Pulsed Raman output coupler for an atom laser

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We theoretically study a pulsed stimulated two-photon Raman outcoupler for an atom laser using a full three-dimensional description. A finite-temperature trapped Bose-condensed atomic gas is treated self-consistently by the Hartree-Fock-Bogoliubov equations. The model is closely related to a recent experiment on optical outcoupling [E.W. Hagley \textit{et al.}, Science \textbf{283}, 1706 (1999)]. We analyze the momentum distribution of the output atoms and show how the output beam may be used as a probe of the quantum state for the trapped atomic gas and how it could be engineered and controlled in a nonlinear way.

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The generation of Bose-Einstein condensates (BECs) of weakly interacting atomic gases has the potential to open a whole new area in atom optics; coherent atom optics. To this end, a well-controlled source of coherent matter waves needs to be developed. Rudimentary coherent matter wave sources (“atom lasers”) have been engineered experimentally \cite{1,2}, but there is a need for sustained development. In this Letter we present a quantum-mechanical, three-dimensional (3D), \textit{ab initio} treatment of a pulsed Raman outcoupler for a BEC. We demonstrate how the output of the atom laser can be engineered and how it may be used as a probe of the quantum and thermal properties of the BEC. We use a well-tested self-consistent field-theoretical treatment to describe the finite-temperature, multi-mode BEC (the analogy of the cavity in a conventional laser), and well controlled and detailed approximations for the output coupling to a quasicontinuum of states.

A fundamental difference between optical and atom lasers is that atoms interact while photons do not. This significantly complicates the theoretical analysis of atom lasers, as compared to optical lasers. Studies using classical mean fields (e.g. Gross-Pitaevskii equation) disregard thermal fluctuations, decoherence, and information about quantum statistics. Due to the inevitable computational difficulty of a complete 3D analysis, there has not existed before a rigorous 3D, multi-mode, finite-temperature description for an outcoupler of an atom laser, despite a notable research activity on atom laser output theory in recent years \cite{6–12}.

We study a pulsed outcoupling which is amenable to a simpler theoretical description than a general continuous coupling, since it allows us to reach the Markovian limit and to ignore any nontrivial coherences between the trapped and free modes. A non-Markovian output may emerge in the continuous case \cite{7,12}, which, in a multimode description for the trapped gas, can lead to very complicated nonequilibrium dynamics.

We start from the Hartree-Fock-Bogoliubov (HFB) \cite{3,4} field theory for the Bose-condensed atoms, harmonically trapped in internal level $|c\rangle$, in thermodynamic equilibrium. The Hamiltonian density for the trapped atomic gas alone reads

\begin{equation}
\mathcal{H}_1 = \psi_c^\dagger (H_{cm}^{(c)} - \mu^{(c)}) \psi_c + \frac{g_c}{2} \psi_c^\dagger \psi_c^\dagger \psi_c \psi_c, \quad (1)
\end{equation}

where $H_{cm}^{(c)} \equiv -\hbar^2 \nabla^2 / (2m) + m \omega^2 r^2/2$ is the center-of-mass Hamiltonian in an isotropic trap with frequency $\omega$. The interaction strength $g_c \equiv 4 \pi a_s \hbar^2 / m$ is given in terms of the s-wave scattering length $a_s$, and $\mu^{(c)}$ denotes the chemical potential. In the HFB theory we decompose the condensate and the noncondensate fractions into mutually orthogonal subspaces and expand the field operator

\begin{equation}
\psi_c(r) = \Phi(r) \alpha_b + \sum_n [u_n(r) \alpha_n - v_n(r) \alpha_n^\dagger], \quad (2)
\end{equation}

where the quasiparticle creation and annihilation operators $\alpha_b^\dagger$ and $\alpha_n$ obey the standard bosonic commutation relations \cite{3}. The BEC wave function $\Phi(r)$ satisfies the generalized Gross-Pitaevskii equation with the thermal population acting as an external potential \cite{4}. We self-consistently solve for the mode functions $u_n$ and $v_n$ and the BEC density profile (as explained in detail in Ref. \cite{4}) in such a way that the Hamiltonian $\mathcal{H}_1$ is diagonal in the Popov approximation (where two-particle correlations due to the anomalous average of the fluctuating field operator are neglected). The HFB-Popov treatment has been exceedingly successful in describing the properties of BECs in both isotropic traps \cite{4} and, at relatively low temperatures, also in anisotropic traps \cite{5}.

We study the outcoupling of the trapped quantum degenerate Bose-Einstein gas from internal sublevel $|c\rangle = |g,m\rangle$ to an untrapped level $|o\rangle = |g,m^{''}\rangle$. We concentrate on a situation where the atoms in level $|o\rangle$ do not experience a magnetic trapping potential. As in the NIST experiments \cite{2} we consider a two-photon output coupling via two noncopropagating lasers: Level $|c\rangle$ is optically coupled to an electronically excited state $|e\rangle = |e,m^{''}\rangle$ by the laser field $E_1$ with the frequency $\Omega_1$, and state $|e\rangle$ is coupled to level $|o\rangle$ by the driving field $E_2$ with the frequency $\Omega_2$. The atomic transition frequencies for $|c\rangle \rightarrow |e\rangle$ and $|e\rangle \rightarrow |o\rangle$ are $\omega_1$ and $\omega_2$.
\( \omega_{oc} \equiv \omega_1 - \omega_2 \), respectively. The lasers are off-resonant, so that the population in level \( |e\rangle \) remains small and spontaneous emission may be ignored. In the rotating wave approximation (rwa) it is useful to define the slowly varying field operator \( \hat{\psi}_o = e^{i(\Omega_1 - \Omega_2)t} \hat{\psi}_o \) for level \( |o\rangle \) in the Heisenberg picture and the positive frequency components \( \hat{E}_o^+ = e^{i\Omega/2} \hat{E}_o^+ \), where \( \hat{E}_o^+ \equiv \hat{E}_o^+ (r,t)/2 \). In the general formalism we allow for the lasers to be focused at the center of the trap, although the main results in this paper are obtained for nonfocused lasers with a Gaussian time profile: \( \hat{E}_o^+ (r,t) \propto e^{-t^2/(2\beta^2)} e^{ikr \cdot r} \). We set the positive \( z \) axis to be the direction of the momentum kick \( q \equiv k_1 - k_2 \). For pulsed outcoupling with strong momentum kick perpendicular to gravity [2], gravity is not expected to play as important a role as in a rf output [1]. Nevertheless, in the diagnostics of the trapped BEC, it would be advantageous to cancel completely the effects of gravity. We assume this is done, e.g., by means of a pulsed magnetic field gradient or an optical potential during the outcoupled pulse and ignore gravity in the following analysis.

We adiabatically eliminate the excited state and keep only the terms of first order in \( 1/\Delta_1 \), the inverse of the one-photon detuning \( \Delta_1 \equiv \omega_1 - \Omega_1 \). With the outcoupling the Hamiltonian density is \( \hat{H}_1 + \hat{H}_2 \), where

\[
\begin{align*}
\hat{H}_2 & = \hat{\psi}_o^\dagger (\hat{H}_{cm}^{(o)} - \mu^{(o)} - \hbar \delta_2 + \hbar \delta_{oc}) \hat{\psi}_o + \hbar \delta_1 \hat{\psi}_o \hat{\psi}_c \left( \hat{\psi}^\dagger \hat{\psi}_c + \text{H.c.} \right) - \hat{\psi}_c^\dagger \hbar \delta_1 \hat{\psi}_c
\end{align*}
\]

and \( \delta_{oc} \equiv \omega_{oc} - \Omega_1 - \Omega_2 \) is the two-photon detuning, \( \hat{H}_{cm}^{(o)} = -\hbar^2 \nabla^2/(2m) \), and the light-induced level shifts \( \delta_l \) and the Rabi coupling \( k \) are defined by

\[
\delta_l (r,t) = \frac{\hat{\psi}_o^\dagger (r,t)^2 \hbar^2}{2\Delta_1}, \quad \kappa (r,t) = \frac{\hat{\psi}_o^\dagger (r,t) \hat{\psi}_c (r,t) \hbar^2}{2\Delta_1}.
\]

Here the reduced dipole matrix element \( d_1 (d_2) \) for the atomic transition \( |c\rangle \rightarrow |e\rangle \) \( (|o\rangle \rightarrow |e\rangle) \) also contains the corresponding nonvanishing Clebsch-Gordan coefficient.

By inserting Eq. (2) into the coupling terms proportional to \( k \) in Eq. (3) we immediately observe that, despite introducing the rwa, the creation of one output atom may be associated with the annihilation \( \hat{\psi}_o^\dagger \alpha_n \) or creation \( \hat{\psi}_c^\dagger \alpha_n \) of a quasiparticle in a trap [11]. Consequently, even at \( T = 0 \), and in the weak coupling limit, the outcoupling of an atom can create elementary excitations in the quasiparticle vacuum demonstrating how the full multimode approach with quasiparticle excitations is unavoidable in obtaining the complete description of the outcoupling process.

We consider a low density for the outcoupled atoms so that \( \mu^{(o)} \simeq 0 \) and we may ignore the term proportional to \( g_o \) in Eq. (3). Moreover, we expand the outcoupling field in the plane wave basis \( \hat{\psi}_o (r) = \sum_k \langle r | k \rangle \hat{\psi}_k \), with \( \langle r | k \rangle \equiv V^{-1/2} \exp (i k \cdot r) \), where \( \alpha_k \) is the annihilation operator for the mode \( k \). We may then obtain the Heisenberg equation of motion for slowly varying output mode operators \( \hat{\psi}_k \equiv \exp [i (\epsilon_k + \omega_{oc})t] \hat{\psi}_k \) with \( \epsilon_k = \hbar k^2 / (2m) 
\]

\[
\hat{\delta}_k = i \sum_{k'} \xi_{kk'} (t) \hat{\psi}_{k'} + i i^{(k')} (t) \psi_{\psi_{k'}} (t) (\epsilon_k + \delta_{oc})t.
\]

We have defined the coupling matrix elements, analogous to the Franck-Condon factors, by \( i^{(k)} (t) \equiv \langle k | \zeta (r,t) f \rangle \) and \( \xi_{kk'} (t) \equiv \langle k | \zeta (r,t) g_{oc} n (r) / \hbar | k' \rangle e^{i (\epsilon_k - \epsilon_{k'}) t} \). The collisional interaction between the trapped and output atoms is treated semiclassically: \( \psi_{\psi_{k'}} \psi_{\psi_{k'}} \rightarrow \psi_{\psi_{k'}} \psi_{\psi_{k'}} \).

For the Markov and Born approximation to be valid we ignore the depletion of the trapped modes, requiring that the light only weakly perturbs them, \( \delta_1 \ll \omega, \) and that we only couple out a small fraction of atoms, \( \beta \delta_1 \ll 1 \). In the first Born approximation we write \( \alpha_n (t) \simeq \alpha_n (\infty) e^{i\omega_{ct} t} \). For a continuous outcoupling the Markovian limit would be more restrictive, since then the atoms should also have the interaction region (the trapped gas of size \( R \)) much faster than any timescale \( \tau \) required to form coherences between the trapped and free modes, indicating \( \tau h / m \gg R \). The timescale \( \tau \) determined by the collision rate \( \tau \sim h / (g_{oc} n (r)) \) or by the coupling rate \( 1/\kappa \) necessitates a very strong laser kick \( q \).

In Eq. (5) \( \xi_{kk'} \) describes the coupling between the different output modes as a result of the laser focusing and the interactions with the trapped atoms. For short pulses, \( \beta \ll h / (g_{oc} n (r)) \), or when \( h^2 q / m \gg R g_{oc} n (r) \), the atomic collisions between the trapped and free atoms can be ignored during the laser coupling, and we may set \( g_{oc} \simeq 0 \) in \( \xi_{kk'} \) [13]. Moreover, if also the spatial variation of the laser intensity is negligible over the atomic cloud (nonfocused case), the coupling between the different \( k \) modes vanishes due to the orthogonality of the plane waves and we obtain \( \xi_{kk'} \simeq \delta_2 (k - k') \). In this case the solution of Eq. (5) reads:

\[
\lim_{t \to \infty} \tilde{\psi}_k (t) / 2i \pi \left[ \sum_n \left[ \alpha_n \upsilon_{n}^{(k)} \delta_{\varphi} (\epsilon_k + \delta_2 + \omega_n) - \alpha_n \upsilon_{n}^{(k)} \delta_{\varphi} (\epsilon_k + \delta_2 + \omega_n) \right] + \alpha_n \upsilon_{n}^{(k)} \delta_{\varphi} (\epsilon_k + \delta_2 + \omega_n) \right].
\]

Here the operators \( \alpha_n \) and \( \alpha_n ^\dagger \) are evaluated at the thermodynamic equilibrium \( t \rightarrow -\infty \) and \( \delta_{\varphi} \equiv \delta_{oc} - \mu^{(c)} / \hbar \). The dynamics of the coupling matrix elements has been factored out: \( \upsilon_{n}^{(k)} = e^{i\beta^2 / \beta^2 \nu^{(k)} (t)} = \nu^{(k)} (0) \). We also introduced a delta-function with a finite width \( \Gamma \):

\[
\delta_{\varphi} (\epsilon_k + \delta_2 + \omega_n) \equiv \Gamma^{-1} e^{-\Gamma^{-1} t - x^2 / 2 \Gamma}.
\]

In the limit of long pulses \( \beta \rightarrow \infty \) the width is zero and \( \delta_{\varphi} (x) \) becomes the Dirac delta function.

In Eq. (6) we have ignored the light-induced level shifts in the delta functions by assuming a weak coupling \( \delta_1 \ll \omega_n, \epsilon_k \). The physical interpretation of Eq. (6) is straightforward. The expression represents the output mode amplitudes after the laser pulses, \( \beta \ll t \). The conservation of the momentum is expressed in the coupling matrix elements \( \nu \) between the modes. The integrand of \( \nu \) is nonnegligible over some finite volume, with the characteristic length scale \( R_0 \), resulting in a momentum uncertainty \( \Delta k \sim 1 / R_0 \). The delta functions dictate the energy conservation. Due to the finite duration of the pulse
the delta functions have acquired a finite width, inversely proportional to the pulse length; again a direct consequence of the Heisenberg uncertainty relation. The sign of the quasiparticle energy in the delta functions is different in the terms proportional to $u_n$ and $v_n^*$ corresponding to the annihilation and creation of an excitation associated with the outcoupling of an atom. In the limit of a short pulse, $\beta^2(k_0 + \delta_{\text{ac}} \pm \omega_n)^2/4 \ll 1$, the delta functions become constant, and we obtain $\langle k_0 \rho \rangle \approx \beta \pi^{1/2} \rho_{\nu_0}(0)$, where $\psi_n$ is evaluated in the initial state $t \rightarrow -\infty$. Then the output after the pulse is proportional to a one-particle correlation function of the trapped atoms:

$$\langle \psi^\dagger_c(r) \psi_c(r') \rangle. \quad (7)$$

For a homogeneous BEC the output mode density can measure a Fourier component of the standard first-order spatial coherence function. Equation (7) should be contrasted to the Bragg spectroscopy which probes two-particle correlations of the trapped atoms [14].

In a more general case the output mode density is obtained from Eq. (6). If the collisions between the trapped and the output atoms cannot be ignored, or if the laser is focused, the result (6) is no longer valid, and we need to solve the coupled set of differential equations (5). In general, evaluating the output field boils down to calculating the dynamics and the coupling matrix elements $\nu_{u_n}$, etc. In a spherically symmetric trap we may expand the mode functions $u_n(r) = \sum_{n\tilde{m}l} \tilde{C}_{n\tilde{m}l} \phi_{n\tilde{m}}(r) Y_{\tilde{m}}(\theta, \phi)$ [similarly for $v_n^*(r)$] in terms of the radially symmetric wave functions $\phi_{n\tilde{m}}(r)$ and the spherical harmonics $Y_{\tilde{m}}(\theta, \phi)$. The symmetry is simplified in the case of nonfocused lasers or when the focused lasers are parallel. Then the outcoupling is cylindrically symmetric around the direction of the momentum kick ($z$ axis).

Consequently, the Rabi frequencies $\kappa(r, \varphi)$ are independent of the polar angle $\varphi$ which can be integrated analytically: $\kappa_{n\tilde{m}l} \equiv \langle \kappa(r, \varphi) u_n \rangle = \langle k_0, k_z | \kappa(r, \varphi) \phi_{n\tilde{m}} \rangle$, where we may write $\langle r, z | k_0, k_z \rangle \equiv A_{n\tilde{m}l} P_{\tilde{m}}^l(z/r) J_{\tilde{m}}(k_0 r) e^{ik_z z}$, with $p^2 = x^2 + y^2$. Here $J_{\tilde{m}}$ denotes the $\tilde{m}$th order Bessel function of the first kind, $P_{\tilde{m}}$ the associated Legendre function, and $A_{n\tilde{m}l}$ complex coefficients. As a result, the momentum distribution of the atom laser is characterized by the two components, $k_z$ and $k_0$, representing the momentum along the laser kick and perpendicular to it.

In the numerical calculations we consider an outcoupling process with nonfocused lasers from an isotropic 3D trap with the trapping frequency $\omega = 2\pi \times 1000$Hz by evaluating the operator expectation values for the output mode density from Eq. (6). The BEC profile, the thermal population, the excitation frequencies $\omega_n$, and the quasiparticle mode functions, $u_n$ and $v_n^*$, are solved for 10000 $^{23}$Na atoms with the scattering length $a = 2.8$nm and the coupling strength $\tilde{\kappa} \equiv \kappa(0, 0) = 0.05\omega$. This involves a self-consistent evaluation of about 1000 excitation frequencies and the respective mode functions [4].

The $(2 \times 2)$D coupling matrix elements $\kappa_{n\tilde{m}l}$ are numerically integrated for $(n, l, \tilde{m})$. With 10000 atoms the number of outcoupled atoms in some examples remains small, but it becomes experimentally reasonable if we scale the results for experiments on several millions of atoms.

At $T = 60$K we obtain 7070 BEC atoms in the trap with the corresponding 2930 atoms in the noncondensate fraction. In Fig. 1 we display the momentum distribution of the outcoupled atoms corresponding to the different coupling processes proportional to $\Phi$, $u_n$, and $v_n^*$. We have chosen the detuning $\delta_{\text{ac}} = -20\omega$, the pulse length $\beta = 0.5/\omega$, and the momentum kick $q = 0.1 \times 4\pi/\lambda$, where $\lambda = 589$nm represents the wavelength of light for typical optical transitions in $^{23}$Na. The momentum distributions corresponding to the three different mechanisms are clearly separated due to the different resonance energies. This results in the spatial separation of the three output fields, which could be used to probe the quantum properties of the trapped gas.

![FIG. 1. The momentum distribution of the outcoupled atoms along the direction of the momentum kick ($k_z$) and perpendicular to it ($k_0$) representing the coherent output from the BEC (left), the annihilation (middle), and the creation (right) of a quasiparticle in the trap. The corresponding processes are peaked at $k_0 \simeq 5.6$, 6.8, and 5.9 resulting in a spatial separation of the propagating output beams.](image-url)
Whether the output distribution is more determined by the optimized energy or momentum conservation depends on the spatial confinement of the trap and on the duration of the laser pulses $\beta$. In Fig. 2 we show the momentum distribution for atoms coupled out of the BEC mode for three different values of $\beta$ describing a transition from well-optimized momentum conservation to well-optimized energy conservation.

As the final example we investigate the number of out-coupled atoms $N_o$. In Fig. 3 we show the number of coherently and incoherently outcoupled atoms as a function of the pulse length for $q = 0.1 \times 4\pi/\lambda$ and for different values of the detuning. We observe that $N_o$ may also decrease with increasing $\beta$. This seemingly counter-intuitive result can be explained by noting that the uncertainty of energy conservation decreases with increasing pulse length. The finite-width delta function in Eq. (6) then approaches the Dirac delta function and a nonresonant coupling becomes suppressed. Even very small changes of the laser detuning $\delta_{oc}$ can result in large modifications of the output. Therefore, e.g., externally induced level shifts could possibly provide a strongly non-linear control of the atom laser output.

The essential features of the coherent output in Fig. 3 may be understood by deriving an approximate expression for the total number of atoms, coherently coupled out from the BEC, by assuming a Gaussian density profile for the BEC with the width $R$ along the laser kick:

$$N_o \sim N\kappa^2 \beta^2 \chi \exp \left[-\chi^2 \beta^2 (\epsilon_q + \delta_{oc})^2 / 2 \right], \tag{8}$$

where $\chi \equiv (qR)^2/[(qR)^2 + 2(\beta\epsilon_q)^2]$. In the derivation of Eq. (8) we assumed that the width of the BEC density profile perpendicular to the laser kick satisfies $R_y/l \gg \beta\omega\chi$. The approximate expression (8) describes well the functional dependence of the output. In Fig. 3 the best fit is obtained by choosing $R \sim 0.5R_{TF}$ for $\delta_{oc} < -10\omega$ and $R \sim 0.6R_{TF}$ for $\delta_{oc} > -10\omega$, where the Thomas-Fermi radius $R_{TF} \equiv (l(15N\kappa_c/l)^{1/3}$. For short pulses $2(\beta\epsilon_q)^2 \ll (qR)^2$, we obtain $\chi \simeq 1$, and the output depends quadratically on the pulse length $N_o \sim N\kappa^2 \beta^2$. For long pulses, $2(\beta\epsilon_q)^2 \gg (qR)^2$, the output is linear with $\beta$: $N_o \sim N\kappa^2 \beta qR/(\sqrt{2}\epsilon_q) \exp \left[-qR(1 + \delta_{oc}/\epsilon_q)/2 \right]^2$. There is also an interesting intermediate regime, where $(qR)^2 \sim 2(\beta\epsilon_q)^2$ and $\chi^2 \sim 1/2$. Provided that the coupling is sufficiently off-resonant, so that we simultaneously have $2 \lesssim \beta(\epsilon_q + \delta_{oc})$, the exponential factor in Eq. (8) becomes important, and the number of output atoms may rapidly decrease as a function of the pulse length for fixed coupling strength. This behavior results from energy conservation and the very narrow energy width of the BEC and is a consequence of the genuine quantum nature of the BEC, as compared to thermal atomic ensembles with a broad energy distribution.

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![FIG. 2. The momentum distribution of the coherently out-coupled atoms from the BEC for three different values of the pulse duration $\epsilon$, with fixed $\delta_{oc} = -10\omega$, $N = 7073$, and $q = 0.05 \times 4\pi/\lambda$. With $\beta\omega = 0.6$ (left) momentum conservation is well-optimized representing a peak at $k_z \simeq q \simeq 2.2/l$. For longer pulses energy conservation becomes better optimized resulting in raising second peak at $k_z \simeq 2q$. With $\beta\omega = 0.8$ (middle) the two peaks exhibit equal heights. With $\beta\omega = 1.0$ (right) energy conservation dominates.

![FIG. 3. The number of coherently out-coupled atoms from the BEC mode (left) as a function of the pulse length $\beta$. Beginning from the lowest curve, the value of the detuning changes evenly from $\delta_{oc}/\omega = -20$ to $-10$ ($\epsilon_q \simeq 10\omega$). The coherent coupling dominates the output close to $T = 0$. The right plot shows the coherently outcoupled atoms (solid lines) and the total number of outcoupled atoms at $T = 60nK$ (dashed lines) for $\delta_{oc}/\omega = -16$, $-14$, and $-10$.

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