RESEARCH OF NONLINEAR VIBRATIONS OF LAMINATED SHALLOW SHELLS WITH CUTOUTS BY R-FUNCTIONS METHOD

In the present work an effective method to research geometrically nonlinear free vibrations of elements of thin-walled constructions that can be modeled as laminated shallow shells with complex planform is applied. The proposed method is based on joint use of R-functions theory, variational methods and Bubnov–Galerkin procedure. It allows reducing an initial nonlinear system of motion equations of a shallow shell to the Cauchy problem. The mathematical formulation of the problem is performed in a framework of the refined first-order theory. The appropriate software is created within POLE–RL program system for polynomial results and using C++ programs for splines. New problems of linear and nonlinear vibrations of laminated shallow shells with cutouts are solved. To confirm reliability of the obtained results their comparison with the ones obtained using spline-approximation and known in literature is provided. Effect of boundary conditions on cutout is studied.

Keywords: R-function theory, Timoshenko’s theory, laminated shallow shells, geometrically nonlinear vibrations.

Introducing

Investigation of geometrically nonlinear vibrations of multi-layered shallow shells with complex planform and a cutout is carried out. The mathematical statement of this class of problems is well developed in literature [1, 2]. However, due to the complexity of the motion equations system, there are practically no works, which contain numerical results for these problems in the case of complex planforms, the presence of cutouts and different types of boundary conditions. Therefore, development of methods solving such problems is an important task. In this paper we propose a method based on the theory of R-functions [3, 4] and variational methods, which allows one to solve such problems.

1 The mathematical statement of the problem

We consider the multi-layered thin shell of the constant thickness \( h \), on the assumption that the slip and separation between the layers are absent. We confine ourselves to the symmetrical structure of layers. The mathematical formulation of the problem is performed via refined theory of multi-layered shells based on the Timoshenko’s shear assumptions. Then the problem of geometrically nonlinear vibrations of shallow shells is reduced to the solution of the system of nonlinear differential equations of motion [5, 6]:

\[
\frac{\partial^2 N_{11}}{\partial x^2} + \frac{\partial^2 N_{12}}{\partial y^2} = m_1 \frac{\partial^2 u}{\partial t^2}, \quad \frac{\partial^2 N_{22}}{\partial x^2} + \frac{\partial^2 N_{11}}{\partial y^2} = m_2 \frac{\partial^2 v}{\partial t^2},
\]

\[
\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + k_1 N_{11} + k_2 N_{22} + N_{11} \frac{\partial^2 w}{\partial x^2} + 2 N_{12} \frac{\partial^2 w}{\partial x \partial y} + N_{22} \frac{\partial^2 w}{\partial y^2} = m_1 \frac{\partial^2 \psi}{\partial t^2};
\]

\[
\frac{\partial M_{11}}{\partial x} + \frac{\partial M_{12}}{\partial y} + Q_x = m_1 \frac{\partial^2 \psi}{\partial t^2};
\]

\[
\frac{\partial M_{22}}{\partial x} + \frac{\partial M_{12}}{\partial y} + Q_y = m_2 \frac{\partial^2 \psi}{\partial t^2};
\]

where \( u(x,y,t), v(x,y,t), w(x,y,t) \) are displacements of the coordinate surface points; \( \psi_x, \psi_y \) are rotation angles of the normal to the coordinate surface; \( N_{11}, N_{12}, N_{22} \) are the in-plane resultants per unit length; \( M_{11}, M_{12}, M_{22} \) are the internal moment resultants per unit length; \( Q_x, Q_y \) are the transverse shear resultants per unit length. Components of these resultants are defined as:

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ψ = ψ ⋅ + ψ ν are the components of the unknown functions in the system (1) and applying the Bubnov-Galerkin procedure, we obtain the nonlinear ordinary differential equation

\[ y''_1(t) + \omega^2 y_1(t) + \beta \cdot y'_1(t) + y \cdot y'_1(t) = 0 \]  \hspace{1cm} (5)

Formulas for the coefficients presented in the equation (5) are given in [8-10].

The solution of the obtained ordinary differential equation may be accomplished through a variety of approximation methods. For example, the Runge-Kutta method, Bubnov-Galerkin and others.

2 Numerical results
To validate the proposed method and created software a number of test problems were solved. The obtained results were compared with the once of other authors [7, 11]. One of the test cases is discussed in Example 1.

Example 1. Consider the problem of free vibrations of a shallow geometrically nonlinear isotropic square shell of constant thickness of double curvature. The following geometric and material parameters are used: \( a/b = 1; R_s/R_e = 1; R_e/R_s = 0; R_p = 10; h = 0.01; v = 0.3 \). The obtained results correspond to a simply-supported edge. Figure 1 shows a comparison of the frequencies ratios dependence on the basic mode amplitude for cylindrical and spherical shells, obtained by RFM with the known results of [11]. To obtain the dependency ratio of the frequencies to the amplitude the Runge-Kutta method was used.

The observed divergence of obtained results with the once of [11] does not exceed 2 %.

Example 2. Consider the problem of a 3-layered cylindrical shallow shell free nonlinear vibration with a square planform and a central square cutout (fig. 2).

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Example 2. Consider the problem of a 3-layered cylindrical shallow shell free nonlinear vibration with a square planform and a central square cutout (fig. 2).
Figure 2 – Geometry of a shell with a central cutout and different boundary conditions

It is assumed that all layers are made of the material with the following characteristics:

Material 1 (M1): \( E_1 = 40E_2; G_{12} = G_{13} = G_{23} = 0.5E_2; \)
\( \nu_{12} = 0.25. \)

Material 2 (M2): \( E_1 = 10E_2; G_{12} = G_{13} = G_{23} = 0.5E_2; \)
\( \nu_{12} = 0.25. \)

The shear correction factors are taken as \( k_1^2 = k_2^2 = 5/6, \) dimensionless parameters of the curvature are defined as \( R_i/h = 300; h/(2a) = 0.01. \) Three types of boundary conditions are investigated: completely clamped edge (CC), simply supported on external edge and clamped cutout (SC), simply supported on external edge and free cutout (SF). Comparison of the obtained fundamental frequencies for two types of materials (M1, M2) with the once in [7] is presented in table 1. Results presented by RFM were obtained both by polynomial approximation (POLY) and splines [12] (SPLI) using mesh of 10x10.

Table 1 – Comparison of the non-dimensional frequency \( \sigma_1 = a^2a^2(\rho/E_2h^3)^{1/2} \) for cross ply (0/90/0) laminated shells with the boundary condition (SF)

| \( c/a \) | M1 | M1 | M2 | M2 |
|----------|----|----|----|----|
| 0        | 29,3565 [7] | 29,481 [POLY] | 23,8253 | 23,933 [POLY] |
| 0.2      | 29,4182 [7] | 30,252 [POLY] | 24,2845 | 24,749 [POLY] |

The observed divergence of obtained results with the once of [7] does not exceed 3%.

Further, new results are presented by using the theory of R–functions.

The amplitude-frequency dependence for cylindrical shells of SC boundary condition with the cutout of size \( c/a = 0.2 \) for two types of materials (M1, M2) is presented in figure 3.

According to the observed curves we can state that the behavior of investigated shell of M1 material is more rigid than the one of M2 with the amplitude increase.

The amplitude-frequency dependence for cylindrical shells of CC boundary condition with the cutout of size \( c/a = 0.2 \) for two types of materials (M1, M2) is presented in figure 4.

According to the observed curves we can state that the investigated shell of M1 material becomes more rigid than the one of M2 when the ratio \( W_{max}/h \) exceeds 1.4.

Conclusions

A proposed numerical-analytically approach based on R–functions theory is used to research free nonlinear vibration problems of laminated shallow shells with cutouts. Three-layered shells made of different materials with different curvatures and square cutout are investigated. Different types of boundary conditions are examined. The amplitude-frequency curves of vibrations of considered...
shells have been constructed using the first-mode approximation by the Runge-Kutta method. A comparison with known results confirms the reliability of the proposed approach.

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