Neutron-antineutron Oscillation and Baryonic Majoron: Low Scale Spontaneous Baryon Violation

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We discuss a possibility that baryon number $B$ is spontaneously broken at low scales, of the order of MeV or even smaller, so that the neutron–antineutron oscillation can be induced at the experimentally accessible level. An associated Goldstone particle – baryonic majoron, can have observable effects in neutron to antineutron transitions in nuclei or dense nuclear matter. By extending baryon number $B - L$ symmetry, baryo-majoron can be identified with the ordinary majoron associated with the spontaneous breaking of lepton number, with interesting implications for neutrinoless $2\beta$ decay with the majoron emission. We also discuss a hypothesis that baryon number can be spontaneously broken by the QCD itself via the six-quark condensates.

I. INTRODUCTION

There is no fundamental principle that can prohibit to neutral particles as are the neutron or neutrinos to have a Majorana mass envisaged long time ago by Ettore Majorana \cite{1}. Nowadays the neutron is known to be a composite fermion having a Dirac nature conserving baryon number. As for the neutrinos, theorists prefer to consider them as Majorana particles though no direct experimental proofs for this were obtained yet (e.g. the neutrinoless double-beta decay). On the other hand, it is not excluded that the neutron $n$, along the Dirac mass term $m \bar{\tilde{n}}$m, with $m \approx$ 940 MeV, has also a Majorana mass term $\epsilon n \bar{\tilde{n}} n + \text{h.c.} = \epsilon \bar{n} \tilde{n} + \text{h.c.}$, with $\epsilon \ll m$, which mixes the neutron and antineutron states (here $C$ is charge conjugation matrix and $\tilde{n} = \bar{\sigma^t} n$ is the antineutron field). This mixing induces a very interesting phenomenon of neutron–antineutron oscillation, $n \leftrightarrow \tilde{n}$ suggested by Kuzmin \cite{2}. First theoretical scheme for $n \leftrightarrow \tilde{n}$ oscillation were suggested in Ref. \cite{3}, followed by other types of models as e.g. \cite{4,4}. Clearly, existence of the Majorana mass of the neutron would violate the conservation of baryon number $B$ by two units (analogously, Majorana masses for neutrinos violate lepton number $L$ by two units). If $B$ and $L$ were exactly conserved, the phenomena like proton decay, $n \leftrightarrow \tilde{n}$ oscillation or neutrinoless $2\beta$ decay would be impossible. Experimental limits on matter stability tell that $B$-violating processes must be very slow: lower bounds on the lifetime of the nucleons (and of stable nuclei) land between $10^{30} - 10^{34}$ yr \cite{7}. On the other hand, we have a strong theoretical argument that baryon number must be indeed violated in some processes – the existence of matter itself. Without $B$-violation no primordial baryon asymmetry could be generated after inflation and so the universe would remain baryon symmetric and thus almost empty of matter. Primordial baryogenesis in the Early Universe maybe related to the same $B$-violating physics that induces neutron-antineutron mixing. As was shown by Sakharov, $B$-violating processes which break also CP and which are out of equilibrium at some early cosmological epoch, can generate non-zero baryon number in the universe \cite{8}. (In modern theoretical scenarios, $B - L$ violation is indispensable and also sufficient \cite{9}. ) It is interesting to note that $n \leftrightarrow \tilde{n}$ oscillation implies breaking of P and CP along with $B - L$ violation, so that two of three Sakharov’s conditions for baryogenesis are automatically satisfied \cite{10}. Hence, discovery of neutron-antineutron oscillation would make it manifest that these underlying physics, based e.g. on models \cite{3,5,6}, contain CP violating terms which could be at the origin of the baryon asymmetry of the Universe.

The structure of the Standard model describing the known particles and their interactions nicely explains why the $B$ and $L$ violating processes are suppressed. Under the standard gauge group $G = SU(3) \times SU(2) \times U(1)$, the left-handed quarks and leptons transform as isodoublets $q_L = (u,d)_L$, $l_L = (\nu,e)_L$ while the right-handed ones are iso-singlets $u_R$, $d_R$, $e_R$. (For simplicity, hereafter we omit the symbols L (left) and R (right) as well as the internal gauge, spinor and family indices; antiparticles will be termed as $\bar{q}$, $\bar{l}$, etc. and the charge conjugation matrix $C$ will be omitted.) As usual, we assign a global lepton charge $L = 1$ to leptons and a baryon charge $B = 1/3$ to quarks, so that baryons composed of three valent quarks have a baryon number $B = 1$.

However, $L$ and $B$ are not perfect quantum numbers. They are related to accidental global symmetries possessed by the Standard Model Lagrangian at the level of renormalizable couplings (no renormalizable coupling can be written that could violate them). However, they can be explicitly broken by higher dimension (non-renormalizable) operators suppressed by large mass scales which may be related to the scales of new physics beyond the Standard Model \cite{11}. E.g., grand unified theories (GUTs) introduce new interactions that transform quarks into leptons and thus induce effective $D = 6$ operators $\frac{1}{M^2} q g q q l l$, etc. which lead to the proton decays like $p \to p \pi^+$, $p \to K \nu$ etc. These decay rates are suppressed by the GUT scale $M \geq 10^{15}$ GeV which makes them compatible with the existing experimental limits \cite{2}.

The lowest dimension operator, D=5, is related to lep-
tons and it violates the lepton number by two units [11]:

\[ \mathcal{O}_5 = \frac{1}{M} \bar{d} \sigma \phi (L = 2) \]  

where \( \phi \) is the Higgs doublet. After inserting the Higgs VEV \( \langle \phi \rangle \), this operator yields small Majorana masses for neutrinos, \( m_\nu \sim \langle \phi \rangle^2 / M \), and induces oscillations between different neutrino flavors. Interestingly, the experimental range of the neutrino mass, \( m_\nu \sim 0.1 \text{ eV} \) or so, also favors the GUT scale \( M \sim 10^{15} \text{ GeV} \) as a natural scale of these operators.

The \( \Delta n \) - antineutron mass mixing, \( \epsilon(\bar{n}n + \text{h.c.}) \), violates the baryon number by two units. It can be related to the effective D=9 operators involving six quarks which in terms of the Standard model fragments \( u = u_R \), \( d = d_R \) and \( q = (u, d)_L \) read as

\[ \mathcal{O}_3 = \frac{1}{M^5} (uddudd + uddqqd + qqdqqd) \]  

where \( M \) is some large mass scale. These operators can have different convolutions of the Lorentz, color and weak isospin indices which are not specified. (Needless to say, the combination \( qq \) in second term in [3] must be in a weak isosinglet combination, \( q = \frac{1}{2} e^{i \alpha} y_\alpha q_\beta = u_L d_L \) where \( \alpha, \beta = 1, 2 \) are the weak \( SU(2) \) indices, while in the third term \( qq \) can be taken in a weak isotriplet combination as well.) More generally, having in mind that all quark families can be involved, these operators give rise to mixing phenomena also for other neutral baryons, e.g. oscillation of the hyperon \( \Lambda \) into the antihyperon \( \bar{\Lambda} \).

If the scale \( M \) is taken of the order of the GUT scale, as one takes for the proton decaying \( D = 6 \) operators \( \mathcal{O}_6 \) or for \( D = 5 \) neutrino mass operator \( \mathcal{O}_5 \), the effects of \( n - \bar{n} \) mixing would become vanishingly small. On the other hand, the GUT scale is not really favored by the primordial baryogenesis. The latter preferably work at smaller scales, in the post-inflation epoch. An adequate scale for baryogenesis in the context of \( \Delta B = 2 \) models can be as small \( M \sim 1 \text{ PeV} \) [6].

Taking into account that the matrix elements of operators \( \mathcal{O}_6 \) between the neutron states are of the order of \( \lambda_{QCD}^3 \sim 10^{-4} \text{ GeV}^6 \), modulo the Clebsch coefficients \( O(1) \), one can estimate:

\[ \epsilon \sim \frac{\lambda_{QCD}^6}{M^5} \sim \left( \frac{1 \text{ PeV}}{M} \right)^5 \times 10^{-25} \text{ eV} . \]  

The coefficients of matrix elements \( \langle \bar{n}|\mathcal{O}_6|n \rangle \) for different Lorentz and color structures of operators [2] were studied in ref. [12] but we do not concentrate here on these particularities and take them as \( O(1) \) factors. In the presence of mixing \( \epsilon(\pi \bar{n} + \text{h.c.}) \), the neutron mass eigenstates become two Majorana states with the masses \( m + \epsilon \) and \( m - \epsilon \), respectively \( n_+ = \sqrt{1/2} (n + \bar{n}) \) and \( n_- = \sqrt{1/2} (n - \bar{n}) \). The characteristic time of \( n \leftrightarrow \bar{n} \) oscillation is related to their mass splitting, \( \tau = \epsilon^{-1} \).

The experimental limit \( \tau > 0.86 \times 10^8 \text{ s} \) (90% C.L.) obtained by a search of \( n - \bar{n} \) oscillation with cold neutrons freely propagating in the conditions of suppressed magnetic field [13] implies \( \epsilon < 7.7 \times 10^{-24} \text{ eV} \). On the other hand, \( n - \bar{n} \) mixing inside the nuclei must destabilize the latter [14]. In fact, operator (2) induces annihilation processes of two nucleons into pions, \( NN \rightarrow \pi \pi \), which transform nucleus with atomic number \( A \) into the nucleons with \( A - 2 \) with emission of pions with total energy roughly equal to two nucleon masses. Interestingly, nuclear stability limits translated to the free \( n - \bar{n} \) oscillation time are not far more stringent than direct experimental limit [15]. E.g. the iron decay limit implies \( \tau > 1.3 \times 10^8 \text{ s} \) [15] while the Oxygen one \( \tau > 2.7 \times 10^8 \text{ s} \) [16]. Hence, one can conclude that \( n - \bar{n} \) oscillation may test the underlying physics up to cutoff scales \( M \sim 1 \text{ PeV} \), also having in mind possible increase of the experimental sensitivity by an order of magnitude. For the discussion of the present status of \( n - \bar{n} \) oscillation and future projects for its search see e.g. refs. [17].

One can envisage a situation when baryon number is broken not explicitly but spontaneously. In particular, one can consider a situation when baryon number associated with an exact global symmetry \( U(1)_B \) is spontaneously broken by a complex scalar field \( \chi \) with \( B = 2 \), which breaking also induces the Majorana mass term for the neutron. Clearly, spontaneous breaking of global \( U(1)_B \) gives rise to a Goldstone boson \( \beta \) which can be coined as the baryonic majoron, or baryo-majoron, in analogy to the majoron associated with the spontaneous breaking of global lepton symmetry \( U(1)_L \) [18] and widely exploited in neutrino physics.

In fact, spontaneous baryon violation in the context of \( n - \bar{n} \) oscillation and the physics of the baryonic majoron was previously discussed in ref. [19], in the context of the model [3]. Spontaneous \( B \)-violation was discussed also in ref. [20], in terms of the operator \( qq \beta \) \((B = 1)\). The associated Goldstone boson was named as bary-axion, for respect of the electroweak anomaly of \( U(1)_B \).

In this paper we discuss the possibility of spontaneous \( B \)-violation at very low scales, \(< 1 \text{ MeV} \) or so, in which case the baryo-Majoron can have observable consequences, inducing nuclear decay via the Majoron emission, related to transition \( n \rightarrow \bar{n} + \beta \). Global baryonic symmetry can be naturally extended to \( U(1)_B \) in which case its spontaneous breaking scale must be relevant also for the neutrino Majorana masses, and the baryonic and leptonic Majorons become in fact the same particle, just the Majoron. In this context, we briefly discuss implications for leptonic sector as e.g. neutrinoless \( 2 \beta \) decay with Majoron emission and astrophysical implications of the Majoron. We shall also discuss a rather unusual possibility when baryon number is broken by six quark condensates \( \langle uddudd \rangle \) and its possible implications.

II. SEEWSAF FOR \( N - \bar{N} \) MIXING

The contact (nonrenormalizable) \( L \) and \( B \) violating terms like [1] and [2] can be induced in the con-
text of renormalizable theories after decoupling of some heavy particles. In particular, lepton-like operator can be induced in the context of seesaw mechanism which involves gauge singlet fermions $N_{(R)}$, so called right-handed (RH) neutrinos, with large Majorana mass terms $\frac{1}{2} M_N N^2 + h.c.$ explicitly violating $L$. Then at energies $E_0 \ll M$, operator emerges from the Yukawa couplings $\phi N + h.c.$ after integrating out the heavy neutrinos $N$, with $M \sim M_N$. Then, modulo Yukawa constants, we have for the Majorana masses of neutrinos

$$m_\nu \sim \frac{\epsilon^2}{M_N} \sim \left( \frac{10^{14} \text{ GeV}}{M_N} \right) \times 0.1 \text{ eV} \quad (4)$$

One can also discuss a simple seesaw-like scenario for generation of terms (2), along the lines suggested in ref. [5]. Let us introduce a new gauge singlet Weyl fermion (or scalars) $N_{(R)}$, sort of “RH neutron”, and a color-triplet scalar $S$, with mass $M_S$, having precisely the same gauge quantum numbers as the right down-quark $d_{(R)}$. Consider the Lagrangian terms

$$SU + S \bar{q} q + S^1 d N + \frac{1}{2} M_N N^2 + h.c. \quad (5)$$

where $qg$ in second term is in a weak isosinglet combination, $qq = \frac{1}{2} e^{\alpha \beta} q_\alpha q_\beta = u_L d_L$ where $\alpha, \beta = 1, 2$ are the weak $SU(2)$ indices (we omit the charge conjugation matrix $C$ and Yukawa constants $\sim 1$). One can prescribe $B = -2/3$ to $S$ and $B = -1$ to $N$, so that the Yukawa couplings in [5] respect the baryon number which is explicitly violated by Majorana mass $M_N$. Then, at energies $E \ll M_S, M_N$, operators $O_9$ of eq. (2) are induced via integrating out the heavy states $S$ and $N$, with $M^2 \sim M_S^2 M_N$ modulo Yukawa constants in [5]. From the model point of view, the scale $M = 1$ PeV accessible via $n - \tilde{n}$ oscillation, may correspond to a democratic choice when $M_N \sim M_S \sim 1$ PeV. However, it can be obtained in different situations, namely (a) light $S$ and heavy $N$, e.g. $M_S \sim 1$ TeV and $M_N \sim 10^{18}$ GeV, or (b) heavy $S$ and light $N$, e.g. $M_S \sim 10^6$ GeV and $M_N \sim 100$ GeV. Hence, for the neutron-antineutron mixing mass we have

$$\epsilon \sim \frac{\Lambda_{QCD}^6}{M_S^3 M_N} \sim \left( \frac{10 \text{ TeV}}{M_S} \right)^4 \left( \frac{10^{14} \text{ GeV}}{M_N} \right) \times 10^{-25} \text{ eV} \quad (6)$$

Let us notice that the “heavy neutrino” $N$ and “heavy neutron” $N'$ cannot be the same particle. Otherwise its exchange would induce also operators like $udd\bar{d}$ with too low cutoff scale which would induce too fast proton decay. If they are singlets, they can be divided by some discrete symmetries. Alternatively, one can consider $N$ as weak isorotplet and $N$ as color octet, in which case no mixed mass terms may exist between $N$ and $N'$ states. (In the case of color-octet $N'$ the scalars $S$ can be taken also as color anti-sixetets). The exchange via color-octet $N$ would generate operators $O_9 \propto (udd)_{S} (udd)_{S}$ with $(udd)_{S}$ in a color octet combination, $(udd)_{S} \sim u d A^\alpha d$ where $A^\alpha$ are the Gell-Mann matrices. Via Fierz Transformation, exchanging $d$ states from the left and right brackets in such $O_9$, the matrix element $\langle n|O_9|\tilde{n}\rangle$ will contribute to the $n - \tilde{n}$ mixing. In the context of supersymmetry, such operators can be easily obtained via $R$-parity breaking terms $u_A d_B d_C$ ($B \neq C$) in the superpotential, where $A, B, C$ are the family indices. Taking e.g. a superpotential term $uds$ involving the up, down and strange RH supermultiplets, one obtains the couplings analogous to $Sud + S^1 d N'$ of [5] with $S$ being the strange squark and $N'$ being gluino with a Majorana mass $M$. This is because the gluino may have flavor-changing coupling between quark and squark states, namely between $d$-quark and $s$-squark. Needless to say, in this scheme somewhat bigger mixing mass would be generated for hyperons, between $\Lambda$ and $\tilde{\Lambda}$, via flavor diagonal gluino coupling between $s$-squark and $s$-squark. However, $\Lambda - \tilde{\Lambda}$ mixing is much more difficult for the experimental detection (though it maybe more efficient in the dense nuclear matter in the neutron stars where hyperons can emerge as natural occupants). In any case, $\Lambda - \tilde{\Lambda}$ mixing would also induce nuclear instability via two nucleon annihilation processes with Kaon emission, $N + N \to K + K$ etc.

The interesting link between seesaw mechanisms for generation of the neutrino and neutron Majorana masses is the following. In parallel to usual leptogenesis scenario due to the heavy neutrino decays $N\to l\phi$ producing lepton number which then is redistributed to baryon number via $B-L$ conserving sphaleron effects, also baryogenesis can take via the heavy ‘neutron’ decays $N'\to udd$ mediated via color-triplet scalar $S$ which can directly produce the baryon number of the universe.

Let us consider now a situation when baryon number is broken not explicitly but spontaneously. Namely, let us assume that baryon number associated with an exact global symmetry $U(1)_B$, and it is spontaneously broken by a complex scalar field $\chi$ $(B = 2)$ once the latter gets a VEV $\langle \chi \rangle = V$. The seesaw Lagrangian [5] in this case is modified as

$$SU + S \bar{q} q + S^1 d N' + \chi N'^2 + h.c. \quad (7)$$

The VEV of $\chi$ induces the Majorana mass to the RH

\[ \text{FIG. 1. Diagram generating } n - \tilde{n} \text{ mixing via exchange of } N' \text{ state which gets a large Majorana mass } M_N \sim \langle \chi \rangle \text{ after } U(1)_B \text{ symmetry breaking.} \]
neutron $\mathcal{N}$ through the Yukawa coupling $\chi \mathcal{N}^2$, with $M_\mathcal{N} \sim V$. Hence, operator $O_9$ emerges after the spontaneous baryon violation as shown on Fig. 1 and $n-\bar{n}$ mixing parameter $\epsilon$ is inversely proportional to the baryon symmetry breaking scale $V$.

The scale $V$ can be related also to the breaking of lepton number if one extends global symmetry $U(1)_B$ to $U(1)_{B-L}$ and assumes that the neutrino Majorana masses emerge from the usual seesaw Lagrangian

$$\phi \mathcal{N} l + \chi \mathcal{N}^2 + \text{h.c.} \quad (8)$$

Since the neutrino masses (4) point twoards $U(1)_{B-L}$ breaking scale $V \sim 10^{14}$ GeV, then $n-\bar{n}$ oscillation, according to (6), can be within the experimental reach if color triplets $S$ have masses in the range $M_S \sim 10$ TeV, potentially within the reach for the LHC run II.

Spontaneous breaking of global $U(1)_B$ or $U(1)_L$ gives rise to a Goldstone boson $\beta$, baryo-majoron or lepto-majoron. These two can be the same particle, simply a majoron, once the global symmetry is promoted to $U(1)_{B-L}$. However, in practice very large scale of symmetry breaking renders such majoron(s) unobservable experimentally and without any important astrophysical consequences. In the following section we discuss models where the global symmetry breaking scale can be rather small, $< 1$ MeV or less, in which case the majoron interactions with the neutron and with neutrinos could have observable experimental and astrophysical consequences.

### III. Low Scale Seesaw Model

Is it possible to build a consistent model in which baryon number, or $B-L$, spontaneously breaks at rather low scales in which case the majoron couplings to the neutrinos and to the neutron can be accessible for the laboratory search? This can be obtained by a simple modification of the above considered model.

Let us introduce along with the Weyl fermion $\mathcal{N}$ with $B = -1$, another guy $\mathcal{N}'$ with $B = 1$. These two together form a heavy Dirac particle with a large mass $M_D$. On the other hand, both $\mathcal{N}$ and $\mathcal{N}'$ can be coupled to scalar $\chi$ ($B = 2$) and get the Majorana mass terms from the VEV of the latter, $M, M' \sim \langle \chi \rangle$, which can be much less than the Dirac Mass $M_D$. The relevant Lagrangian terms now read:

$$Sud + Sqq + S'^{\dagger}d \mathcal{N} + M_D \mathcal{N}\mathcal{N}' + \chi \mathcal{N}^2 + \chi \mathcal{N}'^2 + \text{h.c.} \quad (9)$$

In this way, diagram shown in Fig. 2 after integrating out the heavy fermions $\mathcal{N}+\mathcal{N}'$, induces $D = 10$ operators

$$O_{10} \sim \frac{\chi}{M_D^2 M_S^4} (uddudd + \ldots) \quad (B = 0) \quad (10)$$

where dots stand for other field combinations present in (6). Now, assuming that the field $\chi$ is light, one can consider the matrix element directly of $O_{10}$ between the $n$ and $\bar{n}$ states. Thus, at low energies these operators reduce to the neutron Yukawa couplings with scalar $\chi$,

$$Y_n \chi^\dagger n^T C n + \text{h.c.}, \quad (11)$$

with the coupling constant

$$Y_n \sim \frac{\Lambda^6_{\text{QCD}}}{M_D^2 M_S^4} \sim \left( \frac{100 \text{ TeV}}{M_D} \right)^2 \left( \frac{10 \text{ TeV}}{M_S} \right)^4 \times 10^{-30} \quad (12)$$

Thus, once the baryon number is broken by the VEV $\langle \chi \rangle$, $n-\bar{n}$ mixing emerges with $\epsilon_{n\bar{n}} = Y_n \langle \chi \rangle \sim \Lambda^6_{\text{QCD}} M'/\left( M_D^2 M_S^4 \right)$, or

$$\epsilon_{n\bar{n}} \sim \left( \frac{100 \text{ TeV}}{M_D} \right)^2 \left( \frac{10 \text{ TeV}}{M_S} \right)^4 \left( \frac{M'}{1 \text{ MeV}} \right) \times 10^{-24} \text{ eV}. \quad (13)$$

Taking e.g. $M_S \sim 10$ TeV and $M_D \sim 100$ TeV, then $\epsilon_{n\bar{n}} \sim 10^{-24} \text{ eV}$ would require $\langle \chi \rangle \sim 1 \text{ MeV}$ or so.

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1 The following remark is in order. In general, operators (10) can contain parts which respect and which do not respect $P$-parity. Therefore, taking matrix element $\langle n|O_{10}|\bar{n}\rangle$, in addition to the coupling (11) one can have also $P$-invariant coupling $Y_n \chi n^T C \gamma^5 n + \text{h.c.}$. Only the former term violating $P$ will be relevant for $n-\bar{n}$ oscillation after non-zero $\langle \chi \rangle$ breaks $B$. For the majoron interactions also the latter term would be relevant which now can have both $P$-invariant and $P$-violating couplings. For simplicity we shall not discuss it in the following.
Low scale baryon number violation was suggested in Ref. 5, in a model which was mainly designed for inducing neutron – mirror neutron oscillation \( n \rightarrow n' \). This model treats \( N \) and \( N' \) states symmetrically: their Majorana masses \( \tilde{M} \) and \( \tilde{M}' \) are equal, while in addition to couplings 9, there are terms that couple \( N' \) to \( n' \), \( d' \) and \( S' \) states from hidden mirror sector with a particle content identical to that of ordinary one (for review, see e.g. 22). Namely, the lower diagram of Fig. 2 induces \( n - n' \) mixing with

\[
\epsilon_{nn'} \sim \frac{\Delta_{\text{QCD}}}{M_D M_S^2} \sim \left( \frac{100 \text{ TeV}}{M_D} \right) \left( \frac{10 \text{ TeV}}{M_S} \right)^4 \times 10^{-16} \text{ eV}
\]

(14)

which corresponds to \( n - n' \) oscillation time \( \tau_{nn'} \sim 10 \text{ s} \). Hence, in this case \( n - n' \) mixing should be a dominant effect, conserving a combined baryon number \( B = B - B' \) between ordinary and mirror sectors, while \( n - \tilde{n} \) mixing which breaks \( B \) is suppressed by the ratio \( \tilde{M}/M_D \):

\[
\epsilon_{n\tilde{n}} = \frac{\tilde{M}}{M_D} \epsilon_{nn'}
\]

(15)

As a matter of fact, \( n - n' \) mixing can indeed be much larger than \( n - \tilde{n} \). Existing experimental limits on \( n - n' \) transition 23 allow the neutron–mirror neutrinooscillation time to be less than the neutron lifetime, with interesting implications for astrophysics and particle phenomenology 6, 24.

Let us discuss now the couplings of the majoron \( \beta \) which is a Goldstone component of the \( \chi \) scalar, \( \chi = \frac{1}{\sqrt{2}} (V + \rho) \exp(\text{i}/\sqrt{2}) \), where \( \rho \) denotes a massive (Higgs) mode of \( \chi \) with a mass \( \sim V \). (We assume here that the VEV \( \langle \chi \rangle = V/\sqrt{2} \) emerges via negative mass\(^2\) term in the potential of \( \chi \).) Both \( \rho \) and \( \beta \) are coupled non-diagonally between the \( n \) and \( \tilde{n} \) states, \( g_{\beta nn}(\rho + \text{i} \beta \gamma_5)\tilde{n} + h.c. \), with \( g_{\beta nn} = \frac{1}{\sqrt{2}} Y_n = \epsilon/V \). Observe that the Higgs \( \rho \) is coupled to pseudoscalar combination \( \pi \tilde{n} \) while the majoron \( \beta \) couples to scalar combination \( \chi \tilde{n} \). This is related to the fact that the Majorana mass term \( \sigma \pi \tilde{n} + h.c. \) breaks \( P \) and \( CP \) invariances 10.

In vacuum the transition \( n \rightarrow \tilde{n} + \beta \) is suppressed since \( n \) and \( \tilde{n} \) have equal masses. (We neglect a tiny mass splitting \( \epsilon < 10^{-24} \text{ eV} \) between two Majorana states \( n_+ \) and \( n_- \)... However, in the nuclei the neutron and antineutron have different effective potentials and thus \( n \rightarrow \tilde{n} + \beta \) transition becomes possible which clearly would lead to the nuclear instability. The produced antineutron then annihilates with other spectator nucleons producing pions, thus causing the transition of a nuclei with atomic number \( A \) into a nuclei with \( A - 2 \) and pions with invariant mass which in principle should be less than the mass difference \( M_A - M_{A-2} \) between the initial and daughter nuclei as far as part of the energy will be taken by the majoron. The decay width can be estimated as \( \Gamma = (g_n^2/8\pi)\Delta E \), where \( \Delta E \) is a typical energy budget for this transition which depends on nucleus and which is typically order 10 MeV. Taking into account the the existing experimental limits on the nuclear decay \( \Gamma^{-1} > 10^{32} \text{ yr} \), we get a rough bound \( g_n < 10^{-30} \) or so. Needless to say, the scalar component \( \rho \) with mass order MeV is also relevant for the nuclear transitions \( n \rightarrow \tilde{n} + \rho \).

On the other hand, taking \( \epsilon < 10^{-24} \text{ eV} \), from (13) we see that \( g_n = \epsilon/V \sim 10^{-30} \text{ requires } V < 1 \text{ MeV or so. } \). \( \Gamma^{-1} \sim (V/1 \text{ MeV})^2(10^{-24} \text{ eV}/\epsilon)^2 \times 10^{34} \text{ yr. Taking into account that } \epsilon < 10^{-24} \text{ eV, this exceeds many orders of magnitude the existing experimental limits } \sim 10^{32} \text{ yr unless } V < 1 \text{ MeV or so.} \)

As for the baryo-majoron coupling constant \( g_n = \epsilon/V \), now it can be large enough for making \( n \rightarrow \tilde{n} + \beta \) decay accessible in the experimental search for the nuclear destabilisation. E.g. the nuclear decays at the level \( \Gamma^{-1} \sim 10^{32} \text{ yr can be obtained via } n \rightarrow \tilde{n} \text{ oscillation with } \delta \sim 10^{-24} \text{ eV, or via } n \rightarrow \tilde{n} + \beta \text{ decay with } g_n = \epsilon/V \sim 10^{-30}. \text{ Therefore, if } V < 100 \text{ keV the former mechanism becomes suppressed with respect to the latter which becomes dominant from the perspectives of the experimental search.} \)

Let us remark that in the context of low scale model, with \( f_B \leq 1 \text{ MeV, baryo-majoron could be the same particle as the usual (leptonic) majoron, if one promotes the } \bar{U}(1)_B \text{ symmetry to } \bar{U}(1)_{B-L}, \text{ which is free of anomalies. \text{ Then the Majorana masses of the neutrinos can be induced, along the lines of the model suggested in ref. 25, from the diagram shown in Fig. 2 involving the following Lagrangian terms}} \]

\[
\phi \bar{N} + M_D N N' + \chi N^2 + \chi^2 N'^2
\]

(16)

where \( N, N' \) are the fermion couples, analogous to \( N, N' \), with properly assigned lepton number (or better \( B - L \) and they have large Dirac masses \( M_D \). Then after integrating out of the heavy states, one obtains an operator

\[
O_6 \sim \frac{\chi}{M_D^2} \text{diag}(\phi, \phi)^2
\]

(17)

which at lower energies results in the neutrino Yukawa couplings with the light \( \chi \) scalar, \( Y_\nu \chi^T \bar{C} \nu + h.c. \), where

\[
Y_\nu \sim \frac{v^2}{M_D^2} \sim \left( \frac{100 \text{ TeV}}{M_D} \right)^2 \left( \frac{\langle \chi \rangle}{1 \text{ MeV}} \right) \times 10^{-6}
\]

(18)

Then the neutrino Majorana masses are induced with \( m_\nu = Y_\nu \langle \chi \rangle \), or

\[
m_\nu \sim \frac{v^2}{M_D^2} \langle \chi \rangle \sim \left( \frac{100 \text{ TeV}}{M_D} \right)^2 \left( \frac{\langle \chi \rangle}{1 \text{ MeV}} \right) \times 1 \text{ eV}
\]

(19)

which, taking into account also uncertainties in the Yukawa constants in (16), naturally fall in the experimental mass range of neutrinos when \( M_D \sim 100 \text{ TeV} \)

\[ \text{2 Once again, the “heavy neutrinos” } N, N’ \text{ and “heavy neutrons” } N, N’ \text{ cannot be the same, since in this case their exchange would induce the operators like } \bar{u}d\bar{d}v \text{ with too low cutoff scale which would lead to dramatically fast proton decay.} \]
In this situation, the majoron $\beta$ has large enough Yukawa couplings with the neutrinos [25], with coupling constants $g_{\beta\nu\nu} = m_\nu/V$. Hence, for $V < 1$ MeV the majoron couplings to neutrinos can be rather large, $g_\nu > 10^{-7}$ or so, which could be of interest for searching the neutrinoless 2-beta decay with the majoron emission [26]. The present experimental bound on the majoron coupling to $\nu_e$ reads $g_{\nu e e} < (0.8 - 1.6) \times 10^{-5}$ [27]. In addition, they can bring to interesting effects with interesting applications for astrophysics and cosmology as e.g. matter induced neutrino decay or matter induced decay of the majoron itself [28], blocking of active–sterile oscillations in the early universe by the majoron field [29], etc. Detailed analysis of the astrophysical limits on the neutrino-majoron couplings can be found in [31].

The Majoron coupling constant between the neutron and antineutron is $g_{n\bar{n}} = \epsilon_{n\bar{n}}/V$. Interestingly, the nuclear decays with majoron emission become dominant over majoronless nuclear decays when $V \sim 1$ MeV or smaller. The parallel of such nuclear decays with the neutrinoless 2-beta decays with the majoron emission which also can be observable if $V \leq 1$ MeV is interesting.

One can question the naturality issues when having such a small VEVs, $V \sim 1$ MeV, with respect to the electroweak scale $M_Z \sim 100$ GeV. If scalar $\chi$ gets a VEV from minimization of its Higgs potential with negative mass square, which mass should also be order MeV which gives rise a hierarchy problem, why $V \ll M_Z$. This question can be solved if the scale $V$ is related to some compositeness scale, e.g. if the scalar $\chi$, even being heavy, with mass say $M_\chi \sim 100$ GeV, has the Yukawa couplings with quark-like states, $\chi Q\bar{Q}$ of some hidden sector with a confinement scale order MeV. Then this condensate would induce the non-zero VEV to scalar $\chi$, $\langle \chi \rangle \sim \langle \chi Q\bar{Q} \rangle / M_\chi^2$, and thus the Majorana masses for the neutrinos and neutron.

IV. DISCUSSION AND OUTLOOK

At this point, I am tempted to discuss a less orthodox idea, suggesting that the baryon number could be violated by the Standard Model itself, namely by the strong dynamics of the QCD sector. The conjecture is that along with the basic quark and gluon condensates, $\langle qq \rangle$ and $(G^2)$, or higher order operators $\langle \bar{q}Gq \rangle$, $\langle \bar{q}qq \rangle$, there may exist also a fuzzy six-quark condensates $\langle uudddd \rangle$. These condensates can be built upon different combinations of left and right $u,d$ and perhaps $s$ quarks, and may have different convolutions of the Lorenz and color indices. One could envisage that they might emerge via attractive forces between the quark trilinear in color octet combinations.

One interesting possibility can emerge considering that QCD itself could break baryon number by two units, by forming a six-quark condensate $\langle uudddd \rangle = \lambda_B^0$. Clearly, for experimental compatibility, this condensate must be very fuzzy, with a mass parameter $\lambda_B$ order $1$ MeV or less. This again would create a hierarchy problem, since any condensate in QCD, if it appears, must have a mass scale order QCD scale $\Lambda_{QCD} \sim 200$ MeV. Thus, a fine tuning is required of about twenty orders of magnitude.

Formally, Vafa Witten theorem [32] excludes the possibility of baryon number violating condensates in QCD. However, this theorem is based in some assumptions which leave some loophole. Namely, if quarks have masses (as we know our light quarks $u,d,s$ have masses order few MeV), the prove is formally valid if the vacuum angle $\Theta$ is exactly zero. However, the vacuum angle might be non-zero: the experimental limit on the electric dipole moment of the neutron leads only to a theoretical bound $\Theta < 10^{-10}$ or so. Then one could envisage that in the possible (but not our) world in which $\Theta \sim 1$, the baryon-violating condensates could be formed with $V \sim 100$ MeV, however the continuity hypothesis may imply that in the real world the condensate is suppressed by a factor $\Theta^3 < 10^{-20}$ which can also explain the smallness of the spontaneous breaking scale $V$.

Assuming $ad hoc$ that the six-quark operator $uudddd$ may have non-zero VEV in the QCD vacuum, $\langle uudddd \rangle = B$, then a Goldstone boson $\beta$ should emerge, the baryo-majoron, as a phase of this condensate, $B = \lambda_B^0 \exp(i\beta/f_B)$ where $f_B$ is a respective decay constant.

However now baryo-majoron becomes a composite field, exactly like pions which are the Goldstone modes of the quark condensate $\langle qq \rangle$ that breaks the chiral $SU(2)_L \times SU(2)_R$ symmetry, $\langle qq \rangle = \Sigma \exp(i\tau_3 \pi_3/f_\pi)$ with the typical value $\Sigma \simeq 200$ MeV$^3$ and $f_\pi$ being the pion decay constant. Then one can roughly estimate the mixing mass between $n - \bar{n}$ as $\epsilon \sim B/(1$ GeV)$^3$, by simply

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3 For three light flavors $u,d,s$ the condensate could appear in flavor singlet combination $\langle uudsdu \rangle$, and it would induce the Majorana mass term for the hyperon, i.e., mass mixing between the hyperon and antihyperon states, $\epsilon_\alpha \bar{X}X + h.c.$.
taking scales of the neutron mass and residue and all relevant momenta order 1 GeV and neglecting all combinatorial numerical factors. Therefore, if this six-quark condensate is characterized by a mass scale of the order of current quark masses, say $\lambda_B \sim 0.3$ MeV, then we get $\epsilon \sim 10^{-23}$ eV, which would correspond to $n - \bar{n}$ oscillation time $\tau_{n\bar{n}} \sim 10^8$ s. As for the baryo-majoron, its non-diagonal coupling between $n$ and $\bar{n}$ states is related to the value of $\epsilon$ via Goldberger-Treimann like relation $g_{\beta n} = \epsilon/f_B$. Therefore, for $f_B > 1$ MeV or so, nuclear stability limits versus the neutron decay with the majoron emission, $n \to \bar{n} + \beta$ decay, will be safely respected.

An interesting feature of the dynamical baryon violation by the QCD can be that the order parameter $\lambda_B$ could be different in vacuum and in dense nuclear matter, i.e. in nuclei or in the interiors of neutron stars. In particular, in dense nuclear matter spontaneous baryon violating could occur even if it does not take place in vacuum. Or right the opposite, dense nuclear matter could suppress the baryon violating condensates. In this case, the search of neutron antineutron oscillation with free neutrons and nuclear decay due to neutron antineutron transition become separate issues. Namely, it might be possible that the baryon violating condensates evaporate at nuclear densities and do not lead to nuclear instabilities while for free neutrons propagating in the vacuum they can be at work.

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[1] E. Majorana, Il Nuovo Cimento \textbf{14}, 171 (1937).
[2] V. A. Kuzmin, Pisma Zh. Eksp. Teor. Fiz. \textbf{12}, 335 (1970).
[3] R. N. Mohapatra and R. E. Marshak, Phys. Rev. Lett. \textbf{44}, 1316 (1980) [Erratum-ibid. \textbf{44}, 1643 (1980)].
[4] K. S. Babu and R. N. Mohapatra, Phys. Lett. B \textbf{518}, 269 (2001) [hep-ph/0108089].
[5] Z. Berezhiani and L. Bento, Phys. Rev. Lett. \textbf{96}, 081801 (2006) [hep-ph/0507031].
[6] K. S. Babu \textit{et al}., Phys. Rev. D \textbf{87}, 115019 (2013) [arXiv:1303.6918] [hep-ph].
[7] Particle Data Group, Phys. Rev. D \textbf{86}, 010001 (2012).
[8] A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. \textbf{5}, 32 (1967) [JETP Lett. \textbf{5}, 24 (1967)]; Sov. Phys. Usp. \textbf{34}, 392 (1991) [Usp. Fiz. Nauk \textbf{161}, 61 (1991)].
[9] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B \textbf{155}, 36 (1985).
[10] Z. Berezhiani and A. Vainshtein, [arXiv:1506.05096] [hep-ph].
[11] S. Weinberg, Phys. Rev. Lett. \textbf{43}, 1566 (1979).
[12] S. Rao and R. Shrock, Phys. Lett. B \textbf{116}, 238 (1982).
[13] M. Baldo-Ceolin \textit{et al}., Z. Phys. C \textbf{63}, 409 (1994).
[14] K. G. Chetyrkin M. V. Kazarnovsky, V. A. Kuzmin and M. E. Shaposhnikov, Phys. Lett. B \textbf{99}, 358 (1981).
[15] J. Chung \textit{et al}., Phys. Rev. D \textbf{66}, 032004 (2002) [hep-ex/0205093].
[16] K. Abe \textit{et al}. [Super-Kamiokande Collaboration], Phys. Rev. D \textbf{91}, 072006 (2015) [arXiv:1109.4227] [hep-ex].
[17] D. G. Phillips \textit{et al}., [arXiv:1410.1100] [hep-ex].
[18] Y. Chikashige, R. N. Mohapatra and R. D. Peccei, Phys. Lett. B \textbf{98}, 265 (1981); G. B. Gelmini and M. Roncadelli, Phys. Lett. B \textbf{99}, 411 (1981).
[19] R. Barbieri and R. N. Mohapatra, Z. Phys. C \textbf{11}, 175 (1981).
[20] G. Dvali, hep-th/0507215.
[21] M. Fukugita and T. Yanagida, Phys. Lett. B \textbf{174}, 45 (1986).
[22] Z. Berezhiani, Int. J. Mod. Phys. A \textbf{19}, 3775 (2004) [hep-ph/0312335]; “Through the looking-glass: Alice’s adventures in mirror world,” In “Shifman, M. (ed.) et al.: From fields to strings, vol. 3, pp. 2147-2195 [hep-ph/0508233].
[23] G. Ban \textit{et al}., Phys. Rev. Lett. \textbf{99}, 161603 (2007); A. Serebrov \textit{et al}., Phys. Lett. B \textbf{663}, 161 (2008); I. Altarev \textit{et al}., Phys. Rev. D \textbf{80}, 032003 (2009).
K. Bodek et al., Nucl. Instrum. Meth. A 611, 141 (2009); A. Serebrov et al., ibid. A 611, 137 (2009).

[24] Z. Berezhiani and L. Bento, Phys. Lett. B 635, 253 (2006) [hep-ph/0602227]; R. N. Mohapatra, S. Nasri and S. Nussinov, Phys. Lett. B 627, 124 (2005) [hep-ph/0508109]; Y. N. Pokotilovski, Phys. Lett. B 639, 214 (2006) [nucl-ex/0601017]; Z. Berezhiani, Eur. Phys. J. C 64, 421 (2009) [arXiv:0804.2088 [hep-ph]]; Z. Berezhiani and A. Gazizov, Eur. Phys. J. C 72, 2111 (2012) [arXiv:1109.3725 [astro-ph.HE]]; Z. Berezhiani and F. Nesti, Eur. Phys. J. C 72, 1974 (2012) [arXiv:1203.1035 [hep-ph]].

[25] Z. Berezhiani, A. Y. Smirnov and J. W. F. Valle, Phys. Lett. B 291, 99 (1992) [hep-ph/9207209].

[26] H. M. Georgi, S. L. Glashow and S. Nussinov, Nucl. Phys. B 193, 297 (1981); M. Doi, T. Kotani and E. Takasugi, Phys. Rev. D 37, 2575 (1988).

[27] A. Gando et al. [KamLAND-Zen Collaboration], Phys. Rev. C 86, 021601 (2012) [arXiv:1205.6372 [hep-ex]].

[28] Z. Berezhiani and M. I. Vysotsky, Phys. Lett. B 199, 281 (1987); Z. Berezhiani and A. Y. Smirnov, Phys. Lett. B 220, 279 (1989); Z. Berezhiani and A. Rossi, Phys. Lett. B 336, 439 (1994) [hep-ph/9407265].

[29] K. Choi and A. Santamaria, Phys. Rev. D 42, 293 (1990); Z. Berezhiani, G. Fiorentini, M. Moretti and A. Rossi, Z. Phys. C 54, 581 (1992); Z. Berezhiani, M. Moretti and A. Rossi, Z. Phys. C 58, 423 (1993).

[30] L. Bento and Z. Berezhiani, Phys. Rev. D 64, 115015 (2001) [hep-ph/0108064]; Phys. Rev. D 62, 055003 (2000) [hep-ph/9908211].

[31] G.G. Raffelt, Stars as Laboratories for Fundamental Physics, Chikago Univ. Press, 1996.

[32] C. Vafa and E. Witten, Nucl. Phys. B 234, 173 (1984).