Electroweak Vacuum Stability and Gravitational Vacuum Polarization in Schwarzschild Black-hole Background

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In this paper, we investigate how the gravitational vacuum polarization affects stabilities of the electroweak vacuum in the Schwarzschild black-hole background. By using the renormalized vacuum field fluctuation \(\langle \delta \phi^2 \rangle_{\text{ren}}\) which is approximately expressed by the thermal Hawking temperature \(T_H\) around the black-hole horizon \((r \approx 2M_{BH})\), we discuss the vacuum stability around the Schwarzschild black hole. In particular, we newly investigate the stability of the electroweak vacuum around evaporating primordial black holes (PBHs) by taking into account the back-reaction effects on the effective Higgs potential \(V_{eff}(\phi)\) which was ignored in past works. By incorporating these effects and analyzing the stability of the vacuum, we show that one evaporating black hole does not cause serious problems in vacuum stability of the standard model Higgs and obtain an upper bound on the evaporating PBH abundance \(\beta \lesssim \mathcal{O}(10^{-21})\) \((m_{PBH}/10^9 \text{g})^{3/2}\) not to induce any catastrophes.

I. INTRODUCTION

In the late 1970s, Hawking [1] showed that black holes emit thermal radiation at the Hawking temperature \(T_H = 1/8\pi r_{BH}\) due to the vacuum-polarization effects on the strong gravitational field [2] where \(M_{BH}\) is the black-hole mass. The gravitational polarization around the black hole determines the fate of the evaporating black hole which is still unknown and closely related with the information loss puzzle [3], and furthermore, leads to the spontaneous symmetry restoration [4] or the false vacuum decay around the black hole [5–7], which bring cosmological singular possibility. Thus, the gravitational vacuum-polarization effects on the black hole background have a great impact on the stability of the vacuum.

One of the most curious puzzles of the observed Higgs boson at the LHC experiment [8–11] is that the effective Higgs potential develops an instability at the high scale \(\Lambda_H \approx 10^{11}\) GeV [12] where we assume no corrections of the beyond Standard Model (BSM) and the quantum gravity (QG) [13–16]. Therefore, the current electroweak vacuum state of the Universe seems to be not stable, and finally cause a catastrophic vacuum decay through quantum tunneling [17–19] although the timescale of the vacuum decay is longer than the age of our Universe [20–23]. However, the vacuum fluctuation resulting from the gravitational polarization can affect the fate of the electroweak vacuum. The recent works suggest that the electroweak vacuum becomes unstable on various gravitational or cosmological backgrounds, e.g. during inflation [24–33] corresponding to the de-Sitter spacetime, after inflation [34–39] in particular the preheating stage and on Schwarzschild black-hole background [40–46].

Particularly recent discussion about the stability of the electroweak vacuum around the evaporating black hole has been growing. There are no general mechanisms to prevent the formation of such small black holes which finally evaporate during the history of the Universe. Especially, the primordial black holes (PBH) are formulated by large density fluctuations in the early universe [47–49] instead of the zero-temperature potential in the literature [50, 51]. These phenomena could impose serious constraints on cosmology or beyond Standard Model.

However, there is some controversy about whether the evaporating black hole can be a trigger of the false vacuum decay on the electroweak vacuum. It is because the backreaction of the thermal Hawking radiation can not be ignored [45, 46, 50, 52]. In the literature [40–42], the false vacuum decay in the Schwarzschild black hole has been investigated by the Coleman-De Luccia (CDL) instanton method [53] with the zero-temperature effective Higgs potential. However, it is reasonable intuitively to assume the high-temperature effective Higgs potential [54–58] instead of the zero-temperature potential in the environment of the thermal Hawking flux. The thermal corrections can generally stabilize the effective Higgs potential, and furthermore, the false vacuum decay from strong gravitational field can only happen around the black hole horizon. That is approximately we can obtain the vacuum decay ratio of the Minkowski spacetime only far from the black hole, which is extremely small. From this viewpoint, the probability of vacuum decay around the black hole can be expected to be lower than what was considered in the literature [40–42]. Nevertheless, there are no formal descriptions how the gravitational vacuum polarization like the Hawking radiation modify the effective Higgs potential and definitely affect the stability of the vacuum so far.

Formally, the gravitational vacuum polarization effects can be described by the vacuum expectation value of the energy momentum tensor \(\langle T_{\mu\nu} \rangle\) or the two-point correlation function \(\langle \delta \phi^2 \rangle\) in the quantum field theory (QFT) in curved spacetime. The former \(\langle T_{\mu\nu} \rangle\) provides a exact description of the quantum back-reaction on the geometry.
and it is crucial in order to determine the backreaction of the conformal anomaly or the fate of the evaporating black hole. The two-point correlation function $\langle \delta \phi^2 \rangle$ corresponds to the vacuum field fluctuation and plays the essential role in the vacuum stability. In the present paper, we provide a quantitative description of the vacuum stability on the Schwarzschild background by considering the vacuum field fluctuation $\langle \delta \phi^2 \rangle$, and investigate the electroweak vacuum stability near the black hole. In our analysis, we provide a new description of these phenomena and reach opposite conclusions in past works.

The organization of this paper is as follows. In Sec. II, we introduce renormalized vacuum field fluctuations for various vacua in the Schwarzschild spacetime. In Sec. III, stabilities of the electroweak vacuum around the block hole are discussed. Sec. IV is devoted to our conclusions and future outlooks.

**II. THE RENORMALIZED VACUUM FIELD FLUCTUATION IN SCHWARZSCHILD SPACETIME**

In this section, we consider the renormalized vacuum field fluctuation in Schwarzschild black-hole spacetime. The renormalized expression of the vacuum fluctuation for the massless scalar field has been well-known and analytical estimation is possible. However, the massive case requires complicated numerical calculations. For briefness we consider the renormalized vacuum field fluctuation for the massless scalar field in Boulware, Unruh and Hartle-Hawking vacuum following Candelas work [60]. In Section III, we will discuss the vacuum stability by using the renormalized vacuum field fluctuation.

The metric in the Schwarzschild coordinates where we ignore the quantum backreaction of the scalar field on the geometry can be written by

$$ds^2 = -\left(1 - \frac{2M_{BH}}{r}\right)dt^2 + \frac{dr^2}{1 - \frac{2M_{BH}}{r}} + r^2(d\theta^2 + \sin^2 \theta d\phi^2),$$

which can cover the exterior region $r > 2M_{BH}$ of the spacetime where $M_{BH}$ is the black hole mass. The above singularity at the horizon $r = 2M_{BH}$ can be removed by transforming to Kruskal coordinates. By taking the Kruskal coordinates, we can obtain the following metric

$$ds^2 = \frac{32M_{BH}^3}{r}e^{-r/2M_{BH}}dUdV + r^2(d\theta^2 + \sin^2 \theta d\phi^2),$$

where these coordinates $U$ and $V$ are formally given by

$$U = -M_{BH}e^{-u/4M_{BH}}, \quad V = M_{BH}e^{v/4M_{BH}},$$

$$u = t - r - 2M_{BH} \ln \left(\frac{r}{2M_{BH}} - 1\right),$$

$$v = t + r + 2M_{BH} \ln \left(\frac{r}{2M_{BH}} - 1\right).$$

The Schwarzschild coordinates of Eq. (1) cover only a part of the spacetime, whereas the Kruskal coordinates of Eq. (2) cover the extended spacetime and becomes regular at the black hole horizon. These features of the Schwarzschild geometry are summarized in the Penrose-Carter diagrams as Fig. 1. In the curved spacetime, there are no unique vacua and we must take an appropriate vacuum state. In the Schwarzschild spacetime, there are three well defined vacua, namely: the Boulware vacuum (vacuum polarization around a static star) [61, 62], the Unruh vacuum (black hole evaporation) [63] and the Hartle-Hawking vacuum (black hole in thermal equilibrium) [64] which correspond to the definitions of the normal ordering on the respective coordinates.

The Klein-Gordon equation for the massless scalar field $\phi(t, x)$ can be given by

$$[-\partial_\mu g^{\mu\nu} \sqrt{-g} \partial_\nu] \phi(t, x) = 0,$$

where we drop the curvature term $\xi \nabla^2 g$ because the Ricci scalar becomes $R = 0$ in Schwarzschild spacetime [65] for simplicity, but this approximation may break down when the quantum backreaction on the metric can not be neglected. In the exterior region of Schwarzschild spacetime, the scalar field $\phi(t, r, \theta, \varphi)$ can be decomposed into the form

$$\phi(t, r, \theta, \varphi) = \int_0^\infty dw \sum_{l=0}^\infty \sum_{m=-l}^l \left( a_{\omega lm} u_{\omega lm}^{in} + a_{\omega lm}^{*-1} b_{\omega lm} u_{\omega lm}^{out} \right) + a_{\omega lm}^{*-1} b_{\omega lm} u_{\omega lm}^{out*},$$

where these mode functions $u_{\omega lm}^{in}$ and $u_{\omega lm}^{out}$ defines the vacuum state that $a_{\omega lm}|0\rangle = b_{\omega lm}|0\rangle = 0$ which corresponds to the boundary conditions. In the Schwarzschild spacetime, these mode functions $u_{\omega lm}^{in}$ and $u_{\omega lm}^{out}$ for the
massless scalar field are given by

$$u_{in}^{lm} = (4\pi \omega)^{-1/2} R_{in}^{lm}(r; \omega) Y_{lm}(\theta, \varphi) e^{-i\omega t}, \quad (6)$$

$$u_{out}^{lm} = (4\pi \omega)^{-1/2} R_{out}^{lm}(r; \omega) Y_{lm}(\theta, \varphi) e^{-i\omega t}, \quad (7)$$

where these radial functions $R_{in}^{lm}(r; \omega)$ and $R_{out}^{lm}(r; \omega)$ have the well-known asymptotic forms,

$$R_{in}^{lm}(r; \omega) \simeq \left\{ \begin{array}{ll}
B_l(\omega) r^{-1} e^{-i\omega r} & (r \rightarrow 2M_{BH}) \\
-1 e^{-i\omega r} + A_l^{in}(\omega) r^{-1} e^{i\omega r} & (r \rightarrow \infty)
\end{array} \right.$$ (8)

$$R_{out}^{lm}(r; \omega) \simeq \left\{ \begin{array}{ll}
r^{-1} e^{i\omega r} + A_l^{out}(\omega) r^{-1} e^{-i\omega r} & (r \rightarrow 2M_{BH}) \\
B_l(\omega) r^{-1} e^{i\omega r} & (r \rightarrow \infty)
\end{array} \right.$$ (9)

$A_l^{in}(\omega)$, $A_l^{out}(\omega)$, and $B_l(\omega)$ are the reflection and transmission coefficients [66]. The Boulware vacuum $|0_B\rangle$ is defined by taking ingoing and outgoing modes to be possible with frequency with respect to the Killing vector $\delta_0$ of the Schwarzschild metric [61] and constructed by using the scattering theory interpretation. This state closely reproduces the Minkowski vacuum $|0_M\rangle$ at infinity because $\langle 0_B| \delta_0^2 |0_B\rangle \rightarrow 1/r^2$ in the limit $r \rightarrow \infty$. However, the Boulware vacuum is singular on the event horizons $r = 2M_{BH}$ and hence unacceptable near the black-hole horizon. Therefore, the Boulware vacuum is considered to be the appropriate vacuum state which describes the vacuum polarization around a static star.

The two-point correlation function $\langle \delta \phi^2 \rangle$ related with the Boulware vacuum $|0_B\rangle$ can be given by [60, 67]:

$$\langle 0_B| \delta \phi^2 (x)|0_B\rangle = \frac{1}{16\pi^2} \int_0^\infty d\omega \omega \left[ \sum_{l=0}^{\infty} (2l+1) \left| R_{in}^{lm}(r; \omega) \right|^2 + \left| R_{out}^{lm}(r; \omega) \right|^2 \right], \quad (8)$$

where the sum of these radial functions $R_{in}^{lm}(r; \omega)$ and $R_{out}^{lm}(r; \omega)$ have the asymptotic forms,

$$\sum_{l=0}^{\infty} (2l+1) \left| R_{in}^{lm}(r; \omega) \right|^2 \sim \left\{ \begin{array}{ll}
\frac{\Sigma_{l=0}^{\infty} (2l+1) |B_l(\omega)|^2}{4M_{BH}^2} & (r \rightarrow 2M_{BH}) \\
\frac{4\omega^2}{r^2} & (r \rightarrow \infty)
\end{array} \right.$$ (9)

$$\sum_{l=0}^{\infty} (2l+1) \left| R_{out}^{lm}(r; \omega) \right|^2 \sim \left\{ \begin{array}{ll}
\frac{\Sigma_{l=0}^{\infty} (2l+1) |B_l(\omega)|^2}{r^2} & (r \rightarrow 2M_{BH}) \\
\frac{4\omega^2}{r^2} & (r \rightarrow \infty)
\end{array} \right.$$ (10)

Therefore, the two-point correlation function $\langle \delta \phi^2 \rangle$ of Eq. (8) has clearly UV divergences and must be regularized. There are several regularization methods to eliminate the UV divergences in the quantum field theory (QFT), but the point-splitting regularization is the extremely powerful and standard method to obtain the renormalized expression in the curved spacetime. Let us consider temporarily $\delta \phi^2 (x) \rightarrow \delta \phi (x) \delta \phi (x')$ to remove the divergences and afterwards take the coincident limit $x' \rightarrow x$.

$$\langle \delta \phi^2 (x) \rangle_{ren} = \lim_{x' \rightarrow x} \left[ \langle \delta \phi (x) \delta \phi (x') \rangle - \langle \delta \phi (x) \delta \phi (x') \rangle_{div} \right], \quad (9)$$

where $\langle \delta \phi (x) \delta \phi (x') \rangle_{div}$ express the divergence part and is namely the DeWitt-Schwinger counter-term, which can be generally given by [68]

$$\langle \delta \phi (x) \delta \phi (x') \rangle_{div} = \frac{1}{8\pi^2} \left[ m^2 + \frac{\xi - 1/6}{2} R + \frac{1}{2} \ln \left( \frac{\mu^2}{\sigma} \right) - \frac{1}{2} \left( \frac{m^2}{16\pi^2} + \frac{1}{96\pi^2} R_{\alpha\beta} \sigma^{\alpha\beta} \right) \omega \right], \quad (10)$$

where $\sigma$ is the biscalar associated with the short geodesic, $R$ or $R_{\alpha\beta}$ are respectively the Ricci scalar or tensor and $\gamma$ express the Euler-Mascheroni constant. The renormalization parameter $\mu$ corresponds to the mass $m$ of the scalar field, and therefore, the massless case lead to the well-known ambiguity [69], but the renormalization procedure can eliminate this ambiguity for $(T_{\mu\nu})$ by the cosmological experiment or observation. In the Schwarzschild metric for the massless scalar field where $m = 0$ and $R = 0$, we can simplify the DeWitt-Schwinger counter-term of $\langle \delta \phi (x) \delta \phi (x') \rangle_{div}$ to be

$$\langle \delta \phi (x) \delta \phi (x') \rangle_{div} = \frac{1}{8\pi^2} \sigma. \quad (11)$$

For simplicity we take the time separation as $x = (t, r, \theta, \varphi)$ and $x' = (t + \epsilon, r, \theta, \varphi)$ and the renormalized expression of $\langle \delta \phi^2 \rangle$ in the Boulware vacuum $|0_B\rangle$ can be given by

$$\langle \delta \phi^2 (x) \rangle_{ren} = \frac{1}{16\pi^2} \int_0^\infty d\omega \omega \left[ \sum_{l=0}^{\infty} (2l+1) \left| R_{in}^{lm} (r; \omega) \right|^2 + \left| R_{out}^{lm} (r; \omega) \right|^2 \right] dw - \frac{1}{8\pi^2} \sigma \epsilon^2. \quad (12)$$

By taking a second-order geodesic expansion we can obtain the following expression [68]

$$\sigma \epsilon = -\frac{1}{2} \frac{2M_{BH}/r}{2} \epsilon^2 - \frac{M_{BH}^2 (1 - 2M_{BH}/r)}{24r^4} \epsilon^4 + O (\epsilon^5), \quad (13)$$

where $\epsilon^2$ satisfy the following relation

$$\epsilon^2 = -\int_0^\infty \omega e^{i\omega t} dw. \quad (14)$$

By using Eq. (12), Eq. (13) and Eq. (14), we can obtain the renormalized expression of the Boulware vacuum $|0_B\rangle$ [60],

$$\langle 0_B| \delta \phi^2 (x)|0_B\rangle_{ren} = \frac{1}{16\pi^2} \int_0^\infty d\omega \omega \left[ \sum_{l=0}^{\infty} (2l+1) \left| R_{in}^{lm} (r; \omega) \right|^2 + \left| R_{out}^{lm} (r; \omega) \right|^2 \right] - \frac{M_{BH}^2}{48\pi^2 r^4 (1 - 2M_{BH}/r)} \quad (15)$$

$$- \frac{M_{BH}^2}{48\pi^2 r^4 (1 - 2M_{BH}/r)}.$$
For the Boulware vacuum $|0_B\rangle$ we have the asymptotic expression of the renormalized vacuum field fluctuation $\langle \delta \phi^2 \rangle_{\text{ren}}^{[60]}$,
\[
\langle 0_B | \delta \phi^2 (x) | 0_B \rangle_{\text{ren}} \to \infty \quad (r \to 2M_{\text{BH}}),
\langle 0_B | \delta \phi^2 (x) | 0_B \rangle_{\text{ren}} \to 1/r^2 \quad (r \to \infty),
\]
where the renormalized expression of $\langle \delta \phi^2 \rangle_{\text{ren}}$ is singular on the event horizons $r = 2M_{\text{BH}}$ and ill-defined near the black-hole horizon. In the case of the energy momentum tensor $\langle T_{\mu \nu} \rangle_{\text{ren}}$, the renormalized expression of the energy momentum tensor $\langle T_{\mu \nu} \rangle_{\text{ren}}$ was given by Ref.\[60, 69–74\] and shows similar properties to the renormalized expression of $\langle \delta \phi^2 \rangle_{\text{ren}}^{[75]}$. Therefore, the usual interpretation of the above result is that the Boulware vacuum $|0_B\rangle$ is considered to be the appropriate vacuum state around a static star and not a black hole.

Next, we consider the Unruh vacuum $|0_U\rangle$, which corresponds to the evaporating black hole in the empty space. The Unruh vacuum $|0_U\rangle$ is formally defined by taking ingoing modes to be positive frequency with respect to $\partial \nu$, but outgoing modes to be positive frequency with respect to the Kruskal coordinate $\partial \nu$ [63]. The Unruh vacuum corresponds to the state where the black hole radiates at the Hawking temperature $T_H = 1/8\pi M_{\text{BH}}$ in the empty space, and therefore, the vacuum field fluctuation $\langle \delta \phi^2 \rangle_{\text{ren}}$ approaches the thermal fluctuation near the black-hole horizon as $\langle 0_U | \delta \phi^2 | 0_U \rangle \to \mathcal{O} (T_H^2)$ in the limit $r \to 2M_{\text{BH}}$. Therefore, the Unruh vacuum is considered to be appropriate vacua which describe the evaporating black hole formed by gravitational collapse [60].

For the Unruh vacuum we obtain the two-point correlation functions $\langle \delta \phi^2 \rangle_{\text{ren}}^{[60, 67]}$,
\[
\langle 0_U | \delta \phi^2 (x) | 0_U \rangle = \frac{1}{16\pi^2} \int_0^{\infty} \frac{d\omega}{\omega} \left[ \sum_{l=0}^{\infty} (2l + 1) \left[ |R_{\text{in}}^l (r; \omega)|^2 + \coth \left( \frac{\pi \omega}{\kappa} \right) |R_{\text{out}}^l (r; \omega)|^2 \right] \right],
\]
where we introduce $\kappa = (4M_{\text{BH}})^{-1}$ which is the surface gravity of the black hole and the factor of $\coth \left( \frac{\pi \omega}{\kappa} \right)$ originates from the thermal features of the outgoing modes. The renormalized vacuum field fluctuation in the Unruh vacuum $|0_U\rangle$ can be give by
\[
\langle 0_U | \delta \phi^2 (x) | 0_U \rangle_{\text{ren}} = \frac{1}{16\pi^2} \int_0^{\infty} \frac{d\omega}{\omega} \left[ \sum_{l=0}^{\infty} (2l + 1) \left[ |R_{\text{in}}^l (r; \omega)|^2 + \coth \left( \frac{\pi \omega}{\kappa} \right) |R_{\text{out}}^l (r; \omega)|^2 \right] - \frac{4\omega^2}{1 - 2M_{\text{BH}}/r} \right] \frac{M_{\text{BH}}^2}{48\pi^2 r^4 (1 - 2M_{\text{BH}}/r)}.
\]
For the Unruh vacuum $|0_U\rangle$, we can obtain asymptotic expression of the renormalized vacuum field fluctuation $\langle \delta \phi^2 \rangle_{\text{ren}}^{[60]}$,
\[
\langle 0_U | \delta \phi^2 (x) | 0_U \rangle_{\text{ren}} \to \frac{1}{192\pi^2 M_{\text{BH}}^2} \int_0^{\infty} \frac{d\omega}{\omega} \left[ \sum_{l=0}^{\infty} (2l + 1) |B_l (\omega)|^2 \right] \quad (r \to 2M_{\text{BH}}),
\langle 0_U | \delta \phi^2 (x) | 0_U \rangle_{\text{ren}} \to 1/r^2 \quad (r \to \infty).
\]

The Hartle-Hawking Vacuum $|0_{\text{HH}}\rangle$ is formally defined by taking ingoing modes to be positive frequency with respect to $\partial \nu$, and outgoing modes to be positive frequency with respect to the Kruskal coordinate $\partial \nu$ [64]. In the Hartle-Hawking vacuum $|0_{\text{HH}}\rangle$, we can obtain the two-point correlation functions,
\[
\langle 0_{\text{HH}} | \delta \phi^2 (x) | 0_{\text{HH}} \rangle = \frac{1}{16\pi^2} \int_0^{\infty} \frac{d\omega}{\omega} \left[ \coth \left( \frac{\pi \omega}{\kappa} \right) \sum_{l=0}^{\infty} (2l + 1) \left[ |R_{\text{in}}^l (r; \omega)|^2 + |R_{\text{out}}^l (r; \omega)|^2 \right] \right].
\]

For the Hartle-Hawking vacuum we can get the asymptotic expression of the renormalized vacuum field fluctuation $\langle \delta \phi^2 \rangle_{\text{ren}}^{[60]}$,
\[
\langle 0_{\text{HH}} | \delta \phi^2 (x) | 0_{\text{HH}} \rangle_{\text{ren}} \to \frac{1}{192\pi^2 M_{\text{BH}}^2} \quad (r \to 2M_{\text{BH}}),
\langle 0_{\text{HH}} | \delta \phi^2 (x) | 0_{\text{HH}} \rangle_{\text{ren}} \to T_H^2/12 \quad (r \to \infty),
\]
where the renormalized vacuum field fluctuation $\langle \delta \phi^2 \rangle_{\text{ren}}$ becomes exactly the thermal fluctuation at infinity, i.e $\langle 0_{\text{HH}} | \delta \phi^2 | 0_{\text{HH}} \rangle \to T_H^2/12$ in the limit $r \to \infty$. Therefore, the Hartle-Hawking vacuum corresponds to a black hole in thermal equilibrium at $T_H = 1/8\pi M_{\text{BH}}$.

The analytic approximations of $\langle \delta \phi^2 \rangle_{\text{ren}}$ or $\langle T_{\mu \nu} \rangle_{\text{ren}}$ in the Schwarzschild spacetime for the various vacua (Boulware, Unruh vacuum and Hartle-Hawking) and the various fields of spin 0, 1/2 and 1 can be given by Ref.\[60, 69–74, 76–85\]. The renormalized expression of the various fields are proportional to the inverse of the black-hole mass $M_{\text{BH}}$ near the black-hole horizon and approximately approach the thermal fluctuations with the Hawking temperature $T_H$. This fact means that $\langle \delta \phi^2 \rangle_{\text{ren}}$ and $\langle T_{\mu \nu} \rangle_{\text{ren}}$ originate from the gravitational-polarization effects around the black hole.

### III. THE STABILITY OF THE ELECTROWEAK VACUUM AROUND THE BLOCK HOLE

In this section, we describe how the vacuum field fluctuation affects the stability of the vacuum and then discuss the electroweak vacuum stability around the evaporating block hole where $r \approx 2M_{\text{BH}}$. Now, for briefness we restrict our attention to the scalar field theory and assume a simple scalar potential $V (\phi)$ where the scalar
field \( \phi \) couples the extra scalar field \( \varphi \) with the positive interaction coupling \( g \),

\[
V(\phi) = \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{4}\varphi^4 + \frac{g}{2}\varphi^2\phi^2.
\]

where we assume that the self-interaction coupling \( \lambda \) is positive (\( \lambda > 0 \)). As previously discussed, the renormalized vacuum field fluctuation \( \langle \delta\phi^2 \rangle_{\text{ren}} \) near the black hole can be approximately given by

\[
\langle \delta\phi^2 \rangle_{\text{ren}} \simeq \langle \delta\varphi^2 \rangle_{\text{ren}} \simeq \mathcal{O}(1/M_{\text{BH}}^2).
\]

The vacuum field fluctuation \( \langle \delta\phi^2 \rangle_{\text{ren}} \) from the gravitational vacuum polarization is completely classic and can modify the scalar potential of Eq. (19) as follows:

\[
V(\phi) = \frac{1}{2}(m^2 - \lambda\langle \delta\phi^2 \rangle_{\text{ren}} + g\langle \delta\varphi^2 \rangle_{\text{ren}})\phi^2 - \frac{\lambda}{4}\phi^4,
\]

where we shift the scalar field \( \phi^2 \rightarrow \phi^2 + \langle \delta\phi^2 \rangle_{\text{ren}} \) and \( \varphi^2 \rightarrow \varphi^2 + \langle \delta\varphi^2 \rangle_{\text{ren}} \) in order to take into account the gravitational vacuum polarization. For the relatively large vacuum fluctuation of \( \phi \) to be \( \lambda\langle \delta\phi^2 \rangle_{\text{ren}} \gtrsim g\langle \delta\varphi^2 \rangle_{\text{ren}} \) and \( \langle \delta\phi^2 \rangle_{\text{ren}} \gtrsim m^2/\lambda \), the scalar potential \( V(\phi) \) is destabilized. On the other hand, the scalar potential of Eq. (21) can be stabilized and becomes metastable state when \( \lambda\langle \delta\phi^2 \rangle_{\text{ren}} \lesssim g\langle \delta\varphi^2 \rangle_{\text{ren}} \). However, the vacuum fluctuation can cause directly the false vacuum decay in the metastable vacuum. In fact, when the inhomogeneous and local scalar field described stochastically by \( \langle \delta\phi^2 \rangle_{\text{ren}} \) exceeds the hill of \( V(\phi) \), the localized scalar field can classically form the true vacuum domains or bubbles.

In the case of the electroweak vacuum around the black-hole, the Higgs field fluctuation work to push down the Higgs potential due to the negative running correction of the Higgs self-coupling, whereas the contributions of the gauge bosons or the fermions raise the effective Higgs potential. Therefore, it turns out that the Higgs vacuum stability around the black hole corresponds approximately to the local thermal situations although it is complicated task to investigate the stability of the electroweak vacuum around the black hole.

Let us consider the effective Higgs potential \( V_{\text{eff}}(\phi) \) in the standard model (SM) where \( \phi \) is the Higgs field. The one-loop standard model Higgs potential without the curvature mass \( \xi R \phi^2 \) in the 't Hooft-Landau gauge can be given by [86–88]

\[
V_{\text{eff}}(\phi) = \frac{\lambda_{\text{eff}}(\phi)}{4}\phi^4,
\]

where \( \lambda_{\text{eff}}(\phi) \) is the effective self-coupling of \( \phi \) written by

\[
\lambda_{\text{eff}}(\phi) = \frac{4\rho_\Lambda}{\phi^4} + \frac{2m_\varphi^2}{\phi^2} + \lambda_{\phi} + \sum_{i=W,Z,t,G,H} n_i m_i^4(\phi) \frac{\log \frac{m_i^2}{\mu^2} - C_i}{16\pi^2\phi^4},
\]

where \( \mu \) is the renormalization scale and \( \rho_\Lambda \) is the cosmological constant. The coefficients \( n_i \) and \( C_i \) are given by

\[
\begin{align*}
n_W &= 6, \ n_Z = 3, \ n_t = -12, \ n_G = 3, \ n_H = 1, \\
C_W &= C_Z = 5/6, \ C_t = C_G = C_H = 3/2,
\end{align*}
\]

and the mass terms \( m_i^2(\phi) \) of the W and Z bosons, the top quark, the Nambu-Goldstone bosons, and the Higgs boson can be given by

\[
\begin{align*}
m_W^2 &= \frac{1}{4}g_2^2\phi^2, \ m_Z^2 = \frac{1}{4}[g_2^2 + g_t^2]\phi^2, \ m_t^2 = \frac{1}{2}y_t^2\phi^2, \\
m_G^2 &= m_0^2 + \lambda_{\phi}\phi^2, \ m_H^2 = m_0^2 + 3\lambda_{\phi}\phi^2,
\end{align*}
\]

where \( g, g', y_t \) are the \( SU(2)_L, U(1)_Y \), top Yukawa couplings and \( \lambda_{\phi} \) is the Higgs self-coupling. From the viewpoint of the renormalization group (RG), the effective self-coupling \( \lambda_{\text{eff}}(\phi) \) of Eq. (23) becomes negative when the classic Higgs field is larger than the instability scale to be \( \phi \gtrsim \Lambda_f \) [20].

Now let us improve the effective Higgs potential of Eq. (22) by including \( \langle \delta\phi^2 \rangle_{\text{ren}} \) in the Schwarzschild background. In order to include the back-reaction of the vacuum fluctuation of the Higgs field, let us shift the Higgs field \( \phi^2 \rightarrow \phi^2 + \langle \delta\phi^2 \rangle_{\text{ren}} \). Thus, the effective Higgs potential with the Higgs vacuum fluctuation can be given by

\[
V_{\text{eff}}(\phi) = \frac{\lambda_{\text{eff}}(\phi, \langle \delta\phi^2 \rangle_{\text{ren}})}{4} \langle \delta\phi^2 \rangle_{\text{ren}}\phi^2 + \frac{\lambda_{\text{eff}}(\phi, \langle \delta\phi^2 \rangle_{\text{ren}})}{4} \phi^4,
\]

where the effective self-coupling \( \lambda_{\text{eff}}(\phi, \langle \delta\phi^2 \rangle_{\text{ren}}) \) can be negative and the effective Higgs potential is destabilized when \( \langle \delta\phi^2 \rangle_{\text{ren}}^{1/2} \gtrsim \Lambda_f \). In the Schwarzschild background, the renormalized expression of \( \langle \delta\phi^2 \rangle_{\text{ren}} \) proportional to the inverse of the black-hole mass \( M_{\text{BH}} \) or the Hawking temperature \( T_{\text{H}} \) and therefore, the effective self-coupling \( \lambda_{\text{eff}}(\phi, T_{\text{H}}) \) becomes negative with \( T_{\text{H}} \gtrsim \Lambda_f \). However, the vacuum fluctuation of the W and Z bosons or the top quark can raise the effective Higgs potential, and therefore, the effective Higgs potential \( V_{\text{eff}}(\phi) \) including the vacuum fluctuation of the various fields around the block hole can be written by

\[
V_{\text{eff}}(\phi) \simeq \left( \frac{1}{4}g_2^2 \langle \delta W^2 \rangle_{\text{ren}} + \frac{1}{4}[g_2^2 + g_t^2] \langle \delta Z^2 \rangle_{\text{ren}} \right)
+ \frac{1}{2}y_t^2 \langle \delta t^2 \rangle_{\text{ren}} + \frac{1}{2}\lambda_{\text{eff}} \langle \delta\phi^2 \rangle_{\text{ren}} \phi^2 + \frac{\lambda_{\text{eff}}}{4} \phi^4
\]

\[
\simeq \frac{1}{2}(\lambda_{\text{eff}} T_{\text{H}}^2 + \kappa^2 T_{\text{H}}^4) \phi^2 + \frac{\lambda_{\text{eff}}}{4} \phi^4,
\]

where \( \kappa \) is defined by \( g, g', y_t \) and \( \lambda \), and we assume that the vacuum fluctuation of the various field like the Higgs, W and Z bosons and the top quark approximately approach the Hawking thermal fluctuations near the black.
where the maximal field value \( \phi \).

By using Eq. (26), we can obtain the probability not to affect the false vacuum decay via the vacuum fluctuation on the black-hole background. These estimations are usually calculated by using the CDL instanton method [53] which requires a full numerical analysis for a range of parameter space. In the present paper, we discuss the false vacuum decay by using the stochastic method proposed by Linde [89–91] where it was applied to estimate the probability of the inflationary universe creation. This approach is exactly consistent with the instanton method and extremely simple even in the investigation of the false vacuum decay with taking account of the gravitational effects. Now, we consider the electroweak vacuum stability in the Schwarzschild black-hole background by using the stochastic method. The probability of the local Higgs fields where the vacuum fluctuation \( \langle \delta \phi^2 \rangle \) exists can be given by [32]

\[
P(\phi, \langle \delta \phi^2 \rangle) = \frac{1}{\sqrt{2\pi \langle \delta \phi^2 \rangle}} \exp \left(-\frac{\phi^2}{2 \langle \delta \phi^2 \rangle} \right),
\]

(26)

By using Eq. (26), we can obtain the probability not to exceed the hill of the effective Higgs potential as follows:

\[
P(\phi < \phi_{\text{max}}) = \int_{\phi_{\text{max}}}^{\phi_{\text{max}}} P(\phi, \langle \delta \phi^2 \rangle) d\phi,
\]

(27)

\[
= \text{erf} \left( \frac{\phi_{\text{max}}}{\sqrt{2 \langle \delta \phi^2 \rangle}} \right),
\]

(28)

where we define \( \phi_{\text{max}} \) to be the maximal field value of the effective Higgs potential. The probability that the localized Higgs fields go into the negative Planck-energy true vacuum is estimated to be

\[
P(\phi > \phi_{\text{max}}) = 1 - \text{erf} \left( \frac{\phi_{\text{max}}}{\sqrt{2 \langle \delta \phi^2 \rangle}} \right),
\]

(29)

\[
\simeq \sqrt{\frac{2 \langle \delta \phi^2 \rangle}{\pi \phi_{\text{max}}}} \exp \left(-\frac{\phi_{\text{max}}^2}{2 \langle \delta \phi^2 \rangle} \right),
\]

where the maximal field value \( \phi_{\text{max}} \) can be given by [92].

Then, the constraint from the vacuum decay for the Higgs field is represented by [32]:

\[
\mathcal{V} \cdot P(\phi > \phi_{\text{max}}) \lesssim 1,
\]

(31)

where \( \mathcal{V} \) express the volume factor or the number of the correlated patches. By substituting Eq. (29) into Eq. (31), we can simplify obtain a constraint from the vacuum stability,

\[
\frac{\langle \delta \phi^2 \rangle}{\phi_{\text{max}}} \lesssim \frac{1}{2} (\log \mathcal{V})^{-1},
\]

(32)

in order not to induce a transition due to large vacuum fluctuations. In the inflationary Universe, the volume factor of \( \mathcal{V} \) and the renormalized expression of \( \langle \delta \phi^2 \rangle \) can be given by [32]

\[
\mathcal{V} \sim e^{3N_{\text{hor}}}, \quad \langle \delta \phi^2 \rangle \sim \frac{3H^4}{8\pi^2 m_{\text{eff}}^4},
\]

(33)

where \( N_{\text{hor}} \) is the e-folding number which can be \( N_{\text{hor}} \approx N_{\text{CMB}} \approx 60 \). In the Unruh vacuum \( |\ell_0\rangle \) corresponding to the vacuum state around the evaporating black hole, the renormalized expression of \( \langle \delta \phi^2 \rangle \) approximately approach the value of the the Hawking temperature \( T_H \) near the horizon. But at the infinity \( \langle \delta \phi^2 \rangle \) attenuates rapidly and becomes zero. Therefore, we can summarize the renormalized vacuum fluctuation \( \langle \delta \phi^2 \rangle \) around the evaporating black hole as follows:

\[
\langle \delta \phi^2 \rangle \approx \begin{cases} 
O(T_H^2) & (r \to 2M_{\text{BH}}) \\
0 & (r \to \infty)
\end{cases}
\]

(34)

However, the most uncertain thing in the stochastic formalism is how to determine the volume factor of \( \mathcal{V} \). We took \( \mathcal{V} \) to be the volume of the domains in which the vacuum fluctuation \( \langle \delta \phi^2 \rangle \) governs.

It is obvious that we can not take the entire volume of the Universe as \( \mathcal{V} \) because the vacuum fluctuation of \( \langle \delta \phi^2 \rangle \) approach zero far from the black hole and the large vacuum fluctuation exists only near the black-hole horizon. Therefore, let us assume that the volume factor \( \mathcal{V} \) can be given by \( \mathcal{V} = N_{\text{PBH}} \cdot O(1) \) where \( N_{\text{PBH}} \) is the number of the evaporating or evaporated primordial black holes during the cosmological history of the Universe. By using Eq. (29), Eq. (30) and Eq. (31), we can estimate the constraint of the number on the evaporating primordial black holes as follows:

\[
\mathcal{V} \cdot P(\phi > \phi_{\text{max}}) \approx \frac{N_{\text{PBH}}}{\phi_{\text{max}}} \sqrt{2 \langle \delta \phi^2 \rangle} \exp \left(-\frac{\phi_{\text{max}}^2}{2 \langle \delta \phi^2 \rangle} \right)
\]

\[
\approx N_{\text{PBH}} \cdot e^{-O(100)} \lesssim 1,
\]

(35)

The Eq. (35) on the left side shows the number of the evaporating black holes which cause the Higgs vacuum collapse. Therefore, we can obtain the constraint on the number of the evaporating primordial black holes
as $N_{\text{PBH}} \lesssim \mathcal{O}(10^{43})$ which is extremely huge in order to threaten the Higgs metastable vacuum. Thus, one evaporating black hole can not cause serious problems in vacuum stability of the standard model Higgs case. The total number of the evaporating black hole (or the PBHs) strongly depends on the cosmological models at the early Universe [50, 51], and therefore, we provide the upper bound on the yield of the PBHs $Y_{\text{PBH}}/s \equiv n_{\text{PBH}}/s$ as follows.

$$Y_{\text{PBH}} = \frac{n_{\text{PBH}}}{s} = \frac{N_{\text{PBH}}}{s_0/H_0^2} \lesssim \mathcal{O}(10^{-43}),$$

(36)

where $s_0$ denotes the entropy density at present ($\approx (3 \times 10^{-4} \text{ eV})^3$), and $H_0$ the current Hubble constant ($\approx 10^{-33} \text{ eV}$). Note that $Y_{\text{PBH}}$ is constant from the formation time to the evaporation time.

It is convenient to transform this bound into an upper bound on $\beta$, which is defined by taking values at the formation of the PBH to be

$$\beta \equiv \frac{\rho_{\text{PBH}}}{\rho_{\text{tot}}}_{\text{formation}},$$

(37)

where $\rho_{\text{PBH}}$ and $\rho_{\text{tot}}$ are the energy density of the PBHs and the total energy density of the Universe including the PBHs at the formation, respectively. It is remarkable that $\beta$ means the number of the PBHs per the horizon volume at the formation ($\beta \sim n_{\text{PBH}}/H^3$). Then we have a relation [94],

$$\beta \sim 10^{30} \frac{n_{\text{PBH}}}{s} \left( \frac{m_{\text{PBH}}}{10^{15} \text{g}} \right)^{3/2}.$$  

(38)

Combining this relation with (36), we obtain

$$\beta \lesssim \mathcal{O}(10^{-12}) \left( \frac{m_{\text{PBH}}}{10^{15} \text{g}} \right)^{3/2},$$

$$\lesssim \mathcal{O}(10^{-21}) \left( \frac{m_{\text{PBH}}}{10^9 \text{g}} \right)^{3/2}. \quad (39)$$

This bound can be stronger than the known one for $m_{\text{PBH}} \lesssim 10^9 \text{g}$ [94].

However, at the final stage of the evaporation of the black hole, the black-hole mass $M_{BH}$ becomes extremely small and the Hawking temperature $T_H$ approaches to the Planck scale $M_{Pl} = 2.4 \times 10^{18} \text{ GeV}$ where $M_{Pl}$ is the reduced Planck mass. Therefore, the UV corrections of the beyond Standard Model (BSM) and the quantum gravity (QG) can not be ignored at the last stage of the evaporation and undoubtedly contribute to the Higgs vacuum metastability. In the rest of this section, we discuss how the UV or Planck scale physics affect the electroweak vacuum stability. When the Hawking temperature approaches to the Planck scale as $T_H \to O(M_{Pl})$, the contribution of the Planck physics determines the stability of the vacuum. Therefore, if the UV or Planck scale physics destabilize the effective Higgs potential at the high energy, the catastrophe vacuum decay can happen by even a single evaporating black hole.

Now, we consider the effective Higgs potential with the corrections of the BSM and the QG. For convenience we add two higher dimension operators $\phi^6$ and $\phi^8$ via the Planck scale physics to the effective Higgs potential as follows:

$$V_{\text{eff}}(\phi) = \frac{\lambda_{\text{eff}}}{4} \phi^4 + \frac{\delta \lambda_{\text{bsm}}}{4} \phi^4 + \frac{\lambda_6}{6} \frac{\phi^6}{M_{Pl}^4} + \frac{\lambda_8}{8} \frac{\phi^8}{M_{Pl}^4} + \cdots,$$

(40)

where $\delta \lambda_{\text{bsm}}$ express the running corrections from the BSM, $\lambda_6$ and $\lambda_8$ dimensionless coupling constants. These higher-dimension contributions of $\lambda_6$ and $\lambda_8$ are usually negligible except for the Planck scale excursion $\phi \approx M_{Pl}$. On the other hand, these corrections can affect the false vacuum decay via the quantum tunneling given by the literature [13–16]. In the Schwarzschild black-hole background, these enhancements were discussed in the literature [40–42] although the author entirely neglected the back-reaction of the Hawking thermal radiation.

However, at the final stage of evaporation of the black hole where $T_H \to O(M_{Pl})$, these higher-dimension contributions of $\lambda_6$ and $\lambda_8$ can not be neglected and have a strong impact on the vacuum stability around the black hole. As previously discussed, the effective Higgs potential of Eq. (41) is modified by the Hawking thermal fluctuation around the black hole:

$$V_{\text{eff}}(\phi) = \frac{\lambda_{\text{eff}}}{2} \left( \lambda_{\text{eff}} T_H^2 + \kappa_2 T_H^2 + \frac{\lambda_6 T_H^4}{M_{Pl}^4} + \frac{\lambda_8}{8} T_H^6 + \cdots \right) \phi^2 + \frac{1}{4} \left( \lambda_{\text{eff}} + \delta \lambda_{\text{bsm}} + \frac{\lambda_6 T_H^2}{M_{Pl}^4} + \frac{\lambda_8}{8} T_H^4 + \cdots \right) \phi^4 + \cdots,$$

(41)

where the Planck scale contributions govern the effective Higgs potential at $T_H \to O(M_{Pl})$. If the effective Higgs potential is destabilized by these UV scale corrections via $\langle \phi^2 \rangle_{\text{ren}} \approx \mathcal{O}(T_H^2)$ and the effective Higgs potential becomes negative to be $\delta V_{\text{eff}}(\phi)/\delta \phi < 0$, the local Higgs fields around the black-hole classically roll down into the negative Planck-energy true vacuum and Higgs Anti-de Sitter (AdS) domains whose sizes are about the black-hole horizon are formed. Not all Higgs AdS domains threaten the existence of the Universe, which highly depends on their evolutions (see Ref.[32, 44] for the detail discussions). However, Higgs AdS domains generally expand eating other regions of the electroweak false vacuum and finally consume the entire Universe.

Therefore, even a single evaporating black hole can be completely catastrophic for the stability of the electroweak vacuum via the extremely high Hawking temperature $T_H \to O(M_{Pl})$ although this possibility strongly depends on the BSM or the Planck scale physics and the detail of the evaporation of the black hole.

**IV. CONCLUSION AND OUTLOOK**

In this paper, we have investigated how gravitational vacuum polarization around a Schwarzschild black hole
affects stability of the (electroweak) vacuum. We have discussed the problems by using the renormalized vacuum field fluctuation $\langle \delta \phi^2 \rangle_{\text{ren}}$ describing approximately the Hawking thermal fluctuation near the black-hole horizon ($r \approx 2M_{BH}$). In particular, we have studied the stability of the electroweak vacuum around an evaporating black hole by taking account of the back-reaction effects on the effective Higgs potential $V_{\text{eff}}(\phi)$ which was ignored in previous works of the Higgs vacuum stability.

When we incorporate the back-reaction effects and reanalyze the stability of the vacuum around the black hole, the stability conditions of the vacuum approximately reproduce the ones in the local thermal situations. Therefore, one evaporating black hole does not cause catastrophic problems in vacuum stability of the standard model Higgs, but we have obtained an upper bound on the evaporating PBH abundance $\beta \lesssim \mathcal{O}(10^{-21}) \left( m_{\text{PBH}}/10^9 \text{GeV} \right)^{3/2}$ not to induce any instabilities during the history of the Universe.

However, at the final stage of the evaporation of the black hole, the black-hole mass $M_{BH}$ becomes extremely small and the Hawking temperature reaches the Planck scale where $T_H \to \mathcal{O}(M_P)$. Therefore, the Planck scale physics or the beyond Standard Model (BSM) directly may intervene and have a strong impact on the vacuum stability around the black hole. Our discussion will be changed if the Planck-scale physics or the BSM destabilize the Higgs field.

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The Kretschmann scalar $K$ constructed of two Riemann tensors is non-zero, i.e $K = R^{abcd}R_{abcd} = 48M^2/\rho^3$.

B. S. DeWitt, Phys. Rept. 19, 295 (1975).

S. M. Christensen and S. A. Fulling, Phys. Rev. D15, 2088 (1977).

S. M. Christensen, Phys. Rev. D14, 2490 (1976).

P. R. Anderson, W. A. Hiscock, and D. A. Samuel, Phys. Rev. D51, 4337 (1995).

D. N. Page, Phys. Rev. D25, 1499 (1982).

M. R. Brown, A. C. Ottewill, and D. N. Page, Phys. Rev. D33, 2840 (1986).

V. P. Frolov and A. I. Zelnikov, Phys. Rev. D35, 3031 (1987).

C. Vaz, Phys. Rev. D39, 1776 (1989).

F. A. Barrios and C. Vaz, Phys. Rev. D40, 1340 (1989).

For the Boulware vacuum $|0_B\rangle$ the renormalized energy momentum tensor $\langle T_{\mu\nu}\rangle_{\text{ren}}$ can be obtained as follows:

$$\langle 0_B | T^\mu_{\nu} | 0_B \rangle_{\text{ren}} \longrightarrow - \frac{1}{30 \cdot 2^{21/2}\pi^2 M_{\text{BH}}^3 (1 - 2M_{\text{BH}}/r)^2} \times \left( \begin{array}{cccc} -1 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/3 \end{array} \right) (r \to 2M_{\text{BH}}),$$

$$\langle 0_B | T^\mu_{\nu} | 0_B \rangle_{\text{ren}} \longrightarrow 1/r^6 \quad (r \to \infty),$$

where the renormalized expression of $\langle T^\mu_{\nu}\rangle_{\text{ren}}$ for $|0_B\rangle$ produces a negative energy density divergence at the black-hole horizon $r = 2M_{\text{BH}}$. This originates from the infinite blueshift of the negative energy.
where $\phi_{\text{max}}$ is given as follows

$$\phi_{\text{max}} = \sqrt{-m_{\text{eff}}^2}/\lambda.$$  

In the numerical analysis, we can approximate the maximal field value to be $\phi_{\text{max}} \simeq 10 \cdot m_{\text{eff}}$. 

[93] In the case of the reheating era where the entire Universe is thermalized after inflation, the physical volume of $V$ and the thermal fluctuation of $\langle \delta \phi^2 \rangle_{\text{ren}}$ can be given as follows [35]:

$$V \simeq e^{3N_{\text{hor}}}, \quad \langle \delta \phi^2 \rangle_{\text{ren}} \simeq \frac{T^2}{12} - \frac{m_{\text{eff}} T}{4\pi}$$

(43) where we take the high temperature limit ($m_{\text{eff}} \lesssim T$).

[94] B. J. Carr, K. Kohri, Y. Sendouda, and J. Yokoyama, Phys. Rev. D81, 104019 (2010), arXiv:0912.5297 [astro-ph.CO].

[95] V. Khachatryan et al. (CMS), Phys. Rev. D93, 072004 (2016), arXiv:1509.04044 [hep-ex].

[96] D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio, and A. Strumia, JHEP 12, 089 (2013), arXiv:1307.3536 [hep-ph].

[97] L. Di Luzio and L. Mihaila, JHEP 06, 079 (2014), arXiv:1404.7450 [hep-ph].

[98] A. Andreassen, W. Frost, and M. D. Schwartz, Phys. Rev. D91, 016009 (2015), arXiv:1408.0287 [hep-ph].

[99] A. Andreassen, W. Frost, and M. D. Schwartz, Phys. Rev. Lett. 113, 241801 (2014), arXiv:1408.0292 [hep-ph].

[100] Z. Lalak, M. Lewicki, and P. Olszewski, Phys. Rev. D94, 085028 (2016), arXiv:1605.07613 [hep-ph].

[101] J. R. Espinosa, M. Garny, and T. Konstandin, Phys. Rev. D94, 055026 (2016), arXiv:1607.08432 [hep-ph].

[102] J. R. Espinosa, M. Garny, T. Konstandin, and A. Riotto, Phys. Rev. D95, 056004 (2017), arXiv:1608.06765 [hep-ph].

[103] N. D. Birrell and P. C. W. Davies, Quantum Fields in Curved Space, Cambridge Monographs on Mathematical Physics (Cambridge Univ. Press, Cambridge, UK, 1984).