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Calculation of tooth profile radiuses of curvature into line of contact parameters

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Abstract. Authors created basis formula and algorithm of calculation of tooth profile radiuses of curvature into line of contact parameters. Employment of this basis formula combined with famous Euler-Savary equation allows calculation reduced radius of curvature in any of points on line of contact and contact stress (by Herz) between teeth of gear and wheel without finding of that’s profiles. This greatly simplifies the synthesis of optimal line of contact in cylindrical gears because allows to calculate any of quality indicators in any points on line of contact before finding contacting tooth profiles. And find on fixed plane places with good, acceptable, bad or unacceptable conditionals of tooth contacts There is a given an example of three tasks solutions with using programmes that was created with foundation on basis formula.

1. Statement of the problem and objectives of the investigation

The gears are an important component element of many machines. Quality and marketability of machines are majority depends of gears reliability and long on performance. The most attractive way to up of load possibility is seems in updating of engagement geometry, because it allows taking gears with low dimensions and the weigh for the same materials, precision, oiling and manufacturing costs. Engagements geometry is identified in design phase. Science base of engagement designing and gear tools is Theory of Gearing. Main objects of researching in classic Theory of Gearing is complicated surfaces and tooth profiles that is formed by bending methods, i.e. with complicated relative movements of members in gears and machines engagements.

1.1. Major elements of flat engagement

On Figure 1 is shown elements of flat engagement. It main physical objects is $\Sigma_1$ and $\Sigma_2$ – gear and wheel tooth profiles. Major parameters of contact between $\Sigma_1$ and $\Sigma_2$ are K – point of contact; $\Sigma$ – path of contact, i.e. movement trajectory of point K relative to the column; a and b – starting and ending points of tooth contact, i.e. limit of path of contact $\Sigma$. Points and lines that characterized engagement is W – pitch point, i.e. point where relative speed of two member is zero (it is the instant center of relative speeds); $P_1$ and $P_2$ – centroids, i.e. lines that in members movement is rolling around together without slipping (it’s trajectories of pitch point of engagement W relative system of coordinate for gear and wheel respectively, $P_1$ and $P_2$ is always touching in pitch point). $O_1$ and $O_2$ – centroids curvature centers in point W those touching; $r_{w1}$ and $r_{w2}$ – radiuses of curvature of centroids in point.
W. Important that if $O_1$ and $O_2$ are the centers of rotation of gear and wheel in gearing with constant reduction ratio, that $P_1$ and $P_2$ – cutter pitch circle and its radii are equal $r_{w1}$ and $r_{w2}$.

Vectors and angles in engagement: $n$ – total normal to the profiles $\Sigma_1$ and $\Sigma_2$: that normal is passing into pitch point by basis theorem of flat engagements; $\alpha$ – pressure angle, i.e. angle between $n$ and total tangent to centroids $P_1$ and $P_2$ in pitch pole $W$.

On Figure 1 is shown: four quadrants around pitch point $W$; two paths of contact ($\Sigma$ – interval $ab$ and $\Sigma^*$ – interval $a^*b^*$) for right and left sides of tooth in accordance; general system of coordinate XWY that placed on column with center in pitch $W$. If pitch pole $W$ is stationary, so and system XWY is stationary too. If pitch pole $W$ is moving (for example in non-circular wheel gears), so and XWY is will be move.

Presented detailing exposition of two figures is made with point to bring into focus that researching is executed for total flat engagement and not for only cylindrical spur gears.

![Figure 1. General elements of flat engagement.](image1)

![Figure 2. General system of coordinates XWY.](image2)

1.2. Short review of engagements synthesis methods

There is a two main methodologic of gears geometry synthesis:

1.2.1. Methodology 1

It is based on classical ways of creating the fits engagements, that taking the way from Olivier researches [1] (in [2] is described eight kinds of this ways). In synthesis is setting up the profile (surface) of one of tooth wheels and initial instrumental surface (one or two cutting surface of lines). After that using synthesis method is finding parameters of setting profile (surface) with maximal load possibility of gearing. By the mathematical point of view that kind of synthesis is comes to task of non-linear programming with task to find the vector $x$ with objective function $F(x)\rightarrow_{\text{max}}$ that is confines by limit types of inequalities and equalities. In practice of this type of designing [3-7] is often using special diagrams (limit counters [3] among them dynamical [4, 5]) which including lines of equal level with most important quality indicators. Those diagrams is constructing in system of coordinates of desired (various) parameters. For example is for involute gears in coordinates of desired distance coefficients $x_1$–$x_2$.
1.2.2 Methodology 2

It is special characteristics is consist of using synthesis method for finding not set of optimal parameters $x$, but control function $f(z)$ where one objective function $F(f(z)) \rightarrow \max$ with getting limits. By the mathematical point of view that is the variational problem with burden of limits existence. The point of control function $f(z)$ for the flat engagements is in ruling of one of tooth profiles $\Sigma_1, \Sigma_2$ or the equal of path of action $\Sigma$. It is important if we know one of pathes $\Sigma_1, \Sigma_2$ or $\Sigma$ so it is possible to find other two pathes using methods of Theory of Gearing. In this case if we know $\Sigma_1$ of $\Sigma_2$ we have to calculate algebraically or trigonometrical equation of engagement [8], but if we know $\Sigma$ we have to calculate differential equation by approximation with high precision. Methodology 1 is applying in papers [2–8], methodology 2 is applying in papers [9–12].

1.3 Quality indicators of engagements work

Engagement analysis and synthesis is usually containing calculations of different quality indicators by all of possible points of contact field. Most important of these indicators for flat engagements is radusses of curvature $\rho_1$ and $\rho_2$ of two members profiles. In fits flat engagements $\rho_1$ and $\rho_2$ is combined by famous equation of Euler-Savary:

$$\frac{1}{\rho_1} + \frac{1}{\rho_2} = \left(\frac{1}{\rho_{1+\Sigma W K}} + \frac{1}{\rho_{2+\Sigma W K}}\right) \cdot \sin \alpha$$

(1)

All of parameters that included in (2) is described in part 1.1 and is shown in Figure 1. If one of $\rho_1$ or $\rho_2$ is known then other Is possible to find use (1). After that and reduced radius of curvature $\rho_\Sigma$ too by equations:

$$\rho_\Sigma = \frac{\rho_1 \rho_2}{\rho_1 + \rho_2} ; \; k_{1,2} = \frac{1}{\rho_{1,2}} ; \; k_\Sigma = k_1 + k_2$$

(2)

Important that in over the past years many of Scientists (S A Lagutin, V P Shishov, V I Golfarb, E S Trubachev D T Babichev and others) considering in synthesis how is quality indicators proceed which placed it in standing system of coordinates. With bases on analysis of works [1–14 etc.] and on personal experience is created opinion that methodology 2 is very perspective (by the many reasons that we didn’t illuminate here) with next characters:

1.3.1 Quality indicators and zone of engagement should be placed in coordinates XXY (Figure 2)

1.3.2 With bases on analysis of quality indicators proceeding in system XXY to synthesize optimal path of action $\Sigma$ (Figures 1 and 2) by first and only then to find the tooth profiles.

1.4 Target setting and way of solution.

Step 1.3.2 is possible to realize with condition that we actually know how to calculate radusses of curvature $\rho_1$ for tooth profile $\Sigma_1$ into radusses of curvature $\rho$ or through some another parameters of path of action $\Sigma$. So if along with Euler-Savary equation for pair of tooth profiles $\{\Sigma_1+\Sigma_2\}$ was known and analog of equation (1) for pair of pathes $\{\Sigma+\Sigma_1\}$ either. By the analysis of more than 1000 russian-languages works in [15–16] by Theory of Gearing we have detected on setting and no ways of solution for target of finding the analogue of equation (1) for path $\{\Sigma+\Sigma_1\}$. We have created that equation for pair of pathes $\{\Sigma+\Sigma_1\}$ in flat engagement with progressive advance of one of member. That equation is given below in part 2 with development.

1.5 Object and tasks of researching

Main object of researching is flat engagement with rotation members. In present paper is considered in the main the cylindrical spur gearings with constant of reduce ratio. Task of researching in next: with using of created equation, to design the method of calculation of all radusses of curvature in cylindrical gearing without finding of these tooth profiles; to develop the algorithms and calculation schemes and to get all of needed equations for it; to design interface elements; to realize algorithms and equations in researching types computer programs; to solute some typical task by program.
2. Basis formula for calculation radius of curvature of tooth profile by parameters of path of action

Basis formula is developed with consider that flat engagement is with one progressive moving member, i.e. rack. That ‘rack’ engagement is chooses because there is an equation of relative shift of rack and it is linear [8], and development of formula may be simplify. In believing that engagement is created by the first way of Olivier [2] and after creation of three calculation schemes (is shown on Figure 3-5) was developed basis formula for calculation of radius $\rho$ of curvature of rack profile by parameters $\alpha$ and $\lambda$ of path of action $\Sigma$ and measure $WK$ between contact point $K$ and pitch point $W$:

$$\rho = -WK \cdot \left(1 + \frac{\tan \alpha}{\tan \lambda}\right) \quad (3)$$

In formula (3): $\alpha$ – pressure angle, i.e. angle between the normal to profiles in point $K$ of fits $n$ and axis $XW$: total tangent to centroids $P_1$ and $P_2$ in pitch pole $W$ (see $\alpha$ on Figure 1 and $\alpha_0$ on Figure 3–5); $\lambda$ – angle between contact normal $KW$ and tangent $dL$ to the path of action $\Sigma$ (Figures 3-5). It is angle $\angleNK_0K_1$ by triangle $NK_0K_1$ that is equality $\lambda=\theta_0-\alpha_0$ (see Figures 4 and 5). Angle $\alpha$ is a positive if normal $KW$ is get turned relative to $V_0$ contra clockwise. Angle $\lambda$ is a positive if normal $KW$ is get turned relative to $V_0$ contra clockwise. Radius of curvature $\rho$ is positive if rack profile is convex. I.e. in point motion by cutting profile $\Sigma$ along tangent $\tau_0$ (or along differential $du$) the normal $n$ is turning contra clockwise.

By the development of the formula (3) in case of initial formula was taken equation $\rho= du/d\alpha$, where $du$ is measure between common points on profile (flat line), and $d\alpha$ is an angle between normal (or tangents) to profile on end-points of segment with length $du$. That expression illustrate famous proposition of geometry that single ‘curvature $k=1/\rho$ is equal by angular velocity unit vector of normal to the line (surface) in motion on that line (surface) with linear velocity is equality by 1’.

3. Radii of curvature of gear and wheel tooth profiles.

Reduced radius of curvature in contact. Contact stresses.

Which know that in forming of engagements by first way by Olivier that in gearing $\{\Sigma_1+\Sigma_2\}$ and in two machine engagement $\{\Sigma_0+\Sigma_1\}$ and $\{\Sigma_0+\Sigma_2\}$ path (surface) of action is match and also with relying on formula (3), was created next algorithm of calculation of all curves in all three engagements:

1) To set a point on path of action, i.e. to calculate their coordinates and measure $WK$ and to find the parameters of path of action $\Sigma$ in its point (angles $\alpha$, $\theta$, $\lambda$).

2) To calculate the radius of curvature $\rho_0$ of rack $\Sigma_0$ by formula (3).
3) To find radius of curvature $\rho_1$ of gear teeth by formula of Euler-Savary (1) in machine engagement $\Sigma_0$ with $\Sigma_1$. In additional that in formula (1) $r_w=\infty$ and $\rho_2=\rho_0$:

$$\rho_1 = \frac{r_{w1}\sin\alpha (\rho_0 + \text{sign}(x_K)WK)}{(\rho_0 + \text{sign}(x_K)WK) - r_{w1}\sin\alpha} + \text{sign}(x_K) \cdot WK$$

(4)

4) To find radius of curvature $\rho_2$ of wheel teeth in machine engagement $\Sigma_0$ with $\Sigma_2$ (with $r_w=\infty$ and $\rho_1=\rho_0$):

$$\rho_2 = \frac{r_{w2}\sin\alpha (\rho_0 - \text{sign}(x_K)WK)}{(\rho_0 - \text{sign}(x_K)WK) - r_{w2}\sin\alpha} - \text{sign}(x_K) \cdot WK$$

(5)

5) To calculate the reduced radius of curvature $\rho_5$ and reduced curvature $k_2$ by formulas (2).

6) To find the contact stresses by famous formula of Hertz:

$$\sigma_H = \frac{F (\rho_1+\rho_2) E_1 \cdot E_2}{\pi b_2 \cdot \rho_1 \cdot \rho_2 [E_1 (1-v_2^2)+E_2 (1-v_1^2)]}$$

(6a)

or by

$$\sigma_H = 0.418 \frac{F \cdot E}{b_2 \cdot \rho_5}$$

(6b)

7) To calculate another quality indicators and to construct a graph for it.

4. Development of mathematics models, algorithms, interface and programs

Is defined a list of input and output data, is got a calculating formulas (there is a more then 100 with included part of formulas for calculating of local quality indicators, and all of equations are linear); is developed basis of user interface of programs for analysis and synthesis (partially); is realized a most important programs for analyses that will be included in programs for synthesis. The main input data is close up to data that used for checking calculation. It is the geometrical, kinematical, energy and forces parameters: $a_w$, $i_{12}$, $z_i$, $a_h$, $h_{12}$, $h_a$, $c_0$, $k_s$, $S_{a1}$, $S_{a2}$, $b_2$, $c_2$, $P_1$, $n_1$, $\omega_1$, $[\sigma_0]$, $T_2$, $F_p$. These symbols are generally accepted and standard; is possible to use values by default and by recommends with included calculation and standard; there is a help and explanations marks for inputting data that can appear automatically (on Russian language for the present time). Also is setting data for the calculation and cycles ruling, and parameters of path of action that can consist of three typical lines: segments of line, circles, and involutes.

All of basis segments is setting by identical sets of data: (a) by coordinates of initial point: $x_A$, $y_A$; (b) by tangent $\tau_0$ in initial point (by angle $\theta_0$); (c) by measure of segment $L$. For circle and involute additional are sets (d) radii of curvature $R$ or $R_A$ in initial point $A$. Developed system of input data
allows constructing path of action with several of basis segments. For this is needed to place begin of next segments (point $A_{i+1}$) in end of previous segment (point $B_i$). Also needed to set an angle of kink $\Delta \theta$ in situation when the composing segments without contact. Angle $\Delta \theta > 0$ if $\theta_{A_{i+1}} > \theta_{B_i}$. For all of typical lines is construct a mathematical models (for example to calculation $r_b$ and coordinates of base circle center point $O_1^*$ on Figure 8); is development algorithms and created first computer programs by researching types.

5. Examples of solution for analysis tasks by this programs

Results of two tasks solution is presented on Figures 9 and 10. And each of tasks is a little researching. In task 1 the list of input data is the same of list for gearing analyses and synthesis (see part 4) and data pate is equal: $aw=150$ mm, $i_{12}=2$, $z_1=15$, $ha_1^*=ha_2^*=1$, $P_1=40$ kW, $n_1=600$ min$^{-1}$; wheel face width $b_2=43.6$ mm was adjustment for level of contact stresses by equal $\sigma_H=1000$ MPa with $\alpha_w=200$. Let’s explore the results and some specials of this task solution.

Task 1 – To explore the varieties of curvatures and contact stresses along sett path of action for the pre-pitch cylindrical gearing with path of action $\Sigma$ by type of segment of right line.

It is important that program ‘is didn’t know’ that analyses is made for involute engagement. More of that, the involute tooth profiles were not identify either. Although the program founded point $N_1$ that one of theoretical ending points of path of action. That was founded like a point of path of contact with radius of curvature of pinion gear tooth profile $\rho_1=0$. For the left from point $N_1$ tooth contact is impossible because there is a fits tooth profile of pinion gear is forming into tooth body, and reduced radius of curvature $\rho_\Sigma=0$ – see right and lowest of diagrams on Figure 9. Were made a comparison with results of calculations by other our programs for involute gearing design for all of diagrams that presented on Figure 9. Results by curvatures and of contact stresses are match.

![Engagement zone and path of action](image1)

**Figure 9.** Analyses of work for the pre-pitch cylindrical gearing with path of action $\Sigma$ by type of segment of right line that across the pith point $W$.  

**Contact stresses by Hertz**

![Contact stresses by Hertz](image2)

**Profiles curvature and reduced curvature**

![Profiles curvature and reduced curvature](image3)
Task 2 – To research the gearing with continued self-across path of action that placed in all of four quadrants around the pitch point. Basis formula is simple – there are the only three cofactors. But there are two tangents of angles that can take a value of \(-\infty, 0, +\infty\). By this can take place specialties like \(0, \infty/\infty, 0/0, \infty\) while calculations at some points of possible contacts zone. Particular at some of the points where path of action is passes from one quadrant to another. That can take place in machine engagements (for example at forming of tooth space in all of gears), also at hydro- and pneumatic machines of rotary type (for example in air-blower machine with wheels of Ruth). In task 2 is shown how the function (3) action at close with singular points, is corrected sing in formula (3), is considered calculation error. Path of action \(\Sigma\) was setts not as equation but as lots of points \(\Sigma\) and the tangent \(\tau\) to \(\Sigma\) (angle \(\theta_0\) on Figure. 3) was founded by differencing method. The lots of points was taken with development of rack gearing with tooth profile on a cutting instrument with arc of circle by radius \(\rho_0=30\) mm – see upper diagram in Figure 11. Calculations were made by corrected basis formula:

\[
\rho_0 = -\text{sign}(x \cdot y)\sqrt{x^2 + y^2} \cdot \left(1 + \frac{\tan \alpha}{\tan \lambda}\right)
\]  

(7)

\(x, y\) – coordinates of point on \(\Sigma\) in system of coordinate XWY (see Figure. 2).

Diagrams \(\rho_0=\rho_0(S)\) and calculation error \(\rho_0=f(S)\) (Figure 10) illustrate that getting values of \(\rho_0\) are different from setts radius of curvature of rack profile (it is setts \(\rho_0=30\) mm) by only at two decimal places of micrones. That different can be explained not only by structure of formulas (3) and (7), but by rack

![Rack profile is arc of circle. Path of action is conchoid of Nicomedes](image)

**Error of radius of curvature \(\rho_0\) for rack profile \(\Sigma_0\)**

\(\rho_0\) was founded using parameters of \(\Sigma\): \(\alpha, \lambda\) and \(WK\) by formula (3)

**Radius of curvature \(\rho_0\) for rack and angles:**

\(\alpha\) – for action, \(\theta\) – for turning of \(\Sigma\), \(\lambda\) – between vectors \(n\) and \(\tau\)

![Error of \(\rho_0\) for rack, mm](image)

![Angles in grad; \(\rho_0\) for rack, mm](image)

**Figure 10.** Applicability of basis formula for difficult and self-crossing paths of action and Estimation of the accuracy of calculations using the basic formula.
shift value $DX$ in process of tangent $\tau$ to $\Sigma$ calculation either. Because of reduce $DX$ in 5 times (from 0.0000005 to 0.000001 mm), error of $\rho_0$ is reduced in some about in 4 times (calculation was applied with accuracy of 15 decimal places). At Figure 10 is shown that singular points for basic functions (3) and (7) are points of a crossing between $\Sigma$ and axes of coordinates. Accuracy of calculations near the singular points is takes down deeply, and discontinuity of the second kind are detected in singular points, i.e. functions are approaching to $\pm \infty$. It is a not an bug of basic formula – formula is showing physical reality only. The presence of singular points requires the development of special algorithms for uncovering uncertainties and for ‘smoothing’ of function in and near singular point.

The general conclusion on two tasks in next: basis formulas (3) and (7) are universal and effective for solution of many tasks at analyses and synthesis of different flat gearing. Usefulness of basis formula that was expected at its developments is justified on present time.

In paper is considered effect of path of action parameters only on curvature and contact stresses by Hertz. In present time is continuing the work of calculation and displaying a lots scale of locally and globally indicators of tooth and gears. As in dependence of tooth profile curvature, so and not. Present paper is basing on publications [17-18] which presented: systematization of quality indicators, their scheme of calculations and methods of graphical displaying. Were used and papers [19-21] too which considering of specialties by some calculations of indicators.

6. Discussion and conclusion

1) The diagrams are constructed without finding the tooth profiles $\Sigma 1$ and $\Sigma 2$. (on the Figure 9-10).
2) When calculating the values of qualitative indicators for these diagrams, only data of the gears (see Part 4) and three parameters for each point of the graph are used: two coordinates $x_K$, $y_K$ in the XWY coordinate system (Figures 1 and 2) , and one angle $\theta_0$ of the of the tangent to the path of contact (Figure 3-4).
3) A key factor that made possible the calculation of any local qualitative indicators, depending on the curvature of the not yet found profile of the tooth $\Sigma 1$ and $\Sigma 2$, was the previously unknown basic formula (3).
4) An important part of these calculations is also the algorithm developed and implemented on the computer, given in Part 3.
5) The created methods and programs are a good basis for developing a new methodology of synthesis line of contact in which the optimal line of contact is first synthesized, and only then are the conjugate tooth profile.
6) Such a methodology has the following advantages over traditional:
   - All calculations of any qualitative indicators are unambiguous: they do not need to solve nonlinear equations. Using the traditional methodology to find each point of contact of the conjugate tooth profile it is necessary to solve the non-linear linkage equation. At the same time, the reliability of isolating the desired root of the equation can become a problem.
   - The new methodology makes it possible to create a user-friendly interface with a visual representation of the optimization process and convenient management of this process.
   - The new methodology makes it possible to create a universal package for the analysis and design of all existing and possible types of cylindrical gears. For example, specifying that the path of action $\Sigma$ is a straight line segment, synthesize or modify involute gear. Setting $\Sigma$ in the form of circular arcs, synthesize cycloidal and approximate epicycloidal gearings or other.

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