$Q^2$ evolution studies of nuclear structure function $F_2$ at HERA

S. Kumano and M. Miyama *

Department of Physics
Saga University
Saga 840, Japan

contribution to “Future Physics at HERA”

* Email: kumanos or 96td25@cc.saga-u.ac.jp

Research activities are listed at [ftp://ftp.cc.saga-u.ac.jp/pub/paper/riko/quantum1](ftp://ftp.cc.saga-u.ac.jp/pub/paper/riko/quantum1) or at [http://www.cc.saga-u.ac.jp/saga-u/riko/physics/quantum1/structure.html](http://www.cc.saga-u.ac.jp/saga-u/riko/physics/quantum1/structure.html).
Q² evolution studies of nuclear structure function F₂ at HERA

S. Kumano and M. Miyama
Department of Physics, Saga University, Saga 840, Japan

Nuclear modification of the structure function F₂ has been an interesting topic since the discovery of the EMC effect in 1983. Although most studies discuss x dependence of the modification, Q² dependence becomes increasingly interesting. It is because the NMC measured Q² variations of the ratio F₂^A/F₂^D [1]. Furthermore, it is found recently that there exist significant differences between tin and carbon Q² variations, ∂[F₂^{Sn}/F₂^{C}]/∂[ln Q²] ≠ 0 [1]. However, the NMC data are taken in the limited small Q² range at small x, so that they are not sufficient to test nuclear Q² evolution. The Q² dependence is important for understanding perturbative QCD in nuclear environment, and the future HERA nuclear program can make important contributions to this interesting topic.

The Q² dependence of structure functions can be calculated by using the DGLAP equations. They have been successful in describing many experimental data. However, as it becomes possible to reach the small x region by high-energy accelerators, it is necessary to investigate the details of small x physics. The longitudinal localization size of a parton exceeds the average nucleon separation in a nucleus in the small x region (x < 0.1). It means that partons in different nucleons could interact in the nucleus, and the interaction is called parton recombination (PR). This mechanism is used for explaining nuclear shadowing. There are a number of studies on the recombinations. Among them, we employ the evolution equations proposed by Mueller and Qiu. They investigated gluon-gluon recombination effects on the evolution. The DGLAP and PR evolution equations are given by (see Ref. [3] for the details)

\[
\frac{\partial}{\partial t} q_i (x, t) = \int_x^1 \frac{dy}{y} \left[ \sum_j P_{q_i q_j} \left( \frac{x}{y} \right) q_j (y, t) + P_{q g} \left( \frac{x}{y} \right) g (y, t) \right] + \left( \text{recombination terms} \propto \frac{\alpha_s A^{1/3}}{Q^2} \right), \tag{1a}
\]

\[
\frac{\partial}{\partial t} g (x, t) = \int_x^1 \frac{dy}{y} \left[ \sum_j P_{g q_j} \left( \frac{x}{y} \right) q_j (y, t) + P_{gg} \left( \frac{x}{y} \right) g (y, t) \right] + \left( \text{recombination terms} \propto \frac{\alpha_s A^{1/3}}{Q^2} \right), \tag{1b}
\]

where the variable t is defined by \( t = -(2/\beta_0) \ln[\alpha_s(Q^2)/\alpha_s(Q_0^2)] \). In the PR evolution case, there is an extra evolution equation for a higher-dimensional gluon distribution. The first two terms in Eqs. (1a) and (1b) describe the process that a parton
with the nucleon’s momentum fraction $y$ splits into a parton $p_i$ with the momentum fraction $x$ and another parton. The splitting function $P_{p_ip_j}(z)$ determines the probability that such a splitting process occurs and the $p_j$-parton momentum is reduced by the fraction $z$.

Although the DGLAP equations are well tested by various experimental data, the PR equations are not well established yet. An interesting problem is possible nuclear dependence in the $Q^2$ evolution. There are two possible sources for the nuclear dependence in the evolution equations. One is the input parton distributions, and another is the recombination effects. The modification of the input $x$-distributions in a nucleus affects the $Q^2$ evolution through splitting functions. The recombination contributions enter into the evolution equations as additional higher-twist effects.

In studying the $Q^2$ evolution, it is very important to have correct input distributions. Fortunately, there are many data on the $x$ dependence of $F_A^2/F_D^2$, so that we could have reasonable nuclear input distributions. We employ a hybrid parton model with recombination and $Q^2$ rescaling mechanisms in Ref. [3]. However, it does not matter in the $Q^2$ evolution studies what kind of model is used if it can explain the experimental $x$ dependence of $F_A^2/F_D^2$. In the hybrid model, we first calculate $Q^2$ rescaled valence-quark distributions at $Q^2_0$. Sea-quark and gluon distributions are simply modified by a constant mount so as to satisfy the momentum conservation. Then, obtained distributions are used as input distributions for calculating the recombination effects. In this way, nuclear parton distribution with the rescaling and recombination effects are obtained at $Q^2_0$. Because the recombinations are higher-twist effects, final distributions are very sensitive to the choice of $Q^2_0$. It is fixed so that obtained shadowing agrees with the NMC ratios $F_{Ca}^2/F_{D}^2$ at small $x$.

In the following, we discuss two topics on the $Q^2$ evolution. The first is $Q^2$ variation of $F_{2Ca}^2/F_{2D}^2$ [2, 3] and the second is $\partial[F_{Sn}^2/F_{Ca}^2]/\partial[\ln Q^2]$ [4].

We compare calculated evolution results with the NMC data in Fig. 1 at $x=0.0085$ [3]. The initial distributions at $Q^2_0=0.8$ GeV$^2$ in the nucleon and the calcium nucleus are taken from Ref. [3]. In Fig. 1, the dotted, solid, and dashed curves are obtained in the leading-order (LO) DGLAP, next-to-leading-order (NLO) DGLAP, and NLO evolution equations with parton-recombination contributions respectively ($\Lambda=0.2$ GeV and $N_f=3$). As shown in the figure, NLO and recombination contributions to the ratio are conspicuous at such a small $x$. If we evolve $F_2$ from $Q^2_0=0.8$ GeV$^2$, the recombination effects are larger than the NLO ones. It is interesting to find such large recombination contributions in Fig. 1. However, the recombination cannot be tested at this stage because we do not have the data in the wide $Q^2$ region at small $x$. The future HERA nuclear program should be able to study the large $Q^2$ region, so that the parton recombination mechanism could be tested.

Next, $Q^2$ evolution differences in various nuclei could also be investigated at

![Figure 1: $Q^2$ variation of $F_{2Ca}^2/F_{2D}^2$.](image-url)
HERA. There are significant differences between tin and carbon $Q^2$ variations according to recent NMC analysis. It is the first indication of nuclear effects on the $Q^2$ evolution of $F_2$. The phenomena are worth investigating theoretically.

The $Q^2$ evolution of the structure functions $F_2$ in tin and carbon nuclei is investigated in Ref. [4]. As the input distributions, we employ those in Ref. [3]. $F_2$ is evolved by using LO DGLAP, NLO DGLAP, and PR equations with the help of a computer program in Ref. [2]. Calculated results for $\partial[F_2^{Sn}/F_2^{C}]/\partial[\ln Q^2]$ at $Q^2=5$ GeV$^2$ are compared with the NMC data. The DGLAP evolution curves agree roughly with the experimental tendency, but the PR results are significantly different from the data. However, it does not mean that the recombination mechanism should be ruled out because there exists an unknown parameter $K_{HT}$ associated with the higher-dimensional gluon distribution in the recombination. In order to discuss the validity of the PR evolution, the constant $K_{HT}$ must be evaluated theoretically.

In this way, the NMC experimental result $\partial[F_2^{Sn}/F_2^{C}]/\partial[\ln Q^2] \neq 0$ could be essentially understood by the difference of parton distributions in the tin and carbon nuclei together with the ordinary DGLAP evolution equations. However, we find an interesting indication that “large” higher-twist effects on the $Q^2$ evolution could be ruled out. As shown in Fig. 2, there are large differences among three evolution results at small $x$ ($\approx 10^{-4}$). The future HERA program can study nuclear dependence of the $Q^2$ evolution $(\partial[F_2^{A}/F_2^{D}]/\partial[\ln Q^2])$ in this small $x$ region, and it provides us crucial information on recombination effects and on higher-order $\alpha_s$ effects.

Acknowledgment

This research was partly supported by the Grant-in-Aid for Scientific Research from the Japanese Ministry of Education, Science, and Culture under the contract number 06640406.

References

[1] P. Amaudruz et al. (NMC collaboration), Nucl. Phys. B 441 (1995) 3. Sn/C $Q^2$ variation data are preliminary.
[2] M. Miyama and S. Kumano, Comput. Phys. Commun. 94 (1996) 185.
[3] S. Kumano, Phys. Rev. C48 (1993) 2016; Phys. Rev. C50 (1994) 1247; S. Kumano, M. Miyama, and K. Umekawa, research in progress.
[4] S. Kumano and M. Miyama, Phys. Lett. B 378 (1996) 267.