Excitation of plasma wakefields by proton beam

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A stationary wave solution is obtained for the proton driven plasma wake field accelerator (PDPWFA). The wake field excitation by trains of equidistant proton microbunches produced due to self modulational instability has been discussed. Also, considering the necessity of the external magnetic field to control focusing of the beam, studies on the effect of magnetic field on the wake field structures have been done.

I. INTRODUCTION

Over the last few decades, research on the plasma based acceleration process was mainly focused in creating the wake field by launching a highly relativistic electron beam or an intense laser beam into the plasma. The large electric field excited in the plasma can then be used to accelerate the electrons to high energy which can be achieved over a very much shorter distances compared to the convention RF technology. However, to reach TeV order of energy in the laser pulse or electron bunch driver schemes, it requires multiple stage acceleration. Moreover, the energy gain is limited by the energy carried by the electron driver which is very small (∼ 100 J). To overcome these problems one may use a proton beam to excite the strong plasma wake wave. A proton bunch carrying energy of the order of kJ is capable of producing such energy in a single plasma stage. Because of their higher energy and mass, proton can drive wake fields over a very longer plasma lengths. This proton-driven scheme is therefore much more superior compared to the other accelerators.

Since protons are positively charged and much heavier than electrons, the physics of proton driven wake field accelerator is different from electron beam driven plasma. In case of negatively charged driver, background plasma electrons are repelled to cerate a blow out regime where the wake field is produced. Proton beam, on the other hand, sucks in the plasma electrons towards the propagation axis and creates the wake wave electric field. A trailing witness bunch of electrons then extracts energy from the drive beam and thereby get accelerated to relativistic energies.

The efficient excitation of the wake field and successfully accelerating electrons to high energy depend on some key effects. Unless special care is taken to minimize them, there may be a possibility of the degradation in energy gain. One such effect is the occurrence of ‘phase slippage’ between the proton driving bunch and the electron witness bunch. Proton bunch traveling through the plasma may slow down and the phase relationship with the light electron bunch will begin to change. One way to control this de-phasing is to apply an external magnetic field perpendicular to the particle motion. Also, it may happen that due to some instability caused during the propagation of the beam, the beam head get expanded laterally which can cause the large spread in final accelerated particle energy. The transverse electric field at the wake of the beam as well as the presence of an external magnetic field can minimize this lateral expansion by focusing the propagating beam. Investigation of the effect of magnetic field on the stationary profiles of electric field and electron density is thereby very important.

Proton bunches available today are much more longer in size compared to the plasma wavelength. However, a mechanism called the self modulational instability causes the initially launched long proton bunches to split over a train of short equi-spaced microbunches. In the recent past a numerical analyses has been performed to analyze the excitation mechanism of wake field by such trains of equidistant particle bunches. In our present theoretical study, the field excitation process using multibeam proton has been re-investigated.

There exists a lot of numerical and experimental works performed to understand the basic physics of PDPWFA. It has been shown that Terra electron-volt energy regime can be reached easily by use of proton beam. However, we have not witnessed much theoretical studies made on proton beam driven plasma. In this article such an attempt has been made where incorporating the proton beam density in the Poisson equation the basic fluid Maxwell’s equation have been solved.

In section (II), The basic equations and its solution have been presented to describe the proton beam driven plasma wake field acceleration process in the unmagnetized plasma system. Also, the use of train of proton micro bunches to observe the wake field structures has been analyzed here. Section (III) discusses the results obtained in numerical investigation of magnetized plasma system. The effect of magnetic field strength on the proton beam driven wake field structures is also addressed in this section. The summary and the main conclusion of the work performed here have been made in section (IV).
II. THE BASIC GOVERNING EQUATIONS AND ITS SOLUTION

The basic equations describing proton beam driven nonlinear plasma waves in one dimension are the continuity equations, momentum equations for the electron and ion along with the Poisson’s equation:

\[ \partial_t n_j + \partial_x (n_j v_j) = 0, \]

\[ (\partial_t + v_j \partial_x) (\gamma_j v_j) = q_j E/m_j, \]

\[ \partial_x E = 4\pi \left[ \sum_j q_j n_j + e n_b \right]. \]

All the variables used here have their usual meanings. For simplicity we have considered the motion of the plasma ion to follow nonrelativistic dynamics so that \( \gamma_i \) is taken to be unity.

We have constructed a travelling wave solution of the basic Eqs. 1-3 considering the propagation of longitudinal electrostatic wave motion along \( x \) axis and function of variable \( x = k_p (x - v_{ph} t) \), where \( k_p = \omega_p/v_{ph} \) with \( \omega_p = \sqrt{4\pi n_0/e} \) being the equilibrium plasma density and the phase velocity of the plane wave respectively.

The electron and ion densities normalized by equilibrium density, can be expressed as a function of electrostatic potential as

\[ N_e = \beta_{ph} \gamma^2 \left[ \frac{\varphi_e}{(\varphi_e^2 - \gamma^{-2})^{1/2}} - \beta_{ph} \right], \]

\[ N_i = \frac{\beta_{ph}}{\sqrt{\beta_{ph}^2 + 2\varphi_i}}, \]

where \( \varphi_i = -\mu \varphi \) and \( \varphi_e = 1 + \varphi \), with \( m_e/m_i = \mu \) being the electron to ion mass ratio. Using these expression for species densities, from the Poisson’s equation, we obtain a second order differential equation for \( \varphi \) as,

\[ \frac{d^2 \varphi}{d\xi^2} = -\frac{\beta_{ph}^3}{\sqrt{\beta_{ph}^2 + 2\varphi_i}} + \frac{\beta_{ph}^3 \gamma^2 \varphi_e}{\sqrt{\varphi_e^2 - \gamma^{-2}}} - \beta_{ph}^4 \gamma^2 - \alpha \beta_{ph}^2, \]

where, \( \alpha = n_b/n_0 \) is the normalized proton beam density.

We solve this second order differential equation numerically assuming a rectangular beam profile whose longitudinal extension is

\[ \alpha = \alpha_0 \text{ for } 0 \leq \zeta \leq l_b, \]

\[ = 0, \text{ otherwise}; \]

The solution for the wake field excited inside and behind the single proton bunch as well as corresponding perturbed electron density have been shown in the Fig. 1 and Fig. 2. The transformer ratio (the ratio of the maximum accelerating field behind the beam to the maximum decelerating field inside the beam) which determine the overall energy efficiency of the accelerated particles can be calculated from this field profile.

III. WAKE FIELD EXCITATION BY TRAINS OF PARTICLE BUNCHES

Due to self modulational instability, the long proton bunch can be split into long chain of equispaced microbunches. It is of fundamental interest to see how strong wakefield can be excited behind these multibeams.

\[ \text{FIG. 1: Variation of normalized Electric field in a proton beam driven plasma } [\beta_{ph} = 0.995, \beta_0 = 0.995], \text{ with beam density } [\alpha = 0.5 \text{ for } 0 \leq \xi \leq 5.6\pi \text{ and zero otherwise}] \]

\[ \text{FIG. 2: Variation of normalized perturbed electron density in a beam driven plasma } [\beta_{ph} = 0.995, \beta_0 = 0.995], \text{ with beam density } [\alpha = 0.5 \text{ for } 0 \leq \xi \leq 5.6\pi \text{ and zero otherwise}] \]
The electric field structure and corresponding perturbed electron density profile for the excited wake field by the train of equidistant rectangular particle bunches with peak beam density $0.5n_0$ have been shown in Fig. 3 and Fig. 4 respectively. From the figure, it is observed that electric field amplitude can not grow indefinitely with the increase of the number of beams. Rather, the field saturates due to the amplitude dependent frequency as given in the one dimensional analytical solution reported by Akheizer and Polovin.

$$\tau \simeq \tau_0 \left[1 + \frac{3}{16} \left(\frac{E_m}{E_0}\right)^2\right], \quad (7)$$

where, $E_m$ and $E_0$ are respectively the maximum field amplitude and nonrelativistic wavebreaking limit. The wave period ($\tau$) is seen to increase with the maximum field amplitude. This change in wavelength can cause the bunches to fall under the decelerating field and thereby the field stops growing further.

![Variation of normalized Electric field in absence of magnetic field driven by a train of equidistant particle bunches $[\beta_{ph} = 0.995, \beta_b = 0.995]$, with beam density $[\alpha = 0.5]$](image)

**FIG. 3:** Variation of normalized Electric field in absence of magnetic field driven by a train of equidistant particle bunches $[\beta_{ph} = 0.995, \beta_b = 0.995]$, with beam density $[\alpha = 0.5]$

![Variation of normalized perturbed electron density in absence of magnetic field driven by a train of equidistant particle bunches $[\beta_{ph} = 0.995, \beta_b = 0.995]$, with beam density $[\alpha = 0.5]$](image)

**FIG. 4:** Variation of normalized perturbed electron density in absence of magnetic field driven by a train of equidistant particle bunches $[\beta_{ph} = 0.995, \beta_b = 0.995]$, with beam density $[\alpha = 0.5]$

### IV. MAGNETIZED PROTON BEAM DRIVEN PLASMA

To reduce the driver beam head expansion and particle loss, an external magnetic field is applied in the excitation process of the wake field. In presence of constant magnetic field the wake wave that is excited behind the proton drive beam is nothing but the high frequency upper hybrid wave. We now proceed to find a traveling wave solution for this excited wave.

The basic equations describing the nonlinear relativistic upper hybrid wave generation in the cold magnetized plasma are the fluid Maxwell equations viz. the momentum equations, electron continuity equation, and electric field evolution equations:

$$\frac{\partial \mathbf{p}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{p} = -e\mathbf{E} - e\beta_e \times \mathbf{B}.$$  \quad (8)

$$\nabla \cdot \mathbf{E} = 4\pi e(n_0 - n + n_b). \quad (9)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \quad (10)$$

$$\nabla \dot{\mathbf{B}} = 0.$$

$$\nabla \times \mathbf{B} = -4\pi e(n\beta_e + n_b\beta_b) + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}. \quad (12)$$

where, $\beta_e$ and $\beta_b$ are respectively the electron fluid velocity and ion beam velocity both normalized by free space light speed $c$. The electric field is $\mathbf{E} = E\hat{e}_x$, where $\hat{e}_x$ is the unit vector along the $x$ axis. The external magnetic field is $\mathbf{B} = B_0\hat{e}_z$, where $\hat{e}_z$ is the unit vector along the $z$ axis. Other variables have their usual meanings. The heavier mass ions are assumed to be static for the sake of convenience.

In order to find a travelling wave solution of the excited upper hybrid wave we adopt the same co-ordinate transformation as used in the unmagnetized case. In that coordinate system two momentum equations take the form:

$$\frac{dp_x}{d\xi} = \beta_{ph}\frac{E\sqrt{1 + p_x^2 + p_y^2 + \Omega p_y} - p_x}{\beta_{ph}\sqrt{1 + p_x^2 + p_y^2 - p_x}}.$$  \quad (13)

$$\frac{dp_y}{d\xi} = -\Omega\beta_{ph}\frac{p_x}{\beta_{ph}\sqrt{1 + p_x^2 + p_y^2 - p_x}}.$$  \quad (14)

Combination of Eq. (11) and Eq. (12), gives us the electric field evolution equation:

$$\frac{dE}{d\xi} = -\beta_{ph}\frac{(1 + \alpha)p_x - \alpha\beta_b\sqrt{1 + p_x^2}}{\beta_{ph}\sqrt{1 + p_x^2 - p_x}}.$$  \quad (15)
and \( \beta \) plasma density with and without magnetic field in a beam driven plasma [\( \beta_{ph} = 0.995, \beta_b = 0.995 \)], with beam density [\( \alpha = 0.5 \) for \( 0 \leq \xi \leq 5.6\pi \) and zero otherwise], \( 1 \rightarrow \Omega = 0, 2 \rightarrow \Omega = 0.5, 3 \rightarrow \Omega = 0.9 \).

FIG. 5: Variation of normalized Electric field with and without magnetic field in a beam driven plasma [\( \beta_{ph} = 0.995, \beta_b = 0.995 \)], with beam density [\( \alpha = 0.5 \) for \( 0 \leq \xi \leq 5.6\pi \) and zero otherwise], \( 1 \rightarrow \Omega = 0, 2 \rightarrow \Omega = 0.5, 3 \rightarrow \Omega = 0.9 \).

FIG. 6: Variation of normalized perturbed electron density with and without magnetic field in a beam driven plasma [\( \beta_{ph} = 0.995, \beta_b = 0.995 \)], with beam density [\( \alpha = 0.5 \) for \( 0 \leq \xi \leq 5.6\pi \) and zero otherwise], \( 1 \rightarrow \Omega = 0, 2 \rightarrow \Omega = 0.5, 3 \rightarrow \Omega = 0.9 \).

Here, \( \alpha = n_b/n_0 \) is the normalized proton beam density and \( \beta = eB_0/mc \) is the electron cyclotron frequency. The normalized variables we have used are \( E \rightarrow eE/(m_0\omega_{pe}c), p_x \rightarrow p_x/m_ec, p_y \rightarrow p_y/m_ec, n \rightarrow n/n_0 \) and \( \beta_{ph} = v_{ph}/c \). Also, we defined \( p_x^2 + p_y^2 = p^2 \).

From the electron continuity equation it is easy to show that

\[
n = \frac{\beta_{ph}\sqrt{1 + p^2}}{\beta_{ph}\sqrt{1 + p^2} + p_x}.
\]  

It is very difficult to find an exact analytical solution for the above coupled Equations. \( \text{[K15]} \). We solve these differential equations by \( 4^{th} \) order Runge-Kutta method and obtained the solutions for the wake wave electric field and perturbed electron density. In obtaining those solutions, we have considered the rectangular profile of single proton bunch with a longitudinal variation similar to the unmagnetized case.

Fig.(5) and Fig.(6) show the stationary electric field and perturbed density profiles for different strengths of the applied magnetic fields. Here we see that with increase in the magnetic field strength the maximum electric field behind the pulse gradually increases.

V. CONCLUSION

A one dimensional theoretical investigation has been performed here for the proton beam driven plasma wake field accelerator. Consideration of both single long proton beam or micron sized train of small particle bunches to excite the plasma wake wave have been incorporated in our studies. A recent numerical investigation on the two dimensional plasma wake field excitation by trains of equidistant particle bunches by K. V. Lotov has been the motivation behind the use of such beam train in our analysis. Also, since in order to focus the beam the application of an external magnetic field is necessary, the effect of such magnetic field on the wake field structures has been discussed. The experimental and numerical verification of the results obtained here is needed to be done. Furthermore, our theoretical investigation have some importance in the ongoing AWAKE project at CERN which is aimed to explore the physics of proton driven plasma accelerators.\( \text{[K15]} \).

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