The Odd Story of $\alpha'$-corrections

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Closed strings can wrap non-contractible cycles in space-time giving winding numbers that have no analogue in particle theory.

The existence of both momentum and winding states is the key property of strings that allows T-duality: the complete physical equivalence of string theories on dual backgrounds that have very different geometries.
The T-duality group of compactifications on $T^d$ is $O(d,d|\mathbb{Z})$.

Conserved momentum and winding numbers have associated coordinates

\[
\begin{align*}
    p_i &\leftrightarrow \text{T-duality} &\sim & w^i \\
    \uparrow & &\uparrow \\
    x^i &\leftrightarrow \tilde{x}_i
\end{align*}
\]

String field theory treats momentum and winding rather symmetrically and, as a consequence, expanding the string field gives component fields that depend on both $x$ and $\tilde{x}$.
The complete closed string field theory is exotic and complicated. As a simplification, one can restrict to the set of massless fields.

This leads to Double Field Theory: an $O(D,D)$ invariant reformulation of the supergravity limits of string theory.

DFT is defined on a doubled space $X^M = (\bar{x}_i, x^i)$ but there is a strong constraint

$$\partial_M \partial^M \ldots = 0, \quad \partial_M \ldots \partial^M \ldots = 0.$$
Frame formulation of DFT
D. Geisbuhler, D. Marqués, C.N., V. Penas (2013)

Symmetries:
Global $G = O(D,D)$ with invariant metric $\eta_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \equiv \eta^{MN}$

Local $H = O(d) \times O(d)$ generated infinitesimally by $\Lambda^B_A$ with invariant metrics $\eta_{AB}$, $\mathcal{H}_{AB}$

Generalized diffeomorphisms are generated infinitesimally by $\xi^M = (\tilde{\lambda}_i, \lambda^i)$ through $\mathcal{L}_\xi$

Fields organized in a generalized frame $E_A^M$ and an $O(D,D)$ invariant generalized dilaton $d$

$$\eta_{MN} = E_A^M \eta_{AB} E_B^M \quad \mathcal{H}_{MN} = E_A^M \mathcal{H}_{AB} E_B^M$$
Gauge invariance

- Duality covariant gauge transformations

\[ \delta_\xi E_M^A = \hat{\mathcal{L}}_\xi E_M^A + E_M^B \Lambda_B^A \]

\[ \delta_\xi e^{-2d} = \partial_M (\xi^M e^{-2d}) \]

GR diffeomorphisms

\[
\begin{align*}
G_{ij} \rightarrow & \ G_{ij} + L^\lambda G_{ij}, \\
B_{ij} \rightarrow & \ B_{ij} + L^\lambda B_{ij}, \\
\phi \rightarrow & \ \phi + L^\lambda \phi
\end{align*}
\]

& gauge transformations of the 2-form

\[ B_{ij} \rightarrow B_{ij} + \partial_i \tilde{\lambda}_j - \partial_j \tilde{\lambda}_i \]

- This duality covariant gauge principle fixes the universal gravity action

\[ S_{NSNS} = \int d^D x \sqrt{G} \ e^{-2\phi} \left[ R + 4 \partial^i \phi \partial_i \phi - \frac{1}{12} H_{ijk} H^{ijk} \right] \]
Beyond sugra: α’ corrections

- The string effective actions have higher-derivative corrections

\[ L = e^{-2\phi} \left( R + 4 g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{12} \hat{H}_{\mu\nu\rho} \hat{H}^{\mu\nu\rho} + \frac{(a-b)}{8} F^\alpha_{\mu\nu} F^{\alpha\mu\nu} \right. \]

\[ + \frac{a}{8} R_{\mu\nu\alpha}^{(-) b} R^{(-) \mu\nu}_b \left( a + \frac{b}{8} R_{\mu\nu\alpha}^{(+)} b R^{(+)}_{\mu\nu} a \right) \]

\[ \hat{H}_{\mu\nu\rho} = H_{\mu\nu\rho} + a \Omega(\omega^{(-) b}_{\mu a}) + b \Omega(\omega^{(+)}_{\mu a}) + (a-b) \Omega(A^a_\mu) \]

\[ \omega^{(\pm) a}_{vb} = \omega^{a}_{vb} \pm \frac{1}{2} H_{\mu a}^b \]

Bosonic string \( a = b = -\alpha' \), Heterotic string \( a = -\alpha' \), \( b = 0 \), Type II \( a = b = 0 \)

Can the gauge principle be deformed so that it requires and fixes the higher derivative corrections?
The heterotic string has such a deformation: the Green-Schwarz transformation

$$\delta B_{\mu
u} = \frac{1}{2} \partial_{[\mu} \Lambda_{\nu]}^b \omega_{(\nu)b}^{(-)a}, \quad \omega_{vb}^{(-)a} = \omega_{vb}^a - \frac{1}{2} H_{\mu a}^b$$

This transformation requires and fixes the higher derivative corrections to the three-form field strength

$$\hat{H}_{\mu\nu\rho} = H_{\mu\nu\rho} + \frac{3}{2} \left( \omega_{[\mu a} \partial_{\nu} \omega_{\rho]}^{(-)a} + \frac{2}{3} \omega_{[\mu a} \omega_{vb}^{(-)} \omega_{\rho c}^{(-)} \right)$$

However, the heterotic string contains in addition a Riemann squared term, which is not required by the GS transformations

Since T-duality mixes B and G we look for a duality covariant generalization of the GS transformation.
Duality covariant Green-Schwarz transformation

We turn \( \delta B_{\mu\nu} = \frac{1}{2} \partial_{[\mu} \Lambda_{\alpha}^{\beta} \omega_{\nu]}^{a} \) into a duality covariant expression

\[
\tilde{\delta}_{A} E_{M}^{C} = \left( a \partial_{[M} \Lambda_{A}^{B} \omega_{N]}^{A} + b \partial_{[M} \Lambda_{A}^{B} \omega_{N]}^{A} \right) E^{NC}
\]

\[
V_{M} = P_{M}^{N} V_{N} = \frac{1}{2} (\eta_{M}^{N} - \mathcal{C}_{M}^{N}) V_{N}, \quad V_{M} = \overline{P}_{M}^{N} V_{N} = \frac{1}{2} (\eta_{M}^{N} + \mathcal{C}_{M}^{N}) V_{N}
\]

Then the full duality covariant form of gauge transformations is

\[
\delta E_{M}^{C} = \tilde{\delta}_{A} E_{M}^{C} + E_{M}^{B} \Lambda_{B}^{A} + \tilde{\delta}_{A} E_{M}^{C}
\]

These gauge transformations preserve the constraints and close
The action

- The zeroth order action is
  \[ S_{DFT} = \int dX \ e^{-2d} \ R(E, d) \]

\[ R = S^{AB} \left( 2E_A^M \partial_M F_B - F_A F_B \right) + F_{ABC} F_{DEF} \left[ \frac{1}{4} S^{AD} \eta^{BE} \eta^{CF} - \frac{1}{12} S^{AD} S^{BE} S^{CF} \right] \]

\[ - 2E_A^M \partial^M F_A + F^A F_A - \frac{1}{6} F^{ABC} F_{ABC} \]

with

\[ F_{ABC} = 3\partial_{[A} E_B^N E_C^{N]}, \quad F_A = \partial^B E_B^N E_{AN} + 2E_A^M \partial_M d \]

- The coefficients are fixed by double Lorentz transformations.

- Gauge invariance under generalized GS transformations requires corrections
  \[ \delta_\Lambda R = 0 \quad \text{but} \quad \tilde{\delta}_\Lambda R \neq 0 \]
First order $\alpha'$-corrections

- The four derivative gauge invariant action is fixed

$$S_{DFT} = \int dX \ e^{-2d} \left( \mathcal{R}(E, d) + a \mathcal{R}^(-) + b \mathcal{R}^+) \right)$$

for some $\mathcal{R}^{(\pm)}$ that transform as scalars under diffeomorphisms and under modified doubled Lorentz transformations

$$\tilde{\delta}_\Lambda \mathcal{R} + \delta_\Lambda \left( a \mathcal{R}^(-) + b \mathcal{R}^+) \right) = 0$$
First order $\alpha'$-corrections

- We found

$$\mathcal{R}^{(-)} = \partial_A \partial_B F_{CDE} F_{FGH} \left( P^{CF} P^{DG} \overline{P}^{AE} \overline{P}^{BH} + P^{CF} P^{DG} \overline{P}^{AH} \overline{P}^{BE} \right)$$

$$+ \partial_A F_{BCD} \partial_E F_{FGH} \left( \frac{1}{2} P^{AE} P^{BF} P^{CG} P^{DH} - P^{BF} P^{CG} P^{AD} P^{EH} - \frac{1}{2} P^{BF} P^{CG} P^{AE} P^{DH} \right)$$

$$+ (2\partial_A F_B - F_A F_B) F_{CDE} F_{FGH} P^{CF} P^{AE} P^{DG} P^{BH}$$

$$+ 2\partial_A F_{BCD} F_{FEG} F_E \left( P^{BF} P^{CG} P^{AD} P^{EH} + P^{BF} P^{CG} P^{AH} P^{DE} \right)$$

$$- \partial_A F_{BCD} F_{EFG} F_{HIJ} \left( P^{BH} P^{CI} P^{AE} P^{DF} P^{GJ} + 4P^{BE} P^{CH} P^{FI} P^{AG} P^{DJ} - P^{BE} P^{CF} P^{AH} P^{DI} P^{GJ} \right)$$

$$+ F_{ABC} F_{DEF} F_{GHI} F_{JKL} \left( P^{BG} P^{EJ} P^{AD} P^{HK} \overline{P}^{CL} \overline{P}^{FI} - P^{AD} P^{EJ} P^{HK} P^{BG} \overline{P}^{CF} \overline{P}^{IL} \right)$$

$$+ P^{AD} P^{BE} P^{FK} P^{GJ} P^{CH} P^{IL} + \frac{4}{3} P^{AD} P^{BG} P^{FK} P^{CJ} P^{EH} P^{IL} \right)$$

and

$$\mathcal{R}^{(+)} = \mathcal{R}^{(-)} [P \leftrightarrow \overline{P}]$$
Duality covariance vs gauge covariance

- These generalized GS transformations reproduce the standard GS transformations

- This can be seen parametrizing the G and H-invariant metrics as

\[
\eta_{MN} = \begin{pmatrix} 0 & \delta^\mu_v \\ \delta^\nu_{\mu} & 0 \end{pmatrix} \equiv \eta^{MN}, \quad \eta_{AB} = \begin{pmatrix} g_{ab} & 0 \\ 0 & -g_{ab} \end{pmatrix}, \quad \mathcal{H}_{AB} = \begin{pmatrix} g_{ab} & 0 \\ 0 & g_{ab} \end{pmatrix},
\]

the H-parameter as

\[
\Lambda^{AB}_A = \begin{pmatrix} \Lambda^{(+)}_{ab} \equiv 0 \\ \Lambda^{(-)}_{a} \end{pmatrix}, \quad \Lambda^{(+)} = \Lambda^{(-)} = \Lambda
\]

and the generalized frame and invariant dilaton as

\[
E_{M}^A = \frac{1}{\sqrt{2}} \left( \tilde{e}^\mu_{\alpha} - g^{ab} \tilde{e}^\mu_{\beta} \right), \quad \tilde{e}^a_{\mu} = \tilde{e}_{\mu}^a + g_{ab} \tilde{e}^\rho B_{\rho\mu}, \quad e^{-2d} = \sqrt{-\tilde{g} e^{-2\tilde{\phi}}}, \quad \tilde{g}_{\mu\nu} = \tilde{e}^a_{\mu} g^{ab} \tilde{e}^b_{\nu}
\]
Duality covariance vs gauge covariance

- The double Lorentz transformations of the generalized frame, induce Lorentz transformations of

\[
\delta \tilde{g}_{\mu\nu} = L_\xi \tilde{g}_{\mu\nu} - \frac{a}{2} \omega_{(\mu a}^{(-) b} \partial_{\nu)} \Lambda^a_b - \frac{b}{2} \omega_{(\mu a}^{(+) b} \partial_{\nu)} \Lambda^a_b
\]

\[
\delta \tilde{B}_{\mu\nu} = L_\xi \tilde{B}_{\mu\nu} + 2 \partial_{[\mu} \tilde{\xi}_{\nu]} + \frac{a}{2} \omega_{[\mu a}^{(-) b} \partial_{\nu]} \Lambda^a_b - \frac{b}{2} \omega_{[\mu a}^{(+) b} \partial_{\nu]} \Lambda^a_b
\]

- The non-standard transformation of \( g_{\mu\nu} \) can be removed through a first order non-covariant field redefinition

\[
\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{a}{4} \omega_{\mu a}^{(-) b} \omega_{\nu b}^{(-) a} - \frac{b}{4} \omega_{\mu a}^{(+) b} \omega_{\nu b}^{(+) a}
\]

such that \( \delta g_{\mu\nu} = L_\xi g_{\mu\nu} \).

- Generically for \( B \) this is not possible and gives the GS transformation in the heterotic string \( (a = -\alpha', b = 0) \).
Duality covariance vs gauge covariance

- The generalized frame/metric are not gauge covariant in the standard sense but they are duality covariant.

- The components $\tilde{g}_{\mu\nu}$ and $\tilde{B}_{\mu\nu}$ transform as usual under T-duality, but their induced gauge transformations are deformed.

- The duality covariant fields $\tilde{g}_{\mu\nu}$, $\tilde{B}_{\mu\nu}$ are related to the gauge covariant fields $g_{\mu\nu}$, $B_{\mu\nu}$ through non-covariant field redefinitions.
Rewriting the $\alpha'$-deformed DFT action in terms of the gauge covariant fields we obtain

$$R(E, d) + a R^{(-)} + b R^{(+)} =$$

$$= R + 4 g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{12} \hat{H}_{\mu \nu \rho} \hat{H}^{\mu \nu \rho}$$

$$+ \frac{a}{8} R^{(-)}_{\mu \nu a} R^{(-\mu \nu)} b^a + \frac{b}{8} R^{(+)}_{\mu \nu a} R^{(+\mu \nu)} b^a$$
Scherk-Schwarzing Green-Schwarz

- We now consider compactifications of this action to lower dimensions

- Generalized Scherk-Schwarz compactifications of DFT lead to half maximal gauged supergravities

  G. Aldazablar, W. Baron, D. Marqués, C.N. (2011)

- We expect to obtain the $\alpha'$-corrections to N=4 gauged supergravity

- The procedure has many advantages
  - Internal duality structure is preserved in the process: no need to reorganize dof
  - Provides all covariant field strengths as components of generalized fluxes
  - Allows relaxations of the strong constraint
We extend the duality group to gain generality $G = \text{O}(D,D+N)$

Then we split coordinates into external and internal: $D = n + d$

Organize the dof as $V^M = (\tilde{V}_\mu, V^\mu, V^m)$ where $(\tilde{V}_\mu, V^\mu)$ transform in the fundamental of $G_e = \text{O}(n,n) \in G$ and $V^m$ transforms in the fundamental of $G_i = \text{O}(d,d+N) \in G$

The $H$ group now splits in $H_e = \text{O}(n-1,1) \times \text{O}(1,n-1), H_i = \text{O}(d) \times \text{O}(d+N)$
The generalized frame and the metrics can be parametrized as

\[ E_M^A = \frac{1}{\sqrt{2}} \begin{pmatrix} \tilde{e}_a^\mu & 0 & 0 \\ -\tilde{e}_a^\rho (\tilde{B}_\rho + \frac{1}{2} \tilde{A}^p \tilde{A}_\rho^p) & \tilde{e}_\mu^a & \tilde{A}_\mu^p \tilde{\Phi}_p^\alpha \\ -\tilde{e}_a^\rho \tilde{A}_\rho^m & 0 & \tilde{\Phi}_m^\alpha \end{pmatrix} \]

\[ \eta_{AB} = \begin{pmatrix} 0 & \delta_b^a & 0 \\ \delta_a^b & 0 & 0 \\ 0 & 0 & \kappa_{\alpha\beta} \end{pmatrix}, \quad \Xi_{AB} = \begin{pmatrix} g^{ab} & 0 & 0 \\ 0 & g_{ab} & 0 \\ 0 & 0 & M_{\alpha\beta} \end{pmatrix}, \quad \eta_{MN} = \begin{pmatrix} 0 & \delta^\mu_v & 0 \\ \delta^\nu_\mu & 0 & 0 \\ 0 & 0 & \kappa_{mn} \end{pmatrix} \]

with \( \kappa_{\alpha\beta}, M_{\alpha\beta} \) the H_i invariant metrics and \( \kappa_{mn} \) the G_i invariant metric.

The Green Schwarz transformation induces
\[ \delta \tilde{g}_{\mu \nu} = L_\xi \tilde{g}_{\mu \nu} + \frac{1}{2} \left( a \omega_{(\mu)}^{(-)ab} + b \omega_{(\mu)}^{(+\alpha\beta)} \partial \nu \right) \Lambda_{ab} + \frac{1}{2} \left( a \omega_{(\mu)}^{(-\alpha\beta)} + b \omega_{(\mu)}^{(+\alpha\beta)} \partial \nu \right) \Lambda_{\alpha\beta} \]

where

\[ \omega_{\mu\alpha}^{(-)\beta} = \Phi_{\alpha}^m \nabla_\mu \Phi \beta_m, \quad \omega_{\mu\alpha}^{(+\beta)} = \Phi_{\alpha}^m \nabla_\mu \Phi \beta \bar{m} \]

which requires non-covariant field redefinitions such that

\[ \delta g_{\mu \nu} = L_\xi g_{\mu \nu} \]

and similarly for the other fields.

The generalized Green-Schwarz transformation implies

\[ \hat{H}_{\mu \nu \rho} = H_{\mu \nu \rho} - 3 \Omega_{\mu \nu \rho}^{(g)} - \frac{3}{2} a \Omega_{\mu \nu \rho}^{(e,-)} + \frac{3}{2} b \Omega_{\mu \nu \rho}^{(e,+) -} - \frac{3}{2} a \Omega_{\mu \nu \rho}^{(i,-)} + \frac{3}{2} b \Omega_{\mu \nu \rho}^{(i,+)} \]

\[ \Omega_{\mu \nu \rho}^{(g)} = A_{\mu}^m \partial \nu \omega_{\rho} - \frac{1}{3} f_{\mu \nu \rho} A_{\mu}^n A_{\nu}^p A_{\rho}^l, \quad \Omega_{\mu \nu \rho}^{(e,\pm)} = \omega_{[\mu \alpha} \partial \nu \omega_{\rho]}^{(\pm)a} + \frac{2}{3} \omega_{[\mu \alpha} \omega_{\nu \beta} \omega_{\rho]}^{(\pm)c} \]

\[ \Omega_{\mu \nu \rho}^{(i,\pm)} = \omega_{[\mu \alpha}^{(\pm)\beta} \partial \nu \omega_{\rho]}^{(\pm)\alpha} + \frac{2}{3} \omega_{[\mu \alpha}^{(\pm)\beta} \omega_{\nu \beta}^{(\pm)\gamma} \omega_{\rho]}^{(\pm)\alpha} \]
The action

- The final result is

\[
S = \int d^n X \sqrt{-g} e^{-\phi} \left( R + 4 g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{12} \hat{H}_{\mu\nu\rho} \hat{H}^{\mu\nu\rho} - \frac{1}{4} F_{\mu\nu}^m F^{n\mu\nu}_m M_{mn} + \frac{1}{8} \nabla_\mu M_{mn} \nabla^\mu M^{mn} - V_0 \right) + a L^{(-)} + b L^{(+)}
\]

where \( M_{mn} = \Phi_m^\alpha M^\alpha_{\alpha\beta} \Phi_n^\beta \)

- The first line is the standard form of half-maximal gauged sugra with \( \alpha' \)-corrections in the Chern-Simons terms in \( \hat{H}_{\mu\nu\rho} \)

- The zeroth order potential has the standard form

\[
V_0 = \frac{1}{12} f_{mp}^r f_{nq}^s M^{mn} M^{pq} M_{rs} + \frac{1}{4} f_{mp}^q f_{nq}^p M^{mn} + \frac{1}{6} f_{mnp} f^{mnp}
\]

- This is as expected from SS compactifications of DFT + GS transf.
The action

- The new piece of information is the second line where $L^{(\pm)}$ contain a huge number of terms

- I will discuss some simple special cases

- Taking $n=26$, $d=0$, $N=0$ and $(a,b)=(-\alpha',-\alpha')$ one recovers the bosonic string effective action

$$S_{bos} = \int d^{26}x \sqrt{-g} e^{-2\phi} \left( R + 4 g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi - \frac{1}{12} \hat{H}_{\mu\nu\rho} \hat{H}^{\mu\nu\rho} - \frac{\alpha'}{8} R_{\mu\nu a}^{(-)} R^{(-)\mu\nu}_{b} a - \frac{\alpha'}{8} R_{\mu\nu a}^{(+)} R^{(+)\mu\nu}_{b} a \right)$$
The heterotic string

To recover the heterotic string we take \( n=10, \ d=0, \ N=496 \) and the gaugings \( f_{mnp} \) are the structure constants of \( \text{SO}(32) \) or \( \text{E}_8 \times \text{E}_8 \).

Since there are no scalar fields, we trivialize the frame \( \Phi_m^\alpha = \delta_m^\alpha \).

\( M_{mn} = -\kappa_{mn} \) the Killing metric of the gauge group and \( f_m^q f_n^p = \kappa_{mn} \).

\[
S_{\text{hete}} = \int d^{10}x \sqrt{-g} e^{-2\phi} \left[ R + 4 g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{12} \hat{H}_{\mu\nu\rho} \hat{H}^{\mu\nu\rho} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right. \\
+ \left. \frac{\alpha'}{8} \left( R^{(-)a}_{\mu \nu a} R^{(-)a}_{\mu \nu a} - \frac{1}{2} T_{\mu \nu} T^{\mu \nu} - \frac{3}{2} T_{\mu \nu \rho \sigma} T^{\mu \nu \rho \sigma} \right) \right]
\]

\[
T_{\mu \nu} = F_{\mu}^\rho m F_{\rho \nu m}, \quad T_{\mu \nu \rho \sigma} = F_{[\mu \nu}^m F_{\rho \sigma]m}
\]
\( \alpha' \)-corrections to \( \mathcal{N}=4 \) gauged sugra

- Taking \( b=0 \)

\[
S = \int d^n X \sqrt{-g} e^{-2\phi} \left( R + 4 g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{12} \hat{H}_{\mu \nu \rho} \hat{H}^{\mu \nu \rho} - \frac{1}{4} F_{\mu \nu}^m F^{n \mu \nu} M_{mn} + \frac{1}{8} \nabla_\mu M_{mn} \nabla^\mu M^{mn} - V_0 \right) + a L^{(-)}
\]

we obtain the \( n \)-dimensional effective action of the heterotic string with first order \( \alpha' \)-corrections.

- It is interesting to analyze the structure of the scalar potential.

- From a phenomenological viewpoint, gauged sugras offer the possibility to stabilize all moduli and provide a mechanism of supersymmetry breaking.
\(\alpha'\)-deformations of the moduli space

- A non-zero value of the scalar potential presents a possibility to explain a small value of the cosmological constant.

- However, an accelerating space time solution cannot be obtained from only the lowest order terms in the supergravity action.

- The \(\alpha'\)- corrections to the four dimensional scalar potential, eventually combined with non-perturbative effects, offer the possibility not only to modify the Minkowski minima to a small value but also to stabilize some of the massless modes, modify the flat directions of the lowest order theory or change the slow roll behavior in inflationary models.
Four dimensional de Sitter solutions?

- Four dimensional maximally symmetric de Sitter solutions have been ruled out in the perturbative $\alpha'$- expansion of string theory from generic analysis of both the spacetime and the worldsheet theories.

- However, the $\alpha'$-corrections can be combined with non-perturbative quantum corrections or localized sources to produce solutions with properties that cannot be obtained from two-derivative supergravity.

- There are examples of AdS$_4$ solutions in type IIB or in the heterotic string, in which the leading order Minkowski ground states are broken by higher-derivative terms that generate a nonzero $\Lambda$.

- We have studied the $\alpha'$- corrections to the known Minkowski vacua of $\mathcal{N}=4$ gauged sugra in $n=7$, and found no modifications.
Summary and conclusions

- The traditional formulation of DFT has a duality covariant gauge symmetry principle based on a generalized Lie derivative that determines the two-derivative effective action uniquely.

- Different parametrizations allow to make contact with the standard universal bosonic sector of supergravity and lower-dimensional half-maximal gauged supergravities.

- The duality covariant gauge symmetry principle can be extended to include first-order deformations that account for the first order $\alpha'$-corrections to the bosonic and heterotic string effective actions and to half-maximal gauged sugra in arbitrary dimensions as well as to other duality covariant theories.
One remarkable aspect of the effective action is that the scalar potential receives an unambiguous first order correction.

Understanding how this correction affects the vacuum structure may have interesting phenomenological consequences.

Other promising lines for the future are
- To understand how to incorporate higher order terms in this formalism
- The parameter space can be further constrained by supersymmetry. We expect that only the deformations that correspond to the heterotic string admit supersymmetrization.
- Finding consistent higher-derivative deformations in Exceptional Field Theories (which are S-duality covariant)
- Exploring if the generalized Green-Schwarz transformation, among others, can shed light on the discussion about the geometry of DFT
GRACIAS!
Closure of the algebra

- These gauge transformations preserve the constraints and close

\[
\left[ \delta_{(\xi_1, \Lambda_1)}, \delta_{(\xi_2, \Lambda_2)} \right] = \delta_{(\xi_{21}, \Lambda_{21})}
\]

w.r.t. the modified brackets

\[
\xi_{12}^M = [\xi_1, \xi_2]^M_{(C)} - \frac{a}{2} \Lambda_{[1A}^B \Lambda_{2]}^A + \frac{b}{2} \Lambda_{[1\bar{A}}^B \Lambda_{2]\bar{A}}^A
\]

\[
\Lambda_{12A}^B = 2\xi_1^P \partial_P \Lambda_{2]}^A - 2\Lambda_{[1A}^C \Lambda_{2]}^B
\]

\[
+ a \partial_{[A} \Lambda_{1}^{CD} \partial_{B]} \Lambda_{2DC} + a \partial_{[A} \Lambda_{1}^{CD} \partial_{B]} \Lambda_{2DC}
\]

\[
- ba \partial_{[A} \Lambda_{1}^{CD} \partial_{B]} \Lambda_{2DC} - b \partial_{[A} \Lambda_{1}^{CD} \partial_{B]} \Lambda_{2DC}
\]

\[
[\xi_1, \xi_2]^M_{(c)} = \xi_1^P \partial_P \xi_2^M - \xi_2^P \partial_P \xi_1^M - \frac{1}{2} \xi_1^P \partial^M \xi_{2P} + \frac{1}{2} \xi_2^P \partial^M \xi_{1P}
\]
Scherk-Schwarzing Green-Schwarz

- We proceed as follows

1) Gauge DFT \[ f_{MNP} = f_{[MNP]}, \quad f_{[MN}^R f_{P]R}^Q = 0 \]

2) \[ \hat{S}_\xi V^M \rightarrow \hat{S}_\xi V^M + f_{PQ}^M \xi^P V^Q \]

3) \[ F_{ABC} \rightarrow F_{ABC} + f_{ABC} \]

4) New constraint \[ f_{MN}^P \partial_P \ldots = 0 \]

These components of the spin connection are the only ones completely determined from the conditions of vanishing torsion, compatibility with \( \eta \) & \( \mathcal{H} \) and covariance under generalized diffeomorphisms and double Lorentz transformations.
