Numerical simulation and analysis of energy loss in a nanosecond spark gap switch

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Abstract. A system of differential equations for the RLC circuit of a capacitor-switch assembly was derived being supplemented with an equation for the spark resistance of the switch in accordance with the Braginsky model. The parameters that affect the solutions of equations for the circuit with parallel or series connection of several capacitor-switch assemblies to a common inductive load were determined. Based on numerical solution of the system of equations, a dependence of the energy $E_S$ released in the spark within the first half-period on the discharge circuit and switch parameters was found.

1. Introduction

The spark gap switches are much used for switching high-current pulses over a wide (from subnanoseconds to microseconds) time range [1-4]. This type of switches finds application in different electrophysical experiments: for generation of electron and ion beams [5-7]; excitation of high-power microwaves [8,9]; design of high-intensity X-ray sources [7,10–12]; research in gas discharges [13–15], etc. The challenge of the last years is to create setups capable of transferring megajoule energy to a load in ~100 ns [16], which is required for inertial thermonuclear fusion. In particular, high-current pulse generators serve as drivers in the circuits in which a thermonuclear target is excited with Z-pinch [17–20]. The generation of nanosecond high-power pulses requires multi-module systems in which the energy from each module is transferred through a spark gap switch either to an intermediate energy store [21,22] or to a transmission line [16,23,24]. An important problem in designing this type of setups is to provide efficient use of the energy of the primary store [25] and this involves, among other things, minimization of the energy lost in the switch.

The operation of high-current switches is described on the basis of gas discharge theories. However, despite many works devoted to this problem [13,26,27], the now available discharge theories are still incomplete and contradictory. Nevertheless, the main conclusions following from analysis of rather simple discharge mechanisms are of practical value and are used in designing high-current switches [2].

The operation of a gas-filled spark gap switch results in a current channel of resistance equal to

$$R_s = \frac{l_s}{\pi a^2 \sigma},$$

(1)
where \( l_s \) is the channel length; \( \sigma \) is the channel conductivity; \( a \) is the channel radius. The time dependence of the channel radius and conductivity is determined with the use of various models.

For nanosecond spark gap switches, the best agreement with experiment is ensured by the Braginsky model \([28]\) which describes the development of a spark channel in gas at high pressures and moderate currents. The model assumes that in the gas there arises a rather narrow current-conducting channel in which Joule heat is released resulting in a higher gas pressure and in a shock wave in the gas. It is assumed that the heat from the channel is removed by the radiation absorbed in the shock wave region; in this shock wave region, gas ionization takes place. In the self-similar solution derived for this case, the pressure as well as the temperature and density are constant along the channel radius and the velocity is proportional to the radius. The energy balance equation in this solution has the form:

\[
\frac{I^2}{\sigma} = \frac{\pi^2 \xi}{4 \rho} \left( \frac{da^2}{dt} \right)^3,
\]

where \( \rho \) is the gas density ahead of the shock wave; \( \xi \) is a dimensionless quantity which is generally a function of time.

It is shown that in the range of currents \( I \) in which the model is applicable, the temperature of the expanding channel varies slowly and can thus be taken roughly constant \([28]\). Hence, the channel conductivity and the dimensionless function \( \xi \) are found constant. According to \([28]\), the values of these functions for a discharge in air are \( \sigma = 2 \cdot 10^{14} \text{s}^{-1} \); \( \xi = 4.5 \).

An expression for the channel resistance in the Braginsky model can be derived from (1) in view of (2). This expression has the form:

\[
R_s(t) = \left( \frac{\xi \rho}{4 \pi \sigma^2} \right)^{1/3} \frac{l_s}{\int_0^t I^{2/3} \, dt'}.
\]

The differential form of expression (3) is as follows:

\[
\frac{1}{R_s^2} \frac{dR_s}{dt} = -1 \left( \frac{4 \pi \sigma^2}{\xi \rho} \right)^{1/3} I^{2/3}.
\]

The initial condition for equation (3a) is \( R_s(t = 0) \rightarrow \infty \).

In the paper presented, we perform numerical study of equation (3a), which is based on the Braginsky model, and generator circuit equations for inductive load. Based on the numerical solutions, expressions which determine the energy released in the switch are derived.

2. Dimensionless circuit equations

Let the generator under study be an RLC circuit consisting of \( N \) parallel-connected modules (Figure 1). The model includes RLC circuit equations and equation (3a) describing the switch resistance. The RLC circuit equations for each module have the form (in the SI system):

\[
L_0 \frac{dI}{dt} = U - U_L - IR_s; \quad \frac{dU}{dt} = -\frac{I}{C_0},
\]

where \( I \) and \( U \) are respectively the current in a single circuit module (current through the switch) and the voltage across its capacitor bank; \( U_L \) is the voltage across the load, \( C_0 \) is the capacitance of the capacitor bank of the module, \( L_0 \) is the inductance of the module which comprises the inductances of the capacitor bank, switch, and transmission line.
The initial conditions of equation (3a) are supplemented with two more initial conditions \( I(t = 0) = 0, \ U(t = 0) = U_0, \) where \( U_0 \) is the voltage to which the capacitor bank is charged.

Let us rewrite equations (3a, 4) in the dimensionless form:

\[
\frac{\text{d}i}{\text{d} \tau} = \frac{\pi}{2}(u - u_L - ir_s); \quad \frac{\text{d}u}{\text{d} \tau} = -\frac{\pi}{2}i; \quad \frac{\text{d}r_s}{\text{d} \tau} = -Ar_s^2 \tau^{2/3}. \tag{5}
\]

Here, \( \tau = t/t_0 \) is the dimensionless time; \( i = I/I_0 \) is the dimensionless current; \( u = U/U_0 \) is the dimensionless voltage; \( u_L = U_L/U_0 \) is the dimensionless voltage across the load; \( r_s = R_s/Z_0 \) is the dimensionless switch resistance; \( Z_0 = \sqrt{L_0/C_0} \) is the wave impedance of a single circuit module;

\[
A = \frac{1}{I_0} \left( \frac{8\pi \sigma^2}{\epsilon_0} L_0^2 E_0 \right)^{1/3}
\]

is a dimensionless parameter; \( E_0 = \frac{C_0 U_0^2}{2} \) is the energy stored in the capacitor bank of the module. The time and current scales are determined by the expressions:

\[
t_0 = \frac{\pi}{2} \sqrt{L_0 C_0}, \quad I_0 = \frac{U_0}{Z_0}. \tag{6}
\]

The initial conditions for the system of equations (5) are the following: \( i(\tau = 0) = 0; \ u(\tau = 0) = 1; \) and \( r_s(\tau = 0) = 100. \) The latter initial condition implies that early in the switching, the switch resistance is equal to 100 wave impedances of the module \( Z_0. \) Note that even at \( r_s(\tau = 0) > 10, \) this initial condition hardly affected the processes occurring in the oscillatory circuit.

3. Parallel connection of generator modules

Let us consider the operation of the generator onto an inductive load – inductance \( L_L. \) In this case, the voltage across the load is \( U_L = L_L \frac{\text{d}i}{\text{d}t}, \) where \( N \) is the number of modules in the current generator.

Then, the system of equations (5) takes the form:

\[
(1 + B) \frac{\text{d}i}{\text{d} \tau} = \frac{\pi}{2}(u - ir_s); \quad \frac{\text{d}u}{\text{d} \tau} = -\frac{\pi}{2}i; \quad \frac{\text{d}r_s}{\text{d} \tau} = -Ar_s^2 \tau^{2/3}, \tag{7}
\]
where $B = N L / L_0$ is the dimensionless parameter numerically equal to the ratio between the load inductance multiplied by the number of parallel modules and the inductance of one generator module. Hence, the system of equations (7) is two-parametric. Let us consider solution of the system of equations (7) at different values of the parameters $A$ and $B$.

The energy released in the switch with respect to the energy stored in the capacitor bank of one stage is defined by the expression:

$$\delta(t) = \frac{E_s}{E_0} = \frac{\int_0^t I^2 R(t') \, dt'}{C_0 U_0^2} = \pi \int_0^\tau i^2 r_s(t') \, dt' . \quad (8)$$

Figure 2 shows the dependence of $\Delta = \delta(\tau_s)$ on the parameter $A$ at different values of the parameter $B$. For determination of $\Delta$, the upper limit of integration in (8) was $\tau_s$ taken as the time of completion of the first half-period when the current in the circuit becomes equal to zero. It is seen in figure 2 that the calculated curves of $\Delta$ (solid lines) are rather accurately approximated by the expression:

$$\Delta \approx \frac{7}{A(1 + B)^{3/2}} . \quad (9)$$

The curves corresponding to approximation (9) are shown by dotted lines in figure 2. To the short circuit mode there corresponds the curve $B = 0$. As can be seen from the figure, approximation (9) is asymptotically valid in describing the behavior of $\Delta$ at $A \rightarrow \infty$.

Substitution of the value of the parameter $A$ in (9) gives us the following expression for the relative energy loss in the switch in the case of inductive load:

$$\Delta \approx \left( \frac{\xi \varphi}{8 \pi \sigma^2 L_0^2 E_0} \right)^{1/3} \frac{7 l_s}{(1 + B)^{1/2}} . \quad (9a)$$

Thus the relative loss decreases with increasing the load inductance and number of modules.

Let us rewrite expression (9a) in practical units. We assume that the discharge gap is filled with air whose density at atmospheric pressure is $\rho_0 = 1.29 \times 10^{-3}$ g/cm$^3$, $\sigma = 200 \ \Omega \times$ cm, $\xi = 4.5$ [28, 29]. Then,

$$\Delta \approx 5.8 \times 10^{-5} \left( \frac{p}{L_0^2 E_0} \right)^{1/3} \frac{l_s}{(1 + B)^{1/2}} , \quad (10)$$

where $p$ is the air pressure in atmospheres, inductance is in henrys, energy is in joules, and length is centimeters.

The absolute loss in the switch is defined by the expression:

$$E_s = E_0 \Delta \approx 7 l_s \left( \frac{\xi \varphi}{8 \pi \sigma^2} \right)^{1/3} \left( \frac{E_0}{L_0 + N L L} \right)^{2/3} . \quad (11)$$

Note that the energy lost in the switch can be expressed in the form proposed by Martin, Seaman, and Jobe [29]:
\[ E_s \approx 7I_s \left( \frac{\mu_0}{32\pi \sigma} \right)^{1/3} I_m^{4/3}, \]  

where \( I_m = \frac{U_0}{\sqrt{(1 + B)L_0/C_0}} \) is the current close to the maximum circuit current.

4. **Series connection of generator modules**

Let us consider an RLC circuit consisting of the \( N \) number of series-connected modules. This connection is used, for example, in Arkadiev–Marx generators, see, e.g., Ref. 1. In this case, the RLC circuit equations have the form:

\[ L_N \frac{dI}{dt} = U - U_L - NIR_s; \quad \frac{dU}{dt} = -I / C_N. \]

where \( C_N = \frac{C_0}{N} \), \( C_0 \) is the capacitance of the capacitor bank of one module, \( L_N = L_0N \), \( L_0 \) is the inductance of the module. Division of equations (13) by \( N \) gives a system of equations which differs from (4) only in one term in the first equation; namely, instead of \( U_L \) in (13), we have \( U_L/N \).

Therefore, all results obtained for parallel connection of generator also valid for series connection of the modules if the parameter \( B \) is defined as follows:

\[ B = \frac{L_L}{NL_0} \]

Thus, with parallel connection, increasing the number of generator modules \( N \) decreases the energy lost in the switch, and with series connection, vice versa, increases this loss.

5. **Conclusion**

In the work, the spark channel in the generator discharge circuit is considered as a nonlinear element in which the energy loss in switching may amount to a large portion of the total stored energy and may thus greatly affect the efficiency of the whole generator. The main task of the work was to determine the parameters responsible for the energy loss in the switch and the degree of their influence.

Analysis of the operation of spark gap switches with the Braginsky model confirms the dependences of energy loss in a switch found earlier [2, 3, 29]: the increase in the relative loss \( \Delta \) with an increase in gap width and gas density in the switch; the decrease in \( \Delta \) with an increase in circuit inductance; and the decrease in \( \Delta \) with an increase in capacitor stored energy.

The derived quantitative dependences of relative and absolute losses in the switch on the parameters of the switch and discharge circuit as well as on the number of parallel- or series-connected generator modules can be useful in designing high-current generators of the next generation[16-18]. The reduction of the energy loss to the number of modules, i.e., to the number of capacitor-switch assemblies, allows an optimal choice of the parameters and number of both to minimize the specific and absolute losses.

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