VECTORS MESONS $\rho$, $\rho'$ AND $\rho''$
DIFFRACTIVELY PHOTO- AND LEPTOPRODUCED

G. KULZINGER

Institut für Theoretische Physik der Universität Heidelberg,
Philosophenweg 16, 69120 Heidelberg, Germany

In the framework of non-perturbative QCD we calculate high-energy diffractive production of vector mesons $\rho$, $\rho'$ and $\rho''$ by real and virtual photons on a nucleon. The initial photon dissociates into a $q\bar{q}$-dipole and transforms into a vector meson by scattering off the nucleon which, for simplicity, is represented as quark-diquark. The relevant dipole-dipole scattering amplitude is provided by the non-perturbative model of the stochastic QCD vacuum. The wave functions result from considerations in the frame of light-front dynamics; the physical $\rho'$- and $\rho''$-mesons are assumed to be mixed states of an active $2S$-excitation and some residual rest ($2D$- and/or hybrid state). We obtain good agreement with the experimental data and get an understanding of the markedly different $\pi^+\pi^-$-mass spectra for photoproduction and $e^+e^-$-annihilation.

Keywords: non-perturbative QCD, diffraction, photoproduction, photon wave function, $\rho$-meson, excited vector mesons, hybrid

1. Introduction

Diffractive scattering processes are characterized by small momentum transfer, $-t \lesssim 1 \text{ GeV}^2$, and thus governed by non-perturbative QCD. To get more insight in the physics at work we investigate exclusive vector meson production by real and virtual photons. In this note we summarize recent results from Ref. [1] on $\rho$-, $\rho'$- and $\rho''$-production, see also Ref. [2]. In Refs [1, 3] we have developed a framework which we here can only flash.

We consider high-energy diffractive collision of a photon, which dissociates into a $q\bar{q}$-dipole and transforms into a vector meson, with a proton in the quark-diquark picture, which remains intact. The scattering $T$-amplitude can be written as an integral of the dipole-dipole amplitude and the corresponding wave functions. Integrating out the proton side, we have

$$T^V_{\lambda}(s,t) = \frac{i}{s} \int \frac{dzd^2r}{4\pi} \psi^{\dagger}_V(\lambda) \psi_{\gamma}(Q^2,\lambda)(z,r) J_p(z,r,\Delta T),$$

where $V(\lambda)$ stands for the final vector meson and $\gamma(Q^2,\lambda)$ for the initial photon with definite helicities $\lambda$ (and virtuality $Q^2$); $z$ is the light-cone momentum fraction of the quark, $r$ the transverse extension of the $q\bar{q}$-dipole. The function $J_p(z,r,\Delta T)$ is the interaction amplitude for a dipole $\{z,r\}$ scattering on a proton with fixed momentum transfer $t = -\Delta_T^2$; for $\Delta_T = 0$ due to the optical theorem it is the corresponding total cross section (see below Eq. (4)). It is calculated within non-perturbative QCD:

1 Supported by the Deutsche Forschungsgemeinschaft under grant no. GRK 216/1-96
2 E-mail: G.Kulzinger@thphys.uni-heidelberg.de
Figure 1: Interaction amplitude (arbitrary units) of two colour dipoles as function of their impact (units of correlation lengths $a$). One large $q\bar{q}$-dipole of extension $12a$ is fixed, the second small one of extension $1a$ is, averaged over all its orientations, shifted along on top of the first one. For the $D_1$-tensor structure of the correlator there are only contributions when the endpoints are close to each other, whereas for the $D$-structure large contributions show up also from between the endpoints. This is to be interpreted as interaction with the gluonic string between the quark and antiquark.

In the high-energy limit Nachtmann \cite{Nachtmann} derived a non-perturbative formula for dipole-dipole scattering whose basic entity is the vacuum expectation value of two lightlike Wilson loops. This gets evaluated \cite{Nachtmann} in the model of the stochastic QCD vacuum.

### 2. The model of the stochastic QCD vacuum

Coming from the functional integral approach the model of the stochastic QCD vacuum \cite{Nachtmann} assumes that the non-perturbative part of the gauge field measure, i.e. long-range gluon fluctuations that are associated with a non-trivial vacuum structure of QCD, can be approximated by a stochastic process in the gluon field strengths with convergent cumulant expansion. Further assuming this process to be gaussian one arrives at a description through the second cumulant $\langle g^2 F_{\mu\nu}(x;x_0) F_{\rho\sigma}^{\mu}(x';x_0) \rangle$ which has two Lorentz tensor structures multiplied by correlation functions $D$ and $D_1$, respectively. $D$ is non-zero only in the non-abelian theory or in the abelian theory with magnetic monopoles and yields linear confinement. Whereas the $D_1$-structure is not confining.

The underlying mechanism of (interacting) gluonic strings also shows up in the scattering of two colour dipoles, cf. Fig. 1, and essentially determines the $T$-amplitude if large dipole sizes are not suppressed by the wave functions. To confront with experiment this specific-large distance prediction we are intended to study the broad $\rho$-states and, especially, their production by broad small-$Q^2$ photons. Before we enter the discussion of our results, however, we have to specify these states and have to fix their wave functions as well as that of the photon.

### 3. Physical states $\rho$, $\rho'$ and $\rho''$

Analyzing the $\pi^+\pi^-$-invariant mass spectra for photoproduction and $e^+e^-$-annihilation Donnachie and Mirzaie \cite{Donnachie} concluded evidence for two resonances in the
Figure 2: Mass spectrum of $\pi^+\pi^-$-photoproduction on the proton: The interference in the 1.6 GeV region is constructive in contrary to the case of $e^+e^-$-annihilation into $\pi^+\pi^-$. We display our calculation together with the experimental data [7].

1.6 GeV region whose masses are compatible with the $1^{--}$ states $\rho(1450)$ and $\rho(1700)$. We make as simplest ansatz

$$|\rho(770)\rangle = |1S\rangle,$$
$$|\rho(1450)\rangle = \cos \theta |2S\rangle + \sin \theta |\text{rest}\rangle,$$
$$|\rho(1700)\rangle = -\sin \theta |2S\rangle + \cos \theta |\text{rest}\rangle,$$

where $|\text{rest}\rangle$ is considered to have $|2D\rangle$- and/or hybrid components whose couplings to the photon both are suppressed, see Refs. [8] and [10], respectively. With our convention of the wave functions the relative signs $\{+, -, +\}$ of the production amplitudes of the $\rho$, $\rho'$- and $\rho''$-states in $e^+e^-$-annihilation determine the mixing angle to be in the first quadrant; from Ref. [7] then follows $\theta \approx 41^\circ$. With this value and the branching ratios of the $\rho'$- and $\rho''$-mesons into $\pi^+\pi^-$ extracted in Ref. [7] we calculate the photoproduction spectrum as shown in Fig. 2 with the observed signs pattern $\{+, +, -\}$; for details cf. [1]. We will understand below from Fig. 3 the signs change of the $2S$-production as due to the dominance of large dipole sizes in photoproduction in contrary to the coupling to the electromagnetic current $f_{2S}$ being determined by small dipole sizes.

4. Light-cone wave functions

In the high-energy limit the photon can be identified as its lowest Fock, i.e. $q\bar{q}$-state. The vector meson wave function distributes this $q\bar{q}$-dipole $\{z, r\}$, accordingly.

**Photon:** With mean of light-cone perturbation theory (LCPT) we get explicit expressions for both longitudinal and transverse photons. The photon transverse size which we will see to determine the $T$-amplitude is governed by the product $\varepsilon r$, $\varepsilon = \sqrt{z^2Q^2 + m^2}$ and $r = |r|$. For high $Q^2$ longitudinal photons dominate by a power of $Q^2$; their $z$-endpoints being explicitly suppressed, LCPT is thus applicable. For moderate $Q^2$ also transverse photons contribute which have large extensions because endpoints are not suppressed. For $Q^2$ smaller than 1 GeV$^2$ LCPT definitively breaks down. However, it was shown [11] that a quark mass phenomenologically interpolating between a zero valence and a 220 MeV constituent mass astonishingly well mimics chiral symmetry breaking and confinement. Our wave function is thus given by LCPT with such a quark mass $m(Q^2)$, for details cf. Refs. [1, 3].
Vector mesons: The vector mesons wave functions of the 1S- and 2S-states are modelled according to the photon. We only replace the photon energy denominator \((\varepsilon^2 + k^2)^{-1}\) by a function of \(z\) and \(|k|\) for which ansätze according to Wirbel and Stech \[12\] are made; for the "radial" excitation we account by both a polynomial in \(z\) and the 2S-polynomial in \(k^2\) of the transverse harmonic oscillator. The parameters are fixed by the demands that the 1S-state reproduces \(M_\rho\) and \(f_\rho\) and the 2S-state is both normalized and orthogonal on the 1S-state. For details cf. Ref. \[1\].

5. Results

Before presenting some of our results \[1\] we stress that all calculated quantities are absolute predictions. Due to the eikonal approximation applied, the cross sections are constant with total energy \(s\) and refer to \(\sqrt{s} = 20\) GeV where the proton radius is fixed. (The two parameters of the model of the stochastic QCD vacuum, the gluon condensate \(\langle g^2 F F \rangle\) and the correlation length \(a\), are determined by matching low-energy and lattice results, cf. Ref. \[5\].)

In Fig. 3 we display – for the transverse 2S-state, \(\lambda = T\) – both the functions

\[
J^{(0)}_p(z, r) := \int_0^{2\pi} \frac{d\phi}{2\pi} J_p(z, r, \Delta_T = 0) \tag{3}
\]

\[
r\psi_{V(\lambda)}\psi_{\gamma(Q^2, \lambda)}(r) := \int dz \int_0^{2\pi} \frac{d\phi}{2\pi} |r|\psi_{V(\lambda)}\psi_{\gamma(Q^2, \lambda)}(z, r) \tag{4}
\]

which together, see Eq. (1), essentially determine the leptoproduction amplitude. It is strikingly shown how for decreasing virtuality \(Q^2\) the outer positive region of the wave functions effective overlap \(r\psi_{V(\lambda)}\psi_{\gamma(Q^2, \lambda)}\) wins over the inner negative part due to the strong rise with \(r\) of the dipole-proton interaction amplitude \(J^{(0)}_p\) which itself is a consequence of the string interaction mechanism discussed above. In praxi dipole sizes up to 2.5 fm contribute significantly to the cross section.

![Figure 3: Dipole-proton total cross section \(J^{(0)}_p\) and the effective overlap \(r\psi_{V(T)}\psi_{\gamma(Q^2, T)}\) as function of the transverse dipole size \(r\). The black line is the function \(J^{(0)}_p(1/2, r)\), i.e. the total cross section of a \(q\bar{q}\)-dipole \((z = 1/2, r)\), averaged over all orientations, scattering on a proton; the grey line shows the cross section for a completely abelian, non-confining theory. The \(T\)-amplitude is obtained by integration over the product of \(J_p\) and the overlap function, which essentially is the effective overlap shown for \(Q^2 = 0\), 1 and 20 GeV\(^2\) as short, medium and long dashed curves, respectively.](image-url)
Our results for integrated elastic cross sections as functions of $Q^2$ are given in Fig. 4. For the $\rho$-meson our prediction is about $20 - 30\%$ below the E665-data [13]. However, we agree with the NMC-experiment [14] which measures some definite superposition of longitudinal and transverse polarization, see Table 3 in Ref. [1]. For the 2S-state, due to the nodes of the wave function, we predict a marked structure; the explicit shape, however, strongly depends on the parametrization of the wave functions.

In Fig. 5 we display the ratio $R_{LT}(Q^2)$ of longitudinal to transverse cross sections and find good agreement with experimental data for the $\rho$-state. For the 2S-state

$$R_{LT}(Q^2) = \frac{\sigma^L(Q^2)}{\sigma^T(Q^2)}$$

Figure 5: Ratio of longitudinal to transverse integrated cross sections as function of $Q^2$ both for the $\rho$-meson and the 2S-state. There is only data for $\rho$-production [13].
we again predict a marked structure which is very sensitive to the node positions in
the wave functions.

Further results referring to cross sections differential in $-t$ and the ratio of
$2\pi^+2\pi^-$-production via $\rho'$ and $\rho''$ to $\pi^+\pi^-$-production via $\rho$ are given in Ref. [1].

Acknowledgements

The author thanks H.G. Dosch and H.J. Pirner for collaboration in the underlying
work.

References

[1] G. Kulzinger, H.G. Dosch and H.J. Pirner, hep-ph/9806352, accepted for pub-
lication in Eur.Phys.J. C.
[2] G. Kulzinger, hep-ph/9808440, accepted for publication in Nucl.Phys.B (Proc.
Suppl.).
[3] H.G. Dosch, T. Gousset, G. Kulzinger and H.J. Pirner, Phys.Rev. D55 (1997)
2602.
[4] O. Nachtmann, Ann.Phys. 209 (1991) 436 and Lectures given at Banz (Ger-
many) 1993 and at Schladming (Austria) 1996.
[5] H.G. Dosch, E. Ferreira and A. Krümer, Phys.Rev. D50 (1994) 1992.
[6] H.G. Dosch, Phys.Lett. B190 (1987) 177. H.G. Dosch and Y.A. Simonov, ibid.
B205 (1988) 339.
[7] A. Donnachie and H. Mirzaie, Z.Phys. C33 (1987) 407.
[8] L. Bergström, H. Suellman and G. Tengstrand, Phys.Lett. B80 (1979) 242.
[9] A.B. Clegg and A. Donnachie, Z.Phys. C62 (1994) 455.
[10] F.E. Close and P.R. Page, Nucl.Phys. B443 (1995) 233 and Phys.Rev. D56
(1997) 1584.
[11] H.G. Dosch, T. Gousset, H.J. Pirner, Phys. Rev. D57 (1998) 1666.
[12] M. Wirbel, B. Stech and M. Bauer, Z.Phys. C29 (1985) 637.
[13] E665-collaboration, M.R. Adams et al., MPI-PHE-97-03 Feb 1997, submitted
to Z.Phys. C.
[14] NMC-collaboration, M. Arneodo et al., Nucl. Phys. B429 (1994) 503.
[15] A. Donnachie and P.V. Landshoff, Phys.Lett. B296 (1992) 227.