Numerical analysis on the production of $D^{(*)}(3000)$, $D_{sJ}(3040)$ and their partners through the semileptonic decays of $B_{(s)}$ mesons in terms of the Light Front Quark model

Hao Xu$^{1,2}$, Qi Huang$^{1,2}$, Hong-Wei Ke$^3$ and Xiang Liu$^{1,2,3}$

1School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China
2Research Center for Hadron and CSR Physics, Lanzhou University & Institute of Modern Physics of CAS, Lanzhou 730000, China
3School of Science, Tianjin University, Tianjin, 300072, China

Inspired by the newly observed $D^{(*)}(3000)$, $D_{sJ}(3040)$, in this work we study the production of $D^{(*)}(3000)$, $D_{sJ}(3040)$ and their partners through the semileptonic decays of $B_{(s)}$ mesons, where the Light Front Quark model is applied to the whole calculation. Our numerical results indicate that the $B_{(s)}$ semileptonic decays into the $2\pi$ states of the charmed or charmed-strange meson family have considerable branching ratios, which shows that these discussed semileptonic decays can be accessible at future experiment, especially for LHCb and the forthcoming BelleII.

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I. INTRODUCTION

In the past years, experiment has made big progress on searching for higher charmed or charmed-strange mesons, where more and more charmed or charmed-strange states were reported, which also stimulated theorist’s extensive interest in revealing their underlying properties (see Ref. [1] for brief review).

Among all observed charmed and charmed-strange states, there are three states $D(3000)$, $D^*(3000)$ and $D_{sJ}(3040)$ with the masses around 3 GeV. $D(3000)$ and $D^*(3000)$ were observed by the LHCb Collaboration [2] by measuring the $D^*\pi^-$, $D^0\pi^-$, and $D^{*+}\pi^-$ invariant mass spectra from the inclusive process $pp \rightarrow D^*\pi X$, $pp \rightarrow D^0\pi X$, and $pp \rightarrow D^{*+}\pi X$, where $X$ is a system composed of any collection of charged and neutral particles [2]. $D^{*}(3000)$ appears in the $D^{*}\pi^-$ invariant mass spectrum, while $D(3000)$ exists in the $D^*\pi^-$ invariant mass spectrum. The resonance parameters of $D(3000)$ and $D^*(3000)$ are $m_{D(3000)} = 2971.8 \pm 8.7 \text{ MeV}$, $\Gamma_{D(3000)} = 188.1 \pm 44.8 \text{ MeV}$, $m_{D^*(3000)} = 3008.1 \pm 4.0 \text{ MeV}$, and $\Gamma_{D^*(3000)} = 110.5 \pm 11.5 \text{ MeV}$, which can be as a good platform to study $D(3000)$, $D^*(3000)$ and $D_{sJ}(3040)$.

Before observing $D(3000)$ and $D^*(3000)$, the BaBar Collaboration announced the observation of charmed-strange state $D_{sJ}(3040)$ only in the $D^*K$ invariant mass spectrum in inclusive $e^+e^-$ interactions [3], which has the mass $m_{D_{sJ}(3040)} = 3044 \pm 8(\text{stat})^{+5}_{-5}(\text{syst}) \text{ MeV}$ and the width $\Gamma_{D_{sJ}(3040)} = 239 \pm 35(\text{stat})^{+46}_{-42} \text{ (syst) MeV}$. Here, $D_{sJ}(3040)$ is a good candidate of the first radial excitation of $D_{sJ}(2460)$ just indicated in Ref. [2].

Although there was abundant experimental information of these $D(3000)$, $D^*(3000)$ and $D_{sJ}(3040)$, we notice that these states can be also produced via the semileptonic decays of $B_{(s)}$ theoretically, which are different from these reported production processes of $D(3000)$, $D^*(3000)$ and $D_{sJ}(3040)$, where the semileptonic decays of $B_{(s)}$ can be as a good platform to study $D(3000)$, $D^*(3000)$ and $D_{sJ}(3040)$. Before this work, there were theoretical efforts of studying the production of newly observed charmed or charmed-strange meson through the semileptonic decays of $B_{(s)}$. For example, $B_s \rightarrow D^{(*)}_{sJ}(2317)\ell\nu$, $D_{sJ}(2460)\ell\nu$ were calculated by the QCD Sum rules [7]–[9], the Constituent Quark Meson model [10] and Light-cone QCD Sum rule [11]. In Ref. [12], Li et al. studied the $B_s \rightarrow D^{(*)}_{sJ}(3040)\ell\nu$ semileptonic decays by the covariant Light Front Quark model. These studies show that these semileptonic decays have considerable branching ratios.

In this work, we explore the production of $D^{(*)}(3000)$, $D_{sJ}(3040)$ and their partners through the semileptonic decays of $B_{(s)}$ mesons, which is helpful to estimate the discovery potential of $D^{(*)}(3000)$, $D_{sJ}(3040)$ and their partners via the corresponding semileptonic decays of $B_{(s)}$ mesons. It is obvious that this information is valuable to future experimental search for $D^{(*)}(3000)$, $D_{sJ}(3040)$ and their partners by the $B_{(s)}$ semileptonic decays. As a relativistic quark model, the Light Front Quark model has been applied to investigate the transitions among mesons, where the obtained results agree with the experimental data within the reasonable error tolerance [12]–[52]. Thus, in this work we adopt the Light Front Quark model to calculate the production of $D^{(*)}(3000)$, $D_{sJ}(3040)$ and their partners through the semileptonic decays of $B_{(s)}$ mesons. In the next sections, we will present the calculation details.

This paper is organized as follows. After the introduction, we list the hadronic matrix elements and the corresponding form factors. In Sec. III the numerical results including the obtained form factors and the decay branching ratios are given. The final section is devoted to a summary of our work.

II. THE HADRONIC MATRIX ELEMENTS AND THE CALCULATION OF THE CORRESPONDING FORM FACTORS

In this work, we study the production of $D^{(*)}(3000)$, $D_{sJ}(3040)$ and their partners via the semileptonic decays of $B_{(s)}$ mesons, which is helpful to estimate the discovery potential of $D^{(*)}(3000)$, $D_{sJ}(3040)$ and their partners via the corresponding semileptonic decays of $B_{(s)}$ mesons.
where \( G_F \) is the fermi coupling constant and \( V_{cb} \) denotes the Cabibbo-Kobayashi-Maskawa (CKM) matrix element.

The hadronic matrix elements of \( B^+(0^-) \to D^0_j \) and \( B_0^+(0^-) \to D_s^+ \) decays can be obtained by introducing the form factors, i.e.,

\[
\langle V(P'', s_{''})|V_{cb}|B_{(s)}(P') \rangle = \frac{-1}{m_{B_{(s)}} + m_V} \epsilon_{\mu\nu\rho} \epsilon^{''\nu\rho} p^\mu q^\nu B_{(s)}(q^2),
\]

where \( \epsilon_{\mu\nu\rho} \) denotes the convention \( \epsilon_{0123} = 1 \) are adopted. In addition, there exist two relations among these form factors

\[
A_3^{B_{(s)} \to V}(q^2) = \frac{m_{B_{(s)}} + m_V}{2m_V} A_1^{B_{(s)} \to V}(q^2) - \frac{m_{B_{(s)}} - m_V}{2m_V} A_2^{B_{(s)} \to V}(q^2),
\]

\[
V_3^{B_{(s)} \to A}(q^2) = \frac{m_{B_{(s)}} - m_A}{2m_A} V_1^{B_{(s)} \to A}(q^2) - \frac{m_{B_{(s)}} + m_A}{2m_A} V_2^{B_{(s)} \to A}(q^2),
\]

where we need to specify that these form factors are dimensionless.

Since we adopt the covariant Light Front Quark model to calculate the corresponding form factors in the discussed processes, we first introduce the light-front decomposition of the momentum, i.e., \( P'' = (P'^+, P'^- P_\perp') \), where \( P'^\pm = P'^0 \pm P'^3 \) and \( P'^2 = P'^+ P'^- - P_\perp'^2 \). The initial (final) state meson has the momentum \( P'' = p'^1 + p_2 \) (\( P'' = p'^1 + p'_2 \)) and the mass of \( M'' \) (\( M' \)). Here, the mass and momentum of antiquark inside both the initial and final mesons are \( m_2 \) and \( p_2 \), respectively. The quark in the initial (final) meson has the mass \( m_1^{(')} \) and the momentum \( p'^1_1 \). These momenta are defined by the internal variables \((x_1, p'_\perp)\), i.e.,

\[
p'^{1+}_1 = x_1 P'^+, \quad p'^{1-}_1 = x_1 P'_\perp \pm p'_\perp
\]

with \( x_1 + x_2 = 1 \). Taking \( q^+ = 0 \), with these variables one further defines some useful quantities for the initial state

\[
M_0^2 = (e'_1 + e_2)^2 = \frac{p'^2_1 + m'^2_1}{x_1} + \frac{p'^2_2 + m'^2_2}{x_2},
\]

\[
M'_0 = \sqrt{M_0^2 - (m'^2_1 - m'^2_2)},
\]

\[
e'_i = \sqrt{m'^2_i + p'^2_\perp + p'^2_\perp},
\]

\[
p'_\perp = \frac{x_2 M'_0}{2} - \frac{m'^2_i + p'^2_\perp}{2x_2 M'_0},
\]

where \( e'_i \) is the energy of quark and antiquark. \( M'_0 \) can be interpreted as the kinematic invariant mass in the meson system. With \( p'_\perp = p'_\perp - x_2 q_\perp \), we can define the quantities of the final state meson.

To get the form factor, we first calculate the transition amplitude corresponding to the Feynman diagram depicted in Fig. [1]. The Feynman rules for the corresponding meson-
We first carry out the integrate with respect to the variable. For other quantities, we make the replacements:

\[ N_i^{(m)} \rightarrow N_i^{(m)} = x_i (M_i^{(m)} p_2^2 - M_0^{(m)}), \]

\[ h_M^{(m)} \rightarrow h_M^{(m)}, \quad W_M' \rightarrow W_M', \]

\[ \int \frac{d^3p'}{N_1^2 N_2^2} \tilde{F}^{PV}_{M} \rightarrow -i \int \frac{dx_1 dx_2 d^2p'}{x_2 N_1^2 N_2} h_M' \delta^{PV}_{M}, \]

where \( h'_M \) and \( W'_M \) are given by [18]

\[ h'_p = (M^2 - M_0^2) \sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2M'_p}}, \]

\[ h'_3 = \sqrt{\frac{2}{3}} h'_A = (M^2 - M_0^2)^2 \sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2M'_0}} \frac{M_0^2}{2\sqrt{3}M'_0} \times \phi'_{p}, \]

\[ W'_{A} = M'_0 + m'_1 + m_2, \quad W'_{A} = \frac{M_0^2}{m'_1 - m_2}, \quad W'_{A} = 2.(20) \]

For these quantities in Eq. (13), we have the replacements given by Refs. [15, 18]. For example,

\[ \hat{p}'_{1\mu} \equiv P_{\mu} A_{1}^{(1)} + q_{\mu} A_{1}^{(2)}, \]

\[ \hat{p}'_{1\nu} \equiv g_{\mu\nu} A_{1}^{(2)} + P_{\mu} P_{\nu} A_{2}^{(2)} + (P_{\mu} q_{\nu} + q_{\mu} P_{\nu}) A_{3}^{(2)} + q_{\mu} q_{\nu} A_{4}^{(2)}, \]

where \( A_{1}^{(1)}, A_{1}^{(2)}, A_{2}^{(2)}, A_{3}^{(2)}, A_{4}^{(2)} \) are given in Appendix.

We indicate that the light-front decomposition of these quantities results in a lightlike vector (2, 0, 0, 0) dependence, which obviously makes the Lorentz covariance be violated. However, this term can be cancelled by including so-called zero mode contribution, and finally the violation of the Lorentz covariance can be recovered (more details can be found in Refs. [15, 18]). For readers’ convenience, in Appendix we list the derivations of the relevant form factors in the hadronic matrix element of these discussed semileptonic decays, which were given in literature [15, 18].

III. NUMERICAL RESULTS

A. Form factors

As introduced in Sec. I these experimentally observed states in the charm or charm-strange family, i.e., \( D_{sJ}(3040) \) is explained as the first radial excitation of...
served separately. With these good quantum numbers, we can calculate the form factors, which can be obtained by solving the light degrees of freedom (oscillator form given in Refs. [12, 37] and the modified effective theory, the light degrees of freedom (states in the different wave functions, i.e., the harmonic-light-front wave functions are consistent with the results in [12]. We also notice that the form factor of the semileptonic decay constants given in Ref. [39].

Under the heavy quark effective theory, the light degrees of freedom are decoupled from the heavy quark in a heavy meson system, which makes that the angular momenta of the light degrees of freedom (s) and the heavy quark are conserved separately. With these good quantum numbers, we can categorize the heavy mesons into several doublets. For example, the S doublet is (0.,1.) with s = 3/2, the T doublet is (1.,2.) with s = 3/2. Thus, we can label 1+ states in the S and T doublets as \( P_{1/2} \) and \( P_{3/2} \), respectively, which satisfy the following relations [35]

\[
|P_{1/2}^{3/2}\rangle = \sqrt{\frac{2}{3}} |P_1\rangle + \sqrt{\frac{1}{3}} |P_3\rangle, \\
|P_{1/2}^{1/2}\rangle = \sqrt{\frac{1}{3}} |P_1\rangle - \sqrt{\frac{2}{3}} |P_3\rangle.
\]

We apply the light-front wave functions (LFWF) in calculating the form factors, which can be obtained by solving the realistic bound state equations. Practically, we use the gaussian-type wave function for the convenience, i.e.,

\[
\begin{align*}
\varphi' &= \varphi'(x_2, p'_\perp) = 4 \left( \frac{\pi}{\beta^2} \right)^{3/4} \left( \frac{dp'_\perp}{dx_2} \right) \exp\left( -\frac{p'^2 + p'^2}{2\beta^2} \right), \\
\varphi'_p &= \varphi'_p(x_2, p'_\perp) = \sqrt{\frac{2}{\beta^2}} \varphi', \quad \frac{dp'_\perp}{dx_2} = \frac{e'_1 e_2}{x_1 x_2 M_0}.
\end{align*}
\]

For these discussed 2P states in the \( D \) and \( D_s \) meson families, we adopt two different wave functions, i.e., the harmonic-oscillator form given in Refs. [12, 37] and the modified harmonic-oscillator function as suggested in Ref. [27], which correspond to

\[
\begin{align*}
\varphi'_p &= \varphi'_p(x_2, p'_\perp) \\
&= \sqrt{\frac{2}{\beta^2}} \varphi'_2(x_2, p'_\perp) \\
&= 4 \left( \frac{\pi}{\beta^2} \right)^{3/4} \left( \frac{dp'_\perp}{dx_2} \right) \exp\left( -\frac{p'^2 + p'^2}{2\beta^2} \right) \\
&\times \left( \frac{p'^2 + p'^2}{\beta^2} - \frac{3}{2} \right).
\end{align*}
\]

and

\[
\begin{align*}
\varphi'_{p(M)} &= \varphi'_{p(M)}(x_2, p'_\perp) \\
&= \sqrt{2} \varphi'_2S_M(x_2, p'_\perp) \\
&= 4 \left( \frac{\pi}{\beta^2} \right)^{3/4} \left( \frac{dp'_\perp}{dx_2} \right) \exp\left( -\frac{p'^2 + p'^2}{2\beta^2} \right) \\
&\times \left( \frac{p'^2 + p'^2}{\beta^2} - \frac{3}{2} \right).
\end{align*}
\]

respectively.

In our calculation, the constituent quark mass is taken as \( m_{u,d} = 0.26 \) GeV, \( m_s = 0.37 \) GeV, \( m_c = 1.40 \) GeV and \( m_b = 4.64 \) GeV [18]. In addition, the shape parameter \( \beta \) in LFWF can be determined by the corresponding decay constants [13]. We adopt the lattice results for the charmed meson family, which are fixed by fitting the corresponding results of form factor in the spacelike region (\( q^2 > 0 \)). Thus, we cannot directly apply the obtained form factors to calculate the decay width. Considering the semileptonic decay exists in the timelike region, we need to extrapolate our result to the timelike region, where we use the parametrized formula

\[
F(q^2) = \frac{F(0)}{1 - aq^2/m_B^2 + bq^2/m_B^2}.
\]

for timelike region. \( F(q^2) \) stands for a form factor, while \( a \) and \( b \) are fixed by fitting the corresponding results of form factor in the spacelike region (\(-20 \text{GeV} < q^2 < 0 \text{GeV}\)). Finally, the obtained results are collected in Table [11]. Our obtained results of the form factor of the \( B_s \rightarrow D_{sJ}(3040) \) and \( B_s \rightarrow D_{sJ}(2P_{1/2}) \) transition matrix elements using harmonic-oscillator light-front wave functions are consistent with the results in [12]. We also notice that the form factor of the semileptonic decays of \( B \) meson into the 2P state in the charmed meson family is similar to that of \( B_s \) meson into the 2P state in the charmed-strange meson family, which reflects the \( SU(3) \) flavor symmetry.

B. The semileptonic decay widths

Using these obtained form factors, we can calculate the decay widths of the production of these 2P states in the \( D_{sJ} \)
TABLE I: The experimental values of the masses of $D_{sJ}(3040)$, $D(3000)$, $D'(3000)$ and the theoretical masses of their partners. Here, the values are in unit is of MeV. Here, for distinguishing two $2P_1$ states, we use superscripts $1/2$ and $3/2$ corresponding to quantum numbers $s_f = 1/2$ and $3/2$, respectively.

| $^{nS+1}L_J$ | $D$ meson | $D_s$ meson |
|-------------|-----------|-------------|
|             | Ref. [34] | Ref. [36]   | Expt.  | Ref. [34] | Ref. [36] | Expt.  |
| $2^3P_0$    | 2919      | 2949        | 3008.1 ± 4.0 [2] | 3054 | 3067 | – |
| $2P_1^{1/2}$| 3021      | 3045        | 2971.8 ± 8.7 [2] | 3154 | 3165 | 3044 ± 8.5 [5] |
| $2P_1^{3/2}$| 2932      | 2995        | –             | 3067 | 3114 | – |
| $2^3P_2$    | 3012      | 3035        | –             | 3142 | 3157 | – |

families via the $B_{(s)}$ semileptonic decays. The concrete expressions of these discussed semileptonic decays can be obtained by using the helicity amplitude, i.e., the decay width of scalar $D_{(s)}$ is

$$d\Gamma(B_{(s)} \rightarrow S \ell \bar{\nu})$$

$$= \frac{q^2 - m_{\ell}^2}{q^2} \sqrt{\frac{\lambda(m_{B_{(s)}}, m_{S}^2, q^2)G_F^2 V_{cb}^2}{384m_{B_{(s)}}^2 \pi^3}} + \frac{3m_{\ell}^2(m_{B_{(s)}}^2 - m_{S}^2)^2 [F^{B_{(s)}-S} (q^2)]^2}{q^2} \times \frac{\lambda(m_{B_{(s)}}, m_{S}^2, q^2)[F^{B_{(s)}-S} (q^2)]^2}{384m_{B_{(s)}}^2 \pi^3}$$

(28)

the decay width of axial-vector $D_{(s)}$ reads as

$$d\Gamma(B_{(s)} \rightarrow A \ell \bar{\nu})$$

$$= \frac{d\Gamma_L(B_{(s)} \rightarrow A \ell \bar{\nu})}{dq^2} + \frac{d\Gamma^+(B_{(s)} \rightarrow A \ell \bar{\nu})}{dq^2}$$

$$+ \frac{d\Gamma^-(B_{(s)} \rightarrow A \ell \bar{\nu})}{dq^2}$$

(29)

with

$$d\Gamma_L(B_{(s)} \rightarrow A \ell \bar{\nu})$$

$$= \frac{d\Gamma^+(B_{(s)} \rightarrow A \ell \bar{\nu})}{dq^2} = \frac{d\Gamma^-(B_{(s)} \rightarrow A \ell \bar{\nu})}{dq^2}$$

$$= \left(\frac{q^2 - m_{\ell}^2}{q^2}\right)^2 \sqrt{\frac{\lambda(m_{B_{(s)}}, m_{A}^2, q^2)G_F^2 V_{cb}^2}{384m_{B_{(s)}}^2 \pi^3}}$$

$$\times \left[\frac{3m_{\ell}^2(m_{B_{(s)}}^2 - m_{A}^2)^2 [F^{B_{(s)}-A} (q^2)]^2}{q^2} \times \frac{\lambda(m_{B_{(s)}}, m_{A}^2, q^2)[F^{B_{(s)}-A} (q^2)]^2}{384m_{B_{(s)}}^2 \pi^3}ight]$$

(30)

$$d\Gamma^+(B_{(s)} \rightarrow A \ell \bar{\nu})$$

$$= \left(\frac{q^2 - m_{\ell}^2}{q^2}\right)^2 \sqrt{\frac{\lambda(m_{B_{(s)}}, m_{A}^2, q^2)G_F^2 V_{cb}^2}{384m_{B_{(s)}}^2 \pi^3}}$$

$$\times \left[\frac{3m_{\ell}^2(m_{B_{(s)}}^2 - m_{A}^2)^2 [F^{B_{(s)}-A} (q^2)]^2}{q^2} \times \frac{\lambda(m_{B_{(s)}}, m_{A}^2, q^2)[F^{B_{(s)}-A} (q^2)]^2}{384m_{B_{(s)}}^2 \pi^3}ight]$$

(31)

where $\pm$ and $L$ are the polarizations of the axial-vector $D_{(s)}$ meson. $m_{\ell}$ is the mass of lepton. We define $\lambda(a^2, b^2, c^2) = (a^2 - b^2 - c^2)^2 - 4b^2c^2$.

In Ref. [33], a special way to calculate the semileptonic decay width of $B_{(s)}$ into the tensor $D_{(s)}$ meson was proposed. With the new definition the form factors listed in Eq. (61), we can easily obtain the corresponding decay width [33]:

$$d\Gamma(B_{(s)} \rightarrow T \ell \bar{\nu})$$

$$= \frac{d\Gamma_L(B_{(s)} \rightarrow T \ell \bar{\nu})}{dq^2} + \frac{d\Gamma^+(B_{(s)} \rightarrow T \ell \bar{\nu})}{dq^2}$$

$$+ \frac{d\Gamma^-(B_{(s)} \rightarrow T \ell \bar{\nu})}{dq^2}$$

(32)

with

$$d\Gamma_L(B_{(s)} \rightarrow T \ell \bar{\nu})$$

$$= \frac{1}{2} \frac{\lambda(m_{B_{(s)}}, m_{\ell}^2, q^2) d\Gamma_L(B_{(s)} \rightarrow A \ell \bar{\nu})}{dq^2}$$

(33)

and

$$d\Gamma^+(B_{(s)} \rightarrow T \ell \bar{\nu})$$

$$= \frac{2}{3} \frac{\lambda(m_{B_{(s)}}, m_{\ell}^2, q^2) d\Gamma^+(B_{(s)} \rightarrow A \ell \bar{\nu})}{dq^2}$$

(34)

With the above preparation, we calculate the branching ratios of these discussed semileptonic decays, which are collected in Table III. Here, the obtained breaching ratios for
In general, the decays $B_s \rightarrow D_{sJ}(3040) \ell \nu$ and $B_s \rightarrow D_s(2P_1^{3/2}) \ell \nu$ using the harmonic-oscillator light-front wave functions are consistent with the results given in Ref. [12]. In addition, we also notice that the decay widths of $B_s \rightarrow D_{sJ}(3040) \ell \nu$ are ten times larger than the corresponding decay widths of $B_s \rightarrow D_s(2P_1^{3/2}) \ell \nu$, which is resulted from the mixing angle describing the mixture between $2^3P_1$ and $2^3P_1$ states in the heavy quark limit. From Table III we can learn that the semileptonic decays of

| Case I | $F(q^2=0)$ | $F(q_{max}^2)$ | $a$ | $b$ | $F(q^2=0)$ | $F(q_{max}^2)$ | $a$ | $b$ |
|--------|------------|---------------|-----|-----|------------|---------------|-----|-----|
| $F_0^{B \rightarrow D_s(3000)}$ | 0.37 | 0.34 | -0.42 | 0.295 | $F_0^{B_s \rightarrow D_{sJ}(3040)}$ | 0.41 | 0.37 | -0.47 | 0.37 |
| $F_1^{B \rightarrow D_s(3000)}$ | 0.37 | 0.47 | 1.2 | 0.34 | $F_1^{B_s \rightarrow D_{sJ}(3040)}$ | 0.41 | 0.54 | 1.35 | 0.34 |
| $A_{B \rightarrow D_s(3000)}$ | -0.15 | -0.20 | 1.2 | 0.11 | $A_{B_s \rightarrow D_{sJ}(3040)}$ | -0.17 | -0.22 | 1.27 | 0.16 |
| $V_0^{B \rightarrow D_s(3000)}$ | 0.063 | 0.099 | 2.2 | 1.5 | $V_0^{B_s \rightarrow D_{sJ}(3040)}$ | 0.08 | 0.12 | 2.11 | 1.85 |
| $V_1^{B \rightarrow D_s(3000)}$ | -0.40 | -0.35 | -0.72 | 0.53 | $V_1^{B_s \rightarrow D_{sJ}(3040)}$ | -0.43 | -0.36 | -0.90 | 0.73 |
| $V_2^{B \rightarrow D_s(3000)}$ | -0.16 | -0.2 | 1.2 | 0.2 | $V_2^{B_s \rightarrow D_{sJ}(3040)}$ | -0.18 | -0.23 | 1.22 | 0.19 |
| $A_{B \rightarrow D(2P_1^{3/2})}$ | 0.27 | 0.35 | 1.35 | 0.5 | $A_{B_s \rightarrow D(2P_1^{3/2})}$ | 0.26 | 0.35 | 1.41 | 0.41 |
| $V_0^{B \rightarrow D(2P_1^{3/2})}$ | 0.56 | 0.81 | 1.7 | 0.6 | $V_0^{B_s \rightarrow D(2P_1^{3/2})}$ | 0.57 | 0.80 | 1.65 | 0.47 |
| $V_1^{B \rightarrow D(2P_1^{3/2})}$ | 0.98 | 0.97 | -0.027 | 0.22 | $V_1^{B_s \rightarrow D(2P_1^{3/2})}$ | 1.08 | 1.07 | 0.01 | 0.18 |
| $V_2^{B \rightarrow D(2P_1^{3/2})}$ | -0.12 | -0.086 | 1.1 | 15.1 | $V_2^{B_s \rightarrow D(2P_1^{3/2})}$ | -0.12 | -0.12 | 2.0 | 12.3 |
| $b_0^{B \rightarrow D(3P_2)}$ | 0.022 | 0.032 | 1.87 | 1.22 | $b_0^{B_s \rightarrow D(3P_2)}$ | 0.023 | 0.032 | 1.88 | 1.2 |
| $V_0^{B \rightarrow D(3P_2)}$ | 0.92 | 1.06 | 0.84 | 0.65 | $V_0^{B_s \rightarrow D(3P_2)}$ | 0.95 | 1.13 | 1.03 | 0.67 |
| $V_1^{B \rightarrow D(3P_2)}$ | -0.017 | -0.026 | 1.83 | 1.04 | $V_1^{B_s \rightarrow D(3P_2)}$ | -0.018 | -0.025 | 1.8 | 1.05 |
| $b_0^{B \rightarrow D(3P_2)}$ | 0.024 | 0.033 | 1.65 | 1.21 | $b_0^{B_s \rightarrow D(3P_2)}$ | 0.024 | 0.032 | 1.71 | 1.31 |

Table II: The form factors for the semileptonic decays of $B_{sJ}$ into the corresponding $2P$ states of $D_{sJ}$ meson families. Here, $D'(3000)$, $D(3000)$ and $D^*_s(3040)$ are as $D(2P_0)$, $D(2P_1^{3/2})$ and $D_s(2P_1^{3/2})$, respectively. Case I and case II correspond to the results taking the harmonic-oscillator light-front wave function in Eq. (25) and the modified harmonic-oscillator light-front wave function in Eq. (26), respectively.
B_{c(0)} into the corresponding 2P states of the D_{c(0)} meson families have large branching ratios, which makes that these discussed semileptonic decay can be accessible at future experiments. Just shown in Table IIII the results for case I are similar to the corresponding results for case II, which indicates that taking two difference forms of wave function cannot result in obviously different results.

TABLE III: The branch ratios of the semileptonic decays of B_{c(0)} into the corresponding 2P states of D_{c(0)} meson families. Here, case I and case II correspond to the results taking the harmonic-oscillator light-front wave function in Eq. (26) and the modified harmonic-oscillator light-front wave function in Eq. (29), respectively.

| Case I | \( \ell = e \) | \( \ell = \mu \) | \( \ell = \tau \) |
|--------|----------------|----------------|----------------|
| \( BR(B \to D^*(3000)\bar{\ell}\nu) \) | \( 1.37 \times 10^{-3} \) | \( 1.36 \times 10^{-3} \) | \( 2.7 \times 10^{-4} \) |
| \( BR(B_s \to D_s(2P_{0})\bar{\ell}\nu) \) | \( 1.7 \times 10^{-3} \) | \( 1.7 \times 10^{-3} \) | \( 4.1 \times 10^{-5} \) |
| \( BR(B_s \to D(3000)\bar{\ell}\nu) \) | \( 3.13 \times 10^{-4} \) | \( 3.10 \times 10^{-4} \) | \( 8.64 \times 10^{-6} \) |
| \( BR(B_s \to D_s(3040)\bar{\ell}\nu) \) | \( 3.59 \times 10^{-4} \) | \( 3.55 \times 10^{-4} \) | \( 9.97 \times 10^{-6} \) |
| \( BR(B_s \to D(2P_{1/2})\bar{\ell}\nu) \) | \( 5.01 \times 10^{-4} \) | \( 4.96 \times 10^{-4} \) | \( 1.20 \times 10^{-4} \) |
| \( BR(B_s \to D_s(2P_{1})\bar{\ell}\nu) \) | \( 4.66 \times 10^{-4} \) | \( 4.61 \times 10^{-4} \) | \( 8.87 \times 10^{-5} \) |
| \( BR(B \to D(2P_{2})\bar{\ell}\nu) \) | \( 9.60 \times 10^{-4} \) | \( 9.46 \times 10^{-4} \) | \( 7.55 \times 10^{-6} \) |
| \( BR(B_s \to D_s(2P_{2})\bar{\ell}\nu) \) | \( 8.19 \times 10^{-4} \) | \( 8.07 \times 10^{-4} \) | \( 4.92 \times 10^{-6} \) |

| Case II | \( \ell = e \) | \( \ell = \mu \) | \( \ell = \tau \) |
|--------|----------------|----------------|----------------|
| \( BR(B \to D^*(3000)\bar{\ell}\nu) \) | \( 1.01 \times 10^{-3} \) | \( 9.95 \times 10^{-4} \) | \( 1.86 \times 10^{-5} \) |
| \( BR(B_s \to D_s(2P_{0})\bar{\ell}\nu) \) | \( 1.10 \times 10^{-3} \) | \( 1.09 \times 10^{-3} \) | \( 2.52 \times 10^{-5} \) |
| \( BR(B_s \to D(3000)\bar{\ell}\nu) \) | \( 2.57 \times 10^{-4} \) | \( 2.54 \times 10^{-4} \) | \( 5.23 \times 10^{-6} \) |
| \( BR(B_s \to D_s(3040)\bar{\ell}\nu) \) | \( 2.49 \times 10^{-4} \) | \( 2.46 \times 10^{-4} \) | \( 5.20 \times 10^{-6} \) |
| \( BR(B_s \to D(2P_{1/2})\bar{\ell}\nu) \) | \( 2.72 \times 10^{-3} \) | \( 2.69 \times 10^{-3} \) | \( 6.03 \times 10^{-5} \) |
| \( BR(B_s \to D_s(2P_{1})\bar{\ell}\nu) \) | \( 2.42 \times 10^{-3} \) | \( 2.39 \times 10^{-3} \) | \( 4.35 \times 10^{-5} \) |
| \( BR(B \to D(2P_{2})\bar{\ell}\nu) \) | \( 2.52 \times 10^{-3} \) | \( 2.48 \times 10^{-3} \) | \( 1.92 \times 10^{-5} \) |
| \( BR(B_s \to D_s(2P_{2})\bar{\ell}\nu) \) | \( 1.5 \times 10^{-3} \) | \( 1.48 \times 10^{-3} \) | \( 8.62 \times 10^{-6} \) |

IV. SUMMARY

In the past decade, the charmed and charmed-strange meson families have become more and more abundant with the experimental observations of these higher charmed and charmed-states. Among newly observed charmed and charmed-strange states, there are two charmed states \( D(3000) \) and \( D^*(3000) \) and one charmed-strange state \( D_{sJ}(3040) \) around 3 GeV. These observed \( D(3000) \), \( D^*(3000) \) and \( D_{sJ}(3040) \) can be good candidates of the 2P states in the charmed and charmed-strange meson families [3,6].

At present, \( D^{(*)}(3000) \) was only reported in inclusive processes \( pp \to D^+\pi^-X \), \( pp \to D^{0}\pi^+X \), and \( pp \to D^{*+}\pi^-X \), while \( D_{sJ}(3040) \) was observed in the inclusive \( e^+e^- \) interaction. The \( B_{c(0)} \) semileptonic decays can provide new approach to study these newly observed \( D^{(*)}(3000) \) and \( D_{sJ}(3040) \). For exploring the discovery potential of \( D^{(*)}(3000) \) and \( D_{sJ}(3040) \) via the semileptonic decays of \( B_{c(0)} \), in this work we study the production of \( D^{(*)}(3000) \), \( D_{sJ}(3040) \) and their partners through the semileptonic decays of \( B_{c(0)} \) mesons, where the covariant Light Front Quark model is adopted in the calculation.

Our calculation indicates that the branching ratios of the \( B_{c(0)} \) semileptonic decays into the 2P states of \( D_{c(0)} \) family are considerable. This information shows that experimental search for \( D^{(*)}(3000) \), \( D_{sJ}(3040) \) and their partners via the \( B_{c(0)} \) semileptonic decays is possible at future experiment. Thus, we suggest LHCb and the forthcoming BelleII to carry out the study of \( D^{(*)}(3000) \), \( D_{sJ}(3040) \) and their partners through the \( B_{c(0)} \) semileptonic decays, which is an intriguing and important research topic to further reveal the underlying properties of \( D^{(*)}(3000) \) and \( D_{sJ}(3040) \).

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Appendix: Some useful expressions

With the help of Eqs. (14)-(22), the corresponding form factors in the hadron matrix element of \( B_{c(0)} \to V \) via the loop integration in Eq. (13) [15,18] can be obtained, which include

\[
g(q^2) = \frac{-N_c}{16\pi^3} \int dx_2 d^2p' \frac{2h_{\nu}h_{\nu'}}{x_2 N_i N'_i} \left( x_3 m_1' + x_1 m_2 \\
+ (m'_1 - m_1') \frac{p'_\perp \cdot q_\perp}{q^2} + \frac{2}{w_V} \left( p'_{\perp}^2 + (p'_{\perp} \cdot q_\perp)^2 \right) \right),
\]

(35)
\[ f(q^2) = \frac{N_c}{16\pi^3} \int dx_2 d^2p_1' \frac{h_1'h_2'N_1''}{x_2N_1''} \left( 2x_1(m_2 + m_1')(M'^2_0 + M'^2_0) - 4x_1m_1'M'^2_0 + 2x_2m_1'q \cdot P \right. \\
+ 2m_2q^2 - 2x_1m_2(M'^2 + M'^2) + 2(m_1 - m_2)(m_1' + m_1')^2 + 8(m_1 - m_2) \left[ \frac{p_{1'}^2 + (p_{1'} \cdot q)^2}{q^2} \right] \right. \\
+ 2(m_1' + m_1')(q^2 + q \cdot P) \left( p_{1'}^2 + (p_{1'} \cdot q)^2 \right) \\
\left. \frac{q^2}{q^2 - q \cdot P} \left\{ 2x_1(M'^2 + M'^2) - q^2 - q \cdot P \right. \right. \\
\left. - 2(q^2 + q \cdot P) \frac{p_{1'}^2 + (p_{1'} \cdot q)^2}{q^2} - 2(m_1' + m_1')(m_1' - m_2) \right\}, \quad (36) \]

\[ a_+(q^2) = \frac{N_c}{16\pi^3} \int dx_2 d^2p_1' \frac{2h_1'h_2'N_1''}{x_2N_1''} \left( (x_1 - x_2)(x_2m_1' + x_2m_3') - [2x_1m_2 + m_1' + (x_2 - x_1)m_1']p_{1'}^2 \cdot q_{1'} \right. \\
\frac{q^2}{q^2 + (p_{1'} \cdot q_{1'})^2} \left\{ 2x_1(M'^2 + M'^2) - q^2 - q \cdot P \right. \right. \\
\left. - 2(q^2 + q \cdot P) \frac{p_{1'}^2 + (p_{1'} \cdot q)^2}{q^2} - 2(m_1' + m_1')(m_1' - m_2) \right\}, \quad (37) \]

\[ a_-(q^2) = \frac{N_c}{16\pi^3} \int dx_2 d^2p_1' \frac{h_1'h_2'N_1''}{x_2N_1''} \left( 2(2x_1 - 3)(x_2m_1' + x_2m_3') - 8(m_1' - m_2) \left[ \frac{p_{1'}^2 + (p_{1'} \cdot q)^2}{q^2} \right] \right. \\
\left. - 2x_1q^2 + p_{1'}^2 \cdot q_{1'} \right. \\
\left. - 2(q^2 + q \cdot P) \frac{p_{1'}^2 + (p_{1'} \cdot q)^2}{q^2} - 2(m_1' + m_1')(m_1' - m_2) \right\}, \quad (38) \]

which are related to the form factors in Eqs. (2)-(3), i.e.,

\[ V_{R_0}^{R_0}(q^2) = -(m_B + m_V) g(q^2), \quad (39) \]

\[ A_{1R_0}^{R_0}(q^2) = -f(q^2), \quad (40) \]

\[ A_{2R_0}^{R_0}(q^2) = (m_{B_0} + m_V) a_+(q^2), \quad (41) \]

\[ A_{3R_0}^{R_0}(q^2) - A_{0R_0}^{R_0}(q^2) = \frac{q^2}{2m_V} a_-(q^2). \quad (42) \]

Similarly, the expressions of the form factors for the hadronic matrix elements of \( B_{(o)} \to {}^1A \) and \( B_{(o)} \to {}^1A \) are given, where we only need to make some replacements in \( f(q^2) \), \( g(q^2) \), \( a_+(q^2) \) of Eqs. (35)-(38), i.e.,

\[ \ell_1(q^2) = f(q^2) \quad \text{with} \quad (m_1' \to -m_1', \ h_1' \to h_1', \ w_1' \to w_1'), \quad (43) \]

\[ q_1(q^2) = g(q^2) \quad \text{with} \quad (m_2' \to -m_2', \ h_2' \to h_2', \ w_2' \to w_2'), \quad (44) \]

\[ c_1(q^2) = a_+(q^2) \quad \text{with} \quad (m_1' \to -m_1', \ h_1' \to h_1', \ w_1' \to w_1'), \quad (45) \]

It should be noticed that only \( 1/w'' \) term was left for the \( {}^1A \) charmed meson. And then, the form factors in Eqs. (44)-(47) have the relations

\[ A_{R_0}^{R_0}(q^2) = -(m_{B_0} + m_A) q^4, \quad (46) \]

\[ V_{1R_0}^{R_0}(q^2) = -\frac{q^2}{m_{B_0} - m_A}, \quad (47) \]

\[ V_{2}^{R_0}(q^2) = \frac{q^2}{2m_A} c_A(q^2), \quad (48) \]

\[ V_{3R_0}^{R_0}(q^2) - V_{0R_0}^{R_0}(q^2) = \frac{q^2}{2m_A} c_A(q^2). \quad (49) \]

Analogously, the form factors for the hadronic matrix element of \( B_{(o)} \to S \) read as

\[ u_+(q^2) = \frac{N_c}{16\pi^3} \int dx_2 d^2p_1' \frac{h_1'h_2'N_1''}{x_2N_1''} - x_1(M'^2_0 + M'^2_0) \\
-x_2q^2 + x_2(m_1' + m_1')^2 + x_1(m_1' - m_2)^2 \\
+ x_1(m_1' + m_2)^2, \quad (50) \]
\[u_-(q^2) = \frac{N_c}{16\pi^2} \int dx_2 d\theta' p'_2 \left[ \frac{2\hat{p}'_1 \hat{P}'}{x_2 N'_{1} N_{1}'^*} \left( x_1 M'^2 + p'_1 \right) + m'_2 (m' + m_2) (x_2 m'_1 + x_1 m_2) \right.\]
\[-2 \frac{q \cdot P}{q^2} \left( p'_2 + 2 \frac{(p'_1 \cdot q)}{q^2} \right) - 2 \frac{(p'_1 \cdot q)^2}{q^2} \left. + \frac{p'_1 \cdot q}{q^2} \left[ M'^2 - x_2 (q^2 + q \cdot P) - (x_2 - x_1) M'^2 \right] + 2x_1 M'^2 \right] = 2(m'_2 - m_2) (m'_m - m'_m), \quad (51)\]

which have the relation with these form factors in Eq. (4.37).\]

\[F_2^{B_{o} \to S}(q^2) = -u_+(q^2), \quad (52)\]
\[F_0^{B_{o} \to S}(q^2) = -u_+(q^2) - \frac{q^2}{q \cdot P} u_-(q^2). \quad (53)\]

For the hadronic matrix element of $B_{o} \to T$, the form factors in Eqs. (54-58) can be written as

\[h(q^2) = -g(q^2) \left. \right|_{q \to -q} + \frac{N_c}{16\pi^2} \int dx_2 d\theta' \frac{2h'_1 \hat{P}'}{x_2 N'_{1} N_{1}'^*} \left[ (m'_2 - m'_2) (A_{3}^{(2)} + A_{4}^{(2)}) + (m'_1 + m'_1 - 2m_2) (A_{2}^{(2)} + A_{3}^{(2)}) \right.\]
\[-m'_2 (A_{1}^{(2)} + A_{2}^{(2)}) + \frac{2}{w_{V}'} \left( 2A_{1}^{(2)} + 2A_{3}^{(2)} - A_{2}^{(2)} \right), \quad (54)\]

\[k(q^2) = -f(q^2) \left. \right|_{q \to -q} + \frac{N_c}{16\pi^2} \int dx_2 d\theta' \frac{h'_1 \hat{P}'}{x_2 N'_{1} N_{1}'^*} \left[ 2(A_{1}^{(2)} + A_{2}^{(2)}) (m_2 (q^2 - \hat{N}' - \hat{N}' - m'^2 - m'^2) + 2(m'_1 + m'_1) \left( A_{2}^{(1)} Z_2 + \frac{q \cdot P}{q^2} A_{1}^{(2)} \right) \right.\]
\[+ 16(m_2 - m'_2) (A_{1}^{(2)} + A_{2}^{(2)}) + 4(2m'_1 - m'_1 - m_2) A_{2}^{(2)} + \frac{4}{w_{V}'} \left( M'^2 + M'^2 - q^2 + 2(m'_1 - m_2) (m'_1 + m_2) \right) \times \left( 2A_{1}^{(2)} + 2A_{2}^{(2)} - A_{1}^{(2)} - 2A_{2}^{(2)} \right), \quad (55)\]

\[b_+(q^2) = -a_+(q^2) \left. \right|_{q \to -q} + \frac{N_c}{16\pi^2} \int dx_2 d\theta' \frac{h'_1 \hat{P}'}{x_2 N'_{1} N_{1}'^*} \left[ 8(m_2 - m'_2) (A_{3}^{(2)} + A_{4}^{(2)} + A_{5}^{(2)} - 2m'_1 (A_{1}^{(2)} + A_{2}^{(2)}) \right.\]
\[+ 4(2m'_1 - m'_1 - m_2) (A_{2}^{(2)} + A_{3}^{(2)}) + 2(m'_1 + m'_1) (A_{2}^{(2)} + A_{3}^{(2)} + A_{4}^{(2)}) + \frac{2}{w_{V}'} \left( 2M'^2 + M'^2 - q^2 + 2(m'_1 - m_2) (m'_1 + m_2) \right) \times \left( A_{3}^{(2)} + A_{4}^{(2)} + A_{5}^{(2)} - A_{3}^{(2)} - A_{3}^{(2)} + [q^2 - \hat{N}' - \hat{N}' - (m'_1 + m'_1)^2] (A_{2}^{(2)} + 2A_{2}^{(2)} + A_{4}^{(2)} - A_{1}^{(2)} - A_{1}^{(2)} \right) \right], \quad (56)\]

\[b_-(q^2) = -a_-(q^2) \left. \right|_{q \to -q} + \frac{N_c}{16\pi^2} \int dx_2 d\theta' \frac{h'_1 \hat{P}'}{x_2 N'_{1} N_{1}'^*} \left[ 8(m_2 - m'_2) (A_{4}^{(2)} + A_{5}^{(2)} + A_{6}^{(2)} - 6m'_1 (A_{1}^{(2)} + A_{2}^{(2)}) \right.\]
\[+ 4(2m'_1 - m'_1 - m_2) (A_{3}^{(2)} + A_{4}^{(2)}) + 2(3m'_1 + m'_1 - 2m_2) (A_{2}^{(2)} + A_{3}^{(2)} + A_{4}^{(2)} \right.\]
\[+ \frac{2}{w_{V}'} \left[ 2M'^2 + M'^2 - q^2 + 2(m'_1 - m_2) (m'_1 + m_2) \right] A_{3}^{(2)} + A_{4}^{(2)} + A_{6}^{(2)} - A_{3}^{(2)} - A_{4}^{(2)} - A_{6}^{(2)} + 2Z_2 (3A_{2}^{(2)} - 2A_{6}^{(2)} - A_{4}^{(2)}) \right.\]
\[+ \frac{2q \cdot P}{q^2} \left( 6A_{2}^{(1)} A_{1}^{(2)} - 6A_{2}^{(1)} A_{3}^{(2)} + \frac{2}{q^2} (A_{2}^{(1)} A_{2}^{(2)}) - A_{1}^{(1)} \right) \times \left[ q^2 - 2M'^2 + \hat{N}' - \hat{N}' - (m'_1 + m'_1)^2 + 2(m'_1 - m_2)^2 \right] \times \left( A_{2}^{(2)} + 2A_{3}^{(2)} + A_{4}^{(2)} - A_{1}^{(2)} - A_{1}^{(2)} \right) \right], \quad (57)\]

where $g(q^2)$, $f(q^2)$, $a_+$ and $a_-$ are given in Eqs. (54-58), respectively. With the above results, we can redefine the form factors

\[A_{B_{o} \to T} = -(m_{B_{o}} - m_{T}) h(q^2), \quad (58)\]
\[V_{1}^{B_{o} \to T} = \frac{k(q^2)}{m_{B_{o}} - m_{T}}, \quad (59)\]
\[V_{2}^{B_{o} \to T} = (m_{B_{o}} - m_{T}) b_+(q^2). \quad (60)\]
Although these redefined form factors of the hadronic matrix element of $B(s) \to T$ are not dimensionless, this treatment is convenient to calculate the corresponding decay widths [33], which will be given in Sec. III.

The concrete expressions of $A_1^{(1)}, A_2^{(1)}, A_1^{(2)}, A_2^{(2)}, A_3^{(2)}, A_4^{(2)}, A_3^{(3)}, A_4^{(3)}, A_5^{(3)}, A_6^{(3)}$ and $Z_2$ are

$$A_1^{(1)} = \frac{x_1}{2}, \quad A_2^{(1)} = A_1^{(1)} - \frac{p_\perp \cdot q_\perp}{q^2},$$

$$A_1^{(2)} = -\frac{p_\perp^2 - (p_\perp \cdot q_\perp)^2}{q^2}, \quad A_2^{(2)} = (A_1^{(1)})^2, \quad A_3^{(3)} = A_1^{(1)} A_2^{(2)}, \quad A_2^{(3)} = A_1^{(1)} A_2^{(2)},$$

$$A_3^{(3)} = A_1^{(1)} A_2^{(2)}, \quad A_4^{(3)} = A_1^{(1)} A_2^{(2)}, \quad A_5^{(3)} = A_1^{(1)} A_2^{(2)}, \quad A_6^{(3)} = A_1^{(1)} A_2^{(2)}.$$