CFT adapted approach to massless fermionic fields, AdS/CFT, and fermionic conformal fields

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Abstract

Fermionic totally symmetric arbitrary spin massless fields in AdS space of dimension greater than or equal to four are studied. Using Poincaré parametrization of AdS space, CFT adapted gauge invariant formulation for such fields is developed. We demonstrate that the curvature and radial coordinate contributions to Lagrangian and gauge transformation of the AdS fields can be expressed in terms of ladder operators. Covariant and modified de Donder gauge conditions are proposed. The modified de Donder gauge leads to decoupled equations of motion which can easily be solved in terms of the Bessel function. The AdS/CFT correspondence for conformal current and shadow field and the respective normalizable and non-normalizable modes of fermionic massless AdS field is studied. The AdS field is considered by using the modified de Donder gauge which simplifies considerably the study of AdS/CFT correspondence. We show that on-shell leftover gauge symmetries of bulk massless field are related to gauge symmetries of boundary conformal current and shadow field. We compute the bulk action on solution of the Dirichlet problem and obtain two-point gauge invariant vertex of shadow field. Also we shown that the UV divergence of the two-point gauge invariant vertex gives higher-derivative action of fermionic conformal field.

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1 Introduction

Conjectured duality of conformal SYM theory and superstring theory in $AdS_5 \times S^5$ in Ref.[1] has lead to intensive and in-depth study of various interrelations between AdS field (string) dynamics and CFT. Interesting approach for the studying interrelation between AdS field (string) theories and the corresponding CFT has been proposed in Refs.[2, 3]. In Refs.[2, 3] it has been proposed that AdS field (string) theory action evaluated on the solution of field equations of motion with the Dirichlet problem corresponding to the boundary shadow field can be considered as generating function for correlation functions of the corresponding boundary CFT. In this paper, AdS field theory action evaluated on the solution of field equations of motion with the Dirichlet problem corresponding to the boundary shadow field will be referred to as effective action. Obviously, developing the methods for the computation of effective action for concrete AdS field theories is of crucial importance in studying various aspects of the AdS/CFT correspondence. In general, computation of effective action turns out to be complicated problem. One of ways to simplify analysis of AdS field (string) dynamics, and hence to simplify the computation of effective action, is based on use of the Poincaré parametrization of AdS space\(^1\). As is well known, use of the Poincaré coordinates simplifies considerably analysis of many aspects of AdS field dynamics and this is the reason why these coordinates have extensively been used for the computation of effective action. In Refs.[6, 7], we developed a Lagrangian gauge invariant formulation of arbitrary spin bosonic massless and massive AdS fields which is based on considering of AdS field dynamics in the Poincaré coordinates. Because the formulation developed in Refs.[6, 7] turns out to be very convenient for the studying AdS/CFT formulation for arbitrary spin fields (see Refs.[8, 9, 10]), we refer to this formulation as CFT adapted formulation of AdS field dynamics. The purpose of this paper is to develop CFT adapted formulation for fermionic massless arbitrary spin AdS fields and apply this formulation for the studying AdS/CFT correspondence. Our results can be summarized as follows.

i) Using the Poincaré parametrization of $AdS_{d+1}$ space, we obtain gauge invariant Lagrangian for fermionic free massless arbitrary spin AdS field. The Lagrangian is \emph{explicitly invariant with respect to boundary Poincaré symmetries}, i.e., manifest symmetries of our Lagrangian are adapted to manifest symmetries of boundary CFT. We show that all the curvature and radial coordinate contributions to our Lagrangian and gauge transformation are entirely expressed in terms of two ladder operators that depend on radial coordinate and radial derivative. Besides this, our Lagrangian and gauge transformation are similar to the ones of Stueckelberg formulation of massive field in flat $d$-dimensional space. General structure of the Lagrangian we obtained is valid for any theory that respects Poincaré symmetries. Various theories are distinguished by appropriate ladder operators.

ii) We find two gauge conditions which we refer to as modified de Donder gauge and covariant de Donder gauge. Modified de Donder gauge leads to simple gauge-fixed equations of motion. The surprise is that this gauge gives \emph{decoupled equations of motion}. To our knowledge, the covariant de Donder gauge for arbitrary spin fermionic fields has not been discussed in the earlier literature. Therefore we present our results for the covariant de Donder gauge for fermionic fields. The covariant de Donder gauge respects Lorentz symmetries of AdS space but leads to coupled equations of motion. In contrast to this, our modified de Donder gauge leads to simple decoupled equations which are easily solved in terms of the Bessel function.

iii) We use our CFT adapted formulation for the studying AdS/CFT correspondence between fermionic massless arbitrary spin AdS field and the corresponding arbitrary spin boundary conformal

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\(^1\)Studying $AdS_5 \times S^5$ superstring action [4] in Poincaré parametrization of AdS may be found in Ref.[5].
current and shadow field. Namely, we show that non-normalizable modes of arbitrary spin-$(s + \frac{1}{2})$ massless AdS field are related to arbitrary spin-$(s + \frac{1}{2})$ shadow field, while normalizable modes of arbitrary spin-$(s + \frac{1}{2})$ massless AdS field are related to arbitrary spin-$(s + \frac{1}{2})$ conformal current. We recall that, in earlier literature, the AdS/CFT correspondence between non-normalizable modes of spin-$\frac{s}{2}$ AdS field and the corresponding spin-$\frac{s}{2}$ shadow field was studied in Refs.[11, 12, 13, 14], while the AdS/CFT correspondence between non-normalizable modes of massless spin-$\frac{3}{2}$ AdS field (gravitino field) and the corresponding spin-$\frac{3}{2}$ shadow field was studied in Refs.[15, 16, 17]. The AdS/CFT correspondence for spin-$(s + \frac{1}{2})$ massless AdS field with $s > 1$ and the corresponding spin-$(s + \frac{1}{2})$ conformal current and shadow field has not been considered in the earlier literature.\(^2\)

iv) To compute 2-point effective action we use the modified de Donder gauge. As we have already said, the modified de Donder gauge leads to the simple gauge-fixed decoupled bulk equations of motion which are easily solved. These gauge-fixed equations have on-shell leftover bulk gauge symmetries. We show that these on-shell leftover bulk gauge symmetries are realized as the gauge symmetries of boundary conformal current and shadow field. This is to say that first-order equations of motion for fermionic fields and modified de Donder gauge lead to differential constraints for conformal current and shadow fields. These differential constraints are invariant under gauge transformation of conformal current and shadow field. We find Lagrangian for massless arbitrary spin fermionic fields with proper boundary term and compute the 2-point effective action. The effective action also turns out to be invariant under gauge transformation of shadow field. We give various representations for the effective action.

v) For the case of bosonic fields, it is well known that UV divergence of effective action coincides with higher-derivative action for conformal bosonic fields (for spin-2 field, see Ref.[19], while for arbitrary spin field, see Ref.[8]).\(^3\) We demonstrate that UV divergence of effective action for fermionic fields gives higher-derivative action for conformal fermionic fields. We obtain two representation for the action of conformal fermionic fields.

2 Gauge invariant action of fermionic massless AdS field

In this section, using arbitrary parametrization of AdS space, we review Lagrangian metric-like formulation of fermionic massless arbitrary spin AdS fields developed in Refs.[24, 25].\(^4\) In $(d+1)$-dimensional $AdS_{d+1}$ space, a massless totally symmetric arbitrary spin fermionic field is labelled by one half-integer spin label $s + \frac{1}{2}$, where $s > 0$ is an integer number. To discuss Lorentz covariant and gauge invariant formulation of such field we introduce Dirac complex-valued tensor-spinor field of the $so(d, 1)$ Lorentz algebra,

$$\Psi^{A_1...A_s\alpha},$$

where $A = 0, 1, \ldots, d$ are flat vector indices of the $so(d, 1)$ algebra.

\(^2\)Scaling properties of 2-point effective action for arbitrary spin fermionic field in $AdS_5$ were studied in Ref.[18].

\(^3\)For discussion of $N = 4$ conformal supergravity, see Refs.[20, 21]. In the framework of the AdS/CFT correspondence, recent interesting discussion of conformal fields may be found in Refs.[22, 23].

\(^4\)In the framework of metric-like approach, massless fermionic fields in $AdS_4$ were studied in Ref.[24], while massless fermionic fields in $AdS_{d+1}$, with $d \geq 3$, were studied in Ref.[25]. Frame-like approach to massless fermionic fields was discussed in Ref.[26] (see also Ref.[27]-[31]). In Ref.[32], massless fermionic fields were studied by using the radial dimensional reduction method in Ref.[33]. In the framework of BRST approach, discussion of fermionic fields may be found in Refs.[33]-[39].
The tensor-spinor field $\Psi^{A_1\ldots A_s}$ is subject to the basic algebraic constraint
\begin{equation}
\gamma^A \Psi^{ABBA_4\ldots A_s} = 0 ,
\end{equation}
which tells us that the tensor-spinor field $\Psi^{A_1\ldots A_s}$ is a reducible representation of the Lorentz algebra $so(d, 1)$\textsuperscript{5}. Note that for $s = 0, 1, 2$ the constraint (2.2) is satisfied automatically.

In order to obtain the gauge invariant description of the massless field in an easy-to-use form, let us introduce the creation and annihilation operators $\alpha^A$ and $\bar{\alpha}^A$ defined by the relations\textsuperscript{6}
\begin{equation}
[\bar{\alpha}^A, \alpha^B] = \eta^{AB}, \quad \bar{\alpha}^A |0\rangle = 0 ,
\end{equation}
where $\eta^{AB}$ is the mostly positive flat metric tensor. The oscillators $\alpha^A$, $\bar{\alpha}^A$ transform in the vector representations of the $so(d, 1)$ Lorentz algebra. The tensorial component of the tensor-spinor field (2.1) can be collected into a ket-vector $|\Psi\rangle$ defined by
\begin{equation}
|\Psi\rangle \equiv \frac{1}{s!} \alpha^{A_1} \ldots \alpha^{A_s} \Psi^{A_1\ldots A_s} |0\rangle.
\end{equation}
Here and below spinor indices of ket-vectors are implicit. The ket-vector $|\Psi\rangle$ (2.4) satisfies the constraint
\begin{equation}
(N_\alpha - s)|\Psi\rangle = 0 , \quad N_\alpha \equiv \alpha^A \bar{\alpha}^A ,
\end{equation}
which tells us that $|\Psi\rangle$ is a degree-$s$ homogeneous polynomial in the oscillator $\alpha^A$. We note also, that, in terms of the ket-vector $|\Psi\rangle$ (2.4), the algebraic constraint (2.2) takes the form
\begin{equation}
\gamma \bar{\alpha} \bar{\alpha}^2 |\Psi\rangle = 0 ,
\end{equation}
\begin{equation}
\gamma \bar{\alpha} \equiv \gamma^A \bar{\alpha}^A , \quad \bar{\alpha}^2 \equiv \bar{\alpha}^A \bar{\alpha}^A .
\end{equation}

Action and Lagrangian for the massless fermionic field in $AdS_{d+1}$ space take the form
\begin{equation}
S = \int d^{d+1}x \mathcal{L} ,
\end{equation}
\begin{equation}
i e^{-1} \mathcal{L} = \langle \Psi | E | \Psi \rangle ,
\end{equation}
\begin{equation}
E \equiv E_{(1)} + E_{(0)} ,
\end{equation}
\begin{equation}
E_{(1)} \equiv \mathcal{D} - \alpha \mathcal{D} \gamma \bar{\alpha} - \gamma \alpha \bar{\alpha} \mathcal{D} + \gamma \alpha \bar{\alpha} + \frac{1}{2} \gamma \alpha \bar{\alpha} \alpha^2 + \frac{1}{2} \alpha^2 \gamma \alpha \bar{\alpha} \mathcal{D} - \frac{1}{4} \alpha^2 \mathcal{D} \alpha^2 ,
\end{equation}
\begin{equation}
E_{(0)} \equiv (1 - \gamma \alpha \gamma \bar{\alpha} - \frac{1}{4} \alpha^2 \bar{\alpha}^2 ) e_1^\Gamma ,
\end{equation}
\begin{equation}
e_1^\Gamma \equiv s + \frac{d - 3}{2} ,
\end{equation}
\textsuperscript{5}Constraint (2.2) was introduced in Ref.[40] while study of massless fermionic fields in flat space. This constraint implies that the field $|\Psi\rangle$ being reducible representation of the Lorentz algebra $so(d, 1)$ is decomposed into spin $s + \frac{1}{2}$, $s - \frac{1}{2}$ irreps of the Lorentz algebra. Various Lagrangian formulations in terms of unconstrained fields in flat space and AdS space may be found e.g., in Refs.[41]-[45].
\textsuperscript{6}We use oscillator formulation to handle the many indices appearing for arbitrary spin fields (see e.g., Refs.[46, 47]).
where $\langle \Psi |$ is defined according the rule $\langle \Psi | = (|\Psi \rangle)^\dagger \gamma^0$. We use $\epsilon \equiv \det e^A_\mu$, where $e^A_\mu$ is vielbein of $AdS_{d+1}$ space. We use the notation

$$
\gamma \alpha \equiv \gamma^A \alpha^A, \quad \gamma \bar{\alpha} \equiv \gamma^A \bar{\alpha}^A, \quad \alpha^2 \equiv \alpha^A \alpha^A, \quad \bar{\alpha}^2 \equiv \bar{\alpha}^A \bar{\alpha}^A, \quad (2.14)
$$

$$
\mathfrak{P} \equiv \gamma^A D^A, \quad \alpha D \equiv \alpha^A D^A, \quad \bar{\alpha} D \equiv \bar{\alpha}^A D^A, \quad D_A \equiv e^A_\mu D_\mu, \quad (2.15)
$$

and $e^A_\mu$ stands for inverse vielbein of $AdS_{d+1}$ space, while $D_\mu$ stands for the Lorentz covariant derivative

$$
D_\mu \equiv \partial_\mu + \frac{1}{2} \omega^A_\mu M^{AB}. \quad (2.16)
$$

The $\omega^A_\mu$ is the Lorentz connection of $AdS_{d+1}$ space, while a spin operator $M^{AB}$ forms a representation of the Lorentz algebra $so(d, 1)$:

$$
M^{AB} = M^{AB}_{\text{bos}} + \frac{1}{2} \gamma^{AB}, \quad (2.17)
$$

$$
M^{AB}_{\text{bos}} \equiv \alpha^A \bar{\alpha}^B - \alpha^B \bar{\alpha}^A, \quad \gamma^{AB} \equiv \frac{1}{2} (\gamma^A \gamma^B - \gamma^B \gamma^A). \quad (2.18)
$$

Now we discuss gauge symmetries of the action in (2.8). To this end we introduce parameter of gauge transformations $\Xi^{A_1...A_{s-1}\alpha}$, which is $\gamma$-traceless (for $s' > 0$) Dirac complex-valued tensor-spinor spin-$(s - \frac{1}{2})$ field of the $so(d, 1)$ Lorentz algebra,

$$
\Xi^{A_1...A_{s-1}\alpha}, \quad \gamma^A \Xi^{A_2...A_{s-1}} = 0, \quad \text{for} \quad s > 0. \quad (2.19)
$$

As before to simplify our expressions we use the ket-vector of gauge transformations parameter

$$
|\Xi \rangle \equiv \frac{1}{(s - 1)!} \alpha^{A_1} \ldots \alpha^{A_{s-1}} \Xi^{A_1...A_{s-1}\alpha} |0 \rangle. \quad (2.20)
$$

The ket-vector $|\Xi \rangle$ satisfies the algebraic constraints

$$
(N_\alpha - s + 1) |\Xi \rangle = 0, \quad (2.21)
$$

$$
\gamma \alpha |\Xi \rangle = 0. \quad (2.22)
$$

The constraint (2.21) tells us that the ket-vector $|\Xi \rangle$ is a degree-$(s - 1)$ homogeneous polynomial in the oscillators $\alpha^A$, while the constraint (2.22) respects the $\gamma$-tracelessness of $\Xi^{A_1...A_{s-1}}$ (2.19).

Now the gauge transformations under which the action (2.8) is invariant take the form

$$
\delta |\Psi \rangle = G |\Xi \rangle, \quad (2.23)
$$

$$
G \equiv \alpha D + \frac{1}{2} \gamma \alpha. \quad (2.24)
$$

### 2.1 Covariant de Donder gauge condition

Below, for the study of AdS/CFT correspondence, we will use modified de Donder gauge. In this section we would like to discuss covariant de Donder gauge condition which to our knowledge has
not been discussed in the earlier literature. To this end we consider equations of motion for the fermionic field $|\Psi\rangle$ obtained from Lagrangian given in (2.9),

$$E|\Psi\rangle = 0,$$  \hspace{1cm} (2.25)

where operator $E$ is given in (2.10). It is easy to make sure that equations (2.25) amount to the following equations

$$\hat{E}|\Psi\rangle = 0,$$  \hspace{1cm} (2.26)
$$\hat{A}_e|\Psi\rangle = 0,$$  \hspace{1cm} (2.27)
$$\hat{B}_e|\Psi\rangle = 0,$$  \hspace{1cm} (2.28)

where operator $E$ is given in (2.10). Equations (2.26) turn out to be more convenient for the derivation of second-order equations. This is to say that by acting with operator $\mathcal{D}$ on the l.h.s of equations (2.26) we obtain second-order equations for fermionic fields which can be cast into the form

$$\left(\mathcal{D}^2 - G\tilde{C}_{\text{cov}} - e_i^e e_i^e - \alpha^2\bar{\alpha}^2\right)|\Psi\rangle = 0,$$  \hspace{1cm} (2.32)

where operator $G$ is defined in (2.24). For the reader convenience, we note the relation

$$\mathcal{D}^2|\Psi\rangle = \left(\Box_{\text{AdS}} + s + \frac{d(d+1)}{4} - \gamma\alpha\bar{\alpha}\right)|\Psi\rangle,$$  \hspace{1cm} (2.35)
$$\Box_{\text{AdS}} \equiv D^A D^A + \omega^{AAB} D^B.$$  \hspace{1cm} (2.36)

Gauge invariant equations of motion (2.32) motivate us to introduce a gauge condition which we refer to as covariant de Donder gauge

$$\tilde{C}_{\text{cov}}|\Psi\rangle = 0,$$  \hspace{1cm} (2.37) \hspace{1cm} \text{covariant de Donder gauge,}

where the operator $\tilde{C}_{\text{cov}}$ is defined in (2.33), (2.34).

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7For bosonic fields discussion of the standard de Donder gauge may be found e.g., in Refs.[48]. Recent interesting discussion of modified de Donder gauge may be found in Ref.[49]. We believe that our covariant and modified de Donder gauge conditions will also be useful for better understanding of various aspects of AdS/QCD correspondence which are discussed, e.g., in Refs.[50].
Use of the covariant de Donder gauge (2.37) in gauge invariant equations (2.32) leads to the following second-order gauge-fixed equations

\[
\left( D^2 - e^r_i e^r_i - \alpha^2 \bar{\alpha}^2 \right) \langle \Psi \rangle = 0.
\]  

(2.38)

Thus, we see that the use of covariant de Donder gauge leads to simple gauge-fixed equations of motion. Note however that equations (2.38) are coupled equations. Decoupled equations can be obtained by using CFT adapted approach and modified de Donder gauge which we discuss in next Sections.

The following remarks are in order.

i) We note that gauge-fixed second-order equations of motion (2.38) have on-shell leftover gauge symmetries. These on-shell leftover gauge symmetries can simply be obtained from generic gauge symmetries (2.23) by the substitution \( |\Xi\rangle \rightarrow |\Xi_{lf-ov}\rangle \), where the \(|\Xi_{lf-ov}\rangle\) satisfies the following equations of motion:

\[
\left( \Box_{\text{AdS}} - (s - 1)(s + d - 1) - \frac{d + 1}{4} \right) |\Xi_{lf-ov}\rangle = 0.
\]  

(2.39)

ii) Covariant de Donder gauge condition (2.37) respects algebraic constraint (2.22) only on-shell. In other words, the relation \( \gamma \bar{\alpha} \bar{C}_{\text{cov}} |\Psi\rangle = 0 \) is valid only by using first-order equations of motion (2.28). This can easily be seen by noticing the relation

\[
\gamma \bar{\alpha} \bar{C}_{\text{cov}} |\Psi\rangle = \bar{B}_e |\Psi\rangle.
\]  

(2.40)

It easy to obtain off-shell extension of gauge condition (2.37). This is to say that by considering \( \gamma \)-traceless part of gauge condition (2.37), we obtain the following off-shell covariant de Donder gauge condition:

\[
\bar{C}_{\text{cov}}^{\text{off-sh}} |\Psi\rangle = 0,
\]  

(2.41)

\[
\bar{C}_{\text{cov}}^{\text{off-sh}} \equiv \bar{C}_{\text{cov}} - \frac{1}{2e^r_i} \gamma \bar{\alpha} \bar{B}_e,
\]  

(2.42)

where \( \bar{C}_{\text{cov}} \) is given in (2.33).

### 3 CFT adapted Lagrangian and gauge symmetries

We now discuss CFT adapted approach to fermionic massless arbitrary spin-\((s + \frac{1}{2})\) AdS field. To discuss Lorentz covariant and gauge invariant formulation of such field we introduce Dirac complex-valued tensor-spinor fields of the \( so(d - 1, 1) \) Lorentz algebra \( \psi^{a_1...a_s'\alpha} \), \( s' = 0, 1, \ldots, s \) (where \( a = 0, 1, \ldots, d - 1 \) are flat vector indices of the \( so(d - 1, 1) \) algebra), i.e. we start with the following collection of the tensor-spinor fields:

\[
\sum_{s'=0}^{s} \bigoplus \psi^{a_1...a_{s'}\alpha}.
\]  

(3.1)

\(^8\)Fields in (3.1) are obtained from the field in (2.1) by the invertible transformation.
Note that with respect to the spinor index $\alpha$ we prefer to use nomenclature of $so(d, 1) \,$ algebra. This is to say that the spinor index $\alpha$ takes values $\alpha = 1, \ldots, 2^{[(d+1)/2]} \,$ and the tensor-spinor fields $\psi^{a_1 \ldots a_s' \alpha} \,$ can be presented as 2-vectors

$$\psi^{a_1 \ldots a_s' \alpha} = \begin{pmatrix} \psi_u^{a_1 \ldots a_s' \alpha} \\ \psi_d^{a_1 \ldots a_s' \alpha} \end{pmatrix} , \quad (3.2)$$

where $\psi_u^{a_1 \ldots a_s' \alpha}, \psi_d^{a_1 \ldots a_s' \alpha} \,$ are subject to constraints

$$\begin{align*}
(1 - \gamma^z)\psi_u^{a_1 \ldots a_s'} &= 0 , \\
(1 + \gamma^z)\psi_d^{a_1 \ldots a_s'} &= 0 .
\end{align*} \quad (3.3)$$

We note also that we use $\gamma$-matrices $\gamma^A = \gamma^a, \gamma^z \,$ which are $2^{[(d+1)/2]} \times 2^{[(d+1)/2]} \,$ matrices of $so(d, 1) \,$ algebra.

The tensor-spinor fields $\psi^{a_1 \ldots a_s' \alpha} \,$ are symmetric with respect to vector indices of the $so(d-1, 1) \,$ algebra $a_1 \ldots a_s' \,$ and subject to the basic algebraic constraints

$$\gamma^a \psi^{ab a_1 \ldots a_s'} = 0 , \quad s' = 3, 4, \ldots, s . \quad (3.4)$$

In order to obtain the gauge invariant description of a massless field in an easy–to–use form, let us introduce a set of the creation and annihilation operators $\alpha^a, \alpha^z$ and $\bar{\alpha}^a, \bar{\alpha}^z \,$ defined by the relations

$$[\bar{\alpha}^a, \alpha^b] = \eta^{ab} , \quad [\bar{\alpha}^z, \alpha^z] = 1 , \quad \bar{\alpha}^a |0\rangle = 0 , \quad \bar{\alpha}^z |0\rangle = 0 , \quad (3.5)$$

where $\eta^{ab} \,$ is the mostly positive flat metric tensor. The oscillators $\alpha^a, \bar{\alpha}^a$ and $\alpha^z, \bar{\alpha}^z \,$ transform in the respective vector and scalar representations of the $so(d-1, 1) \,$ Lorentz algebra. Tensorial components of the tensor-spinor fields (3.1) can be collected into a ket-vector $|\psi\rangle \,$ defined by

$$|\psi\rangle \equiv \sum_{s'=0}^s \frac{\alpha_z^{s-s'}}{(s-s')!} |\psi^{s'}\rangle , \quad (3.6)$$

$$|\psi^{s'}\rangle \equiv \frac{1}{s'!} \alpha^a_1 \ldots \alpha^{a_{s'}} \psi^{a_1 \ldots a_{s'} \alpha} |0\rangle . \quad (3.7)$$

The ket-vectors $|\psi^{s'}\rangle \,$ (3.7) satisfy the constraints

$$\begin{align*}
(N_\alpha - s') |\psi^{s'}\rangle &= 0 , \quad s' = 0, 1, \ldots, s , \quad N_\alpha \equiv \alpha^a \bar{\alpha}^a , \quad (3.8) \\
\gamma \bar{\alpha} \bar{\alpha}^2 |\psi^{s'}\rangle &= 0 , \quad s' = 3, 4, \ldots, s . \quad (3.9)
\end{align*}$$

Constraints (3.8) tell us that $|\psi^{s'}\rangle \,$ is a degree-$s'$ homogeneous polynomial in the oscillator $\alpha^a, \,$ while constraints (3.9) amount to the ones in (3.4). Note that for $s' = 0, 1, 2 \,$ the constraints (3.9) are satisfied automatically. In terms of the ket-vector $|\psi\rangle \,$ (3.6), the algebraic constraints (3.8),(3.9) take the form

$$\begin{align*}
(N_\alpha + N_z - s) |\psi\rangle &= 0 , \quad N_\alpha \equiv \alpha^a \bar{\alpha}^a , \quad N_z \equiv \alpha^z \bar{\alpha}^z , \quad (3.10) \\
\gamma \bar{\alpha} \bar{\alpha}^2 |\psi\rangle &= 0 . \quad (3.11)
\end{align*}$$

Equation (3.10) tells us that $|\psi\rangle \,$ is a degree-$s$ homogeneous polynomial in the oscillators $\alpha^a, \alpha^z$. 


Using Poincar’e parametrization of AdS, we find CFT adapted action and Lagrangian for the massless fermionic field in $AdS_{d+1}$.

$$S = \int d^d x dz \mathcal{L} ,$$  
(3.12)

$$i\mathcal{L} = \langle \psi | E | \psi \rangle ,$$  
(3.13)

$$E \equiv E^{(1)} + E^{(0)} ,$$  
(3.14)

$$E^{(1)} \equiv \bar{\phi} - \alpha \partial \gamma \bar{\alpha} - \gamma \alpha \partial \bar{\alpha} + \frac{1}{2} \gamma \alpha \alpha \partial \bar{\alpha}^2 + \frac{1}{2} \alpha^2 \gamma \bar{\alpha} \partial - \frac{1}{4} \alpha^2 \bar{\alpha}^2 ,$$  
(3.15)

$$E^{(0)} = (1 - \gamma \alpha \bar{\gamma} \bar{\alpha} - \frac{1}{4} \alpha^2 \bar{\alpha}^2) e^r_1 + (\gamma \alpha - \frac{1}{2} \alpha^2 \bar{\alpha}) \bar{e}_1 + (\bar{\gamma} \bar{\alpha} - \frac{1}{2} \alpha \bar{\alpha}^2) e_1 ,$$  
(3.16)

$$e^r_1 = e^r_{1,1} (\sigma_- T_{\nu} - \frac{1}{2} - T_{\nu} + \frac{1}{2} \sigma_+ ) ,$$  
(3.17)

$$e_1 = e_{1,1} T_{\nu} - \frac{1}{2} , \quad \bar{e}_1 = T_{\nu} + \frac{1}{2} \bar{e}_{1,1} ,$$  
(3.18)

$$e_{1,1} = - \alpha^s \bar{e}_1 , \quad \bar{e}_{1,1} = - \bar{e}_1 \alpha^s , \quad \bar{\alpha} = \left( \frac{2s + d - 3 - N_z}{2s + d - 4 - 2N_z} \right)^{1/2} ,$$  
(3.19)

$$e^r_{1,1} = \frac{2s + d - 2}{2s + d - 2 - 2N_z} ,$$  
(3.20)

$$T_{\nu} \equiv \partial_z + \nu \frac{z}{s} , \quad \bar{\phi} \equiv \gamma^a \partial^a ,$$  
(3.21)

$$\nu \equiv s + \frac{d - 3}{2} - N_z + \frac{1}{2} \sigma_3 ,$$  
(3.22)

where $\sigma_3$ is the Pauli matrix, while $2 \times 2$ matrices $\sigma_{\pm}$ are defined in (A.9). We note that operator $e^r_1$ (3.17) can be represented as

$$e^r_1 = \sigma_- e^r_{1,1} \left( \partial_z + \frac{1}{s} (s + \frac{d - 3}{2} - N_z) \right) + \sigma_+ e^r_{1,1} \left( - \partial_z + \frac{1}{z} (s + \frac{d - 3}{2} - N_z) \right) ,$$  
(3.23)

**Gauge symmetries.** Now we discuss gauge symmetries of the action given in (3.12). To this end we introduce parameters of gauge transformations $\xi^{a_1...a_{s'}}$, $s' = 0, 1, \ldots, s - 1$ which are $\gamma$-traceless (for $s' > 0$) Dirac complex-valued tensor-spinor fields of the $so(d-1, 1)$ Lorentz algebra, i.e., we start with a collection of the tensor-spinor fields

$$\sum_{s' = 0}^{s-1} \oplus \xi^{a_1...a_{s'}} , \quad \gamma^a \xi^{a_2...a_{s'}} = 0 , \quad \text{for } s' > 0 .$$  
(3.24)

As in (3.2), we assume that the parameter $\xi^{a_1...a_{s'}}$ is presented as 2-vector

$$\xi^{a_1...a_{s'}} \equiv \begin{pmatrix} \xi^{a_1...a_{s'}}_u \\ \xi^{a_1...a_{s'}}_d \end{pmatrix} ,$$  
(3.25)

$$\left( 1 - \gamma^z \right) \xi^{a_1...a_{s'}}_s = 0 , \quad \left( 1 + \gamma^z \right) \xi^{a_1...a_{s'}}_d = 0 .$$  
(3.26)
As before to simplify our expressions we use the ket-vector of gauge transformations parameter

\[ |\xi\rangle \equiv \sum_{s'=0}^{s-1} \frac{\alpha_5^{s-1-s'}}{(s-1-s')!} |\xi^{s'}\rangle, \quad (3.27) \]

\[ |\xi^{s'}\rangle \equiv \frac{1}{s'!} \alpha^{a_1} \ldots \alpha^{a_{s'}} \xi^{a_1 \ldots a_{s'} \alpha} |0\rangle. \quad (3.28) \]

The ket-vector \(|\xi\rangle\) satisfies the algebraic constraints

\[ (N_\alpha + N_\xi - s + 1) |\xi\rangle = 0, \quad (3.29) \]

\[ \gamma \bar{\alpha} |\xi\rangle = 0. \quad (3.30) \]

The constraint (3.29) tells us that the ket-vector \(|\xi\rangle\) is a degree-\((s-1)\) homogeneous polynomial in the oscillators \(\alpha^a, \bar{\alpha}^\alpha\), while the constraint (3.30) respects the \(\gamma\)-tracelessness of \(|\xi\rangle\).

Now the gauge transformations under which the action (3.12) is invariant take the form

\[ \delta |\psi\rangle = G |\xi\rangle, \quad (3.31) \]

\[ G \equiv \alpha \partial - e_1 + \gamma \alpha \frac{1}{2N_\alpha + d - 2} e_1^r - \alpha^2 \frac{1}{2N_\alpha + d} \bar{e}_1, \quad (3.32) \]

where operators \(e_1, \bar{e}_1, e_1^r\) are given in (3.17), (3.18).

### 3.1 Global \(so(d, 2)\) symmetries of CFT adapted action

Relativistic symmetries of \(AdS_{d+1}\) space are described by the \(so(d, 2)\) algebra. In CFT adapted approach, the fermionic massless spin-\((s + \frac{1}{2})\) AdS field is described by the set of the \(so(d - 1, 1)\) algebra fields given in (3.1). Therefore it is reasonable to represent the \(so(d, 2)\) algebra so that to respect manifest \(so(d - 1, 1)\) symmetries. For application to the AdS/CFT correspondence, most convenient form of the \(so(d, 2)\) algebra that respects the manifest \(so(d - 1, 1)\) symmetries is provided by nomenclature of the conformal algebra. This is to say that the \(so(d, 2)\) algebra consists of translation generators \(P^a\), conformal boost generators \(K^a\), dilatation generator \(D\), and generators \(J^{ab}\) which span \(so(d - 1, 1)\) algebra. We use the following normalization for commutators of the \(so(d, 2)\) algebra generators:

\[ [D, P^a] = -P^a, \quad [P^a, J^{bc}] = \eta^{ab} P^c - \eta^{ac} P^b, \quad (3.33) \]

\[ [D, K^a] = K^a, \quad [K^a, J^{bc}] = \eta^{ab} K^c - \eta^{ac} K^b, \quad (3.34) \]

\[ [P^a, K^b] = \eta^{ab} D - J^{ab}, \quad (3.35) \]

\[ [J^{ab}, J^{ce}] = \eta^{bc} J^{ae} + 3 \text{ terms}. \quad (3.36) \]

Requiring \(so(d, 2)\) symmetries implies that the action is invariant with respect to transformation \(\delta_\xi |\phi\rangle = \bar{G} |\phi\rangle\), where the realization of \(so(d, 2)\) algebra generators \(\bar{G}\) in terms of differential operators takes the form\(^9\)

\[ P^a = \partial^a, \quad J^{ab} = x^a \partial^b - x^b \partial^a + M^{ab}, \quad (3.37) \]

\(^9\)In our approach only \(so(d - 1, 1)\) symmetries are realized manifestly. The \(so(d, 2)\) symmetries could be realized manifestly by using ambient space approach (see e.g. Refs.\([51]-[53]\)).
\[ D = x \partial + \Delta, \quad \Delta \equiv z \partial_z + \frac{d}{2}, \quad (3.38) \]
\[ K^a = -\frac{1}{2} x^2 \partial^a + x^a D + M^{ab} x^b + R^a, \quad (3.39) \]

\[ x \partial \equiv x^a \partial^a, \quad x^2 \equiv x^a x^a. \]

In (3.37),(3.39), \( M^{ab} \) is spin operator of the \( so(d - 1, 1) \) algebra. Commutation relations for \( M^{ab} \) and representation of \( M^{ab} \) on space of ket-vector \( |\psi\rangle \) (3.6) take the form

\[ [M^{ab}, M^{ce}] = \eta^{bc} M^{ae} + 3 \text{ terms}, \quad (3.40) \]
\[ M^{ab} = M^{ab}_{\text{bos}} + \frac{1}{2} \gamma^{ab}, \quad (3.41) \]
\[ M^{ab}_{\text{bos}} \equiv \alpha^a \bar{\alpha}^b - \alpha^b \bar{\alpha}^a, \quad \gamma^{ab} \equiv \frac{1}{2} (\gamma^a \gamma^b - \gamma^b \gamma^a). \quad (3.42) \]

Operator \( R^a \) appearing in \( K^a \) (3.39) is given by

\[ R^a = R^a_{(0)} + R^a_{(1)}, \quad (3.43) \]
\[ R^a_{(0)} = r^a_{0,1} \tilde{Y}^a + r_{0,1} \tilde{A}^a, \quad (3.44) \]
\[ R^a_{(1)} = r_{1,1} \partial^a, \quad (3.45) \]

\[ \tilde{A}^a \equiv \alpha^a - \gamma \alpha \gamma^a \quad \frac{1}{2 N_\alpha + d - 2} - \frac{1}{2 N_\alpha + d} \tilde{\alpha}^a, \quad (3.46) \]
\[ \tilde{Y}^a \equiv \gamma^a - \gamma \alpha \quad \frac{\gamma}{2 N_\alpha + d - 2} - \frac{\gamma}{2 N_\alpha + d} \tilde{\alpha}^a, \quad (3.47) \]
\[ r^r_{0,1} = -\frac{i}{2} z e^r_{1,1} \sigma_2, \quad r_{0,1} \equiv z e_{1,1}, \quad \bar{r}_{0,1} = -z \bar{e}_{1,1}, \quad r_{1,1} = -\frac{1}{2} z^2, \quad (3.48) \]

where \( e_{1,1}, \bar{e}_{1,1}, e^r_{1,1} \) are given in (3.19), (3.20). We note the following interesting relations between operators \( e_1, \bar{e}_1, e^r_1 \) and \( r_{0,1}, \bar{r}_{0,1}, r^r_{0,1} \),

\[ r^r_{0,1} = \frac{1}{2} [r_{1,1}, e^r_1], \quad \bar{r}_{0,1} = [\bar{e}_1, r_{1,1}], \quad r_{0,1} = [r_{1,1}, e_1]. \quad (3.49) \]

We see that realization of Poincaré symmetries on bulk AdS fields (3.37) coincide with realization of Poincaré symmetries on boundary CFT operators. Note that realization of \( D^- \) and \( K^a \)-symmetries on bulk AdS fields (3.38),(3.39) coincides, by module of contributions of operators \( \Delta \) and \( R^a \), with the realization of \( D^- \) and \( K^a \)-symmetries on boundary CFT operators. Realizations of the \( so(d, 2) \) algebra on bulk AdS fields and boundary CFT operators are distinguished by \( \Delta \) and \( R^a \). The realization of the \( so(d, 2) \) symmetries on bulk AdS fields given in (3.37)-(3.39) turns out to be very convenient for studying the AdS/CFT correspondence.

### 3.2 Modified de Donder gauge

We begin with discussion of the various forms of equations of motion obtained from Lagrangian given in (3.13). First of all we note that, because operator \( E \) (3.14) respects the constraint given in
(3.11), equations of motion obtained from (3.13) take the form
\[ E|\psi\rangle = 0 . \] (3.50)

It is easy to check that equations (3.50) amount to the following equations
\[ \hat{E}|\psi\rangle = 0 , \] (3.51)
\[ \bar{A}_e|\psi\rangle = 0 , \] (3.52)
\[ \bar{B}_e|\psi\rangle = 0 , \] (3.53)

where we use the notation
\begin{align*}
\hat{E} &= \varnothing - G\gamma\bar{\alpha} + \left(1 + \gamma\alpha\frac{2}{2N_\alpha + d - 2}\gamma\bar{\alpha}\right)e_1^r - \gamma\alpha\frac{2}{2N_\alpha + d - 2}\bar{e}_1 , \tag{3.54} \\
\bar{A}_e &= \bar{A} + \frac{1}{2N_\alpha + d - 2}\left((2N_\alpha + d - 1)\gamma\bar{\alpha} - \frac{1}{2}\gamma\alpha\bar{\alpha}^2\right)e_1^r \\
&\quad + \frac{1}{2}\bar{\alpha}^2e_1 - \frac{1}{2N_\alpha + d - 2}\left(2N_\alpha + d - 2\gamma\alpha\gamma\bar{\alpha} - \frac{1}{2}\alpha^2\bar{\alpha}^2\right)\bar{e}_1 , \tag{3.55} \\
\bar{B}_e &= \bar{B} - \frac{2N_\alpha + d + 2}{2(2N_\alpha + d)}\bar{\alpha}^2e_1^r + \frac{1}{2N_\alpha + d}\left(\gamma\alpha\bar{\alpha}^2 - (2N_\alpha + d - 2)\gamma\bar{\alpha}\right)\bar{e}_1 , \tag{3.56} \\
\bar{A} &\equiv \bar{\alpha}\varnothing - \varnothing\gamma\bar{\alpha} - \frac{1}{2}\alpha\partial\bar{\alpha}^2 , \tag{3.57} \\
\bar{B} &\equiv \bar{\alpha}\partial\gamma\bar{\alpha} - \frac{1}{2}\varnothing\alpha^2 . \tag{3.58} 
\end{align*}

We now discuss second-order equations for fermionic fields. To this end we note that gauge invariant first-order equations (3.50) lead to the following gauge invariant second-order equations of motion:
\[ \left(\Box - M^2 - GC_{\text{mod}}^{\text{on-sh}}\right)|\psi\rangle = 0 , \] (3.59)
\[ M^2 \equiv -\partial_z^2 + \frac{1}{z^2}(\nu^2 - \frac{1}{4}) , \tag{3.60} \]
\[ C_{\text{mod}}^{\text{on-sh}} \equiv \bar{C}_{\text{st}} - \frac{1}{2N_\alpha + d - 2}\left(\gamma\bar{\alpha} + \frac{1}{2}\gamma\alpha\bar{\alpha}^2\right)e_1^r + \frac{1}{2}\bar{\alpha}^2e_1 - \frac{2N_\alpha + d - 4}{2N_\alpha + d - 2}\Pi^{[1,2]}_{\text{bos}}\bar{e}_1 , \tag{3.61} \]
\[ \bar{C}_{\text{st}} \equiv \bar{\alpha}\varnothing - \frac{1}{2}\alpha\partial\bar{\alpha}^2 , \tag{3.62} \]

where the operators \(\nu\) and \(G\) are given in (3.22) and (3.32) respectively, while a projector \(\Pi^{[1,2]}_{\text{bos}}\) is defined in (A.8). Second-order equations (3.59) motivate us to introduce a gauge condition which we refer to as modified de Donder gauge condition,
\[ C_{\text{mod}}^{\text{on-sh}}|\psi\rangle = 0 , \tag{3.63} \]

where operator \(C_{\text{mod}}^{\text{on-sh}}\) is given in (3.61), (3.62). Using the modified de Donder gauge condition (3.63) in gauge invariant equations of motion (3.59) leads to the following surprisingly simple gauge-fixed equations of motion:
\[ \Box_\nu|\psi\rangle = 0 , \tag{3.64} \]

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\[ \square_\nu \equiv \Box + \partial_\nu^2 - \frac{1}{z^2} \left( \nu^2 - \frac{1}{4} \right), \]

where \( \nu \) is defined in (3.22). In terms of tensor-spinor fields (3.1), equations (3.64) can be represented as

\[ \left( \Box + \partial_\nu^2 - \frac{1}{z^2} \left( \nu^2 - \frac{1}{4} \right) \right) \psi^{\alpha_1 \ldots \alpha_{s'}} = 0, \]

(3.66)

\[ \nu_{s'} \equiv s' + \frac{d - 3}{2} + \frac{1}{2} \sigma_3, \]

(3.67)

\( s' = 0, 1, \ldots, s \). Thus, our modified de Donder gauge condition (3.63) leads to decoupled equations of motion (3.66) which can easily be solved in terms of the Bessel function.\(^\text{10}\)

The following remarks are in order.

i) We note that gauge-fixed second-order equations of motion (3.64) have on-shell leftover gauge symmetries. These on-shell leftover gauge symmetries can simply be obtained from generic gauge symmetries (3.31) by the substitution \( |\xi\rangle \rightarrow |\xi_{\text{lf-ov}}\rangle \), where the ket-vector \( |\xi_{\text{lf-ov}}\rangle \) satisfies the following equations of motion:

\[ \Box_\nu |\xi_{\text{lf-ov}}\rangle = 0, \]

(3.68)

where \( \Box_\nu \) is given in (3.65).

ii) We note that our modified de Donder gauge condition (3.63) respects the Poincaré and dilatation symmetries but breaks the conformal boost \( K^\alpha \)-symmetries, i.e., the simple form of gauge-fixed equations of motion (3.64) is achieved at the cost of the \( K^\alpha \)-symmetries.

iii) Modified de Donder gauge condition (3.63) respects algebraic constraint (3.30) only on-shell. In other words, relation \( \gamma \bar{a} \bar{C}_{\text{mod}}^{\text{on-sh}} |\psi\rangle = 0 \) is valid only by using first-order equations of motion (3.53). This can easily be seen by noticing the relation

\[ \gamma \bar{a} \bar{C}_{\text{mod}}^{\text{on-sh}} |\psi\rangle = \bar{B}_e |\psi\rangle. \]

(3.69)

It easy to obtain off-shell extension of gauge condition (3.63). This is to say that by considering \( \gamma \)-traceless part of gauge condition (3.63), we obtain the following off-shell modified de Donder gauge condition

\[ \bar{C}_{\text{mod}}^{\text{off-sh}} |\psi\rangle = 0, \text{ off-shell modified de Donder gauge,} \]

(3.70)

\[ \bar{C}_{\text{mod}}^{\text{off-sh}} \equiv \bar{C}_{\text{mod}}^{\text{on-sh}} - \gamma \alpha \frac{1}{2N_\alpha + d} B_e. \]

(3.71)

Note that operator \( \bar{C}_{\text{mod}}^{\text{off-sh}} \) (3.71) can be represented as

\[ \bar{C}_{\text{mod}}^{\text{off-sh}} = (1 - \gamma \alpha \frac{1}{2N_\alpha + d} \gamma \bar{a} \bar{\alpha} \bar{\alpha}) \bar{\partial} - \frac{1}{2} \left( \alpha \bar{\partial} - \gamma \alpha \frac{1}{2N_\alpha + d} \bar{\partial} \right) \bar{\alpha}^2 \]

\[ - \frac{1}{2N_\alpha + d - 2} (\gamma \bar{a} - \gamma \alpha \frac{1}{2N_\alpha + d} \bar{\alpha}^2) e_1 + \frac{1}{2} \bar{\alpha}^2 e_1 - \frac{2N_\alpha + d - 4}{2N_\alpha + d - 2} \Pi^{[1,3]} e_1, \]

(3.72)

where a projector \( \Pi^{[1,3]} \) is defined in (A.7).

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\(^{10}\) Appearance of Bessel function in the solution of equations of motion for fields in arbitrary background is discussed in Ref.[54].
4 AdS/CFT correspondence. Preliminaries

We now study the AdS/CFT correspondence for free fermionic arbitrary spin massless AdS field and boundary arbitrary spin conformal current and shadow field. To study the AdS/CFT correspondence we use the gauge invariant CFT adapted formulation of massless AdS field and modified de Donder gauge condition we discussed in Sections 3. We emphasize that it is the use of our massless gauge fields and the modified de Donder gauge condition that leads to the surprisingly simple decoupled gauge-fixed equations of motion\textsuperscript{11}. The use of our massless gauge fields and the modified de Donder gauge condition makes the study of AdS/CFT correspondence for fermionic arbitrary spin-\( (s + \frac{1}{2}) \) massless AdS field similar to the one for fermionic spin-\( \frac{1}{2} \) massive AdS field. Owing these properties of our massless gauge fields and the modified de Donder gauge condition, the computation of effective action is considerably simplified. Perhaps, this is the main advantage of our approach.

In our approach to the AdS/CFT correspondence, we have gaugesymmetries not only at AdS side but also at the boundary CFT.\textsuperscript{12} Also, we note that the modified de Donder gauge condition turns out to be invariant under on-shell leftover gauge symmetries of massless AdS field. This is to say that, in the framework of our approach, the study of AdS/CFT correspondence implies the matching of:

i) the bulk first-order equations of motion, the modified de Donder gauge condition for bulk massless field and the corresponding differential constraints for boundary conformal current and shadow field;\textsuperscript{13}

ii) on-shell leftover gauge symmetries of bulk massless field and the corresponding gauge symmetries of boundary conformal current and shadow field;

iii) AdS field action evaluated on the solution of AdS massless field equations of motion with the Dirichlet problem corresponding to the boundary shadow field and the boundary two-point gauge invariant vertex for the shadow field.

4.1 AdS/CFT correspondence for massive spin-\( \frac{1}{2} \) field

As we have already said, the use of our massless gauge fields and the modified de Donder gauge makes the study of AdS/CFT correspondence for arbitrary spin-\( (s + \frac{1}{2}) \) massless AdS field similar to the one for spin-\( \frac{1}{2} \) massive AdS field. Therefore, for the reader convenience, we now briefly recall the AdS/CFT correspondence for the spin-\( \frac{1}{2} \) massive AdS field.

AdS/CFT correspondence for normalizable modes of spin-\( \frac{1}{2} \) massive AdS field and spin-\( \frac{1}{2} \) conformal current. The action of massive spin-\( \frac{1}{2} \) field in AdS\(_{d+1}\) background takes the form

\[
S = \int d^d x dz \mathcal{L},
\]

\textsuperscript{11}Discussion of interesting methods for solving AdS field equations of motion without gauge fixing may be found in Refs.[55].

\textsuperscript{12}For the first time, the gauge invariant approach to bosonic conformal current and shadow fields was developed in Ref.[56], while for anomalous conformal current and shadow field such approach was obtained in Refs.[9, 10]. In these references, by using the AdS/CFT correspondence, we also demonstrated how the gauge invariant approach is related to AdS field dynamics. Note however that, before Refs.[9, 10] gauge invariant approach to anomalous conformal current was obtained in Refs.[57, 58] in the framework of tractor approach. Recent interesting discussion of tractor approach may be found in Refs.[59].

\textsuperscript{13}In the framework of standard CFT, discussion of differential constraints for conformal currents may be found e.g., in Refs.[60]-[62].
where $\Psi = \Psi^\alpha$, $\alpha = 1, \ldots, 2^{[(d+1)/2]}$, is Dirac complex-valued spinor field of $so(d,1)$ algebra.

In the CFT adapted approach, we use complex-valued spinor field $\psi = \psi^\alpha$, where as above the spinor index $\alpha$ takes the values $\alpha = 1, \ldots, 2^{[(d+1)/2]}$ and the field $\psi$ can be presented as 2-vector

$$\psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}, \quad (1 - \gamma^z)\psi_u = 0, \quad (1 + \gamma^z)\psi_d = 0. \quad (4.3)$$

In terms of the field $\psi$, Lagrangian (4.2) takes the form (up to total derivative)

$$iL = \bar{\psi}(\partial / + e_{1}^\nu)\psi, \quad (4.4)$$

$$e_{1}^\nu = \sigma_-(\partial_z + \frac{m}{z}) + \sigma_+(\partial_z - \frac{m}{z}), \quad (4.5)$$

$$\nu = m + \frac{1}{2} \sigma_3, \quad \sigma_3 \equiv \bar{\nu}, \quad (4.6)$$

$$\square_\nu \equiv \partial_z^2 + \frac{\nu}{z}. \quad (4.7)$$

We note that operator $e_{1}^\nu$ (4.5) can be represented as

$$e_{1}^\nu = \sigma_-(\partial_z + \frac{m}{z}) + \sigma_+(\partial_z - \frac{m}{z}). \quad (4.8)$$

The equation of motion obtained from Lagrangian (4.4) is given by

$$\square_\nu \psi = 0. \quad (4.9)$$

It is easy to check that first-order equation (4.9) leads to the following second-order equation

$$\square_\nu \psi = 0, \quad (4.10)$$

$$\square_\nu \equiv \square + \partial_z^2 - \frac{1}{z^2}(\nu^2 - \frac{1}{4}). \quad (4.11)$$

The normalizable solution of Eq.(4.10) takes the form

$$\psi(x,z) = U_{\nu}^{sc}\psi_{cur}(x), \quad (4.12)$$

$$U_{\nu}^{sc} \equiv h_{\nu - 1}\sqrt{zq}J_{\nu}(zq)q^{-\nu + \frac{1}{2}}, \quad (4.13)$$

$$h_{\nu} \equiv 2^{\nu}\Gamma(\nu + 1), \quad q^2 \equiv \square, \quad (4.14)$$

$$\nu_0 = m + \frac{1}{2}, \quad (4.15)$$

where $J_\nu$ stands for the Bessel function. The asymptotic behavior of solution (4.12) is given by

$$\psi(x,z) \xrightarrow{z \to 0} \frac{2^{\nu_0 - 1}\Gamma(\nu_0)}{2^\nu\Gamma(\nu + 1)}z^{\nu + \frac{1}{2}}\psi_{cur}(x). \quad (4.16)$$

From (4.16), we see that the field $\psi_{cur}$ is indeed the asymptotic boundary value of the normalizable solution.
Note that the representation for AdS field as 2-vector in (4.3) implies the corresponding representation for spin-$\frac{1}{2}$ conformal current,

$$\psi_{\text{cur}} = \left( \begin{array}{c} \psi_{\text{cur},u} \\ \psi_{\text{cur},d} \end{array} \right).$$

(4.17)

Using representation of dilatation symmetry on space of AdS field in (3.38) and solution in (4.12), we find realization of the operator of conformal dimension on space of $\psi_{\text{cur}}$,

$$\Delta_{\text{cur}} = \frac{d + 1}{2} + \nu, \quad \nu = m + \frac{1}{2} \sigma_3.$$  

(4.18)

From (4.18), we see that conformal dimensions of the fields $\psi_{\text{cur},u}$ and $\psi_{\text{cur},d}$ are given by

$$\Delta_{\text{cur}}(\psi_{\text{cur},u}) = \frac{d + 2}{2} + m, \quad \Delta_{\text{cur}}(\psi_{\text{cur},d}) = \frac{d}{2} + m.$$  

(4.19)

Note that the choice of normalization factor $h_{\nu_0 - 1}$ in (4.13) is a matter of convenience. Our normalization condition implies the following normalization of asymptotic behavior of the solution in (4.12),

$$\psi_d(x, z) \xrightarrow{z \to 0} z^n \psi_{\text{cur},d}(x).$$  

(4.20)

Matching of bulk equation of motion and boundary constraint. We now demonstrate how differential constraint for the spin-$\frac{1}{2}$ conformal current is obtained from bulk equation of motion. To this end we note the relations for Bessel functions,

$$J_\nu = J_\nu(z), \quad T_\nu J_\nu = J_\nu - 1, \quad T_{-\nu} J_\nu = -J_\nu + 1.$$  

(4.21)

Using (4.21) and solution in (4.12) we find the following relation

$$(\partial / e^{\Gamma_{\text{cur}}}) \psi(x, z) = U_{\nu}^{\text{sc}}(\partial / e^{\Gamma_{\text{cur}}} + \sigma_+ \Box + \sigma_-) \psi_{\text{cur}}(x).$$  

(4.22)

From (4.22), we see that equation of motion (4.9) amounts to the following differential constraint for spin-$\frac{1}{2}$ conformal current

$$(\partial / e^{\Gamma_{\text{cur}}}) \psi_{\text{cur}}(x) = 0,$$  

(4.23)

$$e^{\Gamma_{\text{cur}}} \equiv \sigma_+ \Box + \sigma_-.$$  

(4.24)

Using representation of $\psi_{\text{cur}}$ as 2-vector in (4.17), we see that constraint (4.23) allows us to express $\psi_{\text{cur},u}$ in terms of $\psi_{\text{cur},d}$,

$$\psi_{\text{cur},u} = -\partial \psi_{\text{cur},d}.$$  

(4.25)

In other words, solution to (4.23) is given by

$$\psi_{\text{cur}} = \left( \begin{array}{c} -\partial \psi_{\text{cur},d} \\ \psi_{\text{cur},d} \end{array} \right).$$  

(4.26)

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14 Constraint (4.23) and solution (4.26) were obtained in the framework of tractor approach in Ref.[63]. Our study demonstrates how the constraint (4.23) gives rise in the framework of AdS/CFT correspondence.
AdS/CFT correspondence for non-normalizable modes of spin-$\frac{1}{2}$ massive AdS field and spin-$\frac{1}{2}$ shadow field.\textsuperscript{15} The non-normalizable solution of Eq.\((4.9)\) with the Dirichlet problem corresponding to the boundary shadow field \(\psi_{sh}(x)\) can be presented as

\[
\psi(x, z) = n_\nu \int d^d y \, G_\nu(x - y, z) \psi_{sh}(y), \quad (4.27)
\]

\[
n_\nu \equiv \frac{\Gamma(\nu)}{2\pi \Gamma(\nu_0)}, \quad (4.28)
\]

\[
G_\nu(x, z) = \frac{c_\nu z^{\nu + \frac{1}{2}}}{(z^2 + |x|^2)^{\nu + \frac{d}{2}}}, \quad (4.29)
\]

\[
c_\nu \equiv \frac{\Gamma(\nu + \frac{d}{2})}{\pi^{d/2} \Gamma(\nu)}. \quad (4.30)
\]

The asymptotic behaviors of Green function \((4.29)\) and solution \((4.27)\) are given by,

\[
G_\nu(x, z) \xrightarrow{z \to 0^-} z^{-\nu + \frac{1}{2}} \delta^{d}(x), \quad (4.31)
\]

\[
\psi(x, z) \xrightarrow{z \to 0^-} z^{-\nu + \frac{1}{2}} n_\nu \psi_{sh}(x). \quad (4.32)
\]

Relation \((4.32)\) tells us that solution \((4.27)\) has indeed asymptotic behavior corresponding to the shadow field.

Note that representation of AdS fields as 2-vector in \((4.3)\) implies the corresponding representation of spin-$\frac{1}{2}$ shadow field,

\[
\psi_{sh} = \begin{pmatrix} \psi_{sh,u} \\ \psi_{sh,d} \end{pmatrix}. \quad (4.33)
\]

Using representation of dilatation symmetry on space of AdS field in \((3.38)\) and solution in \((4.27)\), we find realization of the operator of conformal dimension on space of \(\psi_{sh}\),

\[
\Delta_{sh} = \frac{d + 1}{2} - \nu, \quad \nu = m + \frac{1}{2} \sigma_3. \quad (4.34)
\]

From \((4.34)\), we see that conformal dimensions of the fields \(\psi_{sh,u}\) and \(\psi_{sh,d}\) are given by

\[
\Delta_{sh}(\psi_{sh,u}) = \frac{d}{2} - m, \quad \Delta_{sh}(\psi_{sh,d}) = \frac{d + 2}{2} - m. \quad (4.35)
\]

Note that the choice of normalization factor \(n_\nu\) in \((4.27)\) is a matter of convenience. Our normalization condition implies the following normalization of asymptotic behavior of the solution in \((4.27)\),

\[
\psi_u(x, z) \xrightarrow{z \to 0^-} z^{-\nu_0 + \frac{1}{2}} \psi_{sh,u}(x). \quad (4.36)
\]

\textbf{Matching of bulk equation of motion and boundary constraint.} We now demonstrate how differential constraint for the spin-$\frac{1}{2}$ shadow field is obtained from bulk equation of motion. To this end we note the following relations

\[
(\partial^2 + e_v^2)\psi(x, z) = n_\nu \int d^d y \, G_\nu(x - y, z)(\partial^2 + \sigma_+ + \sigma_- \Box)\psi_{sh}(y). \quad (4.37)
\]

\textsuperscript{15}See also Refs.[11]-[14].
From (4.37), we see that equation of motion (4.9) amounts to the following differential constraint for spin-$\frac{1}{2}$ shadow field

$$ (\not\partial + e_1^{\Gamma,sh})\psi_{sh}(x) = 0, \quad (4.38) $$

$$ e_1^{\Gamma,sh} \equiv \sigma_+ + \sigma_- \square. \quad (4.39) $$

Using representation of $\psi_{sh}$ as 2-vector in (4.33), we see that constraint (4.38) allows us to express $\psi_{sh,d}$ in terms of $\psi_{sh,u}$,

$$ \psi_{sh,d} = -\not\partial \psi_{sh,u}. \quad (4.40) $$

In other words, solution to (4.38) is given by

$$ \psi_{sh} = \begin{pmatrix} \psi_{sh,u} \\ -\not\partial \psi_{sh,u} \end{pmatrix}. \quad (4.41) $$

**Matching of effective action and boundary two-point vertex.** To find the effective action we follow the standard procedure. Namely, we plug non-normalizable solution of the bulk equation of motion with the Dirichlet problem corresponding to the boundary shadow field (4.27) into the bulk action for AdS massless field. In other words we are going to compute effective action defined by the relation

$$ \Gamma_{\text{eff}} \equiv \int d^d x dz \mathcal{L}_{\text{on-shell}}, \quad (4.42) $$

where $\mathcal{L}_{\text{on-shell}}$ is the Lagrangian evaluated on non-normalizable solution of the bulk equation of motion with the Dirichlet problem corresponding to the boundary shadow field (4.27). Note however that Lagrangian presented in (4.4) does not involve a proper boundary term. Expression for Lagrangian involving proper boundary term is given by\(^{16}\)

$$ \mathcal{L} = -\bar{\psi}\left(\gamma\overset{<}{\nabla}^a + e_1^{\Psi}\right)\psi, \quad (4.43) $$

$$ e_1^{\Psi} = \sigma_- T_{\nu - \frac{1}{2}} + T_{\nu - \frac{1}{2}} \sigma_+, \quad (4.44) $$

$$ \nu = m + \frac{1}{2}\sigma_3, \quad (4.45) $$

$$ \overset{<}{\nabla}^a \equiv \frac{1}{2}(\overset{<}{\nabla}^a - \overset{<}{\partial}^a), \quad \overset{<}{T}_\nu \equiv \overset{<}{\partial}_\nu + \frac{\nu}{z}, \quad \overset{<}{T}_\nu \equiv \overset{<}{\partial}_\nu + \frac{\nu}{z}. \quad (4.46) $$

We note that operator $\overset{<}{e_1}^{\Psi}$ (4.44) can be represented as

$$ \overset{<}{e_1}^{\Psi} = \sigma_-(\overset{<}{\partial}_z + \frac{m}{z}) + \sigma_+(\overset{<}{\partial}_z + \frac{m}{z}). \quad (4.47) $$

It is easy to see that Lagrangian (4.43) differs from the one in (4.4) by total derivatives $\partial^a, \partial_z$.

Lagrangian (4.43) evaluated on the solution of first-order equations of motion (4.9) takes the form

$$ i\mathcal{L}_{\text{on-shell}} = \frac{1}{2}\partial_z(\bar{\psi}\sigma_1\psi), \quad (4.48) $$

\(^{16}\)Expression for $\mathcal{L}$ in (4.43) is obtained by straightforward application of methods in Refs.[13, 14].
where \( \sigma_1 \) is the Pauli matrix. This implies that effective action defined as in (4.42) takes the form:\(^{17}\)

\[
-i \Gamma_{\text{eff}} \equiv \frac{1}{2} \int d^d x \psi(x, z) \sigma_1 \psi(x, z) \bigg|_{x \to 0}.
\]  

(4.49)

Now, plugging in (4.49) solution to the second-order equations of motion (4.27), we obtain the following three equivalent representations for the effective action:

**1st representation for the effective action**

\[
-i \Gamma_{\text{eff}} = \nu_0 c_{\nu_0} \int d^d x \psi_0 \psi_{s \chi} \left( \sigma_{+} \frac{f_{\nu}}{|x_{12}|^{2\nu+d}} + f_{\nu} \frac{g_{\nu}}{|x_{12}|^{2\nu+d-2}} \right) \psi_{s \chi} \bigg|_{x \to 0}.
\]  

(4.50)

\[
\begin{align*}
&f_{\nu} \equiv \frac{\Gamma(\nu + 1) \Gamma(\nu + \frac{d}{2})}{4^{\nu-\nu} \Gamma(\nu_0 + 1) \Gamma(\nu_0 + \frac{d}{2})}, \\
g_{\nu} \equiv \frac{\Gamma(\nu) \Gamma(\nu + \frac{d}{2} - 1)}{4^{\nu-\nu} \Gamma(\nu_0) \Gamma(\nu_0 + \frac{d}{2} - 1)},
\end{align*}
\]  

(4.51)

**2nd representation for the effective action**

\[
-i \Gamma_{\text{eff}} = \frac{c_{\nu_0}}{4(\nu_0 + \frac{d}{2} - 1)} \int d^d x d^d x_2 \psi_0 \psi_{s \chi} \left( \sigma_{+} \frac{g_{\nu}}{|x_{12}|^{2\nu+d-2}} (\sigma_{+} + \sigma_{-} \square) \psi_{s \chi} \bigg|_{x \to 0} \right),
\]  

(4.52)

**3rd representation for the effective action**

\[
\begin{align*}
&i \Gamma_{\text{eff}} = \frac{c_{\nu_0}}{4(\nu_0 + \frac{d}{2} - 1)} \int d^d x d^d x_2 \psi_0 \psi_{s \chi} \left( \sigma_{+} \frac{g_{\nu}}{|x_{12}|^{2\nu+d-2}} \psi_{s \chi} \bigg|_{x \to 0} \right).
\end{align*}
\]  

(4.54)

Representations for the effective action given in (4.50), (4.52), (4.54) is our solution to problem of 2-point effective action for the case of spin-\( \frac{1}{2} \) AdS field. Advantage of these representation is that, these representations have straightforward generalization to the case of arbitrary spin fields. For the case of spin-\( \frac{1}{2} \) field we consider here, these representations can straightforwardly be related to the one discussed in the earlier literature. All that is required is to plug solution to differential constraint (4.4) into our representations. Doing so, we get result in Refs.[11]-[14].

\[
-i \Gamma_{\text{eff}} = c_{\nu_0} \int d^d x_1 d^d x_2 \psi_{s \chi} \psi_{s \chi, u} \left( \sigma_{+} \frac{f_{\nu}}{|x_{12}|^{2\nu+d}} \psi_{s \chi, u} \bigg|_{x \to 0} \right).
\]  

(4.55)

5. **AdS/CFT correspondence for normalizable modes of massless AdS field and conformal current**

We now ready to consider the AdS/CFT correspondence for the spin-(\( s + \frac{1}{2} \)) massless AdS field and spin-(\( s + \frac{1}{2} \)) conformal current. We begin with the discussion of the normalizable solution of

\(^{17}\)As usually, since solution of the Dirichlet problem (4.27) tends to zero as \( z \to \infty \), we ignore contribution to \( \Gamma_{\text{eff}} \) (4.49) when \( z = \infty \). Note that throughout this paper we use conventions corresponding to the Lorentz signature. For the computation of the effective action, we should use the Euclidean signature. All that is required to cast our results into the form corresponding to the Euclidean signature is to make the following replacement for the effective action:

\[
\Gamma_{\text{Lorentz}}^{\text{Lorentz}} \to \Gamma_{\text{Euclid}}^{\text{Euclid}}.
\]
Eq. (3.64). The normalizable solution of Eq. (3.64) is given by
\[
|\psi(x, z)\rangle = U_\nu |\psi_{\text{cur}}(x)\rangle,
\]
(5.1)
\[
U_\nu \equiv h_{\nu s - 1} (-1)^{N_z} \sqrt{z q} J_\nu(z q) q^{-\left(\nu + \frac{1}{2}\right)},
\]
(5.2)
\[
h_\kappa \equiv 2^\kappa \Gamma(\kappa + 1), \quad q^2 \equiv \Box,
\]
(5.3)
where we do not show explicitly the dependence of \(U_\nu\) on \(z, q,\) and \(\kappa\). The asymptotic behavior of solution (5.1) takes the form
\[
|\psi(x, z)\rangle \xrightarrow{z \to 0} z^{\nu + \frac{1}{2}} \frac{2^{\nu s - 1} \Gamma(\nu s)}{2^\nu \Gamma(\nu + 1)} (-1)^{N_z} |\psi_{\text{cur}}(x)\rangle,
\]
(5.4)
\[
\nu_s \equiv s + \frac{d - 2}{2}.
\]
(5.5)
From (5.4), we see that \(\psi_{\text{cur}}\) is indeed boundary value of the normalizable solution.

Note that representation of AdS fields as 2-vectors in (3.2) implies the corresponding representation of fields of spin-(\(s + \frac{1}{2}\)) conformal current,
\[
\psi_{\text{cur}}^{a_1...a_s \alpha} = \begin{pmatrix} \psi_{\text{cur,u}}^{a_1...a_s \alpha} \\ \psi_{\text{cur,d}}^{a_1...a_s \alpha} \end{pmatrix}, \quad s' = 0, 1, \ldots, s.
\]
(5.6)
Using representation of dilatation symmetry on space of AdS field in (3.38) and solution in (5.1), we find realization of the operator of conformal dimension on space of \(|\psi_{\text{cur}}\rangle\),
\[
\Delta_{\text{cur}} = s + d - 1 - N_z + \frac{1}{2} \sigma_3.
\]
(5.7)
From (5.7), we see that conformal dimensions of the fields in (5.6) are given by
\[
\Delta_{\text{cur}}(\psi_{\text{cur,u}}^{a_1...a_s \alpha}) = s' + d - \frac{1}{2}, \quad \Delta_{\text{cur}}(\psi_{\text{cur,d}}^{a_1...a_s \alpha}) = s' + d - \frac{3}{2}.
\]
(5.8)
Note that the choice of normalization factor \(h_{\nu s - 1}\) in (5.2) is a matter of convenience. Our normalization condition implies the following normalization of asymptotic behavior of the solution for leading rank-s tensor-spinor field in (4.12),
\[
\psi_{d}^{a_1...a_s}(x, z) \xrightarrow{z \to 0} z^{\nu_s - \frac{1}{2}} \psi_{\text{cur,d}}^{a_1...a_s}(x),
\]
(5.9)
where \(\nu_s\) is given in (5.5).

Now we are going to prove the following statements:
i) For normalizable solution (5.1), the first-order equations of motion (3.51) and modified de Donder gauge condition (3.63) lead to the differential constraints of the spin-(\(s + \frac{1}{2}\)) conformal current.

ii) On-shell leftover gauge transformation (3.31) of normalizable solution (5.1) leads to the gauge transformation of the spin-(\(s + \frac{1}{2}\)) conformal current\(^{18}\).

\(^{18}\)Note that gauge transformation given in (3.31) is off-shell gauge transformation. On-shell leftover gauge transformation is obtained from gauge transformation (3.31) by using gauge transformation parameter which satisfies equation (3.68).
To prove these statements we use the following relations for the operator $U_\nu$:

\[ T_{\nu-\frac{1}{2}} U_\nu = U_{\nu-1}, \quad (5.10) \]
\[ T_{-\nu-\frac{1}{2}} U_\nu = -U_{\nu+1} \square, \quad (5.11) \]

which, in turn, can be derived by using the textbook identities for the Bessel function given in (4.21).

**Matching of bulk modified de Donder gauge and boundary constraint.** We now demonstrate how differential constraints for the conformal current are obtained from first-order equations of motion (3.51) and modified de Donder gauge condition (3.63). Using (5.10) and (5.11), we find the important relations

\[ e_1^\Gamma U_\nu = U_\nu e_1^{\Gamma, \text{cur}}, \quad e_1 U_\nu = U_\nu e_1, \quad \bar{e}_1 U_\nu = U_\nu \bar{e}_1^{\text{cur}}, \quad (5.12) \]

where operators $e_1^{\Gamma, \text{cur}}, e_1, \bar{e}_1^{\text{cur}}$ are given below in (5.19)-(5.21). Acting with operators $\hat{E}$ (3.54) and $\hat{C}_{\text{mod}}$ (3.61) on solution $|\psi\rangle$ (5.1) and using (5.12), we obtain the relations

\[ \hat{E}|\psi(x, z)\rangle = U_\nu \hat{E}_{\text{cur}} |\psi_{\text{cur}}(x)\rangle, \quad (5.13) \]
\[ \hat{C}_{\text{mod}}|\psi(x, z)\rangle = U_\nu \hat{C}_{\text{cur}} |\psi_{\text{cur}}(x)\rangle, \quad (5.14) \]

where operators $\hat{E}_{\text{cur}}$ and $\hat{C}_{\text{cur}}$ take the form

\[ \hat{E}_{\text{cur}} \equiv \hat{\psi} - G_{\text{cur}} \gamma \bar{\alpha} + \left(1 + \gamma \alpha \frac{2}{2N_\alpha + d - 2} \bar{\alpha}\right)e_1^{\Gamma, \text{cur}} - \gamma \alpha \frac{2}{2N_\alpha + d - 2} \bar{e}_1^{\text{cur}}, \quad (5.15) \]
\[ \hat{C}_{\text{cur}} \equiv \hat{C}_{\text{st}} - \frac{1}{2N_\alpha + d - 2} \left(\gamma \bar{\alpha} + \frac{1}{2} \gamma \alpha \bar{\alpha}^2\right)e_1^{\text{cur}} + \frac{1}{2} \alpha^2 e_1 - \frac{2N_\alpha + d - 4}{2N_\alpha + d - 2} \Psi_{\text{bos}}^{[1,2]} \bar{e}_1^{\text{cur}}, \quad (5.16) \]
\[ G_{\text{cur}} \equiv \alpha \bar{\alpha} - e_1^{\text{cur}} + \gamma \alpha \frac{1}{2N_\alpha + d - 2} e_1^{\Gamma, \text{cur}} - \alpha^2 \frac{1}{2N_\alpha + d} \bar{e}_1^{\text{cur}}, \quad (5.17) \]
\[ \hat{C}_{\text{st}} \equiv \bar{\alpha} \partial - \frac{1}{2} \alpha \bar{\alpha} \partial^2, \quad (5.18) \]
\[ e_1^{\text{cur}} = e_1^{\text{cur}}(\sigma_- + \sigma_+ \square), \quad (5.19) \]
\[ e_1 = \alpha \bar{\alpha} \bar{\epsilon}_1, \quad \bar{e}_1^{\text{cur}} = -\square \bar{e}_1 \bar{\alpha}^2, \quad (5.20) \]
\[ e_1^{\text{cur}} = \frac{2s + d - 2}{2s + d - 2N_z}, \quad \bar{e}_1 = \left(\frac{2s + d - 3 - N_z}{2s + d - 4 - 2N_z}\right)^{1/2}. \quad (5.21) \]

From (5.13), (5.14), we see that first-order equations of motion (3.51) and our modified de Donder gauge condition (3.63) lead indeed to the differential constraints for the conformal current given by

\[ \hat{E}_{\text{cur}} |\psi_{\text{cur}}\rangle = 0, \quad (5.22) \]

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19 For the case of spin-$\frac{1}{2}$ conformal current, constraints (5.22), (5.23) were obtained in the framework of tractor approach in Ref.[63]. Our discussion provides generalization of the constraints to the case of arbitrary spin conformal current and demonstrates how constraints (5.22), (5.23) give rise in the framework of AdS/CFT correspondence.
As a side of remark we note that constraint (5.22) can be represented as

\[ E_{\text{cur}}|\psi_{\text{cur}}\rangle = 0, \]  
(5.24)

\[ E_{\text{cur}} \equiv E_{\text{cur},(1)} + E_{\text{cur},(0)} \]  
(5.25)

\[ E_{\text{cur},(0)} \equiv (1 - \gamma \alpha \gamma \bar{\alpha} - \frac{1}{4} \alpha^2 \bar{\alpha}^2)e_{1,\text{cur}}^{\gamma} + (\gamma \alpha - \frac{1}{2} \alpha^2 \gamma \bar{\alpha})\bar{e}_{1,\text{cur}} + (\gamma \bar{\alpha} - \frac{1}{2} \bar{\gamma} \alpha \bar{\alpha}^2)e_{1,\text{cur}}, \]  
(5.27)

where the Fang-Fronsdal operator \( E_{(1)} \) appearing in (5.26) is given in (3.15).

**Matching of bulk and boundary gauge symmetries.** We now show how leftover gauge transformation of the massless AdS field is related to gauge transformation of the conformal current. To this end we note that on-shell leftover gauge transformation of massless AdS field is obtained from (3.31) by plugging gauge transformation parameter, which satisfies equation (3.68), into (3.31). The normalizable solution of equation for the gauge transformation parameter (3.68) takes the form

\[ |\xi(x, z)\rangle = U_\nu |\xi_{\text{cur}}(x)\rangle, \]  
(5.28)

where \( U_\nu \) is given in (5.2). On the one hand, plugging (5.28) into (3.31) and using (5.12), we find that bulk on-shell leftover gauge transformation takes the form

\[ \delta|\psi(x, z)\rangle = U_\nu G_{\text{cur}}|\xi_{\text{cur}}(x)\rangle, \]  
(5.29)

where \( G_{\text{cur}} \) is given in (5.17). On the other hand, relation (5.1) leads to

\[ \delta|\psi(x, z)\rangle = U_\nu \delta|\psi_{\text{cur}}(x)\rangle. \]  
(5.30)

Comparing (5.29) and (5.30), we see that gauge transformation of the conformal current takes the form

\[ \delta|\psi_{\text{cur}}\rangle = G_{\text{cur}}|\xi_{\text{cur}}\rangle. \]  
(5.31)

We check that differential constraints (5.22), (5.23) are invariant under gauge transformation (5.31). Thus we see that the on-shell leftover gauge symmetries of solution of the Dirichlet problem for the spin-(s + \( \frac{1}{2} \)) massless AdS field are indeed lead to gauge symmetries of the spin-(s + \( \frac{1}{2} \)) conformal current.

The following remark is in order.

From gauge transformation (5.31), we learn that some fields in (5.6) transform as Stueckelberg fields. These Stueckelberg fields can be gauged away. After this, using differential constraints (5.22), (5.23), we can express all the remaining fields in (5.6) in terms of one rank-s tensor-spinor field \( \psi_{\text{cur},d}^{a_1\ldots a_s} \). Besides this, the field \( \psi_{\text{cur},d}^{a_1\ldots a_s} \) turns out to be \( \gamma \)-traceless and divergence free. Note also that, in view of (5.8), conformal dimension of the spin-(s + \( \frac{1}{2} \)) field \( \psi_{\text{cur},d}^{a_1\ldots a_s} \) is equal to \( s + d - \frac{3}{2} \). This implies that our gauge invariant approach to conformal current is equivalent to the standard CFT.
6 AdS/CFT correspondence for non-normalizable modes of massless AdS field and shadow field

We now discuss the AdS/CFT correspondence for bulk spin-\((s + \frac{1}{2})\) massless AdS field and boundary spin-\((s + \frac{1}{2})\) shadow field. We begin with an analysis of the non-normalizable solution of Eq. (3.64). Solution of Eq. (3.64) with the Dirichlet problem corresponding to the spin-\((s + \frac{1}{2})\) shadow field takes the form

\[
|\psi(x, z)\rangle = n_\nu \int d^d y \, G_\nu(x - y, z)|\psi_{sh}(y)\rangle, \tag{6.1}
\]

\[
n_\nu \equiv \frac{(-)^{N_z} \Gamma(\nu)}{2^{N_z + z}} \frac{\Gamma(\nu_s)}{\Gamma(\nu_s)}, \tag{6.2}
\]

where the Green function is given in (4.29), while \(\nu_s\) is defined in (5.5).

Using asymptotic behavior of the Green function \(G_\nu\) (4.31), we find the asymptotic behavior of our solution

\[
|\psi(x, z)\rangle \xrightarrow{z \to 0} z^{-\nu_s + \frac{1}{2}} n_\nu |\psi_{sh}(x)\rangle. \tag{6.3}
\]

From this expression, we see that solution (6.1) has indeed asymptotic behavior corresponding to the spin-\((s + \frac{1}{2})\) shadow field.\(^{20}\)

Note that representation of AdS fields as 2-vectors in (3.2) implies the corresponding representation of fields of spin-\((s + \frac{1}{2})\) shadow field,

\[
\psi_{sh}^{a_1...a_s\alpha} = \begin{pmatrix} \psi_{sh,u}^{a_1...a_s\alpha} \\ \psi_{sh,d}^{a_1...a_s\alpha} \end{pmatrix}, \quad s' = 0, 1, \ldots, s. \tag{6.4}
\]

Using representation of dilatation symmetry on space of AdS field in (3.38) and solution in (6.1), we find realization of the operator of conformal dimension on space of \(|\psi_{sh}\rangle\),

\[
\Delta_{sh} = 2 - s + N_z - \frac{1}{2} \sigma_3. \tag{6.5}
\]

From (6.5), we see that conformal dimensions of fields in (6.4) are given by

\[
\Delta_{sh}(\psi_{sh,u}^{a_1...a_s\alpha}) = \frac{3}{2} - s', \quad \Delta_{sh}(\psi_{sh,d}^{a_1...a_s\alpha}) = \frac{5}{2} - s'. \tag{6.6}
\]

Note that the choice of normalization factor \(n_\nu\) in (6.1) is a matter of convenience. Our normalization condition implies the following normalization of asymptotic behavior of the solution for leading rank-\(s\) tensor-spinor field in (6.3),

\[
\psi_{u}^{a_1...a_s}(x, z) \xrightarrow{z \to 0} z^{-\nu_s + \frac{1}{2}} |\psi_{sh,u}^{a_1...a_s}(x)\rangle, \tag{6.7}
\]

where \(\nu_s\) is given in (5.5).

Now, we are going to prove the following statements:

i) For solution (6.1), the first-order equations of motion (3.51) and modified de Donder gauge condition (3.63) lead to differential constraints for the shadow field.

---

\(^{20}\)Since solution (6.1) has nonintegrable asymptotic behavior (6.3), such solution is sometimes referred to as the non-normalizable solution.
On-shell leftover gauge transformation (3.31) of solution (6.1) leads to gauge transformation of the spin-$(s + \frac{1}{2})$ shadow field.

action evaluated on solution (6.1) coincides, up to normalization factor, with boundary two-point gauge invariant vertex for the shadow field.

Below we demonstrate how these statements can be proved by using the following relations for the Green function $G_\nu \equiv G_\nu(x - y, z)$:

$$\mathcal{T}_{-\nu + \frac{1}{2}} G_{\nu - 1} = -2(\nu - 1) G_\nu, \quad (6.8)$$

$$\mathcal{T}_{\nu + \frac{1}{2}} G_{\nu + 1} = \frac{1}{2\nu} \Box G_\nu. \quad (6.9)$$

**Matching of bulk modified de Donder gauge and boundary constraint.** We now demonstrate how differential constraints for the shadow field are obtained from first-order equations of motion (3.51) and modified de Donder gauge condition (3.63). To this end we note the relations

$$e_1(n_\nu G_\nu) = (n_\nu G_\nu)(\bar{e}_1 \alpha \bar{e}_1), \quad (6.10)$$

$$\bar{e}_1(n_\nu G_\nu) = (n_\nu G_\nu)(-\bar{e}_1 \bar{e}_1), \quad (6.11)$$

$$e_{1,1}^r(n_\nu G_\nu) = (n_\nu G_\nu)e_{1,1}^r(\sigma_+ + \sigma_0), \quad (6.12)$$

where Laplace operator $\bar{\Box}_y \equiv \frac{\partial}{\partial y^\mu} \frac{\partial}{\partial y^\mu}$ appearing in (6.10) and (6.12) is acting on the Green function $G_\nu = G_\nu(x - y, z)$. Acting with operators $\hat{E}$ (3.54) and $\hat{C}_{\text{mod}}$ (3.61) on solution $|\psi\rangle$ (6.1) and using (6.10)-(6.12), we obtain the relations

$$\hat{E}|\psi\rangle = n_\nu \int d^d y \ G_\nu(x - y, z) \hat{E}_{\text{sh}}|\psi_{\text{sh}}(y)\rangle, \quad (6.13)$$

$$\hat{C}_{\text{mod}}|\psi\rangle = n_\nu \int d^d y \ G_\nu(x - y, z) \hat{C}_{\text{sh}}|\psi_{\text{sh}}(y)\rangle, \quad (6.14)$$

where operators $\hat{E}_{\text{sh}}, \hat{C}_{\text{sh}}$ take the form

$$\hat{E}_{\text{sh}} \equiv \hat{\Box} - G_\nu \gamma \bar{\alpha} + \left(1 + \gamma \alpha \frac{2}{2N_\alpha + d - 2} \gamma \bar{\alpha} \right) e_{1,sh}^r - \gamma \alpha \frac{2}{2N_\alpha + d - 2} \bar{e}_{1,sh}, \quad (6.15)$$

$$\hat{C}_{\text{sh}} \equiv \hat{\Box} - \frac{1}{2N_\alpha + d - 2} \left(\gamma \alpha + \frac{1}{2} \gamma \alpha \bar{\alpha} \right) e_{1,sh}^r + \frac{1}{2} \bar{\alpha}^2 e_{1,sh} - \gamma \alpha \frac{2N_\alpha + d - 4}{2N_\alpha + d - 2} \Box_{\text{bos}} \bar{e}_{1,sh}, \quad (6.16)$$

$$G_\nu \equiv \alpha \partial - e_{1,sh} + \gamma \alpha \frac{1}{2N_\alpha + d - 2} e_{1,sh}^r - \gamma \alpha \frac{1}{2N_\alpha + d} \bar{e}_{1,sh}, \quad (6.17)$$

$$\hat{\Box}_{\text{st}} \equiv \hat{\Box} - \frac{1}{2} \alpha \partial \bar{\alpha}^2, \quad (6.18)$$

$$e_{1,sh}^r = e_{1,1}^r(\sigma_0 - \Box + \sigma_+), \quad (6.19)$$

$$e_{1,sh} = \Box \alpha \bar{e}_1, \quad \bar{e}_{1,sh} = -\bar{e}_1 \bar{\alpha}^2, \quad (6.20)$$

$$e_{1,1}^r = \frac{2s + d - 2}{2s + d - 2 - 2N_z}, \quad \bar{e}_1 = \left(\frac{2s + d - 3 - N_z}{2s + d - 4 - 2N_z}\right)^{1/2}. \quad (6.21)$$
From (6.14), we see that first-order equations of motion (3.51) and our modified de Donder gauge condition (3.63) lead indeed to the differential constraint for the shadow field given by

\[ \hat{E}_{\text{sh}}|\psi_{\text{sh}}\rangle = 0, \tag{6.22} \]

\[ \bar{C}_{\text{sh}}|\psi_{\text{sh}}\rangle = 0. \tag{6.23} \]

As a side of remark we note that constraint (6.22) can be represented as

\[ E_{\text{sh}}|\psi_{\text{sh}}\rangle = 0, \tag{6.24} \]

where the Fang-Fronsdal operator \( E_{(1)} \) appearing in (6.26) is given in (3.15).

**Matching of bulk and boundary gauge symmetries.** We now show how gauge transformation of the shadow field is obtained from the on-shell leftover gauge transformation of the massless AdS field. To this end we note that the corresponding on-shell leftover gauge transformation of massless AdS field is obtained from (3.31) by plugging non-normalizable solution of equation for gauge transformation parameter (3.68) into (3.31). Thenon-normalizable solution of equation (3.68) is given by

\[ |\xi(x,z)\rangle = n_\nu \int d^d y G_\nu(x-y,z)|\xi_{\text{sh}}(y)\rangle, \tag{6.28} \]

where \( n_\nu \) is given in (6.2). We now note that, on the one hand, plugging (6.28) into (3.31) and using (6.1), we can cast the on-shell leftover gauge transformation of \( |\psi\rangle \) into the form

\[ \delta|\psi\rangle = n_\nu \int d^d y G_\nu(x-y,z)G_{\text{sh}}|\xi_{\text{sh}}(y)\rangle, \tag{6.29} \]

where \( G_{\text{sh}} \) is given in (6.17). On the other hand, making use of relation (6.1), we get

\[ \delta|\psi\rangle = n_\nu \int d^d y G_\nu(x-y,z)\delta|\psi_{\text{sh}}(y)\rangle. \tag{6.30} \]

Comparing (6.29) with (6.30), we obtain gauge transformation of the shadow field,

\[ \delta|\psi_{\text{sh}}\rangle = G_{\text{sh}}|\xi_{\text{sh}}\rangle. \tag{6.31} \]

We check that differential constraints (6.22), (6.23) are invariant under gauge transformation (6.31). Thus we see that the on-shell leftover gauge symmetries of solution of the Dirichlet problem for the spin-\((s+\frac{1}{2})\) massless AdS field are indeed lead to gauge symmetries of the spin-\((s+\frac{1}{2})\) shadow field.

**Matching of effective action and boundary two-point vertex.** To find the effective action we follow the standard procedure. Namely, we plug non-normalizable solution of the bulk equation of motion with the Dirichlet problem corresponding to the boundary shadow field (6.1) into the bulk
action for AdS massless field. In other words, we are going to compute an effective action defined by the relation\textsuperscript{21}
\[
\Gamma_{\text{eff}} \equiv \int d^d x dz \mathcal{L}_{\text{on-shell}} ,
\]  
(6.32)
where $\mathcal{L}_{\text{on-shell}}$ is the Lagrangian evaluated on non-normalizable solution of the bulk equation of motion with the Dirichlet problem corresponding to the boundary shadow field (6.1). Note however that the Lagrangian presented in (3.13) does not involve a proper boundary term. We find the following expression for Lagrangian involving the proper boundary term:\textsuperscript{22}
\[
i \mathcal{L} = \langle \psi | \hat{\mathcal{E}} | \psi \rangle ,
\]  
(6.33)
\[
\hat{\mathcal{E}} \equiv \hat{\mathcal{E}}_{(1)} + \hat{\mathcal{E}}_{(0)} ,
\]  
(6.34)
\[
\hat{\mathcal{E}}_{(1)} \equiv \check{\phi} - \alpha \check{\phi} \gamma \bar{\alpha} - \gamma \alpha \check{\phi} \gamma \bar{\alpha} + \gamma \alpha \check{\phi} \gamma \bar{\alpha} + \frac{1}{2} \gamma \alpha \check{\phi} \gamma \bar{\alpha}^2 + \frac{1}{2} \alpha^2 \gamma \bar{\alpha} \check{\phi} \gamma \bar{\alpha} - \frac{1}{4} \alpha^2 \check{\phi} \gamma \bar{\alpha}^2 ,
\]  
(6.35)
\[
\hat{\mathcal{E}}_{(0)} = (1 - \gamma \alpha \gamma \bar{\alpha} - \frac{1}{4} \alpha^2 \gamma \bar{\alpha}^2) \epsilon_1^{\alpha} + (\gamma \alpha - \frac{1}{2} \alpha^2 \gamma \bar{\alpha}) \bar{\epsilon}_1 + (\gamma \bar{\alpha} - \frac{1}{2} \gamma \alpha \bar{\alpha}^2) \check{\epsilon}_1 ,
\]  
(6.36)
\[
\epsilon_1^{\alpha} = \epsilon_{1,1}^{\alpha} (\sigma_+ T_{\nu - \frac{1}{2}} + \bar{T}_{\nu - \frac{1}{2}} \sigma_+ ) ,
\]  
(6.37)
\[
\bar{\epsilon}_1 = \epsilon_{1,1} \bar{T}_{\nu - \frac{1}{2}} , \quad \check{\epsilon}_1 = - \bar{T}_{\nu - \frac{1}{2}} \epsilon_{1,1} ,
\]  
(6.38)
\[
e_{1,1} = - \alpha \epsilon_1 , \quad \bar{\epsilon}_{1,1} = - \bar{\epsilon}_1 \check{\epsilon} , \quad \check{\epsilon}_1 = \left( \frac{2s + d - 3 - N_z}{2s + d - 4 - 2N_z} \right)^{1/2} ,
\]  
(6.39)
\[
e_{1,1}^{\alpha} = \frac{2s + d - 2}{2s + d - 2 - 2N_z} ,
\]  
(6.40)
\[
\check{\delta}^a \equiv \frac{1}{\alpha} (\check{\delta}^a - \check{\phi} \gamma \bar{\alpha}) , \quad \check{T}_{\nu} \equiv \check{\phi} \gamma \bar{\alpha} + \nu \check{\epsilon}_1 , \quad \check{T}_{\nu} \equiv \check{\phi} \gamma \bar{\alpha} + \nu \check{\epsilon}_1 ,
\]  
(6.41)
\[
\nu \equiv s + \frac{d - 3}{2} - N_z + \frac{1}{2} \sigma_3 .
\]  
(6.42)
It is easy to see that Lagrangian (6.33) differs from the one in (3.13) by total derivatives $\partial^a$, $\partial_z$.

Lagrangian (6.33) evaluated on the solution of first-order equations of motion (3.50) takes the form
\[
i \mathcal{L}_{\text{on-shell}} = \frac{1}{2} \check{\partial}_z \left( \langle \psi (x, z) | \mathcal{E}_{\text{eff}} | \psi (x, z) \rangle \right) ,
\]  
(6.43)
\[
\mathcal{E}_{\text{eff}} \equiv (1 - \gamma \alpha \gamma \bar{\alpha} - \frac{1}{4} \alpha^2 \gamma \bar{\alpha}^2) \epsilon_{1,1}^{\alpha} - (\gamma \alpha - \frac{1}{2} \alpha^2 \gamma \bar{\alpha}) \bar{\epsilon}_{1,1} + (\gamma \bar{\alpha} - \frac{1}{2} \gamma \alpha \bar{\alpha}^2) \check{\epsilon}_{1,1} .
\]  
(6.44)
\textsuperscript{21}In this paper, for the study of AdS/CFT correspondence for massless fields, we use Lagrangian approach. Study of the AdS/CFT correspondence by using equations of motion and higher-spin symmetries may be found e.g., in Refs.[64]-[67].

\textsuperscript{22}We note that boundary terms proportional to $\epsilon_{1,1}^{\alpha}$ in (6.33) can be fixed by using the same methods as for spin-$\frac{1}{2}$ field in Refs.[13, 14]. After this, the remaining boundary terms which are proportional to $\epsilon_{1,1}$ and $\bar{\epsilon}_{1,1}$ can simply be fixed by requiring the $\Gamma_{\text{eff}}$ to be invariant under gauge transformation in (6.31). For the case of bosonic fields, interesting discussion of gauge symmetries of boundary terms may be found in Ref.[68].
Taking into account (6.32) we obtain the following expression for the effective action:

\[-i\Gamma_{\text{eff}} \equiv \frac{1}{2} \int d^d x \langle \psi(x,z) | E_{\text{eff}} | \psi(x,z) \rangle |_{x \rightarrow 0} \quad (6.45)\]

Now, plugging in (6.45) solution to the second-order gauge-fixed equations of motion (6.1), we obtain the following three equivalent representations for the effective action:

**1st representation for the effective action**

\[-i\Gamma_{\text{eff}} = \nu_s c_{\nu_s} \int d^d x_1 d^d x_2 \langle \psi_{\text{sh}}(x_1) | \left( (\sigma_+ + (\gamma \alpha - \frac{1}{2} \alpha^2 \gamma \bar{\alpha}) \bar{e}_{1,1} ) \right) \frac{f_\nu}{|x_{12}|^{2\nu+d}}
\]

\[+ \frac{f_\nu}{|x_{12}|^{2\nu+d}} (\sigma_- - (\gamma \alpha - \frac{1}{2} \alpha^2 \gamma \bar{\alpha}) e_{1,1}) \rangle \psi_{\text{sh}}(x_2) \rangle \quad (6.46)\]

\[f_\nu \equiv \frac{\Gamma(\nu+1)\Gamma(\nu + \frac{d}{2})}{4^{\nu_s-\nu} \Gamma(\nu_s+1) \Gamma(\nu_s + \frac{d}{2})} \quad (6.47)\]

\[\nu_s \equiv s + \frac{d-2}{2} \quad (6.48)\]

where \(c_\nu\) and \(\nu\) are given in (4.30) and (6.42) respectively;

**2nd representation for the effective action**

\[-i\Gamma_{\text{eff}} = \frac{c_{\nu_s}}{4(\nu_s + \frac{d}{2} - 1)} \int d^d x_1 d^d x_2 \langle \psi_{\text{sh}}(x_1) | \left( \frac{g_\nu}{|x_{12}|^{2\nu+d-2}} \right) E_{\text{sh},(0)} \psi_{\text{sh}}(x_2) \rangle \quad (6.49)\]

\[g_\nu \equiv \frac{\Gamma(\nu)\Gamma(\nu + \frac{d}{2} - 1)}{4^{\nu_s-\nu} \Gamma(\nu_s+1) \Gamma(\nu_s + \frac{d}{2} - 1)} \quad (6.50)\]

\[E_{\text{sh},(0)} \equiv (1 - \gamma \alpha \gamma \bar{\alpha} - \frac{1}{4} \alpha^2 \gamma \bar{\alpha}) e_{1,sh}^r + (\gamma \alpha - \frac{1}{2} \alpha^2 \gamma \bar{\alpha}) \bar{e}_{1,sh} + (\gamma \bar{\alpha} - \frac{1}{2} \gamma \alpha \bar{\alpha}) e_{1,sh} \quad (6.51)\]

Effective action (6.46), (6.49) is invariant under gauge transformation (6.31) provided the shadow field \(|\psi_{\text{sh}}\rangle\) satisfies the differential constraints (6.22), (6.23). Note also that using constraint (6.24), we can represent (6.49) in terms of the Fang-Fronsdal operator.

**3rd representation for the effective action**

\[i\Gamma_{\text{eff}} = \frac{c_{\nu_s}}{4(\nu_s + \frac{d}{2} - 1)} \int d^d x_1 d^d x_2 \langle \psi_{\text{sh}}(x_1) | \left( \frac{g_\nu}{|x_{12}|^{2\nu+d-2}} \right) E_{\text{sh},(1)} \psi_{\text{sh}}(x_2) \rangle \quad (6.52)\]

where \(E_{\text{sh},(1)} = E_{(1)}\) and the Fang-Fronsdal operator \(E_{(1)}\) is given in (3.15).

To summarize, using CFT adapted action and the modified de Donder gauge, we obtain the 2-point gauge invariant effective action for the spin-\((s + \frac{1}{2})\) shadow field. Representations for the effective action given in (6.46), (6.49), (6.52) is our solution to the problem of 2-point effective action for the case of fermionic arbitrary spin fields.\footnote{In this paper, we discuss 2-point effective action. Recent results on Lagrangian description of interacting AdS fields (see e.g., Refs.[70]-[75]) provides interesting possibility for the studying interaction dependent contributions to the effective action.} Our effective action is gauge invariant under gauge transformation (6.31) and expressed in terms of gauge fields (6.4) which are subject to the differential constraints (6.22), (6.23). By fixing the gauge symmetries in various ways, we can
obtain various new representations for the effective action. For instance, we can use Stueckelberg gauge frame or light-cone gauge frame. In Stueckelberg gauge frame, gauging away Stueckelberg fields and solving the differential constraints (6.22), (6.23), we can express all fields in (6.4) in terms of the \(\gamma\)-traceless rank-\(s\) tensor-spinor field of the \(so(d - 1, 1)\) algebra. This \(\gamma\)-traceless field turns out to be free of differential constraints.\(^{24}\) Plugging such solution for fields (6.4) into (6.46), (6.49), (6.52) provides the representation for the effective action in terms of the \(\gamma\)-traceless rank-\(s\) tensor-spinor field. Such representation gives 2-point function of shadow field in the standard CFT.

In light-cone gauge frame, using light-cone gauge and the differential constraints (6.22), (6.23), we can express all our fields (6.4) in terms of one rank-\(s\) light-cone tensor-spinor fields. Plugging such solution for fields (6.4) into (6.46), (6.49), (6.52) provides light-cone gauge representation for the effective action. In other words, one of advantages of our approach is that our approach gives the possibility for the studying the effective action by using various gauge conditions which might be preferable in various applications.

### 7 Conformal fermionic fields

The kernel of effective action given in (6.46), (6.49), (6.52) is not well-defined when \(d\) is even integer and \(\nu\) takes integer values (see e.g. [69]). However this kernel can be regularized and after that it turns out that the leading logarithmic divergence of the effective action \(\Gamma_{\text{eff}}\) leads to Lagrangian of conformal fermionic fields. To explain what has just been said we note that the kernel of \(\Gamma_{\text{eff}}\) can be regularized by using dimensional regularization. This is to say that using the dimensional regularization and denoting the integer part of \(d\) by \([d]\), we introduce the regularization parameter \(\epsilon\) as

\[
d - [d] = -2\epsilon, \quad [d] - \text{even integer}.
\]

With this notation we note that \(\nu\) (6.42) can be presented as\(^{25}\)

\[
\nu = [\nu] + \frac{d}{2}, \quad [\nu] - \text{integer}.
\]

Now we use the following well know fact. With \(d\) and \(\nu\) given in (7.53) and (7.54) respectively the regularized kernel in (6.49) has the following behavior:

\[
\left|\frac{1}{x}\right|^{2\nu + d - 2} \sim \epsilon \frac{1}{\epsilon} \partial_{\nu - 1}^{\nu - 1} \delta^{(d)}(x),
\]

\[
\partial_{\nu} = \frac{\pi^{d/2}}{4^{\nu} \Gamma(\nu + 1)\Gamma(\nu + \frac{d}{2})}.
\]

Using (7.55) in (6.49), we obtain

\[
\Gamma_{\text{eff}} \sim \epsilon \frac{1}{\epsilon} c_{\nu_s} \nu_s \partial_{\nu_s} \int d^d x \mathcal{L},
\]

where \(\nu_s\) is defined in (6.48) and \(\mathcal{L}\) is a higher-derivative Lagrangian for conformal spin-(\(s + \frac{1}{2}\)) fermionic field. The Lagrangian takes the form

\[
i\mathcal{L} = \langle \psi_{\text{cf}} | \partial^{\nu - 1} E_{\text{cf},(0)} | \psi_{\text{cf}} \rangle,
\]

\(^{24}\)Note that the fact that we can express all fields in (6.4) in terms of the one rank-\(s\) \(\gamma\)-traceless tensor-spinor field of the \(so(d - 1, 1)\) algebra implies that our gauge invariant approach is equivalent to the standard approach to CFT.

\(^{25}\)Operator \([\nu]\) in (7.54) takes the form \([\nu] = s - N_z - \frac{d}{2} + \frac{1}{2}\sigma_3\). Obviously this operator has integer eigenvalues.
\[ E_{\text{cf},(0)} \equiv (1 - \gamma \alpha \gamma \bar{\alpha} - \frac{1}{4} \alpha^2 \bar{\alpha}^2) e_{1,\text{cf}}^{r} + (\gamma \alpha - \frac{1}{2} \alpha^2 \gamma \bar{\alpha}) \bar{e}_{1,\text{cf}} + (\gamma \bar{\alpha} - \frac{1}{2} \gamma \alpha \bar{\alpha}^2) e_{1,\text{cf}}, \tag{7.59} \]

\[ e_{1,\text{cf}}^{r} = e_{1,1}^{r} (\sigma - \Box + \sigma_{+}), \tag{7.60} \]

\[ e_{1,\text{cf}} = \bar{\alpha}^{2} e_{1}, \quad \bar{e}_{1,\text{cf}} = -\bar{e}_{1} \bar{\alpha}^{2}, \tag{7.61} \]

\[ e_{1,1}^{r} = \frac{2s + d - 2}{2s + d - 2 - 2N_{z}}, \quad \bar{e}_{1} = \left( \frac{2s + d - 3 - N_{z}}{2s + d - 4 - 2N_{z}} \right)^{1/2}, \tag{7.62} \]

where we have made the identification for the ket-vector of the conformal field \(|\psi_{\text{cf}}\rangle\),

\[ |\psi_{\text{cf}}\rangle = |\psi_{\text{sh}}\rangle, \tag{7.63} \]

i.e., the ket-vector \(|\psi_{\text{cf}}\rangle\) is represented in terms of tensor-spinor components as

\[ |\psi_{\text{cf}}\rangle \equiv \sum_{s'=0}^{s} \frac{\alpha_{s-s'}}{\sqrt{(s - s')!}} |\psi^{s'}\rangle, \tag{7.64} \]

\[ |\psi^{s'}\rangle \equiv \frac{1}{s!} \bar{\alpha}^{a_{1}} \ldots \bar{\alpha}^{a_{s'}} \psi_{\text{cf}}^{a_{1} \ldots a_{s'}} |0\rangle. \tag{7.65} \]

Using this identification, we note that the differential constraints for the spin-\((s + \frac{1}{2})\) shadow field given in (6.22), (6.23) imply the same differential constraints for the conformal field \(|\psi_{\text{cf}}\rangle\),

\[ \hat{E}_{\text{cf}} |\psi_{\text{cf}}\rangle = 0, \tag{7.66} \]

\[ C_{\text{cf}} |\psi_{\text{cf}}\rangle = 0, \tag{7.67} \]

where

\[ \hat{E}_{\text{cf}} \equiv \hat{\phi} - G_{\text{cf}} \gamma \bar{\alpha} + \left( 1 + \gamma \alpha \frac{2}{2N_{\alpha} + d - 2} \gamma \bar{\alpha} \right) e_{1,\text{cf}}^{r} - \gamma \alpha \frac{2}{2N_{\alpha} + d - 2} \bar{e}_{1,\text{cf}}, \tag{7.68} \]

\[ \hat{C}_{\text{cf}} \equiv \hat{C}_{\text{st}} - \frac{1}{2N_{\alpha} + d - 2} \left( \gamma \bar{\alpha} + \frac{1}{2} \gamma \alpha \bar{\alpha}^2 \right) e_{1,\text{cf}}^{r} + \frac{1}{2} \bar{\alpha}^{2} e_{1,\text{cf}} + \frac{2N_{\alpha} + d - 4}{2N_{\alpha} + d - 2} \Pi_{\text{bos}}^{[1,\alpha]} \bar{e}_{1,\text{cf}}, \tag{7.69} \]

\[ \hat{C}_{\text{st}} \equiv \bar{\alpha} \partial - \frac{1}{2} \bar{\alpha} \partial \bar{\alpha}^2. \tag{7.70} \]

Constraints (7.66), (7.67) are invariant under the gauge transformation

\[ \delta |\psi_{\text{cf}}\rangle = G_{\text{cf}} |\xi_{\text{cf}}\rangle, \tag{7.71} \]

\[ G_{\text{cf}} \equiv \bar{\alpha} \partial - e_{1,\text{cf}} + \gamma \alpha \frac{1}{2N_{\alpha} + d - 2} e_{1,\text{cf}}^{r} - \bar{\alpha}^{2} \frac{1}{2N_{\alpha} + d} \bar{e}_{1,\text{cf}}, \tag{7.72} \]

where ket-vector \(|\xi_{\text{cf}}\rangle\) takes the same form as the ket-vector \(|\xi\rangle\) given in (3.27).\(^{26}\) Also it is easy to check that Lagrangian (7.58) is invariant under gauge transformation (7.71) provided the ket-vector \(|\psi_{\text{cf}}\rangle\) satisfies differential constraints (7.66), (7.67).

The following remarks are in order:

\(^{26}\)Note however that ket-vector \(|\xi\rangle\) given in (3.27) depends on \(x^{a}\) and \(z\), while the ket-vector \(|\xi_{\text{cf}}\rangle\) appearing in (7.71) depends only on \(x^{a}\).
i) Constraint (7.66) can be represented as

\[ E_{\text{cf}}|\psi_{\text{cf}}\rangle = 0, \quad (7.73) \]

\[ E_{\text{cf}} \equiv E_{\text{cf},(1)} + E_{\text{cf},(0)} \quad (7.74) \]

\[ E_{\text{cf},(1)} \equiv E_{(1)}, \quad (7.75) \]

where the Fang-Fronsdal operator \( E_{(1)} \) appearing in (7.75) is given in (3.15), while the operator \( E_{\text{cf},(0)} \) is given in (7.59). Using (7.73), we see that Lagrangian (7.58) can be represented in terms of the Fang-Fronsdal operator,

\[ -i\mathcal{L} = \langle \psi_{\text{cf}} | \Box^{\nu-1} E_{\text{cf},(1)} | \psi_{\text{cf}} \rangle. \quad (7.76) \]

ii) Using differential constraints (7.66), (7.67) and gauging away Stueckelberg fields we can obtain representation for all tensor-spinor fields appearing in (7.64) in terms of one rank-\( s \) \( \gamma \)-traceless tensor-spinor field of \( so(d-1,1) \) algebra which is not subject to any differential constraints. Plugging such representation for the tensor-spinor fields in Lagrangian (7.58), we can express our Lagrangian in terms of the one rank-\( s \) \( \gamma \)-traceless tensor-spinor field.

iii) We note that UV divergence of the effective action leads to higher-derivative action of the conformal fields.\(^{27}\)

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Appendix A  Notation

Vector indices of the \( so(d-1,1) \) algebra take the values \( a, b, c = 0, 1, \ldots, d-1 \), while vector indices of the \( so(d,1) \) algebra take the values \( A, B, C = 0, 1, \ldots, d-1, d \). We use mostly positive flat metric tensors \( \eta^{ab}, \eta^{AB} \). To simplify our expressions we drop \( \eta_{ab}, \eta_{AB} \) in the respective scalar products, i.e., we use \( X^a Y^a \equiv \eta_{ab} X^a Y^b, X^A Y^A \equiv \eta_{AB} X^A Y^B \).

Using the identification \( X^d \equiv X^z \) gives the following decomposition of the \( so(d,1) \) algebra vector: \( X^A = X^a, X^z \). This implies \( X^A Y^A = X^a Y^a + X^z Y^z \).

We use the creation operators \( \alpha^a, \alpha^z \), and the respective annihilation operators \( \bar{\alpha}^a, \bar{\alpha}^z \),

\[ [\bar{\alpha}^a, \alpha^b] = \eta^{ab}, \quad [\bar{\alpha}^z, \alpha^z] = 1, \quad \bar{\alpha}^a | 0 \rangle = 0, \quad \bar{\alpha}^z | 0 \rangle = 0. \quad (A.1) \]

These operators are referred to as oscillators in this paper. The oscillators \( \alpha^a, \bar{\alpha}^a \) and \( \alpha^z, \bar{\alpha}^z \), transform in the respective vector and scalar representations of the \( so(d-1,1) \) algebra and satisfy the hermitian conjugation rules, \( \alpha^{a\dagger} = \bar{\alpha}^a, \alpha^{z\dagger} = \bar{\alpha}^z \). Oscillators \( \alpha^a, \alpha^z \) and \( \bar{\alpha}^a, \bar{\alpha}^z \) are collected into the respective \( so(d,1) \) algebra oscillators \( \alpha^A = \alpha^a, \alpha^z \) and \( \bar{\alpha}^A = \bar{\alpha}^a, \bar{\alpha}^z \).

\( x^A = x^a, z \) denote coordinates in \( d+1 \)-dimensional \( AdS_{d+1} \) space,

\[ ds^2 = \frac{1}{z^2}(dx^a dx^a + dz dz), \quad (A.2) \]

while \( \partial_A = \partial_a, \partial_z \) denote the respective derivatives, \( \partial_a \equiv \partial / \partial x^a, \partial_z \equiv \partial / \partial z \).

We use \( 2^{(d+1)/2} \times 2^{((d+1)/2)} \) Dirac gamma matrices \( \gamma^A \) in \( d+1 \)-dimensions, \( \{ \gamma^A, \gamma^B \} = 2\eta^{AB} \), and adapt the following hermitian conjugation rules for the derivatives, oscillators, and \( \gamma \)-matrices:

\[ \partial^{A\dagger} = -\partial^A, \quad \gamma^{A\dagger} = \gamma^0 \gamma^A \gamma^0, \quad \alpha^{a\dagger} = \bar{\alpha}^a, \quad \alpha^{z\dagger} = \bar{\alpha}^z. \quad (A.3) \]

\(^{27}\)Ordinary-derivative actions for conformal fields are discussed in Refs.[76]-[78].
We use operators constructed out of the derivatives, oscillators, and \( \gamma \)-matrices,

\[
\begin{align*}
\Box & \equiv \partial^a \partial_a, \quad \bar{\partial} \equiv \gamma^a \partial_a, \quad \alpha \partial \equiv \alpha^a \partial_a, \quad \bar{\alpha} \partial \equiv \bar{\alpha}^a \partial_a, \\
\gamma \alpha & \equiv \gamma^a \alpha_a, \quad \gamma \bar{\alpha} \equiv \gamma^a \bar{\alpha}_a, \quad \alpha^2 \equiv \alpha^a \alpha_a, \quad \bar{\alpha}^2 \equiv \bar{\alpha}^a \bar{\alpha}_a, \\
N_\alpha & \equiv \alpha^a \bar{\alpha}_a, \quad N_z \equiv \alpha^z \bar{\alpha}^z, \\
\Pi^{[i,j]} & = 1 - \gamma \alpha \frac{1}{2N_\alpha + d} \gamma \bar{\alpha} - \alpha^2 \frac{1}{2(2N_\alpha + d + 2)} \bar{\alpha}^2, \\
\Pi^{[i,j]}_{\text{bos}} & = 1 - \alpha^2 \frac{1}{2(2N_\alpha + d)} \bar{\alpha}^2.
\end{align*}
\]

The \( 2 \times 2 \) matrices and antisymmetric products of \( \gamma \)-matrices are defined as

\[
\begin{align*}
\sigma_+ & = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad \pi_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \pi_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \\
\gamma^{ab} & = \frac{1}{2} (\gamma^a \gamma^b - \gamma^b \gamma^a), \quad \gamma^{abc} = \frac{1}{3!} (\gamma^a \gamma^b \gamma^c \pm 5 \text{ terms}).
\end{align*}
\]

Notation \( \sigma_1, \sigma_2, \sigma_3 \) stands for the standard Pauli matrices.

The covariant derivative \( D^A \) is given by \( D^A = \eta^{AB} D_B \),

\[
D_A \equiv e^A_\mu D_\mu, \quad D_\mu \equiv \partial_\mu + \frac{1}{2} \omega_\mu^{AB} M^{AB},
\]

\[
M^{AB} \equiv \alpha^A \alpha^B - \alpha^B \alpha^A + \frac{1}{2} \gamma^{AB}, \quad \gamma^{AB} \equiv \frac{1}{2} (\gamma^A \gamma^B - \gamma^B \gamma^A),
\]

\( \partial_\mu = \partial / \partial x^\mu \), where \( e^A_\mu \) is inverse vielbein of AdS\(_{d+1} \) space, \( D_\mu \) is the Lorentz covariant derivative and the base manifold index takes values \( \mu = 0, 1, \ldots, d \). The \( \omega^{AB}_\mu \) is the Lorentz connection of AdS\(_{d+1} \) space, while \( M^{AB} \) is a spin operator of the Lorentz algebra \( so(d, 1) \). Note that AdS\(_{d+1} \) coordinates \( x^\mu \) carrying the base manifold indices are identified with coordinates \( x^A \) carrying the flat vectors indices of the \( so(d, 1) \) algebra, i.e., we assume \( x^\mu = \delta^\mu_A x^A \), where \( \delta^\mu_A \) is Kronecker delta symbol. AdS\(_{d+1} \) space contravariant tensor-spinor field, \( \Psi_{\mu_1 \ldots \mu_s} \), is related with field carrying the flat indices, \( \Psi^{A_1 \ldots A_s} \), in a standard way \( \Psi^{A_1 \ldots A_s} \equiv e_{\mu_1}^{A_1} \ldots e_{\mu_s}^{A_s} \Psi_{\mu_1 \ldots \mu_s} \). Helpful commutators are given by

\[
[D^A, D^B] = \Omega^{ABC} D^C - M^{AB}, \quad [\bar{\alpha} D, \alpha D] = \Box_{\text{AdS}} + \frac{1}{2} M^{AB} M^{AB},
\]

where \( \Omega^{ABC} = -\omega^{ABC} + \omega^{BAC} \) is a contorsion tensor and we define \( \omega^{ABC} \equiv e^{A\mu} \omega_{\mu}^{BC} \).

For the Poincaré parametrization of AdS\(_{d+1} \) space, vielbein \( e^A = e^A_\mu dx^\mu \) and Lorentz connection, \( de^A + \omega^{AB} \wedge e^B = 0 \), are given by

\[
e^A_\mu = \frac{1}{z} \delta^A_\mu, \quad \omega^{AB}_\mu = \frac{1}{z} (\delta^A_\mu \delta^B_\mu - \delta^B_\mu \delta^A_\mu).
\]

With choice made in (A.14), the covariant derivative takes the form \( D^A = z \partial^A + M^A, \partial^A = \eta^{AB} \partial_B \).
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