables. For simplicity, the time-dependence $(t)$ of variables and state will be suppressed if no confusion arises.

4.1. The Open-/Closed-loop Equilibrium

We write the Hamiltonian function $H_1$ for the optimization model in Equations (10) and (11) without government subsidy as follows:
\[ H_1 = (p - \eta z)(a + a_1 z - a_2 p + a_3 x) - b_1 u^2 + b_2(s - s_0) \]
\[ + \lambda_{11} \dot{\sigma} u + \lambda_{12}(k_1(p - \bar{p}) + k_2 z + k_3 u - \dot{x}) \] (12)

where $\lambda_{11}, \lambda_{12}$ denote the dynamic adjoint variables related to their respective state equations $\dot{s}$ and $\dot{x}$ under the case without government subsidies.
From the Hamiltonian function $H_1$ in Equation (12), we get the first conditions for $p, z,$ and $u$ as follows:

$$\frac{\partial H_1}{\partial p} = a + a_1 z - 2a_2 p + a_3 x + a_2 z \eta + \lambda_{11} k_1 = 0$$  \hspace{1cm} (13)

$$\frac{\partial H_1}{\partial z} = (a_2 p - a - a_3 x) \eta + a_1 (p - 2a \eta) + \lambda_{11} k_2 = 0$$  \hspace{1cm} (14)

$$\frac{\partial H_1}{\partial u} = -2b_1 u + \lambda_{11} k_3 + \lambda_{12} \sigma = 0$$  \hspace{1cm} (15)

As mentioned in Section 3, where $a_1, a_2, b_1,$ and $\eta$ are positive parameters, the following sufficient optimality conditions for Equation (12) always hold:

$$\frac{\partial^2 H_1}{\partial p^2} = -2a_2 < 0, \quad \frac{\partial^2 H_1}{\partial z^2} = -2a_1 \eta < 0, \quad \frac{\partial^2 H_1}{\partial u^2} = -2b_1 < 0,$$

$$\frac{\partial^2 H_1}{\partial p \partial z} - \frac{(\partial^2 H_1)}{\partial p \partial u} = -(a_1 - a_2 \eta)^2 \leq 0,$$

$$\frac{\partial^2 H_1}{\partial z \partial u} - \frac{(\partial^2 H_1)}{\partial z \partial u} = 4a_2 b_1 > 0,$$

$$\frac{\partial^2 H_1}{\partial u \partial u} - \frac{(\partial^2 H_1)}{\partial u \partial u} = 4a_1 b_1 \eta > 0.$$

As we know, $\frac{\partial^2 H_1}{\partial p^2} - \frac{(\partial^2 H_1)}{\partial p \partial u} \leq 0$ denotes that the Hamiltonian function $H_1$ has no optimal solution. Therefore there is no open-/closed-loop equilibrium.

### 4.2. The Feedback Equilibrium

Several previous researchers [64] have proved that the feedback equilibrium policy fits the data better than the open-loop ones. Moreover, a feedback solution can better reflect the game dynamics over time. Therefore, in this section, we will perform a feedback equilibrium analysis for the differential game in (10) and (11) without government subsidies.

The Hamilton–Jacobi–Bellman (HJB) equation of the differential game in Equations (10) and (11) is:

$$rV_1 = \max_{p, z, u} \{ V_1 + p \partial V_1 + z \partial V_1 \}
= \max_{p, z, u} \left\{ (p - \eta z)(a + a_1 z - a_2 p + a_3 x) - b_1 u^2 + b_2 (s - s_0) \right\}
+ auV_1 + (k_1 (p - \bar{p}) + k_2 z + k_3 u - \delta \lambda) V_1 \}
= \frac{\partial V_1}{\partial a}, \quad V_1 = \frac{\partial V_1}{\partial a}.$$

From Equation (16), we obtain the following first-order conditions for $p, z,$ and $u$:

$$\begin{cases}
  a + a_1 z + a_2 (z \eta - 2p) + a_3 x + k_1 V_1 = 0, \\
  -a \eta + a_1 (p - 2a \eta) + a_2 p \eta - a_3 x \eta + 2k_2 V_1 = 0, \\
  -2b_1 u + a V_1 + k_3 V_1 = 0.
\end{cases}$$  \hspace{1cm} (17)

Solving Equation (17), we get the optimal feedback equilibrium for $p, z,$ and $u$, denoted by $p^*_F, z^*_F, u^*_F$, which are shown in the following Proposition 1.

**Proposition 1.** Without government subsidies, the optimal feedback equilibrium for $p, z,$ and $u$ are given by

$$p^*_F = \frac{1}{(a_1 - a_2 \eta)} (a_2 \eta (-k_2 V_1 + (a + a_3 x) \eta) - a_1 (k_2 V_1 + (a + 2k_1 V_1 + a_3 x) \eta))$$  \hspace{1cm} (18)

$$z^*_F = \frac{1}{(a_1 - a_2 \eta)} ((a - k_1 V_1 + a_3 x) (-a_1 + a_2 \eta) - 2V_1 (a_1 k_1 + a_2 k_2))$$  \hspace{1cm} (19)

$$u^*_F = \frac{1}{2b_1} (a V_1 + k_3 V_1)$$  \hspace{1cm} (20)
Proposition 2. Without government subsidies, the value function $V_1(x, s)$, and the steady state of variables $p_{F}^{ss}$, $z_{F}^{ss}$, $u_{F}^{ss}$, $s_{F}^{ss}$, and $x_{F}^{ss}$ satisfy the following equations

$$V_1(x, s) = n_0 + n_1 x + n_2 x^2 + \frac{b_2}{r} s$$  \hspace{1cm} (21)

$$p_{F}^{ss} = \frac{a_2\eta (-k_2 (n_1 + n_2 x_{F}^{ss}) + (a + a_3 x_{F}^{ss}) \eta) - a_1 ((n_1 + n_2 x_{F}^{ss}) (k_2 + 2 k_1 \eta) + (a + a_3 x_{F}^{ss}) \eta)}{n_{04}^2}$$  \hspace{1cm} (22)

$$z_{F}^{ss} = \frac{1}{n_{04}^2} (a_2 (a \eta + a_3 x_{F}^{ss} \eta) - a_1 (a + a_3 x_{F}^{ss}) - (n_1 + n_2 x_{F}^{ss}) (a_1 k_1 + 2 a_2 k_2 + a_2 k_1 \eta))$$  \hspace{1cm} (23)

$$u_{F}^{ss} = \frac{1}{2 b_1 r} (k_3 r (n_1 + n_2 x_{F}^{ss}) + b_2 \sigma)$$  \hspace{1cm} (24)

$$s_{F}^{ss} = \frac{1}{2 b_1 r} \sigma (k_3 r (n_1 + n_2 x_{F}^{ss}) + b_2 \sigma) t + C_1$$  \hspace{1cm} (25)

$$x_{F}^{ss} = \frac{r (a_1 k_3 x_{F}^{ss} + b_2 k_1 \beta \eta - b_2 m_0 + a_2 m_1) + b_2 k_3 n_{04}^2 \sigma}{(r (m_2 + m_3 + m_4) + C_2 - \frac{w_{y_{04}} + w_{y_{04}}}{w_{y_{04}} + w_{y_{04}}})}$$  \hspace{1cm} (26)

where:

$$n_0 = \frac{1}{4 b_1 n_{04}^2} (n_{101} + n_{02} + n_{03} - 4 a b_1 n_1 n_{04} n_{05} r^2),$$

$$n_{01} = a_1^2 (k_3^2 n_1^2 r^2 - 4 b_1 r^2 (k_1 n_1 \beta \eta + b_2 s_0) + 2 b_2 k_3 n_1 r \sigma + b_2 \sigma^2)^2;$$

$$n_{02} = -2 a_1 (2 b_1 r^2 (n_{05} - 2 a_2 b_2 s_0 \eta - 2 a_2 k_1 n_1 \beta \eta) + a_2 \eta (k_3 n_1 r + b_2 \sigma)^2);$$

$$n_{03} = a_2 (-4 b_1 r^2 (k_3^2 n_1^2 r^2 + 2 a_1 k_1 n_1 \beta \eta + b_2 s_0) \eta^2 + a_2 \eta^2 (k_3 n_1 r + b_2 \sigma)^2);$$

$$n_{04} = a_1 - a_2 \eta;$$

$$n_{05} = k_2 + k_1 \eta;$$

$$n_1 = \frac{n_{22} n_{04} (2 a b_1 n_{05} r + n_{04} (2 b_1 k_1 \beta \eta - k_3 k_3 \eta))}{r (a_1^2 (k_3^2 n_1^2 r^2 - 2 b_1 (r + \delta) - 2 a_1 n_1 - a_2 n_2))};$$

$$n_{11} = a_2 k_3^2 n_1^2 \eta + a_3 b_1 n_{05} + 2 b_1 (k_1 n_1 + a_2 - (r + \delta) \eta),$$

$$n_{12} = 2 b_1 (2 k_3^2 n_2 - k_2 (a_3 - 2 k_1 \eta) + (a_2 (r + \delta) - a_3 k_1) \eta^2 - a_2 k_3^2 n_2 \eta^2);$$

$$n_2 = a_1^2 k_3^2 - 2 a_1 (a_2 k_3^2 + 2 b_1 k_2 s_0 - a_2 k_1 \eta^2) + a_2 (4 b_1 k_2 s_0 - a_2 k_1 \eta^2);$$

$$m_0 = a b_1 n_{04} n_{05} + a_1 (a_2 k_3 n_1 \eta + 2 b_1 (k_1 n_1 n_{05} - a_2 \beta \eta));$$

$$m_1 = a_2 k_3 n_1 \eta^2 - 2 b_1 (2 k_2 n_1 n_{05} + a_2 k_1 \beta \eta^2);$$

$$m_2 = a_1^2 (2 b_1 \delta - k_3^2 n_2);$$

$$m_3 = 2 a_1 (b_2 k_1 (a_3 + 2 k_1 n_2)) + (a_3 b_1 k_1 + 2 b_1 k_2^2 n_2 + a_2 k_3^2 n_2 - 2 a_2 b_1 \delta) \eta);$$

$$m_4 = a_2 (4 b_1 k_2^2 n_2 - 2 b_2 k_2 (a_3 - 2 k_1 n_2) \eta - (2 a_3 b_1 k_1 + a_2 k_3^2 n_2 - 2 a_2 b_1 \delta) \eta^2).$$

Proof. Substituting the optimal feedback equilibrium in Equations (18)–(20) into the HJB Equation (16) yields:

$$r V_1 = \max_{p_{F}^{ss}, x, s} \left\{ \frac{1}{n_{04}^2} (k_3 V_{1x} + \sigma V_{1s})^2 + a_2 b_2 \eta (s - s_0) (2 a_1 + a_2 \eta) - V_{1x} (k_1 \beta \eta + x \delta) (a_1 \eta + a_2 \eta)^2 + a_2 n_{05} V_{1x} (a + a_3 x) - a_1 V_{1x} n_{05} (a - a_3 x) + V_{1x}^2 n_{05} (a_1 k_1 - a_2 k_2) \right\}$$  \hspace{1cm} (27)

Differentiating the value function in Equation (21) with respect to $s$ and $x$, respectively, gives

$$V_{1s} = \frac{b_2}{r}$$  \hspace{1cm} (28)

$$V_{1x} = n_1 + n_2 x$$  \hspace{1cm} (29)
Substituting Equations (21), (28) and (29) into (27), and equating the coefficients on both sides of Equation (27), we get $n_0, n_1$ and $n_2$.

Substituting Equations (28) and (29) into (18)–(20), we can obtain the steady state of price $p_{F}^\infty$, investment in quality $z_{F}^\infty$, and CSR $u_{F}^\infty$, as shown in Equations (22)–(24).

Substituting Equations (22)–(24) into (11), and solving the differential equations, it yields the steady state of CSR knowledge accumulations $s_{F}^\infty$, corporate goodwill $x_{F}^\infty$, as shown in Equations (25) and (26). □

5. The Case with Government Subsidy

In this section, we will perform the open-loop, closed-loop, feedback equilibrium analysis for the case with government subsidy. Moreover, the subscript FS indicates the feedback equilibrium of variables.

5.1. The Open-/Closed-loop Equilibrium

The Hamilton–Jacobi–Bellman (HJB) equation of the differential game model in Equations (10) and (11) is:

\[ H_2 = (p - \eta z)(a + a_1z - a_2p - a_3x) + b_0u - b_1u^2 + b_2(s - s_0) + \lambda_{21}\sigma u + \lambda_{22}(k_1(p - \bar{p}) + k_2z + k_3u - \delta x) \]  \tag{30}

where $\lambda_{21}, \lambda_{22}$ denote the dynamic adjoint variables related to their respective state equations \( \dot{s} \) and \( \dot{x} \) with government subsidies.

From the Hamiltonian function $H_2$ (30), we get the first conditions for $p, z, u$ as follows:

\[ \frac{\partial H_2}{\partial p} = a + a_1z - 2a_2p + a_3x + a_2\eta + \lambda_{21}k_1 = 0 \]  \tag{31}

\[ \frac{\partial H_2}{\partial z} = (a_2p - a - a_3x)\eta + a_1(p - 2x\eta) + \lambda_{21}k_2 = 0 \]  \tag{32}

\[ \frac{\partial H_2}{\partial u} = b_0 - 2b_1u + \lambda_{21}k_3 + \lambda_{22}\sigma = 0 \]  \tag{33}

As mentioned in Section 3, $a_1, a_2, b_1,$ and $\eta$ are positive parameters, the following sufficient optimality conditions for Model (30) always hold:

\[ \frac{\partial^2 H_2}{\partial p^2} = -2a_2 < 0, \quad \frac{\partial^2 H_2}{\partial z^2} = -2a_1 < 0, \quad \frac{\partial^2 H_2}{\partial u^2} = -2b_1 < 0, \]

\[ \frac{\partial^2 H_2}{\partial p \partial u} - \left( \frac{\partial^2 H_2}{\partial p \partial z} \right)^2 = -(a_1 - a_2\eta)^2 \leq 0, \]

\[ \frac{\partial^2 H_2}{\partial z \partial u} - \left( \frac{\partial^2 H_2}{\partial z \partial z} \right)^2 = 4a_2b_1 > 0, \]

\[ \frac{\partial^2 H_2}{\partial z^2} - \left( \frac{\partial^2 H_2}{\partial u \partial z} \right)^2 = 4a_1b_1\eta > 0. \]

As we know, $\frac{\partial H_2}{\partial p} - \left( \frac{\partial H_2}{\partial u} \right)^2 \leq 0$, which denotes that the Hamiltonian function $H_2$, has no optimal solution. Therefore, there is also no open-/closed-loop equilibrium with government subsidies.

5.2. The Feedback Equilibrium

In the following, we will perform a feedback equilibrium analysis for the differential game model with government subsidies.

The Hamilton–Jacobi–Bellman (HJB) equation of the differential game model in Equations (10) and (11) is:

\[ rV_2 = \max_{p, z, u} \left\{ \pi_2 + \dot{s}V_2 + \dot{x}V_2 \right\} \]

\[ = \max_{p, z, u} \left\{ (p - \eta z)(a + a_1z - a_2p - a_3x) + b_0u - b_1u^2 + b_2(s - s_0) + \sigma uV_2 + (k_1(p - \bar{p}) + k_2z + k_3u - \delta x) V_2 \right\}, \]  \tag{34}
where $V_2 = V_2(x, s)$ denotes the value function with government subsidy, $V_{2x} = \frac{\partial V_2}{\partial x}$, $V_{2s} = \frac{\partial V_2}{\partial s}$.

From the HJB Equation (34), we obtain the following first-order conditions for $p$, $z$, and $u$:

$$\begin{align*}
  &\begin{cases}
  a + a_1 z + a_2 (z \eta - 2p) + a_3 x + k_1 V_{2x} = 0 \\
  -a_1 \eta + a_1 (p - 2z \eta) + a_2 p \eta - a_3 x \eta + k_2 V_{2x} = 0 \\
  b_0 - 2b_1 u + \sigma V_{2s} + k_3 V_{2x} = 0
  \end{cases} \\
  \end{align*}$$

Solving Equation (35), we get the optimal feedback equilibrium for $p$, $z$, and $u$, denoted by $p^*_{FS}$, $z^*_{FS}$, $u^*_{FS}$, which are shown in the following Proposition 3.

**Proposition 3.** With government subsidies, the optimal feedback equilibrium for $p$, $z$, and $u$ are given by

$$p^*_{FS} = \frac{1}{n_{04}^2}(a_2 \eta ((a + a_3) \eta - k_2 V_{2x}) - a_1 (k_2 V_{2x} + (a + 2k_1 V_{2x} + a_3 x) \eta))$$

$$z^*_{FS} = \frac{1}{n_{04}^2}(n_{04} (k_1 V_{2x} - a - a_3 x) - 2V_{2x}(a_1 k_1 + a_2 k_2))$$

$$u^*_{FS} = \frac{1}{2b_1}(b_0 + \sigma V_{2s} + k_3 V_{2x})$$

**Proposition 4.** With government subsidies, the value function $V_2(x, s)$, and the steady state of variables $p^\infty_{FS}$, $z^\infty_{FS}$, $u^\infty_{FS}$, $s^\infty_{FS}$, and $x^\infty_{FS}$ satisfy the following equations

$$V_2(x, s) = n_3 + n_4 x + n_5 x^2 + \frac{b_2}{r}s$$

$$p^\infty_{FS} = \frac{1}{n_{04}^2}(\eta (a + a_3 x^\infty_{FS})(a_1 + a_2 \eta) + (n_4 + n_5 x^\infty_{FS})(2a_1 k_1 \eta - a_2 k_2 \eta - a_1 k_2))$$

$$z^\infty_{FS} = \frac{1}{n_{04}^2}(n_{04} (a + a_3 x^\infty_{FS}) + (n_4 + n_5 x^\infty_{FS})(a_1 k_1 + 2a_2 k_2 + a_2 k_1 \eta))$$

$$u^\infty_{FS} = \frac{1}{2b_1 r}(n_{34} + k_3 n_3 r x^\infty_{FS})$$

$$s^\infty_{FS} = \frac{1}{2b_1 r} \eta(n_{34} + k_3 n_3 r x^\infty_{FS}) + C_3$$

$$x^\infty_{FS} = \frac{m_5 + r(m_6 - m_7 + m_8)}{r(a_1^2 m_9 + m_{10} + m_{11})} + C_4 e^{-\frac{a_1^2 m_9 + m_{10} + m_{11}}{2b_1 r} t}$$
where
\[
\begin{align*}
    n_3 &= \frac{1}{4h_1n_{33}^2}(n_{31} - 2a_1n_{32} + a_2n_{33}), \\
    n_{31} &= a_1(\tilde{r}^2(b_2^2 + 2b_3k_4 + k_3^2n_4^2 - 4b_1(k_1n_4\tilde{p} + b_2s_0)) + b_2(n_34 - 2b_2\sigma), \\
    n_{32} &= 2b_1r^2(a_1n_{35} + k_3^2n_4^2\eta - 2a_2b_2s_0\eta + k_1n_4(k_2n_4 - 2a_2\tilde{p}\eta)) + k_2\eta, \\
    n_{33} &= a_2n_{34}^2\eta^2 - 4b_1r^2(k_2n_4((k_1n_4 - a)\eta + k_2n_4) + (k_1n_4(a_2\tilde{p} - a) + a_2b_2s_0)\eta^2), \\
    n_{34} &= b_0r + k_3n_4r + b_2\sigma, \\
    n_4 &= \frac{1}{r(a_1n_{41} + a_2n_{42} + n_{43})}n_{404}(-2ab_1n_{50}r + n_{404}(b_0k_3r + b_2k_3\sigma - 2b_1k_1\tilde{p})), \\
    n_{41} &= 2b_1(r + 2\delta - k_3^2n_5, \\
    n_{42} &= 2a_1((a_3 + 2k_1n_5)b_1n_{50} - a_2n_{41}\eta), \\
    n_{43} &= a_2(4b_1k_2n_5 - 2b_1k_2(a_3 - 2k_1n_5)^2 + (a_2n_{41} - 2a_3b_1k_1)\eta^2), \\
    n_{5} &= \frac{1}{a_1k_3^2 + a_2k_1\eta(a_2\eta - 2a_1)}4b_1n_{04}(n_{04}(r + 2\delta) + a_2b_1n_{05}), \\
    m_5 &= b_2k_3n_{04}^2\sigma - 2db_1n_{04}n_{50}r, \\
    m_6 &= a_2^2(m_{12} - 2b_1k_1\tilde{p}), \\
    m_7 &= 2a_1(a_2n_{12}\eta - 2b_1k_1(n_4n_{50} - a_2\tilde{p}\eta)), \\
    m_8 &= a_2(2a_2n_{12}\eta^2 - 2b_1(2k_2n_{50} + a_2k_1\tilde{p}\eta^2)), \\
    m_9 &= 2b_1\delta - k_3^2n_5, \\
    m_{10} &= 2b_1(k_2k_3(a_3 - 2k_1n_5) + (a_3b_1k_1 + 2b_1k_1^2n_5 - a_2n_5)n_5), \\
    m_{11} &= a_2(4b_1k_2n_5 - 2b_1k_2(a_3 - 2k_1n_5)\eta - (a_3b_1k_1 - a_2n_5)\eta^2), \\
    m_{12} &= k_3(b_0 + k_3n_4). \\
\end{align*}
\]

**Proof.** Substituting the optimal feedback equilibrium in Equations (36)–(38) into the HJB Equation (45) yields
\[
\begin{align*}
    rV_2 &= \max_{\mu \in \mathcal{M}} \left\{ \frac{1}{\mu_1} \left( k_3V_{2x} + aV_{2x} \right)^2 + \frac{1}{\mu_0} \left( a_2b_2\eta(s - \beta_0)(a_2\eta) - V_{2x}(k_1\tilde{p} + x\delta)(a_1\eta + a_2\eta) + a_2V_{2x}(a + a_3x)(k_1 + k_2\eta) - a_1V_{2x}n_{50}(a - a_3x) + V_{2x}n_{05}(a_1k_1 - a_2k_2) \right) \right\} \\
\end{align*}
\]
Differentiating the value function in Equation (39) with respect to $s$ and $x$, respectively, gives
\[
\begin{align*}
    V_{2s} &= \frac{b_2}{r} \\
    V_{2x} &= n_4 + n_5x
\end{align*}
\]
Substituting Equations (39), (46), and (47) into (45), and equating the coefficients on both sides of Equation (45), we get $n_3$, $n_4$, and $n_5$.

Substituting Equations (46) and (47) into (36)–(38), we can obtain the steady state of control variables $p_{FS}^0$, $a_{FS}^0$, and $w_{FS}^0$, as shown in Equations (40)–(42).

Substituting Equations (40)–(42) into (11), and solving the differential equations, it yields the steady state of state variables $x_{FS}^0$ and $x_{FS}^0$, as shown in Equations (43) and (44).

6. Simulation

For the sake of convenience, we initialize parameters for the proposed model as follows.
\[
\begin{align*}
    \tilde{p} &= 150, \quad \theta_0 = 2, \quad s_0 = 5, \quad \kappa_1 = 0.01, \quad \kappa_2 = 1.8, \quad \kappa_3 = 0.5, \quad \delta = 0.01, \quad b_0 = 0.2, \quad b_1 = 4, \quad b_2 = 3, \quad a = 80, \\
    a_1 &= 2.6, \quad a_2 = 2, \quad a_3 = 2.2, \quad \sigma = 2, \quad r = 0.1, \quad \eta = 3.3. \quad \text{Optimal solutions with/without government subsidies are presented in the following.}
\end{align*}
\]
(i) The case without government subsidies:
\[
\begin{align*}
    p_{FS}(t) &= 7.701919 + 0.398208e^{-1.010689t}, \\
    z_{FS}(t) &= 5.030651 + 0.917139e^{-1.010689t},
\end{align*}
\]
\[ u_F(t) = 0.379791 - 0.030029e^{-1.010689t}, \]
\[ s_F(t) = 0.005942e^{-1.010689t} + 0.075958t + 4.994058, \]
\[ x_F(t) = 4.591009 - 2.591009e^{-1.010689t}, \]
\[ V_1(t) = 11.775529 + 2.489789e^{-1.010689t} + 4.994058, \]
\[ (ii) \text{ The case with government subsidies:} \]
\[ p_{FS}(t) = 7.695105 + 0.636307e^{-1.010689t}, \]
\[ z_{FS}(t) = 5.026388 + 0.912762e^{-1.010689t}, \]
\[ u_{FS}(t) = 0.404778 - 0.029886e^{-1.010689t}, \]
\[ s_{FS}(t) = 0.005914e^{-1.010689t} + 0.080956t + 0.080956, \]
\[ x_{FS}(t) = 4.578642 - 2.578642e^{-1.010689t}, \]
\[ V_2(t) = 13.322593 + 2.466076e^{-2.021379t} + 4.95591e^{-1.010689t} + 2.428666t. \]

In the following, \( t \) varies from 0 to 10 with an increment of 1 in all plots.

6.1. The Optimal Price Levels

Figure 2 presents the evolution trends of the optimal price levels \( p_F \) and \( p_{FS} \) by increasing time \( t \). Figure 2 illustrates that \( p_F \) and \( p_{FS} \) decrease at first, and eventually reach steady levels \( p_\infty^{\infty} = 7.7019 \) and \( p_{FS}^{\infty} = 7.6951 \), respectively. This result shows that the optimal price level with government subsidies is lower than that without government subsidy. Moreover, the effect of government subsidy on the optimal price levels is shown in Figure 3.

6.2. The Optimal Investment Levels in Quality

Figure 4 illustrates the evolution trends of the optimal investment levels in quality \( z_F \) and \( z_{FS} \) by increasing time \( t \). Figure 4 presents that \( z_F \) and \( z_{FS} \) decrease rapidly at the beginning, and eventually reach steady levels \( z_F^{\infty} = 5.0307 \) and \( z_{FS}^{\infty} = 5.0264 \), respectively.
6.2. The Optimal Investment Levels in Quality

Figure 4 illustrates the evolution trends of the optimal investment levels in quality $F_z$ and $F_{Sz}$ by increasing time $t$. Figure 4 presents that $F_z$ and $F_{Sz}$ decrease rapidly at the beginning, and eventually reach steady levels $5.0307 F_z^\infty = 5.0264 F_{Sz}^\infty$, respectively. This result shows that the optimal investment level in quality with government subsidies is lower than the case without government subsidies. Moreover, the effect of government subsidies on the optimal investment levels in quality is shown in Figure 5.

6.3. The Optimal Investment Levels in CER

Figure 6 presents the evolution trends of the optimal investment levels in CER $F_u$ and $F_{Su}$ by increasing time $t$. Figure 6 shows that $F_u$ and $F_{Su}$ increase at first, and eventually reach steady levels $0.3798 F_u^\infty = 0.4048 F_{Su}^\infty$, respectively. Obviously, $F_{Su}^\infty > F_u^\infty$ holds, which means the optimal investment level in CER with government subsidies is higher than that without government subsidies. Moreover, the impact of government subsidies on the optimal investment levels in CER is shown in Figure 7.

This result shows that the optimal investment level in quality with government subsidies is lower than the case without government subsidies. Moreover, the effect of government subsidies on the optimal investment levels in quality is shown in Figure 5.
6.3. The Optimal Investment Levels in CER

Figure 6 presents the evolution trends of the optimal investment levels in CER $u_F$ and $u_{FS}$ by increasing time $t$. Figure 6 shows that $u_F$ and $u_{FS}$ increase at first, and eventually reach steady levels $u_F^\infty = 0.3798$ and $u_{FS}^\infty = 0.4048$, respectively.

Figure 6. Evolutions of the optimal investment levels in CER.
Obviously, $u_{FS}^\infty > u_{F}^\infty$ holds, which means the optimal investment level in CER with government subsidies is higher than that without government subsidies. Moreover, the impact of government subsidies on the optimal investment levels in CER is shown in Figure 7.

6.4. The Optimal CER Knowledge Accumulations Levels

Figure 8 shows the evolution trends of the CER knowledge accumulations levels $s_F$ and $s_{FS}$ by increasing time $t$. Figure 8 illustrates that $s_F$ and $s_{FS}$ are in linear growth because the CER knowledge accumulations function in Equation (4) is linear. Moreover, the impact of government subsidies on the CER knowledge accumulations levels is shown in Figure 9.

6.5. The Optimal Corporate Goodwill Levels

Figure 10 shows the evolution trends of the optimal corporate goodwill levels $F_x$ and $F_{Sx}$ by increasing time $t$. Figure 10 illustrates that $F_x$ and $F_{Sx}$ increase rapidly at the beginning, and eventually reach steady levels $4.5910 F_x^\infty = 4.5786 F_{Sx}^\infty$, respectively.

This result shows that the optimal corporate goodwill level with government subsidies is lower than that without government subsidies. Moreover, the impact of government subsidies on the optimal corporate goodwill levels is shown in Figure 11.
6.5. The Optimal Corporate Goodwill Levels

Figure 10 shows the evolution trends of the optimal corporate goodwill levels \( x_F \) and \( x_{FS} \) by increasing time \( t \). Figure 10 illustrates that \( x_F \) and \( x_{FS} \) increase rapidly at the beginning, and eventually reach steady levels \( x_F^\infty = 4.5910 \) and \( x_{FS}^\infty = 4.5786 \), respectively.

This result shows that the optimal corporate goodwill level with government subsidies is lower than that without government subsidies. Moreover, the impact of government subsidies on the optimal corporate goodwill levels is shown in Figure 11.
Figure 11. Difference in the optimal corporate goodwill levels.

6.6. The Optimal Value Functions

Figure 12 shows the evolution trends of the value functions $V_1$ and $V_2$ by increasing time $t$. Figure 12 illustrates that $V_1$ and $V_2$ are continuously increasing. Moreover, the impact of government subsidies on value functions is shown in Figure 13.

Figure 12. Evolution of the optimal value functions.
6.7. The Effect of Control Variables on Value Functions

The value function reflects the firm’s profits. In this subsection, we simulate the effects of three control variables on value functions as follows.

Figure 14 shows the impact of price and investment in CER on value function $V_1$. The figure illustrates that a 1.85% price decrease and a 2.96% investment in CER increase drive a 172.18% $V_1$ increase.

Figure 15 shows the impact of price and investment in quality on value function $V_1$. The figure illustrates that a 1.85% price decrease and a 6.22% investment in quality decrease drive a 172.18% $V_1$ increase.

Figure 16 shows the effect of investment in CER and quality on value function $V_1$. The figure illustrates that a 6.22% investment in quality decrease and a 2.96% investment in CER increase drive a 172.18% $V_1$ increase.
Figure 14. The effect of price and investment in CER on $V_1$.

Figure 15. The effect of price and investment in quality on $V_1$. The figure illustrates that a 1.85% price decrease and a 6.22% investment in quality decrease drive a 172.18% $V_1$ increase.

Figure 16. The effect of investment in quality and in CER on value function $V_1$. The figure illustrates that a 6.22% investment in quality decrease and a 2.96% investment in CER increase drive a 172.18% $V_1$ increase.

From Figure 14, Figure 15, and Figure 16, we conclude that the first influence factor on $V_1$ is price, the second one is the investment in CER, and the third one is the investment in quality.
Figure 17 shows the influence of price and investment in CER on the value function $V_2$. The figure illustrates that a 1.84% price decrease and a 2.76% investment in CER increase drive a 161.08% $V_2$ increase.

![Figure 17](image1.png)

Figure 17. The effect of price and investment in CER on $V_2$.

Figure 18 shows the influence of price and investment in quality on value function $V_2$. The figure illustrates that a 1.84% price decrease and a 6.2% investment in quality decrease drive a 161.08% $V_2$ increase.

![Figure 18](image2.png)

Figure 18. The effect of price and investment in quality on $V_2$.

Figure 19 shows the influence of investment in quality and CER on value function $V_2$. The figure illustrates that a 6.2% investment in quality decrease and a 2.76% investment in CER increase drive a 161.08% $V_2$ increase.

![Figure 19](image3.png)

Figure 19. The effect of investment in quality and in CER on $V_2$.
According to Figures 17–19, we conclude that the first influence factor on $V_2$ is also price, the second one is also the investment in CER, and the third one is also the investment in quality.

To sum up, whether or not to consider government subsidies, the first influence factor on profit is price, the second one is the investment in CER, and the third one is the investment in quality. The profit with government subsidies is higher than that without government subsidies. However, the growth rate of profit with government subsidies is lower than that without government subsidies.

7. Discussions

Environmentally responsible firms tend to gain better corporate goodwill, while better corporate goodwill helps the enterprise achieve competitive advantages. Since CER is a spontaneous organization behavior, its actual effect is limited. Therefore, government involvement in firms’ CER practice is of great significance. Despite some researchers emphasizing that CER is vital for firms and governments, literature exploring how government subsidies affect firms’ optimal strategies when considering the impacts of price, quality, and CER on corporate goodwill, is scarce. To bridge this literature gap, we developed the monopoly differential game mentioned above to depict a joint optimization of pricing and investing in quality and CER with/without government subsidies. Results reveal that:

1) Government subsidies have adverse effects on the optimal price, investment in quality, and corporate goodwill levels, and positively affect the optimal investment in CER, CER knowledge accumulations levels, and value functions.

2) Considering government subsidies, the monopolist would increase the investment in CER. Comparing Equations (24) and (42), we find the investment increase in CER is only a part of government subsidies, which follows the profit-maximizing hypothesis.

3) Whether or not to consider government subsidies, the first influence factor on profits is the price, the second one is the investment in CER, and the third one is the investment in quality. The profit with government subsidies is higher than that without government subsidies. The growth rate of profit with government subsidies is lower than that without government subsidies.

In this paper, we reveal the relationship between product price, quality, and CER in a monopoly market with/without government subsidies. Our results can guide enterprises in optimizing their overall decisions of product pricing, quality improvement, and investment in corporate environmental responsibility. It can guide enterprises to make rational pricing, continuously improve product quality, and consistently perform CER, which can increase social welfare. Our results also provide theoretical...
support for the government to regulate CER, supervise product quality, regulate pricing, and improve social welfare by using government subsidies.

8. Conclusions

In this paper, we construct a differential game over infinite time, in which a monopolist produces a single product and implements the investment in CER. We then explore an environmentally responsible monopolist’s feedback equilibrium strategies with/without government subsidies. Results show that government subsidies affect a monopolist’s optimal strategies.

The following extensions are of interest for future research:

1. The output of the proposed game is entirely determined by the parameter values and the initial conditions. However, the real world is disturbed by stochasticity. For further development, stochastic models that possess some inherent randomness can be considered, such as a stochastic differential game [65].

2. We leveraged the linear CSR knowledge accumulations function, which can be further improved to a nonlinear function.

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References

1. Abigail, E.H. Corporate Environmental Responsibility. In International Encyclopedia of Geography; Richardson, D., Castree, N., Goodchild, M.F., Kobayashi, A., Liu, W., Marston, R.A., Eds.; John Wiley & Sons, Ltd.: Hoboken, NJ, USA, 2017; pp. 1–7. [CrossRef]
2. Aguado, E.; Holl, A. Differences of Corporate Environmental Responsibility in Small and Medium Enterprises: Spain and Norway. Sustainability 2018, 10, 1877. [CrossRef]
3. Han, H.; Yu, J.; Kim, W. Environmental corporate social responsibility and the strategy to boost the airline’s image and customer loyalty intentions. J. Travel Tour. Mark. 2019, 36, 371–383. [CrossRef]
4. Lee, S.H.; Park, C.H. Eco-Firms and the Sequential Adoption of Environmental Corporate Social Responsibility in the Managerial Delegation. B E J. Theor. Econ. 2019, 19. [CrossRef]
5. Wang, X.Y.; Wang, L.H. State-enterprise relation, local economic priority, and corporate environmental responsibility. Appl. Econ. 2019, 51, 995–1009. [CrossRef]
6. Zou, H.L.; Xie, X.M.; Qi, G.Y.; Yang, M.Y. The heterogeneous relationship between board social ties and corporate environmental responsibility in an emerging economy. Bus. Strategy Environ. 2019, 28, 40–52. [CrossRef]
7. Qin, Y.; Harrison, J.; Chen, L. A framework for the practice of corporate environmental responsibility in China. J. Clean. Prod. 2019, 235, 426–452. [CrossRef]
8. Suganithi, L. Examining the relationship between corporate social responsibility, performance, employees’ pro-environmental behavior at work with green practices as mediator. J. Clean. Prod. 2019, 232, 739–750. [CrossRef]
9. Freeman, R.E. The politics of stakeholder theory: Some future directions. Bus. Ethics Q. 1994, 409–421. [CrossRef]
10. Li, W.; Zhang, R. Corporate Social Responsibility, Ownership Structure, and Political Interference: Evidence from China. J. Bus. Ethics 2010, 96, 631–645. [CrossRef]
11. McWilliams, A.; Siegel, D. Corporate Social Responsibility: a Theory of the Firm Perspective. *Acad. Manag. Rev.* 2001, 26, 117–127. [CrossRef]

12. Babiak, K.; Trendafillova, S. CSR and environmental responsibility: motives and pressures to adopt green management practices. *Corp. Soc. Responsib. Environ. Manag.* 2011, 18, 11–24. [CrossRef]

13. Gregory, A.; Tharyan, R.; Whittaker, J. Corporate Social Responsibility and Firm Value: Disaggregating the Effects on Cash Flow, Risk and Growth. *J. Bus. Ethics* 2014, 124, 633–657. [CrossRef]

14. Statman, M.; Glushkov, D. The wages of social responsibility. *Financ. Anal. J.* 2009, 65, 33–46. [CrossRef]

15. Jo, H.; Harjoto, M.A. Corporate governance and firm value: The impact of corporate social responsibility. *J. Bus. Ethics* 2011, 103, 351–383. [CrossRef]

16. Jo, H.; Harjoto, M.A. The Causal Effect of Corporate Governance on Corporate Social Responsibility. *J. Bus. Ethics* 2012, 106, 53–72. [CrossRef]

17. Eberle, D.; Berens, G.; Li, T. The impact of interactive corporate social responsibility communication on corporate reputation. *J. Bus. Ethics* 2013, 118, 731–746. [CrossRef]

18. Lin-Hi, N.; Blumberg, I. The link between (not) practicing CSR and corporate reputation: Psychological foundations and managerial implications. *J. Bus. Ethics* 2018, 150, 185–198. [CrossRef]

19. Mitra, R. Framing the corporate responsibility-reputation linkage: The case of Tata Motors in India. *Public Relat. Rev.* 2011, 37, 392–398. [CrossRef]

20. Liu, W.; Wei, Q.; Huang, S.Q.; Tsai, S.B. Doing Good Again? A Multilevel Institutional Perspective on Corporate Environmental Responsibility and Philanthropic Strategy. *Int. J. Environ. Res. Public Health* 2017, 14, 1283. [CrossRef]

21. Dang, V.T.; Nguyen, N.; Bu, X.; Wang, J. The Relationship between Corporate Environmental Responsibility and Firm Performance: A Moderated Mediation Model of Strategic Similarity and Organization Slack. *Sustainability* 2019, 11, 3395. [CrossRef]

22. Nerlove, M.; Arrow, K.J. Optimal advertising policy under dynamic conditions. *Econometrica* 1962, 29, 129–142. [CrossRef]

23. Burritt Roger, L. Sustainability accounting and reporting: fad or trend? *Account. Audit. Account. J.* 2010, 23, 829–846. [CrossRef]

24. Schaltegger, S. Is environmental management accounting a discipline? A bibliometric literature review. *Meditari Account. Res.* 2013, 21, 4–31. [CrossRef]

25. Maqbool, S.; Zameer, M.N. Corporate social responsibility and financial performance: An empirical analysis of Indian banks. *Future Bus. J.* 2018, 4, 84–93. [CrossRef]

26. Juan Pablo, S.-I.H.; Benito, Y.-A.; Juan, M.-G. Moderating effect of firm size on the influence of corporate social responsibility in the economic performance of micro-, small- and medium-sized enterprises. *Technol. Forecast. Soc. Chang.* 2019, 119, 774. [CrossRef]

27. Broadstock, D.C.; Matousek, R.; Meyer, M.; Tzeremes, N.G. Does corporate social responsibility impact firms’ innovation capacity? The indirect link between environmental & social governance implementation and innovation performance. *J. Bus. Res.* 2019. [CrossRef]

28. Khan, H.U.R.; Ali, M.; Olya, H.G.T.; Zulqarnain, M.; Khan, Z.R. Transformational leadership, corporate social responsibility, organizational innovation, and organizational performance: Symmetrical and asymmetrical analytical approaches. *Corp. Soc. Responsib. Environ. Manag.* 2018, 25, 1270–1283. [CrossRef]

29. Saha, R.; Shashi; Cercione, R.; Singh, R.; Dahiya, R. Effect of ethical leadership and corporate social responsibility on firm performance: A systematic review. *Corp. Soc. Responsib. Environ. Manag.* 2019, 1–21. [CrossRef]

30. Yoon, B.; Chung, Y. The effects of corporate social responsibility on firm performance: A stakeholder approach. *J. Hosp. Tour. Manag.* 2018, 37, 89–96. [CrossRef]

31. Webb, N.J. Corporate profits and social responsibility: “Subsidization” of corporate income under charitable giving tax laws. *J. Econ. Bus.* 1996, 48, 401–421. [CrossRef]

32. Webb, N.J.; Farmer, A. Corporate Goodwill: A Game Theoretic Approach to the Effect of Corporate Charitable Expenditures on Firm Behaviour. *Ann. Public Coop. Econ.* 2010, 67, 29–50. [CrossRef]

33. Xu, X.D.; Zeng, S.X.; Chen, H.Q. Signaling good by doing good: How does environmental corporate social responsibility affect international expansion? *Bus. Strategy Environ.* 2018, 27, 946–959. [CrossRef]

34. El Ghoul, S.; Guedhami, O.; Kim, H.; Park, K. Corporate Environmental Responsibility and the Cost of Capital: International Evidence. *J. Bus. Ethics* 2018, 149, 335–361. [CrossRef]
35. Zeng, S.H.; Qin, Y.J.; Zeng, G.W. Impact of Corporate Environmental Responsibility on Investment Efficiency: The Moderating Roles of the Institutional Environment and Consumer Environmental Awareness. *Sustainability* 2019, 11, 4512. [CrossRef]

36. Chuang, S.P.; Huang, S.J. The Effect of Environmental Corporate Social Responsibility on Environmental Performance and Business Competitiveness: The Mediation of Green Information Technology Capital. *J. Bus. Ethics* 2018, 150, 991–1009. [CrossRef]

37. De Giovanni, P.; Zaccour, G. Optimal quality improvements and pricing strategies with active and passive product returns. *Omega* 2019, 88, 248–262. [CrossRef]

38. Li, J.; Gómez, M.I.; Brent Ross, R.; Chaddad, F.R. Does passion for wine matter? The effects of owner motivation on pricing and quality decisions in emerging US wine regions. *Agrribusiness* 2019. [CrossRef]

39. Voros, J. An analysis of the dynamic price-quality relationship. *Eur. J. Oper. Res.* 2019, 277, 1037–1045. [CrossRef]

40. Zhao, C.; Zhang, Y. Dynamic quality and pricing decisions in customer-intensive service systems with online reviews. *Int. J. Prod. Res.* 2019, 57, 5725–5748. [CrossRef]

41. Hosseini-Motlagh, S.-M.; Nouri-Harzvili, M.; Choi, T.-M.; Ebrahim, S. Reverse supply chain systems optimization with dual channel and demand disruptions: Sustainability, CSR investment and pricing coordination. *Inf. Sci.* 2019, 503, 606–634. [CrossRef]

42. Khosroshahi, H.; Rasti-Barzoki, M.; Hejazi, S.R. A game theoretic approach for pricing decisions considering CSR and a new consumer satisfaction index using transparency-dependent demand in sustainable supply chains. *J. Clean. Prod.* 2019, 208, 1065–1080. [CrossRef]

43. Jeong, W.S.; Yoon, S. Empirical-causal Approach to the Effects of Corporate Social Responsibility Activities: Is Product Quality Offset by the CSR Image? *J. Prod. Res.* 2014, 32, 83–92. [CrossRef]

44. Gatti, L.; Caruana, A.; Snehota, I. The role of corporate social responsibility, perceived quality and corporate reputation on purchase intention: Implications for brand management. *J. Brand Manag.* 2012, 20, 65–76. [CrossRef]

45. Nie, P.Y.; Wang, C.; Meng, Y. An analysis of environmental corporate social responsibility. *Manag. Decis. Econ.* 2019, 40, 384–393. [CrossRef]

46. Wong, C.W.Y.; Miao, X.; Cui, S.; Tang, Y.H. Impact of Corporate Environmental Responsibility on Operating Income: Moderating Role of Regional Disparities in China. *J. Bus. Ethics* 2018, 149, 363–382. [CrossRef]

47. Jiang, Y.; Xue, X.L.; Xue, W.R. Proactive Corporate Environmental Responsibility and Financial Performance: Evidence from Chinese Energy Enterprises. *Sustainability* 2018, 10, 964. [CrossRef]

48. Lin, L.; Wang, H. Dynamic incentive model of knowledge sharing in construction project team based on differential game. *J. Oper. Res. Soc.* 2019, 70, 2084–2096. [CrossRef]

49. Jiang, K.; You, D.; Li, Z.; Shi, S. A differential game approach to dynamic optimal control strategies for watershed pollution across regional boundaries under eco-compensation criterion. *Ecol. Indic.* 2019, 105, 229–241. [CrossRef]

50. Xin, B.G.; Sun, M.H. A differential oligopoly game for optimal production planning and water savings. *Eur. J. Oper. Res.* 2018, 269, 206–217. [CrossRef]

51. Yang, Y.X.; Xu, X. A differential game model for closed-loop supply chain participants under carbon emission permits. *Comput. Ind. Eng.* 2019, 135, 1077–1090. [CrossRef]

52. Lu, F.; Zhang, J.; Tang, W. Wholesale price contract versus consignment contract in a supply chain considering dynamic advertising. *Int. Trans. Oper. Res.* 2019, 26, 1977–2003. [CrossRef]

53. Wu, C.-H. Price competition and technology licensing in a dynamic duopoly. *Eur. J. Oper. Res.* 2018, 267, 570–584. [CrossRef]

54. Wu, C.-H. Licensing to a competitor and strategic royalty choice in a dynamic duopoly. *Eur. J. Oper. Res.* 2019, 279, 840–853. [CrossRef]

55. Xin, B.; Peng, W.; Sun, M. Optimal Coordination Strategy for International Production Planning and Pollution Abating under Cap-and-Trade Regulations. *Int. J. Environ. Res. Public Health* 2019, 16, 3490. [CrossRef] [PubMed]

56. Esfahani, H. Profitability of horizontal mergers in the presence of price stickiness. *Eur. J. Oper. Res.* 2019, 279, 941–950. [CrossRef]

57. Lu, J.; Zhang, J.; Jia, X.; Zhu, G. Optimal dynamic pricing, preservation technology investment and periodic ordering policies for agricultural products. *Rairo-Oper. Res.* 2019, 53, 731–747. [CrossRef]
58. Kicsiny, R.; Varga, Z. Differential game model with discretized solution for the use of limited water resources. *J. Hydrol.* 2019, **569**, 637–646. [CrossRef]

59. Chan, C.K.; Zhou, Y.; Wong, K.H. A dynamic equilibrium model of the oligopolistic closed-loop supply chain network under uncertain and time-dependent demands. *Transp. Res. Part E-Logist. Transp. Rev.* 2018, **118**, 325–354. [CrossRef]

60. Caulkins, J.P.; Feichtinger, G.; Grass, D.; Hartl, R.F.; Kort, P.M.; Seidl, A. Interaction of pricing, advertising and experience quality: A dynamic analysis. *Eur. J. Oper. Res.* 2017, **256**, 877–885. [CrossRef]

61. Chenavaz, R. Dynamic pricing, product and process innovation. *Eur. J. Oper. Res.* 2012, **222**, 553–557. [CrossRef]

62. Thompson, P. Chapter 10—Learning by Doing. In *Handbook of the Economics of Innovation*; Hall, B.H., Rosenberg, N., Eds.; Elsevier: Amsterdam, The Netherlands, 2010; Volume 1, pp. 429–476.

63. Clarke, F.H.; Darrough, M.N.; Heineke, J.M. Optimal Pricing Policy in the Presence of Experience Effects. *J. Bus.* 1982, **55**, 517–530. [CrossRef]

64. Chintagunta, P.K.; Vlcassim, N.J. An Empirical Investigation of Advertising Strategies in a Dynamic Duopoly. *Manag. Sci.* 1992, **38**, 1230–1244. [CrossRef]

65. Yu, W.; Xin, B. Governance Mechanism for Global Greenhouse Gas Emissions: A Stochastic Differential Game Approach. *Math. Probl. Eng.* 2013, 2013, 312585. [CrossRef]