Dynamical Formation of Spherical Domain Wall
by Hawking Radiation and
Spontaneous Charging-up of Black Hole

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Abstract

We discuss the Hawking radiation in the Higgs-Yukawa system and we show dynamical formation of a spherical domain wall around the black hole. The formation of the spherical wall is a general property of the black hole whose Hawking temperature is equal to or greater than the energy scale of the system. The formation of the electroweak wall and that of the GUT wall are shown as realistic cases. We also discuss a phenomenon of the spontaneous charging-up of the black hole. The Hawking radiation can charge-up the black hole itself by the charge-transportation due to the spherical wall when C- and CP-violation in the wall are assumed. The black hole with the electroweak wall can obtain a large amount of the hyper charge.

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1 INTRODUCTION

The radiation from a black hole known as the Hawking radiation has a thermal spectrum \[1\]. In the case of a Schwarzschild black hole, its temperature, namely, the Hawking temperature is inversely proportional to mass of the black hole. The black hole is losing its mass by the Hawking radiation and it is increasing its Hawking temperature. At the final stage of the radiation, its temperature and its intensity explosively increase, therefore the final stage of the black hole is very interesting to particle physicists [2].

Several authors discussed that the heating-up by the Hawking radiation can thermализе the neighborhood of the black hole and thermalphase transition around the black hole can arise. The QCD phase transition [3] and the electroweak (EW) phase transition [4,5] have been discussed. The localphase-transition around the black hole means the formation of the spherical domain wall which separates region of the symmetric phase and that of the broken-phase. The baryon number production and the electroweak baryogenesis scenario have been proposed as an application of the spherical wall [3,4,5].

The Hawking temperature should be much greater than the critical temperature of the phase transition to consider the formation of the wall with the thermal phase transition. The mean free paths for the radiated particles are much longer than the Schwarzschild radius and we should consider the local thermal equilibrium around the black hole to consider the thermal phase transition. Therefore the Hawking temperature should be much higher than the critical temperature to guarantee the local thermal equilibrium with the critical temperature. However, it is natural to ask if the Hawking radiation whose temperature is equal to or greater than the critical temperature in unesco the Higgs vacuum expectation value (vev) locally.

In this paper, we discuss a Hawking radiation in the gauge-Higgs-Yukawa system, e.g., the Standard Model (SM) and the Grand Unified Theory (GUT), and we show that a structure of the spherical domain wall around the black hole arises by the field dynamics of the system. It seems that we need to consider the full dynamics of the field theory to discuss the phenomenon around the black hole and it will be technically difficult, however, only a part of interaction is relevant to the subject because the mean free paths for the gauge interaction is much longer than the radius of the black hole and it simplifies the analysis. The relevant are interactions between the Hawking-radiated particles and the Higgs vev. The radiated particles can be regard as ballistic because of their long mean-free-paths. By founding on these properties, we propose a effective model with a ballistic approximation. Our model consists from the action for the Higgs field and that for the relativistic point particles radiated from the black hole. We derive an effective action for the Higgs field which determines the structure of the Higgs vev around the black hole. By considering the effective action, the formation of the spherical domain wall around the black hole is shown when the Hawking temperature is equal or greater than the energy scale of the Higgs-Yukawa system. We will call this mechanism the dynamical formation of the spherical wall in distinction from the thermal formation of...
the spherical wall. As realistic cases, the formation of the EW wall and that of the GUT wall are discussed.

We also discuss a mechanism of the spontaneous charging-up of the black hole as one of applications of the spherical wall. We assume the following two conditions: (i) a CP-broken phase in the spherical wall and (ii) a chiral charge assignment of fermions in the field theory, namely, a C-violation of the theory. The Hawking radiation is neutral for the charge, however the spherical wall has a reaction-symmetry for the charge on the assumptions. Therefore a charge-transport mechanism from the wall to the black hole works and it charges-up the black hole. This mechanism is a variant application of the charge transport scenario in the electroweak baryogenesis model proposed by Cohen, Kaplan and Nelson \cite{7,8}. The two assumptions are equivalent to two of the Sakharov’s three conditions for the baryogenesis \cite{9}. In the case of the black hole whose Hawking temperature is EW scale, the black hole can obtain a large amount of the hyper charge. The black hole with GUT temperature can also obtain several charge.

The paper is organized as follows: In Section 2, the formation mechanism of the spherical wall is discussed. In Section 3, a mechanism for the spontaneous charging-up of the black hole is discussed. In section 4 we provide a conclusion and discussions.

2 Dynamical Formation of Spherical Domain Wall

We will consider a Hawking radiation in the gauge-Higgs-Yukawa system like the Standard Model (SM). Especially, we will discuss dynamics of the Higgs field around the black hole whose Hawking temperature is the same energy scale of the system, e.g., the critical temperature. In the situation, the mean free paths from the gauge interactions are much greater than the length scale for the black hole \cite{4}, therefore, most of the gauge interactions are not important. The relevant are the Yukawa interactions for the heavy fermions and the gauge interactions between the Higgs and heavy gauge bosons. These particles obtain their heavy mass from the Higgs vacuum expectation value (vev) through the interactions. Inversely we can expect that the particles radiated from the black hole with high energy and with high density deform the Higgs vev around the black hole. It seems that we need full calculation for the field dynamics to consider the deformation of the Higgs vev exactly. However we can employ the ballistic approximation for the radiated particles because the mean free paths are enough long and the only yukawa-type interactions are relevant. Namely, the deformation of the Higgs vev can be discussed by the relativistic semiclassical kinematics of the radiated particles. The approximated system should have Lorentz invariance and any ballistic particles obtain their mass by the Higgs vev. To discuss the deformation in the ballistic approximation, we propose a kind of Higgs-Yukawa model with relativistic point particles which satisfies the required conditions and describes the
relevant interactions explicitly. Our system is described by the action for a Higgs field \( \phi \) with a Higgs potential \( V(\phi) \) and the action for relativistic point particles \( f_i(y) \). We will use metric convention \( g = \text{diag}(+1; 1; 1; 1) \) and the action is given by a Nambu-Goto type action:

\[
S[\phi; y] = \int d^4x \ (\partial \phi)^2 V(\phi) \ x^2 ds_i Y_1 j (y_i(s_i)) j Y_i(s_i); \tag{1}
\]

where \( y_i = ds_i \) and \( Y_1 \) is a Yukawa coupling constant for the point particle \( i \). The summation in equation (1) takes over all particles. The Higgs potential in the vacuum is given by the double-well form:

\[
V(\phi) = \frac{1}{2} \phi^2 + \frac{1}{4!} \phi^4; \tag{2}
\]

which has a minimum at \( j = v = \frac{\sqrt{2}}{2} \) and the constant \( \frac{1}{4!} > 0 \) is the Higgs mass, therefore mass of the particle \( i \) is given by \( m_i = Y_i v = \frac{\sqrt{2}}{2} \) in the vacuum.

To calculate the effective action for the Higgs, we will x-gauge for the point particles as \( s_i = x^0 = y_i^0(\phi^0) \). Then we have \( Y_i(\phi^0) = 1 \), \( j_i(\phi^0) = 1 \), \( j_1(\phi^0) = 1 \), \( j_2(\phi^0) = 1 \), where \( v_i(\phi^0) \) is 3-velocity for the particle \( i \). The action becomes

\[
S[\phi; y] = \int d^4x \ (\partial \phi)^2 V(\phi) \ x^2 ds_i Y_1 j (y_i(s_i)) j Y_i(s_i); \tag{3}
\]

By using the definition of the 4-momentum for the particle \( i \):

\[
p_i = \frac{\partial L}{\partial \dot{y}_i} = Y_i \ (y_i) \frac{\partial Y_i}{\partial y_i} = Y_i \ (y_i) (L; y_i); \tag{4}
\]

the equation of motion for the particle becomes

\[
p_i + \frac{1}{Y_i} \frac{\partial L}{\partial y_i} = 0; \tag{5}
\]

The propagating modes for the Higgs can be regarded as being taken into the ballistic particles due to the ballistic approximation, therefore the Higgs field \( \phi \) can be considered as the Higgs expectation value and it is independent of time \( x^0 \), namely, \( \theta_0 = 0 \). This assumption simplifies the equation of the motion in equation (3) and

\[
E_i = Y_i; \tag{6}
\]

becomes a constant of the motion. The constant \( E_i \) is the energy of the particle \( i \) at the rest frame. When we put the trajectories for all particles \( f_i(\phi^0) \) with their energy-constants \( E_i \) into the effective action for the Higgs field

\[
S_{\phi} = \int d^4x \ (\partial \phi)^2 V(\phi) \ x^2 ds_i Y_1 j (y_i(s_i)) j Y_i(s_i); \tag{7}
\]

The action for the ordinary relativistic point particle with mass \( m \) is given by \( ds_m \). Our action describes point particles which obtain their mass by the Higgs vev.
Here we adopt the differential particle number-density $dE N_f(x;E)$ for particle species $f$. The effective action for the Higgs field can be written down as

$$S_e[\phi] = \int d^4x \, d^4\phi \, V(\phi) + \int dE \frac{d^3\phi}{E} \, N_f(E;x) \phi^3;$$

where the summation in the equation (8) takes over all particle-species in the theory. Then the effective potential should be

$$V_e(x;\phi) = +\phi^2 \frac{\phi^2}{\sqrt{\phi^2}};$$

where we have defined the effective $\phi$ as

$$\phi(x;\phi) = \phi + \int dE \frac{d^3\phi}{E} \, N_f(E;x).$$

Next we consider that the particle distribution $N_f(E;x)$ is produced by the Hawking radiation from the Schwarzschild black hole with mass $m_{BH}$. By the Hawking process, the black hole radiates all particles in the field theory with Hawking temperature $T_{BH} = \frac{m_{BH}^2}{8\pi m_{BH}}$ from the horizon whose radius is given by the Schwarzschild radius $r_{BH} = \frac{1}{2} \frac{1}{T_{BH}}$. The particle distribution near the black hole is approximately given by

$$dE N_f = \frac{1}{4} \frac{g_r^2}{(\sqrt{2}^2)} f_{r_{BH}}(E) \, 4 \, E^2 dE \, \frac{r_{BH}}{r};$$

where

$$f_{r_{BH}}(E) = \frac{1}{e^{E-r_{BH}} - 1} \tag{12}$$

is Bose-Einstein or Fermi-Dirac distribution function with temperature $T_{BH}$. The leading contribution from the Hawking radiation is given by the equation (13). We have ignored the backreaction from the produced wall into the distribution of the particles, namely, we have ignored the contribution from the non-uniform configuration of the Higgs field which is depending on the distance from the black hole. We note that the ignored contribution to the particle trajectories can be evaluated by solving the equation $\phi_{ij}^f = 1 \, (\phi_i(\phi_i)=E)\phi^2$ derived from equation (5) and it will be a future subject.

By substituting the equation (13) into the equation (14), we obtain

$$\phi(x;\phi) = \phi + \frac{\phi^2}{r^2};$$

where

$$\frac{1}{768} \frac{\phi^2}{r^2} \phi_r.$$
Figure 1: The distribution of the effective Higgs potential $V_e (\phi; r)$ around a black hole. The parameter $r_m$ means the distance from the center of the black hole and $r_{DW}$ is the radius where the sign of $e (r)$ is inverted. The thick curves indicates the value of the Higgs field $\phi_{min} (r)$ which minimizes the effective Higgs potential at the each point $r$. The value $v = \frac{\rho}{2}$ describes the ordinary Higgs vacuum expectation value, namely $\phi_{min} (r = 1) = v = \frac{\rho}{2}$.

is a constant depending on the Higgs theory and we have defined the effective $g_f$ as

$$g_f = \begin{cases} 8 < g_f & (f : \text{boson}) \\ \frac{1}{2} g_f & (f : \text{fermion}) \end{cases} (15)$$

Here we find that the sign of $e (r)$ is reversed at $r_{DW}$, namely, $e (r)$ is negative for $r > r_{DW}$ and $e (r)$ is positive for $r < r_{DW}$. Therefore, the local vacuum for the Higgs field is depending of the distance from the black hole and we can expect the existence of the spherical domain wall with radius $r_{DW}$ when $r_{BH} < r_{DW}$. The distribution of the Higgs potential $V (\phi; r)$ around the black hole is shown in Figure 1. The value of the Higgs field which minimizes the effective potential is

$$\phi_{min} (r) = \begin{cases} 8 \rho \frac{V}{2} \frac{1}{2} \frac{r_{DW}}{r} & (r > r_{DW}) \\ 0 & (r < r_{DW}) \end{cases} (16)$$

The (continuous varying) vacuum depending of the position is one of the distinctive feature in the non-equilibrium system. Such situations and the relation to the formation of the domain wall had been discussed by several authors [3, 4].

The form of the Higgs vev around a black hole, namely, the structure of the domain wall can be calculated as a spherically-symmetric stationary solution of the motion equation of the Higgs field with the effective potential:

$$\phi (r) = \frac{\partial}{\partial r} V_e (\phi; r); (17)$$
where the boundary condition \((r ! 1) = v = \frac{P}{2}\) is required. The equation (17) can be solved numerically and the result is shown in the Figure 2. The form of the Higgs vev is depending on the parameter \(\beta\) which is defined in equation (4). Generally speaking, the curve of the Higgs vev \((r)\) approaches the \(m_{\text{in}}(r)\) when the parameter becomes large. Here we find that the Hawking radiation can form the spherical wall or wall-like structure around the black hole for the non-zero \(\alpha^2\) and the radius of the spherical wall is characterized by the parameter \(r_{DW}\).

![Figure 2](image-url)

**Figure 2:** The profiles of the spherical wall around a black hole. The parameter \(r\) means distance from the center of the black hole and \((r)\) means the expectation value for the Higgs field around the black hole. The thin curves are stationary solutions of the EOM for the Higgs field with the effective potential \(V_e(\phi; r)\) depending on the parameter \(\alpha^2\). The thick curve indicates the value of the Higgs field \(m_{\text{in}}(r)\) which gives the minimum of the effective Higgs potential and the dotted line means the ordinary Higgs vev \(h = \frac{P}{2}\).

The characteristic radius for the wall is given by \(r_{DW} = \) as discussed previously. It should be noticed that the characteristic radius \(r_{DW}\) is not depending on the Hawking temperature then the form of the wall is not so changing after the formation of the wall to
the evaporation of the black hole. The Schwarzschild radius of the black hole $r_{BH}$ should be smaller than the characteristic radius of the wall $r_{DW}$ for the formation of the wall then the condition for the wall-existence is
\[ 1 < \frac{r_{DW}}{r_{BH}} = 4 \frac{T_{BH}}{T_{EW}} : \] (18)

Now we can discuss the formation of the domain wall in the realistic field theories, e.g., the electroweak (EW) theory and the grand unified theory (GUT). In the case of the EW domain wall, the heavy particles which contribute to create the wall are the top quarks, the weak bosons $Z^0, W^+$ and the Higgs bosons, and approximately we have
\[ \frac{2}{EW} \frac{1}{768} \frac{X}{r} Y_{fr}^2 \gamma \neq 2 = 200 : \] (19)

The critical Hawking temperature for the dynamical formation of the spherical EW wall becomes
\[ T_{BH}^{EW} = \frac{4}{T_{EW}} : \] (20)

Therefore the Hawking radiation produces the spherical EW domain wall whose structure is given in the Figure 2 with $\frac{2}{EW} 0.005$ when the Hawking temperature of the black hole is similar to or greater than the EW scale. The characteristic radius of the wall is given by
\[ \frac{r_{EW}}{r_{DW}} = \frac{EW}{r} = 0.07 = \] (21)

In the case of the SU (5)-GUT, there are, at least, lepto-quarks $X, Y$ and 24-Higgs as heavy particles with the GUT scale mass $GUT' 10^{16}$ GeV. The same analysis results
\[ \frac{2}{GUT} \gamma = 70 ; \] (22)
\[ T_{BH}^{GUT} = 0.7GUT ; \] (23)
\[ \frac{r_{GUT}}{r_{DW}} = 0.1 = GUT ; \] (24)

Then the spherical GUT wall around the black hole is also formed when the Hawking temperature is greater than the GUT scale. Finally we can discuss that the dynamical spherical wall around a black hole may be formed in most of the field theory which has a Higgs mechanism when the Hawking temperature is greater than the energy scale of the field theory.

3 Spontaneous Charging-up of the Black Hole

We will discuss that the Hawking radiation can charge up the black hole by the effect of the spherical domain wall which has been discussed. We need two assumptions such that (i) the
domain wall has CP-broken phase and (ii) the field theory has fermions with chiral charge assignment. The Standard Model satisfies the second assumptions because the left-handed quark and the right-handed quark have different hyper charges. When the first assumption is satisfied, the reaction rate on the wall of the left-handed fermions is different from that of the right-handed fermions. Therefore the domain wall has a charge-reaction-asymmetry when both assumptions are satisfied. The Hawking radiation is charge-neutral, however, the black hole obtains net charge by the effect of the domain wall because a part of the reacted particles return into the black hole, namely the net charge is transported to the black hole. This process is similar to the "charge transport mechanism/scenario by the thin wall" in the electroweak baryogenesis proposed by Cohen, Kaplan and Nelson \[7,8\]. In the charge transport scenario of the electroweak baryogenesis, the hyper charge is transported from the thin EW wall to the region of the symmetric phase and it boosts up the baryon number creation by the sphaleron process. On the other hands, our charge-transportation charges up the black hole.

The charging-up-rate for the black hole is given by

\[
\frac{dQ}{dt} = BH \, C_{\text{fs}} F_0 \; ;
\]

(25)

where \( BH \) is the cross section for the absorption to the black hole, \( F_0 \) is the reacted charge flux at the wall and the dimensionless parameter \( C_{\text{fs}} \) is a focusing factor. The absorption-cross-section for the Schwarzschild black hole is given by

\[
BH = \begin{cases} 
\frac{8}{3} \cdot \frac{4 r_B^2}{r_B^2 (r_B < r_D) } & (r_B < r_D) \\
\frac{27}{4} \cdot \frac{r_B^2}{r_D^2} & (r_B > r_D)
\end{cases}
\]

(26)

The cross section is given by the horizon area when the radius of the black hole \( r_B \) is similar to or a little smaller than the characteristic radius of the domain wall \( r_D \). In the case of \( r_B \leq r_D \), the reacted particles at the wall are regarded as the incoming particles from the infinite distance to the black hole, therefore we should adopt the absorption-cross-section as \( BH = \frac{27}{4} r_B^2 \) rather than the horizon area. The difference among these cross sections is only a focusing factor one and the spontaneous charging-up mechanism mainly works for \( r_B < r_D \), then we will use the horizon area as the cross section.

In the spherical reactor, any particles radiated from neighborhood of the center of the reactor return to neighborhood of the center when they are reacted. Then the reacted flux at the horizon is different from the reacted flux at the spherical wall. This is a focusing effect by the spherical reactor. The effect increases the flux at the horizon compared with the flux at the wall \( F_0 \). The flux at the horizon can be written down as \( C_{\text{fs}} F_0 \) by the focusing factor \( C_{\text{fs}} \). We have \( (r_B = r_D) C_{\text{fs}} = 1 \) when the effect maximally works and we have \( C_{\text{fs}} = 1 \) in the absence of the effect.
The charge $\nu$ at the wall is given by

$$F_Q = \sum_{s=2}^X Z_{f_2 \text{Fermions}} \left( E > m_f \right) dE \cdot N_f \left( E; r_{DW} \right) \cdot Q_f \cdot R_f \left( E \right); \quad (27)$$

The summation in the equation is taking over all species of the chiral-charged fermions and the particle species $f$ does not distinguish both the chirality of the particle and the particle/anti-particle. The number $Q_f \cdot Q_{f_L} \cdot Q_{f_R}$ means the difference between the charge of left-handed fermion $f_L$ and that of right-handed fermion $f_R$. These numbers are related to the C-violation for the theory. The value

$$R_f \left( E \right) \quad R_{f_R} ! f_L \left( E \right) \quad R_{f_L} ! f_R \left( E \right) \quad (28)$$

describes the difference of the reaction probabilities, where the reaction probability $R_{f_R} ! f_L \left( E \right)$ means a probability of that the left-handed fermion $f_R$ with energy $E$ is reacted to the right-handed fermion $f_L$ and the reaction probability $R_{f_R} ! f_L \left( E \right)$ means the same for the anti-fermions $f_R ! f_L$. These probabilities are functions for the energy of the particles $E$. The non-zero value of $R_f \left( E \right)$ is related with the CP-broken phase assumed in the wall and can be calculated in the way discussed by the Cohen Kaplan and Nelson [7,8] when we put the profile of the CP-broken phase in the wall.

To evaluate the reaction asymmetry $R_f \left( E \right)$ of the wall around the black hole, we assume the profile of the CP-broken phase in the wall:

$$h \left( r \right) = f \left( r \right) \exp \left( - \frac{f \left( r \right)}{V} \right) \quad (29)$$

where $C_P$ is the amount of the CP-broken phase in the wall and the function $f \left( r \right)$ is the profile of the Higgs field in the wall. The profile function $f \left( r \right)$ is solution of the motion equation of the Higgs field with effective potential in the equation (17) and it is depending of the parameter defined in equation (14). The wall given in equation (29) is defined for $r > r_{BH}$. By substituting equation (13) into equation (27), the charge $\nu$ becomes

$$F_Q = \frac{r_{BH}}{r_{DW}}^2 C_{CS} X_f \cdot g_f \cdot Q_f \cdot m_f^2 \cdot R \left( m_f; r_{BH} \right); \quad (30)$$

where we have defined dimensionless function depending on the wall profile

$$R \left( m_f; r_{BH} \right) = \frac{1}{8} \cdot Z_f^1 \int_{-1}^1 d^2 f_{TBH} \left( m_f \right) \cdot R_f \left( m_f \right); \quad (31)$$

We have numerically evaluated $R_f \left( E \right)$ and $R_f \left( m_f; r_{BH} \right)$ for the wall in equation (29) according to the method proposed by Cohen Kaplan and Nelson [7]. In the case of maximum CP-violated wall $C_P = \frac{1}{8}$, we obtain the dependence of the dimensionless coefficient $R_f \left( m_f; r_{BH} \right)$ in Figure 3. In this calculation we have assumed $m_f = m_f$ for the simplicity.
Figure 3: Numerical results of the dimensionless reflection asymmetry $R (m_f; T_{BH})$ for the wall given in equation (29) with $\alpha^2 = 0.1$ and $\alpha^2 = 0.01$. We have assumed $m_f = m_f$ for simplicity.
The black hole with the Hawking temperature $T_{BH}$ has a finite lifetime $t_{BH} = \frac{20}{g} \frac{m^2_{pl}}{T_{BH}^3}$, where $g$ is the total degree of freedom with a fermion correction. The total charge transported to the black hole in his lifetime by the effect of the wall is

$$Q = \int_0^{Z_{BH}} \frac{dQ}{dt} \cdot \frac{dQ}{dt} = \frac{15}{3g} m^2_{pl} X \frac{g_r}{g} Q_r m^3 f T_{BH}^3 \int_0^{T_{BH}} R (m_f, T);$$

(32)

where we have assumed the maximum focusing effect. The integration in the equation (32) can be performed by using the numerical form of $R$. The numerical results $R$ with $m_f$ in Figure 3 have meaningful value for $T_{BH} >$ and they are exponentially damped for $T_{BH} <$. Therefore we have approximately

$$Q' \approx \frac{3}{3} R (;) \frac{m^2_{pl} X}{f} \frac{g_r}{g} Q_r ;$$

(33)

where we have assumed a simplification $m_f = \ldots$ for all related fermions $f$ and we have assumed the initial Hawking temperature $T_{BH}$ is similar to or smaller than the energy scale of the field theory. Our numerical analysis results $R (;) \approx 10^4$ for $^2 = 1=100$ as the realistic field theories, then we have

$$Q' \approx 10^5 \frac{m^2_{pl} X}{f} \frac{g_r}{g} Q_r ;$$

(34)

Finally we conclude that the spherical wall can charge up the black hole non-trivially when the energy scale of the wall satisfies $< 10^3 m^2_{pl} \approx 10^{16}$ GeV.

In the case of a black hole whose Hawking temperature is the EW scale $\approx 100$ GeV, the spherical EW wall arises and the hypercharge $Y$ is transported to the black hole by assuming CP-broken phase in the wall. Mainly the top quarks, $g_r = 3$ and $Q_r = 1=2$, carry the charge to the black hole. The total transported hypercharge is given by $Q' \approx 10^{27}$, then we consider the spontaneous charging up mechanism of the black hole can work strongly. In the case of a black hole with GUT temperature, several charge can be transported to the black hole. The amount of the charge is depending on the chiral charge assignment $Q_r$ of the GUT.

4 CONCLUSION AND DISCUSSIONS

In this paper the Hawking radiation in a kind of Higgs-Yukawa system is discussed and a dynamical form of the spherical domain wall is shown. An action for the many relativistic point particle (Nambo-Goto like action) with Higgs coupling and the ordinary Higgs action with double well potential are adopted to describe the system. We expect that the system approximately describes the Higgs-Yukawa system which is consist from the ordinary Dirac.
action and the bosonic actions. Our action simplifies the analysis of the Higgs vev structure around a black hole as compared with solving the Dirac/Weyl equations.

The black hole has been assumed as an heat source with Hawking temperature in our analysis, namely, the general relativistic effects are omitted. Reliable treatments for general relativistic corrections for the Hawking-radiated particles do not have been known. For example, the blue-shift effect near the horizon for the radiated particles is arise when we consider that the radiated particles obeys the Schwarzschild metric. The blue-shift effect implies that the energy of the radiated particles near horizon is much higher than the Hawking temperature and we can discuss that any low Hawking-temperature black hole produces the domain wall dynamically or thermally, therefore, the effect is not acceptable widely.

In our calculation, we have evaluated the influence from the radiated particles into the effective potential for the Higgs field, however, the influence from the wall into the motion of the radiated particle, namely, the backreaction is neglected for simplicity. The form of the spherical wall may be slightly deformed by the backreaction. The issue of the backreaction will be future subject.

We have also discussed the mechanism of the spontaneous charging up of the black hole by the effect of the spherical domain wall. The mechanism can work when C-violation of the field theory and CP-broken phase in the wall are assumed. Our analysis results that the mechanism can work when the initial Hawking temperature of the black hole is smaller than about $10^{16}$ GeV. The spherical EW domain wall by the black hole with EW temperature can transport a large amount of hypercharge to the black hole. The black hole with GUT temperature can obtain several charge.

We have discussed the spontaneous charging up of the black hole. On the other hand, a mechanism for the charge loss of the black hole has been discussed. Gibbons first proposed this subject by the semi-classical method [10, 11] and Gabriël discussed that this result can be confirmed recently by the functional method [12]. They discussed that the charged black hole loses its charge by the pair creation of the charged particles because of the strong electric field around the black hole, namely, a kind of Schwinger process works to discharge the black hole. These calculation are reliable only for $r_{BH} > 1/m_e$ where $m_e$ is mass of electron, namely the radius of the black hole should be greater than the Compton wave length of the electron. Several authors discuss the subject for the charge loss of the smaller black hole $r_{BH} < 1/m_e$ [13, 14], however we do not have common understanding for this subject. We have discussed the spontaneous charging-up of the black hole whose radius is smaller than the EW scale, therefore we can not apply these results of the charge-loss directly to our system. This subject may be related to the remnant of the black hole. Zeldovich discussed that a black hole leaves a remnant with a Planck mass scale after the end of its Hawking radiation [15]. Both our mechanism for spontaneous charging-up and some mechanism for the charge-loss may be working at the final stage of the black hole evaporation. We may expect that a remnant with several (hyper) charge will be left after the end of the Hawking radiation.
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