Lagrangian formulation of the massive higher spin $N = 1$ supermultiplets in $AdS_4$ space

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Abstract

We give an explicit component Lagrangian construction of massive higher spin on-shell $N = 1$ supermultiplets in four-dimensional Anti-de Sitter space $AdS_4$. We use a frame-like gauge invariant description of massive higher spin bosonic and fermionic fields. For the two types of the supermultiplets (with integer and half-integer super-spins) each one containing two massive bosonic and two massive fermionic fields we derive the supertransformations leaving the sum of four their free Lagrangians invariant such that the algebra of these supertransformations is closed on-shell.
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1 Introduction

The higher spin theory (see e.g. the reviews [1], [2], [3]) has attracted significant interest for a long time and for many reasons. On the one hand, the theory of massless higher spin fields is a maximal extension of the Yang-Mills gauge theories and gravity including all spin fields. On the other hand, it is closely related to superstring theory which involves an infinite tower of higher spin massive fields. In principle, the higher spin field theory can provide the possibility to study some aspects of string theory in the framework of the field theory. It is also worth pointing out that the construction of Lagrangian formulations for the higher spin field models is extremely interesting itself since it allows to reveal the new unexpected properties to relativistic field theory in general.

Beginning with work [4] it became clear that the nonlinear massless higher spin theory can only be realized in AdS space with non-zero curvature. This raises the interest in studying the various aspects of field theory in AdS space in the context of AdS/CFT correspondence. Taking into account that the low-energy limit of superstring theory should lead to supersymmetric field theory we face the problem of constructing the supersymmetric massive higher spin models in the AdS space. It is expected that the supersymmetry can be an essential ingredient of the consistent theory of all the fundamental interactions including quantum gravity. It is possible that such a theory should also involve the massless and/or massive higher spin fields. This paper is devoted to developing the $N = 1$ supersymmetric Lagrangian formulation of free massive higher spin models in AdS space in the framework of on-shell component formalism.

In supersymmetric theories the massless or massive fields are combined into the corresponding supermultiplets. In the case of free field models containing the different spin fields, it is natural to expect that the Lagrangian should be the sum of the Lagrangians for each concrete spin field. To provide an explicit Lagrangian realization of the free supermultiplet one has to find supertransformations leaving the free Lagrangians invariant and show that the algebra of these supertransformations is closed at least on-shell. In the case of the $N = 1$ supersymmetry the massless higher superspin-s supermultiplets consist of the two massless fields with spins ($s, s + 1/2$). The task of constructing supertransformations for such supermultiplets in four dimensional flat space was completely solved in the metric-like formulation [5] and soon in the frame-like one [6]. In both cases the supertransformations have a simple enough structure and are determined uniquely by the invariance of the sum of the Lagrangians for two free massless fields with spins $s$ and $s + 1/2$. Note that such a requirement allows to find only on-shell supersymmetry when supertransformations are closed on the equations of motion. In order to find off-shell supertransformations, it is necessary to introduce the corresponding auxiliary fields.

A natural procedure to construct off-shell $N = 1$ supersymmetric Lagrangian models is realized in terms of $N = 1$ superspace and superfields (see e.g. [7]), where all the auxiliary fields providing closure of the superalgebra are automatically obtained. In the framework of superfield formulation the $N = 1$ supersymmetric massless higher spin models were constructed in the pioneer papers [8,9]. Later, on the basis of these results, $N = 1$ supersymmetric massless higher spin models were generalized for AdS$_4$ space [10], [11]. In both cases the constructed superfield models, after eliminating the auxiliary fields, reduce to the sum

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$^1$Application of this formulation for quantization of the $N = 1$ higher spin superfield model in AdS$_4$ space
of spin-s and spin-(s + 1/2) (Fang)-Fronsdal Lagrangians \([20,21]\) in flat or \(AdS\) spaces. The
generalization for \(\mathcal{N} = 2\) massless higher spin supermultiplets was given in \([11], [22]\).

There are much fewer results in the case of supersymmetric massive higher spin models
even in the on-shell formalism, the reason being that when moving from the massless compo-
nent formulation to the massive one very complicated higher derivative corrections must be
introduced to the supertransformations. Moreover the higher the spin of the fields entering a
supermultiplet the higher the number of derivatives one has to consider. The problem of the
supersymmetric description of the massive higher spin supermultiplet was only explicitly re-
solved in 2007 for the case of \(N = 1\) on-shell 4D Poincare superalgebra \([23]\) (see also \([24,26]\))
using the gauge invariant formulation for the massive higher spin fields \([27,29]\). In such
a formalism the description for the massive field is obtained in terms of the appropriately
chosen set of the massless ones. It is assumed that the Lagrangian for massive higher spin
supermultiplets is constructed as a sum of the corresponding Lagrangians for massless fields
deformed by massive terms. However, it appeared \([23]\) that to realize such a program one
has to use massless supermultiplets containing four fields \((k - 1/2, k, k', k + 1/2)\) as the
building blocks, where two bosonic fields with equal spins have opposite parities, and this
prevents us from separating them into the usual massless pairs. In \([23]\) it was shown that
to obtain the massive deformation it is enough to add the non-derivative corrections to the
supertransformations for the fermions only. Complicated higher derivative corrections to
the supertransformations reappear when one tries to fix all local gauge symmetries, breaking
the gauge invariance. Note however that in such construction the mass-like terms for the
fermions in the Lagrangian take a complicated non-diagonal form making calculations rather
cumbersome. Surprisingly however, in 4D the above results remain the main results in the
massive supersymmetric higher spin theory until now\(^3\). The aim of this paper is to extend
and generalize the results of \([23]\) to the case of four dimensional \(N = 1\) \(AdS_4\) superalgebra.

We use the gauge invariant description of the massive higher spin bosonic and fermionic
fields but in the frame-like version \([38,39]\). Recall that one of the attractive features of such
a formalism is that it works nicely both in flat Minkowski space as well as in \((A)dS\) spaces.
Our strategy differs from that of \([23]\). For the Lagrangian we take just the sum of four
free Lagrangians for the two massive bosonic and two massive fermionic fields entering the
supermultiplet. Then for each pair of bosonic and fermionic fields (we call it superblock in
what follows) we find the supertransformations leaving the sum of their two Lagrangians
invariant. Next we combine all four possible superblocks and adjust their parameters so that
the algebra of the supertransformations is closed on-shell.

The paper is organized as follows. In section 2 we give all necessary descriptions of
the frame-like formulation of massless bosonic and fermionic higher spin fields and also we
consider the massless higher spin supermultiplets in \(AdS_4\) in such a formalism. Massless
models given in this section will serve as the building blocks for our construction of the massive

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\(^2\)Another gauge invariant approach to Lagrangian formulation of massive higher spin fields is given on
the base of BRST construction \([30,31,32,33]\).

\(^3\)The attempts to developed the off-shell superfield Lagrangian formulation of the massive higher spin
supermultiplets were realized only for some examples in \([34,35,36,37]\). General superfield formulation
is still undeveloped.
higher spin models. In section 3 we give frame-like gauge invariant formulations for free massive arbitrary integer and half-inter spins. In section 4 we consider massive superblocks containing one massive bosonic and one massive fermionic field and find corresponding supertransformations. In section 5 we combine the constructed massive superblocks into one massive supermultiplet.

Notations and conventions. In this work we use a technique of $p$-forms taking the values in the Grassmann algebra. The main geometrical objects are some $p$-forms $\Omega$ ($p=0,1,2,3,4$). They are defined as

$$\Omega = dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_p} \Omega_{\mu_1 \ldots \mu_p},$$

where $\Omega_{\mu_1 \ldots \mu_p}$ is the antisymmetric tensor field. In particular, the partial derivative is defined as one-form $d = dx^\mu \partial_\mu$. In 4D it is convenient to use a frame-like multispinor formalism where all the Lorentz objects have local totally symmetric dotted and undotted spinorial indices (see a description of irreducible representations of 4D Lorentz group in terms of dotted and undotted spinors e.g. in [7]). To simplify the expressions we will use the condensed notations

$$\Omega^{(m)\dot{\alpha}(n)} = \Omega^{(\alpha_1 \alpha_2 \ldots \alpha_m)(\dot{\alpha}_1 \dot{\alpha}_2 \ldots \dot{\alpha}_n)}$$

We also always assume that spinor indices denoted by the same letters and placed on the same level are symmetrized, e.g.

$$\Omega^{(m)\dot{\alpha}(n) \dot{\alpha}} = \Omega^{(m)(\dot{\alpha}_1 \ldots \dot{\alpha}_n \dot{\alpha}_{n+1})}$$

We work in the AdS$_4$ space which is described by pair 1-forms: background frame $e^{\alpha \dot{\alpha}}$ which enters explicitly in all constructions and background Lorentz connections $\omega^{\alpha \beta}, \omega^{\dot{\alpha} \dot{\beta}}$ which are hidden in the 1-form covariant derivative

$$D \Omega^{(m)\dot{\alpha}(n)} = d\Omega^{(m)\dot{\alpha}(n)} + \omega^{\alpha \beta} \wedge \Omega^{(m-1)\beta \dot{\alpha}(n)} + \omega^{\dot{\alpha} \dot{\beta}} \wedge \Omega^{(m)\dot{\alpha}(n-1)\dot{\beta}}.$$  

The covariant derivative satisfies the following normalization conditions:

$$D \wedge D \Omega^{(m)\dot{\alpha}(n)} = -2\lambda^2 (E^{\alpha \beta} \wedge \Omega^{(m-1)\beta \dot{\alpha}(n)} + E^{\dot{\alpha} \dot{\beta}} \wedge \Omega^{(m)\dot{\alpha}(n-1)\dot{\beta}})$$

where $E^{\alpha \beta}, E^{\dot{\alpha} \dot{\beta}}$ are basis elements of 2-form spaces and defined below as the double product of $e^{\alpha \dot{\alpha}}$.

Basis elements of 1, 2, 3, 4-form spaces are:

$$e^{\alpha \dot{\alpha}}, \ E_2^{\alpha \beta}, \ E_2^{\dot{\alpha} \dot{\beta}}, \ E_3^{\alpha \dot{\alpha}}, \ E_4^{\dot{\beta}}$$

They are defined as follows:

$$e^{\alpha \dot{\alpha}} \wedge e^{\beta \dot{\beta}} = \epsilon^{\alpha \beta} E^{\alpha \dot{\beta}} + \epsilon^{\dot{\alpha} \dot{\beta}} E^{\dot{\alpha} \dot{\beta}}$$

$$E^{\alpha \beta} \wedge e^{\gamma \dot{\alpha}} = \epsilon^{\alpha \gamma} E^{\beta \dot{\alpha}} + \epsilon^{\beta \alpha} E^{\dot{\alpha} \dot{\gamma}}$$

$$E^{\dot{\alpha} \dot{\beta}} \wedge e^{\gamma \dot{\alpha}} = -\epsilon^{\dot{\alpha} \gamma} E^{\dot{\beta} \dot{\alpha}} - \epsilon^{\dot{\beta} \dot{\alpha}} E^{\alpha \dot{\gamma}}$$

$$E^{\alpha \dot{\alpha}} \wedge e^{\beta \dot{\beta}} = \epsilon^{\alpha \beta} \epsilon^{\dot{\alpha} \dot{\beta}} E$$

so the Hermitian conjugation laws look like

$$(e^{\alpha \dot{\alpha}})^\dagger = e^{\alpha \dot{\alpha}}, \ (E^{(2)})^\dagger = E^{(2)}, \ (E^{\alpha \dot{\alpha}})^\dagger = -E^{\alpha \dot{\alpha}}, \ E^\dagger = -E$$
We also write some useful relations for these basis elements

\[ e^{\alpha}_{\bar{\beta}} \wedge e^{\beta}_{\bar{\delta}} = 2E^{\alpha}_{\bar{\beta}}, \quad e^{\bar{\alpha}}_{\beta} \wedge e^{\bar{\beta}}_{\delta} = 2E^{\bar{\alpha}}_{\bar{\beta}} \]

\[ E^{\alpha}_{\gamma} \wedge e^{\gamma}_{\tilde{\alpha}} = 3E^{\alpha}_{\bar{\beta}}, \quad E^{\bar{\alpha}}_{\bar{\gamma}} \wedge e^{\alpha}_{\tilde{\gamma}} = -3E^{\bar{\alpha}}_{\bar{\beta}} \]

\[ E^{\alpha}_{\beta} \wedge e^{\beta}_{\delta} = 2\epsilon^{\alpha}_{\beta\delta}E, \quad E^{\alpha}_{\beta} \wedge e^{\beta}_{\delta} = 2\epsilon^{\alpha}_{\beta\delta}E \]

\[ E^{\alpha}_{\beta} \wedge E^{\beta}_{\gamma} = 0, \quad E^{\alpha}_{\gamma} \wedge E^{\beta}_{\gamma} = 3\epsilon^{\alpha}_{\beta\gamma}E \]

The spinor indices are raised and lowered with the help of the antisymmetric Lorentz invariant tensors \( e^{\alpha}_{\beta}, e^{\bar{\alpha}}_{\bar{\beta}} \) and inverse \( \epsilon^{\alpha}_{\beta\gamma}, \epsilon^{\alpha}_{\beta\gamma} \) respectively. All the products of p-forms are understood in the sense of wedge-products. Henceforth the sign of wedge product \( \wedge \) will be omitted.

## 2 Massless higher spin models

In this section we provide all necessary description of the massless bosonic and fermionic higher spin fields as well as massless higher spin supermultiplets in the frame-like multispinor formalism used in this work. In what follows they will serve as building blocks for our construction for massive supermultiplets.

### 2.1 Integer spin \( k \)

In the frame-like formulation a massless spin-\( k \) field \( (k \geq 2) \) is described by the physical one-form \( f^{a(k-1)\hat{a}(k-1)} \) and the auxiliary one-forms \( \Omega^{a(k)\hat{a}(k-2)}, \Omega^{a(k-2)\hat{a}(k)} \), being the higher spin generalization of the frame and Lorentz connection in the frame-like formulation of gravity. Locally they are two-component multispinors symmetric on their dotted and undotted spinorial indices separately. These fields satisfy the following reality condition

\[ (f^{a(k-1)\hat{a}(k-1)})^\dagger = f^{a(k-1)\hat{a}(k-1)}, \quad (\Omega^{a(k)\hat{a}(k-2)})^\dagger = \Omega^{a(k-2)\hat{a}(k)} \]  \( (2.1) \)

The free Lagrangian (a four-form in our formalism) for the massless bosonic field in the four-dimensional \( AdS_4 \) space looks like this:

\[ \frac{(-1)^k}{i}L_k = k\Omega^{a(k-1)\hat{a}(k-2)}E^{\gamma\Omega_{a(k-1)\gamma\hat{a}(k-2)}} - (k - 2)\Omega^{a(k)\hat{a}(k-3)\hat{a}}E^{\gamma\Omega_{a(k)\gamma\hat{a}(k-3)\gamma}} + 2\Omega^{a(k-1)\hat{a}(k-2)e_{\beta}}f_{a(k-1)\hat{a}(k-2)\hat{a}} + 2k\lambda^2 f^{a(k-2)\hat{a}(k-1)}E^{\gamma\gamma}f_{a(k-2)\gamma\hat{a}(k-1)} + h.c. \]  \( (2.2) \)

Here and henceforth \( h.c. \) means hermitian conjugate terms defined by rules \( (2.1) \). This Lagrangian is invariant under the following gauge transformations

\[ \delta\Omega^{a(k)\hat{a}(k-2)} = D\eta^{a(k)\hat{a}(k-2)} + e_{\beta}^{\hat{\alpha}}e_{\beta}^{a(k)\hat{a}(k-3)\hat{a}} + \lambda e_{\beta}^{a(k-1)\hat{a}(k-2)\hat{a}} \]

\[ \delta\Omega^{a(k)\hat{a}(k)} = D\eta^{a(k)\hat{a}(k-2)} + e_{\beta}^{\alpha}e_{\beta}^{a(k)\hat{a}(k-3)\hat{a}} + \lambda e_{\beta}^{a(k-2)\hat{a}(k-1)} \]

\[ \delta f^{a(k-1)\hat{a}(k-1)} = D\xi^{a(k-1)\hat{a}(k-1)} + e_{\beta}^{\hat{\alpha}}\eta^{a(k-2)\hat{a}(k-2)\hat{a}} + e_{\beta}^{\alpha}\eta^{a(k-2)\hat{a}(k-1)\hat{a}} \]  \( (2.3) \)
where zero-forms $\zeta^{(k-1)\dot{a}(k-1)}$ and $\eta^{(k)\dot{a}(k-2)}$ are the gauge parameters for the gauge fields $f^{(k-1)\dot{a}(k-1)}$ and $\Omega^{(k)\dot{a}(k-2)}$. The additional gauge parameter $\zeta^{(k+1)\dot{a}(k-3)}$ leads to the introduction of the so-called extra field $\Upsilon^{(k+1)\dot{a}(k-3)} + \h.c.$ which in turn requires introduction of next extra symmetries and so on. The procedure stops at

$$\Upsilon^{(k+t)\dot{a}(k-t-2)}, \quad \Upsilon^{(k-t)\dot{a}(k+t)}, \quad 1 \leq t \leq k-2$$

These extra gauge fields do not enter the free Lagrangian but play an important role in non-linear higher spin theory.

One of the nice features of the frame-like formulation is that for all fields (physical, auxiliary and extra ones) one can construct a gauge invariant two-form (curvature) generalizing curvature and torsion for gravity. For the physical and auxiliary fields they have the form

$$R^{(k)\dot{a}(k-2)} = D\Omega^{(k)\dot{a}(k-2)} + \lambda e^{(k-1)\dot{a}(k-2)\dot{b}} + e^{(k-1)\dot{a}(k-2)\dot{b}} \Upsilon^{(k)\dot{a}(k-3)}$$

$$R^{(k-2)\dot{a}(k)} = D\Omega^{(k-2)\dot{a}(k)} + \lambda e^{(k-2)\dot{a}(k)\dot{b}} + e^{(k-2)\dot{a}(k)\dot{b}} \Upsilon^{(k-3)\dot{a}(k)}$$

$$T^{(k-1)\dot{a}(k-1)} = Df^{(k-1)\dot{a}(k-1)} + e^{(k-1)\dot{a}(k-1)\dot{b}} \Omega^{(k-1)\dot{a}(k-2)\dot{b}} + e^{(k-1)\dot{a}(k-1)\dot{b}} \Omega^{(k-2)\dot{a}(k-1)\dot{b}}$$

(2.5)

In our construction for the massless and massive supermultiplets we use only the physical and auxiliary fields working in the so-called 1 and 1/2 order formalism which is very well known in supergravity. Namely, we do not consider any variations of the auxiliary fields but all calculations are done using the ”zero torsion condition”:

$$T^{(k-1)\dot{a}(k-1)} \approx 0 \quad \Rightarrow \quad e^{(k-1)\dot{a}(k-1)\dot{b}} \Omega^{(k-1)\dot{a}(k-2)\dot{b}} + e^{(k-1)\dot{a}(k-1)\dot{b}} \Omega^{(k-2)\dot{a}(k-1)\dot{b}} \approx 0$$

(2.6)

At the same time the variation of the Lagrangian (2.2) under the arbitrary variations of the physical fields can be written in the following simple form

$$(-1)^{k} \delta \mathcal{L}_{k} = -i2R^{(k)\dot{a}(k-2)} e^{(k-2)\dot{a}(k-2)\dot{b}} \delta f^{(k-1)\dot{a}(k-1)\dot{b}} + \h.c.$$ 

(2.7)

One can see that in (2.6) and (2.7), the curvatures $R$ enter in such combinations that extra gauge field $\Upsilon$ is dropped out, therefore below they will be omitted.

### 2.2 Half-integer spin $k + 1/2$

In the frame-like formulation, the massless spin-$(k+1/2)$ field ($k \geq 1$) is described by physical 1-forms $\Phi^{(k)\dot{a}(k-1)}$, $\Phi^{(k-1)\dot{a}(k)}$. As in the bosonic case, these two-component multispinors are symmetric on their dotted and undotted spinorial indices separately and satisfy the reality condition

$$\left(\Phi^{(k)\dot{a}(k-1)}\right)^\dagger = \Phi^{(k-1)\dot{a}(k)}$$

The free Lagrangian for such fields in $AdS_{4}$ space looks like this:

$$(-1)^{k} \mathcal{L}_{k + \frac{1}{2}} = \Psi_{\alpha(k-1)\beta\dot{a}(k-1)} e^{\alpha\beta} D\Psi^{(k)\dot{a}(k-1)\dot{b}}$$

$$+ d_{k-1}[(k + 1)\Psi_{\alpha(k-1)\beta\dot{a}(k-1)} E^{\beta\gamma} \Psi^{(k-1)\gamma\dot{a}(k-1)}$$

$$-(k - 1)\Psi_{\alpha(k)\dot{a}(k-2)\dot{b}} E^{\beta\gamma} \Psi^{(k)\dot{a}(k-2)\dot{b}} + \h.c.]$$

(2.8)
and is invariant under the following gauge transformation

\[
\delta \Psi^{\alpha(k)\dot{\alpha}(k-1)} = D\xi^{\alpha(k)\dot{\alpha}(k-1)} + e_{\beta}^\alpha \eta^{\alpha(k)\beta\dot{\alpha}(k-2)} + 2d_{k-1}e_{\beta}^\alpha \xi^{\alpha(k-1)\dot{\alpha}(k-1)\dot{\beta}}
\]

\[
\delta \Psi^{\alpha(k-1)\dot{\alpha}(k)} = D\xi^{\alpha(k-1)\dot{\alpha}(k)} + e_{\beta}^\alpha \eta^{\alpha(k-2)\dot{\alpha}(k)\dot{\beta}} + 2d_{k-1}e_{\beta}^\alpha \xi^{\alpha(k-1)\dot{\alpha}(k-1)}
\]

(2.9)

where

\[
d_{k-1} = \pm \frac{\lambda}{2}
\]

Note that the transformations with the gauge parameters \(\eta^{\alpha(k+1)\dot{\alpha}(k-2)}, \eta^{\alpha(k-2)\dot{\alpha}(k+1)}\) lead to the introduction of extra fields that play the same role as in the bosonic case and do not enter the free Lagrangian. Up to these extra fields the gauge invariant curvatures for the physical fermionic fields have the following form

\[
F^{\alpha(k)\dot{\alpha}(k-1)} = D\Psi^{\alpha(k)\dot{\alpha}(k-1)} + 2d_{k-1}e_{\beta}^\alpha \Psi^{\alpha(k-1)\dot{\alpha}(k-1)\dot{\beta}}
\]

\[
F^{\alpha(k-1)\dot{\alpha}(k)} = D\Psi^{\alpha(k-1)\dot{\alpha}(k)} + 2d_{k-1}e_{\beta}^\alpha \Psi^{\alpha(k-1)\dot{\alpha}(k-1)}
\]

(2.10)

The variation of the free Lagrangian (2.8) under the arbitrary variations of the physical fields can be written as

\[
(-1)^k \delta \mathcal{L}_{k+\frac{1}{2}} = -F^{\alpha(k-1)\dot{\alpha}(k-1)}e_{\beta}^\alpha \delta \Psi^{\alpha(k-1)\dot{\alpha}(k-1)\dot{\beta}} + h.c.
\]

(2.11)

Here the curvature \(F\) also enters in such a combination with the frame \(e\) that the extra fields drop out.

In the following part of this section we combine massless higher spin fermionic and bosonic fields in the \(N = 1\) supermultiplet and construct an explicit form of the supertransformations leaving the sum of the free Lagrangians invariant.

### 2.3 Supermultiplet \((k + 1/2, k)\)

This supermultiplet contains higher half-integer spin \(k + 1/2\) and integer spin \(k\). They are described by \((\Phi^{(k)\dot{\alpha}(k-1)}, h.c.)\) and \((f^{(k-1)\dot{\alpha}(k-2)}, \Omega^{(k)\dot{\alpha}(k-2)}, h.c.)\) respectively. We choose an ansatz for the supertransformations in the following form (as it was already mentioned, we consider supertransformations for the physical fields only):

\[
\delta f^{\alpha(k-1)\dot{\alpha}(k-1)} = \alpha_{k-1} \Phi^{(k-1)\dot{\beta}(k-2)} - \bar{\alpha}_{k-1} \Phi^{(k-1)\dot{\beta}(k-1)}
\]

\[
\delta \Phi^{\alpha(k)\dot{\alpha}(k-1)} = \beta_{k-1} \Omega^{(k)\dot{\alpha}(k-2)} + \bar{\gamma}_{k-1} f^{(k-1)\dot{\alpha}(k-1)}
\]

\[
\delta f^{\alpha(k-1)\dot{\alpha}(k)} = \bar{\beta}_{k-1} \Omega^{(k-2)\dot{\alpha}(k)} + \gamma_{k-1} f^{(k-1)\dot{\alpha}(k-1)}
\]

(2.12)

were we assume that the coefficients \(\alpha_k, \beta_k, \gamma_k\) are complex. The parameters of the supertransformation \(\zeta^\alpha, \zeta^{\dot{\alpha}}\) in \(AdS_4\) satisfy the relations

\[
D\zeta^\alpha = -\lambda e_{\beta}^\alpha \zeta^{\dot{\beta}} , \quad D\zeta^{\dot{\alpha}} = -\lambda e_{\dot{\beta}}^{\dot{\alpha}} \zeta^{\beta}
\]

(2.13)

Using the expressions for Lagrangian variations (2.7) and (2.11) as well as on-shell identity (2.6) the variation for the sum of the bosonic and fermionic Lagrangians can be written as
follows:

\[ \begin{align*}
(-1)^k \delta (L_k + L_{k+1}) &= 4i \alpha_{k-1} \Phi \alpha(k-2)_{\beta} \gamma \alpha(k-1) e^\gamma \mathcal{R} \alpha(k-2)_{\alpha} \dot{\alpha}(k-1) \gamma \zeta^\beta \\
&\quad - (k - 1) \bar{\alpha}_{k-1} \mathcal{F} \alpha(k-1)_{\beta} \bar{\alpha}(k-1) e^\beta \mathcal{R} \alpha(k-2)_{\alpha} \dot{\alpha}(k-1) \beta \zeta^\alpha \\
&\quad - \bar{\gamma}_{k-1} \mathcal{F} \alpha(k-1)_{\beta} \bar{\alpha}(k-1) e^\beta \bar{\alpha} \alpha(k-1) \zeta^\beta \\
&\quad + (k - 1) \mathcal{F} \alpha(k-1)_{\beta} \bar{\alpha}(k-1) e^\beta \bar{\alpha} \alpha(k-1) \dot{\alpha}(k-2) \gamma \zeta^\dot{\alpha} + h.c. 
\end{align*} \]

Note that the invariance of the Lagrangian under the supertransformations can be achieved up to the total derivative only and this leads to a number of useful identities. For example, let us consider

\[ \begin{align*}
0 &\approx D \Phi \alpha(k-1)_{\beta} \bar{\alpha}(k-1) e^\beta \mathcal{R} \alpha(k-2)_{\alpha} \dot{\alpha}(k-1) \beta \zeta^\gamma \\
&\quad + \Phi \alpha(k-2)_{\beta} \gamma \alpha(k-1) e^\beta \mathcal{R} \alpha(k-2)_{\alpha} \dot{\alpha}(k-1) \beta \zeta^\gamma \\
&\quad - 2b_{k-1} \left[ (k + 1) E_\beta \gamma \Psi \alpha(k-2)_{\gamma} \alpha(k-1) \gamma \Omega \alpha(k-2)_{\alpha} \dot{\alpha}(k-1) \beta \zeta^\gamma \\
&\quad - (k - 2) E_\beta \gamma \Psi \alpha(k-3)_{\gamma} \alpha(k-1) \gamma \Omega \alpha(k-2)_{\alpha} \dot{\alpha}(k-1) \beta \zeta^\gamma \\
&\quad + \lambda^2 [(k + 1) E_\beta \gamma \Phi \alpha(k-2)_{\gamma} \alpha(k-1) \gamma \Omega \alpha(k-2)_{\alpha} \dot{\alpha}(k-1) \beta \zeta^\gamma \\
&\quad + \lambda E_\gamma \Phi \alpha(k-1)_{\beta} \bar{\alpha}(k-1) \Omega \alpha(k-2)_{\alpha} \dot{\alpha}(k-1) \beta \zeta^\beta] 
\end{align*} \]

Using the explicit expressions for the bosonic (2.3) and fermionic (2.10) curvatures as well as relation (2.13) we obtain:

\[ \begin{align*}
0 &= \mathcal{F} \alpha(k-1)_{\beta} \bar{\alpha}(k-1) e^\beta \mathcal{R} \alpha(k-2)_{\alpha} \dot{\alpha}(k-1) \beta \zeta^\gamma \\
&\quad - 2b_{k-1} \left[ (k + 1) E_\beta \gamma \Psi \alpha(k-2)_{\gamma} \alpha(k-1) \gamma \Omega \alpha(k-2)_{\alpha} \dot{\alpha}(k-1) \beta \zeta^\gamma \\
&\quad - (k - 2) E_\beta \gamma \Psi \alpha(k-3)_{\gamma} \alpha(k-1) \gamma \Omega \alpha(k-2)_{\alpha} \dot{\alpha}(k-1) \beta \zeta^\gamma \\
&\quad - \lambda^2 [(k + 1) E_\beta \gamma \Phi \alpha(k-2)_{\gamma} \alpha(k-1) \gamma \Omega \alpha(k-2)_{\alpha} \dot{\alpha}(k-1) \beta \zeta^\gamma \\
&\quad + \lambda E_\gamma \Phi \alpha(k-1)_{\beta} \bar{\alpha}(k-1) \Omega \alpha(k-2)_{\alpha} \dot{\alpha}(k-1) \beta \zeta^\beta] 
\end{align*} \]

Similarly, if one considers two relations:

\[ \begin{align*}
0 &\approx D \Phi \alpha(k-1)_{\beta} \bar{\alpha}(k-1) e^\beta \Phi \alpha(k-1) \gamma \alpha(k-1) \beta \zeta^\gamma \\
0 &\approx D \Phi \alpha(k-1)_{\beta} \bar{\alpha}(k-2) \gamma \alpha(k-1) \beta \zeta^\gamma 
\end{align*} \]

then using the explicit expression for the fermionic curvature as well as zero torsion condition one obtains:

\[ \begin{align*}
0 &= \mathcal{F} \alpha(k-1)_{\beta} \bar{\alpha}(k-1) e^\beta \Phi \alpha(k-1) \gamma \alpha(k-1) \beta \zeta^\gamma \\
&\quad - 2b_{k-1} \left[ (k + 1) E_\beta \gamma \Psi \alpha(k-2)_{\gamma} \alpha(k-1) \gamma \Phi \alpha(k-1) \beta \zeta^\gamma \\
&\quad - (k - 1) E_\beta \gamma \Psi \alpha(k-2)_{\gamma} \alpha(k-1) \gamma \Phi \alpha(k-1) \beta \zeta^\gamma \\
&\quad - \lambda^2 [(k + 1) E_\beta \gamma \Phi \alpha(k-2)_{\gamma} \alpha(k-1) \gamma \Phi \alpha(k-1) \beta \zeta^\gamma \\
&\quad + \lambda E_\gamma \Phi \alpha(k-1)_{\beta} \bar{\alpha}(k-1) \Omega \alpha(k-2)_{\alpha} \dot{\alpha}(k-1) \beta \zeta^\beta] 
\end{align*} \]
0 = \mathcal{F}_{\alpha(k-1)\beta\alpha(k-2)\gamma} e^\beta_{\delta} f^{\alpha(k-1)\beta\alpha(k-2)\delta}_{\gamma} \\
-2b_{k-1}[(k+1)E^\delta_{\delta}\Phi^{\alpha(k-1)\beta\alpha(k-2)\gamma}_{\delta} f^{\alpha(k-1)\beta\alpha(k-2)\delta}_{\gamma} \\
-(k-1)E^\delta_{\delta}\Phi^{\alpha(k-2)\alpha(k-1)\gamma}_{\delta} f^{\alpha(k-1)\alpha(k-2)\delta}_{\gamma}] \\
-\frac{3}{2}E^\delta_{\delta}\Phi^{\alpha(k-1)\gamma\alpha(k-2)\gamma}_{\delta} \Omega^{\alpha(k-1)\beta\alpha(k-2)} - (k-2)E^\delta_{\delta}\Phi^{\alpha(k-1)\gamma\alpha(k-2)\gamma}_{\delta} \Omega^{\alpha(k-1)\beta\alpha(k-2)}] \zeta^\gamma \\
+\lambda[E^\delta_{\delta}\Phi^{\alpha(k-1)\gamma\alpha(k-2)\gamma}_{\delta} f^{\alpha(k-1)\gamma\alpha(k-2)\gamma}_{\delta} - E^\delta_{\delta}\Phi^{\alpha(k-1)\gamma\alpha(k-2)\gamma}_{\delta} f^{\alpha(k-1)\gamma\alpha(k-2)\gamma}_{\delta}] \\

Using these identities we obtain from the requirement \(\delta(\mathcal{L}_k + \mathcal{L}_{k+\frac{1}{2}}) = 0:\)

\[\alpha_{k-1} = \frac{i}{4}(k-1)\beta_{k-1}, \quad \gamma_{k-1} = \lambda\beta_{k-1}, \quad 2d_{k-1}\beta_{k-1} = \lambda\beta_{k-1} \quad (2.15)\]

The solution of the last relation depends on the sign of \(d_{k-1} = \pm \frac{1}{2}\). The parameter \(\beta_k\) is real for the "+" sign and imaginary for the "-". These two solutions correspond to the parity-even and parity-odd bosonic fields entering the supermultiplet. This fact will be important for the construction of massive supermultiplets where two bosonic fields must have opposite parities.

### 2.4 Supermultiplet \((k, k - 1/2)\)

This supermultiplet contains higher integer spin \(k\) and half-integer spin \(k - 1/2\). They are described by \((f^{\alpha(k-1)\beta\alpha(k-2)}, \Omega^{\alpha(k-1)\beta\alpha(k-2)}, h.c.)\) and \((\Phi^{\alpha(k-1)\beta\alpha(k-2)}, h.c.)\) respectively. We choose an ansatz for the supertransformations in the following form

\[
\begin{align*}
\delta f^{\alpha(k-1)\beta\alpha(k-1)} &= \alpha_{k-1}' \Phi^{\alpha(k-1)\beta\alpha(k-2)} \zeta^\alpha - \bar{\alpha}_{k-1}' \Phi^{\alpha(k-2)\beta\alpha(k-1)} \zeta^\alpha \\
\delta \Phi^{\alpha(k-1)\beta\alpha(k-2)} &= \beta_{k-1}' \Omega^{\alpha(k-1)\beta\alpha(k-2)} \zeta^\beta + \gamma_{k-1}' f^{\alpha(k-1)\beta\alpha(k-2)} \zeta^\gamma \\
\delta \Omega^{\alpha(k-2)\beta\alpha(k-1)} &= \bar{\beta}_{k-1}' \Omega^{\alpha(k-2)\beta\alpha(k-1)} \zeta^\beta + \bar{\gamma}_{k-1}' f^{\alpha(k-2)\beta\alpha(k-1)} \zeta^\gamma
\end{align*}
\]

Here in the most general case, coefficients \(\alpha_{k}', \beta_{k}', \gamma_{k}'\) are complex. Using the expressions for Lagrangian variations (2.7) and (2.11) as well as on-shell relation (2.6) we get

\[
(-1)^k(\mathcal{L}_k + \mathcal{L}_{k+\frac{1}{2}}) = -4i(k-1)\alpha_{k-1}' \Phi^{\alpha(k-1)\beta\alpha(k-2)} e^\beta_{\delta} R^{\alpha(k-2)\beta\alpha(k-2)} \zeta^\gamma \\
+ \beta_{k-1}' f^{\alpha(k-2)\gamma\alpha(k-2)} e^\gamma_{\delta} \Omega^{\alpha(k-1)\beta\beta\alpha(k-2)} \zeta^\delta \\
+ \gamma_{k-1}' f^{\alpha(k-2)\gamma\alpha(k-2)} e^\gamma_{\delta} f^{\alpha(k-2)\beta\alpha(k-2)} \zeta^\beta + h.c.
\]

As in previous case from two relations

\[
0 \approx D[\Phi^{\alpha(k-2)\gamma\alpha(k-2)} e^\gamma_{\delta} \Omega^{\alpha(k-2)\beta\alpha(k-2)} \zeta^\delta] \\
0 \approx D[\Phi^{\alpha(k-2)\gamma\alpha(k-2)} e^\gamma_{\delta} f^{\alpha(k-2)\beta\alpha(k-2)} \zeta^\beta]
\]
one can derive two identities:

\[ 0 = \mathcal{F}_{\alpha(k-2)\gamma\dot{\alpha}(k-2)} e^{\gamma} \Omega^{\alpha(k-2)\dot{\alpha}(k-2)\dot{\gamma}} \zeta_{\dot{\beta}} + \Phi_{\alpha(k-2)\gamma\dot{\alpha}(k-2)} e^{\gamma} \bar{\mathcal{R}}^{\alpha(k-2)\dot{\alpha}(k-2)\dot{\gamma}} \zeta_{\dot{\beta}} \]

\[ + 2b_{k-2}[k E_{\dot{\gamma}}^\beta \Psi^{\alpha(k-2)\dot{\alpha}(k-2)\dot{\beta}} \Omega^{\alpha(k-2)\dot{\alpha}(k-2)\dot{\beta}} \zeta_{\dot{\beta}}] \]

\[ - (k - 2) E_{\gamma}^{\beta} \Psi^{\alpha(k-3)\gamma\dot{\alpha}(k-1)} \Omega^{\alpha(k-3)\dot{\alpha}(k-1)\dot{\beta}} \zeta_{\dot{\beta}} \]

\[ - \lambda^2[-(k - 2) E_{\gamma}^{\beta} \Phi_{\alpha(k-1)\dot{\alpha}(k-2)} f^{\alpha(k-1)\dot{\alpha}(k-3)\dot{\beta}} + (k + 1) E_{\gamma}^{\beta} \Phi_{\alpha(k-2)\gamma\dot{\alpha}(k-2)} f^{\alpha(k-2)\dot{\beta}(k-2)\dot{\beta}}] \]

\[ - E_{\gamma}^{\beta} \Phi_{\alpha(k-1)\dot{\alpha}(k-2)} f^{\alpha(k-1)\dot{\alpha}(k-2)\dot{\gamma}} \zeta_{\dot{\beta}} \]

\[ + \lambda[E_{\beta}^{\dot{\gamma}} \Phi_{\alpha(k-2)\gamma\dot{\alpha}(k-2)} \Omega^{\alpha(k-2)\dot{\alpha}(k-2)\dot{\beta}} \zeta_{\dot{\gamma}}] \]

Then the invariance of the Lagrangian under the supertransformations requires that

\[ \alpha'_{k-1} = \frac{i}{4(k - 1)} \beta'_{k-1}, \quad \gamma'_{k-1} = \lambda \beta'_{k-1}, \quad 2d_{k-1} \beta'_{k-1} = \lambda \beta'_{k-1} \]  \hspace{1cm} (2.17)

As in the previous case, we see from last relation that parameter \( \beta'_k \) can be real or imaginary. It depends on the sign of \( d_{k-1} = \pm \frac{\lambda}{2} \) and is related to the parity of the fields entering the supermultiplet.

## 3 Massive higher spin fields

In this section we provide frame-like gauge invariant formulation for massive arbitrary integer and half-integer spins \( \text{SS} \) but with the multispinor formalism used for all local indices.

### 3.1 Integer spin \( s \)

In the gauge invariant formalism a massive integer spin-\( s \) field is described by a set of massless fields with spins \( 0 \leq k \leq s \). Frame-like formulation of massive bosonic fields with spins \( k \geq 2 \) were considered above, they are described by one-forms \( f^{\alpha(k-1)\dot{\alpha}(k-1)}, \Omega^{\alpha(k)\dot{\alpha}(k-2)} + h.c. \) while massless spin-1 is described by the physical one-form \( A \) and auxiliary zero-forms \( B^{\alpha(2)}, \bar{B}^{\dot{\alpha}(2)} \), and massless spin-0 is described by physical zero-form \( \varphi \) and auxiliary zero-form \( \pi^{\alpha\dot{\alpha}} \).

The gauge invariant Lagrangian for the massive bosonic field has the form:

\[ \mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{cross}} + \mathcal{L}_{\text{mass}} \]  \hspace{1cm} (3.1)
\[ \frac{1}{i} \mathcal{L}_{\text{kin}} = \sum_{k=2}^{s} (-1)^k \mathcal{L}_k + 4EB_{\alpha(2)}B^{\alpha(2)} + 2E_{\alpha(2)}B^{\alpha(2)}DA + h.c. \]

\[ -6E\pi_{\alpha\bar{\alpha}}\pi^{\alpha\bar{\alpha}} - 12E_{\alpha\bar{\alpha}}\pi^{\alpha\bar{\alpha}}D\varphi \]

\[ \frac{1}{i} \mathcal{L}_{\text{cross}} = \sum_{k=3}^{s} (-1)^{k+1} a_k [E_{\beta(2)}\Omega^{\alpha(k-2)\beta(2)}\dot{\alpha}(k-2)f_{\alpha(k-2)}\dot{\alpha}(k-2) \]

\[ + \frac{(k-2)}{k} E_{\beta(2)}f^{\alpha(k-3)\beta(2)}\dot{\alpha}(k-1)\Omega_{\alpha(k-3)}\dot{\alpha}(k-1) + h.c. ] \]

\[ + a_0 [\Omega^{(2)}E_{\alpha(2)}A - 2B^{\beta\alpha}E_{\beta}\dot{f}_{\alpha\bar{\beta}} + h.c. ] + \bar{a}_0 E_{\alpha\bar{\alpha}}\pi^{\alpha\bar{\alpha}}A \]

\[ \frac{1}{i} \mathcal{L}_{\text{mass}} = \sum_{k=2}^{s} (-1)^k b_k [f^{\alpha(k-2)\beta\dot{\alpha}(k-1)}E_{\beta}^{\gamma}f_{\alpha(k-2)}\gamma\dot{\alpha}(k-1) + h.c. ] \]

\[ + \frac{a_0\bar{a}_0}{2} E_{\alpha\bar{\alpha}}f^{\alpha\bar{\alpha}}\dot{\varphi} + 3a_0^2 E\varphi^2 \]

where

\[ b_k = \frac{2s(s+1)}{k(k-1)(k+1)}M^2, \quad M^2 = m^2 + s(s-1)\lambda^2 \]

\[ a_k^2 = \frac{4(s-k+1)(s+k)}{(k-1)(k-2)}[M^2 - k(k-1)\lambda^2] \]

\[ a_0^2 = 2(s-1)(s+2)[M^2 - 2\lambda^2], \quad \bar{a}_0^2 = 24s(s+1)M^2 \]

Here \( \mathcal{L}_{\text{kinetic}} \) is just the sum of kinetic terms for all fields, that for \( k \geq 2 \) were defined in (2.2). \( \mathcal{L}_{\text{mass}} \) is the sum of the mass terms for them, while \( \mathcal{L}_{\text{cross}} \) contains cross-terms gluing all these fields together. In what follows we assume that all parameters \( a_k, a_0, \bar{a}_0 \) are positive.

Explicit form of the coefficients (3.2) are determined by the invariance of the Lagrangian (3.1) under the following gauge transformations

\[ \delta f^{\alpha(k-1)\dot{\alpha}(k-1)} = D\xi^{\alpha(k-1)\dot{\alpha}(k-1)} + e_\beta^{\dot{\alpha}(k-1)\dot{\beta}(k-2)} + e^{\alpha\beta}\eta^{\alpha(k-2)\dot{\alpha}(k-1)\dot{\beta}} \]

\[ + \frac{(k-1)a_{k+1}}{2(k+1)} e^{\alpha\beta}\eta^{\alpha(k-1)\dot{\alpha}(k-1)\dot{\beta}} + \frac{a_k}{2k(k-1)} e^{\alpha\bar{\alpha}}\xi^{\alpha(k-2)\dot{\alpha}(k-2)} \]

\[ \delta \Omega^{\alpha(k),\dot{\alpha}(k-2)} = D\eta^{\alpha(k),\dot{\alpha}(k-2)} + \frac{a_{k+1}}{2} e^{\alpha\beta}\eta^{\alpha(k)\dot{\alpha}(k-2)\dot{\beta}} + \frac{a_k}{2k(k+1)} e^{\alpha\bar{\alpha}}\eta^{\alpha(k-1)\dot{\alpha}(k-3)} \]

\[ + \frac{b_k}{2k} e^{\alpha\beta}\eta^{\alpha(k-1)\dot{\alpha}(k-2)\dot{\beta}} \]

\[ \delta f^{\alpha\dot{\alpha}} = D\xi^{\alpha\dot{\alpha}} + e_\beta^{\dot{\alpha}\dot{\beta}}\eta^{\alpha\dot{\beta}} + e^{\alpha\beta}\eta^{\dot{\alpha}\dot{\beta}} + \frac{a_3}{6} e^{\alpha\beta}\eta^{\alpha\dot{\beta}} - \frac{a_0}{4} e^{\alpha\dot{\alpha}}\xi \]

\[ \delta B^{\alpha(2)} = \frac{a_0}{2} \eta^{\alpha(2)}, \quad \delta A = D\xi - \frac{a_0}{2} e^{\alpha\dot{\alpha}}\xi \]

\[ \delta \pi^{\alpha\dot{\alpha}} = -\frac{a_0\bar{a}_0}{24} \xi^{\alpha\dot{\alpha}}, \quad \delta \varphi = \frac{\bar{a}_0}{12} \]

Compared to the massless case in the previous section, one can see that we still have all the gauge symmetries that our massless fields possessed modified so as to be consistent with the structure of the massive Lagrangian. Such gauge invariant formulation of the massive
Secondly, in anti-de Sitter space when \( \lambda^2 > 0 \) there is a correct massless limit \( m \to 0 \). Without the gap in the number of physical degrees of freedom. In such a limit our system decomposes into two systems describing the massless spin-\( s \) and the massive spin-(\( s - 1 \)) fields. Lastly, in de Sitter space when \( \lambda^2 < 0 \) one can consider the so-called partially massless limits \( a_k \to 0 \). In such a limit, the system decomposes into the two subsystems describing the partially massless spin-\( s \) field and the massive spin-\( k \) field.

As in the massless case, to construct a complete set of the gauge invariant objects one has to introduce a lot of extra fields which do not, however, enter the free Lagrangian. In the following, we restrict ourselves to the curvatures for the physical and auxiliary fields only.

With the explicit expressions for the gauge transformations at our disposal (3.3), it is rather straightforward to obtain (we omit all terms with the extra fields):

\[
\begin{align*}
\mathcal{T}^{\alpha(k-1)\dot{\alpha}(k-1)} &= Df^{\alpha(k-1)\dot{\alpha}(k-1)} + e_\beta \dot{\alpha} \Omega^{\alpha(k-1)\beta}(k-2) + e^\alpha_\beta \Omega^{\alpha(k-2)\dot{\alpha}(k-1)}
+ \frac{(k-1)a_{k+1}}{2(k+1)}e_\beta \dot{\alpha} f^{\alpha(k-1)\beta}(k-1)\dot{\alpha} + \frac{a_k}{2k(k-1)}e^{\alpha \dot{\alpha}} f^{\alpha(k-2)\dot{\alpha}}(k-2)
\mathcal{R}^{\alpha(k),\dot{\alpha}(k-2)} &= D\Omega^{\alpha(k),\dot{\alpha}(k-2)} + \frac{a_{k+1}}{2}e_\beta \dot{\alpha} \Omega^{\alpha(k)\beta}(k-2)\dot{\alpha} + \frac{a_k}{2k(k+1)}e^{\alpha \dot{\alpha}} \Omega^{\alpha(k-1)\dot{\alpha}(k-3)}
+ \frac{b_k}{2k}e^{\alpha \dot{\alpha}} f^{\alpha(k-1)\dot{\alpha}(k-2)\dot{\alpha}}
\mathcal{R}^{(2)} &= D\Omega^{(2)} + \frac{a_2}{2}e_\beta \dot{\alpha} \Omega^{(2)\beta}\dot{\alpha} + \frac{b_2}{2}e^\alpha_\beta f^{\alpha\beta}\dot{\alpha} + \frac{a_0}{4} E^\alpha_\beta B^{\alpha\beta} + \frac{a_0}{24} E^{a\dot{a}}(2) \varphi
\mathcal{T}^{\alpha\dot{\alpha}} &= Df^{a\dot{a}} + e_\beta \dot{\alpha} \Omega^{\alpha\beta}(k-2)\dot{\alpha} + e^\alpha_\beta \Omega^{\alpha(k-2)\dot{\alpha}(k-1)\dot{\alpha}}
+ \frac{a_0}{4} E^\alpha_\beta B^{\alpha\beta} + \frac{a_0}{24} e^{a\dot{a}}(2) \varphi
\mathcal{C}^{a\dot{a}} &= D\pi^{a\dot{a}} + \frac{a_0}{12} f^{a\dot{a}} - \frac{\tilde{a}_0}{12} (e_\beta \dot{\alpha} B^{\alpha\beta} + e^\alpha_\beta B^{\alpha\beta}) + \frac{a_0^2}{8} e^{a\dot{a}} \varphi
\mathcal{C} &= D\varphi + e_{a\dot{a}} \pi^{a\dot{a}} - \frac{\tilde{a}_0}{12} A
\end{align*}
\]

In our construction of the massive supermultiplets we will consider supertransformations for the physical fields only. However, in all calculations we will heavily use the auxiliary fields equations (on-shell conditions) as well as corresponding algebraic identities:

\[
\begin{align*}
\mathcal{T}^{\alpha(k-1)\dot{\alpha}(k-1)} &\approx 0 \implies e_\beta \dot{\alpha} \mathcal{R}^{\alpha(k-1)\beta}(k-2) + e^\alpha_\beta \mathcal{R}^{\alpha(k-2)\dot{\alpha}(k-1)\dot{\alpha}} \approx 0
\mathcal{R} &\approx 0 \implies E_{a(2)} \mathcal{C}^{a(2)} + E_{\dot{a}(2)} \mathcal{C}^{\dot{a}(2)} \approx 0
\mathcal{C} &\approx 0 \implies e_{a\dot{a}} \mathcal{C}^{a\dot{a}} \approx 0 \implies E^\alpha_\gamma \mathcal{C}^{\beta\gamma} \approx \frac{1}{2} e^{a\beta} E_{\gamma\gamma} \mathcal{C}^{a\dot{a}}
\end{align*}
\]

The variation of the Lagrangian (3.1) under the arbitrary variations for the physical fields
takens the simple form

\[
\delta \mathcal{L} = -2i \sum_{k=2}^{s} (-1)^k R^{\alpha(k-1)\beta \hat{\alpha}(k-2)} e^\beta \delta f_{\alpha(k-1)\hat{\alpha}(k-2)} + 2i E_{\alpha(2)} C^{\alpha(2)} \delta A + 12i E_{\alpha\beta} C^{\alpha\hat{\beta}} \delta \varphi + h.c. \tag{3.6}
\]

Let us stress once again that this expression is such that all extra fields drop out.

### 3.2 Half-integer spin \(s + 1/2\)

In the gauge invariant formalism, the massive half-integer spin-\(s + 1/2\) field is described by the set of massless fields with spins \(1/2 \leq k + 1/2 \leq s + 1/2\). Frame-like formulation for the massless fermionic fields with spins \((k \geq 1)\) were considered above, they are described by the one-forms \((\Phi_{\alpha(k)}^{\hat{\alpha}(k-1)}, h.c.)\), while massless spin-1/2 is described by a physical zero-form \((\phi^\alpha, h.c.)\). The Lagrangian for free massive field in \(AdS_4\) have the form

\[
\mathcal{L} = \sum_{k=1}^{s} (-1)^k \Phi_{\alpha(k-1)\hat{\alpha}(k-1)} e^\beta D\Phi^{\alpha(k-1)\hat{\alpha}(k-1)} - \phi^\alpha E^\alpha_{\hat{\alpha}} D\phi^{\hat{\alpha}} + \sum_{k=2}^{s} (-1)^k c_k \left[ E^{\beta(2)}\Phi_{\alpha(k-2)\hat{\alpha}(2)} \Phi^{\alpha(k-2)\hat{\alpha}(k-1)} \right] + c_0 \Phi^\alpha E^\alpha_{\hat{\alpha}} D\phi^{\hat{\alpha}} + h.c. + \sum_{k=1}^{s} (-1)^k d_k \left[ (k + 1) \Psi_{\alpha(k-1)\hat{\alpha}(k-1)} e^\beta \Psi^{\alpha(k-1)\hat{\alpha}(k-1)} \right] + 2d_1 E\phi^\alpha \phi^\alpha + h.c. \tag{3.7}
\]

where

\[
d_k = \pm \frac{(s + 1)}{2(k + 1)} M_1, \quad M_1^2 = m_1^2 + s^2 \lambda^2, \\
c_k^2 = \frac{(s - k + 1)(s + k + 1)}{k^2} [M_1^2 - k^2 \lambda^2], \tag{3.8} \\
c_0^2 = 2s(s + 2)[M_1^2 - \lambda^2].
\]

In the following, we assume that the parameters \(c_k, c_0, M_1\) are positive.

Explicit form of the coefficients \(3.8\) are determined by the invariance of the Lagrangian under the following gauge transformations

\[
\delta \Phi^{\alpha(k)}_{\hat{\alpha}(k-1)} = D_{\xi}^{\alpha(k)}_{\hat{\alpha}(k-1)} + e_\beta^{\hat{\alpha}} q^{\alpha(k)\beta\hat{\alpha}(k-2)} + 2d_k e^\alpha_{\hat{\beta}} c^{\alpha(k-1)\hat{\beta}(k-1)} + c_k e_{\beta}^{\hat{\alpha}} c^{\alpha(k)\hat{\beta}(k-1)} + \frac{c_k}{(k - 1)(k + 1)} e^{\alpha\hat{\beta}} c^{\alpha(k-1)\hat{\beta}(k-2)} \tag{3.9}
\]

\[
\delta \phi^\alpha = c_0 \xi^\alpha
\]

The general structure of the Lagrangian \(3.7\) is the same as in the bosonic case. The first line is the sum of kinetic terms, the second line contains cross-terms and the last two lines are mass terms. In such a formulation we can take the correct massless limit \(m_1 \to 0\) in \(AdS\).
\[(\lambda^2 > 0)\] and the correct partially massless limits \(c_k \to 0\) in \(dS\) \((\lambda^2 < 0)\). Taking a flat limit \(\lambda \to 0\) we obtain the description of the massive fermionic fields in Minkowski space.

As in the bosonic case, we restrict ourselves with the gauge invariant curvatures for the physical fields only omitting all the extra fields:

\[
F^{\alpha} = D\Phi^{\alpha} + 2d_1e^{\alpha}_\beta \Phi^{\beta} + c_2e^{\alpha}_\beta \Phi^{\alpha\beta} - \frac{c_0}{3}E^{\alpha}_\beta \epsilon^{\beta} + h.c. \tag{3.10}
\]

The variation of the Lagrangian \((3.7)\) under the arbitrary variations of the physical fields has the following form

\[
\delta L = -\sum_{k=1}^{s}(-1)^k F^{\alpha(k-1)}\epsilon^{\beta} \delta \Phi^{\alpha(k-1)} - C^{\alpha}E^{\alpha}_\beta \delta \epsilon^{\beta} + h.c. \tag{3.11}
\]

### 4 Massive higher spin superblocks

There are two types of massive \(N = 1\) supermultiplets, each one containing two massive bosonic fields (with opposite parities) and two massive fermionic ones:

To provide an explicit realization of such supermultiplets one has to find supertransformations connecting each bosonic field with each fermionic field so that: 1) the sum of the four free Lagrangians for these fields is invariant; 2) the algebra of the supertransformations is closed. In this work we use the following strategy. Firstly, for each pair of bosonic and fermionic fields (we call it superblock in what follows) we find the supertransformations leaving the sum of their two Lagrangians invariant. Then we combine all four fields together and adjust parameters of these superblocks so that the algebra of the supertransformations is closed. One can see from the diagrams above that there are only two non-trivial superblocks, namely \((s, s + 1/2)\) and \((s - 1/2, s)\). Such a strategy therefore greatly simplifies the whole construction.

In the gauge invariant formalism the description of massive higher spin fields is constructed out of the appropriately chosen set of massless ones. It seems natural to expect that one can construct a description of massive higher spin supermultiplet out of an appropriately chosen set of massless ones. Indeed, if one decomposes all four massive fields into
their massless components, the resulting spectrum of massless components does correspond to some set of massless supermultiplets. However, the explicit structure of the supertransformations (see below) shows that all massless components still remain connected with all their neighbours so that the whole system looks just like one big massless supermultiplet (similarly to what we obtained in the three dimensional case [40]):

\[
\begin{align*}
\Phi_{k-\frac{1}{2}}, \Psi_{k-\frac{1}{2}} & \quad \Phi_{k+\frac{1}{2}}, \Psi_{k+\frac{1}{2}} \\
\Phi'_{k-1}, \Psi'_{k} & \quad \Phi'_{k+1}, \Psi'_{k+1}
\end{align*}
\]

One can introduce new fermionic variables:

\[
\begin{align*}
\tilde{\Phi}_k &= \cos \Theta_k \Phi_k + \sin \Theta_k \Psi_k \\
\tilde{\Psi}_k &= -\sin \Theta_k \Phi_k + \cos \Theta_k \Psi_k
\end{align*}
\]

and adjust mixing angles \(\Theta_k\) so that the whole system decomposes into the sum of massless supermultiplets containing two bosonic and two fermionic fields:

\[
\begin{align*}
\Phi_{k-\frac{1}{2}}, \Psi_{k-\frac{1}{2}} & \quad \Phi_{k+\frac{1}{2}}, \Psi_{k+\frac{1}{2}} \\
\Phi'_{k-1}, \Psi'_{k} & \quad \Phi'_{k+1}, \Psi'_{k+1}
\end{align*}
\]

The separation of these supermultiplets into the usual pairs is impossible because the bosonic fields have opposite parities. However in this case the structure of cross and mass-like terms in the fermionic Lagrangian cease to be diagonal making the construction of massive supermultiplets more complicated. Note that it is this approach that was used in the previous works of one of the current authors [23].

### 4.1 Supertransformations

We begin with a general discussion valid for the construction of both massive superblocks and consider the most general ansatz for the supertransformations. For the bosonic field variables we choose

\[
\begin{align*}
\delta f^{\alpha(k-1)\dot{\alpha}(k-1)} &= \alpha_{k-1}^{(1)} \Phi^{\alpha(k-1)\dot{\alpha}(k-1)} \zeta_\beta - \bar{\alpha}_{k-1}^{(1)} \Phi^{\alpha(k-1)\dot{\alpha}(k-1)} \bar{\zeta}_\beta \\
&\quad + \alpha'_{k-1}^{(1)} \Phi^{\alpha(k-1)\dot{\alpha}(k-2)} \zeta_\lambda - \bar{\alpha}'_{k-1}^{(1)} \Phi^{\alpha(k-2)\dot{\alpha}(k-1)} \bar{\zeta}_\lambda \\
\delta A &= \alpha_0^{(1)} \Phi^{\alpha} \zeta_\alpha - \bar{\alpha}_0^{(1)} \Phi^{\dot{\alpha}} \bar{\zeta}_\dot{\alpha} + \alpha'_{0}^{(1)} \epsilon_{\alpha\dot{\alpha}}^{(1)} \psi^{\dot{\alpha}} \zeta_\alpha - \bar{\alpha}'_{0}^{(1)} \epsilon_{\alpha\dot{\alpha}}^{(1)} \bar{\psi}^{\dot{\alpha}} \bar{\zeta}_\dot{\alpha} \\
\delta \varphi &= \bar{\alpha}_0^{(1)} \Phi^{\alpha} \zeta_\alpha - \alpha_0^{(1)} \Phi^{\dot{\alpha}} \bar{\zeta}_\dot{\alpha} \quad (4.1)
\end{align*}
\]
and for the fermionic ones

\[
\delta \Phi^\alpha \dot{\alpha} = \beta_{k-1} \Omega^{\alpha(k-1)} \zeta^\alpha + \gamma_{k-1} f^{\alpha(k-1)} \zeta^\alpha + \gamma'_{k-1} f^{\alpha(k-1)} \zeta^\beta
\]

where all coefficients are complex. One can see that the supertransformations for higher spin components are combinations of the massless supertransformations (2.12) and (2.16).

\[
\delta \Phi^\alpha = \beta_0 e^{\alpha \beta} B^{\alpha \beta} \zeta^\beta + \beta_1' \Omega^{\alpha \beta} \zeta^\beta + \gamma_0 A \zeta^\alpha + \gamma_1 f^{\alpha \beta} \zeta^\beta + \gamma_0 e^\alpha \varphi \zeta^\alpha
\]

where all coefficients are complex. One can see that the supertransformations for higher spin components are combinations of the massless supertransformations (2.12) and (2.16).

\[
\Phi_{k-\frac{1}{2}} \quad \Phi_k \quad \Phi_{k+\frac{1}{2}} \quad \Phi_{k+1}
\]

The ansatz for the supertransformations (4.1), (4.2) has the same form for both massive superblocks \((s+1/2, s)\) and \((s, s-1/2)\), the only difference being in the boundary conditions. In the first case we have

\[
\alpha_s = \beta_s = \gamma_s = 0, \quad \alpha'_s = \beta'_s = \gamma'_s = 0
\]

while in the second case

\[
\alpha_{s-1} = \beta_{s-1} = \gamma_{s-1} = 0, \quad \alpha'_s = \beta'_s = \gamma'_s = 0
\]

The variation of the sum of the bosonic and fermionic Lagrangians (3.6), (3.11) under the supertransformations (4.1), (4.2) has the form \(\delta \mathcal{L} + \delta \mathcal{L}'\), where

\[
\delta \mathcal{L} = (-1)^k \sum_{k=2}^{s} \left\{ -(k-1) \beta_{k-1} F_{\alpha(k-1)\beta \dot{\alpha}(k-1)} e^{\beta \dot{\alpha}} \Omega^{\alpha(k-2)\dot{\alpha}(k-2)\dot{\beta} \dot{\beta}} \right. \\
+ 4i \alpha_k \Phi_{\alpha(k-2)\beta \dot{\alpha}(k-1)} e^{\gamma \dot{\gamma}} R^{\alpha(k-2)\dot{\alpha}(k-1)\gamma \dot{\gamma}} \\
- \gamma_{k-1} F_{\alpha(k-1)\beta \dot{\alpha}(k-1)} e^{\beta \dot{\alpha}} \left( f^{\alpha(k-1)\dot{\alpha}(k-1)\dot{\beta} \dot{\beta}} + (k-1) f^{\alpha(k-1)\dot{\alpha}(k-2)\dot{\beta} \dot{\beta}} \right) \\
- \beta_0 E^{\alpha \beta \dot{\alpha}} C_{\alpha \beta} + 4i \alpha_0 E^{\beta \dot{\beta}(2)} F_{\alpha} C^{\beta \dot{\beta}(2)} \zeta^\alpha \\
+ \gamma_0 E^{\alpha \beta \dot{\alpha}} C_{\alpha \beta} + 12i \alpha_0 E^{\beta \dot{\beta} \dot{\alpha} \dot{\alpha}} C^{\beta \dot{\beta} \dot{\alpha} \dot{\alpha}} \zeta^\alpha \\
\left. + \gamma_0 E^{\alpha \beta \dot{\alpha}} F_{\alpha} A \zeta^\alpha + \gamma_0 E^{\alpha \beta \dot{\alpha}} F_{\alpha} \varphi \zeta^\alpha + 2 \gamma_0 E^{\alpha \beta} F_{\alpha} \zeta^\alpha + h.c. \right\}
\]

\[
\delta \mathcal{L}' = \sum_{k=2}^{s} (-1)^k \left\{ \beta'_{k-1} F_{\alpha(k-2)\gamma \dot{\alpha}(k-2)} e^{\gamma \dot{\gamma}} \Omega^{\alpha(k-2)\dot{\alpha}(k-2)\gamma \dot{\gamma}} \right. \\
- 4i (k-1) \beta'_{k-1} \Phi_{\alpha(k-2)\beta \dot{\alpha}(k-1)} e^{\beta \dot{\alpha}} R^{\alpha(k-2)\dot{\alpha}(k-1)\gamma \dot{\gamma}} \\
+ \gamma'_{k-1} F_{\alpha(k-1)\beta \dot{\alpha}(k-1)} e^{\beta \dot{\alpha}} \left( f^{\alpha(k-1)\dot{\alpha}(k-1)\dot{\beta} \dot{\beta}} + (k-1) f^{\alpha(k-1)\dot{\alpha}(k-2)\dot{\beta} \dot{\beta}} \right) \\
- \beta'_0 C_{\alpha} E^{\alpha \dot{\alpha}} C_{\alpha \dot{\alpha}} + 8i \alpha_0' \psi_{\alpha} E^{\beta \dot{\beta}(2)} \zeta^\beta + h.c. \right\}
\]
Here we used the equations for the auxiliary bosonic fields \( (3.5) \). We now proceed as in the massless case, deriving the corresponding identities. Let us recall a general scheme. Lagrangian variation \( (4.5) \) has the structure

\[
\delta \mathcal{L} = (\mathcal{F} \Omega |_{\beta} \oplus \Phi \mathcal{R} |_{\alpha} \oplus \mathcal{F} f |_{\gamma}) \zeta
\]

where \( \Phi, \mathcal{F} \) are sets of all fields and curvatures for the fermion, \( f \) is a set of physical fields for the boson and \( \Omega, \mathcal{R} \) are sets of auxiliary fields and curvatures for the boson. Since a Lagrangian is defined up to a total derivative we have two type of identities \( D[\Phi \Omega \zeta] = 0, D[\Phi f \zeta] = 0 \). They lead to

\[
\mathcal{F} \Omega \zeta = \Phi \mathcal{R} \zeta \oplus \Delta(\Phi \Omega, \Phi f) \zeta \tag{4.6}
\]

\[
\mathcal{F} f \zeta = \Phi \mathcal{R} \zeta \oplus \Delta(\Phi \Omega, \Phi f) \zeta \tag{4.7}
\]

Using the explicit form of identities \( (4.6), (4.7) \) (see Appendix A for details), we obtain expressions for the parameters \( \alpha \) and \( \gamma \) in terms of \( \beta \):

\[
\alpha_{k-1} = i \frac{(k-1)}{4} \bar{\beta}_{k-1}, \quad \alpha'_{k-1} = i \frac{1}{4(k-1)} \bar{\beta}'_{k-1}
\]

\[
\alpha_0 = -i \frac{\bar{\beta}_0}{4}, \quad \tilde{\alpha}_0 = i \frac{\bar{\beta}_0}{24}, \quad \alpha'_0 = -i \frac{\bar{\beta}'_0}{8} \tag{4.8}
\]

and

\[
\gamma_{k-1} = 2d_k \bar{\beta}_{k-1}, \quad \gamma'_{k-1} = 2d_{k-1} \bar{\beta}'_{k-1}
\]

\[
\gamma_0 = -d_1 \bar{\beta}_0, \quad \tilde{\gamma}_0 = -12 \frac{c_0 d_1}{a_0} \bar{\beta}_0, \quad \tilde{\gamma}'_0 = -\frac{1}{8} \bar{a}_0 \bar{\beta}_0 \tag{4.9}
\]

and we also obtain recurrent equations on the parameters \( \beta_k \)

\[
2(k+1) \bar{\beta}_{k-1} c_{k+1} = k \bar{\beta}_k a_{k+1}, \quad 2c_2 \bar{\beta}_0 = a_0 \bar{\beta}_1, \quad 2c_0 \bar{\beta}_0 = -\bar{a}_0 \bar{\beta}_0 \tag{4.10}
\]

\[
\frac{1}{2} \bar{\beta}_{k-1} a_{k+1} = \beta'_1 c_k, \quad \frac{1}{2} a_0 \bar{\beta}'_0 = c_0 \bar{\beta}'_1, \quad \bar{a}_0 \bar{\beta}'_0 = -24 d_1 \bar{\beta}'_0 \tag{4.11}
\]

as well as four independent equations which relate \( \beta \) and \( \beta' \) and the bosonic and fermionic mass parameters:

\[
0 = \frac{\beta'_{k-1} a_k}{(k-1)} - \frac{\beta'_{k-1} a_{k+1}}{2(k+1)} + \lambda \bar{\beta}_{k-1} - \gamma_{k-1} \tag{4.12}
\]

\[
0 = (k-1) \bar{\beta}_{k-1} c_k - \frac{(k-2)}{2} \beta_{k-2} a_k - \lambda \beta'_{k-1} + \gamma'_{k-1} \tag{4.13}
\]

\[
0 = \frac{(k-1)}{2k} \bar{\beta}_{k-1} b_k - 2k d_k \bar{\gamma}_{k-1} + \lambda \bar{\gamma}_{k-1} - \frac{\tilde{\gamma}'_k a_{k+1}}{2k(k+1)} \tag{4.14}
\]

\[
0 = \frac{\bar{\beta}'_{k-1} b_k}{2} - 2(k-1) d_{k-1} \bar{\gamma}'_{k-1} - \lambda \bar{\gamma}'_{k-1} + \tilde{\gamma}'_{k-1} - c_k \tag{4.15}
\]

The explicit solution of these equations depends on the concrete massive superblock. More specifically, it depends on the initial conditions \( (4.3), (4.4) \) and on the sign of \( d_k \), i.e. on the sign before massive terms in Lagrangian for fermions. In the following we present exact solutions for two massive superblocks \( (s+1/2, s) \) and \( (s, s-1/2) \).
4.2 Superblock \((s + 1/2, s)\)

Here we present our results for the massive superblock \((s + 1/2, s)\). The massive boson spin-\(s\) with the mass parameter \(M\), as described in section 3.1, we have

\[
b_k = \frac{2s(s+1)}{k(k-1)(k+1)}M^2, \quad m^2 = M^2 - s(s-1)\lambda^2
\]

\[
a_k^2 = \frac{4(s-k+1)(s+k)}{(k-1)(k-2)}[M^2 - k(k-1)\lambda^2]
\]

\[
a_0^2 = 2(s-1)(s+2)[M^2 - 2\lambda^2], \quad \tilde{a}_0^2 = 24s(s+1)M^2
\]

The massive fermion spin-\((s+1/2)\) with mass parameter \(M_1\) as described in section 3.2, here

\[
d_k = \pm \frac{(s+1)}{2k(k+1)}M_1, \quad m_1^2 = M_1^2 - s^2\lambda^2
\]

\[
c_k^2 = \frac{(s-k+1)(s+k+1)}{k^2}[M_1^2 - k^2\lambda^2]
\]

\[
c_0^2 = 2s(s+2)[M_1^2 - \lambda^2]
\]

Supertransformations for massive superblock \((s + 1/2, s)\) have the form (4.1), (4.2) with the initial conditions (4.3). The parameters \(\alpha_k\) and \(\gamma_k\) are determined by (4.8) and (4.9). From the equation (4.14) one can obtain an important relation on the bosonic and fermionic mass parameters. Indeed, at \(k = s\) we have

\[
(M^2 - M_1^2)\tilde{\beta}_{s-1} = \mp M_1\lambda \beta_{s-1}
\]

where the sign corresponds to that of \(d_k\). So we have four independent cases

\[
M^2 = M_1(M_1 - \lambda), \quad \tilde{\beta}_{s-1} = \pm \beta_{s-1}
\]

\[
M^2 = M_1(M_1 + \lambda), \quad \tilde{\beta}_{s-1} = \mp \beta_{s-1}
\]

The solution of other equations give, for \(M^2 = M_1(M_1 - \lambda)\)

\[
\beta_{k-1} = \sqrt{(s+1+k\lambda)(M_1+k\lambda)}(k-1)^{-1}\beta, \quad \beta'_{k-1} = \sqrt{(k-1)(s-k)(M_1-k\lambda)}\beta
\]

\[
\beta_0 = \sqrt{2(s+2)(M_1+\lambda)}\beta, \quad \beta'_0 = 2\sqrt{s(M_1-\lambda)}\beta, \quad \tilde{\beta}_0 = -\sqrt{6(s+1)M_1}\beta
\]

and for \(M^2 = M_1(M_1 + \lambda)\)

\[
\beta_{k-1} = \sqrt{(s+1+k\lambda)(M_1-k\lambda)}(k-1)^{-1}\beta, \quad \beta'_k = -\sqrt{(k-1)(s-k+1)(M_1+k\lambda)}\beta
\]

\[
\beta_0 = \sqrt{2(s+2)(M_1-\lambda)}\beta, \quad \beta'_0 = -2\sqrt{s(M_1+\lambda)}\beta, \quad \tilde{\beta}_0 = -\sqrt{6(s+1)M_1}\beta
\]

Therefore, we see that in the two cases the parameters \(\beta\) are real and in the other two they are imaginary. This means that one half of the solutions corresponds to the massive superblocks with the parity-even boson while another half corresponds to the massive superblocks with the parity-odd one.
In order to present these four cases for the massive superblock \((s, s + 1/2)\) in a more clear form let us introduce following notations. We denote integer spin \(s\) with the mass parameter \(M\) as
\[
[s]_M^{\pm}
\] (4.16)
here \(\pm\) corresponds to parity-even/parity-odd boson. We also denote half-integer spin \(s + 1/2\) with the mass parameter \(M_1\) as
\[
[s + \frac{1}{2}]^{\pm}_{M_1}
\] (4.17)
here \(\pm\) corresponds to the sign of \(d_k\). In these notations the four solutions for the massive superblock given above look as follows:

1) \(\left( [s]_M^+, [s + \frac{1}{2}]^{+}_{M_1} \right)\), 2) \(\left( [s]_M^-, [s + \frac{1}{2}]^{-}_{M_1} \right)\), 3) \(\left( [s]_{M'}^+, [s + \frac{1}{2}]^{+}_{M_1} \right)\), 4) \(\left( [s]_{M'}^-, [s + \frac{1}{2}]^{-}_{M_1} \right)\).

where
\[
M^2 = M_1(M_1 - \lambda), \quad M'^2 = M_1(M_1 + \lambda)
\]

In the first and third cases we have
\[
\beta_{k-1} = \sqrt{\frac{(s + k + 1)(M_1 \pm k\lambda)}{(k - 1)}} \rho, \quad \beta'_{k-1} = \pm \sqrt{(k - 1)(s - k + 1)(M_1 \mp k\lambda)} \rho
\]
\[
\beta_0 = \sqrt{2(s + 2)(M_1 \pm \lambda)} \rho, \quad \beta'_0 = \pm 2\sqrt{s(M_1 \mp \lambda)} \rho, \quad \tilde{\beta}_0 = -\sqrt{6(s + 1)M_1} \rho
\]
here the upper sign corresponds to 1) and the lower sign corresponds to 3). In the second and fourth cases we have
\[
\beta_{k-1} = i \sqrt{\frac{(s + k + 1)(M_1 \pm k\lambda)}{(k - 1)}} \rho, \quad \beta'_{k-1} = \pm i \sqrt{(k - 1)(s - k + 1)(M_1 \mp k\lambda)} \rho
\]
\[
\beta_0 = i \sqrt{2(s + 2)(M_1 \pm \lambda)} \rho, \quad \beta'_0 = \pm i 2\sqrt{s(M_1 \mp \lambda)} \rho, \quad \tilde{\beta}_0 = -i \sqrt{6(s + 1)M_1} \rho
\]
here the upper sign corresponds to 2) and the lower sign corresponds to 4).

### 4.3 Superblock \((s, s - 1/2)\)

Here we collect our results for the massive superblock \((s, s - 1/2)\). For the massive even or odd spin-s boson with the mass parameter \(M\) we use the same formulation as in the previous subsection, while for the massive spin-\((s - 1/2)\) fermion with the mass parameter \(M_2\) we use its description in section 3.2 with the shift \(s \rightarrow (s - 1)\):

\[
d_k = \pm \frac{s}{2k(k + 1)} M_2, \quad m_2^2 = M_2^2 - (s - 1)^2 \lambda^2
\]
\[
c_k^2 = \frac{(s - k)(s + k)}{k^2} [M_2^2 - k^2 \lambda^2]
\]
\[
c_0^2 = 2(s - 1)(s + 1) [M_2^2 - \lambda^2]
\]
Supertransformations for the massive superblock \((s-1/2, s)\) are the same as in the previous case \((4.1), (4.2)\) but with different initial conditions \((4.4)\). The parameters \(\alpha_k\) and \(\gamma_k\) are still determined by \((4.8)\) and \((4.9)\). From the equation \((4.13)\) one can relate bosonic and fermionic mass parameters, indeed at \(k = s\) we have

\[
(M^2 - M_2^2)\beta'_{s-1} = \pm M_2 \lambda \beta'_{s-1}
\]

here the sign corresponds to that of \(d_k\). So we again have four independent cases

\[
M^2 = M_2(M_2 + \lambda), \quad \beta'_{s-1} = \pm \beta'_{s-1}
\]

\[
M^2 = M_2(M_2 - \lambda), \quad \beta'_{s-1} = \mp \beta'_{s-1}
\]

The solution of other equations gives, for \(M^2 = M_2(M_2 + \lambda)\)

\[
\beta_{k-1} = \sqrt{(s - k)(M_2 - k\lambda)} \beta, \quad \beta'_{k-1} = \sqrt{(k - 1)(s + k)(M_2 + k\lambda)} \beta
\]

\[
\beta_0 = \sqrt{2(s - 1)(M_2 - \lambda)} \beta, \quad \beta'_0 = 2\sqrt{(s + 1)(M_2 + \lambda)} \beta, \quad \tilde{\beta}_0 = -\sqrt{6sM_2} \beta
\]

and for \(M^2 = M_2(M_2 - \lambda)\)

\[
\beta_{k-1} = \sqrt{(s - k)(M_2 + k\lambda)} \beta, \quad \beta'_{k-1} = -\sqrt{(k - 1)(s + k)(M_2 - k\lambda)} \beta
\]

\[
\beta_0 = \sqrt{2(s - 1)(M_2 + \lambda)} \beta, \quad \beta'_0 = -2\sqrt{(s + 1)(M_2 - \lambda)} \beta, \quad \tilde{\beta}_0 = -\sqrt{6sM_2} \beta
\]

Again we see that in two cases the parameters \(\beta\) are real and in other two cases they are imaginary. They correspond to the massive superblocks with the parity-even and parity-odd bosons respectively. Using notations \((4.16), (4.17)\) these four cases for the massive \((s, s-1/2)\) superblock can be presented as

1) \(\left(\begin{array}{c} [s]_M^+ \\ [s-\frac{1}{2}]_{M_2^+} \end{array}\right)\), 2) \(\left(\begin{array}{c} [s]_M^- \\ [s-\frac{1}{2}]_{M_2^-} \end{array}\right)\), 3) \(\left(\begin{array}{c} [s]_{M'}^+ \\ [s-\frac{1}{2}]_{M_2^-} \end{array}\right)\), 4) \(\left(\begin{array}{c} [s]_{M'}^- \\ [s-\frac{1}{2}]_{M_2^+} \end{array}\right)\)

where

\[
M^2 = M_2(M_2 + \lambda), \quad M'^2 = M_2(M_2 - \lambda)
\]

For the first and third cases we have

\[
\beta_{k-1} = \sqrt{(s - k)(M_2 \mp k\lambda)} \rho, \quad \beta'_{k-1} = \pm \sqrt{(k - 1)(s + k)(M_2 \pm k\lambda)} \rho
\]

\[
\beta_0 = \sqrt{2(s - 1)(M_2 \mp \lambda)} \rho, \quad \beta'_0 = \pm 2\sqrt{(s + 1)(M_2 \pm \lambda)} \rho, \quad \tilde{\beta}_0 = -\sqrt{6sM_2} \rho
\]

here the upper sign corresponds to 1) and the lower sign corresponds to 3). In the second and fourth cases we have

\[
\beta_{k-1} = i\sqrt{(s - k)(M_2 \mp k\lambda)} \rho, \quad \beta'_{k-1} = \pm i\sqrt{(k - 1)(s + k)(M_2 \pm k\lambda)} \rho
\]

\[
\beta_0 = i\sqrt{2(s - 1)(M_2 \mp \lambda)} \rho, \quad \beta'_0 = \pm i2\sqrt{(s + 1)(M_2 \pm \lambda)} \rho, \quad \tilde{\beta}_0 = -i\sqrt{6sM_2} \rho
\]

here the upper sign correspond to 2) and the lower sign correspond to 4). In all four cases \(\rho\) is real.
5 Massive higher spin supermultiplets

In the previous section we constructed massive superblocks containing one massive fermion and one massive boson. For each individual superblock, we found supertransformations defined up to a one common parameter \( \rho \). In this section we use these results to construct complete massive supermultiplets. For that we choose appropriate solutions for each superblock and adjust their parameters so that the algebra of these supertransformations is closed. In the next subsection, we consider general properties of such construction and then present our results for the case of integer and half-integer superspins.

5.1 General construction

Any massive \( N = 1 \) supermultiplet contains two massive fermions and two massive bosons. In the notations given in the previous section (4.16), (4.17) they have the following structure

\[
\begin{align*}
\rho_1 & \quad \rho_3 \\
[s + \frac{1}{2}]^+_{M_1} & \quad \rho_2 \\
[s]^M & \quad \rho_4 \\
[s + \frac{1}{2}]^+_{M_2} & \quad \rho_3 \\
[s - \frac{1}{2}]_{M_1} & \quad \rho_2 \\
[s - \frac{1}{2}]^+_{M'} & \quad \rho_4 \\
[s - \frac{1}{2}]_{M_2} & \quad \rho_3 \\
[s - \frac{1}{2}]^-_{M'} & \quad \rho_4
\end{align*}
\]

As already mentioned, the two bosonic fields must have opposite parities and it appears that the two fermionic fields must have opposite signs of the mass terms. Let us introduce notations \((f_+, \Omega_+)\) for the parity-even boson and \((f_-, \Omega_-)\) for the parity-odd one. The fermions we denote as \(\Phi_+, \Phi_-\) according to the sign of \(d_k\).

The ansatz for the supertransformations is a combination of four possible superblocks corresponding to the lines with the parameters \(\rho_{1,2,3,4}\). For example, for the parity-even boson we take:

\[
\begin{align*}
\delta f_+^{\alpha(k-1)\dot{\alpha}(k-1)} &= \alpha_{k-1}|p_1| \Phi_+^{\alpha(k-1)\dot{\alpha}(k-1)} \zeta_\beta - \dot{\alpha}_{k-1}|p_1| \Phi_-^{\alpha(k-1)\dot{\alpha}(k-1)} \dot{\zeta}_\beta \\
&+ \alpha'_{k-1}|p_1| \Phi_+^{\alpha(k-1)\dot{\alpha}(k-2)} \zeta_\beta - \dot{\alpha}'_{k-1}|p_1| \Phi_-^{\alpha(k-2)\dot{\alpha}(k-1)} \zeta_\beta \\
&+ \alpha_{k-1}|p_2| \Phi_-^{\alpha(k-1)\dot{\alpha}(k-2)} \zeta_\beta - \dot{\alpha}_{k-1}|p_2| \Phi_+^{\alpha(k-1)\dot{\alpha}(k-1)} \zeta_\beta \\
&+ \alpha'_{k-1}|p_2| \Phi_-^{\alpha(k-1)\dot{\alpha}(k-2)} \dot{\zeta}_\beta - \dot{\alpha}'_{k-1}|p_2| \Phi_+^{\alpha(k-2)\dot{\alpha}(k-1)} \dot{\zeta}_\beta \\
\delta \Phi_+^{\alpha(k)\dot{\alpha}(k-1)} &= \beta_{k-1}|p_1| \Omega_+^{\alpha(k)\dot{\alpha}(k-2)} \zeta_\beta + \gamma_{k-1}|p_1| f_+^{\alpha(k-1)\dot{\alpha}(k-1)} \zeta_\alpha \\
&+ \beta'_{k-1}|p_1| \Omega_+^{\alpha(k)\dot{\alpha}(k-1)} \dot{\zeta}_\beta + \dot{\gamma}'_{k-1}|p_1| f_+^{\alpha(k-2)\dot{\alpha}(k-1)} \dot{\zeta}_\beta \\
\delta \Phi_-^{\alpha(k)\dot{\alpha}(k-1)} &= \beta_{k-1}|p_2| \Omega_+^{\alpha(k)\dot{\alpha}(k-2)} \zeta_\beta + \gamma_{k-1}|p_2| f_+^{\alpha(k-1)\dot{\alpha}(k-1)} \zeta_\alpha \\
&+ \beta'_{k-1}|p_2| \Omega_+^{\alpha(k)\dot{\alpha}(k-1)} \dot{\zeta}_\beta + \dot{\gamma}'_{k-1}|p_2| f_+^{\alpha(k-2)\dot{\alpha}(k-1)} \dot{\zeta}_\beta
\end{align*}
\]

(and similarly for the lower spin components), while the ansatz for the parity-odd one can be obtained by replacement \(\rho_1 \to \rho_3\) and \(\rho_2 \to \rho_4\).
The commutator of the two supertransformations must produce a combination of translations and Lorentz transformations:

\[
\{Q_\alpha, Q_{\dot{\alpha}}\} \sim P_{\alpha \dot{\alpha}}, \quad \{Q_\alpha, Q_\alpha\} \sim \lambda M_{\alpha \alpha}, \quad \{Q_\alpha, Q_{\dot{\alpha}}\} \sim \lambda M_{\alpha \dot{\alpha}}
\]  

The structure of the mass-shell condition (3.5) shows that, for example, the commutator \( f^{\alpha(k-1)\dot{\alpha}(k-1)}_+ \) must contain fields \( \Omega^{\alpha(k)\dot{\alpha}(k-2)}_+ \), \( \Omega^{\alpha(k-2)\dot{\alpha}(k)}_+ \), \( f^{\alpha(k)\dot{\alpha}(k)}_+ \), \( f^{\alpha(k-1)\dot{\alpha}(k-1)}_+ \) and \( f^{\alpha(k-2)\dot{\alpha}(k-2)}_+ \) only. This gives a number of relations on the parameters:

\[
\begin{align*}
\alpha_{k-1}|\rho_1\beta'_{k}|\rho_1 + \alpha_{k-1}|\rho_2\beta'_k|\rho_2 &= 0, \\
\alpha'_{k-1}|\rho_1\beta_{k-2}|\rho_1 + \alpha'_{k-1}|\rho_2\beta_{k-2}|\rho_2 &= 0 \\
\alpha_{k-1}|\rho_1\beta_{k-1}|\rho_3 + \alpha'_{k-1}|\rho_1\beta'_{k-1}|\rho_3 + \alpha_{k-1}|\rho_2\beta_{k-1}|\rho_4 + \alpha'_{k-1}|\rho_2\beta'_{k-1}|\rho_4 &= 0 \\
\alpha_{k-1}|\rho_1\gamma_{k-1}|\rho_3 - \alpha'_{k-1}|\rho_1\gamma'_{k-1}|\rho_3 + \alpha_{k-1}|\rho_2\gamma_{k-1}|\rho_4 - \alpha'_{k-1}|\rho_2\gamma'_{k-1}|\rho_4 &= 0 \\
\alpha_{k-1}|\rho_1\gamma'_{k-2}|\rho_3 - \alpha'_{k-1}|\rho_1\gamma_{k-2}|\rho_3 + \alpha_{k-1}|\rho_2\gamma_{k-2}|\rho_4 - \alpha'_{k-1}|\rho_2\gamma'_{k-2}|\rho_4 &= 0
\end{align*}
\]

If these relations are fulfilled the resulting expression for the commutator has the form:

\[
[\delta_1, \delta_2]f^{\alpha(k-1)\dot{\alpha}(k-1)}_+ = (\alpha_{k-1}|\rho_1\beta_{k-1}|\rho_1 + \alpha'_{k-1}|\rho_1\beta'_{k-1}|\rho_1 + \alpha_{k-1}|\rho_2\beta_{k-1}|\rho_2 + \alpha'_{k-1}|\rho_2\beta'_{k-1}|\rho_2)\cdot[\Omega^{\alpha(k-1)\dot{\alpha}(k-2)}_+(\zeta_2^\alpha\zeta_2^\dot{\alpha} - \zeta_2^\alpha\zeta_2^\dot{\alpha}) + \Omega^{\alpha(k-2)\dot{\alpha}(k-1)}_+(\zeta_2^\alpha\zeta_2^\dot{\alpha} - \zeta_2^\alpha\zeta_2^\dot{\alpha})]
\]

Let us stress that the coefficients in this expression must be \( k \)-independent. This gives additional restrictions on the parameters and also serves as a quite non-trivial test for our calculations.

Thus to construct massive supermultiplets we start with the four suitable massive superblocks with four free parameters \( \rho_{1,2,3,4} \). Then we require that the commutator of the two supertransformations on the bosonic fields be closed. In the next two subsections we apply this scheme to the massive supermultiplets with half-integer \( Y = s - 1/2 \) and integer \( Y = s \) superspins.

### 5.2 Half-integer superspin \( S - 1/2 \)

The massive superspin-(s-1/2) supermultiplet contains

\[
[s - 1/2]_M \quad \text{and} \quad [s - 1/2]_{M'}
\]

\[
\begin{array}{ccc}
[s - 1/2]_M & \downarrow S - 1/2 & [s - 1/2]_{M'} \\
\rho_1 & \rho_3 \\
[s]_M & \text{or} & [s - 1/2]_{M'} \\
\rho_2 & \rho_4 \\
\end{array}
\]
Firstly, we note that four superblocks give the following relations on the mass parameters:

\[
M^2 = M_1(M_1 + \lambda) \quad M'^2 = M_1(M_1 + \lambda) \\
M^2 = M_2(M_2 - \lambda) \quad M'^2 = M_2(M_2 - \lambda)
\]

Their solution is

\[
M^2 = M'^2 = M_1(M_1 + \lambda), \quad M_2 = M_1 + \lambda \tag{5.2}
\]

All the conditions for the closure of the superalgebra are fulfilled provided:

\[
\rho_1^2 = \rho_2^2 = \rho_3^2 = \rho_4^2, \quad \rho_1 \rho_3 = \rho_2 \rho_4 \tag{5.3}
\]

If the relations (5.3) are satisfied then the commutators of the supertransformations on parity-even spin-\(s\) \(f_+\) and parity-odd spin-(\(s - 1\)) \(f_-\) fields have the same form:

\[
\frac{1}{i\rho_0^2}[\delta_1, \delta_2]f^\alpha_{\pm (k-1)\hat{a}(k-1)} = \Omega_+^{\alpha\gamma}(k-1)\gamma\hat{a}(k-2)\left(\zeta_1^\alpha \zeta_2 - \zeta_2^\alpha \zeta_1\right) + \Omega^{\hat{a} \hat{b}}_{\pm (k-2)\hat{a}}(k-2)\left(\zeta_1^{\hat{a} \hat{b}} \zeta_2 - \zeta_2^{\hat{a} \hat{b}} \zeta_1\right)
\]

\[
\frac{1}{i\rho_0^2}[\delta_1, \delta_2]f^\alpha_{\pm \hat{a}} = \Omega_+^{\alpha \gamma}(\zeta_1^\alpha \zeta_2 - \zeta_2^\alpha \zeta_1) + \Omega^{\hat{a} \hat{b}}_{\pm \hat{a}}(\zeta_1^{\hat{a} \hat{b}} \zeta_2 - \zeta_2^{\hat{a} \hat{b}} \zeta_1)
\]

\[
\frac{1}{i\rho_0^2}[\delta_1, \delta_2]A_{\pm} = -2e_{\beta \gamma}B^{\alpha \beta}_{\pm}(\zeta_1^\gamma \zeta_2 - \zeta_2^\gamma \zeta_1) + e_{\beta \gamma}B^{\hat{a} \hat{b}}_{\pm}(\zeta_1^{\hat{a} \hat{b}} \zeta_2 - \zeta_2^{\hat{a} \hat{b}} \zeta_1)
\]

\[
\frac{1}{i\rho_0^2}[\delta_1, \delta_2]\varphi_{\pm} = \pi_{\pm}^{\alpha \gamma}(\zeta_1^\alpha \zeta_2 - \zeta_2^\alpha \zeta_1)
\]

where \(a_k\) is determined by (3.2) for spin \(s\) and spin \((s - 1)\) respectively and

\[
\rho_0^2 = \frac{s(2M_1 + \lambda)}{2}\rho_1^2
\]

### 5.3 Integer superspin \(S\)

The massive superspin-\(s\) supermultiplet contains

\[
\begin{array}{c}
[s + \frac{1}{2}]_{M}^+ \\
\rho_1 \\
[s]_M^+ \\
S \\
[s]^\dagger_M \\
\rho_3 \\
[s - \frac{1}{2}]_{M}^- \\
\rho_2 \\
\rho_4
\end{array}
\]

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First of all, we note that the four superblocks give the following relations on the mass parameters

\[
M^2 = M_1(M_1 - \lambda) \quad M'^2 = M_1(M_1 + \lambda) \\
M^2 = M_2(M_2 - \lambda) \quad M'^2 = M_2(M_2 + \lambda)
\]

Their solution is

\[
M^2 = M_1(M_1 - \lambda), \quad M'^2 = M_1(M_1 + \lambda), \quad M_2 = M_1
\]  \quad (5.4)

The requirement that the superalgebra be closed again leads to

\[
\rho_1^2 = \rho_2^2 = \rho_3^2 = \rho_4^2, \quad \rho_1 \rho_3 = \rho_2 \rho_4
\]  \quad (5.5)

If the relations (5.5) are satisfied then commutators of supertransformations on parity-even and parity-odd bosonic spin-s fields have the same form:

\[
\frac{1}{i \rho_0^2} [\tilde{\delta}_1, \tilde{\delta}_2] f_{\pm}^{\alpha(k-1)\bar{\alpha}(k-1)} = \epsilon_{\pm}^{\alpha(k-1)\gamma\bar{\alpha}(k-2)} (\zeta_{1,\gamma} - \zeta_{2,\gamma}) + \Omega_{\pm}^{\alpha(k-2)\bar{\alpha}(k-1)\bar{\gamma}} (\zeta_{1,\bar{\gamma}} - \zeta_{2,\bar{\gamma}})

+ \frac{(k-1)a_{k+1}}{2k+1} f_{\pm}^{\alpha(k-1)\gamma\bar{\alpha}(k-1)\bar{\beta}} (\zeta_{1,\gamma} - \zeta_{2,\gamma})

+ \frac{a_k}{2k(k-1)} f_{\pm}^{\alpha(k-2)\bar{\alpha}(k-2)} (\zeta_{1,\bar{\gamma}} - \zeta_{2,\bar{\gamma}})

+ \lambda [f_{\pm}^{\alpha(k-2)\gamma\bar{\alpha}(k-1)} (\zeta_{1,\gamma} - \zeta_{2,\gamma}) + f_{\pm}^{\alpha(k-1)\gamma\bar{\alpha}(k-2)} (\zeta_{1,\gamma} - \zeta_{2,\gamma})]
\]

\[
\frac{1}{i \rho_0^2} [\tilde{\delta}_1, \tilde{\delta}_2] f_{\pm}^{\alpha\bar{\alpha}} = \epsilon_{\pm}^{\alpha\gamma\bar{\alpha}} (\zeta_{1,\gamma} - \zeta_{2,\gamma}) + \Omega_{\pm}^{\alpha\bar{\gamma}} (\zeta_{1,\gamma} - \zeta_{2,\gamma})

+ \frac{a_3}{6} f_{\pm}^{\alpha\gamma\bar{\alpha}} (\zeta_{1,\gamma} - \zeta_{2,\gamma}) - \frac{a_0}{4} A_{\pm} (\zeta_{1,\gamma} - \zeta_{2,\gamma})

+ \lambda [f_{\pm}^{\alpha\gamma\bar{\alpha}} (\zeta_{1,\gamma} - \zeta_{2,\gamma}) + f_{\pm}^{\alpha\bar{\gamma}} (\zeta_{1,\gamma} - \zeta_{2,\gamma})]
\]

\[
\frac{1}{i \rho_0^2} [\tilde{\delta}_1, \tilde{\delta}_2] A_{\pm} = -2e_{\beta\bar{\beta}} B_{\pm}^{\alpha\beta} (\zeta_{1,\beta} - \zeta_{2,\beta}) + e_{\beta\bar{\beta}} B_{\pm}^{\alpha\beta} (\zeta_{1,\beta} - \zeta_{2,\beta})

- \frac{a_0}{2} f_{\pm}^{\alpha\beta} (\zeta_{1,\beta} - \zeta_{2,\beta})
\]

\[
\frac{1}{i \rho_0^2} [\tilde{\delta}_1, \tilde{\delta}_2] \varphi_{\pm} = \pi_{\pm}^{\alpha\bar{\alpha}} (\zeta_{1,\alpha} - \zeta_{2,\alpha})
\]

where \(a_k\) is determined by (3.2) for spin \(s\) and

\[
\rho_0^2 = \frac{(2s + 1) M_1}{2} \rho_1^2
\]

6 Summary

In this paper we have developed the component Lagrangian description of massive on-shell \(N = 1\) supermultiplets with arbitrary (half)integer superspin in four dimensional Anti de Sitter space (AdS\(_4\)). The derivation is based on supersymmetric generalization of frame-like
gauge invariant formulation of massive higher spin fields where massive supermultiplets are described by an appropriate set of massless ones. We show that \( N = 1 \) massive supermultiplets can be constructed as a combination of four massive superbloks, each containing one massive boson and one massive fermion. As a result, we have derived both the supertransformations for the components of the one-shell supermultiplets and the corresponding invariant Lagrangians. Thus, the component Lagrangian formulation of the \( N = 1 \) supersymmetric free massive higher spin field theory on the AdS\(_4\) space can be considered complete.

Let us briefly discuss the possible further generalizations of the results obtained. As we already pointed out, the problem of off-shell supersymmetric massive higher spin theory remains open in general. Only a few examples of such a theory with concrete superspins \([34], [35], [36]\) in flat space have been developed. There are no known examples in the AdS space. There are two possible approaches to study this general problem. One option is to start with on-shell theory and try to find the necessary auxiliary fields closing the superalgebra on the base of Noether’s procedure. Another approach can be based on the use of the superfield techniques from the very beginning. At present, realization of both these approaches seems unclear and will require the development of new methods. Besides, the interesting generalizations of the results obtained can be constructing the partially massless \( N = 1 \) supermultiplets and finding at least on-shell component Lagrangian description for \( N \)-extended massive supermultiplets in flat and AdS spaces. We hope to attack these problems in the forthcoming works.

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**A Identities**

In this appendix we present explicit expression of identities for massive higher spin superbloks.
A.1 Auxiliary bosonic fields

Identities that correspond to (4.6)

\[ 0 = \mathcal{F}_{\alpha(k-2)\gamma\delta\dot{\alpha}(k-2)} e^{\dot{\gamma}\dot{\delta}} \Omega^{\alpha(k-2)\dot{\gamma}(k-2)\dot{\delta}} \zeta^{\alpha} + \Phi_{\alpha(k-2)\beta\gamma\dot{\alpha}(k-1)\dot{\beta}} e^{\dot{\beta}} R^{\alpha(k-2)\dot{\beta}(k-1)\dot{\gamma}} \]

\[ -2d_k [(k + 1) E^{\dot{\gamma}} \Phi^{\alpha(k-2)\gamma\dot{\alpha}(k-1)\dot{\gamma}} \Omega_{\alpha(k-2)\dot{\alpha}(k-1)\dot{\gamma}}]
\]

\[ - (k - 2) E^{\beta} \Phi^{\alpha(k-3)\beta\gamma\dot{\alpha}(k-2) \dot{\gamma}} \Omega_{\alpha(k-2)\dot{\alpha}(k-1)\dot{\gamma}} - E^{\gamma} \Phi^{\alpha(k-2)\beta\dot{\alpha}(k-2) \dot{\gamma}} \Omega_{\alpha(k-2)\dot{\alpha}(k-1)\dot{\gamma}}\]

\[ - \frac{b_k}{2k} [(k + 1) E^{\dot{\beta}} \Phi_{\alpha(k-2)\beta\gamma\dot{\alpha}(k-1)\dot{\beta}} f^{\alpha(k-2)\dot{\delta}(k-1)} - (k - 1) \Phi_{\alpha(k-1)\gamma\dot{\alpha}(k-1)} f^{\alpha(k-1)\dot{\alpha}(k-2)\dot{\beta}}] \zeta^{\gamma} + \lambda \left[ E^{\alpha\beta} \Phi_{\alpha(k-1)\beta\dot{\alpha}(k-1)\dot{\gamma}} \Omega^{\alpha(k-2)\dot{\alpha}(k-1)\dot{\beta}} \zeta^{\gamma} \right]
\]

\[ - c_k + \frac{1}{(k - 1) \delta \Omega^{(k-3)\dot{\alpha}(k-1)\dot{\gamma}} \zeta^{\gamma}}\]

\[ \frac{2}{2k(k + 1)} [(k + 1) E^{\dot{\beta}} \Phi_{\alpha(k-2)\beta\gamma\dot{\alpha}(k-1)\dot{\beta}} \Omega^{(k-3)\dot{\alpha}(k-1)\dot{\gamma}} \zeta^{\gamma}]
\]

\[ 0 = \mathcal{F}_{\alpha(k-2)\gamma\delta\dot{\alpha}(k-2)} e^{\dot{\gamma}\dot{\delta}} \Omega^{(k-2)\dot{\gamma}(k-2)\dot{\delta}} \zeta^{\alpha} + \Phi_{\alpha(k-2)\gamma\dot{\alpha}(k-2)\dot{\gamma}} e^{\dot{\gamma}} R^{\alpha(k-2)\dot{\gamma}(k-2)\dot{\gamma}} \zeta^{\alpha} \]

\[ + 2d_{k-1} [(k + 1) E^{\dot{\gamma}} \Phi^{\alpha(k-2)\gamma\dot{\alpha}(k-1)\dot{\gamma}} \Omega_{\alpha(k-2)\dot{\alpha}(k-1)\dot{\gamma}}]
\]

\[ - (k - 2) E^{\beta} \Phi^{\alpha(k-3)\gamma\dot{\alpha}(k-2) \dot{\gamma}} \Omega_{\alpha(k-2)\dot{\alpha}(k-1)\dot{\gamma}} - E^{\gamma} \Phi^{\alpha(k-2)\beta\dot{\alpha}(k-2) \dot{\gamma}} \Omega_{\alpha(k-2)\dot{\alpha}(k-1)\dot{\gamma}}\]

\[ - \frac{b_k}{2k} [-(k - 2) E^{\dot{\gamma}} \Phi_{\alpha(k-1)\dot{\alpha}(k-2) f^{\alpha(k-1)\dot{\gamma}(k-3)\dot{\gamma}} + (k + 1) E^{\gamma} \Phi_{\alpha(k-2)\gamma\dot{\alpha}(k-2) f^{\alpha(k-2)\dot{\gamma}(k-2)\dot{\beta}}}
\]

\[ - E^{\dot{\beta}} \Phi_{\alpha(k-1)\dot{\alpha}(k-2) f^{\alpha(k-1)\dot{\gamma}(k-2)\dot{\gamma}} \zeta^{\beta}} + \lambda \left[ E^{\dot{\beta}} \Phi^{\alpha(k-2)\dot{\gamma}(k-2) \dot{\beta}} \zeta^{\gamma} \right]
\]

\[ + c_k [E^{\gamma} \Phi^{\alpha(k-2)\beta\dot{\alpha}(k-1)\dot{\beta}} \Omega_{\alpha(k-2)\dot{\alpha}(k-1)\dot{\beta}} \zeta^{\gamma}] + \frac{(k - 2) c_k}{k(k - 2)} [E^{\dot{\gamma}} \Phi_{\alpha(k-2)\dot{\alpha}(k-2) \dot{\gamma}} \Omega_{\alpha(k-2)\dot{\alpha}(k-2) \dot{\gamma}} \zeta^{\gamma}]\]

\[ - \frac{a_k + 1}{(k - 1) \delta \Omega^{(k-3)\dot{\alpha}(k-1)\dot{\gamma}} \zeta^{\gamma}}\]

\[ \frac{2}{2k(k + 1)} [(k + 1) E^{\dot{\gamma}} \Phi_{\alpha(k-2)\gamma\dot{\alpha}(k-2) \dot{\gamma}} \Omega^{(k-3)\dot{\alpha}(k-2) \dot{\gamma}} \zeta^{\gamma}]\]
\[
0 = F_{\alpha\beta\gamma} e^\beta \Omega^\alpha \zeta^\gamma + \Phi_{\beta\gamma\delta} e^\beta R^\alpha \zeta^\gamma \\
- 2d_2 [3E^\beta - E_{\beta}\Phi_{\alpha\gamma\delta} f^{\alpha\beta\gamma\delta} - E_{\beta}\Phi_{\alpha\gamma\delta} f^{\alpha\beta\gamma\delta}] + \lambda [E_{\alpha\beta\gamma\delta} \Phi_{\alpha\beta\gamma\delta}] \\
- c_3 [E_{\beta\gamma\delta} E_{\beta\gamma\delta}(2\Omega_{\alpha\beta\gamma\delta}) - f r a c c_2 [2E_{\beta\gamma\delta} E_{\beta\gamma\delta} + E_{\beta\gamma\delta} E_{\beta\gamma\delta}] \\
- a_3 [E_{\beta\gamma\delta} E_{\alpha\beta\gamma\delta}] + [a_0 / 4 ] (4E^\beta \Phi_{\alpha\beta\gamma\delta} B^{\beta}) - a_0 a_0 / 8 E_{\beta\gamma\delta} E_{\alpha\beta\gamma\delta} \varphi] \zeta^\alpha \\
0 = F_{\alpha\beta\gamma} e^\alpha \Omega^{\beta\gamma} \zeta^\beta + \Phi_{\alpha\beta\gamma} e^\alpha \phi^{\alpha\beta\gamma} \zeta^\beta \\
+ [4d_1 E_{\beta\gamma\delta} E_{\beta\gamma\delta} \zeta^\gamma - c_2 E_{\alpha\beta\gamma} E_{\alpha\beta\gamma} \Omega_{\beta\gamma\delta}] + c_0 E_{\beta\gamma\delta} E_{\beta\gamma\delta} \zeta^\gamma \\
- [a_0 / 2 ] (3E_{\beta\gamma\delta} E_{\alpha\beta\gamma\delta} B^{\beta}) + b_2 / 4 (3E_{\beta\gamma\delta} E_{\alpha\beta\gamma\delta} + B^{\beta}) \\
+ a_0 E_{\beta\gamma\delta} E_{\beta\gamma\delta} B^{\beta} - a_0 a_0 / 8 E_{\alpha\beta\gamma\delta} E_{\beta\gamma\delta} \varphi] \zeta^\gamma + \lambda E_{\beta\gamma\delta} E_{\beta\gamma\delta} \zeta^\gamma \\
0 = E_{\alpha\beta\gamma} F_{\alpha} B^{\beta} \zeta^\gamma - E_{\alpha\beta\gamma} E_{\alpha} \Phi_{\beta} \zeta^\gamma + [4d_1 E_{\alpha\gamma} E_{\alpha} B^{\alpha\gamma}\zeta_{\alpha} - 2c_2 E_{\beta\gamma\delta} E_{\alpha\beta\gamma\delta} B^{\beta\gamma}\zeta_{\alpha}] \\
+ [a_0 / 2 E_{\alpha\beta\gamma} E_{\alpha} \zeta^\gamma - a_0 / 4 E_{\beta\gamma\delta} E_{\alpha\beta\gamma\delta} \zeta^\gamma a] - 2\lambda E_{\beta\gamma\delta} E_{\beta\gamma\delta} \zeta^\gamma \\
0 = C_{\alpha} E_{\alpha} B^{\beta} \zeta^\gamma - \phi_{\alpha} E_{\alpha} \phi_{\alpha} \zeta^\gamma \\
+ [c_0 E_{\alpha\beta\gamma} E_{\alpha\beta\gamma} B^{\alpha\beta\gamma}\zeta_{\gamma} + 4d_1 E_{\alpha\beta\gamma} E_{\alpha\beta\gamma} B^{\alpha\beta\gamma}\zeta_{\gamma}] + [-a_0 / 2 E_{\alpha\gamma} E_{\alpha} \zeta^\gamma + a_0 / 8 E_{\alpha} \phi \pi^{\alpha\beta\gamma}\zeta_{\beta}] \\
0 = -E_{\alpha} C_{\alpha} \phi_{\alpha} \zeta_{\beta} + 1/2 E_{\gamma\alpha} \phi_{\beta} \gamma_{\alpha} \phi_{\alpha} \zeta_{\beta} + 4d_1 E_{\alpha} \phi_{\alpha} \pi^{\alpha} \zeta_{\beta} \\
+ [a_0 a_0 / 24 E_{\alpha} \phi_{\alpha} \phi_{\beta} \pi^{\alpha\beta}\zeta_{\alpha} - a_0 / 12 (2E_{\gamma} \phi_{\gamma} B^{\gamma\beta}) - a_0 / 4 \phi \phi_{\beta} \varphi] \zeta_{\beta} + \lambda E_{\beta\gamma\delta} \pi^{\beta\gamma\delta}\zeta_{\alpha} 
\]
A.2 Physical bosonic fields

Identities that correspond to (L.7)

\[ 0 = \mathcal{F}_{\alpha(k-1)\beta\delta(k-1)} \epsilon^{\beta\gamma} f^{\alpha(k-1)\gamma} \]

\[ -2d_k [(k+1) E_{\alpha(k-1)\beta(k-k)} f^{\alpha(k-1)\gamma} f_{\alpha(k-1)\delta(k-k)}] \]

\[ - (k-1) E_{\beta\gamma} f^{\beta(k-k)\beta(k-k)} f_{\alpha(k-1)\delta(k-k)} \]

\[ - [((k-1) - E_{\beta\gamma} \Phi_{\alpha(k-1)\gamma(k-k)} + E_{\gamma} \Phi_{\alpha(k-1)\gamma(k-k)} + f^{\alpha(k-1)\gamma} \gamma(k-k)] \]

\[ - 2\lambda E_{\gamma} \Phi_{\alpha(k-1)\gamma\delta(k-k)} f^{\alpha(k-1)\gamma\delta(k-k)} \]

\[ - c_{k+1} - E_{\gamma} \Phi_{\alpha(k-1)\gamma\delta(k-k)} f^{\alpha(k-1)\gamma\delta(k-k)} \]

\[ 0 = \mathcal{F}_{\alpha(k-1)\beta\delta(k-1)} \epsilon^{\beta\gamma} f^{\alpha(k-1)\gamma} \]

\[ -2d_k [(k+1) E_{\alpha(k-1)\beta(k-k)} f^{\alpha(k-1)\gamma} f_{\alpha(k-1)\delta(k-k)}] \]

\[ - (k-1) E_{\beta\gamma} f^{\beta(k-k)\beta(k-k)} f_{\alpha(k-1)\delta(k-k)} \]

\[ - [((k-1) - E_{\beta\gamma} \Phi_{\alpha(k-1)\gamma(k-k)} + E_{\gamma} \Phi_{\alpha(k-1)\gamma(k-k)} + f^{\alpha(k-1)\gamma} \gamma(k-k)] \]

\[ - 2\lambda E_{\gamma} \Phi_{\alpha(k-1)\gamma\delta(k-k)} f^{\alpha(k-1)\gamma\delta(k-k)} \]

\[ - c_{k+1} - E_{\gamma} \Phi_{\alpha(k-1)\gamma\delta(k-k)} f^{\alpha(k-1)\gamma\delta(k-k)} \]
\[ 0 = \mathcal{F}_{\alpha(k-2)\gamma\alpha(k-2)}\delta^{\gamma}_{\gamma} f^{\alpha(k-2)\beta\alpha(k-2)}\dot{\zeta}_{\beta} \\
+ 2d_{k-1}[k E^{\gamma}_{\beta} \Phi^{\alpha(k-2)\alpha(k-2)\beta} f_{\alpha(k-2)\beta\alpha(k-2)} \dot{\zeta}_{\beta}] \\
- (k-2) E^{\alpha}_{\gamma} \Phi^{\alpha(k-3)\gamma\alpha(k-2)} f_{\alpha(k-2)\beta\alpha(k-2)} \dot{\zeta}_{\beta}\] \\
- \left[-(k-2) E^{\gamma}_{\beta} \Phi^{\alpha(k-1)\alpha(k-2)} \Omega^{\alpha(k-1)\beta\alpha(k-3)} \dot{\zeta}_{\beta}\right] \\
+ k E^{\gamma}_{\beta} \Phi^{\alpha(k-2)\gamma\alpha(k-2)\beta} \dot{\zeta}_{\beta} + E^{\gamma}_{\beta} \Phi^{\alpha(k-2)\gamma\alpha(k-2)} f^{\alpha(k-2)\beta\alpha(k-2)} \dot{\zeta}_{\gamma}\] \\
+ \lambda[E^{\gamma}_{\beta} \Phi^{\alpha(k-1)\alpha(k-2)} f^{\alpha(k-1)\alpha(k-2)} \dot{\zeta}_{\beta} + E^{\gamma}_{\beta} \Phi^{\alpha(k-1)\alpha(k-2)} f^{\alpha(k-2)\beta\alpha(k-2)} \dot{\zeta}_{\gamma}] \\
+ c_{k} \left[-E^{\gamma}_{\beta} \Phi^{\alpha(k-2)\gamma\alpha(k-1)} f_{\alpha(k-2)\beta\alpha(k-1)} \dot{\zeta}_{\beta}\right] \\
+ \frac{(k-2) c_{k-1}}{k(k-2)} \left[k E^{\gamma}_{\beta} \Phi^{\alpha(k-2)\alpha(k-3)} f_{\alpha(k-2)\beta\alpha(k-2)} \dot{\zeta}_{\beta}\right] \\
- \frac{(k-1) a_{k+1}}{2k(k-1)} \left[-E^{\gamma}_{\beta} \Phi^{\alpha(k-1)\alpha(k-2)} f^{\alpha(k-1)\beta\alpha(k-2)} \dot{\zeta}_{\beta}\right] \\
- \frac{a_{k}}{2k(k-1)} \frac{1}{k} [k E^{\gamma}_{\beta} \Phi^{\alpha(k-2)\gamma\alpha(k-2)} f^{\alpha(k-3)\beta\alpha(k-2)} \dot{\zeta}_{\beta} \\
+ (k-2) E^{\gamma}_{\beta} \Phi^{\alpha(k-2)\gamma\alpha(k-2)} f^{\alpha(k-2)\beta\alpha(k-2)} \dot{\zeta}_{\beta}]
\]

\[ 0 = \mathcal{F}_{\alpha\beta\gamma} e^{\beta}_{\beta} \dot{f}^{\alpha\gamma} \zeta^{\beta} \\
- 2d_{k-2}[3 E^{\gamma}_{\beta} \Phi^{\alpha\alpha\beta} f_{\alpha\alpha} \zeta_{\beta} - E^{\alpha}_{\beta} \Phi^{\beta\alpha\beta} f_{\alpha\alpha} \zeta_{\beta}] \\
- \lambda[E^{\gamma}_{\beta} \Phi^{\alpha\alpha\beta} f_{\alpha\alpha} \zeta_{\beta} - E^{\beta}_{\gamma} \Phi^{\alpha\beta\beta} f_{\alpha\alpha} \zeta_{\beta}] \\
- 2\lambda E^{\gamma}_{\beta} \Phi^{\alpha\beta\beta} f_{\alpha\alpha} \zeta_{\gamma} \\
- c_{2} \left[-E^{\gamma}_{\beta} f_{\alpha\alpha} \Phi^{\alpha\beta\hat{\alpha}} f_{\alpha\hat{\alpha}} \zeta_{\beta}\right] \\
- c_{2} \left[-E^{\gamma}_{\beta} f_{\alpha\alpha} \Phi^{\alpha\beta\hat{\alpha}} f_{\alpha\hat{\alpha}} \zeta_{\beta}\right] - \frac{c_{2}}{3} \left[3 E^{\gamma}_{\beta} \Phi^{\alpha\beta} f_{\alpha\alpha} \zeta_{\gamma} - E^{\beta}_{\gamma} \Phi^{\alpha\beta} f_{\alpha\alpha} \zeta_{\gamma}\right] \\
- \frac{a_{3}}{6} \left[-E^{\gamma}_{\beta} \Phi^{\alpha\beta} f_{\alpha\alpha} \zeta_{\gamma}\right] + \alpha_{0} \left[2 E^{\gamma}_{\beta} \Phi^{\alpha\beta\hat{\alpha}} f_{\alpha\hat{\alpha}} \zeta_{\gamma}\right] \\
+ \frac{a_{3}}{6} \left[-E^{\gamma}_{\beta} \Phi^{\alpha\beta} f_{\alpha\alpha} \zeta_{\gamma}\right] + \alpha_{0} \left[2 E^{\gamma}_{\beta} \Phi^{\alpha\beta\hat{\alpha}} f_{\alpha\hat{\alpha}} \zeta_{\gamma}\right] \\
0 = \mathcal{F}_{\alpha} e^{\alpha}_{\alpha} f^{\beta\alpha} \zeta_{\beta} + \left[4d_{1} E^{\alpha}_{\beta} \Phi^{\gamma\alpha} f^{\gamma\alpha} \zeta_{\beta} - c_{2} E^{\alpha}_{\beta} \Phi^{\alpha\beta\hat{\alpha}} f_{\gamma\hat{\alpha}} \zeta_{\gamma} + c_{2} E^{\alpha}_{\beta} \Phi^{\alpha\beta\hat{\alpha}} f_{\gamma\hat{\alpha}} \zeta_{\gamma}\right] \\
- \left[2 E^{\alpha}_{\beta} \Phi^{\gamma\alpha} f^{\gamma\alpha} \zeta_{\beta} - E^{\alpha}_{\beta} \Phi^{\gamma\alpha} f^{\gamma\alpha} \zeta_{\beta} - \frac{a_{3}}{6} E^{\alpha}_{\beta} \Phi^{\gamma\alpha} f^{\gamma\alpha} \zeta_{\beta} - \frac{a_{0}}{2} E^{\gamma\alpha} \Phi^{\alpha\beta} f^{\gamma\alpha} \zeta_{\gamma}\right] \\
+ \lambda(E^{\alpha}_{\beta} \Phi^{\beta\alpha} f^{\beta\alpha} \zeta_{\beta} + E^{\beta}_{\gamma} \Phi^{\gamma\alpha} f^{\beta\alpha} \zeta_{\alpha}) \]
\[ 0 = -e^\alpha F_\alpha A\zeta^\alpha - [4d_1 E_{\alpha\beta} \Phi^\beta A\zeta^\alpha + c_2 E_{\alpha\beta} \Phi^{\alpha\beta} A\zeta^\beta + c_0 E_{\beta\alpha} \Phi^\beta A\zeta^\alpha] \\
- [4(E_{\alpha\beta} \Phi_{\alpha} B^{(a)\zeta} + E_{\beta\alpha} \Phi_{\beta} B^{(a)\zeta}) - \frac{a_0}{2}(E_{\alpha\beta} \Phi_{\alpha} f^{(a)\zeta} - E_{\alpha\beta} \Phi_{\beta} f^{(a)\zeta})] \\
+ 2\lambda E_{\alpha\beta} \Phi_{\alpha} A\zeta^\beta \\
0 = -E^\alpha C_{\alpha}\varphi\zeta^\alpha - [-c_0 E_{\alpha\alpha} \Phi^\alpha\varphi\zeta^\alpha - 4d_1 E_{\alpha} \varphi\zeta^\alpha] \\
+ [-E_{\alpha} \alpha\pi^{\alpha\zeta} - \frac{a_0}{12} E_{\beta} \phi_{\beta} A\zeta^\beta] - 2\lambda E_{\alpha}\varphi\zeta^\alpha \\
\]

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