On The Cost Distribution of a Memory Bound Function

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Abstract

Memory Bound Functions have been proposed for fighting spam, resisting Sybil attacks and other purposes. A particular implementation of such functions has been proposed in which the average effort required to generate a proof of effort is set by parameters $E$ and $l$ to $E \cdot l$. The distribution of effort required to generate an individual proof about this average is fairly broad. When particular uses of these functions are envisaged, the choice of $E$ and $l$, and the system design surrounding the generation and verification of proofs of effort, need to take the breadth of the distribution into account.

We show the distribution for this implementation, discuss the system design issues in the context of two proposed applications, and suggest an improved implementation.

1 Introduction

Following Abadi et al[1], Dwork et al [2] suggest imposing computational costs on the senders of e-mail by requiring the mail to be accompanied by a proof of effort generated using a class of Memory Bound Functions (MBF). Specifically, they propose a concrete implementation called Algorithm Mbound. The average computational effort, measured in cache misses, it requires to generate a proof of this kind is set by two parameters $E$ and $l$. Generating a proof requires on average $E \cdot l$ cache misses, verifying it requires $l$ cache misses. We measure all costs in cache misses, so a proof costs $E \cdot l$ cache misses to construct and $l$ cache misses to verify. It uses an incompressible fixed public data set $T$ larger than any cache it is likely to meet. A proof generator who must expend effort $E \cdot l$ is given as challenge a nonce $n$ (so that older effort proofs cannot be reused) and the values of $E$ and $l$. In response, the generator must perform a series of pseudorandom walks in the table $T$. Each starting point is determined by an index $s$ which starts at zero and is incremented for each walk, or trial. The walk computes a one-way value $A$ based on $n$, $s$ and the encountered elements of table $T$. The walk is dependent on $n$ and $s$; it is constructed so that the number of encountered elements is $l$, and fetching each encountered element causes an L1 cache miss. Each walk, therefore, causes $l$

1LOCKSS is a trademark of Stanford University.
cache misses.

The generator stops when the value A computed by the walk has 0 bits in its least significant \(e = \log_2 E\) positions. It is thus expected the generator will try on average \(2^e\) walks with different starting positions determined by successive values of \(s\) before finding an appropriate starting position \(i\), costing \(C = E \cdot l\) cache misses.

The \(i\) that yielded the appropriate \(A\) is the effort proof. The verifier need only perform the random walk on \(T\) starting with the \(n\) he chose and the \(i\) sent by the generator, costing \(V = l\) cache misses. If the resulting \(A\) has the proper 0-bits in its last \(e\) positions, the verifier accepts the effort.

There is some non-zero probability that the proof search will take longer than any given number of steps. Dwork et al acknowledge this indirectly when they say:

“If \(i > 2^{2e}\) the [verifier] rejects the message (with overwhelming probability one of the first \(2^{2e}\) trials should be successful).” [2]

Indeed, but what of the poor proof generator who had, using their parameters, expended more than \(2^{42}\) cache misses (about 10 days) and still failed to find a proof? It isn’t just the verifier who must reject extremely long proofs, but also the generator. In any practical implementation of Algorithm MBound, account needs to be taken of the fact that some proportion of the attempts to generate a proof will fail simply because they took too long.

2.2 Use of MBF in Spam Prevention

The idea of using MBF to prevent spam is for a receiver to refuse to accept mail that is not accompanied by “postage” in the form of a proof of effort based on the sender’s alias, the receiver’s alias and the body of the message. The average amount of effort would be set to prevent an individual machine, even the fastest available, sending more than a few thousand mails per day. The goal of the spam adversary would be to reduce the actual effort required to send spam below this average.

2.3 Use of MBF in LOCKSS

The LOCKSS system implements a peer-to-peer network of persistent, mutually suspicious web caches used to preserve access to e-journals. The LOCKSS protocol requires a peer A requesting a service from a peer B to obtain a nonce from B, then provide B with a proof of effort based on that nonce representing more effort than B will need to perform the requested service. B verifies this effort before performing the service and returning the result to A. In the LOCKSS protocol, the service peers provide each other is that of voting on the digest of blocks of content. The proofs of effort prevent a conspiracy of a minority of malign peers overwhelming these votes, or cheaply swamping the system in bogus polls.

3 Distribution of Proof Generation Effort

Figure 1 shows the behavior of Algorithm MBound using the authors’ choice of \(e = 15\). We show histograms of the percentage of proof generations whose search ended after a given number of trials, and of the percentage of the total cost of proof generation due to those proofs. Note the skewed distributions; the majority of proofs take less than expected but, because they are cheap, the majority of the cost is due to proofs taking longer than expected.

From this data we see that:

- Even though the average cost of a proof is 32768 trials, more than 63% of the proofs are found in less than 16384 trials. The median proof lies between 8192 and 16384 trials.
- There is almost 0.1% probability of finding a proof in 32 trials or less.
- There is almost 12% probability of finding a proof in 4096 trials or less.
- 9% of the total cost represents proofs taking
more than four times as long as expected.

- 40% of the total cost represents paths taking more than twice as long as expected.

This behavior is not an artifact of the particular value of \( e \) they choose. Figures 2, 3, 4 show the behavior for \( e = 6, 12, 18 \) respectively.

4 Design Issues

In this section we discuss the issue raised by this distribution of effort proof costs for both the spam prevention and LOCKSS protocol applications.

4.1 What Does The Proof Prove?

At first sight it might appear that after the fact, the verifier of a proof knows how much effort was used to find it. The proof \( i \) in Algorithm MBound is, after all, a count of the number of trials. Although the verifier finds that index \( i \) produces a path terminating with a value with \( e \) low-order zero bits, that does not prove that no path \( j < i \) produces such a path. The verifier does not know that the generator started with index 0.

It is important to observe that the proof shows only that the generator did something that, averaged over a large number of repetitions by a generator using exactly the procedure specified, required approximately \( 2^e \cdot l \) path steps.

The actual number of steps for the proof in question is unknown; it is quite likely to be less than \( 2^{(e-3)} \cdot l \). The average number of steps can be strongly influenced by the point at which the generator abandons the search as taking too long, a parameter that the generator doesn’t reveal to the verifier. This uncertainty about the relationship between the requested and actual effort is a problem for both cases.

4.2 Tails of the Distribution

The left tail of Figure 1 is a region in which the cost of proofs curve is below the number of proofs curve. If a generator could manipulate things to increase the probability of being in this region it could generate proofs more cheaply than the verifier expects. Similarly, the right tail is a region in which the cost of proofs curve is above the number of proofs curve. If a generator could manipulate things to decrease the probability of being in this region it could generate proofs more cheaply than the verifier expects.

Both tails of the cost distribution pose problems for the LOCKSS protocol:

- The significant population of low-cost proofs provides an adversary with a good chance of finding a proof much faster than expected.
- The significant proportion of the cost repre-
sented by the small proportion of very long proofs allows an adversary to roughly halve his cost with little impact on his strategy simply by terminating the search for a proof early and failing to vote.

The LOCKSS protocol uses random challenges which appear to prevent attacks analogous to “chosen plaintext” attacks on encryption systems. Unfortunately, the fault-tolerance of the protocol allows the adversary to selectively refuse to supply expensive proofs and thus bias the “plaintext” towards cheap proofs.

Analogous difficulties arise in the application of Algorithm MBound to spam prevention. At first sight this case is also immune from a “chosen plaintext” attack, since the proof must be based on the text of the e-mail, its source and destination address, and so on. Unfortunately:

- Spammers already perturb the text of their messages with random garbage their victims will ignore but which helps evade signature-based spam filters. They could choose a e-mail text and try many possible starting indices in parallel. If none produced a cheap proof, they could perturb the e-mail text and try again.
- Similarly, no individual recipient of spam is important. By terminating the search for a proof early; the spammer loses a proportion of the intended recipients but reduces his cost much more.

4.3 Adjusting Costs

The LOCKSS protocol needs to vary the additional costs imposed via the MBF scheme over a wide range to match the economic requirements of the protocol to the cost of the underlying hashing.

The LOCKSS application requires considerable freedom in setting costs, whereas for best results Algorithm MBound requires the values of $e$ and $l$ not to be too small, and incrementing $e$ increases the cost by a factor of two. With these restrictions, the choice of $e$ and $l$ can be a problem.

Even in the spam application, setting prices of 0 (from known senders) or 1 (from unknown senders) for all e-mail “postage” lacks flexibility. An ideal design would allow the recipient to accept less “postage” from senders it trusted more.

4.4 Timing Uncertainty

To limit the possibility an adversary could “time-shift” and use a single computer to generate many votes in a single poll, the LOCKSS protocol sets deadlines by which a generator must deliver a proof. To do so, it must adjust for the difference between fast and slow memory systems. Dwork et al report this is a factor of 5. In addition, when using Algorithm MBound it must also adjust for the possibility that the generator may find a proof in many fewer or many more steps than expected. This “slop” in the timing defeats the purpose of the deadlines.

This uncertainty is not a problem in the spam application; since the recipient only sees the fixed time taken to verify the proof.

5 Requirements for MBF

The LOCKSS protocol’s requirements for a cost function were stated [3] as:

“First, it must have an adjustable cost, since different amounts of additional effort are needed at different protocol steps. Second, it must produce effort measurable in the same units as the cost it adjusts (hashing in our case). Third, the cost of generating the effort must be greater than the cost of verifying it.”

To these we can now add:

- The distribution of actual effort involved in generating proofs of effort about the predicted mean cost must be narrow.
- It must not be possible for an effort generator to significantly reduce the average cost of proof generation by failing long proofs selectively.
- The proof must be concise, not requiring much data to be transmitted.
- The effort must be continuously adjustable.

In fact, we would like our proof to prove that the generator performed exactly $E \cdot l$ steps.

6 An Alternative MBF

Consider a mechanism in which the generator is asked to produce not a single index $i$ which
results in a value with \( e \) low-order zeros, but a list of all \( i \) in a specified range which result in values with \( e - m \) low-order zeros, where \( 2^m \) is the expected number of such values. The verifier can cheaply verify some indices randomly chosen from the list, if any are invalid the proof is invalid. The more indices are verified, the more sure the verifier is of the effort expended by the generator. If the verifier considers extra verification effort justified, a search of some randomly chosen intervals between the indices in the list that revealed indices resulting in values with the required number of zeros would also render the proof invalid. The generator doesn't know which indices and ranges the verifier will check, so cannot include bogus entries without risking discovery.

In this way we force the generator to explore an entire range of indices, rather than stopping at the first one. The number of trials needed to generate a proof is fixed, because every index in the requested range must be checked if the generator is unwilling to risk the cheat being discovered. This satisfies the first two of our additional requirements.

There will be considerable variation in the cost of verifying a proof, depending on how sure the verifier wants to be that the generator actually exerted the requested effort.

This mechanism has some attractive features:

- There is no uncertainty as to the amount of effort exerted by the generator; unless it is willing to risk detection it must perform exactly the effort requested.

- The verifier never performs more effort than the generator; it can stop as soon as any trial is invalid.

- The more effort the verifier performs, the better the verification. The amount of this effort can be adjusted to match the verifier’s suspicion of the generator. This property is valuable in both applications we consider:

  - Spammers wish to conceal their true identity behind a screen of proxies, making it unlikely that a recipient would engage in repeated transactions with the same apparent spammer identity. But a recipient normally has repeated transactions with many legitimate e-mail senders; reducing the cost of verifying their identities would reduce the total cost of the system to a recipient.

  - In the LOCKSS case, repeated transactions are the norm and this property would be very useful.

- It provides another parameter with which to adjust the cost of the proofs requested from a generator, namely the size of the range of indices to be searched. In the LOCKSS case, this helps adjust the effort requested to the economic requirements and the underlying hash cost.

- In effect, we have converted the uncertainty in time inherent to Algorithm MBound into uncertainty in size; there is a non-zero probability that every value in the requested range on indices has the required number of low-order zeros and must be included in the proof. There is no upper bound on the time required by Algorithm MBound, there is none on the message size required by our new scheme.

Note that in this scheme there may not be any indices \( i \) in the requested range which result in values with \( e - m \) low-order zeros. Thus an empty list of indices is potentially a valid proof of effort, albeit one that is anomalously expensive to verify. If an empty proof is valid, verifying it takes as much effort as generating it. If it is invalid, the first valid path found in the range shows this. On average this takes \( 1/2^m \) of the effort that should have been used to generate the proof. This poses a problem; an adversary can generate empty proofs free and cause verifiers to waste on average \( 2^{(e-m)} \) trials to reveal they are invalid:

- In the spam case, the recipient should reject all empty proofs. A sender finding an empty proof should perturb the message and try again.

- In the LOCKSS case, the requestee should reject all empty proofs. If the parameters can be set to make empty proofs sufficiently rare, the fault tolerant nature of the protocol means that this will have little effect.
7 Setting Parameters

The authors chose parameters for Algorithm MBound that prove $2^{15}$ paths each $2^{12}$ steps long, and thus $2^{27}$ cache misses. To choose comparable parameters for our scheme, we start by setting $m = 4$ to keep the average proof size to 16 indices or 64 bytes. If we use the same path step mechanism as Algorithm MBound, we also need to set $l = 2048$ and thus need a range of $2^e = 2^{15}$ indices to search. We thus look for paths generating a value with $(e - m) = 11$ low-order zeros. With these choices, the probability that all $2^{15}$ trials fail to find any such path is $(1 - 1/2^4)^{2^{15}}$ or about $10^{-7}$.

8 Future Work

We have yet to implement our new scheme in the LOCKSS protocol. We expect it to resolve many tricky issues that arose in our attempt to use Algorithm MBound. Doubtless, other tricky issues will arise as we proceed. It may well be that the path generation part of Algorithm MBound is not the best choice for generating the larger number of smaller sub-proofs that our scheme needs; this is the subject of further exploration.

9 Conclusion

We have shown that the actual cost, measured in the number of trials, needed to generate a proof of effort using Algorithm MBound has a fairly broad distribution. We have identified a number of design issues this raises for a practical application of this scheme to both its intended spam prevention purpose, and for adjusting the cost of operations in the LOCKSS protocol. We suggest an alternative scheme, in which the cost of generating a proof is fixed and the cost of verifying a proof can be adjusted by the verifier to match his suspicions of the generator.

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Appendix

We used the abstract model implemented by the program below to generate the data for the figures. We had two reasons for this:

- Running large numbers of effort proofs is time-consuming.
- We wished to demonstrate that the problems we identify are a consequence of the way the proof search is organized and not of the particular cryptographic features of Algorithm MBound.

The program generates graphs similar to those of an actual implementation of Algorithm MBound for low $e$.

```java
import java.text.*;
import java.io.*;

public class MBFcost {
    static double[] numTry = new double[32];
    static double[] costTry = new double[32];
```
static double totalCost = 0;
static long e = 0;
static double p = 0.0;
static int maxTries = 0;

public static void main(String args[]) {
    long m = (long)Integer.parseInt(args[0]);
    for (e = 3; e <= m; e += 3) {
        initialize();
        fillHistogram();
        outputHistogram();
    }
}

static void initialize() {
    for (int i = 0; i < numTry.length; i++) {
        numTry[i] = 0.0;
        costTry[i] = 0.0;
    }
    p = 1.0/((double) (1 << e));
    maxTries = (1 << (e + 8));
}

static void fillHistogram() {
    double probSoFar = 1.0;
    for (int i = 1; i < maxTries; i++) {
        int b = bin(i);
        // The probability of a run ending
        // on try i is 1/2^e times probability
        // of not having ended before
        probSoFar *= (1.0 - p);
        double probHere = p * probSoFar;
        numTry[b] += probHere;
        // Thus the cost of a run ending on
        // try i is i/2^e
        double c = ((double) i) * probHere;
        costTry[b] += c;
        totalCost += c;
    }
}

static void outputHistogram() {
    NumberFormat nf = 
        NumberFormat.getNumberInstance();
    nf.setMinimumFractionDigits(2);
    try {
        FileOutputStream numFOS = 
            new FileOutputStream("data/e" + e + "\tries.dat", true);
        PrintWriter numberPW = new PrintWriter(numFOS);
        FileOutputStream costFOS = 
            new FileOutputStream("data/e" + e + "\cost.dat", true);
        PrintWriter costPW = new PrintWriter(costFOS);
        for (int i = 0; i < (e + 8); i++) {
            String pcTry = nf.format(numTry[i] * 100);
            String pcCost = 
                nf.format((100*costTry[i])/(totalCost));
            numberPW.println(i + "\t" + pcTry);
            costPW.println(i + "\t" + pcCost);
        }
        numberPW.close();
        costPW.close();
    }
    catch (FileNotFoundException ex) {
        // No action intended
    }
}

static int bin(int p) {
    int ret = 0;
    if (p < 0)
        p = -p;
    while (p != 1) {
        ret++;
        p >>>= 1;
    }
    return ret;
}

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