Identification of Pulsation Modes in Main Sequence Stars: Potentials and Limits

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Abstract We review the present-day methods of mode identification applied to main sequence pulsators focusing on those that make use of multicolour photometry and radial velocity data. The effects which may affect diagnostic properties of these observables are discussed. We also raise the problem of identification of high $\ell$ modes which can dominate oscillation spectra obtained from space-based projects.

1 Introduction

The accuracy of seismic model increases with the number of well identified pulsation frequencies. Depending on the character of modes we can probe different parts of a star and derive constraints on various parameters of model and theory. Since asteroseismology has entered the space era, the number of detected peaks in the frequency spectra has grown immensely. However, to explore these data, the unequivocal identification of mode geometry is required. In the case of main sequence pulsators this is not an easy task because their oscillation spectra do not exhibit equidistant or regular patterns. An alternative is the usage of the photometric and spectroscopic variations. In this paper, we summarize potentials and limits in using these observables for mode identification.

In Section 2 we recall the photometric diagrams for $\beta$ Cephei, SPB (Slowly Pulsating B-type) and $\delta$ Scuti star models. We mention the most important effects which may affect diagnostic properties of such diagrams, i.e., convection, rotation and model atmospheres. In Section 3 we describe a method which, beside mode identification, can yield constraints on models and theory. A prospect for identification of high $\ell$ modes is discussed in Section 4. The last Section contains conclusions.
2 Mode identification from multicolour photometry

Since pioneering works \cite{10,1,14}, the photometric observables, i.e., photometric amplitudes and phases in various passbands, have become the most often used data for mode identification in main sequence pulsators. If we ignore effects of rotation, these quantities are independent of the intrinsic mode amplitude, \( \varepsilon \), inclination angle, \( i \), and azimuthal order, \( m \). In Fig. 1 we show the position of unstable modes for stellar models with masses of 12, 5 and 2 \( M_\odot \) corresponding to \( \beta \) Cep, SPB and \( \delta \) Sct stars, respectively. In the case of \( \beta \) Cep models we used the Strömgren \( uvy \) filters and in the case of SPB and \( \delta \) Sct models the Johnson \( BR \) filters. The left panels refer to all unstable modes occurring during the main sequence evolution, whereas the right panels contain unstable modes for a model with \( \log T_{\text{eff}} = 4.400, 4.195 \) and 3.909 for the \( \beta \) Cep, SPB and \( \delta \) Sct model, respectively. We considered modes with the degree, \( \ell \), up to 6.

The position of pulsational modes in the photometric diagrams changes if effects of rotation are taken into account. This happens in two cases. The first case is when the frequency difference between modes is of order of the rotational frequency, and the degrees, \( \ell \), differ by 2. Then, such modes are coupled by rotation and photometric diagrams become dependent on the inclination angle and rotational velocity \( \Omega \). The second case is when we deal with slow modes, i.e., modes with frequencies of order of the rotational frequency. Such modes are typical for the SPB pulsators. Then, the perturbation approach fails and another treatment is needed, e.g., traditional approximation. The photometric diagrams become dependent on \( (i, m, V_{\text{rot}}) \) (e.g., \cite{11,8}). In the case of \( \delta \) Sct variables, an additional uncertainty comes from effects of the subphotospheric convection on pulsation. The use of elaborate model atmospheres (including also NLTE effects, see \cite{9}) improves the diagnostic reliability of the photometric diagrams. As an example of the efficiency of the photometric mode identification combined with the exact fitting of observed and theoretical frequencies, we note a study of the \( \delta \) Sct variable 44 Tau in which 15 oscillation frequencies have been detected. For 10 of them the mode degree was uniquely determined, and the fitting of all 15 frequencies was achieved giving strong constraints on the stellar model (\cite{12}, \cite{13}).

3 Including radial velocity variations

The uncertainties coming from pulsation theory can be omitted, if both the mode degree, \( \ell \), and the nonadiabatic \( f \)-parameter are determined from observations (\cite{5}, \cite{6}). The \( f \)-parameter describes the ratio of the radiative flux perturbation to the radial displacement at the photosphere. Fig. 2 shows results for the \( \beta \) Cep star \( \delta \) Ceti. In the case of B-type pulsators, the use of the radial velocity data is essential to get a unique identification of \( \ell \). Moreover, useful constraints on stellar opacities can be derived.

Modelling \( \delta \) Sct stars is problematic due to the subphotospheric convection which interacts with pulsation. Mode identification using the method of simultaneous determination of \( \ell \) and \( f \) is model independent. On the other hand, a com-
Fig. 1 The photometric diagnostic diagrams for the main sequence pulsators: $\beta$ Cep stars (top panels), SPB stars (middle panels) and $\delta$ Sct stars (bottom panels). The left panels correspond to all unstable modes from ZAMS to TAMS for a stellar mass of 12, 5, and 2 $M_\odot$ for $\beta$ Cep, SPB and $\delta$ Sct models, respectively. In the right panels unstable modes of particular models are shown. \cite{3}
parison of the theoretical and empirical values of $f$ provides useful information on the mixing-length parameter of convection as well as on the model atmospheres. In Fig. 3 we present such a comparison for the $\delta$ Sct star $\beta$ Cassiopeiae which pulsates in the dipole mode.

Fig. 2 The $\chi^2$ discriminant as a function of $\ell$ for the $\beta$ Cep star $\delta$ Ceti obtained from the fit of photometric amplitudes and phases without radial velocity data (left panel) and with radial velocity data (right panel). Different lines correspond to models from the center and edges of the error box.

Fig. 3 Comparison of the empirical values of the $f$-parameter inferred from Strömgren photometry for the $\delta$ Sct star $\beta$ Cassiopeiae with the theoretical ones calculated for five values of the MLT parameter, $\alpha$. The empirical values of $f$ were obtained adopting Kurucz models (left panel) and Vienna models (right panel). In the right panel the effect of the microturbulent velocity is also shown.
4 Prospects for extracting high $\ell$ modes

The rich oscillation spectra obtained from space observations are dominated by peaks of very low amplitude. According to the simulations [7], there is a high probability that these peaks are associated to higher $\ell$ modes ($\ell > 6$).

Examples of diagnostic diagrams for a $\beta$ Cep model and pulsational modes with $\ell$ from 7 up to 12 are shown in Fig. 4. The left diagram is calculated for the Strömgren $uv$ passbands. The amplitude ratio for any two photometric passbands of the high $\ell$ modes is around 1, because the geometrical effect, $(1 - \ell)(\ell + 2)$, dominates in the light variations. As we can see there is some mode grouping. This is related to the parity of degree, $\ell$. The odd $\ell$ modes have higher values of the amplitude ratio $A_u/A_y$ than the even $\ell$ modes. The right diagram shows the same modes but using radial velocity variations and the $y$ filter. Here, the mode grouping is twofold. First, the phase differences take the values according to the sign of the disc averaging factor, $b_\ell$. Modes with the phase difference value $\varphi_{V_{\text{rad}}} - \varphi_y$ around 1.5 rad correspond to negative values of $b_\ell$, whereas those with $\varphi_{V_{\text{rad}}} - \varphi_y$ around 4.6 correspond to positive values of $b_\ell$. Second, the values of the amplitude ratio, $A_{V_{\text{rad}}}/A_y$, depend on the parity of $\ell$. The even $\ell$ modes have $A_{V_{\text{rad}}}/A_y < 30$ whereas the odd $\ell$ modes have $A_{V_{\text{rad}}}/A_y > 50$. In both panels of Fig. 4 there is a separation of p and g-modes. The g-modes have negative values of $\varphi_u - \varphi_y$ and larger values of $A_{V_{\text{rad}}}/A_y$.

For much higher $\ell$ mode, $\ell > 20$, the amplitude ratio for any two passbands will be around 1, and the phase difference around 0 for all modes. The amplitude ratios for the photometric passband and radial velocity will be spread from 0 to about 100 and the phase differences will have values of 1.5 or 4.6 rad.
5 Conclusions

In this short review, we mentioned the basic properties and uncertainties of mode identification from multicolour photometry and radial velocity data. We focused on main sequence pulsators because of their irregular oscillation spectra.

Then, we checked a prospect for identification of the high degree modes. Such modes are highly expected in the rich frequency spectra obtained from, e.g., the CoRoT, Kepler and BRITE data. Identification of the high \( \ell \) modes only from multicolour photometry is rather hopeless. However, it turned out that it is possible to get some constraints on intermediate \( \ell \) modes from the diagrams using amplitudes and phases of the photometric and radial velocity changes. Unfortunately, there is no prospect for extracting modes with \( \ell > 30 \) from these data. Here only the analysis of the line-profile variations may help.

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