Method of finding an optimal solution for interval balanced and unbalanced assignment problem

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Abstract. All Assignment problems are a famous subject as well as is second-hand extremely frequently in solving problems of industrial and administration science. Transportation and assignment models playing significant position in logistics and deliver sequence administration for dropping fee and time, for improved facility. The assignment problem is one of the major problems while assigning job to the employee. It is a significant problem in mathematics and is also discuss in actual physical world. In this paper, we planned an original move towards for solving a balanced and unbalanced assignment problem. The grades of original approach are compared through existing approaches. Such parameters are a bunch of devices and these vague parameters are expressed by intervals. Through this involvement we suggest Hungarian method of interval and consider the thinking of interval analysis to solve linear assignment problems. The question of assignment without converting them to traditional assignment issue. In this paper a numerical scheme is investigated in detail.

1. Introduction

The problem of assignment is used internationally to solve real society problems. It acting a significant position in the manufacturing and is used extremely frequently in solving problems of manufacturing, administration science and it has a lot of previous applications. Development organization is planned to organize the resources of an entity on a given position of tasks, in time, fee and excellence. The limited possessions have to be used efficiently so that the best available possessions can be committed for the most part required activities in order to maximize and reduce the income and costs respectively. However, considering the decisions made in the real world anywhere the objectives, constrains or parameters are not accurate. Therefore, a choice is frequently complete on the foundation of unclear information. We classify the assignment matrix, and then we obtain a reduced matrix using determinant representation that has at least one in each row and column. Subsequently through the innovative process, we get the best answer for the problem of assigning intervals by assigning them to every row and column, and after that try to find an absolute assignment to their ones. In a ground-level reality the rate medium entries are not always crusty. In many applications these parameters are uncertain and intervals reflect these vague parameters.

The assignment problem was solved by Lin and Wen [1] using a labeling algorithm with the cost of the Fuzzy interval number. Multi-Objective Assignment (MOAP) problem studied by Bao et al. [2] in a crisp environment. Lin and Wen [3] have suggested a labeling algorithm to solve the problem of assignment with the cost of fuzzy intervals. Kuhn [4] implemented a specially developed algorithm known as Hungarian method for solving AP in a crisp setting. To be sure, cost matrix entries aren't
consistently crisp. Anil D. Gotmare and P.G. Khot [5] used branch and bound techniques to solve fuzzy assignment problem, anywhere the object be a reduce the costs. Marius Posta et al. [6] studied a simple and very effective algorithm to solve the generalized assignment problem. Ventepaka Yadaiah [7] suggested a new approach to finding an optimal solution to an unbalanced problem of assignment where a language search algorithm was used to assign jobs to machines. Sarangam Majumdar [8] solved problems with linear interval assignments using a new approach called the Hungarian interval approach. Ramesh and Ganesan [9] suggested a new computational technique for solving AP at a generalized interval using the Hungarian method. The method of linear assignment of a matrix interval was studied by Ramesh Kumar and Deepa [10] and contrasted with the existing method. Amutha et al. [11] developed a process used for solving the problem of assignment interval extension. In common for the most part of the alive technique afford simply the crisp solution for interval assignment problems. In this paper we suggest a straightforward process for the optimal interval solution of problems with interval assignment without converting to classical linear programming problems.

The remainder of the paper is structured as follows: This is illustrated in the after that segment on interval arithmetic’s. Information of the proposed Hungarian interval approach are described in section 3. In section 4 example problems are solved and results are analyzed. At last conclusions are drawn in section 5.

2. Preliminaries

The plan of this part is to present a few opinions, ideas and comments which are helpful for our more reflection.

2.1. Interval Numbers

Let \( \mathbb{IR} = \{a = [a_1, a_2]: a_1 \leq a_2 \text{ and } a_1, a_2 \in \mathbb{R}\} \) be the set of all proper intervals and \( \overline{\mathbb{IR}} = \{a = [a_1, a_2]: a_1 > a_2 \text{ and } a_1, a_2 \in \mathbb{R}\} \) be all unsuitable intervals set for real line \( \mathbb{R} \). If \( a = a_1 = a_2 = a \), then \( a = [a, a] = a \) is a real number (or a degenerate interval). We shall interchangeably use the terms interval and the interval number. The midpoint and width of an interval number (or half-width) are defined as

\[
m(a) = \frac{a_1 + a_2}{2} \quad \text{and} \quad w(a) = \frac{a_2 - a_1}{2}.
\]

The midpoint and width of an interval number (or half distance) \( \tilde{a} = [a_1, a_2] \) are defined as

\[
m(\tilde{a}) = \frac{a_1 + a_2}{2} \quad \text{and} \quad w(\tilde{a}) = \frac{a_2 - a_1}{2}.
\]

It can also express the interval number in conditions of its average and width as \( \tilde{a} = [a_1, a_2] = \{m(\tilde{a}), w(\tilde{a})\} \).

2.2. Ranking of Interval Numbers

Sengupta and Pal [12] planned a clear and well-organized directory used for comparing a few two intervals on \( \mathbb{IR} \) from side to side decision-makers’ satisfaction. Let \( \preceq \) be an extended order relation between the interval numbers \( \tilde{a} = [a_1, a_2], \tilde{b} = [b_1, b_2] \) in \( \mathbb{IR} \), then for \( m(\tilde{a}) < m(\tilde{b}) \), we construct a premise \( \tilde{a} \preceq \tilde{b} \) which implies that \( \tilde{a} \) is inferior to \( \tilde{b} \) (or \( \tilde{b} \) is superior to \( \tilde{a} \)).

An acceptability function \( A_{\preceq}: \mathbb{IR} \times \mathbb{IR} \to [0, \infty) \) is defined as:

\[
A_{\preceq}(\tilde{a}, \tilde{b}) = A(\tilde{a} \preceq \tilde{b}) = \frac{m(\tilde{b}) - m(\tilde{a})}{w(\tilde{b}) + w(\tilde{a})}, \quad \text{where} \quad w(\tilde{b}) + w(\tilde{a}) \neq 0.
\]
A ≤ can be viewed as acceptability grade of “the first interval number to be inferior to the second interval number”. For every two intervals \( \bar{a} \) and \( \bar{b} \) in \( \mathbb{IR} \) either \( A(\bar{a} \preceq \bar{b}) \geq 0 \) (or) \( A(\bar{b} \succeq \bar{a}) \preceq 0 \) (or) \( A(\bar{a} \preceq \bar{b}) = 0 \) (or) \( A(\bar{b} \succeq \bar{a}) = 0 \) (or) \( A(\bar{a} \preceq \bar{b}) + A(\bar{b} \preceq \bar{a}) = 0 \).

2.3. Arithmetic Operations

Ming Ma et al. [13] suggested a new fuzzy arithmetic based on the position index and the function of the Fuzziness index. The position index number is taken in the ordinary arithmetic, while in the lattice \( \mathcal{L} \) the Fuzziness index functions are supposed to obey the lattice norm, which are the least upper bound and the highest lower bound. That is for \( a, b \in \mathcal{L} \), we define \( a \lor b = \max\{a, b\} \) and \( a \land b = \min\{a, b\} \).

Used for several two intervals \( \bar{a} = [a_1, a_2], \bar{b} = [b_1, b_2] \in \mathbb{IR} \) and for \( * \in \{+, -, \cdot, \div\} \), the arithmetic operations on \( \bar{a} \) and \( \bar{b} \) are defined as:

\[
\bar{a} * \bar{b} = [m(\bar{a}), w(\bar{a})] * [m(\bar{b}), w(\bar{b})] = \left\{ m(\bar{a}) \cdot m(\bar{b}), \max\{w(\bar{a}), w(\bar{b})\} \right\}.
\]

Particularly with interval arithmetic operations such as Addition, Subtraction, Multiplication and Division, we use the following problems here.

3. Main Results

3.1. General Interval Assignment Problems

Let there be \( n \) jobs and \( n \) individuals by means of different skills are available. If \( i \)th person's cost of doing \( j \)th work is \( c_{ij} \). Now the question is which job is to be delegated to whom, in order to minimize the cost of completing the work. We can express the problem in mathematical terms as follows:

4. Algorithms for Hungarian Method

4.1. Example :

Find the following question about linear classification. Allocate the four workers to the three machines so that all costs are reduced.

| Sources | B1 | B2 | B3 | B4 |
|---------|----|----|----|----|
| A1      | 10 | 5  | 10 | 15 |
| A2      | 3  | 9  | 15 | 3  |
| A3      | 10 | 7  | 3  | 2  |
| A4      | 2  | 3  | 2  | 4  |

Solution: Using the above algorithm we got Optimum solution as follows, Then we can delegate the complete assignment and the Zeros and the Optimum solution is \( (A1 \rightarrow B2); (A2 \rightarrow B1); (A3 \rightarrow B4); (A4 \rightarrow B3) \); and minimum cost is 12.

5. Algorithm For Matrix Ones Assignment Problem

Step1. In a minimization (maximization) case, find the minimum (maximum) element of each row in the assignment matrix (say \( a_i \)) and write it on the right hand side of the matrix. Then divide each
element of \( i^{th} \) row of the matrix by \( a_i \). These operations create at least one ones in each rows. In term of ones for each row and column do assignment, otherwise go to step 2.

Step 2. Find the minimum (maximum) element of each column in assignment matrix (say \( b_j \)), and write it below \( j^{th} \) column. Then divide each element of \( j^{th} \) column of the matrix by \( b_j \). These operations create at least one ones in each columns. Make assignment in terms of ones. If no feasible assignment can be achieved from step (1) and (2) then go to step 3.

Step 3. Draw the minimum number of lines to cover all the ones of the matrix. If the number of drawn lines less than \( n \), then the complete assignment is not possible, while if the number of lines is exactly equal to \( n \), then the complete assignment is obtained.

Step 4. If a complete assignment program is not possible in step 3, then select the smallest (largest) element (say \( d_{ij} \)) out of those which do not lie on any of the lines in the above matrix. Then divide by \( d_{ij} \) each element of the uncovered rows or columns, which \( d_{ij} \) lies on it. This operation create some new ones to this row or column. If still a complete optimal assignment is not achieved in this new matrix, then use step 4 and 3 iteratively. By repeating the same procedure the optimal assignment will be obtained.

5.1. Example:
Find the following question about linear classification. Allocate the four workers to the three machines so that all costs are reduced.

### Table 2. Interval assignment table.

| Sources | B1 | B2 | B3 | B4 |
|---------|----|----|----|----|
| A1      | 10 | 5  | 10 | 15 |
| A2      | 3  | 9  | 15 | 3  |
| A3      | 10 | 7  | 3  | 2  |
| A4      | 2  | 3  | 2  | 4  |

**Solution:** Using the above algorithm we got Optimum solution as follows, so maximum assignment is possible and we can assign the Zeros and Optimum solution is \( (A1 \rightarrow B2); (A2 \rightarrow B1); (A3 \rightarrow B4); (A4 \rightarrow B3) \); and minimum cost is 12.

6. Algorithm For Interval Problem By Hungarian Method
An algorithm to resolve task problem using Hungarian approach with comprehensive arithmetic interval:
Step 1: Find out the mid values of each interval in the cost matrix.
Step 2: Subtract the interval which have smallest mid value in each row from all the entries of its row.
Step 3: Subtract the interval which have smallest mid value from those columns which have no intervals contain zero from all the entries of its column.
Step 4: Draw lines through appropriate rows and columns so that all the intervals contain zero of the cost matrix are covered and the minimum number of such lines is used.
Step 5: Test for optimality (i) If the minimum number of covering lines is equal to the order of the cost matrix, then optimality is reached. (ii) If the minimum number of covering lines is less than the order of the matrix, then go to step 6.
Step 6: Determine the smallest mid value of the intervals which are not covered by any lines. Subtract this entry from all un-crossed element elements and add it to the crossing having an interval contain zero. Then go to step 4.

6.1. Example:
Find the following question about linear classification. Allocate the four workers to the three machines so that all costs are high.
Table 3. Matrix Of Costs

| Sources | B1  | B2  | B3  | B4  |
|---------|-----|-----|-----|-----|
| A1      | 10  | 5   | 10  | 15  |
| A2      | 3   | 9   | 15  | 3   |
| A3      | 10  | 7   | 3   | 2   |
| A4      | 2   | 3   | 2   | 4   |

Switch the cost matrix entry now by some sort of interval. Instead we obtain a new matrix of costs as follows.

Table 4. Interval Assignment

| Sources | B1                | B2                | B3                | B4                |
|---------|-------------------|-------------------|-------------------|-------------------|
| A1      | [9,11]            | [4,6]             | [9,11]            | [14,16]           |
| A2      | [2,4]             | [8,10]            | [14,16]           | [2,4]             |
| A3      | [9,11]            | [6,8]             | [2,4]             | [1,3]             |
| A4      | [1,3]             | [2,4]             | [1,3]             | [3,5]             |

At the present we shall give details the similar interval assignment problem given in above table by applying the process proposed in this paper.

Let us express all the interval parameters $\bar{a} = [a_1, a_2]$ in terms of midpoint and width as $\bar{a} = [a_1, a_2] = \langle m(\bar{a}), w(\bar{a}) \rangle$. Now the given interval assignment problem becomes.

Table 5. Midpoint And Width

| Sources | B1                | B2                | B3                | B4                |
|---------|-------------------|-------------------|-------------------|-------------------|
| A1      | < 10, 1>          | < 5, 1>           | < 10, 1>          | < 15, 1>          |
| A2      | < 3, 1>           | < 9, 1>           | < 15, 1>          | < 3, 1>           |
| A3      | < 10, 1>          | < 7, 1>           | < 3, 1>           | < 2, 1>           |
| A4      | < 2, 1>           | < 3, 1>           | < 2, 1>           | < 4, 1>           |

The best arrangement of tasks is:

(A1 $\rightarrow$ B2); (A2 $\rightarrow$ B1); (A3 $\rightarrow$ B4); (A4 $\rightarrow$ B3)

The optimal cost of assignment

= $< 5, 1> + < 3, 1> + < 2, 1 > + < 2, 1 >$

= $< 12, 1 >$

= $[11, 13]$.

Table 6. Balanced optimum solution for three methods is same solution

| Problem       | H-method | MOA-method | IBH-method | Optimum   |
|---------------|----------|------------|------------|-----------|
| Existing method | 12       | 12         | 12         | [8,16]    |
| Proposed method | 12       | 12         | 12         | [11,13]   |

7. Assignment Problem Inbalanced

The above-mentioned assignment method allows for equal column and row numbers in the work matrix. Nevertheless, if the known cost matrix be not a rectangle matrix, then an unbalanced problem is called the problem with the assignment. To make it a square matrix (with zeros as cost elements) a dummy line(s) or column(s) is inserted into the matrix for these situations. Used for illustration, at what time the known cost matrix is of order a dummy column with zero cost dimension will be inserted in that column. If the unbalanced assignment issue is changed into a balanced assignment
problem then be able to adopt the customary algorithm to explain the assignment problem following creation the known cost matrix a rectangle matrix.

7.1. Example:
Consider the following problem regarding linear selection.

Table 7. Interval unbalanced assignment table.

| Sources | B1  | B2  | B3  | B4  |
|---------|-----|-----|-----|-----|
| A1      | 10  | 5   | 10  | 15  |
| A2      | 3   | 9   | 18  | 3   |
| A3      | 10  | 7   | 3   | 2   |

Solution: The problem of unequal assignment is turned into a fair assignment problem

Table 8. Interval unbalanced assignment table.

| Sources | B1  | B2  | B3  | B4  |
|---------|-----|-----|-----|-----|
| A1      | 10  | 5   | 10  | 15  |
| A2      | 3   | 9   | 18  | 3   |
| A3      | 0   | 0   | 0   | 0   |

And the whole assignment is workable and we can assign the zeros. (A1 → B2); (A2 → B1); (A3 → B4); (A4 → B3) and minimum cost 11.

8. Unbalanced Matrix Ones Assignment Problem
Unfair assignment problem is a pure assignment problem. Nevertheless, if the cost matrix given is not a square matrix the assignment problem is called an unbalanced question.

8.1. Example:
Consider the following problem regarding linear classification.

Table 9. Interval unbalanced assignment table.

| Sources | B1  | B2  | B3  | B4  |
|---------|-----|-----|-----|-----|
| A1      | 10  | 5   | 10  | 15  |
| A2      | 3   | 9   | 18  | 3   |
| A3      | 10  | 7   | 3   | 2   |

Solution: The problem of unbalanced assignment is converted into a problem of balanced assignment

Table 10. Interval unbalanced assignment table.

| Sources | B1  | B2  | B3  | B4  |
|---------|-----|-----|-----|-----|
| A1      | 10  | 5   | 10  | 15  |
| A2      | 3   | 9   | 18  | 3   |
| A3      | 1   | 1   | 1   | 1   |

So maximum assignment is possible and we can assign the Zeros and Optimum solution is (A1 → B2); (A2 → B1); (A3 → B4); (A4 → B3) and minimum cost 11.
9. Algorithm For Unbalanced Interval Hungarian Method Problem
The solution above causes the number of columns and rows in the assignment matrix to be equal for the problem of one's assignment interval. Nevertheless, if the given cost matrix is not a square matrix, then the one problem with assigning intervals is called the unbalanced query. In these cases, a dummy row(s) or column(s) is inserted into the matrix (with intervals such as cost elements) to transform it into a square matrix, e.g. if the specified cost matrix is in command, a dummy column with the one-element interval will be added to that column. If the unbalanced one-interval allocation problem is translated into a balanced one-interval allocation problem, then following creation the known cost matrix a square matrix, we can follow the normal algorithm to solve the issue of allocating intervals.

9.1. Example
Find the following problem with respect to linear selection. Allocate the four jobs in the direction of the three machinery so as to reduce the overall cost.

| Sources | B1   | B2   | B3   | B4   |
|---------|------|------|------|------|
| A1      | 10   | 5    | 13   | 15   |
| A2      | 3    | 9    | 18   | 3    |
| A3      | 10   | 7    | 3    | 2    |

**Solution:** Switch the cost matrix entry now by some sort of interval. Then we get a new cost matrix as follows and the problem of unbalanced assignment is transformed into problem of balanced assignment.

| Sources | B1     | B2     | B3     | B4     |
|---------|--------|--------|--------|--------|
| A1      | [9,11] | [4,6]  | [12,14]| [14,16]|
| A2      | [2,4]  | [8,10] | [17,19]| [2,4]  |
| A3      | [9,11] | [6,8]  | [2,4]  | [1,3]  |
| A4      | [1,1]  | [1,1]  | [1,1]  | [1,1]  |

Satisfied with the Hungarian condition, we apply the planned Hungarian method of interval and resolve this problem. At the optimum cycle of assignments we get an ideal assignment

(A1 → B2); (A2 → B1); (A3 → B4); (A4 → B3)

Optimum cost of assignment

= \langle 5, 1 \rangle + \langle 3, 1 \rangle + \langle 2, 1 \rangle + \langle 1,0 \rangle \\
= \langle 11, 1 \rangle \\
= [10, 12]

**Result 13:** Unbalanced problem for three methods is same solution.

| Problem      | H-method | MOA-method | IBH-method | Optimum   |
|--------------|----------|------------|------------|-----------|
| Existing method | 11       | 11         | 11         | [8,14]    |
| Proposed method | 11       | 11         | 11         | [10,12]   |

10. Conclusions
A new approach to solving the assignment problems with generalized arithmetic intervals is suggested in this paper. To illustrate the efficacy of the proposed approach numerical examples are resolved and corresponding findings are compared. Note, our method provides an optimal solution better than the other solution.
11. References

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