IS $M_t \simeq M_W$ RULED OUT?*

HOWARD E. HABER†

CERN, TH-Division
CH-1211 Geneva 23, Switzerland

Abstract

A four generation supersymmetric model is proposed, in which the Tevatron “top-quark” events are reinterpreted as the production of $t'$ which decays dominantly to $bW^+$. In this model, $m_t \simeq m_W$, and $t \rightarrow \tilde{t}\tilde{\chi}_1^0$, with $\tilde{t} \rightarrow c\tilde{\chi}_1^0$. This decay chain, which rarely produces a hard isolated lepton, would have been missed in all previous top quark searches. A narrow region of the model parameter space exists which cannot yet be ruled out by present data. This model predicts a rich spectrum of new physics which can be probed at LEP-II and the Tevatron.

Invited talk presented at the XXXth Rencontres de Moriond, “Electroweak Interactions and Unified Theories”, Les Arcs, Savoie, France, 11–18 March, 1995.

---

* Work supported in part by the U.S. Department of Energy.
† Permanent address: Santa Cruz Institute for Particle Physics, University of California, Santa Cruz, CA 94064 USA.
1. Introduction

Recently, the CDF and D0 Collaborations have announced the discovery of the top quark at the Tevatron,\(^1\) with a measured mass of \(m_t = 176 \pm 8 \pm 10\) GeV and \(m_t = 199^{+19}_{-21} \pm 22\) GeV, respectively. Both measurements are in excellent agreement with the top quark mass deduced by the LEP global analysis of precision electroweak measurements.\(^2\) The LEP determination of \(m_t\) is based on the sensitivity of electroweak observables in \(Z\) decay to virtual top quark exchange, which enters in two distinct ways. First, top quark loops in gauge boson self-energies (the so-called oblique corrections) can directly effect the properties of the \(Z\). The most famous of the oblique corrections is the top-quark contribution to the electroweak \(\rho\) parameter,\(^3\) which is given by \(\rho = 1 + \delta \rho\), where \(\delta \rho \simeq 3GFm_t^2/8\pi^2\sqrt{2}\). Second, virtual top quark exchange can contribute to certain vertex radiative corrections. For example, the one-loop correction to \(Z \to b \bar{b}\) is also quadratically sensitive to the top quark mass. The LEP global fit yields \(m_t = 176 \pm 10^{+17}_{-19}\) GeV, where the second set of errors corresponds to varying the Higgs mass between 60 GeV and 1 TeV (with a central value of 300 GeV).

Clearly a heavy top quark mass has been confirmed. But is there an alternative interpretation? In this paper, I present a model constructed in collaboration with Marcela Carena and Carlos Wagner,\(^4\) in which we explore the possibility of circumventing the apparent ironclad conclusion that \(m_t \gg m_W\).

2. A Four Generation Supersymmetric Model with a Light Top Quark

Consider that the LEP measured rate for \(Z \to b \bar{b}\) differs from the Standard Model prediction by 2.4\(\sigma\). Defining \(R_b \equiv \Gamma(Z \to b \bar{b})/\Gamma(Z \to \text{hadrons})\),\(^2\)

\[
R_b = \begin{cases} 
0.2204 \pm 0.0020, & \text{LEP global fit;} \\
0.2157, & \text{Standard Model prediction.}
\end{cases}
\] (2.1)

Clearly, one does not give up on the Standard Model because of a 2.4\(\sigma\) discrepancy. Nevertheless, it is amusing to note that if one extracts the top quark mass from this measurement alone, one would conclude that \(m_t < m_W\)! We proceed by fixing \(m_t \simeq m_W\) in what follows. Of course, with such a light top quark mass, we must address three obvious questions:

1. Would not a top quark with \(m_t \simeq m_W\) have already been discovered at hadron colliders?
2. What is the particle recently announced by CDF and D0 which is observed to decay into \(bW\)?
3. What is the nature of the new physics that contributes to the oblique corrections and simulates the heavy top quark inferred by the LEP experiments?
If the top quark were sufficiently light, then $W^+ \rightarrow t\bar{b}$ would be kinematically allowed; this would modify the total width of the $W$. But $\Gamma_W$ can be measured at hadron colliders indirectly by studying the ratio of production cross section times leptonic branching ratio of the $W$ and $Z$. The most recent analysis of this kind, reported at this meeting by the D0 collaboration,\textsuperscript{5} finds $m_t > 62$ GeV. Direct searches for the top quark at hadron colliders assume that an observable fraction of top quark decays results in a final state lepton. For example, in ref. 6, the D0 collaboration ruled out the mass range $m_W + m_b \lesssim m_t < 131$ GeV, assuming that the decay $t \rightarrow bW^*$ is not unexpectedly suppressed.\textsuperscript{6} Previous top quark searches at hadron colliders are able to close the window between 62 and 85 GeV, assuming that $t \rightarrow bW^*$ is the dominant top-quark decay mode. However, in this case the final state is three-body since $W^*$ is virtual. If the top quark were to possess any two-body decay modes (due to new physics processes), and if these modes rarely produced leptons, then a top quark in this mass region would not have been detected in any experiment.

An example of such a scenario occurs in supersymmetric models in which the decay $t \rightarrow \tilde{t}\tilde{\chi}_1^0$ is kinematically allowed (where $\tilde{t}$ is the top squark and $\tilde{\chi}_1^0$ is the lightest neutralino). Experimental searches for both $\tilde{t}$ and $\tilde{\chi}_1^0$ place constraints on their masses, but do not rule out the possibility of $M_{\tilde{t}} + M_{\tilde{\chi}_1^0} < m_W$. In particular, the LEP neutralino and chargino searches\textsuperscript{7} obtain a limit on the lightest neutralino mass which typically lies between 20 and 25 GeV. Using this result and the limits on the top squark mass from searches at LEP and at the Tevatron,\textsuperscript{8} one finds that the mass region $42 \lesssim M_{\tilde{t}} \lesssim 60$ GeV cannot be excluded.

To be definite, we choose $m_t \simeq m_W$, $M_{\tilde{t}} \simeq 50$ GeV and $M_{\tilde{\chi}_1^0} \simeq 25$ GeV. Then, the dominant decay chain is $t \rightarrow \tilde{t}\tilde{\chi}_1^0$ followed by $\tilde{t} \rightarrow c\tilde{\chi}_1^0$ through a one-loop process,\textsuperscript{9} which rarely produces a hard isolated lepton. Hence, these events would not have been detected at hadron colliders. But, now we must reconsider to the recent CDF and D0 discoveries and the LEP “measurement” of $m_t$. We propose to account for these results by introducing a fourth generation of quarks (and leptons) plus their supersymmetric partners. Then, $t' \rightarrow bW^+$ can be the source of the CDF and D0 events, while the effects of the third and fourth generation quarks and squarks contributing to the oblique corrections are large enough to be consistent with LEP precision electroweak data.
3. Phenomenological and Theoretical Constraints

The model parameters are determined by imposing the phenomenological and theoretical constraints listed below.

1. In order that the $t'$ be consistent with the CDF and D0 “top-quark” events, its dominant decay must be $t' \to bW^+$. This means that $t' \to b'W^*$ must be a three-body decay. Furthermore, the $t'$-$b$ mixing angle ($V_{t'b}$) must not be too small; otherwise, the latter decay will dominate. We find:

$$\frac{\Gamma(t' \to b'W^*)}{\Gamma(t' \to bW)} = \frac{9G_Fm_{t'}^2}{\pi^2\sqrt{2}|V_{t'b}|^2} \int_0^{1-2\sqrt{x+x}} \frac{z(1-z+x)\sqrt{(1-z+x)^2-4x}}{(1-zm_{t'}/m_{W})^2} dz, \quad (3.1)$$

where $x \equiv m_{b'}/m_{t'}^2$. Since the rate of the CDF and D0 “top-quark” events is consistent with the QCD prediction for $t\bar{t}$ production under the assumption that $BR(t \to bW^+) = 100\%$, a reinterpretation of these events as $t'\bar{t}'$ production (followed by $t' \to bW^+$) requires $BR(t' \to bW^+)$ to be near 1. We assume that $V_{t'b}$ lies between $V_{cb} = 0.04$ and $V_{ud} = 0.2$; for definiteness, we choose $V_{t'b} = 0.1$. Then, if we require $BR(t' \to bW^+) > 0.75$, it follows that we must take $m_{b'} \geq 105$ GeV.

2. In low-energy supersymmetric model building, it is common practice to require that all couplings of the model stay perturbative up to very high energies. Here, we shall insist that the Higgs-quark Yukawa couplings do not blow up below the grand unification (GUT) scale. Then, if we wish to have the $t'$ and $b'$ masses as large as possible, it follows that the corresponding Yukawa couplings will be forced to lie close to their quasi-infrared fixed points.\footnote{For example, if we take $m_{t'} \geq 170$ GeV, then we find that $m_{b'} \leq 110$ GeV. Combined with point 1, we see that the mass of the $b'$ is essentially fixed. Moreover, since we are at the infrared fixed point values of the Yukawa couplings, which depend on the corresponding masses and the ratio of Higgs vacuum expectation values, $\tan \beta$, it follows that $\tan \beta$ is also fixed. In this work, we choose $m_{t'} = 170$ GeV and $m_{b'} = 110$ GeV; for these values $\tan \beta \simeq 1.6$. One can also add in the requirement that the fourth generation leptons lie at their quasi-infrared fixed points (in order to maximize their masses). We assume that the fourth generation neutrino ($N$) is a Dirac fermion. Then, the resulting lepton masses are: $m_{\tau'} \simeq 50$ GeV and $m_N \simeq 80$ GeV. Remarkably, these masses lie above the corresponding bounds from LEP. In addition, it is amusing to note that the above masses are consistent with the unification of all four fermion-Higgs Yukawa couplings at the GUT scale!}
3. In order that $M_{\tilde{t}} < m_t$, there must be substantial $\tilde{t}_L - \tilde{t}_R$ mixing. The squared mass of $\tilde{t}$ is given by the smallest eigenvalue of the matrix

$$
\begin{pmatrix}
M^2_Q + m^2_t + c_L m_Z^2 & m_t(A_t - \mu \cot \beta) \\
m_t(A_t - \mu \cot \beta) & M^2_U + m^2_t + c_R m_Z^2
\end{pmatrix},
$$

where $c_L \equiv \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) \cos 2\beta$, $c_R \equiv \frac{2}{3} \sin^2 \theta_W \cos 2\beta$, $M^2_Q$, $M^2_U$, and $A_t$ are soft-supersymmetry-breaking parameters, and $\mu$ is the supersymmetric Higgs mass parameter. Large mixing requires that the off-diagonal terms above are of the same order as the diagonal terms. If there is large mixing in the third generation squark sector, why not in the fourth generation squark sector as well? In fact, if $A_t' \simeq A_t$, the mixing in the fourth generation squark sector would be too large, driving the smallest eigenvalue of the $\tilde{t}_L - \tilde{t}_R$ squared-mass matrix negative. Remarkably, this does not occur due to the infrared fixed-point behavior of the fourth generation. $A_t'$ is driven to a fixed point that is independent of its high energy value. Roughly, $A_t' \simeq -2m_{1/2}$ where $m_{1/2}$ is the high-energy (GUT-scale) value of the gaugino Majorana mass. In contrast, the top quark is not controlled by the infrared fixed point (since in our model $m_t$ is not large enough); hence, $A_t$ can be chosen large. Moreover, choosing $\mu$ negative enhances the third generation squark mixing while it somewhat suppresses the fourth generation squark mixing.

4. If gaugino Majorana mass parameters are unified with a common GUT-scale mass given by $m_{1/2}$, then the gluino, chargino and neutralino masses are determined by $m_{1/2}$, $\mu$, and $\tan \beta$. Our model prefers the region of parameter space where $m_{1/2} \ll |\mu|$ (with $\mu$ negative). Then, our choice of $M_{\tilde{\chi}^0_1} \simeq 25$ GeV fixes $m_{1/2} \simeq 55$ GeV. Typical values for the masses of the other light chargino and neutralino states are $M_{\tilde{\chi}^\pm_1} \simeq M_{\tilde{\chi}^0_2} \simeq 60$ GeV. The choice of $m_{1/2}$ also fixes the gluino mass; we find $M_{\tilde{g}} \simeq 3m_{1/2} \simeq 165$ GeV. The dominant decay of this gluino would be $\tilde{g} \rightarrow \tilde{t}\tilde{t}$ (or its charge-conjugated state). Such a gluino cannot be ruled out by present Tevatron limits.

We have checked that virtual effects of the light supersymmetric particles do not generate new conflicts with experimental data. For example, because the light chargino is nearly a pure gaugino, the chargino–top squark loop has a negligible effect on the rate for $Z \rightarrow b\bar{b}$. Our model then predicts $R_b = 0.2184$, which is within one standard deviation of the measured LEP value [eq. (2.1)]. The improvement over the Standard Model result is due to the fact that $m_t \simeq m_W$. As a second example, one of the most sensitive tests of the model is to check that its prediction for $b \rightarrow s\gamma$ is consistent with $1.0 \times 10^{-4} \lesssim BR(b \rightarrow s\gamma) \lesssim 4 \times 10^{-4}$, as required by the CLEO measurement. The predictions of our model live comfortably within this bound.
5. The mass of the lightest CP-even Higgs boson should lie above the LEP lower limit. For \( \tan \beta = 1.6 \), the tree-level upper bound on the light Higgs mass is \( m_{h^0} \leq m_Z |\cos 2\beta| = 40 \) GeV, which would have been detected at LEP. However, radiative corrections can raise the upper bound substantially. The bound increases with increasing values of the soft-supersymmetry-breaking parameters which appear in the squark squared-mass matrix [eq. (3.2)]. We find as a typical range of values that \( m_{h^0} \simeq 65–70 \) GeV, above the present LEP limits.

6. The Tevatron may be able to rule out the existence of the \( b' \) with mass \( m_{b'} \simeq 110 \) GeV. If kinematically allowed, the decay \( b' \to \tilde{t}\tilde{\chi}_1^- \) would be the dominant decay mode. If disallowed, there would be a competition between \( b' \to Wc \) (a change of two generations) and \( b' \to W^*\tilde{t} \) (a change of one generation, but suppressed by three-body phase space). If necessary, one can choose \( |V_{b'c}| \ll |V_{t'b}| \) to remove the possibility of \( b' \to Wc \). Then, all \( b' \) decays would result in \( W^*\tilde{e}\tilde{\chi}^0_1 \). There are no published limits that exclude such a \( b' \). However, a dedicated search at the Tevatron should be able to discover or exclude such events.

7. Perhaps the most difficult requirement for our model is to reproduce the oblique electroweak radiative corrections inferred from the precision measurements at LEP. Consider the contributions to \( \delta \rho \). Since in our model, \( m_t \) is less than half of its standard value, the contribution of the \( t-b \) doublet to \( \delta \rho \) is reduced by a factor of 4. This cannot be made up entirely by the contribution of the fourth generation fermions, since the mass of the \( b' \) is not negligible. We find that the contributions of the third and fourth generation fermions make up only half the observed \( \delta \rho \). The remainder must come from the third and fourth generation squarks. This requirement places severe restrictions on the squark parameters [eq. (3.2)]. One must maximize the off-diagonal squark mixing while keeping the diagonal squark mass parameters as small as possible. However, the latter cannot be too small; otherwise the radiative corrections to the light Higgs mass will be reduced leading to a value of \( m_{h^0} \) below the current LEP bound.

It is convenient to parameterize the oblique radiative corrections in terms of the Peskin-Takeuchi variables\(^{14} \) \( S, T \) and \( U \). Here \( T \equiv \alpha^{-1} \delta \rho \) (where \( \alpha^{-1} \simeq 137 \)) is the most sensitive (although some interesting restrictions can be obtained by considering \( S \)). Langacker has performed a global analysis of precision electroweak data,\(^{15} \) assuming that \( m_t = 80 \) GeV and \( m_{h^0} = 65 \) GeV, and extracts values for the oblique parameters. He finds \( T_{\text{new}} = 0.70 \pm 0.21 \), which in our model must arise from the contribution of the fourth generation fermions and the third and fourth generation squarks. (The contributions from other supersymmetric particles are negligible.) We find that the fourth generation fermions yield a contribution of 0.2 to \( T_{\text{new}} \). The contributions of the third and fourth generation squarks depend sensitively on the squark parameters as noted above; a range of parameters can be found that yields a total squark contribution to \( T_{\text{new}} \) that lies between 0.3
and 0.4. This would bring us within one standard deviation of Langacker’s value for $T_{\text{new}}$. To achieve such a value for the squark contribution to $T_{\text{new}}$ requires substantial $\tilde{q}_L-\tilde{q}_R$ mixing in the third generation, which is uncomfortably large and may cause stability problems\cite{16} for the complete scalar potential of the model. Non-negligible mixing in the fourth generation also enhances the fourth generation squark contributions to $T_{\text{new}}$. The maximum effect is limited phenomenologically by a lower bound on the mass of $\tilde{b}'$. In order that $t' \to bW^+$ remain the dominant decay, one must kinematically forbid $t' \to b'\tilde{\chi}_1^+$. Given $M_{\tilde{\chi}_1^+} \simeq 60$ GeV, a value of $M_{\tilde{b}'} \simeq 120$ GeV is a comfortable choice. All the phenomenological constraints have now forced the parameters of the model into a very narrow corner of parameter space.

4. Conclusions

It is still possible that $m_t \simeq m_W$, despite the recent announcement of the top quark discovery by the CDF and D0 collaborations. A model has been exhibited that satisfies all phenomenological constraints and is not ruled out by published data. The most theoretically troubling feature of the model is the large mixing among the third generation squarks that is necessary to ensure a viable prediction for the electroweak $\rho$-parameter.

The model possesses a rich spectrum of new particles that will be accessible to LEP-II and the Tevatron. In particular, eight new particles of this model could be discovered at LEP-II: the $t$-quark, the fourth generation leptons ($\tau'$ and $N$), the light Higgs boson ($h^0$), and four supersymmetric particles ($\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, $\tilde{\chi}_1^\pm$, and $\tilde{t}$). Note that even at the initial run of LEP-II at $\sqrt{s} = 140$ GeV planned for the fall of 1995, all four supersymmetric particles listed above (and the $\tau'$) should be discovered, or else the model would be excluded.

Thus, the fate of this model may be decided before these Proceedings appear in print. Nevertheless, this exercise was useful in demonstrating the difficult in constructing four-generation models of low-energy supersymmetry. In a previous work, Gunion, McKay and Pois\cite{17} attempted to construct four-generation models in the context of minimal low-energy supergravity. They identified the top quark as the state discovered by CDF and D0. In order to keep Higgs-quark Yukawa couplings perturbative up to the GUT scale, they were forced to try to hide the $b'$ and $t'$ in a mass region below $m_t \simeq 175$ GeV. The resulting models were contrived and phenomenologically unappealing. Our approach represents the logical alternative for four-generation low-energy supersymmetric models. If these models are excluded, one will finally be able to state with confidence that in the low-energy supersymmetric approach the number of generations is indeed three!
Acknowledgments

I would like to thank Marcela Carena and Carlos Wagner for an enjoyable and rewarding collaboration. I am also grateful to Jean-Marie Frère for his kind invitation to speak at the Moriond meeting. Finally, I send a special appreciation to Joëlle Raguideau, whose encouragements were a great help to a painful knee. This work was supported in part by a grant from the U.S. Department of Energy.

REFERENCES

1. F. Abe et al. [CDF Collaboration], Phys. Rev. Lett. 74 (1995) 2626; S. Abachi et al. [D0 Collaboration], Phys. Rev. Lett. 74 (1995) 2632.
2. M.G. Alviggi et al. [LEP Electroweak Working Group], LEPEWWG/95-01 (1995); M. Calvi, invited talk at this meeting.
3. M. Veltman, Nucl. Phys. B123 (1977) 89.
4. M. Carena, H.E. Haber and C.E.M. Wagner, CERN preprint in preparation.
5. B. Klima, invited talk at this meeting.
6. S. Abachi et al. [D0 Collaboration], Phys. Rev. Lett. 72 (1994) 2138.
7. Limits on supersymmetric particle masses are summarized in L. Montanet et al. [Particle Data Group], Phys. Rev. D50 (1994) 1173.
8. A. White, invited talk given at the SUSY-95 Conference, 15–19 May 1995, Palaiseau, France.
9. I.I. Bigi and S. Rudaz, Phys. Lett. 153B (1985) 335.
10. C.T. Hill, Phys. Rev. D24 (1981) 691; C.T. Hill, C.N. Leung and S. Rao, Nucl. Phys. B262 (1985) 517.
11. M. Carena, M. Olechowski, S. Pokorski, and C.E.M. Wagner, Nucl. Phys. B419 (1994) 213; B426 (1994) 269.
12. M.S. Alam et al. [CLEO Collaboration], Phys. Rev. Lett. 74 (1995) 2885.
13. H.E. Haber and R. Hempfling, Phys. Rev. Lett. 66 (1991) 1815; Phys. Rev. D48 (1993) 4280; Y. Okada, M. Yamaguchi and T. Yanagida, Prog. Theor. Phys. 85 (1991) 1; Phys. Lett. B262 (1991) 54; J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B257 (1991) 83; B262 (1991) 477; R. Barbieri, M. Frigeni, and F. Caravaglios Phys. Lett. B258 (1991) 167.
14. M.E. Peskin and T. Takeuchi, Phys. Rev. Lett. 65 (1990) 964; Phys. Rev. D46 (1992) 381.
15. J. Erler and P. Langacker, UPR-0632-T (1994) [hep-ph 9411203]; P. Langacker, private communication.
16. J. Ellis, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. 121B (1983) 123; L. Ibáñez and C. Lopez, Phys. Lett. 126B (1983) 54; L. Alvarez-Gaumé, J. Polchinski and M. Wise, Nucl. Phys. B221 (1983) 495.
17. J.F. Gunion, D.W. McKay and H. Pois, Phys. Lett. B334 (1994) 339.