Reheating and Supersymmetric Flat-Direction Baryogenesis

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Abstract

We re-examine Affleck-Dine baryo/leptogenesis from the oscillation of condensates along flat directions of the supersymmetric standard model, which attained large vevs at the end of the inflationary epoch. The key observation is that superpotential interactions couple the flat directions to other fields, which acquire masses induced by the flat-direction vev that may be sufficiently small for them to be kinematically accessible to inflaton decay. The resulting plasma of inflaton decay products then may act on the flat directions via these superpotential Yukawa couplings, inducing thermal masses and supersymmetry-breaking $A$ terms. In such cases the flat directions start their oscillations at an earlier time than usually estimated. The oscillations are also terminated earlier, due to evaporation of the flat direction condensate produced by its interaction with the plasma of inflaton decay products. In these cases we find that estimates for the resulting baryon/lepton asymmetry of the universe are substantially altered. We identify scenarios for the Yukawa couplings to the flat directions, and the order and mass scale of higher-dimensional superpotential interactions that set the initial flat direction vev, that might lead to acceptable baryo/leptogenesis.

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1 Introduction

Initially, one of the most attractive features of Grand Unified Theories (GUTs) \[1\] was the prospect that they might provide an explanation \[2\] for the matter-antimatter asymmetry of the Universe, via their new interactions that violate baryon and/or lepton number. Subsequently, it has been realized that, even in the Standard Model, at the non-perturbative level there are sphaleron interactions that violate both baryon and lepton number. This discovery has given rise to new scenarios for baryogenesis, at the electroweak phase transition \[3\] or via leptogenesis followed by sphaleron reprocessing \[4\]. Supersymmetric extensions of the Standard Model offer yet more scenarios for baryogenesis. For example, they may facilitate electroweak baryogenesis by permitting a first-order electroweak phase transition despite the constraints imposed by LEP \[5\]. There is also the possibility that they may contain perturbative interactions that violate baryon and/or lepton number via a breakdown of $R$ parity, which under certain circumstances \[6\] can induce baryogenesis.

However, perhaps the most attractive mechanism offered by supersymmetry is that proposed by Affleck and Dine, according to which \[7\] a condensate of a combination of squark and/or slepton fields may have formed during an inflationary epoch \[8\] in the early universe, causing the vacuum to carry a large net baryon and/or lepton number, which is then transferred to matter particles when the condensate eventually decays. We recall that the condensate forms along some flat direction of the effective potential of the theory, which we take to be the Minimal Supersymmetric extension of the Standard Model (MSSM) at low energies. In the conventional approach to Affleck-Dine baryogenesis, the condensate is essentially static until a relatively late cosmological epoch, when it starts to oscillate. In turn, the termination of the period of oscillation has been calculated in terms of the magnitudes of the soft supersymmetry-breaking terms present in the effective potential, which become significant only at low temperatures, and of the thermalization effects of inflaton decay \[9\].

The purpose of this paper is to re-examine this Affleck-Dine mechanism by incorporating a more complete treatment of the reheating of the universe after the inflationary epoch. We argue that the flat directions are in general coupled to other fields that are kinematically accessible to inflaton decay. These fields therefore have non-trivial statistical densities, and become thermalized. The couplings of these densities to the flat directions induce effective supersymmetry-breaking masses and $A$ terms for the erstwhile flat fields. As a result, the ‘flat’ directions start oscillating...
earlier than previously estimated. Subsequently, the oscillations also terminate earlier, as the flat-direction condensate interacts with the plasma of inflaton decay products and evaporates. The bottom line is that previous estimates of the resulting baryon/lepton asymmetry of the universe may be substantially altered, and we estimate some orders of magnitude for different representative parameter choices.

2 Flat Directions

The $D$-flat directions of the MSSM are classified by gauge-invariant monomials in the fields of the theory. These monomials have been classified in [10], and, for directions which are also $F$-flat for renormalizable standard model superpotential interactions, the dimension of the non-renormalizable term in the superpotential which first lifts the respective $D$-flat direction has also been derived. Hereafter, we consider only those $D$-flat directions which are not lifted by renormalizable superpotential interactions. These correspond to 14 independent monomials, and each monomial represents a complex $D$-flat direction: one vev magnitude and one phase (all fields in the monomial have the same vev). Since the monomials are gauge-invariant, appropriate gauge transformations generated by non-diagonal generators can be used to remove that part of the $D$-term contribution to the potential which comes from the non-diagonal generators. Also, any relative phase among the fields in the monomial can be rotated away by those gauge transformations which are generated by diagonal generators. There remains only one overall phase, i.e., the phase of the flat direction, which can be absorbed by redefinition of the scalar fields. Note that the $D$-term and $F$-term parts of the scalar potential are invariant under such a redefinition (which is equivalent to a U(1) symmetry transformation) while the soft-breaking terms and fermionic Yukawa terms generally are not. So we can always arrange the vev of the fields in the monomial to be initially along the real axis. We also note that such a non-zero vev breaks spontaneously the MSSM gauge group.

As an explicit example, consider the simplest case, which is the $H_u L$ flat direction. If the $T_3 = \frac{1}{2}$ component of $H_u$ and the $T_3 = -\frac{1}{2}$ component of $L$ have the same vev, then all the $D$ terms from both diagonal and non-diagonal generators of the MSSM are zero. The non-diagonal ones are identically zero and the equality of the vev makes the diagonal ones zero as well. These vev’s can then be chosen along the real axis as noted above. There are eight real degrees of freedom in
the $H^u$ and $L$ doublets. Two of them comprise the flat direction and another three are Goldstone bosons eaten by the gauge fields of the spontaneously-broken symmetries. The remaining three are physical scalars which are coupled to the flat direction, and are massive due to its vev.

Now that all fields in the monomial have the same vev and are real, by an orthogonal transformation we can go to a new basis where there is only one direction with a non-zero vev. Let us label this direction $\alpha$ and the orthogonal directions generically as $\phi$, therefore $\alpha_R \neq 0$ while $\alpha_I = \phi_R = \phi_I = 0$. For the specific $H^u L$ example, these are the following combinations after the Goldstone bosons are absorbed by the Higgs mechanism:

\[
\begin{align*}
\sqrt{2}\alpha_R &= (H_1)_R + (L_2)_R \\
\sqrt{2}\alpha_I &= (H_1)_I + (L_2)_I \\
\sqrt{2}\phi_1 &= (H_1)_R - (L_2)_R \\
\sqrt{2}\phi_2 &= (H_2)_R - (L_1)_I \\
\sqrt{2}\phi_3 &= (H_2)_I + (L_1)_R 
\end{align*}
\]

The $D$ terms from the $T_3$ and $U(1)_Y$ generators give terms $g^2\alpha^2\phi_1^2$ (up to numerical factors) in the potential, whilst those from $T_1$ and $T_2$ give $g^2\alpha^2\phi_2^2$ and $g^2\alpha^2\phi_3^2$ terms (up to numerical factors). It is a generic feature that all fields entering in the flat direction monomial which are left after the Higgs mechanism (except the linear combination which receives the vev after diagonalization) have masses of order $g\alpha$ due to their $D$-term couplings to the flat-direction vev.

We now consider supergravity effects, both in minimal models with soft-breaking terms at the tree level, and in no-scale models \cite{11}, where such terms are absent at tree-level but arise from quantum corrections \cite{12}. The superpotential consists of the tree-level MSSM terms and a series of non-renormalizable terms of successively higher dimension, which are induced in the effective theory by the dynamics of whatever is the underlying more fundamental theory. Without imposing $R$ parity (or any other symmetry) all gauge-invariant terms of higher dimension would exist in the superpotential. We may, however, also wish to impose $R$ parity on the higher-dimensional terms, as we have done on the renormalizable interactions, to prevent substantial $R$-parity violation being fed down from high scales by the renormalization-group running of the soft mass terms \cite{13}. If
we assume that $R$ parity is a discrete gauge symmetry of the theory, then it would be respected by all gauge-invariant superpotential terms of arbitrary dimension. Relevant higher-dimensional superpotential terms which lift the flat direction $\alpha$ are of the form:

$$W \supset \lambda_n \frac{\alpha^n}{n M^{n-3}}$$

where $\lambda_n$ is a number of order one and $M$ is a large mass scale, e.g., the GUT or Planck scale.

During inflation, supersymmetry is strongly broken by the non-zero energy of the vacuum. In minimal models this is transferred to the observable sector through the Kähler potential at tree level [14], while in no-scale models [11] this happens at the one-loop level [15]. Inflation then induces the soft-breaking terms

$$-C_I H_I^2 |\alpha|^2 + a \lambda_n H_I \frac{\alpha^n}{n M^{n-3}} + h.c.$$  \hspace{1cm} (2)

where $C_I$ and $a$ are numbers depending on the sector in which the inflaton lies, and $H_I$ is the Hubble constant during inflation. We shall assume here that $C_I$ is positive and not unnaturally small [14, 15]. In the presence of the $A$ term, the potential along the angular direction has the form $\cos(n \theta + \theta_a)$, where $\theta_a$ is the phase of $a$. Due to its negative mass-squared, the flat direction rolls down towards one of the discrete minima at $n \theta + \theta_a = \pi$ and $|\alpha| = \left( \frac{C_I}{(n-1) \lambda_n} H_I M^{n-3} \right)^{\frac{1}{n-2}}$, and quickly settles at one of the minima ($\frac{C_I}{(n-1) \lambda_n}$ is $O(1)$). Therefore, at the end of inflation $\alpha$ can be at any of the above-mentioned minima.

In the absence of thermal effects, $\alpha$ would track the instantaneous minimum $|\alpha| \sim (H M^{n-3})^{\frac{1}{n-2}}$ from the end of inflation until the time when $H \simeq m_3^2$, where $m_3^2 \sim 1$ TeV is the low-energy supersymmetry-breaking scale [14]. At $H \simeq m_3^2$ the low-energy soft terms

$$m_3^2 |\alpha|^2 + A \lambda_n m_3^2 \frac{\alpha^n}{n M^{n-3}} + h.c.$$  \hspace{1cm} (3)

would take over, with the mass-squared of $\alpha$ becoming positive, and $\alpha$ would then start its oscillations. Also, the minima along the angular direction would then move in a non-adiabatic way, due generally to different phases for $A$ and $a$. As a result, $\alpha$ starts its free oscillations around the origin with an initial vev $\alpha_{osc.} \sim (m_3^2 M^{n-3})^{\frac{1}{n-2}}$ and frequency $m_3^2$ and, at the same time, the torque exerted on it causes motion along the angular direction. In the case that the flat direction
carries a baryon (lepton) number this will lead to a baryon (lepton) asymmetry $n_B$ given by

$$n_B = \alpha_R \frac{\partial \delta_t}{\partial t} - \alpha_I \frac{\partial \delta_R}{\partial t}. \quad \text{At } m_3 t \gg 1$$

the upper bound on $n_B$ may be written

$$n_B \sim \frac{1}{m_3^2 t^2} \alpha_{osc.} \left( \frac{\alpha_{osc.}}{M} \right)^{n-3}$$  \hspace{1cm} (4)$$

which, after transition to a radiation-dominated universe, results in an $\frac{n_B}{s}$ that remains constant as long as there is no further entropy release.

As we will see in subsequent sections, thermal effects of inflaton decay products with superpotential couplings to the flat direction can fundamentally alter the dynamics of the flat direction oscillation, and necessitate revision of the estimates for the resulting baryon/lepton asymmetries produced.

3 Flat-Direction Superpotential Couplings and Finite Temperature Effects

As we have seen, the flat direction $\alpha$ has couplings of the form $g^2 \alpha^2 \phi^2$ to the fields $\phi$ which are in the monomial that represents it. Besides these $D$-term couplings, it also has $F$-term couplings to other fields $\chi$ which are not present in the monomial. These come from renormalizable superpotential Yukawas, and have the form

$$W \supseteq h \alpha \chi \chi$$  \hspace{1cm} (5)$$

which results in a term $h^2 |\alpha|^2 |\chi|^2$ in the scalar potential. Again for illustration, consider the $H^u L$ flat direction: $H^u$ has Yukawa couplings to left-handed and right-handed (s)quarks while $L$ has Yukawa couplings to $H^d$ and right-handed (s)leptons.

In the class of models that we consider, the inflaton is assumed to be in a sector which is coupled to ordinary matter by interactions of gravitational strength only. In this case, the inflaton decay always occurs in the perturbative regime and we need not worry about parametric-resonance decay

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$^1$We note that for $F$-flat directions of the renormalizable piece of the superpotential, which are only lifted by higher-dimensional nonrenormalizable terms, $\alpha$ cannot have such superpotential couplings to $\phi$ fields which appear in the monomial.
The inflaton decay rate is $\Gamma_d \sim \frac{m^3}{M_{Pl}}$, where $m$ is the inflaton mass and $m \leq 10^{13}$ GeV from the COBE data on the CMBR anisotropy [17]. Efficient inflaton decay occurs at the time when $H \simeq \Gamma_d$ and the effective reheat temperature at that time will be $T_R \sim (\Gamma_d M_{Pl})^{\frac{1}{2}}$. For $m \sim 10^{13}$ GeV we get $T_R \sim 10^{10}$ GeV, which is in the allowed range to avoid the gravitino problem [18].

The crucial point to note is that, although inflaton decay effectively completes much later than the start of its oscillation, nonetheless decay occurs throughout this period. In fact, a dilute plasma with temperature $T \lesssim (H\Gamma_d M_{Pl})^{\frac{1}{2}}$ (assuming instant thermalization: we address thermalization below) is present from the first several oscillations, until the effective completion of the inflaton decay [19]. It is easily seen that it has the highest instantaneous temperature at the earliest time, which can reach $T \leq 10^{13}$ GeV. This plasma, however, carries a relatively small fraction of the cosmic energy density, with the bulk still in inflaton oscillations. The dilution of relics produced from this plasma by the entropy release from the subsequent decay of the bulk of the inflaton energy is the reason that it does not lead to gravitino overproduction. It is important to note that the energy density in the plasma may be comparable to the energy density stored in the condensate along a flat direction. As a result, the thermal effects from the plasma may affect the dynamics of flat direction evolution which, as we see below, occurs in many cases.

All fields with mass less than $T$, and gauge interactions with the plasma particles, can reach thermal equilibrium with the plasma. Those fields which are coupled to the flat direction have generically large masses in the presence of its vev, and might not be excited thermally. These include the $\phi$ fields which are gauge-coupled to $\alpha$ and have a mass $g\alpha$ (up to numerical factors of $O(1)$) and many of the $\chi$ fields which have superpotential couplings to $\alpha$, and hence have a mass $h\alpha$ (also up to numerical factors of $O(1)$). For $g\alpha > T$ or $h\alpha > T$, the former or the latter are not in thermal equilibrium, respectively. We recall that, in the presence of Hubble-induced soft-breaking terms, the minimum of the potential for the flat direction determines that $\alpha \sim (HM^{n-3})^{\frac{1}{n-2}}$, and the plasma temperature is $T \sim (H\Gamma_d M_{Pl}^2)^{\frac{1}{4}}$. So a field with a coupling $h$ to the flat direction can be in thermal equilibrium provided that $h\alpha \leq T$, which implies that

$$H^{6-n} \leq \frac{\Gamma_d^{n-2} M_{Pl}^{2(n-3)}}{h^{4(n-2)} M^{4(n-3)}}$$

and similarly for the gauge coupling $g$.  

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The back-reaction effect of the plasma of quanta of this field will then induce a mass-squared $+h^2T^2$ for the flat direction to which it is coupled. If this exceeds the negative Hubble-induced mass-squared $-H^2$, the flat direction starts its oscillation. This happens for $hT \geq H$, i.e., for

$$H^3 \leq h^4 \Gamma_d M_{Pl}^2$$

and similarly for back-reaction from plasma fields with gauge coupling $g$ to the flat direction. Therefore, a flat direction will start its oscillations if both of the above conditions are satisfied simultaneously. We note that the finite-temperature effects of the plasma can lead to a much earlier oscillatory regime for the flat direction, i.e., when $H \gg m_4$.

It is clear that, in order for a plasma of the quanta of a field to be produced, the coupling of that field to a flat direction should not be so large that its induced mass prevents its thermal excitation. On the other hand, in order for its thermal plasma to have a significant reaction back on the flat direction, its coupling to the flat direction should not be so small that the thermal mass-squared induced for the flat direction will be smaller than the Hubble-induced contribution. Therefore, to have significant thermal effects, we need couplings of intermediate strength in order to have both conditions simultaneously satisfied. For the fields $\phi$ which have $D$-term couplings of gauge strength $g$ to flat directions, this is usually not the case: As will be seen shortly, in most cases their couplings are too large to satisfy the equilibrium condition. For the fields $\chi$ which have $F$-term couplings of Yukawa strength $h$ to the flat direction, the existence of significant thermal effects depends on the value of $h$, as well as on the initial value of the flat-direction vev $\alpha$, which in turn depends on the mass scale and the dimension of the higher-dimensional operator which lifts the flatness.

To organize our discussion then, we first assess the typical values of $h$ to be expected for couplings to the flat directions. For these typical values, we then estimate the importance of thermal effects on vev’s determined by higher-dimensional operators ranging over the various different dimensions that can lift the flat direction, for both the case of lifting by the GUT scale: $O(10^{16})$ GeV, and by the Planck scale: $O(10^{19})$ GeV.

We now list the Yukawa couplings of the MSSM. For low $\tan\beta$, the ratio of $H^u$ and $H^d$ vev’s, we have
whilst the $h^u$'s and $h^d$'s tend to be more similar for high $\tan\beta$. The only Yukawa couplings which are significantly different from $O(10^{-2})$ are $h^u_1, h^d_1, h^l_1,$ and $h^u_3$. Only flat directions which include only the left- and right-handed up squark, the left- and right-handed down squark, the left- and right-handed selectron and the left-handed sneutrino will have an $h$ significantly less than $O(10^{-2})$.

For low $\tan\beta$, any flat direction which includes right-handed top squarks has a Yukawa coupling of $O(1)\to$ some $\chi$’s, too large for those $\chi$’s to be in thermal equilibrium, given the expected range of flat direction vev's $\alpha$. The left-handed squarks are coupled to both $H^u$ and $H^d$, so any flat direction which includes a left-handed top squark has a Yukawa coupling of order $10^{-2}$ to $H^d$ as well. For high $\tan\beta$, any flat direction which includes the left- or right-handed top or bottom squarks has a Yukawa coupling of order 1 since the top and bottom Yukawas are of the same order. In general, any flat direction which consists only of the above-mentioned scalars has a Yukawa coupling of order 1 to some $\chi$’s and/or a Yukawa coupling significantly less than $O(10^{-2})$ to other $\chi$’s.

Among all flat directions which are not lifted by the renormalizable superpotential terms, there is only one which allows such a flavor choice: $uude$ with one $u$ in the third generation and all other scalars in the first generation (i.e., $tude$). This exceptional flat direction still has a coupling of $O(10^{-4})$ to some $\chi$ fields, since it includes the right-handed up squark. Taking into account all flavor choices for all flat directions which are not lifted at the renormalizable superpotential level, we can use $h \simeq 10^{-2}$ for the coupling of a generic flat directions to $\chi$ fields. For the above-mentioned exceptional case we shall use $h \simeq 10^{-4}$.

So, for our discussion of the dynamics of flat direction oscillations we will consider three representative cases. We will analyze the dynamics when inflaton decay plasmons are coupled to the flat direction by: gauge couplings with coupling $g \simeq 10^{-1}$, generic Yukawa couplings of order $h \simeq 10^{-2}$, or suppressed Yukawa couplings of order $h \simeq 10^{-4}$. Consideration of these cases should allow us to explore the generic range of physical effects that arise in flat direction oscillations, from a plasma of inflaton decay products.

We now undertake a detailed analysis to determine in which cases a plasma of inflaton decay
products can be produced, and can initiate the flat-direction oscillations by the reaction they induce on the flat direction. Whether this occurs or not depends on the vev of the flat direction, and the strength of the coupling of the plasma quanta to the flat direction. The initial vev of the flat direction is set by both the underlying scale of the physics of the higher-dimensional operators that lift the flat direction, and, for a given flat direction, by the dimension of the gauge-invariant operator of lowest dimension which can be induced by the underlying dynamics to lift the flat direction.

In order to categorize systematically the various cases which arise, we organize them as follows. First, we divide them into two cases, depending on whether the underlying scale of the new physics responsible for the higher-dimensional operators which lift the flat direction and stabilize the vev at the end of inflation are GUT-scale: $O(10^{16})$ GeV, or Planck-scale: $O(10^{19})$ GeV. Each of these cases is subdivided according to whether the coupling between the flat direction and the inflaton decay products is of gauge strength ($g \simeq 10^{-1}$), standard superpotential Yukawa strength ($h \simeq 10^{-2}$), or exceptional suppressed Yukawa strength ($h \simeq 10^{-4}$). As noted above, this covers the generic range of couplings exhibited by fields in flat directions in the supersymmetric standard model. Finally, each of these cases is subdivided and tabulated according to the dimension of the operator that stabilizes the flat-direction vev, setting (given the possibilities listed above for the underlying scale of the new physics responsible for the operators) the initial vev of the flat direction. These higher-dimensional operators are listed by the order of the monomial in the superfields which appears in the superpotential and is responsible for the operator. We tabulate against the order of the higher-dimensional superpotential term the following quantities (in Planck units)\footnote{From now on, we express some dimensionful quantities in Planck units.}: the Hubble constant $H$, the temperature $T$, and the value of the flat-direction vev $\alpha$ at the onset of oscillations, as well as the combination $\frac{\alpha T^2}{H H_\alpha}$ ($\frac{g T^2}{H H_\alpha}$ for the case of the gauge coupling) which will be useful when we discuss the produced baryon asymmetry in the next section. We also explain the reasons for the values of the entries appearing, in the light of the two necessary conditions introduced above for inducing the flat-direction oscillations by plasma effects, i.e., that on the one hand the mass of the plasmon induced by the coupling to the flat direction is small enough that it can be populated in the thermal bath from inflaton decay, and, on the other hand, that the coupling is large enough for back-reaction effects from the plasma to lift the flat direction sufficiently to start oscillation.
the effects of the Hubble-induced mass.

First, let us consider the case that the scale of the new physics that induces operators that stabilize the flat direction is of order the GUT scale: $O(10^{16})$ GeV. We then subdivide this case according to the strength of the coupling of the inflaton decay products to the flat direction. To start, we consider the gauge-coupled case with $g = 10^{-1}$. In this case, it is only for initial flat-direction vev’s fixed by either quartic or quintic higher-dimensional terms in the superpotential that the plasma effects can accelerate the onset of flat direction oscillation, with the results shown in the following Table. Physically, for superpotential monomials of sixth order or higher, the initial flat-direction vev is sufficiently large that the mass generated by its gauge coupling to the prospective inflaton decay products is large enough to prevent them from being kinematically accessible for thermal excitation. In the case of a quintic superpotential monomial this is also initially the case, and it is only after Hubble expansion has reduced $\alpha$, and hence the induced plasmon mass, that thermalized products of inflaton decay can back-react to induce flat-direction oscillation. However, this only occurs for $H < 10^{-16}$, by which time the low-energy soft supersymmetry breaking has already initiated flat-direction oscillation.

$$GUT\ scale\ M = 10^{16}\ GeV,\ gauge\ coupling\ g = 10^{-1}$$

| $n$ | $H$ | $T$ | $\alpha$ | $\frac{qT^4}{\alpha}$ |
|-----|-----|-----|---------|---------------------|
| 4   | $10^{-8}$ | $10^{-\frac{14}{2}}$ | $10^{-\frac{14}{2}}$ | $10^{-\frac{1}{2}}$ |
| 5   | $10^{-18}$ | $10^{-9}$ | $10^{-8}$ | $10^{7}$ |

For the GUT case $M = 10^{16}$ GeV with generic Yukawa coupling $h = 10^{-2}$, we have the results shown in the following Table for lifting of the flat direction by monomials of the orders listed. In the cases that the order of the monomial is four or five we have no difficulty satisfying the condition that $ha \leq T$, so that they are (thermally) populated in the inflaton decay plasma. For monomials of order six, seven or eight, the induced mass of the prospective plasmon is, in fact, of the same order or slightly larger than the instantaneous effective temperature. So thermally they are present, albeit now with some Boltzman suppression. Moreover, we also note that these induced masses are less than the mass of the decaying inflaton, and so they will be produced in the cascade of inflaton decay products, though, as noted above, after complete thermalization they will be subject to some
Boltzmann suppression. In all cases the value of the Hubble constant at the onset of oscillation will be determined by the second condition \( hT \geq H \), which requires that the back-reaction-induced mass overcome the Hubble-induced mass to initiate oscillation. By comparing the results of the Tables for \( g = 10^{-1} \) and \( h = 10^{-2} \), we note that for a general flat direction with \( h = 10^{-2} \) which is lifted at the \( n = 4 \) superpotential level, the values at the onset of oscillations should be taken from the gauge analysis. The reason is that, in this case, the back-reaction of the inflaton decay products which have gauge coupling to the flat direction act at an earlier time than the back-reaction of those decay products which have Yukawa couplings to it.

\[
GUT \text{ scale } M = 10^{16} \text{ GeV, standard Yukawa coupling } h = 10^{-2}
\]

|     | \( H \)    | \( T \)    | \( \alpha \) | \( \frac{hT^2}{M^4} \) |
|-----|------------|------------|---------------|------------------------|
| \( n = 4 \) | \( 10^{-20} \) | \( 10^{-50} \) | \( 10^{-50} \) | \( 10^{-50} \) |
| \( n = 5 \) | \( 10^{-20} \) | \( 10^{-50} \) | \( 10^{-50} \) | \( 10^{-50} \) |
| \( n = 6 \) | \( 10^{-9} \) | \( 10^{-15} \) | \( 10^{-15} \) | \( 10^{-2} \) |
| \( n = 7 \) | \( 10^{-7} \) | \( 10^{-13} \) | \( 10^{-13} \) | \( 10^{-4} \) |
| \( n = 8 \) | \( 10^{-5} \) | \( 10^{-11} \) | \( 10^{-11} \) | \( 10^{-6} \) |

For \( M = 10^{16} \) GeV, \( h = 10^{-4} \), as a function of the order of the superpotential monomial lifting the flat direction we have the results shown in the next Table. For these cases, the flat-direction-induced mass is always less than the instantaneous temperature, due to the weak coupling of the flat direction to the plasmons. The only non-trivial condition now is the second one \( (hT \geq H) \), which determines how long one must wait before the Hubble-induced mass is sufficiently reduced that the back-reaction-induced flat-direction mass can overcome it to initiate oscillation. This fixes the value of \( H \) at the onset of oscillation. Comparing the results of the Tables for \( g = 10^{-1} \) and \( h = 10^{-4} \), we note that for an exceptional flat direction with \( h = 10^{-4} \) which is lifted at the \( n = 4 \) superpotential level, the values at the onset of oscillations should also be taken from the gauge analysis.
We now turn to the case that the underlying scale of new physics responsible for generating the higher-dimensional operators is the Planck scale. This means that the values of the flat direction vev’s after inflation will be larger, raising the mass of prospective plasmons to which they couple, and making it harder to satisfy the constraint that these putative plasmons be generated thermally, or even be kinematically accessible to inflaton decay.

For $M = 10^{19}$ GeV, $g = 10^{-1}$, we have significant effects only for flat directions lifted by superpotential terms arising from quartic or quintic monomials. In all other cases ($n \geq 6$) the flat direction vev is so large that quanta gauge-coupled to it receive sufficiently large masses that they can not be thermally populated at the instantaneous temperature of the inflaton decay products. For the two non-trivial cases we have the results shown in the following Table. Only in the $n = 4$ case can we produce thermally a number of plasma quanta sufficient to induce enough mass for the flat direction to initiate its oscillation at an earlier time. In the $n = 5$ case, back-reaction from the plasma of inflaton decay products only manages to induce flat-direction oscillation after $H \ll 10^{-18}$, by which time the low-energy soft supersymmetry breaking has already acted to start the oscillation and also the inflaton decay has been completed.

**Planck scale $M = 10^{19}$ GeV, gauge coupling $g = 10^{-1}$**

| $n$   | $H$   | $T$   | $\alpha$ | $\frac{gT^2}{H\alpha}$ |
|-------|-------|-------|-----------|------------------------|
| $n = 4$ | $10^{-14}$ | $10^{-8}$ | $10^{-7}$ | $10^4$ |
| $n = 5$ | $10^{-42}$ | $10^{-15}$ | $10^{-14}$ | $10^{25}$ |
For $M = 10^{19}$ GeV and $h = 10^{-2}$, we again have a case where flat directions lifted by superpotential monomials of order six or higher result in such a large flat-direction vev that quanta coupled to it receive too large a mass for them to be thermally excited in the plasma of inflaton decay products. For the cases of quartic or quintic superpotential monomials, we have the results shown in the following Table. We again find that only in the $n = 4$ case can thermal effects actually induce sufficient mass for the flat direction to initiate oscillation earlier. In the $n = 5$ case, back-reaction from the plasma of inflaton decay products only manages to induce flat-direction oscillation after the low-energy soft supersymmetry breaking has already done so, and the inflaton decay has been completed. By comparing the results of the Tables for $g = 10^{-1}$ and $h = 10^{-2}$, we note that, for a generic flat direction, i.e., one with $h = 10^{-2}$, the initial values at the onset of oscillations should be taken from the latter. The reason is that, in this case, the back-reaction of the inflaton decay products which have Yukawa couplings to the flat direction act at an earlier time than the back-reaction of those decay products with a gauge coupling to it.

Planck scale $M = 10^{19}$ GeV, standard Yukawa coupling $h = 10^{-2}$

| $n$ | $H$ | $T$ | $\alpha$ | $\frac{hT}{H_\alpha}$ |
|-----|-----|-----|-------|------------------|
| 4   | $10^{-10}$ | $10^{-7}$ | $10^{-6}$ | $10^{-1}$ |
| 5   | $10^{-30}$ | $10^{-12}$ | $10^{-6}$ | $10^{14}$ |

Finally, in the case $M = 10^{19}$ GeV, $h = 10^{-4}$, for flat directions lifted by monomials higher than sixth order the resulting flat-direction vevs are sufficiently large that quanta coupled to it with this coupling are too massive to be excited at the instantaneous temperature of the inflaton decay products. So the nontrivial cases are those in the following Table. For $n = 6$, we marginally satisfy the requirement that $hT \simeq \alpha$, necessary for thermal production of quanta coupled to the flat direction, while for $n = 4$ and $n = 5$ we do so comfortably. The second condition, that $hT \geq H$ for effective back-reaction, then serves to determine the value of $H$ at the onset of the thermally-induced oscillation. By comparing the results of the Tables for $g = 10^{-1}$ and $h = 10^{-4}$, we note that when the exceptional flat direction with $h = 10^{-4}$ is lifted at the $n = 4$ superpotential level, the values at the onset of oscillations should be taken from the latter. This is because the back-reaction of the inflaton decay products with Yukawa coupling to the flat direction act at an earlier time than the back-reaction of those decay products with gauge couplings to it.
Planck scale $M = 10^{19}$ GeV, exceptional Yukawa coupling $h = 10^{-4}$

|   | $H$   | $T$   | $\alpha$ | $\frac{h T^2}{\alpha}$ |
|---|-------|-------|----------|-----------------|
| $n = 4$ | $10^{-\frac{3}{2}}$ | $10^{-\frac{5}{2}}$ | $10^{-\frac{7}{2}}$ | $10^{-\frac{9}{2}}$ |
| $n = 5$ | $10^{-\frac{3}{2}}$ | $10^{-\frac{5}{2}}$ | $10^{-\frac{7}{2}}$ | $10^{-\frac{9}{2}}$ |
| $n = 6$ | $10^{-\frac{3}{2}}$ | $10^{-\frac{5}{2}}$ | $10^{-\frac{7}{2}}$ | $10^{-\frac{9}{2}}$ |

We noted above that thermal effects from the plasma can be important up to $h \alpha \simeq T$ or even somewhat higher. For $\alpha$ less than this, they change the convexity of the effective potential in the $\alpha$ direction at much earlier times, inducing the onset of flat-direction oscillations. We should note that since $\alpha \sim H^{1-n^2}$ and $T \sim H^{\frac{1}{4}}$, then $\alpha$ decreases at the same rate as, or more slowly than, $T$ for $7 \leq n \leq 9$. This means that if $h \alpha \gg T$ right after the end of inflation, it will remain so for later times as well. Therefore, in the $7 \leq n \leq 9$ cases for $M = 10^{19}$ GeV, the Hubble-induced negative mass-squared is dominant and $\alpha$ will not be lifted until $H \simeq 10^{-16}$, if $h \alpha \gg T$ at $H \approx 10^{-6}$.

In sum, we conclude that for $M = M_{\text{GUT}}$, a general flat direction with $h \simeq 10^{-2}$ starts oscillating at $H \gg 10^{-16}$ in the $4 \leq n \leq 8$ cases. For the exceptional one with $h \simeq 10^{-4}$ it is true in the $n = 9$ case as well. For $M = M_{\text{Planck}}$ in the denominator, only in the $n = 4$ case do oscillations of a general flat direction start at $H \gg 10^{-16}$. In the $5 \leq n \leq 9$ cases, the flat direction is protected from thermal effects because its large vev induces such a large mass for fields coupled to it that they cannot be thermally excited in the plasma of inflaton decay products. For the exceptional flat direction with $h \simeq 10^{-4}$, this protection is weaker because of the smaller Yukawa coupling to $\chi$ (which therefore are lighter and can be excited in thermal equilibrium) and, as a result, oscillations start at $H \gg 10^{-16}$ in the $n = 5, 6$ cases also.

We need to elaborate on the implicit assumption that the $\chi$’s ($\phi$’s) are effectively thermal upon production. In the model that we study, the inflaton decays in the perturbative regime, and the decay products have a momentum less than, or comparable to, the inflaton mass $m \sim 10^{-6}$. The $\chi$’s ($\phi$’s) which are produced in two-body decays have a momentum of order $m$ \footnote{The $\phi$’s generically have larger $\alpha$-induced masses than do the $\chi$’s, so their production may be delayed until $\alpha$ Hubble-dilutes to a smaller value.}. It can easily be seen that the temperature at which oscillations start (assuming thermal equilibrium) is $\approx 10^{-7}$ in all the above cases. Since the momentum of produced particles is greater than the the average thermal
momentum, the dominant process to reach equilibrium is through the decay of $\chi$’s ($\phi$’s) to other particles with smaller momenta. However, the momentum of $\chi$’s ($\phi$’s) is very close to the average thermal momentum. Since thermalization does not change the energy density in the plasma, the number density of $\chi$’s ($\phi$’s) is also close to its thermal distribution. Therefore, the plasma-induced mass-squared $h^2 \frac{n_\chi}{E_\chi} (g^2 \frac{n_\phi}{E_\phi})$ from $\chi$’s ($\phi$’s) is of the same order as $h^2 T^2 (g^2 T^2)$.

4 Thermal $A$ Terms and Baryo/Leptogenesis

Motion along the angular direction is required for the build-up of a baryon or lepton asymmetry. This is possible if a torque is exerted on $\alpha$ or, equivalently, if $\alpha$ is not in one of the discrete minima along the angular direction, when it starts oscillating. These discrete minima are due to the $A$ term part of the potential. Before the start of oscillations, the Hubble-induced $A$ terms are dominant, and the locations of the minima are determined by them. During inflation, $\alpha$ rolls down towards one of these minima and rapidly settles there. After inflation it tracks that minimum and there is no motion along the angular direction [14]. What is necessary then is a non-adiabatic change in the location of the minima, such that at the onset of oscillations $\alpha$ is no longer in a minimum along the angular direction. In the absence of thermal effects, $\alpha$ would start its angular motion (as well as its linear oscillations) at $H \simeq m_3^2$. This occurs as a result of uncorrelated phases of the $A$ terms induced by the Hubble expansion and low-energy supersymmetry breaking. At this time, the latter takes over from the former, and $\alpha$ will in general no longer be in a minimum along the angular direction. This will lead to the generation of a baryon or lepton asymmetry if $\alpha$ carries a non-zero number of either [14].

As we have seen above, due to thermal effects, in many cases the flat directions start oscillating at much larger $H$. At this time the Hubble-induced $A$ terms are still much larger than the low-energy ones from hidden-sector supersymmetry breaking. In order to have angular motion for $\alpha$, another $A$ term of size comparable to the Hubble-induced one, but with uncorrelated phase, is required. Since it is finite-temperature effects from the plasma that produce a mass-squared which dominates the Hubble-induced one, one might expect that the same effects also produce an $A$ term which dominates the Hubble-induced $A$ term. This is the only new effect that could produce such an $A$ term with uncorrelated phase, as the thermal plasma is the only difference from the standard
The simplest such thermal $A$ terms arise at tree-level from cross terms from the following two terms in superpotential

$$h\alpha \chi + \lambda_n \frac{\alpha^n}{nM^{n-3}}$$

which results in the contribution

$$h\lambda_n \frac{\chi^{*2} \alpha^{n-1}}{M^{n-3}} + h.c.$$  \hspace{1cm} (10)

in the scalar potential. In thermal equilibrium, $< \chi^{*2} >$ can be approximated by $T^2$ and therefore the thermal $A$ term is of order

$$h\lambda_n \frac{T^2 \alpha^{n-1}}{M^{n-3}}$$  \hspace{1cm} (11)

There is another thermal $A$ term that arises from one-loop diagrams with gauginos and fermionic partners of $\alpha$. It results in a contribution, in the thermal bath, of order:

$$\lambda_n \left( \frac{gT}{4\pi\alpha} \right)^2 \frac{T \alpha^n}{M^{n-3}}$$  \hspace{1cm} (12)

We have checked that, for the parameter range of interest for this process, this has the same order of magnitude as the tree-level $A$ term. In the following, we use the tree-level term for our estimates.

The ratio of the thermal $A$ term to the Hubble-induced one is $\frac{hT^2}{H\alpha}$. It is clear from the results summarized in the Tables that, at the onset of $\alpha$ oscillations, the thermal $A$ term is weaker than the Hubble-induced one in all cases. Therefore, at this time, the minimum along the angular direction is slightly shifted, the curvature at the minimum is still determined by the Hubble-induced $A$ term, and the force in the angular direction is of order $\lambda_n \frac{hT^2 \alpha^{n-2}}{M^{n-3}}$. The ratio of the thermal $A$ term to the Hubble-induced one grows, however, as $\frac{hT^2}{H\alpha}$ increases in time. Therefore, in what follows we keep both $A$ terms in the equation of motion of $\alpha$.

We consider the case where oscillations start because of the back-reaction of the $\chi$ fields, as is the most common case. The masses of $\alpha_R$ and $\alpha_I$ are then of order $hT$ \footnote{We use the estimate $gT$ for the plasma-induced masses if oscillations start because of the back-reaction of the $\phi$ fields.}. The equation of motion for the flat direction is then:
\[ \ddot{\alpha} + 3H\dot{\alpha} + \frac{h^2T^2\alpha + (n-1)h\lambda_n T^2\alpha^{n-2}}{M^{n-3}} + A\lambda_n \frac{H\alpha^{n-1}}{M^{n-3}} + (n-1)\lambda_n \frac{2|\alpha|^{2(n-2)}}{M^{2(n-3)}} \alpha = 0 \] (13)

At this time, the universe is matter-dominated, by the oscillating inflaton field, and thus \( H = \frac{2}{3t} \). Also, \( T^2 = (H\Gamma_d M_p^4)^{\frac{2}{3}} \sim t^{-\frac{1}{2}} \) for \( H \geq 10^{-18} \). After re-scaling \( \alpha \to (\frac{H_{osc}M^{n-3}}{\lambda_n})^{\frac{1}{n-2}} \alpha \) and \( t \to H_{osc}t \), where \( H_{osc} \) is the Hubble constant at the onset of oscillations, we get the following equations of motion for the real and imaginary components of \( \alpha \):

\[ \begin{align*}
\ddot{\alpha}_R + \frac{2}{t} \dot{\alpha}_R + a\frac{\alpha_R}{t^2} + b|\alpha|^{n-2} \cos((n-2)\theta + \varphi) + A\frac{|\alpha|^{n-1}}{t} \cos((n-1)\theta) + (n-1)|\alpha|^{2(n-2)} \alpha_R &= 0 \\
\ddot{\alpha}_I + \frac{2}{t} \dot{\alpha}_I - a\frac{\alpha_I}{t^2} - b|\alpha|^{n-2} \sin((n-2)\theta + \varphi) - A\frac{|\alpha|^{n-1}}{t} \sin((n-1)\theta) + (n-1)|\alpha|^{2(n-2)} \alpha_I &= 0
\end{align*} \] (14)

Here \( a \equiv \frac{h^2T_{osc}^2}{H_{osc}^2} \), \( T_{osc} = (H_{osc}\Gamma_d M_p^4)^{\frac{1}{2}} \) is the plasma temperature at the onset of oscillations, \( b \equiv (n-1)a\lambda_n (\frac{\alpha_{osc} M^{n-3}}{\lambda_n})^{\frac{n-3}{n-2}} \), and \( \varphi = O(1) \) is the relative phase between the thermal and Hubble-induced \( A \) terms.

The first two terms and the superpotential term in each of these equations are the same as in the equations derived in [14], but there are some important differences. First of all, the flat-direction mass-squared is not the (constant) low-energy value \( m_{\frac{1}{2}}^2 \), but the thermal mass which is redshifted as \( t^{-\frac{1}{2}} \). Also, the Hubble-induced \( A \) term with coefficient \( H \) appears instead of the low-energy one with coefficient \( m_{\frac{1}{2}} \), which is negligible for \( H \gg 10^{-16} \). This explains the \( \frac{1}{t} \) factor in front of the Hubble-induced \( A \) term. Finally, there is another \( A \) term, the thermal one, which is also redshifted as \( t^{-\frac{1}{2}} \), because of its \( T^2 \) dependence.

At the onset of oscillations, \( t = t_i = \frac{2}{3} \) and \( \alpha \) is in one of the minima which are determined by the Hubble soft terms. Therefore, \( |\alpha|_i \), which was \( (\frac{H_{osc}M^{n-3}}{\lambda_n})^{\frac{1}{n-2}} \) before re-scaling, is scaled to \( |\alpha|_i = 1 \), and also \( \theta_i = n\pi \). We have solved these equations numerically for \( \theta_i = 0 \), \( A = (n-1) \) and \( \lambda_n = 1 \), and calculated \( n_B = \alpha \frac{\partial \phi}{\partial t} - \alpha I \frac{\partial \alpha}{\partial t} \). We find that, among the cases listed in the above-mentioned Tables, only for the following ones do we get an \( \frac{n_B}{s} \) of order \( 10^{-11} \) or larger, before any subsequent dilution after reheating.
The value of $\frac{n_B}{s}$ for flat directions which undergo plasma-induced oscillations

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
 & M = 10^{16} \text{ GeV} & & M = 10^{19} \text{ GeV} & \\
\hline
n = 4 & h = 10^{-2} & < 10^{-11} & h = 10^{-2} & < 10^{-11} \\
n = 5 & h = 10^{-4} & 3 \times 10^{-11} & h = 10^{-4} & 3 \times 10^{-9} \\
n = 6 & 10^{-11} & 4 \times 10^{-11} & \text{no plasma effect} & 10^{-7} \\
n = 7 & 4 \times 10^{-11} & 10^{-10} & \text{no plasma effect} & \text{no plasma effect} \\
n = 8 & 10^{-10} & 3 \times 10^{-10} & \text{no plasma effect} & \text{no plasma effect} \\
n = 9 & \text{no plasma effect} & 5 \times 10^{-10} & \text{no plasma effect} & \text{no plasma effect} \\
\hline
\end{array}
\]

We see that, in some cases, $\frac{n_B}{s}$ is near the observed value of $5 \times 10^{-10}$. However, in the most general case, when the standard model gauge group is the only symmetry group, these viable flat directions constitute only a small subset of all flat directions. We also see that $\frac{n_B}{s}$ is larger for the exceptional flat directions, when $M = 10^{19} \text{ GeV}$, and for flat directions which are lifted by terms of higher order $n$. This is easily understandable, as for larger $M$ and $n$, and for smaller $h$, plasma-induced oscillations start later and closer to the efficient reheat epoch $H = 10^{-18}$. Larger $M$ and $n$ lead to a larger vev for the flat direction and, therefore, the condition $h\alpha \leq T$ will be satisfied at a later time. A smaller value of $h$, on the other hand, implies that the condition $hT \geq H$ will be satisfied at a later time. Later oscillations mean less dilution of the generated lepton/baryon asymmetry by the plasma of inflaton decay products (we recall that $s \sim T^3$ is redshifted only as $t^{-\frac{3}{4}}$ for $H \geq 10^{-18}$).

Now we comment how our results may be affected by changes in the model-dependent constants involved in the calculations: the reheat temperature $T_R$ (or equivalently the inflaton decay rate), and the constant $\frac{C_l}{(n-1)\lambda_n}$ which appears in the expression for the flat-direction vev. There are two concerns in this regard. First, whether the two conditions for plasma-induced $\alpha$ oscillations still result in a consistent value for $H_{osc}$ which is greater than $10^{-16}$, and, secondly, what is the corresponding change in the estimated value for $\frac{n_B}{s}$. In our calculations, we have used $T_R \simeq 10^{-9}$ and $\frac{C_l}{(n-1)\lambda_n} \simeq 1$. If we assume instead that $T_R \simeq 10^{-10}$ and $\frac{C_l}{(n-1)\lambda_n} \simeq 10^{-1}$, it turns out that for all cases except the marginal ones (the $n = 6, 7, 8$ cases for $h = 10^{-2}$ and $M = 10^{16} \text{ GeV}$, and the $n = 6$ case for $h = 10^{-4}$ and $M = 10^{19} \text{ GeV}$), plasma effects still trigger the oscillations for $H \geq 10^{-16}$, though at a somewhat smaller value of $H$. Moreover, the value of $\frac{n_B}{s}$ remains the same.
within an order of magnitude. Therefore, the plasma-induced oscillations of the (non-marginal) flat directions, and the resulting value of $\frac{n\beta}{s}$, are rather insensitive to the exact order of magnitude of $T_R$ and $\frac{C_l}{(n-1)\lambda_0}$, at least for our purposes.

5 Evaporation of the Flat Direction

Now let us find the time when the $\alpha$ condensates are knocked out of the zero mode by the thermal bath. For the evaporation to happen, it is necessary that the thermal bath includes those particles which are coupled to $\alpha$. Then, two conditions should be satisfied: first, the scattering rate of $\alpha$ off the thermal bath must be sufficient for equilibration, and secondly, the energy density in the bath must be greater than that in the condensate. The flat direction has couplings both of Yukawa strength $h$ to $\chi$'s and of gauge strength $g$ to $\phi$'s. The conditions for thermal production of $\chi$'s and $\phi$'s are $h\alpha \leq T$ and $g\alpha \leq T$, respectively. Since $h < g$, the $\chi$'s will come to thermal equilibrium at an earlier time. On the other hand, the scattering rate of $\alpha$ off the thermal $\chi$'s is $\Gamma_{\text{scatt.}} \sim h^4T$ while the rate for scattering of $\alpha$ off the thermal $\phi$'s is $\Gamma_{\text{scatt.}} \sim g^4T$. Therefore, $\chi$'s are produced earlier but in general have a smaller scattering rate. The competition between the $\chi$'s and $\phi$'s, and between the ratio of the energy density of the flat direction to the energy density in the plasma will determine whether and how the flat direction evaporates.

First we consider those flat directions which have plasma-induced oscillations. If oscillations start due to the back-reaction of $\chi$’s (which is the situation for most cases) $\Gamma_{\text{scatt.}} \sim h^4T$. For a general flat direction with $h \simeq 10^{-2}$, this is comparable to $H$ at $H \simeq 10^{-17}$, while for the exceptional flat direction with $h \simeq 10^{-4}$ this occurs at a much smaller $H$. However, after $\alpha$ starts its oscillation, it is redshifted as $t^{-\frac{5}{2}}$ while $T$ is redshifted as $t^{-\frac{1}{4}}$. This implies that $\frac{h\alpha}{T}$ decreases rapidly and soon the $\phi$’s will be in thermal equilibrium. The rate for scattering of $\alpha$ off thermal $\phi$’s is $\Gamma_{\text{scatt.}} \sim g^4T$ and $\Gamma_{\text{scatt.}} \geq H$ at $H \leq 10^{-12}$. The energy density in the condensate at the onset of oscillations is $h^2\alpha^2T^2 \leq T^4$ (recall that $h\alpha \leq T$ at this time). The ratio of the two energy densities is further redshifted as $t^{-\frac{7}{2}}$ (for $H \geq 10^{-18}$) which ensures the second necessary condition for the evaporation of condensate, i.e., that the plasma energy density is dominant over the energy density in the condensate. It can easily be checked that the condensate evaporates at $H \gg 10^{-18}$, before the
inflaton decay is completed.\footnote{If $\alpha$ oscillations start due to the back-reaction of $\phi$’s, from the beginning $g\alpha \leq T$ and $\Gamma_{\text{scatt.}} \sim g^4 T$. Therefore, there is no need to wait for further redshift of $\alpha$ and again the condensate evaporates at $H \gg 10^{-18}$.}

In those cases in which the plasma effects do not lead to an early oscillation of the flat direction, oscillations start at $H \simeq 10^{-16}$, when the low-energy supersymmetry breaking takes over the Hubble-induced one. It is important to find the time when the condensate will evaporate in these cases too. For such flat directions, the ratio of the baryon number density to the condensate density is of order one \cite{14}. Therefore, if the condensate dominates the energy density of the universe before evaporation, the resulting $\frac{n_B}{s}$ will also be of order one. Some regulating mechanism is then needed in order to obtain the value for successful big bang nucleosynthesis: $\frac{n_B}{s} \sim 10^{-10}$ \cite{20}.

Now consider a general flat direction with $h \simeq 10^{-2}$. As we showed, in the $5 \leq n \leq 9$ cases for $M = 10^{19}$ GeV, and the $n = 9$ case for $M = 10^{16}$ GeV, plasma effects are not important and the flat direction starts oscillating at $H \simeq 10^{-16}$. By $H \simeq 10^{-18}$ the inflaton has efficiently decayed and $\alpha$ has been redshifted by a factor of $10^{-2}$. From then on, the universe is radiation-dominated, so $\alpha \propto t^{-\frac{1}{4}}$ and $T \propto t^{-\frac{1}{2}}$. Therefore, the energy density in the condensate is redshifted as $t^{-\frac{3}{2}}$ whilst the energy density in radiation is redshifted as $t^{-2}$. If the condensate does not evaporate (or decay) until very late times, its energy density dominates that of the radiation and universe will again be matter-dominated. At the beginning of oscillations, i.e., at $H \simeq 10^{-16}$, $\alpha$ has the largest vev in the $n = 9$ case for $M = 10^{19}$ GeV, which is $\alpha \simeq 10^{-16}$. At $H \simeq 10^{-18}$ this is redshifted to $\alpha \simeq 10^{-30}$ which still leaves $h\alpha > T$, so plasmons with this Yukawa coupling to the flat direction cannot be produced. However, since $\alpha$ redshifts more rapidly than $T$, eventually $h\alpha$ becomes of order $T$, after a time such that

$$T \simeq 10^{-\frac{101}{8}}, \quad \alpha \simeq 10^{-\frac{87}{8}}$$

It is easily seen that at this time the energy density in the condensate and in the radiation are of the same order. Moreover, $\Gamma_{\text{scatt.}} \sim 10^{-8} T \gg H$ and the condensate evaporates promptly. This case is marginal as the condensate almost dominates the energy density of the universe at evaporation.

In the $5 \leq n \leq 8$ cases for $M = 10^{19}$ GeV and the $n = 9$ case for $M = 10^{16}$ GeV the vev is considerably smaller and the energy density in radiation is even more dominant. Therefore, a
general flat direction with \( h \simeq 10^{-2} \) will evaporate before dominating the energy density of the universe. We summarize the situation for a general flat direction with \( h \simeq 10^{-2} \), regarding both the early, i.e., plasma-induced, oscillation, and evaporation, in the following Table.

### Viability of scenarios with generic Yukawa coupling \( h = 10^{-2} \)

| \( n \) | \( M = 10^{16} \text{ GeV} \) | \( M = 10^{19} \text{ GeV} \) |
|-----|------------------|------------------|
|     | Early Oscillation | Evaporation     | Early Oscillation | Evaporation     |
| 4   | √                | √               | √                | √               |
| 5   | √                | √               |                  | √               |
| 6   | marginal         | √               |                  | √               |
| 7   | marginal         | √               |                  | √               |
| 8   | marginal         | √               |                  |                  |
| 9   |                  | √               | marginal         |                  |

For the exceptional flat direction with \( h \simeq 10^{-4} \) the situation is different. Here plasma effects are not important in the \( 7 \leq n \leq 9 \) cases for \( M = 10^{19} \text{ GeV} \). In the \( n = 9 \) case the condition for thermal production of \( \chi \)'s, \( h\alpha = T \) gives

\[
T \simeq 10^{-73}, \quad \alpha \simeq 10^{-45}
\]

which means we do not need as much redshift to reduce \( \alpha \), so \( \chi \)'s are produced earlier and at a higher temperature. However, \( \Gamma_{\text{scatt.}} \sim 10^{-16}T \), which is much smaller than \( H \) at this time. Therefore, the condensate cannot evaporate by scattering off the \( \chi \)'s. It is easily seen that \( hT > m_\chi^2 \simeq 10^{-16} \) when \( h\alpha = T \). This implies that the mass and energy density of the flat direction are \( hT \) and \( h^2\alpha^2T^2 \), respectively, upon thermal production of \( \chi \)'s, and the energy density in the flat direction and the thermal bath are comparable. As long as \( hT \geq m_\chi^2 \), \( \alpha \) and \( T \) are both redshifted as \( t^{-\frac{1}{2}} \). During this interval \( \frac{\alpha}{T} \) remains constant and the flat direction and plasma energy densities remain comparable. Later, when \( T < 10^{-12} \) we have \( hT < m_\chi^2 \), and the energy density in the condensate is \( m_\chi^2\alpha^2 \) and begins to dominate the thermal energy density. At some point, \( g\alpha < T \) and \( \phi \)'s can be produced thermally. The scattering rate of the condensate off the \( \phi \)'s is \( \Gamma_{\text{scatt.}} \sim 10^{-4}T \) which is clearly at equilibrium. However, the energy density in the condensate is now overwhelmingly dominant and evaporation does not occur. For the \( n = 7, 8 \) cases the situation is similar and the
condensate does not evaporate. The summary for the exceptional flat direction, regarding both the early, i.e., plasma-induced, oscillation, and evaporation, is illustrated in the Table below.

Viability of scenarios with exceptional Yukawa coupling $h = 10^{-4}$

|       | $M = 10^{16}$ GeV |       | $M = 10^{19}$ GeV |
|-------|-------------------|-------|-------------------|
|       | Early Oscillation | Evaporation | Early Oscillation | Evaporation |
| $n = 4$ | √                | √                  | √                          | √          |
| $n = 5$ | √                | √                  | √                          | √          |
| $n = 6$ | √                | √                  | marginal                  | √          |
| $n = 7$ | √                | √                  |                            |            |
| $n = 8$ | √                | √                  |                            |            |
| $n = 9$ | √                | √                  |                            |            |

In summary: a general flat direction, i.e., with $h \simeq 10^{-2}$, which does not have plasma-induced early oscillation, does not come to dominate the energy density of the universe (the $n = 9$ case for $M = 10^{19}$ GeV is marginal). For the exceptional flat direction, i.e., with $h \simeq 10^{-4}$, the situation is different and it dominates the energy density of the universe before decay.

6 Discussion

We have found that all flat directions, except those which are lifted by nonrenormalizable superpotential terms of high dimension and with a large mass scale in the denominator, start oscillating at early times due to plasma effects. For a general flat direction with $h \simeq 10^{-2}$ these are the $n = 4$ case for $M = 10^{19}$ GeV and the $4 \leq n \leq 8$ cases for $M = 10^{16}$ GeV (with the $6 \leq n \leq 8$ cases being marginal and sensitive to model-dependent parameters). For the exceptional flat direction with $h \simeq 10^{-4}$ these are the $4 \leq n \leq 6$ cases for $M = 10^{19}$ GeV (with the $n = 6$ case being marginal and sensitive to model-dependent parameters) and all $n$ for $M = 10^{16}$ GeV. In these cases it is difficult to achieve efficient baryon asymmetry generation by the oscillation of the condensate along the flat direction. We showed that a general flat direction, i.e., one with $h \simeq 10^{-2}$, which is not lifted by thermal effects, still evaporates before dominating the energy density of the universe. This is not important for baryogenesis, however, and the resulting dilution by the thermal bath can be used to
regulate the \( \frac{n}{s} \) which is initially of order one. On the other hand, the exceptional flat direction, i.e., one with \( h \simeq 10^{-4} \), which is not lifted by plasma effects, dominates the energy density of the universe before its decay.

For models with supersymmetry breaking via low-energy gauge mediation, on the other hand, the evaporation of the condensate has yet another implication. In such models there is a candidate for cold dark matter, the so called Q-ball [21]. In order to have stable Q-balls as dark matter candidates, some flat directions must dominate the energy density of the universe. This means that any flat direction which is evaporated by the thermal bath cannot be used to form a Q-ball.

Now the question is which flat directions are lifted by \( n > 4 \) terms. A look at [10] reveals that only 18 out of 295 directions which are \( D- \) and \( F- \) flat at the renormalizable level in the MSSM are not lifted at the \( n = 4 \) level. Even a smaller subset of only 2 flat directions are not lifted at the \( n = 6 \) level. If nonrenormalizable terms with \( n = 4 \) and \( n = 5 \) are not forbidden by imposing other symmetries, only a very few flat directions in the MSSM can be used for baryogenesis and even fewer for Q-ball formation (regardless of the mass scale in the denominator or Yukawa couplings of these flat directions). This is if all higher-order terms which respect gauge symmetry exist in the superpotential. With other symmetries (discrete or continuous) imposed on the model, a specific flat direction will, in general, be lifted at a higher level. The initial vev of \( \alpha \) can then be larger and \( \chi \) and \( \phi \) quanta may not be produced thermally, and the standard treatment of the Affleck-Dine baryogenesis may be valid. Model-dependent analysis is needed to identify at which level a given flat direction is actually lifted, in a given model.

Finally, an interesting possibility is the parametric-resonance decay of a supersymmetric flat direction to the fields \( \phi \) to which it is gauge-coupled. The occurrence and implications of a potential parametric resonance are more pronounced for those flat directions which start their oscillations at \( H \simeq 10^{-16} \), as in the standard scenario. They have an incredibly large \( q = \left( \frac{g_\alpha}{2m_{3/2}} \right)^2 \) which could be as large as \( O(10^{20}) \) (the parameter \( q \) determines the strength of resonance [16]). Explosive resonance decay could also prevent these flat directions from dominating the energy density of the universe. However, the situation is too complicated to allow simple estimates based on the results of

\footnote{For those flat directions which have plasma-induced oscillations, \( m_{3/2} \) is replaced by \( hT \) or \( gT \), leading to a considerably smaller \( q \).}
parametric-resonance decay of a real scalar field. First of all, the renormalizable part of the scalar potential (including the $D$-term part which is responsible for parametric-resonance decay to $\phi$'s) is fully known and very complicated. Moreover, the flat direction itself is a complex scalar field. This may result in out-of-phase oscillations in the imaginary part of the flat direction, as well as in other scalar fields which are coupled to the same $\phi$, which can then substantially alter the outcome of simple parametric resonance [22].

7 Conclusion

In conclusion, we have seen that many of the MSSM flat directions may start their oscillations differently than in the standard scenario, where the low-energy supersymmetry breaking determines the onset of oscillations. The two key ingredients for such a different behaviour are: superpotential Yukawa couplings of the flat directions to other fields, and the thermal plasma from partial inflaton decay, whose instantaneous temperature is higher than the reheat temperature. Together, these lead to an earlier start of the oscillations. On the one hand, the masses of those fields which are coupled to the flat direction that are induced by the flat-direction vev are then small enough to be kinematically accessible to inflaton decay and, on the other hand, induce large enough thermal masses for flat directions from the back-reaction of those fields to overcome the negative Hubble-induced mass-squared of the flat directions. Subsequently, thermal masses and $A$ terms may be responsible for baryo/leptogenesis, but typically result in an insufficient baryon/lepton asymmetry of the universe. The oscillations are also terminated earlier, due to evaporation of the flat direction through its interactions with the thermal plasma. It was also shown that even for many flat directions whose oscillations are not initiated by plasma effects, these effects cause them to evaporate before dominating the energy density of the universe.

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