Imaging nanostructured spin textures

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Abstract

One of the key challenges in magnetism remains the determination of the nanoscopic magnetization profile within thick samples, such as bulk ferromagnets. Thanks to the large penetration depth of neutrons, magnetic small-angle neutron scattering (SANS) is a powerful technique to characterize bulk samples with nanometer resolution. The major challenge regarding magnetic SANS is accessing the real-space magnetization vector field from the reciprocal scattering data. In this letter, we apply a fast, iterative algorithm to extract the underlying two-dimensional magnetic correlation functions. We use this approach to analyze the magnetic microstructure of the nanocrystalline model system Nanoperm. We show that the computed correlation functions reflect the projection of the three-dimensional magnetization vector field onto the detector plane. Our results demonstrate that the used methodology can be applied to image nanostructured spin-textures within bulk samples.
Keywords

magnetic imaging, magnetic SANS, magnetization profile, spin texture, nanomagnetism, ferromagnets

Nanostructured magnetic materials attract much interest thanks to the unique magnetic properties that can arise when the structural units (e.g. particles, crystallites, or film layers) are reduced below a characteristic intrinsic magnetic length scale of the system.\cite{1} The prototype of spatially-localized magnetic objects are magnetic nanoparticles, which have a single domain magnetization below a material-specific size of typically a few tens of nanometers.\cite{2} For larger sizes or deviation from spherical shape, even defect-free magnetic nanoparticles can display more complex spin structures such as curling-, flower-, or vortex-states.\cite{3} In general, to fully understand the complex interplay between the structural and magnetic properties of nanostructured magnetic materials, the determination of the internal magnetization profile remains a key challenge.\cite{4} In case of individual, free-standing magnetic structures (such as nanoparticles, or micrometer-sized pillars and discs) the internal magnetization profile can be determined, e.g. by advanced electron microscopy\cite{5–7} or X-ray scattering techniques.\cite{8,9} In particular, X-ray nanotomography enables nowadays the reconstruction of the three-dimensional (3D) magnetization vector field within magnetic microstructures even with temporal resolution in the nanosecond regime.\cite{10} Furthermore, electrons and X-rays can be used to investigate the interparticle moment coupling in planar 2D assemblies of interacting magnetic nanoparticles.\cite{11,12} However, neither electrons nor X-rays are normally suitable to characterize large (i.e. mm-sized) 3D samples and buried magnetic structures due to their small penetration depths, and thus the internal magnetization profile of most bulk magnets or 3D nanoparticle assemblies is not accessible with these techniques. To visualize the complex magnetization profile within 3D magnetic systems small-angle neutron scattering (SANS) can be employed.\cite{13} The general advantage of neutrons as an important probe of magnetism are their large penetration depths, which allows the characterization of bulk materials with thicknesses of up to several millimeters.\cite{14,15}
In Nanoperm, a technologically-relevant nanostructured magnetic alloy, small Fe nanocrystals are embedded in a soft magnetic, amorphous matrix. The Fe nanoparticles are at first approximation single-domain particles and their dipolar stray fields impose a distortion of the magnetization in the surrounding, magnetically softer matrix. In previous SANS studies of Nanoperm it was shown that the magnetodipolar stray fields of the Fe nanocrystals cause characteristic anisotropies in the magnetic SANS patterns. However, in general, the key challenge regarding magnetic SANS remains accessing the real-space magnetization vector field from the reciprocal scattering data. In most studies, data analysis is done by analyzing 1D sectors or radial averages, e.g. by fitting the data to a particular model in reciprocal space, by determining model-independently the real-space 1D correlation functions, or by a qualitative discussion. But due to the anisotropic nature of magnetic scattering, reducing the analysis to 1D essentially means a loss of information. Moreover, in many studies structural form-factor models, adapted from nuclear SANS, are utilized, which fail to account for the existing spin inhomogeneity inside magnetic nanostructured systems. Only recently the analysis of the total (magnetic and/or nuclear) 2D patterns was introduced, either by directly calculating the cross section in reciprocal space or by determining the real-space 2D correlation functions.

Here, we introduce a new method to extract the underlying 2D correlation functions from 2D SANS patterns. We use this approach to analyze the magnetic SANS data of Nanoperm and show that the derived correlation functions nicely reflect the real-space, nanoscale magnetization configuration. With this study we demonstrate that magnetic SANS is a unique and powerful method to visualize the 2D magnetic correlation function, i.e. a projection of the 3D real-space magnetization configuration $M(r)$ into the detector plane.

Details regarding the sample preparation and the SANS experiment can be found in Michels et al. Briefly, the Nanoperm (Fe$_{89}$Zr$_7$B$_3$Cu) sample was prepared by melt spinning and subsequent annealing, and had an average Fe crystallite size of 12 nm according to X-ray diffraction and electron microscopy. The magnetic-field-dependent, unpolarized SANS
measurements were performed at room temperature on the SANS-2 instrument at GKSS, Geesthacht, Germany, using an incident wavelength of \( \lambda = 0.58 \text{ nm} \). In this paper we will focus our analysis on the SANS measurements within a total \( q \)-range of \( q = 0.1 - 1.2 \text{ nm}^{-1} \) (\( q \) is the scattering vector or momentum transfer). The homogeneous magnetic field \( H \parallel |e_z \) was applied normal to the incident neutron beam \( k \parallel |e_x \) and in the plane of the sample. We will focus on the measurements performed at field strengths of \( \mu_0 H = 321, 163, 85, \) and \( 45 \text{ mT} \).

By subtracting the total SANS cross section measured close to saturation (\( \mu_0 H \approx 2 \text{ T} \)) from the data at intermediate field strengths, the residual, purely magnetic SANS cross section

\[
I_m(q) \propto |\tilde{M}_x|^2 + |\tilde{M}_y|^2 \cos^2 \Theta - \left( \tilde{M}_y\tilde{M}_z^* + \tilde{M}_y^*\tilde{M}_z \right) \sin \Theta \cos \Theta,
\]

is obtained\(^{19}\), where the scattering vector \( q \) is defined in the detector \( yz \)-plane. In writing down Eq.\(^1\) it is assumed that the sample is in the approach-to-saturation regime, and that \( M(2 \text{ T}) \approx M_S \), with \( M_S \) being the saturation magnetization; \( \Theta \) is the angle between \( q \) and \( H \), and \( \tilde{M}_x(q), \tilde{M}_y(q), \tilde{M}_z(q) \) are the Fourier transforms of the magnetization components \( M_x(r), M_y(r), M_z(r) \) of the real-space magnetization vector field, where the asterisk '\*' indicates the complex-conjugate. Note that the Fourier components \( \tilde{M}_{x,y,z}(q) \) can be anisotropic, which severely complicates a decoupling of the individual scattering contributions in Eq.\(^1\).

In principle, the real-space 2D magnetic correlation function \( P(r) = rC(r) \), with \( C(r) \) being the autocorrelation function in case of nuclear scattering, can be extracted from the experimental reciprocal scattering data \( I_m(q) \) \textit{via} a direct Fourier transform.\(^{28}\) For the analysis of nuclear scattering patterns, however, usually indirect approaches are applied where the inverse problem is solved,\(^{29,31}\) and which can be readily adapted to magnetic SANS. The challenge is to extract good and robust estimations for \( P(r) \) from the noisy data, also in case of restricted \( q \)-ranges as is usually the case in experiment. An additional problem regarding the evaluation of 2D scattering patterns is the necessary computation time related to processing the large matrices involved (i.e. the data and the 2D correlation function). Here, we
introduce an iterative method (called Kaczmarz’ algorithm\textsuperscript{32}) to solve this ill-conditioned problem, which was already used successfully for the fast analysis of magnetic particle imaging, magnetometry and magnetorelaxometry data of magnetic nanoparticle ensembles.\textsuperscript{33,34}

For $k||\mathbf{e}_x$, the 2D scattering intensity can be written in polar coordinates as $I_m(q_y, q_z) = I_m(q, \Theta)$, with $q = |\mathbf{q}|$ and $\Theta = \arctan(q_y/q_z)$. The 2D scattering pattern has $N$ pixels, and for each pixel ’$i$’ (i.e. data point) it can be expressed as:

$$I(q_i, \Theta_i) = \sum_{j=1}^{K} A_{ij} P(r_j, \varphi_j).$$

(2)

The angle $\varphi$ specifies the orientation of $r$ in the $yz$-plane, and the extracted 2D distribution function $P(r)$ is given by $P(r, \varphi) = C(r, \varphi)r$.\textsuperscript{30} The matrix $A$ in Eq. 2 is the data transfer matrix, which, in case of the 2D indirect Fourier transform, has the elements\textsuperscript{28–31}

$$A_{ij} = \cos (q_ir_j \cos (\Theta_i - \varphi_j)) \Delta r_j \Delta \varphi_j.$$  

(3)

As is typical in such an analysis, we use a linear spacing for the pre-determined $r$- and $\varphi$-vectors. We use the following algorithm from Kaczmarz to update the elements $P(r_j, \varphi_j)$ after each iteration according to:

$$P^{k+1}(r_j, \varphi_j) = P^k(r_j, \varphi_j) + \frac{I(q_i, \Theta_i) - (A_i \cdot P^k(r_j, \varphi_j))}{\sigma ||A_i||^2} A_i,$$

(4)

where $A_i$ is the $i$th row of the matrix $A$, $A_i^\top$ is its transpose, $k$ is the iteration number, and one iteration contains a sweep over all rows $i$. It is important to note that we shuffle randomly through all rows $A_i$, and that we normalized the residuals (i.e. $I(q_i, \Theta_i) - (A_i \cdot P^k(r_j, \varphi_j))$) to $\sigma = \sqrt{I(q_i, \Theta_i)}$, similar to a weighted least-squares fit. The Kaczmarz algorithm can be of course also used to determine the 1D correlation functions from 1D data sets (e.g. the radial average $I(q) = 1/(2\pi) \int_0^{2\pi} I(q, \Theta) d\Theta$ or individual sectors). In this case, Eq. 4 is applied to determine $P^{k+1}(r_j)$ with $A_{ij} = \sin (q_ir_j) / (q_ir_j) \Delta r_j$ being the matrix elements.\textsuperscript{35}
Before focusing on the 2D data, we will first use this approach to analyze the 1D cross section $|\tilde{M}_x|^2$. The transversal magnetization $|\tilde{M}_x|^2$ is of interest because it can be easily extracted from the vertical sectors of the 2D scattering patterns ($\Theta = 90^\circ \pm 10^\circ$, Eq. 1). In Fig. 1(a) we show the field dependence of $|\tilde{M}_x|^2$, whereas Fig. 1(b) displays the 1D correlation functions $P_x(r)$. As can be seen, at the highest field strength $P_x(r)$ exhibits a well pronounced peak for $0 < r < 10$ nm. It is safe to assume that this peak corresponds to the individual Fe crystallites, which have a size of around 12 nm and are in a single-domain state. This peak indicates that at 321 mT the magnetization inside the crystallites slightly deviates from perfect alignment along the field direction, probably due to the local magnetocrystalline anisotropy. With further decreasing field strength the magnitude of the peak increases but its position remains the same indicating a further tilting of the particle moments along the easy axis. Even at the lowest field we still see a shoulder at around 5 nm which is attributed to the single-domain Fe crystallites. In addition to the increase in peak intensity, at decreasing field strength we also observe progressively more deviation of $P_x(r)$ from zero.
for \( r > 10 \text{nm} \). This corresponds to the increased slope we observe for \( |\tilde{M}_x|^2 \) in the low \( q \)-range, and indicates the formation of an inhomogeneous magnetization profile around the crystallites. With decreasing external field strength the perturbations of the magnetization increases within the vicinity of the Fe crystallites.

To verify the strong influence of the stray field on the local magnetization configuration we simulated the magnetic nanostructure of Nanoperm with MuMax3.\textsuperscript{37} Fig. 1(c) shows the squared \( y \)-component of the magnetization vector field at 163 mT (the black arrows indicate the orientation of \( M_y \)). The stray field of the Fe sphere with the functional form

\[
M_y \propto \Delta M \sin \varphi \cos \varphi / r^3. \tag{5}
\]

results in a perturbation of the magnetization of the surrounding matrix picking up the symmetry of the stray field. Thus, the predicted effect of the dipole fields on the Fourier transform of the magnetization is that \( \tilde{M}_y \approx \tilde{M}_y \sin \Theta \cos \Theta \), where \( \tilde{M}_y \) is the angular independent (i.e. isotropic) amplitude of \( \tilde{M}_y \). Moreover, we can assume that for a statistically-isotropic microstructure also \( \tilde{M}_z \) is angular independent (i.e. \( \tilde{M}_z = \tilde{M}_z \)) and therefore we can write for the cross-term in Eq. 1 at first approximation \( \tilde{M}_z \tilde{M}_y \sin^2 \Theta \cos^2 \Theta \).

To describe the scenario depicted in Fig. 1(c), Honecker et al.\textsuperscript{36} developed an analytical theory for \( I_m(q) \) which is valid in the approach to saturation (see Eq. 1). Fig. 2(a) displays the calculated 2D patterns using this model for the field strengths of 321, 163, 85 and 45 mT, and in Fig. 2(b) we plot the corresponding 2D correlation functions which we extracted from the synthetic scattering data using the Kaczmarz algorithm (see Eq. 4). It can be seen that at 321 mT the scattering pattern is dominated by the \( \sin^2 \Theta \cos^2 \Theta \)-term. With decreasing field strength the signature of the \( \sin^2 \Theta \cos^2 \Theta \) term vanishes which indicates an increasing contribution by the \( |\tilde{M}_x|^2 \) and \( |\tilde{M}_y|^2 \) terms (Eq. 1). Consequently, the corresponding 2D correlation functions vary significantly with field strength. Although \( P(r) \) is not directly the autocorrelation function of the real-space magnetization vector field,\textsuperscript{28} the extracted corre-
Theoretical 2D SANS patterns $I_m(q)$ calculated for the four field strengths $\mu_0H = 321, 163, 85, 45$ mT, using the same model and material parameters as in Honecker et al.$^{36}$ The Fe crystallites are assumed to be well-separated spheres and lognormally distributed with a mean size of 10 nm and $\sigma = 0.2$. The plotted $q$-range is $0.1 - 0.4$ nm$^{-1}$, but the total $q$-range we used for the calculations was $0.1 - 1.2$ nm$^{-1}$. (b) Corresponding 2D correlation functions $P(r)$ extracted from the above scattering patterns $I_m(q)$ using the Kaczmarz algorithm ($k = 50$ iterations, see Eq. 4). The patterns $P(r, \varphi)$ were determined for $r = 0 - 100$ nm in 1-nm steps and $\varphi = 0 - 360^\circ$ in 5$^\circ$ steps. The plotted $r$-range is $0 - 80$ nm.

Figure 2: (a) Theoretical 2D SANS patterns $I_m(q)$ calculated for the four field strengths $\mu_0H = 321, 163, 85, 45$ mT, using the same model and material parameters as in Honecker et al.$^{36}$ The Fe crystallites are assumed to be well-separated spheres and lognormally distributed with a mean size of 10 nm and $\sigma = 0.2$. The plotted $q$-range is $0.1 - 0.4$ nm$^{-1}$, but the total $q$-range we used for the calculations was $0.1 - 1.2$ nm$^{-1}$. (b) Corresponding 2D correlation functions $P(r)$ extracted from the above scattering patterns $I_m(q)$ using the Kaczmarz algorithm ($k = 50$ iterations, see Eq. 4). The patterns $P(r, \varphi)$ were determined for $r = 0 - 100$ nm in 1-nm steps and $\varphi = 0 - 360^\circ$ in 5$^\circ$ steps. The plotted $r$-range is $0 - 80$ nm.

Correlation functions clearly reflect the characteristic features of the real-space 3D magnetization profile. At high field strengths, i.e. 321 and 163 mT, $P(r)$ displays a pronounced anisotropy with maxima along $\Theta = 60^\circ$, which follows the dipolar stray field of the Fe crystallites (see also Fig. 1(c)). With decreasing field strength, the anisotropy of $P(r)$ changes and is elongated along the vertical direction ($\Theta = 90^\circ$) in agreement with theory.$^{28}$

In Fig. 3 we show the experimental 2D scattering data for all four field strengths, as well as the corresponding 2D correlation functions. It is evident that the experimentally observed anisotropies are in excellent agreement with the theoretical predictions in Fig. 2. For all four field strengths we obtain basically the identical 2D correlation function. This demonstrates that the Kaczmarz algorithm can be employed to robustly extract the underlying 2D magnetic correlation functions from experimental, noisy magnetic SANS data.
Figure 3: (a) Experimental 2D SANS patterns $I_m(q)$ measured at the four field strengths $\mu_0 H = 321, 163, 85, 45$ mT ($q = 0.1 - 0.4$ nm$^{-1}$). (b) Corresponding 2D correlation functions $P(r)$ extracted from the above scattering patterns $I_m(q)$ using the Kaczmarz algorithm ($k = 50$ iterations, see Eq. [4]). The patterns $P(r, \varphi)$ were determined for $r = 0 - 100$ nm in 1-nm steps and $\varphi = 0 - 360^\circ$ in 5$^\circ$-steps. The plotted $r$-range is $0 - 80$ nm.

To summarize, we have introduced a procedure for the analysis of magnetic SANS data to visualize the magnetization configuration of bulk samples. We applied this approach to characterize the magnetic microstructure of the nanocrystalline ferromagnet Nanoperm. By subtracting the nuclear and magnetic scattering at saturation from the SANS data at intermediate magnetic field strengths, we obtained the purely magnetic SANS intensities $I_m(q)$. We employed Kaczmarz’ algorithm to extract the 2D magnetic correlation functions from the scattering patterns $I_m(q)$. By comparing our results with micromagnetic simulations and theoretical calculations we can show that the extracted correlation functions reflect the real-space magnetization distribution following the dipolar stray fields around Fe nanocrystallites. This study highlights that the 2D correlation functions derived from magnetic SANS data, which contain the projection of the 3D real-space magnetization configuration onto the 2D detector plane, can be used to image nanostructured spin textures within a large variety of samples.
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References

(1) Fischer, P.; Sanz-Hernández, D.; Streubel, R.; Fernández-Pacheco, A. Launching a new dimension with 3D magnetic nanostructures. *APL Mater.* 2020, 8, 010701.

(2) Skomski, R. Nanomagnetics. *J. Phys.: Condens. Matter* 2003, 15, R841.

(3) Pinilla-Cienfuegos, E.; Mañas-Valero, S.; Forment-Aliaga, A.; Coronado, E. Switching the magnetic vortex core in a single nanoparticle. *ACS Nano* 2016, 10, 1764–1770.

(4) Fernández-Pacheco, A.; Streubel, R.; Fruchart, O.; Hertel, R.; Fischer, P.; Cowburn, R. P. Three-dimensional nanomagnetism. *Nat. Commun.* 2017, 8, 1–14.

(5) Phatak, C.; Liu, Y.; Gulsoy, E. B.; Schmidt, D.; Franke-Schubert, E.; Petford-Long, A. Visualization of the magnetic structure of sculpted three-dimensional cobalt nanospirals. *Nano Lett.* 2014, 14, 759–764.

(6) Tanigaki, T.; Takahashi, Y.; Shimakura, T.; Akashi, T.; Tsuneta, R.; Sugawara, A.; Shindo, D. Three-dimensional observation of magnetic vortex cores in stacked ferromagnetic discs. *Nano Lett.* 2015, 15, 1309–1314.

(7) Gatel, C.; Bonilla, F. J.; Meffre, A.; Snoeck, E.; Warot-Fonrose, B.; Chaudret, B.; Lacroix, L.-M.; Blon, T. Size-specific spin configurations in single iron nanomagnet: from flower to exotic vortices. *Nano Lett.* 2015, 15, 6952–6957.
(8) Streubel, R.; Kronast, F.; Fischer, P.; Parkinson, D.; Schmidt, O. G.; Makarov, D. Retrieving spin textures on curved magnetic thin films with full-field soft X-ray microscopies. *Nat. Commun.* **2015**, *6*, 7612.

(9) Donnelly, C.; Guizar-Sicairos, M.; Scagnoli, V.; Gliga, S.; Holler, M.; Raabe, J.; Heyderman, L. J. Three-dimensional magnetization structures revealed with X-ray vector nanotomography. *Nature* **2017**, *547*, 328–331.

(10) Donnelly, C.; Finizio, S.; Gliga, S.; Holler, M.; Hrabec, A.; Odstrčil, M.; Mayr, S.; Scagnoli, V.; Heyderman, L. J.; Guizar-Sicairos, M.; Raabe, J. Time-resolved imaging of three-dimensional nanoscale magnetization dynamics. *Nat. Nanotechnol.* **2020**, 1–5.

(11) Varón, M.; Beleggia, M.; Kasama, T.; Harrison, R.; Dunin-Borkowski, R. E.; Puntes, V. F.; Frandsen, C. Dipolar magnetism in ordered and disordered low-dimensional nanoparticle assemblies. *Sci. Rep.* **2013**, *3*, 1234.

(12) Chesnel, K.; Griner, D.; Smith, D.; Cai, Y.; Trevino, M.; Newbold, B.; Wang, T.; Liu, T.; Jal, E.; Reid, A. H.; Harrison, R. G. Unraveling Nanoscale Magnetic Ordering in Fe$_3$O$_4$ Nanoparticle Assemblies via X-rays. *Magnetochemistry* **2018**, *4*, 42.

(13) Mühlbauer, S.; Honecker, D.; Périto, É. A.; Bergner, F.; Disch, S.; Heinemann, A.; Erokhin, S.; Berkov, D.; Leighton, C.; Eskildsen, M. R.; Michels, A. Magnetic small-angle neutron scattering. *Rev. Mod. Phys.* **2019**, *91*, 015004.

(14) Kardjilov, N.; Manke, I.; Strobl, M.; Hilger, A.; Treimer, W.; Meissner, M.; Krist, T.; Banhart, J. Three-dimensional imaging of magnetic fields with polarized neutrons. *Nat. Phys.* **2008**, *4*, 399–403.

(15) Manke, I.; Kardjilov, N.; Schäfer, R.; Hilger, A.; Strobl, M.; Dawson, M.; Grünzweig, C.; Behr, G.; Hentschel, M.; David, C.; Kupsch, A.; Lange, A.; Banhart, J. Three-dimensional imaging of magnetic domains. *Nat. Commun.* **2010**, *1*, 1–6.
(16) Suzuki, K.; Herzer, G. In Advanced Magnetic Nanostructures; Sellmyer, D., Skomski, R., Eds.; Springer: New York, 2006; pp 365–401.

(17) Vecchini, C.; Moze, O.; Suzuki, K.; Pranzas, P.; Weissmüller, J.; Michels, A. Neutron scattering and modeling of dipole-field-induced spin disorder in Nanoperm. Appl. Phys. Lett. 2005, 87, 202509.

(18) Michels, A.; Vecchini, C.; Moze, O.; Suzuki, K.; Cadogan, J. M.; Pranzas, P.; Weissmüller, J. Dipole-field–induced spin disorder in a nanocomposite soft magnet. Europhys. Lett. 2005, 72, 249.

(19) Michels, A.; Vecchini, C.; Moze, O.; Suzuki, K.; Pranzas, P.; Kohlbrecher, J.; Weissmüller, J. Dipolar correlations in a nanocomposite: A neutron scattering study of Nanoperm Fe$_{89}$Zr$_7$B$_3$Cu. Phys. Rev. B 2006, 74, 134407.

(20) Krycka, K. L.; Booth, R. A.; Hogg, C. R.; Ijiri, Y.; Borchers, J. A.; Chen, W.; Watson, S.; Laver, M.; Gentile, T. R.; Dedon, L. R.; Harris, S.; Rhyne, J. J.; Majetich, S. A. Core-shell magnetic morphology of structurally uniform magnetite nanoparticles. Phys. Rev. Lett. 2010, 104, 207203.

(21) Disch, S.; Wetterskog, E.; Hermann, R. P.; Wiedenmann, A.; Vainio, U.; Salazar-Alvarez, G.; Bergström, L.; Brückel, T. Quantitative spatial magnetization distribution in iron oxide nanocubes and nanospheres by polarized small-angle neutron scattering. New J. Phys. 2012, 14, 013025.

(22) Grutter, A. J.; Krycka, K. L.; Tartakovskaya, E. V.; Borchers, J. A.; Reddy, K. S. M.; Ortega, E.; Ponce, A.; Stadler, B. J. Complex three-dimensional magnetic ordering in segmented nanowire arrays. ACS Nano 2017, 11, 8311–8319.

(23) Bersweiler, M.; Bender, P.; Vivas, L. G.; Albino, M.; Petrecca, M.; Mühlbauer, S.; Erokhin, S.; Berkov, D.; Sangregorio, C.; Michels, A. Size-dependent spatial magne-
zation profile of manganese-zinc ferrite Mn$_{0.2}$Zn$_{0.2}$Fe$_{2.6}$O$_4$ nanoparticles. *Phys. Rev. B* **2019, 100**, 144434.

(24) Bender, P.; Honecker, D.; Fernández Barquín, L. Supraferromagnetic correlations in clusters of magnetic nanoflowers. *Appl. Phys. Lett.* **2019, 115**, 132406.

(25) Alba Venero, D.; Rogers, S.; Langridge, S.; Alonso, J.; Fdez-Gubieda, M.; Svalov, A.; Fernández Barquín, L. Magnetic nanoscopic correlations in the crossover between a superspin glass and a superferromagnet. *J. Appl. Phys.*, **2016, 119**, 143902.

(26) Alves, C.; Pedersen, J. S.; Oliveira, C. L. Calculation of two-dimensional scattering patterns for oriented systems. *J. Appl. Crystallogr.* **2017, 50**, 840–850.

(27) Zákutná, D.; Falke, Y.; Dresen, D.; Prévost, S.; Bender, P.; Honecker, D.; Disch, S. Morphological and crystallographic orientation of hematite spindles in an applied magnetic field. *Nanoscale* **2019, 11**, 7149–7156.

(28) Mettus, D.; Michels, A. Small-angle neutron scattering correlation functions of bulk magnetic materials. *J. Appl. Crystallogr.* **2015, 48**, 1437–1450.

(29) Fritz-Popovski, G. Interpretation of two-dimensional real-space functions obtained from small-angle scattering data of oriented microstructures. *J. Appl. Crystallogr.* **2015, 48**, 44–51.

(30) Fritz-Popovski, G. Two-dimensional indirect Fourier transformation for evaluation of small-angle scattering data of oriented samples. *J. Appl. Crystallogr.* **2013, 46**, 1447–1454.

(31) Bender, P.; Zákutná, D.; Disch, S.; Marcano, L.; Alba Venero, D.; Honecker, D. Using the singular value decomposition to extract 2D correlation functions from scattering patterns. *Acta Crystallogr., Sect. A: Found. Crystallogr.* **2019, 75**.
(32) Kaczmarz, S. Angenäherte auflösung von systemen linearer gleichungen. Bulletin International de l’Academie Polonaise des Sciences et des Lettres 1937, 35, 355–357.

(33) Schmidt, D.; Eberbeck, D.; Steinhoff, U.; Wiekhorst, F. Finding the magnetic size distribution of magnetic nanoparticles from magnetization measurements via the iterative Kaczmarz algorithm. J. Magn. Magn. Mater. 2017, 431, 33–37.

(34) Leliaert, J.; Schmidt, D.; Posth, O.; Liebl, M.; Eberbeck, D.; Coene, A.; Steinhoff, U.; Wiekhorst, F.; Van Waeyenberge, B.; Dupré, L. Interpreting the magnetorelaxometry signal of suspended magnetic nanoparticles with Kaczmarz algorithm. J. Phys. D: Appl. Phys. 2017, 50, 195002.

(35) Bender, P.; Bogart, L. K.; Posth, O.; Szczerba, W.; Rogers, S. E.; Castro, A.; Nilsson, L.; Zeng, L. J.; Sugunan, A.; Sommertune, J.; Fornara, A.; González-Alonso, D.; Fernández Barquín, L.; Johansson, C. Structural and magnetic properties of multi-core nanoparticles analysed using a generalised numerical inversion method. Sci. Rep. 2017, 7, 45990.

(36) Honecker, D.; Dewhurst, C. D.; Suzuki, K.; Erokhin, S.; Michels, A. Analysis of magnetic neutron-scattering data of two-phase ferromagnets. Phys. Rev. B 2013, 88, 094428.

(37) Vansteenkiste, A.; Leliaert, J.; Dvornik, M.; Helsen, M.; Garcia-Sanchez, F.; Van Waeyenberge, B. The design and verification of MuMax3. AIP Adv. 2014, 4, 107133.