High Energy Hadron Spin Flip Amplitude$^{1, 2}$

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Abstract—The high energy part of the hadron spin flip amplitude is examined in the framework of the new high energy general structure (HEGS) model of the elastic hadron scattering at high energies. The different forms of the hadron spin flip amplitude are compared in the impact parameters representation. It is shown that the existing experimental data of the proton-proton and proton-antiproton elastic scattering at high energy in the region of the diffraction minimum and at large momentum transfer give support in the presence of the energy-independent part of the hadron spin flip amplitude with the momentum dependence proposed in the works by Galynskii—Kuraev.

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1. INTRODUCTION

The spin effects play very often the touchstone role for many different theoretical approaches. Especially, it applies to the hadron-hadron elastic processes. The famous experiments carried out by A. Krish at the ZGS to obtain the spin-dependent differential cross sections [1, 2] and the spin correlation parameter $A_N$ [3] and at the AGS [4] to obtain the spin correlation parameter $A_N$ showed the significant spin effects at large momentum transfer. Note that the spin-flip amplitudes, determined by the non-leading Regge exchange, give the large contribution only at small momentum transfer, as the massive non-leading Reggeons have a large slope and decrease faster at large $t$. Of course, many questions arise about the energy dependence of the spin flip amplitudes and the relative phase between the spin non-flip and spin-flip amplitudes. Now there are many different models for the description of the elastic hadrons scattering amplitude [5, 6]. They lead to different predictions of the structure of the scattering amplitude at high energies. The diffraction processes at very high energies, especially in the TeV energy region, are simplified by the asymptotical regime but can display complicated features [7, 8].

According to the standard opinion, the hadron spin-flip amplitude is connected with the quark exchange between the scattering hadrons, and at large energy it can be neglected. Some models, which take into account the non-perturbative effects, lead to the non-dying hadron spin-flip amplitude [9–13]. Another complicated question is related to the difference in phases of the spin-non-flip and spin-flip amplitude.

The description of the high energy processes requires using some unitarization procedures of the Born scattering amplitude. They are related to the different forms of the summation of some sets of diagrams of the tree approximation. The unitarization leads to the asymptotic unitarity bound connected with the so-called Black Disk Limit (BDL), which can leads to different saturation effects [14]. In the partial wave language, we need to sum many different waves with $l \to \infty$ and this leads to the impact parameter representation [15] converting the summation over $l$ into integration over $b$. "$^{1}$ The article is published in the original.

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The differential cross section is
\[ \frac{d\sigma}{dt} = \frac{2\pi}{s^2} \left| \Phi_1^2 + \Phi_2^2 + \Phi_3^2 + 4\Phi_4^2 \right|^2 . \] (3)
The total helicity amplitudes can be written as
\[ \Phi_1(s,t) = \Phi_1^h(s,t) + \Phi_1^{em}(s,t) e^{i\phi(s,t)} , \] (4)
where \( \Phi_1^h(s,t) \) is the pure strong interaction of hadrons, \( \Phi_1^{em}(s,t) \) is the electromagnetic interaction of hadrons and \( \phi(s,t) \) is the electromagnetic–hadron interference phase factor [18, 19]. The corresponding spin-correlation values will be
\[ A_N \frac{d\sigma}{dt} = -\frac{4\pi}{s^2} [ \text{Im}(\Phi_1(s,t) + \Phi_2(s,t)) + \Phi_3(s,t) - \Phi_4(s,t)] \] (5)
and
\[ A_{NN} \frac{d\sigma}{dt} = \frac{4\pi}{s^2} [ \text{Re}(\Phi_1(s,t)\Phi_2^*(s,t)) - \Phi_3^*(s,t)\Phi_4(s,t) + \Phi_4^*(s,t)\Phi_3(s,t)] . \] (6)

The \( s \)-channel factorization together with the experimental information about the spin-correlations effects at high energy and small momentum transfer in the proton–proton elastic scattering suggests that the double helicity flip as a second-order effect and, consequently, the amplitudes \( \Phi_3^h(s,t) \) and \( \Phi_4^h(s,t) \) can be neglected. Furthermore, when the exchange of Regge poles has natural parity, we have for the spin-non-flip amplitudes [20] \( \Phi_3^{h+}(s,t) = \Phi_4^{h+}(s,t) \).

Neglecting the \( \Phi_3(s,t) - \Phi_4(s,t) \) contribution the spin correlation parameter \( A_N(s,t) \) can be written taking into account the phases of separate amplitudes
\[ A_N(s,t) \frac{d\sigma}{dt} = -\frac{4\pi}{s^2} [ (\Phi_{nf}(s,t)) (\Phi_{gf}(s,t)) \sin(\phi_{nf}(s,t) - \phi_{gf}(s,t)) ] . \] (7)
where \( \phi_{nf}(s,t), \phi_{gf}(s,t) \) are the phases of the spin non-flip and spin-flip amplitudes. It is clearly seen that despite the large spin-flip amplitude the analyzing power can be near zero if the difference of the phases is zero in some region of momentum transfer. The experimental data at some point of the momentum transfer show the energy independence of the size of the spin correlation parameter \( A_N(s,t) \) (see Fig. 1). Hence, the small value of the \( A_N(s,t) \) at some \( t \) (for example, very small \( t \)) does not serve as a proof that it will be small in other regions of momentum transfer.

3. ELASTIC NUCLEON SCATTERING (HEGS MODEL)

In some early models of the elastic hadron scattering the hadron was regard as the whole particle and the diffraction occurred on the surface [22, 23] of the hadron. The parton structure of the hadron can be related to the hadron form factors and the elastic scattering amplitude is proportional to these form factors [24]. Some models take into account the parton structure of the hadron as separate interactions of the gluons and quarks [25]. A more complicated picture of the hadron structure appears with introducing the non-forward parton structure—general parton distributions (GPDs) and transfer momentum distributions (TMD). The different moments of GPDs allow us to calculate the different hadron form factors, such as Compton form factors \( (R_c(t), R_T(t), R_A(t)) \), electromagnetic form factors \( (F_1(t), F_2(t)) \), and so-called gravimagnetic form factors \( A(t), B(t) \) [26–28].

Let us use the obtained momentum transfer dependence of GPDs to calculate the electromagnetic and gravimagnetic form factors of the nucleons as the first and second moments of GPDs [29, 30]. The obtained form factors are related to the charge and matter distributions. By fixing the parameters of the obtained form factors the new High Energy General Structure.
(HEGS) model of the elastic proton-proton and proton-antiproton scattering was proposed [31, 32].

The (HEGS) model [32] gives a quantitative description of the elastic nucleon scattering at high energy with only 5 fitting high energy parameters. A successful description of the existing experimental data by the model shows that the elastic scattering is determined by the generalized structure of the hadron. The model leads to a coincidence of the model calculations with the preliminary data at 8 TeV. We found that the standard eikonal approximation [33] works perfectly well from $\sqrt{s} = 9$ GeV up to $\sqrt{s} = 8$ TeV.

In the model the Born term of the elastic hadron amplitude is determined

$$F^\text{Born}_h(s,t) = h_1 F_1^2(t) F_h(s,t) (1 + r_1/\sqrt{s})^3$$

where $F_1(s,t)$ and $F_h(s,t)$ has the standard Regge form

$$F_1(s,t) = s^{\alpha_1} e^{B(t)h_1}; \quad F_h(s,t) = s^{\alpha_2} e^{B(t)h_2}.$$  \hspace{1cm} (9)

The terms proportional to $r_1/\sqrt{s}$ and $r_2/\sqrt{s}$ take into account some possible non-asymptotic contributions.

The size and the energy and momentum transfer dependence of the real part of the elastic scattering amplitude are determined by the complex energy $\hat{s} = s \exp(-i\pi/2)$. Hence, the model does not introduce some special functions or assumptions for the real part of the scattering amplitude. Note that the role of the real part is especially important at low momentum transfer (where the differential cross sections are determined by the Coulomb-hadron interference effects) and in the region of the diffraction minimum (where the imaginary part of the scattering amplitude has the zero, and the size of the diffraction minimum is determined by the real part of the scattering amplitude and the contribution of the spin-flip part of the scattering amplitude).

The hard pomeron contribution was analyzed in the framework of the model [34]. It was shown that such a contribution is very small in the elastic hadron scattering and is not felt in the fitting procedure.

The final elastic hadron scattering amplitude is obtained after unitarization of the Born term. So, first, we have to calculate the eikonal phase. In the most favorable eikonal unitarization scheme the eikonal phase corresponds to the Born term of the scattering amplitude eq.(1) and in the common case corresponds to the spin-dependent potential.

The model description of the differential cross sections in such a huge energy region is reflected brightly in the description of the region of the diffraction minimum (see Fig. 2). In most part, the form and energy dependence of the diffraction region is determined by the real part of the scattering amplitude and possible contributions of the spin-flip amplitude. Note that in the HEGS model only the Born term is determined and the complicated diffractive structure is obtained only after the unitarization procedure.

4. IMPACT OF THE SPIN-FLIP CONTRIBUTION

Usually, one makes the assumptions that the imaginary and real parts of the spin-non-flip amplitude have the exponential behavior with the same slope, and the imaginary and real parts of the spin-flip amplitudes, without the kinematic factor $\sqrt{t}$ [35], are proportional to the corresponding parts of the non-flip amplitude. For example, in [36] the spin-flip amplitude was chosen in the form

$$F_{h'} = \sqrt{-t} m_\rho h_{t'} F_{h''}.$$  \hspace{1cm} (11)

That is not so as regards the $t$ dependence shown in Ref. [37], where $F_{h''}$ is multiplied by the special function dependent on $t$. Moreover, one mostly takes the energy independence of the ratio of the spin-flip parts to the spin-non-flip parts of the scattering amplitude. All this is our theoretical uncertainty [38–40].

In [41] on the basis of generalization of the constituent-counting rules of the perturbative QCD the pro-
ton current matrix elements $J^{\pm s s}_{p}$ for the full set of spin combinations corresponding to the number of the spin-flipped quarks was calculated. It leads to part of the spin-flip amplitude

$$F_{\gamma}^{\pm s s}(t) \sim \sqrt{-t} \left( \frac{4}{9} m_{f}^{2} \right) \sqrt{-t} \left( \frac{4}{9} m_{p}^{2} \right) \sqrt{-t} \left( \frac{4}{9} m_{p}^{2} \right).$$

(12)

Hence, such an amplitude will give large contributions at large momentum transfer. In the HEGS model the calculations are extended up to $-t = 15 \text{ GeV}^2$, and we added the small contribution of the asymptotically independent energy part of the spin-flip amplitude with the kinematical factor Eq. (12). So, the form of the spin-flip amplitude is determined as

$$F_{\gamma}(s,t) = h_{\gamma} q^{3} F_{\gamma}(t) e^{-b_{\gamma} q^{2}}.$$  

(13)

Of course, at lower energy we need to take into account the energy dependence parts of the spin-flip amplitudes. However, it requires including in our examination of additional polarization data and the contributions of the non-leading Reggeons, which essentially complicate the picture. Now it is beyond the scope of this paper. Such a contribution can be made in future works.

It is interesting that the spin-flip amplitude in the form (13) can be compared with the spin-flip amplitude with the standard kinematical factor $\sqrt{t}$ but with the Gaussian form of the slope

$$F_{\gamma}(s,t) = h_{\gamma} q F_{\gamma}(t) e^{-b_{\gamma} q^{2}}.$$  

(14)

Let us compare the spin-flip amplitudes (13) and (14) in the impact parameter representation

$$F_{\gamma}(b,t) = \int dq J_{\gamma}(q,b) F_{\gamma}(s,q).$$  

(15)

The spin-flip amplitudes with the standard kinematical factor, Eq. (13), and with the kinematical factor, Eq. (12), are compared with the spin-flip amplitude, Eq. (14), on Fig. 3a. It can be seen that the standard spin-flip amplitude has more peripheral interactions.

A more obvious result can be obtained by comparing of these amplitude multiplied by the impact parameter $b$ and with the same sizes (we divide the standard amplitude by factor 2 and multiply amplitude Eq. (14) by the same factor). The results present in Fig. 3b. The impact dependence of the amplitudes, Eq. (13), and, Eq. (14), has a small difference. However, the impact dependence of the standard spin flip amplitude with the kinematical factor $\sqrt{t}$ has an essentially different form. It has a more peripheral origin.

As we have already noted that the size of the spin-flip contributions is bounded by the size of the differential cross sections at diffraction minimum of the differential cross sections. In Fig. 2, the model calculations are compared for the energy where the diffraction dip has the minimum value. A similar situation for the proton-proton cross sections occurs at $s = 30 \text{ GeV}^2$ where the real part change its sign at $t = 0$. The model calculations describe the form of the diffraction minimum at this energy very well. Moreover, the model sufficiently well reproduces the energy dependence of the differential cross section in the region of the diffraction dip. Note that in the HEGS model the real part of the elastic scattering amplitude is determined by the complex $\hat{s}$ only. So Fig. 2 shows that the contribution of the spin-flip amplitude is essentially small in this region of the momentum transfer. However, the contributions of the taken form of the energy independent spin-flip amplitude, Eq. (13), allow one to describe the differential cross sections at large momentum transfer. The experimental data on the differential cross section of the elastic proton-proton scattering at large momentum transfer show the small energy dependence (see Fig. 4). The HEGS model calculations reproduce a such small energy dependence of the differential cross sections at large $t$ (see Fig. 4).
Now let us calculate in the framework of the HEGS model the spin correlation parameter \( A_N(s, t) \) in the region of the diffraction minimum. The comparison with the existing experimental data at higher energies, where the experimental data are available, \( p_L = 100 \text{ GeV}, \ p_L = 200 \text{ GeV}, \ p_L = 300 \text{ GeV} \) are shown in Fig. 5. In the last case, the diffraction minimum has a large dip. We can see that both amplitudes, Eq. (13), and, Eq. (14), give a similar picture and qualitatively agree with the existing experimental data. However, for a quantitative description more new more precise experimental data at high energies are needed.

![Fig. 4. Differential cross sections of the proton-proton elastic scattering at large momentum transfer (hard line- the calculations at \( \sqrt{s} = 19.4 \text{ GeV} \), dashed line—the calculations at \( \sqrt{s} = 52.8 \text{ GeV} \)).](image)

![Fig. 5. The spin correlation parameter \( A_N(s, t) \) for the elastic proton proton scattering at \( p_L = 100 \text{ GeV}, \ p_L = 200 \text{ GeV}, \ p_L = 300 \text{ GeV} \). (hard line—model calculations with the spin-flip amplitude, Eq. (13), dashed line—the model calculations with the spin-flip amplitude, Eq. (14), points—the experimental data [42–44].](image)

![Fig. 6. The same as in Fig. 5 (additional points line—the calculation with the spin-flip amplitude, Eq. (13), with the standard factor \( q(t) \) (a) (right panel) at \( \sqrt{s} = 50 \text{ GeV} \), (b) (left panel) at \( \sqrt{s} = 500 \text{ GeV} \).](image)
Our predictions for the spin correlation parameter $A_N$ are presented in Fig. 5 for two energies $\sqrt{s} = 50$ GeV and $\sqrt{s} = 500$ GeV and for all three examined spin-flip amplitudes. We can see that the total behavior is practically the same for all spin-flip amplitudes. The difference in the size of $A_N$ is presented more remarkably at large momentum transfer.

5. CONCLUSIONS

The existing experimental data on the elastic hadron scattering at high energies and large momentum transfer show a small energy dependence of the differential cross sections. In the framework of the Donnachie–Landshoff model [45] such an effect is explained by the energy independent Odderon contribution. In our HEGS model, which describes the maximum experimental data in a wide energy region from $\sqrt{s} = 9$ GeV to $\sqrt{s} = 8$ TeV with minimum fitting parameters, the odderon has the same intercept as the standard soft Pomeron. Taking into account the form of the spin-flip amplitude, proposed in the works of Galynskii–Kuraev [41], the energy independence of the differential cross sections at large momentum transfer is explained by the contribution of the energy independent part of the spin-flip amplitude. The obtained size and momentum transfer dependence of the spin-flip amplitude allow one to describe the differential cross sections in the region of the diffraction minimum and at large momentum transfer.

We show that such a spin-flip amplitude is related to the interaction at a small radius in the hadron. It is found, that such behavior can be modeled by the spin-flip amplitude with the standard kinematical factor proportional to $\sqrt{s}$, but with the Gaussian form of the slope. The spin correlation parameter $A_N(s,t)$ calculated in the framework of the model does not contradict the existing experimental data at high energies. We hope that the forward experiments at a future accelerator can give valuable information for the improvement of our theoretical understanding of the spin-dependent hadron interactions. This is especially true for future experiments at NICA with a polarized beam and target.

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