Time reversal invariance violation in neutron-nucleus scattering

Pavel Fadeev1,∗ and Victor V. Flambaum1,2,†

1Helmholtz Institute Mainz, Johannes Gutenberg University, 55099 Mainz, Germany
2School of Physics, University of New South Wales, Sydney, New South Wales 2052, Australia

(Dated: March 22, 2019)

Planning and interpretation of the experiments searching for the time reversal (T) and parity (P) violation in neutron reactions require values of the matrix elements of the T,P-violating nuclear forces between nuclear compound states. We calculate the root mean square values and the ratio of the matrix elements of the T,P-violating and P-violating interactions using statistical theory based on the properties of chaotic compound states and present the results in terms of the fundamental parameters in four different forms: in terms of the constants of the contact nuclear interaction, meson exchange constants, QCD θ-term and quark chromo-EDMs $d_u$ and $d_d$. Using current limits on these parameters, we obtain the upper bounds on the ratio of the matrix elements.

I. INTRODUCTION

A very popular way to search for T,P-violation and test of the unification theories is based on the measurements of electric dipole moments (EDM) of elementary particles and atomic systems. So far this method produced stringent limits on EDM which excluded or cornered many popular models — see reviews [1–5]. Studies of T,P-violating effects via EDM also give limits on axions interactions [6]. An efficient alternative method is the measurement of T,P-odd effects in neutron-nucleus scattering. The motivation of this method is based on a million times enhancement of parity violation in neutron reactions near $p$-wave nuclear compound resonances which has been predicted in Refs. [7–10]. The first confirmation was obtained in experiments performed at the Joint Institute for Nuclear Research in Dubna [11, 12], then a very extensive experimental study was done in several laboratories, including Joint Institute for Nuclear Research (Dubna), Petersburg Institute of Nuclear Physics, KEK (Tsukuba) and especially in Los Alamos — see reviews [13, 14]. This activity continues now — see, for example, recent experimental paper [15] and references therein. A similar mechanism of enhancement should work for the T,P-odd effects [16–19]. An unusual statistics of P-violating and T,P-violating effects (random-sign observables non vanishing upon averaging) has been demonstrated in Refs. [20, 21]. Experiments searching for T,P-violating effects are in progress in Japan and USA [15, 22–26].

Without any enhancement, effects of P-violation in low-energy nuclear reactions are extremely small, $\sim 10^{-7}$ (e.g. in the proton scattering on hydrogen and helium, neutron radiative capture by proton) [27]. The formula for a P-violation effect near a $p$-wave compound resonance may be presented as [7–10]:

$$P \sim \frac{W_{sp}}{E_s - E_p} \sqrt{\frac{\Gamma^n_s}{\Gamma^n_p}}$$

where $W_{sp}$ is the matrix element of the parity violating interaction mixing $s$ and $p$ resonances, $E_s - E_p$ is the energy interval between these resonances, and $\Gamma^n_s, \Gamma^n_p$ are the neutron widths of these resonances. We see that there are two reasons for the enhancement of P-violation near the $p$-wave compound resonances. Firstly, in a nucleus excited by neutron capture the interval between the chaotic compound states (resonances) of opposite parity is very small, and this enhances by 3 orders of magnitude the mixing of these states by the weak P-violating interaction between nucleons. The second reason: admixture of opposite parity states allows the $s$-wave neutron capture to the $p$-wave resonance. At small neutron energies the $s$-wave amplitude is 3 orders of magnitude larger than the $p$-wave amplitude. As a result of these two $10^3$ factors the P-violating parts reach 1-10% of reactions’ cross sections and become accessible to experimental scrutiny. P,T-violating effects are also produced by the parity violating interaction, therefore Eq. (1) and the enhancement mechanism works for them too [16–19].

For the experiments to produce useful results we need theory for their interpretation. On first impression it seems impossible since chaotic compound states are very complicated. However, chaos allows us to develop a statistical theory, similar to the Maxwell-Boltzmann theory for macroscopic systems which actually gives very accurate predictions. We developed such theory including a method to calculate matrix elements between chaotic states in finite systems (in excited nuclei, atoms and molecules) [28–34]. We briefly present the ideas below.

An increase in the excitation energy of the nucleus increases the number of its active particles $k$ and available orbitals $p$, leads to an exponential increase of the density

∗ pavelfadeev1@gmail.com
† v.flambaum@unsw.edu.au

1 We omit the numerical coefficient which depends on the specific process induced by the neutron capture.
of energy levels \( \sim p! / [(p - k)! k!] \) and brings the system into a state where the residual interaction between particles exceeds the intervals between the energy levels. The eigenstates \( |n\rangle = \sum_i C^m_i |i\rangle \) become chaotic superpositions of thousands or even millions of Hartree-Fock basic states \( |i\rangle \). All medium and heavy nuclei and atoms with an open \( f\)-shell have chaotic excited compound states in the discrete spectrum and/or chaotic compound resonances. The idea of Refs. [28, 29] is to treat the expansion coefficients \( C^m_i \) as Gaussian random variables, with the average values \( \langle C^m_i \rangle = 0 \) and variance:

\[
\langle C^m_i \rangle^2 = \frac{1}{N} \Delta(\Gamma_{spr}, E^n - E_i),
\]

\[
\Delta(\Gamma_{spr}, E^n - E_i) = \frac{\Gamma^2_{spr}/4}{(E^n - E_i)^2 + \Gamma^2_{spr}/4},
\]

where \( \bar{N} = \frac{\pi \Gamma_{spr}}{2d} \) is the normalisation constant found from \( \sum_i \langle C^m_i \rangle^2 = 1 \), \( d \) is the average energy distance between the compound states (resonances) with the same angular momentum and parity, and \( \Gamma_{spr} \) is the spreading width of the component calculated using the Fermi golden rule [35]: \( \bar{N} \) is called the number of principal components.\(^2\)

We have tested this distribution of \( C^m_i \) by the numerical calculations of chaotic compound states in cerium and protactinium atoms [36–42], in highly charged ions with an open \( f\)-shell [43–48], in the two-body random interaction model [49–52] and using an analytical approach [33, 53].

The function \( \langle C^m_i \rangle^2 = \frac{1}{N} \Delta(\Gamma_{spr}, E^n - E_i) \) gives the probability to find the basis component \( |i\rangle \) in the compound state \( |n\rangle \), i.e. it plays the role of the statistical partition function. The difference from the conventional statistical theory is that the partition function depends on the total energy of the isolated system \( E^n \) instead of the temperature of a system in a thermostat (recall the Boltzmann factor exp(\( -E_i/T \))). One may compare this with the microcanonical distribution where the equipartition is assumed within the shell of the states with fixed energy \( E_i \).

Expectation values of matrix elements of any operator \( W \) in a chaotic compound state are found as \[ \langle n | W | m \rangle^2 = \sum_i \langle C^m_i \rangle^2 \langle i | W | j \rangle^2. \] For example, this formula with \( W = a^+_m a^+_n \) (the occupation number operator) gives the distribution of the orbital occupation numbers in finite chaotic systems which replaces the Fermi-Dirac (or Bose-Einstein) distribution.\(^3\)

Average values of the non-diagonal matrix elements are equal to zero, \( \langle n | W | m \rangle = 0 \), while the average values of the squared matrix elements \( W^2 = \sum_i \langle C^m_i \rangle^2 \langle i | W | j \rangle^2 \) are reduced to the sum of matrix elements between the Hartree-Fock basis states \( \langle i | W | j \rangle^2 \), where \( W \) is any perturbation operator. The distribution of the matrix elements \( \langle n | W | m \rangle \) is Gaussian with the variance given by the \( W^2 \).

For the correlator between two different operators (e.g. \( P\)-violating and \( T,P\)-violating) we obtain \[ \langle n | W_p | m \rangle \langle m | W_{T,P} | n \rangle = \sum_{i,j} \langle C^m_i \rangle^2 \langle C^m_j \rangle^2 \langle i | W_p | j \rangle \langle j | W_{T,P} | i \rangle \] [28–32]. Note that our theory predicts the results averaged over several compound resonances.

We have done many tests, comparing the statistical theory results for electromagnetic amplitudes [40], electron recombination rates [43–48, 54] and parity violation effects [28, 29] with the experimental data and with numerical simulations. For example, we obtained a thousand times enhancement of the electron recombination rate with many highly charged tungsten ions due to the very dense spectrum of chaotic compound resonances [44–48, 54]. The results agree with all available experimental data and predict recombination rates for ions with a high ionization degree, where the experimental results cannot be obtained with existing experimental techniques. These results are important for the thermonuclear reactors which are made from tungsten. Tungsten ions contaminate plasma and significantly affect the energy output.

Using the theory of chaotic nuclear compound resonances, we calculate in this paper the ratio \( w/v \) of the root mean squared values of the matrix elements of the \( T,P\)-odd \( (w) \) and \( P\)-odd \((v)\) matrix elements. We present the results in terms of the fundamental parameters in four different forms: in terms of the constants of the contact nuclear interaction, meson exchange constants, QCD \( \theta\)-term and quark chromo-EDMs \( d_u \) and \( d_d \). Using latest bounds on \( \theta, d_u \) and \( d_d \) we arrive at bounds on the magnitude of possible \( T\)-violation.

II. \( P\)- AND \( T,P\)- VIOLATING INTERACTIONS

The ratio of the time reversal invariance violating (TRIV) and parity violating (PV) parts of the neutron nuclear cross sections induced by mixing of \( s\) - and \( p\)-wave nuclear compound resonances, \( \Delta \sigma_{PT}/\Delta \sigma_P \), can be expressed as [15, 55, 56]:

\[
\frac{\Delta \sigma_{PT}}{\Delta \sigma_P} = \kappa \frac{\langle \psi_p | W_{PT} | \psi_s \rangle}{\langle \psi_p | W_P | \psi_s \rangle},
\]

where the factor \( \kappa \) includes amplitudes of the partial neutron widths which depend on spin channels \( J = I \pm 1/2 \), where \( I \) is the spin of the target nucleus and \( J \) is the spin of the compound resonance. For example, in the case of

\(^2\) Basis states \( |i\rangle \) with shell model energies \( E_i \) close to the energy of a compound state \( E^n \) (with the spreading width \( \Gamma_{spr} \)) have the highest weight (\( \sim 1/N \)) and dominate in the normalization sum \( \sum_i \langle C^m_i \rangle^2 = 1 \). The number of such states is \( N \).

\(^3\) However, numerical calculations [36, 43, 44, 49] give occupation numbers which are close to the Fermi-Dirac distribution.
J = 0, one obtains κ = 1, as in such a case κ does not depend on neutron partial widths [16–18].

The ratio Δσ_{PT}/Δσ_P for the neutron-deuteron scattering has been calculated in Ref. [57]. However, the experiments are planned for heavier nuclei where we expect a million times enhancement of the T,P-odd and P-odd effects.

In the short-range interaction limit the PV operator \( W_P \) and TRIV operator \( W_{PT} \) are:

\[
W_P = \frac{G g}{2\sqrt{2m}} \{(\sigma p), \rho\},
\]

\[
W_{PT} = \frac{G \eta}{2\sqrt{2m}} (\sigma \nabla) \rho.
\]

Here \( G \) is the weak interaction Fermi constant, \( m \) is the nucleon mass, \( p \) and \( \sigma \) are its momentum and spin correspondingly, and \( \rho \) is the nucleon density. Nucleon dimensionless constants \( g_{p,n} \) and \( \eta_{p,n} \) characterize the strength of the interactions. Note that in the standard definition of angular wavefunctions the matrix element of \( W_P \) between bound states is imaginary (since the momentum operator \( p = -i \nabla \) and the matrix element of TRIV operator \( W_{PT} \) is real.

We define \( v^2 \) to be the average of the absolute value of the squared PV matrix element, and \( w^2 \) to be the average value of the squared TRIV matrix element between the \( s \) and \( p \) compound resonances, such that:

\[
v = \sqrt{\langle \psi_p | W_P | \psi_s \rangle \langle \psi_s | W_P | \psi_p \rangle},
\]

\[
w = \sqrt{\langle \psi_p | W_{PT} | \psi_s \rangle \langle \psi_s | W_{PT} | \psi_p \rangle}.
\]

Correlations might exist in the matrix elements of PV and TRIV interactions. The quantity parametrizing such correlations, the correlator, is defined as:

\[
C = \frac{\langle \psi_p | W_P | \psi_s \rangle \langle \psi_s | W_{PT} | \psi_p \rangle}{v \cdot w}.
\]

The correlator, which takes values between zero and one, can be useful to deduce the values and signs of TRIV effects, since a lot is already known about the PV effects. The correlator \( C \) was calculated by the same technique as the mean squared matrix element, and was found to be [32]:

\[
| C | \approx 0.1.
\]

This result tells us that the correlations in the matrix elements are relatively small.

### A. Rough estimate of \( w/v \)

Naively one would expect from Eqs. (5) and (6) the following relation: \( \frac{w}{v} \sim \frac{g}{\rho} \). However, this ratio is actually \( A^{1/3} \) times smaller than the ratio of interaction constants [31], where \( A \) is the number of nucleons. Indeed, \( \nabla \rho \) in Eq. (6):

\[
\nabla \rho \sim \frac{\rho}{R_N} \sim \frac{\rho}{r_0 \cdot A^{1/3}},
\]

where \( r_0 \) is the inter-nucleon distance, and \( R_N = r_0 \cdot A^{1/3} \) is the nuclear radius. The momentum in Eq. (5) is approximated as \( p \sim p_F \sim \hbar/r_0 \). Thus, the ratio of matrix elements is smaller than the ratio of interaction constants in Eqs. (5) and (6) by a factor of \( A^{1/3} \):

\[
\frac{w}{v} \sim \frac{\eta}{gA^{1/3}}.
\]

For elements with the number of nucleons in the range 100–250, \( A^{1/3} \approx 5 \). A detailed discussion of this suppression factor including many-body effects can be found in Ref. [31].

### B. Dependence of the matrix elements on the nucleon interaction constants

A general expression for the root mean squared value of the matrix element \( v \) of PV operator (and the matrix element \( w \) of TRIV operator) has been derived in Ref. [29]:

\[
v = \frac{1}{\sqrt{N}} \left\{ \sum_{abcd} \nu_a (1 - \nu_b) \nu_c (1 - \nu_d) \frac{1}{4} | V_{ab,cd} - V_{ad,eb} |^2 \Delta (\Gamma_{spr}, \epsilon_a - \epsilon_b + \epsilon_c - \epsilon_d) \right\}^{1/2}.
\]

Here \( \nu \) are the orbital occupation numbers given by the Fermi-Dirac distribution in excited nucleus, numerical values of the matrix elements of the two-nucleon interaction \( V_{ab,cd} \) (see Fig. 1) are presented in Refs. [29, 32],
FIG. 1. Possible configurations of weak interactions $V_{ab,cd}$ [29] within the nucleus between protons (p) and neutrons (n). In each diagram, the upper vertex is P-violating. Part (a): interactions between two protons; part (b): interactions between two neutrons; parts (c) and (d) show interactions between protons and neutrons, which contribute to the squared PV matrix element $v^2$ by $(V_{np} + V_{pn})^2 = V_{np}^2 + V_{pn}^2 + 2V_{np}V_{pn}$. When summing matrix elements in parts (c) and (d), the terms $V_{np}V_{pn}$ have random signs and the result is much smaller than the sums of $V_{np}^2$ and $V_{pn}^2$.

The dependence of $v$ and $w$ on the nucleon interaction constants $g$ and $\eta$ (which appear in Eqs. (5,6)) can be presented in the following form [29, 32]:

$$v = \frac{1}{\sqrt{N}} \sqrt{\left(\Sigma_{pp}^{(P)} g_p^2\right)^2 + \left(\Sigma_{nn}^{(P)} g_n^2\right)^2 + \left(\Sigma_{pn}^{(P)} g_p g_n\right)^2},$$

$$w = \frac{1}{\sqrt{N}} \sqrt{\left(\Sigma_{pp}^{(PT)} \eta_p^2\right)^2 + \left(\Sigma_{nn}^{(PT)} \eta_n^2\right)^2 + \left(\Sigma_{pn}^{(PT)} \eta_p \eta_n\right)^2},$$

(14)

(15)

where $g_p$ and $g_n$ are proton and neutron weak constants — they characterize the strength of the P-odd weak potential, $\eta_p, \eta_n$ are constants that characterize the strength of the T,P-odd potential, and $\Sigma$ are sums of the weighted squared matrix elements of the weak interaction between nucleon orbitals defined in Eq. (13). Contributions of the cross terms $\Sigma_{pn}^{(P)} g_p g_n$ and $\Sigma_{pn}^{(PT)} \eta_p \eta_n$ are small since they contain products of different matrix elements which have random signs, while in the terms containing squared interaction constants all contributions are positive.

Therefore we can present $v$ and $w$ in the following form:

$$v = K_P \sqrt{g_p^2 + k g_p^2},$$

$$w = K_{PT} \sqrt{\eta_p^2 + k \eta_p^2}.$$  

(16)

(17)

The coefficient $k$ should be slightly smaller than 1 since in heavy nuclei the number of neutrons $N = 1.5Z$, where $Z$ is the number of protons (i.e. the nuclear charge). To make a simple estimate of the sensitivity of $v$ and $w$ to changes in the interaction constants, we assume in the next step that $\Sigma$ from Eqs. (14) and (15) are proportional to the number of interaction terms in the nucleus. There are $Z^2/2$ interaction terms between protons, $N^2/2$ such terms between neutrons, and $ZN$ terms between a proton and a neutron (Fig. 1). Thus we can write:

$$k = \frac{Z^2 + 2ZN}{N^2 + 2ZN} = 0.76.$$  

(18)

Numerical calculations of $v$ and $w$ have been done in Refs. [29–32] for specific values of the interaction constants $g_p, g_n, \eta_p$ and $\eta_n$. The values of these constants have been updated since those calculations. Therefore, we would like to find updated values of these constants to insert into Eqs. (16) and (17). The general expressions for $g_p$ and $g_n$ are [58–61]:

$$g_p = 2 \cdot 10^5 W_p \left[ \frac{176 W_\pi}{W_p} f_\pi - 19.5 h_0^p - 4.7 h_1^p + 1.3 h_2^p - 11.3 (h_0^p + h_1^p) \right],$$

(19)

$$g_n = 2 \cdot 10^5 W_p \left[ -118 \frac{W_\pi}{W_p} f_\pi - 18.9 h_0^p + 8.4 h_1^p - 1.3 h_2^p - 12.8 (h_0^p + h_1^p) \right],$$

(20)

where $f$ and $h$ are the weak NN-meson couplings, and $W_\pi$ and $W_p$ are constants which account for the repulsion between nucleons at small distances as well as for the finite range of the interaction potential. We take $W_p = 0.4$ and $W_\pi = 0.16$ as in [59, 61].

For the choice of constants $g_p = 4, g_n = 1$ [62] numerical calculations give [29]:

$$v = K_P \sqrt{1 + 16k} = 2.08 \text{ meV}.$$  

(21)

We calculate updated values for $g_p$ and $g_n$, using the best values of the constants $h$ from DDH [60] with an updated $f_\pi \equiv h_0^1$, which was recently derived by lattice QCD methods [63, 64]. Such calculations give $g_p = 2.6, g_n = 1.5$ (Table I). Using these values with Eq. (21), we have:

$$v^{updated} = 2.08 \text{ meV} \frac{\sqrt{1.5^2 + 2.6^2k}}{\sqrt{1 + 16k}} = 1.56 \text{ meV},$$

(22)

where in the last step we used $k = 0.76$. This theoretical estimate is in excellent agreement with the experimental value $1.39^{+0.55}_{-0.38}$ meV [65, 66].

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4 For $k = 1$ we would get 1.51 meV. Thus we see that our result is not sensitive to the value of $k$. 

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Numerical calculations have been done for $\eta_p = \eta_n$ and gave $w = 0.2|\eta_n|$ meV [32]. Using this result and Eq. (17) we obtain:

$$w_{\text{updated}} = 0.15 \text{meV} \sqrt{\eta_n^2 + 0.76\eta_p^2}. \quad (23)$$

| Refs. | $g_p$ | $g_n$ |
|-------|-------|-------|
| DDH (1980) [60, 61] | 4.5 | 0.2 |
| ND (1986) [29, 62] | 4 | 1 |
| DZ (1986) [67] | 2.4 | 1.1 |
| FCDH (1991) [68] | 2.7 | −0.1 |
| Wasem (2012) [64] | 2.6 | 1.5 |

**TABLE I:** Values of $g_p$ and $g_n$ (two columns on the right) based on the meson exchange constants from different publications (the left column). Refs. [60, 67], and [68] quoted in the table derive the $h$ values which we used in equations Eqs. (19) – (20) to obtain $g_p$ and $g_n$. In the line of Wasem (2012) (Ref. [64]) we used the best DDH values for all the values of $h$ except $f_{\pi} = h_\pi^1$, which was recently derived by the lattice QCD methods [63, 64] to be $h_\pi^1 = 1.1 \cdot 10^{-7}$.

C. **The ratio $w/v$ expressed via meson exchange constants, QCD $\theta$-term and quark chromo-EDMs $d_u$ and $d_d$**

Now we can express the ratio $w/v$ in four different ways: as a function of $\eta_p, \eta_n$; by $\pi_0$ meson exchange coupling constants with the nuclei; by QCD CP violation parameter $\theta$; and finally by quark chromo-EDMs.

First, to express the ratio $w/v$ as a function of $\eta$, we use Eqs. (22) and (23) to obtain:

$$w \overline{v} = 0.10 \sqrt{\eta_n^2 + 0.76\eta_p^2}. \quad (24)$$

If, following Refs. [32, 69], we take $|\eta_p| = |\eta_n|$, the ratio in Eq. (24) becomes:

$$w \overline{v} = 0.13|\eta_n|. \quad (25)$$

Secondly, the T,P-odd nuclear forces are dominated by $\pi_0$ meson exchange. Such an exchange is described by the interaction [70–72]:

$$W(r_1 - r_2) = -\frac{\bar{g}}{8\pi m_N} \left[ \Gamma_1 \left( \frac{e^{-m_\pi r_{12}}}{r_{12}} \right) \right] \left\{ (\sigma_1 - \sigma_2) \cdot \left[ \bar{g}_0 (\tau_1 \cdot \tau_2 + \bar{g}_2 (\tau_1 \cdot \tau_2 - 3\tau_1z\tau_2z)) \right] + \bar{g}_1 (\tau_1z\sigma_1 - \tau_2z\sigma_2) \right\}, \quad (26)$$

where $\bar{g} = 13.6$ is the strong force T,P-conserving $\pi NN$ coupling constant, $\bar{g}_0, \bar{g}_1$ and $\bar{g}_2$, are the strengths of the isoscalar, isovector, and isotensor T,P-violating couplings respectively, $m_N$ is the nucleon mass, $m_\pi$ is the pion mass, $\sigma$ is the nucleon spin, $\tau$ is the nucleon Pauli isospin matrix in vector form, and $r_{12}$ is the separation between nucleons. The coupling constants $\eta$ can be expressed in terms of $\bar{g}$ [69]:

$$-\eta_p = \eta_n = 5 \cdot 10^6 \bar{g} (\bar{g}_1 + 0.4\bar{g}_2 - 0.2\bar{g}_0). \quad (27)$$

Then we have:

$$w \overline{v} = 0.13|\eta_n| = |6.5 \cdot 10^{-5} \bar{g} (\bar{g}_1 + 0.4\bar{g}_2 - 0.2\bar{g}_0)|. \quad (28)$$

Thirdly, using previous results $\bar{g}\bar{g}_0 = -0.37\theta$ [73], where $\theta$ is the QCD CP violation parameter, and $\bar{g}\bar{g}_1 = \bar{g}\bar{g}_2 = 0$, we can write the ratio $w/v$ as a function of $\theta$:

$$w \overline{v} = 4.8 \cdot 10^4|\theta|. \quad (29)$$

Using updated results [4, 74]:

$$\bar{g}\bar{g}_0 = -0.2108\theta, \quad (30)$$

$$\bar{g}\bar{g}_1 = 46.24 \cdot 10^{-3}\theta. \quad (31)$$

we can write, still with $\bar{g}\bar{g}_2 = 0$,

$$w \overline{v} = 5.7 \cdot 10^4|\theta|. \quad (32)$$

Using the current limit on $\theta$, obtained from constraints on neutron EDM, $|\theta| < 10^{-10}$ [4], we obtain:

$$w \overline{v} < 10^{-5}. \quad (33)$$

Lastly, we can connect our result to the quark chromo-EDM $\tilde{d}$ [2]:

$$\bar{g}\bar{g}_1 = 4 \cdot 10^{15} \left( \tilde{d}_u - \tilde{d}_d \right) / \text{cm}, \quad (34)$$

Then:

$$w \overline{v} = |6.5 \cdot 10^{20} \left( 4 \left( \tilde{d}_u - \tilde{d}_d \right) - 0.16 \left( \tilde{d}_u + \tilde{d}_d \right) \right) | / \text{cm}. \quad (35)$$

Using the current limits (Table IV in [75], see also [76]):

$$|\tilde{d}_u - \tilde{d}_d| < 6 \cdot 10^{-27} \text{cm}, \quad (36)$$

$$|\frac{1}{2} \tilde{d}_u + \tilde{d}_d| < 3 \cdot 10^{-26} \text{cm}, \quad (37)$$

we obtain:

$$w \overline{v} < 2 \cdot 10^{-5}. \quad (38)$$

**III. CONCLUSION**

Using the bound in Eq (38), and assuming $\kappa \approx 1$ in Eq (4) (which is reasonable [55] and matches experimental results [15, 26]), we arrive at:

$$\Delta \sigma_{PT} \over \Delta \sigma_P \sim 2 \cdot 10^{-5}. \quad (39)$$
Current expected experimental sensitivity is [26, 77]:

$$\frac{\Delta \sigma_{PT}}{\Delta \sigma_{F \text{ exp.sensitivity}}} < 10^{-6}. \quad (40)$$

Thus we confirm that the expected experimental sensitivity may be sufficient to improve the limit on the TRIV interactions, or possibly detect it.

ACKNOWLEDGEMENTS

This work is supported by the Australian Research Council and Gutenberg Fellowship.

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