Influence of oscillatory phenomena in vertical well on volume of outflow from production string

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Abstract. During well operation, there is a leaky cement ring, which is due to various reasons, the main of which are the technology of casing production strings and the used grouting materials that do not regard the actual geological and technical conditions of wells. Thus, the most important task is to ensure high-quality annulus insulation during well cementing. Theoretical studies which can be used to calculate the volume of fluid outflowing from a cemented and uncemented casing regarding the pressure distribution in a vertical well on conditions that data on the characteristics of the casing and working fluid is known are an urgent issue.

1. Introduction
When the vertical wellhead is closed by a valve preventing the liquid from flowing out, a pressure $p_0$ exceeding the atmospheric pressure $p_a$ is applied to the surface of the liquid from the side of the valve. After the instantaneous valve opening, part of the liquid will pour out due to an increase in its volume caused by a decrease in density, and to the production string narrowing caused by a decrease in pressure in the liquid [1-11]. Let us calculate the spilled volume under the assumption that the fluid is homogeneous and its compressibility does not depend on pressure.

2. Materials and methods
Let us introduce the following notation: H is the height of the column with liquid in a stress-free state; $r$ is the inner radius of the column in a stress-free state; $R$ is the outer radius of the column in a stress-free state; $\delta$ is a column wall thickness; $\beta$ is a fluid compressibility ratio; $E$ is elastic modulus of the column material; $\rho$ is liquid density at an arbitrary pressure $p$; $\rho_a$ is liquid density at atmospheric pressure $p_a$; z is a coordinate axis descending the borehole axis.

3. Results
Let us use the obtained equation relating pressure and density:
$$\beta \rho dp = d \rho.$$
Taking into account the condition $p=p_a$ with $\rho=\rho_a$ according to (1) we obtain the following:
$$\rho = \rho_a e^{\beta(p-p_a)}.$$
Knowing the pressure \( p = p_0 \) at the wellhead with \( z = 0 \), according to (2) we obtain the liquid density at the level \( z=0 \):

\[
\rho_0 = \rho_a e^{\beta(p_0 - p_a)}.
\]

At an arbitrary depth \( z \) we select an infinitely small area \( dz \). Along this interval, the pressure of the liquid column changes by the value \( dp(z) = \rho(z)g dz \). Upon substituting this value in (1) and integrating, we obtain the dependence of the following fluid density on the depth:

\[
\rho(z) = \frac{\rho_0}{1 - \beta \rho_0 g z},
\]

where \( \rho_0 \) is the density of the liquid at a valve level, i.e. at \( z=0 \).

Let us find the dependence of the fluid pressure on the depth. Equality (1) can be rewritten as

\[
\beta dp = d(ln \rho).
\]

Having integrated regarding the conditions \( p = p_0, \rho = \rho_0 \) with \( z = 0 \), we obtain the following:

\[
p(z) = p_0 - \frac{1}{\beta} \ln(1 - \beta \rho_0 g z).
\]

This internal pressure expands the pipe, causing its radius to increase. Let us find the inner radius under the assumption that there is no external influence on the pipe. Increasing the radius means increasing the circumference. In turn, the increase in circumference is due to circumferential tensile stress in the pipe material. At depth, the stress in a thick-walled pipe caused by fluid pressure \( p(z) \) is as follows:

\[
\sigma(z) = \frac{R^2 + r^2}{(R + r)\delta} p(z).
\]

Let us designate the initial circumference \( l = 2\pi r \), herewith, \( \Delta l(z) \) is the increase in this length. Let us derive an equation relating specific elongation with the stress and we get the following:

\[
\frac{\Delta l(z)}{l} = \frac{\sigma(z)}{E}.
\]

Hence, we obtain the following equation:

\[
\Delta l(z) = \frac{l}{E} \sigma(z).
\]

Therefore, the new circumference is equal to the following:

\[
L(z) = l + \Delta l(z) = l \left[ 1 + \frac{\sigma(z)}{E} \right].
\]

Pipe radius at depth \( z \) is as follows:

\[
r(z) = \frac{L(z)}{2\pi} = r \left[ 1 + \frac{\sigma(z)}{E} \right],
\]

cross-sectional area is equal to:

\[
S(z) = \pi r^2(z) = \pi r^2 \left[ 1 + \frac{\sigma(z)}{E} \right]^2.
\]

The mass of the compressed fluid is determined by the formula given below:

\[
m = \int_{(V)} \rho(z) dV = \int_{0}^{H} \rho(z) S(z) dz.
\]
Let us apply subintegral functions:

\[ m = \pi^2 \rho_0 \frac{H}{1 - \beta \rho_0 g z} \left\{ 1 + \frac{R^2 + r^2}{(R + r)E \delta} \left[ p_0 - \frac{1}{\beta} \ln(1 - \beta \rho_0 g z) \right] \right\}^2 dz. \]

Calculation of the integral leads to the following expression:

\[ m = \frac{\pi^2}{3Ag} \left\{ \left[ 1 + Ap_0 - \frac{A}{\beta} \ln(1 - \beta \rho_0 g H) \right]^3 - (1 + Ap_0)^3 \right\}, \]

which designates the following:

\[ A = \frac{R^2 + r^2}{(R + r)E \delta}. \]

We have found a mass of liquid compressed in a pipe. Now let us open the latch. The pressure \( p_0 \) will decrease to atmospheric \( p_a \), part of the liquid will pour out and the following mass will remain in the pipe:

\[ m_a = \frac{\pi^2}{3Ag} \left\{ \left[ 1 + Ap_a - \frac{A}{\beta} \ln(1 - \beta \rho_a g H) \right]^3 - (1 + Ap_a)^3 \right\}. \]

Consequently, the mass of the poured liquid \( \Delta m = m - m_a \) is equal to the following:

\[ \Delta m = \frac{\pi^2}{3Ag} \left\{ \left[ 1 + Ap_0 - \frac{A}{\beta} \ln(1 - \beta \rho_0 g H) \right]^3 - (1 + Ap_0)^3 \right\} - \left[ \left[ 1 + Ap_a - \frac{A}{\beta} \ln(1 - \beta \rho_a g H) \right]^3 + (1 + Ap_a)^3 \right]. \]

At atmospheric pressure, this mass outside the pipe will have a density \( \rho_a \). Therefore, the spilled volume \( \Delta V = \Delta m/\rho_a \) is as follows:

\[ \Delta V = \frac{\pi^2}{3Ag \rho_a} \left\{ \left[ 1 + Ap_0 - \frac{A}{\beta} \ln(1 - \beta \rho_0 g H) \right]^3 - (1 + Ap_0)^3 \right\} - \left[ \left[ 1 + Ap_a - \frac{A}{\beta} \ln(1 - \beta \rho_a g H) \right]^3 + (1 + Ap_a)^3 \right]. \]

\( E \to \infty \) corresponds to the case when there is a cement stone behind the production casing which does not allow the pipe to expand. Then \( A \to 0 \), and, using the approximations we obtain the following:

\[ (1 + Ap)^3 \to 1 + 3Ap, \]

\[ \left[ 1 + Ap - \frac{A}{\beta} \ln(1 - \beta \rho g H) \right]^3 \to 1 + 3Ap - \frac{3A}{\beta} \ln(1 - \beta \rho g H), \]

Out of the last equality we find the following:
\[ \Delta V_c = \frac{\pi^2}{\beta \rho g} \ln \frac{1 - \beta \rho_a g H}{1 - \beta \rho_0 g H}. \]

As you can see, the volume \( \Delta V_c \) pouring out of the cemented column does not depend on the characteristics of the column \( E \) and \( \delta \).

The table for pipes of various diameters shows the calculated values of the outflowing volume obtained with the following data:

\[
E = 2.1 \cdot 10^{11} \text{ Pa}, \quad \delta = 7 \cdot 10^{-3} \text{ m}, \quad H = 2000 \text{ m}, \quad g = 9.81 \text{ m/s}^2, \quad \beta = 4 \cdot 10^{-10} \text{ Pa}^{-1}, \quad p_a = 10^6 \text{ Pa}, \quad p_o = 3 \cdot 10^7 \text{ Pa}, \quad \rho = 1250 \text{ kg/m}^3. 
\]

**Table 1.** Values of outflowing volume are calculated for different diameters

| Column outer diameter, mm | Outflowing volume, l from an uncemented column | Outflowing volume, l from cemented column |
|---------------------------|----------------------------------------------|----------------------------------------|
| 60.3                      | 77                                           | 69                                     |
| 73                        | 116                                          | 102                                    |
| 89                        | 176                                          | 151                                    |

4. Conclusion

Formulas enabling to determine the density of the liquid and the pressure in it with regards to the depth and the elastic properties of the liquid are obtained. Formulas enabling to calculate the volume of outflowing fluid from cemented and uncemented casings under the situation of known data on the casing and working fluid characteristics are obtained.

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