Dilaton and Moduli Fields in $D$-term inflation

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Abstract

We investigate the possibility of $D$-term inflation within the framework of type I string-inspired models. Although $D$-term inflation model has the excellent property that it is free from the so-called $\eta$- problem, two serious problems appear when we embed $D$-term inflation in string theory, the magnitude of FI term and the rolling motion of the dilation. In the present paper, we analyze the potential of $D$-term inflation in type I inspired models and study the behavior of dilaton and twisted moduli fields. Adopting the non-perturbative superpotential induced by gaugino condensation, the twisted moduli can be stabilized. If the dilaton is in a certain range, it evolves very slowly and does not run away to infinity. Thus $D$-term dominated vacuum energy becomes available for driving inflation. By studying

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the density perturbation generated by the inflation model, we derive the constraints on model parameters and give some implications on $D$-term inflation in type I inspired models.
1 Introduction

In the success of slow-roll inflation, the sufficient flat potential for the inflaton, which is the inflation driving scalar field, is required. However, it is not so easy to construct a model of inflation where the corresponding scalar field has a very flat potential in the framework of supergravity. Usually, the flatness of the potential is expressed as

\[ \epsilon \equiv \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2 \ll 1, \]  
\[ \eta \equiv M_p^2 \frac{V''}{V} \ll 1, \]  

in terms of the slow roll parameters, where the prime denotes the derivative with respect to the inflaton and \( M_p \) is the reduced Planck mass. In the case that the vacuum energy is dominated by the \( F \)-term during the inflation, supergravity effects produce the soft mass of inflaton field whose magnitude is the same order of \( H \) and spoil the flatness of potential. In other words, such large inflaton mass gives \( \eta \sim 1 \) and violates the slow roll condition, which is often referred to as the \( \eta \) problem.

From this point of view, the \( D \)-term inflation is one of attractive inflation scenarios within the framework of supersymmetric models [1, 2],\(^3\) because the inflaton does not acquire the mass squared term of the order of \( H^2 \) and the flatness of the potential is preserved. In the \( D \)-term inflation scenario, Fayet-Iliopoulos (FI) term is dominant during the inflation and this vacuum energy causes the accelerating expansion. The FI term comes from the anomalous \( U(1) \) symmetry. In fact, most of 4D string models have anomalous \( U(1) \) symmetry for both heterotic models [4, 5] and type I models [6]. These anomalies can be cancelled by the Green-Schwarz mechanism, where certain fields transform non-trivially under anomalous \( U(1) \) symmetries [7]. Such role is played by the dilaton field \( S \) in heterotic models and the twisted moduli fields \( M \) in type I models. That generates the FI term. Thus, the \( D \)-term inflation seems possible to be realized in string models.

However, there are difficulties in the realization of the \( D \)-term inflation based on heterotic models. The first problem is the energy scale of inflation.

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\(^3\)See Ref. [3] for early study on an inflation model relevant to the \( D \)-term.
The magnitude of the FI term is given as

\[ \xi = \frac{g^2}{192\pi^2} \text{Tr}(Q)M_p^2, \tag{3} \]

where \( g^2 \) is the gauge coupling and \( \text{Tr}(Q) \) is a model dependent constant of the order of \( \mathcal{O}(10) - \mathcal{O}(10^2) \) [4]. Eq. (3) reads \( \sqrt{\xi}/M_p = \mathcal{O}(10^{-1}) - \mathcal{O}(10^{-2}) \). On the other hand, the CMB anisotropy requires \( \xi^{1/2} \approx 10^{15} \) GeV.

We find that the theoretical prediction is too larger to meet the observational estimation. This inconsistency is still a serious problem, although several studies to construct a model with effectively suppressed FI terms have been done in heterotic models [8, 9]. The second problem is due to the dilaton dependence of the FI term and the anomalous \( U(1) \) gauge coupling. The presence of the dilaton \( S \) and several types of moduli fields is one of important features in superstring theory. Vacuum expectation values (VEVs) of these fields determine the magnitudes of all the couplings, e.g. gauge couplings and Yukawa couplings. The dilaton field prevents the realization of \( D \)-term inflation. Since \( g^2 \propto 1/\text{Re}S \), the \( D \)-term scalar potential, \( V_D = g^2 \xi^2/2 \), is in proportion to \( (\text{Re}S)^{-3} \). Hence the dilaton rolls down the potential to the infinity, \( \text{Re}S \to \infty \), and the \( D \)-term potential energy goes to zero, \( V_D \to 0 \).

This phenomenon is the same as the dilaton runaway problem in generic string models and such runaway behavior of the dilaton and moduli fields prevents a viable inflation [10]. Even if we adopt type I models rather than heterotic models, the FI term is dependent on the field which plays a role in the Green-Schwarz anomaly cancellation mechanism for the anomalous \( U(1) \). Hence, if these fields run away, the \( D \)-term inflation can not occur in 4D string models.

In this paper we study behavior of dilaton and moduli fields in the \( D \)-term inflation scenario with the above two problems in mind. The purpose of the present paper is to explore a way to avoid these problems at the same time. Actually, studies on the runaway problem have been done for heterotic models in Refs. [11, 12]. Now, in particular, we will consider type I string-inspired models. The \( D \)-term inflation in type I inspired model has been studied in Ref.[13] and it was shown that the magnitude of FI-term is reducible to a desired value. This result arise from the facts that the FI-term is determined by the expectation value of the twisted moduli and the string scale \( M_s \) is independent of the Planck scale \( M_p \) [14] in type I string models. In fact, the twisted moduli field plays a role in the Green-Schwarz
anomaly cancellation in 4D type I string models. However, in Ref. [13] the stabilization of the twisted moduli and the runway problem have not been discussed. These issues might be not trivial. Indeed, stabilization of the twisted moduli fields was studied and different aspects from those of the dilaton field was revealed [15]. As we will show, the twisted moduli field cannot stabilize with non-vanishing vacuum energy in only $D$-term potential and any vacuum energy driving inflation does not appear, unless we assume a specific Kähler potential. Therefore, stabilization of the twisted moduli is also an important issue for the inflation in Type I models.

This paper is organized as follows. In the next section, we review briefly the $D$-term inflation. In section 3, we study behavior of dilaton and twisted moduli fields. It will be shown that the twisted moduli field is stabilized during the inflation. In section 4, dynamics of the inflaton and dilaton fields are studied, and we will derive constraints on parameters in our models. Section 5 is devoted to conclusions and discussions.

2 $D$-term inflation

Here we give a brief review on the $D$-term inflation [1, 2]. We consider the N=1 supersymmetric model with $U(1)$ gauge symmetry and the non-vanishing FI term $\xi$. The model includes three matter fields $X$ and $\phi_{\pm}$. The fields $\phi_{\pm}$ have $U(1)$ charges $\pm 1$ while $X$ has no $U(1)$ charge. The $U(1)$ $D$-term is written as

$$D = \xi + |\phi_+|^2 - |\phi_-|^2. \quad (4)$$

Hereafter we take the charge assignment such that $\xi > 0$. Suppose that the following superpotential,

$$W = \lambda X \phi_+ \phi_- \quad (5)$$

Then, the scalar potential is written as

$$V = \sum_i |\partial_i W|^2 + \frac{g^2}{2} D^2, \quad (6)$$

$$= \lambda^2 |X|^2(|\phi_-|^2 + |\phi_+|^2) + \lambda^2 |\phi_- \phi_+|^2 + \frac{g^2}{2}(\xi + |\phi_+|^2 - |\phi_-|^2)^2. \quad (7)$$

\[^4\text{See also Refs. [16, 17].}\]
The true vacuum of this potential corresponds to

\[ X = \phi_+ = 0, \quad |\phi_-| = \sqrt{\xi}, \]  

(8)

and supersymmetry (SUSY) is not broken.

For a value of \( X \) fixed, we analyze the minimum of \( V \). We define

\[ X_c \equiv \frac{g}{\lambda} \sqrt{\xi}. \]  

(9)

For \(|X| < X_c\), the minimum corresponds to

\[ |\phi_-|^2 = \xi - \frac{\lambda^2 X^2}{g^2}, \quad |\phi_+| = 0. \]  

(10)

On the other hand, for \(|X| > X_c\), the minimum corresponds to

\[ |\phi_\pm| = 0. \]  

(11)

In this case, the vacuum energy is obtained as

\[ V = \frac{g^2}{2} \xi^2, \]  

(12)

and the radial part of \( X \) is identified with the inflaton. Although the mass of \( X \) vanishes at the tree-level, since the supersymmetry breaking by the non-vanishing D-term generates the mass difference between the masses of \( \phi_\pm \),

\[ m_\pm^2 = \lambda^2 |X|^2 \pm g^2 \xi \]  

(13)

and the masses of those fermionic partners, then the one-loop effective potential is given as

\[ V_{1\text{-loop}} = \frac{g^2}{2} \xi^2 \left( 1 + \frac{g^2}{16\pi^2} \ln \frac{\lambda^2 |X|^2}{\Lambda^2} \right), \]  

(14)

where \( \Lambda \) is the renormalization scale. That generates the mass term of \( X \) and the potential is slightly lifted. Then, the inflaton slowly rolls down the potential. From the following equation,

\[ \eta = -\frac{g^2}{8\pi^2} \frac{M_p^2}{|X|^2}, \]  

(15)
we find that the slow roll condition is violated ($\eta \sim 1$) when the inflaton reach at
\[ |X_f|^2 = \frac{g^2}{8\pi^2} M_p^2, \] 
and the inflation ends.

We have reviewed on the simplest $D$-term inflation model. However, as noted in Introduction, the magnitude of FI term $\xi$ in such simplest model is required to be much smaller than the values of $\xi$ derived from 4D heterotic models. Ref. [8] has proposed the mechanism in heterotic models to reduce effectively the FI term compared with its original value. Here we also give a brief review on such models. Instead of the single field $X$, we consider two fields $X_\pm$. Generic string models have several $U(1)_a$ symmetries. Some of them are anomalous and others are anomaly-free. In Ref. [8] 4D heterotic models were studied and the FI term was considered for only one of $U(1)$ symmetries. That is because only one $U(1)$ symmetry can be anomalous in a 4D heterotic model. However, we will study inflation models inspired by type I string models, where two or more $U(1)$ symmetries can be anomalous. Thus, here we extend the model of Ref. [8] into the case with many FI terms, $\xi_a$, while $\xi_a = 0$ for anomaly-free $U(1)_a$. Suppose that $X_\pm$ have $U(1)_a$ charges $\pm Q_a$. Then the $D$-term is written as
\[ D_a = \xi_a + Q_a (|X_+|^2 - |X_-|^2) + \cdots, \] (17)
where the ellipsis denotes contributions due to other chiral matter fields. We consider large field values for $X_\pm$, $X_\pm \gg \sqrt{\xi_a}$ as before and assume that the other fields gain mass terms, e.g.,
\[ W = \lambda \frac{X_+ X_-}{M} \phi_+ \phi_. \] (18)
Then, the matter fields other than $X_\pm$ vanish for $X_\pm \gg \sqrt{\xi_a}$, and the $D$-term is dominated by $X_\pm$. In this case, the scalar potential is written as
\[ V_D = \sum_a \frac{g_a^2}{2} (\xi_a + Q_a (|X_+|^2 - |X_-|^2))^2. \] (19)
The direction $|X_+| = |X_-|$ corresponds to the inflaton. The minimum of $V_D$ is obtained as
\[ (V_D)_{min} \equiv \frac{g_{eff}^2}{2} \xi_{eff}^2 = \sum_a \frac{g_a^2}{2} \xi_a^2 - \frac{(\sum_a g_a^2 Q_a \xi_a)}{\sum_a g_a^2 Q_a^2}. \] (20)
for

\[ |X_+|^2 - |X_-|^2 = \frac{\sum_a g_a^2 Q_a \xi_a}{\sum_a g_a^2 Q_a^2}. \]  

(21)

In a certain situation (maybe with fine-tuning), this vacuum energy \((V_D)_{\text{min}}\) becomes much smaller than \(O(g_a^2 \xi_a^2)\). Furthermore, in Ref. [9] more fields \(X_i\) relevant to the inflaton have been introduced and in such case an effective value of FI term can be reduced.

### 3 Behavior of dilaton and twisted moduli fields

In this section we study behavior of dilaton and twisted moduli fields in type I string-inspired \(D\)-term inflation model. We consider the case that the anomalous \(U(1)\) is originated from the \(D9\) brane. In this case the gauge kinetic function is obtained as [18]

\[ f_9 = S + \sigma \frac{M}{M_s}, \]  

(22)

where \(\sigma\) is a model-dependent constant and we define \(\bar{\sigma} = \sigma / M_s\). The corresponding gauge coupling \(g_9\) is obtained as \(g_9^2 = 1 / \text{Re}(f_9)\). The Kähler potential of the dilaton field \(S\) is written as

\[ K(S, \bar{S}) = -\ln(S + \bar{S}). \]  

(23)

On the other hand, the Kähler potential of the twisted moduli field \(K(M, \bar{M})\) is not clear. For small values of \(M\), it can be expanded as [19]

\[ K(M, \bar{M}) = \frac{1}{2} (M + \bar{M})^2 + \cdots. \]  

(24)

However, the reliability of this form is not clear for large values of \(M\), i.e. \(M \geq O(1)\).

The twisted moduli field plays a role in the 4D Green-Schwarz anomaly cancellation mechanism, and the FI term is obtained as

\[ \xi = \delta_{\text{GS}} K'(M, \bar{M}), \]  

(25)

where \(\delta_{\text{GS}}\) is a model-dependent constant. Thus the \(D\)-term scalar potential is obtained during the inflation,

\[ V_D = \frac{[\delta_{\text{GS}} K'(M, \bar{M})]^2}{S + S + \bar{\sigma}(M + \bar{M})}. \]  

(26)
First, let us study behavior of $S$ during the inflation. If $S$ run away into infinity, the $D$-term inflation can not succeed. Here, we introduce the canonically normalized dilaton field $\phi$, which is defined as
\[
\phi = M_p \frac{1}{\sqrt{2}} \ln \frac{S + \bar{S}}{2}.
\] (27)
The first and second derivatives of $V_D$ by $\phi$ are obtained as
\[
\frac{\partial V_D}{\partial \phi} = -\frac{\sqrt{2} (S + \bar{S})}{S + S + \bar{S}} \frac{V_D}{M_p},
\] (28)
\[
\frac{\partial^2 V_D}{\partial \phi^2} = -\frac{2 \sqrt{2} (S + \bar{S})}{S + S + \bar{S}} \frac{V_D}{M_p^2} \left( 1 - \frac{2(S + \bar{S})}{S + S + \bar{S}} \right).
\] (29)
Suppose the twisted moduli $M$ is stabilized somehow such that $S + \bar{S} \ll \tilde{\sigma}(M + \bar{M})$. Then, we obtain
\[
\left( \frac{1}{V_D} \frac{\partial V_D}{\partial \phi} \right)^2 \simeq 2 \left( \frac{S + \bar{S}}{\tilde{\sigma}(M + \bar{M})} \right) \ll 1,
\] (30)
\[
\frac{1}{V_D} \frac{\partial^2 V_D}{\partial \phi^2} \simeq \frac{2 \sqrt{2} (S + \bar{S})}{\tilde{\sigma}(M + \bar{M})} \ll 1,
\] (31)
that is, the slow roll condition is satisfied for the dilaton field and it does not run away. Note that this result is independent of the form of $K(M, \bar{M})$.

We have considered the case that the $U(1)$ gauge multiplet is originated from the $D9$ brane. We can easily extend the above analysis into another case. For example, when the $U(1)$ gauge multiplet is originated from the $D5$ brane wrapping a 2D torus of the 6D compact space. In this case, the gauge kinetic function is obtained as
\[
f_{5i} = T_i + \tilde{\sigma}(M + \bar{M}),
\] (32)
where $T_i$ is the moduli field whose VEV determines the volume of the 2D torus. Its Kähler potential is the same as the dilaton field, i.e.,
\[
K(T_i, \bar{T}_i) = -\ln(T_i + \bar{T}_i).
\] (33)
Hence, the $D$-term scalar potential during the $D$-term inflation is written as
\[
V_D = \frac{[\delta_{GS} K'(M, \bar{M})]^2}{T_i + \bar{T}_i + \tilde{\sigma}(M + \bar{M})}.
\] (34)
In a similar way to the dilaton field, the moduli field $T_i$ does not run away if $M$ is stabilized such that $T_i + \bar{T}_i \ll \tilde{\sigma}(M + \bar{M})$.

Next, we study the stabilization of $M$ in the model with $V_D$, Eq. (26), that $U(1)$ is originated from the $D9$ brane. For $S + \bar{S} \ll \tilde{\sigma}(M + \bar{M})$, the potential reduces to

$$V_D = \frac{[\delta_{GS} K'(M, \bar{M})]^2}{\tilde{\sigma}(M + \bar{M})}. \quad (35)$$

Unfortunately, only the term $V_D$ does not stabilize the twisted moduli $M$ at a finite value with non-vanishing vacuum energy, unless we choose a special form of $K(M, \bar{M})$. In fact, the minimum in this potential is located at $M + \bar{M} = 0$ and $V_D|_{M + \bar{M} = 0} = 0$ for the Kähler potential, $K(M, \bar{M}) = (M + \bar{M})^2/2$. Hence, another term is necessary for stabilization of $M$ such that the $D$-term inflation can be realized. We assume that gaugino condensation of another non-abelian gauge group generates the following non-perturbative superpotential,

$$W_{np} = de^{-\Delta(S + \bar{\sigma}^\prime M)}, \quad (36)$$

where $\Delta = 24\pi^2/(-b)$ and $b$ is the one-loop beta-function coefficient of the gauge coupling. In general, the constant $\sigma'$ is different from $\sigma$. Anyway one needs a stabilization mechanism of $S$ as well as $M$ in the true vacuum after the inflation ends. It is the usual approach to stabilize $S$ by this type of non-perturbative superpotential. Thus, it is natural to assume the above superpotential. For stabilization of $S$ in the true vacuum, double or more gaugino condensations are often used in the so-called race-track model. Here, for simplicity, we consider the case that only the single gaugino condensation potential is dominant during the inflation. As we did in estimation of $V_D$, we assume $S + \bar{S} \ll \tilde{\sigma}(M + \bar{M})$ during inflation, that is, we have $W_{np} = de^{-\Delta^\prime M}$. In this case, the scalar potential in the global SUSY is written as

$$V = (\Delta^\prime)^2 d^2 e^{-\Delta^\prime m} + \frac{\delta_{GS}^2}{\tilde{\sigma}^2} m, \quad (37)$$

where $m \equiv M + \bar{M}$. We need the explicit form of $K(M, \bar{M})$ to discuss the stabilization of $M$. Here and hereafter we take

$$K(M, \bar{M}) = \frac{1}{2}(M + \bar{M})^2. \quad (38)$$

\textsuperscript{5}When we study the scalar potential within the framework of supergravity theory, we obtain the qualitatively same result.
The stationary condition $\frac{\partial V}{\partial m} = 0$ is satisfied by
\begin{equation}
\langle M + \bar{M} \rangle_{inf} = \frac{1}{\Delta \tilde{\sigma}'} \ln \left( \frac{\tilde{\sigma} d^2 (\Delta \tilde{\sigma}')^3}{\delta^2_{GS}} \right),
\end{equation}
where $\langle \ldots \rangle_{inf}$ denotes the expectation value during inflation. If the effective mass squared during inflation,
\begin{equation}
\left. \frac{\partial^2 V}{\partial m^2} \right|_{m=(M+\bar{M})_{inf}} \simeq \frac{\Delta \tilde{\sigma}}{\langle M + M \rangle_{inf}} V_D = 3 \frac{\Delta \tilde{\sigma}}{\langle M + M \rangle_{inf}} M_p^2 H^2,
\end{equation}
is enough larger than $H^2$ around this stationary point, the twisted moduli can be stabilized. The condition is satisfied in the present model because of $\Delta \tilde{\sigma} M_s^2 / \langle M + \bar{M} \rangle_{inf} \gg 1$ and $M_p \sim M_s^6$, as we will see soon. Note that the key-point for stabilization is the polynomial form of the Kähler potential, in particular the canonical form. When we consider additional higher terms $(M + \bar{M})^n \ (n > 2)$ in the Kähler potential, we obtain qualitatively similar result for stabilization. However, if the twisted moduli field has the logarithmic form of Kähler potential, this type of stabilization can not be realized.

Now let us estimate $\langle M + \bar{M} \rangle_{inf}$. If $b/\sigma' = \mathcal{O}(1)$, we estimate $(\Delta \sigma')^{-1} = \mathcal{O}(10^{-3}) - \mathcal{O}(10^{-2})$, because of $(\Delta \sigma')^{-1} = 0.0004 \times b/\sigma'$. For such small value of $(\Delta \sigma')^{-1} = \mathcal{O}(10^{-3}) - \mathcal{O}(10^{-2})$, the factor $(\Delta \sigma')^3$ inside of the logarithmic function enlarges $\langle M + \bar{M} \rangle_{inf}$ by a factor of $\mathcal{O}(10)$. Also, the GS coefficient $\delta_{GS}$ may enlarge $\langle M + \bar{M} \rangle_{inf}$ by a few factor for $\delta_{GS} \ll 1$. Then we can estimate
\begin{equation}
\frac{\langle M + \bar{M} \rangle_{inf}}{M_s} = \mathcal{O}(10^{-2}) - \mathcal{O}(10^{-1}).
\end{equation}
Fig. 1 shows $V$ against $m$ for $\Delta \sigma' = 100$ and $\delta^2_{GS}/\sigma d^2 = 0.01$. The form of Kähler potential, Eq.(38) is not reliable for a large value of $\langle M + \bar{M} \rangle_{inf}$, i.e. $\langle M + \bar{M} \rangle_{inf}/M_s \geq \mathcal{O}(1)$. From this point, the above result that $M$ is stabilized as a small value is favorable. Another nice point of the result is that the stabilized value satisfies
\begin{equation}
\langle M + \bar{M} \rangle_{inf} \gg \frac{1}{\Delta \sigma'},
\end{equation}
This relation is derived in the next section.
when the value inside of the logarithmic function is large. Note that $F$-term and $D$-term parts in the scalar potential satisfy
\[ \frac{V_D}{V_F} \approx \Delta \tilde{\sigma}' \langle M + \bar{M} \rangle_{inf}. \] (43)

Eqs. (42) and (43) imply the following inequality,
\[ V_F \ll V_D, \] (44)
that is, with this stabilized value of $\langle M + \bar{M} \rangle_{inf}$ the $D$-term scalar potential is dominant. That allows to realize the $D$-term inflation. Therefore, the vacuum energy during inflation is estimated as
\[ V = \frac{\delta_{GS}^2}{\sigma} \langle M + \bar{M} \rangle_{inf}. \] (45)

In explicit type I string models through type IIB orientifold construction, the GS coefficient $\delta_{GS}$ is calculated in the unit of $M_s$ as $\delta_{GS}/M_s = O(10^{-1}) - O(10^{-3})$ [20]. It is not completely clear what is a natural value of $\delta_{GS}$ from the viewpoint of generic type I models except type IIB construction. Furthermore, as reviewed in the previous section, the original value of the FI term appears as the inflation vacuum energy in the simplest model, but it can be effectively reduced in the models with more fields and $U(1)$ symmetries. As another remark, if $d$ is suppressed by any mechanism, the stabilized value $\langle M + \bar{M} \rangle_{inf}/M_s$ becomes small like $O(10^{-2}) - O(10^{-3})$.

4 Dynamics and density perturbation

Now let us discuss the dynamics in this inflation model. The relevant potential including one-loop correction during inflaton is given by
\[
V = \frac{\delta_{GS}^2 \langle M + \bar{M} \rangle_{inf}^2}{S + S + \tilde{\sigma} \langle M + \bar{M} \rangle_{inf}} \times \left[ 1 + \frac{1}{16\pi^2} \frac{2}{S + S + \tilde{\sigma} \langle M + \bar{M} \rangle_{inf}} \ln \frac{\lambda^2 |X|^2}{(S + S)\Lambda^2} \right]. \] (46)

Here the $F$-term contribution is negligibly smaller than the $D$-term in potential, as already mentioned. On the other hand, one should notice that the
stabilization of twisted moduli is achieved by the $F$-term potential and we can replace $M + \bar{M}$ with $\langle M + \bar{M} \rangle_{inf}$. Furthermore, hereafter we will consider only the region, $S + \bar{S} \ll \bar{\sigma} \langle M + \bar{M} \rangle_{inf}$ in order to avoid the runway motion of the dilaton and obtain a successful inflation. In this sense, the initial value problem of the dilaton, why the dilaton takes such an initial value, remains unsolved.

From
\[ \eta_{XX} = -\frac{1}{8\pi^2 \bar{\sigma} \langle M + \bar{M} \rangle_{inf}} \frac{2}{X^2}, \]  \hspace{1cm} (47)

we find that when the inflaton, $X$,\(^7\) reaches at the critical point, $X_f$, where
\[ X^2_f = \frac{1}{8\pi^2 \bar{\sigma} \langle M + \bar{M} \rangle_{inf}} \frac{2M_p^2}{X^2}, \]  \hspace{1cm} (48)

the slow roll condition is violated. On the other hand, $X_c$ is expressed as
\[ X^2_c \approx \frac{2}{\bar{\sigma} \langle M + \bar{M} \rangle_{inf}} \frac{\delta_{GS} \langle M + \bar{M} \rangle_{inf}}{\lambda^2}. \]  \hspace{1cm} (49)

\(^7\)Hereafter, we omit the absolute value symbol, $\|$. 

Figure 1: The scalar potential (37) of $m$ for $\Delta \sigma' = 100$ and $\delta_{GS}^2/\sigma d^2 = 0.01$. 

Hence, if the inequality
\[
\frac{M_p^2}{8\pi^2} > \frac{\delta_{GS} \langle M + \bar{M} \rangle_{inf}}{\lambda^2}
\]
hold, inflationary expansion terminates when inflaton reaches at \(X_f\) rather than \(X_c\). This inequality holds for \(\lambda \sim 1\).

Then the equations of motion for the homogeneous part of scalar fields, \(X\) and \(\phi\), are respectively given by
\[
\ddot{X} + 3H\dot{X} = -\partial_X V \\
\simeq -\frac{\delta_{GS}^2}{\sigma^2} \frac{1}{8\pi^2} \frac{2}{X},
\]
and
\[
\ddot{\phi} + 3H\dot{\phi} = -\partial_\phi V \\
\simeq \frac{\delta_{GS}^2}{\sigma^2} \frac{2^{3/2}}{M_p} e^{\sqrt{2}\phi/M_p},
\]
where we ignored the dilaton dependence in the radiative correction part because of the weakness of the dependence. The solutions under the slow roll approximation are
\[
X^2 = X_e^2 + \frac{\delta_{GS}^2}{\sigma^2} \frac{1}{2\pi^2} \frac{H(t_e - t)}{3H^2},
\]
\[
e^{-\sqrt{2}\phi/M_p} = e^{-\sqrt{2}\phi_e/M_p} + \frac{4\delta_{GS}^2}{\sigma^2} \frac{H(t_e - t)}{3M_p^2 H^2},
\]
with \(X_e = X(t_e)\) and \(\phi_e = \phi(t_e)\), where \(t_e\) denotes the time of the end of the inflation. By using the definition of the number of e-folds \(-dN \equiv H dt\), Eq.(54) is rewritten as
\[
\left. \frac{S + \bar{S}}{\sigma (M + \bar{M})_{inf}} \right|_{t_e} = \frac{1}{\frac{\sigma (M + \bar{M})_{inf}}{S + \bar{S}} - 2N}.
\]
Since the l.h.s. must be less than the order of \(10^{-1}\), we find that \(\sigma (M + \bar{M})_{inf}/(S + \bar{S}) \sim 10^2\) for the present horizon scale. Similarly, from Eq.(53) we obtain
\[
X^2 = X_e^2 + \frac{N}{2\pi^2\sigma (M + \bar{M})_{inf}} M_p^2,
\]
13
we unfortunately found the somewhat large field value region, \( X = \mathcal{O}(1) M_p \), is used in this inflation model. This is the undesirable feature in this model. Here we estimate the following quantity

\[
\left( \frac{\dot{\phi}}{X} \right)^2 = \left( 8 \pi^2 \tilde{\sigma} \langle M + \bar{M} \rangle_{inf} \right)^2 \left( \frac{S + S}{\tilde{\sigma} (M + M)_{inf} \Gamma} \right)^2 \frac{X^2}{2M_p^2} \]

\[
\simeq 10^{-1} \left( \frac{\tilde{\sigma} (M + \bar{M})_{inf}}{10^{-1}} \right) \left( \frac{S + S}{\tilde{\sigma} (M + M)_{inf} \Gamma} \right)^2 \frac{1 + 2H (t_e - t)}{\left( 1 + \frac{S + S}{\tilde{\sigma} (M + M)_{inf} \Gamma} \right) 2H (t_e - t)}^2, \tag{57}
\]

where we adopted \( X_f \) as \( X_e \) and suppose \( \sigma \sim 1 \). Thus we find that the ratio of kinetic energies of these scalar fields takes a value of \( \mathcal{O}(10^{-1}) - \mathcal{O}(10^{-2}) \), because the final part in this expression is almost unity during the inflation.

Next, we turn to the density perturbation generated by this inflation model. The WMAP data shows that the adiabatic fluctuation is favored [21, 22]. On the other hand, in general, a inflation model which contains several evolving scalar fields produces also the isocurvature fluctuation. Accordingly, we estimate the contribution from the isocurvature mode and confirm this model to be consistent with the observations. The perturbed Einstein equation in Fourier space reads

\[
\ddot{\delta \phi} + 3H \dot{\delta \phi} + \left( \frac{k^2}{a^2} + V_{,\phi\phi} \right) \delta \phi + V_{,\phi X} \delta X = -4\dot{\phi} \dot{\Phi} + 2V_{,\phi} \Phi, \tag{58}
\]

\[
\ddot{\delta X} + 3H \dot{\delta X} + \left( \frac{k^2}{a^2} + V_{,XX} \right) \delta X + V_{,X\phi} \delta \phi = -4\dot{X} \dot{\Phi} + 2V_{,X} \Phi, \tag{59}
\]

\[
\dot{\phi} + H \Phi = -\frac{1}{2M_p^2} (\dot{\delta \phi} + \dot{X} \delta X), \tag{60}
\]

where \( k \) is the wave number, \( a \) is the scale factor, the comma denotes the derivative of \( V \) with respect to the fields \( (X, \phi) \), \( \delta \phi \) and \( \delta X \) represent the perturbations of scalar fields \( \phi \) and \( X \), respectively, and \( \Phi \) is the curvature perturbation variable in the notation of Ref. [23]. These equations have analytic solutions for a long wavelength under the slow-roll approximation.
\[ \delta X = (\ln V)_X \left[ Q_1 + Q_3 \int_{t_k}^t (\ln V)_X J d\phi \right], \quad (61) \]
\[ \delta \phi = (\ln V)_\phi \left[ Q_1 + Q_3 - Q_3 \int_{t_k}^t (\ln V)_\phi J dX \right], \quad (62) \]

with
\[ J = \exp \left( - \int_{t_k}^t [ (\ln V)_X d\phi + (\ln V)_\phi dX ] \right). \quad (63) \]

Here \( t_k \) denotes the horizon crossing time for a wave number \( k \), \( k = a(t_k)H \).

In addition, \( Q_1 \) and \( Q_2 \) are integration constants and expressed as
\[ Q_1 = - \frac{H^2}{\sqrt{2k^3}} e_X(k) \bigg|_{t_k}, \quad (64) \]
\[ Q_3 + Q_1 = - \frac{H^2}{\sqrt{2k^3}} e_\phi(k) \bigg|_{t_k}, \quad (65) \]

where \( e_X(k) \) and \( e_\phi(k) \) are classical random quantities which are normalized as
\[ \langle e_X(k) e_X^*(k') \rangle = \langle e_\phi(k) e_\phi^*(k') \rangle = \delta^{(3)}(k - k'). \quad (66) \]

The appearance of \( Q_3 \) is due to the existence of isocurvature perturbation. Furthermore, as we already mentioned, since the dilaton dependence in the radiative correction part is negligible, the potential, Eq.(46), seems a separated form \( V = V_\phi(\phi) V_X(X) \). Since \( J = 1 \) in this case, Eqs.(61) and (62) can be rewritten as
\[ \delta X = \frac{V_X}{V} Q_1, \quad (67) \]
\[ \delta \phi = \frac{V_\phi}{V} (Q_1 + Q_3). \quad (68) \]

Then, the curvature perturbation is expressed as
\[ \Phi = - \frac{\dot{H}}{H^2} Q_1 + \frac{(V_\phi)^2}{2V^2} Q_3 \]
\[ = \frac{\dot{H}}{H^2} \frac{H^2}{\sqrt{2k^3}} e_X(k) \bigg|_{t_k} + \frac{(V_\phi)^2}{2V^2} \left( \frac{H^2}{\sqrt{2k^3}} e_X(k) - \frac{H^2}{\sqrt{2k^3}} e_\phi(k) \right) \bigg|_{t_k} \quad (70) \]
From $\dot{X}^2 \gg \dot{\phi}^2$, we find the first term is much larger than the second term and the primary contribution to the density perturbation comes from the adiabatic fluctuation generated by the inflaton. Hence, we find that the density perturbation in this inflation model is almost adiabatic with small isocurvature fluctuations.

Here we shall introduce the so-called Bardeen parameter which is defined by

$$\zeta \equiv \Phi + \frac{2}{3} \frac{\Phi + H^{-1} \dot{\Phi}}{1 + w},$$

which is conserved in the superhorizon scale, $k \ll aH$, if the fluctuation is adiabatic [25]. Here $w$ denotes the ratio of pressure to energy density. However, the present model contains two slow-rolling scalar fields and may produce the isocurvature fluctuation. In this case, the time variation of $\zeta$ is caused by the isocurvature fluctuation and the Bardeen parameter is no longer conserved quantity on superhorizon scales. Indeed, the time variation of $\zeta$ is expressed as

$$\frac{3}{2} H (1 + w) \dot{\zeta} = -c_s^2 \frac{k^2}{a^2} \Phi - \frac{\rho}{2} \Gamma,$$

where $c_s^2 = \dot{\rho}/\rho$ and the following quantity

$$\Gamma = \frac{\delta p}{\rho} - \frac{c_s^2}{w} \frac{\delta \rho}{\rho}$$

represents the amplitude of an entropy perturbation. In the case that the universe is filled with two component scalar fields $(X, \phi)$, Eq.(72) is rewritten as [26]

$$\dot{\zeta} = \frac{H k^2}{2} \Phi + \frac{H}{2} \frac{\partial}{\partial t} \left( \frac{\dot{X}^2 - \dot{\phi}^2}{X^2 + \phi^2} \right) \left( \frac{\delta X}{X} - \frac{\delta \phi}{\phi} \right).$$

In a single field inflation model, either $\dot{X}$ or $\dot{\phi}$ goes to zero and the second term in r.h.s of Eq.(74) vanishes. Thus, $\zeta$ becomes the conserved quantity on superhorizon scales in that case. However, we need take the variation of $\zeta$ into account, because there are two evolving scalar fields in the present model. The spectrum of the density perturbations, $\zeta$, at the horizon crossing time is

$$P_{\zeta} \left| _{t_k} \right. = \left( \frac{H^2}{2 \pi |X|} \right)^2 \left[ 1 - \frac{1}{2} \frac{\epsilon_{\phi}}{\epsilon_X} \right] \left| _{t_k} \right.$$

$$= \left( \frac{H^2}{2 \pi |X|} \right)^2 \left[ 1 - \frac{1}{2} \frac{\epsilon_{\phi}}{\epsilon_X} \right] \left| _{t_k} \right.$$. 

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up to the first order of $\dot{\phi}^2/\dot{X}^2$, where $\epsilon_\phi = \dot{\phi}^2/(2V)$ and $\epsilon_X = \dot{X}^2/(2V)$ are slow roll parameters. From Eq.(74), the variation of the Bardeen parameter after the horizon crossing is estimated as

$$\Delta \zeta \equiv \int_{t_k}^{t_e} \dot{\zeta} dt = Q_3 \left( \frac{\dot{\phi}^2}{X^2} |_{t_k} - \frac{\dot{\phi}^2}{X^2} |_{t_e} \right).$$

(76)

Furthermore, the power spectrum of the density perturbations at the end of inflation [27] is estimated as

$$P_{\zeta} |_{t_e} = \left( \frac{H^2}{2\pi |X|} \right)^2 \left| 1 - 2 \frac{\epsilon_\phi}{\epsilon_X} |_{t_e} + \left( \frac{\epsilon_\phi/\epsilon_X |_{t_e} |}{\epsilon_\phi/\epsilon_X |_{t_k}} \right)^2 \right|.$$  

(77)

We find that the variation of $\zeta$ on super-horizon scales by the isocurvature perturbation is suppressed by the power of $\dot{\phi}^2/\dot{X}^2 = O(10^{-1}) - O(10^{-2})$. The amplitude of curvature perturbation is rewritten as

$$\frac{H^2}{2\pi |X|} |_{t_k} = 2 \frac{\delta_{GS} (M + \bar{M})_{inf}}{M_p^2} \sqrt{\frac{N}{6}}.$$  

(78)

and we obtain the constraint on the model parameters

$$\frac{\delta_{GS} (M + \bar{M})_{inf}}{M_p^2} \simeq 10^{-5}$$  

(79)

for the present horizon scale. For the GS coefficient $\delta_{GS}$, a suppressed value is required like the heterotic case. Explicit models through type IIB orientifold construction seem to derive $\delta_{GS}/M_s = O(10^{-1}) - O(10^{-3})$ [20]. Anyway, the GS coefficient $\delta_{GS}$ is model-dependent. If we find the model that $\delta_{GS}$ is small enough like $\delta_{GS}/M_s \sim 10^{-3}$, the string scale can be almost comparable with $M_p$ or slightly smaller. Otherwise, if the original FI terms are not small enough, we would need the mechanism to reduce the effective FI term as mentioned in Section 2 or a lower string scale.

On the other hand, the spectrum of gravitational wave perturbation produced by the inflation is given by $P_g = |H/(2\pi M_p)|^2$ and its observational results show $P_g \leq 10^{-10}$. This constraint is expressed as $V \leq 10^{-8}M_p^4$ in term of the potential. Therefore we obtain

$$V = \frac{\delta_{GS}^2 (M + \bar{M})_{inf}}{\sigma} \leq 10^{-8}M_p^4.$$  

(80)
By combining Eqs.(79) and (80), we obtain $\tilde{\sigma}\langle M + \bar{M} \rangle_{inf} \geq 10^{-2}$. That can be satisfied by a stabilized value (41) of $\langle M + \bar{M} \rangle_{inf}$, e.g. $\langle M + \bar{M} \rangle_{inf} \sim 0.1 \times M_s$ for $\sigma \sim 1$.

Finally we discuss the evolution of the universe after inflation. The relevant potential after inflation is given by

$$V = (\text{gaugino condensation potential}) + \lambda^2 |X|^2 |\phi_-|^2 + \frac{(\delta_{GS} K'(M, \bar{M}) - |\phi_-|^2)^2}{S + S + \tilde{\sigma}(M + \bar{M})}. \quad (81)$$

After the inflation, scalar fields, $X$ and $\phi_-$ oscillate around their corresponding minimum $X = 0$ and $\langle \phi_- \rangle = \sqrt{\xi}$, respectively. Here we should notice that $\xi$ is function of the twisted moduli $M$ and still a time variant in the reheating stage, because the twisted moduli does not yet reach at the true vacuum at this moment. This is a feature which the simplest model of $D$-term inflation does not possess.

Now, we consider the reheating process according as Ref. [28]. Since there are fields which are charged under both anomalous $U(1)$ and each of the subgroups of the standard model, the $D$-term receives a contribution from these field. As the result, the $D$-term takes the form of

$$D = g \left( |\phi_+|^2 - |\phi_-|^2 + \sum_i q_i |Q_i|^2 + \xi \right), \quad (82)$$

$$\xi = \delta_{GS} K'(M, \bar{M}), \quad (83)$$

where $Q_i$ are charged fields for both anomalous $U(1)$ and gauge groups of the standard model and $q_i$ are their positive charge. In the case that fields $\varphi_i$ of the minimal supersymmetry standard model belong to the fields $Q_i$, the $\phi_-$ field couples with them by the interaction,

$$\mathcal{L}_{int} = g^2 \sqrt{\xi} \sum_i q_i |\varphi_i|^2 (\delta \phi_- + \delta \phi_-^\dagger), \quad (84)$$

where $\delta \phi_- \equiv \phi_- - \langle \phi_- \rangle$. Hence the $\phi_-$ field would immediately decay through this gauge interaction. On the other hand, the decay of $X$ would delay. The mass of $\phi_-$ is $m_{\phi_-} = (g/\lambda) m_X$. Since $g > 1$ in our model at the time of the end of inflation, the decay of $X$ to $\phi_-$ would be forbidden kinematically. As a results, $X$ decays to light matter through higher-dimensional interactions and the reheating can be completed with an enough high reheating temperature.
In this scenario, huge entropy after the reheating by the inflaton is produced by the decay of the dilaton and moduli fields in late time. Therefore, this $D$-term inflation model should be incorporated with the Affleck-Dine baryogenesis [29]. The Affleck-Dine baryogenesis with the dilution by the dilaton and moduli decay have been investigated [30, 31]. However, the actual estimation of the baryon to entropy ratio is beyond the scope of this paper, because their full potential strongly depend on the form of the Kähler potential of twisted moduli and the condensation gauge groups [15].

Furthermore, it is pointed out that cosmic strings generated after $D$-term inflation can modify the CMB anisotropy significantly [32]. Huge entropy production in a late time is desirable from this point of view, because the dilution by the entropy production is one possible way to remove the effect by cosmic strings. Moreover there is one thing to note. The FI term is given by Eq.(25) and its magnitude is parameterized by the expectation value of the twisted moduli. Therefore the tension of cosmic strings in late time is not necessarily the same as the scale of FI term during inflation, namely, $\langle M + \bar{M} \rangle_{inf} \neq \langle M + \bar{M} \rangle$, where $\langle ... \rangle$ denotes the VEV. This feature might change the situation of the effect by cosmic strings.

5 Conclusions and Discussions

We have studied the $D$-term inflation within the framework of type I string-inspired models. In this case, the twisted moduli field plays a significant role. For stabilization of the twisted moduli, the polynomial form of the Kähler potential is important. The stabilization of the twisted moduli during inflation is achieved with the help of the gaugino condensation potential and this stabilization mechanism does not spoil the situation that the potential is dominated by $D$-term. Furthermore it is a favorable property that the expectation value of the twisted moduli during inflation is enough smaller than the unity where the Kähler potential $K = (M + \bar{M})^2/2$ is valid, while this result leads to one shortcoming that the inflaton $X$ must take a somewhat large value $X > M_p$ during inflation.

At first, we found that the magnitude of FI term would be reducible to a desirable value within this framework even if we do not assume a hierarchically large gap between the string scale $M_s$ and the Planck scale $M_p$.

Concerning the second problem, or dilaton runaway problem, we found
the condition to avoid this difficulty for the initial value of the dilaton. In
the potential with the fixed twisted moduli, we could find that the field
region where the dilaton does not run away, \( S + \bar{S} \ll \tilde{s}(M + \bar{M})_{inf} \). If the
dilaton takes such initial value at the pre-inflation stage, it is possible that
the universe undergoes the quasi de-Sitter expansion, because the dilaton can
evolve slowly. Of course, at the moment, we cannot answer the question why
the dilaton takes such initial value. The point we would like to emphasize in
the present paper is that \( D \)-term inflaton is possible in type I string-inspired
model under certain conditions. On the other hand, the small expectation
value of the dilaton during inflation might be a good point from another point
of view. In studies on the evolution of the dilaton in string cosmology, the
initial condition that the dilaton takes a smaller value than the VEV at the
beginning, \( (S + \bar{S})|_{t_i} < \langle S + \bar{S} \rangle \), has been adopted [10, 31, 33]. Therefore,
the initial value of the dilaton set by this inflation is consistent with the
initial condition in these studies. Although these are based on a heterotic
model, the situation would not change essentially. That might lead to the
dilaton stabilization at the correct vacuum.

Moreover, the ingenious point in this model is that the condition which
prohibits the dilaton from rolling down the potential also suppresses the
growth of the unnecessary isocurvature fluctuation. Thus, we can obtain the
almost adiabatic density perturbation.

What we have to investigate further is the late time evolution of the
universe. In the present paper, we have not estimated the baryogenesis and
the effect of cosmic strings and so on, because they would involve us in other
factors such as a concrete potential for the supersymmetry breaking than the
potential of inflation. These are important issues in future works.

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