Covariances for cosmic shear and galaxy–galaxy lensing in the response approach

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ABSTRACT

In this study, we measure the response of matter and halo projected power spectra $P^{XY}_{2D}(k)$ ($X, Y$ are matter and/or halos), to a large-scale density contrast, $\delta_b$, using separate universe simulations. We show that the fractional response functions, i.e., $d \ln P^{XY}_{2D}(k)/d \delta_b$, are identical to their respective three-dimensional power spectra within simulation measurement errors, as long as we can ignore redshift evolution of the power spectra over the redshift range of projection. Then, using various $N$-body simulation combinations (small-box simulations with periodic boundary conditions and sub-volumes of large-box simulations) to construct hypothetical observations of mock projected fields, we study how super-survey modes, in both parallel and perpendicular directions to the projection direction, affect the covariance matrix of $P^{XY}_{2D}(k)$, known as super-sample covariance (SSC). Our results indicate that the SSC term provides dominant contributions to the covariances of matter-matter and matter-halo spectra at small scales but does not provide significant contributions in the halo-halo spectrum. We observe that the large-scale density contrast in each redshift shell, i.e., the trace of second-derivative tensor of the large-scale gravitational potential field, causes most of the SSC effect, and we did not observe a SSC signature arising from large-scale tidal field, within the levels of measurement accuracy. We also develop a response approach to calibrate the SSC term for cosmic shear correlation function and galaxy–galaxy weak lensing, and validate the method by comparison with the light-cone, ray-tracing simulations. Our method provides a reasonably accurate, albeit computationally inexpensive, way to calibrate the covariance matrix for clustering observables available from wide-area galaxy surveys, without the need to run light-cone simulations.

Key words: gravitational lensing: weak – large-scale structure of Universe – cosmology: theory

1 INTRODUCTION

Weak gravitational lensing is a powerful cosmological probe to constrain the nature of dark matter and dark energy (e.g., Hoekstra & Jain 2008; Munshi et al. 2008; Kilbinger 2015; Mandelbaum 2017, for a review). Measuring angular correlations between distant galaxy shapes enables the mapping of foreground matter distribution (i.e., cosmic shear). Similarly, stacking shapes of background galaxies with respect to foreground large-scale structure tracers, such as galaxies and clusters, provides reconstructions of the averaged matter distribution around the tracers (i.e., galaxy–galaxy lensing or stacked cluster lensing). The weak lensing observables are sensitive to cosmological parameters such as a combination of the cosmological matter density ($\Omega_m$) and the current amplitude of density fluctuations at $8h^{-1}$Mpc scale ($\sigma_8$). Several galaxy imaging surveys have put stringent constraints on $\Omega_m$ and $\sigma_8$ from their cosmic shear measurements such as the results from the Canada-France-
The purpose of this study is to develop a SSC calibration method for projected matter-matter and matter-halo power spectra. To relevant for cosmic shear and galaxy–galaxy weak lensing, respectively. To develop this method, we first recognize the response functions of the projected power spectra to the large-scale density contrast using separate universe simulations (Sirkó 2005; Li et al. 2014a,b; Baldauf et al. 2011; Wagner et al. 2015; Baldauf et al. 2016; Barreira & Schmidt 2017; Chiang et al. 2017; Schmidt et al. 2018), where the effects of large-scale density contrast are absorbed by changes in background cosmological parameters used in $N$-body simulations. We study whether the response functions of the projected power spectra differ from those of their respective three-dimensional power spectra. In observing these spectra, we pay particular attention to the manner in which super-survey modes, both in parallel and perpendicular directions to the projection direction, affect the projected power spectra using mock catalogues of projected fields that are constructed from a combination of different $N$-body simulations. Subsequently, we develop a method to calibrate SSC contributions for the cosmic shear correlation function and the halo-convergence cross-correlation function using response functions calibrated from separate universe simulations. We validate this method by comparing it with the results from light-cone ray-tracing simulations. Then,
we discuss the approach to calibrate the covariance matrix by combining the actual data, the response approach, and numerous simulations.

Throughout this study, we adopt the flat-geometry ΛCDM (Lambda cold dark matter) model that is consistent with the Planck 2015 results (Planck Collaboration 2016, hereafter, Planck). The cosmological parameters are as follows: the matter density, $\Omega_m = 1 - \Omega_\Lambda = 0.3156$, the baryon density, $\Omega_b = 0.0492$, the Hubble parameter, $h = H_0/(100\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}) = 0.6727$, the present amplitude of density contrast at $8\,h^{-1}\,\text{Mpc}$, $\sigma_8 = 0.831$, and the spectral index, $n_s = 0.9645$.

## 2 POWER SPECTRUM COVARIANCE OF PROJECTED FIELD IN A FINITE SURVEY AREA

### 2.1 Projected halo number density field

In this subsection, we define the statistical quantities of the halos. We assume that halos in a survey region are identifiable via observables such as optical richness and X-ray observables; we also assume that mass and redshift of each halo are available. However, the following discussion can extend to a general case in which only their proxies are available.

The number density field for halos, in the mass range of $[M, M + dM]$, at a position $\mathbf{r}$ and redshift $z$, is defined as

$$n_h(\mathbf{r}, M, z) = \frac{dn}{dM}(M, z)[1 + \delta_h(\mathbf{r}; M, z)]$$  \hspace{1cm} (1)$$

where $dn/dM$ is the (ensemble-averaged) number density of halos with mass $[M, M + dM]$, $\delta_h(\mathbf{r}; z, M)$ is the three-dimensional number density fluctuation field for halos at a redshift of $z$ with mass $M$, and $\mathbf{r}$ is the three-dimensional position vector given by the comoving radial distance, $\chi$, and the two-dimensional position vector $\mathbf{x}$ perpendicular to the line-of-sight direction, i.e., $\mathbf{r} = (\chi, \mathbf{x})$. Here, $\chi$ is given as a function of redshift via the distance-redshift relation, $\chi = \chi(z)$, for the underlying true cosmological model. Note that we can infer the projected position vector via $\mathbf{x} = \chi(z)\mathbf{\theta}$ for a flat geometry universe, where $\mathbf{\theta}$ is the angular position vector.

By denoting a radial weight function, $f_h(\chi)$, and a survey window function, $W(\mathbf{x})$, we can define the projected number density field for halos, integrated over a range of redshift and halo masses, in terms of the three-dimensional density field, $n_h(\mathbf{r}, M, z)$, as follows:

$$n_{h,\text{2D}}^\text{SW}(\mathbf{x}) = \int d\chi \, f_h(\chi) \int dM \, W(\mathbf{x}) \, n_h(\mathbf{r}, M, \chi).$$

$$= \int d\chi \, f_h(\chi) \int dM \, W(\mathbf{x}) \frac{dn}{dM}(M, \chi)[1 + \delta_h(\mathbf{x}, \chi; M)].$$  \hspace{1cm} (2)

Here, $f_h(\chi)$ is the radial weight or selection function, generally given as a function of halo mass and redshift (or $\chi$), e.g., $f_h(\chi) \neq 0$, if $\chi$ is inside the redshift range of a survey, otherwise $f_h(\chi) = 0$. Similarly, the survey-window function, $W(\mathbf{x})$, is defined to satisfy the condition $W(\mathbf{x}) = 1$ if the position vector $\mathbf{x}$ is inside a survey region, otherwise $W(\mathbf{x}) = 0$. Throughout this paper, we use the subscript “W” to denote the field measured in a survey region, and use the superscript “2D” to denote the projected quantities. Note that, if $f_h(\chi)$ is dimensionless, $n_{h,\text{2D}}^\text{SW}$ has a dimension of $\text{(length)}^{-2}$. We use a distant observer approximation so that the survey boundary does not depend on the redshift. This is a good approximation for most cases of interest in, e.g., galaxy–galaxy lensing, where the lensing galaxies are taken from a narrow redshift width. This approximation is also valid for a general case such as cosmic shear if radial integration is replaced by a discrete summation on thin lens shells (i.e., $\int d\chi \approx \sum_i \Delta\chi$, where $\Delta\chi$ is the thickness of the lens shell) for each shell that we can use the approximation, $\mathbf{x} = \chi_i\mathbf{\theta}$, where $\chi_i$ is the mean distance to the $i$-th lens shell. The projected survey area, $S_W$, is given by

$$S_W = \int d^2\mathbf{x} W(\mathbf{x}).$$  \hspace{1cm} (3)

An estimator of the mean projected halo number density, in a survey volume, is defined as:

$$\bar{n}_{h,\text{2D}} = \frac{1}{S_W} \int d^2\mathbf{x} n_{h,\text{2D}}^\text{SW}(\mathbf{x}).$$

$$= \int d\chi \, f_h(\chi) \int dM \frac{dn}{dM}(M, \chi) \left[1 + \frac{1}{S_W} \int d^2\mathbf{x} W(\mathbf{x}) \delta_h(\mathbf{x}, \chi; M)\right].$$

$$= \int d\chi \, f_h(\chi) \int dM \frac{dn}{dM}(M, \chi) \left[1 + b(M, \chi) \frac{1}{S_W} \int d^2\mathbf{x} W(\mathbf{x}) \delta_{\text{m,lin}}(\mathbf{x}, \chi)\right].$$

$$\approx \bar{n}_{h,\text{2D}} \left[1 + \bar{b}_{\text{h}} \bar{\delta}_h\right].$$  \hspace{1cm} (4)

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where
\[ \delta_{b}^{2D} = \int d\chi f_{b}(\chi) \int dM \frac{dn}{dM}(M, \chi), \]
\[ \delta_{h} = \frac{1}{\bar{d} M} \int dM' \frac{dn}{dM'}(M', \chi) \int dM \frac{dn}{dM}(M, \chi) b(M), \]
\[ \delta_{h} = \frac{1}{\int d\chi' f_{h}(\chi') \int d^2x' W(x') \int d^2x W(x) \delta_{m_{lin}}(x, \chi'). \]  
\[ (5) \]

Here, we use a halo density fluctuation field at large scales, which is given as \( \delta_{h} = b(M)\delta_{m_{lin}} \), where \( b(M) \) is the linear halo bias parameter with mass \( M \) and \( \delta_{m_{lin}} \) is the linear mass density fluctuation field. We also assume that both the halo mass function and halo bias are not rapidly varying functions of redshift; therefore, we ignore their redshift dependences in the radial integration. In the above equations, \( \delta_{b}^{2D} \) is the ensemble average of projected number density, \( \delta_{h} \) is the mean halo bias in the sample, and \( \delta_{b} \) is the average density contrast within the survey volume. Equation (4) indicated that the number of halos in a survey region is generally modulated by the large-scale density contrast, \( \delta_{b} \), and up-weighted by the halo bias, on the basis of an individual survey region (Hu & Kravtsov 2003; Takada & Bridle 2007), where \( \delta_{h} \) and \( \delta_{b} \) are constants for a given realisation.

Following the formulation in Takada & Hu (2013), the projected halo density fluctuation field is defined as
\[ \delta_{b}^{2D}(x) = \int d\chi f_{b}(\chi) W(x) \delta_{h}(x; \chi; M). \]  
\[ (6) \]

Similarly, we can define the projected mass density fluctuation field as
\[ \delta_{m}^{2D}(x) = \int d\chi f_{m}(\chi) W(x) \delta_{m}(x, \chi). \]  
\[ (7) \]

where \( f_{m}(\chi) \) is the radial weight function that depends only on \( \chi \) (\( f_{m} \) may have a dimension, but here we keep it general). If we take an appropriate form of \( f_{m}(\chi) \), we can express the weak lensing or cosmic shear field using the form above. Unlike in equation (6), we do not normalise the projected matter field by the local mass density contrast, \( \delta_{b} \), because weak lensing measured from the lensing distortion effects on galaxy shapes arises from gravitational potential fields in large-scale structures, related to matter density fluctuation fields with respect to the global background mean mass density (see Takada & Hu 2013; Li et al. 2014a, for details).

### 2.2 An estimator of projected power spectrum

From Eqs. (6) and (7), we can define a general form to express the projected field of matter or halos as
\[ \delta_{X}^{2D}(x) = \int d\chi F_{X}(\chi) W(x) \delta_{X}(x, \chi). \]  
\[ (8) \]

where the subscript “\( X \)” denotes either matter or halo \( (X = m \text{ or } h) \), and \( F_{X}(\chi) \) is a radial function, e.g., \( F_{X}(\chi) = f_{m}(\chi) \) or \( F_{X}(\chi) = f_{b}(\chi)/\bar{d}_{b}^{2D} \). The Fourier transform of the projected field is represented as
\[ \delta_{X}^{2D}(k_{\perp}) = \int d\chi F_{X}(\chi) \int \frac{d^2q_{\perp}}{(2\pi)^{2}} \int \frac{dq_{\parallel}}{2\pi} e^{-i(k_{\perp}q_{\perp} + k_{\parallel}q_{\parallel})} \delta_{X}(k_{\perp} = q_{\perp}; k_{\parallel}; \chi) \tilde{W}(q_{\parallel}) e^{i q_{\parallel} x}, \]  
\[ (9) \]

where quantities with tilde symbols, “\( \sim \)”, denote the Fourier transforms, \( k_{\perp} \) and \( q_{\perp} \), which are two-dimensional wavevectors in two-dimensional space aligned perpendicular to the line-of-sight direction, and \( k_{\parallel} \) is the parallel component. We often ignore the subscript, “\( \sim \)”, to denote the perpendicular vector for notational simplicity.

Let us then define a power spectrum estimator of the projected field as follows:
\[ \tilde{P}^{2D}_{XY}(k) = \frac{1}{S_{k}} \int |k'| e^{i k'} \text{Re} \left[ \frac{\tilde{P}^{2D}_{XY}(k')}{\tilde{P}^{2D}_{XY}(-k')} \right], \]  
\[ (10) \]

where we integrate over a circular annulus of width \( \Delta k \), around the radius \( k \) in two-dimensional \( k \)-space, and \( S_{k} \equiv \int |k'| e^{i k'} \bar{S}_{k} \approx 2\pi k \Delta k \) for \( k \gg 1/\bar{S}_{k} \). The ensemble average of the power spectrum estimator (equation (10)) is computed as
\[ \langle \tilde{P}^{2D}_{XY}(k) \rangle = \frac{1}{S_{k}} \int |k'| e^{i k'} \int \frac{d^2q}{(2\pi)^{2}} |\tilde{W}(q)|^{2} \tilde{P}^{2D}_{XY}(|k' - q|). \]  
\[ (11) \]

Here, \( P^{2D}_{XY}(k) \) is the projected filed power spectrum that is expressed in terms of the underlying three-dimensional power spectrum using Limber’s approximation (Limber 1954):
\[ P^{2D}_{XY}(k) = \int d\chi F_{X}(\chi) F_{Y}(\chi) P_{XY}(k; \chi), \]  
\[ (12) \]

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where \( P_{XY}(k; \chi) \) is the three-dimensional power spectrum of the underlying fields, defined as

\[
\langle \delta_X(k, \chi_i) \delta_Y(k', \chi_j) \rangle = \frac{1}{(2\pi)^3} \delta_D^2(k + k') \delta_D(k_{ij}) P_{XY} \left( \sqrt{k^2 + k'^2}; \chi \right),
\]

where \( \delta_D(k) \) is the Dirac delta function. In equation (12), we assume that the projected field covariance originates predominantly from correlations in underlying three-dimensional fields at equal times of \( \chi = \chi' \) such that Fourier modes are perpendicular to the line-of-sight direction and that effects from the radial Fourier mode are negligible. This is a good approximation for weak lensing statistics (Vale & White 2003), including galaxy–galaxy lensing or stacked cluster lensing. To include the effects from the radial mode, formulations in this study are extendable, e.g., by incorporating a formula in Kitching & Heavens (2017).

In this study, we are particularly interested in the effects that global survey geometry has on the covariance of the projected power spectrum and not in the effects of masked regions at small scales. If we focus on the wavenumber modes, which satisfy \( k \gg S_W^{-1} \), we observe that the power spectrum estimator (equation (10)) is unbiased.

\[
\langle \hat{P}_{XY}(k) \rangle = \frac{1}{S_W} \int d^2k' S_k \hat{P}_{XY}^{2D}(k') \int d^2q (2\pi)^2 |\tilde{W}(q)|^2 \approx \hat{P}_{XY}^{2D}(k) = \frac{1}{S_W} \int d^2q (2\pi)^2 |\tilde{W}(q)|^2.
\]

Here, we assume that \( \hat{P}_{XY}^{2D}(k - q) = \hat{P}_{XY}^{2D}(k) \) over an integration range of \( q \), and we also assume that \( \hat{P}_{XY}^{2D}(k) \) is not a rapidly varying function within the \( k \)-bin. In the above equation, we have used \( S_W = \int |\tilde{W}(q)|^2 d^2q/(2\pi)^2 \) for the window function.

### 2.3 Covariance of the projected power spectrum

The covariance matrix of the projected power spectrum can be formally expressed as

\[
\text{Cov}[P_{XY}(k), P_{XY}(k')] = \begin{bmatrix} \hat{P}_{XY}(k) & \hat{P}_{XY}(k') \end{bmatrix} \begin{bmatrix} \hat{P}_{XY}(k) & \hat{P}_{XY}(k') \end{bmatrix} = \text{Cov}^{G}(k, k') + \text{Cov}^{cNG}(k, k') + \text{Cov}^{SSC}(k, k').
\]

The covariance matrix is broken down into three parts: the Gaussian (“G”) covariance, the connected non-Gaussian (“cNG”) covariance and the super-sample covariance (“SSC”). The Gaussian and cNG terms scale with the survey area \( (S_W) \) as \( S_W^{-1} \), but the SSC term does not follow this simple scaling.

Next, we derive the expression for the SSC term using the response function of the three-dimensional (3D) power spectrum to the large-scale density contrast along the line-of-sight direction. We express the radial integration in the projected power spectrum (equation (12)), using a discrete summation of the \( N_{ab} \) radial shells, as

\[
\hat{P}_{XY}^{2D}(k) = \sum_{i=1}^{N_{ab}} \Delta \chi F_X(\chi_i) F_Y(\chi_i) \hat{P}_{XY}(k; \chi_i),
\]

where \( \chi_i \) is the radial distance to the \( i \)-th redshift shell (e.g., the distance to the central redshift of the shell), \( \Delta \chi \) is the width of the shell, and we assume that all shells have the same width in the comoving length. For both the power spectrum and the radial weight, which are not a rapidly varying functions of redshift within the shell, a discrete summation is a good approximation. In the presence of super-survey modes in each shell, the power spectrum estimator modulates to:

\[
\hat{P}_{XY}^{2D}(k) = \sum_{i=1}^{N_{ab}} \Delta \chi F_X(\chi_i) F_Y(\chi_i) \hat{P}_{XY}(k; \delta_{bi}, \chi_i)
\]

\[
\approx \sum_{i=1}^{N_{ab}} \Delta \chi F_X(\chi_i) F_Y(\chi_i) \left[ \hat{P}_{XY}(k; \delta_{bi} = 0, \chi_i) + \frac{\partial P_{XY}(k; \delta_{bi}, \chi_i)}{\partial \delta_{bi}} \right].
\]

where \( \partial P_{XY}/\partial \delta_{bi} \) is the power spectrum response to large-scale density modes (Takada & Hu 2013; Li et al. 2014a), and \( \delta_{bi} \) is the super-survey mode of the \( i \)-th shell. For each particular survey realisation, each \( \delta_{bi} \) has a particular value. We calculate the SSC contribution to the projected power spectrum covariance with the following equation:

\[
\text{Cov}^{SSC} \left[ \hat{P}_{XY}^{2D}(k), \hat{P}_{XY}^{2D}(k') \right] = \sum_{i,j} (\Delta \chi)^2 F_X(\chi_i) F_X(\chi_j) F_Y(\chi_i) F_Y(\chi_j) \frac{\partial P_{XY}(k; \chi_i)}{\partial \delta_{bi}} \frac{\partial P_{XY}(k'; \chi_j)}{\partial \delta_{bj}} \langle \delta_{bi} \delta_{bj} \rangle
\]

\[
\approx \sum_{i=1}^{N_{ab}} (\Delta \chi)^2 F_X^2(\chi_i) F_Y^2(\chi_i) \frac{\partial P_{XY}(k; \chi_i)}{\partial \delta_{bi}} \frac{\partial P_{XY}(k'; \chi_i)}{\partial \delta_{bj}} \sigma_b^2(z_i),
\]

where we assume \( \langle \delta_{bi} \rangle = 0 \) and \( \langle \delta_{bi} \delta_{bj} \rangle = \sigma_b^2(z_i) \delta_{ij} \). For the latter condition, we assume that the super-survey modes between
Table 1. The \(N\)-body simulation parameters listed for the fiducial cosmological model (second row) and SU simulations (third and fourth rows). The first column denotes the local density contrast at \(z = 0\), \(\delta_b\), in the SU simulations \((\delta_b = 0\) for the fiducial model). The second to the ninth columns list the simulation box size \((L_W)\), the number of particles \((N_p)\), the Hubble parameter \((h_{\text{b}})\), the matter density parameter \((\Omega_{\text{m}})\), the cosmological constant \((\Omega_{\Lambda})\), the curvature parameter \((\Omega_{K})\), the softening length \((r_{\text{soft}})\), and the number of realisations \((N_t)\), respectively. The corresponding values in the second row are parameters for the global background simulations. We determined that the simulation box size, \(L_W\), is the same as that in the global background while converting it to a length scale in units of \([h^{-1}\text{Mpc}]\): i.e., \(L_W[h^{-1}\text{Mpc}] = L_{\text{W}}/h = 250[h^{-1}\text{Mpc}] = L[h^{-1}\text{Mpc}]\).

| \(\delta_b\) | \(L_W[h^{-1}\text{Mpc}]\) | \(N_p\) | \(h_{\text{b}}\) | \(\Omega_{\text{m}}\) | \(\Omega_{\Lambda}\) | \(\Omega_{K}\) | \(r_{\text{soft}}[h^{-1}\text{kpc}]\) | \(N_t\) |
|---|---|---|---|---|---|---|---|---|
| 0 | 250 | 512\(^3\) | 0.6727 | 0.3156 | 0.6848 | | 24.4 | 100 |
| +0.01 | 249.16 | 512\(^3\) | 0.67045 | 0.3177 | 0.6890 | -0.0067 | 24.3 | 100 |
| -0.01 | 250.83 | 512\(^3\) | 0.67494 | 0.3135 | 0.6799 | 0.0066 | 24.5 | 100 |

Different shells are uncorrelated with each other. In other words, we ignore radial large-scale modes that cause correlations between the different \(\delta_{bi}\). We will verify this assumption in Section 3.2.

When we appropriately consider redshift evolution within the SSC computation, we use Limber’s equation to derive a line-of-sight integration from the discrete summation formula above:

\[
\sum_{i=1}^{N_{\text{sh}}} (\Delta k)^2 F^2_X(k_i) F^2_Y(k_i) \frac{\partial P_X(k; \chi)}{\partial \delta_b} \frac{\partial P_Y(k'; \chi)}{\partial \delta_b} \sigma^2_g(z_i) = \int d\chi F^2_X(k) F^2_Y(k') \frac{\partial P_X(k; \chi)}{\partial \delta_b} \frac{\partial P_Y(k'; \chi)}{\partial \delta_b} \sigma^2_g(\chi) \Delta \chi. \tag{19}
\]

Following Takada & Spergel (2014, the discussion near equation (33) in their study), we find that:

\[
sigma^2_g(\chi) \Delta \chi = \left( \frac{\langle \delta^2 \rangle}{\langle \delta \rho \rangle} \right) \Delta \chi. \tag{20}
\]

Hence, we are able to recover the standard SSC term expression for the projected power spectrum (Sato et al. 2009; Takada & Hu 2013; Takada & Spergel 2014; Schaan et al. 2014). The SSC term depends solely on the survey window originating from \(\sigma^2_g\).

For a sufficiently wide survey area we can use the linear matter power spectrum or Gaussian simulations to accurately compute \(\sigma^2_g\) for a given survey window. We suggest that the SSC formulae are good approximations as far as the effects of the radial Fourier modes on the projected power spectrum are negligible. We will perform various simulations to validate the SSC formulae.

3 TESTING SSC OF PROJECTED POWER SPECTRUM WITH \(N\)-BODY SIMULATIONS

In this section, we compare the covariance matrices from our analytical formula with those from the \(N\)-body simulations. To do this, first, we numerically evaluate the power spectrum response to large-scale density contrasts using separate universe (SU) simulations (Section 3.1), and subsequently study SSC of projected power spectrum by using a sufficient number of \(N\)-body realisations (Section 3.2).

3.1 Power spectrum response

In this subsection, we numerically calculate the power-spectrum response to large-scale density contrasts using SU simulations. In this method, we absorb the large-scale density contrast, \(\delta_b\), into changes in cosmological parameters and subsequently study the effects on large-scale structures using \(N\)-body simulations under the changed cosmological background.

We employ the growth-dilation method in Li et al. (2014a) to implement the SU simulations. Hereafter, quantities with the subscript, \("W\", denote quantities in the SU simulation whose value is different from that in the global background. We
perform 100 SU simulations for each super-survey mode, \( \delta_b(z = 0) = \pm 0.01 \) (i.e., 100 paired simulations), where \( \delta_b \) evolves according to the spherical collapse dynamics. The simulation and cosmological parameters are summarised in Table 1. We used a fixed number of \( N \)-body particles, 512\(^3\), but we changed the box size for each SU simulation \( (\delta_b = \pm 0.01) \) so that the corresponding comoving lengths in the global background were identical: \( L_W (h_W^{-1} \text{Mpc}) = 250 \ (h^{-1} \text{Mpc}) \). We adopted an identical initial seed for each paired SU \( (\delta_b = \pm 0.01) \) simulation to reduce contamination from sample variance. We employed the second-order Lagrangian perturbation theory (2LPT; Crocce et al. 2006; Nishimichi et al. 2009) to compute initial displacements at a redshift of \( z = 49 \). We obtain the evolution of the scale factor, \( a_W(t) \), in the SU comoving frame as a function of the global scale factor by solving the spherical collapse dynamics in the initial condition generator. Here, we do not linearise the equation so that it can still compute large \( \delta_b \) values in the \( \Lambda \)CDM global cosmological model. The first- and second-order growth factors for the density perturbations are consistently computed in the SU background and subsequently are used to compute particle displacements. Outputs of simulation data are dumped at two redshifts, \( z_W \), for each SU simulation corresponding to \( z = 0.20 \) and 0.55 in the global background. We followed the gravitational evolution of particles using Gadget2 (Springel et al. 2001; Springel 2005), which we slightly modified to take into account the differences between local and global densities. The modified code reads a data table of the Hubble expansion rate, \( H_W(t) \), in the SU background, and uses it to compute the friction term in the force calculation.

We also identify halos in an \( N \)-body simulation output using the public code Rockstar (Behroozi et al. 2013), which we also slightly modified to make the resultant halo catalogues consistent among different frames. Density thresholds to define the halo masses and the Friends-of-Friends linking length are set to be the same when translated into the global frame. Throughout this study, we adopt \( M_{200m} \) for the definition of halo mass, where the spherical region defines each halo. Inside the spherical region, the inner mean mass density is 200 times overdense compared with the global background matter density. In the following, we consider two halo samples, which were selected by their sharp mass thresholds, \( M_{200m} > 10^{12} \) or \( > 10^{13} h^{-1} M_\odot \), respectively.

For each SU simulation realisation, we project the particle and halo distributions into two-dimensional \( xY \), \( yZ \) or \( zX \)-plane. Then, we compute the projected density fluctuation fields of matter or halo as

\[
\tilde{\delta}_{mW}^2(\mathbf{x}_W) = \frac{P_{mW}^2(\mathbf{x}_W)}{P_{mW}} - 1, \quad \tilde{\delta}_{hW}^2(\mathbf{x}_W) = \frac{P_{hW}^2(\mathbf{x}_W)}{P_{hW}} - 1, \tag{22}
\]

where \( \mathbf{x}_W \) is the two-dimensional vector (in the \( xY \), \( yZ \) or \( zX \)-plane). Then, we compute the Fourier components, \( \delta_{mW}^2(\mathbf{k}_W) \) and \( \delta_{hW}^2(\mathbf{k}_W) \), by performing a two-dimensional (2D) Fourier transform\(^9\) with 16384\(^2\) grids. The power spectrum estimator is defined as

\[
P_{XYW}(k) = \frac{1}{N_k} \sum_k \Re \left[ P_{XYW}^2(k) \tilde{\delta}_{XYW}^2(k) \right], \tag{23}
\]

which gives the matter power spectrum if the subscripts are \( X = Y = m \), the halo power spectrum if \( X = Y = h \), and the halo-matter cross-power spectrum if \( X = h \) and \( Y = m \). The summation is done over the circular annulus \( (k - \Delta k / 2, k + \Delta k / 2) \) with a bin-width of \( \Delta k \) in 2D Fourier space, and \( N_k \) is the number of modes in the \( k \)-bin, \( N_k = 2 \pi k \Delta k / (2 \pi / L)^2 \simeq k \Delta k S_W / (2 \pi) \) for \( k \gg 1 / L \). For the halo auto-power spectrum, we subtract the shot noise from the measured \( P_{hW}^2(k) \) in equation \( (23) \). We also calculate the 3D power spectrum estimator, \( P_{XYW}(k) \), following a similar procedure as described above.

We briefly mention a procedure to evaluate the 3D \( P(k) \) response function following the methods described in Li et al. (2014a). We use the 3D power spectrum measured in each SU simulation to infer the power spectrum in the global background via the relation

\[
P_XY(k) k^3 = (1 + \delta_b)^n P_{XYW}(kW) k_W^3, \tag{24}
\]

where \( k_W = (1 - \delta_b) / 3 \) \( k \) is a wavenumber measured in the SU simulation, and \( n = 2.1 \) and 0 when \( XY = mm, hh \) and \( hh \), respectively. The extra factor \( (1 + \delta_b)^n \) is necessary when converting matter density fluctuation fields, defined with respect to the SU local density, to that with respect to the global background density (de Putter et al. 2012): \( 1 + \delta_{\text{mglobal}} = (1 + \delta_b)(1 + \delta_{\text{SU}}) \). The different definition of the fluctuation fields for matter and halos is motivated by the following observables: weak lensing or cosmic shear arises from matter fluctuation fields with respect to the global mean density, while clustering statistics of large-scale structure tracers, such as galaxies and galaxy clusters, are measured with respect to the local mean density in a

\[9\] We used the public code FFTW3 (Fast Fourier Transform in the West) at http://www.fftw.org/.
Redshift  | \( M (h^{-1} M_{\odot}) \) | \( b_1 \) | \( b_1(T10) \) | \( b_2 \) | \( b_2(L16) \) | \( b_2(H17) \) \\
--- | --- | --- | --- | --- | --- | --- \\
0.55 | \( > 10^{12} \) | \( 1.30 \pm 0.02 \) | 1.22 | \(-0.42 \pm 0.09 \) | \(-0.72 \) | \(-0.61 \) \\
 | \( > 10^{13} \) | \( 2.00 \pm 0.05 \) | 1.97 | \( 0.15 \pm 0.44 \) | 0.13 | 0.14 \\
0.20 | \( > 10^{12} \) | \( 1.09 \pm 0.01 \) | 1.02 | \(-0.51 \pm 0.07 \) | \(-0.74 \) | \(-0.61 \) \\
 | \( > 10^{13} \) | \( 1.58 \pm 0.03 \) | 1.56 | \(-0.33 \pm 0.21 \) | \(-0.45 \) | \(-0.39 \)

Table 2. The first- and second-order halo bias parameters, \( b_1 \) and \( b_2 \), estimated from SU simulations for each halo sample with \( M > 10^{12} \) and \( > 10^{13} h^{-1} M_{\odot} \) at \( z = 0.20 \) and 0.55, respectively (see text for details). The mean, with error, is evaluated from 10 SU realisations.

survey region (Li et al. 2014a). Hence, we compute the \( P_{XY}(k) \) response function from equation (24) as:

\[
\frac{\mathrm{d} \ln P_{XY}(k)}{\mathrm{d} \ln b} = n - 1 + \frac{1}{2} \frac{\ln P_{XY}(k) |_{n}}{k}, \\
\]

\[
\approx n - 1 + \frac{\ln P_{XYW}(\delta(k_{W(n)})) - \ln P_{XYW}(-\delta(k_{W(-n)}))}{0.02 \times D(z)},
\]

where \( P_{XYW}(\delta) \) is measured in the SU simulations with \( \delta = \pm 0.01 \) at \( k_{W(n)} = (1 \pm \delta n)/3 \). The power spectrum has unit of \( h^{-3} \)Mpc\(^3\), not \( h_{\mathrm{Mpc}}^{-3} \). We numerically evaluate the derivative in the second line. Thus, the SU simulation technique provides a computationally inexpensive method to calibrate the power spectrum response. In principle, we only need a one-paired simulation to obtain the response if we use identical initial seeds to reduce the sample variance contamination (Li et al. 2014a). This also allows easy computation of the power spectrum response for different cosmological models. In this study, we estimate the mean response from 100-paired SU simulations for the fiducial model.  

Now we consider the projected power spectrum. As described above, the box size for the SU simulations are identical when converted to the global background. For example, for the \( \delta = \pm 0.01 \) case, \( L_{W} = 249.16 \times h_{\mathrm{Mpc}}^{-1} = 249.16 \times (h/h_{\odot}) h^{-1} \)Mpc = 250 \( h^{-1} \)Mpc = \( L \), found in Table 1. In other words, we determined the SU simulation box size to satisfy this condition. In this study, we consider the same projection thickness as that in the global background to compute the projected density fields. Since the 2D power spectrum, \( P_{XYW}^{2D}(k) \), relates to the corresponding 3D spectrum, \( P_{XYW}(k) \), via \( P_{XYW}^{2D} = P_{XY}/L \) and \( P_{XYW}^{2D} = P_{XY}/L \) ( \( L \) is the width of projection length or the box size), the same relation found in equation (24) holds for the 2D power spectrum:

\[
P_{XYW}^{2D}(k) \left( k^{3} = (1 + \delta b) n P_{XYW}(k_{W}) \right) k^{3}. \\
\]

We also obtain the \( P_{XYW}^{2D}(k) \) response in a manner that is identical to equation (25) by replacing \( P_{XY} \) with \( P_{XYW}^{2D} \). We calculate the mean response from 300-paired SU samples (≈ 3 projection directions × 100 realisations). Similarly, predictions from perturbation theory, for the 2D response, are obtained by replacing \( P_{XY} \) with \( P_{XYW}^{2D}(k) \). Note that the above formula is different from equation (16) in Takada & Jain (2009), where they employed Limber’s approximation, and subsequently performed the azimuthal angle average in the long mode direction to derive the response function. In this study, we ignore any effects that large-scale tidal fields may have on the covariance (Akitsu et al. 2017; Akitsu & Takada 2018; Barreira et al. 2017).

To validate the simulation results at large scales (small \( k \)), we compare them with perturbation theory predictions for the 3D fields (Baldauf et al. 2016):

\[
\frac{\mathrm{d} \ln P_{mm}(k)}{\mathrm{d} \ln b} = \frac{47}{21} - \frac{1}{3} \frac{\ln P_{m(m)}(k)}{\ln k}, \\
\frac{\mathrm{d} \ln P_{mn}(k)}{\mathrm{d} \ln b} = \frac{47}{21} + \frac{b_2}{b_1} - \frac{1}{3} \frac{\ln P_{m(m)}(k)}{\ln k}, \\
\frac{\mathrm{d} \ln P_{nn}(k)}{\mathrm{d} \ln b} = \frac{47}{21} + \frac{b_2}{b_1} - \frac{2}{3} \frac{\ln P_{m(m)}(k)}{\ln k},
\]

where \( b_1 \) and \( b_2 \) denote the first- and second-order halo biases. We also use the same SU simulations to estimate these bias parameters (Baldauf et al. 2016; Lazeyras et al. 2016; Li et al. 2016). For this, we use 10 paired realisations from extended SU simulations with larger \( \delta b \) values, i.e., \( \delta b(z) = 0 \pm 0.02, 0 \pm 0.04, 0 \pm 0.1, \) and \( 0 \pm 0.2 \) in addition to \( 0 \pm 0.1 \), to estimate the nonlinear \( b_2 \) parameter (Lazeyras et al. 2016; Paranjape & Padmanabhan 2017). The number of halos heavier than \( M \) found in the SU simulation box can be expanded in terms of \( \delta b \) with the following equation:

\[
N_b(> M, z; \delta b = 0) = N_b(> M, z; \delta b = 0) \left[ 1 + b_1^0(> M, z) \delta b + \frac{1}{2!} b_1^1(> M, z) \delta b^2 + \frac{1}{3!} b_1^2(> M, z) \delta b^3 + \cdots \right],
\]

10 How many paired SU simulations are necessary to evaluate the response functions depends on the simulation box size \( L \) because the measurement error scales as \( L^{-3/2} \). In our case of \( L = 250 h^{-1} \)Mpc, we use the 100-paired simulations to obtain accurate results. If we adopt larger box simulations such as \( L \geq 1 h^{-1} \)Gpc, a single pair will be sufficient.
Figure 1. Fractional response functions of power spectra to the large-scale density contrast \( \delta_b \), \( \frac{\mathrm{d} \ln P_X(k)}{\mathrm{d} \ln \delta_b} \), for the matter-matter auto-spectrum, matter-halo cross-spectrum and halo-halo auto-spectrum from panels on the left to the right at \( z = 0.55 \) (in the upper row) and \( z = 0.20 \) (in the lower row), respectively. In this fractional form, we expect that the response functions, for the three-dimensional and projected power spectra, are identical: \( \frac{\mathrm{d} \ln P_X(k)}{\mathrm{d} \ln \delta_b} = \frac{\mathrm{d} \ln P_X^{2D}(k)}{\mathrm{d} \ln \delta_b} \) (see text for details). The large dark-coloured symbols in each panel display the response functions of projected power spectra estimated from the SU simulations, while the corresponding small light-coloured symbols are those for the 3D power spectra. The plus and cross symbols in the middle and right panels are results for halos with masses \( M > 10^{12} h^{-1} M_\odot \) and \( M > 10^{13} h^{-1} M_\odot \), respectively. The solid red curves in each panel are predictions from the perturbation theory (equation (27)), for which we take into account the \( k \)-binning used in the simulations and use bias parameters, \( b_1 \) and \( b_2 \), estimated from halo abundance in the SU simulations (see text for details).

where the coefficients are the Lagrangian biases. Here, we fit these bias parameters up to the third order. The Eulerian biases are \( b_1 = b_1^{(1)} + 1 \) and \( b_2 = b_2^{(1)} + (8/21)b_1^{(1)} \) (Cooray & Sheth 2002; Baldauf et al. 2016). Table 2 lists the measured halo biases, \( b_1 \) and \( b_2 \), from the 10 realisations, compared with results from previous studies.\(^{11}\) Our results for \( b_1 \) are consistent with those of Tinker et al. (2010) within 10%, while our results for \( b_2 \) which are noisy, agree well with those of Lazeyras et al. (2016) and Hoffmann et al. (2017) within \( |\Delta b_2| = 0.2 \). Hoffmann et al. (2017) has better agreement but predicts slightly smaller values than those in this study. The \( b_2 \) for halos with \( M > 10^{12} h^{-1} M_\odot \) at \( z = 0.55 \) exhibits a sizable difference from previous studies, but the reason for this is beyond the scope of this study.

Figure 1 shows the response functions for matter-matter (left panel), halo-matter (middle), and halo-halo (right) spectra. First, response functions have large amplitudes, with an order of matter auto-, matter-halo cross- and halo auto-spectra. Large amplitudes for matter spectra compared with the halo spectra are due to the use of the global mean mass density in the definition of the mass density fluctuation field (the term of \( n - 1 \) in equation (25)). The large dark-coloured symbols in each panel are the response functions for the 2D power spectra, while the corresponding small light-coloured symbols are the response functions for the 3D spectra. The comparison clearly shows that response functions for the 2D and 3D spectra agree with each other or are identical to within scatter in both the linear and nonlinear regimes, validating the arguments near equation (26). Simulation results are consistent with theoretical predictions in the linear regime. Note that,

\(^{11}\) Based on previous studies, the mean halo biases for halos heavier than \( M \) were calculated as follows:

\[
b_1(M, z) = \frac{1}{n_h(M, z)} \int_{M}^{\infty} \frac{\mathrm{d} M'}{\mathrm{d} M} (M', z) b_1(M', z), \quad b_2(M, z) = \frac{1}{n_h(M, z)} \int_{M}^{\infty} \frac{\mathrm{d} M'}{\mathrm{d} M} (M', z) b_2(b_1(M', z)),
\]

where \( n_h(M, z) = \int_{M}^{\infty} \mathrm{d} M' [\ln(1 + \delta_M)] (M', z) \) is the cumulative halo number density. We used the fitting functions from Tinker et al. (2008, 2010) for the halo mass function and linear halo bias as well as functions from Lazeyras et al. (2016) and Hoffmann et al. (2017) for the second-order halo bias, \( b_2 = b_2(b_1) \).
projection length of a flat plane perpendicular to the line-of-sight direction for a hypothetical observer. For simplicity, we ignore redshift evolution; we use the small and large boxes, respectively. We divide each large-box simulation into small-box and subvolume realisations. In all of the below cases (i)-(iii), we use a survey area of \( 2048^2 \) (\( h^{-1} \text{Mpc} \))^2, where spatial resolution remains the same. Each column provides the simulation box size (\( L \)), the number of particles (\( N_p \)), and the number of realisations (\( N_r \)), respectively. We use simulations at the two redshifts, \( z = 0.20 \) and 0.55.

| Name       | SSC term (equation) | Projection length (\( h^{-1} \text{Mpc} \)) | Perpendicular & parallel super-survey modes | Characteristics                                                                 |
|------------|---------------------|---------------------------------------------|---------------------------------------------|------------------------------------------------------------------------------|
| Case (i)   | (33)                | 1000                                        | Included & No                               | Use \( 250^2 \times 1000 \) (\( h^{-1} \text{Mpc} \))^2 rectangular box for the projection. |
| Case (iia) | (34)                | \( 4 \times 250 \)                         | Included & No                               | Use the same rectangular box in Case (i), but divide it into four cubic boxes before projection. |
| Case (iib) | (34)                | \( 4 \times 250 \)                         | Included & Included                         | Similar to Case (iia), but use four cubic boxes randomly taken from different large-box realisations. |
| Case (iii) | No                  | \( 4 \times 250 \)                         | No & No                                     | Similar to Case (iia,b), but use four cubic small-boxes with periodic boundary conditions. |

Figure 2. An illustration of the configuration of the \( N \)-body simulation boxes used for hypothetical experiments of projected power spectrum measurements. We consider that \( \Delta \chi = 250 \ h^{-1} \text{Mpc} \) for a subvolume thickness used to define the projected field in each plane, \( L_l = 1000 \ h^{-1} \text{Mpc} \) for the total projection thickness and \( S_W = 250^2 \) (\( h^{-1} \text{Mpc} \))^2 for the square-shaped survey area in the two-dimensional flat plane perpendicular to the line-of-sight direction for a hypothetical observer. For simplicity, we ignore redshift evolution; we use the \( N \)-body data at the same redshift, \( z = 0.20 \) or 0.55.

Table 3. \( N \)-body simulation parameters used to test and validate the covariance matrix of projected power spectra for matter-matter, matter-halo and halo-halo spectra. To do this, we use two different box-size \( N \)-body simulations, i.e., small-box and large-box simulations, whereas spatial resolution remains the same. Each column provides the simulation box size (\( L \)), the number of particles (\( N_p \)), and the number of realisations (\( N_r \)), respectively. We use simulations at the two redshifts, \( z = 0.20 \) and 0.55.

Table 4. A summary of our numerical experiments in Section 3.2. All the cases use the simulations, each of which has a total volume of \( 250^2 \times 1000 \) (\( h^{-1} \text{Mpc} \))^2, to generate the projected fields, where \( 250^2 \) (\( h^{-1} \text{Mpc} \))^2 is the projected area (see Fig. 2).

for perturbation theory predictions, we take into account \( k \)-binning used in the simulation results; we smooth out perturbation theory predictions within a given \( k \)-bin, which smears baryon acoustic oscillation (BAO) features and provides better matches to the simulation results. The measurement errors of the \( P(k) \) responses among the 100 realisations are smaller for \( P_{\text{mm}} \) than for \( P_{\text{hm}} \) than for \( P_{\text{hh}} \).

3.2 Covariances of the power spectra

In this section, we analyse the covariance matrices of projected power spectra using a sufficient number of \( N \)-body simulation realisations. We ran the following two types of simulations for the fiducial cosmological model (as given in the first line of Table 1): simulations with small boxes, side length \( L = 250 \ h^{-1} \text{Mpc} \) with \( 512^3 \) particles, and those with large boxes, \( L = 1000 \ h^{-1} \text{Mpc} \) with \( 2048^3 \) particles, summarised in Table 3. The number of realisations performed is 100 and 20 for the small and large boxes, respectively. We divide each large-box simulation into \( 4^3 = 64 \) cubic subvolumes with side lengths of \( 250 \ h^{-1} \text{Mpc} \), with 1280 subvolumes in total. To study the impact of super-survey modes on the projected power spectra, we perform the following numerical experiments using simulations constructed from different combinations of the large-box, small-box and subvolume realisations. In all of the below Cases (i)-(iii), we use a survey area of \( S_W = 250^2 \) (\( h^{-1} \text{Mpc} \))^2 and a projection length of \( L_l = 1000 \ h^{-1} \text{Mpc} \) (see also the simulation box configuration in Fig. 2).
Figure 3. The square root of the variance of projected power spectra at each $k$ bin, for matter-matter, matter-halo, and halo-halo from the left to right panels, respectively. The variances are estimated from hypothetical measurements in the light-cone realisations of $N$-body simulations that have total volumes of $250^3 \times 1000 (h^{-1}\text{Mpc})^3$, as illustrated in Fig. 2. By doing this, we consider the following four different cases that all perform the light-cone realisation: Case (i) is a projection of the whole rectangular volume, and Cases (iia, b)-(iii) are from superpositions of the four small-boxes, each of which has a volume of $250^3 (h^{-1}\text{Mpc})^3$. Case (iia) has contributions from super-survey modes in the perpendicular direction, while Case (iib) has super-survey modes in both parallel and perpendicular directions. Case (iii) has small boxes with periodic boundary conditions (i.e., no super-survey mode effects). We show all the results relative to expectations from the Gaussian variance for Case (i). We expect that the Gaussian variance for Cases (iia, b) and (iii) are smaller than that for Case (i) by a factor of 2, as denoted by the two horizontal dashed lines. The solid red curves in each panel are the results obtained by adding the SSC contribution to Case (iii) (the circle symbols), which efficiently reproduces the results of Cases (iia, b). The upper (lower) panels show results for halos with $M > 10^{12} (10^{13}) h^{-1} M_{\odot}$ at $z = 0.55 (0.20)$. The black arrows, in the right panels, indicate scales where $P_{\text{halo}}^2(k)$ equals the shot noise. The $k$-bin width, which the Gaussian error expectation depends on, is $\Delta \log k = 0.2$.

- **Case (i)** We use a rectangular-shaped subvolume of $250^3 \times 1000 (h^{-1}\text{Mpc})^3$ cut out of a large-box realisation to create projected matter and halo fields with square-shaped areas of $250^2 (h^{-1}\text{Mpc})^2$. We use a direction length of $1000 h^{-1}\text{Mpc}$ for the projection (line-of-sight) direction. Then, we compute the projected power spectra from the two-dimensional data by FFT. The power spectra measured in this manner include the effects of super-survey modes that are perpendicular to the line-of-sight direction. Note that, in this case, the density fluctuation field is continuous and obeys periodic boundary conditions in the line-of-sight direction. Since we generate the projected field from the $N$-body simulations, at the same redshift output, the density fields have no redshift (line-of-sight) evolution in a statistical sense, and the power spectrum of the projected fields is simply

$$P_{XY}^2(k) = \frac{1}{L_\parallel} P_{XY}(k),$$

where $L_\parallel = 1000 (h^{-1}\text{Mpc})$ is the projection width and $P_{XY}(k)$ is the 3D power spectrum. This is found by setting $F_X(\chi) = F_Y(\chi) = 1/L_\parallel$ in equation (12).

- **Case (iia)** The same rectangular-shaped volumes as those in Case (i) are used. In this case, however, we first divide each realisation into four cubic subvolumes of $250^3 (h^{-1}\text{Mpc})^3$ each and then project the field along the line-of-sight direction in each subvolume. Then, we estimate the projected power spectrum in each subvolume and finally average the projected power spectra to estimate the projected spectrum for the original rectangular volume, as shown in Fig. 2. Similarly, by setting $F_X(\chi) = F_Y(\chi) = 1/L_\parallel$ in equation (12), we can express the projected power spectrum estimator using a discrete summation.
as

\[ \hat{P}_{XY}^{2D}(k) = \frac{\Delta k^2}{(L/\Delta k)^2} \sum_{i=1}^{N_{sub}} \hat{P}_{XY(i)}^{2D}(k) = \frac{\Delta k^2}{(L/\Delta k)^2} N_{sub} \sum_{i=1}^{N_{sub}} \hat{P}_{XY(i)}^{2D}(k; \delta_{bi}). \]  

(30)

where \( \Delta k = 250 \, h^{-1} \text{Mpc} \) is the width of each subvolume, and \( \hat{P}_{XY(i)}^{2D}(k) \) is the projected power spectrum estimator in the \( i \)-th subvolume. The number of subvolumes is \( N_{sub} = L/\Delta k = 4 \). As mentioned previously, the local mean density of halos in the survey area is used to estimate the halo power spectra (\( \hat{P}_{XY(i)}^{2D}(k) \) and \( \hat{P}_{X(i)}^{2D}(k) \)). In the above equation, we explicitly denote that the power spectrum, in each subvolume, has a modulation due to the super-survey mode, \( \delta_{bi} \). Similarly, the ensemble average of the above estimator is identical to Case (i):

\[ \left\langle \hat{P}_{XY}^{2D}(k) \right\rangle = \frac{\Delta k^2}{(L/\Delta k)^2} \sum_{i=1}^{N_{sub}} \left\langle \hat{P}_{XY(i)}^{2D}(k) \right\rangle = \frac{\Delta k^2}{(L/\Delta k)^2} N_{sub} \hat{P}_{XY}(k) = \frac{1}{N_{sub}} P_{XY}(k). \]  

(31)

where we used \( \left\langle \hat{P}_{XY(i)}^{2D}(k) \right\rangle = P_{XY}(k)/\Delta k \). Note that this case preserves a continuous radial mode in each realisation.

- **Case (iiib)** This case is similar to Case (iia), but we randomly choose four cubic subvolumes of \( 250^3 \, (h^{-1} \text{Mpc})^3 \) each, which are taken from the different large-box simulations to construct each rectangular-shaped realisation of \( 250^2 \times 1000 \, (h^{-1} \text{Mpc})^3 \). These are placed into the four subvolumes along the line-of-sight direction (as illustrated in Fig. 2). Then, we estimate the projected power spectrum in the same manner as Case (iia). This case includes super-survey modes over subvolumes in both perpendicular and parallel directions to the line-of-sight direction, and the density fluctuations are discontinuous between different subvolumes. Comparing the results of Cases (i), (iia) and (iiib) reveals a radial super-survey mode effect.

- **Case (iii)** This is similar to Cases (iia,b), but we use four small-box realisations, each of which has a volume of \( 250^3 \, (h^{-1} \text{Mpc})^3 \), obeying the periodic boundary conditions, to construct rectangular-shaped realisations of \( 250^2 \times 1000 \, (h^{-1} \text{Mpc})^3 \). This case does not include super-survey modes that are both perpendicular and parallel to the line-of-sight direction (no modes beyond the small-box size).

We briefly summarise the characteristics of the above cases in Table 4. Below, we use 960 realisations (=4² patches × 3 projections × 20 large-box realisations) for Cases (i)-(iia,b), where we use \( 4^2 = [1000 \, h^{-1} \text{Mpc}]/(250 \, h^{-1} \text{Mpc})^2 \) patches for each \( x, y \) or \( z \)-direction projection, respectively. For Case (iii) we use 75 realisations (=100/4 subsets of 100 small-box realisations × 3 projections). Here, we show results at \( z = 0.20 \) and 0.55. As described above, the ensemble average of the projected power spectrum estimator should be identical for all Cases (i)-(iii) (or designed to satisfy this condition). Then, we use these realisations to estimate the covariance matrices of projected power spectra for all these cases.

We discuss theoretical expectations for the covariance matrices of Cases (i)-(iii). First, we consider the Gaussian covariance contribution. Since we estimate the projected power spectrum from four cubic subvolumes for Cases (ii)-(iii), we obtain a different Gaussian covariance contribution compared with Case (i), which estimates the projected power spectrum for the whole rectangular volume. From equation (30), we can derive the Gaussian covariance matrix for Cases (ii)-(iii):

\[ \text{Cov}_{XY(i)}^{G}(k, k') = \frac{\Delta k^4}{(L/\Delta k)^2} \frac{1}{N_{mode}(k)} \sum_{i=1}^{N_{sub}} \left[ \hat{P}_{XY(i)}^{2D}(k) + \hat{P}_{XY(i)}^{2D}(k') \right] \delta_{kk'} \]

\[ = \frac{\Delta k^2}{(L/\Delta k)^2} \frac{N_{sub}}{N_{mode}(k)} \left[ \hat{P}_{XY}^{2D}(k) + \hat{P}_{XY}^{2D}(k') \right] \delta_{kk'} \]

\[ = \frac{1}{N_{sub}} \text{Cov}_{XY(i)}^{G}(k, k'), \]

(32)

where we used \( \hat{P}_{XY(i)}^{2D}(k) = (L/\Delta k) \hat{P}_{XY}^{2D}(k) = 4 \hat{P}_{XY}^{2D}(k) \) (because \( N_{sub} = L/\Delta k = 4 \)) and we assumed that the density fields, in different slices (i.e., subvolumes), are independent of each other (see Fig. 2). \( N_{mode}(k) \) is the number of modes used to estimate the projected power spectrum at the \( k \)-bin, \( N_{mode}(k) = k_0 \Delta k S_0^2/(2\pi) \) for \( k \gg S_0^{-1} \). In the last line, \( \text{Cov}_{XY(i)}^{G} \) is the Gaussian covariance for Case (i), showing that the Gaussian covariance for Cases (ii)–(iii) is smaller in amplitude than that of Case (i) by a factor of \( N_{sub} = 4 \).

Next, we consider SSC contribution that affects the results for Cases (i)–(ii). By inserting \( F_X(k) = F_Y(k) = 1/L_{\parallel} \) into equation (18), the SSC contribution for Case (i) is

\[ \text{Cov}_{XY(i)}^{SSC}(k, k') = \frac{\partial \hat{P}_{XY}^{2D}(k)}{\partial b_i} \frac{\partial \hat{P}_{XY}^{2D}(k')}{\partial b_j} \sigma_b^2(V_{tot}). \]

(33)

where \( V_{tot} = 250^2 \times 1000 \, (h^{-1} \text{Mpc})^3 \), and we use the fact that the density field does not evolve along the projection direction.
Similarly, we solve for the SSC covariance for Cases (iia) and (iib):

\[
\text{Cov}_{XY(i)(j)}^{\text{SSC}}(k,k') = (\Delta k)^2 \sum_{i=1}^{N_{\text{sub}}} \frac{\partial P_{XY(i)}^{2D}(k)}{\partial \delta_b} \frac{\partial P_{XY(j)}^{2D}(k')}{\partial \delta_b} \sigma_b^2(\delta_{\text{sub}}) \\
= (\Delta k)^2 \sum_{i=1}^{N_{\text{sub}}} \frac{\partial P_{XY(i)}^{2D}(k)}{\partial \delta_b} \frac{\partial P_{XY(j)}^{2D}(k')}{\partial \delta_b} \sigma_b^2(\delta_{\text{sub}}) \\
= \frac{\partial P_{XY(i)}^{2D}(k)}{\partial \delta_b} \frac{\partial P_{XY(j)}^{2D}(k')}{\partial \delta_b} \frac{1}{N_{\text{sub}}} \sigma_b^2(\delta_{\text{sub}}) \\
= \text{Cov}_{XY(i)(j)}^{\text{SSC}}(k,k').
\]

(34)

where \( \delta_{\text{sub}} = 250^3 \, (h^{-1}\text{Mpc})^3 \), and the last equality originates from the following equality for large-scale density variance:

\[
\sigma_b^2(\delta_{\text{tot}}) = \frac{1}{N_{\text{sub}}} \sigma_b^2(\delta_{\text{sub}}).
\]

(35)

The above relation was checked for validity, by comparing the variances of rectangular volume with those of subvolume taken from the large- and small-box simulations. We expect that SSC covariance is similar to Cases (i)–(ii), which we will verify below with simulations.

Fig. 3 displays the square root of the variance, i.e., the diagonal covariance elements for the projected power spectra of matter-matter, matter-halo and halo-halo. For illustrative purposes, we show the variance relative to the Gaussian error of Case (i). The triangle, plus, cross and circle symbols, in each panel, show the variances measured from simulations for Cases (i), (iia), (iib), and (iii), respectively. All results are near unity or 0.5 at \( k \leq 0.2 \, h/\text{Mpc} \), i.e., consistent with Gaussian error expectations, where the Gaussian error for Cases (iia,b) and (iii) should be smaller than that of Case (i) by a factor of 2, as shown by equation (32). On the other hand, non-Gaussian contributions greatly exceed Gaussian error for the matter auto-spectrum and the halo-matter cross-spectra in the nonlinear regime of \( k \geq 0.2 \, h/\text{Mpc} \), while non-Gaussian error for the halo auto-spectra does not appear to be significant because shot noise is dominant at such small scales. Comparing the results for Cases (iia,b) with Case (iii), we observe that SSC provides a dominant contribution to non-Gaussian errors, while results that deviate from 0.5 for Case (iii) show the contribution of connected non-Gaussian error from equation (15) because Case (iii) uses simulations with periodic boundary conditions (i.e., there are no super-survey modes beyond the area in the projected plane). In fact, the solid red curve in each panel shows results obtained by summing the variance for Case (iii) and the SSC term (equation (34)), where we used response functions measured from SU simulations (in Section 3.1), and directly estimated \( \sigma_b \) from rectangular realisations (from variations of \( N \)-body particles in the realisations). The solid curves agree remarkably with results for Case (iia,b) over the wide range of scales as well as results for Case (i) in the nonlinear regime. From these results, we observe no strong evidence of large-scale tidal field effects, which we ignored in the SSC calibration method. This does not seem consistent with results from Barreira et al. (2017), who claimed that large-scale tidal fields cause a sizable, additional contribution to the SSC term in cosmic shear covariance (which corresponds to the results for matter-matter covariance in Fig. 3). Large SSC contributions for matter spectra relative to matter-halo cross-spectra or for matter-halo spectra relative to halo auto-spectra are due to large amplitudes of response functions, as shown in Fig. 1. Furthermore, results for Case (iib) do not show any sizable deviation from Case (iia) (or Case (i) in the nonlinear regime), indicating that the large-scale mode parallel to the projection direction has a negligible impact on the covariance of projected power spectra.

Fig. 4 displays results that are similar to Fig. 3; however, these results are for off-diagonal covariance components. This figure confirms the results shown in Fig. 3; the non-Gaussian and SSC contributions are significant in the nonlinear regime, especially for the matter-matter or matter-halo spectra. The results (circle symbols) for Case (iii) (periodic boundary condition) show a deviation from Cases (i) and (iia,b) at \( k \leq 0.1 \, h/\text{Mpc} \) in the linear regime due to the effect of window function; the cutout of \( 250^2 \, (h^{-1}\text{Mpc})^2 \) patches from larger-area simulation in Cases (i) and (iia,b) modifies the projected power spectra at small \( k \)-bins.

4 COMPARISON WITH RAY-TRACING SIMULATION RESULTS

So far, we have studied the response functions and SSC contributions for matter-matter, matter-halo and halo-halo projected power spectra using hypothetical simulations. In this section, we develop a method to calibrate covariances for cosmic shear and galaxy–galaxy lensing by combining the model ingredients that we have computed. We consider the covariances of their correlation functions, instead of the power spectra, because the correlation functions are more commonly used in actual measurements. We first derive the response functions of the correlation functions to \( \delta_b \), and subsequently test the method by comparing the covariances estimated based on this method, with those estimated using light-cone ray-tracing simulations of weak lensing and halo fields.
Fourier transforms of the power spectra give their respective correlation functions for the two- and three-dimensional fields:

\[ \xi^{2D}_{XY}(r) = \frac{1}{2\pi} \int dk \, J_0(kr) P^{2D}_{XY}(k), \]

\[ \xi^{3D}_{XY}(r) = \frac{1}{2\pi^2} \int dk \, k^2 \frac{\sin(kr)}{kr} P_{XY}(k), \]  

where \( J_0 \) is the zeroth-order Bessel function. We note that the 2D and 3D power spectra follow a simple scaling relation, i.e., 
\[ P^{2D}_{XY}(k) = P^{3D}_{XY}(k)/L \]  (where \( L \) is the projection length), but the correlation functions do not obey this relation, \( \xi^{2D}_{XY}(r) \neq \xi^{3D}_{XY}(r)/L \) from equation (36). Instead, \( \xi^{2D}_{XY}(r) \) simply scales as \( \xi^{3D}_{XY}(r) \propto L^{-1} \). As we showed, the perturbation theory gives a validation of the response functions at least in the linear regime, so it is useful to compare with the simulation results. Performing the 2D Fourier transformation of the perturbation theory response of \( P_{XY}(k) \) (equation (27)) yields expressions for the response function of \( \xi^{2D}_{XY}(r) \) to \( \delta_b \):

\[ \frac{d\xi^{2D}_{mm}(r)}{d\delta_b} = \frac{61}{21} \xi^{2D}_{m,\text{lin}}(r) + \frac{1}{3} \frac{d\xi^{2D}_{m,\text{lin}}(r)}{d\ln r}, \]

\[ \frac{d\xi^{2D}_{m\delta}(r)}{d\delta_b} = \frac{61}{21} \frac{\delta}{b_1} b_1 - b_1 \xi^{2D}_{m,\text{lin}}(r) + \frac{b_1}{3} \frac{d\xi^{2D}_{m,\text{lin}}(r)}{d\ln r}, \]

\[ \frac{d\xi^{2D}_{\delta\delta}(r)}{d\delta_b} = \left( \frac{61}{21} + 2 \frac{\delta}{b_1} - 2 b_1 \right) \xi^{2D}_{m,\text{lin}}(r) + \frac{b_1^2}{3} \frac{d\xi^{2D}_{m,\text{lin}}(r)}{d\ln r}. \]  

**Figure 4.** Similar to Fig. 3. However, here we show the off-diagonal elements of the covariance matrices. For illustrative purposes, we display the \( \text{Cov}[P^{2D}_{XY}(k_1), P^{2D}_{XY}(k_2)]/[P^{2D}_{XY}(k_1)P^{2D}_{XY}(k_2)] \) in units of \( 10^{-3} \) for a fixed \( k_2 \) value with varying \( k_1 \) along the x-axis. The vertical dashed line in each panel denotes the chosen \( k_2 \) value. The upper two-row panels show results at \( z = 0.20 \). Panels labelled as M13 and M12 are for halo samples with \( M > 10^{13} \) and \( > 10^{12} h^{-1} M_{\odot} \), respectively. The solid red curve in each panel shows the Ss response added to Case (iii) (from small boxes with the periodic-boundary condition).
Figure 5. Responses of the projected correlation functions to large-scale density, $\delta_b$, for matter-matter and halo-matter at $z = 0.55$ (upper panel) and $z = 0.20$ (lower), respectively. Here, $r \times d\xi_{\rm XY}^{\bf 2D}(r)/d\delta_b$ is plotted for illustrative purposes. For the matter-halo response, we show the results for halos with $M > 10^{13} h^{-1} M_\odot$ in the middle and right panels, respectively. The circle, cross and plus symbols are results obtained from 2D Fourier transformations of the 2D power spectrum responses estimated from the SU simulations in Fig. 1. For comparison, the solid-red curve show the perturbation theory predictions (equation (37)) where we properly employ the minimum wavenumber $k_{\min} = 2\pi/L$ with $L = 250 h^{-1}\text{Mpc}$, which is the box size of SU simulations, for the $k$-integration (labelled as “finite box”). The dashed-orange curves are the results for $k_{\min} = 0$.

where $\xi_{\rm m,lin}^{\bf 2D}(r)$ is the linear projected correlation function of matter (obtained by plugging the linear matter power spectrum $P_{m,\text{lin}}(k)$ into equation (36)). The above equation is similar to the response function for the 3D correlation function from Sherwin & Zaldarriaga (2012) (see also Baldauf et al. 2016), except for numeric coefficients in the first term, i.e. 68/21 for the 3D correlation function instead of 61/21.

Fig. 5 displays the $\xi_{\rm XY}^{\bf 2D}(r)$ responses obtained by performing 2D Fourier transformations of the $P_{\chi \chi}^{\bf 2D}(k)$ response functions in Fig. 1. The solid-red and dashed-orange curves are the perturbation theory predictions (equation (37)), which differ in the minimum wavenumber $k_{\min}$ in the integration when computing the Fourier transform. For the solid-red curves, we employed $k_{\min} = 2\pi/L$, where $L = 250 h^{-1}\text{Mpc}$ is the box size of SU simulations, while we set $k_{\min} = 0$ for the dashed-orange curves. Here we take into account a smoothing of the theoretical prediction due to the $r$-binning ($\Delta \log r = 0.2$ in the figure). Clearly, the solid curves give a better match to the simulation results at large separations. Since the response function of halo-halo power spectrum is noisy, owing to shot noise contamination, we did not find a reliable $k$-integration results; thus, it is not shown here. This agreement is not possible if we employ 68/21 instead of 61/21 for the coefficient of the first term in the perturbation theory prediction. This is also confirmed by Fig. 6, which shows responses for the 3D correlation functions. Here, we performed 3D Fourier transforms of the 3D $P_{\chi \chi}(k)$ response functions from Fig. 1. Perturbation theory predictions, with the coefficient 68/21, agree well with SU simulation results at large separations.

4.2 SSC calibration for cosmic shear correlation functions

In this section, we consider an application of our method to calibrate the covariance matrix of cosmic shear correlation function. Cosmic shear or cosmological weak lensing is the distortion effect on the shapes of distant galaxies, caused by intervening mass distribution in large-scale structures. The lensing convergence field characterises the cosmic shear effect, i.e., the weighted mass distribution integrated along the line-of-sight direction (e.g., Bartelmann & Schneider 2001). The convergence field in an angular direction $\theta$ is expressed based on the form of equation (7):

$$\kappa(\theta; z_\alpha) = \int_0^{r_\chi} d\chi f_\chi(\chi) \delta_m(\chi, \theta).$$

(38)
Variance of the large-scale density contrast in the shell-volume along the line-of-sight direction, Figure 7. Similar to Fig. 6.

Figure 6. Similar to Fig. 5, but showing the responses of the 3D correlation functions. Comparing this with the previous figure indicates that the amplitude of the 2D response is smaller than that of the 3D response by \( \xi^2M(r) = L^{-1} \), where \( L \) is the projection width.

Figure 7. Variance of the large-scale density contrast in the shell-volume along the line-of-sight direction, \( \sigma_b^2(z_i) \), for a fixed survey area, computed from the realisations of light-cone simulations (see text for details). Here, the shell-volume element at \( z_i \) is given by \( dV(z_i) = S_W \chi_i^2 \Delta \chi \), where \( \chi_i \) is the radial distance to the \( i \)-th redshift and \( \Delta \chi = 150 h^{-1} \text{Mpc} \) is the radial thickness fixed for all shells. The different symbols show the results for \( S_W = 54, 215 \) and 859 sq. degrees, respectively. We plot \( S_W \times \sigma_b^2(z_i) \) so that the results for different areas have similar amplitudes. There are 16 lens shells from \( z = 0 \) to 1.

by choosing:

\[
    f_e(\chi) = \frac{3H_0^2 \Omega_m}{2 - a^{-2}(\chi)X} \left( 1 - \frac{\chi}{\chi_s} \right)
\]

for \( f_m(\chi) \) (we use the notational convention, \( f_e \), instead of \( f_m \)). Here, \( \chi_s \) is the distance to source galaxies, and we consider a single source redshift (\( \chi_s = 1.033 \)) for simplicity. It is straightforward to include the source redshift distribution in the following
Covariances for the projected correlations in the response approach

Figure 8. Left panel: The square root of the variance of cosmic shear two-point correlation function, i.e., the diagonal components of the covariance matrix. Here, source galaxies are at a single source redshift, $z_s = 1.033$. The plus, cross and circle symbols are the results estimated from numerous realisations of a hypothetical cosmic shear survey, which were constructed from full-sky ray-tracing simulations (see text for details). The different symbols vary in the assumed survey areas as indicated, and the survey geometry for each realisation is defined based on the Healpix subdivisions of the sky (see text for details). Simulation results include SSC contributions. We plot the results for $\Delta S^2 \times S^{1/2}$, where $\Delta S^2 \equiv \text{Cov}^{1/2}$ and $S_W$ is the survey area, such that all results appear to have similar amplitudes. The dashed red curves show the Gaussian error predictions (equation (43)). The solid curves show results obtained by adding the SSC contribution to the dashed Gaussian for each survey area, where we used $\sigma^2_s(z_s)$ from Fig. 7 and equation (42). The SSC results, even though inexpensively computed, exhibit excellent agreement with the simulation results. Middle and right panels: Similar results but for the off-diagonal components of the covariance matrices, $\text{Cov}[\xi_{ss}^x(\theta_1), \xi_{ss}^x(\theta_2)]$, for a given $\theta_2$ (as indicated) with varying $\theta_1$ in the x-axis. We plot the results multiplied by $\theta_1 \theta_2 \times S_W$ for illustrative purposes. The SSC calibration method displays excellent agreement with the simulation results.

discussion. Using Limber’s approximation, the angular power spectrum of the cosmic shear field is expressed as

$$C^x_{\ell}(z_s) = \int_0^\chi f^2_k(\chi) P_{\ell mm}(k = \chi z_s).$$

where $\ell$ is an angular multipole. Since the angular correlation function of the cosmic shear field is given by the 2D Fourier transform of $C^x_{\ell}$, under the flat-sky approximation, we can express its form via a discrete summation of thin-shell integration, similarly to equation (16):

$$\xi^x_{2D}(\theta; z_s) = \int \frac{dk}{2\pi} C^x_{\ell}(z_s) J_0(k \chi \theta),$$

$$= \sum_{i=1}^{N_\ell} f_k(\chi) \Delta \chi J_0(k \chi \theta),$$

$$\approx \sum_{i=1}^{N_\ell} f_k(\chi) \Delta \chi J_0(k \chi \theta),$$

$$= \sum_{i=1}^{N_\ell} f_k(\chi) \Delta \chi J_0(k \chi \theta),$$

in which we use the relation $P_{\ell mm}(k; z_s) = P_{\ell mm}(k; z_s) / \Delta \chi$ that is valid for a sufficiently thin shell around $\chi$ (or such a narrow shell with a width of $\Delta \chi$ should be used), and we assume that the radial thickness, $\Delta \chi$, is the same for all shells. Summation continues to the $N_\ell$-th shell corresponding to the source redshift. For consistency with the notation used thus far, we use the superscript “2D” to denote the projected correlation function of cosmic shear, $\xi^x_{2D}$. Therefore, the SSC term in the $\xi^x_{2D}$ covariance is

$$\text{Cov}_{\ell}^{\text{SSC}}[\xi^x_{2D}(\theta_1; z_s), \xi^x_{2D}(\theta_2; z_s)] = \sum_{i} f_k(\chi) \Delta \chi \frac{\partial^2 C_{\ell mm}(\theta_1, z_s)}{\partial \theta_1^2} \frac{\partial^2 C_{\ell mm}(\theta_2, z_s)}{\partial \theta_2^2} \sigma^2_s(z_s).$$

Note that only the large-scale density variance, $\sigma^2_s(z_s)$, depends on the survey-window function, whereas the response function is not dependent. To compute the above equation numerically, we first measured the responses from SU simulations at each
of the 16 redshifts and subsequently interpolated them to estimate the response at an arbitrary redshift. Redshift evolution and k-dependence of the response functions are all smooth, thus the interpolation is efficient. As given in Table 1, we use Planck cosmology for the SU simulations, while the light-cone ray-tracing simulations from Takahashi et al. (2017), which we use for comparison, are based on the nine-year WMAP cosmology (Hinshaw et al. 2013) (hereafter, WMAP). The correlation function for the Planck model is 12% – 23% higher in amplitude than that for the WMAP model at θ = 1’ – 100’. To correct for differences due to the different cosmological models, we multiply the Planck-based SSC prediction (equation (42)) by a factor of [Ckk(θ1; z1)2Ckk(θ2; z2)]WMAP/[Ckk(θ1; z1)2Ckk(θ2; z2)]Planck.

To compute the total power of the covariance matrix, we include the Gaussian covariance contribution but ignore the connected non-Gaussian term for simplicity. One of the main reasons is we do not have sufficient simulation realisations to compute the connected non-Gaussian part (or equivalently, contributions from the connected parts of the four-point functions). This also requires the development of a method to combine all pieces of the different contributions to estimate the total power. This is one of our on-going research, but in this study, we model the covariance by summation of the Gaussian contribution (CovG) and the SSC term (CovSSC) to demonstrate a proof-of-concept. To do this, we use an analytical formula to compute the Gaussian covariance (Joachimi et al. 2008):

\[
Cov^G[κ_1^2(θ; z_1), κ_2^2(θ; z_2)] = \frac{1}{S_0} \int dθ \frac{J_0(θ_1) J_0(θ_2)}{π} \left[ C_κ^2(z_0) \right]^2,
\]

where \(S_0\) is the survey area in units of steradians. To compute this equation, we first employ the revised halo fit (Smith et al. 2003; Takahashi et al. 2012) to compute the nonlinear matter power spectrum for the WMAP cosmology and subsequently perform the k- and χ-integration to obtain the Gaussian error prediction. The Gaussian covariance scales with the survey area as \(1/S_0\).

To test this SSC calibration method (equation (42)), we use full-sky ray-tracing simulations developed in Takahashi et al. (2017) to estimate the full covariance matrix. The full-sky simulation map (lensing fields) is given in the Healpix pixelisation (Górski et al. 2005), in which the sphere is divided by \(12 × N_{side}^2\) equal-area pixels. We use maps with \(N_{side} = 4096\) and 8192, corresponding to a pixel size of 0.43 (\(N_{side}/8192\))^-1 arcmin. In this study, we use 108 full-sky lensing maps for source galaxies at a single redshift, \(z = 1.033\). For each realisation, the halo catalogue containing information on mass and radial and angular positions for each halo is available, and we use these lensing maps and halo catalogues to construct hypothetical measurements of galaxy–galaxy lensing (shear-halo lensing) for the source galaxy sample (Shirasaki et al. 2017; Shirasaki & Takada 2018). Each full-sky map contains multiple spherical-lens planes that are placed according to the spacing of the \(Δ_θ = 150 h^{-3}\)Mpc up to \(z = 1.033\). Equivalently, the projected density field on each lens plane is computed from the radial projection of matter distribution within a thickness of \(Δ_θ = 150 h^{-3}\)Mpc. Each full-sky map contains 16 lens shells up to \(z = 1.033\) in total. To increase the number of realisations for hypothetical cosmic shear measurements as well as to study the survey area dependence, we divide each full-sky map into 48,192 and 768 equal-area subregions according to the Healpix pixelisation, corresponding to 859, 215, and 54 square degrees for each subregion area, respectively. By considering each subregion to be one cosmic shear survey region, we have 5184, 20736, and 82944 samples from the 108 full-sky map realisations in total, which is sufficient to precisely estimate the covariance matrix of cosmic shear correlation functions. Since cosmic shear correlation functions are affected by super-survey modes, which exist across each subregion, the covariance matrix estimated in this manner should include all the contributions of Gaussian, connected non-Gaussian and SSC terms. When estimating the cosmic shear correlation function from a given survey region, we use the following estimator:

\[
κ_1^2(θ; z_1) = \frac{1}{N_p} \sum_{|θ_1 - θ_i| < θ + Δθ/2} k(θ_1; z_1) k(θ_2; z_2),
\]

where \(k(θ_1; z_1)\) is the convergence field at \(θ_1\) on the spherical surface, and \(N_p\) is the number of pairs within \(|θ_1 - θ_2| < θ + Δθ/2\) with a bin width of \(Δθ\). In the following, we ignore the shape noise contribution. Takahashi et al. (2017) verified that the full-sky simulations provide accurate estimations of the cosmic shear correlation function and agree with analytical model predictions to within 5% at \(θ > 1’\) (see Section 3.2 of their paper).

Another model ingredient that is necessary in the SSC calibration method is the variance of super-survey modes, which exist across each subregion. To evaluate this variance, we need to properly use the survey-window function (angular and radial windows) at each redshift, \(z_0\). Provided that a survey area is sufficiently wide, \(σ^2_θ\) arises from the mass fluctuations and we can use the linear mass power spectrum to compute \(σ^2_θ\) in equation (2) (see also Takada & Hu 2013; Li et al. 2014a). To minimize any possible uncertainty in the comparisons, we use the same subregions in the full-sky simulations to estimate \(σ^2_θ(zi)\) from the variance of the surface mass density on each shell, for a given

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12 \(z = 0, 0.05, 0.10, 0.15, 0.20, 0.25, 0.31, 0.36, 0.42, 0.48, 0.55, 0.62, 0.69, 0.77, 0.85,\) and 0.93.

13 The full-sky lightcone simulation data (lensing fields and halo catalogues) are freely available for download at http://cosmo.phys.hirosaki-u.ac.jp/takahasi/allsky_raytracing/.

14 These halos are identified by the halo finder Rockstar (Behroozi et al. 2013) from the N-body simulations.

15 The original full-sky maps that are publicly available have the lens planes up to the last scattering surface (\(z = 1100\)).
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Figure 9. Similar to Fig. 8, but this figure shows results for the covariance of the halo-convergence correlation function, which is relevant for galaxy–galaxy weak lensing, for a halo sample with $M > 10^{13} h^{-1} M_\odot$ in the redshift range of $0.47 < z < 0.59$. We employed $z_s = 1.033$ for the source redshift. Note that, in this study, we do not consider the shape noise contribution. The dashed red curves are the analytical Gaussian error prediction, the dotted orange curves include the one-halo term of trispectrum for the connected non-Gaussian covariance, and the solid curves further include the SSC contribution. On the $x$-axis, the angular separation, $\theta$, corresponds to the transverse comoving distance, $= 0.4 h^{-1} \text{Mpc}(\theta/1')$.

survey area. Fig. 7 displays $\sigma_0^2(z) \times S_W$ as a function of redshift. Note that the redshift binning is fixed at $\Delta z = 150 h^{-1} \text{Mpc}$ as used in the full-sky ray-tracing simulations. After multiplying $S_W$, all of the results for the different survey areas appear to be similar, although results do not exactly agree with each other, especially at low redshifts, because survey area dependence of $\sigma_0$ originates from the shape of the linear mass power spectrum (Takada & Hu 2013; Takahashi et al. 2014). We use the results of Fig. 7 in equation (42) to estimate SSC contributions.

Now we compare the covariance matrix estimated from the light-cone simulation realisations with that computed from the SSC calibration method. Fig. 8, in the left panel, displays the square-of-root of the variance, i.e., the diagonal covariance components. The different symbols represent the different survey areas, and we plot the results of $(\text{Cov} \times S_W)^{1/2}$ such that all results have similar amplitudes. The plot clearly shows that non-Gaussian error is dominant at small scales, $\theta < 10'$, which is well explained by the SSC term in this method. While a close look reveals that the green curve is larger than the blue and purple curves, this is expected to be due to $\sigma_0^2(z)$ at $z \lesssim 0.2$, as shown in Fig. 7. At large scales, $\theta > 10'$, simulation results are consistent with the Gaussian prediction. Deviation of the simulation results from the Gaussian error at large separations, particularly for small area cases, is due to boundary effects in the finite survey area that are missing in the analytical calculation in equation (43) (see Appendix A in Sato et al. 2011). The middle and right panels display the off-diagonal components of the covariance matrix for a chosen $\theta_2$, but with varying $\theta_1$ in the $x$-axis. For $\theta_2 = 2.5'$, non-Gaussian errors appear dominant but can similarly be explained by addition of the SSC term. Note that these results vary with the source redshift or, more generally, the distribution of source redshifts, but it is straightforward to include these effects in the SSC calibration method.

4.3 SSC calibration for halo-convergence correlation functions

In this section, we discuss the covariance for the angular correlation functions of foreground halos and the background convergence fields that is closely related to that measured in galaxy–galaxy lensing. We consider a sample of halos selected by mass above a given threshold, $M > M_{th} = 10^{13} h^{-1} M_\odot$, in a redshift range of $0.47 < z < 0.59$, which mimics halos hosting BOSS CMASS galaxies (e.g. Alam et al. 2015; More et al. 2015). The number of halos is approximately $2.2 \times 10^6$ in the all sky, and the comoving number density $n_h \approx 3.2 \times 10^{-4} \ h^3 \text{Mpc}^{-3}$, which is similar to the number density of CMASS galaxies.

The halo-convergence angular correlation function for the mass limited sample is defined by equations (6) and (7) as

\begin{equation}
\Delta \xi_{bh}(\theta) = \xi_{bh}(\theta) - \xi_{bh}(\theta) \bigg|_{\text{Gaussian}} \times S_W \times \chi(\theta),
\end{equation}

where $\Delta \xi_{bh}(\theta)$ is the difference between the covariance of halo-convergence correlation function, and the Gaussian prediction, $\xi_{bh}(\theta)$.

\begin{equation}
\text{Cov} \left[ \Delta \xi_{bh}(\theta_1, \theta_2), \Delta \xi_{bh}(\theta_1', \theta_2') \right] = \text{Cov} \left[ \xi_{bh}(\theta_1, \theta_2), \xi_{bh}(\theta_1', \theta_2') \right] \times S_W \times \chi(\theta_1, \theta_2, \theta_1', \theta_2'),
\end{equation}

where $\text{Cov} \left[ \Delta \xi_{bh}(\theta_1, \theta_2), \Delta \xi_{bh}(\theta_1', \theta_2') \right]$ is the covariance of the halo-convergence correlation function, $\xi_{bh}(\theta_1, \theta_2)$.

\begin{equation}
\text{Cov} \frac{\Delta \xi_{bh}(\theta_1, \theta_2), \Delta \xi_{bh}(\theta_1', \theta_2')}{\xi_{bh}(\theta_1, \theta_2)} \times \chi(\theta_1, \theta_2, \theta_1', \theta_2').
\end{equation}

Where $\text{Cov} \left[ \xi_{bh}(\theta_1, \theta_2), \xi_{bh}(\theta_1', \theta_2') \right]$ is the covariance of the halo-convergence correlation function, $\xi_{bh}(\theta_1, \theta_2)$.
follows:

\[ \xi_{\text{hh}}^{2D}(\theta; z_{1}, M_{\text{th}}) = \langle \xi_{h}(\theta_{1}; z_{1}, M_{\text{th}}) \kappa(\theta_{2}; z_{1}) \rangle \bigg|_{\theta_{1} = \theta_{2} = \theta}, \]

\[ = \frac{1}{N_{\text{pair}}^{2D}} \sum_{i} \int d\chi_{1} f_{h}(\chi_{1}) f_{h}(\chi_{2}) \int_{M_{\text{th}}}^{\infty} \frac{dM}{dM} \left( \frac{dN}{dM}(M, \chi) \left( \delta_{h}(\chi_{1} \theta_{1}, \chi_{1}; M) \delta_{h}(\chi_{2} \theta_{2}, \chi_{2}) \right) \right) \bigg|_{\theta_{1} = \theta_{2} = \theta}, \]

\[ = \frac{1}{N_{\text{pair}}^{2D}} \sum_{i} \int d\chi_{1} f_{h}(\chi_{1}) f_{h}(\chi_{2}) \int_{M_{\text{th}}}^{\infty} \frac{dM}{dM} \left( \frac{dN}{dM}(M, \chi) \Delta \chi \right) \frac{kdk}{2\pi} P_{h}(k; \chi) J_{0}(k\theta), \]

\[ = \frac{1}{N_{\text{pair}}^{2D}} \sum_{i} \left[ f_{h}(\chi_{1}) f_{h}(\chi_{2}) \delta_{h}(M_{\text{th}}; \chi_{1}) (\Delta \chi)^{2} \right] \xi_{\text{hm}}^{2D}(\chi_{1}; \theta_{1}; M_{\text{th}}), \]

where \( f_{h}(\chi) = \chi^{2} \) if \( \chi \) is in the range of 0.47 < \( \chi < 0.59 \), otherwise \( f_{h}(\chi) = 0 \), \( \delta_{h} \) denotes the redshift range of lensing halos, and the projected halo number density, \( \tilde{n}_{h} \), given in equation (5), has the dimensions of angular number density, i.e., [rad\(^{-2}\)]. We also introduce the following quantities for notational simplicity:

\[ \xi_{\text{hh}}^{2D}(r; \chi, M_{\text{th}}) = \frac{1}{\tilde{n}_{h}(M_{\text{th}}; \chi)} \int_{M_{\text{th}}}^{\infty} \frac{dM}{dM} (M, \chi) \xi_{\text{hm}}^{2D}(r; \chi, M), \]

\[ \tilde{n}_{h}(M_{\text{th}}; \chi) = \int_{M_{\text{th}}}^{\infty} \frac{dM}{dM} (M, \chi). \]

Similarly to the previous case, we multiply the ratio \[ \{\xi_{\text{hh}}^{2D}(\theta_{1}; z_{1}) \xi_{\text{hh}}^{2D}(\theta_{2}; z_{1})\} \text{WMAP}/[\xi_{\text{hh}}^{2D}(\theta_{1}; z_{1}) \xi_{\text{hh}}^{2D}(\theta_{2}; z_{1})]_{\text{Planck}} \] by the SSC above, to account for the differences in the cosmological model.

The Galactic covariance was derived by Jeong et al. (2009) (also see Oguri & Takada 2011) as

\[ \text{Cov}_{\text{SSC}}^{\text{NG}}[\xi_{\text{hh}}^{2D}(\theta_{1}; z_{1}), \xi_{\text{hh}}^{2D}(\theta_{2}; z_{1})] = \frac{1}{S_{W}} \int \frac{d\theta}{\theta} J_{0}(\theta_{1}) J_{0}(\theta_{2}) \left( C_{\text{gh}}^{\text{th}}(z_{1}) + \frac{1}{\tilde{n}_{2D}} \right) C_{\text{gg}}^{\text{NS}}(z_{1}) \xi_{\text{hh}}^{2D}(\theta_{1}; z_{1}). \]

To compute this contribution, we use the halo model (Cooray & Sheth 2002) to evaluate the halo auto-power spectrum, \( C_{\text{th}}^{\text{th}} \), and the halo-convergence cross-power spectrum, \( C_{\text{th}}^{\text{NS}} \). We adopted the same model parameters that were used in Oguri & Hamana (2011). Takahashi et al. (2017) verified that the halo model agreed with the full-sky simulation results for the halo auto-correlation function and the halo-convergence cross-correlation function (see Sections 3.3 and 3.4 of their paper).

To obtain a better agreement between the SSC calibration method and the light-cone simulations, as we show below, we also include the connected non-Gaussian term, given in Sato et al. (2011) as follows:

\[ \text{Cov}_{\text{SSC}}^{\text{NG}}[\xi_{\text{hh}}^{2D}(\theta_{1}; z_{1}), \xi_{\text{hh}}^{2D}(\theta_{2}; z_{1})] = \frac{1}{S_{W}} \int \frac{d\theta}{\theta} J_{0}(\theta_{1}) J_{0}(\theta_{2}) \left( C_{\text{gh}}^{\text{th}}(z_{1}) + \frac{1}{\tilde{n}_{2D}} \right) C_{\text{gg}}^{\text{NS}}(z_{1}) \xi_{\text{hh}}^{2D}(\theta_{1}; z_{1}). \]

where \( \tilde{T}_{\text{halo}} \) is the angle-averaged triangle of halo-convergence-halo-convergence. We again employ the halo model given in Appendix A in Krause & Eifler (2017) to compute the trispectrum. In this study, we consider only the 1-halo term.

To test the SSC calibration method, we use light-cone weak lensing maps and halo catalogues constructed from the respective same \( N \)-body simulations (Takahashi et al. 2017). Here, we used 40 full-sky maps with \( N_{\text{side}} = 8192 \) (corresponding to ~0.43 arcmin for the angular resolution). We use the following halo-matter cross-correlation estimator:

\[ \xi_{\text{hm}}^{2D}(\theta; z_{1}, M_{\text{th}}) = \frac{1}{N_{h}} \sum_{i=1}^{N_{h}} \sum_{j=1}^{N_{\text{pair},i}} \kappa(\theta_{j}; z_{1}) \bigg|_{\theta_{j} = \theta}, \]

where \( N_{h} \) is the number of halos in the survey region, the summation, \( \sum_{i} \), runs over pairs between the \( i \)-th halo and the \( j \)-th pixel, satisfying the angular separation condition, \( |\theta_{ij} - \theta_{ij}| \leq \theta \), to within a bin width of \( \Delta \theta \), and \( N_{\text{pair},i} \) is the number of pairs for the \( i \)-th halo. Here, the halos are taken from the survey region, whereas the convergence fields are taken from all the sky (including outside the survey region). As indicated by Singh et al. (2017) (also see Shirasaki et al., 2017), using a sufficient number of random catalogues is essential to have an unbiased estimate of the halo-convergence cross-correlation as well as the correct covariance matrix. We use 10 times the number of random points than the number of halos in each realisation.

Fig. 9 shows the SSC calibration method for the covariance of \( \xi_{\text{hh}}^{2D}(\theta) \). First, we find that, compared to Fig. 8, the non-Gaussian covariance contribution is small and is significant only at small angular scales, \( \theta < 5' \). The figure shows that the
Covariances for the projected correlations in the response approach

Figure 10. Probability distribution of $\hat{\xi}_{hk}(\theta)$ at $\theta = 10'$ (left panel) and 100' (right) for the three survey areas. The PDF amplitudes are arbitrarily scaled. In each panel, we also show the mean with 1σ errors (the vertical lines with horizontal bars in the upper part) as well as the skewness, defined as $S_3 = \langle (\hat{\xi}_{hk} - \langle \hat{\xi}_{hk} \rangle)^3 \rangle / \langle (\hat{\xi}_{hk} - \langle \hat{\xi}_{hk} \rangle)^2 \rangle^{3/2}$, which is exactly zero if the PDF follows the Gaussian distribution. For large $\theta$, the distribution is highly, positively skewed particularly for small survey areas.

SSC calibration method fairly well reproduces the covariance from the ray-tracing simulations at the small scales, if the connected non-Gaussian term is added. The one-halo term provides a moderate contribution, which is expected from the top-middle panel of Fig.3, showing the connected non-Gaussian error is significant at small scales (i.e. the circles are much larger than the horizontal dashed line). But it is not the case in the left panels of the same figure (the matter-matter components, corresponding to cosmic shear). The analytical Gaussian covariance prediction appears to over-estimate covariance amplitudes at large scales, which are more clearly seen in the middle and right panels. This is owing to analytical calculations that do not consider survey geometry effects, which can be easily taken into account using similar methods found in Sato et al. (2011) (see also Shirasaki et al. 2017; Murata et al. 2018; Mandelbaum et al. 2018). If there are a sufficient number of independent modes in a survey area, the distribution of $\hat{\xi}_{hk}$ approaches the Gaussian owing to the central limit theorem. However, in our halo sample, there are less modes in the smaller survey areas at large scales, which cause the distribution to be highly skewed (see Fig. 10). This figure clearly shows that skewness causes a smaller mean and variance especially for the smaller survey area. However the scope of this study is the non-Gaussian covariance; therefore, this is not further explored.

5 DISCUSSION AND CONCLUSIONS

In this study, we have studied the response functions for the projected power spectra of matter and halos to large-scale density contrast ($\delta_b$) and the covariance matrices for the projected power spectra. First, by using paired SU simulation realisations, we calibrated the response functions of the projected power spectra and subsequently compared the results with the response functions of their respective three-dimensional power spectra as well as with perturbation theory predictions. We showed that the fractional response functions, $d \ln P_{XY}^{2D}(k)/d\delta_b$, are identical to their respective three-dimensional power spectra. In other words, the line-of-sight projection does not change the form of the response function, contrary to claims in analytical calculations based on Limber’s approximation (Takada & Jain 2009). Simulation results match perturbation theory predictions at small $k$-bins, supporting this finding. Second, we studied the covariance matrices for the projected power spectra (i.e., matter-matter, matter-halo and halo-halo) using a sufficient number of realisations of hypothetical projected fields constructed from $N$-body simulations. To do this, we used the following sets of projected field realisations: one set of realisations follow the periodic boundary conditions (i.e., no super-survey mode) and another set has super-survey mode contributions that are in both parallel and perpendicular directions to the line-of-sight (projection) direction, and the final
set has only the super-survery modes in the perpendicular direction. We showed that the SSC calibration method is capable of reproducing covariance for the projected power spectra, in which we calibrate the covariance by adding the SSC contribution based on the response function to the covariance from simulations with periodic boundary conditions. This is analogous to what was shown in Li et al. (2014a) for three-dimensional matter power spectrum. We did not observe a clear signature of the super-survery mode effect in the line-of-sight direction or the super-survery tidal effect, within the statistical accuracy of current simulation realisations. This is not consistent with recent claims made by Barreira et al. (2017). A further, careful study is needed to resolve this inconsistency. Third, we used the response calibration method to calibrate the covariances for the cosmic shear correlation function and the halo-convergence cross-correlation function, in which the latter is essential for galaxy–galaxy weak lensing. We showed that the response calibration method sufficiently reproduces the covariance obtained from the light-cone ray-tracing simulations.

Thus, the response calibration method developed in this paper offers an efficient technique to calibrate covariance matrices for the projected field, without running light-cone ray-tracing simulations. To obtain a sufficiently accurate calibration of the covariance matrix for ongoing and upcoming wide-area galaxy surveys, the following can be a practically useful method:

- **Gaussian covariance contribution** (Cov\(^G\)) – This term is the simplest and is naively considered to be the easiest to calibrate. However, calibration requires precise care because one needs properly take into account observational effects such as survey geometry/masks and the intrinsic distribution of galaxies used in the sample. To do this, we should use the real galaxy catalogue as much as possible, as several studies have already done (Shirasaki et al. 2017; Murata et al. 2018; Mandelbaum et al. 2018). For example, for the lensing field, the real catalogue of source galaxies should be used as follows: (1) Randomly rotate galaxy ellipticity for each galaxy to erase real lensing effects. (2) Apply actual lensing measurement pipeline to the modified galaxy catalogue to estimate correlation functions such as the cosmic shear correlation and galaxy–galaxy lensing. (3) Repeat the first and second procedures, and then estimate the covariance from mock realisations. The estimated covariance matrix includes all of the observational effects, such as survey masks, intrinsic shape distribution and photometric redshift distribution. In particular, this covariance can take into account the correlated shape noise, which arises because ellipticities of the same source galaxies, i.e., galaxies with relatively large intrinsic ellipticities, are used multiple times in the correlation function estimation. To further include the Gaussian sample variance, which arises from products of the power spectrum or two-point correlation function, we could use the method in Appendix A of Sato et al. (2011); they developed a method to estimate the Gaussian sample variance taking into account actual pairs used in the analysis. It would be straightforward to extend this method to including cross terms between the shape noise and the sample variance. Or one could use the Gaussian realisations of lensing fields to estimate the Gaussian sample variance, and then add contribution to the shape noise covariance. This would be our future work.

- **Connected non-Gaussian contribution** (Cov\(^{NG}\)) – This term becomes non-negligible only over the transition regime (which is a narrow scale range) between the Gaussian covariance and the SSC contribution. In addition, survey mask effects can be ignored in this contribution if separations are much smaller than the survey-window size. Hence, to calibrate this term, one could use a set of N-body simulations with periodic boundary conditions as we have done in this paper, or use the halo model predictions as done in Sato et al. (2009) (see also Cooray & Hu 2001; Takada & Hu 2013; Li et al. 2014a; Krause & Eifler 2017).

- **Super-sample covariance contribution** (Cov\(^{SSC}\)) – SSC contribution can be significant at small scales. As we have shown in this study, we can use the response function approach to calibrate the SSC term. To obtain the response function for a given observable, we use SU simulations, which are quite efficient at reducing sample variance contamination. In principle, one could use only a paired simulation to calibrate the response function as a function of redshifts and separation scales for a given cosmological model. This is quite inexpensive for current computer resources. Then, one could compute the SSC contribution from the line-of-sight integration of the response function, weighted by the proper radial functions (e.g., see equations (42) and (47)). The SSC term also originates from the line-of-sight integral of the variance of density contrast averaged in the survey area, \(\sigma^2_S(z; W)\), at each redshift. Only this term depends on the survey window (survey masks) and can be computed from the integration of linear power spectrum or the linear Gaussian realisations (see also Takada & Hu 2013; Li et al. 2014a, for the similar discussion). Thus, this method does not require light-cone simulations.

Thus, by combining the three contributions, one could calibrate the full covariance matrix for a given observable. This is a computationally inexpensive method, and it allows one to include the cosmological dependence of the covariance matrix in the parameter estimation. To do this, we need to calibrate the response function of a given observable as a function of cosmological parameters. Nishimichi et al. (in prep.) are now constructing an emulator to output the power spectra for matter and halos as functions of halo mass, redshift and cosmological model. They are planning to include the response functions in the emulator, which could allow one to calibrate the covariance matrices for matter and halo observables. However, we do not wish to claim that light-cone simulations are no longer needed (Takahashi et al. 2017). Such simulations are still required to make mock catalogues of a galaxy survey, which are useful for testing systematic errors and analysis pipelines and for calibrating the covariance matrix. However, since making a large number of light-cone simulations for different cosmological models is still computationally expensive, these methods are complementary to each other.
In this paper, we have ignored the effects of large-scale tidal fields on the projected power spectra. Although we did not find a clear signature of these effects, it is interesting to further explore whether the large-scale tidal fields affect the projected power spectra because this carries independent information about large-scale gravitational fields. As pointed out by Akitsu et al. (2017) (see also Akitsu & Takada 2018), large-scale tidal fields directly affect the galaxy redshift-space power spectra, or, more generally, the redshift-space clustering features. It would be straightforward to extend the method in this paper to develop a method of calibrating the covariance of the redshift-space clustering power spectra based on the response approach. However, in this case, the large-scale tidal field is a tensor, i.e. depends on directions and vary with the degree of alignment with the line-of-sight direction. When we need to consider a wide-area galaxy survey, effects of spherical curvature needs to be taken into account because the directions of the tidal tensor relative to the line-of-sight direction would vary with redshift. These would be interesting subjects and worth exploring.

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