Fulde-Ferrell-Larkin-Ovchinnikov phase and Symmetry of Fermi Surface in Two-dimensional Spin-Orbit Coupled Fermi Gas

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Abstract. We show that the combination of spin-orbit coupling and in-plane Zeeman field in a two-dimensional degenerate Fermi gas can lead to a larger parameter region for Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phases than that using spin-imbalanced Fermi gases. The resulting FFLO superfluids are also more stable due to the enhanced energy difference between FFLO and conventional Bardeen-Cooper-Schrieffer (BCS) excited states. We clarify the crucial role of the symmetry of Fermi surface on the formation of finite momentum pairing. The phase diagram for FFLO superfluids is obtained in the BCS-BEC crossover region and possible experimental observations of FFLO phases are discussed.
Fulde-Ferrell-Larkin-Ovchinnikov phase and Symmetry of Fermi Surface in Two-dimensional Spin-Orbit Coupled Superconductors

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1. Introduction

In 1964, just shortly after the great success of Bardeen-Cooper-Schrieffer (BCS) theory for superconductivity [1], Fulde and Ferrell (FF) [2], and Larkin and Ovchinnikov (LO) [3, 4] independently demonstrated that a new type of superconducting state, which is characterized by Cooper pairs with nonzero total momentum, may exist in certain regime of a clean superconductor under a strong magnetic field. The order parameters in real space for these two superconductors read as

\[ \Delta_{\text{FF}}(x) = \Delta e^{iQ \cdot x}, \quad \Delta_{\text{LO}}(x) = \Delta \cos(Q \cdot x). \]  

The superconducting state is now known as the FFLO superconductor or inhomogeneous superconductor. For conventional BCS superconductors [1], the pairing takes place between electrons with opposite momentum and opposite spin, i.e., \( k \uparrow \) and \( -k \downarrow \). Therefore when the magnetic field exceeds certain critical value, the superconductivity is destroyed due to Pauli paramagnetic depairing effect. As a consequence, magnetism and superconductivity generally cannot coexist for the BCS type-I superconductor. The physics is totally different for FFLO phases because these two different orders naturally coexist; more precisely, the FFLO phase arises from the interplay between magnetism and superconductivity. This important feature makes the FFLO phase a central concept for understanding many exotic phenomena in different physics branches, ranging from unconventional solid state superconductors (e.g., layered [5, 6], heavy-fermion [7, 8, 9], organic [14, 15] superconductors, etc.), to chiral quark matter in quantum chromodynamics (QCD), and to neutron star glitches in astrophysics [16, 17]. In the past several decades, great efforts have been made to unveil this novel quantum phase, and a lot of exotic signatures that may be related to the FFLO phase have been observed, for instance, the anisotropic thermal conductivity [18], specific heat [9], nuclear magnetic resonance [10, 11, 12], and ultrasound velocities [13] have been ascribed to the formation of FFLO superconductor in the heavy fermion superconductor CeCoI\(_5\). However, until now, clear, unambiguous and direct experimental evidences for the existence of FFLO phases are still lacking [16, 19]. There are several reasons for that: the existence of FFLO phase requires very stringent conditions; the direct probing of periodic oscillation of the order parameter is challenging; and the disorder effects in the superconductor induce strong scattering between different momenta that destroys the superconducting pairing [20, 21].

The recent experimental advances of population-imbalanced ultracold Fermi gases may have the potential to elucidate this long-sought problem. The ultracold atomic system possesses some remarkable advantages over their counterpart in solid state systems due to its high controllability and tunability [22, 23, 24]. The experimental parameters in ultracold atoms can be tuned in realistic experiments. Furthermore, the system can be made disorder free, which if necessary, can be introduced to the system in a controllable manner [25, 26, 27]. On the experimental side, the superfluidity of the Fermi gas can be characterized by the generation of vortices when the gas is rotated [28],
and the momentum of the Cooper pair in the FFLO phase can be directly probed using the time-of-flight imaging \[29, 30\], while in solid state the direct observation of the FFLO phase and its Cooper pair momentum is challenging. Unfortunately, this system still have two major obstacles hinder the observation of FFLO phase in recently experiments. Firstly, the FFLO phase only exists in an extremely narrow parameter regime in 2D and 2D (see Fig. 3a) degenerate Fermi gases \[29, 31\], therefore in experiments the FFLO phase is generally missed out. For instance, in recent experiments with population-imbalanced Fermi gases \[32, 33\] only the phase transition from BCS superfluids to normal gases has been observed. While in another experiment \[34\] the phase separation phase, which is also known as the breached pair \[35\], has been observed. Secondly, the energy difference between FFLO ground state and BCS excited state is much smaller than the temperature, therefore even the parameters for the FFLO state have been reached, the Fermi gas is still too hot to reach the ground state.

The above two obstacles can be overcome using spin-orbit (SO) coupled degenerate Fermi gases with an in-plane Zeeman field. Here the SO coupling is a central ingredient in modern physics, because it is essential to a number of important concepts in condensed matter physics, ranging from spin Hall effect \[36, 37\], anomalous quantum Hall effect \[38, 39\], and topological insulators \[40, 41, 42\]. In solid materials, the SO coupling is induced by inversion symmetry of bulk or structure \[43\]. However, in cold atom systems, the SO coupling is induced by Raman coupling between hyperfine states \[44, 45, 46, 47, 48, 49, 50\], therefore in principle, different types of SO coupling can be created by carefully choosing different laser configurations. Experimentally, the one dimensional SO coupling has been realized using Raman coupling between hyperfine states for both Bose and Fermi gases \[51, 52, 53, 54, 55, 56\], while the in-plane Zeeman field naturally exists in this system. Here we show that the combination of a Rashba-type of SO coupling and an in-plane Zeeman field can support FFLO superfluids with a unique FFLO vector in a 2D degenerate Fermi gas. The required Zeeman field or the population imbalance can be extremely small with realistic experimental parameters. The driving mechanism for the FFLO superfluids is the interplay between deformation of Fermi surface and superconducting order \[57\], thus should be in stark constrast to the physics in original FFLO superconductor \[2, 3, 4\]. Recently, there are already several related works showing that the FFLO superfluids can be observed with different SO coupling and Zeeman fields \[58, 59, 60, 61, 62, 63\], and all of them belong to the scope of this new driven mechanism. Here in this work, we provide a comprehensive understanding for the formation of FFLO superfluids in the SO coupled Fermi gas from the symmetry of Fermi surface.

The rest of this work is organized as following. We present our mean field treatment of the SO coupled degenerate Fermi gas with an in-plane Zeeman field in Sec. 2 and we discuss the basic particle-hole symmetry of the effective Hamiltonian in Sec. 3. The rotational symmetry breaking due to the in-plane Zeeman field is presented in Sec. 4. The numerical details for the FFLO superfluid are presented in Sec. 5. We plot the phase diagram and discuss its basic properties in Sec. 6 and we plot the free energy
landscape in Sec. 7. We discuss the measurement of the FFLO phase in Sec. 8. The notable differences between cold atom systems and solid materials are discussed in Sec. 9. At last we conclude in Sec. 10.

2. Physical Model

We consider a 2D degenerate Fermi gas with Rashba-type SO coupling and an in-plane Zeeman field. The 2D degenerate Fermi gases can be constructed by applying a strong standing wave along the third direction, and have been realized in recent experiments [64]. The 2D SO coupled Fermi gases can be described as [70, 71]

\[ H = \sum_{\mathbf{k},\sigma\sigma'} c_{\mathbf{k},\sigma}^\dagger [\xi_{\mathbf{k}\sigma} + \alpha (k_x \sigma_y - k_y \sigma_x) - \hbar \sigma_x] c_{\mathbf{k},\sigma'} + V_{\text{int}}, \]

where \(\alpha\) is the SO coupling strength, \(\sigma_x\) and \(\sigma_y\) are the Pauli operators, \(\xi_{\mathbf{k}\sigma} = \frac{\mu^2}{2m} - \mu\), and \(\mathbf{k} = (k_x, k_y)\). The one dimensional SO coupling has been realized in Fermionic cold atoms [55, 56]. The schemes to the realization of two dimensional SO coupling are similar, but requires more complicated laser beams, see recent works [47, 48, 49].

The last term corresponds to the s-wave scattering interaction

\[ V_{\text{int}} = g \sum_{\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}_3 + \mathbf{p}_4} c_{\mathbf{p}_1,\uparrow}^\dagger c_{\mathbf{p}_2,\downarrow}^\dagger c_{\mathbf{p}_3,\downarrow} c_{\mathbf{p}_4,\uparrow}, \]

where \(\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}_3 + \mathbf{p}_4\) due to the conservation of the total momentum during the scattering process, \(g\) is the scattering interaction strength.

When atoms form Cooper pairs with a finite total momentum \(\mathbf{Q}\), the scattering process in Eq. 3 can be simplified with \(\mathbf{p}_1 = \mathbf{k} + \mathbf{Q}/2, \mathbf{p}_2 = -\mathbf{k} + \mathbf{Q}/2, \mathbf{p}_3 = \mathbf{p} + \mathbf{Q}/2,\) and \(\mathbf{p}_4 = -\mathbf{p} + \mathbf{Q}/2\). When \(\mathbf{Q} = 0\), the Cooper pairs are formed between two atoms with opposite momentum and opposite spin, and we recover the conventional BCS type pairing. Denote \(\beta_{\mathbf{p}} = gc_{\mathbf{p}+\mathbf{Q}/2,\uparrow} c_{\mathbf{-p}+\mathbf{Q}/2,\downarrow}\), the interaction term can be written as

\[ V_{\text{int}} = g \sum_{\mathbf{k},\mathbf{p}} c_{\mathbf{k}+\mathbf{Q}/2,\uparrow}^\dagger c_{-\mathbf{k}+\mathbf{Q}/2,\downarrow}^\dagger c_{\mathbf{p}+\mathbf{Q}/2,\downarrow} c_{-\mathbf{p}+\mathbf{Q}/2,\uparrow} = \sum_{\mathbf{k},\mathbf{p}} \frac{\beta_{\mathbf{k}}^\dagger \beta_{\mathbf{p}}}{g}. \]

This interaction term can be decoupled using the standard mean-field method

\[ \beta_{\mathbf{k}}^\dagger \beta_{\mathbf{p}} \rightarrow \langle \beta_{\mathbf{k}}^\dagger \rangle \beta_{\mathbf{p}} + \beta_{\mathbf{k}}^\dagger \langle \beta_{\mathbf{p}} \rangle - \langle \beta_{\mathbf{k}}^\dagger \rangle \langle \beta_{\mathbf{p}} \rangle, \]

where the order parameter in the momentum space \(\Delta = \sum_{\mathbf{p}} g \langle c_{\mathbf{p}+\mathbf{Q}/2,\uparrow} c_{-\mathbf{p}+\mathbf{Q}/2,\downarrow} \rangle\). The interaction term now reduces to

\[ V_{\text{int}} = \sum_{\mathbf{k}} \Delta \beta_{\mathbf{k}}^\dagger + \Delta^* \beta_{\mathbf{k}} - \frac{|\Delta|^2}{g}. \]

Notice that \(c_{\mathbf{p}\sigma} = \int d\mathbf{x} c_{\sigma}(\mathbf{x}) e^{-i\mathbf{p} \cdot \mathbf{x}},\) thus we have the paring in the real space,

\[ \langle c_{\mathbf{p}\uparrow}(\mathbf{x}) c_{\mathbf{p}'\downarrow}(\mathbf{y}) \rangle = \int d\mathbf{p} d\mathbf{p}' \langle gc_{\mathbf{p}\uparrow} c_{\mathbf{p}\downarrow} \rangle e^{i(\mathbf{p} \cdot \mathbf{x} + \mathbf{p}' \cdot \mathbf{y})} = \Delta \delta(\mathbf{x} - \mathbf{y}) e^{i\mathbf{Q} \cdot \mathbf{x}}. \]
We see the finite momentum pairing in the momentum space corresponds to a spatially modulated pairing in the real space. The $\delta$-function arises from the contact interaction. The above pairing breaks the time-reversal symmetry, which is inconsistent with our model because a Zeeman field is applied.

Using the standard Bogoliubov transformation, the Hamiltonian can be written as

$$H = \frac{1}{2} \sum_k \psi^\dagger_k \mathcal{Q} H_{\text{eff}} \psi_k \mathcal{Q} - \frac{|\Delta|^2}{g} + \frac{1}{2} \sum_{k, \sigma} \xi_{k\sigma},$$

(8)

where the effective Hamiltonian reads as,

$$H_{\text{eff}} = \begin{pmatrix} \mathcal{K}(k) & \Delta I_{2 \times 2} \\ \Delta^\dagger I_{2 \times 2} & -\sigma_y \mathcal{K}^*(-k)\sigma_y \end{pmatrix},$$

(9)

with

$$\mathcal{K}(k) = \begin{pmatrix} \xi_{k+\mathcal{Q}/2,\uparrow} & h - \alpha R(k) \\ h - \alpha R^*(k) & \xi_{k+\mathcal{Q}/2,\downarrow} \end{pmatrix},$$

(10)

$$R(k) = (k + \mathcal{Q}/2)_x + i(k + \mathcal{Q}/2)_y, \text{ and } I_{2 \times 2} = \text{diag}(1, 1).$$

The basis defined in Eq. 8 is $\psi_{k,\mathcal{Q}} = (c_{k+\mathcal{Q}/2,\uparrow}, c_{k+\mathcal{Q}/2,\downarrow}, c_{-k+\mathcal{Q}/2,\downarrow}^\dagger, -c_{-k+\mathcal{Q}/2,\uparrow}^\dagger)^T$. The minus sign in the last term of the basis is used to achieve the $\Delta I_{2 \times 2}$ type off-diagonal term in Eq. 9.

The thermodynamical potential at zero temperature reads as

$$\Omega = -\frac{\Delta^2}{g} + \frac{1}{2} \sum_{k\sigma} \xi_{k\sigma} + \frac{1}{2} \sum_{k, \lambda} E_\lambda \Theta(-E_\lambda),$$

(11)

where the Heaviside step function

$$\Theta(x) = \begin{cases} 
1, & x \geq 0 \\
0, & x < 0
\end{cases}.$$  

(12)

$E_\lambda, \lambda = 1, 2, 3 \text{ and } 4,$ are the eigenvalues of the effective Hamiltonian $H_{\text{eff}},$ whose exact expressions are too complex to be presented here. We therefore see the standard mean-field decoupling used in Eq. 5 actually corresponds to the Hubbard-Stratanovich transformation in the quantum field theory. We use the mean field theory as the main theoretical tool in this work because it provides a more transparent description of the FFLO physics.

The effective scattering interaction $g$ in Eq. 11 in a 2D Fermi gas should be regularized through

$$\frac{1}{g} = -\sum_k \frac{1}{k^2/m + E_b},$$

(13)

where the binding energy $E_b$ can be tuned by varying the $s$-wave scattering length through Feshbach resonance \cite{22, 23, 24}. 

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*Fulde-Ferrell-Larkin-Ovchinnikov phase and Symmetry of Fermi Surface in Two-dimensional Spin-Orbit Coupling.*
3. Particle-Hole Symmetry

The particle-hole operator for the SO coupled Fermi gases can be written as $\Sigma = \Lambda K$, where $\Lambda = \sigma_y \tau_y$ ($\sigma_y$ is the Pauli spin matrix and $\tau_y$ is the Nambu particle-hole matrix), and $K$ represents the complex conjugate operator. Obviously, $\Lambda$ is a unitary operator, and $\Lambda = \Lambda^{-1}$. It is easy to check $\Sigma^2 = \Lambda K \Lambda^* \Lambda = \Lambda \Lambda^* = \Lambda^2 = 1$. Moreover, the formation of FFLO phase breaks the time-reversal symmetry, therefore the system belongs to the symmetry class $D$, see Ref. [66]. The particle-hole operator has the following basic property,

$$\Sigma H_{\text{eff}}(k) \Sigma^{-1} = \Lambda H_{\text{eff}}^*(k) \Lambda$$

$\Sigma$ establishes a one-to-one correspondence between $k$ and $-k$, therefore if $\psi_k = (u(k), v(k))^T$ is an eigenvector of the Hamiltonian $H_{\text{eff}}(k)$ with energy $E(k)$, then $\psi'_k = \Sigma \psi_k = (\sigma_y v^*(k), -\sigma_y u^*(k))^T$ is the eigenvector of the Hamiltonian $H_{\text{eff}}(-k)$ with energy $-E(k)$ (see in Fig. 1). The particle-hole symmetry does not automatically ensure that the eigenvalues appear in pairs $(E, -E)$ because the system lacks the chiral
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This is a direct consequence of the inversion symmetry breaking (see Sec. 4) of our model in the presence of an in-plane Zeeman field. The trace of the effective Hamiltonian gives,

$$\text{tr}(H_{\text{eff}}(k)) = \sum_{\lambda} E_{\lambda} = \frac{2k \cdot Q}{m},$$

(15)

therefore for the FFLO phase, the eigenvalues never appear with pairs. Eq. 15 is essential for us to understand the band structures of the FFLO superfluid. With only in-plane Zeeman field, only trivial phase can be observed, see discussion in Ref. 67.

4. Symmetry of Fermi Surface

The symmetry of Fermi surface is essential to understand the properties of different quantum phases and their signature in the time-of-flight imaging, which is the basic motivation of this work. The Rashba type SO coupling, $V_{\text{so}} = \alpha(k_x \sigma_y - k_y \sigma_x)$, is invariant under the simultaneous rotation of the momentum and spin in the $xy$ plane,

$$
\begin{pmatrix}
-k'_y \\
k'_x
\end{pmatrix} = U 
\begin{pmatrix}
-k_y \\
k_x
\end{pmatrix},
$$

$$
\begin{pmatrix}
\sigma'_x \\
\sigma'_y
\end{pmatrix} = U 
\begin{pmatrix}
\sigma_x \\
\sigma_y
\end{pmatrix},
$$

(16)

where $U$ is the SO(2) rotation matrix,

$$
U = \begin{pmatrix}
\cos(\theta) & \sin(\theta) \\
-\sin(\theta) & \cos(\theta)
\end{pmatrix}.
$$

(17)

The SO(2) rotation matrix does not change the magnitude of the momentum, thus $\xi_{k'\sigma} = \xi_{k\sigma}$ is also invariant under this transformation. Meanwhile, by defining $\sigma'_z = \sigma_z$, the new Pauli matrices $\sigma'_{x,y,z}$ satisfy the standard commutation relation

$$
[\sigma'_{a}, \sigma'_{b}] = 2i \sum_c \varepsilon_{abc} \sigma'_{c},
$$

$$
\{\sigma'_{a}, \sigma'_{b}\} = 2\delta_{ab},
$$

(18)

with $\varepsilon_{abc}$ the Levi-Civita symbol and $\delta_{ab}$ the Kronecker delta.

The SO(2) symmetry may breakdown in the present of both Rashba and Dresselhaus SO coupling. However, in this case, the Fermi surface still has inversion symmetry, which means that the eigenvalues of single particle Hamiltonian have the basic property $E_{k\sigma} = E_{-k\sigma}$ for any $k$ and $\sigma$. This symmetry is unbroken by out-of-plane Zeeman field. The inversion symmetry of Fermi surface is most relevant to the physics in this work, and it is exact this symmetry ensures that the BCS phase instead of FFLO phase is more energetically favorable in the present of out-of-plane Zeeman field. An intuitive understanding of this result is that for any state with momentum $k$, we can always find another degenerate state with opposite momentum at the same band, thus we have BCS phase. The SO coupling here plays the role of inducing pairing at the same band.
The inversion symmetry is broken by in-plane Zeeman field because the rotational in Eq. [17] results in the following transformation,

$$\sigma_x \rightarrow \cos(\theta)\sigma_x + \sin(\theta)\sigma_y.$$  

(19)

Physically, it means that it is impossible to find two degenerate states with opposite momentum at the same band. This anisotropy effect also lead to a unique FFLO vector $Q$ for the FFLO superfluid, which is one of the key point of our proposal in Ref. [57]. The unique $Q$ makes the detection of the FFLO vector much easier in realistic experiments, see more discussions in Sec. 8. This picture is quite general and for this basic reason, the FFLO phase in this work can also be realized using other types of SO coupling [58, 59, 60, 61, 62, 63].

The symmetry breaking has a direct consequence on the formation of FFLO superfluids. Before the presentation of our numerical results, we first illustrate the basic physical picture for the formation of FFLO superfluids. For the Fermi gas with only Zeeman field, see Fig. 2a, the two mismatched Fermi surfaces always form concentric circles, therefore for the s-wave pairing, the up- and down-spins acquire different Fermi
momentum, i.e., $k + Q/2, \uparrow$ and $-k + Q/2, \downarrow$, with $\uparrow$ and $\downarrow$ spins in the pseudospin representation and $Q$ as the total momentum of the Cooper pairs. The free energy of the system satisfies the following basic property,

$$F(Q) = F(|Q|).$$

(20)

Here the Zeeman field only fixes the direction of the spin, but does not fix the direction of the momentum axis, therefore the free energy should be invariant under rotation of the momentum $Q$. Mathematically, it can also be understood from the fact that the total free energy depends on $k^2$, $Q^2$ and $k \cdot Q$, thus the summation over $k$ should be independent of the direction of $Q$, see also Ref. [20] for more details. Physically, Eq. 20 means that the total momentum of the Cooper pair can take any direction by spontaneous symmetry breaking, therefore the ground state FFLO phase is infinity-fold degenerate. Generally in the numerical simulation, we artificially set $Q$ along a particular direction and demonstrate that the FFLO phase indeed has a lower energy than the regular BCS superfluid ($Q = 0$). Due to the Pauli paramagnetic depairing effect, the FFLO phase only survives in a very narrow parameter regime, see also the numerical results in Fig. 3a. In realistic system, any weak scattering induced by disorder effect can lead weak coupling between the degenerate ground states manifold, making the LO superfluids, which can be regarded as a superposition of the two FF superfluids with total momentum $Q$ and $-Q$, as the true ground states, and the LO superfluids still respect the basic symmetry argument in Eq. 20.

The physical picture is totally different when the SO coupling is presented, as schematically shown in Fig. 2b. In this case the Fermi surface is deformed and the center of the Fermi surface is no longer located at $k = 0$, therefore breaks the inversion symmetry. Here we should notice that the deformation of the Fermi surface depends strongly on the direction of the SO coupling and Zeeman field. For the model we consider here, the deformation is along the $y$ direction. In the pseudospin representation (the eigenstates of single particle Hamiltonian), we have both singlet pairing and triplet pairing, where the triplet pairing will not be destroyed by strong Zeeman field, thus the FFLO phase can be observed in a much larger parameter regime. The deformation of the Fermi surface makes the FFLO phase always energetically favorable even with a small Zeeman field. In our numerics, we find that the FFLO vector $Q$ is along the deformation direction of the Fermi surface. The inversion symmetry breaking directly lead to $F(Q) \neq F(-Q)$, which stabilize the FF superfluids against the formation of LO superfluids phase.

Generally, the mismatch of the Fermi surface is the basic route to the FFLO phase, and such mismatched Fermi surface can be created by population imbalance [32, 33, 34], Zeeman field [30] or mass imbalance [68, 69]. In this work, together with our previous work [57], we demonstrate that the FFLO phase can be created more efficiently through the deformation of the Fermi surface, which can be constructed by SO coupling, or, non-Abelian gauge field [44, 45, 46, 47, 48, 49, 50], and Zeeman field. Notice that the generation of non-Abelian gauge fields is a subject of intensive investigations in
ultracold atoms in the past decade, see a recent review in Ref. [50]. For this new route, the Zeeman field is still needed. Otherwise the system has the time-reversal symmetry and the band structure should satisfy $E_{k\uparrow} = E_{-k\downarrow}$, which means that two Fermions with opposite momentum on the Fermi surface can always form BCS Cooper pairs efficiently (the pairing is not necessary in the singlet channel), leading to BCS superfluids, instead of FFLO phases. Our route here, however, shows that the FFLO phase may be observed even with small Zeeman field (thus small population imbalance). It therefore represents a new driven mechanism for FFLO superfluid.

5. Numerical Details

The order parameter $\Delta$, chemical potential $\mu$, and the FFLO momentum $Q$ should be solved self-consistently due to the conservation of atom number, i.e.,

$$\frac{\partial \Omega}{\partial \mu} = -n, \quad \frac{\partial \Omega}{\partial \Delta} = 0, \quad \frac{\partial \Omega}{\partial Q} = 0.$$  \hspace{1cm} (21)

Here $\Delta$ and $Q$ are used to minimize the thermodynamical potential $\Omega$. We consider three different quantum phases: the normal phase with $\Delta = 0$ and $Q = 0$ (for the normal gas $Q$ does not enter the effective free energy, thus can be any value. We force $Q = 0$); The BCS-type of superfluid with $Q = 0$ but $\Delta \neq 0$; and the FFLO phase with $Q \neq 0$ and $\Delta \neq 0$. When we fix $Q = 0$ then only BCS type of superfluid phase and the normal gas can be obtained. Throughout this work, we use two different strategies to check the influence of $Q$ on the formation of the FFLO phase. In the first strategy, we enforce $Q = 0$ while in the other strategy, we let $Q$ as a free parameter. For the results at $Q \neq 0$, these two strategies yield the energy difference between the FFLO ground state and the possible BCS superfluid excited state, which is crucial for the stability of the FFLO phase at finite temperature. Throughout this work, $Q_c = 10^{-3}K_F$ is used.

Because the Zeeman field is applied along the $x$-axis, the population imbalance should be defined using the eigenstates of $\sigma_x$, instead of $\sigma_z$. Since $\langle \sigma_x \rangle = \sum_k \langle c_{k\uparrow}^\dagger c_{k\downarrow} + \text{h.c} \rangle$, we have

$$P = \frac{\langle \sigma_x \rangle}{n} = \frac{\sum_k \langle c_{k\uparrow}^\dagger c_{k\downarrow} + \text{h.c} \rangle}{n} = \frac{1}{2n} \sum_{k,\lambda} \psi_{k,\lambda}^\dagger \begin{pmatrix} \sigma_x & 0_{2\times2} \\ 0_{2\times2} & -\sigma_x \end{pmatrix} \psi_{k,\lambda}. \hspace{1cm} (22)$$

Here $\psi_{k,\lambda}$ is the eigenstate of the effective Hamiltonian $H_{\text{eff}}$, i.e., $H_{\text{eff}} \psi_{k,\lambda} = E_{\lambda} \psi_{k,\lambda}$.

In our calculation, we choose the energy unit as the Fermi energy $E_F$ of the system without interaction, Zeeman field and SO coupling. The corresponding length scale $K_F^{-1}$ is defined through the Fermi momentum $K_F$. At finite temperature, the 2D system does not have the long-range order due to the phase fluctuation and the relevant physics is the Kosterlitz-Thouless transition [72]. In this paper, we restrict to the physics at zero temperature, where the mean-field theory is still valid. For this specific model, we find $Q = (0, Q)$, which means that the FFLO momentum is along the Fermi surface direction, see Fig. 2. We notice that the direction of the FFLO vector $Q$ is also consistent with the results in solid state systems with weak SO coupling, see Ref. [79].
6. Phase diagram

We first present the phase diagram with different SO coupling strength and Zeeman field in Fig. 3. Without SO coupling, see Fig. 3a, we see that the FFLO phase only exists in an extremely narrow parameter regime. When $E_b \geq 0.7E_F$, the FFLO phase disappears, thus such a phase can be only observed in the weak binding energy regime, for instance, $E_b \in (0.15, 0.7)E_F$. Similarly, the FFLO phase can also be observed in the 3D system, see Ref. [57]; however, the FFLO phase in 3D Fermi gases can only be observed near the unitary regime within a small parameter region, and the small FFLO regime can be easily missed out in realistic experiments, which is also one of the main reasons why the FFLO phases cannot be observed in recent experiments in 3D Fermi gases [32, 33, 34]. With an increasing SO coupling strength, see Fig. 3b for $\alpha K_F = 0.5 E_F$ and Fig. 3c for $\alpha K_F = 1.0 E_F$, we find that the FFLO phase regime is greatly enlarged. In the strong
Figure 4. (Color online). Evolution of chemical potential (a), order parameter (b), and FFLO vector $Q$ (c) as a function of the binding energy. In (d) we plot $(F_{\text{FFLO}} - F_{\text{BCS}})/nE_F$ vs. $E_b$. In all calculations we set $h = 0.8E_F$. The solid line, dashed line and dash-dotted line correspond to $\alpha K_F = 0.0$, $0.5$ and $1.0$, respectively.

Figure 5. (a) FFLO momentum $Q$ as a function of Zeeman field (solid line), and the dashed line is the linear fitting at small Zeeman field regime, which give $Q = 0.1434h$. (b) Population imbalance $P$ (Eq. 22) as a function of SO coupling strength. The parameters are: (a) $E_b = 0.4E_F$, $\alpha K_F = 1.0E_F$; (b) $E_b = 0.4E_F$. 
SO regime in Fig. 3, we even observe that the phase diagram is almost fully filled by the FFLO phase, while the BCS superfluid phase is greatly suppressed and only survives in a very small regime. To see the impact of SO coupling more clearly, we plot in Fig. 3 the phase diagram in the $h - \alpha K_F$ plane with $E_b = 0.4 E_F$. We define the boundary between BCS superfluid and FFLO phase as $h_1$ and the boundary between FFLO phase and normal gas as $h_2$ for convenience, see Fig. 3. We observe $h_1$ decreases while $h_2$ increases with the increasing SO coupling strength, therefore the FFLO phase is greatly enlarged in the strong SO coupling regime. It should be noticed that in 3D Fermi gases $h_2$ slightly decreases with the increasing SO coupling strength \cite{57}. In the strong SO coupling region, $h_1$ becomes very small, but never becomes zero because the the Zeeman field is essential for the FFLO phase, which breaks the time-reversal symmetry.

We plot the evolution of chemical potential, order parameter and $Q$ as a function of binding energy in Fig. 4 where the Zeeman field is fixed to $h = 0.8 E_F$. As we decreases the binding energy, we observe a sudden drop of the order parameter in Fig. 4 at zero SO coupling strength due to the Pauli paramagnetic depairing effect, following which there is a small regime that supports FFLO phase, see also the solid line in Fig. 4.
$Q \neq 0$. With the increasing SO coupling strength, we see that the change of $\Delta$ becomes a smooth function of $E_b$, and in a much larger parameter regime we can observe the FFLO phase with non-zero $Q$. The results in Fig. 4c clearly demonstrate the enlargement of FFLO superfluid phases observed in Fig. 3.

The FFLO superfluids in our model may be directly observed at finite temperature. We denote $F_{\text{FFLO}}$ as the free energy obtained by letting $Q$ as a free parameter, while $F_{\text{BCS}}$ as the free energy by enforcing $Q = 0$. In the FFLO phase regime, $F_{\text{BCS}}$ represents the free energy of BCS excited states, therefore the energy difference per particle between $F_{\text{FFLO}}$ and $F_{\text{BCS}}$, i.e.,

$$\delta F = (F_{\text{FFLO}} - F_{\text{BCS}})/nE_F,$$

which directly characterizes the stability of the FFLO phase (i.e., the larger $|\delta F|$, the more stable FFLO phase). Obviously, when $Q = 0$, $\delta F = 0$. The numerical results are presented in Fig. 4d, where we clearly see the enhancement of $\delta F$ due to the SO coupling. However, in 2D Fermi gases the enhanced factor is about two order of magnitude smaller than that in SO coupled 3D Fermi gases.

In Fig. 4b, we see that in the BCS superfluid regime ($E_b > 0.7E_F$), the order parameter decreases with the increasing SO coupling, which is in sharp contrast to that for SO coupled BEC-BCS crossover with Z direction Zeeman field. Generally, with the Z direction Zeeman field, the SO coupling plays the role of increasing the density of states near the Fermi surface, which increases the order parameter as well as the critical temperature. With an in-plane Zeeman field, the SO coupling plays a totally different role. Firstly, the in-plane Zeeman field deforms the Fermi surface, thus any small deformations leads to small finite momentum $Q$, as shown in Fig. 5a. In the small Zeeman field regime, the momentum $Q \propto h$, while in the large Zeeman field regime, it become a nonlinear behavior. Secondly, the SO coupling can enhance the population imbalance, see Fig. 5b, thus renders the decrease of the order parameter as observed in Fig. 4b. In the FFLO phase regime, the order parameter increases with the increasing SO coupling strength due to the formation of the FFLO phase. Notice that in our model, the FFLO superfluid can appear with extremely small population imbalance, thus it is driven by the interplay between the deformation of Fermi surface and the superconducting order, instead of the original idea of FFLO superfluid which arises from the interplay between magnetism and superconducting order. The new driven mechanism represents a more efficient way to create the FFLO superfluid.

7. Free Energy Landscape

To understand the results more clearly, we plot the free energy per particle in the $Q - \Delta$ plane, where the global minimum of the free energy marked by the cross symbol in each panel corresponds to the self-consistent solution of Eq. 21. The left to right shows the influence of the SO coupling on the formation of the FFLO phase, while the top to down shows the influence of the Zeeman field. Note that without SO coupling, the free energy is a symmetric function of $Q$, therefore in Fig. 6a2 there are two degenerate
Figure 7. (a) Free energy as a function of $Q$, where $F(Q) \neq F(-Q)$ stabilize the FF superfluids against the formation of LO superfluids. (b-d) $(F(-Q) - F(Q))/nE_F$, $\Delta/E_F$ and $Q/K_F$ as a function of SO coupling, Zeeman field and binding energy, respectively. The parameters used in all figures are: (a): $E_b = 0.4E_F$, $\alpha K_F = 1.0$, $h = 0.8E_F$; (b) $E_b = 0.4E_F$, $\alpha K_F = 1.0$; (c) $E_b = 0.4E_F$, $h = 0.5E_F$; and (d) $\alpha K_F = 1.0E_F$, $h = 0.5E_F$.

FFLO ground states at $\pm Q$ because we have assumed $Q$ is along the $y$ direction in our numerical simulation. This is not true in the real system where the free energy only depends on the magnitude of $|Q|$ due to the rotation symmetry, see Eq. [20]. The free energy is still a symmetric function with respect to $Q$ when $h = 0$. However it becomes an asymmetric function when both Zeeman field and SO coupling strength become non-zero, thus only one global minimum can be found in the $Q - \Delta$ plane in Figs. 6b2, c2, c3, and the ground state is unique (hence $Q$ is unique). The plot of the free energy in the $Q - \Delta$ plane directly reflects the effect of the rotational symmetry breaking of the effective Hamiltonian. With the increasing Zeeman field, the order parameter decreases due to the formation of the FFLO phase. Due to the increases of $h_2$, the boundary between FFLO phase and normal gas, we observe the FFLO phase in the strong Zeeman field and strong SO coupling region in Fig. 6c3.

It is well known in solid materials that the LO superfluids, which is the superposition
of FF superfluids with total momentum $Q$ and $-Q$, is more energetically favorable in realistic systems. The basic reason is that the deformation of Fermi surface is very small, thus $F(Q) \approx F(-Q)$. As a results, the coupling between FF superfluids with momentum $Q$ and $-Q$ lead to the formation of LO superfluids with slightly lower energy. The coupling between different FF superfluids is even significant in the degenerate ground states manifolds. So it means that the energy difference between $F(Q)$ and $F(-Q)$, where $Q$ is the FFLO superfluids momentum obtained using the previous procedures, can be used to qualify whether FF superfluids is more stable than the LO superfluids. Hence we define

$$\delta F_Q = (F(-Q) - F(Q))/nE_F.$$  

(24)

Obviously, when $Q = 0$, $\delta F_Q = 0$. On the other hand, $\delta F_Q = 0$ when $\Delta = 0$. Note that

\textbf{Figure 8.} (color online). Eigenvalues $E_\lambda (\lambda = 1, 2, 3, 4)$ of the SO coupled degenerate Fermi gas. (a), (b) correspond to the typical eigenvalues $E_\lambda$ for the BCS superfluid with parameters $h = 0.2$, $E_b = 0.4$, $\alpha k_F = 1.0$. (c), (d) correspond to the typical eigenvalues $E_\lambda$ for the FFLO superfluid with parameters $h = 0.4$, $E_b = 0.4$, $\alpha k_F = 1.0$. (e), (f) correspond to the typical eigenvalues $E_\lambda$ for the normal gas with parameters $h = 1.0$, $E_b = 0.4$, $\alpha k_F = 1.0$. The first column shows the results along the $x$ direction, while the second column shows the results along the $y$ direction. The energies are in unit of $E_F$. In each panel, the dash-dotted line represent $\text{Tr}(H_{\text{eff}}(k))$, see Eq. 15.
δF defined in Eq. 23 and δF_Q defined above have totally different physical meanings, and should not be mixed up. In Fig. 7a, we show that the breaking of inversion symmetry readers $F(Q) \neq F(-Q)$. We also plot $\delta F_Q$, $\Delta/E_F$ and $Q/K_F$ as a function of SO coupling, Zeeman field and binding energy in Fig. 7b-d. We see that the increase of SO coupling monotonically increase $Q$ and hence $\delta F_Q$ also increase monotonically. However, when $\Delta$ or $Q$ has a sudden jump at some point, see Fig. 7b, d, then $\delta F_Q$ may take a maximum at these points. In the condition with strong Zeeman field or large binding energy, the FFLO phase is suppressed, therefore $\delta F_Q$ approaches zero as the increase of these parameters. Here the most interesting result we observed is that $\delta F_Q$ can be as large as 0.1, and this large energy difference ensures that the FF superfluid phase has much lower energy than the LO superfluid phase. Similar result can not be observed in solid materials.

8. Measurement of the FFLO phase

The three different phases have different properties which can be used for the identification of these phases. In Fig. 8 we plot the typical band structures $E_{\lambda}$, $\lambda = 1$, 2, 3, 4, for the BCS superfluid, the FFLO phase and the normal gas. Due to the rotational symmetry breaking, we have to plot the dispersions along the $k_x$ and $k_y$ axes, respectively. For a typical BCS superfluid ($Q = 0$) in Fig. 8a and Fig. 8b, we see that the system is always gapped and the band structure is always symmetric about $E_{\lambda}$. In 2D system the linear dispersion is essential to make the FFLO phase robust against low-energy fluctuations.
Figure 10. (color online). Momentum distributions $n_\sigma(k) = \langle c_{k\sigma}^\dagger(k)c_{k\sigma}(k) \rangle$ and $n = n_\uparrow + n_\downarrow$ for different quantum phases. Other parameters are exactly the same as that in Fig. 8.

$k = 0$ for the dispersion along $k_x$. While along the $k_y$ axis, such symmetry is absent. In fact we can verify exactly that the BCS superfluid is always gapped. However for the FFLO phase, the superfluid becomes gapless along both $k_x$ and $k_y$ axes. Along the $k_x$ axis the band structure is symmetric about $k = 0$, but along the $k_y$ direction such symmetry is broken. For the FFLO phase we observe $\sum_\lambda E_\lambda \neq 0$ because $Q \neq 0$ (see numerical results in Fig. 8), which is consistent with our symmetry analysis in sec. 4.

Note that the gapless excitation is a typical feature of the FFLO phase, as have been pointed out in literature [29]. In the vicinity of the gapless excitation, see Fig. 3 the dispersion becomes linear which is essential to ensure that the FFLO phase is robust against the low-energy fluctuations. Here we should emphasize that not all FFLO phases are gapless. The FFLO state may become gapless only when $Q$ is relatively large, while for a small $Q$ (near the boundary between FFLO and BCS superfluid) the FFLO phase is still gapped, similar to that in the BCS superfluid. For the normal gas the band structure also shows strong deformation along the $k_y$ axis, as seen in Fig. 8b and Fig. 8.

The corresponding momentum distributions $n_\sigma = \langle c_{k\sigma}^\dagger c_{k\sigma} \rangle$ and $n = n_\uparrow + n_\downarrow$ provide
an important tool to detect the properties of the FFLO state because they can be directly measured via free expansion of the atomic cloud. We plot the momentum distributions in Fig. 10 for three different phases presented in Fig. 8 at zero temperature. The dispersion properties of the band structures can be directly reflected on the corresponding momentum distributions. We see that for three different quantum phases, the momentum distributions are always symmetric about \( k = 0 \) along the \( k_x \) direction, while show strong asymmetric along the \( k_y \) direction. However, the sum of the momentum distributions \( n \) for spin up and spin down components still shows perfect symmetry about \( k = 0 \) along both \( k_x \) and \( k_y \) directions. Therefore detecting the asymmetry of the superfluid is not sufficient for the identification of the FFLO phase. To identify the superfluid nature of the FFLO phase, we have to rotate the sample to create vortices, which is a direct evidence of superfluidity. Near the boundary between different phases, the fluctuation effect may become significant thus the phase boundary region is not suitable for the observation of vortices. With the large FFLO phase region in our model we can safely choose some parameters in the middle of the FFLO phase region where the fluctuation effect should be minimized. The large FFLO superfluid phase ensures that it will not be missed out in future realistic experiments.

The properties of the FFLO phase may be measured using a number of methods developed in ultracold atom systems, for instance, shot-noise correlation [74] and density-density correlation measurement [75, 76], which shows a peak at the Cooper pair momentum \( Q \). After released from a trapping potential, the free expansion of the Fermi cloud has a peak at \( r = hQ t / m \), therefore the direct measurement of the FFLO momentum \( Q \) is possible [77]. In our model when \( Q \) is unique, repeated measurement to determine the FFLO momentum becomes possible, see Fig. 11. In the FFLO phase without SO coupling, the ground state is independent of the direction of \( Q \), thus only a
Fulde-Ferrell-Larkin-Ovchinnikov phase and Symmetry of Fermi Surface in Two-dimensional Spin-Orbit Coupled Circle with radius $\hbar|Q|t/m$ can be observed, see Fig. 11. So the time-of-flight imaging provides the most convenient way to probe the symmetry effect of the degenerate Fermi gas. In other words, the time-of-flight imaging directly reflect the deformation direction of the Fermi surface. The FFLO phase can also be measured using the Fourier sampling of time-of-flight images proposed by Duan [78]. The gapless excitations in the FFLO phase may be observed using the Bragg spectroscopy [77].

9. Comparison between cold atom and solid materials

We notice that the similar model (Rashba SO coupling, in-plane Zeeman field, etc.) has been discussed in condensed matter physics in the context of noncentrosymmetric superconductors [80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91], and our observation that $Q$ perpendicular to the direction of Zeeman field as well as the SO interaction significantly broadens the FFLO phase in parameter space are consistent with that in solids[79]. It does not means that the physics in solids and cold atoms are similar or identical. In the following, we summarize some of the notable differences between these two totally different systems. Some of these differences have been briefly discussed in our previous work[57].

9.1. Different driving mechanism for FFLO phases

The driving mechanisms for FFLO phases in solid state systems and cold atom systems are quite different. In solid state systems, Zeeman field and SO energy are generally much smaller than the Fermi energy, the asymmetry of the Fermi surface is still very small, and the FFLO phase is mainly induced by the interplay between magnetic and superconducting order. In our work[57], we propose a totally different route for the creation of FFLO phase. In cold atom system, the Fermi energy, SO coupling energy and Zeeman field energy are at the same order, the deformation of the Fermi surface becomes coupling energy at the order of the Fermi energy, the deformation of the Fermi surface becomes significant, therefore the FFLO phase can still be observed even for system with small population imbalance. In this new mechanism, the FFLO phase is induced by the interplaying between asymmetry of Fermi surface and superconducting order. Note that in Ref. [86], the ratio between SO coupling energy and Fermi energy is of the order of $0.1 - 0.5$ in the SrTiO$_3$/LaAlO$_3$ oxide interface, and we expect this new mechanism applies to this solid material. Here we need to emphasize that the basic mechanism for finite momentum pairing in some of the solid materials are still not well theoretically understood due to the complicated spin fluctuating effect, multi-band structure, magnetism, Fermi surface nesting and other uncontrollable interactions. The physics in cold atom system is extremely clear in this sense.

The different driving mechanisms lead to completely different physical results. In solid materials[79, 81, 82, 83, 84], the broadening of the FFLO phase in discussed in the temperature - magnetic field ($H - T$) plane. In Ref. [79] the broadening of FFLO phases
mainly comes from the increase of \(H_{2c}\) (the second critical field between superconductor and normal states) in solid state materials. In contrast, the broadening of FFLO phases in our work comes from the decrease of the critical Zeeman field between BCS superfluids and FFLO superfluids, see Fig. 3d and Fig. 1d in Ref. [57].

9.2. Different dimensionality

In solid materials, the FFLO phase can only be observed in low dimensional systems, in which an external magnetic field parallel to the sample surface can effectively suppress the orbital effect. The disadvantage is that the fluctuating effect is also significant in low dimensional systems, which is probably one of the basic reasons that why FFLO phase is not observed in practical experiments. In cold atom systems, the orbital effect is independent of the dimensionality of the system because the Zeeman field is purely induced by Raman coupling and detuning of hyperfine states. Therefore the FFLO phase can not only be observed in low dimensional systems, but also in three dimensional system[57]. The cold atom platform has the additional advantage of disorder free.

9.3. BCS limit versus BCS-BEC crossover physics

In solid materials, \(h \ll E_F\), and the relevant FFLO physics occurs in the BCS limit. In cold atom systems for the parameter regime \(h \sim E_F\), the FFLO phase can be observed in the strong coupling regime, which can be tuned by Feshbach resonance (\(E_b\) in this work, and scattering length in Ref. [57]). As a result, the relevant interesting physics in cold atom system is the BEC-BCS crossover.

9.4. Different realistic experimental conditions and concerns

The solid state systems and ultra-cold atom systems have very different constraints for the experimental realization and observation of FFLO phases. The following is a comparison: Solid state systems: 1) Temperature is not an issue because of the large Fermi energy; 2) Disorder is very important and its role is still not well understood (see Ref. [86] for the influence of disorder on FFLO states); 3) The FFLO phase is hard to observe directly; 4) The scattering effect and associated lifetimes are crucial for FFLO phase. Cold atomic systems: 1) Temperature is important because current experimentally reachable temperature is around \(0.05E_F\). Therefore the energy difference between FFLO state and BCS excited state is important, which is shown to be large in our scheme and is one major advantage of our proposal; 2) Disorder free; 3) The FFLO phase can be observed directly in time of flight images; 4) The system is very stable, and the lifetime issue is unimportant. The experimental tools in these materials are also quite different. In solids, the thermal conductivity [18], specific heat[9], nuclear magnetic resonance[10,11,12], and ultrasound velocities[13] are generally used to study the anomalous properties of FFLO phase, while in cold atoms, the time-of-flight imaging
can be directly used to probe the pairing momentum and associated Fermi surface asymmetry. In this sense, the cold atom platform may provide the most convincing evidence for the formation of finite momentum pairing.

10. Conclusion

To summarize, in this paper we study the possible FFLO phase in SO coupled degenerate Fermi gases with in-plane Zeeman fields. We show that the parameter region for the FFLO phase can be greatly enlarged due to the deformation of the Fermi surface. The emergence of the FFLO phase is explained from different angles. The properties of the BCS superfluid, FFLO phase and normal gas have also been discussed and their measurement through the time-of-flight imaging is presented. Our results indicate that the deformation of the Fermi surface provides a more efficient method to generate the FFLO phase. Because the SO coupling has been realized in Bose [51, 52, 53, 54] and Fermi [55, 56] cold atom gases in experiments, where the in-plane Zeeman field can be naturally created [51, 52, 53, 55, 56] and tuned, we expect the idea in this work may provide a path for elucidating the long-standing problem about FFLO phases in experiments in the near future.

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