Partial-wave analysis of $K^+$ nucleon scattering

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Abstract

We have performed a partial-wave analysis of $K^+$-nucleon scattering in the momentum range from 0 to 1.5 GeV/c addressing the uncertainties of the results and comparing them with several previous analyses. It is found that the treatment of the reaction threshold behavior is particularly important.

We find a $T=0$ scattering length which is not consistent with zero, as has been claimed by other analyses. The $T=0$ phase shifts for $\ell > 0$ are consistent with a pure spin-orbit potential. Some indications for the production of a $T=0$ pentaquark with spin-parity $D^6_5/2^+$ are discussed.

I. INTRODUCTION

The interaction of the $K^+$ meson with the nucleon has a number of interesting features. Because of lack of annihilation of the meson anti-quark there is no three-quark intermediate state possible so 3-q resonances are not possible. This lack of 3-body states leads to a feeble interaction, among the smallest among the strong forces.

This fact has been useful for probing the possible changes in nucleon structure and/or $K^+N$ interaction in the nuclear medium \[1, 2, 3, 4, 5, 6, 7, 8\]. Following the measurement of the ratio of total cross sections \[4, 5\], a number of suggestions were made to explain the results, among the principal ones being a partial deconfinement of quarks, an interaction with exchanged mesons in the nucleus and a modification of the $K^+N$ interaction through exchanged mesons. In order to understand which of these possible scenarios might be the right one, a detailed comparison with experiment is in order. One element in making calculations of the multiple scattering corrections which are crucial for this comparison is the availability of reliable phase shifts.

Of equal importance is the use of the $KN$ system to test fundamental theories of hadronic interaction.

There have been a number of theoretical studies of the $K^+N$ system using a variety of approaches \[9, 10, 11, 12, 13, 14\]. The phase shifts from $K^+N$ scattering provide the principal data with which they compare.

There have been very little data taken (but see \[4, 5\]) in an ordinary sense since the most recent partial-wave analyses \[15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28\]. The suggestion of the existence of a possible pentaquark resonance (see Ref. \[29\] for a list of experiments claiming to see it, some of which have been withdrawn) might change the point of view of any analysis. The existence of the pentaquark is now considered doubtful by many.

A partial-wave analysis is particularly interesting in the region of the reaction threshold. The production of pions in the collision of two nucleons is often calculated with the “re-scattering” diagram which describes the formation of the $\Delta^{33}$ resonance by the exchange of one pion. In the case of $K^+$ scattering this simple re-scattering process does not exist since the $K^+$ cannot emit a single pion, and other mechanisms must be considered. The excitation by exchange of a $\rho$ meson might be a natural process to consider, at least from the point of view of the exchange of heavy mesons. Additional information on this process can be obtained from a phase shift analysis which indicates which partial waves are participating in the production.

Section II describes our fitting method, Section III gives a description of the $T=1$ analysis, Section IV discusses the results of the $T=1$ analysis, and Section V gives the method for the $T=0$ analysis. Section VI discusses the results of the $T=0$ analysis and Section VII gives an overall summary and discussion of the results.
II. FITTING PROCEDURE

In the heart of any amplitude analysis lies the search for the (or a) minimum in a \( \chi^2 \) (or similar) measure for the best fit to the data. It is normally assumed that the numerical procedure for this minimization is straightforward and does not pose any problems, but that may not be the case. The technique that we use is very pedestrian but seemingly very sure. The \( \chi^2 \) is minimized on each parameter in turn sweeping through a significant number of them (88 for the \( T=1 \) case and 61 for the \( T=0 \) case including data normalizations). To minimize \( \chi^2 \), each parameter is stepped by a fixed interval until the value of \( \chi^2 \) increases. At this point in the search three values of \( \chi^2 \) are known at three values of the parameter. A parabola is then passed through the three points and the position of the minimum is predicted. This procedure, in some form, is common to most methods of minimization although several methods treat the full parameter space as a vector. There is a difficulty that arises due to the fact that \( \chi^2 \) as a function of the parameter is very often not a perfect parabola. This means that the prediction of the minimum position (and hence \( \chi^2 \) value) is not the true minimum in this region and the value of \( \chi^2 \) predicted may exceed the one at the central (lowest) point. Since the middle point is very often the value obtained in the search on the previous parameter in the sequence, if the predicted value is always accepted, the "current minimum" \( \chi^2 \) will increase in some cases. In our algorithm we test the predicted value of \( \chi^2 \) against the central value and if it is greater, the central (previous best) value is used instead. This incorrect prediction is not a rare occurrence. We observed that it happened about 4% of the time when the search was far from the minimum and up to 40%-50% of the time when it was close to the minimum. If one decreases the step size, the deviation from a parabolic shape can be lessened, but there is a limit to this process since the difference between values of \( \chi^2 \) needed to predict the new minimum becomes small compared with their values and with finite precision another source of error becomes important, even with double precision (which we use). Hence, there is an inherent limit to how well the minimum can be found.

This limit depends on the details of the method used, of course, but also on the parameterization of the phase shifts. Different parameterizations will have different functions to replace the parabola, or perhaps more practically, will have a different importance of third order terms.

This inability to find perfectly the minimum (or minima) translates to a dependence of the final result on the starting values. Tests of the dependence on starting point were made by perturbing some of the parameters (typically two or three) until the \( \chi^2 \) was very large. For \( \chi^2 \) of the order of 100,000 the search usually was unable to find a sensible minimum but for \( \chi^2 \) in the range 10,000 to 20,000 a minimum was found near, but not identical, to the principal fit. A large number of sweeps through the parameters was needed (several tens of thousands). The results of these tests are given in the discussions of the \( T=1 \) and \( T=0 \) analyses.

A \( \chi^2 \) corresponding to each normalization was included in the total \( \chi^2 \) of the fit by adding

\[
\frac{(N-1)^2}{\Delta N}
\]

for each normalizing parameter, \( N \), to the \( \chi^2 \) coming from the individual data points. The value of 0.03 was chosen for \( \Delta N \) since it is a typical value for normalization errors.

The philosophy behind this procedure is that the normalization should be treated as an independent data point. In a typical model experiment the number of counts is registered in a set of counters and then those counts are multiplied by a normalization factor determined by the effective number of target particles, the beam flux etc. The errors in this normalization factor will be common to the cross sections for every counter. While this would appear to be the right statistical procedure, some care is needed in the interpretation of the results. Even if a significant discrepancy occurs in the normalization, it can be hidden if one simply looks at the total \( \chi^2 \) per data point in the case that there are a large number of points which accompany a single normalization. The large number of points dilute the total \( \chi^2 \) so that it is only slightly larger than the number of degrees of freedom. A distinction based solely on total \( \chi^2 \) requires the separation of a value of the reduced \( \chi^2 \) closer and

\[
\frac{(N-1)^2}{\Delta N}
\]
closer to unity from unity. Practical difficulties arise since one can assume that the experimental errors have been estimated correctly only up to a certain level (experience would suggest something of the order of 5%). Hence the square of the errors in the denominator of the definition of $\chi^2$ leads to a sufficient error in its value that this method is impractical. This does not mean that a problem with the normalizations cannot be found, however. It simply means that one must examine the normalizations separately to verify that the the $\chi^2$ from the normalizations alone is not excessive.

III. $K^+$ PROTON ($T=1$) ANALYSIS

The $T=1$ data base used consists of 1880 data points including 1501 angular distribution points, 91 total cross section points, 265 polarization points and 23 reaction cross section points. Much of these data are available from compilations [30, 31, 32] but we try to give references to the original data as much as possible [33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53].

Of these points, four were removed because they were outliers. These were: two points from Ref. [38] at 0.686 and 0.717 GeV/c, and two from Ref. [34] at 0.713 and 1.029 GeV/c. The error on Ref. [33] polarization at 1.330 GeV/c and $\cos \theta = 0.242$ was doubled. The errors were doubled on the polarization set at 1.430 GeV/c [33] since the variation from point to point is greater than the errors quoted.

Partial waves through $F7/2$ were considered. The $G$ and $H$ waves were not needed to get a good fit. We write the partial-wave amplitudes as

$$F_{\ell \pm} = \frac{(S_{\ell \pm} - 1)e^{2i\sigma_{\ell}}}{2i}; \quad S_{\ell \pm} = \eta_{\ell \pm}e^{2i\delta_{\ell \pm}}$$

(2)

where the sign $\pm$ corresponds to $j = \ell \pm \frac{1}{2}$ and $\sigma_{\ell}$ is the Coulomb phase shift. No “inner” Coulomb corrections were made, the Coulomb effect being very small for the energies considered here.
TABLE I: Parameters for the representation of the inelasticity, $\eta_{\ell,j}$ for $T=1$ using the form of Eq. 7 except for the $D3/2$ wave where Eq. 8 is used.

| $LJ$ | $q_R$ (GeV/c) | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ | Equation |
|------|---------------|-------------|-------------|-------------|----------|
| $S1/2$ | 0.4134 | 0.0858 | 0.5030 | | 7 |
| $P1/2$ | 0.3858 | 0.1811 | 0.0163 | | 7 |
| $P3/2$ | 0.4809 | 1.0794 | 2.9043 | 3.9024 | 7 |
| $D3/2$ | 0.6758 | 2.1264 | | | 3 |
| $D5/2$ | 0.4304 | 0.3604 | 0.2723 | | 7 |
| $F5/2$ | 0.5286 | 0.1778 | | | 7 |

TABLE II: Parameters for the representation of the phase shifts for $T=1$ using the form of Eq. 4 for the $S$ and $P$ waves, Eq. 5 for the $D$ waves and Eq. 6 for the $F$ waves.

| $LJ$ | $a$ (GeV/c)$^{-2(\ell+1)}$ | $b_{1,\ell\pm}$ (GeV/c)$^{-2}$ | $b_{2,\ell\pm}$ (GeV/c)$^{-4}$ | $b_{3,\ell\pm}$ (GeV/c)$^{-6}$ | Equation |
|------|------------------------|------------------------|------------------------|------------------------|----------|
| $S1/2$ | $-1.562$ | $-1.108$ | 0.217 | | 3 |
| $P1/2$ | $-12.002$ | $31.139$ | | | 3 |
| $P3/2$ | $13.357$ | $126.76$ | $-666.951$ | $1276.123$ | 3 |
| $D3/2$ | $-2.984$ | $4.119$ | | | 3 |
| $D5/2$ | $-1.702$ | $6.571$ | $-7.462$ | | 3 |
| $F5/2$ | $-0.415$ | | | | 6 |
| $F7/2$ | 0.089 | | | | 6 |

The differential cross section, polarization and total cross section are expressed in terms of the amplitudes in the standard manner (see e.g. Hashimoto [15]) and will not be repeated here. The reaction cross section (essentially pion production) can be written as

$$\sigma_R = \frac{10\pi}{q_f^2} \sum_\ell \left[ \ell(1-\eta_{\ell-}^2) + (\ell+1)(1-\eta_{\ell+}^2) \right]$$

so is independent of the phase shifts and can be particularly important in determining the $\eta_{\ell\pm}$. Here, and in what follows, $q$ denotes the momentum in the center of mass (in GeV/c unless otherwise noted) and $q_f$ is $q/\hbar c$, i.e., in femtometers.

We have taken for the $S$- and $P$-wave phase shifts the form

$$\delta_{\ell\pm} = \tan^{-1} \left[ \frac{a_{\ell\pm} q^{2\ell+1}}{1 + \sum_\ell b_{\ell\pm} q^{2\ell}} \right].$$

The index, $j$ runs from 1 to 2 for the $S$ wave and 1 to 3 for the $P3/2$ wave. There is only one term in the sum for the $P1/2$ wave.

For the $D$ waves a simple polynomial is used

$$\delta_{2\pm} = q^3 (c_{\pm} + d_{\pm} q^2 + e_{\pm} q^4)$$

and only the lowest order was used for the $F$ waves

$$\delta_{3\pm} = q^7 f_{\pm}$$
The $\eta_{\pm}$ were parameterized allowing a different threshold, $q_R$, for each partial wave. Below the threshold the values were unity and above threshold the form

$$\eta = 1 - \sum_{i=1}^{n} \gamma_i \left( \frac{q}{q_R} - 1 \right)^i$$

was used except for the D3/2 partial wave which was represented by

$$\eta = \cos \left[ \sum_{i=1}^{n} \gamma_i \left( \frac{q}{q_R} - 1 \right)^i \right].$$

The highest power of $j$ varied with partial wave, the greatest being 3 in the case of the P3/2 wave. For the F7/2 wave $\eta$ was taken as unity. The values of the parameters for our best fit are given in Tables I and II.

The normalizations of 55 of the data sets was varied in the fitting procedure and their distribution is given in Fig. 1. The standard deviation of the normalizations was 2.2%, less than the 3% used for $\Delta N$ in Eq. 1. The mean normalization was 1.0004 and only two normalizations gave an adjustment greater than 5%. The total $\chi^2$ is 2051 for a $\chi^2$ per data point of 1.09.

The phase shifts for the best fit are shown in Fig. 2 compared with three other analyses discussed below. At low energy (below threshold) the results are very similar to those of Martin [19].

Aside from the main fit, we performed tests for the degree of uniqueness of the minimum. Values of the parameters were altered in varying degrees such that the $\chi^2$ values were very large (generally
FIG. 3: $T=1$ phase shifts and $\eta$'s obtained in the present work (solid line) compared with the variations. The solid line is the principal fit, the dashed lines correspond to fits a, b, c, e, and f, and the dash-dot line to the poorer fit d. The designations of the cases correspond to those in Table III.

of the order of 10,000) and the minimum search was restarted. This was done 6 times and the values of $\chi^2$ at the minima found are given in table III. The values of $\chi^2$ were about 9 above the best fit except for case d where $\chi^2$ was about 20 above the best fit.

The fits found at these other minima are very similar to the original fit. The comparison is shown in Fig. 3. There it is seen that the fits are nearly identical below threshold (with the possible exception of case d) but above threshold there is a noticeable variation. In principle one can eliminate all fits but the principal solution on the grounds that the difference in $\chi^2$ is considerably larger than unity. In practice, however, since one can never be sure that the “true” minimum has been found, we consider the variation among the fits (excluding fit d) as a conservative estimate of the error in the determination of the phase shifts and inelasticities.

A. Other Work for $T=1$

Leaving aside many of the earlier analyses [16, 17, 18, 23, 24, 25, 26, 27, 28], there are three modern analyses with which we compare.

1. Analysis of Hashimoto

Hashimoto [15] performed an energy-independent analysis for momenta from 0.6 to 1.5 GeV/c. Such an analysis has the advantage that no theoretical prejudices are inserted in the parameterization of the energy dependence but the disadvantage that S-matrix elements obtained at one energy do not share information from nearby energies which can result in large fluctuations from one energy to another.
FIG. 4: Top: Contribution of the various partial waves to the reaction cross section. Bottom: Comparison of the reaction cross section from the present work with that of the VPI group and Martin.
TABLE III: Values of $\chi^2$ for the best fit and the variations made in the present work for T=1. The columns labeled P1/2 and P3/2 contain scattering volumes.

| Case  | $\chi^2$ | $\chi^2/N$ | Scatt. Len. (fm) | P1/2 (fm$^3$) | P3/2 (fm$^3$) |
|-------|----------|------------|-----------------|---------------|---------------|
| Best Fit | 2031.05  | 1.080 | $-0.308$ | $-0.092$ | 0.103 |
| a | 2042.47  | 1.086 | $-0.311$ | $-0.055$ | 0.046 |
| b | 2039.94  | 1.085 | $-0.310$ | $-0.067$ | 0.058 |
| c | 2040.00  | 1.085 | $-0.310$ | $-0.066$ | 0.057 |
| d | 2051.98  | 1.091 | $-0.313$ | $-0.029$ | 0.009 |
| e | 2039.67  | 1.085 | $-0.308$ | $-0.084$ | 0.094 |
| f | 2039.02  | 1.085 | $-0.311$ | $-0.060$ | 0.051 |

Observables sometimes have to be grouped to have enough data at a given energy. When amplitudes are slowly changing this does not lead to problems but at the threshold for pion production, where some of the amplitudes change rapidly, it can. Structure was seen in this analysis in the P3/2 and other partial waves.

2. Analysis of the VPI Group

The VPI group [20, 21, 22] has published three analyses. In these fits the S-matrix elements for each partial wave were parameterized in the form of a K-matrix with one elastic and one inelastic channel. Structure is seen in these fits in the P3/2 wave very similar to that of Hashimoto. This led the group to suggest that there was a resonance in this wave and two analyses give the mass pole at 1.780 and 1.796 GeV/c², corresponding to beam momenta of 0.971 and 1.005 GeV/c.

The onset of one-pion inelasticity was assumed to come about by the production of intermediate particles which then decayed to the KN$\pi$ system. Such a model is made very plausible by the experimental fact that pion production does not start at its threshold but some 200 MeV/c higher in momentum (see Figs. 4 and 5). Since the $\Delta$ threshold is 340 MeV/c above pion threshold and the delta has considerable width, so that the production can start below that, it was the prime candidate. The $K^*(892)$ was considered as well but it has a higher threshold.

The VPI work [21, 22] allowed renormalization in much the same way as described above in the present work. However, unlike the present case, substantial renormalization occurred with a number of the data sets being renormalized by more than 5%. We inserted the VPI solution (including the G and H waves) into our program to compare with the data base used here to find a $\chi^2$ of 2810 for the 1880 points (a ratio of 1.49), which implies a fit very similar in quality to the original fit which was 1.73 or 1.25 depending on the case. Two points in the threshold of the reaction cross section (at 0.735 and 0.785 GeV/c) contribute 270 to the $\chi^2$. If we remove these two points the ratio drops to 1.35. One cannot expect a closer agreement since their parameters were not fit to our data base and the phase shifts taken from the paper [22] (or the web site) are quoted with a limited accuracy.

3. Analysis of Martin

Martin performed an analysis in which the partial-wave amplitudes were parameterized directly. The result is equivalent to the partial-wave expansion in terms of $\delta_{\ell\pm}$ and $\eta_{\ell\pm}$ discussed above and the correspondence is easily made.
FIG. 5: Partitioning of the $T=1$ reaction cross sections among partial waves for Hyslop et al. \cite{22} and Martin \cite{19}.
FIG. 6: Argand plot of the T=1 partial waves.

Suppressing the partial-wave index, the Martin amplitudes have the form

\[ F = \frac{C(q)}{A(q) + i[1 + \theta(q - q_0)R^2(q)]C(q)} \]  

(9)

\[ C_{\ell\pm} = \frac{q^{2\ell+1}}{1 + (q/q_0)^{2\ell+1}}. \]  

(10)

\( A(q) \) was approximated by a polynomial in \( q/q_m \) and \( R(q) \) a polynomial in \( \left( \frac{q - q_0}{q_m - q_0} \right) \) where \( q_0 \) is the reaction (pion production) threshold (0.3106 GeV/c in the center of mass) and \( q_m \) is the maximum C. M. momentum considered in the fit. Thus, the inelasticity (expressed in terms of \( R \)) has a form similar to that used in the present work but only a single threshold \( (q_0) \) was used for all partial waves.

In his fit Martin used \( G \) and \( H \) waves which were estimated by means of a dispersion relation calculation \[54\]. He did not give these values and they are no longer available so we were forced to set them to zero. These waves are only important at the higher energies so did not affect the calculations done for the nuclear scattering \[1, 2\]. However, for momenta above \( \approx 1.2 \) GeV/c the Kaon-nucleon amplitude can not be accurately obtained from Martin’s phase shifts alone.

Martin listed several re-normalizations chosen by hand. The average value of these re-normalizations was 0.98 and the standard deviation was 0.067. Since these normalizations were changed by hand, no
\( T = 1 \) P3/2 phase shift in the region just above threshold. The vertical line marks the \( \Delta \) threshold. The chain-dash curve, taken from the paper by Wyborny et al. [56] and is the result of a calculation of the final-state interaction between the \( K^+ \) and the \( \Delta \). The other curves have the same meaning as in Fig. 2.

\( \chi^2 \) was accorded to them by Martin. With a fixed error of 0.03 they would have contributed 64.7 to the \( \chi^2 \).

The Martin solution was also compared with our data base and we found a \( \chi^2 \) of 3023 for a ratio of 1.61. The same two reaction cross section points which contributed large \( \chi^2 \) in the case of the VPI group give a contribution to the \( \chi^2 \) of 446. If we remove these two points, the ratio becomes 1.37. Since the normalizations for the data were refit the test is not completely valid. The major discrepancy occurs for large beam momenta where the lack of \( G \) and \( H \) partial waves used in the original fit is most important. Removing the two reaction cross section points and all data above 1.2 GeV/c, we find a \( \chi^2 \) of 1372 for the 1132 points of the reduced data set or a ratio of 1.21 so that the quality of the fit approaches that quoted in the original paper which was 1.08.

IV. DISCUSSION FOR \( T = 1 \)

The values of the phase shifts and inelasticities for the obtained partial waves are given, compared with other analyses, in Fig. 2. The behavior of the amplitudes is also given in an Argand diagram in Fig. 6. We note that there is no counter clockwise behavior in the P3/2 partial wave as was observed in Nakajima et al. [25] or Arndt et al. [20, 21].

A. Reaction Cross Section

The partitioning of the reaction cross section among the partial waves is given in Fig. 4. The lower part of Fig. 4 gives the comparison of the reaction cross sections calculated with those from Arndt et al. [20] and Martin [19]. One notices that just above the threshold for pion production the reaction cross section remains very small for about 200 MeV/c. A similar thing is seen in nucleon-nucleon scattering and the usual interpretation is that pion production proceeds primarily by \( \Delta \) production with the threshold for the reaction cross section being governed by the mass of the \( \Delta \).

In the present reaction, the threshold behavior can perhaps be explained again by \( \Delta \) production...
although the exchange of a single pion to form the \( \Delta \) is not possible since a pion cannot couple to the kaon. Another possibility is for the reaction to proceed by the excitation of the \( K^*(892) \). The VPI group \cite{21} used these two intermediate states to model the threshold behavior. As can be seen from the insert in Fig. 4 they obtain a rise in the reaction cross section very similar to that obtained by Martin, neither of which is sufficiently rapid. In the case of the VPI group \cite{21} this is because the \( K^*(892) \) has a higher threshold (1.077 GeV/c in the laboratory) and, while the \( \Delta \) has about the right threshold (0.886 GeV/c in the Lab), it has a significant width which causes the onset of the reaction cross section to be gradual. However, Oset and Vicente Vacas \cite{55} had a reasonable success in reproducing the data by combining this mechanism with a chiral symmetry calculation.

Wyborny et al. \cite{56} included final-state interactions between the \( \Delta \) and kaon in the pion production channel and were able to produce a maximum in the \( P3/2 \) phase shift very similar to the one observed. Their result gives a shape closer to our results (or Martin’s) than to those of Hashimoto \cite{15} or Watts et al. \cite{28} with which they compared. Figure 7 shows the comparison of their calculation with the four sets of phase shifts considered here.

In order to have a rapid onset for the reaction cross section through the mechanism of an intermediate particle production, that particle must have a small width. An interesting possibility is pion production by the intermediate step of \( K + N \rightarrow \theta^+ + \pi \) where \( \theta^+ \) is the much discussed pentaquark. Hyodo et al. \cite{57} have considered pion production in this reaction by this mechanism. The mass of the proposed pentaquark is thought to be around 1.54 GeV/c\(^2\) and its width very small. The threshold
for production by this means (0.758 GeV/c in the Lab) is slightly lower than the Δ threshold. The diagrams for these three possibilities are given in Fig. 8.

The separation of the reaction cross section into its partial-wave components may be of some help in sorting out the reaction mechanisms (see Figs. 4 and 5). If one assumes that the intermediate particle is produced at rest in the center of mass then it should be in a relative s-wave. For the case of the Δ (with spin-parity 3/2+) in an s-wave, the only incident partial wave allowed is D3/2. It is interesting to note that in the present work, and in Martin’s analysis, this partial wave has very little reaction cross section and the VPI analysis [21] has only a moderate contribution. One possible explanation is that the production mechanism does not allow the formation of the Δ-kaon final state in a relative s-wave. If we assume that it is produced in a relative p-wave then possibilities for the initial partial wave are P1/2, P3/2 and F5/2. We see that the P1/2 partial wave contributes a significant reaction cross section and the P3/2 partial wave also contains strength, although at slightly higher energies.

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Bland et al. [47] measured the angular dependence of the pion production directly and found that Δ production is in the P1/2 and P3/2 waves. They attempted a partial-wave analysis of the production although it was only possible to include a limited number of amplitudes. Their analysis indicated that the two P waves contributed about equally. They did not include the D5/2 wave in their study. In this same work [47] a model of Δ formation by ρ exchange was also presented which indicated that the expected ratio of P3/2 to P1/2 formation was 5:1, similar to what we find (Fig. 4).

Bland et al. [47] also found strong evidence for interference with an even parity partial wave. This effect became much larger for low beam momenta and invariant masses away from the central mass of the Δ. This even partial wave was only present in the final charge states K0pπ+ and K+nπ+ and not in the state K+pπ0 where the kaon-nucleon system has isospin unity. Also observed in this paper was a strong asymmetry in the Dalitz plot which, again, was not seen in the K+pπ0 final state. These observations suggest that the asymmetry may be linked to the production of a T=0 KN final state, again consistent with the presence of an isospin 0 particle. They [47] also studied the K∗(892) production and were able to say that it occurred in a low angular momentum state consistent with our observation of a rise in the s-wave contribution in the region of the K∗(892) threshold.

### B. Scattering Length

The value of the T=1 scattering length has been very stable over the years. Lévy-Leblond and Gourdin [58] obtained a scattering length of –0.34 fm in what was probably the first analysis. Hyslop et al. [22] obtained a value of –0.33 fm, Cutkosky et al. [27] found –0.28 ± 0.06 fm and Martin [19] found –0.32 fm.

We obtain a central value of the s-wave scattering length of –0.308 fm from the best fit. One can estimate from the variation of minima in Table III an error of ± 0.002 fm. Fixing the scattering length at various values and re-fitting the rest of the parameters, one finds again an error of ± 0.002 fm. Adding the two errors in quadrature we can quote a value of –0.308 ± 0.003 fm. While this error is that obtained with the present data set, it is so small that one would have to expect that the addition of new data would lead to a change of the same order or larger. We note that the average from Refs. [22], [27] and [19] is –0.301 fm so our central value is what might be expected.

On the theory side, Barnes and Swanson [9] obtained, in a quark Born approximation, a scattering length of –0.35 fm. When this value was corrected for unitarity, by solving for scattering from a potential, the value became –0.22 fm.
C. Scattering Volumes

The scattering volumes have always been estimated to be very small and are poorly determined. In early work, the p-waves were often neglected as input to the $T=0$ determinations. Cutkosky et al. [27] obtained $-0.04 \pm 0.03 \text{ fm}^3$ for the $P_{1/2}$ wave and $0.02 \pm 0.02 \text{ fm}^3$ for the $P_{3/2}$ wave, for example. The values obtained in the present work are very dependent on the minimum found, unlike the scattering length. For the higher values of $\chi^2$ the scattering volumes are small, in agreement with Cutkosky et al. [27] but for the best fit, the volumes are somewhat larger in magnitude.

![Graph](https://via.placeholder.com/150)

**FIG. 9:** Distribution of the values of $\chi^2$ for isospin 0 for all data.

V. ANALYSIS FOR $T=0$

A. Data Treatment

The treatment of the isospin zero amplitude is more difficult since there exist no free neutron targets. What is normally used is scattering from the neutron contained in the deuteron. Modern analyses [59, 60] suggest that scattering from the meson cloud is not a problem. There are only slightly more than half of the number of data points compared to the $T=1$ case and the reliability of the data is less, given that it must be extracted from deuteron data with corrections. Again most of the data can be obtained from compilations [30, 31, 32] but we try to give references to original papers [28, 45, 46, 53, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72]. Normally the neutron result is obtained assuming that the interactions of the kaon with the proton and neutron are independent. This is usually a good approximation except at low energies. However, the $T=1$ reaction dominates the deuteron (for example the total cross section on the deuteron is $\frac{1}{2}\sigma(T = 1) + \frac{1}{2}\sigma(T = 0)$) so a good knowledge of the $T=1$ amplitudes is needed. We used $T=1$ amplitudes from our best fit as input to the $T=0$ determination.

The data used in our standard fit are all from incoherent $K^+$ deuteron scattering and $K^+$ deuteron total cross sections. There are 92 points of total cross section data, 17 points of one and two pion
FIG. 10: Partitioning of the $T=0$ reaction cross sections among partial waves for the present work with the standard reaction data.

production data, 336 points of elastic cross sections from the neutron, 43 points of elastic polarization from the neutron, 657 points of charge exchange cross section data and 44 points of charge exchange polarization data for a total of 1189 points.

As might be expected, the pruning of the $T=0$ data is more significant and problematic than the $T=1$ case. In order to choose which points to eliminate, a preliminary fit was made with all data included. By binning the values of $\chi^2$ (bins of unit size were chosen) a distribution was obtained. This distribution, along with the expected distribution obtained from a $\chi^2$ distribution with one degree of freedom is shown in Fig. 9. We see that the expected number of counts in a bin of unit size falls to 0.1 around $\chi^2=15$. For this reason, the 19 points with $\chi^2 \geq 15$ were removed leaving 1170 data points to be fit. It is clear that there is still an excess of points below $\chi^2=15$ but it is impossible to tell which points to remove so no further pruning was done and we must expect to have a higher $\chi^2$. However, we did make one run with all points giving a $\chi^2 > 10$ being removed. The results of this fit are shown in Table II.

B. Double Scattering

At very low energies, double scattering in the deuteron gives an important contribution to the total cross section. Other observables are not sufficiently well measured that it will have a significant effect.

The double scattering amplitude at $0^\circ$ is given by

$$f_D(\theta = 0) = f_D(k, k) = \frac{1}{2\pi^2} \int \frac{dq f_d(q, k) f_u(k, q)}{q^2 - k^2 - i\epsilon} z(|k - q|)$$

where $k$ is the initial and final (on-shell) momentum of the scattering meson, $z(p)$ is the two-body...
FIG. 11: Partitioning of the $T=0$ reaction cross sections among partial waves for the present work with alternate reaction data.

The form factor, and $f(k, q)$ and $f(q, k)$ are half-off-shell basic scattering amplitudes.

For the scattering amplitudes we write the off-shell dependence as

$$f(q, q') = f_{av}(q)v(q')$$

where the form

$$v(q) = \left(\frac{k^2 + \Lambda^2}{q^2 + \Lambda^2}\right)^2$$

will be assumed.

$$f_D(\theta = 0) = \frac{ik}{4\pi} \int d\Omega q f_{b}(q, k) f_{a}(k, q) z(|k - q|) + \frac{1}{2\pi^2} \mathcal{P} \int \frac{d\omega q q^2 dq f_{b}(q, k) f_{a}(k, q)}{q^2 - k^2} z(|k - q|)$$

(12)

(13)

We will consider only the s- and p-waves for this correction. The isospin 1 s-wave is the strongest so we consider the double scattering between it and the s- and p-waves of the neutron. With these assumptions

$$f_D(\theta = 0) = \frac{ik f_{p} f_{n}^*}{4\pi} \int d\Omega q z(|k - q|) + \frac{f_{p} f_{n}^*}{2\pi^2} \mathcal{P} \int \frac{d\omega q q^2 dq}{q^2 - k^2} z(|k - q|)$$

$$+ \frac{ik f_{p} f_{n}^*}{4\pi} \int d\Omega q x z(|k - q|) + \frac{f_{p} f_{n}^*}{2\pi^2} \mathcal{P} \int \frac{d\omega q q^2 dq x}{q^2 - k^2} z(|k - q|)$$

(14)

where $x$ is the cosine of the angle between $q$ and $k$. The double scattering contribution to the total cross section will be

$$\sigma_T = 2\text{Im} \left[ if_{p} f_{n}^* \int d\Omega q z(|k - q|) + \frac{2f_{p} f_{n}^*}{\pi k} \mathcal{P} \int \frac{d\omega q q^2 dq}{q^2 - k^2} z(|k - q|) \right]$$
\[+i f_0^p f_1^n \int d\Omega_q x z (|k - q|) + \frac{2 f_0^p f_1^n}{\pi k} \mathcal{P} \int \frac{d\omega_p q^2 dq x}{q^2 - k^2} z (|k - q|)\]  \hspace{1cm} (15)

Where the factor of 2 comes from the fact that there are two orders of scattering possible. To include charge-exchange scattering we can replace

\[f_0^p f_1^n \rightarrow f_0^p f_1^n - \frac{1}{2} f_0^p f_0^p\]  \hspace{1cm} (16)

Near zero energy the amplitudes become real (so the principal value terms become very small) and the p-wave amplitude goes to zero so that only the first term remains and gives a contribution of \(8\pi f_0^p f_0^n\). Since the fourth term is small for both reasons we have neglected it. The function \(z\) was computed using the one-pion-exchange deuteron wave function \([74]\) which gives a good representation of the momentum distribution in the deuteron \([75]\). While we believe that this correction is needed to get a proper fit, the results with it being left out are shown in Table IV.

C. Reaction Cross Sections

Aside from the values given by Hirata et al. \([64]\), the principal reaction cross sections (one and two pion production) are from Giacomelli et al. \([62]\). They used their deuteron and proton pion production data to extract the neutron data. A more consistent way for us is perhaps to use our fit to the proton results with their deuteron production data to obtain directly the \(T=0\) reaction cross section. These can be obtained directly in the single scattering impulse approximation from the following equations.

\[\sigma_R(T = 1) = \sigma_R(\text{proton}); \quad \sigma_R(\text{deuteron}) = \sigma_R(\text{proton}) + \sigma_R(\text{neutron}) = \frac{3}{2}\sigma_R(T = 1) + \frac{1}{2}\sigma_R(T = 0)\]  \hspace{1cm} (17)
TABLE IV: Values of $\chi^2$ for the best fit and the variations made in the present work for $T=0$. Cases a-f show different fits obtained with the full code. Case “No Krauss/Weiss” corresponds to a fit in which the data of Refs. [4] and [5] were left out. In case “No Double” the double scattering correction was omitted and in case “Alter. Reaction” the reaction data was replaced by the average as explained in the text. The case “Stenger Full” gives the results with the full Stenger data [76] included and case “Stenger Partial” the results where the most forward points were omitted. For the case “No Damerell” the Damerell et al. data [77] were included in the fit.

| Case                  | $\chi^2$ | $N$ | $\chi^2/N$ | Scatt. Len. (fm$^3$) | P1/2 (fm$^3$) | P3/2 (fm$^3$) |
|-----------------------|----------|-----|-------------|----------------------|---------------|---------------|
| Best                  | 1670.561 | 1170| 1.428       | −0.1048              | 0.183         | −0.029        |
| a                     | 1671.214 | 1170| 1.428       | −0.1015              | 0.183         | −0.028        |
| b                     | 1670.561 | 1170| 1.428       | −0.1053              | 0.182         | −0.029        |
| c                     | 1670.758 | 1170| 1.428       | −0.1055              | 0.182         | −0.029        |
| d                     | 1671.085 | 1170| 1.428       | −0.1055              | 0.183         | −0.030        |
| e                     | 1675.274 | 1170| 1.432       | −0.1067              | 0.181         | −0.022        |
| f                     | 1675.301 | 1170| 1.432       | −0.1003              | 0.181         | −0.022        |
| $\chi^2 < 10$         | 1540.480 | 1162| 1.326       | −0.1027              | 0.182         | −0.028        |
| No Krauss/Weiss       | 1649.073 | 1155| 1.428       | −0.0988              | 0.177         | −0.027        |
| No Double             | 1662.132 | 1170| 1.421       | −0.1166              | 0.200         | −0.038        |
| Alter. Reaction       | 1665.105 | 1170| 1.423       | −0.0957              | 0.183         | −0.021        |
| Stenger Full          | 1714.224 | 1190| 1.441       | −0.1096              | 0.174         | −0.036        |
| Stenger Partial       | 1692.632 | 1186| 1.427       | −0.1036              | 0.181         | −0.029        |
| No Damerell           | 1284.174 | 1008| 1.274       | −0.0997              | 0.205         | −0.025        |
| Kbp                   | 1895.322 | 1319| 1.437       | −0.1069              | 0.173         | −0.032        |

TABLE V: Parameters for the representation of the inelasticity, $\eta_{l,j}$ for $T=0$ using the form of Eq. [5] for all the partial waves except the F5/2 wave which uses Eq. [7]. The threshold for the D5/2 wave is $P_{Lab}=1.55$ GeV/c so the $\eta$ for this partial wave can be taken as unity over our fitted range.

| $LJ$ | $q_R$ (GeV/c) | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ | Equation |
|------|---------------|-------------|-------------|-------------|----------|
| S1/2 | 0.4350        | 3.581       | −4.349      | [5]         |
| P1/2 | 0.2515        | 0.822       | 0.822       | [5]         |
| P3/2 | 0.5303        | −9.101      | 46.593      | −79.735     | [8]      |
| D3/2 | 0.3351        | 0.887       | [5]         |
| D5/2 | 0.7120        | [5]         |
| F5/2 | 0.6352        | 1.485       | [7]         |

With not much to choose between the two one might hope to improve the errors by averaging the two determinations. Figures 10 and 11 show the result of a fit with the original reaction data and the alternate reaction data. Table IV shows the effect of using this alternative reaction data.

D. Normalizations

We maintained the constant error of 3% for most of the normalizations although, for the reasons mentioned above, we might expect it to be larger. Figure 12 shows the distribution of normalizations. The distribution is wider than for the $T=1$ case and there is a slight bias. The calculated width of


\[ a (\text{GeV}/c)^{-2} \times 2 \ell + 1 \]  

\[ b_{1,\ell\pm} (\text{GeV}/c)^{-2} \]  

\[ b_{2,\ell\pm} (\text{GeV}/c)^{-4} \]  

\[ b_{3,\ell\pm} (\text{GeV}/c)^{-6} \]  

Table VI: Parameters for the representation of the phase shifts for \( T=0 \) using the form of Eq. 4 for the \( S \) waves, Eq. 5 for the \( D \) waves and Eq. 6 for the \( F \) waves.

the distribution is influenced by one outlier from the Damerell et al. elastic data [72].

E. Other Work

1. Analysis of Hashimoto

Hashimoto [15] observed a resonance-like structure in the \( T=0 \) \( D_{3/2} \) wave which had been seen before [19, 25, 26]. While we see a loop in the Argand diagram, it is less pronounced than what he saw.

2. Analysis of Martin

Martin’s database was somewhat smaller than the one used here. He used, in addition to the direct deuteron data, real parts of the amplitude obtained from a dispersion relation analysis using the \( K^+ \) and \( K^- \) total cross sections. The dispersion relations were once subtracted with the subtraction point was taken at zero energy which means that the scattering lengths were input. He assumed that the isospin 0 scattering length was \( 0 \pm 0.04 \) fm based on previous analyses. His fit resulted in a scattering length of \(-0.035 \) fm in agreement with his small input value.

We inserted Martin’s solution in our code to compare with our database. Since he had a smaller amount of data and the parameters were not fit to the present data base, one cannot expect a reduced \( \chi^2 \) very close to what he obtained. We find (for 1170 points) a \( \chi^2 \) of 2849 or a ratio of 2.44. for the full data set. Restricting the comparison to data below 1.2 GeV/c (because of the missing high partial-waves problem mentioned before) we find a \( \chi^2 \) of 1430 for 827 points for a ratio of 1.73. His original fit (number 2) found a \( \chi^2 \) of 924.5 for 760 points or 1.22.

3. Analysis of the VPI Group

Hyslop et al. [22] use \( \Delta \) production as a model for inelasticity in spite of the fact that the \( K^+ \Delta \) final state is forbidden in the \( T=0 \) channel. Comparing their amplitudes with our data base we find a \( \chi^2 \) of 2776 on 1170 points for a ratio of 2.37. They found 3181 for 1746 data points for a ratio of 1.82. Again, we remark that their parameters were not adjusted to our data base and phase shifts of only limited accuracy are available so the same value cannot be expected.
FIG. 13: Partitioning of the $T=0$ reaction cross sections among partial waves for Hyslop et al. [22] and Martin [10].
FIG. 14: $T=0$ phase shifts and $\eta$'s obtained in the present work (solid line) compared with VPI [22] (dotted line), Hashimoto [15] (dots) and Martin [19] (dashed line). The vertical lines show the relevant thresholds, dash-dot: pion production threshold, dashed: threshold for production of a $K^+$. 

VI. RESULTS FOR $T=0$

Figure 14 shows the phase shifts obtained for the $T=0$ fit compared with three other analyses and Tables V and VI show the parameters which give the phase shifts and inelasticities. Figure 15 shows the behavior of the $T=0$ phase shifts in Argand plots. There is some structure in the plots but nothing that can surely be associated with resonances.

A. Reaction Channels

The isospin 0 channel is quite interesting from the point of view of the mechanism for pion production. Because of isospin, $\Delta$ production is not allowed, nor is the production of the isoscalar $\theta^+$. Indeed, the reaction cross section is seen to be smooth in the region of these two thresholds, unlike the $T=1$ case. The $K^*(892)$ production is permitted and one does observe a rapid rise around where it would be expected (Figs. 10 and 11). This is particularly true of our results but is consistent with the reaction cross sections of the VPI group and Martin as is seen in Fig. 13. Since the $K^*(892)$ has spin-parity $1^-\, \, \frac{1}{2}^-$, if it is produced with the nucleon in a relative $s$-state, the spin-parity values possible are $1/2^-$ and $3/2^-$ which correspond to the incident waves of $S_{1/2}$ and $D_{3/2}$. Indeed, we see the $S_{1/2}$ wave giving an important contribution to the reaction cross section at the threshold and the $D_{3/2}$ wave is the largest single contributor. Somewhat surprising is the dominance (or at least importance) of the $D_{3/2}$ channel. This is the partial wave expected for the (isospin forbidden) production of a $\Delta$ in the $s$-wave. However, this wave may be simply the dominant wave for the non-isobar production as well as receiving a contribution for the $K^*(892)$ production.
FIG. 15: Argand plots of the real and imaginary parts of $\frac{S-1}{2i}$ for $T=0$.

**B. Scattering Length**

The $T=0$ scattering length has a rather checkered history. Lévy-Leblond and Gourdin [58] obtained a value of $-0.05$ fm, with large but unspecified errors. Stenger et al. [76] found a value of $+0.04$ fm (a value which was commonly used in dispersion relation work [17]). There were also suggestions that it might be positive and large [54]. Presumably based on this previous work, Martin [19] set the scattering length to zero with an error of $0.04$ fm as the subtraction point for his dispersion relation constraint. Martin’s fit gives $-0.035$ fm, although he states that it has a large error. Later Martin gives [18] a value of $0.02$ fm and then in still later work [23] he found $-0.23\pm0.18$ fm.

Barnes and Swanson [9] obtained a theoretical scattering length of $-0.12$ fm from a Born quark model. To compare with the experimental value they performed their own extrapolation to zero energy based on single-energy analyses and found $-0.09$ or $-0.17$ fm depending on which analysis they used.

From Hyslop et al. we extrapolate a value of $-0.019$ although they quote in the paper a value of zero. It can be seen from Fig. [14] that the behavior of the isospin 0 s-wave in their fit is rather different from ours and has a great deal of variation in the low-energy region where there are no data. It can only be assumed that the variation in this case is a result of a fit at higher energies. It is seen that the trend of the curve above 1.1 GeV/c is noticeably different from the other determinations.
FIG. 16: T=0 phase shifts and η’s obtained in the present work (solid line) compared with the variations. The solid line is the best fit, the dashed lines correspond to fits a, b, c, and d, and the dash-dot lines to the poorer fits e and f where the labeling of the cases are indicated in Table IV.

For the T=0 s-wave scattering length we adopt for the central value our best fit value of $-0.105 \text{ fm}$ from Table IV. The error for the uncertainty in the minimum is estimated from Table IV to be 0.002 fm. In order to estimate the statistical error the fit was redone for several fixed values of the scattering length varying all other parameters. From the resulting $\chi^2$ curve the error can be estimated to be 0.01 fm. For values of scattering length close to the central value, the $\chi^2$ curve is symmetric but for larger deviations it is not, rising steeply for small values. For the value of $-0.035 \text{ fm}$ given by Martin [19], the $\chi^2$ corresponds to 8.6 standard deviations from our central value. Since the statistical error dominates, we take the scattering length as $-0.105 \pm 0.01 \text{ fm}$. This error does not include possible errors from the variations in the database. Observations on the change in value from the omission of data sets as shown in Table IV suggest that the error from this source can be expected to be of the same order or slightly smaller.

C. Scattering Volumes

The $P_{1/2}$ scattering volume is well determined by the fit to be $0.183 \pm 0.005 \text{ fm}$ where the error comes from an examination of Table IV assuming that there is no reason to exclude the double scattering correction or the Damerell data.

The $P_{3/2}$ scattering volume is smaller and more poorly determined. Again from Table IV we take the value of $-0.029 \pm 0.008 \text{ fm}$.
FIG. 17: Square-well spin-orbit model. The solid line gives the results of the present amplitude analysis, the dotted lines the model described in the text without a central term and the dash-dot lines the model with the central potential which gives the correct s-wave scattering length.

D. Spin-orbit Splitting

The phase shifts for $\ell > 0$ shown in Fig. 14 display a remarkable symmetry below threshold. All phase shifts for $j = \ell - \frac{1}{2}$ are positive and all of those for $j = \ell + \frac{1}{2}$ are negative. In order to see how far a pure spin-orbit interaction would go toward explaining this behavior, we calculated a simple model consisting of scattering from a square well potential with strength $V$ where

$$V = V_0 L \cdot S = \begin{cases} \frac{1}{2} V_0 \ell & j = \ell + \frac{1}{2} \\ -\frac{1}{2} V_0 (\ell + 1) & j = \ell - \frac{1}{2} \end{cases}$$  \tag{18}$$

The radius of the well was taken to be $R = 0.85$ fm (corresponding to an rms radius of 0.66 fm), and the strength, $V_0$ was chosen to be 0.36 GeV. The results are shown in Fig. 17 with the dashed line. The angular momentum barrier changes a great deal from one value of $\ell$ to another and the potential strength also changes over a significant range. The rather remarkable agreement indicates that for $\ell > 0$ the phase shifts in the lower energy region are described by a pure spin-orbit interaction.

Such a potential gives no contribution to the s-wave. If we introduce a central potential (independent of $\ell$) in all partial waves of strength 0.04 GeV we obtain an s-wave scattering length of $-0.11$ fm (in
agreement with our determination from the data). The result for the higher partial waves is shown
in Fig. [17] by the dash-dot lines. Thus, including a central potential of sufficient strength to give the
moderate s-wave scattering length does not destroy the good agreement seen before.

E. Variations in the fit

As in the $T=1$ case, a number of different minima were found corresponding to different starting
points. The basic properties of the different fits are given in Table [IV] (cases a-f) and the variations of
the phase shifts are shown in Fig. [16].

The elastic data by Stenger et al. [76] was not included in the general fit. These data were among
the first to find a very small scattering length for the $T=0$ channel. They consist of charge exchange
and elastic scattering from the deuteron. In the elastic scattering there was no separation of scattering
from the neutron or proton or, indeed, coherently from the deuteron (leaving it intact). Thus, for
the elastic scattering the cross section from the proton should be added to that of the neutron and
coherence must be taken into account as well. The contamination from coherent scattering is largest
for small angles. For the charge-exchange cross section small angle scattering (small momentum
transfer) tends (without spin-flip) to leave two protons in a triplet s-wave state. Since this state is
blocked by the Pauli principle and the spin-flip amplitude is small at small angles, this effect leads to
a very large suppression of the charge-exchange cross section such that it is far from charge exchange
on a free neutron. The data were taken in large angle bins ($0.4 \cos \theta$). In order to estimate the
effect of leaving out this data set we made two runs, one in which the two lowest energies were fully
included in the fit and one in which the most forward points (at $\cos \theta = 0.8$) were excluded. The
results are summarized in Table [IV].

There are two modern data sets of total cross section data, those of Krauss et al. [4] and Weiss et
al. [5]. In the fitting process these data suffer a renormalization (down) of about 4% which is greater
than might be expected. To see the effect of these data on the fit, a run was made with them left out.
The result is shown in Table [IV].

Martin [19] comments that the Damerell et al. [72] data fit poorly with the rest of the data base.
For this reason we made a fit with this data removed. Since there is no a priori reason to mistrust
these data they were used in all of the other fits. The results of this fit are shown in Table [IV].
The normalization of the elastic scattering ($K^+n$) is greater than 20% and there is some shift in the
low-energy parameters obtained.

Two experiments have been performed [77, 78] using the inverse charge-exchange reaction on proton
targets with $K$-long beams. It has been recognized for some time that there is great difficulty in
controlling the normalization of these beams because of regeneration of $K$-short mesons. For this
reason we did not use these data in the principal fit. However, we did include one of the sets [77] in
a run to see the possible effect. The results are shown in Table [IV]. Table [VII] gives the normalization
factors which result from the fit. The normalization errors in the fit were taken as 0.1 through 0.95
GeV/c and 0.2 above that as suggested in the experimental paper [77].

VII. SUMMARY AND CONCLUSIONS

We have presented an easy-to-use parameterization of the $K^+$ Nucleon amplitudes. As mentioned
in the introduction, there is a need for reliable amplitudes for several purposes.

Our representation for s- and p-waves is equivalent to the effective-range expansion which, for
s-waves, reads

$$k \cot \delta = 1/a + \frac{1}{2} r_0 k^2 \ldots$$  (19)

This form was derived originally for a potential interaction but was shown to be valid for an effective
field theory [73, 80]. More recently it has been shown to arise from renormalization group calculations.
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| $P_{Lab}$ (GeV/c) | Normalization Factor |
|------------------|-----------------------|
| 0.65             | 0.769                 |
| 0.75             | 1.025                 |
| 0.85             | 0.684                 |
| 0.95             | 0.667                 |
| 1.05             | 0.695                 |
| 1.15             | 0.881                 |
| 1.25             | 1.082                 |
| 1.35             | 1.210                 |
| 1.45             | 1.356                 |

TABLE VII: Normalization factors for the data of Armitage et al. [77].

Since one would tend to believe that this is an appropriate expansion it would seem to be ill advised to set the scattering length to zero.

We now discuss several points which are particularly interesting for their physics potential.

1) The ratio of the scattering length for $T=1$ to that of $T=0$ may be an interesting quantity. For example, in the work of Barnes and Swanson [9] this ratio depends only on the ratio of the sizes of the kaon and nucleon and the strange quark mass, being independent of the absolute size of the hadronic systems. While this is only approximately true in their work (i.e. only in Born approximation), it indicates that this may be a quantity which is sensitive to only a restricted set of physical parameters. Our value for this ratio is $2.9 \pm 0.3$.

2) The reaction cross section in the $T=0$ state may provide an interesting piece of data for the calculation of pion production. Here the usual dominant mechanism ($\Delta$ production) is isospin forbidden so that other mechanisms will be more apparent. Calculations of the type of Oset and Vicente Vacas [55] might be interesting for the $T=0$ channel.

3) The simple form of the $\ell > 0$ phase shifts for the $T=0$ amplitude is remarkable. To date, no theoretical model has been able to reproduce this feature although the work of Büttgen et al. [56] was able to get a moderately good representation of the data with some degree of phenomenology.

4) The presence of a narrow pentaquark state would facilitate the understanding of pion production in the $T=1$ channel. In fact, the best way to look for a narrow resonance may be to produce it and look for a sudden change in the inelastic cross section. This was the way in which the existence of the $J/\Psi$ was first indicated. If this is indeed the explanation of the rapid rise in pion production and if the final $\Theta - \pi$ state has relative angular momentum zero, then the spin-parity of the $\Theta$ must be $D5/2^+$. Thus, the resonance, as seen directly in the $T=0$ channel, would be in the $F5/2$ partial wave. We have seen that this partial wave is very attractive with a large angular momentum barrier so that a narrow “molecular” state is possible.

We thank Jean-Pierre Dedonder for comments after a careful reading of the manuscript.

This work was supported by the National Science Foundation under contract PHY-0099729.

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