MULTIOBJECTIVE MATHEMATICAL MODELS AND SOLUTION APPROACHES FOR HETEROGENEOUS FIXED FLEET VEHICLE ROUTING PROBLEMS

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Abstract. In this paper, we study three types of heterogeneous fixed fleet vehicle routing problems, which are capacitated vehicle routing problem, open vehicle routing problem and split delivery vehicle routing problem. We propose new multiobjective linear binary and mixed integer programming models for these problems, where the first objective is the minimization of a total routing and usage costs for vehicles, and the second one is the vehicle type minimization, respectively. The proposed mathematical models are all illustrated on test problems, which are investigated in two groups: small-sized problems and the large-sized ones. The small-sized test problems are first scalarized by using the weighted sum scalarization method, and then GAMS software is used to compute efficient solutions. The large-sized test problems are solved by utilizing the tabu search algorithm.

1. Introduction. The vehicle routing problem (VRP) is a well-known combinatorial optimization problem. The problem aims to find optimal routes with minimum costs, for the given fleet of vehicles, which depart from a depot toward a set of customers, by considering the vehicles’ capacities and the customers’ demands.

Vehicle routing problems are first considered by Dantzig and Ramser [6]. There are several variants of the problem, such as capacitated vehicle routing problem (CVRP), open vehicle routing problem (OVRP), vehicle routing problem with time windows, vehicle routing problem with split deliveries (SDVRP), vehicle routing problem with multiple depots, vehicle routing problem with backhauls etc. One of the important variants of VRP is a heterogeneous fleet vehicle routing problem (HFVRP), in which customers are served by a fleet of vehicles with possibly different capacities. The heterogeneous fleet VRP is first studied in a structured way by Golden [15]. The heterogeneous fixed fleet vehicle routing problem (HFFVRP) is an important extension of the VRP with various vehicle types and a fixed number of vehicles per type. HFFVRP is first studied by Fisher and Jaikumar [10]. They proposed an integer programming formulation and a new heuristic to solve this problem with minimum fleet size.

The OVRP consists of designing the routes for a set of vehicles such that, each route starts at depot and finishes at one of the customer locations. The important

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feature of this problem is that, the vehicles do not have to return to depot. This problem is first considered by Schrage [30], where customer demands have to be satisfied and all customers have to be visited exactly once. The heterogeneous fixed fleet open vehicle routing problem (HFFOVRP) is an important extension of the OVRP which was first considered by Li et al. [24]. The HFFOVRP has numerous applications in industrial and logistics problems and the OVRP can be obtained from HFFOVRP by removing the constraint of vehicle heterogeneity [34].

SDVRP is one of the most studied variants of the CVRP. In real life problems, it may not be possible to deliver customer demands by one vehicle which means that customers can be served by more than one vehicle. Splitting customer demands in optimal way is an important decision in this problem. SDVRP was first considered by Dror and Trudeau [7]. After then, Dror et al. [8] presented an integer programming formulation for SDVRP.

There are different solution methods for different types of vehicle routing problems. Taillard [31] presented a heuristic column generation method for the heterogeneous fleet vehicle routing problem. Gendreau et al. [13] studied tabu search algorithm, Renaud and Boctor [29] studied a heuristic based sweep algorithm for the heterogeneous fleet vehicle routing problem. Tarantilis et al. [32] studied a threshold accepting metaheuristic algorithm for the heterogeneous fixed fleet vehicle routing problems. Li et al. [23] had a research about the record to record travel algorithm for the HFVRP. Hybrid tabu search algorithm is studied by Euchi and Chabchoub [9] for the heterogeneous fixed fleet vehicle routing problem. Liu [25] focused on the hybrid genetic algorithm for the heterogeneous vehicle routing problems.

Brandao [3] studied tabu search algorithm, Yu et al. [35] focused on hybrid genetic and tabu search algorithm, and Fleszar et al. [11] applied variable neighborhood search algorithm for the open vehicle routing problem. Heterogeneous fleet is not considered in these papers. For the heterogeneous fixed fleet open vehicle routing problems, a multi start adaptive memory-based tabu search algorithm is applied by Li et al. [24]. Yousefikhoshbakht et al. [34] presented a combined metaheuristic algorithm for the heterogeneous fixed fleet open vehicle routing problems.

For the split delivery vehicle routing problem, Dror and Trudeau [7] studied savings algorithm. Archetti et al. [1] and Bolduc et al. [2] studied on tabu search algorithm for the split delivery vehicle routing problem. Jin et al.[18] focused on column generation method. Ho and Haugland [16] developed a tabu search algorithm for the vehicle routing problems with time windows and split deliveries. For the heterogeneous fleet split delivery vehicle routing problems, Belfiore and Yoshizaki [4] proposed a scatter-search algorithm. Chen et al. [5] implemented a novel approach for this problem using a priori split strategy which splits customer demands in advance.

In this paper, we study three types of heterogeneous fixed fleet vehicle routing problems, which are heterogeneous fixed fleet capacitated, heterogeneous fixed fleet open and heterogeneous fixed fleet split delivery vehicle routing problems. We propose new multiobjective linear binary and mixed integer programming models for these problems, with two objective functions each. The first objective function is related to the minimization of sum of the total routing cost and the usage cost for vehicles, and the second objective is the vehicle type minimization. To the best of our knowledge, the second objective function is not considered in the VRP
literature previously. Earlier, the type minimization was considered in the literature, for cutting and assortment problems [12] and [19].

The important feature of this study is considering the vehicle type minimization in different types of heterogeneous fixed fleet VRPs. It is clear that the use of many (different) types of vehicles with different capacities may lead to “easy” minimization of shipping costs in VRP. If the number of vehicle types is numerous, the model can easily minimize the shipping costs by using more vehicle types, because in such a case the choosing of the corresponding types of vehicles with suitable capacities may lead to a totally lesser number of vehicles, with a large number of vehicle types. Since the objective function of VRP does not include the usage costs related to the different types of vehicles, this can be deceptive. There is a real impact of high transportation costs on transportation budgets when using too many vehicle types. There are several costs such as hiring, maintenance and holding costs, costs for garage, taxes, organization and vehicle managing costs, etc., which increase the total costs when different vehicle types are used. Hence, using a minimum number of vehicle types becomes an important issue in logistic problems (see e.g. [28] and [26]). For example, it would be easy (and cheaper) to hire 1 type of vehicles and organize their management, mainanence, garage arrangement and holding costs, than to manage 2 or more types of vehicles.

The mathematical models studied in this paper are illustrated on test problems and the obtained computational results, are interpreted. These test problems are considered in two groups: the small-sized test problems with 10 cities, and the large-sized ones. The small-sized test problems are first scalarized by using the weighted sum scalarization method, and then GAMS software is used to compute efficient solutions. The large-sized test problems are solved by utilizing the tabu search algorithm.

To emphasize the role of the second objective function, which is related to the vehicle type minimization, some test problems for heterogeneous capacitated problem, are analyzed in two settings: first as a multiobjective problem, and second, in the form of a single-objective problem with bounded number of vehicle types. The computational results demonstrate that, the multiobjective problem setting leads to a lower total travel cost (see Section 3.1).

The rest of the paper is outlined as follows. Section 2 presents mathematical models for multiobjective heterogeneous fixed fleet capacitated, open and split delivery vehicle routing problems. Section 3 presents an analysis of computational results for small-sized problems. In Section 4, tabu search algorithm for multiobjective mathematical models is explained in detail and the computational results for large-sized test problems are interpreted. Section 5 presents some conclusions.

2. Problem formulations and the mathematical models. This section presents multiobjective mathematical models for capacitated, open and split delivery vehicle routing problems with heterogeneous fixed fleet.

2.1. A mathematical model for the multiobjective heterogeneous fixed fleet capacitated vehicle routing problem. First we present the mathematical model for the heterogeneous fixed fleet capacitated vehicle routing problem. The following notations are used throughout the section.

Sets and Parameters

- $n$: is the total number of customers,
- $i, j$: will be used to denote customer indices, $i, j = 1, \ldots, n$ (1 is the depot),
$t$: will be used to denote the vehicle type, $t = 1, 2, ..., T$,
$A_t$: is the number of vehicles of type $t$,
$v$: will be used to denote the vehicle index, $v = 1, ..., A_t$.
$Q_t$: is the capacity of vehicle of type $t$,
$c_{ij}$: is the distance from customer $i$ to customer $j$,
$d_i$: is the demand of customer $i$, it is assumed that $d_1 = 0$,
$M_{tv}$: is the cost of using the vehicle $v$ of type $t$,
$p_t$: is the routing cost for the vehicle type $t$, per unit distance,
$u_i$ and $u_j$ are positive variables which are used in the subtour elimination constraints.

**Decision Variables**

$$x_{ijtv} = \begin{cases} 1, & \text{if vehicle } v \text{ of type } t \text{ travels from customer } i \text{ to customer } j, \\ 0, & \text{otherwise.} \end{cases}$$

$$f_{tv} = \begin{cases} 1, & \text{if vehicle } v \text{ of type } t \text{ is used,} \\ 0, & \text{otherwise.} \end{cases}$$

$$h_t = \begin{cases} 1, & \text{if vehicle type } t \text{ is used,} \\ 0, & \text{otherwise.} \end{cases}$$

**Objective functions**

We consider two objective functions. The first objective function is the sum of the total routing costs and the costs of using vehicles. The cost of using vehicles means the different costs such as the total cost of vehicles for leaving depot, the depreciation cost, the possible lease cost of vehicle and so on.

$$z_1 = \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} \sum_{t=1}^{T} c_{ij} p_t x_{ijtv} + \sum_{t=1}^{T} \sum_{v=1}^{A_t} M_{tv} f_{tv}. \quad (1)$$

The second objective function is the the number of vehicle types used.

$$z_2 = \sum_{t=1}^{T} h_t. \quad (2)$$

Under these notifications, the multiobjective binary integer linear programming model for the heterogeneous fixed fleet capacitated vehicle routing problem is formulated as follows.

$$\min [z_1, z_2] \quad (3)$$

subject to

$$\sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} d_j x_{ijtv} \leq Q_t, \quad t = 1, ..., T, v = 1, ..., A_t, \quad (4)$$

$$\sum_{j=1}^{A_t} \sum_{v=1}^{A_t} x_{ijtv} \leq A_t, \quad t = 1, ..., T, \quad (5)$$

$$\sum_{i=1}^{n} x_{iktv} - \sum_{j=1, j \neq k}^{n} x_{kjtv} = 0, \quad k = 2, ..., n, t = 1, ..., T, v = 1, ..., A_t, \quad (6)$$
Constraint set (4) ensures the capacity constraint which means that total demand of a route can not exceed the capacity of the vehicle serving this route. Since the demand of depot is assumed to be zero, when some vehicle will travel from some customer \( i \) to depot (where \( j = 1 \)), then this situation will not violate the constraint. Constraint set (5) states that there is a limited number of vehicles for each vehicle type \( t \), which can leave the depot. For example, if there are 5 vehicles of vehicle type 1, the model does not allow using 6 vehicles of vehicle type 1. Constraint set (6) is a typical vehicle flow constraint that ensures the continuity of each vehicle route. If a vehicle travels from customer \( i \) to some customer \( k \), it must leave the customer \( k \) to travel to another customer, which may be depot. Constraints (7) state that each customer \( j \) should be served by only one vehicle. Relations (8) are the Miller-Tucker-Zemlin subtour elimination constraint [27]. Constraint set (9) ensures that if a vehicle \( v \) of type \( t \) is used, this leads to \( f_{tv} = 1 \) and the vehicle must return to depot. Constraints (10) are connectivity constraints establishing relations between the decision variables \( x \) and \( f \), where \( M \) is a sufficiently large number. If a vehicle \( v \) of type \( t \) is used, constraint set (10) forces \( f_{tv} = 1 \). In this case, this constraint allows the vehicle \( v \) of type \( t \) to be used in some route, but it is not compulsory. In the case if \( f_{tv} = 0 \), this means that the left hand side of (10) has to be 0, and the vehicle \( v \) of type \( t \) should not be used in any route. On the other hand, there is no need to use the inverse inequality, because the minimization of the first objective function expressing the cost of using the vehicles, will force the number of different vehicles to decrease. Constraint set (11) states that, if some vehicle type \( t \) is used then the left hand side of (11) may become positive but it is not compulsory. Again the inverse inequality is not used because the minimization of vehicle type is considered in the second objective function.

2.2. A mathematical model for the multiobjective heterogeneous fixed fleet open vehicle routing problem. In this section, we present the multiobjective mathematical model for a heterogeneous fixed fleet open vehicle routing problem. The same notations of Section 2.1 are used for this model too. The additional binary decision variable used in this model, is:
The description of the objective functions are the same as in the mathematical model of Section 2.1

The first objective function is:

\[ z_1 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{t=1}^{T} \sum_{v=1}^{A_t} c_{ij} p_t x_{ijtv} + \sum_{t=1}^{T} \sum_{v=1}^{A_t} M_{tv} f_{tv} \]  

(13)

The second objective function is:

\[ z_2 = \sum_{t=1}^{T} h_t. \]  

(14)

Under these notifications, the multiobjective binary integer linear programming model for the heterogeneous fixed fleet open vehicle routing problem is formulated as follows.

\[ \text{min } [z_1, z_2] \]  

(15)

subject to

\[ \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} d_{ij} x_{ijtv} \leq Q_t, \quad t = 1, \ldots, T, v = 1, \ldots, A_t, \]  

(16)

\[ \sum_{j=2}^{n} \sum_{v=1}^{A_t} x_{1jtv} \leq A_t, \quad t = 1, \ldots, T, \]  

(17)

\[ \sum_{t=1}^{T} \sum_{v=1}^{A_t} y_{itv} = 1, \quad i = 1, \ldots, n, \]  

(18)

\[ \sum_{i=1}^{n} x_{ijtv} = y_{jtv}, \quad j = 1, \ldots, n, t = 1, \ldots, T, v = 1, \ldots, A_t, \]  

(19)

\[ \sum_{j=1, j \neq i}^{n} x_{ijtv} \leq y_{itv}, \quad i = 1, \ldots, n, t = 1, \ldots, T, v = 1, \ldots, A_t, \]  

(20)

\[ u_i - u_j + n \sum_{t=1}^{T} \sum_{v=1}^{A_t} x_{ijtv} \leq n - 1, \quad i, j = 2, \ldots, n, i \neq j, \]  

(21)

\[ \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} x_{ijtv} \leq Mf_{tv}, \quad t = 1, \ldots, T, v = 1, \ldots, A_t, \]  

(22)

\[ \sum_{v=1}^{A_t} f_{tv} \leq Mh_t, \quad t = 1, \ldots, T, \]  

(23)

\[ x_{ijtv}, f_{tv}, h_t \in \{0, 1\}, \quad i, j = 1, \ldots, n, t = 1, \ldots, T, v = 1, \ldots, A_t. \]  

(24)

Constraint set (16) ensures the capacity constraint. Constraint set (17) states that, there is a limited number of vehicles for each vehicle type \( t \) which can leave the depot. Constraint set (18) indicates that every customer must be visited exactly
once. Constraint (19) means, if some vehicle serves customer $j$, then this vehicle has to travel to customer $j$ from another customer. Constraint set (20) states that, if customer $i$ is visited by the vehicle $v$ of type $t$, then this vehicle may travel or may not travel to another customer. Note that because of this situation, the given model is called the open VRP, since the vehicles do not have to return to depot. Relations (21) are the subtour elimination constraints. Constraints (22) are connectivity constraints establishing relations between the decision variables $x$ and $f$. $M$ is a sufficiently large positive number. Constraints (23) are connectivity constraints establishing relations between the decision variables $f$ and $h$. The constraints (22) and (23) have similar explanations given for constraints (10) and (11) of Section 2.1.

2.3. A mathematical model for the multiobjective heterogeneous fixed fleet split delivery vehicle routing problem. In this section, we present the multiobjective mathematical model for a heterogeneous fixed fleet split delivery vehicle routing problem. The same notations of Section 2.1 are used for this model too.

The new positive variable used in this model is $r_{itv}$:

$r_{itv}$: the fraction of the demand of customer $i$, delivered by vehicle $v$ of type $t$.

The description of the objective functions are the same as in the mathematical model of Section 2.1.

The first objective function is:

$$z_1 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{t=1}^{T} \sum_{v=1}^{A_t} c_{ij} p_t x_{ijtv} + \sum_{t=1}^{T} \sum_{v=1}^{A_t} M_{tv} f_{tv}$$

(25)

The second objective function is:

$$z_2 = \sum_{t=1}^{T} h_t.$$  

(26)

Under these notifications, the multiobjective mixed integer linear programming model for the heterogeneous fixed fleet split delivery vehicle routing problem, is formulated as follows.

$$\min [z_1, z_2]$$

(27)

subject to

$$\sum_{i=2}^{n} d_i r_{itv} \leq Q_{t}, \quad t = 1, \ldots, T, v = 1, \ldots, A_t,$$

(28)

$$\sum_{j=2}^{n} \sum_{v=1}^{A_t} x_{1jtv} \leq A_t, \quad t = 1, \ldots, T,$$

(29)

$$\sum_{i=1, i \neq k}^{n} x_{iktv} - \sum_{j=1, j \neq k}^{n} x_{kjtv} = 0, \quad k = 2, \ldots, n, t = 1, \ldots, T, v = 1, \ldots, A_t,$$

(30)

$$r_{jtv} \leq \sum_{i=1}^{n} x_{ijtv}, \quad j = 2, \ldots, n, t = 1, \ldots, T, v = 1, \ldots, A_t,$$

(31)

$$r_{jtv} \geq \varepsilon \sum_{i=1}^{n} x_{ijtv}, \quad j = 2, \ldots, n, t = 1, \ldots, T, v = 1, \ldots, A_t,$$

(32)
where $\varepsilon > 0$ is sufficiently small positive number,

\begin{equation}
\sum_{t=1}^{T} \sum_{v=1}^{A_t} r_{itv} = 1, \quad i = 2, \ldots, n, \tag{33}
\end{equation}

\begin{equation}
-u_i - u_j + n \sum_{t=1}^{T} \sum_{v=1}^{A_t} x_{ijtv} \leq n - 1, \quad i, j = 2, \ldots, n, i \neq j \tag{34}
\end{equation}

\begin{equation}
\sum_{i=2}^{n} x_{i1tv} = f_{tv}, \quad t = 1, \ldots, T, v = 1, \ldots, A_t, \tag{35}
\end{equation}

\begin{equation}
\sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} x_{ijtv} \leq Mf_{tv}, \quad t = 1, \ldots, T, v = 1, \ldots, A_t, \tag{36}
\end{equation}

\begin{equation}
\sum_{v=1}^{A_t} f_{tv} \leq Mh_t, \quad t = 1, \ldots, T, \tag{37}
\end{equation}

\begin{equation}
0 \leq r_{itv} \leq 1, \quad i = 1, \ldots, n, t = 1, \ldots, T, v = 1, \ldots, A_t, \tag{38}
\end{equation}

\begin{equation}
x_{ijtv}, f_{tv}, h_t \in \{0, 1\}, \quad i = j = 1, \ldots, n, t = 1, \ldots, T, v = 1, \ldots, A_t. \tag{39}
\end{equation}

Constraints (28) ensure that vehicle capacities are never exceeded. Constraints (29) mean that, there is a limited number of vehicles for each type $t$ which can leave the depot. Constraints (30) ensure the vehicle flow constraint. Constraint set (31) ensure that if customer $j$ is not visited by vehicle $v$ of type $t$, then the fraction of the demand of this customer delivered by this vehicle, should be zero. This fraction may be positive, only if the customer is visited by this vehicle. The strong positivity of this fraction in such case is guaranteed by the constraint set (32), where the number $\varepsilon$ determines the threshold value for the lower bound of the fraction that can be delivered. In practice it may be required that this fraction is positive only for the given lower bound. Constraint set (33) specifies that some fraction demands should be positive and that the demand of every customer should entirely be satisfied. Constraints (34) are subtour elimination constraints. Constraints (35) and (36) are connectivity constraints establishing relations between the decision variables $x$ and $f$. Constraints (37) are connectivity constraints establishing relations between the decision variables $f$ and $h$. For the explanations on the constraints (35)–(37), see explanations on formulas (9)–(11) of Section 2.1. Constraints (38) and (39) are set constraints for the decision variables.

3. **Computational results obtained for small-sized problems.** This section illustrates and analyzes the efficiency and performances of the proposed models, by using the so-called small-sized test problems. The multiobjective mathematical models for these test problems are first scalarized using the weighted sum scalarization method (for scalarization methods, see e.g. [17, 20, 21, 22, 33]), and then the scalarized (single-objective) problems are solved by applying the GAMS (General Algebraic Modeling System) software. Documentation and information about GAMS is available via World Wide Web at the URL: www.gams.com. GAMS is implemented on ASUS Intel Core with i7 processor, 16 GB RAM. To illustrate the multiobjective mathematical models from Sections 2.1 and 2.2, we have generated new test problems by slightly changing the data of Taillard’s test problem 13 [31]
(see Table 7 for the data description). This test problem is generated by including only first 10 customers (first coordinate is the depot location and the rest of 9 coordinates are customer locations). We have also generated data for the split delivery vehicle routing problem (see Section 2.3). Note that, the optimal solutions are found in 15 minutes for all the test problems.

The following notations are used to summarize the results obtained:
- $w_1$ and $w_2$ are positive weights for objective functions 1 and 2 respectively.
- $z_1$ and $z_2$ are values for objective functions 1 and 2 respectively.

### 3.1. Results for the multiobjective heterogeneous fixed fleet capacitated vehicle routing problem

The results obtained for the heterogeneous fixed fleet capacitated vehicle routing test problems, are summarized in Table 1. In this table, for weight values $w_i \in \{1, 2, \ldots, 9\}$, $i = 1, 2$ and $w_1 + w_2 = 10$, the corresponding optimal values for objective functions, the optimal routes and the selected vehicles are depicted. As it is expected, for a greater weight value of some objective function, a smaller optimal value is obtained. Moreover, the results given in Table 1, demonstrate the following hypothesis emphasized in Section 1: “If the number of vehicle types is numerous, the model can easily minimize the shipping costs by using more vehicle types, because in such a case the choosing of the corresponding types of vehicles with suitable capacities may lead to a totally lesser number of vehicles, with a large number of vehicle types.” As it can be seen from this table, for the weight values from $(w_1, w_2) = (1, 9)$ to $(3, 7)$, four vehicles were selected, all of them are of type 3. While for weight values from $(w_1, w_2) = (4, 6)$ to $(9, 1)$, only three vehicles were selected by using two vehicle types.

#### 3.1.1. Particular case related to the fixed number of vehicle types

In this section, we consider a particular case of the multiobjective heterogeneous fixed fleet capacitated vehicle routing problem described above, related to the situation when the decision maker is interested in choosing only a few number of vehicle types among all the possible ones. In this case the problem becomes to determine which vehicle types and how many of each type should be hired (determined) in order to minimize the first objective function (1) on total routing costs and the costs of using vehicles, and to satisfy all the standard constraints (4)–(12), plus the new constraint on vehicle types. For this model the maximal number of different vehicle types is treated as a fixed parameter.

Let $S$ be the maximal number of different types of vehicles to be selected from $T$ possible vehicle types. The formulation of the mathematical model corresponding to this case is given below.

\[
\begin{align*}
\min \quad & z_1 \\
\text{subject to} \quad & \sum_{t=1}^{S} h_{it} \leq T, \\
(x, f, u) & \in X = \{ x = (x_{ijt}), u = (u_i), f = (f_{tv}) : i, j = 1, \ldots, n; t = 1, \ldots, T; \\
& v = 1, \ldots, A_t; \text{ constraints (4)–(12) are all satisfied} \}.
\end{align*}
\]

Note that this problem can be considered as a problem obtained from the two-objective one, by transforming the second objective function $z_2$ to the constraint set by using a fixed (upper) bound for it. Since this is a single-objective problem,
Table 1. GAMS results for the multiobjective heterogeneous fixed fleet capacitated vehicle routing problem.

| $w_1$ | $w_2$ | $z_1$ | $z_2$ | Routes | Vehicles |
|-------|-------|-------|-------|--------|----------|
| 1     | 9     | 523.81| 1     | 1-2-10-6-1 vehicle 1 of type 3 |
|       |       |       |       | 1-4-9-1   vehicle 2 of type 3 |
|       |       |       |       | 1-5-3-1   vehicle 3 of type 3 |
|       |       |       |       | 1-7-8-1   vehicle 4 of type 3 |
| 2     | 8     | 523.81| 1     | 1-3-5-1   vehicle 1 of type 3 |
|       |       |       |       | 1-6-10-2-1 vehicle 2 of type 3 |
|       |       |       |       | 1-8-7-1   vehicle 3 of type 3 |
|       |       |       |       | 1-9-4-1   vehicle 4 of type 3 |
| 3     | 7     | 523.81| 1     | 1-3-5-1   vehicle 1 of type 3 |
|       |       |       |       | 1-6-10-2-1 vehicle 2 of type 3 |
|       |       |       |       | 1-8-7-1   vehicle 3 of type 3 |
|       |       |       |       | 1-4-9-1   vehicle 4 of type 3 |
| 4     | 6     | 468.91| 2     | 1-4-3-6-1 vehicle 2 of type 4 |
|       |       |       |       | 1-5-10-1  vehicle 1 of type 1 |
|       |       |       |       | 1-7-8-9-2-1 vehicle 1 of type 4 |
| 5     | 5     | 468.91| 2     | 1-5-10-1  vehicle 1 of type 1 |
|       |       |       |       | 1-6-3-4-1 vehicle 1 of type 4 |
|       |       |       |       | 1-7-8-9-2-1 vehicle 2 of type 4 |
| 6     | 4     | 468.91| 2     | 1-2-9-8-7-1 vehicle 1 of type 4 |
|       |       |       |       | 1-5-10-1  vehicle 1 of type 1 |
|       |       |       |       | 1-6-3-4-1 vehicle 2 of type 4 |
| 7     | 3     | 468.91| 2     | 1-2-9-8-7-1 vehicle 1 of type 4 |
|       |       |       |       | 1-6-3-4-1 vehicle 2 of type 4 |
|       |       |       |       | 1-10-5-1  vehicle 1 of type 1 |
| 8     | 2     | 468.91| 2     | 1-5-10-1  vehicle 1 of type 1 |
|       |       |       |       | 1-6-3-4-1 vehicle 1 of type 4 |
|       |       |       |       | 1-7-8-9-2-1 vehicle 2 of type 4 |
| 9     | 1     | 468.91| 2     | 1-2-9-8-7-1 vehicle 1 of type 4 |
|       |       |       |       | 1-4-3-6-1 vehicle 2 of type 4 |
|       |       |       |       | 1-5-10-1  vehicle 1 of type 1 |

its optimal solutions may or may not be Pareto efficient for the initial two-objective one (see e.g. the similar results obtained in [12, Problem (9)–(12)]).

Computational results for this problem, corresponding to five different values of the parameter $S$, are presented in Table 2. It is noticeable that only the solution presented at the 1st line of Table 2, is Pareto efficient, which is reported in the Table 1 and found for the weights $w_1 = 1, w_2 = 9$. The comparison of the solutions given in Tables 1 and 2, easily demonstrates that all the other solutions found for the particular problem, are not Pareto efficient. For example, the solution reported in second row of the Table 2 (for the two vehicle types), is not Pareto efficient, because it is worser than the value of the first objective function’s value (468.91) reported in the Table 1 obtained for the case when two vehicle types (1 and 4) were chosen (see rows 4 to 9). The same interpretation can be made also for the rows 3–5 of the Table 2, where both the objective functions values are worser than those obtained in the Table 1.

3.2. Results for the multiobjective heterogeneous fixed fleet open vehicle routing problem. The results obtained for the multiobjective mathematical
Table 2. GAMS results for the single-objective heterogeneous fixed fleet capacitated vehicle routing problem

| $z_2$ | $z_1$ | Routes                  | Vehicles       |
|-------|-------|-------------------------|----------------|
| $z_2 \leq 1$ | 523.81 | 1-2-10-6-1 | vehicle 1 of type 3 |
|       |       | 1-4-9-1     | vehicle 2 of type 3 |
|       |       | 1-5-3-1     | vehicle 3 of type 3 |
|       |       | 1-7-8-1     | vehicle 4 of type 3 |
| $z_2 \leq 2$ | 505.78 | 1-7-9-4-3-10-6-1 | vehicle 1 of type 5 |
|       |       | 1-2-8-5-1  | vehicle 1 of type 3 |
| $z_2 \leq 3$ | 494.54 | 1-6-5-1    | vehicle 1 of type 2 |
|       |       | 1-7-8-9-4-3-2-1 | vehicle 1 of type 5 |
|       |       | 1-10-1    | vehicle 1 of type 1 |
| $z_2 \leq 4$ | 480.63 | 1-2-3-10-6-1 | vehicle 1 of type 4 |
|       |       | 1-4-9-1     | vehicle 1 of type 3 |
|       |       | 1-7-5-1     | vehicle 1 of type 2 |
|       |       | 1-8-1      | vehicle 1 of type 1 |
| $z_2 \leq 5$ | 480.63 | 1-2-3-10-6-1 | vehicle 1 of type 4 |
|       |       | 1-4-9-1     | vehicle 1 of type 3 |
|       |       | 1-7-5-1     | vehicle 1 of type 2 |
|       |       | 1-8-1      | vehicle 1 of type 1 |

model of heterogeneous fixed fleet open vehicle routing problem obtained by using GAMS, are summarized in Table 3. As it can be seen from this table, in the situation when the minimization of the second objective is much more preferable than the minimization of first one, i.e. when $w_1 = 1$, $w_2 = 9$, the only one type has been chosen by the model. In this situation the value of the first objective function related to the total cost is very large, because all the efforts of the model are used to minimize the second objective function related to a total number of vehicle types decided to be used. It is remarkable that the optimal value of the first objective function decreases with increasing its “weight” (the value of $w_1$) in the total weighted sum whereas the number of vehicle types increases.

3.3. Results for the multiobjective heterogeneous fixed fleet split delivery vehicle routing problem. The data related to the split delivery vehicle routing problem, is given in Tables 4 and 5. To calculate the distances between the customers, Euclidean distance formula is used. Customer 1 represents the depot location.

The results obtained for the multiobjective mathematical model of heterogeneous fixed fleet split delivery vehicle routing problem, are summarized in Table 6. In this table, for weight values $w_1 = 5$ and $w_2 = 5$, the demand of customer 5 is splitted. 10 units of customer 5 is satisfied in the first route by vehicle 1 of type 1; the remained 4 units of the demand of customer 5 is satisfied in the third route by vehicle 2 of type 3. If $w_1 = 8$ and 9, and $w_2 = 2$ and 1 respectively, which means that the total routing cost is more preferable than the vehicle type minimization, the optimal solution is $z_1 = 770.60$ and $z_2 = 3$, which can be interpreted in the following form: all the efforts of the model are used to minimize the first objective function, by choosing three vehicle types. By calculating the total cost as a sum of the routing cost (770.60) and the vehicle type cost for all three ones (100 + 150 + 200 = 450), we get the amount of 1220. In this case, the demand of customer 4 is splitted: 10 units of its demand is satisfied in the first route by vehicle 1 of type 1; the remained
Table 3. GAMS results for the multiobjective heterogeneous fixed fleet open vehicle routing problem.

| $w_1$ | $w_2$ | $z_1$ | $z_2$ | Routes | Vehicles |
|-------|-------|-------|-------|--------|----------|
| 1     | 9     | 408.43| 1     | 1-2-6-10 vehicle 1 of type 3 |
|       |       |       |       |        | 1-5-3 vehicle 2 of type 3 |
|       |       |       |       |        | 1-7-8 vehicle 3 of type 3 |
|       |       |       |       |        | 1-9-4 vehicle 4 of type 3 |
| 2     | 8     | 408.43| 1     | 1-7-8 vehicle 1 of type 3 |
|       |       |       |       |        | 1-2-6-10 vehicle 2 of type 3 |
|       |       |       |       |        | 1-5-3 vehicle 3 of type 3 |
|       |       |       |       |        | 1-9-4 vehicle 4 of type 3 |
| 3     | 7     | 408.43| 1     | 1-2-6-10 vehicle 1 of type 3 |
|       |       |       |       |        | 1-5-3 vehicle 2 of type 3 |
|       |       |       |       |        | 1-7-8 vehicle 3 of type 3 |
|       |       |       |       |        | 1-9-4 vehicle 4 of type 3 |
| 4     | 6     | 352.76| 2     | 1-2-9-8 vehicle 1 of type 4 |
|       |       |       |       |        | 1-3-4 vehicle 2 of type 4 |
|       |       |       |       |        | 1-5 vehicle 1 of type 1 |
|       |       |       |       |        | 1-6 vehicle 3 of type 4 |
|       |       |       |       |        | 1-7 vehicle 4 of type 4 |
|       |       |       |       |        | 1-10 vehicle 2 of type 1 |
| 5     | 5     | 352.76| 2     | 1-2-9-8 vehicle 1 of type 4 |
|       |       |       |       |        | 1-5 vehicle 1 of type 1 |
|       |       |       |       |        | 1-6-3-4 vehicle 2 of type 4 |
|       |       |       |       |        | 1-7 vehicle 3 of type 4 |
|       |       |       |       |        | 1-10 vehicle 2 of type 1 |
| 6     | 4     | 352.76| 2     | 1-2-9-8 vehicle 1 of type 4 |
|       |       |       |       |        | 1-5 vehicle 1 of type 1 |
|       |       |       |       |        | 1-6-3-4 vehicle 2 of type 4 |
|       |       |       |       |        | 1-7 vehicle 3 of type 4 |
|       |       |       |       |        | 1-10 vehicle 2 of type 1 |
| 7     | 3     | 352.76| 2     | 1-2-9-8 vehicle 1 of type 4 |
|       |       |       |       |        | 1-5 vehicle 1 of type 1 |
|       |       |       |       |        | 1-6-3-4 vehicle 2 of type 4 |
|       |       |       |       |        | 1-7 vehicle 3 of type 4 |
|       |       |       |       |        | 1-10 vehicle 2 of type 1 |
| 8     | 2     | 352.76| 2     | 1-2-9-8 vehicle 1 of type 4 |
|       |       |       |       |        | 1-5 vehicle 1 of type 1 |
|       |       |       |       |        | 1-6-3-4 vehicle 2 of type 4 |
|       |       |       |       |        | 1-7 vehicle 3 of type 4 |
|       |       |       |       |        | 1-10 vehicle 2 of type 1 |
| 9     | 1     | 347.01| 3     | 1-2-9 vehicle 1 of type 2 |
|       |       |       |       |        | 1-5 vehicle 1 of type 1 |
|       |       |       |       |        | 1-6-3-4 vehicle 1 of type 4 |
|       |       |       |       |        | 1-7 vehicle 2 of type 1 |
|       |       |       |       |        | 1-8 vehicle 3 of type 1 |
|       |       |       |       |        | 1-10 vehicle 4 of type 1 |

Table 4. Data related to customers for the multiobjective heterogeneous fixed fleet split delivery vehicle routing problem.

| customer | x coordinate | y coordinate | demand |
|----------|--------------|--------------|--------|
| 1        | 145          | 215          | 0      |
| 2        | 151          | 264          | 10     |
| 3        | 159          | 261          | 8      |
| 4        | 130          | 254          | 12     |
| 5        | 128          | 252          | 14     |

2 units is satisfied in the third route by vehicle 1 of type 3. If $w_1 = 1$ and $w_2 = 9$, the corresponding result is $z_1 = 26.40$ and $z_2 = 1$ (only type 3 is selected), which clearly demonstrates the trade-off between the two objectives, and leads to the
Table 5. Data related to vehicles for the multiobjective heterogeneous fixed fleet split delivery vehicle routing problem.

| Vehicle type | Vehicle capacity | Number of vehicles | Routing Cost | Usage Cost | Type cost |
|--------------|------------------|--------------------|--------------|------------|-----------|
| 1            | 10               | 1                  | 1            | 20         | 100       |
| 2            | 15               | 2                  | 1.1          | 35         | 150       |
| 3            | 20               | 3                  | 1.2          | 50         | 200       |

Total cost 926.40 + 200 = 1126.40 which is less than the total cost corresponding to the case of $w_1 = 8$ and $w_2 = 2$. This experiment demonstrates the strength and importance of considering the second objective function, and obviously illustrates that the only considering the first objective function, may really be deceptive.

Table 6. GAMS results for the multiobjective heterogeneous fixed fleet split delivery vehicle routing problem.

| $w_1$ | $w_2$ | $z_1$ | $z_2$ | Routes | Vehicles |
|-------|-------|-------|-------|--------|----------|
| 1     | 9     | 926.40| 1     | 1-4-1  | vehicle 1 of type 3 |
|       |       |       |       | 1-2-3-1| vehicle 2 of type 3 |
|       |       |       |       | 1-5-1  | vehicle 3 of type 3 |
| 2     | 8     | 926.40| 1     | 1-2-3-1| vehicle 1 of type 3 |
|       |       |       |       | 1-4-1  | vehicle 2 of type 3 |
|       |       |       |       | 1-5-1  | vehicle 3 of type 3 |
| 3     | 7     | 926.40| 1     | 1-4-1  | vehicle 1 of type 3 |
|       |       |       |       | 1-5-1  | vehicle 2 of type 3 |
|       |       |       |       | 1-2-3-1| vehicle 3 of type 3 |
| 4     | 6     | 926.40| 1     | 1-4-1  | vehicle 1 of type 3 |
|       |       |       |       | 1-2-3-1| vehicle 2 of type 3 |
|       |       |       |       | 1-5-1  | vehicle 3 of type 3 |
| 5     | 5     | 812.40| 2     | 1-5(10)-1| vehicle 1 of type 1 |
|       |       |       |       | 1-2-3-1| vehicle 1 of type 3 |
|       |       |       |       | 1-5(4)-4-1| vehicle 2 of type 3 |
| 6     | 4     | 812.40| 2     | 1-5(10)-1| vehicle 1 of type 1 |
|       |       |       |       | 1-2-3-1| vehicle 1 of type 3 |
|       |       |       |       | 1-5(4)-4-1| vehicle 2 of type 3 |
| 7     | 3     | 812.40| 2     | 1-5(10)-1| vehicle 1 of type 1 |
|       |       |       |       | 1-5(4)-4-1| vehicle 1 of type 3 |
|       |       |       |       | 1-3-2-1| vehicle 1 of type 3 |
| 8     | 2     | 770.60| 3     | 1-4(2)-1| vehicle 1 of type 1 |
|       |       |       |       | 1-5-1  | vehicle 1 of type 2 |
|       |       |       |       | 1-3-2-4(10)-1| vehicle 1 of type 3 |
| 9     | 1     | 770.60| 3     | 1-4(10)-1| vehicle 1 of type 1 |
|       |       |       |       | 1-5-1  | vehicle 1 of type 2 |
|       |       |       |       | 1-4(2)-2-3-1| vehicle 1 of type 3 |

4. **Tabu search algorithm.** Tabu search (TS) algorithm was first studied by Glover [14] and since then it has been utilized to investigate solutions of a number of combinatorial optimization problems including the vehicle routing problems. TS is a single solution based metaheuristic algorithm which tries to escape from the local optimal solutions. The algorithm permits deterioration in the objective function to prevent cycles (returning to previously visited solutions). TS algorithm uses memory structures for storing the attributes of the currently accepted solutions in the tabu list. In this paper, we utilize the tabu search algorithm to solve the
mathematical models presented. To solve the multiobjective models, a penalty cost is used to minimize the vehicle types (second objective function). The detailed information about the TS algorithm used in this paper, is given in sections 4.1 and 4.2 below.

4.1. Finding initial solutions. The following algorithm is used to find initial solutions.

**Initialization.** Calculate the total customer demand by summing all demands for all existing customers. Choose the vehicle type with maximum amount of capacity and the maximum number of vehicles. The two cases should be considered. The total amount of capacity of some type(s) of vehicles is greater than the total customer demand, or total amount of capacities of all vehicle types are less than the total customer demand. In the first case, if the total amount of capacity of some type is greater than the total customer demand, then choose this type. If there are more than one such types, choose the one with minimal total vehicle type cost. In the case, if no vehicle type satisfies the total customer demand, first choose the type whose total capacity is closest to the total customer demand, calculate the remaining unsatisfied customer demand and repeat the above procedure for the remaining part of vehicle types.

In this paper, for generating an initial route, two algorithms are used: the nearest neighbourhood heuristic (NNH) algorithm, and the random neighborhood heuristic (RNH) algorithm.

**NNH for finding an initial solution.** Step 1) For the chosen vehicle (type), and starting from the depot, under the capacity and the demand constraints, the closest customer to depot is added to the current route. The vehicle capacity is updated by subtracting the demand of the added customer.
Step 2) The closest customer to the last added one in the route, is inserted to current route by checking the customer demand and the vehicle capacity constraints. If the current capacity is less than the closest customer’s demand, consider the next closest customer. If the capacity constraint does not hold for all remaining customers, return to the depot.
Step 3) Check whether there is a customer with unsatisfied demand. If no customer remained then STOP, otherwise choose the next vehicle and go to Step 1.

**RNH algorithm for finding an initial solution.**
Step 1) Let n be the number of customers, where n = 1 denotes the depot. An array with 2, 3, . . . , n members is randomly generated.
Step 2) Choose the first element of the array. For the chosen vehicle (type), update the vehicle capacity, by subtracting the demand of the added customer, and update the array by deleting the customer added to the route. If the current capacity is less than the next customer’s demand, consider the following customer. If the capacity constraint does not hold for all remaining customers, return to the depot.
Step 3) Check whether there is a customer in the array (with unsatisfied demand). If no customer remained then STOP, otherwise choose the next vehicle and go to Step 2.

4.2. Main algorithm. After generating an initial solution, neighbourhood is generated using the swap operator. For the two randomly selected routes, two customers are exchanged. In the tabu search algorithm, the swapped customers are kept in the tabu list. Tabu customers are extracted from the tabu list after some...
iterations by aspiration criteria. By using aspiration criteria, search space can be diversified. In the literature, main sources used for the diversification, are strategic oscillation, shaking of a good solution, the change of neighborhood or tabu tenure. For the intensification, main sources are restarting the procedures after a given number of iterations or post optimization procedures. In our algorithm, tabu tenure and restarting procedures are used for diversification and intensification operations, respectively. Algorithm works until the stopping criteria, which is defined as a maximum iteration number.

The only difference arises in application of the main algorithm in solving the multiobjective heterogeneous fixed fleet split delivery vehicle routing problem, whose steps are explained as follows.

Step 1) The Initialization Step given above is applied to determine the vehicle type(s).

Step 2) Apply the NNH algorithm to find an initial solution in the following form. For the chosen vehicle (type), and starting from the depot, under the capacity and the demand constraints, the closest customer to depot is added to the current route. The vehicle capacity is updated by subtracting the demand of the added customer. The closest customer to the last added one in the route, is inserted to current route by checking the customer demand and the vehicle capacity constraints. If the current capacity is less than the closest customer’s demand, then the current capacity is divided to the customer’s demand and the resulting quotient is compared to the threshold value \( \varepsilon \) (see the constraint (32)). In this paper, \( \varepsilon = 0.2 \) is used for the threshold value. If the quotient is greater than 0.2, the splitting can be done, that is the current customer is added to the route, for which not all but the remaining capacity amount of the vehicle is used to satisfy the fraction of the demand of this current customer. After which this vehicle returns back to the depot and the remaining part of the total demand is satisfied by a new vehicle. Otherwise, if the quotient is less than 0.2, the vehicle returns to the depot and the next customer is served by another vehicle.

4.3. Computational results obtained by using the tabu search algorithm.

In this section, the tabu search algorithm described above, is applied to test problems of Taillard [31], from 13 to 20. These problems were originally created by Golden [15]. Taillard modified these 8 problems to find good tours by a given fleet. The number of customers in the test problems solved, are 50, 75 and 100. The number of vehicle types are between 3 and 6, and the number of vehicles from each type is not too many so that almost all vehicle types and almost all vehicles are used in the final solutions of original test problems (by Taillard). The number of vehicles of all types in these problems is determined so that, the total capacity of all vehicles is approximately equal to the total customers’ demand. Therefore, all vehicle types have to be used in order to meet the total demand. In order to emphasize the importance and the role of the second objective function, which minimizes the vehicle type used, we have changed the original data by taking 10 vehicles from each type (these data will be denoted as TK data, in the sequel). In such a case the total capacity of all vehicles becomes substantially greater than the total demand, which enables the model, to choose only the optimal vehicle types. The data for the test problems used, are given in Table 7, where third column gives the original data of Taillard, related to the number of vehicles for each type, and the fourth column gives the number of vehicles for types, changed by the authors, which is denoted by TK data.
The algorithm is coded in C#. In the algorithm, the values 7 and 10 are used for the tabu list size, and the values 5 and 10 are used for the tabu tenure parameter. Initial solutions used in the algorithm, for heterogeneous fixed fleet capacitated and open vehicle routing mathematical models, are calculated by using both the algorithms: the nearest neighbourhood heuristic, and the random neighborhood heuristic. The results obtained for all test problems with different initial solution algorithms, are illustrated in separate tables for every mathematical model. Initial solutions for heterogeneous fixed fleet split delivery vehicle routing problems, are calculated by using only the nearest neighbourhood heuristic algorithm. The termination criterion is taken as 1000 iterations. Each run takes between 15 (for the problems with 50 customers) and 40 minutes (for the problems with 100 customers). Note that the run time does not depend on the parameter values such as the tabu list size and the tabu tenure parameter.

Table 7. Vehicle type data for tabu search algorithm

| Problem no | vehicle type | Taillard’s original vehicle number data | TK data | vehicle type cost (penalty cost) |
|------------|--------------|----------------------------------------|---------|----------------------------------|
| 13         | 1            | 4                                      | 10      | 10                               |
| 13         | 2            | 2                                      | 10      | 15                               |
| 14         | 3            | 4                                      | 10      | 20                               |
| 15         | 4            | 4                                      | 10      | 35                               |
| 16         | 5            | 2                                      | 10      | 60                               |
| 17         | 6            | 1                                      | 10      | 100                              |
| 18         | 1            | 4                                      | 10      | 10                               |
| 19         | 2            | 2                                      | 10      | 20                               |
| 20         | 3            | 1                                      | 10      | 40                               |

The computational results obtained for the heterogeneous fixed fleet capacitated vehicle routing test problems using the TS algorithm, are summarized in Tables 8 and 9. Table 8 presents results of test problems, obtained using the initial solutions calculated by the NNH algorithm, and Table 9 presents results where the initial solutions are created using the RNH algorithm.

The computational results obtained for the heterogeneous fixed fleet open vehicle routing test problems using the TS algorithm, are summarized in Tables 10 and 11. Table 10 presents results of test problems, obtained using the initial solutions calculated by the NNH algorithm and Table 11 presents results where the initial solutions are created using the RNH algorithm.
The computational results obtained for the heterogeneous fixed fleet split delivery vehicle routing test problems using the TS algorithm, are summarized in Table 12, where the initial solutions are obtained applying NNH algorithm.

In all tables from 8 to 12 the following notations are used. First column represents the instance number, second column $n$ denotes the number of customers, third column shows the tabu size, fourth column shows the tabu tenure, columns 5, 6 depict the computational results obtained for the test problems with Taillard’s original data and the columns 7, 8, 9 depict the computational results obtained for the test problems with TK data (see Table 7). The columns 5 and 7 give the objective functions values obtained using the algorithm, where the objective function includes the sum of total routing cost, the vehicle usage costs and the costs related to the vehicle types used. Finally the last columns of all tables represent the vehicle types obtained using the algorithm for the TK data. Note that in the solutions obtained for Taillard’s data, all types of vehicles were used, therefore the corresponding data is not included in the tables.

It is remarkable that, the solutions obtained for all test problems with TK data, use just one vehicle type and lesser number of vehicles, where the exceptions are the only solutions obtained for the heterogeneous fixed fleet open vehicle routing problem’s instance 14, see Table 10 and the solutions obtained for the heterogeneous fixed fleet split delivery vehicle routing problem’s instance 14, see Table 12. For example, the first row of Table 8, shows results obtained for the test problem 13, where the solution obtained for the Taillard’s data, use all vehicle types from 1 to 6 and all 17 vehicles. While the solution obtained for TK data uses just 1 type, that is type 6 and only 6 vehicles (of 10).

For some instances we get solutions where the total cost for the TK data is less than the one for Taillard’s data, see for example solutions obtained for the instance 18 in Table 8; solutions obtained for the instances 16, 17, 18, 19 in Table 9, solutions obtained for the instance 18 in Table 10 and solutions obtained for the instances 16, 18, 19 in Table 11.

The computational results also demonstrate how the tabu size and tabu tenure parameters affect the obtained solutions. For example, consider Table 9, problem number 13 : if tabu size is 7 and tabu tenure is 5, the objective function value for TK data is 5390.14. But, if tabu size equals 7 and tabu tenure is 10, the objective function value is found equal to 6127.04.

5. **Conclusions.** In this paper, three types of heterogeneous fixed fleet vehicle routing problems are studied. These are the capacitated, the open and the split delivery vehicle routing problems. New multiobjective linear binary and mixed integer programming models are proposed for these problems, where the first objective is the minimization of a total routing and usage costs for vehicles, and the second one is the vehicle type minimization, respectively. The proposed mathematical models and the solution approaches are all illustrated on test problems. Small-sized test problems are first scalarized by applying the weighted sum scalarization method and then, the scalarized problems are solved by using the GAMS software. The large-sized test problems are solved by utilizing the tabu search algorithm. To illustrate the role of the second objective function, new problem data are generated and a computational experiment is provided. All the obtained results are interpreted.
Table 8. Computational results for heterogeneous fixed fleet capacitated VRP, with initial solutions obtained using the NNH algorithm.

| Pr. no | n   | tabu size | tabu tenure | Taillard's data | TK data | vehicle types used |
|--------|-----|-----------|-------------|----------------|---------|-------------------|
|        |     |           |             | number of vehicles | obj. value | n. of vehicles |          |
| 13     | 50  | 7         | 5           | 4962.84         | 17       | 5596.35         | 6         | type 6 |
| 13     | 50  | 7         | 10          | 4962.84         | 17       | 5596.35         | 6         | type 6 |
| 13     | 50  | 10        | 5           | 4962.84         | 17       | 5596.35         | 6         | type 6 |
| 14     | 50  | 7         | 5           | 11717.23        | 17       | 15277.12        | 4         | type 3 |
| 14     | 50  | 7         | 10          | 11717.23        | 17       | 15277.12        | 4         | type 3 |
| 14     | 50  | 10        | 5           | 11717.23        | 17       | 15277.12        | 4         | type 3 |
| 15     | 50  | 7         | 5           | 4109.82         | 9        | 4411.77         | 9         | type 2 |
| 15     | 50  | 7         | 10          | 4109.82         | 9        | 4411.77         | 9         | type 2 |
| 15     | 50  | 10        | 5           | 4109.82         | 9        | 4411.77         | 9         | type 2 |
| 16     | 50  | 7         | 5           | 4967.63         | 9        | 5014.15         | 6         | type 3 |
| 16     | 50  | 7         | 10          | 4967.63         | 9        | 5014.15         | 6         | type 3 |
| 16     | 50  | 10        | 5           | 4967.63         | 9        | 5014.15         | 6         | type 3 |
| 17     | 75  | 7         | 5           | 3581.81         | 11       | 3819.1          | 8         | type 3 |
| 17     | 75  | 7         | 10          | 3581.81         | 11       | 3819.1          | 8         | type 3 |
| 17     | 75  | 10        | 5           | 3581.81         | 11       | 3819.1          | 8         | type 3 |
| 17     | 75  | 10        | 10          | 3581.81         | 11       | 3819.1          | 8         | type 3 |
| 18     | 75  | 7         | 5           | 8251.13         | 14       | 7535.78         | 10        | type 4 |
| 18     | 75  | 7         | 10          | 8251.13         | 14       | 7535.78         | 10        | type 4 |
| 18     | 75  | 10        | 5           | 8251.13         | 14       | 7535.78         | 10        | type 4 |
| 18     | 75  | 10        | 10          | 8251.13         | 14       | 7535.78         | 10        | type 4 |
| 19     | 100 | 7         | 5           | 12635.68        | 9        | 12712.45        | 8         | type 2 |
| 19     | 100 | 7         | 10          | 12635.68        | 9        | 12712.45        | 8         | type 2 |
| 19     | 100 | 10        | 5           | 12635.68        | 9        | 12712.45        | 8         | type 2 |
| 19     | 100 | 10        | 10          | 12635.68        | 9        | 12712.45        | 8         | type 2 |
| 20     | 100 | 7         | 5           | 6982.42         | 13       | 8497.79         | 8         | type 3 |
| 20     | 100 | 7         | 10          | 6982.42         | 13       | 8497.79         | 8         | type 3 |
| 20     | 100 | 10        | 5           | 6982.42         | 13       | 8497.79         | 8         | type 3 |
| 20     | 100 | 10        | 10          | 6982.42         | 13       | 8497.79         | 8         | type 3 |

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Table 9. Computational results for heterogeneous fixed fleet capacitated VRP, with initial solutions obtained using the RNH algorithm.

| Pr. no | n  | tabu size | tabu tenure | Taillard's data | TK data |
|--------|----|-----------|-------------|----------------|---------|
|        |    |           |             | obj. value      | n. of vehicles | n. of vehicles | vehicle types used |
|        |    |           |             | n. of vehicles |           |               |                   |
| 13     | 50 | 7         | 5           | 5226.86        | 17         | 5390.14       | 6                   |
| 13     | 50 | 7         | 10          | 5438.5         | 17         | 6127.04       | 6                   |
| 13     | 50 | 10        | 5           | 5322.39        | 17         | 6085.09       | 6                   |
| 13     | 50 | 10        | 10          | 4979.67        | 17         | 5859.03       | 6                   |
| 14     | 50 | 7         | 5           | 11655.46       | 7          | 15133.31      | 4                   |
| 14     | 50 | 7         | 10          | 11747.49       | 7          | 15168.62      | 4                   |
| 14     | 50 | 10        | 5           | 11780.2        | 7          | 15392.41      | 4                   |
| 14     | 50 | 10        | 10          | 11755.49       | 7          | 15125.57      | 4                   |
| 15     | 50 | 7         | 5           | 4108.74        | 9          | 4141.99       | 9                   |
| 15     | 50 | 7         | 10          | 4371.32        | 9          | 4149.24       | 9                   |
| 15     | 50 | 10        | 5           | 3945.91        | 9          | 4242.02       | 9                   |
| 15     | 50 | 10        | 10          | 4175.87        | 9          | 4224.99       | 9                   |
| 16     | 50 | 7         | 5           | 5042.91        | 9          | 5146.22       | 6                   |
| 16     | 50 | 7         | 10          | 4651.12        | 9          | 5028.29       | 6                   |
| 16     | 50 | 10        | 5           | 4471.94        | 9          | 5521.87       | 6                   |
| 16     | 50 | 10        | 10          | 4887.57        | 9          | 4845.65       | 6                   |
| 17     | 75 | 7         | 5           | 3668.52        | 11         | 4018.59       | 8                   |
| 17     | 75 | 7         | 10          | 3170.15        | 11         | 3566.41       | 8                   |
| 17     | 75 | 10        | 5           | 3722.46        | 11         | 3694.85       | 8                   |
| 17     | 75 | 10        | 10          | 3749.49        | 11         | 3772.36       | 8                   |
| 18     | 75 | 7         | 5           | 7012.94        | 14         | 6661.09       | 10                  |
| 18     | 75 | 7         | 10          | 8142.29        | 14         | 6977.01       | 10                  |
| 18     | 75 | 10        | 5           | 7304.11        | 14         | 6080.45       | 10                  |
| 18     | 75 | 10        | 10          | 7370.3         | 14         | 6405.93       | 10                  |
| 19     | 100| 7         | 5           | 14452.39       | 9          | 13371.45      | 8                   |
| 19     | 100| 7         | 10          | 13523.82       | 9          | 12986.54      | 8                   |
| 19     | 100| 10        | 5           | 14516.23       | 9          | 15678.2       | 8                   |
| 19     | 100| 10        | 10          | 15692.56       | 9          | 16994.23      | 8                   |
| 20     | 100| 7         | 5           | 7140.38        | 13         | 8527.07       | 8                   |
| 20     | 100| 7         | 10          | 7267.18        | 13         | 8396.23       | 8                   |
| 20     | 100| 10        | 5           | 7816.72        | 13         | 7918.24       | 8                   |
| 20     | 100| 10        | 10          | 7329.74        | 13         | 8531.93       | 8                   |

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Table 10. Computational results for heterogeneous fixed fleet open VRP, with initial solutions obtained using the NNH algorithm.

| Pr. no | n tabu size | tabu tenure | Taillard’s data OBJ value | n. of vehicles | TK data OBJ value | n. of vehicles | vehicle types used |
|--------|-------------|-------------|---------------------------|----------------|-------------------|----------------|-------------------|
| 13     | 50          | 7           | 4381.93                   | 14             | 6376.82           | 6              | type 6            |
| 13     | 50          | 10          | 4386.98                   | 14             | 6376.82           | 6              | type 6            |
| 13     | 50          | 10          | 4381.93                   | 14             | 6376.82           | 6              | type 6            |
| 14     | 50          | 7           | 12609.85                  | 7              | 10864.35          | 9              | type 1            |
| 14     | 50          | 10          | 12609.85                  | 7              | 10864.35          | 9              | type 1            |
| 14     | 50          | 10          | 12609.85                  | 7              | 10864.35          | 9              | type 1            |
| 15     | 50          | 7           | 3878.93                   | 9              | 10054.3           | 9              | type 1            |
| 15     | 50          | 7           | 3878.93                   | 9              | 10054.3           | 9              | type 1            |
| 15     | 50          | 10          | 3878.93                   | 9              | 10054.3           | 9              | type 1            |
| 16     | 50          | 7           | 4473.47                   | 9              | 4845.49           | 6              | type 3            |
| 16     | 50          | 10          | 4473.47                   | 9              | 4845.49           | 6              | type 3            |
| 16     | 50          | 10          | 4473.47                   | 9              | 4845.49           | 6              | type 3            |
| 17     | 75          | 7           | 3125.69                   | 11             | 3652.77           | 8              | type 3            |
| 17     | 75          | 7           | 3125.69                   | 11             | 3652.77           | 8              | type 3            |
| 17     | 75          | 10          | 3125.69                   | 11             | 3652.77           | 8              | type 3            |
| 17     | 75          | 10          | 3125.69                   | 11             | 3652.77           | 8              | type 3            |
| 18     | 75          | 7           | 7803.44                   | 14             | 6062.63           | 10             | type 4            |
| 18     | 75          | 7           | 7803.44                   | 14             | 6062.63           | 10             | type 4            |
| 18     | 75          | 10          | 7803.44                   | 14             | 6062.63           | 10             | type 4            |
| 18     | 75          | 10          | 7803.44                   | 14             | 6062.63           | 10             | type 4            |
| 19     | 100         | 7           | 12274.38                  | 10             | 12405.72          | 8              | type 2            |
| 19     | 100         | 7           | 12274.38                  | 10             | 12405.72          | 8              | type 2            |
| 19     | 100         | 10          | 12274.38                  | 10             | 12405.72          | 8              | type 2            |
| 19     | 100         | 10          | 12274.38                  | 10             | 12405.72          | 8              | type 2            |
| 20     | 100         | 7           | 6782.57                   | 13             | 8497.79           | 8              | type 3            |
| 20     | 100         | 7           | 6782.57                   | 13             | 8497.79           | 8              | type 3            |
| 20     | 100         | 10          | 6782.57                   | 13             | 8497.79           | 8              | type 3            |
| 20     | 100         | 10          | 6782.57                   | 13             | 8497.79           | 8              | type 3            |

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Table 11. Computational results for heterogeneous fixed fleet open VRP, with initial solutions obtained using the RNH algorithm.

| Pr. no | n   | tabu size | tabu tenure | Taillard’s data | TK data |
|--------|-----|-----------|-------------|-----------------|---------|
|        |     |           |             | obj. value      | n. of vehicles | obj. value | n. of vehicles | vehicle types |
| 13     | 50  | 7         | 5           | 4100.25         | 14       | 6211.45     | 6           | type 6        |
| 13     | 50  | 7         | 10          | 4215.89         | 14       | 6541.47     | 6           | type 6        |
| 13     | 50  | 10        | 5           | 4390.47         | 14       | 6622.15     | 6           | type 6        |
| 13     | 50  | 10        | 10          | 4412.69         | 14       | 6542.12     | 6           | type 6        |
| 14     | 50  | 7         | 5           | 15463.88        | 7        | 16250.75    | 4           | type 3        |
| 14     | 50  | 7         | 10          | 16213.24        | 7        | 16350.2     | 4           | type 3        |
| 14     | 50  | 10        | 5           | 15478.23        | 7        | 16272.54    | 4           | type 3        |
| 14     | 50  | 10        | 10          | 16932.3         | 7        | 16897.56    | 4           | type 3        |
| 15     | 50  | 7         | 5           | 5988.45         | 9        | 9556.47     | 9           | type 1        |
| 15     | 50  | 7         | 10          | 3995.64         | 9        | 10056.23    | 9           | type 1        |
| 15     | 50  | 10        | 5           | 3654.21         | 9        | 9854.12     | 9           | type 1        |
| 15     | 50  | 10        | 10          | 3875.46         | 9        | 9932.41     | 9           | type 1        |
| 16     | 50  | 7         | 5           | 4852.77         | 9        | 4912.55     | 6           | type 3        |
| 16     | 50  | 7         | 10          | 4744.46         | 9        | 4753.21     | 6           | type 3        |
| 16     | 50  | 10        | 5           | 4715.23         | 9        | 4655.12     | 6           | type 3        |
| 16     | 50  | 10        | 10          | 4879.56         | 9        | 4899.52     | 6           | type 3        |
| 17     | 75  | 7         | 5           | 3478.56         | 11       | 3678.99     | 8           | type 3        |
| 17     | 75  | 7         | 10          | 3541.72         | 11       | 3755.17     | 8           | type 3        |
| 17     | 75  | 10        | 5           | 3655.77         | 11       | 3547.89     | 8           | type 3        |
| 18     | 75  | 7         | 5           | 7653.45         | 14       | 7563.33     | 10          | type 4        |
| 18     | 75  | 7         | 10          | 7664.52         | 14       | 7895.41     | 10          | type 4        |
| 18     | 75  | 10        | 5           | 7569.44         | 14       | 7754.13     | 10          | type 4        |
| 18     | 75  | 10        | 10          | 8004.56         | 14       | 7965.52     | 10          | type 4        |
| 19     | 100 | 7         | 5           | 16542.33        | 10       | 17841.22    | 8           | type 2        |
| 19     | 100 | 7         | 10          | 15478.99        | 10       | 16984.53    | 8           | type 2        |
| 19     | 100 | 10        | 5           | 14563.01        | 10       | 16547.99    | 8           | type 2        |
| 19     | 100 | 10        | 10          | 15879.22        | 10       | 15642.33    | 8           | type 2        |
| 20     | 100 | 7         | 5           | 6961.08         | 13       | 7918.54     | 8           | type 3        |
| 20     | 100 | 7         | 10          | 6742.13         | 13       | 7654.13     | 8           | type 3        |
| 20     | 100 | 10        | 5           | 6830.15         | 13       | 7326.58     | 8           | type 3        |
| 20     | 100 | 10        | 10          | 6955.41         | 13       | 7456.23     | 8           | type 3        |

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Table 12. Computational results for heterogeneous fixed fleet split delivery VRP, with initial solutions obtained using the NNH algorithm.

| Pr. no | n  | tabu size | tabu tenure | Taillard’s data | TK data |
|--------|----|-----------|-------------|----------------|---------|
| 13     | 50 | 7         | 5           | 4248.33        | 17      |
| 13     | 50 | 7         | 10          | 4248.33        | 17      |
| 13     | 50 | 10        | 5           | 4236.18        | 17      |
| 13     | 50 | 10        | 10          | 4236.18        | 17      |
| 14     | 50 | 7         | 5           | 12264.91       | 7       |
| 14     | 50 | 7         | 10          | 12264.91       | 7       |
| 14     | 50 | 10        | 5           | 12255.03       | 7       |
| 14     | 50 | 10        | 10          | 12255.03       | 7       |
| 15     | 50 | 7         | 5           | 3750.95        | 9       |
| 15     | 50 | 7         | 10          | 3750.95        | 9       |
| 15     | 50 | 10        | 5           | 3750.95        | 9       |
| 15     | 50 | 10        | 10          | 3750.95        | 9       |
| 16     | 50 | 7         | 5           | 4047.47        | 9       |
| 16     | 50 | 7         | 10          | 4047.47        | 9       |
| 16     | 50 | 10        | 5           | 3988.54        | 9       |
| 16     | 50 | 10        | 10          | 3988.54        | 9       |
| 17     | 75 | 7         | 5           | 2850.35        | 11      |
| 17     | 75 | 7         | 10          | 2850.35        | 11      |
| 17     | 75 | 10        | 5           | 2839.2         | 11      |
| 17     | 75 | 10        | 10          | 2839.2         | 11      |
| 18     | 75 | 7         | 5           | 5121.54        | 14      |
| 18     | 75 | 7         | 10          | 5121.54        | 14      |
| 18     | 75 | 10        | 5           | 5094.1         | 14      |
| 18     | 75 | 10        | 10          | 5094.1         | 14      |
| 19     | 100| 7         | 5           | 11492          | 10      |
| 19     | 100| 7         | 10          | 11492          | 10      |
| 19     | 100| 10        | 5           | 11484.76       | 10      |
| 19     | 100| 10        | 10          | 11484.76       | 10      |
| 20     | 100| 7         | 5           | 5606.26        | 13      |
| 20     | 100| 7         | 10          | 5606.26        | 13      |
| 20     | 100| 10        | 5           | 5541.51        | 13      |
| 20     | 100| 10        | 10          | 5541.51        | 13      |

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