Classically Conformal Radiative Neutrino Model

with

Gauged $B - L$ Symmetry

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We discuss a classically conformal radiative neutrino model with gauged B–L symmetry, in which the B–L symmetry breaking can occur through the Coleman-Weinberg mechanism. As a result, Majorana mass term is generated and EW symmetry breaking also occurs. We show some allowed parameters to satisfy several theoretical and experimental constraints. Theoretical constraints are inert conditions and Coleman-Weinberg condition. Experimental bounds are lepton flavor violation (especially $\mu \rightarrow e\gamma$), the current bound on the $Z'$ mass at LHC, in additions to the neutrino oscillations.

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I. INTRODUCTION

The standard model (SM) has to be still extended so as to include massive neutrinos and dark matter (DM), even though the SM Higgs has been discovered. One of the elegant solutions to resolve this issue is known as radiative seesaw models, in which active neutrino masses are generated at multi-loop level and exotic fields are naturally introduced in order to get such radiative masses. Such an exotic field can be often identified as a DM candidate. In this sense, one might finds that neutrinos has a strong correlation to the DM candidate. There exist the vast numbers of papers along the though of this line [1–68]. Especially, Ma model [5] is known as a minimal radiative seesaw model including fermionic and bosonic DM candidates.

On the other hand the hierarchy problem arises in the SM. One solution of the hierarchy problem is supersymmetry. This is a beautiful theory however there are no signals in the LHC experiments. In these days, the alternative solutions are discussed [69–83]. In this paper, we take another approach to the hierarchy problem following Bardeen’s argument [84]. Bardeen has argued that once the classically conformal symmetry and its minimal violation by quantum anomalies are imposed on the SM, it may be free from quadratic divergences. The models based on this idea are called classically conformal models [49, 85–111]. The classical Lagrangian for these models has no mass terms and all dimensional parameters are dynamically generated. The models need an absence of intermediate scales between the TeV scale and Planck scale. Then the Planck scale physics is directly connected to the electroweak (EW) physics.

We consider the classically conformal Ma model which is combined Ma model and the classically conformal model. This model connects tiny neutrino mass scale and Planck scale. However the minimal classically conformal Ma model doesn’t realize for following two reasons. First the EW symmetry does’t occur for large top Yukawa coupling. Second the classically conformal symmetry forbids Majorana mass term. The Majorana mass plays an important role in Ma model. We need the extended model. The minimal extension is $B-L$ extended model. In this model, EW symmetry breaking is triggered by $B-L$ symmetry breaking and Majorana mass term is generated by $B-L$ symmetry breaking.

This paper is organized as follows. In Sec. II, we show our model building including neutrino mass. In Sec. III, we show our numerical results. We conclude in Sec. VI.
II. THE MODEL

| Fermion       | $L_L$  | $e_R$  | $N_R$ |
|---------------|--------|--------|-------|
| $(SU(2)_L, U(1)_Y)$ | $(2, -1/2)$ | $(1, -1)$ | $(1, 0)$ |
| $U(1)_{B-L}$  | $-1$   | $-1$   | $-1$  |
| $Z_2$         | $+$    | $+$    | $-$   |

TABLE I: $L_L$, $e_R$, and $N_R$ have three generations, which is abbreviated.

| Boson       | $\Phi$  | $\eta$  | $\varphi$ |
|-------------|---------|---------|-----------|
| $(SU(2)_L, U(1)_Y)$ | $(2, 1/2)$ | $(2, 1/2)$ | $(1, 0)$ |
| $U(1)_{B-L}$  | $0$     | $0$     | $2$      |
| $Z_2$         | $+$     | $-$     | $+$      |

TABLE II: The particle contents for bosons.

In this section, we devote to explain our model based on one of the radiative neutrino models, Ma model [5, 27]. We discuss the one-loop induced radiative neutrino model with gauged $U(1)_{B-L}$ symmetry containing the DM candidates: the lightest field of $N_R$ and $\eta$ which $Z_2$ odd are assigned. The particle contents are shown in Tab. I and Tab. II. We add three $SU(2)_L$ singlet Majorana fermions $N_R$ with $-1$ charge under the $B - L$ symmetry to the SM fields. For new bosons, we introduce a $SU(2)_L$ doublet scalar $\eta$ with zero charge under the $B - L$ symmetry, and a neutral $SU(2)_L$ singlet scalar $\varphi$ with $2$ charge under the $B - L$ symmetry to the SM fields. We assume that the SM-like Higgs $\Phi$ and $\varphi$ have respectively vacuum expectation value (VEV); $v/\sqrt{2}$ and $v'/\sqrt{2}$. The $Z_2$ symmetry assures the stability of these DM candidates.

The relevant Lagrangian for Yukawa sector and scalar potential under these assignments are given by

$$-\mathcal{L}_Y = (y_\ell)_a \bar{L}_a \Phi e_R \eta N_R + \frac{1}{2} y_N \varphi \bar{N}_R \varphi N_R + \text{h.c.}$$

$$\mathcal{V} = \lambda_{\Phi} |\Phi|^4 + \lambda_\eta |\eta|^4 + \lambda_{\varphi} |\varphi|^4 + \lambda_{\Phi \eta} |\Phi|^2 |\eta|^2 + \lambda'_{\Phi \eta} |\Phi^\dagger \eta|^2 + \lambda''_{\Phi \eta} |(\Phi^\dagger \eta)^2 + \text{c.c.}} + \lambda_{\varphi \eta} |\varphi|^2 |\eta|^2 |\varphi|^2 ,$$

where mass terms are forbidden by the conformal symmetry, $a = 1-3$, and the first term of $\mathcal{L}_Y$ can generates the (diagonalized) charged-lepton masses. Without loss of generality,
we here work on the basis that the third term of $L_Y$ is diagonalized and of $y_N$ is real and positive.

A. Symmetry breakings

In this subsection, we discuss the symmetry breakings in our model. The RGEs are given in the Appendix. We assume the classically conformal symmetry and the EW symmetry breaking doesn’t occur by negative mass parameter. The symmetry breaking is occurred by radiatively.

We assume the following conditions at the Planck scale for simplicity,

$$\lambda_{\Phi\eta} = \lambda'_{\Phi\eta} = \lambda_{\Phi\varphi} = \lambda_{\eta\varphi} = 0.$$  \hspace{1cm} (II.3)

Under this assumption, these couplings are generated by quantum correction. As a result, the couplings are very small at low energy scale. Therefore we can consider the SM with inert doublet sector and the B–L sector separately.

First, we consider the B–L sector. The B–L symmetry is broken by the Coleman-Weinberg mechanism[112]. In this scenario, the running coupling $\lambda_{\varphi}$ should satisfy the following relation at the symmetry breaking scale,

$$\lambda_{\varphi}(\mu = v') \sim \frac{3}{4\pi^2} \left( g_{B-L}^4 - \frac{1}{96} Tr \left[ y_N^\dagger y_N y_N^\dagger y_N \right] \right).$$  \hspace{1cm} (II.4)

The $\Phi$ mass can be obtained by the following form,

$$m_{\varphi}^2 = -4\lambda_{\varphi}v'^2.$$  \hspace{1cm} (II.5)

Once the B–L symmetry is broken, the SM Higgs doublet mass is generated through the mixing term between the SM Higgs and B–L breaking scalar in the potential. The effective tree-level mass squared is induced. If $\lambda_{\Phi\varphi}$ is negative, the EW symmetry breaking occurs as usual in the SM. Under our assumption($\lambda_{\Phi\varphi}(M_{pl}) = 0$), $\lambda_{\Phi\varphi}$ becomes negative because of positive RGE(see Eq. (A.13)). Inserting the tadpole condition, $\lambda_{\Phi} = -\lambda_{\Phi\varphi}v'^2/(2v^2)$, the SM Higgs mass is given by

$$m_h^2 = -\lambda_{\Phi\varphi}(\mu = v')v'^2.$$  \hspace{1cm} (II.6)
B. Scalar sector

After the EW symmetry breaking, the scalar fields can be parameterized as

$$\Phi = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix}, \quad \eta = \begin{bmatrix} \eta^+ \\ \eta^0 \end{bmatrix}. \quad (\text{II.7})$$

The neutral components of the above fields and the singlet scalar fields can be expressed as

$$\phi^0 = \frac{1}{\sqrt{2}}(v + h), \quad \varphi = \frac{1}{\sqrt{2}}(v' + \rho), \quad (\text{II.8})$$

where $\Phi$ is the SM-like Higgs, and $v$ is its VEV, which is related to the Fermi constant $G_F$ by $v^2 = 1/(\sqrt{2}G_F) \approx (246 \text{ GeV})^2$.

$\eta$ is the inert doublet and the mass of $\eta$ should be positive. In our model, $\eta$ mass is generated by the mixing between $\eta$ and $\varphi$. Consequently, the mixing should be positive at the symmetry breaking scale,

$$\lambda_{\eta\varphi} > 0. \quad (\text{II.9})$$

And the quartic couplings satisfy the following inert conditions,

$$\lambda_{\Phi} > 0, \quad \lambda_{\eta} > 0, \quad \lambda_{\Phi\eta} + \lambda'_{\Phi\eta} - |\lambda''_{\Phi\eta}| > -2\sqrt{\lambda_{\Phi}\lambda_{\eta}}. \quad (\text{II.10})$$

The mass matrix of the neutral component of $h$ and $\rho$ is given by

$$m^2(h, \rho) = \begin{pmatrix} -\lambda_{\Phi\varphi}v'^2 & \lambda_{\Phi\varphi}vv' \\ \lambda_{\Phi\varphi}vv' & -4\lambda_{\varphi}v'^2 \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} m_h^2 & 0 \\ 0 & m_H^2 \end{pmatrix} \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}, \quad (\text{II.11})$$

where $h$ implies SM-like Higgs and $H$ is an additional Higgs mass eigenstate. The mixing angle $\alpha$ is given by

$$\tan 2\alpha = \frac{-2\lambda_{\Phi\varphi}vv'}{v'^2(4\lambda_{\varphi} - \lambda_{\Phi\varphi})}. \quad (\text{II.12})$$

The Higgs bosons $h$ and $\rho$ are rewritten in terms of the mass eigenstates $h$ and $H$ as

$$h = h \cos \alpha + H \sin \alpha,$$

$$\rho = -h \sin \alpha + H \cos \alpha. \quad (\text{II.13})$$

The other scalar masses are found as

$$m_{\eta}^2 \equiv m^2(\eta^+) = \frac{1}{2}(\lambda_{\Phi\eta}v^2 + \lambda_{\eta\varphi}v'^2), \quad (\text{II.14})$$

$$m_H^2 \equiv m^2(\text{Re } \eta^0) = \frac{1}{2}\left[(\lambda_{\Phi\eta} + \lambda'_{\Phi\eta} + 2\lambda''_{\Phi\eta})v^2 + \lambda_{\eta\varphi}v'^2\right], \quad (\text{II.15})$$

$$m_I^2 \equiv m^2(\text{Im } \eta^0) = \frac{1}{2}\left[(\lambda_{\Phi\eta} + \lambda'_{\Phi\eta} - 2\lambda''_{\Phi\eta})v^2 + \lambda_{\eta\varphi}v'^2\right]. \quad (\text{II.16})$$
Notice here that there exists a constraint between \( m_\eta \) and \( m_I \) that comes from the \( S-T-U \) parameter.

C. Neutrino mass matrix

The neutrino mass matrix can be obtained at one-loop level as follows \cite{5, 27}:

\[
(M_\nu)_{ab} = \frac{(y_\eta)_{ak}(y_\eta)_{bl} M_k}{(4\pi)^2} \left[ \frac{m_R^2}{m_R^2 - M_k^2} \ln \frac{m_R^2}{M_k^2} - \frac{m_I^2}{m_I^2 - M_k^2} \ln \frac{m_I^2}{M_k^2} \right],
\]

where \( M_i \equiv (y_N)_{i} v'/\sqrt{2} \). In this form, observed neutrino mass differences and their mixings are obtained \cite{27}, when the mixing matrix of the charged-lepton is diagonal basis. \( Y_\eta \) can generally be written as

\[
Y_\eta = U_{\text{MNS}} \begin{pmatrix} m_1^{1/2} & 0 & 0 \\ 0 & m_2^{1/2} & 0 \\ 0 & 0 & m_3^{1/2} \end{pmatrix} OR^\perp,
\]

where \( U_{\text{MNS}} \) is the MNS matrix, \( m_i \)'s are neutrino masses, \( O \) is an complex orthogonal matrix and \( R \) is the following diagonal matrix,

\[
R_{ii} = M_i \left( \frac{m_R^2}{m_R^2 - M_i^2} \ln \frac{m_R^2}{M_i^2} - \frac{m_I^2}{m_I^2 - M_i^2} \ln \frac{m_I^2}{M_i^2} \right).
\]

We use this formula. We assume the lightest neutrino mass is zero and the neutrino mass spectrum is normal hierarchy. In this case, the complex orthogonal matrix \( O \) can be written as

\[
O = \begin{pmatrix} 0 & 0 & 1 \\ \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \end{pmatrix},
\]

where \( \alpha \) is complex parameter.

III. NUMERICAL RESULTS

We numerically solve the RGEs and find parameters that satisfy the inert conditions, Eq. (II.9), (II.10).

\[1\] We assume \( m_\eta \) is lighter than \( m_{\eta R} \), i.e., \( \chi_\eta^0 \) is positive.
First we calculate $\alpha = 0$ (in Eq. (II.20)) case. We use the following parameters at the Planck scale,

\[
\lambda_\Phi = 0.01, \quad \lambda_\eta = 0.09, \quad \lambda_\varphi = 0.011, \quad \lambda_{\Phi\eta}'' = 10^{-9}, \quad g_{B-L} = 0.17, \quad y_m = 0.2. \tag{III.1}
\]

The RG flows of the quartic couplings are depicted in Fig. 1, Fig. 2, Fig. 3. In Fig. 1, $\lambda_\varphi$ becomes negative and satisfies Coleman-Weinberg condition (Eq. (II.4)) at $v' = 10.9$ TeV. At that scale, other couplings satisfy inert conditions. In this case, $Z'$ mass is 3.7 TeV. The experimental search for the $Z'$ boson at LHC gives the limit on $Z'$ boson mass, $m_{Z'} \geq 3$ TeV \cite{114,115}.  

**FIG. 1:** Running for quartic couplings. Black solid line is $\lambda = 0$ axis.

**FIG. 2:** Running for mixings between $B-L$ Higgs and doublets.

**A. $\alpha = 0$ case**
FIG. 3: Running for mixings between two doublets.

We investigate Lepton Flavor Violation (LFV) under the flavor structure Eq. (II.18). The most stringent experimental upper bounds of the branching ratio is \( \text{Br}(\mu \to e\gamma) \leq 5.7 \times 10^{-13} \) \([116]\). In our model, \( \text{Br}(\mu \to e\gamma) = 4.6 \times 10^{-14} \) for the former parameters.

**B. \( \alpha \neq 0 \) case**

Next we calculate \( \alpha \neq 0 \) case. In \( \alpha = 0 \) case, the quartic coupling \( \lambda''_{\Phi \eta} \) becomes very small. If \( \lambda''_{\Phi \eta} \) becomes larger, \( y_\eta \) becomes smaller and \( \lambda''_{\eta \phi} \) becomes negative at TeV scale. In this instance, \( \eta \) isn’t inert doublet and has non-zero VEV. In \( \alpha \neq 0 \) case, however, \( y_\eta \) becomes large and \( \lambda''_{\eta \phi} \) becomes positive, even if \( \lambda''_{\Phi \eta} \) is \( O(10^{-2}) \).

We use the following parameters at the Planck scale,

\[
\lambda_\Phi = 0.01, \quad \lambda_\eta = 0.1, \quad \lambda_\phi = 0.019, \quad \lambda''_{\Phi \eta} = 0.01, \quad g_{B-L} = 0.27, \quad y_m = 0.5 \quad (\text{III.2})
\]

The RG flows of the couplings are depicted in Fig.4 These parameters satisfy inert conditions and B–L symmetry breaking is realize at TeV scale.

In this case, \( \text{Br}(\mu \to e\gamma) = 2.2 \times 10^{-12} \). The branching ratio becomes very large because \( y_\eta \) becomes larger than \( \alpha = 0 \) case.

**IV. CONCLUSIONS**

We have investigated a classically conformal radiative neutrino model with gauged B–L symmetry, in which we have successfully obtained the B–L symmetry breaking through
the Coleman-Weinberg mechanism. As a result, Majorana mass term is generated and
EW symmetry breaking occurs. We have also shown some allowed parameters to satisfy
several constraints such as inert conditions, Coleman-Weinberg condition, lepton flavor vi-
olation (especially $\mu \to e\gamma$), the current bound on the $Z'$ mass at LHC, and so on as well as
the neutrino oscillations experiments.

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Appendix A: RGE

In this section, we analyze the RGEs at one-loop level. The covariant derivative can be
written as

$$D_\mu = \partial_\mu - ig' Q^Y B_\mu - i \left(g_{\text{mix}} Q^Y + g_{B-L} Q^{B-L} \right) B'_\mu - ig^3 \frac{g^\alpha}{2} W^\alpha_\mu - ig_3 T^\alpha G^\alpha_\mu, \quad (A.1)$$

where $B_\mu$ and $B'_\mu$ are gauge bosons of $U(1)_Y$ and $U(1)_{B-L}$, and $Q^Y, Q^{B-L}$ are their charge
operators. The RGE formulae for the gauge couplings are

$$(4\pi)^2 \frac{dg'}{dt} = 7g^3, \quad (A.2)$$
The RGE formulae for the quartic couplings are given by

\[ (4\pi)^2 \frac{dg}{dt} = -3g^3, \quad (A.3) \]
\[ (4\pi)^2 \frac{dg_3}{dt} = -7g_3^3, \quad (A.4) \]
\[ (4\pi)^2 \frac{dg_{B-L}}{dt} = g_{B-L} \left( 12g_{B-L}^2 + \frac{32}{3} g_{B-L} g_{\text{mix}} + 7g_{\text{mix}}^2 \right), \quad (A.5) \]
\[ (4\pi)^2 \frac{dg_{\text{mix}}}{dt} = 12g_{B-L}^2 g_{\text{mix}} + \frac{32}{3} g_{B-L} \left( g_{\text{mix}}^2 + g^2 \right) + 7g_{\text{mix}} \left( g_{\text{mix}}^2 + 2g^2 \right). \quad (A.6) \]

The RGE formulae for the quartic couplings are given by

\[ (4\pi)^2 \frac{\lambda_\phi}{dt} = 24\lambda_\phi^2 + 2\lambda_{\phi\eta}^2 + \lambda_{\phi\eta}^2 + 4\lambda_{\phi\eta}^2 + 2\lambda_{\phi\eta} \lambda_{\phi\eta} + \lambda_{\phi\eta}^2 \]
\[ + \frac{3}{8} \left[ 2g^4 + \left( g^2 + g^2 + g_{\text{mix}}^2 \right)^2 \right] - 3\lambda_\phi \left[ 3g^2 + g^2 + g_{\text{mix}}^2 \right] - 6y_t^4 + 12\lambda_\phi y_t^2, \quad (A.7) \]
\[ (4\pi)^2 \frac{\lambda_\eta}{dt} = 24\lambda_\eta^2 + 2\lambda_{\phi\eta}^2 + \lambda_{\phi\eta}^2 + 4\lambda_{\phi\eta}^2 + 2\lambda_{\phi\eta} \lambda_{\phi\eta} + \lambda_{\phi\eta}^2 \]
\[ + \frac{3}{8} \left[ 2g^4 + \left( g^2 + g^2 + g_{\text{mix}}^2 \right)^2 \right] - 3\lambda_\eta \left[ 3g^2 + g^2 + g_{\text{mix}}^2 \right] - 2T r \left[ y_N^\dagger y_N y_N^\dagger y_N \right] + 4\lambda_\eta T r \left[ y_N^\dagger y_N \right], \quad (A.8) \]
\[ (4\pi)^2 \frac{\lambda_\phi}{dt} = 20\lambda_\phi^2 + 2(\lambda_{\phi\phi}^2 + \lambda_{\eta\phi}^2) + 96\lambda_{\phi\phi} g_{B-L}^2 - 48\lambda_{\phi\phi} g_{B-L}^2 \]
\[ - T r \left[ y_N^\dagger y_N y_N^\dagger y_N \right] + 2\lambda_{\phi\phi} T r \left[ y_N^\dagger y_N \right], \quad (A.9) \]
\[ (4\pi)^2 \frac{\lambda_{\phi\eta}}{dt} = \lambda_{\phi\eta} \left[ 4\lambda_{\phi\eta} + 12\lambda_\phi + 12\lambda_\eta + 2T r \left[ y_N^\dagger y_N + y_N^\dagger y_N \right] - 3 \left( 3g^2 + g^2 + g_{\text{mix}}^2 \right) + 6y_t^2 \right] \]
\[ + 2\lambda_{\phi\phi} \lambda_{\eta\phi} + 4\lambda_\eta \lambda_{\phi\eta} + 4\lambda_\phi \lambda_{\phi\eta} + 2\lambda_{\phi\phi} + 8\lambda_{\phi\phi} + \frac{3}{4} \left( 2g^4 + \left( g^2 + g^2 + g_{\text{mix}}^2 \right)^2 \right) \]
\[ - 4T r \left[ y_N^\dagger y_N y_N^\dagger y_N \right], \quad (A.10) \]
\[ (4\pi)^2 \frac{\lambda_{\phi\phi}}{dt} = \lambda_{\phi\phi} \left[ 4\lambda_\phi + 4\lambda_\eta + 8\lambda_{\phi\eta} + 4\lambda_{\phi\phi} + 2T r \left[ y_N^\dagger y_N + y_N^\dagger y_N \right] + 6y_t^2 - 3 \left( 3g^2 + g^2 + g_{\text{mix}}^2 \right) \right] \]
\[ + 16\lambda_{\phi\phi}^2 + 3g^2 \left( g^2 + g_{\text{mix}}^2 \right) + 4T r \left[ y_N^\dagger y_N y_N^\dagger y_N \right], \quad (A.11) \]
\[ (4\pi)^2 \frac{\lambda_{\eta\phi}}{dt} = 4\lambda_{\phi\phi} \left[ \lambda_\phi + \lambda_\eta + 2\lambda_{\phi\eta} + 3\lambda_{\phi\phi} + \frac{1}{2} T r \left[ y_N^\dagger y_N + y_N^\dagger y_N \right] + \frac{3}{2} y_t^2 - \frac{3}{4} \left( 3g^2 + g^2 + g_{\text{mix}}^2 \right) \right], \quad (A.12) \]
\[(4\pi)^2 \frac{\lambda_{\Phi \phi}}{dt} = 4 \lambda_{\Phi \phi}^2 + 12 \lambda_{\Phi \phi} \lambda_{\phi} + (4 \lambda_{\Phi \eta} + 2 \lambda_{\Phi \eta}) \lambda_{\eta \phi} + 8 \lambda_{\Phi \phi} \lambda_{\phi} + 12 g_{\text{mix}}^2 g_{B-L}^2 \]
\[+ \lambda_{\Phi \phi} \left[ 6 y_t^2 + Tr \left[ y_N^\dagger y_N \right] - \frac{3}{2} \left( 3g^2 + g'^2 + g_{\text{mix}}^2 \right) - 24 g_{B-L}^2 \right], \quad (A.13)\]

\[(4\pi)^2 \frac{\lambda_{\eta \phi}}{dt} = 4 \lambda_{\eta \phi}^2 + 12 \lambda_{\eta \phi} \lambda_{\phi} + (4 \lambda_{\Phi \eta} + 2 \lambda_{\Phi \eta}) \lambda_{\eta \phi} + 8 \lambda_{\Phi \phi} \lambda_{\phi} + 12 g_{\text{mix}}^2 g_{B-L}^2 - 4Tr \left[ y_N^\dagger y_N \right], \]
\[+ \lambda_{\eta \phi} \left[ 6 y_t^2 + Tr \left[ y_N^\dagger y_N \right] - \frac{3}{2} \left( 3g^2 + g'^2 + g_{\text{mix}}^2 \right) - 24 g_{B-L}^2 \right]. \quad (A.14)\]

The RGE for the Yukawa couplings are given by [105?]

\[(4\pi)^2 \frac{dy_t}{dt} = y_t \left[ \frac{3}{2} y_t^\dagger y_N + \frac{1}{2} y_t^\dagger y_t + Tr \left[ y_t^\dagger y_t \right] - \frac{3}{4} \left( g^2 + g_{\text{mix}}^2 \right) - \frac{9}{4} g^2 - 6 g_{B-L}^2 - 3 g_{B-L} g_{\text{mix}} \right], \quad (A.15)\]

\[(4\pi)^2 \frac{dy_t}{dt} = y_t \left[ \frac{3}{2} y_t^\dagger y_t + \frac{1}{2} y_N^\dagger y_N + Tr \left[ y_t^\dagger y_N \right] - \frac{15}{4} \left( g^2 + g_{\text{mix}}^2 \right) - \frac{9}{4} g^2 - 6 g_{B-L}^2 - 9 g_{B-L} g_{\text{mix}} \right], \quad (A.16)\]

\[(4\pi)^2 \frac{dy_t}{dt} = y_t \left[ \frac{9}{2} y_t^2 - 8 g_3^2 - \frac{9}{4} g^2 - \frac{17}{12} \left( g^2 + g_{\text{mix}}^2 \right) - \frac{2}{3} g_{B-L}^2 - \frac{5}{3} g_{\text{mix}} g_{B-L} \right], \quad (A.17)\]

\[(4\pi)^2 \frac{dy_N}{dt} = y_N \left[ y_N^\dagger y_N + \frac{1}{2} Tr \left[ y_N^\dagger y_N \right] - 6 g_{B-L}^2 \right]. \quad (A.18)\]

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