Selective Transparence of Single-Mode Waveguides with Surface Scattering

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A random surface scattering in a one-mode waveguide is studied in the case when the surface profile has long-range correlations along the waveguide. Analytical treatment of this problem shows that with a proper choice of the surface, one can arrange any desired combination of transparent and non-transparent frequency windows. We suggest a method to find such profiles, and demonstrate its effectiveness by making use of direct numerical simulations.

Recently much attention is paid to 1-dimensional solid state models with a correlated disorder. The interest to this subject is due to the results that demonstrate a direct relevance of anomalous transport properties to particular pair correlations in the cite potential. Specifically, it was shown that any wanted combination of transparent and non-transparent frequency intervals can be realized by a proper construction of potentials with long-range correlations. Experimental realization of such potentials for single-mode waveguides with inserted delta-like scatters has proved a possibility to construct devices that are transparent for any given range of frequency.

On the other hand, there is a well developed theory of a wave propagation through surface-disordered waveguides (see, e.g., \([5,7]\) and references therein). This subject is important both from the theoretical viewpoint, and for experimental applications, such as the optical fibers, remote sensing, radio wave propagation, shallow water waves etc., \([4]\). One should note, that main results in this field refer to random surfaces with a fast decay of correlations along a scattering surface. It is now of great interest to explore the role of slowly decaying correlations that are expected to result in anomalous properties of surface scattering.

We would like to stress that the problem of a correspondence between surface and bulk scattering still deserves a detailed study. In this respect, one should mention recent results \([3]\) where specific properties of a surface scattering have been discovered, that are different from those known in standard quasi-1D models with random potentials \([4]\).

In this Letter we analyze a surface scattering in the case of one open channel, assuming long-range correlations in the surface potential. Our main interest is to explore a possibility to construct such surfaces that result in windows of transparency in dependence on the frequency of incoming waves.

We consider a 2D waveguide of the length \(L\) along the \(x\)-axis, with a flat upper surface \(z = d\) and a rough (corrugated) lower surface \(z = \xi(x)\), see also \([3,4]\). The random function \(\xi(x)\) is characterized as follows:

\[
\langle \xi(x) \rangle = 0, \quad \langle \xi(x)\xi(x') \rangle = \sigma^2 \mathcal{W}(|x - x'|). \tag{1}
\]

The angular brackets stand for a statistical average over the ensemble of realizations of \(\xi(x)\) and the root-mean-square \(\sigma\) determines the roughness strength. The binary correlator \(\mathcal{W}(|x|)\) decreases with a typical scale \(R_c\) which is of the order of a mean length of surface irregularities.

Keeping in mind the relevance of a surface scattering in our model to the Anderson localization, in what follows we consider a single-mode waveguide when there is the only propagating mode with a real longitudinal wave number \(k_1 = \sqrt{(\omega/c)^2 - (\pi/d)^2}\). All other modes are evanescent, therefore, the width \(d\) is restricted by the conditions, \(0 < k_1 d/\pi < \sqrt{3}\). Note that the assumed condition \(\sigma \ll d\) leads to the inequality \(k_1 \sigma \ll 1\).

It can be shown \([3]\) that the corresponding wave equation takes the form,

\[
\left[ \frac{d^2}{dt^2} + (k_1 R_c)^2 \right] \Psi(t) = \frac{2}{\pi} \frac{\sigma}{R_c} \left( \frac{\pi R_c}{d} \right)^3 \varphi(t) \Psi(t) \tag{2}
\]

with \(t = x/R_c\). Here the function \(\varphi(t)\) is determined by the relation \(\xi(x) = \sigma \varphi(x/R_c)\). One can see that the problem of surface scattering in a single-mode waveguide is reduced to a 1D-model with random potential \(\varphi(t)\).

The expression \(L_b\) for the backscattering length \(L_b\) is given by the following expression,

\[
L_b^{-1} = \frac{4\sigma^2}{\pi^2} \left( \frac{\pi}{d} \right)^6 W(2k_1) \left( \frac{2k_1}{(2k_1)^2} \right)^2. \tag{3}
\]

Here \(W(k_x)\) is the Fourier transform of a binary correlator \(\mathcal{W}(|x|)\), \(W(|x|) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \exp(ik_x x) W(k_x)\). Note that \(W(k_x)\) is a positive function of the order of \(R_c\), which decreases in dependence on \(|k_x|\) with a typical scale \(R_c^{-1}\).

The expression \(L_b\) for the backscattering length \(L_b\) gives a complete information about the transmission through the waveguide. In particular, it determines the conductance and its fluctuations for any ratio \(L/L_b\), see
As one can see, the binary correlator of the surface profile defines all transport properties of a one-mode waveguide. In particular, if $W(2k_1)$ vanishes within some interval of the wave number $k_1$, the waveguide is fully transparent. Below, we show how to construct surface profiles that result in a complete transparency in a given range of $k_1$.

To do this, we generalize the approach developed for the Kronig-Penney model with the correlated disorder. This model can be treated as a particular case of the equation when the function $\varphi(t)$ is given in the form of periodic delta-kicks with random amplitudes. In this case, the construction is possible in the case of a weak disorder, $\sigma \ll d$, this is why only binary correlator is involved in the reconstruction of $\varphi(t)$. For this reason, the above solution is not unique since higher correlators are not controlled. Below we demonstrate the suggested approach by considering two simple cases of a correlated surface profile.

Let us first consider the case when the waveguide is completely transparent for $k_1 > 1/2R_c$. In this case one can get the following expressions for the binary correlator and its Fourier transform,

$$W_1(|x|) = \frac{\sin(x/R_c)}{x/R_c}; \quad W_1(k_x) = \pi R_c \Theta(1 - |k_x|R_c) \quad (6)$$

with $\Theta(x)$ as the unit-step function, $\Theta(0) = 1/2$. This kind of a $\Theta$-like dependence of $W(k_x)$ was recently analyzed in [3]. The surface profile that has the above correlations, is described by the function,

$$\varphi_1(t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dt' Z(t - t') \frac{\sin t'}{t'} \quad (8)$$

In this case the inverse backscattering length is

$$\frac{1}{L_{b1}(k_1)} = \frac{4}{\pi R_c} \left( \frac{\sigma}{R_c} \right)^2 \left( \frac{\pi R_c}{d} \right)^6 \Theta(1 - 2k_1 R_c)/(2k_1 R_c)^2 \quad (9)$$

Therefore, with an increase of the wave number $k_1$ the backscattering length smoothly increases, and after, goes abruptly to infinity for $k_1 > 1/2R_c$. Such a behavior can be observed if the transition point $k_1 = 1/2R_c$ is located inside an allowed single-mode region, for $12(\pi R_c/d)^2 > 1$, see above. In order to observe this effect for finite waveguides, one needs to assume that at the transition point the regime of a strong localization holds,

$$\frac{L}{L_{b1}(1/2R_c)} = \frac{2}{\pi} \left( \frac{\sigma}{R_c} \right)^2 \left( \frac{\pi R_c}{d} \right)^6 \frac{L}{R_c} \gg 1. \quad (10)$$

In this case the average transmittance is expected to be exponentially small due to a strong localization for $k_1 < 1/2R_c$. In contrast, in the interval $1 < (2k_1 R_c)^2 < 12(\pi R_c/d)^2$ a ballistic regime occurs with a perfect transparence.

The second case refers to a complimentary situation when for $k_1 < 1/2R_c$ the waveguide is transparent and for $k_1 > 1/2R_c$ is non-transparent. One can find that the corresponding expressions for $W(|x|)$ and $W(k_x)$ are

$$W_2(|x|) = \pi \delta(x/R_c) - \frac{\sin(x/R_c)}{x/R_c}; \quad (11)$$

$$W_2(k_x) = \pi R_c \Theta(|k_x|R_c - 1). \quad (12)$$

In this case the corrugated surface is described by a superposition of the white noise and the roughness of the first type. As a result, the surface-profile potential $\varphi(t)$ takes the form,

$$\varphi_2(t) = \sqrt{\pi} Z(t) - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dt' Z(t - t') \frac{\sin t'}{t'} \quad (13)$$

Correspondingly, the inverse backscattering length is expressed by

$$\frac{1}{L_{b2}(k_1)} = \frac{4}{\pi R_c} \left( \frac{\sigma}{R_c} \right)^2 \left( \frac{\pi R_c}{d} \right)^6 \Theta(2k_1 R_c - 1)/(2k_1 R_c)^2 \quad (14)$$

Therefore, in contrast to the first case, the backscattering length $L_{b2}(k_1)$ is equal to infinity below the transition point $k_1 = 1/2R_c$. With further increase of $k_1$, it smoothly increases starting from the value $L_{b2}(1/2R_c)$. Again, the transition point $k_1 = 1/2R_c$ is supposed to be inside the single-mode interval. In addition, we assume a strong localization to be retained at the upper point $k_1 = \pi\sqrt{3}/d$ of the single-mode region, i.e.,

$$\frac{L}{L_{b2}(\pi\sqrt{3}/d)} = \frac{4}{3\pi} \left( \frac{\sigma}{R_c} \right)^2 \left( \frac{\pi R_c}{d} \right)^4 \frac{L}{R_c} \gg 1. \quad (15)$$

In this case the ballistic transparence is abruptly replaced by a strong localization at the transition point $k_1 = 1/2R_c$. 
Let us now demonstrate the above predictions by direct numerical simulations. Keeping in mind the relation $4L_b = l_\infty$ between the backscattering length $L_b$ and the localization length $l_\infty$ in infinite waveguides (for $L \to \infty$), we compute $l_\infty$ by the method described in [1]. For this, we approximate the continuous function $\varphi(t)$ in Eq.(2) by the sum of delta-kicks with the spacing $\delta$ chosen much smaller that any physical length scale in our model. In this way one can write a Hamiltonian map which can be used to find the localization length via the Lyapunov exponent $\lambda = l_\infty^{-1}$ of a dynamical problem associated with this map (see details in [1,2]).

Numerical data are reported in Fig.1 where the normalized Lyapunov exponent $\Lambda = c_0 \lambda K^2$ is plotted against the normalized wave vector $K = 2k_1 R_c$ for the range $0 < K < 2$ which corresponds to the single-mode interval. In these units one can clearly see a non-trivial dependence of $\lambda$ on the wave vector, which is due to specific binary correlations in the potential $\varphi(t)$.

The potential $\varphi(t)$ was constructed according to discrete versions of the expressions (3) and (4) which determine complimentary dependences of the backscattering length $L_b(k_1)$, see (5) and (6). The data clearly demonstrate a drastic dependence of $\lambda$ on the wave vector when crossing the transition point $K = 1$. By taking the size $L$ in accordance with the expressions (10) and (15), one can arrange the selective transparence of waveguides as is predicted by the theory.

In conclusion, we study the possibility to construct one-mode waveguides with a selective transparence in dependence on the wave vector of an incoming wave. Analytical treatment shows that this can be done by a proper choice of random surfaces with specific long-range correlations along waveguides. Numerical data for two cases with complimentary dependences of the backscattering length on the wave vector demonstrate the effectiveness of the theoretical predictions. The results presented here may be used for experimental realizations of waveguides with a desired selectivity of the transmission. Note that random surfaces with discontinuous dependence of $W(k_x)$ have been recently fabricated in the experimental study of backscattering enhancement [10].

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