The Solar Cycle: a new prediction technique based on logarithmic values

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Abstract A new prediction technique based on logarithmic values is proposed to predict the maximum amplitude ($R_m$) of a solar cycle from the preceding minimum aa geomagnetic index ($aa_{\text{min}}$). The correlation between $\ln R_m$ and $\ln aa_{\text{min}}$ ($r = 0.92$) is slightly stronger than that between $R_m$ and $aa_{\text{min}}$ ($r = 0.90$). From this method, cycle 24 is predicted to have a peak size of $R_m(24) = 81.7 (1 \pm 13.2 \%)$. If the suggested error in $aa$ before 1957 is corrected, the correlation coefficient between $R_m$ and $aa_{\text{min}}$ ($r = 0.94$) will be slightly higher, and the peak of cycle 24 is predicted much lower, $R_m(24) = 52.5 \pm 13.1$. Therefore, the prediction of $R_m$ based on the relationship between $R_m$ and $aa_{\text{min}}$ depends greatly on the accurate measurement of $aa$.

Keywords Space weather · The Sun · The Solar Cycle

1 Introduction

Predicting the strength of an upcoming solar cycle ($R_m$) is important in both solar physics and space weather. A variety of methods have been used to do so, of which some are based on statistics and some others are related to physics (Kane 2007; Cameron and Schüssler 2007; Pesnell 2008; Hiremath 2008; Tlatov 2009; Messerotti et al. 2009; Petrovay 2010; Du and Wang 2010). A reliable prediction of $R_m$ may test models for explaining the solar cycle (Pesnell 2008; Choudhuri et al. 2007) have been proposed to explain the solar cycle but the predictive skill of $R_m$ needs to be checked in future (Cameron and Schüssler 2007; Pesnell 2008; Du 2011a). Based on the Solar Dynamo Amplitude (SODA) index, Schatten (2005) predicted that the peak sunspot number of the current cycle (24) will be low, at $\sim 80$. Dikpati et al. (2006) predicted that the peak size of cycle 24 will be $30\%–50\%$ higher than that of cycle 23 based on a modified flux-transport dynamo model. In contrast, Choudhuri et al. (2007) predicted that the peak size of cycle 24 will be $30\%–50\%$ lower than that of cycle 23 based on a flux-transport dynamo model.

Since Ohl (1966) found a high correlation between the minimum aa geomagnetic activity ($aa_{\text{min}}$) in the declining phase of a solar cycle and the maximum sunspot number of the succeeding cycle ($R_m$), a great many papers related to this finding have been published over the past decades (Brown and Williams 1969; Kane 2010; Wilson 1990; Hathaway and Wilson 2006; Charvátová 2009; Wang and Sheeley 2009). The level of geomagnetic activity near the time of solar activity minimum has been shown to be a good indicator for the amplitude of the following solar activity maximum (Ohl 1976; Wilson 1990; Layden et al. 1991; Thompson 1993; Hathaway and Wilson 2006; Kane 2010). This method based on a solar dynamo concept that the geomagnetic activity during the declining phase of the preceding cycle or at the cycle minimum provides approximately a measure of the poloidal solar magnetic field that generates the toroidal field for the next cycle (Schatten et al. 1978).

When geomagnetic precursor methods are applied to the current cycle (24), some discrepancies are shown for different authors. Hathaway and Wilson (2006) predicted $R_m(24) = 160 \pm 25$ using the I component of $aa$ by subtracting the linear $R$ component with $R_m$ from $aa$ (Feynman 1982). Dabas et al. (2008) employed the number of geomag-
netic disturbed days prior to the minimum of the sunspot cycle, and predict \( R_m(24) = 124 \pm 23 \). Wang and Sheeley (2009) predicted \( R_m(24) = 97 \pm 25 \) based on the total open flux at sunspot minimum, which is derived from the historical \( aa \) index by removing the contribution of the solar wind speed.

The correlation coefficients between \( R_m \) and the geomagnetic-based parameters are usually very high, from 0.8 (Ohl 1976; Wilson 1990; Thompson 1993; Shastri 1998; Kane 2010) up to 0.97 (Layden et al. 1991; Lantos and Richard 1998; Hathaway et al. 1999; Dabas et al. 2008). However, a high correlation coefficient does not always yield an accurate prediction, such as in the case of cycle 23 (Kane 2007). It was found that the correlation coefficient between \( R_m \) and \( aa_{\text{min}} \) varies roughly in a cycle of about 44-year and that the prediction error based on this method when the correlation coefficient decreases is much larger than that when the correlation coefficient increases (Du et al. 2009; Du 2011a).

Conventionally, the correlation between \( R_m \) and \( aa_{\text{min}} \) is analyzed by a linear relationship (Sect. 2), and cycle 19 is viewed as anomalous or an ‘outlier’ due to the very great \( R_m \) ever seen. However, by analyzing the relationship between the logarithms of \( R_m \) and \( aa_{\text{min}} \) in Sect. 3, cycle 19 is no longer anomalous from the scatter points of \( \ln R_m \) versus \( \ln aa_{\text{min}} \). Whether correcting the suggested error (3 nT) in \( aa \) before 1957 has great influences on the prediction of \( R_m \) based on the relationship between \( R_m \) and \( aa_{\text{min}} \) (Sect. 4). The results are briefly discussed and summarized in Sect. 5.

2 Linear relationship between \( R_m \) and \( aa_{\text{min}} \)

This study uses the annual values of geomagnetic \( aa \) index computed from the 3-hourly \( K \) indices at two near-antipodal midlatitude stations (Mayaud 1972; Love 2011) since 1868 and the equivalent ones from measurements taken in Finland from 1844 to 1867 (Nevanlinna and Kataja 1993; Nevanlinna 2004), and the annual values of the International sunspot number (\( R_s \)) since 1844 produced by the Solar Influences Data Analysis Center (SIDC), World Data Center for the Sunspot Index, at the Royal Observatory of Belgium. The maximum amplitude of sunspot cycle (\( R_{\text{m}} \)) and the preceding \( aa \) minimum (\( aa_{\text{min}} \)) are listed in Table 1.

Conveniently, one employs the linear relationship between \( R_m \) and \( aa_{\text{min}} \) to predict the former. Figure 1 depicts the scatter plot between \( R_m \) and \( aa_{\text{min}} \) (triangles). The dotted line indicates the linear fit of \( R_m \) to \( aa_{\text{min}} \).

\[
R_m = 12.9 \pm 14.7 + (7.84 \pm 1.06) aa_{\text{min}}, \tag{1}
\]

\( \Delta R_m(19) = 42.0 \). Therefore, cycle 19 is often called as an ‘outlier’ (Kane 2007) and this point may, where the values following \( \pm \) indicate the standard deviation. The correlation coefficient between \( R_m \) and \( aa_{\text{min}} \) is very high, \( r = 0.90 \) at the 99% level of confidence. From this equation and \( aa_{\text{min}}(24) = 8.7 \), the peak size of cycle 24 is predicted to be \( R_m(24) = 81.2 \pm 16.2 \), where \( \sigma = 16.2 \) is the standard deviation of fitting, defined by

\[
\sigma = \sqrt{\frac{\sum_{i=9}^{23} (R_f(i) - R_m(i))^2}{N - 1}}, \tag{2}
\]

Table 1

| \( n \) | Parameters | From \( \ln aa_{\text{min}} \) | Corrected \( aa \) |
|---|---|---|---|
| \( aa_{\text{min}} \) | \( R_m \) | \( R_f \) | \( |\epsilon| \% \) | \( R_p \) | \( |\Delta R_p| \) |
| 9 | 14.1 | 124.7 | 122.8 | 1.5 | 132.7 | 8.0 |
| 10 | 10.3 | 95.8 | 94.2 | 1.6 | 96.4 | 0.6 |
| 11 | 16.0 | 139.0 | 136.6 | 1.7 | 150.8 | 11.8 |
| 12 | 7.0 | 63.6 | 68.0 | 6.7 | 64.9 | 1.3 |
| 13 | 10.7 | 85.1 | 97.3 | 13.4 | 100.2 | 15.1 |
| 14 | 6.0 | 63.5 | 59.7 | 6.1 | 55.4 | 8.1 |
| 15 | 8.6 | 103.9 | 80.9 | 25.0 | 80.2 | 23.7 |
| 16 | 10.1 | 77.8 | 92.7 | 17.5 | 94.5 | 16.7 |
| 17 | 13.3 | 114.4 | 116.9 | 2.1 | 125.0 | 10.6 |
| 18 | 16.3 | 151.5 | 138.8 | 8.8 | 153.7 | 2.2 |
| 19 | 17.2 | 189.8 | 145.2 | 26.8 | 162.3 | 27.5 |
| 20 | 14.0 | 105.9 | 122.1 | 14.2 | 103.1 | 2.8 |
| 21 | 19.9 | 155.3 | 164.3 | 5.6 | 159.4 | 4.1 |
| 22 | 18.5 | 157.8 | 154.5 | 2.1 | 146.0 | 11.8 |
| 23 | 16.1 | 119.5 | 137.4 | 13.9 | 123.1 | 3.6 |
| \( |\epsilon| \) | 13.2 | 116.5 | 115.4 | 9.8 | 116.5 | 9.9 |
| 24 | 8.7 | ? | ?81.7 | ?13.2 | ?52.5 | ?13.1 |

\( ^a \)From Fig. 2 and (3) and (5)

\( ^b \)From Fig. 3 and (6) and (7)

\[ a = 17.8 \pm 17.8, \quad b = 41.0 \pm 41.0, \]

Fig. 1 Scatter plot of \( R_m \) vs. \( aa_{\text{min}} \) (triangles)
Now, we analyze the scatter plot between the logarithms of $R$ and $aa_{\min}$, as shown in Fig. 2 (triangles).

The correlation coefficient of $\ln R_m$ with $\ln aa_{\min}$ is $r = 0.92$ at the 99% level of confidence, slightly stronger than that of $R_m$ with $aa_{\min}$ (0.90). The least-squares-fit regression equation (dotted line) is

$$\ln R_f = 2.58 \pm 0.26 + (0.84 \pm 0.10) \ln aa_{\min},$$

where $\ln R_f$ denotes the fitted value of $\ln R_m$. The standard deviation of fitting is $\sigma_f = 0.132$. This equation is equivalent to the form of a power-law,

$$R_f = e^{2.58} aa_{\min}^{0.84},$$

implying that $R_m$ does not depend completely linearly on $aa_{\min}$. The error of $\ln R_f$ is

$$\varepsilon = \Delta \ln R_f = \ln R_f - \ln R_m.$$

The values of $R_f$ and $\varepsilon$ are listed in Table 1.

One can see from Table 1 that the maximum relative error occurs in cycle 19, $|\varepsilon(19)| = 26.8\%$, which is only slightly larger than that in cycle 15, $|\varepsilon(15)| = 25.0\%$. Therefore, cycle 19 seems to be not an ‘outlier’ as cycle 15 in view of the relative error. In fact, Ramesh and Lakshmi (2011) proved, through a thorough analysis of the linear relationship between $R_m$ and the preceding sunspot minimum ($R_{\min}$), that cycle 19 is not an outlier—it is more appropriate to be called as an anomalous.

From (3), the peak sunspot number for cycle 24 can be predicted: $\ln R_f(24) = 4.403 \pm 0.132$ or $R_f(24) = 81.7(1 \pm 13.2\%)$, close to that by the linear relationship (81.2).

4 Using the corrected $aa$

The $aa$ index was suggested to exist an error and should be increased by 3 nT before 1957 (Nevanlinna and Kataja 1993; Lukianova et al. 2009; Svalgaard et al. 2004). In this section, we corrected the suggested error in $aa$ by adding 3 nT to $aa_{\min}$ for cycles 9–19 and re-examine the previous results. The scatter plot between $R_m$ and the corrected $aa_{\min}$ is shown in Fig. 3 (triangles).

The correlation coefficient between $R_m$ and $aa_{\min}$ is now $r = 0.94$ at the 99% level of confidence, slightly stronger than that using the un-corrected $aa_{\min}$ (0.90). The linear regression equation (dotted line) is

$$R_m = -30.5 \pm 15.8 + (9.54 \pm 1.00) aa_{\min},$$

and the standard deviation of fitting is $\sigma = 13.1$. Table 1 lists the fitted result ($R_p$) and the prediction error of $R_p$.

$$\Delta R_p = R_p - R_m.$$

From (6), the peak sunspot number for the next cycle (24) is predicted to be $R_m(24) = 52.5 \pm 13.1$, much lower than that using the un-corrected $aa_{\min}$ in Sect. 2 (81.2 ± 16.2). Therefore, the prediction of $R_m$ based on the relationship between $R_m$ and $aa_{\min}$ depends greatly on the accurate measurement of $aa$, that is, whether correcting the suggested error in $aa$ before 1957.

5 Discussions and conclusions

Studying the variations in the 11-yr solar cycle may help to understand the formation and dynamo mechanism of the cycle (Parker 1955; Babcock 1961). It has long been noted that the 11-yr Schwabe cycle is close to the synodic period of the co-alignments of the Earth, Venus, and Jupiter (Wood 1975; Grandpierre 1996). However, it is uncertainty whether the planetary tidal force can trigger the dynamo mechanism (Grandpierre 1996) as the acceleration due to planetary tidal force is much smaller than the observed acceleration at the level of tachocline (De Jager and Versteegh 2005). In the dynamo mechanism, the differential rotation in the solar convective envelope transforms the poloidal magnetic field structure into toroidal magnetic field structure which leads to the formation of sunspots due to Coriolis
force (Parker 1955; Babcock 1961; Schatten et al. 1978; Dikpati et al. 2006; Choudhuri et al. 2007). Dynamo models can reproduce certain features of the cycle (e.g., sunspot butterfly diagrams), but the predictive skill of $R_m$ has not been checked so far (Cameron and Schüssler 2007; Pessnell 2008; Du 2011a). As the actual observational time series of poloidal field (available only since the mid-1970s) is not long (Choudhuri et al. 2007), the geomagnetic activity around the cycle minimum is used as a measure to estimate the poloidal solar magnetic field (Schatten et al. 1978). Javaraiah (2008) found that $R_m$ is well correlated with the sum of the sunspot group areas in the $0^\circ$–$10^\circ$ latitude interval both of the Sun’s northern hemisphere near the minimum of the previous cycle ($r = 0.95$) and of the southern hemisphere just after the time of the maximum of the previous cycle ($r = 0.97$). Recently, Tlatov (2009) suggested that the parameter $G = \Sigma (1/N_a)^2$, defined by the number of sunspot groups $N_a \geq 1$, may be useful for calibration of the residual magnetic poloidal fields, as the amplitude of $G$ is highly correlated with $R_m$ at one and a half solar cycles later ($r = 0.96$).

Conventionally, the relationship between $R_m$ and $aa_{\min}$ is analyzed linearly. The upcoming $R_m$ is predicted by extrapolating the linear regression equation from a least-squares-fit algorithm ($R_1$), and its uncertainty is estimated by the standard deviation ($\sigma$) as the actual prediction error ($\Delta R = R_1 - R_m$) has not been known until the cycle is over. Thus, the prediction is usually expressed in the form of $R_m = R_p \pm \sigma$ ($2\sigma$), regarding the 68% (95%) level of confidence.

Giving the uncertainty in an absolute measure ($\sigma$) is enough in most circumstances. If a prediction error ($\Delta R_m$) is less than 20, this prediction is usually thought as a successful one. In some cases, however, it may alternately be better to describe the uncertainty in a relative form. Suppose that the prediction errors are the same for two predictions ($R_{m1}$, $R_{m2}$). $\Delta R_m = 20$. If $R_{m1} > 100$, this prediction is rather successful as its relative prediction error is less than 20%. However, if $R_{m2} < 50$, this prediction is terrible as its relative prediction error is larger than 40%. Therefore, showing the prediction error in a relative form is a better choice, particularly in comparison with two or more predictions.

This study examined the relationship between $\ln R_m$ and $\ln aa_{\min}$, with a correlation coefficient ($r = 0.92$) slightly higher than that for the linear relationship between $R_m$ and $aa_{\min}$ ($r = 0.90$). The standard deviation of fitting so obtained ($\sigma_i$) refers directly to the relative standard deviation. From this method, the peak sunspot number for cycle 24 is predicted to be $R_m(24) = 81.7 (1 \pm 13.2\%)$, near to that from a modified Gaussian function (72 $\pm$ 11, Du 2011d), that from the sunspot minimum (85 $\pm$ 17, Ramesh and Lakshmi 2011), and that from the sum of the sunspot group areas in the $0^\circ$–$10^\circ$ latitude interval of the previous cycle (87 $\pm$ 7, Javaraiah 2008). In fact, the logarithmic $R_2$ was often used in the studies of Waldmeier (1939).

If the suggested error in $aa$ (3 nT) before 1957 (Nevalinna and Kataja 1993; Lukianova et al. 2009; Svalgaard et al. 2004) is corrected, the correlation coefficient between $R_m$ and $aa_{\min}$ ($r = 0.94$) will be slightly higher than that using the un-corrected $aa_{\min}$ ($r = 0.90$). From this method, the peak sunspot number for the next cycle (24) is predicted to be $R_m(24) = 52.5 \pm 13.1$. It is close to that from long-term trends of sunspot activity (55.5, Tan 2011), lower than both that by an autoregressive model (110 $\pm$ 11, Hirrmann 2008) and that by the $G$ parameter (135 $\pm$ 12, Tlatov 2009). The prediction (52.5) is lower than that using the total open flux derived from the $aa$ index (97 $\pm$ 25, Wang and Sheeley 2009), that using the number of geomagnetic disturbed days (124 $\pm$ 23, Dabas et al. 2008), and that using the I component of $aa$ (160 $\pm$ 25, Hathaway and Wilson 2006). It should be pointed out that the prediction using the corrected $aa$ (52.5), which is close to that by Kane (58.0 $\pm$ 25.0, 2010), is much lower than that using the uncorrected $aa$ (81.2 $\pm$ 16.2). Therefore, the accurate measurement of $aa$ is crucial to predict $R_m$ when using the relationship between $R_m$ and the preceding $aa_{\min}$. Whether correcting the suggested error in $aa$ before 1957 may lead to great discrepancies in the prediction of $R_m$ by using the above relationship.

Accurately predicting the peak size of a upcoming sunspot cycle is a difficulty task as the monthly sunspot numbers may have systematic uncertainties of about 25% (Vitinskij et al. 1986). In fact, the relationship between $aa$ and $R_2$ is very complex and the current $aa$ value may be related to the past solar activities (Du 2011b, 2011c), reflecting long-term evolution characteristics of the Sun’s magnetic field (Lockwood et al. 1999; Tlatov 2009).

Main conclusions can be drawn as follows.

1. The correlation between $\ln R_m$ and $\ln aa_{\min}$ ($r = 0.92$) is slightly stronger than that between $R_m$ and $aa_{\min}$ ($r = 0.90$). From this method, the $R_m$ for cycle 24 is predicted to be $R_m(24) = 81.7 (1 \pm 13.2\%)$.

2. The prediction of $R_m$ based on the relationship between $R_m$ and $aa_{\min}$ depends greatly on the accurate measurement of $aa$. If the suggested error in $aa$ (3 nT) before 1957 is corrected, the correlation coefficient between $R_m$ and $aa_{\min}$ ($r = 0.94$) will be slightly higher, and the peak size of cycle 24 will be predicted much lower, $R_m(24) = 52.5 \pm 13.1$.

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