Realizing High-order Bessel Vortex Beams with High-level Phase Holograms

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Abstract. High-order Bessel beams are widely used in laser processing and precision measurement. For the realization of high-order Bessel beams, a spatial light modulator is used to load the high-order phase hologram instead of the real spiral phase plate and the axicon. The distribution of light field in two modes of axicon combination and phase superposition is analyzed theoretically. The beam modulation in two modes is designed and compared experimentally. The results show that the modulation effect of the phase superposition method is better than that of the axicon combination method. Meanwhile, the operation is flexible and the manufacture errors can be reduced. It is an effective method for realizing high-order Bessel beams.

1. Introduction
The high-order Bessel beam is a kind of vortex beam. Its wavefront array is spiral, and the central light intensity is zero[1]. These unique characteristics make high-order Bessel beams widely used in laser processing, precision measurement, information coding and transmission, micro-particle manipulation and optical tweezers[2-4]. Common methods for realizing high-order Bessel beams are active and passive[5]. The active mode is to generate high-order Bessel-Gauss beams directly from the laser through a resonator with a specific structure[6], while the passive mode is to convert other beams into high-order Bessel-Gauss beams[7-8].

This paper focuses on the realization method of high-order Bessel beams. It analyzes the light field distribution in the two modes of axicon combination and phase superposition theoretically, and compares the beam modulation in the two modes. From the modulation effect and flexible operation, the phase superposition method is superior to the axicon combination method.

2. Theory analysis

2.1. Characteristics of higher-order Bessel beams
The ideal high-order Bessel beams carry infinite energy, which violates the law of conservation of energy. So it is difficult to achieve ideal high-order Bessel beams in practice. In order to overcome the difficulty of realizing high-order Bessel in physics, people have tried to add another modulation of Gauss profile distribution to the high-order Bessel beam, so that the generation of shigh-order Bessel-Gauss beam can be realized. Its expression is as follows:
Where $\omega_z$ is the waist radius of the Gauss beams, $m$ is the order of the higher-order Bessel-Gauss beam and the number of topological charges of the vortex beams. Equation (1) shows that the field expression of High-order Bessel-Gauss beams contains $\exp(im\theta)$, which is a spiral phase factor. Therefore, high-order Bessel-Gauss beams are also a kind of vortex beams. The helical wavefront is singular, and the light interferes and cancels at the center to form a dark core, which causes the light intensity to disappear and the center’s light intensity to be zero.

2.2. Combination of SPP and axicon

High-order Bessel beams have both spiral and axicon phases. In the experiment, the Gauss beams can be transformed into vortex beams through the spiral phase plate (SPP), then the high-order Bessel beams are generated by an axicon with vertical incident light, as shown in figure 1.

The ideal SPP is smooth. Because of the manufacturing process, a stepped SPP is often used in practice. But manufacturing errors are inevitable. Because spatial light modulator (SLM) can modulate the amplitude and phase of the beam by using the photoelectric effect of liquid crystal, SLM can simulate the SPP by loading high-level phase hologram, realizing the generation of vortex beam.

Then the distribution of the light field after the incident of Gauss beam on the SLM is as follows:

$$E(r,\theta,z,t) = J_m(\alpha r) \exp(im\theta) \exp\left[i\left(\beta z - \omega t\right)\right] \exp\left(-r^2/\omega_z^2\right)$$

(1)

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$$E_1(\rho,\varphi,z) = \int_0^R \exp\left[\frac{ik}{2z}\left(\rho^2 + \rho^2\right) - ik(n_1 - 1)\gamma r\right]$$

(2)

$$\times \left(\frac{r}{\omega_1}\right)^n \exp\left(-\frac{r^2}{\omega_1^2}\right) \exp(in\varphi) i^n J_n\left(\frac{kr}{z}\right) rdr$$

(3)

Where the polar coordinates of observation plane are $\rho$ and $\varphi$, the radius of beam is $R$, the refractive index of axicon is $n_1$, the angle of axicon is $\gamma$, and the Bessel function of order $n$ is $J_n$.

It can be seen from equation (3) that in the light field distribution of the final emergent light, the vortex term of the vortex beam does not change, and the background light field has a higher-order Bessel function term relative to the original. It also indicates high-order Bessel beams belong to the vortex beam.

2.3. Superposition of vortex phase and axicon phase

The SLM is flexible and convenient. By inputting appropriate parameters, arbitrary vortex beams can be realized. Because of the manufacturing errors of the real axicon, the SLM can also be used to
simulate the real axicon to reduce the manufacturing errors. Therefore, the vortex phase and the axicon phase are superimposed to generate a new phase hologram, as shown in figure 2.

\[ \text{vortex phase} + \text{axicon phase} = \text{holograms} \]

Figure 2. Superposition of vortex phase and axicon phase

The transmittance function of the new phase hologram is

\[ T(r, \theta) = \exp\left(-\frac{ir2\pi}{r_0}\right) \exp(il\theta) \]  

(4)

Where \( r, \theta \) are independent variables of holographic phase plane polar coordinate system, \( r_0 \) are variable constants and \( l \) are topological charges.

Assuming that the Gauss beam is modulated by the SLM loaded with a new hologram, the Fresnel diffraction integral in cylindrical coordinates gives the field \( E_z \) as

\[ E_z(\rho, \phi, z) = \frac{1}{ikz} \exp\left(\frac{ik\rho^2}{2z}\right) \exp(ikz) \exp\left[il\left(\phi - \frac{\pi}{2}\right)\right] \]

\[ \times \int_0^\infty \exp\left(-\frac{r^2}{\omega_0^2}\right) \exp\left(-\frac{r^{2\pi}}{r_0}\right) \exp\left(-\frac{kr\rho}{z}\right) J_l\left(\frac{kr\rho}{z}\right) r dr \]  

(5)

Where the waist radius of the Gauss beams is \( \omega_0 \), and the Bessel function of order \( l \) is \( J_l \).

It can also be seen from equation (5) that the field distribution of the light emitted by the new phase hologram also contains the terms of vortex and higher-order Bessel functions, which can also indicate that the higher-order Bessel beam is one of the vortex beam.

3. Experimental results and analysis

By loading the vortex phase from topological charge 2 to 4 to SLM, the corresponding vortex beams are realized. Then the generated vortex beams are incident vertically to the real axicon. The high-order Bessel beams are realized as shown in figure 3. The central region of the beam is concentric rings, which conforms to the intensity distribution of Bessel beam. The inner ring is a hollow ring. With the increase of order, the radius of the hollow ring increases, which satisfies the characteristics of high-order Bessel beam.

\[ \text{(a)} \quad \text{(b)} \quad \text{(c)} \]

Figure 3. Axicon combination method. (a)m=2; (b)m=3; (c)m=4

Similarly, the new phase holograms with topological charges of 2 to 4 is loaded into the SLM. Since the new phase hologram has an axicon phase, it is not necessary to let the emitted light pass through the axicon and can be directly received by CCD. The high-order Bessel beam realized is
shown in figure 4. Like the high-order Bessel beams realized by the combination of the real axicon, the generated beams also satisfy the intensity distribution and shape characteristics of the high-order Bessel beams.

![Figure 4. Phase superposition method. (a)m=2; (b)m=3; (c)m=4](image)

The experimental results of figure 3 and figure 4 show that both axicon combination and phase superposition can realize high-order Bessel beams. However, due to the different generation modes, the high-order Bessel beams realized in the two modes are also different. From the modulation effect of the incident light, the high-order Bessel beams realized by the phase superposition method are better than that realized by the axicon combination method.

At the same time, SLM can not only simulate the phase distribution of the SPP and the axicon, but also adjust the topological charge of the spiral phase plate and the angle of the axicon. It can realize the required high-order Bessel beams more flexibly and reduce the low quality of the high-order Bessel beams caused by manufacturing errors.

4. Conclusions
In this paper, theory analysis of high-order Bessel beams in two modes of axicon combination and phase superposition is presented. The experiments are designed to realize high-order Bessel beams in two modes. By comparing the experimental results, it can be found that the modulation effect of phase superposition method is better than that of axicon combination method. Meanwhile, the operation is flexible and the processing error can be reduced. It is an efficient method for generating high-order Bessel beams. Therefore, using SLM to load holograms to realize the required high-order Bessel beams, which is of great significance in laser manufacturing, laser precision measurement and other fields.

Acknowledgments
This work was supported by the National Natural Science Foundation of China (Nos. 51575164, 51405143), the Science and Technology Research Project of Department of Education of Hubei Province (No. D20161406), the Open Fund of State Key Laboratory of Precision Measurement and Instrument (No PIL1602).

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