Research Article

An In-Depth Study on Load Distribution Characteristics of the Planetary Threaded Roller Bearing

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Received 30 June 2020; Revised 21 September 2020; Accepted 28 October 2020; Published 10 November 2020

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To lay a solid base for the property analysis and structural optimization on the high-strength planetary threaded roller bearing (PTRB) which is commonly used in mountain-walking robotic platforms, this research mainly focuses on characterizing the load distribution on the bearing under different loading conditions. An in-depth analysis was performed on the structural components and the contact characteristics of the PTRB. Based on the space meshing and Hertz contact theories, the elastic deformation of both inner and outer rings and the threaded roller was evaluated when the bearing was subjected to an axial or radial load. Meanwhile, a calculation method was proposed to obtain the load distribution coefficients of the PTRB according to the compatibility equations. In this way, an analysis on the load distribution of the PTRB was provided on the basis of the theoretical models. The results indicated that the effect of the applied load was very limited on the load distribution coefficients under an axial loading condition. The maximum value was found at the first thread of the inner ring-threaded roller contact pair, while no obvious effect was found when the bearing was carrying a radial load. It was indicated that the uniformity of the load distribution was effectively improved for axial and radial loading conditions through adjusting the distance between threads and increasing the threaded roller numbers, respectively. Therefore, both the rated load and the life cycle of the bearing can be further improved, while the friction torque can be minimized within a limited space. The research provides an important guidance for the property analysis, design, and optimization of the PTRB.

1. Introduction

Load distribution within the roller bearing is used to characterize the load differences between different contact points [1], which is one of the most important indexes for assessing the performance of the roller bearing since it has a direct effect on the rated load, life cycles, and friction torque of the bearing. And to improve the working performances of the mechanical systems, a study for the load distribution of the bearing is very helpful [2, 3]. The contact characteristics of the PTRB can be obtained via both numerical calculation and experimental investigation based on Hertz contact theory and elastic mechanics [4, 5]. Combined with the method of high calculation accuracy proposed by Jing Liu, load distribution and related contact characteristics can be analyzed [2, 3].

Guo and Li [6] and Cheng and Wang [7] investigated the load distribution within the tapered roller bearing and single-/double-row angular contact ball bearing, respectively. The relationship between the load distribution and the applied load was obtained, which provided a theoretical base for further investigations on the load distribution and structural optimization. However, the size deviation of the roller was neglected in these studies, which was different from the practical cases. Chen et al. [8] proposed a model to calculate the load distribution which took account of the effects of roller size deviation. Based on the size deviation induced by the manufacturing process, an in-depth research was carried out by Zhou and Xu [9] on the effects of size deviation magnitudes, the bearing clearance, the number of size-deviated rollers, and their locations on the load
distribution within the bearing, which provide an essential support for the investigations on both static and fatigue behavior of the roller bearing.

Based on the structural features of the PTRB, the load distribution is not uniform between the thread teeth in the threaded connection [10]. Extensive studies, including both numerical and experimental ones, have been conducted in this field to discuss the effects of this ununiform load distribution [11, 12]. Kenny and Patterson [13] studied the load distribution within the thread teeth via photoelastic tests and compared the experimental results with the theoretical model. Wang and Marshek [14] proposed a modified spring group model for the threaded connection and characterized the load distribution by using second-order difference equations. Sun and Liao [15] analyzed the axial load distribution within the low-sliding deformed thread connection via a symmetrical model and discussed the effects of the mesh density on the axial load distribution. In addition, Guo and Huang [16] investigated the load distribution on the thread teeth through a spring system with multi-degree-of-freedom. A comparative study was performed on the load scenario on both nuts and screws under different loading conditions. A mathematical model was developed by Zhou and Sun [17, 18] by using a discretization method, and the axial load distribution on the thread teeth was numerically studied. Chen and Zeng [19] proposed a finite element model for characterizing the load distribution within the thread connection. The results on the load ratio between different thread teeth were validated by the experimental data. According to the structural characteristics of the casing thread joints, Xi and Nie [20] theoretically analyzed the effects of the pitch differences between inner and outer threads on the load transfer.

As a novel roller bearing, PTRB achieves a higher rated load and life cycle while lower friction torque in comparison to the traditional configuration. However, the ununiform load distribution during the loading process is still a limitation, which introduces difficulties in the property investigations. The research in this paper mainly analyzed the load distribution within the PTRB and put forward the analysis and calculation method of load distribution, which is commonly used in mountain-walking robotic platforms, under axial and radial loading conditions, which provide a theoretical support for the property studies on rated load, life cycles, and friction torque.

The outline of this paper is summarized as follows: Section 2 introduces the structural components and contact characteristics of the PTRB. The load distribution models for the axial and radial loading conditions are proposed in Sections 3 and 4, respectively. A detailed analysis on the load distribution of the PTRB is carried out in Section 5. Section 6 draws the main conclusions.

2. Mechanical Structure and Contact Characteristics

The mechanical structure of the PTRB is shown in Figure 1. The thread rollers are uniformly distributed outside of the inner ring throughout the entire process via matching of the holes designed on the planet carrier. The thread profile of the thread roller and of the inner (outer) ring are double circular and trapezoid, respectively. The thread roller meshes with the inner and outer ring. All these three parts have multiple annular threads, i.e., the thread stroke is equal to zero, with the same space.

PTRB can bear both axial and radial loads, as shown in Figure 2. When the PTRB bears an axial load, the side of the outer ring is fixed, and the inner ring bears the axial pressure transmitted by the actuator. Through the mutual contact of the threaded rollers with the outer ring and the inner ring, this axial load is transferred from the outer ring to the inner ring and eventually to the housing. The radial load is transmitted to the outer ring and the housing via the force thread rollers. When the PTRB bears an axial load, each thread roller is stressed, and each thread tooth of each thread roller is only stressed on one side of the thread surface. When the PTRB bears a radial load, only part of the threaded roller is stressed, and each thread tooth of the stressed thread roller is stressed on both sides of the thread surface.

The inner and outer ring-threaded roller contact pair were generated at the contact points. According to Hertz theory, two vertical planes can be obtained through the contact point of each contact pair, and both of them are perpendicular to the common tangent surface [21]. The radius of curvature of the inner ring-threaded roller contact pair in principle plane 1 is denoted as $R_{R11}$ and $R_{R12}$, while that in principle plane 2 is $R_{R12}$ and $R_{R22}$. Similarly, the radius of curvature of the outer ring-threaded roller contact pair in principle plane 1 is denoted as $R_{O11}$ and $R_{O21}$, while that in principle plane 2 is $R_{O12}$ and $R_{O22}$. Figure 1 depicts the aforementioned parameters [22].

The contact characteristics of the bearing are directly linked to its structural parameters. The main structural parameters of the proposed PTRB are summarized in Table 1 and are marked in Figure 3.

3. Load Distribution under the Axial Load

The analysis on the local stress and deformation of each component in the PTRB is the premise of the investigation of the load distribution. This section mainly presents a study on the load distribution between thread teeth when the PTRB is subjected to an axial load according to the load-deformation relationship of the inner and outer rings and the threaded roller.

Despite of the effects of the applied load, the load distribution within the bearing is also influenced by the centrifugal force of the threaded roller and the gyroscopic moment during the running process. However, this influence can be neglected in this specific research owing to the low rotation velocity of the PTRB. That is to say, only the load distribution at static- or low-velocity running conditions is considered.

The following assumptions are made: (a) the load distribution analysis is performed considering only the axial loading condition. (b) Only elastic deformation takes place at the loading process, which satisfies Hooke’s law. (c) The axial load is uniformly distributed at each threaded roller. (d)
The contact surfaces are completely smooth, and the contact force is vertical to the surfaces.

3.1. Force Analysis and Elastic Deformation. A force decomposition was conducted at the contact points, as shown in Figure 4. It is obtained that the force components at the axial and vertical directions for the contact load $F_{On}$ and $F_{In}$, respectively, are expressed as

$$
\begin{align*}
F_{Oai} &= F_{Oni} \sin \alpha, \\
F_{Oni} &= F_{Oni} \cos \alpha, \\
F_{Iai} &= F_{Ini} \sin \alpha, \\
F_{Iri} &= F_{Ini} \cos \alpha.
\end{align*}
$$

(1)

The axial load applied at the PTRB is $F_a = Z \sum_{i=1}^{n} F_{Oni} = Z \sum_{i=1}^{n} F_{Ini}$ where $\alpha$ is the contact angle and $Z$ is the number of threaded rollers.

As indicated in Figure 4, the thread teeth of the roller are compressed and deformed by contacting those of both the inner and outer rings, which mainly present as the deformation of the thread teeth, the deformation of the shaft segment, and the contact deformation of the thread teeth at the contact points. These three typical deformations have a direct link to the ununiform load distribution at each contact point.

The first one represents the compressive or shear deformation of the thread teeth from the inner and outer rings and threaded roller caused by the applied load. The
deformation of the shaft segment is the compressive deformation of this structure under the compression load.

| Parameter | Meaning of the parameters | Value |
|-----------|--------------------------|-------|
| \( B_i \) | Thread bottom thickness of the inner ring | 1.08 mm |
| \( B_O \) | Thread bottom thickness of the outer ring | 1.08 mm |
| \( B_R \) | Thread bottom thickness of the threaded roller | 1.05 mm |
| \( s \) | Thread ridge thickness | 0.6 mm |
| \( h \) | Dedendum | 0.24 mm |
| \( d_1 \) | Pitch diameter of the bearing inner ring | 36.5 mm |
| \( d_2 \) | Pitch diameter of the bearing outer ring | 48.5 mm |
| \( d_r \) | Pitch diameter of the threaded roller | 6 mm |
| \( D_0 \) | Outside diameter of the PTRB | 55 mm |
| \( D_0 \) | Inner diameter of the PTRB | 30 mm |
| \( P \) | Distance between threads | 1.2 mm |
| \( \alpha \) | Contact angle | 45° |
| \( \beta \) | Tooth shape half-angle | 45° |
| \( E \) | Elastic modulus | 212 GPa |
| \( d_{rac} \) | Addendum circle diameter of the threaded roller | 6.4 mm |
| \( \mu \) | Poisson’s ratio | 0.29 |
| \( D_{pw} \) | Pitch diameter of the thread pair | 42.5 mm |
| \( r_1 \) | Raceway radius of the bearing inner ring | \( \infty \) |
| \( r_0 \) | Raceway radius of the bearing outer ring | \( \infty \) |
| \( R_r \) | Arc radius of the threaded roller | 5.1 mm |
| \( Z \) | Number of threaded rollers | 11 |
| \( n \) | Number of thread teeth per threaded roller | 10 |

Note that the shaft segments are a ring cylinder for both inner and outer rings, while they are a solid cylinder for the threaded roller. The last one stands for the Hertz contact deformation caused by the contact force between the thread teeth. As a consequence, the total axial deformation \( \delta_{X_{Tai}} \) at a specific point is the sum of the aforementioned three components:

\[
\delta_{X_{Tai}} = \delta_{XTai1} + \delta_{XTai2} + \delta_{X_{Tai}3} + \delta_{XTai4} + \delta_{XTai5}
\]

\[
= (1 - \mu^2) \frac{3 w_{Xi} \cos \beta}{4E} \left\{ \left[ 1 - \left( 2 - \frac{s}{B_X^2} \right)^2 + 2 \ln \left( \frac{B_X}{s} \right) \right] \cot^3 \beta - 4 \left( \frac{h}{B_X^2} \right)^2 \tan \beta \right\} + (1 + \mu) \frac{6 w_{Xi} \cos \beta}{5E} \ln \left( \frac{B_X}{s} \right) \cot \beta + (1 - \mu^2) \frac{12 h w_{Xi} \cos \beta}{\pi E B_X^2} \left( \frac{h}{2} \tan \beta \right) + \left( 1 - \mu^2 \right) \frac{2 \pi w_{Xi} \cos \beta}{\pi E} \left[ \frac{P}{B_X} \ln \left( \frac{P + (B_X/2)}{P - (B_X/2)} \right) + 2 \ln \left( \frac{4 P^2}{B_X^2} - 1 \right) \right] + \delta_{XTai5}^\prime
\]

where \( w_{Xi} \) and \( \beta \) are the unit force of the normal load at the contact point and the half-angle of the thread teeth, \( \mu \) is Poisson’s ratio, \( P \) is the distance between threads, \( E \) is the elastic modulus, \( h \) is the dedendum, and \( s \) is the thread ridge thickness.

For different thread configurations, the deformation of the tooth teeth induced by the radial force components is different. The deformation for the inner ring with outer threads is [23]

\[
\delta_{Trai5} = \left( d_1^2 + d_0^2 - \mu \right) \frac{d_t w_{Xi} \sin \beta}{2PE} \tan \beta.
\]

The deformation for the outer ring with inner threads is [23]

\[
\delta_{Otra5} = \left( D_0^2 + d_0^2 - \mu \right) \frac{d_t w_{Xi} \sin \beta}{2PE} \tan \beta.
\]
3.3. Deformation of the Shaft Segment. Owing to the assumptions on continuity, uniformity, isotropy, and small deformation, the deformation of the shaft segment can be obtained by using the tension-compression stiffness formula in material mechanics [24]:

\[
\delta_{X_{Ai}} = \frac{F_{X_{Ai}} P}{EA_x}
\]  

(7)

Note that the number for the contact points in the inner and outer ring and the threaded roller is increasing from left to right. Consequently, the numbers for the contact points in the inner and outer rings are from 1 to \(n\), while for the threaded roller, they are from 1 to \(2n\). The components are divided into several segments by the contact points. The corresponding numbers for the segments of the inner and outer ring are from 0 to \(n\), while for those of the threaded roller are from 0 to \(2n\). The axial load and cross-sectional area for each segment of the inner ring can be expressed as

\[
F_{Iai} = \begin{cases} 
\frac{F_a}{Z}, & i = 0, \\
-\frac{F_a}{Z} + \sum_{j=1}^{i} F_{Iaj}, & i > 0,
\end{cases}
\]  

(8)

\[
A_I = \frac{\pi \left[(d_i - 2h)^2 - d_0^2\right]}{4Z}.
\]  

These values for the segments of the outer ring are

\[
F_{Oai} = \begin{cases} 
0, & i = 0, \\
-\sum_{j=1}^{i} F_{Oaj}, & i > 0,
\end{cases}
\]  

(9)

\[
A_O = \frac{\pi \left[D_0^2 - (d_O + 2h)^2\right]}{4Z}.
\]
These values for the segments of the threaded roller are

\[
F_{Ra_i} = \begin{cases} 
0, & i = 0, \\
-F_{Ta_{i+1}}, & i = 1, \\
\sum_{j=1}^{k} (F_{Oaj} - F_{Iaj}), & i = 2k (k > 0), \\
\sum_{j=1}^{k} (F_{Oaj} - F_{Iaj}) - F_{Ta(i+1)}, & i = 2k + 1 (k > 0),
\end{cases}
\]

\[
A_R = \frac{\pi (d_R - 2h)^2}{4},
\] (10)

where \(d_R\) is the pitch diameter of the threaded roller and \(Z\) is the number of threaded rollers.

3.4. Contact Deformation of Thread Teeth. Owing to the mismatch of the thread teeth of the threaded roller (double-circular shape) and of the inner and outer rings (trapezoidal shape), it is point contact for the surfaces of both inner and outer ring-threaded roller pairs. The contact deformation is lying at the normal line direction of their common tangent surface.

The curvature is the reciprocal of the curvature radius. Therefore, according to Figure 1, the curvature at the contact point of the inner ring-threaded roller pair is obtained as [1]

\[
\rho_{111} = \frac{1}{R_{R11}} = \frac{1}{R_{rr}},
\]

\[
\rho_{112} = \frac{1}{R_{R12}} = \frac{2 \cos \alpha}{d_R},
\]

\[
\rho_{121} = \frac{1}{R_{I21}} = \frac{1}{r_1},
\]

\[
\rho_{122} = \frac{1}{R_{I22}} = \frac{2 \cos \alpha}{d_1}.
\] (11)

Similarly, the curvature value of the outer ring-threaded roller pair is

\[
\rho_{011} = \frac{1}{R_{R11}} = \frac{1}{R_{rr}},
\]

\[
\rho_{012} = \frac{1}{R_{R12}} = \frac{2 \cos \alpha}{d_R},
\]

\[
\rho_{021} = \frac{1}{R_{O21}} = \frac{1}{r_O},
\]

\[
\rho_{022} = \frac{1}{R_{O22}} = \frac{2 \cos \alpha}{d_O}.
\] (12)

where \(R_{rr}\) is the raceway radius of the bearing inner ring, and \(r_O\) is the raceway radius of the bearing outer ring.

The sum of the main curvature values at the contact point is an intrinsic characteristic of a contact pair. The sum of the main curvature for both inner \((\sum \rho_{IR})\) and outer \((\sum \rho_{OR})\) ring-threaded roller pairs can be calculated by using equations (10) and (11), respectively. The main curvature function for two types of contact pairs is [1]

\[
F_{IR}(\rho) = \left| \frac{\rho_{111} - \rho_{112} + (\rho_{121} - \rho_{122})}{(\sum \rho_{IR})} \right|,
\]

\[
F_{OR}(\rho) = \left| \frac{\rho_{011} - \rho_{012} + (\rho_{021} - \rho_{022})}{(\sum \rho_{OR})} \right|.
\] (13)

Based on the Hertz contact formula, the contact deformation of thread teeth can be obtained by projecting the deformation at the contact point of thread teeth pairs under the normal contact load to the axial direction [1]:

\[
\delta_{XCA} = \frac{2K(e)}{\pi n_{mX}} \sqrt{\frac{9}{8} \left( \frac{1 - \mu^2}{E} \right)^2 F_{XCA} \left( \sum \rho \right)} \sin \alpha,
\] (14)

where the value of the contact elliptic coefficient \(2K(e)/\pi n_{mX}\) can be obtained by using the lookup method based on the main curvature function.

3.5. Compatibility Equations. According to the equilibrium condition of forces, the equilibrium equations of the PTRB at the axial direction are denoted as

\[
\begin{cases} 
Z \sum F_{Ta} = F_a, \\
Z \sum F_{Oa} = F_a.
\end{cases}
\] (15)

Assuming that each contact point keeps contacting throughout the loading process, the deformation of the thread teeth for the inner ring-threaded roller pair is

\[
(\delta_{TAd})_{i} + (\delta_{TId})_{i} - (\delta_{Ta_{i+1}}) + (\delta_{Ica})_{i} - (\delta_{Ica_{i+1}}) = (\delta_{RAd})_{2i-1} + (\delta_{RAd})_{2i} - (\delta_{RId})_{2i-1} - (\delta_{RId})_{2i+1}.
\] (16)

Similarly, the deformation of the thread teeth for the inner ring-threaded roller pair is

\[
(\delta_{OAd})_{i} - (\delta_{OId})_{i} + (\delta_{OId})_{i+1} - (\delta_{Oca})_{i} + (\delta_{Oca})_{i+1} = (\delta_{RAa})_{2i} + (\delta_{RAa})_{2i+1} - (\delta_{RAa})_{2i+2} - (\delta_{RAa})_{2i+1}.
\] (17)

There are 2\(n\) unknown contact forces at the inner and outer rings contacted by a threaded roller. Owing to the two equilibrium equations in equation (15), there are 2\(n\)–2 compatibility equations in total in equations (16) and (17). Combining the equations from (1) to (17), we can use MATLAB to solve the equations to solve the load distribution of the PTRB along the axial direction.

For the sake of the modelling and analysis on other indexes, such as rated load, friction, and life cycle, the axial
load distribution coefficient $f_{ai}$ is defined as the ratio between the axial load of each thread and the averaged axial load:

$$f_{ai} = \frac{nZF_{xai}}{F_a},$$  \hspace{1cm} (18)

where $n$ is the number of thread teeth per threaded roller.

### 4. Load Distribution under the Radial Load

Once the PTRB is only subjected to a radial load, it is necessary to conduct an analysis on the load distribution due to different deformations caused by different contact forces at different positions of the threaded roller. This section mainly presents a study on the load distribution between thread teeth when the PTRB is subjected to a radial load according to the load-deformation relationship of the inner and outer rings and the threaded roller, which provides a support for the structural design and property investigations. Note that the assumptions made for analysis under the axial load are also provided in this section for the specific loading condition, i.e., radial load.

#### 4.1. Force Analysis and Elastic Deformation

As depicted in Figures 2 and 5, only half of the threaded roller are subjected to the contact force under the radial load at the vertical direction. The part that bears contact force is defined as an effective threaded roller. Therefore, the number of the effective threaded rollers ($Z_e$) during the rotation process is

$$Z_e = \left\lfloor \frac{Z - 1}{2} \right\rfloor,$$

or

$$Z_e = \left\lfloor \frac{Z + 1}{2} \right\rfloor.$$  \hspace{1cm} (19)

The symbol $\lfloor \rfloor$ in equation (19) is a Gauss mark, that is, $[x]$ can be used to represent the largest integer not exceeding $x$, for example, $[1.2] = 1$.

Defining the angle between the first effective threaded roller from the left side and the horizontal line is $\theta_0$, the argument ($\varphi_i$) of each effective threaded roller is denoted as

$$\varphi_i = \theta_0 + (i - 1) \frac{2\pi}{Z}, \hspace{0.5cm} \theta_0 \in \left(0, \frac{2\pi}{Z}\right].$$  \hspace{1cm} (20)

The force equilibrium equation can be obtained via Newton’s third law:

$$2n \sum_{i=1}^{Z_e} F_{ri} \sin(\varphi_i) = F_r.$$  \hspace{1cm} (21)

Similar to the axial loading condition, the bearing deformation consists of the deformation of the contact points and the thread teeth under the radial load. The deformation of the shaft segment can be neglected owing to its significant high stiffness.

#### 4.2. Elastic Deformation of Thread Teeth

According to the analysis on Figure 2, owing to the geometric symmetry, the deformation induced by bending, shear, rotation, and sliding are all offset when the PTRB is subjected to a radial load. Therefore, only the compressive deformation $\delta_{T_{\text{Tri}}}$ caused by the radial force component is considered.

For the inner threads at the inner ring, the deformation is expressed as [23]

$$\delta_{T_{\text{Tri}}} = \left( \frac{d_1^2 + d_0^2}{d_1^2 - d_0^2} - \mu \right) \frac{d_1 w'_{xi} \sin \beta}{2PE}. $$  \hspace{1cm} (22)

For the outer threads at the outer ring [23],

$$\delta_{O_{\text{Tri}}} = \left( \frac{D_0^2 + d_0^2}{D_0^2 - d_0^2} + \mu \right) \frac{d_0 w'_{xi} \sin \beta}{2PE}. $$  \hspace{1cm} (23)

For the outer threads at the threaded roller [23],

$$\delta_{R_{\text{Tri}}} = (1 - \mu) \frac{d_R w'_{xi} \sin \beta}{2PE}. $$  \hspace{1cm} (24)

Thus, the total deformation is denoted as

$$\delta_T = \delta_{T_{\text{Tri}}} + \delta_{O_{\text{Tri}}} + 2\delta_{R_{\text{Tri}}}. $$  \hspace{1cm} (25)

#### 4.3. Contact Deformation of Thread Teeth

Similar to the deformation occurring in the thread teeth under the axial load, the contact deformation can be obtained by projecting the deformation at the contact point of thread teeth pairs under the normal contact load to the radial direction [1]:

$$\delta_{X_{\text{Gri}}} = \frac{2K(e)}{\pi m_X} \left[ 9 \left( 1 - \mu^2 \right)^2 \frac{F_n}{E \cos \alpha} \left( \sum \rho \right) X \cos \alpha \right]. $$  \hspace{1cm} (26)

Then, the total contact deformation of the thread teeth is

$$\delta_{C_T} = \delta_{R_{\text{Gri}}} + \delta_{O_{\text{Gri}}}.$$ \hspace{1cm} (27)

#### 4.4. Compatibility Equations

As indicated in Figure 5, the vertical deformations of two adjacent effective threaded rollers are equal, and we can get
Load distribution coefficient of the inner ring-threaded roller contact pair

\[
\frac{\delta_{i1}}{\sin \phi_1} = \frac{\delta_{i2}}{\sin \phi_2} = \frac{\delta_{i3}}{\sin \phi_3} = \frac{\delta_{i4}}{\sin \phi_4} = \frac{\delta_{i5}}{\sin \phi_5}
\]  (28)

Figure 6: Evolution of the axial load distribution coefficient with the axial load.

5. Analysis on the Load Distribution

5.1. The Axial Load Case. Based on the theoretical analysis, the evolution of the load distribution as a function of the axial load was obtained by using MATLAB. As indicated in Figure 6, the load distribution did not show a significant change when increasing the applied load, which implied that the axial load distribution coefficient can be regarded as an inherent characteristic of the PTRB with a certain structural configuration. However, the load distribution coefficients of inner and outer ring-threaded roller pairs showed different magnitudes and contradictory evolution trends, ranging from 0.94~1.12 for the former and 0.97~1.05 for the latter. It can be seen that the inner ring-threaded roller pair showed less uniformity in the load distribution.

As a result of the further analysis, when given an axial load of 30 kN, the spacing between every two adjacent contact points was leveling at around 0.1 μm as comparing to the difference in the tooth spacing (1.2 mm), which indicated that the ununiform load distribution was caused by this tiny difference in the tooth spacing. Therefore, one can adjust the spacing between two contact points according to the deformation coordination, thus minimizing the ununiformity of the load distributed on the thread teeth. This will be done in the future work.

5.2. The Radial Load Case. Similarly, the evolution of the load distribution as a function of the radial load was obtained by using MATLAB.

As illustrated in Figure 7, the maximum load distribution coefficient on the radial direction of the PTRB was 0.373. The applied load had little effect on the load distribution, which indicated that the radial load distribution coefficient can also be regarded as an inherent characteristic of the PTRB with a certain structural configuration.

Figure 8 illustrates the evolution of the radial load distribution as a function of the number of the threaded rollers on the basis of a fixed structural configuration of the PTRB. When the number of the threaded rollers increased from 9 to 11, the maximum load distribution coefficient decreased from 0.460 to 0.373, which revealed that a higher radial load uniformity can be achieved by increasing the number of thread rollers. From a viewpoint of structural
design, given an increased threaded roller number, the load at each contact point is reduced, which tends to improve the rated load and life cycles for the bearing. In addition, an empirical study indicates that the friction torque can be lower combined with a neglected influence on the original bearing structure when properly increasing the number of threaded rollers, which therefore should be considered in priority [25].

However, it should be noted that the number of threaded rollers is limited by the installation space and the installation methods. Figure 9 demonstrates the PTRB with the most threaded rollers inserted. There are 11 threaded rollers in total which are uniformly distributed at the circumference of the inner ring, which results in an angle $\lambda$ in between each threaded roller pair.

The relationship between the maximum number of the threaded rollers and the PTRB structural parameters is expressed as

$$Z \leq \left\lfloor \frac{\pi}{2} \arcsin \left( \frac{d_{rac}}{D_{pw}} \right) + 1 \right\rfloor,$$  \hspace{1cm} (31)

where $d_{rac}$ is the diameter of the addendum circle in the threaded roller.

### 6. Conclusions

Load distribution within the PTRB has vital importance on its property analysis and structural optimization. The research in this paper mainly focuses on the load distribution of the PTRB used in the mountain-walking robotic platform.

1. An introduction was provided for the mechanical structures and the load transfer within the PTRB. A detailed analysis was performed regarding the contact patterns of both inner and outer ring-threaded roller pairs. A mathematical model was proposed on the basis of the elastic deformation and the force nonlinearity of each major load-carrying part, providing a support for the load distribution analysis.

2. The load distribution coefficients of the PTRB were calculated by using the compatibility equations and MATLAB. The load distribution analysis results indicated that the applied load had neglected effect on the load distribution coefficients, which indicated that a set of approximation values can be regarded as the coefficients for a PTRB with a fixed structural configuration. The axial load distribution coefficients showed different magnitudes and contradictory trends for the inner and outer ring-threaded roller pairs. The values on the radial direction were different but radially symmetrical at different locations of the threaded roller.

3. The uniformity of the load distribution on the axial and radial directions can be improved by adjusting
the distance between threads and increasing the number of threaded rollers, respectively. When the roller number was increased to 11, the maximum load coefficient decreased to 0.373.

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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