Tachyon Solution in Cubic Neveu-Schwarz String Field Theory

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Abstract: A class of exact analytic solutions in the modified cubic fermionic string field theory with the $GSO(-)$ sector is presented. This class contains the $GSO(-)$ tachyon field and reproduces the correct value for the nonBPS D-brane tension.

Keywords: String Field Theory, Tachyon Condensation
1. Introduction

Tachyon condensation [1, 2] and rolling tachyon solutions [3, 4] attract a lot of attention in the last years, in particular, in a context of cosmological applications [5]. They are nonperturbative effects in string theory and it is reasonable to study these phenomena within string field theory (SFT) [6].

A characteristic feature of the tachyon condensation of fermionic string is that the tachyon belongs to the $GSO^-(\cdot)$ sector and the original superstring field theory has to be enlarged to include the $GSO^-(\cdot)$ sector [6, 8]. This concerns the cubic SSFT [6, 10] as well as the nonpolynomial SSFT [11].

Tachyon condensation within the level truncated method [12] for cubic bosonic SFT has been a subject of numerous studies [2, 13, 14, 15] and Sen’s conjecture has been supported with impressive precision.

Tachyon condensation within the level truncated method for cubic fermionic SFT has been investigated in [8, 16] and Sen’s conjectures have been proved up to few lower levels.

Condensation of some auxiliary fields in the cubic superstring SFT [8, 10] (i.e. in the theory containing only the $GSO^{(+)}$ sector) has been obtained in [17], where it has been interpreted as a supersymmetry breaking solution.

Several attempts have been performed to find analytical solutions to SFT. Finally, Schnabl found an analytical solution in the open bosonic Witten’s SFT and proved analytically the tachyon condensation [18]. His solution and related problems have been examined in a series of recent works and its algebraic structure has been studied in detail [19, 21, 22, 23, 24, 25]. Construction of lower D-branes solutions and the analytical proof of the relations between D-brane tensions are also a subject of a recent interest.

The purpose of this paper is to find an analytical tachyon condensation solution in the ABKM cubic Neveu-Schwarz SFT [8]. The important property of this solution is a presence of the tachyon field leaving in the $GSO^{(+)}$ sector. It occurs that only a part of the constructed solution contributes to the ABKM action and this part coincides with the solution of the cubic $GSO^{(+)}$ NS SFT constructed recently by Erler [27]. This fact explains a puzzle that the $GSO^{(+)}$ solution without any tachyon part reproduces the correct value for the nonBPS D-brane tension.

The paper is organized as follows. In Section 2 we remind notations. In Section 3 we present a formal construction which is a generalization of the pure-gauge solution of the bosonic case. In Section 4 we present the tachyon solution and discuss it in the algebraic formalism. In section 5 we calculate the action on the constructed solution.
2. Set up

The unique (up to rescaling of fields) gauge invariant action unifying the GSO(+) and GSO(−) sectors is \[ S[\Phi, \Psi] = -\frac{1}{g^2} \left[ \frac{1}{2} \langle \langle Y_{-2} | \Phi, Q \Phi \rangle \rangle + \frac{1}{3} \langle \langle Y_{-2} | \Phi, \Phi, \Phi \rangle \rangle + \frac{1}{2} \langle \langle Y_{-2} | \Psi, Q \Psi \rangle \rangle - \langle \langle Y_{-2} | \Phi, \Psi, \Psi \rangle \rangle \right]. \tag{2.1} \]

Here the factors in front of the odd brackets are fixed by the constraint of the gauge invariance. It is important for the following that \( \Phi \) is Grassman odd and \( \Psi \) is Grassman even. A variation of this action with respect to \( \Phi, \Psi \) yields the following equations of motions (we assume that L.H.S. is zero modulo a kernel of \( Y_{-2} \), see for details \[28\])

\[
Q \Phi + \Phi * \Phi - \Psi * \Psi = 0, \tag{2.2}
\]

\[
Q \Psi + \Phi * \Psi - \Psi * \Phi = 0. \tag{2.3}
\]

Let us note that this system of equations of motion admits a pure GSO(+) solution i.e. \( \Psi = 0 \) and \( \Phi = \Phi_+ \) is a subject of the following equation

\[ Q \Phi_+ + \Phi_+ * \Phi_+ = 0. \tag{2.4} \]

Just a solution of this equation of motion has been found in \[27\].

However if we take a pure GSO(−) solution, i.e. \( \Phi = 0 \), we have to assume that \( Q \Psi_- = 0 \) and \( \Psi_- * \Psi_- = 0 \).

3. Construction of Pure-gauge Solutions

3.1 Zero Curvature and Pure-gauge Solutions

Let us remind that in the bosonic SFT as well as in the cubic supersymmetric SFT the equations of motion have the form of the zero curvature and therefore a formal solution of the equation of motion \[2.4\] can be find in a pure-gauge form

\[ \Phi_+ = \Omega^{-1} * Q \Omega \tag{3.1} \]

In particular one can take

\[ \Phi_+ = Q \omega \frac{1}{1 - \omega}, \tag{3.2} \]

where we omit the \(*\) multiplication between string fields.

This construction can be checked explicitly.
3.2 Construction of Pure-gauge Solutions

We start with solving equations of motion (2.2), (2.3) perturbatively in a parameter \( \lambda \). Let us introduce \( \phi_n \) and \( \psi_n \) as follows:

\[
\Phi_\lambda = \sum_{n=1}^{\infty} \lambda^n \phi_n, \quad \Psi_\lambda = \sum_{n=1}^{\infty} \lambda^n \psi_n.
\] (3.3)

Equations (2.2), (2.3) take the form:

\[
Q\phi_n + \sum_{p+q=n} (\phi_p \phi_q - \psi_p \psi_q) = 0, \quad (3.4)
\]
\[
Q\psi_n + \sum_{p+q=n} (\phi_p \psi_q - \psi_p \phi_q) = 0. \quad (3.5)
\]

At the first order in \( \lambda \) we have:

\[
Q\phi_1 = 0, \quad Q\psi_1 = 0. \quad (3.6)
\]

A partial solution of (3.6) reads:

\[
\phi_1 = Q\phi, \quad (3.7)
\]
\[
\psi_1 = Q\psi. \quad (3.8)
\]

At the second order in \( \lambda \) we find\(^1\)

\[
Q\phi_2 + \phi_1 \cdot \phi_1 - \psi_1 \cdot \psi_1 = 0. \quad (3.9)
\]

For \( \phi_1, \psi_1 \) given by (3.7) and (3.8) one gets taking into account the grassman parities:

\[
Q\phi_2 + Q\phi \cdot Q\phi - Q\psi \cdot Q\psi = Q\phi_2 - Q(Q\phi \cdot \phi) - Q(Q\psi \cdot \psi) = Q(\phi_2 - Q\phi \cdot \phi - Q\psi \cdot \psi) = 0. \quad (3.10)
\]

To solve this equation we put:

\[
\phi_2 = Q\phi \cdot \phi - Q\psi \cdot \psi = 0. \quad (3.11)
\]

Following the same way we get for \( \psi_2 \):

\[
\phi_2 = Q\phi \cdot \phi + Q\psi \cdot \psi, \quad (3.12)
\]
\[
\psi_2 = Q\phi \cdot \psi + Q\psi \cdot \phi. \quad (3.13)
\]

Then, at the third order in \( \lambda \) we find

\[
\phi_3 = Q\phi \cdot (\phi^2 + \psi^2) + Q\psi \cdot (\phi \cdot \psi + \psi \cdot \phi),
\]
\[
\psi_3 = Q\psi \cdot (\phi^2 + \psi^2) + Q\phi \cdot (\phi \cdot \psi + \psi \cdot \phi). \quad (3.14)
\]

\(^1\)Here \( \cdot \) is Witten’s star \( \star \).

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Let us introduce new variables

\[ A_i = \phi_i + \psi_i, \]
\[ B_i = \phi_i - \psi_i \]

and

\[ a = \phi + \psi, \]  \hspace{1cm} (3.15)
\[ b = \phi - \psi, \]  \hspace{1cm} (3.16)

then

\[ A_1 = \phi_1 + \psi_1 = Q\phi + Q\psi = Qa, \]
\[ A_2 = \phi_2 + \psi_2 = Q\phi \cdot (\phi + \psi) + Q\psi \cdot (\phi + \psi) = Qa \cdot a, \]
\[ A_3 = \phi_3 + \psi_3 = Q\phi \cdot (\phi + \psi)^2 + Q\psi \cdot (\phi + \psi)^2 = Qa \cdot a^2, \]  \hspace{1cm} (3.17)
\[ A_4 = \phi_4 + \psi_4 = Q\phi \cdot (\phi + \psi)^3 + Q\psi \cdot (\phi + \psi)^3 = Qa \cdot a^3 \]

and so on.

So our partial solution reads:

\[ \phi_n + \psi_n = A_n = Qa \cdot a^{n-1}. \]  \hspace{1cm} (3.18)

If we put \( A_\lambda \equiv \Phi_\lambda + \Psi_\lambda, (B_\lambda \equiv \Phi_\lambda - \Psi_\lambda) \) one gets

\[ A_\lambda = \Phi_\lambda + \Psi_\lambda = \sum_{n=1}^{\infty} \lambda^n (\phi_n + \psi_n) = \sum_{n=1}^{\infty} \lambda^n A_n \]
\[ = \sum_{n=1}^{\infty} \lambda^n Qa \cdot a^{n-1} = \lambda Qa \cdot \frac{1}{1 - \lambda a}. \]  \hspace{1cm} (3.19)

Similarly

\[ B_1 = \phi_1 - \psi_1 = Q\phi - Q\psi = Qb, \]
\[ B_2 = \phi_2 - \psi_2 = Q\phi \cdot (\phi - \psi) - Q\psi \cdot (\phi - \psi) = Qb \cdot b, \]
\[ B_3 = \phi_3 - \psi_3 = Q\phi \cdot (\phi - \psi)^2 - Q\psi \cdot (\phi - \psi)^2 = Qb \cdot b^2, \]  \hspace{1cm} (3.20)
\[ B_4 = \phi_4 - \psi_4 = Q\phi \cdot (\phi - \psi)^3 - Q\psi \cdot (\phi - \psi)^3 = Qb \cdot b^3, \]

and we find:

\[ \phi_i - \psi_i = B_i = Qb \cdot b^{i-1}, \]  \hspace{1cm} (3.21)

that gives

\[ B_\lambda = \Phi_\lambda - \Psi_\lambda = \sum_{n=1}^{\infty} \lambda^n (\phi_n - \psi_n) = \sum_{n=1}^{\infty} \lambda^n B_n \]
\[ = \sum_{n=1}^{\infty} \lambda^n Qb \cdot b^{n-1} = \lambda Qb \cdot \frac{1}{1 - \lambda b}. \]  \hspace{1cm} (3.22)
Equations of motion (2.2) and (2.3) have the following nice form in terms of $A_\lambda$ and $B_\lambda$
\[
QA_\lambda + B_\lambda \cdot A_\lambda = 0,
QB_\lambda + A_\lambda \cdot B_\lambda = 0
\] (3.23)
and their “pure-gauge” solutions are
\[
A_\lambda = \lambda Q a \cdot \frac{1}{1 - \lambda a},
B_\lambda = \lambda Q b \cdot \frac{1}{1 - \lambda b}.
\] (3.24)

It is now straightforward to check that $A_\lambda$ and $B_\lambda$ given by (3.24) satisfy eq. (3.23).
Indeed, since
\[
Q \frac{1}{1 - \lambda b} = \lambda \frac{1}{1 - \lambda a} Q b \frac{1}{1 - \lambda b},
Q \frac{1}{1 - \lambda a} = \lambda \frac{1}{1 - \lambda b} Q a \frac{1}{1 - \lambda a},
\] (3.25)

$QA_\lambda$ and $QB_\lambda$ for $A_\lambda$ and $B_\lambda$ given by (3.19) and (3.22) read
\[
QA_\lambda = \lambda Q a \frac{1}{1 - \lambda a} = -\lambda^2 Q b \frac{1}{1 - \lambda b} Q a \frac{1}{1 - \lambda a} = -B_\lambda \cdot A_\lambda.
QB_\lambda = \lambda Q b \frac{1}{1 - \lambda b} = -\lambda^2 Q a \frac{1}{1 - \lambda a} Q b \frac{1}{1 - \lambda b} = -A_\lambda \cdot B_\lambda.
\] (3.26)

Turning back to the original variables we have the solutions:
\[
\Phi_\lambda = \frac{\lambda}{2} Q (\phi + \psi) \frac{1}{1 - \lambda (\phi + \psi)} + \frac{\lambda}{2} Q (\phi - \psi) \frac{1}{1 - \lambda (\phi - \psi)},
\] (3.27)
\[
\Psi_\lambda = \frac{\lambda}{2} Q (\phi + \psi) \frac{1}{1 - \lambda (\phi + \psi)} - \frac{\lambda}{2} Q (\phi - \psi) \frac{1}{1 - \lambda (\phi - \psi)}.
\] (3.28)

These solutions are parameterized by a set of two string fields $\{\phi, \psi\}$. It is instructive to consider two particular cases.

- $\psi = 0$. In this case $a = b$ and
  \[
  A_\lambda = \lambda Q \phi \frac{1}{1 - \lambda \phi} = B_\lambda,
  \] (3.29)
it gives
  \[
  \Phi_\lambda = \frac{1}{2} (A_\lambda + B_\lambda) = A_\lambda,
  \Psi_\lambda = \frac{1}{2} (A_\lambda - B_\lambda) = 0.
\] (3.30)
This is a solution of equations of motion (2.2) and (2.3) reduced by $\Psi = 0$ to pure "bosonic" form
\[
Q \Phi + \Phi^2 = 0.
\] (3.31)
The solution of this equation was built in [27], with $\phi = B_1^1 c_1 |0\rangle$. 

\[ \text{– 6 –} \]
\* \( \phi = 0 \). In this case

\[
A_\lambda = \lambda Q \psi \frac{1}{1 - \lambda \psi},
\]

\[
B_\lambda = -\lambda Q \psi \frac{1}{1 + \lambda \psi},
\]

and therefore

\[
\Phi_\lambda = \lambda^2 Q \psi \frac{\psi}{1 - \lambda^2 \psi^2},
\]

\[
\Psi_\lambda = \lambda Q \psi \frac{1}{1 - \lambda^2 \psi^2}.
\]

(3.32)

4. Tachyon Condensation Solution

4.1 Algebraic Construction of Solutions

Our starting formula for the tachyon condensation solution are given by \((3.27)\) and \((3.28)\). Now we have to take a choice for particular \(\phi\) and \(\psi\). From the level truncation method we know that the tachyon condensation solution starts from the tachyon field that is \[|\text{fermion tachyon}\rangle = \gamma_\frac{1}{2} |0\rangle. \]

(4.1)

In a similar way the bosonic tachyon condensation solution starts from the tachyon field that for the bosonic string is

\[|\text{boson tachyon}\rangle = c_1 |0\rangle. \]

(4.2)

In the boson string the construction of the Schnabl solution starts from \(18, 19\)

\[\phi_{\text{boson}} = B_1^T c_1 |0\rangle. \]

(4.3)

By the analogy with this solution we can start from

\[
\phi = B_1^T c_1 |0\rangle,
\]

(4.4)

\[
\psi = B_1^T \gamma_\frac{1}{2} |0\rangle.
\]

(4.5)

To construct solutions via formula \((3.27)\) and \((3.28)\) it is convenient to use the following equivalent representations

\[
\Phi_\lambda = \frac{\lambda}{2} \sum_{n=0}^{\infty} \lambda^n (Qa \cdot a^n + Qb \cdot b^n),
\]

(4.6)

\[
\Psi_\lambda = \frac{\lambda}{2} \sum_{n=0}^{\infty} \lambda^n (Qa \cdot a^n - Qb \cdot b^n).
\]

(4.7)

In what follows using formula \((4.6)\) and \((4.7)\) and initial fields \((4.4)\) and \((4.5)\) we are going to find an explicit form of \(\Phi_\lambda\) and \(\Psi_\lambda\).
To this purpose let us at first calculate \( Qa \) and \( Qb \):

\[
Qa = Q(\phi + \psi) = Q(B_1^L c_1 |0\rangle + B_1^L \gamma_1 |0\rangle) = Q(|0\rangle - B_1^R c_1 |0\rangle - B_1^R \gamma_1 |0\rangle)
\]

\[
= -QB_1^R c_1 |0\rangle - QB_1^R \gamma_1 |0\rangle = -K_1^R c_1 |0\rangle + B_1^R Qc_1 |0\rangle - K_1^R \gamma_1 |0\rangle + B_1^R Q \gamma_1 |0\rangle
\]

\[
= -K_1^R (c_1 + \gamma_1) |0\rangle - B_1^R (c_0 c_1 + \gamma_2 \gamma_1) |0\rangle + B_1^R (c_1 \gamma_2 - \frac{1}{2} c_0 \gamma_2) |0\rangle,
\]

(4.8)

here we use the equations from Appendix.

Next we find:

\[
Qb = Q(\phi - \psi) = Q(B_1^L c_1 |0\rangle - B_1^L \gamma_1 |0\rangle) = Q(|0\rangle - B_1^R c_1 |0\rangle + B_1^R \gamma_1 |0\rangle)
\]

\[
= -QB_1^R c_1 |0\rangle + QB_1^R \gamma_1 |0\rangle = -K_1^R c_1 |0\rangle + B_1^R Qc_1 |0\rangle + K_1^R \gamma_1 |0\rangle - B_1^R Q \gamma_1 |0\rangle
\]

\[
= -K_1^R (c_1 - \gamma_1) |0\rangle - B_1^R (c_0 c_1 + \gamma_2 \gamma_1) |0\rangle - B_1^R (c_1 \gamma_2 - \frac{1}{2} c_0 \gamma_2) |0\rangle.
\]

(4.9)

Now we can calculate \( a^n, b^n \). At first we calculate \( a^2 \) and \( b^2 \)

\[
a^2 = \phi^2 + \psi^2 + \phi \psi + \psi \phi
\]

(4.10)

\[
\phi^2 = B_1^L c_1 |0\rangle \ast B_1^L c_1 |0\rangle = (1 - B_1^R c_1 |0\rangle \ast B_1^L c_1 |0\rangle = |0\rangle \ast B_1^L c_1 |0\rangle - B_1^R c_1 |0\rangle \ast B_1^L c_1 |0\rangle
\]

\[
= |0\rangle \ast B_1^L c_1 |0\rangle - c_1 |0\rangle \ast B_1^L B_1^L c_1 |0\rangle = |0\rangle \ast B_1^L c_1 |0\rangle = |0\rangle \ast \phi,
\]

(4.11)

\[
\psi^2 = B_1^L \gamma_1 |0\rangle \ast B_1^L \gamma_1 |0\rangle = -B_1^R \gamma_1 |0\rangle \ast B_1^L \gamma_1 |0\rangle = -\gamma_1 |0\rangle \ast B_1^L B_1^L \gamma_1 |0\rangle = 0.
\]

(4.12)

\[
\phi \psi = B_1^L c_1 |0\rangle \ast B_1^L \gamma_1 |0\rangle = (1 - B_1^R c_1 |0\rangle \ast B_1^L \gamma_1 |0\rangle = |0\rangle \ast B_1^L \gamma_1 |0\rangle - B_1^R c_1 |0\rangle \ast B_1^L \gamma_1 |0\rangle
\]

\[
= |0\rangle \ast B_1^L \gamma_1 |0\rangle - c_1 |0\rangle \ast B_1^L B_1^L \gamma_1 |0\rangle = |0\rangle \ast B_1^L \gamma_1 |0\rangle = |0\rangle \ast \psi,
\]

(4.13)

\[
\psi \phi = B_1^L \gamma_1 |0\rangle \ast B_1^L c_1 |0\rangle = -B_1^R \gamma_1 |0\rangle \ast B_1^L c_1 |0\rangle = -\gamma_1 |0\rangle \ast B_1^L B_1^L c_1 |0\rangle = 0.
\]

(4.14)

So

\[
a^2 = |0\rangle \ast \phi + |0\rangle \ast \psi = |0\rangle \ast a
\]

(4.15)

and similarly

\[
b^2 = |0\rangle \ast \phi - |0\rangle \ast \psi = |0\rangle \ast b.
\]

(4.16)

Therefore by the associativity of the star product we get \( n > 0 \)

\[
a^n = \underbrace{|0\rangle \ast |0\rangle \ast \ldots \ast |0\rangle}_{n-1} \ast a = |n\rangle \ast a = |n\rangle \ast B_1^L c_1 |0\rangle + |n\rangle \ast B_1^L \gamma_1 |0\rangle,
\]

(4.17)

\[
b^n = \underbrace{|0\rangle \ast |0\rangle \ast \ldots \ast |0\rangle}_{n-1} \ast b = |n\rangle \ast b = |n\rangle \ast B_1^L c_1 |0\rangle - |n\rangle \ast B_1^L \gamma_1 |0\rangle.
\]
By trivial substitution

\[ Qa \cdot a^n = Qa \right| n \rangle \right| a = Qa \right| n \rangle \right| B_1^c c_1 \right| 0 \rangle + Qa \right| n \rangle \right| B_1^c \gamma_{\frac{1}{2}} \right| 0 \rangle \]

\[ = (-K_1^R(c_1 + \gamma_{\frac{1}{2}})) \right| 0 \rangle - B_1^R(c_0 c_1 + \gamma_{\frac{1}{2}} \gamma_{\frac{1}{2}}) \right| 0 \rangle + B_1^R(c_1 \gamma_{\frac{1}{2}} - \frac{1}{2} c_0 \gamma_{\frac{1}{2}}) \right| 0 \rangle \right| n \rangle \right| B_1^c c_1 \right| 0 \rangle \]

\[ + (-K_1^R(c_1 + \gamma_{\frac{1}{2}})) \right| 0 \rangle - B_1^R(c_0 c_1 + \gamma_{\frac{1}{2}} \gamma_{\frac{1}{2}}) \right| 0 \rangle + B_1^R(c_1 \gamma_{\frac{1}{2}} - \frac{1}{2} c_0 \gamma_{\frac{1}{2}}) \right| 0 \rangle \right| n \rangle \right| B_1^c \gamma_{\frac{1}{2}} \right| 0 \rangle \]

\[ = -K_1^R(c_1 + \gamma_{\frac{1}{2}}) \right| 0 \rangle \right| n \rangle \right| B_1^c c_1 \right| 0 \rangle - K_1^R(c_1 + \gamma_{\frac{1}{2}}) \right| 0 \rangle \right| n \rangle \right| B_1^c \gamma_{\frac{1}{2}} \right| 0 \rangle \]

\[ = (c_1 + \gamma_{\frac{1}{2}}) \right| 0 \rangle \right| n \rangle \right| K_1^c B_1^c c_1 \right| 0 \rangle + (c_1 + \gamma_{\frac{1}{2}}) \right| 0 \rangle \right| n \rangle \right| K_1^c B_1^c \gamma_{\frac{1}{2}} \right| 0 \rangle \]

(4.18)

and

\[ Qb \cdot b^n = Qb \right| n \rangle \right| b = Qb \right| n \rangle \right| B_1^c c_1 \right| 0 \rangle - Qb \right| n \rangle \right| B_1^c \gamma_{\frac{1}{2}} \right| 0 \rangle \]

\[ = (-K_1^R(c_1 - \gamma_{\frac{1}{2}})) \right| 0 \rangle - B_1^R(c_0 c_1 + \gamma_{\frac{1}{2}} \gamma_{\frac{1}{2}}) \right| 0 \rangle - B_1^R(c_1 \gamma_{\frac{1}{2}} - \frac{1}{2} c_0 \gamma_{\frac{1}{2}}) \right| 0 \rangle \right| n \rangle \right| B_1^c c_1 \right| 0 \rangle \]

\[ - (-K_1^R(c_1 - \gamma_{\frac{1}{2}})) \right| 0 \rangle - B_1^R(c_0 c_1 + \gamma_{\frac{1}{2}} \gamma_{\frac{1}{2}}) \right| 0 \rangle - B_1^R(c_1 \gamma_{\frac{1}{2}} - \frac{1}{2} c_0 \gamma_{\frac{1}{2}}) \right| 0 \rangle \right| n \rangle \right| B_1^c \gamma_{\frac{1}{2}} \right| 0 \rangle \]

\[ = -K_1^R(c_1 - \gamma_{\frac{1}{2}}) \right| 0 \rangle \right| n \rangle \right| B_1^c c_1 \right| 0 \rangle + K_1^R(c_1 - \gamma_{\frac{1}{2}}) \right| 0 \rangle \right| n \rangle \right| B_1^c \gamma_{\frac{1}{2}} \right| 0 \rangle \]

\[ = (c_1 - \gamma_{\frac{1}{2}}) \right| 0 \rangle \right| n \rangle \right| K_1^c B_1^c c_1 \right| 0 \rangle - (c_1 - \gamma_{\frac{1}{2}}) \right| 0 \rangle \right| n \rangle \right| K_1^c B_1^c \gamma_{\frac{1}{2}} \right| 0 \rangle \]

(4.19)

Therefore for \( n > 0 \)

\[ Qa \cdot a^n + Qb \cdot b^n = (c_1 + \gamma_{\frac{1}{2}}) \right| 0 \rangle \right| n \rangle \right| K_1^c B_1^c c_1 \right| 0 \rangle + (c_1 + \gamma_{\frac{1}{2}}) \right| 0 \rangle \right| n \rangle \right| K_1^c B_1^c \gamma_{\frac{1}{2}} \right| 0 \rangle \]

\[ + (c_1 - \gamma_{\frac{1}{2}}) \right| 0 \rangle \right| n \rangle \right| K_1^c B_1^c c_1 \right| 0 \rangle - (c_1 - \gamma_{\frac{1}{2}}) \right| 0 \rangle \right| n \rangle \right| K_1^c B_1^c \gamma_{\frac{1}{2}} \right| 0 \rangle \]

\[ = 2c_1 \right| 0 \rangle \right| n \rangle \right| K_1^c B_1^c c_1 \right| 0 \rangle + 2 \gamma_{\frac{1}{2}} \right| 0 \rangle \right| n \rangle \right| K_1^c B_1^c \gamma_{\frac{1}{2}} \right| 0 \rangle \]

(4.20)

The net result for \( \Phi \) from the GSO(+) sector is

\[ \Phi_\lambda = \sum_{n=0}^{\infty} \lambda^{n+1} \phi_n' \],

(4.21)

\[ \phi_n' = \left( -K_1^R c_1 - B_1^R(c_0 c_1 + \gamma_{\frac{1}{2}}^2) \right) \right| 0 \rangle \right| n \rangle \right| \]

(4.22)

\[ \phi_n' = c_1 \right| 0 \rangle \right| n \rangle \right| K_1^c B_1^c c_1 \right| 0 \rangle + \gamma_{\frac{1}{2}} \right| 0 \rangle \right| n \rangle \right| K_1^c B_1^c \gamma_{\frac{1}{2}} \right| 0 \rangle \]

(4.23)

Performing the similar calculation we get the field \( \Psi \) from the GSO(−) sector

\[ \Psi_\lambda = \sum_{n=0}^{\infty} \lambda^{n+1} \psi_n' \]

(4.24)

\[ \psi_n' = \left( -K_1^R \gamma_{\frac{1}{2}} + B_1^R(c_1 \gamma_{\frac{1}{2}} - \frac{1}{2} c_0 \gamma_{\frac{1}{2}}) \right) \right| 0 \rangle \right| n \rangle \right| \]

(4.25)

\[ \psi_n' = \gamma_{\frac{1}{2}} \right| 0 \rangle \right| n \rangle \right| K_1^c B_1^c c_1 \right| 0 \rangle + c_1 \right| 0 \rangle \right| n \rangle \right| K_1^c B_1^c \gamma_{\frac{1}{2}} \right| 0 \rangle \]

(4.26)

Note that the zero order terms (4.22) and (4.25) can be obtained as a limit \( n \to 0 \) of the corresponding terms (4.21) and (4.24) for \( n > 0 \). It is interesting that only the first term in formula (4.23) coincides with the similar term in the pure GSO(+) solution [27].
4.2 Explicit Check of (4.21) and (4.22)

Here we present some steps of a proof that (4.21)-(4.23) and (4.24)-(4.26) satisfy the E.O.M. This proof is analogous to calculations performed by Okawa [19] for the bosonic string and is based on equations from Appendix.

We calculate:

\[ Q \phi_n' = -c_0 c_1(0) \ast |n\rangle \ast K^L_1 B^L_1 c_1(0) - c_1(0) \ast |n\rangle \ast (K^L_1)^2 c_1(0) - c_1(0) \ast |n\rangle \ast K^L_1 B^L_1 c_0 c_1(0) \]
\[ \quad - \gamma_1 \gamma_1(0) \ast |n\rangle \ast K^L_1 B^L_1 c_1(0) - c_1(0) \ast |n\rangle \ast K^L_1 B^L_1 \gamma_1 \gamma_1(0) + c_1 \gamma_1(0) \ast |n\rangle \ast K^L_1 B^L_1 \gamma_1 \gamma_1(0) \]
\[ \quad - \frac{1}{2} c_0 \gamma_1(0) \ast |n\rangle \ast K^L_1 B^L_1 \gamma_1(0) + \gamma_1(0) \ast |n\rangle \ast (K^L_1)^2 \gamma_1(0) - \gamma_1(0) \ast |n\rangle \ast K^L_1 B^L_1 c_1 \gamma_1(0) \]
\[ \quad + \frac{1}{2} \gamma_1(0) \ast |n\rangle \ast K^L_1 B^L_1 c_0 \gamma_1(0), \quad (4.27) \]

\[
\sum_{m=1}^{n-1} \phi_m \phi_{n-m} = -K^R_1 c_1(0) \ast B^L_1 c_1(0) \ast |n\rangle \ast K^L_1 c_1(0) + K^R_1 c_1(0) \ast |n\rangle \ast B^L_1 c_1(0) \ast K^L_1 c_1(0) \\
\quad - \sum_{m=1}^{n-1} \left( K^R_1 c_1(0) \ast |m+1\rangle \ast B^L_1 \gamma_1(0) \ast |n-m\rangle \ast K^L_1 \gamma_1(0) \right) \\
\quad + K^R_1 \gamma_2(0) \ast |m\rangle \ast B^L_1 \gamma_2(0) \ast |n-m+1\rangle \ast K^L_1 c_1(0), \quad (4.28) \]

\[
\phi_0' \ast \phi_n' = c_1(0) \ast |n+1\rangle \ast (K^L_1)^2 c_1(0) + K^R_1 c_1(0) \ast B^L_1 c_1(0) \ast |n\rangle \ast K^L_1 c_1(0) \\
\quad - K^R_1 c_1(0) \ast B^L_1 \gamma_2(0) \ast |n\rangle \ast K^L_1 \gamma_2(0) \\
\quad + c_0 c_1(0) \ast |n+1\rangle \ast B^L_1 K^L_1 c_1(0) + \gamma_2 \gamma_2(0) \ast |n+1\rangle \ast B^L_1 K^L_1 c_1(0), \quad (4.29) \]

\[
\phi_n' \ast \phi_0' = -K^R_1 c_1(0) \ast |n\rangle \ast B^L_1 c_1(0) \ast K^L_1 c_1(0) + c_1(0) \ast |n+1\rangle \ast K^L_1 B^L_1 c_0 c_1(0) \\
\quad + c_1(0) \ast |n+1\rangle \ast K^L_1 B^L_1 \gamma_2 \gamma_2(0) - K^R_1 \gamma_2(0) \ast |n\rangle \ast B^L_1 \gamma_2(0) \ast K^L_1 c_1(0), \quad (4.30) \]

\[
\sum_{m=1}^{n-1} \psi_m \psi_{n-m} = -K^R_1 \gamma_2(0) \ast B^L_1 c_1(0) \ast |n\rangle \ast K^L_1 \gamma_2(0) + K^R_1 \gamma_2(0) \ast |n\rangle \ast B^L_1 c_1(0) \ast K^L_1 \gamma_2(0) \\
\quad - \sum_{m=1}^{n-1} \left( K^R_1 \gamma_2(0) \ast |m+1\rangle \ast B^L_1 \gamma_2 \ast |n-m\rangle \ast K^L_1 c_1(0) \right) \\
\quad + K^R_1 c_1(0) \ast |m\rangle \ast B^L_1 \gamma_2 \ast |n-m+1\rangle \ast K^L_1 \gamma_2(0) \right), \quad (4.31) \]

\[
\psi_0' \ast \psi_n' = K^R_1 \gamma_2(0) \ast B^R_1 \gamma_2(0) \ast |n\rangle \ast K^L_1 c_1(0) - K^R_1 \gamma_2(0) \ast B^R_1 c_1(0) \ast |n\rangle \ast K^L_1 \gamma_2(0) \\
\quad + c_1 \gamma_2(0) \ast |n+1\rangle \ast B^L_1 K^L_1 \gamma_2(0) - \frac{1}{2} c_0 \gamma_2(0) \ast |n+1\rangle \ast B^L_1 K^L_1 \gamma_2(0), \quad (4.32) \]

\[
\psi_n' \ast \psi_0' = -K^R_1 \gamma_2(0) \ast |n\rangle \ast B^L_1 c_1(0) \ast K^L_1 \gamma_2(0) + K^R_1 \gamma_2(0) \ast |n+1\rangle \ast B^L_1 c_1 \gamma_2(0) \\
\quad - \frac{1}{2} K^R_1 \gamma_2(0) \ast |n+1\rangle \ast B^L_1 c_0 \gamma_2(0) - K^R_1 c_1(0) \ast |n\rangle \ast B^L_1 K^L_1 \gamma_2(0), \quad (4.33) \]
Gathering the pieces together we get

$$Q\phi'_{n+1} = -\sum_{m=0}^{n} \phi'_{n-m} \star \phi'_{m} + \sum_{m=0}^{n} \psi'_{n-m} \star \psi'_{m}. \quad (4.34)$$

These equations are satisfied for whole range $n \geq 0$.

The check for eq. (4.24) is the same.

### 4.3 Solutions in the Split String Formalism

Formula (4.21) and (4.24) in split notation [19, 23] can be presented as

$$\Phi = Fc \frac{KB}{1 - F^2} \frac{cF}{F^2} + F\gamma \frac{KB}{1 - F^2} \frac{\gamma F}{F^2}, \quad (4.35)$$

$$\Psi = Fc \frac{KB}{1 - F^2} \frac{\gamma F}{F^2} + F\gamma \frac{KB}{1 - F^2} \frac{cF}{F^2}. \quad (4.36)$$

These fields satisfy the algebraic relations,

$$[B, K] = 0, \quad [B, \gamma^2] = 0, \quad [B, \gamma] = 0, \quad [c, \gamma^2] = 0, \quad [c, \gamma] = 0, \quad \{B, c\} = 1, \quad B^2 = c^2 = 0. \quad (4.37)$$

The actions of $d \equiv Q$ on $c, \gamma, \gamma^2, B$ and $K$ are given by

$$dc = cKc - \gamma^2, \quad d\gamma = cK\gamma - \frac{1}{2} \gamma Kc, \quad d\gamma^2 = cK\gamma^2 - \gamma^2 Kc, \quad (4.38)$$

$F$ is the square root of the $SL(2, \mathbb{R})$ vacuum $F \equiv e^{\frac{\pi}{4} K} = \Omega^{\frac{1}{4}}$.

### 4.4 Singular pieces

As it now well understood [19, 22, 27] to calculate the action on a solution one has to provide the validity of the equation of motion in a strong sense. In other words one has to regulates the constructed formal solution. We can use the regularization similar to the regularization in the pure $GSO(+)\text{ sector}$ [27]

$$\frac{K}{1 - F^2} = \lim_{N \to \infty} \left[ \sum_{n=0}^{N} K F^{2n} - \left( 1 + \frac{1}{2} K \right) F^{2N} \right]. \quad (4.39)$$

This gives

$$\Phi = \lim_{N \to \infty} \left[ \sum_{n=0}^{N} \phi^'_{n} - \varphi_N - \frac{1}{2} \varphi^'_{N} \right] + \lim_{N \to \infty} \left[ \sum_{n=0}^{N} \chi^'_{n} - \chi_N - \frac{1}{2} \chi^'_{N} \right], \quad (4.40)$$

$$\Psi = \lim_{N \to \infty} \left[ \sum_{n=0}^{N} \xi^'_{n} - \xi_N - \frac{1}{2} \xi^'_{N} \right]. \quad (4.41)$$
where

\[ \varphi_n = FcF^{2n}BcF \]  
\[ \varphi'_n = \frac{1}{2} \frac{d}{dn} \varphi_n \]  
\[ \chi_n = F\gamma F^{2n}B\gamma F \]  
\[ \chi'_n = \frac{1}{2} \frac{d}{dn} \chi_n \]  
\[ \xi_n = F(cF^{2n}B\gamma + \gamma F^{2n}B)cF \]  
\[ \xi'_n = \frac{1}{2} \frac{d}{dn} \xi_n \]  

\[ (4.42) \]
\[ (4.43) \]
\[ (4.44) \]
\[ (4.45) \]
\[ (4.46) \]
\[ (4.47) \]

5. Action

To prove the Sen’s conjecture we have to calculate the action on the classical solution. It is interesting to note that in this case \( \Psi \) does not contribute explicitly to the action.

Indeed, let us put the \( Q\Psi \) from the second equation of motion into the action \( (2.1) \):

\[ S[\Phi, \Psi] = -\frac{1}{g_0^2} \left[ \frac{1}{2} \langle \langle Y-2| \Phi, Q\Phi \rangle \rangle + \frac{1}{3} \langle \langle Y-2| \Phi, \Phi, \Phi \rangle \rangle \right], \]

\[ (5.1) \]

here we use the cyclicity property

\[ \langle \langle Y-2| \Phi, \Psi, \Psi \rangle \rangle = -\langle \langle Y-2| \Psi, \Phi, \Psi \rangle \rangle = \langle \langle Y-2| \Psi, \Psi, \Phi \rangle \rangle. \]

\[ (5.2) \]

So the solution from \( GSO(-) \) sector doesn’t make a contribution to the action as it must be for the fields with a pure quadratic action.

Let us also note that we can represent the \( GSO(+) \) part of the solution to \( (2.2) \) as

\[ \Phi = \Phi_+ + \Phi' \]

\[ (5.3) \]

where \( \Phi_+ \) is a solution to \( (2.4) \) and \( \Phi' \) is a solution to

\[ Q\Phi' + \Phi' \star \Phi' + \Phi_+ \star \Phi' + \Phi' \star \Phi_+ - \Psi \star \Psi = 0. \]

\[ (5.4) \]

Substituting this decomposition into \( (5.1) \) we get

\[ -g_0^2 S[\Phi_+ + \Phi', \Psi] = \frac{1}{2} \langle \langle Y-2| \Phi_+, Q\Phi_+ \rangle \rangle + \frac{1}{3} \langle \langle Y-2| \Phi_+, \Phi_+, \Phi_+ \rangle \rangle \]

\[ + \langle \langle Y-2| \Phi', Q\Phi_+ \rangle \rangle + \langle \langle Y-2| \Phi', \Phi_+, \Phi_+ \rangle \rangle \]

\[ + \frac{1}{2} \langle \langle Y-2| \Phi', Q\Phi' \rangle \rangle + \langle \langle Y-2| \Phi', \Phi', \Phi_+ \rangle \rangle + \frac{1}{3} \langle \langle Y-2| \Phi', \Phi', \Phi' \rangle \rangle. \]

\[ (5.5) \]

Taking into account E.O.M. \( (2.4) \) we get

\[ -g_0^2 S[\Phi_+ + \Phi', \Psi] = \frac{1}{2} \langle \langle Y-2| \Phi_+, Q\Phi_+ \rangle \rangle + \frac{1}{3} \langle \langle Y-2| \Phi_+, \Phi_+, \Phi_+ \rangle \rangle \]

\[ + \frac{1}{2} \langle \langle Y-2| \Phi', Q\Phi' \rangle \rangle + \langle \langle Y-2| \Phi', \Phi', \Phi_+ \rangle \rangle + \frac{1}{3} \langle \langle Y-2| \Phi', \Phi', \Phi' \rangle \rangle. \]

\[ (5.6) \]
Formula (1.21) gives representation (5.3), where $\Phi_+$ is the Erler’s solution [27]

$$\Phi_+ = \sum_{n=0}^{\infty} \lambda^{n+1} c_1 |0\rangle \star |n\rangle \star K_1^L B_1^L c_1 |0\rangle + \lambda B_1^L \gamma \frac{1}{2} \gamma \frac{1}{2} |0\rangle$$  \hspace{1cm} (5.7)

and $\Phi'$ is given by

$$\Phi' = \sum_{n=1}^{\infty} \lambda^{n+1} \gamma \frac{1}{2} |0\rangle \star |n\rangle \star K_1^L B_1^L \gamma \frac{1}{2} |0\rangle.$$  \hspace{1cm} (5.8)

Since the action $S[\Phi_+]$ reproduces the correct D – brane tension to check the Sen conjecture we have to check that the contribution from the $\Phi'$ component is zero, i.e.

$$\frac{1}{2} \langle \langle Y^2 | \Phi', Q \Phi' \rangle \rangle + \langle \langle Y^2 | \Phi', \Phi', \Phi_+ \rangle \rangle + \frac{1}{3} \langle \langle Y^2 | \Phi', \Phi', \Phi' \rangle \rangle = 0,$$  \hspace{1cm} (5.9)

The terms in (5.8) being the second order on $\gamma$ do not contribute to the action and we conclude only the first term in the decomposition does contribute to the action.

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A. Appendix

In this appendix we collect all algebraic formula used in the text.

The set of formula collect the relevant properties properties of the BRST charge $Q$, the $K$ operator and the corresponding ghost $B$ and the Witten’s $\star$ multiplication ( for definitions see [19]).

\begin{align}
Q(\phi_1 \star \phi_2) &= (Q \phi_1) \star \phi_2 + (-1)^{\phi_1} \phi_1 \star (Q \phi_2), \quad (A.1) \\
Q^2 &= 0, \quad (A.2) \\
Q|0\rangle &= 0, \quad (A.3) \\
Q c_1|0\rangle &= - c_0 c_1|0\rangle - \gamma_{\frac{1}{2}} \gamma_{\frac{1}{2}}|0\rangle, \quad (A.4) \\
Q \gamma_{\frac{1}{2}}|0\rangle &= c_1 \gamma_{-\frac{1}{2}}|0\rangle - \frac{1}{2} c_0 \gamma_{\frac{1}{2}}|0\rangle, \quad (A.5) \\
K_1 &\equiv L_1 + L_{-1}, \quad (A.6) \\
B_1 &\equiv b_1 + b_{-1} = B^R_1 + B^L_1, \quad (A.7) \\
[B_1, \gamma_{\frac{1}{2}}] &\equiv 0, \quad (A.8) \\
(B^R_1 \phi_1) \star \phi_2 &= - (-1)^{\phi_1} \phi_1 \star (B^L_1 \phi_2), \quad (A.9) \\
(B^L_1)^2 &= (B^R_1)^2 = 0, \quad (A.10) \\
(B^L_1 + B^R_1)|0\rangle &= 0, \quad (A.11) \\
(B^L_1 + B^R_1) c_1|0\rangle &= |0\rangle, \quad (A.12) \\
(K^R_1 \phi_1) \star \phi_2 &= - \phi_1 \star (K^L_1 \phi_2), \quad (A.13) \\
(K^L_1 + K^R_1)|0\rangle &= 0, \quad (A.14) \\
\{Q, B^L_1\} &= K^L_1, \quad (A.15) \\
\{Q, B^R_1\} &= K^R_1, \quad (A.16) \\
[Q, K^L_1] &= 0, \quad (A.17) \\
[B^L_1, K^L_1] &= 0 \quad (A.18)
\end{align}

for any string fields $\phi_1$ and $\phi_2$. Here the subscript $L$ ($R$) denotes the left (right) half of the corresponding charge. We also have

\begin{align}
B^L_1 |0\rangle \star |0\rangle &= |0\rangle \star B^L_1 |0\rangle, \quad (A.19) \\
K^L_1 |0\rangle \star |0\rangle &= |0\rangle \star K^L_1 |0\rangle. \quad (A.20)
\end{align}

For the wedge states we have

\begin{align}
|\phi\rangle \star |n\rangle &= e^{-\pi(n-1)K^R_1}|\phi\rangle \quad (A.21) \\
|n\rangle \star |\phi\rangle &= e^{\pi(n-1)K^L_1}|\phi\rangle \quad (A.22)
\end{align}
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