ACCELERATING COSMOLOGIES WITH AN ANISOTROPIC EQUATION OF STATE

TOMI KOIVISTO
Helsinki Institute of Physics, P.O. Box 64, FIN-00014 Helsinki, Finland

AND

DAVID F. MOTA
Institute for Theoretical Physics, University of Heidelberg, 69120 Heidelberg, Germany

Received 2007 October 4; accepted 2008 February 8

ABSTRACT

If the dark energy equation of state is anisotropic, the expansion rate of the universe becomes direction dependent at late times. We show that such models are not only cosmologically viable but that they could explain some of the observed anomalies in the cosmic microwave background. The possible anisotropy can then be constrained by studying its effects on the luminosity distance–redshift relation inferred from several observations. A vector field action for dark energy is also presented as an example of such a possibility.

Subject headings: cosmic microwave background — cosmology: miscellaneous — cosmology: observations — cosmology: theory — large-scale structure of universe

Online material: color figures

1. INTRODUCTION

It is not clear whether the large-angle anomalies in the observed cosmic microwave background (CMB) are of a cosmological origin and not due to systematics (Eriksen et al. 2004; Land & Magueijo 2005; Copi et al. 2006). Nevertheless, it is tempting to associate the apparent statistical anisotropy with dark energy, since the anomalies occur at the largest scales, and these enter inside the horizon at the same epoch that the dark energy dominance begins.

The paramount characteristic of dark energy is its negative pressure. We examine then the possibility that this pressure varies with the direction. Then, the universal acceleration also becomes anisotropic, and one would indeed see otherwise unexpected effects at the smallest multipoles of CMB. At the background level we will find a similar CMB pattern as in a universe which is ellipsoidal at the era of last scattering (Campanelli et al. 2006). We then show that the supernovae could be used to distinguish these different scenarios and to constrain the possible anisotropic properties of dark energy.

There are several motivations for anisotropic models of dark energy. For instance, many dark energy models are in principle compatible with the FLRW metric, but exhibit anisotropic stresses at the perturbative level, including nonminimally coupled fields, viscous fluids, and modified gravity models (Koivisto & Mota 2006, 2007a, 2007b; Schimd et al. 2005; Mota et al. 2007; Clifton et al. 2005). It is possible that a more accurate description of such models should take into account the anisotropic effects on the background expansion, which then breaks the statistical isotropy of perturbations too (as appears to have happened in the CMB; Eriksen et al. 2004; Land & Magueijo 2005; Copi et al. 2006). In addition, considering dark energy as an effective description of a back-reaction of sizable inhomogeneities in dark matter (Buchert 2007; Behrend et al. 2008), the validity of a perfect fluid description could seem dubious.

To have a general description of an anisotropic dark energy component, we consider a phenomenological parameterization of dark energy in terms of its equation of state (w) and two skewness parameters (δ and γ) and also include a coupling term (Q) between dark energy and a perfect fluid (dark matter). Previous studies of anisotropic dark energy have mainly considered the anisotropic properties of the inhomogeneous perturbations (Koivisto & Mota 2006; Battye & Moss 2006), whereas our approach here is to focus on a smooth cosmology with the anisotropic pressure field. Whereas in a previous approach the anisotropy was only weakly constrained (Mota et al. 2007), we find that in the present description only a narrow parameter range can survive the observational tests. Similar anisotropic inflation has been considered in the early universe (Burd & Lidsey 1991; Ford 1989). Therefore, we focus in the present paper more on the dark energy era, where the qualitative differences are that matter cannot be neglected and that the cosmologies do not isotropize. As a proof of a concept, we also write an explicit field theory example where a vector field drives the anisotropic acceleration of the universe.

An anisotropic expansion is not compatible with the Robertson-Walker (RW) metric. Hence, we use the Bianchi type I (BI) metric which generalizes the flat RW metric and may be employed to obtain limits on cosmological skew pressures from the CMB (Barrow 1997). The line element of a BI universe is

\[ ds^2 = -dt^2 + a^2(t)dx^2 + b^2(t)dy^2 + c^2(t)dz^2. \]

There are thus three scale factors and, consequently, three expansion rates. In principle, all these could be different, and in the limiting case that all of them are equal, one recovers the RW case. It is useful to express the mean expansion rate as an average Hubble rate \( H \) (where an overdot means derivative with respect to \( t \)),

\[ H \equiv \frac{1}{3} \left( \frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c} \right), \]

and then to express the differences of the expansion rates as the Hubble-normalized shear \( R \) and \( S \) (Barrow 1997),

\[ R \equiv \frac{1}{H} \left( \frac{\dot{a}}{a} - \frac{\dot{b}}{b} \right), \quad S \equiv \frac{1}{H} \left( \frac{\dot{a}}{a} - \frac{\dot{c}}{c} \right). \]
We consider a universe filled with a perfect fluid having the energy-momentum tensor \( T_{\mu \nu} = \text{diag}(-1, w_m, w_m, w_m) \rho_m \), and dark energy, which we allow to have the most general energy-momentum tensor compatible with the metric from equation (1),

\[
T_{\mu \nu} = \text{diag}(-1, w, w + 3\delta, w + 3\gamma)\rho.
\] (4)

The generalized Friedmann equation may be written as

\[
H^2 = \frac{8\pi G}{3} \left( \frac{\rho_m + \rho}{1 - (1/9)(R^2 + S^2 - RS)} \right).
\] (5)

We let the two components also interact. The continuity equations are then

\[
\dot{\rho}_m + 3H(1 + w_m)\rho_m = QH \rho, \quad \rho_m \quad (6)
\]

\[
\dot{\rho} + 3 \left( (1 + w)H + \frac{\dot{b}}{b} + \frac{\gamma}{c} \right) \rho = -QH \rho, \quad \rho \quad (7)
\]

where \( Q \) determines the coupling. If it vanishes, it follows that \( \rho_m \sim (abc)^{-1+w} \) and \( \rho \sim (abc)^{-1+b(1/9)2^{3/6}} \). Defining \( x \equiv \frac{1}{4} \log \left( -g \right) \), where the metric determinant \( g = -abc \), one notes that \( H = x \). We will use \( x \) as our time variable rather than \( t \). Derivative with respect to \( x \) is denoted by an asterisk. We also define the dimensionless density fractions

\[
\Omega_m \equiv \frac{8\pi G \rho_m}{3 \, H^2}, \quad U \equiv \frac{\rho}{\rho_m + \rho}.
\]

Using \( R, S, \) and \( U \) as our dynamical variables, the system can finally be written as

\[
U^* = U(U - 1)[\gamma(3 + R - 2S) + 6(3 - 2R + S) + 3(w - w_m)] - UQ,
\]

\[
S^* = \frac{1}{6}(R - 2R^2 + RS - S^2) \times x[S[U(\delta + \gamma + w - w_m) + w_m - 1] - 6\gamma U],
\]

\[
R^* = \frac{1}{6}(R - 2R^2 + RS - S^2) \times x[R[U(\delta + \gamma + w - w_m) + w_m - 1] - 6\delta U].
\] (8)

Note the coupling term \( Q \) appears only in the evolution equation for \( U \). Nevertheless, its presence can change the dynamics completely (Koivisto & Mota 2008).

2. VECTOR FIELD

As a proof of concept, we present an explicit field theory for the anisotropically stressed dark energy. Consider the vector field action

\[
S = \int d^4x \sqrt{-g} \left( \frac{R}{16\pi G} - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - V + qL_m \right),
\] (9)

where the kinetic term involves \( F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \). The potential \( V \) and the possible couplings \( q \) are understood as functions of the quadratic term \( A^2 = A_\mu A^\mu \). The field equations are \( G_{\mu \nu} = 8\pi G (T_{\mu \nu} + T_{\mu \nu}^A) \), where the energy-momentum tensor of the vector field follows by varying with respect to the metric

\[
T_{\mu \nu}^A = F_{\mu \nu} F^\nu - 2V'A_\mu A_\nu - \left( \frac{1}{4} F_{\alpha \beta} F^{\alpha \beta} + V \right) g_{\mu \nu}.
\] (10)

A prime denotes derivative with respect to \( \lambda^A \). With the metric from equation (1), the equation of motion for the \( \lambda_0 \) component of the field dictates that \( q(L_m + V')A_0 = 0 \). Thus, we confine our study to purely spatial vector fields. The spatial components of the energy-momentum tensor are

\[
T_i^{\lambda A} = \frac{1}{a_i^2} (\dot{A}_i A_j + 2V'A_i A_j) + \left( \frac{3}{2} \sum_{k=1}^{3} \frac{\dot{A}_k^2}{a_k^2} - V \right) \delta_{ij}.
\]

The off-diagonal terms should vanish. Thus, one is restricted to consider only vectors which are parallel to one of the coordinate axes. There, however, could be many such fields.

In the RW case, all the diagonal components of \( T_i^\lambda \) should be equal, which indeed requires three vector fields, one in each coordinate direction, of exactly equal magnitudes, as the triads of Armendariz-Picon (2004). With the metric from equation (1), the only constraint is that every vector field should be along a coordinate axis. Any system of these vector fields is thus required to have an anisotropic equation of state, unless it reduces to the very special triad case. Let us therefore briefly look at the case of a single vector field with \( A_1 = A_2 = 0 \). Notice that, in analogy to the system from equation (8), the coupling \( Q \) is now related to the function \( q \). If we define the additional dimensionless parameters

\[
\frac{X}{bH} = \frac{Y}{H^2}, \quad Y_1 = 2 \frac{V'A_2}{b^2H^2},
\]

one can relate the present case to the general notation by noting that \( H^2 \rho = X^2/2 + Y \), and that if \( S = 0 \), then

\[
w = \frac{X^2 - 2Y}{X^2 + 2Y}, \quad \delta = - \frac{X^2 - Y_1}{(3/2)X^2 + 3Y}.
\] (11)

We see that the anisotropy is naturally small if the field is either subdominant or near its minimum. Such models, even though perturbatively close to standard cosmology, would be excluded by imposing the RW symmetry. Viable models with large anisotropy do also exist as shown below. An extensive investigation of specific models will be undertaken elsewhere (Koivisto & Mota 2008).

3. SCALING SOLUTIONS

(IN THE AXISYMMETRIC CASE)

To begin, we will assume for simplicity that (1) the perfect fluid is minimally coupled dark matter, \( w_m = Q = 0 \), and further that (2) the skewness together with the equation of state of dark energy is constant, \( \dot{w} = \dot{\delta} = \dot{\gamma} = 0 \) (see Koivisto & Mota 2008 for an extensive analysis with these assumptions relaxed). In our model, the universe is then initially (close to) isotropic, but is driven to anisotropic expansion by the skewness of dark energy. Under the additional simplifying assumption of axial symmetry \( (S = \gamma = 0) \), in Figure 1 we indicate the final possible stages of such a universe, corresponding to the fixed points of the system from equation (8).

The universe will then end up in three possible scenarios:

1. The isotropic RW case corresponding to

\[
R = 0, \quad U = 0.
\] (12)

2. An anisotropic dark energy–dominated solution with

\[
R = \frac{6\delta}{w + \delta - 1}, \quad U = 1.
\] (13)
3. A scaling solution,

\[ R = \frac{3(w + \delta)}{2\delta}, \quad U = \frac{-(w + \delta)}{3\delta^2 - 2\delta w - w^2}. \tag{14} \]

Given that \( w \) is negative, this solution accelerates \( w_{\text{eff}} \equiv -\left(\frac{2}{3}\right)(H^2/H) - 1 < -1/3 \), if \( w < \delta < w/3 \) or \(-w/5 < \delta < -w/3 \). Note that in the \( R = 0 \) case, scaling solutions could only be found for coupled components.

Notice that within the RW universe it has proven difficult to address the coincidence problem by finding a model entering from a matter-dominated scaling solution to an accelerating scaling solution. Allowing for the presence of three expansion rates opens up the possibility of describing a universe entering from a perfect fluid-dominated scaling to an anisotropically accelerating scaling era. This might eventually help to understand the coincidence problem, since then matter and dark energy would have had similar energy densities both in the past and in the future (in the past the dark energy fraction, if constant, should not exceed about 1/10; Doran & Robbers 2006).

We now study the observational implications of models with nonzero skewness, relaxing assumption 2. The relevant case is then a universe entering from the RW solution from equation (12) to an anisotropically accelerating universe (which generalizes eq. [13] if \( \gamma \neq 0 \)).

4. CMB ANISOTROPY

The BI model reproduces the predictions of the concordance model at small scales, while featuring anomalies at large angles, since the (originally statistically isotropic) CMB field experiences an (statistically) anisotropic integrated Sachs-Wolfe effect in the nowadays ellipsoidal universe. Furthermore, it is plausible that these anomalies can match the observed ones, since a BI model can be effectively described as a BI model with an additional anisotropic energy source.\(^1\) A detailed calculation is postponed into future studies, as it requires taking into account the inhomogeneous perturbations of these models with imperfect fluid sources.\(^2\)

As a first step, we check whether the background from equation (1) is compatible with the available cosmological data. There are a number of calculations of CMB anisotropies in general Bianchi umbrellas already available in the literature (Hawking 1969; Barrow et al. 1985; Pontzen & Challinor 2007). Here we consider the dark energy–dominated universe at small scales (Jaffe et al. 2006).

The BI model reproduces the predictions of the concordance model at small scales, while featuring anomalies at large angles, since the (originally statistically isotropic) CMB field experiences an (statistically) anisotropic integrated Sachs-Wolfe effect in the nowadays ellipsoidal universe. Furthermore, it is plausible that these anomalies can match the observed ones, since a BI model can be effectively described as a BI model with an additional anisotropic energy source.\(^1\) A detailed calculation is postponed into future studies, as it requires taking into account the inhomogeneous perturbations of these models with imperfect fluid sources.\(^2\)

As a first step, we check whether the background from equation (1) is compatible with the available cosmological data. There are a number of calculations of CMB anisotropies in general Bianchi umbrellas already available in the literature (Hawking 1969; Barrow et al. 1985; Pontzen & Challinor 2007). Here we consider the dark energy–dominated universe at small scales (Jaffe et al. 2006).

By considering the geodesic equation for photons, one can derive an equation for the redshift \( z \) of a photon arriving from the direction \( \hat{p} \)

\[ 1 + z(\hat{p}) = \frac{1}{a} \sqrt{1 + \hat{p}^2 c^2 e_r^2 + \hat{p}_r^2 e_z^2} \tag{15} \]

in terms of the eccentricities

\[ e_r^2 = \left(\frac{a^2}{b^2}\right)^2 - 1, \quad e_z^2 = \left(\frac{a^2}{c^2}\right)^2 - 1. \tag{16} \]

Note that the scale factors and eccentricities here are evaluated at the time of last scattering in the case that the scale factors are all normalized to unity today. If \( T_\| \) is the temperature at decoupling, the temperature field is given by \( T(\hat{p}) = T_\| /\sqrt{1 + z(\hat{p})} \), and its spatial average is \( 4\pi \bar{T} = \int d\Omega_\hat{p} T(\hat{p}) \). The anisotropy field is then

\[ \frac{\delta T(\hat{p})}{T} = 1 - \frac{T(\hat{p})}{\bar{T}}. \tag{17} \]

The coefficients in the spherical expansion of this anisotropy field are called \( a_{\ell m} \) and, due to the orthogonality of spherical harmonics \( Y_{\ell m} \), are given by

\[ a_{\ell m} = \int d\Omega_\hat{p} \frac{\delta T(\hat{p})}{T} Y^*_{\ell m}. \tag{18} \]

The multipole spectrum is described by

\[ Q_\ell = \frac{1}{2\pi} \frac{\ell (\ell + 1)}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2. \tag{19} \]

\(^1\) One may obtain the BIUI model from BI by adding anisotropic curvature. To obtain the BIUI model one should also add an isotropic curvature similar to that present in BV models.

\(^2\) Because of the small complication, it is not feasible to address the stability of the most general (inhomogeneous) perturbations here.
Expanding the redshifts from equation (15) in powers of the equation of state of the universe $w$, and the dash-dotted line describes the evolution of eccentricity, $E = 900 x^2$. The potential is a double power law $V = m A^2 + A^{-6}$. The dynamics of the field is such that although there are significant anisotropies, the eccentricity at the present is close to zero. This may be achieved with different power-law potentials, but requires fine-tuning of the mass scales $m$ and $x$. [See the electronic edition of the Journal for a color version of this figure.]

\begin{equation}
\begin{aligned}
& a_{20} = \frac{1}{3} \sqrt{\frac{\pi}{5}} (2 e_z^2 - e_y^2), \quad a_{21} = a_{2-1} = 0, \\
& a_{22} = a_{2-2} = -\sqrt{\frac{\pi}{30}} c_z \Rightarrow Q_2 = \frac{2}{5\sqrt{3}} \sqrt{e_y^4 + e_z^4 - e_x^2 e_y^2}.
\end{aligned}
\end{equation}

The observed value of $Q_2$ is lower than the concordance model predicts (Hinshaw et al. 2007). It has been suggested in previous works that this discrepancy could be explained by an ellipsoidality of the universe (Campanelli et al. 2006; Jimenez & Mota 2007). This would require that the anisotropy of the background is suitably oriented with respect to the intrinsic quadrupole and cancels its power to a sufficient amount. Too large an anisotropy would of course only make the situation worse regardless of the orientation. Depending on the cosmological model, one should have $Q_2 \leq 2 \times 10^{-3}$ to be consistent with observations taking into account the cosmic variance. The constraints this implies on the skewness of dark energy are very tight. However, we remark that in more general models, in particular with time-varying $\delta$ and $\gamma$, one could allow more anisotropy. It is in principle possible for arbitrarily anisotropic expansion to escape detection from CMB (considering only the effects from the background), as long as the expansion rates evolve in such a way that $e_x = e_y = 0$. In other words, the (background) quadrupole vanishes, if each scale factor has expanded—no matter how anisotropically—the same amount since the last scattering. An example of such a scenario, derived from the action from equation (9), is shown in Figure 2.

5. TYPE Ia SUPERNOVA LUMINOSITIES

The luminosity-redshift relationship of the Type Ia supernova (SNIa) data could be used to probe the possible anisotropies in the expansion history. This is a useful complementary probe, since these objects are observed at the $z < 2$ region, whereas CMB comes from much further away at $z \sim 1100$. The luminosity distance at the redshift $z$ in the direction $\hat{p}$ is now given by (Koivisto & Mota 2008)

\begin{equation}
d_L(z, \hat{p}) = (1 + z) \int_0^{\theta(z)} \frac{dt}{\sqrt{\hat{p}_x^2 a^2 + \hat{p}_y^2 b^2 + \hat{p}_z^2 c^2}}.
\end{equation}

To test this prediction with the data, we apply equation (15) for each observed redshift of a supernova and match its luminosity distance inferred from the observation to the one computed from equation (21). In addition, we also have to take into account the angular coordinates of each individual supernova in the sky to fix $\hat{p}$ for each object. In our analysis we use the GOLD data set (Riess et al. 2007), which consists of five subsets of data. We marginalize over the directions in the sky and over the present value of the Hubble constant. The results are summarized in Figures 3 and 4. The best-fit anisotropic models are only slightly preferred over the $\Lambda$CDM, the difference being $\Delta \chi^2 \approx 1$. Because of the additional parameters in the anisotropic models, the reasonable interpretation of these statistics is that the SNIa data favors isotropic expansion. However, the SNIa data constrains the skewness parameters much looser than the CMB quadrupole. The amount anisotropy that is present in CMB is yet practically undetectable from the SNIa data, which therefore cannot be used to rule out such anisotropies.

3 Note that the apparent lack of symmetry in the expression for $Q_2$ is due to our choice of $x$ as a reference axis.

4 SNIa angular coordinates of each of the 182 GOLD supernovae can be found in Riess et al. (2007; Astier et al. 2006) and at http://cfa-www.harvard.edu/supernova/SNgroup.html.
We would also like to make the point that this applies only if the skewness parameters are constant. In specific models with time-evolving CMB and SNIa constraints from CMB and from SNIa can be of comparable magnitude and allow anisotropy to an interesting degree. This means that the even if the CMB formed isotropically at early time, it could be distorted by the acceleration of the later universe in such a way that it appears to us anomalous at the largest scales.\footnote{Generating these effects at low redshift has the advantage that it relaxes constraints which would otherwise come from the CMB polarization (Pontzen & Challinor 2007) and could be strong for a given temperature anisotropy in isotropizing models because of the significant polarization anisotropy at last scattering. However, anisotropic dark energy could evade this, since the optical depth to z \approx 1 is very small.} The future SNIa data with considerably improved error bars (e.g., from the SNAP experiment\footnote{See http://snap.lbl.gov/}) might be used to rule out this possibility and to distinguish whether the possible statistical anisotropy was already there at last scattering or whether it is due to dark energy.

6. CONCLUSIONS

In conclusion, dark energy with an anisotropic equation of state might be the culprit for both the cosmic acceleration and the large-angle anomalies in the CMB. This might also be the key to understanding the coincidence problem. The present SNIa data allows anisotropic acceleration, but SNAP could set things straight about the skewness of dark energy and hence of its nature. Such a possibility would open a completely new window not only on the nature of the CMB anomalies but also into high-energy physics models beyond the usual isotropic candidates of dark energy such as scalar fields or the cosmological constant.

We thank Y. Gaspar, S. Hervik, and the anonymous referee for useful comments. T. K. acknowledges support from the Magnus Ehrnrooth Foundation, the Finnish Cultural Foundation, and EU FP6 Marie Curie Research and Training Network “UniverseNet” (MRTN-CT-2006-035863). D. F. M. acknowledges support from the A. Humboldt Foundation and the Research Council of Norway through project 159637/V30.