Surprises in relativistic matter in a magnetic field

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Abstract

A short review of recent advances in understanding the dynamics of relativistic matter in a magnetic field is presented. The emphasis is on the dynamics related to the generation of the chiral shift parameter in the normal ground state. We argue that the chiral shift parameter contributes to the axial current density, but does not modify the conventional axial anomaly relation. The analysis based on gauge invariant regularization schemes in the Nambu-Jona-Lasinio model suggests that these findings should be valid also in gauge theories. It is pointed out that the chiral shift parameter can affect observable properties of compact stars and modify the key features of the chiral magnetic effect in heavy ion collisions.

1 Introduction

Many recent theoretical studies [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14] of relativistic matter under extreme conditions, e.g., those realized inside compact stars and/or in heavy ion collisions, revealed that matter in a strong magnetic field may hold some new surprises. (For lattice studies of relativistic models in strong magnetic fields, see Refs. [15, 16, 17].) Here we discuss one of them, which was triggered by a finding that a topological contribution to the axial current is already induced at the lowest Landau level (LLL) in the free theory in a magnetic field [2]. This led us to propose that in the realistic case, with interactions, the ground state of such a matter is characterized by a chiral shift parameter $\Delta$ [4], which enters the effective Lagrangian density through the $\Delta \bar{\psi} \gamma^3 \gamma^5 \psi$ term and is generated at all Landau levels. The value of $\Delta$ determines a relative shift of the longitudinal momenta in the dispersion relations of opposite chirality fermions, $k^3 \rightarrow k^3 \pm \Delta$, where $k^3$ is the momentum along the magnetic field. This conclusion is approximately valid even in the case of massive particles, if the (ultra-)relativistic regime is realized. This is achieved, for example, in matter at a sufficiently high density (i.e., $\mu \gg m$, where $\mu$
is the chemical potential and $m$ is the mass of fermions), or a sufficiently high temperature ($T \gg m$) [13].

The chiral shift parameter is even under the parity transformation $\mathcal{P}$ and the charge conjugation $\mathcal{C}$, but breaks the time reversal $\mathcal{T}$ and the rotational symmetry $SO(3)$ down to $SO(2)$. Since the global symmetries of dense relativistic matter in an external magnetic field are exactly the same, the generation of the chiral shift parameter is expected even in perturbation theory [4].

2 Main results

Model. Here we briefly review the dynamics responsible for the generation of the chiral shift parameter. While we use a simple Nambu-Jona-Lasinio model (NJL), we also envision the generalization of the main results to gauge theories. Keeping this in mind, we will utilize a gauge-invariant regularization scheme in the analysis below.

The Lagrangian density of the model reads

$$\mathcal{L} = \bar{\psi} \left( iD_\nu + \mu_0 \delta^0_\nu \right) \gamma^\nu \psi - m_0 \bar{\psi} \psi + \frac{G_{\text{int}}}{2} \left[ (\bar{\psi} \gamma_0 \psi)^2 + (\bar{\psi} i \gamma_5 \psi)^2 \right],$$

(1)

where $m_0$ is the bare fermion mass, $\mu_0$ is the chemical potential, and $G_{\text{int}}$ is a dimensional coupling constant. By definition, $\gamma^5 \equiv i\gamma^0 \gamma^1 \gamma^2 \gamma^3$. The covariant derivative $D_\nu = \partial_\nu - ieA_\nu$ includes the external gauge field $A_\nu$.

The structure of the (inverse) fermion propagator is given by the following ansatz:

$$iG^{-1}(u, u') = \left[ (i\partial_t + \mu)\gamma^0 - (\pi_\perp \cdot \gamma) - \pi^3 \gamma^3 + i\tilde{\mu}\gamma^1 \gamma^2 + \Delta \gamma^3 \gamma^5 - m \right] \delta^4(u - u'),$$

(2)

where $u = (t, \mathbf{r})$ and the canonical momenta are $\pi^k_\perp = i\partial^k + eA^k$ (with $k = 1, 2$) and $\pi^3 = i\partial^3 = -i\partial_3$. While the spatial components of the gradient $\nabla$ are given by covariant components $\partial_k$, the spatial components of the vector potential $A$ are identified with the contravariant components $A^k$. We choose the vector potential in the Landau gauge, $A = (0, xB, 0)$, where $B$ is the strength of the magnetic field pointing in the $z$-direction.

In Eq. (2), in addition to the usual tree level terms, two new dynamical parameters ($\tilde{\mu}$ and $\Delta$) are included. From the Dirac structure, it should be clear that $\tilde{\mu}$ plays the role of an anomalous magnetic moment and $\Delta$ is the chiral shift parameter. In the mean-field approximation, there are no solutions with a nontrivial $\tilde{\mu}$ [13]. So, we take $\tilde{\mu} \equiv 0$ below. Note that in 2 + 1 dimensions (without $z$ coordinate), $\Delta \gamma^3 \gamma^5$ would be a mass term that is odd under time reversal. This mass is responsible for inducing the Chern-Simons term in the effective action for gauge fields [19], and it plays an important role in the quantum Hall effect in graphene [20].

Gap equation. In the mean-field approximation, the gap equation is equivalent to the following set of three equations [18, 21]:

$$\mu = \mu_0 - \frac{1}{2}G_{\text{int}} \langle j^0 \rangle, \quad m = m_0 - G_{\text{int}} \langle \bar{\psi} \psi \rangle, \quad \Delta = -\frac{1}{2}G_{\text{int}} \langle j^3_5 \rangle,$$

(3)

which are solved to determine the three dynamical parameters $\mu$, $m$, and $\Delta$. Here we will not discuss the vacuum solution, realized at small values of the chemical potential ($\mu_0 \lesssim m_{\text{dyn}}/\sqrt{2}$) as the result of the magnetic catalysis [22], but concentrate exclusively on the normal ground state with $\Delta \neq 0$, which occurs at nonzero fermion density.

Let us start by analyzing the equation for $\Delta$ in perturbation theory. In the zero order approximation, $\mu = \mu_0$ and $\Delta = 0$, while the fermion number density $\langle j^0 \rangle$ and the axial current density $\langle j^3_5 \rangle$ are nonzero. In particular, as discussed in Ref. [2], $\langle j^3_5 \rangle_0 = -eB\mu_0/(2\pi^2)$. (Our convention is such that the electric
charge of the electron is $-e$ where $e > 0$.) To the leading order in the coupling constant, one finds from Eq. (3) that $\Delta \propto G_{\text{int}}(\langle j^{3}_5 \rangle)_0 \neq 0$ and $\mu - \mu_0 \propto G_{\text{int}}(\langle j^{0}_5 \rangle)_0 \neq 0$. The latter implies that $\mu$ and $\mu_0$ are nonequal (in the model at hand, this is a consequence of the Hartree contribution to the gap equation). More importantly, we find that a nonzero $\Delta$ is induced. The same conclusion is also reached in a more careful analysis of the gap equations, utilizing a proper-time regularization [18]. This finding has interesting implications for theory and applications.

**Axial current density.** As pointed out in Refs. [1, 2], the structure of the topological axial current, induced at the LLL, is intimately connected with the axial anomaly [23]. Then, the important question is whether the form of the induced axial current $\langle j^{3}_5 \rangle$ coincides with the result in the theory of noninteracting fermions [1, 2, 5, 24], or whether it is affected by interactions.

We find that the dynamical generation of the chiral shift parameter $\Delta$ does modify the ground state expectation value of the axial current density [4, 18, 21]. The corresponding correction to the current density was calculated using several different regularization schemes (including the gauge invariant proper-time and point-splitting ones [18, 21]). It reads $\langle j^{3}_5 \rangle - \langle j^{3}_5 \rangle_0 \propto a\Lambda^2\Delta$, where $\Lambda$ is a cut-off parameter and $a$ is a dimensionless constant of order 1. Formally, this contribution to the axial current appears to be quadratically divergent when $\Lambda \to \infty$. However, it is finite because the solution for $\Delta$ itself is inversely proportional to $\Lambda^2$. After taking this into account, one finds that the axial current density is finite in the continuum limit. The same is expected in renormalizable gauge theories, in which $\Delta$ will be a running parameter that falls off quickly enough in ultraviolet to render a finite (or, perhaps, even vanishing) correction to the axial current.

**Axial anomaly relation.** The above result for the axial current density, which gets a correction due to the chiral shift parameter, brings up the question whether the conventional axial anomaly relation [23] is affected in any way. This issue was studied in Ref. [21], using a gauge invariant point-splitting regularization scheme, and it was found that the chiral shift parameter does not modify the axial anomaly. This is in agreement with the findings of Refs. [1, 2].

### 3 Applications

**Fermi surface.** The immediate implication of a nonzero chiral shift parameter in dense magnetized matter is the modification of the quasiparticle dispersion relations. These relations can be used to determine the Fermi surface in the space of the longitudinal momentum $k^3$ and the Landau index $n$. In relativistic dense matter ($\mu \gg m$), the corresponding states at the Fermi surface can be approximately characterized by their chiralities. Taking this into account, it is possible to define quasiparticles at the Fermi surface, which are predominantly left-handed or right-handed. Without loss of generality, let us assume that $s_\perp = \text{sgn}(eB) > 0$. Then, the Fermi surface for the *predominantly left-handed* particles is given by

\[ n = 0 : \quad k^3 = +\sqrt{(\mu - s_\perp \Delta)^2 - m^2}, \quad (4) \]
\[ n > 0 : \quad k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}, \quad (5) \]
\[ k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}, \quad (6) \]

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and the Fermi surface for the predominantly right-handed particles is

\begin{align}
  n = 0 : \quad & k^3 = -\sqrt{(\mu - s_\perp \Delta)^2 - m^2}, \\
  n > 0 : \quad & k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}, \\
  & k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}.
\end{align}

In the massless case, of course, this correspondence becomes exact. Then, we find that the Fermi surface for fermions of a given chirality is asymmetric in the direction of the magnetic field. In the left panel in Fig. 1, we show a schematic distribution of negatively charged fermions and take into account that the parameter \(s_\perp \Delta\) has the same sign as the chemical potential. (A similar distribution is also valid for positively charged fermions, but the left-handed and right-handed fermions will interchange their roles.) For the fermions of a given chirality, the LLL and the higher Landau levels give opposite contributions to the overall asymmetry of the Fermi surface. For example, the left-handed electrons in the LLL occupy only the states with positive longitudinal momenta (pointing in the magnetic field direction). The spins of the corresponding LLL electrons point against the magnetic field direction. In the higher Landau levels, while the left-handed electrons can have both positive and negative longitudinal momenta (as well as both spin projections), there are more states with negative momenta occupied, see Fig. 1. If there are many Landau levels occupied, which is the case when \(\mu \gg \sqrt{|eB|}\), the relative contribution of the LLL to the whole Fermi surface is small, and the overall asymmetry is dominated by higher Landau levels. In the opposite regime of super-strong magnetic field (if it can be realized in compact stars at all), only the LLL is occupied and, therefore, the overall asymmetry of the Fermi surface will be reversed. In the intermediate regime of a few Landau levels occupied, one should expect a crossover from one regime to the other, where the asymmetry goes through zero.

**Compact stars.** The asymmetry with respect to longitudinal momentum \(k^3\) of the opposite chirality fermions in the ground state of dense magnetized matter, discussed above, may have important physical consequences. For example, the fact that only the left-handed fermions participate in the weak interactions means that the neutrinos will scatter asymmetrically off the matter, in which the chiral shift parameter is nonvanishing.

By making use of this observation, a qualitatively new mechanism for the pulsar kicks [25] was proposed in Ref. [4]. It can be realized in almost any type of relativistic matter inside a protoneutron star (e.g., the electron plasma of the nuclear/hadronic matter, or the quark and electron plasma in the deconfined quark matter), in which a nonzero chiral shift parameter \(\Delta\) develops.

When trapped neutrinos gradually diffuse through the bulk of a protoneutron star, they can build up an asymmetric momentum distribution as a result of their multiple elastic scattering on the nonisotropic state of left-handed fermions (electrons or quarks). This is in contrast to the common dynamics of diffusion through an isotropic hot matter, which leads to a very efficient thermal isotropization and, therefore, a wash out of any original nonisotropic distribution of neutrinos [26, 27].

It appears also very helpful for the new pulsar kick mechanism that the chiral shift parameter is not much affected even by moderately high temperatures, \(10\,\text{MeV} \lesssim T \lesssim 50\,\text{MeV}\), present during the earliest stages of protoneutron stars [28]. Indeed, as our findings show, the value of \(\Delta\) is primarily determined by the chemical potential and has a weak/nonessential temperature dependence when \(\mu \gg T\). In the stellar context, this ensures the feasibility of the proposed mechanism even at the earliest stages of the protoneutron stars, when there is sufficient amount of thermal energy to power the strongest (with \(v \gtrsim 1000\,\text{km/s}\)) pulsar kicks observed [25]. Alternatively, the constraints of the energy conservation would make it hard, if not impossible, to explain any sizable pulsar kicks if the interior matter is cold (\(T \lesssim 1\,\text{MeV}\)).

Let us also mention that the robustness of the chiral shift in hot magnetized matter may be useful
to provide an additional neutrino push to facilitate successful supernova explosions as suggested in Ref. [29]. The specific details of such a scenario are yet to be worked out.

**Heavy Ion Collisions.** It is natural to ask whether the chiral shift parameter can have any interesting implications in the regime of relativistic heavy ion collisions, where sufficiently strong magnetic fields may exist [30]. The examples of the recently suggested phenomena, that appear to be closely related to the generation of the chiral shift, are the chiral magnetic effect [3, 5], the chiral magnetic spiral [8, 9, 10], and the chiral magnetic wave [31].

As was recently revealed in Ref. [18], at high temperatures, i.e., in the regime relevant for relativistic heavy ion collisions, the chiral shift parameter is not suppressed and generated for any nonzero chemical potential. However, its role is not as obvious as in the case of stellar matter. At high temperatures, the Fermi surface and the low-energy excitations in its vicinity are not very useful concepts any more. Instead, it is the axial current itself that is of interest. The chiral shift should induce a correction to the topological axial current, see the middle panel in Fig. 1. However, unlike the topological term, which is also proportional to the chemical potential, the dynamical one contains an extra factor of the coupling constant. Therefore, only at relatively strong coupling, which can be provided by QCD interactions, the effect of the chiral shift parameter on the axial current can be substantial.

Following the ideas similar to those that were used in the chiral magnetic effect [3, 5, 32], we have recently suggested [18] that the axial current by itself can play an important role in hot matter produced by heavy ion collisions and lead to a modified version of the chiral magnetic effect, which does not rely on the initial topological charge fluctuations [3]. An initial axial current generates an excess of opposite chiral charges around the polar regions of the fireball. Then, these chiral charges trigger two “usual” chiral magnetic effects with opposite directions of the vector currents at the opposite poles (see the right panel in Fig. 1). The inward flows of these electric currents will diffuse inside the fireball, while the outward flows will lead to a distinct observational signal: an excess of same sign charges going back-to-back. A numerical estimate of the modified chiral magnetic effect has been recently done in Ref. [33].

Concerning the regime of hot relativistic matter, let us also mention that it will be of interest to
extend our analysis of magnetized relativistic matter to address the properties of collective modes, similar to those presented in Ref. [31], by studying various current-current correlators.

4 Outlook

The present analysis was performed in the framework of the NJL model. It would be important to extend it to realistic field theories, QED and QCD. In connection with that, we would like to note the following. The expression for the chiral shift parameter, $\Delta \sim g \mu e B / \Lambda^2$, obtained in the NJL model implies that both fermion density and magnetic field are necessary for the generation of $\Delta$. This feature should also be valid in renormalizable theories. As for the cutoff $\Lambda$, it enters the results only because of the nonrenormalizability of the NJL model.

Similar studies of chiral symmetry breaking in the vacuum ($\mu_0 = 0$) QED and QCD in a magnetic field show that the cutoff scale $\Lambda$ is replaced by $\sqrt{|eB|}$ there [34]. Therefore, one might expect that in QED and QCD with both $\mu$ and $B$ being nonzero, $\Lambda$ will be replaced by a physical parameter, such as $\sqrt{|eB|}$. This in turn suggests that a constant chiral shift parameter $\Delta$ will become a running quantity that depends on the longitudinal momentum $k^3$ and the Landau level index $n$.

Another important feature that one could expect in QCD in a magnetic field is a topological contribution in the baryon charge [7] connected with collective massless fermion excitations in the phase with spontaneous chiral symmetry breaking. This feature could dramatically change the properties of that phase [5].

It is clear that dynamics in dense relativistic matter in a magnetic field is rich and sophisticated. In particular, one could expect surprises in studies of the phase diagram of QCD in a magnetic field [12, 13, 14, 16].

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