Abstract: This work presents the benefits of using an adaptive model predictive control approach for controlling an ESP-lifted oil well system. The idea behind the proposed scheme is the successive linearization of the plant nonlinear model and incorporate it into an infinite-horizon MPC formulation, which handles feasibility issues by considering slacked terminal constraints. Also, suitable use of a zone control strategy to deal with time-varying ESP operating envelope constraints (downthrust and upthrust) becomes the approach implementable in practice. Nonlinear plant-model mismatch scenarios, including simulation of unmeasured disturbances and the tracking of ESP operation economic targets, show the effectiveness of the proposed controller concerning different operating conditions of the oil production process equipped with ESP.

Keywords: Artificial Lift Methods, Electric Submersible Pump (ESP), Infinite-Horizon Model Predictive Control, Adaptive Control.

1. INTRODUCTION

Electric submersible pump (ESP) is an artificial lift method for oil wells that aims to provide additional pressure to the fluid for its elevation to the surface. The ESP system consists of a multi-stage centrifugal pump installed between hundreds to thousands of meters below the surface, providing an increase in pressure oil wells and making it possible to produce flow rates suitable for economic purposes. Therefore, this method is usually used when it is desired to produce significant volumes of oil (Liang et al., 2015).

The operation of the ESP consists of adjusting the pump rotational speed and the opening of the production valve (choke) at the wellhead that feeds the manifold. Moreover, the safe and stable operation of ESP-lifted oil wells is carried out by the so-called ESP operating envelope-like set (Takacs, 2009), which comprises time-variant constraints (upthrust and downthrust) that rely on the phase portrait related to the flow rate and head of ESP.

In the last years, some researchers have spent considerable efforts to develop algorithms of model predictive control (MPC) applied to ESP systems, given that they can systematically deal with multivariable features, system constraints, as well as economic targets. The milestone was the work of Pavlov et al. (2014), which focused on an MPC controller implementation in an ESP-lifted oil well test facility system. The controller formulation dealt with a step-response linear model, aimed at the tracking of the ESP intake pressure, while the minimization of the ESP power consumption was obtained by settling explicitly a production choke opening target in the objective function of the controller, along with the operational envelope constraints. Using a similar MPC formulation to the aforementioned one, Binder et al. (2014) investigated aspects of the implementation of an embedded MPC on a programmable logic controller, performing hardware-in-the-loop simulations. However, the ESP power consumption was minimized through the regulation of the motor current and the operational envelope constraints problem was replaced by the tracking of the ESP reference flow rate. Then, Krishnamoorthy et al. (2016) designed an MPC controller with the same control objectives as in Pavlov et al. (2014) but the system representation used in the controller was based on a linearized model obtained from a high fidelity simulator for ESP installations producing heavy viscous crude oil.

Aiming at control performance improvements, Binder et al. (2019) evaluated the beneficial effects of feed-forward actions using the MPC formulation described in Pavlov et al. (2014); Binder et al. (2014) by considering reservoir pressure-like measured disturbances of the ESP-lifted oil well system. Concerning the first practical implementation of an MPC controller in a real oilfield, the seminal work by Patel et al. (2019) tested the effectiveness of the linear model-based controller for different control structures, namely three sets of controlled variables using as
manipulated variables: rotational speed, choke valve opening, and ESP volt at 60 Hz (representing the volts/speed ratio). The successful implementation of the tested control strategies near nominal operating conditions (regions in which step-response-oriented linear models are identified) brought a power consumption saving between 10% and 20% in the oilfield operation.

In the above-mentioned papers, the authors focused only on conventional MPC approaches, and closed-loop system stability and feasibility are neglected. In this sense, Fontes et al. (2020) recently proposed the application of a stabilizing infinite-horizon based MPC (IHMPC) for oil production wells with ESP installations. The tracking for maximizing the ESP oil production is properly designed within an implementable target zone scheme, including explicitly the associated downthrust and upthrust constraints, besides the optimizing target related to the ESP intake pressure. This control zone scheme softens, only when necessary, the typical conflict among the output constraints of the ESP-lifted oil well system by the use of the slacked terminal constraints-type endpoint constraints, preserving the stabilizing properties of the IHMPC control law and making it implementable in practice as well.

Despite the advances in MPC strategies for ESP-lifted oilfield operations, the formulations studied so far work well near the operating conditions in which the models used in the controller are identified. The linear models vary significantly depending on, for instance, the choke opening. To work around this, Delou et al. (2019) proposed an adaptive MPC control law in such a way that widens the ESP-lifted oil production operating range with switching step-response linear models. The hindrance behind the aforementioned technique is the fact that it requires a careful adjustment in the tuning parameters of the adaptive strategy. Under this circumstance, we extend the work of Fontes et al. (2020) by considering a zone control IHMPC formulation with an adaptive strategy for oil production wells with ESP installations, which aims to expand the ESP-lifted oil production operating range. The model for controller prediction is obtained from the linearization at each instant of sampling of a nonlinear phenomenological model of the system.

2. THE ESP MODEL

Figure 1 describes a typical ESP-lifted oil well system, highlighting its main operating variables as well as ESP operating envelope-like limiting operating conditions. The downthrust and upthrust conditions are time-varying, which becomes the challenger system operation, whose use of MPC-type multivariable control strategies can aid in this realistic and critical scenario. Oil from the reservoir passes through the ESP, which provides additional pressure to the fluid, enabling the flow up to the surface installations. The operation of an ESP-lifted oil well usually has two degrees of freedom, namely the pump rotational speed \( f \) and the production choke valve opening \( z_c \). Also, this oil production process is subject to some typical operating disturbances, such as fluid composition, variations in manifold pressure \( p_{in} \), reservoir pressure \( p_r \), among others.

\[
\begin{align*}
\dot{p}_{wh} &= 1.54 \times 10^8 (q_p - q_c) \\
\dot{p}_{bh} &= 0.8584 (p_r - p_{bh}) - 3.7 \times 10^8 q_p \\
\dot{q}_p &= 5.02 \times 10^{-9} \left[ p_{bh} - p_{wh} - 6.30 \times 10^8 q_p^1.75 \
&+ 9.32 \times 10^4 (H - 1 \times 10^3) \right] \\
q_c &= 2 \times 10^{-3} z_c \sqrt{p_{wh} - p_{bh}} \\
p_{wh} &= p_{bh} - 1.85 \times 10^8 q_p^1.75 - 1.9 \times 10^6 \\
H &= 0.2664 f^2 + 133.69 f q_p - 1.41 \times 10^6 q_p^2 
\end{align*}
\]

where \( p_{wh}, p_{bh}, p_{mn} \), and \( p_r \) are the wellhead, bottom hole, intake and reservoir pressures, respectively; \( q_p \) and \( q_c \) are the average flow rate of the production column and flow rate in the production choke, respectively; and \( H \) is the pump head.

3. PROPOSED ADAPTIVE MPC SCHEME FOR ESP

The MPC-based solution proposed here for an ESP-oriented oil production process, as described in (1), has not yet been applied to this type of system. The idea behind the proposed control law is to design an adaptive scheme concerning an infinite-horizon MPC (IHMPC) formulation, hitherto unexplored in literature, which makes use of the successive update of an output predicted-oriented model (OPOM).

OPOM is a linear state-space model synthesized from an analytical expression of the step-response of the system, in symbols (González and Odloak, 2009):

Fig. 1. Typical scheme of an oil production process with an ESP installation.

Here, the third-order dynamic model proposed by Pavlov et al. (2014), including its parameter values and assumptions, will be used for dynamic simulation of the ESP-lifted oil well. The model written by a set of ordinary differential equations is summarized as follows.
In the state equation defined in (2), the state component \( x^s \) corresponds to the integrating poles produced by the incremental form of inputs, and \( x^{st} \) corresponds to the system modes. \( F^{st} \) is related to the system poles, while \( B^s \) and \( B^{st} \) correspond to the partial fraction expansion coefficients; more details can be found in (Odo10). In the proposed control scheme, therefore, OPOM will be updated at each time step \((A_k, B_k, C_k)\) from the nonlinear plant model linearization described in (1).

Aiming at the practical case, following Fontes et al. (2020), the adaptive IHMPC will deal with zone control to handle feasibility issues of the ESP-lifted oil well system, downthrust and upthrust constraints, and with optimizing target associated with more profitable ESP operations. Figure 2 sketches the proposed application, where the controlled variables are the pump head and intake pressure whereas rotational speed and choke valve opening are the manipulated variables.

The adaptive IHMPC control law aims, therefore, to solve the following infinite-horizon optimization problem at each time step \( k \):

**Problem 1.**

\[
\min_{\Delta u_k, y_{sp,k}} \sum_{j=1}^{n_u} \left[ \int_{t_k}^{t_{k+j}} \left( \bar{Q}_y (y_{sp,k} - y_{k+1})^2 + \bar{Q}_u \Delta u_k^2 \right) \right] \frac{\mathrm{d}t}{T_{cost}}
\]

subject to (2) and:

\[
\begin{align*}
\|u_{min} & \leq u(k - 1) + \sum_{j=0}^{m-1} \Delta u(k + j) \leq u_{max} \\
-\Delta u_{max} & \leq \Delta u(k + j) \leq \Delta u_{max}, \quad j = 0, \ldots, m - 1 \\
\|\Delta y_k, \Delta u_k \|_{\mathbb{R}^{n_y}} & \leq \bar{y}_{y,k} + \bar{u}_{u,k} \quad \bar{y}_{y,k} + \bar{u}_{u,k}
\end{align*}
\]

where \( m \) is the control horizon, \( \Delta u(k+j) \) are increments of manipulated variables and \( y(k+j) \) are predictions of controlled variables at time step \( k+j \) given the current information of the plant at time step \( k \); \( y_{sp,k} \) are artificial set-points of controlled variables within the zone control scheme; \( u_{min}, u_{max}, \Delta u_{max}, \Delta u_{min}, y_{max} \) and \( y_{min} \) are the constraints of manipulated variables, increments of manipulated variables, and controlled variables, respectively; \( \Delta u_k = [\Delta u(k)\top, \ldots, \Delta u(k+m-1)\top] \top \) is the vector of control actions; \( Q_y \in \mathbb{R}^{ny} \) and \( R \in \mathbb{R}^{nu} \) are weighting matrices of controlled \((ny)\) and manipulated \((nu)\) variables, respectively. Note that \( \Delta u_k, y_{sp,k}, \Delta y_k \) and \( \delta_y \) and \( \delta_u \) are the decision variables of the optimization problem, in particular, \( \delta_y \) and \( \delta_u \) are the slack variables in order to guarantee the feasibility of the controller in the realistic scenario of plant-model mismatch, \( S_u \) and \( S_y \) are weighting matrices of the slack variables; \( u_{tg} \) are input targets and \( Q_u \) is their respective weighting matrix. Since the prediction model (2) has integrating modes, terminal constraints ((5) and (6)) must be added to prevent the cost from becoming unbounded. In addition, the terminal cost \( T_{cost} \) is an essential component in order to make it the infinite prediction horizon-based MPC to be implementable in practice, where \( Q_u \) is the terminal weighting matrix calculated from the Lyapunov equation of the system, at each output from OPOM in the adaptive scheme, i.e.:

\[
\bar{Q}_u - (F_k^{st})^\top \bar{Q}_u F_k^{st} = (\Psi_k^{st})^\top Q_y \Psi_k^{st}.
\]

It is worth mentioning that the time-varying ESP operating envelope constraints, described here through the pump head \((H)\), are treated in the proposed zone control by (4), so that the output tracking corresponding to the intake pressure is easily achieved in this formulation by collapsing its lower and upper bounds to a given set-point, thus reducing to a conventional set-point tracking. About more profitable ESP operating conditions, one can define optimizing targets \( u_{tg} \) associated with the manipulated variables, e.g. \( u_{tg} = z_c = 100% \) on the production choke valve opening, which would be related to the power consumption minimization. As will be shown in the next section, in case of no use of optimizing target on the rotational speed, one can configure a zero-weight \((q_u(f) = 0)\) related to it. Furthermore, a state estimator is necessary to accommodate the model-plant mismatch existing between the plant described by model (1) and the artificial states \((x(k))\) from OPOM used internally in the proposed adaptive IHMPC controller.

### 4. SIMULATION RESULTS

This section presents the benefits of the proposed zone control-oriented adaptive IHMPC strategy, namely feasibility of the resulting optimization problem in a realistic scenario of constraint violation and performance tracking in different operating conditions, in a nonlinear ESP-lifted oil well system described as in (1). Its results are compared...
to ones obtained from the application of a conventional stabilizing IHMPC, Problem 1 based only on a linearized model, which is designed to operate in a typical point of maximization of oil production, namely at $z_c = 99\%$ and $f = 65\text{Hz}$. The simulated scenario starts from an initial condition related to the operational point $z_c = 50\%$ and $f = 50\text{Hz}$, and two set-point tracking cases are evaluated. The first set-point applied to intake pressure is defined a few far from the linearization region corresponding to the typical operating condition, namely 90 bar ($t \in [0, 200]\text{s}$), whereas the second one is set close to it, 42 bar ($t \in [200, 400]\text{s}$). Furthermore, some unmeasured disturbances are applied at time instants, $t = 100\text{s}$ and $t = 300\text{s}$. The tuning parameters of the IHMPC controllers was configured as: $m = 3$, $Q_y = \text{diag}([10, 1])$, $R = \text{diag}([1, 1])$, $Q_u = \text{diag}([0, 1])$, $S_u = \text{diag}([0, 100])$, $S_y = \text{diag}([1, 1] \times 10^6)$, $u_{\text{min}} = [35 \text{Hz}, 0 \%]$, $u_{\text{max}} = [65 \text{Hz}, 100 \%]$, $\Delta u_{\text{max}} = [0.5 \text{Hz}, 1 \%]$.

Figure 3 reveals that both controllers can lead the intake pressure to its first set-point, by decreasing the rotational frequency and choke valve opening. Nevertheless, since we are in a limiting operating condition that can occur in practice, and it is far from the assumed linearization point for the design of the linear model-based conventional IHMPC, this controller provides an oscillatory response whereas the proposed adaptive IHMPC remains stable in the desired operational condition even with an unmeasured disturbance. In fact, with respect to the rejection of unmeasured disturbances, the feasibility property of both controllers was achieved by virtue of a suitable set of slack variables. There, as the adaptive IHMPC uses less its slack variables, it can bring back the ESP pump to the desired envelope faster than the conventional IHMPC, thus showing its better performance. It is still worth mentioning concerning this operating condition that the target corresponding to $u_{\text{tg}} = 100\%$ (cf. Figure 6) cannot be reached owing to the prioritization ($Q_y \geq Q_u$) of both controllers is primarily the controlled variables (zone and set-point).

On the other hand, when the set-point is set to close to the linearization point used to obtain the model incorporated internally in the conventional stabilizing IHMPC, operating condition to be usually sought in oil production wells with ESP installations, it has a slightly better performance than the adaptive IHMPC ones concerning the intake pressure, keeping it in the desired operation point, even in a presence of unmeasured disturbance. This behavior is also observed about the target tracking related to the production choke. Note that at this operating condition both controllers can perform such a task quite well, and it is more evident when the controllers compensate the unmeasured disturbance injected at $t = 300\text{s}$, so that the available degrees of freedom, namely ESP motor rotational speed, suffer a significant change in its steady-state so as to maintain the choke valve opening in its target. Fig. 3. Dynamic of the intake pressure for both IHMPC controllers.

Although it would be expected the IHMPC controller based on only a linear model to have superior performance, even slightly, than the one corresponding to the adaptive IHMPC controller near the typical operating condition, the latter controller yielded a global better performance (all manipulated and controlled variables) because of its effectiveness in the rejection of disturbance case concerning the ESP head (see Figure 4). It is clearly seen in Figure 7, where one presents the cost-functions of both controllers, that the adaptive IHMPC cost function (global performance) reaches zero faster than the conventional IHMPC one. It shows therefore that the successive linearization used in the proposed adaptive MPC strategy (parameters update of the linear model) can also be as useful as a linear-based MPC controller operating near the nominal operating conditions.

Since it has shown the effectiveness of the proposed adaptive IHMPC strategy with respect to the linear IHMPC one, so that it can be used in a wide operating condition range, we will proceed now towards to evaluate its robustness in a scenario in which the plant is corrupted by measurement noises. Also, aiming at the economic tracking of the system, one evaluates the effect of not considering...
Slacks associated with the conventional IHMPC controller. 

Slacks associated with the adaptive IHMPC controller. 

(a) Slacks associated with the conventional IHMPC controller. 

(b) Slacks associated with the adaptive IHMPC controller. 

Fig. 5. Behavior of slack variables.

Fig. 6. Signal of the manipulated variables for both IHMPC controllers.

Simulating the same output tracking cases for intake pressure, initial condition, as well as unmeasured disturbances, as in the case earlier, but considering now measurement noises in the controlled variables (with Gaussian distribution $N(0, W)$, and $W = \text{diag}(\{1.1 \text{ bar}^2 \ 105.2 \text{ m}^2\}))$, inserted in the plant model (1), the results are summarized in figures 8, 9 and 10. The IHMPC$_{\text{target off}}$ formulation refers to the proposed adaptive control scheme with $q_u(z_c) = 0$. It is possible to see that both formulations can perform their respective tasks quite well even in the noisy scenario. The difference between them lies in the fact of (non)tracking the economic target, leading them to distinct steady-states. While the intake pressure reaches the same steady-states for both control formulations, the ESP head differs from each other under (non)compensating the choke valve opening target. If one considers the calculation of the ESP power consumption through $P = C_P P_0 \left( \frac{f}{f_0} \right)^3$, where $P_0$ and $f_0$ are the reference power and frequency and $C_P$ is the viscosity correction factor, it can be shown that the economic formulation (IHMPC$_{\text{target on}}$) yields a reduction of 5% in the ESP power consumption. In this way, as the proposed adaptive IHMPC strategy assures the feasibility of the optimization problem for any of its formulation, the use of the optimizing target should be then required, given that, from a practical standpoint, it brings the underlying economic benefits whenever possible.
5. CONCLUSION

This work presented an adaptive MPC strategy that has not yet been explored in oil production processes with ESP installations. The proposed control law uses successive linearization from the ESP-lifted oil well system model to update the model internally used in the proposed controller, which also considers optimizing targets and zone control. From the practical point of view, the resulting control scheme accommodates the guarantee of feasibility by considering slacked terminal constraints-type endpoint constraints, whereas the zone control approach handles successfully the downthrust and upthrust ESP envelope operating constraints by incorporating artificial set-points, thus assuring that the controller is implementable in practice.

The simulated results showed the practical benefits of the proposed adaptive IHMPC scheme when compared to the one formulation that uses only a step-response-like linear model, as has been usually used in literature. The linear model-based IHMPC achieves great performance near the nominal operating condition, as would be expected, whereas the adaptive controller can widen the ESP-lifted oil well-operating ranges, which is quite useful in several scenarios of perturbations that an oil production process is subjected, including measurement noises that were evaluated here as well.

Finally, given the feasibility guarantee of the proposed controller for different arrangements, the option of including economic targets (e.g., keeping the production choke valve as open as possible) should be tracked constantly.

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