Temporal evolution of two quantum radiators at interaction with electromagnetic cavity field

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Abstract. In this paper, we study a new quantum reversible effect for a three-level atom which interacts with electromagnetic cavity field. We investigate the reversible conditions for a three-level atom that is in resonance with two cavity modes and in off resonance with a single mode of cavity. The possibility of the reversible factorization between the atom and cavity radiation is the subject of this work. It is found the reversible condition for which these quantum subsystems can (or not) restore their diagonal moments.

1. Introduction
Recently, considerable attention in quantum physics has been devoted to interaction between the atoms and electromagnetic cavity field [1 – 3]. One of this, which is very popular and in great demand for realization of quantum computer represents the quantum time separation of two quantum subsystems. The quantum collapse and revival realization of two radiators is an interesting problem due to the possibilities of restoration of initially separated states of atom and electromagnetic field. There are a lot of works in the literature devoted to the revival problem of two quantum subsystems in interaction [4 – 6].

In the first part of this paper, it is investigated the reversible condition for a non-equidistant three-level atom in interaction with two cavity modes. We analyze a general case in that the atom is prepared in a superposition of all three states and the cavity field is regarded as a finite superposition of Fock states. Taking into account the various possibilities to restore the initial state, we put the reversible problem of this system in general case. The recursion relation between the coefficients of bimodal cavity field is obtained. From this relation it is found the particular cases for that the coherent trapping conditions are realized in squeezed field proposed in [4]. We propose a new reversible condition, taking into account the coherent trapping effect [4] and dynamic trapping effect [5] as particular cases. This condition has direct connection with entangled states and is reduced to quantum regular nutation between the excited and ground states of three-level atom. The temporal behavior and the properties of these new states are investigated.

In the second part, it is analyzed the quantum mixing, when we have a detuning from resonance between the radiator and single mode cavity field. From quantum mechanical points of view we put the problem of reversibility in such interaction. In order to obtain this condition it is established the recursion relation between the decomposition coefficients of wave functions on the Fock states of atom and cavity field. We analytically analyze the possibility of states restoration of two quantum oscillators in off resonance interaction. As follows from our results,
in the off-resonance case the full reversibility is impossible. We find the conditions for which the atom and cavity field restore all diagonal elements of density operator. This is possible only for the case in which the transit time through the micro-cavity strongly satisfies the reversible condition.

2. Quantum reversible condition for a three-level atom in resonance with bimodal cavity field

In this section we analyzed the reversible condition for a non-equidistant three-level atom in interaction with two cavity modes. We consider a cavity with a high quality factor, into which a flux of three-level atoms with a very low rate is pumped, so that during the flying time, $\tau$, only one atom in the cavity is considered and the field damping time is larger than the atomic transit time through the cavity. Preparing in the superpositions of all three-level states (see Figure 1) in classical Ramsay zone[6], at the time moment $t$ the atom enters into the cavity. Let us consider the situation when the states of the atomic and the field are considered totally separated, so that the wave function can be represented as

$$|\Psi(t)\rangle = (\alpha |e\rangle + \beta |i\rangle + \gamma |g\rangle) \otimes \sum_{n,m=Nd,Md} S_{n,m} |n\rangle_a |m\rangle_b.$$  

Figure 1. Configuration of the energy levels of a three-level atom.

In the expression (1), $Nd$, $Md$ and $Nu$, $Mu$ represent the bottom and upper limits. We emphasize that the energies of excited and ground states are measured relatively to the position of the intermediate state, so that $E_e = h\omega_a$, $E_g = h\omega_b$ and $E_i = 0$. The frequencies $\omega_a$ and $\omega_b$ correspond to the resonance cavity modes described by annihilation $a$, $b$ and creation $a^+$, $b^+$ field operators.

In the time moment, $t + \tau$, the atom enters into an interaction with the cavity field and the system is described by the Hamiltonian

$$H = H_0 + H_i,$$  

(2)
with the following free and interaction parts
\[ H_0 = \hbar \omega_a |e\rangle \langle e| - \hbar \omega_b |g\rangle \langle g| + \hbar \omega_\alpha a^\dagger a + \hbar \omega_\beta b^\dagger b, \]
\[ H_i = \hbar \alpha_a \left( a^\dagger |i\rangle \langle e| + a |e\rangle \langle i| \right) + \hbar \alpha_b \left( b^\dagger |g\rangle \langle i| + b |i\rangle \langle g| \right). \]

The coefficients \( \hbar \alpha_a \) and \( \hbar \alpha_b \) describe the coupling energies between the modes \( a \), \( b \) and level transitions \( i \leftrightarrow e, i \leftrightarrow g \).

Taking into account the commutative propriety between the free and interaction parts of the Hamiltonian, \( [H_0, H_I] = 0 \), the wave function solution in the interaction picture \[ \tilde{\Psi}(t + \tau) = \exp(\frac{\hbar}{2} H_I \tau) |\Psi(t + \tau)\rangle \] of the Schrodinger equation \( i\hbar \partial_\tau |\Psi(t + \tau)\rangle = H_I |\Psi(t + \tau)\rangle \) can be exactly solved

\[ |\tilde{\Psi}(t + \tau)\rangle = |\tilde{\Psi}_e(t + \tau)\rangle + |\tilde{\Psi}_g(t + \tau)\rangle + |\tilde{\Psi}_i(t + \tau)\rangle, \quad \text{(3)} \]

where

\[ |\tilde{\Psi}_e(t + \tau)\rangle = \sum_{n,m=N_d M_d}^{N_u M_u} \left\{ S_{n,m} \left[ -2\lambda_a^2 \frac{n + 1}{\Omega_{n+1,m+1}^2} \sin^2(\Omega_{n+1,m+1} \tau/2) + \alpha \right] \right. \]
\[ - S_{n+1,m+1} 2\gamma \lambda_a \lambda_b \frac{(n+1)(m+1)^{1/2}}{\Omega_{n+1,m+1}^2} \sin^2(\Omega_{n+1,m+1} \tau/2) \]
\[ - S_{n+1,m} i\beta \lambda_a \lambda_b \frac{(n+1)^{1/2}}{\Omega_{n+1,m+1}} \sin(\Omega_{n+1,m+1} \tau) \} \}
\[ \{ |e\rangle \langle n| \langle m| \}, \]

\[ |\tilde{\Psi}_g(t + \tau)\rangle = \sum_{n,m=N_d M_d}^{N_u M_u} \left\{ -S_{n-1,m-1} 2\alpha \lambda_a \lambda_b \frac{(nm)^{1/2}}{\Omega_{n,m}^2} \sin^2(\Omega_{n,m} \tau/2) \right. \]
\[ + S_{n,m} \left\{ \gamma - 2\gamma \lambda_a^2 \frac{m}{\Omega_{n,m}^2} \sin^2(\Omega_{n,m} \tau/2) \right. \}
\[ - S_{n,m-1} i\beta \lambda_a \lambda_b \frac{m^{1/2}}{\Omega_{n,m}} \sin(\Omega_{n,m} \tau) \} \}
\[ \{ |g\rangle \langle n| \langle m| \}, \]

\[ |\tilde{\Psi}_i(t + \tau)\rangle = \sum_{n,m=N_d M_d}^{N_u M_u} \left\{ -S_{n-1,m} i\alpha \lambda_a \lambda_b \frac{n^{1/2}}{\Omega_{n,m+1}} \sin(\Omega_{n,m+1} \tau) + S_{n,m} \beta \cos(\Omega_{n,m+1} \tau) \right. \]
\[ - S_{n,m+1} i\gamma \lambda_a \lambda_b \frac{(m+1)^{1/2}}{\Omega_{n,m+1}} \sin(\Omega_{n,m+1} \tau) \} \}
\[ \{ |i\rangle \langle n| \langle m| \}. \]

In order to obtain the recursion relation between the coefficients, \( S_{n,m} \), we must require the probability amplitudes for the ground, \( \gamma S_{n,m} |g\rangle \langle n| \langle m| \}, \) excited \( \alpha S_{n,m} |n\rangle \langle m| \langle e| \), and intermediate \( -\beta S_{n,m} |i\rangle \langle n| \langle m| \) states to remain unchanged after the flying time, \( \tau \). As follows, all three reversible conditions are satisfied by the same recursion relation

\[ S_{n,m} = -\frac{\alpha \lambda_a}{\gamma \lambda_b} \left( \frac{n}{m} \right)^{1/2} S_{n-1,m-1} - \frac{\beta}{\gamma \lambda_b} \frac{\Omega_{n,m}}{m^{1/2}} \cot(\Omega_{n,m} \tau/2) S_{n,m-1}, \quad \text{(4)} \]

where \( \Omega_{n,m} = \sqrt{\lambda_a^2 n + \lambda_b^2 m} \).
In the case when, $\lambda_a = \lambda_b = \lambda$, and $\alpha = 0$ (or $\gamma = 0$), three-level system is reduced to a two-level quantum oscillator relative to the ground and intermediate states (or intermediate and excited states), so that the recursion relation (4) passes into the same recursion relation proposed in [5].

For simplicity, considering the intermediate level is not populated, $\beta = 0$, we can obtain a similarity between expression (4) and the coherent trapping effect studied in the literature. In this case, the upper and bottom limits, are considered finite and the class of the biharmonic cavity field which satisfies the recursion relation is

$$\sin \left( \left( \lambda_a^2 n + \lambda_b^2 m \right)^{1/2} \frac{\tau}{2} \right) \left( S_{n,m} + \alpha \lambda_a \gamma \lambda_b \left( \frac{n}{m} \right)^{1/2} S_{n-1,m-1} \right) = 0. \quad (5)$$

In the case when the bottom limits for both modes are the vacuum states $N_a \to 0$, $M_a \to 0$, and the upper limits are infinite $N_u \to \infty$, $M_u \to \infty$. If the number of photons in both modes are equal, $m = n$, and $S_{n,m} = \delta_{n,m} S_{n,m}$, from relation (5) we can use only the second factor, $S_{n,m} - AS_{n-1,m-1} = 0$, for which the inversion is not changed (here $A = -\alpha\lambda_a/\gamma\lambda_b$). In such situation, the EM field state in which this effect is realized can be regarded as squeezed states of two cavity modes

$$|\Psi\rangle = \sqrt{1 - A^2} \exp \left( Aa^\dagger b^\dagger \right) |0\rangle_a |0\rangle_b.$$  \quad (6)

Let us consider the situation for which the number of photons in both modes are equal, $m = n$, and the upper states are finite $n \leq N_u$, $m \leq M_u$. In this case when, $S_{N_u+1,N_u+1} = 0$, the second part of relation (5) is truncated, $AS_{N_u,N_u} \neq S_{N_u+1,N_u+1}$, from which follows

$$\sin \left( \frac{k\pi}{2} \left( (N_u + 1) (\lambda_a^2 + \lambda_b^2) \right)^{1/2} \right) = 0,$$

$$\tau_k = 2k\pi / \left( (N_u + 1) (\lambda_a^2 + \lambda_b^2) \right)^{1/2}, \quad (k \text{ integer}). \quad (7)$$

The relation (7) represents the reversible condition which describes the atom flying time, $\tau_k$, through the cavity.

From a physical point of view, the reversible condition (7) becomes more clear, if we analyze the following partial cases:

a. Reversible condition on the vacuum state of EM cavity field, when, $N_a = N_u$, $M_a = M_u$ and $N_u = M_u = 0$. Considering, $\sin \left( \frac{k\pi}{2} (\lambda_a^2 + \lambda_b^2) \right)^{1/2} = 0$, we can drop the above recursion relation, $S_{1,1} - AS_{0,0} \neq 0$, so that only one element is non-equal with zero, $S_{00} = 1$. From this condition we obtain the following discrete values for reversible time, $\tau_k$

$$\tau_k = \frac{2k\pi}{\left( \lambda_a^2 + \lambda_b^2 \right)^{1/2}}. \quad (8)$$

In accord with flying time (8), the populations on the excited and intermediate states oscillate with vacuum Rabi frequency, $\bar{n}_e = |\alpha|^2 \cos^4 \left( \frac{k\pi}{2} (\lambda_a^2 + \lambda_b^2) \right)^{1/2}$, and $\bar{n}_i = |\alpha|^2 \sin^2 \left( \frac{k\pi}{2} (\lambda_a^2 + \lambda_b^2) \right)^{1/2} / 2$.

b. Entangled bimodal field, when $N_u = M_u = 1$, and $N_d = M_d = 0$. As follows, the recursion relation (5) is realized only between the coefficients, $S_{2,2}$, and $S_{1,1}$. Choosing, $S_{2,2} - AS_{1,1} \neq 0$, and $\sin \left( \frac{k\pi}{2} \left( 2(\lambda_a^2 + \lambda_b^2) \right)^{1/2} \right) = 0$, we obtain the following expression for flying time $\tau_k = \sqrt{2k\pi} / \sqrt{\lambda_a^2 + \lambda_b^2}$. 
When, $\lambda_a = \lambda_b = \lambda$, the EM wave function in the time moment, $\tau = \tau_k$, corresponds to the entangled state of photons

$$|\tilde{\Psi}_{EM}(t + \tau_k)\rangle = \left(1 + \frac{\alpha^2}{\gamma^2}\right)^{-1/2} \left[|0\rangle_a |0\rangle_b - \frac{\alpha}{\gamma} |1\rangle_a |1\rangle_b\right]. \quad (9)$$

3. Quantum mixing for off resonance case

In this section, we investigate the quantum mixing between a three-level atom and one cavity mod. It is consider the case when these oscillators are not in resonance each other (see Figure 2).

Assuming the energy is measured from the intermediate level of the atom, we study the influence of the detuning from resonance of its state. The general Hamiltonian of the system can be written

$$H = H_0 + H_i, \quad (10)$$

where

$$H_0 = \hbar \omega a^+ a + \hbar \omega |e\rangle \langle e| - \hbar \omega_g |g\rangle \langle g|,$$

$$H_i = \hbar \Delta (|e\rangle \langle e| + |g\rangle \langle g|) + \hbar \lambda_1 (|i\rangle \langle g| a + a^+ |g\rangle \langle i|) + \hbar \lambda_2 (|e\rangle \langle i| a + a^+ |i\rangle \langle e|)$$

are the free and interaction parts of the Hamiltonian; $E_g = -\hbar \omega_g$, $E_e = \hbar \omega_e$ represent the energetically positions of ground and excited levels of the atom. In the Hamiltonian (10) the frequency $\omega$ corresponds to the resonance cavity mode described by annihilation $a$, and creation $a^+$ field operators. The coefficients $\hbar \lambda_1 = \langle d_{i,g}, \bar{g}(\omega) \rangle$ and $\hbar \lambda_2 = \langle d_{i,e}, \bar{g}(\omega) \rangle$ represent the coupling energies between the cavity mode, level excitations $|i\rangle \langle g|$, $|e\rangle \langle i|$ and relaxation $|g\rangle \langle i|$, $|i\rangle \langle e|$ transitions. $d_{i,g}$ and $d_{i,e}$ are the transitions of dipole matrix elements; $\bar{g}(\omega)$ is the strength constant.

**Figure 2.** Energy levels relevant to the two photons laser with atomic polarization.

Let us considering that at the time, $\tau = 0$, the atom enters in the cavity and the atomic and cavity field states are totally factorized

$$|\Psi(0)\rangle = \sum_{n=N_0}^{N_a} (\alpha |e\rangle + \beta |i\rangle + \gamma |g\rangle) \otimes S_n |n\rangle. \quad (11)$$

After interaction, at the time moment, $\tau$, the wave function of these two oscillators can be written in the following way
\[
\Psi(\tau) = \sum_{n=N_d}^{N_s} \{ \left[ \alpha \frac{\lambda_2^2 (n+1)}{\Theta_{n+1}} \right] e^{-i\Delta \tau/2} \cos \left( \Omega_{n+1} \frac{\tau}{2} \right) - i \frac{\Delta}{\Omega_{n+1}} e^{-i\Delta \tau/2} \sin \left( \Omega_{n+1} \frac{\tau}{2} \right) - e^{-i\Delta \tau} S_n - 2\beta i \lambda_2 \sqrt{\frac{n+1}{\Omega_{n+1}}} e^{-i\Delta \tau/2} \sin \left( \Omega_{n+1} \frac{\tau}{2} \right) S_{n+1} \\
+ \gamma \frac{\lambda_1 \lambda_2}{\Theta_{n+1}} e^{-i\Delta \tau/2} \sin \left( \Omega_{n+1} \frac{\tau}{2} \right) \left[ e^{-i\Delta \tau/2} \cos \left( \Omega_{n+1} \frac{\tau}{2} \right) - i \frac{\Delta}{\Omega_{n+1}} e^{-i\Delta \tau/2} \sin \left( \Omega_{n+1} \frac{\tau}{2} \right) - e^{-i\Delta \tau} \right] S_{n+2} \} \langle e \rangle + \{ -2i \alpha \frac{\lambda_2 \sqrt{n}}{\Theta_{n-1}} e^{-i\Delta \tau/2} \sin \left( \Omega_{n-1} \frac{\tau}{2} \right) S_{n-1} + \beta e^{-i\Delta \tau/2} \left[ \cos \left( \Omega_{n-1} \frac{\tau}{2} \right) + i \frac{\Delta}{\Omega_{n-1}} \sin \left( \Omega_{n-1} \frac{\tau}{2} \right) - e^{-i\Delta \tau} \right] S_n \\
- 2i \gamma \frac{\lambda_1 \sqrt{n}}{\Theta_{n-1}} e^{-i\Delta \tau/2} \sin \left( \Omega_{n-1} \frac{\tau}{2} \right) S_{n-1} + \{ \alpha \frac{\lambda_2 \sqrt{(n-1)}}{\Theta_{n-2}} e^{-i\Delta \tau/2} \sin \left( \Omega_{n-2} \frac{\tau}{2} \right) S_{n-2} - 2\beta i \lambda_1 \sqrt{\frac{n}{\Omega_{n-2}}} e^{-i\Delta \tau/2} \sin \left( \Omega_{n-2} \frac{\tau}{2} \right) S_{n-1} \\
+ \gamma \left[ \frac{\lambda_1^2 \sqrt{n}}{\Theta_{n-1}} e^{-i\Delta \tau/2} \cos \left( \Omega_{n-1} \frac{\tau}{2} \right) - i \frac{\Delta}{\Omega_{n-1}} \sin \left( \Omega_{n-1} \frac{\tau}{2} \right) - e^{-i\Delta \tau} \right] \} \langle g \rangle \} |n\rangle,
\]

where \( \Omega_n = \sqrt{\Delta^2 + 4 \lambda_1^2 (n+1) + \lambda_2^2 n} \) is the Rabi frequency and \( \Theta_n = (\Omega_n^2 - \Delta^2) / 4 \).

During the interaction process the atom and cavity field are coupled in according with the solution (12). The aim of this paper consists in the finding of quantum field properties for which the atomic population and photon number distributions on the Fock states after the transit time, \( \tau \), restore their initial properties (11). In other words the population probabilities of the field on the Fock states \( |n\rangle \), \( \langle \Psi(\tau)| |n\rangle \rangle = \langle S_n \rangle^2 \), atom in the ground, \( \langle \Psi(\tau) | |g\rangle \rangle = \langle \alpha \rangle^2 \), intermediate, \( \langle \Psi(\tau) | |i\rangle \rangle = \langle \alpha_i \rangle^2 \) and excited \( \langle \Psi(\tau) | |e\rangle \rangle = \langle \alpha_e \rangle^2 \) states become reversible. Such field must have special coefficients \( S_n \), for which the reversibility is possible. Indeed, according to the wave function (12) this requirement can be satisfied for the evolution of the ground state amplitude during the transit time, \( \tau \). Thus, after the interaction the amplitude on the ground and excited states remain as in the initial state (11), but with other phase : \( \gamma(l) \langle \tau | S_n^{\dagger} | i \rangle \rightarrow \gamma \langle \tau | S_n e^{-i\Delta \tau} \rangle, \alpha' \langle \tau | S_n^{\dagger} | e \rangle \rightarrow \alpha S_n e^{-i\Delta \tau} \).

As follows, we obtain the same recursion relation between the coefficients \( S_n \)

\[
S_n = \frac{1}{2\beta \Theta_n} \left[ i \Omega_n e^{-i\Delta \tau/2} - \cos \left( \Omega_n \frac{\tau}{2} \right) \sin \left( \Omega_n \frac{\tau}{2} \right) - \Delta \right] \left[ \alpha \lambda_2 \sqrt{n} S_{n-1} + \gamma \lambda_1 \sqrt{n+1} S_{n+1} \right].
\]

The recursion relation (13) must be satisfied for all three-states of the atom at the same time. Introducing the coefficient, \( S_n \), from this relation in the amplitude of the intermediate atomic state, \( |i\rangle \), from solution (12) we obtain that the amplitude of the intermediate state also pass in the initial amplitude, but with a phase which depends on the field state numbers

\[
\beta' \langle \tau | S_n^{\dagger} | i \rangle |n\rangle = -\beta e^{-i\Delta \tau} \frac{Z(n)}{Z^*(n)} S_n |i\rangle |n\rangle = -\beta e^{-i\Delta \tau + 2i \phi(n)} Z S_n |i\rangle |n\rangle.
\]

Here the notation, \( Z(n) = \cos \left( \Omega_n \tau / 2 \right) + i \Delta / \Omega_n \sin \left( \Omega_n \tau / 2 \right) \), represents the complex number. The module \( |Z(n)| \) and argument \( \phi(n) = \arctan \left( \Delta \tan \Omega_n / \Omega_n \right) \) depend on the Fock state number.
As a conclusion, we observe that the recursion relation (13) will be satisfied for all atomic states and in the time moment, \( \tau \), the wave function of the system evaluates in a quasifactorised state with the same probability amplitude as in the initial state

\[
|\Psi(\tau)\rangle = e^{-i\Delta\tau} \sum_{n=N_d}^{N_u} S_n [\alpha |e\rangle + \gamma |g\rangle - \beta \frac{Z(n)}{Z^*(n)} |i\rangle |n\rangle. \tag{15}
\]

The reversibility between two quantum oscillators can be analyzed in two cases:

- **Full separation**, that corresponds to exact resonance case, \( \Delta = 0 \). In this situation the wave function which describes the atomic and field subsystem is separated \( \Psi(\tau) = \Psi_A(\tau) \otimes \Psi_{EM}(\tau) \). In accordance with the expression (15), the coefficient \( \frac{Z(n)}{Z^*(n)} \) represents the probability to find the atom and field in the same initial Fock state \( |n\rangle \), where \( k \equiv e, i, g \). Using the definition (14), we observe that the phase factor \( \phi(n) \) does not change the probability to find the atom and field in the Fock states \( |k\rangle \) and \( |n\rangle \): \( |a_k| S_n e^{i\phi(n)}|^2 = |a_k|^2 |S_n|^2 \).

- **Partial separation**, that corresponds to \( \Delta \neq 0 \). In this case, the coefficient, \( |a_k|^2 |S_n|^2 \), represents the probability to find the atom and field in the same initial Fock state \( |k\rangle |n\rangle \), where \( k \equiv e, i, g \). Using the definition (14), we observe that the phase factor \( \phi(n) \) does not change the probability to find the atom and field in the Fock states \( |k\rangle \) and \( |n\rangle \): \( |a_k| S_n e^{i\phi(n)}|^2 = |a_k|^2 |S_n|^2 \).

As follows from both definitions the amplitude phases do not affect the diagonal matrix elements of the system after the interaction process. According to second case, we observe that the wave function remains unfactorized. This effect can be regarded as a partial factorization and becomes more clear, introducing the reduced density operator for atomic states \( W_A = Sp \{|\Psi\rangle \langle \Psi|\}_{ph} \):

\[
W_A = [\alpha |e\rangle + \gamma |g\rangle] [\alpha^* \langle e| + \gamma^* \langle g| + |\beta|^2 |i\rangle \langle i| \]
\[ - \beta^* (\alpha |e\rangle + \gamma |g\rangle) \langle i| \sum_{n=N_d}^{N_u} |S_n|^2 \frac{Z^*(n)}{Z(n)} \]
\[ - \beta [\alpha^* < e| + \gamma^* < g||i\rangle \sum_{n=N_d}^{N_u} |S_n|^2 \frac{Z(n)}{Z^*(n)} . \tag{16}
\]

In analogical way, we can write the reduced density matrix for the cavity field

\[
W_{EM} = Sp_A |\Psi(\tau)\rangle \langle \Psi(\tau)| = \sum_{n=N_d}^{N_u} \sum_{m=M_d}^{M_u} \left( 1 + |\beta|^2 \frac{Z_m Z_n}{Z_m Z_n} - 1 \right) S_n S_m |n\rangle \langle m|. \tag{17}
\]

As follows from definitions (16) and (17) after the transit time, \( \tau \), the diagonal matrix elements coincide with their initial values, but the mixing phases between the atom and field of non-diagonal matrix elements remain coupled.

4. Conclusions

In this paper we have found the reversible conditions for a three-level atom flying through the cavity in the resonance case and in the absence of it. In the case of non-vacuum state the decompose coefficients of Fock states, \( S_{nm} \), must comply with the reversible conditions described by recursion relations (4) and (7). Here, we proposed a new reversible condition which not coincides with earlier discussed trapping conditions [4 – 6], but in particular case this general condition contains the reversibilities discussed in above papers. We consider that this reversibility open new experimental [8] and theoretical realizations in quantum mechanics.
In the off resonance case is demonstrated that the phases coupling between the cavity field and atomic inversion remain unchanged. As follows from reduced density operator, after the interaction process the atom and cavity subsystem can restore only their initial diagonal matrix elements (atomic population, mean number of photons and its fluctuations), but the nondiagonal elements remain coupled. This effect was named partial restoration of two quantum interaction subsystems.

The effect discussed here gives us the possibility to use the prepared atomic inversion of three-level atom flying through the cavity in quantum computing. Choosing the flying time accordingly to the reversible condition (7) and (8) one can obtain the factorized wave function of two subsystems: atom and cavity vacuum after interaction. Quantum computer describes the evolution of two or more subsystems in which the reversible transition from non-entangled to the entangled state and vice versa represents an essential feature [9]. This manipulation with quantum phase can be used in logical gates of information processing. Quantum logic gates are the basic units of quantum control. For example, if the system remains in the same factorized state the quantum control is realized (or not realized). It is equivalent to the controlled-NOT gate operation proposed in paper [10].

5. References

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