Extended Antifield–Formalism

Friedemann Brandt a †, Marc Henneaux b,c ‡ and André Wilch b §

a Departament d’Estructura i Constituents de la Matèria, Facultat de Física, Universitat de Barcelona, Diagonal 647, E–08028 Barcelona, Spain.
b Faculté des Sciences, Université Libre de Bruxelles, Campus Plaine C.P. 231, B–1050 Bruxelles, Belgium.
c Centro de Estudios Científicos de Santiago, Casilla 16443, Santiago 9, Chile

The antifield formalism is extended so as to incorporate the rigid symmetries of a given theory. To that end, it is necessary to introduce global ghosts not only for the given rigid symmetries, but also for all the higher order conservation laws, associated with conserved antisymmetric tensors $j^{\mu_1\ldots\mu_k}$ fulfilling $\partial_\nu j^{\nu\mu_1\ldots\mu_k} \approx 0$. Otherwise, one may encounter obstructions of the type discussed in [13]. These higher order conservation laws are shown to define additional rigid symmetries of the master equation and to form — together with the standard symmetries — an interesting algebraic structure. They lead furthermore to independent Ward identities which are derived in the standard manner, because the resulting master (“Zinn-Justin”) equation capturing both the gauge symmetries and the rigid symmetries of all orders takes a known form. Issues such as anomalies or consistent deformations of the action preserving some set of rigid symmetries can be also systematically analysed in this framework.

I. INTRODUCTION

Gauge theories may possess global (= rigid) symmetries in addition to their local (= gauge) symmetries. Through Noether’s first theorem, non-trivial rigid symmetries of the classical action correspond to non-trivial conserved currents,

$$\partial_\mu j^\mu \approx 0 .$$  \hspace{1cm} (1.1)

Now, (1.2) is actually only a special case of the more general conservation law

$$\partial_\mu j^{\mu_1\ldots\mu_k} \approx 0$$  \hspace{1cm} (1.2)

where $j^{\mu_1\ldots\mu_k}$ is completely antisymmetric and the symbol $\approx$ denotes weak (i.e. on-shell) equality. Non-trivial solutions of (1.2) define what we call non-trivial conservation laws of order $k$. Although it has been proved under fairly general conditions that non-trivial conservation laws of higher order $k > 1$ are absent for theories without gauge invariance [1] [2], they may be present in the case of gauge theories. Examples are given by $p$-form gauge theories which admit non-trivial conserved antisymmetric tensors of rank $p + 1$ [4]. These conservation laws play an important role in supergravity (see e.g. [5]).

The quantum mechanical implications of the rigid symmetries that are associated with ordinary conserved currents (1.1) are well understood. If these symmetries are non-linear, they get renormalized. The most expedient way to derive the corresponding Ward identities is to introduce sources for the composite operators representing the variations of the fields $\delta$. In order to avoid an infinite number of such sources (one for the first variation, one for the second...)

---

*This paper supersedes “Ward Identities for Rigid Symmetries of Higher Order”, [hep-th/9611056]
†E-mail: brandt@ecm.ub.es
‡E-mail: henneaux@ulb.ac.be
§E-mail: awilch@ulb.ac.be

1A conserved antisymmetric tensor is trivial if it is of the form $j^{\mu_1\ldots\mu_k} \approx \partial_\nu k^{\nu\mu_1\ldots\mu_k}$ where $k^{\nu\mu_1\ldots\mu_k}$ is also completely antisymmetric.
variation etc. \[1\], constant ghosts are introduced of ghost number 1 and of Grassmann parity opposite to that of the symmetry parameter \[1\]. The Ward identities then follow by solving an extended master equation \[10\], the explicit form of which will be given below. This approach, which works even if the gauge-fixing procedure does not preserve manifest invariance under the rigid symmetry, has proved useful in the investigation of the renormalization and anomaly problems in globally supersymmetric models \[11\].

However, it has been shown that the construction of a local solution of the extended master equation of \[8\] may get obstructed – already at the classical level – in presence of higher order non-trivial conservation laws \[13\]. When this occurs, it is not possible to incorporate the standard rigid symmetries along the lines of \[8\] and the procedure breaks down.

The main purpose of this paper is to show that the obstructions can be avoided and that locality can be recovered if one extends the approach of \[8\] by properly including in the formalism also the higher order conservation laws. Together with the original symmetries, they form a rich algebraic structure involving structure constants of increasing order, which fulfill generalized Jacobi identities. We introduce global ghosts for each independent rigid symmetry of any order and we write down the corresponding form of the master equation (equation (4.1) below). We then prove the existence of a local solution of the master equation, which is our main result.

Because the master equation incorporating all the gauge and rigid symmetries takes a form that is very similar to the standard one, one can derive, by the familiar procedure of differentiating it with respect to the sources, the Ward identities for the Green functions. In particular, since the rigid symmetries of higher order come with their own ghosts, they lead to independent Ward identities. Similarly, the analysis of anomalies in both the rigid and gauge symmetries may still be formulated as a cohomological problem, with a differential extending the standard BRST operator.

We finally describe the consistent deformations of the action in this context.

II. HIGHER ORDER CONSERVATION LAWS AS HIGHER ORDER SYMMETRIES

One could of course attempt to regard the higher order conserved antisymmetric tensors \(j_{\mu_1...\mu_k}\) as ordinary Noether currents parametrized by further indices. There are many good reasons for not doing this. One of them is that this approach does not yield the appropriate notion of “triviality” because it does not take properly into account all the antisymmetry properties of \(j_{\mu_1...\mu_k}\). Viewed as a higher order conservation law, the equation \(\partial_\mu j_{\mu_1...\mu_k} \approx 0\) is trivial if and only if \(j_{\mu_1...\mu_k} \approx \partial_\mu k_{\mu_1...\mu_k}\), where \(k_{\mu_1...\mu_k}\) is also completely antisymmetric. But if one views \(\partial_\mu j_{\mu_1...\mu_k} \approx 0\) as a collection of ordinary conservation laws parametrized by further indices \(\mu_2, ..., \mu_k\), triviality holds under the weaker condition \(j_{\mu_1...\mu_k} \approx \partial_\nu S_{\mu_1...\mu_k}\), where \(S_{\mu_1...\mu_k}\) is only required to be antisymmetric in its first two indices \(\nu\) and \(\mu_1\).

That these two notions are inequivalent is best illustrated in the case of the free \(n\)-dimensional Maxwell theory, for which \(\partial_\mu F^{\mu_1\mu_2} \approx 0\), \(F^{\mu_1\mu_2} = -F^{\mu_2\mu_1}\). The relation \(\partial_\mu F^{\mu_1\mu_2} \approx 0\) is a non-trivial conservation law of order 2 because there is no completely antisymmetric \(k_{\mu_1\mu_2}\) which is local (i.e. polynomial in derivatives) and satisfies \(F^{\mu_1\mu_2} \approx \partial_\mu k_{\mu_1\mu_2}\), even if one allows \(k_{\mu_1\mu_2}\) to depend explicitly on \(x^\mu\) \[3\]. In contrast to that, the \(F^{\mu_1\mu_2}\) become trivial when they are regarded as a set of \(n\) Noether currents, one for each value of \(\mu_2\),

\[
F^{\mu_1\mu_2} \approx \partial_\mu S^{\mu_1\mu_2}, \quad S^{\mu_1\mu_2} = \partial^\lambda F^{\mu_1\lambda} = -S^{\mu_2\mu_1}.
\]

(2.1)

Accordingly, the conservation law \(\partial_\mu F^{\mu_1\mu_2} \approx 0\) does not correspond to a nontrivial rigid symmetry of the Maxwell action. Rather, it is associated with the shift symmetry \(A_\mu \to A_\mu + \epsilon_\mu (\epsilon_\mu = \text{constant})\) which is a trivial symmetry as it is just a special gauge transformation of \(A_\mu\) with parameter \(x^\mu \epsilon_\mu\).

An extension of the Noether theorem that does take into account complete antisymmetry has been proposed in the interesting work \[14\]. We shall not follow that approach here, but rather, we shall directly relate the higher order conservation laws to rigid symmetries of the solution of the master equation. This point of view turns out to be particularly convenient for the quantum theory.

Our starting point is thus the solution \(S = S[\Phi^a, \Phi^a_\ast]\) of the master equation for the gauge symmetries \[15\],

\[
(S, S) = 0,
\]

(2.2)

where \((\cdot, \cdot)\) is the standard antibracket. The \(\{\Phi^a\}\) are the fields (classical fields \(\phi^i\), ghosts for the gauge symmetries \(C^a\), ghosts of ghosts if necessary, antighosts, Nakanishi-Lautrup auxiliary fields) while the \(\{\Phi^a_\ast\}\) are the corresponding antifields. The master equation (2.2) always admits a solution \(S\) which is a local functional \[16\].

As shown in \[3\], one can associate with each conservation law \(\partial_\mu j_{\mu_1...\mu_k} \approx 0\) of order \(k_A\) a local functional \(S_A[\Phi^a, \Phi^a_\ast]\) which (i) has ghost number \(-k_A\); and (ii) is BRST-invariant. Since the BRST transformation \(\delta\) is generated in the antibracket by \(S\), (ii) means
\[ s \mathcal{S}_A \equiv (\mathcal{S}_A, \mathcal{S}) = 0. \]  
(2.3)

But this condition expresses at the same time that the solution \( S \) of the master equation is invariant by the canonical transformation generated in field-antifield space by \( S_A \).

\[ \delta \mathcal{S}_A \equiv (\mathcal{S}, \mathcal{S}_A) = 0. \]  
(2.4)

Consequently, each conservation law defines indeed a symmetry of \( \mathcal{S} \).

The relationship between the conserved antisymmetric tensors and the generators \( S_A \) has been given in \([3]\), since \( S_A = \int dx m_A \) is BRST invariant\(^2\), its integrand satisfies

\[ sm_A + \partial_\mu m_A^{\mu} = 0. \]  
(2.5)

If \( S_A \) has ghost number \(-1\) (corresponding to ordinary rigid symmetries), \( m_A^{\mu} \) has ghost number zero and the antifield independent part of (2.5) reproduces (1.1) because the antifield independent part of \( \mathcal{S} \) possesses an interesting algebraic structure. Assume that \( \{S_A\} \) is a basis of symmetry generators, i.e. of the cohomology \( H^{-k}(s) \) in the space of local functionals \( \Gamma[\Phi, \Phi^*] \) at all negative ghost numbers \((-k)\). This means that any \( s \)-closed local functional \( \Gamma[\Phi, \Phi^*] \) with negative ghost number is a linear combination of the \( S_A \) up to a \( s \)-exact term,

\[ s\Gamma[\Phi, \Phi^*] = 0 \quad \text{and} \quad gh(\Gamma) < 0 \iff \Gamma[\Phi, \Phi^*] = \lambda^A S_A[\Phi, \Phi^*] + s\tilde{\Gamma}[\Phi, \Phi^*], \]  
(3.1)

and no non-vanishing linear combination of the \( S_A \) is \( s \)-exact in the space of local functionals \( \Gamma[\Phi, \Phi^*] \),

\[ \lambda^A S_A[\Phi, \Phi^*] = s\tilde{\Gamma}[\Phi, \Phi^*] \iff \lambda^A = 0 \quad \forall A. \]  
(3.2)

Since the antibracket of two \( S_A \) is BRST-closed (one has \((S,(S_A,S_B)) = 0 \) by the Jacobi identity for the antibracket), it must be of the form

\[ (-1)^{\varepsilon_A} (S_A, S_B) = f^D_{AB} S_D + (S,S_{AB}) \]  
(3.3)

for some constants \( f^D_{AB} \) and some local functionals \( S_{AB} \) (\( \varepsilon_A + 1 \) denotes the Grassmann parity of \( S_A \); phases and factors are introduced for later convenience). Taking the antibracket of this expression with \( S_C \) and using the Jacobi identity for the antibracket then leads to

\[ S_E f^E_{D(A} f^D_{BC]} = \left( S, (-)^{\varepsilon_B} (S_{[B}, S_{CA]} - S_{D[A} f^D_{BC]} \right) \]  
(3.4)

where \([\ ]\) denotes graded antisymmetrization. According to (3.2), both sides of (3.4) have to vanish separately. This yields the Jacobi identity for the structure constants \( f^C_{AB} \) and - due to (3.1) - the additional identity

---

\(^2\)Throughout the paper we use \( \int dx \) to indicate integration over an \( n \)-dimensional base manifold ("spacetime"), and call a functional BRST invariant if the BRST variation of its integrand is a total derivative.
for some second order structure constants $f_{ABC}^D$ and some local functionals $S_{ABC}$. If there does not exist any higher order conservation law (and thus no $S_A$ with $gh(S_A) < -1$), then higher order structure constants like $f_{ABC}^D$ cannot occur. This follows from a mere ghost number counting argument. However, in the presence of higher order symmetries, terms of the form $f_{ABC}^D S_D$ are allowed and indeed do occur in explicit examples (see [13] and the example treated below).

The above construction can be continued, defining further local functionals $S_{A_1 \ldots A_r}$ and structure constants $f_{A_1 \ldots A_r}^{C_1 \ldots C_r}$ that will satisfy generalized higher order Jacobi identities. As an illustration, we just provide the next step, leading to the local functionals $S_{ABC}^{D}$ and the structure constants $f_{ABC}^E$. By taking the antibracket of (3.5) with $S_D$ and using the Jacobi-identity, one obtains

\[
\frac{1}{r} (-)^r (S_{ABC}) = S_{D[C} f_{AB]}^D + \frac{1}{2} f_{ABC}^D S_D + \frac{1}{2} (S, S_{ABC}) \tag{3.5}
\]

and

\[
\frac{1}{r} (-)^r (S_{ABC}) + \frac{1}{r} (-)^{r+1} (S_{A_1 \ldots A_r}) + \frac{1}{2} f_{ABC}^D S_D + \frac{1}{2} (S, S_{ABC}) = 0 \tag{3.6}
\]

and

\[
\frac{1}{r} (-)^r (S_{ABC}) + \frac{1}{r} (-)^{r+1} (S_{A_1 \ldots A_r}) + \frac{1}{2} f_{ABC}^D S_D + \frac{1}{2} (S, S_{ABC}) = 0 . \tag{3.7}
\]

Eq. (3.7) is a generalized Jacobi identity for the higher order structure constants $f_{ABC}^E$. Both the usual Jacobi identity and Eq. (3.7) can be written as

\[
\sum_{r=2}^{p-1} \frac{1}{r! (p-r)!} f_{A_i+1 \ldots A_p}^D f_{A_1 \ldots A_r}^C = 0 \tag{3.8}
\]

where $p = 3$ and $4$. It turns out that the Jacobi identities for the subsequent structure constants are also given by (3.8). Note that (3.3) contains the commutation relations of the standard rigid symmetries (for $k_A = k_B = 1$), and that (3.8) includes the Jacobi identities for the corresponding structure constants.

Below, we shall set up the extended antifield formalism such that it automatically incorporates this algebra to all orders through a modified master equation. Algebraic structures similar to the ones appearing here have been analyzed in [24].

A subset of symmetry generators $S_A$ defines a subalgebra if and only if the relations to which they lead never involve the other symmetry generators $S_\Delta, A = (\alpha, \Delta)$. An equivalent condition is that the structure constants $f_{A_1 \alpha_1}^\Delta, f_{A_1 A_2 \alpha_1}^\Delta \ldots$ all vanish. The subset $\{ S_A, S_{\alpha_1 \alpha_2}, S_{\alpha_1 \alpha_2 \alpha_3}, \ldots \}$ is then a closed set for the generating equations (3.3) and the subsequent ones. As shown in [23] and in the example below, the set of all symmetry generators of order one (standard rigid symmetries) may not form a subalgebra in the above sense.

IV. EXTENDED MASTER EQUATION

The most expedient way of generating all the local functionals $S_{A_1 \ldots A_r}$ and structure functions $f_{A_1 \ldots A_r}^{C_1 \ldots C_r}$ appearing in the algebra described above, is through an extended master equation. Another motivation for using the master-equation approach has to do with the quantum theory. The fact that all the conservation laws, including the higher-order ones, appear as symmetries of the solution of the usual master equation, makes it possible to investigate in a unified manner the corresponding Ward identities. Since the transformations generated by the $S_A$ may be non-linear, they may get renormalized in the quantum theory. To cope with this feature, we extend the approach of [24] and introduce, besides the standard antifields and local ghosts associated with the gauge symmetry, constant ghosts $\xi^A$ for all independent local conservation laws. The constant ghosts are assigned opposite ghost number and the same Grassmann parity as the corresponding generator $S_A$. Consequently, the ghost number of $\xi^A$ equals the order $k_A$ of the corresponding conservation law,

\[
gh(\xi^A) = - gh(S_A) = k_A .
\]

For instance, constant ghosts corresponding to Noether currents carry ghost number 1, as one expects since these ghosts correspond to global symmetries of the classical action.

\footnote{It is crucial, in order to avoid the obstructions, to take the higher order conservation laws into account as done here, with constant ghosts of ghost number $k$. It would not work to treat the higher order conservation laws [1, 2] as ordinary conservation laws parametrized by further indices and to associate with them constant ghosts $\xi^{\mu_1 \ldots \mu_k}$ of ghost number one.}
We then add the term $S_A^A\xi^A$ to $S$ and search for a solution $S[\Phi, \Phi^*, \xi]$ of the extended master equation

$$(S, S) + 2 \sum_{r \geq 2} \frac{1}{r!} \partial^R \partial^B f^B_{A_1 \cdots A_r} \xi^A_r \cdots \xi^A_1 = 0$$

(4.1)

of the form

$$S = S + S_A^A \xi^A + \sum_{r \geq 2} \frac{1}{r!} S_{A_1 \cdots A_r} \xi^A_r \cdots \xi^A_1,$$

(4.2)

where the $f^B_{A_1 \cdots A_r}$ are the structure constants and the $S_{A_1 \cdots A_r}$ are the local functionals of the symmetry algebra described above, and still need to be constructed.

The existence-proof of $S$, to be given in the next section, becomes straightforward if constant antifields $\xi^A_A$ conjugate to the constant ghosts $\xi^A$ are introduced through

$$S' = S + \sum_{r \geq 2} \frac{1}{r!} \xi^A_A f^B_{A_1 \cdots A_r} \xi^A_r \cdots \xi^A_1.$$

(4.3)

These additional antifields have the usual properties,

$$gb(\xi^A_A) = -gb(\xi^A) - 1 = -k_A - 1$$

and

$$\varepsilon(\xi^A_A) = \varepsilon(\xi^A) + 1 = \varepsilon_A.$$

The extended master equation (4.1) now takes the familiar form

$$(S', S')' = 0,$$

(4.4)

where the extended antibracket $(\cdot, \cdot)'$ is given by

$$(X, Y)' = \frac{\partial^R X}{\partial \tilde{\xi}_A^*} \frac{\partial^L Y}{\partial \xi_A^*} - \frac{\partial^R X}{\partial \xi_A^*} \frac{\partial^L Y}{\partial \tilde{\xi}_A^*} + \int dx \left[ \frac{\delta^R X}{\delta \Phi^a(x)} \frac{\delta^L Y}{\delta \Phi^a(x)} - \frac{\delta^R X}{\delta \Phi_a^*(x)} \frac{\delta^L Y}{\delta \Phi^a(x)} \right].$$

V. EXISTENCE OF $S$

In order to prove that there always exists a solution of Eq. (4.4), we shall follow the method of [18]. For this purpose we shall extend the definition of the Koszul-Tate differential $\delta$ appropriately and use an expansion of (4.4) according to the antighost number ($agh$) defined by

$$agh(\Phi^a) = agh(\xi^A) = 0, \quad agh(\Phi_a^*) = -gh(\Phi_a^*), \quad agh(\xi_A^*) = -gh(\xi_A^*) = k_A + 1.$$ 

Before we do this, we recall some standard results [16, 18] on the cohomology of the Koszul-Tate operator in the space $F$ of local functionals $\Gamma[\Phi, \Phi^*]$. On the fields $\Phi^a$ and their antifields $\Phi^*_a$, $\delta$ is defined through [18]

$$\delta \Phi^a = 0, \quad \delta \phi^*_i = -\frac{\delta^L S_0}{\delta \phi^*_i}, \quad \cdots$$

(5.1)

where $S_0$ is the classical action. Now, in $F$, the cohomologies of $s$ and $\delta$ are isomorphic at all negative ghost numbers ($-k$) and positive antighost numbers $k$ respectively, $H^{-k}(s, F) \simeq H_k(\delta, F)$ for $k > 0$ (superscript and subscript of $H$ denote the ghost number and antighost number respectively). The representatives $S^0_A$ of $H_k(\delta, F)$, $k > 0$ can be chosen so as not to depend on the ghost fields. The corresponding representatives $S_A$ of $H^{-k}(s, F)$ are BRST-invariant extensions of the $S^0_A$,

$$S_A[\Phi, \Phi^*] = S^0_A[\phi, \Phi^*] + \text{ghost-terms.}$$

(5.2)

Note that the part of $S_A$ that does not involve the ghosts satisfies
\[ gh(S_A^0) = -agh(S_A^0) = -k_A. \]

The \( S_A^0 \) fulfill therefore requirements analogous to (3.1) and (3.2), i.e.

\[ \delta \Gamma[\Phi, \Phi^*] = 0 \text{ and } agh(\Gamma) > 0 \iff \Gamma[\Phi, \Phi^*] = \lambda^A S_A^0[\phi, \Phi^*] + \delta \Gamma[\Phi, \Phi^*], \quad (5.3) \]
\[ \lambda^A S_A^0[\phi, \Phi^*] = \delta \Gamma[\Phi, \Phi^*] \iff \lambda^A = 0 \quad \forall A. \quad (5.4) \]

Now we define the above-mentioned extension of \( \delta \). It applies to the functional space to which \( S' \) belongs, namely the vector space \( \mathcal{E} \) of functionals \( \mathcal{A} \) defined through

\[ \mathcal{A} \in \mathcal{E}: \Rightarrow \mathcal{A} = \Gamma[\Phi, \Phi^*, \xi] + \lambda^A(\xi) \xi_A^*, \quad (5.5) \]

where \( \Gamma = \int \omega_a \) is an integrated local volume form which does not involve the \( \xi_A^* \) (it may depend polynomially on the \( \xi^A \)) and \( \lambda^A(\xi) \) is a polynomial in the constant ghosts. Note that functionals in \( \mathcal{E} \) depend on the \( \xi_A^* \) at most linearly via non-integrated terms \( \lambda^A(\xi) \xi_A^* \).

To define \( \delta \) in \( \mathcal{E} \) appropriately, we extend its definition to the global ghosts and antifields via

\[ \delta \xi^A = 0, \quad \delta \xi_A^* = S_A^0[\phi, \Phi^*] \quad (5.6) \]

where \( S_A^0 \) is the ghost independent part of \( S_A \), see above. On the \( \Phi^0 \) and \( \Phi_A^* \), \( \delta \) is defined as before in (5.1). Note that \( \delta \) is well-defined in \( \mathcal{E} \), as \( \mathcal{A} \in \mathcal{E} \Rightarrow \delta \mathcal{A} \in \mathcal{E} \).

With these definitions, \( \delta \) is nilpotent due to \( \delta^2 \xi_A^* = \delta S_A^0 = 0 \). Furthermore, by construction, it is acyclic in \( \mathcal{E} \) at positive antighost number, i.e. \( H_k(\delta, \mathcal{E}) = 0 \) for \( k > 0 \). Indeed, \( \delta \mathcal{A} = 0 \) is equivalent to \( \delta \Gamma + \lambda^A S_A^0 = 0 \) which implies \( \lambda^A = 0 \) for \( agh(\mathcal{A}) > 0 \) due to (5.1) and thus also \( \delta \Gamma = 0 \). From this we conclude, using (5.3) and (5.6), \( \mathcal{A} = \Gamma = \delta \Gamma + \lambda^A S_A^0 = \delta(\Gamma + \lambda^A \xi_A^*) \) and thus

\[ \delta \mathcal{A} = 0, \quad agh(\mathcal{A}) > 0, \quad \mathcal{A} \in \mathcal{E} \]
\[ \Rightarrow \mathcal{A} = \delta \mathcal{A}, \quad \delta \mathcal{A} \in \mathcal{E}. \quad (5.7) \]

The construction of solutions to Eq.(4.4) now follows almost word for word the standard pattern of homological perturbation theory (18) (section 10.5.4). The sought \( S' \) is expanded according to the antighost number, \( S' = S_0 + S_1 + S_2 + \cdots, agh(S_k) = k, \quad S_k \in \mathcal{E}. \quad (5.8) \)

Here \( S_0 \) is the classical action and we require \( S' \) to contain the piece \( S_A^0 \xi^A \), i.e.

\[ \frac{\partial^R S'}{\partial \xi^A} = S_A^0 + O(k_A + 1) \quad (5.9) \]

where \( O(k) \) denotes collectively terms with antighost numbers \( \geq k \). Together with the standard conditions (“properness”) on the solution \( S \) of the usual master equation in gauge theories, (3.3) fixes the boundary conditions that we impose on \( S \) in order to guarantee that it encodes indeed all the local conservation laws. This fixes in particular \( S_1 \) to the form

\[ S_1 = \phi^*_i R^{i} C^n + \sum_{A,k_A=1} S_A^0 \xi^A \quad (5.10) \]

where the first term encodes the gauge symmetries of \( S_0 \) (we used De Witt’s notation) and the second term contains its global symmetries (for \( k_A = 1 \) one has \( S_A^0 = \phi^*_i (\delta_A \phi^i) \) in the notation of (3.3) where \( \delta_A \) are the global symmetries). The invariance of \( S_0 \) under the gauge and global symmetries encoded in \( S_1 \) then ensures that \( S_0 + S_1 \) fulfills the extended master-equation up to terms of antighost number \( \geq 1 \). Suppose now that \( S^k = S_0 + S_1 + \cdots + S_k \in \mathcal{E} \) had been constructed so as to satisfy (4.4) and (5.3) up to terms of antighost number \( \geq k \),

\[ (S^k, S^k)' = R_k + O(k + 1), \quad agh(R_k) = k, \quad (5.11) \]
\[ \frac{\partial^R S^k}{\partial \xi^A} = S_A^0 + O(k_A + 1) \quad \forall A: k_A \leq k. \quad (5.12) \]

Taking the extended antibracket of (5.11) with \( S^k \) and using the Jacobi identity for the extended antibracket as well as (5.12), it is possible to infer that \( R_k \) is \( \delta \)-closed, \( \delta R_k = 0 \). Furthermore we have \( R_k \in \mathcal{E} \) since the vector space \( \mathcal{E} \)
is invariant under the extended antibracket \((A, B) \in \mathcal{E} \Rightarrow (A, B)' \in \mathcal{E}\). Due to \(k > 0\), \(\mathcal{E}\) therefore guarantees that \(R_k\) is \(\delta\)-exact in \(\mathcal{E}\),

\[
R_k = -2\delta S_{k+1}, \quad S_{k+1} \in \mathcal{E},
\]

for some \(S_{k+1}\). This in turn implies that \(S^{k+1} = S_0 + \ldots + S_{k+1}\) satisfies the extended master equation up to terms of antighost number \(\geq (k+1)\),

\[
(S^{k+1}, S^{k+1})' = R_{k+1} + O(k+2)
\]

where \(\text{agh}(R_{k+1}) = k + 1\). Note that \((5.13)\) determines \(S_{k+1}\) only up to a \(\delta\)-closed functional in \(\mathcal{E}\). In particular one can always add to \(S_{k+1}\) a term of the form \(\sum_{A,k=0}^k S^0_A \xi^A\) without violating \((5.13)\). Hence, the “boundary conditions” \((5.9)\) can always be fulfilled. Since the arguments apply to all \(k > 0\), we have indeed proved the existence of a solution \(S\) to the extended master equation of the form \((2.3)\) with the required properties. In other words, the inclusion of global ghosts for the rigid symmetries of higher order eliminates the obstructions found in \([13]\).

For the sake of completeness we remark that the boundary conditions \((5.9)\) guarantee that the part of \(S\) which is linear in \(\xi^A\) is indeed of the form \(S^0_A \xi^A\) where \(S^0_A\) is a BRST-invariant completion of \(S^0_A\), cf. \((2.2)\). Indeed, one has

\[
(S', S')' = 0 \Rightarrow (\partial^R S'/\partial \xi^A, S')' = 0 \Rightarrow (S_A, S) = 0
\]

where

\[
S_A = \frac{\partial S'}{\partial \xi^A} |_{\xi=0} = \frac{\partial S}{\partial \xi^A} |_{\xi=0}.
\]

In \((5.15)\) we first differentiated the extended master equation with respect to \(\xi\) and set all the \(\xi^A\) to zero afterwards. \((2.3), (5.13), (5.16)\) and \((5.9)\) show that the part \(S_A\) of \(S\) which is linear in \(\xi^A\) is indeed a BRST-invariant completion of \(S^0_A\), as promised.

A remarkable feature of the extended master equation \((1.1)\) or \((4.4)\) is that it encodes the structure constants of the algebra of rigid symmetries described above. Indeed, given \(S\) and the generators \(S_A \xi^A\) (or actually, just their pieces \(S^0_A \xi^A\) linear in the ghosts), the higher order functionals \(S_{A_1 A_2} \xi^{A_1 A_2}\), and for the structure constants \(f^B_{A_1 A_2}\), are recursively determined by the demand that the extended master equation be satisfied. For instance, one gets relation \((3.3)\) by collecting in Eq \((1.4)\) all the terms that contain three global ghosts and no global antifield. The relation \((3.3)\) then corresponds to the part containing four global ghosts and no \(\xi^*\). The terms involving \(\xi^*\) provide the Jacobi identities \((3.8)\).

The fact that the extended master equation captures the complete algebraic structure of the gauge and the rigid symmetries parallels the property of the usual master equation that encodes all the information on the algebra of gauge transformations, including the Jacobi identities of first and higher order \([12, 13]\). When there are only standard rigid symmetries, Eq. \((1.1)\) reduces to the extended master equation of \([8]\),

\[
(S, S) + \frac{\partial R S}{\partial \xi^A} f^B_{BA} \xi^A \xi^B = 0.
\]

In the absence of rigid symmetries of any order, the extended master equation reduces, of course, to \((2.2)\).

VI. WARD IDENTITIES

From the extended master equation \((4.1)\), the Ward identities for the Green functions can be derived straightforwardly. Since the extended master equation \((4.1)\) is similar to the extended master equation \((5.17)\) of \([8]\), the procedure follows the familiar pattern and we sketch only the main steps.

The generating functional for the Green functions of the theory is given by the path integral

\[
Z_{J,K,\xi} = \int [D\Phi] \exp i\{S^\Psi[\Phi, K, \xi] + \int dx J_a(x) \Phi^a(x)\}.
\]

The functional \(S^\Psi[\Phi, K, \xi]\) appearing in \(Z_{J,K,\xi}\) is obtained from \(S[\Phi, \Phi^*, \xi]\) by making the transformation \(\Phi^*_a = K_a + \frac{\delta}{\delta \Phi^a}\), where the gauge-fixing fermion \(\Psi[\Phi]\) is chosen such that \(S^\Psi[\Phi, 0, 0]\) is completely gauge-fixed. The functional \(S^\Psi[\Phi, K, \xi]\) obeys the same equation \((4.1)\) – with \(\Phi^*\) replaced by \(K\) – as \(S[\Phi, \Phi^*, \xi]\), because the transformation from
Φ* to K is a canonical transformation that does not involve the ξ^A. The fields J_a(x) and K_a(x), as well as the constant ghosts ξ^A, are external sources not to be integrated over in the path integral. Now, perform in Z_{J,K,ξ} the infinitesimal change of integration variables

Φ_a \to Φ_a + (Φ_a, S^Φ) = Φ_a + \frac{δL S^Φ}{δK_a}.

(6.2)

Using (4.1) and assuming the measure to be invariant 4, the following Ward identity results for Z_{J,K,ξ}:

\int dx J_a(x) \frac{δL Z_{J,K,ξ}}{δK_a(x)} - \sum_{r \geq 2} \frac{1}{r!} \frac{∂^R Z_{J,K,ξ}}{∂ξ^B} f^B_{A_1...A_r} ξ^{A_r} ... ξ^{A_1} = 0.

(6.3)

Since Eq. (6.3) is a linear functional equation on Z_{J,K,ξ}, the generating functional W = −i ln Z_{J,K,ξ} for the connected Green functions obeys the same identity. Performing the standard Legendre transformation

Γ[Φ^c, K, ξ] = W[J, K, ξ] - \int dx J_a(x) Φ^c_a(x), \quad Φ^c_a(x) = \frac{δL W}{δJ_a(x)}, \quad J_a(x) = -\frac{δR Γ}{δΦ^c_a(x)}

(6.4)

one finds that the effective action Γ fulfills a Ward identity of the same form as (4.1),

\int dx \frac{δR Γ}{δΦ^c_a(x)} \frac{δL Γ}{δK_a(x)} + \sum_{r \geq 2} \frac{1}{r!} \frac{∂^R Γ}{∂ξ^B} f^B_{A_1...A_r} ξ^{A_r} ... ξ^{A_1} = 0.

(6.5)

The Ward identities (6.3, 6.5) capture the consequences of both the local and the global symmetries for the generating functionals. They hold even when the gauge fixing fermion is not invariant under the rigid symmetries, provided there are no anomalies (see below). Indeed, no condition was ever assumed on the gauge fixing fermion, except that it should fix the gauge. This is of course, as it should, since the (BRST-invariant) physical subspace yields a true representation of the symmetry [25].

The identities on the Green functions are obtained in the usual manner, by differentiating (6.3, 6.5) with respect to the sources and setting these sources equal to zero afterwards. In particular, since the rigid symmetries of higher order have their own rigid ghosts, they lead to independent identities.

VII. EXAMPLE

In order to illustrate the above formulas, consider the simple case of a 2-form abelian gauge field B_{μν} with classical action

S_0 = \int dx \left( -\frac{1}{12} F^μνρ F^μνρ \right), \quad F^μνρ = \partial_ρ B^μν.

(7.1)

This theory is invariant not only under the well known first stage reducible gauge transformations, but also (among other rigid symmetries) under the following two global transformations:

B_{μν} \to B_{μν} + a^σ \partial_σ B_{μν}, \quad a^μ = constant

(7.2)

4If the measure is not invariant, one must replace the extended master equation (4.1) by the quantum extended master equation, which reads

(\mathcal{S}, \mathcal{S}) + 2i ∆ S + 2 \sum_{r \geq 2} \frac{1}{r!} \frac{∂^R S}{∂ξ^B} f^B_{A_1...A_r} ξ^{A_r} ... ξ^{A_1} = 0

where of course a meaningful (regularized) definition has to be given to ∆. The Ward identities (6.3) and (6.5) are unchanged in absence of anomalies.
and
\[ B_{\mu \nu} \rightarrow B_{\mu \nu} + b_{\mu \nu \rho \sigma} x^\sigma, \quad b_{\mu \nu \rho \sigma} = b_{[\mu \nu \rho \sigma]} = \text{constant}. \quad (7.3) \]

This exactly parallels the abelian 1-form case treated in \[13\].

The solution of the master-equation incorporating only the gauge-symmetries reads
\[ S = \int dx \left\{ -\frac{1}{12} F_{\mu \nu \rho} F^{\mu \nu \rho} + B^{\ast \mu \nu} (\partial_\mu C_\nu - \partial_\nu C_\mu) + C^{\ast \mu} \partial_\mu C \right\}, \quad (7.4) \]

where the \( C_\mu \) are the ghosts corresponding to the gauge-symmetries and \( C \) stands for the second-order ghost related to the reducibility of the gauge-symmetries. In order to find the generators of the two global symmetries, one just has to calculate the BRST invariant extension of the corresponding ghost-independent pieces \( S^0_\sigma = \int dx B^{\ast \mu \nu} \partial_\sigma B_{\mu \nu} \) and \( S^0_{\mu \nu \sigma} = \int dx B^{\ast [\mu \nu \sigma]} \). The result is
\[ S_\sigma = \int dx \left\{ B^{\ast \mu \nu} \partial_\sigma B_{\mu \nu} + C^{\ast \mu} \partial_\sigma C_{\mu} + C^{\ast} \partial_\sigma C \right\} \quad (7.5) \]

and
\[ S^{\mu \nu \sigma} = \int dx \ B^{\ast [\mu \nu \sigma]}. \quad (7.6) \]

In addition to the global symmetries discussed above, the model possesses a conservation law of order three which reads explicitly \( \partial_\mu F^{\mu \nu} \approx 0 \) and corresponds to the symmetry of the master-equation generated by
\[ S_{C^\ast} = \int dx \ C^\ast. \quad (7.7) \]

In fact, \( S_{C^\ast} \) can be regarded as the representative of the non-trivial BRST-cohomology at ghost-number \(-3\).

\( S_{C^\ast} \) has to be taken into account because otherwise the algebra generated by \( S_\sigma \) and \( S^{\mu \nu \sigma} \) would not close. Indeed one gets the following antibrackets:
\[ (S_\sigma, S^{\mu \nu \rho}) = (S, S^{\mu \nu \rho}), \]
\[ (S_\sigma, S^{\mu \nu \rho}) = (S, S_{\lambda\rho}), \]
\[ (S_\sigma, S^{\mu \nu \rho}_{\lambda \tau}) = \delta^\mu_\rho \delta^\nu_\sigma \delta^\rho_\tau S_{C^\ast}. \quad (7.8) \]

where
\[ S^{\mu \nu \rho}_\sigma = -\frac{1}{2} \int dx \ C^\ast [\mu \nu \rho] \delta^\sigma_\delta, \]
\[ S^{\mu \nu \rho}_{\lambda \tau} = \frac{1}{2} \int dx \ C^\ast [\mu \nu \rho] \delta^\tau_\lambda \delta^\sigma_\delta. \]

All the antibrackets of \( S^{\mu \nu \rho}, S^{\mu \nu \rho}_\sigma, S^{\mu \nu \rho}_{\sigma \lambda} \) and \( S_{C^\ast} \) vanish because these generators involve only the antifields, but not the fields. Together with \((S_\sigma, S_{C^\ast}) = 0\) we therefore get a closed algebra of the type described in section \[11\] with
\[ \{S_{\alpha_1}\} \equiv \{S_\sigma, S^{\mu \nu \rho}, S_{C^\ast}\}, \]
\[ \{S_{\alpha_1, \alpha_2}\} \equiv \{S^{\mu \nu \rho}_\sigma\}, \quad \{S_{\alpha_1, \alpha_2, \alpha_3}\} \equiv \{S^{\mu \nu \rho}_{\sigma \lambda}\} \quad (7.9) \]

and nonvanishing structure constants \( f^\beta_{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \) of fourth order. Note that it is the last identity in \(7.8\) which makes it necessary to include \( S_{C^\ast} \).

Introducing global ghosts \( \xi^\sigma, \xi_{\mu \nu \rho} \) and \( \xi \) corresponding to \( S_\sigma, S^{\mu \nu \rho} \) and \( S_{C^\ast} \) respectively, the extended master-equation \( (4.1) \) has the following form:
\[ (S, S) + \int dx \ C^\ast \xi_{\mu \rho \sigma} \xi^\rho \xi^\sigma \xi^\tau = 0. \quad (7.10) \]

All the ghosts have odd Grassmann-parity. \( \xi^\sigma \) and \( \xi_{\mu \rho \sigma} \) have ghost-number 1 while \( \xi \) has ghost-number 3. The solution \( S \) reads
\[ S = \int dx \left\{ -\frac{1}{4} F_{\mu
u} F^{\mu\nu} + B^{*\mu\nu} (\partial_\mu C_\nu - \partial_\nu C_\mu) + C^{*\mu} \partial_\mu C \right. \\
+ B^{*\mu\nu} (\partial_\mu B_\nu - \partial_\nu B_\mu) \xi^\sigma + x^\sigma \xi_{\mu\sigma}) + C^{*\mu} \partial_\sigma C_\mu \xi^\sigma + C^{*} (\partial_\sigma C \xi^\sigma + \xi) \right. \\
\left. - \frac{1}{2} C^{*\mu} x^\nu \xi_{\mu\nu\sigma} \xi^\sigma + \frac{1}{2} C^* x^\mu \xi_{\mu\rho\sigma} \xi^\sigma \right\} \]

The master-equation can be cast in the standard form \((\text{4.4})\) by adding to \(S\) a term depending on the global antifield \(\xi^*\):

\[ S' = S + \frac{1}{2} \xi^* \xi_{\mu\rho\sigma} \xi^\rho \xi^\sigma. \]

The Ward identities associated with the higher order conservation laws may yield useful information on the Green functions. For instance, in the more general case of an interacting 2-form abelian gauge field \(B_{\mu\nu}\) with Lagrangian

\[ \mathcal{L} = \mathcal{L}(F_{\lambda\mu\nu}, \partial_\rho F_{\lambda\mu\nu} \ldots), \]

the conservation law of order three \(\partial_\alpha(\delta \mathcal{L}/\delta F_{\alpha\mu\nu}) \approx 0\) survives. Also the global invariance Eq. (7.2) remains valid, in contrast to the second global symmetry (7.3) which is not a symmetry of \(\mathcal{L}\) in general. Dropping the latter symmetry, the analogous extended solution reads

\[ S = \int dx \left\{ \mathcal{L} + B^{*\mu\nu} (\partial_\mu C_\nu - \partial_\nu C_\mu) + C^{*\mu} \partial_\mu C \right. \\
+ B^{*\mu\nu} \partial_\sigma B_{\mu\nu} \xi^\sigma + C^{*\mu} \partial_\sigma C_\mu \xi^\sigma + C^{*} (\partial_\sigma C \xi^\sigma + \xi) \right\} \]

and the Ward identity (6.3) then implies

\[ \int dx J_{(C)}(x) (\partial_\sigma C(x) \xi^\sigma + \xi) = 0. \]

The \((\xi\text{-dependent part of})\) identity (6.3) for the effective action implies

\[ \int dx \frac{\delta \Gamma}{\delta C(x)} = 0, \]

where \(\Gamma = \Gamma(K = 0, \xi = 0)\). The identity (7.10) expresses the invariance of the effective action \(\Gamma\) under constant shifts of the ghost of ghost \(C(x)\).

\section*{VIII. ANOMALIES}

Let us finally discuss some applications of the resulting extended antifield formalism that parallel analogous applications of the usual BRST formalism. Consider for instance the problem of anomalies in the rigid symmetries. They can be analysed along the algebraic lines initiated in the pioneering work [22]. The procedure is explained in [7,8], and we just recall here the main arguments, taking into account the higher order symmetries. An anomaly appears as a violation of the master equation (6.3) for the regularized \(\Gamma\) and must fulfill, to lowest loop order, the generalized Wess-Zumino consistency condition [23]

\[ D \int dx a = 0 \]

where the extended BRST differential \(D\) is defined (in the functional space \(E\)) by

\[ DX \equiv (X, S')' \Rightarrow D^2 = 0. \]

and takes into account all symmetries. For local functionals not involving the \(\xi_\lambda\), like \(\int dx a, D\) takes the form

\[ X = \int dx a \Rightarrow DX = (X, S) + \sum_{r \geq 2} \frac{1}{r!} \frac{\partial^r X}{\partial \xi_1^{A_1} \ldots \xi_1^{A_r}} \xi_1^{A_1} \ldots \xi_1^{A_r}. \]

An anomaly (for the local or global symmetries, or combinations thereof) is a solution of (8.3) that is non-trivial, i.e. not of the exact form \(D \int dx b\). The investigation of the possible anomalies in the local and global symmetries of all orders is accordingly equivalent to the problem of computing the cohomology of the extended BRST differential \(D\) at ghost number one. Terms proportional to the rigid ghosts would define anomalies in the corresponding rigid symmetries.

The problem of computing the cohomology of \(D\) in the space of local functionals is equivalent to the problem of computing the cohomology of \(D\) modulo the spacetime exterior derivative in the space of local volume forms, provided the fields decrease fast enough at infinity. (This excludes instanton-like configurations, [24, 28]; see also [29].)
IX. DEFORMATIONS

Consider next the question of whether it is possible to deform a given action continuously such that it remains invariant under (possibly deformed) gauge and global symmetries. This problem can be efficiently and systematically studied in the antifield formalism along the lines of [19,20] by looking for a solution to the extended master equation of the form

\[ S'_\tau = S' + \tau S^{(1)'} + \tau^2 S^{(2)'} + \ldots \]  

(9.1)

where \( S' \) is the original (undeformed) solution to the extended master equation containing the gauge symmetries and the global symmetries in question, and \( \tau \) is a constant deformation parameter. To first order in \( \tau \) the extended master equation for \( S'_\tau \) then requires

\[ (S^{(1)'}, S')' \equiv D S^{(1)'} = 0. \]  

(9.2)

The first order deformation \( S^{(1)'} \) is thus invariant under the extended BRST operator \( D \). This allows to classify the possible deformations in question (to first order in \( \tau \)) through investigating the \( D \)-cohomology at ghost number 0. Note that we have used the extended master equation in its compact form (4.4). This is particularly useful in this context because it automatically takes into account that some of the structure constants \( f^B_{A_1 \cdots A_r} \) may get deformed too.

In general it will be neither possible nor desirable to promote all the global symmetries of the original action to symmetries of a nontrivially deformed action. Therefore in general one would actually include only a physically important subset of the global symmetries in \( S \). An instructive example is the deformation of free abelian gauge theories to Yang–Mills theories which was discussed in [21] along the lines of [19]. Clearly Yang–Mills theories have less global symmetries than the corresponding free abelian gauge theories (which have in fact infinitely many nontrivial global symmetries), showing that indeed not all the global symmetries can be maintained. However, it is evidently possible to keep at least the physically important Poincaré symmetries.

A similar application is the construction and classification of actions which are invariant under prescribed gauge and global symmetries whose commutator algebra closes off-shell. This problem can be studied through the \( D \)-cohomology at ghost number 0 too. An example for such an investigation (rigid N=1 supersymmetry in four dimensions) can be found in [12].

X. CONCLUSIONS

We have developed in this paper the master equation formalism for both local and rigid symmetries. The key to overcoming difficulties encountered in the past is to introduce global ghosts for all the rigid symmetries, and not just for those of first order. We have shown explicitly how to incorporate in an appropriately extended master equation the higher order rigid symmetries associated with higher order conservation laws \( \partial_{\mu_1 j} \cdots \cdots \approx 0 \). This leads to new Ward identities and, more importantly, avoids the obstructions that may be encountered when trying to construct a solution of the extended master equation (5.17) that does not take these higher order conservation laws into account. While (4.4) is never obstructed, (5.17) may fail to have local solutions in the presence of higher order symmetries. Of course, (5.17) will not be obstructed if a subset of first order conservation laws is used that defines a subalgebra in the above sense. But otherwise obstructions can – and do – arise [13]. Furthermore, the Ward identities associated with the higher order conservation laws may yield useful information on the Green functions. Finally, the extended antifield approach turns out to be useful in the systematic analysis by cohomological techniques of deformation problems or anomaly issues.

Acknowledgements: This work has been supported in part by research funds from the F.N.R.S. (Belgium) and research contracts with the Commission of the European Community. F.B. has been supported by the Spanish ministry of education and science (MEC).

[1] A. M. Vinogradov, Sov. Math. Dokl. 18 (1977) 1200; Sov. Math. Dokl. 19 (1978) 144; Sov. Math. Dokl. 20 (1979) 985; J. Math. Anal. Appl. 100 (1984) 1.
[2] R.L. Bryant and P.A. Griffiths, J. Amer. Math. Soc. 8 (1995) 507.
[3] G. Barnich, F. Brandt and M. Henneaux, Commun. Math. Phys. 174 (1995) 57.
[4] M. Henneaux, B. Knaepen and C. Schomblond, Commun. Math. Phys. 186 (1997) 137.
[5] D. N. Page, Phys. Rev. D 28 (1983) 2976.
[6] J. Zinn-Justin, Renormalisation of Gauge Symmetries, in: Trends in Elementary Particle Theory, Lecture Notes in Physics 37, Springer, Berlin, Heidelberg, New York 1975; Quantum Field Theory and Critical Phenomena, 2nd Edition, Clarendon Press, Oxford 1993.
[7] G. Bonneau and F. Delduc, Nucl. Phys. B 266 (1986) 536; articles in Renormalization of Quantum Field Theories with Non-linear Field transformations, P. Breitenlohner, D. Maison and K. Sibold eds., Lecture Notes in Physics 303, Springer Verlag, Berlin (1988).
[8] A. Blasi and R. Collina, Nucl. Phys. B 285 (1987) 204; C. Becchi, A. Blasi, G. Bonneau, R. Collina and F. Delduc, Commun. Math. Phys. 120 (1988) 121; see also contribution of A. Blasi to second ref.
[9] O. Piguet and S. P. Sorella, Algebraic Renormalization, Lecture Notes in Physics m28, Springer Verlag, Berlin, Heidelberg, 1995.
[10] F. Delduc, N. Maggiore, O. Piguet and S. Wolf, Phys. Lett. B 385 (1996) 132.
[11] L. Bonora, P. Pasti and M. Tonin, Phys. Lett. B 156 (1985) 341;
C. Becchi, A. Blasi, G. Bonneau, R. Collina and F. Delduc, Commun. Math. Phys. 120 (1988) 121;
R. Kaiser, Z. Phys. C 39 (1988) 585;
P. Altevogt and R. Kaiser, Z. Phys. C 43 (1989) 455;
P. Howe, U. Lindstrom and P. White, Phys. Lett. B 246 (1990) 430;
L. Baulieu, M. Bellon, S. Ouvry and J.-C. Wallet, Phys. Lett. B 252 (1990) 387;
J. A. Dixon, Commun. Math. Phys. 140 (1991) 169;
F. L. White, Class. Quant. Grav. 9 (1992) 413, 1663;
F. Brandt, Phys. Lett. B 320 (1994) 57;
G. Bonneau, Phys. Lett. B 333 (1994) 46; Helv. Phys. Acta 67 (1994) 930, 954;
N. Maggiore, O. Piguet and S. Wolf, Nucl. Phys. B 458 (1994) 403;
N. Maggiore, Int. J. Mod. Phys. A 10 (1995) 3781, 3937;
N. Maggiore, O. Piguet and M. Ribordy, Helv. Phys. Acta 68 (1995) 264.
[12] F. Brandt, Nucl. Phys. B 392 (1993) 428.
[13] F. Brandt, M. Henneaux and A. Wilch, Phys. Lett. B 387 (1996) 320.
[14] B. Julia, Effective Gauge Fields and Generalized Noether Theorem, preprint LPTENS 80/14, published in Johns Hopkins Workshop 1980 (QCD161:J55:1980) 295.
[15] I. A. Batalin and G. A. Vilkovisky, Phys. Lett. B 102 (1981) 27, Phys. Rev. D 28 (1983) 2567, Phys. Rev. D 30 (1984) 508.
[16] M. Henneaux, Commun. Math. Phys. 140 (1991) 1.
[17] C. Lucchesi, O. Piguet and K. Sibold, Int. J. Mod. Phys. A 2 (1987) 329.
[18] M. Henneaux and C. Teitelboim, Quantization of Gauge Systems, Princeton University Press 1992.
[19] G. Barnich and M. Henneaux, Phys. Lett. B 311 (1993) 123.
[20] J. Stasheff, h-alg/9702012.
[21] G. Barnich, M. Henneaux and R. Tatar, Int. J. Mod. Phys. D 3 (1994) 139.
[22] C. Becchi, A. Rouet and R. Stora, Ann. Phys. 98 (1976) 287.
[23] J. Wess and B. Zumino, Phys. Lett. B 37 (1971) 95.
[24] J. de Azcárraga and J. Pérez Bueno, Commun. Math. Phys. 184 (1997) 669.
[25] M. Henneaux, Nucl. Phys. B 308 (1988) 619.
[26] S. Weinberg, The Quantum Theory of Fields, Cambridge University Press 1996.
[27] A. Schwarz, Quantum Field Theory and Topology, Springer 1993.
[28] G. ’t Hooft, Phys. Rev. Lett. 37 (1976) 8.
[29] R. Amorim and N. Braga, hep-th/9612118.