How particle collisions increase the rate of accretion from the cosmological background onto primordial black holes in braneworld cosmology

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It is shown that, contrary to the widespread opinion, particle collisions considerably increase accretion rate from the cosmological background onto 5D primordial black holes formed during the high-energy phase of the Randall-Sundrum Type II braneworld scenario. Increase of accretion rate leads to much tighter constraints on initial primordial black hole mass fraction imposed by the critical density limit and measurements of high-energy diffuse photon background and antiproton excess.

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It is well known that primordial black holes (PBHs) open up a way to obtain information on earliest era of the Universe [1,2]. Recent works [3-7] show that PBHs can also be a probe of braneworld cosmologies. The Hawking temperature drop [3] and considerable PBH mass growth due to radiation dominated background accretion occur [4] in the simplest braneworld cosmology of the Randall-Sundrum Type II (RS2). These modifications of PBH properties provide new constraints on the initial PBH mass fraction [8]. PBH mass growth is exponentially sensitive to the accretion efficiency. However, authors of [6, 7, 8] considered the accretion efficiency as a free parameter. In fact, they started with accretion efficiency as a free parameter. In fact, they started with accretion rate sub-

In fact, authors of [6, 7, 8] considered the accretion efficiency as a free parameter. In fact, they started with the condition $F \ll l$ presently bound [8] by condition $l \leq 0.2 \text{mm}$. A black hole (BH) behaves as an essentially 5D object described by the metric [9, 10, 11, 12] 

$$ds^2 = -(1 - r_{BH}^2 / r^2) dt^2 + \frac{1}{(1 - r_{BH}^2 / r^2)} dr^2 + r^2 d\Omega_2^2,$$ 

where $d\Omega_2^2$ is the line element on a unit 2-sphere, if its modified Schwarzschild radius

$$r_{BH} = \sqrt{\frac{8}{3\pi}} \sqrt{\int t_4 \sqrt{M_4 t_4}}$$ 

is much smaller than $l$. We use the units where $c = G = 1$ and introduce 4D Planck scale quantities $t_4$, $M_4$ and $t_4$ for convenience. The high energy brane regime is characterized [8] by scale factor, energy density, horizon mass, and duration

$$a = a_h \left( \frac{t}{t_h} \right)^{1/2}, \quad \rho = \frac{3}{32\pi} \frac{M_4}{t_c},$$

$$M_h = 8M_4^2 t_c^2, \quad t_c = t_4 \frac{l}{2t_4},$$

where $t$ is cosmic time, and $t_h < t_c$. It is usually assumed [1, 2, 3, 4, 5, 6, 7, 8] that at formation time $t_i$ the PBH mass is

$$M_i = f M_h(t_i).$$

The condition $r_{BH} \ll l$ is fulfilled if $f \sim 1$ and $t_i < t_c$. The accretion rate

$$dM = F A r_{BH}^2 \rho(t)$$

and energy density (3) lead to exponential BH mass growth

$$M(t) = M_i \left( \frac{t}{t_i} \right)^{2F/\pi}$$

at $t < t_c$ [8]. In fact, Eq. (5) is justified [8] only in the case of collisionless relativistic particles when accretion

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efficiency $F$ equals 1. However, a simple comparison of the scattering length $l_{sc}$ with the radius (2) demonstrates that the cosmological background cannot be considered as a collisionless particle medium in the case of PBH masses most relevant for observational constraints. In fact, the opposite condition $l_{sc} \ll r_{BH}$ is true, in general, for such PBHs. It allows one to treat the accreting cosmological background as a continuous medium. Therefore, to obtain $F$ one should evaluate the accretion rate (5) in hydrodynamic approximation.

Since radiation dominated background is characterized by an indefinite particle number, the momentum conservation equation

\[(\rho + P)u_i k u^k = -P_{,i} - u_{,i} P_k u^k,\]  

should be used \[14\] together with the entropy conservation equation

\[(\sigma u^k)_{,k} = 0.\]  

Here comma and semicolon denote usual and covariant derivatives; $\rho$, $\sigma$, $P$ and $u^i$ are energy and entropy densities, pressure and 4-velocity of accretion flow, respectively. In the metric (1) adiabatic accretion equations (7) and (8) take the form

\[uu' + \frac{r_{BH}^2}{r^3} + \frac{P'}{\rho + P} \left( 1 + u^2 - \frac{r_{BH}^2}{r^2} \right) = 0\]  

and

\[\frac{\sigma'}{\sigma} + \frac{u'}{u} + \frac{2}{r} = 0\]  

where $u = u^1$ is the radial component of 4-velocity and primes denote the derivative with respect to $r$. The radiation dominated background is described by equations \[15\]

\[\rho(T) = 3P(T) = \frac{\pi^2}{30} g_{eff} T^4\]  

and

\[\sigma(T) = \frac{2\pi^2}{45} g_{eff} T^3,\]  

where $g_{eff}$ is the number of relativistic particle species at temperature $T$. The value $g_{eff} \simeq 100$ will be used below because $T \geq 1TeV$ at $t < t_c$. Equations (11) and (12) allow to rewrite Eqs. (9) and (10) in terms of temperature

\[uu' + \frac{r_{BH}^2}{r^3} + \frac{T'}{T} \left( 1 + u^2 - \frac{r_{BH}^2}{r^2} \right) = 0,\]  

\[\frac{3T'}{T} + \frac{u'}{u} + \frac{2}{r} = 0.\]  

Solving Eqs. (13) and (14) for $u'$ and $T'$, one obtains (compare with \[9\])

\[u' = \frac{D_1}{D}, \quad T' = -\frac{D_2}{D},\]  

where

\[D = \frac{2u^2 - 1 + r_{BH}^2/r^2}{uT},\]  

\[D_1 = \frac{2u^2 + 2 - 5r_{BH}^2/r^2}{rT}\]  

and

\[D_2 = \frac{2u^2 - r_{BH}^2/r^2}{ru}.\]  

To avoid singularities in the accretion flow, the solution of Eqs. (15) must pass through a "critical" or "sound" point $r = r_s$ where $D = D_1 = D_2 = 0$. Solving equations $D = D_1 = D_2 = 0$, one obtains

\[r_s = \sqrt{2} r_{BH}\]  

and

\[u_s = u(r_s) = \frac{1}{2}\]  

in light speed units.

Integration of Eq. (13) leads to a conservation law

\[\left( 1 + u^2 - \frac{r_{BH}^2}{r^2} \right) T^2 = \text{const} = T_b^2;\]  

where $T_b$ is the background temperature defined by expressions (3) and (11) for energy density \[8\]. Substituting Eqs. (19) and (20) into Eq. (21) one obtains

\[T_s = T(r_s) = \frac{2}{\sqrt{3}} T_b.\]  

Direct integration of Eq. (10) shows that the term $r^2 \sigma(r) u(r)$ does not depend on $r$, while Eqs. (19), (20), and (22) allow to evaluate its value

\[r^2 \sigma(r) u(r) = r_s^2 \sigma(r_s) u(r_s).\]  

Applying the latter equation sufficiently far from BH, where $T(r) \to T_b$, and using Eqs. (12), (19), (20), and (22), one obtains

\[u(r) \approx \frac{8}{3\sqrt{3}} \times \frac{r_{BH}^2}{r^2}, \quad r \gg r_{BH}.\]  

Substituting $r \gg r_{BH}$, $T(r) \approx T_b$, $\rho(r) \approx \rho(T_b) \equiv \rho(t)$ and Eq. (24) into the formula

\[\frac{dM}{dt} = 4\pi r^2 \rho(r) u(r)\]  

we finally come to the accretion rate (5) with the accretion efficiency

\[ F = \frac{8}{3\sqrt{3}} \simeq 1.54 \]  

(26)
describing radiation dominated background accretion in hydrodynamic approximation applicable in the limit of frequent particle collisions. Since (26) exceeds 1, particle collisions, in fact, increase both accretion rate (5) and PBH mass growth coefficient (6).

Let us now demonstrate that continuous approximation is indeed valid for the most of the high-energy phase. Since the characteristic space scale (19) of accretion flow is determined by \( r_{BH} \), one should compare \( r_{BH} \) with the scattering length \( l_{sc} = l_{sc}(t) \simeq 1/\sigma n \). Here \( n = n(T_b) \sim \rho(T_b)/3T_b^3 \) is the particle number density and \( \sigma = \sigma(T_b) \simeq \alpha^2/T_b^2 \) is the scattering cross section in which \( \alpha = \alpha(T_b) \) is the interaction constant.

Let us remind the reader that a substantial mass growth follows from the absence of excessive large-angle microwave anisotropy while the upper one

\[ M_{min} \leq \dot{M}_i < M_{max}. \]  

(27)
The lower limit

\[ M_{min} = 2 \cdot 10^6 \left( \frac{l}{l_4} \right)^{-1/3} M_4 \]  

(28)
follows from the absence of excessive large-angle microwave anisotropy while the upper one

\[ M_{max} \simeq \left( \frac{l}{l_4} \right) M_4 \]  

(29)
follows from both the condition \( r_{BH} = l \) and Eq. (4) at \( t = t_c \). Note that BHs with larger masses and radii are the usual 4D BHs which do not experience considerable accretion mass growth.

For simplicity we will completely neglect scattering taking \( F = 1 \) if \( r_{BH} < l_{sc} \) and treat the radiation dominated background as a continuous medium taking \( F \simeq 1.54 \) if \( r_{BH} > l_{sc} \). Let us introduce the initial PBH masses \( M_{i1} \) and \( M_{i2} \) for which the condition \( r_{BH} = l_{sc} \) fulfills, respectively, at the final moment \( t = t_c \) of the high-energy phase and at the moment \( t = t_i \) of the PBH formation. These masses allow us to separate the mass intervals corresponding to different scenarios of PBH mass growth (see Fig. 1), namely, the interval \( M_{min} < \dot{M}_i < M_{i1} \) where \( l_{sc} > r_{BH} \) and \( F = 1 \), the interval \( M_{i2} < \dot{M}_i < M_{max} \) where \( l_{sc} < r_{BH} \) and \( F \simeq 1.54 \) and the mixed interval \( M_{i1} < \dot{M}_i < M_{i2} \) for which a moment \( t_i < t_s < t_c \) exists at which \( r_{BH}(t_s) = l_{sc}(t_s) \) and one, correspondingly, has \( r_{BH} < l_{sc}, F = 1 \) at \( t_i < t_s < t_c \) and \( r_{BH} > l_{sc}, F \simeq 1.54 \) at \( t_s < t < t_c \). Equation (6) can now be directly applied to each interval with constant \( F \) giving a composite formula

\[ \frac{M(t_c)}{M_i} = \begin{cases} \left( \frac{l_c}{l_i} \right)^{2/\pi}, & M_{min} < \dot{M}_i < M_{i1}; \\ \left( \frac{l_c}{l_i} \right)^{2/\pi} \left( \frac{l_c}{l_i} \right)^{16/(3\sqrt{3}\pi)}, & M_{i1} < \dot{M}_i < M_{i2}; \\ \left( \frac{l_c}{l_i} \right)^{16/(3\sqrt{3}\pi)}, & M_{i2} < \dot{M}_i < M_{max} \end{cases} \]  

(30)
for the coefficient of accretion mass growth during the high-energy phase. The solid lines on Fig. 1 show the coefficient of accretion mass growth as a function of \( M_i \) for curvature radii \( l/l_4 = 10^{21}, 10^{26}, \) and \( 10^{31} \) and \( f = 0.1 \). Long dashed and short dashed lines correspond, respectively, to the values \( F = 1 \) and \( F = 0.5 \) used in [3, 4, 5, 6, 7, 8]. From Fig. 1 one can see that Eq. (30) predicts considerably larger mass growth than Eq. (6) with \( F = 1 \) and 0.5. One can also see that the region \( M_{i2} < M_i < M_{max} \) in which Eq. (30) reduces to Eq. (6) with \( F = 1 \) is quite wide.

Let us keep in mind that the curvature radius is experimentally bound from above by the value \( b_{nm} \sim 10^{31} l_4 \) [13], while its lower limit is determined by the choice of 5D BH evaporation time [3, 4, 5, 6, 7, 8]

\[ \frac{t_{\text{evap}}}{l_4} \simeq g^{-1/2} \left( \frac{M(t_c)}{M_4} \right)^2, \]  

(31)
where \( g = 0.023 \). Since the most direct ways of PBH search are connected with PBHs completing their evaporation now or at a future time, we limit \( l \) by \( l_{\text{min}} = 10^{20} l_4 \) which follows from (31) at \( t_{\text{evap}} = t_0 = 13.7 \text{Gyr} \) and \( M(t_c) = M_{\text{max}} \). Note that at \( l/l_4 < 10^{20} \), one will not see any difference from the 4D case.

The accretion efficiency (26) allows us to tighten the constraints imposed on the initial PBH mass fraction [6].

![Fig. 1: Accretion mass growth coefficient vs the initial PBH mass for choices \( l/l_4 = 10^{21}, 10^{26}, \) and \( 10^{31} \) (from bottom to top). Solid lines correspond to the PBH mass growth coefficient (30), while long dashed and short dashed lines are obtained from Eq. (6) with fixed accretion efficiencies \( F = 1 \) and \( F = 0.5 \), respectively.](image)
where $\rho_{PBH,M_i}(t_i)$ is the mass density of PBHs with formation mass on the order of $M_i$ and $\rho_{rad}(t_i)$ is the radiation energy density at the moment $t_i$ of such PBH formation. The mass $M_i$ should be evaluated substituting the mass $M(t_0)$ found from Eq. (31) at given $l$ and $t_{\text{evap}} = t_0$ into Eq. (30). The PBH mass fraction grows with time starting from the initial value (32) proportionally to $a(t)M(t)$ at $t_i < t < t_e$ and proportionally to $a(t)$ at $t_e < t < t_0$. \cite{4}. Describing the PBH mass growth by Eq. (30) and restricting the present PBH mass density, by 30\% of the critical density one obtains new constraints on $\alpha_i$ presented by the solid line in Fig. 2. The latter are much tighter than both the 4D constraint $\alpha_i < f^{-1/2}10^{-18}$ and the old 5D constraints \cite{6} illustrated by long dashed and short dashed lines evaluated using Eq. (6) with $F = 1$ and $F = 0.5$, respectively. The dotted lines both in Fig. 2 and in Figs. 3, 4 below are evaluated using the third line of Eq. (30) in order to illustrate the degree of violation of hydrodynamical approximation in the region $l/l_4 \geq 10^{30}$ where the accretion actually begins in a collisionless regime and only later switches over into a hydrodynamical one being, thus, adequately described by the second line of Eq. (30).

\begin{align}
\alpha_i = \alpha_i(M_i) = \frac{\rho_{PBH,M_i}(t_i)}{\rho_{rad}(t_i)},
\end{align}

the curvature radius variation from $l/l_4 = 10^{31}$ to $10^{20}$ \cite{5}. The diffuse photon spectrum can be estimated \cite{4} by the formula

\begin{align}
I(E) = \frac{c}{4\pi E} n(t_0, M_i(M^*))\end{align}

where $c$ is the speed of light and

\begin{align}
n(t_0, M_i(M^*)) &= \frac{3\alpha_i(M_i)}{2^{17/4}\pi^{5/4}} \frac{a^3(t_{eq})}{a^3(t_0)} l^{1/8} l^{-3/8} t_{eq}^{-3/2} M_i^{9/8}(M^*)\end{align}

is the present number density of PBHs with mass of the order $M^*$, where $t_{eq} = 72.6$ kyr \cite{8} is the time of matter-radiation equality and the mass $M_i$ should be found by substituting $M(t_e) = M^*$ into Eq. (30). In particular, one can find that initial masses of PBHs completing their evaporation at present are equal to $M_i = 10^{-4}$ and $10^2$ g for $l/l_4 = 10^{31}$ and $10^{20}$, respectively. Comparing (33) with the observational data collected in \cite{7}, one obtains new constraints presented in Fig. 3 by a solid line. These constraints are much tighter than both the 4D constraint $\alpha_i < 10^{-27}$ and the old 5D ones, presented by dashed lines.

\begin{align}
\Phi_F \propto \frac{dM_i}{dM^*} \frac{M_{GRV}^2}{M_i M^*} G n(t_0, M_i(M^*))\end{align}

where $M_{GRV} = M_{GRV}(l)$ is the mass of a PBH with Hawking temperature $T_{BH} \sim 1\text{ GeV}$ and the factor $G \sim 10^5$ represents the galactic matter density enhancement at the solar neighborhood. Equation (35) differs from equations of ref. \cite{5}, combined together in a similar way, solely by the relation (30) of $M^* = M(t_e)$ with $M_i$. This ”small” difference, however, allows us to considerably restrict the "most important result" of Ref. \cite{5} concerning the $\bar{p}$ flux dependence on $l$. Indeed, using the

![Graph](image-url)

**FIG. 2:** Constraints on initial PBH mass fraction imposed by the total matter density. Dotted, long dashed and short dashed lines are obtained using PBH mass growth coefficient (6) with $F = 1.54$, 1 and 0.5, respectively.

**FIG. 3:** Constraints on initial PBH mass fraction imposed by the high-energy diffuse photon background. Dotted, long dashed and short dashed lines are obtained using Eq. (6) with $F = 1.54$, 1 and 0.5, respectively.
PBH mass growth coefficient (6), one obtains from (35) that \( \Phi_p \propto l^p \) with exponent
\[
p = \frac{40F - 13\pi}{16(\pi - F)}.
\]
(36)

Since \( p < 0 \) at \( F \leq 1 \) it was argued in \([8]\) that the \( \bar{p} \) flux is a decreasing function of \( l \) (see both dashed lines in Fig. 4) and should be used to set a lower bound on \( l \). However the new value \( F \simeq 1.54 \) leads to the positive exponent \( p \approx 0.81 \) corresponding to increasing \( \bar{p} \) flux (35) and decreasing constraints on \( G_{\alpha} \) (solid line in Fig. 4) up to \( l/l_4 \simeq 10^{30} \), where the hydrodynamic approximation is violated. These constraints are again much tighter than both the 4D constraint \( \alpha_i < 10^{-27} \) and the old 5D ones, presented by dashed lines.

Because of violation of hydrodynamic approximation at \( l/l_4 > 10^{30} \) and in the corresponding photon energy region \( E \leq 500keV \) all the considered constraints reach their maxima at \( l/l_4 \approx 10^{30} \). Figures 2-4 demonstrate that the tightest constraints following from the critical density limit and measurements of the high-energy diffuse photon background and antiproton excess at \( l/l_0 \approx 10^{30} \) amount, respectively, to \( \alpha_i \approx 10^{-31}, 10^{-39} \) and \( 10^{-36} \). Since the tightness of all the constraints increases with the curvature radius \( l \), all of them can be used to set an upper bound on \( l \) at \( l/l_4 \leq 10^{30} \).

To summarize, particle collisions make hydrodynamic approximation adequate to describe the accretion of the radiation dominated background onto PBHs during most of the high-energy phase of the RS2 braneworld scenario. Continuous background is characterized by 54% higher accretion efficiency onto PBHs than that of collisionless relativistic particles. This higher accretion efficiency provides larger PBH mass growth during the high-energy phase and makes it possible to obtain much tighter constraints on initial mass fraction than those found in \([8]\) as well as than the constraint equal to \( 10^{-27} \) obtained in the standard 4D cosmology. The tightness of the obtained constraints mostly increases with curvature radius allowing to restrict it from above.

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[1] Ya. B. Zel’dovich, I. D. Novikov, Astron. Zh. (USSR) 43, 758 (1966) [Sov. Astron. 10, 602 (1967)].
[2] S.W. Hawking, Mon. Not. R. Astron. Soc. 152, 75 (1971).
[3] R. Guedens, D. Clancy and A. R. Liddle, Phys. Rev. D 66, 043513 (2002).
[4] A. S. Majumdar, Phys. Rev. Lett. 90, 031303 (2003).
[5] R. Guedens, D. Clancy and A. R. Liddle, Phys. Rev. D 66, 083509 (2002).
[6] D. Clancy, R. Guedens and A. R. Liddle, Phys. Rev. D 68, 023507 (2003).
[7] Y. Sendouda, S. Nagataki and K. Sato, Phys. Rev. D 68, 103510 (2003).
[8] Y. Sendouda, K. Kohri, S. Nagataki and K. Sato, Phys. Rev. D 71, 063512 (2005).
[9] S. Shapiro and S. Teukolsky, Black Holes, White Dwarfs and Neutron Stars (Wiley, New York, 1983).
[10] Ya. B. Zel’dovich, I. D. Novikov, Relativistic Astrophysics, Vol.I: Stars and Relativity (University of Chicago Press, Chicago, 1971).
[11] H. Bondi, Mon. Not. Roy. Astron. Soc. 112, 195 (1952).
[12] F.C. Michel, Astrophys. Space Sci., 15, 153, (1972).
[13] J.C. Long et al. Nature (London) 421, 922 (2003).
[14] L.D. Landau and E.M. Lifshitz, Fluid Dynamics (Pergamon Press, Oxford, 1987), 2nd ed.
[15] H.V. Klapdor-Kleingrothaus and K. Zuber, Teilchenastrophysik (B.G. Teubner GmbH, Stuttgart, 1997).