Significance of an external magnetic field on two-phonon processes in gated lateral semiconductor quantum dots

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Theoretical and numerical calculations of two-phonon processes on gated lateral semiconductor quantum dots (QDs) are outlined. A heterostructure made with two laterally coupled QDs, in the presence of an external magnetic field, has been employed in order to study the electron scattering rate due to two-phonon processes. The formalism is based on the acoustic phonon modes via the unscreened deformation potential and the piezoelectric interaction whenever the crystal lattice lacks a center of inversion symmetry. The rates are calculated by using second order perturbation theory. The strong dependence of the scattering rate on the external magnetic field, lattice temperature and QDs separation distance is presented.

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I. INTRODUCTION

The gated lateral semiconductor quantum dots (QDs) in a quantum well (QW), in which the growth direction (z direction, or vertical direction) confinement is due to the higher bandgap of the barrier materi,

I. INTRODUCTION

We consider a heterostructure composed of two laterally coupled QDs. In order to calculate the electron states within the coupled system, we have used a one-band ef-
fective mass approximation. The Hamiltonian which describes the single-electron motion, which is confined in laterally coupled QDs is given by
\[ \hat{H} = \hat{H}_\parallel + \hat{H}_z \] (1)
where the lateral motion of electron is decoupled from the one along the quantum well growth (z-axis). The external magnetic field is applied along the z-axis \( B = B\hat{e}_z \), and as a result the magnetic vector potential \( \mathbf{A} \) could be given as
\[ \mathbf{A} = B ( -y\hat{e}_x + x\hat{e}_y ) / 2 \] (2)
The Hamiltonian operators for the lateral directions and z-direction have been considered as
\[ \hat{H}_\parallel = \frac{\hat{p}_x^2}{2m^*} + \frac{\hat{p}_y^2}{2m^*} + \frac{\hat{p}_z^2}{2\omega^2} + \frac{1}{2}\mathbf{L}_z \] (3)
\[ \hat{H}_z = -\frac{\hbar}{2} \frac{1}{m^*(z)} \frac{\partial}{\partial z} + V_0 \Theta ( |z| - L_0 ) . \] (4)
where \( \mathbf{L}_z \) is the operator of the z component of the angular momentum, \( m^*(z) \) is the electron effective mass, \( V_0 \) is the offset between the band edges of the GaAs well and the AlGaAs barrier, \( \Theta \) is the Heaviside step function, \( \hat{p}_z \) is the quantum mechanical operator of momentum, \( \omega^2 = \omega_0^2 + (\omega_c/2)^2 \).

According to Eq. (1), electron wavefunction can be given by the following envelope function,
\[ \psi(r) = \psi_\parallel (r_\parallel) \psi_z (z) \] (5)

In our investigation, we have only considered the ground state wavefunction along the QW growth and the wavefunction along the lateral direction is given by Fock-Darwin states. Following the same procedure as 9, we have considered that the external confining potential for the electron within two QDs structure is given by
\[ V_c = \frac{1}{2} m^* \omega_0^2 \min\{ (x - \alpha)^2 + y^2, (x + \alpha)^2 + y^2 \} \] (6)
where \( \alpha \) is the separation distance of the dots. The electron wavefunction of the coupled QD structure could be described by
\[ \Psi(r) = \Psi_\parallel (r_\parallel) \psi_z (z) \] (7)
where the single electron wavefunction for the parallel plane is given by:
\[ |\Psi_\parallel > = \sum_k C_k |\psi_\parallel^k, L > + D_k |\psi_\parallel^k, R > \] (8)

A numerical scheme has been employed in order to calculate the total wavefunction in the parallel plane of the coupled dot system. In low dimensional structures, the electrons interact with acoustical and optical phonons. The optical phonons do not have any contribution to electron scattering rates due to the small electron energy splitting. Therefore, only the acoustical phonons contribute to the relaxation rates. In this work, we calculate the electron scattering rate which is caused due to deformation potential and piezoelectric acoustic phonon interaction. The Hamiltonian which describes these interactions is given by:
\[ H = \sum_q \left( \frac{\hbar^2}{2\rho_m V \omega_q} \right)^{1/2} \mathcal{M}(q) \rho(q) (a_q + a^\dagger_{-q}) , \] (9)
The term \( \mathcal{M}(q) \), which includes both the deformation and the piezoelectric interaction for zinblende crystals, is defined by
\[ \mathcal{M}(q) = D |q| + i M^\rho \delta (q) \] (10)
with
\[ M^\rho_q = 2\epsilon_{14} (q_x q_y \xi_z + q_y q_z \xi_x + q_z q_x \xi_y) \] (11)

In Eqs. (9-11), \( \rho_m \) is the mass density of the host material, \( \omega_q \) is the frequency of the phonon mode with wavevector \( q \), \( V \) is the volume of the sample, \( a_q \) and \( a^\dagger_{-q} \) are phonon annihilation and creation operators, \( \rho(q) \) is the electron density operator, \( D \) denotes the deformation potential, \( \epsilon_{14} \) is the piezoelectric constant and \( \xi \) is the polarization vector. All values of the above mentioned parameters used in our calculations have been taken from Ref. 18.

The last part of our theoretical formalism is the calculation of the electron scattering rates due to two-phonon processes. Considering only LA phonons, the scattering rates (second order perturbation theory) are given by the following equations
\[ \Gamma_{++} = \frac{\pi}{\hbar} \sum_q \sum_k \sum_s \left( \frac{M^s q \delta k}{E_i - E_s - E_q} + \frac{M^s q f \delta k}{E_i - E_s - E_k} \right)^2 \] \( (N_q + 1) (N_k + 1) (N_q - E_f - E_q) \) (12)
\[ \Gamma_{+-} = \frac{2\pi}{\hbar} \sum_q \sum_k \sum_s \left( \frac{M^s q \delta k}{E_i - E_s - E_q} + \frac{M^s q f \delta k}{E_i - E_s - E_k} \right)^2 \] \( N_k (N_q + 1) (E_f - E_i - E_q + E_k) \) (13)
\[ \Gamma_{-+} = \frac{\pi}{\hbar} \sum_q \sum_k \sum_s \left( \frac{M^s q \delta k}{E_i - E_s + E_q} + \frac{M^s q f \delta k}{E_i - E_s + E_k} \right)^2 \] \( N_q N_k (E_f - E_i + E_q + E_k) \) (14)
where the indices ++, −−, +−, −+ represent the emission of two phonons (LA+LA), the absorption of two phonons (LA-LA) and the emission of one phonon and absorption of one phonon (LA-LA or -LA+LA) respectively. \( M^s \) stands for the electron-phonon matrix elements where
the index \(i\) (f) corresponds to qubit electron first excited state (ground state) and \(s\) stands for the intermediate electronic states. The other elements are taken by changing the proper suffixes. \(N_k\) \((N_q)\) is the Bose distribution function referring to phonons with energy \(E_k = \hbar \omega_k\) \((E_q = \hbar \omega_q)\). Note that the summation over \(s\) excludes the initial and final states. The integrals which are included in Eqs. (12-14) by transforming the summations to integrations, have been calculated by Monte Carlo code.

### III. RESULTS

Fig. 1 shows all possible scattering processes concerning the electron transitions due to second order contributions associated to acoustic phonons. The transitions described by eqs. (12) and (13) are presented in Fig. 1-I (Fig. 1-b) and Fig. 1-III (Fig. 1-c) respectively. It is worth mentioning that eq. (13) creates two different transitions as illustrated in Fig. 1-II (Fig. 1-b) and Fig. 1-IV (Fig. 1-d). Using the results of the second order perturbation theory (Eqs. 12 14), we estimate the relaxation rates for an electron which relaxes to ground state via the two phonon processes.

In Fig. 2 we present the relaxation rates for the case of the emission of a LA phonon and the absorption of a LA phonon (LA-LA), as a function of an external magnetic field. Increasing the magnetic field in the range of 0-12 T, the electron wavefunctions get the largest value (resonance value) at \(B = 3.7\) T and as a result the matrix elements involved in the two-phonon scattering process increase. Furthermore, the increasing number of phonon modes (for \(B = 0-4\) T) that can be involved in the relaxation process, increases the scattering rates. For larger values of the magnetic field \((B = 4-12\) T) the rates decrease due to the fact that the electron wavefunctions move away from the resonance value and due to the decreasing number of phonon density of phonons\(9\). The dependence of matrix elements on wavevector is related to \(\sqrt{q}\) for the deformation potential, while for the piezoelectric coupling it is related to \(1/\sqrt{q}\). It is worth mentioning that although we have included both deformation and piezoelectric interactions in our phonon description, it is impossible to separate their contribution to the electron relaxation rates though two-phonon processes because of the dependence of matrix elements on the interaction strength (eq.10).

The dependence of the electron scattering rates on the half of the interdot separation distance for a range of magnetic field values is presented in Fig. 3. It is obvious that for \(B = 4\) T the rates have the largest value because the electron-phonon matrix elements get the highest value at magnetic field 3.7 T. Increasing the interdot distance, the rates emerge peaks due to the increasing number of phonon density of phonons and due to the matrix elements enhancement. For large interdot distance, the scattering rates decrease because of the decreasing number of phonon modes that can be involved in the scattering rates.

Fig. 4 shows the calculated relaxation rates as a function of the lattice temperature for all possible two phonon processes for a non zero applied external magnetic field. The first feature is that there are crossovers between the rates of different phonon processes due to different phonon population factors related to Bose distribution function (see Eqs. (12 14). Secondly, the rates increase rapidly by increasing the lattice temperature from low temperatures-which are generally the operating temperature of the devices made with low dimensional structures.
FIG. 3: Relaxation rates of an electron due to multiphonon process LA-LA versus the half interdot distance $\alpha$. The lattice temperature is fixed to $T = 1 \text{ K}$ and QW width is fixed to $2L_z = 10 \text{ nm}$.

FIG. 4: Relaxation rates of an electron due to four different multiphonon processes versus the lattice temperature. The magnetic field is fixed to $B = 8\text{ T}$ confinement strength is $\hbar \omega = 3 \text{ meV}$, half interdot distance $\alpha = 25 \text{ meV}$ and QW width $2L_z = 10 \text{ nm}$.

up to $300 \text{ K}$. The rise of the rates reaches almost 8 order of magnitude difference (-LA+LA) for the examined edges of temperature interval. This type of behavior is because of differing phonon population factors as the lattice temperature increases. Lastly, at room temperature the processes +LA+LA and -LA-LA have almost the same scattering rates due to the same phonon distribution. The same feature also exists for the rates corresponding to the +LA-LA and -LA+LA processes for very large temperature.

IV. CONCLUSIONS

We have researched the decoherence channel due to electron-phonon interaction by studying the two-phonon processes on gated lateral semiconductor QDs. We have studied the electron coupling to acoustical phonons through a deformation potential and piezoelectric interaction and found a strong dependence of relaxation rates on the external magnetic field, the separation distance and the lattice temperature. Although the electron relaxation rates can have very tiny values for the $mK$ range, the increase of temperature can increase the rates to large values. Lastly, according to the best of the authors knowledge, such experiments related to charge decoherence for our geometric configuration in the presence of an external magnetic field and large operating temperature have not been reported in order to compare them with our theoretical results.

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