Generalized parton distributions and spin structures of light mesons from a light-front Hamiltonian approach

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Basis Light-Front Quantization (BLFQ)

- Nonperturbative approach
- Solve the light-front eigenvalue equation: \( H_{\text{LF}} | \psi \rangle = M^2 | \psi \rangle \)
- General BLFQ algorithm:
  
  1. Formulate \( H_{\text{LF}} \)
  2. Construct basis states
  3. Calculate matrix elements
  4. Diagonalize
  5. Evaluate observables

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Effective Hamiltonian: BFLQ-NJL Model

\[ H_{\text{eff}} = \frac{\vec{k}^{\perp 2} + m_q^2}{x} + \frac{\vec{k}^{\perp 2} + m_{\bar{q}}^2}{1 - x} + \kappa^4 x (1 - x) \vec{r}^{\perp 2} + \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x \left(x(1 - x) \partial_x \right) + H_{\text{NJL}} \]

\[ | \pi \rangle_{\text{phys}} = a | q\bar{q} \rangle + b | q\bar{q}g \rangle + c | q\bar{q}q\bar{q} \rangle + \ldots \]

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[1] S. Jia and J. P. Vary, Phys. Rev. C 99, 035206 (2019).

[2] S. Klimt, M. F. M. Lutz, U. Vogl and W. Weise, Nucl. Phys. A 516, 429-468 (1990).
Meson Light Front Wave Functions (LFWFs)

The valence LFWFs in orthonormal bases [1]

\[ \psi_{rs}(x, \vec{k}_\perp) = \sum_{n,m,l} \langle n, m, l, r, s | \psi \rangle \times \phi_{nm}(\vec{k}_\perp) \chi_l(x) \]

- transverse direction
  \[ \phi_{nm}(\vec{k}_\perp) \sim (|\vec{k}_\perp|^m) \times \exp(-\vec{k}_\perp^2) L_n^m(\vec{k}_\perp^2) \]

- longitudinal direction
  \[ \chi_l(x) \sim x^{\beta/2} (1 - x)^{\alpha/2} P_l^{(\alpha, \beta)}(2x - 1) \]

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Y. Li, P. Maris and J. P. Vary, Phys. Rev. D 96, 016022 (2017).
BLFQ-NJL model parameters

- Parameters are fixed to
  - reproduce the ground state masses
  - experimental charge radii of $\pi^+$ and the $K^+$ \cite{1}

- Successfully applied to
  - compute the PDAs and the EMFFs \cite{1}
  - PDFs for the pion and the kaon and pion-nucleus induced Drell-Yan cross sections \cite{2,3}

- Summary of the model parameters \cite{1}

| Valence flavor | $N_{\text{max}}$ | $L_{\text{max}}$ | $\kappa$(MeV) | $m_q$(MeV) | $m_{\bar{q}}$(MeV) |
|----------------|-----------------|-----------------|--------------|------------|------------------|
| $ud$           | 8               | 8–32            | 227          | 337        | 337              |
| $u\bar{s}$     | 8               | 8–32            | 276          | 308        | 445              |

\cite{1} S. Jia and J. P. Vary, Phys. Rev. C 99, 035206 (2019).
\cite{2} J. Lan, C. Mondal, S. Jia, X. Zhao and J. P. Vary, Phys. Rev. D 101, 034024 (2020).
\cite{3} J. Lan, C. Mondal, S. Jia, X. Zhao and J. P. Vary, Phys. Rev. Lett. 122, 172001 (2019).
Generalized Parton Distributions (GPDs) : Spin-0 Meson

two independent GPDs at leading twist

\[ H_P(x, \zeta, t) = \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle \mathcal{P}(P') | \bar{\Psi}_q(0) \gamma^+ \Psi_q(z) | \mathcal{P}(P) \rangle |_{z^+ = z^\perp = 0} \]

\[ i\epsilon^{ij} q_i^\perp \frac{E_T^P}{2M_P} (x, \zeta, t) = \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle \mathcal{P}(P') | \bar{\Psi}_q(0) i\sigma^j + \gamma_5 \Psi(z)_q | \mathcal{P}(P) \rangle |_{z^+ = z^\perp = 0} \]

- \( H \Rightarrow \) chirally even unpolarized quark GPD
- \( E_T \Rightarrow \) chirally odd transversely polarized quark GPD
- \( P(P') \) denotes the meson momentum of initial (final) state of the meson \( (\mathcal{P}) \).  
- We choose \( A^+ = 0 \) and the kinematical region: \( 0 < x < 1 \) at zero skewness \( (\zeta = 0) \).
Results: Pion GPDs

- $H^\pi_u (x, 0) \Rightarrow$ symmetric with peak at $x = 0.5$
- $E^{\pi}_{Tu} (x, 0) \Rightarrow$ asymmetric with peak below $x = 0.5$
- peak in the GPDs shift towards higher values of $x$
- oscillations are numerical artifacts due to longitudinal cutoff $L_{\text{max}}$
Results: Kaon GPDs

\[ H^K_u(x, t) \]

\[ H^K_s(x, t) \]

\[ E^K_Tu(x, t) \]

\[ E^K_Ts(x, t) \]
The Mellin moments of the valence GPDs give the generalized FFs

\[ A_{n0}^q(t) = \int_0^1 dx \, x^{n-1} \, H^q(x, 0, t) \]
\[ B_{Tn0}^q(t) = \int_0^1 dx \, x^{n-1} \, E_T^q(x, 0, t) \]

- \( H^q(x, 0, t) \) \( \xrightarrow{\text{first moment}} \) \( A_{10}^q(t) \) electromagnetic FF of an unpolarized quark

- \( E_T^q(x, 0, t) \) \( \xrightarrow{\text{first moment}} \) \( B_{T10}^q(t) \) tensor FF when the quark is transversely polarized

- The second moments \( \rightarrow \) gravitational FFs of the quarks
The EMFF $A_{10}(t)$ of the pion is compared with the experimental data and the lattice QCD result [1].

The gravitational FF $A_{20}(t)$ is compared with the parameterization of lattice QCD simulations at $\mu^2 = 4 \text{ GeV}^2$ [2].

$B_{T10}(t)$ and $B_{T20}(t)$ are compared with lattice QCD and the $\chi_{QM}$ results at the same scale $\mu^2 = 4 \text{ GeV}^2$.

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[1] D. Brömmel et al. [QCDSF/UKQCD], Eur. Phys. J. C 51, 335-345 (2007).
[2] D. Brömmel et al. [QCDSF/UKQCD], Phys. Rev. Lett. 101, 122001 (2008).
[3] S. i. Nam and H. C. Kim, Phys. Lett. B 700, 305-312 (2011).
Results: Kaon Generalized Form Factors

(a) \( A^{K,u}_{10}, A^{K,u}_{20}, A^{K,s}_{10}, A^{K,s}_{20} \) (this work)

(b) \( A^K_{10} = e_u A^{K,u}_{10} + e_s A^{K,s}_{10} \) (EMFF data)

(c) \( B^{K,u}_{10}, B^{K,u}_{20}, B^{K,s}_{10}, B^{K,s}_{20} \)
Spin densities in Impact Parameter Space

The density of quarks with transverse spin $\vec{s}^\perp$

$$\rho^n(\vec{b}^\perp, \vec{s}^\perp) = \frac{1}{2} \left[ A_{n0}^q(\vec{b}^\perp) - \frac{\vec{s}_i^\perp \epsilon_{ij} \vec{b}_j^\perp}{M_P} B_{Tn0}^q(\vec{b}^\perp) \right]$$

unpolarized quark $\rightarrow \vec{s}^\perp = 0$

transversely polarized quark $\rightarrow \vec{s}^\perp = (+1, 0)$
probability densities as a function of $b_y$ at fixed $b_x = 0.15$ fm

results are found to be consistent with the results of lattice QCD [1] and the $\chi$QM [2].

[1] D. Brömmel et al. [QCDSF/UKQCD], Phys. Rev. Lett. 101, 122001 (2008).
[2] S. I. Nam and H. C. Kim, Phys. Lett. B 700, 305-312 (2011).
Spin densities in Impact Parameter Space: Kaon

\[ \rho(\bar{b}^1) [\text{fm}^{-2}] \]

\[ \rho_T(\bar{b}^1) [\text{fm}^{-2}] \]

\[ \rho(\bar{b}^\perp) [\text{fm}^{-2}] \]

\[ \rho_T(\bar{b}^\perp) [\text{fm}^{-2}] \]
Spin densities in Impact Parameter Space: Kaon

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**Figure 1:**
- **Top Left:** Graph showing spin density $\rho(b^+)$ as a function of $b^y$ for $b^x = 0.15$ fm, $u$ quark.
- **Top Right:** Graph showing spin density $\rho_T(b^+)$ as a function of $b^y$ for $b^x = 0.15$ fm, $u$ quark.
- **Bottom Left:** Graph showing spin density $\rho(b^+)$ as a function of $b^y$ for $b^x = 0.15$ fm, $s$ quark.
- **Bottom Right:** Graph showing spin density $\rho_T(b^+)$ as a function of $b^y$ for $b^x = 0.15$ fm, $s$ quark.

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**Legend:**
- **BLFQ - NJL**
- **$\chi$QM**

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**Units:**
- $\rho(b^+)$ and $\rho_T(b^+)$ are measured in $[\text{fm}^{-2}]$.
Average transverse shift $\langle b_y^\perp \rangle_n$

Average transverse shift of the peak position along $b_y$ direction [1]

$$\langle b_y^\perp \rangle_n = \frac{\int d^2\vec{b} \cdot b_y^\perp \rho_n(\vec{b}^\perp, \vec{s}^\perp)}{\int d^2\vec{b}^\perp \rho_n(\vec{b}^\perp, \vec{s}^\perp)} = \frac{1}{2M_P} \frac{B^q_{Tn0}(0)}{A^q_{n0}(0)}$$

| Approach                  | $\langle b_y^\perp \rangle_1^{q/P}$ fm | $\langle b_y^\perp \rangle_2^{q/P}$ fm | $\langle b_y^\perp \rangle_1^{u/K}$ fm | $\langle b_y^\perp \rangle_2^{u/K}$ fm | $\langle b_y^\perp \rangle_1^{s/K}$ fm | $\langle b_y^\perp \rangle_2^{s/K}$ fm |
|---------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|
| BLFQ-NJL (this work)     | 0.162 $\pm$ 0.003                      | 0.131 $\pm$ 0.003                      | 0.164 $\pm$ 0.003                      | 0.141 $\pm$ 0.002                      | 0.114 $\pm$ 0.002                      | 0.114 $\pm$ 0.002                      |
| Lattice QCD [1]          | 0.151 $\pm$ 0.024                      | 0.106 $\pm$ 0.028                      | ...                                    | ...                                    | ...                                    | ...                                    |
| $\chi$QM [2]             | 0.152                                  | ...                                    | ...                                    | ...                                    | ...                                    | ...                                    |
| $\chi$QM (model I) [3]   | ...                                    | ...                                    | 0.168                                  | 0.166                                  | ...                                    | ...                                    |
| $\chi$QM (model II) [3]  | ...                                    | ...                                    | 0.139                                  | 0.100                                  | ...                                    | ...                                    |
| CCQM [4]                 | 0.090 $\pm$ 0.001                      | 0.080 $\pm$ 0.001                      | ...                                    | ...                                    | ...                                    | ...                                    |
| NJL model [5]            | ...                                    | ...                                    | 0.116                                  | 0.083                                  | ...                                    | ...                                    |

[1] D.Brömmel et al. [QCDSF/UKQCD], Phys. Rev. Lett. 101, 122001 (2008).
[2] S. i. Nam and H. C. Kim, Phys. Lett. B 700, 305-312 (2011).
[3] S. i. Nam and H. C. Kim, Phys. Lett. B 707, 546-552 (2012).
[4] C. Fanelli, E. Pace, G. Romanelli, G. Salme and M. Salmistraro, Eur. Phys. J. C 76, 253 (2016).
[5] J. L. Zhang and J. L. Ping, Eur. Phys. J. C 81, 814 (2021).
Transverse squared radius $\langle b_{\perp}^2 \rangle_q(x)$

Squared radius of the quark density [1]

$$\langle b_{\perp}^2 \rangle_q(x) = \frac{\int d^2 \vec{b}_{\perp} (\vec{b}_{\perp})^2 q(x, \vec{b}_{\perp})}{\int d^2 \vec{b}_{\perp} q(x, \vec{b}_{\perp})};$$

$$\langle b_{\perp}^2 \rangle = \sum_q e_q \frac{1}{N_q} \int_0^1 dx H^q(x, 0, 0) \langle b_{\perp}^2 \rangle_q(x)$$

![Graph showing the transverse squared radius for pions and kaons.](image)

- $\langle b_{\perp}^2 \rangle_{\pi} = 0.285 \text{ fm}^2$
- $\langle b_{\perp}^2 \rangle_{K} = 0.223 \text{ fm}^2$
- $\langle b_{\perp}^2 \rangle_{\pi}^{\exp} = 0.301 \pm 0.014 \text{ fm}^2$ [2]
- $\langle b_{\perp}^2 \rangle_{K}^{\exp} = 0.209 \pm 0.047 \text{ fm}^2$ [2]

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[1] R. Dupre, M. Guidal and M. Vanderhaeghen, Phys. Rev. D 95, 011501 (2017).

[2] M. Tanabashi et al. [Particle Data Group], Phys. Rev. D 98, 030001 (2018).
Conclusion

- We investigated the valence quark GPDs of the light pseudoscalar mesons in the BLFQ-NJL model.

- We also calculated the spin densities of the unpolarized and transversely polarized quark inside the pion and the kaon.

- Our results were found to be consistent with lattice QCD and $\chi$QM results.

Thank You