Spin rotation as an element of polarization experiments on elastic electron-proton scattering

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The validity of some rules in the classical theory of spin, which are followed by the Bargmann–Michel–Telegdi formula for a relativistic particle spin rotation in a constant homogeneous magnetic field, is analyzed. In the framework of the quantum theory, we give examples where these rules are violated. Consequences for polarization experiments are discussed.

1. Introduction

In the 2000s, a series of unique experiments on elastic electron-proton scattering have been carried out at Jefferson Laboratory [1]–[7]. A substantial discrepancy in the values of the ratio of electric $G_E$ and magnetic $G_M$ formfactors of the proton is found for two different techniques of extracting them from experimental measurements. In one of the approaches, the values of the quantity $R = \mu_p G_E / G_M$ (with $\mu_p$ being the magnetic moment) is extracted from polarization experiments [1]–[5] using the Akhiezer–Rekalo formula [8]. They decrease almost linearly from unity to approximately 0.3 when the squared transferred momentum $Q^2$ increases from 0 to 5.6 (GeV/c)$^2$. In the other approach, the values of $R$ are obtained from the Rosenbluth formula [9] at processing of the high-precision unpolarized experiments [6], [7]. These values are close to unity in the same interval of $Q^2$.

The results of old numerous calculations of radiative corrections to the Rosenbluth formula, which are summarised in Ref. [10], have a rather small influence on the values of $R$ [11]. A new analysis [12]–[16] of the possible size of two-photon exchange contributions to the Rosenbluth and Akhiezer–Rekalo formulae shows, according to the dominant opinion, that the discrepancy between the mentioned values of $R$ can be reduced to some extent, but not eliminated at all.

In view of the above, we find it necessary to carefully reconsider the theoretical basis used in deriving the final results in the experimental works [1]–[7]. The essential ingredients of this basis are (i) assuming the proton to be a Dirac particle, that realizes in the general form of the nucleon electromagnetic current [17] and, further on, in the Rosenbluth and Akhiezer–Rekalo formulae; (ii) using of the Bargmann–Michel–Telegdi (BMT) formula [18] to describe the relativistic proton spin rotation in a constant homogeneous magnetic field; and (iii) modelling the azimuthal asymmetry in the angular distribution of the protons after their secondary scattering on the carbon target as a result of spin-orbital interaction [19].

A strict analysis of some consequences of changing the general form of the nucleon electromagnetic current, caused by refusal of the description of the proton and the neutron by a Dirac spinor, is presented in Ref. [20]. It is established that this change in itself cannot cause the appearance of the discussed contradictions in the values of $R$. Namely, the obtained formulae for the angular distribution of electrons elastically scattered on the protons, and, also for the ratio of the transversal and longitudinal components of the recoil proton polarization vector come accordingly to the Rosenbluth and Akhiezer–Rekalo formulae, irrespective of the representation of the proper Lorentz group $L^\uparrow_+$ attributed to the proton as a particle with the rest spin $1/2$.

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Let us proceed now with discussing the quantum theoretical validity of preconditions of the BMT formula.

2. Some rules of the classical theory of spin and their violation in the quantum theory

Bargmann, Michel, and Telegdi [18] have obtained their formula assuming that, in the framework of classical theory, it is admissible to describe the spin with an axial 4-vector $s^\alpha$ (it was introduced by Tamm [21]), whose time component in the particle rest frame is zero

$$s^\alpha u_\alpha = 0,$$

where $u^\mu$ is the particle 4-velocity. On the other hand, the equation for evolution of the spin at motion of a particle in an electromagnetic field has been originally proposed by Frenkel [22] and was expressed in terms of the properly introduced antisymmetric tensor of spin $s^{\mu\nu}$. According to Frenkel’s rule, the tensor of spin has only space-space components in the particle rest frame, i.e.

$$s^{\mu\nu} u_\nu = 0.$$  

In the classical theory, as we will see later, the rule (2) is the necessary condition, first, for deriving the spin rotation formula based on the spin tensor $s^{\mu\nu}$, and, second, for the consistent description of spin with 4-vector $s^\alpha$, including the equality (1) which plays the key role in deriving the BMT formula in Ref. [18]. Since the spin is a group-theoretical concept, all our subsequent discussions and conclusions about the consistency of this double description of the spin in the classical theory, as well as the possibility to fulfill Eq. (2) will be based on the group-theory definitions and exact solutions in the quantum field theory. Note that substantiating the BMT equation for a particle with rest spin $1/2$ and arbitrary anomalous magnetic moment on the basis of a quantum description of that particle with Dirac equation have been attempted in Refs. [23], [24]. This substantiation cannot be said convincing, because of many assumptions introduced in the mathematical calculations.

As it is well known, in the nonrelativistic quantum mechanics, the concept of spin is a group-theoretical analogue of the orbital angular momentum. Namely, both the spin and orbital momenta associate with antisymmetric tensor operators, which are identified with the generators of the rotation group, the latter being, however, realized in essentially different ways in the spaces of group representations nonequivalent in their structure. The orbital momentum corresponds to the realization of such generators as operators of the form $-i(x^i \partial / \partial x^j - x^j \partial / \partial x^i)$ in the space of coordinate functions. The spin refers initially to a matrix realization of the generators $L^{ij}$ of the rotation group in the space of finite-component vectors. Then, for a given particle state, the spin is described with the mean values of these generators forming an antisymmetric tensor $s^{ij}$ of the rotation group or an equivalent three-dimensional vector $s = \{s^{23}, s^{31}, s^{12}\}$. A natural extension of the above definition of the spin to the relativistic quantum theory consists in its association with a matrix realization of the generators $L^{\mu\nu}$ of the proper Lorentz group in the space of any representation we are interested in. (Note that, sometimes, even in relativistic quantum theory as, for example, within the framework of describing the irreducible representations of the group $L^1_\uparrow$, the term of “spin” is connected only with the rotation group [25], [26].) The mean values of these generators can be interpreted for a given particle state as components of the antisymmetric tensor of spin $s^{\mu\nu}$ in the classical theory. As far as the 4-vector velocity $u^\mu$ is concerned, it has to be regarded in the quantum theory as the mean value of the 4-momentum operator $P^\mu$ divided by the particle mass. For an arbitrary particle state not possessing a definite value of 4-momentum, the rest system is given by the quantum mechanical value $u = 0.$
The quantum description of the spin given above and the classical description of the spin which follows from that is logically simple and consecutive. It is suitable for any particles and for any their states, and so, it should be considered as the primary or standard description. Prior to employing the axial 4-vector \( s^\alpha \) in the classical theory of spin, we should be convinced, that it is consistent with the standard description for all possible states under consideration. Namely, we have to make sure that, in all cases, there is one-to-one correspondence between the tensor \( s^{\mu \nu} \) and the 4-vector \( s^\alpha \). Otherwise, the same word of "spin" will mean different physical entities, that can lead to those or other contradictions.

The axial 4-vector \( s^\alpha \) results from the antisymmetric tensor \( s^{\mu \nu} \) by invoking the 4-velocity \( u^\mu \) as

\[
s^\alpha = \frac{1}{2} \varepsilon^{\mu \nu \rho \alpha} s_{\mu \nu} u^\rho.
\]

From here, Eq. (1) follows in particular.

Consider now the relation (3) as a system of four linear equations with given values of quantities \( s^\alpha \) and \( u^\mu \) and with unknown values of six components of the antisymmetric tensor \( s^{\mu \nu} \). We will be assuming that the quantities \( s^\alpha \) satisfy to the condition (1), otherwise, the system (3) would be controversial. Then, one of the four equations in (3) is linearly expressed through three other. Thus, a general solution of this system for the unknown \( s^{\mu \nu} \) contains three arbitrary constants and can be written as

\[
s^{\mu \nu} = \varepsilon^{\mu \nu \rho \alpha} s^\alpha u^\rho + u^\mu a^\nu - u^\nu a^\mu,
\]

where the 4-vector \( a^\mu \), being arbitrary and noncollinear to \( u^\mu \) (\( a^\mu \neq \lambda u^\mu \)), is determined by the three constants mentioned above. It is easy to see that the tensor \( s^{\mu \nu} \) is uniquely determined by the 4-vector \( s^\alpha \), i.e. that \( a^\mu = 0 \), if and only if the relation (2) is fulfilled.

There is no physical reasoning for any concrete quantum description of the spin with the axial 4-vector \( s^\alpha \). All the suggestions on the choice of the operator associated with the 4-vector \( s^\alpha \) were only motivated by appealing to the transformation properties of that operator under the orthochronous Lorentz group.

Let us dwell in brief on the interpretation [18] of the axial 4-vector of spin \( s^\alpha \) in the classical theory as the mean value of operator

\[
W^\alpha = \frac{1}{2m} \varepsilon^{\mu \nu \rho \alpha} L_{\mu \nu} P^\rho,
\]

which was used by Bargmann and Wigner [27] to classify irreducible representations of the Poincaré group. (We would like to draw attention to that in Ref. [27] the antisymmetric tensor operator in matrix realization rather than the operator \( W^\alpha \) is put in correspondence to the spin.) For consequences of the relation (5) an essential role is played by the fact that the mean of a product of two operators is equal to the product of their mean values only when a state, on which averaging is carried out, is the proper state for one of these operators, and besides, maybe, in some exceptional cases. Therefore, from formula (5) for operators the relation (3) for classical quantities follows up only for states with definite value of the 4-momentum, and, maybe, for some special states. For all other states, the operator relation (5) does not lead to the c-numerical relation with the structure given by formula (4), also as the operator relation \( W^\alpha P_\alpha = 0 \), received from (5), does not lead to the relation (1). We will demonstrate it later on two examples showing that the quantity \( s^0 \), as mean value of the operator \( W^0 \) (5), is not equal to zero in the quantum mechanical rest system. Let us note also that, considering the relation (5) as the equation concerning the unknown antisymmetric operator \( L^{\mu \nu} \), we receive the solution having the structure of type (1) and containing an arbitrary 4-vector operator.

Another available interpretation [23], [24] of the spin 4-vector \( s^\alpha \) assigned to a Dirac particle as the mean value of the operator \( (1/2) \gamma^5 \gamma^\alpha \) is, most likely, only restricted to free particle states.
with definite 4-momentum, when it is consistent, by direct calculations, with the standard description of the spin. It is easy to be convinced that, for the written below states (6)–(8) and (12)–(14), this treatment of the spin results in breaking the rule (1).

It is worth saying in addition, that specifying of matrix 4-vector operators in the quantum theory for a broad set of representations of the proper Lorentz group is non-unique. Indeed, if such an operator couples more than two irreducible representations of the group \( L_+ \), then it contains arbitrary constants which cannot be eliminated by any normalization [25], [26].

Let us now turn to the problem of feasibility of the equality (2).

For the state (6)–(8), the mean values of the momentum operator, of the generator \( c^{03} \) of the proper Lorentz group is zero, i.e. the relation (2) holds.

It has been demonstrated in [20] that, for a broad class of finite- or infinite-dimensional representations of the group \( L_+ \), which correspond to the wave vectors of a particle, the following statement is valid. If the rest state of a free particle, described by the vector \( \psi(x) = \exp(-imt)\psi(p_0) \), possesses definite parity (it is true for all states obeying this or that relativistic invariant equation), then the mean value of generators \( L^{0i} \) \((i = 1, 2, 3)\) of the proper Lorentz group is zero, i.e. the relation (2) holds.

Consider now the state of a free Dirac particle, representing a wave packet in the momentum space

\[
\psi = (2\pi)^{-3/2}N^{1/2} \int \exp(-iEt + ip\mathbf{r}) \left[ c_{+1}(\mathbf{p})\psi_{+1}(\mathbf{p}) + c_{-1}(\mathbf{p})\psi_{-1}(\mathbf{p}) \right] d\mathbf{p},
\]

where \( N \) is the normalization coefficient, \( c_{\pm 1}(\mathbf{p}) \) are some arbitrary functions of momentum, and \( \psi_{\pm 1}(\mathbf{p}) \) are the Dirac spinors, normalized to unity, which have in the spherical coordinate system of the momentum space the following form

\[
\psi_{+1}(\mathbf{p}) = \frac{1}{\sqrt{2m}} \begin{pmatrix}
\sqrt{E + m\cos(\theta/2)} \exp(-i\phi/2) \\
\sqrt{E - m\cos(\theta/2)} \exp(i\phi/2) \\
\sqrt{E - m\sin(\theta/2)} \exp(-i\phi/2) \\
\sqrt{E + m\sin(\theta/2)} \exp(i\phi/2)
\end{pmatrix},
\]

\[
\psi_{-1}(\mathbf{p}) = \frac{1}{\sqrt{2m}} \begin{pmatrix}
\sqrt{E + m\sin(\theta/2)} \exp(-i\phi/2) \\
\sqrt{E - m\sin(\theta/2)} \exp(-i\phi/2) \\
\sqrt{E - m\cos(\theta/2)} \exp(-i\phi/2) \\
\sqrt{E + m\cos(\theta/2)} \exp(i\phi/2)
\end{pmatrix}.
\]

For the state (6)–(8), the mean values of the momentum operator, of the generator \( c^{03} \) of the group \( L_+ \), and of the operator \( W^0 = \Sigma \mathbf{P}/2 \) are, respectively:

\[
m\mathbf{u} = \int \bar{\psi} \left( -i \frac{\partial}{\partial \mathbf{r}} \right) \psi d\mathbf{r} = N \int \mathbf{p} \left[ |c_{+1}(\mathbf{p})|^2 + |c_{-1}(\mathbf{p})|^2 \right] d\mathbf{p},
\]

\[
s^{03} = \int \bar{\psi} \left( \frac{i}{2} \gamma^{03} \right) \psi d\mathbf{r} = \frac{N}{m} \int |\mathbf{p}| \sin \theta \im \left[ c_{+1}^*(\mathbf{p})c_{-1}(\mathbf{p}) \right] d\mathbf{p},
\]

and

\[
s^0 = \int \bar{\psi} \left( -i \frac{\Sigma}{2} \frac{\partial}{\partial \mathbf{r}} \right) \psi d\mathbf{r} = \frac{N}{2m} \int |\mathbf{p}| \left[ |c_{+1}(\mathbf{p})|^2 - |c_{-1}(\mathbf{p})|^2 \right] d\mathbf{p}.
\]

For the sake of simplicity of the integrand expressions in (9)–(11), let us take the functions \( c_{\pm 1}(\mathbf{p}) \) such that \( c_{+1}(\mathbf{p}) = ic_{-1}(\mathbf{p})/2 = |c(p)| \). Then we conclude from (9) that, for such a choice of the free state, \( \mathbf{u} = 0 \), i.e. the particle is at rest in the quantum mechanical sense. At the same time it follows from (10) and (11) that neither the Frenkel’s rule (2) nor the rule (1) are fulfilled because both the \( s^{03} \) component of the spin tensor and the \( s^0 \) component of the 4-vector are not zero.

Another example of a state for which the relation (2) is violated concerns to a Dirac particle interacting with an external field. Recall the well-known solution of a problem of motion of an electron in a constant homogeneous magnetic field, whose induction \( \mathbf{B} \) is directed along
the axis $Z$ (see, e.g., [28]). For a fixed value of the momentum projection $p_z$, the electron energy levels form a discrete spectrum with non-degenerate ground state $E_0 = \sqrt{m^2 + p_z^2}$ and twice-degenerate excited states $E_n = \sqrt{m^2 + p_z^2 + 2ne_0B}$ (where $e_0 = |e|$, and $n = 1, 2, \ldots$). Consider the electron state with the $Y$ and $Z$ projections of the momentum exactly equal to zero, $p_y = 0, p_z = 0$, and with energy $E_1 = \sqrt{m^2 + 2e_0B}$, which has the form

$$\Psi = N_1^{1/2} \exp(-iE_1t) [a_{+1}\Psi_{+1} + a_{-1}\Psi_{-1}], \quad (12)$$

where

$$\Psi_{+1} = \exp(-\xi^2/2) \begin{pmatrix} (E_1 + m) \\ 0 \\ 0 \\ 2i\sqrt{e_0B}\xi \end{pmatrix}, \quad (13)$$

$$\Psi_{-1} = \exp(-\xi^2/2) \begin{pmatrix} (E_1 + m)\xi \\ 0 \\ -i\sqrt{e_0B} \\ 0 \end{pmatrix}, \quad (14)$$

$$\xi = \sqrt{e_0Bx}, \quad N_1 \text{ is the normalization coefficient. We take the constants } a_{+1} \text{ and } a_{-1} \text{ such that } \text{Re}(a_{+1}a_{-1}^*) \neq 0 \text{ and } \text{Im}(a_{+1}a_{-1}^*) \neq 0.$$  

For the state (12)–(14), the mean values of the momentum projection onto the axis $X$, of the generator $\sigma^{03}/2$ of the group $L_+^\uparrow$, and of the operator $W^0 = \Sigma P/2$ are, respectively:

$$m_{ux} = \int_{-\infty}^{+\infty} \bar{\psi} \left(-i \frac{d}{d\xi}\right) \psi \, d\xi = 0, \quad (15)$$

$$s^{03} = (e_0B)^{-1/2} \int_{-\infty}^{+\infty} \bar{\psi} \left(\frac{i}{2} \gamma^0 \gamma^3\right) \psi \, d\xi = 2\sqrt{\pi}(E_1 + m)N_1 \text{Re}(a_{+1}a_{-1}^*) \neq 0, \quad (16)$$

and

$$s^0 = \int_{-\infty}^{+\infty} \bar{\psi} \left(-i \frac{d}{d\xi}\right) \psi \, d\xi = \sqrt{\pi}(E_1 + m)N_1 \text{Im}(a_{+1}a_{-1}^*) \neq 0. \quad (17)$$

It follows from here that, for the chosen state of an electron in the magnetic field, the relations (2) and (1) are not fulfilled.

Thus, if a quantum mechanical state possesses a definite value of the 4-momentum, then the spin tensor $s^{\mu\nu}$ and the spin 4-vector $s^\alpha$, received with averaging respectively generators $L^{\mu\nu}$ of the proper Lorentz group and operators $W^\alpha$ (5), satisfy the rules (2) and (1) in classical theory of spin and, consequently, give the equivalent description of the spin. The use of one or another description is determined only by reasons of simplicity in this or that situation. It is plausible that in all other states the quantum description of the spin by operators $L^{\mu\nu}$ and/or $W^\alpha$ breaks the rules (2) and/or (1). It means, in particular, that in the quantum theory of spin it is necessary to return to the derivation of the BMT formula, expressed either in terms of 4-vector $s^\alpha$ or tensor $s^{\mu\nu}$.

3. Modification of the spin rotation formula having regard to the violation of Frenkel’s rule
Let us discuss now the derivation of BMT formula for the spin rotation in a constant homogeneous electromagnetic field $F$. In terms of the axial 4-vector $s^\alpha$, this formula looks

$$
\frac{ds^\alpha}{d\tau} = \frac{ge}{2m} F^{\alpha\beta} s_\beta + \frac{e}{m} \left( \frac{g}{2} - 1 \right) s^\beta F_{\beta\gamma} u^\gamma u^\alpha,
$$

whereas in terms of the spin tensor $s^{\mu\nu}$ it has the form

$$
\frac{ds^{\mu\nu}}{d\tau} = \frac{ge}{2m} (s^{\mu\rho} F_{\nu\rho} - s^{\nu\rho} F^{\mu\rho})
+ \frac{e}{m} \left( \frac{g}{2} - 1 \right) (u^\mu s^{\nu\rho} - u^\nu s^{\mu\rho}) u^\sigma F_{\sigma\rho},
$$

where $\tau$ is the particle proper time, and $e$, $m$, and $g$ are the charge, mass, and gyromagnetic ratio, respectively. Each of the formulae (18) and (19) can be obtained from the other one through the relations (3) and (4) by re-expressing the quantities $s^\alpha$ and $s^{\mu\nu}$ in terms of one another (taking $a^\mu = 0$).

When deriving the BMT formula, it is assumed that, in the rest frame of a particle being in a constant homogeneous magnetic field $B$, the three-dimensional vector of spin $s$ obeys the usual equation of motion

$$
\frac{ds}{d\tau} = \frac{ge}{2m} (s \times B).
$$

Let us pass in Eq. (20) from the three-dimensional vectors $s$ and $B$ to their components constituting the antisymmetric tensors of the rotation group $s^{ij}$ and $F^{ij}$, respectively. Replace in the obtained equation the three-dimensional vector indices with four-dimensional Lorentz ones and add with arbitrary coefficients $C_1$ and $C_2$ two new terms vanishing in the particle rest frame where there is only a constant homogeneous magnetic field. Then

$$
\frac{ds^{\mu\nu}}{d\tau} = \frac{ge}{2m} (s^{\mu\rho} F_{\nu\rho} - s^{\nu\rho} F^{\mu\rho}) + C_1 (u^\mu s^{\nu\rho} - u^\nu s^{\mu\rho}) u^\sigma F_{\sigma\rho}
+ C_2 (s^{\mu\rho} F_{\nu\sigma}^{\prime} - s^{\nu\rho} F_{\mu\sigma}^{\prime}) u_\rho u_\sigma.
$$

Let us look now, what occurs if to consider that the Frenkel’s rule (2) is true. First of all, because of it, the term with coefficient $C_2$ of the relation (21) has to be omitted as equal to zero. Besides that, taking the derivative of both sides of (2) with respect to the time $\tau$ and keeping in mind the classical equation of motion for a charged particle in an external electromagnetic field

$$
\frac{du^\mu}{d\tau} = \frac{e}{m} F^{\mu\nu} u_\nu,
$$

we get

$$
\frac{ds^{\mu\nu}}{d\tau} u_\nu + \frac{e}{m} s^{\mu\rho} F_{\nu\rho} u^\rho = 0.
$$

Substituting here the expression given by the right-hand side of (21) for $ds^{\mu\nu}/d\tau$, we find that

$$
C_1 = \frac{e}{m} \left( \frac{g}{2} - 1 \right),
$$

i.e. the formula (19) generates.

The derivation of the formula (18) for a 4-vector $s^\alpha$, being founded on the rule (1), is similar to what was written for the tensor $s^{\mu\nu}$.

Since, as it has been shown above, in the quantum theory the situations are frequent when Frenkel’s rule (2) is not fulfilled, we believe it to be highly probable that this rule is violated at passing of a particle through an external magnetic field. This calls in question the lawfulness of the description of the spin rotation of a relativistic proton with the formula (19) (or (18)). At a
logic level, it is necessary to replace this formula by the modified equation (21) with unknown coefficients $C_1$ and $C_2$, which are, most likely, some functions of the invariants made up of tensors $s^{\mu\nu}$, $F^{\mu\nu}$, and 4-vector $u^\mu$, and which can appear essentially different for the charged leptons and the baryons. Expressing the Eq. (21) in terms of an axial 4-vector at violation of the Frenkel’s rule (2) is inadmissible.

4. On the experimental tests of Bargmann–Michel–Telegdi formula

In the absence of suitable exact solutions of the quantum theory, no opportunity is seen to say something certain about the quantities $C_1$ and $C_2$ in Eq. (21). In such a situation, an experimental research of whether the BMT formula (19) (or (18)) is a good or bad approximation for relativistic particles of this or that sort, is very important. It could be realized by the means of additional procedures in polarization experiments on elastic electron-proton scattering.

The main task of these experiments is in extracting the ratio of the transverse $P_t$ (in the plane of all the particle momenta) and longitudinal $P_l$ components of the recoil proton polarization vector (which is twice the rest spin vector). The secondary scattering on a carbon target is only sensitive to the transverse component of the incident particle polarization. Therefore, the protons are passed through a magnetic dipole before getting to a carbon target. As a result of the spin rotation in magnetic field, the initial longitudinal component of the recoil proton polarization vector contributes to the values of the transverse components of polarization of the proton on its output from a dipole.

At the practical use of the formula (19) (or (18)), it is accepted to take the induction $B$ of a magnetic field, the particle velocity $v$ and the time $t$ in the laboratory frame and, using of Eq. (2) (or (1)), to express the components of the spin tensor (or 4-vector) through its space-space part $s$ in the particle instant rest frame. As a result, the evolution of the rest spin of a particle is described by the following equation (see, e.g., [29])

$$\frac{ds}{dt} = \frac{e}{m\gamma} s \times \left[ \frac{g}{2} B + \left( \frac{g}{2} - 1 \right) (\gamma - 1) B_\perp \right],$$

where $\gamma = (1 - v^2)^{-1/2}$, and $B_\perp$ is the induction component perpendicular to the velocity at the given instant.

In case of violation of the Frenkel’s rule for a relativistic particle in a magnetic field, the formula (19) should be replaced with the formula (21), in which, generally speaking, $C_1 \neq (e/m)(g/2 - 1)$ and $C_2 \neq 0$. But introducing now the description of spin tensor in the particle instant rest frame is inexpedient, because it will be not more simple, than in the laboratory frame. Both in that, and in other frame of reference, generally speaking, all or some of the components $s^{0i}$ ($i = 1, 2, 3$) of the spin tensor are nonzero. Thus, besides the uncertainty in the coefficients $C_1$ and $C_2$, the formula (21) bears other complication, namely, it is not reducible to a system of three first order differential equations, which would be in some sense similar to the system (equation) (25).

Our proposition about checking the standard spin rotation formula, expressed now by Eq. (25), consists in obtaining final results of polarization experiments on elastic electron-proton scattering for various vectors of the induction $B$ of a magnetic field.

In Ref. [4] describing the polarization experiment at Jefferson Laboratory, a sketch of the experimental set-up is presented showing the mutual arrangement of the velocity vector $v$, the transverse component of the polarization $P_t$ and the magnetic field induction $B$ when the recoil proton enters the magnetic dipole: $v \perp B$, $P_t \parallel B$. It would be expedient to have a set of magnetic dipoles with different directions of the induction vector $B$ with respect to $P_t$. 


and to $v$ and to change the strength of the induction in a wide enough interval. If the BMT formula is true for relativistic protons, then, at the given proton energy and the given angle of departure of the recoil proton, the extracted ratio $P_t/P_l$ will not depend on the direction and the size of the induction vector in the dipole. Otherwise, it will be possible to declare that Eq. (19) (or (18)) is unsuitable for the description of the relativistic proton spin rotation, and that the theoretical basis of the polarization experiments being carried out does not give any opportunity to extract the ratio $P_t/P_l$ and, hence, the ratio of the proton formfactors $G_E/G_M$.

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