Berry phase theory of the Anomalous Hall Effect: Application to Colossal Magnetoresistance Manganites

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We show that the Anomalous Hall Effect (AHE) observed in Colossal Magnetoresistance Manganites is a manifestation of Berry phase effects caused by carrier hopping in a non-trivial spin background. We determine the magnitude and temperature dependence of the Berry phase contribution to the AHE, finding that it increases rapidly in magnitude as the temperature is raised from zero through the magnetic transition temperature $T_c$, peaks at a temperature $T_{\text{max}} > T_c$ and decays as a power of $T$, in agreement with experimental data. We suggest that our theory may be relevant to the anomalous Hall effect in conventional ferromagnets as well.

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The Anomalous Hall Effect (AHE) is a fundamental but incompletely understood aspect of the physics of metallic ferromagnets \textsuperscript{7}. The Hall effect is the development of a voltage which is transverse to an applied current; the constant of proportionality is the Hall resistivity $\rho_H$. In non-magnetic materials, $\rho_H$ is proportional to the magnetic induction $B$ and its sign is determined by the carrier charge. Many ferromagnets however exhibit an anomalous contribution to $\rho_H$ which is proportional to the magnetization $M$, thus

$$\rho_H = R_0 B + R_s M \quad (1)$$

The definition of $R_s$ implies a sample with demagnetization factor $N \approx 1$ so that $M$ represents the spin polarization in the material and the physical dipolar magnetic field caused by the ferromagnetically aligned spins cancels. The AHE thus involves a coupling of orbital motion of electrons to the spin polarization and must involve spin-orbit coupling.

The conventional theoretical understanding of $R_s$ is based on a skew scattering mechanism which is a third order process involving interference between spin-orbit coupling (to first order) and spin flip scattering (to second order) \textsuperscript{4}.\textsuperscript{7}. In conventional ferromagnets, this theory yields values of $R_s$ two orders of magnitude smaller than experimental data \textsuperscript{3}. Also, some papers including Ref. \textsuperscript{6} use a spin-orbit term involving coupling to the dipole fields produced by the spins which would apparently vanish for demagnetization factor $N = 1$.

Recently, several groups measured the Hall resistivity $\rho_H$ of epitaxial films \textsuperscript{6} and single crystals \textsuperscript{7} of the ‘colossal magnetoresistance’ (CMR) material $La_{0.7}Ca_{0.3}MnO_3$. These materials involve carriers derived from Mn $e_g$ symmetry d-levels which may move through the lattice but are strongly ferromagnetically coupled to localized ‘core spins’ derived from Mn $t_{2g}$ symmetry orbitals. The coupling is so strong that it may be taken to be infinite: a carrier on site $i$ must have its spin parallel to the core spin on site $i$. The spin of the mobile carrier is thus quenched, but its amplitude to hop from site $i$ to site $j$ is modulated by a factor involving the relative spin states of core spins on the two sites. This physics is called ‘double-exchange’ \textsuperscript{6}.

The Hall effect measurements found that in CMR materials $\rho_H$ was of the form of Eq.\textsuperscript{1} with $R_0$ hole-like and $R_s$ electron-like. $R_s$ becomes evident above 100K, increases sharply around $T_c$, peaks at a temperature $T_{\text{max}} \approx T_c + 30K$ and decreases slowly at high temperatures. $R_s$ is proportional to the zero-field resistivity from 200K to 360K. This cannot be explained via the conventional skew scattering theory because the quenching of the carrier spin means the required spin-flip process cannot occur. That the sign of $R_s$ is opposite to $R_0$, and that $R_s$ peaks above $T_c$ are also surprising.

In this paper, we present a new theory for anomalous Hall effect. It is inspired by the physics of CMR, but we suggest that a simple generalization could apply to conventional ferromagnets as well. Our mechanism is based on the observation that a carrier moving in a topologically non-trivial spin background acquires a ‘Berry phase’ \textsuperscript{8} which affects the motion of electrons in the same way as does the phase arising from a physical magnetic field \textsuperscript{9} and has been argued to influence the Hall effect in high-$T_c$ superconductors \textsuperscript{10}. We shall show that this Berry phase can in the presence of spin-orbit coupling give rise to an AHE similar in magnitude and temperature dependence to that observed in CMR materials. This idea was advanced in a preprint \textsuperscript{11}. The present paper treats skyrmion physics and the high tem-
perature phase in more detail, and presents a more gen-
eral treatment valid for any half-metallic ferromagnet. It
supercedes the previous work.

After the work presented here was completed two
preprints appeared presenting a discussion of the physics
introduced in [1] along with new data which are argued
to confirm the basic picture [2].

We begin our analysis by writing down the DE model
for electrons hopping on a lattice, coupled ferromagneti-
cally by atomic exchange $J_R$ to core spins $S_c$ and subject
to a spin-orbit coupling $H_{so}$ and to other interactions
represented by · · ·:

$$H = -t \sum_{<ij>,\alpha} (d_{i\alpha}^\dagger d_{j\alpha} + h.c.) - \mu \sum_{<ij>,\alpha} d_{i\alpha}^\dagger d_{j\alpha}$$

$$-J_H \sum_i S_{ic} \cdot d_{i\alpha}^\dagger \sigma_{\alpha\beta} d_{i\beta} + H_{so} + \cdots$$  \hspace{1cm} \text{(2)}

The orbital indices of two $e_g$ orbitals are suppressed in
Eq. 2 because they are not essential in this paper [3].
$H_{so}$ in Eq.3 arises from the fundamental spin-orbit coupl-
ing $H_{so} \sim \langle \vec{k} \times \vec{\sigma} \rangle \cdot \nabla V_c$ ($V_c$ is the crystalline potential and
$\vec{\sigma}$ are the Pauli spin matrices). The projection of
$H_{so}$ onto tight binding bands leads to many terms [4];
the one of relevance here is

$$H_{so} = \frac{i \lambda_{so}}{4} \sum_i \epsilon^{abc} \sigma_{\alpha\beta}^i d_{i\beta}^\dagger (d_{i\alpha}^\dagger + b_{\alpha\beta} + d_{i\beta} - d_{i\alpha})$$  \hspace{1cm} \text{(3)}

The crucial physics of the manganites is a strong
Hunds coupling $J_H \gg t$. In the following, we study the
$J_H/t \rightarrow \infty$ limit, and comment on the $J_H < t$
in the conclusion. It is convenient to parameterize $S_{ic}$ by polar
angles $\theta_i, \phi_i$ and to express the operator as

$$d_{i\alpha} = d_{i\alpha} \tilde{z}_{i\alpha}, \quad \tilde{z}_{i\alpha} = |\tilde{n}_i| > = \left( \begin{array}{l}
\cos \frac{\theta_i}{2} e^{i\phi_i} \\
\sin \frac{\theta_i}{2}
\end{array} \right)$$  \hspace{1cm} \text{(4)}

where $\tilde{z}_{i\alpha}$ is the $SU(2)$ coherent spin state along $\tilde{n}_i =
\epsilon_{i\alpha\beta}^\dagger \tilde{\sigma}_{\alpha\beta} \tilde{z}_{i\beta}$. Using Eq.4 in the presence of an external magnetic
field $\vec{H} = \nabla \times \vec{A}$, we can write the action

$$S = \int d^3x \left[ \sum_i i \epsilon_{i\alpha\beta} a_{i\beta}^\dagger A_{\alpha\beta} + \sum_i d_{i\alpha}^\dagger (\partial_\tau - i \alpha_0 - \mu) d_i - t \sum_{<ij>,\alpha} \left( \frac{1 + \tilde{n}_i, \tilde{n}_j}{2} \right)^{1/2} [e^{i(\alpha_0 \tau + \frac{1}{2} \partial_\tau A_{i\alpha})} d_i^\dagger d_{i+\delta} + h.c.] + g \mu_B \vec{H} \cdot \sum_i \tilde{n}_i (S_{ic} + \frac{1}{2} d_i^\dagger d_i) \right] + H_{so}$$  \hspace{1cm} \text{(5)}

where $n_c = 2S_c$, $\mu$ is the chemical potential fixing the
electron density $< d_i^\dagger d_i > = x$, $a$ is the lattice constant,
$\alpha_0 = iz^\dagger \partial_z z$ and $\alpha_{i,\delta}$ (defined below) are the internal
gauge fields generated by the spin configurations.

The term involving $t$ shows explicitly how the electron
hopping is affected by the nearest neighbor spin overlap
factor $z_{i\alpha}^\dagger z_{j\alpha} = < \tilde{n}_i, \tilde{n}_j > \sim \frac{e^{i(\tilde{n}_i, \tilde{n}_j)/2}}{1 + \tilde{n}_i, \tilde{n}_j}^{1/2}$. The phase factor $\Phi(\tilde{n}_i, \tilde{n}_j) = a_{i,\delta}$ ($j = i + \delta$) is the solid
angle subtended by the three unit vectors $\tilde{n}_i, \tilde{n}_j$ and $\tilde{z}$
on the unit sphere. In the continuum $a_{i,\delta} \rightarrow z^\dagger \partial_z z$. $a_{i,\delta}$
affects the motion of an electron just as does a external
electromagnetic field [9]. A time dependent $\tilde{a}$ was shown
by Nozieres and Lewiner [10] to lead to a time dependent
AHE when the magnetization precessed. Here we show
leads to a static AHE when topologically nontrivial spin
configurations are considered.

We now use Eq. 3 to calculate $R_s$, considering the low,
high and critical temperature regimes separately.

The low temperature regime: here the core spins are
slowly varying on the lattice scale and the system is
metallic, so we can take the continuum limit, treat the
core spins classically and obtain

$$S = \beta \int d^3x \left[ \frac{\rho_s}{2} (\partial_\tau \tilde{n})^2 - M_0 \tilde{H} \cdot \tilde{n} + S_{cl} + H_{so} \right]$$  \hspace{1cm} \text{(6)}

where $\rho_s \sim txa^{2-d}$ is the spin stiffness, $M_0 = g \mu_B |S_c +
\frac{1}{2} x|a^{-d}$ is the magnetization, and $S_{cl}$ is the action of spin-
less fermions moving in the band structure defined by $t$
in Eq.3 and coupled to a gauge field $\tilde{a} + \frac{e}{c} \varepsilon_{\alpha\beta} \tilde{A}_\alpha$ to spin fluctu-
tations and to other interactions not explicitly written.
It is important to stress that in the CMR case consid-
ered here, the core spins and electrons are independent
fields and the internal gauge field is simply a representa-
tion of topologically non-trivial configuration of the core
spins instead of an independent field. However, in high $T_c$
cuprates, the gauge fields are independent fields related
to spin-charge separation [11].

Taking the continuum limit of Eq.3 and neglecting
terms involving electron currents yields

$$H_{so} = i \lambda_{so} x a^2 e^{i\epsilon_{\alpha\beta} (\partial_\tau + ia_\alpha) z_{\alpha\beta}^\dagger (\partial_\tau - ia_\alpha) z_{\beta}$$  \hspace{1cm} \text{(7)}

Integrating the conduction electrons out of Eq.3 and eval-
huating $H_{so}$ using Eqs.4,5 yields

$$S = \beta \hbar \int d^4x \left[ \frac{\rho_s}{2} (\partial_\tau \tilde{n})^2 - M_0 \tilde{H} \cdot \tilde{n} + \frac{\chi_F}{2} \left( \frac{2\pi}{\Phi_0} \tilde{H} + \tilde{b} \right)^2 \right]$$

$$+ \lambda_{so} \frac{x}{2a} \int d^4x \tilde{n} \cdot \tilde{b}$$  \hspace{1cm} \text{(8)}

where $\chi_F \sim ta x^{1/3}$ is the electron diamagnetic suscepti-
bility, $\Phi_0 = \frac{\Phi_0}{2a} = 4.1358 \times 10^{-7} \text{ gauss } \cdot \text{ cm}^2$ is the funda-
mental flux quantum and $b_i = e \epsilon_{ijk} \partial_j a_k = e^2 \epsilon_{ijk} \tilde{H} \cdot (\partial_j \tilde{n} \times
\partial_k \tilde{n})$ is the internal magnetic field arising from the gauge
field $\tilde{a}$.

A non-zero $b$ arises from topologically non-trivial spin
configurations. In two spatial dimensions, these are the
skyrmions [12] which are important in quantum Hall fer-
romagnets [13]. In three dimensions, the objects are
skyrmion strings (dipoles) which begin at monopoles and
end at an anti-monopoles. In the ordered phase, these
have finite creation energies and are exponentially suppressed at low T, but can proliferate near $T_c$. Their behavior has been studied numerically [12].

It is known that at zero external magnetic field, the core energy of a dipole separated by distance $d$ is $E_c = 4\pi \rho_a d$ (i.e. string tension $\sim \rho_a$). To relate $E_c$ to experimental parameters we note that according to the numerical analysis of Ref. [10], $T_c = 1.45\rho_a$, while in the experiment of Ref. [3], $T_c = 265 K$. We find the core energy for a dipole separated by a lattice constant $a$ is $E_c = 2295 K$. Following Ref. [13], we find only $2-3\%$ increase of $E_c$ in an external magnetic field $H = 10T$, therefore, we can simply neglect the core energy dependence of $H$. The core energy $E_c$ is independent of the angle $\phi$ which defines the global orientation of the XY spin component. This $U(1)$ invariance implies the existence of a family of very soft twist modes. Dilatation modes have a small energy gap but are also likely to be thermally excited. These soft modes mean that at $T > 0$ each skyrmion carries a large entropy.

In the absence of spin-orbit coupling, dipoles are randomly oriented, leading to vanishing average $b$. In the presence of spin-orbit coupling and a non-zero magnetization, the dipoles are preferentially oriented, leading to a nonzero average $b$. To see this we rewrite Eq. 3 in terms of the polarization $\vec{P} = Q\vec{l}$ where $Q = \pm 1$ is the charge of a monopole and $\vec{l}$ is the vector connecting the monopole to anti-monopole which are the two end points of the dipole. We find, for $\vec{M}$ parallel to $z$

$$M_\chi \int dz \int dx dy dz = M_\chi \int dz Q_z = \frac{\lambda_{ox} x}{a} M_o^3 \cdot \vec{P} \quad (9)$$

where $M_\chi = \frac{\lambda_{ox} x}{a} \frac{M_o^3}{g\mu_B}$.

At low $T$, numerics Ref. [16] shows that the monopole and anti-monopole are very dilute and are tightly bound into dipoles of size of one lattice constant. We thus can treat them as independent classical particles. The density $(n_\pm)$ of (anti-)skyrmions is

$$n_\pm = \alpha e^{-\beta(E_\pm M Q a)} \quad (10)$$

where $\alpha$ is the entropy per dipole. Refs. [16] find $\alpha \sim 320$. We believe this large factor comes from the twist and dilatation modes mentioned above.

The average internal magnetic field $< b > = -Q(n_+ - n_-)/a^2$ is thus

$$< b > = -Q \frac{\lambda_{ox} x}{a^2} e^{-\frac{E_0}{k_B T}} \sinh(\frac{Q\lambda_{ox} x M_o^3}{k_B T g\mu_B}) \quad (11)$$

An internal field $b$ produces the same change in the phase of an electron as would be produced by a physical field of magnitude $\frac{b}{e^2}$. The equivalent physical field is large because the appropriate dimensionless coupling (fine structure constant) for the internal field is 1, rather than the $1/137$ relevant for physical fields.

The internal magnetic field produces a Hall effect in the usual way. Writing the Hall resistivity as $\rho_H = \frac{1}{n_e f \epsilon_c (H + \frac{Q}{2\pi} (b))}$, and linearizing the sinh, we find

$$\frac{R_s}{R_0} = -\alpha \frac{\lambda_{ox} Q^2}{2\pi k_B T g\mu_B} e^{-\frac{E_0}{k_B T}} \quad (12)$$

$Q = 1$ is the charge of a monopole, $a = 3.92 A$ is the lattice constant of La$_{1-x}$Ca$_x$MnO$_3$. At $T = 200K$, the experimental value $\frac{R_s}{R_0}$ is $-13.6$, implying $\lambda_{ox} Q^2 = 2.5 \times 10^{-2}$ so $\lambda_{ox} \sim 5K$ which justifies the linearization used in Eq. 12. Because the on-site spin-orbit interaction is quenched by the cubic crystal field, $\lambda_{so}$ is dominated by inter-ion hopping and is difficult to determine, but a rough estimate can be obtained by combining the dimensionless coupling appropriate for d-orbitals $(\frac{Ze^2}{2mC_{ao}}) (a_0 \sim 0.5A$ is the d-orbital size) with the band kinetic energy $\frac{k^2}{2m}$ yielding $\lambda_{so} \sim 2K$.

The sign of $R_s$ relative to $R_0$ depends on the sign of the spin orbit coupling. The physically reasonable sign (usual precession of charge around field of core spin) leads to an internal field $b$ which acts to cancel the applied field, implying opposite signs for $R_s$ and $R_0$.

The high temperature regime: here $k_BT_c g\mu_B H, \ll k_BT \ll J_H$, so the core spins vary on the lattice scale, the continuum action Eq. 3 does not apply and the skyrmion is not well defined. We calculate the average of the $z$-component of the gauge invariant internal flux $\Phi_\Delta = \epsilon^{abc}n_{i \pm x}h_{i \pm y}e_{i \pm z}$ through a triangle of adjacent sites $i \pm \hat{x}, i, i + \hat{y}$ to leading order in $\lambda_{so}/k_BT$ using the lattice action Eq. 3 finding

$$\Phi_\Delta \sim \frac{\lambda_{so} \chi H}{k_BT} \sum_a \langle d_{i \pm x a}^\dagger d_{i \pm y a} \rangle \quad (13)$$

where $\chi$ is the spin susceptibility.

Further, the La$_{1-x}$Ca$_x$MnO$_3$ materials of main experimental interest are in a highly resistive ‘polaron hopping’ regime which we model by assuming $t \ll k_BT$. In the original lattice model with only nearest neighbor hopping the fermion expectation value is of second order in $t/k_BT$, and may be computed, leading to

$$\frac{R_s}{R_0} = -\frac{\lambda_{so}}{k_BT} \frac{t}{k_BT}^2 x(1-x)(1-2x)C_3 \quad (14)$$

where $C_3$ is a constant of order unity. (Recall $R_s$ is the term in the Hall resistivity proportional to $M \sim \chi H$.)

The difficulty of unambiguously extracting the term in the Hall conductivity proportional to the spin susceptibility increases as $T$ is increased further above $T_c$, so a detailed study of the high-$T$ $R_s$ is probably not warranted.

The critical regime: For $T$ near $T_c$, a theory of the topological defects has not been constructed, because in the 3d Heisenberg model, there is no decoupling between spin-wave fluctuations and the topological defects,
so spin waves cannot simply be integrated out. Ref. [16] simulated the behavior of the topological defects near $T_c$ at zero external magnetic field. The dipole density $n$ was found to increase sharply as $T$ passes through $T_c$; there is a derivative discontinuity at $T_c$ which is controlled by the specific heat exponent $\alpha$ [15], (as is also the case for the resistivity). The singularity at $T_c$ may be difficult to observe: it is cut off by the field needed to observe an AHE at $T > T_c$ and the AHE has also an amplitude involving the field and temperature dependence of the magnetization.

From the low temperature and critical regime calculations, we find $R_s$ increases as $T$ is increased, and $dR_s/dT > 0$ at $T_c$. The high temperature expansion yields a $dR_s/dT < 0$. Therefore, $R_s$ has a maximum at some $T_{\text{max}} > T_c$. The decrease for $T > T_{\text{max}}$ has two causes–strong thermal spin fluctuations disrupt the local correlations needed to produce a $b$, and also the fermion correlator in Eq [3] decreases. Our knowledge of CMR materials is insufficient to allow a quantitative theory of the region near $T_{\text{max}}$.

To summarize: we have constructed a new theory of the anomalous Hall effect in CMR manganites. Our results are shown in the Figure, and account naturally for the order of magnitude, the sign (relative to the conventional Hall effect), and the peak at $T > T_c$ found experimentally [3].

![Fig 1](image)

**Fig 1** Calculated temperature dependence of anomalous Hall coefficient

Our results for $R_s$ have same qualitative behavior as (and identical critical behavior to) the longitudinal resistivity, but we have no argument for the very close correspondence between the two found experimentally [3].

This paper has assumed a large $J_H$ limit, so the conduction band is completely spin-polarized. An extension to the smaller $J_H$ limit, with a partially-polarized conduction band would be of interest in connection with the AHE in conventional magnets. In this case, conventional skew scattering would also contribute and indeed would dominate at very low $T$ because it varies as a power of $T$. Recently, a very large AHE was found in Co oxides [17]. These materials involve additional physics, including high-spin/low-spin transitions and inhomogeneous states, whose incorporation in our theory would be of interest.

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[1] The Hall effect and its Applications, edited by C. L. Chien and C. R. Westgate (Plenum Press, 1979)
[2] C. M. Hurd, The Hall effect in metals and alloys (Plenum Press, New York, 1972).
[3] F. E. Maranzana, Phys. Rev. 160, 421 (1967)
[4] P. Nozières and C. Lewiner, J. Phys. (Paris) 34, 901 (1973).
[5] P. Matl, N. P. Ong, Y. F. Yan, Y. Q. Li, D. Studebaker, T. Baum and G. Doubinina, Phys. Rev. B 57, 10248, (1998).
[6] S. H. Chun, M. B. Salamon and P. D. Han, Phys. B59 11155 (1999).
[7] P. W. Anderson and H. Hasegawa, Phys. Rev. 100, 675 (1955).
[8] For a derivation from microscopics and references to earlier work see H. J. Schulz, Phys. Rev. Lett. 65 2462 (1990).
[9] L. B. Ioffe and A. I. Larkin, Phys. Rev. B 39, 8988, (1989); N. Nagaosa and P. A. Lee, Phys. Rev. Lett. 64, 2450 (1990); Jinwu Ye and Subir Sachdev, Phys. Rev. B 44, 10173, (1991).
[10] L. B. Ioffe, V. Kalneymer and P. B. Weigmann, Phys. Rev. B43 1219-22 (1991); L. B. Ioffe, G. B. Lesovik and A. J. Millis, Phys. Rev. Lett. 77, 1584 (1996).
[11] Y. B. Kim, P. Majumdar, A. J. Millis and B. I. Shraiman, [cond-mat/9803551](cond-mat/9803576).
[12] Y. Lyanda-Geller, P. M. Goldbart, S. H. Chun and M. B. Salamon, unpublished [cond-mat/9904331] and S. H. Chun, M. B. Salamon, P. D. Han, Y. Lyanda-Geller and P. M. Goldbart, unpublished [cond-mat/9904332].
[13] The quantum phase associated with the orbital degree of freedom may also in some circumstances lead to an AHE; see e.g. Y.B. Kim et. al., op. cit.
[14] Gauge Fields and Strings A. M. Polyakov, (Harwood Academic Publishers: 1987); see also L. B. Ioffe and A. I. Larkin, Int. J. Mod. Phys. B2 203 (1988).
[15] S. L. Sondhi, A. Karlhede, S. A. Kivelson and E. H. Rezayi, Phys. Rev. B 47, 16419, (1993); R. Cote, et al, Phys. Rev. Lett. 78, 4825 (1997).
[16] C. Holm and W. Janke, J. Phys. A27 2553 (1994), M. H. Lau and C. Dasguptha, Phys. Rev. B 39, 7212 (1989); M. Kamal and G. Murthy, Phys. Rev. Lett. 71, 1911 (1993).
[17] A. V. Samoilov, G. Beach, C. C. Fu, N. C. Yeh and R. P. Vasquez, Phys. Rev. B 57, R14032, (1998)