Bounds on the origin of extragalactic ultrahigh energy cosmic rays from the IceCube neutrino observations

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(Dated: September 11, 2014)

We study general implications of the IceCube observations in the energy range from $10^6$ GeV to $10^{10}$ GeV for the origin of extragalactic ultrahigh energy cosmic rays assuming that high energy neutrinos are generated by the photomeson production of protons in the extragalactic universe. The observed neutrino flux at the energy of PeV gives strong bounds on the photomeson-production optical depth of protons in their sources and the intensity of the proton component of extragalactic cosmic rays. The neutrino flux implies that extragalactic cosmic-ray sources should have the optical depth greater than $\sim 0.01$ and contribute to more than a few percent of the observed bulk of cosmic rays at 10 PeV. If the spectrum of cosmic rays from these extragalactic sources extends further well beyond 1 EeV, extragalactic cosmic rays are dominant in the observed total cosmic-ray flux at 1 EeV and above, favoring the dip transition model of cosmic rays. The cosmic-ray sources are also required to be efficient neutrino emitters with the optical depth close to unity in this case.

The highest energy cosmic-ray ($\sim 10^{11}$ GeV) sources should not be strongly evolved with redshift to account for the IceCube observations, suggesting that any cosmic-ray radiation scenario involving distant powerful astronomical objects with strong cosmological evolution are strongly disfavored. These considerations conclude that none of the known extragalactic astronomical objects can be simultaneously a source of both PeV and trans-EeV energy cosmic rays. We also discuss a possible effect of cosmic-ray propagation in magnetized intergalactic space to the connection between the observed total cosmic-ray flux and neutrino flux.

PACS numbers: 98.70.Sa, 95.85.Ry

I. INTRODUCTION

The detection of PeV-energy neutrinos and the follow-up analysis by the IceCube Collaboration opened high-energy neutrino astronomy. These detected neutrinos are difficult to be reconciled with cosmogenic neutrinos (but see also Ref. [4]), and therefore revealed the existence of an astrophysical “on-source” neutrino flux in the energy range from sub-PeV to PeV. The bulk of these ultrahigh energy (UHE) neutrinos are expected to originate from the interactions of UHE cosmic ray (UHECR) protons with matter ($pp$ interactions) or photons ($\gamma p$ interactions) in their sources in situ, and thus carries information about the origin of UHECRs, which has been a longstanding mystery in astroparticle physics. Although the arrival directions of the detected UHE neutrinos are consistent with an isotropic distribution and exhibit no statistically significant correlation with any particular astronomical objects discussed as possible UHECR-driven neutrino emitters, the bulk intensity of the observed neutrinos, $E_\nu^2 d^2 J_{\nu_e+\nu\bar{\nu}_\mu}/dE_\nu \sim 3.6 \times 10^{-8}$ GeV cm$^{-2}$ s$^{-1}$ sr$^{-1}$, offers an important clue to understand the general characteristics of UHECR sources through the link between the cosmic-ray flux and the neutrino emissivity mainly determined by interaction target density.

The neutrino intensity is also considered as an index to probe a redshift distribution of UHECR sources because neutrinos can penetrate over cosmological distance. Neutrino intensity averaged over the sky is a consequence of the integral of neutrino emission from high redshift epochs to the present cosmic time. It traces the cosmic evolution of a neutrino emission rate per comoving volume. This advantage is especially prominent in 100 PeV – 10 EeV (EeV = $10^3$ PeV) range where the “GZK cosmogenic” neutrinos are expected to be dominant in the neutrino sky, which are secondary neutrinos produced by the Greisen-Zatsepin-Kuz’mín (GZK) process, i.e., the photomeson production of UHECRs with the cosmic microwave background (CMB) photons. Since the intensity of UHECRs and the evolution of CMB over the cosmological history are well understood, the most uncertain parameter to determine the cosmogenic neutrino intensity is the source distribution function (SDF) of UHECRs in the redshift space. The upper limit of neutrino intensity with energies beyond 100 PeV can hence be converted into the constraints on the SDF of UHECRs. The null observation of neutrinos above 100 PeV by the IceCube experiment has indeed put the most stringent bound on the cosmological evolution of UHECR emission rate.

In this work, we study implications for general characteristics of extragalactic UHECR sources obtained by

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neutrino observation with the IceCube observatory. Extragalactic on-source neutrino production through photomeson interactions \((\gamma p)\) is considered as a working hypothesis. We derive an analytical formula to calculate the intensity of on-source astrophysical neutrinos. This is a formulation extended from the analytical formula to estimate the cosmogenic neutrino flux \[11\]. The connection between the observed neutrino flux and UHECR proton flux allows for constraining the optical depth of photomeson collision in UHECR sources. Then, we bound the cosmological evolution of the UHECR sources by the measured neutrino flux at PeV energies \[2\] as well as by the upper limit of neutrino intensity in the energy region above 100 PeV \[11\].

This paper is laid out as follows. The analytical formula for diffuse neutrino intensity is proposed in Section II. In Section III, we derive general constraints on UHECR sources from IceCube observations and their possible links to the total cosmic-ray flux. We interpret the constraints derived in Section III and discuss some effects to the constraints, including a propagation effect in magnetized intergalactic space, in Section IV and then summarize this study in Section V. The standard ΛCDM cosmology with \(H_0 = 73.5 \text{ km s}^{-1} \text{ Mpc}^{-1}\), \(\Omega_M = 0.3\), and \(\Omega\Lambda = 0.7 \ [12\] is assumed throughout the paper, which are the Hubble parameter, the matter density normalized by the critical density, respectively.

II. ANALYTICAL FORMULA FOR ESTIMATING ON-SOURCE \(\nu\) INTENSITY

The intensities of neutrinos from in situ photomeson production in UHECR sources are calculated by an analytical manner with several approximations. The analytical formulation in this study follows that for GZK cosmogenic neutrinos which has already been published \[10\]. Key strategies are the approximations that the kinematics of photomeson production is treated as single-pion production and that the \(\Delta\) resonance governs the interaction. A power-law distribution of target photons is adopted for on-source neutrinos, instead of the CMB spectrum for GZK cosmogenic neutrinos. The analytical formula allows us to constrain astrophysical parameters concerning UHECR emission from the observational results of IceCube without intensive computation. In this section, we describe the essence of deriving the analytical formulation and resultant equations to represent diffuse on-source neutrino intensity. The details in the formulation are found in Ref. \[10\].

The neutrino number intensity \(dJ_\nu/dE_\nu\) can be calculated by

\[
\frac{dJ_\nu}{dE_\nu}(E_\nu) = \frac{c n_\nu}{4\pi} \int_0^{z_{\text{max}}} \int_0^{1+z} \frac{d\psi(z)(1+z)}{dz} \frac{d^2 N_\nu}{dE_\nu dE^\nu_\pi}(E_\nu^\nu, z),
\]

where \(d^2 N_\nu/dE_\nu dE^\nu_\pi\) is the neutrino yield of a UHECR source located at the redshift of \(z\) and \(c\) is the speed of light. The superscript \(s\) represents physical quantities measured at a redshift \(z\). Throughout this paper, we call this frame the source frame. The comoving number density of UHECR sources, i.e., neutrino sources, is represented as \(n_\nu(z)\) from the neutrino source number density in the local universe \(n_0\) and its cosmological evolution factor \(\psi(z)\). The evolution factor \(\psi(z)\) is parametrized as \((1+z)^m\) such that the parameter \(m\) represents the “scale” of the cosmological evolution often used in the literature. Together with the maximum redshift of UHECR sources \(z_{\text{max}}\), the index describes a general characteristic of UHECR sources.

The neutrino yield is given by a convolution of the number of UHECRs, the number density of target photons, and the energy distribution of generated neutrinos per interaction. Cosmic rays are assumed to be distributed isotropically in the rest frame of the flow of plasma moving towards the observer with the Lorentz factor of \(\Gamma\). Cosmic-ray distribution in the plasma rest frame (simply called the rest frame below) is Lorentz-transformed to that in the source frame where cosmic rays are beamed. Now that cosmic rays are ultrarelativistic, neutrinos are well approximated to be produced in the same momentum directions as those of the cosmic rays, and then neutrinos are beamed. For convenience, we define the neutrino yield as an isotropic-equivalent form as

\[
\frac{d^2 N_\nu}{dE_\nu dE^\nu_\pi}(E_\nu^\nu) = 4\pi \frac{d^2 N_\nu}{dE_\nu dE^\nu_\pi d\Omega_\nu}(E_\nu^\nu; \mu^s = 1),
\]

where \(d^2 N_\nu/dE_\nu dE^\nu_\pi d\Omega_\nu\) is an angular-dependent neutrino yield and \(\mu^s\) is the cosine of the angle between the direction of the plasma flow and the momentum of neutrinos. The angular-dependent neutrino yield with \(\mu^s = 1\) can be evaluated as a convolution of the number of cosmic rays moving along with the plasma motion, the number density of target photons, and the energy distribution of produced neutrinos per reaction. The energy distribution of cosmic rays towards the observer is represented as a power-law function if the spectrum of cosmic rays is described by a power-law function in the rest frame. Thus, we denote the spectrum of these cosmic rays in the source frame \(d^2 N_{\text{CR}}/dE^{\text{CR}}\) as \(d^2 N_{\text{CR}}/dE^{\text{CR}} = \kappa_{\text{CR}}(E^\text{CR}_0/E^\text{CR}_0)^{-\alpha}\). Here, \(E^\text{CR}_0\) is the reference energy of cosmic rays in the source frame and will be convenient to set \(E^\text{CR}_0 = 10\ \text{PeV}\) when we consider PeV-energy neutrinos. The number of cosmic rays in the emission region can be estimated by using the escape timescale of cosmic rays from the region \(t^\text{esc}\) as \(dN_{\text{CR}}/dE^{\text{CR}} = t^\text{esc}d^2 N_{\text{CR}}/dt^s dE^{\text{CR}}\). Note that this escape timescale is transformed into that in the rest frame \(t^\text{esc}\) by \(t^\text{esc} = \Gamma t^\text{esc}\). Primed (‘) characters represent quantities measured in the rest frame below.

The energy distribution of neutrinos generated from an interaction is

\[
Y(E_\nu^\nu; E^\text{CR}_\pi, s) = \int dE_\pi \frac{d\sigma_{\nu\pi}}{dE^\nu_\pi}(E^\nu_\pi; E^\text{CR}_\pi, s) \frac{d\rho_{\nu\pi}}{dE^\nu_\pi}(E^\nu_\pi; E^\nu_\pi),
\]

where

\[
da_{\nu\pi}/dE^\nu_\pi(E^\nu_\pi; E^\text{CR}_\pi, s) = \int dE_\pi \frac{d\sigma_{\nu\pi}}{dE^\nu_\pi}(E^\nu_\pi; E^\text{CR}_\pi, s) \frac{d\rho_{\nu\pi}}{dE^\nu_\pi}(E^\nu_\pi; E^\nu_\pi),
\]

and

\[
\rho_{\nu\pi}(E^\nu_\pi; E^\text{CR}_\pi, s) = \int dE_\pi \frac{d\sigma_{\nu\pi}}{dE_\pi}(E_\pi; E^\text{CR}_\pi, s) \frac{d\rho_{\nu\pi}}{dE_\pi}(E_\pi; E^\nu_\pi),
\]
where \( \frac{d\sigma_{p\gamma}}{dE_{\gamma}'} \) is the differential cross-section of charged-pion production as a function of the energy of a generated charged pion \( E_{\gamma}' \), \( d\rho_{p\gamma\gamma}/dE_{\gamma}' \) is the energy distribution of neutrinos produced by the decay of charged pions with the energy of \( E_{\gamma}' \), and \( s \) is a Mandelstam variable or the square of invariant mass of the cosmic-ray nucleon and a target photon in the photonson collision with the cross section of \( \sigma_{p\gamma} \). The approximated expressions of both the functions are given in Ref. [10]. In this approximation, the energy distribution of neutrinos per interaction depends on only \( E_{\gamma}^p \) and \( s \), not being dependent on the energy of a target photon \( E_{\gamma}' \) and its direction \( \mu^s \) explicitly. Furthermore, if we evaluate the convolution of the neutrino energy distribution and the distribution of target photons at the \( \Delta \) resonance, this convolution can be simplified to the multiplication of the energy distribution at \( s = s_R(\approx 1.5 \text{ GeV}^2) \) and the inverse of the mean free path of cosmic rays with the energy of \( E_{\gamma} \), \( \lambda_{p\gamma}(E) \). Since the inverse of \( \lambda_{p\gamma}(E) \) is the probability of interactions per unit path length, the optical depth of \( \gamma p \) interactions, \( \tau(E) = c t_{\text{esc}}/\lambda_{p\gamma}(E) \), explicitly appears in the convolution of the target photon number density and neutrino distribution. Recalling that \( \lambda_{p\gamma}(E) \) is transformed similarly to \( t_{\text{esc}} \), the optical depth can be evaluated in either the source frame or the rest frame. Thus, we evaluate the mean free path of \( \gamma p \) interactions in the frame where target photons are distributed isotropically for simplicity.

Typically, there are two cases of target photon distribution. One is the case that photons are isotropically distributed in the rest frame of the plasma moving with the Lorentz factor \( \Gamma \), i.e., the photons are distributed similarly to cosmic rays. This case corresponds to the situation that target photons are provided by electrons accelerated in the same place as cosmic rays, as mainly discussed for gamma-ray bursts and relatively weak blazars, that is, BL Lac objects. The other case is that photons are isotropically distributed in the source frame. This treatment is appropriate if target photons are mainly provided outside the relativistic plasma flow. This situation is often realized in quasars. Below we derive the analytical expression of diffuse neutrino intensity in the first case, but can also obtain that in the second case by replacing several quantities, as will be shown.

The mean free path of \( \gamma p \) interactions is given by

\[
\frac{1}{\lambda_{p\gamma}(E_{\gamma}^p)} = \int ds \frac{dn_{\gamma}/ds}{\sigma_{p\gamma}(s)}. \quad (4)
\]

We approximate the energy distribution of isotropically-distributed target photons by \( dn_{\gamma}/dE_{\gamma}' = \kappa' E_{\gamma}'^{-\gamma} \) in the range of \( E_{\gamma}^{\text{min}} \leq E_{\gamma}' \leq E_{\gamma}^{\text{max}} \) for a model-independent approach. The energy range of protons contributing to photonomes production is typically \((s_R - m_p^2)/4E_{\gamma}^{\text{max}} \leq E_{\gamma}^p \leq (s_R - m_p^2)/4E_{\gamma}^{\text{min}} \), where \( m_p \) is the proton mass. Since the amount of cosmic rays with the energy of \( E_{\gamma}^p > (s_R - m_p^2)/4E_{\gamma}^{\text{min}} \) is small due to their steep spectrum, this energy range is negligible in the integrations over \( E_{\gamma}^p \). On this assumption, the spectral number density of photons is written as

\[
dn_{\gamma}/ds = \frac{\kappa'}{4E_{\gamma}^p} \left( \frac{s - m_p^2}{2E_{\gamma}^p} \right)^{-\gamma} \frac{2^{\gamma+1}}{\gamma + 1}, \quad (5)
\]

as long as \( E_{\gamma}^p > \Gamma(s_R - m_p^2)/4E_{\gamma}^{\text{max}} \). Otherwise, the number density is zero. Here, we use \( E_{\gamma}^p = \Gamma E_{\gamma}^p \), which is justified by the fact that we focus on only cosmic rays directed to the observer. Thus, the mean free path is represented as

\[
\frac{1}{\lambda_{p\gamma}(E_{\gamma}^p)} = \frac{\kappa' 2^{\gamma+1}}{4E_{\gamma}^p \gamma + 1} \int ds \sigma_{p\gamma}(s) \left( \frac{s - m_p^2}{2E_{\gamma}^p} \right)^{-\gamma}. \quad (6)
\]

This indicates that the energy dependence of the optical depth is

\[
\tau_{p\gamma}(E_{\gamma}^p) = \tau_0 \left( \frac{E_{\gamma}^p}{E_0^p} \right)^{\gamma - 1}, \quad (7)
\]

with \( \tau_0 \), the optical depth at \( E_{\gamma}^p = E_0^p \), assuming that the escape timescale is independent of the energy of cosmic rays. Note that even if the escape timescale depends on the cosmic-ray energy with a power-law form, equation (7) can be applied after re-interpreting \( \gamma \) as the sum of the cosmic-ray spectral index and the index of the escape timescale. Using the expression of the optical depth, the isotropic equivalent neutrino yield of a UHECR source is

\[
\frac{d^2N_{\nu}}{dt^2dE_\nu} \approx \int dE_{\gamma}^p \kappa_{\nu} \left( \frac{E_{\gamma}^p}{E_0^p} \right)^{-\alpha} Y(E_\nu; E_{\gamma}^p, s_R) \tau(E_{\gamma}^p). \quad (8)
\]

The intensity of diffuse neutrinos is obtained by substituting the neutrino yield into equation (11). If we focus on leading terms for simplicity after integrating the formula by \( z \) with the approximation in Appendix A the neutrino intensity is

\[
\frac{dJ_{\nu}}{dE_\nu} (E_\nu) \simeq \frac{n_0 \kappa_{\nu} \tau_0}{(\alpha + 1 - \gamma)^2} \frac{c}{H_0} \frac{1}{\sqrt{(s_R + m_\pi^2 - m_p^2)^2 - 4s_R m_\pi^2}} \frac{3}{1 - \tau_0} E_\nu^{\alpha+1-\gamma} \zeta, \quad (9)
\]

where \( r_\pi = m_\pi^2/m_\pi^2 \simeq 0.57 \) is the muon-to-pion mass-squared ratio, and the factor of three corresponds to the
number of neutrinos produced from the $\pi$ meson and $\mu$ lepton decay chain. The factor $\zeta$ in Eq. (9) is the term which accounts for the redshift dependence and given by,

$$
\zeta = I(z_{\text{down}}, z_{\text{up}}) \left( \frac{E_{\nu}}{x_R^+(1-r_\pi)} \right)^{-(\alpha+1-\gamma)} I(z_{\text{down}}, z_{\text{down}}) \left( \frac{E_{\nu}}{x_R^- (1-r_\pi)} \right)^{-(\alpha+1-\gamma)} + \frac{2}{2m - 1} \left[ \frac{m_p^2}{4E_{\gamma}^{\text{max}}} \left( \frac{\Gamma \left[ \frac{s_R - m_p^2}{4E_{\gamma}^{\text{max}}} \right]}{x_R^-(1-r_\pi)} \right)^{-(\alpha+1-\gamma)} - \frac{1}{\Omega_M} \right],
$$

$$
I(z_1, z_2) = \frac{2}{2(m - \alpha + \gamma) - 3} \Omega_M^{m - \alpha + \gamma} \left[ \{ \Omega_M (1 + z_2)^3 + \Omega_\Lambda \}^{\frac{m - \alpha + \gamma}{4}} - \{ \Omega_M (1 + z_1)^3 + \Omega_\Lambda \}^{\frac{m - \alpha + \gamma}{4}} \right],
$$

where $x_R^\pm$ are the maximal and minimal bound of the relative energy of emitted pion normalized by the parent cosmic-ray energy. These are given by a kinematical relation,

$$
x_R^\pm = \frac{(s_R + m_\pi^2 - m_p^2) \pm \sqrt{(s_R + m_\pi^2 - m_p^2)^2 - 4s_R m_\pi^2}}{2s_R}. \tag{13}
$$

It is worth noting that the main energy range of neutrinos produced by an UHECR nucleon with energy of $E_{CR}$ is given by

$$
E_{CR} x_R^- (1 - r_\pi) \lesssim E_\nu \lesssim E_{CR} x_R^+(1 - r_\pi). \tag{14}
$$

The characteristic redshift $z_{\text{up}}$, $z_{\text{down}}$, and $z_{\text{down}}$ in Eqs. (10) and (12) are the bounds of the redshifts associated with the range of redshift in which UHECR sources are distributed, but also depend on neutrino energies $E_\nu$ due to kinematics of $\pi$-decay and the redshift energy loss. The expression of $z_{\text{up}}$ is given by

$$
1 + z_{\text{up}} = \frac{1 + z_{\text{max}}}{\frac{s_R - m_p^2}{4E_{\gamma}^{\text{min}}} \frac{\Gamma}{E_{\gamma}^{\text{max}}} x_R^- (1-r_\pi) \frac{(s_R - m_p^2)}{4(1+z_{\text{max}})} \frac{\Gamma}{E_{\gamma}^{\text{min}}} x_R^- (1-r_\pi) + \frac{1}{4}}.
$$

The other two characteristic redshifts can be obtained as follows; $z_{\text{down}}$ is also given by Eq. (15) replacing $(x_R^+, E_{\gamma}^{\text{max}})$ by $(x_R^-, E_{\gamma}^{\text{max}})$, and $z_{\text{down}}$ is given by Eq. (15) replacing $E_{\gamma}^{\text{max}}$ by $E_{\gamma}^{\text{min}}$.

The $E_{\gamma}^{\text{max/min}}$ and $x_R^\pm$ dependence of $\zeta$ reflects the association between the parent cosmic-ray energy $E_{CR}$ and the target photon energy $E_\gamma$ in the photon-meson collision. When $E_\nu \gtrsim (s_R - m_p^2) \Gamma x_R^- (1-r_\pi)/4E_{\gamma}^{\text{min}}$, then $\zeta \sim 0$ and the neutrino flux thus becomes null. This is a consequence of the fact that there is no target photons which can interact with cosmic rays through the $\Delta$ resonance.

The energies of target photons are within the photon-meson collision range in all sources distributed from $z = 0$ to $z = z_{\text{max}}$ when the neutrino energy satisfies the following condition:

$$
\frac{s_R - m_p^2}{4E_{\gamma}^{\text{max}}} \Gamma x_R^- (1-r_\pi) \lesssim E_\nu \lesssim \frac{s_R - m_p^2}{4E_{\gamma}^{\text{min}}} \Gamma x_R^- (1-r_\pi). \tag{16}
$$

The neutrino flux given by Eq. (9) follows the simple power law of $\sim E_\nu^{\alpha+\gamma-1}$ in this case. If we further assume $\gamma = 1$, i.e., the optical depth does not depend on cosmic-ray energies (see Eq. (7)), the on-source neutrino flux can be represented by the simpler form:
\[
\frac{dJ}{dE}(E) \simeq \frac{2n_0\kappa_{\text{CR}}}{\alpha^2 \alpha_0} \frac{e}{H_0} \frac{s_R}{\sqrt{(s_R + m_\pi^2 - m_\pi^2)^2 - 4s_Rm_\pi^2}} \frac{3}{2(m - \alpha)} \frac{E_0^{\alpha}(1 - e^{-\gamma_0})}{(m - \alpha + 1)} \frac{\Omega_M}{\Omega}\frac{\tau_0}{2(1 - \tau_0)}
\]

\[
\times \left[ \{\Omega_M(1 + z_{\text{max}}) + \Omega_\Lambda \}^{\alpha - \frac{\alpha}{\tau} - \frac{1}{\tau}} - 1 \right] \left( \frac{E}{x_R^{\frac{\alpha}{\tau_0}(1 - \tau_0)}(1 - \tau_0)} \right)^{-(\alpha + 1 - \gamma)}.
\]

(17)

Here, we replace \(\tau_0\) by \(1 - e^{-\gamma_0}\) to treat the case with \(\tau_0 \sim 1\). This equation corresponds to an advanced formulation of the pioneering work [13] based on the simplified energetics argument to estimate an upper bound of the on-source neutrino flux. It is worth noticing that the expressions of the diffuse neutrino intensity, i.e., Eqs. (9) and (17), does not strongly dependent on the Lorentz factor \(\Gamma\) explicitly because the effects of \(\Gamma\) are included in the definitions of some parameters, such as \(\kappa_{\text{CR}}\).

So far, we have considered the case that target photons are isotropically distributed in the rest frame of the plasma moving with the Lorentz factor \(\Gamma\). In the case that photons are isotropically distributed in the source frame, the diffuse neutrino intensity can be obtained by removing \(\Gamma\) in Eqs. (10), (12), and (15) and replacing \(E_{\gamma,\text{min/max}}\) into \(E_{\gamma,\text{min/max}}\), which are the minimum and maximum energy of target photons in the source frame.

Figure 4 displays the neutrino fluxes calculated by the present analytical equation for the on-source neutrinos and the formula in Ref. [10] for the GZK cosmogenic neutrinos. Here, \(\alpha = 2.3, \gamma = 1, \tau_0 = 0.1, \Gamma = 1, z_{\text{max}} = 3, E_{\gamma,\text{min}} = 0.3 \text{ eV}\) are assumed. The absolute intensity determined by \(n_0\) and \(\kappa_{\text{CR}}\) was calculated from the observed UHECR flux described in Ref. [10] for illustrative purposes. The on-source fluxes have a smooth cutoff at around 3 PeV in this particular set of realizations because \(E_{\gamma,\text{min}} = 0.3 \text{ eV}\) implies that a major population of neutrinos is in energy range of \(E_\nu \lesssim (s_R - m_\pi^2)^\frac{\Gamma_x^R}{4(1 + z_{\text{max}})} E_{\gamma,\text{min}} \sim 3 \text{ PeV}\) as described by Eq. (16). In the present formulation, the parameters \(E_{\gamma,\text{min}}\) and \(\Gamma\) control the maximal energy of neutrinos. Other factors such as synchrotron cooling of secondary muons could set the maximal energy bound as well, although they are not taken into account in our calculations. However, the mechanism setting the maximal energy of neutrinos is irrelevant in constraining UHECR source characteristics if the only arguments are based on the total neutrino intensity and the event rate seen by IceCube. It is insensitive to the detailed spectral shape of neutrinos such as the cut-off structure.

Hereafter we fix \(\Gamma = 1\) and \(\gamma = 1\) for simplicity and clearness. The overall conclusion remains valid in changing these parameters.

### III. RESULTS

#### A. The constraints on the optical depth and extragalactic UHECR nucleon flux

The final formula of a neutrino flux given by Eq. (9) is linked with the intensity of extragalactic cosmic rays through the optical depth \(\tau_0\). The UHECR intensity is calculated by integrating a UHECR spectrum over all the sources in the redshift space as

\[
\frac{dJ}{dE}(E) = n_0\kappa_{\text{CR}} \int_0^{z_{\text{max}}} (1 + z)^{1-\alpha} \psi(z) \frac{dt}{dz} e^{-\tau_0} \left( \frac{E}{E_0} \right)^{-\alpha},
\]

(18)
with neglecting intergalactic magnetic fields and the energy-loss in the CMB field during UHECR propagation. The convolution of Eq. (9) with the effective area of IceCube [1, 2, 11] predicts a neutrino event rate. Compared with the allowed range of the number of signal neutrino events in the IceCube observations estimated by the Feldman-Cousins (FC) upper and lower bounds [14], the predicted event rate places constraints on extragalactic UHECR intensity and the optical depth \( \tau_0 \).

First, we constrain these parameters at 10 PeV, i.e., \( 10^7 \) GeV, the representative cosmic-ray energy to produce PeV-energy neutrinos by \( \gamma p \) interactions. The range of the number of astrophysical neutrino events \( N_{\nu} \) at the FC 90% confidence level is \( 8.0 \leq N_{\nu} \leq 26.0 \) for the IceCube observation of the 28 event detection with the expected background of 12.0 in the energy region of \( E_{\nu} \gtrsim 3 \times 10^4 \) GeV [2]. The intensity of neutrinos per flavor is treated as one-third of the all-flavor neutrino intensity estimated by the analytical function, assuming the full mixing in the standard neutrino oscillation scenario. Figure 2 displays the resultant constraints on the optical depth and the UHECR intensity for several values of \( \alpha \), all of which are consistent with the IceCube observation at the present statistics [2]. The parameters concerning the UHECR SDF, \( m \) and \( z_{\text{max}} \), are assumed to follow the star formation rate [10] here and parameterized as \( \psi(z) \propto (1 + z)^m \) where \( \psi(z) \propto \begin{cases} (1 + z_{\nu})^{3.4} & (0 \leq z_{\nu} \leq 1) \\ \text{constant} & (1 \leq z_{\nu} \leq 4) \end{cases} \) (19).

We should remark that the constraints do not strongly depend on an assumed SDF because the cosmological evolution factor appears both in the neutrino and UHECR fluxes. The allowed region in the parameter space is rather small and is almost independent of the spectral index of cosmic rays \( \alpha \). The optical depth of \( \gamma p \) interactions needed to reproduce the PeV-energy neutrino detection is \( \tau_0 \gtrsim 10^{-2} \). The optical depth should be less than unity for UHECRs producing the PeV-energy neutrinos to also contribute to the observed total cosmic-ray flux. Under this requirement, extragalactic cosmic rays occupy at least a few percent of the observed total cosmic-ray flux.

Next, we investigate the corresponding bounds at the cosmic-ray energy of 1 EeV, i.e., \( 10^9 \) GeV, shown in Fig. 3. These can constrain the nature of the origin of UHECRs in the case that the energy spectrum of UHECR protons emitted from the sources extends to much higher energies than the observed neutrino energies. The allowed regions in the parameter space become much smaller than for the constraints for \( E_0 = 10 \) PeV because the spectrum of the total cosmic rays is steeper than that applied for the calculations. Note that the optical depth has been constrained for PeV energy protons following the events detected by IceCube, however we assume that this optical depth is energy independent and therefore can be applied to EeV energies.

The dumping factor \( e^{-\tau_0} \) (see Eq. 18) is taken into account to calculate UHECR flux at 1 EeV in the left panel. In other words, we consider the situation that \( \gamma p \) interactions also occur for UHECRs with the energies of EeV, that is, EeV-energy neutrinos are also produced. In this case it is likely to require the synchrotron cooling of \( \gamma/\mu \) in a strongly magnetized region to suppress the resultant neutrino flux at EeV to account for the fact that IceCube has not seen EeV-energy neutrinos in spite of its larger effective area than that at PeV-energies [1]. The other extreme case that the \( \gamma p \) collision does not occur in the EeV range is displayed in the right panel of Fig. 3 where the beam dump factor is dropped from the UHECR flux calculation. In either case, the constraints suggest that the optical depth of protons for PeV-energy neutrinos is rather high, \( \tau_0 \gtrsim 0.2 \), and also that a major fraction of UHECRs in the EeV region is extragalactic protons. It is supportive to the “dip” transition model [17] where the ankle structure of the cosmic-ray spectrum, which appears at 3 to 10 EeV, is caused by the energy-loss of UHECR protons by the Bethe-Heitler pair production with CMB photons. The dip transition model also predicts high cosmogenic neutrino flux up to \( 10^8 \) GeV [8, 9].
The observed all-particle cosmic-ray flux $10^{-8} \cdots 10^{-1}$

B. The constraints on the UHECR source evolution

The expected event rates with the IceCube neutrino observatory by using the derived analytical formula provides constraints on the UHECR source evolution. In the energy region above 100 PeV, the GZK cosmogenic neutrino production is considered as a “guaranteed” mechanism to yield neutrinos. The null detection of neutrinos above 100 PeV by IceCube has put solid bounds on the source evolution parameters $m$ and $z_{\text{max}}$ [11] with the analytical formulation of the cosmogenic neutrino flux [10]. The blue curves in Fig. 4 shows the obtained bounds assuming all the UHECRs in the highest energy region are protons.

Here we further constrain the evolution parameter space assuming that the sources of the highest energy cosmic rays also produce the on-source PeV-energy neutrinos that IceCube has detected. The normalization factor of UHECR flux, $\kappa_{\text{CR}}$, is determined by the observed integral flux at $E_{\text{CR}}$ of 100 EeV as in Ref. [10] in this case study. With this condition, the present analytical formula of neutrino flux gives the event rate as a function of $m$ and $z_{\text{max}}$ which can be compared with the observed event rate allowed by the FC statistical confidence level as stated in the previous section. The shaded area in Fig. 4 is the allowed region in the parameter space obtained from this comparison under two representative $\gamma p$ optical depth $\tau_0$ = 0.1 and 0.01. The red curve is the boundary for the calculated cosmic-ray flux not to exceed the total cosmic-ray flux at 1 EeV.

Fig. 4 indicates that the cases of $\tau_0 = 10^{-2}$ are already ruled out by the constraints from the intensity of cosmic-ray flux at 1 EeV because strong cosmological evolution, i.e., large $m$ with high $z_{\text{max}}$ is required. About a half of this parameter space is also disfavored by the null detection of cosmogenic neutrinos by IceCube. An optically-thicker source class with $\tau_0 = 0.1$ is consistent with these two constraints. Thus, if the sources of PeV-energy neutrinos are directly connected to the sources of the highest energy cosmic rays, they have relatively large $\gamma p$ optical depth to produce PeV-energy neutrinos and relatively weak cosmological evolution (see Section IV A for discussions on specific source classes). Note that the results depend on the spectral index $\alpha$.

IV. DISCUSSION

A. Case study for specific astronomical objects

We have obtained the general constraints on the optical depth of $\gamma p$ interactions to produce PeV-energy neutrinos. The connection between the IceCube neutrino flux and cosmic-ray flux at 10 PeV indicates the $\gamma p$ optical depth of $\tau_0 \gtrsim 10^{-2}$ and the fraction of extragalactic cosmic rays in the observed total cosmic ray intensity above a few percent (Fig. 2). If the spectrum of cosmic rays producing the PeV-energy neutrinos extends up to 1 EeV, the constraints on the optical depth becomes more stringent, $\tau_0 \gtrsim 0.2$ and more than ten percent extragalactic cosmic rays are required at 1 EeV (Fig. 3). Furthermore, if the cosmic-ray spectrum extends to the highest energies and explains the observed flux of UHECRs with the energies of 100 EeV, i.e., the spectrum is normalized at this energy, the cosmological evolution of UHECRs is constrained on the $m - z_{\text{max}}$ plane (Fig. 4). In this section we survey PeV-energy neutrino emis-
FIG. 4. Constraints on the UHECR source evolution index \(m\) and the maximum redshift of the sources \(z_{\text{max}}\) with the IceCube observations. The area above the blue curves is excluded with the null detection of neutrinos with energies above 100 PeV at the quoted confidence levels [11] by requiring the GZK cosmogenic neutrino flux to be consistent with the null detection. The shaded region displays the allowed parameter space by the detection of PeV-energy neutrinos assuming that the sources of the highest energy cosmic rays also produce the PeV-energy neutrinos. The two representative cases \((\tau_0 = 0.1 \text{ and } 0.01)\) are shown for \(\alpha = 2.5\). The area below the red curve is consistent with the total cosmic-ray flux at 1 EeV.

AGN are one of the promising sites to accelerate UHECRs [18] and therefore are expected to produce high-energy neutrinos. Several classical predictions of neutrino flux have been already ruled out by AMANDA-II and IceCube [19]. Surviving models are mainly related to blazars, which are radio-loud AGN with relativistic jets directed toward us. Hence, here we focus on blazars as sources of high-energy neutrinos.

Blazars are classified into two categories by their activity: BL Lac objects and flat spectrum radio quasars (FSRQs). In both cases, particles are believed to be accelerated in inner jets, subparsec-to-parsec scale from the central supermassive black hole. FSRQs have accretion disks with activity higher than BL Lac objects, and therefore the particle acceleration region of FSRQs is filled by photons from the accretion disks, broad line regions, and dusty tori. These are the main target photons for photons to emit synchrotron radiation in gamma-ray bands. However, due to small electron-synchrotron photon density, proton synchrotron radiation dominantly contribute to gamma rays, and therefore the optical depth \(\tau_0\) is still \(\sim 10^{-7}\) for Mrk 421, and even considering a relatively bright BL Lac object, PKS 0716+714, i.e., a member of low-frequency-peaked BL Lac objects (LBLs), the optical depth \(\tau_{\gamma\gamma}\) reaches \(\sim 3 \times 10^{-4}\) [23]. Hence, the optical depth is much smaller than the requirement displayed in Fig. [2], indicating that BL Lac objects do not contribute to the PeV-energy neutrino flux.

FSRQs can realize a much larger \(\gamma p\) optical depth because of strong external photon fields and their own high luminosity. Especially photons in broad line regions contribute to the production of PeV neutrinos as long as a proton acceleration site is inside these regions, as they are at ultraviolet wavelengths, and resultant \(\gamma p\) optical depth for PeV-energy neutrino production is \(\gtrsim 10^{-2}\) [23] [20]. However, several gamma-ray observations have indicated that particle acceleration happens outside broad line regions [27] (but see [24]). In that case, UHE protons mainly interact with infrared photons of \(E_{\nu} \sim 0.1\) eV from dusty tori and the neutrinos have higher energies than PeV (Eq. [10]), although this photon field is dense enough to produce neutrinos. Hence, as long as protons are accelerated in broad line regions, FSRQs can satisfy the constraint of \(\gamma p\) optical depth from the IceCube flux at 10 PeV (Fig. [2]).

FSRQs are also capable of streaming out UHECR protons with enough intensity to yield the detected PeV neutrinos. The luminosity function of AGN is generally described by a broken power-law function with a luminosity-dependent density evolution model [29]. Around the break at which the isotropic equivalent gamma-ray luminosity of \(L_{\gamma} \sim 10^{48}\) erg s\(^{-1}\), the
gamma-ray luminosity function of FSRQs is cosmologically evolved as $\propto (1 + z)^{6.51\pm1.97}$ up to $z \sim 1.4$ based on FSRQs detected by Fermi Large Area Telescope or $\propto (1 + z)^{4.23\pm0.39}$ up to $z \sim 1.9$ [31], where these indices are consistent within the large errors. Since FSRQs with $L_\gamma \sim 10^{48}$ erg s$^{-1}$ (called typical FSRQs below) are energetically dominant, FSRQ contribution to the total UHECRs can be estimated from this luminosity and these cosmological evolution. With $\psi(z) \propto (1 + z)^4$ up to $z = 2$, the local UHECR luminosity density is $\sim 10^{46}$ erg Mpc$^{-3}$ yr$^{-1}$ from the IceTop cosmic-ray flux measurement. This is compatible to the $\gamma$-ray luminosity density of typical FSRQs. Since the gamma-ray luminosity may be comparable with cosmic ray luminosity, FSRQs can provide a major fraction of the UHECR flux at 10 PeV, fulfilling the requirements displayed in Fig. 2.

However, if the spectrum of UHECRs producing PeV-energy neutrinos extends up to EeV energies, only bright FSRQs such as $L_\gamma \gtrsim 10^{50}$ erg s$^{-1}$ can satisfy the constraint on the $\gamma p$ optical depth, $\tau_0 \gtrsim 0.2$ shown in Fig. 3. Such bright FSRQs are too rare to energetically reproduce the observed UHECR flux at 1 EeV.

Furthermore, if the PeV neutrino sources are also responsible for the highest energy cosmic rays, UHECR emission from FSRQs are also unlikely by their strong cosmological evolutions. The allowed parameter space of the evolution for an object with $\tau_0 \sim 10^{-2}$, the typical optical depth of FSRQs [28], is in the upper right shaded region in Fig. 4. However, the allowed region is already ruled out by the UHECR flux at 1 EeV and partially by the non-detection of GZK cosmogenic neutrinos. Thus, FSRQs cannot simultaneously be the sources of the highest energy cosmic rays and those of PeV-energy neutrinos.

2. Gamma-ray bursts

GRBs are another strong candidate of UHECR acceleration sites and therefore high-energy neutrino production sites. Here we introduce four representative sites to produce high-energy neutrinos from literature.

Internal shocks are the most popular sites to produce high-energy neutrinos [32, 33]. Particle acceleration at internal shocks is believed to produce prompt emission of GRBs with short timescale variability $\sim 0.01$ s. The internal shocks are generated at the place well close to the central engine of GRBs, such as core collapse of massive stars for long duration GRBs or the mergers of compact objects for short duration GRBs, and therefore ultraviolet-to-soft X-ray photons in the rest frame, which are target photons to produce PeV neutrinos, exist densely. The benchmark model of a recent study for long duration GRBs [32] indicates $\tau_0 \sim 0.1$. The internal shocks of low-luminosity GRBs, which are one or two orders of magnitude dimmer but about two orders of magnitude more frequent than high-luminosity or regular GRBs, are also promising sites for the production of PeV-energy neutrinos [34]. Although their darkness provides less target photons, the $\gamma p$ optical depth of $\sim 10^{-2} - 10^{-3}$ can be still achieved, depending on the dissipation radius.

The second candidate site is external shocks, leading to GRB afterglow [35, 36]. Target photons are the synchrotron radiation of electrons accelerated at the shocks interacting with circumstellar medium. The relatively large spatial scale of the external shocks results in small photon density, and then the optical depth is $\tau_0 \sim 10^{-3}$ for photons to produce PeV-energy neutrinos even considering an early phase [36]. Furthermore, magnetic fields lower than those in internal shocks lead to less synchrotron cooling of charged decay products, i.e., charged pions and muons, and therefore the energy spectrum of neutrinos peaks at high energies $\sim 10^9 - 10^{10}$ GeV.

The third possible site is inside progenitor stars. Some long duration GRBs with jets of relatively small Lorentz factor may fail because relativistic jets cannot penetrate progenitor stars for the success of GRBs. Protons can be accelerated in choked jets inside the stars and then produce high-energy neutrinos via photomeson production interactions with dense stellar matter [37]. In such dense environments, the efficiency of $p\gamma$ interactions is extremely high, $\tau_0 \gg 1$, which means that all the proton’s energy is converted into neutrinos, i.e., the observed UHECRs and neutrinos are not directly connected.

Neutron star binary mergers are believed to be the origin of short GRBs. The mergers also eject significant masses with subrelativistic velocity [38]. The ejecta interact with interstellar medium and produce remnants like supernova remnants [39, 40]. The remnants can accelerate protons up to $\gtrsim 100$ PeV [41], and therefore are a potential source of PeV neutrinos. However, target photons for $p\gamma$ interactions, synchrotron radiation of electrons accelerated at the same place, are sparse, and the optical depth for PeV neutrino production is $\tau_0 \sim 10^{-7} - 10^{-5}$.

Among these four possibilities, the internal shock model of regular GRBs and the choked jet model are consistent with the constraint on $\tau_0$ from the UHECR flux at 10 PeV. The internal shock model of low-luminosity GRBs may be also possible. However, their energetics may be problematic. The typical gamma-ray energy output of a regular GRB is $\sim 10^{52}$ erg and the local occurrence rate of long GRBs is $\sim 1$ Gpc$^{-3}$ yr$^{-1}$ [41], indicating the local CR luminosity of $\sim 10^{44}$ ([$\eta_\gamma/(10)$] erg Mpc$^{-3}$ yr$^{-1}$ where $\eta_\gamma$ is the ratio of UHECR output and gamma-ray output known as baryon loading factor. The luminosity is about two orders of magnitude smaller than that of UHECRs at 10 PeV and thus too low to satisfy the requirements on the UHECR flux for $\tau_0 \gtrsim 10^{-2}$ ($\sim 10\%$ of the total cosmic-ray flux; Fig. 2) unless $\eta_\gamma \gtrsim 10^3$. The situation is similar for low-luminosity GRBs because their total energy budget is comparable with that of regular GRBs. Choked jets do not provide UHECRs observed at the earth because of extremely high $\gamma p$ optical depth, and therefore are acceptable if such failed GRBs exist.
enough.

If the spectrum of UHECRs producing the PeV neutrinos extends to EeV and beyond, the optical depth must be \( \tau_0 \gtrsim 0.2 \) (Fig. 3), leading to the low-luminosity GRBs out of the candidate sources.

It is still valid that GRBs can be the sources of the highest energy cosmic rays with energies of 10-100 EeV. The cosmological evolution of GRBs, although it is still uncertain, is not as strong as FSRQs. In Ref. [53], three redshift evolution models are discussed. Two of them are motivated by cosmic star formation rate and can be approximated by the same functional form as Eq. (19), but 2.5 and 3.8 instead of 3.4. The other model mildly evolves to more distant Universe, \( \alpha (1+z)^{2.1} \) (\( z < 3 \)) and \( \alpha (1+z)^{-1.7} \) (\( z \gtrsim 3 \)). All of these three evolution models are consistent with the constraints by the cosmogenic neutrinos and UHECR intensity shown in Fig. 4.

Note that the break redshift of the star formation rate evolution can be regarded as our \( z_{\text{max}} \) as a good approximation in terms of contribution to neutrinos and UHECRs because the star formation rate to neutrinos or evolved or negatively evolved above the break redshift. The local UHECR luminosity to reproduce UHECR flux above 10 EeV is \( \sim (0.6 - 2) \times 10^{44} \text{ erg Mpc}^{-3} \text{ yr}^{-1} \) [42], which can be marginally provided by regular GRBs. Therefore, regular GRBs can be the origin of the highest energy cosmic rays.

B. Magnetic horizon

We have constrained the optical depth of \( \gamma p \) interactions in the PeV-energy neutrino sources and their cosmological evolution by using a connection between the observed neutrinos and UHECR fluxes. Our estimation of UHECR flux from PeV-energy neutrino sources, i.e., Eq. (18), is valid if the sources are distributed uniformly. However, if UHECR sources are discretely distributed and the Universe is magnetized, low energy cosmic rays cannot reach the Earth within the age of the Universe even from the nearest source, and therefore UHECR flux at low energies is suppressed without the suppression of neutrino flux [43][44]. This magnetic horizon effect could affect the constraints.

The flux suppression of arrival UHECRs happens below the energy at which the diffusion length of UHECRs within the energy-loss time of cosmic rays \( t_{\text{loss}} \) is less than the typical separation between the observer and the nearest source \( r_{\text{sep}} \) [44]. The diffusion length is \( \sim \left( D l_{\text{loss}} \right) \) on the assumption of the Kolmogorov diffusion \( D(E) = c_{\ell} r_{L}(E)/l_{\ell} \) where \( l_{\ell} \) is the coherent length of average extragalactic magnetic fields and \( r_{L}(E) \) is the Larmor radius of protons with the energy of \( E \) in the extragalactic magnetic field with the strength of \( B \). The separation is related to the comoving (local) number density of UHECR sources \( n_{s} \).

The number density \( n_{s} \sim 10^{-4} \text{ Mpc}^{-3} \) is motivated by the anisotropy of UHECRs above \( 6 \times 10^{18} \text{ GeV} \) [45], which is estimated in small-deflection cases, i.e., proton-dominated composition or extremely weak extragalactic magnetic fields, but there is no trivial reason to stick to this number density for the sources of PeV neutrinos. Since the acceleration of cosmic rays up to \( \sim 10 \) PeV may be much easier than that to \( \sim 10^{30} \text{ eV} \) and much more source candidates are allowed following the Hillas criterion [46], a larger number density is possible. Note that if many sources contribute to PeV neutrinos but are localized in rare cosmic structures such as clusters of galaxies \( (n_{s} \lesssim 10^{-6} \text{ Mpc}^{-3}) \), the effective number density of the sources is determined by the number density of the rare structures.

The properties of extragalactic magnetic fields are poorly known. A reliable upper limit of the properties originates from the Faraday rotation measurements of polarized radio emission from distant objects, \( B \sqrt{l_{\ell}} \lesssim (1 \text{ nG}) \sqrt{\text{Mpc}} \) [48]. The properties of average extragalactic magnetic fields evaluated from recent numerical simulations, e.g., [19], are \( B \sqrt{l_{\ell}} \sim 0.3 - 1.0 \text{ nG Mpc}^{3/2} \) [50], which is close to the upper limit obtained by Faraday rotation measurements.

Figure 5 demonstrates the magnetic horizon effect in UHECR spectra above \( 10^{7} \text{ GeV} \) with the recent IceTop spectrum [51] for normalization. The theoretical spectra are calculated following a formulation by Ref. [52] with \( B = 1 \text{ nG} \) and \( l_{\ell} = 1 \text{ Mpc} \), but their comoving source term is extended with the cosmological evolution of UHECR sources, \( Q(E, z) \propto \psi(z) E^{-\alpha} \). The cosmological evolution is implemented as pure luminosity evolution without changing the comoving density of
UHECR sources. The universal spectrum, i.e., the spectrum of UHECRs from uniformly distributed sources, corresponds to Eq. (13). Sources are located at the three dimensional lattice points \((x, y, z) = (D_{\text{sep}}(i + 1/2), D_{\text{sep}}(j + 1/2), D_{\text{sep}}(k + 1/2))\) for integers \(i, j, k\) for discrete sources with the observer at the origin of the coordinates, where \(D_{\text{sep}} = (3/4\pi n_s)^{1/3}\). The distance to the nearest source(s) is actually \(r_{\text{sep}} = \sqrt{3}D_{\text{sep}}/2 = 13.4(n_s/10^{-4}\text{ Mpc}^{-3})^{-1/3}\text{ Mpc}\). The comoving luminosity density of UHECRs with energies between \(10^7\) GeV and \(10^{10}\) GeV is \(L_0 = 1 \times 10^{46}\text{ erg Mpc}^{-3}\text{ yr}^{-1}\) in the local Universe to suit the universal spectrum to the IceTop spectrum at \(10^9\) GeV. The input spectrum has the index of \(\alpha = 2.5\) and is truncated at \(10^{10}\) GeV.

The magnetic horizon effect, which appears in the difference between the universal spectrum and a spectrum calculated from discretely distributed sources, is not significant above \(10\) PeV for \(n_s = 10^{-3} - 10^{-5}\text{ Mpc}^{-3}\) (the propagation theorem). In these cases, the constraints on \(\gamma p\) optical depth, i.e., Fig. 2 and 3 do not change.

If \(n_s\) is very small, the magnetic horizon effect appears. In the case of \(n_s = 10^{-6}\text{ Mpc}^{-3}\), UHECR flux at \(10\) PeV is suppressed by a factor of 2.5 compared to the universal spectrum under the same cosmic ray luminosity density, and therefore, the constraint on \(\tau_0\) (Fig. 2) is relaxed. UHECR flux suppression is found also at \(1\) EeV, but by \(20\%\). Thus, the constraint from UHECR flux at \(1\) EeV (Fig. 3) is nearly unchanged. These magnetic horizon effects to the constraints on \(\tau_0\) are summarized in Fig. 6.

In the case of \(n_s = 10^{-7}\text{ Mpc}^{-3}\), the magnetic horizon effect becomes much larger. UHECR fluxes at \(10\) PeV and \(1\) EeV are suppressed by a factor of 300 and 3 compared to the universal spectrum, respectively (see Fig. 6 again). These flux suppressions relax the constraints, and therefore a much smaller \(\gamma p\) optical depth can be allowed as long as source candidates can provide UHECRs enough to produce PeV-energy neutrinos.

The flux suppression mainly depends on the properties of extragalactic magnetic fields. Thus, suppression factors for \(\alpha = 2.3\) and 2.7 are close to those discussed for \(\alpha = 2.5\).

The magnetic horizon effect can affect some results discussed in the previous subsection. For example, LBLs can satisfy a constraint on \(\tau_0\) at the cosmic-ray energy of \(10\) PeV. A recent gamma-ray population study revealed that the cosmological evolution of LBLs is weakly positive in number density, but can be well modeled by pure luminosity evolution with the local number density of \((3 - 5) \times 10^{-7}\text{ Mpc}^{-3}\) [54]. In this case, the allowed region in the left panel of Fig. 6 is between the green region and light-green region, and the estimated optical depth, \(\tau_0 \sim 3 \times 10^{-4}\), becomes consistent with the constraint thanks to the magnetic horizon effect. However, LBLs are not energetically enough to contribute to a dominant fraction of extragalactic cosmic rays at \(10\) PeV, which is also required from the left panel of Fig. 6.

Finally, we point out that effects similar to the magnetic horizon effect discussed above should be considered in PeV-energy neutrino sources. UHECRs linking to neutrinos must escape from neutrino production sites and their host galaxies with magnetic fields and/or dense target fields. The spectrum of UHECRs may be deformed through interactions with them, but the distortion is not trivial and depends on individual source classes. Since the aim of this study is to obtain general constraints on

FIG. 6. Same as Figs. 2 and 3 but the magnetic horizon effect to the constraints is demonstrated. Blue lines show the constraints without extragalactic magnetic fields which are exactly the same as Figs. 2 and 3. The green lines and light-green lines represent constraints including the magnetic horizon effect for \(n_s = 10^{-6}\text{ Mpc}^{-3}\) and \(n_s = 10^{-7}\text{ Mpc}^{-3}\), respectively. The cases of \(n_s \geq 10^{-5}\text{ Mpc}^{-3}\) are the same as the blue lines because the magnetic horizon effect is quite small. Note that the shades are the parameter space disfavored by the neutrino detection and the total cosmic-ray flux for \(n_s = 10^{-6}\text{ Mpc}^{-3}\).
Also, a useful application for this study is the discussions on individual source classes are beyond the scope of this paper.

C. pp interactions

We have examined neutrinos by $\gamma p$ interactions, but pp interactions also can produce neutrinos. The hadronic interactions become important in the systems where there is no dense photon fields, but UHECRs are in abundance and efficiently confined, such as starburst galaxies [55, 56] and clusters of galaxies [57]. Remarkable differences of $pp$ interactions from $\gamma p$ interactions are a much lower energy threshold and no prominent resonant structure in the cross-section of $pp$ interactions. As a result, non-thermal protons distributed over a wide energy range produce secondary neutrinos and gamma rays also over a wide energy range. Ref. [56] showed that the secondary gamma-ray emission can violate the flux of diffuse gamma-ray emission detected by the Fermi Large Area Telescope unless the spectrum of cosmic-ray protons is rather hard. Also, such a hard spectrum may easily violate the indicated IceCube cutoff at several PeV energies in the near future [2]. Thus, $pp$ sources will be strongly constrained in future multi-messenger studies.

V. SUMMARY

We have developed an analytical formula for the intensity of PeV-energy neutrinos, and have derived the general constraints on the optical depth of $\gamma p$ interactions in the neutrino sources, required fractions of extragalactic cosmic rays, and on their cosmological evolution, assuming that the photomeson production of extragalactic cosmic rays in the sources provides the bulk of PeV-energy neutrinos detected by IceCube. The connection between the IceCube neutrino flux and cosmic-ray flux at 10 PeV, which corresponds to the energy of protons producing the IceCube neutrinos, indicates that the $\gamma p$ optical depth is $\gtrsim 10^{-2}$ and that extragalactic component of UHECRs contributes to more than a few percent of cosmic ray flux at 10 PeV. These constraints becomes more stringent if the PeV-energy neutrino sources are also responsible for EeV-energy cosmic rays, favoring the dip transition model of cosmic rays. Furthermore, if these sources are also the emitters of the highest energy cosmic rays with energies reaching to 100 EeV, their redshift distribution should not be strongly evolved with cosmic time. Among the source candidates of the PeV-energy neutrinos discussed in this paper, FSRQs satisfies the conditions as sub-PeV to PeV energy cosmic ray and neutrino emitters, but they cannot be simultaneously emitters of the highest energy cosmic rays due to their strong cosmological evolution. The internal shocks at regular GRBs can account to the neutrino detection only if the rather extremely large baryon loading takes place in the GRB emission. None of the known extragalactic objects can be found to function as an origin of both PeV-energy neutrinos and the highest energy cosmic rays. The obtained constraints are valid even in the magnetized universe unless the UHECR sources are very rare in space, such as their number density is smaller than $10^{-6}$ Mpc$^{-3}$.

ACKNOWLEDGMENTS

We wish to acknowledge M. Ahlers, K. Ioka, and K. Murase for useful discussions in preparations for this work. We also thank M. Relich for helpful comments on the manuscript. This work is supported by the JSPS Grants-in-Aid for Scientific Research 25105005, 25220706 (S. Y.) and 24·9375 (H. T.).

Appendix A: Integration on source redshift

We approximately integrated the neutrino yield over redshift in Section II. The treatment of the integration follows that in Ref. [10]. If $\Omega_M(1+z)^3 \gtrsim 1$, which is valid in the wide range treated in this study, we can derive the following expression by using $I_t \equiv \sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda}$,

\begin{equation}
I_z = \int \frac{(1+z)^d z}{\sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda}} \simeq \frac{2}{2x-1} \Omega_M^{-\frac{x-1}{3}} \{\Omega_M(1+z)^3 + \Omega_\Lambda\}^{\frac{3}{2}} \cdot \log(1+z) - \frac{2}{2x-1} I_x, \tag{A1}
\end{equation}

Also, a useful application for this study is

\begin{equation}
\int \frac{(1+z)^d z}{\sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda}} \simeq \frac{2}{2x-1} \Omega_M^{-\frac{x-1}{3}} \{\Omega_M(1+z)^3 + \Omega_\Lambda\}^{\frac{3}{2} - \frac{1}{2}} \log(1+z) - \frac{2}{2x-1} I_x, \tag{A2}
\end{equation}

if the derivative of equation (A1) is applied.

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