Abstract

Three years after the completion of the next-to-leading order calculation, the status of the theoretical estimates of $\epsilon'/\epsilon$ is reviewed. In spite of the theoretical progress, the prediction of $\epsilon'/\epsilon$ is still affected by a $100\%$ theoretical error. In this paper the different sources of uncertainty are critically analysed and an updated estimate of $\epsilon'/\epsilon$ is presented. Some theoretical implications of a value of $\epsilon'/\epsilon$ definitely larger than $10^{-3}$ are also discussed.
1 Introduction

After many years, the direct $CP$ violation in $K^0$ decays, parametrized by $\epsilon'$, is still an open issue. The last generation experiments have found \[1, 2\]

\[
\frac{\epsilon'}{\epsilon} = \begin{cases} 
(7.4 \pm 5.9) \times 10^{-4} & \text{E731} \\
(20 \pm 7) \times 10^{-4} & \text{NA31}
\end{cases}
\]  

(1)

From these results no definite conclusions can be drawn on the $CP$ property of the $K^0$ decay vertices, namely on whether $\epsilon'$ is vanishing.

Theoretically, the Standard Model makes precise assumptions on the mechanism that generates the $CP$ violation. Indeed the only source of $CP$ violation is the free phase which appears in the Cabibbo-Kobayashi-Maskawa (CKM) matrix \[3\] with three quark generations. This choice implies a non-vanishing $\epsilon'$, unless some dynamical cancellation occurs. The $CP$-violating phase appears in the $K^0$ decays through the so-called penguin diagrams. However other choices are possible, for example the superweak model \[4\], which predicts strictly $\epsilon' = 0$.

To clarify this issue, a new generation of experiments is going to be built, achieving a sensitivity on $\epsilon'/\epsilon$ at the level of $1–2 \times 10^{-4}$ \[5\]. On the theoretical side, the problem is giving a reliable estimate of $\epsilon'/\epsilon$, including the theoretical error. The task is not easy: physics from many scales effectively contribute to $\epsilon'/\epsilon$, from the top mass down to the strange mass, including important non-perturbative effects. Nevertheless all recent analyses agree on predicting $\epsilon'/\epsilon = \text{few} \times 10^{-4}$, with roughly a 100% relative error \[6, 7\].

In the following we present an updated prediction of $\epsilon'/\epsilon$, giving an account of the procedure and the different sources of theoretical uncertainty. We also discuss the dependence of $\epsilon'/\epsilon$ on some critical non-perturbative parameters.

2 $\epsilon'/\epsilon$ in a few steps

The essential theoretical tool for the calculation of $\epsilon'/\epsilon$ is the $\Delta S = 1$ effective Hamiltonian, which allows the separation of the short- and long-distance physics. Using the effective Hamiltonian, one obtains an expression of $\epsilon'/\epsilon$ that involves CKM parameters, Wilson coefficients and local operator matrix elements. Therefore the evaluation of $\epsilon'/\epsilon$ requires essentially three steps, namely (1) the phenomenological determination of the CKM parameters, (2) the calculation of the Wilson coefficients at a next-to-leading order (NLO) and (3) the determination of the matrix elements of the local operators appearing in the $\Delta S = 1$ effective Hamiltonian.
2.1 Basic formulae

The NLO $\Delta S = 1$ effective Hamiltonian at a scale $m_b > \mu > m_c$ can be written as

$$\mathcal{H} = -\frac{\lambda_u G_F}{\sqrt{2}} \left\{ (1 - \tau) \left[ C_1(\mu) \left( Q_1(\mu) - Q_1^\dagger(\mu) \right) + C_2(\mu) \left( Q_2(\mu) - Q_2^\dagger(\mu) \right) \right] + \tau \sum_{i=1}^{9} C_i(\mu) Q_i(\mu) \right\},$$

(2)

where $G_F$ is the Fermi constant, $\lambda_q = V_{qd} V_{qs}^*$ and $\tau = -\lambda_t/\lambda_u$, $V_{q_i q_j}$ being the CKM matrix elements. The $CP$-conserving and $CP$-violating contributions are easily separated, the latter being proportional to $\tau$.

The operator basis includes eleven dimension-six local four-fermion operators$^1$. They are given by

$$Q_1 = (\bar{s}_\alpha d_\alpha)_{(V-A)} (\bar{u}_\beta u_\beta)_{(V-A)}$$
$$Q_2 = (\bar{s}_\alpha d_\beta)_{(V-A)} (\bar{u}_\beta u_\alpha)_{(V-A)}$$
$$Q_1^c = (\bar{s}_\alpha d_\alpha)_{(V-A)} (\bar{c}_\beta c_\beta)_{(V-A)}$$
$$Q_2^c = (\bar{s}_\alpha d_\beta)_{(V-A)} (\bar{c}_\beta c_\alpha)_{(V-A)}$$
$$Q_{3,5} = (\bar{s}_\alpha d_\alpha)_{(V-A)} \sum_q (\bar{q}_\beta q_\beta)_{(V+A)}$$
$$Q_{4,6} = (\bar{s}_\alpha d_\beta)_{(V-A)} \sum_q (\bar{q}_\beta q_\alpha)_{(V+A)}$$
$$Q_{7,9} = \frac{3}{2} (\bar{s}_\alpha d_\alpha)_{(V-A)} \sum_q e_q (\bar{q}_\beta q_\beta)_{(V+A)}$$
$$Q_8 = \frac{3}{2} (\bar{s}_\alpha d_\beta)_{(V-A)} \sum_q e_q (\bar{q}_\beta q_\alpha)_{(V+A)} ,$$

(3)

where $(\bar{q}_\alpha q_\beta)_{(V\pm A)} = \bar{q}_\alpha \gamma_{\mu} (1 \pm \gamma_5) q_\beta$, $\alpha$ and $\beta$ are colour indices, and the sum index $q$ runs over $\{d, u, s, c\}$. Operators $Q_3$–$Q_6$ are generated by the insertion of the tree level operator $Q_2$ into the strong penguin diagram, while $Q_7$–$Q_9$ come from the electromagnetic penguin diagrams. As we will see, the two classes of operators are both relevant for $\epsilon'/\epsilon$. Further details on the NLO $\Delta S = 1$ effective Hamiltonian can be found in ref. $^8$

From the definition of $\epsilon'$ and using eq. (2), one readily obtains

$$\epsilon' = i e^{i(\delta_2 - \delta_0)} \frac{\omega}{\sqrt{2} \text{Re} A_0} \left[ \omega^{-1}(\text{Im} A_2)' - (1 - \Omega_{IB}) \text{Im} A_0 \right],$$

(4)

$^1$One more operator must be included if $\mu > m_b$ is considered.
where, as usual, $A_I e^{i\delta_I} = \langle \pi\pi(I)|\mathcal{H}|K^0 \rangle$, $\Omega_{IB}$ is the isospin breaking contribution due to the $\pi-\eta-\eta'$ mixing \cite{9}, $\omega = \text{Re} A_2 / \text{Re} A_0$, and

$$
\text{Im} A_0 = -G_F \text{Im} \left( V_{ts} V^\dagger_{td} \right) \times \left\{ - \left( C_6 B_6 + \frac{1}{3} C_5 B_5 \right) Z + \left( C_4 B_4 + \frac{1}{3} C_3 B_3 \right) X \\
+ C_7 B_7^{1/2} \left( \frac{2Y}{3} + \frac{Z}{6} + \frac{X}{2} \right) + C_8 B_8^{1/2} \left( 2Y + \frac{Z}{2} + \frac{X}{6} \right) \\
- C_9 B_9^{1/2} \frac{X}{3} + \left( \frac{C_1 B_1}{3} + C_2 B_2 \right) X \right\},
$$

$$
(\text{Im} A_2)' = -G_F \text{Im} \left( V_{ts} V^\dagger_{td} \right) \times \left\{ C_7 B_7^{3/2} \left( \frac{Y}{3} - \frac{X}{2} \right) + C_8 B_8^{3/2} \left( Y - \frac{X}{6} \right) \\
+ C_9 B_9^{3/2} \frac{2X}{3} \right\},
$$

(5)

The relevant operator matrix elements are given in terms of the $B$-parameters as follows:

$$
\langle \pi\pi(0)|Q_i|K \rangle = B_i^{1/2} \langle \pi\pi(0)|Q_i|K \rangle_{VIA} \\
\langle \pi\pi(2)|Q_i|K \rangle = B_i^{3/2} \langle \pi\pi(2)|Q_i|K \rangle_{VIA},
$$

(6)

where the subscript $VIA$ means that the matrix elements are calculated in the vacuum insertion approximation. VIA matrix elements can be calculated and expressed in terms of the three quantities

$$
X = f_\pi \left( M_K^2 - M_\pi^2 \right), \\
Y = f_\pi \left( \frac{M_K^2}{m_s(\mu) + m_d(\mu)} \right)^2 \sim 12 X \left( \frac{150 \text{ MeV}}{m_s(\mu)} \right)^2, \\
Z = 4 \left( \frac{f_K}{f_\pi} - 1 \right) Y.
$$

(7)

Notice that, contrary to $X$ and $Z$, $Y$ does not vanish in the chiral limit. This reflects the different chiral properties of the operators $Q_7$ and $Q_8$.

### 2.2 CKM matrix elements

In order to estimate $\epsilon'/\epsilon$, we need $\text{Im} \left( V_{ts} V^\dagger_{td} \right)$. This requires a complete knowledge of the CKM matrix $V$. For example, in the Wolfenstein parametrization \cite{10}, up to $O(\lambda^3)$,

$$
V = \begin{pmatrix}
1 - \lambda^2 & \lambda & A\lambda^2 (\rho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^2 (1 - \rho + i\eta) & -A\lambda^2 & 1
\end{pmatrix}
$$

(8)
Figure 1: Constraints on the CP-violating phase and the unitarity triangle. The \( \cos \delta \) distributions are plotted above. The dashed distribution includes the constraint coming from \( \Delta M_{B_d} \) and the lattice determination of \( B_B \sqrt{f_b} \). Below, the same constraints are shown as contour plots in the \( \rho - \eta \) plain. The solid, dashed and dotted lines correspond to the 5\%, 68\% and 95\% of the generated configurations respectively. In this plane the allowed region determines the third vertex of the unitarity triangle \( \sum_{q=\{u,c,t\}} V^*_{qB} V_{qd} = 0 \), the others being (0,0) and (1,0).

and

\[
\text{Im} \left( V_{tq}^* V_{td} \right) = -A^2 \lambda^5 \eta = -A^2 \lambda^5 \sigma \sin \delta ,
\]

(9)

using the definition of the CP-violating phase \( \delta \) given by \( \sigma e^{i\delta} = \rho + i\eta \). While \( \lambda \) is well known, \( A \) and \( \sigma \) can be extracted from the measurements of the \( B \) lifetime and the semileptonic decay rates, respectively. The remaining task is the determination of the CP-violating phase. The CP-violating parameter \( \epsilon \) in the \( K^0-\bar{K}^0 \) mixing is the obvious tool. It is given by

\[
|\epsilon|_{\xi=0} = C_\epsilon B_K A^2 \lambda^6 \sigma \sin \delta \left\{ F(x_c, x_t) + F(x_t) \right\} = F(x_c)\right] - F(x_c) \right\},
\]

(10)
Table 1: Input parameters assumed to be constants in the analysis

| Parameter | Value                        |
|-----------|------------------------------|
| $G_F$     | $1.16639 \times 10^{-5}$ GeV$^{-2}$ |
| $m_c$     | 1.5 GeV                      |
| $m_b$     | 4.5 GeV                      |
| $M_W$     | 80.32 GeV                    |
| $M_B$     | 5.279 GeV                    |
| $M_K$     | 498 MeV                      |
| $\Delta M_K$ | $3.495 \times 10^{-12}$ MeV |
| $M_\pi$  | 140 MeV                      |
| $f_\pi$  | 132 MeV                      |
| $f_K$    | 160 MeV                      |
| $\text{Re}A_0$ | $2.7 \times 10^{-7}$ GeV   |
| $\omega$ | 0.045                        |
| $\lambda = \sin \theta_c$ | 0.221                        |
| $\epsilon^{\text{exp}}$ | $2.284 \times 10^{-3}$      |
| $\mu$    | 2 GeV                        |

where $F(x_i)$ and $F(x_i, x_j)$ are the Inami-Lim functions [11], including the QCD corrections [12], and

$$C_\epsilon = \frac{G_F^2 f_K^2 M_K M_W^2}{6\sqrt{2}\pi^2 \Delta M_K}.$$  \hspace{1cm} (11)

The comparison of the previous expressions with the measured values of $\epsilon$ allows the extraction of $\cos \delta$. However, since eq. (10) is a quadratic function of $\cos \delta$, one obtains two different solutions, corresponding roughly to $\cos \delta$ positive and negative. The solid-line distribution of $\cos \delta$ in fig. is the result of this analysis. The distribution is obtained by generating randomly the various parameters according to the values and errors listed in tables 1 to 3.

To further constrain the $CP$-violating phase, one can exploit the $B_d - \bar{B}_d$ mass difference, given by

$$\Delta M_{B_d} = \frac{G_F^2 M_W^2 M_B^2}{6\pi^2} \frac{f_B^2}{M_B} B_B A^2 \lambda^2 \left(1 + \sigma^2 - 2\sigma \cos \delta\right) F(x_t),$$ \hspace{1cm} (12)

where $B_B$ is the $B$-parameter associated to the $\Delta B = 2$ operator $(\bar{b}d)(V-A)(\bar{b}d)(V-A)$. Recent lattice results give quite large values of $B_B \sqrt{f_B}$ [13]. We use $B_B \sqrt{f_B} = 210 \pm 35$ MeV. The requirement of compatibility between $B_B \sqrt{f_B}$ extracted from eq. (12) and the lattice value results in an effective selection of the positive values of $\cos \delta$; see the dashed $\cos \delta$ distribution in fig. [14]. Therefore we can quote the value

$$\cos \delta = 0.38 \pm 0.23,$$ \hspace{1cm} (13)
Table 2: Input parameters assumed to be uniform in the analysis

| Parameter          | Value         |
|--------------------|---------------|
| $\Lambda_{QCD}^{(4)}$ | $330 \pm 100$ MeV |
| $\Omega_{IB}$        | $0.25 \pm 0.10$ |
| $(f_B B_B^{1/2})_{th}$ | $210 \pm 35$ MeV |
| $B_K^{RG-inv}$       | $0.75 \pm 0.15$ |
| $B_1^{c}$            | $0 - 0.15^{(*)}$ |
| $B_{3,4}$            | $1 - 6^{(*)}$ |
| $B_{5,6}$            | $1.0 \pm 0.2$ |
| $B_{7-8-9}^{(1/2)}$  | $1^{(*)}$     |
| $B_{7}^{(3/2)}$      | $0.6 \pm 0.1$ |
| $B_{8}^{(3/2)}$      | $0.8 \pm 0.15$ |
| $B_{9}^{(3/2)}$      | $0.62 \pm 0.10$ |

where the error is the variance of the dashed distribution in fig. [1].

2.3 Wilson coefficients

The Wilson coefficients $C_i(\mu)$ appearing in eq. (6) can be calculated using the renormalization group improved perturbation theory, provided that $\mu$ is a scale large enough for the perturbation theory to be reliable. Indeed their $\mu$ dependence is controlled by the renormalization group equation

$$\mu^2 \frac{d}{d\mu^2} C_i(\mu) = \frac{1}{2} \sum_j \hat{\gamma}_{ij} C_j(\mu) ,$$

(14)

where $\hat{\gamma}$ is the anomalous dimension matrix of the operators in eq. (4). The NLO calculation of these coefficients is discussed in ref. [8]. To our end, it suffices to recall that, for any suitable value of $\mu$, the Wilson coefficients are a known set of real numbers, which however still depend on the scheme chosen to renormalize the local operators. The typical relative error on the coefficients relevant for $\epsilon'/\epsilon$ is 10–20%. For more details and numerical values, see for example ref. [7].

2.4 Local operator matrix elements

The calculation of the matrix elements of the local operators appearing in eq. (2) requires the use of a non-perturbative technique. Indeed these matrix elements contain the low-energy QCD dynamics, from the scale $\mu$ downward. Besides the $\mu$ dependence, at the NLO they also depend on the operator regularization scheme. Both these dependences must be matched with
the corresponding dependence in the Wilson coefficients, in order to have a scale- and scheme-independent physical prediction. Therefore only a non-perturbative approach which allows a full control over the renormalization scale and scheme dependences can be consistently used. Furthermore this technique should allow choosing the renormalization scale $\mu$ large enough for the perturbative calculation of the Wilson coefficients to be reliable. As far as $\epsilon'/\epsilon$ is concerned, the only known non-perturbative approach that fulfills these requirements is lattice QCD. Several $B$-parameters appearing in eq. (6) have been computed on the lattice [14]. In particular $B_6$ and $B_8^{(3/2)}$, which turn out to be the most important numerically, are known. For the others, we use the vacuum insertion approximation, namely $B = 1$ at $\mu = 2$ GeV. There are two exceptions: first, $B_3$ and $B_4$ are allowed to be as large as 6, considering that the penguin operator matrix elements may be at least partially responsible for the $\Delta I = 1/2$ rule. The other is $B_{1,2}^c$, which strictly speaking cannot even be defined, since $\langle \pi\pi | Q_{1,2}^c | K^0 \rangle_{VIA} = 0$. However, a small contribution is expected beyond the VIA and we parametrize it by assuming the VIA matrix elements to be equal to those of $Q_{1,2}$ and introducing a small $B$-parameter. Table 3 contains the values of the $B$-parameters used in the numerical analysis.

The relevant $B$-parameters are known with an error of about 20%. We will see that unfortunately they produce a larger error in $\epsilon'/\epsilon$ because of a partial cancellation between different terms. However, there is another source of uncertainty stemming from the normalization of the matrix elements, namely the value of the running strange mass, see eqs. (3)–(5). The former good agreement between lattice and QCD sum rules on this mass [13, 14] has been questioned by a recent lattice result, in which the extrapolation to the continuum limit is attempted [18]. We will quantitatively discuss this issue in the next section; however, it is worth noting that this uncertainty comes only from the choice of normalizing the lattice results to the vacuum insertion approximation. In the future the possibility of normalizing lattice results to a better known quantity should be considered.

| $m_t^{\overline{MS}}(m_t)$ | 167 ± 8 GeV |
|---------------------------|-------------|
| $|V_{cb}| = A\lambda^2$ | 0.040 ± 0.003 |
| $|V_{ub}/V_{cb}| = \lambda\sigma$ | 0.080 ± 0.015 |
| $\tau_B$ | 1.56 ± 0.06 ps |
| $\Delta M_{B_d}^{exp}$ | 0.464 ± 0.018 ps$^{-1}$ |
| $m_t^{\overline{MS}}(2\text{ GeV})$ | 128 ± 18 MeV |

Table 3: Input parameters assumed to be Gaussian in the analysis
The “best” estimate

Having the necessary ingredients, we can put them together and produce an estimate of $\frac{\epsilon'}{\epsilon}$. Varying randomly the parameters in tables 2 and 3, we obtain the distribution of fig. 2, from which we estimate

$$\frac{\epsilon'}{\epsilon} = (4.6 \pm 3.0 \pm 0.4) \times 10^{-4}.$$  \hspace{1cm} (15)

Again the first error is the variance of the distribution, while the second one refers to the residual scheme dependence coming from higher order in perturbation theory and it is obtained by using two different renormalization schemes.

Figure 2 also shows the probability distribution $P(\epsilon'/\epsilon > x)$ as a function of $x$. For instance, given our choice of the input parameters and distributions, one obtains $P(\epsilon'/\epsilon > 2 \times 10^{-4}) \sim 0.8$.

Why is the theoretical error 100% or even larger? Wilson coefficients and $B$-parameters have errors $\sim 20\%$ or less. One can argue that there are many contributions, but fig. 3 shows
Figure 3: Relative contributions of the different terms appearing in eq. (6) with respect to $\epsilon'/\epsilon$. The largest terms are those containing $B_6$ and $B_{8}^{(3/2)}$. A partial cancellation occurs between them.

that actually the bulk of the result can be obtained retaining only two operators. On the other hand the two main terms, say $B_6$ and $B_{8}^{3/2}$, partially cancel each other, lowering the central value of the prediction and increasing the relative error. The effectiveness of this cancellation depends not only on the top mass, which nowadays is a well-known quantity, but also on the isospin-breaking parameter $\Omega_{IB}$, of which we have only quite old theoretical estimates \cite{3}. Contributions to the error also come from some overall factors, $\sin \delta$ and $m_{s}^{-2}$ which appears in front of the largest terms. All these effects sum up to give the large error of the final result.

## 3 The future of $\epsilon'/\epsilon$

Given the theoretical difficulties in reducing the error on $\epsilon'/\epsilon$, we may wonder what we will learn from the next generation of experiments, with an expected sensitivity at the level of $2 \times 10^{-4}$.

### 3.1 What the present estimate is good for

According to present theoretical estimates, the Standard Model can accommodiate $\epsilon'/\epsilon$ values between 0 and $10^{-3}$. If the present ambiguity on the experimental value of $\epsilon'/\epsilon$ will be solved,
confirming the E731 figure, then the new measurement will hardly contribute to improving our knowledge of the Standard Model and the QCD dynamics. Indeed, even if a non-vanishing value of $\epsilon'/\epsilon$ would rule out the superweak model allowed at present, a measurement of $\epsilon'/\epsilon < 10^{-3}$ can be accommodated by the Standard Model with no pain, weakly constraining the different parameters that enter the theoretical expression of $\epsilon'/\epsilon$.

On the contrary a measurement of $\epsilon'/\epsilon > 10^{-3}$ would be a fruitful surprise. Its explanation would require either new physics at work or an improved understanding of the QCD dynamics.

3.2 What does $\epsilon'/\epsilon > 10^{-3}$ imply?

In the following let us assume that indeed the future measurements of $\epsilon'/\epsilon$ give a value definitely larger than $10^{-3}$. New sources of $CP$ violation beyond the Standard Model can change the prediction in eq. (15) and accommodate such a large value. However the task is not trivial. For instance supersymmetric effects on $\epsilon'/\epsilon$ are small in comparison with the large error, in the Minimal Supersymmetric Standard Model [17].

Anyway, let us ignore here the possibility of new physics effects and stick to the Standard Model. One or more input parameters must deviate from the values listed in tables 1–3 to account for the increase of $\epsilon'/\epsilon$ in this case. We believe that these tables collect the “best” set of input parameters allowed by our present knowledge of the CKM matrix and the QCD dynamics. However, one can still consider the effect on $\epsilon'/\epsilon$ caused by assuming that some critical parameter is outside its allowed range. This is a particularly reasonable speculation in the case of the $B$-parameters and the hadronic quantities in general. Indeed the systematic errors of the lattice QCD results are not completely under control at present and may be underestimated in some cases. Hopefully new developments of the lattice QCD techniques, such as non-perturbative renormalization and unquenching, will lead to a better control of the systematic errors in the future.

Let us consider two scenarios leading to a large $\epsilon'/\epsilon$: first a small strange mass, then anomalous $B_6$ and $B_8^{3/2}$.

The possibility of a small strange mass, at the level of 50–90 MeV, has been suggested by a recent compilation of lattice results [18]. A previous lattice analysis, done at finite lattice spacing, gave a running mass $m_s^{\overline{MS}}(2\text{ GeV}) = 128 \pm 18$ MeV [15], in agreement with the QCD sum rules determination [16]. The new analysis of ref. [18] extrapolates the value of the strange mass to zero lattice spacing, finding $m_s^{\overline{MS}}(2\text{ GeV}) = 90 \pm 15$ MeV in the quenched
approximation. The inclusion of dynamical fermions further decreases $m_s$ and they quote $m_s^{\overline{MS}}(2 \text{ GeV}) = 70 \pm 11 \text{ MeV}$ for two generations of dynamical fermions. This result indicates at least that the error associated to the finite lattice spacing was probably underestimated in the previous analysis. However, we believe that it is premature to accept the new figures for the running strange mass, until the discrepancy with the QCD sum rules result is understood.

Anyway it is an easy exercise to see how $\epsilon'/\epsilon$ changes in this small-$m_s$ scenario, since the largest terms are proportional to $m_s^{-2}$. We obtain

$$\frac{\epsilon'}{\epsilon} = (14 \pm 8) \times 10^{-4},$$

for $m_s^{\overline{MS}}(2 \text{ GeV}) = 70 \pm 11 \text{ MeV}$. Thus a small strange mass can push $\epsilon'/\epsilon$ up to some units in $10^{-3}$. Notice however that the relative error is roughly unchanged and a vanishing $\epsilon'/\epsilon$ is still well inside the allowed range. We stress again that, as far as $\epsilon'/\epsilon$ is concerned, the problem of $m_s$ appears as the consequence of an inappropriate choice of the $B$-parameter normalization.

In another conceivable scenario, one can take $B_6$ to be large and at the same time a small $B_8^{3/2}$. This choice tends to spoil the cancellation between strong- and electro-penguin operators, thus increasing $\epsilon'/\epsilon$. Indeed a recent result gives a new value $B_8^{3/2} = 0.81 \pm 0.03^{+0.03}_{-0.02}$ smaller than the previous ones [18, 19]. To our knowledge, no new results are available for $B_6$. We can explore an extreme scenario where we choose

$$B_6 = 1.50 \pm 0.15 \text{ and } B_8^{3/2} = 0.50 \pm 0.05.$$  

In this case, we obtain

$$\frac{\epsilon'}{\epsilon} = (12 \pm 4) \times 10^{-4}$$

Again $\epsilon'/\epsilon$ larger than $10^{-3}$ is predicted. The relative error is smaller than before because of the spoiling of the cancellation between $B_6$ and $B_8^{3/2}$. Now $\epsilon' = 0$ is only marginally compatible with the prediction. Notice, however, that precisely the very peculiar situation of a large $B_6$ and a small $B_8^{3/2}$ must be realized in order to enhance $\epsilon'/\epsilon$.

The previous examples show that a measurement of $\epsilon'/\epsilon$ larger than $10^{-3}$ would indeed have non-trivial implications, forcing us either to reconsider some non-perturbative results or to call for new physics.

Acknowledgements

I benefited from comments by E. Franco, W.J. Marciano, G. Martinelli and J.L. Rosner.
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