Can annihilating Dark Matter be lighter than a few GeVs?

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We estimate the gamma ray fluxes from the residual annihilations of Dark Matter particles having a mass \(m_{dm} \in [\text{MeV},O(\text{GeV})]\) and compare them to observations. We find that particles lighter than \(O(100 \text{ MeV})\) are excluded unless their cross section is S-wave suppressed.

Introduction

The accurate measurement of galactic rotation curves, the CMB spectrum, the primordial abundances of light elements, together with our understanding of structure formation provide convincing evidence in favor of the existence of Dark Matter (DM) \(^1\). While the MA-CHOs searches \(^2\) indicate that an astrophysical solution is rather unlikely, most efforts are now concentrated on searches for Weakly Interacting Massive Particles (WIMPs) \(^3\). These particles would belong to the Cold Dark Matter scenario (CDM); they would annihilate and suffer from negligible damping effects at a cosmological scale. Considering fermions only and assuming Fermi interactions, it was concluded \(^4\) that the relic density argument constrains the DM mass \((m_{dm})\) to be greater than a few GeVs, as is quite naturally predicted within the framework of supersymmetry. Nevertheless searches for very massive particles \((m_{dm} \gtrsim O(\text{GeV}))\) remain unsuccessful \(^5\), so there is still room for new suggestions.

Alternative DM scenarios have been proposed in response to the discrepancy between observations and CDM numerical simulations on small scales (which notably predict cuspy haloes \(^6\)). The most “robust” one, the Warm Dark Matter scenario (WDM), involves non-annihilating particles and a very narrow range for the DM mass \(^7\), obtained by requiring that the free-streaming length be of the order of the smallest primordial scale one wants to be compatible with. For example, a typical warmon mass is \(m_{dm} \sim O(\text{keV})\) for \(\sim 100 \text{ kpc}\) \(^8\) but this scenario of light and non-annihilating DM, although not excluded as yet, also gives cuspy haloes \(^9\).

In this letter we propose another model, somehow intermediate between CDM and WDM. We shall indeed consider annihilating DM particles having a mass \(m_{dm} \in [\text{MeV},O(\text{GeV})]\). This has rarely been studied, probably because of the Lee-Weinberg argument that we shall evade here by considering bosonic (in fact scalar) candidates (for \(m_{dm} \lesssim \text{GeV}\). Some of these particles could perhaps turn out to be Warm not because of their mass but because of their collisions with relativistic species \(^10\). If not excluded by any cosmological/astrophysical arguments, they could compete with the collisionless WDM and CDM scenarios. On the other hand, they are likely to fail to give flat galactic cores at \(\sim 1 \text{ kpc}\) despite their quite large annihilation rate and a possible significant damping mass. Interestingly enough, they should escape present DM direct detection experiments (which so far are only sensitive to masses greater than \(\sim O(\text{GeV})\), as well as accelerator experiments, as briefly discussed in the next section. These particles would be compatible with the blackbody spectrum measurement and will not yield any \(^4\)He photodissociation (for \(m_{dm} > 26 \text{ MeV}\) provided their (s-wave) cross section satisfies the relation \((m_{dm}/\text{MeV}) > 5 \times 10^{-27} \text{ cm}^2/\text{s}\) (\(\Omega_{dm} h^2\))\(^2\) (assuming self-conjugate DM particles, \(H = 100h \text{ km/s/Mpc}\) and using the \(D\) measurement only \(^11\).)

Since light particles could yield gamma rays at energies that have been already probed experimentally, we mainly focus on their indirect detection signature to determine whether or not it is reasonable to consider them. We find that the gamma ray fluxes associated with particles lighter than \(O(100 \text{ MeV})\) are in conflict with observations unless the \(v^2\)-dependent term in the annihilation cross section \(i\) is much larger than the S-wave term (developed at the first order) and \(ii\) satisfies the relic density requirement. Radio fluxes also validate this conclusion.

Acceptable values of the cross sections

Relic density calculations provide a strong constraint on any DM candidate. When DM particles are able to annihilate \((i.e.\text{, when their non-relativistic transition occurs before their thermal decoupling})\), one obtains a simple relationship, independent of \(m_{dm}\), between the DM cosmological parameter \(\Omega_{dm} h^2\) and the total annihilation cross section \(^12\). By requiring that \(\Omega_{dm} h^2\) matches the observed value \((\sim 0.1, \text{see } \Omega_{dm} h^2)\), one gets the following approximate annihilation cross section

\[
\langle \sigma v \rangle_{\text{ann}} \simeq 7 \times 10^{-27} \frac{x_F}{\sqrt{g_*}} \left( \frac{\Omega_{dm} h^2}{0.1} \right)^{-1} \text{ cm}^3/\text{s} \tag{1}
\]

with \(x_F = m_{dm}/T_F \simeq 17.2 + \ln(g/\sqrt{g_*}) + \ln(m_{dm}/\text{GeV}) + \ln \sqrt{x_F} \in [12-19]\) for particles in the MeV-O(GeV) range \((g \text{ and } g_* \text{ being the number of internal and relativistic degrees of freedom respectively})\). We write the cross section as \(\sigma v \sim a + b(v/c)^2\) where \(a\) and \(b\) are some constants.
related to the S and P-wave terms (here $v$ is the DM velocity and $v_r$ the relative velocity). We shall assume first that $a \geq b$ (and take $c = 1$).

Eq. (11) sets the maximum value of $\langle \sigma v_r \rangle_{\text{ann}}$ one can use to compute gamma ray and radio fluxes from DM residual annihilations. Larger cross sections are possible if $n_{\text{dm}} \neq n_{\text{dm}}$ (but residual annihilations are unlikely or even impossible since no, or few, anti-particles would be left after the DM freeze-out, $t_{f_0}$) or if the co-annihilation mechanism (which involves DM and X particles) is at work [14]. In this case, however, the relation $\langle \sigma v \rangle_{\text{ann}} \propto \left( \frac{m_{\text{dm}}}{m_{\text{dm}}} \right)^{3/2} \times_{\text{ann}} \times_{\text{ann}}$ indicates that $m_X$ should be close to $m_{\text{dm}}$, which is actually excluded if $m_{\text{dm}} < O(\text{GeV})$ and if X is a charged particle. Thus, unless there exists a neutral (long-lived) particle X, the maximum annihilation cross section (times relative velocity) into ordinary particles (e.g. $\gamma, e^-$) that is legitimate to consider is about $10^{-26} \text{cm}^3 \text{s}^{-1}$.

**The least massive annihilating WIMP**

Since eq. (11) is almost independent of the DM mass (the only dependence in $m_{\text{dm}}$ being the logarithm which is “hidden” in $x_F$), even light candidates are expected to be allowed. However, when dealing with the range $m_{\text{dm}} \lesssim O(\text{GeV})$, having $\langle \sigma v_r \rangle_{\text{ann}} \sim 10^{-26} \text{cm}^3 \text{s}^{-1}$ rather favors particles with an annihilation cross section independent of the DM mass or, for which the annihilations rely on the exchange of light neutral particles. Taken at face value, the first point strongly suggests that light DM should be made of scalars (which is in agreement with the second) and with non chiral couplings [17]. One now has to check that their damping scale is not too large.

In contrast with the free-streaming length ($l_{fs}$) of non-annihilating DM particles which depends only on $m_{\text{dm}}$, the scale $l_{fs}$ of interacting particles in the [MeV, O(GeV)] mass range depends on both the DM mass and interaction rate $\langle \sigma v \rangle_{\text{ann}}$. By imposing $l_{fs} \lesssim 100 \text{ kpc}$ (the scale of the smallest galaxies) and assuming that interactions are weak enough to imply a decoupling in the radiation dominated era, one finds that particles with $m_{\text{dm}} \lesssim O(\text{MeV})$ ($n_{\text{dec}}/10^{-4}$) induce a cut-off in the matter power spectrum at $\sim 100 \text{ kpc}$ ($a_{\text{dec}}$ being the scale-factor at the DM thermal decoupling). Nevertheless, in a realistic particle physics model, the thermal decoupling (based on the estimate of the DM-e elastic scattering cross section) is seen to occur around 1 MeV ($a_{\text{dec}} \sim 10^{-10}$), so the mass range above 1 MeV should not modify the matter power spectrum at cosmological scales. In fact, even if the DM thermal decoupling was in the matter dominated era, only masses of $m_{\text{dm}} \lesssim 15 \text{ MeV}$ ($\Omega_{\text{dm}} h^2 / 0.1$) would affect the $10^9 M_\odot$ scale. The limit on $m_{\text{dm}}$ would actually get even smaller if DM was thermally decoupling after the non-linear collapse 10 (c.f. self-interacting DM for instance [16], which now appears unlikely [17]), as the primordial fluctuations “disappear” to form objects. Collisional damping due to $\nu$-DM interactions may also contribute but is expected to yield a damping mass $< 10^9 M_\odot$ so light annihilating candidates certainly deserve to be studied.

We note that low DM masses are not constrained by direct detection experiments which are only sensitive to masses greater than $\sim 7 \text{ GeV}$ [3] (except for e.g. CRESST [18], SAGE3, ROSEBUD, Tokyo [19] which are or will be able to go as low as $\sim 1 \text{ GeV}$). In fact, exploring the low DM mass region should be a problem for cryogenic detectors since the detection mechanism they currently use (based on nucleus recoil) does not allow for the detection of particles much lighter than $\sim 1 \text{ GeV}$ without a significant effort (even by using the lightest possible nucleus). Light scalars could escape searches in $e^+e^-$ colliders even when their production is based on the exchange of massive fermions (with a mass $\gtrsim 100 \text{ GeV}$) as their cross section for anomalous single photon events is still below (albeit very close) the sensitivity of past experiments [17]. Moreover, if they are able to annihilate into photons, the most interesting signature $e^+e^- \rightarrow e^+e^- \nu \nu$ ($\nu$ denoting the DM particles i.e. some missing energy) at the $\alpha^4 \alpha^2$ order should also be invisible because the electrons should remain mainly in the beam pipe, as one can infer from a kinematic analysis. Nevertheless, even if there exists a deviation large enough to be detected, it is unlikely that PETRA or LEP experiments, for instance, got enough sensitivity. Hence, particle physics experiments still allow for the range we consider. Note that in any case DM should not have a coupling to the Z boson (otherwise it would have been detected in accelerator experiments) but since bino particles or right-handed neutrinos, for instance, do not have this coupling either, this assumption seems reasonable.

**Indirect detection during the recombination epoch**

Let us check if DM annihilations at the recombination epoch yield enough redshifted photons at an energy $E_\gamma \in [\text{MeV}, 10 \text{ MeV}]$ to be detected nowadays. Assuming $a \gtrsim 1$, one gets the following gamma-ray number density $n_{\text{rec}}^{\gamma_{\nu_{\gamma}}} \approx 2 \langle \sigma v_r \rangle_{\text{ann}} (n_{\text{rec}}/10^{-4})^2 t_{\text{rec}}$ (with $n_{\text{rec}}$ and $t_{\text{rec}}$ the DM number density and Hubble time at the recombination epoch). This yields the present day flux $\Phi \sim \left( \frac{n_{\text{rec}}^{\gamma_{\nu_{\gamma}}}}{0.12} \right)^2 \langle \sigma v_r \rangle_{\text{ann}} m_{\text{MeV}}^{-2} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$ using $\Phi = c n_{\gamma} / 4 \pi$, $\langle \sigma v_r \rangle_{\text{ann}} = \langle \sigma v_r \rangle_{\text{ann}} / 10^{-26} \text{ cm}^3 \text{s}^{-1}$ and $m_{\text{MeV}} = m_{\text{dm}} / (\text{MeV}/c^2)$. P-wave annihilation cross sections would decrease this flux 20 but the latter is already overestimated since $O(\text{MeV})$ photons can lose some energy by scattering on remnant free electrons and by reionizing atoms [21]. Comparing $\Phi$ with the much higher observed fluxes, namely $\Phi_{\text{obs}}^{11-30\text{keV}} (> \langle \sigma v \rangle_{\text{ann}} m_{\text{MeV}}^{-2} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$), which now appears unlikely [17], as the primordial fluctuations “disappear” to form objects. Collisional damping due to $\nu$-DM interactions may also contribute but is expected to yield a damping mass $< 10^9 M_\odot$ so light annihilating candidates certainly deserve to be studied.

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$E_{\text{min}} \sim 20(E_{\text{min}}/\text{keV})^{-0.4}/(\text{cm}^2\text{s sr})$, $\Phi_{\text{obs}}^{[0.1, 10]} \text{MeV} (> E_{\text{min}}) \sim 3 \times 10^{-3}(E_{\text{min}}/\text{MeV})^{-1.5}/(\text{cm}^2\text{s sr})$ and $\Phi_{\text{obs}} \approx (4.15) \times 10^{-5}/(\text{cm}^2\text{s sr})$ for the range [30, 100] MeV (and $> 100$ MeV respectively) $^{24}$, we find that such residual annihilations are not ruled out by recent data.

**Dark Matter haloes**

| $\alpha$ | $\beta$ | $\gamma$ | $r_s$ | $F(\theta)$ | $\Phi/((\sigma v_r)_{26} m_{\text{GeV}}^2)$ |
| --- | --- | --- | --- | --- | --- |
| kpc | $^\circ$ | $^\circ$ | $^\circ$ | cm$^{-2}$s$^{-1}$ |
| NFW | 1 | 3 | 1 | 25 | 0.077 | 0.62 | 1.7 | 5.9 $\times 10^{-6}$ |
| KRA | 2 | 3 | 0.2 | 11 | 1.7 $\times 10^{-4}$ | 0.014 | 0.15 | 7.5 $\times 10^{-8}$ |
| ISO | 2 | 2 | 0 | 4 | 1.2 $\times 10^{-4}$ | 0.011 | 0.08 | 1.8 $\times 10^{-7}$ |
| BE | 1 | 3 | 0.3 | 4 | 1.2 $\times 10^{-4}$ | 0.004 | 0.01 | 4.1 $\times 10^{-6}$ |

| $\delta$ | $\gamma$ | $r_s$ | $F(\theta)$ | $\Phi/((\sigma v_r)_{26} m_{\text{GeV}}^2)$ |
| --- | --- | --- | --- | --- |
| kpc | $^\circ$ | $^\circ$ | $^\circ$ | cm$^{-2}$s$^{-1}$ |
| NFW | 1 | 3 | 1 | 25 | 0.077 | 0.62 | 1.7 | 5.9 $\times 10^{-6}$ |
| KRA | 2 | 3 | 0.2 | 11 | 1.7 $\times 10^{-4}$ | 0.014 | 0.15 | 7.5 $\times 10^{-8}$ |
| ISO | 2 | 2 | 0 | 4 | 1.2 $\times 10^{-4}$ | 0.011 | 0.08 | 1.8 $\times 10^{-7}$ |
| BE | 1 | 3 | 0.3 | 4 | 1.2 $\times 10^{-4}$ | 0.004 | 0.01 | 4.1 $\times 10^{-6}$ |

TABLE II: Expected fluxes from the Coma (C) and Virgo (V) cluster for different DM profiles $^{24}$. For the $\beta$-profile of Virgo, only the flux within 1 Mpc is given. $h = 0.7$.

- The total gamma ray flux from the DM halo of a galaxy cluster located at distance $D$ is well approximated by $\Phi_{\text{cl}} = F D n_\gamma (\nu)/(4\pi D^2)$. We list it for different halo profiles of two nearby clusters (Table I $^{10}$ $^{24}$. These values have to be compared with $\Phi_{\text{obs}} (100 \text{MeV}) < 4 \times 10^{-8} \text{cm}^{-2} \text{s}^{-1}$ at 2 sigma for these two clusters $^{24}$.

By comparing the numbers in Table I with $\Phi_{\text{obs}}$, we conclude that the annihilation cross section into $^2\gamma$ (times relative velocity) of particles lighter than $O(100)$ MeV should be much below $(\sigma v_r)_{\text{ann}} \sim 10^{-26} \text{cm}^3\text{s}^{-1}$. This is clearly in conflict with the relic abundance requirement unless one assumes that i) $\sigma v_r$ is dominated by $bv^2$ at the freeze-out epoch and ii) the $a$-term is smaller or equal to $10^{-31} - 10^{-28} \text{cm}^3\text{s}^{-1}$ for $m_{\text{dm}} \in [1, 100] \text{MeV}$ respectively (i.e. $\lesssim 10^{-5} - 10^{-2}$ times the $b$-term at the freeze-out epoch). On the other hand, heavy particles annihilating into photons and with $m_{\text{dm}} \gtrsim O( \text{GeV})$ appear quite compatible with observations even when $\sigma v_r \propto \text{const}$ (regardless of the profile used).

If the DM residual annihilations mainly proceed into leptons, then electrons could be detectable via synchrotron emission in a magnetic field $B$ $^{27}$. For the galactic centre only, the flux $F_{\nu} (\nu < 0.1 \text{eV}) \approx 5861.04 Jy (\nu/\text{GHz})^{-0.5} m_{\text{GeV}} (\sigma v_r)_{26} (B/\mu G)^{-0.5}$ is compatible with the observed radio flux of Sgr A (of the order of 360 Jy at $\nu = 330 \text{ MHz}$) provided $m_{\text{dm}} \gtrsim 10$ GeV (assuming that all the annihilations proceed into $e^+ e^-$, a NFW profile and $B \sim 1 \mu G$). However, masses $m_{\text{dm}} \lesssim 10$ GeV are OK if $\sigma v_r \gtrsim bv^2|_{\text{low}}$. Majorana fermions that would annihilate into $e^+ e^-$ through the exchange of scalar particles may actually have this property (if they are chirally coupled and for some value of $m_{\text{dm}}$) but they would fail to give the correct relic density if $m_{\text{dm}} \lesssim O(\text{GeV})$. In contrast, scalars coupled to fermions have the correct relic density but not the correct radio flux. But these particles satisfy both criteria if they exchange a light gauge boson. For Coma (Virgo being swamped with emission from M87) only $m_{\text{dm}} \gtrsim 10$ GeV produces radio emission at observed frequencies but the latter is (depending on $B \in [0.1, 10] \mu G$) of the same order of magnitude as the observed flux for $m_{\text{dm}} \sim 10$ GeV only (it is less for higher masses).

**Discussion and conclusion**

We now discuss three specific mass interval which have not been studied significantly before. They represent a “hole” in the investigations of the DM parameter space and deserve to be studied at least to probe whether or not there exist alternatives to heavy DM particles.

$m_{\text{dm}} \in [1 \text{ MeV}, m_{\nu}]$. Since the annihilation cross section of a pair of scalars into $e^+ e^-$ (via a fermion exchange $F$ and non-chiral couplings) is expected to be free of $m_{\text{dm}}$ (and potentially of the order of eq. (11), see e.g. (28)), one can discuss the case of light scalar DM...
candidates. However, to be compatible with the galactic centre COMPTEL/EGRET data, their annihilation cross section should be dominated by a term in $r^2$ at $t_{fr}$. This is actually in conflict with the $F$ exchanges (needed to get the correct relic density) but one can postulate an asymmetry between the DM and anti-DM number densities or assume that annihilations mainly proceed through the exchange of either a light neutral fermionic WIMP or gauge boson. In any case, a more careful study is needed to ensure that the particles introduced satisfy all experimental constraints. Such light DM particles could perhaps be detected or excluded by using their interactions with ordinary matter or searching for the particles supposed to be exchanged in the annihilation process.

$$m_{dm} \in [m_{\mu}, m_{\tau}]$$

They may be compatible with observed fluxes and relic density even for $a \gtrsim b$ but the production of $D^+He$ tends however to favor an annihilation cross section dominated by the term in $r^2$.

$$m_{dm} \in [m_{\tau}, O(10\text{GeV})]$$

They seem in agreement with observations even when $a \sim 10^{-26}\text{cm}^3\text{s}^{-1}$ but, if they mainly annihilate into $e^+e^-$, radio fluxes tend to favor $\sigma v \propto b^2|t_{fr}|$. If they mainly annihilate into a pair $\tau^{-}\bar{\tau}$, one expects a soft gamma ray emission (notably from $\pi$ decays) plus an excess of positrons (due to decays) plus an excess of positrons (due to their interactions with nuclei, giving them a potential signature).

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