Neutrino Mass and Lepton Number Violation with Charged Scalars

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Abstract

If there exist only three light neutrinos below the weak scale, the neutrino masses can arise by breaking the lepton number \( L \). One example is the Zee mechanism for the neutrino masses. We study phenomenological implications of neutrino masses arising from the \( L \) violation through introduction of \( SU(2) \times U(1) \) singlet scalars with \( Q_{em} = 1 \) and 2.

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I. INTRODUCTION

Historically, neutrinos contributed significantly in the development of particle theory. Firstly, Pauli postulated it to satisfy the energy conservation and rotational invariance. Then weak interaction with neutrinos was described by Fermi’s four fermion interaction which was known to be nonrenormalizable. Thus the current renormalizable theory of weak interactions, standard model (SM), can be traced to the origin where the introduction of the electron type neutrino, \( \nu_e \), was crucial. However, neutrinos have been elusive, which made it difficult to find their properties such as the masses and magnetic moments.

We are now in the new era of neutrinos with the accumulating evidence on their oscillation [1–4]. In particular, we have some information on \( \Delta m^2_{ij} \) and mixing angles. These important data, especially the maximal mixing of the muon type neutrino \( \nu_\mu \) with a neutrino other than \( \nu_e \),

\[
a \text{mass eigenstate} \approx \frac{1}{\sqrt{2}} \nu_\mu + e^{i \alpha} \frac{1}{\sqrt{2}} \sum_i c_i \nu_i, \quad (i \neq \nu_e, \nu_\mu)
\]

may hint a new particle(s) or a new theory.

Because it has been turned out that the mixing angles are almost maximal, it is worthwhile to study the neutrino mass generating mechanisms without using the see-saw mechanism. To pose the problem within the zoo of the known SM fermions, we assume that for weakly interacting fermions there exist only the SM fermions below the electroweak scale. But we will introduce scalars if needed. [With supersymmetry, the fermions, axino [5] and gravitino [6], can be light since their interactions are much weaker than the weak interaction.] Namely we are assuming three light neutrinos: \( \nu_e, \nu_\mu \) and \( \nu_\tau \). We do not assume light sterile neutrinos. Then there does not exist a Dirac mass term for neutrinos. Neutrino masses must be of the Majorana type,

\[
-m_{ij} \nu^T_i \nu_j, \quad (i, j = e, \mu, \tau)
\]

where the matrix \( C^{-1} \) is omitted for the simplicity of notation. The Majorana neutrino mass term violates the lepton number \( L \). Thus the neutrino masses can arise only if the theory violates the lepton number. The well known see-saw mechanism [7] violates \( L \) through the Majorana mass term at high energy scale which is the source for the light neutrino masses.

In this paper, we study phenomenological implications of the neutrino mass of a model which does not have a true Goldstone boson. Toward a model with this property, we will not introduce a neutral scalar field. Then in the vacuum respecting \( U(1)_{em} \) invariance, there is no place for a Goldstone boson. Of course, with neutral scalars introduced, the potential can have appropriate parameters so that the additional hypothetical neutral scalars do not develop VEV’s. We simply do not bother to worry about to find this limited region of the parameter space.

In Sec. II, we introduce a few models in which neutrino masses have been generated. In this short review, we emphasize the symmetry argument and relate it to Feynman diagrams. In Sec. III, we discuss a model without neutral Higgs scalar field to avoid possible problems with Goldstone bosons. In this model, we present the limits of the parameters introduced from various experimental data. Sec. IV is a conclusion.
II. A SHORT REVIEW ON NEUTRINO MASS WITH SCALARS

For the neutrino mass, it is important to pinpoint how the $L$ symmetry is broken in the model. In the literature, already there exist numerous studies on the $L$ symmetry violation through singly charged scalars. In this section, we briefly comment on the diagrammatic symmetry argument in these models.

A. Triplet Higgs Scalar

Majorana neutrino mass can be generated with a triplet Higgs scalar $\xi = (\xi^+ , \xi^+ , \xi^0)$ through the interactions

$$ f_{ij} \left[ \xi^0 \nu_i \nu_j + \xi^+ (\nu_i l_j + l_i \nu_j)/\sqrt{2} + \xi^{++} l_i l_j + {\rm h.c.} \right], $$

(3)

where $f_{ij}$ is symmetric under the exchange of indices. If we assume $L$ symmetry, $L(\xi) = -2$. To generate neutrino mass $L$ must be broken spontaneously, i.e. $\langle \xi^0 \rangle \neq 0$. We take the following potential for the triplet and one Higgs doublet $H = (H^+, H^0)$

$$ V = m^2 H^+ H + M^2 (\xi^+ \xi + \frac{1}{2} \lambda_1 (H^+ H)^2 + \frac{1}{2} \lambda_2 (\xi^+ \xi)^2 + \lambda_3 (H^+ H)(\xi^+ \xi)). $$

(4)

where we have not allowed the cubic term,

$$ \mu (\xi^0 H^0 H^0 + \sqrt{2} \xi^- H^+ H^0 + \xi^{--} H^+ H^+ + {\rm h.c.}). $$

(5)

With the potential Eq. (4), the vacuum expectation values (VEV) of $\langle H^0 \rangle$ and $\langle \xi^0 \rangle$ can be developed. The presence of non–zero VEV of the neutral triplet Higgs boson modifies the $\rho$ parameter as follows

$$ \rho \equiv \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = \frac{1 + 2 \langle \xi^0 \rangle^2 / \langle H^0 \rangle^2}{1 + 4 \langle \xi^0 \rangle^2 / \langle H^0 \rangle^2}. $$

(6)

From $\rho_{\exp} = 1.00412 \pm 0.00124$, one can obtain $\langle \xi^0 \rangle \leq 4 \text{ GeV}$. To introduce an eV range neutrino mass, $\langle \xi \rangle \sim O(\text{eV})$. This means that the parameter $M^2$ must be fine–tuned to equal to $(\lambda_3/\lambda_1) m^2 \sim \text{eV}^2$. This is unnatural. Moreover, this model contains the massless CP–odd field (majoron) which corresponds to the Goldstone boson due to the spontaneous breaking of $L$ and CP–even field which has a small mass proportional to $\langle \xi^0 \rangle$. This leads to the $Z$ decay into the majoron and the light scalar with a decay width of two neutrino flavors, which is ruled out because of the LEP results of $N_{\nu} = 2.994 \pm 0.011$ and $\Gamma_{\text{inv}} = (500.1 \pm 1.9) \text{ MeV}$. But this model with the cubic terms given in Eq. (5), which violates $L$ explicitly, can be considered natural since the vacuum expectation value is of order $\langle \xi^0 \rangle \sim \mu \langle H^0 \rangle^2 / M^2$ in the limit $\langle H^0 \rangle / M \ll 1$ and consistent with the LEP data because all physical neutral scalars become heavy.
B. The Zee Model

Another $L$ violating model was proposed by Zee \cite{15},
\begin{equation}
\mathcal{L} = \mathcal{L}_{\text{SM}} + \epsilon_{\alpha\beta} l^T_{\alpha} l_{\beta} \phi^+ + \epsilon_{\alpha\beta} H_1 \alpha H_2 \beta \phi^+ + \text{h.c.}
\end{equation}

where $l_i$ is the $i$\textsuperscript{th} lepton doublet, $H_1$ with $Y = -1/2$ is the Higgs doublet present in the SM Lagrangian $\mathcal{L}_{\text{SM}}$, $H_2$ with $Y = -1/2$ is the Higgs doublet not coupled to fermions, $\phi^+$ is a singly charged $SU(2)$ singlet scalar field, $\epsilon_{\alpha\beta}$ is the $SU(2)$ Levi-Civita symbol, and the couplings are suppressed. The coupling $l^T_{\alpha} l_{\beta} \phi^+$ is antisymmetric in $(i, j)$. The SM Lagrangian gives $H_1$ the lepton number $L = 0$. The $ll\phi$ coupling defines $L = -2$ for $\phi^+$. The $H_1 H_2 \phi^+$ coupling defines $L = +2$ for $H_2$. Thus the above Lagrangian does not violate the $L$ number. But $L$ can be broken by the vacuum expectation value(s) of the $L$ carrying neutral Higgs field(s). Indeed, there exist one such component in this theory, the neutral element of $H_2$. Thus, $\langle \tilde{H}_0 \rangle \equiv v_{LV} = 0$ breaks $L$ spontaneously, and this theory predicts a Goldstone boson. Neutrinos obtain mass at one loop-level, as shown in Fig. 1. Note that Fig. 1 includes all the couplings and the VEV needed to break $L$ as discussed above. To give the Goldstone boson a mass, the Lagrangian should violate the lepton number explicitly. It can be done by introducing $m_{12}^2 \tilde{H}_1 \tilde{H}_2 + \text{h.c.}$ where $\tilde{H}_1 = i\tau_2 H_1^\dagger$. Then the neutrinos get masses through the two-loop diagrams as in Fig. 2 even with $\langle \tilde{H}_0^0 \rangle = 0$. In any case, the diagonal elements of the mass matrix vanish.

The phenomenological consequences of the model has been studied extensively in the literature \cite{16,17}. One simple feature is that the neutrino mass matrix has vanishing diagonal elements in the flavor basis, which can lead to a nontrivial prediction on the neutrino oscillation phenomenology.

C. Supersymmetry with R-parity Violation

In this section, we briefly comment on the roles of scalars(mainly the sleptons) in the R-parity violating models \cite{18,19}. But for the baryon number conservation, we impose the baryon parity ($B$-parity). Then there exist the $L$ violating terms in the superpotential in the minimal supersymmetric standard model (MSSM),
\begin{equation}
\mu_i \tilde{H}_2 \tilde{L}_i + \frac{1}{2} \lambda_{ijk} \tilde{L}_i \tilde{L}_j \tilde{E}_k^c + \lambda'_{ijk} \tilde{L}_i \tilde{Q}_j \tilde{D}_k^c.
\end{equation}

These interactions lead to the neutrino mass through the tree and one-loop level as shown in Fig. 3 with the appropriate gaugino mass term $M_Z \tilde{Z} \tilde{Z}$ and the soft terms $A_i \tilde{e} H_1^0 \tilde{e}^c$ and $A_i \tilde{d} H_1^0 \tilde{d}^c$.

In the tree level diagram, the gaugino mass term and the gaugino interaction term with the neutrino and sneutrino are needed to define the appropriate lepton number of the sneutrino field. The gaugino mass term defines $L(\tilde{Z}) = 0$; and hence $L(\tilde{\nu}) = 1$. Thus $\Delta L = 2$ results from two insertions of $\langle \tilde{\nu} \rangle$. Fig. 3(a) contains all these information. Being the gauge interaction, the $\tilde{Z}$ coupling is universal (democratic), and the mass matrix arising from this diagram contains the common entry to every elements. Thus, only one state obtain mass by Fig. 3(a).
Similarly, the loop diagrams present in Fig. 3(b)(c) violate the lepton number and contribute to the neutrino masses. At one-loop level, \( A^{(l,d)} \) term and the mass term of the charged lepton (down-type quark) are needed to define the lepton numbers of the \( H_0^0, \tilde{e}, \) and \( \tilde{\nu}_c \) fields. In the case of \( \lambda_{ijk} \) as shown in Fig. 3(b), the charged lepton mass term and \( A \) term give \( L(H_0^0) = 0 \) and \( L(\tilde{e}) + L(\tilde{\nu}_c) = 0 \), respectively. This lepton number assignment leads to \( |\Delta L| = 2 \) interactions through \( \nu_e \tilde{\nu}_c \) and \( \nu_e \tilde{\nu} \) interactions. This explains that at least, four kinds of interactions are involved to generate the Majorana neutrino mass and the induced mass is proportional to the internal fermion mass and \( A \) term. Similar arguments can be applied for the case of \( \lambda'_{ijk} \) case, viz. Fig. 3(c). Since the couplings appearing in these diagrams are the Yukawa couplings, the mass matrix arising from the one-loop is quite general and all three neutrinos can obtain masses.

III. A MODEL WITH \( \phi^+ \) AND \( \Phi^{++} \)

In the remainder of this paper, we study another interesting model, violating \( L \) with scalars, singly charged \( \phi_a^+ \) \((a = 1, 2)\) and doubly charged \( \Phi^{++} \) [20]. This model is free from the problem of a Goldstone boson since there is no extra neutral scalar field. We can generate the neutrino masses with just one singly charged scalar \( \phi_1^+ \) and one doubly charged scalar \( \Phi^{++} \). Then the resulting neutrino mass matrix is not general enough, and hence we introduce an additional singly charged scalar

\[
\mathcal{L} = \mathcal{L}_{SM} + f_{ij}^a \epsilon_{\alpha \beta} l_{\alpha i}^T l_{\beta j} \phi_a^+ + \mu_{ab} \phi_a^+ \phi^+_b \Phi^{--} + \lambda_{ij}^{--} e_i^c e_j^c \Phi^{--} + h.c. \tag{9}
\]

where \( a, b = 1 \) or 2, \( i \) is the family number index \( i = 1, 2, 3 \), and \( e^c \) is the \( SU(2) \) singlet charged anti-lepton field. In a theory with only one singly charged \( SU(2) \) singlet scalar, we have \( a = 1 \) only. Note that we introduced an \( SU(2) \) singlet doubly charged scalar \( \Phi^{++} \). Note that coupling matrix \( f^a \) is an antisymmetric matrix and \( \mu_{ab} \) is symmetric. The \( f \) coupling defines the \( L \) number for \( \phi_a^+ \) as \(-2\). The \( \mu \) coupling defines the \( L \) number for \( \Phi^{++} \) as \(-4\). But the \( \lambda \) coupling defines the \( L \) number of \( \Phi^{++} \) as \(-2\). Thus, the interaction terms give inconsistent \( L \) numbers of \( \Phi^{++} \), i.e. the Lagrangian does not respect the \( L \) symmetry. Since \( L \) is not a symmetry of \( \mathcal{L} \), neutrino masses can arise at higher orders, here at a two-loop level. At least the two-loop as shown in Fig. 4 is needed to include \( f, \mu \) and \( \lambda \) couplings, which are the requisite for the violation of \( L \). Because \( L \) is explicitly broken in the Lagrangian, there does not exist a Goldstone boson in this model.

From the above Lagrangian and Fig. 4, we estimate the two-loop neutrino mass as

\[
\left( m_{2\text{-loop}}^{\nu} \right)_{im} \approx \sum_{a,b,j,k} \lambda_{jk}^{--} f_{ij}^a f_{mk}^b \frac{\mu_{ab} m_j m_k}{(8\pi^2)^2 m_{\phi^{--}}} \tag{10}
\]

where \( m_j \) denotes the mass of the charged lepton and we assume \( \Phi^{--} \) is heavier than \( \phi_a^+ \). This two-loop neutrino mass matrix is symmetric in flavor basis if \( \lambda_{jk}^{--} \) is symmetric. This means that only the symmetric part of \( \lambda_{jk}^{--} \) contributes the Majorana neutrino mass. From now on, we assume \( \lambda_{jk}^{--} \) is symmetric.

Now, let’s estimate the size of \( m_{2\text{-loop}}^{\nu} \) of Eq. (10).
\[(m_{\nu}^{2-\text{loop}})_{\text{im}} \approx 0.5 \sum_{a,b,j,k} \lambda_{-j}^{\mu} f_{ij}^{a} f_{jk}^{b} \left( \frac{m_{j} m_{k}}{m_{\tau}} \right) \left( \frac{\mu_{ab}}{1 \text{ TeV}} \right)^{2} \text{keV}. \tag{11}\]

For example, neglecting the electron mass and assuming universal \(\mu_{ab}, \lambda_{ij}^{\mu}\) and \(f_{ij}^{a}\), the neutrino mass matrix is

\[m_{\nu}^{2-\text{loop}} \approx 2 \omega \lambda_{-} f^{2} \begin{pmatrix} 1 + 2 r_{\mu \tau} + r_{\mu \tau}^{2} & 1 + r_{\mu \tau} & r_{\mu \tau} + r_{\mu \tau}^{2} \\ 1 + r_{\mu \tau} & 1 & r_{\mu \tau} \\ r_{\mu \tau} + r_{\mu \tau}^{2} & r_{\mu \tau} & r_{\mu \tau}^{2} \end{pmatrix} \text{keV}, \tag{12}\]

where \(r_{\mu \tau} = m_{\mu}/m_{\tau} \approx 0.056\), \(\lambda_{-}\) and \(f\) denote the universal values of \(\lambda_{ij}^{\mu}\) and \(f_{ij}^{a}\), respectively, and the normalization factor \(\omega = (\mu_{ab}^{*}/1 \text{ TeV}) \cdot (1 \text{ TeV}/m_{\Phi-})^{2}\). Thus, it is impossible to explain the large mixing between \(\nu_{\mu}\) and \(\nu_{\tau}\) with universal \(f_{ij}^{a}\) and \(\lambda_{ij}^{\mu}\). The hierarchies between \(f_{ij}^{a}\)'s and \(\lambda_{ij}^{\mu}\)'s are needed to accommodate the large mixing angle solution of \(\nu_{\mu}\) for the atmospheric neutrino data.

Since the large (1,1) component of \(m_{\nu}\) is not desirable to explain the deficit of \(\nu_{\mu}\) in Super Kamiokande data as oscillation of \(\nu_{\mu} \rightarrow \nu_{\tau}\) \cite{16}, we need tunings between couplings \(\lambda_{-}^{ij}\)'s and \(f_{ab}\)'s. For example, we can take \(\mu_{ab}\)'s as universal parameters. To suppress the large (1,1) component of \(m_{\nu}\), we require the following relations between couplings :

\[
\begin{align*}
\lambda_{23}^{23} & \equiv \lambda, & \lambda_{-22}^{22} & \equiv r_{\mu \tau}^{N} \lambda, & \lambda_{-33}^{33} & = -2 r_{\mu \tau} \lambda, \\
 f_{12} & \equiv - f / r_{\mu \tau}, & f_{13} & = - f / r_{\mu \tau}, & f_{23} & \equiv f \gamma, \tag{13}
\end{align*}
\]

where we take \(N\) as positive integer and neglect the electron mass. Note that \(\lambda_{-33}^{33} = -2 r_{\mu \tau} \lambda_{-22}^{23}\) and \(f_{12} = f_{13}\). Then, the mass matrix is given by

\[m_{\nu}^{2-\text{loop}} \approx 2 \omega \lambda f^{2} g \begin{pmatrix} r_{\mu \tau}^{N} / g & 1 & 1 + r_{\mu \tau}^{N+1} \\
1 & -2 r_{\mu \tau} g & - r_{\mu \tau} g \\
1 + r_{\mu \tau}^{N+1} & - r_{\mu \tau} g & r_{\mu \tau}^{N+2} g \end{pmatrix} \text{keV}. \tag{14}\]

With this mass matrix, we obtain for \(N = 2\)

\[
\begin{pmatrix} \nu_{e} \\ \nu_{\mu} \\ \nu_{\tau} \end{pmatrix} \approx \begin{pmatrix} \frac{-1}{\sqrt{2}} + \frac{g r_{\mu \tau}}{4} \\ \frac{1}{2} + \frac{3 \sqrt{2} g r_{\mu \tau}}{8} \\ \frac{1}{2} - \frac{\sqrt{2} g r_{\mu \tau}}{8} \end{pmatrix} \begin{pmatrix} \nu_{1} \\ \nu_{2} \\ \nu_{3} \end{pmatrix}, \tag{15}\]

with

\[
\begin{align*}
m_{1}^{2} & \approx (2 \omega \lambda f^{2} g)^{2} \left( 2 + 2 \sqrt{2} g r_{\mu \tau} \right) \text{keV}^{2}, \\
m_{2}^{2} & \approx (2 \omega \lambda f^{2} g)^{2} \left( 2 - 2 \sqrt{2} g r_{\mu \tau} \right) \text{keV}^{2}, \\
m_{3}^{2} & = 0, \tag{16}
\end{align*}
\]

The mixing matrix and eigenvalues are independent of \(N\) if \(N > 1\) up to \(O(r_{\mu \tau})\). Therefore, to give \(\Delta m_{\text{atm}}^{2} = |m_{3}^{2} - m_{1}^{2}| = |m_{3}^{2} - m_{2}^{2}| \approx (0.5 - 6) \times 10^{-3} \text{eV}^{2}\), the combination of the couplings and mass parameters satisfies the relation
\[ \omega \lambda f^2 g \simeq (0.8 - 2.7) \times 10^{-5}, \]  
\[ \Delta m_{\text{sol}}^2 = |m_2^2 - m_1^2| = 2\sqrt{2}\Delta m_{\text{atm}} g r_{\mu \tau} \simeq (0.8 - 9.5) \times 10^{-5} \left( \frac{g}{1/10} \right) \text{eV}^2. \]

We observe that this model can explain the atmospheric neutrino data and accommodate the large angle MSW resolution of the solar neutrino data.

Now let us consider the phenomenological constraints on \( f \)'s and \( \lambda \)'s. The current experimental data of the muon decay \( \mu \to \nu_e e \bar{\nu}_e \) and radiative decay \( \mu \to e \gamma \), whose constraints \( f_{12}, f_{13}, \) and \( f_{23} \), are not enough to determine these parameters \([16,17]\). The constraints on \( \lambda_{23}^{23}, \lambda_{22}^{22}, \) and \( \lambda_{33}^{33} \) come from the tau decay \( \tau \to 3\mu \) and radiative decay \( \tau \to \mu \gamma \). The decay \( \tau \to 3\mu \) can be described by the following four-fermion effective Lagrangian induced by the doubly charged boson exchange (after appropriate Fiertz transformation):

\[ \frac{\lambda_{ij}^{kl} \lambda_{kl}^{rs}}{2m_F^{2-2}} \bar{e}_k \gamma^\mu P_R e_i \bar{e}_l \gamma_\mu P_R e_j. \]

Since the doubly charged scalar interacts only with the right–handed charged leptons, the effective Lagrangian has \( V + A \) form. From this effective Lagrangian we obtain \([21]\),

\[ B(\tau \to 3\mu) \approx B(\tau \to \nu_e e \bar{\nu}_e) \left( \frac{\lambda_{33}^{33} \lambda_{22}^{22}}{4 G_F m_F^{2-2}} \right)^2. \]

where \( G_F \) is the Fermi coupling constant. From the upper bound on the decay mode \( \tau \to 3\mu \) and the branching ratio \( B(\tau \to \nu_e e \bar{\nu}_e) \) \([22]\), we obtain

\[ \frac{\lambda_{33}^{33} \lambda_{22}^{22}}{m_F^{2-2}} \lesssim 1.3 \times 10^{-2} G_F. \]

This constraints can be easily satisfied without affecting the results from the neutrino data if, for example, we take large enough \( N \) even with \( \lambda \approx O(1) \), viz. Eq. (13). For \( \tau \to \mu \gamma \) through the one–loop diagram with the doubly charged scalar \([16]\),

\[ B(\tau \to \mu \gamma) \approx B(\tau \to \nu_e e \bar{\nu}_e) \frac{\alpha}{3072\pi} \left( \frac{\lambda_{33}^{33} \lambda_{23}^{23}}{G_F m_F^{2-2}} \right)^2. \]

From the upper bound on the decay mode \( \tau \to \mu \gamma \) \([22]\) and using the Eq. (13),

\[ \frac{r_{\mu \tau} \lambda^2}{m_F^{2-2}} \lesssim 2.4 G_F. \]

This allows \( \lambda \approx O(1) \). Therefore, there are no significant constraints on \( \lambda_{23}^{23}, \lambda_{22}^{22}, \) and \( \lambda_{33}^{33} \) at present. But the product of couplings \( \omega, \lambda, f, \) and \( g \) must satisfy the relation (17).
IV. CONCLUSION

We studied a neutrino mass generating model with a doubly charged scalar $\Phi^{++}$ and singly charged scalars $\phi^+_a,b$. This has been motivated from the observation that the neutrino mixing angles are large, which is not easily incorporated in the see-saw mechanism. In this study, we emphasized the importance of the consideration of the lepton number $L$.

However, the parameters introduced in this model through Eq. (9) are restricted as Eq. (17) to explain $\Delta m^2_{\text{atm}}$. This constraint is not very strong since it is a constraint on the product of four coupling constants.

Generalizing the doubly charged scalar idea to GUT models such as $SU(5)$ and $SO(10)$ may not be easy. However, it is not impossible. For example, it can be introduced in higher dimensional representations which couple to 5 of $SU(5)$. In an $SU(5) \times U(1)$ model, one can introduce a singlet which is charged. In this case, one does not need high dimensional representations. However, the GUT inclusion of a doubly charged scalar field is premature to study at present, without a detailed knowledge of light particles below the GUT scale.

Distinguishing the doubly charged scalar idea from the see-saw mechanism can be achieved from the study of rare processes occurring through the exchange of the doubly charged scalars as studied in this paper. Also, if its mass is below the threshold of future accelerators, it can be easily identified through its decay to four leptons, $\Phi^{--} \rightarrow l^-l^-\nu\nu$.

While we were finishing this manuscript, we received a preprint by Joshipura and Rindani [23] considering the neutrino masses and mixings in a Zee model extended by the doubly charged scalar field. In this model, the neutrino mass matrix of this model has contributions from both one– and two–loop diagrams because the model has one more SM $SU(2)_L$ doublet comparing with the model considered in Sec. III.

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FIG. 1. The one-loop neutrino mass in the Zee model.
FIG. 2. The two-loop neutrino mass in the Zee model.
FIG. 3. The neutrino mass in the MSSM with R-parity violation.
FIG. 4. The two-loop neutrino mass in the model with $\phi^+$ and $\Phi^−$. 