Robust Dynamic Average Consensus for a Network of Agents with Time-varying Reference Signals

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Abstract—This paper presents a continuous dynamic average consensus (DAC) algorithm for a group of agents to estimate the average of their time-varying reference signals cooperatively. We propose a consensus algorithm that is robust to agents joining and leaving the network, at the same time, avoid the chattering phenomena and guarantee zero steady-state consensus error. Our algorithm is an edge-based protocol with smooth functions in its internal structure to avoid the chattering effect. Furthermore, each agent can only perform local computations and can only communicate with its local neighbors. For a balanced and strongly connected underlying communication graph, we provide the convergence analysis to determine the consensus design parameters that guarantee the estimate of the average to asymptotically converge to the average of the time-varying reference signals. We provide simulation results to validate the proposed consensus algorithm and perform a performance comparison of the proposed algorithm to existing algorithms in the literature.

Index Terms—Robust dynamic average consensus, Time-varying signals, Multi-agent systems, Zero steady-state error, Chattering effect, Switching topology, Networked control systems

I. INTRODUCTION

Recently, triggered by advances in control techniques, communication and computation capabilities, there has been a great deal of interest in using multiple cooperating agents to perform complicated tasks such as formation control [1]–[5], deployment of autonomous vehicles in adversarial environments for Internet of Battle Things [6]–[8], disaster-area management [9], cooperative search and coverage [10], [11], sensor-fusion [12], distributed tracking [13], cooperative tasking [14]–[16], distributed optimization [17], resource allocation [18], network connectivity maintenance [19], and many others. In many of these tasks and related application areas of multi-agent systems and distributed networked systems, consensus protocols become the backbone of control and decision-making algorithms [20], [21]. Consensus protocols are the rules through which the agents in a networked system interact to reach an agreement on quantities of interest through local communications with their neighbors. Quantities of interest might take different forms, such as the average, the minimum, the maximum, and the min-max of local information at every agent in the network, based on which many researchers design various average consensus, min-consensus, max-consensus, and min-max consensus algorithms, respectively [22].

In this paper, we focus on the average consensus problem—the problem of reaching an agreement on the average of local reference signals at each agent in a distributed way. The reference signals at each agent could be sensor measurements, local computations, states of the agents, and states of the leader, to name a few. Commonly, an average consensus problem has two forms: the static average consensus and the dynamic average consensus. In the static average consensus, the agents use the reference signals only to initialize the consensus iteration algorithm. Then, the agents use linear weighting to update the estimate of the average iteratively [23]. In contrast, in the dynamic consensus, each agent’s signals derive the consensus iteration algorithm continuously [24]. Focusing on the Dynamic average consensus, many authors have studied dynamic average tracking (DAT) problems [25]–[28] and dynamic average consensus (DAC) problems [24], [29]–[33] extensively. DAT problems involve the design of combined estimator and controller algorithms to track a time-varying average of agents’ reference signals. On the other hand, in DAC problems, the estimator and control design problems are considered separately.

In the DAC, agents in the network estimate the average of local time-varying signals at each agent in the network via local communications with its neighboring agents. The authors in [24] attempted to address this problem by proposing a distributed robust consensus algorithm for reaching an average consensus on the average of signals with constant values in the presence of non-uniform delays in the network. In [34], proportional and proportional-integral estimators are proposed to estimate the average of slowly varying signals. However, the performance of the estimators in [24] and [34] deteriorate when the reference signals are fast time-varying signals. In [35], the authors extended the proportional integrator estimator to estimate the average of multiple time-varying signals, including some classes of polynomial and sinusoidal signals, at each agent in the network. Leveraging the singular perturbation theory, the authors in [36] proposed two DAC algorithms with exponential convergence for any initial conditions. However, the implementation of consensus algorithms in [35] and [36] requires the knowledge of the model of reference signals and the first and second derivatives of the reference signals, respectively.

Recently, employing a switching control, powerful non-linear consensus protocols that converge in finite time are proposed in [28], [32], [33]. However, the consensus algorithm in [28] is not robust to topology changes introduced...
by agents leaving and joining the network. In contrast, the DAC algorithms in [32], [33] are robust to topology changes and do not assume access to derivatives of reference signals except to determine or reset consensus gains. However, due to the discontinuity introduced by the switching control, the consensus algorithms in [32], [33] suffer from the chattering effect. The chattering phenomenon is undesired behavior, and reducing its effects require very small integration time-steps. Therefore, a discrete-time implementation of algorithms with switching control requires high sampling rates to decrease the adverse effects of the chattering phenomena. In fact, authors in [32], [33] were aware of the adverse effect of the chattering phenomena and proposed a boundary-layer approximation (approximating the discontinuous switching signal by a smooth function inside a boundary layer) to remove the chattering effect. However, the boundary layer approximation guarantees only the convergence of the consensus error to an $\epsilon$-neighborhood of the origin in finite time. In [37], a new continuous dynamic consensus algorithm that avoids the chattering phenomena and, at the same time, does not require the knowledge of the derivatives of reference signals is proposed. However, the convergence of the consensus algorithm in [37] is only limited to guaranteeing a bounded steady-state error.

In this paper, we introduce a continuous robust DAC algorithm for a network of dynamic agents with time-varying reference signals where the underlying network topologies are balanced and strongly connected bidirectional graph. The algorithm is an edge-based algorithm where the edges capture the disagreement between agent $i$ and its neighboring agents. We design the internal structure of the consensus estimator based on the edges via a smooth function. In our algorithm, unlike in many DAC algorithms (for example, see [28], [36]), we utilize the knowledge of the derivatives of reference signals only when it is available. In other words, we can convert the proposed algorithm to an equivalent protocol that does not require the knowledge of the derivatives of reference signals through a coordinate transformation when the knowledge of the derivatives of reference signals is not available. Also, the proposed consensus protocol i) is robust to topology changes, ii) does not suffer from the chattering phenomena, and iii) guarantees zero steady-state error.

Compared to methods in [32], [33], our algorithm does not involve discontinuous switching signal. In practice, it is impossible to implement the discontinuous switching signal precisely, and therefore, methods in [32], [33] guarantees only the convergence of the consensus error to an $\epsilon$-neighborhood of the origin. However, our algorithm guarantees asymptotic convergence of the consensus error to the origin versus the algorithm in [37] that only guarantees a bounded steady-state error and the methods in [32], [33] that guarantee a small steady-state error in reality. Simulation results are provided to compare the performance of the proposed consensus algorithm in this paper with [33].

The organization of the rest of this paper is as follows. In Section II, the DAC problem is formulated. In Section III, we propose a robust DAC estimator to estimate the average of local time-varying signals at each agent in the network via only local communications with its neighboring agents. In this Section, we also discuss the detailed stability and convergence analyses of the proposed consensus algorithm. In Section IV, we provide detailed simulation results to evaluate the proposed estimators numerically. Finally, concluding remarks are provided in Section V.

II. MATHEMATICAL PRELIMINARIES AND PROBLEM FORMULATION

For a bidirectional graph $\mathcal{G}(t) = (\mathcal{V}(t), \mathcal{E}(t))$, where $\mathcal{V}(t) \triangleq \{v_1(t), \cdots, v_N(t)\}$ is the set of agents and $\mathcal{E}(t) = \{\mathcal{E}_1(t), \cdots, \mathcal{E}_k\} \subseteq \mathcal{V}(t) \times \mathcal{V}(t) - \{(v_i(t), v_j(t)) | v_i(t) \in \mathcal{V}(t)\}$ is the set of communication links among the agents. The set of neighbors $\mathcal{N}_i(t)$ of agent $i$, $i = \{1, \cdots, N\}$ is given by:

$$\mathcal{N}_i(t) = \{j \in \mathcal{V}(t) : (j, i) \in \mathcal{E}(t)\}. \quad (1)$$

Let $\mathcal{A}(t) = [a_{ij}] \in \{0, 1\}^{N \times N}$ be the adjacency matrix of graph $\mathcal{G}(t)$, where $a_{ij} = 1$ if $(v_i(t), v_j(t)) \in \mathcal{E}(t)$ and $a_{ij} = 0$ otherwise. Then, the degree matrix and the laplacian matrix of graph $\mathcal{G}(t)$ is given by $\Delta(t) = \text{diag}(\mathcal{A}\mathcal{L}_N)$ and $\mathcal{L}(t) = \Delta(t) - \mathcal{A}(t)$, respectively. The notation $\mathcal{L}_N$ denotes $N$-dimensional vector of all ones. The incidence matrix of graph $\mathcal{G}(t)$ is given by $\mathcal{B}(t) = [b_{ij}], \in \{-1, 0, 1\}^{N \times \mathcal{E}}$, where $b_{ij} = -1$ for the outgoing communication link from agent $i$, $b_{ij} = 1$ for the incoming communication link to agent $i$, and $b_{ij} = 0$ otherwise. The following two lemmas will be used in the ensuing sections.

**Lemma 1** [32] Let $M \triangleq (I_N - \frac{1}{N} \mathcal{L}_N^T \mathcal{L}_N)$. For any balanced and strongly connected bidirectional graph $\mathcal{G}(t)$, the Laplacian matrix $\mathcal{L}(t)$ and the incidence matrix $\mathcal{B}(t)$ satisfy $M = \mathcal{L}(t)^{+} = (\mathcal{B}(t) \mathcal{B}(t)^T(t))(\mathcal{B}(t) \mathcal{B}(t)^T(t))^+$, where $(\cdot)^+$ is the generalized inverse.

From Lemma 1 it is also clear that the Laplacian matrix $\mathcal{L}(t)$ and the incidence matrix $\mathcal{B}(t)$ of graph $\mathcal{G}(t)$ are related as $\mathcal{L}(t) = \mathcal{B}(t) \mathcal{B}(t)^T(t)$.

**Lemma 2** [38] Laplacian matrix of a balanced and strongly connected bidirectional graph $\mathcal{G}(t)$ is a positive definite matrix with an eigenvalue at zero corresponding to right and left eigenvectors $\mathcal{L}_N$ and $\mathcal{L}_N^T$, respectively.

**Lemma 2** describes that $\forall x \in \mathbb{R}^N, x^T \mathcal{L}(t)x \geq 0$, $\mathcal{L}_N^T \mathcal{L}(t) = 0$, $\mathcal{L}_N^T \mathcal{B}(t) = 0$, $\mathcal{B}(t) \mathcal{B}(t)^T(t)$, $\mathcal{L}(t) \mathcal{L}_N = 0$, $\mathcal{B}(t) \mathcal{B}(t)^T(t)$, $\mathcal{L}_N \mathcal{L}_N^T = 0$, and $\mathcal{L}_N \mathcal{L}_N^T = 0$.

Now, consider a network of $N$ agents, where each agent $i$, $i = \{1, \cdots, N\}$, computes or measures a time-varying reference signal $z_i(t) \in \mathbb{R}$ with bounded first derivative $\dot{z}_i(t)$, i.e., $\sup_{t \geq t_0} \|\dot{z}_i(t)\|_{\infty} \leq \psi_i$, where $\psi_i$ is a positive constant. Let the underlying network topology at time $t$ be given by a balanced and strongly connected bidirectional graph $\mathcal{G}(t)$, and let the average of the agents’ time-varying reference signals be: $\bar{z}(t) = \frac{1}{N} \sum_{i=1}^{N} z_i(t) = \frac{1}{N} \mathcal{L}_N^T z(t)$, where $\bar{z}(t) = \{z_1, \cdots, z_N\}^T$. Our main objective is to design a consensus algorithm that the agents use to cooperatively estimate a time-varying average signal $\bar{z}(t)$ in a distributed fashion through local communications with their respective
neighbors in the network. More precisely, we state the main problem as follows.

**Problem 1** Consider a network of $N$ agents where the underlying network topology at time $t$ is given by a balanced and strongly connected bidirectional graph. Let $\gamma_i(t)$ be agent $i$’s estimate of the average signal $\bar{z}(t)$, where $i = \{1, \ldots, N\}$. Let agent $i$’s estimation error and agent $j$’s disagreement with agent $j$ are computed as $\hat{\gamma}_i(t) = \gamma_i(t) - \bar{z}(t)$, and $\bar{\gamma}_i(t) = \gamma_i(t) - \gamma_j(t)$, where $j \in \mathcal{N}_i(t)$, respectively. Then, design a consensus protocol such that $\lim_{t \to \infty} \hat{\gamma}_i(t) \to 0$, that is, i) $\sum_{i=1}^{N} \gamma_i(t) = 0$, and ii) $\lim_{t \to \infty} (\hat{\gamma}_i(t) - \bar{\gamma}_j(t)) \to 0$, $\forall(i, j) \in \mathcal{E}(t)$, $t \geq t_0$.

III. THE PROPOSED ROBUST DAC

In this section, we present a DAC algorithm to solve problem 1. We employ edge-based approaches to design the consensus protocol, where, unlike the node-based approaches, there exist multiple internal states per each node in the network.

A. Robust DAC algorithm

In this section, we propose an enhanced DAC algorithm that leverages the knowledge of the derivative of reference signals, if available. When the derivative of reference signals is not available, we convert the proposed algorithm into another DAC protocol utilizing coordinate transformation. In this case, the transformed DAC algorithm does not require the knowledge of the derivative of reference signals. Let agent $i$’s estimation error and agent $i$’s disagreement with agent $j$ are computed as $\tilde{\gamma}_i(t) = \gamma_i(t) - \bar{z}(t)$, and $\bar{\gamma}_i(t) = \gamma_i(t) - \gamma_j(t)$, where $j \in \mathcal{N}_i(t)$, respectively. Now, let agent $i, i = \{1, \ldots, N\}$, implements a DAC algorithm of the form

\[
\dot{\eta}_i^+(t) = -\alpha(\bar{z}_i(t) - \bar{z}_j(t)) - \rho \tanh(c(\gamma_i(t) - \gamma_j(t)))
\]

\[
\dot{\eta}_i^-(t) = -\alpha(\bar{z}_i(t) - \bar{z}_j(t)) - \rho \tanh(c(\gamma_i(t) - \gamma_j(t)))
\]

\[
\gamma_i(t) = \sum_{j \in \mathcal{N}_i} \eta_i^+(t) - \sum_{j \in \mathcal{N}_i} \eta_i^-(t) + z_i(t)
\]

\[
\eta_i^+(t_0) = \eta_{i,j_0}, \eta_i^-(t_0) = \eta_{i,j_0}, c \geq 1, j \in \mathcal{N}_i,
\]

where $\eta_i = [\eta_i^+, \eta_i^-] \in \mathbb{R}^{2N}$, is the internal estimator state; $\alpha, \rho \in \mathbb{R}$ and $c \in \mathbb{R}$ are the global design parameters, and $\gamma_i(t) \in \mathbb{R}$ is the estimate of the average. The edge dynamics is captured via the internal state dynamics of the estimator. From (2), it is clear that the edge dynamics captures the state of the disagreement between agent $i$ and agent $j$. This approach makes the protocol robust to agents joining or leaving the network, and communication link failures among the agents. In a vector notation, (2) can be written as

\[
\dot{\eta}(t) = -\alpha \mathbf{B}^T(t) \dot{z}(t) - \rho \tanh(c \mathbf{cB}^T(t) \gamma(t)),
\]

\[
\gamma(t) = \mathbf{B}(t) \eta(t) + z(t), \quad \eta(t_0) = \eta_0,
\]

where $\eta(t) = [\eta_1, \ldots, \eta_N]^T$, $\gamma(t) = [\gamma_1, \ldots, \gamma_N]^T$, $z(t) = [z_1, \ldots, z_N]^T$, $\bar{z}(t) = [\bar{z}_1, \ldots, \bar{z}_N]^T$, $\mathbf{B}(t)$ is the incidence matrix, and the $\tanh(.)$ is defined component wise. The $\tanh(.)$ function in the proposed the DAC algorithm empowers the consensus protocol to avoid the chattering phenomena.

Let $\xi(t) = \eta(t) + \alpha \mathbf{B}^T z(t)$. Then, the DAC algorithm (3) can be transformed to the following equivalent algorithm

\[
\xi(t) = -\rho \tanh(c \mathbf{cB}^T(t) \gamma(t)), \quad \xi(t_0) = \xi_0,
\]

\[
\gamma(t) = \mathbf{B}(t) \xi(t) + (I - \alpha \mathbf{B}(t) \mathbf{B}^T(t)) z(t),
\]

which can be implemented without the knowledge of derivative information of reference signals. The agent-wise representation of the consensus algorithm (4) is given as

\[
\xi_{i,j}^+(t) = -\rho \tanh(c(\gamma_{i,j}(t) - \gamma_{j,i}(t)))
\]

\[
\xi_{i,j}^-(t) = -\rho \tanh(c(\gamma_{j,i}(t) - \gamma_{i,j}(t)))
\]

\[
\gamma_i(t) = \sum_{j \in \mathcal{N}_i} \xi_{i,j}^+(t) - \sum_{j \in \mathcal{N}_i} \xi_{i,j}^-(t) + (1 - \alpha d_i) z_i(t)
\]

\[
\xi_{i,j}^+(t_0) = \xi_{i,j_0}, \xi_{i,j}^-(t_0) = \xi_{i,j_0}, c \geq 1, j \in \mathcal{N}_i,
\]

where $d_i$ is the degrees of agent (node) $i$ in the network. The proposed consensus protocols (3) and (5) are guaranteed to asymptotically converge to the average $\bar{z}(t)$ with zero-steady-state error. This claim is formally stated in the following theorem.

**Theorem 1** For the balanced and strongly connected bidirectional graph $\mathcal{G}(t)$ and time-varying reference signals $z_i(t), i = \{1, \ldots, N\}$ with bounded first derivatives, the robust DAC algorithm in (3) guarantees that the consensus error $\tau(t) = |\gamma(t) - \frac{1}{N} \mathbf{1}^T \bar{z}(t)|$, asymptotically converges to zero for any $\eta_0$ if and only if $\rho > 0, c \geq 1$, and $\alpha I - (\mathbf{B}(t) \mathbf{B}^T(t))^T > 0$.

**Proof:** From (3), the consensus error and the consensus error dynamics can be written as

\[
\gamma(t) = \gamma(t) - \frac{1}{N} \mathbf{1}^T \bar{z}(t) = \mathbf{B}(t) \eta(t) + \mathbf{M} z(t)
\]

\[
\dot{\gamma}(t) = \mathbf{B}(t) \dot{\eta}(t) + \dot{M} z(t)
\]

From the consensus error dynamics, we have $\sum_{i=1}^{N} \gamma_i(t) = \mathbf{1}_N^T \gamma(t) = 0, 1 \mathbf{1}_N^T \dot{\gamma}(t) = 0$, as $1 \mathbf{1}_N \dot{\eta}(t) = 0$ and $1 \mathbf{1}_N \mathbf{M} z(t) = 0$ for all $t \geq t_0$. Therefore, the consensus protocol in (3) does not require special initialization requirement.
To prove the convergence of consensus protocol in (3), we consider a candidate Lyapunov function
\[ V = \frac{1}{2} \tilde{\gamma}^T(t)\tilde{\gamma}(t). \]  
Taking a derivative of \( V \), we have
\[ \dot{V} = \dot{\tilde{\gamma}}^T(t)B(t)\dot{\tilde{\gamma}}(t) \]
\[ + \tilde{\gamma}^T(t)B(t)B^T(t)(B(t)B^T(t))^+ \dot{\tilde{\gamma}}(t). \]  
Letting \( \dot{\gamma}(t) = B^T(t)\tilde{\gamma}(t) = B^T(t)\gamma(t) \) and using \( \tilde{\gamma} \) from (3), we have
\[ \dot{V} = -\rho \dot{\gamma}^T(t) \tanh(c\dot{\gamma}(t)) \]
\[ - \dot{\gamma}^T(t)B^T(t)(\alpha I - (B(t)B^T(t))^+)\dot{\gamma}(t) \]  
Now, \( \dot{V} < 0 \) if \( \rho > 0 \), \( \alpha \) are selected as
\[ \alpha I - (B(t)B^T(t))^+ > 0. \]  
Therefore, the consensus protocol (3) guarantees the asymptotic convergence of the estimate of the average \( \gamma(t) \) to \( \tilde{\gamma}(t) \).

Note that we can also conclude condition (10) for consensus protocol (4) by utilizing Lyapunov function (7). The proposed consensus protocol is also robust to topology changes where the communication link failures due to agents leaving and joining the network switch the underlying communication graph. We summarize this claim in the following proposition.

**Proposition 1** If the proximity based graph \( G_t \) is switching, balanced and strongly connected, for time-varying reference signals \( z_i(t), i = \{1, \cdots, N\} \) with bounded first derivatives, the robust DAC algorithm in (3) guarantees that the consensus error \( \tilde{\gamma}(t) = |\gamma(t) - \frac{k_B z(t)}{N}| \) asymptotically converges to zero for any \( \eta_0 \) if and only if \( \rho > 0, c \geq 1, \) and \( \alpha I - (B(t)B^T(t))^+ > 0. \)

**Proof:** The proof is similar to that of Theorem 1 and is hence omitted.

**IV. Simulation Results**

In this section, we provide the simulation results to verify the performance of the proposed consensus algorithm. Consider a network of nine agents with the underlying communication graph given in Figure 2. Let the agents’ time-varying reference signals, \( z_i(t), i = \{1, \cdots, 9\} \) be given as
\[ z_1 = 5 \cos(t), z_2 = 4 \cos(t), z_3 = 3 \cos(t), z_4 = 2 \cos(t), \]
\[ z_5 = \cos(t), z_6 = -\cos(0.01t), z_7 = -\cos(0.01t), \]
\[ z_8 = -3 \cos(0.01t), z_9 = -\cos(0.01t). \]  
We let the agent 2 to fail to communicate with its neighbours for all times after 2\( s \) as shown in Figure 2 to verify the robustness of the proposed algorithm to agents leaving the network. This action will create two sub-networks. The first sub-network has only a node and no edges, while the other sub-network has the remaining 8-nodes and the edges among them. We present the estimate of the average of the agents’ time-varying signals implementing the proposed consensus algorithm (5). The consensus algorithm (5) guarantees the asymptotic convergence of the consensus error. Choosing the parameters of the consensus algorithm (5) as \( c = 4, \alpha = 0.16, \) and \( \rho = 4.1 \) with sampling time of 0.01s, the distributed estimate of the average of the agents’ time varying reference signals and the consensus estimation errors are presented in Figures 3a and 3b, respectively.

We compare our results to the robust discontinuous consensus protocol in [33] which is proved to converge in finite time. However, the precise implementation of the discontinuous switching estimator input signal requires very tiny sampling time and it is impossible to guarantee zero steady-state error in practice. Now, choosing the parameters of the consensus algorithm in [33] as \( \alpha_1 = 10 \) with sampling time of 0.0001s, the estimate of the average and the consensus estimation errors are presented in Figures 3c and 3d, respectively. The results show that unlike the consensus algorithm proposed in this paper, the consensus algorithm in [33] requires very low sampling times. For comparably higher sampling times, the results of the consensus algorithm in [33] deteriorates as demonstrated in Figures 3e-3f. The simulation results in Figures 3e-3f are generated by choosing the parameters of the consensus algorithm as \( \alpha_1 = 5.7, \alpha_2 = 4.6, \alpha_3 = 3.4, \alpha_4 = 2.3, \alpha_5 = 1.2, \alpha_6 = 1.2, \alpha_7 = 2.3, \alpha_8 = 3.4, \) and \( \alpha_9 = 4.6 \) with the sampling time of 0.01s. In fact, the authors in [33] observed these issues in their algorithm and proposed a boundary layer approximation to circumvent the problem. Nevertheless, with the boundary layer approximation, the convergence proof will only guarantee a bounded steady-state error in a finite time. However, designed by similar approaches to [33], our algorithm guarantees asymptotic convergence of the average consensus error while avoiding any chattering effects.
Fig. 3: Performance comparisons of the proposed DAC algorithm with the DAC algorithm proposed in [33]: (a) Estimate of average of multiple time-varying signals using our proposed consensus algorithm (5), (b) Log plot of estimation error of our proposed consensus algorithm (5), (c) Estimate of average of multiple time-varying signals using the consensus algorithm in [33] with sampling time of 0.0001s, (d) Log plot of estimation error of the consensus algorithm in [33] with sampling time of 0.0001s, (e) Implementation of the consensus algorithm in [33] with sampling time of 0.01s, and (f) Log plot of estimation error of the consensus algorithm in [33] implemented with sampling time of 0.01s.

V. CONCLUSION

In this paper, we proposed a DAC algorithm that allows a network of agents to estimate the average of their time-varying reference signals cooperatively. The algorithm is robust to agents joining and leaving the network, at the same time, remove the chattering phenomena that arise in many non-linear consensus protocols. Further, we provided the convergence and robustness analysis of the proposed consensus protocol utilizing Lyapunov functions. The convergence analysis shows that the algorithm guarantees asymptotic convergence to zero steady-state error. We also provided a discrete-time implementation and demonstrated a simulation example to show the effectiveness of the proposed consensus protocol. Future work focuses on extending the algorithm to directed graph topology in the presence of delays, and sensor and model uncertainties.

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