Kac-Moody structure of chiral gravity in the light-cone gauge

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ABSTRACT

We study the residual symmetry $SL(2, R) \otimes U(1)$ of the chiral gravity in the light-cone gauge. Quantum gravitational effects renormalize the Kac-Moody central charge and introduce, through the Lorentz anomaly, an arbitrary parameter. Due to the presence of this free parameter the Kac-Moody central charge has no forbidden range of values, and the strong gravity regime is open to investigations.
Recently much work has been devoted to the study of quantum two dimensional gravity both in connection with strings out of critical dimensions and as a useful “laboratory” to get insight into realistic four dimensional gravity. The advantage of two dimensional gravity is that it is exactly solvable either in the context of random surfaces theory [1], matrix models [2], conformal field theory [3], or in a perturbative approach [4,5]. The choice of the light-cone gauge is one of the main ingredients in two dimensional gravity calculations. This gauge choice has at least two distinctive advantaged: first, light-cone coordinates led to the discovery of underlying Kac-Moody algebra of residual gauge symmetry; secondly, it provided a satisfactory regulator for perturbative calculations [6].

The above mentioned approaches displayed a remarkable result: the renormalized Kac-Moody central charge can attain complex value where physical information is lost. As a result, the weak gravity \( c \leq 1 \) and strong gravity \( c \geq 25 \) regimes (\( c \) denotes the total number of matter fields) are separated by a “phase transition”, which forbids a satisfactory investigation of the strong gravity domain. A more promising framework is provided by \( N = 2 \) super-gravity models where there is no gap at all between the weak and strong gravity regions [7]. Unfortunately, \( N = 2 \) models have unphysical signature (++)−−), and their physical relevance is presently unclear, although some effort has been devoted to provide a physical justification for them [8].

In this letter, we are going to study a chiral, induced, quantum gravity in the light-cone gauge. Our main interest in this model concerns the presence of
anomalies which cannot be removed by local counter-terms, and therefore naturally incorporate regularization dependent free parameter [9]. Though anomalous, chiral theories can achieve consistency at least in the case of the chiral Schwinger model [10]. In a similar spirit we shall investigate the Kac-Moody structure of chiral gravity in light-cone gauge with the hope that the presence of additional dynamical (Lorentz) degree of freedom may improve, or possibly avoid forbidden regions of values for renormalized Kac-Moody central charge. This hope is sustained by the known fact that the Kac-Moody central charge corresponding to an abelian symmetry (which, in our case, is described by Lorentz local invariance) does not get renormalized by quantum effects [11]. In this case all one needs to calculate is the Kac-Moody central charge corresponding to residual diffeomorphisms invariance (as in the case of non-chiral gravity) by taking into account the additional Lorentz contribution.

The dynamics of chiral induced gravity can be encoded into the symmetric action [12]

$$S = \int d^2x \sqrt{-g} \left[ \tilde{R} \frac{1}{\sqrt{g}} \tilde{R} + a' \omega^2 \right], \quad \tilde{R} = \alpha R + \beta \nabla \omega,$$

(1.1)

where: $\alpha$ and $\beta$ are constants related to the sum and the difference $n_{\pm}$ of the number of left and right chirality components of matter fermions, in a manner which we shall describe later; $\nabla_\mu$ is the Christoffel generally covariant derivative.

The action (1.1) displays the dependence from the spin connection $\omega_\mu$, acting as the gauge field of the local Lorentz symmetry. The non-local term in $\omega_\mu$ is the origin of the Lorentz anomaly, which cannot be removed by any choice of the coefficient $a'$ in front of the local term $\omega^2$. Strictly speaking, eq.(1.1) describes a
whole family of actions, each member of the family being labelled a different value of $a'$ corresponding to a different choice of the regularization scheme. Finally, the metric tensor $g_{\mu\nu}$ is actually a composite object, built up form the more fundamental zweibein field $e^a_\mu(x)$. Accordingly, the energy-momentum current should be properly defined as the response of the action under zweibein variation. However, we find more comfortable to work with the energy-momentum tensor defined as

$$T_{\mu\nu} \equiv -\frac{e^a_\mu}{\sqrt{-g}} \frac{\delta S}{\delta e^a_\nu}$$

$$= 2\alpha \nabla_\mu \nabla_\nu \phi - \nabla_\mu \phi \nabla_\nu \phi + \beta (\omega_\mu \nabla_\nu \phi + \omega_\nu \nabla_\mu \phi)$$

$$- a' \omega_\mu \omega_\nu - g_{\mu\nu} \left(2\alpha \nabla^2 \phi - \frac{1}{2} \nabla^\rho \phi \nabla_\rho \phi + \beta \omega^\rho \nabla_\rho \phi - \frac{a'}{2} \omega^2\right)$$

$$- 2\beta \left[\epsilon_{\mu\nu} \nabla^2 \phi + \epsilon_{\nu\rho} \nabla^\rho \nabla_\mu \phi\right] + 2a' \left[\epsilon_{\mu\nu} \nabla \omega + \epsilon_{\nu\rho} \nabla^\rho \omega_\mu\right],$$

where we introduced the auxiliary, scalar, field $\phi$, which allows to write the action (1.1) in a local form, and is related to $\tilde{R}$ by $\nabla^2 \phi = \tilde{R}$.

We shall be working in the light-cone gauge where the metric reads

$$ds^2 = dx^+dx^- + h_{++}dx^+dx^+,$$

and the spin-connection components can be written in terms of $h_{++}$, and the Lorentz degree of freedom $L$, as

$$\omega_+ = \left[\partial_+ L + 2\partial_- h_{++}\right],$$

$$\omega_- = \partial_- L.$$

Furthermore, $L$ is defined in terms of the zweibein components through

$$e^L = e^+ e^-.$$

Lorentz indices in eq.(1.5) are denoted by a hat to distinguish them from (unhatted) world indices.
Since the action (1.1) is only diffeomorphism invariant, general covariance is the only symmetry one can use to eliminate redundant degrees of freedom. We choose to eliminate $h_{--}$ and $S = \frac{1}{2}(h_{+-} + h_{-+})$, while $h_{++}$ and $L$(Lorentz degree of freedom) remain as dynamical variables. Therefore, various components of $T_{\mu\nu}$ have distinct role. $T_{--}$ and $\epsilon^{\mu\nu}T_{\mu\nu}$ couple to dynamical degrees of freedom and will generate equations of motion governing dynamics of chiral gravity. These components are given by

$$T_{--}(\phi) = \left[ 2(\alpha + \beta)\partial_-^2 \phi - (\partial_- \phi)^2 + 2\beta \omega_- \partial_- \phi - 2a' \partial_- \omega_- - a' \omega_-^2 \right] , \quad (1.6)$$

and

$$\epsilon^{\mu\nu}T_{\mu\nu} = -2\alpha \beta R + 2 \left( a' - \beta^2 \right) \nabla \omega . \quad (1.7)$$

Resulting equations of motion are

$$- \left[ (\alpha \pm \beta)^2 + (a' - \beta^2) \left( 1 \mp \frac{\alpha \beta}{a' - \beta^2} \right)^2 \right] \partial_+^2 h_{++} - \alpha \beta \partial_-^2 A_+ = 0 , \quad (1.8)$$

$$(a' - \beta^2) \partial_- A_+ = 0 , \quad (1.9)$$

where we have introduced convenient redefinition

$$A_+ = D_+ L + \left( 1 - \frac{\alpha \beta}{a' - \beta^2} \right) \partial_- h_{++} \quad (1.10)$$

which gives (1.9) a simple looking form, and decouples fields in the lagrangian (1.1)
which then written in the light-cone gauge is:

\[
L = \left[ (\alpha \pm \beta)^2 + (a' - \beta^2) \left( 1 \mp \frac{\alpha \beta}{a' - \beta^2} \right) \right]^2 \partial_- h_{++} \frac{1}{D_+} \partial_- h_{++} - (a' - \beta^2) \partial_- A_+ \frac{1}{D_+} A_+ .
\]

(1.11)

The new derivative \( D_+ \) is defined as \( D_+ L = \partial_+ L - h_{++} \partial_- L \). On the other hand, components of the energy-momentum tensor \( T_{++} \) and \( T_{+-} \) that couple to the gauge degrees of freedom \( h_{--} \) and \( S (=\text{Weyl degree of freedom}) \) are the generators of the residual symmetry of the invariant line element (1.2) in the light-cone gauge [13], and weakly vanishing condition must be imposed in order to preserve the residual symmetries at the quantum level. It is worth mentioning that the Lorentz symmetry (or its absence) has no influence on the line element and serves only to induce dynamics for the Lorentz degree of freedom through (1.7). These components of the energy-momentum tensor are

\[
T_{++} = \left[ -(\partial_- h_{++})^2 + 2h_{++} \partial_-^2 h_{++} - 2Q_1 \partial_- \partial_+ h_{++} \right] + \left[ A_+^2 - 2Q_2 \partial_+ A_+ \right] \quad (1.12)
\]

where we have conveniently rescaled fields according to

\[
h_{++} \rightarrow \left[ -(\alpha \pm \beta)^2 - (a' - \beta^2) \left( 1 \mp \frac{\alpha \beta}{a' - \beta^2} \right) \right]^2 \left( \frac{1}{2} \right)^{1/2} h_{++} ,
\]

\[
A_+ \rightarrow (a' - \beta^2)^{1/2} A_+ ,
\]

(1.13)

with the constants \( Q_1, Q_2 \) defined as

\[
Q_1 \equiv \left[ -(\alpha \pm \beta)^2 - (a' - \beta^2) \left( 1 \mp \frac{\alpha \beta}{a' - \beta^2} \right) \right]^2 \left( \frac{1}{2} \right)^{1/2} ,
\]

\[
Q_2 \equiv (a' - \beta^2)^{1/2} \left( 1 + \frac{\alpha \beta}{a' - \beta^2} \right) .
\]

(1.14)

We would reasonably expect the rescalings (1.13) to be real in order to preserve the
physical character of the corresponding fields. But, this condition is not guaranteed by eqs.(1.13) and must be imposed as a constrain on the parameters. Reality of the first rescaling in eqs.(1.13) gives

$$\frac{1}{4} (|\alpha + \beta| + |\alpha - \beta|)^2 < a' - \beta^2 < \frac{1}{4} (|\alpha + \beta| - |\alpha - \beta|)^2 ,$$  

(1.15)

and, unavoidably, leads to a purely imaginary rescaling of \( A_+ \). As a consequence, the kinetic term of \( A_+ \) flips from the correct to the “wrong” sign, and the rescaled field becomes a ghost-like object. Alternatively, one could maintain \( A_+ \) real at the expense of assigning \( h_{++} \) a ghost-like character. In what follows we shall adhere to the former choice. The impossibility of having simultaneously both \( h_{++} \) and \( A_+ \) in the physical sector, is the light-cone gauge analogue of a similar result obtained in the conformal gauge, where, in order to have a physical Liouville field, the Lorentz degree of freedom must belong to the ghost-like sector [14].

As we mentioned earlier, constants \( \alpha, \beta \) are related to the number of left and right chirality component of matter in the following way [15,12]

\[
\begin{align*}
\alpha^2 + \beta^2 &= \frac{N}{96\pi} \\
\alpha\beta &= \frac{\Delta N}{192\pi} \\
\hat{a} &= 192\pi (a' - \beta^2) \\
N &= n_+ + n_- \\
\Delta N &= n_+ - n_- \\
(\alpha \pm \beta)^2 &= \frac{n_\pm}{48\pi}
\end{align*}
\]

(1.16)

Eqs.(1.16) give the link among the parameter in our symmetric action (1.1) and the parameters, in the asymmetric action for chiral gravity, that usually appears
in the literature. Therefore, eqs.(1.16) provide the translation code among our formulae and those in other papers [15,16].

To complete our calculation, it is further necessary to account for the ghosts corresponding to the light-cone gauge choice, as well as for the non-trivial Jacobian following from the field redefinition (1.10). It is possible to give a unique formula containing all these contributions by noticing that either ghosts or the Jacobian are expressed in terms of the generalized derivative

\[ D_{(q)}^+ = \partial_+ - h_{++} \partial_- - q \partial_- h_{++} \]  

(1.17)

where \( q \) is the Lorentz weight of the field, the operator \( D_{(q)}^+ \) is acting on. Therefore, one can find the following result

\[ \left( \det D_{(q)}^+ \right)^{(-n)^{s+1}} = \exp \left[ \frac{(-1)^s n}{24\pi} (6q^2 - 6q + 1) \int d^2x \partial^2 h_{++} \frac{1}{D^+_+ \partial_- h_{++}} \right] \]  

(1.18)

where \( s \) denotes the statistics of the involved fields, and \( n \) is either 1 or 1/2 depending whether field variables are complex or real. \( T_{++} \) in eq.(1.12) does not contain quantum gravitational contributions. Its Virasoro central charge is given by

\[ c_{\text{grav.}} = 28 - n_- \]  

\[ c_{\text{grav.}} = 2 + 48\pi \left( Q_1^2 + Q_2^2 \right) \]  

(1.19)

This the same result obtained in the conformal gauge [3] with the DDK argument of the vanishing of the total central charge. It is worth mentioning that the Virasoro central charge is independent of the free parameter due to the explicit cancellation
between $Q_1^2$ and $Q_2^2$. The solutions of the equations of motion (1.7), (1.9) are:

\[
\begin{align*}
    h_{++}(x) &= J_{++}(x^+) - 2x^- J_{+}(x^+) + (x^-)^2 J_0(x^+) , \\
    A_{+}(x) &= \tilde{J}_{+}(x^+) .
\end{align*}
\] (1.20)

With the help of the anomaly equations (1.6), (1.7) it is possible to obtain Ward identities relating various multipoint functions [4] which in our case read

\[
\begin{align*}
    \left\langle h_{++}(x)h_{++}(x_1)\ldots h_{++}(x_n) \right\rangle &= \sum_{i=1}^{n} \left\{ 4\pi Q_1^2 \left( \frac{x^- - x_i^-}{x^+ - x_i^+} \right)^2 \left\langle h_{++}(x)\ldots \tilde{h}_{++}(x_i)\ldots h_{++}(x_n) \right\rangle \right. \\
    &\left. + \left[ \frac{(x^- - x_i^-)^2}{x^+ - x_i^+} \partial_{x_i} + 2\frac{x^- - x_i^-}{x^+ - x_i^+} \right] \left\langle h_{++}(x)\ldots h_{++}(x_i)\ldots h_{++}(x_n) \right\rangle \right\} \\
    (1.21)
\end{align*}
\]

\[
\begin{align*}
    \left\langle A_{+}(x)A_{+}(x_1)\ldots A_{+}(x_n) \right\rangle &= 4\pi Q_2^2 \sum_{i=1}^{n} \frac{1}{(x^+ - x_i^+)^2} \left\langle A_{+}(x)\ldots \tilde{A}_{+}(x_i)\ldots A_{+}(x_n) \right\rangle , \\
    (1.22)
\end{align*}
\]

where we have conveniently rescaled fields as $h_{++} \rightarrow (1/4\pi Q_1^2)h_{++}$, and $A_{+} \rightarrow (1/\sqrt{2(a' - \beta^2)})2\pi Q_2 A_{+}$, and "hat" means omission of that term. From eqs.(1.21), (1.22) and (1.20) one finds the OPE for various currents

\[
\begin{align*}
    J^a(x^+)J^b(x^+) &= -\frac{K}{2} \frac{\eta^{ab}}{(x^+ - y^+)^2} + \frac{f^{ab}}{2} J^c(y^+) + \text{reg.} , \\
    \tilde{J}(x^+)\tilde{J}(x^+) &= -\frac{K_{U(1)}}{2} \frac{1}{(x^+ - y^+)^2} + \text{reg.} , \\
    J^a(x^+)\tilde{J}(x^+) &= \text{reg.} ,
\end{align*}
\] (1.23)

where $K$ and $K_{U(1)}$ are given by
K^{cl.} = -8\pi Q_1^2, \quad (1.24)
K^{cl.}_{U(1)} = -8\pi Q_2^2.

and \ f_{abc} \ and \ \eta^{ab} \ are \ structure \ constants \ and \ metric \ of \ the \ SL(2, R) \ current \ algebra.

Therefore, we see that in the case of chiral gravity the underlying Kac-Moody structure is \ SL(2, R) \ \otimes \ U(1). \ The \ solutions (1.20) allow to write the quantum version of the Sugawara energy-momentum tensor

\[ T_{++} = -\frac{1}{K + 2} \eta^{ab} : J_a J_b : -\partial_+ J_+ - \frac{1}{K_{U(1)}} : J_+^2 : -\partial_+ \tilde{J}_+ : \quad (1.25) \]

Various OPE’s of such \( T_{++} \) with other fields are determined to be

\[ T_{++}(x)A_+(y) = \frac{K_{U(1)}}{(x^+ - y^+)^3} + \frac{A_+(y)}{(x^+ - y^+)^2} + \frac{\partial_+ A_+(y)}{x^+ - y^+} + \text{reg.}, \quad (1.26) \]

\[ T_{++}(x)h_{++}(y) = -\frac{2y^+ K}{(x^+ - y^+)^3} + \frac{(2h_{++}(y) - y^- \partial_- h_{++})}{(x^+ - y^+)^2} + \frac{\partial_+ h_{++}(y)}{x^+ - y^+} + \text{reg.}, \quad (1.27) \]

\[ T_{++}(x)T_{++}(y) = \frac{1}{2} \frac{3K + 2 - 6K + 1 - 6K_{U(1)}}{(x^+ - y^+)^4} + \frac{2T_{++}(y)}{(x^+ - y^+)^2} + \frac{\partial_+ T_{++}(y)}{x^+ - y^+} \]

\[ -\frac{x^- - y^-}{x^+ - y^+} \left[ \frac{2\partial_- T_{++}(y)}{(x^+ - y^+)^2} + \frac{\partial_+ \partial_- T_{++}(y)}{x^+ - y^+} \right] + \text{reg.}, \quad (1.28) \]

\[ T_{++}(x)T_{+-}(y) = \frac{2T_{+-}(y)}{(x^+ - y^+)^2} + \frac{\partial_+ T_{+-}(y)}{x^+ - y^+} + \text{reg.}, \]

\[ T_{+-}(x)T_{+-}(y) = \text{reg.}. \]

From eqs.(1.26),(1.27) we see that \( A_+ \) and \( h_{++} \) are not primary fields. Eqs.(1.28) show OPE of the quantum generators of the residual symmetries in the light-cone
gauge. Symmetry condition is equivalent to the weakly vanishing of $T_{++}$ and $T_{+-} \propto J_0$ which leads to the vanishing of the total Virasoro central charge

$$n_+ - 28 + c_{grav.} = 0,$$
$$c_{grav.} = \frac{3K}{K+2} - 6K + 1 + 48\pi Q^2_2.$$  \hspace{1cm} (1.29)$$

Comparing eq.(1.29) to (1.19) shows that these are the same equations resulting from the vanishing of the total Virasoro charge. The difference is that in eq.(1.29) we have exploited the quantum relation between Virasoro and Kac-Moody charge $c = \frac{K \cdot \text{dim} G}{K+2}$ [3], where $G$ is appropriate symmetry group, and, therefore, eq.(1.29) is an equation for the renormalized Kac-Moody central charge.

$$K + 2 = -\frac{1}{12} \left[ A \pm \sqrt{(A-12)(A+12)} \right]$$  \hspace{1cm} (1.30)$$

where $A = -12 - 48\pi Q^2_1$ and we have exploited the property of the $U(1)$ Kac-Moody central charge of being not renormalized at the quantum level [11]. Eq.(1.30) displays dependence of the renormalized Kac-Moody central charge on the free parameter, while Virasoro central charge of gravity is independent of such a parameter as visible from eq.(1.29) or (1.19). The later result has been obtained also in the conformal gauge while the former is visible only indirectly in this gauge as explained later in the conclusions of this letter.

Now, the reality condition (1.15) can be written as a constraint over the allowed values $\hat{a}$:

$$(\sqrt{n_+} - \sqrt{n_-})^2 < -\hat{a} < (\sqrt{n_+} + \sqrt{n_-})^2.$$  \hspace{1cm} (1.31)$$

Once $\hat{a}$ is chosen inside the range defined by (1.31) the square root in eq.(1.30) is real, and the renormalized $SL(2, R)$ Kac-Moody central charge is real too, without
any restriction over $n_{\pm}$. This is our main conclusion. The above result has fulfilled our hope that the presence of Lorentz anomaly (and therefore of a free parameter) improves the value of the renormalized Kac-Moody central charge with respect to the non-chiral gravity, where exist regions of complex values of the renormalized Kac-Moody charge. It is therefore reasonable to expect that the physical parameters such as conformal dimensions and string susceptibility will depend on the free parameter as well. The latter is expressed in terms of the renormalized Kac-Moody central charge by: $\Gamma = K + 3$. If we use eq. (1.30) to compute $\Gamma$, we reproduce a result previously found in the conformal gauge [17].

In this letter we have considered the Kac-Moody structure of the chiral induced gravity in the light-cone gauge. We have shown that the presence of the Lorentz degree of freedom produces results different from non-chiral situation. First of all, Kac-Moody algebra is enlarged by the Lorentz $U(1)$ factor; secondly, renormalized $SL(2, R)$ Kac-Moody central charge has no forbidden regions thanks to the presence of an arbitrary parameter. Our results agree with those obtained in conformal gauge by the DDK method [18]. Although comparison between the two gauges is possible, as in [19], $SL(2, R)$ symmetry is not directly visible in the conformal gauge but only through the comparison of the expressions for invariant quantities such as string susceptibility and scaling dimensions. Details of this comparison will be given elsewhere, but we anticipated the above relation for the string susceptibility proving agreement to the conformal gauge. The role of the regularization parameter has also been considered in [20], where it has been claimed that the light-cone analysis selects the value $\hat{a} = -(n_+ - n_-)/192\pi$ for the regularization parameter. This choice does not seem to be justified neither in our analysis, nor
in conformal gauge. The result proposed in ref.[20] is equivalent, in our language, to setting $Q_2 = 0$. But, we find no physical reason for such a choice.

Finally, as far as the presence of a ghost-like field is concerned, we recall that a mechanism to decouple the Lorentz ghost from the physical sector has been proposed for the model in the conformal gauge [14]. It is based on the usual treatment of ghosts in the BRST formalism. The same decoupling mechanism works in the light-cone gauge as well, therefore resolving the problem of unitarity in this model.

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