Stability Analysis of Half-rotating Walking Mechanism with Self-balancing Configuration

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Abstract. Aiming at the problem of dynamic stability of half-rotating walking mechanism, a walking mechanism with self-balancing configuration is proposed. Using D-H method, the transformation matrix between coordinate systems is obtained. Based on the structural and kinematic characteristics of the half-rotating walking mechanism, a new evaluation method for the kinematic stability of the half-rotating walking mechanism is proposed: the stable triangle method. A stable walking strategy of half-rotating walking mechanism is proposed to keep the minimum stable angle as the maximum in the process of motion. Adams motion simulation shows the strategy adopted is reasonable and effective. The stability analysis based on the minimum stability angle shows that the half-rotating walking mechanism can walk stably, and the faster the speed of the half-rotating mechanism is, the weaker the stability of the mechanism is.

1. Introduction

The motion stability of walking robot is the key to ensure it to complete the target task, and it is also a hot issue concerned by scholars at home and abroad. Whether the mobile robot has enough anti overturning ability is very important, and it is also an important index to evaluate the obstacle surmounting performance of the robot [1]. In conclusion, it is necessary to establish a reasonable stability evaluation standard for the structure design and motion control of the mobile system. At present, there are many methods to judge the overturning stability of mobile system. Static stability judgment methods [2-3] mainly include center of gravity projection method and static stability boundary method. When a robot performs any kind of motion, due to the dynamic effect produced by its motion, the static stability index fails to judge its stability. Only the dynamic stability criterion can perform better [4]. Dynamic stability judgment methods [5-8] mainly include pressure center method (COP), zero moment point method (ZMP), force angle stability measurement (FASM) and stability cone method.

Based on the motion principle of half-rotating mechanism [9], the research and innovation team of Anhui University of technology has constructed a half-rotating walking mechanism [10]. Aiming at the problem of its walking stability, a self-balancing mechanism configuration is proposed, and a new evaluation method of motion stability of half-rotating walking mechanism is proposed: stable triangle method. The stability of the half-rotating walking mechanism is evaluated based on the minimum stability angle. Finally, the simulation experiment is carried out to verify and evaluate the stability.
2. Self-balancing configuration of half-rotating mechanism
Aiming at the problem of dynamic stability of half-rotating mechanism, a half-rotating walking mechanism with self-balancing configuration is proposed, as shown in figure 1. In this configuration, the balancing device is a counterweight, which is placed in the vertical guide groove in the middle of the central axis of the mechanism. When the walking mechanism is moving, the half-rotating leg rotates around the central axis (frame). At the same time, the motor drives the balance weight to move in a straight line in the guide rail slot to adjust the overall center of mass of the walking mechanism, so as to ensure that the center of mass of the half-rotating mechanism is always in the stable area, and then realize the stable walking of the half-rotating mechanism.

3. Kinematics analysis of half-rotating walking mechanism
In order to calculate the homogeneous transformation matrix of the foot relative to the central coordinate system, the coordinate system should be established first, as shown in figure 2. According to the coordinate system in figure 2, the kinematic D-H parameters of the half-rotating walking mechanism can be obtained, and the relevant parameters are shown in table 1. In table 1, only the D-H parameters of the right part are listed. Because the half-rotating walking mechanism is symmetrical, for the coordinate system transformation of the left straddle bar, it is only necessary to change the value of joint offset \(d\) along the Z-axis to negative.

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### Table 1. D-H parameters of half-rotating walking mechanism coordinate system.

| Coordinate system | Joint angle \(\theta\)/rad | Joint offset \(d\)/mm | Length of connecting rod \(a\)/mm | Angle of torque \(\alpha\)/rad |
|-------------------|----------------------------|------------------------|-------------------------------|-----------------------------|
| \(R \rightarrow A_i\) | \(-\pi/2\)                  | \(d_{RA}\)             | \(a_{RA}\)                     | 0                           |
| \(A_i \rightarrow R_i\) | \(-\theta\)                | \(d_{AB}\)             | \(a_{AB}\)                     | 0                           |
| \(B_{i+1} \rightarrow C_{i+1}\) | \((\pi + \theta)/2\) | \(d_{BC}\)             | \(a_{BC}\)                     | 0                           |
| \(C_{i+1} \rightarrow D_{i+1}\) | \((\theta - \pi)/4\)  | \(d_{CD}\)             | \(a_{CD}\)                     | 0                           |

According to Euler kinematics, the homogeneous transformation matrix from the origin \(R\) of the central coordinate system of the mechanism to the end point of the step bar (taking the end point of \(D_{i+1}\) as an example) is as follows
Where, $\theta_1 = -\theta$, $\theta_2 = \frac{\pi + \theta}{2}$, $\theta_3 = -\frac{\pi}{4} + \frac{\theta}{4}$, $s_1 = \sin \theta_1$, $s_{12} = \sin(\theta_1 + \theta_2)$, $s_{123} = \sin(\theta_1 + \theta_2 + \theta_3)$, $\theta_1 = -\theta$, $\theta_2 = (\pi + \theta)/2$, $\theta_3 = (\theta - \pi)/4$, $c_1 = \cos \theta_1$, $c_{12} = \cos(\theta_1 + \theta_2)$, $c_{123} = \cos(\theta_1 + \theta_2 + \theta_3)$. Because the guide rail where the counterweight is located coincides with the x-axis of the central coordinate system, so the coordinates of the center of mass of the counterweight can be expressed as follows

$$x_p y_p z_p = [l 0 0]^T$$

4. Stability analysis of half-rotating walking mechanism

4.1. Stable triangle method

According to the structure and motion characteristics of half-rotating mechanism, a new evaluation method is proposed for the stability analysis of half-rotating mechanism: stable triangle method. The stable triangle method is a simplification of the stable cone method, as shown in figure 3. The principle of stable triangle method is the same as that of stable cone method. In essence, it is to remove the consideration of stability of some side lines by person. The above consideration is to eliminate the instability consideration of angle point tipping and left and right side line overturning. It can not only effectively evaluate the motion stability of the half-rotating walking mechanism, but also reduce a lot of operations.

According to the stable cone method, the center of mass (CM) of the half-rotating walking mechanism is the vertex of the stable cone.

$$r_{CM} = \frac{\sum_{i=1}^{k} m_i r_i}{\sum_{i=1}^{k} m_i}$$

Where, $r_{CM}$ is the vector of the center of mass of the half-rotating walking mechanism in the global coordinate system, $r_i$ is the vector of the center of mass of each component of the half-rotating walking mechanism in the global coordinate system, $m_i$ is the corresponding mass of each component, and $k$ is the number of components.

4.2. Angle of stability

Stability angle is an important index to measure the stability performance of walking mechanism. The schematic diagram of the side line tipping angle of the half-rotating walking mechanism is shown in figure 4, where $a$ is the half length of the foot. According to figure 4, it can be analyzed that the sideline tipping stable angle $\gamma$ of the half-rotating walking mechanism is as follows

$$\gamma = \arctan\left(\frac{a - r_{CMx}}{r_{CMy}}\right) \quad (i=1,2)$$

Considering the global stability of the system, the stability angle is taken

$$\theta = \min \gamma_i$$

When $\theta < 0$, it means that the equivalent force on the center of mass of the mechanism is outside the stable cone, and the walking mechanism is unstable and will overturn; when $\theta = 0$, the walking mechanism is unstable; when $\theta > 0$, the equivalent force on the center of mass of the mechanism is within the stable cone, the walking mechanism is in a stable state. $\theta$ represents the stability of the walking mechanism. The larger the $\theta$ is, the more stable the walking mechanism will be, and the
dynamic stability of the walking mechanism will become the problem of finding a larger tipping angle.

4.3. Stable walking strategy of half-rotating walking mechanism

It is expected that by adjusting the motion of the balance weight in real time, the half-rotating walking mechanism can complete the stable stride action, and keep the pitch angle of the central axis of the mechanism to 0, so as to realize the stability control of the half-rotating walking mechanism. Therefore, the control of the stable walking of the half-rotating walking mechanism is transformed into the position control of the counterweight drive.

According to the bar constitution of the half-rotating walking mechanism, the centroid of the half-rotating walking mechanism can be calculated according to equations (1) and (3).

\[
 r_{CM} = \frac{2(4m_j + 2m_k + m_\omega)\gamma_{02} + 2m_y \gamma_{06} + m_x \gamma_{05} + m_p \gamma_{03}}{2(4m_j + 2m_k + m_\omega) + m_x + m_p} 
\]

(6)

Where, \( m_j, m_k, m_\omega, m_y, m_x, m_p \) represents the mass of foot sole, stride rod, secondary rotating arms, first rotating arms, central axis of mechanism and balance mechanism, respectively.

According to the above analysis of the motion characteristics of the half-rotating mechanism, only the forward and backward tilting of the half-rotating mechanism is considered, that is, only the coordinates in the XY plane are considered. In the mechanism center coordinate system \( R \), the centroid of the half-rotating walking mechanism can be calculated according to equation (6).

\[
 r_{CM} = \left[ \begin{array}{c} m_1 \frac{m}{m} + \frac{2m_y}{m} a_{AZ} - a_{AB} \\ - \left( m_1 \frac{m}{m} a_{AB} + \frac{2m_y}{m} a_{AZ} - a_{AB} \right) c_1 + a_{BC} c_{12} + a_{CD} c_{123} + m_p l e_\phi \\ - \left( m_1 \frac{m}{m} a_{AB} + \frac{2m_y}{m} a_{AZ} - a_{AB} \right) s_1 - a_{BC} s_{12} - a_{CD} s_{123} - m_p l s_\phi \\ - \left( m_1 \frac{m}{m} a_{AB} + \frac{2m_y}{m} a_{AZ} - a_{AB} \right) c_1 + a_{BC} c_{12} + a_{CD} c_{123} + m_p l e_\phi \\ - \left( m_1 \frac{m}{m} a_{AB} + \frac{2m_y}{m} a_{AZ} - a_{AB} \right) s_1 - a_{BC} s_{12} - a_{CD} s_{123} - m_p l s_\phi \end{array} \right] 
\]

(7)

Where, \( m = 2(4m_j + 2m_k + m_\omega) + m_x + m_p \), \( m_1 = 2(4m_j + 2m_k + m_\omega) \), \( \omega = -90^\circ \).

Then, the attitude transformation matrix is used to project the equation (7) into the follow-up coordinate system \( R' \).

\[
 r_{CM} = \left[ \begin{array}{c} c_\phi z \ - s_\phi z \\ s_\phi z \ c_\phi z \end{array} \right] \psi_{CM} = \left[ \begin{array}{c} \frac{m_1}{m} a_{AB} + \frac{2m_y}{m} a_{AZ} - a_{AB} \\ - \left( m_1 \frac{m}{m} a_{AB} + \frac{2m_y}{m} a_{AZ} - a_{AB} \right) c_1 + a_{BC} c_{12} + a_{CD} c_{123} + m_p l e_\phi \\ - \left( m_1 \frac{m}{m} a_{AB} + \frac{2m_y}{m} a_{AZ} - a_{AB} \right) s_1 - a_{BC} s_{12} - a_{CD} s_{123} - m_p l s_\phi \\ - \left( m_1 \frac{m}{m} a_{AB} + \frac{2m_y}{m} a_{AZ} - a_{AB} \right) c_1 + a_{BC} c_{12} + a_{CD} c_{123} + m_p l e_\phi \\ - \left( m_1 \frac{m}{m} a_{AB} + \frac{2m_y}{m} a_{AZ} - a_{AB} \right) s_1 - a_{BC} s_{12} - a_{CD} s_{123} - m_p l s_\phi \end{array} \right] \psi_{CM} 
\]

(8)

As shown in figure 5, to control the balance weight to move along the guide rail groove of the central axis of the mechanism, so that the center of mass of the half-rotating walking mechanism is directly above the center of the supporting polygon at all times.
According to equation (8), the following expression can be obtained.

$$0 = \left( \frac{m_1}{m} a_{AB} + \frac{2m_1}{m} a_{AZ} - a_{AB} \right) s_{\phi_{s+1}} - a_{BC} s_{\phi_{s+12}} - a_{CD} s_{\phi_{s+123}} + \frac{m_p}{m} l c_{\phi_z}$$

(9)

In general, the central axis attitude of the mechanism will be pitched at a small angle, and the desired attitude is zero, that is, $\phi_z = 0$. According to formula (9), the following expression can be obtained.

$$l = m \left[ \frac{m_1}{m} a_{AB} + \frac{2m_1}{m} a_{AZ} - a_{AB} \right] s_{\phi_{s+1}} - a_{BC} s_{\phi_{s+12}} - a_{CD} s_{\phi_{s+123}}$$

(10)

Where, $l$ in equation (10) is the displacement of the counterweight along the guide rail groove on the central axis of the mechanism during the movement of the mechanism.

5. Simulation verification of motion stability of half-rotating walking mechanism

5.1. Motion simulation of half-rotating mechanism

Take the parameters of the half-rotating walking mechanism as shown in table 2, establish the three-dimensional model of the half-rotating walking mechanism in ADAMS software, and complete the relevant conditions setting. Next, the simulation experiment of the half-rotating walking mechanism is carried out. Figure 6 shows the movement simulation of the half-rotating walking mechanism in a step cycle. The simulation results show that the half-rotating walking mechanism can realize stable walking. The simulation results verify the feasibility of the proposed self-balancing walking mechanism and the rationality and effectiveness of the stable walking strategy.

| Component name       | Length/mm | Quality/g |
|----------------------|-----------|-----------|
| First-order rotating arm | 70        | 43        |
| Second-order rotating arm | 210       | 89        |
| Stride rod           | 260       | 78        |
| Central axis         | 400       | 118       |
| Sole of the foot     | 50        | 0.6       |
| Balance block        | /         | 4000      |
| Guide slot           | 500       | /         |
| Links lateral deviation | 20       | /         |

5.2. The influence of motion speed on stability

When the half-rotating walking mechanism runs at different driving speeds, the change of pitch angle
of the central axis of the mechanism is different under the action of various forces. In order to ensure the walking quality, the pitch angle is only allowed to fluctuate in a small range near the equilibrium position ($\phi_2=0$). If the pitch angle is too large, the tilting of the walking mechanism will easily occur.

Figure 7 shows the fluctuation of the elevation angle of the central axis of the half-rotating mechanism under different first-order rotating arm speeds. It can be seen from the figure that the faster the rotation speed of the first-order rotating arm is, the greater the change of the pitch angle of the central axis of the mechanism is, and the more unstable the motion of the walking mechanism is. In the low-speed state, the variation range of the central axis elevation angle $\phi_z$ of the configurations is $\pm 0.5^\circ$, and the motion stability of the half-rotating walking mechanism is better. With the increase of the rotational speed, the motion stability of the half-rotating walking mechanism is poor.

5.3. Simulation of minimum stable angle
Figure 8 shows the curve of the minimum stability angle of the half-rotating walking mechanism with self-balancing configuration in a motion cycle at a fixed speed measured by the proposed stable triangle method, and its theoretical value is determined by equation (4) and (5). It can be seen that the minimum stable angle of the walking mechanism is less than its theoretical value. This is because the mechanism is inevitably affected by the measurement error, control error, inertial force and external interference in the movement process. In addition, the fluctuation of the minimum stable angle increases with the increase of the running speed of the mechanism.

6. Conclusion
Based on the motion principle of the two-stage half-rotating mechanism, a half-rotating walking mechanism with self-balancing function is designed.

The kinematics analysis of the half-rotating mechanism is carried out, and the position matrix of
the foot end and balance weight is obtained relative to the central coordinate system. Based on the stable triangle method, a stable walking strategy of half-rotating walking mechanism is proposed, which makes the whole center of mass of the half-rotating walking mechanism directly above the center of the supporting polygon.

The simulation results of Adams show that the strategy is reasonable and effective. The stability analysis based on the minimum stability angle shows that the half-rotating walking mechanism can walk stably, and the faster the speed of the half-rotating mechanism is, the weaker the stability of the mechanism is.

7. Conflict of Interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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