Decentralized Goal Assignment and Trajectory Generation in Multi-Robot Networks

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Abstract—This paper considers the problem of decentralized goal assignment and trajectory generation for multi-robot networks when only local communication is available, and proposes an approach based on methods related to switched systems and set invariance. A family of Lyapunov-like functions is employed to encode the (local) decision making among candidate goal assignments, under which the agents pick the assignment which results in the shortest total distance to the goals. An additional family of Lyapunov-like barrier functions is activated in the case when the optimal assignment may lead to colliding trajectories, maintaining thus system safety while preserving the convergence guarantees. The proposed switching strategies give rise to feedback control policies which are scalable and computationally efficient as the number of agents increases, and therefore are suitable for applications including first-response deployment of robotic networks under limited information sharing. Simulations demonstrate the efficacy of the proposed method.

I. INTRODUCTION

Task (target) assignment problems in multi-agent systems have received great interest within the robotics and controls communities in the past couple of years, in part because they encode the accomplishment of various objectives in, among others, surveillance, exploration and coverage applications.

A common thread in such problems is the development of algorithms which assign targets to agents by optimizing a predefined criterion and while meeting certain performance guarantees. The interest of researchers often focuses on the optimization aspects of the problem and the associated computational complexity under (quite) relaxed assumptions, which may not be acceptable in realistic settings. For instance, first-response or search-and-rescue missions using robotic agents (such as aerial robots) typically involve multiple tasks that, on one hand, may need to be performed as quickly as possible, yet on the other hand they typically can be accomplished only when information sharing is available, and under tight safety guarantees. In the interest of space, here we can not provide an overview of relevant formulations and approaches on target assignment problems in multi-robot networks. A more detailed introduction and literature review is provided in our previous work [1]; the reader is also referred to [2]–[7] and the references therein.

This paper builds upon our previous work and proposes algorithms which concurrently address the problems of:

(P1) Assigning goal locations (targets) to agents by minimizing a cost function, which is defined as the total distance to the goals.

(P2) Designing feedback control policies which guarantee:

(i) the convergence of the agents to their assigned goals,

(ii) that the resulting trajectories are collision-free.

The key specifications in the proposed formulation are:

(S1) Agents and goal locations are interchangeable, which means that the mission is considered accomplished when each agent has converged to some goal location, no matter which one.

(S2) Information exchange between a pair of agents is reliable only when they lie within a certain communication range, which means that the decision making on the optimal goal assignment can be performed only locally, i.e., in a decentralized fashion.

(S3) Agents are modeled as non-point robots, which not only means that avoiding collisions is a non-negligible objective, but is rather of top priority even in the expense of resorting to suboptimal paths, if necessary, to the goals.

To this end, we formulate the goal assignment and trajectory generation problems into a control theoretic framework which is related to switched systems theory [8] and set-theoretic methods in control [9]. More specifically, we build our approach based on ideas and tools that rely on multiple Lyapunov-like functions [10].

We first encode the decision making on the optimal goal assignment (which in the sequel we call the Optimal Goal Assignment (OGA) policy) as a state-dependent switching logic among a family of candidate Lyapunov-like functions. Each Lyapunov-like function encodes the cost-to-go under a candidate goal assignment, i.e., the sum of distances to the goals. The switching logic dictates that, when (a subset of) agents become(s) connected at some time instant $t$, they decide to switch to the Lyapunov-like function of minimum value at time $t$. This further reads that they adopt the goal assignment which corresponds to the shortest paths to the interchangeable goals at time $t$. We show that this decision making gives rise to a Globally Asymptotically Stable (GAS) switched system which furthermore does not suffer from Zeno trajectories.

Then, we demonstrate via a simple example that the OGA policy is not sufficient for guaranteeing that the resulting trajectories are always collision-free, since the Lyapunov-like functions do not encode any information on inter-agent distances. Thus, based on our recent work [11], we build an additional state-dependent switching logic, which employs a family of Lyapunov-like barrier functions encoding both inter-agent collision avoidance and convergence to the goal locations determined by the OGA policy. This control policy (in the sequel called the Last Resort (LR) policy) provides
sufficient conditions on determining in a reactive manner whether the OGA policy is safe, and furthermore serves as a supervisor that takes action only when safety under the OGA policy is in stake. We show that the switching between the OGA policy and the LR policy results in stable and safe trajectories for the multi-robot system, in the expense of possibly resorting to suboptimal paths; this situation appears only in the cases when the LR policy forces the agents to deviate from their shortest paths to the goals, in order to maintain system safety. In any other case, the proposed approach renders stable and safe trajectories, which evolve along the shortest paths to the goals.

The paper is organized as follows: Section II gives the mathematical formulation of the considered problem, while Section III gives an overview of the theoretical tools from switched systems theory that are used throughout our analysis. The proposed goal assignment and trajectory generation policies, along with the mathematical proofs that verify their correctness are given in Sections IV and V. Simulation results to evaluate their efficacy are included in Section VI, while our conclusions and thoughts on future research are summarized in Section VII.

II. PROBLEM FORMULATION

Assume N agents $i$ and equal number of goal locations $G, i \in \mathcal{N} = \{1, 2, \ldots, N\}$. The motion of each agent $i$ is governed by single integrator dynamics:

$$\dot{r}_i = u_i,$$

where $r_i = [x_i \ y_i]^T$ is the position vector of agent $i$ with respect to (w.r.t.) a global cartesian coordinate frame $G$, and $u_i$ is its control vector comprising the velocities $u_{xi}, u_{yi}$ w.r.t. the frame $G$.

We assume that each agent $i$: (i) has access to its position $r_i$ and velocities $u_i$ via onboard sensors, (ii) can reliably exchange information with any agent $j \neq i$ which lies within its communication region $C_i : \{r_i \in \mathbb{R}^2, r_j \in \mathbb{R}^2 \mid \|r_i - r_j\| \leq R_c\}$, where $R_c$ is the communication range. In other words, a pair of agents $(i, j)$ is connected as long as the distance $d_{ij} = \|r_i - r_j\| \leq R_c$.

A. Encoding Assignments

The task is considered completed as long as each agent has converged to some goal location, i.e., that the goals are interchangeable for each agent. This specification defines $N!$ possible goal assignments $k \in \{1, 2, \ldots, N\}$. Each goal assignment can be encoded via an assignment matrix:

$$A_k = \{\alpha_{im}\} = \begin{bmatrix} \alpha_{11} & \ldots & \alpha_{1N} \\ \vdots & \vdots & \vdots \\ \alpha_{N1} & \ldots & \alpha_{NN} \end{bmatrix},$$

with the rows representing agents and the columns representing goal locations. For agent $i \in \{1, \ldots, N\}$ assigned to goal $G_m, m \in \{1, \ldots, N\}$, one has $\alpha_{im} = 1$ and the remaining elements of the $i$-th row equal to zero. For instance, for 2 agents and 2 goal locations, the possible $N! = 2$ goal assignments are encoded via the assignment matrices:

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$  

In this case, the matrix $A_1$ encodes that agent 1 goes to $G_1$ and agent 2 goes to $G_2$, while the matrix $A_2$ encodes that agent 1 goes to $G_2$ and agent 2 goes to $G_1$.

Denote $r_{Gm} = [x_{Gm} \ y_{Gm}]^T$ the position vectors of the goal locations $G_m \text{ w.r.t.} \text{ the global frame}$. Then, the assigned goal position for agent $i$ under the $A_k$ assignment matrix (or under the $k$-th goal assignment) can be expressed as:

$$r_{gi}^{(k)} = \sum_{m=1}^{N} \alpha_{im}^{(k)} r_{Gm},$$

where $\alpha_{im}^{(k)}$ is the $i$-th row of $A_k$.

Lemma 1: The position trajectories $r_i(t)$ of agent $i$ are Globally Exponentially Stable (GES) w.r.t. the $k$-th goal assignment under the control law:

$$u_i^{(k)} = -\lambda_i \left( r_i - r_{gi}^{(k)} \right), \quad (2)$$

where $i \in \{1, \ldots, N\}, \lambda_i > 0$.

Proof: Consider the candidate Lyapunov function $V_i^{(k)}$ for each agent $i$ under the $k$-th assignment as:

$$V_i^{(k)} = \|r_i - r_{gi}^{(k)}\|,$$

which encodes the distance of agent $i$ w.r.t. its goal $r_{gi}^{(k)}$ under the goal assignment $k$. Given that the goal locations are stationary, i.e., $\dot{r}_{Gm} = 0$, the control law $u_i^{(k)}$ taken out of (2) renders the time derivative $\dot{V}_i^{(k)}$ of the corresponding Lyapunov function along the trajectories of agent $i$ equal to:

$$\dot{V}_i^{(k)} = -\lambda_i \|r_i - r_{gi}^{(k)}\|^2 \leq -\lambda_i V_i^{(k)},$$

yielding that the position trajectories $r_i(t)$ of each agent $i$ are GES w.r.t. the assigned goal $r_{gi}^{(k)}$.

Furthermore, by considering the Lyapunov function:

$$V^{(k)} = \sum_{i=1}^{N} V_i^{(k)},$$

equencing the sum of distances of the agents to their assigned goals, one has that the control laws (2) render its time derivative:

$$\dot{V}^{(k)} = \sum_{i=1}^{N} -\lambda_i V_i^{(k)} \leq -\min_{i} \{\lambda_i\} \sum_{i=1}^{N} V_i^{(k)} = -\min_{i} \{\lambda_i\} V^{(k)},$$

which implies that the sum of distances of agents $i$ to their assigned goals $r_{gi}^{(k)}$ is GES to zero. ■
B. Problem Description

We are interested in the development of decentralized algorithms which will allow a pair of agents \((i, j)\) to coordinate its motion locally, i.e., when agent \(i\) lies in the communication region of agent \(j\) and vice versa, and are therefore able to exchange information on their position and velocity vectors. Based on the updated information, the agents make a decision on whether they keep their current goal assignment or swap goals. The criterion on the decision making is to choose the assignment \(k \in \{1, \ldots, N\} \) which minimizes the sum of total distance (i.e., results in the shortest paths) to the goal locations. In the sequel, we refer to this assignment as the optimal assignment. The challenge in the case of non-point robots is to furthermore ensure that the optimal assignment results not only in stable, but also in collision-free trajectories w.r.t. the goal locations.

III. NOTIONS FROM SWITCHED SYSTEMS THEORY

To facilitate the analysis, let us review some basic results on the stability of switched systems. Following [10], let us consider the prototypical example of a switched system

\[
\dot{x}(t) = f_k(x(t)), \quad k \in \mathcal{K} = \{1, 2, \ldots, K\},
\]

where \(x(t) \in \mathbb{R}^n\), each \(f_k\) is globally Lipschitz continuous and the \(k\)'s are picked in such a way that there are finite switches in finite time. Consider a strictly increasing sequence of times:

\[
T = \{t_0, t_1, \ldots, t_n, \ldots\}, \quad n \in \mathbb{N},
\]

the set: \(I(T) = \bigcup_{n \in \mathbb{N}}[t_{2n}, t_{2n+1}]\) denoting the interval completion of the sequence \(T\), and the switching sequence:

\[
\Sigma = \{x_0; (k_0, t_0), (k_1, t_1), \ldots, (k_n, t_n)\},
\]

where \(t_0\) is the initial time, \(x_0\) is the initial state, \(\mathbb{N}\) is the set of nonnegative integers and \(k_0 \in \mathcal{K}\). The switching sequence \(\Sigma\) along with (3) completely describes the trajectory of the switched system according to the following rule: \((k_p, t_p)\) means that the system evolves according to:

\[
\dot{x}_{k_p}(t) = f_{k_p}(x_{k_p}(t), t)
\]

for \(t_p \leq t < t_{p+1}\). Equivalently, for \(t \in [t_p, t_{p+1})\) one has that the \(k_p\)-th subsystem is active. We assume that the switching sequence is minimal in the sense that \(k_p \neq k_{p+1}\). For any \(m \in \mathcal{K}\), denote:

\[
\Sigma \mid m = \{t_{m_1}, t_{m_1+1}, t_{m_2}, t_{m_2+1}, \ldots, t_{m_n}, t_{m_n+1}, \ldots\}
\]

the sequence of switching times when the \(m\)-th subsystem is switched on or switched off. Hence, \(I(\Sigma \mid m)\) is the set of times that the \(m\)-th subsystem is active.

Denote \(E(T) = \{t_0, t_2, t_4, \ldots\}\) the even sequence of \(T\); then, \(E(\Sigma \mid m) = \{t_{m_1}, t_{m_2}, \ldots, t_{m_n}, \ldots\} \in \mathbb{N}\) denotes the sequence of the switched on times of the \(m\)-th subsystem.

Definition 1: [10] A \(C^1\) function \(V : \mathbb{R}^n \to \mathbb{R}^+\) with \(V(0) = 0\) is called a Lyapunov-like function for a vector field \(f(\cdot)\) and the associated trajectory over a strictly increasing sequence of times \(T\) if:

\[
\begin{align*}
(i) \quad & \dot{V}(x(t)) \leq 0, \quad \forall t \in I(T), \\
(ii) \quad & V \text{ is monotonically nonincreasing on } E(T).
\end{align*}
\]

Theorem 1: [10] If for each \(k \in \mathcal{K}\), \(V_k\) is a Lyapunov-like function for the \(k\)-th subsystem vector field \(f_k(\cdot)\) and the associated trajectory over \(\Sigma \mid k\), then the origin of the system is stable in the sense of Lyapunov.

If in addition, for any \(t_p, t_q \in E(\Sigma \mid m), \) where \(p < q,
\]

\[
V_{k(t_q)}(x(t_{q+1})) - V_{k(t_p)}(x(t_{p+1})) \leq -\rho \|x(t_{p+1})\|^2,
\]

(4) holds for some constant \(\rho > 0\), then the origin of the system is GAS [12].

The theorem essentially states that: Given a family of functions \(\{V_k \mid k \in \mathcal{K}\}\) such that the value of \(V_k\) decreases on the time interval when the \(k\)-th subsystem is active, if for every \(k\) the value of the function \(V_k\) at the beginning of such interval exceeds the value at the beginning of the next interval when the \(k\)-th system becomes active (see Fig. 1), then the switched system is GAS.

Fig. 1. Branicky’s “decreasing sequence” condition. Image from [13].

A fundamental assumption of the theorem is the decreasing condition of \(V\) on \(E(T)\). This is quite conservative and, in general, hard to check [14]. The concept of generalized Lyapunov-like functions relaxes this condition.

Definition 2: [14] A \(C^0\) function \(V : \mathbb{R}^n \to \mathbb{R}^+\) with \(V(0) = 0\) is called a generalized Lyapunov-like function for a vector field \(f(\cdot)\) and the associated trajectory over a strictly increasing sequence of times \(T\) if there exists a function \(h : \mathbb{R}^+ \to \mathbb{R}^+\) satisfying \(h(0) = 0\), such that:

\[
V(x(t)) \leq h(V(x(t_n))), \quad \text{for all } t \in (t_n, t_{n+1}) \text{ and all } n \in \mathbb{N}.
\]

Theorem 2: [14] Consider the prototypical switched system (3) and assume that for each \(k \in \mathcal{K}\) there exists a positive definite generalized Lyapunov-like function \(V_k(x)\) w.r.t. \(f_k(x(t))\) and the associated trajectory. Then:

(i) The origin of the switched system is stable if and only if there exist class \(G\) functions \(\alpha_k(\cdot)\) satisfying

\[
V_k(x(t_{k_q+1})) - V_k(x(t_{k_q})) \leq \alpha_k(\|x_0\|),
\]

(5) \(q \geq 1, \quad k = 1, 2, \ldots, K\).

(ii) The origin of the switched system is asymptotically stable if and only if (5) holds and there exists \(k\) such that \(V_k(x(t_{k_q})) \to 0\) as \(q \to \infty\).
Theorem essentially states that stability of the switched system is ensured as long as $V_k(x(t_{k+1})) - V_k(x(t_k))$, which is the change of $V_k$ between any *switched on* time $t_{k+1}$ and the first active time $t_k$, is bounded by a class $\mathcal{K}$ function, regardless of where $V_k(x(t_k))$ is.

Remark 1: The condition (5) is equivalently rewritten as:

$$
\sum_{q=1}^{n} (V_k(x(t_{k+1})) - V_k(x(t_k))) \leq \alpha_k(||x_0||),
$$

for any integer $n \geq 1$ and any $k \in \mathcal{K}$. Note that here $V_k(x(t_{k+1})) - V_k(x(t_k))$ stands for the change of $V_k(x)$ at two adjacent "switched on" times. Thus, the condition means that $V_k$ is allowed to grow on $E(\Sigma | k)$ but the total such change on any finite time interval should be bounded from above by a class $\mathcal{K}$ function.

Remark 2: Theorem 2 relaxes the decreasing requirement of $V_k(x(t))$ on the corresponding active intervals. Instead, we only need that $V_k(x(t))$ on an active interval does not exceed the value of some function of $V_k$ at the "switched on" time.

IV. THE OPTIMAL GOAL ASSIGNMENT POLICY

A. The $N = 2$ Robot Case

For simplicity, consider $N = 2$ agents $i$, $j$ which need to safely move towards goal locations $G_1$, $G_2$, and let us build a switching logic realizing the local decision making on the OGA.

Assume that the agents initiate at $t_0 = 0$ such that $d_{ij}(t_0) > R_c$ under a random assignment $k \in \{1, 2\}$ (not necessarily the optimal one) and move under the control laws (2). If the inter-agent distance remains always greater than the communication range $R_c$, i.e., if $d_{ij}(t) > R_c$, $\forall t \in [0, \infty)$, we say that the agents are never involved in a meeting event, and as such, no decision on the goal assignment needs to be made. Out of Lemma 1, the position trajectories $r_i(t)$, $r_j(t)$ are GES w.r.t. the assigned goal locations. Clearly, inter-agent collisions do not occur.

Assume now that the agents initiate at $t_0 = 0$ such that $d_{ij}(t_0) \leq R_c$, or that at some time instant $t_d > 0$ they lie within distance $d_{ij}(t_d) \leq R_c$ under some goal assignment $k \in \{1, 2\}$. We say that the agents are involved in a meeting event and a decision regarding on the goal assignment has to be made. The agents:

1) Exchange information on their current positions $r_i(t_d)$, $r_j(t_d)$ and goal locations $r_{g_i}(t_d)$, $r_{g_j}(t_d)$.
2) Compare the total cost-to-go through the values of the Lyapunov functions $V^{(k)}(t_d)$ for each possible assignment $k \in \{1, 2\}$.
3) Move under: $V(t_d) = \min \{V^{(k)}(t_d)\}$, $k \in \{1, 2\}$.

In other words: Without loss of generality, let $k = 1$ be the goal assignment before the decision making. The agents implement the following switching logic:

- If $V^{(1)}(t_d) \leq V^{(2)}(t_d)$, then: $V(t_d) = V^{(1)}(t_d)$, i.e., they keep the same goal assignment,
- If $V^{(1)}(t_d) > V^{(2)}(t_d)$, then: $V(t_d) = V^{(2)}(t_d)$, i.e., they switch goal assignment.

In the sequel, we refer to the decision making based on the logic described above as to the OGA policy.

Problem 1: We would like to establish that the OGA policy renders stable trajectories for the multi-robot system, i.e., that each agent does converge to some goal location.

B. STABILITY ANALYSIS ON THE SWITCHED MULTI-ROBOT SYSTEM

To this end, we resort to control analysis tools for switched systems. The closed-loop dynamics of agent $i$ read:

$$
\dot{r}_i(t) = u_i^{(k)} = -\lambda_i \left( r_i(t) - r_{g_i}^{(k)} \right),
$$

while similarly one may write the closed-loop dynamics for the trajectories $r_j(t)$ of agent $j$. The OGA policy gives rise to switched dynamics for each agent, in the sense that for $k = 1$ each agent moves under assignment matrix $A_1$, while for $k = 2$ each agent moves under assignment matrix $A_2$.

Denote $\mathbf{r} = [r_i^T, r_j^T]^T$ the state vector of the multi-robot system, which is governed by the switched dynamics:

$$
\dot{\mathbf{r}}(t) = \mathbf{f}_k(\mathbf{r}(t)),
$$

where $\mathbf{f}_k = \begin{bmatrix} u_i^{(k)} \\ u_j^{(k)} \end{bmatrix}$, $k \in \mathcal{K} = \{1, 2\}$.

Assumption 1: We assume for now that there are only a finite number of switches per unit time.

Denote $n \in \mathbb{N}$ and consider the sequence of switching times $T = \{t_0, t_1, t_2, t_3, \ldots, t_n, \ldots\}$ and the switching sequence:

$$
\Sigma = \{r_0; (k_0, t_0), (k_1, t_1), \ldots, (k_n, t_n), \ldots\}, k_n \in \mathcal{K}.
$$

Assuming that the switching sequence is minimal, it follows that $k_n \neq k_{n+1}$. Furthermore, without loss of generality, assume that $k_0 = 1$. Then the (minimal) switching sequence reads: $\Sigma = \{r_0; (1, t_0), (2, t_1), (1, t_2), (2, t_3), \ldots\}$.

Theorem 3: The trajectories $\mathbf{r}(t)$ of the switched multi-robot system (7) are GAS w.r.t. the goal assignment $k$ under the OGA policy. This implies that agents $i$, $j$ converge to their goal locations $r_{g_i}^{(k)}$, $r_{g_j}^{(k)}$.

Proof: We consider the candidate Lyapunov-like functions $V^{(k)}$, $k \in \{1, 2\}$, encoding the motion of the agents under goal assignment $k$. Out of Lemma 1 one has that each individual $k$-th subsystem, i.e. the motion of the multirobot system under the $k$-th assignment, is GES i.e., each $V^{(k)}$ is decreasing on the time intervals that the $k$-th subsystem is active. Furthermore, the OGA policy dictates that at the switching times $\{t_0, t_1, t_2, t_3, \ldots\}$ one has:

$$
V^{(1)}(t_0) > V^{(2)}(t_1) > V^{(1)}(t_2) > V^{(2)}(t_3) > \ldots,
$$

i.e., the value of each Lyapunov-like function $V^{(k)}$ at the beginning of the time intervals when the $k$-th subsystem becomes active satisfies the decreasing condition. This proves that the switched system is stable.

To draw conclusions on asymptotic stability, consider any pair of switching times $t_{k_1} < t_{k_2}$ when the $k$-th subsystem becomes active, the corresponding time intervals $[t_{k_1}, t_{k_2})$. 

The decision making does not result in goal swap; no switching occurs
which imply that: 
$$V(k)(t_{k+1}) < V(k)(t_{k+1}),$$ i.e., that: 
$$V(k)(t_{k+1}) = \rho_1 V(k)(t_{k+1}), \quad 0 < \rho_1 < 1.$$ Then: 
$$V(k)(t_{k+1}) - V(k)(t_{k+1}) = -(1 - \rho_1) V(k)(t_{k+1}) = -(1 - \rho_1) \|r(t_{k+1})\|^2,$$
where $1 - \rho_1 > 0.$ Therefore, the switched system is GAS.

C. Avoiding Zeno Behavior

Given the sequence $T = \{t_0, t_1, \ldots, t_n, \ldots\}, n \in \mathbb{N},$ of switching times, a switched system is Zeno if there exists some finite time $t_Z$ such that:
$$\lim_{n \to \infty} t_n = \infty \quad \text{and} \quad \lim_{n \to \infty} (t_{n+1} - t_n) = t_Z.$$ In simpler words, Zeno behavior means that the switching times have a finite accumulation point, i.e., that infinite amount of switchings occurs in a finite time interval. In general, the task of detecting possible Zeno trajectories and extending them beyond their accumulation points is far from trivial [8] and depends on the problem at hand.

The definition above results in two qualitatively different types of Zeno behavior. A Zeno switched system is [15]:

(i) Chattering Zeno if there exists a finite $C \in \mathbb{N}$ such that $t_{n+1} - t_n = 0$ for all $n > C.$
(ii) Genuinely Zeno if $t_{n+1} - t_n > 0$ for all $n \in \mathbb{N}.$

The difference between these is prevalent especially in their detection and elimination. Chattering Zeno behavior results from the existence of a switching surface on which the vector fields “oppose” each other. This behavior can be eliminated by defining either (i) Filippov solutions which force the flow to “slide” along the switching surface; this results in a sliding mode behavior, or (ii) hysteresis switching, in order to, not only approximate a sliding mode, but also to maintain the property that two consecutive switching events are always separated by a time interval of positive length.

The proposed OGA policy dictates that a switch among vector fields $f_k$ may occur when $\|r(t) - r_j(t)\| \leq R_e,$ i.e., when the multi-robot system trajectories $r(t)$ hit the surface $S^e(t) : \{r \in \mathbb{R}^2 \mathbb{N} | \|r(t) - r_j(t)\| = R_e\}.$ Denote $t_d$ the time instant when the system trajectories lie on the surface $S^e(t_d)$ and the agents involved in the decision making, $t_d^l, t_d^u$ the time instants before and after the decision making, respectively, and $k(t_d^l), k(t_d^u)$ the assignment before and after the decision making, respectively. The decision making on $S^e(t_d)$ results in two different cases:

1) The agents decide to keep their goal assignment, i.e. $k(t_d^l) = k(t_d^u),$ in which case no switching occurs.
2) The agents decide to swap goals, i.e. $k(t_d^l) \neq k(t_d^u)$ and a switching occurs.

Theorem 4: The switched multi-robot system (7) under the OGA policy does not suffer from Zeno points.

Proof: We employ the results in [16], Theorem 2. Let us assume that the switched multi-robot system (7) has a Zeno point $r.$ Then it holds that: $r \in S^e(t_d), r$ is an accumulation point of the set $S = \{r \in S^e(t_d) : f_k(t_d^l) = f_k(t_d^u)\},$ and satisfies: $\nabla S^e(r)f_k(t_d^l) = \nabla S^e f_k(t_d^u) = 0,$ where $t_d$ is a decision making and switching time instant.

Denote $r_{G_1} = [x_{G_1}, y_{G_1}]^T, r_{G_2} = [x_{G_2}, y_{G_2}]^T$ where $r_{G_1} \neq r_{G_2}$ the position vectors of the goal locations $G_1, G_2,$ assigned to the agents $i, j$ respectively, at time $t_d.$ The agents implement the OGA policy at time $t_d$ and decide to swap goals. Then, the corresponding vector fields read:
$$f_k(t_d^l) = \begin{bmatrix} -\lambda_i(r_i - r_{G_1}) \\ -\lambda_j(r_j - r_{G_2}) \end{bmatrix}, f_k(t_d^u) = \begin{bmatrix} -\lambda_i(r_i - r_{G_2}) \\ -\lambda_j(r_j - r_{G_1}) \end{bmatrix},$$
while $\nabla S^e(t_d) = [r_{ji}^T - r_{ji}^T],$ where $r_{ji} = (r_i - r_j).$ The set $S = \emptyset,$ since $f_k(t_d^l) = f_k(t_d^u) \Rightarrow r_{G_1} = r_{G_2},$ a contradiction. Thus, the set of accumulation points of $S$ is empty, which implies that no Zeno points $r$ are contained there, a contradiction. Thus, the OGA policy does not suffer from Zeno points.

This result establishes that the decision making under the OGA renders trajectories that do not accumulate on, i.e., always escape, the switching surface $S^e(t_d).$ The switched vector fields before and after the decision making will qualitatively look: (i) like the blue-green pair in Fig. 2, i.e., the vector fields $f_k(t_d^l), f_k(t_d^u)$ point in the same direction relatively to $S^e,$ and therefore the trajectories cross the switching surface, (ii) or like the blue-red pair in the same Fig., i.e., the vector fields $f_k(t_d^l), f_k(t_d^u)$ both point towards $S^e;$ in this case, the solution should be interpreted in the sense of Filippov, resulting in a sliding mode behavior along the switching surface $S^e.$ The sliding mode behavior is, typically, undesirable from an applications’ standpoint, since
The procedure generates a piecewise constant signal \( r \) unit time. Therefore, to maintain the property that there always will be a positive time interval \( \tau > 0 \) between consecutive switching times we resort to a hysteresis-like switching logic; the idea is that after the \((i, j)\) decision making at time \( t_d \) and while connected, the agents \( i, j \) do not get involved in a new \((i, j)\) decision making, unless the agents have become disconnected first.

We formalize this logic by additionally defining the switching surfaces \( S^1_i = \{ \| r_i - r_j \| = (R_c + \delta_c) \} \), \( S^2_i = \{ \| r_i - r_j \| = (R_c - \delta_c) \} \), where \( \delta_c > 0 \) and implementing the results in [8]. More specifically, the state space is decomposed into the operating regions: \( \Omega_1 = \{ r \in \mathbb{R}^4 : d_{ij} > R_c + \delta_c \} \), \( \Omega_{12} = \{ r \in \mathbb{R}^4 : R_c + \delta_c \leq d_{ij} < R_c \} \), \( \Omega_{21} = \{ r \in \mathbb{R}^4 : R_c \leq d_{ij} < R_c - \delta_c \} \), \( \Omega_2 = \{ r \in \mathbb{R}^4 : R_c - \delta_c \leq d_{ij} \} \). To orchestrate the switching between the operating regions we introduce a discrete state \( \sigma \), whose evolution is described as follows.

For \( t = t_0 \):
1) If \( \| r_i(t_0) - r_j(t_0) \| > R_c \), then set:
\[ \sigma(t_0) = 1 \text{ and } k(t_0) = 1, \text{ without loss of generality.} \]
2) If \( \| r_i(t_0) - r_j(t_0) \| \leq R_c \), then set:
\[ \sigma(t_0) = 2 \text{ and } k(t_0) = k^*, \text{ where } k^* \text{ is taken out of OGA.} \]

For each \( t > t_0 \):
1) If \( r(t) \in \Omega_1 \) and \( \sigma(t^-) = 1 \), then:
\[ \sigma(t) = 1 \text{ and } k(t) = k(t^-). \]
2) If \( r(t) \in \Omega_{12} \) and \( \sigma(t^-) = 1 \), then:
\[ \sigma(t) = 1 \text{ and } k(t) = k(t^-). \]
3) If \( r(t) \in \Omega_{21} \) and \( \sigma(t^-) = 1 \), then:
\[ \sigma(t) = 2 \text{ and } k(t) = k^*, \text{ where } k^* \text{ is taken out of OGA.} \]
4) If \( r(t) \in \Omega_{12} \) and \( \sigma(t^-) = 2 \), then:
\[ \sigma(t) = 2 \text{ and } k(t) = k(t^-) = k^*. \]
5) If \( r(t) \in \Omega_1 \) and \( \sigma(t^-) = 2 \), then:
\[ \sigma(t) = 1 \text{ and } k(t) = k(t^-) = k^*. \]
6) If \( r(t) \in \Omega_{21} \) and \( \sigma(t^-) = 2 \), then:
\[ \sigma(t) = 2 \text{ and } k(t) = k(t^-) = k^*. \]
7) If \( r(t) \in \Omega_2 \) and \( \sigma(t^-) = 2 \), then:
\[ \sigma(t) = 2 \text{ and } k(t) = k(t^-) = k^*. \]

This procedure generates a piecewise constant signal \( \sigma : \mathbb{R}^{2N} \to \{1, 2\} \) which is continuous from the right, implying thus that there are only a finite number of switches per unit time.\(^1\) Note also that the hysteresis logic renders our Assumption (1) valid.

\(^1\)Note also that under the hysteresis switching, the closed loop system is a hybrid system, with \( \sigma \) being its discrete state. This is because the discrete part has “memory”: the value of \( \sigma \) is not determined by the current value of state \( r \) alone, but depends also on the previous value of \( \sigma \).

D. The N > 2 Robot Case

The results on the stability and non-Zeno behavior of the multi-robot system naturally extend to the case of \( N > 2 \) agents. To see how, let us consider \( N \) agents that need to converge to \( N \) goal locations. This gives rise to \( N! \) possible goal assignments \( k \in \{1, 2, \ldots, N!\} \), or in other words, \( N! \) candidate switched subsystems \( f_k \) for the switched multi-robot system:

\[
\dot{r}(t) = f_k(r(t)), \tag{8}
\]

where: \( r = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix}, \quad f_k = \begin{bmatrix} u_1^{(k)} \\ \vdots \\ u_N^{(k)} \end{bmatrix}, \quad k \in K = \{1, \ldots, N!\}. \)

For \( N > 2 \) agents the decision making on the optimal goal assignment involves the \( N \) connected agents at time \( t_d \). The agents exchange information on their positions and goal locations at time \( t_d \), compare the cost-to-go through the \( N! \) Lyapunov functions \( V^{(k)}(t_d) \), where \( k \in K \), and pick the goal assignment \( k \) which corresponds to \( \min \{V^{(k)}(t_d)\} \). The \( N! \) combinations of robots to goals result in intractable enumeration for all but the smallest problems but fortunately, there exists the Hungarian Algorithm [17], a \( O(N^3) \) algorithm from the Operations Research community which optimally solves this problem in a centralized manner. Recent extensions [18] modify this approach for distributed systems.

1) Stability Analysis: Consider the sequence of switching times \( T = \{t_0, t_1, \ldots, t_n, \ldots\} \) and the switching sequence \( \Sigma = \{r_0; (k_0, t_0), (k_1, t_1), \ldots, (k_n, t_n), \ldots\} \), \( k_n \in K \). Then, the reasoning and analysis followed in Theorem 3 apply to the \( N > 2 \) case as well.

Theorem 5: The trajectories \( r(t) \) of the switched multi-robot system (8) are GAS w.r.t. the goal assignment \( k \).

Proof: We consider the candidate Lyapunov-like functions \( V^{(k)}, k \in \{1, \ldots, N!\} \), encoding the motion of the agents under goal assignment \( k \). Out of Lemma 1, each individual \( k \)-th subsystem, i.e., the motion of the multi-robot system under the \( k \)-th assignment, is GES. This implies that each \( V^{(k)} \) is decreasing on the time intervals that the \( k \)-th subsystem is active. Furthermore, the OGA policy dictates that at the switching times \( \{t_0, t_1, t_2, t_3, \ldots, \} \) one has:

\[
V^{(k_0)}(t_0) > V^{(k_1)}(t_1) > V^{(k_2)}(t_2) > V^{(k_3)}(t_3) > \ldots,
\]

i.e., the value of each Lyapunov-like function \( V^{(k)} \) at the beginning of the time intervals when the \( k \)-th subsystem becomes active satisfies the decreasing condition, i.e., the switched system is Lyapunov stable. To draw conclusions on the asymptotic stability, consider any pair of switching times \( t_{k_1} < t_{k_2} \) when the \( k \)-th subsystem becomes active, the corresponding time intervals \( [t_{k_1}, t_{k_1}+1), [t_{k_2}, t_{k_2}+1) \) and...
follow the reasoning as in Theorem 2, i.e.:

\[ V^{(k)}(t_{k_1}) > V^{(k)}(t_{k_1+1}), \quad \text{since the subsystem } k \text{ is active on } [t_{k_1}, t_{k_1+1}] \text{ and GES} \]

\[ V^{(k)}(t_{k_1+1}) \geq V^{(l)}(t_{k_1+1}), \quad \text{out of the OGA Policy, } l \neq k \]

\[ V^{(l)}(t_{k_1+1}) \geq V^{(k)}(t_{k_2}), \quad \text{out of the OGA Policy} \]

\[ V^{(k)}(t_{k_2}) > V^{(k)}(t_{k_2+1}), \quad \text{since the subsystem } k \text{ is active on } [t_{k_2}, t_{k_2+1}] \text{ and GES} \]

which imply that: \( V^{(k)}(t_{k_2+1}) < V^{(k)}(t_{k_1+1}) \), i.e., that:

\[ V^{(k)}(t_{k_2+1}) - V^{(k)}(t_{k_1+1}) = -(1 - \rho_k) V^{(k)}(t_{k_1+1}) \]

\[ = -(1 - \rho_k) \| \hat{r}^{(k)}(t_{k_1+1}) \|^2, \]

where \( 1 - \rho_k > 0 \). Therefore, the switched system is GAS.

2) Non-Zeno Behavior: Finally, following the same pattern as for the \( N = 2 \) case, one may verify that:

**Theorem 6:** The switched multi-robot system \((8)\) under the OGA policy does not suffer from Zeno behavior.

**Proof:** Assume that the switched system \((8)\) has a Zeno point \( \bar{r} \). Then, \( \bar{r} \in S^c(t_d) \) and \( \bar{r} \) is an accumulation point of the set \( S = \{ r \in S^c(t_d) : \bar{f}_k(t_d) = f_k(t_d^+) \} \), where \( S^c(t_d) \) is the switching surface at the decision and switching time \( t_d \). The OGA policy dictates that at least one pairwise goal swap occurs at time \( t_d \), since if the agents decided to keep the goals they had at time \( t_d^* \), then \( t_d \) would not have been a switching time, a contradiction. Since at least one pair of agents \((i, j)\) switches goal locations, denoted as \( r_{G_i}, r_{G_j} \), the condition \( f_k(t_d^*) = f_k(t_d^+) \) on the switched vector fields holds true only when \( r_{G_i} = r_{G_j} \), see the analysis in Theorem 4, i.e., by contradiction. Thus, the set \( S = \emptyset \), which furthermore implies that the set of its accumulation points is empty, implying that no Zeno points can be contained there. Thus, the switched multi-robot system \((8)\) does not exhibit Zeno behavior.

\[ \square \]

V. A Switching Logic on Collision Avoidance

The OGA policy does not ensure that inter-agent collisions, realized as keeping \( d_{ij}(t) \geq \partial r_0 \), \( \forall t \in [0, \infty) \), are always avoided. Consider the scenario in Fig. 3 and assume that the agents move under \( k = 1 \), with agent \( i \) assigned to goal \( A \) and agent \( j \) assigned to goal \( B \). At some time instant \( t_d \), one has \( d_{ij}(t_d) = \| r_i(t_d^*) - r_j(t_d^*) \| = R_c \), and the agents run the OGA policy. The total distance-to-go under \( k = 1 \) is \( \delta(t_d) = \delta_i(t_d) + \delta_j(t_d) \), while under \( k = 2 \) the total distance-to-go is \( \delta^2(t_d) = \delta_i^2(t_d) + \delta_j^2(t_d) \). Since \( \delta(t_d) > \delta^2(t_d) \) the agents switch to \( k = 2 \), with agent \( j \) moving to goal \( A \) and agent \( i \) moving to goal \( B \). If, however, \( j \) moves relatively fast to its goal location compared to agent \( i \), collision may occur.

**Problem 2:** We would like to establish (sufficient) conditions under which the OGA policy is collision-free.

A. Detecting Conflicts

Recall that we are referring to time \( t > t_d \), i.e. after \( N_c \leq N \) connected agents have decided on a goal assignment \( k \) based on the OGA policy, and move towards their goal locations \( r_{G_i}(k) \). In the sequel we drop the notation \( \cdot(k) \), in the sense that the goal assignment \( k \) is kept fixed.

We would first like to identify a metric (a "supervisor") determining online whether the OGA policy results in collisions. Let us consider the collision avoidance constraint:

\[ c_{ij}(r_i, r_j) = (x_i - x_j)^2 + (y_i - y_j)^2 - \Delta^2 > 0, \quad (9) \]

encoding that the inter-agent distance \( d_{ij} = \| r_i - r_j \| \) should always remain greater than \( \Delta \).

To facilitate the analysis using Lyapunov-like approaches, we first need to encode the constraint \((9)\) as a Lyapunov-like function. Inspired by interior point methods \([19]\), let us first recall that a barrier function is a continuous function whose value on a point increases to infinity as the point approaches the boundary of the feasible region; therefore, a barrier function is used as a penalizing term for violations of constraints. The concept of *recentered barrier functions* in particular was introduced in \([20]\) in order to not only regulate the solution to lie in the interior of the constrained set, but also to ensure that, if the system converges, then it converges to a desired point.

In this respect, we first define the logarithmic barrier function \( b_{ij}(\cdot) : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R} \) for the constraint \((9)\) as:

\[ b_{ij}(r_i, r_j) = -\ln(c_{ij}(r_i, r_j)), \quad (10) \]

which tends to \( -\infty \) as \( c_{ij} \to 0 \), i.e. as \( d_{ij} \to \Delta \). The recentered barrier function of \((10)\) is defined as: \([20]\):

\[ r_{ij} = b_{ij}(r_i, r_j) - b_{ij}(r_g, r_j) - \nabla b_{ij} r_g |_{r_g = r_g}, \quad (11) \]

where \( \nabla b_{ij} = \left[ \frac{\partial b_{ij}}{\partial x_i}, \frac{\partial b_{ij}}{\partial y_i} \right]^T \) is the gradient vector of the function \( b_{ij}(\cdot) \),
and \( \nabla b_{ij}^T \big|_{r_g} \) is the transpose of the gradient vector evaluated at the goal position \( r_g \). By construction, the recentered barrier function (11):

(i) is non-zero everywhere except for the goal location \( r_g \).

(ii) tends to \( +\infty \) as the distance \( d_{ij} \) tends to \( \Delta \).

These characteristics of recentered barrier functions (11) are suitable for encoding both collision avoidance of each agent \( i \) w.r.t. an agent \( j \neq i \), and convergence of agent \( i \) to its assigned goal \( r_g \). To ensure that we have a nonnegative function encoding these objectives, so that it can be used as a Lyapunov-like function, for each agent \( i \) we define [11]:

\[
    w_{ij}(\cdot) = (r_{ij}(r_i, r_j, r_g))^2,
\]

which now is a positive definite function. More specifically:

\[
    w_{ij}(r_i, r_j, r_g) \text{ is zero only at the goal position } r_g, \text{ and furthermore tends to infinity as } d_{ij} \to \Delta.
\]

To ensure that the position trajectories of agent \( i \) remain bounded in a prescribed region (for reasons that will be explained in the technical analysis later on), for each agent \( i \) we define the workspace constraint:

\[
    c_{00}(r_i, r_0) = R_0^2 - (x_i - x_0)^2 - (y_i - y_0)^2 > 0,
\]

which encodes that the position \( r_i \) should always lie in the interior of a circle of center \( r_0 = [x_0 \ y_0]^T \) and radius \( R_0 > R_c \). This can be thought as the workspace where the agents operate. Then, in the same reasoning followed before, we define the barrier function:

\[
    b_{00}(r_i, r_0) = -\ln(c_{00}(r_i, r_0)),
\]

and its corresponding recentered barrier function:

\[
    r_{i0} = b_{00}(r_i, r_0) - b_{00}(r_g, r_0) - \nabla b_{00}^T \big|_{r_{i0}} (r_i - r_g),
\]

which vanishes only at the goal location \( r_g \), and tends to infinity as the position \( r_i \) tends to the workspace boundary.

To get a positive definite function we consider:

\[
    w_{i0}(\cdot) = (r_{i0}(r_i, r_0, r_g))^2.
\]

Therefore, an encoding that agent \( i \) stays \( \Delta \) apart w.r.t. all of its neighbor agents, while staying within the bounded workspace, can be now given by an approximation of the maximum function of the form [11]:

\[
    w_i = \left( (w_{i0})^\delta + \sum_{j \in N_c} (w_{ij})^\delta \right)^{1/\delta},
\]

where \( \delta \in [1, \infty) \), and \( N_c \subseteq N \) is the set of connected (neighbor) agents \( j \neq i \) of agent \( i \). The function \( w_i \) vanishes

at the goal location \( r_g \), and tends to \( +\infty \) as at least one of the terms \( w_{i0}, w_{ij} \) tends to \( +\infty \), i.e., as at least one of the inter-agent distances \( d_{ij} \) tends to \( \Delta \), or as agent \( i \) tends to the workspace boundary. Finally, to ensure that we have a Lyapunov-like function for agent \( i \) which uniformly attains its maximum value on constraints’ boundaries we take:

\[
    W_i = \frac{w_i}{1 + w_i},
\]

which is zero for \( w_i = 0 \), i.e., at the goal position \( r_g \), of agent \( i \), and equal to 1 as \( w_i \to \infty \). For more details on the analytical construction the reader is referred to [2], [11].

The Lyapunov-like function (18) can now be used as a (sufficient) criterion on determining whether the control inputs \( u_i, u_j \) of the OGA policy jeopardize safety. Let us consider the time derivative of (18):

\[
    \dot{W}_i = \left[ \frac{\partial W_i}{\partial x_i} \right] \dot{x}_i + \sum_{j \in N_c} \left( \frac{\partial W_i}{\partial x_j} \dot{x}_j \right) = \zeta_i^T u_i + \sum_{j \in N_c} (\zeta_{ij}^T u_j),
\]

where \( \zeta_i = \left[ \frac{\partial W_i}{\partial x_i}, \frac{\partial W_i}{\partial y_i} \right]^T \), \( \zeta_{ij} = \left[ \frac{\partial W_i}{\partial x_j}, \frac{\partial W_i}{\partial y_j} \right]^T \). The evolution of the time derivative \( \dot{W}_i \) along the trajectories of agent \( i \) depends not only on its own motion (through \( u_i \)), but also on the motion of its neighbor agents \( j \neq i \) through their velocities \( u_j \). This time derivative provides sufficient conditions on establishing that the OGA policy is safe. To see how, let us consider the following Lemma:

**Lemma 1:** If the control inputs \( u_i, u_j \), where \( j \in N_c \), out of the OGA policy satisfy the following condition for all \( i \in \{1, \ldots, N\} \):

\[
    \zeta_i^T u_i + \sum_{j \in N_c} (\zeta_{ij}^T u_j) < 0,
\]

then the OGA policy is asymptotically stable w.r.t. the current goal assignment \( k \) and furthermore collision-free.

**Proof:** To verify the argument, consider the properties of the Lyapunov-like function (18) and denote the constrained set for each agent \( i \) as \( K_i = \{ r \in \mathbb{R}^{2N} | c_{ij}(\cdot) \geq 0 \} \), where \( j \in N_c \cup \{0\} \). The set \( K_i \) is by construction compact (i.e., closed and bounded), with the level sets of \( W_i \) being closed curves contained in the set \( K_i \). Then, the condition (20) implies that the system trajectories \( r_i(t) \) under the OGA control input \( u_i \) evolve downwards the level sets of \( W_i \), i.e., always remain in the interior of the constrained set \( K_i \), which furthermore reads that the inter-agent distances \( d_{ij}(t) \) never violate their critical value \( \Delta \).

**Problem 3:** We need to establish a LR policy ensuring that inter-agent distances \( d_{ij} \) remain greater than a critical distance \( \Delta \), and will be active only when the OGA policy is about to result in colliding trajectories.

**B. Resolving Conflicts**

The condition (20) gives rise to furthermore determining a LR control policy in the case that the OGA policy is about
to violate it. Let us denote the switching surface:

$$Q_i = \zeta_i^T u_i + \sum_{j \in \mathcal{N}_i} \left( \zeta_{ij}^T u_j \right).$$

(21)

Then for $Q_i < 0$ one has out of Lemma 1 that the OGA policy is both GAS and collision-free. For $Q_i > 0$ one has $W_i > 0$, which implies that the position trajectories $r_i(t)$ evolve upwards the level sets of the Lyapunov-like function $W_i$. This condition may jeopardize safety and dictates the definition of an additional, LR policy, which will ensure that the trajectories $r_i(t)$ remain in the constrained set $K_i$.  

**Lemma 2:** The LR control policy for agent $i$, realized via:

$$u_{ib} = \left[ -k_i \frac{\partial W_i}{\partial x_i} - \sum_{j \in \mathcal{N}_i} \frac{\partial W_i}{\partial y_j} u_{ij} - k_i \frac{\partial W_i}{\partial y_i} - \sum_{j \in \mathcal{N}_i} \frac{\partial W_i}{\partial y_j} u_{ij} \right]$$

where $k_i > 0$, renders the trajectories of the multi-robot system collision-free in the set $Q_i > 0$.

**Proof:** To verify the argument, consider that the time derivative (19) under the proposed control inputs (22) reads:

$$\dot{W}_i = -k_i \left( \frac{\partial W_i}{\partial x_i} u_{ix} + \frac{\partial W_i}{\partial y_i} u_{iy} \right)^2,$$

which implies that the position trajectories $r_i(t)$ move downwards the level sets of the Lyapunov-like function (18).

C. Ensuring both Stability and Safety

The switching between the OGA policy and LR policy across the switching surface $Q_i = 0$ gives rise to closed-loop dynamics with discontinuous right-hand side for each agent $i$, reading:

$$\dot{r}_i(t) = v_i^{(p)},$$

(23)

where $p \in \mathcal{P} = \{1, 2\}$. Thus, for $p = 1$, agent $i$ moves under the OGA policy, that is, $v_i^{(1)} = u_i$, while for $p = 2$ agent $i$ moves under the LR policy, that is, $v_i^{(2)} = u_{ib}$. To avoid sliding-mode behavior that would result from solutions in the Filippov sense, we formalize the switching logic between the closed-loop vector fields $u_i, u_{ib}$ via a hysteresis technique, similar to the one used in Section IV-C. The hysteresis logic gives rise to a switching sequence of times $T = \{\tau_1, \tau_2, \tau_3, \ldots, \tau_n, \ldots\}$, where the time interval $\tau_n - \tau_{n-1}$ is finite $\forall n \in \mathbb{N}$, excluding thus any sliding-like behavior, and to the switching sequence:

$$\Sigma^* = \{r_i(\tau_0), (\tau_1), (\tau_2), \ldots, (\tau_{2q}), (\tau_{2q+1}), \ldots, \},$$

where $\tau_0 > t_d, q \in \mathbb{N}, \tau_{2q}$ are the time instants when the subsystem $p = 1$ (OGA policy) is "switched on", $\tau_{2q+1}$ are the time instants when the subsystem $p = 2$ (LR policy) is "switched on".

**Problem 4:** We finally need to establish that the proposed switching strategy $\Sigma^*$ between the OGA policy and the LR policy renders the multi-robot trajectories stable w.r.t. to the assigned goals and also collision-free. This switching strategy makes it rather difficult to check Branicky’s decreasing condition on the “switched-on” time intervals of each individual subsystem. For this reason we resort to results that remove this condition in order to draw conclusions on (asymptotic) stability.

**Theorem 7:** The trajectories of the switched multi-robot system under the switching logic $\Sigma^*$ are (i) collision-free, and (ii) asymptotically stable w.r.t. the assigned goals.

**Proof:** The first argument is proved in Lemmas 1, 2. The second argument can be verified by directly applying Theorem 2. Lemmas 1, 2 imply that the functions $V_i, W_i$ for each agent $i$ serve as generalized Lyapunov-like functions for the individual subsystems $p \in \{1, 2\}$, respectively. The stability condition (5) for each subsystem $p$ is satisfied out of the boundedness of the solutions $r_i(t)$ within the constrained set $K_i$, which is by construction closed and bounded. Therefore, the switched system is stable. To furthermore establish asymptotic stability, it should hold that for at least one of the individual subsystems $p$, the value of the corresponding generalized Lyapunov-like function decreases along the sequence of switching times. Let us assume that this is not the case; then this would imply that the closed-loop switched vector fields $v_i^{(p)}$, $p \in \{1, 2\}$, "oppose" each other and cancel out on the switching surface $Q_i$, forcing the system trajectories to get stuck there. Note that the vector field $v_i^{(2)} = u_{ib}$ is by construction parallel to the gradient vector of the function $W_i$, which is by construction transverse to the boundary of the constrained set $K_i$. Then, this assumption would furthermore imply that the vector field $v_i^{(1)} = u_i$ of the OGA policy forces the system trajectories of agent $i$ towards a goal on an intersecting path w.r.t. the neighbor agent; a contradiction, since the OGA does not result in intersecting paths [1]. Therefore, the switched multi-robot system under the switching strategy $\Sigma^*$ is asymptotically stable.

VI. SIMULATION RESULTS

Simulation results are provided to evaluate the performance of the switched multi-robot system under the proposed switching logic and control policies. Let us here consider a typical scenario involving $N = 10$ agents with initial and goal locations in very close proximity such that the OGA policy is not sufficient to ensure there are no collisions, see Fig. 4. The agents start from the initial conditions marked with red squares towards goal locations that do not necessarily correspond to the optimal assignment. The paths from the starting locations to the initially assigned goals are depicted as the blue lines. Communications range is denoted by the cyan ring around each agent and in this case $R_c = 5R$. During this simulation, at least one subgroup of robots using...
using the LR control policy for 21% of the duration of the simulation. Fig. 5 shows the minimum clearance between any two robots. As this is always positive, there is never a collision between any robots.

The number of re-plan operations depends greatly on the number of robots, the communications range, initial distribution, and quality of initial assignments. The time for a re-plan operation also directly depends on the number of agents in the connected component. Planning times on simulated teams of robots using a single computer take approximately $10^{-7} N_c^3$ seconds such that a component of 100 robots takes about 0.1 seconds to plan. This is more than sufficient for the proposed applications.

![Fig. 4. A simulation with $N = 10$ robots. The paths followed are straight when in the OGA segments, but are potentially curved when using the LR control law.](image)

![Fig. 5. The minimum clearance between any two robots for a simulated trial with $N = 10$, where clearance is defined as the free space between robots. Note that as this is always positive, there were never any collisions between robots.](image)

VII. DISCUSSION AND CONCLUDING REMARKS

We presented a switched systems approach on the decentralized concurrent goal assignment and trajectory generation for multi-robot networks which guarantees safety and global stability to interchangeable goal locations. The proposed switching logic relies on multiple Lyapunov-like functions which encode goal swap among locally connected agents based on the total traveled distance, avoidance of inter-agent collisions and convergence to the assigned goal locations. As such, the proposed methodology renders feedback control policies with local coordination only, and therefore is suitable for applications such as in first-response deployment of robotic networks under limited information sharing. Our current work focuses on the consideration of agents with more complicated dynamics, as well as the robustness of our algorithms under communication failures and uncertainty.

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