Universal Fermi liquid crossover and quantum criticality in a mesoscopic system

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Quantum critical systems derive their finite-temperature properties from the influence of a zero-temperature quantum phase transition. The paradigm is essential for understanding unconventional high-$T_c$ superconductors and the non-Fermi liquid properties of heavy fermion compounds. However, the microscopic origins of quantum phase transitions in complex materials are often debated. Here we demonstrate experimentally, with support from numerical renormalization group calculations, a universal crossover from quantum critical non-Fermi liquid behaviour to distinct Fermi liquid ground states in a highly controllable quantum dot device. Our device realizes the non-Fermi liquid two-channel Kondo state, based on a spin-1/2 impurity exchange-coupled equally to two independent electronic reservoirs. On detuning the exchange couplings we observe the Fermi liquid scale $T^*$, at energies below which the spin is screened conventionally by the more strongly coupled channel. We extract a quadratic dependence of $T^*$ on gate voltage close to criticality, and validate an asymptotically exact description of the universal crossover between strongly correlated non-Fermi liquid and Fermi liquid states.

A conventional second-order quantum phase transition (QPT) features quantum mechanical fluctuations of a classical order parameter. Some second-order QPTs in heavy fermion materials, notably CeCu$_6$Au and YbRh$_2$Si$_2$, defy easy description in this scheme, and their quantum critical behaviour instead appears to be related to the breakdown of Kondo screening. Distinctive non-Fermi liquid behaviours appear above a so-called Fermi liquid (FL) scale that vanishes at the quantum critical point (QCP); away from the QCP, a crossover from non-FL to FL behaviour is observed at low energies. A diverging effective mass $m^*$ at the QCP, seen in both materials, signifies the absence of quasiparticles at the Fermi surface.

In many heavy fermion materials and in high-$T_c$ superconductors, the relevant degrees of freedom and the effective Hamiltonian can be controversial. We aim to understand quantitatively a second-order QPT outside the usual order-parameter-fluctuation description. Quantum dots provide an experimental framework for realizing known quantum impurity Hamiltonians that can feature tunable second-order QPTs. However, QCPs are challenging to reach even in engineered systems, since perturbations that steer away from quantum criticality may be inherently uncontrollable, as in two-impurity Kondo experiments to date.

At the QCP of a two-channel Kondo (2CK) system, a single overscreened spin yields a non-FL state with no quasiparticles (that is, only collective excitations) at the Fermi surface. The overscreened spin gives rise to a decoupled Majorana mode at the impurity site, to which the non-FL behaviour has been attributed. An order parameter is typically not invoked. A FL scale $T^*$ results from several relevant perturbations: Zeeman splitting, difference in exchange couplings and charge transfer between the two channels. Requiring that all these perturbations be small would seem to diminish prospects for observing the QCP in bulk systems. Nonetheless, 2CK physics has been invoked to explain experiments on heavy fermion materials and two-level tunnelling centres.

A 2CK state has been predicted and observed in a quantum dot tunnel-coupled to leads and to a ‘metallic grain’, an electron reservoir big enough to have a small, ideally metallic level spacing $\Delta \ll kT$, but small enough to retain a charging energy $E_C \gg kT$ at temperatures of interest. The leads and the grain serve as two independent screening channels for the dot spin. Simply having two conventional leads coupled to the dot is not enough to realize the 2CK state, as charge transfer between the leads cannot be avoided. In contrast, the grain’s measured between source and drain leads (blue). The four grey stars indicate additional ohmic contacts which are floated during measurement.

**Figure 1 | Device and model.** a, SEM micrograph of a device nominally identical to the device studied. The five brighter features seen coming in from the left are metal bridges suspended above the sample surface. b, Schematic of the device with labelled gate electrodes. Gates BWT, BP and BWB define the grain (red) along with LBT and LBB; the last two also control the dot–grain coupling. Gates LWT, LP and LWB define the dot (green), along with LBT and LBB. Gates BR are used to isolate the dot measurement circuit. Other gates are held at a fixed voltage throughout the experiment. Conductance is measured between source and drain leads (blue). The four grey stars indicate additional ohmic contacts which are floated during measurement. c, Model of the system used for the NRG calculations. $I_{c,i}$ is the dot–grain coupling; $I'$ is the total dot–lead coupling (sum of couplings to source and drain leads, $I_{c,i}$ and $I_{c,dr}$, respectively). The source and drain leads together act as one channel in the spin 2CK regime, and the Coulomb-blockaded grain acts as an independent channel. The full Hamiltonian is given in Methods section ‘Hamiltonian’.

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charging energy strongly suppresses charge transfer with the other leads. Spin exchange with the dot remains possible, so the grain and leads compete and overcompensate the dot spin. In ref. 3, the resulting non-FL behaviour was observed, as were conventional FL single-channel Kondo states far from the QCP. However, $T^*$ was not identified and the non-FL to FL crossover was not seen. In particular, how the FL scale $T^*$ vanishes on approach to the QCP is an important hallmark of the phase transition, with associated critical exponents. Recently, in addition to prior numerical22 and analytical22 descriptions of the crossover, a new description has been found using Abelian bosonization and conformal field theory (CFT) methods, yielding asymptotically exact predictions for conductance in the regime where $eV_{\text{SD}}$, $kT$, $kT^* \ll kT_K$, with $T_K$ the 2CK temperature and $V_{\text{SD}}$ the bias voltage on a weakly-coupled probe23. This description has the decoupled Majorana mode at the impurity site coupling to degrees of freedom in the reservoirs for non-zero $T^*$.

In this work, we show how fine control over the 2CK state in a mesoscopic device allows direct comparison to exact results in the crossover regime, yielding $T^*$ as a function of gate detuning away from the QCP. The device (Fig. 1a) is fabricated by lithographically patterned gate electrodes on a GaAs/Al0.3Ga0.7As heterostructure hosting a two-dimensional electron gas (2DEG). In this experiment, two-terminal conductance $I/V$ measurements may be important elsewhere25. To directly compare to the experimentally measured conductance data of Fig. 2c, Fig. 2b adjusts the parameters. We focus on the spin $2CK$ regime, though charge fluctuations are expected instead of isolated QCPs2,23,24. These are closely related to the finite size spectrum. These diagonal elements relate to the spectral functions collapse onto the QCP. The device (Fig. 1a) is fabricated by lithographically patterned gate electrodes on a GaAs/Al0.3Ga0.7As heterostructure hosting a two-dimensional electron gas (2DEG). A plan of the device is given in Fig. 1a. Despite the number of gates, the device is conceptually simple (Fig. 1c): a metallic grain (red) and two leads (blue) are each tunnel-coupled to a quantum dot (green) at rates $\Gamma_G$ and $\Gamma_I$, respectively. The total dot–lead tunnel rate $\Gamma = \Gamma_G + \Gamma_I$, the sum of tunnel rates to the source and drain leads. The charging energy is $U (E_C)$ for the dot (grain), and the full Hamiltonian is given in Methods section ‘Hamiltonian’. In this experiment, two-terminal conductance $G(dI/dV_{\text{SD}})$ is measured between the pair of leads (Methods section ‘Measurements’). We use gate voltage $V_{\text{BWT}} (V_{\text{LP}})$ to tune the grain level $\phi$ (dot level $\varepsilon$) (gates are named in Fig. 1b).

We first identify the set of QCPs in the $(-\varepsilon/U, -\phi/E_C)$ plane for fixed $\Gamma, \Gamma_P$. For our model Hamiltonian, quantum critical ‘2CK lines’ periodic in the grain charge are expected instead of isolated QCPs2,23,24. Figure 2a shows the 2CK lines overlaid on numerical renormalization group (NRG) calculations of $G(-\varepsilon/U, -\phi/E_C)$ using realistic device parameters. We focus on the spin 2CK regime, though charge fluctuations may be important elsewhere25. To directly compare to the experimentally measured conductance data of Fig. 2c, Fig. 2b adjusts the NRG calculations of Fig. 2a to account for the cross-capacitance between $V_{\text{LP}}$ and the grain.

To identify transport signatures of quantum criticality along the 2CK line, we look for the characteristic square-root scaling of $G(V_{\text{SD}}, T)$ derived from the CFT of ref. 26. The CFT yields temperature-dependent spectral functions $A_{2CK}(\omega, T, \delta_P)$, where $\delta_P$ is a phase shift from potential scattering. These are closely related to $G(V_{\text{SD}}, T)$ for $h\omega \rightarrow -eV_{\text{SD}}$ (Methods section ‘Relationship of $G(V_{\text{SD}}, T)$ to spectral functions’). A scaling collapse of $G(V_{\text{SD}}, T)$ is expected:

$$\frac{G(0, T) - G(V_{\text{SD}}, T)}{\sqrt{T}} \propto \frac{1}{T_K^{1/2}} Y_{2CK}(-eV_{\text{SD}}/kT, \delta_P)$$

(1)

where $T_K$ (Kondo temperature) is a scale below which the 2CK physics is observed and $Y_{2CK}(-eV_{\text{SD}}/kT, \delta_P)$ a universal function closely related to $A_{2CK}(\omega, T, \delta_P)$ (Methods section ‘Fitting expressions for 2CK’).

Figure 2d shows spectral functions $A(\omega, T)$ calculated by NRG. Importantly, the spectral functions collapse onto $A_{2CK}(\omega, T, \delta_P)$, with the horizontal axis scaled to emphasize the $\omega^{1/2}$ behaviour for large $\omega/T$. Measured $G(V_{\text{SD}}, T)$ on or very near the 2CK line (Fig. 2e) collapse similarly, except for the 20 mK and 40 mK traces at positive $V_{\text{SD}}$. This deviation could result from a small $T^* \lesssim T_\sigma$, the base electron temperature. Data taken at more negative $V_{\text{LP}}$ show very clear 2CK scaling (Methods section ‘Scaling along the quantum critical lines’) but are less suitable for analysing the crossover. Experimental $kT_K \approx 50 \mu$eV should only be trusted up to factors of order unity: in equation (1), $T_K$ enters only as a scale factor, and other scale factors like source–drain coupling asymmetry must be estimated. We estimate that $T_\sigma/T_K \approx 0.15$, and consider the source lead weakly coupled (Methods section ‘Measurements’).

Having identified the 2CK lines in Fig. 2, we consider how to perturb the quantum critical state. In the 2CK model, a single FL scale $T^*$ suffices to describe any combination of symmetry-breaking perturbations22. The limit $\hbar \omega, kT, kT^* \ll kT_K$ permits an exact expression for the scattering $T$ matrix in the low-temperature 2CK crossover23. In our experimental configuration, the $T$ matrix is diagonal:

$$\pi n T_{2CK}(\omega, T, \delta_P) = 1 - e^{2ih\sigma} \sum_{\alpha, \alpha} G(\hbar \omega/kT^* \cdot T)$$

(2)

with the universal complex-valued function $G(\hbar \omega/kT^* \cdot T)$. $T_{2CK}$ encoding the crossover physics. These diagonal elements relate to $A(\omega, T)$ and thus to experimental $G = dI/dV_{\text{SD}}$ for highly asymmetric source–drain coupling. $v$ is the bare density of states per spin in the leads, $\sigma$ is the spin.
index, and $\alpha = 1 (-1)$ labels electrons in the leads (grain). The $S$ matrix gives a (spin and channel dependent) scattering phase shift that is a function of the relative strengths of any perturbations present. Negligible charge transfer between channels and zero magnetic field yields $S_{nx, n\pi} = \pm \alpha$, with $+(-)$ indicating the dot is more strongly exchange-coupled to the grain (leads). The factor $e^{2\delta_p}$ accounts for additional spin-independent phase shifts from potential scattering. We fix $S_{nx, n\pi} = \alpha$ and let $\delta_p$ jump by $\pi/2$ to account for sign changes.

To observe the FL crossover experimentally, we fix $V_{\text{LP}} = -236\text{ mV}$ (dashed line in Fig. 2c) and detune the exchange couplings using $V_{\text{BWT}}$. Moving slightly away from the QCP so that $T^* \approx T_\star$, we still measure a $T^{1/2}$ scaling collapse for $T > 50\text{ mK}$ (Fig. 3a). These high-$T$ data are fitted nicely using the ref. 26 CFT result with small $\alpha$ (black line). The clear scaling behaviour at high $T$ can only be observed for $V_{\text{BWT}}$ in a small neighbourhood around the QCP. Below 50 mK, prominent deviations from 2CK scaling develop, which we attribute to a crossover into a FL state where the grain screens the dot spin. Near zero bias these low-$T$ traces are fitted by the crossover theory with similar, small $\delta_p$ (Fig. 3b).

We stress this is a non-trivial regime since $T^* \approx T_\star$ asymptotics of the FL fixed point are insufficient to describe the observed deviations from $2CK$ scaling develop, which we attribute to a crossover to fitting range.”.

Near the QCP, $T^*$ should depend quadratically on the strength of symmetry-breaking perturbations$^{5,6,22}$ (Fig. 3c). At $B = 0$, we expect $T^* \propto |J_1 - J_2|^2$, where $J_{1(2)}$ is the exchange coupling for channel 1(2), and the critical exponent $\xi_2 = 2$ (see ref. 1 for definition of $\alpha$ and $\nu$). As is generically true for QPTs, the constant of proportionality is non-universal and may differ on either side of the QCP$^4$. Measured $G(V_{\text{SD}}, V_{\text{BWT}})$ reveals periodic zero bias dips that transition sharply to zero bias peaks as $V_{\text{BWT}}$ is increased (Fig. 3d, top). The zero bias dip (peak) corresponds to a $T = 0$ ground state where the grain (lead) screens the dot spin; these are separated by a QCP. In Fig. 3d (middle), $T^*$ depends quadratically on $V_{\text{BWT}}$ away from the QCP, although the curvature differs between the two sides of the QCP, which have different ground states. The phase shift $\delta_p = 0$ on one side of the QCP, and appears to approach $\pi/2$ on the other (Fig. 3d, bottom). The $\pi/2$ shift reflects a sign flip in $S_{nx, n\pi}$ between distinct FL ground states, where either the grain or leads screen the dot spin$^{27}$. Between QCPs, $\delta_p$ varies smoothly. $T^*$ and $\delta_p$ are not plotted directly to the right of each QCP, reflecting the ambiguity of fitting a small crossover peak on top of the 2CK peak. Both $T^*$ and $\delta_p$ are insensitive to small changes in the range of $V_{\text{SD}}$ used for fitting (Methods section ‘Sensitivity of $T^*$ and $\delta_p$ to fitting range’).

A remarkable conclusion of these measurements is that, contrary to prevailing belief, the neighbourhood of a 2CK QCP can be quite large.

**Figure 3 | Crossover from quantum criticality to a Fermi liquid.**

$V_{\text{LP}} = -236.0\text{ mV}$ for experimental data. a, Measured $G(V_{\text{SD}}, T)$. At $V_{\text{BWT}} = -376.4\text{ mV}$, a thermally broadened spectral function from the 2CK CFT ($\delta_p = -0.022\text{ mV}$, solid black line) describes the high-$T$ data. b, Subset of data shown in a, with fits $G(V_{\text{SD}})$ at low energies is fitted to thermally broadened spectral functions from the crossover theory (top, 20 mK; bottom, 40 mK; $\delta_p = -0.045\text{ mV}$, $T^* = 0.5\text{ mK}$). Fitting details in Methods. c, Quantum criticality (QC) occurs for energies above the Fermi liquid scale $T^*$ (grey parabolid), which should depend quadratically on the coupling asymmetry $J_1 - J_2$ between the two channels as well as on the Zeeman splitting $E_Z$. We vary $T^*$ by tuning $J_1 - J_2$ (cut along red parabola). d, Extraction of $T^*$ and $\delta_p$ from measurements. The triangle denotes $V_{\text{BWT}}$ for a and b. Top, $G(V_{\text{SD}}, V_{\text{BWT}})$ ($T = 20\text{ mK}$). Middle, $T^*$ from crossover theory fits to the experimental $G(V_{\text{SD}}, T)$. Red traces are parabolas with $T^* = 0$ at the QCP and unequal scale factors on either side of the QCP. The largest $T^*$ values may not be much less than $T_\star$, so the crossover theory is not strictly valid for all $V_{\text{BWT}}$. Labels indicate approximate QCP locations. Bottom, $\delta_p$ from the crossover theory fits. Error bars, 1 s.d. confidence intervals from the fits. e, Extraction of $T^*$ and $\delta_p$ from NRG calculations. Parameters as in Fig. 2. Top, $G(-\hbar\omega, -\psi/E_C)$, rescaled to match maximum $G$ of d. Middle, $T^*$. Bottom, $\delta_p$. 

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The 7\(^g\) parabolas span most of the range in \(V_{\text{NWT}}\), so fine tuning is not needed to reach a state within the influence of a QCP. Also, since \(T^* \propto |J_1 - J_2|^2\), the measurements imply the exchange couplings depend linearly on gate voltage, which is not usually anticipated. Prior theoretical studies on the model dot–grain Hamiltonian have not noted either of these features. Fitting the crossover theory to spectral functions from NRG yields conductance via equation (3) (Methods section ‘Relationship of \(G(V_{\text{SN}}, T)\) to spectral functions’). The NRG conductance (Fig. 3e, top) shows zero bias dips transitioning into peaks, as well as the shift of the peak towards positive \(-\hbar \omega\), as in transport spectroscopy (Fig. 3d). Importantly, the \(\phi\) dependence of \(T^*\) (Fig. 3e, middle) shows extended parabolas like in the measurements, and the extracted \(\delta_\phi\) (Fig. 3e, bottom) reproduces the rapid \(\pi/2\) phase shift across a QCP, with an otherwise smooth \(\phi\) dependence. A perfect correspondence between experiment and NRG calculations should not be expected, since not all parameters may be extracted directly from measurements, and the dot–grain Hamiltonian is an idealization (Methods sections ‘Hamiltonian’ and ‘Extracting device parameters’). Yet the qualitative numerical reproduction of key experimental features helps to validate our surprising conclusions.

The experimental and numerical corroboration of analytical results in the vicinity of a QCP is a milestone in our understanding of correlated electron systems, with implications for other systems that may be influenced by a QCP, such as high-\(T_c\) superconductors and heavy fermion materials. An essentially identical universal crossover (same \(G(\hbar \omega/KT^*, T/T^*)\), but different symmetry-breaking perturbations) is expected for the two-impurity Kondo model. Future work will address the full phase diagram of the device, which may host charge 2\(C_2\) and SU(4) Kondo regimes\(^{29,30}\). Our device geometry could enable Aharonov–Bohm interference measurements to probe phase coherence of low-lying excitations in the non-FL 2\(C_2\) state\(^{27,31}\), giving new insight into the nature of a local non-FL.

**Online Content** Methods, along with any additional Extended Data display items and Source Data, are available in the online version of the paper; references unique to these sections appear only in the online paper.

**Received 11 March; accepted 28 July 2015.**

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**Acknowledgements** We are grateful to S. Amasha, Y. Oreg, A. Carmi, E. Sela, A. K. Mitchell and M. Heiblum for discussions; H. K. Choi, Y. Chung and J. MacArthur for electronics expertise; M. Heiblum for use of his laboratory during initial device characterization; H. Inoue, N. Olek, D. Raslin and E. Weisz for fabrication guidance; F. B. Anders, E. Lebanon and the late A. Schiller for their calculations which guided prior experimental work; and M. Stopa for his SETE software for electrostatic quantum dot modelling. The device was fabricated in the Braun Submicron Center at the Weizmann Institute of Science, with final fabrication steps done at Stanford Nano Shared Facilities (SNSF) at Stanford University. This work was supported by the Gordon and Betty Moore Foundation grant no. GBMF3429, the Hungarian National Scientific Research grant OTKA K105149, the Polish National Science Centre project no. DEC-2013/10/E/ST3/00213, EU grant no. CGG-280484, the National Science Foundation grant no. DMR-0705682, and the US-Israel BSF grant no. 2008149. A. J. K. and L. P. were supported by a Stanford Graduate Fellowship. SETE calculations were run on the Odyssey cluster supported by the FAS Division of Science, Research Computing Group at Harvard University. NRG calculations were performed at the Poznan Supercomputing and Networking Center.

**Author Contributions** A. J. K., G. Z. and D. G.-G. designed the experiment. A. J. K. and L. P. performed the measurements. I. W., C. P. M. and G. Z. performed the NRG calculations. C. P. M. and I. W. contributed equally to the theoretical analysis. A. J. K., L. P., C. P. M., I. W., G. Z. and D. G.-G. analysed the data. A. J. K. designed and fabricated the devices, with e-beam lithography from D. M., using heterostructures grown by V. U. A. J. K. and L. P. wrote the paper with critical review provided by all other authors.

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METHODS

Device. The 2DEG is 50 nm below the surface and has an electron density \( n = 3.3 \times 10^{11} \) cm\(^{-2}\) and mobility \( \mu = 1.2 \times 10^{5} \) cm\(^2\) V\(^{-1}\) s\(^{-1}\).

Figure 1a shows a top-down SEM micrograph of the device. This view is appropriate for labelling the gate electrodes and explaining the function of each gate, but the air bridges are hard to see. In Extended Data Fig. 1 we show a view of the device at a 40° tilt with respect to normal incidence. The five air bridges clearly rise above the gate electrodes underneath. The device is rotated approximately 180° with respect to the orientation of Fig. 1a.

As initially fabricated, the bridges did not make good electrical contact to the gates. This problem was remedied with an in situ platinum deposition procedure. The SEM images of Fig. 1a and Extended Data Fig. 1 were obtained before the platinum deposition.

Measurements. The measurements are performed in the mixing chamber of a wet dilution refrigerator (Oxford Kelvinox TLM) with a base electron temperature \( T_{\text{e}} = 20 \) mK, verified by Coulomb blockade thermometry. The device was cooled down with \( +300 \) mV bias on all gates to enhance charge stability by reducing the range of voltage needed for operation. Remaining charge instability manifested in discrete jumps of features in gate voltage every few hours, which were compensated by simply relocating those features with other parameters (for example, temperature) fixed. For all measurements we use an SR830 lock-in amplifier with a 1 \( \mu \)V excitation at 33 Hz and a custom 108 VA \( \times 10 \) V source (see related publication34.) A custom voltage source \( V_{\text{LWT}} \) and observe a decrease in the overall conductance.

HAMILTONIAN AND QUANTUM NUMERICAL SIMULATION

In equilibrium, the conduction electrons’ scattering matrix is proportional to the self-energy. In case of the quantum dot system considered here, the latter quantity translates to the Green’s function of the \( d \)-level of the small dot. This allows us to use the exact 5 matrix at the 2CK fixed point and express the equilibrium spectral function of the small dot in the limit \( kT^* \ll h \omega_0, kT \ll kT_{\text{F}} \) as

\[
A(\alpha, T) = A_{\text{2CK}}(\alpha, T, \delta_p) = \text{Im} \left\{ 1 - 3 e^{2 h \alpha_0} \sqrt{\frac{\pi}{2}} \right\} \frac{1}{T_x} \text{Re} \left[ \sum_{m=1}^{3} \frac{1}{F(3, 2, 1; 1 - \text{coth} x)} \right] (5)
\]

where \( x = u - \frac{h \omega_0}{kT} \) and \( \delta_p \) is the scattering phase shift. We fix the dimensionless parameter \( \delta = -0.09 \) so that the spectral function drops to half of its \( \alpha = 0 \) value at \( h \omega_0 = kT_{\text{F}} \) in the limit \( T \rightarrow 0 \) (refs 21, 36). Equation (5) immediately implies that \( A_{\text{2CK}}(0, T, \delta_p) / A_{\text{2CK}}(0, T, \delta_p) / \sqrt{T_x/T} \) is a universal function of \( h \omega_0/kT \), which when convolved with a Fermi function gives the function \( V_{\text{2CK}}(eV_{\text{F}}/kT, \delta_p) \) of equation (1). We stress that this \( h \omega_0/kT \) scaling is a special property of the 2CK fixed point. When fitting the experimental data, we shall assume an asymmetric coupling to the leads (see previous section).

Fitting expressions for crossover. At frequencies and temperatures far below the 2CK temperature \( T_{\text{F}} \), we can use the crossover form of the \( T \) matrix derived in refs 5 and 6 to express the \( d \)-level’s equilibrium spectral function. Here we obtain the expression

\[
A(\alpha, T) = A_{\text{2CK}}(\alpha, T, \delta_p) \approx \text{Im} \left\{ 1 - e^{2 h \alpha_0} G(\tilde{c}(\alpha, T)) \right\} (6)
\]

where \( \delta_p = 0 (\delta_p = \pi/2) \) in the case when the dot is coupled more strongly to the grain (leads), and

\[
G(\tilde{c}(\alpha, T)) = \frac{-i}{\sqrt{2\pi\hbar \Gamma}} \exp \left\{ \frac{i}{2\hbar} \frac{1}{\Gamma} \left( 1 + 1 + \frac{1}{2\piT} \right) \right\} \left\{ \sinh x \right\} (7)
\]

is a universal function of rescaled energy \( \tilde{c} = h \omega_0/kT^* \) and temperature \( T_{\text{F}} = T/\Gamma^* \). For equation (7) only, \( \Gamma^* \) is the gamma function, not a tunnel rate. Again, when fitting to experimental data, the spectral function must be thermally broadened (see Methods section ‘Relationship of \( G(\text{VSD}, T) \) to spectral functions’).

Fitting range. When fitting the crossover theory to experimental data, we fit \( G(\text{VSD}, T) \) only in a small window of \( \text{VSD} \) of \( \pm 6 \mu \text{V} \) around zero, regardless of temperature. A priori \( T^* \) is unknown and it only makes sense to fit \( d \text{VSD}/dT \text{SD} \) in this window.

Additionally, thermal broadening of high energy features can in principle spoil the scaling of the low energy features, even for otherwise sensible ranges of \( \text{VSD} \). At minimum the 20 mK and 40 mK traces are used for fitting, but sometimes also the 52 mK and possibly the 70 mK traces, provided \( T \leq T^* \) (the fitting process is somewhat iterative in this respect). Once the temperatures to be used in fitting are decided for a given value of \( \text{VSD} \), the fitting considers data from all of those temperatures simultaneously. Fitting the crossover theory to NRG calculations is done analogously (a window of \( h \omega_0 \approx 5 \mu \text{eV} \) about zero).

Sensitivity of \( T^* \) and \( \delta_p \) to fitting range. In Fig. 3 we use the crossover CFT to fit experimental data and thereby extract the Fermi liquid scale \( T^* \) and the scattering phase shift \( \delta_p \). The fitting procedure uses a limited range of \( \text{VSD} \) (6 mV) and it is argued that this is a conservative approach.

In Extended Data Fig. 2 we show that the fitting is insensitive to small changes in the fitting range. At each value of \( \text{VSD} \), we try and extract \( T^* \) and \( \delta_p \) for nine different ranges of bias voltage, which we obtain by starting with \( (6, +6 \mu \text{V}) \) and adding or subtracting a point on either end, for example, \( (7.5, 6), (6, -6), (6.5, -7.5), (-4.5, 6) \) and so on. It is important not to add so many points that data outside the validity of the theory are included. However, subtracting too many points may degrade the fit quality.

After finding \( T^* \) and the error in \( T^* \) reported by the fits, we consider all nine fitting ranges to give independent estimates of \( T^* \), and find the weighted mean \( T^* \), weighted by the errors from each fit. The error bars show the standard deviation of the weighted mean, and indicate the spread of \( T^* \) values returned by the fits. We do the same for \( \delta_p \). Varying the fitting range by small amounts does not seem to contribute significantly to the uncertainty in \( T^* \) and \( \delta_p \).
Scaling along the quantum critical lines. In Fig. 2e we demonstrated that measured \( G(V_{SD}, T) \) falls onto a scaling curve derived from the CFT results of ref. 26. However, at the lowest temperatures there are deviations from the expected scaling curve. Here we justify attributing the deviations to a small \( T^* \leq T_c \). For clarity we break out the data of Fig. 2e into separate panels for each temperature in Extended Data Fig. 3.

For \( T \gtrsim 80 \text{ mK} \), the data appear to collapse onto the scaling curve. Deviations appear for \( V_{SD} > 0 \) and become increasingly prominent as temperature decreases. Focusing on the 20 mK trace, the difference between the data and scaling curve vanishes by construction \( V_{SD} = 0 \), but becomes quite large for small positive \( V_{SD} \). The difference between the scaling curve and the data actually decreases at larger \( V_{SD} \). These observations are at least qualitatively consistent with the effect of a small \( T^* \), which should result in corrections at low energies. It is perhaps surprising that a small \( T^* \leq T_c \) could result in substantial deviations from 2CK scaling, but we estimate from figure 7 of ref. 6 that even for \( T^* = 10T_c^* \), signs of the crossover may be easily seen up to \( eV_{SD} \approx 30kT^* \), and perhaps even factors of a few higher than that.

It is notable that the deviations from 2CK scaling appear significant only for \( V_{SD} > 0 \), which would seem to imply a scattering phase that is not quite \( n/2 \). Another possibility to explain the asymmetry about \( V_{SD} = 0 \) would be that true zero \( V_{SD} \) drifted over the course of the measurement. Typically some small applied \( V_{SD} \) is required to compensate for an offset voltage at the current amplifier input, but at base temperature the quality of the scaling collapse is sensitive to errors of even a few times this magnitude.

Another possibility to explain the asymmetry about \( V_{SD} = 0 \) is that less than an order of magnitude in temperature is considered. No obvious Hamiltonian.

Data Fig. 5d).

The tunneling matrix elements between the leads (grain) and the dot are denoted by \( t_{ij} \) and are assumed to be independent of momentum. The strengths of the couplings are given by \( F_{G} = \pi v_{FG}(\varepsilon_{G}) \) and \( F_{S} = \pi v_{FS}\varepsilon_{S} \), respectively, where \( v_{FG}(\varepsilon_{G}) \) is the density of states for lead \( G \) (grain). In the NRG calculations the energy spectrum of the grain is assumed to be continuous and the densities of states for leads and grain and taken to be constant and equal: \( v_{FG} = v_{SG} = 1/(2D) \), with \( D = 1 \) being the half bandwidth used as the energy unit in NRG calculations.

In the Hamiltonian (equation 9) we have neglected the dot–grain capacitive coupling, which can give rise to a term of the form, \( V_{DG}(n_D - N_c) \), where \( n_D = n_1 + n_2 \). An estimate for \( V_{DG} \) is extracted experimentally in the Methods section 'Extracting device parameters', and \( V_{DG} \) is thought to play no role for the purpose of the present analysis.

Several other Hamiltonians have been proposed to realize 2CK physics in quantum dot based systems36,39.

NRG calculations. To solve the Hamiltonian (equation 9) we use the NRG method40 (open access Budapest code is available at http://www.phy.bme.hu/~dmngr). First, we introduce the collective charge operators (bosonic operators) for the grain32:

\[
\hat{N} = \sum_{m = -\infty}^{\infty} |m}\langle m| \quad \text{and} \quad \hat{N}^\dagger = \sum_{m = -\infty}^{\infty} |m+1\rangle\langle m| \quad \text{(14)}
\]

Strictly speaking, the identity, \( \hat{N} = n_0 \) must be fulfilled, but within the NRG approach this can be relaxed by treating \( \hat{N} \) as an independent quantity. This is possible as the spectral properties of the system are not sensitive to the exact number of conduction electrons present in the grain in the limit of infinitely small level spacing. To extract the finite site spectrum and determine the location of the 2CK lines, however, a projection to the physical subspace was necessary. In our calculations we took into account seven charges in the grain.

Using the above charge operators, the grain part of the Hamiltonian (11) can be rewritten as

\[
H_{\text{grain}} = \sum_{p,s} \varepsilon_p a_p^\dagger a_p + \frac{E_c}{2}(\hat{N} - N_0)^2 + \phi(\hat{N} - N_0) \quad \text{(15)}
\]

The \( \hat{N}^\dagger \) operators capture the charging transitions of the grain and enter explicitly in the tunnelling Hamiltonian (equation (13)), which now reads

\[
H_{\text{tunnelling}} = \frac{2F_{G}}{\pi} \sum_{p,s} \left( \hat{N}^\dagger a_p^\dagger d_s + d_s^\dagger a_p^\dagger \hat{N} \right) + \frac{2F_{S}}{\pi} \sum_{k,\sigma} \left( \varepsilon_k a_k^\dagger d_s^\dagger + d_s^\dagger a_k^\dagger \right) \quad \text{(16)}
\]

The first term in equation (16) describes the dot–grain tunnelling, while the second term accounts for the dot–leads coupling. This second term is obtained by performing an orthogonal transformation42 from the two-lead basis to an effective single lead with resultant coupling \( \Gamma_* \). The resulting Hamiltonian consists then of two conduction bands coupled to a complex impurity composed of the grain (\( N \)) and dot degrees of freedom.

The core of the NRG procedure is the logarithmic discretization of the conduction band with discretization parameter \( \lambda \) and mapping of the conduction band onto a semi-infinite chain with exponentially decreasing hoppings. The resulting Hamiltonian is then diagonalized iteratively in an NRG framework. In our calculations we used discretization parameter \( \lambda = 2 \) and kept 4,000 states at each iteration. We also exploited the SU(2) symmetry of the total spin and two U(1) symmetries for \( N_1 = n_1 + n_3 + N_0 \) and \( N_2 = n_2 - N_0 \), where \( n_0 \) is the electron number operator in the first conduction channel (leads coupled to the dot) and \( n_0 \) is the electron number operator in the second channel (the grain). We performed the full density-matrix NRG calculations (fDM-NRG41,42), employing the Budapest Flexible DM-NRG code43, to compute the normalized dimensionless spectral function, \( A(\varepsilon, T) = \pi (\Gamma_0 + \Gamma_* )A_0(\varepsilon, T) \), where \( A_0(\varepsilon, T) \) is the spectral function of a single channel (the grain). We performed the full density-matrix NRG calculations (fDM-NRG41,42), employing the Budapest Flexible DM-NRG code43, to compute the normalized dimensionless spectral function, \( A(\varepsilon, T) = \pi (\Gamma_0 + \Gamma_* )A_0(\varepsilon, T) \), where \( A_0(\varepsilon, T) \) is the spectral function of a single channel (the grain).
function for the $d'_n$ operators that describe the dot level. The linear conductance through the small dot can be then determined with the equation

$$G = \frac{2e^2}{h} \left( \frac{\mathcal{T}D}{\mathcal{T}S + \mathcal{T}D} \right)^2 \left[ \delta \left( \frac{\epsilon(t_0, kT) + 1}{\delta} \right) \right] A(\epsilon_0, T)$$

(17)

where $\delta(\epsilon, T)$ is the Fermi–Dirac distribution function.

**Shifting of NRG calculations in Fig. 2b.** In Fig. 2b we incorporate a linear $\epsilon$-dependent shift into $\phi$ to obtain agreement between NRG calculations and experiment (Fig. 2c). The agreement is obtained by first rescaling the NRG calculations so that the maximum value of $G$ is the same. The global scaling takes into account the source–drain coupling asymmetry, as explained elsewhere in Methods. Then, the sharp features in the cut taken at $V_{LP} = -260$ mV are compared with NRG calculations to establish $V_{LP} = -260$ mV $\approx -\epsilon/U = 0.55$. Finally, another cut for fixed $V_{LP}$ is taken to establish a linear relationship between $V_{LP}$ and $-\epsilon/U$. The two points give a linear relationship between $V_{LP}$ and $-\epsilon/U$. One global offset in $-\epsilon/U$ then suffices to give good agreement everywhere. Using this method we find $-\epsilon/U = -\epsilon/U + 3.1(\approx -U + 1.5)$, where $-\epsilon/U$ is the vertical axis of Fig. 2b. Physically, the linear dependence of $\epsilon$ on $-\epsilon/U$ can be understood as a consequence of the indirect capacitive coupling between $V_{LP}$ and the grain.

**Extracting device parameters.** In this section we describe how model parameters are extracted from measurements, and comment on which of the parameters should be considered free. We also determine bounds on any dot–grain charging energy $U_{cg}$ neglected in the model.

The dot charging energy $U = 2.9$ meV is determined from source–drain bias spectroscopy of the dot in the Coulomb blockade regime (Extended Data Fig. 6). In previous cooldowns $U$ has varied between 1 and 3 meV, perhaps owing to how $U$ depends sensitively on the number of electrons in the few electron regime. We use $U = 2$ meV as the model parameter in NRG calculations, and note that the calculations should be relatively insensitive when $U > D = 1$ meV, the electronic half bandwidth used in calculations. This value of $D$ corresponds roughly to the internal level spacing on the small dot, providing a high energy cut-off.

The grain charging energy $E_G = e^2/C = 160$ meV is measured by source–drain bias spectroscopy of the grain (Extended Data Fig. 7). We compare this measurement to geometric estimates. A common rule of thumb is that upon gate depletion, the extent of the depletion region extends as far from the gate laterally as the 2DEG shaded area in Extended Data Fig. 7a is 1.2 nm, keeping in mind that the width of the Fermi–Dirac distribution is approximately 3.5 nm. This gives an expected $E_G = 280$ μeV, which is within a factor of two of the measurement. In a previous cooldown of the same device we measured $E_G = 150$ μeV, which we use as the model parameter in NRG calculations.

In designing the device we aimed for as large an $E_G$ as possible while still being able to imagine a near continuum of states in the grain. The level spacing may be estimated by considering a particle in a 2D box. The level spacing $D = \hbar^2/2m a^2$, where $a$ is the area of the box and $m = 0.067m_e$ is the effective mass in GaAs. Using the design area $A = 1.2$ μm$^2$ we find $D = 4.6$ μeV $= 2.6kT_e$, where $T_e = 20$ μK. If we instead take $A = 0.93$ μm$^2$ inferred from measurement of $E_G$ and the approximation for the capacitance, we find $D = 6.0$ μeV $= 3.4kT_e$. In either case, $D$ is no more than factors of a few times $T_e$ keeping in mind that the width of the Fermi–Dirac distribution is approximately 3.5$kT_e$. This implies that the grain is indeed acting as a metallic grain at all measured temperatures. In Extended Data Fig. 7, it appears that the typical level spacing (spacing in $\delta\epsilon$ between diagonal lines) is larger than anticipated. The peak conductance can differ significantly for each level, which reflects a distribution in source–drain coupling asymmetry from level to level. Some levels may not be visible if their source–drain coupling asymmetry is strong.

The dot level $\epsilon$ may be tuned by changing the voltage applied to gate LP. We typically tune $V_{LP}$ by tens of mV, and only a change of 1.9 mV in $V_{BW,T}$ is needed to add an electron to the grain, so even a weak capacitance of gate LP to the grain may be important. Over a small range in $V_{BW,T}$, the converse effect of $V_{BW,T}$ on the dot can be safely ignored. We think of $\phi$ as a function of both $V_{LP}$ and $V_{BW,T}$, and as a function of $V_{LP}$.

The dependence of $\phi$ on $V_{BW,T}$ may be determined easily. By tuning $V_{LP}$ towards $-1$ using $V_{LP}$, the increasingly sharp features as a function of $V_{BW,T}$ (Fig. 2b, c) should be spaced by $E_G$ in units of energy. These sharp features are rather generic as a function of all the other parameters. We already know $E_G$ from a direct measurement, and therefore we have a conversion between $V_{BW,T}$ and $\phi$. The dependence of $\phi$ on $V_{LP}$ can be estimated by looking at the overall skew of features in Fig. 2c, and estimating some $\Delta V_{BW,T}/\Delta V_{LP}$ to describe the skew. Estimating the skew is not strictly a well defined procedure, but the skew itself is well defined as the ratio of capacitances between gate LP and the grain and gate BWT and the grain. Since the dependence of $\phi$ on $V_{BW,T}$ is known explicitly and there is a good strategy for estimating the dependence on $V_{LP}$, we do not consider $\phi$ a free parameter.

The dependence of $\epsilon$ on $V_{LP}$ is hard to determine explicitly from measurements, although they should be proportional. Unlike how we calibrated $V_{BW,T}$, there are no consistent sharp features as a function of $V_{LP}$ which are independent of the other parameters. We believe we can identify to within the order of ten per cent the constant of proportionality just using the broad features observed in both experiments and calculations.

The tunnel couplings $t_S$ and $t_D$ are essentially free parameters in that we have not identified a way to extract them experimentally. Only an even combination of the source and drain leads will couple to the dot, so it makes more sense to think about the tunnel coupling for this even combination, $t$, and a source–drain asymmetry parameter, $\alpha$, rather than $t_S$ and $t_D$. Source–drain coupling symmetry, in the limit of zero source–drain bias, just acts like a global scale factor. Apart from our intuition developed from experimental measurements, we estimate our source–drain coupling asymmetry by how much we need to scale the NRG calculations to match the experiment.

In summary, it would seem there are three free parameters: $t$, $t_S$, $\alpha$. The last free parameter is a trivial scale factor. The other parameters can either be explicitly measured, or are linearly related to gate voltages and may be estimated well. We have tried exploring the $(t\alpha)$ plane and cannot obtain much better agreement.

Finally, measurements of $G(V_{BW,T}, V_{LP})$ yield $U_{cg} = 21$ μeV (Extended Data Fig. 8). This analysis considers the dot–grain system as a capacitively-coupled double quantum dot. The $U_{cg}$ we extract should be thought of as an upper bound—the gate voltages are set to a very different regime where $G_c$ is negligible, unlike the situation in the work we report here. When tuning between this regime and the regime where $G_c = G$, it appears as if the splitting of the lines in Extended Data Fig. 8 goes to zero long before $G_c$ becomes a significant fraction of $G$, implying that $U_{cg} \rightarrow 0$.

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Extended Data Figure 1 | SEM micrograph of a device nominally identical to the device studied. Acceleration voltage in the SEM was 5 kV. The device is tilted 40° with respect to normal incidence.
Extended Data Figure 2 | Sensitivity of $T^*$ and $\delta_P$ to fitting range. Top panel, $G(V_{SD}, V_{BWT})$ at $T = 20$ mK, exactly as in Fig. 3. Middle and bottom panels, $T^*$ (middle) and $\delta_P$ (bottom) as functions of $V_{BWT}$. Black points and red curves are exactly as in Fig. 3; the blue points correspond to the weighted mean of extracted $T^*$ and $\delta_P$ for an ensemble of fitting ranges, and error bars on the blue points correspond to the s.d. of the weighted mean.
Extended Data Figure 3 | Measured \((G(0, T) - G(V_{SD}, T))/kT^{1/2}\) (symbols) and CFT fit (solid lines) of Fig. 2e broken out into separate panels for each \(T\). Temperature \(T\) in mK is shown centrally in each panel. The range in measured \(V_{SD}\) is from \(-31.5\) to \(28.5\) \(\mu\)V, resulting in a decreasing range on the \((eV_{SD}/kT)^{1/2}\) axis as \(T\) is increased. The single fit in Fig. 2e is plotted against the measured data for each \(T\).
Extended Data Figure 4  |  Two-channel Kondo scaling.  Top left, measured $G(V_{LP}, V_{BWT})$ from Fig. 2c. Panels at right, measured $(G(0, T) - G(V_{SD}, T))/(kT)^{1/2}$ at six points on 2CK lines in the $(V_{LP}, V_{BWT})$ plane; points are indicated by coloured stars. Black lines are fits to thermally broadened spectral functions from ref. 26 with small phase shifts from potential scattering.
Extended Data Figure 5 | Coulomb blockade thermometry. a, Measured $G(V_{SD}, V_{LP})$ reveals two prominent linear features, the slopes of which are labelled as $m$ and $n$. b, Measured $G(V_{SD} = 0, T)$ (crosses) and fits using equation (8) (lines). Every measurement is the average of 20 successive traces. c, Power-law fit of peak height to extracted electron temperature yields an exponent of $-1.04(1)$. d, Residuals of the fit shown in c are all less than $0.001e^2/h$. 

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Extended Data Figure 6 | Measurement of $U$. Within each Coulomb diamond we label the number of electrons on the dot ($N$) as determined by charge sensing techniques. The intersection of the lines indicate $U = 2.9$ meV.
Extended Data Figure 7 | Measurement of $E_C$.  

**a.** Measurement scheme. $G(V_{SD} - V_{BWT})$ is measured using the grain’s own pair of measurement leads (red pads), which are isolated from the measurement leads of the dot by depleting gate BR. Gate BL is depleted to avoid shorting conductance through the channel just left of the grain. The current path is the red dashed line. The grey stars indicate ohmic contacts which are floated during measurement.

**b.** $G(V_{SD} - V_{BWT})$ through the grain in the Coulomb blockade regime ($T = 20 \, \text{mK}$). The intersections of the lines indicate $E_C = 160 \, \text{µeV}$. 

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Extended Data Figure 8 | Bounding \( U_{\text{dg}} \) from measurements in the Coulomb blockade regime. a, \( G(V_{\text{SD}}, V_{\text{LP}}) \) through the dot in the Coulomb blockade regime, with both the dot and grain formed. Here \( V_{\text{BWT}} \) is such that the grain is Coulomb-blockaded. From the slopes of the dashed lines overlaid on the peaks in the data, we determine lever arms \( a_{\text{LP}} = 0.081 \) and \( a_{\text{SD}} = 0.194 \). b, \( G(V_{\text{BWT}}, V_{\text{LP}}) \) through the dot at zero \( V_{\text{SD}} \). Peaks in \( G \) correspond to Coulomb blockade on the dot being lifted; the splitting implies finite \( U_{\text{dg}} \). For fixed \( V_{\text{BWT}} \) the difference in peak positions (blue horizontal bar) gives the dot–grain charging energy \( U_{\text{dg}} = (e)(a_{\text{LP}})(\Delta V_{\text{LP}}) = 0.081 \times 0.26 \text{ meV} = 21 \text{ meV} \). Dot–grain tunnelling is negligible in this limit. c, Conductance through the grain appears where expected given the interpretation of b. The conductance is measured with gates BL and BR depleted, measuring through the two point contacts formed by gate pairs LBB/BWB and LBT/BWT.