GRBs Neutrinos as a Tool to Explore Quantum Gravity induced Lorentz Violation

Uri Jacob♣ and Tsvi Piran♠

Racah Institute for Physics, The Hebrew University, Jerusalem, 91904, Israel

♣ uriyada@phys.huji.ac.il
♠ tsvi@phys.huji.ac.il

Received __________________; accepted __________________
ABSTRACT

Lorentz Invariance Violation (LIV) arises in various quantum-gravity theories [1, 2]. As the typical energy for quantum gravity is the Planck mass, $M_{pl}$, LIV will, most likely, be manifested at very high energies that are not accessible on Earth in the foreseeable future. One has to turn to astronomical observations [2-12]. Time of flight measurement from different astronomical sources set current limits on the energy scale of possible LIV to $>0.01M_{pl}$ (for n=1 models) and $>10^{-9}M_{pl}$ (for n=2). According to current models Gamma-Ray Bursts (GRBs) are accompanied by bursts of high energy ($\sim 100\text{TeV}$) neutrinos [13, 14]. At this energy range the background level of currently constructed neutrino detectors is so low that a detection of a single neutrino from the direction of a GRB months or even years after the burst would imply an association of the neutrino with the burst and will establish a measurement of a time of flight delay. Such time of flight measurements provide the best way to observe (or set limits) on LIV. Detection of a single GRB neutrino would open a new window on LIV and would improve current limits by many orders of magnitude.

History tells us that symmetries, observed over a large range of parameters and believed to be fundamental properties of our physical world, may lose their significance later when observations are made over a larger range of parameters and when a new physical understanding arises. Such apparent symmetries often emerge as leading order approximations of more complex symmetries, found to describe more accurately the larger range of observations.

Lorentz invariance might be such a symmetry. A growing number of speculations suggest that Lorentz invariance might be violated or deformed at very high energies. These speculations arise in
some theories of Quantum Gravity in which Lorentz violation appears (see e.g. [1, 2] for a review). They are motivated, independently, by attempt [15-19] to resolve the GZK paradox [20, 21]: the arrival on Earth of ultra high energy cosmic rays at energies above the expected GZK threshold. LIV would also explain the observations of 20TeV photons from Mk 501 (a BL-Lac object at distance of 150 Mpc) [22, 23, 19]. These photons should have been annihilated via pair creation with the IR background.

We consider here a simple phenomenological approach [19] for LIV with a symmetry breaking energy scale $\xi M_{pl}$. Taking into account only the leading order correction we expect, for particles with $E \ll \xi M_{pl}$, a generic approximate dispersion relation:

$$E^2 - p^2 c^2 - m^2 c^4 \simeq \pm E^2 \left( \frac{E}{\xi_n E_{pl}} \right)^n. \quad (1)$$

We consider a single $\xi$ for all particles as such correction is the simplest and it arises naturally from any theory in which the modification arises from a small scale structure of spacetime. Assuming the standard relation $v = dE/dp$ holds the $+$ ($-$)$^1$ sign accounts for superluminal (infraluminal) motion.

The most generic attempts to constrain the LIV scale are based on the energy dependent delay$^2$ in arrival of high energy particles [2]. As no delays have been observed in GRB [3-6], flaring AGN [7], or TeV emission from the Crab pulsar [8] we have only lower bounds: $\xi_1 \gtrsim 0.01$ from GRBs and $\xi_2 \gtrsim 10^{-9}$ from flaring AGN [7]. Limits for higher values of $n$ are too small to be of any relevance. Stronger bounds can be obtained under more specific assumptions on the nature of LIV (see e.g.

$^1$The increase in the reactions’ thresholds that resolve the GZK and the TeV photon paradoxes requires the $-\$ sign.

$^2$Loosely speaking we use the term delay to imply both a delay (corresponding to a $-$ sign in Eq. 1, and an early arrival (corresponding to a $+$ sign).
We focus here on time delays as these provide the most model independent LIV test [24].

The time delay of a particle with energy, $E$, arriving from a source at a distance $d$, is of order$^3$:

$$\Delta t \approx \frac{1 + n}{2} \left( \frac{d}{c} \right) \left( \frac{E}{\xi_n M_{pl}} \right)^n.$$

To improve current limits one needs a more distant source, an observation at higher energies or an improved temporal resolution (provided that all particles are emitted simultaneously at the source). However, pair production on the IR background limits the distances that high energy photons can travel. The lower photon number fluxes at higher energies limit, further, the possible time resolution [25]. Very high energy neutrino [26] provide an alternative that overcomes these problems.

Practically all current GRB models (see [27] for a review) predict bursts of very high energy neutrinos, with energy ranging from 100TeV to $10^4$TeV (and possibly up to $10^6$TeV), that should accompany GRB [13, 14]. As the energy of these neutrinos are orders of magnitude higher than the energies of photons observed from cosmological distances, the corresponding time delays are longer and can open a new window on the LIV parameter space.

The 1637 bursts detected over an effective exposure time of 2.62 years and recorded in the BATSE 4B Catalog have an average fluence of $1.2 \times 10^{-5}$ ergs/cm$^2$. Assuming that the emitted neutrinos fluence is one tenth that observed in photons (a reasonable assumption concerning the relevant interactions), we obtain an average GRB induced $\nu$ flux of $5 \times 10^{18}$ eV/(km$^2$yr). Using the most likely value of $E_\nu \approx 100$TeV and a detection probability of $10^{-4}$ in a km$^3$ detector, we estimate a detection rate of 5 events per year. This rough estimate is in agreement with several model dependent calculation [28-30] that find a detection rate of a few to a few dozen events per year. The

$^3$See Eq. 3 below for an exact formula.
increasing sensitivity of the detectors with the neutrino energy [29, 31] compensates somewhat over a decreasing flux (for a given fixed total emitted $\nu$ flux) and the detected flux decrease only like $E^{-0.5}$ for $E \gtrsim 100\text{TeV}$. Thus, for Icecube only neutrinos up to $10^4\text{TeV}$ are relevant.

The detection probability of neutrinos from a given burst is small $\sim 10^{-2}$. It is, therefore, unlikely that two neutrinos will be detected from the same burst and a direct comparison between the arrival time of two neutrino [26] cannot be done. LIV measurements will have to depend on the time delay between a single detected neutrino and the prompt low energy GRB photons.

The LIV time delay of a high energy neutrino with an observed energy, $E$, emitted at redshift $z$ is:

$$\Delta t = \frac{1}{H_0} \int_0^z \left(\frac{1 + n}{2} \left(\frac{E}{\xi E_{\text{pl}}}\right)^n (1 + z')^n\right) \frac{dz'}{\sqrt{\Omega_m (1 + z')^3 + \Omega_\Lambda}}. \quad (3)$$

Delays of order of hours are expected for a $100\text{TeV} \nu$ from a $z = 1$ burst with $\xi_1 = 1$ (or with $\xi_2 = 10^{-7}$). These values should be compared with the time interval, $t_b(E, p)$, in which (at a given confidence level, $p$) no background neutrino from the same direction in the sky is expected.

The dominant background arises from muons produced by atmospheric neutrinos. Using the atmospheric neutrino spectrum [29] ($\propto E^{-\beta}$) and the probability that a $\nu_\mu$ generates a detectable $\mu$ [29] ($\propto E^\alpha$) we estimate the number of background events detected in a detector of size $A$, from a solid angle $\Omega$ and during a time interval $\Delta t$ as:

$$N_{bg} \simeq 5 \times 10^{-17} A \cdot \Omega \cdot \Delta t \int_{E}^\infty d\tilde{E}_{\nu} \tilde{E}_{\nu}^{\alpha-\beta} \quad (4)$$

where $\tilde{E}_{\nu} \equiv E_{\nu}/100\text{TeV}$, $(\alpha = 1, \beta = 3.7)$ for $E_{\nu} < 100\text{TeV}$ and $(\alpha = 0.5, \beta = 4)$ for $E_{\nu} > 100\text{TeV}$.

---

4The detection probability may reach 0.1 in an extremely bright burst

5We safely neglect the neutrino mass that adds, at these energies, an negligible delay.
The currently constructed IceCube is designed to determine the direction of muons with sub-degree accuracy [31] corresponding to $\Omega \approx 10^{-3}$ square radians. Fig. 1 depicts $t_b(E, 0.01)$, the interval in which we expect $10^{-4}$ background events (corresponding to false alarm of 1%).

![Graph showing $t_b(E, 0.01)$ and LIV time delays](image)

Fig. 1.— $t_b(E, 0.01)$ (solid red line) the time interval for $10^{-4}$ background events (corresponding to false positive of 1%) and the LIV time delays, $\Delta t$, from $z = 1$ and $\xi_1 = 1$ (long dashed blue line) or $\xi_2 = 10^{-7}$ (short dashed purple line).

An observed neutrino can be associated with a burst (and interpreted as a positive detection of a time delay) if $t_b(E, p) > \Delta t$. As the detector is extremely quiet at these energies a neutrino arriving months or even years after the burst can be associated with the burst. Specifically, for $\xi \gtrsim 1$ ($\xi_2 \gtrsim 10^{-7}$) the background does not pose any problem. For $n=1$ one can explore using Grb $\nu$’s the parameters up and above to the Planck scale, a region that cannot be explored in any other way today [25].
Fig. 2.— The range of $\xi_1$ that can be explored for LIV delays using GRB $\nu$'s from $z = 1$ as a function of $E_{\nu}$. The colored regions indicate the range where $1000\text{sec} < \Delta t < t_b(E, 0.01)$. The additional condition $\Delta t < 1\text{year}$ is imposed in the upper (dark blue) range. The dashed bold line describes the lower limit that can be obtained from a simultaneous (within 30sec) detection of a high energy neutrino and the prompt GRB. Also marked are current limit obtained using various photon sources.

The range of LIV scale parameters, $\xi_n$, that can be detected using LIV time delays of GRB $\nu$'s is limited by additional considerations. The duration of long GRBs can be of order 1000sec and it is not clear whether the $\nu$'s will accompany the prompt emission or the early afterglow. We impose, therefore, a conservative minimal delay of 1000sec. Additionally we set an arbitrary practical bound on the maximal delay as a year. Figs. 2 and 3 depict the region of $\xi_{1,2}$ that can be determined by delayed detection of GRB $\nu$s. This range is many orders of magnitude above current limits. Detection of neutrinos coinciding with the prompt GRB emission will set new upper limits to the
Fig. 3.— The range of $\xi_2$ that can be explored for LIV delays using GRB $\nu$’s from $z = 1$ as a function of $E_\nu$. The colored regions indicate the range where $\text{1000 sec} < \Delta t < t_b(E, 0.01)$. The additional condition $\Delta t < \text{1 year}$ is imposed in the upper (dark blue) range. The dashed bold line describes the lower limit that can be obtained from a simultaneous (within 30 sec) detection of a high energy neutrino and the prompt GRB. Also marked are current limit obtained using various photon sources.

LIV scale (see Figs. 2 and 3). No detection at all will, of course, send us back to revise current GRB models.

Given the incomplete sky coverage of GRB detectors we expect a few associations between a GRB and a $\nu$ event per year. Clearly no matter what is the statistical significance a single detection won’t convince a skeptic observer. However, repeated detections over several years of $\nu$’s associated with GRBs with compatible time delays and observed energies might do so.
This research was supported by a US-Israel BSF grant.
REFERENCES

1. G. Amelino-Camelia, Nature, 408, 661 (2000).

2. G. Amelino-Camelia, J. Ellis, N. E. Mavromatos, et al., Nature, 393, 763 (1998).

3. J. Ellis, N. E. Mavromatos, D. V. Nanopoulos and A. S. Sakharov, Astron. Astrophys. 402, 409 (2003).

4. S. E. Boggs, C. B. Wunderer, K. Hurley and W. Coburn, ApJL 611, L77 (2004).

5. M. Rodríguez Martínez, T. Piran and Y. Oren, JCAP 0605, 017 (2006).

6. J. Ellis, N. E. Mavromatos, D. V. Nanopoulos, et al., APh, 25, 402 (2006,)

7. S.D. Biller et al., Phys. Rev. Lett. 83, 2108 (1999).

8. P. Kaaret, Astron. Astrophys. 345, L32 (1999).

9. T. Jacobson, S. Liberati and D. Mattingly, Nature, 424, 1019, (2003)

10. T. Jacobson, S. Liberati and D. Mattingly, Phys. Rev. Lett. 93, 021101 (2004)

11. T. Jacobson, S. Liberati, Mattingly and F. W. Stecker Phys. Rev. Lett. 93, 021101, (2004)

12. F. W. Stecker and S. T. Scully , APh, 23, 203, (2005)

13. E. Waxman and J. Bahcall, Phys. Rev. Lett. 78, 2292, (1997)

14. M. Vietri, ApJL 507, 40, (1998)

15. L. Gonzalez-Mestres, AIP Conf. Proc., 433, 148 (1998).

16. S. Coleman and S. L. Glashow, Phys. Rev. D 59, 116008 (1999).
17. R. Aloisio, P. Blasi, P. L. Ghia and A. F. Grillo, *Phys. Rev. D* **62**, 053010 (2000).

18. O. Bertolami and C. S. Carvalho, *Phys. Rev. D* **61**, 103002 (2000).

19. G. Amelino-Camelia and T. Piran, *Phys. Rev. D* **64**, 036005 (2001).

20. K. Greisen, 1966, *Phys. Rev. Lett.*, **16**, 748 (1966).

21. G. T. Zatsepin and V. A. Kuzmin, *JETP Lett.* **4**, 78 (1966).

22. T. Kifune, *ApJL* **518**, L21 (1999).

23. R. J. Protheroe and H. Meyer, *Phys. Lett. B* **493**, 1 (2000).

24. J. Ellis, N. E. Mavromatos, D. V. Nanopoulos and A. S. Sakharov, *Nature*, **428**, 2481 (2004)

25. M. Rodríguez Martínez and T. Piran, *JCAP* **0604**, 006 (2006).

26. S. Choubey and S.F. King, *Phys. Rev. D* **67**, 073005 (2003).

27. T. Piran, *Rev. Mod. Phys.* **76**, 1143 (2004).

28. E. Waxman, *Phys. Scr.* **T121**, 147 (2005).

29. D. Guetta et al., *Astropart. Phys.* **20**, 429 (2004).

30. J. Alvarez-Muñiz, F. Halzen and D. W. Hooper, *Phys. Rev. D* **62**, 093015 (2000).

31. F. Halzen, *EPJ C* **46**, 669 (2006).

This manuscript was prepared with the AAS LATEX macros v4.0.