Estimation and Optimization for System Availability Under Preventive Maintenance

YASHPAL SINGH RAGHAV1, MRADULA2, RAHUL VARSHNEY2, UMAR MUHAMMAD MODIBBO3,4, ABDULLAH ALI H. AHMADINI1, AND IRFAN ALI3

1Department of Mathematics, Jazan University, Jazan 45142, Saudi Arabia
2Department of Statistics, School of Physical and Decision Sciences, Babasaheb Bhimrao Ambedkar University, Lucknow 226025, India
3Department of Statistics and Operations Research, Aligarh Muslim University, Aligarh 202002, India
4Department of Statistics and Operations Research, Modibbo Adama University, Yola 2076, Nigeria

Corresponding author: Irfan Ali (irfii.st@amu.ac.in)

ABSTRACT Engineers design systems to be reliable and work to fulfil their missions without failure for a specific period. However, the system components deteriorate with time and lead to its failures. A frequent system failure increases the management costs, hence posing a challenge to decision-makers. Therefore, for the avoidance of frequent system failures, preventive maintenance is necessary. The objective of any manufacturing firm is to maximize profit and minimize costs. The interval for preventive maintenance can be optimized if the system’s availability is maximized and its cost function minimized. This study evaluates the availability and cost function for a continuous operating series-parallel system under a fixed time environment. A multiobjective model is formulated to maximize the availability and minimize the cost function of the system. The study illustrated a numerical example and solved using goal programming (GP), fuzzy goal programming (FGP), genetic algorithm (GA), and particle swarm optimization (PSO) techniques. The results are compared using a robust statistical test and, the PSO proves to be better. A simulation study was carried out further to evaluate the availability and cost function using R and MATLAB packages.

INDEX TERMS Availability constraints, cost function, exponential distribution, series-parallel system, particle swarm optimization, preventive maintenance policy.

I. INTRODUCTION
System’s reliability and availability play an essential role in many functional processes like industrial systems, power plants, cloud technology structures, telecommunication networks, manufacturing systems, etc. The availability of the system is considered as an important factor for optimization design chosen for such systems which consists of higher reliable components. A reliable structural design of a system or its components, are critical to improve the reliability of such systems at a higher level. Formulating the appropriate mathematical programming model of such scenario has been useful in the system reliability determination. Thus the objective of such problems is to increase the system’s availability with some constraints like time, weight, cost, etc. Therefore, several studies have been carried out in this area. Cox [1] has explained the conditions for the finite optimum solution and [2] suggested the best solution to minimize the expected cost. Wang [3] focused on the series-parallel system and [4] dealt with personal computer design in reliability optimization. Klutke et al. [5] derived the availability by exogenous random environment for an inspected system. Also, they defined a relationship between remaining life, deterioration and repair by the Markovian method. One of the most contributing factors to workout availability is the failure rate of component. According to [6] availability of extensive data partly led to the creation of predictive maintenance. In the literature of availability, there are two types of the failure rates of components which are considered in general. One is the constant failure rate and other is the time-dependent failure rate; these are mainly based on lifetime distributions like exponential distribution, Weibull distribution, etc. When
the failure rates of components remains constant, then the reliability function may be obtained by using appropriate statistical relationship.

According to the periodic inspection policy, Kiessler et al. [7] proposed the non-self-announcing failures in which a Markov chain governs the rate of deterioration for lifetime distribution. Cui and Xie [8] carried out the study by considering two models, in first one the system is taken to be as good as new after the end of the evaluation or repair, while in the other system is considered when no maintenance or corrective action requires at the time of inspection if the system is still running. Also, the state of the system is assumed to be same as before the inspection. Under the presumption of random repair or replacement time, both models are presented. Several authors have worked on repairable series-parallel system such as [9], [10], [11], and [12], etc. After that, [13] and [14] have used the semi Markov approach to analyze the steady-state availability of repairable mechanical systems with opportunistic maintenance. Wang and Pham [15], [16] proposes a quasi renewal process and discusses its applications in maintenance theory. They used the method to obtain the expected maintenance cost rate and availability.

According to [17], more than 40 mathematical models for imperfect maintenance have been studied over the last 30 years. Erkoyuncu et al. [18] examined a process to estimate the performance which was based on supporting contract costs and attributed to corrective maintenance. Li et al. [19] studied an equipment fault prediction and joining equipment maintenance decision in urban transportation using big data and internet of things concept. Ivezic et al. [20] defined availability by using the time coefficient, which includes the time state with a fuzzy expert model. After that [21] used Markov models for availability and reliability function when the failure, imperfect repair and replacement rates are available. Also, they presented a retrial system with mixed standby and unreliable repair facility for availability. In this strategy, [21] obtained mean time to failure of the system by using the Laplace transform technique.

Moreover, Mellal et al. [22] has also suggested the cuckoo optimization technique for repairable systems to find out the minimum cost. The problem is formulated by considering the constraints of availability requirement and involved three aspects as component failure rates, repair rates and redundancy allocation. Further, [23] worked on the steady-state availability of the system for general time distributions. Zheng et al. [24] proposed the availability measures for smart electric power grid systems and analyzed parametric sensitivities by using binary decision diagrams for fault trees. [25] studied selective maintenance allocation problem as a bi-level programming using the nonlinear optimization.

Many authors published works on optimization problems with different techniques in recent times. Several reliability and maintenance models were suggested and discussed with their optimal approaches [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37], [38]. Functional dependency has been considered in remaining useful life prediction and predictive maintenance strategies for multi-state manufacturing systems [39]. The authors applied the proposed model in serial manufacturing system and concluded that the systems can simultaneously complete production tasks with high quality product, and reduce the maintenance cost in the production cycle. A novel evaluation methodology combining Markov model and dynamic Bayesian networks has been developed to assess systems’ resilience under various fixed external disasters [40]. Norelfath et al. [41] proposed a combined method based on Markov processes, GA and universal moment generating function to calculate the availability of the multistate system. Reference [42] discussed a methodology to solve the multiobjective reliability optimization model. In their study, the parameters of model are considered imprecise in triangular interval data. They converted the uncertain multiobjective optimization model to a deterministic form and used PSO and GA to solve these problems. Garg [43] proposes PSO-GA as a hybrid technique for solving constrained non-linear optimization problem. Adhikary et al. [44] used a multiobjective GA to solve a series-parallel system with a preventive maintenance (PM) scheduling model that does not provide PM with an off-working time. Wang et al. [45] uses a numerical algorithm and the PSO to derive an optimal imperfect PM interval considering maximal two-dimensional warranty product availability.

Many techniques are available to solve optimization problems but PSO technique gives the appropriate, convergent and feasible solutions in comparison to other techniques. It is computationally inexpensive in terms of time, memory and speed. Due to its flexibility, the PSO technique is used instead of other available techniques. PSO is considered as a potential competitor to other promising techniques like GA and GP techniques, etc. This paper proposed availability and cost model for series-parallel systems with components which are periodically inspected and managed subject to some maintenance strategy. The objective is to optimize the maintenance policy for each part of a program and maximizing the availability limit’s cost function. The solution procedures are explained by PSO technique. A comparative study is also included by considering some other optimization techniques. Series and parallel systems structures are the most widely and fundamentally used in representing systems structures in a classical reliability theory [46]. Next subsections discusses series, parallel, and series-parallel systems briefly as an over view because of their usefulness in the theory of reliability.

A. SERIES SYSTEM
According to [47], a series system is one of the most important and common systems in reliability theory and applications. In a series system, the components or subsystems are arranged and connected in series, and all must work before the system function. In other words, if a component fails, then the system fails as well. In a series system, the total failure rates of its components equal the system’s failure rate. Similarly, the series system’s lifetime equal to a minor lifetime of its components. It is the simplest form of a system.
The function of a series system is given as:

\[ \xi(x) = \prod_{i=1}^{n} x_i = \min\{x_1, x_2, \ldots, x_n\}. \]  (1)

**B. PARALLEL SYSTEM**

Unlike a series system, a parallel system is one that the components or subsystems are arranged and connected in parallel. The system fails only if all its components are not functional. It works if at least one component is working. In a series system, there is only one way for the system to work properly, hence it does not require redundancy. Unlike the parallel system, there are \(2^n - 1\) ways the system can work differently; hence a redundancy can be added to strengthen the system structure. The function is given in Eqn. (2).

\[ \xi(x) = \left(1 - \prod_{i=1}^{n} (1 - x_i)\right) = \max\{x_1, x_2, \ldots, x_n\}. \]  (2)
C. SERIES-PARALLEL SYSTEM

This system consists of \( m \) disjoint subsystems (modules) connected in series and each module \( i \) (subsystem) has components of \( n_i \), connected in parallel. Subsystems can be identical and independently distributed with same size or otherwise. The availability function is giving in Eqn. (3).

\[
A_s(T) = \prod_{i=1}^{n} \left( 1 - (1 - A_i)^{m_i} \right) \quad (3)
\]

\[
A_i = \frac{MTBF}{MTTR + MTBF} \quad (4)
\]

where \( A_s(T) \) is the system availability at period \( T \), and \( A_i \) is the \( i^{th} \) subsystem availability which is a function of mean time to repair (MTTR) and mean time between failures (MTBF), given in Eq. (4).

Note: The components can be independent or identical and independently distributed (i.i.d). However, in this study it is assumed to be i.i.d with the same module sizes. Also, for illustration purpose, the failure rates assumed to follow an exponential distribution.

The article is organized as follows: Section I introduces the work and discusses its background. Some relevant literature are reviewed and summarized in Table 1 of Section II. Section III discusses the methodology and techniques used in this research. In Section V, the availability and cost functions of the series-parallel system are discussed and the problem formulated. Section VI illustrated the formulated model numerically. Section VII discusses the results and managerial implications, and the article is concluded in Section VIII. A further direction for future research also highlighted.

II. LITERATURE REVIEW AND RESEARCH GAP

This section reviewed and presented the related work in series-parallel system. GP technique and its variants such as FGP have been used widely in several discipline for decision-making processes, ranging from financial management, social science to engineering design. For more details see [48], [49], [50], [51], [52], [53], [54], [55], [56], and [57]. Similarly, the GA has gained a huge concentration as a metaheuristic algorithm in solving complex and hard problems. A number of researchers applied the GA methodology in a variety of areas ranging from lecture time-tabling problem, supply chain management, to system reliability and availability analysis in and science and engineering. As a reference, refer to some publications on GA as [58], [59], [60], [61], [62], [63], [64], [65], [66], [67]. As GA based its concept on the evolutionary algorithms. PSO on the other hand based its concept on the social behaviour of some animals such as bees and fish etc. and are generally termed as nature inspired algorithm. It has been applied continously in many areas of human endeavours, ranging from healthcare system, education, military, engineering, actuarial sciences, artificial intelligence and robotics. Some applications of PSO appeared in [68], [69], [70], [72], [73], [74], [75], [76], [77], [78], and [79]. The optimization techniques, the rates of failures such as recurrence, constants, time-dependent, repair time, etc., and the concepts used in the system reliability and availability studies as discussed in Section I are summarized in compacted form in Table 1. The research gap and contribution of the present study can be identified from the Table at a glance. Also, list of some notations and abbreviations used in this paper are presented in a Tabular form in II-A

A. LIST OF SOME NOTATIONS AND ABBREVIATIONS

System Parameters:

\[
A_s(T) : \text{ is the system availability at time } T;
\]

\[
C_{\text{max}} : \text{ is the maximum allowable system cost};
\]

\[
T_s : \text{ is the system time}
\]

\[
T_{\text{max}} : \text{ is the total maximum time.}
\]

\[
N_1(T) : \text{ Represents the number of failures occurring during } (0, t)
\]

\[
N_2(T) : \text{ Represents the number of non failed units occurring within } (0, t).
\]

\[
F(t) : \text{ Identical Failure distribution}
\]

\[
AIC : \text{ Akaike’s information criterion}
\]

\[
BIC : \text{ Bayesian information criterion}
\]

\[
MTTR : \text{ Mean time to repair}
\]

\[
MTBF : \text{ Mean time between failures}
\]

\[
WRV : \text{ is the worst reliability value of the system;}
\]

Decision Variables:

\[
A_i: \text{ is the } i^{th} \text{ subsystem availability}
\]

\[
C_i(T_0, T) : \text{ is the cost value at time } T;
\]

III. METHODOLOGY

This section presents and discusses the optimization techniques used in solving the availability problems in this study. They include PSO, GA, GP and FGP. It also discusses some lifetime distributions for which some availability functions data follows in real-life situations. The method of estimating parameters of the distribution are also presented.

A. ESTIMATION AND BEST-FITTING PROCEDURE FOR PROBABILITY DISTRIBUTIONS

System availability is the probability that a system can perform its mission within a specific time frame without failure [28]. In other words, it is a probability that such a system will not fail in a given time. In a real-life engineering problem such as design and preventive maintenance, data related to the failure rate function, mean time to system failure (MTSF), the median time to system failure (MdTSF), mean time between failures (MTBF), mean time to repair (MTTR), etc., follows a particular lifetime distribution. The failure rate can be constant or time-dependent, and they usually are estimated based on the distribution they follow through simulation studies. Before a particular distribution is selected, the data can be fit to observe which probability distribution best fit the data. For so doing, Akaike’s information criterion (AIC) and Bayesian information criterion (BIC) techniques help identify the best-fitted model for the failure data set. The
| Authors | Rates | Description | Solving Method |
|---------|-------|-------------|----------------|
| [1]     | Recurrence times | Probabilistic models of failure and strategies of replacement | Poisson process |
| [2]     | Constant failure rate | Mathematical model for reliability | Renewal theory process |
| [3]     | Failures, phased mission systems | Availability demonstration and estimation | Petri nets and dynamic FTA |
| [4]     | Constant failure rate | Computer design model for reliability optimizing | GA |
| [5]     | Constant failure rate | Stationary availability | Markovian method |
| [26]    | Constant repair time | Reliability optimization of a series system | Branch and bound algorithm |
| [90]    | Availability of a regularly inspected system under maintenance | Limiting average availability | |
| [7]     | Non self announcing deficiencies for the rate of degradation | Markov chain control | |
| [9]     | Birnbaum importance factor | Minimized under given availability constraint | GA |
| [8]     | Constant repair rate | Instantaneous and the steady-state availability | |
| [10]    | Markov Models | Availability for repairable parallel systems | Continuous time markov chain |
| [11]    | Constraints as random for repair and replace time | System availability with probabilistic maintenance time | Chance constrained programming |
| [12]    | Equivalent of the availability of multiple designs of a repairable series parallel device | | Warm and cold duplication methods |
| [13]    | Constant rate | Steady-state availability with repairable mechanical systems | Markov model GA |
| [44]    | Continuous operating series systems | Availability and preventive maintenance scheduling | Multiobjective GA |
| [81]    | Minimizes the execution time and maximizes the reliability | BAT algorithm | |
| [44]    | Time dependent | Reliability of the system | Benchmark problem |
| [18]    | Maintenance and availability at the level of equipment form | Benchmarking | |
| [82]    | Three types of failures | Availability and cost functions | Fuzzy model |
| [20]    | Fuzzy expert model for availability evaluation | | Lagrange multiplier, penalty function methods |
| [21]    | General distribution | Availability and reliability of a parallel system | Markov theory |
| [83]    | Constant failure rate | Steady state availability of repairable series-parallel system | Efficient cuckoo optimization algorithm |
| [22]    | Redundancy allocation, failure, and repair rates | Availability and cost of repairable systems | Artificial bee colony |
| [84]    | General time | Availability for general time distribution | PSO |
| [85]    | Soft and hard failure model | System availability modelling | |
| [86]    | Constant failure and repair rate | Availability equivalence and repairable bridge network system | Laplace transforms and the Crammer’s rule. |
| [23]    | Constant failure rate | Availability for a stochastic system | PSO, GA, GP and FGP |

**TABLE 1. Literature review.**
AIC measures the quality of the statistical model relative to each other. Because whenever data are plotted on a particular distribution, some information losses due to error; therefore, AIC try to estimate the relative amount of loss information and select the model having a minimum loss as the best model. The typical AIC model can be computed as:

\[ AIC = 2k - 2 \log(\hat{L}) \]  

(5)

On the other hand, the BIC is used for best statistical model selection based on likelihood function like AIC; however, it resolves data overfitting in a particular distribution by introducing a more significant parameter than AIC. The BIC model is computed as

\[ BIC = k \log n - 2 \log(\hat{L}) \]  

where, 

\[ \hat{L} \] is the maximized likelihood function value for the model.

\[ n \] is the sample size (number of data points)

\[ k \] the number of parameters in the model under consideration to be estimated freely.

The model(s) that has a minimum value(s) of the AIC and BIC is selected as the best-fitted data.

In the literature, failure time data are mainly analysed using the proportional reversed hazard rate (PRHR) function proposed by [27] and the generalized life distribution model [Eqn. (8)]. According to [28], the cumulative distribution function (CDF) of the PRHR model is:

\[ F(x) = [G(x)]^\theta, \quad \theta > 0, \]  

(7)

where, the baseline distribution function is denoted as \( G(x) \) and the unknown parameter with \( \theta > 0 \). Several well-known distributions covered by this distributions family.

While the generalized life distribution function is given by probability model as:

\[ f(x) = \frac{[y(x)]^{a-1}y'(x)}{\theta a \Gamma(a)} \exp \left( -\frac{y(x)}{\theta} \right), \quad x > 0, \]  

(8)

here \( a \) and \( \theta \) represent parameters, \( y(x) \) is an increasing function of \( x \) that is real positive valued, with \( y(0)=0 \) and \( y'(x) \) is the derivatives of \( y(x) \). For more details see [28].

According to [28], the PRHR model [Eqn. (7)] and the probabilistic model [Eqn. (8)] reduced to some life distributions shown in Table 2 and 3, respectively.

For the purpose of illustration in this study, an exponential distribution is used throughout.

After identifying the most suitable and appropriate distribution best-fitted by the failure data, the parameters for that distribution can be estimated. There are several techniques and methods for this purpose. Recently, [28] proposed two new models for estimating system availability as well as parameters that follow a particular lifetime distribution based on “the maximum likelihood estimators (MLEs) and uniformly minimum variance unbiased estimators (UMVUEs)”.

\[ \hat{\Lambda}_M^U = \prod_{i=1}^{n} \left[ 1 - \left( \frac{\hat{\Lambda}_i^M(t)}{\hat{\Lambda}_i^U(t)} \right)^{\alpha_i} \right]. \]  

(9)

\[ \hat{\Lambda}_U^U = \prod_{i=1}^{n} \left[ 1 - \left( \frac{\hat{\Lambda}_i^U(t)}{\hat{\Lambda}_i^U(t)} \right)^{\alpha_i} \right]. \]  

(10)

where \( \hat{\Lambda}_M^M \) and \( \hat{\Lambda}_U^U \) are the availability function which can be estimated differently based on MLEs and UMVUE estimators [see [28]].

\[ \text{TABLE 2. Reduced life time distributions by PRHR model.} \]

| S. No. | G(x) | Distribution |
|-------|------|-------------|
| 1     | x, \( x \in (0,1) \) | Power function |
| 2     | \( (1 - e^{-x}) \), \( x \in (0,\infty) \) | Generalized exponential |
| 3     | \( (1 - e^{-x^2}) \), \( x \in (0,\infty) \) | Generalized Rayleigh |
| 4     | \( (1 - e^{-x^p}) \), \( x \in (0,\infty), p > 0 \) | Exponentiated -Weibull |
| 5     | \( (2x - x^2), x \in (0,1) \) | Topp-Leone |

\[ \text{TABLE 3. Reduced life time distributions by probabilistic model.} \]

| S. No. | h(x) | \( \alpha \) Values | Distribution |
|-------|------|---------------------|-------------|
| 1     | x    | \( \alpha = 1 \)   | One parameter exponential with parameter \( \theta \) |
| 2     | x    | \( \alpha > 0 \)   | Gamma |
| 3     | \( x^p, p > 0 \) | \( \alpha > 1 \) | Generalized gamma |
| 4     | \( x^p, p > 0 \) | \( \alpha = 1 \) | Weibull |
| 5     | \( x^2 \) | \( \alpha = 1/2 \) | Half normal |
| 6     | \( x^2 \) | \( \alpha = 1 \) | Rayleigh |
| 7     | \( \frac{x^2}{\alpha} \) | \( \alpha = 3/2 \) | Maxwell’s |
| 9     | \log (1+ x^p), p > 0 | \( \alpha = 1 \) | Burr |

The MLE and UMVUEs for the system availability are given in Eqn. (9) and (10).

\[ \text{B. MULTIOBJECTIVE OPTIMIZATION} \]

Optimization, in simple terms, is finding the best possible desired result(s) out of many available feasible solutions. In an optimization problem, the objective could be single or multiple. A multiobjective problem has more than one objective or goal desired to be achieved in some kind. It can be linear or nonlinear function(s) with some constraints or limitations, which can also be linear or nonlinear. For instance, cost minimization and benefit, profit, or performance maximization. It could be a mixture of both minimization and maximization.

In engineering design, an engineer may wish to maximize system availability and, in addition, minimize cost functions, volume or weight of the system. These objectives might be conflicting, and a single optimal value cannot satisfy all the objectives. Here, the designer faces the problem of optimizing all the objectives simultaneously. In a single objective optimization, an optimal solution is possible depending on
the problem nature; however, in a multiobjective optimization problem (MOOP), it is impossible to obtain an optimal solution to all the objectives since they could be conflicting. Therefore, a Pareto or nondominated or a compromise solution is possible. There are different types of models and solutions obtainable in MOOP. The MOOP can be linear or nonlinear depending on the problem nature and constraints. However, the general MOOP model is presented below:

Let a multiobjective programming problem (MOPP) with \( j \) objectives functions be given as:

\[
\text{Optimize } (Z_1(X), Z_2(X), \ldots Z_j(X))
\]

\[
\text{subject to : } g_i(x)(\leq, =, \geq) b_i, \quad i = 1, 2, \ldots, m; \quad x \geq 0.
\]

(11)

where \( Z_j \) is the set of objectives, \( g_i(x)(\leq, =, \geq) b_i \) are \( m \) set of constraints for which \( b_i \) is the \( i^{th} \) resources. Many techniques and approaches exist for solving MOPP model Eqn. (11), among them are the GP and its variants such as FGP.

C. GOAL PROGRAMMING

Charnes and Cooper [29] introduce the GP in the early 60s as simple linear programming. The same authors [30], [31] demonstrated the GP model applications in several areas, after which numerous applications follow. The technique helps solve multiple criteria, multiple objective optimization problems. GP is an efficient and feasible methodology that can apply to a variety of targets in decision-making problems. GP techniques appeared to be a commanding methodology to overcome decision-making problems with multi-criteria. The goal is to minimize the variation between the successes of objectives. In GP, decision-makers desires are characterized as ‘goals’ or ‘targets’ that must be met as closely as possible under some conditions. The goals could be a specific value(s) or an interval (range) of these values. In reality, it is challenging to achieve a target value(s) exactly. In some cases, they could be overachievement or underachievement. The GP model tries to minimize the total deviations from the desired goal. The procedure solves the individual objectives. In each case, the unwanted deviations are penalized in the objective function. The goal function becomes an additional constraint, and the new objective function minimizes the total deviations.

A typical GP model is given in Eqn. (12).

\[
\min \quad Z = \sum_{i=1}^{n} (\delta_i^+ + \delta_i^-)
\]

\[
\text{subject to : } \sum_{i=1}^{n} a_{ij}x_j - \delta_i^+ + \delta_i^- = g_i, \quad j = 1, 2, \ldots, m,
\]

\[
\delta_i^+, \delta_i^- \geq 0, \quad i = 1, 2, \ldots, n.
\]

(12)

Here, \( x_j \) is the \( j^{th} \) decision variable and \( a_{ij} \) its coefficient, \( \delta_i^+, \delta_i^- \) are the unwanted deviations, and \( g_i \) is the \( i^{th} \) goal value. The problem has \( n \) goals with \( m \) variables.

Note: Suppose some or all parameters of the goals or the constraints are unknown or involve some vagueness or imprecision. In that case, the concept of FGP developed by [32] is helpful in this sense to address the vagueness or the fuzziness in the model parameters. The next subsection discusses the FGP.

In goal programming formulation, the objective function does not contain decision variables; instead, it includes the deviational variables \( (\delta_i^+, \delta_i^-) \), representing each type of goal or sub-goal. A deviational variable is typically represented in the objective function as a combination of overachieving and underachieving the current goal [see Eqn. (12)]. Deviational variables show the possible deviation below and above the target values. The negative deviation is the deviation for a given goal by which it is less than the aspiration level. The positive deviation is the amount of deviation to a particular goal by which it exceeds the aspiration level.

D. FUZZY GOAL PROGRAMMING

As previously discussed, some real-life decision-making processes involve imprecision. The decision-makers goal value may have some incomplete information or vagueness, and a decision must be taken in such a scenario. Fuzzy sets deal with such goals’ parameters or values that are imprecise. The FGP concept is applied to the theory of fuzzy set. This real-life modelling concept is traceable from the Zadeh’s work [33]. The first application of fuzzy programming in solving MOOP appeared in [32]. An FGP function is represented generally as:

Find a vector \( X = [x_1, x_2, \ldots, x_n]^T \), that satisfy

\[
Z_k(X)(\geq, =, \leq) G_k, \quad k = 1, 2, 3, \ldots, K.
\]

\[
AX \leq b_i, \quad i = 1, 2, \ldots, m,
\]

\[
X \geq 0,
\]

(13)

Here, \( G_k \), represents goals vectors, \( b_i \), represent \( m \) resources vector, \( A \) represent decision variables, coefficient. The symbol \( \geq \) represent fuzzy-maximization objective-type, \( = \) represent fuzzy-minimization objective-type and \( \leq \) represent fuzzy-equality constraint-type. \( Z_k \) denotes the \( k^{th} \) objective and \( X \) represent \( n \)-dimensional vector of decision variables.

The fuzzy-minimization-type membership is given as

\[
\mu_{Z_k}(X) = \begin{cases} 
1, & \text{if } Z_k(X) \geq G_k, \\
\frac{Z_k(X) - L_k}{G_k - L_k}, & \text{if } L_k \leq Z_k(X) \leq G_k, \\
0, & \text{if } Z_k(X) \leq L_k.
\end{cases}
\]

(14)

The fuzzy-maximization-type membership is given as

\[
\mu_{Z_k}(X) = \begin{cases} 
1, & \text{if } Z_k(X) \leq G_k, \\
\frac{U_k - Z_k(X)}{U_k - G_k}, & \text{if } G_k \leq Z_k(X) \leq U_k, \\
0, & \text{if } Z_k(X) \geq U_k.
\end{cases}
\]

(15)
The fuzzy-equality-type linear-membership is given by

\[
\mu_{Z_k}(X) = \begin{cases} 
0, & \text{if } Z_k(X) = U_k, \\
Z_k(X) - L_k, & \text{if } L_k \leq Z_k(X) < G_k, \\
G_k - L_k, & \text{if } L_k \leq Z_k(X) \leq G_k, \\
U_k - Z_k(X), & \text{if } G_k < Z_k(X) \leq U_k, \\
U_k - G_k, & \text{if } G_k < Z_k(X) < U_k, \\
0, & \text{if } Z_k(X) > U_k,
\end{cases}
\]

(16)

where \( U_k \) is the upper limit and \( L_k \) the lower limit, and \( G_k \) is the aspirational levels given by the DM for the \( k \)th goal.

1) STEPWISE PROCEDURE FOR FGP

There is no developed algorithm for solving multiple objective problems straightforwardly, without some transformations. Therefore, such a problem(s) may be transformed into single objective for a proper and easy solution using compromise rules. The FGP solution procedure is as follows:

Step 1: To get the solution of the multiobjective nonlinear programming problem, consider the single-objective problem utilizing just one objective at once and disregard the other objective functions and received the optimum solution for every characteristic as the ideal solution.

Step 2: From the afferent of Step 1, decide the relating values for each objective at each solution acquired. Let \((x_{11}, x_{22}, \ldots, x_{ij})\) be the ideal solutions of the objective functions \((Z_1, Z_2, \ldots, Z_K)\).

Step 3: Step 2, define the payoff matrix utilizing the ideal solutions and discover the upper and lower values for each objective function corresponding to the set of solutions as \(U_k\) and \(L_k\) for the \(k\)th objective function \(Z_k(X)\).

Step 4: Use the membership functions defined in Eqn. (14), (15), and (16) for the given problem type, respectively.

Step 5: Then using max-min operator, to obtain max \(\{\mu_1, \mu_2, \ldots, \mu_k\}\), then

\[
\text{Maximize } \lambda \\
\mu_1 \geq \mu \\
\mu_2 \geq \mu \\
\vdots \\
\mu_k \geq \mu,
\]

where \(\mu_k = \min \{\mu_k(Z_k); k = 1, 2, \ldots, K\}\) and \(\mu \in [0, 1]\). Finally, the FGP problem to be solve is:

\[
\text{Maximize } \lambda \\
\text{subject to } Z_1 - \mu(U_1 - L_1) \geq L_1 \\
Z_2 - \mu(U_2 - L_2) \geq L_2 \\
\vdots \\
Z_k - \mu(U_k - L_k) \geq L_k \\
\mu \in [0, 1], k = 1, 2, \ldots, K.
\]

(17)

E. GENETIC ALGORITHM

A GA is a search method based on probabilistic norms to solve an optimization problem. It adapts the probabilistic approach on the basis and principle of natural evolution. It adapts the probabilistic approach on the basis and principle of natural evolution. The approach can be applied for complex combinatorial problems effectively and provides a heuristic solution. Several researchers used the concept in solving a reliability optimization problem. The most widely used GA is considered to solve optimization problems and first implemented at Michigan University by [34]. The GA algorithm combined the better population with its best solution in each iteration and repeated more the procedures in the next generation until the suitability of each solution or stopping criteria is realized. This process will reach an optimal solution.

1) ALGORITHM STEPS

Step 1: Create a random population, including \(n\) chromosome or initial solution.

Step 2: Establish in the population the fitness role of each chromosome.

Step 3: Building a new population-based on the selection of parent chromosomes by selective methods such as roulette wheel, match, random, competitive, etc.

Mentioning an absolute value for the likelihood of a crossover operator and then conducting a combination procedure on parents to create offspring and assuming a specific value for the mutation operator’s. Using this procedure into establish a new chromosome shift one or more genes from parent’s chromosome.

Step 4: Replacing new offspring in the new population.

The pseudo code for GA generation is given in Table 4.

| TABLE 4. Pseudo code of genetic algorithm. |
|-------------------------------------------|
| Algorithm 1: Pseudo code of GA. |
| 1. Define fitness function. |
| 2. Initialize the initial random population. |
| 3. Set the value of parameters, population size, crossover and mutation rate. |
| 4. While number of generation < maximum generation. |
| Do |
| 5. Evaluate fitness value. |
| 6. Update the population for evaluation value. |
| 7. Update the selection value, crossover operator and mutation operator for feasibility. |
| 8. Update the population for the next generation. |
| End while one of the stopping criteria is met |
| Output the optimal solution of fitness function. |

F. PARTICLE SWARM OPTIMIZATION

There are several meta-heuristics techniques, PSO is one of the newest techniques that basis on bird flocking behaviour synchrony developed [35]. The optimal solution in PSO has rooted among the distance neighbours appropriately. It is a population-based algorithm with random initialization similar to GA in terms of global optima searching in a successive iterations. The subsequent generation relied on the nearest
neighbour velocity and its current position in the search direction, \[36\]. Unlike GA, the PSO neither mutate nor undergo crossover; however, the particles move within the feasible solution space in search for the global best solution from the current optimum particles. The objective of using PSO is to emulate cooperative foraging living organisms and social behaviors such as birds flock, fish school, bees swarm, etc. In PSO, the population is called a swarm and any variable of the swarm is taken as particle. The particles of the swarm head, in the search space, have the limited speed, so as to reach the desired goal.

Using their cognitive and collective ability the search particles may be able to adjust their best position and it may be explored by a swarm respectively. Reference \[35\] developed an Eqn. (18) to modify each particle’s velocity and thus modified the individual particle’s position according to this modified velocity in Eqn. (19).

\[
V_p(t) = V_p(t-1) + C_1 \cdot rand(.) \cdot (X_p^{\text{best}} - X_p(t-1)) + C_2 \cdot rand(.) \cdot (X_s^{\text{best}} - X_p(t-1)) \tag{18}
\]

\[
X_p(t) = X_p(t-1) + V_p(t), \tag{19}
\]

where, \(V_p(t) \in [V\text{min}, V\text{max}]\) represents individual particle velocity \((p = 1, 2, \ldots, N)\) at \(t^{th}\) iteration. \(C_1\) and \(C_2\), respectively, denote particle cognitive and collective ability, called coefficients of acceleration. \(\text{Rand}(. )\) is a random value between 0 and 1. \(X_p\) represents the particle position \(p\), best of \(X_p^{\text{best}}\) and best of \(X_s^{\text{best}}\) are used to denote the current best location (up to \(t^{th}\) iterations) found so far for an individual particle and the entire swarm (global best), respectively. The pseudo code for generating the PSO algorithm is shown in Table 5, and the flow chart of PSO technique is shown in Figure 1.

| TABLE 5. Pseudo code for particle swarm optimization. |
|-------------------------------------------------------|
| **Algorithm 2:** Pseudo code for PSO. |
| **For each particle \(p\) in a swarm population size \(N\):** |
| 1: Define objective function. |
| 2: Initialize value of parameters \(w\), \(C_1\), \(C_2\) and iteration. |
| 3: Initialize the position \(X_p\) and velocity of particles \(V_p\). |
| 4: Initialize particle best value (\(p\text{best}\)) and swarm best value (\(s\text{best}\)). |
| 5: Define fitness function. |
| **Repeat until a stopping criterion is satisfied for each particle \(p\):** |
| 6: Update particle velocity \(V_p(t)\) according to Eqn. (18). |
| 7: Update particle position \(X_p(t)\) according to Eqn. (19). |
| 8: Particle best value (\(p\text{best}\)), swarm best value (\(s\text{best}\)) and weight coefficient. |
| **End** |
| Output the optimal value of fitness function. |

IV. SYSTEM DESCRIPTION

When corrective maintenance is performed on a unit after it has failed, it may take a long time and be expensive. Particularly, the number of system failures should be reduced in order to minimize the downtime of systems like computers, plants, radar etc. Preventive maintenance is necessary for this situation to maintain a unit and avoid failures, but from the perspectives of cost and dependability, it shouldn’t be performed too frequently.

Aircraft, computer networks, and other large-scale, complex systems all significantly impact society. In maintaining these systems, maintainability theory is crucial. An excellent maintenance strategy development entails mathematical maintenance rules with a focus on preventive maintenance that has primarily been established in the research field of operations research. Designing a maintenance strategy with two maintenance options, preventive replacement and corrective replacement, is the most significant challenge in mathematical maintenance strategies. When a system or unit is changed as part of preventative maintenance, it is done before it breaks down. On the other hand, with the corrective replacement, it is the failed unit that is replaced.

Practically important preventive maintenance optimization models that involve age replacement and block replacement are reviewed in the well-known renewal reward argument framework. Some extensions to these basic models and the corresponding discrete-time models are also introduced with the aim of applying the theory to practice.

V. MODEL FORMULATION

This section discusses system cost function and availability. It presents the model formulation of system cost and availability functions for a series-parallel system as a multiobjective optimization. It further uses the concept to demonstrate the solution approaches discussed in Section III for numerical illustrato. Next, cost function is discussed.

A. COST FUNCTION

The cost model suggested by \[37\] is reconsidered. The use of this model is to reduce the average cost per unit time and the cost of repaired (or replaced or failed) units. Suppose each unit has an identical failure distribution of \(F(t)\) with a finite mean and the cost of \(c_2(<c_1)\) is accumulated for each unit that is transmitted without failure. In addition, let \(N_1(T)\) represents the number of failures occurring during \([0, t]\) and \(N_2(T)\) represents the number of non-failed units occurring within \([0, t]\). So, the estimated cost, for \([0, t]\), is given by

\[
c_1E(N_1(T)) + c_2E(N_2(T)) = c_1M(T) + c_2.
\]

where \(M(T)\) is the mean number of failures over the interval \([0, t]\).

The period of one cycle is considered from one replacement to the next replacement. The time and cost pairs are distributed independently and identically for each cycle and both have finite means. So, for an infinite time, the estimated cost per-unit of time is

\[
\rho(T) = \lim_{T \to \infty} Z(T) = \frac{\text{Expected cost of cycle}}{\text{Mean time of cycle}}. \tag{20}
\]

Then Eqn. (20) becomes:

\[
C(T) = \frac{Z(T)}{T} = \frac{[c_1M(T) + c_2]}{T}.
\]
Barlow et al. [2] have established the policy of periodic replacement when period $kT (k = 1, 2, \ldots)$ is still replaced by a cycle but a failure does not replace and thus for failure period the expected cost rate is

$$
\frac{c_3}{T} \int_0^{T-T_0} G(t) dt,
$$

(21)

where $c_3$ is downtime cost from a failure to its recognition. By Eqns. (20) and (21) the expected cost rate is

$$
C(T_0, T) = \frac{1}{T} \left[ c_1 M(T) + c_2 + c_3 \int_0^{T-T_0} G(t) dt \right].
$$

(22)

The Eqn. (22) is used for age replacement. In this case, the system is regularly replaced at $kT$ and repaired/replaced at the failed unit up to $t$. Where, $T$ is the total system time; $c_1$ be the cost of replacement for a failed unit; $c_2$ be the cost of the planned replacement.

During the time cycle $E(N_1(T)) = M(T) = \sum_{n=1}^{\infty} F^{(n)}(T)$ represents the mean number of failures over $(0, T)$ (renewal function) and $F^{(n)}(T)$ convolution of the $n$-fold lifetime distribution.

$$
F^{(n)}(T) = \int_0^T F^{(n-1)}(t-u) dF(u), \quad n = 1, 2, \ldots
$$

B. COST FUNCTION FOR SERIES-PARALLEL SYSTEM

The number of units must be specified in series-parallel system. The independent and identically exponential distribution is used for its practical utility. The cost value with a fixed time period is given as

$$
C(T_0, T) = \frac{1}{T} \left[ c_1 M(T) + c_2 + c_3 \int_0^{T-T_0} G(t) dt \right].
$$

Let $F(t) = 1 - \exp(-\lambda t)$, then

$$
M(T) = \sum_{n=1}^{\infty} F^{(n)}(T) = \sum_{n=1}^{\infty} \left[ 1 - \sum_{i=0}^{n-1} \frac{(\lambda t)^i}{i!} \right] \exp(-\lambda t).
$$

The unit replaces by another when its failure occurs at $T \to \infty$, then $M(T) = T\lambda$.

And value of

$$
\int_0^{T-T_0} G(t) dt = \int_0^{T-T_0} \exp(-\mu t) dt,
$$

or

$$
\int_0^{T-T_0} G(t) dt = \frac{1 - \exp(-T_0 \mu - T \mu)}{\mu}.
$$

Then,

$$
C(T_0, T) = \frac{1}{T} \left[ Tc_1 \lambda + c_2 + c_3 \frac{1 - \exp(-T_0 \mu - T \mu)}{\mu} \right].
$$

C. AVAILABILITY FUNCTION

The $n$ subsystems are considered that are connected in series, for $i = 1, 2, \ldots, n$ and each $i$ subsystem has $m_i$ parallel connected components. The parallel subsystem works by using the standard series-parallel configuration when at least one of its components are operated and the whole system operates if and only if all subsystems operate. The components are independent in each subsystem $i (i = 1, 2, \ldots, n)$ and distributed identically and independently for failure and repair rate. The $X$ failure rate is considered to be independent and has an equal $F(t)$ distribution with finite mean and the $Y$ repair rate is also independent and has an equal finite mean distribution of $G(t)$. Let $A(T)$ be the availability of the subsystem at time $T$. Then,

$$
A(T) = \frac{E(X_k)}{E(X_k) + E(Y_k)}
$$

Many authors are worked on optimum PM such as [2] and [37] and others. This model is similar to [37], which describes the assumptions of system repair and failure problem. If a device fails, it undergoes under maintenance immediately and is restored to the operational state after repaired. Repair time is divided into two parts; one is before time $T$ having $Y_1$ distribution. The repair time is independent and has a finite mean $G_1(t)$. If a unit’s operating time is already known and its failure rate rises over time, it might be prudent to maintain it at the time $T$ preventively until its operating time failure. Other repair time is after time $T$ having distribution of $Y_2$. Time distribution of $Y_2$ to PM completion time distribution is $G_2(t)$ with a finite mean, which could be lower than the repair time of $Y_1$. A new unit begins to work at $t = 0$. We describe one process from the beginning of service until PM or repair is complete. Then the loss of one cycle is given by:

$$
E \left( Y_1 I_{(X < T)} + Y_2 I_{(X \geq T)} \right) = \frac{F(T)}{\mu_1} + \frac{F(T)}{\mu_2},
$$

where $X_k$ and $Y_k$ ($k = 1, 2, \ldots$) as referring to uptime and downtime and $A(T)$ is the likelihood that the device will work at the time of $T$, respectively. Hence the availability of the state is

$$
A(T) = \frac{\int_0^T F(T)^{D} dt}{\int_0^T F(T)^{D} dt + \frac{F(T)}{\mu_1} + \frac{F(T)}{\mu_2}}.
$$

Now, this model is defined as a series-parallel system. Let $A_{ij}$ be the availability of the component $j$ ($i = 1, 2, \ldots, m_i$) in subsystem $i (i = 1, 2, \ldots, n)$ and let $A_i$ be the availability of the subsystem $i$. That is $A_{ij}$ and $A_i$ can be expressed for series-parallel system, such as

$$
A_i = 1 - \prod_{j=1}^{m_i} (1 - A_{ij}).
$$

The system consists of $n$ subsystems connected in series and each subsystem $i$ has $m_i$ components, connected in parallel for $i = 1, 2, \ldots, n$. Then the cost of the series-parallel system is given as Eqn. (24)
then the system availability is

\[ A_s(T) = \prod_{i=1}^{n} [1 - (1 - A_i)^{m_i}] . \]

Suppose failure and repair time follow an exponential distribution with different parameters. Then the availability of the series-parallel system is given as Eqn. (23)

It has been assumed that the time negligible for replaced at periodic times and at any time, there is an unlimited supply of units available for replacement.

**D. MULTIOBJECTIVE MODEL FORMULATION FOR AVAILABILITY AND COST FUNCTION**

Here, the series-parallel system cost function and availability formulated in Eqn. (24) and (23), as shown at the bottom of the next page, are brought together as a MOOP under some constraints related to system worst reliability value, cost, and time. The optimization model maximizes the system's availability and minimizes its cost function during given time period. It is thus, formulated as follows:

Maximize \( A_s(T) \)  
minimize \( C_s(T_0, T) \)  
subject to \( A_s(T) \geq \operatorname{WRV}(A_0) \)  
\( C_s(T_0, T) \leq C_{\max} \)  
\( T_s \leq T_{\max} \)  
and \( T_{\max} \geq 0, A_0 \geq 0, C_{\max} \geq 0, C_s(T_0, T) \geq 0, \) \( (25) \)

where \( A_s(T) \) is the system availability at time \( T \); \( C_s(T_0, T) \) is the cost value at time \( T \); \( \operatorname{WRV} \) is the worst reliability value of the system; \( C_{\max} \) is the maximum allowable system cost; \( T_s \) is the system time, and \( T_{\max} \) is the total maximum time.

**Note:** We consider the combined model of periodic replacement and no replacement at failure. Failed units are replaced with a new time duration \( (0, T_0] \) and after \( T_0 \) if a failure occurs in an interval \( (T_0, T) \), then the replacement is not made in this interval and the unit remains failed until the planned time \( T \). Using the results of renewal theory the expected cost rate is obtained, and the optimum \( T_0 \) and \( T \) to minimize it are analytically derived.

A unit is replaced at a planned time \( T \). If a unit fails during \( (0, T_0] \) for \( 0 \leq T_0 \leq T \) then it is replaced with a new one, if it fails in an interval \( (T_0, T) \), then it remains failed for the time interval from its failure to time \( T \).

Where \( T_0 \) limit between \( 0 \leq T_0 \leq T \).

**VI. NUMERICAL STUDY**

This section presents the numerical example to illustrate the models formulated in this study. It validates the usefulness of all the techniques discussed in this research and established the best among the four methods. For this purpose, a case study is designed based on military operations in a war zone. The next section presents the case study.

**TABLE 6.** Data simulation by R software from exponential distribution for different parameters.

| \( i \) | \( \lambda_i \) | \( \mu_1 \) | \( \mu_{11} \) | \( \mu_{21} \) | \( c_{11} \) | \( c_{21} \) | \( c_{31} \) | \( T \) (Hours) |
|---|---|---|---|---|---|---|---|---|
| 1 | 1.78811239 | 5.4037288890 | 0.37562002 | 1.494384 | 2 | 1 | 3 | 1350 |
| 2 | 0.01909341 | 0.62901464 | 0.2403815 | 1.442109 | 3 | 2 | 4 | 1350 |
| 3 | 0.09763477 | 6.4458548851 | 1.1246886 | 0.182228 | 4 | 3 | 5 | 1350 |
| 4 | 1.93696814 | 1.9918158221 | 2.7208015 | 0.546778 | 5 | 4 | 6 | 1350 |
| 5 | 0.27380822 | 0.2128789000 | 2.64490963 | 1.093491 | 6 | 5 | 7 | 1350 |

**A. CASE STUDY**

It is a requirement for a military system to complete a series of flight missions in a war. However, it is impossible without an inevitable break between missions to maintain some components at period \( T \) that might fail during the operation. However, not every component may require such maintenance action. Therefore, to enhance the system availability, let us assume the system is composed of \( n \) subsystems in series, each having \( m \) components connected in parallel. The subsystems’ components assumed further to be identical and independently distributed, and supplied from the same manufacturer. Then, the MTBF and MTTR for every failed component are independent of one another for the mission period \( T \). Let \( A_{ij} \) be the \( j \)th component availability in the \( i \)th system, given by Eqn. (3).

Furthermore, if data for the MTBF and MTTR are available, then the procedure discussed in Section III will be used to identify the best-fitted distribution using Eqn. (5) and (6), respectively. Moreover, the identified best distribution parameters will be estimated using the likelihood function either by MLEs or UMVUEs models proposed by [28]. The MTBF and MTTR data are assumed to follow a lifetime distribution with exponential properties to demonstrate this case study.

For illustrating the above case study numerically, the data are simulated by using exponential distribution using R software and information regarding the parameters have been summarized in Table 6 with cost and time values.

**B. SOLUTION USING GP APPROACH**

Here, the general MOOP for series-parallel system formulated in Section V-D is presented using the concept of GP discussed in Section III-C. For all the \( p \) functions, the problem can be reported separately as:

Firstly, Eqn. (26), as shown at the bottom of the next page, is solved for objective function \( Z_1 \) ignoring the \( Z_2 \) subject to set of feasible constraints. The optimal individual solution for objective function \( Z_1 \) is obtained as \( Z_1^* \). It is referred to as goal value for \( Z_1 \). Now, an additional constraint with a deviational variable can be defined as

\[ Z_1 - \delta_1 \leq Z_1^* \]

Similarly, for the second objective function \( Z_2 \), the additional constraint can be defined as:

\[ Z_2 + \delta_2 \geq Z_2^* \]

Finally, the GP model is defined as: The value \( \sum_{i=1}^{p} \delta_i \) will give us the total deviations in objective values by not using the
individual allocations. To solve the following GP problem

$$\min \sum_{j=1}^{2} \delta_j$$

subject to

$$C_s(T_0, T) - \delta \leq 153.8694$$
$$A_i(T) + \delta \geq 0.9923$$
$$T_s \leq 1350$$
and $$T_{\text{max}} \geq 0, A_0 \geq 0, C_{\text{max}} \geq 0, C_s(T_0, T) \geq 0.$$

C. SOLUTION USING FGP TECHNIQUE

The FGP procedure discussed in section III-D is used to solve the first objective as follows:

$$\min Z_1 = C_s(T_0, T)$$

subject to

$$A_i(T) \geq 0.85$$
$$C_s(T_0, T) \leq 150$$
$$T_s \leq 1350$$
$$T_{\text{max}} \geq 0, A_0 \geq 0, C_{\text{max}} \geq 0, C_s(T_0, T) \geq 0.$$

The individual solution is obtained as

$$n_1 = 2, n_2 = 2, n_3 = 5, n_4 = 2, n_5 = 1, Z_2 = 159.978.$$

Similarly, the second objective (27) is solved. The solution obtained as

$$n_1 = 4, n_2 = 1, n_3 = 1, n_4 = 2, n_5 = 5, Z_1 = 0.99486,$$

From the above solutions, formulate the payoff matrix by identifying the upper and lower value of each objective function as

$$Z^L = 0.9923, Z^U = 0.9948, Z^L = 172.096, Z^L = 159.976.$$
TABLE 7. Formulate the payoff matrix using the ideal solutions.

|   | $Z_{1k}$ | $Z_{2k}$ |
|---|---|---|
| $n^{(1)}$ | 0.9948 | 0.9923 |
| $n^{(2)}$ | 172.096 | 159.976 |

Where $n^{(j)}$ are the ideal solutions for the objective functions $Z_{jk}$; $j = 1, 2$, respectively. Then the above payoff matrix defines upper and lower tolerance limits of each objective functions are: $L_j = \min Z_j(n^j) = 0.9923$ and $U_j = \max Z_j(n^j) = 172.096$. Construct the membership function as follows:

\[
\mu_1(d^1) = \begin{cases} 
0 & \text{if } Z_1(n_{1k}) \geq 0.9948 \\
0.0025 & \text{if } 0.9923 \leq Z_1(n_{1k}) \leq 0.9948 \\
1 & \text{if } Z_1(n_{1k}) \leq 0.9923 
\end{cases}
\]

\[
\mu_2(d^2) = \begin{cases} 
0 & \text{if } Z_2(n_{2k}) \geq 172.096 \\
12.12 & \text{if } 159.976 \leq Z_2(n_{2k}) \leq 172.096 \\
1 & \text{if } Z_2(n_{2k}) \leq 159.976 
\end{cases}
\]

On applying the min-max addition operator, objective function reduces to the problem as

\[
\min \left\{ 412.1193 - \left( \frac{Z_1(n_{1k})}{0.0025} + \frac{Z_2(n_{2k})}{12.12} \right) \right\}
\]

In order to maximize the above problem, with subject to constraints as described:

\[
\begin{align*}
\max & \quad \lambda \\
\text{subject to} & \quad Z_1 - 0.0025\mu_1 \geq 0.9923 \\
& \quad Z_2 - 12.12\mu_2 \leq 159.976 \\
& \quad T_S \leq 1350 \\
& \quad T_S \geq 0,
\end{align*}
\]

where $\mu \in [0, 1]$.

Using MATLAB software the solution to the above problem is obtained as: $n_1 = 2$, $n_2 = 3$, $n_3 = 1$, $n_4 = 5$, $n_5 = 4$, with cost value $153.9672$, and availability $= 0.9977$.

D. SOLUTION USING GA TECHNIQUE

The formulated problem is solved like other techniques using the GA procedure discussed in Section III-E with the help of an inbuilt function of a MATLAB version. Table 8 presents the results.

E. SOLUTION USING PSO TECHNIQUE

There is need to define optimal values of several components and cost value in a series-parallel system. The problem is considered as a multiobjective optimization model. It has two objective functions: first is to maximize the availability and second is to minimize the cost function. To solve such a complex problem, it can be divided into two groups:

- (i) Weighted sum approach by assigning weights to each feature to turn the multiobjective into a single objective problem.
- (ii) Optimize goal function with some constraints. The objective functions are transformed to a single objective function as follows.

Using the values of Table 6, model (29), as shown at the bottom of the next page, is solved using PSO technique to find out optimum solution.

From Figure 2, it is easy to see that as time increases, availability value decreases.

F. STATISTICAL ANALYSIS

Statistical analysis play a vital role in testing validity of arguments. There are different statistical tools for analysing a comparison. Since this study compares four different techniques, it is necessary to analyse the comparison statistically. Since the population considered is small (less than 30), a t-test can be used to analyse the comparison of methods employed in this study. The t-test conducted on the GA, FGP and GP with PSO technique at $\alpha = 0.05$ significance level that there is no difference in their population means, and populations have equal variances. Using the pooled t-test for a null hypothesis, the result are presented in Tables 9 and 10, respectively.

VII. RESULTS AND DISCUSSION

The four techniques discussed in methodology Section III have been all used to solve the multiobjective optimization problem of a series-parallel system. Table 8 summarises the results of the solution methods. The aim is to maximize the system’s availability and minimize its costs to optimize the interval of preventive maintenance of the components of the subsystems. It can be observed from Table 8 that on comparing these four results, goal
programming have the same value of system availability with particle swarm optimization result. However, it is not the best because it violets the cost constraints, which must not exceed 150. Both GP and FGP have nearly the same maximum cost value and hence are not the best. On the other hand, the genetic algorithm technique gave a very minimum cost value compared to all other three methods, satisfying the cost limitation. In contrast, it has a lower system availability value among the remaining methods. Therefore, it is not the desired solution as well. Only the PSO has the best possible compromise solution of all the four methods, satisfying both the system availability and cost goals. Therefore, it can be concluded that the PSO technique has outperformed the other techniques in terms of generating the desired compromise solution of a multiobjective optimization problem relating to a series-parallel system. Further statistical analysis conducted to compare the four techniques using the pooled t-test under two cases.

Furthermore, a statistical test conducted to compare the four techniques using the pooled t-test under two cases. In case one, it assumed that two samples have equal variances for the system cost function. In case two, two samples assumed to have equal variances for the system availability. The objective is to test the null hypothesis that no differences in the population means of the problem and the population and have equal variances at a 95% significance level of $\alpha$. Tables 9 and 10 shows that the two types of means differ significantly, and the differences are statistically significant. The statistical test also reveals that the PSO is better than the GP, FGP and GA techniques because the PSO has minimum variance and population mean in both cases.

**A. MANAGERIAL IMPLICATIONS**

System maintenance issues are relevant to this study, be it production, manufacturing, or operational system. Components unreliability poses challenges to the field operators and top management; hence maintenance action is imperative. This study will help decision-makers integrate maintenance decision to correctly estimate the effects of component failures and optimize the system availability and costs. It brought the most suitable valuable technique to achieving the desired compromise solution when confronted with multiple objectives, seeking to generate a possible and plausible result in an engineering or operational system. In deteriorating systems, component failure is inevitable, but maintenance action prevents unnecessary faults. However, frequent system maintenance increases costs and is not desirable; hence maximizing the system’s availability and minimizing its cost function will optimize the interval for the preventive system maintenance. Therefore, this is precisely the present study’s features, and it is the most desired goal of decision-makers and field operators in several systems, including military-based operations.

Additionally, researchers and practitioners will use the present study as a directional guide for adjusting and furthering its applicability in industries where systems deteriorate.

**TABLE 9. Two samples assuming equal variances for cost function.**

|       | GA      | GP      | FGP     | PSO     |
|-------|---------|---------|---------|---------|
| Mean  | 190.8586| 152.3006| 188.2218| 149.8492|
| Variance | 755.3194| 26.93697| 252.4255| 3.436041|
| Observations | 25       | 25      | 25      | 25      |
| Pooled variance | 379.0042| 14.81299| 126.2226|         |
| Hypothesized mean difference | 0       | 0       | 0       |         |
| Degree of freedom | 48       | 48      | 48      |         |
| $t$ statistic | 7.339641| 1.70581 | 58.9198 |         |
| $P(Tc<0)$ one-tail | $1.11 \times 10^{-6}$ | 0.047254 | 9.89 $\times 10^{-47}$ |         |
| $t$ critical one-tail | 1.677224 | 1.677224 | 1.677224 |         |

**TABLE 10. Two samples assuming equal variances for availability function.**

|       | GA      | GP      | FGP     | PSO     |
|-------|---------|---------|---------|---------|
| Mean  | 0.996336| 0.988516| 0.996677| 0.989255|
| Variance | 5.86 $\times 10^{-6}$ | 5.53 $\times 10^{-6}$ | 9.4 $\times 10^{-6}$ | 2.59 $\times 10^{-6}$ |
| Observations | 25       | 25      | 25      | 25      |
| Pooled variance | 4.23 $\times 10^{-6}$ | 9.06 $\times 10^{-6}$ | 2.99 $\times 10^{-6}$ |         |
| Hypothesized mean difference | 0       | 0       | 0       |         |
| Degree of freedom | 48       | 48      | 48      |         |
| $t$ statistic | 12.19585 | 1.87285 | 15.18888 |         |
| $P(Tc<0)$ one-tail | $1.3 \times 10^{-10}$ | 0.033593 | 2.91 $\times 10^{-20}$ |         |
| $t$ critical one-tail | 1.677224 | 1.677224 | 1.677224 |         |

$$
\begin{align*}
\min Z(A, C_s) = 0.5 \sum_{i=1}^{n} \left[ & \frac{1}{T} \left( Tc_{1i} \lambda_{1i} + c_{2i} + c_{3i} \frac{1 - \exp(T_{0i} \mu_{1i} - T \mu_{1i})}{\mu_{1i}} \right) \right] m_{1i} \nonumber \\
- 0.5 \prod_{i=1}^{n} \left[ & 1 - \left( \frac{1 - \exp(-\mu_{1i} T)}{\mu_{1i}} \right) \right] m_{1i} \nonumber \\
\text{subject to} & \prod_{i=1}^{n} \left[ 1 - \left( \frac{1 - \exp(-\mu_{1i} T)}{\mu_{1i}} \right) \right] m_{1i} \geq \text{WRV}(A_0) \nonumber \\
\sum_{i=1}^{n} \left[ \frac{1}{T} \left( Tc_{1i} \lambda_{1i} + c_{2i} + c_{3i} \frac{1 - \exp(T_{0i} \mu_{1i} - T \mu_{1i})}{\mu_{1i}} \right) \right] m_{1i} \leq C_{max} \\
\text{and } & T_i \leq T_{max} \\
& T_{max} \geq 0, A_0 \geq 0, C_{max} \geq 0.
\end{align*}
$$
and characterize with random component failures and repairs for upgrading the availability in system design. The study can be helpful in automobile industries, manufacturing and robotics design. This study proposes a framework for analyzing system availability and cost function using different techniques. It also presents a procedure for identifying best-fitted failure rate distribution for proper estimating of the parameters.

VIII. CONCLUSION
Modelling and optimizing system availability and cost simultaneously is a challenging task in engineering designs. The maintenance of subsystem components is one of the essential aspects in preventing system failure. This study discusses the procedure on how to identify a suitable distribution for which system failure rates follows and estimate its parameters. The study further proposes a multiobjective system availability optimization problem for the continuous operating series-parallel system. For planning and scheduling effectively, both the availability and cost functions are considered simultaneously. This study discusses four techniques: goal programming, fuzzy goal programming, genetic algorithms, particle swarm optimization, and their algorithms. The model demonstrated the algorithm’s applications using numerical results. The results show that the model increases availability while decreasing maintenance costs. The results obtained from the different techniques are compared. Hence, based on comparisons made in Table 8, the study shows that the proposed model may improve availability and reduce maintenance costs by using the PSO technique, unlike the other techniques. Comparatively, the PSO technique demonstrates the effective and efficient result for global compromise solutions compared to other optimization approaches. From the statistical analysis viewpoint, it may be deduced easily that the variance of design availability and cost by PSO are minimum as compared to those values obtained by other techniques as Shown in Table 9 and 10, respectively. Thus, the study concludes that the PSO technique can be helpful most appropriately for formulating and analyzing a mathematical model of the system availability problem. Some limitations of the study include the use of simulated data based on exponential distribution. If accurate failure data exist, it can be used to ascertain the actual distribution it follows using the procedures discussed in this paper. There are numerous extensions of the study. For instance, parameters such as component failure detection rates, utilization of resources rates, and factors related to component quality can be investigated and considered in future work.

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YASHPAL SINGH RAGHAV was born in Bulandshahr, India, in 1983. He received the degree from the Department of Statistics and Operations Research, Aligarh Muslim University, in 2002, and the M.Sc., M.Phil., and Ph.D. degrees in statistics from the Department of Statistics and Operations Research, Aligarh Muslim University, in 2004, 2008, and 2012, respectively.

After completion of the Ph.D., he had worked in various reputed institutions and universities. He is currently working with the Department of Mathematics, Jazan University, Saudi Arabia. He has published more articles in national and international journals of repute. He is involved in various projects. His research interests include various topics in statistics, sample surveys, mathematical programming, reliability optimization, and time series.

YASHPAL SINGH RAGHAV

MRADULA was born in Meerut, Uttar Pradesh, India, in August 1990. She received the B.Sc. and M.Sc. degrees in statistics from Chaudhary Charan Singh University, Meerut, in 2010 and 2012, respectively, and the M.Phil. degree in statistics and the Ph.D. degree in applied statistics from the Baba Saheb Bhimrao Ambedkar University, Lucknow, India, in 2016 and 2021, respectively.

In 2021, she joined the Ministry of Health and Family Welfare, New Delhi, as a Bio Statistician.

Her research interests include the area of statistical modeling, sampling theory, and optimization tools. She has collaborated actively with researchers in several other disciplines of mathematical science fields.

MRADULA

RAHUL VARSHNEY received the Ph.D. degree from Aligarh Muslim University, Aligarh. The area of expertise is in sampling theory, mathematical programming, and optimization.

He is currently working as an Assistant Professor with the Department of Statistics, Babasaheb Bhimrao Ambedkar University, Lucknow. He has published more than 17 research papers in various journal of repute. He is a Life Member of the International Indian Statistical Association (IISA), a member of the Indian Society for Probability and Statistics (ISPS) (ID: LM/2010/49/833), a member of the Kerala Statistical Association, a member of the Operational Research Society of India (ORSI) (ID: 1128/R/15/ML), and a Life Member of The Indian Society for Medical Statistics.

RAHUL VARSHNEY

UMAR MUHAMMAD MODIBBO received the B.Tech. and M.Tech. degrees in operations research from the Federal University of Technology, Yola, Nigeria (Now The Modibbo Adama University, Yola), in 2010 and 2016, respectively.

He is currently a Lecturer with the Modibbo Adama University, Yola, Nigeria. He is also working as a Research Scholar with the Aligarh Muslim University, Aligarh, India. He has nine years experience in teaching, research, and community services. He has published more than 20 research articles in journals of national and international repute, and attended many conferences and workshops in his domain area. His research interests include mathematical programming and its applications, reliability optimization, fuzzy programming, multi-objective optimization, inventory and supply chain management, and sustainable development goals.

Dr. Modibbo was a recipient of University Grant to study M.Sc. Operations Research, in 2014, and a Nigerian Tertiary Education Trust Fund (TETFund) to study Ph.D. Operations Research, in 2018. He was a recipient of the Young Researcher Award and the Research Excellence Award from Institute of Scholars (InSc), India, in 2020. He is an Associate-Fellow, and the President of the Institute for Operations Research of Nigerian [INFORM] Membership No. AF18017 and, a life-time Member the African Federation of Operations Research Societies [AFROS] and the International Federation of Operational Research Societies [IFORS]. He is a Reviewer of many journals, including IEEE Access.

UMAR MUHAMMAD MODIBBO

ABDULLAH ALI H. AHMADINI

IRFAN ALI

IRFAN ALI received the B.Sc., M.Sc., M.Phil., and Ph.D. degrees from Aligarh Muslim University.

He is currently working as a Faculty Member with the Department of Statistics and Operations Research, Aligarh Muslim University. He has supervised M.Sc., M.Phil., and Ph.D. students in operations research. He has completed a Research Project UGC–Start-Up Grant Project,UGC, New Delhi, India. He has published more than 100 research articles in SCI/SCIE and other reputed journals and serves as a reviewer for several journals. He has published some edited books for Taylor France and Springer Nature publishers, and some are in the process of publication. He has published a textbook, “Optimization with LINGO-18: Problems and Applications”. This book is useful for academicians, practitioners, students, and researchers in the field of OR. His research interests include applied statistics, survey sampling, reliability theory, supply chain networks and management, mathematical programming, fuzzy optimization, and multiobjective optimization.

He received the Post Graduate Merit Scholarship Award during the M.Sc. degree in statistics and the UGC-BSR Scholarship Award during the Ph.D. degree in statistics Program. He is a life-time member of various professional societies: the Operational Research Society of India, the Indian Society for Probability and Statistics, the Indian Mathematical Society, and the Indian Science Congress Association. He has delivered invited talks in several universities and institutions. He also serves as an Associate Editor and the Guest Editor of SCI/SCIE for some journals.

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