String Scattering from D-branes in Type 0 Theories

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ABSTRACT

We derive fully covariant expressions for all two-point scattering amplitudes involving closed string tachyon and massless strings from Dirichlet brane in type 0 theories. The amplitude for two massless D-brane fluctuations to produce closed string tachyon is also evaluated. We then examine in detail these string scattering amplitudes in order to extract world-volume couplings of the tachyon with itself and with massless fields on a D-brane. We find that the tachyon appears as an overall coupling function in the Born-Infeld action and conjecture form of the function.

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1 Introduction

Recent years have seen dramatic progress in the understanding of non-perturbative aspects of string theory\cite{1}. With these studies has come the realization that extended objects, other than just strings, play an essential role. An important tool in these investigations has been Dirichlet brane\cite{2}. D-branes are non-perturbative states on which open string can live, and to which various closed string states can couple.

In perturbative type II superstring theories, scattering amplitudes describing the interaction of two massless closed string states with unexcited D-branes, and one massless closed string with two open strings were studied extensively in \cite{3,4} and \cite{5,6}, respectively. Perturbative spectrum of bosonic type 0 theories have the same massless states as the bosonic part of type II, but with the doubled set of Ramond-Ramond (RR) states\cite{7}. Hence the elementary D-branes of the theory carry simultaneously two RR charges\cite{8,9}. One can use the same techniques as in type II theories to evaluate various string scattering with the D-branes in type 0 theories. Most of these amplitudes are already presented in above citations. At the massless level, scattering amplitude describing interaction of D-brane with two different RR states is the only amplitude that should be evaluated. The bosonic type 0 theories have closed string tachyon($\tau$) in their spectrum as well\cite{7}. Therefore, scattering amplitudes involving this tachyon cannot be address from previous work on type II theories either. The present paper provide calculations on these amplitudes.

The paper is organized as follows. In the following section we describe the calculations of the scattering of two closed string tachyon, one tachyon and one closed string massless state, and two different massless RR states from D-brane using conformal field theory techniques. The open string poles of these amplitudes are consistent with the GSO projection of the open strings attached to the the D-branes\cite{10,11}. In section 3 we evaluate the scattering amplitude of one closed string tachyon and two open string states. Then, in section 4 we examine in detail these string amplitudes in order to find the world-volume coupling of tachyon to itself and to the massless fields on the D-branes. We find that the tachyon does not have world-volume coupling to RR fields. However, the couplings of tachyon to other massless fields and to itself are consistent with an extension of the Born-Infeld action. In particular, the tachyon appears as an overall coupling function in the Born-Infeld action. We provide more evidence in support of this action by studying some simple string amplitudes using the conformal field theory with background fields. We conclude with a brief discussion of our results in section 5.

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1Sectors of type 0 theories are

\begin{align*}
\text{type } 0A : & \quad (NS_-, NS_-) \oplus (NS_+, NS_+) \oplus (R_+, R_-) \oplus (R_-, R_+)
\text{type } 0B : & \quad (NS_-, NS_-) \oplus (NS_+, NS_+) \oplus (R_+, R_+) \oplus (R_-, R_-)
\end{align*}
Before continuing with our calculations, let us make a comment on conventions. In the scattering amplitudes below, we will set $\ell_s^2 = \alpha' = 2$. Our index conventions are that lowercase Greek indices label vector in the entire ten-dimensional spacetime, e.g., $\mu, \nu = 0, 1, \ldots, 9$; early Latin indices label vector in the world-volume, e.g., $a, b, c = 0, 1, \ldots, p$; middle Latin indices label vector in the transverse space, e.g., $i, j = p+1, \ldots, 8, 9$; early capital Latin indices label spinor in the ten-dimensional spacetime, e.g., $A, B, C = 1, 2, \ldots, 32$; and middle capital Latin indices label vector in $U(N)$ gauge space, e.g., $I, J = 1, 2, \ldots, N^2$.

2 Two-point amplitudes

Scattering amplitudes describing interaction of strings with a fixed D-brane at tree level are calculated as the string vertex operator insertions on a disk (world-sheet) with Dirichlet and Neumann boundary conditions. In conformal field theory frame, effect of these boundary conditions appears in the world-sheet propagators. Alternatively, one may use a doubling trick to convert the propagator to the standard form of open string propagator and shift the effect to the momentum and polarization of vertex operators\[4\]. Hence, in order to find scattering amplitude of strings with D-brane, one may replace these modified momenta and polarizations into the appropriate known scattering amplitudes of open strings\[4, 5\]. In \[4, 5\], using this idea, the authors were able to find various massless superstring scattering with D-brane in type II theory by relating them to the known amplitudes of type I theory. However, type I theory does not have open string tachyon, so the scattering amplitudes describing interaction of the closed string tachyon with D-branes in type 0 are not related to amplitudes in type I theory.

In all the string scattering amplitudes appearing in this paper, we use the doubling trick and then explicitly evaluate correlation functions and take the integrals appearing in the amplitudes.

2.1 Tachyon and Tachyon amplitude

We begin by calculating the amplitude describing the scattering of two closed string tachyon from a D-brane. It is given by the following correlation function:

$$A^{\tau,\tau} \sim \int d^2z_1 d^2z_2 < V^\tau(p_1, z_1, \bar{z}_1) V^\tau(p_2, z_2, \bar{z}_2) >$$

where the tachyon vertex operators are

$$V^\tau(p_1, z_1, \bar{z}_1) = : V_0(p_1, z_1) :: V_0(p_1 \cdot D, \bar{z}_1) :$$

$$V^\tau(p_2, z_2, \bar{z}_2) = : V_{-1}(p_2, z_2) :: V_{-1}(p_2 \cdot D, \bar{z}_2) :$$
where \( p_i^2 = 1 \) and \( D_{\mu\nu} \) is a diagonal matrix with values +1 on the world-volume and -1 in the transverse space \([4]\). In the vertex operators, subscripts 0 and -1 refer to their superghost charges which must be added to -2 for the disk world-sheet. Here we have also used the doubling trick to write the closed string vertex operator in terms of only holomorphic components. The holomorphic components \( V_0 \) and \( V_{-1} \) are given by

\[
V_0(p, z) = i p \cdot \psi(z) e^{i p \cdot X(z)} \\
V_{-1}(p, z) = e^{-\phi(z)} e^{i p \cdot X(z)}
\]

Using the standard world-sheet propagators of type I theory, one can evaluate the correlations in (1) and show that the integrand is SL(2,R) invariant. We refer the reader to ref. \([4]\) for the details of fixing this symmetry and performing the integrals, and simply state the final result

\[
A(\tau, \tau) = -i \frac{\kappa^2 T_p}{2} \frac{\Gamma(-t/2) \Gamma(-2s)}{\Gamma(-1 - t/2 - 2s)}
\]

where \( t = -(p_1 + p_2)^2 = -2 - 2p_1 \cdot p_2 \) is the momentum transfer to the D-brane and \( s = -(p_1 a p_1^a) = -(p_2 a p_2^a) \) is the momentum flowing parallel to the world-volume of the brane. We also normalized (3) and subsequent amplitudes in this section by \(-i \kappa^2 T_p/2\), the same factor as for the massless closed string scattering amplitudes\([4]\). From the gamma function factors in (3), we see that the amplitude contain two infinite series poles corresponding to closed and open string states in \( t \)-channel and \( s \)-channel, respectively, with

\[
m^2_{\text{closed}} = 4n/\alpha', \quad m^2_{\text{open}} = m/\alpha'
\]

where \( n, m = 0, 1, 2, \ldots \). The closed string states in the \( t \)-channel belong only to the \((\text{NS}_+, \text{NS}_+)\) sector of the theory. Here one concludes that either the coupling of the \((\text{NS}_-, \text{NS}_-)\) states to the D-branes or to the two tachyons in the bulk is zero. From the amplitudes in sections 2.2 and 2.4, one finds that the \((\text{NS}_-, \text{NS}_-)\) stat es do couple with the D-branes. Hence one concludes that the coupling of two tachyons and the \((\text{NS}_-, \text{NS}_-)\) states is zero, e.g., coupling of three tachyons is zero in the bulk space \([4]\). The open strings in the \( s \)-channel are in the \( \text{NS}_+ \) sector. This is consistent with the GSO projection for the open string states on the D-brane \([4]\). However this amplitude does not rule out the \( \text{NS}_- \) sector, because the coupling of closed string tachyon and \( \text{NS}_- \) states is zero, e.g., because of the world-sheet fermions in the vertex operator of open string tachyon, the world-volume coupling of open and closed tachyon is zero.

## 2.2 Tachyon and NS-NS amplitudes

Next, we evaluate the amplitudes describing the scattering of one tachyon and one massless NS-NS(graviton, dilaton or Kalb-Ramond(antisymmetric tensor)) states from the D-
branes. The amplitude is given by
\[ A^{,NSNS} \sim \int d^2\bar{z}_1 d^2z_2 < \mathcal{V}^{,NSNS}(\varepsilon_2, p_2, z_2, \bar{z}_2) > \]
where the tachyon vertex operator is given in (2) and the NS-NS vertex operator is
\[ V^{NSNS}(\varepsilon_2, p_2, z_2, \bar{z}_2) = (\varepsilon_2 \cdot D)_{\mu\nu} : V^{\mu}_{\nu}(p_2, z_2) \cdot V^{\nu}_{\mu}(p_2, \bar{z}_2) : \]
where \( p_2^2 = 0 \). Here \( \varepsilon_2^{\mu\nu} \) is the polarization of NS-NS states which is traceless and symmetric (antisymmetric) for graviton(Kalb-Ramond) and
\[ \varepsilon_2^{\mu\nu} = \frac{1}{\sqrt{8}} (\eta^{\mu\nu} - \ell^\mu p_2^\nu - \ell^\nu p_2^\mu) ; \quad \ell \cdot p_2 = 1 \]
for dilaton. Again, we refer the reader to refs. [3, 4, 6] for the details of the calculations. The final result in this case is
\[ A = -\frac{i n T_p^2}{2} \left( \text{Tr}(\varepsilon_2 \cdot D) \frac{\Gamma(-t/2 + 1/2) \Gamma(-2s)}{\Gamma(1/2 - t/2 - 2s)} \right. \]
\[ \left. + p_1 \cdot \varepsilon_2 p_1 \frac{\Gamma(-1/2 - t/2) \Gamma(-2s)}{\Gamma(1/2 - t/2 - 2s)} - p_1 \cdot D \cdot \varepsilon_2 D \cdot p_1 \frac{\Gamma(-t/2 + 1/2) \Gamma(-2s)}{\Gamma(1/2 - t/2 - 2s)} \right) \]
where \( t = -(p_1 + p_2)^2 = -1 - 2p_1 \cdot p_2 \). As a check of our calculations, we inserted the dilaton polarization (4) into (5) and found that, as expected, the auxiliary vector \( \ell^\mu \) disappears from the amplitude.

The \( t \)- and \( s \)-channel poles of the amplitude indicates that the mass of the closed and open string in these channels are
\[ m^2_{closed} = \frac{(4n - 2)}{\alpha'} , \quad m^2_{open} = \frac{m}{\alpha'} . \]
The closed strings in \( t \)-channel belong to the \((NS_-, NS_-)\) sector. This indicates that closed string state in this sector couple to the D-branes, e.g., the closed string tachyon couple to the branes [6]. The poles of \( t \)-channel also indicate that the coupling of one tachyon, one massless NS-NS state and any of the \((NS_+, NS_+)\) states is zero in the bulk, e.g., the coupling of two massless NS-NS and one tachyon is zero in the bulk space [6]. Again the open strings in the \( s \)-channel is consistent with the GSO projection.

2.3 Tachyon and RR amplitude

Type 0 theories have two set of RR states, each made of direct product of two open string state in R sector. At the massless level, they are
\[ \text{type 0A} : \quad (8_- \otimes 8_+) \oplus (8_+ \otimes 8_-) = (8_1 + 56_3) + (8_1 + 56_3) \]
\[ \text{type 0B} : \quad (8_- \otimes 8_-) \oplus (8_+ \otimes 8_+) = (1_0 + 28_2 + 35^{(+)}_4) + (1_0 + 28_2 + 35^{(-)}_4) \]
where symbols indicate the irreducible representations and the subscripts 0, 1, 2, 3 and 4 indicate a scalar, vector, antisymmetric 2-, 3- and 4-tensor, respectively. Here, the subscript $(\pm)\, (\pm)$ means that for this tensor the field strength satisfies a self-(anti-self)-duality condition. The vertex operator corresponding to each of these states is also tensor product of two open string vertex operators with an appropriate polarization tensor. Scattering amplitude of one of these states, the tachyon and D-brane is given by

$$A^{\tau, RR} \sim \int d^2 z_1 d^2 z_2 < V^\tau (p_1, z_1, \bar{z}_1) V^{RR}(\varepsilon_2, p_2, z_2, \bar{z}_2) > \quad (6)$$

where the RR and tachyon vertex operators are

$$V^{RR}(\varepsilon_2, p_2, z_2, \bar{z}_2) = (P_\mp \Gamma_{2(n)} M_\mu)^{AB} : V_{-1/2A}(p_2, \bar{z}_2) :: V_{-1/2B}(p_2 \cdot D, \bar{z}_2) : \quad (7)$$

$$V^\tau (p_1, z_1) = : V_0(p_1, z_1) :: V_{-1}(p_1 \cdot D, \bar{z}_1) : .$$

The two different chiral projection operators, $P_\pm$, refers to the two different set of RR states $(c$ and $\bar{c})$. Here we have chosen the holomorphic and anti-holomorphic parts of the tachyon vertex operator in two different picture in order for saturating the superghost charge of the world-sheet. We refer the reader to ref. [4] for our conventions. In evaluating the correlators in (6), one needs the correlation of two right-handed or left-handed spin operators and one world-sheet fermion that is given by (see, e.g., [11])

$$<: S_A(z_1) :: S_B(z_2) :: \psi^\mu :> = 2^{-1/2} (\gamma^\mu)_{AB} \epsilon_{12}^{-3/4} (z_1 z_2) ^{-1/2} .$$

Other correlators in (6) can easily be evaluated. The final result is

$$A(\tau, c) = \frac{\kappa^2 T_p}{2\sqrt{2}} \text{Tr} (P_\mp \Gamma_{2(n)} M_\mu \gamma \cdot p_1) \frac{\Gamma(-t/2) \Gamma(-2s)}{\Gamma(-t/2 - 2s)}$$

and an identical amplitude for $A(\tau, \bar{c})$. Poles of this amplitude are at

$$m_{\text{closed}}^2 = 4n / \alpha', \quad m_{\text{open}}^2 = m / \alpha'$$

The $t$-channel poles indicate that there is no coupling between one tachyon, one massless RR and one of the $(NS_-, NS_-)$ states, e.g., there is no coupling between two tachyons and one massless $RR$ field in the bulk space. Moreover, the massless pole in this channel is consistent with having the coupling $\tau c \bar{c}$ in the bulk[9]. The massless pole in $s$-channel means that the tachyon couple to the open string scalar fields on the world-volume of the D-brane.

### 2.4 RR and RR amplitude

The amplitudes describing the scattering of two $c$’s or two $\bar{c}$’s with D-brane is exactly the same as the corresponding amplitudes in type II theories[4]. Whereas, the scattering amplitudes for $c, \bar{c}$ and D-brane is different and given by

$$A^{c, \bar{c}} \sim \int d^2 z_1 d^2 z_2 < V^c(\varepsilon_1, p_1, z_1, \bar{z}_1) V^{\bar{c}}(\varepsilon_2, p_2, z_2, \bar{z}_2) > \quad (8)$$
where the vertex operators are given in \((1)\). In this case, in order to calculate the amplitude \((3)\), one needs the correlation function between two right-handed and two left-handed spin operators which is given by (see, \(e.g.,\) \([1]\))

\[
< S_A(z_1)S_B(z_2)S_C(z_3)S_D(z_4) > = \frac{1}{2}\left< (\gamma_\mu)_{AB}(\gamma^\mu)_{CD}(z_{13}z_{14}z_{23}z_{24})^{-1/4}(z_{12}z_{34})^{-3/4}
+ C_{AC}C_{BD}(z_{12}z_{34})^{1/4}(z_{14}z_{23})^{-1/4}(z_{13}z_{24})^{-5/4}
- C_{AD}C_{BC}(z_{12}z_{34})^{1/4}(z_{13}z_{24})^{-1/4}(z_{14}z_{23})^{-5/4}
\right>
\]

where \(C\) is the charge conjugation matrix (see, \(e.g.,\) \([1]\)). Using this correlator and performing the others, one arrives at the following final result:

\[
A(c, \bar{c}) = -\frac{ie^2 t_p}{2}\left(\frac{1}{2}\left< \mathrm{Tr}(P_+\Gamma_{1(n)}M_p\gamma^\mu)\right>\mathrm{Tr}(P_+\Gamma_{2(m)}M_p\gamma^\mu)\left< \frac{\Gamma(-t/2 + 1/2)\Gamma(-2s)}{\Gamma(1/2 - t/2 - 2s)}\right>\right.
- \left< \mathrm{Tr}(P_-\Gamma_{1(n)}C^{-1}\Gamma_{2(m)}C)\right>\frac{\Gamma(-t/2 - 1/2)\Gamma(-2s + 1)}{\Gamma(1/2 - t/2 - 2s)}
- \left< \mathrm{Tr}(P_-\Gamma_{1(n)}M_p\Gamma_{2(m)}M_p)\right>\frac{\Gamma(-t/2 + 1/2)\Gamma(-2s + 1)}{\Gamma(3/2 - t/2 - 2s)}\right)\]

where \(t = -(p_1 + p_2)^2 = -2p_1\cdot p_2\). This amplitude has the pole structure

\[
m_{\text{closed}}^2 = (4n - 2)/\alpha' , \quad m_{\text{open}}^2 = m/\alpha'
\]

The closed string channel indicates that there is a coupling between \(c, \bar{c}\) and \((NS_-, NS_-)\) states, \(e.g.,\) the \(\tau\bar{c}\bar{c}\) coupling in the bulk is non-zero. It also tells us that the coupling of \(c, \bar{c}\) and \((NS_+, NS_+)\) is zero, \(e.g.,\) there is no coupling between \(c, \bar{c}\) and the massless NS-NS states in the bulk.

The amplitude \((4)\) is consistent with the GSO projection of the open strings, \(i.e.,\) it has no pole corresponding to on-shell propagation of \(NS_-\) strings. Alternatively, the amplitude \((4)\) indicates that there is no coupling between massless RR and \(NS_-\) strings on the D-brane world-volume. For example, consider the coupling of the open string tachyon (a state belonging to the \(NS_-\) sector) to the RR state on the world-volume of the D-brane. The coupling is given by

\[
A^{t, RR} \sim \int dx d^2z < V'(2k, x)V^{RR}(\varepsilon, p, z, \bar{z}) >
\]

where the open string tachyon vertex operator \(V'(2k, x) = V_{-1}(2k, x)\) and the RR vertex operator is given in \((7)\). Performing the correlators in above amplitude, one finds

\[
A(t, c) \sim E_{a_0...a_p}(e^\nu)^{a_0...a_p}
\]

and similar result for \(A(t, \bar{c})\) which are zero, because there is no \(p\)-form RR potential in the type 0 theory.
3 Closed-Open-Open amplitudes

Scattering amplitudes with two massless open strings and one closed string can be read from the corresponding amplitudes in type II theories\cite{5}. The amplitudes describing interaction of one closed string tachyon and two massless open strings is given by

$$A_{NS,NS,\tau} \sim \int dx_1 dx_2 d^2 z \text{Tr} < V^{NS}(\zeta_1, k_1, x_1) V^{NS}(\zeta_2, k_2, x_2) V^{\tau}(p_3, z_3) >$$

where the open string vertex operators are

$$V^{NS}(\zeta_i, k_i, x_i) = \zeta_i^\mu : V_{\mu}^{-1}(2, k_i, x_i) : T_i$$

for i = 1, 2 and the tachyon vertex operator is given by (2). Here we have defined $\zeta_i T^I \equiv \zeta_i T_i$ where T’s are the $N \times N$ generators of the $U(N)$ gauge symmetry of the coincident D-branes. Using appropriate world-sheet propagators from \cite{4}, one can evaluate the correlations above and show that the integrand in $SL(2,\mathbb{R})$ invariant. Gauging this symmetry by fixing $z = i$ and $x_2 = \infty$, one arrives at

$$A \sim 2^{-2s-2} \int dx_1 \left( (2s + 1)\zeta_1 \cdot \zeta_2 - \frac{i\zeta_1 \cdot p_3 \zeta_2 \cdot x_1 - i}{x_1} - \frac{i\zeta_1 \cdot p_3 \zeta_2 \cdot x_1 + i}{x_1} \right) (x_1^2 + 1)^s \text{Tr}[T_1 T_2]$$

where the integral is taken from $-\infty$ to $+\infty$, and $s = -p_3 a p_3^a = -(k_1 + k_2)^2 = -2k_1 \cdot k_2$. This integral is doable and the result is

$$A = -\frac{i\kappa}{2} \left( \frac{s}{2} \zeta_1 \cdot \zeta_2 + p_3 \cdot \zeta_1 p_3 \cdot \zeta_2 + 1 \leftrightarrow 2 \right) \frac{\Gamma(-2s)}{\Gamma(-s + 1) \Gamma(-s)} \text{Tr}[T_1 T_2]$$

We have also normalized the amplitude at this point by $-i\kappa/2\pi$. Here the D-brane coupling constant $T_p$ cancels with the normalization of open string states \cite{12}. This amplitude has the same pole structure as the corresponding amplitude for massless closed string states\cite{6}, i.e., $m_{open}^2 = (2n + 1)/\alpha'$. 

4 World-Volume interactions

To extract appropriate world-volume coupling of open and closed string states from string theory amplitudes, one should expand the massive poles of string amplitudes and then write a field theory for massless fields and tachyon, if any, to reproduce those results. In general, the field theory should contain infinite terms to reproduce effect of the massive poles of string amplitudes. However, at low energy, one should not consider the terms which have large number of momentum in the expansion. Using this idea, in \cite{12} we extracted world-volume interaction of different massless open and closed string states from string
amplitudes. We then showed that they are consistent with Born-Infeld and Chern-Simons action provided the closed string states treated as functionals of the nonabelian scalar fields describing transverse fluctuations of the Dp-brane. In the present case, the world-volume interaction of massless fields remain unchanged. The only difference is that now we have two set of RR fields.

Now we do the same calculation to extract world-volume interaction of tachyon and massless fields. We start with the simple case of two open and one closed string scattering amplitude. This amplitude (10) has no massless pole, so at low energy it gives only contact terms, that is,

\[ A(a,a,\tau) = -\frac{i\kappa}{4} \left( f_{1ab} f_{2}^{ab} \text{Tr}[T_1 T_2] + O(k^4) \right) \tag{11} \]

\[ A(\lambda,\lambda,\tau) = \frac{i\kappa}{2} \left( (k_{1a} \zeta_1 k_{2}^{a} \zeta_2^i + p_3 i \zeta_1^i p_3 j \zeta_2^j) \text{Tr}[T_1 T_2] + O(k^4) \right) \tag{12} \]

and \( A(a,\lambda,\tau) \) is zero. Here we have suggestively introduced \( f_{iab} = i(k_{ia} \zeta_{ib} - k_{ib} \zeta_{ia}) \). These terms are reproduced in field theory by

\[ L = -\frac{\kappa}{2} \text{Tr} \left( T_0 \tau(X) + \frac{1}{2}(\partial \lambda)^2 \tau(X) + \frac{1}{4}(f)^2 \tau(X) + \cdots \right) \tag{13} \]

where \( X^i = \lambda^i / \sqrt{T_p} \). Taylor expansion of the first term above reproduces the second term of (12), the second and last term of (13) reproduces the first term of equations (12) and (11), respectively. The first term in (13) is consistent with the result basing on the cylinder amplitude [9].

Now to find non-derivative world-volume interaction of the tachyon to itself and to other massless closed string fields, we expand their corresponding string amplitudes and ignore the terms which have two and more momenta, that is,

\[ A(\tau,\tau) = -i\kappa^2 T_p \left( 1 + 2s \frac{(p_1 p_2) \cdot \varepsilon}{4s} + \frac{3}{4} + O(p^2) \right) \tag{14} \]

\[ A(\tau,h) = \frac{i\kappa^2 T_p}{2} \left( \frac{2p_1 \cdot \varepsilon}{t+1} \cdot \varepsilon \cdot p_1 + \frac{p_1 i \varepsilon_{a}^{i} p_{2a} + 2p_1 \varepsilon_{a}^{i} p_{a} - \varepsilon_{2a}^{a} + O(p^2)}{s} \varepsilon \right) \]

\[ A(\tau,\phi) = \frac{i\kappa^2 T_p}{4\sqrt{2}} \left( \frac{2}{t+1} + \frac{p_1 i \varepsilon_{a}^{i} (p-3)}{s} - (p-3) + O(p^2) \right) \]

and \( A(\tau,b) \) is zero. Similar expansion can be done for \( A(\tau,c) \) and \( A(c,\bar{c}) \). These tachyonic and massless poles should be reproduced in field theory by suitable world-volume and bulk actions in Einstein frame metric. The appropriate actions in the world-volume are the Born-Infeld, the Chern-Simons and eq. (13), and in the bulk is\(^{3}\)[9]

\[ S = \int d^{10}x \sqrt{-g} \left[ \frac{1}{2\kappa^2} R - \frac{1}{2}(\nabla \phi)^2 - \frac{1}{2}(\nabla \tau)^2 + \frac{1}{2} \tau^2 e^{\phi} - \frac{16}{5!} F_{5}^{(+)} F_{5}^{(-)}(2\kappa \tau) \right] \]

\(^{3}\)Note that the fields in this paper have the normalization of the vertex operators [12].
\[
- \frac{8}{n!} (F(n) \cdot F(n) + \tilde{F}(n) \cdot \tilde{F}(n) + 4\kappa F(n) \cdot \tilde{F}(n) \tau e^{\frac{\kappa}{2} (5-n) \phi})
\]

where the tachyon coupling function is

\[
f(2\kappa \tau) = 1 + 2\kappa \tau + \frac{1}{2} (2\kappa \tau)^2 + \cdots .
\]

We use these actions to reproduce the tachyon and massless poles of the string amplitudes \((14)\), and then compare them with the string theory results to find the new contact terms of the world-volume field theory. We will present the details of the calculations for scattering with two tachyons. We begin by calculating the massless \(t\)-channel amplitude

\[
A'_t(\tau, \tau) = i \tilde{S}_\phi \tilde{G}_\phi \tilde{V}_{\phi \tau_1 \tau_2} + i (\tilde{S}_h)^{\mu \nu} (\tilde{G}_h)_{\mu \nu}^{\alpha \beta} (\tilde{V}_{h \tau_1 \tau_2})_{\alpha \beta}
\]

where we used the fact that in the bulk both dilaton and graviton couple to two tachyons. Their corresponding vertex functions are derived from \((13)\)

\[
(\tilde{V}_{h \tau_1 \tau_2})^{\mu \nu} = -2i\kappa p_1^{\mu} p_2^{\nu} + i\kappa \eta^{\mu \nu} (p_1 \cdot p_2 + 1)
\]

\[
\tilde{V}_{\phi \tau_1 \tau_2} = \frac{i\kappa}{\sqrt{2}}.
\]

the dilaton and graviton sources and their corresponding propagators can also be derived from \((13)\)(they appear in \([4]\)). Now substituting these into \((17)\), one finds

\[
A'_t(\tau, \tau) = -i\kappa^2 T_p \left( \frac{1 + 2s}{t} \right)
\]

The first term above reproduces the \(t\)-channel massless pole of string amplitude \((14)\). This insures that the normalization of string amplitude \((3)\) is consistent with normalization of bulk action \((13)\). Now, the \(s\)-channel amplitude corresponding to propagation of massless open string on the D-brane world-volume can be evaluated as:

\[
A'_s(\tau, \tau) = \tilde{V}^i_{\tau_1 \lambda} \tilde{G}^{ij}_{\lambda} \tilde{V}^j_{\lambda \tau_2}
\]

where \(\tilde{G}^{ij}_{\lambda} = iN^{ij} / s\) is the scalar propagator, and the vertex function is derived from the Taylor expansion of the first term in eq. \((13)\)

\[
\tilde{V}^j_{\lambda \tau_k} = \frac{\kappa}{2} \sqrt{T_p P^j_k}.
\]

for \(k = 1, 2\) and \(P^j_k\) is momentum of external tachyon. The \(s\)-channel amplitude then becomes

\[
A'_s(\tau, \tau) = i\kappa^2 T_p \frac{p_1 \cdot p_2}{4s}.
\]
This amplitude reproduces the \( s \)-channel pole of (14). Hence, normalization of (13) is consistent with (3) and subsequently with (15). Now subtracting these field theory amplitudes (19) and (20) from (14), one finds:

\[
A(\tau, \tau) - A'_{t}(\tau, \tau) - A'_{s}(\tau, \tau) = -i\kappa^2 T_p \left( \frac{3}{4} + O(p^2) \right)
\]

\[
A(\tau, h) - A'_{t+1}(\tau, h) - A'_{s}(\tau, h) = -i\kappa^2 T_p \left( \frac{p - 3}{4\sqrt{2}} + O(p^2) \right)
\]

\[
A(\tau, \phi) - A'_{t+1}(\tau, \phi) - A'_{s}(\tau, \phi) = -i\kappa^2 T_p \left( \frac{p - 3}{4\sqrt{2}} + O(p^2) \right)
\]

\[
A(\tau, c) - A'_{t}(\tau, c) - A'_{s}(\tau, c) = \kappa^2 T_p O(p^2)
\]

\[
A(c, \bar{c}) - A'_{t+1}(c, \bar{c}) - A'_{s}(c, \bar{c}) = \kappa^2 T_p O(p^2)
\]

Here we have listed the results for the other closed string modes as well. These contact terms are reproduced in field theory by the following action:

\[
L = -\frac{\kappa^2 T_p}{2} \left( \tau h^a + \frac{p - 3}{2\sqrt{2}} \tau \phi + \frac{3}{4} \tau^2 + \cdots \right)
\]

(21)

Now, it is not difficult to see that eqs. (13) and (21) are consistent with the following generalized Born-Infeld action:

\[
S_{BI}^{\tau} = -T_p \int d^{p+1} \sigma \left( g(\kappa \tau) e^{\frac{\kappa \phi}{2\sqrt{2}}} - \det(g_{ab} - 2\kappa \tilde{b}_{ab} e^{-\frac{\kappa \phi}{2\sqrt{2}}} + \frac{1}{T_p} f_{ab} e^{-\frac{\kappa \phi}{2\sqrt{2}}}) \right)
\]

(22)

where the closed string tachyon coupling function is

\[
g(\kappa \tau) = 1 + \frac{1}{2} \kappa \tau + \frac{3}{8}(\kappa \tau)^2 + \cdots
\]

(23)

and the dots represents possible higher order terms which can not be read from the scattering amplitudes considered in this paper. We refer the reader to [12] for details of expanding the square root in eq. (22).

Now we give more evidence in support of eq. (22). We evaluate some more world-volume interactions in the presence of constant background field and show that they are consistent with the generalized Born-Infeld action (22). We consider constant background Kalb-Ramond field and (or) constant gauge field strength, \( i.e., F^{12} = -F^{21} \equiv F \). We begin with evaluating the world-volume coupling of one closed string tachyon and one massless open string NS state. At the world-sheet level, this coupling is given by

\[
A^\tau_{\tau,NS} \sim \int dx dz \text{Tr} < V^\tau(p_1, F, z_1, \bar{z}_1)V^{NS}(k_2, \zeta_2, F, x_2) >
\]
Using the doubling trick, one may write the tachyon and the open string vertex operators as

\[ V^\tau(p_1, F, z_1, \bar{z}_1) = : V_0(p_1, z_1) :: V_{-1}(p_1, D_F, \bar{z}_1) : \]
\[ V^{NS}(k_2, \zeta_2, F, x^2) = \zeta_2 \cdot G_{\mu} : V^\mu(k_2 + k_2 \cdot D_F, x^2) : T_2 \]

where \( G^{ab} = (\eta^{ab} + D^{ab}_F)/2 \) for the gauge fields and \( G^{ij} = N^{ij} \) for scalar fields. And the \( D^{ab}_F \) is a diagonal matrix given in [13]. Performing the correlators above, one finds

\[ A^{\tau,NS}_F \sim \zeta_2 \cdot G \cdot p_1 \text{Tr}[T_2]. \]

For the scalar and gauge states, it becomes

\[ A_F(\tau, \lambda) \sim \zeta_2 p_1^i \text{Tr}[T_2] \]
\[ A_F(\tau, a) \sim \frac{f_{2ab} F^{ab}}{1 + F^2} \text{Tr}[T_2] \]

First term above is reproduced by Taylor expansion of the linear tachyon in (22), and the second term by expanding the square root in (22) around the background field [13]. This gives evidence that the tachyon has linear coupling to the Born-Infeld terms. To show that the tachyon has quadratic coupling to the Born-Infeld terms as well, we consider scattering amplitude of two tachyon and one massless open string NS state with D-brane. This amplitude is given by

\[ A^{\tau,\tau,NS}_F \sim \int dxdz_1 dz_2 \text{Tr} < V^\tau V^\tau V^{NS} >. \]

We are not going to make efforts here to evaluate this amplitude explicitly. However, from the Taylor expansion of the quadratic tachyon in (21), one may conclude that the scattering amplitude above has the following contact term:

\[ A^{\tau,\tau,NS}_F \sim (\zeta_3 \cdot G \cdot p_1 + \zeta_3 \cdot G \cdot p_2) \text{Tr}[T_3] + \cdots \]

For the gauge fields, this amplitude becomes

\[ A_F(\tau, \tau, a) \sim \frac{f_{3ab} F^{ab}}{1 + F^2} \text{Tr}[T_3] + \cdots. \]

Again this contact term is reproduced by the action (22). Hence two tachyons couple with the entire Born-Infeld terms. This ends our evidences in support of the action (22).

5 Discussion

In this paper we have evaluated the amplitudes describing interaction of two closed string tachyons, one tachyon and one massless state, and two different RR states with D-branes in
type 0 theories. We have also evaluated scattering amplitude of one closed string tachyon and two massless open string states. We then examine these amplitudes in detail in order to extract their world-volume couplings. We found that the tachyon dose not couple to the RR fields on the D-branes. We have also found that the coupling of tachyon to other massless fields can be described by some extension of the Born-Infeld action \( i.e., \) eq. (22).

Evaluation of bulk effective actions for tachyon based on on-shell scattering amplitudes calculations is uncertain. This stem from the fact that it is hard to distinguish between \( \tau^2 \) and \( \tau \partial_\mu \partial^\mu \tau \), \( i.e., \) in the scattering amplitude it becomes \( p^2 = 1 \). However, this is not the case for the world-volume effective action. In this case, derivatives in the D-brane world-volume and the transverse spaces, \( i.e., \partial_a \) and \( \partial_i \), appear differing in the world-volume action. Hence the effective action (22) is unambiguous.

From the tachyon coupling to the gauge and to the scalar fields in (13), one concludes that this field couples with the 10-dimensional Yang-Mills theory reduced to the \( p+1 \) dimensional world-volume. Hence the action (13) or (22) is consistent with the T-duality.

The extended Born-Infeld action (22) in terms of the standard normalized fields \( i.e., 2\kappa \tau = T \) and see [12] for other fields) and in the string frame is

\[
S = -T_p \int d^{p+1} \sigma \ g(T/2) e^{-\Phi} \sqrt{-det(\tilde{G}_{ab} + \tilde{B}_{ab} + 2\pi \ell_s F_{ab})}.
\]

From this action, one finds that the effective Yang-Mills coupling is\(^4\)

\[
\frac{1}{g^2_{YM}} \sim (1 + \frac{T}{4} + \frac{3T^2}{32} + \cdots) e^{-\Phi}
\]

This is consistent with the power expansion of the following function

\[
\frac{1}{g^2_{YM}} \sim \frac{e^{-\Phi}}{\sqrt{1 - T/2}}
\]

It would be interesting to find cubic coupling of tachyon to the D-branes and see if it is consistent with the above conjectured function.

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\(^4\)In the original version of ref. [14] the form of the Yang-Mills coupling was conjectured to be \( 1/g^2_{YM} \sim (1+T)e^{-\Phi} \). However, later on they found independently that the coupling should have structure \( 1/g^2_{YM} \sim (1+T/4)e^{-\Phi} \).
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