TRANSVERSE MOMENTUM DIFFUSION AND
BROADENING OF THE BACK-TO-BACK
DI-HADRON CORRELATION FUNCTION

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Abstract

We extend the Gyulassy-Levai-Vitev reaction operator approach to multiple elastic scattering
of fast partons traversing dense nuclear matter to take into account the leading power corrections
due to the medium recoil and to derive the change in the partons’ longitudinal momentum. We
employ a boost-invariant formalism to generalize previous treatments of the problem, which were
specific to the target rest frame. We apply the transverse momentum diffusion results in a simple
analytic model to evaluate the broadening of the back-to-back di-hadron correlation function in
d + Au reactions.

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I. INTRODUCTION

New experiments at the Relativistic Heavy Ion Collider (RHIC) will provide high statistics measurements of photons, leptons and hadrons at large transverse momentum ($p_T$) in high energy nucleus-nucleus collisions [1]. Particle production from a single hard scattering with momentum exchange much larger than $1/$fm should be localized in space-time. It is multiple parton scattering before or after the hard collision that is sensitive to the properties of the nuclear matter [2, 3, 4]. By comparing the high-$p_T$ observables [5, 6, 7] in $p + p$, $d + A$ and $A + A$ reactions, we are able to study the strong interaction dynamics of QCD in the vacuum, cold nuclear matter and hot dense medium of quarks and gluons, respectively.

In this Letter, we extend the Gyulassy-Levai-Vitev (GLV) reaction operator approach to multiple elastic scatterings [8] to take into account the longitudinal momentum reduction for a hard jet ($\sqrt{p^2}/p_0 \approx 0$) that propagates through dense nuclear matter due to the medium recoil. At the leading power approximation in $O(1/P)$ for a fast parton of momentum $p_\mu = (P, 0_\perp, P)$ elastic scattering in nuclear matter introduces transverse momentum broadening without changing its “+” light-cone component. By including leading power corrections due to the medium response, we evaluate the parton’s longitudinal momentum shift, $-\Delta p_\parallel = \langle p^2_\perp\rangle/2P$, that ensures 3D momentum conservation in an elastic collision up to corrections of $O((\langle p^2_\perp\rangle^2/P^2)$. In our derivation we introduce a boost invariant formulation of the problem of multiple jet interactions that generalizes the Gyulassy-Wang picture of static or massive scattering centers [9] employed in recent studies [3, 4, 8].

We apply the transverse momentum diffusion results in a simple analytic model to evaluate the nuclear modification of the back-to-back di-hadron correlation function $C(\Delta \phi) = N^{h_1,h_2}(\Delta \phi)/N_{tot}^{h_1,h_2}$ in $d + Au$ reactions at RHIC using cold nuclear matter transport coefficients [5] extracted from low energy $p + A$ data [10]. We find a small, weakly dependent on centrality, increase of the far-side ($\Delta \phi > \pi/2$) width $\sigma_{Far}$ of $C(\Delta \phi)_{dAu}$. Such behavior is distinctly different from the reported disappearance of the back-to-back correlations in central $Au + Au$ reactions [11], which can be understood in terms of strong final state radiative energy loss [12] and subsequent redistribution of the lost energy in the parton system [13].
II. MULTIPLE ELASTIC SCATTERING IN THE GLV REACTION OPERATOR APPROACH

The GLV reaction operator formalism was developed for calculating the induced radiative energy loss of hard quarks or gluons when they pass through a dense medium \[4\]. In this approach the multiparton dynamics is described by a series expansion in \( \chi = \int \sigma_{el}(z)\rho(z)dz = L/\lambda \), the mean number of interactions that a fast projectile undergoes along its trajectory. Each interaction is represented by a reaction operator that summarizes the unitarized basic scattering between the propagating system and the medium. The summation to all orders in \( \chi \) is achieved by a recursion of the reaction operators and is given in a closed form in \[4, 8\].

In this Letter we focus on the case of multiple elastic (no-radiation) scattering of a jet of momentum \( p \) in nuclear matter. Let \( M_0 = ie^{ip\cdot z_0}j(p)1_{dR\times dR} \) be the amplitude of the parton in color representation \( R \) of dimension \( d_R \) prepared at a position \( z_0 = [z_0^+, z_0^-, z_0^\bot] \), where the light-cone coordinates are defined as \( z^\pm = (z^0 \pm z^3)/\sqrt{2} \). The unperturbed inclusive distribution of jets in the wave packet is given by \[8\]:

\[
d^3N^i = \text{Tr} |M_0|^2 \frac{d^3\vec{p}}{(2\pi)^3 2p^0} = |j(p)|^2 d_R \frac{dp^+d^2p_\perp}{2p^+ (2\pi)^3} \ . \tag{1}
\]

For the instructive case of a normalized forward monochromatic beam (\( p_0 = p_\parallel \equiv P \))

\[
\frac{d^3N^i}{dp^+d^2p_\perp} \bigg|_{p^-=p_\perp^2} = \delta(p^+ - \sqrt{2}P)\delta^2(p_\perp) \ . \tag{2}
\]

In Eq.(2) the constraint on the parton’s “−” light-cone component from the \( p^2 = 0 \) on-shell condition is also shown.

In the presence of nuclear matter the multiple jet interactions are modeled by scattering in the presence of an external non-Abelian field \( V^{\mu,c}(q) \) which satisfies the condition: \( q_\mu V^{\mu,c}(q) = 0, \ c \) being the color index. When the parton energy is much larger than the typical momentum scale of the medium, \( P^2 \gg |q|^2 \), we have

\[
V^{\mu,c}(q) = n^\mu 2\pi \delta(q^+) V^c(q) e^{iq^\bot} , \quad g_s V^c(q) \equiv v(q) T^c(T) \ , \tag{3}
\]

where \( g_s \) is the strong coupling constant, the four-vector \( n^\mu = \delta^{\mu,\perp} = [0, 1, 0_\perp] \) and \( q^\mu = [q^+ = 0, q^-, q_\perp] \) is the momentum exchange with the medium. The phase in Eq.(3) keeps track of the position of the scatterer relative to a fixed space-time point, e.g. the hard collision vertex. The color matrix \( T^c(T) \) in Eq.(3) represents the non-Abelian charge of the
FIG. 1: In-medium interaction of fast almost on-shell \( p^2 \simeq 0 \) quarks and gluons with an external color field \( V^\mu(q) T^c \) located at position \( z \). The momentum flow for the elastic scattering subprocess is shown in the diagrams.

Field radiated by a quark (or a gluon) with \( T = \) fundamental (or adjoint) representation of \( SU_c(N) \) of dimension \( d_T \). The Fourier transform of the non-Abelian field is given by \( v(q) \) and, similarly to the Gyulassy-Wang model \([9]\), we employ the color-screened Yukawa type but with Lorentz boost invariance:

\[
v(q) \equiv \frac{4\pi\alpha_s}{-q^2 + \mu^2} = \frac{4\pi\alpha_s}{q_{\perp}^2 + \mu^2} = v(q_{\perp}),
\]

where we have used the \( q^+ = 0 \) choice of frame. This specific form of \( v(q) = v(q_{\perp}) \) is particularly useful since in-medium interactions in both hot and cold nuclear matter are of finite range \( r_{int.} = \mu^{-1} \) and we shall assume that \( \lambda\mu \gg 1 \), where \( \lambda \) is the parton’s mean free path.

Figure (a) represents the simplest case of a quark or gluon jet with a large forward momentum \( p^+ \) scattering in the medium. For initial state interactions, the elastic scattering amplitude for the subprocess with momentum flow illustrated in Fig. (a) is given by:

\[
M^{\alpha}_{cl} = \int \frac{d^4q}{(2\pi)^4} \frac{1}{2} \text{Tr} \left[ \frac{ig_s\gamma^\mu T^c(F)}{(p+q)^2 + i\epsilon} \right] V^\mu_{\perp}(q) V^\perp_{\perp}(q) e^{iq^- (z-z_0)^+} e^{-iq_{\perp}(z_{\perp} - z_{0\perp})} T^c(F) T^c(T),
\]

where \( T^c(F) \) is the color matrix in the quark representation. Before giving the corresponding matrix element for the gluon case in Fig. (b) we specify the use of the light-cone gauge \( A_{\mu} n^\mu = A^+ = 0 \), a “physical” gauge for a system moving very fast along the “+” direction.
In this gauge we evaluate the gluon scattering amplitude:

\[
M_{el}^q = \int \frac{d^4q}{(2\pi)^4} \left[ \left( -g_{s\alpha}q^\alpha \right) \left( g_{\mu\beta}(p - q)_\beta + g_{\mu\beta}(2q - p)_\alpha + g_{\beta\alpha}(-2p - q)_\mu \right) \frac{i}{(p + q)^2 + i\epsilon} \right] \\
\times \left[ -g^{\beta\gamma} + \frac{n^\beta(p + q)^\gamma + n^\gamma(p + q)^\beta}{(p + q) \cdot n} \right] \left( V_{\mu}(q)M_{2,\beta}(p + q)M_{1,\alpha}(p) \right) \\
\approx \int \frac{dq}{2\pi} \frac{d^2q_{\perp}}{(2\pi)^2} \left\{ \frac{2p^+ v(q_{\perp})}{(p + q)^2 + i\epsilon} \right\} e^{iq^- (z - z_0^+)} e^{-i\cdot(q_{\perp} \cdot (z_{\perp} - z_{0,\perp}))} T^c(R = F, A) T^c(T) \\
\times (M_{2,\alpha}M_{1}^\alpha) .
\]

In the derivation of Eq.(5) we have used the approximations \( |q_{\perp}| \ll p^+ \) and \((p \pm q)^\mu M_\mu(p) \approx 0\). The long range color interference has been neglected, allowing to factor out \( M_{1}^\alpha M_{2,\alpha} \), and \(-if_{abc} = [T^c(A)]_{ba}\) is the color matrix in the adjoint representation. We note that for small angle scattering of high-energy partons the amplitudes \( M_{el}^q, M_{el}^g \), Eqs.(5,6), differ only by a color factor.

The remaining \( q^- \) integral can be performed by closing the contour in the lower half-plane \((z^+ - z_0^+ < 0)\) and picking the contribution at \( q^- = q_{\perp}^2/2p^+ - i\epsilon\), corresponding to the pole in the propagator of the scattered parton of momentum \( p + q \). The factorizable elastic scattering amplitude becomes:

\[
M_{el}^{q,g} = -i\theta(z_0^+ - z^+) \int \frac{d^2q_{\perp}}{(2\pi)^2} v(q_{\perp}) e^{iq^- (z - z_0^+)} e^{-i\cdot(q_{\perp} \cdot (z_{\perp} - z_{0,\perp}))} T^c(R = F, A) T^c(T) .
\]

Similarly, for final-state elastic scattering with the “observed” parton of momentum \( p^\mu \), we get the same scattering amplitude except the pole of the \( q^- \) integration is in the upper half-plane, \( q^- = -q_{\perp}^2/2p^+ + i\epsilon\), and the argument of the \( \theta \)-function in Eq.(7) is reversed to \( z^+ - z_0^+ \).

In computing the elastic scattering cross section in matter one takes the average over the distribution of scattering centers in the transverse plane of area \( A_{\perp} \), \( 1/A_{\perp} \int d^2z_{\perp} \langle \cdots \rangle \), which diagonalizes the squared amplitudes in the \( q_{\perp}, q'_{\perp} \) variables [4, 8]:

\[
\langle e^{-i (q_{\perp} - q'_{\perp}) \cdot (z_{\perp} - z_{0,\perp})} \rangle_{A_{\perp}} \approx \frac{T(z_{0,\perp})}{N} (2\pi)^2 \delta^2(q_{\perp} - q'_{\perp})
\]

in the \( \sqrt{A_{\perp}} \gg \mu^{-1} \) limit. \( T(z_{0,\perp}) = \int dz \rho(z, z_{0,\perp}) \) is the Glauber thickness function at impact parameter \( z_{0,\perp} \) and \( 1/A_{\perp} = T(z_{0,\perp})/N \) with \( N \) being the number of scattering centers. The corresponding differential elastic scattering cross section per unit partonic scattering
(T(z_{0\perp})/N = 1) is found to be:

\[ \frac{d\sigma_{el}(R,T)}{d^2q_{\perp}} = \frac{C_2(R)C_2(T)}{d_A} \frac{|v(q_{\perp})|^2}{(2\pi)^2} = \frac{C_2(R)C_2(T)}{d_A} \frac{4\alpha_s^2}{(q_{\perp}^2 + \mu^2)^2} \]  

(9)

for a fast parton in color representation $R$ and the non-Abelian field radiated from a parton in color representation $T$. For example, the color factor $C_2(R)C_2(T)/d_A = 2/9, 1/2$, and $9/8$ for the scattering of $qq$, $qg$, and $gg$, respectively. A real nuclear medium can be a mixture of soft quarks and gluons with a “mean” color factor $\langle C_2(R)C_2(T)/d_A \rangle$. For $\mu = 0$, Eq.(9) is effectively the small angle, $t = -q_{\perp}^2$ and $u \approx s$, limit of the exact $qq \rightarrow qq$, $qg \rightarrow qg$ and $gg \rightarrow gg$ elastic parton-parton scattering cross sections.

In deriving Eq.(9) the $O(1/p^+)$ correction, $\propto (q_{\perp}^2/2p^+ - q_{\perp}^2/2p^+)$, from the phase factors in Eq.(7) vanishes due to Eq.(8), $\delta^2(q_{\perp} - q_{\perp}')$. Thus, interference effects for an energetic jet are largely suppressed, but they become important for soft gluon radiation [3, 4]. In the case of multiple elastic scatterings the proper inclusion of unitarity corrections and their derivation has been discussed in Ref. [8]. We present here the final result, noting that there is once again cancellation of the interference effects controlled by a phase $\propto (q_{\perp} + q_{\perp}')^2/2p^+$ due to the forward weight, $\delta^2(q_{\perp} + q_{\perp}')$, of the virtual scattering:

\[ \frac{d\sigma_{u.c.}(R,T)}{d^2q_{\perp}} = -\left[ \int d^2\xi_{\perp} \frac{C_2(R)C_2(T)}{d_A} \frac{|v(\xi_{\perp})|^2}{(2\pi)^2} \right] \delta^2(q_{\perp}) = -\sigma_{el} \delta^2(q_{\perp}) \]  

(10)
i.e. the total scattering cross section in the forward $q_{\perp} = 0$ direction.

The full solution for the momentum distribution of a jet that has traversed nuclear matter of longitudinal extent $L$ and opacity $\chi = L/\lambda = \rho L\sigma_{el} = T(z_{0\perp})\sigma_{el}$ can be obtained using the Gyulassy-Levai-Vitev reaction operator approach [8]:

\[ \frac{d^3N^f(p^+, p_{\perp})}{dp^+ d^2p_{\perp}} \bigg|_{p^- = \frac{p^2}{2p^+}} = \sum_{n=0}^{\infty} \frac{\chi^n}{n!} \int \prod_{i=1}^{n} d^2q_{i\perp} \left[ \frac{1}{\sigma_{el}} \frac{d\sigma_{el}(R,T)}{d^2q_{i\perp}} \left( e^{-q_{i\perp} \cdot \vec{v}_{p_{\perp}}} - 1 \right) \right] \times \frac{d^3N^l(p^+, p_{\perp})}{dp^+ d^2p_{\perp}} \bigg|_{p^- = \frac{p^2}{2p^+}} \]  

(11)

for the leading power in $q_{\perp}^2/P^2$. Eq.(11) shows that in the asymptotic limit, $q_{\perp}^2/P^2 \approx 0$, elastic multiple parton scattering in nuclear matter introduces transverse momentum broadening to the jet without changing its “$+$” momentum. The average “$-$” momentum component becomes $p^- = \langle p_{\perp}^2 \rangle/2p^+$ from the on-shell constraint but 3D momentum is not conserved: $|\vec{p}^2| = \langle p_{||}^2 \rangle + \langle p_{\perp}^2 \rangle \approx (P^2 - \langle p_{\perp}^2 \rangle/2) + \langle p_{\perp}^2 \rangle \neq P^2$.  
The constant “+” momentum component is a direct consequence of the leading power approximation, which results in the momentum constraint \( \delta(p'^+ - p^+) = \delta(q^+) \) in Eq. (3). We here focus on the \( q_\perp^2/P^2 \) power correction due to the medium recoil because it gives a leading contribution to the change of the fast parton’s “+” momentum. In a simple model of elastic jet scattering off a non-Abelian field radiated by a parton with momentum \( k' = k'^0 = (\sqrt{2}P)n' \) the correction from the medium response replaces the momentum constraint at the leading power by \( \delta(p'^+ - p^+ (1 - k'^+ / p^+)) \) for recoil momentum \( k'^+ \). We find \( k'^+ = (q\perp^2/(p + k)^2)p^+ = \frac{1}{2}(q\perp^2/\sqrt{2}P) \) if the scattering puts the recoil parton on-shell \((k'^2 = 0)\). By including this longitudinal momentum reduction Eq. (11) is modified to:

\[
\frac{d^3N_f(p^+, \mathbf{p}_\perp)}{dp^+d^2\mathbf{p}_\perp} \bigg|_{p^+ = \frac{p^2}{2p^+}} = \sum_{n=0}^{\infty} \frac{\chi^n}{n!} \int \prod_{i=1}^n d^2\mathbf{q}_i \int \prod_{i=1}^n \frac{d\sigma_{el}(R, T)}{d^2\mathbf{q}_i} \left( e^{-\mathbf{q}_\perp \cdot \mathbf{p}_\perp} e^{+\frac{1}{2}(q^2_\perp/\sqrt{2}P)\mathbf{q}_\perp^\perp} \right) \left( e^{-Q_i \cdot \mathbf{b}_\perp} e^{+Q_i \cdot \mathbf{q}_\perp^\perp} \right) \left( e^{-\mathbf{b}_\perp \cdot \mathbf{b}_\perp} e^{+\mathbf{b}_\perp \cdot \mathbf{q}_\perp^\perp} \right)
\]

For any initial jet flux, Eq. (1), the opacity series in Eq. (12) is most easily resummed in the impact parameter space \((b^-, \mathbf{b}_\perp)\) conjugate to \((p^+, \mathbf{p}_\perp)\). We substitute in Eq. (12) the illustrative example of an initial parton momentum distribution, Eq. (2), which gives

\[
\left[ \frac{d^3\tilde{N}_f}{dp^+d^2\mathbf{p}_\perp} \right] (b^-, \mathbf{b}_\perp) = \frac{e^{-\chi} e^{i\sqrt{2}Pb^-}}{(2\pi)^2} \frac{2\pi}{2\pi} \exp \left[ \chi \int d^2\mathbf{q}_\perp \frac{1}{\sigma_{el}} \frac{d\sigma_{el}}{d^2\mathbf{q}_\perp} e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} e^{+i\frac{1}{2}(q^2_\perp/\sqrt{2}P)b^-} \right]
\]

\[
\frac{d^3\tilde{N}_f}{dp^+d^2\mathbf{p}_\perp} \bigg|_{p^+ = \frac{p^2}{2p^+}}.
\]

While it is difficult to find a closed form for the integral in Eq. (13) even for simple forms of the differential elastic scattering cross section, the first correction for large longitudinal momentum can be evaluated by expanding \( e^{-i\frac{1}{2}(q^2_\perp/\sqrt{2}P)b^-} = 1 - i\frac{1}{2}(q^2_\perp/\sqrt{2}P)b^- + \cdots \). The azimuthal integral in Eq. (13) yields \( J_0(b\perp q_\perp) \) and the first two terms in the expansion of the exponent integrated over the Yukawa potential give:

\[
\int_0^\infty dq_\perp q_\perp^2 \frac{2\mu^2}{(\mu^2 + q^2_\perp)^2} J_0(b\perp q_\perp) = b\perp \mu K_1(b\perp \mu) ,
\]

\[
-\frac{1}{2} \frac{b^-}{\sqrt{2}P} \int_0^\infty dq_\perp q_\perp^3 \frac{2\mu^2}{(\mu^2 + q^2_\perp)^2} J_0(b\perp q_\perp) = \left( -\frac{1}{2} \frac{b^-}{\sqrt{2}P} \right) \mu^2(2K_0(b\perp \mu) - b\perp \mu K_1(b\perp \mu)) .
\]

\[
(14)
\]

The key to evaluating the average \( p\perp \)-broadening and the shift in the “+” momentum of the partons is the small \( b\perp \mu \) expansion in Eq. (14): \( b\perp \mu K_1(b\perp \mu) = 1 -
\[ (b_\perp^2 \mu^2 / 2) [\ln(2e^{-\gamma E / (b_\perp \mu)}) + 1/2] + \mathcal{O}(b_\perp^4 \mu^4), \quad K_0(b_\perp \mu) = \ln(2e^{-\gamma E / (b_\perp \mu)}) + \mathcal{O}(b_\perp^2 \mu^2). \]

Keeping terms \( \propto \chi \mu^2 \xi \), where \( \xi = \ln(2e^{-\gamma E / (b_\perp \mu)}) \gtrsim \mathcal{O}(1) \) we find:

\[
\left[ \frac{d^3 \bar{N}_f}{dp^+ d^2 \mathbf{p}_\perp} \right]_{b^-} (b_\perp, b_\perp) = \frac{1}{(2\pi)^3} e^{-\frac{1}{2} \chi \mu^2 \xi} b_\perp^2 e^{i \left( \sqrt{2} p^- - \frac{1}{2} \frac{2 \chi \mu^2 \xi}{\sqrt{2} P^2} \right)} b^- .
\]

(15)

Because of the power behavior of \( b_\perp^2 \), we treat \( \xi \) as approximately constant when Fourier transforming Eq.(15) back to momentum space:

\[
\frac{d^3 N_f}{dp^+ d^2 \mathbf{p}_\perp} \bigg|_{p^-=p^+_2} = \frac{1}{2\pi} e^{-\frac{1}{2} \chi \mu^2 \xi} \delta \left[ p^+ - \left( \sqrt{2} P - \frac{1}{2} \frac{2 \chi \mu^2 \xi}{\sqrt{2} P} \right) \right].
\]

(16)

The broadening of the parton beam, approximated in Eq.(16) by a Gaussian, induces a negative light-cone component via the poles of the projectile propagators (see e.g. Eqs.(5,6)) that ensure on-shellness at all intermediate stages. The power corrections from the target recoil lead to a reduction of the large “+” momentum component: \( p^+ \rightarrow p^+ - \langle p_\perp^2 \rangle / 2p^+ \), where \( \langle p_\perp^2 \rangle = \int d^2 \mathbf{p}_\perp \left( d^2 N_f / d^2 \mathbf{p}_\perp \right) / \int d^2 \mathbf{p}_\perp \left( d^2 N_f / d^2 \mathbf{p}_\perp \right) = 2 \chi \mu^2 \xi \) is evaluated from Eq.(16). We note that the recoil power corrections to \( p^- \) are \( \mathcal{O}(\langle p_\perp^2 \rangle^2 / (p^+)^2) \) and hereby neglected. To the order to which we computed the final state distribution \( \langle p_\perp^2 \rangle = 2 \chi \mu^2 \xi \), \( \langle p^+ \rangle = \sqrt{2} P - \frac{1}{2} \frac{2 \chi \mu^2 \xi}{\sqrt{2} P} \) and \( \langle p^- \rangle = \frac{1}{2} \frac{2 \chi \mu^2 \xi}{\sqrt{2} P} \). We emphasize that proper inclusion of the target response ensures \( p^0 = P, \sqrt{p^2} = \langle p^2 \rangle = \langle p_\perp^2 \rangle = P^2 \) energy and 3D momentum conservation in our final result Eq.(16).

One important consequence of the formalism that we have presented is the longitudinal momentum backward shift:

\[
- \frac{dp^\parallel}{dz} \approx - \frac{\Delta p^\parallel}{L} = \frac{\mu^2 (2 \xi)}{\lambda_{q,g} \sqrt{2} p^\parallel} = \left( \frac{\mu^2}{\lambda_{q,g}} \right)_{\text{eff}} \frac{1}{2p^\parallel},
\]

(17)

that couples to the transverse momentum broadening, and may mimic small elastic energy loss if the full structure of \( d^3 N_f / dp^+ d^2 \mathbf{p}_\perp \) is not observed. We note that the results on elastic transverse momentum diffusion presented in this Section are rather general, for example, the same expression is obtained for electron scattering via photon exchange or the scattering of a fast nucleon in nuclear matter via pion exchange.
III. APPLICATION TO THE BROADENING OF THE BACK-TO-BACK DI-
HADRON CORRELATION FUNCTION

As an application of the multiple initial and final state elastic scattering formalism elaborated here we consider the nuclear induced broadening of the back-to-back jet correlations associated with hard QCD \( ab \to cd \) partonic subprocesses. We will limit the discussion to the Gaussian 2D random walk approximation, Eq.(16), to make use of its additive dispersion property. Beyond this approximation the power law corrections to the tails of the distribution can be numerically evaluated as in [8].

For \( p + p \) collisions discussion of the jet acoplanarity resulting from vacuum radiation is given in [15]. Experimental test of the theoretical estimates is performed through measurements of the near-side and away-side di-hadron correlations in a plane perpendicular to the collision axis [11, 16]. In the absence of soft gluon bremsstrahlung and deviations from the double collinear pQCD approximation the back-to-back leading hadrons are expected to be coplanar. Two major effects contribute to the observed acoplanarity in \( p + p \): the non-perturbative fragmentation process, where final state hadrons pick up momentum \( j_T \) perpendicular to the thrust axis, and the \( k_T \) dependence in the unintegrated parton distributions \( \phi_{\alpha/h}(x, k_T) \) [17]. Phenomenologically extracted \( \langle k_T^2 \rangle_{pp} \) vacuum broadening can be implemented in the pQCD hadron production formalism as in [15, 18] via a normalized 2-dimensional Gaussian. The \( j_T \) dependence of \( D_{h/\alpha}(z, j_T) \) has not been discussed in the context of single inclusive particle production.

Measurements of intra-jet correlations find an approximately Gaussian jet cone shape. If one defines \( \langle |j_{Ty}| \rangle \) to be the average particle transverse momentum relative to the hard scattered parent parton in the plane normal to the collision axis, it can be related to the width \( \sigma_{\text{Near}} \) of the near-side \( (\Delta \phi < \pi/2) \) di-hadron correlation function \( C(\Delta \phi) \approx \frac{N_{h_1,h_2}}{N_{\text{tot},h_1}h_2} \) as follows: \( \langle |j_{Ty}| \rangle = \langle |p_T| \rangle \sin(\sigma_{\text{Near}}/\sqrt{\pi}) \). Results from the CCOR collaboration (consistent with preliminary PHENIX measurements) [16] give a value for \( \langle |j_{Ty}| \rangle \approx 400 \text{ MeV} \) that is roughly independent of \( \sqrt{s} \) and the \( p_T \) of the trigger particle. This allows for the approximate separation of short and long distance dynamics in QCD in accord with the factorization theorem [19] for inclusive hadron production with momentum
exchange much larger than $\langle |j_{Ty}| \rangle$ and $\langle |k_{Ty}| \rangle$:

$$E \frac{d\sigma^h}{dp^T} \propto \sum_{ab,c} \phi(x_a, k_{Tb}) \otimes \phi(x_b, k_{Tb}) \otimes \frac{d\sigma^{ab\rightarrow cd}}{dt} \otimes D_{c/h}(z_c, j_T) \; ,$$

(18)

where "$\otimes$" denotes standard integral convolution over the parton momentum fractions. Moreover, a lack of statistically significant broadening of the near-side di-hadron correlations in $Au + Au$ can be interpreted as a signal for fragmentation sufficiently outside of the interaction region where the hot and dense quark-gluon plasma [14] is expected to be formed. The reported consistency in the shape of the near-side peak of $C(\Delta \phi)$ in $p + p$, $d + Au$, and $Au + Au$ for $p_T \gtrsim 2$ GeV [11, 16] puts strong constraints on non-fragmentation and short formation time hadronic scattering models [20] that, similarly to hydrodynamics [21], are free of jet-like correlations.

The out of trigger plane momentum component of the far-side correlated hadron is given by [15]:

$$\langle |p_{Tout}|^2 \rangle = \langle |j_{Ty}|^2 \rangle + x_E^2 \langle |j_{Ty}|^2 \rangle^2 + 2 \langle |k_{Ty}|^2 \rangle \; .$$

(19)

In Eq. (19) $x_E = -p_T^h \cdot p_{T\text{trig}}/|p_{T\text{trig}}|^2 \approx -\cos(|\Delta \phi|)$, where $\Delta \phi$ is the angle between the approximately back-to-back hadrons $h_1, h_2$ and $\langle |\Delta \phi| \rangle = \sqrt{2/\pi} \sigma_{\text{Far}}$. Expressing also $\langle |p_{Tout}| \rangle = \langle p_T^h \rangle \sin(|\Delta \phi|)$ and eliminating it in Eq. (19) for $\langle |\Delta \phi| \rangle$, $\langle p_T^h \rangle \approx \langle p_{T\text{trig}} \rangle = \langle |p_T| \rangle$ the following approximate relation is found:

$$\langle |k_{Ty}| \rangle = \langle |p_T| \rangle \cos \left( \frac{\sigma_{\text{Near}}}{\sqrt{\pi}} \right) \sqrt{\frac{1}{2} \tan^2 \left( \sqrt{\frac{2}{\pi}} \sigma_{\text{Far}} \right)} - \tan^2 \left( \frac{\sigma_{\text{Near}}}{\sqrt{\pi}} \right) \; .$$

(20)

In the presence of nuclear matter initial state (IS) and final state (FS) transverse momentum diffusion add a large $\langle \Delta k_T^2 \rangle \propto A^{1/3}$ term to the vacuum parton broadening [2, 8], as confirmed by Fermilab experiments on the nuclear $A$-dependence of di-jet acoplanarity [22]. For $d + Au$ reactions at RHIC in the IS before the hard collision only the partons from the incoming deuteron scatter multiply on the nucleus: $\langle \Delta k_T^2 \rangle_{IS} = (\mu^2/\lambda)_{eff} \langle L \rangle_{IS}$. The gluon scattering dominated transport coefficient $\mu^2/\lambda \simeq 0.14$ GeV$^2$/fm is constrained [5] from existing low energy $p + A$ data [10] and consistent with $\mu^2/\lambda_q = 0.047 \pm 0.035$ GeV$^2$/fm found in [23] when one takes into account that $\lambda_q/\lambda_g = C_A/C_R = 2.25$. After the hard collision vertex in the FS both outgoing jets scatter in the medium to acquire $\langle \Delta k_T^2 \rangle_{FS}$. Projection on the plane of measurement with which $k_{TFS}$ forms an angle $\alpha_{FS}$, $\langle \cos^2 \alpha_{FS} \rangle = 1/2$, reduces the FS effect by a factor of two. Comparison of nuclear broadening in Drell-Yan and di-jet
production indicate that the strength of FS scattering may be bigger in the IS\cite{22}. This can be modeled by $K_{FS} = (\mu^2/\lambda)_{effFS}/(\mu^2/\lambda)_{effIS}$. Lacking precise experimental data, we naturally choose $K_{FS} = 1$ to be the default value of this parameter. The total vacuum+nuclear induced broadening in the plane perpendicular to the collision axis in $d + Au$ is given by:

$$\langle k_T^2 \rangle = \langle k_T^2 \rangle_{vac} + 1_{jet} \left( \mu^2/\lambda \right)_{eff} \langle L \rangle_{IS} + 2_{jets} \left( 1/2 \right)_{projection} K_{FS} \left( \mu^2/\lambda \right)_{eff} \langle L \rangle_{FS}. \quad (21)$$

To relate the effective opacity (or mean number of scatterings) $\chi = \bar{n} = \langle L \rangle/\lambda_{eff}$ for the IS and the FS it is useful to look in the rest frame of the nucleus where the back-to-back midrapidity hard scattered partons move in the forward direction at an angle $\theta \approx \gamma^{-1}$ relative to the incident beam axis, $\gamma = E_N/m_N = \sqrt{s}/(2m_N)$ being the Lorentz boost factor. For RHIC $\gamma = 106$ and uniform distribution of the hard scatter in the nucleus $\chi_{IS} = \chi_{FS}$.

The estimate for $\langle |k_T| \rangle_{vac} = 0.75$ GeV per parton is taken from \cite{16} (shown by dot-dashed line in Fig. 2) and the di-hadron correlation function is approximated by near-side and far-side Gaussians. Such simplification does not take into account the large angle production...
FIG. 3: Elastic broadening of the back-to-back di-hadron correlation function in minimum bias and central \( d + Au \) reactions relative to the vacuum \( p + p \) result (here represented by a shaded area to guide the eye) for \( \langle |p_T| \rangle = 4 \text{ GeV} \). \( C(\Delta \phi) \) is modeled via near-side and far-side Gaussians with away-side width evaluated from Eqs. (20,21) and \( K_{FS} = 1 \). Scaled STAR correlation data \( [11] \) for \( \frac{1}{2}(h^+ + h^-) \) in \( p + p \) is also shown.

multiple scattering and fragmentation and the growth of the background in the intermediate \( \Delta \phi \sim \pi/2 \) region. The combined vacuum+nuclear matter induced IS and FS transverse momentum diffusion for minimum bias \( d + Au \) reactions is evaluated from Eq. (21). Noting that for two Cartesian components \( \langle |k_{Ty}| \rangle = \sqrt{\langle k_T^2 \rangle/\pi} \), in the top panel of Fig. 2 \( \sim 30\% \) enhancement of \( \langle |k_{Ty}| \rangle \) that measures the increased acoplanarity due to \( k_T \)-broadening is predicted. The lower panel illustrates the increased width, \( \sigma_{Far} \), of the away-side correlation function \( C(\Delta \phi)_{dAu} \) obtained as a solution to Eq. (20).

Figure 3 shows the change in the shape of the far-side di-hadron correlations from transverse momentum broadening and the coupled 20% reduction of their maximum strength (at \( \Delta \phi = \pi \)). In central \( d + Au \) reactions the effect of nuclear matter induced acoplanarity is slightly larger. We emphasize that even a significant increase in the opacity of the medium will lead to only a subtle change in \( \sigma_{Far} \) since its observable growth is no faster than \( \sqrt{\chi} \). Triggering on very high-\( p_T \) hadrons will bias the measurement toward quark jets and may lead to a reduction of the estimated broadening.

For \( Au + Au \) collisions one has to account for the IS broadening of both partons and replace
the FS diffusion in cold nuclear matter with quark-gluon plasma induced broadening, which for
the Bjorken expansion scenario \[21\] at midrapidity has the form:

\[
\langle \Delta k_T^2 \rangle_{FS} = C_2(R) \frac{3 \pi \alpha_s^2}{2} \frac{1}{A_\perp} \frac{dN^g}{dy} 2 \xi \ln \frac{\langle L_T \rangle}{\tau_0}, \quad C_2(R) = 4/3 \quad \text{(3) for quarks (gluons).}
\]

In Eq. (22) \(dN^g/dy\) is the initial gluon rapidity density, \(\langle L_T \rangle\) is the transverse size of the
medium and \(\tau_0\) is the initial equilibration time \[21\].

**IV. CONCLUSIONS**

We have extended the GLV reaction operator formalism to multiple elastic scatterings \[8\] to take into account the energy-momentum conservation for the incident parton flux and the
recoil of the target partons along the projectile path. The resulting coupling between the
forward and backward light-cone and transverse momentum components of the distribution
of jets that have traversed dense nuclear matter is non-negligible for small jet energies. We
have introduced a boost invariant formulation of the problem of multiple parton interactions
that generalizes the Gyulassy-Wang model of dense nuclear matter \[9\]. Relaxing the
assumptions of static or very massive scattering centers in the target simplifies the calcula-
tion, the transition between different reference frames and also allows to analytically include
corrections, accurate to \(O(1/p^+)\) in Eq. (16), to the leading power approximation.

In a practical application of the transverse momentum diffusion results we evaluate the
nuclear-induced acoplanarity of the hard-scattered parton pair, Eq. (21), and relate it to the
expected broadening of the back-to-back di-hadron correlation function in \(d + Au\) reactions
at RHIC. We find that scattering in cold nuclear matter, which results in a predicted small
20-30% Cronin enhancement \[5\], also leads to \(\sim 25 - 30\%\) increase of the far-side width
\(\sigma_{Far}\) of \(C(\Delta \phi)_{dAu}\) in minimum bias \(d + Au\) reactions for \(\langle |p_T| \rangle = 3 - 4\ GeV\). The centrality
dependence of \(\sigma_{Far}\) is shown to be weak with \(\sim 35 - 40\%\) growth of the far-side width in
central \(d + Au\) relative to \(p + p\). In summary, di-hadron correlations for scattering in cold
nuclear matter are predicted in Fig. 3 to be qualitatively similar to the \(p + p\) case.

*Note added in proof:* During the completion of this manuscript data on the nuclear
modifications of hadron production and back-to-back di-hadron correlations in \(d + Au\)
at \(\sqrt{s_{NN}} = 200\ GeV\) became available \[24\]. The moderate-\(p_T\) enhancement observed in
minimum bias \(d + Au\) versus the jet quenching established in central and semi-central
\(Au + Au\) are in quantitative agreement with the predictions in \[1, 24\]. \(C(\Delta \phi)_{dAu}\) and \(C(\Delta \phi)_{pp}\) have the same qualitative behavior in both the near-side and far-side regions. Further analysis of the experimental data is needed to clarify the centrality and rapidity dependence \[5, 25\] of the Cronin effect at RHIC and to quantify the broadening of the far-side width, \(\sigma_{\text{Far}}\), discussed here.

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