Dynamics of Extended Objects: the Einstein-Hilbert Drop

U. Khanal

Central Department of Physics, Tribhuvan University, Kirtipur, Kathmandu, NEPAL

(October 27, 2018)

Abstract

The Einstein-Hilbert worldspace action is used to investigate the dynamics of extended object. In the Robertson-Walker worldspace, this is seen to introduce a pressureless density which could contribute to dark matter. Such pressureless energy density, present from the very beginning, should have enormous consequences on large scale structure formation in the early Universe. Generalizing the idea to complexified internal co-ordinates, it becomes possible to gauge the action with U(1) symmetry. A trivial solution of this theory is Einstein’s general relativity and source free Maxwell theory. Generally, the equations of motion of the gauge fields are Maxwell equations with source terms that include these fields themselves. The internal co-ordinates, under vacuum domination with negative pressure, obey an EOM that is a hyperbolic wave equation of a charged scalar field that interacts with the gauge fields and gravity in a disperso-conductive medium; under matter domination with positive pressure however, it is an elliptic potential equation. Since the hyperbolic to elliptic transition can be made by introducing imaginary time, this result supports the view that time is actually complex, becoming Minkowskian in vacuum and Euclidean in matter. A supersymmetric version of the action can also be immediately written down.
I. INTRODUCTION

In the quest for a unified theory to describe all physical interactions of nature, the idea of the pointlike fundamental structure of matter is being abandoned in favour of extended objects now called p-branes. Indeed, most of the singularities in quantum theory that require renormalization and effective subtraction of infinities, can be traced to the assumption of pointlike structure. At present, the one-dimensional string leads the race towards the theory of everything, leaving the higher dimensional contenders far behind. But it is just as important to investigate different higher dimensional incarnations, even if only for elimination, as it is to develop a viable string theory. A particularly attractive proposition is the three-dimensional drop evolving with time, as the space-time with which we are familiar is (3+1) dimensional. Furthermore, both electromagnetic waves and gravitation cannot propagate in a space of less than three dimensions. The author has been contending that the Einstein-Hilbert (EH) worldcurvature action provides a more appropriate and general means, than the Nambu-Goto (NG) worldvolume action, of studying the gravitodynamics of extended objects. [1] The NG brane that can be considered a special constant curvature case of the EH one, can also be included as a cosmological term to constrain the extremization of worldcurvature with simultaneously extremized worldvolume.

The n+1 dimensional internal co-ordinates $X^a$, $0 \leq a \leq n$, are functions of the p+1 dimensional worldspace co-ordinates $\xi^\alpha$, $0 \leq \alpha \leq p < n$. $\xi^0 = \tau$ is assumed to be timelike, and so is at least one internal co-ordinate $X^0 = t$. The line element $ds^2 = \eta_{ab} dX^adX^b = g_{a\beta}d\xi^a d\xi^\beta$, with the internal space metric tensor $\eta_{ab}$ that need not necessarily be flat, shows that the induced metric tensor is $g_{a\beta} = \partial_a X^\alpha \partial_\beta X^b \eta_{ab}$, where $\partial_a = \partial / \partial \xi^a$. The worldspace EH action is

$$I = -\frac{1}{2\kappa} \int d^{p+1} \xi \sqrt{g} R,$$  \hspace{1cm} (1)

where $R$ is the worldcurvature scalar, $g = - \det g_{a\beta}$ and $\kappa$ is the gravitational constant appropriate to the p-dimensional space. When $R$ is constant, Eq.(1) is just the NG action.
Using the well known result \[2\]

\[
\delta I = \frac{1}{2\kappa} \int d^{p+1}\xi \sqrt{g} G^{\alpha\beta} \delta g_{\alpha\beta} =
\]

\[
\frac{1}{2\kappa} \int d^{p+1}\xi \sqrt{g} G^{\alpha\beta} \left[ 2\partial_\alpha X_a \partial_\beta \delta X^a + \partial_\alpha X^c \partial_\beta X^b \frac{\partial \eta_{cb}}{\partial X^a} \delta X^a \right] = 0, \tag{2}
\]

where \(G^{\alpha\beta}\) is the world Einstein tensor. The equation of motion (EOM), found from the last line of Eq.(2) by performing an integration by parts of the first term in the square bracket, throwing away the surface integral with the requirement that the variations \(\delta X\) vanish at the boundary, and then demanding that the integrand of the volume integral be zero for arbitrary \(\delta X\), is

\[
(\sqrt{g})^{-1} \partial_\alpha \left[ \sqrt{g} G^{\alpha\beta} \partial_\beta X_a \right] - \frac{1}{2} G^{\alpha\beta} \partial_\alpha X^c \partial_\beta X^b \frac{\partial \eta_{cb}}{\partial X^a} = 0. \tag{3}
\]

As \(G^{\alpha\beta}\) contains up to second derivatives of the induced metric, Eq.(3) contains up to the fourth derivative of the internal co-ordinates and is also highly non-linear. Henceforth in this paper, we will use the locally Minkowskian internal system with \(\eta_{ab} = \text{diag}(-1, 1, 1, 1, ...), \) whence the second term in the left hand side of Eq.(3) vanishes, making it the same as that discussed in Ref.(1). It reduces to the wave equation in any worldspace where \(G^{\alpha\beta} \propto g^{\alpha\beta}\), like in the constant curvature worldspace. In this case, the solutions for \(X\) ’s are harmonic functions of the \(\xi\) ’s. If we further reparametrize the wordspace in co-ordinates \(\xi\) that are themselves harmonic, we may write the EOM as \(g^{\alpha\beta} \partial_\alpha \partial_\beta X = 0\). In this form, the EOM is just of second order in derivatives, but still non-linear. A trivial solution of Eq.(3), \(G^{\alpha\beta} = 0\), are just the Einstein equations for free space. Depending on the signature of \(G^{\alpha\beta}\), particularly the relative signs of \(G^{00}\) and the \(G^{ii}\) ’s, Eq.(3) is generally found to assume the hyperbolic form if the two signs are different and the elliptic form if they are the same. These two cases can be interpreted as vacuum or matter domination respectively. As the hyperbolic to elliptic transition can be achieved by making time imaginary, this result can be taken to mean that time is actually complex, becoming Minkowskian in vacuum and Euclidean in matter. The solution, found in Ref.(1) for a vacuum dominated case, exhibits an unexpected superluminality due to the dependence of density on the world space-time. The
matter dominated case was also solved [3]. It now appears quite certain that the Universe is endowed with some form of matter that exerts negative pressure, a prominent candidate being the cosmological constant that represents the vacuum energy density of space-time. The extremely high matter pressure of the early Universe decreases with expansion, and eventually becomes dominated by however small a cosmological constant there may be. Such a matter-vacuum transition should have a bearing on some of the unexplained features of the Universe, particularly in relation to the formation of structures. A further application of this method to the string with gaussian density and tension [4] showed that the vibration of the open ends of the string can be controlled with heavy ends.

In the next Section, this method is applied to the Robertson-Walker (RW) worldspace. 

Section III describes a possible method of introducing gauge interaction in the general relativistic EH brane and looks into some consequences. The final Section makes some concluding remarks on the results and points towards some directions in which the theory can be further developed.

II. ROBERTSON-WALKER DROP

In the first step towards the interacting EH drop, we generalize the action of Eq.(1) to

\[ J = -\frac{1}{2\kappa} \int d^{p+1}\xi \sqrt{g} R - \rho_V \int d^{p+1}\xi \sqrt{g} + I_M \] (4)

where the cosmological term involving the vacuum energy density \( \rho_V \) is just the NG part and \( I_M \) is the action of matter. Requiring \( J \) to be stationary with respect to variations of the internal co-ordinates gives

\[ \delta J = \frac{1}{2\kappa} \int d^{p+1}\xi \sqrt{g} \left[ G^{\alpha\beta} - \kappa \left( \rho_V g^{\alpha\beta} - T_M^{\alpha\beta} \right) \right] \delta g_{\alpha\beta} = \frac{1}{\kappa} \int d^{p+1}\xi \sqrt{g} \left[ G^{\alpha\beta} - \kappa \left( \rho_V g^{\alpha\beta} - T_M^{\alpha\beta} \right) \right] \partial_\alpha \delta X^\alpha \partial_\beta X_a = 0. \]

Integrating the last expression by parts and then discarding the surface integral leads us to the EOM,

\[ (\sqrt{g})^{-1} \partial_\alpha \left[ \sqrt{g} h^{\alpha\beta} \partial_\beta X_a \right] = 0, \] (5)

where \( h^{\alpha\beta} = G^{\alpha\beta} - \kappa \rho_V g^{\alpha\beta} - T_M^{\alpha\beta} \), and \( T_M^{\alpha\beta} \) is the energy-momentum tensor of matter. Generally, Eq.(5) is fourth order non-linear partial differential equation that will prove to
be quite difficult to solve. But a trivial solution, $h^{\alpha\beta} = 0$, just reproduces the Einstein field equations $G^{\alpha\beta} - \kappa \rho_V g^{\alpha\beta} = -\kappa T^{\alpha\beta}_M$. Thus we can interpret $h_{\alpha\beta}$ as a tensor that describes the deviation of the spacetime from that of general relativity.

To look into some simple consequences, let us identify the internal and world times by requiring $\tau = \xi^0 = X^0 = t$, in which case Eq.(5) for $X^0$ becomes $\partial_\alpha \left[ \sqrt{g} h^{\alpha 0} \right] = 0$. If $h^{\alpha\beta}$ is a diagonal tensor, then the solution is $h^{00} = f(r, \theta, \phi)/\sqrt{g}$. In RW worldspace which is homogeneous and isotropic, the solution is $R^3 h^{\tau}_\tau = f(\xi)$ where $R$ is the scale factor. As the left hand side is only $\tau$ dependent, $f(\xi)$ has to be a constant which can be written as $\frac{\kappa}{3} \rho_B (R_0)^3$. Thus we can write the energy conservation equation in the usual Friedmann form as

$$\left( \frac{\dot{R}}{R} \right)^2 + \frac{k}{R^2} = \frac{\kappa}{3} \left[ \rho_V + \rho_M + \rho_B \left( \frac{R_0}{R} \right)^3 \right]$$  \hspace{1cm} (6)

with $\dot{R} = \partial R/\partial \tau$, whence $\rho_B$ can be understood as a dusty, pressureless, uniform background density contributed by the internal co-ordinates. The space-space part $h^{ij}_j = \left[ 2 \frac{\dot{R}}{R} + \left( \frac{\ddot{R}}{R^2} + \frac{\dot{R}^2}{R^2} + \frac{\kappa}{3} (P_M + P_V) \right) g^{ij}_j \right]$, where $P_M$ is the matter pressure and $P_V = -\rho_V$ is the vacuum pressure, can be shown to vanish by differentiating $R^3 \times \text{Eq.}(6)$ with respect to $\tau$ to determine $\dot{R}$, using the energy-momentum conservation equation $\partial (R^3 \rho)/\partial R = -3PR^2$, and making appropriate substitution from Eq.(6). Then the other independent equation also takes on the familiar form

$$\frac{\ddot{R}}{R} = -\frac{\kappa}{6} \left[ 3P + \rho \right]$$  \hspace{1cm} (7)

here the total density is $\rho = \rho_M + \rho_V + \rho_B R_0^3/R^3$ and the total pressure $P = P_M - \rho_V$ as the term with $\rho_B$ does not contribute any pressure.

Such a background $\rho_B$ could constitute a cold dark matter that was present from the very beginning. The presence of this pressureless energy density that behaves as cold dark matter from the initial time, should play a very important role in the evolution of the Universe, and on structure formation. Without the presence of pressure to counteract gravity, this energy density could easily undergo gravitational collapse from the outset. Such regions in space could form the seeds around which other matter accrete to form the structures we see.
Under these conditions the EOM of the extra internal co-ordinates, Eq.(5), are found to be \( \partial_r^2 X_r = 0, r > 3 \). These have solutions that are linear in \( \tau \), viz.,

\[
X_r(\tau, \xi) = A_r(\xi) + B_r(\xi) \times \tau,
\]

where \( A \) and \( B \) are arbitrary functions of spatial co-ordinates that have to be fixed by boundary conditions.

### III. INTERACTING EINSTEIN-HILBERT DROP

To introduce gauge field interaction into the theory, we have to first complexify the internal co-ordinates \( X \) and write \( g_{\alpha \beta} = \frac{1}{2} [\partial_\alpha X^* a \partial_\beta X_a + (\alpha \leftrightarrow \beta)] \), where \( X^* a \) are complex conjugates of \( X^a \). We are motivated to complexify the internal co-ordinates by the discussion in Sec.I that the time (and also space) co-ordinate may actually be complex. Under a global phase change of \( X \) by a factor \( e^{iq\phi} \), where \( q \) is the charge and \( \phi \) is a constant phase angle, \( g_{\alpha \beta} \) remains invariant and so does \( J \), provided \( I_M \) is also invariant. This invariance will generate a conserved charge current.

To make the theory invariant under a local U(1) transformation when \( \phi \) is a function of the \( \xi^\alpha \)'s, we can use our knowledge of electrodynamics to introduce real gauge potentials \( A_\alpha \) with minimal coupling to write

\[
g_{\alpha \beta} = \frac{1}{2} [(\partial_\alpha X^* a + i q A_\alpha X^* a) (\partial_\beta X_a - i q A_\beta X_a) + (\alpha \leftrightarrow \beta)].
\]

It can be easily checked that \( g_{\alpha \beta} \) is invariant under the simultaneous gauge transformation

\[
X \rightarrow X' = e^{iq\phi} X,
A_\alpha \rightarrow A'_\alpha = A_\alpha + \partial_\alpha \phi.
\]

Explicitly, \( \partial_\alpha X' - i q A'_\alpha X' = e^{iq\phi} [\partial_\alpha X - i q A_\alpha X] \), and the phase factor will vanish on multiplying by the conjugate. Hence, any quantity calculated from \( g_{\alpha \beta} \), like \( g \) and \( R \), are invariant. The field strength tensor \( F_{\alpha \beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha \) is also seen to be invariant under the gauge
transformation. For a gauge invariant action, we have to include the Lagrangian due to the
gauge fields, $-\frac{1}{4} F_{\alpha\beta} F^{\alpha\beta}$, and modify the action $J$ of Eq.(4) to

$$K = -\frac{1}{2\kappa} \int d^{p+1} \xi \sqrt{g} R - \rho V \int d^{p+1} \xi \sqrt{g} - \frac{1}{4} \int d^{p+1} \xi \sqrt{g} F_{\alpha\beta} F^{\alpha\beta} + I_M. \quad (10)$$

Imposing the stationarity of this action, we have

$$\delta K = \frac{1}{2\kappa} \int d^{p+1} \xi \sqrt{g} H^{\alpha\beta} \delta g_{\alpha\beta} - \int d^{p+1} \xi \sqrt{g} F^{\alpha\beta} \delta A_{\alpha} = 0, \quad (11)$$

where $H^{\alpha\beta} = G^{\alpha\beta} - \kappa (\rho V g^{\alpha\beta} - T_{M}^{\alpha\beta} - T_{F}^{\alpha\beta})$ has been modified from $h^{\alpha\beta}$ of Eq.(5) to include the energy-momentum tensor of the gauge field $T_{F}^{\alpha\beta} = F^{\alpha\mu} F_{\beta}^{\mu} - \frac{1}{4} g^{\alpha\beta} F_{\mu\nu} F^{\mu\nu}$. Now, $\delta g_{\alpha\beta} = \frac{1}{2} [\partial_\alpha \delta X^* a + iq A_\alpha \delta X^* a + iq \delta A_a X^* a] (\partial_\beta X_a - iq A_\beta X_a) + (\partial_\alpha X^* a + iq A_\alpha X^* a) (\partial_\beta X_a - iq A_\beta X_a - iq \delta A_\beta X_a) + (\alpha \leftrightarrow \beta)$. For the EOM of $X$ we take the variations in Eq.(11) to be $\delta X^* \neq 0$, $\delta X = \delta A_a = 0$, perform an integration by parts and discard the surface integral to find

$$(\sqrt{g})^{-1} (\partial_\alpha - iq A_\alpha) [\sqrt{g} H^{\alpha\beta} (\partial_\beta - iq A_\beta) X_a] = 0. \quad (12)$$

Obviously, $X^*$ will satisfy the conjugate of Eq.(12). Next, for the EOM of $A_\alpha$, we take the variations $\delta X^* = \delta X = 0, \delta A_\alpha \neq 0$. Then an integration by parts of the last integral in Eq.(11) and throwing away the surface part leaves us with the inhomogeneous Maxwell equations

$$(\sqrt{g})^{-1} \partial_\alpha [\sqrt{g} F^{\alpha\beta}] = -\frac{1}{\kappa} H^{\alpha\beta} j_\alpha \quad (13)$$

where the $p+1$ charge current of the internal co-ordinates are $j_\alpha = -\frac{iq}{2} [(\partial_\alpha X^* a + iq A_a X^* a) X_a - X^* a (\partial_\alpha X_a - iq A_a X_a)]$. The homogeneous Maxwell equations,

$$\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0, \quad (14)$$

are automatically satisfied from the definition of $F_{\alpha\beta}$. So the action $K$ generates both gravitation and electromagnetism.
As in the non-interacting case of Eq.(5), a trivial solution of Eq.(12), $H^{\alpha\beta} = 0$, is just the Einstein field equations

$$G^{\alpha\beta} - \kappa \rho_V g^{\alpha\beta} = -\kappa \left( T_M^{\alpha\beta} + T_F^{\alpha\beta} \right).$$

(15)

In this case, Eq.(13) becomes source-free. Thus, general relativity is included in this theory of interacting EH drop. General solution of Eq.(12) should describe new effects. Another trivial solution of Eq.(12) can be used to write the gauge fields as one of the internal co-ordinates. In particular, $(\partial_\beta - iq A_\beta) X_n = 0$ is solved by

$$X_n = L_n \exp \left[ iq \int A_\alpha d\xi^\alpha \right],$$

(16)

where $L_n$ is a constant. In essence, this is an equivalent way of looking at the Kaluz-Klein idea of getting electromagnetism from extra dimension. Another simple solution of Eq.(12),

$$H^{\alpha\beta} (\partial_\beta - iq A_\beta) X_a = 0,$$

(17)

is solved by all $X_a$’s having the form of Eq.(16) for all non-singular $H^{\alpha\beta}$.

For other solutions of Eq.(12) we particularly need a knowledge of $T_M^{\alpha\beta}$. As an example, let us consider a workspace of constant curvature with $C^{\alpha\beta} = -\frac{p(p+1)}{2} C g^{\alpha\beta}$ where $C$ is the curvature constant and $\rho_C = \frac{p(p+1)}{2\kappa} C$ may be considered the energy density due to curvature. Isotropic, perfect fluid matter in its co-moving co-ordinates has $T_M^{\alpha\beta} = diag(-\rho_M, P_M, P_M, \ldots)$. Assuming the electric and magnetic fields to be isotropic as well, we may write $T_F^{\alpha\beta} = diag(-\rho_F, \rho_F/p, \rho_F/p, \rho_F/p, \ldots)$ where the radiation pressure is $P_F = \rho_F/p$. Specializing further to the $p = 3$ case, we find that $H^{\alpha\beta} = \kappa \ diag(-\rho, P, P, P)$, where the total density is $\rho = \rho_V + \rho_C + \rho_M + \rho_F$ and the total pressure is $P = P_M + \rho_F/3 - \rho_V - \rho_C$. Then Eq.(13) can be explicitly written down in terms of the familiar electric and magnetic field vectors $\mathbf{E}$ and $\mathbf{B}$ and the charge four current of the internal co-ordinates $j^\alpha = (j^0, \mathbf{j})$ as

$$\nabla \cdot \mathbf{E} = -\rho j^0 \quad \text{and}$$

$$\nabla \times \mathbf{B} - \frac{1}{\sqrt{g}} \partial_\tau (\sqrt{g} \mathbf{E}) = P \mathbf{j}.$$
Eq. (18) shows coupling of the charge density of the internal co-ordinates \( j^0 \) with the total energy density at that point to produce the source for the electric field. Hence, the electric field contributes to its own source through its energy density \( \rho_F = (E^2 + B^2)/2 \), making the Poisson equation, Eq. (18), non-linear. Similar is the case with Ampere’s law, Eq. (19), where the magnetic field also contributes to its own source. In this simplified scenario, Eq. (12) can be written as

\[
\frac{1}{\sqrt{g}} \left( \frac{\partial}{\partial \tau} - iqA_0 \right) \left( \sqrt{g} \rho \left( \frac{\partial}{\partial \tau} - iqA_0 \right) X \right) + (\nabla - iqA) \cdot [P(\nabla - iqA) X] = 0.
\]

In vacuum with \( \rho = -P \) = positive constant, Eqs. (18)-(20) are exactly the same as those for the electrodynamics of a massless scalar field. Assuming positive energy density, Eq. (20) is hyperbolic under vacuum domination with negative pressure and elliptic under matter domination with positive pressure. Thus in the general case where \( \rho \) and \( P \) are both space-time dependent, the theory described here for the bosonic internal co-ordinates, is found to generalize scalar electrodynamics to a disperso-conductive medium.

A supersymmetric version of the theory, capable of generating fermionic interaction as well, can be written down just by supersymmetrizing the worldspace metric to \( g_{\alpha \beta} = \frac{1}{2} \left[ \Pi_\alpha^a \Pi_\beta^b \eta_{ab} + (\alpha \leftrightarrow \beta) \right] \) with \( \Pi_\alpha^a = (\partial_\alpha - iqA_\alpha) X^a + \theta \Gamma^a (\partial_\alpha - iqA_\alpha) \theta \) where \( \theta \) are the fermionic co-ordinates and the \( \Gamma \)'s are the \( n+1 \) dimensional Dirac matrices. The possibility of gauging such a supersymmetric theory with U(1), or with even larger symmetries like \( SU(n+1) \) or a \( n+1 \)dimensional Poincarè type is also open. The mechanism for an effective breaking of this large symmetry into a 3+1 Lorentz group, low energy representation of the remaining components, the verifiable predictions, dimensionality of the internal space, the results of quantization and supersymmetrization, etc., are some open problems.

**IV. CONCLUSIONS**

In this paper, some results of generalizing the brane action from NG to EH action of the worldspace has been investigated. The NG action is contained as a special, constant-
curvature, case in this general relativistic theory. The NG part can also be included in the general action as an effective cosmological term, with the cosmological constant appearing as a Lagrange multiplier that constrains the extremization of the curvature simultaneously with extremized volume. Applying this method to the RW space gives an EOM that is exactly similar to what we are familiar with, except that a pressureless matter is introduced. This pressureless background density, contributed by the internal co-ordinates that grow linearly with time, is present from the very beginning. So it could account significantly as a cold dark matter that is present throughout the evolution of the Universe.

Complexifying the internal co-ordinates allows us to introduce the charge and consider interaction amongst the internal co-ordinates. The knowledge of electrodynamics was used as hindsight to gauge this theory of interacting EH drop with U(1) symmetry. The equations of motion that result are generalized partial differential equation of either the hyperbolic or elliptic type for the internal co-ordinates, and Maxwell equations with the gauge field self-interaction contributions to the source terms for the gauge fields. Under simplifying assumptions like constant-curvature worldspace and isotropic energy-momentum tensors of matter and electromagnetic field in $p = 3$ worldspace, the EOM of the internal co-ordinates reduce to very familiar form that combines two important equations of physics into one. In a worldspace that is dominated by vacuum, with the total pressure negative, the equations are just those of scalar electrodynamics, albeit, modified a little with temporal damping due to time dependence of the density, and spatial dispersion due to pressure gradient. The internal co-ordinates behave as charged, massless scalar field in a disperso-conductive medium, interacting with the electromagnetic field and gravity. Under matter domination with positive total pressure, this EOM becomes an elliptic type. As the hyperbolic to elliptic transition can be made by making the worldtime imaginary, these results further support the author’s proposition that time is in fact complex, becoming Minkowskian under vacuum domination and Euclidean under matter domination. In an earlier investigation of the non-interacting EH drop in Ref.(1), the author had shown that the conductivity that arises due to the time dependence of the density of the medium interacts with the internal
co-ordinate field to give it a tachyonic behaviour. When the worldtime is made imaginary, the conductivity also becomes imaginary. Thus the imaginary conductivity will behave as mass of the scalar field. These conclusions also hold in the theory of the interacting general relativistic drop presented in this paper. In the complexified worldtime plane, the real axis represents the vacuum dominated Minkowskian time, while the imaginary axis represents matter dominated Euclidean time.

Another important result is that the EOM of this theory, Eq.(12), has a trivial solution Eq.(15) which is general relativity. In this case the Maxwell equations become source-free, indicating that general relativity is the vacuum solution of this theory of interacting EH drop. Application of this theory to any other situation besides this trivial case will describe physics at a level that is wider than general relativity and electrodynamics. Another trivial solution can be used to relate one of the internal co-ordinates to the gauge potentials. Supersymmetric generalization is also immediate, but the necessary Wess-Zumino and other terms, as well as the consequences have to be worked out. Only the classical theory has been described here, so its consistency under quantization, as well as the quantum effects, have to be investigated.

In this paper, the theory has just been gauged with U(1) symmetry. It is also possible to gauge it with any other relevant group like SU(n+1) or a generalized n+1 dimensional Poincarè type group. The symmetry breaking mechanism, resultant low energy representations, and further predictions of this theory will be investigated in future work.
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