Intrinsic shapes of very flat elliptical galaxies

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ABSTRACT

Photometric data from the literature are combined with triaxial mass models to derive the variation in the intrinsic shapes of the light distribution of elliptical galaxies NGC 720, 2768 and 3605. The inferred shape variation is given by a Bayesian probability distribution, assuming a uniform prior. The likelihood of obtaining the data is calculated by using an ensemble of triaxial models. We apply the method to infer the shape variation of a galaxy, using the ellipticities and the difference in the position angles at two suitably chosen points from the profiles of the photometric data. The best-constrained shape parameters are found to be the short-to-long axial ratios at small and large radii, and the absolute values of the triaxiality difference between these radii. The elliptical galaxies of our present investigation are very flat, with ellipticity typically around 0.3 or more. We find that the expectation values of the short-to-long axial ratio of these galaxies are around 0.5.

Key words: galaxies: photometry – galaxies: structure.

1 INTRODUCTION

The intrinsic shapes of individual elliptical galaxies have been investigated by Binney (1985), Tenjes et al. (1993), Statler (1994a,b), Bak & Statler (2000), Statler (2001) and Statler et al. (2004). These authors have used kinematical data and photometric data, and have used triaxial models with the density distribution $\rho (m^2)$, where $m^2 = x^2 + y^2/p^2 + z^2/q^2$ with axial ratios $p$ and $q$. Here, $(x, y, z)$ are the usual Cartesian coordinates, oriented such that the $x$-axis (z-axis) lies along the longest (shortest) axis of the model. It was shown analytically that the projected density of such a distribution $\rho (m^2)$ with constant $(p, q)$ is stratified on similar and co-aligned ellipses (Stark 1977; Binney 1985). Statler (1994a) used (apart from the kinematical data) a constant value of ellipticity, which is an average over a suitably chosen range of radial distances, for the shape estimates. The shape estimates are robust and are described by a pair of shape parameters, namely the short-to-long axial ratio, $c_L$, of the light distribution and the triaxiality, $T_M$, of the mass distribution.

A complementary problem was attempted by Chakraborty, Singh & Gaffar (2008, hereinafter C08), wherein variation in the intrinsic shapes of the light distribution of elliptical galaxies was investigated by using triaxial models, which exhibit ellipticity variation and position angle twist. These models are fixed by assigning the values of axial ratios $(p_0, q_0)$ and $(p_\infty, q_\infty)$ at small and large radii, respectively. These axial ratios are related to triaxialities $T_0$ and $T_\infty$, respectively, at small and large radii. We use Bayesian statistics and obtain the variation in the shape, following the methodology described in Statler (1994a). We find that the marginal posterior density (MPD) is likelihood dominated, so that it is relatively insensitive to the unknown prior density. We use a flat prior and a large ensemble of models, so that the shape estimates may be model-independent.

The basic ingredients of our method are the same as in Statler (1994a) and we adopt all the necessary alterations described in C08. We use $(q_0, T_0, q_\infty, T_\infty)$ as the shape parameters, and the ellipticities $\epsilon_{in}$ and $\epsilon_{out}$ and the position angle difference $\Theta_{out} - \Theta_{in}$ at two suitably chosen points $R_{in}$ and $R_{out}$ from the profiles of the photometric data of the galaxies. We find that the best-constrained shape parameters are $q_0, q_\infty$ and the absolute value of the triaxiality difference, $T_\Delta$, defined as $|T_\infty - T_0|$. C08 have estimated the shapes of 10 elliptical galaxies, which are comparatively rounder, with ellipticities $< 0.3$. We now investigate the shapes of three more galaxies, namely NGC 720, 2768 and 3605. These are very flat galaxies with ellipticity around 0.3 or more. We find that the expectation values of the short-to-long axial ratio of these galaxies are around 0.5. We use triaxial models, which are very flat. We take models with the lower limit of $(q_0, q_\infty) \sim 0.3$ for our shape investigation. We find that a class of very flat triaxial models develop several undesirable features (Section 2 and Appendix A) and are not employed in the present shape estimates.

Determination of the intrinsic shape using photometry is important, because the number of galaxies with good photometry is much higher than those with good kinematics. Besides this, the results obtained by alternative models and techniques can be used for a comparison. Photometry constrains the flattening $(q_0, q_\infty)$, but cannot constrain $(T_0, T_\infty)$. Thus, our work is complementary and not contradictory to that of Statler and his coworkers.

The structure of the text is as follows. Section 2 presents the models. The necessity for the choice of small values of the lower limits of $(q_0, q_\infty)$ and the intrinsic shapes of the galaxies are presented in Section 3. Section 4 is devoted to results and a discussion.

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2 MODEL

We use models which are triaxial generalizations of the spherical $\gamma$ models of Dehnen (1993), with density $\rho$ given by

$$\rho(r) = \frac{M_0(3-\gamma)b}{4\pi r^\gamma(b+r)^{\gamma+4}},$$

(1)

where $M_0$ is the mass of the model, $r$ is the radial coordinate, $0 \leq \gamma < 3$ and $b$ is the scalelength. The models have a cusp at the centre and the density decreases as $r^{-\gamma}$ at large radii. Dehnen’s models are the generalization of the well-studied models of Jaffe (1983) and Hernquist (1990), corresponding to $\gamma = 2$ and 1, respectively. The projected surface density of the model of Dehnen, corresponding to $\gamma = 1.5$, most closely resembles the de Vaucouleurs $R^{1/4}$ law. Presently, we concentrate on $\gamma = 1.5$ models only.

A triaxial generalization of (1) is presented in Chakraborty (2004), which is modified in C08. The model is the density distribution of the same form as (1) with $r$ replaced by $M$, where

$$M^2 = x^2 + \frac{y^2}{P^2} + \frac{z^2}{Q^2},$$

(2)

with varying axial ratios

$$P^{-2}(M) = \frac{\beta b^2 p_0^{-2} + M^2 p_\infty^{-2}}{\beta b^2 + M^2},$$

(3)

and

$$Q^{-2}(M) = \frac{\beta b^2 q_\infty^{-2} + M^2 q_\infty^{-2}}{\beta b^2 + M^2}.$$  

(4)

The axial ratios ($P, Q$) reduce to ($p_0, q_0$) at small radii and to ($p_\infty, q_\infty$) at large radii. $\beta > 0$ is a parameter which for a choice of ($p_0, q_0, p_\infty, q_\infty$) alters $P$ and $Q$ in the intermediate region. The models are fixed once the axial ratios ($p_0, q_0, p_\infty, q_\infty$) are chosen. The triaxialities $T_0$ and $T_\infty$ are related to the axial ratios at small and large radii by

$$T_0 = \frac{1-p_0^2}{1-q_0^2} \quad \text{and} \quad T_\infty = \frac{1-p_\infty^2}{1-q_\infty^2},$$

(5)

respectively. To fix the scalelength $b$ of the triaxial models, we consider $\gamma = 1.5$ and use the value of the effective radius $R_e = 1.28b$ of the spherical model. The effective radius of the triaxial models depends on the axial ratios, as well as on the viewing angles. However, such changes are small for $\gamma$ models (de Zeeuw & Carollo 1996) and are neglected.

The constant $\rho$ surfaces are coaxial ellipsoids. Projection of these models on a plane perpendicular to the line of sight and therefore the calculation of the ellipticity and position angle are performed numerically. We refer to these models as $M^2$ models.

Another form of triaxial generalization of (1) is investigated by de Zeeuw & Carollo (1996), where two more terms are added to equation (1); each one of these is a suitable radial function multiplied by spherical harmonics of low order. The models provide a simple analytical representation of the observed surface brightness of triaxial elliptical galaxies. However, for large values of flattening, the models become peanut shaped and are not used in the present investigation. Very flat de Zeeuw–Carollo models are discussed in Appendix A.

3 INTRINSIC SHAPES

The galaxies chosen here are very flat. The morphological classification of NGC 720, 2768 and 3605 is E5/E3, E6/E5 and E4/E5, respectively, from the RC2 (de Vaucouleurs, de Vaucouleurs & Corwin 1976) catalogue. The apparent flattening of an elliptical galaxy depends on the intrinsic flattening and the orientation. Further, the MPD $P$ of the Bayesian estimate is obtained by integrating the posterior density over all viewing angles. To gain some insight into the possible values of the intrinsic shape, which will be obtained by the Bayesian method, we perform the following numerical experiments. The objective of these experiments is to find suitable limits of the axial ratios ($q_0, q_\infty$) for the plots of $P$. In the plots of $P$ in C08, $q_0$ and $q_\infty$ extend from 0.5 to 1.0. Statler (1994a) chose $0.4 \leq c_1 \leq 1.0$.

Fig. 1 shows the plot of the number $N$ of viewing angles ($\theta', \phi'$) versus the axial ratio $q$, which gives ellipticity $0.50 \leq \epsilon \leq 0.55$ (panel a) and $0.07 \leq \epsilon \leq 0.17$ (panel b). The number of viewing angles is counted between 0.0 and 90.0 at an interval of 1.0, for both $\theta'$ and $\phi'$. The total number of viewing angles in this numerical experiment is 8100. The axial ratio $p$ is taken as 0.9. Here, we use the...
Stark model. We find that a higher value of ellipticity is produced by flatter models and a lower value of ellipticity is produced by rounder models, over a larger number of viewing angles. Therefore, the Bayesian estimate should pick up a flat model to represent the shape of the galaxies of our present investigation. It is interesting to note that plots (a) and (b) show a maximum, which lies at $q \sim 0.44$ for Fig. 1(a) and at $q \sim 0.82$ for Fig. 1(b).

Fig. 1 and its inferences are based on applying the Stark model, which has constant values of the axial ratio ($p$, $q$). However, in our shape estimates we use models with varying axial ratios. So we re-examine the results of these plots by considering $M^2$ models.

Fig. 2 presents the MPD $\mathcal{P}$ as a function of $(q_0, q_\infty)$, summed over various values of $(T_0, T_\infty)$ for NGC 720. We use the observational data as shown in Table 1. We choose the values of both $q_0$ and $q_\infty$ from the region $0.1 \leq (q_0, q_\infty) \leq 0.9$. We use the $M^2$ models with $\beta = 1.0$. The probability of the shape is plotted in dark grey shade: the darker is the shade, the higher is the probability. The white contour encloses the region of 68 per cent highest posterior density (HPD), which may be interpreted as a 1σ error bar. This figure indicates that a higher probability region is confined approximately between 0.3 and 0.8 of $(q_0, q_\infty)$. Hence, it is more appropriate to choose the lower and the upper limits of both $q_0$ and $q_\infty$ as 0.3 and 0.8 for the shape estimates of very flat galaxies. This is discussed further in Section 3.1.

For the same choice of the values of $(q_0, q_\infty)$, Fig. 3 shows the shape $\mathcal{P}(q_0, q_\infty)$ of a rounder galaxy, NGC 3379. We use the observational data $\epsilon_{in} = 0.078, \epsilon_{out} = 0.133$ at $R_{in} = 15.7$ arcsec and $R_{out} = 49.3$ arcsec. The effective radius $R_e$ of NGC 3379 is 37.5 arcsec. We find that the HPD region is confined between $(q_0, q_\infty) \geq 0.4$ and the highest values of $(q_0, q_\infty)$ allowed in this plot.

3.1 NGC 720

The observed data of NGC 720 are taken from $R$-band surface photometry of Peletier et al. (1990). The ellipticity $\epsilon$ increases monotonically from 0.315 at $R_{in} = 8.5$ arcsec to 0.442 at $R_{out} = 51.8$ arcsec. In this range, the position angle decreases by 3.5. We consider the uncertainty in the ellipticity as 0.02 and in the position angle as 1°, both at $R_{in}$ and at $R_{out}$. These are the typical errors in observations (de Carvalho, Djorgovski & da Costa 1991; Penereiro et al. 1994). The effective radius of the galaxy is 52.0 arcsec. We use the ensemble of models, as described in Section 2, with $\beta = 5.0, 2.5, 1.0, 0.5$ and 0.2. Taking the sum of the MPD over all possible values of $T_0$ and $T_\infty$, and taking the unweighted sum over all the models, we obtain the shape estimate $\mathcal{P}$ as a function of $(q_0, q_\infty)$.

Fig. 4 presents the shape estimate $\mathcal{P}(q_0, q_\infty)$ of NGC 720, wherein we have allowed the limits 0.5–1.0 both for $q_0$ and for $q_\infty$. We find that the 1σ region is very narrow, which should be the consequence of the choice of the limits of $q_0$ and $q_\infty$. Examining this limit in Fig. 1, we find that this choice falls in the region where high values of the ellipticity will not be reproduced. Therefore, we need to go to smaller values of $(q_0, q_\infty)$ to obtain higher ellipticities, which may be close to the observed ellipticities of NGC 720.

Fig. 5 presents the shape $\mathcal{P}(q_0, q_\infty)$, wherein we have allowed the limits 0.3 and 0.8 for $q_0$ and $q_\infty$, respectively. The 1σ region is wider now (but narrow enough to satisfy the requirement of the likelihood-dominated shape estimate). Although it is the plot of MPD $\mathcal{P}$ as a function of shape parameters, which constitute the Bayesian estimate of the shape, a statistical summary of the shape will prove very helpful in its description. The expectation values $(q_0, q_\infty)$ and location of the peak values $q_{\rho}, q_{\sigma}$ are such quantities. Table 2 provides such a summary. The expectation values of the flattening at small and large radii are $(q_0) = 0.64$ and $(q_\infty) = 0.43$, respectively.

In Figs 4 and 5, we choose the interval between higher and lower limits of $(q_0, q_\infty)$ as 0.5. This is basically to save computer time, but maintain the reliability of the results. Shape calculation requires a very large number of projections, which need to be calculated...
Figure 4. Plot of unweighted sum of MPD ($P$) as a function of $q_0, q_\infty (= q)$, for NGC 720 using the limits 0.5–1.0, for both $q_0$ and for $q_\infty$. The sum is taken over the $M^2$ models with $\beta = 5.0$, 2.5, 1.0, 0.5 and 0.2. The plus symbol marks the location of the maximum probability.

Figure 5. As Fig. 4, but for NGC 720 using the limits 0.3–0.8, for both $q_0$ and for $q_\infty$.

numerically. We divide the parameter space of $(q_0, q_\infty)$ into $48 \times 48$ square bins of equal size and calculate the likelihood at the centre of each bin. The bin size is small enough that the calculated likelihood can be regarded as a continuum function of $(q_0, q_\infty)$, and at the same time the number of bins is small enough that the computer time is not unmanageable.

Fig. 6 shows the three-dimensional intrinsic shape of NGC 720 as a function of $q_0, q_\infty$ and $|T_d|$. We cut a total of 16 sections, each perpendicular to the $|T_d|$ axis, and arrange these sections in the form of a two-dimensional array. The value of $|T_d|$ is constant in each section and is shown in the plot. We find that the $1\sigma$ region occupies a larger area in the sections with smaller values of $|T_d|$. Furthermore, in each section of constant $|T_d|$, a $1\sigma$ region occupies a small area of the $(q_0, q_\infty)$ plane. We find that higher $P$ is concentrated in sections with $|T_d|$ between 0.28 and 0.47. The expectation value of $\langle|T_d|\rangle = 0.41$.

3.2 Intrinsic shapes of NGC 2768 and 3605

The observational data used in the models of these galaxies are presented in Table 1. Here also, the data are obtained from $R$-band surface photometry of Peletier et al. (1990).

Figs 7 and 8 present the plots of $P$ of NGC 2768 and 3605 as functions of $(q_0, q_\infty)$. The lower and upper limits of $q_0$ and $q_\infty$ are taken as 0.25 and 0.75, respectively. The HPD region shows that these galaxies are very flat. The expectation values are $\langle q_0 \rangle = 0.63$ and $\langle q_\infty \rangle = 0.32$ for NGC 2768, and $\langle q_0 \rangle = 0.62$ and $\langle q_\infty \rangle = 0.42$ for NGC 3605. We find that these galaxies are also intrinsically very flat. Figs 9 and 10 present the three-dimensional plots of $P$ of NGC 2768 and 3605 as a function of $q_0, q_\infty$ and $|T_d|.

A statistical summary of the intrinsic shapes of all three flat galaxies, NGC 720, 3605 and 2768, is presented in Table 3. Here, the values are taken from the three-dimensional shape estimates. The expected and the peak values of $q_0$ and $q_\infty$ as obtained from two-dimensional estimates $P(q_0, q_\infty)$ are reported in Table 2. These values are quite close, but not exactly the same as those reported in Table 3. The differences may be attributed to ‘resolution’; in

| Galaxy    | $q_0$ | $q_\infty$ | $\langle q_0 \rangle$ | $\langle q_\infty \rangle$ |
|-----------|-------|------------|------------------------|--------------------------|
| NGC 720   | 0.68  | 0.48       | 0.64                   | 0.43                     |
| NGC 2768  | 0.65  | 0.29       | 0.63                   | 0.32                     |
| NGC 3605  | 0.72  | 0.49       | 0.62                   | 0.42                     |
Shapes of elliptical galaxies

4 RESULTS AND DISCUSSION

We have presented the intrinsic shapes of three very flat galaxies. A specific feature of these estimates is the choice of the lower limit of $q_0$ and $q_\infty$. The lower limit was chosen as 0.5 by C08 and as 0.4 by Statler (1994a,b) in their investigation of galaxies which are comparatively rounder, without mentioning any specific reason for these choices. Through the plots of Fig. 1 and the detailed discussion of the plots of $P$ for NGC 720, we have justified the choice of very small values of $(q_0, q_\infty)$ as their lower limits for very flat elliptical galaxies. We have taken the lower limit as either 0.25 or 0.3.

A summary of the intrinsic shapes of these very flat galaxies is presented in Tables 2 and 3. We find that these galaxies are little rounder inside (average value of $\langle q_0 \rangle \sim 0.6$), but very flat outside (average value of $\langle q_\infty \rangle \sim 0.4$). Following the nomenclature introduced in C08, these galaxies may be termed as RF type.

The intrinsic shapes of elliptical galaxies have implications for their formation and evolution. As the galaxies studied here are very flat, we have given emphasis to the shape $P(q_0, q_\infty)$ and information about the flattening at inner and outer radii.


**Table 3.** Statistical summary of the three-dimensional shape estimates $T(q_0, q_{\infty}, |T_0|)$ of the galaxies.

| Galaxy   | $q_0$ | $q_{\infty}$ | $|T_0|$ | $T_0$ | $q_{\infty}$ | $|T_1|$ |
|----------|-------|---------------|--------|-------|--------------|--------|
| NGC 720  | 0.68  | 0.38          | 0.41   | 0.56  | 0.40         | 0.41   |
| NGC 2768 | 0.68  | 0.28          | 0.22   | 0.62  | 0.33         | 0.59   |
| NGC 3605 | 0.68  | 0.43          | 0.16   | 0.60  | 0.41         | 0.37   |

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**APPENDIX A: VERY FLAT DE ZEEUW–CAROLLO MODELS**

A simple family of triaxial models, with ellipticity variation and position angle twist, was presented by de Zeeuw & Carollo (1996) with density distribution

$$
\rho(r, \theta, \phi) = f(r) - g(r)Y_0^0(\theta) + h(r)Y_1^0(\theta),
$$

(A1)

where $f(r)$ is the same as (1), $g(r)$ and $h(r)$ are two radial functions, and $Y_0^0$ and $Y_1^0$ are the usual spherical harmonics. Here $(r, \theta, \phi)$ are the standard polar coordinates. The projected surface density of (A1) can be calculated easily and often analytically, $g(r)$ and $h(r)$ are fixed by assigning axial ratios $(p_0, q_0)$ and $(p_{\infty}, q_{\infty})$, respectively, at small and large radii, where constant-$\rho$ surfaces are approximately ellipsoidal. A numerical distribution function was shown to exist for prolate triaxials, $(p, q) = (0.65, 0.60)$, and for oblate triaxials, $(p, q) = (0.95, 0.65)$, wherein it is assumed that $q_0 = q_{\infty} = q, p_0 = p_{\infty} = p$ and $\gamma = 1$. Such models were studied by Chakraborty & Das (2003).

Table A1. Regions of negative $\rho$ for $\theta = 0.0$ and different values of $q$.

| $p = p_0 = p_{\infty} = 0.9, \gamma = 1.5, \theta = 0.0, \phi = \frac{\pi}{2}$ | $r_{low}/b$ | $r_{high}/b$ |
|------------------------------------------------------------------------|-------------|-------------|
| 0.55                                                                   | 1.93        | > 6.40      |
| 0.56                                                                   | 2.31        | 6.27        |
| 0.57                                                                   | 3.08        | 4.48        |

Table A2. Regions of negative $\rho$ for $q = 0.57$ and different values of $\theta$.

| $p = p_0 = p_{\infty} = 0.9, \gamma = 1.5, \phi = \frac{\pi}{2}, q = 0.57$ | $r_{low}/b$ | $r_{high}/b$ |
|------------------------------------------------------------------------|-------------|-------------|
| 0.0                                                                    | 3.08        | 4.48        |
| 1.0                                                                    | 3.08        | 4.48        |
| 2.0                                                                    | 3.21        | 4.36        |
| 3.0                                                                    | 3.46        | 3.85        |

$\geq 4.0$ $\rho$ is positive for all $r$

Triaxial models of a similar form to (A1) were proposed by Schwarzschild (1979) as a numerical model, where $f(r)$ is taken as the modified Hubble density distribution. Later, it was put into an analytical form by de Zeeuw & Merritt (1983). Projected properties of such triaxial-modified Hubble models were studied by Chakraborty & Thakur (2000). We now find that sufficiently flat versions of triaxial-modified Hubble models also exhibit regions of negative $\rho$.

We find that the appearance of negative-$\rho$ regions is correlated with the ‘dimpledness’ of constant-$\rho$ surfaces. We examine an extended version of models (A1), which includes terms with high-order spherical harmonics $Y_0^0, Y_2^0$, and $Y_2^0$. Such models were studied by Chakraborty & Das (2003). It was found that the dimpledness reduces, that is, the models become more ellipsoidal-like. We now find that for the same choice of the parameters as in Table A1, the interval $(r_{high} - r_{low})$ of negative $\rho$ decreases.

It will be interesting to extend studies of the particle orbit and numerical distribution function of Thakur et al. (2007) to the models that exhibit regions of negative $\rho$.

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