Performing in this paper the duality and statefinder diagnostic for a class of Quintom models of Dark Energy which is constructed by the spinor field, we study the possible connections among different Spinor Quintom models and differentiate the Quintom dark energy in spinor scenario from the others. A class of evolutionary trajectories of these Quintom models in Spinor field are plotted in the statefinder parameter planes. We also find that the power-law-like potential also plays important role in this class of models from the statefinder viewpoint.

I. INTRODUCTION

There is mounting datum from type Ia supernovae and cosmic microwave background (CMB) radiation and so on[1, 2, 3, 4] have provided strong evidences for the present spatially flat and accelerated expanding universe which is dominated by dark sectors. Combined analysis of the above cosmological observations support that the energy of our universe is occupied by dark energy (DE) about 73%, dark matter about 23% and usual baryon matter only about 4% which can be described by the well known particle theory. In the context of Friedmann-Robertson-Walker (FRW) cosmology, this acceleration is attributed to an exotic form of negative pressure, the so-called DE. So far, the nature of dark energy remains a mystery. Theoretically, the obvious candidate for such a component is a small cosmological constant \( \Lambda \) (or vacuum energy) with equation of state \( w = -1 \), while the cosmological model that consists of a mixture of Vacuum energy and cold dark matter (CDM) is called LCDM (or \( \Lambda \) CDM), but it is at the expense of the difficulties associated with the fine tuning and the coincidence problems. The inspiration from inflation has suggested the idea that dark energy is due to a dynamical component with one- or multi-scalar fields, such as the Quintessence[1, 2], Phantom[3], K-essence[4, 5]. However, a positive kinetic term in the Lagrangian of Quintessence dark energy model may violate the strong energy condition. On the other hand, the negative kinetic term in Phantom scenario leads to some quantum instabilities such as the violation of the dominant energy condition, as well as the occurrence of the phenomenon of Big Rip[6]. There are also other models, including Chaplygin gas[7], braneworld models[8, 9], holographic models[10, 11, 12, 13], and so on, which are resorted to accounting for the present cosmic accelerating expansion.

Although the recent fits to the data in combination of WMAP[14, 15], the recently released 182 SNIa Gold sample[16] and also other cosmological observational data show remarkably the consistence of the cosmological constant, it is worth of noting that a class of dynamical models with the equation-of-state (EoS) across \( -1 \) Quintom is mildly favored[20, 21, 22, 23, 24]. In the literature there have been a lot of theoretical studies of Quintom-like models. For example, motivated from string theory, the authors of Ref. 25 realized a Quintom scenario by considering the non-perturbative effects of a generalized DBI action. Moreover, a No-Go theorem has been proved to constrain the model building of Quintom[26], and according to this No-Go theorem there are models which involve higher derivative terms for a single scalar field[27], models with vector field[28], making use of an extended theory of gravity[29], non-local string field theory[30], and others (see e.g. [31, 32, 33, 34, 35, 36, 37, 38, 39]). The similar work applied in scalar-tensor theory is also studied in Ref. 40.

Previously, it has been considered that a Quintom dark energy model and its combination with Chaplygin gas fluid can be realized with non-regular spinor matter[41]. Interestingly, we find that this type of model can realize many kinds of Quintom scenario with transforming the form of potential of the Spinor. To understand the possible combinations among different types of Quintom model in spinor field, in this paper we study the implications of cosmic duality with this class of models. Cosmic duality is a mathematical feature which origins from string cosmology[42, 43] and was later considered to link the standard cosmology and phantom cosmology together, see Refs. 44, 45, 46, 47. In Ref. 48, it was pointed out that there is a behavior of dual between the models of Quintom-A and Quintom-B. By studying the behavior of the energy density and pressure in spinor field we find a dual of the Quintom-A and Quintom-B. Meanwhile, we realize additional Quintom models by the aid of this property. Since more and more dark energy models have been proposed to explain the current cosmic acceleration, a method to discriminate between the various contenders in a model independent manner was introduced by Sahni in 49. The new cosmological diagnostic pair \( \{r, s\} \), called the statefinder, is a geometrical diagnostic which is algebraically related to the higher derivatives of the scale factor \( a \) with respect to time and is a natural next step beyond the Hubble parameter \( H \equiv \frac{\dot{a}}{a} \) and the deceleration parameter \( q \) which depends on \( a \). In this way, the dark energy can be distinguished more universally.
than the model-dependent physical variables depending on the properties of physical fields describing the dark energy. We plot the trajectories in the $r-s$ plane corresponding to these kind of Quintom dark energy model. Departure from the fixed point $(r,s) = (1,0)$, for one given Quintom model in spinor field, in correspondence with the spatially flat LCDM scenario provides a nice way of setting up the distance from LCDM.

This letter is organized as follows: In section 2, we investigate the implications of the cosmic duality in Spinor Quintom models of dark energy. In section 3, we apply the statefinder diagnostic to the Spinor Quintom dark energy models. Section 4 contains discussions and conclusions.

II. DUALITY OF SPINOR QUINTOM UNIVERSES

To begin with the discussion, we consider a universe filled with Quintom dark energy in spinor field, neglecting the contributions of the components of matter and radiation. And we deal with the homogeneous and isotropic Friedmann-Robertson-Walker (FRW) space-time, assuming the space-time metric as

$$ds^2 = dt^2 - a^2(t) dx^2.$$  

In the aid of the dynamics of a spinor field which is minimally coupled to Einstein’s gravity, we can write down the following Dirac action in a curved space-time background

$$S_\psi = \int d^4x \ e \left[ \frac{i}{2} (\bar{\psi} \Gamma^a D_a \psi - D_a \bar{\psi} \Gamma^a \psi) - V \right]$$
$$= \int d^4x \ e \ L_\psi,$$  

Here, $e$ is the determinant of the vierbein $e^\mu_a$ and $V$ stands for any scalar function of $\psi, \bar{\psi}$ and possibly additional matter fields. We will assume that $V$ only depends on the scalar bilinear $\bar{\psi}\psi$. For a gauge-transformed homogeneous and a space-independent spinor field, the equation of motion of spinor reads

$$\dot{\psi} + \frac{3}{2} H \psi + i \gamma^0 V' \psi = 0,$$  

$$\dot{\bar{\psi}} + \frac{3}{2} H \bar{\psi} - i \gamma^0 V' \bar{\psi} = 0,$$  

where a dot denotes a time derivative while a prime denotes a derivative with respect to $\psi \bar{\psi}$, and $H$ is Hubble parameter. To take a further derivative, we can obtain the solution of equation of motion:

$$\ddot{\psi} \bar{\psi} = \frac{N}{a^3},$$  

where $N$ is a positive time-independent constant and we define it as present value of $\psi \bar{\psi}$.

In this section we study the duality of the Quintom universe with spinor matter. Firstly, with the Lagrangian given by Eq. (2), we review the connection between Quintom-A and Quintom-B with the duality which has been studied by Ref. [53] with two fields consists of quintessence-like and phantom-like. Here, we give the new dualities between the two models. Furthermore, the duality of Quintom model will be extended to the Spinor Quintom twice crossing $-1$.

In the framework of FRW cosmology, the Friedmann equation reads

$$H^2 = \frac{1}{3} \rho,$$  

where we use units $8\pi G = \hbar = c = 1$ and all parameters are normalized by $M_p = 1/\sqrt{8\pi G}$ in the letter.

Following the work of Ref. [54, 55] there is a form-invariant transformation by defining a group of quantities as follow:

$$\bar{\rho} = \bar{\rho} (\rho),$$
$$\bar{H} = -\left(\frac{\bar{\rho}}{\rho}\right)^{\frac{1}{2}} H.$$  

Under this transformation, we obtain the corresponding changes for the pressure $p$ and the EoS $w$,

$$\bar{p} = -\bar{\rho} - \frac{3}{2} \bar{\rho} (\rho + p) \frac{dp}{d\rho},$$
$$\bar{w} = -1 - \frac{3}{2} \bar{\rho} \frac{d\rho}{dp} (1 + w).$$  

Taking $\bar{\rho} = \rho$ in Eqs. (9) and (10) as an example of detailed discussion without loss of the generality of the physical conclusion and information, we can get the dual transformation:

$$\bar{H} = -H,$$  

$$\bar{\rho} = -2p - p = -V' \bar{\psi} \psi - V,$$  

$$\bar{w} = -2 - w = 1 - \frac{V' \bar{\psi} \psi}{V}.$$  

Consequently, the dual form of Lagrangian reads

$$\bar{L} = \frac{i}{2} (D_\mu \bar{\psi} \Gamma^\mu \psi - \bar{\psi} \Gamma^\mu D_\mu \psi) - V.$$  

Contrasting the Lagrangian derived from Eq. (2) and its dual form in (14), we may see, with the dual transformation, if the original Lagrangian is for a Quintom-A model the dual one is for a Quintom-B one, and vice versa. With this property, it is possible that the early Universe can be linked to other epochs of the universe. While from the Eq. (13), one can expect a symmetrical

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1 We notice such a duality has been investigated in Ref. [50], however, in phantom cosmology.
evolutions of the EoS comparing with its dual:

(i). there is

\[ V' < 0 \rightarrow V' > 0 , \]

which gives a Quintom-A scenario by describing the universe evolves from Quintessence-like phase with \( w_\psi > -1 \) to Phantom-like phase with \( w_\psi < -1 \);

(ii). there is

\[ V' > 0 \rightarrow V' < 0 , \]

which gives a Quintom-B scenario for which the EoS is arranged to change from below \(-1\) to above \(-1\);

For our detailed discussions, we will consider three kind of special forms of power-law-like potentials and perform its semi-analytic solution to present the dual characteristics, then we take special form of potentials to study its numerical solutions.

In the first instance, we take the potential as \( V = V_0[(-\bar{\psi}\psi - b)^2 + c] \) which can realize Quintom-A scenario, and the detailed discussion can be found in Ref. [41]. Its dual form of the solution is a description of universe in the case of Quintom-B. According to Eq. (3), one finds that \( \bar{\psi}\psi \) is decreasing along with an increasing scale factor \( a \) during the expansion of the universe. From the formula of \( V' \), we deduce that at the beginning of the evolution the scale factor \( a \) is very small, and so \( \bar{\psi}\psi \) becomes very large and make sure \( V' > 0 \) at the beginning. Then \( \bar{\psi}\psi \) decreases along with the expanding of \( a \). At the moment of \( \bar{\psi}\psi = b \), one can see that \( V' = 0 \) which results in the EoS \( w_\psi = -1 \). After that \( V' \) becomes less than 0, so the universe enters a Phantom-like phase. Finally the universe approaches to a de-Sitter space-time in the Quintessence Phase in the future. Accordingly, the EoS of the dual form evolves from the \( w < -1 \) and crosses \(-1\) by \( t \rightarrow 0 \) from below \(-1\) to above \(-1\) then increases and declines finally approaches to the cosmological constant when \( t \rightarrow +\infty \). It is shown that either Quintom-A or Quintom-B will avoid a big rip when \( w < -1 \). In Fig. 1 we plot the concrete picture of this dual pair. One can find this two models dual to each other rigorously.

In succession, if \( V = V_0[(-\bar{\psi}\psi - b)\bar{\psi}\psi + c] \), one can obtain a Quintom-B model (see Ref. [41]). Take its dual form theoretically given by the above transformation, we can present the numerical solution in Fig. 2. Clearly, the duality of this Spinor Quintom model show the evolutionary picture of Quintom-A.

These two class of models describe different behaviors of the cosmological evolution with one in the expanding phase while the other in the contracting one depending on the potential and initial conditions we choose. It is found that Quintom model and its dual form are symmetric around \( w = -1 \).

For an extending investigation, we take \( V = V_0[(\bar{\psi}\psi - b)^2\bar{\psi}\psi + c] \), which can realize a picture across \( w = -1 \) twice, for discussion. In Fig. 3 one can see its dual model that evolves from its symmetrical side with respect to \(-1\) and crosses \(-1\) two times and ultimately approaches to the cosmological constant boundary when \( t \rightarrow +\infty \).

From the above analysis, we may investigate the connections among different periods or phases of our universe with the help of this character. As is known that with different type of dark energy, the fate of the universe will be different. Our study in this section helps understand the properties of various dark energy models and their connections to the evolution and the fate of the Universe.

![Fig. 1: Plot of the evolution of the EoS of Quintom-A and its dual Quintom-B as a function of time for \( V = V_0[(\bar{\psi}\psi - b)^2 + c] \). In the numerical calculation we take \( V_0 = 1.0909 \times 10^{-117} \). For the model parameters we choose \( b = 0.05, c = 10^{-3} \). For the initial conditions we take \( (\bar{\psi}\psi)_0 = 0.051 \).](image1)

![Fig. 2: Plot of the evolution of the EoS of Quintom-B and its dual Quintom-A as a function of time for \( V = V_0[-(\bar{\psi}\psi - b)\bar{\psi}\psi + c] \). In the numerical calculation we take \( V_0 = 1.0909 \times 10^{-117} \). For the model parameters we choose \( b = 0.05, c = 10^{-3} \). For the initial conditions we take \( (\bar{\psi}\psi)_0 = 0.051 \).](image2)
Moreover, the past and future properties can be understood by studying the above characters. One application of combining these properties together is Quintom-like bouncing cosmology, which has been intensively studied in Refs. [35, 36, 37]. In the meanwhile, we also realize three additional Quintom models.

III. STATEFINDER DIAGNOSTIC TO SPINOR QUINTOM MODELS

Based on the above discussions, it can be seen that there are so many dark energy models proposed to explain the cosmic acceleration, thus how to secrern these models become a widely attentional issue. With this regards, Sahni proposed a geometrical–constructed from space-time metric directly– statefinder diagnostic pair \{r, s\}, which is defined as

\[ r = \frac{\ddot{a}}{aH^2}, \quad s = \frac{r - 1}{3(q - \frac{1}{2})}. \]  

(15)

Here \( q \) is the deceleration parameter with the definition

\[ q = -\frac{\ddot{a}}{aH^2}. \]  

(16)

Accordingly, by showing different evolutionary trajectories qualitatively in the \( r - s \) and \( r - q \) diagram this statefinder pair can distinguish different dark energy models from the others. Hitherto, some dark energy models, such as Quintessence, Phantom, the Chaplygin gas, braneworld models, holographic models, and interacting and coupling dark energy models, have been perfectly differentiated, correlative works have been performed by Ref. [59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71]. In what follows we will apply the statefinder diagnostic to three Quintom models in spinor field—Quintom-A, Quintom-B, and the Quintom model crossing \(-1\) two times. We will use the form of the statefinder parameter written by pressure and energy density in the following text,

\[ r = 1 + \frac{9(\rho + p)}{2\rho p}, \quad s = \frac{(\rho + p)p}{\rho \rho}. \]  

(17)

where the energy density and pressure are given by Ref. [41].

Taking components of dark matter and dark energy into account in a spatially flat universe, we can write down the Friedmann equation:

\[ H^2 = \frac{1}{3}(\rho_\psi + \rho_m). \]  

(18)

Here \( \rho_m \) is the energy density of dark matter with EoS \( p_m = (\gamma_m - 1)\rho_m \). Ignoring the interaction between the two dark sectors, we can see the energy both of dark energy and of dark matter are conserved and satisfy its continuity equation respectively,

\[ \dot{\rho}_\psi + 3H(\rho_\psi + p_\psi) = 0, \]  

(19)

\[ \dot{\rho}_m + 3H\rho_m = 0. \]  

(20)

As a result, we obtain the following expressions,

\[ r = 1 + \frac{9}{2}w_\psi(1 + w_\psi)\Omega_\psi + \frac{9}{2}w_\psi^2\Omega_\psi, \]  

(21)

\[ s = 1 + w_\psi + \frac{w'}{w_\psi}, \]  

(22)

and the deceleration parameter

\[ q = \frac{1}{2} + \frac{3}{2}w_\psi\Omega_\psi, \]  

(23)

where \( \Omega_\psi = \frac{\rho_\psi}{\rho} \) is the fraction of dark energy density, and \( w' \) is the derivative of equation of state with respect to \( \psi\psi \).

Hereinafter, we will study the statefinder diagnostic for the Spinor Quintom models with three different potentials. Firstly, we discuss the Quintom-A model with the form of potential \( V = V_0[(\psi\psi - b)^2 + c] \), where \( V_0, b, c \) are undefined parameter. In Fig. 3 and Fig. 5 we show the time evolution of statefinder pair \{r, s\} and \{r, q\}. One can see in \( r - s \) diagram the trajectory of this case will be close to the LCDM fixed point.

Next, the trajectories of Spinor Quintom-B model with potential \( V = V_0[-(\psi\psi - b)\psi\psi + c] \) will be plotted. In numerical calculations, we take the same value with Quintom-A model. It can be seen that the evolutionary graphics of Quintom-B is roughly opposite to that of
FIG. 4: The $r$-$s$ diagram with the form of potential $V = V_0[\overline{\psi} \psi - b]^2 + c]$. Evolution trajectories of $r(s)$ in the numerical calculation we take $V_0 = 1.0909 \times 10^{-117}$. For the model parameters we choose $b = 0.05, c = 10^{-3}$. For the initial conditions we take $(\overline{\psi} \psi)_0 = 0.051$. 

FIG. 6: The $r$-$s$ diagram with the form of potential $V = V_0[-(\overline{\psi} \psi - b) \overline{\psi} \psi + c]$. Evolution trajectories of $r(s)$ in the numerical calculation we take $V_0 = 1.0909 \times 10^{-117}$. For the model parameters we choose $b = 0.05, c = 10^{-3}$. For the initial conditions we take $(\overline{\psi} \psi)_0 = 0.051$.

Quintom-A in both $\{r, s\}$ and $\{r, q\}$ diagrams, see Fig. 6 and Fig. 7 for a clear cognition.

Finally, we turn to the case of crossing the cosmological boundary two times. The phase portraits of $\{r, s\}$ and $\{r, q\}$ are presented in Fig. 8 and Fig. 9 respectively, where the values of parameters are also the same with those of the case of Quintom-A.

We can see that in $r-s$ diagram either of the portraits is very close to the fixed point $\{r, s\} = \{1, 0\}$ corresponding to the LCDM with FRW cosmological model at some period but not passing it. In the case of Quintom-A, it is found that the $r-s$ phase portrait is always in the region of negative $s$ and positive $r$. On the contrary, most of the $r(s)$ trajectories of Quintom-B lies in the opposite locations. While the third case gives another evolutionary portraits.
In conclusion, we have investigated the dynamics of Spinor Quintom dark energy models by using the new geometrical diagnostic method—statefinder pair \{r, s\}. The Ref. [64] has applied this method to Quintom model and successfully differentiate this class of models with other dark energy models, but it is seem not useful to discriminate Quintom models with different potentials. However, as can be seen in this section, the statefinder diagnostic is able to differentiate different Quintom models with diverse kinds of power-law potential in the spinor scenario, as well as distinguish the Spinor Quintom models with other dark energy models.

IV. CONCLUSION AND DISCUSSIONS

To summarize, since more and more Quintom DE models are proposed, we established the connections among these models by studying the cosmic duality which connects the two totally different scenarios of universe evolution keeps the energy density of the Universe and Einstein equations unchanged, but transforms the Hubble parameter. Besides, applying the new geometrical diagnostic method—statefinder pair \{r, s\} to the Spinor Quintom model, we differentiate different Quintom models with different kind of power-law potentials in the spinor scenario, and distinguish the Spinor Quintom models with other dark energy models, as well.

Acknowledgements

It is a pleasure to thank Yi-Fu Cai, Tao-Tao Qiu and Xinmin Zhang for enlightening discussions and cooperations at the beginning. We also thank Chao-Jun Feng for helpful discussions. This work is supported by Hebei Natural Science Foundation Project under Grant No. A2008000136.

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