Masses and magnetic moments of hadrons with one and two open heavy quarks: heavy baryons and tetraquarks

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In this work, we compute masses and magnetic moments of the heavy baryons and tetraquarks with one and two open heavy flavors in a unified framework of MIT bag model. Using the parameters of MIT bag model, we confirm that an extra binding energy, which is supposed to exist between heavy quarks ($c$ and $b$) and between heavy and strange quarks in literatures, is required to reconcile light hadrons with heavy hadrons. Numerical calculations are made for all light mesons, heavy hadrons with one and two open heavy flavors, predicting the masses of doubly charmed baryons to be $M(\Xi_{cc}^+) = 3.604$ GeV, $M(\Xi_{cc}^*) = 3.714$ GeV, and that of the strange isosinglet tetraquark $ud\bar{s}\bar{c}$ with $J^P = 0^+$ to be $M(ud\bar{s}\bar{c}, 0^+) = 2.934$ GeV. The state mixing due to chromomagnetic interaction is shown to be sizable for the strange scalar tetraquark $nn\bar{s}\bar{c}$.

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I. INTRODUCTION

Four years ago, the LHCb Collaboration at CERN discovered the first doubly charmed baryon $\Xi_{cc}^+$ with $J^P = 1/2^+$ and measured its mass to be $3621.40 \pm 0.78$ MeV [1]. Later, the $\Xi_{cc}^+$ state was confirmed in the decay to $\Xi_c^+\pi^+$[2] and its lifetime, mass and production cross-section were subsequently measured[3, 4]. Containing two charmed quarks, such a baryon provide a unique probe for quantum chromodynamics(QCD), the gauge theory of strong interactions. In addition, the observation provides a useful experimental information about strength of interaction between two heavy quarks and enables us to further explore tetraquarks $QQ'\bar{q}\bar{q}$ containing two open heavy quarks, which is allowed by QCD. See Refs. [5–7] for instance. Recently, LHCb Collaboration reported the first exotic state $X_0(2900)$ with open heavy flavors and mass $2866 \pm 7$ MeV[8], which is interpreted to be an isosinglet tetraquark $cs\bar{u}\bar{d}$ in Ref. [9]. More recently, observation of a dou-
bly charmed tetraquark $T_{cc}^{++}$ is reported also by LHCb Collaboration[10]. These findings, among others, make it of interest to explore doubly heavy(DH) hadrons in details. There exist extensive studies of DH hadrons with various approaches, including potential quark model and bag model [11–14, 16–21], AdS/QCD approach [22–26] and relativistic quark model [27]. See Refs [28, 29] for recent reviews.

In identifying and/or finding these DH hadrons experimentally, it is helpful to have a systematic estimate of masses and other properties of them within an unified framework. For instance, a mass predictions [14, 16] of the doubly charmed baryon $\Xi_{cc}$, which are larger about 100 MeV than that measured in 2002 by the SELEX Collaboration at Fermilab [30](awaiting confirmation), helps LHCb Collaboration to search the $\Xi_{cc}^{++}[1]$ eventually.

In this work, we apply MIT bag model [31, 32] with chromomagnetic interaction and a strong coupling $\alpha_s$ running with the bag radius to systematically study the open heavy baryons and tetraquarks with one and two heavy flavors and compute the masses and other static properties(magnetic moments, electric charge radii) of them. It is confirmed that an extra binding energy between heavy quarks ($c$ and $b$) and between heavy and strange quarks is required to reconcile light hadrons with heavy hadrons. Computed results are compared to other calculations and in consistent with the measured masses and other properties of light hadrons and singly heavy baryons in their ground states(except for $\pi$). For the $J^P = \frac{1}{2}^+$ states of heavy baryons $\Xi_c$, $\Xi_b$, $\Xi_{bc}$, $\Omega_{bc}$ and the heavy tetraquarks, the chromomagnetic mixing is taken into account, and the respective mass splittings are computed variationally.

It is well known that bag model [31] embodies two primary features of quantum chromodynamics (QCD): asymptotic freedom at short distance and confinement at long distance. The simple structure of the model enables us to describe mesons($q\bar{q}$), baryons($qqq$) and even hadrons made of multiquarks. In the past few decades, bag model has been applied to describe the doubly heavy baryons[11–13] and multiquark hadrons, including light exotic baryons with five and seven nonstrange quarks [33]. In order to evaluate the masses of doubly heavy baryons, a large running strong coupling $\alpha_s$ was applied in Ref. [12]. A Coulomb-like interaction is derived between heavy quarks in a bag in Refs. [34, 35].

This paper will be organized as follows. In Sec. II, we review some basic relations of MIT bag model, including chromomagnetic interaction (CMI) among the quarks in bag. In Sec. III, a systematic numerical calculation is performed for the established light and singly heavy(SH) baryons, with the optimal set of parameters obtained and the results for masses and other properties reproduced. In Sec. IV, we present detailed predictions for masses and other properties for doubly heavy baryons and the tetraquarks with one and two open heavy quarks. The paper ends with summary and conclusions in Sec. V.

II. METHOD FOR MIT BAG MODEL WITH CMI

A. Mass Formula

Treating hadron as a spherical bag, MIT bag model provides an approach to estimate masses and other properties of hadrons in their ground states[31, 32], in which the chromomagnetic interaction is derived from the energy of a sphere-like gluon field interacting with quark fields in bag.
The mass formula of hadron in MIT bag model is,

\[ M(R) = \sum_{i=n,s,c,b} n_i \omega_i + \frac{4}{3} \pi R^3 B - \frac{Z_0}{R} + \langle \Delta H \rangle, \tag{1} \]

where the first term is the kinematic energy of all quarks in bag with radius \( R \), the second is the volume energy of bag with bag constant \( B \), the third is the zero-point-energy (ZPE) with coefficient \( Z_0 \), and \( \langle \Delta H \rangle \) is the short-range interaction among quarks in bag, which we will address in this work. Here in Eq. (1), \( n_i \) is number of quark or antiquark in bag with mass \( m_i \) and flavor \( i \), where \( i \) can be the light nonstrange quarks \( n = u, d \), the strange quark \( s \), the charm quark \( c \) and the bottom quark \( b \). The value of \( R \) is to be determined variationally, and the dimensionless parameters \( x_i = x_i(mR) \) are related to the bag radius \( R \) by an transcendental eigen-equation

\[ \tan x_i = \frac{x_i}{1 - m_iR - \left( m_i^2 R^2 + x_i^2 \right)^{1/2}}. \tag{3} \]

The interaction energy \( \langle \Delta H \rangle = B_{EB} + M_{CMI} \) is composed of two energy terms:

1. The spin-independent binding energy \( B_{EB} \), due mainly to the short-range chromoelectric interaction between quarks (and/or antiquarks). Owing to its smallness for the relativistic light quarks \( n(= u, d) \), this energy, scales mainly as \(- \sum \alpha_s/r_{ij} \), becomes sizable when both of two quarks \( i \) and \( j \) are massive and moving nonrelativistically. In present work, we treat this energy as sum of the pair binding energies \( B_{QQ'} (B_{Qs}) \) between heavy quarks and between heavy quark \( Q \) and strange quark \( s \) \[14, 36, 37\]. The net effect for this chromoelectric interaction amounts to introduction of five binding energies \( B_{cs}, B_{cc}, B_{bs}, B_{bb} \) and \( B_{bc} \) for any quark pair in color configuration \( \bar{3}_c \), which are extractable from heavy mesons and can be scaled to other color configurations.

2. The chromomagnetic interaction energy, due to perturbative gluon exchange between quarks (antiquarks) \( i \) and \( j \),

\[ M_{CMI} = - \sum_{i<j} (\lambda_i \cdot \lambda_j) (\sigma_i \cdot \sigma_j) C_{ij}, \tag{4} \]

with \( \lambda_i \) the Gell-Mann matrices, \( \sigma_i \) the Pauli matrices, and \( C_{ij} \) the CMI parameter. In MIT bag model, the parameters \( C_{ij} \) are given by

\[ C_{ij} = 3 \frac{\alpha_s(R)}{R^3} \tilde{\mu}_i \tilde{\mu}_j I_{ij}, \tag{5} \]

\[ \tilde{\mu}_i = \frac{R \cdot 4 \alpha_i + 2 \lambda_i - 3}{6 \cdot 2 \alpha_i (\alpha_i - 1) + \lambda_i}, \tag{6} \]

\[ I_{ij} = 1 + 2 \int_0^R \frac{d \bar{r}}{r^3} \tilde{\mu}_i \tilde{\mu}_j = 1 + F(x_i, x_j), \tag{7} \]

where \( \alpha_i \equiv \omega_i R, \lambda_i \equiv m_i R, \alpha_s(R) \) is the running strong coupling, \( \tilde{\mu}_i \) is the reduced magnetic moment without electric charge, and \( I_{ij} \) and \( F(x_i, x_j) \) are rational functions of \( x_i \) and \( x_j \), given
explicitly by [31].

\[
F(x_i, x_j) = \left( x_i \sin^2 x_i - \frac{3}{2} y_i \right)^{-1} \left( x_j \sin^2 x_j - \frac{3}{2} y_j \right)^{-1}
\]

\[
\left\{ \frac{-3}{2} y_i y_j - 2 x_i x_j \sin^2 x_i \sin^2 x_j + \frac{1}{2} x_i x_j [2 x_i \text{Si}(2 x_i) \\
+ 2 x_j \text{Si}(2 x_j) - (x_i + x_j) \text{Si}(2(x_i + x_j)) \\
- (x_i - x_j) \text{Si}(2(x_i - x_j))] \right\},
\]

where \( y_i = x_i - \cos(x_i) \sin(x_i) \), \( x_i \) is the root of Eq. (3) for a given \( m_i R \), and

\[
\text{Si}(x) = \int_0^x \frac{\sin(t)}{t} dt.
\]

FIG. 1: Four running behaviors of strong coupling. Bag radius \( R \) ranges from 3 GeV\(^{-1}\) to 6 GeV\(^{-1}\), and the standard radius is set to be 5 GeV\(^{-1}\) (\( \approx 1 \) fm) for checking. The solid line represents our result (11) while the dashed line corresponds to Eq. (10) with \( \gamma = 2.847 \), \( \Lambda_{QCD} = 0.281 \) GeV and \( w = 2\pi n/9 \). The dotted line shows Eq. (10) with \( \gamma = 1 \), \( \Lambda_{QCD} = 0.281 \) GeV and \( w = 2\pi n/9 \). The dotdashed line indicates that of Ref. [12]. All four behaviors adopt \( n = 1 \).

In some applications of bag model [11, 12, 34, 35, 38], the parameter \( \alpha_s \) takes a logarithmic form

\[
\alpha_s(R) = \frac{w}{\ln \left[ \gamma + (RA_{QCD})^{-n} \right]},
\]

where \( w \) and \( n \) (\( n = 1 \) or 2) are the parameters, \( \Lambda_{QCD} \) is the QCD scale (0.2 ~ 0.5 GeV), and \( \gamma \) is prefactor used to avoid infrared divergence. Similar to Ref. [12], we take \( w = 0.296 \), \( \Lambda_{QCD} = 0.281 \) GeV, \( \gamma = 1 \) and \( n = 1 \) to set

\[
\alpha_s(R) = \frac{0.296}{\ln \left[ 1 + (0.281R)^{-1} \right]},
\]
which is plotted in Fig. 1. Among four lines showing the running of $\alpha_s$ in the plot, the solid line shows notable variation and corresponds to a relative lower value of $\alpha_s$.

### TABLE I: Bag radius $R$ (in GeV$^{-1}$) and mass prediction $M_{bag}$ (in GeV) obtained from this work ($Z_0 = 1.83$) and original MIT bag model for light hadrons, compared to the measured mass $M_{exp}$ (in GeV) being isospin-averaged.

| State | MIT bag$^{[32]}$ | This work | $M_{exp}$$^{[39]}$ |
|-------|-----------------|------------|-----------------|
| $N$   | 5.00 0.938      | 5.22 0.932 | 0.939           |
| $\Delta$ | 5.48 1.233      | 5.33 1.241 | 1.232           |
| $\Lambda$ | 4.95 1.105      | 5.26 1.096 | 1.116           |
| $\Sigma$ | 4.95 1.144      | 5.22 1.137 | 1.193           |
| $\Xi$ | 4.91 1.289      | 5.27 1.282 | 1.318           |
| $\Sigma^*$ | 5.43 1.382      | 5.38 1.383 | 1.385           |
| $\Xi^*$ | 5.39 1.529      | 5.42 1.529 | 1.533           |
| $\Omega$ | 5.35 1.672      | 5.46 1.677 | 1.672           |
| $\pi$ | 3.34 0.280      | 4.31 0.348 | 0.137           |
| $\omega$ | 4.71 0.783      | 4.55 0.776 | 0.783           |
| $K$ | 3.26 0.497      | 4.34 0.561 | 0.496           |
| $K^*$ | 4.65 0.928      | 4.63 0.918 | 0.894           |
| $\phi$ | 4.61 1.068      | 4.70 1.064 | 1.019           |

Given the parameter values of the quark mass $m_i$, bag constant $B$, the ZPE coefficient $Z_0$ and strong coupling constant $\alpha_s(R)$ depending on bag radius $R$, one can apply variational method to determine the respective bag radius $R$ for each hadron and the respective $x_i$ through Eq. (3). Then, it is straightforward to use Eqs. (1),(2) and (4) to compute the ground-state masses and other static properties (magnetic moments, the charge radius) of the hadrons ranging from the light hadrons to heavy tetraquarks. The computed results for the light hadrons are listed in Table I, compared to that predicted by original MIT bag model.

We stress that for a given hadronic state there is in principle a unique set of the solution $x_i$ and $R$ corresponding to respective bag dynamics, as indicated by our computation. Owing to $(x_i,R)$-dependence of the $\langle \Delta H \rangle$, a simple and analytic mass formula is lacking for the hadrons with chromomagnetic-mixing since for that purpose one has to first diagonalize the CMI matrices before the variational analysis, which amounts to a higher-order algebraic equations.

#### B. Chromomagnetic Interaction

In evaluating the spin-dependent mass due to the CMI (4), in which $\lambda_i$ should be replaced by $-\lambda_i^*$ for an antiquark, one has to diagonalize the CMI matrix for given hadron multiplets with
certain spin-parity \( J^P \) to give the respective mass splittings [7] within the multiplets. For this, we list all the flavor-spin-color wavefunctions of hadrons including tetraquarks considered in this work, and present relevant formulas of the color and spin factors for them.

**Mesons:** The color wavefunction \( \phi^M = |q_1 \bar{q}_2 \rangle \) can be one of two spin states (of vector and scalar like):

\[
\chi^M_1 = |q_1 \bar{q}_2 \rangle_1, \quad \chi^M_2 = |q_1 \bar{q}_2 \rangle_0, \tag{12}
\]

where subscript \( J = 0 \) or 1 outside the bracket denotes the total spin of hadron. The spin-color wavefunctions with spin \( J = 0 \) and 1 are then

\[
\phi^M \chi^M_1 = |q_1 \bar{q}_2 \rangle_1, \quad \phi^M \chi^M_2 = |q_1 \bar{q}_2 \rangle_0. \tag{13}
\]

**Baryons:** The color wavefunctions \( \phi^B = \left[(q_1 q_2)^3 q_3 \right] \) can be in one of three spin states

\[
\chi^B_1 = \left|(q_1 q_2)_1 q_3 \right\rangle_{3/2}, \\
\chi^B_2 = \left|(q_1 q_2)_1 q_3 \right\rangle_{1/2}, \\
\chi^B_3 = \left|(q_1 q_2)_0 q_3 \right\rangle_{1/2}, \tag{14}
\]

where \( (q_1 q_2) \) stands for a diquark with spin \( J = 1 \) or 0 in color configuration \( 3_c \).

To write wavefunction for a hadron, the flavor symmetry has to be considered. For a flavor-symmetric wavefunction of \( (q_1 q_2) \), with isospin \( I = 1 \) or identical flavors, we use a symbol \( \delta^S_{12} = 1 \). For a flavor-asymmetric wavefunction with \( I = 0 \), a symbol \( \delta^A_{12} = 1 \) will be used. For two quarks \( q_1 \) and \( q_2 \) with different flavors which goes beyond isospin symmetry, one can use \( \delta^S_{12} = \delta^A_{12} = 1 \). With the help of Pauli principle, one can write three flavor-spin-color wavefunctions for baryons

\[
\phi^B \chi^B_1 = \left|(q_1 q_2)^3 q_3 \right\rangle_{3/2} \delta^S_{12}, \\
\phi^B \chi^B_2 = \left|(q_1 q_2)^3 q_3 \right\rangle_{1/2} \delta^S_{12}, \\
\phi^B \chi^B_3 = \left|(q_1 q_2)^3 q_3 \right\rangle_{1/2} \delta^A_{12}. \tag{15}
\]

Owing to the non-diagonal chromomagnetic interaction (4), some hadronic states with same \( J^P \) but different spin-color wavefunctions can mix(CMI mixing). For example, for the doubly heavy baryons \( \Xi_{bc} \) and \( \Xi'_{bc} \) with flavor structure \( (bc) \) the use of \( \delta^S_{12} = \delta^A_{12} = 1 \) is not enough to distinguish the two configurations \( \phi^B \chi^B_2 \) and \( \phi^B \chi^B_3 \) solely in terms of their \( J^P \) quantum numbers. Thus, the physical state must be one of the mixing states of them. See Sect. IV for the details of chromomagnetic mixing.

**Tetraquarks:** A tetraquark can have the color structure of whether \( 6_c \otimes \bar{6}_c \) or \( 3_c \otimes 3_c \), with the respective color wavefunctions,

\[
\phi^T_1 = \left|(q_1 q_2)^6 (\bar{q}_3 \bar{q}_4)^6 \right\rangle, \quad \phi^T_2 = \left|(q_1 q_2)^3 (\bar{q}_3 \bar{q}_4)^3 \right\rangle, \tag{16}
\]

and it can be one of the following six states

\[
\chi^T_1 = \left|(q_1 q_2)_1 (\bar{q}_3 \bar{q}_4)_1 \right\rangle_2, \quad \chi^T_2 = \left|(q_1 q_2)_1 (\bar{q}_3 \bar{q}_4)_1 \right\rangle_1, \\
\chi^T_3 = \left|(q_1 q_2)_1 (\bar{q}_3 \bar{q}_4)_0 \right\rangle_0, \quad \chi^T_4 = \left|(q_1 q_2)_1 (\bar{q}_3 \bar{q}_4)_0 \right\rangle_1, \\
\chi^T_5 = \left|(q_1 q_2)_0 (\bar{q}_3 \bar{q}_4)_1 \right\rangle_1, \quad \chi^T_6 = \left|(q_1 q_2)_0 (\bar{q}_3 \bar{q}_4)_0 \right\rangle_0. \tag{17}
\]
which lead to twelve basis wavefunctions

\[
\begin{align*}
\phi_1^T \chi_1^T &= \left( \langle q_1 q_2 \rangle_1^{(0)} (\bar{q}_3 \bar{q}_4) \right)^{\gamma_1}_2 \delta_{12} \delta_{34}, \\
\phi_2^T \chi_1^T &= \left( \langle q_1 q_2 \rangle_1^{(3)} (\bar{q}_3 \bar{q}_4) \right)^{\gamma_1}_2 \delta_{12} \delta_{34}, \\
\phi_1^T \chi_2^T &= \left( \langle q_1 q_2 \rangle_1^{(6)} (\bar{q}_3 \bar{q}_4) \right)^{\gamma_1}_2 \delta_{12} \delta_{34}, \\
\phi_2^T \chi_2^T &= \left( \langle q_1 q_2 \rangle_1^{(6)} (\bar{q}_3 \bar{q}_4) \right)^{\gamma_1}_2 \delta_{12} \delta_{34}, \\
\phi_1^T \chi_3^T &= \left( \langle q_1 q_2 \rangle_1^{(6)} (\bar{q}_3 \bar{q}_4) \right)^{\gamma_1}_2 \delta_{12} \delta_{34}, \\
\phi_2^T \chi_3^T &= \left( \langle q_1 q_2 \rangle_1^{(6)} (\bar{q}_3 \bar{q}_4) \right)^{\gamma_1}_2 \delta_{12} \delta_{34}, \\
\phi_1^T \chi_4^T &= \left( \langle q_1 q_2 \rangle_1^{(6)} (\bar{q}_3 \bar{q}_4) \right)^{\gamma_1}_2 \delta_{12} \delta_{34}, \\
\phi_2^T \chi_4^T &= \left( \langle q_1 q_2 \rangle_1^{(6)} (\bar{q}_3 \bar{q}_4) \right)^{\gamma_1}_2 \delta_{12} \delta_{34}, \\
\phi_1^T \chi_5^T &= \left( \langle q_1 q_2 \rangle_1^{(6)} (\bar{q}_3 \bar{q}_4) \right)^{\gamma_1}_2 \delta_{12} \delta_{34}, \\
\phi_2^T \chi_5^T &= \left( \langle q_1 q_2 \rangle_1^{(6)} (\bar{q}_3 \bar{q}_4) \right)^{\gamma_1}_2 \delta_{12} \delta_{34}, \\
\phi_1^T \chi_6^T &= \left( \langle q_1 q_2 \rangle_1^{(6)} (\bar{q}_3 \bar{q}_4) \right)^{\gamma_1}_2 \delta_{12} \delta_{34}, \\
\phi_2^T \chi_6^T &= \left( \langle q_1 q_2 \rangle_1^{(6)} (\bar{q}_3 \bar{q}_4) \right)^{\gamma_1}_2 \delta_{12} \delta_{34}.
\end{align*}
\]

We list all relevant color wavefunctions in Appendix A and spin wavefunctions in Appendix B. With them, one can evaluate the color and spin factors in Eq. (4) with the help of the following formulas

\[
\langle \lambda_i \cdot \lambda_j \rangle_{nm} = \sum_{a=1}^{8} Tr \left( c_i^a \lambda^a c_j \right) Tr \left( c_j^a \lambda^a c_i \right),
\]

\[
\langle \sigma_i \cdot \sigma_j \rangle_{xy} = \sum_{a=1}^{3} Tr \left( \chi_i^a \sigma^a \chi_j^a \right) Tr \left( \chi_j^a \sigma^a \chi_i^a \right),
\]

where \(n, m\) and \(x, y\) indicate the specific color and spin states respectively, \(i\) and \(j\) are the indexes of quarks (antiquarks), and the functions \(c\) and \(\chi\) are the respective basis vectors in the color and spin spaces. Table II lists a set of non-mixed hadronic states with their respective CMI’s.

Now, we are in the position to construct the matrix formula of CMI energy (4) and diagonalize it so as to minimize the obtained mass formula. Adding the binding energy (for heavy quark pair and for a pair of one heavy quark and one strange quark) to the bag energy, one can solve the dynamical parameters \(x_i\) and \(R\), and thereby obtain the wavefunctions of a given hadron.

### C. Hadronic Properties

Given the parameters \(x_i\) and \(R\) describing a hadronic state, mass and other properties (e.g., the charge radius and magnetic moment) can be evaluated. Following the standard method, one can firstly calculate the contribution of a quark or an antiquark \(i\) with electric charge \(Q_i\) to charge
TABLE II: Chromomagnetic interactions for the non-mixing hadrons with respective wavefunctions. $C_8$ follows Eq. (5) with subscripts corresponding to quark or antiquark.

| State | Wave Function | CMI | State | Wave Function | CMI |
|-------|---------------|-----|-------|---------------|-----|
| $\pi$ | $\phi_{A_2}^M$ | $-16C_{nn}$ | $\omega$ | $\phi_{A_1}^M$ | $\frac{16}{3}C_{nn}$ |
| $K$ | $\phi_{A_2}^M$ | $-16C_{sn}$ | $K^*$ | $\phi_{A_1}^M$ | $\frac{16}{3}C_{sn}$ |
| $D$ | $\phi_{A_2}^M$ | $-16C_{cn}$ | $D^*$ | $\phi_{A_1}^M$ | $\frac{16}{3}C_{cn}$ |
| $D_s$ | $\phi_{A_2}^M$ | $-16C_{cs}$ | $D^*_s$ | $\phi_{A_1}^M$ | $\frac{16}{3}C_{cs}$ |
| $\eta_c$ | $\phi_{A_2}^M$ | $-16C_{cc}$ | $J/\psi$ | $\phi_{A_1}^M$ | $\frac{16}{3}C_{cc}$ |
| $N$ | $\phi_{A_3}^B$ | $-8C_{nn}$ | $\Delta$ | $\phi_{A_1}^B$ | $8C_{nn}$ |
| $\Lambda$ | $\phi_{A_3}^B$ | $-8C_{nn}$ | $\Lambda$ | $\phi_{A_1}^B$ | $8C_{nn}$ |
| $\Sigma$ | $\phi_{A_3}^B$ | $\frac{8}{3}C_{nn} - \frac{22}{3}C_{sn}$ | $\Sigma^*$ | $\phi_{A_1}^B$ | $\frac{8}{3}C_{nn} + \frac{16}{3}C_{sn}$ |
| $\Lambda_c$ | $\phi_{A_3}^B$ | $-8C_{nn}$ | $\Sigma_c$ | $\phi_{A_1}^B$ | $\frac{8}{3}C_{nn} + \frac{16}{3}C_{cn}$ |
| $\Xi$ | $\phi_{A_3}^B$ | $\frac{8}{3}C_{ss} - \frac{22}{3}C_{sn}$ | $\Xi'$ | $\phi_{A_1}^B$ | $\frac{8}{3}C_{ss} + \frac{16}{3}C_{sn}$ |
| $\Xi_c$ | $\phi_{A_3}^B$ | $-8C_{sn}$ | $\Xi_c'$ | $\phi_{A_1}^B$ | $\frac{8}{3}C_{sn} - \frac{16}{3}C_{cn} + \frac{16}{3}C_{cs}$ |
| $\bar{ss}\bar{cc}$ | $\phi_{A_3}^T$ | $\frac{8}{3}C_{ss} - \frac{16}{3}C_{cs} + \frac{8}{3}C_{cc}$ | $ss\bar{cc}$ | $\phi_{A_1}^T$ | $\frac{8}{3}C_{ss} + \frac{16}{3}C_{cs} + \frac{8}{3}C_{cc}$ |
| $(nn\bar{c})^{I=1}$ | $\phi_{A_3}^T$ | $\frac{8}{3}C_{nn} - \frac{16}{3}C_{cn} + \frac{8}{3}C_{cc}$ | $(nn\bar{c})^{I=1}$ | $\phi_{A_1}^T$ | $\frac{8}{3}C_{nn} + \frac{16}{3}C_{cn} + \frac{8}{3}C_{cc}$ |

radius $[31]$

$$\langle r_E^2 \rangle_i = Q_i R^2 \frac{\alpha_i \left[ 2 \alpha_i (\alpha_i - 1) + 4 \alpha_i + 2 \lambda_i - 3 \right]}{3 \alpha_i^2 \left[ 2 \alpha_i (\alpha_i - 1) + \lambda_i \right]} - Q_i R^2 \frac{\lambda_i \left[ 4 \alpha_i + 2 \lambda_i - 2 \lambda_i^2 - 3 \right]}{2 \alpha_i^2 \left[ 2 \alpha_i (\alpha_i - 1) + \lambda_i \right]}.$$  \hspace{1cm} (21)

The sum of Eq. (21) then gives the charge radius of a hadronic state $[40]$

$$r_E = \left| \sum_i \langle r_E^2 \rangle_i \right|^{1/2}.$$  \hspace{1cm} (22)

We note that Eq. (22) also holds true for the chromomagnetic-mixing systems having the identical quark constituents.

For magnetic moment, the following equations $[31, 41]$, which are computed relative to the magnetic moment of proton and has the unit of $\mu_p$, are useful:

$$\mu_i = Q_i \mu_i = Q_i R \frac{4 \alpha_i + 2 \lambda_i - 3}{6 \alpha_i (\alpha_i - 1) + \lambda_i},$$  \hspace{1cm} (23)

$$\mu = \psi_{spin} \left| \sum_i g_i \mu_i S_{iz} \psi_{spin} \right|.$$  \hspace{1cm} (24)
TABLE III: Sum rule for magnetic moments of spin states of mesons \((q_1\bar{q}_2)\), baryons \((q_1q_2)q_3\), and tetraquarks \((q_1q_2)(\bar{q}_3\bar{q}_4)\) and their spin-mixed systems.

| \(\psi_{\text{spin}}\) | \(\mu\) |
|----------------------|--------|
| \(X_1^M\)           | \(\mu_1 + \mu_2\) |
| \(X_2^M\)           | 0      |
| \(X_1^B\)           | \(\mu_1 + \mu_2 + \mu_3\) |
| \(X_2^B\)           | \(\frac{1}{2}(2\mu_1 + 2\mu_2 - \mu_3)\) |
| \(X_3^B\)           | \(\mu_3\) |
| \(C_1X_1^B + C_2X_3^B\) | \(C_1^2\mu(X_1^B) + C_2^2\mu(X_3^B) + \frac{2}{\sqrt{3}}C_1C_2(\mu_2 - \mu_1)\) |
| \(X_1^T\)           | \(\mu_1 + \mu_2 + \mu_3 + \mu_4\) |
| \(X_2^T\)           | \(\frac{1}{2}(\mu_1 + \mu_2 + \mu_3 + \mu_4)\) |
| \(X_3^T\)           | 0      |
| \(X_4^T\)           | \(\mu_1 + \mu_2\) |
| \(X_5^T\)           | \(\mu_3 + \mu_4\) |
| \(X_6^T\)           | 0      |
| \(C_1X_1^T + C_2X_6^T\) | 0 |
| \(C_1X_2^T + C_2X_4^T\) | \(C_1^2\mu(X_2^T) + C_2^2\mu(X_4^T)\) |
| \(C_1X_2^T + C_2X_4^T + C_3X_5^T\) | \(C_1^2\mu(X_2^T) + C_2^2\mu(X_4^T) + C_3^2\mu(X_5^T) + \sqrt{2}C_1C_2(\mu_3 - \mu_4) + \sqrt{2}C_1C_3(\mu_2 - \mu_1)\) |
| \(C_1X_2^T + C_2X_4^T + C_3X_5^T\) | \(C_1^2\mu(X_2^T) + C_2^2\mu(X_4^T) + C_3^2\mu(X_5^T) + \sqrt{2}C_1C_2(\mu_2 - \mu_1) + \sqrt{2}C_1C_3(\mu_3 - \mu_4)\) |

where \(g_i = 2\), and \(S_{i\bar{i}}\) is the third component of spin for an individual quark or antiquark. In all Tables for the results of magnetic moments, obtained from Eq. (23) and Eq. (24), we transform them into that in the unit of the nuclear magneton \(\mu_N\), with the help of the measured data \(\mu_p = 2.79285\mu_N\). If the chromomagnetic mixing enters, the total spin wavefunction becomes

\[
|\psi_{\text{spin}}\rangle = C_1\chi_1 + C_2\chi_2,
\]

by which Eq. (24) gives

\[
\mu = C_1^2\mu(\chi_1) + C_2^2\mu(\chi_2) + 2C_1C_2\mu^{\text{tr}}(\chi_1, \chi_2),
\]

with \(\mu^{\text{tr}}\) the cross-term standing for transition moment [13] and \((C_1, C_2)\) the eigenvector of the given mixing state. We list all spin wavefunctions in Appendix B, and derive magnetic moments for them and their possibly-mixed systems involved in this work. The results for the spin wavefunctions and the respective magnetic moments are listed in Table III collectively.

Note that the cross-terms in CMI-mixing systems are not symmetric under the exchange between quarks \(q_1\) and \(q_2\) or, between \(q_3\) and \(q_4\) in the flavor space. While the expression of cross-term for the diquark \((ud)\) differs a sign for \((ud)\) and \((du)\) within the symmetric or asymmetric flavor wavefunctions when \(I_3 = 1\) or -1 in isospin space, respectively, the explicit computation via these wavefunctions can offset such cross-term. Similar conclusions also apply as the hadron systems respect the \(SU(2)\) isospin symmetry.
III. DETERMINATION OF PARAMETERS

In MIT bag model, the parameters (nonstrange \( m_n \) and strange \( m_s \) quark masses, \( B, Z_0 \) and \( \alpha_s \)) are determined based on the mass spectra of the light hadrons \( N, \Delta, \omega \) and \( \Omega \) in their ground states. The results read [31]

\[
\begin{align*}
 m_n &= 0, \quad m_s = 0.279 \text{ GeV}, \\
 Z_0 &= 1.83, \quad B^{1/4} = 0.145 \text{ GeV}, \quad \alpha_s = 0.55.
\end{align*}
\]

We choose Eq. (27) to be the parameters applying to both of light and heavy hadrons, with one exception that the strong coupling \( \alpha_s \) changes with the size \( R \) of hadron around 0.55, as given by Eq. (11). To fix the model parameters it remains two tasks yet.

The first task is to extract the heavy quark masses \( m_c \) and \( m_b \). Given Eq. (27), one can apply Eqs. (1) and (4) to the heavy-light mesons \( D^* \) and \( B^* \) to fix numerically \( m_c \) and \( m_b \), respectively. The results are

\[
\begin{align*}
 m_c &= 1.641 \text{ GeV}, \quad m_b = 5.093 \text{ GeV}.
\end{align*}
\]

The second is to fix the binding energy \( B_{QQ} \) (and \( B_{QQ'}, Q, Q' = s, c, b \) here), which is proposed in Ref. [14] to occur in charmed-strange hadrons, bottom-strange hadrons and heavy quadrennia. It can be due to nontrivial short-range interaction between two heavy quarks and between heavy and strange quarks[14, 36, 37]. Applied to the strange heavy mesons\( (Q\bar{s} \text{ and } Q\bar{Q}r) \), this binding enters the mass formula through

\[
egin{align*}
 M(Q\bar{s}) &= \omega_Q + \omega_s + \frac{4}{3} \pi R^3 B - \frac{Z_0}{R} + \langle H_{\text{CMI}} \rangle + B_{Q\bar{s}}, \\
 M(Q\bar{Q}r) &= \omega_Q + \omega_{Q'} + \frac{4}{3} \pi R^3 B - \frac{Z_0}{R} + \langle H_{\text{CMI}} \rangle + B_{Q\bar{Q}r}.
\end{align*}
\]

Applying to the case of the vector mesons \( D_s^* = c\bar{s} \), this allows one to solve the binding term,

\[
 B_{c\bar{s}} = M(D_s^*) - \omega_c - \omega_s - \frac{4}{3} \pi R^3 B + \frac{Z_0}{R} - \langle H_{\text{CMI}} \rangle,
\]

with the bag radius \( R \) solved variationally for the \( D_s^* \) mesons. Numerically, one finds, \( B_{c\bar{s}} = -0.050 \text{ GeV} \) by Eq. (30). Note that the short-range color interaction of the quark pair \( c\bar{s} \) in color singlet(1\(_c\)) in heavy meson \( D_s^* \) can be related to that of the \( cs \) pair in color antitriplet(\( \bar{3}_c \)) in a heavy baryon \( nsc \) by the factor 1/2, one can reasonably assume, in the short range, that the strength of the \( cs \) interaction in \( \bar{3}_c \) is half that of \( c\bar{s} \) in \( 1_c \). This follows that \( B_{cs} = B_{c\bar{s}}/2 = -0.025 \text{ GeV} \). Here, the factor half can be extracted from the ratio of the color factor \( -8/3 \) in Eq. (A4) for \( 1_c \) and \( -16/3 \) Eq. (A5), evaluated in Appendix A:1/2 = (-8/3)/(-16/3). The same holds true also for the quark pair \( Q\bar{Q}r \) in the heavy mesons \( B^*_r, J/\psi, \ Upsilon \) and \( B^*_c \). Thus, for the quark pair\( (Qs \text{ an } QQr) \) in heavy baryon, we choose

\[
 B_{Qs} = B_{Q\bar{s}}/2, \ B_{QQr} = B_{Q\bar{Q}r}/2,
\]

for the quark pair in \( \bar{3}_c \) and the quark-antiquark pair in \( 1_c \), and solve the model (1) for the heavy mesons \( B^*_r, J/\psi, \ Upsilon \) and \( B^*_c \), obtaining, by Eqs. (29) and (31),

\[
\begin{align*}
 B_{cs} &= B_{c\bar{s}}/2 = -0.025 \text{ GeV}, \quad B_{cc} = B_{c\bar{c}}/2 = -0.077 \text{ GeV}, \\
 B_{bs} &= B_{b\bar{s}}/2 = -0.032 \text{ GeV}, \quad B_{bb} = B_{b\bar{b}}/2 = -0.128 \text{ GeV}, \\
 B_{bc} &= B_{b\bar{c}}/2 = -0.101 \text{ GeV},
\end{align*}
\]
where the results for the $c\bar{s}$ pair is also included. Here in computation, we have used the mass $M(B_c^*) = 6.332$ GeV of the heavy meson $B_c^*$ in Ref. [42], due to lacking of the measured $B_c^*$.

![Diagram](image_url)

**FIG. 2:** Binding energy $B_{QQ'}$ (solid line) in Eq. (32) as a function of the reduced mass $\mu_{QQ'}$ of two quarks $Q$ and $Q'$. Circles correspond to the pair data $(\mu_{QQ'}, B_{QQ})$ with the respective $QQ' = sc, sb, cc, cb, bb$.

It can be seen from Eq. (32) that $B_{QQ'}$ depends monotonically on the reduced mass $\mu_{QQ'} = m_Qm_{Q'}/(m_Q+m_{Q'})$ of two involved quarks $Q$ and $Q'$. The dependence (FIG. 2) can be approximated by

$$B_{QQ'}(\bar{3}_c) = 0.274 \text{ GeV} - 0.3604 \text{(GeV}^{7/8})\mu_{QQ'}^{1/8}. \quad (33)$$

**TABLE IV:** Computed masses (in GeV), magnetic moments (in $\mu_N$) and charge radii (in fm) of light ground-states mesons, compared to the measured data. The blank cells indicate the values same with the above.

| State | $R_0$(GeV$^{-1}$) | $M_{bag}$ | $M_{exp}$ [39] | $\mu_{bag}$ | $r_E$ (fm) | $r_E$ (fm)[39] |
|-------|----------------|------------|----------------|-------------|------------|---------------|
| $\pi^+$ | 4.31 | 0.348 | 0.140 | - | 0.62 | 0.66 |
| $\omega$ | 4.55 | 0.776 | 0.783 | 0 | 0 | - |
| $K^+$ | 4.34 | 0.561 | 0.494 | - | 0.61 | 0.56 |
| $K^0$ | 4.498 | - | 0.13 | 0.28 |
| $K^{*+}$ | 4.63 | 0.918 | 0.892 | 2.30 | 0.65 | - |
| $K^{*0}$ | 0.896 | -0.18 | 0.15 | - |
| $\phi$ | 4.70 | 1.064 | 1.019 | 0 | 0 | - |

One can scale Eq. (32) for the pair in $\bar{3}_c$ to the pair $QQ'$ in other color configurations. This can be done by computing the explicit ratios of the color factors in Eq. (19) (evaluated in Appendix
TABLE V: Computed masses (in GeV), magnetic moments (in $\mu_N$) and charge radii (in fm) of heavy mesons in their ground-states. The blank cell follows 5.325 GeV indicates the values same with the above.

| State | $R_0$(GeV$^{-1}$) | $M_{bag}$ | $M_{exp}$ | $\mu_{bag}$ | $r_E$(fm) |
|-------|-----------------|----------|-----------|-------------|-----------|
| $D^+$ | 3.63            | 1.835    | 1.870     | -           | 0.46      |
| $D^0$ |                 | 1.865    | -         | -           | 0.25      |
| $D^{*+}$ | 4.09        | 2.009[input] | 2.010 | 1.21 | 0.51      |
| $D^{*0}$ |              | 2.007    | -0.98     | 0.29        |           |
| $B^+$ | 3.14            | 5.248    | 5.279     | -           | 0.42      |
| $B^0$ |                 | 5.280    | -         | 0.17        |           |
| $B^{*+}$ | 3.47         | 5.325[input] | 5.325 | 1.32 | 0.46      |
| $B^{*0}$ |              | -0.53    | 0.19      |             |           |
| $D^+_s$ | 3.77          | 1.961    | 1.968     | -           | 0.46      |
| $D^{*_s}$ | 4.17        | 2.112[input] | 2.112 | 1.08 | 0.51      |
| $B^+_s$ | 3.35           | 5.346    | 5.367     | -           | 0.16      |
| $B^{*_s}$ | 3.62        | 5.415[input] | 5.415 | 1.01 | 0.17      |
| $\eta_c$ | 3.15        | 3.002    | 2.984     | -           | 0         |
| $J/\psi$ | 3.54         | 3.097[input] | 3.097 | 0    | 0         |
| $B^+_c$ | 2.53           | 6.273    | 6.274     | -           | 0.29      |
| $B^{*_c}$ | 2.81        | 6.332[input] | 6.332 | 0.52 | 0.32      |
| $\eta_b$ | 1.59         | 9.396    | 9.399     | -           | 0         |
| $\Upsilon$ | 1.80        | 9.460[input] | 9.460 | 0    | 0         |

A). The scale factor $g([QQ']_R)(=$ ratio of the color factor for representation $R$ and the color factor for $\bar 3_c$) for pair $QQ' (= bb, cc, bc, bs, cs)$ can be given explicitly by

\[
\begin{align*}
g([QQ'_{1_c}) & = 2, \\
g([b\bar s]_{6_c}) & = g([c\bar s]_{6_c}) = 5/4, \\
g([QQ'_{6_c}) & = -1/2, \\
g([b\bar s]_{\bar 3_c}) & = g([c\bar s]_{\bar 3_c}) = 1/2,
\end{align*}
\]

(34)

and the binding energy between the pair $QQ'$ in $R$ is then

\[
B([QQ']_R) = g([QQ']_R)B_{QQ'}.
\]

(35)

with $B_{QQ'} \equiv B([QQ']_{\bar 3_c})$ given in Eq. (32). For instance, for a pair $QQ'$ in $R = 1_c$, the scaled result for the binding energy $B([QQ'_{1_c}) = 2B_{QQ'}$. For a pair in $R = 6_c$, it is $-B_{QQ'}/2$. For $Q\bar s$ in tetraquark $\bar q\bar sQQ'$, the scaled binding energy is $5B_{QQ'}/4$ for $Q\bar s$ in $6_c \otimes \bar 6_c$, and is $B_{QQ'}/2$ for $Q\bar s$ in $3_c \otimes \bar 3_c$. The total binding energy of the baryons and tetraquark systems are given by the sum of all pair binding energies and can be found in Eqs. (C8), (C9), (C10) in Appendix C.

Given these values for the $QQ'$ binding energies and the expressions for the CMI matrices in Eq. (4) with the coefficients derived in Appendices A and B, one can numerically solve the MIT model (1) via the variational method for all established hadrons in their lowest-lying states and thereby compute masses, magnetic moments and charge radii for them. The results are listed in Tables IV, V, VI, VII and Table IX for the states without state mixing due to the CMI. The results for the DH baryons are also presented in Tables IX.
TABLE VI: Computed mass (in GeV) of ground-state light baryons. $M_{\text{exp}}$ and $\mu_{\text{exp}}$ are the observed values of mass and magnetic moments (all in $\mu_N$), respectively.

| State | $R_0$(GeV$^{-1}$) | $M_{\text{bag}}$ | $M_{\text{exp}}$ [39] | $\mu_{\text{bag}}$ | $\mu_{\text{exp}}$ [39] | $r_E$(fm) |
|-------|------------------|------------------|------------------------|------------------|------------------------|----------|
| $p$   | 5.22             | 0.932            | 0.938                  | 2.79             | 2.79                   | 0.75     |
| $n$   |                  | 0.940            | -1.86                  | -1.91            | 0                      |          |
| $\Delta^{++}$ | 5.33            | 1.241            | 1.231                  | 5.70             | 6.14                   | 1.08     |
| $\Delta^+$ |                | 1.235            | 2.85                   | 2.7              | 0.77                   |          |
| $\Delta^0$ |                | 1.233            | 0                      | -                | 0                      |          |
| $\Delta^-$ |                |                  | -2.85                  | -                | 0.77                   |          |
| $\Lambda$ | 5.26            | 1.096            | 1.116                  | -0.71            | -0.61                  | 0.17     |
| $\Sigma^+$ |                | 5.22             | 1.137                  | 1.189            | 2.72                   | 2.46     |
| $\Sigma^0$ |                | 1.193            | 0.86                   | -                | 0.17                   |          |
| $\Sigma^-$ |                | 1.197            | -1.01                  | -1.16            | 0.73                   |          |
| $\Sigma^{++}$ | 5.38           | 1.383            | 1.383                  | 3.11             | -                      | 0.79     |
| $\Sigma^{*0}$ |               | 1.384            | 0.23                   | -                | 0.18                   |          |
| $\Sigma^{*-}$ |               | 1.387            | -2.64                  | -                | 0.75                   |          |
| $\Xi^0$ | 5.27            | 1.282            | 1.315                  | -1.58            | -1.25                  | 0.25     |
| $\Xi^-$ |                | 1.322            | -0.64                  | -0.65            | 0.72                   |          |
| $\Xi^{*0}$ | 5.42            | 1.529            | 1.532                  | 0.48             | -                      | 0.26     |
| $\Xi^{*-}$ |                | 1.535            | -2.43                  | -                | 0.74                   |          |
| $\Omega$ | 5.46            | 1.677            | 1.672                  | -2.20            | -2.02                  | 0.72     |

Some remarks are in order: (i) Some of the computed masses $M_{\text{bag}}$ in Table I deviate from the measured masses about $30 \sim 40$ MeV; (ii) The anti-particles of mesons are not listed in Table IV and V as they share the same masses, charge radii but minus magnetic moments in comparison with the mesons in Tables. The anti-particles of the heavy tetraquarks are ignored as well; (iii) In Table IX, our mass predictions $3.714$ GeV(for $\Xi^{*+}_{cc}$) are comparable to the quark-model prediction $M(\Xi^{*+}_{cc}) = 3.727$ GeV [16], and also to $3706 \pm 28$ MeV and $3692 \pm 28$ MeV by the lattice QCD [43, 44] respectively. Table X shows comparison of our predictions with other works for DH baryons. The prediction $3.604$ GeV for the $\Xi_{cc}$ is in consistent with the measured mass $3.621$ GeV, considering the simplicity of the model. (iv) The predicted magnetic moments in Table VI are in good agreement with the measured values, from which the magnetic moments for heavy baryons and tetraquarks are predicted; (v) Our prediction $0.75$ fm for the proton charge radius is slightly lower than the newly-measured value $0.83$ fm[45, 46], similar to original MIT bag model [31, 32].

IV. BARYONS AND TETRAQUARKS

A. Heavy Baryons including the CMI Mixing

Hadrons containing a diquark or antiquark with different flavors, may not respect flavor-symmetry of wavefunction for involved light quark pairs. As such, the states with same $J^P$ but...
TABLE VII: Computed mass (in GeV), magnetic moments (all in $\mu_N$) and charge radii of ground-state SH baryons (non-mixed). $M_{\text{exp}}$ stands for the observed mass isospin-averaged [39].

| State | $R_0$(GeV$^{-1}$) | $M_{\text{bag}}$ | $M_{\text{exp}}$ [39] | $\mu_{\text{bag}}$ | $\mu$ [27] | $r_E$(fm) |
|-------|-------------------|------------------|----------------------|--------------|-------------|----------|
| $\Lambda_c$ | 4.86 | 2.270 | 2.286 | 0.49 | 0.42 | 0.60 |
| $\Sigma_c^{++}$ | 4.82 | 2.411 | 2.454 | 2.13 | 1.76 | 0.92 |
| $\Sigma_c^+$ | 2.453 | 0.41 | 0.36 | 0.60 |
| $\Sigma_c^0$ | 2.454 | -1.31 | -1.04 | 0.35 |
| $\Sigma_c^{++}$ | 5.01 | 2.512 | 2.518 | 4.07 | - | 0.95 |
| $\Sigma_c^+$ | 2.518 | 1.39 | - | 0.62 |
| $\Sigma_c^0$ | 2.518 | -1.29 | - | 0.37 |
| $\Omega_c$ | 4.93 | 2.680 | 2.695 | -1.07 | -0.85 | 0.28 |
| $\Omega_c^*$ | 5.10 | 2.764 | 2.766 | -0.90 | - | 0.29 |
| $\Lambda_b$ | 4.60 | 5.648 | 5.620 | -0.09 | -0.06 | 0.25 |
| $\Sigma_b^+$ | 4.64 | 5.835 | 5.811 | 2.23 | 2.07 | 0.71 |
| $\Sigma_b^0$ | - | 0.58 | 0.53 | 0.26 |
| $\Sigma_b^-$ | - | 5.816 | -1.07 | -1.01 | 0.62 |
| $\Sigma_b^{++}$ | 4.73 | 5.872 | 5.830 | 3.29 | - | 0.73 |
| $\Sigma_b^{*0}$ | - | 0.76 | - | 0.26 |
| $\Sigma_b^{*-}$ | - | 5.835 | -1.77 | - | 0.63 |
| $\Omega_b$ | 4.77 | 6.080 | 6.046 | -0.86 | -0.82 | 0.60 |
| $\Omega_b^*$ | 4.84 | 6.112 | -1.43 | - | 0.60 |

different spin-color wavefunctions may mix due to the CMI (4), as mentioned in Sect. II (B). To begin with, we first consider the system of baryons with $J^P = 1/2^+$ in which two spin-color states $(\phi^B \chi^B_2, \phi^B \chi^B_3)$ can mix. The associated baryons are the $\Xi_c$, the $\Xi'_c$, the $\Xi_b$ and the $\Xi'_b$.

In terms of the wavefunctions in color and spin space (Appendix A and B), one can compute the CMI matrices in the degenerate subspace of the spin-color basis $\phi^B \chi^B$ when the chromomagnetic mixing occurs (Appendix C). These CMI matrices depend upon $C_{ij}$ with the subscripts $(i, j)$ of $C_{ij}$ denote the flavor constituents. One can diagonalize the CMI matrix, say (C1), to write mass formulas of the baryons using Eq. (1). This is done by solving the eigenvalues and eigenvectors of the matrix (C1) analytically and using the later to identify (denote) the mixed states. Of course, the relevant binding energies ($B_{cs}$, $B_{bs}$ and $B_{bc}$) are included in the mass formulas.

In Table VIII, we list our computed results of masses and other properties for the CMI-mixed systems of heavy baryons. The net effects of the state mixing (the second column of Table) are not so significant in general and they are somehow negligible in the case of singly heavy baryons. This can be due to the higher $SU(3)$ flavor symmetry and heavy quark symmetry which suppress the off-diagonal elements in matrix (C1). For this reason, we employ still the normal notations of the states for the SH baryons. The computed masses of the SH baryons $\Xi_c$, the $\Xi'_c$, the $\Xi_b$ and the $\Xi'_b$ are comparable with the measured data, as seen in the fifth column with reasonable errors. The magnetic moments for $\Xi_c^+$, $\Xi'_c$, $\Xi_b^0$ and $\Xi'_b$ are predicted to be 0.37$\mu_N$, 0.50$\mu_N$, -0.12$\mu_N$ and -0.08$\mu_N$ which are comparable to 0.35$\mu_N$, 0.50$\mu_N$, -0.045$\mu_N$ and -0.08$\mu_N$ in Ref. [47], respec-
TABLE VIII: Predicted masses (in GeV), magnetic moments (in $\mu_N$) and charge radii of heavy baryons. $M_{\text{exp}}$ is the observed mass isospin-averaged [39]. Magnetic moments and charge radii are organized in the order of $I_3 = \frac{1}{2}, -\frac{1}{2}$ for $I = \frac{1}{2}$. Bag radius $R_0$ is in GeV$^{-1}$.

| State | Eigenvector | $R_0$ (GeV$^{-1}$) | $M_{\text{bag}}$ | $M_{\text{exp}}$ [39] | $\mu_{\text{bag}}$ | $r_E$ (fm) |
|-------|-------------|-------------------|-----------------|-----------------|-----------------|----------|
| $(\Xi_c', \Xi_c)$ | (0.05, 1.00) | 4.89 | 2.436 | 2.469 | 0.37, 0.50 | 0.63, 0.32 |
| $\Xi_c^*$ | 1.00 | 5.06 | 2.636 | 2.646 | 1.61, -1.10 | 0.65, 0.33 |
| $(\Xi_b', \Xi_b)$ | (0.01, 1.00) | 4.64 | 5.805 | 5.794 | -0.12, -0.08 | 0.30, 0.60 |
| $\Xi_b^*$ | 1.00 | 4.79 | 5.991 | 5.954 | 0.96, -1.61 | 0.31, 0.62 |
| $(\Xi_{bc}', \Xi_{bc}^*)$ | (0.39, 0.92) | 4.22 | 7.015 | - | 1.48, -0.33 | 0.58, 0.19 |
| $(\Omega_{bc}', \Omega_{bc}^*)$ | (0.40, 0.92) | 4.29 | 7.117 | - | -0.20, 0.09 | 0.56, 0.19 |

TABLE IX: Computed masses (in GeV), magnetic moments (all in $\mu_N$) and charge radii of the ground-state doubly heavy baryons (non-mixed states). The magnetic moments by Ref. [27] are listed for comparison.

| State | $R_0$(GeV$^{-1}$) | $M_{\text{bag}}$ | $\mu_{\text{bag}}$ | $\mu$ [27] | $r_E$(fm) |
|-------|-----------------|-----------------|-----------------|-------------|----------|
| $\Xi_{cc}^{++}$ | 4.42 | 3.604 | 0.12 | 0.13 | 0.78 |
| $\Xi_{cc}^+$ | 4.91 | 0.72 | - | 0.45 |
| $\Xi_{cc}^{++}$ | 4.64 | 3.714 | 2.64 | - | 0.82 |
| $\Xi_{cc}^+$ | 0.16 | - | 0.47 |
| $\Xi_{bb}^0$ | 3.71 | 10.311 | -0.55 | -0.53 | 0.29 |
| $\Xi_{bb}^-$ | 0.11 | 0.18 | 0.45 |
| $\Xi_{bc}^0$ | 3.87 | 10.360 | 1.21 | - | 0.30 |
| $\Xi_{bc}^-$ | -0.86 | - | 0.47 |
| $\Omega_{cc}^+$ | 4.49 | 3.726 | 0.86 | 0.67 | 0.48 |
| $\Omega_{cc}^*$ | 4.69 | 3.820 | 0.33 | - | 0.50 |
| $\Omega_{bb}^0$ | 3.83 | 10.408 | 0.07 | 0.04 | 0.45 |
| $\Omega_{bb}^-$ | 3.97 | 10.451 | -0.75 | - | 0.47 |

respectively. The magnetic moments for $\Xi_{c}^{++}, \Xi_{c}^{+}, \Xi_{b}^{0}$ and $\Xi_{b}^{-}$ are $1.61\mu_N, -1.10\mu_N, 0.96\mu_N$ and $-1.61\mu_N$ comparable to $1.68\mu_N, -0.68\mu_N, 0.50\mu_N$ and $-1.42\mu_N$ in Ref. [48], respectively.
combination (φφ before evaluating the hadron mass. Note that diagonalization should be applied to the sum of the interaction matrices diagonal in mass formula since the mixed states have two color configurations while spin states are orthogonal. Let us consider the strange tetraquarks nn¯c containing one heavy quark, one strange quark and two nonstrange light quarks. In such a case, the CMI mixing happens if J ≠ 2. We use a combination of the spin-color basis functions φTχT to denote the mixed states. For instance, the combination (φTχT, φTχT) stands for a mixed state c1φTχT + c2φTχT for (JP, I)=(0+, 1). Similarly, other mixed states can be denoted as (φTχT, φTχT) for (JP, I)=(0+, 0), as (φTχT, φTχT, φTχT) for (JP, I)=(1+, 1) and (φTχT, φTχT, φTχT) for (Jp, I)=(1+, 0). The binding energy matrices become diagonal in mass formula since the mixed states have two color configurations while spin states are orthogonal. Note that diagonalization should be applied to the sum of the interaction matrices before evaluating the hadron mass.

Following the variational principle, we diagonalize the 2 × 2 matrix to solve two analytical eigenvalues and construct the mass formula as usual. Application of the same procedure to the 3 × 3 matrix is, however, not straightforward, for which the eigenvalues are some roots of a cubic equation. For this, we scan three sets of xi and R to solve the cubic equation numerically so that one can obtain the minimized masses within three root eigenvalues.

Our numerical results are shown in Table XI, with a notable tetraquark of an isosinglet mn¯c with Jp = 0+, which has two masses 2.934 GeV and 2.513 GeV for its two mixed states. Comparing with the measured mass 2866 ± 7 MeV of X0(2900) reported by LHCb[8] and the quark model prediction 2863.4 ± 12 MeV[9], our prediction 2.934 GeV is larger even if the model error 40 MeV is subtracted. If we rather, as Karlner suggested for the color 3 ⊗ 3 configuration, ignore the CMI mixing and evaluate directly the masses of the φTχT and φTχT states, the resulted masses lie around 2.7 GeV, away from the LHCb reported mass of the X0(2900). Our calculation suggests that chromomagnetic mixing is strong for the strange tetraquark mn¯c with Jp = 0+ and yields a mass splitting as large as 420 MeV.

### Table X: Computed mass and other calculations cited of doubly heavy baryons, all in GeV.

| State | J   | This work | [14] | [15] | [16] | [17] | [18] | [19] | [20] |
|-------|-----|-----------|------|------|------|------|------|------|------|
| Ξc    | 1/2 | 3.604     | 3.627- | -    | 3.620| 3.676| 3.612| 3.547| 3.557|
| Ξc+   | 3/2 | 3.714     | 3.690- | 3.72 | 3.727| 3.753| 3.706| 3.719| 3.661|
| Ξc-   | 1/2 | 3.726     | -     | -    | 3.778| 3.815| 3.702| 3.648| 3.710|
| Ξc'   | 3/2 | 3.820     | -     | 3.78 | 3.872| 3.876| 3.783| 3.770| 3.800|
| Ξbb   | 1/2 | 10.311    | 10.162-| -    | 10.202|10.340|10.197|10.185|10.062|
| Ξbb+  | 3/2 | 10.360    | 10.184-| 10.3 | 10.237|10.367|10.236|10.216|10.101|
| Ξbb-  | 1/2 | 10.408    | -     | -    | 10.359|10.454|10.260|10.271|10.208|
| Ξbc   | 1/2 | 10.451    | -     | 10.4 | 10.389|10.486|10.297|10.289|10.244|
| Ξbc+  | 3/2 | 6.953     | 6.914- | -    | 6.933|7.011 |6.919 |6.904 |6.846 |
| Ξbc-  | 1/2 | 7.015     | 6.933- | -    | 6.963|7.047 |6.948 |6.920 |6.891 |
| Ξbc'  | 3/2 | 7.044     | 6.969- | 7.2  | 6.980|7.074 |6.986 |6.936 |6.919 |
| Ξc'   | 1/2 | 7.064     | -     | -    | 7.088|7.136 |6.986 |6.994 |6.999 |
| Ξc'   | 1/2 | 7.116     | -     | -    | 7.116|7.165 |7.009 |7.005 |7.036 |
| Ξc'   | 1/2 | 7.142     | -     | 7.35 | 7.130|7.187 |7.046 |7.017 |7.063 |

### B. Singly Heavy Tetraquarks

Let us consider the strange tetraquarks mn¯c and mn¯b containing one heavy quark, one strange quark and two nonstrange light quarks.
TABLE XI: Computed mass (in GeV), magnetic moments (in $\mu_N$) and charge radii of singly heavy tetraquarks $nn\bar{s}\bar{c}$ and $nn\bar{s}\bar{b}$. Magnetic moments and charge radii are organized in the order of $I_3 = 1, 0, -1$ for $I = 1$. Bag radius $R_0$ is in GeV$^{-1}$.

| State  | $J^P$ | Eigenvector | $R_0$ | $M_{bag}$ | $\mu_{bag}$ | $r_E$ (fm) |
|--------|-------|-------------|-------|-----------|-------------|------------|
| $(nn\bar{s}\bar{c})^{I=1}$ | 0$^+$ | (0.54, 0.84) | 5.73  | 3.218     | -           | 0.91, 0.38, 0.73 |
|        |       | (-0.84, 0.55) | 5.39  | 2.776     | -           | 0.85, 0.35, 0.69 |
|        |       | (0.81, 0.58, 0.10) | 5.46  | 3.001     | 3.48, 1.54, -0.40 | 0.86, 0.36, 0.70 |
|        | 1$^+$ | (0.25, -0.49, 0.84) | 5.57  | 3.154     | 1.03, 0.23, -0.57 | 0.88, 0.37, 0.71 |
|        |       | (-0.54, 0.65, 0.54) | 5.38  | 2.846     | 1.67, 0.04, -1.60 | 0.85, 0.35, 0.69 |
|        | 2$^+$ | 1.00        | 5.64  | 3.075     | 4.27, 1.25, -1.77 | 0.89, 0.37, 0.72 |
| $(nn\bar{s}\bar{c})^{I=0}$ | 0$^+$ | (0.63, 0.77) | 5.56  | 2.934     | -           | 0.37 |
|        |       | (-0.78, 0.63) | 5.19  | 2.513     | -           | 0.34 |
|        |       | (0.77, 0.07, 0.64) | 5.40  | 2.895     | 0.54         | 0.35 |
|        | 1$^+$ | (-0.23, -0.90, 0.36) | 5.57  | 3.056     | 1.24         | 0.37 |
|        |       | (-0.60, 0.43, 0.68) | 5.35  | 2.674     | 0.04         | 0.35 |
|        | 2$^+$ | 1.00        | 5.66  | 3.063     | 1.26         | 0.37 |
| $(nn\bar{s}\bar{b})^{I=1}$ | 0$^+$ | (0.53, 0.85) | 5.53  | 6.580     | -           | 1.07, 0.71, 0.36 |
|        |       | (-0.85, 0.53) | 5.28  | 6.202     | -           | 1.02, 0.68, 0.34 |
|        |       | (0.66, 0.75, 0.06) | 5.35  | 6.419     | 3.59, 1.37, -0.86 | 1.03, 0.69, 0.35 |
|        | 1$^+$ | (0.36, -0.38, 0.85) | 5.43  | 6.554     | 1.33, 0.72, 0.12 | 1.05, 0.70, 0.35 |
|        |       | (-0.66, 0.55, 0.52) | 5.22  | 6.228     | 1.99, 0.55, -0.89 | 1.01, 0.67, 0.34 |
|        | 2$^+$ | 1.00        | 5.47  | 6.446     | 4.72, 1.79, -1.13 | 1.05, 0.70, 0.36 |
| $(nn\bar{s}\bar{b})^{I=0}$ | 0$^+$ | (0.66, 0.75) | 5.41  | 6.327     | -           | 0.69 |
|        |       | (-0.75, 0.66) | 5.14  | 5.980     | -           | 0.66 |
|        |       | (0.66, -0.23, 0.72) | 5.33  | 6.322     | 0.71         | 0.68 |
|        | 1$^+$ | (-0.46, -0.88, 0.14) | 5.38  | 6.454     | 1.31         | 0.69 |
|        |       | (-0.60, 0.43, 0.68) | 5.14  | 6.038     | 0.61         | 0.66 |
|        | 2$^+$ | 1.00        | 5.48  | 6.431     | 1.80         | 0.70 |

C. Doubly Heavy Tetraquarks

Now, let us consider the doubly heavy tetraquarks $qq\bar{Q}\bar{Q}$ with strangeness $S \leq 2$. In this case, hadrons consist of the nonstrange tetraquarks $nn\bar{Q}\bar{Q}$ and the strange tetraquarks $ns\bar{Q}\bar{Q}$ and $ss\bar{Q}\bar{Q}$. They lie in a larger (compared to baryons) space spanned by more configurations (bases) in which the CMI mixing occurs variously. For the isotriplet tetraquarks with $J^P = 0^+$, the general ground state can be the mixed one, with the wavefunction $(\phi^T_{\chi_3} \chi_6^T, \phi^T_{\chi_1} \chi_6^T)$. For isosinglet tetraquark $nn\bar{Q}\bar{Q}$ with $J^P = 1^+$, the wavefunction has the form of $(\phi^T_{\chi_5} \chi_3^T, \phi^T_{\chi_1} \chi_3^T)$. In the case of the strange tetraquark $ns\bar{Q}\bar{Q}$ with $J^P = 1^+$, the wavefunction can be of $(\phi^T_{\chi_3} \chi_5^T, \phi^T_{\chi_1} \chi_5^T)$ and the wavefunction of the tetraquark $nn\bar{c}\bar{b}$ is similar to that of $nn\bar{s}\bar{c}$. Note that the strange DH states $ns\bar{c}\bar{b}$ with mixing among six spin-color states are not considered for simplicity.

The computation of the mass and other properties of these DH tetraquarks is similar to that for
TABLE XII: Computed mass (in GeV) and other properties of doubly heavy tetraquarks \( nnn\bar{c}, nn\bar{b}\bar{b} \) and \( nn\bar{c}\bar{b}. \) Magnetic moments (in \( \mu_N \)) and charge radii are organized in the order of \( I_3 = 1, 0, -1 \) for \( I = 1. \) Bag radius \( R_0 \) is in GeV\(^{-1}. \) The mass and magnetic moment of \( T_{cc}^+ \) are predicted to be 3.925 GeV and 0.88\(\mu_N \) which is comparable to 0.66\(\mu_N \) in Ref. [49].

| State | \( J^P \) | Eigenvector | \( R_0 \) | \( M_{bag} \) | \( \mu_{bag} \) | \( r_E \) (fm) |
|-------|---------|-------------|--------|-----------|-------------|------------|
| \( nnn\bar{c} \) | \( I^=1 \) | \( 0^+ \) | (0.40, 0.92) | 5.40 | 4.342 | - | 0.56, 0.54, 0.94 |
| | | | (-0.91, 0.41) | 5.04 | 4.032 | - | 0.52, 0.50, 0.88 |
| | | | 1.00 | 5.22 | 4.117 | 1.36, -0.03, -1.43 | 0.54, 0.52, 0.91 |
| | | | 1.00 | 5.32 | 4.179 | 2.80, -0.05, -2.90 | 0.55, 0.53, 0.93 |
| \( nnn\bar{c} \) | \( I^=0 \) | \( 1^+ \) | (0.97, 0.25) | 5.15 | 3.925 | -0.88 | 0.51 |
| | | | (-0.24, 0.97) | 5.30 | 4.205 | 0.83 | 0.53 |
| \( nn\bar{b}\bar{b} \) | \( I^=1 \) | \( 0^+ \) | (0.17, 0.99) | 4.90 | 11.092 | - | 0.92, 0.59, 0.38 |
| | | | (-0.98, 0.18) | 4.77 | 10.834 | - | 0.90, 0.58, 0.37 |
| | | | 1.00 | 4.83 | 10.854 | 1.81, 0.52, -0.78 | 0.91, 0.58, 0.38 |
| | | | 1.00 | 4.88 | 10.878 | 3.65, 1.04, -1.57 | 0.92, 0.59, 0.38 |
| \( nn\bar{b}\bar{b} \) | \( I^=0 \) | \( 1^+ \) | (1.00, 0.08) | 4.76 | 10.654 | 0.18 | 0.57 |
| | | | (-0.08, 1.00) | 4.83 | 10.982 | 0.86 | 0.58 |
| \( nn\bar{c}\bar{b} \) | \( I^=1 \) | \( 0^+ \) | (0.30, 0.95) | 5.16 | 7.714 | - | 0.78, 0.25, 0.70 |
| | | | (-0.95, 0.31) | 4.91 | 7.438 | - | 0.74, 0.23, 0.67 |
| | | | (0.65, 0.76, 0.09) | 5.04 | 7.509 | 2.33, 0.21, -1.90 | 0.76, 0.24, 0.68 |
| | | | (0.13, -0.22, 0.97) | 5.10 | 7.699 | -0.17, -0.32, -0.47 | 0.77, 0.24, 0.69 |
| | | | (-0.75, 0.61, 0.24) | 4.96 | 7.465 | 2.57, 0.82, -0.93 | 0.75, 0.24, 0.67 |
| | | | 1.00 | 5.12 | 7.531 | 3.24, 0.50, -2.24 | 0.78, 0.25, 0.69 |
| \( nn\bar{c}\bar{b} \) | \( I^=0 \) | \( 0^+ \) | (0.93, 0.37) | 4.96 | 7.502 | - | 0.24 |
| | | | (-0.38, 0.93) | 4.84 | 7.260 | - | 0.23 |
| | | | (0.91, -0.36, 0.21) | 4.93 | 7.518 | 0.55 | 0.23 |
| | | | (-0.39, -0.92, 0.10) | 5.07 | 7.605 | 0.50 | 0.24 |
| | | | (-0.16, 0.18, 0.97) | 4.93 | 7.288 | -0.33 | 0.23 |
| | | | 1.00 | 5.14 | 7.483 | 0.50 | 0.25 |

heavy baryons discussed in Sect. IV(A). Our numerical results for DH tetraquarks are listed in Table XII for the \( nn\bar{Q}\bar{Q}r \), in Table XIII for the \( ss\bar{Q}\bar{Q}r \) and Table XIV for the \( ns\bar{Q}\bar{Q}r \). Our results for the mass predictions of the DH tetraquarks are summarized in Table XV and compared with some other calculations cited.

V. SUMMARY AND DISCUSSIONS

In this work, we have studied systematically masses and other properties of hadrons with one and two open heavy quarks within an unified framework of MIT bag model with chromo-magnetic interaction. Masses, magnetic moments and charge radii of heavy baryons and heavy
TABLE XIII: Computed mass (in GeV), magnetic moments (in $\mu_N$) and charge radii of doubly heavy tetraquarks $ss\bar{c}\bar{c}$, $ss\bar{b}\bar{b}$ and $ss\bar{c}\bar{b}$. Bag radius $R_0$ is in GeV$^{-1}$.

| State   | $J^P$ | Eigenvector          | $R_0$ | $M_{bag}$ | $\mu_{bag}$ | $r_E$ (fm) |
|---------|-------|----------------------|-------|-----------|-------------|------------|
| $ss\bar{c}\bar{c}$ | 0$^+$ | $(0.49, 0.87)$      | 5.48  | 4.521     | -           | 0.92       |
|         |       | $(-0.87, 0.50)$     | 5.13  | 4.300     | -           | 0.87       |
|         | 1$^+$ | 1.00                 | 5.30  | 4.382     | -1.22       | 0.89       |
|         | 2$^+$ | 1.00                 | 5.39  | 4.433     | -2.46       | 0.91       |
| $ss\bar{b}\bar{b}$ | 0$^+$ | $(0.24, 0.97)$      | 5.01  | 11.232    | -           | 0.32       |
|         |       | $(-0.97, 0.25)$     | 4.88  | 11.078    | -           | 0.31       |
|         | 1$^+$ | 1.00                 | 4.94  | 11.099    | -0.60       | 0.31       |
|         | 2$^+$ | 1.00                 | 4.98  | 11.119    | -1.20       | 0.32       |
| $ss\bar{c}\bar{b}$ | 0$^+$ | $(0.40, 0.92)$      | 5.26  | 7.875     | -           | 0.67       |
|         |       | $(-0.91, 0.40)$     | 5.01  | 7.693     | -           | 0.64       |
|         |       | $(0.70, 0.71, 0.11)$| 4.98  | 7.757     | -1.54       | 0.64       |
|         | 1$^+$ | $(0.17, -0.32, 0.93)$| 5.06  | 7.858     | -0.48       | 0.64       |
|         |       | $(-0.69, 0.63, 0.35)$| 4.88  | 7.716     | -0.66       | 0.62       |
|         | 2$^+$ | 1.00                 | 5.20  | 7.779     | -1.84       | 0.66       |

TABLE XIV: Computed mass (in GeV) and other properties of doubly heavy tetraquarks $ns\bar{c}\bar{c}$, $ns\bar{b}\bar{b}$. Magnetic moments (in $\mu_N$) and charge radii are organized in the order of $I_3 = 1/2$, $-1/2$ for $I = 1/2$. Bag radius $R_0$ is in GeV$^{-1}$.

| State   | $J^P$ | Eigenvector          | $R_0$ | $M_{bag}$ | $\mu_{bag}$ | $r_E$ (fm) |
|---------|-------|----------------------|-------|-----------|-------------|------------|
| $ns\bar{c}\bar{c}$ | 0$^+$ | $(0.44, 0.90)$      | 5.44  | 4.429     | -           | 0.51, 0.93 |
|         |       | $(-0.89, 0.45)$     | 5.09  | 4.165     | -           | 0.48, 0.88 |
|         |       | $(0.99, -0.07, 0.09)$| 5.16  | 4.247     | 0.32, -1.33 | 0.49, 0.89 |
|         | 1$^+$ | $(-0.11, -0.28, 0.95)$| 5.23  | 4.314     | 0.86, -1.58 | 0.49, 0.90 |
|         |       | $(0.04, 0.96, 0.29)$| 5.04  | 4.091     | -0.95, -1.03| 0.48, 0.87 |
|         | 2$^+$ | 1.00                 | 5.36  | 4.305     | 0.19, -2.68 | 0.50, 0.92 |
| $ns\bar{b}\bar{b}$ | 0$^+$ | $(0.20, 0.98)$      | 4.96  | 11.160    | -           | 0.62, 0.35 |
|         |       | $(-0.98, 0.20)$     | 4.83  | 10.955    | -           | 0.60, 0.34 |
|         |       | $(1.00, -0.01, 0.03)$| 4.79  | 10.974    | 0.65, -0.68 | 0.60, 0.34 |
|         | 1$^+$ | $(-0.03, -0.09, 1.00)$| 4.77  | 11.068    | 1.02, -1.50 | 0.60, 0.34 |
|         |       | $(0.01, 1.00, 0.10)$| 4.66  | 10.811    | 0.14, 0.16  | 0.58, 0.33 |
|         | 2$^+$ | 1.00                 | 4.93  | 10.997    | 1.25, -1.39 | 0.62, 0.35 |

tetraquarks are computed systematically, including the predictions $M(\Xi_{cc}, 1/2^+) = 3.604$ GeV, $M(\Xi_{cc}', 3/2^+) = 3.714$ GeV, and $M(ud\bar{s}\bar{c}, 0^+) = 2.934$ GeV for the strange isosinglet tetraquark $ud\bar{s}\bar{c}$. The state mixing due to chromomagnetic interaction is shown to be sizable for the strange scalar tetraquark $ns\bar{s}\bar{c}$, giving mass splitting as large as 420 MeV roughly, while it is small for
other heavy hadrons.

We also confirm that a term of extra binding energy $B_{QQ'}$, proposed previously to exist among heavy quarks ($c$ and $b$) and between heavy and strange quarks \cite{14}, is required to reconcile light hadron with heavy hadrons, with a useful formula provided for $B_{QQ'}$. This binding effect may rise from the enhanced short-range interaction between two relatively heavy quarks and makes the mass pattern and other properties of heavy hadrons differing from that in light sector. We have also employed a slowly-running strong coupling $\alpha_s(R)$ to reflect its dependence upon the hadron sizes proportional to the average distance between two interacted quarks (or antiquark) in a hadron. The strong coupling $\alpha_s(R)$ runs from 0.4 to 0.6 as the bag radius $R$ varies between $3 \sim 6 \text{ GeV}^{-1}$.

We remark that the MIT bag model can reproduce the measured masses of heavy hadrons within the accuracy of 40-50 MeV, from which we proceed to predict the masses and other properties of the tetraquarks with one and two open heavy quarks. For the DH tetraquarks, we reduce the error limit to about 40 MeV and exclude $X_0(2900)$ to be an isosinglet tetraquark of $nn\bar{s}\bar{c}$ due to the mismatch with the measured data as high as 70 MeV.

Owing to the uncertainty of model computations, we are not able to discuss the near-threshold effect. The mismatch of our predictions with the measured data may come from the limitations of bag model in this work: (1) the bag may deform into elliptic shape in the case of the DH hadrons, and (2) the constant approximation of the short-range binding energy may not be sufficient as the later may depend upon hadrons size $R$ implicitly, for instance, in the form of a Coulomb-like $\sim 1/R$, and needs to be determined variationally. These effects go beyond the scope of this work and await the further exploration in the future.

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**Appendix A**

For meson $q_1\bar{q}_2$(denoted by $M$), baryon $(q_1q_2)q_3$ (denoted by $B$), and tetraquark systems $q_1q_2\bar{q}_3\bar{q}_4$(denoted by $T$), the full color wavefunctions, which respect $SU(3)_c$ symmetry, can be written as

\begin{equation}
\phi^M = \frac{1}{\sqrt{3}} (r\bar{r} + g\bar{g} + b\bar{b}), \quad (A1)
\end{equation}

\begin{equation}
\phi^B = \frac{1}{\sqrt{6}} (gbr - bgr + brg - rgb + rgb - grb), \quad (A2)
\end{equation}
TABLE XV: Comparison of calculated mass (in GeV) among different calculations for double heavy tetraquarks. The masses before and after slash stand for that of the color states split chromo-magnetically. Refs. [7] and [50] employ the CMI model.

| State | $J$ | This work | Scheme 1[7] | Scheme 2[7] | [50] | [51] |
|-------|-----|-----------|-------------|-------------|-----|-----|
| $(nn\bar{c}\bar{c})^{I=1}$ | 0   | 4.032/4.342 | 4.078/4.356 | 3.850/4.128 | 4.195/4.414 | 4.056 |
|       | 1   | 4.117      | 4.201       | 3.973       | 4.268  | 4.079 |
|       | 2   | 4.179      | 4.271       | 4.044       | 4.318  | 4.118 |
| $(nn\bar{c}\bar{c})^{I=0}$ | 0   | 3.295/4.205 | 4.007/4.204 | 3.779/3.977 | 4.041/4.313 | 3.935 |
| $(nn\bar{b}\bar{b})^{I=1}$ | 0   | 10.834/11.092 | 10.841/10.937 | 10.637/10.734 | 10.765/11.019 | 10.648 |
|       | 1   | 10.854     | 10.875      | 10.671      | 10.779  | 10.657 |
|       | 2   | 10.878     | 10.897      | 10.694      | 10.799  | 10.673 |
| $(nn\bar{b}\bar{b})^{I=0}$ | 0   | 10.654/10.982 | 10.686/10.821 | 10.483/10.617 | 10.550/10.951 | 10.502 |
| $(nn\bar{c}\bar{b})^{I=1}$ | 0   | 7.438/7.714  | 7.457/7.643  | 7.241/7.428  | 7.519/7.740  | 7.383 |
|       | 1   | 7.465/7.509  | 7.473/7.548  | 7.258/7.332  | 7.537/7.561  | 7.396/7.403 |
|       | 2   | 7.699       | 7.609       | 7.393       | 7.729    |
| $ns\bar{c}\bar{c}$ | 0   | 4.165/4.429  | 4.236/4.514  | 3.933/4.210  | 4.323/4.512  | 4.221 |
|       | 1   | 4.091/4.247  | 4.225/4.363  | 3.921/4.060  | 4.232/4.394  | 4.143/4.239 |
|       | 2   | 4.305       | 4.434       | 4.131       | 4.440    |
| $ns\bar{b}\bar{b}$ | 0   | 10.955/11.160 | 10.999/11.095 | 10.707/10.804 | 10.883/11.098 | 10.802 |
|       | 1   | 10.811/10.974 | 10.911/11.010 | 10.619/10.718 | 10.734/10.897 | 10.706/10.809 |
|       | 2   | 10.997     | 11.060      | 10.769      | 10.915    |
| $ss\bar{c}\bar{c}$ | 0   | 4.300/4.521  | 4.395/4.672  | 4.016/4.293  | 4.417/4.587  | 4.359 |
|       | 1   | 4.382       | 4.526       | 4.146       | 4.493    |
|       | 2   | 4.433       | 4.597       | 4.218       | 4.536    |
| $ss\bar{b}\bar{b}$ | 0   | 11.078/11.232 | 11.157/11.254 | 10.777/10.875 | 10.972/11.155 | 10.932 |
|       | 1   | 11.099     | 11.199      | 10.820      | 10.986    |
|       | 2   | 11.119     | 11.224      | 10.844      | 11.004    |
| $ss\bar{c}\bar{b}$ | 0   | 7.693/7.875  | 7.774/7.960  | 7.394/7.581  | 7.735/7.894  | 7.673 |
|       | 1   | 7.716/7.757  | 7.793/7.872  | 7.414/7.493  | 7.752/7.775  | 7.683/7.684 |
|       | 2   | 7.779       | 7.908       | 7.529       | 7.798    |
|       |     |             |             |             |          | 7.701 |
\[ \phi_1^T = \frac{1}{\sqrt{6}} (rr\bar{r} + gg\bar{g} + bb\bar{b}) + \frac{1}{2\sqrt{6}} (rb\bar{r} + br\bar{r}) \\
+ gr\bar{g} + rg\bar{r} + gb\bar{g} + bg\bar{g} + gr\bar{g} + rg\bar{r} + gb\bar{g} + bg\bar{g} \\
+ bg\bar{g} + rb\bar{r} + br\bar{r}), \]
\[ \phi_2^T = \frac{1}{2\sqrt{3}} \left( rb\bar{r} - br\bar{r} - gr\bar{g} + rg\bar{r} + gb\bar{g} - bg\bar{g} \\
+ gr\bar{g} - rg\bar{r} - gb\bar{g} + bg\bar{g} - rb\bar{r} + br\bar{r} \right), \]  
respectively. Here, the wavefunction \( \phi_1^T \) in Eq. (16) corresponds to the configuration \( 6_c \otimes \bar{6}_c \) while \( \phi_2^T \) there corresponds to \( 3_c \otimes \bar{3}_c \).

Using the color wavefunctions above and Eq. (19), one can compute the matrices of color factors. The results can be given explicitly by

\[ \langle \lambda_1 \cdot \lambda_2 \rangle = -\frac{16}{3}, \quad \text{(A4)} \]

for meson with the wavefunction \( (\phi^M) \) and

\[ \langle \lambda_1 \cdot \lambda_2 \rangle = \langle \lambda_1 \cdot \lambda_3 \rangle = \langle \lambda_2 \cdot \lambda_3 \rangle = \frac{8}{3}, \quad \text{(A5)} \]

for baryon with \( (\phi^B) \). For tetraquarks with the two-components wavefunctions \( (\phi_1^T, \phi_2^T) \), the matrices of color factors are

\[ \langle \lambda_1 \cdot \lambda_2 \rangle = \langle \lambda_3 \cdot \lambda_4 \rangle = \begin{bmatrix} \frac{4}{3} & 0 \\ 0 & -\frac{8}{3} \end{bmatrix}, \]
\[ \langle \lambda_1 \cdot \lambda_3 \rangle = \langle \lambda_2 \cdot \lambda_4 \rangle = \begin{bmatrix} -\frac{10}{3} & 2\sqrt{2} \\ 2\sqrt{2} & -\frac{4}{3} \end{bmatrix}, \]
\[ \langle \lambda_1 \cdot \lambda_4 \rangle = \langle \lambda_2 \cdot \lambda_3 \rangle = \begin{bmatrix} -\frac{10}{3} & -2\sqrt{2} \\ -2\sqrt{2} & -\frac{4}{3} \end{bmatrix}, \]  
all of which are 2 \times 2 matrices in the space of the two-components wavefunction \( (\phi_1^T, \phi_2^T) \).

**Appendix B**

For meson \( q_1\bar{q}_2 \) (denoted by \( M \)), baryon \( (q_1 q_2)q_3 \) (denoted by \( B \)), and tetraquark systems \( q_1 q_2 \bar{q}_3 \bar{q}_4 \) (denoted by \( T \)), one can write the spin wavefunctions for them, with the help of the Clebsch-Gordan coefficients. The results are

\[ \chi_1^M = \uparrow\uparrow, \quad \chi_2^M = \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow), \quad \text{(B1)} \]

for the mesons, and

\[ \chi_1^B = \uparrow\uparrow\uparrow, \]
\[ \chi_2^B = \sqrt{\frac{2}{3}} \uparrow\uparrow\downarrow - \frac{1}{\sqrt{6}} (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow), \]
\[ \chi_3^B = \frac{1}{\sqrt{2}} (\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow), \quad \text{(B2)} \]
for the baryons. For the tetraquark there are six of spin wavefunctions,

\[
\chi_T^1 = \uparrow \uparrow \uparrow \uparrow, \\
\chi_T^2 = \frac{1}{2} (\uparrow \uparrow \downarrow \downarrow + \uparrow \downarrow \uparrow \downarrow - \downarrow \uparrow \uparrow \downarrow - \downarrow \downarrow \uparrow \uparrow), \\
\chi_T^3 = \frac{1}{\sqrt{3}} (\uparrow \downarrow \downarrow \downarrow), \\
\chi_T^4 = \frac{1}{\sqrt{2}} (\uparrow \downarrow \uparrow \uparrow - \uparrow \uparrow \downarrow \downarrow), \\
\chi_T^5 = \frac{1}{\sqrt{2}} (\uparrow \uparrow \uparrow \downarrow - \downarrow \uparrow \uparrow \uparrow), \\
\chi_T^6 = \frac{1}{2} (\uparrow \downarrow \downarrow \downarrow - \downarrow \uparrow \uparrow \downarrow + \uparrow \downarrow \uparrow \downarrow + \downarrow \uparrow \downarrow \uparrow), \\
\]

which correspond to the states (12), (14) and (17), respectively.

Given the spin wavefunctions above, one can also compute the matrices of spin factors with the help of Eq. (20). There is one spin matrix

\[
\langle \sigma_1 \cdot \sigma_2 \rangle = \begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix}, \\
\]

for meson in \((\chi_1^M, \chi_2^M)\) space, and three spin matrices

\[
\langle \sigma_1 \cdot \sigma_2 \rangle = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \\
\langle \sigma_1 \cdot \sigma_3 \rangle = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & -\sqrt{3} \\ 0 & -\sqrt{3} & 0 \end{bmatrix}, \\
\langle \sigma_2 \cdot \sigma_3 \rangle = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & \sqrt{3} \\ 0 & \sqrt{3} & 0 \end{bmatrix}, \\
\]

for baryon in \((\chi_1^B, \chi_2^B, \chi_3^B)\) space. In the case of tetraquark, there are six spin matrices,

\[
\langle \sigma_1 \cdot \sigma_2 \rangle = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3 \end{bmatrix}, \\
\]

(B3)
in the subspace of $\{\chi_1^T, \chi_2^T, \chi_3^T, \chi_4^T, \chi_5^T, \chi_6^T\}$.

One can then use these factors of color and spin space to find the matrix representation of the CMI in Eq. (4) for a given hadronic state. The color and spin factors are the diagonal elements of the matrices in Eq. (A4-A6) and Eq. (B4-B13), respectively. The off-diagonal elements of these matrices lead to chromomagnetic mixing of these basis functions given in Appendix A and at the beginning of this section.
Appendix C

Based on Appendices A and B, one can use Eq. (4) to calculate the matrices of the CMI in the hadronic basis of the wavefunctions involved in this work. We list some of them whose non-diagonal elements are nonvanishing. For instance, for the \((\phi^B \chi^B_2, \phi^B \chi^B_3)\) mixed state of baryons, the CMI matrix is
\[
\begin{pmatrix}
  \frac{8}{3} C_{12} - \frac{16}{3} C_{13} - \frac{4}{3} C_{23} & -\frac{8}{3} \sqrt{3} C_{13} + \frac{4}{3} \sqrt{3} C_{23} \\
  -\frac{8}{3} \sqrt{3} C_{13} + \frac{4}{3} \sqrt{3} C_{23} & -8 C_{12}
\end{pmatrix},
\]
(C1)
and for the \((\phi^T \chi^T_3, \phi^T \chi^T_6)\) state of tetraquarks, it is
\[
\begin{pmatrix}
  \frac{8}{3}(\alpha - \beta) & 2 \sqrt{6} \beta \\
  2 \sqrt{6} \beta & 4 \alpha
\end{pmatrix},
\]
(C2)
For other cases of tetraquarks, the CMI matrices can be obtained similarly. They are
\[
\begin{pmatrix}
  -\frac{8}{3} \theta & -2 \sqrt{2} \beta \\
  -2 \sqrt{2} \beta & -\frac{4}{3} \eta
\end{pmatrix},
\]
(C3)
for the tetraquark wavefunction \((\phi^T \chi^T_5, \phi^T \chi^T_4)\) and
\[
\begin{pmatrix}
  -\frac{8}{3}(\alpha + 5 \beta) & 2 \sqrt{6} \beta \\
  2 \sqrt{6} \beta & -8 \alpha
\end{pmatrix},
\]
(C4)
for the tetraquark \((\phi^T \chi^T_3, \phi^T \chi^T_6)\). In the case of three-dimensional subspace, one can find the CMI matrix to be
\[
\begin{pmatrix}
  \frac{4}{3}(2 \alpha - \beta) & 4 \frac{\sqrt{3}}{3} \delta & 4 \delta \\
  4 \frac{\sqrt{3}}{3} \delta & \frac{8}{3} \eta & -2 \sqrt{2} \beta \\
  4 \delta & -2 \sqrt{2} \beta & \frac{4}{3} \beta
\end{pmatrix},
\]
(C5)
for the mixed state of \((\phi^T \chi^T_2, \phi^T \chi^T_4, \phi^T \chi^T_5)\) of tetraquark and
\[
\begin{pmatrix}
  \frac{4}{3}(2 \alpha - \beta) & -\frac{4}{3} \gamma & -4 \gamma \\
  -\frac{4}{3} \gamma & -\frac{8}{3} \theta & -2 \sqrt{2} \beta \\
  -4 \gamma & -2 \sqrt{2} \beta & -\frac{4}{3} \eta
\end{pmatrix},
\]
(C6)
for the tetraquark state of \((\phi^T \chi^T_2, \phi^T \chi^T_5, \phi^T \chi^T_4)\), in additional to
\[
\begin{pmatrix}
  -\frac{2}{3}(2 \alpha + 5 \beta) & \frac{10}{3} \sqrt{3} \delta & 4 \delta \\
  \frac{10}{3} \sqrt{3} \delta & -\frac{4}{3} \eta & -2 \sqrt{2} \beta \\
  4 \delta & -2 \sqrt{2} \beta & \frac{8}{3} \theta
\end{pmatrix},
\]
(C7)
for the tetraquark state of \((\phi^T \chi^T_2, \phi^T \chi^T_4, \phi^T \chi^T_5)\). In these matrices, one used \(\alpha = C_{12} + C_{34}\), \(\beta = C_{13} + C_{14} + C_{23} + C_{24}\), \(\gamma = C_{13} + C_{14} - C_{23} - C_{24}\), \(\delta = C_{13} - C_{14} + C_{23} - C_{24}\), \(\eta = C_{12} - 3 C_{34}\) and \(\theta = 3 C_{12} - C_{34}\), from Ref. [7].
In the following, we list the expressions for overall binding energy of the hadrons involved in this work. They are

\[ B_{12} + B_{13} + B_{23}, \]  

(C8)

for the baryons described by \( \phi^B \). For the tetraquarks, the overall binding energy is

\[ -\frac{1}{2}B_{12} + \frac{5}{4}B_{13} + \frac{5}{4}B_{14} + \frac{5}{4}B_{23} + \frac{5}{4}B_{24} - \frac{1}{2}B_{34}, \]  

(C9)

for the configuration \( \phi^T_1 \), and

\[ B_{12} + \frac{1}{2}B_{13} + \frac{1}{2}B_{14} + \frac{1}{2}B_{23} + \frac{1}{2}B_{24} + B_{34}, \]  

(C10)

for the configuration \( \phi^T_2 \), respectively. Here, the notation \( B_{ij} \) (\( i, j = 1, 2, 3, 4 \) corresponding to \( b, c, s \) and \( n = u, d \)) stands for the binding energies in Eq. (32), and is assumed to be vanish if \( ij = nn, sn \) or \( ss \), in which case there is no short-distance binding in hadrons [14].
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