An efficient cooling of the quantized vibration by a four-level configuration

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Cooling vibrational degrees of freedom down to ground states is essential to observation of quantum properties of systems with mechanical vibration. We propose two cooling schemes employing four internal levels of the systems, which achieve the ground-state cooling in an efficient fashion by completely deleting the carrier and first-order blue-sideband transitions. The schemes, based on the quantum interference and Stark-shift gates, are robust to fluctuation of laser intensity and frequency. The feasibility of the schemes is justified using current laboratory technology. In practice, our proposal readily applies to an nano-diamond nitrogen-vacancy center levitated in an optic trap or attached to a cantilever.

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Introduction.- To observe quantum characteristics of some systems with spin-vibration coupling, e.g., trapped ions or nanomechanical cantilevers, we have to eliminate the thermal phonons intrinsically owned by the systems. As such, cooling a system down to ground states of the vibrational degrees of freedom is usually a prerequisite of executing quantum tasks, such as quantum computing and ultra-precision measurements. Despite great success so far in cooling of trapped ions or atoms, cooling solid-state systems down to vibrational ground states is still tough experimentally.

Heating during the cooling process originates from two aspects. One is from the environment. This heating in solid-state systems can be suppressed by reducing the surface area of the system (or say, enhancing the quality factor Q), decreasing the work temperature of the surrounding environment and/or dynamically controlling the dissipation. The other is from some unexpected processes in the cooling, such as the carrier and the blue-sideband transitions. Suppressing those undesired transitions is an effective way to the cooling acceleration. As such, modified cooling schemes using quantum interference, such as the electromagnetic induced transparency (EIT), can largely suppress the blue-sideband transitions and work more efficiently than the original idea of sideband cooling. In addition, a proposal involving Stark-shift gate can suppress both carrier and blue-sideband transitions by steering the system to red-sideband transitions.

In the present letter, we propose two efficient cooling schemes by suppressing the unexpected processes as mentioned above, through employing four internal levels of the systems. As clarified below, the schemes can readily apply to vibrational systems involving nano-diamond nitrogen-vacancy (nNV) centers. Due to large mass and special characteristic, the nNV center system cannot be cooled down by simply merging previous cooling ideas, such as the scheme with EIT plus Stark-shift-gate for cooling trapped ions. Magnetic field gradient, in addition to laser irradiation, is required to couple the internal to the vibrational degrees of freedom of the solid-state system. Besides, due to existence of an additional decay to a metastable level in the nNV center, a pumping process in addition to usual cooling operations is demanded. Even with all these considerations, however, a three-level structure employed in a nNV center is proven to be not qualified for a perfect cooling since the first-order blue sideband transition cannot be fully eliminated.

In contrast, the two schemes proposed in the present letter employ four internal levels of the nNV center, which can well accomplish the cooling by completely eliminating the carrier and the first-order blue sideband transitions. One of our schemes with a modified Λ-type configuration is based on a dynamical EIT plus a Stark-shift gate, which is called briefly as the asymmetric cooling method. The other, called shortly as the symmetric cooling method, combines the Λ-type with V-type configurations, which yield double Stark-shift gates. As we know, most of the cooling schemes proposed so far are based on Λ-type three-level systems, rather than the V-type three-level configuration, due to the fact that the latter with two upper levels is more susceptible to dissipation. However, as shown below, a better cooling could be achieved if we elaborately combine the Λ-type and V-type configurations and have them interfered with each other. The interference enhanced by the Stark-shift gates yields a dark state and a Stark-shift-gate point, which help for an efficient cooling.

The systems. - We exemplify two systems to clarify our schemes. One is an nNV center levitated in an optic trap (see Fig. 1(a)), which is promising for detecting quantized gravity, preparing large distance superpositions and building matter-wave interferom-
eters [20]. The other is the nNV center attached to a cantilever [27] (see Fig. 1b)), which has potential applications in observation of phononic Mollow triplet [28], ultra-sensitive measurements [4], quantum information processing [29, 30] and biological sensing [31]. To achieve the objectives, the vibrational degrees of freedom in these systems are required to be cooled down to ground states which are very challenging with current laboratory technologies. We show below that our schemes can accomplish the cooling of the above systems in an efficient and robust way.

FIG. 1: (Color online) (a, b) Schematic illustrations of a single nNV center levitated in an optic trap and attached at the head of a cantilever. A magnetic field gradient is required to couple the internal state of the nNV center to its own vibration in (a) or to the cantilever’s vibration in (b). (c) The level configuration employed in the asymmetric cooling method. The two levels \( \pm 1 \) are coupled to the excited level \( |A_2 \rangle \) by a pair of laser beams with equal Rabi frequencies \( \Omega \) but different detunings \( \Delta_\pm \). A microwave resonantly couples the lowest state \( |0 \rangle \) to \( |+1 \rangle \) with a Rabi frequency \( \tilde{\Omega}_g \). (d) The level configuration employed in the symmetric cooling method. The two levels \( \pm 1 \) are coupled to \( |A_2 \rangle \) by a pair of laser beams with Rabi frequency \( \Omega \) and detuning \( \Delta \), and to \( |0 \rangle \) by a pair of microwaves with Rabi frequency \( \Omega_g \) and detuning \( \Delta_g \).

The nNV center owns multiple levels and a small surface area, which ensure it to be operated more flexibly and with lower heating from the environment than other solid-state candidates. Besides, the internal electronic states of the nNV center and the corresponding vibrational states can be coupled strongly under a modest magnetic field gradient (10^6 T/m) [22]. Provided that the trapping frequency of the nNV center (or the vibrational frequency of the cantilever) is \( \omega_k \) and a magnetic field gradient along the nNV center axis couples the nNV electron spin \( (S = 1) \) to its vibration, such a system can be described in units of \( \hbar = 1 \) as \( H_0 = \omega_k a^\dagger a + \delta_B(|+1\rangle\langle+1| - |−1\rangle\langle−1|) + \lambda(a^\dagger + a)(|+1\rangle\langle+1| - |−1\rangle\langle−1|) \), where \( a (a^\dagger) \) is the annihilation (creation) operator of the vibration, \( \delta_B = g_e\mu_B B(0) \) is the energy difference between \( | \pm 1 \rangle \) induced by the magnetic field and the coupling strength is \( \lambda = g_e\mu_B B'(0) \) with the zero-point fluctuation amplitude \( \delta_B = \sqrt{\hbar/(2M\omega_k)} \), the mass of the NV center \( M \), the Lande factor \( g_e \), the Bohr magneton \( \mu_B \), and the magnetic field gradient \( B'(0) \) along the nNV center axis. The ground state \( |m_\sigma \rangle \) with \( m_\sigma = 0, \pm 1 \) represents the Zeeman sublevels of the spin \( S = 1 \). Due to intrinsic spin-spin coupling properties, there exists a zero-field splitting of 2\( \pi \times 2.88 \) GHz between \( | + 1 \rangle \) and \( | 0 \rangle \). [23, 32].

The asymmetric cooling method. - The nNV center is driven by external light fields as in Fig. 1c). To carry out the cooling scheme, we first employ a dark state, which, under the condition of \( \Delta = \Delta_+ + \Omega_g/2 \), is \( |d \rangle = (|+1 \rangle \langle +1| - |−1\rangle\langle−1|)/\sqrt{3} \). Correspondingly, we define the states \( |b \rangle = (|+1 \rangle \langle +1| + |0 \rangle \langle 0| - 1)/\sqrt{6} \) and \( |Y \rangle = (|+1 \rangle \langle +1| - |0 \rangle \langle 0|)/\sqrt{2} \). The steady state, obtained from the first-order expansion of the parameter \( \eta = \lambda/\omega_k \) [16] can be written as

\[
|\varphi \rangle = |d \rangle |0 \rangle_n - \eta \sqrt{2}(|b \rangle - |Y \rangle)|1 \rangle_n,
\]

(1)

where \( |j \rangle (j = 0, 1) \) denotes the phonon state. Normalization factor is omitted in Eq. 1 for simplicity, but considered in calculations below. The effective Hamiltonian contributing to the cooling is \( H^a = H_f + H_{\text{EIT}} + H_{\text{Stark}} \) with

\[
H_f = \omega_k a^\dagger a - \Delta_\pm |A_2 \rangle \langle A_2|,
\]

\[
H_{\text{EIT}} = \frac{\sqrt{6}\Omega}{4} \sigma_x A_{2,b}^a + \frac{\sqrt{2}}{2} \lambda(a + a^\dagger) \sigma_x b^a d^a,
\]

\[
H_{\text{Stark}} = -\Omega_g |Y \rangle \langle Y| + \frac{\sqrt{2}\Omega}{4} \sigma_x A_{2,Y}^a + \frac{\sqrt{6}}{6} \lambda(a + a^\dagger) \sigma_x d^a Y,
\]

where \( \sigma_x^{i,j} = |\langle i | k \rangle + |\langle j | k \rangle| \) with \( j, k = A_2, Y, d, b \). \( H_f \) denotes the free energy term, and \( H_{\text{EIT}} \) and \( H_{\text{Stark}} \) are terms employed in the EIT cooling and the Stark-shift-gate cooling, respectively. So it is clear that the asymmetric cooling method works with a combination of the EIT cooling and the Stark-shift-gate cooling. For the steady state \( |\varphi \rangle \), we have \( H^a |\varphi \rangle = \sqrt{3} \eta (\omega_k/3 - \Omega_g)|Y \rangle |1 \rangle_n \), which remains invariant under the condition

\[
3 \Omega_g = 4 \omega_k.
\]

(3)

This condition, also called Stark-shift-gate point, implies that all the terms in \( H^a \) act on the steady state by a destructive interference and thus the steady state does not suffer from any loss due to spontaneous emission.

The symmetric cooling method. - With respect to the asymmetric cooling method, we apply an additional microwave to couple the ground state \( |0 \rangle \) to the ground state \( | - 1 \rangle \), as shown in Fig. 1d). The level configuration in this case is actually a combination of the \( \Lambda \)-type and \( \Gamma \)-type structures, which looks graphically symmetric. In this case, it is easily proven that \( |D \rangle = (| + 1 \rangle \langle + 1| - | - 1 \rangle \langle - 1|)/\sqrt{2} \) is the dark state under the double resonant conditions. The Hamiltonian here is written as
$H^s = H^s_0 + H_{\text{Stark}} + H'_{\text{Stark}}$, with
\[ H^s_0 = \omega_k a^\dagger a + \lambda(a + a^\dagger)(|B\rangle \langle D| + |D\rangle \langle B|), \]
\[ H_{\text{Stark}} = -\Delta(|A_2\rangle \langle A_2| - |B\rangle \langle B|) + \frac{\sqrt{2}\Omega}{2} \sigma_x a^2 B, \]
\[ H'_{\text{Stark}} = (\Delta_0|0\rangle \langle 0| - \Delta|B\rangle \langle B|) + \frac{\sqrt{2}\Omega}{2} \sigma_x^0 B, \] (4)
where the state $|B\rangle = (|+1\rangle + |-1\rangle)/\sqrt{2}$. Eq. (4) indicates that the symmetric cooling method works with two Stark-shift-gate cooling processes collaboratively. The first-order expansion of $\eta$ for the steady state $|\phi\rangle$ (omitting the normalization factor) [37] is $|\phi\rangle = |D\rangle|0\rangle_n - \sqrt{2}\omega_k/\Omega_0|1\rangle_n$. Similar to the asymmetric cooling method, we can have a simplified expression from $H^s$ applied on $|\phi\rangle$, $H^s|\phi\rangle = -\sqrt{2}\omega_k/\Omega_0(\omega_k + \Delta_0)|0\rangle|1\rangle_n$ which is invariant once the detuning $\Delta_0$ meets the condition of Stark-shift-gate point,
\[ \Delta_0 = -\omega_k. \] (5)

The cooling effects. - We first solve the cooling coefficients by the fluctuation spectrum, which is given by [37]
\[ S(\omega) = \frac{1}{2M\omega} \int_0^\infty dt e^{i\omega t} \langle F(t) F(0) \rangle_{ss}, \] (6)
where $\langle \cdots \rangle_{ss}$ implies the expectation value over the steady state. In the Schrödinger representation, the operator $F = -(d/dz)H_{\text{int}}|z=0$ with $z = z_0(a + a^\dagger)$ and $H_{\text{int}} = \lambda(a + a^\dagger)(|+1\rangle \langle +1| - |1\rangle \langle -1|)$. For the steady state $|\varphi\rangle$ in the asymmetric cooling scheme, we find two forces, $F_{\text{EIT}} = -\eta \omega_k/\sqrt{2} \sigma_x^{0,b}$ and $F_{\text{Stark}} = -\eta \omega_k/\sqrt{2} \sigma_x^{D,d}$, contributing to the cooling. These two forces split the fluctuation spectrum $S^a(\omega)$ into three parts, that is, $S_{\text{EIT}}(\omega)$, $S_{\text{Stark}}(\omega)$ and their interaction term $S_{\text{int}}(\omega)$ [38]. The heating coefficient can be obtained by the fluctuation spectrum as $A^s = 2\text{Re}[S^s(-\omega_k)] = \Gamma n\omega_k^2 \Omega^2 (3\Omega_2 - 4\omega_k^2) / 24 |M(-\omega_k)|^2$ with $M(\omega) = (3\Omega_2^2/4 + 2\lambda \omega + 2\omega_k^2)(\Omega_2 + \omega) - \omega \Omega_k^2/4 + i\Gamma \omega(\omega + \Omega_2)$ [38]. Under the condition in Eq. (3), we have $A^s_{\text{EIT}} = 0$ and zero values in fluctuation spectra at the points $\omega/\omega_k = -1, 0$ (see Fig. 2 a), which means that quantum interference induced by the EIT and the Stark-shift gate fully eliminates heating from the first-order blue sideband and carrier transitions. At the detuning $\Delta = (3\Omega_2^2/\Omega_k) - \omega_k$, the cooling coefficient reaches the peak value
\[ A^s = 2\text{Re}[S^s(\omega_k)] = \frac{48\lambda^2 \Omega^2}{49\Gamma \omega_k^2}, \] (7)
which is similar to the square-law style of the cooling laser strength in the EIT cooling scheme [18].

Similar to the asymmetric cooling scheme, the fluctuation spectra in symmetric counterpart are obtained and plotted in Fig. 2 b. The cooling coefficient in this scheme reaches zeros if Eq. (5) is satisfied. When we choose $\Omega_2 = 2\omega_k$, a maximal cooling coefficient is obtained as
\[ A^s = 2\text{Re}[S^s(\omega_k)] = \frac{2\Gamma \lambda^2}{\Omega^2}, \] (8)
which evidently obeys the inverse square-law as in [18, 19].

FIG. 2: (Color online) (a) and (b) Fluctuation spectra for the asymmetric and symmetric cooling schemes, respectively. The dashed (solid) curves are plotted with $\Omega/\omega_k = 2.0(8.0)$ and $\eta = 0.1$.

FIG. 3: (Color online) (a) Sketch for working regions of typical cooling schemes in the change of $\lambda/\omega_k$ under the assumption of appropriate parameter values. (b) and (c) Dynamics of $(n(t))$ in execution of these schemes in different regions, where (b) $\eta = \lambda/\omega_k = 1/80$ with $\Omega/\omega_k = 5.0$ and (c) $\eta = 1/20$ with $\Omega/\omega_k = 1.5$. In each panel, the dashed-dotted, dotted, dashed and solid curves represent the asymmetric cooling, EIT [18], Stark-shift-gate [18] and the symmetric cooling schemes, respectively. Here we suppose zero-temperature environment and the decay rate of the internal levels to be $\Gamma = 150\lambda$. Other parameters are chosen to satisfy the destructive interference and the resonance condition. Values would be changed for a finite-temperature environment, but physical essence as expressed here remains.

To demonstrate the cooling effects more specifically,
we simulate the dynamics of the system under a zero-
temperature environment by the Lindblad master equation
dp/dt = L_0 \rho 
where
\[ L_0 = -i[H, \rho] + \sum_{i=\pm 1,0} \frac{\Gamma_i}{2} \mathcal{D}[i\gamma_i (A_2, \rho)] \]
with \( \mathcal{D}[\rho] = 2\alpha^2 \rho - \alpha^2 \rho - \rho^2 \). We find working
regions of different cooling schemes, as sketched in Fig. 3. Although the
asymmetric cooling scheme favors a weaker coupling (i.e., smaller values of \( \lambda \)), stronger lasers
(i.e., larger values of \( \Omega \)) are necessary in that case. As
plotted in Fig. 4, the asymmetric scheme can achieve the
lowest final phonon number among the four typical
schemes when \( \eta < 0.05 \). In contrast, if we have a stronger
coupling (e.g., with bigger magnetic field gradient), we
may achieve the cooling by the symmetric scheme using
weaker lasers (see Fig. 5). An evident reason for this
difference between the two schemes is due to different forms of \( A_- \) as in Eqs. (7) and (8) as well as the relation of the
cooling coefficient with the final phonon number, i.e., \( \langle n \rangle_{ss} \sim 1/A_- \) in the case of \( A_- = 0 \). Deeper physics
for this difference is reflected in different properties be-
tween the EIT cooling and the Stark-shift-gate cooling.
In the asymmetric scheme, the \( \Lambda \)-type structure plus the
Stark-shift gate constitutes an enhanced EIT cooling that
is steered to a better cooling rate under the condition of
\( \Omega \sim \sqrt{\Gamma \omega_k/\eta} \). In contrast, the symmetric cooling is essen-
tially an enhanced Stark-shift cooling, in which the
better cooling occurs if \( \Omega \sim \sqrt{\eta/\Gamma \omega_k} \). Since \( \eta \) is always
smaller than 0.5, stronger lasers are definitely required in the
asymmetric cooling scheme. Due to the differences in physics, once we switch from the asymmetric scheme to
the symmetric one by applying an additional microwave, although we may have a sudden transition from \( H^a \) to
\( H^s \), cooling rate would significantly drop down unless other conditions, such as the appropriate laser strength,
are also satisfied.

Under a realistic circumstance, the nNV center is in-
fluenced by a finite-temperature environment, where the phonon occupations in the vibrational degrees of freedom satisfy Boltzmann distribution as \( N(\omega_k) = \frac{1}{\exp(\omega_k/T) - 1}^{-1} \) in units of \( k_B = 1 \). As such, an additional Lindblad operator \( L_1 \) should be involved in the Lindblad master equation with
\[ L_1 \rho = \frac{N(\omega_k)\gamma_k}{2} \mathcal{D}[b^\dagger, \rho] + \frac{[N(\omega_k) + 1]\gamma_k}{2} \mathcal{D}[b, \rho]. \]
Considering the vibrational decay rate \( \gamma_k = \omega_k/Q \) with the quality factor \( Q \), the final average number
of phonons \( \langle n \rangle_{ss} = [A_+ + N(\omega_k)\gamma_k]/(W + \gamma_k) \) with the cooling rate \( W = A_- - A_+ \) can be obtained by solving the rate equation \ref{eq:rate_equation}. The term \( N(\omega_k)\gamma_k \)
turns to be most important in determining \( \langle n \rangle_{ss} \) when \( A_+ = 0 \). Ideally, the final average phonon number is \( \langle n \rangle_{ss} = A_+/W \), which vanishes if \( A_+ = 0 \).

Robustness. - A working scheme for cooling should be highly robust to the parameter fluctuation, which is es-
tential to experimental implementation. In our case, the
deviations from the conditions of Eqs. (3) and (5) yield
second-order effects away from the ideal final average
phonon numbers, i.e., \( \delta_{\langle n \rangle_{ss}} \propto (\delta_{\Omega_{ss}})^2 \) and \( \delta_{\langle n \rangle_{ss}} \propto (\delta_{\omega_s})^2 \). In contrast, the EIT cooling and the Stark-shift-
gate cooling are more sensitive to such deviations, whose
effect is reflected in the first-order expansion. Further
considering the inaccuracy of \( \Omega \) in our schemes, we find \( \delta_{\langle n \rangle_{ss}} \propto \delta_{\Omega_{ss}}(\delta_{\Omega_{ss}})^2 \) and \( \delta_{\langle n \rangle_{ss}} \propto \delta_{\Omega_{ss}}(\delta_{\omega_s})^2 \), i.e., only the
third-order effect in the cooling.

Experimental feasibility. - For a nNV center levitated in an optic trap \cite{23, 25, 26}. the diameter of the nNV sphere is about 20 - 200 nm, the vibrational frequency is \( \omega_k/2\pi \approx 100 \sim 500 \text{ kHz} \), and the magnetic field gradient \( G \approx 10^3 \sim 10^5 \text{ T/m} \) and the coupling strength \( \lambda/2\pi \leq 100 \text{ kHz} \) are available. Moreover, the mechanical quality factor \( Q \) is relevant to the pressure of the vacuum and can be as high as \( 3 \times 10^{12} \) for a pressure \( P = 10^{-10} \text{ Torr} \) \cite{41}. The decay from \( |A_2 \rangle \) is \( \Gamma/2\pi = 15 \text{ MHz} \) and the environment can be at room temperature \( 20 \text{ K} \) or cryogenic (such as 1 mK) \cite{23}. For a specific calculation, we choose \( \omega_k/2\pi = 500 \text{ kHz} \) and \( \lambda/2\pi = 50 \text{ kHz} \), implying with the working region of the symmetric cooling scheme. Provided that the environment is at room temperature \( T=300 \text{ K} \), \( Q = 10^{10} \) and \( \Omega/2\pi = 1.5 \text{ MHz} \), we may achieve \( \langle n \rangle_{ss} \approx 0.027 \) after cooling for 100 \( \mu \text{s} \), better than the cooling by the Stark-shift-gate scheme (only reaching \( \langle n \rangle_{ss} \approx 0.082 \)).

With respect to the symmetric cooling scheme, the asym-
tmetic scheme favors a smaller \( \eta \) and meanwhile saves a microwave irradiation. For a nNV center at-
tached at the end of a cantilever, since the vibrational frequency of the cantilever is usually of the order of few
MHz or larger, the asymmetric scheme applies to such
a case. Provided that the environmental temperature \( T=20 \text{ mK} \), \( \omega_k/2\pi = 8 \text{ MHz} \), \( Q = 10^9 \), \( \lambda/2\pi = 500 \text{ kHz} \) and \( \Omega/2\pi = 40 \text{ MHz} \), we may cool the cantilever’s vibration by the asymmetric scheme down to \( \langle n \rangle_{ss} \approx 0.013 \) after cooling for 90 \( \mu \text{s} \).

Discussion. - Two special characteristics of the NV cen-
ter, as the difference from the atoms’, should be men-
tioned. First, there is a leaking channel out of the four-
level configuration employed in Fig. \ref{fig:schematic}(c, d), i.e., from \( |A_2 \rangle \) to \( |1 \rangle \), and finally down to \( |0 \rangle \). For the three-level structure involving \( | \pm 1 \rangle \) and \( |A_2 \rangle \), a pumping from \( |0 \rangle \) back to \( |1 \rangle \) is required in order to accomplish the desired cooling \cite{18}. In contrast, there is no need to additionally consider such a pumping in the present schemes since \( |0 \rangle \) is involved in the four levels. Second, the detrimental in-
fluence from nuclear spin bath of \(^{14} \text{N} \) and \(^{13} \text{C} \) should be seriously considered in the nNV center. This unexpected effect is neglected in above treatment, but actually leads to slight deviation from the dark-state condition and to inefficiency in removing the first-blue sideband transition. Simple estimate involving the nuclear spin noise can be found in the Supplementary Material \cite{38}. It is evident that this noise is beyond the scope of our schemes and should be suppressed by other approaches \cite{42, 43}.
on quantum interference enhanced by the Stark-shift gates, can achieve a very low final average phonon number in some hot-topic systems with nNV phonon centers. To the best of our knowledge, the schemes are the first proposal for cooling solid-state systems by completely eliminating heating effects from carrier transitions and the first-order blue sideband transitions. The two schemes favor slightly different conditions, and if employed appropriately, can be very general to achieve cooling for systems with four internal levels and a wide range of vibrational frequencies. Besides the two systems exemplified above, our cooling schemes can readily apply to other spin-vibration coupling systems involving four internal levels.

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