The effect of the nuclear equation of state on the direct flow in heavy ion collision reaction by using quantum molecular dynamics model

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Abstract. This work concentrated on the effect of the nuclear equation of state on the direct flow ($v_1$) in heavy ion collision reaction by using the quantum molecular dynamics (QMD) model. In addition, the proton direct flow is a function of the centrality transverse component of the 4-velocity ($u_{t0}$) at incident energy 0.09 A GeV and impact parameter between 0.25 and 0.45 fm by using the nuclear equation of state (soft and hard EoS), computed and compared with FOPI experiments. The calculated results show that the proton direct flow as a function of $u_{t0}$ is dependent appreciably upon the nuclear equation of state (EoS). The proton direct flow as a function of $u_{t0}$ is calculated with soft EoS that is consistent with the experimental data and the nuclear equation of state. This can describe the behavior of matter in high density and high pressure in soft EoS, whereas the discrepancy between the results obtained with hard EoS and experimental data that goes up by increasing the centrality transverse component $t$ of the 4-velocity.

1. Introduction

In Relativistic Heavy-Ion collisions at high energy as the specific method for the studied behavior of nuclear matter at high density, it is an important and challenging way to examine and exhibit the behavior of nuclear matter at the high temperature and high density. Furthermore, it is also useful in understanding astrophysical phenomena such as the Evolution of the early universe, Neutron star simulator and an Evolution of Supernova [1]. Each as abovementioned is all about the properties of nuclear matter under the extreme conditions. Additionally, it is worthwhile in understanding astrophysical phenomena such as the evolution of the early universe, the collapsed core of a constellation and the evolution of supernova. Each as aforesaid is all about the properties of nuclear matter under the extreme conditions [2].

According to the previous research [3], it measured elliptic flow in a collision reaction of $^{197}$Au + $^{197}$Au at 0.09 - 1.49 A GeV. This reveals that it is the first occurrence of an elliptic
Another study experimented the elliptic flow from the collision reaction of $^{40}_{20}$Ca + $^{40}_{20}$Ca at 30-100 A MeV by using the isospin dependent transport model [4]. This experiment indicates that the size of the flow depends on the nuclear equation of state and nucleon-nucleon scattering cross section. Also, the result of this experiment is associated with Soft EoS. Another research on anisotropy [5] studies anisotropic flow from a reaction of $^{197}_{79}$Au + $^{197}_{79}$Au at 40-150 A MeV. Researchers find that the direct flow of the participle having $z < 2$ at 100 A MeV will be promising, compared with the result from the FOPI experiment. Nonetheless, this direct flow will changes and shows the negative state at 50-60 A MeV. When it is compared with reaction level, it can be measured after heavy ions collision at a high energy level. An amount of Hadron is explained as Fourier equation. Consequently, the research “The effect of nuclear equation of state on the Direct flow in heavy ion collision reaction by using quantum molecular dynamics model” aims at investigating the influence on nuclear state affecting direct flow ($v_1$) from collision reaction of $^{197}_{79}$Au + $^{197}_{79}$Au at 0.09 A GeV using impact parameter ($b_0$) at 0.25-0.45, and quantum molecular dynamics model. Subsequently, it is compared with soft EoS and hard EoS. Finally, the result of investigating is theoretically calculated and compared with the result from FOPI experiments [5].

2. Theories
2.1. The Quantum Molecular Dynamics(QMD) Model
The nuclear equation of state has described the possibility of compressing nuclear matter. The quantum molecular dynamics in which each nucleon was represented by a coherent state of the form

$$\psi(\mathbf{r}, \mathbf{p}_0, t) = \frac{exp\left[i\mathbf{p}_0 \cdot (\mathbf{r} - \mathbf{r}_0)^2\right]}{(2\pi L)^{3/2}} \cdot exp\left[-(\mathbf{r} - \mathbf{r}_0)^2/4L\right],$$

(1)

where $\mathbf{r}_0$ is the center of a Gaussian wave pocket and $L = 1.08 \text{ fm}^2$ is the width of the wave pocket. Consequently, the density of the system with $N$ nucleons in coordinate space is given as follows:

$$\rho(\mathbf{r}, t) = \sum_{i=1}^{N} \frac{1}{(2\pi L)^{3/2}} \cdot exp\left[-(\mathbf{r} - \mathbf{r}_{i0})^2/2L\right].$$

(2)

The time evolution of the $N$-body distribution is determined by the motion of the centroid of Gaussian $\{\mathbf{r}_{i0}, \mathbf{p}_{i0}\}$, which are propagated by the Poisson brackets,

$$\dot{\mathbf{r}}_{i0} = \{\mathbf{p}_{i0}, H\},$$

(3)

$$\dot{\mathbf{p}}_{i0} = \{\mathbf{r}_{i0}, H\},$$

(4)

with $H$ is the nuclear Hamiltonian

$$H = \sum_{i} \sqrt{\mathbf{p}_{i0}^2 + m_i^2} + \sum_{i<j} (U_{ij}^{Str} + U_{ij}^{Coul}).$$

(5)

Here $U_{ij}^{Str}$ is a nuclear mean field, $U_{ij}^{Coul}$ is the Coulomb interaction.

The strength of the nuclear compression is quoted normally in terms of the incompressibility by value constant $K$ (compressibility). A soft EoS is represented by a value of $K = 200$ MeV, while a hard EoS is represented by the value of $K = 380$ MeV as below:
\[ K = 9 \rho^2 \frac{\partial}{\partial \rho^2} \left( \frac{E}{A} \right). \] (6)

**Table 1.** Showing parameter in equation (6) for the soft and hard equation of state (EoS).

| K (MeV) | α  | β  | γ   | EoS   |
|--------|----|----|-----|-------|
| 200    | -356 | 303 | 7/6 | Soft  |
| 380    | -124 | 70.5| 2   | Hard  |

2.2. **Direct flow**

The phenomenon of collective flow\[6\] could be quantitatively described in terms of anisotropies of the azimuthal emission pattern, expressed by a Fourier series

\[ \frac{dN}{d\phi} \propto [1 + 2v_1 \cos(\phi) + 2v_2 \cos(\phi) + ...], \] (7)

whereas \( \phi \) is the azimuthal angle of the outgoing particle with respect to the reaction plane. The first order Fourier coefficient, \( v_1 \) describes the collective sideward ‘directed flow’ as shown in figure 1 below:

![Figure 1. Showing the direct flow of proton (v₁) on rectangular coordinate x-y system.](image-url)
2.3. The centrality transverse component $t$ of the 4-velocity

The 4-velocity is 4-vector replaced by the magnitude of 4-velocity and the $4^{th}$ dimension is also the speed of light explained by

$$ u_{t0} = \frac{u_t}{u_p}, \quad (8) $$

where $u_{t0}$ is the velocity of reference frame on the transverse coordinates, related to

$$ u_t = \beta_t \gamma_t, \quad (9) $$

$$ \beta_t = \frac{P_t}{E}, \quad (10) $$

$$ \gamma_t = \frac{1}{\sqrt{1-\beta_t^2}}, \quad (11) $$

and $u_t$ can be proved by $\gamma_t$ and $\beta_t$ which $\beta_t$ is the velocity value of transverse plane and where $u_p$ is the velocity of reference frame on the reaction coordinates, related to

$$ u_p = \beta_p \gamma_p, \quad (12) $$

$$ \beta_p = \frac{P_p}{E}, \quad (13) $$

$$ \gamma_p = \frac{1}{\sqrt{1-\beta_p^2}}, \quad (14) $$

and $u_p$ can be verified by $\gamma_p$ and $\beta_p$ which $\beta_p$ is the velocity value of reaction plane.

Both of equation have $\gamma$, that is the Lorenz factor, which helps to connect two experiments frame of Physics particle system together.

3. Method

1. Simulation collision at incident energy 0.09 A GeV, within the Quantum Molecular Dynamics (QMD) model.
2. Calculate direct flow ($v_1$) of the proton as a function of $u_{t0}$ from QMD model with

$$ v_1 = \left< \frac{P_x}{P_t} \right> = \langle \cos(\phi) \rangle, \quad (15) $$

$$ \frac{1}{N} \frac{dN}{d\phi} = \frac{1}{2\pi} \left\{ 1 + \sum_n 2v_n \cos(n(\phi - \Psi_R)) \right\}. \quad (16) $$

3. Compute and compare the proton production direct flow ($v_1$) as a function of the centrality transverse component $t$ of the 4-velocity ($u_{t0}$) with FOPI data.
4. Results and Discussion

Figure 2 shows the direct flow of proton ($v_1$) as a function of the centrality transverse component $t$ of the 4-velocity ($u_{t0}$) at incident energy $0.09$ A GeV and impact parameter between $0.25$ and $0.45$ from collision by using the equation of state (EoS; soft and hard). The results of the theoretical calculation are soft EoS and tend to be consistent with the FOPI data. This indicates that the soft EoS should be taken into account in the theoretical simulation of the proton direct flow in heavy ion collisions in order to reasonably describe the experimental data.

**Figure 2.** shows the direct flow of proton ($v_1$) as a function of the centrality transverse component $t$ of the 4-velocity ($u_{t0}$) at incident energy $0.09$ A GeV and impact parameter between $0.25$ and $0.45$ from collision by using the equation of state (EoS; soft and hard).

| $u_{t0}$ | $v_1$  | $v_{1error}$  |
|---------|--------|---------------|
| 0.603   | 0.0506 | 3.437x10^{-6} |
| 0.678   | 0.06194| 3.471x10^{-6} |
| 0.753   | 0.0770 | 3.586x10^{-6} |
| 0.828   | 0.08179| 3.820x10^{-6} |
| 0.903   | 0.08900| 4.176x10^{-6} |
| 0.978   | 0.09674| 4.667x10^{-6} |
| 1.053   | 0.10480| 5.335x10^{-6} |
| 1.128   | 0.11500| 6.212x10^{-6} |
| 1.203   | 0.12530| 7.468x10^{-6} |
| 1.277   | 0.13540| 9.128x10^{-6} |
| 1.352   | 0.13970| 1.150x10^{-5} |
| 1.427   | 0.14790| 1.446x10^{-5} |
| 1.502   | 0.14760| 1.900x10^{-5} |
| 1.577   | 0.15560| 2.569x10^{-5} |
| 1.652   | 0.15540| 3.474x10^{-5} |
| 1.727   | 0.15650| 4.947x10^{-5} |
| 1.802   | 0.15030| 6.563x10^{-5} |

**Table 2.** showing the results of hard EoS

| $u_{t0}$ | $v_1$  | $v_{1error}$  |
|---------|--------|---------------|
| 0.603   | 0.04033| 3.311x10^{-6} |
| 0.678   | 0.04947| 3.400x10^{-6} |
| 0.753   | 0.05256| 3.567x10^{-6} |
| 0.828   | 0.05860| 3.859x10^{-6} |
| 0.903   | 0.06615| 4.283x10^{-6} |
| 0.978   | 0.07143| 4.903x10^{-6} |
| 1.053   | 0.08201| 5.714x10^{-6} |
| 1.128   | 0.08494| 6.916x10^{-6} |
| 1.203   | 0.09432| 8.450x10^{-6} |
| 1.277   | 0.09499| 1.059x10^{-5} |
| 1.352   | 0.09945| 1.373x10^{-5} |
| 1.427   | 0.10700| 1.811x10^{-5} |
| 1.502   | 0.10900| 2.256x10^{-5} |
| 1.577   | 0.10700| 3.528x10^{-5} |
| 1.652   | 0.10850| 4.894x10^{-5} |
| 1.727   | 0.10780| 7.060x10^{-5} |
| 1.802   | 0.09763| 1.036x10^{-5} |

**Table 3.** showing the results soft EoS
Tables 2 and 3 are results of theoretical calculation on flow of $u_{t0}$ function obtained from model of Au+Au collision at the energy 0.09 A MeV with the quantum molecular dynamical (QMD) model.

5. Conclusions
The reaction is simulated by using the quantum molecular dynamics (QMD) model. The result of the direct flow of proton ($v_1$) as a function of the centrality transverse component of the 4-velocity ($u_{t0}$) is dependent appreciably upon the nuclear equation of state (EoS). The proton direct flow as a function of $u_{t0}$ is calculated with soft EoS, which is consistent with the experimental data and the nuclear equation of state. This can describe the behavior of matter in high density and a high pressure in soft EoS, whereas a discrepancy between the results obtained with hard EoS and experimental data going up by increasing the centrality transverse component $t$ of the 4-velocity.

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