Quark Confinement Physics from Quantum Chromodynamics

H. Suganuma\textsuperscript{a}, K. Amemiya\textsuperscript{a}, H. Ichie\textsuperscript{b} and A. Tanaka\textsuperscript{a}

\textsuperscript{a}Research Center for Nuclear Physics (RCNP), Osaka University
Ibaraki, Osaka 567-0047, Japan

\textsuperscript{b}Department of Physics, Tokyo Institute of Technology
Ohokayama 2-12-1, Meguro, Tokyo 152-8551, Japan

We show the construction of the dual superconducting theory for the confinement mechanism from QCD in the maximally abelian (MA) gauge using the lattice QCD Monte Carlo simulation. We find that essence of infrared abelian dominance is naturally understood with the off-diagonal gluon mass $m_{\text{off}} \simeq 1.2\text{GeV}$ induced by the MA gauge fixing. In the MA gauge, the off-diagonal gluon amplitude is forced to be small, and the off-diagonal gluon phase tends to be random. As the mathematical origin of abelian dominance for confinement, we demonstrate that the strong randomness of the off-diagonal gluon phase leads to abelian dominance for the string tension. In the MA gauge, there appears the macroscopic network of the monopole world-line covering the whole system. We investigate the monopole-current system in the MA gauge by analyzing the dual gluon field $B_\mu$. We evaluate the dual gluon mass as $m_B = 0.4 \sim 0.5\text{GeV}$ in the infrared region, which is the lattice-QCD evidence of the dual Higgs mechanism by monopole condensation. Owing to infrared abelian dominance and infrared monopole condensation, QCD in the MA gauge is describable with the dual Ginzburg-Landau theory.

1. QCD and Dual Superconducting Theory for Confinement

Since 1974, quantum chromodynamics (QCD) has been established as the fundamental theory of the strong interaction, however, it is still hard to understand the nonperturbative QCD (NP-QCD) phenomena such as color confinement and dynamical chiral-symmetry breaking, in spite of the simple form of the QCD lagrangian

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{2} \text{tr} G_{\mu\nu} G^{\mu\nu} + \bar{q}(i\gamma_\mu D^\mu - m_q)q.$$  \hfill (1)

In particular, to understand the confinement mechanism is one of the most difficult problems remaining in the particle physics. As the hadron Regge trajectory and the lattice QCD simulation show, the confinement force between the color-electric charges is characterized by the \textit{one-dimensional squeezing} of the color-electric flux and the universal physical quantity of the \textit{string tension} $\sigma \simeq 1\text{GeV/fm}$.

As for the confinement mechanism, Nambu first proposed the \textit{dual superconducting theory} for quark confinement, based on the electro-magnetic duality in 1974.\textsuperscript{1} In this theory, there occurs the one-dimensional squeezing of the color-electric flux between quarks by the
dual Meissner effect due to condensation of bosonic color-magnetic monopoles. However, there are two large gaps between QCD and the dual superconducting theory.\(^2\)

1. The dual superconducting theory is based on the abelian gauge theory subject to the Maxwell-type equations, where electro-magnetic duality is manifest, while QCD is a nonabelian gauge theory.

2. The dual superconducting theory requires condensation of color-magnetic monopoles as the key concept, while QCD does not have such a monopole as the elementary degrees of freedom.

These gaps can be simultaneously fulfilled by the use of the MA gauge fixing, which reduces QCD to an abelian gauge theory. In the MA gauge, the off-diagonal gluon behaves as a charged matter field similar to \(W_\pm\) in the Standard Model and provides a color-electric current in terms of the residual abelian gauge symmetry. As a remarkable fact in the MA gauge, color-magnetic monopoles appear as topological objects reflecting the nontrivial homotopy group \(\Pi_2(\text{SU}(N_c)/U(1)^{N_c-1}) = \mathbb{Z}^{N_c-1}\), similarly in the GUT monopole.\(^3\-\!^6\)

Thus, in the MA gauge, QCD is reduced into an abelian gauge theory including both the electric current \(j_\mu\) and the magnetic current \(k_\mu\), which is expected to provide the theoretical basis of the dual superconducting theory for the confinement mechanism.

2. MA Gauge Fixing and Extraction of Relevant Mode for Confinement

In the Euclidean QCD, the maximally abelian (MA) gauge is defined by minimizing\(^2,^6\)

\[
R_{\text{off}}[A_\mu(\cdot)] \equiv \int d^4x \text{tr} [\hat{D}_\mu, \vec{H}][\hat{D}_\mu, \vec{H}]^\dagger = \frac{e^2}{2} \int d^4x \sum_\alpha |A_\mu^\alpha(x)|^2, \tag{2}
\]

with the SU\((N_c)\) covariant derivative operator \(\hat{D}_\mu \equiv \hat{\partial}_\mu + ieA_\mu\) and the Cartan decomposition \(A_\mu(x) = \tilde{A}_\mu(x) \cdot \vec{H} + \sum_\alpha A_\mu^\alpha(x)E^\alpha\). In the MA gauge, the off-diagonal gluon components are forced to be as small as possible by the SU\((N_c)\) gauge transformation. Since the covariant derivative \(\hat{D}_\mu\) obeys the adjoint gauge transformation, the local form of the MA gauge fixing condition is derived as \(^2,^6\)

\[
[\vec{H}, [\hat{D}_\mu, [\hat{D}_\mu, \vec{H}] ]] = 0. \tag{3}
\]

(For \(N_c = 2\), this condition is equivalent to the diagonalization of \(\Phi_{\text{MA}} \equiv [\hat{D}_\mu, [\hat{D}_\mu, \tau^3]]\), and then the MA gauge is found to be a sort of the ’t Hooft abelian gauge\(^3\).) In the MA gauge, the gauge symmetry \(G \equiv \text{SU}(N_c)_{\text{local}}\) is reduced into \(H \equiv U(1)^{N_c-1}_{\text{local}} \times \text{Weyl}_\text{global}\), where the global Weyl symmetry is the subgroup of SU\((N_c)\) relating the permutation of the \(N_c\) bases in the fundamental representation.\(^2,^6\)

We summarize abelian dominance, monopole dominance and extraction of the relevant mode for NP-QCD observed in the lattice QCD in the MA gauge.

(a) Without gauge fixing, all the gluon components equally contribute to NP-QCD, and it is difficult to extract relevant degrees of freedom for NP-QCD.
(b) In the MA gauge, QCD is reduced into an abelian gauge theory including the electric current \( j_\mu \) and the magnetic current \( k_\mu \). The diagonal gluon behaves as the abelian gauge field, and the off-diagonal gluon behaves as the charged matter field. In the MA gauge, the lattice QCD shows abelian dominance for NP-QCD (confinement\(^6,7\), chiral symmetry breaking\(^8\), instantons\(^9\)) : only the diagonal gluon is relevant for NP-QCD, while off-diagonal gluons do not contribute to NP-QCD. In the lattice QCD, there appears the global network of the monopole world-line covering the whole system in the MA gauge. (See Fig.3(a).)

(c) The diagonal gluon can be decomposed into the “photon part” and the “monopole part”, corresponding to the separation of \( j_\mu \) and \( k_\mu \). In the MA gauge, the lattice QCD shows monopole dominance for NP-QCD: the monopole part \((k_\mu \neq 0, j_\mu = 0)\) leads to NP-QCD, while the photon part \((j_\mu \neq 0, k_\mu = 0)\) seems trivial like QED and does not contribute to NP-QCD. Thus, by taking the MA gauge, the relevant collective mode for NP-QCD can be extracted as the color-magnetic monopole.\(^2,10\)

3. Essence of Abelian Dominance : Off-diagonal Gluon Mass in MA Gauge

In this section, we study essence of abelian dominance for NP-QCD in the MA gauge in terms of the generation of the effective mass \( m_{\text{off}} \) of the off-diagonal (charged) gluon by the MA gauge fixing. In the SU(2) QCD partition functional, the mass generation of the off-diagonal gluon \( A^\pm_\mu \equiv (A^1_\mu \pm iA^2_\mu)/\sqrt{2} \) in the MA gauge is expressed as\(^2\)

\[
Z_{\text{QCD}}^{\text{MA}} = \int DA_\mu \exp\{iS_{\text{QCD}}[A_\mu]\} \delta(\Phi_{\text{MA}}^\pm[A_\mu]) \Delta_{\text{FP}}[A_\mu] = \int DA^3_\mu \exp\{iS_{\text{eff}}^A[A_\mu]\} \int DA^3_\mu \exp\{i \int d^4x m_{\text{off}}^2 A^\pm_\mu A^\mp_\mu \} F[A_\mu], \tag{4}
\]

with \( \Phi_{\text{MA}} \equiv [\hat{D}_\mu, [\hat{D}_\mu, \tau_3]] \), the Faddeev-Popov determinant \( \Delta_{\text{FP}} \), the abelian effective action \( S_{\text{eff}}[A^3_\mu] \) and a smooth functional \( F[A_\mu] \).

To investigate the off-diagonal gluon mass \( m_{\text{off}} \), we study the Euclidean gluon propagator \( G^{ab}_{\mu\nu}(x-y) \equiv \langle A^a_\mu(x)A^b_\nu(y) \rangle \) in the MA gauge, using the SU(2) lattice QCD.\(^2\) As for the residual U(1)\(_3\) gauge symmetry, we impose the U(1)\(_3\) Landau gauge fixing to extract most continuous gauge configuration under the MA gauge constraint and to compare with the continuum theory. The continuum gluon field \( A^a_\mu(x) \) is extracted from the link variable as \( U_\mu(s) = \exp(\imath e A^a_\mu(s) \tau^a_2) \). Here, the scalar-type gluon propagator \( G^a_{\mu\nu}(r) \equiv \sum_{\mu=1}^4 \langle A^a_\mu(x)A^a_\nu(y) \rangle \) is useful to observe the interaction range of the gluon, because it depends only on the four-dimensional Euclidean distance \( r \equiv \sqrt{(x_\mu - y_\mu)^2} \).

We show in Fig.1(a) \( G^3_{\mu\nu}(r) \) and \( G^+_{\mu\nu}(r) \equiv \sum_{\mu=1}^4 \langle A^+_{\mu}(x)A^-_{\nu}(y) \rangle = \frac{1}{2} \{G^1_{\mu\nu}(r) + G^2_{\mu\nu}(r)\} \) in the MA gauge using the SU(2) lattice QCD with \( 2.2 \leq \beta \leq 2.4 \) and the various lattice size \((12^3 \times 24, 16^3, 20^3)\). Since the massive vector-boson propagator with the mass \( M \) takes a Yukawa-type asymptotic form as \( G_{\mu\nu}(r) \sim \frac{M^{1/2}}{2\pi^2} \exp(-Mr) \), the effective mass \( m_{\text{off}} \) of the off-diagonal gluon \( A^\pm_\mu(x) \) can be evaluated from the slope of the logarithmic plot of \( r^{3/2}G^\pm_{\mu\nu}(r) \sim \exp(-m_{\text{off}}r) \) as shown in Fig.1(b). The off-diagonal gluon \( A^\pm_\mu(x) \) behaves as the massive field with \( m_{\text{off}} \simeq 1.2 \) GeV in the MA gauge for \( r \gtrsim 0.2 \) fm.

We perform also the standard mass measurement for the off-diagonal gluon in the similar manner to the hadron mass measurement in the lattice QCD. We calculate \( \Phi^\pm_\mu(\tau) \equiv \int d\epsilon \langle A^\pm_\mu(x)A^\mp_\nu(y) \rangle \exp(-\imath \epsilon \tau_3) \) as the Euclidean gluon propagator. The effective mass \( m_{\text{eff}} \) is obtained from the slope of the logarithmic plot of \( r^{-1}G_{\mu\nu}(r) \sim \exp(-m_{\text{eff}}r) \) as shown in Fig.1(c). The off-diagonal gluon \( A^\pm_\mu(x) \) behaves as the massive field with \( m_{\text{eff}} \simeq 1.2 \) GeV in the MA gauge for \( r \gtrsim 0.2 \) fm.
within the short range as

\[ \text{r} \]

and (c), the effective mass of the off-diagonal gluon \( A_{\mu}^\pm \) can be estimated as \( m_{\text{off}} \simeq 1.2 \text{GeV} \).

\[ \int d\vec{x} A_\mu^\pm (\vec{x}, \tau), \]

and measure the temporal correlation of \( \Gamma_{\mu\mu}^{\pm-} (\tau) \equiv \langle \Phi_\mu^+ (\tau) \Phi_\mu^- (0) \rangle \) in the MA gauge with the \( U(1)_3 \) Landau gauge. We obtain the off-diagonal gluon mass \( m_{\text{off}} \simeq 1.2 \text{GeV} \) again from the slope of the logarithmic plot of \( \Gamma_{\mu\mu}^{\pm-} (\tau) \) as the function of the temporal distance \( \tau \) in the lattice QCD with \( 2.3 \leq \beta \leq 2.35 \) with \( 16^3 \times 32 \) and \( 12^3 \times 24 \). From the slope of the dotted lines in (b) and (c), the effective mass of the off-diagonal gluon \( A_{\mu}^\pm \) can be estimated as \( m_{\text{off}} \simeq 1.2 \text{GeV} \).

Thus, essence of infrared abelian dominance in the MA gauge can be physically interpreted with the effective off-diagonal gluon mass \( m_{\text{off}} \) induced by the MA gauge fixing. \(^2\) Due to the effective mass \( m_{\text{off}} \simeq 1.2 \text{GeV} \), the off-diagonal gluon \( A_{\mu}^\pm \) can propagate only within the short range as \( r \lesssim m_{\text{off}}^{-1} \simeq 0.2 \text{fm} \), and cannot contribute to the infrared QCD physics in the MA gauge, which leads to abelian dominance for NP-QCD. \(^2\)

4. Randomness of Off-diagonal Gluon Phase as the Mathematical Origin of Abelian Dominance for Confinement

In the lattice QCD, the SU(2) link variable is factorized as \( U_\mu (s) = M_\mu (s) u_\mu (s) \), according to the Cartan decomposition \( \text{SU}(2)/U(1)_3 \times U(1)_3 \). Here, \( u_\mu (s) \equiv \exp \{ i \tau_3 \theta_\mu (s) \} \in U(1)_3 \) denotes the abelian link variable, and the off-diagonal factor \( M_\mu (s) \in \text{SU}(2)/U(1)_3 \) is parameterized as

\[ M_\mu (s) \equiv e^{i \tau_\mu (s)} e^{i \theta_\mu (s)} = \begin{pmatrix} \cos \theta_\mu (s) & -\sin \theta_\mu (s) e^{-i x_\mu (s)} \\ \sin \theta_\mu (s) e^{i x_\mu (s)} & \cos \theta_\mu (s) \end{pmatrix}. \tag{5} \]

In the MA gauge, the diagonal element \( \cos \theta_\mu (s) \) in \( M_\mu (s) \) is maximized by the SU(2) gauge transformation and the “abelian projection rate” becomes almost unity as \( R_{\text{Abel}} = \langle \cos \theta_\mu (s) \rangle_{\text{MA}} \simeq 0.93 \) at \( \beta = 2.4 \). Using the lattice QCD simulation, we find the two remarkable features of the off-diagonal element \( e^{i x_\mu (s)} \sin \theta_\mu (s) \) in \( M_\mu (s) \) in the MA gauge. \(^2,6\)
(1) The off-diagonal gluon amplitude $|\sin \theta_\mu(s)|$ is forced to be minimized in the MA gauge, which allows the approximate treatment on the off-diagonal gluon phase.

(2) The off-diagonal phase variable $\chi_\mu(s)$ is not constrained by the MA gauge-fixing condition at all, and tends to be random.

Therefore, $\chi_\mu(s)$ can be regarded as a random angle variable on the treatment of $M_\mu(s)$ in the MA gauge in a good approximation.

Now, we show the analytical proof of abelian dominance for the string tension or the confinement force in the MA gauge, within the random-variable approximation for $\chi_\mu(s)$ or the off-diagonal gluon phase.\(^\text{2,6}\) Here, we use $\langle e^{i\chi_\mu(s)} \rangle_{\text{MA}} \approx \int_0^{2\pi} d\chi_\mu(s) \exp\{i\chi_\mu(s)\} = 0$.

In calculating the Wilson loop $\langle W_C[U] \rangle \equiv \langle \text{tr}\Pi_C U_\mu(s) \rangle = \langle \text{tr}\Pi_C \{M_\mu(s)u_\mu(s)\} \rangle$, the off-diagonal matrix $M_\mu(s)$ is simply reduced as a $c$-number factor, $M_\mu(s) \to \cos \theta_\mu(s)$ 1, and then the SU(2) link variable $U_\mu(s)$ is reduced to be a diagonal matrix,

$$U_\mu(s) \equiv M_\mu(s)u_\mu(s) \to \cos \theta_\mu(s)u_\mu(s),$$

after the integration over $\chi_\mu(s)$. For the $R \times T$ rectangular $C$, the Wilson loop $W_C[U]$ in the MA gauge is approximated as

$$\langle W_C[U] \rangle \approx \langle \text{tr}\Pi_{L} \{M_\mu(s)u_\mu(s)\} \rangle \simeq \langle \Pi_{L} \cos \theta_\mu(s_i) \cdot \text{tr}\Pi_{L} u_\mu(s_j) \rangle_{\text{MA}}$$

$$\simeq \langle \exp\{\Sigma_{i=1}^{L} \ln(\cos \theta_\mu(s_i))\} \rangle_{\text{MA}} \langle W_C[u] \rangle_{\text{MA}},$$

with the perimeter length $L \equiv 2(R+T)$ and the abelian Wilson loop $W_C[u] \equiv \text{tr}\Pi_{L} u_\mu(s_i)$. Replacing $\Sigma_{i=1}^{L} \ln(\cos \theta_\mu(s_i))$ by its average $L\langle \ln(\cos \theta_\mu(s)) \rangle_{\text{MA}}$ in a statistical sense, we derive a formula for the off-diagonal gluon contribution to the Wilson loop as\(^\text{2,6}\)

$$W_C^{\text{off}} \equiv \langle W_C[U] \rangle / \langle W_C[u] \rangle_{\text{MA}} \approx \exp\{L\langle \ln(\cos \theta_\mu(s)) \rangle_{\text{MA}}\},$$

which provides the relation between the macroscopic quantity $W_C^{\text{off}}$ and the microscopic quantity $\langle \ln(\cos \theta_\mu(s)) \rangle_{\text{MA}}$. Using the lattice QCD, we have checked this relation for large loops, where such a statistical treatment is accurate.\(^\text{2,6}\)

In this way, the off-diagonal gluon contribution $W_C^{\text{off}}$ obeys the perimeter law in the MA gauge, and then the off-diagonal gluon contribution to the string tension vanishes as

$$\sigma_{\text{SU(2)}} - \sigma_{\text{Abel}} \simeq -2\langle \ln(\cos \theta_\mu(s)) \rangle_{\text{MA}} \lim_{R,T \to \infty} \frac{R + T}{RT} = 0.$$\(^\text{(9)}\)

Thus, abelian dominance for the string tension, $\sigma_{\text{SU(2)}} = \sigma_{\text{Abel}}$, can be demonstrated in the MA gauge within the random-variable approximation for the off-diagonal gluon phase. Also, we can predict the deviation between $\sigma_{\text{SU(2)}}$ and $\sigma_{\text{Abel}}$ as $\sigma_{\text{SU(2)}} > \sigma_{\text{Abel}}$, due to the finite size effect on $R$ and $T$ in the Wilson loop.\(^\text{2,6}\)

5. The Structure of QCD-Monopoles in terms of the Off-diagonal Gluon

Let us compare the QCD-monopole with the point-like Dirac monopole. There is no point-like monopole in QED, because the QED action diverges around the monopole. The QCD-monopole also accompanies a large abelian action density inevitably, however, owing to cancellation with the off-diagonal gluon contribution, the total QCD action is kept finite even around the QCD-monopole.\(^\text{2,6}\)
Figure 2. (a) The total probability distribution $P(S)$ on the whole lattice and (b) the probability distribution $P_k(S)$ around the monopole for SU(2) action density $S_{SU(2)}$ (dashed curve), abelian action density $S_{Abel}$ (solid curve) and off-diagonal gluon contribution $S_{off}$ (dotted curve) in the MA gauge at $\beta = 2.4$ on $16^4$ lattice. Around the QCD-monopole, large cancellation between $S_{Abel}$ and $S_{off}$ keeps the total QCD-action small. (c) The schematic figure for the QCD-monopole structure in the MA gauge. The QCD-monopole includes a large amount of off-diagonal gluons around its center.

To see this, we investigate the structure of the QCD-monopole in the MA gauge in terms of the action density using the SU(2) lattice QCD. From the SU(2) plaquette $P_{\mu\nu}^{SU(2)}(s)$ and the abelian plaquette $P_{\mu\nu}^{Abel}(s)$, we define the “SU(2) action density” $S_{SU(2)}^{\mu\nu}(s) \equiv 1 - \frac{1}{2}\text{tr} P_{\mu\nu}^{SU(2)}(s)$, the “abelian action density” $S_{Abel}^{\mu\nu}(s) \equiv 1 - \frac{1}{2}\text{tr} P_{\mu\nu}^{Abel}(s)$ and the “off-diagonal gluon contribution” $S_{off}^{\mu\nu}(s) \equiv S_{SU(2)}^{\mu\nu}(s) - S_{Abel}^{\mu\nu}(s)$. In the lattice formalism, the monopole current $k_\mu(s)$ is defined on the dual link, and there are 6 plaquettes around the monopole. Then, we consider the average over the 6 plaquettes around the dual link,

$$S(s, \mu) \equiv \frac{1}{12} \sum_{\alpha,\beta,\gamma} \sum_{m=0}^{1} |\epsilon_{\mu\alpha\beta\gamma}| S_{\alpha,\beta}(s + m\hat{\gamma}). \quad (10)$$

We show in Fig.2(b) the probability distribution of the action densities $S_{SU(2)}$, $S_{Abel}$ and $S_{off}$ around the QCD-monopole in the MA gauge. We summarize the results on the QCD-monopole structure as follows.

1. Around the QCD-monopole, both the abelian action density $S_{Abel}$ and the off-diagonal gluon contribution $S_{off}$ are largely fluctuated, and their cancellation keeps the total QCD-action density $S_{SU(2)}$ small.
2. The QCD-monopole has an intrinsic structure relating to a large amount of off-diagonal gluons $A_{\mu}^{\pm}$ around its center, similar to the ’t Hooft-Polyakov monopole.
3. At the large-distance scale, off-diagonal gluons inside the QCD-monopole become invisible, and the QCD-monopole can be regarded as the point-like Dirac monopole.
4. From the concentration of off-diagonal gluons around QCD-monopoles in the MA gauge, we can naturally understand the local correlation between monopoles and instantons. In fact, instantons tend to appear around the monopole world-line in the MA gauge, because instantons need full SU(2) gluon components for existence.
6. Lattice-QCD Evidence of Infrared Monopole Condensation

In the MA gauge, there appears the global network of the monopole world-line covering the whole system as shown in Fig.3(a), and this monopole-current system (the monopole part) holds essence of NP-QCD\(^2,8^{-10}\). We finally study the dual Higgs mechanism by monopole condensation in the NP-QCD vacuum in the MA gauge.

Since QCD is described by the “electric variable” as quarks and gluons, the “electric sector” of QCD has been well studied with the Wilson loop or the inter-quark potential, however, the “magnetic sector” of QCD is hidden and still unclear. To investigate the magnetic sector directly, it is useful to introduce the “dual (magnetic) variable” as the dual gluon field \(B^\mu\), which is the dual partner of the diagonal gluon and directly couples with the magnetic current \(k^\mu\).

Due to the absence of the electric current \(j^\mu\) in the monopole part, the dual gluon \(B^\mu\) can be introduced as the regular field satisfying \((\partial \wedge B)_{\mu \nu} = *F_{\mu \nu}\) and the dual Bianchi identity, \(\partial^{\mu}(\partial \wedge B)_{\mu \nu} = j^\nu = 0\). By taking the dual Landau gauge \(\partial^\mu B^\mu = 0\), the field equation is simplified as \(\partial^2 B^\mu = k^\mu\), and therefore we obtain the dual gluon field \(B^\mu\) from the monopole current \(k^\mu\) as

\[
B^\mu(x) = (\partial^{-2} k^\mu)(x) = -\frac{1}{4\pi^2} \int d^4y \frac{k^\mu(y)}{(x-y)^2}.
\]  

In the monopole-condensed vacuum, the dual gluon \(B^\mu\) is to be massive, and hence we investigate the dual gluon mass \(m_B\) as the evidence of the dual Higgs mechanism.

First, we put test magnetic charges in the monopole-current system in the MA gauge, and measure the inter-monopole potential \(V_M(r)\) to get information about monopole condensation. Since the dual Higgs mechanism is the screening effect on the magnetic flux, the inter-monopole potential is expected to be short-range Yukawa-type. Using the dual Wilson loop \(W_D\) as the loop-integral of the dual gluon,

\[
W_D(C) \equiv \exp\left\{i\frac{e}{2} \oint_C dx^\mu B^\mu\right\} = \exp\left\{i\frac{e}{2} \iint d\sigma_{\mu \nu} *F^{\mu \nu}\right\},
\]  

the potential between the monopole and the anti-monopole can be derived as

\[
V_M(R) = -\lim_{T \to \infty} \frac{1}{T} \ln \langle W_D(R, T) \rangle.\]  

Here, \(W_D(C)\) is the dual version of the abelian Wilson loop \(W_{Abel}(C) \equiv \exp\left\{i\frac{e}{2} \oint_C dx^\mu A^\mu\right\} = \exp\left\{i\frac{e}{2} \iint d\sigma_{\mu \nu} F^{\mu \nu}\right\}\) and we have set the test monopole charge as \(e/2\).

We show in Fig.3(b) the inter-monopole potential \(V_M(r)\) in the monopole part in the MA gauge.\(^2\) Except for the short distance, the inter-monopole potential can be almost fitted by the Yukawa potential \(V_M(r) = -(e/2)^2 e^{-m_B r}/r\), after removing the finite-size effect of the dual Wilson loop. In the MA gauge, the dual gluon mass is estimated as \(m_B \simeq 0.5\) GeV from the infrared behavior of \(V_M(r)\).

Second, we investigate also the scalar-type dual gluon propagator \(\langle B^\mu(x) B^\mu(y) \rangle_{MA}\) as shown in Fig.3(c), and estimate the dual gluon mass as \(m_B \simeq 0.4\) GeV from its large-distance behavior.

From these two tests, the dual gluon mass is evaluated as \(m_B = 0.4 \sim 0.5\) GeV, and this can be regarded as the lattice-QCD evidence for the dual Higgs mechanism by monopole condensation at the infrared scale.
Figure 3. The SU(2) lattice-QCD results in the MA gauge. (a) The monopole world-line projected into $\mathbb{R}^3$ on the $16^3 \times 4$ lattice with $\beta = 2.2$ (the confinement phase). There appears the global network of monopole currents covering the whole system. (b) The inter-monopole potential $V_M(r)$ v.s. the 3-dimensional distance $r$ in the monopole-current system on the $20^4$ lattice. The solid curve denotes the Yukawa potential with $m_B = 0.5 \text{GeV}$. The dotted curve denotes the Yukawa-type potential including the monopole-size effect. (c) The scalar-type dual-gluon correlation $\ln(r_{E}^{3/2} \langle B_{\mu}(x)B_{\mu}(y) \rangle_{\text{MA}})$ as the function of the 4-dimensional Euclidean distance $r_{E}$ on the $24^4$ lattice.

To summarize, the lattice QCD in the MA gauge exhibits infrared abelian dominance and infrared monopole condensation, and therefore the dual Ginzburg-Landau (DGL) theory can be constructed as the infrared effective theory directly based on QCD in the MA gauge.².

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