Analysis of angular deflection of bearing node in machine 
with toothed transport belt

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Abstract. The available CAE solutions are a significant assistance when carrying out 
engineering works. They facilitate conceptual design, design and carrying out strength 
calculations. These tools are very popular because of the simplicity of use and the ability to 
obtain quite accurate distributions of structural stress and strain. Unfortunately, when designing 
shafts the rotation angle values on supports are equally important. The present article draws 
attention to the essential problem of the misalignment of roller bearing assemblies caused, among 
others, by exceeded allowable angles of rotation. The article furthermore presents the results of 
the analysis which aimed to determine the influence of selected geometrical characteristics of 
the shaft as well as the bearing span on the value of the shaft angle of rotation at the supports.

1. Introduction

In analyzing a technical issue it is oftentimes necessary to determine not just the values of stress present 
in a beam under load, but also the deflection values, i.e. the vertical displacement of individual points 
of the beam axis. Identifying these values allows to describe the rigidity of the designed component, and 
in some cases serves as the dominant criterion used to select the desired material and geometrical 
characteristics of the construction. Among the inseparable parameters related to the deflection value of 
the beam is the deflection angle. It is of particular importance when designing drive shaft bearing nodes. 
Its value determines not only the possible type of bearing to use, but is as important as the stress value 
and rigidity from the standpoint of shaft design [1, 2].

Design is a time-consuming process which often demands a significant level of knowledge and 
experience from the constructor [3–7]. Therefore, all kinds of software tools to assist us in the design 
process are employed with much eagerness, including dedicated computer-assisted engineering 
solutions. Modern systems for computer-assisted engineering works allow to build virtual, three-
dimensional models of devices. The ability to visualize the concept in such a way provides a unique 
perspective and insight. All of a sudden, the verification of the geometrical parameters of the assembly 
is no longer problematic for less experienced constructors. It is easier to verify the risk of collision 
between the individual components. Furthermore, making design documentation becomes much easier 
and introducing changes becomes easier and a matter of a few moments.

An important aspect of computer-assisted engineering is the possibility to carry out simulations and 
numerical calculations [8–10]. These enable to analyze the kinematics of the assembly in motion, for 
determination of parameters such as the travel path, velocity, acceleration, but also the forces and 
 moments occurring in the kinematic nodes. The data obtained in this way can be used in structural 
strength calculations, both in the analytical and numerical approach, e.g. employing the finite element 
method. A major advantage of some of the software is the ability to carry out parametric analyses. These
allow determining the influence of geometric and material characteristics on the strength characteristics of a given assembly.

However, all the available software are but tools in the hands of engineers who still need to interpret the results and make appropriate decisions. In many cases, the obtained results are not interpreted correctly, incorrect input data are introduced, or the entire model is not correctly designed. As a consequence, the machine may suffer a premature fault or a catastrophic incident.

The software can never replace the knowledge and experience of an engineer. It is only a tool which is often used incorrectly. We focus on the obtained results of equivalent stress distribution but fail to account for the rigidity of the construction, and when it comes to shaft design, we tend to forget about deflection angles.

2. Misalignment of roller bearing assemblies

The use of shafts with insufficient rigidity, and at the same time excessive angles of deflection at supports causes misalignment which is one of the major causes of damage to the roller bearing. Consequently, it may result in the cracking of the bearing case which results in seizing or incorrect distribution of loads in the bearing, significantly reducing its life. In a damaged bearing, it is usually easy to diagnose misalignment by examining the so called travel path of the rolling component on the outer side of the rings. As the bearing rotates, the rolling components generate a wear mark on the outer and inner raceway. If the bearing is aligned correctly, the wear path is located in the middle of the inner and outer ring, whereas misalignment produces uneven marks.

| Bearing type       | Allowable misalignment angles (rad) | Shigley’s [11] | Mazanek [12] |
|--------------------|------------------------------------|----------------|--------------|
| Tapered roller     |                                    | 0.0005–0.0012  | 0.00058      |
| Cylindrical roller |                                    | 0.0008–0.0012  | 0.00116      |
| Deep-groove ball   |                                    | 0.001–0.003    | 0.0023–0.0047|
| Spherical ball     |                                    | 0.026–0.052    | 0.035–0.07   |
| Self-align ball    |                                    | 0.026–0.052    | 0.07         |
| Uncrowned spur gear|                                    | <0.0005        | -            |

The construction of the bearing only allows for a slight misalignment. While the actual value depends on a given source in subject literature, the values nonetheless do not exceed 0.003 Radian (10 minutes of angle) for ball bearings and 0.0012 Radian (4 minutes of angle) for tapered and cylinder roller bearings. However, the maximum values depend mostly on the manufacturer and the radial bearing slackness values. Table 1 presents a breakdown of allowable misalignment angles derived from different sources.

The allowable misalignment depends not only on the type and construction of the bearing, or the used radial bearing slackness, but also on the method of assembly. Information available at the SKF manufacturer website indicates that for single-stack tapered roller bearings, the allowable misalignment when mounting bearing in a divergent system (O) is approx. 2 minutes of angle, whereas in a convergent system (X) approx. 4 minutes of angle.

3. The system under examination

This article presents an analysis of the influence of the geometrical parameters of the shaft on the values of rotation angles. The works carried out were to select the correct geometry of the drive shaft for the toothed transmission belt. The considered system is analogous to solutions employed in belt conveyors,
the difference being that the shaft bearing is only mounted on the one side of the toothed belt for easy removal. Shaft length is 1200 mm, whereas the width of the toothed belt is 1000 mm. For the sake of simplification, a constant shaft diameter was assumed along its entire length. Figure 1 provides an example view of the assembly longitudinal cross section. In the course of the consideration, both solid shafts with outer diameter between $D = 20$ and $D = 50$ mm were considered, as well as hollowed shafts with bore diameter $d$ equal to 25 %, 50 % and 75 % of the shaft outer diameter.

The selection of maximum outer shaft diameter was determined by the assembly construction, i.e. the ability to mount selected gear wheel for the belt, for which the diameter of the installation opening must not exceed 50 mm. For the purpose of calculations, steel material was used with Young’s modulus value of $E = 2.1 \cdot 10^{11}$ Pa. Two variants of bearing spacing were assumed: $l = 20$ mm and $l = 180$ mm. Load was equal to $Q = 250$ N accounting for the weight of the toothed belt as well as wheels mounted on the shaft.

Figure 1. Transverse cross-section view of the analyzed assembly; 1 – body, 2 – bearings, 3 – belt toothed wheels, 4 – shaft, 5 – toothed belt.

Figure 2. Schematic diagram of shaft support and load.

The analyzed construction was modeled as a beam placed on two supports, fixed and movable (figure 2). The load is distributed continuously on the right side of the beam, protruding beyond the movable support. The formulas used in the calculation are listed in table 2, whereas figures 3–5 presents the results of the performed analyses.
Table 2. List of formulas used in the analytical calculations [13].

|                      | Solid shaft                                      | Hollow shaft                                   |
|----------------------|--------------------------------------------------|-----------------------------------------------|
| Cross-section area   | \( A_p = \frac{\pi \cdot D^2}{4} \)             | \( A_d = \frac{\pi \cdot (D^2 - d^2)}{4} \) |
| Volume               | \( V_p = A_p \cdot L \)                         | \( V_d = A_d \cdot L \)                       |
| Mass                 | \( m_p = \rho \cdot V_p \)                       | \( m_d = \rho \cdot V_d \)                    |
| Moment of inertia    | \( J_p = \frac{\pi \cdot D^4}{64} \)            | \( J_d = \frac{\pi \cdot (D^4 - d^4)}{64} \) |
| Deflection angle     | \( \theta_A = \frac{qla^2}{12 \cdot E \cdot J_p} \) | \( \theta_A = \frac{qla^2}{12 \cdot E \cdot J_d} \) |
| Deflection angle     | \( \theta_B = \frac{qla^2}{6 \cdot E \cdot J_p} \) | \( \theta_B = \frac{qla^2}{6 \cdot E \cdot J_d} \) |

Figure 3. Graph showing the change of deflection angle depending on the outer shaft diameter.
Figure 4. Graph showing the change of deflection angle depending on the inner shaft diameter for $l = 20\,\text{mm}$.

Figure 5. Graph showing the change of deflection angle depending on the inner shaft diameter for $l = 180\,\text{mm}$.
4. Conclusion

Figures 3 and 4 demonstrate the results of carried out analyses. These indicate that in the considered construction solution, the highest values of deflection angle occur with movable support (from the side of the shaft under load). Furthermore, we notice the major influence of spacing between supports. Higher spacing increases the deflection angle value, which is consistent with the formulas. However, the less obvious relationship can be seen in figure 3. The size of shaft hollowing has lower influence on the deflection angle than the spacing between supports. For a solid shaft, it is observed that the deflection angle values are nearly identical to the hollow shaft with bore diameter 25% – 50% of the outer shaft diameter. Only when the bore diameter reaches approx. 75% of the outer shaft diameter, we observe a major difference in the obtained values of deflection angle (figures 4 and 5). This is significant from the standpoint of shaft design aiming to reduce the weight of the assembly. Hollowing out the shaft can reduce the weight by approx. 33% without affecting the criteria for bearing selection.

5. References

[1] Dudziak M, Kołodziej A, Domek G and Talaśka K 2017 Multi-angularity-identification of parameters and compatibility conditions of the axisymmetric connection with form deviations Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture 231 1333–1344
[2] Kołodziej A, Dudziak M and Talaśka K 2018 Cooperation of axisymmetric connection elements under dynamic load MATEC Web of Conferences 157 02017
[3] Talaśka K, Malujda I and Wilczyński D 2016 Agglomeration of natural fibrous materials in perpetual screw technique – a challenge for designer Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture 230 1225–1236
[4] Górecki J, Malujda I, Wilczyński D and Wojtkowiak D 2019 Influence of the face surface shape of the piston on the limit value of compaction stress in the process of dry ice agglomeration, MATEC Web of Conferences 254 06001
[5] Wojtkowiak D and Talaśka K 2019 Determination of the effective geometrical features of the piercing punch for polymer composite belts The International Journal of Advanced Manufacturing Technology 104 315–332 https://doi.org/10.1007/s00170-019-03746-7
[6] Wojtkowiak D, Talaśka K, Malujda I and Domek G 2018 Estimation of the perforation force for polymer composite conveyor belts taking into consideration the shape of the piercing punch The International Journal of Advanced Manufacturing Technology 98 2539–2561
[7] Wojtkowiak D and Talaśka K 2019 The influence of the piercing punch profile on the stress distribution on its cutting edge MATEC Web of Conferences 254 02001
[8] Talaśka K and Wojtkowiak D 2018 Modelling a mechanical properties of the multilayer composite materials with the polyamide core MATEC Web of Conferences 157 02052
[9] Wilczyński D, Malujda I, Górecki J and Jankowiak P 2019 Research on the process of biomass compaction in the form of straw MATEC Web of Conferences 254 05015
[10] Wilczyński D, Malujda I, Talaśka K and Długi R 2017 The study of mechanical properties of natural polymers in the compacting process Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture 231 1333–1344
[11] Budynas R G and Nisbett J K 2015 Shigley’s Mechanical Engineering Design (New York)
[12] Mazanek E 2008 Examples of calculations from the basics of machine construction (Warszawa: WNT) (in Polish)
[13] Niezgodziński M E and Niezgodziński T 2004 Formulas, Charts and Strength Tables (Warszawa: WNT) (in Polish)