On the control of global modes in swirling jet experiments

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Abstract. The aim of this work is to control the self-excited global mode that concomitants vortex breakdown in turbulent swirling jets. This mode is characterized by a co-rotating counter winding single helical instability wave that originates from the jet center. Experiments show that the amplitude of this global mode is effectively reduced by exciting a double-helical mode in the outer shear layer. This mode is shown to be convective unstable at growth rates that are well predicted by spatial linear stability analysis. The dampening of the global mode occurs through an energy transfer between the inner and the outer shear layer.

In preparation of closed-loop experiments, a reduced order model of the flow dynamics is developed based on five leading POD modes. The model is calibrated to flow–transients recorded via time-resolved PIV. A state estimator is designed that predicts the flow state from two hot-wire probes. The performance of the estimator is validated in open-loop experiments. First results support the design of the model and state estimator.

1. Introduction

A turbulent swirling jet undergoing vortex breakdown is investigated. Breakdown is known to occur when the ratio of azimuthal to axial velocity exceeds a certain threshold. It is characterized by the appearance of a stagnation point on the jet centerline and the creation of an internal recirculation zone. For comprehensive reviews on the vortex breakdown phenomenon the reader is referred to Leibovich (1978) and Lucca-Negro & O’Doherty (2001).

Swirling jets undergoing vortex breakdown are commonly used to improve the efficiency of so-called swirl-stabilized combustors. Thereby, the internal recirculation zone deals as a flame anchor and the enhanced turbulent mixing leads to an reduction of NOx emissions. As a drawback, swirl stabilized (in particularly lean premixed) combustors are susceptible to self-excited combustion oscillations (see review article of Huang & Yang (2009)). The driving mechanism of these so-called thermo-acoustic instabilities is the dynamic coupling of heat release and acoustic pressure oscillations. This feedback cycle can be interrupted by controlling flow oscillations caused by large scale flow structures, also referred to as coherent structures. Their occurrence in swirling jets has been intensively studied within the last decade. The most prominent structure that is consistently found in experimental and numerical investigations is a self-excited large scale oscillation that is associated with an unstable global mode (Ruith et al. (2003); Liang & Maxworthy (2005); Gallaire et al. (2006); Oberleithner et al. (2011); see Huerre & Monkewitz (1990) and Chomaz (2005) for reviews on local/global instability concepts). It
is characterized by a co-rotating, counter-winding instability wave with azimuthal wavenumber \( m = 1 \) that originates from a pocket of absolute unstable flow in the region of reversed flow (Ruith et al., 2003; Gallaire et al., 2006). Within our group, the shape of this global mode was predicted by computing the most convectively unstable mode based on the measured turbulent mean flow. Its agreement to phase-averaged measurements was found to be surprisingly well in the shear layers surrounding the region of reversed flow (Oberleithner et al., 2011). The applicability of the convective analysis is of great importance for efficient flow control. It implies the validity of the signaling problem, meaning that the outer shear layer acts as an amplifier to upstream perturbations. To recall, the convective unstable nature of axisymmetric (Crighton & Gaster, 1976; Cohen & Wygnanski, 1987) and plane shear layers (Oster & Wygnanski, 1982) have enabled efficient jet-noise and separation control by means of periodic excitation (Greenblatt & Wygnanski, 2000).

In the present work, we investigate experimentally the controllability of the aforementioned global mode \( m = 1 \) that dominates the strongly swirling jet. Periodic excitation is applied at the nozzle lip where the outer shear layer is most receptive to external forcing (Cohen & Wygnanski, 1987; Long & Petersen, 1992). The flow response is quantified using stereo-PIV and hot-wire anemometry. The outline of the manuscript is as follows: The experimental setup and procedures are described in §2. The main features that characterize the natural flow are summarized in §3. In section §4, open-loop controlled experiments are described, containing forcing mode, amplitude, and frequency variations. The section §5 summarizes our first steps to model-based closed-loop experiments, including system identification, calibration and estimator design. The main observations are summarized in the conclusions (§6).

2. Experimental facility and procedure

2.1. Setup
A schematic view of the experimental setup is shown in figure 1. The swirling jet is generated by an axial and a tangential stream that merge in the swirler. The amount of swirl is adjusted by changing the ratio of azimuthal to axial volume flux, which are measured upstream of the swirler via sharp orifices. Downstream of the swirler, the jet is guided through a perforated plate, a tube, and a contraction before it emanates into the quiescent surrounding fluid. At the nozzle lip, eight loudspeakers are mounted circumferentially around the orifice. An acoustic wave guide from each speaker terminates in a rectangular duct leading to a narrow slot that does not interfere with the jet flow when the speakers are inactive. Eight speakers enable to force modes at azimuthal wavenumbers \(|m| \leq 4\). For real-time state-estimation, two hot-wires are placed near the nozzle exit in the shear layers of the jet. For future closed-loop experiments, the bridge-current of the anemometer may than be fed to a real-time controller which is connected with the speakers amplifiers. In the present manuscript no closed-loop experiments are presented and the hot-wire measurements are solely used to validate the performance of the state estimator.

2.2. PIV Measurements
Stereoscopic Particle Image Velocimetry (Stereo-PIV) is used to measure the flow field. It consists of velocity measurements of particles going through a laser sheet generated by a double pulsed Nd:Yag laser at 532 nm and 160 mJ. Two CCD cameras with a resolution of 2048 × 2048 pixels are used. Both cameras are positioned at a 45° angle in back-scattering mode in order to measure all three velocity components in a 2D-plane. As indicated in figure 1, data is taken in the plane along the jet axis. Each ensemble of PIV snapshots consists of 900 events captured at approximately 6 Hz. The camera view angle allows to acquire data even inside the nozzle which is necessary to have reliable data at \( x/D = 0 \).
2.3. Coordinate system

In consistency with previous investigations (Oberleithner et al., 2011) we use cylindrical coordinates \((x,r,\theta)\) and Cartesian coordinates \((x,y,z)\). We further decompose all flow variables into three parts (Hussain & Reynolds, 1970): a steady mean flow, a coherent component, and stochastic fluctuations, yielding \(v(x,t) = V(x) + v^c(x,\tau) + v^s(x,t)\), with the phase-locked averaged velocity \(v^p = V(x) + v^c(x,\tau)\). Table 1 summarizes the symbols used to distinguish between the different parts of the velocity components in their corresponding coordinate systems.

**Figure 1.** Sketch of experimental facility including the future closed-loop control arrangement.
Table 1. Nomenclature of velocity components used in this work

|        | Cartesian | Cylindrical |
|--------|-----------|-------------|
| $x$    | $(x,y,z)$ | $(x,r,\theta)$ |
| $V$    | $(V_x,V_y,V_z)$ | $(V_x,V_r,V_\theta)$ |
| $v^c$  | $(v^c_x,v^c_y,v^c_z)$ | $(v^c_x,v^c_r,v^c_\theta)$ |
| $v^s$  | $(v^s_x,v^s_y,v^s_z)$ | $(v^s_x,v^s_r,v^s_\theta)$ |

2.4. Characteristic numbers

The global parameters that characterize swirling jets are defined by

$$Re_D = \frac{DV}{\nu}, \quad S = \frac{\dot{G}_\theta}{D/2G_x}, \quad \text{and} \quad St = \frac{fD}{V}$$

The Reynolds number $Re_D$ is based on the nozzle diameter $D$ and on the bulk velocity $V$ which is derived from the mean mass flow rate. It is set to $V = 5.8$ m/s throughout this investigation, yielding $Re_D \approx 20,000$. The Strouhal number $St$ characterizes the forcing frequency $f$ and is based on the nozzle diameter $D$ and the bulk velocity $V$. The swirl number $S$ is defined as the ratio between the axial flux of angular momentum $\dot{G}_\theta$ and the axial flux of axial momentum $\dot{G}_x$. Since the axial momentum flux must be conserved in axial direction, the swirl number $S$ must remain constant with axial distance. As shown in an earlier investigation (Oberleithner et al., 2010), the commonly used expressions for the axial flux of momenta that are

$$\dot{G}_\theta = 2\pi \rho \int_0^\infty (V_xV_\theta + v_x^s v_\theta^s)r^2 dr \quad \text{and} \quad \dot{G}_x = 2\pi \rho \int_0^\infty \left(V_x^2 - \frac{V_\theta^2}{2}\right)r dr$$

are inaccurate in the region of vortex breakdown. The underlying boundary layer approximations are invalid in the vicinity of vortex breakdown due to the strong jet divergence. Hence, it is necessary to account for all terms of the equations of motion (Rajaratnam, 1976), yielding the more complex expressions for the axial flux of momenta

$$\dot{G}_\theta = 2\pi \rho \int_0^\infty (V_xV_\theta + v_x^s v_\theta^s)r^2 dr$$

and

$$\dot{G}_x = 2\pi \rho \int_0^\infty \left(V_x^2 - \frac{V_\theta^2}{2} - \frac{(v_x^s)^2 + (v_\theta^s)^2}{2}\right)r dr + \left(V_x \frac{\partial V_x}{\partial x} + V_r \frac{\partial V_r}{\partial r} + \frac{\partial}{\partial x} v_x^s v_\theta^s \right) \frac{r^2}{2} dr .$$

Figure 2 shows the simplified and full swirl number of the present flow configuration, derived from the PIV measurements. In the region upstream of $x/D = 2$, the appearance of vortex breakdown and the associated jet divergence falsifies the swirl number based on the boundary layer approximations. Using the full equations, the swirl number of the present investigation is approximately $S \approx 1$ throughout the measurement domain.
Figure 2. Swirl number versus axial distance. In the region of vortex breakdown ($x/D < 2$) the simplified swirl number differs strongly from the one based on the full equations.

2.5. Coherent velocity extraction
In order to decompose the velocities into a mean, a coherent, and a fluctuating part, a phase-averaging method is applied that is explained in detail in Oberleithner et al. (2011). Accordingly, proper orthogonal decomposition (POD) is used to derive the phase angle of the dominant coherent structure of each uncorrelated PIV snapshot which allows to perform a phase-average a posteriori. This method is applicable whenever the coherent structure is solely characterized by two POD modes that span the traveling wave pattern. Throughout this investigation the consistency of the phase-averaging procedure is guaranteed by either checking the phase portraits (in case of the unforced, natural flow) or by comparing the results with classically derived phase-averaged data (in case of forced flow). For the classic approach, the phase angle is derived from the excitation signal that is recorded together with the PIV acquisition time-stamp.

3. Description of the natural flow
The flow under consideration is dominated by an unstable global mode. A detailed spatio-temporal description of this mode is given in Oberleithner et al. (2011). It is described as a helical instability wave with azimuthal wavenumber $m = 1$ rotating in the same direction as the base flow but winding in opposite direction. The authors show that this mode arises from a super-critical Hopf bifurcation when the critical swirl number $S = 0.88$ is exceeded. Hence, for the presently considered swirl number $S = 1$, the global mode oscillates at its limit-cycle.

The most noticeable features of the natural flow are summarized in figure 3. The mean flow, shown in frame (a), inhibits a large recirculation region due to vortex breakdown. The streamlines and swirl velocity contours indicate the strong divergence of the jet upstream of the first internal stagnation point. The energy distribution of the global mode, shown in frame (b), indicates that the mode originates from the jet center upstream of the first internal stagnation point. The energy distribution peaks quite exact on the center of the inner shear layer (inner black thick line) upstream of the maximum diameter of the bubble. The phase-locked azimuthal vorticity distribution, shown in frame (d), represents an instantaneous snapshot of the global mode. Sinusoidal perturbation arise from the jet center and excite instabilities in the outer shear layer that are convected downstream. The turbulent kinetic energy, shown in frame (c),
Figure 3. Natural flow at $S = 1$ and $Re_D = 20,000$: mean (a), coherent (b), stochastic (c), and phase-locked (d) flow; 2D-streamlines from $V_x$ and $V_y$; gray lines represent $V_x$–profiles; thick black lines indicate center of shear layers; phase-locked vorticity $\omega_p^z$ is shown at arbitrary phase.
peaks on the jet axis upstream of the internal stagnation point. These fluctuations are mainly caused by an randomly fluctuation of the vortex breakdown location. The high axial and radial gradients at this location generates high RMS values.

4. Open-loop control

The following section describes how the swirling jet at $S = 1$ responds to sinusoidal excitation applied at $x = 0$. The experimental facility allows to force axisymmetric and azimuthal modes. To demonstrate the effect of forcing we reprint data from a previous investigation where measurements were conducted along the crosswise plane (figure 4). The flow forced at $m = 1$ differs only marginally from the natural flow, showing a precessing vortex core near the jet center. By forcing the flow at higher modes, helical waves amplify in the outer shear layer and double- and triple-spiral vortices are generated. The jet core is formed to an elliptical ($m = 2$) or triangular ($m = 3$) shape. Hence, by forcing $m = 1$, lock-in to the natural helical instability is achieved. Details to the lock-in characteristic of mode $m = 1$ may be found in Oberleithner et al. (2011), revealing typical oscillator dynamics. By forcing $m > 1$, instabilities are amplified that evolve primarily in the outer shear layer. In the following we will focus on the flow forced at $m = 2$, as this is the configuration that is chosen for flow control studies.

![Figure 4. Cross-section of the jet undergoing vortex breakdown in the center of the recirculation zone. Contours show the phase-averaged axial velocity at arbitrary phase angle. 2D-streamlines are computed from $v_c^x$ and $v_c^z$. The black dashed circle indicates the diameter of the nozzle exit. All modes rotate clockwise in the same direction as the base flow and are forced at the same frequency. Details can be found in Oberleithner et al. (2009).](image)

4.1. Amplitude variation of mode $m = 2$

The nature of the excited instability is of great importance for flow control applications. The question is, whether the forced mode $m = 2$ is convective or absolute unstable. According to linear stability theory, shear flows that are convective unstable everywhere, act as linear amplifier to infinitesimal small disturbances. This stands in contrast to absolute unstable flows which behave as oscillators (Huerre & Monkewitz, 1990).

Figure 5 shows the streamwise energy distribution of modes $m = 1$ and $m = 2$ while the latter being forced at varying amplitudes. Recall that mode $m = 1$ can only be derived when its energy is high enough to perform a reliable phase-average based on POD. For $A_{spk} = 0$, no forcing is applied and mode $m = 1$ is dominant. Its energy saturates at $x/D ≈ 1$, which coincides approximately with the maximum diameter of the recirculation bubble. Forcing mode $m = 2$ at $A_{spk} = 25$ mV does not influence mode $m = 1$ and there is hardly any energy of $m = 2$ detectable. At $A_{spk} > 25$ mV, mode $m = 2$ amplifies upstream of $x/D ≈ 1$ and mode $m = 1$ gets successively damped. For $A_{spk} > 75$ mV mode $m = 1$ is too small to be accurately
derived. Note that for $A_{\text{spk}} = 75$ mV, mode $m = 1$ peaks at $x/D \approx 2$, indicating that the forcing dampens and displaces the global mode.

Figure 6 (a) shows the amplitude of mode $m = 2$, integrated across the shear layers, at $x/D = 0.6$ versus speaker input voltage $A_{\text{spk}}$. At this axial location the modes grow exponentially in downstream direction. The linear relation between forcing amplitude and mode amplitude proves the validity of the signaling problem for mode $m = 2$. Thus, the downstream exponential growth of mode $m = 2$ can be predicted by convective linear stability analysis. This important finding enables to apply flow control at small forcing amplitudes in order to effectively...
control the flow further downstream. Figure 6 (b) shows that the saturation amplitude correlates linearly with the square root of the forcing amplitude, yielding \( \int |v_c| r dr \propto \sqrt{A_{spk} - A_{spk,0}} \), with \( A_{spk,0} = 24.7 \text{ mV} \) being the critical forcing amplitude above which \( m = 2 \) is amplified. The saturation of the convective unstable modes is apparently a non-linear process. As shown by Freymuth (1966), the saturation amplitude of a forced instability is independent of the forcing amplitude provided that the mean flow remains unchanged. Hence, the presently observed dependence of the saturation amplitude on the forcing amplitude must result from a significant manipulation of the mean flow.

**Figure 7.** Results from linear stability analysis employing the natural mean flow. Left: Contours of spatial amplification rate \( \alpha_i D \) of mode \( m = 2 \); dashed lines indicate the forcing frequencies of the experiments; right: overall amplification of forced mode \( m = 2 \).

**Figure 8.** Streamwise development of coherent energy \( |v_c|^2 \) of the natural mode \( m = 1 \) (left frame) and the mode \( m = 2 \) forced at \( A_{spk} = 100 \text{ mV} \) (right frame)
4.2. Frequency variation of mode $m = 2$
Within the framework of convective linear stability analysis, modes forced at different frequencies undergo different amplification cycles. In figure 7 (a), contours of the spatial amplification rate $\alpha_i$ of mode $m = 2$ is shown. It is derived by numerically solving the dispersion relation of the Orr-Sommerfeld operator applied to the natural mean flow. A detailed description of the stability analysis of swirling jets can be found in Oberleithner et al. (2011). Figure 7 (b) displays the integral $\int_0^x \alpha_i dx$, which represents the overall amplification of mode $m = 2$ forced at a constant frequency St. For the four St considered, the computations predict the highest overall amplification for $St = 0.22$, followed by $St = 0.44, 0.66$, and $0.88$. This contradicts the measured overall amplification shown in figure 8. Experiments assign mode $m = 2$ forced at $St = 0.44$ to reach the highest overall energy, causing the strongest suppression of mode $m = 1$. Forcing at $St = 0.88$ does not show any significant impact on the global mode, at all. Apparently, the linear stability analysis based on the natural mean flow is not capable to model the non-linear saturation that occurs downstream of $x/D = 0.8$. There, the inner and the outer shear layers interact which causes the dampening of the global mode and a significant change of the mean flow. Qualitatively speaking, one may derive the following conclusions: Forcing at $St = 0.44$ is most effective because it goes through its complete amplification cycle before it interacts non-linearly with the global mode. This is not the case for $St = 0.22$. Contrarily, instabilities forced at $St = 0.88$ grow rapidly, saturate and decay before they interact with the global mode. This scenario is supported by the fundamental derivations of Pier (2009). He demonstrates that flows with absolute unstable regions are controlled by upstream harmonic perturbations provided that the forced mode reaches non-linear saturation at sufficiently high amplitude upstream of the absolute unstable regime.

4.3. Impact of forcing on the flow characteristic
As mentioned above, that fact that the saturation amplitude of $m = 2$ scales (non-linearly) with forcing amplitude indicates that the forcing alters significant the mean flow and its stability. The mean flow, the coherent structures, and the turbulent intensity distribution is shown in figure 9 exemplary for the flow forced at mode $m = 2$, $St = 0.44$, and $A_{spk} = 100 \text{ mV}$. It may be compared to figure 3 which shows the same flow quantities of the corresponding natural flow. The streamlines in frame (a) indicate a significant reduction and a downstream displacement of the recirculation bubble which goes in hand with an decreased jet divergence. This causes the swirl velocity to decay slower in axial direction. In contrast to the natural flow, the coherent kinetic energy (frame (b)) shows high values in the outer shear layer close to the nozzle exit. The phase-locked vorticity (frame (d)) shows the roll-up of the outer shear layer to double-helical vortices. The maximum coherent energy in the outer shear layer is located at $0.7 < x/D < 1$ which agrees quite well with the theoretically predicted overall amplitude (confirm with left frame of figure 7). In that region coherent energy is transfered to the inner shear layer—the source of the global mode—presumably leading to the nonlinear saturation discussed in figure 6 (b). Hence, the forced mode $m = 2$ originates in the outer shear layer and spreads to the inner one, whereas for the natural flow, the mode $m = 1$ originates from the jet center and spreads to the outer shear layer.

Finally, we consider the impact of the open-loop forcing on the location and strength of vortex breakdown. In vortex breakdown studies it is a challenging task to define a global variable that characterizes the state of vortex breakdown. An intuitive quantity might be the size and location of the recirculation bubble. Figure 10 displays the mean and RMS-fluctuations of the upstream and downstream stagnation points for the natural and the forced flow. For the previous, the upstream stagnation point is located at $x/D \approx 0.5$ with an RMS of 0.15. The corresponding downstream stagnation point is located at $x/D \approx 1.5$ and shows much higher fluctuations. Forcing at mode $m = 2$ at moderate amplitudes shifts the upstream end of the recirculation
Figure 9. Flow forced at $m = 2$ and $St = 0.44$: mean (a), coherent (b), stochastic (c), and phase-locked (d) flow; 2D–streamlines from $V_x$ and $V_y$; gray lines represent $V_x$–profiles; thick black lines indicate center of shear layers; phase-locked vorticity $\omega_p^z$ is shown at arbitrary phase.
bubble downstream and enhances its fluctuation.

5. First steps to model based closed-loop control

This section describes the first approache for a closed-loop control of the swirling jet. The accomplished steps are the modeling of the main flow dynamics trough POD modes, and the design of a state estimator based on this model. The flow model and the estimator are designed to capture the natural flow, the actuated flow (with mode $m = 2$) and the flow states in between these two limit states. The aim of this work is the estimation of the global modes Fourier amplitudes with time resolved point measurements (similar to Seidel et al. (2009); Luchtenburg et al. (2010)).

5.1. Calculation of POD Modes

As data basis for the model calibration a time resolved PIV measurement of the transient between the natural and actuated state is used. The flow is actuated with mode $m = 2$ as this mode has prior proven its good capability in dampening the natural mode $m = 1$. The measurement consists of a snapshot time series (3.2k samples at acquisition frequency 500 Hz) during which the actuation is switched on and off four times. To calculate the steady state POD modes for the actuated and natural flow a subset of the measurement is used. These subsets are the parts of the transients where the natural/actuated mode reach its saturation amplitude (also referred to as limit-cycle or steady-state). From both steady state subsets a POD is calculated providing spatial modes for each flow state. The modes that are used for the reduced order model (ROM) are: two modes for actuated oscillation, two modes for natural oscillation and one shift mode. The selected spatial modes are illustrated in Figure 11. They are merged to an orthogonal basis with Gram-Schmidt algorithm. Finally, the temporal POD amplitudes (also called POD Fourier amplitudes) are calculated by projecting the entire snapshot time series on the selected spatial
Figure 11. POD modes selected as basis for the flow model. Each row constitutes the contour plot of a flow component (indicated at the right side), while each column represents a POD mode (1, 2 - actuated mode; 3, 4 - natural mode; 5 - shift mode).

The resulting POD amplitudes are depicted in Figure 12. For a detailed description of the POD procedure see e.g. Oberleithner et al. (2011).

5.2. Reduced Order Model (ROM)

The POD Fourier amplitudes were modeled with a generalized mean-field model (Luchtenburg et al., 2009) which characteristics fit remarkably well to the calculated POD amplitudes. The
Figure 12. POD amplitudes and simulated ROM model, only the normalized amplitude of the oscillatory natural and actuated mode is depicted (see eq. (2)). The actuation was turned on at $0 < t < 0.8$.

The model describes the flow as two oscillating pairs of modes, one for each flow state, and an additional shift mode for the mean flow variations:

$$
\frac{d}{dt} \begin{bmatrix} a_{n,1} \\ a_{n,2} \\ a_{a,1} \\ a_{a,2} \\ a_s \end{bmatrix} = \begin{bmatrix} \tilde{\sigma}^n & -\tilde{\varphi}^n & 0 & 0 & 0 \\ \tilde{\varphi}^n & \tilde{\sigma}^n & 0 & 0 & 0 \\ 0 & 0 & \tilde{\sigma}^a & -\tilde{\varphi}^a & 0 \\ 0 & 0 & \tilde{\varphi}^a & \tilde{\sigma}^a & 0 \\ 0 & 0 & 0 & 0 & \tilde{\sigma}^s \end{bmatrix} \begin{bmatrix} a_{n,1} \\ a_{n,2} \\ a_{a,1} \\ a_{a,2} \\ a_s \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \kappa & -\lambda \\ \lambda & \kappa \\ 0 & 0 \end{bmatrix} b \quad (1)
$$

Where $a_{n,1}a_{n,2}$ are the Fourier coefficients of the natural POD modes, $a_{a,1}a_{a,2}$ of the actuated POD modes, and $a_s$ is the shift mode amplitude. The corresponding oscillation amplitudes are:

$$
\hat{a}_n = \sqrt{a_{n,1}^2 + a_{n,2}^2}
$$

$$
\hat{a}_a = \sqrt{a_{a,1}^2 + a_{a,2}^2}
$$

(2)

The vector $b$ contains the sine and cosine of the periodic forcing signal. All amplitudes are normalized with the saturation amplitudes (amplitudes at the steady-states). The coefficients of the ROM are:

$$
\tilde{\sigma}^n = \sigma^n(1 - \hat{a}_n^2) - \sigma^{n,a}\hat{a}_a^2
$$

$$
\tilde{\sigma}^a = \sigma^a
$$

$$
\tilde{\sigma}^s = \sigma^s(\hat{a}_a^2 - \hat{a}_n^2 - a_s)
$$

$$
\tilde{\varphi}^n = \varphi_n = \text{const.}
$$

$$
\tilde{\varphi}^a = \varphi_a = \text{const.}
$$

(3)
The frequencies $\varphi_n, \varphi_a$ are calibrated through a Fourier transformation of the POD amplitudes at steady state. The amplification rates are determined with least square fit at the transient amplitudes. The behavior of the resulting system together with the POD amplitudes are shown in Figure 12. The POD amplitudes are scattered in contrast to the model dynamic. This is due to unresolved POD modes and turbulent fluctuations. Note that the investigated POD modes contain only 10% of the turbulent kinetic energy in the flow.

This special system is not necessarily selected with respect to flow physics, but more to fit the amplitude dynamics extracted with the POD. The model can be understood as two oscillators for $m=1$ and $m=2$ around the steady state frequencies and a low pass filter which separates the slow mean flow variations (shift mode). The shift mode has no influence on the dynamics of the oscillating modes. It is essential for the state estimation as it captures the hot-wire offsets caused by the changing mean field and axial breakdown fluctuations.

5.3. State Estimation

The aim of the state estimation is the calculation of the complete flow state (POD Fourier coefficients) from the limited information of time resolved point measurements (so called observers). Therefore, the relationship between POD amplitudes and the observer signal $c = h(a)$ are derived from spatial POD modes. Generally speaking, the shortcoming of the model due to insufficient resolved flow features as well as the random turbulent noise cause uncertain state estimation. This is taken into consideration by adding noise terms to equation (1), which yields the following system:

\[
\frac{da}{dt} = g(a, b) + n_g, \quad (4)
\]

\[
c = h(a) + n_h, \quad (5)
\]

where $g$ is a nonlinear function (here the ROM-model) describing the flow of the system state $a$ with forcing $b$ and $h$ is the observation function describing the relationship of $a$ to the observed velocities $c$. $n_g$ and $n_h$ are the system and observation noise. If noise is assumed to be Gaussian this estimation problem can be solved with an extended Kalman filter (EKF). First numerical investigations using the EKF showed a miserable performance when the noise level exceeded a certain threshold. Much better results are achieved with the unscented Kalman filter (UKF) which uses a statistical approach to calculate the system covariance instead of the Jacobian of $g$ (see Julier et al. (2004) for details).

A critical point for the state estimation is the correct placement of the observers. A position is wanted with high signal level and a low signal to noise ratio. The sate estimation setup is arranged with two hot-wire probes placed symmetrical with respect to the jet axis, which allows the observer to distinguish between the symmetric mode $m = 2$ and the antisymmetric mode $m = 1$ (compare Fig. 11). A satisfying position for the observer is at $x/D = y/D = 0.4$. As can be seen in Fig. 3 and 9 both modes have a sufficiently large amplitude and the turbulent noise is reasonably low.

To evaluate the estimation performance PIV measurements are carried out and the velocity is measured with two hot-wires simultaneously. This evaluation measurement is performed for different open-loop actuation amplitudes (each 1k samples at acquisition frequency 6 Hz). From these measurements the POD Fourier coefficients are calculated from the PIV snapshots and estimated from the hot-wire measurements with using the UKF. To validate the state estimation the correlation $R$ of the estimated signals with the measured POD amplitudes is used, yielding:

\[
R = \frac{\langle a_{POD,a_{UKF}} \rangle}{\| a_{POD} \| | a_{UKF} \|}, \quad (6)
\]
Figure 13. Estimator evaluation for different actuation amplitudes. Top plot shows the correlation between estimated and true POD Fourier coefficients (see eq. (6)). Bottom plot shows the average estimated and true amplitudes.

where $\langle \cdot, \cdot \rangle$ indicates a temporal scalar product and $|\cdot|$ the associated norm. The results show only moderate agreement between the amplitudes calculated through POD and the estimated amplitudes (figure 13 top). The best results are achieved for each mode at its limit state. These lacks of accuracy occur mainly due to the high noise level of the hot-wire signal (same magnitude as system dynamics). The same result can be seen in figure 13. Accordingly, the accuracy of the estimation depends strongly on the amplitude of the corresponding mode. In future experiments this problem can be overcome using more sensors.
6. Summary and conclusion

In this work, the control of a turbulent swirling jet undergoing vortex breakdown is envisaged. This flow configuration is dominated by a self excited global mode with azimuthal wavenumber $m = 1$. Experiments are conducted at $S = 1$ and $Re_D = 20,000$, where the global mode oscillates at its limit-cycle oscillation. The phase-averaged measurements of the natural flow reveal that global oscillations originate from the jet center, upstream of the internal recirculation zone. Further downstream, energy is transferred to the outer shear layer and large scale coherent structures evolve.

By means of sinusoidal forcing at the nozzle lip, instability waves are excited in the jet periphery that grow exponentially in downstream direction. Forcing mode $m = 1$ causes the global mode to lock-in to the actuation. Forcing at mode $m = 2$ excites a double-helical instability wave and suppresses the global mode. Its amplitude scales linearly with the actuation amplitude, revealing that mode $m = 2$ is convectively unstable. Hence, the outer shear layer acts as a linear amplifier for upstream perturbations – a necessity for efficient active flow control. Downstream of $x/D = 0.8$ the forced instability saturates non-linearly. In this axial region, coherent energy is transferred from the inner to the outer shear layer (natural flow) and from the outer to the inner shear layer (actuated flow). The frequency dependence of the forced mode $m = 2$ and its effectiveness in suppressing the global mode is related to this interaction process. Waves that reach their maximum amplitude (neutral instability) right upstream of $x/D = 0.8$ reach the highest amplitude and are most effective in suppressing the global mode. This scenario is supported by recent theoretical considerations of Pier (2009).

The impact of the forcing on the mean flow is validate by tracking the location of the recirculation bubble for different forcing amplitudes. The excitation of instabilities in the outer shear layer drastically shortens the recirculation zone and displaces it downstream. As commonly know, a region of reversed flow is closely related to absolute instability (e.g. Monkewitz (1988); Huerre & Monkewitz (1990); Gallaire & Chomaz (2003); Gallaire et al. (2006)). Furthermore, a necessary condition for an globally unstable flow is a sufficiently large pocket of absolutely unstable flow. Hence, a shortening of the recirculation zone reduces the temporal growth rate of the global mode until it eventually reaches zero amplification and the global mode is completely dampened. This scenario can only be proven by an spatio/temporal analysis of the forced flow and remains tentative within this work.

Finally, an outlook for future closed-loop experiments is given in this article, including the development of a reduced order model based on POD that captures the dynamics of the natural flow, the actuated flow, and the transient flow in between. Although, the POD decomposition of the natural and actuated flow provides the dominant modes for both flow states, the modes capture only a small amount of the total turbulent kinetic energy. In consequence a large portion of the flow dynamics is ascribed to turbulent noise. The interaction of the identified modes can be described through the selected mean-field model, but a close inspection reveals that the POD amplitudes at the steady-states are not really steady. These shortcomings are either be attributed to an insufficient resolution of flow features or to stochastic turbulent fluctuations. With the proposed state estimator it is theoretically possible to extract the states of the dominant modes, but the high level of turbulent noise, corrupting the observers signal, hampers an exact prediction of the flow state. An escape would be the use of more sensors, as they should cancel out the noise trough averaging.

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