Policy Improvement from Multiple Experts

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Abstract

Despite its promise, reinforcement learning’s real-world adoption has been hampered by its need for costly exploration to learn a good policy. Imitation learning (IL) mitigates this shortcoming by using an expert policy during training as a bootstrap to accelerate the learning process. However, in many practical situations, the learner has access to multiple suboptimal experts, which may provide conflicting advice in a state. The existing IL literature provides a limited treatment of such scenarios. Whereas in the single-expert case, the return of the expert’s policy provides an obvious benchmark for the learner to compete against, neither such a benchmark nor principled ways of outperforming it are known for the multi-expert setting. In this paper, we propose the state-wise maximum of the expert policies’ values as a natural baseline to resolve conflicting advice from multiple experts. Using a reduction of policy optimization to online learning, we introduce a novel IL algorithm MAMBA, which can provably learn a policy competitive with this benchmark. In particular, MAMBA optimizes policies by using a gradient estimator in the style of generalized advantage estimation (GAE). Our theoretical analysis shows that this design makes MAMBA robust and enables it to outperform the expert policies by a larger margin than IL state of the art, even in the single-expert case. In an evaluation against standard policy gradient with GAE and AggreVaTeD, we showcase MAMBA’s ability to leverage demonstrations both from a single and from multiple weak experts, and significantly speed up policy optimization.

1 Introduction

Reinforcement learning (RL) promises to bring self-improving decision-making capability to many applications, including robotics [1], computer systems [2], recommender systems [3] and user interfaces [4]. However, deploying RL in any of these domains is fraught with numerous difficulties, as vanilla RL agents need to do a large amount of trial-and-error exploration before discovering good decision policies [5]. This inefficiency has motivated investigations into training RL agents with domain knowledge, an example of which is having access to expert policies in the training phase.

The broad class of approaches that attempt to mimic or improve upon an available expert policy is known as imitation learning (IL) [6]. Generally, IL algorithms work by invoking an expert policy through demonstrations to guide an RL agent towards promising states and actions as per the expert. As a result, expert-level performance can be achieved without global exploration, thus avoiding RL’s main source of high sample complexity. For IL with a single expert policy, the expert policy’s return provides a natural benchmark for the agent to match or outperform. Most existing IL techniques assume this single-expert setting, with a good but possibly suboptimal expert policy. Behavior cloning [7] learns a policy from a fixed batch of trajectories in a supervised way by treating expert actions as labels. Inverse reinforcement learning uses recorded expert trajectories to infer the expert’s reward function [8–11]. Interactive IL [12, 13] assumes the learner can actively ask an expert policy for a demonstration starting at the learner’s current state. When reward information of the original RL problem is available, IL algorithms can outperform the expert policy [14–16].
In this paper, we ask the question: how should an RL agent leverage domain knowledge encoded in more than one (potentially suboptimal) expert policies available to the learner? We study this question in the aforementioned interactive IL setting. Having multiple expert policies is quite common in practice. For instance, consider the problem of minimizing task processing delays via load-balancing a network of compute nodes. Existing systems and their simulators have a number of human-designed heuristic policies for load balancing that can serve as experts [18]. Likewise, in autonomous driving, available experts can range from PID controllers to human drivers [19]. In these examples, each expert has its own strengths and can provide desirable behaviors for different situations.

Intuitively, because more expert policies can provide more information about the problem domain, the RL agent can learn a good policy faster than using a single expert. However, in reality, the agent does not know the properties of each expert. What it sees is instead the conflicting demonstrations from the expert policies. Resolving this disagreement can be non-trivial, because there may not be a single expert comprehensively outperforming the rest, and the quality of each expert policy is unknown. Recently, several IL and RL works have started to study this practically important class of scenarios. InfoGAIL [20] conditions the learned policy on latent factors that motivate demonstrations of different experts. AC-Teach [21] models each expert with a set of attributes and relies on a Bayesian approach to decide which action to take based on their demonstrations. OIL [19] tries to identify and follow the best expert in a given situation.

However, all existing approaches to IL from multiple experts sidestep two fundamental questions. What is a reasonable benchmark for policy performance in these settings, analogous to the single-expert policy quality in conventional IL? Is there a systematic way to stitch together several suboptimal experts into a stronger baseline that we can further improve upon?

We provide answers to these questions, making the following contributions:

1. We identify the state-wise maximum of expert policies’ values as a natural benchmark for learning from multiple experts. We call it the max-aggregated baseline and propose policy improvement from it as a natural strategy to combine these experts together, which creates a new policy that is uniformly better than all the experts in every state. These insights establish the missing theoretical foundation for designing algorithms for IL with multiple experts.

2. We propose a novel IL algorithm MAMBA (Max Aggregation of Multiple Baselines) to learn a policy that is competitive with the max-aggregated baseline by a reduction of policy optimization to online learning [13, 22]. MAMBA is a first-order algorithm based on a new IL gradient estimator designed in the spirit of generalized advantage estimation (GAE) [17] in the RL literature. Like some prior works in IL, MAMBA interacts with the expert in a roll-in/roll-out format [13, 15] and does not assume access to expert actions.

3. We provide regret-based performance guarantees for MAMBA. In short, MAMBA generalizes a popular single-expert IL algorithm AggreVaTeD [13, 14] to learn from multiple experts and to achieve larger improvements from suboptimal experts. Empirically, we evaluate MAMBA against the IL baseline (AggreVaTeD [14]) and direct RL (GAE policy gradient [17]). Fig. 1 highlights the experimental results, where MAMBA demonstrates the capability to bootstrap demonstrations from multiple weak experts to significantly speed up policy optimization.
2 Background: Episodic Interactive Imitation Learning

Markov decision processes (MDPs). We consider finite-horizon MDPs with state space \( \mathcal{S} \) and action space \( \mathcal{A} \). Let \( T, d_0(s), \mathcal{P}(s'|s,a), \) and \( r : \mathcal{S} \times \mathcal{A} \to [0,1] \) denote the problem horizon, the initial state distribution, the transition dynamics, and the reward function, respectively. We assume that \( d_0, \mathcal{P}, \) and \( r \) are fixed but unknown. Given a class of state-dependent policies \( \Pi \), our goal is to find a policy \( \pi \in \Pi \) that maximizes the \( T \)-step return with respect to the initial state distribution \( d_0 \):

\[
V^\pi(d_0) := \mathbb{E}_{s_0 \sim d_0} \mathbb{E}_{a_0 \sim \pi(s_0)} \left[ \sum_{t=0}^{T-1} r(s_t,a_t) \right],
\]

where \( \rho^\pi(s_t|s_0) \) denotes the distribution over trajectory \( \xi_t = s_1, \ldots, s_{T-1}, a_{T-1} \) generated by running policy \( \pi \) starting from the state \( s_0 \) at time 0 to the problem horizon. To compactly write down non-stationary processes, we structure the state space \( \mathcal{S} \) as \( \mathcal{S} \times \{0, T-1\} \), where \( \mathcal{S} \) is some basic state space; thus, \( \mathcal{P} \) and \( r \) can be non-stationary in \( \mathcal{S} \). We allow \( \mathcal{S} \) and \( \mathcal{A} \) to be either discrete or continuous. We use the subscript of \( t \) to emphasize the time index. When writing \( s_t \) we assume it is at time \( t \), and every transition from \( s_t \) to \( s_{t+1} \) via \( \mathcal{P}(s_{t+1}|s_t, a_t) \) increments the time index by 1.

State distributions and value functions. We let \( d_T^\pi \) stand for the state distribution at time \( t \) induced by running policy \( \pi \) starting from \( d_0 \) (i.e. \( d_T^\pi = d_0 \) for any \( \pi \)), and define the average state distribution as \( d_T := \frac{1}{T} \sum_{t=0}^{T-1} d_t^\pi \). Sampling from \( d_T \) returns \( s_t \), where \( t \) is uniformly distributed. Therefore, we can re-cast a policy’s \( T \)-step return in (1) as \( V^\pi(d_0) = T \mathbb{E}_{s \sim d_T} \mathbb{E}_{a \sim \pi(s)} [r(s,a)] \). With a slight abuse of notation, we denote by \( V^\pi : \mathcal{S} \to \mathbb{R} \) as the value function of policy \( \pi \), which satisfies \( V^\pi(d_0) = \mathbb{E}_{s \sim d_T} [V^\pi(s)] \). Given a function \( f : \mathcal{S} \to \mathbb{R} \) such that \( f(s_T) = 0 \), we define the \( Q \)-function w.r.t. \( f \) as \( Q^f(s,a) := r(s,a) + \mathbb{E}_{s' \sim \mathcal{P}|s,a} [f(s')] \) and the advantage function w.r.t. \( f \) as

\[
A^f(s,a) := Q^f(s,a) - f(s) = r(s,a) + \mathbb{E}_{s' \sim \mathcal{P}|s,a} [f(s')] - f(s) \tag{2}
\]

When \( f = V^\pi \), we also write \( A^V = A^\pi \) and \( Q^V = Q^\pi \), which are the standard advantage and \( Q \)-functions of a policy \( \pi \). We write \( f \)’s advantage function under a policy \( \pi \) as \( A^f(s,\pi) := \mathbb{E}_{a \sim \pi(s)} [A^f(s,a)] \) and similarly \( Q^f(s,\pi) \) and \( f(d) := \mathbb{E}_{s \sim d} [f(s)] \) given a state distribution \( d \). We refer to functions that index \( Q \) or \( A \) functions as baseline value functions, because we aim to improve upon the value they provide in each state.

Definition 1. We say a baseline value function \( f \) is improvable w.r.t \( \pi \) if \( A^f(s,\pi) \geq 0 \), \( \forall s \in \mathcal{S} \).

Policy optimization with multiple expert policies. The setup above describes a generic episodic RL problem, where the agent faces the need to perform strategic exploration and long-term credit assignment. A common approach to circumvent the exploration challenge in practice is by leveraging an expert policy. In this paper, we assume access to multiple (potentially suboptimal) expert policies during training, and leverage episodic interactive IL to improve upon them. We suppose that the learner (i.e. the agent) has access to a set of expert policies \( \Pi^e = \{\pi^k\}_{k \in [K]} \). During training, the learner can interact with the experts in a roll-in-roll-out (RIRO) paradigm to collect demonstrations. In each episode, the learner starts from an initial state sampled from \( d_0 \) and runs its policy \( \pi \in \Pi \) up to a switching time \( t_e \in [0, T-1] \); then the learner asks an expert policy \( \pi^k \in \Pi^e \) to take over and finish the trajectory. At the end, the learner records the entire trajectory, including reward information. Note that we do not assume that expert actions are observed. In addition, as sampled rewards are available here, the learner can potentially improve upon the expert policies.

3 A Conceptual Framework for Learning from Multiple Experts

In this paper, we focus on the scenario where the set \( \Pi^e = \{\pi^k\}_{k \in [K]} \) contains more than one expert policy. Having multiple expert policies offers an opportunity to gain more information about the problem domain. Each expert may be good at different situations, so the learner can query suitable experts at different states for guidance. But how exactly can we leverage the information from multiple experts to learn more efficiently than from any single one of them?

Some natural baselines. One approach for leveraging multiple experts is to combine them into a single expert, such as by using a fixed weighted mixture [23], or multiplying their action probabilities
in each state \([24]\). But the first attempt can be quite bad even if only one expert is bad, and the second fails to combine two deterministic experts. Another alternative is to evaluate each expert and run a single-expert IL algorithm with the one with the highest return. In Appendix A, however, we show an example where two experts have identical returns, but switching between them results in the optimal behavior. We wonder if there is a general principle for combining multiple experts. For instance, if we seek to switch amongst the multiple experts, natural questions are how such a switching point should be chosen and whether we can learn a rule for doing so reliably.

In this section, we show that the issues mentioned above can be addressed by performing policy improvement upon the state-wise maximum over the experts’ values, i.e. the max-aggregated baseline. We describe two conceptual algorithms: one based on the perfect knowledge of the MDP and the experts’ value functions, while the other builds on this using online learning to handle an unknown MDP and expert values in the interactive IL setup. The insights gained from these two conceptual algorithms will be used to design their practical variation, MAMBA, in Section 4.

3.1 Max Aggregation with Policy Improvement

To illustrate the key idea, let us first suppose that perfect knowledge of the MDP and the experts’ value functions is available. In this idealized setting, the IL problem can be rephrased as follows: find a policy that is at least as good as all the expert policies in complexity independent of the problem horizon. The restriction on the complexity is important; otherwise we can just use the MDP knowledge to construct the optimal policy of the problem.

How do we solve this idealized IL problem? When there is only a single expert in \(\Pi^e\), say \(\pi^e\), a natural solution is the policy given by one-step policy improvement from \(\pi^e\), i.e. the policy \(\pi^e N\) that acts according to \(\arg\ max_{s,a} A(s,a) + E_{s',d^e} [V^N(s')]\). It is well known that this policy \(\pi^e N\) is uniformly better than \(\pi^e\) for all the states, i.e. \(V^e N(s) \geq V^e (s)\) (cf. [25] and Corollary 1 below). However, this basic approach no longer applies when multiple experts are in \(\Pi^e\), and some direct efforts to invoke a single-expert algorithm do not work in general as discussed earlier.

A max following approach A simple way to remedy the failure mode of uniformly mixing the expert policies is to take a non-uniform mixture that is aware of the quality of each expert. If we have the value function of each expert, we have a natural measure of their quality. With this intuition, for the \(k\)-th expert policy \(\pi^k\) in \(\Pi^e\), let us write \(V^k = V^e N\). A natural candidate policy based on this idea is the greedy policy that follows the best expert in any given state:

\[
\pi^*(a|s) := \pi^{k_s}(a|s), \quad \text{where} \quad k_s := \arg\ max_{k \in [K]} V^k(s)
\]  

Imitating a benchmark similar to \(\pi^*\) was recently proposed as a heuristic for IL with multiple experts in [19]. Our first contribution is a theoretical result showing that the intuition behind this heuristic holds mathematically: \(\pi^*\) indeed satisfies \(V^\pi^*(s) \geq \max_{k \in [K]} V^k(s)\). To show this, we construct a helper corollary based on the useful Performance Difference Lemma (Lemma 1)\(^2\).

**Lemma 1.** [26, 27] Let \(f : S \rightarrow \mathbb{R}\) be such that \(f(s_T) = 0\). For any MDP and policy \(\pi\), it holds

\[
V^\pi(d_0) - f(d_0) = T \mathbb{E}_{s \sim d^\pi} [A^f(s, \pi)].
\]  

**Corollary 1.** If \(f\) is improvable w.r.t. \(\pi\), then \(V^\pi(s) \geq f(s), \forall s \in S\).

Corollary 1 implies that a policy \(\pi\) has a better performance than all the experts in \(\Pi^e\), if there is a baseline value function \(f\) that is improvable w.r.t. \(\pi\) (i.e. \(A^f(s, \pi) \geq 0\)) and dominates the value functions of all expert policies everywhere (i.e. \(f(s) \geq V^k(s), \forall k \in [K], s \in S\)).

This observation suggests a natural value baseline for studying IL with multiple experts:

\[
f^{\max}(s) := \max_{k \in [K]} V^k(s).
\]  

Below we prove that this max-aggregated baseline \(f^{\max}\) in (5) is improvable with respect to \(\pi^*\). Together with Corollary 1, this result implies that \(\pi^*\) is a valid solution to the idealized IL problem with multiple experts. We write the advantage \(A^{\max}\) with respect to \(f^{\max}\) in short as \(A^{\max}\).

**Proposition 1.** \(f^{\max}\) in (5) is improvable with respect to \(\pi^*\), i.e. \(A^{\max}(s, \pi^*) \geq 0\).

\(^1\)These approaches are proposed for supervised learning and not specifically IL.

\(^2\)Lemma 1 is an adaptation of the standard Performance Difference Lemma to using \(f\) that is not necessarily the value function of any policy. We provide proofs of Lemma 1 and Corollary 1 in Appendix C for completeness.
A degeneracy of max following. The policy $\pi^*$ above, however, suffers from a degenerate case: when there is one expert in $\Pi^e$ that is uniformly better than all the other experts (say $\pi^e$), we have $\pi^* = \pi^e$, whereas we know already $\pi^e$ is a uniformly better policy that we can construct using the same information. In this extreme case, the $\pi^*$ would simply return the suboptimal expert back in the standard IL setting with one expert.

A max aggregation approach. Having noticed the failure mode of $\pi^*$, we obtain a natural fix by combining the same value baseline (5) with the standard policy improvement operator. We define

$$
\pi^{\text{max}}(s|a) := \delta_a = a_s, \quad \text{where} \quad a_s := \arg\max_{a \in \mathcal{A}} A^{\text{max}}(s, a),
$$

and $\delta$ denotes the delta distribution. In contrast to $\pi^*$, $\pi^{\text{max}}$ looks one-step ahead and takes the action with the largest advantage under $f^{\text{max}}$, which is not necessarily the same as following highest-valued expert in the current state. Since $\pi^{\text{max}}$ satisfies $A^{\text{max}}(s, \pi^{\text{max}}) \geq A^{\text{max}}(s, \pi^*) \geq 0$, by Corollary 1, $\pi^{\text{max}}$ is also a valid solution to the idealized IL problem with multiple experts.

The use of $\pi^{\text{max}}$ is novel in IL to our knowledge, though $\pi^{\text{max}}$ is called Multiple Path Construction Algorithm in controls [28, Chapter 6.4.2]. Corollary 1 provides a simple proof of why $\pi^{\text{max}}$ works.

In general, $V^{\pi^{\text{max}}}(s)$ and $V^{\pi^*}(s)$ are not comparable. But, crucially, in the degenerate case above we see that $\pi^{\text{max}}$ reduces to $\pi^e$, and therefore would perform better than $\pi^*$, though in Appendix A we also show an MDP where $\pi^*$(s) is better. Intuitively, this happens as $f^{\text{max}}$ implicitly envisions using a single expert for the remaining steps, but both $\pi^*$ and $\pi^{\text{max}}$ re-optimize their expert choice at every step whereby their relative quality can be arbitrary. While both $\pi^*$ and $\pi^{\text{max}}$ improve upon all the experts, in this paper, we choose $\pi^{\text{max}}$ as our imitation benchmark, because it is consistent with prior works in the single-expert case and does not require observing the experts’ actions in IL unlike $\pi^*$.

3.2 Max Aggregation with Online Learning

The previous section shows that improving from the max-aggregated baseline $f^{\text{max}}$ in (5) is a key to reconciling the conflicts between expert policies. However doing so requires the knowledge of the MDP and the experts’ value functions, which are unavailable in the episodic interactive IL setting.

To compete with $f^{\text{max}}$ yet without the assumption above, we design an IL algorithm by a reduction to online learning [29], a technique used in many prior works in the single-expert setting [12–16, 30]. To highlight the main idea, we first assume that the experts’ value functions are still given, but only the MDP is unknown. We then show how to handle unknown value functions. For clarity, we use the subscript in $\pi_n$ to index the learner policy in $\Pi$ generated in the $n$-th round of online learning, while using the superscript in $\pi^k$ to index the expert policy in $\Pi^e$.

Ideal setting with known values. If the MDP dynamics and rewards are unknown, we can treat $d^{\pi_n}$ as the adversary in online learning and define the online loss in the $n$-th round as

$$
\ell_n(\pi) := -\mathbb{E}_{s \sim d^{\pi_n}} [A^{\text{max}}(s, \pi)].
$$

By Lemma 1, making $\ell_n(\pi_n)$ small ensures that $V^{\pi_n}(d_0)$ is not much worse than $f^{\text{max}}(d_0)$. Formally, averaging this argument over $N$ rounds of online learning, we obtain

$$
\frac{1}{N} \sum_{n=1}^N V^{\pi_n}(d_0) = f^{\text{max}}(d_0) + \Delta_N - \epsilon_N(\Pi) - \frac{\text{Regret}_N}{N},
$$

where we define Regret$_N := \sum_{n=1}^N \ell_n(\pi_n) - \min_{\pi \in \Pi} \sum_{n=1}^N \ell_n(\pi)$,

$$
\Delta_N := \frac{1}{N} \sum_{n=1}^N \ell_n(\pi^{\text{max}}), \quad \text{and} \quad \epsilon_N(\Pi) := \min_{\pi \in \Pi} \frac{1}{N} \left( \sum_{n=1}^N \ell_n(\pi) - \sum_{n=1}^N \ell_n(\pi^{\text{max}}) \right).
$$

The regret characterizes the learning speed of an online algorithm, while $\epsilon_N(\Pi)$ captures the quality of the policy class. If $\pi^{\text{max}} \in \Pi$, then $\epsilon_N(\Pi) = 0$; otherwise, $\epsilon_N(\Pi) \geq 0$. Furthermore, we have $\Delta_N \geq 0$ because we showed $A^{\text{max}}(s, \pi^{\text{max}}) \geq 0$ in Section 3.1. Thus, when $\pi^{\text{max}} \in \Pi$, running a no-regret algorithm to solve this online learning problem will guarantee producing a policy whose performance at least $\mathbb{E}_{s \sim d_0} \max_{k \in [K]} |V^k(s)| + \frac{\Delta_N}{N} + o(N)$ after $N$ rounds.

The above reduction in (7) generalizes AggreVaTE [13] from using $f = V^{\pi^{\text{max}}}$ in $A^f$ to define the online loss for the single expert case to $f = f^{\text{max}}$ that is also applicable to multiple experts. When an expert in $\Pi^e$ dominates the others, (7) is the same as the online loss in AggreVaTE.
We propose MAMBA as a practical realization of the first-order reduction idea in Theorem 1 (shown in (1)). As discussed, obtaining a good sample estimate of (10) is nontrivial. As a workaround, we design MAMBA based on an alternate online loss estimates of the form (10).

Algorithm 1 MAMBA for IL with multiple experts

Input: Initial learner policy \(\pi_1\), expert polices \(\{\pi^k\}_{k \in [K]}\), function approximators \(\{\hat{V}^k\}_{k \in [K]}\).

Output: The best policy in \(\{\pi_1, \ldots, \pi_N\}\).

1: for \(n = 1, \ldots, N - 1\) do
2: \quad Uniformly sample \(t_n \in [T - 1]\) and \(k \in [K]\).
3: \quad Roll-in \(\pi_n\) up to \(t_n\) and switch to \(\pi^k\) to complete the remaining trajectory to collect data \(D_n\).
4: \quad Update \(\hat{V}^k\) using \(D_n\) (e.g. using Monte-Carlo estimates).
5: \quad Roll-in \(\pi_n\) for the full \(T\)-horizon to collect data \(D_n'\).
6: \quad Compute the sample estimate \(g_n\) of \(\nabla \hat{\ell}_n(\pi; \lambda)\) (14) using \(D_n'\) and \(\hat{f}_{\max}(s) = \max_{k \in [K]} \hat{V}^k(s)\).
7: \quad Update \(\pi_n\) to \(\pi_{n+1}\) by giving \(g_n\) to a first-order online learning algorithm (e.g. mirror descent).
8: end for

Effect of approximate expert values

Recall that for the above derivation we assumed expert policy values (and hence \(f_{\max}\)) are given. In practice, \(f_{\max}\) is unavailable and needs to be approximated by some \(\hat{f}_{\max}\). Let \(\hat{A}\) denote the shorthand of \(A^{\hat{f}_{\max}}\). We can treat the approximation error as bias and variance in the gradient signal, such as the sample estimate of the gradient below

\[
\nabla \hat{\ell}_n(\pi) = -T E_{s \sim \pi^*} E_{a \sim \pi(s)} \left[ \nabla \log \pi(a|s) \hat{A}(s, a) \right],
\]

(10)

where \(\nabla\) is with respect to the policy. We summarize the approximation effects as a meta theorem.

Theorem 1. Suppose a first-order online algorithm that satisfies Regret \(R_N \leq O(\beta N + \sqrt{\nu N})\) is adopted, where \(\beta\) and \(\nu\) are the bias and the variance in the gradient, respectively. Then

\[
E[\max_{n \in [N]} V^{\pi_n}(d_0)] \geq E_{s \sim d_0} \left[ \max_{k \in [K]} V^k(s) \right] + E[\Delta N - \epsilon N(\Pi)] - O(\beta + \sqrt{\nu N^{-1/2}})
\]

(11)

where the expectation is over the randomness in feedback and the online algorithm.

Theorem 1 describes considerations of using \(\hat{f}_{\max}\) in place of \(f_{\max}\). For the single-expert case, \(\hat{f}_{\max}\) can be an unbiased Monte-Carlo estimate (i.e. \(\nabla \hat{\ell}_n = \nabla \ell_n\)) but the sample estimate of \(\nabla \ell_n(\pi_n)\) suffers from a variance that is \(T\)-times larger than the Monte-Carlo estimate of policy gradient. Using function approximators [14] as \(f_{\max}\) shifts the variance from the gradient estimate to learning \(\hat{f}_{\max}\). In this case, (10) becomes akin to the actor-critic policy gradient. But when the accuracy of the value estimate \(\hat{f}_{\max}\) is bad, the bias in (10) can also compromise the policy learning.

For the multi-expert case, unbiased Monte-Carlo estimates of \(f_{\max}\) are infeasible, because \(f_{\max}(s) = V^{\pi^k}(s)\) and \(\pi^k\) is unknown (i.e. we do not know the best expert policy at state \(s\)). Therefore, \(\hat{f}_{\max}\) in (10) must be a function approximator. But, due to the max operator in \(f_{\max}\), learning \(\hat{f}_{\max}\) becomes challenging as all the experts’ value functions need to be approximated uniformly well.

4 MAMBA: An Approach for Improving upon Multiple Experts

We propose MAMBA as a practical realization of the first-order reduction idea in Theorem 1 (shown in Algorithm 1). As discussed, obtaining a good sample estimate of (10) is nontrivial. As a workaround, we will design MAMBA based on an alternate online loss \(\ell_n(\pi; \lambda)\) that shares the same property as \(\ell_n(\pi)\) in (7) but has a gradient expression with tunable bias-variance trade-off. Specifically, for \(\lambda \in [0, 1]\), we define the online loss in the \(n\)-th round alternatively as

\[
\ell_n(\pi; \lambda) := - (1 - \lambda) T E_{s \sim \pi^*} [A^{\max, \pi}_{\lambda}(s, \pi)] - \lambda E_{s \sim d_0} [A^{\max, \pi}_{\lambda}(s, \pi)]
\]

(12)

where we define a \(\lambda\)-weighted advantage

\[
A^{\max, \pi}_{\lambda}(s, a) := (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i A^{\max, \pi}(s, a)
\]

(13)

by combining various \(i\)-step advantages:

\[
A^{\max, \pi}_{(i)}(s_t, a_t) := E_{s_{t+i} \sim \pi^*}[r(s_t, a_t) + \ldots + r(s_{t+i}, a_{t+i}) + f_{\max}(s_{t+i+1})] - f_{\max}(s_t).
\]

\(^3\)As \(V^{\pi^k}\) is not the value of \(\pi_n\) but \(\pi^k\), computing an unbiased estimate of \(\nabla \ell_n(\pi_n)\) requires uniformly selecting the switching time \(t_e \in \{0, \ldots, T - 1\}\) in the RIRO setting, which amplifies the variance by \(O(T)\).
When there is a single expert (i.e. \( f^{\text{max}} = V^\pi \)), we can interpret the online loss (12) in terms of known IL objectives. In (12), the first term is the \( \lambda \)-weighted version of the AggreVaTe loss [13], and the second term is the \( \lambda \)-weighted version of the THOR loss [31] (but these prior IL algorithms did not use \( \lambda \) weighted advantages.)\(^4\) The \( \lambda \) parameter in (12) controls the strength of the online adversary: When \( \lambda = 0 \) (i.e. AggreVaTe), we view the state distribution \( d^n \) as an adversary. In another extreme, when \( \lambda = 1 \) (i.e. the original RL), the adversary \( d^n \) disappears and (12) is the same for every round.

While the online loss \( \ell_n(\pi; \lambda) \) in (12) appears complicated, interestingly, its gradient \( \nabla \ell_n(\pi; \lambda) \) has a very clean expression: in Theorem 2 below, we prove that approximating \( f^{\text{max}} \) with \( \hat{f}^{\text{max}} \) in the gradient \( \nabla \ell_n(\pi; \lambda) \) leads to a gradient estimator:

\[
\nabla \hat{\ell}_n(\pi; \lambda) = -T \mathbb{E}_{s \sim d^n} \mathbb{E}_{a \sim \pi_n}[\nabla \log \pi(a|s) \hat{A}^\pi_\lambda(s, a)],
\]

where \( \hat{A}^\pi_\lambda \) is defined by replacing \( f^{\text{max}} \) in (13) with the approximation \( \hat{f}^{\text{max}} \). The gradient estimator in (14) is reminiscent of the GAE for policy gradient [17] but now it is now applied to IL.

At a high level, we see that \( \nabla \hat{\ell}_n(\pi; \lambda) \) in (14) replaces \( \hat{A} \) in \( \nabla \hat{\ell}_n(\pi) \) in (10) with the \( \lambda \)-weighted version \( \hat{A}^\pi_\lambda \) to achieve tunable bias-variance trade-off. Controlling \( \lambda \) regulates the effects of the error \( \hat{f}^{\text{max}} - f^{\text{max}} \) on the difference \( \hat{A}^\pi_\lambda - A^\pi_\lambda \), similar to the properties of the GAE policy gradient [17]. We recover\(^4\) the gradient in (10) when \( \lambda = 0 \) and the policy gradient when \( \lambda = 1 \).

But we emphasize that \( \nabla \hat{\ell}_n(\pi; \lambda) \) in (14) is not an approximation of \( \nabla \ell_n(\pi) \) in (10), because generally \( \nabla \ell_n(\pi_n; \lambda) \neq \nabla \ell_n(\pi_n) \) even when \( \hat{f}^{\text{max}} = f^{\text{max}} \), except for \( f^{\text{max}} = V^\pi_n \) (the policy gradient). Therefore, while the GAE policy gradient [17] is an approximation of the policy gradient, \( \nabla \ell_n(\pi) \) and \( \nabla \hat{\ell}_n(\pi; \lambda) \) are gradient approximations of different online loss functions in (7) and (12).

We justify the validity of learning with \( \nabla \ell_n(\pi_n; \lambda) \) in (14). We remark that the first equality part of the theorem below holds for any baseline value function \( f \), not just limited to \( f^{\text{max}} \).

**Theorem 2.** For \( \lambda \in [0, 1] \), it holds that \( V^\pi_n(d_0) - f^{\text{max}}(d_0) = -\ell_n(\pi_n; \lambda) \) and \( \nabla \ell_n(\pi; \lambda) \) is given by the expression in (14) with \( \hat{f}^{\text{max}} \) replaced by \( f^{\text{max}} \). Consequently, performing online learning w.r.t. (12) has the guarantee in Theorem 1, where \( \Delta_N \geq 0 \) and \( \epsilon_N(\Pi) \) can be negative when \( \lambda > 0 \).

Theorem 2 shows that learning with (14) has a similar performance guarantee to using (10) in Theorem 1, but with one important exception: now \( \epsilon_N(\Pi) \) can be negative (which is in our favor), because \( \pi^{\text{max}} \) may not be the best policy for the multi-step advantage in \( \ell_n(\pi; \lambda) \) when \( \lambda > 0 \). This means that, when using \( \lambda > 0 \) in MAMBA, larger improvements can be made from the expert polices.

5 Experiments and Discussion

We corroborate our theoretical discoveries with simulations of IL from multiple experts. We compare MAMBA with two representative algorithms: GAE Policy Gradient [17] (denoted as PG-GAE with \( \lambda = 0.9 \)) for direct RL and AggreVaTeD [14] for IL with a single expert. Because we can view these algorithms as different first-order oracles for policy optimization, comparing their performance allows us to study two important questions: 1) whether the proposed GAE-style gradient in (14) is an effective update direction for IL and 2) whether using multiple experts helps the agent learn faster.

To facilitate a meaningful comparison, we let these three algorithms use the same first-order optimizer\(^5\), train the same initial neural network policies, and share the same random seeds. In each training iteration, an algorithm would perform 8 rollouts following the RIRO paradigm (see also Algorithm 1). Two continuous-state-and-action environments (CartPole and DoubleInvertedPendulum) in OpenAI Gym [34] based on DART physics engine [35] are used, and each expert policy here is a partially trained, suboptimal neural network. Please see Appendix D for implementation details. The codes are provided at https://github.com/chinganc/mamba.

Effects of \( \lambda \)-weighting \(^6\) First we consider in Fig. 7e the single-expert IL setting with the expert that has the highest return. We see that with the help of the expert policy, AggreVaTeD (which is MAMBA

\(^4\)We have the identities \( \hat{A}^\pi_0 = \hat{A}^\pi_{\infty} = \hat{A} \) and \( \hat{A}^\pi_{\infty} = \hat{A}^\pi_{\infty} = \hat{A}^\pi_{\infty} = A^\pi \).

\(^5\)ADAM [32] for CartPole and Natural Gradient Descent [33] for DoubleInvertedPendulum.
with $\lambda = 0$) improves faster than PG-GAE. However, while AggreVaTeD learns to significantly outperform the expert (which has a return of 89), it does not reach the optimal performance (1000), like PG-GAE. To fix this issue, we use MAMBA with $\lambda > 0$ to learn from the same suboptimal expert. As we showed in Theorem 2 using a positive $\lambda$ can increase the amount of improvement that can be made from the expert’s performance compared with $\lambda = 0$. In addition, by properties of GAE [17], we know that increasing $\lambda$ would reduce the bias due to function approximation error at the cost of increasing the gradient variance. The trend in Fig. 7e supports these theoretical intuitions, where high $\lambda = 0.9$ allows the learner to reach higher return than the low $\lambda$s. In this particular experimental setup, using the middle $\lambda = 0.5$ gives the worst performance, likely because it settles badly in the trade-off of properties. From now on, we will use $\lambda = 0.9$ in the following experiments for MAMBA as it performs the best in this test.

**Effects of multiple experts** After validating our GAE-style gradient for IL, we show the effects of using multiple experts in Figs. 2c and 7a. Here we run MAMBA with $\lambda = 0.9$ with 1 to 8 experts. We index these experts in a descending order of their performance; e.g., MAMBA-0.9-max(1) uses the best expert and MAMBA-0.9-max(2) uses the top two. Interestingly, by including more but strictly weaker experts, MAMBA starts to improve the performance of policy optimization. In CartPole (Fig. 7a), we see the performance strictly improves when more experts are used, and MAMBA is able to reach 1000 by IL from 8 weak experts (recall that the best expert has a return of only 89). In DoubleInvertedPendulum (Fig. 2c), we see a similar trend, but the improvement speed of MAMBA slows down when using more than 4 experts. This is because DoubleInvertedPendulum is a harder domain for exploration than CartPole; although using more experts can potentially yield higher performance, the learner also needs to spend more time to learn the experts’ value functions.

**Summary** We conclude this paper by revisiting Fig. 1, which showcases the best multi-expert settings in Figs. 2c and 7a and an additional heuristic extension of MAMBA that replaces the max-aggregated baseline ($f_{\text{max}}$ in line 6 of Algorithm 1) with a mean-aggregated baseline (i.e. $\frac{1}{K} \sum_k V_k$). Overall these results support the benefits of IL from multiple experts and the new GAE-style IL gradient in (14). Interestingly we found that replacing the max-aggregated baseline with the mean-aggregated baseline in MAMBA can also improve the results from AggreVaTeD, which uses the single best expert. While our current theoretical results do not fully explain this phenomena, a plausible hypothesis is the use of GAE-style gradient and that we are averaging values, instead of actions. Therefore, as long as most experts have similar values in a state, the mean-aggregated baseline can still provide a meaningful direction for policy improvement.

In conclusion, this paper proposes a novel theoretical foundation and algorithm MAMBA for IL with multiple expert policies. We study how the conflicts between different experts can be resolved through the max-aggregated baseline and propose a new GAE-style gradient for the IL setting, which can also be used to improve the robustness and performance of existing single-expert IL algorithms. We provide regret-based theoretical guarantees on MAMBA and demonstrate its properties empirically. The experimental results show that MAMBA is able to improve upon multiple, very suboptimal expert policies to achieve the optimal performance, faster than both the pure RL method (PG-GAE [17]) and the single-expert IL algorithm (AggreVaTeD [14]).
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Appendix

A MDP Examples for Section 3

Problem with selecting experts based on initial value. In the example of Figure 3, each expert $\pi_\ell$ and $\pi_r$, individually gets same the suboptimal reward of $1/2$. Alternatively, we can switch between the experts once to get a reward of $3/4$, and twice to get the optimal reward of $1$.

Figure 3: An example MDP for illustrative purposes. All terminal states not shown give a reward of 0 and intermediate states have no rewards. Two expert policies $\pi_\ell$ and $\pi_r$ choose the left and right actions respectively in each state. The optimal terminal state is outlined in bold.

Ordering of $\pi^*$ and $\pi^{\text{max}}$. Consider the example MDP of Figure 3 again. In the state $s_0$, the policy $\pi^*$ selects the expert with largest value in $s_0$ and goes left (suppose we adopt a tie breaking rule that chooses the left action over the right action when there is a tie). It subsequently selects the right expert in $s_1$ and left in $s_4$ to get the optimal reward. $\pi^{\text{max}}$ on the other hand chooses between the left and right actions in $s_0$ based on $f^{\text{max}}(s_1) = 0.7$ and $f^{\text{max}}(s_2) = 3/4$. Consequently it goes right and eventually obtains a suboptimal reward of $3/4$. In this case, we see that $\pi^*$ is better than $\pi^{\text{max}}$. On the other hand, if we swap the rewards of $s_7$ and $s_{11}$, then $\pi^*$ chooses the right action in $s_0$ and gets a suboptimal reward. Further swapping the rewards of $s_9$ and $s_{10}$ makes $\pi^{\text{max}}$ pick the left action in $s_0$ and it eventually reaches the optimal reward. This illustrates clearly that $\pi^{\text{max}}$ and $\pi^*$ are incomparable in general.

B Additional Notes on Related Work

Several prior works proposed empirical approaches to IL settings with multiple experts. InfoGAIL [20] is an extension of GAIL [36] that aims at automatically identifying semantically meaningful latent factors that can explain variations in demonstrations across experts. It assumes that demonstrations come from a mixture-of-experts policy, where each demonstration is generated by sampling a value of the latent factors from a prior and using it to condition an expert’s action choices. InfoGAIL tries to recover this expert mixture. In contrast, MAMBA can be viewed as choosing actions based on learned estimates of experts’ value functions without imitating any single expert or their mixture directly. In multi-modal IL [37], latent factors conceptually similar to those in InfoGAIL correspond to different skills being demonstrated by the experts, and Tamar et al. [38]’s approach focuses on settings where these factors characterize different experts’ intentions. OIL [19] is more similar to MAMBA: like MAMBA, it uses individual expert policies’ state values to decide on an action in a given state. However, OIL does so by using the best-performing expert in a given state as the learner’s “critic” and doesn’t justify its approach theoretically.

At least two algorithms have used a Bayesian approach to decide which expert to trust in a multiple-expert setting. AC-Teach [21] models each expert with a set of attributes and relies on a Bayesian approach to decide which action to take based on their demonstrations. Gimelfarb et al. [39] assume that experts propose reward functions, some of which are inaccurate, and uses Bayesian model...
We observe that where the last step follows since \( \pi \). Theorem 1.

Corollary 1. If \( C.2 \) Proof of Corollary 1

Combing the two equalities shows the result.

Because Lemma 1 holds for any MDP, given the state \( s \) and properly adapt the problem horizon. Then Corollary 1 follows directly from applying Lemma 1 to this new MDP.

C.3 Proof of Proposition 1

Proposition 1. \( f^{\text{max}} \) in (5) is improvable with respect to \( \pi^\star \), i.e. \( A^{\text{max}}(s, \pi^\star) \geq 0 \).

Proof. Let us recall the definition (3) of \( k_s \) and let us assume without loss of generality that \( k_s = 1 \). We observe that

\[
A^{\text{max}}(s, \pi^\star) = r(s, \pi^\star) + E_{a \sim \pi^\star} E_{s' \sim P_s} [f^{\text{max}}(s')] - f^{\text{max}}(s)
\]

\[
\geq r(s, \pi^\star) + E_{a \sim \pi^\star} E_{s' \sim P_s} [V^1(s')] - V^1(s) = A^{V^1}(s, \pi^1) \geq 0
\]

where the last step follows since \( \pi^1(a|s) = \pi^k(a|s) = \pi^1(a|s) \) and the advantage of a policy with respect to its value function is always 0.

C.4 Proof of Theorem 1

Theorem 1. Suppose a first-order online algorithm that satisfies \( \text{Regret}_N \leq O(\beta N + \sqrt{\nu N}) \) is adopted, where \( \beta \) and \( \nu \) are the bias and the variance in the gradient, respectively. Then

\[
E[\max_{n \in [N]} V^{\pi^a}(d_0)] \geq E_{s \sim d_0} \left[ \max_{k \in [K]} V^k(s) \right] + E[\Delta_N - \epsilon_N(\Pi)] - O(\beta + \sqrt{\nu N}^{-1/2})
\]

where the expectation is over the randomness in feedback and the online algorithm.

Proof. By using (8) and the assumption on the first-order algorithm, we can write

\[
E \left[ \frac{1}{N} \sum_{n \in [N]} V^{\pi^a}(d_0) \right] = f^{\text{max}}(d_0) + E \left[ \Delta_N - \epsilon_N(\Pi) - \frac{\text{Regret}_N}{N} \right]
\]

\[
\geq f^{\text{max}}(d_0) + E \left[ \Delta_N - \epsilon_N(\Pi) \right] - O \left( \beta + \sqrt{\nu N} \right)
\]

Finally, using \( \frac{1}{N} \sum_{n \in [N]} V^{\pi^a}(d_0) \leq \max_{n \in [N]} V^{\pi^a}(d_0) \) and the definition of \( f^{\text{max}} \), we have the final statement.
C.5 Proof of Theorem 2

**Theorem 2.** For \( \lambda \in [0, 1] \), it holds that \( V^{\pi_n}(d_0) - f^{\max}(d_0) = -\ell_n(\pi_n; \lambda) \) and \( \nabla \ell_n(\pi; \lambda) \) is given by the expression in (14) with \( \hat{f}^{\max} \) replaced by \( f^{\max} \). Consequently, performing online learning w.r.t. (12) has the guarantee in Theorem 1, where \( \Delta_N \geq 0 \) and \( \epsilon_N(\Pi) \) can be negative when \( \lambda > 0 \).

**Proof.** The first equality part of the theorem holds for any baseline value function \( f : S \to \mathbb{R} \) beyond the special choice \( f^{\max} \) of interest here. To state more general version, let us first formally extend the definition of (13) to use a general baseline value function \( f \),

\[
A^f_{\lambda}(s, a) := (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i A^f_{(i)}(s, a)
\]

where we define the \( i \)-step advantage accordingly as

\[
A^f_{(i)}(s_t, a_t) := \mathbb{E}_{t\sim\rho} [r(s_t, a_t) + \cdots + r(s_{t+i}, a_{t+i}) + f(s_{t+i+1})] - f(s_t).
\]

We see that \( A^{\lambda, \pi}_{\max} \) in (13) is a special case of \( A^f_{\lambda, \pi} \) in (15) when \( f = f^{\max} \).

The proof of Theorem 2 is based on two non-trivial technical propositions of this general \( \lambda \)-weighted advantage.

**Proposition 2** (\( \lambda \)-weighted Performance Difference Lemma). For any policy \( \pi \), any \( \lambda \in [0, 1] \) and any baseline value function \( f : S \to \mathbb{R} \), it holds

\[
V^{\pi}(d_0) - f(d_0) = (1 - \lambda)T \mathbb{E}_{s\sim d^\pi} \left[ A^f_{\lambda}(s, \pi) \right] + \lambda \mathbb{E}_{s\sim d_0} \left[ A^f_{\lambda}(s, \pi) \right]
\]

**Proposition 3.** For any \( \lambda \in [0, 1] \), any baseline value function \( f : S \to \mathbb{R} \), any policy \( \pi \), and the average state distribution \( d^\pi \) of a policy \( \mu \), define

\[
\ell(\pi; \lambda) := (1 - \lambda)T \mathbb{E}_{s\sim d^\pi} \left[ A^f_{\lambda}(s, \pi) \right] + \lambda \mathbb{E}_{s\sim d_0} \left[ A^f_{\lambda}(s, \pi) \right]
\]

Then it holds

\[
\nabla \ell(\pi; \lambda)|_{\mu=\pi} = T \mathbb{E}_{s\sim d^\pi} \mathbb{E}_{a\sim\pi|s} \nabla \log \pi(a|s) A^f_{\lambda}(s, a)
\]

We shall delay the proofs of Propositions 2 and 3 to the later part of this section. For now suppose they are true. Then setting \( f = f^{\max} \) in Propositions 2 and 3 proves the first part of Theorem 2: \( V^{\pi_n}(d_0) - f^{\max}(d_0) = -\ell_n(\pi_n; \lambda) \) and \( \nabla \ell_n(\pi; \lambda) \) is given by the expression in (14) with \( \hat{f}^{\max} \) replaced by \( f^{\max} \).

To prove the second part, we can then write down an equality like (8) by the equality \( V^{\pi_n}(d_0) - f^{\max}(d_0) = \ell_n(\pi_n; \lambda) \) we just obtained:

\[
\frac{1}{N} \sum_{n \in [N]} V^{\pi_n}(d_0) = f^{\max}(d_0) + \Delta_N - \epsilon_N(\Pi) - \frac{\text{Regret}_N}{N}
\]

where Regret, \( \Delta_N \), and \( \epsilon_N(\Pi) \) are now defined with respect to the \( \lambda \)-weighted online loss \( \ell_n(\pi; \lambda) \). Therefore, running a no-regret algorithm with respect to the approximate gradient (14) of this online loss function \( \ell_n(\pi; \lambda) \) would imply a similar performance guarantee shown in Theorem 1 (see the proof Theorem 1).

Finally, to justify the use of (14), what remains to be shown is that the term \( \Delta_N - \epsilon_N(\Pi) \) behaves similarly as before. First we notice that, because \( \pi^{\max} \) may not be the best policy for the multi-step advantage in the online loss \( \ell_n(\pi; \lambda) \), \( \epsilon_N(\Pi) \) now be negative (which is in our favor). Next we show that \( \Delta_N \geq 0 \) is true by Proposition 4 (whose proof is again postponed). These results conclude the proof of Theorem 2.

**Proposition 4.** It holds \(-\ell_n(\pi^{\max}; \lambda) \geq 0\).
C.5.1 Proof of Proposition 2

Proposition 2 (λ-weighted Performance Difference Lemma). For any policy \( \pi \), any \( \lambda \in [0, 1] \) and any baseline value function \( f : \mathcal{S} \to \mathbb{R} \), it holds

\[
V^\pi(d_0) - f(d_0) = (1 - \lambda) T \mathbb{E}_{s \sim d^\pi} \left[ A^{f, \pi}_{\lambda}(s, \pi) \right] + \lambda \mathbb{E}_{s \sim d_0} \left[ A^{f, \pi}_{\lambda}(s, \pi) \right]
\]

(17)

Proof. The proof is uses a new generalization of the Performance Difference Lemma (Lemma 1), which we state generally for non-Markovian processes. A similar equality holds for the infinite-horizon discounted problems.

Lemma 2 (Non-even performance difference lemma). Let \( \pi \) be a policy and let \( f \) be any function that is history dependent such that \( \mathbb{E}_{h_T \sim d_T} [f(h_T)] = 0 \). Let \( \tau_0, \tau_1, \tau_2, \ldots, \tau_I \) be monotonically increasing integers where \( \tau_0 = 0 \) and \( \tau_I = T \). For any non-Markovian decision process, it holds that,

\[
V^\pi(d_0) - f(d_0) = \sum_{k=0}^{l-1} \mathbb{E}_{h_{\tau_k} \sim d_{\tau_k}} \left[ A^{f, \pi}_{\lambda}(h_{\tau_k}, \pi) \right]
\]

where \( i_k = \tau_{k+1} - \tau_k - 1 \).

Proof of Lemma 2. By definition,

\[
V^\pi(d_0) = \sum_{t=0}^{T-1} \mathbb{E}_{h_t \sim d_t} [r(h_t, \pi)] = \sum_{k=0}^{l-1} \mathbb{E}_{h_{\tau_k} \sim d_{\tau_k}} \mathbb{E}_{h_{\tau_k} \sim d_{\tau_k}} \left[ \sum_{t=\tau_k}^{\tau_{k+1}-1} r(h_t, \alpha_t) \right]
\]

On the other hand, we can write \(-f(d_0) = \sum_{k=1}^{l} f(d_{\tau_k}^{-}) - \sum_{k=0}^{l-1} f(d_{\tau_k}^{+})\). Combing the two equalities shows the result.

Now return to the Markovian case. Using Lemma 2, we derive a \( \lambda \)-weighted Performance Difference Lemma (Proposition 2). A history dependent (discounted) version can be shown similarly. To simplify writing, we let \( \Theta = V^\pi(d_0) - f(d_0) \) and \( A_{(i)} = A^{f, \pi}_{\lambda}(i) \) as shorthands, and we will omit the dependency on random variables in the expectation. Using Lemma 2, we can write

\[
\Theta = \sum_{t=1, \ldots, T-1} E_{d_t} E_{\pi} [A_{(0)}]
\]

\[
2\Theta = \sum_{t=0, 2, 4, \ldots} E_{d_t} E_{\pi} [A_{(1)}] + \left( E_{d_0} E_{\pi} [A_{(0)}] + \sum_{t=1, 3, \ldots} E_{d_t} E_{\pi} [A_{(1)}] \right)
\]

\[
= E_{d_0} E_{\pi} [A_{(0)}] + \sum_{t=0}^{T-1} E_{d_t} E_{\pi} [A_{(1)}]
\]

\[
3\Theta = \sum_{t=0, 3, 6, \ldots} E_{d_t} E_{\pi} [A_{(2)}] + \left( E_{d_0} E_{\pi} [A_{(0)}] + \sum_{t=1, 4, \ldots} E_{d_t} E_{\pi} [A_{(2)}] \right)
\]

\[
+ \left( E_{d_0} E_{\pi} [A_{(1)}] + \sum_{t=2, 5, \ldots} E_{d_t} E_{\pi} [A_{(2)}] \right)
\]

\[
= E_{d_0} E_{\pi} [A_{(0)}] + E_{d_0} E_{\pi} [A_{(1)}] + \sum_{t=0}^{T-1} E_{d_t} E_{\pi} [A_{(2)}]
\]

\[
\vdots
\]

Applying a \( \lambda \)-weighted over these terms, we then have

\[
(1 - \lambda)(1 + 2\lambda + 3\lambda^2 + \ldots)\Theta = T E_{d_T} E_{\pi} \left[ (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i A_{(i)} \right] + \lambda \sum_{i=0}^{\infty} \lambda^i E_{d_0} E_{\pi} [A_{(i)}]
\]
Then it holds 
\[ \lambda \text{write the} \]
\[ \text{where in the last equality we use the fact} \]
\[ \nabla \text{implies that} \]
\[ \Theta = V^\pi(d_0) - f(d_0) = (1 - \lambda)T \mathbb{E}_{d^\pi} \mathbb{E}_\pi \left[ (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i A(i) \right] + \lambda (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i \mathbb{E}_{d_0} \mathbb{E}_\pi \left[ A(i) \right] \]

The above derivation implies that

C.5.2 Proof of Proposition 3

**Proposition 3.** For any \( \lambda \in [0, 1] \), any baseline value function \( f : S \to \mathbb{R} \), any policy \( \pi \), and the average state distribution \( d^t \) of a policy \( \mu \), define

\[ \ell(\pi; \lambda) := (1 - \lambda)T \mathbb{E}_{s \sim d^t} \left[ A^f_{\lambda}(s, \pi) \right] + \lambda \mathbb{E}_{s \sim d_0} \left[ A^f_{\lambda}(s, \pi) \right] \]

Then it holds

\[ \nabla \ell(\pi; \lambda)|_{\mu=\pi} = T \mathbb{E}_{s \sim d^t} \mathbb{E}_{a \sim \pi|s} \nabla \log \pi(a|s) A^f_{\lambda}(s, a) \]

**Proof.** We first show the gradient expression in the second term in \( \ell(\pi; \lambda) \).

**Lemma 3.**

\[ \nabla \mathbb{E}_{s \sim d_0} \left[ A^f_{\lambda}(s, \pi) \right] = \sum_{t=0}^{T-1} \lambda^t \mathbb{E}_{s_t \sim d_t^\pi} \mathbb{E}_{a_t \sim \pi|s_t} \left[ \nabla \log \pi(a_t|s_t) A^f_{\lambda}(s_t, a_t) \right] \]

**Proof of Lemma 3.** Define \( Q_{(i-t)}^{f,\pi}(s_t, a_t) := \mathbb{E}_{p^\pi|s_t, a_t} \left[ \sum_{t=0}^{i-1} r(s_{t+\tau}, a_{t+\tau}) + f(s_{i+1}) \right] \). By using the definition of \( i \)-step advantage function \( A^f_{(i)} \) in (16), we can first rewrite the desired derivative as

\[ \nabla \mathbb{E}_{s \sim d_0} \left[ A^f_{(i)}(s, \pi) \right] = \nabla \mathbb{E}_{s \sim d_0} \mathbb{E}_{p^\pi|s} \left[ r(s_0, a_0) + r(s_1, a_1) + \cdots + r(s_i, a_i) + f(s_{i+1}) \right] - f(s_i) \]

where in the last equality we use the fact \( \nabla \mathbb{E}_{a \sim \pi|s} [f(s)] = 0 \) for any \( f : S \to \mathbb{R} \). Therefore, we can write the \( \lambda \)-weighted version as follows:

\[ \nabla \mathbb{E}_{s \sim d_0} \left[ A^f_{\lambda}(s, \pi) \right] = (1 - \lambda) \sum_{i=0}^{\infty} \nabla \mathbb{E}_{s \sim d_0} \left[ \lambda^i A^f_{(i)}(s, a) \right] \]

\[ = (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i \mathbb{E}_{s_t \sim d_t^\pi} \mathbb{E}_{a_t \sim \pi|s_t} \left[ \nabla \log \pi(a_t|s_t) A^f_{(i-t)}(s_t, a_t) \right] \]

\[ = (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i \mathbb{E}_{s_t \sim d_t^\pi} \mathbb{E}_{a_t \sim \pi|s_t} \left[ \nabla \log \pi(a_t|s_t) A^f_{(i-t)}(s_t, a_t) \right] \]

\[ = \sum_{t=0}^{T-1} \lambda^t \mathbb{E}_{s_t \sim d_t^\pi} \mathbb{E}_{a_t \sim \pi|s_t} \left[ \nabla \log \pi(a_t|s_t) A^f_{\lambda}(s_t, a_t) \right] \]
With this intermediate result, we can further derive the gradient expression in the first term in $\nabla \ell(\pi; \lambda)$ when $\mu = \pi$:

$$
T \mathbb{E}_{s \sim d^n} \left[ \nabla A_{\lambda}^f(s, \pi) \right] = \sum_{t=0}^{T-1} \mathbb{E}_{s_t \sim d_t} \nabla \left[ A_{\lambda}^f(s_t, \pi) \right] = \sum_{t=0}^{T-1} \sum_{r=t}^{T-1} \lambda^{r-t} \mathbb{E}_{s_r \sim d_r} \mathbb{E}_{a_r \sim \pi|s_r} \left[ \nabla \log \pi(a_r|s_r) A_{\lambda}^f(s_r, a_r) \right] = \sum_{t=0}^{T-1} \mathbb{E}_{s_t \sim d_t} \mathbb{E}_{a_t \sim \pi|s_t} \left[ \nabla \log \pi(a_t|s_t) A_{\lambda}^f(s_t, a_t) \right] \left( \sum_{r=t}^{T-1} \lambda^{r-t} \right)
$$

Finally, combining the two equalities, we arrive at a very clean expression:

$$
\nabla \ell(\pi; \lambda)|_{\mu=\pi} = (1 - \lambda)T \mathbb{E}_{s \sim d^n} \left[ \nabla A_{\lambda}^f(s, \pi) \right] = \sum_{t=0}^{T-1} \mathbb{E}_{s_t \sim d_t} \mathbb{E}_{a_t \sim \pi|s_t} \left[ \nabla \log \pi(a_t|s_t) A_{\lambda}^f(s_t, a_t) \right] \left( (1 - \lambda) \frac{1 - \lambda^{t+1}}{1 - \lambda} + \lambda \cdot \lambda' \right)
$$

C.5.3 Proof of Proposition 4

**Proposition 4.** It holds $-\ell_n(\pi_{\text{max}}; \lambda) \geq 0$.

**Proof.** We first prove a helpful lemma.

**Lemma 4.** For $\pi_{\text{max}}$, it holds that $A_{\lambda}^\text{max}(s, \pi_{\text{max}}) \geq 0$.

**Proof of Lemma 4.** Without loss of generality, take $s = s_0$. First we arrange

$$
A_{\lambda}^\text{max}(s_0, \pi_{\text{max}}) = \mathbb{E}_{r^{\text{max}}|s_0} \left[ r(s_0, a_0) + r(s_1, a_1) + \cdots + r(s_t, a_t) + V_{\pi_{\text{max}} + 1}(s_{t+1}) \right] - V_{\pi_{\text{max}}}(s_0)
$$

where we have the inequality

$$
Q_{\text{max}}(s_i, \pi_{\text{max}}) := \mathbb{E}_{a_i \sim \pi_{\text{max}}|s_i} \left[ r(s_i, a_i) + \mathbb{E}_{s_{i+1} \sim p|s_i, a_i} \left[ V_{\pi_{\text{max}} + 1}(s_{i+1}) \right] \right] \geq \mathbb{E}_{a_i \sim \pi_{\text{max}}|s_i} \left[ r(s_i, a_i) + \mathbb{E}_{s_{i+1} \sim p|s_i, a_i} \left[ V_{\pi_{\text{max}} + 1}(s_{i+1}) \right] \right] \geq \mathbb{E}_{a_i \sim \pi_{\text{max}}|s_i} \left[ r(s_i, a_i) + \mathbb{E}_{s_{i+1} \sim p|s_i, a_i} \left[ V_{\pi_{\text{max}}}(s_{i+1}) \right] \right] \geq V_{\pi_{\text{max}}}(s_i)
$$

By applying this inequality recursively, we get

$$
A_{\lambda}^\text{max}(s_0, \pi_{\text{max}}) \geq V_{\pi_{\text{max}}}(s_0) - V_{\pi_{\text{max}}}(s_0) \geq 0
$$

The lemma above implies $A_{\lambda}^\text{max}(s, \pi_{\text{max}}) \geq 0$ for $\lambda \geq 0$ and therefore we have

$$
-\ell_n(\pi_{\text{max}}; \lambda) = (1 - \lambda)T \mathbb{E}_{s \sim d^n} \left[ A_{\lambda}^\text{max}(s, \pi_{\text{max}}) \right] + \lambda \mathbb{E}_{s \sim d_0} \left[ A_{\lambda}^\text{max}(s, \pi_{\text{max}}) \right] \geq 0
$$

\[17]
D  Experiment Details and Additional Results

In this section we describe the details of MAMBA and additional experimental results.

D.1 Implementation Details of MAMBA

MAMBA is based on running a first-order algorithm with unbiased estimates of the approximate \(\lambda\)-weighted gradient

\[
\nabla \hat{G}_{n}(\pi; \lambda) = -T E_{s \sim d_{\pi}} E_{a \sim \pi_n | s} \left[ \nabla \log \pi(a | s) \hat{A}^x_{\lambda}(s, a) \right] \tag{14}
\]

where we recall that

\[
\hat{A}^x_{\lambda}(s, a) := (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i \hat{A}^x_{\lambda}^{(i)}(s, a)
\]

\[
\hat{A}^x_{\lambda}^{(i)}(s, a) := E_{\xi_i \sim \rho_{\pi | s_i}} [r(s_t, a_t) + \cdots + r(s_{t+i}, a_{t+i}) + \hat{f}^{\max}(s_{t+i+1})] - \hat{f}^{\max}(s_t)
\]

While the above equation helps in understanding the properties of MAMBA, it does not give a useful expression for implementation because of the infinite sum. Here we provide an alternative, but equivalent, formula of \(\hat{A}^x_{\lambda}(s, a)\), by which unbiased estimates of \(\hat{A}^x_{\lambda}(s_t, a_t)\) can be computed by rolling out a single trajectory of \(\pi\).

**Lemma 5.**

\[
\hat{A}^x_{\lambda}(s_t, a_t) = E_{\xi_t \sim \rho_{\pi | s_t}} \left[ \sum_{\tau=t}^{T-1} \lambda^{\tau-t} \hat{A}(a_{\tau}, s_{\tau}) \right] \tag{20}
\]

where we recall \(\hat{A}(s, a) = r(s, a) + E_{s' | s, a} [\hat{f}^{\max}(s')] - \hat{f}^{\max}(s)\).

**Proof.** This equality can be derived as follows:

\[
\hat{A}^x_{\lambda}(s_t, a_t) = (1 - \lambda) E_{\xi_t \sim \rho_{\pi | s_t}} \left[ \sum_{i=0}^{\infty} \lambda^i \left( \sum_{\tau=t}^{T-1} r(s_{\tau}, a_{\tau}) + \hat{f}^{\max}(s_{t+i+1}) \right) \right] - \hat{f}^{\max}(s_t)
\]

\[
= E_{\xi_t \sim \rho_{\pi | s_t}} \left[ \sum_{\tau=t}^{T-1} r(s_{\tau}, a_{\tau}) \left( (1 - \lambda) \sum_{i=\tau-t}^{\infty} \lambda^i \right) + (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i \hat{f}^{\max}(s_{t+i+1}) \right] - \hat{f}^{\max}(s_t)
\]

\[
= E_{\xi_t \sim \rho_{\pi | s_t}} \left[ \sum_{\tau=t}^{T-1} \lambda^{\tau-t} r(s_{\tau}, a_{\tau}) + \sum_{\tau=t}^{T-1} (1 - \lambda) \lambda^{\tau-t} \hat{f}^{\max}(s_{\tau+1}) \right] - \hat{f}^{\max}(s_t)
\]

\[
= E_{\xi_t \sim \rho_{\pi | s_t}} \left[ \sum_{\tau=t}^{T-1} \lambda^{\tau-t} r(s_{\tau}, a_{\tau}) + \sum_{\tau=t}^{T-1} \lambda^{\tau-t} \hat{f}^{\max}(s_{\tau+1}) - \sum_{\tau=t+1}^{T-1} \lambda^{\tau-t} \hat{f}^{\max}(s_{\tau}) \right] - \hat{f}^{\max}(s_t)
\]

\[
= E_{\xi_t \sim \rho_{\pi | s_t}} \left[ \sum_{\tau=t}^{T-1} \lambda^{\tau-t} \left( r(s_{\tau}, a_{\tau}) + \hat{f}^{\max}(s_{\tau+1}) - \hat{f}^{\max}(s_{\tau}) \right) \right] - \hat{f}^{\max}(s_t)
\]

\[
= E_{\xi_t \sim \rho_{\pi | s_t}} \left[ \sum_{\tau=t}^{T-1} \lambda^{\tau-t} \hat{A}(s_{\tau}, a_{\tau}) \right]
\]

\[\blacksquare\]

With (20), we provide the details of MAMBA in Algorithm 1 below:

The above description closely follow Algorithm 1 with a few minor, practical modifications which we describe below:

---

6We describe the version that samples one trajectory per each iteration; in practice, it can be modified directly to collect multiple trajectories.
We provide additional details of the experimental setup below.

Algorithm 2: Implementation details of MAMBA for IL with multiple experts

**Input:** Initial learner policy \(\pi_1\), expert policies \(\{\pi^k\}_{k \in [K]}\), function approximators \(\{\hat{V}^k\}_{k \in [K]}\).

**Output:** The best policy in \(\{\pi_1, \ldots, \pi_N\}\).

1: For each \(k \in [K]\), collect dataset \(D^k\) by rolling out \(\pi^k\) for the full problem horizon.
2: Update value models \(\hat{V}^k = \text{MonteCarloRegression}(D^k)\) for \(k \in [K]\).
3: \(\pi_1 = \text{UpdateInputWhitening}(\pi_1, D^1)\)
4: for \(n = 1 \ldots N - 1\) do
5: Sample one trajectory using \(\pi_n\) to collect data \(D'_n\).
6: Let \(t_e = \text{SampleSwitchTime}(t_{avg}) \in [T - 1]\), where \(t_{avg}\) is the average trajectory length of \(D'_n, n=1\) (for the first iteration set \(t_{avg} = 0\)). Sample a RIRO trajectory using \(\pi_n\) and then \(\pi^k\) after \(t \geq t_e\) to collect data \(D_{\text{RIRO}}\), where \(k\) is uniformly sampled in \([K]\). If the sampled trajectory in \(D_{\text{RIRO}}\) is longer than \(t_e\), aggregate the trajectory after \(t_e\) in \(D_{\text{RIRO}}\) into \(D^k\). Otherwise, aggregate \(D_{\text{RIRO}}\) into \(D'_n\) for gradient computation.
7: Update value model \(\hat{V}^k = \text{MonteCarloRegression}(D^k)\).
8: Let \(\pi'_n = \text{UpdateInputWhitening}(\pi_n, D'_n)\) and compute the sampled gradient based on \(D'_n\) with one-step importance sampling as

\[
g_n = - \sum_{t=0}^{T-1} \nabla \log \pi'_n(a_t|s_t) \pi'_n(a_t|s_t) \left( \sum_{\tau=0}^{T-1} \lambda^{T-t} \left( r(s_t, a_t) + \hat{f}^\text{max}(s_{\tau+1}) - \hat{f}^\text{max}(s_\tau) \right) \right)
\]

where the recursion starts with \(\hat{V}^k(s_T) = 0\) and \(\hat{f}^\text{max}(s) = \max_{k \in [K]} \hat{V}^k(s)\).
9: Update the policy \(\pi_{n+1} = \text{MirrorDescent}(\pi'_n, g_n)\).
10: end for

- The UpdateInputWhitening function keeps a moving average of the first and the second moment of the states it has seen, which is used to provide whitened states (by subtracting the estimated mean and dividing by the estimated standard deviation) as the input to the neural network policy. When UpdateInputWhitening is called, the perceived feature of the neural network policies change.
- In Algorithm 2, \(t_c = \text{SampleSwitchTime}(t_{avg})\) samples \(t_c\) based on a geometric distribution whose mean is \(t_{avg}\), because in the learner might not always be able to finish the full \(T\) steps. If the agent unfortunately still cannot finish up \(t_c\), we use that trajectory for the gradient computation in the iteration.

Apart from these two changes, Algorithm 2 follows the pseudo-code in Algorithm 1.

We provide additional details of the experimental setup below.

- Time is appended as a feature in addition to the raw state, i.e. \(s = (t, \tilde{s})\).
- The policy is a Gassian with mean modeled by a \((128, 128)\) FCNN (fully connected neural network), and the standard deviation is diagonal, learnable and independent of the state. The value function is estimated by a \((256, 256)\) FCNN. In these FCNNs, the activation functions are \(\tanh\) except the last layer is linear. The policy and the value networks are parameterized independently.
- MonteCarloRegression performs least-squares regression by first whitening the input and then performing 100 steps of ADAM with a batchsize of 128 samples and step size 0.001. The target is constructed by Monte-Carlo estimate and one-step importance sampling.
- MirrorDescent is set to be either ADAM [32] or Natural Gradient Descent (NGD) [33]. We adopt the default hyperparameters of ADAM \((\beta_1 = 0.9, \beta_2 = 0.99)\) and a stepsize 0.001. For NGD, we adopt the ADAM-style adaptive implementation described in [40, Appendix C.1.4] using \(\beta_2 = 0.99\) and a stepsize of 0.1.
- \(D^k\) is limited to data from the past 100 iterations.
- All MAMBA, PG-GAE, and AggreVaTeD follow the protocol in Algorithm 2. In the pre-training phase (lines 1-3), each expert policy (or the learner policy for PG-GAE) would perform 16 rollouts to collect the initial batch of data to train its value function estimator. In every iteration, each algorithm would perform 8 rollouts: For MAMBA and AggreVaTeD, 4 rollouts are used to collect data for updating the value functions (line 6) and 4 rollouts (line
5) are for computing the gradient. For PG-GAE, all 8 rollouts are performed by the learner; they are just used to compute the gradient and then update the value function (so there is no correlation).

- Additional 8 rollouts are performed to evaluate the policy’s performance, which generate the plots presented in the paper.

- Both MDPs, CartPole and DoubleInvertedPendulum, have a problem horizon of 1000 steps and continuous states and actions. For CartPole, the dimensions of the state and the action spaces are 4 and 1, respectively. For DoubleInvertedPendulum, the dimensions of the state and the action spaces are 8 and 1, respectively. The maximal return for both cases are 1000 and 10000, respectively.

Computing Infrastructure  All the experiments were conducted using Azure Standard F64s v2, which was based on the Intel Xeon Platinum 8168 processor with 64 cores (base clock 3.4 GHz; turbo clock 3.7 GHz) and 128 GB memory. No GPU was used. The operating system was Ubuntu 18.04.4 LTS. The prototype codes were based on Python and Tensorflow 2. Using a single process in the setup above, a single seed of the CartPole experiment (100 iterations) took about 27 minutes to 45 minutes to finish, whereas a single seed of the DoubleInvertedPendulum experiment (200 iterations) took about 110 minutes to 125 minutes to finish.

Hyperparameter Selection  We only performed a rough search of the step sizes of ADAM (0.01 vs 0.001) and Natural Gradient Descent (0.1 vs 0.01). We conducted experiments with different λ and number of experts in order to study their effects on MAMBA. The main paper presents the results of the pilot study: we first investigated the effect of λ by comparing MAMBA with AggreVaTeD and concluded with a choice of λ = 0.9; using this λ value, we then studied the effects of the number of experts. For completeness, we present and discuss results of all the hyperparameters below.

D.2 Additional Experimental Results of Hyperparameter Effects

In this section, we include in Figs. 4 to 7 additional experimental results of CartPole and DoubleInvertedPendulum environments. The purpose of these extra results is to provide a more comprehensive picture of the properties of MAMBA under various hyperparameter settings.

Setup  For each of the environments (CartPole and DoubleInvertedPendulum), we conduct experiments with bad experts (Figs. 4 and 6) and mediocre experts (Figs. 5 and 7), where the results of the bad experts are the ones presented in the main paper. In each experiment, we run MAMBA with λ = 0, 0.1, 0.5 and 0.9, and with the number of experts varying between 1, 2, 4 and 8. In addition, we run AggreVaTeD with each of the experts (whereas the main paper only presents the results of the experts with the highest return). Finally, for baselines, we include the learning curve of PG-GAE as well as the return of each expert. Recall that the experts are indexed in a descending order of their returns, which are estimated by performing 8 rollouts.

D.2.1 Brittleness of AggreVaTeD

First, the experiments of AggreVaTeD highlight that performing IL via policy improvement\(^7\) from the best expert (in terms of the return) does not always lead to the best learner policy, as consistently shown in Figs. 4 to 7. In general there is no upper bound on the amount of performance change that policy improvement can make, because the policy improvement operator is myopic, only looking one step ahead. As a result, running AggreVaTeD with the best expert does not always give the best performance that can be obtained with an expert chosen in the hindsight. Another cause of the differences between the best foresight and hindsight experts is that the return of each expert is only estimated by 8 rollouts here.

Our experimental results show that such sensitivity is reduced in MAMBA: even in the single-expert setting, using a \(\lambda > 0\) in MAMBA generally leads to more robust performance than AggreVaTeD using the same, best expert, which is MAMBA with \(\lambda = 0\). We should remark that this robustness is not fully due to the bias-variance trade-off [17], but also largely attributable to the incorporation of the multi-step information in online loss in (12) (cf. Theorem 2). By using \(\lambda > 0\), MAMBA can

\(^7\)AggreVaTeD is an approximate policy improvement algorithm [14].
start to see beyond one-step improvement and becomes less dependent on the expert quality. In the
experiments, we observe by picking a large enough $\lambda$, MAMBA with a single expert usually gives
comparable if not better performance than AggreVaTeD with the best policy chosen in the hindsight.

D.2.2 Effects of $\lambda$-weighting

Beyond the single-expert scenario discussed above, we see consistently in Figs. 4 to 7 that using a
non-zero $\lambda$ improves the performance of MAMBA. The importance of $\lambda$ is noticeable particularly
in setups with bad experts, as well as in the experiments with the higher-dimensional environment
DoubleInvertedPendulum. Generally, when the experts are bad (as in Figs. 4 and 6), using a larger
$\lambda$ provides the learner a chance to outperform the suboptimal experts as suggested by Theorem 2,
because the online loss function in (12) starts to use multi-step information. On the other hand, when
the top experts’ performance is better and the state space is not high-dimensional, as in CartPole with
mediocre experts in Fig. 5, the effects of $\lambda$ is less prominent. The usage of $\lambda > 0$ also helps reduce
the dependency on function approximation error, which is a known GAE property [17], as we see in
the experiments with DoubleInvertedPendulum in Figs. 6 and 7.

---

Figure 4: Performance of the best policies in CarlPole with bad experts. (a)-(d) MAMBA with $\lambda =
0, 0.1, 0.5, 0.9$ (e) AggreVaTeD with different experts. (f) The return of each expert policy. A curve shows an
algorithm’s median performance across 8 random seeds. The center and right figures use the same line colors for
all methods. The shaded area shows 25th and 75th percentiles.
Figure 5: Performance of the best policies in CarlPole with mediocre experts. (a)-(d) MAMBA with $\lambda = 0, 0.1, 0.5, 0.9$ (e) AggreVaTeD with different experts. (f) The return of each expert policy. A curve shows an algorithm’s median performance across 8 random seeds. The center and right figures use the same line colors for all methods. The shaded area shows 25th and 75th percentiles.

D.2.3 Effects of multiple experts

Using more than one experts generally lead to better performance across Figs. 4 to 7. In view of Theorem 2, using more experts can improve the quality of the baseline value function, though at the cost of having a higher bias in the function approximators (because more approximators need to be learned). We see that the benefit of using more experts particularly shows up when higher values of $\lambda$ are used; the change is smaller in the single-expert settings.

However, in the settings with mediocre experts in Figs. 5 and 7, increasing the number of experts beyond a certain threshold degrades the performance of MAMBA. Since a fixed number of rollouts are performed in each iteration, having more experts implies that the learner would need to spend more iterations to learn the expert value functions to a fixed accuracy. In turn, this extra exploration reflects as slower policy improvement. Especially, because using more experts here means including strictly weaker experts, this phenomenon is visible, e.g., in Fig. 7.

D.3 Additional Experimental Results of Expert Ordering

In all the previous experiments, we order the experts based on the their performance in terms of their return. However, these return estimates are only empirical and do not always correspond to the true ordering of the experts, as we discussed in Appendix D.2.1. To study the robustness to expert
selection, here we randomly order the experts before presenting them to the IL algorithms (MAMBA and AggreVaTeD) and repeat the controlled experiment of testing the effects of $\lambda$-weighting and the number of experts in Fig. 2. The results of random expert ordering are presented in Fig. 8; because of this extra randomness we inject in expert ordering, we use more seeds in these experiments. First, we see in Fig. 8a that using the random ordering degrades of the performance of the single-expert setup. This is reasonable because there is a high chance of selecting an extremely bad expert (see Fig. 4 for the expert quality). Nonetheless, the usage of $\lambda > 0$ still improves the performance: the learning is faster and converges to higher performance, though it is still slower than PG-GAE because of the extremely bad experts.

But interestingly once we start to use multiple experts in Fig. 8b, MAMBA starts to significantly outperform AggreVaTeD and PG-GAE. By using more than one expert, there is a higher chance of selecting one reasonable expert in the set filled with bad candidates. In addition, the diversity of expert behaviors also helps to strengthen the baseline value function (cf. Theorem 2). Thus, overall we observe that MAMBA with $\lambda > 0$ and multiple experts yields the most robust performance.
Figure 7: Performance of the best policies in DoubleInvertedPendulum with mediocre experts. (a)-(d) MAMBA with $\lambda = 0, 0.1, 0.5, 0.9$ (e) AggreVaTeD with different experts. (f) The return of each expert policy. A curve shows an algorithm’s median performance across 8 random seeds. The center and right figures use the same line colors for all methods. The shaded area shows 25th and 75th percentiles.

Figure 8: Performance of the best policies with random orderings of experts in CartPole with bad experts. (a) shows the single-expert setup comparing MAMBA with different $\lambda$ values. (b) show MAMBA with different number of experts ($\lambda = 0.9$). A curve of MAMBA and AggreVaTeD shows the performance across 32 random seeds. The curves of PG-GAE shows the performance across 8 random seeds. The shaded area shows 25th and 75th percentiles.