Neutral-current Neutrino-nucleus Scattering in Quasielastic Region

K. S. Kim\textsuperscript{1)}, B. G. Yu\textsuperscript{1)}, M. K. Cheoun\textsuperscript{2)}, T. K. Choi\textsuperscript{3)}, and M. T. Chung\textsuperscript{4)}

\textsuperscript{1)}School of Liberal Arts and Science, Korea Aerospace University, Koyang 200-1, Korea
\textsuperscript{2)}Department of Physics, Soongsil University, Seoul, 156-743, Korea
\textsuperscript{3)}Department of Physics, Yonsei University, Wonju, 220-710 Korea
\textsuperscript{4)}Department of Radiation, Dongsin University, Naju, 520-714 Korea

The neutral-current neutrino-nucleus scattering is calculated through the neutrino-induced knocked-out nucleon process in the quasielastic region by using a relativistic single particle model for the bound and continuum states. The incident energy range between 500 MeV and 1.0 GeV is used for the neutrino (antineutrino) scattering on $^{12}$C target nucleus. The effects of the final state interaction of the knocked-out nucleon are studied not only on the cross section but also on the asymmetry due to the difference between neutrinos and antineutrinos, within a relativistic optical potential. We also investigate the sensitivity of the strange quark contents in the nucleon on the asymmetry.

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I. INTRODUCTION

Neutrino-nucleus scattering has become to be widely interested in different fields of physics such as astrophysics, cosmology, particle, and nuclear physics. Such interests are found not only in the nuclear astrophysics but also in the nuclear physics itself. In particular, the neutral-current scattering of neutrinos and antineutrinos on nuclei is used to determine the structure of hadronic weak neutral currents. Along this line, Brookhaven National Laboratory (BNL)\textsuperscript{1)} reported that the value of a strange axial vector form factor of the nucleon does not have zero. The primary goal of the BooNE experiment\textsuperscript{2)} was to search for neutrino oscillation to test neutrino mass and then detected the first anti-neutrino events in January of 2006. The CNGS project\textsuperscript{3)} was proposed to detect the $\tau$ production through $(\nu_\tau, \tau^-)$ or $(\bar{\nu}_\tau, \tau^+)$ reactions, which will send a neutrino beam from CERN to the
At intermediate energies, there are many theoretical works \cite{4, 5, 6, 7, 8, 9, 10, 11} for the neutrino-nucleus scattering. The relativistic Fermi gas (RFG) model in Refs. \cite{5, 6} was applied to study the contribution of the strange quark to the nucleon form factor in the neutrino-induced knocked-out nucleon process. In Ref. \cite{7}, the relativistic plane wave impulse approximation (RPWIA) calculations were compared with the RFG calculations using a relativistic shell model. In particular, Ref. \cite{8} suggested a method of identifying neutrinos and antineutrinos by estimating the polarization asymmetry stemming from the differences of the intrinsic helicities of them. The calculation is carried out within a nonrelativistic nuclear shell model under a Woods-Saxon potential for the bound states.

For the final state interaction (FSI) of the knocked-out nucleon, the authors in Ref. \cite{9} showed that there is no flux loss to take into account the complex optical potential in the charged-current reaction. In Ref. \cite{10}, the importance of the FSI and the contribution of the strange quark content were shown in the neutral-current reaction by using a relativistic optical potential. Ref. \cite{11} presented the comparison of the relativistic distorted wave impulse approximation (RDWIA) by Madrid group \cite{12} with the relativistic multiple scattering Glauber approximation developed by Ghent group \cite{13}.

On the other hand, Ohio University group \cite{14, 15} calculated inclusive $(e, e')$ and exclusive $(e, e'p)$ reactions in quasielastic region using partial wave expansions of the electron wave functions in the distorted wave Born approximation (DWBA) under the presence of the electron Coulomb distortion due to the target nucleus. However, the DWBA calculations do not allow a separation of the cross section into a longitudinal part and a transverse part and are numerically challenging, and computational time increases rapidly with higher incident electron energies. In order to avoid such difficulties of the DWBA calculations, Kim and Wright \cite{16, 17} developed an approximate treatment of the electron Coulomb distortion which does allow the separation of the cross section into a longitudinal part and a transverse part.

In addition, Ref. \cite{16} showed a very good description of quasielastic scattering processes using a relativistic single particle model which requires the wave functions of bound and continuum nucleons and a transition current operator. The bound state wave functions are obtained from solving the Dirac equation in the presence of the strong vector and scalar potentials \cite{18}. For the inclusive $(e, e')$ reaction where the knocked-out nucleons are not
observed, the continuum wave functions are solutions to a real potential so as not to lose any flux. This ansatz guarantees the current conservation and gauge invariance. For the exclusive \((e,e'p)\) reaction, Kim and Wright [17] used the relativistic optical potential to solve the wave function of the knocked-out proton, which is generated by Ohio State University group [19]. These theoretical results explained experimental data very well with no free parameters except an overall scale factor, called the spectroscopic factor.

In this paper, we present the neutral-current neutrino-nucleus scattering in the quasielastic region, where the inelastic processes like pion production and delta resonance are excluded. The incident neutrino (antineutrino) energies are concerned in intermediate ranges (between 500 MeV and 1.0 GeV). The relativistic bound state wave functions are obtained from solving a Dirac equation in the presence of strong scalar and vector potentials based on the \(\sigma - \omega\) model [18]. In order to investigate the effect of the FSI we take account into the relativistic optical potential [19]. This nuclear model is the same model as used in Ref. [17] except the electron Coulomb distortion. Furthermore, the effects of the strangeness in the axial form factor of the weak current operator are also studied.

The outline of this paper is as follows. In Sec. II we address the formalism and the numerical results are shown in Sec. III. Our summary of the results and conclusions are given in Section IV.

## II. FORMALISM

The neutrino-nucleus scattering is described by the connection of the electromagnetic interaction and the weak interaction. The four-momenta of the incident and outgoing neutrinos (antineutrinos) are labelled \(p_i^\mu = (E_i,p_i)\) and \(p_f^\mu = (E_f,p_f)\). We choose the nucleus fixed frame where the target nucleus is seated at the origin of the coordinate system. \(p_A^\mu = (E_A,p_A)\), \(p_{A-1}^\mu = (E_{A-1},p_{A-1})\), and \(p_\mu = (E_p,p)\) represent the four-momenta of the target nucleus, the residual nucleus, and the knocked-out nucleon, respectively. In the laboratory frame, the inclusive cross section is given by the contraction between the lepton tensor and the hadron tensor [11]

\[
\frac{d\sigma}{dE_f} = 4\pi^2 M_N M_{A-1} (2\pi)^3 M_A \int \sin \theta_L d\theta_L \int \sin \theta_P d\theta_P p f_{rec}^{-1} \tilde{Z} \sigma_M [v_L R_L + v_T R_T + h v'_T R'_T],
\]

\(1\)
where $M_N$ is the nucleon mass, $\theta_l$ denotes the scattering angle of the lepton, and $h = -1$ ($h = +1$) corresponds to the helicity of the incident neutrino (antineutrino). The squared four-momentum transfer is given by $Q^2 = q^2 - \omega^2$. $\sigma^Z_M$ is defined by

$$\sigma^Z_M = \frac{G_F \cos(\theta_l/2) E_f M_Z^2}{\sqrt{2\pi}(Q^2 + M_Z^2)},$$

(2)

where $G_F$ is the Fermi constant given by $G_F \simeq 1.16639 \times 10^{-11}$ MeV$^{-1}$ and $M_Z$ is the rest mass of Z-boson. The recoil factor $f_{rec}$ is given by

$$f_{rec} = \frac{E_{A-1}^{pquer}}{M_A} \left| 1 + \frac{E_p}{E_{A-1}} \left[ 1 - \frac{q \cdot P}{p^2} \right] \right|.$$  

(3)

For the neutral-current reaction, the coefficients $v$ are given by

$$v_L = 1, \quad v_T = \tan^2 \frac{\theta_l}{2} + \frac{Q^2}{2q^2}, \quad v'_T = \tan \frac{\theta_l}{2} \left[ \tan^2 \frac{\theta_l}{2} + \frac{Q^2}{q^2} \right]^{1/2}.$$  

(4)

The corresponding response functions are given by

$$R_L = \left| J^0 - \frac{\omega}{q} J^1 \right|^2, \quad R_T = |J^x|^2 + |J^y|^2, \quad R'_T = 2 \text{Im}(J^x J^y).$$  

(5)

The nucleon current $J$ represents the Fourier transform of the nucleon current density written as

$$J^\mu = \int \bar{\psi}_p \hat{J}^\mu \psi_b e^{iq \cdot r} d^3r,$$  

(6)

where $\hat{J}^\mu$ is a free nucleon current operator, and $\psi_p$ and $\psi_b$ are the wave functions of the knocked-out and the bound state nucleons, respectively. For a free nucleon, the current operator comprises the weak vector and the axial form factors related to the electromagnetic current given by

$$\hat{J}^\mu = F^V_1(Q^2) \gamma^\mu + F^V_2(Q^2) \frac{i\kappa}{2M_N} \sigma^{\mu\nu} q_\nu + G_A(Q^2) \gamma^\mu \gamma^5 + \frac{1}{2M_N} G_F(Q^2) q^\mu \gamma^5,$$  

(7)

where $\kappa$ is the anomalous magnetic moment. The weak vector form factors for protons ($F^V_{1,p}(Q^2)$) and neutrons ($F^V_{1,n}(Q^2)$), by the conservation of the vector current (CVC) hypothesis with inclusion of an isoscalar strange quark ($F^s_i$) contribution, are given by

$$F^V_{i, p(n)} = \left( \frac{1}{2} - 2 \sin^2 \theta_W \right) F^p_{i,n} - \frac{1}{2} F^{n(p)} - \frac{1}{2} F^s_i,$$  

(8)

where $\theta_W$ is the Weinberg angle given by $\sin^2 \theta_W = 0.2224$. 


The strange vector form factors are expressed as:

\[
F_s^1(Q^2) = \frac{F_s^1 Q^2}{(1 + \tau)(1 + Q^2/M_V^2)^2}, \quad F_s^2(Q^2) = \frac{F_s^2(0)}{(1 + \tau)(1 + Q^2/M_V^2)^2},
\]

where \(\tau = Q^2/(4M_N^2)\), \(M_V = 0.843\) GeV, \(F_s^1 = -<r_s^2>/6 = 0.53\) GeV\(^{-2}\), and \(F_s^2(0) = \mu_s\) is a strange magnetic moment given by \(\mu_s = -0.4\).

The axial form factor is given by:

\[
G_A = \frac{1}{2}(\mp g_A + g_A^s) G,
\]

where \(g_A = 1.262, G = 1/(1 + Q^2/M^2)\) with \(M = 1.032\) GeV, and \(g_A^s = -0.19\), which represents the strange quark contents on the nucleon \([21]\). \(-(+)\) coming from the isospin dependence denotes the knocked-out proton (neutron), respectively.

The induced pseudoscalar form factor from the Goldberger-Treiman relation is parametrized as:

\[
G_P(Q^2) = \frac{2M_N}{Q^2 + m^2} G_A,
\]

where \(m_\pi\) is the pion mass. The contribution of the pseudoscalar form factor vanishes for the neutral-current reaction because of the final lepton mass participating in this reaction.

### III. RESULTS

We calculate the neutral-current neutrino-nucleus scattering for \(^{12}\)C nucleus in the quasielastic region, where the inelastic processes like pion production, Delta resonance, and etc are excluded. Within the relativistic framework, the wave functions of the bound state are generated by the \(\sigma - \omega\) model \([18]\) and the continuum states of the knocked-out nucleon are obtained by solving a Dirac equation in the presence of the phenomenological relativistic optical potentials generated by EDAD1 \([19]\). We use two incident neutrino (antineutrino) energies, 500 MeV and 1.0 GeV, where the inelastic contributions are as small as negligible.

In Figs. 1 and 2, we show the inclusive cross sections for the neutrino-nucleus and antineutrino-nucleus scattering of the neutral-current reactions as a function of the knocked-out nucleon kinetic energy \(T_p\) at incident neutrino energies \(E = 500\) MeV and 1.0 GeV. Solid curves present the results for the cross sections, and dashed and dotted lines show the contributions of the knocked-out protons and the knocked-out neutrons, respectively. Thick (thin) curves are the results with (without) the optical potential. The effects of
the FSI produce a reduction of the cross section around 50%, which is the same result as Refs. [10, 11]. Therefore the FSI of the knocked-out nucleon is confirmed again to be a vital ingredient in the neutrino-nucleus reactions. The knocked-out neutrons contribute to the cross section larger than the knocked-out protons. The ratios of the contribution between the knocked-out protons and neutrons are similar with/without the optical potential.

Figures 3 and 4 exhibit the effect of a strange quark contribution to the axial form factor for the neutrino and the antineutrino. The kinematics are the same as in Fig. 1. Thick (thin) curves are the results without (with) the strange quark contribution. Note that the relativistic optical potentials are used in these cases. For the incident neutrino, the effect of the strange quark reduces the cross section by the amount of 5% for 500 MeV and 2% for 1.0 GeV. The effect of the strange quark for the knocked-out protons is increased about 15% for 500 MeV and 20% for 1.0 GeV, but for the corresponding knocked-out neutrons decreased about 30% for 500 MeV and 20% for 1.0 GeV. This different behavior is due to the sign of $g_A$ in Eq. (10).

For the incident antineutrino, the effect of the strange quark enhances the cross section by 10% for 500 MeV and 5% for 1.0 GeV, contrary to the neutrino case. For the knocked-out neutrons, the effect constricts the results by 10% for 500 MeV and by 12% for 1.0 GeV and enhances them by 40% for 500 MeV and 35% for 1.0 GeV for the protons. From these results, we learn that the effect of the strange quark contribution changes the magnitude of the cross section while the position of the peaks and the shape are not affected. The effect of the strange quark contributes more to the knocked-out protons than the neutrons in the corresponding cross sections.

The intrinsic polarization of the neutrino and the antineutrino, which is the difference between the neutrino and the antineutrino, is reflected only on the third term in Eq. (1) due to its helicity dependence. Using the helicities, the asymmetry for the neutrino and the antineutrino is written as

$$A_l = \frac{\sigma(h = -1) - \sigma(h = +1)}{\sigma(h = -1) + \sigma(h = +1)},$$

(12)

where $\sigma$ denotes the cross section in Eq. (1) and $h = -1$ ($h = +1$) presents the helicity of the incident neutrino (antineutrino).

In Fig. 5, we study the effect of the FSI using the asymmetry of the neutrino and the antineutrino at the incident energies $E = 500$ MeV and 1.0 GeV. Solid and dashed lines represent the results with and without the FSI, respectively. The effects of the FSI on
the asymmetry appear to be much smaller than those on the corresponding cross sections. The asymmetry itself does not give any drastic effects coming from the FSI. But the effect increases with larger kinetic energies of the knocked-out nucleon.

With the same method in Fig. 5, the effect of the strange quark contribution is investigated in Fig. 6. Solid (dashed) curves represent the results without (with) the strange quark contribution. Surprisingly, the effect appears to be very large comparing with the corresponding cross sections. The effect also becomes larger as the kinetic energies of the knocked-out nucleon increase. Therefore the asymmetry might be an effective test for the strange quark contribution on the axial vector form factor according to these calculations. Note that the FSI is included through the optical potentials in these calculations.

IV. SUMMARY AND CONCLUSION

We present the calculations of the neutral-current reaction for the neutrino-nucleus scattering of $^{12}\text{C}$ in quasielastic region. The incident neutrino energies are used in intermediate energies, 500 MeV and 1.0 GeV. The wave functions for bound state are generated by the strong scalar and vector potentials based on $\sigma – \omega$ model, and for the knocked-out nucleon by the phenomenological optical model.

The FSI associated with the optical potential produces a large reduction of the cross section and influences differently on the ejecting proton and neutron. The effect appears to be larger on the neutrino than on the antineutrino. On the other hand, by the effect of the strange quark contribution, the cross sections are reduced for the neutrino and enhanced for the antineutron. The strange quark contribution shows a bigger contribution to the reaction by the antineutrino than by the neutrino. For the asymmetry of the intrinsic polarization of the neutrino and the antineutrino, the effects of the FSI and the strange quark increase with higher knocked-out nucleon kinetic energies. According to Ref. [10], the effect of the strange quark is negligible small but in our calculation it is not small for the asymmetry.

In conclusion, the effects of the final state interaction and the strange quark contribution on the intrinsic polarization asymmetry exhibit very different behavior on the corresponding cross sections. It needs to investigate the effects on various structure functions extracted from the methods commonly used in electron scattering like $(e, e'p)$ and $(e, e')$ reactions. As one of next works, it will be possible to include the Coulomb distortion of the final leptons
using charged-current neutrino-nucleus scattering.

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FIG. 1: Neutral current $^{12}\text{C}(\nu, \nu')$ cross sections as a function of the knocked out nucleon kinetic energy $T_p$ at incident neutrino energies $E = 500\text{ MeV}$ and $1.0\text{ GeV}$. Solid curves are the results for the cross sections, dashed and dotted lines are the contributions of the proton and the neutron, respectively. Thick and thin lines are calculations with the relativistic optical potential and with no any potential of the knocked-out nucleon.
FIG. 2: The same as in Fig. 1 but for the antineutrino.
FIG. 3: Neutral current $^{12}$C$(\nu, \nu')$ cross section as a function of the knocked out nucleon kinetic energy $T_p$ at incident neutrino energy $E = 500$ MeV and 1.0 GeV. Solid curves are the results for the cross sections, dashed and dotted lines are the contributions of the proton and the neutron, respectively. Thick (thin) lines are calculations without (with) the strange quark contribution. The optical potential of the knocked-out nucleon is used for the final state interaction.
FIG. 4: The same as in Fig. 3 but for the antineutrino.
FIG. 5: The asymmetry as a function of the kinetics energy of the knocked-out nucleon. Solid and dashed curves represent the results with and without the final state interaction, respectively.
FIG. 6: The asymmetry as a function of the kinetics energy of the knocked-out nucleon. Solid (dashed) curves represent the results without (with) the strange quark contribution.