QRATPre+: Effective QBF Preprocessing via Strong Redundancy Properties

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Abstract. We present version 2.0 of QRATPre+, a preprocessor for quantified Boolean formulas (QBFs) based on the QRAT proof system and its generalization QRAT\(^+\). These systems rely on strong redundancy properties of clauses and universal literals. QRATPre+ is the first implementation of these redundancy properties in QRAT and QRAT\(^+\) used to simplify QBFs in preprocessing. It is written in C and features an API for easy integration in other QBF tools. We present implementation details and report on experimental results demonstrating that QRATPre+ improves upon the power of state-of-the-art preprocessors and solvers.

1 Introduction

The application of preprocessing prior to the actual solving process is crucial for the performance of most of the quantified Boolean formula (QBF) solvers \(^{16,18,19}\). Preprocessors aim at decreasing the complexity of a given formula with respect to the number of variables, the number of clauses, or the number of quantifier blocks. Contrary to complete QBF solvers, preprocessors are incomplete but can detect redundant parts of the given formula by applying resource-restricted reasoning. Bloqqer \(^6\) and HQSpre \(^{24}\) are leading preprocessors which show their power in the yearly QBFEVAL competitions in potentially almost doubling the number of instances solved by certain state-of-the-art solvers. These tools apply a diverse set of redundancy elimination techniques.

We present QRATPre+ 2.0\(^4\), a QBF preprocessor based on the QRAT\(^+\) proof system \(^7,9\). QRATPre+ processes QBFs in prenex conjunctive normal form (PCNF) and eliminates redundant clauses and universal literals. Redundancy checking relies on redundancy properties defined by the QRAT\(^+\) proof system, which is a generalization of the QRAT (quantified resolution asymmetric tautology) proof system \(^7,9\). QRAT is a lifting of (D)RAT \(^{11,23}\) from the propositional to the QBF level, and it simulates virtually all simplification rules applied in state-of-the-art QBF preprocessors. This is made possible by strong redundancy properties.

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\(^3\) http://www.qbflib.org/index_eval.php

\(^4\) QRATPre+ is licensed under GPLv3: https://lonsing.github.io/qratpreplus/
However, the strong redundancy properties of QRAT have not been applied to preprocess QBFs so far. With QRATPre+ we close this gap and leverage the power of QRAT and QRAT+ for QBF preprocessing. Compared to the initially released version 1.0, version 2.0 comes with a more modularized code base and a C API that allows to easily integrate QRATPre+ in other tools. QRATPre+ currently applies only rewrite rules of the QRAT and QRAT+ proof systems that remove either clauses or universal literals from a PCNF. That is, it does not attempt to add redundant parts with the aim to potentially enable further simplifications later on. Despite this fact, experimental results with benchmarks from QBFEVAL’18 clearly indicate the effectiveness of QRATPre+. It improves on state-of-the-art preprocessors, such as Bloqer and HQSpre, and solvers in terms of formula size reduction and solved instances, respectively.

2 QRAT+ Redundancy Checking for Preprocessing

QRATPre+ eliminates redundant clauses and universal literals within clauses from a QBF in PCNF. Redundancy checking in QRATPre+ relies on redundancy properties defined by the QRAT+ proof system. We present QRAT+ only informally and refer to related work instead [17].

Let $\phi := \Pi.(\psi \land (C' \cup \{l\})$ be a PCNF with prefix $\Pi := Q_1B_1 \ldots Q_nB_n, \ldots \ldots B_n, B_n$, where $Q_iB_i$ are quantifier blocks (qblocks) consisting of a quantifier $Q_i \in \{\forall, \exists\}$ and a block (i.e., set) $B_i$ of variables. We write $Q(B_1 \ldots B_n)$ for $Q(B_1 \ldots \cup B_n)$. Index $i$ is the nesting level of qblock $Q_iB_i$ and of the variables in $B_i$. Formula $\psi \land (C' \cup \{l\})$ is in CNF, where $(C' \cup \{l\})$ is a clause in $\phi$ containing literal $l$. We consider only PCNFs without tautological clauses of the form $(C \cup \{p\} \cup \{\bar{p}\})$ for some propositional variable $p$.

Given a clause $C := (C' \cup \{l\})$ in $\phi$, checking whether $C$ or the literal $l \in C$ is redundant requires to consider all clauses $D_i$ in the resolution neighborhood $\text{RN}(C, l) := \{D_i | D_i \in \phi, l \in D_i\} \cup \{C\}$ of $C$ with respect to $l$, cf. [11,12]. This is illustrated in Fig. 1. Given $C$ and some $D_j \in \text{RN}(C, l)$, we first compute the outer resolvent $\text{OR}_j \subset (C \cup D_j)$ of $C$ and $D_j$ on literal $l$ [9]. Then we determine the maximum nesting level $i := \max(\text{levels}(\Pi, \text{OR}_j))$ of variables appearing in $\text{OR}_j$. Based on $i$ we construct the PCNF $\text{Abs}(\Pi, \psi, i) := \exists(B_1 \ldots B_i)Q_{i+1}B_{i+1} \ldots Q_nB_n, \psi$ by converting all universal quantifiers in the subprefix $Q_1B_1 \ldots Q_iB_i$ to existential ones. The resulting PCNF $\text{Abs}(\Pi, \psi, i)$ is an abstraction of $\Pi.\psi$.

We add the negation $\overline{\text{OR}}_j$ of the outer resolvent $\text{OR}_j$, which is a set of unit clauses, to the abstraction $\text{Abs}(\Pi, \psi, i)$ to obtain the formula $\text{Abs}(\Pi, (\psi \land \overline{\text{OR}}_j), i)$. Finally, we check whether a conflict, i.e., the empty clause $\emptyset$, is derived by applying QBF unit propagation (QBCP) [3,5,11,23] to $\text{Abs}(\Pi, (\psi \land \overline{\text{OR}}_j), i)$. If so, which we denote by $\text{Abs}(\Pi, (\psi \land \overline{\text{OR}}_j), i) \vdash \emptyset$, then the current outer resolvent $\text{OR}_j$ has the quantified asymmetric tautology (QAT) [17] redundancy property.

If all possible outer resolvents of $C := (C' \cup \{l\})$ and clauses $D$ in the resolution neighborhood $\text{RN}(C, l)$ of $C$ with respect to literal $l$ have the QAT property, then clause $C$ has the QRAT+ redundancy property on literal $l$. In this case, either $C$ or $l$ is redundant, depending on whether $l$ is existential or universal, respectively.
$$D_1 = D'_1 \cup \{\overline{l}\} \quad \ldots \quad D_j = D'_j \cup \{\overline{l}\} \quad \ldots \quad D_n = D'_n \cup \{\overline{l}\}$$

$$\text{OR}_j \subset (C \cup D_j)$$

$$i := \max(\text{levels}(\Pi, \text{OR}_j))$$

$$\Pi := Q_1B_1 \ldots Q_iB_i \ldots Q_nB_n$$

$$\text{Abs}(\Pi, \phi, i) := \exists (B_1 \ldots B_i)Q_{i+1}B_{i+1} \ldots Q_nB_n.\phi$$

Check for conflict: $$\text{Abs}(\Pi, (\phi \land \text{OR}_j), i) \vdash \forall \emptyset$$

$$C = C' \cup \{l\}$$

Fig. 1. Redundancy checking of clause $$C := (C' \cup \{l\})$$ based on QRAT+. The resolution neighborhood $$\text{RN}(C, l)$$ of $$C$$ consists of the clauses $$D_i$$ with $$\overline{l} \in D_i$$ shown on top. The boxes indicate the pair of clauses for which the current outer resolvent OR$_j$ is computed.

Note that in general, for every outer resolvent OR$_j$, the index $$i$$ for which the abstraction $$\text{Abs}(\Pi, (\psi \land \overline{\text{OR}_j}), i)$$ is constructed may be different.

Eliminating redundant clauses or universal literals via the above workflow is denoted by QRATE$^+$ and QRATU$^+$, respectively. In QRATPre+, we apply the QRATE$^+$ and QRATU$^+$ rewrite rules for preprocessing. The QRAT redundancy property [9] differs from QRAT$^+$ [17] in that for QRAT always a full abstraction $$\text{Abs}(\Pi, (\psi \land \overline{\text{OR}_j}), i)$$ with $$i := n$$ is constructed, regardless of the actual maximum nesting level of variables in the current outer resolvent OR$_j$. Moreover, QRAT$^+$ relies on QBCP which includes the universal reduction operation [13] to temporarily shorten clauses during propagation. This way, potentially more conflicts are derived. In contrast to that, QRAT applies propositional unit propagation to abstractions where all variables are existential. Due to that, the QRAT$^+$ redundancy property is more general and stronger than QRAT.

3 Implementation Details

Algorithm 1 shows the high-level workflow implemented in QRATPre+. To limit the computational costs of the relative expensive techniques QRATE$^+$ and QRATU$^+$, which are based on QBCP, we apply cheaper ones first to reduce the formula size upfront. In quantified blocked clause elimination (QBCE) [1], which is a restriction of QRATE$^+$, it is checked whether every outer resolvent (cf. Fig. 1) contains a pair of complementary literals. Blocked literal elimination (BLE) [8] is a restriction of QRATU$^+$ and, like QBCE, checks for complementary literals in outer resolvents. Before checking whether some clause $$C$$ has the QRAT$^+$ property, we check whether it has the QAT property. This is done analogously to checking whether an outer resolvent has the QAT property in QRAT$^+$ testing (cf. Section 2). QAT checking is necessary in the workflow since it is not subsumed by QRATE$^+$.

During clause elimination, all clauses found redundant in the current PCNF $$\phi'$$ are cleaned up lazily in one pass after an application of a technique. In
literal elimination, redundant literals are cleaned up eagerly since clauses are shortened, which increases chances to detect conflicts in QBCP.

If the QRAT+ redundancy check fails for some clause $C$ with literal $l \in C$ and clause $D_i \in \text{RN}(C, l)$ in the resolution neighborhood of $C$, we mark $D_i$ as a witness for that failure. If a witness $D$ is found redundant then all clauses in $\text{RN}(D, l)$, for all literals $l \in D$, are scheduled for being checked in the next iteration as $D$ has prevented at least one of these clauses from being detected before. This witness-based scheduling potentially avoids superfluous redundancy checks. In our experiments the median number of clauses being checked in a run of QRATPre+ on a given PCNF was only by a factor of 3.3 larger than the initial number of clauses in the PCNF.

We maintain the index $i$ indicating the maximum nesting level of variables in the current outer resolvent being tested (cf. Fig. 1). Any variable with an index smaller than $i$ is treated as an existential one during QBQP. This way, abstractions $\text{Abs}(\Pi, \psi, i)$ are constructed implicitly. For QBQP, we implemented standard two-literal watching [4]. However, when assignments are retracted after deriving a conflict, then in general the literal watchers have to be restored to literals which are existential in the input PCNF rather than in the current abstraction. Restoring literal watchers in our implementation is necessary to maintain certain invariants, in contrast to, e.g., QBQP in QCDCL solvers.

Compared to version 1.0, version 2.0 of QRATPre+ comes with a C API that allows for easy integration in other tools. The API provides functions to import and export PCNFs and to configure the preprocessing workflow. Options include switches to toggle the individual redundancy tests in the main loop, user-defined limits, and shuffling the orderings in which clauses are tested. Shuffling may affect the result of preprocessing since the QAT, QRATE+ and QRATU+ rewrite rules are not confluent. In the default configuration, QRATPre+ does not shuffle clauses and applies rewrite rules until saturation.

4 Experiments

QRATPre+ improves on state-of-the-art preprocessors and solvers in terms of formula size reduction and solved instances. We ran experiments with the preprocessors Bloqper v37 [6] and HQSpre 1.3 [24], and the solvers CAQE (commit-ID 9b95754 on GitHub) [21,22], RAReQS 1.1 [10], Ijtihad v2 [2], Qute 1.1 [20], and DepQBF 6.03 [15]. The solvers implement different solving paradigms, e.g., QCDCL (Qute and DepQBF), expansion (RAReQS and Ijtihad), and clausal abstraction (CAQE).
The following experiments were run on a cluster of Intel Xeon CPUs (E5-2650v4, 2.20 GHz) running Ubuntu 16.04.1. We used the 463 instances from the PCNF track of QBFEVAL’18. In all our experiments, we allowed 600s CPU time and 7 GB of memory for each call of Bloqqer, HQSpre, or QRATPre+. For formulas where Bloqqer or HQSpre exceeded these limits, we considered the original, unpreprocessed formula. In contrast to Bloqqer and HQSpre, we implemented a soft time limit in QRATPre+ which, when exceeding the limit, allows to print the preprocessed formula with identified redundant parts being removed.

Table 1 shows the effect of preprocessing by QRATPre+ (Q), Bloqqer (B), HQSpre (H), and combinations, where QRATPre+ is called before (QB, QH) or after (BQ, HQ) Bloqqer or HQSpre. The table shows average numbers of clauses (#cl), qblocks (#qb), existential (#∃l) and universal literals (#∀l) as a percentage relative to the original benchmark set. Except for qblocks, QRATPre+ considerably further reduces the formula size when combined with both Bloqqer (B vs. QB and BQ) and HQSpre (H vs. QH and HQ). These results include solved instances: QRATPre+, Bloqqer, and HQSpre solve 18, 74, and 158 original instances, respectively. The preprocessors timed out on 12 (HQSpre), 38 (Bloqqer), and 59 (QRATPre+, soft time limit) original instances.

Table 1. Effect of preprocessing.

|      | Q | B | QB | BQ | QH | HQ |
|------|---|---|----|----|----|----|
| #cl  | 79| 78| 23 | 70 | 69 | 18 |
| #qb  | 92| 22| 17 | 21 | 22 | 59 |
| #∃l  | 82| 85| 27 | 80 | 77 | 20 |
| #∀l  | 73| 95| 74 | 82 | 86 | 44 |

The application of computationally inexpensive techniques like QBCE and BLE to shrink the PCNF before applying more expensive ones like QRATE+ and QRATU+ pays off. With the original workflow (Algorithm 1 and column Q in Table 1), QRATPre+ spends 96s on average per instance, compared to 120s when disabling QBCE and BLE, where it exceeds the time limit on 77 instances.

Shuffling the ordering of clauses before applying QRATE+ and QRATU+ in Algorithm 1 based on five different random seeds hardly had any effect on the aggregate data in column Q in Table 1 except for an increase in eliminated universal literals by one percent. When using the redundancy property of QRAT, which is weaker than the one of QRAT+, by constructing full abstractions (cf. Section 2), we observed a moderate decrease of all four metrics by one percent each.

Tables 2a to 2f show the numbers of instances solved after preprocessing with different combinations of QRATPre+, Bloqqer, and HQSpre. We used a limit of 1800s CPU time and 7 GB of memory for solving. Times for preprocessing are not included in the times reported in the tables. Preprocessing by QRATPre+ increases the number of solved instances in most cases, except for Ijtihad on instances preprocessed with HQSpre and QRATPre+ (Tables 2a and 2b). Similar to formula size reduction shown in Table 1, the ordering of whether to apply QRATPre+ before or after Bloqqer or HQSpre has an impact on solving performance. Interestingly, for all solvers the combination BQ (Bloqqer before QRATPre+) results in a decrease of solved instances compared to QB (QRATPre+ before Bloqqer). We made similar observations for combinations with HQSpre (QH and HQ) 5.

5 We refer to the appendix for results with combinations BQ and QH.
Table 2. QBFEVAL’18: solved instances (S), unsatisfiable (⊥), satisfiable (⊤), and uniquely solved ones (U), and total CPU time in kiloseconds (K) including time outs.

(a) Original instances.

| Solver | S | ⊥ | ⊤ | U | Time |
|--------|---|---|---|---|------|
| CAQE   | 151| 107| 44| 11| 586K |
| DepQBF | 149| 87 | 62|  50| 592K |
| RAReQS | 147| 115| 32| 4 | 588K |
| Ijtihad| 132| 111| 21|  2| 609K |
| Qute   |  98|  79| 19| 6 | 665K |

(b) QRATPre+ only (Q).

| Solver | S | ⊥ | ⊤ | U | Time |
|--------|---|---|---|---|------|
| CAQE   | 211| 135| 76| 29| 487K |
| RAReQS | 178| 120| 58| 9 | 533K |
| DepQBF | 165|  83|82 | 33| 562K |
| Ijtihad| 156| 111|45 | 1 | 562K |
| Qute   | 137|  92|45 | 9 | 598K |

(c) Bloqqer only (B).

| Solver | S | ⊥ | ⊤ | U | Time |
|--------|---|---|---|---|------|
| CAQE   | 269| 150|119| 24| 383K |
| RAReQS | 258| 159| 99|7 | 399K |
| Ijtihad| 200| 128|72 | 4 | 482K |
| DepQBF | 198|  99|99 |21| 501K |
| Qute   | 189| 114| 75|  2| 512K |

(d) QRATPre+ and Bloqqer (QB).

| Solver | S | ⊥ | ⊤ | U | Time |
|--------|---|---|---|---|------|
| CAQE   | 290| 169|121| 37| 347K |
| RAReQS | 260| 163| 97|  5| 390K |
| Ijtihad| 216| 140|76 |  2| 456K |
| DepQBF | 210| 107|103|21 | 481K |
| Qute   | 197| 117|78 |  2| 493K |

(e) HQSpre only (H).

| Solver | S | ⊥ | ⊤ | U | Time |
|--------|---|---|---|---|------|
| CAQE   | 322| 183|139| 21| 290K |
| RAReQS | 292| 180|112|  2| 317K |
| DepQBF | 270| 160|110|20 | 364K |
| Qute   | 253| 161| 92|  3| 390K |
| Ijtihad| 249| 167| 82|  0| 394K |

(f) HQSpre and QRATPre+ (HQ).

| Solver | S | ⊥ | ⊤ | U | Time |
|--------|---|---|---|---|------|
| CAQE   | 325| 188|137| 14| 279K |
| RAReQS | 303| 189|114|  3| 304K |
| DepQBF | 271| 158|113|20 | 362K |
| Qute   | 263| 170| 93|  2| 377K |
| Ijtihad| 245| 166| 79|  0| 407K |

5 Conclusion

We presented version 2.0 of QRATPre+, a preprocessor for QBFs in PCNF that is based on strong redundancy properties of clauses and universal literals defined by the QRAT+ proof system [17]. QRAT+ is a generalization of the QRAT proof system [7,9]. QRATPre+ is the first implementation of the QRAT and QRAT+ redundancy properties for applications in QBF preprocessing. As such, the techniques implemented in QRATPre+ are orthogonal to techniques applied in state-of-the-art preprocessors like Bloqqer and HQSpre. Our experiments demonstrate a considerable performance increase of preprocessing and solving. QRATPre+ comes with a C API that allows easy integration into other tools.

We observed a sensitivity of solvers to the ordering in which QRATPre+ is coupled with other preprocessors. To better understand the interplay between redundancy elimination and the proof systems implemented in solvers, we want to further analyze this phenomenon. We used a simple but effective witness-based scheduling to avoid superfluous redundancy checks. However, with more sophisticated watched data structures the run time of QRATPre+ could be optimized. To enhance the power of redundancy checking, it could be beneficial to selectively add redundant formula parts to enable additional simplifications afterwards.
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A Appendix

A.1 Additional Experimental Data

Table 3. QBFEVAL’18: solved instances (\(S\)), unsatisfiable (\(\bot\)), satisfiable (\(\top\)), and uniquely solved ones (\(U\)), and total CPU time in kiloseconds (K) including time outs.

(a) Bloqqer and QRATPre+ (BQ).

| Solver   | S | \(\bot\) | \(\top\) | U | Time |
|----------|---|--------|--------|---|------|
| CAQE     | 284 | 165   | 119    | 35 | 358K |
| RAReQS   | 257 | 161   | 96     | 5  | 399K |
| DepQBF   | 204 | 102   | 102    | 17 | 490K |
| Ijtihad  | 197 | 127   | 70     | 3  | 490K |
| Qute     | 184 | 111   | 73     | 1  | 515K |

(b) QRATPre+ and HQSpre (QH).

| Solver   | S | \(\bot\) | \(\top\) | U | Time |
|----------|---|--------|--------|---|------|
| CAQE     | 308 | 183   | 125    | 16 | 303K |
| RAReQS   | 283 | 178   | 105    | 7  | 340K |
| DepQBF   | 251 | 149   | 102    | 23 | 398K |
| Qute     | 244 | 159   | 85     | 3  | 410K |
| Ijtihad  | 231 | 159   | 72     | 2  | 428K |