Thermal Diffusive Free Convective Radiating Flow Over an Impulsively Started Vertical Porous Plate in Conducting Field

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Abstract

In this manuscript we have studied the laminar convective heat and mass transfer flow of an incompressible, viscous, electrically conducting fluid over a fluid over an impulsively started vertical plate with conduction-radiation embedded in a porous medium in the occurrence of transverse magnetic field. An exact solution is derived by solving the dimensionless main coupled partial differential equations using Laplace transform technique. The properties of important physical parameters on the velocity, temperature, concentration, Skin friction, Sherwood number and Nusselt number have been studied through graphs.

Keywords: MHD; Porous medium; Thermal diffusion; Thermal radiation; Shear stress; Nusselt number and Sherwood number

Nomenclature

\( C^i \): Species concentration fluid
\( C_p \): Specific heat at constant pressure
\( C_w^i \): Concentration of the fluid for away from the plate
\( C_n^i \): Concentration level near the plate/wall
\( D \): Chemical molecular diffusivity
\( g \): Acceleration due to gravity
\( q \): Radiative heat flux
\( Gr \): Thermal Grashof number
\( Gm \): Modified Grashof number
\( K_r \): Permeability parameter
\( M \): Hartmann number
\( Nu \): Nusselt number
\( P_r \): Prandtl number
\( S_o \): Soret number
\( S_h \): Sherwood number
\( T_w \): Fluid temperature at the surface
\( u \): Dimensional velocity components
\( S_c \): Schmidt number
\( T^\circ \): Temperature
\( u_c \): Plate velocity
\( \beta \): Coefficient of volume expansion for mass transfer
\( \kappa \): Thermal conductivity
\( \rho \): Density
\( \tau \): Shearing stress
\( W \): Condition on the wall
\( \infty \): Free stream condition

Introduction

Several transport processes exist in industries and technology where the transfer of heat and mass occurs simultaneously as an outcome of thermal diffusion and diffusion of chemical species. Natural convection induced by the simultaneous achievement of buoyancy forces resulting from thermal and mass diffusion is of considered interest in nature and in many industrial applications such as cosmic fluid dynamics, meteorology, chemical industry, cooling of nuclear reactors, magneto hydrodynamics power generators and the earth's core. Bharat et al.[1] investigated the effects of mass transfer on MHD free convective radiation flow over an impulsively started vertical plate embedded in a porous medium. Ahmed et al. [2] discussed convective laminar radiating flow over an accelerated vertical plate embedded in a porous medium with an external magnetic field. Chamka et al. [3] studied thermal radiation and buoyancy effects on hydro magnetic flow over an accelerating porous surface with heat source or sink. Ahmed et al. [4] examined Non-linear magneto hydrodynamic flow more an impulsively started vertical plate in a saturated porous regime Laplace and Numerical approach. Ravi Kumar et al. [5] examined MHD

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double diffusive and chemically reactive flow through porous medium bounded by two vertical plates. Palani et al. [6] studied free convection MHD flow with thermal radiation from an impulsively-started vertical plate. Ravi Kumar et al. [7] discussed heat and mass transfer effects on MHD flow of viscous fluid through non-homogeneous porous medium in occurrence of temperature dependent heat source. Chen et al. [8] discussed heat and mass transfer in MHD flow by ordinary convection from a permeable, inclined surface with variable wall temperature and concentration. Ravi Kumar et al. [9] discussed combined effects of heat absorption and MHD on convective Rivilin-Ericksen flow past a semi-infinite perpendicular porous plate with variable temperature and suction. Ahmed et al. [10] examined Numerical/Laplace transform investigation for MHD radiating heat/mass transport in a Darcian porous regime bounded by an oscillating vertical surface. Kumar et al. [11] discussed thermal radiation and mass transfer effects on MHD flow past a vertical oscillating plate among variable temperature effects variable mass diffusion. Hossain et al. [12] studied radiation effect on mixed convection along a perpendicular plate with uniform surface temperature. Ibrahim et al. [13] examined similarity solution of heat and mass transfer for normal convection over a moving vertical plate with internal heat generation and a convective boundary state in the presence of thermal radiation, viscous dissipation, and chemical reaction. Pradyumna Kumar et al. [14] examined analytical solution of magnetic hydro magnetic free convective flow through porous media with time dependent temperature and concentration. Das et al. [15] discussed mass transfer effects on MHD flow and heat transfer past a vertical porous plate throughout a porous medium below oscillatory suction and heat source [16]. Seth et al. [17] studied effects of thermal radiation and rotation on unsteady hydro magnetic free convection flow past an impulsively moving vertical plate with ramped temperature in a porous medium. Das et al. [18] discussed heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in porous medium. Kumar et al. [19] examined magnetic field effect on transient free of charge convection flow through porous medium past an impulsively started vertical plate with fluctuating temperature and mass diffusion. Mamtha et al. [20] discussed thermal diffusion effect on MHD mixed convection unsteady flow of a non-Newtonian liquid past a semi-infinite vertical porous plate with radiation and mass transfer. Reddy et al. [21] examined unsteady MHD free convection flow of a Kuvshinski fluid past a vertical porous plate in the presence of chemical reaction and heat source/sink. Kumar et al. [22] investigated theoretical investigation of an unsteady magnetic hydro magnetic free convection heat and mass transfer flow of a non-Newtonian liquid flow past a permeable moving perpendicular plate in the presence of thermal diffusion and heat sink. Reddy et al. [23] discussed mass transfer and heat generation effects on magnetic hydro magnetic free convection flow past an inclined vertical surface in a porous medium. Senapati et al. [24] examined magnetic effects on mass and heat transfer of hydrodynamics flow past an oscillating vertical plate in presence of chemical reaction. Raju et al. [25] investigated MHD convective flow through porous medium in a vertical channel with insulated and impermeable base wall in the presence of viscous dissipation and joule heating. The Effects of mass transfer on MHD free convective radiation flow over an impulsively started vertical plate embedded in a porous medium was studied by Bharat and Nityananda [1]. We have extended this work by including the thermal diffusion effect. Though it is an extension to the previous work, it will differ in several aspects like governing equations, non-dimensional parameters, figures etc. The novelty of this study is the investigation of various physical parameter on the flow quantities in the presence of thermal diffusion.

**Mathematical Formulations**

The laminar convective heat as well as mass transfer flow of an incompressible, viscous, electrically conducting fluid over an impulsively started vertical plate among conduction-radiations embedded in a porous medium in presence of transverse magnetic field has been studied. The $x'$ axis is taken the length of plate in the vertical upward direction and the $y'$ axis is taken normal to the plate. A transverse magnetic field of identical strength $B_0$ is assumed to be applied normal to the plate. It is also implicit that the thermal radiation along the plate and viscous dissipation is implicit to be negligible. The induced magnetic field and viscous dissipation is understood to be negligible. Initially it is assumed that the plate and fluid are at same temperature $T_0$ in the stationary situation with concentration level $C_0$. It may be expressed as a linear function of the temperature, after neglecting higher order terms.

\[
\frac{\partial u}{\partial t} = gB(T' - T_0) + gD\left(C' - C_0\right) + \frac{1}{\rho} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho} \frac{\partial T'}{\partial y} \quad (1)
\]

\[
\rho C_p \frac{\partial T'}{\partial t} = \kappa \frac{\partial^2 T'}{\partial y^2} - \frac{\partial q_r}{\partial y} \quad (2)
\]

\[
\frac{\partial C'}{\partial t} = D \frac{\partial^2 C'}{\partial y^2} + D_h \frac{\partial^2 T'}{\partial y^2} \quad (3)
\]

The initial and boundary conditions are

\[
t' \leq 0; \quad u' = 0, \quad T' = T_0', \quad C' = C_0' \quad \text{for ever} \quad y
\]

\[
t' > 0; \quad u' = u_0, \quad T' = T_0', \quad C' = C_0' \quad \text{at} \quad y = 0
\]

\[
t' > 0; \quad u' \to 0, \quad T' \to T_0', \quad C' \to C_0' \quad \text{at} \quad y \to 0
\]

The radiation heat flux term is simplified by making use of the Rosseland approximation [16] as

\[
q_r = \frac{-4\sigma' \frac{\partial T'}{\partial y}}{3 \quad a'} \quad (4)
\]

Where $\sigma'$ and $a'$ are the Stefan-Boltzmann steady and the mean absorption coefficient respectively. It should be noted that by using the Rosseland approximation, we limit our investigation to optically thick fluids. It temperature differences within the flow are sufficiently small, such that $T'$ may be expressed as a linear function of the temperature, then the Taylor's series for $T'$ and $T_0'$, after neglecting higher order terms is given by

\[
T' \approx 4T_0' T_0' - 3T_0'
\]

Substitute (5) and (6) in (2) we have

\[
\rho C_p \frac{\partial T'}{\partial t} = k \frac{16 \sigma' \frac{\partial T'}{\partial y}}{3 \quad a'} \frac{\partial^2 T'}{\partial y^2} \quad (7)
\]

Let us introduce the following non-dimensional terms in (1), (7) and (3)
Taking Laplace transform, the equations (9), (10) and (11) reduce to the following form after applying Laplace transform.

\[ \theta = \text{erfc}\left(\frac{\sqrt{s}}{2\sqrt{t}}\right) \]  \hspace{1cm} (20)

\[ C = \text{erfc}\left(\frac{\sqrt{s}C}{2\sqrt{t}}\right) - L_0\text{erfc}\left(\frac{\sqrt{s}C}{2\sqrt{t}}\right) + L_0\text{erfc}\left(\frac{\sqrt{s}C}{2\sqrt{t}}\right) \]  \hspace{1cm} (21)

The transformed initial and boundary conditions are

\[ \theta(0, y) = 0, \quad C(0, y) = 0 \quad \text{for every} \quad y \]

\[ \theta(\infty, y) = 0, \quad C(\infty, y) = 0 \quad \text{as} \quad y \to \infty \]  \hspace{1cm} (22)

The Skin friction at the surface of the plate is given by

\[ \tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{1}{\sqrt{\pi t}} \left(\frac{\partial u}{\partial y}\right)_{y=0} \]  \hspace{1cm} (23)

The Nusselt number and Sherwood number at the plate are respectively

\[ Nu = \left(\frac{\partial \theta}{\partial y}\right)_{y=0} = \sqrt{\frac{s}{\pi t}} \]  \hspace{1cm} (24)

\[ Sh = \left(\frac{\partial C}{\partial y}\right)_{y=0} = \sqrt{\frac{sC}{\pi t}} - L_0\sqrt{\frac{sC}{\pi t}} + L_0\sqrt{\frac{sC}{\pi t}} \]  \hspace{1cm} (25)

Result and Discussion

To discuss the physical implication of various parameters involved in the results (20) - (25), the numerical calculation has been carried out for the distributions of velocity, temperature, concentration, Skin friction, Nusselt number and Sherwood number. The effects of various physical parameters on these flow quantities such as Hartmann number M, Prandtl number Pr, Soret number S0, Schmidt number Sc. A comparison of curves in the figure shows a decrease in the concentration as the Soret number S0 increases on the species concentration profiles. From this figure, it is noticed that an increase in the concentration number Sc results in an increase in the concentration.
profiles. Figure 3 reveals the temperature profiles for different values of Prandtl number Pr. It is observed that the temperature decreases as an increase in the values of Prandtl number Pr. The reason is that smaller values of Prandtl number are equivalent to an increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface extra rapidly for higher values of Pr (Appendix). Hence, in the case of larger Prandtl number the thermal boundary layer is thinner and the rate of heat transfer is reduced. Figure 4 shows the temperature profile for different values of Radiation Parameter Na. From this figure it is noticed that velocity decreases with an increase in Na. Figure 5 depicts the effect of Grashof number Gr on velocity. It is observed that an increase in Gr leads to an increase in the velocity boundary layer. Figure 6 shows the velocity profile for different values of modified Grashof number. From this figure it is observed that an increase in the values of modified Grashof number results in an increase in the velocity profiles. Figure 7 shows the velocity profiles for different values of radiation parameter Na. From this figure it is noticed that velocity decreases with an increase in Na. Figure 8 reveals the effect of Prandtl number Pr on the velocity.
which intern increases the velocity of a fluid. Figure 11 illustrates the velocity profiles for different values of Hartmann number M. From this figure it is notice that velocity decreases with an increase in Hartmann number M. Figure 12 reveals the effect of time t on the transient velocity profiles. It is evident from the figure that the velocity decreases with increase in t. The velocity profiles are plotted in Figure 13 for various values of permeability parameter $K_r$. From this figure, it is notice that the velocity increases with the increase in the values of the permeability profile. It is evident from the figure that the velocity decreases with an increase in Pr. Figure 9 illustrates the velocity profiles for different values of Schmidt number Sc. It has been observed that the velocity decreases with increase in Sc. Figure 10 shows the velocity profiles for different values of Soret number $S_o$. It was found that an increase in the value of $S_o$ leads to an increase in the velocity distribution across the boundary layer. This is true, as the $S_o$ increases, small light molecules and large heavy molecules get separated under a temperature gradient.
parameter $K$. Physically, an increase in the permeability of porous medium leads to rise, in the flow of fluid during it. When the holes of the porous medium become large, the resistance of the medium may be neglected. A similar approach is noticed with Raju et al. [25]. Figure 14 shows Sherwood number is presented against time $t$ for different values of Schmidt numbers. We observed that Sherwood number increases with increasing Schmidt numbers $Sc$. Figure 15 shows the Sherwood number ($Sh$) on the porous plate for different values of Soret number $So$. The result display that an increase in the value of $So$ results an decrease in the Sherwood number. Figure 16 presents the variation of the Nusselt number $Nu$ against time $t$ for various values of Prandtl number $Pr$. From this figure we notice that Nusselt number increases when the values of Prandtl number $Pr$ increase. Figure 17 illustrates the Nusselt number for different values of radiation parameter $Na$. From this figure it is noticed that Nusselt number increases with increase in $Na$. Figures 18 and 19 depicts skin-friction against time $t$ for different values of Grashof number $Gr$ and modified Grashof number $Gm$. From these figures it is notice that Skin-friction increases with an increase $Gr$ and $Gm$. Figure 20 depict skin-friction against time $t$ for different values of Hartmann number $M$. It is observed that the skin friction decreases with increase in Hartmann number $M$. Figures 21-23 depicts skin-friction against time $t$ for different values of Permeability parameter $K_r$. Physically, an increase in the permeability of porous medium leads to rise, in the flow of fluid during it.
$K_r$, Schmidt number $Sc$ and Prandtl number $Pr$. From these figures it notice that skin-friction increases with an increases $K_r$, $Sc$ and $Pr$.

**Conclusion**

In this paper a theoretical examination has been carried out to study thermal diffusion on MHD free convective radiating flow more an impulsively started vertical plate embedded in a porous medium. Solutions for the model has been derived by using Laplace transform technique. Some conclusions of the study are as follow:

- Concentration distributed is observed to decrease with increase in Schmidt number and it increases with increase in Soret number.
- Temperature decreases with increase in $Pr$ and $Na$.
- Velocity increases with increase in $Gr$, $Gm$, $S_0$ and $K_r$ while it decreases with increase in $Na$, $Pr$, $Sc$, $M$ and $t$.
- Sherwood number increase with increase in $Sc$ and decrease with increase $S_0$.
- Nusselt number increases with increase in $Pr$ and $Na$.
- Skin-friction increases with an increase in $Gr$, $Gm$, $K_r$, $Sc$ and $Pr$ and decreases with increase in $M$.

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