Abstract

Decay of $B_s$ meson into the dileptonic channels of $e^+e^-$ and $\mu^+\mu^-$ are suppressed in the QCD corrected standard model due to the vector and axial nature of the coupling to lepton bilinears. We show that in SUSY theories with large $\tan\beta$, contributions to the amplitude arising out of exchange of neutral Higgs Bosons considerably enhances the decay of $B_s$ into $l^+l^-$. In some region of parameter space, the enhancement is more than two orders of magnitude, bringing it well within experimental possibilities in the near future.
Flavor changing neutral current (FCNC) decays of the neutral B-mesons, $B^0$ and $B^0_s$, provide unique testing grounds of the Standard Model (SM) improved by QCD-corrections via the operator product expansions [1]. During the last few years, considerable theoretical attention has therefore been focussed on decays like $B \to K^\ast \gamma$, $B \to X_s \gamma$, $B \to X_d l^+ l^-$, $B_s \to \gamma \gamma$ and $B_s \to \gamma l^+ l^-$ in view of the planned experiments at B-factories, which are likely to measure branching fractions as low as $10^{-8}$ [1]. The calculations of the branching ratios of FCNC B-decay processes are sensitive to new physics beyond SM. In particular calculations based on Minimal Supersymmetric extension of the Standard Model (MSSM) for many of FCNC processes have been done [2],[3]. Nothing spectacularly different from SM happens to the theoretical predictions but sizable changes in the branching ratios can occur for some regions of the parameter space in MSSM [2],[3].

The MSSM and in general any two Higgs-doublet model has in addition to the unknown masses of the new particle content over and above SM, the ratio of the vacuum expectation values of the two neutral Higgs fields, $\tan\beta = v_2/v_2$, as parameters [4]. From available data, one can only constrain the parameter space within certain regions and typically $\tan\beta$ can be constrained to be $\tan\beta \leq 50$, i.e. it can be large [2],[3].

In some recent works [4], it has been pointed out that the processes $B \to X_s \tau^+ \tau^-$ and $B \to X_s \mu^+ \mu^-$ provide unique opportunities to distinguish SUSY models with large $\tan\beta$. Exchange of Neutral Higgs Boson (NHB) gives rise to new amplitude which were negligible in SM but which for large $\tan\beta$ can enhance these processes by as much as 200%. The purpose of this note is to show that in SUSY-models with large $\tan\beta$ NHB exchange contributions causes a sizable enhancement of processes like $B_s \to \mu^+ \mu^-$, as much as by two orders of magnitude in some allowed region of parameter space.

Let us start by recalling the result for $B_s \to \mu^+ \mu^-$ in QCD-improved Standard Model [5]. The effective Hamiltonian describing this process is:

$$H_{\text{eff}} = \frac{\alpha G_F}{\sqrt{2} \pi} \lambda \left[ C_{9}^{\text{eff}} \left( \bar{s} \gamma^\mu \gamma_5 \gamma_5 P_L b \right) (\bar{l} \gamma^\mu l) + C_{10} \left( \bar{s} \gamma^\mu P_L b \right) (\bar{l} \gamma_\mu l) + \frac{2C_{7}^{tb} m_b}{p^2} \left( \bar{s} \not{p} \gamma^\mu P_R b \right) (\bar{l} \gamma^\mu l) \right]$$

(1)

with $\lambda \equiv V_{tb} V_{ts}^*$, $P_{L,R} = \frac{1}{2} \left( 1 \mp \gamma_5 \right)$ and $p = p_+ + p_-$, the sum of the
momentum of $\mu^+$ and $\mu^-$. $C_7(m_b)$, $C_{eff}^7(m_b)$ and $C_{10}(m_b)$ are the Wilson Co-efficients, whose values are given by Misiak [8]. Since we are considering $B_s$, the matrix element of $\mathcal{H}_{eff}$ is to be taken between vacuum and $|B_s^0\rangle$ state. This can be expressed in terms of the $B_s^0$ decay constant $f_{B_s}$ [7].

\begin{align}
\langle 0 | \bar{s} \gamma^\mu \gamma_5 b | B_s^0 \rangle &= - f_{B_s} p_B^\mu \\
\langle 0 | \bar{s} \gamma_5 b | B_s^0 \rangle &= - f_{B_s} m_{B_s}
\end{align}

and

\begin{equation}
\langle 0 | \bar{s} \sigma^{\mu\nu} P_R b | B_s^0 \rangle = 0
\end{equation}

Since $p_B^\mu = p_{s}^\mu + p_l^\mu$, the $C_9$ term in eqn(1) gives zero on contraction with the lepton bilinear, $C_7$ gives zero by (4) and the remaining $C_{10}$ term gets a factor of 2 $m_l$.

Thus

\begin{equation}
\langle 0 | \mathcal{H}_{eff} | B_s \rangle = \frac{\alpha G_F}{\sqrt{2\pi}} \lambda \frac{1}{2} f_{B_s} C_{10} p_B^\mu \bar{l} \gamma_5 l
\end{equation}

Decay rate of $B_s \to \mu^+ \mu^-$ is

\begin{equation}
\Gamma(B_s \to l^+ l^-) = \frac{\alpha^2 G_F^2 f_{B_s}^2 m_{B_s} m_l^2}{16\pi^3} |V_{tb} V_{ts}^*|^2 C_{10}^2
\end{equation}

The presence of the factor $m_l^2$ makes (6) almost vanish for $e^+ e^-$ and give rise to a small Branching ratio of $2.6 \times 10^{-9}$ for $\mu^+ \mu^-$.  

Consider now the MSSM. $l^+ l^-$ production via the neutral Higgs bosons give rise to additional amplitudes and these become important for large $\tan \beta$ [4]. The neutral Higgs bosons coupling to b-quark and $l^+$ proportional to $m_t \tan \beta$ and $m_l \tan \beta$ respectively for large $\tan \beta$ [4]. As pointed out in [4] $m_l \tan \beta$ can be large $\sim m_b$ or more. In addition, as has been pointed out in [4], the $b \to s$ transition via chargino-stop loop gives rise to another $\tan \beta$ factor, so that the $b \to s \ l^+ \ l^-$ amplitude goes like $m_b m_l (\tan \beta)^3$ for large $\tan \beta$. Thus these additional amplitude effectively do not suffer from the $m_l$ suppression of expression (3). The additional diagrams can be taken care of through and additional effective Hamiltonian[4]

\begin{equation}
\mathcal{H}_{eff}^{NHB} = \frac{\alpha G_F}{\sqrt{2\pi}} \lambda \left[ C_{Q_1} (\bar{s} P_R b) (\bar{l} l) + C_{Q_2} (\bar{s} P_R b) (\bar{l} \gamma_5 l) \right]
\end{equation}
The co-efficients $C_{Q_1}, C_{Q_2}$ are Wilson co-efficients. There values at $M_W$ scale can be calculated by evaluating the Feynman diagrams of NHBs contributions. The result is

\[
C_{Q_1}(m_W) = \frac{m_b m_\mu}{4m_{\rho_0}^2 \sin^2 \theta_W} \tan^2 \beta \left\{ \left( \sin^2 \alpha + h \cos^2 \alpha \right) \left( \frac{1}{x_{Wt}} (f_1(x_{Ht}) - f_1(x_{Wt})) \right) + \sqrt{2} \sum_{i=1}^{2} \frac{m_{\chi_i}}{m_W} \frac{U_{i2}}{\cos \beta} \left( -V_{i1} f_1(x_{\chi_i q}) + \sum_{k=1}^{2} \Lambda(i,k) T_{k1} f_1(x_{\chi_i i_k}) \right) + (1 + \frac{m_{H_u}^2}{m_W^2}) f_2(x_{Ht}, x_{Wt}) \right\} 
+ 2 \sum_{i'i'} \left\{ B_1(i,i') \Gamma_1(i,i') + A_1(i,i') \Gamma_2(i,i') \right\} \quad (8)
\]

\[
C_{Q_2}(m_W) = -\frac{m_b m_\mu}{4m_{\rho_0}^2 \sin^2 \theta_W} \tan^2 \beta \left\{ \left( \sin^2 \alpha + h \cos^2 \alpha \right) \left( \frac{1}{x_{Wt}} (f_1(x_{Ht}) - f_1(x_{Wt})) + 2 f_2(x_{Ht}, x_{Wt}) \right) + \sqrt{2} \sum_{i=1}^{2} \frac{m_{\chi_i}}{m_W} \frac{U_{i2}}{\cos \beta} \left( -V_{i1} f_1(x_{\chi_i q}) + \sum_{k=1}^{2} \Lambda(i,k) T_{k1} f_1(x_{\chi_i i_k}) \right) + 2 \sum_{i'i'} \left\{ -U_{i'2} V_{i1} \Gamma_1(i,i') + U_{i'2} V_{i'1}^* \Gamma_2(i,i') \right\} \right\} \quad (9)
\]

where

\[
B_1(i,i') = -\frac{1}{2} U_{i'1} V_{i2} \sin 2\alpha (1 - h) + U_{i'2} V_{i1} (\sin^2 \alpha + h \cos^2 \alpha)
\]

\[
A_1(i,i') = -\frac{1}{2} U_{i1}^* V_{i'2} \sin 2\alpha (1 - h) + U_{i1}^* V_{i'2} (\sin^2 \alpha + h \cos^2 \alpha)
\]

\[
\Gamma_1(i,i') = m_{\chi_i} m_{\chi_{i'}} U_{i2} \left( -\frac{1}{m^2} f_2(x_{\chi_i q}, x_{\chi_{i'} q}) V_{i'1} + \sum_{k=1}^{2} \frac{1}{m_{i_k}^2} \Lambda(i',k) T_{k1} f_2(x_{\chi_i i_k}) \right)
\]

\[
\Gamma_2(i,i') = U_{i2} (-f_2(x_{\chi_i q}, x_{\chi_{i'} q}) V_{i'1} + \sum_{k=1}^{2} \Lambda(i',k) T_{k1} f_2(x_{\chi_i i_k}))
\]

\[
\Lambda(i,k) = V_{i1} T_{k1} - V_{i2} T_{k2} \frac{m_t}{\sqrt{2} m_W \sin \beta}
\]

\[
f_1(x_{ij}) = 1 - \frac{x_{ij}}{x_{ij} - 1} \ln x_{ij} + \ln x_{Wj}
\]

\[
f_2(x, y) = \frac{1}{x - y} \left( -\frac{x}{x - 1} \ln x - \frac{y}{y - 1} \ln y \right)
\]

\[
x_{ij} = m_i^2 / m_j^2 \quad ; \quad x_{Wj} = m_W^2 / m_j^2
\]
In (8),(9), the various m’s represent the masses of the corresponding particle and $h = m_{h^0}^2/m_{H^0}^2$. U and V represent the matrices which diagonalises the chargino masses. T is the matrix used for diagonalisation of stop mass matrix. $\tilde{m}$ is the average mass of the first two generation of squarks. Starting with $C_{Q_i}(m_W)$, there values at a scale $m_b$ can be evaluated by solving the RGE equations. The result is

$$C_{Q_i}(m_b) = \eta^{-\gamma_{Q_i}/\beta_0}C_{Q_i}(M_W)$$

(11)

where $\eta = \alpha_s(m_b)/\alpha_s(M_W) \approx 1.8, \gamma_Q = -4$ and $\beta = 11 - (2/3)n_f = 9$.

Taking the matrix elements of (7) between vacuum and $|B_0\rangle$ as before, we see in effect that $C_{Q_1}$ and $C_{Q_2}$ enter multiplied by $m_{B_s}$ unlike $C_{10}$ in (8) which entered as $m_{l}C_{10}$. For large $\tan\beta$, thus, the decay $B_s \rightarrow \mu^+ \mu^-$ will be dominated by (7) and we get:

$$\Gamma^{SUSY}(B_s \rightarrow \mu^+ \mu^-) = \frac{\alpha^2 G_F^2 f_B^2 m_{B_s}}{16\pi^3} |V_{tb}V_{ts}^*|^2 \left[ \frac{C_{Q_1}^2 m_{B_s}^2}{4} \right]$$

$$+ \left( m_{\mu}C_{10} + \frac{m_{B_s}}{2C_{Q_2}} \right)^2$$

(12)

The co-efficient $C_{10}$ in (12) has been evaluated in NLO approximation by Misiak in SM. We use his value since SUSY contributions are expected to change this value only slightly and also because in the region of large $\tan\beta$, the RHS of (12) will be dominated by the $C_{Q_1}, C_{Q_2}$ term. As we can see from (8)-(11) the co-efficients $C_{Q_1}$ and $C_{Q_2}$ depend, in addition to SM parameters, to the MSSM parameters. The most economical version of MSSM is the one where the spontaneous breaking of $SU(2) \otimes U(1)$ is achieved through radiative effects. Further at Unification scale of $M_{GUT}(\sim 10^{16}$ GeV) scalars have a common soft breaking mass term m and Gauginos also have a unified soft breaking mass M. Thus at scale $M_{GUT}$, MSSM in addition of SM parameters have five parameters: m,M, the ratio of the vacuum expectation values of the two Higgs fields $\tan\beta$, the trilinear soft breaking scalar term coefficient A and the bilinear soft breaking term coefficient B. The values of these parameters at scale $\sim m_b$ relevant to us can be related to their values at $M_{GUT}$ through RGE which has been extensively discussed in literature. As pointed out in [13] to suppress the large contribution to $K^0 - \bar{K}^0$ mixing it is sufficient to require degeneracy of the soft SUSY breaking mass in squark sector. Thus, the strict universality of all scalar masses is not necessarily required in context of SUGRA model. We thus consider in our estimate of
study $B_s \rightarrow \mu^+\mu^-$, in regions of parameter space where the condition of universality of scalar masses is relaxed.

The parameter space of MSSM is constrained by several pieces of known experimental results, the $W^\pm, Z$ one loop mass corrections, $(g - 2)$ of the $\mu$ meson and most importantly by the lower and upper bounds to $b \rightarrow s\gamma$ decay rate \[13\] :

\[
B(b \rightarrow s\gamma) = (0.6 - 5.4) \times 10^{-4}
\]

MSSM parameter space in this context has been extensively analyzed by Lopez et.al \[10\], Goto et.al \[14\]. We present in Fig(3) the rate of $\Gamma^{SUSY}(B_s \rightarrow \mu^+\mu^-)$ \[12\] relative to their SM values for some of the allowed region of the parameter space for large values of $\tan\beta$ in SUGRA model where the universality of scalar masses in SUGRA model is relaxed. The SM branching ratio of $B_s \rightarrow \mu^+\mu^-$ is $2.6 \times 10^{-9}$, so for large values of $\tan\beta$ the branching ratio can be as large as $\sim 10^{-6}$. This is well within the experimental reach of the B-factories expected soon. The $B_s \rightarrow \mu^+\mu^-$ observation or otherwise will thus serve as a very useful constraint on the parameter space of MSSM.

We present the result of SUGRA model in which we are assuming the universality of scalar masses in Fig(4). We also present the result of a more predictive class of Supergravity (SUGRA) theories: the Moduli (or No-scale) scenario ($m = A = 0$ at unification scale) and the dilaton scenario ($m = M/\sqrt{3}, A = -1$) in Fig(4). We see that with the allowed choice of parameters the $B_s \rightarrow \mu^+\mu^-$ rate depend crucially on the value of the Higgs scalars and pseudo-scalars. For low values of those masses with not too small splitting of the stops, the rate can be enhanced by more than two orders of magnitude (Fig 3). Thus $B_s \rightarrow \mu^+\mu^-$ data together with estimates of masses of superpartners will be very useful in cross-checking these models with experimental results.

We next turn to the decay $B_s \rightarrow l^+ l^- \gamma$, which was studied in the framework of QCD corrected effective SM Hamiltonian by Eilam et.al [11]. As is pointed out there, the radiative process unlike the non-radiative channel does not face the helicity suppression factor proportional to $m_l$. The net result that one ends up is paradoxical in that the Branching ratio for the radiative mode turns out to be higher than the one for the non-radiative mode notwithstanding the fact that the former has an extra factor of $\alpha$. This is very easily seen by considering the set of graphs where the photon line is hooked on to the various external lines of the non-radiative decay process. Graphs when the photon line is hooked on to the lepton gets a factor
of \(m_l\) and so can be ignored. The two graphs with the photon hooked on to the b and s quark give rise to the following amplitude for the process \(B_s(p_{ Bs}) \rightarrow \gamma(p_\gamma) + l^+(p_+) + l^- (p_-)\) [11]

\[
A(B_s \rightarrow l^+ l^- \gamma) = \frac{\alpha G_F}{\sqrt{2\pi}} \lambda \epsilon_\nu \left[ \gamma^\nu \frac{\not{p}_\gamma - \not{p}_s + m_s}{-2p_s.p_\gamma} \gamma^\mu P_L \cdot Q_s 
\right. \\
+ P_R \gamma^\mu \frac{\not{p}_b - \not{p}_s + m_b}{-2p_b.p_\gamma} Q_b \left. \left[ C_9^{\text{eff}} \bar{l} \gamma_\mu l + C_{10} \bar{l} \gamma_\mu \gamma_5 l \right] 
\right]
\]

Another contribution can be written in terms of \(C_{Q_1}\) and \(C_{Q_2}\) terms. So \(A(B_s \rightarrow l^+ l^- \gamma)\) is:

\[
A(B_s \rightarrow l^+ l^- \gamma) \approx \frac{e \alpha G_F f_{B_s} \lambda}{12\sqrt{2\pi}} \left( \frac{m_s}{m_b} \right) \left( p_\gamma \cdot \epsilon_\nu - (p_\gamma)_\nu \epsilon^\mu \right) \\
\times \left[ C_0 - \frac{2C_7 m^2_{B_s}}{p^2} \left( \bar{l} \gamma_\mu l + C_{10} \bar{l} \gamma_\mu \gamma_5 l \right) \right]
\]

Where once again we have used eqns(2),(3),(4). Using the values of \(C_0, C_7\) and \(C_{10}\) the branching ratio into the \(B^0_s \rightarrow \mu^+ \mu^- \gamma\) mode is estimated at \(4.6 \times 10^{-9}\) [11], almost twice the branching ratio of \(B^0_s \rightarrow \mu^+ \mu^-\).

Let us now examine the effect of the NHB exchange diagram in SUSY models on the amplitude \(A(B_s \rightarrow l^+ l^- \gamma)\). The contribution can be written in terms of \(C_{Q_1}\) and \(C_{Q_2}\) terms. So \(A(B_s \rightarrow l^+ l^- \gamma)\) is:

\[
A^{NHB}(B_s \rightarrow l^+ l^- \gamma) \approx \frac{\alpha G_F}{12\sqrt{2\pi}} f_{B_s} \lambda \epsilon_\nu \bar{s} \left[ \gamma^\nu \frac{\not{p}_\gamma}{m_s p_{BS} \cdot p_\gamma} P_R \right] b \left[ C_{Q_1} (\bar{l} l) + C_{Q_2} (\bar{l} \gamma_5 l) \right]
\]

where we have made the same approximations as in deriving eqn(13) i.e. retaining only the leading power of \((m_s/m_b)\).
However:

\[ \epsilon_\nu \langle 0 | \bar{s}_\gamma P_R b | B_s^0 \rangle = \frac{1}{2} \epsilon_\nu p_\nu \langle 0 | \bar{s} P_R b | B_s^0 \rangle \]

\[ + \frac{1}{2} \epsilon_\nu p_\mu \langle 0 | \bar{s} [\gamma_\nu, \gamma_\mu] P_R b | B_s^0 \rangle \]

\[ = 0 \]

(16)

from eqns.(14) and the transversality condition for real photon \( \epsilon_\nu p_\gamma = 0 \).

Thus, the NHB terms are negligible even for large \( \tan \beta \) co-efficients \( C_9 \) and \( C_{10} \) do not change much in MSSM. Changes in \( C_7 \) in SUSY are constrained by \( b \rightarrow s \gamma \) data. This change too is less than a factor of two. Thus the SM estimate of (14) effectively still holds in MSSM also.

We thus can have a change in the pattern of conclusion of [11] when the Neutral Higgs Boson contribution are included with the large \( \tan \beta \) depending upon SUSY and most importantly upon the values of Higgs masses and \( \tan \beta \). The radiative mode \( B_s^0 \rightarrow l^+ l^- \gamma \) branching ratio stay put at about \( 4.6 \times 10^{-9} \), whereas the non-radiative (suppressed earlier without NHB) can jump by a factor of the order \( \sim 10^2 \) to a value \( \sim 10^{-7} \). With the flux of \( B_s^0 \) expected to go up to observe such branching ratios, one thus has a concrete and interesting situation to test SUSY-models with large \( \tan \beta \) in studying the radiative and non-radiative dileptonic decays of \( B_s^0 \).

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FIGURE CAPTIONS

**Figure 1**: Wilson coefficient $C_{Q_1}$ as a function of light chargino mass in SUGRA model without nonuniversality of scalar masses (pseudo-scalar higgs mass is taken to be in range $80 < m_A < 700$), $m = M$, $A = -1/2$.

**Figure 2**: Wilson coefficient $C_{Q_2}$ as a function of light chargino masses in SUGRA model without nonuniversality of scalar masses with same parameters as above. Parameters are $80 < m_A < 700$, $m = M, A = -1/2$.

**Figure 3**: Ratio of $\Gamma^{SUSY}/\Gamma^{SM}$ (SUSY rate normalized to SM rate) as a function of light chargino mass in SUGRA model without nonuniversality of scalar masses. Parameters are $80 < m_A < 700$, $m = M, A = -1/2$.

**Figure 4**: $\Gamma^{SUSY}/\Gamma^{SM}$ as a function of light chargino mass in

(a) Dilaton ($m = M/\sqrt{3}$, $A = -1$)

(b) SUGRA ($m = M$, $A = -1/2$)

(c) Moduli ($m = A = 0$)

Points with dots (.) are consistent with allowed parameter space whereas points marked with plusses (+) are those excluded by our choice of parameter space (including $b \rightarrow s\gamma$ constraints).
Figure 1:
Figure 2:
Figure 3:
Figure 4: