The correlations and anticorrelations in QPO data

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Received; accepted; published online

Abstract. Double peak kHz QPO frequencies in neutron star sources varies in time by a factor of hundreds Hz while in microquasar sources the frequencies are fixed and located at the line \( \nu_2 = 1.5 \nu_1 \) in the frequency-frequency plot. The crucial question in the theory of twin HFQPOs is whether or not those observed in neutron-star systems are essentially different from those observed in black holes. At first sight, this seems not to be the case for neutron stars. For each individual neutron star, the upper and lower kHz QPO frequencies, \( \nu_2 \) and \( \nu_1 \), are linearly correlated, \( \nu_2 = A \nu_1 + B \), with the slope \( A < 1.5 \), i.e., the frequencies definitely are not in a 1.5 ratio. In this contribution we show that when considered jointly on a frequency-frequency plot, the data for the twin kHz QPO frequencies in several (as opposed to one) neutron stars uniquely pick out a certain preferred frequency ratio that is equal to 1.5 for the six sources examined so far.

Key words: QPOs – Neutron stars
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1. Introduction

It has been recognized some time ago [Bursa 2002] that the pairs of high-frequency QPOs observed in all neutron star sources can be fitted by a single linear relation

\[ \nu_2 = 0.89 \nu_1 + 375 \text{ Hz}, \]

where \( \nu_2 \) and \( \nu_1 \) refers to the upper and lower observed kHz QPO frequency. The linear correlation between the pair frequencies becomes even more apparent if we restrict ourselves to individual sources, though the coefficients of the relation \( \nu_2 = A \nu_1 + B \) slightly differ from source to source. A particular example is the case of Sco X-1, where the observed frequencies are well fitted with the linear dependence \( \nu_2 = (3/4) \nu_1 + 450 \text{ Hz} \) with the accuracy better than one percent.

Abramowicz et al. [2003] pointed out that the distribution of frequency ratios is strongly peaked near the 1.5 value, but made no statement about the actual correlation of frequencies. A concern has been raised, whether their result is a statistical illusion [Belloni et al. 2003].

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In this contribution we examine the linear frequency-frequency correlations for six neutron-star sources: 4U 1820–30, 4U 1728–34, 4U 0614+09, 4U 1608–52, 4U 1735–44 and 4U 1636–53, and show that within errors the correlations are consistent with the following statement: the plots of the six \( \nu_2 = A \nu_1 + B \) relations intersect in one point \( [N_1, N_2] \), with \( N_1 \approx 600 \text{ Hz} \), and the ratio of the frequencies in the intersection point is \( N_2/N_1 = 1.5 \) to high accuracy. To demonstrate this, we plot the five pairs of coefficients, \( A \) and \( B \), and show that they are linearly anticorrelated.

2. Anticorrelation between the slope and shift

We examine the six individual neutron star sources by fitting each of them with a linear formula

\[ \nu_2 = A \nu_1 + B , \]

where the coefficients \( A \) and \( B \) are referred to as the slope and shift, respectively. The resulting values of the slope and shift and of corresponding errors for each source are summarized.
Table 1. Best linear fits and their errors for the frequency–frequency correlation.

| Source   | $A$  | $\Delta A$ | $B$ [Hz] | $\Delta B$ [Hz] |
|----------|------|------------|----------|-----------------|
| 4U 1636  | 0.58 | 0.03       | 622      | 27              |
| 4U 1608  | 0.76 | 0.03       | 456      | 16              |
| 4U 1820  | 1.02 | 0.09       | 255      | 66              |
| 4U 1735  | 0.61 | 0.05       | 593      | 39              |
| 4U 0614  | 0.79 | 0.02       | 420      | 5               |
| 4U 1728  | 0.98 | 0.02       | 330      | 5               |

in Table I and plotted in Figure II showing the slope–shift plane. The dependence $A = A(B)$ strongly suggests that the two quantities are anticorrelated. The linear fit for the anticorrelation gives

$$A = (1.50 \pm 0.03) - (0.0016 \pm 0.0001) \, \text{Hz}^{-1} \, B.$$  \hspace{1cm} (3)

As it was mentioned above, this result is consistent with the statement that the six linear plots of the $\nu_2 - \nu_1$ relations intersect in one point $[N_2, N_1]$. The frequencies $N_2$ and $N_1$ can be determined from the coefficients of the anticorrelation \(\ref{eq:anticorrelation}\), from which we obtain

$$N_2 = (940 \pm 80) \, \text{Hz}, \quad N_1 = (625 \pm 40) \, \text{Hz}.$$  \hspace{1cm} (4)

The ratio of frequencies $N_2/N_1$ at the intersection point equals to 1.5 with the accuracy of 15%. Hence, it is remarkable that neutron stars QPOs pick up the same 3:2 ratio common for the black-hole sources. We can rephrase the equation \(\ref{eq:anticorrelation}\) to the form

$$A = \frac{3}{2} - \frac{B}{625 \, \text{Hz}}.$$  \hspace{1cm} (5)

The data points of Figure 1 have been obtained through a shift-and-add technique, as described in Barret et al. (2005) applied to the whole archival RXTE data available to date. The technique relies firstly on estimating the QPO frequency down to the shortest timescales permitted by the data statistics, and secondly on identifying the QPOs for which the shift-and-add can be applied (generally the lower QPOs). Having a set of frequencies, the frequencies are then binned (typically with a bin of 10 to 20 Hz), and the individual PDS shifted to the mean frequency within the bin. The averaged PDS so-obtained is then searched for QPOs, and when both QPOs are present, their frequencies are obtained with a Lorentzian fit. A linear fit is then applied to the lower and upper QPO frequencies so obtained. This method has been applied to 4 sources shown in Figure 1 (4U1820; 4U1608, 4U1735, 4U1636), whereas for 0614 and 1728, the linear fit has been applied to the frequencies measured with a shift and add technique applied to a continuous segment of observation (of typical duration close to the orbital period of the RXTE spacecraft). Both methods should yield consistent results: the smaller error bars the points of 0614 and 1728 can be explained by a larger number of frequencies involved in the linear fit. A more careful comparison of the results of the two methods (with estimates of the potential biases, such as only the narrower QPOs are detected, hence with small errors on the frequency determination) is clearly required and will be part of a forthcoming paper (Barret et al. 2005b in preparation).

3. A possible explanation

It has been suggested that QPO arises from a resonant interaction between the radial and vertical oscillation modes in relativistic accretion flows. The strongest possible resonance occurs at the radius, where the radial and vertical epicyclic frequencies are in the 3:2 ratio.

In general, the frequency and the amplitude of non-linear oscillations are not independent. In the lowest order of approximation the observed frequencies $\nu$ differ from the eigenfrequencies $\nu_0$ of oscillators by corrections proportional to the squared amplitudes. We consider system having two oscillation modes, whose eigenfrequencies are $\nu_{1,0}$ and $\nu_{2,0}$. The frequencies of non-linear oscillations may be written in the form

$$\nu_1 = \nu_{1,0} + \Delta \nu_1, \quad \nu_2 = \nu_{2,0} + \Delta \nu_2.$$  \hspace{1cm} (6)

The frequency corrections $\Delta \nu_1$ and $\Delta \nu_2$ are proportional to the squared amplitudes

$$\Delta \nu_1 = \kappa_1 a_1^2 + \kappa_2 a_2^2, \quad \Delta \nu_2 = \lambda_1 a_1^2 + \lambda_2 a_2^2,$$  \hspace{1cm} (7)
where $\kappa_1$, $\kappa_2$, $\lambda_1$ and $\lambda_2$ are constants depending on non-linearities in the system.

Let us suppose that for some reason the two amplitudes $a_1$ and $a_2$ are correlated. Thus, one may consider the amplitudes as functions of a single parameter $s$,

$$a_1 = a_1(s), \quad a_2 = a_2(s).$$

(8)

Such relations can be considered as a natural consequence of an interplay between the resonance excitation mechanism and the dissipation of the energy in the system.

It follows that the frequencies of non-linear oscillations $\nu_1$ and $\nu_2$ are correlated as well. Up to the linear order in $s$, we obtain from equation (6)

$$\nu_1 = \nu_{1,0} + \nu_{1,0} F_1 s + O(s^2),$$

(9)

$$\nu_2 = \nu_{2,0} + \nu_{2,0} F_2 s + O(s^2),$$

(10)

where the coefficients $F_1$ and $F_2$ are given in terms of constants $\kappa_1$, $\kappa_2$, $\lambda_1$ and $\lambda_2$ of frequency corrections (7) and of the derivatives $\partial a_1/\partial s$ and $\partial a_2/\partial s$ at the point $s = 0$.

Isolating the parameter $s$ from equations (9) and (10) we get the linear correlation between the observed frequencies

$$\nu_2 = A \nu_1 + B,$$

(11)

where the slope and shift are respectively given as

$$A = \frac{\nu_{2,0}}{\nu_{1,0}} Q,$$

(12)

$$B = \frac{\nu_{2,0}(1 - Q)}{\nu_{1,0}}$$

(13)

and we define $Q$ as $Q \equiv F_2/F_1$.

The observed frequencies $\nu_1$ and $\nu_2$ of systems with different amplitude prescription (8) are correlated in a different way. Any particular value of $Q$ leads to particular values of the slope and the shift. However, if the eigenfrequencies of the systems are similar, the slope and shift are necessarily anticorrelated. Solving the equations (12) and (13) for the parameter $Q$, we obtain

$$A = \frac{\nu_{2,0}}{\nu_{1,0}} \frac{1}{\nu_{1,0}} B.$$

(14)

From the resonance condition it follows that the eigenfrequency ratio $\nu_{2,0}/\nu_{1,0}$ is approximately 3/2. Therefore we arrive at

$$A = \frac{3}{2} \frac{1}{\nu_{1,0}} B.$$

(15)

4. Discussion and Conclusions

In black-hole sources the observed QPO frequencies are fixed and always have the ratio 3/2. It has been recognized that their actual frequencies scales inversely with mass $M$ assuming a similar value of the spin (McClintock & Remillard 2005 [van der Klis], [Török]). In neutron-star sources the frequencies are not fixed, but their distribution seems to cluster around a single line for each individual source. By linear fitting the observed data, we have found out that these lines intersect around a single point [1, N2], which have coordinates given by equation (14). The fact that the frequencies are close to 3:2 ratio supports the idea that there is a similar mechanism at work in both classes of sources. Moreover, if we extend the black-hole $1/M$ scaling law up to the frequency of the intersection point, we obtain a mass of order of one solar mass (assuming zero angular momentum for neutron stars), which provides an additional hint.

Assuming that the $1/M$-scaling can be adopted also to the neutron-star QPOs, the fact that the individual positions of sources in the $A$–$B$ plane do not strictly follow the anticorrelation line can be attributed to small differences in neutron-star masses. By scaling the 614 Hz frequency of equation (4), we find that the $A$–$B$ is steeper or softer for more massive or less massive sources, respectively. This is demonstrated by the shaded region in Figure 4. Under this assumption the deviation in the masses of examined neutron stars should not be greater than $\sim 20\%$.

Acknowledgements. It is a pleasure to acknowledge the hospitality of Sir Franciszek Oborski, the master of the Wojnowice Castle, where a large part of this work was done.

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