Heuristic model of teaching

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A simple physical model differentiates effective from ineffective teaching and identifies elements that make teaching productive, with specific implications concerning training of teachers.

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I. INTRODUCTION

It is reasonable to attempt to describe the process of teaching in terms of a model because it is known that models can lead to improvement in our understanding of natural phenomena and the understanding may allow us to distinguish useful from useless action. A physical model proposed in this article provides a picture in which effective teaching is clearly separated from ineffective teaching. The model picture also allows for identification of elements which lead to teaching that may be called productive. These insights have implications for the training of teachers.

Section II discusses the reasons why a model of teaching is needed and sets the stage for the next two sections, in which the model proposed in this article is described. Section III describes a kinematical part of the model that is sufficient to identify the difference between effective and ineffective teaching. Section IV describes a dynamical part of the model that leads to the concept of productive teaching. Conclusions are in Section V.

II. THE NEED FOR A MODEL

Building models is a time-honored practice to organize thinking about concepts in physics. For example, the concept of the atom has been discussed since ancient times. About a century ago, Bohr was equipped with enough data on atomic phenomena to conceive and describe a concrete image of an atom [1]. Bohr’s model allowed physicists to focus their attention on the dynamical issues in the physics of atoms and gradually replace the model with quantum mechanics. The latter provided a mathematical basis for the progress that followed [2].

Thus, one can say that physical models may be useful in two ways. One way is that a model provides a context in which a concept can be spoken about in concrete terms. The concreteness of the context helps eliminate confusion and allows researchers to focus on the dynamics of the observed phenomena. Another way is that a model can be wrong in the sense that it disagrees with results of experiments or observations. Such findings provide the basis for seeking a better next-generation model. The same methodology may be applied to teaching. There is no reason to limit this methodology to physics teaching; it applies also to teaching in other disciplines.

A simple physical model of teaching (MOT, or just “model”) described in this paper incorporates the concepts of a teacher and a student through the concept of observers. Two observers can communicate with each other about phenomena they observe and one can help the other in understanding what happens. In a preliminary way, the model also allows for incorporation of the concept of forces that drive this process. Recognition of the existence of relevant forces, and how they manifest themselves in a teacher-student relationship, leads to a description of the concept of productive teaching. Since the same model structure appears valid in all contexts in which the process of teaching may occur, the scope of the model is not limited to the case of a teacher teaching a student in a classroom at a school or university.

The claim that a physical MOT may be formulated in a simple way requires explanation because it is known from physics education research [3, 4, 5] that the process of teaching is not simple. Procedures used by teachers to specifically check what students actually learn in a physics course involve complex physical notions and as such are not simple [6]. One may expect that a process of teaching is even more complex than in physics when the subject matter involves phenomena that cannot be explained simply in terms of physics.

From a psychological point of view, the process of learning cannot be separated from the process of learning and the latter is not commonly defined in simple terms [7, 8, 9, 10, 11, 12, 13]. From a neuroscientific point of view, the process of learning by the brain is a subject of intense study, and complex phenomenological and theoretical models are needed to describe how individual neurons and networks of neurons work [14, 15, 16, 17, 18, 19]. Besides the brain, the process of learning involves a student’s body and the body is also complex. For example, the involvement of hands in the process of learning is not simple [20]. The role that hands play in learning is related to the complex process of evolution of species [21], including the ability to learn as a means of increasing the chances of survival and reproduction. Highly-evolved learning abilities in humans emerged from processes that correlate the behavior of individuals with the environment they live in, and the latter influences individuals in very complex ways (for a popular explanation of how advanced human behaviors, such as altruism of teachers, could emerge in the evolution, see [22]; cf. [23, 24]).

At the level of contemporary society, the processes of
teaching and learning in a large system of education can be seen as exceedingly complex and may fascinate physicists. The need to corral the complexity of the process of teaching provides perhaps the strongest motivation for building a suitable physical model. Without a MOT, the purpose of educational reform is ambiguous.

A physical MOT can be simple only in the sense that the basic elements of the process of teaching can be identified using physical concepts such as an observer and a frame of reference. The complexity of teaching comes from the complexity of the events that observers communicate about and from the complexity of the observers themselves, including the forces that drive their learning and ultimately determine what teaching may accomplish. Given this starting point, a simple model will be obtained in the next two sections.

III. KINEMATICAL PART: EFFECTIVE TEACHING

In order to kinematically describe what happens in the process of teaching, one first postulates that every student has a mental “dictionary” that associates specific word definitions with specific concepts, such as the sequences of words associated with “velocity” or “acceleration”. A growing body of research on misconceptions in physics shows how bizarre the word entries for these two concepts can be in a student’s mental dictionary. The source of this variety of entries is that every student’s dictionary results from his or her prior learning history and every student has a different history.

By definition, in the process of teaching a teacher communicates with a student and, as a result, the student’s dictionary grows and improves in accuracy. However, one should remember that students who take a course have different mental dictionaries concerning concepts to be taught, such as in physics, and faculty members would only deceive themselves if they believed that every student has the same interpretation for the language that they use, especially when the students are only beginning to learn this language.

It is postulated that the words and concepts learned by a student are stored in his or her brain in a coded form that ultimately amounts to sequences of coordinates (numbers) in a multidimensional space. For brevity, this space will be called the student’s space of knowledge. Nothing more needs to be said about this space except that the model will require an additional structure to provide a representation of what a word or a concept means to the student. This additional structure will be introduced soon.

In support of the concept of coordinates in the student’s space of knowledge one might recall mathematical considerations which imply that statements in any logical system can be represented in terms of numbers. Similarly, neural science suggests that learning and memory storage processes can be understood in terms of numbers that represent in specific units how networks of neurons work, such as voltages, currents, densities of ions, etc. Psychological studies also have a history of attempts to describe the meaning of words and concepts to individuals in terms of numbers. Attempts to develop artificial intelligence presume that the human brain can be understood as an information-processing machine that ultimately operates only with numbers.

However, the MOT described here postulates the student’s space of knowledge not for the purpose of explaining, but for the purpose of circumventing, the issues of complexity and lack of understanding of the human brain, its development and function. The goal is to model concepts of teaching in such basic terms that a person interested in these concepts can form suitable entries for them in his or her own dictionary without the necessity of first engaging in life-long studies of what is currently known about the human brain and its psychology.

Thus, the changes in a student’s mental dictionary are represented in the model by transformations of the coordinates of words and concepts in the student’s space of knowledge. Teaching a subject matter to a student is meant to cause the coordinates of words and concepts pertaining to the subject matter in the student’s space of knowledge to form the intended relations among each other, thereby forming an accurate dictionary.

It follows that a teacher and a student need to communicate well with each other in order for the teaching process to stay on track. The communication must be good in both directions (not only from the teacher to the student but also from the student to the teacher) because the only person who knows what the student’s knowledge consists of after interpretation using the student’s dictionary, is the student.

The internal mental processes in the student’s brain enable the student to produce conversations or questions using his or her own dictionary. These internal processes involve the coordinates of the words and concepts in the student’s brain but the student is not conscious of what the coordinates are and what their numerical values are. What the student can be conscious of regarding his or her mental dictionary is, for example, a sense of confusion about what the teacher is saying, or a visible disagreement between a prediction the student makes using his or her words and concepts about a possible result of an observation or experiment and a result he or she actually obtains.

When the student undertakes a conscious effort to make a change in his or her space of knowledge and asks questions, he or she needs to get useful answers from the teacher, which means answers that suggest to him or her what needs to be changed and how. In order to be able to help the student, the teacher must know what the student thinks. Using the model, one can say that the task of the teacher is to establish (as best as one can) the coordinates of relevant words and concepts in
the student’s brain, identify the transformation that will reduce or remove the confusion that stems from the improper relations among these coordinates, and then use the words and concepts already known to the student to provide him or her with information about what needs to be reconsidered in order to deal with the confusion effectively, the effect being the required change in the student’s dictionary.

A student may learn without constant help from a teacher, or people who play this role, directly or through textbooks and other means. However, by definition, the model presented here pertains to a process of teaching that involves a teacher who helps a student [39].

Further description of the model concerns a representation of the meaning of words and concepts and how the meaning is conveyed from person to person. Both aspects are incorporated into the model using the concept of observers and how they communicate about events.

A. Students and teachers as observers

Consider a student to be an observer whose brain registers events and stores information about them in memory. Every event is registered at some place and time and has some content. It is reasonable to postulate that the content of an event registered in a brain ultimately amounts to a sequence of parameters. (A concrete way of imagining these parameters is described in Appendix A.) Essentially, the parameters describe activity in all parts of the brain involved in the process of registration. The parameters are treated in the model as coordinates in the space of events. If the content of every event registered in a brain is assumed equivalent to a sequence of values of n parameters, the number of dimensions in the space of events is \( N = 4 + n \); 4 for place and time and n for content. This space of events is used below to model the process of transfer of information about the meaning of words and concepts between a teacher and a student.

Firstly, it is postulated that the meaning of a word (or concept) is represented in the student’s brain in the form of memory about a set of events that are associated in the brain with the word (or concept). The model is not meant to explain the neurobiological nature of how the memory functions so that the association can be formed. The association is taken for granted in order to represent the fact that concrete examples allow people to quickly identify the intended meaning of words and concepts.

Secondly, it is postulated that a student creates and changes his or her own dictionary of words and concepts on the basis of observation of correlations between two sets of events: one set that the student associates with a word definition and another set that the student associates with a concept. The identification of these correlations does not have to be fully conscious. But it is not necessary to determine the extent to which the identification of correlations is conscious or not conscious for the model to provide a picture of what happens in the process of teaching, including the transfer of meaning of words and concepts.

In the process of teaching, a teacher introduces a set of events considered relevant and provides students with information about them, using words (defined to a various degree of precision) to describe the events and related concepts. The relevant events are, for example, experimental demonstrations of phenomena. According to the model, a student’s dictionary changes as a result of the student’s attempts to correlate demonstrated events with their description by the teacher. The teacher is another observer who helps the student understand the observed events using the introduced concepts. The student tries to improve his or her own dictionary by making it, from the student’s point of view, resemble the one of the teacher. More generally, teachers and students become familiar with many events in their lives and teaching often refers to and certainly draws on this experience instead of using direct experimental demonstrations.

B. Communication between observers

Once the kinematics of teaching is reduced to communication between observers about events, further description of the model concerns the process of communication.

In physics, effective communication between two observers (meaningful, precise, and fast) uses coordinates in their respective frames of reference. The use of frames of reference is a hallmark of physics [40]. Each observer builds her or his own frame of reference according to universal rules. Two observers can effectively exchange information using coordinates of events as soon as they establish and know (understand and remember) how their frames are related. The relationship is found in physics using the following procedure [40, 41].

The observers make use of a set of physically well-defined events that each of them can unambiguously recognize (in the sense of classical physics). For example, an event in which two initially distinct material points meet each other is a physically well-defined event. The coordinates of the two points coincide in all frames of reference when the points coincide. Having introduced a suitable set of well-defined events, both observers can find the coordinates of these events in their respective frames. Knowing the coordinates in both frames, the observers can figure out the rules of correspondence between their coordinates for all events in the set. Subsequently, they can extrapolate this correspondence to a general relationship between their frames of reference that is supposed to connect their coordinates for all events. They inspect many cases and, if they gain confidence in the extrapolated relationship they use, they consider it valid.

For example, if the correspondence in the set of well-defined events is figured out to be a linear one in an N-dimensional vector space, a set of only N well-defined and linearly independent events (in terms of their coordinates) is sufficient to find all parameters of a general
linear relationship between the two frames of reference. If the
required relationship is not linear, a more complex,
case-specific reasoning is required and a simple, gener-
ally valid relationship is not guaranteed. Note that Ap-
pendix A suggests significant topological complexity of
the space of events registered in a student’s brain. The
same concerns the teacher’s space of events. A linear rela-
tionship between the frames of reference in the student’s
and teacher’s spaces of events is unlikely.

In the case of teaching introductory physics, the set
of well-defined events may take the form of tabletop ex-
periments that students carry out with their own hands
and whose results they first analyze each from her or his
own point of view and in a small group. They then dis-
cuss their observations, confusions, discoveries, and con-
cclusions with an instructor. Such discussions create and
change the dictionaries students have for thinking and
communicating about physical phenomena.

Since the process of teaching is postulated to involve
the space of knowledge and the space of events, it fol-
 lows that the process of communication between brains
requires the establishment of frames of reference in two
spaces per brain. Indeed, the model associates such two
frames with every brain. Thus every word, concept, and
event becomes equivalent in a brain to some sequence
of co-ordinates, and discussions about events using all
kinds of languages are reduced to a transfer of sequences
of co-ordinates from one brain to another. Since it is ob-
vious from the point of view of physics that a meaningful
exchange of sequences of co-ordinates between observers
must be preceded by establishment of a relationship be-
tween their frames of reference, it is now also clear that
the process of establishing a dictionary must involve the
processes of building frames of reference and finding re-
lationships between them.

However, the process of communication between two
brains cannot proceed in as simple and direct a way as
the process of communication between two observers in
physics. The postulated reason is that we do not have
access through thinking to the values of co-ordinates that
our brains assign to events they register and to words
and concepts we use in communication about the events.
A teacher may imagine the coordinates and relation-
ships among them in a student’s brain for the purpose of
teaching. But neither the teacher nor the student knows
the coordinates with which the student’s brain registers
events or stores words and concepts. Therefore, one has
to accept that the process of associating words and con-
cepts with events must exhibit a large degree of ambi-
guity, which cannot be eliminated by precise matching
and changing of the numerical values of coordinates
of words and concepts on the basis of directly measuring
and comparing them.

Instead of the direct measurement and comparison of
coodinates, the model postulates that the creation and
development of a dictionary in the student’s space of
knowledge is essentially based on a process of trial and
error. A word or concept is unconsciously assigned its
initial coordinates in the space of knowledge. These co-
ordinates are associated with the coordinates of concrete
events stored in memory. The initial association can hap-
pen quickly but it is not certain and requires testing be-
fore it is promoted to a more stable memory. One may
postulate that the initial association depends on the un-
conscious structural and functional features of the brain.
One is also free to postulate that the association involves
feedback from complex processes that include both short-
and long-term memory and conscious thinking. But the
nature of the association does not matter for the model.

Of importance is the postulate that the initial associa-
tion between coordinates in both spaces is modified and
stabilized in the brain via the process of registration of
new events and communication about them. The growth
and change in the brain tissue and its functioning that
result from this process are accepted in the model as an
underlying neurobiological realization of learning. How-
ever, precise knowledge of the neurobiological realiza-
tion is not needed to understand the model. For example,
the meaning of a word is postulated to become eventually
established in the following way.

The coordinates initially assigned to a word in the
space of knowledge are unconsciously correlated in the
brain with coordinates of some events stored in memory.
The initial association is not entirely random, neither is
it precise. Then the word is used in thinking, absorb-
ing information, and communication about events. This
process uncovers defects in the initial association through
identification of misunderstandings. The association
is changed to a new one that appears to reduce the confu-
sion and is used in further thinking, experimenting, ob-
ervation, and communication. Eventually, subsequent
uses of the word cease to lead to confusions that re-
quire modification of the association and the meaning
of the word is obtained in the form of a stable associa-
tion between the coordinates of the word in the space of
knowledge and the coordinates of corresponding events
in memory. A similar, multistage process is postulated
for concepts. Identification of the meaning of concepts in
terms of events precedes the introduction of their names
in the dictionary.

Ultimately, the rank and order in the space of knowl-
edge is postulated in the model to be brought about by
observing events, and communicating about them with
other observers. Thus, a language is built and learned,
and the words in it and concepts required to understand
and predict events are tuned with increasing accuracy.

Whether one communicates about current events or
events that are only remembered, or merely imagined
on the basis of memory of the actual events (such as
the motion of a point along a straight line, Einstein’s
gedanken experiment, or an event described
in a book or shown in a film), is of secondary importance.
The same scheme is expected to apply to teaching in all
disciplines, not just physics.
C. Effective teaching

What follows is the model description of the process referred to as effective teaching. For the purpose of differentiating between effective teaching and another process that may be referred to as ineffective teaching, the latter will be presented first. This order of presentation brings out the conceptual contrast between the two processes, providing an example of the utility of the model.

Consider a teacher, for brevity called $T$, and a student, for brevity called $S$. $T$ is supposed to teach $S$.

In the case of ineffective teaching, $T$ tries to elicit from $S$ a reproduction of a sequence $t$ of words (or other symbols) that $T$ presents to $S$. Eventually, $S$ reproduces $t$, from memory, but without having much of an idea why and what $T$ wanted to convey by $t$. The sequence $t$ may be similar to some sequences that $S$ already knows, but whether these sequences correspond to what $T$ meant by $t$ and what was the goal of talking about $t$ remains unclear to $S$. $S$ will soon forget the sequence $t$ because there is no reason to remember it.

Such ineffective teaching does not include the process of establishing the meanings of words and concepts that $T$ uses and $S$ is supposed to learn about. In the model picture of ineffective teaching, what $T$ does is, in essence, supplying $S$ with a sequence of co-ordinates valid in the frame of reference of $T$ without trying to find out how the frame of reference of $S$ is related to the frame of reference of $T$. Then $T$ expects to obtain the same sequence back, but has no way to establish what this sequence means in the $S$’s frame of reference. Thus, the model makes it obvious that the process of ineffective teaching, based on transferring information only one way, is fundamentally inadequate. At the same time it becomes clear that ineffective teaching is characterized by arbitrary assumptions about how $S$’s brain associates meaning with words and concepts in terms of events. It is assumed that $S$ does it in the same way as $T$ so that the dictionaries of $T$ and $S$ are the same. $T$’s dictionary is assumed to be well-calibrated with respect to the world, including $S$ as a part of the world. It is also assumed that $S$ can be considered well-taught if he or she repeats the statements that $T$ wants him or her to repeat. This is what is ordinarily described as rote memorization.

The model thus predicts that ineffective teaching leads to a lack of communication between $T$ and $S$. It generates confusion and correlates word definitions with names of concepts in the mind of $S$ in a disordered fashion, devoid of meaning. On the surface, things appear not that bad but only because the ambiguities of thinking and language hide the imperfections of the communication.

Further, the model implies that when $T$ interacts ineffectively with a whole group of students instead of just one, most of them are confused and each of them in a different way. Having no chance to understand what is going on, the students lose interest. $T$ is not able to regain the students’ attention and disciplinary measures are likely to dominate other aspects of teaching. An educational system may be unable to identify dysfunctional situations of ineffective teaching and students may be required to pass tests. They are then judged on the basis of the numerical values of their scores. The problem is that these values reflect not whether the students learn but how the system functions.

In the case of effective teaching the situation is remarkably different. According to the model, the initial task of $T$ is to establish how the frame of reference of $S$ is oriented with respect to the frame of reference of $T$. This means that $T$ first tries to find out what statements $S$ already knows regarding the subject of study and what $S$ means by them, i.e., what events correspond to these statements according to $S$. $T$ uses these events to estimate how the dictionaries of $T$ and $S$ are related to each other. Further tuning of communication between $T$ and $S$ involves hands-on activity during which concrete events are associated with concrete words and concepts.

When key parts of the dictionaries of $T$ and $S$ are already tuned, which is described in the model by saying that the relation between $T$’s and $S$’s frames of reference is approximately known to $T$ and $T$ knows how to use the dictionaries, $T$ becomes able to try to understand what to do in order to convey information contained in the sequence $t$ to $S$ in terms of sequences that $S$ already understands and can think and talk about with confidence. Eventually, $T$ finds the sequence $s$ that conveys to $S$ the same information about the world that the sequence $t$ corresponds to in the dictionary of $T$. If $S$ represents some important insight, $S$ will use it and not forget. This experience paves the way for further steps in building $S$’s dictionary with $T$’s help. $T$ can always arrange a dialogue around issues that are comprehensible to $S$, and the dialog helps $T$ identify issues that still confuse $S$.

In the case of a group of students, a discussion is led by $T$ in order to identify a set of issues that all members of the group find unclear. In terms of the model, this discussion allows $T$ to approximately understand orientation of the students’ frames of reference and co-ordinates of entries in their dictionaries. This understanding allows $T$ to help the students in beginning a coherent study of the identified issues. Thanks to the meaningful communication, $T$ is welcomed by students to play the role of an advisor and as such can transparently influence the course of study so that it may reach its stated goals. Teaching of this kind usually leads students to discoveries of new aspects of the subject matter. Students may subsequently consider the new aspects important and learn more about them. Students may also realize the value of the process of clear communication about the subject matter, which $T$ enables them to practice.

D. Training in effective teaching

Teachers can incorporate effective teaching into their work with students if they are trained to do so. Both
pre-service and in-service teachers need training. The training must itself be an example of effective teaching so that the trainees can experience the process first from the standpoint of students. The resulting appreciation of the great value of effective teaching in comparison with ineffective teaching, the value explained here in physical terms using the MOT, motivates teachers to incorporate effective teaching into their work with students.

McDermott and the Physics Education Group at the University of Washington have developed a program for training science teachers [3,12,13]. The model concept of effective teaching suggests the possibility of similar elements also being useful in training in other disciplines. Indeed, one can compare effective teaching as defined by the MOT with the teaching of actors that was described by Stanislavski in 1936 [48] (his three-volume textbook has had more than 40 editions since then). Such a comparison may initially appear pointless to a reader focused on physics education. But, when equipped with the model, the reader will find that the same elements are present in both cases, even though they concern different subject matters and different dictionaries of words and concepts. Another example, in which the same elements of teaching can be identified by a trainee equipped with the model, is provided by the program of Clay [19,51]. Clay’s program concerns teaching young children who have extreme difficulty learning to read and write (it is now in use in thousands of schools).

The point of these comparisons is that the MOT allows trainees to notice common aspects of teaching in all disciplines. This in turn allows them to properly identify the elements that they could otherwise misinterpret (associate exclusively with their specialty) if they thought that such elements occur only in their discipline. MOT implies that teaching in all disciplines requires the establishment of relationship between the frames of reference of T and S before a meaningful communication between them can begin. In all disciplines, T must understand S’s initial vocabulary before T becomes able to help S in building a proper dictionary. Thus, MOT predicts that the same basic elements must be present in training of teachers in all disciplines.

A chief example of a universal feature is that training requires time. It must extend over the period that the trainees need to work out hands-on examples which enable them to discover the meanings of relevant words and concepts. The required amount of time can only be decided by research. Training that does not provide the required time cannot accomplish the mission [52]. MOT makes these predictions physically obvious.

For illustration, consider the case of in-service science teachers. The minimal period of training required for acquisition of physical concepts of mass, force, energy, electric current, and magnetic field, can be estimated as not shorter than about two months of study every day of the week [53,54]. At least a year of further study should follow in which trainees attempt to implement effective teaching in their work at school.

The implementation will encounter difficulties. For example, one can now predict that it will be impossible for a science trainee to achieve success with students if the time allocated in a school program for teaching the words and concepts of physics is much shorter than the time required by a typical human brain for learning these words and concepts in an effective way. Note, however, that this and other inconsistencies between training in effective teaching and known practices of schooling are not specific to the subject matter and stem instead from the way the educational system is organized. This brings the development of MOT offered here to the key point where it becomes clear that the process of training of teachers cannot be considered complete unless it addresses the issue of success with students and what is meant by the success.

The issue of a trainee’s success or failure in implementing what he or she learns in the process of training requires an extended discussion. The discussion begins here and extends into Section IV where it proceeds to the issues of dynamics of teaching and leads to the concept of productive teaching.

When a teacher steps in front of a class of students and begins to teach, the concept of effective teaching in the teacher’s brain is soon confronted with challenging, unavoidable questions: Do the students want to learn what they are taught? What if they don’t? Should they be forced? What will the students actually learn by going through an obligatory course if they are not interested in the subject and all they want is a passing grade required for graduation? Will they ever appreciate the subject if they are not interested in it and do not want to learn? How to find out what they really think? What is the teacher to do? The training of teachers is incomplete if it does not address these questions. These questions pertain to teaching in all disciplines and are not specific to any one subject matter.

A trainer of teachers is confronted with similar questions. This is exemplified by replacing the words: teacher, student, and passing grade required for graduation, with the words: trainer, trainee, and career incentive, respectively. The resulting questions lead to a new one: How to design a training program, including selection of candidates to be trained and trainers to train them, so that a large percentage of trainees will afterwards achieve success in working with their students? The question is important because teachers cannot consciously engage in a long-term process of improvement of teaching in schools unless they are successful with students. Such success confirms for them that they know what to do and how to do it, and the clarity of purpose enables them to continue their work every day.

That a MOT should address the questions stated above is also suggested by results of a longitudinal study of the same program that was used earlier to estimate the minimal period of training for science teachers [53]. In addition to other findings of interest to educators, the study
found that a year after completion of the training in basic physics almost three quarters of the program participants eagerly responded to mail surveys concerning their subsequent experiences in teaching. However, only about one third responded after four years. It was not found why the number dropped so much and the drop could have occurred for multiple reasons. On the other hand, teachers’ commitment to enhance the quality of their work can be sustained and reinforced for many years if they achieve success with students. One may not exclude that the drop in the number of responses occurred because the majority of trainees were not sufficiently motivated by the results of their work with students to stay in touch with and provide data to the program that trained them.

Since the concept of effective teaching does not answer the questions of practice with students, it is postulated that the process of training teachers of all kinds and levels misses something essential if it is limited to the concept of effective teaching. A MOT would be most useful if it could help in the identification and inclusion of the missing elements. A heuristic point of view toward this goal is discussed in the next section.

IV. DYNAMICAL PART: PRODUCTIVE TEACHING

Section III describes the process of teaching as the sequence of changes in a student’s dictionary that a teacher helps the student make in order to enable the student to interpret events that belong in the subject area, and communicate about them in terms of a language suitable for the subject. Effective teaching corresponds to the well-defined sequence that is described kinematically in Section III. It includes establishment of how the frames of reference of $T$ and $S$ in their spaces of knowledge and events are related to each other, establishing what are the co-ordinates of relevant words, concepts, and events, which are used in communication, and establishing new entries in the relevant dictionaries on the basis of observation, experiment, and exchange of information.

In the context of training of teachers, Section III shows that the kinematical concept of effective teaching is not sufficient to address the issue of teachers’ success in working with students. Students respond to teaching in various ways and a model of teaching must be able to incorporate the dynamics of the teacher-student relationship. This section discusses the dynamics. The discussion addresses the issue of success with students and leads to the concept of productive teaching.

A. Extension of the dictionary

The dictionary of words and concepts in the space of knowledge (of a student or a teacher) introduced in Section III is limited to the subject matter. For example, in the case of physics, the dictionary includes words such as “velocity” and “acceleration” or concepts such as “mass” and “force.” But Section III shows that the concept of teaching involves words and concepts such as “success with students” or “wanting to learn,” and these suggest a host of other related words and concepts that are all absent in the dictionaries for specific subjects such as physics. Therefore, a complete MOT must include an extended dictionary that by definition includes specific dictionaries for all disciplines and the additional words and concepts that pertain to the process of teaching irrespective of the discipline. This extended dictionary will be called the meta-dictionary of teaching. The meta-dictionary is required to discuss the dynamics of teaching. The following examples illustrate what kind of words and concepts must be included in the meta-dictionary (again, $S$ means a student and $T$ a teacher).

When $T$ is to teach a subject matter that is important for $S$ to know but $S$ does not understand why he or she needs to know the subject, is not interested in learning, and does not want to do what $T$ tells $S$ to do, it is clear that there exists a barrier in communication between $T$ and $S$ through which $T$ must break. The key step is to explain to $S$ how the subject to be taught is related to $S$’s future (this will be clarified within the model picture in Sections IV D and IV E). The dictionary of the subject itself is of marginal importance in this step because $S$ does not know it yet. $T$ must instead refer to the concepts and words that $S$ already knows, understands, and appreciates regarding $S$’s own future. This is why the meta-dictionary needs to contain the concept of starting from where $S$ is in a broader sense than in effective teaching, where the corresponding concept is limited to starting where $S$ is only with respect to the dictionary of a particular subject matter. The broader concept means that $T$ must assess and utilize a variety of dictionaries that already exist in $S$’s space of knowledge. Unless $T$ starts from where $S$ is in this broader sense and explains why the subject to be taught is important for $S$, it is likely that $S$ will not be interested in learning the subject.

A primary example of obligatory courses that students may be not interested in is mathematics. The lack of appreciation is reflected in their performance. The percentage of students performing at required levels drops down from grade to grade (from above 60% in 4th grade to below 50% in 12th grade), being lowest at the time of graduation from high school. The drop occurs despite the fact that results of math tests count toward admission to college. Recent publicly-discussed data on students’ performance in mathematics in grades 3 to 8 reflect the same trend. The percentage of students who score at or above proficiency levels drops from about 80% in third grade to below 60% in eighth grade. If the drop is interpreted as a consequence of students losing interest in what math is about and what it is needed for, one has to conclude that teaching of math is inadequate. The inadequacy is not describable in terms of the language of mathematics alone. One needs an extended dictionary
that can be used in discussing the relationship between mathematics and students, students and their future in society, and thus also mathematics and society.

When $S$ thinks that teaching is not adequate, he or she should be able to say so. For example, it may be unclear to $S$ why knowledge of the mathematical concept of integration is required. $T$ must respond to $S$’s concern using the dictionary that already exists in $S$’s space of knowledge. Note, however, that an entirely new concept emerges. $S$ may be afraid to ask about the purpose of lessons on integration due to a fear of consequences if the question is interpreted as unintelligent or violating the school order. Such fear prevents $S$ from learning what it means to speak up on matters that concern $S$ while the way the teaching process is conducted influences the development of $S$’s brain and determines what $S$ knows and understands about the meaning and significance of the taught subject. Thus, the way $S$ is treated in the process of teaching determines $S$’s competence and self-image as a member of society. The apparently narrow problem with teaching math is ultimately related to the concept of educating a citizen, which includes experience of social control, democracy, freedom, and critical thinking.

These aspects are not specific to any area of study and play an important role in the dynamics of teaching (see next sections). They require a dictionary of words and concepts that are fundamental to the well-being of society. The corresponding meta-dictionary of teaching is much richer than the dictionary-to-be-taught concerning just mathematics, or any other discipline taken out of its relevant context. In the case of mathematics, $T$ may help the class understand not only why integration is taught but also why $S$’s question about the purpose of lessons and the way $T$ answers this question are all important. Subsequently, the obligatory lesson of mathematics and the role of $T$ will appear justified instead of arbitrary from the point of view of $S$ and the class.

Of course, students will not voice doubts about the adequacy or purpose of the process of teaching unless they have a feeling of safety with $T$. Otherwise, the fear of negative judgment by $T$ and hence uncertainty of the future will silence students. But to gain the trust of students and achieve openness in communication with them so that real issues on their minds have a chance to get resolved, $T$ needs to think in terms far broader than the subject matter alone. The corresponding meta-dictionary of teaching must contain concepts of safety and trust in communication between people, in addition to the words and concepts limited to the dictionaries of a specific subject and judgment of students’ progress in learning the specific subject.

Safety and trust are important for asking questions about the subject matter. Asking a question that discloses confusion may imply that $S$ does not know or does not understand something in a situation where others appear to already know and understand. Such exposure could put $S$ at a disadvantage in an already stressful situation. Without feelings of safety and trust, $S$ may refrain from asking questions.

Teaching does not have to become inadequate and misguided in purpose if $T$ starts from where the students are and takes advantage of their curiosity. Curiosity is a central concept in the meta-dictionary of teaching.

A curious $S$ eagerly confronts a problem to be solved. Finding it difficult, $S$ is glad to receive help from $T$, and appreciates $T$’s contribution. In order to contribute to the self-motivated study, $T$ must be interested in and capable of helping $S$ in the learning process. The self-motivated learning is hard for $S$ because it involves unlearning, a change in brain structure that was in place, and learning anew, creating a new structure. This requires that $S$ overcomes the stress that is associated with the changes and performs the work that is needed to make the changes happen. Curiosity can take $S$ through the difficulties. So, $T$ should sustain $S$’s curiosity, a condition valid irrespective of the subject of study.

### B. Inclusion of internal events

Effective teaching concerns a dictionary of a concrete subject matter and uses events observed by $S$ and $T$ as the vehicle that conveys between them the meaning of words and concepts in the dictionary. For example, $S$ and $T$ discuss well-defined events on a laboratory table.

A complete MOT contains the meta-dictionary described in Section IV A. This dictionary extends beyond the subject matter itself and includes “new” words and concepts such as success with students, wanting to learn, starting from where the student is, educating a citizen, safety, trust, and curiosity. One can also talk about attitude, attention, engagement, etc. According to Section III in order to identify the meanings of these words and concepts, one should associate them with concrete events. However, the required events are definitely not of the kind that happen on a laboratory table.

In the case of new words and concepts, the events that matter happen inside $S$ or inside $T$ (see below), instead of outside of them, as is the case with the events on a laboratory table. Therefore, the space of events that is needed in a complete MOT must include events inside an observer. Such events will be called internal. Their description in the model is provided in a few detailed steps below. The same details will be useful later in the discussion of the concept of productive teaching. More precisely, they will be used in the model to show that the concept of productive teaching is not merely an addition to the concept of effective teaching but underlies the latter as a foundation of the whole process of teaching.

Firstly, consider $T$ teaching $S$ some motor skill, such as riding a bike, catching or throwing a ball, or pressing strings on the fingerboard of a violin; or a mental skill, such as the ability to focus, be patient, contain emotion, or perform a gedanken experiment. The process actively involves $S$ and $T$ as performers of the skill and thus in-
volves phenomena that happen inside $S$ or $T$ in ways not reducible to learning a dictionary for description of events outside observers. Namely, $S$ must focus on what happens within the body and mind of $S$ when performing a skill. $T$ judges $S$’s performance from outside but focuses on what happens within the body and mind of $S$. Moreover, the analysis of $S$’s performance that $T$ makes is based on what $T$ knows is happening in the body and mind of $T$ when performing the same skill. It should be clarified that learning a skill includes a buildup of a dictionary concerning the skill. One cannot fully comprehend concepts and words used in communicating about the skill unless one is able to perform the skill in a way corresponding to the meanings that count.

Secondly, consider the model’s definition of the space of events registered in a brain that is provided in Appendix A. The definition makes it clear that the events registered in a brain are determined not only by physical events outside an observer but also by a multitude of processes that happen concurrently in the observer’s brain. These processes depend on the neurophysiological history and state of the brain as a central organ of a living person, which includes how the brain functions using the senses, memory, and thinking, both subconsciously and consciously. It is clear that many physical events outside an observer are not registered in the observer’s brain because the input they provide is below the threshold for altering the concurrent activity in the brain.

Thirdly, while registration of events is associated with changes in an observer’s brain, a change in the functioning of the observer’s brain is not necessarily expressed in the observer’s overt behavior. If it is not overtly and clearly expressed, the change cannot be registered and unambiguously interpreted by another observer.

In essence, using the model of Appendix A one can say that events registered in a brain can be approximately divided into three classes according to their dominant coordinates. Some events have dominant coordinates associated with what is happening outside of an observer. These will be called external events. Other events have dominant coordinates associated with what is happening inside of the observer. These will be called internal events. The third class contains events that have their dominant coordinates associated both with what happens inside and outside of the observer. For brevity, these will be called engaging events.

In terms of the model nomenclature, one can say that the concept of effective teaching involves external events, such as an event on a laboratory table; engaging events, such as those that cause changes in $S$’s dictionary concerning what happens on a laboratory table; and internal events, such as thinking about an event on a laboratory table, experiencing confusion regarding the subject, or formulating questions about it. The concept of effective teaching ignores and obscures all internal events that are not directly related to the improvement of $S$’s dictionary for the taught subject.

The ignored internal events include, for example, wandering of thoughts away from the taught subject toward issues of main interest to $S$, or $S$’s plain feeling of aversion toward learning the subject. The possibility that $S$ sees the subject as having no relevance to $S$’s way of life and future is also overlooked. Phenomena such as boredom, a sense of violation by enforcement of classroom lessons or homework, feeling insecure in the presence of $T$, or not trusting $T$ enough to share important thoughts or information with $T$, are not included in the model picture of effective teaching. All internal events that contribute to (positive) attitudes and manifest themselves in $S$’s attention or engagement, are considered given, or stimulated and reinforced by effective teaching, although no reason is identified in the kinematical part of the model for the necessity of their occurrence. Internal events that contribute to the phenomenon of curiosity are not explicitly considered. Instead, curiosity for the subject matter is assumed to be always in place, as it by definition is assumed to be in place in the case of $T$, who is also assumed to be interested in teaching the subject to the students. In summary, the kinematical concept of effective teaching in the model does not explicitly include a host of feelings, thoughts, attitudes, wants, and expectations that are important for the course and outcome of the process of teaching.

The above examples make it clear that the concept of effective teaching described in Section III omits the internal events that provide meaning to the meta-dictionary described in Section III. Of course, ignoring the internal events does not change the fact that the behavior of $S$ hinges on them and the process of teaching cannot be successful if it is incompatible with the internal events. A realistic MOT must account for their significance.

C. Communication about internal events

The issues that require the meta-dictionary of Section IV A cannot be discussed between $T$ and $S$ without communication about internal events. But the communication cannot proceed according to the scheme of effective teaching described in Section IIIB.

In effective teaching, the dictionary of words and concepts concerning a subject of study is built, used, and changed with the help of a process of communication based on a set of well-defined events; both $T$ and $S$ observe and discuss these events as two different observers who use two different frames of reference to describe the same phenomena. Such a set of well-defined events does not exist for communication between observers about events that happen inside one of them. The difficulty is that, by definition, only one observer has access to what happens in an internal event.

Since the standard methodology adopted in effective teaching does not apply, the question arises how $T$ and $S$ may proceed. The model asserts that the situation is not hopeless because $T$ and $S$ may rely on their own internal events in concrete situations and each can try to
find similarities in what the other says regarding these examples. The model’s description of what may happen in a dialog between T and S when they attempt to establish orientation of their frames of reference with respect to each other, including internal events [57], is provided below in two parts. The first part describes only what may happen and does not address the issue of why the dialog may proceed in the described way. The second part, which contains a heuristic point of view toward the origin of forces that can keep the dialog on track, is provided in the next subsection.

Suppose the observer of an internal event is S (an event happens inside S). T can infer what happens but only from the overt, nonverbal behavior of S, and from the verbal information provided by S in terms of S’s dictionary. In order to decode such information, T must hypothesize about what happens inside S and about the meaning of the words S uses. T must conduct the dialog using and expressing the hypotheses so that the dialog can lead to improvement of the hypotheses, or to making new ones. However, it is up to S whether the required dialog with S will proceed.

In particular, the dialog’s prospects depend on how S perceives T. S will judge how T comes across according to the rules chosen by S, not by T. The judgment will include the utility of the relationship between S and T for the purpose of dealing with the internal events in S by S (see next subsection). No quick fix or superficial verbal assurance can change the judgment that S builds over time. Instead, overt behavior of T in response to overt behavior of S is registered in the brain of S and this extended process contributes to S’s judgment of T.

If S judges T as helpful, the dialog will proceed because S expects a benefit. If T is judged as not helpful, S will not be motivated to cooperate. These regularities in behavior are observed in countless examples [13].

The key to S’s willingness to cooperate with T is that T comes across as somebody who wants and is able to help S unfold the potential that S believes himself or herself to have; potential that is not reducible to any subject matter or dictionary. Inclusion of this regularity in communication between T and S about internal events is a challenge for every MOT. It will be addressed in the present MOT in the next sections (including relevant references). The goal is to capture in the MOT that T’s authentic respect for the potential of S to develop as a learning human being and the interest of T in S having a chance to realize his or her true potential are both seen and appreciated by S in the process of building up their interpersonal relationship. Eo ipso the cooperation happens voluntarily on both their parts. Only a conscious process supported from both sides allows T and S to build the required meta-dictionary. This mutually supported process is identified in the model as the way around the basic difficulty of an absence of well-defined events: wanting allows the participants to continue despite misunderstandings. This is also why the significance of safety and trust was stressed in Section [4A] even though every normal human being is able to identify the significance of safety and trust in their experience.

Since the task of establishing meaningful communication between two brains is at the discretion of the brains that are unique as individuals, and since this task depends on the interpersonal relationship between them and the context of their communication that are not fully predictable, it cannot be carried out in steps enforced by a predetermined curriculum according to a rigid schedule. The magnitude of complexity involved in communication between two brains about internal events boggles the mind and requires study. In fact, science is yet to discover a precise methodology [68] to deal with this complexity because the standard procedure based on well-defined events is not available. The fact that the present MOT eventually produces such clear conclusion in disagreement with most common educational practices, is an example of the power of models that is needed in the theory of teaching according to Section [11].

This means that teaching involves an area of communicating despite ambiguities that cannot be systematically eliminated by any simple procedure. Performance of this art requires extensive preparation and continuous training such as performance in other disciplines [69]. The issue of required teacher selection and training is taken up in Section [1.V3].

Even if the buildup of the meta-dictionary depends on the internal events, which depend in turn on the interpersonal relationship between T and S and complex contexts of their interactions, all of these elements being unpredictable to such an extent that teaching cannot be reduced to simple procedures, the model must now provide a dynamical idea for what drives the process of teaching in the right direction. The next subsection incorporates the required concepts of “dynamics” and “right direction” into the model.

D. A heuristic view toward the origin of forces

Challenges for a MOT stated in the previous sections must be approached with the realization that the advanced processes that go on in the brain cannot be ultimately explained today in terms of their physics, strictly speaking, because they are too complex and too little is known and understood about them. Approaches of a far less precise nature than physics, such as Freud’s attempts [70] to understand psychology and Hebb’s idea [71] of explaining the same in terms of neurophysiology, are still far away from reaching conclusion [72]. Moreover, all these approaches concern just one brain. The process of a teacher teaching students is of a much finer kind because it involves at least two brains in interaction in a complex environment and it causes complex changes in them in complex ways.

But the current lack of fundamental understanding of the forces relevant to the MOT does not imply that such
forces do not exist, that one may succeed in teaching by acting against them, or that one has a chance to seriously tackle problems of education being ignorant of their role and strength. The situation is like one with the force of gravity that causes stones to roll only down a slope irrespective of whether one understands the law of gravity or not.

This section employs the model picture of teaching developed in the previous sections to describe a heuristic point of view toward the origin of the relevant forces. The model view suggests that the same basic forces always determine the course and outcome of the process of teaching and that attempts to teach in ways that act against these forces cannot lead to success. As a result of recognizing the depth of the relevant forces’ origin, and thus also their strength and significance, the next subsection will introduce the concept of productive teaching.

To begin with, the concept of “force” as a dynamical agent in the process of teaching should be distinguished from other known concepts that are associated with the word “force” in teaching. For example, there exist forces that act in the daily practice of teaching and do not directly refer to the buildup of dictionaries through interactions between brains. This concept of forces includes systemic rules mandated by law or economic and social pressures that act on teachers, students, and other stake-holders in the system. Such forces qualify as constraints or influences on what happens in schools (and other places where teaching occurs) rather than causes of the process of teaching that the model is about.

Another important concept is that the force of teaching is the teacher. In a trivial way, this may be interpreted to mean that the teacher is the person who forces students to learn, no matter how it is done. An opposite meaning associated with the concept of teacher as a force of teaching is that the teacher is the identifier, facilitator, creator, and guardian of contexts in which students learn. A vast difference between these two interpretations demonstrates the magnitude of ambiguity that needs to be eliminated. A more subtle ambiguity is contained in popular statements that students must realize that “only they can learn” and that “it depends on them how much they learn,” which seem to imply that the teacher plays only a secondary role. On the other hand, it is also said that teachers need to “create a desire to learn” in students. The latter statement can be confronted with the fact that most children are eager to learn. Evidently, there is a need for clarification of the concept of forces in the process of teaching.

Consider first a few examples which show that the action of forces relevant in teaching manifests itself in recognizable psychological phenomena. These phenomena may lead to unproductive outcomes of teaching in major ways even though the subject has been learned by S sufficiently well to pass some test. Consider that S has questions concerning his or her self-image in the context of learning a dictionary. For example, if S feels judged as “not gifted enough to learn physics” and never talks about it with T, S’s entire life may later be tainted by efforts to overcome the feeling of inferiority. The other highly unproductive outcome would be that S resists to the feeling of inferiority and never becomes a fully developed person because of the internal feeling of failure. An opposite case of concern is an overconfident S who will have to deal with consequences of serious errors resulting from the illusion of competence. A common psychological effect of school or college is that alumni avoid contact with subjects that were forced upon them during their academic training. This effect negatively influences the perception of arts and sciences in society.

To be more specific, it is known that test anxiety in students has major implications for their learning and for measurements of what they learn. In an environment that stimulates test anxiety, many individuals cannot learn effectively and their actual knowledge cannot be reliably assessed. Of course, the anxiety is not invented by testers: some natural inner forces lead to anxiety in some conditions. Although the forces that cause anxiety are not precisely known, it is known that anxiety has devastating consequences for teaching. If, instead of being ignored and blindly caused to backfire, the same underlying dynamics were better understood and taken advantage of in the process of teaching, it would be possible to create and maintain conditions that do not stimulate the destructive anxiety in students who are prone to it, and do not induce a whole spectrum of unproductive consequences of such anxiety in their future behavior. One could employ the dynamics to instead stimulate the same students’ curiosity about the world and useful dictionaries, with an entirely different spectrum of consequences in their future behavior.

In order to locate forces in a potentially complete physical MOT (even if the forces are not fully understood), one needs to delineate a plausible way in which known physical processes might in principle be responsible for the existence of these forces. Once room is made for these processes in the model structure, and thus the basic (i.e., following from physical laws) origin of the dynamics of teaching is incorporated (even if not directly because of the absence of required details), a model definition of the concept of productive teaching will be offered.

In the model, T is considered an observer who communicates with S as another observer of the same world. There is no postulate in the model of any fundamental difference between observers. However, T is a more experienced observer than S and has two special features. One feature is that T wants to possess expertise about the world’s mechanisms and share it with S. Another feature is that T knows what to do in order to share this expertise with S. Among the mechanisms of the world, T understands and appreciates the role of personal growth in human life. On the other hand, S needs and wants T’s help in learning about the world because S is convinced that T possesses the expertise S wants to gain and S sees that T wants and knows how to share it with S.

A heuristic point of view toward the physical origin of
these features of $T$ and $S$ involves several observations put together. These observations concern an individual and a large number of generations of many individuals. There is nothing new about these observations except for their inclusion into the description of a physical MOT. The purpose is to make the heuristic point of view as concrete as possible within as small an amount of text as possible. From a vast relevant literature, only a few references are selected that seem to most accurately put the observations in the model’s context.

The first observation is that humans learn eagerly from the moment of birth. Children are curious, explore, and ask questions of adults. That adults are motivated to explore is seen in their tendency to travel, especially after they retire and are free to make choices about how they spend time. Apparently, humans need to explore to feel well because this is the only way available to them for discovering regularities of the environment in which they live. They need to know these regularities to successfully operate in the environment. Similarly, all anomalies (news) catch their attention because humans need to figure out the new elements in order to know how to deal with them. A brain must feel pleasure when exploring, uncovering regularities, and explaining surprises, in order to develop strategies for survival and apply them in an ever-changing and unpredictable environment. As suggested already by Hebb [80], the feeling of pleasure coming from exploration, observing regularities, and solving puzzles, can be associated with “directed growth or development in cerebral organization.”

The second observation is that a human being is the product of evolution [21]. This means that the “growth or development in cerebral organization” in a human brain and the associated feeling of pleasure are a result of a chain of physical events that took many millions of years. A huge amount of information accumulated in this chain of events is encoded in the capabilities of the genetic material of individuals [81]. In the course of the life of a single individual, the individual’s genetic code is expressed in varying conditions in different ways to various degrees [82, 83]. This process results in the learning brain. Interactions between different brains are critical to the continuation of the chain because they are involved in the transfer and survival of the genetic material. Ultimately, the formation of genetic codes and their expression in processes that produce functioning human brains happen according to the dynamical laws of physics [84]. But there is no simple path that connects the basic laws of physics to a single living human brain. Instead, a human brain is built and functions in the ways that result from and are informed by an unimaginably large number of intermediate structural and functional steps that involve simpler structures involved in simpler functions. As a result, many layers of complexity are superimposed in the brain dynamics [85, 86]. An ultimate understanding of the brain is a fascinating prospect and one of, if not the greatest, challenges to science.

The second observation means that the forces which determine how brains function and interact result from and are tested by such a long “battle of life” [21] that they will not yield to any artificial concept of teaching. Instead, teaching must be adjusted to them. For example, there is no point in artificially suppressing the role of feelings because of the assumption that they only “jumble” rigorous thinking, or in artificially suppressing rigorous thinking as “secondary” to the essence of humanity [87]. In fact, feelings and thinking are coupled into an inseparable whole. One cannot argue that logic is more important than emotion or that emotion is more important than logic. The only form of the human brain that passes the test of evolution involves both. Attempts at artificial separation will encounter resistance since only a whole brain can function as a learner and a teacher.

The third observation is that teaching (such as parents’ teaching of children, or teaching of less experienced group members by more experienced members) helps individuals develop values in behavior and knowledge, including the development of language [21]. This is why individuals can grow far beyond what any one of them could ever accomplish from scratch or separately. The ultimate recursion is that the increasing capability for teaching using increasingly advanced language allows us to describe the world we live in to new generations with increasing precision. The new generations are thus equipped to achieve greater scope, accuracy of knowledge and understanding than previous ones, achieving greater capability to take action.

In the context of dynamics of the model of teaching developed here, these three observations are interpreted as inescapably suggesting that the origin of internal events which manifest themselves as wanting to learn, wanting to teach, and wanting to take action, is of the same depth as the origin of man. Therefore, it is postulated that the dominant internal events in the process of teaching are related to who $S$ and $T$ are and may become as human beings. Each has a strategy for this purpose. The evolutionary perspective makes it obvious that disclosure of such a strategy to a stranger is associated in a brain with danger. On the other hand, the disclosure may lead to success thanks to cooperation. Both $S$ and $T$ judge and choose the depth of communication that suits them.

In the process of teaching, when $S$ and $T$ encounter difficulties in communication due to ambiguities in their respective dictionaries and when they need to share information about internal events in order to explain what happens, they operate near the limits of communication that they set previously as useful and safe. Pushing these limits, with some necessary elements of curiosity, pleasure, and risk [88], is what breaks new ground in the process of teaching according to the present model. The concepts of a “right direction” and “dynamics” of this process can now be included in the model.

The process of teaching is essentially “blind” [89] in the same sense, in which the underlying laws of physics and the resulting process of evolution are blind, and it is driven by the same forces that drive evolution. But since
it is the highest existing form of transfer of information from generation to generation (actually, from a brain to a brain) and as such includes the transfer of values that allow humans to operate at the level we have currently reached, many characteristics of teaching appear to have a purpose. This is viewed as a misconception analogous to suggesting that cows have milk so that people may milk them.

Instead, the model postulates that the concept of “right direction” can be associated with the increase of options that individuals in principle have for choosing their own steps into the future as their understanding of the world increases. (Examples from modern history that illustrate the complexity of the phenomena that contribute to this process can be found in Refs. [90, 91, 92].) But “in principle” does not imply that every individual is equally aware of the possibilities and limitations, has equal access to the resources and equally contributes to their replenishment, can benefit from the development in the same way as others and provides others with equal services, etc. The concept of “right direction” that T may adopt in teaching S is defined in the model by saying that T chooses steps that lead S to greater awareness of possibilities and limitations, more efficient access to resources and participation in their replenishment, wiser consumption of benefits and more competent service to others, etc. Most succinctly, T and S share an understanding of their world and values.

The concept of “dynamics” which causes T and S to make concrete choices in interacting with each other is reduced in the model to the statement that the phenomenon of teaching is shaped through evolution. It takes forms corresponding to the qualities and knowledge of its participants and how they communicate. Most succinctly, T and S cooperate with each other in agreement with their needs and understanding of the world and values.

As a result of gathering enough experience and insight, T becomes consciously aware of the process of creation and transfer of knowledge and values, and realizes that this process includes T’s own communication with students. From that moment on, T can consciously work on better understanding how to communicate with students and transfer to them foundations of the meaning of what T has comprehended, so that the values and knowledge the students learn in the process help them move into their futures.

Self-consistency of the model dynamics requires that the process of buildup and transfer of the meta-dictionary of teaching is perpetual. This means that the transfer happens in agreement with the existing values and knowledge and is carried on and evolves as these imply, from generation to generation. So, through participation in the process, T’s students become familiar with the mission T serves. Appreciation of T’s work by S signifies that T’s mission is accomplished in the case of S.

That the model dynamics may resemble what currently happens only in a minority of schools (some examples are described in Refs. [94, 95, 96]) is a separate issue of great practical significance. An in-depth discussion of this issue is beyond the scope of this article. But if the process of teaching in every school and the model picture of teaching were similar today, it would mean that we have already entered the era of 

E. Productive teaching

Teaching may be called productive when it includes all the elements of effective teaching found in Section III C uses the meta-dictionary of teaching to solve the problems presented in Section III D (as described in Sections IV A-C), and proceeds as a result of action of the forces discussed in Section IV D.

In other words, the process of productive teaching is not blindly enforced by T but stems and branches from the will of S with the help of T. T helps S understand what is relevant and motivates S to participate in making the teaching process happen in terms of engaging events. In particular, S feels free to exchange information about internal events with T (and other members of the group when more people are involved). This freedom results from and contributes to the buildup of an interpersonal relationship between S and T and becomes palpable along the establishment of the meta-dictionary that they use to solve problems they encounter.

While effective teaching discussed in Section III is a concept that may be tied to some subject of study (and measured solely in terms of knowledge and understanding of the specific subject), productive teaching produces a person who finds pleasure in learning and wants and knows how to learn more, where the word “more” includes future subjects that are not known and not predictable by T during the process of teaching. No matter what subjects are covered in the process of productive teaching, S learns to use the inner forces of learning that manifest themselves in S’s curiosity, wanting to learn, and joy, even if it gets hard to make progress. The open state of mind of S that results from productive teaching is not measurable in terms of knowledge or understanding of the subject matter that S is taught. One may say that productive teaching concerns teaching a person, not a subject matter. A person thus educated knows the forces that drive learning and how to use them.

In particular, the new insights, and observation of many instances of T’s helpful behavior in the interpersonal relationship with S (and with others as witnessed by S), allows T to realize that the approach of productive teaching, adopted by T, is driven by the respect that T has for S as another human being in need of learning. This instructive interpersonal relationship, created by T and perceived by S as a fruitful one, shows S the necessity and value of T’s work [92].

Appreciation of T’s work by S is what completes the
The definition of productive teaching according to the model. The outcome of productive teaching is not only the mastery of subjects and skills but also appreciation by $S$ of the value of understanding among people, how it happens and bears fruits. Through this appreciation and understanding, productive teaching prepares $S$ for further learning and making choices for action.

The concept of productive teaching is closely related to the concept of context of productive learning that was introduced by Sarason [73, 74] but so far has not been discussed in terms of a model. Using the model, one can now say that productive teaching proceeds through employment of contexts of productive learning. The insight provided by the physical MOT is threefold: 1) effective communication between $T$ and $S$ requires comprehension of the relationship between the frames of reference in their spaces of knowledge and events, which implies that $T$ and $S$ have to start from finding out what this relationship is and learning how to use it. 2) The internal events determine the course and outcome of the teaching process and $T$ and $S$ need to communicate about them in order to keep the teaching process on track, and 3) teaching becomes productive when it is driven by the forces that characterize $T$ and $S$ as human beings in pursuit of their goals. One can now also say that the context of productive learning is a context in which issues of importance to $S$ are the center stage and subject of effective communication with $T$ so that the three elements identified in the model are present and $S$ can learn with the help of $T$. This means that teaching begins with and proceeds in concrete, real-life contexts that $S$ and $T$ identify together and in which $T$ facilitates $S$’s learning, creating opportunities for $S$ to discover new concepts and guarding $S$’s process of learning from derailment. In the process of teaching, $T$ also learns: constantly studying how students learn, building a model of how to teach, and seeking validation for the model, aiming at improvement.

Documented examples that illustrate the concept of a context of productive learning, including its long-term consequences [94, 95, 96], also help illustrate in practical terms what kind of teaching is described by the physical MOT offered here. In particular, they indicate what may happen in the dialog between $S$ and $T$ (see Refs. [97, 98]). Examples that illustrate the same concepts and can be seen in popular cinema, are described in Refs. [13, 99]. An example from a program of teaching science [53] based on Ref. [42], can be found in Ref. [100].

A few examples given below further illustrate what typically happens in the context of productive learning and should occur in practice of productive teaching according to the model. $S$ thinks it is best to say, “I do not know” when he or she does not know how to find an answer to a question or solve a problem. $S$ is confident that this is the best way to respond when missing a point because $S$ knows that the shortest path to learning is to take advantage of communication with $T$ and other people in the process of filling gaps and seeking connections. Learning is impeded by hiding gaps in knowledge and pretending to understand. Similarly, if a personal image and opinion are more important than actual learning, disclosure of shortcomings in knowledge or understanding is out of the picture.

In the process of productive teaching, $S$ has no reason to be afraid to say, “I do not know,” because $S$ knows that $T$ understands what such a statement means, listens to $S$ carefully, and helps. So, $S$ does not hesitate to say, “Please explain this and that because I do not understand” when he or she is confused, or, “I do not remember” when memory fails, or, “Why do you say such and such?” if what $T$ says does not sound clear to $S$, etc.

In order to appreciate how difficult it is to create and sustain contexts of productive learning (especially by the unprepared for the unprepared), these apparently simple examples can be compared not only with what happens today in many classrooms, but also with how people communicate in other contexts. Consider the contexts of discussion between a teacher and a principal, a principal and a superintendent, an employee and a boss, or a citizen and a government official. In these and other contexts, the subject matter can suddenly become secondary in importance to the issue of dominance of one person’s position in the system. At this point, communication about the subject matter breaks down and there is no room for communication about internal events. Instead, the only course of action for the person in a weaker position in the system is to adjust to the decision of the person in a stronger position in the system. In different circumstances, the hierarchy of their positions may change and a reaction based on memory takes place.

In the process of productive teaching, there is no need for blind measures of discipline and judgment. Instead, the values transferred in the process bond $S$ to the idea of participation. Testing of $S$ by $T$ is essentially replaced by gathering of information by $T$ about what $S$ does while learning. This information is the basis for advice that $T$ gives to $S$ so that $S$ can improve the process and reach the intended results [101] (see Section IV F).

In summary, one may say that productive teaching differs from effective teaching by the dynamical condition that the relevant sequence of changes in $S$’s dictionary occurs as a result of the action of the natural forces of learning in $S$. Therefore, $S$ learns what the word “learn” means and becomes a learner for the rest of his or her life.

Learning in agreement with the natural forces that $S$ is equipped with as a human being, allows $S$ to discover the meaning of words and concepts such as human rights, law, democracy, achievement, and other entries of fundamental value in $S$’s dictionary. For example, since the rules of communication between $T$ and $S$ are not decided solely by $T$, the issues of motivation and power in the process of teaching get discussed between $S$ and $T$. Decisions are made in ways that do not subject $S$ to the will of $T$. Instead, $S$ is engaged in making decisions in the contexts familiar to $S$. If $T$ must decide because $S$ does
not have the required knowledge, T must also explain to S why T is responsible for making the decision, and why T considers a particular decision to be the right one.

The model described here helps in identifying several meanings of the word “productive” in connection with teaching. For example, many correct entries are produced in the multiple dictionaries concerning subject matters that belong to S’s space of knowledge. The large number may be achieved because the process of teaching takes advantage of the forces that drive learning and they accelerate it to large speeds. Since the entries are correct and understood, they are useful in thinking about new problems and become stable (are not forgotten, as useless information typically is). When teaching takes advantage of self-motivated learning, it produces entries in S’s dictionary that are basic to the well-being of S as an individual, such as inquiry, discovery, and study. By learning how to use inner forces, a person may become creative. A rich meta-dictionary is produced in S for communication about internal events. This dictionary contains entries for the values that are fundamental to S’s future as a member of a society, indispensable for S becoming a conscious citizen. The teaching produces a person who carries these concepts and values on in relation to other people. The overarching meaning is that the teaching is productive when it contributes to the student’s future as a fully developing person. This short statement is itself a sequence of words whose meaning cannot be grasped without a proper dictionary.

F. Training in productive teaching

Training in productive teaching is more difficult and time consuming than training in effective teaching (see Section III D) because productive teaching requires the buildup of many words and concepts in the meta-dictionary of teaching on top of a dictionary of specific discipline(s). However, in a system in which basic competence of trainees in a subject matter is itself obtained as a result of productive teaching, the difficulty and time consumption can be predicted considerably smaller than in a system in which a basic familiarity with the same subject matter is obtained by the trainees without the first-hand experience of productive teaching (first as students, and only then as teachers). Let the productive system be called P, and the questionable one Q.

In the case of P, teachers are trained in the subject matter in ways that create entries in the meta-dictionary in parallel to the creation of entries in the dictionary for a discipline. In the case of Q, the dictionary for a discipline remains uncorrelated with the meta-dictionary. Moreover, the dictionary for a discipline is plagued with wrong entries, such as the illusion that science results from pure logic, the false impression that art results only from emotion, the mistaken belief that interpersonal relationships are irrelevant to the development of science, and the invalid assumption that the context of discoveries and creations is not important for how they occur. Q compounds the difficulties of training in productive teaching because the trainees first have to unlearn what was inculcated in them during their training in the subject matter. Then they can learn anew. But it is much harder to change an existing structure in a brain than to build a new one from scratch. Unfortunately, the Q-like systems greatly outnumber P-like systems (see below).

In a system of the dominant type (Q), almost all practical suggestions for training in productive teaching originate one way or another in a small number of examples that have already been mentioned (some only through references) during construction of the model in previous sections. A few new sources are included in this section as particularly relevant to the training of teachers. One can say that this section illustrates that a model, which organizes thinking about teaching, leads also to a selection of suggestions for training.

The model principle of beginning from establishment of relationship between frames of reference and starting where the trainees are implies that the events suitable for beginning a buildup of the meta-dictionary of teaching, irrespective of the discipline, should involve a subject matter that is familiar to the trainees. The more familiar the subject matter the easier it is for a trainee to focus on the issues of teaching rather than on the issues of the subject matter [102]. For example, consider a physicist to be trained in productive teaching (a professional in another discipline could just as easily be considered instead of a physicist). Let the subject matter be reading. Instead of reading Shakespeare or Goethe, however, consider the case of teaching young children who have difficulty learning to read. Imagine a visit by the physicist (as a trainee in productive teaching) to a training session for teachers of reading in a program called Reading Recovery [51, 103]. The physicist witnesses how the trainees observe what happens between a child learning to read and a veteran Reading Recovery teacher who helps the child overcome the difficulties the child encounters. The required setup includes a sound-proof room with a microphone and a one-way mirror so that the trainees (and the physicist) can hear from a loudspeaker and observe through the mirror what happens in the room between the child and teacher without being seen or heard by the child.

Since the physicist and all trainees already know and understand what it means to read, they are able to focus on what the teacher does in response to the difficulties that the child encounters. The whole group observes how the teacher works with the child and they discuss what they see, among themselves and with an instructor who observes the reading room with them and explains what happens when the trainees have questions or confusions arise in their discussion. Using the model, one can say that the observed events allow the trainees to build entries in their meta-dictionaries of teaching without unnecessarily focusing their attention on the subject matter, which would impede the process of building the
right entries.

The physicist sees that the teacher behind the mirror is focused on finding out what blocks the child in reading a story composed of a few simple words. The teacher discusses with the child the meaning of the individual words and the whole story; illustrated with a picture, and helps the child understand how to self-correct mistakes the child makes [104]. It becomes clear to the physicist that what the teacher does is not focused on some abstract concepts of reading but on discovering how to help a unique child solve a concrete problem. Once the child finds a solution, the teacher uses it as an example and helps the child understand how to seek solutions to similar problems with concrete letters, words, or groups of words, explaining through practice the role of comprehension of the text and the methods of self-correction. This is a complex interpersonal process, immersed in a context in which the child wants to learn to read.

The physicist also observes the behavior of the teacher trainees and their instructor as they follow the lesson through the mirror and loudspeaker. It becomes clear to the physicist that the instructor and trainees do not talk about meanings of the read words or sentences. These are understood well and do not require special attention. They talk about concepts that matter in helping the child succeed in becoming a learner. This is a very complex issue, incomparably more complex than the simple sentence the child tries to read. The other clear lesson is that in order to grasp the meaning of words and concepts in the meta-dictionary of teaching one must observe events such as those behind and in front of the one-way mirror.

The next suggestion about training in productive teaching concerns reading documents that illustrate what productive teaching can accomplish. Consider the experiment carried out by Schaefer-Simmern in the nineteen forties [105]. He demonstrated in a number of cases ranging from mental defectives to people in business and the professions that one can teach a person to see art as experience [106]. His students unfolded their creativity in visual art. Schaefer-Simmern concluded from his experiment with mental defectives that [107]: the experiment “seems to confirm the fact that creative activity in the visual arts can be unfolded and developed in mentally defective persons to a degree analogous to that of their mental potentialities. Real difficulties appeared usually only with individuals with higher IQ’s who had previously received art instruction based on copying. With such a background, feeble-minded persons cling to a technique and slavish imitation which are in no way related to their stage of visual comprehension. Nevertheless, the imitative, memorized picture seems to give them a certain security. Any attempt to lead such persons back to their own stage of visual conceiving is usually resented vigorously because of their anxiety over losing that sterile mental possession and being thrown back into a state of uncertainty.” The above is only one of many lessons from Schaefer-Simmern’s experiments. Dewey stressed in his foreword to Ref. [105] that Schaefer-Simmern’s experiments provided an “effective demonstration of what is sound and alive in theoretical philosophies of art and of education.” Trainees in productive teaching may be asked if they see parallels between Schaefer-Simmern’s experiments and what they think is possible to achieve in their disciplines.

Other examples of literature that reports on the practice of productive teaching in schools are Refs. [94, 95] (see Section IV E). Trainees in productive teaching can observe exemplary ongoing programs in action and learn from individuals working in these programs. Experienced individuals can play the role of mentors to the trainees. The trainees may learn from their mentors how the latter achieved understanding of productive teaching with the help of their own mentors of a previous generation. What were the breakthrough events that helped the mentors get on their paths to productive teaching? How long did it take and how did it happen that as a result of these events they learned how to create and sustain contexts of productive learning for their students? Such contexts are certainly not limited to the classroom in the cases described in Refs. [94, 95]. A challenge to training programs is identifying people who can be mentors, and the schools that are suitable for study and accessible to the trainees [108]. Currently, the concept of apprenticeship in productive teaching as a common path to professionalism in education can only be considered a concept of the future [109]. What the physical MOT provides here is a logical structure which explains why the above examples of recommendations for training are not merely items in another trade book in education but a necessity for building a culture of a meaningful communication between teachers and students as learning observers of the world.

In the current circumstances, one of the primary objectives of training is that the trainees learn the difference between the classroom regularities in systems of type $Q$ and $P$. In $P$, students may study the neighborhood of the school and spend a lot of their time outside the school building, and in $Q$ they may almost always stay in the building. But the trainees need to know and understand the concept of regularities in the process of teaching. Some regularities will agree with the MOT, and some will not.

An example of a study of classroom regularities that qualifies for incorporation in a training program in productive teaching is Susskind’s study of question asking [110, 111]. Susskind found that teachers often think that they ask approximately the same number of questions as students do and that they expect that only a slightly larger number of students’ questions would be better. In fact, teachers ask about 25 times more questions per period than all students in a classroom. They receive less than one fifth the rate of students’ questions that they estimate as occurring and as desirable. The bottom line is that on average a student asks one question per month in all his or her social studies classes combined (assuming four 45-minute social studies lessons per
week and realistic estimates of time available for asking questions), whereas teachers ask about one question per minute. The main reasons for students to not ask questions are identified by Susskind to be yelling by teachers and laughing by peers. Students appear to assume and adjust to a rule that only teachers are to ask questions, while the duty of students is to provide answers.

Susskind reported on rates of asking low-level questions (related to memorization, i.e., of the type: Who, what, where?) and high-level questions (such as concerning causes of wars) by teachers. He observed that these two rates are about the same on average. But the students of teachers who ask less low-level and more high-level questions ask more questions than the students of teachers who ask more low-level and less high-level questions. Susskind’s study also includes a category of competitive questions that have a particularly negative impact on students’ interest in asking about anything.

In addition, Susskind showed that a measurable change in the classroom frequencies of question-asking can occur as a result of the following sequence of events (see Susskind’s work for important procedural details, such as anonymity of records): 1) teachers make predictions of the numbers of questions that will be found in their classrooms, 2) researchers measure what happens and demonstrate the results to the teachers, 3) the results are discussed at a few seminar meetings, at which the researchers and teachers exchange ideas about the origin of differences between expectations and facts. Such discussions stimulate teachers to try different approaches and improve the situation. After the seminars, teachers are observed again. It is found that on average the number and types of questions teachers ask before and after the sequence of events 1, 2, 3 do not differ much, although some differences are discernible (for example, the average number of questions asked by teachers dropped by about 10%; while the average percentage of high-level questions increased from about 50 by about 2 and the average percentage of competitive questions decreased from about 8 by about 2, with all these changes being comparable with the magnitudes of corresponding standard deviations but correlated in ways that nevertheless allowed Susskind to draw his conclusions). On the other hand, the observed change in the behavior of students of the teachers who went through the sequence 1, 2, 3, is dramatic: the rate of their students’ question-asking doubles. Susskind’s work warns readers that there may be many reasons for why the seminars could help teachers to change their classroom behaviors and thus also change the students’ behavior (see also two paragraphs below).

The MOT suggests that when the human brain sees a conflict between an assumption or prediction it makes, especially about itself, and actual results of observation and experiment, it attempts to make changes in its space of knowledge in order to become better prepared to deal with new events (see Sections III A and IV D). It tries out new ideas. This is precisely what Susskind says the teachers did as a result of participation in the seminars. Although no simple explanation is offered by Susskind, the correlations he observed in the study (see the original articles) still allow him to make a few recommendations.

Susskind’s recommendations for teachers are: to reduce the number of questions teachers ask, to ask questions of greater complexity, and to write the best questions asked by students on the blackboard. Furthermore, if teachers ask questions about students’ personal experiences, students feel encouraged to ask questions concerning issues that truly interest them.

Susskind also recommends videotaping of classrooms as a means of creating a reliable source of information for teachers about how they and their students behave (the same recommendation is made by many authors [112]). Unfortunately, studies of classroom regularities are rare (on the order of 10 per 50 years before Susskind’s [110], cf. [109]). Today’s technology provides many means for recording (including self-recording) and discussing classroom regularities by the trainers and trainees that are significantly better than the ones that existed at the time of Susskind’s studies. The virtue of a film record is that a trainee can see how his or her action appeared from outside while the trainee also knows what internal events in the trainee accompanied what happened.

An example of observations that trainees need to learn to make is that Susskind’s study describes only correlations among measured numbers of various types of questions asked by teachers and students (statistically analyzed assuming that systematic errors cannot be large if various observers agree in their counting and judgment of the type of the questions). There is no claim of observing or understanding causal relationships or dynamical origins of the observed correlations, even though researchers are tempted to think that they see some causality [113].

A description of what is observed must be distinguished by trainees from an understanding of the dynamical origin of what is observed. Using the analogy of Bohr’s model (see the beginning of Section II), one may say that the description of the observed atomic spectra by Bohr’s model was an essential step on the way to the discovery of the underlying dynamics of the atom, but by itself did not provide an understanding of this dynamics. Moreover, trainees need to realize that understanding the dynamics of behavior of teachers and students in interaction is incomparably more difficult than just observing and describing their behavior. This realization can motivate the trainees to seek a better understanding of the dynamics of their own interactions with students.

Studies by Csikszentmihalyi et al. [114] used modern remote communication technology (such as beepers) to monitor students’ behavior during various activities around the clock (the technique is called experience sampling method). Results of these studies agree with the model assumption stated in Section IV A and elaborated on in Section IV D that students are strongly motivated by the visions they have of their own future [113]. On the other hand, the personal desire for and enjoyment of participation in activities is associated with the concept that
Dewey described (in the context of artistic expression) as follows [116]: “Because of this wholeness of artistic activity, because the entire personality comes into play, artistic activity which is art itself is not an indulgence but is refreshing and restorative, as is always the wholeness that is health. There is no inherent difference between fullness of activity and artistic activity; the latter is one with being alive.” Csikszentmihalyi calls such ultimate involvement in an activity “flow” [118], since one can say that a person feels flowing fully immersed in the activity and ceases to sense the flow of time. The same phenomenon is known to exist in all disciplines, including science [119], art, sport, and teaching [120]. The training of teachers should include the experience of flow in the process of teaching. Productive teaching is a source of immense joy for a teacher who sees how a student moves on in the process of learning, and crosses otherwise insurmountable barriers. This feature is heuristically included in the dynamical part of MOT discussed in Section [IVD].

The training of teachers must include basic issues of communication and cooperation with parents. Numerous examples illustrate that teachers and parents must communicate [121]. Parents are major players in the student’s life [122]. They can help a teacher understand where the student is coming from and how to help a student in getting on the path of learning, which are indispensable elements in productive teaching according to the MOT. On the other hand, the model suggests that communication between teachers and parents as observers of the same reality from different frames of reference encounters the same type of difficulties as communication between teachers and students. Of course, the case of teachers and parents involves significant internal events (most parents deeply care about their children) and needs an extended meta-dictionary (for parents, the subject matter is of secondary importance to how their children fare). The MOT predicts and explains why a clear communication in these circumstances must be difficult and requires training, see Section [IVC].

The degree of success in training of a pool of teachers in productive teaching will depend on the selection of candidates [123]. Ideally, a selection process should be based on strengths that the candidates exhibit in their records and during “auditions” in the role of a teacher. As in the case of good schools of music, where it is not enough to just know how to hold an instrument and play a sequence of notes, so in the case of teaching it is not enough to know something about the subject matter and spell it out in front of the class (e.g., see the MOT predictions concerning difficulties in communication about internal events described in Section [IVC]). The analogy with music makes it clear that people who start learning how to teach early, for example, in scouts, can develop their skills far beyond average before they come to the “audition” appropriate for their admission to the school of productive teaching as defined by the MOT.

If someday training in productive teaching becomes a regularity of the educational system, one can imagine that promising students will be immersed in the contexts of productive learning by giving them responsible teaching and administrative roles in their schools and they will be helped in practicing their skills as part of their work. The help they may provide in return in teaching younger students and running the school could in principle reduce the burdens on teachers and provide the teachers with more time than they have now for their own professional development [124]. The MOT provides physical arguments for the necessity of increase in the amount of time made available to teachers for training.

It is clear at this point that the heuristic concept of productive teaching defined by the MOT implies a long list of suggestions for training of teachers and a corresponding list of questions that require answers. With current capabilities for research on teaching and learning it should be possible to begin systematic studies on the applicability of the concepts of productive teaching and training in practice.

It is expected that a trainee teacher who learns the concept of productive teaching will voluntarily express gratitude to a trainer for providing the lesson. The reason is that if the trainer works on the lesson and the trainee understands what the result is supposed to be, it will be clear to the trainee familiar with the MOT that confirmation of the result is what the trainer needs to obtain in order to know the degree to which the task is accomplished. Observing how the trainee behaves in contact with students under supervision (or aware of being observed or filmed) is not sufficient because a trainee may behave like an actor, who knows how to follow instructions given by a director and perfectly fakes a character while not actually becoming one. Since the concept of productive teaching is learned inside the trainee, only the trainee can provide the information that the lesson is learned. There is no point in asking the trainee if he or she has understood the concept before it is observed as emitted by the trainee. When it occurs, the information is provided by the trainee in a voluntary expression of appreciation of the concept (the concept includes the appreciation of the work of the trainer and understanding by the trainee that the voluntary expression of this appreciation is the way by which the trainer knows the result). Honest, self-motivated discussion of the concept cannot be faked since it can only be based on concrete events in which the trainee behaves in a sincere way and thus learns from them. If that is not the case, the trainee only pretends comprehension of the concept. Attempts to hide confusion and maintain superficial claims to understanding are not difficult to identify. When the trainer points out a problem related to the trainee’s conception of productive teaching, the trainee’s reaction is itself a measure of the depth of understanding of the concept. So, it is relatively easy to know whether a trainee is still in the woods. But it is not possible to tell that the training is completed until the trainee voluntarily provides the evidence.

Everyone who works in the capacity of a teacher does
so in large part as a result of prior training. If that training was not based on the principles of communication between brains that are identified in the model, the principles may remain unknown to the person. It is then not the person’s fault that she or he is not able to create a productive teaching environment for learning by others. Such cases can be gradually eliminated. According to the MOT, the elimination requires an extension of the concept of teaching from effective to productive. Those who are to train teachers would have to demonstrate understanding of the difference before becoming trainers.

V. CONCLUSION

The MOT proposed here provides a description of the concepts of effective and productive teaching that are not at all new in their meanings. Authors of great influence on our thinking have discussed these meanings extensively. The original aspect of the model is its simplicity. Namely, it sketches a picture of these concepts in simple physical terms.

This is useful for seeking systemic solutions to problems in Education since one has to define the goals of the system before one attempts to figure out how to create systemic conditions in which these goals can be achieved. It is clear that the system must be designed in such a way that the forces pushing toward the goal are not counter-balanced by other forces that might also act in a system and win if the system is designed without the necessary understanding. If the goal is productive teaching defined in the MOT, the system forces must be arranged in such a configuration that the basic forces that drive productive teaching can function and perform their work.

However, the major problem of figuring out a suitable arrangement is not solved here. For example, the question of how to train people who are to legislate, administer, and judge the work of teachers is not resolved. This problem can be called the system government problem. It is clear that it depends on the competence of the system government if teachers (appropriately trained) can perform in agreement with their training in the conditions created for their work (see Section III D). The system government problem also means that the search for systemic solutions cannot be limited to the educational system alone. The bedrock assumption is that in all circumstances where teaching occurs in a system, both teacher and student are learning and they learn from each other in identical ways as observers who exchange information about the same reality.

The insights the model offers regarding effective teaching (Section III) and productive teaching (Section IV) help in comprehending Sarason’s early prediction of the failure of educational reforms. Sarason’s prediction is based on his concept of a context of productive learning. Until now, there existed no simple description of what this concept means. According to the model, a context of productive learning exists for a student in the process of productive teaching (see Section IV E). A reform effort cannot be successful in creating contexts of productive learning for students unless the processes of productive teaching are center stage in the system. Thus, if an expert looks at a reform plan, including plans to monitor and document the performance of the system in the long run, and sees that nowhere in it the key processes are paid due attention, the prediction of failure may be made long before the reform is implemented. According to Sarason, Bensman and Levine provided credible descriptions of reform efforts that come the closest to incorporating contexts of productive learning in school practice.

The model picture also sheds some light on the belief that a rigorous scheme for training teachers in science (and through them their students) can by itself stimulate interest in learning. The model suggests instead that the kinematical image of the process of science must result from its proper dynamics (whatever it may be). When the image is artificially created through special constraints and incentives, neither teachers nor students can learn what the relevant forces are (whatever they may be). A less arbitrary alternative for the design of reforms is to secure facilitation of action of the natural forces of productive teaching: strengths of teachers and students. For this purpose, the knowledge of teachers needs to include the meta-dictionary of teaching. The model provides a preliminary definition of the meta-dictionary, showing that a dictionary that is limited to subject matter alone is insufficient to discuss educational reform.

When teachers and students are seen as observers of the world from different frames of reference and the kinematics and dynamics of their communication about what they observe are analyzed in simple terms of a physical model, it becomes clear that the interpersonal relationship between a teacher and a student is the key to productive teaching defined in the MOT. According to the model, productive teaching is not only effective in terms of mastery of the subject of study. It also helps a student to understand the role the teacher plays in the process, a facilitator of learning rooted in the individual’s strengths. Through appreciation of this role, a student becomes a person who understands the principles and virtues of communication between people. This is why the model concept of productive teaching appears to provide a picture of what systemic solutions need to support in order to make a transition to a society in which words and ideas can help people in carrying out the work that is necessary for their well-being.

However, the model also says that the number of dimensions in the space of events registered in a brain is large (see Appendix A). Therefore, the corresponding relationships between frames of reference of different brains are complex, nonlinear, and topologically nontrivial. MOT predicts through these circumstances that an exchange of sequences of coordinates without paying much attention to their proper interpretation in different frames of reference may be easily mistaken for the con-
cept of communication between brains. But if teaching is equated to handing down to a student a sequence of coordinates \( t \) formed using a language known in fact only to the teacher, and then checking if the sequence \( t \) is correctly handed back, not only is the teaching ineffective and students unfairly judged, but also a new phenomenon is created. Every sequence \( t \) can be handed around without clarity or need for clarity of its meaning in terms of real events. In such case, everybody can “teach” because there is no need to know either the content or how to transfer the content. The model predicts in simple terms that teaching cannot be improved by streamlining such a mindless process of handing down statements without content. The prediction holds no matter how much money may be spent on such a process because the process has nothing to do with productive teaching defined in the model. Thus, the model supports a claim that this process will not lead to transition toward new forms of society such as the one envisioned by Drucker [128].

According to the MOT, productive teaching demands preparation of teachers who can communicate with students in many more dimensions than only those of the subject matter (Section III). Such advanced preparation requires an apprenticeship system in which novices are taken care of by mentors over extended periods of time and may stay in touch with the mentors during their subsequent professional activities. Certainly, such preparation of teachers resembles processes of induction or mediated entry in professional disciplines and as such implies an open-ended self-correcting evolutionary process of change in the discipline of teaching based on ongoing research and development of the highest standards. By comparison with contemporary systems of education, such an evolutionary process of change demands new organization because the existing organization is focused too much on handing down sequences of words without paying due attention to their meanings. This illustrates how potentially far-reaching predictive power physical models of teaching may have.

Understood as a process based on interaction between different brains, teaching cannot be described by models that are limited to learning by only one brain and do not involve the transfer and processing of information in interaction between different brains. However, models of how a single brain learns are very important for teaching despite their limitations [14, 15, 129, 130]. For example, consider that the process of gene expression in the formation of an individual’s memory [83, 131] (and perhaps also in other Hebbian processes) depends on factors such as attention [82]. When combined with the MOT, this consideration suggests that an educational system has to incorporate a number of ideas that originate in various arts and sciences into a physically-motivated structure. Such attempts are in need of building bridges between different disciplines; for example, between psychology and physics in terms of neural science. It is encouraging that physicists and neural scientists are already involved in discussions [132, 133, 134], and that neural scientists appear to be in communication with psychologists [17]. Even though a physical MOT may in principle offer a framework in which all required elements have a chance to eventually find their place, it is predicted that much more precise physical models of teaching will be needed than the very preliminary one described here. Nevertheless, because this preliminary MOT has a simple physical structure with many concrete implications regarding otherwise complex issues of teaching, it can be predicted on general grounds [135] that physical MOTs will be useful in guiding reform of educational systems.

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APPENDIX A: SPACE OF EVENTS REGISTERED IN A BRAIN

The word “event” is associated in physics with what happens in a set of points in space-time. But a learning human brain obtains information about events only in the form of input it can receive. A heuristic idea of parameterization of the information about events registered in a brain is used below to define and visualize the concept of space of events that is used in Sections III and IV.

Consider how a snapshot of a learning individual’s environment forms a record of an event and how such a record is received as input in the individual’s brain. The input comes from the visual system and causes changes in the brain. Complete description of the changes undoubtedly requires a large set of parameters. But even without knowing the appropriate set of parameters and their values one may postulate that the changes ultimately result from the interplay of three elements: 1) The light actually received in the eyes, 2) performance of the system of vision that provides input to the brain, and 3) the activity in the brain at the moment of reception of the input from the visual system.

The first two elements contribute to the process of registration of events by the individual’s brain in familiar ways. For example, a student may be observing what happens on a laboratory table and, perhaps, must be wearing corrective lenses in order to see with sufficient precision. The third element, i.e., the ongoing brain activity whose change constitutes registration of an event, is least well-understood. Obviously, such brain activity is
not based solely on the momentary input from the visual system. It depends also on the input from all other senses and from a large number of body organs. Ultimately, the processes that occur in the brain itself determine how events are registered. Factors such as attention and interest (or lack thereof for various, complex reasons) play significant roles. Also, since the brain develops over time, its activity at every moment depends on its biophysical history and the current structure this history has produced, including memory. It is clear that the brain’s reaction to every momentary input it receives is not a simple function of this input. Thus, the concept of space of events registered in a brain requires a definition.

It is postulated that at every moment in time a brain located at any place in space consists of particles and fields in some state that in principle may be described using a very large number of physical parameters. A considerably smaller number of parameters describes the biophysical state of the brain, which corresponds to the underlying state of particles and fields. These parameters change over time. A change in this set of parameters over a short period will be called a raw representation of an event in the brain. The length of the suitable period is of secondary importance. For the MOT, periods on the order of one hundredth of a second are perhaps appropriate since a human eye cannot discern images that appear with frequency larger than on the order of one hundred per second, ears cannot discern words spoken faster than about ten per second, typical muscular reaction time is on the order of one fifth of a second, etc.

The raw representations of events are somehow processed in a brain so that the results can be stored in memory, recalled, and recognized. These results are called events registered in a brain and, by definition, they are elements of the space of events registered in the brain.

The space of events registered in a brain is certainly not isomorphic with the space of physical events outside the volume swept in space-time by the brain because the events registered in the brain depend on processes that involve physical events inside this volume. The latter are specific to the brain in which they happen. It is clear that events registered in different brains cannot be compared in any simple way.

Studies of the nervous systems of simple organisms \[72, 82, 83, 131\] indicate how the raw representations of events may be processed and how the results of this processing may be registered in an organism’s brain. The human brain is so complex that attempts to fully explain how it works are unlikely to succeed in the near future \[15\]. Nevertheless, one may imagine the space of events registered in a human brain using the concept of coordinates. This coordinate picture is referred to in Sections \[11, 15\] and helps in appreciation of the magnitude of dimensionality of the events that underlie the process of productive teaching discussed in Section \[15\].

In order to introduce the relevant coordinates, one may start from the postulate that the human brain is built from units that are connected by links. The smallest unit to think about would be a neuron and the smallest link would be a synapse. A more suitable level of analysis is that neurons work in groups and these groups may be considered units that are connected by complex links. A candidate to consider for a unit would be Hebb’s assembly \[14\]. Fortunately for the purposes of the model, one may postulate the existence of units and links without specifying precisely their nature, except that there are many units and many more links among them. The physical arrangement of units and links in the brain, such as their apparent spatial extent or overlap, is irrelevant.

In order to quickly imagine the coordinates of events, it is best to consider first only two units connected by just one link. It is postulated that the magnitude of activity in the link at any moment and any position of the brain can be described by one number. This number is treated as a coordinate of content of the event registered in the brain. Note that the coordinate most likely describes a compact dimension since the link ceases to function properly and is turned off when the activity increases above a certain threshold. Thus, a coordinate above threshold does not exist and the threshold value is considered equivalent to zero \[136\].

Since the brain is considered to be built from many units and the number of just pair-wise links between \(m\) units, denoted by \(n\), is much larger than \(m\), \(n = m(m - 1)/2\), the content of an event registered in the brain can be identified with an element in an \(n\)-dimensional space with a large \(n\). It is postulated that \(n\) is fixed and the same for all brains. More complex alternatives do not need to be discussed here. Including the time and place of registration, the number of dimensions in the space of events registered in a brain is \(N = 4 + n\), a much larger number than 4 that applies only to space-time. The magnitude of \(N\) suggests greater difficulties with precise communication between observers about events their brains register than the observers typically encounter already in communication about point-like events in four-dimensional spaces.

\[1\] N. Bohr, Philosophical Magazine 26, 1, 476, 857 (1913).
\[2\] P. A. M. Dirac, Mathematical Foundations of Quantum Theory, ed. A. R. Marlow (Academic Press, 1978).
\[3\] A. B. Arons, A guide to Introductory Physics Teaching (Wiley, 1990).
\[4\] L. C. McDermott, E. F. Redish, Am. J. Phys. 67, 755 (1999).
\[5\] L. C. McDermott, Am. J. Phys. 74, 758 (2006).
\[6\] L. Bao, E. F. Redish, Phys. Rev. ST-PER 2, 010103 (2006).
\[7\] Plato, Republic, e.g., trans. R. E. Allen (Yale University Press, 2006); Symposium, e.g., trans. R. Waterfield
(Oxford University Press, 1994).
[8] J. J. Rousseau, Emile, ed. P. D. Jimack (Dent, 1993).
[9] M. Montessori, The Montessori Method (Cosmo, 2006).
[10] J. Dewey, Democracy and Education (Dover, 2004).
[11] J. Piaget, Psychology of Intelligence (Routledge, 2001).
[12] S. B. Sarason, And What Do You Mean by Learning? (Heinemann, 2004).
[13] S. D. Glazek, S. B. Sarason, Productive Learning (Corwin Press, 2006).
[14] D. O. Hebb, The Organization of Behavior (Wiley, 1949).
[15] D. Marr, Proc. Roy. Soc. London B 176, 161 (1970).
[16] P. Rakic, Science 241, 170 (1988).
[17] E. R. Kandel, J. H. Schwartz, T. M. Jessel, Principles of Neural Science (McGraw-Hill, 2000).
[18] C. Wieman, APS News, November 2007 (Vol. 16, No. 10); see also www.cwsei.ubc.ca/resources.
[19] Words are used to describe and to name concepts.
[20] C. Darwin, The Descent of Man and What Do You Mean by Learning? (Heinemann, 2004).
[21] L. M. McDermott, P. S. Shaffer and the Physics Education Group at the University of Washington, Tutorials in Introductory Physics (Prentice Hall, 1998).
[22] See Ref. [12] for examples of meanings people associate with the word learning that has the highest count in the literature on education.
The process of teaching cannot be understood here as limited to what does because works in a system and the system may dictate norms that generate anxiety. E.g., see Ref. [79].

Physical laws lead to the formation of the genetic code and its expression in ways that may be not understandable using classical concepts, because these concepts do not apply in description of quantum phenomena.

For example, in terms of renormalization group, suitably generalized, a long chain of steps connects the complex dynamics of complex effective degrees of freedom, such as entire brains, with the simple dynamics of basic initial degrees of freedom, such as particles and fields.

K. G. Wilson, in Nobel Lectures (World Scientific, 1993).

Suitable generalization of Renormalization Group, see Ref. [47].

The word “blind” is borrowed from R. Dawkins, The Blind Watchmaker (Norton, 1996).

T. P. Hughes, American Genius (Penguin, 1989).

M. J. Bennett, When Dreams Came True (Brassey’s, 1996).

In rare cases, individuals are able to learn using information from others almost entirely without direct personal contact (e.g., only reading books), and the teaching they receive this way may be effective in the sense of learning a specialty. Productive teaching includes personal contacts in order to avoid unproductive consequences for the learner of learning almost entirely without personal contacts.

D. Bensman, Central Park East and its graduates (Teachers College Press, 2000).

E. Levine, One kid at a time: Big lessons from a small school (Teachers College Press, 2001).

G. Mortenson and D. O. Relin, Three cups of Tea (Penguin, 2007).

Ref. [95], pp. 102-104.

Ref. [96], pp. 299-300.

Ref. [10], pp. 32, 35-36, and 56; cf. Ref. [98], pp. 150-151.

Ref. [13], pp. 216-218; cf. pp. 191-193 etc.

Clay identified in clear terms the difference between measuring how students learn and testing them.

Wieman’s observation that teaching may be not as productive for students as a teacher thinks, appears related to a possibility that it is difficult for a person to pay attention to issues of teaching when the person needs to focus on the subject matter.

The author observed training sessions for teachers and teacher leaders in the Reading Recovery program led by G. S. Pinnell at The Ohio State University, in 1996.

The same need for self-correction based on comprehension is evident in students of physics who mechanically transform equations without realizing what they are doing and cannot see and correct errors they make.

H. Schaefer-Simtrum, The Unfolding of Artistic Activity (University of California Press, 1961).

J. Dewey, Art as Experience (Penguin, 1980). See also Ref. [69], Chapter 7.

See Report on Selma, Ref. [103], pp. 55-61; and p. 183.

For example, could the teachers in the private Shady Hill School, Cambridge, Massachusetts (www.shs.org), be able and willing to provide mentoring to the trainees in the Boston area, nationally, or internationally?

D. C. Lortie, The school teacher (University of Chicago Press, 2002).

E. C. Susskind, in Community Mental Health Monograph of Psycho-Educational Clinic, Eds. F. Kaplan and S. B. Sarason, Vol. 4 (1969), pp. 130-151; see also E. C. Susskind, Questioning and curiosity in the elementary school classroom, Ph.D. Thesis, Yale University, 1969.

E. Susskind, J. of Clinical Child Psychology, Summer 1979, pp. 101-106.

Ref. [44] can be studied with the help of a documentary film High School II by F. Wiseman, photography by R. Leiterman; see C. James, Film Review; 25 Years Later, Wiseman Goes Back to School, New York Times, July 6, 1994.

This reservation is made explicitly in Refs. [110], [111].

M. Csikszentmihalyi, K. Rathunde, S. Whalen, M. Wong, Talented teenagers (Cambridge University Press, 1997).

Ref. [114], pp. 211-212.

See Dewey’s foreword to Schaefer-Simtrum’s work in Ref. [103], pp. 296.

M. Csikszentmihalyi, Flow (HarperCollins, 1991).

Hadamard reports on similar phenomenon in mathematics, see Ref. [47].

Ref. [114], Chap. 9. The chapter is focused on teachers providing conditions for flow to students and suggests that teachers can do it on the basis of their own experience of flow.

E.g., see Ref. [94], p. 103; Ref. [93], pp. 97, 137; Ref. [13], pp. 64, 67.

Ref. [12], pp. 39-67; Ref. [114], pp. 152-176.
[123] Ref. [69], pp. 96-113.
[124] Ref. [27], pp. 202-204.
[125] System dynamics occurs at the end of the chain [85].
[126] Perhaps a better name is “the system brain problem,” since it reflects the need for learning by a complex system that is to evolve consciously.

[127] S. B. Sarason, Educational Reform: a self-scrutinizing memoir (Teachers College Press, 2002).
[128] P. F. Drucker, Post-Capitalist Society (Harper Business, 1993); pp. 6-9, 201-204, and Chap. 12.
[129] J. J. Hopfield, Proc. Nat. Acad. Sci. USA 79, 2554 (1982), ibid. 81, 3088 (1984).

[130] T. J. Sejnowski, O. Paulsen, J. Neurosci. 26, 1673 (2006).
[131] C. H. Bailey, E. R. Kandel, K. Si, Neuron 44, 49 (2004).
[132] L. Borg-Graham, Nature Neurosci. 3, 1191 (2000).
[133] J. J. Hopfield, Nature Neurosci. 3, 1204 (2000).
[134] J. Knight, Nature 419, 244 (2002).
[135] E. M. Rogers, Diffusion of Innovations (Free Press, 2003); e.g., see p. 257.
[136] On a circle, as example of a compact dimension, an angle can be chosen as a co-ordinate: it ranges from 0 to $2\pi$, and the point with co-ordinate $2\pi$ is the same as the one with co-ordinate 0.