On the strength assessment of a small liquid petroleum gas carrier in oblique waves by 1D and 3D-FEM analyses

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Abstract. The research from this paper is focused on the strength assessment in oblique design waves of a small LPG liquid petroleum gas carrier, with maximum length of 125.6 m. As loading conditions, full cargo and ballast cases are selected. Because the LPG carrier has an unrestricted maritime navigation class, the maximum height of the oblique quasi-static wave is 12 m, in hogging and sagging conditions, with a height step of 1 m. The LPG-oblique quasi-static waves heading angle is in the range 0 – 75 (360) degrees, with step 15 degrees, where the maximum structural response is expected to occur. For the numerical analyses two classes of structural models have to be considered, equivalent 1D-beam and 3D-FEM, both extended over the whole LPG length, on both sides. The 1D model is implemented in the eigen code P_QSW for the LPG and oblique waves equilibrium parameters and sectional efforts computation, by a non-linear iterative three parameters procedure. For detailed structural analysis the LPG 3D-FEM model is used, developed by Femap/NX Nastran program. Combining the numerical results by 1D and 3D models, the safety polar diagrams, in terms of limit oblique quasi-static waves' height, for the small LPG carrier strength assessment are obtained.

1. Introduction

Standard strength analysis of ship structures involves only head equivalent design wave’s condition [1, 2]. For an extended strength analysis of the maritime ships, including the vertical, horizontal and torsional structural responses, local and global criteria, the design environmental conditions have to be modelled by equivalent quasi-static oblique waves [3, 4].

In this research, the strength analysis of a small LPG 10000 cbm liquid petroleum gas carrier [5] is approached, with structural independent tanks for gas, placed in four cargo holds with different capacity. The structure has special strengthen elements for the double bottom and double side panels, with a dome structure above the main deck level, extended over the four cargo holds.

For the structural analysis of the small LPG carrier, 1D equivalent beam and 3D-FEM models are developed, using an own non-linear three parameters algorithm for the LPG hull and oblique waves equilibrium computation, implemented in P_QSW code [6]. Based on user functions implemented in the Femap/NX Nastran program [7], the link between the 1D and 3D structural models is ensured [6].

Combined limit criteria are considered: minimum freeboard and admissible sectional efforts for the 1D model, yielding stress limit and admissible vertical deflection for the 3D-FEM model [1, 2], delivering the safety operation polar diagrams for the LPG, formulated as limit height of the waves.
2. Bases of the algorithm for ship’s structural analysis in oblique waves

For the ship’s strength analysis in oblique equivalent quasi-static waves, I have developed an own algorithm that combines 1D equivalent beam and 3D-FEM structural models, full extended over the ship’s length and on both sides [6], with improvements for ships having slender hull shape.

The LPG 1D model has a 3D external shape for the offset line formulation, with the mass diagram and strength characteristics distribution over the length imported from the full 3D-FEM model. The 1D model is developed by equivalent beam approach [3]. The LPG 3D-FEM model includes structural details, so that global and local response can be obtained. The 3D-FEM model is developed by Femap/NX Nastran program [7], with triangle membrane-plate elements (shell), Mindlin formulation [8, 9], for the whole steel hull structure, and lumped mass elements for the onboard masses, including the internal independent gas tanks and the ballast water masses. The 1D equivalent beam model requires no boundary conditions. For the 3D-FEM the boundary conditions are specific for models extended on the entire hull structure, applied at aft and fore parts (figure 4) [6], for global and local strength approach. The small LPG models’ characteristics are presented in the next section (table 1).

The algorithm for LPG strength analysis in oblique design waves includes the following steps:

- the development of the 1D and 3D-FEM structural models for the small LPG carrier and setup of the loading case, full cargo and ballast, by specific distribution of the onboard masses, with $\Delta$, $x_c$, $y_c=0$ displacement and gravity centre position into coordinates system from figure 1;
- the oblique quasi-static wave and LPG equilibrium position computation by 1D model, incrementing the environmental parameters: $h_w$, $h_c=12$ m the wave height, step $\partial h_w=1$ m, on sagging and hogging wave conditions, and $\mu=0-75$ (360) deg the ship-wave heading angle, step $\partial \mu=15$ deg;

For each environmental state $(h_w, \mu)$, a non-linear iterative three parameters procedure is applied, implemented in own P_QSW code [6], including cycles on transversal trim angle $\phi$, sinkage $d_m$ and longitudinal trim angle $\theta$, that define the oblique quasi-static wave median plane position on base plane ($z=0$) reference (figure 1). The free surface of the oblique quasi-static wave has equation (1).

$$\zeta_w(x, y) = d_m + (x-x_f)\theta + (y-y_f)\gamma \varphi \pm a_w \cos k\xi; \quad a_w = h_w/2; \quad \xi = x + y \cdot 1/g\mu; \quad k = 2\pi/L$$  

(1)

where: $a_w$ the wave amplitude; $L$ the LPG length; $x_f$, $y_f$ the wave medium plane centre position; $\pm$ sagging and hogging cases; $x, y$ coordinates into the ship’s reference system (figure 1).

For the three equilibrium parameters $d_m, \theta, \varphi$, the algorithm convergence criteria are in equation (2).

$$V \rightarrow \Delta p_w ; \quad x_G \rightarrow x_G ; \quad y_G \rightarrow y_G$$  

(2)

where: $V$ the buoyancy volume, $x_G$, $y_G$ the buoyancy centre position in reference system (figure 1).

- the application of the oblique quasi-static waves pressure from equation (3), on hull sides, for each environmental state $(h_w, \mu)$, with user function from equation (4) implemented in the FEM program [7];

$$p_w(x, y, z) = \rho_w \cdot g \cdot [\zeta_w(x, y) - z] ; \quad x, y, z \in S_{\text{hull external shell}}$$  

(3)

where: $\rho_w$ the water density and $g$ the gravity acceleration.

$$\text{Load / Elemental / Pressure: max}(0.00;\{(R^*\cdot\text{ZEL}(/EL))+(T^\text{AX}(/EL))\cdot(F^*D)\}+\{(YEL(/EL)^*G)*E^\text{AX}\cdot(\text{ZEL}(/EL)+B^\text{AX}

(4)

where for each element placed on the LPG hull external shell the centre position is obtained by program functions $XEL, YEL, ZEL$ [7], and the pressure is applied only if the computed value $p_w$ is positive, corresponding to the immerse condition of the current element from the 3D-FEM model
- the assessment of the LPG carrier using 1D and 3D models by the next criteria: minimum freeboard $F$, as in equation (5); admissible vertical bending moment $\text{VBM}$, vertical shear force $\text{VSF}$, horizontal bending moment $\text{HBM}$, horizontal shear force $\text{HSF}$ and torsion moment $\text{MT}$, as in equation (6); admissible equivalent von Mises stress $\sigma_{\text{equM}}$ and vertical deflection $w$, as in equation (7) and table 1.
\[ F_{PS, SB} = H_{SB, mid, fore} - \zeta_w \left( x_{SB, mid, fore}, y_{PS, SB} \right) \geq F_{min} ; \quad x_{SB, mid, fore} = 0, L/2, L ; \quad y_{PS, SB} = \pm B/2 \]  
(5)

\[ VBM_{max} \leq VBM_{adm} ; \quad VSF_{max} \leq VSF_{adm} ; \quad HBM_{max} \leq HBM_{adm} ; \quad HSF_{max} \leq HSF_{adm} ; \quad MT_{max} \leq MT_{adm} \]  
(6)

\[ \sigma_{VBM_{max}} \leq \sigma_{adm} ; \quad w_{max} \leq w_{adm} \]  
(7)

Based on combined limit criteria by 1D and 3D models, the safety polar diagrams as oblique design wave’s height limits \( h_{w,lim}(\mu, \text{load}) \) are obtained, pointing out the differences between the two models.

3. The small liquid petroleum gas carrier structural models

This study first involves the development of the 1D equivalent beam and 3D-FEM structural models of a small LPG 10000 cbm carrier, with the main data presented in table 1. The equivalent beam elements have the length between 0.3-0.7 m, corresponding to the requirements for global strength analysis by 1D model [3]. The shell elements average size is 0.6 m, thus making possible to analysis by 3D-FEM model (figure 1) the global and the local strength of the LPG carrier [1]. As loading conditions two relevant are selected, full cargo and ballast cases (table 1). According to the rules [1], the reference design wave height is 8.45 m, but for extended analysis, the maximum wave height is considered 12 m. Both structural models are completely balanced by the algorithm from section 2.

Table 1. The characteristics of the small LPG 10000 cbm carrier 1D and 3D models [5].

| Main dimensions | 3D-FEM model | Nodes | Elements | F_{min} (m) | x_{SB} (m) | x_{mid} (m) | x_{fore} (m) | y_{PS, SB} (m) | VBM_{adm} (kNm) | VSF_{adm} (kN) | HBM_{adm} (kNm) | HSF_{adm} (kNm) | MT_{adm} (kNm) |
|-----------------|--------------|-------|----------|-------------|------------|-------------|--------------|----------------|----------------|----------------|----------------|----------------|---------------|
| L (m) | 125.600 | 37443 | 97943 | 0.300 | 0 | 62.800 | 125.600 | 0.209 | 10.250 | 5.36E+05 | 1.71E+04 | 1.93E+05 | 5.12E+03 | 9.74E+04 |
| B (m) | 20.500 | 191 | 110 | 0 | 66.000 | 120.500 | 10.250 | -10.250 | 5.36E+05 | 1.71E+04 | 1.93E+05 | 5.12E+03 | 9.74E+04 |
| H_{SB, mid} (m) | 13.000 | | | | | | | | | | | | | |
| H_{SB, fore} (m) | 16.500 | | | | | | | | | | | | | |
| Full cargo loading case | | | | | | | | | | | | | | |
| \( \Delta \) (t) | 13194.62 | | | | | | | | | | | | | |
| \( x_{SB} \) (m) | 61.867 | | | | | | | | | | | | | |
| \( y_{SB} \) (m) | 0 | | | | | | | | | | | | | |
| \( d_{SB, mid} \) (m) | 7.640 | | | | | | | | | | | | | |
| \( d_{SB, fore} \) (m) | 7.640 | | | | | | | | | | | | | |
| Ballast loading case | | | | | | | | | | | | | | |
| \( \Delta \) (t) | 8412.47 | | | | | | | | | | | | | |
| \( x_{SB} \) (m) | 58.662 | | | | | | | | | | | | | |
| \( y_{SB} \) (m) | 0 | | | | | | | | | | | | | |
| \( d_{SB, mid} \) (m) | 6.500 | | | | | | | | | | | | | |
| \( d_{SB, fore} \) (m) | 4.000 | | | | | | | | | | | | | |

Material steel A

\( E (N/mm^2) \) | 2.10E+05 | | | | | | | | | | | | | |
| \( R_{dy} (MPa) \) | 255 | | | | | | | | | | | | | |
| \( \rho_{water} (t/m^3) \) | 7.85 | | | | | | | | | | | | | |

Figure 1. 3D-CAD/FEM model of the small LPG carrier, (a) general view and (b) bottom view.
Figure 2. 3D-CAD/FEM model of the small LPG carrier, amidships cargo holds M2-M3 section view.

Figure 3. The small LPG transversal section.

Figure 4. The LPG 3D-FEM boundary conditions.

Figure 5. The small LPG, wave pressure, sagging, $\mu=45$ deg, $h_w=8.45$ m, full cargo loading case.

Figure 6. The small LPG, wave pressure, hogging, $\mu=60$ deg, $h_w=8.45$ m, ballast loading case.
Figures 1, 2, 3 present the 3D-CAD/FEM model of the small LPG liquid petroleum gas carrier, full extended over the whole structure, from aft to fore and both sides. The small LPG carrier has slender shapes at aft and fore parts, for more than 0.4 L length, and prismatic shape amidships (figure 1).

Figure 4 presents the boundary conditions for the 3D-FEM model: at fore on centre line all three displacements are restraint (DOF 1, DOF 2, DOF 3), at aft on both sides the vertical displacements are restraint (DOF 3), at aft on centre line the transversal displacement is restraint (DOF 2).

Figures 5, 6 present the oblique quasi-static wave pressure acting on the external shell of the small LPG carrier, on both sides, with rules wave height $h_w^r=8.45m$ [1], for full cargo loading case at sagging $\mu=45$ deg condition (figure 5) and ballast loading case at hogging $\mu=60$ deg condition (figure 6).

4. The small LPG strength analysis by 1D and 3D models in oblique waves, full cargo case

For each loading condition (table 1), full cargo or ballast, a number of 157 environmental cases ($h_w^r, \mu$) are analysed by the 1D-beam and 3D-FEM models of the small LPG carrier, with the method presented in section 2. The three parameters non-linear equilibrium algorithm, based on the 1D-beam model by program P_QSW [6], for one environmental case, requires a maximum of 500 iterations for sinkage $d_m$, 1000 iterations for longitudinal trim $\theta$ and 300 iterations for transversal trim $\phi$ variables.

This section presents the main results for the analysis of the full cargo case of the small LPG carrier (table 1) in oblique quasi-static waves loading conditions.

The structural analysis by 1D model on full cargo case of small LPG carrier leads to the next selection of the main results:
- Table 2 presents the three equilibrium parameters obtained for reference wave heights $h_w^r = 4, 8.45, 12$ m, heading angle $\mu=0\text{-}75$ deg, sagging and hogging oblique waves and full cargo case. Those parameters have significant changes due to the environmental conditions ($h_w^r, \mu$) variation.
- Figures 7, 8 present the sectional efforts ($VBM, VSF, HBM, MT$) selected for reference oblique quarter-sea wave ($\mu=45$ deg), sagging condition, for whole wave height range $h_w^r=0\text{-}12$ m and full cargo loading case.
- Figures 11, 12 present the maximum values for the sectional efforts ($VBM, VSF, HBM, MT$), for the whole range of oblique wave height $h_w^r=0\text{-}12$ m and significant heading angles $\mu=0\text{-}75$ deg, on full cargo case. The sagging wave condition exceeds in many cases the admissible values (table 1).

### Table 2. The equilibrium parameters of the small LPG in oblique waves by 1D model, full cargo case.

| $\mu$ (deg) | $h_w^r$ (m) | Still water | Hogging wave condition | Sagging wave condition |
|-------------|-------------|-------------|------------------------|-----------------------|
|             |             | 4           | 8.45                   | 12                    |
|             |             | 4           | 8.45                   | 12                    |
| 0           | $d_m$ (m)   | 7.640       | 7.126                  | 6.412                 | 5.738                 | 8.018                 | 8.324                 | 8.491                 |
| 0           | $\theta$ (rad) | 0.00000    | -0.00292               | -0.00190              | -0.00278              | 0.00980               | 0.01962               | 0.02553               |
| 0           | $\phi$ (rad) | 0.00000    | 0.00000                | 0.00000               | 0.00000               | 0.00000               | 0.00000               | 0.00000               |
| 15          | $d_m$ (m)   | 7.640       | 7.131                  | 6.424                 | 5.774                 | 8.013                 | 8.313                 | 8.476                 |
| 15          | $\theta$ (rad) | 0.00000    | -0.00288               | -0.00185              | -0.00274              | 0.00970               | 0.01936               | 0.02498               |
| 15          | $\phi$ (rad) | 0.00000    | -0.00142               | -0.00175              | -0.00225              | 0.00231               | 0.00439               | 0.00529               |
| 30          | $d_m$ (m)   | 7.640       | 7.153                  | 6.466                 | 5.829                 | 7.993                 | 8.273                 | 8.421                 |
| 30          | $\theta$ (rad) | 0.00000    | -0.00275               | -0.00167              | -0.00258              | 0.00935               | 0.01846               | 0.02368               |
| 30          | $\phi$ (rad) | 0.00000    | -0.00307               | -0.00385              | -0.00498              | 0.00489               | 0.00928               | 0.01127               |
| 45          | $d_m$ (m)   | 7.640       | 7.206                  | 6.573                 | 5.970                 | 7.943                 | 8.173                 | 8.281                 |
| 45          | $\theta$ (rad) | 0.00000    | -0.00241               | -0.00118              | -0.00212              | 0.00847               | 0.01634               | 0.02065               |
| 45          | $\phi$ (rad) | 0.00000    | -0.00530               | -0.00694              | -0.00915              | 0.00804               | 0.01530               | 0.01893               |
| 60          | $d_m$ (m)   | 7.640       | 7.358                  | 6.883                 | 6.383                 | 7.804                 | 7.886                 | 7.874                 |
| 60          | $\theta$ (rad) | 0.00000    | -0.00145               | 0.00034               | -0.00038              | 0.00616               | 0.01116               | 0.01340               |
| 60          | $\phi$ (rad) | 0.00000    | -0.00900               | -0.01302              | -0.01768              | 0.01193               | 0.02259               | 0.02891               |
| 75          | $d_m$ (m)   | 7.640       | 7.966                  | 8.232                 | 6.704                 | 7.254                 | 6.757                 | 7.575                 |
| 75          | $\theta$ (rad) | 0.00000    | 0.00313                | 0.00752               | 0.00129               | -0.00167              | -0.00376              | 0.00894               |
| 75          | $\phi$ (rad) | 0.00000    | -0.01409               | -0.02708              | -0.02275              | 0.01017               | 0.01660               | 0.03175               |
Figure 7. Small LPG 1D, (a) $VBM (\text{kNm})$ and (b) $VSF (\text{kN})$, sagging, $\mu=45^\circ$, $h_w=0-12$ m, full cargo.

Figure 8. Small LPG 1D, (a) $HBM (\text{kNm})$ and (b) $MT (\text{kNm})$, sagging, $\mu=45^\circ$, $h_w=0-12$ m, full cargo.

Figure 9. Small LPG 3D, stress $\sigma_{\text{vonM}} (\text{MPa})$, oblique wave, sagging, $\mu=45^\circ$, $h_w=8.45$ m, full cargo.

Figure 10. Small LPG 3D, deflection $w (\text{m})$, oblique wave, sagging, $\mu=45^\circ$, $h_w=8.45$ m, full cargo.
Table 3. The maximum values for the small LPG 1D/3D assessment in oblique waves, full cargo case.

| Criteria | $h_s$ (m) | Hoggling wave condition | 0 | 15 | 30 | 45 | 60 | 75 | 0 | 15 | 30 | 45 | 60 | 75 |
|----------|-----------|-------------------------|---|----|----|----|----|----|---|----|----|----|----|----|
| VBM (kNm) | 8.45 | 3.6E+5 | 3.6E+5 | 3.5E+5 | 5.2E+5 | 7.2E+5 | 8.6E+5 | 7.6E+5 | 4.5E+5 | 1.1E+6 | 3.2E+5 | 6.2E+4 | 6.2E+4 |
| $\mu$ (deg) | 66.9% | 66.9% | 64.6% | 64.6% | 48.6% | 13.5% | 46.4% | 83.6% | 83.1% | 76.9% | 59.8% | 11.5% |
| $b_{limi}$ | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 9.613 | 9.616 | 9.869 | 10.563 |
| VSF (kN) | 8.45 | 1.1E+4 | 1.1E+4 | 1.1E+4 | 9.8E+3 | 7.7E+3 | 5.2E+3 | 1.4E+4 | 1.3E+4 | 1.2E+4 | 9.8E+3 | 3.6E+3 |
| $\mu$ (deg) | 64.7% | 64.7% | 62.2% | 57.6% | 30.7% | 80.6% | 80.0% | 77.8% | 72.4% | 57.4% | 21.2% |
| $b_{limi}$ | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 10.338 | 12.00 | 12.00 |
| HBM (kN) | 8.45 | 0.0E+0 | 1.6E+4 | 3.4E+4 | 9.4E+3 | 1.3E+4 | 5.0E+3 | 8.4E+4 | 1.1E+5 | 12.00 | 0.0E+0 | 2.3E+4 | 4.9E+4 | 9.3E+4 | 1.1E+5 |
| $\mu$ (deg) | 61.0% | 61.0% | 57.5% | 41.2% | 18.8% | 88.6% | 64.0% | 103.2% | 182.1% |
| $b_{limi}$ | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 |
| HSF (kN) | 8.45 | 0.0E+0 | 1.1E+3 | 2.3E+3 | 3.8E+3 | 4.8E+3 | 4.6E+3 | 1.5E+3 | 4.1E+3 | 12.00 | 0.0E+0 | 1.6E+3 | 3.4E+3 | 6.9E+3 | 9.3E+3 |
| $\mu$ (deg) | 60.9% | 60.9% | 64.6% | 93.0% | 134.0% | 0.0% | 13.2% | 28.5% | 49.0% | 129.3% |
| $b_{limi}$ | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 |
| MT (kNm) | 8.45 | 0.0E+0 | 1.1E+3 | 2.3E+3 | 4.0E+3 | 6.7E+3 | 1.1E+4 | 3.8E+3 | 4.6E+3 | 12.00 | 0.0E+0 | 1.4E+3 | 3.1E+3 | 5.3E+3 | 9.3E+3 |
| $\mu$ (deg) | 60.9% | 60.9% | 64.6% | 93.0% | 134.0% | 0.0% | 13.2% | 28.5% | 49.0% | 129.3% |
| $b_{limi}$ | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 |
| $\sigma_{conn}$ (MPa) | 8.45 | 164.1 | 171.7 | 178.9 | 185.1 | 187.7 | 209.3 | 164.9 | 177.1 | 195.6 | 218.7 | 252.1 | 290.0 |
| $\mu$ (deg) | 86.4% | 86.4% | 94.2% | 97.4% | 98.8% | 110.2% | 86.8% | 93.2% | 103.0% | 115.1% | 132.7% | 152.6% |
| $b_{limi}$ | 12.00 | 185.1 | 187.2 | 195.4 | 201.9 | 210.1 | 234.2 | 247.7 | 255.8 | 266.4 | 289.7 | 334.4 | 367.8 |
| W (m) | 8.45 | 0.062 | 0.063 | 0.066 | 0.072 | 0.088 | 0.150 | -0.078 | -0.079 | -0.081 | -0.085 | -0.098 | -0.137 |
| $\mu$ (deg) | 29.6% | 30.1% | 31.5% | 34.4% | 42.0% | 71.7% | 37.3% | 37.7% | 38.7% | 40.6% | 46.8% | 65.4% |
| $b_{limi}$ | 12.00 | 0.078 | 0.080 | 0.084 | 0.091 | 0.112 | 0.192 | -0.123 | -0.124 | -0.126 | -0.132 | -0.149 | -0.206 |
| $\sigma_{conn}$ (MPa) | 8.45 | 37.3% | 38.2% | 40.1% | 43.5% | 53.5% | 91.5% | 58.8% | 59.2% | 60.2% | 63.1% | 71.2% | 98.2% |
| $b_{limi}$ | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 | 12.00 |

Figure 11. Small LPG 1D, maximum (a) VBM (kNm) and (b) VSF (kN), sagging waves, full cargo.
The structural analysis by 3D-FEM model on full cargo case of the small LPG carrier leads to the next selection of the main results:

- Figures 9, 10 present the von Mises stress distribution and the vertical deflection over the amidships cargo holds ($x=40-95$ m) for oblique wave $\mu=45$ deg wave and height $h_w=8.45$ m, sagging condition, full cargo case. At the transversal bulkheads, the maximum stress hot-spots are recorded. The vertical displacement is maximum at amidships.

- Figure 14 presents the maximum values of von Mises stress and vertical deflection over the amidships cargo holds ($x=40-95$ m) for wave height $h_w=0-12$ m and heading angles $\mu=0-75$ deg, sagging wave condition, representing the extreme response values for the full cargo case.

Table 3 presents the maximum values obtained by 1D and 3D models for strength assessment of the small LPG carrier on full cargo case and oblique wave’s conditions. The limit criteria (table 1) are checked for wave height $h_w=8.45$ m (the rules reference [1]) and maximum condition $h_w=12$ m.
Figure 13 presents the minimum freeboard values, at port-side (PS), aft and fore, by 1D and 3D-FEM models, for all the environmental conditions \( h_w, \mu \), on full cargo case, which are not exceeding the minimum limit.

5. The small LPG strength analysis by 1D and 3D models in oblique waves, ballast case

In this section, the main results for the analysis of the ballast loading case of the small LPG carrier (table 1) in oblique quasi-static wave conditions are presented.

For the structural analysis by 1D-beam model of the ballast loading case of the small LPG carrier the next selection of the main results is obtained:
- Table 4 presents the three equilibrium parameters obtained for reference wave heights \( h_w = 4, 8.45, 12 \) m, heading angle \( \mu = 0-75 \) deg., sagging and hogging oblique waves, on ballast case. Analogous to full cargo, due to the environmental conditions \( h_w, \mu \) the three parameters are significantly changing.
- Figures 15, 16 present \( V_{BM}, V_{SF}, H_{BM}, M_T \) selected for oblique \( \mu = 60 \) deg wave, hogging condition, wave height range \( h_w = 0-12 \) m and ballast loading case.
- Figures 19, 20 present the maximum values for \( V_{BM}, V_{SF}, H_{BM}, M_T \) for all the environmental conditions in oblique waves, on ballast loading case. The hogging wave condition is selected, when in some cases the admissible values (table 1) are exceeded.

Table 4. The equilibrium parameters of the small LPG in oblique waves by 1D model, ballast case.

| \( \mu \) (deg) | \( h_w \) (m) | Still water | Hogging wave condition | Sagging wave condition |
|---------------|--------------|-------------|------------------------|------------------------|
| \( d_m \) (m) | \theta (rad) | \phi (rad) | \( h_w \) | \( h_w \) | \( h_w \) | \( h_w \) | \( h_w \) | \( h_w \) |
| 0 | 5.250 | -0.01978 | 0.00000 | 4 | 8.45 | 12 | 4 | 8.45 | 12 |
| 0 | 4.621 | -0.02176 | 0.00000 | 5.775 | 6.209 | 6.464 | 0.00000 | 0.00000 | 0.00000 |
| 15 | 6.493 | -0.03016 | 0.00000 | -0.04453 | -0.01319 | 0.00006 | 0.00930 | 0.00000 | 0.00000 | 0.00000 |
| 30 | 6.500 | -0.04456 | -0.00178 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 45 | 6.566 | -0.01328 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 60 | 6.700 | -0.01359 | 0.00000 | -0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 75 | 6.777 | -0.02175 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |

Figure 15. Small LPG 1D, (a) \( V_{BM} \) (kNm) and (b) \( V_{SF} \) (kN), hogging, \( \mu = 60 \) deg, \( h_w = 0-12 \) m, ballast.
Figure 16. Small LPG 1D, (a) $HBM$ (kNm) and (b) $MT$ (kNm), hogging, $\mu=60\,\text{deg}$, $h_w=0$–12 m, ballast.

Figure 17. Small LPG 3D, stress $\sigma_{vonM}$ (MPa), oblique wave, hogging, $\mu=60\,\text{deg}$, $h_w=8.45$ m, ballast.

Figure 18. Small LPG 3D, deflection $w$ (m), oblique wave, hogging, $\mu=60\,\text{deg}$, $h_w=8.45$ m, ballast.

Figure 19. Small LPG 1D, maximum (a) $VBM$ (kNm) and (b) $VSF$ (kN), hogging waves, ballast.
### Figure 20
Small LPG 1D, maximum (a) \( HBM \) (kNm) and (b) \( MT \) (kNm), hogging waves, ballast.

#### Table 5
The maximum values for the small LPG 1D/3D assessment in oblique waves, ballast case.

| Criteria                  | Hoggling wave condition | Sagging wave condition |
|---------------------------|-------------------------|------------------------|
| \( \mu \) (deg)           |                         |                        |
| 0                         | 15                      | 30                     | 45         | 60         | 75         | 0          | 15         | 30         | 45         | 60         | 75         |
| \( h, (m) \)              |                         |                        |
| VBM (kNm)                 |                         |                        |
| % adm                     | 78.9%                   | 78.5%                  | 77.0%      | 73.3%      | 62.9%      | 26.8%      | 48.0%      | 47.4%      | 45.3%      | 40.3%      | 27.1%      | 28.2%      | 15.6E+05 | 1.5E+05 |
| \( h_{limit} \)           | 12.000                  | 12.000                 | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     |
| VSF (kN)                  |                         |                        |
| % adm                     | 73.1%                   | 72.8%                  | 71.8%      | 69.2%      | 61.1%      | 34.6%      | 49.1%      | 48.5%      | 46.6%      | 43.1%      | 35.8%      | 26.7%      | 8.45    | 3.4E+5  |
| \( h_{limit} \)           | 12.000                  | 12.000                 | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     |
| HBM (kNm)                 |                         |                        |
| % adm                     | 84.5%                   | 83.8%                  | 81.6%      | 78.2%      | 69.2%      | 35.1%      | 82.1%      | 81.3%      | 78.2%      | 79.0%      | 53.3%      | 25.2%      | 8.45    | 4.9E+5  |
| \( h_{limit} \)           | 12.000                  | 12.000                 | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     |
| HSF (kN)                  |                         |                        |
| % adm                     | 0.0%                    | 5.3%                   | 11.4%      | 19.4%      | 31.6%      | 49.0%      | 4.8%       | 10.2%      | 17.0%      | 26.6%      | 36.9%      | 36.9%      | 8.45    | 4.9E+5  |
| \( h_{limit} \)           | 12.000                  | 12.000                 | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     |
| MT (kNm)                  |                         |                        |
| % adm                     | 0.0%                    | 13.8%                  | 29.3%      | 48.8%      | 69.8%      | 92.1%      | 0.0%       | 4.8%       | 10.2%      | 17.0%      | 26.6%      | 36.9%      | 8.45    | 4.9E+5  |
| \( h_{limit} \)           | 12.000                  | 12.000                 | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     |
| \( \sigma_{eq,M} \) (MPa) |                         |                        |
| % adm                     | 130.0%                  | 0.0%                   | 0.0%       | 12.4%      | 26.1%      | 42.8%      | 64.0%      | 118.7%     | 120.0%     | 120.0%     | 120.0%     | 120.0%     | 8.45    | 4.9E+5  |
| \( h_{limit} \)           | 12.000                  | 12.000                 | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     |
| \( \tau \) (m)            |                         |                        |
| % adm                     | 11.9%                   | 25.5%                  | 43.9%      | 77.4%      | 136.8%     | 0.0%       | 24.6%      | 51.5%      | 83.0%      | 121.4%     | 124.3%     | 124.3%     | 8.45    | 4.9E+5  |
| \( h_{limit} \)           | 12.000                  | 12.000                 | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     | 12.000     |

\( \text{Max. HBM (kNm)} \) Ballast LPG 10000cbm EDW-Hogging (1D)  
\( \text{Max. MT (kNm)} \) Ballast LPG 10000cbm EDW-Hogging (1D)
Max. Faft_SB(m) Ballast LPG 10000cbm EDW-H&S (1D/3D)

Max. Ffore_SB(m) Ballast LPG 10000cbm EDW-H&S (1D/3D)

Figure 21. Small LPG, maximum (a) $F_{aft\ SB}(m)$ and (b) $F_{fore\ SB}(m)$, hogging & sagging waves, ballast.

Max. $\sigma_{vonM}(\text{MPa})$ Ballast LPG 10000cbm EDW-Hogging (3D)

Max. $w$ (m) Ballast LPG 10000 cbm EDW-Hogging (3D)

Figure 22. Small LPG 3D, maximum (a) $\sigma_{vonM}(\text{MPa})$ and (b) $w$ (m), hogging waves, ballast.

For the structural analysis by 3D-FEM model of the ballast loading case of the small LPG carrier the next selection of the main results is obtained:

- Figures 17, 18 present the von Mises stress distribution and the vertical deflection for oblique $\theta=60$ deg wave, height $h_w=8.45$ m, hogging condition, ballast case, at amidships cargo holds ($x=40-95$ m). Analogous to full cargo, for ballast case significant stress hot-spots at the transversal bulkheads are obtained and at amidships the vertical deflection is maximum.

- Figure 22 presents the maximum values of von Mises stress and vertical deflection, on ballast loading case and whole environmental sets, hogging wave condition, at the amidships cargo holds ($x=40-95$ m), when the maximum values are obtained.

Table 5 presents the maximum values obtained by 1D and 3D models for strength assessment of the small LPG carrier on ballast condition in oblique waves. As reference, the rules wave height $h_w=8.45$ m [1] and also the maximum wave height $h_w=12$ m are considered.

Figure 21 is presenting the minimum freeboard values, at starboard (SB), aft and fore parts, by 1D-3D models, on ballast loading case, pointing out that the minimum limit is satisfied.

6. Discussions
For the small LPG 10000 cbm carrier (section 3), analysed by 1D equivalent beam and 3D-FEM full models, with the advanced method from section 2, in synthesis results the cumulative safety operation polar diagrams from figures 23, 24. Table 6 presents the oblique quasi-static wave height limits on hogging and sagging wave conditions, for full cargo and ballast loading cases.

Due to the slender shapes of the small LPG carrier at aft and fore parts, representing more than 40% from the length, the variation of the environmental sets ($h_w, \theta$) leads to significant changes of the three equilibrium parameters, more on ballast loading case (tables 2, 4), in compare to a prismatic barge type shape [10, 11]. For the full cargo case, the parameters changes are (table 2): $c_w=5.758 - 8.491$ m (-24.64 % to 11.13% from 7.64 m, still water reference), $\theta = -0.167 - 1.452$ deg and $\varphi =$...
-1.552 – 1.819 deg. For the ballast case, the parameters changes are (table 4): \(d_w = 2.965 – 6.464 \text{ m} \) (-43.64 % to 23.12% from 5.25 m, still water reference), \(\theta = -2.558 – 0.533 \text{ deg} \), \(\varphi = -2.339 – 2.318 \text{ deg} \).

A first evaluation of the small LPG strength has been done by 1D model sectional efforts (figures 7, 8, 15, 16). The maximum vertical bending and torsion moments are obtained amidships. The vertical shear forces and horizontal bending moments are maxim at quarter and three quarter of the length. The maximum values of the sectional efforts (tables 3, 5 and figures 11, 12, 19, 20) are pointing out the environmental conditions \((h_w, \mu)\) when the admissible limits (table 1) are not satisfied.

For the rules reference height \(h_w = 8.45 \text{ m} \) [1], on full cargo case the limits are exceeded only for HSF 29.3-34.0% (\(\mu = 75 \text{ deg, hogg. & sagg.}\)), MT 12.0% (\(\mu = 75 \text{ deg, hogg.}\)) criteria; on ballast case the limit criteria are satisfied. For the maximum height \(h_w = 12 \text{ m} \) more limit criteria are not satisfied, on full cargo: VBM 16.1-30.3% (\(\mu = 0-45 \text{ deg, sagg.}\)), VSF 5.0-17.4% (\(\mu = 0-45 \text{ deg, sagg.}\)), HSF 3.2-88.1% (\(\mu = 45-75 \text{ deg, hogg. & sagg.}\)), MT 42.4-62.8% (\(\mu = 60-75 \text{ deg, hogg. & sagg.}\)); on ballast case: HSF 18.7-30.0% and MT 24.3-36.8% (\(\mu = 75 \text{ deg, hogg. & sagg.}\)). For both cases, by 1D model, the most restrictive limit criteria in oblique waves are the horizontal shear force HSF and torsion moment MT.

A second evaluation of the small LPG strength has been done by 3D-FEM full-extended model, on both sides, equivalent von Mises stress and vertical deflection distributions (figures 9, 10, 17, 18). The admissible vertical deflection criterion is satisfied for all the environmental conditions on both loading cases (tables 3, 5 and figures 14, 22). The admissible von Mises criterion, due to the stress hot-spots recorded at the bulkheads and frames, for rules wave height \(h_w = 8.45 \text{ m} \) [1] is exceeded on full cargo case by 3.0-52.6% (\(\mu = 30-45 \text{ deg, sagg. & hogg.}\)) and on ballast case by 4.5-24.0% (\(\mu = 75 \text{ deg, sagg. & hogg.}\)). For maximum wave height \(h_w = 12 \text{ m} \) the von Mises stress criterion is exceeded up to 93.6% (full cargo) and 116.8% (ballast), mainly on sagging condition for \(\mu = 0-75 \text{ deg}\) (tables 3, 5 and figures 14, 22). The yielding stress limit 3D criterion is also exceeded for oblique waves in sagging condition (\(\mu = 0-75 \text{ deg}\)), starting from wave height \(h_w = 6.361 \text{ m} \) (full cargo) and \(h_w =9.600 \text{ m} \) (ballast).

The last criterion for the small LPG carrier assessment is the minimum freeboard limit, which is satisfied for all the environmental and loading cases (tables 3, 5 and figures 13, 21).

**Figure 23.** Small LPG 10000 cbm carrier, \(h_{\text{limit}}(\mu)\) polar diagram in oblique quasi-static waves, by 1D equivalent beam model, full cargo and ballast loading cases.

**Figure 24.** Small LPG 10000 cbm carrier, \(h_{\text{limit}}(\mu)\) polar diagram in oblique quasi-static waves, by 3D-FEM detailed structural model, full cargo and ballast loading cases.
Table 6. Comparative $h_{\text{limit}}$ (m) for the small LPG carrier in oblique waves, by 1D and 3D models.

| Load      | $\mu$ (deg) | 0   | 15  | 30  | 45  | 60  | 75  | 0   | 15  | 30  | 45  | 60  | 75  |
|-----------|-------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|           | 1D/3D       | 1D  | 1D  | 1D  | 1D  | 3D  | 3D  | 1D  | 1D  | 3D  | 3D  | 3D  | 3D  |
| Full      |             |     |     |     |     |     |     |     |     |     |     |     |     |
| cargo     |             | 12.000 | 12.000 | 12.000 | 11.110 | 9.038 | 6.275 | 9.613 | 9.616 | 9.869 | 10.563 | 8.784 | 6.463 |
| Ballast   |             | 12.000 | 12.000 | 12.000 | 12.000 | 12.000 | 12.000 | 12.000 | 12.000 | 12.000 | 12.000 | 12.000 | 12.000 |

Comparing the limit results by the two structural models (table 6 and figures 23, 24), the 3D-FEM model imposes more restriction, $h_{\text{limit}}=4.456$ m (full cargo) and $h_{\text{limit}}=5.553$ m (ballast), than the 1D-beam model $h_{\text{limit}}=6.275$ m (full cargo) and $h_{\text{limit}}=8.775$ m (ballast), due to the local stress hot-spots by 3D-FEM approach. Due to the slender shape of the hull and non-uniform mass distributions of the small LPG carrier (figure 1), the maximum restrictions are recorded for beam sea condition (figures 23, 24), with a very different behaviour in compare to the case of a prismatic barge type shape [10].

From the polar diagrams (figures 23, 24) results that in oblique waves by 1D and 3D models, on both loading cases, the strength criteria are imposing significant restrictions for $\phi=15-165$ (195-345) deg, which justifies the necessity of the oblique waves instead of head waves [1, 5] strength analyses.

7. Conclusions

For the strength assessment in oblique quasi-static waves of ships with slender shape, by 3D-FEM and 1D-beam models, the three parameters approach (section 2), with non-linear iterative procedures, applied for prismatic barge type shape models [10], has been improved in this study, so that the own code P_QSW has a higher sensitivity for modelling the hull geometric gradients.

In this study, the improved approach for ship’s strength assessment in oblique quasi-static waves is applied on a small LPG 10000 cbm carrier. The numerical analyses require the development of a 3D-FEM model, full extended over the whole hull structure, and a 1D-beam equivalent model (1D structural, 3D external geometric shape) for the small LPG carrier (section 3). Customized numerical procedures were developed for the data exchange between 3D and 1D models, in order to ensure the best correlation of the two structural models and the accuracy of the improved approach.

Extended numerical strength analyses in oblique quasi-static waves of the small LPG carrier have been developed for two reference loading cases, full cargo (section 4) and ballast (section 5), covering the main ship’s heading angles and a maximum 12 m wave height.

The numerical results of the LPG 10000 cbm carrier study case (sections 4, 5) have been analysed by a combined set of admissible structural criteria, as in equations (6), (7), and also by the minim freeboard criteria, as in equation (5), with the synthetize in section 5.

In compare to the floating structures with prismatic shapes, the small LPG carrier with geometric nonlinearities provides significant changes concerning the predicted most restrictive environmental cases, shifting from head and follow waves [10] to beam-oblique quasi-static waves condition (figures 23, 24), combined with major changes of the ship and wave reference equilibrium position. The minimum freeboard criterion involves no operation restriction for the analysed ship (sections 4, 5).

Comparing the operation limits in oblique quasi-static waves by 3D and 1D structural models of the small LPG carrier, the strength criteria are leading to significant differences, due to the 3D-FEM model local structure details that induce stress hot-spots which are not provided by the 1D-equivalent beam model (table 6 and figures 23, 24).

The supplementary weight and the external quasi-static oblique wave pressure in the full cargo case (section 4) in compare to ballast case (section 5), for the small LPG 10000 cbm carrier, lead to more restrictive operation limits on whole range of the ship heading angle (table 6 and figures 23, 24).

In conclusion, in this study the own code for ship and oblique quasi-static wave equilibrium computation has been updated for ships with slender hull shape. The numerical analyses made possible
to assess by structural criteria the operation capabilities of a small LPG 10000 cbm carrier, under design oblique waves loads, according to the international classification societies requirements [1, 2].

Although only two loading conditions for the small LPG 10000 cbm carrier have been analysed (sections 4, 5), this study shall continue with other specific loading cases, in order to obtain a whole range of polar diagrams in oblique quasi-static waves for the evaluation of the operation safety limits by global and local strength criteria.

In addition, further studies must continue the strength assessment in oblique quasi-static waves for other ship types, with different ship hull shapes and operation capacities, in order to analyse the sensitivity of the method from section 2.

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Acknowledgements
The author wishes to express his thanks to ICEPRONAV Galati, who has granted the small LPG 10000 cbm liquid petroleum gas carrier technical data. This research and the program P_QSW are developed in the frame of Naval Architecture Research Centre, from the “Dunarea de Jos” University of Galati.