Aeroacoustic noise calculation of noncompact bodies with time-domain scattered green’s function

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Abstract. In view of the scattering effect induced by non-compact bodies, and the growing time cost of the noise calculation with high-order methods, tailored green function methods gradually become effective means to study acoustic noise of noncompact bodies. With the help of free-space green function and convective wave equation, time-domain integral method is presented to calculate scattered green function under arbitrary boundary condition. In order to verify the present method, the cylinder noise under solid wall condition is firstly calculated with stationary point source. The scattered green functions are in good agreement with Gloerfelt's analytical solution and Takaishi's frequency method. The pressure distribution demonstrates that the pressure appears as circle shape at low frequency but petal-like pattern at high frequency. Furthermore, point sources in parabolic moving form is chosen to study the influence of scattering effect to far field noise. Numerical results illustrate that the scattering effect is closely related to cylinder radius and the location of sound sources, large body size and sound sources adjacent closely to body surface could lead to serious scattered noise. Besides, time-domain method overcomes the limit that noise calculations have to be performed in relative coordinate system, and it can calculate scattered noise accurately and instantaneous.

1. Introduction
Numerical simulations of flow induced noise has recently received much more attention in aviation and aerospace fields, civil aviation, underwater engineering, et al, many researchers presented a series of methods to study noise and its production mechanism. Lighthill analogy theory[1] is a fundamental method for most research, Ffowcs Williams and Hawkings[2] presented a mathematical model for moving bodies in combination with free-space green function, and FW-H equation[3] was used extensively and effectively in the field of aeroacoustics. Meanwhile, Howe[4] and Powell[5] proposed the vortex sound theory which revealed the cause of noise production, and they explained why the vortices generate noise in the fluid flow. Under the premise of accurate fluid simulation, the above two methods could be directly used to perform aeroacoustic calculation. However, high-order schemes which need enough fine grids especially in three-dimensional simulation will bring great inconvenience in practical engineering. Another alternative to the aeroacoustic problem is the Kirchhoff approach[6,7] which assumes that the noise propagation is governed by the linear wave equation. The main shortcoming of this approach is its validity just in regions where the linear wave equation can describe the flow accurately, therefore it considers the characters of wave propagation but ignores the nature of noise production. Except for studying the mechanism of flow induced noise, the
scattering effect of noncompact bodies gradually becomes the key problem of numerical simulations because the influence induced by noise/solid-body interaction changes the noise field, especially for large-scale bodies and complex-shaped bodies.

At present, the noise calculation of noncompact bodies is an urgently solved problem. Hu and Jones\cite{8,9} presented scattered time-domain and frequency-domain green function satisfied solid wall condition along normal direction of body surface, whereby, basic functions and free-space green function are respectively used to construct scattered green function. There exists difficulties that the second derivatives for each discrete point have to be computed during acoustic calculation, and their solutions are only useful for solid walls. Because constructed green function is time-consuming, Scharm\cite{3} proposed the definition of pressure decomposition in which the pressure fluctuation is divided into the sum of hydrodynamic pressure fluctuation and acoustic pressure fluctuation. Under the assumption that body surfaces satisfy solid wall condition, scattered sources distributed on body surface should be firstly computed before far field noise calculation. Above two methods have advantage over high-order methods but cannot be applied for other cases with different boundary condition, where time-domain method is much convenient for moving bodies. Antes\cite{10} applied time-domain wave equation to perform numerical simulations for high-speed train. It illustrates that time-domain method could describe the characteristics of noise propagation in detail and overcome the limit of employing relative coordinate system in frequency calculation.

In order to study noise induced by moving bodies, a time-domain scattered green function is presented to perform noise calculation for arbitrary boundary condition and motion forms, the emphasis is put on calculating the scattered noise of noncompact bodies in time domain. The free-space green function is used to construct the integral solution of scattered green function under different arbitrary boundary conditions, and boundary element method (BEM)\cite{11,12} is adopted to develop numerical calculations. Two dimensional stationary circular cylinder is chosen as the computational model to investigate noise propagation. The integral solution is derived in Sec. II. In Sec. III, numerical simulation is developed. Stationary point source is firstly considered as the radiated source to inspect scattered noise, which is compared with Gloerfelt's analytical solution\cite{13} and Takaishi's frequency solution\cite{14}. Then, numerical simulations are performed to calculate scattered noise with point sources in moving form with uniform parabolic motion. The conclusion is drawn in Sec. IV.

2. Integral solution for time-domain scattered green function

The wave equation for scattered green function under arbitrary boundary condition is given as

$$
\frac{1}{c^2} \left( \frac{\partial}{\partial t} + U \cdot \nabla \right)^2 g(x, y, t, \tau) - \nabla^2 g(x, y, t, \tau) = \delta(t - \tau) \delta(x - y)
$$

$$
\left[ \alpha(y, t) g + \beta(y, t) \frac{\partial g}{\partial n} \right] = f(y, t)
$$

where \(x\) and \(y\) are the location of observation point and point source, respectively. \(g(x, y, t, \tau)\) satisfied given condition is the scattered green function, describing the noise propagation from \(y\) to \(x\). \(\alpha(y, t)\), \(\beta(y, t)\), \(f(y, t)\) are relative functions with variable \(y\). \(n\) is the normal vector of body surface, \(c\) is the wave speed.

The wave equation with convection effect of free-space green function \(g_0(z, y, t', \tau)\) is written as

$$
\frac{1}{c^2} \left( \frac{\partial}{\partial t} - U \cdot \nabla \right)^2 g_0(z, y, t', \tau) - \nabla^2 g_0(z, y, t', \tau) = \delta(t - \tau) \delta(z - y)
$$

where \(z\) is a point in the acoustic field. Eq. 2 can be used to describe the propagation of outgoing wave, and a point source moves from \(y\) to \(z\) with constant velocity \(U\).

In combination with Eq. 2, the convective wave equation related to \(g(x, y, t, \tau)\) can be cast into a formulation
\[ g(x, z, t, t') = g_0(x, z, t, t') - \frac{1}{c^2} \int \int v_n (g \frac{\partial g_0}{\partial \tau} - g_0 \frac{\partial g}{\partial \tau}) ds d\tau \\
- \int \int (g \frac{\partial g_0}{\partial n} - g_0 \frac{\partial g}{\partial n}) ds d\tau \]  \tag{3}

Where the green function satisfies \[ g_0|_r = 0, \frac{\partial g_0}{\partial n}|_r = 0 \] and Sommerfeld condition [15] in the far field.

Under the premise of known boundary condition, the scattered green function can be calculated. The left hand side of Eq.3 expresses scattered green function of noncompact bodies, the right hand side are the radiation source term, the scattered source terms induced by moving boundary and noncompact body surface, respectively. According to Eq.3, the observation points receive both radiated noise and scattered noise in the space containing noncompact bodies, it means that acoustic fluctuation produced by noise propagation also has impact on the far field pressure, and the numerical simulation will be developed in the next section.

3. Numerical simulations

The boundary condition \[ \frac{\partial g}{\partial n} \bigg|_r = 0 \] is given to verify the time domain integral solution in Eq.3. And the following formulation is obtained

\[ g(x, z, t, t') = g_0(x, z, t, t') - \int \int g \frac{\partial g}{\partial n} ds d\tau \]  \tag{4}

The scattering sources are assumed to locate on the solid wall, then Eq.8 is transformed into another formula

\[ C(y')g(x, y^t, t, t') = g_0(x, y^t, t, t') - \sum_{i=1}^{N} \sum_{\nu = 1}^{m} g(x, y^t, t, \tau) \frac{\partial g_0(y, y^t, \tau, t')}{\partial n} \Delta s \Delta t \]  \tag{5}

where, \( y^t \) represents the scattering source on solid wall, \( S'\setminus\{y'\} \) is the residue part when \( y^t \) is removed from solid surface \( S \), and \( C(y') \) is the solid angle function for source point \( y' \).

Henceforth, scattered green function of arbitrary point can be computed in combination with Eq. 4 and Eq. 5 when the two-dimensional cylinder is chosen as mathematical model. The stationary point source is firstly used as the radiation source; then the scattered green function of point sources with constant velocity is studied. Besides, the variations of acoustic field for three motion forms are compared and analyzed.

3.1. Scattered green function of circular cylinder

At first, the scattered green function induced by stationary point source is studied. Under the premise of acoustic calculation with Eq.8 and Eq.9, the Fourier modes of scattered green function are computed in contrast with Takaishi’s integral solution and Gloerfelt’s analytical solution. As sketched in figure 1, the cylindrical coordinate system is adopted. Gloerfelt’s scattered green function of two-dimensional cylinder under solid wall condition is expressed as

\[ G(x, y, \omega) = \frac{j}{4} \sum_{m=0}^{\infty} e_m \cos m \theta H_0^m(kr)[J_m(kr_0) - \alpha_m H_0^m(kr_0)] \]  \tag{6}

where \( \theta = \theta_s - \theta_s', e_m = 1 \) for \( m = 0 \); \( e_m = 2 \) for \( m > 0 \), \( \alpha_m = \frac{J_{m+1}^{(1)}(kr) - \alpha_{m+1}^{(1)}(kr)}{H_{m+1}^{(1)}(kr) - H_{m+1}^{(1)}(kr)} \) with \( r = |x - y| \).
Meanwhile, the frequency scattered green function presented by Takaishi is given as

$$C(\omega)G(x, z, \omega) = G_c(x, z, \omega) - \int_0^\infty G(x, y, \omega) \frac{\partial G_c(y, z, \omega)}{\partial n} \, ds$$  \hspace{1cm} (7)

where $\omega$ is the angular frequency, $G_c(x, z, \omega)$ is the free-space green function in frequency domain. When the cylindrical radius is given $R=0.025m$, the scattered green functions are calculated in two cases:

1) case1: a point source is located at $y=(0.04,0)$, and an observation point is located at $x=(2,0)$
2) case2: a point source is located at $y=(0.05,0.05)$, and an observation point is located at $x=(0,2)$

On the basis of time-domain calculation completion, 300 samples are continuously chosen to execute FFT method with frequency resolution $df=166.7Hz$. Firstly, the numerical comparison of three methods at discrete frequency is shown in figure 2. It demonstrates that the present method agrees well with other two methods at low and medium frequencies among three methods for two cases.

![Figure 2. Scattered green function for stationary point source at discrete frequency.](image)

Secondly, the far field directivity is investigated. The location of point source is at $y=(0.05,0.05)$, and there are 90 observation points located uniformly on a circle with radius $R_f=2$. As shown in figure 3, the directivities change from circular shape to petal-like pattern, and the amplitudes reduce gradually with frequency. This conclusion is also coincided with wave propagation rule.

![Figure 3. Directivity pattern for scattered green function.](image)
Besides, there is significant difference existed between the present method and other two methods especially at high frequencies as shown in figure 2 and figure 3. This difference mainly caused by retarded time calculation and sampling of FFT could be solved by extending calculation time and increasing number of samples. As observed above, the difference is negligible compared with green function’s amplitude. It demonstrates that time-domain integral solution and algorithm can be used to implement acoustic calculation.

3.2. Scattered green function of moving sources
The point source is moving with uniform parabolic motion, and the curve is expressed as

\[ y_1 = v(t - 0.5) \]
\[ y_2 = h[2 - \frac{v^2(t - 0.5)^2}{4r^2}] \]

where, the cylinder radius is \( r = 0.5 \) m, \( v = 2 \) m/s, \( h = 0.3 \). A point source is located at \( y_0 = (-0.1 \) m, 0.03 m) at the initial time. Figure 4 gives the motion curve of the point source.
Assuming time step is given as $\Delta t=0.001s$, the observation point is located at $x=(100r,0)$. Figure 5 shows green function in a short period from the initial time to several observing times, where the horizontal axis represents the retarded time, the vertical axis represents the amplitude of green function. The variation of green function, the scattering effect and the contribution of scattered noise to the far field are investigated and discussed. For example, in Fig.5 (a), $t=0.3s$, represents the scattered green function of $X$ is from reception time to $t=0.3s$, and the retarded time belong to $[0.15s, 0.3s]$. As observed in figure 5(a)-(d), the conclusion that all of the noise reduces on the whole is identical with the uniform linear motion. However, it is different from cylinder with radius $r=0.025m$ that the contribution of scattered noise to total noise is enhanced because of serious scattering effect induced by noncompact bodies. Besides, the contributions of radiated noise and scattered noise are variable with the location of a point source. Figure 5(c)-(d) show that scattered noise is a little larger than radiated noise when the point source gets close to body surface, and the far field noise depends on both radiated and scattered noise.

**Figure 4.** The motion curve of the Point source with uniform parabolic motion.

**Figure 5.** Point source with uniform parabolic motion: the scattered green function of point X at different reception time.
In order to investigate the physical characters of scattered noise during the noise propagation, Fig. 6 demonstrates the noise sending out at different time with point source moving in parabolic path. As Fig. 6 (a)-(b) shown, the total noise mainly depends on the radiated noise at \( t=0.01 \text{s} \) and \( t=0.05 \text{s} \). It can be explained detailely that all of the sound wave do not reach the body surface after the point source gets away from \( y_0 \) for a short time. However, the scattering sources gathered on body surface gradually increase when the point source get close to body surface. Fig. 6(c)-(e) illustrate that the scattering effect grows seriously and scattered noise becomes the main part within a short time. Then the scattering effect gradually reduces when the point source leaves far away from body surface, and the radiated noise becomes the main part as shown in Fig. 6(g)-(h).
Figure 6. Point source with uniform parabolic motion: the scattered green function of point X at different emission time.

In figure 6, the horizontal axis represents the retarded time, the vertical axis represents the amplitude of green function. For example, in the figure 6 (h), t=0.6s, represents the scattered green function of X sent out at t=0.6s, and the propagation time belong to [0.2s, 0.9s].

4. Conclusions

This paper presents time-domain integral method of green function for noncompact bodies under arbitrary boundary conditions in consideration of scattering effect induced by wave/solid-wall interaction. Under the premise of solid wall condition, the noise produced by stationary and moving point sources around cylinder are studied. The following conclusions are drawn:

(1) The numerical results obtained by the time-domain scattered green function are in good agreement with Gloerfelt's analytical solution and Takaishi's frequency method for stationary point source. Although the present method is agree well with Gloerfelt's analytical solution at low and medium frequencies, there exists little numerical differences at high frequencies which could be greatly improved by extending the retarded time and increasing number of samples.

(2) The time-domain scattered green function has advantage over other methods in that it can describe the noise propagation and the influence produced by noncompact bodies instantly, especially for moving cases. The noise propagation with point source in parabolic motion demonstrates that the scattering effect varies with retarded time and the location of sound sources. In the process of the acoustic energy attenuation, the scattered green function reduces continuously with propagation time.

(3) This time-domain integral method is helpful to study physical characters of scattered noise and its scattering effect in detail. Compared with frequency-domain method, this method overcome the limit that the aeroacoustic computations of moving bodies have to be performed in relative coordinate system, and it can be used to investigate noise propagation randomly and accurately.

5. References

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