Simple solution to the optimal deployment of cooperative nodes in two-dimensional TOA-based and AOA-based localization system

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Abstract

Cellular-based cooperative communication is a promising technique that allows cooperation among mobile devices not only to increase data throughput but also to improve localization services. For a certain number of cooperative nodes, the geometry plays a significant role in enhancing the accuracy of target. In this paper, a simple solution to the deployment of cooperative nodes aiming at the lowest geometric dilution of precision (GDOP) is proposed suitable for both time of arrival-based cooperative localization system and angle of arrival-based cooperative localization system. Inertia dependence factor is suggested to reveal the relationship among the optimal positions of each cooperative node, which extracts the inertia and the recursiveness of deployment. It is shown in simulations that the proposed solution almost achieves the same GDOP as that of global exclusive method but with less complexity.

Keywords: GDOP, Optimal placement, Cooperative node, TOA, AOA

1 Introduction

With the proliferation of mobile communication technology, wireless positioning has become an increasingly important issue and drawn plenty of attention [1]. Over the past several decades, a variety of location-based services emerged in the fields such as emergency medical systems, asset monitoring and tracking, location sensitive billing, fraud protection, fleet management, and even also including intelligent transportation systems [2, 3]. However, global positioning system (GPS) and cellular positioning system (CPS), two representative positioning technologies applied in many scenarios of daily life, still have some drawbacks [4–7]. On the one hand, in outdoor environment, GPS and CPS signals are sensitive to the interference from space weather, transformer substations, geomagnetic radiation, and etc., which leads to the degradation of the accessibility and continuity of GPS and CPS [8–11]. On the other hand, in indoor environment, GPS and CPS face a challenging radio propagation environment, including mainly multi-path effect and none-line-of-sight (NLOS) effect [12–15].

Indoor wireless communication (IWC) technology, represented by ultrasonic (US), wireless sensor networks (WSN), Bluetooth (BT), radio frequency identification (RFID) and ultra wide band (UWB), provides a novel kind of approach to the data interaction and burgeons rapidly in the last decade. Owing to their low cost, an ocean of IWC devices has been placed in indoor environment to supply context-awareness service for the target. Various positioning systems have also been proposed and unfolded satisfactory performance [16–20]. Meanwhile, thanks to the continual miniaturization of the microelectronic technology, cutting-edge cell phones equipped with IWC modules have been committed to remote control, intelligent entrance guard, instant payment, and tag identification via the connection with surrounding IWC devices. In this situation, if we regard all or part of the surrounding IWC devices that could provide spatial characteristic about the cell phone as cooperative partners, a cooperative localization system on the basis of traditional CPS or GPS can be established, which is...
reasonable to expect a higher location precision and a better stability [21–23].

As shown in Fig. 1, a typical cellular-based cooperative localization system consists of five fundamental components including general nodes (GNs), landmark nodes (LN)s, cooperative nodes (CNs), positioning targets (PTs), and info-station (IS). The GNs are the base stations (BSs) that enable to capture the external spatial characteristic and provide general CPS service to the PTs. The LN$s represent the infrastructures equipped with IWC modules like RFID readers, infrared receivers, Bluetooth receivers and WiFi access points which can interact data with PTs and afford the internal spatial characteristic about the PTs. Exclusively, we term the LN$s that participate in the localization of the PTs and serve like the partner of GNs as CNs. The PTs not only limit to the cell phones but also include other intelligent mobile devices equipped with IWC modules as well as CPS. The IS is a server connecting to both the CN and the GN to converge external and internal spatial characteristic about the PTs. Exclusively, the spatial characteristic is expressed in the form of the time stamp change, phase variation, and amplitude attenuation of the carrier signal. Triangulation, involving measuring principles such as time of arrival (TOA), angle of arrival (AOA), time difference of arrival (TDOA), and phase of arrival (POA), is one of the positioning algorithms that extracts the geometric properties of triangles from the spatial characteristic so as to estimate the target location [24]. Targeting different applications or services, these principles have unique advantages and disadvantages. Hence, using more than one type of principles at the same time could get better performance. In this paper, our attention focuses on TOA and AOA for their extensive applications and convenient implementations.

The process of tracking a single PT in cooperative localization can be briefly described as follows. First, after moving to indoor environments from the outside, the PT begins to broadcast interrogation signal and monitors the reply back from LN$s while maintaining the CPS service. Second, taking no account of accidental interference, the interrogation signal would be heard as long as the PT gets into the surveillance region of any LN. Each LN appraises its computational resource such as power dissipation capacity, arithmetic speed, and dynamic link stability. If the computational resource meets the scheduled requirements of cooperative localization, the LN transmits the acknowledgement signal to the PT and prepares for the cooperation. Third, after verifying the whole acknowledgement signals, the PT chooses a part of LN$s, labels them as CNs, and sends ranging signal to them. IS discriminates the ranging signal’s difference between the transmitter and receiver, and extracts effective indoor spatial characteristic by leveraging measuring principles such as TOA or AOA. Finally, the location of the PT is estimated by converging the indoor spatial characteristic from the CN and the outdoor spatial characteristic from the GN.

Evidently, a larger number of CNs ensure a higher precision of location but at the cost of more computational complexity and more power consumption involved [25]. For a certain number of CNs, the geometry plays a vital part in the performance of the cooperative positioning system. CNs cited in different position may result in different positioning accuracy. Geometric dilution of precision (GDOP), a relatively convenient indicator calculated on the basis of Cramer-Rao bound (CRB), is commonly used to describe the influence of geometry on the relation between the position determination error and the measurement error [26–31]. Low GDOP is a guarantee for all highly accurate wireless location systems. D. J. Torrieri firstly defined GDOP and analysed the performance of the two most important passive location systems for stationary transmitters [26]. N. Levanon examined the position determination in two-dimensional TOA scenarios and proved that the lowest possible GDOP attainable from range or pseudo-range measurements to

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**Fig. 1** The components of a typical cellular-based cooperative localization system.
optimal located points is $2/\sqrt{N}$ [27]. A.G. Dempster obtained the expression of GDOP for two-dimensional AOA scenarios [28]. In [29], P. Deng et al. studied the relationship between GDOP and GN number at the centre of a polygon with and without the correlation between TDOA measurements considered. A GDOP method for computing TDOA and TDOA/DAOA under non-line-of-sight environment was also presented. Quan investigated GDOP in absolute range-based wireless location systems with emphasis on its low bounds and introduced angle of coverage as a parameter to describe the geometry between a target and the measuring points [30].

In addition, an efficient deployment scheme of CNs in TOA positioning systems, which aims to minimize the GDOP, is proposed on the basis of generalized eigenvalue decomposition of fisher information matrix (FIM) [31]. In this paper, we termed it as eigenvalue-based approach for ease of illustration. The eigenvalue-based approach, which could obtain optimal performance for one CN and suffer only small loss of performance for more CNs, can dramatically reduce the computational complexity compared with the exhaustive method. Developing a practical arrangement out of the eigenvalue-based approach, however, entails some challenges. First, the eigenvalue-based approach only describes the procedure of scheme rather than theoretically infers the expression of the eigenvalue and eigenvector. Variation of each CN has not been thoroughly investigated and introduced angle of coverage as a parameter to describe the geometry between a target and the measuring points [30].

In this paper, we propose another solution to the deployment of CN suitable for both TOA-based cooperative localization system and AOA-based cooperative localization system. Optimal placement mechanism based on partial differentiation, termed as OPMPD, is suggested to expose the optimal position of CN and achieve a more explicit solution. Inertia dependence factor is also introduced to reveal the relationship among the optimal position of each CN, which extracts the inertia and the recursiveness of deployment.

The rest of the paper is organized as follows. CRB based on FIM is reviewed in “Section 2.” In “Section 3,” we respectively deduce the GDOP expressions of two-dimensional TOA-based cooperative localization system and AOA-based cooperative localization system. Our new solution aiming at the lowest GDOP is detailed in “Section 4.” Simulation results are illustrated in “Section 5,” which proves the effectiveness and the efficiency of the proposed approach. Finally, the conclusion of our work is presented in “Section 6.”

2 Cramer-Rao bound based on FIM

The CRB, which can be derived by the inverse of the FIM, provides the lowest limit on estimation accuracy of any unbiased estimator. Suppose a parameter vector to be estimated is $\theta = [\theta_1, \theta_2, \ldots \theta_p]^T$, $\theta_p$ stands for the $p$th unknown parameter, $p \in [1, P]$, $P$ denotes the number of parameters to be estimated. The FIM is then

$$J(\theta) = -E\left\{(\frac{\partial \ln f(\Lambda|\theta)}{\partial \theta})(\frac{\partial \ln f(\Lambda|\theta)}{\partial \theta})^T\right\}$$

(1)

where $J(\theta)$ is a $P \times P$ matrix, $f(\Lambda|\theta)$ is the likelihood function of observation vector $\Lambda$, and $E$ represents the expectation operation. Assume that $f(\Lambda|\theta)$ follows $N$ dimensional Gaussian distribution denoted as

$$f(\Lambda|\theta) = \frac{1}{\sqrt{(2\pi)^N|Q|}} e^{-\frac{1}{2}(\Lambda-\mu(\theta))^TQ^{-1}(\Lambda-\mu(\theta))}$$

(2)

where $N$ is the scale of $\Lambda$, $Q$ is the covariance matrix with $\theta$, $\mu(\theta)$ is the expectation of $\Lambda$, and $(\cdot)^T$ and $(\cdot)^{-1}$ respectively represent the transpose and inverse of the matrix. Consider $H = \frac{\partial \mu(\theta)}{\partial \theta}$ as the Jacobian matrix and substitute (2) into (1), $J(\theta)$ will be updated to $J(\theta) = H^TQ^{-1}H$, and the CRB of $\theta$ can be expressed as

$$\text{var} \left( \hat{\theta} \right) \geq \text{diag} \left( J^{-1}(\theta) \right) = \text{diag} \left( (H^TQ^{-1}H)^{-1} \right)$$

(3)

where var is the variance matrix of unbiased estimators $\hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \ldots \hat{\theta}_p]^T$ and diag denotes the diagonal elements of a matrix.

3 GDOP of two-dimensional localization systems

3.1 GDOP of typical localization system

Consider a two-dimensional typical localization system with a single PT, $N$ GNs. Let $\theta = [x, y]^T$ implies the unknown location of PT and $(x_n, y_n)$ denotes the known location of the $n$th GN. $n \in [1, N]$. Accordingly, observation vector $\Lambda$ in TOA measurement is $[d_1, d_2, \ldots d_N]^T$ in which $d_n$ denotes the measured distance between the PT and the $n$th GN given by

$$d_n = ct = \sqrt{(x-x_n)^2 + (y-y_n)^2} = \overline{d_n} + e_n$$

(4)

Here, $c$ denotes the light speed. $t$ denotes the absolute arrival time. $\overline{d_n}$ stands for the theoretical value. $e_n$ stands for the measurement error that follows the Gaussian
distribution with zero mean and $\sigma_n$ root-mean-square error. Given the assumption that the measured values in $\Lambda$ are mutually independent, the cross-covariance matrix can be rewritten as

$$\text{var}(\boldsymbol{\theta}) = \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix}$$  \hspace{1cm} (5)$$

where $\sigma_x^2$ and $\sigma_y^2$ represent the variance of $x$ and $y$, respectively.

According to the definition in [26], the GDOP of TOA system can be expressed as

$$\text{GDOP} = \frac{\sqrt{\sigma_x^2 + \sigma_y^2}}{\sigma_D}$$  \hspace{1cm} (6)$$

with $\sigma_D = \frac{1}{N} \sum_{n=1}^{N} \sigma_n$, where $\sigma_D$ is the root-mean-square ranging error of system. Suppose $\sigma_1 = \sigma_2 = \sigma_3 = \ldots = \sigma_n = \sigma_n$, (6) can be simplified as $\text{GDOP} = \sqrt{G_{11} + G_{22}}$, where $G_{11}$ and $G_{22}$ are the diagonal elements of $G$ given by

$$G = \frac{1}{\sigma_D} (H^T \mathbf{Q}^{-1} H)^{-1}$$  \hspace{1cm} (7)$$

Considering that all the measurement errors are independent, we have $\mathbf{Q} = \sigma_n^2 \mathbf{I}$ with an $N \times N$ identity matrix $\mathbf{I}$. Then the $G$ will become

$$G = (H^T H)^{-1}$$  \hspace{1cm} (8)$$

Correspondingly, the Jacobian matrix $H$ of TOA is

$$H_{\text{TOA}} = \begin{bmatrix} x-x_1 & y-y_1 \\ d_1 & d_1 \\ x-x_2 & y-y_2 \\ d_2 & d_2 \\ x-x_N & y-y_N \\ d_N & d_N \end{bmatrix} = \begin{bmatrix} -\cos \alpha_1 & -\sin \alpha_1 \\ -\cos \alpha_2 & -\sin \alpha_2 \\ \vdots & \vdots \\ -\cos \alpha_N & -\sin \alpha_N \end{bmatrix}$$  \hspace{1cm} (9)$$

where $\alpha_n$ is the angle of the ray from the PT to the $n$th GN relative to the positive $X$-axis.

Substituting (9) into (8), we have

$$G_{\text{TOA}} = \begin{bmatrix} \sum_{n=1}^{N} \sin^2 \alpha_n & \sum_{n=1}^{N} \cos \alpha_n \sin \alpha_n \\ \sum_{n=1}^{N} \cos \alpha_n \sin \alpha_n & \sum_{n=1}^{N} \cos^2 \alpha_n \end{bmatrix} \sum_{n=1}^{N-1} \sum_{m=n+1}^{N} \sin^2(\alpha_m-\alpha_n)$$  \hspace{1cm} (10)$$

Therefore, the GDOP of TOA can be derived as

$$\text{GDOP}_{\text{TOA}} = \frac{N}{\sqrt{\sum_{n=1}^{N-1} \sum_{m=n+1}^{N} \sin^2(\alpha_m-\alpha_n)}}$$  \hspace{1cm} (11)$$

Similarly, the Jacobian matrix and GDOP of AOA can be obtained with observation vector $\Lambda$ replaced by $[\alpha_1, \alpha_2, \ldots, \alpha_N]^T$

$$H_{\text{AOA}} = \begin{bmatrix} -\frac{y-y_1}{d_1} & \frac{x-x_1}{d_1} \\ -\frac{y-y_2}{d_2} & \frac{x-x_2}{d_2} \\ \vdots & \vdots \\ -\frac{y-y_N}{d_N} & \frac{x-x_N}{d_N} \end{bmatrix} = \begin{bmatrix} \frac{\sin \alpha_1}{d_1} & \frac{-\cos \alpha_1}{d_1} \\ \frac{\sin \alpha_2}{d_2} & \frac{-\cos \alpha_2}{d_2} \\ \vdots & \vdots \\ \frac{\sin \alpha_N}{d_N} & \frac{-\cos \alpha_N}{d_N} \end{bmatrix}$$  \hspace{1cm} (12)$$

$$\text{GDOP}_{\text{AOA}} = \frac{\sum_{n=1}^{N} d_n^{-2}}{\sqrt{\sum_{n=1}^{N-1} \sum_{m=n+1}^{N} d_m^{-2} d_n^{-2} \sin^2(\alpha_m-\alpha_n)}}$$  \hspace{1cm} (13)$$

3.2 GDOP of cooperative localization system

The key idea of cooperative localization systems is improving the accuracy of PTs by introducing more LNs and filtering out CNs from them. Theoretically, the more CN employed, the lower GDOP received. In this section, assume that there are totally $M$ CNs to be allocated to a single PT for reducing the GDOP. After the implementation of assignment, the Jacobian matrices of the observation vector of TOA-based and AOA-based cooperative localization systems are respectively refreshed to

$$H_{\text{C-TOA}} = \begin{bmatrix} H_{\text{TOA}}^T & H_{\text{C-TOA}}^T \end{bmatrix}^T$$  \hspace{1cm} (14)$$

$$H_{\text{C-AOA}} = \begin{bmatrix} H_{\text{AOA}}^T & H_{\text{C-AOA}}^T \end{bmatrix}^T$$  \hspace{1cm} (15)$$

with

$$H_{\text{C-TOA}} = \begin{bmatrix} x-x_1 & y-y_1 \\ r_1 & r_1 \\ x-x_2 & y-y_2 \\ r_2 & r_2 \\ \vdots & \vdots \\ x-x_M & y-y_M \\ r_M & r_M \end{bmatrix} = \begin{bmatrix} -\cos \beta_1 & -\sin \beta_1 \\ -\cos \beta_2 & -\sin \beta_2 \\ \vdots & \vdots \\ -\cos \beta_M & -\sin \beta_M \end{bmatrix}$$  \hspace{1cm} (16)$$
where $\beta_m$ is the angle of the ray from the PT to the $m$th CN relative to the positive $X$-axis, $(x_{cm}, y_{cm})$ denotes the coordinate of the $m$th CN, and $r_m$ denotes the distance between the PT and the $m$th CN. Geometry of the nodes in two-dimensional cooperative localization scenario is illustrated in Fig. 2.

Hence, the GDOP with respect to $M$ CNs can be calculated as

$$GDOP_{\text{TOA}} = \sqrt{\frac{N+M}{\sum_{n=1}^{N} \sum_{m=1}^{M} \sin^2(\beta_m - \alpha_n) + \sum_{n=1}^{N} \sum_{m=1}^{M} \sin^2(\alpha_m - \beta_n) + \sum_{n=1}^{N} \sum_{m=1}^{M} \sin^2(\alpha_n - \beta_m)}}$$  \hspace{1cm} (18)

and

$$GDOP_{\text{AOA}}^{\text{M}} = \sqrt{\frac{N \sum_{n=1}^{N} d_n^2 + M \sum_{m=1}^{M} r_m^{-2}}{\Sigma_1 + \Sigma_2 + \Sigma_3}}$$  \hspace{1cm} (19)

4 OPMPD approach

From (18) and (19), we could find that the GDOP of two-dimensional TOA-based cooperative localization systems is affected by the azimuth of the CNs, whereas the GDOP of the AOA-based is relying on both the azimuth and the distance between the PT and the CN. Theoretically, the deployment of cooperative nodes is a multiple parameter optimization problem with high complexity, especially for the large-scale CNs instances. In order to simplify the multiple parameter optimization problem, the OPMPD approach employs stepwise strategy to obtain the optimal position of each CN step by step just as eigenvalue-based approach. Whereas, OPMPD approach has three major advantages. First, OPMPD approach is deduced on the basis of partial differentiation which provides a more explicit scheme. Since there is no need to calculate the generalized eigenvalue and eigenvector of FIM, unnecessary computation is avoided. Second, inertia dependence factor is introduced to indicate the relationship among the optimal position of each CN, which suggests a discipline of the deployment. Third, OPMPD approach is suitable for both two-dimensional TOA-based cooperative localization system and AOA-based cooperative localization system.

4.1 Objective function based on stepwise strategy

In OPMPD approach, the optimal position of the CN to be assigned depends on not only the geometry of the PT and the GN but also the location of the CN already deployed since each CN is placed in sequence. To begin with, assume the first CN shall be placed, the GDOP of the TOA-based cooperative system is found to be

$$GDOP_{\text{C-TOA}} = \sqrt{\frac{N+1}{\sum_{n=1}^{N} \sin^2(\beta_1 - \alpha_n) + \sum_{n=1}^{N-1} \sum_{m=n+1}^{N} \sin^2(\beta_n - \alpha_m)}}$$  \hspace{1cm} (20)

Obviously, the current GDOP is only relevant to $\beta_1$ since $\alpha_1, \alpha_2, ..., \alpha_N$ is known. Considering that the minimization of GDOP is equivalent to the maximization of $\sum_{n=1}^{N} \sin^2(\beta_1 - \alpha_n)$, we extract the component relevant to $\beta_1$ and define the initial observation function as

\[
H_{\text{C-AOA}} = \begin{bmatrix}
\frac{y - y_{c1}}{r_1} & \frac{x - x_{c1}}{r_1} & \cdots & \frac{y - y_{cM}}{r_M} & \frac{x - x_{cM}}{r_M}
\end{bmatrix} = \begin{bmatrix}
\frac{\sin \beta_1}{r_1} & \frac{-\cos \beta_1}{r_1} & \cdots & \frac{-\cos \beta_M}{r_M}
\end{bmatrix}
\]

(17)
\[ Q(\beta) = \sum_{n=1}^{N} \sin^2(\beta - \alpha_n) \quad (21) \]

Accordingly, if the former \((m-1)\) CNs have been placed, the observation function with respect to the \(m\)th CN will be updated to

\[ Q(\beta_m) = \sum_{n=1}^{N} \sin^2(\beta_m - \alpha_n) + \sum_{j=1}^{m-1} \sin^2(\beta_m - \beta_{OJ,T}) \quad (22) \]

where \(\beta_{OJ,T}\) denotes the best azimuth of the \(j\)th CN that has been acquired.

Similarly, the GDOP of the AOA cooperative system can be expressed as

\[
G\text{DOP}_{\text{C-\text{AOA}}} = \sqrt{\frac{r_1^2 + \sum_{n=1}^{N} d_n^{-2}}{r_1^2 \sum_{n=1}^{N} d_n^{-2} \sin^2(\beta_1 - \alpha_n) + \sum_{n=1}^{N} \sum_{m=1+1}^{N} d_n^{-2} d_m^{-2} \sin^2(\alpha_n - \alpha_m)}} \quad (23)
\]

Suppose each \(r_m\) is a fixed value, then we have

\[ Q(\beta_1) = \sum_{n=1}^{N} d_n^{-2} \sin^2(\beta_1 - \alpha_n) \quad (24) \]

\[ Q(\beta_m) = \sum_{n=1}^{N} d_n^{-2} \sin^2(\beta_m - \alpha_n) + \sum_{j=1}^{m-1} r_j^{-2} \sin^2(\beta_m - \beta_{OJ,T}) \quad (25) \]

In addition, we particularly discuss the effect of \(r_m\) in section 4.3 to fully analyze the performance of GDOP of AOA-based cooperative localization system.

### 4.2 OPMPD approach in TOA-based cooperative localization system

The core concept of OPMPD approach is to get the extremum of \(Q(\beta_m)\) by calculating the partial derivative with respect to \(\beta_m\). Above all, we built the first-order partial differential equations to search the arrest points. For TOA-based cooperative localization systems, let \(\partial Q(\beta_1)/\partial \beta_1 = 0\) and define \(\beta^1_{1,T}\) and \(\beta^2_{1,T}\) as the arrest points of \(Q(\beta_1)\), then we have

\[
\begin{align*}
\sin 2\beta^1_{1,T} & = \cos 2\beta^1_{1,T} \sum_{n=1}^{N} \cos 2\alpha_n = \sum_{n=1}^{N} \sin 2\alpha_n \\
\sin 2\beta^2_{1,T} & = \cos 2\beta^2_{1,T} \sum_{n=1}^{N} \cos 2\alpha_n = \sum_{n=1}^{N} \sin 2\alpha_n
\end{align*}
\]

(26)

Suppose \(\sum_{n=1}^{N} \cos 2\alpha_n \neq 0\), \(\cos 2\beta^1_{1,T} \neq 0\), and \(\cos 2\beta^2_{1,T} \neq 0\), (26) can be refreshed according to the equal ratios theorem

\[
\frac{\sin 2\beta^1_{1,T}}{\cos 2\beta^1_{1,T}} = \frac{\sin 2\beta^2_{1,T}}{\cos 2\beta^2_{1,T}} = \sum_{n=1}^{N} \frac{\sin 2\alpha_n}{\cos 2\alpha_n} \quad (27)
\]

Assume \(\beta^2_{1,T} > \beta^1_{1,T}\), then

\[
\begin{align*}
\beta^1_{1,T} & = \frac{1}{2} \arctan \left( \sum_{n=1}^{N} \frac{\sin 2\alpha_n}{\cos 2\alpha_n} \right) \in \left[ -\frac{\pi}{4}, \frac{\pi}{4} \right] \quad (28) \\
\beta^2_{1,T} & = \beta^1_{1,T} + \frac{1}{2} \pi \in \left[ \frac{\pi}{4}, \frac{3\pi}{4} \right]
\end{align*}
\]

For the depiction and distinction, we term \(\beta^1_{1,T}\) and \(\beta^2_{1,T}\) in (28) as positive azimuth factor and negative azimuth factor of the first CN, respectively.

Substitute \(\beta^1_{1,T}\) and \(\beta^2_{1,T}\) into the second-order partial derivative \(Q^\prime\) respectively to find which azimuth factor would obtain the maximum of observation function, then we have

\[
Q^\prime (\beta^1_{1,T}) = 2|A_1||\beta^1_{1,T}||\cos g^1; Q^\prime (\beta^2_{1,T}) = 2|A_1||\beta^2_{1,T}||\cos g^2
\]

(29)

with

\[
\begin{align*}
\bar{A}_1 & = \mathfrak{R} + jk = \sum_{n=1}^{N} \cos 2\alpha_n + j \sum_{n=1}^{N} \sin 2\alpha_n, \\
B^1_{1,T} & = \cos 2\beta^1_{1,T} + j \sin 2\beta^1_{1,T}, \\
B^2_{1,T} & = \cos 2\beta^2_{1,T} + j \sin 2\beta^2_{1,T}
\end{align*}
\]

where \(\bar{A}_1\) is the azimuth vector of system, and \(\mathfrak{R}\) is the cosine azimuth coefficient. \(B^1_{1,T}\) and \(B^2_{1,T}\) are defined as the positive azimuth vector and negative azimuth vector of the first CN, \(j\) denotes the imaginary unit, \(g^1\) denotes the angle between \(\bar{A}_1\) and \(B^1_{1,T}\) and \(g^2\) denotes the angle between \(\bar{A}_1\) and \(B^2_{1,T}\).

From (27), (28), (29) and (30), we could draw three useful conclusions. First, the value of \(\cos(g^1)\) is either 1 or \(-1\). So does \(\cos(g^2)\). Second, the direction of \(B^1_{1,T}\) is opposite to that of \(B^2_{1,T}\). Third, if \(\mathfrak{R} < 0\), the azimuth of \(\bar{A}_1\) is confined from \(\pi/2\) to \(3\pi/2\), thereby leading to \(G\text{DOP}_{\text{TOA}}\) achieving the minimum value with \(\cos g^1 = -1\) and \(Q^\prime (\beta^1_{1,T}) < 0\). Conversely, if \(\mathfrak{R} > 0\), the azimuth of \(\bar{A}_1\) is confined from \(-\pi/2\) to \(\pi/2\), which means that \(G\text{DOP}_{\text{TOA}}(\beta^2_{1,T})\) would get the minimum value with \(\cos g^2 = -1\) and \(Q^\prime (\beta^2_{1,T}) < 0\).
Consequently, a deployment mechanism of the first CN can be summarized as

$$\beta_{O,1,T} = \begin{cases} 
\text{random if } \Re = 0 \\
\beta_{1,T}^1 \text{ if } \Re < 0 \\
\beta_{1,T}^2 \text{ if } \Re > 0 
\end{cases}$$  \hspace{0.5cm} (31)

where $\beta_{O,1,T}$ represents the optimal azimuth corresponding to the least GDOP and $B_{O,1,T} = \cos 2\beta_{O,1,T} + j \sin 2\beta_{O,1,T}$ represents the optimal azimuth vector of the first CN.

After the first CN is arranged at the optimum position, both the azimuth vector of system and the objective function will be refreshed to

$$\bar{A}_T = \Re + j\kappa = \left(\cos 2\beta_{O,1,T} + \sum_{n=1}^{N} \cos 2\alpha_n\right) + j\left(\sin 2\beta_{O,1,T} + \sum_{n=1}^{N} \sin 2\alpha_n\right)$$  \hspace{0.5cm} (32)

$$Q(\beta_2) = \sum_{n=1}^{N} \sin^2(\beta_2 - \alpha_n) + \sin^2(\beta_2 - O,1,T)$$  \hspace{0.5cm} (33)

Define $\beta_{2,T}^1$ and $\beta_{2,T}^2$ as the arrest points of $Q(\beta_2)$, then we have

$$\beta_{2,T}^1 = \beta_{1,T}^1, \hspace{0.2cm} \beta_{2,T}^2 = \beta_{1,T}^2$$  \hspace{0.5cm} (34)

since

$$\frac{\sin 2\beta_{2,T}^1}{\cos 2\beta_{2,T}^1} = \frac{\sin 2\beta_{2,T}^2}{\cos 2\beta_{2,T}^2}$$

\[= \sum_{n=1}^{N} \frac{\sin 2\alpha_n + \sin 2\beta_{O,1,T}}{\cos 2\alpha_n + \cos 2\beta_{O,1,T}} \]

\[= \sum_{n=1}^{N} \sin 2\alpha_n \}

\[= \sum_{n=1}^{N} \cos 2\alpha_n \}

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The optimal azimuth of the second CN can be determined as $\Re$ updated to $\sum_{n=1}^{N} \cos 2\alpha_n + \cos 2\beta_{O,1,T}$. From (35), it is clear that either the positive azimuth factor or the negative azimuth factor of each successive CN equals to that of the first CN. Hence, we define $\beta_{1,T}^1$ and $\beta_{1,T}^2$ as the positive azimuth factor and negative azimuth factor of the system where $\beta_{1,T}^1 = \beta_{1,T}^1, \beta_{1,T}^2 = \beta_{1,T}^1$.

Suppose the absolute value of initial cosine azimuth coefficient is larger than 2, the absolute value of cosine azimuth coefficient refreshed will decrease as the CN is introduced since the directions of $B_{O,1,T}$ and $B_{O,2,T}$ are opposite to that of initial $A_T$. That is

$$\left| \cos 2\beta_{O,1,T} + \cos 2\beta_{O,2,T} + \sum_{n=1}^{N} \cos 2\alpha_n \right| < \left| \cos 2\beta_{O,1,T} + \sum_{n=1}^{N} \cos 2\alpha_n \right| \hspace{0.5cm} (36)$$

Moreover, if initial cosine azimuth coefficient is even greater, there exists a theoretical quantity of the CNs termed as $Z_T$ satisfying

$$\beta_{O,Z_T+1,T} \neq \beta_{O,Z_T,T} \ldots = \beta_{O,2,T} = \beta_{O,1,T}$$  \hspace{0.5cm} (37)

which means that the optimal azimuth of the former $Z_T$ CNs equal to $\beta_{O,1,T}$ rather than $\beta_{O,Z_T+1,T}$.

For simplicity, we define $Z_T$ as inertia dependency factor and estimate it by

$$\begin{cases} 
(Z_T-1) \cdot \cos 2\beta_{O,1,T} + \sum_{n=1}^{N} \cos 2\alpha_n \sum_{n=1}^{N} \cos 2\alpha_n > 0 \\
Z_T \cos 2\beta_{O,1,T} + \sum_{n=1}^{N} \cos 2\alpha_n \sum_{n=1}^{N} \cos 2\alpha_n < 0 
\end{cases}$$  \hspace{0.5cm} (38)

From (38), we have

$$\begin{cases} 
1 + (Z_T-1) V > 0 \\
Z_T V + 1 < 0 
\end{cases}$$  \hspace{0.5cm} (39)

with

$$V = \frac{\sin 2\beta_{O,1,T}}{\sum_{n=1}^{N} \sin 2\alpha_n} = \frac{\cos 2\beta_{O,1,T}}{\sum_{n=1}^{N} \cos 2\alpha_n}$$  \hspace{0.5cm} (40)

Hence,

$$Z_T = \sqrt{\left(\sum_{n=1}^{N} \sin 2\alpha_n \right) + \left(\sum_{n=1}^{N} \cos 2\alpha_n \right)^2}$$  \hspace{0.5cm} (41)

where \(\lceil \cdot \rceil\) denotes the operation of rounding up.

The effect of the magnitude of $M$ on the deployment is also taken into consideration in our research. Assume the initial value of $\Re$ is less than zero. If $Z_T \geq M$, the relationship among the optimal azimuth of each CN is found to be

$$\beta_{O,M,T} \ldots = \beta_{O,2,T} = \beta_{O,1,T} = \beta_{T}$$  \hspace{0.5cm} (42)

On the contrary, if $Z_T < M$, it is necessary to explore the optimal azimuth of CN whose index is larger than $Z_T$. For the $(Z_T + 1)th$ CN to be placed, $\Re$ is updated to
\[ \mathcal{R} = Z_T \cos 2\beta_T + \sum_{n=1}^{N} \cos 2\alpha_n \] (43)

Since \( Z_T \cos 2\beta_T + \sum_{n=1}^{N} \cos 2\alpha_n < 0 \), we have

\[ \beta_{O,Z_1+1,T} = \beta_T^2 \] (44)

Then, for the \((Z_T + 2)\)th CN to be placed, we have

\[ \mathcal{R} = Z_T \cos 2\beta_T^1 + \sum_{n=1}^{N} \cos 2\alpha_n \]
\[ + \cos 2\beta_{O,Z_T+1,T} \]
\[ = (Z_T - 1) \cos 2\beta_T^1 + \sum_{n=1}^{N} \cos 2\alpha_n \] (45)

So

\[ \beta_{O,Z_{t+2}} = \beta_T^1 \] (46)

Consequently, the relationship among the optimal azimuth of each CN is found to be

\[ \beta_{O,Z_{t+2k,T}} = \ldots = \beta_{O,Z_{t+4,T}} = \beta_{O,Z_{t+2,T}} \]
\[ = \beta_{O,Z_{t,T}} = \ldots = \beta_{O,Z_{t+2,T}} = \beta_{O,Z_{t+1,T}} \]
\[ = \beta_{O,Z_{t+2k-1,T}} = \ldots = \beta_{O,Z_{t+1,T}} \]
\[ = \beta_{O,Z_{t+1,T}} = \beta_T^2 \] (47)

with

\[ K = \lfloor 0.5(M + 1 - Z_T) \rfloor \] (48)

where \( \lfloor \cdot \rfloor \) denotes the operation of rounding down. Similar conclusion can also be derived in the case of the initial value of \( \mathcal{R} \) real or greater than zero.

Hence, a solution to the optimal deployment of cooperative nodes in two-dimensional TOA-based localization system aiming at the lowest GDOP can be summarized as follows.

Step 1: According to the spatial relationship between the GN and PT, establish the initial observation function and calculate \( \beta_T^1, \beta_T^2, Z_T \) and the initial \( \mathcal{R} \).

Step 2: Determine the optimal azimuth of the first CN from \( \beta_T^1 \) and \( \beta_T^2 \), according to the polarity of \( \mathcal{R} \).

Step 3: If \( Z_T < M \), deploy the whole CNs in the same azimuth as the first CN assigned.

Step 4: If \( Z_T \geq M \), deploy the former \( Z_T \) CNs in the same azimuth as the first CN assigned while place the \((Z_T + 1)\)th CN in the azimuth vertical to that of the first CN. Then, \( \beta_T^1 \) and \( \beta_T^2 \) are alternatively chosen as the optimal azimuth of the rest CNs.

### 4.3 OPMPD approach in AOA-based cooperative localization system

As mentioned in the initial portion of the “Section 4,” in two-dimensional AOA-based localization system, both the azimuth and distance of each CN exert an influence on the performance of the GDOP.

First of all, we analyze the influence of \( \beta_1 \) on the value of \( \text{GDOP}_{1, \text{AOA}}^1 \) depicted in (23) assuming each \( r_m \) is a fixed value. With the help of a method similar to that in TOA-based system, the positive azimuth factor and negative azimuth factor of the system can be derived as

\[ \beta_A = \beta_{1,a} = 0.5 \times \arctan \left( \frac{\sum_{n=1}^{N} d_{n}^2 \sin 2\alpha_n}{\sum_{n=1}^{N} d_{n}^2 \cos 2\alpha_n} \right) \]
\[ \beta_A = \beta_{1,y} = \beta_{1,a} + 0.5\pi \] (49)

where \( \beta_{1,a} \) and \( \beta_{1,y} \) denote the arrest points of the observation function depicted in (24). The azimuth vector of system can also be obtained by

\[ \bar{A} = \mathcal{R} + \mathcal{J} = \sum_{n=1}^{N} d_{n}^2 \cos 2\alpha_n + \sum_{n=1}^{N} d_{n}^2 \sin 2\alpha_n \] (50)

Accordingly, the optimal azimuth of the first CN is updated to

\[ \beta_{O,1,a} = \begin{cases} \text{random} & \text{if } \mathcal{R} = 0 \\ \beta_{1,a} & \text{if } \mathcal{R} < 0 \\ \beta_{1,y} & \text{if } \mathcal{R} > 0 \end{cases} \] (51)

where \( \beta_{O,1,a} \) represents the optimal azimuth corresponding to the least GDOP.

Secondly, we further discuss the influence of \( r_1 \) on \( \text{GDOP}_{1, \text{AOA}}^1 \). Suppose the scope of \( r_m \) follows

\[ r_{1, \text{min}} \leq r_m \leq r_{1, \text{max}} \] (52)

where \( r_{1, \text{min}} \) and \( r_{1, \text{max}} \) denote the minimum and maximum. For simplicity, let

\[ C = r_T^2, D = \sum_{n=1}^{N} d_{n}^2, E = \sum_{n=1}^{N} d_{n}^2 \sin^2 (\beta_1 - \alpha_n), F = \sum_{n=1}^{N} d_{n}^2 d_{m}^2 \sin^2 (\alpha_n - \alpha_m) \] (53)

The first-order derivative of \( \text{GDOP}_{1, \text{AOA}}^1 \) with respect to \( r_1 \) is expressed as follows:
\[ \frac{\partial \text{GDOP}_{\text{AOA}}(r_1)}{\partial r_1} = \frac{1}{2 \text{GDOP}^2_{\text{AOA}}} \frac{-2(F-DE)}{r_1^3 (F + CE)^2} \]  

(54)

Here, we pay our attention to the polarity of \((F - DE)\) to investigate the monotone property of \(\text{GDOP}^3_{\text{AOA}}\). Based on (53), we have

\[ 2E-D = \left( \cos 2\beta \sum_{n=1}^{N} \cos 2\alpha_n d_n^{-2} + \sin 2\beta \sum_{n=1}^{N} \sin 2\alpha_n d_n^{-2} \right) \]

\[ = \sqrt{\left( \sum_{n=1}^{N} d_n^{-4} \right) + 2 \sum_{n=1}^{N} \sum_{m=n+1}^{N} d_n^{-2} d_m^{-2} (1 - 2 \sin^2 (\alpha_n - \alpha_m))} \]  

(55)

\[ D^2 - 4F = \left( \sum_{n=1}^{N} d_n^{-2} \right)^2 - 4 \sum_{n=1}^{N} \sum_{m=n+1}^{N} d_n^{-2} d_m^{-2} \sin^2 (\alpha_n - \alpha_m) \]

\[ = \left( \sum_{n=1}^{N} d_n^{-4} \right) + 2 \sum_{n=1}^{N} \sum_{m=n+1}^{N} d_n^{-2} d_m^{-2} (1 - 2 \sin^2 (\alpha_n - \alpha_m)) \]  

(56)

\[ 2F - D^2 = - \left( \sum_{i=1}^{N} d_i^{-4} \right) + 2 \sum_{i=1}^{N} \sum_{j=i+1}^{N} d_i^{-2} d_j^{-2} (\sin^2 (\alpha_i - \alpha_j) - 1) < 0 \]  

(57)

From (55), (56), and (57), we have

\[ \sqrt{D^2 - 4F} = 2E - D \geq 0 \geq \frac{2F - D^2}{D} \]  

(58)

Furthermore, we have

\[ F - DE \leq 0 \]  

(59)

Obviously, \(\text{GDOP}^3_{\text{AOA}}\) is an increasing function with respect to \(r_1\), which means \(r_{min}\) is the optimal distance of the first CN. Similar conclusion is available for the other CN. After the former \(m\) CN have been placed, we have

\[ \mathcal{R} = \sum_{j=1}^{m} r_{j,\text{min}}^2 \cos 2\beta_{n,\text{OA}} + \sum_{n=1}^{N} d_n^{-2} \cos 2\alpha_n \]

\[ \beta_{n,\text{OA},m+1} = \begin{cases} \text{random} & \text{if } \mathcal{R} = 0 \\ \beta^1_{n,\text{OA}} & \text{if } \mathcal{R} < 0 \\ \beta^2_{n,\text{OA}} & \text{if } \mathcal{R} > 0 \end{cases} \]  

(60)

\[ (Z_A - 1) r_{\text{min}}^{-2} \cos 2\beta_{n,\text{OA},A} + \sum_{n=1}^{N} d_n^{-2} \cos 2\alpha_n \geq 0 \]

\[ Z_A r_{\text{min}}^{-2} \cos 2\beta_{n,\text{OA},A} + \sum_{n=1}^{N} d_n^{-2} \cos 2\alpha_n < 0 \]  

(61)

\[ Z_A = \sqrt{\left( \sum_{n=1}^{N} r_{\text{min}}^{-2} \sin \alpha_n \right)^2 + \left( \sum_{n=1}^{N} r_{\text{min}}^{-2} \cos \alpha_n \right)^2} \]  

(62)

Finally, the solution to the optimal deployment of cooperative nodes in two-dimensional AOA-based localization system can be generalized as follows.

Step 1: According to the spatial relationship between the GN and PT, establish the initial observation function and calculate \(\beta^1_{\text{OA}}, \beta^2_{\text{OA}}\), and the initial \(\mathcal{R}\).

Step 2: Choose the lower bound of each \(r_m\) as the the optimal distance for each CN.

Step 3: Determine the optimal azimuth of the first CN from \(\beta^1_{\text{OA}}\) and \(\beta^2_{\text{OA}}\), according to the polarity of \(\mathcal{R}\).

---

![Fig. 3](image-url)  

Fig. 3: The layout of the three different simulation environments. (a) Environment 1. (b) Environment 2. (c) Environment 3.
Step 4: If $r_{1,\text{min}} = r_{2,\text{min}} = \ldots = r_{M,\text{min}} = r_{\text{min}}$ is not satisfied, determine the optimal azimuth of the other CN according to the polarity of $\Re$ updated.

Step 5: If $r_{1,\text{min}} = r_{2,\text{min}} = \ldots = r_{M,\text{min}} = r_{\text{min}}$ is satisfied, calculate $Z_A$. If $Z_A < M$, deploy the whole CNs in the same azimuth as the first CN assigned. If $Z_A \geq M$, deploy the former $Z_A$ CNs in the same azimuth as the first CN assigned while place the $(Z_A+1)$th CN in the azimuth vertical to that of the first CN. Then, $\beta^1_A$ and $\beta^2_A$ are alternatively chosen as the optimal azimuth of the rest CNs.

5 Simulation

In this section, we establish three quite different simulation environments to evaluate the performance of OPMPD approach and compare it with partial exhaustive method, global exhaustive method, and eigenvalue-based approach. It indicates that our approach could provide a rapid and accurate deployment of CNs for TOA-based and AOA-based localization system and is superior to the other approaches.

5.1 Simulation set up

These simulations are implemented in MATLAB language and tested on a PC with an Intel Core i5-3470 CPU of 3.20 GHz and DDR3 SDRAM of 8 GB. Suppose there are totally 6 CNs to be placed to improve the accuracy of a PT that serviced by 6 GNs. Simulation data are collected from three environments whose details depicted in Fig. 3 and Table 1. As reported in (31) and (49), if $\Re = 0$, any azimuth can be regarded as the best choice. In our test, for the case of $\Re = 0$, we respectively choose $\pi/5$, $\pi/6$, and $\pi/4$ as the optimal azimuth of CN in environment 1, environment, 2 and environment 3 for easy of analysis. In addition, we set the domain of $\beta_{O,m}$ as $[-\pi/4, 3\pi/4]$, considering the periodicity of the $\mathcal{Q}(\beta_m)$ and the scope configured in (28).

5.2 Deployment accuracy

Because OPMPD approach estimate the optimal position of each CN based on the partial differentiation and the stepwise strategy, both partial exhaustive method and global exhaustive method are chosen to evaluate the accuracy. The significant difference between them is that the optimum of the CNs in partial exhaustive method is decided step by step just like eigenvalue-based approach and our approach while the optimum of the CNs in global exhaustive method determined all at once. For the two kinds of exhaustive method, the granularity of

| Table 1 Optimal performance obtained by OPMPD approach |
|-----------------------------------------------|
| Environment 1 | Environment 2 | Environment 3 |
| $m$ | 1 | 2 | 3 | 4 | 5 | 6 | 1 | 2 | 3 | 4 | 5 | 6 |
| $r_{\text{min}}$ | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 |
| $r_{\text{max}}$ | 40 | 40 | 40 | 40 | 40 | 40 | 40 | 40 | 40 | 40 | 40 | 40 |

| Table 2 Optimal performance obtained by OPMPD approach |
|-----------------------------------------------|
| Environment 1 | Environment 2 | Environment 3 |
| $m$ | $\Re$ | $\beta_{O_{\text{mT}}}$ | GDOP$^{n}_{C_{\text{TOA}}}$ | $Z_T$ | $\Re$ | $\beta_{O_{\text{mT}}}$ | GDOP$^{n}_{C_{\text{TOA}}}$ | $Z_T$ | $\Re$ | $\beta_{O_{\text{mT}}}$ | GDOP$^{n}_{C_{\text{TOA}}}$ | $Z_T$ |
| 1 | 0 | 0.6283 | 0.7683 | 1 | 2.8974 | 3.8644 | 2 | 2.9044 | 2.9044 | 0.8000 | 0.9675 | 2.0051 | 2.0051 |
| 2 | 0.309 | 2.1991 | 0.7071 | 2 | 2.3794 | 2.9044 | 0.8000 | 0.9675 | 2.0051 | 2.0051 |
| 3 | 0 | 0.6283 | 0.6708 | 3 | 1.8794 | 0.9444 | 0.7004 | 0.3217 | 0.9675 | 2.0051 | 0.6677 |
| 4 | 0.309 | 2.1991 | 0.6325 | 4 | 1.3794 | 0.9444 | 0.6425 | 0.3217 | 0.9675 | 2.0051 |
| 5 | 0 | 0.6283 | 0.5055 | 5 | 0.8794 | 0.9444 | 0.6425 | 0.3217 | 0.9675 | 2.0051 |
| 6 | 0.309 | 2.1991 | 0.5774 | 6 | 0.3794 | 0.9444 | 0.6425 | 0.3217 | 0.9675 |

| (a) TOA-based |
|---------------|
| $m$ | $\Re$ | $\beta_{O_{\text{mA}}}$ | GDOP$^{n}_{C_{\text{AOA}}}$ | $Z_A$ | $\Re$ | $\beta_{O_{\text{mA}}}$ | GDOP$^{n}_{C_{\text{AOA}}}$ | $Z_A$ | $\Re$ | $\beta_{O_{\text{mA}}}$ | GDOP$^{n}_{C_{\text{AOA}}}$ | $Z_A$ |
| 1 | 0.0015 | 2.1057 | 26.3141 | 2 | 0.0002894 | 2.0944 | 45.8142 | 1 | 0.0024 | 1.7526 | 27.4444 |
| 2 | 0.0002527 | 2.1057 | 22.5644 | 2 | 0.0002894 | 2.0944 | 26.8686 | 1 | 0.002859 | 1.7526 | 22.0765 |
| 3 | −0.000948 | 0.5349 | 19.2462 | 3 | 0.0002894 | 2.0944 | 22.8770 | 1 | 0.0041 | 0.1868 | 14.8350 |
| 4 | 0.0002527 | 2.1057 | 17.5197 | 4 | 0.0002894 | 2.0944 | 19.4544 | 1 | 0.0052 | 1.7526 | 13.9309 |
| 5 | −0.000948 | 0.5349 | 15.9077 | 5 | 0.0002894 | 2.0944 | 17.6657 | 1 | 0.0057 | 1.7526 | 12.8015 |
| 6 | 0.0002527 | 2.1057 | 14.8611 | 6 | 0.0002894 | 2.0944 | 16.0377 | 1 | 0.0057 | 1.7526 |

| (b) AOA-based |
|---------------|
| $m$ | $\Re$ | $\beta_{O_{\text{mA}}}$ | GDOP$^{n}_{C_{\text{AOA}}}$ | $Z_A$ | $\Re$ | $\beta_{O_{\text{mA}}}$ | GDOP$^{n}_{C_{\text{AOA}}}$ | $Z_A$ | $\Re$ | $\beta_{O_{\text{mA}}}$ | GDOP$^{n}_{C_{\text{AOA}}}$ | $Z_A$ |
| 1 | 0.0015 | 2.1057 | 26.3141 | 2 | 0.0002894 | 2.0944 | 45.8142 | 1 | 0.0024 | 1.7526 | 27.4444 |
| 2 | 0.0002527 | 2.1057 | 22.5644 | 2 | 0.0002894 | 2.0944 | 26.8686 | 1 | 0.002859 | 1.7526 | 22.0765 |
| 3 | −0.000948 | 0.5349 | 19.2462 | 3 | 0.0002894 | 2.0944 | 22.8770 | 1 | 0.0041 | 0.1868 | 14.8350 |
| 4 | 0.0002527 | 2.1057 | 17.5197 | 4 | 0.0002894 | 2.0944 | 19.4544 | 1 | 0.0052 | 1.7526 | 13.9309 |
| 5 | −0.000948 | 0.5349 | 15.9077 | 5 | 0.0002894 | 2.0944 | 17.6657 | 1 | 0.0057 | 1.7526 | 12.8015 |
| 6 | 0.0002527 | 2.1057 | 14.8611 | 6 | 0.0002894 | 2.0944 | 16.0377 | 1 | 0.0057 | 1.7526 | 11.6386 |
Fig. 4 Performance of partial exhaustive method towards TOA-based system. a Environment 1. b Environment 2. c Environment 3.

Fig. 5 Performance of partial exhaustive method towards AOA-based system in environment 1. a Introduction of the first CN. b Introduction of the second CN. c Introduction of the third CN. d Introduction of the fourth CN. e Introduction of fifth CN. f Introduction of sixth CN.
Fig. 6 Performance of partial exhaustive method towards AOA-based system in environment 2. a Introduction of first CN. b Introduction of second CN. c Introduction of third CN. d Introduction of fourth CN. e Introduction of fifth CN. f Introduction of sixth CN.

Fig. 7 Performance of partial exhaustive method towards AOA-based system in environment 3. a Introduction of first CN. b Introduction of second CN. c Introduction of third CN. d Introduction of fourth CN. e Introduction of fifth CN. f Introduction of sixth CN.
azimuth is set $\pi/3600$ rad and the granularity of distance is set 0.5 m.

Table 2 depicts the optimal performance obtained by OPMPD approach. As revealed in Table 2(a), 0.6283, 2.9044, and 2.0051 are chosen as the optimal azimuth of the first CN in the three environments. Compared to the performance depicted in Fig. 4, the optimum of each CN achieved by OPMPD nearly equals to the result afforded by partial exhaustive method, which proves the correctness of theoretical derivation in OPMPD approach. Similar conclusion about azimuth toward AOA-based localization system can be derived from Table 2(b) and Figs. 5, 6, and 7. Besides, it is showed that the GDOP of AOA-based cooperative localization system decreases as the distance of the CN shrinks, which agrees with the conclusion in (59).

The accuracy comparison with global exclusive method is illustrated in Figs. 8 and 9. It can be seen that the GDOP of OPMPD approximately equals to that of global exclusive method with occasionally a little larger. Whereas, it is important to note that the OPMPD approach is much less complex.

5.3 Deployment efficiency

Running time is chosen to evaluate the efficiency of the OPMPD approach and the eigenvalue-based approach. For the TOA-based cooperative system, the procedure of the eigenvalue-based approach consists of three steps. First, the symmetric matrix $G_{TOA}$ would be calculated. Second, the eigenvalue and eigenvector of $G_{TOA}$ should be determined. Third, the optimal position of the CN to be placed is decided between the two eigenvector by contrasting the fore-and-aft eigenvalue. On the other, the procedure in OPMPD approach mainly comprises two steps. First, the positive azimuth factor, the negative azimuth factor, the initial cosine azimuth coefficient, and the inertia dependence factor are calculated. Second, the optimal position of each CN can be obtained according to the polarity of the initial cosine azimuth coefficient and the value of the inertia dependence factor. Figures 10 and 11, respectively, shows the pseudocode for the procedure in eigenvalue-based approach and in OPMPD approach towards TOA-based localization system. The data of running time is collected on the basis of 10,000 times Monte Carlo simulation. As illuminated in Table 3,
the efficiency of the OPMPD approach is nearly two
times higher than that of the eigenvalue-based approach.
One reason is that the complex eigen-decomposition of
the matrix has been replaced by the straightforward
arithmetic operation.

5.4 Requirement of inertia dependence factor

Inertia dependence factor that reveals the inertia and the
recursiveness of deployment also accounts for the efficiency
improvement. For TOA-based localization system, the
requirement of inertia dependence factor is that the CN
can be placed in any azimuth. In environment 2, since
$Z_T = 6$, assigning the total CNs in the azimuth
2.9044 rad would achieve the minimum value of
GDOPTOA = 0.5775. For AOA-based localization
system, besides azimuth, the requirement of inertia
dependence factor includes that the lower bound of the
distance to the PT of each CN should be identical to
the others. In environment 1, since $r_{1,\text{min}} = r_{2,\text{min}} = \ldots = r_{6,\text{min}} = 20$, $Z_A = 2$, thereby assigning the former two

\begin{verbatim}
Procedure \alpha = [\alpha_1, \alpha_2, \ldots, \alpha_n];

\begin{align*}
\text{Initialize optimal azimuth } & \beta_{0,m} (m = 1, 2, 3, \ldots, M) \text{ set to 0; } \\
H_{\text{TOA}} & = \begin{bmatrix} -\cos(\alpha) & -\sin(\alpha) \end{bmatrix}; \\
G_{\text{TOA}} & = \left( H_{\text{TOA}}^T H_{\text{TOA}} \right)^{-1}; \\
\text{Acquire eigenvalue } & \lambda_1, \lambda_2 \text{ and eigenvector } e_1, e_2 \text{ from } G \text{ where } \lambda_1 < \lambda_2; \\
\beta_{0,t} & = \arctan(e_t); \\
\text{for } i = 2, \ldots, M & \\
\text{if } \lambda_1 + 1 < \lambda_2 & \\
\beta_{0,t} & = \arctan(e_t); \\
\lambda_1 & = \lambda_1 + 1; \\
\text{else} & \\
\beta_{0,t} & = \arctan(e_t); \\
\lambda_2 & = \lambda_2 + 1;
end
end
end procedure

Fig. 10 Pseudocode for the deployment procedure in eigenvalue-based approach towards TOA-based localization system
\end{verbatim}
CNs in the azimuth 2.1057 rad and alternately choosing 0.5349 rad and 2.1057 rad as the optimal azimuth of each remaining CN would achieve the minimum value of GDOP$_{\text{C,AOA}} = 14.8611$. But in environment 3, as the lower bound of the distance of the CNs differ from each other, inertia dependence factor is unavailable which renders the optimal azimuth of each CN decided by the polarity of the updating cosine azimuth coefficient.

### 6 Conclusions

In this paper, our research is to solve the optimal deployment of cooperative nodes in two-dimensional localization system. To achieve the deployment with the least GDOP, the GDOP expressions of TOA-based and AOA-based cooperative localization systems are derived on the basis of Cramer-Rao bound. By examining the partial differentiation of the GDOP and leveraging stepwise strategy, an approach termed as OPMPD is suggested to expose the optimal position for each CN. Inertia dependence factor is also deducted which reveals the relationship among the optimal positions of each CN. Simulation results prove that our solution almost achieves the same GDOP to that of global exclusive method but with less time complexity. It is noted that the solution proposed in this paper are only suitable for TOA system and AOA system, not for TDOA system. The reason for this is that the idea of TDOA is to determine the relative position of the mobile transmitter by examining the difference in time at which the signal arrives at multiple measuring units, rather than the absolute arrival time of TOA. The Jacobian matrix of TDOA is quite different from those of TOA and AOA. Hence, our future work is to study the optimal deployment of cooperative nodes in other more complex localization systems such as the TOA-based, the TDOA/TOA-based, and the TDOA/DAOA-based.

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### Authors’ contributions

WS and XQ conceived and designed the study. JL, SY and LC performed the simulation experiments. YY and XF wrote the paper. WS and XQ reviewed and edited the manuscript. All authors read and approved the manuscript.

### Competing interests

The authors declare that they have no competing interests.

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