Selftuning and its footprints

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ABSTRACT: We re-consider the self tuning idea in brane world models of finite volume. We notice that in a large class of self tuning models, the four dimensional physics is sensitive to the vacuum energy on the brane. In particular the compactification volume changes each time the tension of the brane is modified: consequently, observable constants, as the effective Planck mass or masses of matter fields, change as well. We notice that the self tuning mechanism and the stabilization mechanism of the size of the extra dimensions are generically in apparent conflict. We focus on a self tuning model in six spacetime dimensions to analyze how the above considerations are explicitly realized.
1. Introduction

The cosmological constant problem is one of the most serious manifestations of the difficulty to join together a quantum theory of fields with a theory of gravitation. In the presence of a non-zero cosmological constant, it is not possible to find solutions that preserve Poincaré symmetry; in particular, even a relatively small contribution to the energy momentum tensor in the form of vacuum energy curves the space-time significantly, in deep conflict with observations (for a review, see [1]).

To reconcile the particle physics models, whose predictions for the amount of cosmological constant are of the order of $M_{Pl} \simeq 10^{18}$ GeV, with the quite small amount of the
currently observed vacuum energy, of the order of $10^{-3}$ eV [2], one has consequently to rely on an extreme fine tuning at each order in perturbation theory. In addition, the above fine tunings have to be repeated from scratch every time that a phase transition in the cosmological history takes place.

_Four dimensional approaches to the cosmological constant problem_

Various approaches have been considered to attack the problem. The most natural is maybe to look for a _powerful unbroken symmetry_, such as supersymmetry or conformal symmetry, that cancels the quantum contributions to the vacuum energy. However, the observed world, at least up to the electroweak scale, does not respect any of these exact symmetries: thus, they must be broken at some scale bigger than a scale of the order of 100 GeV. This is then the scale one expects as natural for the cosmological constant: it is clear that, in this picture, the problem can be ameliorated, but it is still far from being solved.

As an alternative to exact symmetries, in the eighties some effort had been devoted in building the so-called _adjustment mechanisms_ [3]. In these mechanisms, a scalar field is suitably coupled to gravity and matter: its dynamics should respond to any contribution to the vacuum energy, adjusting its value to zero, corresponding to the minimum of the scalar potential. There has not been, however, a successful realization of this idea. Weinberg [1], moreover, provided a no-go theorem that, under some general assumptions, states that the potential for the compensator field, which should adjust the vacuum energy to zero, has a runaway behavior. That is, there is no stationary point for the potential of the scalar that should realize the adjustment, and thus the mechanism cannot work.

Other quite interesting possibilities rely on the observation that the (expected large) value of the cosmological constant can be compensated by some _non-dynamical_ field, like a 4-form $F_4$ in four dimensions. The constant value of this field (or, in some approaches, of a large number of these fields [1]) changes via a process of membrane nucleation [4, 5], up to a value that compensates the cosmological constant to a degree sufficient to render it compatible with observations. This value is generally fixed by anthropic considerations, based on Weinberg’s arguments on galaxy formation [7]; alternatively, the desired value of $F_4$, that compensates the cosmological constant, can be obtained considering tunneling effects between metastable vacua, each corresponding to a different value of the 4-form [8].

_Self tuning from higher dimensions_

More recently, the possibility that our world is confined on a hypersurface (called _brane_) embedded in a higher dimensional spacetime (called _bulk_), suggested to physicists new possibilities to tackle the problem. Suppose, for example, that we live on a three-brane (i.e. a brane with three spatial dimensions) located on a singular region of a higher dimensional background. All the fields of the standard model (SM) are, by some mechanism, localized
on the brane, and only gravity (besides other fields which are either heavy or couple sufficiently weakly to the SM ones) can propagate through the entire higher dimensional space. One could imagine a model in which any contribution to the cosmological constant on the brane (that can be regarded in this context as the tension of the same brane), is transmitted to bulk parameters, like integration constants, in such a way that a four dimensional observer does not realize any change in the four dimensional geometry. In particular, suppose that it is possible to find solutions in which the four dimensions, that correspond to the observed ones, preserve Poincaré symmetry, regardless of the value of the brane tension. Of course, a solution of the cosmological constant problem is interesting when this transmission from brane to bulk occurs automatically, without the necessity to re-tune bulk quantities by hand every time the vacuum energy in four dimensions changes. Models that realize this fascinating idea are called self tuning models.

Let us note here that as we have defined the self tuning mechanism, we are interested on solutions that the Poincaré vacuum is preserved irrespectively of the brane tension. However, this does not exclude that there are nearby curved solutions. In other words, there can in principle exist solutions for neighboring values of some bulk parameters which result in a curved four dimensional space. A completely satisfactory solution to the cosmological constant problem should forbid such curved solutions to exist close to the Poincaré vacuum. Nevertheless, in the following we follow a less ambitious road and content ourselves with finding vacua which realize self tuning as defined above even if there exist nearby curved solutions.

A self tuning model can realize a compensation of the brane tension via bulk quantities in various ways. In the discussion of the present paper, we would like to separate self tuning models in two classes.

The first class contains models in which a change in the brane tension is accompanied by a change of integration constants relative to fields that live in the bulk (like, for example, scalars or gauge fields), in such a way that the higher dimensional geometry is not modified with respect to the original one. To this class belong, for example, the earlier attempts to construct self tuning models [9, 10], although a careful analysis of these examples revealed that a fine tuning is actually required to compensate the brane tension in a consistent way [11].

The second class is more general, and includes models in which a change in the brane tension can be compensated by integration constants that govern the geometry of the bulk; in this case, it is possible to relate a change in the brane tension to a modification of the background geometry. Examples that belong to this class include recent attempts to construct self tuning models in six dimensions [12, 13, 14, 15, 16]: in these models, a three dimensional brane is located on a conical singularity of a six dimensional space, and any change in the brane tension is compensated by a change in the deficit angle of the cone, in such a way to maintain a Poincaré invariant four dimensional subspace.
Consequences for four dimensional physics

The question that we would like to address in the present paper is the following: does the compensation of the brane tension via higher dimensional parameters leave some effect at the level of four dimensional physics? In other words, is the low energy, four dimensional observer somehow sensitive to the mechanism of cancellation of the cosmological constant?

In general, when a compensation mechanism of a change in the brane tension presents fine tuning at the higher dimensional level, the same characteristic is expected to be inherited also at the four dimensional effective level, when the procedure of dimensional reduction is properly done.

When, instead, a self tuning mechanism can be realized in a higher dimensional system, we will show that, in general, well defined, measurable quantities in the effective model, like the effective Planck mass, explicitly depend on the brane tension: if the tension of the brane changes, measurable quantities change accordingly. This is due to the fact that when a change in the brane tension is compensated by a (self tuned) modification in the brane geometry, at the same time the volume of the extra dimensions changes its size. Since there exist four dimensional quantities, like the Planck mass, that are proportional to this volume, we can find, after determining the dependence of the volume on the brane tension, the remaining effects of the self tuning mechanism in the four dimensional physics. In particular, we can determine in which cases the effects of the self tuning mechanism influence in a drastic way four dimensional quantities, providing new phenomenological constraints for these models.

Generalizing this observation, we point out what we can call a conflict between the self tuning mechanism and a mechanism that stabilizes the size of the extra dimensions. Indeed, the condition to have stabilized extra dimensions contrasts with the requirements at the basis of each self tuning example.

Consequently, of particular interest is the analysis of how this conflict manifests itself at the four dimensional effective level, for models that automatically incorporate a stabilization mechanism. We will explicitly discuss this issue for a six dimensional model, and provide the effective dimensionally reduced description of the self tuning mechanism. We identify the four dimensional parameters that compensate a change in the vacuum energy, and discuss how the four dimensional remnant of the stabilization constraint, that corresponds to the quantization condition of a non-dynamical 4-form field, influences the cancellation mechanism.

Organization of the paper

We start with a general discussion on how self tuning models can be in principle realized in an extra dimensional setup. Then, we examine various examples of fine tuned and self tuning models, observing that the available self tuning models present a variation of the
compactification volume as one changes the tension of the brane we are interested in. We focus on the six dimensional example and we discuss the consequences of the variation of the compactification volume, namely the variation of the gravitational constant, or the masses of matter fields, as a function of the brane tension. We continue discussing under which conditions contributions to the brane tension lead to drastic effects at the four dimensional level. To conclude, we provide a four dimensional interpretation of the self tuning mechanism in terms of a non-dynamical field, pointing out the conflict between self tuning and the stabilization mechanism, as soon as the quantization condition of the relevant form is taken into account. At the end we present our conclusions and we comment on the prospects of the self tuning models in extra spacetime dimensions.

2. Self tuning from higher dimensions

In this section we will discuss, in a well determined framework, the idea of self tuning from higher dimensions, and how it can be realized. Our aim is to present a setup that is sufficiently general to comprehend the various features that characterize self tuning models. In particular, our framework allows us to explicitly discuss, at the end of the section, the consequences of self tuning mechanisms at the level of four dimensional physics.

2.1 The setup

Consider a $D$-dimensional background that corresponds to a solution of the equations of motion relative to the following action

$$S_D = \int d^D x \sqrt{-g_D} \left[ M_f^{D-2} \mathcal{R}_D - \mathcal{L}(\phi, g^M_D) \right], \quad (2.1)$$

where $\mathcal{L}$ is a Lagrangian that describes the $D$-dimensional fields that we denote with $\phi^1$. We now concentrate on solutions in which the metric presents a symmetrically warped form:

$$ds^2_D = e^{2W(y)} g_{\mu\nu}(x) dx^\mu dx^\nu + \gamma_{mn}(y) dy^m dy^n, \quad (2.2)$$

where the $x$ coordinates span a four dimensional subspace and the $y$ coordinates a $d$ dimensional compact internal space ($D = d + 4$). Additionally, we will assume that the bulk fields depend only on the $d$ extra dimensions $y$. We consider the metric of the 4-dimensional space, $g_{\mu\nu}$, given as an initial and fixed input (it can be flat or curved space).

It is important to notice here that in general the knowledge of the $D$-dimensional action is not enough to completely fix integration constants relative to bulk fields (for example $\phi(y_0)$) or to metric components. This is generally a welcome feature for the self tuning models: these integration constants will be fixed only after specifying suitable conditions at the boundary and on the singular regions of the space time.

\footnote{We always work in the Einstein frame in the $D$-dimensional action.}
Let us consequently add, to the above $D$-dimensional action, localized terms, that describe three dimensional branes (labeled by an index $i = 0, 1, \ldots n$), placed in singular surfaces of the bulk background geometry \footnote{One can generalize our subsequent arguments for the case when branes of different dimensionality than three are also present.}. Their action is given by (notice that we use the projected metric in the four dimensional space):

$$S_4 = - \sum_{i=0}^{n} e^{4W(y_i)} e^{\sigma_i \phi(y_i)} T_i \int d^4x \sqrt{-g(x)}.$$ \hspace{1cm} (2.3)

For our arguments, we will only consider branes that contain tension, labeled by $T_i$. We also allow a direct coupling between bulk scalars and brane tensions, and we model this coupling with exponentials. The inclusion of these localized terms in general fixes completely the integration constants of the general bulk solution.

### 2.2 Self tuning at work

In this setup, the idea of the self tuning approach is the following. Let us suppose we live on the brane situated at $y = y_0$, with tension $T_0$. Then, it is possible to find solutions of the equations of motion in the bulk, that respect the conditions given by boundary terms and maintain the same metric $g_{\mu\nu}$ for the four dimensional slice, in such a way that there is no fine tuning between $T_0$, the tension of any other brane $T_i$, and/or higher dimensional parameters explicitly appearing in $\mathcal{L}$.

This means that we can freely change the tension of our brane $T_0$, with the only consequence that integration constants relative to bulk fields or to metric components change accordingly in such a way that the observed geometry in four dimensions, described by $g_{\mu\nu}$, is not modified. The simplest case corresponds to the Minkowski metric $\eta_{\mu\nu}$: this is the case in which we will concentrate in the following, unless otherwise stated.

Since we are looking for solutions with vanishing vacuum energy, the numerical value of the total action, evaluated on the solution, should be zero: indeed, integrating out the extra dimensions, it is easy to see that this number corresponds to the value of the vacuum energy from the four dimensional effective point of view. Let us rewrite the total action in a way that facilitate our following discussion:

$$0 = S_{tot} = S_D + S_4$$

$$= S^{reg}_D + M_f^{D-2} \sum_{i=0}^{n} e^{4W(y_i)} R_i^{sing} \int d^4x \sqrt{-g} - \sum_{i=0}^{n} e^{4W(y_i)} e^{\sigma_i \phi(y_i)} T_i \int d^4x \sqrt{-g} \hspace{1cm} (2.4)$$

In the last equality of the previous formula, the bulk action has been split into a regular part that describes the higher dimensional curvature scalar and the bulk fields, and a singular part on the scalar curvature, which is present since the branes are located at singular points of the geometry. For simplicity, we will limit the following analysis to cases in which up
to two branes are present in the localized part of the action. We have written the singular part of the curvature scalar as \( R_{i}^{sing} = \Delta^{(D-4)}(y - y_i) \) where \( \Delta^{(D-4)}(y - y_i) \) is the generalized delta function in the curved space \(^3\). As we have noted, the numerical value of the total action \( S_{tot} \) cannot vary keeping the Minkowski vacuum as solution in four dimensions: this should remain true also after modifying \( T_0 \). Thus, there ought to be cancellations between the above terms in the action when \( T_0 \) changes. We will divide our discussion into two cases, regarding how these cancellations occur. The first case corresponds to the special situation in which the bulk part \( S_D \) (that includes also the singular part of the curvature) and the brane part \( S_4 \) remain separately invariant after a change in \( T_0 \). The second case describes the more general situation in which each of them can vary, but in such a way that their sum cancels to zero.

### 2.2.1 Inter-brane compensation

Let us start considering the possibility that both \( S_D \) and \( S_4 \), in \(^2\), are separately invariant when the brane tension is modified. In such a case, a change in \( T_0 \) (the tension of the brane in which the SM fields live) should be entirely compensated by brane quantities, that is, quantities that appear in \( S_4 \). In particular, we ask that the integration constants that compensate a change in the brane tension do not modify the bulk action \( S_D \), and consequently do not change the bulk geometry.

The natural quantity that acts as compensator of a change in \( T_0 \) is the value of \( \phi(y_0) \), that is generally determined up to some integration constant. The integration constants relative to the scalar, in this case, change in such a way that they compensate a change in \( T_0 \), without changing the value of \( S_D \). However, the scenario is potentially problematic, since in general a change on the integration constants of the bulk fields require, for the consistency of the system, a corresponding change in the tension \( T_1 \) in a fine tuned way.

This is precisely what happens in the first attempts to realize this idea, in the five dimensional dilatonic models of \(^3\). As we will show in detail in the next section (also more complex frameworks share the same problem), a change in the integration constants, that allows to compensate a change in the brane tension \( T_0 \), requires also a change on the tension of the second brane \( T_1 \) \(^3\). We are not aware of a complete, consistent model, belonging to this class, that presents a self tuning behavior.

### 2.2.2 Brane-bulk compensation

An alternative way to realize self tuning is the following. Imagine that any variation of the brane tension \( T_0 \) is accompanied by a variation of the bulk action \( S_D \) in \(^2\), in such a way that the resulting value of the total action remains null. If this condition can be satisfied without fine tuning, and preserve a flat four dimensional slice, one obtains a self

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\[^3\text{This function satisfies the following identity: } \int \sqrt{\gamma} \Delta^{(D-4)}(y - y_i) = 1.\]
tuning mechanism to cancel the cosmological constant *via* a change in the geometry of the internal dimension.

We can have the case in which both the *regular* and the *singular* parts of the bulk action change with the brane tension. This is actually what happens in the Randall-Sundrum models [20], that however compensate the cosmological constant at the price of fine tuning bulk parameters. Another example is the model of [21], where a 3-form gauge field is considered, with Lagrangian of the specific form $1/H^2$, where $H$ denotes the 4-form field strength of that field. In this case, the brane contribution is canceled by a correlated change of the bulk geometry and of an integration constant of the bulk gauge field solution, and no fine tuning of parameters is apparently required.

In another situation, the regular part of the bulk action remains unchanged after a change on $T_0$, and the *singular part of the bulk action alone* compensates a change in the brane tension. As a consequence, only the global properties of the geometry result modified, while locally the geometry remains the same. Examples that belong to this class are constructed in a six dimensional background, that is the only dimensionality in which the idea can be applied. Indeed, Einstein equations (see Appendix A for details) imply the following equality, when the regular part of the Einstein tensor cancels exactly the bulk part of the energy momentum tensor:

$$M_D^{D-2} R_i^{\text{sing}} = \frac{4}{D-2} \epsilon_{\sigma_i \phi(y_i)} T_i.$$  \hspace{1cm} (2.5)

Then, for the cancellation to be realized, one should have [12]

$$\frac{4}{D-2} = 1 \quad \Rightarrow \quad D = 6.$$ \hspace{1cm} (2.6)

[In general if the $x$-coordinates span a ($p+1$)-dimensional space, the requirement would have been that $D = (p+1) + 2$.]

Since it is possible to analyze quite generally how the compensation applies in this specific six dimensional situation, let us end this subsection with a discussion of how self tuning is in principle realized in this case, showing explicitly how the global properties of the background change.

Since the mechanism of cancellation relies only on bulk quantities, we can, for the sake of simplicity, neglect the presence of other singular objects apart from our brane. In the simplest situation, let us impose that the six dimensional space time presents an *azimuthal symmetry* around an axis. An ansatz for the metric that preserves the four dimensional Poincaré symmetry is

\[\text{[Note: We do not mean that the presence of other branes is not important for the consistency of the model. Simply, they are not essential for the features of the self tuning mechanism we intend to discuss.]}\]
\[ ds_6^2 = e^{2W(r)} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + \rho^2(r) d\phi^2, \]  
where, in the previous formula, \( r \) is a radial variable, while \( \phi \) an angular variable with values \( \phi \in [0, 2\pi) \).

Assuming that \( \rho(r) \propto r \) for \( r \to 0 \), the point \( r = 0 \) corresponds to a single point in the transverse space. A three-brane of tension \( T_0 \), that extends along the directions \( x^\mu \) and is located in \( r = 0 \), induces a conical singularity with deficit angle \( \delta \), related to the brane tension via the equality

\[ \delta = \frac{T_0}{2M_4^f}. \]  

At this point, consider a change in the brane tension \( T_0 \). A new solution that takes into account this modification, and that preserves the required azimuthal symmetry, differs from the previous one just by a change in the value of the deficit angle \( \delta \), that adjusts to the new value of the tension. A change on this deficit angle, indeed, represents a modification of the global geometry. If this change does not imply a fine tuning on the parameters appearing in the action, a self tuning mechanism is realized.

### 2.3 What happens to four dimensional physics?

After our general discussion of how self tuning can in principle be realized, let us ask the following question: does the higher dimensional mechanism to cancel the cosmological constant have some effects at the level of four dimensional physics?

In general, one expects that some remnant of the higher dimensional mechanism should be present in the effective four dimensional description. Indeed, conceptually, the cosmological constant problem is present also at low scales, even lower than the typical energy scales in which higher dimensional physics manifests itself. Consequently, for this range of energies, one expects to find a four dimensional effective description for the cancellation mechanism, although maybe quite non standard.

When the cosmological constant is compensated, in the higher dimensional framework, by some symmetry, one expects that some remnant of this symmetry is left at the four dimensional level, and the mechanism of cancellation can be effectively understood in terms of this inherited symmetry. Instead, when the responsible of the cancellation is a dynamical field in the bulk, one expects some very light degree of freedom to be present at the effective four dimensional level, that dynamically compensates the cosmological constant. Alternatively, the higher dimensional cancellation mechanism can be ultimately due to some non-dynamical parameter in the bulk, whose value compensates the brane tension. In this case, one expects that, at the four dimensional level, the cancellation mechanism can be described in terms of some non-dynamical field.

As we have seen, in general the self tuning mechanisms that we have outlined are based on the last possibility: the brane tension is compensated by non-dynamical parameters in
the bulk, like integration constants. Naturally, the determination of the effective four dimensional description of the mechanism is not an easy task. For this reason, in the following we concentrate our discussion on the effects of the higher dimensional mechanism at the level of the four dimensional effective theory. Once these remaining effects are identified, the connection between higher and lower dimensional descriptions should become clearer.

In general, one expects that models that require a fine tuning at the level of higher dimensional parameters, present a fine tuning in the four dimensional effective action, after the procedure of dimensional reduction is performed: the four dimensional model presents, at the effective level, the cosmological constant problem in its traditional form.

On the other hand, when we look at four dimensional effects of the most promising self tuning models, belonging to the class discussed in Section (2.2.2), we know that they require a modification of the bulk geometry. We observe that in this case a dimensionful parameter of the four dimensional action varies when the brane tension changes: this is the effective Planck mass in four dimensions. This is due to the fact that, in the examples we discuss, a change of the geometry implies a modification in the volume of the internal dimensions. In general, the volume is proportional to the effective Planck mass in four dimensions, and this quantity changes accordingly with the brane tension: as a consequence, an analysis of these models should take into account also phenomenological constraints related to variations of the Planck mass.

In more general terms, in order for an extra dimensional model to satisfy phenomenological constraints on the behavior of gravitational interactions, it should incorporate a mechanism that stabilizes the size of the extra dimensions (at least at the present cosmological epoch). The self tuning models we consider require just the opposite, since the volume of the extra dimensions must change in order to compensate a change in the brane tension. As we will see analyzing explicit examples, the requirement to maintain a fixed volume for the extra dimensions imposes an additional constraint that ruins the self tuning mechanism. This means that whenever the higher dimensional system includes a stabilization mechanism, there is a conflict between self tuning and stabilization. This will be particularly transparent in a six dimensional example, on which we will focus our attention in Section (4), presenting also a four dimensional description of this fact.

3. Explicit examples

In this section, we present examples of higher dimensional models, already known in the literature, in which the general arguments we have discussed so far find an explicit realization.

We start with a model in which the cancellation occurs between the brane part of the action, that is, a model that belongs to the class discussed in Section (2.2.1). We
continue with three models that realize a compensation using quantities related to the bulk geometry, and that consequently belong to the class of Section (2.2.2).

Finally, somehow outside the discussion of the previous section, we present a five dimensional self tuning model whose bulk metric does not satisfy the ansatz (2.2) but is instead *asymmetrically warped*, in a sense we will define below. Also in this case the compensation occurs due to bulk quantities relative to the higher dimensional geometry, and the bulk volume changes.

### 3.1 A five dimensional dilatonic model

In this subsection, we discuss one of the dilatonic models of \[9, 10\]. In this approach, a cancellation mechanism is realized exploiting the properties of an integration constant relative to a bulk scalar field, without involving quantities relative to the bulk geometry. The model belongs to the class discussed in Subsection (2.2.1), and we will see how the arguments developed in that context apply here.

The five dimensional background consists of a four dimensional Minkowski space plus a compact extra dimension, modded by a $\mathbb{Z}_2$ symmetry with two fixed points (located at the positions 0 and $y_s$) on which two three-branes are located. We parameterize the five dimensional metric in the following way:

$$ds_5^2 = e^{2W(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2.$$  \hspace{1cm} (3.1)

The model contains gravity, a bulk scalar field, and two branes, characterized by their tension and conformally coupled to the bulk scalar. The action is given by:

$$S_{\text{tot}} = M_f^3 \int_{-y_s}^{y_s} dy d^4x \sqrt{-g_5} [R_5 - \frac{4}{3} (\partial \phi)^2] - T_0 e^{-\frac{4}{3} \phi(0)} \int \sqrt{-g} - T_1 e^{-\frac{4}{3} \phi(y_s)} e^{4W(y_s)} \int \sqrt{-g}$$ \hspace{1cm} (3.2)

where we have normalized $W(0) = 0$. The solution that maintains a four dimensional Poincaré vacuum, results:

$$W = \frac{1}{4} \ln \left| \frac{T_0 e^{-\frac{4}{3} \phi_0}}{3M_f^3} \right| y| - 1 \right|$$ \hspace{1cm} (3.3)

and

$$\phi = \frac{3}{4} \ln \left| \frac{T_0 e^{-\frac{4}{3} \phi_0}}{3M_f^3} \right| y| - 1 \right| + \phi_0.$$ \hspace{1cm} (3.4)

Notice that the solution for the scalar field is defined up to an integration constant $\phi_0$: this observation is crucial for the cancellation mechanism we are going to discuss. The solution has a singularity in $y = y_s = \frac{3M_f^3}{T_0} e^{\frac{4}{3} \phi_0}$, and the rôle of the second brane in $y_s$ is just to screen the singularity \[11\] (for a general discussion about the presence of background singularities in self tuning models see also \[19\]). The boundary conditions on the second brane force to choose $T_1 = -T_0$. 

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**Note:** The text above is a direct transcription from the provided image, ensuring the accurate representation of the content. Further adjustments may be necessary for complete clarity and readability.
Suppose, now, that we live on the brane at the origin, and we vary the tension $T_0$. To have a solution that, after dimensional reduction, gives us a flat four dimensional geometry, we must ask that the value of $S_{\text{tot}}$ does not change varying $T_0$, and in particular it keeps a vanishing value.

Looking at the second term in the action (3.2), we see that any change in $T_0$ like $T_0 \to e^{\lambda}T_0$ can be compensated by a change in the integration constant $\phi_0 \to \phi_0 + \frac{3}{4}\lambda$. A change in $\phi_0$ does not affect the bulk part of the action, that is, the first term in (3.2). However, it is clear that the value of the third term changes after this procedure, unless we also impose by hand the additional condition $T_1 \to e^{\lambda}T_1$. Thus, we must re-fine tune a parameter that appears explicitly in the action to maintain a Poincaré invariant four dimensional subspace, and the model presents an apparent fine tuning at the five dimensional level.

### 3.2 The Randall-Sundrum model

In the (one brane) Randall-Sundrum model [20], a three-brane is located at the fixed point (located at the origin) of a $\mathbb{Z}_2$ symmetric $AdS_5$ background. The action for the model, that describes gravity, bulk cosmological constant, and brane tension, is given by

$$S_{\text{tot}} = \int_{-\infty}^{+\infty} dy d^4x \sqrt{-g_5} (M_5^3 R_5 - \Lambda_B) - T_0 \int d^4x \sqrt{-g}. \quad (3.5)$$

Considering formula (3.1) as ansatz for the metric, a solution that preserves four dimensional Poincaré invariance for any choice of the brane tension $T_0$ is given by $W(y) = -ky$, with the parameter $k$ given by $k = \sqrt{-\frac{\Lambda_B}{12M_f^3}}$. It is well known, however, that this mechanism of cancellation requires a fine tuning between brane tension and bulk cosmological constant. Indeed, the following condition must be imposed between brane and bulk cosmological constant

$$T_0 = \sqrt{-12 \Lambda_B M_f^3}. \quad (3.6)$$

Any change of $T_0$ must be accompanied, to maintain a four dimensional Poincaré space, by a change of the five dimensional bulk cosmological constant, changing the value of the regular part of the bulk action in (3.3). Consequently, this model, that gives a (fine tuned) compensation of the cosmological constant, belongs to the class discussed in Section (2.2.2).

Notice also that this is the first model in which the higher dimensional mechanism to cancel the cosmological constant, although it requires fine tuning, presents a clear footprint at the level of four dimensional physics, since it requires a modification of the bulk volume. Indeed, in this model, the four dimensional Planck mass is given by

$$M_{\text{Pl}}^2 = \frac{M_f^3}{k} = \frac{12M_f^6}{T_0}, \quad (3.7)$$

and a change in $T_0$ implies a change in the Planck mass.
3.3 A six dimensional model

We would like to discuss now the six dimensional model of [13] (see also [14, 15, 22]). In this model, a three-brane is located on the singular point (a conical singularity) of a six dimensional background containing gravity, a 2-form field strength, and cosmological constant $\Lambda$. The action one considers is

$$S_{\text{tot}} = \int d^6x \sqrt{-g_6} \left[ M_4^2 R_6 - \Lambda_B - \frac{1}{4} F_{MN}^2 \right] - T_0 \int d^4x \sqrt{-g}. \tag{3.8}$$

Looking for un-warped solutions that maintain a four dimensional subspace with Poincaré symmetry, one finds as a solution for the metric with a locally spherical geometry in the two extra dimensions

$$ds_6^2 = \eta_{\mu\nu} dx^\mu dx^\nu + R_0^2 \left( d\theta^2 + \alpha^2 \sin^2 \theta d\phi^2 \right), \tag{3.9}$$

while a solution for the 2-form is given by the following expression

$$F_{\theta\phi} = f \epsilon_{\theta\phi}. \tag{3.10}$$

In the last formula, $f$ is a constant, while $\epsilon_{\theta\phi}$ is the volume form of the two extra dimensions. In the metric solution (3.9), to obtain a flat four dimensional background one must impose

$$\frac{1}{R_0^2} = \frac{f^2}{2M_f^4}, \quad \Lambda_B = \frac{f^2}{2}. \tag{3.11}$$

The important observation, in this context, is that the formulas (3.11) do not depend on the brane tension. The only dependence on $T_0$, in formulas (3.9)-(3.11), is limited to the size of the deficit angle, which is related to $\alpha$. Indeed, one finds that the sphere presents conical singularities at the poles $\alpha$, with deficit angle $\delta$ given by

$$\delta \equiv 2\pi (1 - \alpha) = \frac{T_0}{2M_f^4}. \tag{3.12}$$

As a consequence, changing the tension $T_0$, one finds a new solution simply changing accordingly the deficit angle, following formula (3.12). Only the singular part of the curvature changes, and the model belongs to the class discussed in Section (2.2.2). In this way, the curvature of the four dimensional subspace remains invariant, and, apparently, no fine tuning is required on quantities that explicitly appear in the bulk action.

At the four dimensional level, the self tuning mechanism has an important effect. Indeed, as we mentioned in Section (2.3), a change in the deficit angle implies a change in the bulk volume. Consequently, the effective, four dimensional Planck mass changes: this observable quantity results to depend on the brane tension as

$\text{Note:}$

$^5$The system can be generalized including also scalar fields: see [23, 16].

$^6$One of the two conical singularities can be discarded by imposing a $Z_2$ symmetry at the equator of the sphere.
where in the last equality we defined, using also (3.12), the effective Planck mass in absence of branes on the singular points. From the previous expression, it is apparent that if one imposes to the system the additional condition to maintain the same Planck mass after changing the brane tension (that is, keeping the volume fixed), one must also change the radius $R_0$ of the sphere, and consequently also the bulk cosmological constant $\Lambda_B$ (see formula (3.11)). In other terms, one ends with a fine tuning à la Randall-Sundrum.

Another important observation concerning this model, that as we will see is related to the issue of the variation of the Planck mass, has been made in [22] (see also [15]). There, it has been noticed that, for consistency of the model, the monopole charge must satisfy a quantization condition that ruins the self tuning property: the brane tension can acquire only well defined, quantized values, and cannot vary in a continuous way.

Both these two observations, and an interesting relation between them, will be reconsidered and expanded later on, in Section (4).

3.4 A five dimensional 3-form model

The next model we will discuss, that has a self tuning behavior, is the one presented in [21]. It uses a 3-form field in five dimensions that are compactified on an $\mathbb{R}/\mathbb{Z}_2$ orbifold. The action for the 3-form field has the unconventional form of $1/H^2$ where $H_{MNKL}$ is the field strength of the 3-form field. Since a 4-form field (here $H$) in five dimensions is dual to a scalar field $\sigma$, we can work with the equivalent action for that dual scalar field (as done in the second reference of [21]):

$$S_{\text{tot}} = \int_{\infty}^{-\infty} dy d^4x \sqrt{-g_5} \left[ M_f^3 R_5 - \left( \partial_M \sigma \partial^M \sigma \right)^{1/3} - \Lambda \right] - T_0 \int d^4x \sqrt{-g}. \quad (3.14)$$

The solution that presents a four dimensional Poincaré vacuum is the following, in terms of the ansatz of formula (3.1):

$$W(y) = \frac{1}{4} \ln \left[ \frac{\cosh c}{\cosh(4k|y| + c)} \right], \quad (3.15)$$

while a solution for the scalar is

$$\sigma'(y) = \frac{216k^3 M_f^{9/2}}{\cosh^3(4k|y| + c)}, \quad (3.16)$$

with $k = \sqrt{-\frac{\Lambda}{12M_f^2}}$, $c = \tanh^{-1} \left( \frac{T_0}{12kM_f^2} \right)$, and where we have normalized to $W(0) = 0$. This model belongs to the class discussed in (2.2.2), and a variation of the brane tension $T_0$ is compensated by a change in the integration constant $c$, without, at least apparently, involving a fine tuning of other parameters.
Also this model, in any case, realizes self tuning at the price of changing the volume of the extra dimensions, and consequently of the Planck mass. This quantity is given by the following formula, and depends on \( T_0 \) since it depends on \( c \):

\[
M_{Pl}^2(T_0) = 2M^3_f \sqrt{\cosh c} \int_0^\infty \frac{dy}{\sqrt{\cosh(4ky + c)}}
\]

\[
= 2M^3_f \int_0^\infty \frac{dy}{\sqrt{\cosh 4ky + \sinh 4ky \tanh c}} \tag{3.17}
\]

Let us end this subsection noticing that the present model is quite unconventional, since, for example, the term of the action that contains the derivatives of the scalar contains a fractional power of a quantity that is not necessarily positive definite.

### 3.5 An asymmetrically warped model

To conclude our presentation of higher dimensional examples to cancel the cosmological constant, we present another case of self tuning model in five dimensions. We consider a bulk geometry of the form \([24, 25]\):

\[
ds_5^2 = -h(r)dt^2 + \frac{d^2r}{h(r)} + r^2d\Omega^2_{3,k}.
\]

where \( d\Omega^2_{3,k} \) denotes a maximally symmetric three spatial surface of constant curvature \( k \), where the SM fields are supposed to live. This form for the geometry in five dimensions naturally appears when we consider backgrounds that contain horizons covering naked singularities, a situation that we consider here in order to avoid the introduction of a second brane. The model is called asymmetrically warped since time and the three spatial dimensions are differently warped.

Since this form for the metric is different from the one discussed in the previous section, this example requires a separate discussion. In any case, we will see that, also in this context, self tuning is obtained at the price of changing the Planck mass when the brane tension changes.

In this background, a brane-world is defined in the following way. Let us consider a hypersurface that resides on a fixed point of a \( \mathbb{Z}_2 \) symmetry, and that extends along the three spatial dimensions \( \Omega_{3,k} \). Its position on the other coordinates \( t \) and \( r \) is given in terms of the proper parameter \( \tau \):

\[
X^\mu = (t(\tau), r(\tau), x_1, x_2, x_3) .
\]

Defining in a suitable way the normal to the brane, and writing the projected metric in the hypersurface, one can show that the following FRW form of the metric is obtained in four dimensions

\[
d_{\text{induced}}^2 = -d\tau^2 + r(\tau)^2d\Omega^2_{3,k} . \tag{3.20}
\]
The previous form for the projected metric suggests an interesting geometrical picture for the evolution of our four dimensional universe. Since the scale factor in four dimensions \( r(\tau) \) corresponds precisely to the position of the brane in the bulk, the motion of the brane along the bulk is reflected in the expansion (or contraction) of the corresponding four dimensional geometry.

Let us consider a brane that contains tension \( T_0 \). Without entering into details, it is known that in order to have a static brane, that is \( r(\tau) = r_0 \), the junction conditions at the position of the brane require, in suitable units, that

\[
T_0^2 - \frac{h(r_0)}{r_0^2} = 0, \quad T_0^2 - \frac{h'(r_0)}{2r_0} = 0. \tag{3.21}
\]

Once these conditions are fulfilled, one obtains a static four dimensional model that presents the usual four dimensional gravity.

Now, the idea of [26] (for other examples see [27]) is essentially the following: if the bulk metric presents integration constants contained in its metric coefficient \( h(r) \), these integration constants can be used to compensate, without fine tuning, the tension of the brane in formula (3.21).

It has been proved in [28] that, imposing that the singularity is covered by an event horizon, and with natural requirements on the bulk field content, this idea cannot be realized in the case of flat \((k = 0)\) or negatively \((k = -1)\) curved three dimensional subspace. This means that we cannot obtain a static flat solution in four dimensions, but (possibly) only a static, positively curved solution that does not depend on the value of the brane tension \( T_0 \). In the case \( k = 1 \), indeed, the self tuning mechanism works already with the simple Schwarzschild black hole, and we will limit our discussion on this case. Taking

\[
h(r) = 1 - \frac{2M}{r^2}, \tag{3.22}
\]

one fulfills the requirements (3.21) if

\[
T_0^2 = \frac{1}{2r_0^2} \quad \text{and} \quad \frac{4M}{r_0^2} = 1. \tag{3.23}
\]

Changing the brane tension, the mass \( M \) of the black hole (which is an integration constant) and the parameter \( r_0 \) change in such a way that the expressions in (3.23) are satisfied; at the same time, the volume of the extra dimensions varies its size. Consequently, it is easy to see (see for example [26]) that in this model the effective four dimensional Planck mass depends on the brane tension via the following expression

\[
M^2_{Pl}(T_0) = c \frac{M_f^6}{T_0}, \tag{3.24}
\]

where \( c \) is a(n unimportant) numerical factor, and \( M_f \) is the fundamental scale in five dimensions. Also in this model, as in the previous ones, a self tuning mechanism is obtained at the price of changing the Planck mass at the four dimensional level whenever the brane tension changes.
4. Self tuning in six dimensions and four dimensional physics

After a survey of various examples that provide higher dimensional mechanisms to cancel the cosmological constant, and after commenting on their effects at the level of four dimensional effective physics, we dedicate this section to a more detailed analysis of the six dimensional model we described in Subsection (3.3).

This six dimensional approach has various features that render it particularly interesting. First, it is possible to obtain, without invoking unconventional physics, four dimensional effective models that present flat three dimensional subspace ($k = 0$), a geometry that is favored, for example, by the recent WMAP observations [29]. Second, it is particularly suitable for interesting generalizations that allow embedding in supergravity frameworks, as in [15, 23, 16].

In the first subsection, we will explicitly discuss how four dimensional physics depends on the brane tension, showing that, by a suitable Weyl rescaling, it is possible to transfer the dependence on $T_0$ from the Planck mass to other four dimensional quantities. In the second subsection, we will present a qualitative discussion of the possible contributions, from both brane and bulk physics, to the brane tension.

The third subsection, finally, is dedicated to discuss the issue of the quantization condition that the form field, present in this model, must satisfy. We interpret this condition as a stabilization mechanism of the extra dimensions, that, as expected, results in conflict with the self tuning mechanism. Integrating out the extra dimensions, we present a four dimensional effective description of the self tuning model, based on a non-dynamical 4-form field with a quantized vacuum expectation value.

4.1 Variation of constants

In this section, we will discuss in more detail the dependence of four dimensional quantities on the brane tension $T_0$. As we will see, a Weyl rescaling of the metric allows to transfer the $T_0$ dependence of the Planck mass to other four dimensional quantities potentially measurable.

We know that the six dimensional action (3.8) contains a bulk part that describes gravity, a 2-form field, and a bulk cosmological constant. In addition, there is a localized, brane part that is generally modelled as pure tension. In this section, we are interested in the four dimensional effective action, obtained after integrating out the heavy Kaluza-Klein modes, and keeping only the fields that couple to physics localized on the brane. Imposing a suitable compensation between higher dimensional quantities (see (3.11)), that ensure a flat four dimensional background, we can model the four dimensional effective action in the following way

$$S_{4d} = M_{Pl}^2 (T_0) \int d^4x \sqrt{-g} R_4 + \int d^4x \sqrt{-g} L_4$$  \hspace{1cm} (4.1)
where $M_{Pl}(T_0)$ is the effective four dimensional Planck mass, and we introduced a Lagrangian $\mathcal{L}_4 \equiv \mathcal{L}_4(\phi, g_{\mu\nu})$. This describes fields localized on the brane, that we generally denote with $\phi$, that contribute, via their dynamics, to the brane tension $T_0$. The action explicitly depends on the brane tension through the four dimensional Planck mass, as

$$M^2_{Pl}(T_0) = \left[1 - \frac{T_0}{4\pi M^4_f}\right] M^2_{Pl}(0).$$

($4.2$)

$[M_{Pl}(0)$ is the Planck mass in absence of branes$]$

The previous formula connects, in a well defined way, the self tuning process to an observable phenomenon like the variation of the Planck mass. However, it is interesting to observe that a Weyl transformation allows to transmit all the dependence on $T_0$ from the Planck mass to pure four dimensional quantities contained in $\mathcal{L}_4$. Indeed, consider the Weyl rescaling of the metric $g_{\mu\nu} = \frac{1}{\alpha}\tilde{g}_{\mu\nu}$, with $\alpha$ given by

$$\alpha = 1 - \frac{T_0}{4\pi M^4_f}.$$  

($4.3$)

Then the action becomes

$$S^{4d}_{tot} = M^2_{Pl}(0) \int d^4x \sqrt{-\tilde{g}} \tilde{R}_4 + \int d^4x \sqrt{-\tilde{g}} \tilde{\mathcal{L}}_4(\alpha).$$

($4.4$)

It is clear that all the dependence on $\alpha$ (and consequently on $T_0$), has been transmitted to $\tilde{\mathcal{L}}_4$, that is, to the fields localized on the brane.

Interestingly, whenever $\tilde{\mathcal{L}}_4$ results independent on $\alpha$, the four dimensional action does not depend on the brane tension. But this happens only when the original $\mathcal{L}_4$ is scale invariant, in which case the cosmological constant problem does not appear at all.

Choosing a particular $\mathcal{L}_4$, for the sake of definiteness, we can understand more explicitly what is going on. Consider a simple model in which a gauge symmetry is spontaneously broken by the vev of some scalar. A possible Lagrangian, that includes also fermionic fields, is the following

$$\mathcal{L}_4 = -\frac{1}{4g^2_e} F^2_{\mu\nu} - \bar{\psi}i\gamma^\mu(\partial_\mu - iA_\mu)\psi - |(\partial_\mu - iA_\mu)\phi|^2 - \xi_0(\phi^2 - v^2_0)^2 - \lambda_0 \phi \bar{\psi}\psi.$$  

($4.5$)

where $\phi$, $\psi$ and $A_\mu$ are a Higgs scalar, a spinor and a gauge boson respectively, $g_e$ is the gauge coupling, $\xi_0$ and $v_0$ the Higgs quartic coupling and vev, and $\lambda_0$ the Yukawa coupling.

Performing the abovementioned conformal transformation, the four dimensional brane action, after rescaling the fields $\psi = \alpha^{3/4} \tilde{\psi}$ and $\phi = \sqrt{\alpha} \tilde{\phi}$ in such a way that they have canonical kinetic terms, becomes

$$\tilde{\mathcal{L}}_4 = -\frac{1}{4g^2_e} F^2_{\mu\nu} - \bar{\tilde{\psi}}i\gamma^\mu(\partial_\mu - iA_\mu)\tilde{\psi} - |(\partial_\mu - iA_\mu)\tilde{\phi}|^2 - \xi \left(\tilde{\phi}^2 - v^2(\alpha)\right)^2 - \lambda \tilde{\phi} \bar{\tilde{\psi}}\tilde{\psi},$$

($4.6$)
where the new constants appearing in the Lagrangian are:

\[ \xi = \xi_0, \quad \lambda = \lambda_0, \quad v(\alpha) = \frac{v_0}{\sqrt{\alpha}}. \]  

(4.7)

Thus, after the Weyl rescaling, various four dimensional “constant” quantities acquire a dependence on \( \alpha \), and, consequently, on \( T_0 \). The masses \( m \) of the scalar, the spinor and the vector fields depend on \( \alpha \) as:

\[ m = \frac{m_0}{\sqrt{\alpha}} \]  

(4.8)

where \( m_0 \) denotes the value obtained with \( T_0 = 0 \). For example, let us consider how one of the the masses varies with the brane tension. We obtain the formula

\[ \frac{\delta m_\psi}{m_\psi} = \frac{\delta T_0}{8\pi M_f^3} \frac{1}{1 - \frac{T_0}{4\pi M_f^2}}. \]  

(4.9)

### 4.2 Contributions to the vacuum energy

After the discussion of the previous subsection, that indicates how four dimensional quantities depend on the brane tension \( T_0 \) in this model, the next natural question to ask is which kind of contributions the brane tension can receive. When the contributions are too large, one indeed expects visible effects at the four dimensional level.

We will not consider a precise model of fields localized on the brane that, \textit{via} their dynamics, contribute to the brane tension: we will limit the discussion to a qualitative level. Nevertheless, our considerations allow to draw some conclusions on the characteristics of the models that can be phenomenologically acceptable.

*Contributions from quantum corrections*

A discussion of the contributions due to quantum corrections is necessarily model dependent, in the sense that it relies on the matter content of the brane and on the symmetries that one imposes on the system, both at the brane and at the bulk level.

In general, one expects that one loop contributions to the brane tension that involve brane fields, or Kaluza-Klein (KK) modes, result of the order of \( M_f \). Possible loop factors can in principle lower this quantity, but usually, the presence of a large number of fields brings back the total contributions to larger values; therefore, one should study in great detail quantum corrections in each specific model to calculate the precise amount of contributions to the brane tension. In any case, it is important to notice that when the size of the contributions becomes too large (close to \( 4\pi M_f^2 \)), formula (4.2) tells us that we reach a regime with strongly coupled gravitational field, that is not possible to manage.

Imposing additional symmetries to the system, like \textit{e.g.} supersymmetry, it is conceivable that quantum contributions to \( T_0 \) can be lowered to a small scale. For example, consider a supersymmetric model on the brane, in which supersymmetry is broken to a
scale $M_{\text{susy}} \ll M_f$. This condition ensures that loop contributions, involving fields localized on the brane, maintain the scale of $T_0$ much below the scale $M_f$, at the price, however, of introducing new scales in the model. However, loop contributions involving Kaluza-Klein (KK) modes of bulk fields are still potentially dangerous, since their expected corrections are still of order $M_f$, the fundamental scale in the bulk. These contributions can in principle be limited if one manages to maintain some powerful symmetry in the bulk, like supersymmetry and/or some form of scale invariance \cite{15, 17}. In this situation, indeed, the higher dimensional symmetry reduces the contributions to the brane tension due to loops involving KK fields.

We see in this way that, at least conceptually, it is possible to maintain the contributions to $T_0$ below dangerous values. The price to pay is the requirement to understand how the delicate interplay between brane and bulk symmetries can effectively be realized: further analysis on specific models are surely required for a complete comprehension of this point.

Contributions from phase transitions

Other possible contributions to the vacuum energy come from phase transitions that occur during the cosmological evolution of the Universe. It is a well known fact that a phase transition transmits to the vacuum energy a contribution proportional to the scale in which it occurs. Although the explicit study of cosmology in this class of six dimensional models is still in its infancy \cite{30}, we can nevertheless draw some qualitative observations.

If the phase transition happens at a scale comparable with the fundamental scale of the theory, $M_f$, one expects a drastic change in the Planck mass (see formula (4.2)). If, instead, one constructs models in which the typical scale for phase transitions is much lower $M_f$, the change in the Planck mass is safely small.

4.3 Quantization conditions, stabilization, and four dimensional effective model

In the previous section we have outlined how it is possible to conceive self tuning models that, although requiring a change in the volume of the extra space, can limit the effects to observable physics to an acceptable amount. In this section, we consider an additional aspect of the six dimensional model we are examining, related to the issue of stabilization of the extra dimensions.

In \cite{22} (see also \cite{15}), it has been noticed that the charge of the monopole, relative to the 2-form, must satisfy a quantization condition, that ruins the properties of the bulk action which are used to implement the idea of self tuning \footnote{The conflict between quantization conditions and self tuning models have been already discussed, in a different context, in \cite{31}.}. This fact \textit{per se} is not surprising: indeed, it is well known, and largely used in string theory, that fluxes of form fields through compact spaces can provide efficient stabilization mechanisms of the extra
dimensional space. Consequently, the six dimensional model presents in its own definition a natural stabilization mechanism of the volume, provided by the quantization condition, that contrasts with the requirement of the self tuning model.

Similar observations have been already pointed out, in a different context, in the discussion of four dimensional models, that rely on the introduction of a non-dynamical 4-form field, whose value cancels pure cosmological constant contributions \[1, 5\]. Let us briefly summarize the model and these observations, since they turn out quite useful to understand the (dimensionally reduced) six dimensional model.

The 4-form mechanism in four dimensions

The action for the four dimensional system we consider is

\[ S = \int d^4x \sqrt{-g} \left[ M^2_{Pl} R_4 - \frac{1}{48} F^2_4 - \Lambda_{bare} \right] + \frac{1}{6} \int d^4x \partial_\mu (\sqrt{-g} F^{\mu\nu\rho\sigma} A_{\nu\rho\sigma}) , \quad (4.10) \]

where \( \Lambda_{bare} \) represents the various contributions to the cosmological constant. In the above action we have added a boundary term, necessary to render the action stationary under variations that leave \( F_4 \) fixed on the boundary. The solution of the equation of motion for the 4-form reads

\[ F_{\mu\nu\rho\sigma} = c \epsilon_{\mu\nu\rho\sigma} , \quad (4.11) \]

where \( c \) is a constant, and \( \epsilon_{\mu\nu\rho\sigma} \) is the four dimensional volume density. Since \( F_4 \) is a non dynamical field, inserting back this value in the action one realizes that it is possible to compensate the value of \( \Lambda_{bare} \) via a careful choice of \( c \), in such a way that the effective cosmological constant results as

\[ \lambda_{eff} = \Lambda_{bare} + \frac{c^2}{2} . \quad (4.12) \]

However, it has been observed that, embedding the model in a higher dimensional theory like string/M-theory, the value of the four form field is quantized \[6\], and that the quantization condition is generally related to the stabilization of the moduli \[32, 33, 34\]. One finds in particular that \( c_n = e_4 n \), where \( e_4 \) is the gauge coupling of the four-form in four dimensions, and \( n \) an integer number. Consequently, only the part of brane cosmological constant that satisfies the following quantization condition can be compensated:

\[ \Lambda_{bare} = -n^2 \frac{e_4^2}{2} . \quad (4.13) \]

When the steps between two quantized values of \( c \) are sufficiently small, one can in principle obtain a degree of cancellation that brings \( \lambda_{eff} \) inside the interval fixed by experimental bounds. This goal, however, is quite difficult to obtain, and in general requires some form of anthropic considerations \[1, 32, 33, 34\] (see, however, also \[8\]).
A four dimensional description of the self-tuning model

The connection between the 4-form approach in four dimensions and the 2-form model in six dimensions, that we have just outlined, represents more than a simple analogy. Let us, indeed, recall the six dimensional action for the model of Section (3.3)

\[ S_{\text{tot}} = \int d^6 x \sqrt{-G} \left[ M_4^2 R_6 - \Lambda_B - \frac{1}{48} \hat{F}_4^2 \right] - T_0 \int d^4 x \sqrt{-g} + \int d^4 x \sqrt{-g} \mathcal{L}_4, \quad (4.14) \]

where we additionally add matter Lagrangian on the brane (consider, as an example, the Lagrangian of (4.5)), while we understand from now on possible surface terms like the ones present in (4.10). Let us write the previous action in terms of the dual of the 2-form, that in six dimensions corresponds to a 4-form, \( F_4 \):

\[ S_{\text{tot}} = \int d^6 x \sqrt{-G} \left[ M_4^2 R_6 - \Lambda_B - \frac{1}{48} \mathcal{F}_4^2 \right] - T_0 \int d^4 x \sqrt{-g} + \int d^4 x \sqrt{-g} \mathcal{L}_4, \quad (4.15) \]

The two systems described by (4.14) and (4.15) are equivalent [35, 6]. We find more comfortable to work, in this context, with the 4-form field \( F_4 \).

At this point, let us dimensional reduce this system from six to four dimensions integrating out the extra dimension. According to [31] the only relevant degrees of freedom at low energies are the ones of the massless four dimensional graviton (notice that there is no massless field associated to the deficit angle). Taking into account also the the non-dynamical 4-form field, one obtains the following dimensionally reduced action:

\[ S_4 = \int d^4 x \sqrt{-g} \alpha \left[ M_{P_4}(0) R_4 - \frac{1}{48} \hat{F}_4^2 - \Lambda \right] + \int d^4 x \sqrt{-g} \mathcal{L}_4, \quad (4.16) \]

where \( \Lambda = 4\pi R_0^2 \left( \Lambda_B - \frac{2M_4^2}{R_6} \right) \), and we define \( \hat{F}_4 \equiv 2\sqrt{\pi} R_0 F_4 \). Notice that the previous effective action depends on \( \alpha \) via the explicit overall factor, and it contains the non-dynamical field \( \hat{F}_4 \). The Weyl rescaling we have presented in section (4.1), that is \( g_{\mu\nu} = \frac{1}{\alpha} \hat{g}_{\mu\nu} \), allows to rewrite the action (4.16) in the following form, that removes the \( \alpha \) dependence from the effective Planck mass:

\[ S_4 = \int d^4 x \sqrt{-\hat{g}} \alpha \left[ M_{P_4}(0) R_4 - \frac{1}{48} \hat{F}_4^2 - \Lambda(T_0) \right] + \int d^4 x \sqrt{-\hat{g}} \hat{\mathcal{L}}_4(\alpha). \quad (4.17) \]

In the previous formula, we define \( \hat{F}_4 = \alpha^{3/2} \hat{F}_4 \) to obtain a canonically normalized field kinetic term, and in addition \( \Lambda(T_0) = \hat{\Lambda} = \frac{4\pi R_0^2}{\alpha} \left( \Lambda_B - \frac{2M_4^2}{R_6} \right) \). The Lagrangian \( \hat{\mathcal{L}}_4 \) is the one given in (4.18), and the parameters appearing in it depend on the brane tension.

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8See also [34] for a discussion of this issue in this particular system. We thank H. M. Lee and S. Förste for valuable conversations regarding this point.
At this point, comparing the first part of (4.17) with the action in (4.10), it is easy to notice the relation between the lower dimensional description of the six dimensional mechanism, and the classical 4-form model. Also in this case, keeping fixed the parameter $\Lambda_B$, a change on $T_0$ must be compensated by a change in the integration constant relative to the non-dynamical $F_4$ field. However, this form must obey a quantization condition that ultimately corresponds a stabilization condition for the volume of the two extra dimensions. For our 4-form background $\tilde{F}_{\mu\nu\rho\sigma} = E \epsilon_{\mu\nu\rho\sigma}$ (and accordingly $\tilde{F}_{\mu\nu\rho\sigma} = \frac{E}{\sqrt{\alpha_\Lambda}} \tilde{\epsilon}_{\mu\nu\rho\sigma}$), the quantization condition can be read from the quantization of the dual 2-form, expressed in [22]. It is given by

$$E_n = \frac{\sqrt{\pi \epsilon_6}}{R_0 \alpha n}$$

(4.18)

where $e_6$ is the gauge coupling of the 4-form in six dimensions (and it is related to the gauge coupling of the 4-form in four dimensions by $e_4 = \frac{\sqrt{\pi \epsilon_6}}{R_0 \alpha_\Lambda}$).

The effective cosmological constant then has a different dependence on the bare cosmological constant (here tension of the brane) than the one noted in (4.12). In this case we obtain

$$\lambda_{\text{eff}} = \frac{1}{\alpha} \left( \Lambda + \frac{E_n^2}{2} \right),$$

(4.19)

where the tension dependence (through $\alpha = 1 - \frac{T_0}{4\pi M_P^2}$) enters both as an overall factor and in the value of $E_n$. It would be interesting to construct cosmological scenarios based on this quantized model, on the lines of the cosmology of the 4-form field [6, 32, 34].

5. Conclusions

We have explored the self tuning idea in brane world models based on extra spacetime dimensions. These are models where solutions with a Poincaré invariant vacuum can be found irrespectively of the brane vacuum energy (tension), when keeping all the other bulk and brane parameters fixed. We have analyzed the various ways that self tuning can be realized and reviewed explicit examples where this can be achieved.

The important message of the present paper is that, in general, the four dimensional physics is affected by the modification of the brane vacuum energy. Indeed, all the available examples present a variation of the compactification volume as the tension of the brane changes. This means that, although solutions can be found where the curvature of the four dimensional space remains zero as the brane tension changes, the effective Planck mass or the masses of matter fields on the brane vary. Consequently, it is important to determine, in discussing a specific model, whether the variation of observable quantities can be limited in order to satisfy phenomenological bounds.

In addition, it is observed that a generic feature of the self tuning mechanism is the variation of the compactification volume as a function of the brane tension. In the examples we discussed, any mechanism that would be used to stabilize the size of the extra dimensions...
will be in apparent conflict with self tuning: once the extra dimension is stabilized the brane vacuum energy cannot vary without re-introducing fine tuning.

The most promising example of self tuning that we have singled out in the paper is the one with three-branes embedded in six dimensions. An interesting property of co-dimension two branes, firstly noticed in [12], makes this model particularly attractive. However, the quantization condition of the gauge field used in the particular setup, that essentially corresponds to a stabilization condition for the extra dimensions, disturbs the self tuning. For the particular model, we show that the six dimensional self tuning resembles a higher dimensional realization of a well known mechanism to compensate the cosmological constant in four dimensions, based on a non-dynamical field.

We would like to stress that the self tuning models have opened a new point of view to the cosmological constant problem, but still there are various issues that deserve further investigations. The variation of four dimensional constants should be treated with caution since, as we noted, in some cases one needs to keep the contributions to the vacuum energy smaller than the higher dimensional fundamental scale. This could imply the necessity to introduce new scales on the model, or invoking additional symmetries. In addition, the apparent conflict between self tuning and stabilization should be taken into account when considering, in a specific model, a mechanism to stabilize the extra dimensions. Furthermore, the question of how to eliminate nearby curved solutions remains an important challenge.

Concluding, the self tuning brane world models provide a new insight to the cosmological constant problem and are accompanied with very interesting four dimensional signatures, as well as potential disturbing problems. A more careful study of these models, that includes also an analysis of the dynamics of matter localized on the brane, may shed light on many of the unsettled aspects of these models.

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A. The singular part of the scalar curvature

In this appendix we present the relation of the singular part of the curvature scalar to the brane tension in $D$ dimensions, obtained by the Einstein equations. Suppose we start from
an action of the form:

\[ S_{\text{tot}} = M^{D-2}_f \int d^D x \sqrt{-g_D} R_D + S_{B,br} \quad (A.1) \]

where \( S_{B,br} \) is the action of the bulk and the branes present in the model. The Einstein equations derived from the above action are:

\[ R^D_{MN} - \frac{1}{2} g^D_{MN} R^D = \frac{1}{2M^{D-2}_f} T_{MN} \quad (A.2) \]

and their trace is:

\[ M^{D-2}_f R_D = -\frac{1}{D-2} T^M_M \quad (A.3) \]

Suppose the \( D \)-dimensional metric has the form (2.2):

\[ ds^2_D = e^{2W(y)} g_{\mu\nu}(x) dx^\mu dx^\nu + \gamma_{mn}(y) dy^m dy^n \quad (A.4) \]

where we have in general the \( x^\mu \) coordinates to span a \((p+1)\)-dimensional subspace. If the model has only \( p \)-branes along the \( x^i \) directions, then the energy momentum tensor is:

\[ T_{MN} = T^{(B)}_{MN} - \sum_i e^{2W(y)} g_{\mu\nu}(x) e^{\sigma_i \phi(y)} T_i \Delta^{(d)}(y - y_i) \quad (A.5) \]

where \( T^{(B)}_{MN} \) is the bulk contribution to the energy-momentum tensor and the next terms are the ones of the \( p \)-branes coupled to a scalar field \( \phi \). We denote by \( \Delta^{(d)}(y - y_i) \) the generalized delta function with the property \( \int \sqrt{\gamma} \Delta^{(d)}(y - y_i) = 1 \) [17]. Taking the trace we find:

\[ T^M_M = T^{(B)}_M - (p + 1) \sum_i e^{\sigma_i \phi(y)} T_i \Delta^{(d)}(y - y_i) \quad (A.6) \]

where the factor \( (p + 1) \) appears because the trace of the induced metric in the \((p + 1)\) longitudinal dimensions of the \( p \)-brane has been performed. Combining the above equality with (A.3), we find that the singular part of the higher dimensional curvature is:

\[ M^{D-2}_f R^{\text{sing}}_D = \frac{p + 1}{D-2} \sum_i e^{\sigma_i \phi(y)} T_i \Delta^{(d)}(y - y_i) \quad (A.7) \]

Hence, the quantities \( R^{\text{sing}}_i \) defined in section 2.3 are:

\[ M^{D-2}_f R^{\text{sing}}_i = \frac{p + 1}{D-2} e^{\sigma_i \phi(y_i)} T_i \quad (A.8) \]

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