Structural Damage Detection Using Changes in Natural Frequencies: Theory and Applications

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Abstract. A vibration-based method that uses changes in natural frequencies of a structure to detect damage has advantages over conventional nondestructive tests in detecting various types of damage, including loosening of bolted joints, using minimum measurement data. Two major challenges associated with applications of the vibration-based damage detection method to engineering structures are addressed: accurate modeling of structures and the development of a robust inverse algorithm to detect damage, which are defined as the forward and inverse problems, respectively. To resolve the forward problem, new physics-based finite element modeling techniques are developed for fillets in thin-walled beams and for bolted joints, so that complex structures can be accurately modeled with a reasonable model size. To resolve the inverse problem, a logistical function transformation is introduced to convert the constrained optimization problem to an unconstrained one, and a robust iterative algorithm using a trust-region method, called the Levenberg-Marquardt method, is developed to accurately detect the locations and extent of damage. The new methodology can ensure global convergence of the iterative algorithm in solving under-determined system equations and deal with damage detection problems with relatively large modeling error and measurement noise. The vibration-based damage detection method is applied to various structures including lightning masts, a space frame structure and one of its components, and a pipeline. The exact locations and extent of damage can be detected in the numerical simulation where there is no modeling error and measurement noise. The locations and extent of damage can be successfully detected in experimental damage detection.

1. Introduction
Most damage in a structure, such as cracks, fatigue, corrosion, and loosening of bolted joints, manifests itself as a stiffness reduction. Assuming that mass changes due to damage are negligible, the linear vibration characteristics of a structure, such as natural frequencies, will change due to stiffness reduction. Damage at different locations with different extent can lead to different patterns in the changes in the natural frequencies. Hence one can use the changes in the natural frequencies to detect the locations and extent of damage by solving an inverse problem. Because the natural frequencies of a structure can be more easily measured than the mode shapes, and the measurement error is usually less than 1%, it would be easier to use the changes in the natural frequencies to detect damage in a structure than the mode shapes. The authors’ group developed an iterative damage detection method
that can detect the locations and extent of damage in simple structures such as beams [2], using changes in the natural frequencies of the first several modes. However, to apply the method to complex engineering structures such as space frame structures, there are two major challenges that need to be addressed.

The first challenge is accurate modeling of structures. Because the damage detection method is model-based, an accurate model of a test structure is required to establish the relationship between the natural frequencies of the structure and its stiffness. The model must be physics-based so that it can simulate real changes in the natural frequencies caused by particular damage in the structure. Developing accurate models for complex structures, especially those with bolted joints, is more challenging than that for simple beams. In addition, the model size and computation efficiency must be at an acceptable level so that the models can be used with an iterative damage detection algorithm.

The second challenge is from the inverse analysis, which is the development of a robust damage detection algorithm. The changes in the natural frequencies due to damage include the effects of modeling error and measurement noise. Since it is difficult to develop an accurate model for a complex structure, the effect of modeling error can be relatively large in damage detection of complex structures. The presence of large modeling error and measurement noise can significantly affect the accuracy of the damage detection algorithm, and the level of modeling error and measurement noise determines the size of minimum detectable damage [2]. Hence the damage detection algorithm should have sufficient robustness to deal with relatively large modeling error and measurement noise [3].

With the development of modeling techniques for two common features in complex structures: fillets in thin-walled beams [4] and bolted connections [5], the main challenge associated with the modeling of complex structures can be resolved. By applying the modeling technique for fillets in thin-walled beams [4], thin-walled beams can be accurately modeled using shell and beam elements. The higher mode vibrations of a filleted thin-walled beam, which usually cannot be captured by beam elements due to the plate-like deformations of the thin walls, can be accurately predicted by a shell and beam element model. The bolted connection modeling technique developed in Ref. [5] provides a simple, yet efficient, solution for the modeling of bolted joints. A bolted connection is modeled by a solid cylinder; the radius and elastic modulus of the cylinder can be determined by solving a contact problem using an intensive solid element model for the bolted connection. It was found that the radius of the cylinder, which represents the contact area between the two clamped components, is the most critical parameter in the modeling of the bolted connection. The model developed for a bolted joint is physics-based; it can accurately predict the natural frequencies and mode shapes of a structure with tightened or loosened bolted connections as long as the vibrations of the structure are within a linear range. Because the sizes of the models for fillets and bolted joints are relatively small and only regular elements are used in the models, the modeling techniques can be easily applied to an assembled structure with thin-walled beams and bolted joints.

The inverse analysis of the vibration-based damage detection method is essentially solving a nonlinear least-square problem [3,6,7]. Because the number of unknown stiffness parameters to be determined is usually greater than that of the natural frequencies used in damage detection, the nonlinear least-square problem is under-determined. In Ref. [2], an iterative line-search method, which is essentially the Gauss-Newton method [6], was used to solve the under-determined nonlinear least-square problem [3]. To obtain a unique solution at each iteration and ensure convergence of the iterations for an under-determined system, the Moore-Penrose inverse, which provides the minimum-norm solution [6,7], was used to solve for the search step at each iteration [2]. Using the Gauss-Newton method along with the Moore-Penrose inverse to solve an under-determined nonlinear least-square problem cannot guarantee that the iterations will converge to a stationary point of the objective function [3,8], and the Moore-Penrose inverse will amplify the effects of modeling error and measurement noise when the condition number of the Jacobian matrix is large [3,7].

To improve the robustness of the damage detection algorithm, a trust-region search method, called the Levenberg-Marquardt (LM) method [6], is used to solve the nonlinear least-square problem [3]. By
combing the trust-region search strategy and a regularization method that can reduce the condition number of the Jacobian matrix \cite{6}, the LM method can restrict the effects of modeling error and measurement noise to an acceptable level in each iteration. The LM method can also ensure global convergence of the iterations, i.e., convergence to a stationary point of the objective function \cite{3,6}.

With the challenges associated with the modeling and inverse analysis resolved, the vibration-based damage detection method was applied to detect different types of damage, including loosening of bolted joints, in slender and three-dimensional structures, including lightning masts \cite{9}, a space frame structure \cite{3} and one of its components \cite{9}, and a pipeline \cite{10}. The exact locations and extent of damage can be detected in the numerical simulation where there is no modeling error and measurement noise, and the locations and extent of damage were successfully detected in experimental damage detection.

2. Damage Detection Algorithm

2.1 Nonlinear Least-Square Problem

To illustrate the damage detection methodology, consider a cantilever beam shown in Fig. 1. The beam is divided into \( n \) groups, and the stiffness of the \( i \)th group can be represented by a dimensionless stiffness parameter \( 0_i G \), where \( 1, 2, \ldots, n \) and \( G_i = 0 \) indicating that the \( i \)th group is undamaged and fully damaged, respectively. When all the stiffness parameters are identified, the locations and extent of damage are known. While smaller groups can provide a more accurate representation of damage, the size of a group cannot be too small since the minimum size of detectable damage depends on the damage information contained in the natural frequencies used in damage detection and decreasing the size of a group will increase the number of stiffness parameters to be identified.

A damage detection algorithm is to identify what changes in \( G_i \) can cause the changes in the natural frequencies due to damage. The \( m \) natural frequencies of a structure that are used in damage detection are denoted by \( F = [f_1, f_2, \ldots, f_m] \), where \( f_j \) (\( j = 1, 2, \ldots, m \)) are functions of the dimensionless stiffness parameters \( G = [G_1, G_2, \ldots, G_n] \). Note that it is difficult to obtain the analytical expressions of \( f_j \) in terms of \( G \) unless the structure is relatively simple; the nonlinear relationships between \( f_j \) and \( G \) are usually numerically represented by a finite element (FE) model of the test structure. The measured natural frequencies of the damaged structure are denoted by \( F^d = [f^d_1, f^d_2, \ldots, f^d_m] \); the changes in the natural frequencies caused by damage are \( r = [f_1 - f^d_1, f_2 - f^d_2, \ldots, f_m - f^d_m] \). To identify what changes in \( G \) can cause the changes in the natural frequencies indicated by \( r \), a nonlinear least-square problem that minimizes the objective function

\[
Q(G) = \frac{1}{2} \sum_{j=1}^{m} (f_j(G) - f^d_j)^2 = \frac{1}{2} r^T r
\]

in the domain of \( G \) can be formulated. For a solution of the nonlinear least-square problem, \( G^* \), each entry in \( G^* \) is the remaining stiffness of the structure at the corresponding location; if the value is less than one, there is possible damage at the corresponding location.
2.2 LM Method with a Logistic Function Transformation

The LM method only accepts search steps whose lengths are less than the radii of prescribed trust regions in the search domain [6]. If the length of a search step calculated by the Gauss-Newton method is greater than the radius of the trust region, the search step is rejected, and a new search step $\delta \mathbf{G}$ is calculated by solving

$$
\begin{cases}
-\mathbf{J}^T \mathbf{r} = (\mathbf{J}^T \mathbf{J} + \lambda \mathbf{I}) \delta \mathbf{G} \\
\|\delta \mathbf{G}\| = \Delta
\end{cases}
$$

(2)

where $\lambda$ is a regularization parameter, and $\Delta$ is the radius of the trust region. For a given radius $\Delta$, $\lambda$ and $\delta \mathbf{G}$ are simultaneously calculated from Eq. (2). The radius of the trust region is selected to ensure that approximating the nonlinear objective function in Eq. (1) by its second-order Taylor series expansion is acceptable within the trust region in the search domain. The radius of the trust region will be updated before each iteration based on the descent made by the previous iteration [6]. Since introducing the regularization parameter $\lambda$ can reduce the condition number of $\mathbf{J}^T \mathbf{J} + \lambda \mathbf{I}$ in Eq. (2), the error amplification effect of the Moore-Penrose inverse [7] can be significantly reduced in the LM method.

The LM method mentioned above is for unconstrained nonlinear least-square problems. Because the dimensionless stiffness parameters $\mathbf{G}$ are bounded between 0 and 1, the nonlinear least-square problem associated with vibration-based damage detection is a constrained problem. In order to apply the LM method to damage detection, a logistic function transformation is used to convert the constrained problem into an unconstrained one:

$$
G_i = \frac{1}{1 + e^{-\alpha S_i}}
$$

(3)

where $S_i$ ($i = 1,2,\ldots,n$) are the transformed dimensionless stiffness parameters, and $\alpha > 0$ is an index of the transformation, which can be adjusted to change the convergence speed of the iterations [3,9]. With the logistic function transformation, the damage detection algorithm will search in the domain of $\mathbf{S} = [S_1 \ S_2 \ \cdots \ S_n]$, whose entries vary from $-\infty$ to $\infty$.

3. Applications and Results

3.1 Detecting Damage in Lightning Masts

The vibration-based damage detection method was first used to detect damage in lightning masts. A lightning mast in an electrical substation is shown in Fig. 2(a), which consists of two pipes connected by bolted flanges and a rod-type spike. An impact hammer was used to excite the lighting mast at the ground level, and the natural frequencies of the mast were measured by two accelerometers attached to the lower pipe of the mast in two perpendicular radial directions of the cross-section. Because the shell-like deformations of the upper pipe wall caused by the vibration of the spike cannot be neglected...
[9], the upper and lower pipes in the lightning mast are modeled by shell elements using SDTools [11] (Fig. 2(b)). The spike of the lightning mast is modeled by beam elements. The steel cap at the top of the upper pipe and the mass of the brackets at the upper joint are included in the FE model (Fig. 2). The model developed can accurately predict the natural frequencies of the lightning mast; the maximum error between the first eight calculated and measured natural frequencies is 1.69%.

To apply the damage detection algorithm developed, the lightning mast in Fig. 2, including the spike, is evenly divided into 20 groups with a dimensionless stiffness parameter assigned to each group (Fig. 3(a)). The first six measured natural frequencies were used in damage detection. The lightning mast was essentially undamaged since there was no significant stiffness reduction in the damage detection results in Fig. 3(b).

The damage detection algorithm was also applied to detect introduced damage in a scale model of the lightning mast in Fig. 2(a) (Fig. 4(a)). A section from 0.0923 m to 0.1348 m from the fixed end and 0.000285 m deep was machined from the surface of the lower pipe, and another section from 0.8959 m to 0.9471 m from the fixed end and 0.000355 m deep was machined from the surface of the upper pipe; they represent a 30% and a 55% stiffness reduction, respectively. The natural frequencies of the mast were measured by a laser vibrometer to avoid mass loading.

Since the shell-like deformation of the upper pipe is negligible for the scale mast [9], the scale mast is modeled mainly using beam elements (Fig. 4(b)); only the flanges and the base plate are modeled by shell elements (Fig. 4(b)). The stiffness reductions caused by the machined sections are taken into account in the FE model by reducing the elastic modulus of the beam elements at the corresponding locations. The mainly beam element model can accurately predict the natural frequencies of the scale mast. The maximum error between the first 15 calculated and measured natural frequencies is 1.88%; the machined sections introduce a maximum change of 3.57% in the first 15 calculated natural frequencies.

To apply the damage detection algorithm, the lower pipe, the upper pipe, and the spike are evenly divided into 20, 14, and 4 groups, respectively (Fig. 5(a)). The elastic modulus of each group is represented by a dimensionless stiffness parameter. The first six measured natural frequencies were used to detect the introduced damage. The locations and extent of damage were successfully detected (Fig. 5(b)); the maximum error between the first six calculated and measured natural frequencies of the damaged mast is 1.73%. In the numerical simulation where there is no modeling error and measurement noise, the exact locations and extent of damage were detected (Fig. 5(b)). Note that the use of the logistic function transformation can significantly increase the convergence speed of the iterative algorithm; it reduces the number of iterations from 1300 to 13 [9].
3.2 Detecting Damage in a Space Frame Structure

To study damage detection of space frame structures, an aluminium three-bay space frame structure with L-shaped beams and bolted joints was fabricated (Fig. 6). Twenty seven accelerometers were used to measure the natural frequencies of the structure and to match the calculated and measured mode shapes. Two accelerometers were placed at each of the 12 bolted joints that connect the L-shaped beams and the horizontal beams, to measure the vibrations of the structure in two perpendicular horizontal directions. Two accelerometers were placed in the middle of two perpendicular horizontal beams at the top of the structure, and the other accelerometer was placed on the bottom plate (Fig. 6). An impact hammer was used to excite the vibrations of the structure in two perpendicular horizontal directions. The measured data were collected and analyzed using a 36-channel LMS spectrum analyzer.

An accurate FE model of the space frame structure in Fig. 6 was developed using SDTools [11] (Fig. 7(a)) [3]. The modeling techniques for fillets [4] and bolted joints [5] are used to model the L-shaped beams and bolted joints in the structure. The horizontal beams with rectangular cross-sections are modeled by shell elements. The fixed boundaries of the structure are modeled by shell and solid elements [3]. The maximum error between the first 30 calculated and measured natural frequencies is 1.86%. When eight bolted connections in a bolted joint are loosened to hand-tight (Fig. 7(b)), by reducing the shear moduli of the solid cylinders corresponding to the loosened bolted connections to 3.41% of their original values, the maximum error between the first 31 calculated and measured natural frequencies is 1.82%.

The vibration-based damage detection method was used to detect various types of damage in the space frame structure in Fig. 6 [3]. The structure is divided into 39 groups in the FE model (Fig. 7(c)). Each solid cylinder that models a bolted connection connecting the bottom plate to the ground is represented by a group (groups 1-4). The bottom plate is represented by a group (group 5). Each of the L-shaped beams and the horizontal and diagonal beams is represented by a group.
(groups 6-8, 11-13, 16-18, 21-23, 26-39). The eight solid cylinders representing the eight bolted connections that connect the L-shaped beams and the bracket, in each of the eight bolted joints in the middle of the structure, are grouped together and represented by a stiffness parameter (groups 9, 10, 14, 15, 19, 20, 24, 25). A loosened bolt connection is modeled by a reduced shear modulus of the solid cylinder [5].

The experimental damage detection was first conducted to detect damage in a beam member (Fig. 8). The damage was introduced to the structure in Fig. 6 by almost fully loosening the upper bolted connection of a diagonal beam (group 26 in Fig. 7(c)); the diagonal beam did not provide any stiffness to the space frame structure, but retained its mass effect. The first 14 measured natural frequencies were used to detect the introduced damage. The maximum change in the first 14 natural frequencies caused by the damage was 4.18%. The location and extent of the damage were detected after 14 iterations; the maximum error between the calculated and measured natural frequencies of the first 14 modes of the damaged structure was 0.29%.

The second case was to detect loosening of a bolted joint (Fig. 9). The damage was introduced to the structure in Fig. 6 by loosening eight bolted connections in a bolted joint (group 9 in Fig. 7(c)) to hand-tight (Fig. 7(b)); note that the two bolted connections that connect the two horizontal beams to the bracket and the L-shaped beam remained tightened, and loosening them would reduce the stiffness effects of the two horizontal beams. The first 10 measured natural frequencies were used to detect the damage. The maximum change in the first 10 natural frequencies caused by the damage was 8.36%. The location and extent of the introduced damage were detected after 15 iterations; the maximum error between the calculated and measured natural frequencies of the first 10 modes of the damaged structure was 1.09%.

The third case was to detect damage at a lower boundary of the structure (Fig. 10). The damage was introduced to the structure in Fig. 8 by loosening one of the four bolted connections that connected the bottom plate to the ground (group 4 in Fig. 7(c)) to hand-tight. The first nine measured natural frequencies were used to detect the damage. The maximum change in the first nine natural frequencies caused by the damage was 4.35%. The location and extent of the introduced damage were detected after 12 iterations; the maximum error between the first nine calculated and measured natural frequencies of the damaged structure was 1.82%. When a similar damage was introduced to the FE model of the space frame structure in the numerical simulation, the exact location and extent of damage was detected in each case (Figs. 8-10).
3.3 Detecting Loosened Bolted Connections in a Component of the Space Frame Structure

The damage detection method was also used to detect loosened bolted connections in a component of the space frame structure in Fig. 6, with free boundaries (Fig. 11) [9]. Four of the ten bolted connections in the bolted joint were loosened to hand-tight, which causes a maximum change of 4.31% in the natural frequencies of the first 16 elastic modes [9]. The first 16 elastic modes were used to detect the damage. The portion of each L-shape beam that is not in contact with the bracket is evenly divided into 10 groups (groups 1 to 10 for one L-shaped beam and groups 18 to 27 for the other L-shaped beam) in the FE model, the elements representing the bracket are grouped together as one group (group 11), and the elements representing the L-shaped beams in contact with the bracket are grouped together as one group (group 17). The elastic modulus of each group is represented by a dimensionless stiffness parameter. Each pair of the bolted connections that are symmetric about the corner of the bracket, which is assumed to have the same tightness, is grouped together as one group (groups 12 to 16), and the corresponding shear modulus of the cylinders is represented by a dimensionless stiffness parameter. The loosened bolted connections were successfully detected, and the maximum error between the calculated and measured natural frequencies of the first 16 elastic modes of the damaged component is 1.50%. The exact locations and extent of damage were detected in the numerical simulation (Fig. 11).

3.4 Detecting Loosening of Bolted Connections in a Pipeline

The last application was to detect loosening of bolted connections in a pipeline [10]. A full-size steel pipeline, consisting of two pipes, each welded to a steel flange through a circumferential weld, and eight bolted connections that bolt the two flanges together, was put on two air beds to simulate the free boundaries (Fig. 12). An impact hammer was used to excite the pipeline. Three accelerometers were used to measure the bending/breathing and torsional vibrations of the pipeline: two accelerometers were placed inside the pipeline in two perpendicular radial directions of the cross-section, and one accelerometer was attached along the circumferential direction of the cross-section through a steel bracket that is fixed to the outer wall of the pipeline (Fig. 12). The measured natural frequency of the highest rigid body mode was 7.74% of that of the first elastic mode, which validated the free boundaries of the pipeline [12].

A FE model of the pipeline was developed using SDTools [11] (Fig. 13). The pipes and flanges can be easily modeled by shell elements; the difficulty lies in the modeling of the bolted connections. The modeling technique developed in Ref. [3] cannot be directly applied here due to the relatively thick flanges and the gasket between them. It was observed from the numerical simulation [10] that the pressure generated by a large clamping force widely spreads out in the relatively thick clamped component, and due to the presence of the gasket between the flanges, the flanges will be fully in
contact with the gasket under a large clamping force. Hence the flanges cannot be assumed to be in contact within an area around a bolted connection, as the case in Secs. 3.2 and 3.3, and the technique of modeling a bolted connection by determining the radius of the effective area cannot be applied here. However, each bolted connection can still be modeled by a solid cylinder connecting the shell elements that represent the flanges (Fig. 13), but unlike modeling a bolted connection in Secs. 3.2 and 3.3, in which a solid cylinder models the equivalent stiffness and mass effects of a bolted connection, a solid cylinder here only represents a bolt. The radius of the solid cylinder is set to be that of the shaft calculated by setting the mass of the cylinder to be that of the bolt. The shell elements that represent each flange are put on the surface of the flange that is in contact with the gasket. The gasket is modeled by solid elements that are fully in contact with the shell elements representing the flanges (Fig. 13(b)). Rigid links are used to connect the shell elements representing a flange and those representing the connecting pipe (Fig. 13).

The natural frequencies of each bolt, and its material properties are set to be those of the bolt as well. The density of the solid cylinder is of the pipeline can be accurately modeled; the maximum error between the calculated and measured natural frequencies of the first 21 elastic modes is 1.82%. When all the bolted connections in the pipeline are slightly loosened, which causes a maximum change of 16.54% in the natural frequencies of the first 21 elastic modes, the model developed can also be used to model the damaged pipeline. When the elastic and shear moduli of the solid elements representing the bolted connections and the rubber gasket in the FE model are reduced to 0.12 times their original values, the maximum error between the calculated and measured natural frequencies of the first 21 elastic modes of the damaged pipeline is 1.75%.

The vibration-based damage detection method was used to detect the loosened bolted connections. Each pipe is evenly divided into 20 groups (groups 1-20 and 22-41) (Fig. 14). The solid elements representing the bolted connections and the rubber gasket are grouped together as one group (group 21). The measured natural frequencies of the first six even elastic modes were first used to detect damage. The location and extent of damage can be successfully detected (Fig. 15(a)), but the result indicates that there is a relatively large stiffness reduction (over 15%) in the 31st group. The maximum error between the calculated and measured natural frequencies is 1.48% for the first four even elastic modes and the sixth even elastic mode, but the error is 3.08% for the fifth even elastic mode. The error in the damage extent was mainly caused by the relatively large modeling error of the fifth even elastic mode. When the measured natural frequencies of the first four even elastic modes and the sixth even elastic mode were used to detect damage, an excellent result was obtained (Fig. 15(b)). The maximum error between the calculated and measured natural frequencies of the first six even elastic modes of the damaged pipeline is 1.99%. The exact location and extent of damage were detected in the numerical simulation (Fig. 15(c)).

4. Conclusions
The creation of an accurate physics-based model for both the undamaged and damaged states of a structure and the development of a robust iterative algorithm for an under-determined system are the keys to implementing the vibration-based damage detection method using changes in the natural frequencies of the structure. New modeling techniques are developed to accurately model tightened
and loosened bolted connections in the space frame structure and the pipeline. The LM method can resolve the error amplification problem in solving an under-determined nonlinear least-square problem, and improve the robustness of the iterative damage detection algorithm. With the logistic function transformation, which converts the constrained optimization problem to an unconstrained one, the damage detection algorithm is ensured to converge to a stationary point of the objective function. The logistic function transformation can also significantly increase the convergence speed of the iterative algorithm. The new methodology can successfully detect various types of damage in the scale mast, the space frame structure and one of its components, and the pipeline, including loosening of bolted joints or connections, where the maximum modeling error is less than 2%, and the maximum change in the natural frequencies due to damage can be less than 4%. The exact locations and extent of damage can be detected in the numerical simulation where there is no modeling error and measurement noise.

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References
[1] Wong, C.N., Zhu, W.D., and Xu, G.Y., “On an Iterative General-Order Perturbation Method for Multiple Structural Damage Detection,” Journal of Sound and Vibration, Vol. 273, 2004, pp. 363-386.
[2] Xu, G.Y., Zhu, W.D., and Emory, B.H., “Experimental and Numerical Investigation of Structural Damage Detection Using Changes in Natural Frequencies,” Journal of Vibration and Acoustics, Vol. 129, Dec. 2007, pp. 686-700.
[3] He, K., and Zhu, W. D., “Detection of Damage in Space Frame Structures with L-shaped Beams and Bolted Joints Using Changes in Natural Frequencies,” Proceedings of the IMAC – XXIX, Jacksonville, FL, Jan. 2011; also Journal of Vibration and Acoustics, accepted for publication with some revision.
[4] He, K., and Zhu, W. D., “Modeling of Fillets in Thin-walled Beams Using Shell/Plate and Beam Finite Elements,” Journal of Vibration and Acoustics, Vol. 131, 2009a, 051002 (16 pages).
[5] He, K., and Zhu, W.D., “Finite Element Modeling of Structures with L-shaped Beams and Bolted Joints,” Proceedings of the ASME Biennial Conference on Mechanical Vibration and Noise, San Diego, CA, Sept. 2009b; also Journal of Vibration and Acoustics, Vol. 133, 011011 (13 pages), 2011.
[6] Nocedal, J. and Wright, S. J., Numerical Optimization, New York, Springer-Verlag, 1999, pp. 252-270.
[7] Friswell, M. I., and Mottershead, J. E., Finite Element Model Updating in Structure Dynamics, Netherlands, Kluwer Academic Publishers, 1995.
[8] Galantai, A., "The Theory of Newton's Method," Journal of Computational and Applied Mathematics, Vol. 124, 2000, pp. 25-44.
[9] K. He, and W.D. Zhu, "Detection of Damage in Lightning Masts and Loosening of Bolted Connections in Structures Using Changes in Natural Frequencies," ASNT Materials Evaluation, Vol. 68, No. 6, 2010.
[10] K. He, and W.D. Zhu, "Detecting Loosening of Bolted Connections in a Pipeline Using Changes in Natural Frequencies," Experimental Mechanics, in revision.
[11] Bâlmes, E., Bianchi, J. P., and Leclère, J. M., Structural Dynamics Toolbox, Users Guide, Version 6.1, Paris, France, Scientific Software Group, 2009.
[12] D. J. Ewins, Modal Testing: Theory, Practice and Application, 2nd Edition, Research Studies Press Ltd., Baldock, Hertfordshire, UK, 2000.