Dynamic behaviour of a full-load cavitation vortex in a Francis turbine draft tube excited at its eigenfrequencies

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Abstract. The operating range of Francis turbines is limited at full-load conditions by the formation of a cavitation vortex rope that may enter self-oscillations under certain conditions. This induces severe pressure pulsations in the entire system, as well as output power swings putting at risk the integrity of the electro-mechanical components. The understanding of the underlying physical mechanisms and the prediction of the stability of hydropower units at full-load conditions are therefore crucial to ensure a safe extension of their operating range. In the present paper, the dynamic behaviour of a stable cavitation vortex rope at full load is investigated by high-speed visualizations while the test rig is excited at its first and second hydroacoustic eigenfrequencies. It is first demonstrated that the cavitation volume and the pressure in the draft tube are more likely to oscillate at the first eigenfrequency, in agreement with the observations of self-excited oscillations at the first eigenfrequency of the cavitation vortex rope during unstable full-load conditions. In addition, it is observed that the amplitude of both the cavitation volume and pressure fluctuations in the draft tube reach a limit value when the amplitude of the excitation is further increased. Further investigations will determine if this behaviour can be generalized to any full-load conditions and will focus on the determination of the hydro-acoustic parameters of the draft tube cavitation flow based on the behaviour of the vortex rope during forced oscillations.

1. Introduction

Francis turbines operating at full-load conditions, i.e. with a flow rate higher than the value at the Best Efficiency Point (BEP), are subject to the formation of a swirling flow in their draft tube, leading to the development of an axisymmetric cavitation vortex rope. Under certain conditions, the cavitation volume enters self-excited oscillations, inducing severe pressure pulsations in the whole hydraulic system [1] and power swings that can reach more than 10% of the rated power of the hydropower unit [2]. The prediction of the stability of hydropower units operating at full-load conditions is therefore crucial to ensure a safe extension of their operating range. In the past decades, several researches focused on the investigation of the full-load surge mechanisms by numerical simulations [3, 4, 5] or experimental methods [6, 7]. One-dimensional models of the draft tube cavitation flow were also introduced and improved [8] to investigate the stability of full-load draft tube flow [9, 10]. The presence of a gaseous phase in the draft tube introduces
additional compliance and mass flow gain, which are usually modelled by the so-called cavitation compliance and mass flow gain factor [11]. The value of these parameters can be quantified on the reduced scale model of Francis turbines by performing modal analysis of the complete hydraulic circuit [12] or by means of high-speed visualizations of the draft tube flow [13].

In the present paper, the dynamic behaviour of the cavitation flow in the draft tube of a Francis turbine reduced scale model operating at full-load is investigated by both modal analysis and high-speed visualizations. The first and second eigenfrequencies of the test rig are determined at one stable full-load operating point by using a flow excitation system whose frequency excitation linearly increases from 0 to 8 Hz. The dynamic behaviour of the cavitation volume in the draft tube and the response of the test rig when excited at its first and second eigenfrequencies is then studied by high-speed visualizations and pressure measurements along the complete hydraulic circuit. The influence of the excitation amplitude on the cavitation volume and pressure fluctuations are highlighted to further derive the additional dissipation, mass flow gain factor and cavitation compliance introduced by the presence of a cavitation vortex rope at full-load conditions.

Figure 1: Reduced scale physical model of a Francis turbine installed on the EPFL LMH test rig PF1, together with the flow excitation system and the location of the pressure sensors (locations 1 to 16)

2. Test case

2.1. Experimental set-up

The test case is a reduced scale physical model of a Francis turbine with a specific speed of \( \nu = 0.33 \) and a reference diameter of \( D = 0.35 \) m. The reduced scale model is installed on EPFL LMH test rig PF1, as shown in Figure 1. A 300kV DC generator connected to the model runner regulates the rotational speed of the turbine and a vertical 900 kW pump generates the specific hydraulic energy in the closed-loop circuit.

A flow excitation system is connected to the test rig between the reduced scale model and the upstream reservoir, see Figure 1. The flow excitation system is composed of a rotating valve
with a variable frequency ranging from 0 to 15 Hz, a variable-speed pump and an accumulator
to reduce the interference of the variable-speed pump. A complete description of the flow
excitation system and its application to another test-case can be found in the reference [12].
The excitation system injects periodical discharge pulsations into the test rig, whose frequency
and amplitude can be adjusted by changing the rotating valve frequency and the pump rotational
speed, respectively. By covering a large range of excitation frequency, it is possible to identify
the eigenfrequencies of the test rig at a given operating point.

A total of 10 flush-mounted piezo-resistive sensors (sensors 5 to 14 in Figure 1) are installed
along the whole test rig to identify the eigenmode shape. Four sensors regularly spaced by 90°
sensors 1 to 4) are installed in one cross-section of the draft tube cone. In addition, two sensors
(sensors 15 and 16) are installed along the pipe upstream the rotating valve to compute the
discharge pulsations injected into the test rig by the flow excitation system.

2.2. Investigated operating conditions

One full-load operating point with a discharge factor $Q_{ED} = Q/D^2\sqrt{E}$ corresponding to 105% of
the value $Q_{ED}$ at the BEP is investigated in the present paper. The Thoma number $\sigma$ is
equal to 0.07. The operating conditions are summarized in Figure 1. At this operating point,
the draft tube features a stable axisymmetric cavitation vortex rope, with low-amplitude and
stochastic pressure and cavitation volume fluctuations, as shown in Figure 2. Low-amplitude
pressure and cavitation volume fluctuations can be observed at a frequency of $f = 2.6$ Hz, which
corresponds to the first eigenfrequency of the test rig, as shown in Section 3.

Table 1: Investigated operating parameters (* indicates the value at the BEP)

| OP   | $n_{ED} / n_{*ED}$ | $Q_{ED} / Q_{*ED}$ | $n$  | $\sigma$ |
|------|--------------------|--------------------|------|----------|
| 1 (stable) | 1.0               | 1.05               | 16.66| 0.07     |
| 2 (unstable) | 1.0               | 1.13               | 16.66| 0.05     |

For the sake of comparison, the results obtained at a higher discharge factor $Q_{ED} / Q_{*ED} = 1.13$ and
deeper Thoma number $\sigma = 0.05$ are given in Figure 3. At this operating point, the cavitation vortex rope enters self-oscillation and both pressure and cavitation volume fluctuations in the draft tube feature periodical fluctuations at a low frequency $f = 1.5$ Hz.

3. Dynamic modal analysis of the test rig at OP1

3.1. Methodology for the dynamic modal analysis

The first and second eigenfrequencies of the test rig are determined by applying the method of
dynamic modal analysis proposed by Favrel et al. [14]. The rotational frequency of the valve is
linearly increased from 0 to 7 Hz, with a slope of $v_{ramp} = 0.028 \text{ Hz.s}^{-1}$. The excitation frequency is
determined by measuring the instantaneous rotating valve frequency with an inductive sensor
placed on the shaft of the rotating valve. The amplitude of the excitation discharge pulsations
injected into the test rig is estimated by applying the pressure-time method to the pressure
sensors 15 and 16 located upstream the rotating valve [15].

As shown in [14], the method of dynamic modal analysis provides similar results compared
with the conventional stationary method used in [12], i.e. by changing step by step the frequency
of the flow excitation system.
3.2. Identification of the test rig eigenfrequencies

Modal analysis is applied when the test rig is operating at the full-load operating point OP1. During the excitation frequency ramp, pressure signals are recorded synchronously with a sampling rate of 1'000 Hz. To determine the evolution of the test rig response during the excitation frequency ramp, the individual pressure signals are split into time windows of $T = \sqrt{2/v_{\text{ramp}}} = 12$ s, following the criterion proposed in [14]. The Fast Fourier Transform (FFT) of each signal is then computed for each individual time window.

The evolution of the FFT of the pressure signals measured at the locations 6 and 12 during the excitation frequency ramp is shown in Figure 4 as a function of the excitation frequency. The amplitude of the pressure signals is strongly amplified when the excitation frequency is equal to about $f_0 = 2.6$ Hz and $f_1 = 5.9$ Hz for both pressure signals.

The frequencies $f_0 = 2.6$ Hz and $f_1 = 5.9$ Hz are identified as the first and second eigenfrequencies of the test rig, which is confirmed by the shape of the test rig response and the evolution of the phase shift along the test rig when excited at both frequencies, see Figure 5. In the latter, $L$ represents the pipes length between the draft tube outlet and the sensors location and is made dimensionless by $L_{\text{max}}$ corresponding to the value of $L$ at the location 13. When the test rig is excited at $f_0$, the pressure signals measured along the first part of the test rig,
i.e. from $L/L_{max} = 0$ to 0.5, are in phase, and a slight phase shift of $\pi/3$ is observed between the first and second part of the test rig. In addition, one pressure anti-node is observed around $L/L_{max} = 0.2$.

On the contrary, when the test rig is excited at $f_1$, a phase change of $\pi$ is observed at the middle of the test rig, which typically corresponds to a second eigenmode. The position of the corresponding pressure node can however not be precisely determined due to the location of the sensors, but it is presumably located between $L/L_{max} = 0.3$ and $L/L_{max} = 0.7$.

![Figure 4: Evolution of the FFT of pressure signals during excitation frequency ramp measured at the locations 6 (a) and 12 (b) (stable point, OP1)](image)

![Figure 5: Amplitude and phase of the test rig response when excited at its eigenfrequencies $f_0$ and $f_1$ (stable point, OP1)](image)

4. Dynamic behaviour of the cavitation vortex rope excited at the test rig eigenfrequencies at OP1

4.1. Influence of the excitation amplitude on test rig response

In the following, the test rig operating at the stable full-load operating point OP1 is excited at its first eigenfrequency and then at the second one. For each frequency, different pump rotational speeds are tested to investigate the effect of the excitation amplitude on both the dynamic response of the test rig and the fluctuations of the cavitation volume in the draft tube. The parameters of the excitation system tested in the following are summarized in Table 2.
The influence of the excitation amplitude on the amplitude and phase of the response of the test rig when excited at its first and second eigenfrequencies is given in Figure 6 and Figure 7, respectively. The distribution of phase along the test rig is not modified when the amplitude of the excitation is increased, while the amplitude of the test rig response increases as expected.

### Table 2: Tested parameters of the flow excitation system at OP1

| $f_{exc}$ (Hz) | Pump rotational speed (rpm) | excitation amplitude (m$^3$ s$^{-1}$) |
|----------------|-----------------------------|-------------------------------------|
| $f_0 = 2.6$ Hz | 960                         | 0.00156                             |
|                | 1080                        | 0.00182                             |
|                | 1200                        | 0.00273                             |
| $f_1 = 5.9$ Hz | 960                         | 0.00118                             |
|                | 1080                        | 0.00249                             |

Figure 6: Amplitude and phase of the test rig response when excited at its first eigenfrequency $f_0$ with different flow excitation amplitude (stable point, OP1)

4.2. Influence of the excitation amplitude on the cavitation volume and pressure fluctuations

While the test rig operating at the stable full-load operating point OP1 is excited at one of its eigenfrequencies, the draft tube flow is visualized by means of a Photron Fastcam Mini AX100 with a sampling rate of 1'000 Hz. A good contrast between the water and vapour cavity is provided by a LED screen. An image processing similar to the one used in [13] is then applied to the resulting images to estimate the instantaneous cavitation volume, which is considered axisymmetric.

Images of the vortex rope at two different instants of the excitation period $T_{exc} = 1/f_{exc} = 1/f_0 = 0.38s$ are provided in Figure 8. The evolution of the pressure and cavitation volume in the draft tube cone and the corresponding FFT when $f_{exc} = f_0 = 2.6$ Hz and $f_{exc} = f_1 = 5.9$ Hz is given in Figure 9 and Figure 10, respectively. The rotational speed of the excitation system pump is equal to $N_{pump} = 960$ rpm in both cases. When the test rig is excited at its first eigenfrequency $f_0 = 2.6$ Hz, large periodical fluctuations at $f_0$ of both the pressure and cavitation volume are observed. On the contrary, when the excitation frequency is set at the second test
rig eigenfrequency $f_1$, the amplitude of both the pressure and cavitation volume fluctuations are much lower. This observation is in agreement with the tendency of the cavitation vortex rope to enter self-oscillations at the first test rig eigenfrequency and not at one higher-order eigenfrequency during unstable full-load operations.

This observation is confirmed by the influence of the excitation amplitude on the amplitude of both the pressure and cavitation volume fluctuations for both cases $f_{exc} = f_0$ and $f_{exc} = f_1$, see Figure 11. The excitation amplitude is expressed as the amplitude of the discharge pulsations injected into the test rig computed by the pressure-time method (locations 15 and 16). For any value of the excitation amplitude, the fluctuations of pressure and cavitation volume are much higher in the case $f_{exc} = f_0$.

In the case $f_{exc} = f_0$, the amplitude of the cavitation volume and pressure fluctuations in the draft tube increases when the amplitude of the excitation is increased from $\Delta Q_{exc} = 1.56 \times 10^{-3} \text{ m}^3\text{s}^{-1}$ to $\Delta Q_{exc} = 1.82 \times 10^{-3} \text{ m}^3\text{s}^{-1}$ and then remains almost constant if the amplitude of the excitation is further increased. These results suggest that the oscillations of the draft tube cavitation flow are reaching a limit, which is supported by Figure 12. In the latter, the phase shift between the pressure and cavitation volume fluctuations at the excitation frequency is plotted against the excitation amplitude. The pressure and cavitation volume fluctuations are almost synchronous at $\Delta Q_{exc} = 1.56 \times 10^{-3} \text{ m}^3\text{s}^{-1}$. The value of the phase

Figure 7: Amplitude and phase of the test rig response when excited at its second eigenfrequency $f_1$ with different flow excitation amplitude (stable point, OP1)

Figure 8: Visualization of the cavitation vortex rope in the draft tube - $f_{exc} = f_0$, $N_{pump} = 960$ rpm (stable point, OP1)
shift then decreases as the excitation amplitude is increased and seems to tend to the value observed at unstable full-load conditions (OP2 in Table 1). However, this behaviour remains to be confirmed by performing additional measurements at other full-load operating conditions with a wider range of excitation amplitude.

![Figure 9](image1.png)

Figure 9: Evolution of the pressure and cavitation volume in the draft tube (a) and corresponding FFT (b) when the test rig is excited at $f_0$ with $N_{\text{pump}} = 960$ rpm (stable point, OP1)

![Figure 10](image2.png)

Figure 10: Evolution of the pressure and cavitation volume in the draft tube (a) and corresponding FFT (b) when the test rig is excited at $f_1$ with $N_{\text{pump}} = 960$ rpm (stable point, OP1)

4.3. Evolution of $\Delta V_{\text{cav}}$ as a function of $\Delta p$ and $\Delta Q_{\text{exc.}}$ over one excitation cycle

For the cases $f_{\text{exc}} = f_0$, the phase averaged signals of the pressure in the draft tube, the cavitation volume and the excitation discharge pulsations are computed by using the procedure proposed in [13]. The consecutive periods of the fluctuations are first identified and then split into 90 phase windows. All the data corresponding to the same phase window are then averaged together, giving rise to 90 values representing the pure periodic fluctuations of the signals at the excitation frequency, as illustrated in Figure 13.

The phase-averaged values of the cavitation volume fluctuations are plotted against the phase-averaged values of pressure and excitation discharge over one complete cycle of flow
excitation in Figure 14. At low excitation amplitude, i.e. $N_{\text{pump}} = 960$ rpm, the cavitation volume is in phase with the pressure and the excitation amplitude: the maximum cavitation volume is observed when both the excitation discharge and the pressure in the draft tube are at their maximum. This would suggest that the cavitation volume is mainly driven by the flow rate, and therefore by the intensity of the swirl at the runner outlet. At full-load conditions, both tangential and axial velocity components at the runner outlet increase as the discharge is increased. The swirl intensity first increases as the discharge is increased and tends to a maximum value if the discharge is further increased. This might explain the fact that the amplitude of the cavitation volume and pressure fluctuations do not further increase when the excitation amplitude is increased beyond a certain value, as observed in Figures 11 and 12.

As the excitation amplitude is increased, a phase shift between the cavitation volume, the pressure in the draft tube and the excitation discharge appears, giving rise to a cycle featuring elliptical $\Delta V_{\text{cav}}-\Delta \tilde{p}$ and $\Delta V_{\text{cav}}-\Delta \tilde{Q}_{\text{exc}}$ curves. Further investigations will attempt to determine if a limit cycle is observed as the excitation amplitude is further increased and if this behaviour can be generalized to any full-load operating condition.

5. Conclusion and perspectives
In this paper, the response of a reduced scale physical model of a Francis turbine operating at full-load conditions to an excitation at the first and second eigenfrequencies of the test rig is investigated. The tested operating condition features an axisymmetric stable cavitation vortex in the draft tube, together with stochastic low-amplitude pressure pulsations. The first and second

Figure 11: Influence of the excitation amplitude on the amplitude of the pressure and cavitation volume fluctuations in the draft tube at $f_0$ and $f_1$ (stable point, OP1)

Figure 12: Influence of the excitation amplitude on the phase shift between the pressure and cavitation volume fluctuations in the draft tube at $f_0$ and $f_1$ (stable point, OP1)
eigenfrequencies of the test rig are first determined by dynamic modal analysis. The behaviour of the draft tube cavitation flow when excited at the first and second eigenfrequencies is then studied by high-speed visualizations, together with pressure measurements along the complete test rig. It is first demonstrated that the cavitation volume and the pressure in the draft tube are more likely to oscillate at the first eigenfrequency of the hydraulic circuit. In addition, the amplitude of both the cavitation volume and pressure fluctuations in the draft tube seems to reach a limit value when the amplitude of the excitation discharge injected into the test rig is increased. As the excitation amplitude is increased, a phase shift between both quantities appears and tends to the value observed at an unstable full-load operating point. This behaviour remains to be further confirmed and generalized to other full-load operating conditions. The results presented in this paper and further measurements will enable the identification of the hydro-acoustic parameters of the vortex rope at full-load and the prediction of the stability limits of the Francis turbine reduced scale model.

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