HYPER and gravitational decoherence

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We study the decoherence process associated with the scattering of stochastic backgrounds of gravitational waves. We show that it has a negligible influence on HYPER-like atomic interferometers although it may dominate decoherence of macroscopic motions, such as the planetary motion of the Moon around the Earth.

I. INTRODUCTION

Decoherence is a general phenomenon which occurs for any physical system coupled to any kind of environment. It plays an important role in the transition between microscopic and macroscopic physics by washing out quantum coherences on a time scale which becomes extremely short for systems with a large degree of classicality or, in other words, by suppressing superpositions of different quantum states when the latter have sufficiently different classical properties.

For large macroscopic masses, say the Moon orbiting around the Earth, decoherence is in fact so efficient that the classical description of the motion is sufficient. Precisely, the decoherence time scale is so short that the observation of any quantum coherence is impossible. For microscopic masses in contrast, decoherence is expected to be so inefficient that we are left with the ordinary quantum description of the system. If we consider for example electrons orbiting inside atoms, the decoherence time scale is so long that decoherence can be forgotten.

Decoherence has been observed in a few experiments only and this can be understood from the simple arguments sketched in the previous paragraphs. Decoherence can only be seen by dealing with ‘mesoscopic’ systems for which the decoherence time is neither too long nor too short. The micro/macro transition has then to be assessed by following the variation of this decoherence time with some parameter measuring the degree of classicality of the system. These experimental challenges have been met with microwave photons stored in a high-Q cavity or trapped ions. In such model systems where the fluctuations are particularly well mastered, the quantum/classical transition has been shown to fit the predictions of decoherence theory.

It has been suggested that matter-wave interferometers could reveal the existence of intrinsic spacetime fluctuations through decoherence processes. The effect has not been seen in existing matter-wave interferometers, but more sensitive instruments are now being developed, like the atomic interferometer HYPER designed to measure the Lense-Thirring effect in a space-borne experiment and it is important to obtain quantitative estimates of the effect of decoherence associated with spacetime fluctuations for such instruments.

The perturbations of interest in the study of an atomic interferometer correspond to frequencies much smaller than Planck frequency. At such frequencies, general relativity is an accurate effective description of gravitation, although it is certainly not the final word. It follows that the intrinsic spacetime fluctuations which constitute our gravitational environment are essentially the free solutions of general relativity, that is also the gravitational waves predicted by the linearized form of the theory. This linearized form is widely used for studying propagation of gravitational waves and their interaction with the presently developed interferometric detectors.

In the present paper, we will study the decoherence of atomic interferometers due to their interaction with the stochastic background of gravitational waves emitted by astrophysical or cosmological processes. We will show that this scattering does not lead to an appreciable decoherence for the atomic interferometers presently studied, HYPER being chosen as the typical example. Incidentally, this ensures that HYPER will not have its interference fringes destroyed by decoherence and will therefore be able to measure Lense-Thirring effect. We will contrast this answer with recent results showing that the gravitational decoherence is the dominant decoherence mechanism, and an extremely efficient one, for macroscopic motions such as the motion of the Moon around the Earth.

This contrast is directly connected to a dimensional argument: since gravity is coupled to energy, the associated decoherence effects are certainly more efficient for...
macroscopic masses than for microscopic ones. In particular, the mass of the Moon is larger than Planck mass by orders of magnitude whereas the atomic probes used in HYPER have their mass much smaller than Planck mass. Of course, this scaling argument is not by itself sufficient to answer quantitative questions about the decoherence rates. We will give below precise estimations of the gravitational decoherence effect which depend not only on the mass of the atoms, but also on their velocity, on the geometry of the interferometer and on the noise spectrum characterizing the gravitational fluctuations in the relevant frequency range.

II. GRAVITATIONAL WAVES AND ATOMIC INTERFEROMETERS

Gravitational waves, as well as other gravitational perturbations such as the Lense-Thirring frame-dragging effect, dephase the matter waves in the two arms of the interferometer and thus affect the interference fringes. As a consequence of their stochastic character, the dephasings result in a loss of contrast of the interference fringes when averaged over the integration time of the measurement. We will use these ideas below to evaluate the effect of decoherence and write it in terms of properties of the interferometer and on the noise spectrum characterizing the gravitational fluctuations in the relevant frequency range.

We now use these reminders to describe in simple words the coupling of the interferometer to stochastic gravitational waves. These waves are consequences in our local environment of the motion of masses in the Galaxy or, more generally, in the Universe. In fact, they are the radiation fields, freely propagating far from their sources, originating from the gravitomagnetic fields present in the vicinity of the sources. Their effect on the gyrometer may be written analogously to the Sagnac dephasing $\Phi_{\text{Sagnac}}$

$$\Phi_{\text{Sagnac}} = \frac{2m_{\text{at}} A}{\hbar} \Omega_{\text{Sagnac}} \quad (1)$$

The area is given by the length $L$ of the rhomb side and the aperture angle $\alpha$ (see Fig. 1)

$$A = L^2 \sin \alpha = v_{\text{at}}^2 \tau_{\text{at}}^2 \sin \alpha \quad (2)$$

Alternatively, $L$ may be substituted by the product $v_{\text{at}} \tau_{\text{at}}$ where $v_{\text{at}}$ is the atomic velocity and $\tau_{\text{at}}$ the time of flight on one rhomb side.

The Sagnac effect measures the rotation frequency of the atomic interferometer with respect to the inertial frame as it is defined at its location. In general relativity, this local inertial frame differs from the celestial frame determined by the ‘fixed stars’ as a consequence of the dragging of inertial frames by the rotation of nearby bodies. This gravitomagnetic Lense-Thirring effect can be observed by comparing the local inertial measurement performed by the atoms to the indication of the star tracker pointing to a star. It can be described by a dephasing $\Phi_{\text{LT}}$ written analogously to the Sagnac dephasing $\Phi_{\text{Sagnac}}$

$$\Phi_{\text{LT}} = \frac{2m_{\text{at}} A}{\hbar} \Omega_{\text{LT}} \quad (3)$$

The rotation frequency $\Omega_{\text{LT}}$ measures the frame dragging induced, for the HYPER project, by the rotation of the Earth. It is given by general relativity and depends on the position of the satellite on its orbit but it does not vary with time at a fixed spatial location.

We now use these reminders to describe in simple words the coupling of the interferometer to stochastic gravitational waves. These waves are consequences in our local environment of the motion of masses in the Galaxy or, more generally, in the Universe. In fact, they are the radiation fields, freely propagating far from their sources, originating from the gravitomagnetic fields present in the vicinity of the sources. Their effect on the gyrometer may be written analogously to the Lense-Thirring dephasing

$$\delta \Phi_{\text{gr}} = \frac{2m_{\text{at}} A}{\hbar} \delta \Omega_{\text{gr}} \quad (4)$$

This analogy must not be pushed too far: the Lense-Thirring field is a quasistatic near field while the gravitational waves are radiated far fields. The dephasing $\delta \Phi_{\text{gr}}$ and frequency $\delta \Omega_{\text{gr}}$ are stochastic variables representing a time-dependent dragging of the local inertial frame.

To be more precise, we may write the dephasing $\delta \Phi_{\text{gr}}$ due to the effect of gravitational waves as they are registered by the atoms along the two arms of the interferometer. We disregard most perturbing effects which play a role in the real interferometer [18]. We treat only
the dominant contribution to the dephasing $\delta \Phi_{\text{gr}}$, that is the difference between the phases accumulated on the two arms 1 and 2 by slow atoms because of the geodesic perturbation.

$$\delta \Phi_{\text{gr}}(t) = \frac{m_{\text{at}}}{2\hbar} \left[ \int_1 h_{ij}(t') \dot{w}^i(t') \dot{w}^j(t') dt' - \int_2 h_{ij}(t') \dot{w}^i(t') \dot{w}^j(t') dt' \right] (5)$$

Here the metric components are evaluated in a specific transverse traceless gauge (see below). The dephasing can also be written in an explicitly gauge invariant manner by taking into account all the components of the interferometer, in particular the mirrors. For the purpose of the present paper, we do not need to enter into these subtle descriptions.

Using the symmetry of the rhomb, it is easy to rewrite this expression as in equation (4) with the equivalent rotation frequency $\delta \Omega_{\text{gr}}$ obtained from the derivative of the metric component $h_{12}$

$$\delta \Omega_{\text{gr}}(t) = -\frac{1}{2} \frac{d(t_{12})}{dt} (6)$$

The directions 1 and 2 correspond to the spatial plane defined by the interferometer. The averaged quantity $\overline{h_{12}}$ is obtained from the metric component $h_{12}$ through a convolution

$$\overline{h_{12}}(t) = \int h_{12}(t-\tau) g(\tau) d\tau (7)$$

The linear filtering function $g$ is represented on Fig. 2 with a triangular shape which reflects the distribution of the time of exposition of atoms to gravitational waves inside the rhombic shape of the interferometer. It differs from 0 for values of $\tau$ having a modulus smaller than $\tau_{\text{at}}$ and has an integral normalized to unity. Its Fourier transform describes the linear filtering in frequency space

$$\tilde{g}[\omega] = \left( \frac{\sin \frac{\omega \tau_{\text{at}}}{2}}{\omega \tau_{\text{at}}/2} \right)^2 (8)$$

The square of this function is the apparatus function discussed in [28].

### III. GRAVITATIONAL WAVE BACKGROUND

We now explain how we describe the fundamental fluctuations of space-time and their effect on the motion of matter. The basic idea is that the frequency range of interest lies far below Planck frequency, for all systems of current experimental interest. At these frequencies, general relativity is an accurate description of gravitational phenomena, and this statement is essentially independent of the modifications of the theory which will have to take place in a complete theory of quantum gravity [33].

![FIG. 2: Filtering function associated with the atomic gyrometer of Fig. 4; it describes the averaging of the rotation frequency associated with the finite time of flight of atoms.](image)

It follows that the intrinsic spacetime fluctuations which constitute our gravitational environment are simply the gravitational waves predicted by the linearized version of Einstein theory of gravity and which are thoroughly studied in relation with the ongoing experimental development of gravitational wave detectors. In this context, the stochastic backgrounds of gravitational waves emitted by astrophysical or cosmic processes are of particular interest. These backgrounds might be treated in a gauge-invariant manner by introducing the correlation functions describing the fluctuations of curvature. For the sake of simplicity, we will present here evaluations in a specific gauge. We choose the transverse traceless (TT) gauge which is tangent to the proper frame of the atomic interferometer at the time $t$ of measurement. In this gauge, often used in the studies of gravitational wave detectors, metric components vanish as soon as they involve a temporal index

$$h_{00} = h_{0i} = 0 (9)$$

$i,j=1,2,3$ stand for the spatial indices whereas 0 will represent the temporal index); the spatial components $h_{ij}$ of the metric tensor are directly connected to the Riemann curvature or, equivalently in free space, to the Weyl curvature

$$\frac{d^2 h_{12}}{dt^2} = -2 R_{1020} = -2 W_{1020} (10)$$

so that the trace of the spatial components vanish

$$h_{ii} = 0 (11)$$

Then the gravitational waves are conveniently described through a mode decomposition

$$h_{ij}(x) = \int \frac{d^4 k}{(2\pi)^4} h_{ij}[k] e^{-i \mathbf{k} \cdot \mathbf{x}} (12)$$

Each Fourier component is a sum over the two circular polarizations $h^+$ and $h^-$

$$h_{ij}[k] = \Sigma_\pm \left( \frac{\epsilon_1^\pm x_1^\pm}{\sqrt{2}} \right) h^\pm[k] (13)$$

The gravitational polarization tensors are obtained as products of the polarization vectors $\epsilon_1^\pm$ well-known from
The gravitational waves correspond to wavevectors $k$ lying on the light cone ($k^2 = 0$), they are transverse with respect to this wavevector component, say polarized and isotropic backgrounds. Then, a given metric gravitational wave frequency $f$ in the $10\,\text{Hz}$-$10\,\text{kHz}$ range responds to the space project LISA with its optimal sensitivity, the 3 curves on the right correspond to detectors (GEO, LIGO, VIRGO) and in space (LISA); the solid line describes the binary confusion background and the dotted lines represent potential cosmic backgrounds with different parameters.

We consider for simplicity the case of stationary, unpolarized and isotropic backgrounds. Then, a given metric component, say $\eta_{12}$, evaluated at the center of the interferometer as a function of time $t$, is a stochastic variable entirely characterized by a noise spectrum $S_h$

$$
(h(t) h(0)) = \int \frac{d\omega}{2\pi} S_h[\omega] e^{-i\omega t}
$$

$S_h$ is the spectral density of strain fluctuations considered in most papers on gravitational wave detectors (see for example [24]). It has the dimension of an inverse frequency. It can be written in terms of the mean number $n_{gr}$ of gravitons per mode

$$
S_h = \frac{16G}{5c^3} \hbar \omega n_{gr}
$$

or, equivalently, of a noise temperature $T_{gr}$

$$
S_h = \frac{16G}{5c^3} k_B T_{gr}
$$

with $k_B$ the Boltzmann constant and $G$ the Newton constant.

We have represented on Fig.3 a part of the information available from the studies devoted to interferometric detectors of gravitational waves (see for example [24]). The dashed lines represent the sensitivity curves for detectors: the 3 curves on the right correspond to detectors presently built on ground with their optimal sensitivity in the $10\,\text{Hz}$-$10\,\text{kHz}$ range; the curve in the central part corresponds to the space project LISA with its optimal sensitivity in the $10^{-4}$-$10^{-3}\,\text{Hz}$. The solid line on the left part represents the ‘binary confusion background’, that is the estimated level for the background of gravitational waves emitted by unresolved binary systems in the galaxy and its vicinity. This ‘binary confusion background’ corresponds to a nearly flat function $S_h$, that is also to a nearly thermal spectrum, in the $\mu\text{Hz}$ to $10\,\text{mHz}$ frequency range

$$
10^{-6}\,\text{Hz} < \frac{\omega}{2\pi} < 10^{-4}\,\text{Hz} \quad S_h \sim 10^{-34}\,\text{Hz}^{-1}
$$

With the conversion factors given above, this corresponds to an extremely large equivalent noise temperature

$$
T_{gr} \sim 10^{41}\,\text{K}
$$

It is worth stressing that $T_{gr}$ is an effective noise temperature, that is an equivalent manner for representing the noise spectrum $S_h$, but certainly not a real temperature. The value obtained here for this temperature is much higher than the thermodynamical temperature associated with any known phenomenon. It is even larger than Planck temperature ($\sim 10^{22}\,\text{K}$), which emphasizes its unconventional character from the thermodynamical point of view. In fact, the motion of matter is so weakly coupled to the gravitation that it remains always far from the thermodynamical equilibrium.

The estimations discussed here correspond to the confusion background of gravitational waves emitted by binary systems in our Galaxy or its vicinity. They may be treated as stochastic variables because of the large number of unresolved and independent sources. As a consequence of the central limit theorem, they may even be considered to obey a gaussian statistics, a property which will be used later on. Since they rely on the laws of physics and astrophysics as they are known in our local celestial environment, they may be considered as granted sources of gravitational waves. There also exist predictions for gravitational backgrounds associated with a variety of cosmic processes [27]. These predictions are represented by the dotted lines on Fig.3. They depend on the parameters used in the cosmic models and have a more speculative character than local astrophysical predictions. The associated temperatures vary rapidly with frequency but they are usually thought to be dominated by the confusion binary background in the frequency range considered thereafter.

### IV. Gravitational decoherence of atomic interferometers

We come now to the evaluation of decoherence of the interferometer due to the scattering of stochastic gravitational waves. Here again, we choose to present a simple description. Decoherence will be understood as a loss of fringe contrast resulting from the averaging of stochastic dephasings. Precisely, stochastic gravitational waves with frequencies higher than the inverse of the averaging time will be identified with the unobserved degrees of freedom which are usually traced over in decoherence.

![FIG. 3: Variation of the square root $\sqrt{S_h}$ of the spectral density of strain fluctuations (measured in Hz$^{-\frac{1}{2}}$) versus gravitational wave frequency $f = \frac{\omega}{2\pi}$ (measured in Hz) : the dashed lines represent the sensitivity curves of detectors on ground (GEO, LIGO, VIRGO) and in space (LISA); the solid line describes the binary confusion background and the dotted lines represent potential cosmic backgrounds with different parameters.](image)
theory (see [3] and references therein). The phase dispersion approach used in the present paper is known to be equivalent (see for example [26]) to the other approaches to decoherence and it is obviously well adapted to the description of interferometers where the phase is the natural variable.

The evaluation of decoherence is presented in a more detailed manner in [28]. Here we merely consider the degradation of fringe contrast obtained by averaging over stochastic dephasings. Since $\delta \Phi_{gr}$ is a gaussian stochastic variable, the degraded fringe contrast is read as

$$\langle \exp (i \delta \Phi_{gr}) \rangle = \exp \left( -\frac{\Delta \Phi_{gr}^2}{2} \right)$$

(19)

where $\Delta \Phi_{gr}^2$ is the variance of $\delta \Phi_{gr}$

$$\Delta \Phi_{gr}^2 = \langle \delta \Phi_{gr}^2 \rangle$$

(20)

Using the expression of $\delta \Phi_{gr}$ in terms of the averaged time derivative of $h_{12}$ we write the variance $\Delta \Phi_{gr}^2$ as an integral over the noise spectrum $S_h$

$$\Delta \Phi_{gr}^2 = 4\mu_{at}^2 \int \frac{d\omega}{2\pi} S_h [\omega] \left( 1 - \cos (\omega \tau_{at}) \right)^2$$

(21)

We have introduced a parameter $\mu_{at}$ which has the dimension of a frequency and is essentially determined by the kinetic energy of the atoms and the aperture angle of the interferometer

$$\mu_{at} = \frac{2m_{at} A}{\hbar \tau_{at}} = \frac{2m_{at} v_{at}^2 \sin \alpha}{\hbar}$$

(22)

Using this integral expression, we can calculate the variance $\Delta \Phi_{gr}^2$ for an arbitrary noise spectrum $S_h$. For the purpose of the present paper, we obtain interesting results by considering the special case where the spectrum $S_h$ is approximately flat. This corresponds to the assumption of a thermal spectrum which, as already discussed, is met by the binary confusion background on a significant frequency range. With this kind of white noise assumption, the variance is found to be proportional to the constant value of the noise spectrum $S_h$. It is read as

$$\Delta \Phi_{gr}^2 = \mu_{at}^2 S_h 2\tau_{at}$$

(23)

where $\tau_{at}$ is the time of exposition of atoms to gravitational waves and $\mu_{at}$ the typical frequency scale already discussed.

After the substitution in this estimation of the numbers corresponding to HYPER [17], we deduce that the decoherence of the interferometer due to the scattering of gravitational waves is completely negligible

$$\Delta \Phi_{gr}^2 \sim 10^{-20} \ll 1$$

(24)

This number corresponds only to the direct effect of gravitational waves on the matter waves involved in the atomic interferometer. We have also to take into account the dephasings which are picked up by the laser fields involved in the stimulated Raman processes used for building up beam splitters and mirrors for matter waves.

These fields register gravitational waves on their flights from the lasers to the atoms. The calculation of the corresponding dephasings is discussed in [28] and is not repeated here. The result of this calculation can be written under the same form as previously

$$\Delta \Phi_{gr}^2 [\text{phot}] \simeq \mu_{phot}^2 S_h 2\tau_{phot}$$

(25)

$\mu_{phot}$ is the laser frequency, that is also the frequency scale corresponding to the kinetic energy of one photon; $\tau_{phot}$ is of the order of the time of flight of photons from the lasers to the atoms (see a more precise discussion in [28]), that is also the time of exposition of photons to gravitational waves.

Using the numbers corresponding to HYPER [17], it turns out that the contribution to decoherence of optical dephasings largely dominates the contribution of atomic dephasings

$$\Delta \Phi_{gr}^2 [\text{phot}] \gg \Delta \Phi_{gr}^2 [\text{at}]$$

(26)

while nevertheless remaining negligible

$$\Delta \Phi_{gr}^2 \sim 10^{-12} \ll 1$$

(27)

As already evoked in the Introduction, this is good news for HYPER: intrinsic spacetime fluctuations do not have the ability to wash out the interference fringes. Should we have found a variance $\Delta \Phi_{gr}^2$ of the order of unity, or greater, it would have been difficult to vary this value while controlling the associated effects in HYPER.

The negligible effect of decoherence can be discussed in an interesting alternative manner. The phase noise level associated with gravitational waves may be measured as an equivalent displacement $\delta q$ of the mirrors which reflect the lasers. The noise spectrum corresponding to this equivalent noise is simply approximated as

$$S_q [\omega] \sim S_h L_{phot}^2 \sim 10^{-34} \left( \text{m/}\sqrt{\text{Hz}} \right)^2$$

(28)

We have again used the numbers of HYPER with the length $L_{phot}$ of the optical arms of the order of 1m. This corresponds to a noise level $\sqrt{S_q} \sim 10^{-17} \text{m/}\sqrt{\text{Hz}}$ which is far beyond the vibration noise level $\sqrt{S_q} \sim 10^{-12} \text{m/}\sqrt{\text{Hz}}$ which is the target of the HYPER instrument. This means that the phase noise induced by the scattering of gravitational waves is completely negligible with respect to the phase noise corresponding to mechanical vibrations of the mirrors. In the real instrument, decoherence is expected to be induced by the latter instrumental fluctuations rather than by the former fundamental fluctuations.
V. GRAVITATIONAL DECOHERENCE OF PLANETARY MOTIONS

This does not mean that the scattering of gravitational waves always has a negligible contribution to decoherence. To make this point clear, we now consider the case of macroscopic motions, say the planetary motion of the Moon around the Earth. This case is often chosen in introductory discussions on decoherence as the archetypical system for which decoherence is so efficient that quantum fluctuations can certainly not be observed. In these discussions, decoherence is often attributed to collisions of residual gas, to radiation pressure of solar radiation or, even, to the scattering of electromagnetic fluctuations in the cosmic microwave background. In fact, as discussed here, the decoherence of planetary motions is dominated by the scattering of stochastic gravitational waves present in our galactic environment.

The Earth-Moon system can be thought of as a giant gyroscope and its motion is thus sensitive to the stochastic dragging of the inertial frame already discussed in the previous sections. For the sake of simplicity, we consider only the case of a circular planetary orbit in the plane $x_1, x_2$. We introduce the reduced mass $m = \frac{m_a m_b}{m_a + m_b}$ and the total mass $M = m_a + m_b$, which are defined in terms of the masses $m_a$ and $m_b$ of the two bodies. The radius $\rho$, that is the constant distance between the two masses, and the orbital frequency $\Omega$ are related to the masses by the third Kepler law

$$\rho^3 \Omega^2 = GM$$  \hspace{1cm} (29)

We may also use as characteristic parameters the tangential velocity $v = \rho \Omega$ and the normal acceleration $a = \rho \frac{\Omega^2}{\rho^2}$.

For planetary systems, the effect of gravitational waves is conveniently described as a perturbation coupling the quadrupole momentum of the system to the Weyl curvature tensor $W_{00}$. Using this description, it can be shown that the effect of gravitational waves is in fact a Brownian force acting along the mean circular motion. As a consequence, the Moon undergoes a momentum diffusion with the variance $\Delta p^2$ of the transferred momentum varying linearly with the time of exposition $\tau$ to gravitational waves

$$\Delta p^2 = 2 D_{gr} \tau$$  \hspace{1cm} (30)

The momentum diffusion coefficient $D_{gr}$ is obtained as

$$D_{gr} = m \Gamma_{gr} k_B T_{gr}$$  \hspace{1cm} (31)

$T_{gr}$ is the effective noise temperature of the gravitational background, evaluated at twice the orbital frequency; $\Gamma_{gr}$ is the damping rate associated with the emission of gravitational waves by the planetary system

$$\Gamma_{gr} = \frac{32 G m a^2}{5 c^5}$$  \hspace{1cm} (32)

where $a$ is the normal acceleration on the circular orbit.

These formulas bring together the Einstein fluctuation-dissipation relation on Brownian motion and the Einstein quadrupole formula for gravitational wave emission. As it is well known, the gravitational damping is so small for the Moon that it has a negligible effect on its mean motion

$$\Gamma_{gr} \approx 10^{-34} \text{ s}^{-1}$$  \hspace{1cm} (33)

It is only for strongly bound binary systems that gravitational damping has a noticeable effect. For the Moon, it is not only small but much smaller than the damping due to the scattering of electromagnetic radiation pressure or to the interaction between Earth and Moon tides which is the dominant contribution to damping

$$\Gamma_{em} < \Gamma_{gr} < \Gamma_{tides}$$  \hspace{1cm} (34)

However, we show now that the gravitational mechanism dominates the decoherence process.

In order to evaluate decoherence, we consider two neighbouring motions on the circular orbit of the Moon around the Earth. More precisely, we consider two motions characterized by the same spatial geometry but slightly different values of the epoch - i.e. the time of passage at a given space point. For simplicity, we measure the difference by the spatial distance $\Delta x$ between the two motions which is constant on a circular orbit (see Fig.4).

As the gravitational wave perturbation depends on time, these two motions undergo different diffusion processes. This differential effect has been evaluated by and we reproduce here the result of this evaluation. Should we associate a quantum phase to a motion of the Moon, the two neighbouring motions would suffer a differential dephasing characterized by an exponential $e^{i \delta \Phi_{moon}}$. We can then average this quantity over the stochastic effect of gravitational waves, still supposed to obey gaussian statistics. We obtain in this manner a decoherence factor

$$\langle e^{i \delta \Phi_{moon}} \rangle = \exp \left( - \frac{\Delta \Phi_{moon}^2}{2} \right)$$  \hspace{1cm} (35)
which can be expressed in terms of the momentum diffusion coefficient, of the time of exposition and of the distance between the two motions

\[ \Delta \Phi_M^2 = \frac{2D_{\text{gr}} \Delta x^2 \tau}{\hbar^2} \]  

(36)

This is just the result expected from general discussions on decoherence with decoherence efficiency increasing exponentially fast with \( \tau \) and \( \Delta x^2 \).

For the sake of comparison with atomic interferometers, we rewrite this formula by using (16)

\[ \Delta \Phi_M^2 = \mu_M^2 S_h \frac{2 \tau}{x^2} \]  

\[ \mu_M = \frac{2m v^2}{h} \sin \alpha \]  

\[ \sin \alpha = \frac{\Delta x}{2 \rho} \]  

(37)

\( \mu_M \) is a frequency determined by the kinetic energy of the Moon and \( \sin \alpha \) is the aperture angle of the equivalent interferometer. With the numbers corresponding to the Moon, we find

\[ D_{\text{gr}} \approx 10^{75} \text{s}^{-1} \text{m}^{-2} \]  

(38)

This corresponds to an extremely short decoherence time, even for ultrasmall distances \( \Delta x \). To fix ideas, the time lies in the 10\( \mu \)s range for \( \Delta x \) of the order of the Planck length.

In fact, the gravitational contribution to decoherence is found to be much larger than the contributions associated with tide interactions and electromagnetic scattering

\[ D_{\text{gr}} \gg D_{\text{tides}} > D_{\text{em}} \]  

(39)

The reversal of roles is due to the huge effective temperature of the gravitational environment. To be more specific, the ratio \( \frac{\tau_{\text{gr}}}{\tau_{\text{tides}}} \) of the damping constants associated with gravitational waves and tides is a very small number of the order of \( 10^{-16} \). But, at the same time the ratio \( \frac{\tau_{\text{em}}}{\tau_{\text{tides}}} \) is an extremely large number of the order of \( 10^{38} \). It follows that the ratio \( \frac{\tau_{\text{em}}}{\tau_{\text{tides}}} \) is itself very large so that the gravitational contribution to decoherence is found to dominate the other contributions.

This entails that the ultimate fluctuations of the motion of the Moon, and the associated decoherence mechanisms, are determined by the classical gravitation theory which also explains its mean motion. In other words, the environment to be considered when dealing with macroscopic motions consists in the gravitational waves of the confusion binary background. This background is naturally defined in the reference frame of the galaxy if it is dominated by galactic contributions or in a reference frame built on a larger region of the universe if extragalactic contributions have to be taken into account.

### VI. GRAVITATIONAL DECOHERENCE AND PLANCK SCALES

The results obtained for gravitationally induced decoherence are reminiscent of the qualitative discussions of the Introduction. For macroscopic bodies, such as the Moon orbiting around the Earth, decoherence is extremely efficient with the consequence that potential quantum coherences between different positions can never be observed. For microscopic probes, such as the atoms or photons involved in HYPER, decoherence is so inefficient that it can be ignored with the consequence that ordinary quantum mechanics can be used.

We remark that the Planck mass, that is the mass scale which can be built up on the constants \( h, c \) and \( G \), lies on the borderland between microscopic and macroscopic masses

\[ m_p = \sqrt{\frac{hc}{G}} \sim 22 \mu g \]  

(40)

In other words, microscopic and macroscopic values of mass \( m \) may be delineated by comparing the associated Compton length \( \ell_C \) to the Planck length \( \ell_P \)

\[ m \ll m_p \iff \ell_P \ll \ell_C = \frac{h}{mc} \]  

(41)

It is then tempting to consider that this coincidence is not just accidental but that it might be a consequence of the existence of fundamental gravitational fluctuations. The idea was already present in the Feynman lectures on gravitation and it was developed and popularized by a number of authors, for example [41, 42, 43]. The results obtained for gravitationally induced decoherence allow one to test quantitatively this idea.

To this aim, we rewrite the phase variance which determines decoherence in all the systems studied in this paper as

\[ \langle e^{i\delta \Phi} \rangle = \exp \left( \frac{-\Delta \Phi^2}{2} \right) \frac{\Delta \Phi^2}{2} \sim \mu^2 S_h \tau \]  

(42)

Introducing the squared Planck time

\[ t_P^2 = \frac{\hbar G}{c^3} = \left( \frac{\hbar}{m_p c^2} \right)^2 \]  

(43)

we express the gravitational spectral density \( S_h \) as

\[ S_h \approx \Theta_{\text{gr}} t_P^2 \]  

(44)

where \( \Theta_{\text{gr}} \) is a frequency measuring the temperature of the background

\[ \Theta_{\text{gr}} \approx \frac{k_B T}{\hbar} \approx 10^{52} \text{s}^{-1} \]  

(45)

Note that terms of order of unity are disregarded in these scaling arguments.

Collecting these relations, we obtain

\[ \frac{\Delta \Phi^2}{2} \approx \left( \frac{mv^2 \sin \alpha}{m_p c^2} \right)^2 \Theta_{\text{gr}} \tau \]  

(46)
The ratio $\frac{m_2^2}{m_1^2}$ which appears in this expression suggests that the Planck mass effectively plays a role in the definition of the borderland between microscopic and macroscopic masses. However, the presence of the other terms in the formula implies that the scaling argument on masses is not sufficient for obtaining quantitative estimations. The phase variance also depends on the ratio of the probe velocity over the velocity of light, on the equivalent aperture angle $\alpha$ and on the frequency $\Theta$ which measures gravitational noise level at the frequency of interest for the motion under study. This last quantity has an enormous value, so that the transition between quantum and classical regimes could in principle be observed for masses smaller than Planck mass. Note that the parameter to be compared with Planck mass $m_\nu c^2$ is the kinetic energy $m_\nu c^2$ of the probe rather than its mass energy $mc^2$.

Formula (40) can be used to answer the question whether or not it is possible to find systems on which the quantum/classical transition induced by intrinsic gravitational fluctuations can be observed experimentally. In order to approach the transition region $\Delta \Phi^2 \sim 1$, one has to consider heavy and fast enough particles in a matter-wave interferometer. Interference patterns have already been observed on fullerene molecules [44]. The kinetic energy of these molecules, the area and aperture angle of the interferometer are such that the gravitational decoherence has a negligible effect in these experiments, as in HYPER. Increasing these numbers so that the transition could be approached appears to be a formidable experimental challenge with current technology. An alternative approach is to look at interferometers using quantum condensates (see for example [45, 46] for suggestions along these lines) but this requires new experimental developments (see for example [47, 48, 49, 50] as well as new theoretical ideas.

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