On some weak-gravitational effects

Jian-Qi Shen

E-mail address: jqshen@coer.zju.edu.cn
Zhejiang Institute of Modern Physics and Department of Physics, Zhejiang University, Spring Jade,
Hangzhou 310027, P. R. China
(December 24, 2018)

ABSTRACT

Some physically interesting weak-gravitational effects and phenomena are reviewed and briefly discussed: particle geometric phases due to the time-dependent spin-rotation couplings, non-inertial gravitational wave in rotating reference frame, hyperbolic geometric quantum phases and topological dual mass as well.

I. INTRODUCTION

On the basis of the theoretical and experimental works concerning the spin-rotation coupling by Mashhoon et al. [1,2], we investigate some relativistic quantum gravitational effects associated with gravitomagnetic fields: the geometric quantum phase factor of a spinning particle interacting with time-dependent gravitomagnetic fields and the non-inertial gravitational wave in time-dependent rotating reference frame. By means of neutron-gravity interferometry experiment, this geometric phase factor due to time-dependent spin-rotation coupling can be applied to obtaining information on small fluctuations of Earth’s rotating frequency [3]. Mashhoon’s spin-rotation coupling is a static case, while the interaction of the intrinsic spin of a particle with the non-inertial gravitational wave is its mobile realization. With the improvements in precise-measuring instruments, particularly the laser-interference technology, it becomes possible for investigating quantum mechanics in weak-gravitational fields, which provides tests of Einstein theory of general relativity in quantum regime.

Both geometric phases [4,5] of wave function in Quantum Mechanics and gravitomagnetic charge (topological dual charge of mass) [6–9] in the general theory of relativity reveal nature’s geometric or global properties. Differing from the dynamical phase, geometric phase depends only on the geometric nature of the pathway along which the quantum system evolves [10,11]. Here we show the existence of the hyperbolic geometric quantum phase that is different from the ordinary trigonometric geometric quantum phase [12]. Gravitomagnetic charge (dual mass) is the gravitational analogue of magnetic monopole in Electrodynamics [13]; but, as we will show here, it possesses more interesting and significant features, e.g., it may constitute the dual matter that has different gravitational properties compared with mass. In order to describe the space-time curvature due to the topological dual mass, we construct a dual Einstein’s tensor. Further investigation shows that gravitomagnetic potentials caused by dual mass are respectively analogous to the trigonometric and hyperbolic geometric phase. The study of the geometric phase and dual mass provides a valuable insight into the time evolution of quantum systems and the topological properties in General Relativity.

II. GRAVITOMAGNETIC FIELDS, SPIN–ROTATION COUPLINGS AND GEOMETRIC PHASES

According to the equivalence principle in general relativity, the nature of spin-rotation coupling is merely the interaction between the gravitational spin-magnetic moment and the gravitomagnetic field. By considering the Doppler effect of a light signal in the non-inertial reference frame rotating relative to the fixing reference frame, Mashhoon obtained the Hamiltonian of the spin-rotation coupling, which is of the form $H = \vec{\omega} \cdot \vec{S}$ with $\vec{\omega}$ and $\vec{S}$ respectively denoting the rotating frequency of the rotating frame and the intrinsic spin of a photon. In the framework of general relativity, however, we transformed the Kerr metric from the fixing reference frame to the rotating frame, and therefore obtained the exterior gravitomagnetic potential (one of the metric components $g_{0\varphi}$) of the spherically symmetric rotating body as follows [3]

$$g_{0\varphi} \simeq \frac{2aGM}{c^2 r} + \frac{2\omega r^2 \sin^2 \theta}{c}, \quad (2.1)$$

which is calculated by using the weak-field approximation, where $c$ and $G$ are the speed of light and the gravitational constant, respectively; $a$ is so defined that $a/c$ represents the angular momentum per unit mass of this gravitating body, and $(r, \theta, \varphi)$ stands for the displacements of the spherical coordinate system fixed in the rotating reference frame. It follows that the first term on the right handed side in Eq.(2.1) is related to the gravitational constant $G$ and is vanishing when the radial coordinate $r \to \infty$, whereas the second term tends to infinity rather than vanishes since it arises from the coordinates transformation in terms of the equivalence principle. It is of interest that the second term in Eq.(2.1) also gives rise to a gravitomagnetic field whose strength is just the rotating frequency, $\vec{\omega}$, of the rotating frame with respect to the fixing reference system. It is well known that the
magnetic field results in the Lorentz force acting on a charged particle, in the similar fashion, the non-inertial gravitomagnetic field $\hat{\omega}$ also yields a fictitious Coriolis force acting on a moving particle. In view of above discussion, one can also obtain the same Hamiltonian of spin-rotation coupling by using the Dirac equation with spin connection in curved spacetimes. Essentially, we can draw a conclusion that the interaction of the gravitational magnetic moment and the gravitomagnetic field involves spin-rotation coupling. The spin-rotation coupling leads to the inertial effects of the intrinsic spin of a particle, for instance, although the equivalence principle still holds, the universality of Galileo’s law of freely falling particles is violated, provided that the spin is polarized vertically up or down in the non-inertial frame [1].

Since there exists observational evidence for the coupling of spin-$\frac{1}{2}$ particle with the rotation of the Earth, we suggest a geometric effect in the coupling of the neutron spin with the time-dependent rotation of the Earth. Since the analogy can be drawn between gravity and electromagnetic force in some aspects, Aharonov and Carmi proposed the geometric effect of vector potentials of gravity, and Anandan, Dresden and Sakurai et al. proposed the quantum-interferometry effect associated with gravity [14–17]. What they investigated is now termed the quantum-interferometry effect associated with gravitomagnetic field, i.e. gravitomagnetic force in some aspects, Aharonov and Carmi effect have the same origin, namely, both arise from the presence of the Coriolis force (i.e., the interaction of the motion of the particle with the non-inertial gravitomagnetic field, $\hat{\omega}$), we argue that the Aharonov-Carmi effect mentioned above does not comprise the new geometric effect due to the time-dependent rotation of the non-inertial frame.

It is well known that the geometric phase factor appears in the quantum systems whose Hamiltonian explicitly depends on time or possesses evolution parameters. Differing from the dynamical phase which is related to the energy, frequency or velocity of a particle or a quantum system, geometric phase is dependent only on the geometric nature of the pathway along which the system evolves, which reflects the global and topological properties of evolution of the quantum systems. By making use of the Lewis-Riesenfeld invariant theory and the invariant-related unitary transformation formulation [18], we compute the geometric phase that results from the neutron spin-rotation coupling in which the rotating frequency is time-dependent. The result may be written as follows:

$$\phi_{\pm} = \pm \frac{1}{2} \int_{0}^{\pi} \frac{d\varphi(t')}{dt'} [1 - \cos \theta(t')] dt'$$

(2.2)

with $(\theta, \varphi)$ being the angle displacements over the parameters space of the conserved invariant, an operator whose eigenvalue is time-independent; $\pm$ corresponding to the neutron spin polarized vertically up and down. For the case of the adiabatic limit in which $\theta$ is constant, the geometric phase in one cycle over the parameter space is then $\phi_{\pm} = \pm \frac{1}{2} \cdot 2\pi(1 - \cos \theta)$, where $2\pi(1 - \cos \theta)$ is the expression for the solid angle which presents the geometric properties of time evolution of this spin-rotation system.

At present, investigation of geometric phase factor is an important direction in atomic and molecular physics, quantum optics, condensed matter physics and molecular reaction chemistry as well. Geometric phase factor has many applications in various branches of physics, say, in the case of this spin-rotation coupling, a potential application can be suggested. Since this geometric phase reveals the time evolution of $\hat{\omega}(t)$, information on the Earth’s variations of rotation will be obtained by measuring the geometric phase of the oppositely polarized neutrons through the neutron interferometry experiment.

### III. NON-INERTIAL GRAVITATIONAL WAVE IN ROTATING REFERENCE OF FRAME

Note that Mashhoon’s spin-rotation or spin-gravity coupling is only confined within the static case in which the rotating frequency is independent of time. Here we propose another interaction where the intrinsic spin of a particle is coupled to a propagating gravitomagnetic field. Einstein theory of General Relativity predicts that the accelerated mass produces the propagation of the space-time curvature which is termed the gravitational wave. Detecting and investigating these space-time ripples is one of the leading areas in astrophysics and cosmology. Because of the weakness of the gravitational radiation, gravitational wave has not been detected up to now even by means of both resonant-mass detectors and laser-interferometric detectors. In this paper, however, we suggest the concept of non-inertial gravitational wave, which does not relate to the smallness of the gravitational constant. Further detailed reasons are illustrated in the following. It follows from Eq.(2.1) that, in the rotating reference frame, the rotation yields additional space-time curvature (expressed by $\frac{2\omega r^2\sin^2 \theta}{c}$) which is independent of the gravitational constant, $G$. If, therefore, the rotating frequency $\hat{\omega}$ varies with time, the variation of the space-time curvature (the disturbance of the gravitational field) in the rotating reference frame propagates outwards in the form of wave motion. Such a propagating disturbance is referred to as a non-inertial gravitational wave (NGW). By ignoring some negligible
and higher-order terms, the wave equation of motion involving source terms is readily obtained

\[
\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) h_{mn} + \frac{16\pi G}{c^4} S_{mn} = \eta_{hk} \frac{\partial^2}{\partial t^2} (a^i_m a^n_k),
\]

where \( h_{mn} \) and \( \eta_{hk} \) denote the amplitudes of gravitational wave and the metric tensor of the flat Minkowski spacetime, respectively; \( a^i_m \) and \( a^n_k \) are the matrix elements of coordinate transformation from the fixing frame to the rotating frame; \( S_{mn} = T_{mn} - \frac{1}{2} \eta_{mn} T \) with \( T_{mn} \) being the energy-momentum tensor of matter and \( T = \eta^{mn} T_{mn} \). By using the low-motion weak-field approximation, further analysis shows that there exist only two kinds of amplitudes of NGW, i.e., \( h_{00} \) and \( h_{0\varphi} \) in the time-dependent rotating reference frame, and the expressions of the source term on the right handed side of Eq. (3.1) are respectively of the form \( -r^2 \sin^2 \theta \frac{\partial^2}{\partial \varphi^2} \left( \frac{z^2}{r^2} \right) \) and \( r^2 \sin^2 \theta \frac{\partial^2}{\partial \varphi^2} \left( \frac{z}{r} \right) \). Note, however, that this NGW depends on the non-inertial reference frame and differs from the standard gravitational wave predicted by Einstein.

Experimental evidence for spin-rotation coupling exists in the microwave and optical regimes via the phenomenon of frequency shift of polarized radiation. It is believed that with the foreseeable improvements in detecting technology, the interaction between the intrinsic spin of fundamental particles and NGW can also be detected. Since the existence of spin-rotation coupling can be observed only from the non-inertial frame, NGW is thus defined to be such space-time ripples associated with the non-inertial reference frame. It is therefore apparent that investigation of NGW is of much physical interest since its intensity is no longer dependent on the gravitational wave.

Although it possesses the non-inertial properties, this NGW can be coupled to matter. It is shown in what follows that the coupling coefficient depends on the gravitational constant, \( G \).

In London’s electrodynamics of superconductivity, the velocity \( \mathbf{v} \) of the superconducting electrons and the magnetic potentials \( \mathbf{A} \) satisfies \( \frac{\partial}{\partial t} \mathbf{v} = - \frac{e}{m_e} \frac{\partial}{\partial t} \mathbf{A} \) which leads to the self-induced charge current and thus provides photons field with an effective mass term in field equation or Lagrangian of electrodynamics. This procedure is equivalent to the mechanism of spontaneously symmetrical breaking. In the theory of gravity, the similar phenomenon exists since the momentum density \( \rho_m \mathbf{v} \) is conserved before and after the particles scattering, which is in analogy with the case in the superconductor where the electric current density is also conserved as the scattering is inhibited for Cooper pairs. Note that the Einstein’s equation of gravitational field under the low-motion weak-field approximation is formally analogous to the Maxwell equation of electromagnetic field, then in the case of gravitating matter, one can arrive at the similar relation \( \frac{\partial}{\partial t} \mathbf{v} = - \frac{e}{m_e} \mathbf{A} \) between the velocity of particles and the gravitomagnetic vector potentials \( \mathbf{h} \), where \( \mathbf{h} = (g^{01}, g^{02}, g^{03}) \). From the point of view of the low-motion weak-field approximation, the self-induced mass current results from this coupling of gravity to matter and then enables the gravitational field to possess an effective rest mass squared, \( m_e^2 = \frac{\hbar^2}{8\pi G \rho_m} \), where \( \hbar \) and \( \rho_m \) denote the Planck constant and the mass density of matter, respectively. Apparently, the NGW is not merely the non-inertial effect, for the effective mass of NGW is related to the gravitational constant, \( G \).

IV. HYPERBOLICAL GEOMETRIC QUANTUM PHASES AND TOPOLOGICAL DUAL MASS

Geometric phase exists in time-dependent quantum systems or systems whose Hamiltonian possesses evolution parameters [18,19]. As is well known, the dynamical phase of wave function in Quantum Mechanics is dependent on dynamical quantities such as energy, frequency, coupling coefficients and velocity of a particle or a quantum system, while the geometric phase is immediately independent of these physical quantities. When Berry found that the wave function would give rise to a non-integral phase (Berry’s phase) in quantum adiabatic process [4], geometric phase attracts attention of many physicists in various fields such as gravity theory [20,21], differential geometry [11], atomic and molecular physics [22], nuclear physics [23], quantum optics [24–26], condensed matter physics [27–30] and molecular-reaction chemistry [22] as well. In many simple quantum systems such as an electron possessing intrinsic magnetic moment interacting with a time-dependent magnetic field (or a neutron spin interacting with the Earth’s rotation [3]), a photon propagating inside the curved optical fiber [12,26], and the time-dependent Jaynes-Cummings model describing the interaction of the two-level atom with a radiation field [31], geometric phase is often proportional to \( 2\pi (1 - \cos \theta) \), which equals the solid angle subtended by a curve with respect to the origin of parameter space\(^1\). This, therefore, implies that geomet-

\(^1\)The solid angle subtended by the curve trace on the cone, at the center, shows the topological meanings of geometric phase. Geometric phase is absent in the quantum system when its Hamiltonian is independent of time. Geometric phase of many simple physical systems in the adiabatic process can be expressed in terms of the solid angle over the parameter space, e.g., the wave function of a photon propagating inside the non-coplanar curved optical fiber obtains this topological phase, where the parameter space is just the momentum (i.e., velocity) space of the photon [26].
ric phase differs from dynamical phase and it involves
global and topological information on the time evolution
of quantum systems. In addition to this trigonometric
geometric phase, there exists the so-called hyperbolical
geometric phase that is expressed by $2\pi(1 - \cosh \theta)$ with
the hyperbolical cosine $\cosh \theta = \frac{1}{2}(\exp(\theta) + \exp(-\theta))$ in
some time-dependent quantum systems, e.g., the two-
level atomic system with electric dipole-dipole inter-
action and the harmonic-oscillator system [32]. It is verified
that the generators of the Hamiltonians of these quantum
systems form the $SU(1,1)$ Lie algebra. Further analysis
indicates that quantum systems, which possess the non-
compact Lie algebraic structure (whose group parameters
can be taken to be infinity) will present the hyperbolical
geometric phase, while quantum systems with compact
Lie algebraic structure will give rise to the trigonometric
geometric phase.

Since Lorentz group (describing the boosts of refer-
ence frames in the space direction) in the special the-
ory of relativity is also a non-compact group, this leads
us to consider the topological properties associated with
space-time. We take into account the gravitational ana-
logue to the magnetic charge [13], i.e., gravitomagnetic
charge that is the source of gravitomagnetic field just as
the case that mass (gravitoelectric charge) is the source of
gravitoelectric field (i.e., Newtonian gravitational field in
the sense of weak-approximation). In this sense, gravito-
magnetic charge is also termed the **dual mass**. It should
be noted that the concept of the ordinary mass is of
no physical significance for the gravitomagnetic charge;
it is of interest to investigate the relativistic dynamics
and gravitational effects as well as geometric properties
of this **topological dual mass** (should such exist). From
the point of view of differential geometry, matter may
be classified into two categories: gravitoelectric matter
and gravitomagnetic matter. The former category pos-
sesses mass and constitutes the familiar physical world,
while the latter possesses dual mass that would cause the
non-analytical property of space-time metric. Einstein’s
field equation of gravitation in general theory of relativity
governs the couplings of gravitoelectric matter (which
possesses mass) to gravity (space-time): accordingly, we
should have a field equation governing the interaction of
dual matter with gravity. By making use of the varia-
tional principle, the gravitational field equation of gravit-
omagnetic matter can be obtained where the dual Ein-
stein’s tensor is denoted by $[8] \varepsilon^{\alpha\lambda\sigma\tau} R^\beta_{\lambda\sigma\tau} - \varepsilon^{\beta\lambda\sigma\tau} R^\alpha_{\lambda\sigma\tau}$ with
$\varepsilon^{\alpha\lambda\sigma\tau}$ and $R^\beta_{\lambda\sigma\tau}$ being the four-dimensional Levi-
Civita completely antisymmetric tensor and the Riemann
curvature tensor that describes the space-time curvature,
respectively. By exactly solving this field equation, one
can show that the topological property of the solution
$g_{\mu\nu}(r, \theta)$ can be illustrated as follows: solid angle sub-
tended by the curve shows the topological property of the
gravitomagnetic vector potential of a static gravit-
omagnetic charge at the origin of the spherical coordinate system. Take the gravitomagnetic vector potentials
$\mathbf{g} = \left(0, 0, \frac{2\mu}{4\pi r} \left(1 - \cos \theta\right)\right)$ in spherical coordinate system,
then the loop integral, $\oint_{\text{curve}} \mathbf{g} \cdot d\mathbf{l} = \mu(1 - \cos \theta)$, is pro-
portional to the solid angle subtended by the curve with
respect to the origin. The same situations arise in the
adiabatic quantum geometric phase (Berry’s quantum
phase), which reflects the global or topological properties
of time evolution (or parameters evolution) of quantum
systems. Such property is in analogy with that of the
geometric quantum phase in the time-dependent spin-
gravity coupling (**i.e.**, the interaction between a spinning
particle with gravitomagnetic field [3]) and other quan-
tum adiabatic processes [12,21]. The topological prop-
erties of gravitomagnetic charge (dual mass) may be shown
in terms of the global features of geometric quantum
phase. It follows that the expression of gravitomagnetic
potential, $g_{\mu\nu}(r, \theta)$, due to dual mass is exactly analogous
to that of the trigonometric geometric phase. In the sim-
ilar fashion, it is readily verified that the gravitomagnetic
potential, $g_{\mu\nu}(r, t)$, is similar to that of the hyperbolical
geometric phase. This feature originates from the fact
that the Lorentz group is a non-compact group.

Although there is no evidence for the existence of this
topological dual mass at present, it is still essen-
tial to consider this topological or global phenomenon
in General Relativity. It is believed that there would
exist formation (or creation) mechanism of gravitomag-
netic charge in the gravitational interaction, just as some
prevalent theories provide the theoretical mechanism of
existence of magnetic monopole in various gauge interac-
tions [33,34]. Magnetic monopole in electrodynamics and
gauge field theory has been discussed and sought after
for decades, and the existence of the ‘t Hooft-Polyakov
monopole solution [33] has spurred new interest of both

---

2 Analogy between **topological dual mass** and **geometric quantum phase** are as follows: the dynamical correspondences to both of them are respectively the **mass** and the **dynamical phase**; gravitomagnetic moment results from the mass cur-
rent, which is also the dynamical physical quantity; both re-
veal the global properties of physical systems. Comparison
between gravitomagnetic charge and geometric phase enables
to show the topological properties of the former. The rea-
son why the topological property is important lies in that the
global description of the physical phenomena is essential to
understand the world. It is of interest that dual matter may
constitute a dual world where dual mass abides by their own
dynamical and gravitational laws, which is somewhat differ-
ent from the laws in our world; for example, dual mass is acted upon by a gravitomagnetic Lorentz force in Newton’s
gravitational (gravitoelectric) field, and the static dual mass
produces the gravitomagnetic field rather than the Newton’s
gravitoelectric field.
theorists and experimentalists [34–36]. As the topological gravitomagnetic charge in the curved space-time, dual mass is believed to give rise to such interesting situation similar to that of magnetic monopole. If it is indeed present in universe, dual mass will also lead to significant consequences in astrophysics and cosmology. We emphasize that although the gravitomagnetic vector potential produced by the gravitomagnetic charge is the classical solution to the field equation, this kind of topological gravitomagnetic monopoles may arise not as fundamental entities in gravity theory, e.g., it will behave like a topological soliton.

Gravitomagnetic charge has some interesting relativistic quantum gravitational effects [1,3,8], e.g., the gravitational anti-Meissner effect, which may serve as an interpretation of the smallness of the observed cosmological constant. In accordance with quantum field theory, vacuum possesses infinite zero-point energy density due to the vacuum quantum fluctuations; whereas according to Einstein’s theory of general relativity, infinite vacuum energy density yields the divergent curvature of space-time, namely, the space-time of vacuum is extremely curved. Apparently it is in contradiction with the practical fact, since it follows from experimental observations that the space-time of vacuum is asymptotically flat. In the context of quantum field theory a cosmological constant corresponds to the energy density associated with the vacuum and then the divergent cosmological constant may result from the infinite energy density of vacuum quantum fluctuations. However, a diverse set of observations suggests that the universe possesses a nonzero but very small cosmological constant [37–40]. How can we give a natural interpretation for the above paradox? Here, provided that vacuum matter is perfect fluid, which leads to the formal similarities between the weak-gravity equation in perfect fluid and the London’s electrodynamics of superconductivity, we suggest a potential explanation by using the cancelling mechanism via gravitational anti-Meissner effect\(^3\): the gravitoelectric field (Newtonian field of gravity) produced by the gravitoelectric charge (mass) of the vacuum quantum fluctuations is exactly cancelled by the gravitoelectric field due to the induced current of the gravitomagnetic charge of the vacuum quantum fluctuations; the gravitomagnetic field produced by the gravitomagnetic charge (dual mass) of the vacuum quantum fluctuations is exactly cancelled by the gravitomagnetic field due to the induced current of the gravitoelectric charge (mass current) of the vacuum quantum fluctuations. Thus, at least in the framework of weak-field approximation, the extreme space-time curvature of vacuum caused by the large amount of the vacuum energy does not arise, and the gravitational effects of cosmological constant is eliminated by the contributions of the gravitomagnetic charge (dual mass). If gravitational Meissner-anti-Meissner effect is of really physical significance, then it is necessary to apply this effect to the early universe where quantum and inflationary cosmologies dominate the evolution of the Universe.

\(^3\)Note that in London’s electrodynamics for superconductivity, the equation governing the magnetic fields in superconductors is of the type: \(\nabla^2 \mathbf{B} = \lambda^2 \mathbf{B} \) with \(\lambda\) being a real number. In cosmological perfect fluid, however, the equation governing the gravitomagnetic fields in fluid is of the type: \(\nabla^2 B_\mu = -\lambda_\mu^2 B_\nu\), which means the absence of the gravitational analogue to the superconducting Meissner’s effect. We can refer to this phenomenon as the gravitational (gravitomagnetic) anti-Meissner effect due to the minus sign on the right-handed side of the latter equation.

V. CONCLUDING REMARKS

For the present, it is possible to investigate quantum mechanics in weak-gravitational fields, with the development of detecting and measuring technology such as laser-interferometer technology and so on. These investigations enable physicists to test validity or universality of fundamental laws and principles of General Relativity in, for instance, the microscopic quantum regimes. The above-mentioned effects associated gravitomagnetic fields reflect the relativistic quantum gravitational properties of matter-gravity couplings in weak-gravitational fields. It is believed that these effects will give rise to further interest of investigation, since there exist many physically potential applications of these relativistic quantum gravitational effects in quantum optics, gravity theory, quantum theory, cosmology, applied physics and other related areas as well.

Study of the geometric property in quantum regimes is an interesting and valuable direction. Since it reveals the global and topological properties of evolution of quantum systems, geometric phase has many applications in various branches of physics, say, in the coupling of neutron spin to the Earth’s rotation [1,2], a potential application may be suggested where the information on the Earth’s variations of rotating frequency will be obtained by measuring the geometric phase of the oppositely polarized neutrons through the neutron-gravity interferometer experiment. The topological charge in curved space-time also deserves further investigation, since it reflects plentiful global or geometric properties hidden in the gravity theory. It is believed that both theoretical and experimental interest in this direction may enables people to understand the global phenomena of the physical world better.
[1] Mashhoon, B. Gravitational couplings of intrinsic spin. Class. Quant. Grav. 17, 2399-2410 (2000).
[2] Mashhoon, B. On the spin-rotation-gravity coupling. Gen. Rel. Grav. 31, 681-691 (1999).
[3] Shen, J. Q., Zhu, H. Y., Shi, S. L. & Li J. Gravitomagnetic field and time-dependent spin-rotation coupling. Phys. Scr. 65, 465-468 (2002).
[4] Berry, M.V. Quantal phase factor accompanying adiabatic changes. Proc. R. Soc. Lond. A 392, 45-57 (1984).
[5] Berry, M. V. Interpreting the anholonomy of coiled light. Nature 326, 277-278 (1987).
[6] Bell, D. L. & Zonoz, M. N. Classical monopole: Newton, NUT space, gravomagnetic lensing and atomic spectra. Rev. Mod. Phys. 70, 427-445 (1998).
[7] Zonoz, M. N. Cylindrical analogue of NUT space: spacetime of a line gravomagnetic monopole. arXiv: gr-qc/9706015 (1997).
[8] Shen, J. Q. Gravitational Analogues, Geometric Effects and Gravitomagnetic Charge. Gen. Rel. Grav. 34, 1423-1435 (2002).
[9] Shen, J. Q. Dynamics of Gravitomagnetic Charge. arXiv: gr-qc/0301100 (2003).
[10] Robinson, L. An optical measurement of Berry’s phase. Science 234, 424-426 (1986).
[11] Simon, B. Holonomy, the quantum adiabatic theorem, and Berry’s phase. Phys. Rev. Lett. 51, 2167-2170 (1983).
[12] Chiao, R. & Wu, Y. S. Manifestations of Berry’s phase for the photon. Phys. Rev. Lett. 57, 933-936 (1986).
[13] Dirac, P. A. M. Quantized singularities in the electromagnetic field. Proc. R. Soc. Lond. A 133, 60-71 (1931).
[14] Anandan, J. Gravitational and rotational effects in quantum interference. J. Phys. Rev. D 15, 1448-1457 (1977).
[15] Dresden, M. & Yang, C. N. Phase shift in a rotation neutron or optical interferometer. Phys. Rev. D 20, 1846-1848 (1979).
[16] Overhauser, A. W. & Colella, R. Experimental test of gravitationally induced quantum interference. Phys. Rev. Lett. 33(20), 1237-1239 (1974).
[17] Werner, S. A., Staudenmann, J. L. & Colella, R. Effect of earth rotation on the quantum mechanical phase of the neutron. Phys. Rev. Lett 42(17), 1103-1106 (1979).
[18] Gao, X. C., Xu, J. B. & Qian, T. Z. Geometric phase and the generalized invariant formulation. Phys. Rev. A 44, 7016-7021 (1991).
[19] Yang, L. G. & Yan, F. L. The area theorem of the Berry’s phase for the time-dependent externally driven system. Phys. Lett. A 265, 326-330 (2000).
[20] Furtado, C. & Bezerra, V. B. Gravitational Berry’s quantum phase. Phys. Rev. D 62, 045003-(1-5) (2000).
[21] Shen, J. Q., Zhu, H. Y. & Li, J. Exact solutions to the intersection between neutron spin and gravitation by using invariant theory. Acta Phys. Sin. 50, 1884-1887 (2001).
[22] Wu, Y. S. M. & Kuppermann, A. Prediction of the effect of the geometric phase on product rotational state distributions and integral cross sections. Chem. Phys. Lett. 201, 178-186 (1993) and references therein.
[23] Wagh, A. G. et al. Neutron polarimetric separation of geometric and dynamical phases. Phys. Lett. A 268, 209-216 (2000).
[24] Sanders, B. C. et al. Geometric phase of three-level sys-