A reduced physical optics model for electromagnetic scattering problem

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Abstract. The ultimate simplified point scatterer model contains no object’s geometry informations, so a more complex prediction model containing geometrical informations is needed. The Physical Optics (PO) approximation has been widely used and considered as a good approximation of the far field electromagnetic scattering. Here, a forward model of bistatic scattering by PEC infinite elliptic cylinder based on the Physical Optics approximation is presented. The aim is to develop a simple scattering model for 2D targets illuminated by a monochromatic wave bistatic Radar. The reduced model is obtained by applying the stationary phase method to the PO integrals in both TE and TM wave cases. A parameter estimation procedure is also presented in order to examine the efficiency of the presented model.

1. Introduction
The objective of the presented study is to develop a simple scattering model for 2D PEC objects to extract geometrical informations (as shape and size) using Radar’s data, which is not available by some classical models as the point scatterer model [1].

The extraction of the approximated forward model is briefly presented in Section 2. The stationary phase method is applied to the PO integral equations which yields an approximated physical optics far-field direct scattering model. In Section 3, the validity of the presented model is confirmed by the comparison between the approximated model and results based on the Method of Moments (MoM) [2]. In Section 4, the parametric model is exploited in the parameter estimation methods, Adaptive Matched Filter (AMF) [3] and Minimum Mean Square Error (MMSE) [4] estimator, in order to extract the object’s geometrical characterizations. Section 5, presents an application of presented method which allows to characterize the geometrical parameters of the objects.

2. Forward model extraction
Considering the geometry shown in Fig.1, where the surface $S$ of scatterer is shift invariant along the $z$-axis, the surface integral of the EFIE and the MFIE, can be reduced to [2]

$$A = A^{inc} + \oint_C \left[ A \frac{\partial G}{\partial m} - G \frac{\partial A}{\partial m} \right] dl$$  \hspace{1cm} (1)

where $C$ is the cross-sectional contour of $S$ and $\hat{n}$ is the surface unit outward normal.
The parameter $A$ is replaced by $E(\rho, \phi)$ or $H(\rho, \phi)$ in cylindrical coordinates and the time
dependence of the field is assumed to be \( \exp(-j \omega t) \), where \( \omega \) is the angular frequency. The function \( G \) is the well known two dimensional frequency dependent Green’s function in free space, which satisfies the Helmholtz equation and has the form \[5\]

\[
G(r, r') = -\frac{j}{4} H_2^0(k \|r - r'\|), \quad r \neq r' \tag{2}
\]

where \( k \) is the free space wave number and \( H_2^0 \) is the Hankel function of the second kind of zero order.

The electric and magnetic fields of the incident plane wave are considered as parallel to the axis of the cylinder, at the observation cylindrical coordinates \( r' \) they are expressed as follows

\[
P_z^i(L_T, r') = -\frac{j}{4} H_2^0(k \|L_T - r'\|) \tag{3}
\]

where \( P_z^i = E_z \) for TM wave, \( P_z^i = H_z \) for TE wave and \( L_T \) is the vector representing the position of the transmitter. For significant values of \( \{k \|L_T - r\|, k \|r - r'\|\} \gg 0 \), Hankel function in (2) and (3) can be replaced by its large argument asymptotic form [6]

\[
H_0^2(x) \sim \sqrt{\frac{2}{\pi x}} \exp(-jx + \frac{\pi}{4}) \quad \text{as} \quad |x| \to \infty \tag{4}
\]

The Kirchhoff approximation [7], also known as the Physical-Optics, assumes that the surface \( S \) is replaced at each point by its tangent plane. This approximation is valid for smooth objects with radius of curvature \( \kappa \) larger than the investigating wavelength \( \lambda \).

Assuming \( \{\|L_R\|, \|L_T\|\} \gg \|r'\| \) (\( L_R \) represents the position of the receptor and \( r' \) is on the geometrically illuminated side of the scatterer denoted by \( C_+ \)), i.e far-field measurement and \( \lambda \ll \kappa \), high-frequency measurement, yields

\[
E_{TM}^i(L_R) = E_0 \rho_0 e^{j \phi_0} \int_{C_+} \hat{n} \hat{L}_T e^{jk(L_T+\hat{L}_R)r'} \Gamma(r')dr' \tag{5}
\]

\[
H_{TE}^i(L_R) = H_0 \rho_0 e^{j \phi_0} \int_{C_+} \hat{n} \hat{L}_R e^{jk(L_T+\hat{L}_R)r'} \Gamma(r')dr' \tag{6}
\]

where \( \int_{C_+} \) denotes an integral over the illuminated portion of the target with infinitesimal part of the curve \( \Gamma(r')dr' \) and \( \rho_0 e^{j \phi_0} = \frac{8\pi \sqrt{\|L_T\|\|L_R\|}}{\sqrt{\pi}} \).
The stationary phase method [8] states that these oscillatory integrals have an evaluation containing the significant non-zero contributions that occur in regions of the integration range where rapidly-varying phase function vanishes, i.e. at points of stationary phase. It states that an integral of the form:

\[ I = \int_{-\infty}^{+\infty} \rho(r)e^{jk\Phi(r)} \, dr \]  \hspace{1cm} (7)

has an approximation containing contributions from the critical points inside the boundary of integration

\[ I \sim \left(\frac{2\pi}{k}\right)^{\frac{1}{2}} \sum_{i=1}^{n} e^{\text{sign}(\Phi''(r_i))\frac{\pi}{2}} \rho(r_i) e^{j\Phi(r_i)} \]  \hspace{1cm} (8)

where \( r_i \) is a point of stationary phase which satisfy the condition \( \Phi'(r) = 0 \).

The scatterer is an infinite PEC elliptic cylinder of radius \( a \) and \( b \), and we suppose \( E_0 = H_0 = 1 \). Then using the stationary phase evaluation of scattered field for both TM and TE waves, yields

\[ A_{TM,TE} = \rho_0 \rho_p e^{j(\phi_0 + \phi_p - \pi/4)} \]  \hspace{1cm} (9)

where \( \rho_p \) and \( \phi_p \) are function of \( (\theta_T, \theta_R, a, b) \),

\[ \rho_p = \frac{ab(1 + \cos \theta_T \cos \theta_R + \sin \theta_T \sin \theta_R)}{(a^2 \cos \theta_T + \cos \theta_R)^2 + b^2(\sin \theta_T + \sin \theta_R)^2)^{3/4}} \]  \hspace{1cm} (10)

and

\[ \phi_p = k \left( a^2 \cos \theta_T + \cos \theta_R ight)^2 + b^2(\sin \theta_T + \sin \theta_R)^2)^{1/2} \]  \hspace{1cm} (11)

\( \theta_R \) and \( \theta_T \) are the reception and transmission azimuth.

In the general case, where the semi-axis of cylinder have the angle \( \gamma \) with \( x, y \) axis, (9) can be used by the transformation \( \theta_T \rightarrow (\theta_T - \gamma) \) and \( \theta_R \rightarrow (\theta_R - \gamma) \).

3. Forward Model validity

The numerical solution provided by MoM for TM polarization, is used as reference to verify the model, where 800 of segments on the object’s surface ensures the segment length \( \Delta l \leq \lambda/9 \) of the object’s circumference.

Fig. 2 shows the comparison between the MoM and stationary phase model for an elliptic cylinder of dimensions \( a = 5 \lambda, \ b = 20 \lambda \), and simulation conditions given in Table 1. In this case, the validity range is limited (due to PO approximation) to \( 112^\circ \), i.e. \( |\theta_R - \theta_T| < 112^\circ \), for \( \text{Phase-error}_{\text{max}} = \pi/4 \) and \( \text{Amplitude-error\text{-}}_{\text{max}} = 10\% \). It shows that the model is a good approximation of scattered field outside of the forward-scattering region, \( |\theta_R - \theta_T| \rightarrow 180^\circ \). Fig. 2 indicates the validity of the present approximation for the scattering by an elliptic cylinder in the mentioned hypothetical contexts of measurement. The phase prediction is more accurate than the amplitude due to stationary phase approximation. In spite of the reduced range of validity, the model behavior remains well approximated.
4. Parameter estimation methods

By adaptive matching, the measured signal is correlated with the parameterized filter, here the target position is considered as known:

\[ I_{AMF}(a_i, b_j) = \| \sum_{\theta_R} S(\theta_R) A^*(a_i, b_j, \theta_R) \|, \quad [\hat{a}, \hat{b}] = \{a, b | I_{AMF}(a_i, b_j) \text{ is max}\} \quad (12) \]

where, \( S \) is the measured signal and \( A^* \) is the complex conjugate of \( A \) (eq. 9)

The product \( SA^* \), is the effect of parameterized filter on the signal and is to be maximized in order to maximize the likelihood ratio. The approach consists of numerically searching the maximum value of two dimensional function \( I_{AMF} \) in the space of \((a, b)\), where \( a \) and \( b \) are positive variables.

The Minimum mean square error estimator searches the parameters minimizing the Mean Square Error (MSE) between the measured signal and the response predicted by the model

\[ I_{MMSE}(a_i, b_j) = \sum_{\theta_R} \| S(\theta_R) - A(a_i, b_j, \theta_R) \|^2, \quad [\hat{a}, \hat{b}] = \{a, b | I_{MMSE}(a_i, b_j) \text{ is min}\} \quad (13) \]

Values of MSE are used to comparative purpose and the model with lower MSE is considered as the best predictor model.

5. Application

The numerical results are obtained for the TM synthetic signal \( S(\theta_R) \), generated by Method of Moments then corrupted by a circular complex additive white Gaussian noise (SNR = 5 dB). Here, the electromagnetic interactions between the cylinders are neglected. The simulation conditions and scene configuration are listed in the Tables 1 and 2.

Table 3 represent the estimated radius of the simulation scene for MMSE and AMF methods. A comparison between the estimated parameters, indicate no remarkable difference between AMF and MMSE in this application and there is no performance degradation as a function of SNR, until the false alarms appears about SNR = 0 dB.

6. Conclusion

We have presented an approximated analytical scattering model of a PEC infinite elliptical cylinder. The model is valid under far-field and high-frequency measurement and it is used to generate signal model in the AMF and MMSE estimation methods.
Table 1. Simulation conditions

| Frequency (GHz) | $f_0$ | 14.3 |
|-----------------|-------|------|
| Transmitter position | $L_T$ | $500\,\lambda$ |
| $\theta_T$ | 0° |
| Receptor positions | $L_R$ | $500\,\lambda$ |
| $\theta_R$ | 0°.. 360° |
| $\Delta \theta_R$ | 1° |

Table 2. Objects and scene configuration

| Object | Dimension a | Dimension b | Position (x, y) |
|--------|-------------|-------------|-----------------|
| A      | 20 $\lambda$ | 10 $\lambda$ | (10 $\lambda$, 15 $\lambda$) |
| B      | 10 $\lambda$ | 10 $\lambda$ | (10 $\lambda$, -10 $\lambda$) |
| C      | 5 $\lambda$ | 20 $\lambda$ | (-20 $\lambda$, 7.5 $\lambda$) |

Table 3. MMSE/AMF objects radius estimation; SNR = 5 dB

| Objects | Radius $\hat{a}$ | Radius $\hat{b}$ |
|---------|------------------|------------------|
| MMSE    | 19.93 $\lambda$ | 10.28 $\lambda$ |
| AMF     | 19.59 $\lambda$ | 10.04 $\lambda$ |

In spite of reduced range of model’s validity near the forward scattering region, the parameter estimation procedure works fine and with this approach the geometrical parameters of elliptical cylinders are estimated.

As a future development of this work, the electromagnetic interaction between targets will be taken into account and we will investigate the possibility of using this model in the reconstruction methods. Although the objects with circular and elliptical cross sections are common in the nature, as trees, other geometrical forms, like triangular, are not studied here and the influence of diffraction by corners could also be taken in account in future work.

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