CP violation in the 2HDM and EFT: the $ZZZ$ vertex

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1 Motivation, C2HDM

2 Calculation setup
   - Couplings & Propagators
   - The diagrams
   - Result for $ZZZ$ in C2HDM
   - Discussion

3 Comparison with $ZZZ$ in SM-EFT
   - Generalities
   - Matching with EFT
   - Identifying the operator(s) in the SM-EFT
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4 Summary
Motivation for 2HDM and $ZZZ$ vertex

- **2HDM**: simple SM extension realized by some motivated BSM models: e.g. type-II by SUSY models, other types in composite Higgs models (see [Stefano Moretti’s talk]).

- Amongst $\neq$ signatures, a possible one concerns deviations in $ZZ$ production via contributions from $ZZZ$ vertex (see [Grządkowski–2016]).

- The $ZZZ$ tensor structure contains an observable CP-odd part.

- Comparing wrt. an EFT model-matching and an SM-EFT approach (top-down vs. bottom-up), allowing us to understand how well NP can be described with EFT & how much information is lost (see also [HBM–2016]).
Complex 2HDM in a nutshell \cite{Gunion–1989, Branco–2011} \(1/3\)

- **Original Lagrangian** \(\mathcal{L}\) with two scalar doublets \(\Phi_{1,2}\) (VEVs: \(v_{1,2}/\sqrt{2}\)).
- \(\mathbb{Z}_2\) symm. imposed to avoid FCNCs at tree-level: \((\Phi_1, \Phi_2) \rightarrow (\Phi_1, -\Phi_2)\).
- 4 types of 2HDM depending on \(H_i\) couplings to fermions.

“Higgs basis” \cite{Lavoura–1994}: only 1\(^{st}\) one has a VEV, \(v = \sqrt{v_1^2 + v_2^2}\).

\[
\Phi_{1,2} \rightarrow H_1 = \left( \frac{1}{\sqrt{2}} \begin{pmatrix} -iG^+ \\ (v+h+iG^0) \end{pmatrix} \right), \quad H_2 = \left( \frac{1}{\sqrt{2}} \begin{pmatrix} H^+ \\ (R+iI) \end{pmatrix} \right).
\]

\(\mathcal{L}\) then becomes \cite{Bernon–2015} \((Y_3, Z_{5,6,7} \in \mathbb{C})\):

\[
\mathcal{L} = \mathcal{L}^{\text{no Higgs}}_{\text{SM}} + |D_\mu H_1|^2 + |D_\mu H_2|^2 + \mathcal{L}_Y - V_H, \quad -\mathcal{L}_Y = Y_f \bar{f} R H_1^\dagger f L + \frac{\eta_f}{t_\beta} Y_f \bar{f} R H_2^\dagger f L + \text{h.c.},
\]

\[
V_H = Y_1 |H_1|^2 + Y_2 |H_2|^2 + (Y_3 H_1^\dagger H_2 + \text{h.c.}) + \frac{Z_1}{2} |H_1|^4 + \frac{Z_2}{2} |H_2|^4 + Z_3 |H_1|^2 |H_2|^2
\]

\[
+ Z_4 (H_1^\dagger H_2)(H_2^\dagger H_1) + \left\{ \frac{Z_5}{2} (H_1^\dagger H_2)^2 + (Z_6 |H_1|^2 + Z_7 |H_2|^2)(H_1^\dagger H_2) + \text{h.c.} \right\}
\]

(Sum over \(f = u, d, l\), and \(\eta_f = 1\) or \(-t_\beta^2\) depending on the 2HDM type.)
Complex 2HDM in a nutshell \[\text{[Gunion–1989, Branco–2011]} \ (2/3)\]

- Neutral scalars \(\{h, R, I\}\) “Higgs basis” \(\rightarrow\) neutral mass-eigenstate scalars \(\{h_1, h_2, h_3\}\) via rotation matrix \(T\) \[\text{[Branco–2011, Fontes–2014]}\].

- E.g. for the real 2HDM, couplings to vector bosons \(g_{h_iVV}\) are \(\propto \sin\beta - \alpha\) for \(h_1\), \(\propto \cos\beta - \alpha\) for \(h_2\) and none for \(h_3\) (\(\alpha, \beta\): rotation angles).

**Two limits (similar to R2HDM)**

- **Alignment limit**: \(\cos\beta - \alpha \ll 1\), i.e. \(h_1\) lives in \(H_1\) and corresponds to the SM Higgs boson, while \(h_{2,3}\) can be almost degenerated.

- **Decoupling limit**: \(Y_2 \gg v^2\), so that \(m_{h_{2,3}, H^\pm} \gg m_{h_1}\).

**Stationarity conditions (\(\leftarrow\) potential minimization)**

\[
Y_1 = -\frac{Z_1 v^2}{2}, \quad \Rightarrow \text{Only } Z_5, Z_6, Z_7 \text{ are independently complex.}
\]

\[
Y_3 = -\frac{Z_6 v^2}{2}. \quad \Rightarrow \text{The invariants source of CP violation must be related to}
\]

\[
\text{Im}(Z_7 Z^*_6), \text{Im}(Z^2_6 Z^*_5) \text{ and } \text{Im}(Z^2_7 Z^*_5) \quad \text{[Lavoura–1994]}.
\]
From the rotation matrix $T$:

- $h$ couples to gauge bosons, coupling coincides with the SM one $g_{hVV}^{SM}$
  
  $\Rightarrow g_{h_iVV} = T_{1i} \times g_{hVV}^{SM}$.

- $T$ is orthogonal $\Rightarrow$ **Sum rule:** $\sum_i |g_{h_iVV}|^2 = |g_{hVV}^{SM}|^2$.
  
  Ensures that each $|g_{h_iVV}|$ is always $< |g_{hVV}^{SM}|$ (generalizes for any $n$-HDM).

**Nota bene**

Finding a $|g_{h_iVV}| > |g_{hVV}^{SM}|$ would exclude SM and all of these models.

Experimental measurements consistent with SM predictions $\rightarrow$ the mixing angles in $T$ are in the *alignment limit* and $h_1 \sim h$ in the "Higgs basis".
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**ZZZ vertex structure** \([\text{Hagiwara–1986, Gounaris–1999}]\) \((1/2)\)

\[ Z_\alpha, Z_\beta \text{ on-shell; } Z_\mu \text{ off-shell; } q = p_1 + p_2. \]

**CP properties via:**
- \(C : Z_\mu \to -Z_\mu\), and
- \(P : (Z_0, Z_i) \to (Z_0, -Z_i)\) and \((\partial_0, \partial_i) \to (\partial_0, -\partial_i)\).

Lorentz + Bose symmetries constrain \(Z^3\) vertex function \(\Gamma_{\mu\alpha\beta}\):

\[
i\Gamma_{\mu\alpha\beta} = -e \frac{q^2 - m_Z^2}{m_Z^2} f_4^Z(q^2) (\eta_{\mu\alpha} p_{1,\beta} + \eta_{\mu\beta} p_{2,\alpha}) - e \frac{q^2 - m_Z^2}{m_Z^2} f_5^Z(q^2) \epsilon_{\mu\alpha\beta\rho}(p_1 - p_2)^\rho
\]
\[+ \tilde{f}_1(q^2) (\eta_{\mu\alpha} p_{2,\beta} + \eta_{\mu\beta} p_{1,\alpha}) + \tilde{f}_2(q^2) \eta_{\alpha\beta} q_\mu + \tilde{f}_3(q^2) q_\mu p_{1,\beta} p_{2,\alpha}
\]
\[+ \tilde{f}_4(q^2) q_\mu p_{1,\alpha} p_{2,\beta} + \tilde{f}_5(q^2) q_\mu (p_{1,\alpha} p_{1,\beta} + p_{2,\alpha} p_{2,\beta}).\]

\(f_4^Z(q^2)\) term is CP-odd: e.g. effective interaction \(\tilde{\kappa}_{ZZZ}^Z m_Z^2 \partial_\mu Z_\nu \partial^\mu Z_\rho \partial_\rho Z^\nu\) provides \(f_4^Z(q^2) = \tilde{\kappa}_{ZZZ}^Z\).

\(f_5^Z(q^2)\) term is CP-even.
**ZZZ vertex structure** [Hagiwara–1986, Gounaris–1999] (2/2)

\[ i\Gamma_{\mu\alpha\beta} = -\frac{q^2 - m_Z^2}{m_Z^2} f_4^Z(q^2) \left( \eta_{\mu\alpha} p_{1,\beta} + \eta_{\mu\beta} p_{2,\alpha} \right) -e\frac{q^2 - m_Z^2}{m_Z^2} f_5^Z(q^2) \epsilon_{\mu\alpha\beta\rho}(p_1 - p_2)_{\rho} \]
\[ + \tilde{f}_1(q^2) \left( \eta_{\mu\alpha} p_{2,\beta} + \eta_{\mu\beta} p_{1,\alpha} \right) + \tilde{f}_2(q^2) \eta_{\alpha\beta} q_{\mu} + \tilde{f}_3(q^2) q_{\mu} p_{1,\beta} p_{2,\alpha} \]
\[ + \tilde{f}_4(q^2) q_{\mu} p_{1,\alpha} p_{2,\beta} + \tilde{f}_5(q^2) q_{\mu} (p_{1,\alpha} p_{1,\beta} + p_{2,\alpha} p_{2,\beta}) \]  

**Remarks:**

- \( f_4^Z(q^2) \) and \( f_5^Z(q^2) \) are related to observables, the \( \tilde{f}_i(q^2) \) are not.
- Example of \( \bar{f}f \to ZZ \) with a \( Z^* \) in s-channel.
- The \( \tilde{f}_i(q^2) \) may be gauge-dependent in specific calculations.
Couplings & Propagators

Vertices [Fontes–2017] (momenta incoming, Feynman rules’ ‘i’ included):\(^1\)

\[
[h_i, h_j, Z^\mu] = \frac{g}{2c_W} (p_i - p_j)^\mu \epsilon_{ijk} x_k , \quad [Z^\mu, G^0, h_i] = \frac{g}{2c_W} (p_i - p_0)^\mu x_i ,
\]

\[
[h_i, Z^\mu, Z^\nu] = i \frac{g}{c_W} m_Z g^{\mu\nu} x_i , \quad \text{where:} \quad x_i \equiv T_{1i} = \frac{g_{h_iVV}^{SM}}{g_{hVV}} , \quad c_W \equiv \cos \theta_W .
\]

In generic \( R_\xi \) gauge, Goldstone \( G^0 \) and \( Z \) propagators read [Romao–2012]:

\[
[G^0, G^0] = \frac{i}{p^2 - \xi m_Z^2 + i\epsilon} , \quad [Z^\mu, Z^\nu] = \frac{-i}{k^2 - m_Z^2 + i\epsilon} \left[ g^{\mu\nu} - (1 - \xi) \frac{k^\mu k^\nu}{k^2 - \xi m_Z^2} \right].
\]

Calculations performed with Mathematica and package FeynCalc [Mertig–1990, Shtabovenko–2016], cross-checked with Package-X [Patel–2015].

Loop-functions conventions from LoopTools [Hahn–1998].

\(^1\)The gauge couplings convention \( D_\mu = \partial_\mu + igA_\mu \) is used. If \( D_\mu = \partial_\mu - igA_\mu \) is used instead, the sign of \([h_i, h_j, Z^\mu]\) and \([Z^\mu, G^0, h_i]\) is flipped and the \( Z^3 \) form factor picks up an overall minus sign.
Each scalar in the loop is different (← $\epsilon_{ijk}$ in the couplings).

\[
e_{q^2 - m_Z^2} \frac{f^Z_{hhh}}{m_Z^2} = -\frac{8}{16\pi^2} \left( \frac{g}{2c_W} \right)^3 x_1 x_2 x_3 \sum_{i,j,k} \epsilon_{ijk} C_{001} (q^2, m_Z^2, m_Z^2, m_i^2, m_j^2, m_k^2).
\]
The Goldstone can be on each of the internal lines. All combinations of $h_i, h_j$ with $i \neq j$ appear.

\[
e \frac{q^2 - m_Z^2}{m_Z^2} f_{4}^{Z,hhG} = \frac{8}{16\pi^2} \left( \frac{g}{2c_W} \right)^3 x_1 x_2 x_3 \sum_{i,j,k} \epsilon_{ijk} \left[ C_{001}(q^2, m_Z^2, m_Z^2, m_i^2, m_j^2, \xi m_Z^2) + C_{001}(q^2, m_Z^2, m_Z^2, \xi m_Z^2, m_j^2, m_k^2) + C_{001}(q^2, m_Z^2, m_Z^2, m_i^2, \xi m_Z^2, m_k^2) \right]
\equiv F_4^{Z,hhG}(\xi).
\]
The $Z$ can be on each of the internal lines. All combinations of $h_i, h_j$ with $i \neq j$ appear.

\[
e^q - m_Z^2 \frac{f_4^{Z,hhZ}}{m_Z^2} = F_4^{Z,hhG}(1) - F_4^{Z,hhG}(\xi) - \frac{8}{16\pi^2} \left( \frac{g}{2c_W} \right)^3 x_1 x_2 x_3 m_Z^2 \sum_{i,j,k} \epsilon_{ijk} C_1(q^2, m_Z^2, m_Z^2, m_i^2, m_Z^2, m_k^2).
\]
The result (1-loop) – C2HDM

- \[ f_4^Z = f_4^{Z,hhh} + f_4^{Z,hhZ} + f_4^{Z,hhG} \]: the \( \xi \)-dependent parts cancel out each other: Result is gauge-invariant.
- Due to the antisymmetric \( \epsilon_{ijk} \) the UV-divergences of the PaVe \( C_{001} \) cancel out: Result is finite.

\[
e \frac{q^2 - m_Z^2}{m_Z^2} f_4^Z(q^2) \left[ \frac{1}{16\pi^2} \left( \frac{g}{c_W} \right)^3 x_1 x_2 x_3 \right]^{-1} \equiv \hat{f}_4^Z = \\
\sum_{i,j,k} \epsilon_{ijk} \left[ -C_{001}(q^2, m_Z^2, m_Z^2, m_i^2, m_j^2, m_k^2) + C_{001}(q^2, m_Z^2, m_Z^2, m_i^2, m_j^2, m_k^2) \\
+ C_{001}(q^2, m_Z^2, m_Z^2, m_Z^2, m_j^2, m_k^2) + C_{001}(q^2, m_Z^2, m_Z^2, m_i^2, m_Z^2, m_k^2) \\
- m_Z^2 C_1(q^2, m_Z^2, m_Z^2, m_i^2, m_Z^2, m_k^2) \right].
\]

(Note: Each diagram agrees with [Grządkowski–2016] when \( \xi = 1 \).)
Phenomenological discussion (1/2)

**Figure:** $|f_4|$ scatter plots in C2HDM for two $\neq$ CM energies, satisfying theoretical (unitarity, $V_H$ bounded from below) and experimental (LHC Higgs, EDM, EW precision meas.) constraints.

- $|f_4^Z|$ can reach values of $\mathcal{O}(10^{-5})$ in realistic parameter space of C2HDM.
- Compare with recent ATLAS [Aaboud–2017] and CMS [Sirunyan–2017] analyses of $ZZ$ production at the LHC: upper bound on $|f_4^Z|$ (assumed $\Re$) of $\mathcal{O}(10^{-3})$. 
However when considering a generic BSM framework, one must check whether effects other than $f_4^Z$ may contribute to the actual experimental observable being measured (and from which $f_4^Z$ is inferred): example with $h \rightarrow ZZ$ production:

- Not a problem with SM Higgs: $\approx 5\%$ contribution to $\sigma_{ZZ}$; for measuring $f_4^Z$ each $Z$ in final state is required to have a $m_Z \in [60; 120]$ GeV.

- Problem happens if heavier Higgs decays to $ZZ$. Mitigated in $\mathbb{C}2$HDM because: 1) from $h_{125} \rightarrow ZZ$ measurements the corresponding coupling in $\mathbb{C}2$HDM lies very close to SM value ($\rightarrow$ alignment limit), 2) $\mathbb{C}2$HDM sum rule guarantees that any heavier scalar has a very small coupling to $ZZ$. 
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EFT intro (simplified!)

- Suppose new degrees of freedom @ high energy
  ⇒ Separation of scales: $m(\text{NP}) \gg m(\text{EW})$.

- At lower energies, NP modifies interactions of SM fields (modify SM predictions). Formally: NP fields are integrated out, generation of non-renormalizable dim. $\geq 5$ effective operators.

$$L_{\text{eff}} = L_{\text{SM}} + \sum_{d \geq 5} \frac{C^{(d)}}{\Lambda_{\text{NP}}^{d-4}} \mathcal{O}^{(d)}(\{\text{SM fields}\}) = L_{D=5} + L_{D=6} + \ldots ,$$

- $L_{\text{SM}}$: the Standard-Model Lagrangian.
- $\Lambda_{\text{NP}}$: energy scale(s) of NP;
- $C^{(d)}$: dimensionless effective coupling ("Wilson coefficient");
- $\mathcal{O}^{(d)}$: effective operator, function of SM fields only.
- $L_{D=5}$ ("Weinberg operator"): masses for neutrinos.
- $L_{D \geq 6}$: the part of interest!
Matching \(C2HDM\) result with EFT

“Naive” expansion of loop functions in terms of \(1/m_H\)?
\[\rightarrow\] Not tractable due to complicated form and non-analytic behaviour.

\(\Rightarrow\) Method of regions [Beneke–1997]

In our 1-loop integrals case with two \(\neq\) mass scales \(m_{\text{light}} \ll m_{\text{heavy}}\):
1) expand integrand for soft momenta and compute integral;
2) expand integrand for hard momenta and compute integral, and
3) sum both contribs. together.

With \(m_1 = m_h = 125\,\text{GeV}, m_2 = m_H\) and \(m_3 = \sqrt{m_H^2 + \delta}\) with \(\delta \sim v\), in decoupling limit \(m_h \ll m_H\) (and \(q^2 \ll m_H^2\)), we find:

- leading contributions are \(\mathcal{O}(m_H^{-4})\), from diagrams with 1 heavy scalar and 2 SM particles \((h, Z, G^0)\) in the loop, correspond to the soft region \((k \ll m_H)\) of the integrals;

- other regions / diagrams are \(\mathcal{O}(m_H^{-6})\) or higher.
Matching result

The form of the expansion is found to be (when $m_h \ll m_H$ and $q^2 \ll m_H^2$):

$$e f_4^Z(q^2) \sim \frac{\delta^2 x_1 x_2 x_3}{m_H^4} \left( \frac{g}{c_W} \right)^3 \times \text{func}(q^2, m_h, m_Z),$$

where $\text{func}(q^2, m_h, m_Z)$ is some complicated kinematical function. In the decoupling limit the Higgs mixing angles are also suppressed:

$$\delta^2 x_1 x_2 x_3 \approx \frac{v^6}{2m_H^4} \text{Im}(Z_5^* Z_6^2).$$

$$e f_4^Z(q^2) \sim \text{Im}(Z_5^* Z_6^2) \frac{v^6}{2m_H^8} \left( \frac{g}{c_W} \right)^3 \times \text{func}(q^2, m_h, m_Z).$$
Comparing exact vs. matched EFT (1/2)

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Comparing exact vs. matched EFT (1/2)
Comparing exact vs. matched EFT (2/2)

- |$f_4^{\text{Exact}}$| (red) versus |$f_4^{\text{EFT}}$| (blue)

- $\sqrt{q^2} = 200$ (GeV)

- |$\text{Re}(f_4)$| (red) versus |$\text{Re}(f_4)$| (blue)

- $\sqrt{q^2} = 1000$ (GeV)

- |$\text{Im}(f_4)$| (red) versus |$\text{Im}(f_4)$| (blue)

- $\sqrt{q^2} = 1000$ (GeV)
Prerequisites for SM-EFT

**General assumptions**

- The operators are $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant.
- The 125 GeV Higgs boson $h_1$ belongs to the Higgs scalar $SU(2)$ doublet $H$ that transforms as $(1, 2)_{1/2}$ of $G_{SM}$ and acquires a VEV $v$. (OK since we already work with such doublets in the 2HDM.)

Start from $\mathcal{L}_{C2HDM}$ (terms not relevant here are dropped) and work in the “Higgs basis” where $\langle H_1 \rangle = v/\sqrt{2}$ while $\langle H_2 \rangle = 0$, ⊕ Stationarity conditions:

$$\mathcal{L}_{C2HDM} \supset |D_\mu H_1|^2 - Z_1 \frac{|H_1|^2 - v^2}{2} |H_1|^2 + |D_\mu H_2|^2 - Y_2 |H_2|^2 - Z_3 |H_1|^2 |H_2|^2$$

$$- Z_4 (H_1^\dagger H_2)(H_2^\dagger H_1) - \left\{ \frac{Z_5}{2} (H_1^\dagger H_2)^2 + Z_6 X_0 (H_1^\dagger H_2) + \text{h.c.} \right\} + \ldots ,$$

where: $X_0 = |H_1|^2 - \frac{v^2}{2}$. 
Procedure

Write EOM for $H_2^{(†)}$ & search for perturbative solution $H_2 = \sum_{n=1}^{+\infty} Y_2^{-n} H_2^{(n)}$; $Y_2 \equiv$ large mass$^2$ scale $\Lambda_{NP}$ (we implicitly suppose the decoupling limit):

$$Y_2 H_2 + D^2 H_2 + Z_6^* X_0 H_1 + Z_5^* (H_2^{\dag} H_1) H_1 + \cdots = 0.$$  

$\Rightarrow$ Recursive equations:

$$H_2^{(1)} = -Z_6^* X_0 H_1, \quad H_2^{(n+1)} = -D^2 H_2^{(n)} - Z_5^* (H_2^{(n)} \dag H_1) H_1 + \cdots,$$

and we need to go up to $n = 4$. Replace all the $H_2^{(n)}$ values recursively into the expanded ansatz and back into $\mathcal{L}_{C2HDM}$, to obtain a tree-level-generated EFT expressed only in terms of the $H_1 \equiv H$ doublet and $D_\mu$:

$$\mathcal{L}_{C2HDM}^{EFT} = \mathcal{L}_{SM} + \sum_{n=1}^{+\infty} Y_2^{-n} \mathcal{L}^{(2n+4)}.$$
Identifying the operator(s) in the SM-EFT (1/2)

Examples (note: \(X_0 = |H_1|^2 - \frac{v^2}{2}\)):

| Operator | Properties |
|----------|------------|
| \(\mathcal{L}^{(6)} \supset |Z_6|^2 X_0^2 |H|^2\) | Shifts triple-\(h\) coupling. |
| \(\mathcal{L}^{(8)} \supset |Z_6 D_\mu (X_0 H)|^2\) | Renormalizes \(h\) kinetic term. |
| \(\mathcal{L}^{(10)} \supset \propto D_\mu (H^\dagger X_0) D_\mu (X_0 |H|^2 H) + \text{h.c.}\) | CP-odd interactions |

And (red: term that generates CP-violating interactions):
\[
\mathcal{L}^{(12)} \supset \frac{-Z_5^* Z_6^2}{m_H^8} \left[ D^2 (H^\dagger X_0) D^2 (X_0 |H|^2 H) + \left( D^2 (H^\dagger X_0) H \right)^2 / 2 \right] + \text{h.c.},
\]
leading to (using classical EOM for \(h\): \(v \Box h = \ldots\)):
\[
\mathcal{L}^{(12)} \supset \frac{\text{Im}(Z_5^* Z_6^2)}{m_H^8} \frac{gv^6}{2c_W} Z^\nu \partial_\nu h \Box h + \mathcal{O}(Z h^3)
\]
\[
\rightarrow \frac{\text{Im}(Z_5^* Z_6^2)}{m_H^8} \frac{gv^5}{2c_W} Z^\nu \partial_\nu h (m_Z^2 Z_\mu Z^\mu + 2m_W^2 W_\mu^+ W^- \mu).
\]
Identifying the operator(s) in the SM-EFT (2/2)

\[ \text{Im}(Z^*_5 Z^2_6) \left( \frac{g}{c_W} \right)^3 \frac{v^7}{8m_H^8} Z^\nu \partial_\nu h Z_\mu Z^\mu \text{ at } d = 12 \text{ and is CP-odd.} \]

**Figure:** 1-loop diagram contributing to the \( Z^3 \) vertex in EFT, with insertion of the \( d = 12 \) operator. (+ 2 other diags. with permutations of external legs.)

Personal comment! Alternative computation: use “Universal 1-Loop Effective Action” (UOLEA) technique, extended at \( d = 12 \) and including light/heavy fields mixing? ([Cheyette,Gaillard (1980); Henning, Lu, Murayama (2014); Drozd, Ellis, Quevillon, You (2014-2015), + Zhang (2017); et al.], and [HBM talk @ 2HDM-Workshop 2016].)
In $\mathbb{C}^2$HDM the $ZZZ$ vertex arises from a $d = 12$ operator inserted at 1-loop level.

While $ZZZ$ cannot be generated at $d = 6$, it could be a priori generated at $d \geq 8$, e.g. $\mathcal{L}_{d=8} = \frac{ic_8}{\Lambda^4} B_{\mu\nu} B^{\mu\rho} H^\dagger \{D^\nu, D_\rho\} H$ (and $B_{\mu\nu} \rightarrow W^i_{\mu\nu}$) [Degrande–2013], and contribute to $f_4^Z$.

However these cannot be generated in the $\mathbb{C}^2$HDM at 1-loop, because all the CP-violating effects are $\propto$ to the Jarlskog-type invariant (see [Lavoura–1994]) $J_{CP} = \frac{(m_{h_3}^2 - m_{h_2}^2)(m_{h_3}^2 - m_{h_1}^2)(m_{h_2}^2 - m_{h_1}^2)}{m_{h_1}^2 m_{h_2}^2 m_{h_3}^2} x_1 x_2 x_3 \propto \text{Im}(Z_5^* Z_6^2)$.

$h$ power-counting (see refs. in [HBM–2018]) show that the $d = 12$ operator is allowed within the $\mathbb{C}^2$HDM at tree-level, while the $d = 8$ one cannot appear before 3-loop level in the matching.
Motivation, C2HDM

Calculation setup
- Couplings & Propagators
- The diagrams
- Result for $ZZZ$ in C2HDM
- Discussion

Comparison with $ZZZ$ in SM-EFT
- Generalities
- Matching with EFT
- Identifying the operator(s) in the SM-EFT
- Discussion

Summary
Summary

- The CP-violating $ZZZ$ vertex has been studied in the C2HDM and in its matching within the SM-EFT framework.

- The CP-odd form-factor $f^Z_4$ has been evaluated at 1-loop in $R_\xi$ gauge and is gauge-independent; the leading contribs. arise from triangle diagrams with SM particles and heavy Higgses.

- It probes one of the Jarlskog $J_{CP}$ invariants in the extended Higgs sector.

- Using the $f^Z_4$ approximation in decoupling limit we found the dominant diagrams and operator responsible for CP-violating $ZZZ$ vertex in the low-energy EFT where the heavy scalars are integrated out.

- Via power-counting and $J_{CP}$, we confirmed that the operator appears in the EFT at $d = 12$ in the matching at 1-loop. $\Rightarrow$ CP-violating effects in $ZZ$ production are extremely suppressed when $v \ll m_H$.

Thank you for your attention!
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