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Numerical simulation of multicellular natural convection in air-filled vertical cavities

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Abstract. The paper deals with 2D laminar natural convection in vertical air-filled cavities of aspect ratio 20, 30 and 40 with differentially heated sidewalls. The airflow and heat transfer were simulated numerically with an in-house Navier-Stokes code SINF. The focus is on the appearance of stationary vortex structures, “cat’s eyes”, and their transition to unsteady regime in the Rayleigh number range from $4.8 \times 10^3$ to $1.3 \times 10^4$. The dependence of the predicted flow features and the local and integral heat transfer on the aspect ratio value is analysed.

1. Introduction

Transient natural convection in differentially heated infinite vertical fluid layers confined between two rigid boundaries is a classical problem in fluid dynamics considered first analytically by Gershuni [1]. Since that, numerous analytical, experimental and numerical contributions have been published considering convection of different fluids with wide range of the Prandtl number, and detailed reviews could be found in the books by Gershuni et al. [2] and by Lappa [3]. The instability scenario is as follows. The primary flow with one-dimensional odd cubic velocity and linear temperature profiles (the so called conduction regime) at a certain Rayleigh number is disturbed and a multicellular “cats-eye” steady-state pattern of two-dimensional (2D) transverse rolls is superimposed on the basic flow. At higher $Ra$ the cells become unstable, and the oscillatory regime occurs.

For engineering applications, a slot of finite height is of interest, and a lot of work by different authors has been devoted to the study of this problem. In the applications related to building engineering, isotope separation, cooling of electronic devices etc., the slot aspect ratio $A$ is large, and the multicellular “cats-eye” instability has been analysed by many authors. The paper by Vest and Arpaci [4] is an example of early experimental works. Natural convection of silicon oil with very high $Pr$ for $A = 20$ was visualized there for $Ra$ of $3.7 \times 10^5$. The experiment detected stable cats-eye pattern.

An example of a well-documented experimental study of natural convection in a vertical cavity is the paper by Wright at al. [5]. Natural convection of air ($Pr = 0.7$) in the cavity of $A = 40$ was studied using smoke patterns and interferometry. The range of the Rayleigh number considered ($Ra = 4.85 \times 10^3 \div 5.48 \times 10^4$) covered different flow regimes. Remarkably, secondary cells were not perfectly steady at any value of $Ra$ considered.

Flow patterns of two-dimensional natural convection in a vertical air-filled tall cavity with differentially heated sidewalls were predicted numerically in [6]. For aspect ratios from 10 to 24, various cellular structures characterized by the number of secondary cells were clarified. The onset of
the steady-state regime with the secondary cells was detected at the critical $Ra$ of $8.5 \times 10^3$ for $A = 11.5$. It was found that with an increase of the aspect ratio the critical $Ra$ decreases, and eventually becomes constant: for $A > 20$ $Ra_{crit} \approx 7 \times 10^3$.

Transition from unicellular to multicellular flow in differentially heated cavities with $A = 20$ was revisited in [7] where the transition between uni- and multicellular flows was explained by thermodynamic analysis. The detailed scenario of transition was proposed, and the value of $Ra_{crit} \approx 5.8 \times 10^3$ was reported. Numerical results for natural convection in vertical cavities filled with air were obtained also in [8] ($A = 12 \div 20$, the effect of layer inclination was also studied in the paper). It was reported that at $Ra = 1.1 \times 10^4$ the “cat’s eye” instability arises for $A > 12$.

Recently, 2D laminar natural convective transient flow characteristics in a differentially heated air-filled tall cavity with gradual heating were investigated both experimentally and numerically in [9]. The results showed that for $Ra < 7.4 \times 10^3$ the flow pattern was unicellular, when $Ra$ increases to about $6 \times 10^3$, the flow became multicellular, and for $Ra > 1.18 \times 10^4$ the flow pattern became unstable.

The current study considers the convective cells onset in air-filled vertical cavities for the same values of $Ra$ that were investigated experimentally in [5]. Numerical simulation of 2D transient natural convection were performed, with the focus on the accurate resolution of the region near the end walls as the effect of the end walls could be important even for large $A$ values.

2. Problem definition and computational aspects

Figure 1 shows the vertical cavity with the height $H$ much higher than the distance between the vertical walls, $L$. The aspect ratio $A = H/L$ is varied, and three values of $A = 20$, 30, and 40 are considered in the current paper. The cavity is supposed to be long in the $z$-direction, so that 2D formulation is used.

The vertical plates are isothermal. The left plate is heated, with the temperature $T_h$, while the right plate is cooled, with the temperature $T_c$ that is treated as the reference temperature. The reference temperature difference is defined as $\Delta T_{ref} = T_h - T_c$. The top and bottom boundaries are thermally insulated. All boundaries are solid walls with the no-slip boundary condition.

![Figure 1. Computational domain and details of the coarse grid; the grid plot is scaled in x-direction](image)

The buoyancy-induced transient flow is simulated using the 2D continuity, Navier-Stokes and energy equations. The incompressible fluid with constant physical properties is considered. The buoyancy effects are represented with the Boussinesq approximation. Assuming that the buoyancy
velocity $V_b = (g\beta \Delta T_{ini} L^3)^{1/2}$ is the velocity scale, here $g$ is the gravity acceleration and $\beta$ is the thermal expansion coefficient, and the ratio $L/V_b$ is the time scale, the governing equations in the non-dimensional form are written as:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\sqrt{Pr / \Delta}}{Ra} \Delta u,$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\sqrt{Pr / \Delta}}{Ra} \Delta v + T,$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\sqrt{Pr \Delta}} \Delta T$$

where $u$ and $v$ are the horizontal and vertical velocity components, respectively, $p$ is the pressure and $T$ is the temperature. Two non-dimensional parameters are included into the equations: the Rayleigh number, $Ra = g \beta \Delta T_{ini} L^3/va$, and the Prandtl number, $Pr = va$, here $v$ is the kinematic viscosity and $a$ is the thermal diffusivity. In the current study the Prandtl number value is fixed, $Pr = 0.7$, while the Rayleigh number is varied in the range from $4.8 \times 10^3$ to $1.3 \times 10^4$.

The computations were performed using the in-house 3D steady/unsteady Navier-Stokes code SINF (Supersonic to INcompressible Flows) being under development since 1993 [10-12]. The structured version of the code used in the current study operates with the second-order finite-volume spatial discretization using the cell-centered variable arrangement and body-fitted block-structured grids [10]. The artificial compressibility (AC) technique and/or the SIMPLEC method are used for the pressure-velocity coupling in the case of incompressible fluid flows (see [11] for details). The viscous fluxes are evaluated on the basis of central differences. Weighted corrections are calculated on the basis of the Rhie and Chow approach. Three-layer second-order scheme is implemented for physical time stepping. The code is parallelized based on the MPI standard and the domain decomposition strategy according to the grid block structure [12].

In the current contribution, the QUICK scheme was applied to compute the convective terms, and the coupled AC/SIMPLEC algorithm was activated (the AC-parameter value of seven was chosen, the Courant number was equal to five). A mesh dependency study was performed for the case of $A = 30$ using several successively refined grids, the meshes were clustered to the walls with special attention to the endwall regions, an illustration of a coarse mesh is given in figure 1. The solutions obtained with the mesh of $838 \times 101$ nodes (83,700 cells) were accepted as mesh-independent.

To simulate unsteady flow regimes, the time step of one time unit was set based on several tests with various time step values. Special attention was paid also on the effect of solution convergence at each time step, and after several additional tests it was concluded that 50 iterations per time step is enough to provide sufficient convergence. To control the convergence in case of a steady-state solution and local characteristics evolution in case of an unsteady solution, data from the monitoring point $M (x = 0.25; y = 5.006)$ were used, see figure 1.

3. Results and discussion

Formation of the buoyancy-induced flow in the cavity with $A = 30$ is illustrated by the pathline patterns shown in figure 2. The figure gives the plots for the central part of the cavity, far from the endwalls, at six values of $Ra$. Note that the pathline plots illustrate the flow field qualitatively, as the pathlines are issued manually. At the lowest $Ra = 4850$, a steady-state solution is obtained, and the unicellular flow without any secondary cells is detected (figure 2a). A steady-state solution is obtained also at $Ra = 7300$, though the pathlines given in figure 2b are curved slightly that indicates formation of a weak secondary flow with narrow cells extended in the vertical direction. Note that experimental smoke patterns presented in [5] point to transition to secondary flow at $Ra \approx 6800$, and the secondary cells detected in [5] were always unsteady, with slow movement downward.
Pronounced secondary cells are visible in the plots corresponding to higher $Ra$ values (figure 2c-f). The flow at $Ra \geq 8600$ is unsteady, and the plots give instantaneous flow fields. The width of the co-rotating cells increases with an increase in $Ra$, while the vertical length of the cells does not change much. The downward movement of the cells detected in all unsteady cases was very slow, e.g., at $Ra = 8600$ the period of high-amplitude velocity fluctuations at point $M$ was about 270 time units. Instantaneous flow patterns at two successive time instants are given in figure 3 ($Ra = 8600$). Time interval between two plots is 105 time units. The distance travelled downward by the cell structure during this time interval is about one length unit.
The wave length of the cells, $\lambda$, is defined as the distance between the centres of two neighbouring cells (see figure 3a), and for the case considered $\lambda \approx 2.7$. According to [9], the wave number $\alpha = 2\pi \lambda$ can be also introduced to characterize the cells. It was found that the influence of the aspect ratio on the wave number value is significant. E.g., for the highest $Ra = 12600$ the wave number $\alpha = 2.71$ at $A = 20$, $\alpha = 2.33$ at $A = 30$, and $\alpha = 2.31$ at $A = 40$. At the lowest $A = 20$ the wave number values depend also on the $Ra$ values: $\alpha = 2.63$ at $Ra = 8600$. Thus, at $A = 20$ the wave number increases slightly with the Rayleigh number growth, and the increment in the $Ra$ range considered is about 3%. Stronger wave number dependence on $Ra$ is reported in [9] for the cavity of $A = 16$, but the paper considered the wave number behavior at lower $Ra$ values correspondent to the transitional range ($\alpha = 2.77$ at $Ra = 7153$). At higher aspect ratio values considered the wave number does not depend on $Ra$ in the range considered.

The periodical structure of the cells immanent in the central part of the layer is destroyed near the endwalls. It is illustrated in figure 4 where the flow patterns at the bottom end region are given for three $Ra$ values. In all three cases shown the flow patterns are skewed in the vicinity of the endwalls. Non-regular cells of smaller size are detected if the flow regime is unsteady. Note that the cells origin and unsteady behavior are very sensitive to the spatial resolution not only in the central region of the cavity where the cells are detected, but also near the endwalls.

The “cat’s eye” convective cells play an important role in heat transfer. Figure 5 gives the distributions of the local Nusselt number, $Nu$, over the hot isothermal wall; $Nu = q_wL/k\Delta T_{eq}$, $q_w$ is the local heat flux, $k$ is the thermal conductivity. If there is only primary upward-downward flow ($Ra \leq 6800$), the heat transfer is dominated by conduction with $Nu = 1$ everywhere except near-endwall regions. As $Ra$ increases, the convective cells form, and $Nu$ should grow locally from unity. The local effects of convection are significant, especially at higher $Ra$; e.g., at $Ra = 12600$ the peak values of $Nu$ exceed 1.7. The number of peaks and the distance between the neighboring maximum values in the $Nu$ distribution correspond to the above discussion of $A$ influence on the wave number.

![Figure 5. Local Nusselt number distributions over a vertical wall: (a) various $Ra, A = 30$; (b, c) various $A$, (b) $Ra = 8600$ and (c) $Ra = 10500$](image)

**Table.** Computed $\langle Nu \rangle$ values for different $A$ in comparison with correlation [13].

| $Ra$   | 4850 | 6220 | 6800 | 7300 | 8600 | 9600 | 10500 | 12600 |
|--------|------|------|------|------|------|------|-------|-------|
| $\langle Nu \rangle, A = 20$ | 1.18 | 1.23 | 1.27 | 1.29 | 1.35 | 1.39 | 1.43 | 1.50 |
| $\langle Nu \rangle, A = 30$ | 1.12 | 1.17 | 1.17 | 1.19 | 1.23 | 1.26 | 1.32 | 1.39 |
| $\langle Nu \rangle, A = 40$ | 1.09 | 1.12 | 1.15 | 1.17 | 1.21 | 1.26 | 1.28 | 1.34 |
| $\langle Nu \rangle$ [13]    | 1.052 | 1.094 | 1.113 | 1.133 | 1.194 | 1.250 | 1.301 | 1.402 |

The computed area-averaged Nusselt number values, $\langle Nu \rangle$, are compared in the table with the values estimated with the empirical correlation of Wright [13] valid for $A > 40$. As expected, the effect
of the endwalls is pronounced in the transitional $Ra$ range, $Ra \leq 10^3$: at lower $A$ values the $<Nu>$ values are higher, and even at $A = 40$ the computed values still exceed slightly the empirical data. As reported in [13], at $Ra > 10^4$ a new segment of correlation started that could be attributed to some 3D effects that start to develop. That could be the reason of slight underestimation of the $Nu$ values at $Ra = 10500$ and $Ra = 12600$. It should be concluded that to predict accurately chaotic convective cell behavior at these $Ra$ values 3D unsteady formulation is necessary.

4. Conclusions

Natural convection in vertical air-filled cavities of aspect ratio 20, 30 and 40 with differentially heated sidewalls was simulated numerically using an in-house Navier-Stokes code SINF. The focus is on the appearance of the convective vortex structures, “cat’s eyes”, and their transition to unsteady regime in the Rayleigh number range from $4.8 \times 10^3$ to $1.3 \times 10^4$.

It was proved that the cells origin and unsteady behavior are sensitive to the spatial resolution not only in the central region of the cavity where the cells are detected, but also near the endwalls. If sufficient spatial resolution in the vicinity of the endwalls is provided, the computed results correlate with the literature experimental data. It was found that the wave number of the convective cells and the integral Nusselt number values depend on the cavity aspect ratio up to $A = 40$.

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