Searching for New Physics in Rare $B \to \tau$ Decays

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Abstract

The rare decays $B^- \to \tau \bar{\nu}$, $B \to \tau^+ \tau^-$, $b \to X \nu \bar{\nu}$ and $b \to X \tau^+ \tau^-$ all contain third generation leptons in the final state, and hence are sensitive to new physics that couples more strongly to the third family. We present model independent expressions for these decays that can be useful to study several types of new physics effects. We concentrate on supersymmetric models without R-parity and without lepton number. We also assume a horizontal $U(1)$ symmetry with fermion horizontal charges chosen to explain the magnitude of fermion masses and quark mixing angles. This allows us to estimate the order of magnitude of the new effects, and to derive numerical predictions for the various decay rates and for the forward-backward asymmetry and the $\tau$ polarization components measurable in $b \to X \tau^+ \tau^-$. In some cases the branching ratios are enhanced by more than one order of magnitude, rendering foreseeable their detection at upcoming $B$-factories. We also discuss how a measurement of asymmetries in $b \to X \tau^+ \tau^-$ can be crucial in distinguishing between different sources of new physics.

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I. INTRODUCTION

The standard model (SM) of the strong and electroweak interactions provides a successful description of all the phenomena involving the known elementary particles. However, experimental results involving fermions of the third generation are far less precise than for the first two generations. From the theoretical point of view, several models of new physics predict larger deviations from the SM for processes involving third generation fermions [1]. This is also the case in a class of supersymmetric (SUSY) models without R-parity [2] where violation of lepton (L) and baryon (B) number couples more strongly to the heavier fermions [3–6].

In recent years, the experiments at the CERN $e^+e^-$ collider LEP have provided us with most of the results on $b$ decays into the third generation leptons $\tau$ and $\nu_\tau$. This is because the LEP environment has the advantage over symmetric $B$-factories (like CLEO) or hadron colliders (like CDF) of allowing accurate measurements of the missing energy associated with primary $b \to \nu_\tau$ or secondary $b \to \tau \to \nu_\tau$ final state neutrinos. In this way, decay modes yielding a missing energy spectrum harder than the usual semileptonic decay can be effectively measured or constrained. At LEP, measurements of an excess of events over the semileptonic background with missing energy between 10 and 30 GeV [7–9] was interpreted as the signature of the decay $B \to X_c \tau \bar{\nu}_\tau$ followed by $\tau \to \nu X$. This yielded $\text{BR}(b \to X_c \tau \bar{\nu}_\tau) = 2.68 \pm 0.34\%$ in agreement with the SM prediction $\text{BR}(b \to X_c \tau \bar{\nu}_\tau) = 2.30 \pm 0.25\%$ [10]. Using a large missing energy tag the L3 Collaboration set the 90% confidence level upper limit on the exclusive leptonic decay $B^- \to \tau \bar{\nu}$ [11]

$$\text{BR}(B^- \to \tau \bar{\nu}) < 5.7 \times 10^{-4}. \quad (1.1)$$

In [12] it was discussed how similar analyses can yield a limit on the flavor changing decay $b \to X \nu \bar{\nu}$. Based of the full LEP–I data sample, the ALEPH Collaboration derived a preliminary 90% confidence level limit on this decay mode [13]

$$\text{BR}(b \to X \nu \bar{\nu}) < 7.7 \times 10^{-4}. \quad (1.2)$$

Being only one order of magnitude above the SM predictions, the limits (1.1) and (1.2) imply strong constraints on several models of new physics [12,14].

The tight limits on very large missing energy events ($E_{\text{miss}} > 35$ GeV ) in $b$ decays reported by the ALEPH collaboration [7] allowed to estimate order of magnitude bounds on $\text{BR}(B_d \to \tau^+ \tau^-)$ and $\text{BR}(B_s \to \tau^+ \tau^-)$ [14]. In Ref. [14] it was also argued that a weak upper limit on the branching ratio for $b \to X \tau^+ \tau^-$ of the order of the semitauonic branching ratio is implied by the LEP missing energy measurements. The limits were estimated as [15]

$$\text{BR}(B_d \to \tau^+ \tau^-) < 1.5 \times 10^{-2}$$
$$\text{BR}(B_s \to \tau^+ \tau^-) < 5.0 \times 10^{-2}$$
$$\text{BR}(b \to X \tau^+ \tau^-) < 5.0 \times 10^{-2}. \quad (1.3)$$

Even if several orders of magnitude above the SM rates, these figures still yield bounds on some new physics parameters which are unconstrained by other processes [15].

In the near future, experiments at $B$-factories will reach a much higher sensitivity in the study of $b \to \tau, \nu_\tau$ decays. A measurement of $B^- \to \tau \bar{\nu}$ appears to be accessible even
at the low SM rate. With refined experimental techniques and after few years of run, the experimentally very challenging decay $b \to X \nu \bar{\nu}$ could also be measured, at least in some exclusive decay channel. Because of the even lower rates and of the expected low efficiency in $\tau$ identification, the decays $B_d \to \tau^+ \tau^-$ and $b \to X \tau^+ \tau^-$ might be out of the reach of experiments like BaBar and BELLE if their rates are at the SM level. However, as we will discuss, some new physics models predict decay rates more than one order of magnitude above the SM. Therefore, dedicated studies of $b \to \tau, \nu$ decays at $B$-factories represent a powerful tool for detecting signals of new physics.

The paper is organized as follows. In section II we introduce general four fermion amplitudes for the decays $B^- \to l \bar{\nu}$, $B_q \to l^+ l^-$ and $b \to X_q \nu \bar{\nu}$ ($q = d, s$) and we give the results for the various branching ratios. Since our approach is essentially model independent, it is well suited to study different types of new physics contributions.

In section III we concentrate on the decay $b \to X l^+ l^-$. To take into account the effects of new physics, the standard basis of operators contributing to the effective Hamiltonian of the decay \cite{16} has to be enlarged. We generalize it by introducing a set of operators for the right-handed flavor changing current $\bar{s}_R \gamma_\mu b_R$ together with a set of new scalar operators. These new effective operators arise in several new physics models, like SUSY models without R-parity \cite{2}, models with leptoquarks \cite{17}, left-right symmetric models \cite{18}, etc. We study the various observables measurable in the decay: the inclusive rate, the forward-backward asymmetry and the $\tau$ polarization asymmetries. Since for the decay channel $b \to X \tau^+ \tau^-$ the effect of the $\tau$ mass is non negligible and the average energy of the final hadronic system is not very large, in our computation we retain all the fermion masses. We next apply our results to the study of SUSY models without R-parity and without $L$ number. The theoretical framework is presented in section IV. Since in these models the values of the various R-parity violating couplings is not determined, without further theoretical input no numerical prediction of the corresponding effects on $B$ decays is possible. In order to estimate the new physics effects, we appeal to models where the magnitude of the fermion masses and CKM mixing angles is explained by assuming some horizontal $U(1)$ symmetry. This framework provides us with additional theoretical constraints yielding a set of numerical predictions for the various $L$ violating couplings, and allowing for order of magnitude estimates of the various decay rates.

In section V we present a numerical analysis of two representative models and we discuss the results. In particular, we compare our estimates for decays involving the transition $b \to \tau$ with the corresponding decays involving muons, and we confront the predictions of our new physics models with the SM. Finally, section VI contains the summary and our conclusions. The set of input parameters used in the numerical analysis is collected in an Appendix.

II. THE DECAYS $B^- \to \tau \bar{\nu}$, $B \to \tau^+ \tau^-$ AND $b \to X \nu \bar{\nu}$.

The most general non-derivative effective four-fermion interaction involving a $b$ quark, a $q = d, s$ or $u$ quark, and a pair of leptons $\ell$ and $\ell'$ can be written in the form

$$\mathcal{H}_{\text{eff}}^{qb} = -G_F \sum_a \left( \bar{q} \Gamma_a b \right) \left( \bar{\ell} \left[ C_a \Gamma_a + C'_a \gamma_5 \Gamma_a \right] \ell' \right)$$

(2.1)
where $\Gamma_a = \{I, \gamma_5, \gamma^\mu, \gamma^\mu \gamma_5, \sigma_{\mu\nu}\}$ with $a = \{S, P, V, A, T\}$ the standard basis of operators of the Clifford algebra. In (2.1) we have factored out the Fermi constant $G_F$ so that all the coefficients $C_a$ and $C'_a$ are dimensionless. Even in the presence of new physics, most of the rare $B$ decays depend only on a subset of the operators in (2.1). This is due to the fact that for purely leptonic $B$ decays, several matrix elements of the quark operators vanish. Assuming that neutrinos are described by two component left-handed spinor fields, the number of relevant operators is further reduced when neutrinos appear in the final state.

We will now list the general expressions for the different decays.

### A. The decay $B^- \to l \bar{\nu}$.

The decay $B^- \to l \bar{\nu}$ is described by the effective Hamiltonian (2.1) with $q = u, \ell = l_i$ and $\ell' = \nu_j$ where $i, j = 1, 2, 3$ correspond to the different lepton flavors. In the presence of new physics (for example in SUSY models without R-parity) $i \neq j$ is an open possibility. Since final state neutrinos are not detected, in these cases a sum has to be taken over all the allowed decay modes.

The general amplitude for this decay involves a set of matrix elements $\langle 0 | \bar{u} \gamma^\mu \gamma_5 b | B^- \rangle$. They vanish for the parity-even operators $\Gamma_S = I$ and $\Gamma_P = \gamma^\mu$ due to the pseudoscalar nature of the $B$ meson. The tensor operator $\Gamma_T = \sigma_{\mu\nu}$ is antisymmetric in the Lorentz indices, and hence its matrix element must vanish as well, since the only available four-vector is the momentum $p_B^\mu$ of the $B$ meson. Therefore, only the matrix elements of the pseudoscalar and axial-vector operators contribute. They are given by the PCAC (partial conserved axial current) relations

\[
\langle 0 | \bar{u} \gamma^\mu \gamma_5 b | B^- \rangle = i f_B p_B^\mu, \\
\langle 0 | \bar{u} \gamma_5 b | B^- \rangle = -i f_B \frac{m_B^2}{m_b + m_u} \simeq -i f_B \frac{m_B^2}{m_b}.
\]

Under the assumption of two-component left-handed neutrinos $\nu_L = P_L \nu$, with $P_L = \frac{1}{2}(1 - \gamma_5)$, we further have $\bar{l} \gamma_5 \nu_L = -\bar{l} \nu_L$ and $\bar{l} \gamma^\mu \gamma_5 \nu_L = -\bar{l} \gamma^\mu \nu_L$. For on-shell final state leptons, the latter operator contracted with the $B$ meson four-momentum $p_B^\mu = k_l^\mu + k_\nu^\mu$ yields $p_B^\mu (\bar{l} \gamma^\mu \nu_L) = m_l (\bar{l} \nu_L)$. Hence the amplitude for the $B^- \to l \bar{\nu}$ decay reads

\[
\mathcal{A}^\nu = i f_B m_B G_F \left( (C_A^{d\bar{\nu}} - C_A^{d\bar{\nu}}') \frac{m_l}{m_B} - (C_P^{d\bar{\nu}} - C_P^{d\bar{\nu}}') \right) (\bar{l} \nu_L).
\]

The corresponding expression for the decay rate is

\[
\text{BR}(B^- \to l \bar{\nu}) = f_B^2 \frac{G_F^2 m_B m_l^2}{16 \pi} \left[ 1 - \frac{m_l^2}{m_B^2} \right]^2 \sum_i \left| (C_A^{d\bar{\nu}} - C_A^{d\bar{\nu}}') - \frac{m_B}{m_l} (C_P^{d\bar{\nu}} - C_P^{d\bar{\nu}}') \right|^2, \quad (2.4)
\]

\*We assume that even in the presence of new physics, the neutrinos produced in $B$ decays are mainly the SM ones. This is not a strong assumption, since light right-handed neutrinos are theoretically disfavored. If the ‘SM neutrinos’ have non-vanishing masses, operators that vanish in the massless limit are suppressed at least as $m_\nu/m_B$ and hence always negligible.
where the sum over the index $i$ accounts for possible $\nu_i \neq \nu_j$ decay channels. For $l = \tau$ new physics can induce sizeable enhancements over the SM rate (this can occur for example in multi Higgs doublet models where new contributions arise from charged Higgs exchange diagrams [13]). However, a large theoretical uncertainty in predicting the branching ratios is associated with the present poor knowledge of $f_B$. Therefore, it could be difficult to identify unambiguously new physics effects in this decay.

In the SM $C_P^q = C_P^{q'} = 0$ and

$$\left[ C_A^q - C_A^{q'} \right]_{\text{SM}} = -\sqrt{2} V_{ub}.$$  \hspace{1cm} (2.5)$$

Using the set of reference parameters listed in the Appendix, for the SM branching ratio we find $\text{BR}^\text{SM}(B^- \to \tau \bar{\nu}) = 7.1 \times 10^{-5}$.

\section*{B. The decay $B_q \to l^+ l^-$}

A detailed analysis of the decay $B_q \to l^+ l^-$ and of the possible types of new physics contribution was presented in [13]. For on shell $\tau$'s, $p_B^\mu (\bar{l} \gamma_\mu l) = (k_{\tau^+} + k_{\tau^-})(\bar{l} \gamma_\mu l) = 0$ so that also the contribution of the axial-vector operator $\langle 0| \bar{q} \gamma^\mu \gamma_5 b | B \rangle$ vanishes when contracted with the leptonic vector current. The general form of the amplitude reads

$$A_l^q = i f_{B_q} m_B G_F \left[ \left( C_P^q - \frac{2 m_l}{m_B} C_A^q \right) (\bar{l} \gamma_5 l) + C_P^{q'} (\bar{l} l) \right], \hspace{1cm} (2.6)$$

and the corresponding branching ratio is

$$\text{BR}(B_q \to l^+ l^-) = f_{B_q}^2 \tau_B m_B^2 G_F^2 \frac{C_F^q}{8 \pi} \sqrt{1 - \frac{4 m_l^2}{m_B^2}} \left[ \left| C_P^q - \frac{2 m_l}{m_B} C_A^q \right|^2 + \left( 1 - \frac{4 m_l^2}{m_B^2} \right) \left| C_P^{q'} \right|^2 \right]. \hspace{1cm} (2.7)$$

In the SM, $C_P^{q'}$ and $C_P^q$ arise from penguin diagrams with physical and unphysical neutral scalar exchange, and are suppressed as $\sim (m_b/m_W)^2$ [20]. The decay rate is then determined by

$$\left[ C_A^q \right]_{\text{SM}} = \frac{\alpha V_{tb} V_{tq}}{\sqrt{8 \pi \sin^2 \theta_W}} Y_0(x_t), \hspace{1cm} (2.8)$$

where $x_t = m_t^2/m_W^2$, and at leading order [21]

$$Y_0(x) = \frac{x}{8} \left[ \frac{x - 4}{x - 1} + \frac{3x}{(x - 1)^2} \ln x \right]. \hspace{1cm} (2.9)$$

Using the parameters listed in the Appendix, we find $\text{BR}^\text{SM}(B_q \to \tau^+ \tau^-) = 9.1 \times 10^{-7} |V_{tq}/V_{ts}|^2$. We stress that as for $B^- \to \tau \bar{\nu}$, also for this decay theoretical predictions are plagued by the large uncertainty in the value of $f_{B_q}$, which can easily mask new physics effects.
C. The decay \( b \to X_q \nu \bar{\nu} \).

The decay \( b \to X_q \nu \bar{\nu} \) was thoroughly studied in \([12]\). In the presence of new physics, the flavor of the two final state neutrinos can differ. Still, under the only assumption that neutrinos are purely left-handed and effectively massless, the general form of the amplitude has the remarkably simple form \([12]\)

\[
A^{qij} = G_F \left\{ C_L^{qij} (\bar{q}_L \gamma_\mu b_L) \left( \bar{\nu}_L^\mu \nu_L^\mu \right) + C_R^{qij} (\bar{q}_R \gamma_\mu b_R) \left( \bar{\nu}_L^\mu \nu_L^\mu \right) \right\}.
\]

(2.10)

In terms of the coefficients in \((2.1)\) we have \(C_{L,R} = [(C_{V} - C_{V}') \pm (C_{A} - C_{A}')] / 4\). Summing over the undetected neutrino flavors the branching ratio normalized to the semileptonic decay reads

\[
\text{BR}(b \to X_q \nu \bar{\nu}) = \frac{\sum_{ij} \left( |C_L^{qij}|^2 + |C_R^{qij}|^2 \right)}{8|V_{cb}|^2 f_{PS}(m_e^2/m_b^2)} \text{BR}(b \to X_e e \bar{\nu}),
\]

(2.11)

where \(f_{PS}(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x \approx 0.5 \) for \(x = m_e^2/m_b^2\) is the phase space factor for the semileptonic decay. In the SM the decay proceeds via W box and Z penguin diagrams, and only one operator contributes to the decay: \(O_{SM}^L = (\bar{q}_L \gamma_\mu b_L) \sum_i (\bar{\nu}_L^\mu \nu_L^\mu)\). The corresponding coefficient reads

\[
\left[ C_L^g \right]_{SM} = \frac{\sqrt{2} \alpha V_{tb} V_{ts}}{\pi \sin^2 \theta_W} X_0(x_t),
\]

(2.12)

where \([21]\)

\[
X_0(x) = \frac{x}{8} \left[ \frac{2 + x}{x - 1} + \frac{3x - 6}{(x - 1)^2} \ln x \right].
\]

(2.13)

The additional \(1/m_b^2\) and \(\alpha_s\) corrections to this result can be found in \([12, 22, 23]\). In contrast to the previous decays, theoretical predictions for \(b \to X_q \nu \bar{\nu}\) are remarkably free from uncertainties. In fact all the parameters entering in \((2.11)\) and \((2.12)\) are known with good accuracy (the main uncertainty comes from \(m_t\)), there are no long distance effects and QCD corrections are small \([22]\). From the theoretical point of view, new physics affecting this decay could be identified in a very clean way. At leading order, the SM prediction for the branching ratio is \(\text{BR}^{SM}(b \to X_q \nu \bar{\nu}) = 4.4 \times 10^{-5} \left| V_{ts}/V_{ts} \right|^2\).

III. THE DECAY \( b \to X^+ \nu \bar{\nu} \).

A. General operator basis.

In the SM, the effective Hamiltonian for the weak decay \( b \to X_q \nu \bar{\nu} \) is defined in terms of a set of ten effective operators \(O_1 - O_{10} \) \([10, 1]\). At leading order, and neglecting small

\(^1\)For simplicity we will restrict ourselves to the case when the final hadronic system carries strangeness \((X = X_s)\). Generalization to the case \(X = X_d\) requires introducing the additional operators \(O_1^d = (\bar{d}_L^\gamma \bar{\mu}_b^d) (\bar{u}_L^\delta \gamma_\mu u_L^d)\) and \(O_2^d = (\bar{d}_L^\gamma \bar{\mu}_b^d) (\bar{u}_L^\delta \gamma_\mu u_L^d)\), keeping the terms proportional to \(V_{ub}\) and including new long distance effects.
contributions induced only through operator mixing, the operator basis can be truncated to the following set \[24\]

\[
\begin{align*}
O_1 &= (\bar{s}_L^2 \gamma_\mu b^\alpha_R) (\bar{c}_L^3 \gamma_\mu c^\alpha_L) \\
O_2 &= (\bar{s}_L^2 \gamma_\mu b^\alpha_R) (\bar{c}_L^3 \gamma_\mu c^\alpha_L) \\
O_7 &= (\bar{s}_5^\sigma \sigma^{\mu\nu} [m_\mu P_L + m_\nu P_R] b^\alpha) F^{\mu\nu} \\
O_9 &= (\bar{s}_L^2 \gamma_\mu b^\alpha_R) (\bar{l}_\gamma \mu l) \\
O_{10} &= (\bar{s}_L^2 \gamma_\mu b^\alpha_R) (\bar{l}_\gamma \mu \gamma_5 l) .
\end{align*}
\]

(3.1)

In the models we want to study, a larger set of non-renormalizable operators arises. As it will become clear in the next section, after Fierz transformation squark exchange induces at the tree level the new operators \(O_2', O_9', O_{10}'\) which are analogous to the corresponding operators in (3.1) with the replacement \((\bar{s}_L \gamma_\mu b_L) \rightarrow (\bar{s}_R \gamma_\mu b_R)\). While \(O_9'\) and \(O_{10}'\) contribute directly to the decay, \(O_2'\) enters only at the one loop level, so that the corresponding short distance contribution is small. However, \(O_2'\) induces also new long distance effects associated with \(\bar{c}c\) resonances which can further enhance the decay rate above the SM. For this reason we include \(O_2'\) in our set. At the new physics scale \(\tilde{m} \gtrsim 100\) GeV where these operators are generated, other operators from new physics appear only at the loop level. Since they give only suppressed short distance contributions we set to zero the corresponding high energy coefficients. However, in the evolution from the scale \(\tilde{m}\) down to \(m_b\) operator mixing occurs, and from the point of view of the low energy theory a clear distinction between tree-level and loop-level contributions is lost. This forces us to extend the basis to the following set

\[
\begin{align*}
O_1' &= (\bar{s}_R^\alpha \gamma_\mu b^\alpha_R) (\bar{c}_L^3 \gamma_\mu c^\alpha_L) \\
O_2' &= (\bar{s}_R^\alpha \gamma_\mu b^\alpha_R) (\bar{c}_L^3 \gamma_\mu c^\alpha_L) \\
O_7' &= (\bar{s}_5^\sigma \sigma^{\mu\nu} [m_\mu P_L + m_\nu P_R] b^\alpha) F^{\mu\nu} \\
O_9' &= (\bar{s}_R^\alpha \gamma_\mu b^\alpha_R) (\bar{l}_\gamma \mu l) \\
O_{10}' &= (\bar{s}_R^\alpha \gamma_\mu b^\alpha_R) (\bar{l}_\gamma \mu \gamma_5 l) .
\end{align*}
\]

(3.2)

In addition to the new set \(\{O_1'\}\), the exchange of sleptons induces at the tree level new scalar operators which also contribute directly to the decay

\[
\begin{align*}
O_9^S &= (\bar{s}_R^\alpha b^\alpha_L) (\bar{\bar{l}} l) \\
O_{10}^S &= (\bar{s}_R^\alpha b^\alpha_L) (\bar{\bar{l}} \gamma_5 l) \\
O_9'^S &= (\bar{s}_R^\alpha b^\alpha_L) (\bar{l} l) \\
O_{10}'^S &= (\bar{s}_R^\alpha b^\alpha_L) (\bar{l} \gamma_5 l) .
\end{align*}
\]

(3.3)

These operators do not mix with \(\{O_i\}\) and \(\{O_i'\}\), neither with new loop-induced four-quarks scalar operators, which at lowest order do not contribute to the decay. Hence we can truncate the basis to the subset of operators listed in (3.1), (3.2) and (3.3).

Including the new physics, we can now write the Hamiltonian density as

\[
\mathcal{H}_{\text{eff}} = -G_F \left\{ (\bar{s} \gamma_\mu (C_9 P_L + C_9' P_R) b) (\bar{l} \gamma^\mu l) + (\bar{s} \gamma_\mu (C_{10} P_L + C_{10}' P_R) b) (\bar{l} \gamma^\mu \gamma_5 l) \right\}
\]
\[-2i\frac{g^2}{q^2} \left( \bar{s} \sigma_{\mu\nu} \left[ (C_7P_R + C'_7P_L)m_b + (C_7P_L + C'_7P_R) m_s \right] b \right) \left( \bar{l}\gamma^\mu l \right) \]
\[+ \left( \bar{s} (C_9^S P_L + C'^S_9 P_R) b \right) \left( \bar{l}\bar{l} \right) + \left( \bar{s} (C_{10}^S P_L + C'^S_{10} P_R) b \right) \left( \bar{l}\gamma_5 l \right) \right], \tag{3.4} \]

where we have factored out the Fermi constant \( G_F \) so that the coefficients \( C \) are dimensionless.

The SM contributions to (3.4) enter through the coefficients
\[
C_i = \frac{\alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* c_i \quad (i = 7, 10), \quad \text{and} \quad C_9 = \frac{\alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* c_9^{eff}, \tag{3.5} \]

where \( c_i \) are the usual QCD improved Wilson coefficients \[24\]. In the leading logarithmic approximation, and neglecting small operator mixings, we have \[24, 25\]
\[
c_1(m_b) = \frac{1}{2} \left[ \eta^{\frac{23}{12}} - \eta^{-\frac{23}{12}} \right] c_2(M_W) \\
c_2(m_b) = \frac{1}{2} \left[ \eta^{\frac{23}{12}} + \eta^{-\frac{23}{12}} \right] c_2(M_W) \\
c_7(m_b) = \eta^{\frac{23}{12}} \left\{ c_7(M_W) - \frac{58}{135} (\eta^{-\frac{23}{12}} - 1) + \frac{29}{189} (\eta^{-\frac{23}{12}} - 1) \right\} c_2(M_W) \\
c_9(m_b) = c_9(M_W) - \frac{4\pi}{\alpha_s(M_W)} \left[ \frac{4}{33} (1 - \eta^{\frac{23}{12}}) - \frac{8}{87} (1 - \eta^{\frac{23}{12}}) \right] c_2(M_W) \\
c_{10}(m_b) = c_{10}(M_W) \tag{3.6} \]

where \( \eta = \alpha_s(M_W)/\alpha_s(m_b) \) and at the renormalization point \( \mu \sim M_W \), \( c_1(M_W) = 0 \) and \( c_2(M_W) = 1 \). The remaining three coefficients are functions of \( m_b/M_W \) and their explicit expressions can be found in \[24\]. Using \( m_t = 176 \text{ GeV} \) and \( \sin^2 \theta = 0.23 \) (where \( \theta \) is the weak mixing angle) we obtain \( c_7(M_W) = -0.195 \), \( c_9(M_W) = 2.056 \) and \( c_{10}(M_W) = -4.415 \) (with \( c_9(M_W) \) defined according to the prescription given in \[24\]). The coefficient \( c_9^{eff} \) appearing in (3.3) includes the \( s \) dependence induced by the one-loop matrix element of the four-quark operators, and reads \[24\]
\[
c_9^{eff} = c_9(m_b) + [3c_1(m_b) + c_2(m_b)] g(\hat{m}_c, \hat{s}). \tag{3.7} \]

where \( \hat{m}_c = m_c/m_b \) and \( \hat{s} = s/m_b^2 \). In our numerical analysis, we use the expression for \( g(\hat{m}_c, \hat{s}) \) given in \[24\]. An additional contribution to this decay mode comes from the long distance effects associated with on-shell and off-shell \( c\bar{c} \) resonances. There are six known resonances that can contribute. They generate an additional term which has the same structure as \( O_9 \). Hence, it is convenient to include the resonance contributions directly into \( c_9^{eff} \) by making the replacement \[27\]
\[
g(\hat{m}_c, \hat{s}) \to \tilde{g}(\hat{m}_c, \hat{s}) = g(\hat{m}_c, \hat{s}) - \frac{3\pi}{\alpha^2} \sum_{V=J/\psi, \psi', \ldots} \frac{\kappa_V \hat{M}_V}{\hat{s} - M_V^2 + iM_V \Gamma_V}, \tag{3.8} \]

where \( \hat{\Gamma}_V \) and \( \hat{\Gamma}(V \to l^+ l^-) \) are respectively the total and partial decay width of the vector meson resonances normalized to \( m_b \), while \( \hat{M}_V \) is the normalized vector meson mass. The
values of the masses, widths and leptonic branching ratios of the six $\bar{c}c$ resonances can be found in [28], while the phenomenological factor $\kappa_V \sim 2.3$ is calculated by fitting the $B \to J/\psi K^*$ amplitude to the experimental rate [29]. The replacement (3.8) to model the long distance contributions from off-shell resonances is not a rigorous procedure [30]. Here we adopt this simple prescription in order to compare the long distance effects with the new physics short distance contributions.

The low energy coefficients of the new operators in (3.2) can be determined in the same way as the SM coefficients. Due to the vectorlike nature of QCD, the matrix of anomalous dimensions for the set $\{O'_i\}$ is the same as for the standard basis. Moreover, in the models that we will study in the next section, operators induced by squark exchange at the scale $m_{\tilde{q}}$ do not renormalize. Therefore, to take into account the QCD effects on the new physics operators, the only new scale that needs to be introduced is $\tilde{m} = m_{\tilde{t}}$. We fix $\tilde{m} = 100$ GeV.

We note that in this case the renormalization group evolution is still controlled by the QCD $\beta$-function with five active flavors. Modifications to account for the case $m_{\tilde{t}} > m_t$ are straightforward. In the models discussed in the next section, $C'_1(\tilde{m}) = 0$. Since new physics generates $O'_2$ only at the loop level, we will also set $C'_2(\tilde{m}) = 0$ and $C'_7(\tilde{m})$ arises only from operator mixing. Then the set of equations that determines the low energy coefficients $C'_i(m_b)$ is the same as [3.4] with the replacements $c_7(M_W) \to C'_7(\tilde{m}) = 0$ and $\eta \to \eta' = \alpha_s(\tilde{m})/\alpha_s(m_b)$. In particular, we have included in $C'_9(m_b)$ also the additional long distance contributions induced by $O'_1$ and $O'_2$ in the same way as for $\epsilon_{ij}^f$ in (3.7).

Finally, the evolution of the coefficients of the scalar operators in (3.3) is controlled by the anomalous dimensions of $O^S_9, O^S_{10}, O^S_9$ and $O^S_{10}: \gamma_{O^S} = -4$ [31].

**B. Inclusive rate and various observables.**

Neglecting non-perturbative ($\sim 1/m_b^2$) corrections [32], the inclusive $b \to X_s l^+ l^-$ decay width as a function of the invariant mass of the lepton pair $q^2 = m_{l^+ l^-}^2$ is given by

$$
\frac{d\Gamma(\hat{s})}{d\hat{s}} = \frac{G_F^2 m_b^5}{384 \pi^3} \lambda^{1/2}(1, \hat{s}, \hat{m}_s^2) \sqrt{1 - \frac{4 \hat{m}_i^2}{\hat{s}}} \Sigma(\hat{s}),
$$

where $\hat{s} = q^2/m_b^2$, $\hat{m}_i = m_i/m_b$, $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ac)$ and

$$
\Sigma(\hat{s}) = 4 \left( 1 + \frac{2 \hat{m}_i^2}{\hat{s}} \right) \left[ \frac{1}{\hat{s}} \left( |C'_7|^2 + |C'_9|^2 \right) F_1(\hat{s}, \hat{m}_s^2) + \frac{4 \hat{m}_s^2}{\hat{s}} \text{Re} (C'_7 C'_7^*) F_2(\hat{s}, \hat{m}_s^2) 
+ 3 \text{Re} (C'_7 C'_9 + C'_7 C'_9^*) F_3(\hat{s}, \hat{m}_s^2) - 3 \hat{m}_s \hat{s} \text{Re} (2C'_7 C'_9 + 2C'_7 C'_9^* + C'_9 C'_9^*) \right] 
+ \left( |C'_9|^2 + |C'_9|^2 + |C'_10|^2 + |C'_10|^2 \right) F_4(\hat{s}, \hat{m}_s^2, \hat{m}_{\tilde{t}}^2) - 12 \hat{m}_s \hat{s} \left( 1 - \frac{6 \hat{m}_t^2}{\hat{s}} \right) \text{Re}(C'_9 C'_9^* ) 
+ 6 \hat{m}_t^2 \left( |C'_9|^2 + |C'_9|^2 - |C'_9|^2 - |C'_9|^2 \right) F_5(\hat{s}, \hat{m}_s^2) 
+ \frac{3}{2} \hat{s} \left[ \left( 1 - \frac{4 \hat{m}_t^2}{\hat{s}} \right) \left( |C'_9|^2 + |C'_9|^2 \right) + \left( |C'_9|^2 + |C'_9|^2 \right) \right] F_5(\hat{s}, \hat{m}_s^2)
$$

9
\[ + 6 \hat{s} \hat{m}_s \left[ \left( 1 - \frac{4 \hat{m}_t^2}{\hat{s}} \right) \Re \left( C_9^{S*} C_9^{dS} \right) + \Re \left( C_{10}^{sS*} C_{10}^{dS} \right) \right] \]

\[ + 6 \hat{m}_t \Re \left( C_{10}^{sS*} C_{10} + C_{10}^{S*} C_{10}^{d} \right) \left( F_5(\hat{s}, \hat{m}_s^2) - 2 \hat{m}_s^2 \right) \]

\[ - 6 \hat{m}_t \hat{m}_s \Re \left( C_{10}^{sS*} C_{10} + C_{10}^{S*} C_{10}^{d} \right) \left( F_5(\hat{s}, \hat{m}_s^2) - 2 \right) . \]  \hspace{1cm} (3.10)

The functions \( F_i \) read

\[
F_1(\hat{s}, \hat{m}_s^2) = 2(1 + \hat{m}_s^2)(1 - \hat{m}_s^2) - \hat{s}(1 + 14\hat{m}_s^2 + \hat{m}_s^4) - \hat{s}^2(1 + \hat{m}_s^2), \\
F_2(\hat{s}, \hat{m}_s^2) = 2(1 - \hat{m}_s^2)^2 - \hat{s}(4 + 4\hat{m}_s^2 + \hat{s}), \\
F_3(\hat{s}, \hat{m}_s^2) = (1 - \hat{m}_s^2)^2 - \hat{s}(1 + \hat{m}_s^2), \\
F_4(\hat{s}, \hat{m}_s^2, \hat{m}_t^2) = (1 - \hat{m}_s^2)^2 + \hat{s}(1 + \hat{m}_s^2) - 2\hat{s}^2 + \lambda(1, \hat{s}, \hat{m}_s^2) \frac{2\hat{m}_t^2}{\hat{s}}, \\
F_5(\hat{s}, \hat{m}_s^2) = 1 - \hat{s} + \hat{m}_s^2. \]  \hspace{1cm} (3.11)

The forward-backward asymmetry \( A_{FB} \) is defined with respect to the angular variable \( c_\theta = \cos \theta \) where \( \theta \) is the angle of the lepton \( l^- \) with respect to the \( b \)-direction in the \( l^+l^- \) center-of-mass system:

\[
A_{FB}(\hat{s}) \equiv \frac{1}{d\Gamma(\hat{s})/d\hat{s}} \left[ \int_0^1 d\hat{c}_\theta \frac{d^2}{d\hat{s} d\hat{c}_\theta} \Gamma(\hat{s}, \hat{c}_\theta) - \int_{-1}^0 d\hat{c}_\theta \frac{d^2}{d\hat{s} d\hat{c}_\theta} \Gamma(\hat{s}, \hat{c}_\theta) \right]. \]  \hspace{1cm} (3.12)

We obtain:

\[
A_{FB}(\hat{s}) = 3 \lambda^{1/2}(1, \hat{s}, \hat{m}_s^2) \sqrt{1 - \frac{4\hat{m}_t^2}{\hat{s}} \frac{\Delta(\hat{s})}{\Sigma(\hat{s})} \left[ \left( 1 - \frac{4\hat{m}_t^2}{\hat{s}} \right) \Delta(\hat{s}) \right] .} \]  \hspace{1cm} (3.13)

where

\[
\Delta(\hat{s}) = \hat{s} \Re \left( C_9^SC_{10} - C_9^{dS} C_{10}^d \right) + 2(1 + \hat{m}_s^2) \Re \left( C_7^SC_9 + C_9^{dS} C_7^d \right) \\
4\hat{m}_s \Re \left( C_7^SC_{10} - C_{10}^{dS} C_7^d \right) + \hat{m}_t \Re \left[ \left( C_9^{S*} + \hat{m}_s C_9^{dS*} \right) C_7^d \right] \\
+ \hat{m}_t \Re \left[ \left( C_9^{S*} + \hat{m}_s C_9^{dS*} \right) \left( 2C_7 + C_9 \right) \right] . \]  \hspace{1cm} (3.14)

Recently, polarization measurements of final state \( \tau \) leptons in the decay \( b \rightarrow X_s \tau^+ \tau^- \) have been proposed as a useful tool in discerning physics beyond the SM. Experimental studies of the \( \tau \) polarization have been carried out by the four LEP collaborations, and from the analysis of the distributions of the \( \tau \) decay products in different decay modes, the \( \tau \) polarization was determined with an error of about 10% \[36\]. For \( B \)-factory experiments the task of reconstructing the \( \tau \) polarization appears more challenging. For example, in contrast to the LEP environment, the energy of the decaying \( \tau \) is not a-priori known. However, if a comparable sensitivity can be reached, then a measurement of the \( \tau \) polarization components could provide crucial informations for distinguishing among different sources of new physics. Following Ref. \[35\] we define the inclusive lepton polarization components by introducing three orthogonal unit vectors

\[
e_L = \frac{\mathbf{p}_-}{\| \mathbf{p}_- \|}, \quad e_N = \frac{\mathbf{p}_s \times \mathbf{p}_-}{\| \mathbf{p}_s \times \mathbf{p}_- \|}, \quad e_T = e_N \times e_L, \]  \hspace{1cm} (3.15)
where \( p_\perp \) and \( p_s \) are the three-momenta of \( l^- \) and of the \( s \) quark in the c.m. frame of the \( l^+l^- \) system. The differential decay rate for \( b \to X_s l^+l^- \) for any given spin direction \( n \) of the lepton \( l^- \) (\( n \) being a unit vector in the \( l^- \) rest frame) can be written as

\[
\frac{d\Gamma(\hat{s}, n)}{d\hat{s}} = \frac{1}{2} \left( \frac{d\Gamma(\hat{s})}{d\hat{s}} \right)_{\text{unpol}} \left[ 1 + \left( \mathcal{P}_L(\hat{s}) e_L + \mathcal{P}_N(\hat{s}) e_N + \mathcal{P}_T(\hat{s}) e_T \right) \cdot n \right],
\]

(3.16)

where \( \mathcal{P}_L \), \( \mathcal{P}_N \) and \( \mathcal{P}_T \) give the longitudinal, normal and transverse components of the \( l^- \) polarization as a function of \( \hat{s} \). The polarization asymmetries \( \mathcal{P}_i(\hat{s}) \) (\( i = L, N, T \)) are obtained by evaluating

\[
\mathcal{P}_i(\hat{s}) = \frac{d\Gamma(e_i, \hat{s})/d\hat{s} - d\Gamma(-e_i, \hat{s})/d\hat{s}}{d\Gamma(e_i, \hat{s})/d\hat{s} + d\Gamma(-e_i, \hat{s})/d\hat{s}}
\]

(3.17)

for \( e_i = e_L, e_N, e_T \). For the general form of the interaction (3.4) we obtain

\[
\mathcal{P}_L(\hat{s}) = \frac{1}{\Sigma(\hat{s})} \sqrt{1 - \frac{4\hat{m}_s^2}{\hat{s}}} \left\{ 12 \text{Re} \left( C_7^* C_{10} + C_7^* C_{10}' \right) \left[ (1 - \hat{m}_s^2)^2 - \hat{s}(1 + \hat{m}_s^2) \right] \\
+ 2 \text{Re} \left( C_9^* C_10 + C_9^* C_10' \right) \left[ (1 - \hat{m}_s^2)^2 + \hat{s}(1 + \hat{m}_s^2) - 2\hat{s}^2 \right] \\
- 12 \hat{m}_s \hat{s} \text{Re} \left( 2 C_7^* C_{10} + 2 C_7^* C_{10} + C_9^* C_{10} + C_9^* C_{10} \right) \\
- 3\hat{s} \left[ (1 - \hat{s} + \hat{m}_s^2) \text{Re} \left( C_9^* C_{10} + C_9^* C_{10} \right) - 2\hat{m}_s \text{Re} \left( C_9^* C_{10} + C_9^* C_{10} \right) \right] \\
- 6 \hat{m}_l (1 - \hat{s} - \hat{m}_s^2) \text{Re} \left( C_9^* C_{10} + C_9^* C_{10} \right) \\
- 6 \hat{m}_l \hat{m}_s (1 + \hat{s} \hat{m}_s^2) \text{Re} \left( C_9^* C_{10} + C_9^* C_{10} \right) \right\},
\]

(3.18)

\[
\mathcal{P}_N(\hat{s}) = \frac{1}{\Sigma(\hat{s})} \frac{3\pi}{4} \lambda^{1/2}(1, \hat{s}, \hat{m}_s^2) \sqrt{1 - \frac{4\hat{m}_s^2}{\hat{s}}} \left\{ 2 \text{Im} \left( -C_7^* C_{10} + C_7^* C_{10} \right) \hat{m}_s \\
+ \text{Im} \left( C_7^* C_{10} - C_7^* C_{10} \right) (1 + \hat{m}_s^2) \right\} + 2 \hat{m}_l \sqrt{\hat{s}} \text{Im} \left( C_9^* C_{10} - C_9^* C_{10} \right) \\
- \sqrt{\hat{s}} \text{Im} \left( C_9^* \left( 2 C_7 + C_9 \right) + C_9^* \left( 2 C_7 + C_9 \right) + \left( C_9^* C_{10} + C_9^* C_{10} \right) \right) \\
- \hat{m}_s \sqrt{\hat{s}} \text{Im} \left( C_9^* \left( 2 C_7 + C_9 \right) + C_9^* \left( 2 C_7 + C_9 \right) + \left( C_9^* C_{10} + C_9^* C_{10} \right) \right) \right\},
\]

(3.19)

\[
\mathcal{P}_T(\hat{s}) = \frac{1}{\Sigma(\hat{s})} \frac{3\pi}{2\sqrt{\hat{s}}} \lambda^{1/2}(1, \hat{s}, \hat{m}_s^2) \left\{ \hat{m}_l \left[ -\frac{4}{\hat{s}} \left( |C_7|^2 - |C_7'|^2 \right) \right] (1 - \hat{m}_s^2)^2 \\
+ \text{Re} \left( C_9^* C_10 + C_9^* C_10' \right) (1 - \hat{m}_s^2) - 8 \hat{m}_s \text{Re} \left( C_7^* C_9 - C_7^* C_9' \right) - \hat{s} \left( |C_9|^2 - |C_9'|^2 \right) \\
+ 4 \text{Re} \left( C_9^* C_9 - C_9^* C_9' \right) (1 + \hat{m}_s^2) + 2 \text{Re} \left( C_7^* C_{10} + C_7^* C_{10} \right) (1 - \hat{m}_s^2) \right\} \\
+ \frac{\hat{s}}{2} \left[ \left( 1 - \frac{4\hat{m}_s^2}{\hat{s}} \right) \text{Re} \left( C_9^* C_{10} + C_9^* C_{10}' \right) + \hat{m}_s \text{Re} \left( C_9^* C_{10} + C_9^* C_{10}' \right) \right] \\
+ \hat{m}_s \text{Re} \left( C_9^* \left( 2 C_7 + C_9 \right) + C_9^* \left( 2 C_7 + C_9 \right) \right) \\
+ \hat{m}_s \text{Re} \left( C_9^* \left( 2 C_7 + C_9 \right) + C_9^* \left( 2 C_7 + C_9 \right) \right) \right\}.
\]

(3.20)
The possibility of measuring the longitudinal polarization of the $\tau$ in the decay $b \to X_s \tau^+ \tau^-$ was first proposed in [34]. This analysis was extended to include the other two polarization components $P_N$ and $P_T$ in [33]. The SM results are recovered from our formulæ by setting the $C_i'$ and the $C_{9,10}'$ coefficients to zero. In this limit our expressions for $P_L(s)$ and $P_N(s)$ agree with the results given by Krüger and Sehgal [32], and when we set $m_s = 0$ in (3.18) we find agreement with $P_L$ as given in [34]. However, the SM limit of $P_T$ disagrees with eq. (5.5) in [35] for the factor of two multiplying the term $\text{Re}(C_7 C_{10})$ in the third line of (3.20).

To derive numerical results for the inclusive branching ratios and for the averaged values of the asymmetries, we minimize long distance effects by imposing cuts on the dilepton invariant mass. For the muon channel we select the region below the resonances $\hat{s} < 0.4$, while for the tau channel we require $\hat{s} > 0.6$, above the $\psi$. The remaining effects of the four additional $\bar{c}c$ resonances in the tail of the invariant mass distribution for $b \to X_s \tau^+ \tau^-$ are not very large.

At leading order, the SM results for the various quantities are as follows. The SM inclusive branching ratio for muons is predicted to be $\text{BR} (b \to X_s \mu^+ \mu^-)_{\hat{s} < 0.4} = 4.3 \times 10^{-6}$. The purely short distance contribution yields in the same region $\text{BR}^{sd} (b \to X_s \mu^+ \mu^-)_{\hat{s} < 0.4} = 3.9 \times 10^{-6}$. Due to phase space suppression and to the different cut, the tau production rate is more than one order of magnitude smaller. We find $\text{BR} (b \to X_s \tau^+ \tau^-)_{\hat{s} > 0.6} = 1.5 \times 10^{-7}$ and a similar number for the purely short distance contribution. Below our cut, the average value of the $\mu$ forward-backward asymmetry is rather small $\langle A_{\text{FB}}^\mu \rangle_{\hat{s} < 0.4} = -0.01$. This is due to the fact that the asymmetry changes sign in this region, and in taking the average large cancellations occur. For the $\tau$ lepton the asymmetry is larger $\langle A_{\text{FB}}^\tau \rangle_{\hat{s} > 0.6} = -0.13$. Only the longitudinal polarization asymmetry $P_L$ is significant for the $\mu$: $\langle P_L^\mu \rangle_{\hat{s} < 0.4} = -0.57$ while all the three components are sizeable for $\tau$. We find the following average values $\langle P_L^\tau \rangle_{\hat{s} > 0.6} = -0.34$, $\langle P_T^\tau \rangle_{\hat{s} > 0.6} = -0.40$, $\langle P_N^\tau \rangle_{\hat{s} > 0.6} = 0.05$.

Notice that the normal component $P_N$, though small, is a T-odd quantity and it is considerably larger than the corresponding normal polarization of leptons in $K_L \to \pi^+ \mu^- \bar{\nu}$ or $K^+ \to \pi^+ \mu^+ \mu^- [37]$. However, this should not be interpreted as signaling $CP$ violation. In fact, since we assume real couplings, the leading contribution comes from the absorptive part of the effective coupling $C_9$ (and $C_9'$ in the new physics analysis), which are dominated by the $\bar{c}c$ real intermediate states (cf. (3.8)). It is in principle possible to remove this background to $CP$ violating effects by measuring the difference between the $P_N$ components for example in $B^+$ and $B^-$ decays. This is because $CP$ violating phases yield asymmetries with the same sign, while phases originating from strong interactions cancel off [38].

IV. SUSY WITHOUT R-PARITY

In this section we present a short introduction to SUSY models where R-parity is not imposed, and $L$ is (mildly) violated already at the renormalizable level. Next we will embed these models in the framework of an Abelian horizontal symmetry [39] that will provide us with the additional theoretical constraints needed to derive numerical predictions. As it was discussed in [3], this procedure has the additional advantage of ensuring that in the
particular models we will discuss, the present constraints on the R-parity violating couplings are satisfied.

A. R-parity violating couplings

The field content of the SM together with the requirement of $SU(2)_L \times U(1)_Y$ gauge invariance, implies that at the renormalizable level the most general Lagrangian possesses additional accidental $U(1)$ symmetries, corresponding to conserved baryon and lepton flavor ($L_i$) quantum numbers. The conservation of $B$, $L_i$ and hence of total lepton number ($L = \sum L_i$) naturally explains nucleon stability as well as the non observation of $L$ and $L_i$ violating transitions. In SUSY extensions of the SM, additional gauge and Lorentz invariant terms are allowed, which violate $B$, $L_i$ and $L$. Denoting collectively by $\hat{H}_\alpha$ ($\alpha = 0, 1, 2, 3$) the supermultiplets containing the down-type Higgs and the left-handed lepton doublets, which transform in the same way under the gauge group, the following $L_i$ and $L$ violating superpotential terms arise

$$W_L = \mu_\alpha \hat{H}_\alpha \hat{H}_u + \lambda_{\alpha\beta\gamma} \hat{H}_\alpha \hat{H}_\beta \hat{l}_\gamma^c + \lambda'_{\alpha\beta\gamma} \hat{H}_\alpha \hat{Q}_\beta \hat{d}_\gamma^c + \ldots$$

(4.1)

Here $\hat{Q}_i$ and $\hat{d}_i^c$ denote the quark doublet and down-quark singlet superfields, $\hat{l}_i^c$ are the lepton singlets and $\hat{H}_u$ contains the up-type Higgs field. There are also renormalizable terms which violate $B$, $W_B = \lambda''_{ijk} \hat{u}_i^c \hat{d}_j^c \hat{d}_k^c$, and physics at some large scale $M_\Lambda$ can induce additional dimension 5 $B$ and $L$ violating terms like $(\Gamma'_{\alpha\beta\gamma}/M_\Lambda) \hat{H}_\alpha \hat{Q}_i \hat{Q}_j \hat{Q}_k + \ldots$

To forbid the dangerous dimension 4 terms, a parity quantum number $R = (-1)^{3B+L+2S}$ ($S$ being the spin) is assigned to each component field, and invariance under R transformations is imposed. However, even if suppressed by the Plank mass, the R-parity conserving dimension 5 terms can still induce too fast proton decay unless $\Gamma' \lesssim 10^{-8}$ etc. From a phenomenological point of view, the first priority is to ensure the absence of operators leading to fast nucleon decay, and in this respect other discrete symmetries can be more effective than $R$. These interesting alternatives forbid dimension 4 and 5 $B$ violating terms but do not imply the same for the $L$ non-conserving terms. Since a mild violation of $L$ can be phenomenologically tolerated, SUSY extensions of the SM with highly suppressed $B$ violation but without R parity and without $L$ number, represent interesting alternatives to the Minimal Supersymmetric Standard Model (MSSM). We henceforth assume that $B$ is effectively conserved, and we concentrate only on the $L$-violating terms contained in (4.1).

The first term in (4.1) can mix the fermions with the Higgsinos, resulting in too large neutrino masses [13]. In Ref. [3] this problem was solved by assuming that the down-type Higgs transforms differently from the lepton doublets under a horizontal symmetry. The different charge assignments can generate enough suppression of the lepton mixing with $H_u$. However, this does not occur in the models discussed in the next section (and in Ref. [4]). In fact in these models the down-type Higgs and the $\tau$ doublets have the same horizontal charge, and in the model labeled below as Model II, this is true also for the muon doublet. We can still avoid generating too large neutrino masses under the assumption that the soft SUSY breaking terms are universal. As it is discussed in [13], this assumption implies the following:

(i) Modulo small violation of the universality conditions at the electroweak scale induced by the renormalization group running of the soft SUSY breaking parameters, the
combination $H_d \equiv \mu_\alpha H_\alpha / \sqrt{\mu_\alpha \mu_\alpha}$ corresponds to the down-type Higgs. Namely its scalar component is the only other field, besides $H_u$, acquiring a non-vanishing vacuum expectation value.

(ii) In the basis $\{\hat{H}_d, \hat{L}_i\}$ (where $\hat{L}_i$ with $i = 1, 2, 3$ denote the three combinations orthogonal to $\hat{H}_d$) the couplings $\lambda_{0ij}$ and $\lambda'_{0ij}$ are respectively the Yukawa couplings $Y^l_{ij}$ of the leptons and $Y^d_{ij}$ of the down-type quarks.

As it will become clear in the next subsection, this short discussion is relevant only for the subset of the $\hat{H}_\alpha$ multiplets that carry the same minimum charge. For doublets having horizontal charge assignments different from the minimum one (as for example for the electron doublet) the horizontal symmetry by itself defines to a very good approximation the physical fields.

Once the fields are rotated to the physical basis, the $L$ violating trilinear terms contained in (4.1) read

$$\lambda_{ijk} \hat{L}_i \hat{L}_j \hat{L}_k + \lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{d}_k,$$

(4.2)

where $\lambda_{ijk} = -\lambda_{jik}$ due to the antisymmetry in the $SU(2)$ indices. Several of the $\lambda$ and $\lambda'$ couplings are strongly constrained by the existing phenomenology [4]. The best limits are for couplings involving fermions of the first two generations ($i, j, k = 1, 2$) while for couplings involving more than a single third generation field the existing limits are much weaker, and in some cases no bounds exist to date. This situation is interesting since in general models that can explain the observed fermion mass hierarchy also predict that R-parity violating couplings involving third generation fields are the largest ones.

### B. R-parity violation in the framework of horizontal symmetries

In order to evaluate the effects of the new R-parity violating interactions, we need to estimate quantitatively the coefficients $\lambda$ and $\lambda'$. We work in the framework of the supersymmetric models with horizontal symmetries that have been thoroughly investigated in [39]. These models successfully predict the order of magnitude of the fermion masses and CKM angles, and can also explain the suppression of $L$ violation [1] and $B$ violation [4] in SUSY models without R-parity. Assuming a horizontal $U(1)$ symmetry allows us to estimate the size of the $L$ violating couplings, and to work out numerical predictions for various observables measurable in $B$ decays.

In the models we are interested in, there are no additional fields in the low energy spectrum with respect to minimal SUSY. However, a charge $H(\psi)$ of an Abelian horizontal symmetry $\mathcal{H} = U(1)_H$ is assigned to each supermultiplet $\psi$. $\mathcal{H}$ is explicitly broken by a small parameter $\varepsilon$ with charge $H(\varepsilon) = -1$ giving rise to a set of selection rules for the effective couplings of the low energy Lagrangian [39]. If we assume that each of the lepton, quark and Higgs superfields carries positive or zero charge, the selection rule relevant for the present discussion is that the effective coupling $g_{abc}$ for a general trilinear superpotential term $\hat{\psi}_a \hat{\psi}_b \hat{\psi}_c$ is of order $g_{abc} \sim \varepsilon^{H(\hat{\psi}_a) + H(\hat{\psi}_b) + H(\hat{\psi}_c)}$. Therefore the leptons and down-type quarks Yukawa couplings are respectively of order $Y^l_{ij} \sim \varepsilon^{H(\Phi_d) + H(L_i) + H(l_j)}$ and $Y^d_{ij} \sim \varepsilon^{H(\Phi_d) + H(Q_i) + H(d_j)}$ (rotation to the exact quark mass eigenstate basis does not affect these order of magnitude
estimates \[3\]). Most of the \(^{L}\)-violating couplings in (4.2) are further suppressed with respect to the corresponding Yukawa couplings. They can be estimated as

\[
\lambda_{kij} \sim Y_{ij}^{l} H(L_{k}) - H(\Phi_{d}) \sim \left( \frac{2\sqrt{2}G_{F}}{\cos^{2}\beta} \right)^{1/2} m_{\ell_{i}} \varepsilon^{H(l_{j}^{c}) - H(l_{j}^{c}) + H(L_{k}) - H(\Phi_{d})},
\]

(4.3)

and

\[
\lambda'_{kij} \sim Y_{ij}^{d} H(L_{k}) - H(\Phi_{d}) \sim \left( \frac{2\sqrt{2}G_{F}}{\cos^{2}\beta} \right)^{1/2} m_{\tilde{\ell}_{i}} \varepsilon^{H(d_{j}^{c}) - H(d_{j}^{c}) + H(L_{k}) - H(\Phi_{d})}.
\]

(4.4)

These equations show that

(i) the couplings \(\lambda\) and \(\lambda'\) involving fermions of the third generation

are respectively enhanced by \(m_{\tau}\) and \(m_{b}\);

(ii) like the lepton and down quark Yukawa couplings, the \(\lambda, \lambda'\) couplings increase with \(\tan \beta\).

In order to give a numerical estimate of the couplings, we need a set of charges and a value for the \(\mathcal{H}\)-symmetry breaking parameter \(\varepsilon\). In the model discussed in \[39\], \(\varepsilon \approx 0.22\) is fixed by the magnitude of the Cabibbo angle, while the quark, lepton and Higgs charges are chosen to reproduce the values of the fermion masses and CKM mixing angles. Besides reproducing the measured values, the model has some predictivity in the quark sector \[39\], it yields estimates for ratios of neutrino masses \[4,6\], and most important in the present context, it ensures that the \(^{L}\)-violating couplings in (4.2), (4.3) and (4.4) are safely suppressed below the present experimental limits \[3\]. The following \(\mathcal{H}\)-charge assignments \[39\] fit the order of magnitude of all the quark masses and CKM mixing angles

\[
\begin{align*}
\hat{Q}_{1} & \quad \hat{Q}_{2} & \quad \hat{Q}_{3} & \quad \hat{d}_{1}^{c} & \quad \hat{d}_{2}^{c} & \quad \hat{d}_{3}^{c} & \quad \hat{u}_{1}^{c} & \quad \hat{u}_{2}^{c} & \quad \hat{u}_{3}^{c} & \quad \tilde{\Phi}_{d} & \quad \tilde{\Phi}_{u} \\
(3) & \quad (2) & \quad (0) & \quad (3) & \quad (2) & \quad (2) & \quad (3) & \quad (1) & \quad (0) & \quad (0) & \quad (0).
\end{align*}
\]

(4.5)

For the leptons, we will use two different sets of charges that fit well the order of magnitude of the charged lepton masses. We also use a different value for the squark masses \(m_{\tilde{q}}\) for each set :

\[
\begin{align*}
\hat{L}_{1} & \quad \hat{L}_{2} & \quad \hat{L}_{3} & \quad \hat{l}_{1}^{c} & \quad \hat{l}_{2}^{c} & \quad \hat{l}_{3}^{c} & \quad m_{\tilde{\ell}_{1}} & \quad m_{\tilde{\ell}_{1}} (\text{GeV}) & \quad m_{\tilde{\ell}_{1}} (\text{GeV}) \\
\text{Model I} : & \quad (4) & \quad (2) & \quad (0) & \quad (4) & \quad (3) & \quad (3) & \quad 100 & \quad 170 \\
\text{Model II} : & \quad (3) & \quad (0) & \quad (0) & \quad (5) & \quad (5) & \quad (3) & \quad 100 & \quad 350.
\end{align*}
\]

(4.6)

The charge assignments and the sfermion mass values listed in (4.5) and (4.6), together with \(\varepsilon \approx 0.22\) completely define the two models and allow us to estimate the order of magnitude of the various decay rates. The charges of Model I coincide with the charges of the “master model” of \[4\]. While in this model new physics effects are induced dominantly by the new operators \(\{O'\}\) in \(3.2\) arising from squark exchange, in Model II the leading effects are due to the scalar operators \(\{O^{S}\}\) in \(3.3\) induced by slepton exchange. The choices (4.5) and (4.6) for the horizontal charges are not unique. Since the Yukawa interactions are invariant under a set of \(U(1)\) symmetries such as hypercharge or lepton number, it is always possible to shift the \(H\)-charges of any amount proportional to one of the corresponding \(U(1)\) quantum
numbers, without affecting the predictions for the masses and mixing angles. In particular, a shift proportional to $L: H (L_i) \rightarrow H (L_i) + n, H (l_i^c) \rightarrow H (l_i^c) - n$ and $H (\bar{\psi}) \rightarrow H (\bar{\psi})$ for all the other fields, has the effect of suppressing (for $n > 0$) all the $L$ violating couplings in (4.2), (4.3) and (4.4) by a factor of $\varepsilon^n$. We have found that already for $n = 1$ the couplings are enough suppressed so that no signal of new physics can be detected in the experimental quantities we are considering in this paper. Notice also that Model II can be derived from Model I by means of shifts proportional to lepton flavor numbers: $n_e = -1, n_\mu = -2, n_\tau = 0$. This has the effect of enhancing some of the $\lambda$ couplings without affecting the charged lepton masses. Of course, in Model II the predictions for neutrino masses and mixings will differ from the predictions of Model I [4].

C. Coefficients and rates for the various decays

In SUSY models without R-parity, $b \rightarrow \tau \bar{\nu}$ decays can proceed through the exchange of sleptons and/or squarks, yielding significant enhancements over the SM rates. In this section we give the expressions for the various coefficients of the effective operators contributing to the decays $B^+ \rightarrow \tau \bar{\nu}, B \rightarrow \tau^+ \tau^-, b \rightarrow X_{q} \nu \bar{\nu}$ and $b \rightarrow X_s \tau^+ \tau^-$ including the new physics contributions.

For the decay $B^+ \rightarrow \tau \bar{\nu}$ the coefficients appearing in (2.3) and (2.4) read

$$C^{\tau \bar{\nu}}_A - C'^{\tau \bar{\nu}}_A = \frac{\lambda^*_{31k} \lambda_{13k}}{4 G_F m_{\tilde{t}_k}^2} - \sqrt{2} V_{ub}$$

$$C^{\tau \bar{\nu}}_P - C'^{\tau \bar{\nu}}_P = \frac{\lambda^*_{k33} \lambda_{k3k}}{2 G_F m_{\tilde{l}_k}^2} \quad (k \neq i)$$

(4.7)

where a sum over the repeated index $k$ is left understood. The index $i$ refers to the final state neutrino flavor which, as already said, can be different from $\nu_\tau$.

For the decay $B_q \rightarrow \tau^+ \tau^-$ the coefficients in (2.6) and (2.7) read

$$C^q = \frac{\lambda_{k33} \lambda^*_{k3q} + \lambda_{k33} \lambda^*_{k3q}}{4 G_F m_{\tilde{t}_k}^2}$$

$$C'^q = \frac{\lambda_{k33} \lambda^*_{k3q} - \lambda_{k33} \lambda^*_{k3q}}{4 G_F m_{\tilde{l}_k}^2}$$

$$C^q_A = \frac{\lambda^*_{3k3} \lambda_{k3q}}{8 G_F m_{\tilde{q}_k}^2} + \left[ C^q_A \right]_{SM}$$

where $\left[ C^q_A \right]_{SM}$ is given in (2.8).

For the decay $b \rightarrow X_q \nu_i \bar{\nu}_j$ the coefficients appearing in (2.10) and (2.11) read

$$C^q = \frac{\lambda^*_{3k3} \lambda_{3kq}}{8 G_F m_{\tilde{q}_k}^2} + \left[ C^q_A \right]_{SM}$$

(4.8)

In models where the $H$ symmetry is gauged, shifts of this kind cannot be arbitrary but must respect the constraints from anomaly cancellation.
\[ C_{L}^{qij} = \frac{\lambda'_{ij3k} \lambda'_{jk3k}}{2 G_F m_d^2} + \frac{1}{3} \left[ C_{L}^{q} \right]_{\text{SM}} \delta_{ij} \]

\[ C_{R}^{qij} = \frac{\lambda'_{ik3} \lambda'_{jk3}}{2 G_F m_d^2} \]  

(4.9)

where \( \left[ C_{L}^{q} \right]_{\text{SM}} \) is given in (2.12). Finally, for the decay \( b \to X_s \tau^+ \tau^- \), the coefficients of the new operators \( \{ O' \} \) \( (i = 2, 7, 9, 10) \) in (3.2) and \( \{ O^S \} \) in (3.3) at the scale \( \tilde{m} = m_{\tilde{t}} \) are

\[
C_{2}^{'}(\tilde{m}) = \frac{\lambda'_{k2k}^* \lambda_{k22}}{2 G_F m_{t_k}^2} \\
C_{9}^{'}(\tilde{m}) = \frac{\lambda'_{3k3} \lambda_{3k2}}{4 G_F m_{u_k}^2} \\
C_{9}^{S}(\tilde{m}) = C_{10}^{S}(\tilde{m}) = \frac{\lambda_{k32} \lambda_{k33}^*}{2 G_F m_{u_k}^2} \quad (k \neq 3) \\
C_{9}^{S'}(\tilde{m}) = -C_{10}^{S'}(\tilde{m}) = \frac{\lambda_{k23} \lambda_{k33}^*}{2 G_F m_{u_k}^2} \quad (k \neq 3) .
\]  

(4.10)

As regards the coefficients of the standard operators in (3.1), they are affected by the new physics only at the loop level. Since these are subleading effects we will neglect them.

Few comments are in order. The antisymmetry in the first two indices of the \( \lambda \) couplings forces \( k \neq 3 \) in the coefficients of the scalar operators. In Model I, because of the lepton charge assignments (4.6) this results in a strong suppression of the scalar couplings. We have for example \( C_{9}^{S} \sim \varepsilon^4 C_{9}^{'} \) so that the leading new physics effects are due to the operators induced by squark exchange. In Model II, being \( H(\hat{L}_2) = H(\hat{L}_3) = 0 \) there is no suppression of the scalar couplings from the \( H \)-symmetry. Then the large value of the squark masses \( m_{\tilde{q}} = 350 \text{ GeV} \) suppresses \( C_{9}^{'} \) and \( C_{10}^{'} \) down to \( \sim \varepsilon C_{9}^{S} \) so that in this model slepton exchange gives the dominant effects. As regards the \( b \to X_s \mu^+ \mu^- \) decay channel, the charge difference \( H(\hat{L}_2) - H(\hat{L}_3) = 2 \) of Model I implies a strong suppression of the \( \lambda' \lambda' \) couplings. In contrast, for the couplings involving the combination \( \lambda' \lambda \) the antisymmetry now allows \( k = 3 \), with the noticeable result that the scalar couplings in the \( \mu \) channel are enhanced with respect to the \( \tau \) channel. Still the enhancement is only of about a factor \( \varepsilon^{-2} \) so that we cannot expect particularly large new physics effects. In Model II there is no suppression of the squark exchange operators for \( b \to X_s \mu^+ \mu^- \) with respect to the \( \tau \) channel. Thus we can expect that \( C_{9}^{'} \) and \( C_{10}^{'} \) will still produce observable signals of new physics. Finally, \( C_{2}^{'} \) that controls the relevance of the new long distance contributions, is rather small in both models. This is due to the charge \( H(Q_2) = 2 \) which yields a relative factor \( \varepsilon^4 (m_{\tilde{q}}/m_{\tilde{t}})^2 \sim 0.001 \) (Model I)–0.03 (Model II) with respect to the \( C_{9}^{S} \) short distance contribution. Therefore we can expect that the resonance peaks will be smeared off by the dominant new physics short distance effects.
V. RESULTS

The main results of our analysis consist of a set of numerical predictions for several observables in the presence of new physics from SUSY models without R-parity. We recall that the theoretical framework we adopted is a straightforward extension of successful models for fermion masses and CKM mixing angles, and gives predictions for the R-parity breaking couplings which are consistent with all the present experimental constraints [3].

We have studied several observables measurable in $b \to \tau, \nu$ and $b \to \mu$ transitions within two different models, that were defined in (4.3) and (4.4). In Model I, the horizontal charges coincide with the charges of the “master model” discussed in [4,39]. The sfermions within two different models, that were defined in (4.5) and (4.6). In Model I, the horizontal charges (4.6). The sfermion masses are $m_{\tilde{q}} = 100$ GeV and $m_{\tilde{q}} = 170$ GeV. In this model the dominant new physics effects come from squark exchange which generate the leading new effective operators. Model II was introduced in order to study the effects of sleptons. It is defined in terms of a different set of lepton horizontal charges (4.5). The sfermion masses are $m_{\tilde{q}} = 100$ GeV and $m_{\tilde{q}} = 350$ GeV.

Varying the values of the new physics parameters results in the following scaling behaviors of the branching ratios:

(i) In both models, the branching ratios scale as $(\tan \beta)^4$ and as $\varepsilon^4 n$, where $n$ is an arbitrary shift of the $H$-charges of the leptons proportional to lepton number (see the discussion at the end of section IV-B). Our results are given for $n = 0$ and $\tan \beta = 1$.

(ii) In Model I the branching ratio for $b \to X_s \tau^+ \tau^-$ scales as $(170 \text{ GeV}/m_{\tilde{q}})^4$. Hence, for light squarks ($m_{\tilde{q}} \approx 100$ GeV) and moderate values of $\tan \beta$ ($\gtrsim 2$) Model I can predict values of the branching ratio up to few $\times 10^{-5}$. In Model II the rate for $b \to X_s \tau^+ \tau^-$ scales as $(100 \text{ GeV}/m_{\tilde{q}})^4$ so that the main enhancement with respect to our predictions (see table I) can only come from $\tan \beta > 1$.

Our results are collected in tables I and II, and in figures 1-8.

Table I lists the predictions for decays involving the transition $b \to \tau$, $\nu$. The first five lines give the results for the decays $B^\rightarrow \tau \bar{\nu}$, $B_q \to \tau^+ \tau^-$ and $b \to X \nu \bar{\nu}$, while the results for the branching ratio, forward-backward asymmetry and longitudinal polarization asymmetries in $b \to X_s \tau^+ \tau^-$ are given in the remaining entries. Table II collects the results for the corresponding processes with final state muons. In both tables, the first column lists the SM predictions for the various observables (computed in the leading order approximation) the second column lists the predictions of Model I while the results for Model II are listed in the third column.

Since the two decays $b \to X_s \mu^+ \mu^-$ and $b \to X_s \tau^+ \tau^-$ are affected by large long distance effects, to single out the short distance contributions we have applied cuts on the dilepton invariant mass. We study $b \to X_s \tau^+ \tau^-$ in the region above the $\psi^\prime$ ($s > 0.6$) while $b \to X_s \mu^+ \mu^-$ is analyzed below the resonance region ($s < 0.4$). A comparison between the total inclusive branching ratios $\text{BR}(b \to X_s l^+ l^-)_{\text{cut}}$, the branching ratio in the region within the cuts and the kinematic limits $\text{BR}(b \to X_s l^+ l^-)_{s<0.4}(\hat{s}>0.6)$ and the short distance contribution in the same region $\text{BR}^{sd}(b \to X_s l^+ l^-)$ shows the effects of the cuts on the total rates, and their effectiveness in isolating the interesting contributions.

From the results in table I, it is apparent that in most cases the decays $B^\rightarrow \tau \bar{\nu}$ and $b \to X_q \nu \bar{\nu}$ are not very sensitive to the sources of new physics we are analyzing here. In both our models these decays do not show any significant enhancement with respect to the SM rates. New physics from Model I can enhance the rates for $B_s \to \tau^+ \tau^-$ and $B_d \to \tau^+ \tau^-$. 

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However, the rates remain very small, and because of the large theoretical uncertainty related to $f_B$ it is not obvious that these signals could be unambiguously identified. In Model II the branching ratio for $B_s \to \tau^+ \tau^-$ increases by two orders of magnitudes, up to $\sim 10^{-4}$. This process cannot be searched for at $B$-factories running at the $\Upsilon(4S)$. However, hadron colliders might be able to detect this enhancement, depending on the efficiency in identifying the $\tau$'s. In this model we also observe a two orders of magnitude enhancement for $B_d \to \tau^+ \tau^-$ which appears more promising for new physics searches at future $B$-factories.

In the muon channel, the corresponding decays $B_q \to \mu^+ \mu^-$ are also sensitive to new physics effects from both models. However, even if the decay rates are enhanced by two orders of magnitude, the branching ratios are still only at the level of $\sim 7 \times 10^{-7}$ for $B_s$ and $\sim 3 \times 10^{-8}$ for $B_d$.

As regards the decay $b \to X_s \mu^+ \mu^-$, we see that no signal of new physics is expected in Model I. The rate, the forward-backward asymmetry and the longitudinal polarization asymmetry remain at their SM values. This decay is somewhat more sensitive to new physics from Model II, which induces a factor of two enhancement of the short distance contributions. However, as it clear also by inspecting figs. 6 - 8, below the cut $\hat{s} < 0.4$ the overall effects are very likely too small to be unambiguously identified above the theoretical and experimental uncertainties.

We turn now to the decay $b \to X_s \tau^+ \tau^-$. In both our models the branching ratio for this decay is enhanced by more than one order of magnitude, at a level that could be observable with a good $\tau$ identification efficiency. Notice that with our choice of new physics parameters, Model I and Model II both predict very similar rates (see also fig. 1) even if the respective enhancements are induced by effective operators of quite a different nature. Therefore, even if such a large signal of new physics will be observed, it would not be possible to identify which kind of new physics is producing the effect just from a measurement of the decay rate. In contrast, we see that the forward-backward asymmetry $A_{FB}$ (fig. 2) and the longitudinal polarization asymmetry $P_L$ (fig. 3) could provide the additional information needed to disentangle the different effects.

In Model I $A_{FB}$ is almost at the 20% level, and opposite in sign with respect to the SM in the whole kinematic region. In contrast, in Model II this asymmetry is vanishingly small. On the other hand in Model II the longitudinal component $P_L$ is rather large, about twice the SM prediction, while in Model I it remains close to the SM value.

As regards the other two polarization components, in both models the transverse asymmetry $P_T$ is about a factor of three smaller than in the SM. The T-odd component $P_N$ which in the SM is at the 5% level, is practically zero in both our models. This can be traced back to the fact that the new (real) short distance contributions dominate over the absorptive part of the decay rate related to on shell $\bar{c}c$ intermediate states.

VI. CONCLUSIONS

In this paper we have studied rare $b$ decays into leptons of the third generation. These decay modes are well suited to study sources of new physics which couple more strongly to third family. We have first discussed a general framework for studying effects beyond the SM, and we have introduced general four-fermion amplitudes for the decays $B^- \to \tau \bar{\nu}$,
$B_q \to \tau^+ \tau^-$ and $b \to X_q \nu \bar{\nu}$. We have also defined the effective Hamiltonian for the decay $b \to X_s \tau^+ \tau^-$ in terms of an enlarged operator basis.

We have applied our results to the study of SUSY models without R-parity and without $L$ number. In these models new contributions to the decays appear already at the tree level, through new effective operators generated by squark and slepton exchange having a different structure than the SM ones. In order to derive numerical predictions for the various observables, we have embedded SUSY without R-parity in the framework of models for fermion masses based on Abelian horizontal symmetries. This allowed us to estimate the order of magnitude of the various R-parity violating couplings.

We have carried out a numerical study of two representative models, in which new physics effects arise from two different sets of effective operators, induced respectively by squarks and by sleptons exchange. We found that the most sensitive among the processes involving the $b \to \tau$ transition are the decays $B_q \to \tau^+ \tau^- \ (q = d, s)$ and $b \to X_s \tau^+ \tau^-$, that can be enhanced up to two orders of magnitude over the SM rates. If such an enhancement is observed, additional measurements of the forward-backward asymmetry and of the $\tau$ longitudinal polarization in $b \to X_s \tau^+ \tau^-$ can be very helpful in identifying the kind of underlying new physics. We have confronted the predictions for decays into final state taus with the corresponding results for decays into muons. The decay $b \to X_s \mu^+ \mu^-$ is not sensitive to this kind of new physics, and we found that only the $B_q \to \mu^+ \mu^-$ decay modes show enhancements comparable to the $\tau$ channel. However, even with the new physics contributions, the overall rates for these decays remain rather small.

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APPENDIX A: INPUT PARAMETERS

\[ m_b = 4.8 \text{ GeV}, \ m_c = 1.4 \text{ GeV}, \ m_s = 0.2 \text{ GeV}, \ m_t = 176 \text{ GeV}, \]
\[ m_\mu = 0.106 \text{ GeV}, \ m_\tau = 1.777 \text{ GeV}, \ M_W = 80.2 \text{ GeV}, \]
\[ V_{tb} = 1, \ V_{ts} = V_{cb} = -0.040, \ V_{td} = 0.009, \ V_{ub} = 0.003, \]
\[ f_{B_d} = 200 \text{ MeV}, \ f_{B_s} = 230 \text{ MeV}, \ m_B = 5.3 \text{ GeV}, \ \tau_B = 1.6 \text{ ps}, \]
\[ \Lambda_{QCD} = 225 \text{ MeV}, \ \mu = m_b, \ \alpha(M_Z) = 1/129, \ \sin^2 \theta_W = 0.23, \]
\[ \text{BR}(B \to X_c l\bar{\nu}_l) = 10.4\%. \]
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TABLE I. Predictions for the various decay rates and asymmetries for $b \rightarrow \tau$ transitions in the standard model and in the R-parity violating models discussed in the text. Model I is sensitive to operators generated by squark exchange. The lepton horizontal charges are $H(\tilde{L}) = (4, 2, 0)$, $H(\tilde{c}) = (4, 3, 3)$ while the SUSY masses are $m_{\tilde{t}} = 100$ GeV and $m_{\tilde{q}} = 170$ GeV. Model II is sensitive to operators generated by slepton exchange, with horizontal charges $H(\tilde{L}) = (3, 0, 0)$, $H(\tilde{c}) = (5, 5, 3)$ and SUSY masses $m_{\tilde{t}} = 100$ GeV and $m_{\tilde{q}} = 350$ GeV. In both models the value of the horizontal symmetry breaking parameter is $\varepsilon = 0.22$.

| Process                     | Standard Model | Model 1 | Model 2 |
|-----------------------------|----------------|---------|---------|
| $\text{BR} (B^- \rightarrow \tau^- \bar{\nu})$ | $7.1 \times 10^{-5}$ | $7.2 \times 10^{-5}$ | $7.4 \times 10^{-5}$ |
| $\text{BR} (B_s \rightarrow \tau^+\tau^-)$   | $9.1 \times 10^{-7}$ | $5.7 \times 10^{-6}$ | $1.8 \times 10^{-4}$ |
| $\text{BR} (B_d \rightarrow \tau^+\tau^-)$   | $4.3 \times 10^{-8}$ | $1.9 \times 10^{-7}$ | $6.3 \times 10^{-6}$ |
| $\text{BR} (b \rightarrow X_{\tilde{q}} \nu \bar{\nu})$ | $4.4 \times 10^{-5}$ | $6.7 \times 10^{-5}$ | $5.0 \times 10^{-5}$ |
| $\text{BR} (b \rightarrow X_{\tilde{d}} \nu \bar{\nu})$ | $2.7 \times 10^{-6}$ | $3.9 \times 10^{-6}$ | $3.0 \times 10^{-6}$ |
| $\text{BR}(b \rightarrow X_{\tilde{q}} \tau^+\tau^-)_{\text{no-cut}}$ | $4.9 \times 10^{-6}$ | $9.6 \times 10^{-6}$ | $1.0 \times 10^{-5}$ |
| $\text{BR}(b \rightarrow X_{\tilde{d}} \tau^+\tau^-)_{\text{j>0.6}}$ | $1.5 \times 10^{-7}$ | $4.1 \times 10^{-6}$ | $4.6 \times 10^{-6}$ |
| $\text{BR}^{sd}(b \rightarrow X_{\tilde{q}} \tau^+\tau^-)_{\text{j>0.6}}$ | $1.6 \times 10^{-7}$ | $4.1 \times 10^{-6}$ | $4.6 \times 10^{-6}$ |
| $\langle A_{FB} \rangle_{\text{j>0.6}}$ | $-0.13$ | $0.18$ | $-0.03$ |
| $\langle P_{L} \rangle_{\text{j>0.6}}$ | $-0.34$ | $-0.40$ | $-0.68$ |
| $\langle P_{T} \rangle_{\text{j>0.6}}$ | $-0.40$ | $-0.13$ | $-0.14$ |
| $\langle P_{N} \rangle_{\text{j>0.6}}$ | $0.05$ | $0.00$ | $0.01$ |

TABLE II. Predictions for the various decay rates and asymmetries for $b \rightarrow \mu$ transitions in the standard model and in the R-parity violating models discussed in the text. Model I is sensitive to operators generated by squark exchange. The lepton horizontal charges are $H(\tilde{L}) = (4, 2, 0)$, $H(\tilde{c}) = (4, 3, 3)$ while the SUSY masses are $m_{\tilde{t}} = 100$ GeV and $m_{\tilde{q}} = 170$ GeV. Model II is sensitive to operators generated by slepton exchange, with horizontal charges $H(\tilde{L}) = (3, 0, 0)$, $H(\tilde{c}) = (5, 5, 3)$ and SUSY masses $m_{\tilde{t}} = 100$ GeV and $m_{\tilde{q}} = 350$ GeV. In both models the value of the horizontal symmetry breaking parameter is $\varepsilon = 0.22$.

| Process                     | Standard Model | Model 1 | Model 2 |
|-----------------------------|----------------|---------|---------|
| $\text{BR} (B^- \rightarrow \mu^- \bar{\nu})$ | $3.2 \times 10^{-7}$ | $3.2 \times 10^{-7}$ | $3.3 \times 10^{-7}$ |
| $\text{BR} (B_s \rightarrow \mu^+\mu^-)$   | $4.3 \times 10^{-9}$ | $7.9 \times 10^{-7}$ | $7.2 \times 10^{-7}$ |
| $\text{BR} (B_d \rightarrow \mu^+\mu^-)$   | $2.1 \times 10^{-10}$ | $2.9 \times 10^{-8}$ | $2.7 \times 10^{-8}$ |
| $\text{BR}(b \rightarrow X_{\tilde{q}} \mu^+\mu^-)_{\text{no-cut}}$ | $3.1 \times 10^{-4}$ | $3.1 \times 10^{-4}$ | $3.4 \times 10^{-4}$ |
| $\text{BR}(b \rightarrow X_{\tilde{d}} \mu^+\mu^-)_{\text{j<0.4}}$ | $4.3 \times 10^{-6}$ | $4.5 \times 10^{-6}$ | $8.3 \times 10^{-6}$ |
| $\text{BR}^{sd}(b \rightarrow X_{\tilde{q}} \mu^+\mu^-)_{\text{j<0.4}}$ | $3.9 \times 10^{-6}$ | $4.1 \times 10^{-6}$ | $7.7 \times 10^{-6}$ |
| $\langle A_{FB}^{\mu} \rangle_{\text{j<0.4}}$ | $-0.01$ | $0.00$ | $0.08$ |
| $\langle P_{L}^{\mu} \rangle_{\text{j<0.4}}$ | $-0.57$ | $-0.56$ | $-0.73$ |
FIG. 1. Predictions for the differential branching ratio $\text{BR}(b \to X_s \tau^+ \tau^-)$ as a function of $\hat{s}$ in the standard model (solid), in Model I (dashed) and in Model II (dash-dotted) discussed in the text, including the long distance contribution. Model I is sensitive to operators generated by squark exchange. The lepton horizontal charges are $H(\hat{L}) = (4, 2, 0), H(\hat{\ell}^c) = (4, 3, 3)$ while the SUSY masses are $m_{\tilde{l}} = 100 \text{ GeV}$ and $m_{\tilde{q}} = 170 \text{ GeV}$. Model II is sensitive to operators generated by slepton exchange, with horizontal charges $H(\hat{L}) = (3, 0, 0), H(\hat{\ell}^c) = (5, 5, 3)$ and SUSY masses $m_{\tilde{l}} = 100 \text{ GeV}$ and $m_{\tilde{q}} = 350 \text{ GeV}$. In both models the value of the horizontal symmetry breaking parameter is $\varepsilon = 0.22$. 
FIG. 2. Predictions for the forward-backward asymmetry $A_{FB}$ for the $\tau$ lepton as a function of $\hat{s}$ in the standard model (solid), in Model I (dashed) and in Model II (dash-dotted) discussed in the text. The new physics model parameters are as in fig. 1.

FIG. 3. Predictions for the longitudinal polarization $P_L$ for the $\tau$ lepton as a function of $\hat{s}$ in the standard model (solid), in Model I (dashed) and in Model II (dash-dotted) discussed in the text. The new physics model parameters are as in fig. 1.
FIG. 4. Predictions for the transverse polarization $P_T$ for the $\tau$ lepton as a function of $\hat{s}$ in the standard model (solid), in Model I (dashed) and in Model II (dash-dotted) discussed in the text. The new physics model parameters are as in fig. 1.

FIG. 5. Predictions for the normal polarization $P_N$ for the $\tau$ lepton as a function of $\hat{s}$ in the standard model (solid), in Model I (dashed) and in Model II (dash-dotted) discussed in the text. The new physics model parameters are as in fig. 1.
FIG. 6. Predictions for the differential branching ratio $\text{BR}(b \rightarrow X_s \mu^+ \mu^-)$ as a function of $\hat{s}$ in the standard model (solid), in Model I (dashed) and in Model II (dash-dotted) discussed in the text, including the long distance contribution. The new physics model parameters are as in fig. 1.

FIG. 7. Predictions for the forward backward asymmetry $A_{FB}$ for the $\mu$ lepton as a function of $\hat{s}$ in the standard model (solid), in Model I (dashed) and in Model II (dash-dotted) discussed in the text. The new physics model parameters are as in fig. 1.
FIG. 8. Predictions for the longitudinal polarization $P_L$ for the $\mu$ lepton as function of $\hat{s}$ in the standard model (solid), in Model I (dashed) and in Model II (dash-dotted) discussed in the text. The new physics model parameters are as in fig. 1.