A Spatial Basis Coverage Approach For Uplink Training And Scheduling In Massive MIMO Systems

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Abstract

Massive multiple-input multiple-output (massive MIMO) can provide large spectral and energy efficiency gains. Nevertheless, its potential is conditioned on acquiring accurate channel state information (CSI). In time division duplexing (TDD) systems, CSI is obtained through uplink training which is hindered by pilot contamination. The impact of this phenomenon can be relieved using spatial division multiplexing, which refers to partitioning users based on their spatial information and processing their signals accordingly. The performance of such schemes depend primarily on the implemented grouping method. In this paper, we propose a novel spatial grouping scheme that aims at managing pilot contamination while reducing the required training overhead in TDD massive MIMO. Herein, user specific decoding matrices are derived based on the columns of the discrete Fourier transform matrix (DFT), taken as a spatial basis. Copilot user groups are then formed in order to obtain the best coverage of the spatial basis with minimum overlapping between decoding matrices. We provide two algorithms that achieve the desired grouping and derive their respective performance guarantees. We also address inter-cell copilot interference through efficient pilot sequence allocation, leveraging the formed copilot groups. Various numerical results are provided to showcase the efficiency of the proposed algorithms.

This research has been supported by the European project One5G.
I. INTRODUCTION

Wireless networks are under the strain of exponentially increasing demand for higher data rates. In order to provide the required performance leap, several new physical layer technologies can be relied upon. One of the most promising technologies is, without doubt, massive MIMO [1]. By leveraging a large number of antennas at the BSs, massive MIMO is able to provide considerable improvement in the network’s spectral and energy efficiencies with simplified transceiver design [2],[3]. This promoted massive MIMO to be a key enabler of future generation networks [4].

However, massive MIMO gains depend heavily on acquiring accurate CSI estimates at the BSs. In TDD systems, CSI estimation is performed through uplink (UL) training, leveraging channel reciprocity [5]. Owing to the limited coherence interval, the training resources are restricted and the same pilot sequences need to be reused resulting in pilot contamination [5]. Addressing this issue has lead to the development of numerous CSI estimation methods that exploit different channel statistics in order to mitigate copilot interference and enhance CSI accuracy. We refer to the survey [6] for more information. Several of these methods leverage spatial division multiplexing (SDM).

In [7], the authors proposed a coordinated approach to channel estimation. They proved that exploiting covariance information, under certain conditions, can lead to a complete removal of pilot contamination when the number of antennas grow very large. The authors showed that the channel estimation performance is a function of the degree to which the eigenspaces of the desired and interference signals overlap with each other. In [8], Joint spatial division and multiplexing (JSDM) for multi-user MIMO downlink (DL) was investigated. JSDM is a scheme that aims to serve users by clustering them such that users within a group have approximately similar channel covariances, while users across groups have near orthogonal covariance eigenspaces. JSDM has been designed, originally, for FDD MIMO systems without considering inter-cell interference. In [9], an adaptation of JSDM to the TDD case was proposed, taking into consideration inter-cell interference. In [9], density-based spatial clustering of applications with noise (DBSCAN) algorithm was used instead of $K$-mean in order to avoid the otherwise additional requirement of estimating the number of eigenspace-based clusters $K$. In [10], user grouping based on channel direction was proposed. The authors proposed to schedule copilot user such that their channels are semi-orthogonal. This results in enhancing CSI accuracy which, consequently, improve the achievable spectral efficiency (SE). In [11], a unified scheme for TDD/FDD massive MIMO
systems was proposed. Based on a spatial basis expansion model (SBEM), CSI estimation overhead was reduced and pilot contamination relieved. Each user is associated with a spatial signature that groups the index set of nonzero DFT points of its channel. Users are then grouped such that Users in the same group have non-overlapping spatial signatures. Each group is then allocated a pilot sequence and signals are discriminated based on the different spatial signatures.

Previous works on spatial multiplexing proposed to group users based on their channel co-variance eigenspace [8], [12], when the channel covariances are known, or simply based on their signals mean direction of arrivals [11]. The performance of such methods is determined by the user grouping scheme. In [8], a chordal distance based $K$-mean clustering was proposed for FDD systems with JSDM. In [12], the authors investigated a wide range of similarity measures such as weighted likelihood, subspace projection and Fubini-Study based similarity measures. The authors introduced two clustering methods namely, hierarchical and $K$-medoids clustering for user grouping with the aforementioned proximity measures. A comparison of the proposed grouping methods was performed and the combination that achieves the largest capacity was derived. In [11], a greedy user scheduling algorithm was implemented in order to partition users into copilot groups based on their spatial signatures. Although a considerable increase in SE was recorded, these methods come with a number of shortcomings. In fact, in order to achieve a good user clustering, the $K$-medoids and $K$-mean clustering methods require a prior estimation of the parameter $K$. In addition, these methods use an averaging in order to derive the group specific eigenspace matrix which can lead to a substantial overlapping between clusters. Practically, users might have similar but not necessarily identical second order channel statistics. This dictates the need to consider individual user spatial information. In addition, the efficiency of leveraging the totality of the available degrees of freedom (DoF) should be more emphasized.

In this work, we propose a novel spatial user grouping scheme. We consider a multi-cell TDD massive MIMO system, in which, spatial diversity is exploited in order to allow for a more pilot reuse, within each cell, while mitigating copilot interference. This allows to increase in the number of scheduled users for the same training overhead while improving SE. Since a multi-cell system is considered, both intra and inter-cell pilot contamination are tackled. We choose to decouple these two problems and address them successively. In order to deal with intra-cell interference, we propose a spatial grouping and scheduling scheme. We construct copilot groups based on the users spatial signatures. In each cell, any given copilot group is formed such that it contains users with minimum overlapping in their signals spatial signatures.
and that provide a maximum coverage of the signal spatial basis. The proposed approach is referred to as \textit{Spatial basis coverage copilot user selection}. The idea is to associate each user with a set of beams that concentrate a large amount of its channel power. Since uniform linear arrays (ULAs) are considered, the columns of a unitary DFT matrix are used as spatial basis \cite{8},\cite{11}. After obtaining the users specific decoding matrices, the BSs derive copilot groups. Each group provides a maximum coverage of all available independent DFT beams with minimum overlapping between users specific beam matrices. This approach enables also to couple the problems of user grouping and scheduling which reduces the complexity of the network management. We provide two formulation of the copilot grouping problem and we propose two grouping algorithms accordingly. First, the problem of copilot user selection is formulated as a \textit{maximum coverage problem} \cite{13}. This formulation enables to derive a low complexity algorithm that provides at least an \((1 - (\frac{1}{\tau} - 1))\cdot (1 - \frac{1}{e})\)-approximation of the optimal grouping. In the second case, the problem of copilot group generation is formulated as a \textit{Generalized maximum coverage problem} \cite{14}. We propose a low complexity algorithm that provides at least an \((1 - (\frac{1}{\tau} - 1))\cdot \frac{3}{4} - \frac{1}{e}\)-approximation of the optimal user grouping.

Based on the constructed copilot groups, we address inter-cell copilot interference through an efficient cross-cell pilot allocation. We propose a graphical framework based on the copilot groups spatial signature. Using this information, the network is able to allocate specific UL training sequences to copilot groups, such that the previously defined spatial receivers can manage cross-cell copilot interference. The resulting pilot allocation problem is formulated as a \textit{max-\tau-cut problem} \cite{15}, which enables to use a low complexity algorithm that provides a \((1 - \frac{1}{\tau})\)-approximation of the optimal solution.

\textbf{Notations:} We use boldface small letters (\(a\)) for vectors, boldface capital letters (\(A\)) for matrices. The notations \(A^\dagger\) and \(\text{Tr}(A)\) are used for Hermitian transpose and trace of matrix \(A\), respectively. \(|a|\) denotes the euclidean norm of vector \(a\). \(I_n\) is used to denote the \(n \times n\) identity matrix and \(|A|^2 = \text{Tr}(A^\dagger A)\) denotes the square of the Frobenius norm of \(A\).

\section{System Model and Preliminaries}

We consider a multi-cell, multi-user massive MIMO network operating in TDD mode. The system is composed of \(N_c\) cells containing, each, a BS that is equipped with a large \(M\)-element ULA. Each BS is serving \(K\) single omni-directional antenna users such that \(K \gg M\). Users
are randomly distributed in each cell. As introduced above, in this paper we focus on a TDD massive MIMO system, where the entire frequency band is used for DL and UL transmission by all BSs and users. Since a TDD system is considered, the focus will be on addressing the UL training bottleneck. Considering flat fading channels, the channel vector between user $i$ in cell $b$ and the BS of the $r^{th}$ cell, $\mathbf{g}_{ib}^{[r]}$ is composed of an arbitrary number of i.i.d. $P$ rays ($P >> 1$)[7]. Hence, the UL channel $\mathbf{g}_{ib}^{[r]}$ is given by the following multi-path model

$$\mathbf{g}_{ib}^{[r]} = \frac{1}{\sqrt{P}} \sum_{p=1}^{P} \mathbf{a}(\theta_{ib}^{[r,p]})\gamma_{ib}^{[r,p]},$$

(1)

Here, $\gamma_{ib}^{[r,p]}$ represents the complex gain of the $p^{th}$ ray from user $i$ in cell $b$ and the BS of the $r^{th}$ cell and follows a $\mathcal{CN}(0, \mu_{ib}^{[r]} )$ distribution where $\mu_{ib}^{[r]}$ denotes the average attenuation of the channel. $\theta_{ib}^{[r,p]}$ denotes the direction of arrival (DOA) of the $p^{th}$ ray from user $i$ in cell $b$ and the BS of the $r^{th}$ cell. Moreover, $\mathbf{a}(\theta_{ib}^{[r,p]}) \in \mathbb{C}^{M \times 1}$ is the array manifold vector which is given by:

$$\mathbf{a}(\theta_{ib}^{[r,p]}) = \left[ 1, e^{j2\pi d \sin(\theta_{ib}^{[r,p]} )}, \ldots, e^{j2\pi d \sin(\theta_{ib}^{[r,p]} ) (M-1)} \right],$$

(2)

where $\lambda$ denotes the signal wavelength, $d$ refers to the antenna spacing such that $d \leq \frac{\lambda}{2}$. As in [7] and [11], the incident angles of each user, with mean DOA $\theta_{ib}^{[r]}$, are considered to be restrained in a narrow angular range $[\theta_{ib}^{[r]} - \Delta \theta_{ib}^{[r]} , \theta_{ib}^{[r]} + \Delta \theta_{ib}^{[r]} ]$. Within this range $\mathbf{a}(\theta_{ib}^{[r,p]}), p = 1, \ldots, P, \forall i, b, r$ are mutually correlated. Consequently, the covariance matrix of each channel $\mathbf{g}_{ib}^{[r]}$, which is given by $\mathbf{R}_{ib}^{[r]} = \mathbb{E} \left[ \mathbf{g}_{ib}^{[r]} \mathbf{g}_{ib}^{[r]} \right]$, possesses a low-rank property.

The BSs acquire CSI estimates using orthonormal training sequences (i.e., pilot sequences) in the UL. For that, we consider a set of orthonormal training sequences, that is, sequences $q_i \in \mathbb{C}^{\tau \times 1}$ such that $q_i^\dagger q_j = \delta_{ij}$ (with $\delta_{ij}$ the Kronecker delta). In this paper, we consider an aggressive pilot reuse approach. In fact, in addition to reusing the same set of orthogonal pilot sequences in every cell, we consider that the same sequences are reused even within one cell. Consequently, the channel estimates are corrupted by both inter and intra-cell pilot contamination.

III. SPATIAL DIVISION MULTIPLEXING BASED USER SCHEDULING

A. Spatial basis in massive ULAs

Massive MIMO systems provide a substantial SE gain by spatially multiplexing a large number of mobile devices. This greater number of served devices requires higher signaling
or feedback overhead in order to obtain CSI estimates. This issue promoted many research work which resulted in designing new transmission strategies for massive MIMO leveraging low-rank approximation of the channel covariance matrix [7], [8], [12]. Indeed, based on the fact that the incident signals at the BSs are characterized by narrow angular spread, it was proven that the effective channel dimension can be reduced with negligible capacity loss. Exploiting the channel low-rank property in massive MIMO transmission strategies proved to provide non-negligible gains in performance for both TDD and FDD systems [8], [11], [20]. In FDD mode, spatial division multiplexing allows to reduce the CSI feedback overhead while incurring no capacity loss [8]. In TDD, such methods enable to reduce training overhead while mitigating the impact of pilot contamination [11]. These gains are mainly due to the capacity of spatial division methods to utilizes the independent spatial spaces of different users in order to discriminate between their signals. Spatial division based methods rely on efficient spatial information-based user grouping. Several works proposed to perform this grouping using the $K$-mean algorithm with different proximity measures [8], [12]. $K$-medoids and hierarchical clustering have also been proposed [12]. A DFT-based greedy user grouping was also considered in SBEM [11].

In this paper, we provide an alternative low-rank CSI estimation approach, leveraging the characteristics of ULAs. Indeed, in this case, it was proven that a unitary DFT matrix constitute a good spatial basis for the signal [11], [21]. This means that, without accurate estimates of the channel covariance matrices, spatial division multiplexing can be implemented using a unitary DFT matrix. For each user, it is sufficient to derive decoding matrices based on the DFT matrix vectors that span the majority of its channel power [11]. The proposed spatial grouping approach differs from previously propose ones, [8], [11], [12], as it takes into consideration the efficiency of the spatial space coverage. The main idea is to group users according to their spatial signatures and allocate pilot sequence such that copilot users are spatially separated (i.e., their spatial signatures span independent subspaces). Copilot user grouping is performed based on two criterion. First, users are grouped such that their spatial signatures’ overlap is minimal. Second, the users in each group provide maximum coverage of all DFT vectors. It is worth mentioning that the latter criterion was arguably under-leveraged in previous work.

B. Obtaining spatial information

Low-rank CSI estimation methods rely on different knowledge levels of the channel statistics. In our case, the proposed method requires the knowledge of each user spatial signature (i.e. the
main DoAs of each user signal). This information needs to be estimated. Owing to the slow varying nature of spatial information, an efficient estimation can be achieved with low signaling overhead. Indeed, the spatial signatures can be obtained using an UL preamble or DL training followed by feedback. We shed some light on how spatial signatures are obtained.

UL preamble[11] can be used in order to obtain the users spatial signatures. In this case, we need to include an additional UL spatial training period. This should require a minimum of $K$ training resource elements. During the UL preamble, the BSs receive the spatial training signal and derives the DFT-based decoding matrix for each user. In this work we choose to include a design parameter $0 < \alpha < 1$ in order to quantify the importance of each DoA for each channel. Particularly, for each user $i,b$ the DFT-based decoding matrix $F_{ib}^{[b]}$ is obtained as follows

$$F_{ib}^{[b]} = \{ f_s \in F, \| g_{ib}^{[b]} f_s \|^2 \geq \alpha \},$$

(3)

$F_{ib}^{[b]}$ will be used as the bases in which the user’s signal is detected and will henceforth be refereed to as spatial signature of user $ib$.

The spatial signature of each user can be also inferred from DL spatial training followed by feedback [10]. In this case, users quantify the power of their channel along each direction and feedback the indexes of the chosen DFT vectors according to (3). The spatial signature of each user forms a subspace that concentrate a large percentage of its channel power. Consequently, allocating the same copilot sequence to users with minimum overlapping in their spatial signature (3) enables to discriminate between their signals since the power of their channels is concentrated in different subspaces. In order to gain more insight on the needed criterion to achieve efficient spatial grouping, we analyze the power of the desired and interference signals, when the considered DFT-based decoding matrices are used.

C. Achievable Performance With Adaptive Spatial Division Based User Scheduling

For every user $i,b$, we consider the matrix $F_{ib}$ formed by the vectors of the DFT matrix according to (3). During the UL training period, active users send their pilot sequences. For analytical simplicity, we consider that UL training sequences have the same reuse factor within all cells. We consider that the network schedules $N_p$ users to use any given pilot sequences in
each cell. During UL training, the received pilot signal \( Y_{p}^{[b]} \) at BS \( b \) is given by:

\[
Y_{p}^{[b]} = \sqrt{\rho_p} \sum_{r=1}^{N_c} \sum_{l=1}^{\tau} \sum_{i \in \Sigma(l,r)} g_{ir}^{[b]} q_l^\dagger + W_{p},
\]

where \( W_p \in \mathbb{C}^{M \times 1} \) refers to an additive white Gaussian noise matrix with i.i.d. \( \mathcal{CN}(0,1) \) entries and \( \rho_p \) denote the pilot transmit power. \( \Sigma(l,r) \) denotes the set of user in cell \( r \) that are using pilot sequence \( l \) during UL training. The \( b^{th} \) BS then uses the orthogonality of the training sequences in order to obtain the Least square (LS) estimate of the channel of user \( i, b \). The BS estimates the channel of each user \( i, b \) after projecting the received signal on \( F_{ib}^{[b]} \) as

\[
\tilde{g}_{ib}^{[b,ib]} = F_{ib}^{[b]} \left( \frac{Y_{p}^{[b]} q_{\chi(i,b)}}{\sqrt{\rho_p}} \right),
\]

where \( \chi(i, b) \) denote the index of the training sequence used by user \( i, b \). Note that indexes of the projection matrix \( F_{ib}^{[b]} \) are added to the channel estimate since its distribution depends on user \( i, b \) spatial signature. During UL data transmission, BS \( b \) receives the following data signal

\[
Y_{u}^{[b]} = \sqrt{\rho_u} \sum_{r=1}^{N_c} \sum_{l=1}^{\tau} \sum_{i \in \Sigma(l,r)} g_{ir}^{[b]} d_{ir} + w_u,
\]

where \( w_u \in \mathbb{C}^{M \times 1} \) refers to an additive white Gaussian noise vector with i.i.d. \( \mathcal{CN}(0,1) \) entries and \( \rho_u \) denotes the UL data transmission power. We consider linear detection where, the signal of each user \( i, b \) is estimated using a precoded maximum ratio combining receiver. In order to detect the signal of user \( i, b \), BS \( b \) uses \( F_{ib}^{[b]} || g_{ib}^{[b,ib]} || \) as receive filter. The estimate of the signal of user \( i, b \) can be decomposed as follows:

\[
\hat{d}_{ib} = \frac{g_{ib}^{[b,ib]} || g_{ib}^{[b,ib]} ||}{DS_{ib}^{[b]}} d_{ib} + \sum_{r=1}^{N_c} \sum_{u \in \Sigma(\chi(i,b),r)} g_{ir}^{[b,ib]} g_{ur}^{[b,ib]} d_{ur} + \frac{\sum_{r=1}^{N_c} \sum_{u \in \Sigma(\chi(i,b),r)} g_{ir}^{[b,ib]} g_{ur}^{[b,ib]} \sqrt{\rho_u}}{I_{ib}^{[b,C]}} w_u + \sum_{r=1}^{N_c} \sum_{u \in \Sigma(\chi(i,b),r)} g_{ir}^{[b,ib]} g_{ur}^{[b,ib]} d_{ur},
\]

where \( g_{ib}^{[b,ib]} \) represents the channel estimation error \( \forall i, b \) which is defined as \( g_{ib}^{[b,ib]} = g_{ib}^{[b,ib]} + \tilde{g}_{ib}^{[b,ib]} \). Here \( DS_{ib}^{[b]} \), \( I_{ib}^{[b,C]} \), \( N_{ib}^{[b]} \) and \( I_{ib}^{[b,NC]} \) refer to the desired signal, the impact of copilot user interference, additive noise and non-coherent interference, respectively. As we can see in (7), the power of copilot interference depends on to which degree the spatial signatures of
copilot users overlap. Although increasing the pilot reuse factor increases the number of active users, intra-cell copilot interference can be completely mitigated if the same pilot sequence is allocated to users with non-overlapping spatial signatures. Scheduling an excess of users with non-overlapping spatial signatures implies that all the system DoFs need to be exploited when possible. Consequently, selecting copilot users, in each cell, is then of paramount importance. In the next section, we address the problem of intra-cell copilot interference through appropriate spatial signature-based user grouping.

**IV. AN ALTERNATIVE APPROACH TO SPATIAL USER GROUPING: A SPATIAL BASIS COVERAGE PROBLEM**

Although previously proposed grouping approaches get the work done and provide considerable performance increase for both FDD and TDD modes, they suffer, nevertheless, from a range of shortcomings that may limit the potential of spatial division multiplexing. Indeed, maximum coverage of all the available DoFs should be emphasized. When applied to the spatial division problem, classical clustering approaches concentrate on the mutual distance between user channels subspaces with little regard to the final coverage of independent streams. This means that, although the condition of independent spatial information is met, the DoFs that the massive MIMO system provides can be underexploited. Addressing these shortcomings can help boost the performance of spatial division multiplexing methods.

In this paper, we propose a method that, in addition to the requirement of independent spatial subspaces, emphasizes on leveraging all DoFs of massive MIMO system. The same pilot sequence is allocated to users with minimum spatial signature overlapping which constitute a copilot group within each cell. In addition, the users within each copilot group achieve a maximum coverage of all interdependent DoAs. Consequently, in each cell, a total of \( \tau \) copilot groups need to be constructed, each of which is associated with a distinct training sequence. In what follows, we formulate the spatial basis coverage copilot user selection problem. We then provide efficient algorithms that enables to solve it. Two formulations are considered.

The first approach is power agnostic. This means that it neglects the channel power and concentrates only on minimum spatial signature overlapping and maximum coverage without discriminating users. The second approach is power aware. This means that users are prioritized based on their achievable channel gains in addition to the criterion considered in the power agnos-
tic approach. The differences between the two approaches lie mainly in complexity, achievable SE gains and fairness. These differences will be discussed in more details further in this paper.

![Figure 1: Example of spatial basis coverage for $M = 100$](image)

A. Power Agnostic Spatial Basis Coverage

In this subsection, we focus on solving the spatial basis coverage problem in the power agnostic approach. In this case, the BSs know only the set of DFT beams that concentrate a large percentage of channel power (i.e., $F_b^{[i]}, \forall i, b$). The user-beam association is performed as specified in the previous section thanks to DL training (3) [10] or using UL preamble [11]. As previously discussed, the BSs perform spatial basis coverage based copilot user selection in order to schedule users for UL training. This is done in order to deal with intra-cell copilot interference. The out of cell copilot interference will be addressed later in this paper. In the power agnostic case, the BSs do not take into consideration the achievable gain along each beam. Consequently, in this case, the problem reduces to scheduling users with minimum spatial signature overlapping and maximum coverage of the DFT beams. This actually simplifies the problem at hand and enables to derive the desired grouping with low complexity. The power agnostic approach is also characterized by the upside of fairness since it does not discriminate scheduled users based on their channel gain. However, this means that more flexibility should be allowed when constructing copilot users. In fact, since we cannot prioritize users based on their channel gains, it may be wise to allow for some spatial overlapping. We also allow for another degree of flexibility in this problem, namely, pilot reuse in each cell. In fact, in this work, we
consider that the reuse factor of each pilot sequence can vary from one cell to the other. This implemented in order to allow for a more flexible specific training sequences allocation to the copilot groups when dealing with inter-cell copilot interference. The main principle of the spatial basis coverage problem is depicted in Figure (1).

We consider \( \tau \) copilot groups (covers) per cell \( C^{[b]}_k, k = 1, \ldots, \tau, b = 1, \ldots, N_c \). Each copilot group in each cell will be associated with a distinct pilot sequence. We start by defining \( x^{[k]}_{i,b}, \forall i, b, k \) and \( y^{[k]}_{s,b}, \forall s = 1, \ldots, M, b = 1, \ldots, N_c, k = 1, \ldots, \tau \), which are given by

\[
x^{[k]}_{i,b} = \begin{cases} 
1 & \text{if user } i, b \text{ is selected in copilot group } C^{[b]}_k. \\
0 & \text{otherwise.}
\end{cases}
\]

\[
y^{[k]}_{s,b} = \begin{cases} 
1 & \text{if beam } f_s \text{ is covered in cell } b \text{ and copilot group } C^{[b]}_k. \\
0 & \text{otherwise.}
\end{cases}
\]

Formally, under the power agnostic approach, the spatial basis coverage based copilot UE selection problem can be formulated as follows:

\[
\max_y \sum_{k=1}^{\tau} \sum_{b=1}^{C_c} \sum_{s=1}^{M} y^{[k]}_{s,b}
\]

subject to

\[
\sum_{i} x^{[k]}_{i,b} \leq U^{[k]}_b \quad \forall k = 1, \ldots, \tau, \forall b = 1, \ldots, N_c
\]

\[
\sum_{i, f_s \in F_{ib}} x^{[k]}_{i,b} \geq y^{[k]}_{s,b} \quad \forall k = 1, \ldots, \tau, \forall b = 1, \ldots, N_c
\]

\((10a)\) guarantees that the number of users in a given copilot group \( C^{[b]}_k, k = 1, \ldots, \tau, b = 1, \ldots, N_c \), is upper bounded by \( U^{[k]}_b \). \( U^{[k]}_b \) is a design parameter that defines the reuse factor of a given pilot sequence in each cell. Depending on the considered setting, \( U^{[k]}_b \) can be the same or differs from one cell to the other. \((10b)\) guarantees that, for any covered beam \( f_s \) in cell \( b \), in copilot group \( C^{[b]}_k \), at least one user \( i, b \) with \( f_s \in F^{[b]}_ib \) is scheduled for UL training in copilot group \( C^{[b]}_k \). We start by showing the computational intractability of problem \((9)\).

**Lemma 1.** The spatial basis coverage based copilot UE selection problem \((9)\) is NP-hard.

**Proof.** For \( C = 1 \) and \( \tau = 1 \), \((9)\) is equivalent to a **maximum coverage problem** which is known to be NP-hard [14]. Consequently, \((9)\) is also NP-hard. \(\square\)
In order to obtain an efficient suboptimal solution of (9), we use two nested greedy phases. In the upper phase, the algorithm produces $\tau$ maximum coverages of the DFT matrix vectors ($F$), in each cell. The maximum covers $C^{[b]}_k, k = 1..\tau, b = 1,..,N_c$, are computed successively in a greedy manner. Each of the maximum covers is computed using another greedy method that goes as follows. For each $C^{[b]}_k, k = 1..\tau, b = 1,..,N_c$, the set of uncovered beams is initialized as the vectors of the DFT matrix. Then users are added to $C^{[b]}_k$ successively while selecting, at each iteration, the user with the spatial signature that cover a maximum of the uncovered DFT columns. This procedure is repeated until attaining the reuse constraint $U^{[k]}_b$ for each copilot group, in each cell. Different from previously proposed algorithms, the present approach enables to satisfy the spatial independence requirements within each copilot group while offering a maximum utilization of the excess of DoFs. The detailed algorithm is given in table I. We denote by $\Gamma(b)$ the set of users in cell $b = 1,\ldots,N_c$.

| Initialize: Copilot groups sets $C^{[b]}_k = \emptyset, k = 1,\ldots,\tau, b = 1,\ldots,N_c$ |
| User specific beam matrices $F_{ib}, \forall i \in \Gamma(b), b = 1,\ldots,N_c$ |
| 1. For $b = 1 : N_c$ do: |
| 2. For $k = 1 : \tau$ do: |
| 3. Define the set $Un = F$ as the set of uncovered beams. |
| 4. For $j = 1 : U^{[k]}_b$ do: |
| 5. $i^* \leftarrow \arg \max_{i \in \Gamma(b)} |F_{ib} \cap Un|$ |
| 6. $Un \leftarrow Un \setminus \{F_{i^*b} \cap Un\}$ |
| 7. $C^{[b]}_k \leftarrow C^{[b]}_k \cup i^*$ |
| 8. End for |
| 9. End for |
| 10. End for |

Table I: Power Agnostic Spatial Basis coverage based copilot UE selection

The algorithm in table I produces $\tau$ copilot user groups in each cell. Each copilot group maximizes the coverage of the DFT vectors while minimizing the overlapping between copilot users spatial signatures. Consequently, the CSI estimation of each user can be enhanced by a simple linear projection. Note that the proposed algorithm allows for some subspace overlapping. This is actually needed since users are not prioritized based on their channel gains. We now
proceed by deriving the performance guarantee of the proposed algorithm.

**Theorem 1.** The algorithm in table I provides an \((1 - (\frac{\tau - 1}{\tau})\tau)(1 - \frac{1}{e})\)-approximation of the optimal solution of problem (9).

**Proof.** See VIII. \(\square\)

**B. Power Aware Spatial Basis Coverage**

In the power aware approach, users are prioritized based on the power of their signals along each direction. The resulting problem provides a more efficient grouping since it takes into consideration the overlapping between copilot users spatial signatures, the coverage of the signal space and the power of each user channel. Such grouping provides a higher SE gain. However, this improvement comes at the price of augmented complexity since users are discriminated based on their channel gains. In this case, we define a different value for each beam depending on which user is covering it. For each user \(i, b\), the value associated with beam \(f_s, s = 1, \ldots, M\) is given by \(\zeta_{ib}^s\), where \(\zeta_{ib}^s\) is the power of user \(i, b\) channel along \(f_s, s = 1, \ldots, M\). This consideration changes the formulation of the spatial basis coverage based copilot UE selection problem (9). The main idea of providing maximum coverage of the DFT beams, in each cell and for each pilot sequence, still holds but the actual gain associated with each beam will also be taken into consideration. The resulting combinatorial optimization problem can be formulated as follows

\[
\max_y \sum_{k=1}^{\tau} \sum_{b=1}^{N_c} \sum_{i \in \Gamma(b)} \sum_{f_s \in \mathcal{F}} \zeta_{ib}^s y_{i,b}^{[s,k]} \quad (10)
\]

subject to

\[
\sum_{i \in \Gamma(b), f_s \in \mathcal{F}_{ib}} y_{i,b}^{[s,k]} \leq 1 \quad \forall k = 1, \ldots, \tau, \quad \forall b = 1, \ldots, N_c \quad (11a)
\]

\[
\sum_{i \in \Gamma(b), f_s \in \mathcal{F}_{ib}} x_{i,b}^{[k]} \geq y_{i,b}^{[s,k]} \quad \forall k = 1, \ldots, \tau, \quad \forall b = 1, \ldots, N_c \quad (11b)
\]

\[
\sum_{i \in \Gamma(b)} x_{i,b}^{[k]} \leq U_{ib}^{[k]} \quad \forall k = 1, \ldots, \tau, \quad \forall b = 1, \ldots, N_c \quad (11c)
\]

The constraints, in (11a), guarantees that each beam is covered by at most one user. (11b) guarantees that, for any covered beam \(f_s\) in cell \(b\), in copilot group \(\mathcal{C}_k^{[b]}\), at least one user \(i, b\) with \(f_s \in \mathcal{F}_{ib}\) is scheduled for UL training in copilot group \(\mathcal{C}_k^{[b]}\). (11c) guarantees that the total number of the users associated with a given pilot sequence in a given cell is bounded. The difference between (9) and (10) is mainly the fact that the actual gain along each DFT beam is
taken into consideration. This means that, the BS can optimize its pilot allocation accordingly with the final aim of maximizing the total weight of the covered DFT beams. This means that, in addition to reducing copilot interference thanks to the non-overlapping spatial signatures, the users are selected such that the achievable gain along all the available independent beams is maximized. We start by showing the computational intractability of problem (10).

Lemma 2. The spatial basis coverage based copilot UE selection problem (10) is NP-hard.

Proof. For \( C = 1 \) and \( \tau = 1 \), the optimization problem (10) is equivalent to a Generalized Maximum Coverage Problem (GMC) which is known to be NP-hard. Consequently, (10) is also NP-hard.

The proof of computational intractability provides us with insight on how to tackle problem (10) efficiently. Indeed, in order to obtain an efficient suboptimal solution for (10), we adopt a successive coverage approach as the previous algorithm. The difference here comes in the construction of each copilot group where the actual power along each beam needs to be considered. To this end, \( \forall k = 1..\tau, b = 1,..,N_c \), a GMC problem is solved. The proposed approach is based on a modification of the coverage algorithm in [14]. While, in [14], a GMC is solved leveraging a greedy procedure and Dynamic programming for Knapsack problems, we propose a two nested greedy phases in addition to replacing Dynamic programming with the greedy algorithm for Knapsack problems. Replacing Dynamic programming by the greedy algorithm for Knapsack problems results in reducing the complexity of practical implementation.

The present approach enables to satisfy the spatial independence requirements within each copilot group while offering a maximum utilization of the excess of DoFs. Since it takes into consideration the actual power of users signals, the present approach enables also to discriminate between users based on their channel gain in each direction. This, ultimately, results in more efficient utilization of the system’s DoFs by prioritizing users with high signal power in each direction.

Before providing the detailed algorithm that addresses (10), some definitions are now in order. We define a user allocation \( A \) as a triple \( A = (\phi, \xi, h) \), where \( \phi \) represents the set of selected users, \( \xi \) denotes the set of corresponding covered beams and \( h \) is an assignment from \( \xi \) to \( \phi \) such that \( \forall f_s \in \xi, h(f_s) \) denotes the user covering beam \( f_s \). For a given allocation \( A \), we define
\[ V(A) = \sum_{f_s \in \xi} c_{h(f_s), b} \] as the value of \( A \) and \( W(A) = |\phi| \) as its weight. We also define the residual value of \(((i, b), f_s)\) with respect to \( A \) as follows

\[
V_A((i, b), f_s) = \begin{cases} 
  c_{i, b}^{[s]} & \text{if } f_s \text{ is not covered by } A. \\
  c_{i, b}^{[s]} - \zeta_{h(f_s), b}^{[s]} & \text{otherwise.}
\end{cases}
\]  

(11)

The detailed algorithm is presented in tables II and III.

| 1. For \( b = 1 : N_c \) do: |
|-----------------------------|
| 2. For \( k = 1 : \tau \) do: |
| 3. \( A_g \leftarrow \text{Greedy}(S = \{i, i \in \Gamma(b)\}, b, k) \) |
| 4. Find a single user \( i^*, b \) with the highest power along its covered DFT vectors. |
| 5. \( V(A_s) \leftarrow \sum_{f_s, f_s \in F_i, b} c_{i, b}^{[s]} \) |
| 6. If \( V(A_g) \geq V(A_s) \): |
| 7. \( C_k^{[b]} \leftarrow (A_g) \) |
| 8. Else |
| 9. \( C_k^{[b]} \leftarrow (A_s) \) |
| 10. End for |
| 11. End for |

Table II: Power Aware Spatial Basis coverage based copilot UE selection
1. $j \leftarrow 0$, $A = \emptyset$

2. **While** new DFT vectors with positive residual value can be added to $A$ without violating the cardinality constraint $U_j^k$ ($W(A) \leq U_j^k$)**do:**

3. Use the greedy algorithm for Knapsack problems in order to select a user $i^*$ and a subset of DFT vectors from $F_{i^*,r}$ which has the maximum density (Each DFT vector is given a weight of 1 if it is not covered in $A$).

4. $A \leftarrow A \oplus ([i^*, r], F_{i^*,r})$

5. **For** $u \not\in A$ **do:**

6. **If** $W(A \oplus ([u, r], F_{ur})) \leq U_j^k$ **and** $\forall f \in F_{ur}, V_{A\oplus([u,r],F_{ur})}([u, r], f) > 0$ **do:**

7. $A \leftarrow A \oplus ([u, r], F_{ur})$

8. **End for**

9. $j \leftarrow j + 1$

10. **End While**

11. **Return**($A$)

---

**Table III: Greedy(S,r,k)**

The algorithm consists of solving a generalized maximum coverage problem for each copilot group $C_{k}^{[b]}$, $k = 1, \ldots, \tau$, $b = 1, \ldots, N_c$, successively. The main idea of the algorithm is to use two nested greedy phases. In the upper phase, the maximum coverages $C_{k}^{[b]}$, $k = 1, \tau$, $b = 1, \ldots, N_c$ are computed successively in a greedy manner. In order to obtain each coverage, in the lower phase, the algorithm uses the residual value (11) in a greedy procedure so as to choose a subset of DFT beams that are part of a given user spatial signature with the highest density. Users are then added to the selection as long as the residual value of their associated DFT vectors is positive and the pilot reuse constraint is not violated. When the greedy phase ends, its resulting selection is compared with the highest value that can result from selecting a single user. The selection with the best coverage is then returned. This procedure is repeated in the each cell $b = 1, \ldots, N_c$, for each copilot group $k = 1, \tau$. We now derive the performance guarantee of the proposed algorithm.

**Theorem 2.** The proposed algorithm in tables II and III provides an $(1 - (\frac{\tau - 1}{\tau})^\tau) \frac{2 - e^{-2}}{1-e^{-2}}$-approximation of the optimal solution of problem (10).
The proposed algorithm in Table II provides \( \tau \) covers of the DFT matrix beams in each cell. This results in \( \tau \) copilot user groups, in each cell, that fully exploit all available DoFs with minimum overlapping between the beam sets of each user. This leads to an efficient reduction of intra-cell copilot interference. The main differences between the power agnostic and aware cases are performance guarantees and complexity. Indeed, the simplified power agnostic case enables to achieve the desired grouping with good performance guarantee (see Theorem 1) and low complexity. The power aware case provides a more efficient grouping since it takes into consideration the users channel gains. Nevertheless, this comes with a penalty of a higher computational complexity. Constructing the copilot user groups enables to efficiently address the intra-cell interference. Nevertheless, further SE gain can be achieved by addressing the issue of inter-cell copilot interference. This will be the focus of the next section.

V. CROSS CELL PILOT ALLOCATION: A GRAPHICAL APPROACH

A. Cross Cell Pilot allocation problem

In this paper, we focus on mitigating copilot interference. Constructing copilot groups across cells, in order to address the problems of intra-cell and inter-cell interference simultaneously, proves to be quite complex. This is due to the complexity of defining a proper grouping metric that is based on the spatial signatures of both useful and interference links. This fact motivated the present approach of dealing with interference through two consecutive subproblems addressing intra and inter-cell copilot interference, respectively. A major advantage of such division is the reduction of complexity. Indeed, since copilot user groups have already been constructed in each cell, addressing out-of-cell copilot interference reduces to allocating specific training sequences to copilot groups. If copilot interference is to be addressed from the beginning as a whole, it will result in a complex pilot allocation problem among all users in all cells which proves to be complicated. Practically, complete removal of interference is not physically possible. In addition, copilot user grouping was performed with the clear goal of managing intra-cell interference. Consequently, when dealing with inter-cell copilot interference, previously formed copilot groups should be maintained. We propose a scheme in which pilot allocation is done such that high interference links are suppressed when spatial signature based receivers are used. In this section, we address this problem using an intelligent pilot assignment scheme. The basic idea is to infer
inter-cell copilot interference from the spatial signatures of interference links. A training phase to obtain spatial information of the interference links is required. This can be implemented without a large signaling overhead owing to the slow changing spatial information. We now consider that, each user $i, b, i = 1...K, b = 1...N_c$ is associated with $N_c$ matrices $\{F_{ib}^{[r]}, r = 1...N_c\}$. Each matrix $F_{ib}^{[r]}$ is constructed in a similar manner to (3) as follows

$$F_{ib}^{[r]} = \{ f_s \in F, \frac{\|g_{ib}^{[r]} f_s\|^2}{Tr(R_{ib}^{[r]})} \geq \alpha \},$$

(12)

B. Graphical Modeling and proposed solution

The first step to manage inter-cell copilot interference is to construct an interference graph that corresponds to the considered system setting. We construct an undirected interference graph $G(C, E)$. Each node $C_{k}^{[b]} \in C, k = 1...\tau, b = 1...N_c$ represents copilot user group of index $k$ in a give cell $b$. Each edge in $e_{C_{k}^{[j]} , C_{l}^{[k]}} \in E$ represents an interference link and is associated with a given weight $w_{C_{k}^{[j]} , C_{l}^{[k]}}$. We propose a method for determining the edge weight without accurate SINR measurements since it cannot be obtained before pilot allocation. In this work, we propose to infer interference levels form spatial information. This consideration is due to the practical low signaling overhead that is required.

Since the weight of each edge quantifies the level of interference between two copilot groups, an appropriate measure needs to be considered. This task is not an easy one since the weight of the edge between two different copilot groups should properly characterize the levels of resulting interference. The research papers that investigated spatial division multiplexing proposed different metrics to characterize subspace distances. The most used one is chordal distance. Such metric has the considerable downside of neglecting the actual signal power in each subspace. In addition, the present framework implies that the weight of each edge need to characterize the mutual interference between two groups of users. Consequently, defining a distance measure between copilot groups is a major issue in our case. In order to solve this issue, we call upon hierarchical clustering where measuring distances between groups is commonly encountered. We adopt a linkage method in hierarchical clustering [12], namely weighted average linkage. To quantify interference on each link, we use spatial signature overlapping between the users forming the
two copilot groups which is obtained using the chordal distance between spatial signatures. The weight of each edge $w_{C_b^j, C_l^k}$ is then given by

$$w_{C_b^j, C_l^k} = \min_{y \in C_b^j, z \in C_l^k} \left\{ \frac{1}{2} \| F_{y_b} F_{y_b}^\dagger - F_{z_l} F_{z_l}^\dagger \|^2_F + \frac{1}{2} \| F_{y_b}^\dagger F_{y_b} - F_{z_l}^\dagger F_{z_l} \|^2_F \right\}, \quad (13)$$

Here $\| F_{y_b} F_{y_b}^\dagger - F_{z_l} F_{z_l}^\dagger \|^2_F$ represents the chordal distance between the spatial signatures of the useful signal of user $y \in C_b^j$ and the interference generated by $z \in C_l^k$. $\| F_{y_b}^\dagger F_{y_b} - F_{z_l}^\dagger F_{z_l} \|^2_F$ denotes the chordal distance between the spatial signatures of the useful signal of user $z \in C_l^k$ and the interference generated by $y \in C_b^j$.

The weight expression (13) captures the minimum chordal distance between the spatial signatures of the interference and useful signals for all users in the two copilot groups and is inspired by the single and weighted average linkage, commonly used in hierarchical clustering [12]. In each cell, users from the same copilot group are the only devices allowed to transmit the same UL training sequence. Consequently, during pilot allocation, we need to make sure that any given pilot sequence $q_l, l = 1 \ldots \tau$ should be allocated to only one copilot group in each cell. In order to do so, the weight of the links between copilot groups from the same cell will be given a very large value $w_{\infty}$, because intra-cell interference between copilot groups must be avoided. Using this metric we are able to construct the interference graph $G(C, E)$, which is a first step in the proposed pilot allocation scheme. An illustration of $G$ is presented in figure 2 for the case of $N_c = 2$ and $\tau = 2$.

The considered problem is closely related to MAX-CUT problem [15]. Indeed, the task of interference management reduces to suppressing high pilot contamination between copilot groups. This can be performed by allocating the same training sequence to copilot groups with minimum mutual interference weights. In the considered graphical framework, this task is equivalent to partitioning the interference graph into $\tau$ subgraphs where the copilot groups in each subgraph will be allocated the same training sequence. In the graph theory, a cut is a partition of the vertices of the graph into multiple sets. In weighted graphs, the size of the cut is the sum of weights of the edges that cross the cut. A cut is said to be maximal if the size of the cut is not smaller than the size of any other cut. By generalizing this notion to $\tau$ cuts, the MAX-$\tau$-CUT problem is to find a set of $\tau$ cuts that are not smaller in size than any other $\tau$ cuts. Consequently, pilot allocation is equivalent to a MAX-$\tau$-CUT problem on the interference graph and can be stated as follows:

**Pilot sequence allocation problem:** Given the interference graph $G(C, E)$ with $\tau \times N_c$ nodes
and edge weight \( w_{C_i^{[j]}, C_i^{[k]}} \) for each edge \( e_{C_i^{[j]}, C_i^{[k]}} \in \mathcal{E} \), partition the graph into \( \tau \) disjoint sets \( P_g, g = 1, \ldots, \tau \), such that

\[
\sum_{C_i^{[j]} \in P_g, C_i^{[k]} \in P_{g'}} w_{C_i^{[j]}, C_i^{[k]}} \quad \text{is maximized.}
\]

The training sequence length constraint is already taken into consideration by the definition of the number of resulting sets \( \tau \). Since interference links between copilot users in the same cell was assigned a large weight \( w_{\infty} \), we are sure that all copilot groups within a given cell will be allocated to different sets. The \( \max-\tau \)-cut algorithm assigns different training sequences to copilot groups with strong spatial signatures overlapping between the useful and interference signals. The complexity of the proposed pilot allocation algorithm depends on the number of copilot groups, edges and training sequences.

A remark on the complexity of this algorithm is now in order. Proceeding to sequence allocation, once copilot groups are formed, results in a substantial simplification of the problem. Instead of processing each user individually, the proposed method exploits the formed copilot groups in order to reduce running time of the pilot allocation algorithm. This impact becomes very interesting in an IoT communication scenario.

The Pilot sequence allocation problem is NP-hard [19], meaning that the optimal solution is computationally prohibitive to obtain. Consequently, we use the low complexity algorithm in
The heuristic algorithm, in [15], provides an approximate solution that achieves at a ratio of $(1 - \frac{1}{\tau})$ of the optimal one for a general MAX-$\tau$-CUT problem, given that all weights in the graph are nonnegative integers. Since all weights in the considered interference graph are positive, using the heuristic from [15] provides us with a $(1 - \frac{1}{\tau})$-approximation of the optimal solution for the considered problem. The detailed algorithm for cross cell Pilot assignment is given in table IV.

| Initialize: intra-set weights $W_g = 0, \forall g = 1, \ldots, \tau$ |
|---------------------------------------------------------------|
| 1. Assign the $\tau$ copilot groups in cell 1 to different pilot sets |
| 2. Randomly order the rest of copilot groups. |
| 3. Select the next copilot group $v$ and assign it to set $g^*$ for which $W^v_{g^*}$ is minimized where $W^v_g = \sum_{u \in P_g} W_{v,u}$ |
| 4. Update the Average weight of group $g^*$ such that $W_{g^*} = W_{g^*} + W^v_{g^*}$ |
| 5. Repeat steps 3 – 4 until all copilot groups are assigned. |

**Table IV: Cross cell Pilot assignment algorithm**

**VI. Numerical Results and Discussion**

In this section, we provide numerical results demonstrating the performance of the proposed spatial basis coverage copilot user selection. We compare the proposed approach with a conventional TDD massive MIMO system where all scheduled, within each cell, are given orthogonal training sequences. We then extend the simulation results to include MAX-$\tau$-CUT pilot allocation. We consider a network constituted of $N_c = 4$ hexagonal cells. Each cell has a radius 0.5 Km from center to vertex. Each cell contains a massive MIMO BS at its center, equipped with $M = 128$ equally spaced isotropic antennas. The minimum distance between antenna elements is equal to $\frac{\lambda}{2}$. Each cell contains $K = 25$ users with randomly generated mean direction of arrivals. The channel vectors of the different users are generated according to (1) where $P = 100$. Each coefficient $\mu_{ib}^{[r]}$, $\forall i, b, r$ denotes the path-loss between the user and the target BS. The path-loss coefficient is 3.5. For each user $i, b$, the angles of its rays $\theta_{ib}^{[r,p]}$, $p = 1, \ldots, P$ are uniformly distributed in the interval $[\theta_{ib}^{[r]} - \Delta\theta_{ib}^{[r]}, \theta_{ib}^{[r]} + \Delta\theta_{ib}^{[r]}]$ where the AS is supposed to be the same for all users with $\Delta\theta_{ib}^{[r]} = \Delta = 4^\circ$. The coherence interval is set to $T_s = 128$ samples, split between training and data transmission. We take $\alpha = 0.05$. In order
to assess the accuracy of channel estimation, we take as metric the average individual mean square error (MSE).

Figure 3: Comparison of uplink channel estimation MSE with $\tau = 8$ and $U_b^{[k]} = 3, \forall k, b$

Figure (3) illustrates a comparison of MSE performance of UL channel estimation, as a function of transmit signal to noise ratio (TX SNR). Figure (3) shows that UL channel estimation is improved when using the two proposed spatial basis coverage algorithms in the low SNR range (up to approximately 7.5dB for the power agnostic approach and up to 15 dB for the power aware approach ). It shows that power aware spatial basis coverage outperform the power agnostic approach in MSE. This is mainly due to the fact that the power aware approach takes into consideration the channel gain when forming copilot groups. In addition, the power aware approach allow for less overlapping in spatial signature when compared to the power agnostic one. Figure (3) shows also that, as SNR increases, the performances of the two proposed algorithms reach two distinct error floors. This phenomenon is due to the truncation error that results from projection on the users specific spatial subspaces. These error floors depend on the rank of the user’s spatial signatures and can be reduced by decreasing $\alpha$ (i.e. considering the DFT vectors that concentrate lower levels of the user channel power, see 3). Note that, in the conventional approach, all scheduled users, in each cell, are allocated orthogonal training sequences. Consequently, the conventional CSI estimation uses $U_b^{[k]} \times \tau$ orthogonal pilot sequences. On the other hand, the proposed algorithms use training sequences of length $\tau$. This explains, in part, MSE performance in the high SNR range where the conventional CSI estimation approach can outperform the proposed algorithms. However, our method enables to reduce the
needed UL training resources which will result, ultimately, in improving SE.

Nevertheless, we should emphasize on the fact that the presented performances are attained with UL training overheads of $\tau$ and $U_b^{[k]} \times \tau$, for the proposed algorithms and the conventional approach respectively. This means that, for a reasonable SNR range, we are able to improve CSI estimation accuracy while reducing the UL training overhead.

![Figure 4: Comparison of the achievable average SE with $\tau = 10$ and $U_b^{[k]} = 2, \forall k, b$](image)

Figure (4) illustrates a comparison of the achievable SE between the proposed spatial basis coverage algorithms and a conventional TDD massive MIMO system, as a function of the SNR. Figure (4) shows that, for an SNR of 0 dB, the power aware and the power agnostic spatial basis coverage approaches achieve 124.75 bits/Hz/s and 93.095 bits/Hz/s respectively. This represent gains of 41.983bits/Hz/s and 10.328 bits/Hz/s, respectively, in comparison with a conventional TDD massive MIMO system. As SNR increases the gain in spectral efficiency that the proposed spatial basis coverage approaches provide also increases. This is mainly due to the reduced impact of additive noise since the system becomes interference limited which emphasizes the ability of the proposed schemes to mitigate intra-cell copilot interference.

Figure (5) illustrates a comparison of CDFs of the achievable spectral efficiency between the proposed beam coverage algorithms and a conventional TDD mMIMO system, for different for different $\tau$ and $U_b^{[k]}$ values. The proposed algorithms are applied in the same network setting with $\tau = 10$, $U_b^{[k]} = 2 \forall k, b$ and $\tau = 5$, $U_b^{[k]} = 4 \forall k, b$, respectively. Figure (5) shows that, for $\tau = 10$ and $U_b^{[k]} = 2 \forall k, b$, the power aware and the power agnostic spatial basis coverage approaches achieves 5% outage rate around 132 bit/s/Hz and 113 bit/s/Hz, respectively. For $\tau = 5$ and
Figure 5: Comparison of CDFs of achievable SE for different $\tau$ and $U_b^{[k]}$ values with $SNR = 10$ dB

$U_b^{[k]} = 4 \forall k, b$, 5% outage rate is attained around 186 bit/s/Hz and 126 bit/s/Hz, respectively. This gain in performance is mainly due to the spared training resources when decreasing $\tau$ from 10 to 5. Figure (5) shows that the impact of increased intra-cell copilot interference, due to more pilot reuse, can be efficiently mitigated through spatial basis coverage and the spared training resources.

Figure 6: Comparison of CDFs of achievable SE for $\tau = 4$, $U_b^{[k]} = 5$ and with $SNR = 10$ dB

Figure (6) illustrates the impact of the proposed max-$\tau$-cut pilot allocation algorithm. Figure (6) shows that addressing the issue of inter-cell copilot interference through efficient pilot
sequence allocation results in an improvement in the system SE. Indeed, while the power aware spatial basis cover approach achieves 5% outage rate around 168 bit/s/Hz, the combination with the max-$\tau$-cut pilot assignment algorithm achieves 5% outage rate around 191 bit/s/Hz. Consequently, after constructing copilot user groups based on the spatial basis approach, the same diversity in spatial signatures can be leveraged in order to address the problem of inter-cell copilot interference. Although complete removal of interference is still not possible, especially since copilot groups are constructed based on the useful links spatial information, non-negligible performance improvement can be achieved by efficient pilot sequence allocation across cells.

VII. Conclusion

In this paper, we have studied user scheduling and pilot allocation based on spatial division multiplexing for TDD massive MIMO systems. We proposed a copilot user grouping approach based on DFT basis coverage. After associating each user with the DFT vectors that concentrate the majority of its channel power, users are assigned to copilot groups in order to achieve a maximum coverage of the DFT vectors per group with minimum overlapping in their respective spatial signatures. The proposed approach enables to increase the spectral efficiency while reducing the required training overhead. This is achieved by enabling accurate CSI estimation leveraging spatial diversity. Various numerical results were provided to demonstrate the effectiveness of the proposed grouping approach. In order to efficiently manage inter-cell copilot interference, further optimization is performed. We have proposed a graphical approach for training sequence allocation across cells. Leveraging the interference links spatial information and the previously constructed copilot groups, training sequence allocation is formulated as a max-cut problem. Thus enabling the utilization of a low complexity algorithm for training sequences allocation. Although the proposed approach does not remove entirely copilot interference, it provides a practical method to meet essential requirements for 5G and beyond networks, namely, increasing spectral efficiency and connection density for the same training overhead.

VIII. Appendix

Appendix A proof of Theorem 1:

In order to derive a tight bound on the performance of the proposed algorithm in table I, we consider the worst case behavior of the greedy heuristic as in [17]. However in our case, the
The optimization problem (9) can be decomposed into \( N_c \) independent problems, each of which is defined in a given cell \( b = 1, \ldots, N_c \). We start by defining \( C_k^{[b]}, k = 1, \ldots, \tau, b = 1, \ldots, N_c \) as the maximum coverage at iteration \( k \) of the approximate algorithm in table \( I \). We define \( C_{k_{opt}}^{[b]}, k = 1, \ldots, \tau, b = 1, \ldots, N_c \) as the optimal maximum coverage that can be obtained at iteration \( k \). We also consider \( C_{opt}^{[b]} \) as the optimal solution the problem of (9) defined in each cell \( b = 1, \ldots, N_c \):

\[
V(C_{opt}^{[b]}) = \max_y \sum_{k=1}^{\tau} \sum_{s=1}^{M} y_{s,k}^{[b]}
\]

\[
\text{s.t.} \quad \sum_i x_{\{i,b\}}^{[k]} \leq U_b^{[k]} \quad \forall k = 1, \ldots, \tau
\]

\[
\sum_{i,f \in F_{ib}} x_{\{i,b\}}^{[k]} \geq y_{s,k}^{[b]} \quad \forall k = 1, \ldots, \tau,
\]

where \( V(C_{opt}^{[b]}) \) represents the value of the coverage \( C_{opt}^{[b]}, b = 1, \ldots, N_c \). The objective function in (14) is modular. Consequently, the following property holds \( \forall b = 1, \ldots, N_c \):

\[
V(C_{opt}^{[b]}) \leq \sum_{k=1}^{t-1} V(C_{k_{opt}}^{[b]}) + \tau V(C_{t_{opt}}^{[b]}), \forall t = 1, \ldots, \tau,
\]

In order to derive a bound on the achievable performance of the proposed algorithm, we consider the worst case behavior of the greedy heuristic. Doing so is equivalent to solving the following linear problems \( \forall b = 1, \ldots, N_c \):

\[
P(b) = \min \sum_{k=1}^{t-1} \frac{V(C_{k_{opt}}^{[b]})}{V(C_{opt}^{[b]})}
\]

\[
\text{s.t.} \quad \sum_{k=1}^{t-1} \frac{V(C_{k_{opt}}^{[b]})}{V(C_{opt}^{[b]})} + \tau \frac{V(C_{t_{opt}}^{[b]})}{V(C_{opt}^{[b]})} \geq 1 \quad \forall t = 1, \ldots, j
\]
Where the constraint \((16a)\) is obtained from \((15)\). Since \((16)\) is a linear problem, we can solve it by considering its dual \(\forall b = 1, ..., N_c\) which can be written as follows

\[
D(b) = \max \sum_{t=1}^{j+1} v_t
\]

\[
\text{s.t } \tau v_k + \sum_{t=k+1}^{j+1} v_t = 1, \forall k = 1 \ldots j
\]

\[
v_t \geq 0 \ \forall t = 1 \ldots j + 1
\]

We now proceed by solving \((17)\), and, consequently, by linear programming duality, \((16)\). Let \(v = 1 - v_{j+1}\), where \(v_{j+1}\) is defined in \((17)\). Consequently, \(\forall k = 1 \ldots j\), we have:

\[
\tau v_k + \sum_{t=k+1}^{j} v_t = v \quad \text{and} \quad v_k = \frac{v - \sum_{t=k+1}^{j} v_t}{\tau}
\]

Then \(v_k\) are calculated iteratively \(\forall k = 1 \ldots j\). Indeed,

\[
v_j = \frac{v}{\tau}, \quad v_{j-1} = \frac{v - v_j}{\tau} = \frac{v - \frac{v}{\tau}}{\tau} = \frac{v}{\tau} \left( \frac{\tau - 1}{\tau} \right), \ldots, \quad v_1 = \frac{v - \sum_{t=2}^{j} v_t}{\tau} = \frac{v}{\tau} \left( \frac{\tau - 1}{\tau} \right)^{j-1}.
\]

Consequently, \(v_k, \forall k = 1 \ldots j\) are given by

\[
v_k = \frac{v}{\tau} \left( \frac{\tau - 1}{\tau} \right)^{j-k}
\]

Therefore, finding \(D(b), \forall b = 1, ..., N_c\) is equivalent to the following :

\[
D(b) = \max_{0 \leq v \leq 1} \left( \sum_{t=1}^{j+1} v_t \right) = \max_{0 \leq v \leq 1} \left( v(1 - \left( \frac{\tau - 1}{\tau} \right)^j) \right)
\]

which is achieved for \(v = 1\). It follows that, \(P(b) = 1 - \frac{j}{\tau} \left( \frac{\tau - 1}{\tau} \right)^j\). Taking \(j = \tau\), we obtain:

\[
P(b) = 1 - \left( \frac{\tau - 1}{\tau} \right)^\tau
\]

Consequently, the obtained solution using a greedy sequential maximum coverage verifies:

\[
\frac{V(C_{opt}^{[b]}) - \sum_{k=1}^{\tau} V(C_{kopt}^{[b]})}{V(C_{opt}^{[b]})} \leq \left( \frac{\tau - 1}{\tau} \right)^\tau
\]

In order to derive the performance guarantee of the algorithm in table \(I\), the approximation ratio of the used algorithm to perform maximum coverage, at each iteration, needs to be accounted for. We use the following result which is based on the approximation ratio given in [13].
Lemma 3. For any given cell index \( b = 1, \ldots, N_c \) and iteration \( k = 1, \ldots, \tau \), the implemented algorithm provides a \( (1 - \frac{1}{e}) \) approximation of the optimal cover, i.e \( C_k^{[b]} \geq (1 - \frac{1}{e})C_{k_{\text{opt}}}^{[b]} \), \( k = 1, \ldots, \tau, b = 1, \ldots, N_c \).

Since at each iteration of the algorithm, in table I, we obtain a \( (1 - \frac{1}{e}) \) approximation of the optimal maximum coverage, we obtain the following for \( b = 1, \ldots, N_c \)

\[
\sum_{k=1}^{\tau} V(C_k^{[b]}) \geq (1 - \frac{1}{e}) \sum_{k=1}^{\tau} V(C_{k_{\text{opt}}}^{[b]})
\]

we can deduce that the algorithm in table I provides a \( (1 - (\frac{\tau-1}{\tau})^{\tau}) (1 - \frac{1}{e}) \)-approximation for each subproblem of (9). Taking the sum over \( b = 1, \ldots, N_c \) finishes the proof.

Appendix B proof of Theorem 2:

The proof for the performance bound of the proposed algorithm in table II follows the same reasoning as the proof of Theorem 1. The main idea is also to consider the worst case behavior of the greedy heuristic with a change in the achievable approximation ratio at each iteration. The optimization problem (10) can be decomposed into \( N_c \) independent problems, each of which is defined in a given cell \( b = 1, \ldots, N_c \). We start by defining \( C_k^{[b]}, k = 1, \ldots, \tau, b = 1, \ldots, N_c \) as the maximum coverage at iteration \( k \) of the algorithm in table II. We define \( C_{k_{\text{opt}}}^{[b]}, k = 1, \ldots, \tau, b = 1, \ldots, N_c \) as the optimal maximum coverage that can be obtained at iteration \( k \). We also consider \( C_{\text{opt}}^{[b]} \) as the optimal solution the problem of (10) defined in each cell \( b = 1, \ldots, N_c \):

\[
V(C_{\text{opt}}^{[b]}) = \max_Y \sum_{k=1}^{\tau} \sum_{i \in \Gamma (b)} \sum_{f_s \in F} \zeta_{ib}^{[s,k]} y_{i,b}^{[s,k]}
\]

subject to

\[
\sum_{i \in \Gamma (b), f_s \in F} y_{i,b}^{[s,k]} \leq 1 \quad \forall k = 1 \ldots \tau,
\]

\[
\sum_{i \in \Gamma (b), f_s \in F} x_{i,b}^{[k]} \geq y_{i,b}^{[s,k]} \quad \forall k = 1 \ldots \tau
\]

\[
\sum_{i \in \Gamma (b)} x_{i,b}^{[k]} \leq U_b^{[k]} \quad \forall k = 1 \ldots \tau,
\]

The objective function in (25) is modular. Consequently, the following property holds \( \forall b = 1, \ldots, N_c \):

\[
V(C_{\text{opt}}^{[b]}) \leq \sum_{k=1}^{t-1} V(C_{k_{\text{opt}}}^{[b]}) + \tau V(C_{k_{\text{opt}}}^{[b]}) , \forall t = 1, \ldots, \tau,
\]
In order to derive a bound on the achievable performance of the proposed algorithm, we consider the worst case behavior of the greedy heuristic. Doing so is equivalent to solving the following linear problems \( \forall b = 1, \ldots, N_c \):

\[
P(b) = \min \sum_{k=1}^{j} \frac{V(C_{kopt}^{[b]})}{V(C_{opt}^{[b]})} \tag{27}
\]

\[
s.t \sum_{k=1}^{t-1} \frac{V(C_{kopt}^{[b]})}{V(C_{opt}^{[b]})} + \tau \frac{V(C_{kopt}^{[b]})}{V(C_{opt}^{[b]})} \geq 1 \forall t = 1 \ldots j \tag{27a}
\]

Where the constraint (27a) is obtained from (26). Here also (27) is solved using its dual. It follows that, \( P(b) = 1 - \frac{j}{\tau} \left( \frac{\tau - 1}{\tau} \right)^j \). Taking \( j = \tau \), we obtain:

\[
P(b) = 1 - \left( \frac{\tau - 1}{\tau} \right)^\tau, \forall b = 1, \ldots, N_c. \tag{28}
\]

Consequently, the obtained solution using a greedy sequential maximum coverage verifies:

\[
\frac{V(C_{kopt}^{[b]}) - \sum_{k=1}^{\tau} V(C_{kopt}^{[b]})}{V(C_{opt}^{[b]})} \leq \left( \frac{\tau - 1}{\tau} \right)^\tau \tag{29}
\]

In order to derive the performance guarantee of the proposed algorithm in table II, the approximation ratio of the used algorithm , at each iteration, needs to be considered. We use the following result which is based on the approximation ratio given in [14].

**Lemma 4.** For any given cell index \( b = 1, \ldots, N_c \) and iteration \( k = 1, \ldots, \tau \), using an algorithm of approximation ratio \( \beta \) at step 3 of the algorithm in table III, the implemented algorithm provides a \( 1 + \beta - \beta e^{-\frac{\beta}{1-e^{-\frac{1}{\tau}}}} \) approximation of the optimal cover, i.e \( C_k^{[b]} \geq \frac{1 + \beta - \beta e^{-\frac{\beta}{1-e^{-\frac{1}{\tau}}}}}{1-e^{-\frac{1}{\tau}}} C_{kopt}^{[b]}, k = 1, \ldots, \tau, b = 1, \ldots, N_c \).

In this work, we use the greedy algorithm for knapsack problems in step 3 of the algorithm in table III. Consequently, in our case, \( \beta = \frac{1}{2} \) and the approximation ratio, at each iteration, becomes \( \frac{3 - e^{-2}}{1-e^{-\frac{1}{\tau}}} \).

Since at each iteration of the algorithm, in table II, we obtain a \( \frac{3 - e^{-2}}{1-e^{-\frac{1}{\tau}}} \) approximation of the optimal maximum coverage, we obtain the following for \( b = 1, \ldots, N_c \)

\[
\sum_{k=1}^{\tau} V(C_{kopt}^{[b]}) \leq \frac{3 - e^{-2}}{2} \sum_{k=1}^{\tau} V(C_{kopt}^{[b]}) \tag{30}
\]

we can deduce that the algorithm in table II provides a \( \left( 1 - \left( \frac{\tau - 1}{\tau} \right) \right)^\tau \frac{3 - e^{-2}}{1-e^{-\frac{1}{\tau}}} \)-approximation for each subproblem of (10). Taking the sum over \( b = 1, \ldots, N_c \) finishes the proof.
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