Effective Theory of the Color-Flavor-Locked Phase

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We explain how an effective theory of the CFL phase can be used to study the effect of the strange quark mass on the ground state and the excitation spectrum. We also apply the effective theory to the problem of neutrino emission from a CFL quark core inside a neutron star.

1. Introduction

What is the ground state of ordinary hadronic matter if it is compressed to baryon densities several times larger than nuclear matter saturation density? This theoretical question, which has important applications to the physics of neutron stars, has led to the discovery of several novel and unusual phases of QCD. At zero baryon density chiral symmetry is broken by a condensate of quark-anti-quark pairs. If the baryon density is large then condensation in the quark-anti-quark channel is suppressed. Instead, attractive interactions between two quarks in a color anti-symmetric wave function lead to diquark condensation and color superconductivity [1–3].

The global symmetries of color superconducting quark matter depend on the density, the number of quark flavors, and their masses. In the physically relevant case of three light quark flavors a particularly symmetric phase exists, color-flavor-locked (CFL) quark matter [4]. This phase is believed to be the true ground state of strange quark matter at very large density. The CFL phase is characterized by the order parameter

\[ \langle q_{L,s}^a C q_{L,j}^b \rangle = -\langle q_{R,s}^a C q_{R,j}^b \rangle = \phi \left( \delta_i^a \delta_j^b - \delta_i^b \delta_j^a \right) . \]

At realistic baryon densities flavor symmetry breaking due to the quark masses \( m_s \neq m_d \neq m_u \) and non-zero lepton chemical potentials will lead to distortions of the ideal CFL state [5–14]. In this contribution we show how to address this problem using the effective chiral theory of the CFL phase [15].

2. Chiral Effective Theory

For excitation energies smaller than the gap the only relevant degrees of freedom are the Goldstone modes associated with the breaking of chiral symmetry and baryon number. The interaction of the Goldstone modes is described by the effective Lagrangian

\[ L_{eff} = \frac{f^2}{4} \text{Tr} \left[ \nabla_0 \Sigma \nabla_0 \Sigma^\dagger - v^2 \partial_i \Sigma \partial_i \Sigma^\dagger \right] + \left[ C \text{Tr}(M \Sigma^\dagger) + h.c. \right] \]
Figure 1. Figure a) shows schematically how to construct fluctuations of the CFL order parameter that have the quantum numbers of the $K^0$. Fig. b) shows the leading order contribution to the $\langle j^L_{\mu}j^R_{\mu}\rangle$ correlation function which determines $f_\pi^2$ in the CFL phase. The open and solid squares are insertions of $\langle \bar{\psi}_L \psi_L \bar{\psi}_R \psi_R \rangle$ and $\langle \bar{\psi}_L \psi_L \bar{\psi}_R \psi_R \rangle$. Note that the graph involves $XY^\dagger X^\dagger Y \sim \Sigma \Sigma^\dagger$.

$$K^0 \sim \epsilon^{abc} \epsilon^{ade} (\bar{u}_b^b C \bar{s}_c^c)(d^d_L C u^e_L)$$

Here $\Sigma = \exp(i\phi^a \lambda^a / f_\pi)$ is the chiral field, $f_\pi$ is the pion decay constant and $M$ is the mass matrix. The field $\phi^a$ describes pion, kaon, and eta collective modes in the CFL phase. The microscopic nature of these modes is not relevant for our discussion. It is nevertheless useful to have a physical picture of these excitations, see Fig. 1a. In the CFL phase the flavor and color orientation of the left handed condensate $X_i^a = \epsilon_{ijk} \epsilon^{abc} \langle (\bar{\psi}_L)^b C(\psi_L)^c \rangle$ are locked, $X_i^a \sim \delta_i^a$. The same is true for the right handed condensate $Y_i^a = \epsilon_{ijk} \epsilon^{abc} \langle (\bar{\psi}_R)^b C(\psi_R)^c \rangle$. Low energy excitations of the CFL phase correspond to small fluctuations of $X$ and $Y$ around their equilibrium values. Because color gauge invariance is broken colored excitations acquire a mass via the Higgs mechanism. The true low energy modes are color neutral fluctuations of $X$ relative to $Y$, parameterized by $\Sigma = XY^\dagger$. For example, a low energy mode with the quantum numbers of the $K^0$ is given by $K^0 \sim \epsilon^{abc} \epsilon^{ade} (\bar{u}_b^b C \bar{s}_c^c)(d^d_L C u^e_L)$.

At very high baryon density the effective coupling is weak and the coefficients $f_\pi^2, C, A_i$ can be determined in perturbative QCD. The pion decay constant is given by (see Fig. 1b)

$$f_\pi^2 = \frac{21 - 8 \log(2)}{18} \frac{(p_F^2)}{2\pi^2} .$$

(3)

The coefficient $C$ is related to instantons and was computed in [17]. At large baryon density $C \sim (\Lambda_{QCD}/p_F)^8$ and the linear mass term is not important. The coefficients $A_i$...
of the quadratic mass terms are given by \[16,18\]

\[ A_1 = -A_2 = \frac{3\Delta^2}{4\pi^2}, \quad A_3 = 0. \] (4)

Finally, the covariant derivative \(\nabla_\mu \Sigma\) was determined in \[9\]. The temporal component \(\nabla_0 \Sigma\) contains the quark mass matrix,

\[ \nabla_0 \Sigma = \partial_0 \Sigma + i \left( \frac{MM^\dagger}{2p_F} \right) \Sigma - i \Sigma \left( \frac{M^\dagger M}{2p_F} \right). \] (5)

We note that equ. (5) is completely fixed by the symmetries of the theory. In addition to that, using the power counting proposed in \[9\] we find that the low energy constants \(A_i\) are of natural magnitude. This suggests that even though equ. (3-5) were obtained using weak coupling methods, the results are more general.

3. Kaon Condensation

The effective chiral lagrangian equ. (2) determines the masses and interactions of the CFL Goldstone bosons. In addition to that, the effective theory also determines the structure of the ground state as a function of the quark masses and external fields that couple to the Goldstone modes \[9,10\]. The effective potential is given by

\[ V_{\text{eff}}(\Sigma) = \frac{f_\pi^2}{2} \text{Tr} \left[ X_L \Sigma X_R \Sigma^\dagger \right] + A \left[ \text{Tr}(M \Sigma^\dagger) \text{Tr}(M \Sigma^\dagger) - \text{Tr}(M \Sigma^\dagger M \Sigma^\dagger) \right] + \text{h.c.}, \] (6)

with \(X_L = MM^\dagger/(2p_F)\) and \(X_R = M^\dagger M/(2p_F)\). For small \(M\) the minimum of \(V_{\text{eff}}\) is at \(\Sigma = 1\). If the mass is increased a number of different phases can occur. In practice we are mostly interested in the case \(m_s \gg m_d \simeq m_u\). In this case there is an instability towards kaon condensation. The simplest ansatz for a \(K^0\) condensed ground state is given by \(\Sigma = \exp (i\alpha \lambda_4)\). With this ansatz the vacuum energy is

\[ V(\alpha) = -f_\pi^2 \left( \frac{1}{2} \left( \frac{m_s^2 - m_u^2}{2p_F} \right)^2 \sin(\alpha)^2 + (m_K^0)^2 \right)^2 (\cos(\alpha) - 1), \] (7)

where \((m_K^0)^2 = (4A/f_\pi^2)m_{u,d}(m_{u,d} + m_s)\) is the \(O(M^2)\) kaon mass in the limit of exact isospin symmetry. Minimizing the vacuum energy we obtain \(\alpha = 0\) if \(m_s^2/(2p_F) < m_K^0\) and \(\cos(\alpha) = (m_K^0)^2/\mu_s^2\) with \(\mu_s = m_s^2/(2p_F)\) if \(\mu_s > m_K^0\). The hypercharge density is given by

\[ n_Y = f_\pi^2 \mu_s \left( 1 - \frac{(m_K^0)^4}{\mu_s^4} \right). \] (8)

We observe that within the range of validity of the effective theory, \(\mu_s < \Delta\), the hypercharge density satisfies \(n_Y < \Delta p_F^2/(2\pi^2)\). This means that the number of condensed kaons is bounded by the number of particles contained within a strip of width \(\Delta\) around the Fermi surface. It also implies that near the unlocking transition, \(\mu_s \sim \Delta\), the CFL state is significantly modified. In this regime, of course, we can no longer rely on the effective theory and a more microscopic calculation is necessary.
4. Summary and Outlook

We have studied the ground state of CFL quark matter for non-zero quark masses. We have argued that there is a new scale \( m_s^2/(2p_F) \ll (\Delta/p_F) \) which corresponds to the onset of kaon condensation. For larger values of the strange quark mass, \( m_s^2/(2p_F) \sim 1 \), color-flavor-locking breaks down. Using weak coupling methods, we can determine the critical \( m_s \) for kaon condensation to occur. We find \( m_s|_{crit} \simeq 3(m_{u,d}\Delta^2)^{1/3} \). This result suggests that for values of the strange quark mass and the gap that are relevant to compact stars CFL matter is likely to support a kaon condensate.

The effective lagrangian equ. (4) can also be used to compute many physical properties of the CFL phase. In recent work [19,20] we considered the specific heat, neutrino emission and neutrino scattering in the CFL phase. At energies \( E < \Delta \) these processes are dominated by Goldstone modes. We find that the long term emissivity of the CFL phase is very low, and that the CFL phase contributes little to the long term cooling of a neutron star with a CFL core. Interaction in the CFL phase are important in determining the neutrino opacity at early times [19].

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