Model of two-fluid reconnection

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A theoretical model of quasi-stationary, two-dimensional magnetic reconnection is presented in the framework of incompressible two-fluid magnetohydrodynamics (MHD). The results are compared with recent numerical simulations and experiment.

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INTRODUCTION

Magnetic reconnection is a fundamental physical process of topological rearrangement of magnetic field lines in magnetized plasmas during which magnetic energy is converted into kinetic and thermal energy. Reconnection is of particular importance in the solar atmosphere, the Earth’s magnetosphere, and in laboratory plasmas [1, 2, 3, 4]. In hot plasmas, because of the low electrical resistivity, magnetic reconnection due to resistive dissipation alone is very slow. As a result, a simple single-fluid MHD description of the plasma is generally believed to be insufficient for the theoretical explanation of fast reconnection events. Instead, a two-fluid MHD approach has been frequently used in recent studies of fast reconnection [1, 2, 3, 4]. Most of these studies have been numerical and experimental, while an ultimate goal of construction a comprehensive theoretical model of two-fluid reconnection has not yet been achieved. In this Letter we present a model of two-fluid magnetic reconnection, which serves this goal.

TWO-FLUID MHD EQUATIONS

We use physical units in which the speed of light $c$ and four times $\pi$ are replaced by unity, $c = 1$ and $4\pi = 1$. To rewrite our equations in the Gaussian centimeter-gram-second (CGS) units, one needs to make the following substitutions: magnetic field $\mathbf{B} \rightarrow \mathbf{B}/\sqrt{4\pi}$, electric field $\mathbf{E} \rightarrow c\mathbf{E}/\sqrt{4\pi}$, electric current $\mathbf{j} \rightarrow \sqrt{4\pi}\mathbf{j}/c$, electrical resistivity $\eta \rightarrow \eta c^2/4\pi$, electrical current $\mathbf{j} \rightarrow \sqrt{4\pi}\mathbf{j}/c$, electrical resistance $\eta \rightarrow \eta c^2/4\pi$, electrical current $\mathbf{j} \rightarrow \sqrt{4\pi}\mathbf{j}/c$.

We consider an incompressible, non-relativistic and quasi-neutral plasma, composed of electrons and protons. Using standard notation, the equations of motion for the electrons and protons are [1, 2, 3, 4, 5]

\begin{align}
ne\left[\partial_t \mathbf{u}^e + (\mathbf{u}^e \cdot \mathbf{V}) \mathbf{u}^e\right] &= -\nabla P_e - ne(\mathbf{E} + \mathbf{u}^e \times \mathbf{B}) - \mathbf{K}_e, \\
nm_p\left[\partial_t \mathbf{u}^p + (\mathbf{u}^p \cdot \mathbf{V}) \mathbf{u}^p\right] &= -\nabla P_p + ne(\mathbf{E} + \mathbf{u}^p \times \mathbf{B}) + \mathbf{K}_p,
\end{align}

where $n$ is the (constant) number density, and the subscripts and superscripts $e$ and $p$ refer to electrons and protons respectively. Here, $\mathbf{K}$ is the resistive frictional force due to electron-proton collisions that can be approximated as $\mathbf{K} = n^2 e^2 \pi (\mathbf{u}^e - \mathbf{u}^p) = -\eta ne \mathbf{j}$, where $\eta$ is the electrical resistivity [1, 2, 3]. For simplicity, we neglect proton-proton and electron-electron collisions and the corresponding viscous forces. We also introduce the electric current $\mathbf{j} = ne(\mathbf{u}^e - \mathbf{u}^p)$ and the plasma velocity $\mathbf{V} = n(m_p \mathbf{u}^p + m_e \mathbf{u}^e)/\rho$, where $\rho = n(m_p + m_e)$ is the plasma density. Taking into account $m_e \ll m_p$, we find $\mathbf{u}^p = \mathbf{V} + m_e \mathbf{j}/nem_p$ and $\mathbf{u}^e = \mathbf{V} - \mathbf{j}/ne$. We substitute these expressions and $\mathbf{K} = -\eta ne \mathbf{j}$ into Eqs. (1) and (2). We also substitute the electric field $\mathbf{E}$ from Eq. (1) into Eq. (2). We obtain

\begin{align}
\mathbf{E} &= \eta \mathbf{j} - \mathbf{V} \times \mathbf{B} + \mathbf{j} \times \mathbf{B}/ne - \left[\nabla P_e - (\mathbf{d}_e^2/\rho_p)\nabla \rho_p \right]/ne \\
+ \mathbf{d}_e^2 \left[\partial_t \mathbf{j} + (\mathbf{V} \cdot \nabla)\mathbf{j} + (\mathbf{j} \cdot \mathbf{V})\mathbf{V} - (1/ne)(\mathbf{V} \cdot \nabla)\mathbf{j}\right],
\end{align}

\begin{align}
nm_p\left[\partial_t \mathbf{V} + (\mathbf{V} \cdot \nabla)\mathbf{V} - \nabla P + \mathbf{j} \times \mathbf{B} - \mathbf{d}_e^2 (\mathbf{V} \cdot \nabla)\mathbf{j},
\end{align}

where $P = P_e + P_p$ is the total pressure, $d_e = (m_e/ne)^{1/2}$ and $d_p = (m_p/ne)^{1/2}$ are the electron and proton inertial lengths. Eq. (3) is the generalized Ohm’s law describing the motion of the electrons. Eq. (4) is the plasma momentum equation describing the motion of the protons. We note that the electron inertia terms, proportional to $d_e^2$, enter both Ohm’s law and the momentum equation. Although these terms are essential for fast two-fluid reconnection (as we shall see presently), they have been frequently neglected in the momentum equation before. We also note that $\nabla \cdot \mathbf{V} = 0$ and $\nabla \cdot \mathbf{j} = 0$ for incompressible and non-relativistic plasmas.

We consider two-fluid magnetic reconnection in the classical Sweet-Parker-Petschek geometry, shown in Fig. 1. The reconnection layer is in the $x$-$y$ plane with the $x$- and $y$-axes perpendicular to and along the layer respectively and all $z$ derivatives are zero. The thickness of the reconnection current layer is $2\delta$, which is defined in terms of the out-of-plane current ($j_z$) profile across the layer [2, 3]. It can be shown that $2\delta$ turns out to be also the thickness of the electron layer, where the electrons are decoupled from the field lines. The length of the electron (current) layer is defined as $2L$. The proton layer, where the protons are decoupled from the field lines, has thickness $2\Delta$ and length $2L_{ext}$, which can be much larger than $2\delta$ and $2L$ respectively. The values of the reconnecting field outside the electron layer (at $x \approx \delta$) and outside the proton layer (at $x \approx \Delta$) are about the same, $B_y \approx B_{ext}$ up to a factor of order unity. The out-of-plane field $B_z$ is assumed to have a quadrupole structure (see Fig. 1) [2, 3, 4, 5]. Also, the re-
connection layer is assumed to have a point symmetry with respect to its geometric center \( O \) (see Fig. 1) and reflection symmetries with respect to the \( x \)- and \( y \)-axes. Thus, \( V_z(\pm x, \mp y) = \mp V_z(x, y) \), \( V_y(\pm x, \mp y) = \pm V_y(x, y) \), \( V_z(\pm x, \mp y) = V_z(x, y) \), \( B_x(\pm x, \mp y) = \mp B_x(x, y) \), \( B_y(\pm x, \mp y) = \pm B_y(x, y) \), \( B_z(\pm x, \mp y) = -B_z(x, y) \), \( j_z(\pm x, \mp y) = \mp j_z(x, y) \), \( j_y(\pm x, \mp y) = \pm j_y(x, y) \) and \( j_x(\pm x, \mp y) = j_x(x, y) \). The derivations below extensively exploit these symmetries and are similar to [6, 7].

**SOLUTION FOR TWO-FLUID RECONNECTION**

We make the following assumptions for the reconnection process. First, \( \eta \) is constant and small. Second, the reconnection process is quasi-stationary, so that we can neglect time derivatives. This assumption is satisfied if there are no plasma instabilities in the reconnection layer, and the reconnection rate is sub-Alfvénic, \( E_z \ll V_y B_{\text{ext}} \). Here \( V_y \equiv B_{\text{ext}}/\sqrt{\mu n e} \) is the Alfvén velocity. Third, the pressure tensors are isotropic, so that the pressure terms in Eqs. (3) and (4) are scalars.

Using Ampère’s law and neglecting the displacement current, we find the current components to be \( j_x = \partial_y B_z \), \( j_y = -\partial_x B_z \) and \( j_z = \partial_x B_y - \partial_y B_x \). The \( z \)-component of the current at the central point \( O \) is

\[
j_o \equiv (j_z)_o = (\partial_x B_y - \partial_y B_x)_o \approx B_{\text{ext}}/\delta . \tag{5}
\]

where we use the estimates \( (\partial_y B_z)_o \ll (\partial_x B_y)_o \) and \( (\partial_x B_y)_o \approx B_{\text{ext}}/\delta \) at the point \( O \). The last estimate follows directly from the definition of \( \delta \).

Equation (4) for the plasma (proton) acceleration along the reconnection layer gives

\[
nm_p(V \nabla)V_y + d^2_e (j \nabla)j_y = -\partial_y P + j_y B_x - j_x B_z . \tag{6}
\]

The \( y \) derivative of this equation at the point \( O \) gives

\[
nm_p(\partial_y V_y)_o \gamma (1 + d^2_e \beta^2 / \delta^2) \approx 2 B_{\text{ext}}^2 / L^2 + j_o (\partial_y B_z)_o . \tag{7}
\]

Here we introduce a useful dimensional parameter

\[
\gamma \equiv (\partial_{xy} B_z)_o / ne (\partial_y V_y)_o , \tag{8}
\]

which measures the relative strength of the Hall term \( (j \times B)_z / ne \) and the ideal MHD term \( (V \times B)_z \) inside the electron layer. In the derivation of Eq. (7) we use the estimate \( (\partial_{yy} P)_o \approx (\partial_{yy} B_y^2/2)_{\text{ext}} \approx -2 B_{\text{ext}}^2 / L^2 \), which follows from the force balance condition for the slowly inflowing plasma across the layer [4].

Faraday’s law \( \nabla \times \mathbf{E} = -\partial_t \mathbf{B} \) for the \( x \) - and \( y \)-components of a quasi-stationary magnetic field in two dimensions gives \( \partial_y E_z = -\partial_t B_x \approx 0 \) and \( \partial_x E_z = \partial_t B_y \approx 0 \). Thus, \( E_z \) is approximately constant in space, and from the generalized Ohm’s law [3] we obtain

\[
E_z = \eta j_z - V_x B_y + V_y B_x + (j_x B_y - j_y B_z)/ne + d^2_e [j_x \partial_y V_x + j_y \partial_y V_x + V_x \partial_x j_y + V_y \partial_y j_z - (j_x \partial_x j_x + j_y \partial_y j_y)/ne] \approx \text{constant} . \tag{9}
\]

The reconnection rate is determined by the value of \( E_z \) at the point \( O \), namely \( E_z = \eta j_o \). We estimate \( j_o \) below.

Taking the second derivatives of the \( z \)-component of Eq. (4) with respect to \( x \) and \( y \) at the point \( O \), we find \( (\partial_{xx} V_z)_o = - (\epsilon^2/\eta)(\partial_y B_x)_o - d^2_e (\partial_x j_z)_o \) and \( (\partial_{yy} V_z)_o = (\epsilon^2/\eta)(\partial_y B_y)_o + d^2_e (\partial_y j_z)_o \). Using these expressions, we calculate the second derivatives of Eq. (9) with respect to \( x \) and \( y \) at the point \( O \) and obtain

\[
\eta j_o / \delta^2 \approx 2 (\partial_y V_y)_o j_o [1 + d^2_e / \delta^2] \times [1 + \gamma (1 - d^2_e \beta^2 / \delta^2)] , \tag{10}
\]

\[
\eta j_o / L^2 \approx 2 (\partial_y V_y)_o [(\partial_y B_x)_o - d^2_e j_z / L^2] \times [1 + \gamma (1 - d^2_e \beta^2 / \delta^2)] , \tag{11}
\]

where we use \( (\partial_x V_z)_o = - (\partial_y V_y)_o \), \( (\partial_y B_x)_o \approx j_o \), \( (\partial_{xx} V_z)_o \approx \gamma j_o / \delta^2 \) and \( (\partial_{yy} V_z)_o \approx - j_o / \delta^2 \).

In Eq. (10), the electric field \( E_z \) is balanced by the MHD and Hall terms outside the electron layer, where the electron inertia terms are unimportant. Therefore,

\[
E_z \approx - V_x B_y (1 - j_z / ne V_z) \approx (\partial_y V_y)_o j_o B_{\text{ext}} (1 + \gamma) , \tag{12}
\]

\[
E_z \approx V_y B_x (1 - j_y / ne V_y) \approx (\partial_y V_y)_o (\partial_y B_x)_o L^2 (1 + \gamma) , \tag{13}
\]

at the points \( (x \approx \delta, y = 0) \) and \( (x = 0, y \approx L) \) respectively. Here we use the estimates \( j_x \approx (\partial_{xy} B_z)_o \delta \), \( j_y \approx - (\partial_{xy} B_z)_o L \), \( V_x \approx - (\partial_y V_y)_o \delta \), \( V_y \approx (\partial_y V_y)_o L \), \( B_z \approx (\partial_y B_y)_o L \) and \( B_y \approx B_{\text{ext}} \).

The ratio of Eqs. (10) and (11) gives \( (\partial_B B_z)_o \approx (\delta^2 j_o / L^2) (1 + 2d^2_e / \delta^2) \), while the ratio of Eqs. (12) and (13) gives \( (\partial_B B_z)_o \approx B_{\text{ext}} / L^2 \approx \delta j_o / L^2 \), where we use Eq. (3). Comparing these two estimates, we find \( \delta \gtrsim d^2_e / \delta \); therefore, \( j_o \gtrsim B_{\text{ext}} / d_e \) and \( E_z \gtrsim \eta B_{\text{ext}} / d_e \).

Next, we use the \( z \)-component of Faraday’s law: \( 0 \approx - \partial_z B_z = \partial_t E_y - \partial_y E_z \). Taking the \( \partial_y \) derivative of this equation at the point \( O \), and using Eq. (8) for \( E_y \) and \( E_z \), after tedious but straightforward derivations, we obtain

\[
0 \approx - \eta [ (\partial_{yy} B_z)_o + (\partial_{yy} B_z)_o ] + (1 - d^2_e \gamma / d^2_p) \times [(\partial_y B_x)_o (\partial_y j_z)_o + (\partial_y B_y)_o (\partial_y j_z)_o ] / ne \\
\approx \eta \eta e (\partial_y V_y)_o \gamma \beta^2 - (1 - d^2_e \gamma / d^2_p) \times [(\partial_y B_x)_o (\partial_y j_z)_o / \delta^2 + j_z^2 / L^2] / ne . \tag{14}
\]
\( (\partial_{x^2}j_0)_{\parallel} \approx -j_0/\delta^2, \ (\partial_{yy}j_0)_{\parallel} \approx -j_0/L^2, \ (\partial_zB_y)_{\parallel} \approx j_0. \) Note that Eq. (12) results in \( \gamma < d_p^2/d_e^2. \)

We estimate the proton layer half-thickness \( \Delta \) as follows. Outside the electron layer the electron inertia and magnetic tension terms can be neglected in Eq. (12), and we have \( n m_p (V \nabla) V_y \approx -\partial_y V_y \). Taking the \( y \) derivative of this equation at \( y = 0 \), we obtain \( n m_p [V_x(\partial_x V_y) + (\partial_y V_y)] \approx - (\partial_y P)_{\parallel} \approx 2B_{xy}^2/L^2. \) Here the term \( V_x(\partial_x V_y) \) is about of the same size as the term \( (\partial_y V_y) \). Therefore, we find that \( (\partial_y V_y)_{\parallel} \approx V_y/L \) outside the electron layer (but inside the proton layer). Next, in the upstream region outside the proton layer ideal single-fluid MHD applies. As a result, at \( x \approx \Delta \) and \( y = 0 \), Eq. (12) reduces to \( E_z = \eta j_0 \approx -V_y B_y \approx (\partial_y V_y)\Delta B_{ext} \approx \Delta B_{ext} V_A/L. \) Thus,

\[
(\partial_y V_y)_{\parallel} \approx V_A/L, \quad \Delta \approx \eta j_0 L/V_A B_{ext}. \tag{15}
\]

Now we solve equations (6), (7), (8), (10)–(15) for unknown quantities \( j_0, \delta, \Delta, \gamma, (\partial_y V_y)_{\parallel}, (\partial_y B_2)_{\parallel} \), and \( (\partial_y B_x)_{\parallel} \). We neglect factors of order unity, and we treat the external field \( B_{ext} \) and scale \( L_{ext} \) as model parameters. Recall that parameter \( \gamma \), given by Eq. (8), measures the relative strength of the Hall term and the ideal MHD term in the \( z \)-component of the Ohm’s law. Depending on the value of \( \gamma \), we have the following solutions and the corresponding reconnection regimes.

**Sweet-Parker reconnection.** When \( \gamma \lesssim 1 \), both the Hall current and electron inertia are negligible, and the electrons and protons flow together. In this case, we obtain the Sweet-Parker solution:

\[
E_z = \eta j_0 \approx V_A B_{ext}/\sqrt{\delta^2 + 1}, \quad \delta \approx \Delta \approx L_{ext}/\sqrt{\delta^2 + 1}, \quad L_{ext} \approx \gamma S d_p^2, \quad (\partial_y V_y)_{\parallel} \approx (\partial_y V_y)_{\parallel} \approx V_A/L_{ext}, \quad (\partial_y B_x)_{\parallel} \approx B_{ext}/L_{ext} \sqrt{\delta^2 + 1}, \quad (\partial_y B_2)_{\parallel} \approx B_{ext}/L_{ext} \Delta
\]

where \( S = (V_A L_{ext})/\eta \) is the Lundquist number. Condition 1 \( \lesssim 1 \) gives \( S \approx L_{ext}^2/d_p^2. \) Therefore, Sweet-Parker reconnection occurs when \( dp \) is less than the Sweet-Parker layer thickness, \( dp \lesssim L_{ext}/\sqrt{\delta^2 + 1} \) when \( \gamma \lesssim 1 \). We essentially treated as a fixed parameter. Here, we take a different approach and make a conjecture that the Hall reconnection regime describes a transition from the slow Sweet-Parker reconnection to the fast collisionless reconnection (presented below). Numerical simulations and experiment have demonstrated that this transition occurs when \( dp \approx L_{ext}/\sqrt{S \delta^2 + 1} \). Therefore, our conjecture implies that the Hall reconnection solution is

\[
S \approx L_{ext}^2/d_p^2, \quad L_{ext} \geq L \approx d_p L_{ext}/dp, \quad j_0 \approx B_{ext} L_{ext}/d_p L, \quad E_z \approx (d_p/L) V_A B_{ext}, \quad \delta \approx d_p L_{ext}/d_p L, \quad \Delta \approx d_p L_{ext}/d_p L, \quad (\partial_y V_y)_{\parallel} \approx V_A/L_{ext}, \quad (\partial_y B_x)_{\parallel} \approx B_{ext}/d_p L_{ext} \approx B_{ext}/L_{ext} d_p L_{ext}, \quad (\partial_y B_2)_{\parallel} \approx B_{ext}/L_{ext} d_p L_{ext}^2.
\]

As the electron layer length \( L \) decreases from its maximal value \( L = L_{ext} \) to its minimal value \( L = d_p L_{ext}/d_p L_{ext} \), this solution changes from the slow Sweet-Parker solution to the fast collisionless reconnection solution that is presented next.

**Collisionless reconnection.** When \( dp/d_p \ll 1 \), the electron inertia and the Hall current are important inside the electron layer and the proton layer respectively. In this case, the solution is

\[
\begin{align*}
\delta & \approx d_p, \\
E_z & = \eta j_0 \approx \eta B_{ext}/d_p \approx (L_{ext}/d_p S) V_A B_{ext}, \\
L & \approx V_A d_p d_p /\eta = S d_p d_p /L_{ext}, \\
d_p/d_p & \ll \gamma < d_p^2/d_e^2, \\
(\partial_y V_y)_{\parallel} & \approx \eta d_p^2 \gamma = V_A L_{ext}/S d_p^2, \\
(\partial_y V_y)_{\parallel} & \approx \gamma/d_p = B_{ext} L_{ext}^2 /S^2 d_p^2, \\
(\partial_y B_x)_{\parallel} & \approx B_{ext} L_{ext}^2 /S^2 d_p^2, \\
(\partial_y B_x)_{\parallel} & \approx B_{ext} \gamma /V_A d_p d_p = B_{ext} L_{ext}^2 /S^2 d_p^2. \tag{23}
\end{align*}
\]

Apart from the definition of the reconnecting field \( B_{ext} \), Eqs. (17)–(23) essentially coincide with the results obtained in [8] for an electron MHD (EMHD) reconnection model. Note that the value of \( \gamma \) or, alternatively, the value of the proton acceleration rate \( (\partial_y V_y)_{\parallel} \) at the point \( O \) cannot be determined exactly. This is because in the plasma motion equation (4), the magnetic tension and pressure forces are balanced by the electron inertia term \( (\partial_y V_y)_{\parallel} \) inside the electron layer. The proton inertia term \( n m_p (V \nabla) V_y \) can be of the same order or smaller, resulting in the upper limit \( (\partial_y V_y)_{\parallel} \lesssim V_A/L \), which inside the electron layer the magnetic energy is converted into the kinetic energy of the electrons (and into Ohmic heat), while the proton kinetic energy can be much smaller. However, in the downstream region \( y \lesssim L_{ext}, \) as the electrons gradually decelerate, their kinetic energy is converted into the proton kinetic energy. As a result, the eventual proton outflow velocity becomes \( \approx V_A \). These results emphasize the critical role that electron inertia plays in the plasma momentum equation (4).

The collisionless reconnection rate given by Eq. (17), although being proportional to resistivity [24], is much faster than the Sweet-Parker rate \( E_z \approx V_A B_{ext}/\sqrt{S} \) as long as \( S < L_{ext}^2/d_p^2 \). The solution (17–23) is valid provided \( L_{ext}/d_p < S \approx L_{ext}^2/d_p d_p \), which is obtained from conditions \( E_z \approx V_A B_{ext} \) and \( L_{ext} \approx L_{ext} \). Thus, both fast collisionless and slow Sweet-Parker reconnection regimes can exist simultaneously, as found in simulations [13]. These simulations also found a hysteresis for transition between slow and fast reconnection regimes. This implies...
that the transition Hall regime, during which the electron layer length $L$ decreases, may occur at Lundquist numbers other than $S \approx L_{\text{ext}}^2/d_e^2$, depending on the past history. Unfortunately, our stationary model cannot describe time-dependent transition processes.

It is known that the single-fluid MHD reconnection becomes much faster when resistivity $\eta$ is anomalously enhanced by being a physical mechanism of very fast reconnection. Eq. (17) shows that resistivity enhancement can considerably increase the collisionless reconnection rate as well. This enhancement can occur after the electric current value ($j_\rho$) jumps up during the transition from the Sweet-Parker to the collisionless regime at $d_p \approx L_{\text{ext}}/\sqrt{S}$, and could be a physical mechanism of very fast reconnection.

**DISCUSSION**

Let us compare theoretical results [10]–[23] for collisionless reconnection with numerical simulations and experiment. The estimates $\Delta \approx d_p$ for the proton layer thickness, $\delta \approx d_e$ for the electron layer thickness, $B_z \approx (\partial_{xy} B_z)_0 \delta L \approx B_{\text{ext}}$ for the quadrupole field, and $u'_y \approx -j_y/ne \approx (\partial_{xy} B_z)_0 L/ne \approx V_A \equiv B_{\text{ext}}/\sqrt{ne}\nu_e$ for the electron outflow velocity agree with simulations [2, 3, 6, 14, 16, 17, 18]. The estimates $\Delta \approx d_p$ and $B_z \approx B_{\text{ext}}$ also agree with experiment [3]. However, the experimentally measured electron layer thickness is about eight times larger than the model and simulations predict [10]–[20]. Three-dimensional geometry effects and plasma instabilities make direct comparison and exact interpretation of the experimental results difficult [3, 20].

Our theoretical results are qualitatively consistent with recent numerical findings of an inner electron dissipation layer and of electron outflow jets that extend into the proton layer [12, 16, 17, 18] in the electron layer thickness $L \propto d_e \propto m_{e}^{1/2}$ decreases with the electron mass, as in simulations [18], but the scaling law observed in these simulations was slightly different, $L \propto m_{e}^{3/8}$. Length $L \approx Sd_p/d_{\text{ext}}$ is generally much larger than both $\delta \approx d_e$ and $\Delta \approx d_p$, consistent with simulations [15, 16, 17]. However, if resistivity $\eta$ becomes anomalous and enhanced over the Spitzer value so much that $S \approx L_{\text{ext}}/d_e$, then $L$ can theoretically become of order of $d_p$, as in simulations [12, 15].

Our theoretical results for the proton layer thickness $V_A$ agree with simulations [17], which found the proton outflow velocity to be significantly less than $V_A$ and also found accelerating protons in the decelerating electron jets. Unfortunately, detailed quantitative comparison of our results to the results of kinetic numerical simulations is hindered because the simulations do not explicitly specify resistivity $\eta$. Also, in the simulations the electron pressure tensor anisotropy was found to play a critical role inside the electron layer and jets [17, 18], while in this study an isotropic pressure is assumed and the electrons are coupled to the field lines inside the electron outflow jets. Thus, in our model the electric field $E_z$ is supported by the Hall term $(j \times B)_{2}/ne$ in the downstream region $y \gtrsim L$. As a result, there are Hall-MHD Petschek shocks attached to the ends of the electron layer [25], as observed in numerical simulations [21]. However, in these simulations a spatially localized anomalous resistivity was prescribed, resulting in a short layer length, while here resistivity $\eta$ is assumed to be constant.

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