A possibility on prohibition of Higgs mass by the extended Lorentz transformation in noncommutative geometry

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**Abstract**

In this letter, we propose the extended Lorentz transformation in noncommutative geometry, as a possibility on prohibition of the Higgs mass.

Since it is difficult to build the symmetry between the connections $A_\mu$ and $H$, the transformation is defined for the differential two-forms. The parameter of the transformation $\omega$ changes a two-form into other two-forms. Comparing the coefficients of the two-forms, the transformations are translated to those of the product fields $F_{\mu\nu}, D_\mu H$ and $HH^\dagger$. It shows the invariance of the bosonic Lagrangian explicitly.
1 Introduction

The Higgs model in the noncommutative geometry (NCG), proposed by Connes and Lott [1], is an interesting possibility of the explanation of the Higgs boson. In this picture, the Higgs boson is interpreted as a gauge boson along the discrete fifth dimension, which has the noncommutative differential algebra. This concept is applied in various theories, e.g., grand unified theory (GUT) [2–4], ideas related to the extra dimensions [5], supersymmetry [6], and so on.

A crucial problem in this model is that the Higgs mass cannot be forbidden by the gauge invariance. As a straightforward idea, a deformed five dimensional Lorentz symmetry should play such a role. In this letter, we propose the extended Lorentz transformation in noncommutative geometry, as a possibility on prohibition of the Higgs mass. Since it is difficult to build the symmetry between the connections $A_\mu$ and $H$, the transformation is defined for the differential two-forms. The parameter of the transformation $\omega$ changes a two-form into other two-forms. Comparing the coefficients of the two-forms, the transformations are translated to those of the product fields $F_{\mu\nu}, D_\mu H$ and $HH^\dagger$. It shows the invariance of the bosonic Lagrangian explicitly.

For the Lagrangian of fermions, the appropriate matrix representation of this transformation is not be found [1]. The form of the transformation indicates that the representation space of fermions is twice larger than one of gauge connections. Such a representation might be realized by the real structure $\Psi = (\psi, \psi^c)$ [7, 8] or related concepts.

Finally, we comment on the Coleman–Mandula theorem [9]. The theorem is usually interpreted as prohibiting the symmetry between the Minkowski space and the internal space. However, the extended Lorentz transformation is defined in the five dimensional noncommutative space. It should be broken (for example, at the Planck scale) to the direct product of the Lorentz group and the gauge groups to produce the finite Higgs mass. Then, the symmetry does not contradict to the theorem which is applied in the broken phase of the extended Lorentz symmetry. Similar discussions can be found in the graviweak theory [10].

2 Higgs model in the noncommutative geometry

To describe this Higgs model, there are several formalizations to represent the noncommutative differential algebra. The original formalism [1] and its succeeding papers [11–13] (and for reviews, Refs. [14–16]) utilizes the “universal differential algebra”, which is a generalization of usual differential forms. Meanwhile, some of other formalizations are based on a more simple algebra, such as $dyy = -ydy$ [17–22].

Here we shortly review the theory with the latter algebra in the simplest $M^4 \times Z_2$ model. The extended exterior derivative is defined as

$$df \equiv df + d_5 f \equiv \partial_\mu f dx^\mu + [M, f]dy^5,$$

(1)

\[\text{In the first version of ArXiv, we suggest the invariance of the Lagrangian of fermions by a gamma matrix-like operator } \hat{\gamma}^M \text{ whose action is defined by the commutation relation, } \hat{\gamma}^M X = [\gamma^M, X].\]
where \( M_{nm}(n, m = L, R) \) is the distance matrix which defines vacuum expectation value (vev) and the mass of the Higgs boson. Since \( M_{nm} \) is arbitrary parameters, the model still works with \( M_{nm} = 0 \). It leads to the Higgs boson without vev and mass \[23\]. From now on, \( M = 0 \) and \( d = d \) is assumed. The nilpotency of \( d \) is manifest.

The wedge products of one-forms \( dx^\mu \) and \( dy \) are given by \[20\],

\[
(dx^\mu \wedge dx^\nu = -dx^\nu \wedge dx^\mu, \quad dx^\mu \wedge dy^5 = -dy^5 \wedge dx^\mu, \quad dy^5 \wedge dy^5 \neq 0. \tag{2}
\]

The generalized connection \( A(x) \) is defined to be

\[
A(x) = \begin{pmatrix}
A_L(x) dx^\mu & H^5(x) dy^5 \\
H^5(x) dy^5 & A_R(x) dx^\mu
\end{pmatrix}
\equiv \begin{pmatrix}
A_L & H^5 \\
H^5 & A_R
\end{pmatrix}, \tag{3}
\]

where \( dy^5 = -dy^5 \) and \( H^5 = -H^5 \) with \( g_{MN} = (+, -, -, -) \). The gauge transformation of \( A \) is given by

\[
A' = \begin{pmatrix}
G_L & 0 \\
0 & G_R
\end{pmatrix} \begin{pmatrix}
A_L & H^5 \\
H^5 & A_R
\end{pmatrix} \begin{pmatrix}
G^\dagger & 0 \\
0 & G^\dagger_R
\end{pmatrix} + \begin{pmatrix}
dG_L \cdot G^\dagger_L & 0 \\
0 & dG_R \cdot G^\dagger_R
\end{pmatrix}. \tag{4}
\]

where \( G_{L,R}(x) = \exp[it^a \alpha^a_{L,R}(x)] \) are the unitary matrices of gauge transformations. Note that the Higgs field \( H^5(x) \) transform as a bifundamental field in this model. Henceforth, we omit the argument \( x \) if there is no confusion.

The extended field strength is defined as

\[
F = dA + A \wedge A = \begin{pmatrix}
F_L + H^5 H^5, & dA_L(x) \wedge A_R(x) \\
D_\mu H^5 dx^\mu \wedge dy^5 & F_R + H^5 dx^\mu \wedge dy^5
\end{pmatrix}. \tag{5}
\]

Here, \( F_{L,R} = dA_{L,R} + A_L(x) \wedge A_R \) and \( D^\mu H^5 = \partial^\mu H^5 + A_L H^5 - H^5 A_R \). In order to build the gauge-invariant Lagrangian, we use the following inner products of two-forms \[24, 25\]

\[
\langle dx^\mu \wedge dx^\nu, dx^\rho \wedge dx^\sigma \rangle = g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}, \tag{6}
\]

\[
\langle dx^\mu \wedge dy^5, dx^\nu \wedge dy^5 \rangle = -\alpha^2 g^{\mu\nu}, \tag{7}
\]

\[
\langle dy^5 \wedge dy^5, dy^5 \wedge dy^5 \rangle = 2\beta^4, \tag{8}
\]

while other products between the two-forms to be vanish. Summarizing these results, the bosonic Lagrangian is found to be

\[
\mathcal{L}_B = -\text{Tr}(F, F)
= -\frac{1}{2} \text{tr}[F_{L\mu
u} F^{\mu\nu}_L + F_{R\mu
u} F^{\mu\nu}_R] + \text{tr}[2\alpha^2 |D_\mu H^5|^2 - 4\beta^4 |H^5|^2], \tag{9}
\]

where \( F_{(L,R)\mu
u} = (\partial_\mu A_{(L,R)\nu} - \partial_\nu A_{(L,R)\mu} + [A_{(L,R)\mu}, A_{(L,R)\nu}]) \). Tr and tr denote the trace over the external linear space and internal gauge spaces, respectively. The gauge coupling constants are introduced by rescaling of fields, c.f. \( A_{\mu} \rightarrow ig_L A_{\mu} \).
A crucial problem in this model is that the Higgs mass cannot be forbidden by the
gauge invariance: In other words,
\[
\mathcal{L}'_B = \mathcal{L}_B + m^2 \text{tr}[H_S^\dagger H^5],
\]
is also invariant under the gauge transformation. It is desirable if some symmetry prohibits
the Higgs mass. As a straightforward idea, a deformed five dimensional Lorentz symmetry
should play such a role in noncommutative geometry.

3 Extended Lorentz transformation

In this letter, we propose the infinitesimal transformation of such a symmetry, the extended
Lorentz transformation. At first, we attempted to consider the following transformation:

\[
A' = \begin{pmatrix} 1 & \omega \\ -\omega^\dagger & 1 \end{pmatrix} \begin{pmatrix} A_L & H \\ H^\dagger & A_R \end{pmatrix} \begin{pmatrix} 1 & -\omega \\ \omega^\dagger & 1 \end{pmatrix}
\]
\[
= \begin{pmatrix} A_L & H \\ H^\dagger & A_R \end{pmatrix} + \begin{pmatrix} \omega H^\dagger + H \omega^\dagger & -\omega A_R + A_L \omega \\ -\omega^\dagger A_L + A_R \omega^\dagger & -\omega^\dagger H - H^\dagger \omega \end{pmatrix},
\]
or, in components,
\[
A'_{\mu} = \begin{pmatrix} A_L & H^5 \\ H_5^\dagger & A_R^\mu \end{pmatrix} + \begin{pmatrix} \omega_5^\mu H_5^\dagger + H^5 \omega_{\mu 5}^\dagger & -\omega_5^\mu A_R^\mu + A_L \omega_{\mu 5}^\mu \\ -\omega_{5 \mu}^\dagger A_L^\mu + A_R^\mu \omega_{5 \mu}^\dagger & -\omega_{5 \mu}^\dagger H^5 - H_5^\dagger \omega_{5 \mu}^\dagger \end{pmatrix}.
\]

In order to match the gauge transformation of each matrix element, the parameter \(\omega_5^\mu\)
should be a matrix \(\omega_5^\mu = \omega_5^b \delta^b_{ij}\). The matrix \(\omega_5^\mu\) is transformed as \(\omega_{5 \mu}^\dagger = (G_5 \omega_5^\mu G_5^\dagger)_{ij}\)
under the gauge transformation \(\mathcal{G}\). However, Eq. (13) does not seem to make the La-
grangian invariant, because the anti-symmetrization of Eq. (13) \(F_{MN} = \partial_M A'_{N} + A'_{M} A'_{N} - (M \leftrightarrow N)\) can not be written by \(F_{\mu \nu}\) and \(D_{\mu} H^5\).

Then, we try to consider a similar transformation for the extended field strength \(F\). Here, the shortened notation is used
\[
F \equiv \begin{pmatrix} F_L + H \wedge H^\dagger & DH \\ DH^\dagger & -H^\dagger \wedge H + F_R \end{pmatrix},
\]
with change of the sign of \(H^\dagger \wedge H\) in the 22 element. Although it seems \(ad \ hoc\), the later
formula shows that this correction is required from consistent transformation of the Higgs
field. The transformation of \(F\) is defined to be
\[
F' = \begin{pmatrix} 1 & \omega \\ -\omega^\dagger & 1 \end{pmatrix} \begin{pmatrix} F_L + H \wedge H^\dagger & DH \\ DH^\dagger & -H^\dagger \wedge H + F_R \end{pmatrix} \begin{pmatrix} 1 & -\omega \\ \omega^\dagger & 1 \end{pmatrix},
\]
\[
\delta F = \begin{pmatrix} \omega DH^\dagger + DH \omega^\dagger \\ -\omega^\dagger (F_L + H \wedge H^\dagger) + (-H^\dagger \wedge H + F_R) \omega^\dagger \end{pmatrix} \begin{pmatrix} \omega (-H^\dagger \wedge H + F_R) - (F_L + H \wedge H^\dagger) \omega \\ -\omega^\dagger DH - DH^\dagger \omega \end{pmatrix}.
\]
If this kind of transformation exists, the invariance of the bosonic Lagrangian can be shown easily by the trace cyclicity:

\[
\delta \mathcal{L} = - \text{Tr}(\delta F, F) - \text{Tr}(F, \delta F) = -2 \text{Tr}(\delta F, F),
\]

(17)

\[
\text{Tr}(\delta F, F) = \text{tr}(\langle (\omega DH^\dagger + DH \omega^\dagger), (F_L + H \wedge H^\dagger) \rangle
+
\text{tr}(\omega(F_R - H^\dagger \wedge H) - (F_L + H \wedge H^\dagger) \omega, DH^\dagger)
+
\text{tr}(\langle -\omega^\dagger(F_L + H \wedge H^\dagger) + (F_R - H^\dagger \wedge H) \omega, DH \rangle
+
\text{tr}(\langle -\omega^\dagger DH - DH^\dagger \omega), (F_R - H^\dagger \wedge H) \rangle = 0.
\]

(18)

Since the transformation (16) is defined for the differential forms, the action of $\omega$ expected to be

\[
\omega(dx^\mu) = \omega_5^\mu dy^5, \quad \omega(dy^5) = -\omega_5^\mu dx_\nu.
\]

(19)

However, when this relation is applied for the two-forms, anti-symmetric $dx^\mu \wedge dy^5$ will mapped to non anti-symmetric $dy^5 \wedge dy^5$. In order to correct this point, the action of $\omega$ is defined

\[
\omega(A \wedge B) = (\omega A) \wedge B + (-1)^{\partial A} A \wedge (\omega B),
\]

(20)

where $\partial A$ is $Z_2$ parity of form $A$ ($\partial(dx^\mu) = 1, \partial(dy) = -1$). The action of $\omega$ is also symbolically represented by

\[
\omega = \omega_{\mu5}(dy^5 dx^{\mu*} - (-1)^{\partial A} dx^\mu dy^5),
\]

(21)

where $dx^{\mu*}$ and $dy^5$ are the dual of each one-forms $dx^{\mu*} dx_\nu = \delta^{\mu\nu}, dy^5 dy_5 = 1$. Specifically, the action for the two-forms are calculated as

\[
\omega C_{\mu\nu} dx^\mu \wedge dx^\nu = \omega_5^\mu C_{\mu\nu} dy_5 \wedge dx^\nu + \omega_5^\nu C_{\mu\nu} dx^\mu \wedge dy^5,
\]

(22)

\[
\omega C_{\mu5} dx^\mu \wedge dy^5 = \omega_5^\mu C_{\mu5} dy_5 \wedge dy^5 + \omega_5^\mu C_{\mu5} dx^\mu \wedge dx^\nu,
\]

(23)

\[
\omega C_{55} dy^5 \wedge dy^5 = \omega_5^\mu C_{55} dx^\mu \wedge dy^5 - \omega_5^\mu C_{55} dy^5 \wedge dx^\nu.
\]

(24)

The action from righthand side is defined similarly:

\[
(A \wedge B)\omega = (A\omega) \wedge B + (-1)^{\partial A} A \wedge (B\omega).
\]

(25)

By these definitions, components of Eq. (16) found to be

\[
(D_\mu H^5 dx^\mu \wedge dy_5)\omega = D_\mu H^5 \omega^{\mu5} dy_5 \wedge dy_5 - D_\mu H^5 \omega_5^\mu dx^\mu \wedge dx^\nu,
\]

(26)

\[
\omega^\dagger(D_\mu H^5 dx^\mu \wedge dy_5) = \omega^{\mu5} D_\mu H^5 dy_5 \wedge dy_5 - \omega_5^\mu D_\mu H^5 dx^\mu \wedge dx^\nu,
\]

(27)

\[
\omega(D_\mu H^5 dx^\mu \wedge dy^5) = \omega_{\mu5} D^\mu H^5 dy^5 \wedge dy^5 - \omega_5^\mu D_\mu H^5 dx^\mu \wedge dx^\nu,
\]

(28)

\[
(D_\mu H^5 dx^\mu \wedge dy^5)\omega = D_\mu H^5 \omega_{\mu5} dy^5 \wedge dy^5 - D_\mu H^5 \omega_5^\mu dx^\mu \wedge dx^\nu,
\]

(29)
and
\[
\omega^\dagger (H \wedge H^\dagger) = -2\omega^{\dagger}_{\mu_5} H^5 H^5_{\mu} dx^\mu \wedge dy^5, \quad (H \wedge H^\dagger) \omega = -2H^5 H^5_5 \omega^{\dagger}_{\mu_5} dx^\mu \wedge dy^5, \quad (30)
\]
\[
\omega(-H^\dagger \wedge H) = -2\omega^{\dagger}_{\mu_5} H^5 H^5_5 dx^\mu \wedge dy^5, \quad (-H^\dagger \wedge H) \omega^\dagger = -2H^5_5 H^5 \omega^{\dagger}_{\mu_5} dx^\mu \wedge dy^5. \quad (31)
\]
and
\[
F_L \omega = 2 \frac{1}{2} F_{L\mu \nu} \omega^{\dagger}_\nu dx^\mu \wedge dy^5, \quad \omega^\dagger F_L = 2 \omega^{\dagger}_{\mu_5} \frac{1}{2} F_{L\mu \nu} dx^\mu \wedge dy^5, \quad (32)
\]
\[
\omega F_R = 2 \omega^{\dagger}_{\mu_5} \frac{1}{2} F_{R\mu \nu} dx^\mu \wedge dy^5, \quad F_R \omega^\dagger = 2 \frac{1}{2} F_{R\mu \nu} \omega^{\dagger}_\nu dx^\mu \wedge dy^5. \quad (33)
\]
Comparing the coefficients of the same two-forms, the components in \( \delta F \) are found to be
\[
\delta(\frac{1}{2} F_{L\mu \nu}) = -D_{\mu} H^5 \omega^{\dagger}_{\nu_5} - \omega^{\dagger}_{\nu_5} D_{\mu} H^5_{\nu}, \quad \delta(\frac{1}{2} F_{R\mu \nu}) = +\omega^{\dagger}_{\nu_5} D_{\mu} H^5 + D_{\mu} H^5_{\nu_5} \omega^{\dagger}_{\nu}, \quad (34)
\]
\[
\delta(H^5 H^5_{\mu_5}) = D^\mu H^5 \omega^{\dagger}_{\mu_5} + \omega^{\dagger}_{\mu_5} D^\mu H^5_{\mu_5}, \quad \delta(-H^5_{\mu_5} H^5) = -\omega^{\dagger}_{\mu_5} D^\mu H^5 - D^\mu H^5_{\mu_5} \omega^{\dagger}_{\mu_5}, \quad (35)
\]
and
\[
\delta(D_{\mu} H) = 2 \left( \omega^{\dagger}_{\nu_5} \frac{1}{2} F_{R\mu \nu} + \omega^{\dagger}_{\mu_5} H^5_5 H^5_{\mu_5} - \frac{1}{2} F_{L\mu \nu} \omega^{\dagger}_{\nu_5} + H^5 H^5_{\mu_5} \omega^{\dagger}_{\mu_5} \right), \quad (36)
\]
\[
\delta(D_{\mu} H^\dagger) = 2 \left( \frac{1}{2} F_{R\mu \nu} \omega^{\dagger}_{\nu_5} + H^5_5 H^5_5 \omega^{\dagger}_{\mu_5} \frac{1}{2} - \omega^{\dagger}_{\nu_5} F_{R\mu \nu} + \omega^{\dagger}_{\mu_5} H^5 H^5_5 \right). \quad (37)
\]
Substituting these relation into Eq. (18), we can show the bosonic Lagrangian is invariant. The condition \( \delta \mathcal{L} = 0 \) requires \( \beta^4 = \alpha^2 = 1 \) in Eqs. (34-35). It is natural relation for the inner products with five dimensional Lorentz symmetry.

Actually, the Lagrangian with \( \beta^4 = \alpha^2 = 1 \)
\[
\mathcal{L}_B = -\text{Tr}(\mathbf{F}^\dagger, \mathbf{F})
\]
\[
= -\frac{1}{2} \text{tr}[F_{L\mu \nu} F_{L\mu \nu} + F_{R\mu \nu} F_{R\mu \nu}] + \text{tr}[2|D_{\mu} H|^2 - 2|H H^\dagger|^2 - 2|H^\dagger H|^2]. \quad (38)
\]
is invariant under the transformation (34-57) by the trace cyclicity:
\[
\delta \mathcal{L} = -\text{tr}[\delta F_{L\mu \nu} F_{L\mu \nu}^\dagger] - \text{tr}[\delta F_{R\mu \nu} F_{R\mu \nu}^\dagger] + 2\text{tr}[\delta(D_{\mu} H) D^\mu H^\dagger] + 2\text{tr}[D_{\mu} H \delta(D^\mu H^\dagger)]
\]
\[
- 4\text{tr}[\delta(H^\dagger H) H H^\dagger] - 4\text{tr}[\delta(H H^\dagger) H^\dagger H]
\]
\[
= + \text{tr}[2(D_{\mu} H^5 \omega^{\dagger}_{\nu_5} + \omega^{\dagger}_{\nu_5} D_{\mu} H^5_{\nu}) F_{L\mu \nu}] - \text{tr}[2(\omega^{\dagger}_{\nu_5} D_{\mu} H^5 + D_{\mu} H^5_{\nu_5} \omega^{\dagger}_{\nu}) F_{L\mu \nu}]
\]
\[
+ 2\text{tr}[(\omega^{\dagger}_{\nu_5} F_{R\mu \nu} - F_{R\mu \nu} \omega^{\dagger}_{\nu_5}) D^\mu H^\dagger] + 2\text{tr}[D_{\mu} H (F_{R\mu \nu} \omega^{\dagger}_{\nu_5} - \omega^{\dagger}_{\nu_5} F_{L\mu \nu})]
\]
\[
+ 4\text{tr}[-\omega^{\dagger}_{\nu_5} H^5_{\mu_5} H^5 + H^5 H^5_{\mu_5} \omega^{\dagger}_{\mu_5}] + 4\text{tr}[D_{\mu} H (-H^5_{\mu_5} \omega^{\dagger}_{\mu_5} + \omega^{\dagger}_{\mu_5} H^5 H^5_{\mu_5})]
\]
\[
- 4\text{tr}[(D^\mu H^5 \omega^{\dagger}_{\mu_5} + \omega^{\dagger}_{\mu_5} D^\mu H^5_{\mu_5}) H H^\dagger] + 4\text{tr}[(\omega^{\dagger}_{\mu_5} D^\mu H^5 + D^\mu H^5_{\mu_5} \omega^{\dagger}_{\mu_5}) H H^\dagger] = 0. \quad (40)
\]
Then, the bosonic Lagrangian \( L'_{B} \)

\[
L'_{B} = -\text{Tr}(F', F') = L_{B},
\]

is invariant under the transformation.

In the usual NCG theory, Higgs mass can not be prohibited by the symmetry of theory. This problem originates that the field strength \( F_{\mu} \) with the Higgs mass term

\[
\tilde{F} = \left( \begin{array}{cc}
F_{L} + H^{5}H_{5}^{\dagger}dy_{5} \wedge dy_{5} & D_{\mu}H^{5}dx^{\mu} \wedge dy_{5} \\
D_{\mu}H_{5}^{\dagger}dx^{\mu} \wedge dy_{5} & F_{R} - H_{5}^{\dagger}H^{5}dy_{5} \wedge dy_{5}
\end{array} \right)
\]

is also gauge invariant under Eq. (4), with the constant vector \( m_{\mu} \). The extended Lorentz transformation Eq. (15) or Eqs. (34-37) (with gauge invariance) clearly prohibit the second term in Eq. (43). Indeed, Eq. (35) shows the Higgs mass term will be transformed as

\[
m^{2}\text{tr}[\delta(H^{5}H_{5}^{\dagger})] = m^{2}\text{tr}[\delta(H_{5}^{\dagger}H^{5})] = m^{2}\text{tr}[D_{\mu}H^{5}\omega_{\mu5}^{\dagger} + \omega_{\mu5}^{\dagger}D_{\mu}H_{5}^{5}],
\]

and it is not invariant. Note that the change of the sign in Eq. (14) is required from the consistency of \( \text{tr}[\delta(H^{5}H_{5}^{\dagger})] = \text{tr}[\delta(H_{5}^{\dagger}H^{5})] \).

3.1 Discussions

The transformations Eqs. (34-37) are defined for the products fields. However, it is not clear whether Eqs. (34-37) can be rewritten to the transformation of single fields \( A'_{\mu} \) and \( H_{5}^{5} \). The problem is that the uniqueness of the transformation is somehow lost. When we show the invariance in Eq. (40), the potential term of Higgs is rewritten as

\[
2\text{tr}(|HH^{\dagger}|^{2}) \rightarrow |HH^{\dagger}|^{2} + |H^{\dagger}H|^{2}
\]

in Eq. (38). This modification is required by the invariance. However, the following inequality holds

\[
\text{tr}[\delta(H^{5}H_{5}^{\dagger})H^{5}H_{5}^{\dagger}] = \text{tr}[(\omega_{\mu}^{5}D_{\mu}H_{5}^{\dagger} + D_{\mu}H^{5}\omega_{\mu5}^{\dagger})H^{5}H_{5}^{\dagger}] \\
\neq \text{tr}[(\omega_{\mu5}^{\dagger}D_{\mu}H_{5}^{\dagger} + D_{\mu}H^{5}\omega_{\mu5})H^{5}H_{5}^{\dagger}] = \text{tr}[\delta(H_{5}^{\dagger}H^{5})H_{5}^{\dagger}H_{5}^{\dagger}],
\]

and then the uniqueness of the transformation for \( |H^{5}H_{5}^{\dagger}|^{2} \) is lost.

One solution of this point is redefining the transformation (37) to that of the product field \( |H^{5}H_{5}^{\dagger}|^{2} \):

\[
2\delta \text{tr}(|H^{5}|^{4}) \equiv \text{tr}[\delta(H^{5}H_{5}^{\dagger})H^{5}H_{5}^{\dagger}] + \text{tr}[\delta(H_{5}^{\dagger}H^{5})H_{5}^{\dagger}H^{5}],
\]

because the transformation is defined for product fields in the first place.

Finally, we comment on the Coleman–Mandula theorem [9]. The theorem is usually interpreted as prohibiting the symmetry between the Minkowski space and the internal
space. However, the extended Lorentz transformation is defined in the five dimensional noncommutative space. It should be broken (for example, at the Planck scale) to the direct product of the Lorentz group and the gauge groups to produce the finite Higgs mass. Then, this symmetry does not contradict to the theorem which is applied in the broken phase of the extended Lorentz symmetry. Similar discussions can be found in the graviweak theory [10].

4 Conclusions and Discussions

In this letter, we proposed the extended Lorentz transformation in noncommutative geometry, as a possibility on prohibition of the Higgs mass. Since it is difficult to build the symmetry between the connections $A_\mu$ and $H$, the transformation is defined for the differential two-forms. The parameter of the transformation $\omega$ changes a two-form into other two-forms. Comparing the coefficients of the two-forms, the transformations are translated to those of the product fields $F_{\mu\nu}, D_\mu H$ and $HH^\dagger$. It shows the invariance of the bosonic Lagrangian explicitly.

For the Lagrangian of fermions, the appropriate matrix representation of this transformation is not be found. The form of the transformation indicates that the representation space of fermions is twice larger than one of gauge connections. Such a representation might be realized by the real structure $\Psi = (\psi, \psi^c)$ [7, 8] or related concepts.

Acknowledgements: This study is financially supported by the Iwanami Fujukai Foundation, and Seiwa Memorial Foundation.

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