Renormalization Effect on Large
Neutrino Flavor Mixing in the
Minimal Supersymmetric Standard Model

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ABSTRACT

In the minimal supersymmetric standard model, we have studied the evolution of the neutrino flavor mixing by using the renormalization group equation (RGE) with the Georgi-Jarlskog texture for the Yukawa coupling matrices. For the large Yukawa coupling of the charged lepton, i.e., $\tan \beta \gg 1$, the neutrino flavor mixing increases significantly with running down to the electroweak scale by the RGE. If one wishes to get the large neutrino flavor mixing $\sin \theta_{23}$ at the electroweak scale, which is suggested by the muon neutrino deficit in the atmospheric neutrino flux, the initial condition $\sin \theta_{23} \geq 0.27$ is required at the GUT scale. Combined with the see-saw enhancement of the neutrino flavor mixing, the large mixing is naturally reproduced by setting the reasonable initial condition.

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The recent observed solar neutrino deficit[1] and muon neutrino deficit in the atmospheric neutrino flux[2] stimulate the systematic study of the neutrino flavor mixings[3]. In the standpoint of the quark-lepton unification in most GUT groups, the Dirac mass matrix of neutrinos is similar to the one of quarks, therefore, the neutrino flavor mixings turn out to be typically of the same order of magnitude as the quark mixings. However, some authors pointed out that the large neutrino flavor mixing could be obtained in the see-saw mechanism[4] as a consequence of certain structure of the right-handed Majorana mass matrix[5,6,7]. That is the so called see-saw enhancement[7] of the neutrino flavor mixing due to the cooperation between the Dirac and Majorana mass matrices. The see-saw enhancement of the mixing gives a clue to solve the problem of the muon neutrino deficit in the atmospheric neutrino flux[2] since the deficit may be derived from the large mixing between the muon neutrino and other neutrino.

The see-saw enhancement condition[7] given by Smirnov was modified in presence of the right-handed phases of the Dirac mass matrix and the Majorana mass matrix[8]. It was pointed out that the see-saw enhancement could be obtained even if the Majorana matrix is proportional to the unit matrix, i.e., there is no hierarchy in the Majorana mass matrix. This enhancement is caused by the right-handed phases, which never appear in the case of the quark mixing. Thus, it seems that the large neutrino flavor mixing is naturally reproduced in some GUT models with the see-saw mechanism.

There may be another enhancement mechanism of the neutrino flavor mixing. Recently, the renormalization group equation(RGE) of the see-saw neutrino mass operators with dimension-5 has been investigated by some authors[9,10]. Babu, Leung and Pantaleone pointed out that the neutrino flavor mixing is enhanced by the RGE in the minimal supersymmetric standard model(MSSM)[10]. They have shown that even
resonant mixing occurs between the GUT scale and the electroweak scale for the large
Yukawa coupling of the charged lepton, which corresponds to the case of \( \tan \beta \gg 1 \)
in the MSSM, where \( \tan \beta \equiv v_2/v_1 \), \( v_1 \) and \( v_2 \) being of the vacuum expectation values
with \( v_1^2 + v_2^2 = v^2 = (174 \text{GeV})^2 \). Thus, it is important to investigate the enhancement
of the neutrino flavor mixing by the RGE in the case of the large Yukawa couplings
of the charged lepton. In this paper, we study how large the neutrino flavor mixing is
enhanced by the RGE in the realistic model.

We start with writing the RGE for the Yukawa couplings \( Y_U, Y_D, Y_E \) and \( Y_N \),
which are \( 3 \times 3 \) matrices in the generation space. Since we are concerned with the
case of the large Yukawa couplings, we neglect the first generation. Therefore, those
Yukawa coupling matrices shall be considered to be \( 2 \times 2 \) matrices. We have the simple
RGE at one-loop approximation in the MSSM as follows[11,12]:

\[
8\pi^2 \frac{d}{dt}(Y_U Y_U^\dagger) = \left\{ -\sum_i c_U^i g_i^2 + 3(Y_U Y_U^\dagger) + \text{Tr}[3(Y_U Y_U^\dagger) + (Y_N Y_N^\dagger)] \right\}(Y_U Y_U^\dagger) \\
\quad + \frac{1}{2}\{(Y_D Y_D^\dagger)(Y_U Y_U^\dagger) + (Y_U Y_U^\dagger)(Y_D Y_D^\dagger)\}, \\
8\pi^2 \frac{d}{dt}(Y_D Y_D^\dagger) = \left\{ -\sum_i c_D^i g_i^2 + 3(Y_D Y_D^\dagger) + \text{Tr}[3(Y_D Y_D^\dagger) + (Y_E Y_E^\dagger)] \right\}(Y_D Y_D^\dagger) \\
\quad + \frac{1}{2}\{(Y_U Y_U^\dagger)(Y_D Y_D^\dagger) + (Y_D Y_D^\dagger)(Y_U Y_U^\dagger)\}, \\
8\pi^2 \frac{d}{dt}(Y_N Y_N^\dagger) = \left\{ -\sum_i c_N^i g_i^2 + 3(Y_N Y_N^\dagger) + \text{Tr}[3(Y_U Y_U^\dagger) + (Y_N Y_N^\dagger)] \right\}(Y_N Y_N^\dagger) \\
\quad + \frac{1}{2}\{(Y_E Y_E^\dagger)(Y_N Y_N^\dagger) + (Y_N Y_N^\dagger)(Y_E Y_E^\dagger)\}, \\
8\pi^2 \frac{d}{dt}(Y_E Y_E^\dagger) = \left\{ -\sum_i c_E^i g_i^2 + 3(Y_E Y_E^\dagger) + \text{Tr}[3(Y_D Y_D^\dagger) + (Y_E Y_E^\dagger)] \right\}(Y_E Y_E^\dagger) \\
\quad + \frac{1}{2}\{(Y_N Y_N^\dagger)(Y_E Y_E^\dagger) + (Y_E Y_E^\dagger)(Y_N Y_N^\dagger)\}, \\
\]

(1)

with

\[
t = \ln(\mu),
\]

(2)

where the coefficients \( c_U^i, c_D^i, c_N^i \) and \( c_E^i \) \((i = 1, 2, 3)\) are given to be \((13/15, 3, 16/3), \)
(7/15, 3, 16/3), (3/5, 3, 0) and (9/5, 3, 0), respectively in the MSSM.

However, the neutrino Yukawa coupling \( Y_N \) decouples from the RGE in eq.(1) below the right-handed Majorana mass scale, in which the heavy right-handed neutrinos decouple, and then, the evolution equation of the neutrino Yukawa coupling \( Y_N \) becomes meaningless. Recently, the RGE of the see-saw neutrino mass operator with the dimension-5 have been investigated by some authors. Below the right-handed Majorana mass scale, we use the RGE of the neutrino mass operator given by Babu, Leung and Pantaleone[10] in the MSSM, which is written as follows:

\[
8\pi^2 \frac{d}{dt} \kappa_N = \left\{ -\left( \frac{3}{5} g_1^2 + 3g_2^2 \right) + \text{Tr}[3Y_U Y_U^\dagger] \right\} \kappa_N + \frac{1}{2} \left\{ (Y_E Y_E^\dagger) \kappa_N + \kappa_N (Y_E Y_E^\dagger)^T \right\} ,
\]

where

\[
\kappa_N \simeq Y_N M_R^{-1} Y^T_N ,
\]

where \( M_R \) is a right-handed Majorana matrix, which is for a while assumed to be proportional to the unit matrix such as \( M_R = M_R I \). Hereafter, \( M_R \) refers to the scale of the right-handed Majorana Mass. Then, the neutrino mixing angle \( \theta_{23} \) for the third generation and the second one is determined by the evolution equation,

\[
16\pi^2 \frac{d}{dt} \sin^2 2\theta_{23} = -2 \sin^2 2\theta_{23} (1 - \sin^2 2\theta_{23}) \frac{Y_{E3}^2 - Y_{E2}^2}{(\kappa_N)_{33} + (\kappa_N)_{22}} (\kappa_N)_{33} - (\kappa_N)_{22} ,
\]

where \( Y_{E3} \) and \( Y_{E2} \) are the Yukawa couplings of the third generation charged lepton and the second generation one, respectively. It is noticed that \( \sin^2 2\theta_{23} \) increases with running down from the GUT scale \( (M_{GUT}) \) to the electroweak scale \( (M_Z) \). In particular, the evolution of the mixing angle is remarkable if \( Y_{E3} \) is large. For example, if \( Y_{E3} = 3 \) is fixed between the \( M_{GUT} \) scale and the \( M_Z \) one, \( \sin^2 2\theta_{23} \) can reach the maximal value 1 on the way to the \( M_Z \) scale[10].

However, the magnitude of the Yukawa coupling of the charged lepton rapidly decreases in the evolution of the RGE as seen in eq.(1) even if the large Yukawa
coupling is put at the $M_{GUT}$ scale. We show the evolution of $Y_{E3}$ for $Y_{E3} = 3, 2, 1, 0.5$ at the $M_{GUT}$ scale in fig.1, where we have neglected Yukawa couplings of the second generation and we have taken GUT conditions $Y_{D3} = Y_{E3}$ and $Y_{U3} = Y_{N3} = 3$ with $M_{GUT} = 3 \times 10^{16} \text{GeV}$, $M_R = 3 \times 10^{14} \text{GeV}$ and $\alpha_s(M_Z)=0.125[13]$. The dashed-lines denote evolutions from $M_{GUT}$ to $M_R$ and the solid lines denote evolutions from $M_R$ to $M_Z$. Thus, it is not clear whether the neutrino flavor mixing is enough enhanced. Therefore, we study quantitatively the enhancement of $\sin \theta_{23}$ due to the large Yukawa coupling of the charged lepton in the specified model of the Yukawa coupling matrices.

In our studies, we take the Georgi-Jarlskog texture[14] for the Yukawa coupling matrices at the $M_{GUT}$ scale since this texture is intensively investigated in the quark- and the lepton-mixings[3,15]. It was concluded that the acceptable solution of the quark mixings lie at the edges of allowed regions in this texture. The Yukawa coupling matrices are written in the three generation space as follows:

$$
Y_U = \begin{pmatrix}
0 & C_U & 0 \\
C_U & 0 & B_U \\
0 & B_U & A_U 
\end{pmatrix},
Y_D = \begin{pmatrix}
0 & C_D & 0 \\
C_D & B_D & 0 \\
0 & 0 & A_D 
\end{pmatrix},
Y_E = \begin{pmatrix}
0 & C_D & 0 \\
C_D & -3B_D & 0 \\
0 & 0 & A_D 
\end{pmatrix},
$$

(6)

for the up-quarks, down quarks and the charged leptons. In this texture with $SU(5)$, the neutrino Yukawa couplings are not related to the up-quark ones. In $SO(10)$, the neutrino Yukawa couplings could be related to the up-quark ones. Then, the neutrino Yukawa coupling matrix has same form as the up-quark one in eq.(6). However, the magnitudes of the matrix elements depend on the multiplets of the Higgs fields such as $10$ or $126$. If the vacuum expectation values of both multiplets contribute to the matrix elements, the magnitudes of the matrix elements are independent each other.
in the up-quark sector and the neutrino sector. In general, the matrix of the Yukawa coupling is written as,

\[
Y_N = \begin{pmatrix}
0 & C_N & 0 \\
C_N & 0 & B_N \\
0 & B_N & A_N
\end{pmatrix}.
\] (7)

Hereafter, we consider only the third generation and the second one in the matrices of eqs.(6) and (7) since our concern is the mixing angle in the case of the large Yukawa couplings. Then, the free parameters are matrix elements \(A_U, B_U, A_N, B_N, A_D\) and \(B_D\), and the right-handed Majorana mass \(M_R\).

At first, the parameter \(A_D\) is taken to be as possible as large in order to study how large the neutrino flavor mixing is enhanced by the evolution of the RGE in the case of the large Yukawa coupling of the charged lepton. In the framework of the pertubative unification, we take a possible large value \(A_D\), while the large top quark mass favours the large \(A_U\). Then, \(A_N\) is also expected to be large due to the quark-lepton symmetry. For definiteness, we take the following values in our calculation:

\[A_U = A_N = A_D = 3.\] (8)

The unification of \(b\)-quark and \(\tau\)-lepton masses at the \(M_{\text{GUT}}\) scale, where the three gauge couplings unify, is one of crucial issues of the Grand Unification. In the MSSM, there is a successful gauge coupling unification[16] and also the \(b-\tau\) unification is possible. Vissani and Smirnov[12] studied the \(m_b/m_\tau\) ratio in the standpoint of the \(b-\tau\) unification in the MSSM. They showed that the ratio depends crucially on \(\alpha_s(M_Z)\) and \(M_R\) as well as the Yukawa couplings. The increase of \(A_U\) or \(A_D\) tends to decrease the ratio. The gauge coupling \(\alpha_s(M_Z)\) works in opposite direction. The neutrino renormalization also results in increase of the ratio. In other words, the decrease of \(M_R\) and the increase of \(A_N\) tend to increase the ratio. Our input values in eq.(8) predict the rather small \(m_b/m_\tau\) ratio. Fixing \(M_{\text{GUT}} = 3 \times 10^{16}\text{GeV},\)
$M_R \simeq 3 \times 10^{14}\text{GeV}$ and $\alpha_s(M_Z) = 0.125$, our predicted ratio is $m_b/m_\tau = 1.54$ at the $M_Z$ scale. This value corresponds to the pole mass of the $b$-quark $m_b^\text{pole} \simeq 4.75\text{GeV}$, which is derived by using the two-loop standard model RGE with $\alpha_s(M_Z) = 0.125$ and $m_\tau = 1.7771\text{GeV}[17]$. This value of the pole mass is consistent with the experimental value. Actually, Dominguez and Paver have given $m_b^\text{pole} \simeq 4.72\pm0.05\text{GeV}[18]$ although Titard and Ynduráin have obtained the rather large value $4.9\text{GeV}[19]$. It may be useful for readers to add a following comment: one could get $m_b^\text{pole} = 4.85\text{GeV}$ by changing the parameters such as $A_N = 3 \rightarrow 5$ or $\alpha_s(M_Z) = 0.125 \rightarrow 0.127$. 

Now, the flavor mixings of the quark sector and the lepton sector can be investigated by the use of the parameters $B_U$ and $B_N$. By taking $B_U = 0.086$, which gives $\sin \theta_{23} = 0.029$ at $M_{\text{GUT}}$, we get the quark mixing, $\sin \theta_{23}(M_Z) = 0.051$, which may lie just at the edge of the experimentally allowed region of $V_{cb}[20]$ as discussed in ref.15. On the other hand, $B_N$ is a free parameter for the present. In order to investigate the evolution of the neutrino flavor mixing, we take $B_N = 0.65, 0.83, 1.05$, which correspond to the neutrino flavor mixing $\sin \theta_{23}(M_{\text{GUT}}) = 0.2, 0.25, 0.3$, respectively. We show the evolution of the neutrino flavor mixing $\sin^2 2\theta_{23}$ in fig.2, in which $M_{\text{GUT}} = 3 \times 10^{16}\text{GeV}$, $M_R = 2.2 \times 10^{14}\text{GeV}$ and $\alpha_s(M_Z) = 0.125$ are taken. Then, we get $m_t(M_Z) = 177\text{GeV}$ and $\tan \beta = 60.8$, which are given by

$$m_t(M_Z) = Y_t(M_Z) \frac{\tan \beta}{\sqrt{1 + \tan^2 \beta}} v, \quad m_\tau(M_Z) = Y_\tau(M_Z) \frac{1}{\sqrt{1 + \tan^2 \beta}} v. \quad (9)$$

The predicted $m_t(M_Z)$ corresponds to $m_t^\text{pole} = 188\text{GeV}$, which is consistent with the recent experimental measurements[21]. The increases of $\sin^2 2\theta_{23}$ are $0.08(53\%), 0.11(48\%)$ and $0.14(43\%)$ at the $M_Z$ scale for each initial condition $\sin^2 2\theta_{23} = 0.16, 0.24$ and $0.33$, respectively, at the $M_{\text{GUT}}$ scale. The evolution is remarkably rapid near the $M_{\text{GUT}}$ scale, but becomes rather mild below the $M_R$ scale. This behaviour is un-
derstandable if one finds out the behaviour of the evolution of the Yukawa coupling of the charged lepton in fig.1. Thus, it is impossible to get the remarkable enhancement of the neutrino mixing to reproduce $\theta_{23}(M_Z) = \pi/4$ unless we set the large neutrino flavor mixing as an initial condition at $M_{GUT}$ in the MSSM.

However, our result may present a clue to solve the problem of the muon neutrino deficit in the atomospheric neutrino flux. The atomospheric neutrino deficit data presented by KAMIOKANDE and IBM[2] have given under the assumption of the dominant mixing between the muon neutrino and the tau neutrino as follows:

$$\Delta m_{23}^2 = (0.3 \sim 3) \times 10^{-2} \text{eV}^2, \quad \sin^2 2\theta_{23} = 0.4 \sim 1 ,$$

(10)

where rather conservative bounds are taken. In fig.2, the predicted mass of the tau neutrino is fixed such as $m_{\nu_\tau} = 0.1\text{eV}$ by taking $M_R = 2.2 \times 10^{14}\text{GeV}$. In order to get $\sin^2 2\theta_{23} \geq 0.4$ at $M_Z$, one should prepare the initial condition $\sin^2 2\theta_{23}(M_{GUT}) \geq 0.27$.

Here, we comment on our prediction of the muon neutrino mass $m_{\nu_\mu}$. The predicted mass is $3.2 \times 10^{-3}\text{eV}$ in the case of $\sin^2 2\theta_{23} \simeq 0.4$ at the $M_Z$ scale. This predicted value is consistent with the MSW solution of the solar neutrino data(small mixing angle solution) [1,22],

$$\Delta m_{12}^2 = (0.5 \sim 1.2) \times 10^{-5}\text{eV}^2 .$$

(11)

The mixing angle between the muon neutrino and the electron neutrino is beyond the scope of our model because the first generation is neglected.

In above calculations, the parameter $B_N$ is a free parameter. However, this parameter may be related with the one in the quark sector. Here, we study the specific
cases of $B_N = B_U = 0.086$ and $B_N = 0.64$. The former case denotes the quark-lepton unification of the mass matrix elements, and the latter one means $\sin \theta_{23} = 0.2$, i.e., around the Cabibbo angle at the $M_{GUT}$ scale. Of course, these values of $B_N$ are not enough large to get $\sin^2 2\theta_{23} \geq 0.4$ at the $M_Z$ scale. Until now, we have assumed that the right-handed Majorana matrix $M_R$ is proportinal to the unit matrix. However, this assumption may be modified simply in the two generation space as follows:

$$
M_R = \begin{pmatrix} M_{R2} & 0 \\ 0 & M_{R3} \end{pmatrix} = M_R \begin{pmatrix} \epsilon_R & 0 \\ 0 & 1 \end{pmatrix}, \tag{12}
$$

where $M_R \equiv M_{R3}$ and $\epsilon_R \equiv M_{R2}/M_{R3}$. The parameter $\epsilon_R$ is a complex number due to the unknown right-handed phase in general. Of course, this matrix could have off-diagonal elements, but we do not discuss the case. The diagonal matrix in eq. $(12)$ is enough to investigate the see-saw enhancement. Then, at the GUT scale the light neutrino flavor mixing angle is given as follows:

$$
\tan 2\theta_{23} = \frac{2A_NB_N}{A_N^2 - B_N^2 + \frac{B_N^2}{\epsilon_R}}. \tag{13}
$$

The see-saw enhancement is obtained if $\epsilon_R \simeq -B_N^2/A_N^2$ is taken\[7,8\]. Now, we study the RGE evolution of the neutrino flavor mixing with the see-saw enhancement.

Let us begin with discussing the case of $B_N = B_U = 0.086$. If we take $\epsilon_R = 1$, we get the very small initial value $\sin \theta_{23} = 0.029$ at the $M_{GUT}$ scale. It is impossible to obtain $\sin^2 2\theta_{23} \geq 0.4$ at the $M_Z$ scale by using this initial condition. Therefore, we need the significant see-saw enhancement in the initial condition. We show the evolution with $\epsilon_R = -1/800$, $-1/950$, $-1/1050$ in fig.3(a), in which $M_R = 3 \times 10^{13}\text{GeV}$ is taken. The initial conditions are $\sin^2 2\theta_{23}(M_{GUT}) = 0.027, 0.064, 0.150$ for $\epsilon_R = -1/800$, $-1/950$, $-1/1050$, respectively. If we take $\epsilon_R = -1/1050$, we get $\sin^2 2\theta_{23}(M_Z) = 0.48$, which is much enhanced from the initial value. The neutrino masses are $m_3 = 9.6 \times 10^{-2}\text{eV}$ and $m_2 = 1.5 \times 10^{-2}\text{eV}$, in which $m_3$ is consistent with
the atmospheric neutrino data in eq.(10), however, $m_2$ is too large to explain the solar neutrino data in eq.(11). This enhancement of the neutrino flavor mixing is caused by the see-saw mechanism at the initial condition and then, the enhanced mixing increases by the RGE.

The large mass ratio $|M_{R3}/M_{R2}| \simeq 1000$ may not be reasonable in a realistic model. In the case of $B_N = 0.64$, we do not need such large mass ratio in the Majorana mass matrix. We show the evolution with $\epsilon_R = -1/2, -1/4, -1/5$ in fig.3(b), in which $M_R = 3.2 \times 10^{14}$GeV is taken. The enhanced initial conditions are $\sin^2 2\theta_{23}(M_{GUT}) = 0.196, 0.234, 0.256$ for $\epsilon_R = -1/2, -1/4, -1/5$, respectively. If we take $\epsilon_R = -1/5$, $\sin^2 2\theta_{23}$ increses to 0.47 at the $M_Z$ scale. The neutrino masses are $m_3 = 5.5 \times 10^{-2}$eV and $m_2 = 3.5 \times 10^{-3}$eV, which lie at the edges of allowed regions eqs.(10) and (11). If we get $\sin^2 2\theta_{23}(M_Z) \geq 0.47$ by taking $|\epsilon_R| \leq 1/5$, the muon neutrino mass $m_2$ becomes comparable to the tau neutrino mass $m_3$. Then, the solar neutrino data cannot be explained. Since the mass ratio $|M_{R3}/M_{R2}| = 2 \sim 5$ is a reasonable one in the right-handed Majorana mass matrix, this case may be a realistic one.

The summary is given as follows. In the MSSM, we have studied the evolution of the neutrino flavor mixing by using the RGE with the Georgi-Jarlskog texture for the Yukawa coupling matrices. For the large Yukawa coupling of the charged lepton, i.e., $\tan \beta \gg 1$, the neutrino flavor mixing increases significantly with running down to the electroweak scale from the GUT scale by the RGE. However, the maximal flavor mixing could not be reproduced unless the large mixing is set as an initial condition at the GUT scale. If one wishes to get the large neutrino flavor mixing at the $M_Z$ scale,
which is suggested by the muon neutrino deficit in the atmospheric neutrino flux, the initial condition $\sin \theta_{23}(M_{\text{GUT}}) \geq 0.27$ is required at the GUT scale. Combined with the see-saw enhancement of the neutrino flavor mixing, the large mixing is reproduced by the evolution of the RGE using the input of the realistic parameters such as $A_N/B_N \leq 3/0.64$ at the GUT scale and $|M_{R3}/M_{R2}| \geq 5$. Although our analyses have been done by using the Georgi-Jarlskog texture for the Yukawa coupling matrices, our qualitative conclusion does not change even if other texture will be used. The RGE evolution of the neutrino flavor mixing should be taken into consideration seriously in building the model with the large neutrino flavor mixing.

Acknowledgments

This research is supported by the Grant-in-Aid for Science Research, Ministry of Education, Science and Culture, Japan (No. 07640413).
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Figure Captions

Fig.1:

Running of the Yukawa coupling of the charged lepton. The initial values are set to be 3, 2, 1 and 0.5, respectively. The dashed-lines denote evolutions from $M_{GUT}$ to $M_R$ and the solid lines denote evolutions from $M_R$ to $M_Z$.

Fig.2:

Running of the neutrino flavor mixing. The initial values are set to be $\sin \theta_{23} = 0.3$, 0.25, and 0.2, respectively at the $M_{GUT}$ scale. The notations are same as in fig.1.

Fig.3:

Running of the neutrino flavor mixing. The parameter $\epsilon_R$ is taken to be (a) $-1/1050$, $-1/950$ and $-1/800$, and (b) $-1/5$, $-1/4$ and $-1/2$, respectively. The notations are same as in fig.1.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9508247v1
This figure "fig1-2.png" is available in "png" format from:

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