Efficient homogeneously weighted dispersion control charts with an application to distillation process

Saddam Akber Abbasi, Sara Hussam Nassar, Mashael Mohammed Aldosari, Olatunde A. Adeoti

Abstract
Monitoring disturbances in process dispersion using control chart is mostly based on the assumption that the quality characteristic follows normal distribution, which is not the case in many real-life situations. This paper proposes a set of new dispersion charts based on the homogeneously weighted moving average (HWMA) scheme, for efficient detection of shifts in process standard deviation ($\sigma$). These charts are based on a variety of $\sigma$ estimators and are investigated for normal as well as heavy tailed symmetric and skewed distributions. The shift detection ability of the charts is evaluated using different run length characteristics, such as average run length (ARL), extra quadratic loss (EQL), and relative ARL measures. The performance of the proposed HWMA control charts is also compared with the existing EWMA dispersion charts, using different design parameters. Furthermore, an illustrative example is presented to monitor the vapor pressure in a distillation process.

Keywords
control chart, process dispersion, HWMA, average run length, extra quadratic loss

1 INTRODUCTION

Statistical Process Control (SPC) is a tool-kit to enhance process performance by monitoring changes in process parameters. Walter A. Shewhart originally proposed the SPC concept in the 1920s. One of the most popular tools of SPC is control chart. The main aim of using control chart is to improve the quality of the process by detecting shifts occurring in either the process mean or/and the process variance as soon as possible to prevent them from occurring again.

There are two types of control charts, which are memory-type control charts and memory-less-type control charts. In recent years, the focus toward memory-type control charts has increased massively because of their high ability to detect small-to-moderate shifts. Page and Roberts proposed the CUSUM and the EWMA charts, for quick detection of small shifts in process parameters, respectively. These are the two most commonly used memory-type control charts for process monitoring because of their simplicity and good performance. These charts make use of current and previous sample information by accumulating small continual shifts and are helpful for reducing large financial loss, by quick detection of
small shifts. The well-known Shewhart chart structures are only based on current sample information and hence are only suitable for handling large process shifts. More details about basic control chart designs can be found in Montgomery.³

In process monitoring, the dispersion parameter is of more immediate concern. Indeed, there is an inverse relation between the improvement of the process and the process variance. Huwangetal⁴ stated that monitoring shifts in process dispersion is more important compared to the monitoring of process mean. Many researchers, in recent years, devoted their studies on monitoring process dispersion using memory control charts. For instance, Abbas et al,⁵–⁷ Abbasi and Miller,⁶ Zhou et al,⁹ Ahmed et al,¹⁰ Castagliola et al,¹¹ Huang et al,¹² Haq,¹³ Hossain et al,¹⁴ Rajmanya and Ghute,¹⁵ Osei-Aning and Abbasi,¹⁶ Lee and Khoo,¹⁷,¹⁸ Zaman et al,¹⁹ and the references cited, therein.

The distributional assumption of most control chart structures for statistical process monitoring is that the quality characteristic follow the normal distribution; however, in practice, this assumption is mostly violated. Abbasi and Miller²⁰ indicated that control charts should be based on appropriate parent distribution of the process rather than just relying on normality assumption. Many authors have proposed EWMA control charts for monitoring process parameter when the assumption of normality is violated. Maravelakis et al²¹ investigated robustness to nonnormality on EWMA dispersion charts based on individual measurements. Abbas et al²² investigated several EWMA dispersion charts for monitoring processes following normal and a variety of nonnormal distributions. Recently, Abbas²³ proposed a homogeneously weighted moving average (HWMA) control chart for monitoring small shifts in process mean. The HWMA chart assigns a specific weight to the current observation while the remaining weights are assigned equally to the past observations.

This study presents the HWMA dispersion control chart based on a set of σ estimators. These estimators are sample range (R), standard deviation (SD), inter quartile range (Q), Downton’s estimator (D), mean absolute deviation from median (MD), and two estimators based on interpoint distances, namely, Sₙ and Qₙ. We have investigated processes that follow normal and nonnormal parent distributions. The distributions considered are the normal, two heavy-tailed symmetric and five skewed distributions. The distributions and the dispersion estimates investigated in this study are the same as the ones investigated by Abbasi et al²² for comparison purposes. Thus, for every distribution, we investigated the Rₙ chart based on R estimator, Sₙₙ chart based (SD), Qₙₙ chart based on (Q), Dₙₙ chart based on (D), MDₙₙ chart based on (MD), MADₙₙ chart based on (MAD), Sₙₙ and Qₙₙ charts based on Sₙ and Qₙ estimators, respectively.

The remainder of the paper is organized as follows: The different dispersion estimates used in the study are given in Section 2. The design of the proposed HWMA dispersion chart is presented in Section 3 while Section 4 highlights the performance assessment and the steps taken in the simulation study. A comparison between the proposed HWMA and the EWMA dispersion control charts is given in Section 5 and an illustrative example is reported in Section 6. Finally Section 7 presents the conclusions of the study.

## 2 DISPERSION ESTIMATORS

This section describes a variety of σ estimators as described in the previous section. Suppose X is the quality characteristic of interest and x₁, x₂, . . . , xₙ be a random sample of size n. Moreover, suppose xᵢ be the ith order statistic (lowest to highest), ̄x be the sample mean, |X| be the absolute value of X and ̃x be the sample median. The different dispersion estimates are briefly described below:

(i) Sample range (R) based on order statistics: This is defined as

\[ R = x_{(n)} - x_{(1)}. \]  

(1)

It is the most widely used dispersion estimator. It is the difference between the highest and lowest observations in the sample. R is too sensitive for extreme values and the normality requirement is not satisfied.

(i) Standard deviation (S) based on squared deviations from the mean: it is defined as

\[ S = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}. \]  

(2)
Comparing $S$ estimator with other estimators shows that $S$ is the most efficient dispersion estimator for normal distribution but it is sensitive to outliers.

(i) Interquartile range ($Q$) based on order statistics: It is given as

$$Q = (Q_3 - Q_1) / 1.34898,$$

where $Q_3$ and $Q_1$ are the upper and lower quantiles. For large sample size, $Q$ loses its efficiency while it is used as a robust measure of scale with a breakdown point of 25%. Abbasi and Miller\textsuperscript{20} showed that for some nonnormal distributions, the $Q$ chart performed better than $R$ and $S$ charts.

(i) Downton’s Estimator ($D$) based on order statistics: It is defined as:

$$D = \frac{2\sqrt{\pi}}{n(n-1)} \sum_{i=1}^{n} \left[ i - \frac{1}{2}(n+1) \right] x_{(i)}.$$

For normally distributed processes, $D$ is an unbiased estimator of $\sigma$. Control charts based on $D$ showed superiority over $R$- and $S$-based charts considering nonnormal processes.\textsuperscript{20,24}

(i) Mean absolute deviation (MD) based on absolute deviations from median: This is defined as

$$MD = \frac{1}{n} \sum_{i=1}^{n} |x_i - \bar{x}|.$$

Abbasi and Miller\textsuperscript{20} showed that $MD$ is an efficient estimator for nonnormal distributions such as $t$ and gamma distributions. Riaz and Saghir\textsuperscript{25} found that the $MD$ chart is more sensitive than $R$ and $S$ charts.

(i) Median absolute deviation (MAD) based on absolute deviations from median: This is defined as

$$MAD = 1.4826 \text{med} \frac{1}{n} |x_i - \bar{x}|.$$

MAD is a very robust estimator of $\sigma$ since it has the highest possible breakdown point (50%). However, its reliance on the distribution being symmetric and low Gaussian efficiency (36.74%) are the two main drawbacks.\textsuperscript{26}

(i) $\sigma$ estimators ($S_n$ and $Q_n$)

$S_n$ and $Q_n$ estimators uses interpoint distances to estimate $\sigma$. $S_n$ is given as

$$S_n = 1.1926 \text{med}_i \left\{ \text{med}_i |x_i - x_j| ; i \neq j \right\},$$

where the outer and inner medians represent $[(n+1)/2]$th and $[(n/2)+1]$th ranked statistic, respectively. These medians can be termed as the “high” and “low” medians.\textsuperscript{26}$Q_n$ is defined as:

$$Q_n = 2.2219 \left\{ x_i - x_j ; i < j \right\},$$

where $p = \left( \frac{n+1}{2} \right)$

$Q_n$ is defined as the $p$th ranked statistic of the $n$-choose-2 interpoint distances. $Q_n$ performs better than $S_n$ and MAD estimates for symmetric and skewed distributions.\textsuperscript{26}

The proposed HWMA charts will use all the above estimates to identify the best chart for normal and nonnormal process environments.
3 | PROPOSED HWMA DISPERSION CHART

Let $T_i (i = 1, 2, \ldots)$ represent a sequence of the observed values of an estimator $T$ at time $t$, computed from a sample of size $n$. Here, $T$ represents any of the dispersion estimators, mentioned in Section 2. The mean of the process is assumed to be stable as we are only interested in monitoring the process dispersion. Let the plotting statistic for HWMA dispersion charts be defined as:

$$H_i = \lambda T_i + (1 - \lambda) T_{i-1},$$

(9)

where $\lambda$ is the smoothing parameter and it is selected to be between zero and one, that is, $0 < \lambda \leq 1$. $T_{i-1}$ is the average of previous $(i - 1)$ dispersion statistics and is given as:

$$T_{i-1} = \frac{\sum_{j=1}^{i-1} T_j}{i-1}.$$  

(10)

The initial value ($T_0$) is taken to be equal to the target mean of $T$, that is, $\mu_0$ (if target mean is unknown). Further, let $Z$ defines relative dispersion statistic (ie, $Z = \frac{T}{\sigma}$). Let $E(Z) = t_2$ and $\sigma_Z = t_3 \cdot t_2$, and $t_3$ are entirely dependent on sample size for a specific combination of $T$ and parent distribution (see………).

Following Abbas, we can show that:

$$E(H_i) = E(T_i)$$

$$V (H_i) = \begin{cases} 
\lambda^2 \sigma_i^2, & \text{if } t = 1 \\
\lambda^2 \sigma_i^2 + \frac{(1 - \lambda)^2 \sigma_i^2}{(t-1)}, & \text{if } t > 1
\end{cases}.$$  

(11)

where $E(T_i) = t_2 \sigma$ and $\sigma_i$ can be replaced by its estimator $\hat{\sigma}_i = \frac{\bar{T}}{t_2}$. Using these notations, the time varying upper control limit of the HWMA chart for the statistic $T$ is defined as:

$$UCL_t = \begin{cases} 
T + L_T \frac{t_3 \bar{T}}{t_2} \sqrt{\lambda^2}, & \text{if } t = 1 \\
T + L_T \frac{t_3 \bar{T}}{t_2} \sqrt{\frac{\lambda^2 + (1 - \lambda)^2 \sigma_i^2}{t-1}}, & \text{if } t > 1
\end{cases}.$$  

(12)

where $L_T$ is the width of the control limit and it is adjusted to achieve the desired in-control average run length (ARL$_0$). Furthermore, only the UCL is considered in this study as we are interested in detecting an upward shift in the process dispersion. Abbasi et al$^{22}$ reported the $t_2$ and $t_3$ values for different parent normal and nonnormal distributions, considering a variety of sample sizes.

4 | PERFORMANCE EVALUATION OF HWMA CHART

The performance of control charts is usually evaluated using run length characteristics. The most common performance measures of control charts is the average run length (ARL). It is the average number of samples plotted before the detection of an out-of-control point. The in-control and the out-of-control average run lengths are denoted as ARL$_0$ and ARL$_1$, respectively. For a comparison between control charts, the ARL$_0$ is fixed at a desired level and ARL$_1$ is compared, the chart with the lowest ARL$_1$ value has the best performance in detecting shifts. The distribution of ARL is mostly highly skewed; hence few researchers recommended to use some other run length measures such as median run length and standard deviation of the run length distribution.$^{22}$ So, some other performance measures have been proposed to evaluate the overall effectiveness of HWMA charts over the entire range of shifts. In this study, some useful measures to assess the performance of the proposed HWMA dispersion control charts are the average run length (ARL), standard deviation of the
run length (SDRL), median run length (MDRL), extra quadratic loss (EQL), and the relative average run length (RARL). Here, we used $10^4$ Monte Carlo simulation runs developed in R language. Note that the results and report of study are based on subgroups size $n = 5$ and $n = 7$ using $\lambda = 0.05$ and $\lambda = 0.25$, respectively, for each HWMA dispersion chart with normal and nonnormal distributions. The main aim of this study is to investigate the performance of the HWMA charts and identify a chart (or group of charts) that is superior in the monitoring of dispersion for normal and nonnormal distribution processes. The nonnormal symmetric distributions are: logistic and Student’s $t$. The skewed distributions are Weibull, chi-square, gamma and exponential, the extremely skewed distribution is log-normal. Furthermore, the density functions and the parameter values for different parent distributions used in this study are given below:

Normal ($\mu, \sigma^2$), $-\infty < \mu < \infty$,

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

(13)

Logistic ($\mu, k$), $-\infty < \mu < \infty$,

$$f(x) = \frac{e^{-\frac{(x-\mu)^2}{k}}}{k(1 + e^{-\frac{(x-\mu)}{k}})^2}, \quad -\infty < x < \infty,$$

(14)

Student’s $t(k)$, $k > 0$

$$f(x) = \frac{\Gamma[(k+1)/2]}{\sqrt{k\pi\Gamma(k/2)}} \left(1 + \frac{x^2}{k}\right)^{-\frac{(k+1)}{2}}, \quad -\infty < x < \infty,$$

(15)

Weibull($\alpha, \beta$), $\alpha > 0, \beta > 0$

$$f(x) = \frac{\alpha}{\beta} x^{\alpha-1} e^{-x^\beta/\beta}, \quad x \geq 0,$$

(16)

Chi-square($k$), $k > 0$

$$f(x) = \frac{x^{(k/2)-1} e^{-x/2}}{2^{k/2}\Gamma(k/2)}, \quad x > 0,$$

(17)

Gamma($\alpha, \beta$), $\alpha > 0, \beta > 0$

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \quad x > 0,$$

(18)

Exponential($\lambda$), $\lambda > 0$

$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0,$$

(19)

Log-normal ($\mu, \sigma^2$), $-\infty < \mu < \infty$,

$$f(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} e^{-\frac{(\ln(x)-\mu)^2}{2\sigma^2}}, \quad x \geq 0.$$

(20)

Without loss of generality, we considered $\mu = 0$ and $\sigma = 1$ for normal distribution, $\mu = 0$ and $k = 1$ for logistic distribution, $k = 5$ for Student’s $t$-distribution, $\alpha = 1.5$ and $\beta = 1$ for Weibull distribution, $\alpha = 2$ and $\beta = 1$ for gamma distribution, $\lambda = 1$ for exponential distribution and $\mu = 0$ and $\sigma = 1$ for the log-normal distribution.

The extra quadratic loss (EQL) and relative average run length (RARL) measures are used to evaluate the overall shift detection ability of the proposed HWMA charts. These are briefly defined below:
4.1 | Extra quadratic loss (EQL) and relative ARL (RARL)

Reynolds Jr. and Stoumbos\textsuperscript{27} and Wu et al\textsuperscript{28} recommended the use of extra quadratic loss (EQL) and relative ARL (RARL) measures to evaluate the overall detection ability of control charts over the entire shift domain. EQL uses a weight $\delta^2$ to compute the weighted average of ARL over the whole shift range. It is defined as:

$$\text{EQL} = \frac{1}{\delta_{\max} - \delta_{\min}} \int_{\delta_{\min}}^{\delta_{\max}} \delta^2 \text{ARL}(\delta) \, d\delta,$$

where $\text{ARL}(\delta)$ is the ARL value of a specific chart at shift $\delta$. A chart with the smallest EQL is considered better than competing charts.

RARL is defined as the ratio of the ARL value of a specific chart at a given shift $\delta$ and the ARL of benchmark chart. Mathematically, it is given by

$$\text{RARL} = \frac{1}{\delta_{\max} - \delta_{\min}} \int_{\delta_{\min}}^{\delta_{\max}} \frac{\text{ARL}(\delta)}{\text{ARL}_{\text{benchmark}}} \, d\delta,$$

where $\text{ARL}(\delta)$ is the ARL value of the chart for a specific shift $\delta$ and $\text{ARL}_{\text{benchmark}}$ is the ARL value of the benchmark chart, is mostly taken as a chart with the lowest EQL value.

5 | PERFORMANCE COMPARISON

In this section, the ARL, SDRL, MDRL as well as the EQL and RARL performance measures of the proposed HWMA charts are compared for processes following normal and nonnormal distributions. Also, the performance of the proposed HWMA charts is compared with their respective EWMA dispersion charts. Table 1 reports the run length characteristics (ARL, MDRL, and SDRL) of the HWMA dispersion charts under the idea assumption of normality. To save space, only the ARL results are presented in Tables 2–9, for processes following other parent nonnormal distributions. In these tables, results are only reported for $n = 7$ and $\lambda = 0.25$. The control chart multipliers for each chart are given to achieve a desired $\text{ARL}_0$ set at 200 in the tables. The overall performance measures EQL and RARL for the different HWMA dispersion charts are reported in Table 9 (using $n = 7$ and $\lambda = 0.25$) and Table 10 (using $n = 5$ and $\lambda = 0.05$). To save space, we did not provide detailed results for other combinations of $n$ and $\lambda$. The RARL comparison of the proposed charts is provided in Figure 1, considering using $n = 5$ and $\lambda = 0.05$. The control chart is said to be superior if its ARL values are lower and have the minimum EQL compared to other competing charts. Furthermore, the sensitivity of the proposed HWMA chart for different smoothing constant $\lambda$ and varying sample sizes $n$ are evaluated for some special cases.

5.1 | Proposed HWMA chart

Normal distribution: The ARL performance of a variety of HWMA dispersion charts is compared for normally distributed quality characteristics in Table 1 whereas EQL and RARL comparison is provided in Figure 1 and Tables 9–10. We can clearly observe that the $S_{H}$ chart has the lowest ARL values among the other competing charts, indicating that it has the best performance. Moreover, the $S_{H}$ chart has the lowest EQL and RARL values, which shows that the overall shift detection ability of $S_{H}$ chart is also better, compared to other charts. The $D_{H}$ chart is considered the second best control chart, as the results were close to $S_{H}$ chart, followed by the $K_{H}$ and the $MD_{H}$ charts. Furthermore, the $MAD_{H}$, $SN_{H}$, and $QN_{H}$ have the worst performance as their ARLs, EQLs, and RARLs are high and thus their abilities of detecting shifts are low. The SDRL and MDRL criteria gave the same conclusions as the ARL. The MDRL values for all charts are the same when the shift size is $3 \leq \delta \leq 4$. We conclude that for the normal distribution, $S_{H}$ has the best performance while $D_{H}$ is close competitor with $S_{H}$. $MAD_{H}$, $SN_{H}$, and $QN_{H}$ have the worst performance in detecting shifts whereas $R_{H}$ and $MD_{H}$ charts lie between the best and worst chart performances. Also, we have computed the ARL values for all
TABLE 1  Run length profiles of the HWMA dispersion chart for normal distribution when $\lambda = 0.25$, $n = 7$, and ARL0 = 200

| $\delta$ | $R_H$ | $S_H$ | $Q_H$ | $D_H$ | $MD_H$ | $MAD_H$ | $SN_H$ | $QN_H$ |
|---------|-------|-------|-------|-------|--------|---------|-------|-------|
| 1.0     | 201.70| 200.67| 201.48| 200.22| 201.54 | 199.39  | 200.11| 199.36|
|         | 191.50| 187.65| 185.92| 188.04| 186.66 | 183.39  | 184.74| 186.80|
|         | 145.00| 146.00| 147.00| 145.00| 146.00 | 146.00  | 144.00| 145.00|
| 1.1     | 34.38 | 29.78 | 47.33 | 30.75 | 33.02  | 55.31   | 46.44 | 47.06 |
|         | 28.42 | 24.39 | 39.45 | 25.56 | 26.59  | 47.54   | 39.50 | 39.13 |
|         | 27.00 | 23.00 | 36.00 | 24.00 | 27.00  | 42.00   | 36.00 | 37.00 |
| 1.2     | 13.64 | 11.87 | 20.87 | 12.20 | 13.21  | 25.25   | 20.53 | 20.59 |
|         | 10.28 | 8.88  | 16.15 | 9.06  | 9.86   | 20.00   | 16.38 | 16.45 |
|         | 11.00 | 10.00 | 17.00 | 10.00 | 11.00  | 20.00   | 16.00 | 16.00 |
| 1.3     | 7.80  | 6.80  | 12.13 | 7.11  | 7.55   | 14.98   | 11.94 | 12.03 |
|         | 5.66  | 4.81  | 9.02  | 5.05  | 5.33   | 11.57   | 9.10  | 9.01  |
|         | 7.00  | 6.00  | 10.00 | 6.00  | 6.00   | 12.00   | 10.00 | 10.00 |
| 1.4     | 5.26  | 4.62  | 8.21  | 4.74  | 5.17   | 10.13   | 8.10  | 8.05  |
|         | 3.60  | 3.17  | 6.01  | 3.32  | 3.58   | 7.64    | 5.99  | 5.93  |
|         | 5.00  | 4.00  | 7.00  | 4.00  | 4.00   | 8.00    | 7.00  | 7.00  |
| 1.5     | 3.93  | 3.52  | 6.04  | 3.55  | 3.77   | 7.57    | 5.97  | 6.04  |
|         | 2.66  | 2.33  | 4.35  | 2.39  | 2.53   | 5.59    | 4.34  | 4.38  |
|         | 3.00  | 3.00  | 5.00  | 3.00  | 3.00   | 6.00    | 5.00  | 4.00  |
| 1.6     | 3.12  | 2.79  | 4.80  | 2.87  | 3.02   | 5.94    | 4.77  | 4.77  |
|         | 2.03  | 1.85  | 3.37  | 1.85  | 2.01   | 4.34    | 3.42  | 3.40  |
|         | 3.00  | 3.00  | 4.00  | 3.00  | 3.00   | 5.00    | 4.00  | 3.00  |
| 1.8     | 2.22  | 2.04  | 3.35  | 2.08  | 2.18   | 4.15    | 3.55  | 3.30  |
|         | 1.39  | 1.27  | 2.23  | 1.30  | 1.36   | 2.94    | 2.37  | 2.26  |
|         | 2.00  | 2.00  | 3.00  | 2.00  | 2.00   | 4.00    | 3.00  | 2.00  |
| 2.0     | 1.78  | 1.62  | 2.63  | 1.68  | 1.76   | 3.27    | 2.63  | 2.60  |
|         | 1.07  | 0.95  | 1.73  | 0.99  | 1.05   | 2.27    | 1.80  | 1.74  |
|         | 1.00  | 1.00  | 2.00  | 1.00  | 1.00   | 3.00    | 2.00  | 1.00  |
| 2.5     | 1.28  | 1.23  | 1.77  | 1.23  | 1.28   | 2.15    | 1.80  | 1.76  |
|         | 0.62  | 0.55  | 1.09  | 0.55  | 0.61   | 1.40    | 1.13  | 1.10  |
|         | 1.00  | 1.00  | 1.00  | 1.00  | 1.00   | 2.00    | 1.00  | 1.00  |
| 3.0     | 1.11  | 1.09  | 1.42  | 1.10  | 1.11   | 1.71    | 1.46  | 1.42  |
|         | 0.38  | 0.34  | 0.78  | 0.34  | 0.39   | 1.04    | 0.84  | 0.78  |
|         | 1.00  | 1.00  | 1.00  | 1.00  | 1.00   | 1.00    | 1.00  | 1.00  |
| 3.5     | 1.05  | 1.04  | 1.24  | 1.05  | 1.05   | 1.45    | 1.30  | 1.24  |
|         | 0.25  | 0.22  | 0.57  | 0.23  | 0.24   | 0.82    | 0.65  | 0.58  |
|         | 1.00  | 1.00  | 1.00  | 1.00  | 1.00   | 1.00    | 1.00  | 1.00  |
| 4.0     | 1.02  | 1.01  | 1.15  | 1.02  | 1.02   | 1.30    | 1.19  | 1.16  |
|         | 0.15  | 0.14  | 0.45  | 0.15  | 0.17   | 0.65    | 0.51  | 0.46  |
|         | 1.00  | 1.00  | 1.00  | 1.00  | 1.00   | 1.00    | 1.00  | 1.00  |
| $L$     | 2.780 | 2.660 | 3.000 | 2.680 | 2.722  | 2.950   | 2.780 | 2.840 |

charts when $n = 5$ and $\lambda = 0.05$; however, due to limitation of space, the result is not provided but available upon request from the authors. We found that $S_H$, $D_H$, $R_H$, and $MD_H$ charts have the best performance compared to other charts as their ARL values are the lowest for all shift sizes, following them is the $SN_H$ and $QN_H$ charts. The $MAD_H$ and $Q_H$ charts have the worst performance, as their ARL values are the highest. The EQL and RARL results for this case are shown in Table 10, $S_H$ has the lowest values and thus it is the best chart and $Q_H$ has the highest values and thus it is has the worst performance.
TABLE 2  Average run length profiles of the HWMA dispersion chart for logistic distribution when $\lambda = 0.25$, $n = 7$, and $\text{ARL}_0 = 200$

| $\delta$ | $R_H$ | $S_H$ | $Q_H$ | $D_H$ | $MD_H$ | $MAD_H$ | $SN_H$ | $QN_H$ |
|---------|-------|-------|-------|-------|--------|--------|-------|-------|
| 1.0     | 200.53| 201.74| 199.63| 199.14| 200.19 | 201.41 | 200.88| 200.56|
| 1.1     | 52.42 | 46.00 | 53.73 | 42.94 | 42.13  | 63.04  | 56.01 | 56.79 |
| 1.2     | 22.14 | 19.35 | 23.96 | 17.59 | 17.34  | 29.66  | 25.33 | 25.45 |
| 1.3     | 12.63 | 10.96 | 14.08 | 10.01 | 11.69  | 17.45  | 14.51 | 14.74 |
| 1.4     | 8.40  | 7.29  | 9.33  | 6.65  | 6.61   | 11.89  | 9.83  | 9.97  |
| 1.5     | 6.10  | 5.33  | 6.86  | 4.89  | 4.93   | 8.87   | 7.25  | 7.35  |
| 1.6     | 4.84  | 4.09  | 5.41  | 3.88  | 3.85   | 6.84   | 5.66  | 5.69  |
| 1.8     | 3.29  | 2.85  | 3.79  | 2.72  | 2.71   | 4.81   | 3.94  | 4.00  |
| 2.0     | 2.53  | 2.25  | 2.90  | 2.10  | 2.12   | 3.58   | 3.07  | 3.04  |
| 2.5     | 1.68  | 1.51  | 1.92  | 1.46  | 1.45   | 2.35   | 2.04  | 2.01  |
| 3.0     | 1.32  | 1.22  | 1.53  | 1.21  | 1.21   | 1.81   | 1.59  | 1.55  |
| 3.5     | 1.17  | 1.11  | 1.31  | 1.09  | 1.10   | 1.55   | 1.39  | 1.34  |
| 4.0     | 1.09  | 1.06  | 1.20  | 1.05  | 1.05   | 1.36   | 1.26  | 1.23  |
| $L$     | 3.190 | 3.066 | 3.030 | 2.950 | 2.920  | 3.050  | 2.900 | 3.010 |

TABLE 3  Average run length profiles of the HWMA dispersion chart for Student’s $t$ distribution when $\lambda = 0.25$, $n = 7$, and $\text{ARL}_0 = 200$

| $\delta$ | $R_H$ | $S_H$ | $Q_H$ | $D_H$ | $MD_H$ | $MAD_H$ | $SN_H$ | $QN_H$ |
|---------|-------|-------|-------|-------|--------|--------|-------|-------|
| 1.0     | 200.87| 200.98| 199.92| 200.91| 199.66 | 199.18 | 201.22| 201.01|
| 1.1     | 81.97 | 77.61 | 58.10 | 64.48 | 56.75  | 64.93  | 61.99 | 62.78 |
| 1.2     | 39.09 | 36.48 | 26.24 | 28.23 | 24.86  | 31.36  | 28.45 | 28.36 |
| 1.3     | 22.39 | 20.30 | 15.46 | 15.63 | 13.90  | 18.78  | 16.75 | 16.72 |
| 1.4     | 14.71 | 12.98 | 10.16 | 10.28 | 8.98   | 12.98  | 11.23 | 11.12 |
| 1.5     | 10.43 | 9.08  | 7.53  | 7.43  | 6.48   | 9.40   | 8.07  | 8.20  |
| 1.6     | 7.75  | 6.92  | 5.94  | 5.68  | 5.04   | 7.35   | 6.35  | 6.29  |
| 1.8     | 5.17  | 4.58  | 4.06  | 3.82  | 3.46   | 5.11   | 4.35  | 4.34  |
| 2.0     | 3.77  | 3.36  | 3.11  | 2.85  | 2.63   | 3.87   | 3.40  | 3.37  |
| 2.5     | 2.31  | 2.08  | 2.04  | 1.83  | 1.70   | 2.49   | 2.19  | 2.16  |
| 3.0     | 1.73  | 1.56  | 1.58  | 1.41  | 1.36   | 1.92   | 1.69  | 1.66  |
| 3.5     | 1.42  | 1.31  | 1.37  | 1.21  | 1.19   | 1.62   | 1.44  | 1.41  |
| 4.0     | 1.24  | 1.17  | 1.23  | 1.11  | 1.09   | 1.42   | 1.31  | 1.27  |
| $L$     | 3.710 | 3.630 | 3.148 | 3.400 | 3.249  | 3.220  | 3.138 | 3.166 |

Nonnormal symmetric distributions (logistic, and Student’s t): From Tables 2 and 3, it can be observed that the $MD_H$ and $D_H$ charts for both parent distributions have the best performance in terms of their ARL values, which indicate that less samples are required to detect shifts and thus their abilities of detecting shifts are high, in addition, they have the least EQL and RARL values, as it is seen from Table 9. The $R_H$ and $S_H$ charts are less efficient for the Student’s $t$ distribution, compared with the logistic distribution. The $MAD_H$, $SN_H$, and $QN_H$ are the worst performing charts. For $n = 5$ and $\lambda = 0.05$, considering logistic distribution, both $D_H$ and $MD_H$ charts have the best performance in detecting small shifts ($1.1 \leq \delta \leq 1.6$), while for Student’s $t$ distribution, they are sensitive in detecting large shifts only and the $Q_H$ chart is sensitive in detecting small shifts. The $R_H$, $S_H$, $SN_H$, and $QN_H$ lie between the superior and inferior charts, and the $MAD_H$ charts for both distributions have the worst performance in terms of all the run length measures. Furthermore, for both distributions, $MD_H$ chart is the best in terms of their EQL and RARL values as can be seen from Table 10, while $MAD_H$ chart is the worst.

Skewed distributions (Weibull, chi-square, gamma, and exponential): From Tables 4–7 and 9, we can see that the performance of $MD_H$ and $D_H$ charts are the best, as compared to all other competing charts. The $S_H$, $R_H$, and $Q_H$ charts have almost the same performance among the four distributions but are more efficient for parent Weibull distribution, compared to the other distributions, in terms of their ARL. Again, the $MAD_H$, $SN_H$, and $QN_H$ were the least efficient charts as their ARLs, EQLs, and RARLs are high. The ARL values for $n = 5$ and $\lambda = 0.05$ were also computed and they gave similar
| δ  | RH | SH | QH | DH | MDH | MADH | SNH | QNH |
|----|----|----|----|----|-----|------|------|------|
| 1.0 | 199.99 | 199.70 | 201.97 | 199.90 | 200.80 | 199.62 | 201.35 | 199.41 |
| 1.1 | 48.81 | 47.47 | 56.07 | 40.52 | 42.38 | 62.07 | 55.13 | 54.67 |
| 1.2 | 20.86 | 19.76 | 25.22 | 16.93 | 17.62 | 29.19 | 25.27 | 24.55 |
| 1.3 | 11.90 | 11.20 | 14.96 | 9.86 | 10.03 | 17.43 | 14.87 | 14.39 |
| 1.4 | 7.77 | 7.45 | 10.03 | 6.50 | 6.79 | 11.91 | 9.89 | 9.54 |
| 1.5 | 5.73 | 5.46 | 7.45 | 4.82 | 4.98 | 8.79 | 7.37 | 7.08 |
| 1.6 | 4.50 | 4.24 | 5.76 | 3.77 | 3.94 | 6.88 | 5.85 | 5.55 |
| 1.8 | 3.12 | 3.00 | 4.02 | 2.67 | 2.78 | 4.74 | 4.10 | 3.83 |
| 2.0 | 2.41 | 2.30 | 3.07 | 2.13 | 2.19 | 3.64 | 3.14 | 2.96 |
| 2.5 | 1.60 | 1.57 | 2.05 | 1.46 | 1.49 | 2.37 | 2.06 | 1.95 |
| 3.0 | 1.30 | 1.26 | 1.59 | 1.20 | 1.23 | 1.84 | 1.63 | 1.54 |
| 3.5 | 1.14 | 1.13 | 1.36 | 1.10 | 1.11 | 1.55 | 1.41 | 1.33 |
| 4.0 | 1.07 | 1.07 | 1.23 | 1.05 | 1.06 | 1.37 | 1.28 | 1.22 |
| L   | 3.100 | 3.062 | 3.190 | 2.922 | 2.944 | 3.120 | 3.030 | 3.040 |

Table 5 Average run length profiles of the HWMA dispersion chart for chi-square distribution when \( \lambda = 0.25 \), \( n = 7 \), and ARL0 = 200

| δ  | RH | SH | QH | DH | MDH | MADH | SNH | QNH |
|----|----|----|----|----|-----|------|------|------|
| 1.0 | 200.86 | 200.96 | 200.78 | 199.21 | 201.72 | 201.11 | 201.07 | 201.13 |
| 1.1 | 59.21 | 56.46 | 60.50 | 47.66 | 47.78 | 64.85 | 60.00 | 59.05 |
| 1.2 | 26.48 | 24.66 | 27.79 | 21.01 | 20.15 | 30.98 | 27.15 | 26.39 |
| 1.3 | 15.19 | 14.21 | 16.40 | 11.67 | 11.40 | 18.42 | 16.04 | 15.24 |
| 1.4 | 10.00 | 9.32 | 10.84 | 7.77 | 7.63 | 12.58 | 10.83 | 10.29 |
| 1.5 | 7.25 | 6.91 | 8.04 | 5.70 | 5.63 | 9.24 | 8.01 | 7.62 |
| 1.6 | 5.68 | 5.29 | 6.28 | 4.49 | 4.44 | 7.26 | 6.29 | 5.84 |
| 1.8 | 3.84 | 3.63 | 4.28 | 3.12 | 3.09 | 5.03 | 4.32 | 4.04 |
| 2.0 | 2.90 | 2.76 | 3.30 | 2.42 | 2.42 | 3.82 | 3.33 | 3.12 |
| 2.5 | 1.86 | 1.80 | 2.12 | 1.63 | 1.59 | 2.48 | 2.16 | 2.04 |
| 3.0 | 1.46 | 1.39 | 1.66 | 1.30 | 1.30 | 1.90 | 1.68 | 1.59 |
| 3.5 | 1.24 | 1.21 | 1.40 | 1.16 | 1.16 | 1.60 | 1.43 | 1.37 |
| 4.0 | 1.13 | 1.11 | 1.27 | 1.08 | 1.08 | 1.42 | 1.29 | 1.23 |
| L   | 3.310 | 3.225 | 3.161 | 3.050 | 3.001 | 3.160 | 3.088 | 3.100 |

results to that of \( n = 7 \) and \( \lambda = 0.25 \) and their EQL and RARL values are given in Table 10, where it shows that \( D_H \) chart is the best among all the dispersion charts.

**Extremely skewed distribution (log-normal):** From Table 8, we can observe that the MADH, SNH, and QNH charts have the least ARLs, thus they are the best charts in detecting shifts in the process dispersion. The QH, MDH, and DH charts by order come right after the best charts. Both SH and RH charts have the worst performance as all their run length measures are higher than other charts. For \( n = 5 \) and \( \lambda = 0.05 \), the SNH, QNH, and MADH performances are considered superior in terms of their ARLs, EQLs, and RARLs. Both SH and RH charts have the worst performances as their ARL and other performance measures.

### 5.2 Effect of sample size (n)

We evaluated the performance of \( S_H \) and \( D_H \) charts for different levels of sample size \( n = 5, 7, 10, 12, \) and 15, when \( \lambda = 0.05 \). Table 11 reports the ARL values of the proposed \( S_H \) chart for normally distributed quality characteristic whereas for \( D_H \) chart, the ARL comparison is presented graphically in Figure 2. It can be observed from Table 11 and Figure 2 that the
Table 6  Average run length profiles of the HWMA dispersion chart for gamma distribution when $\lambda = 0.25$, $n = 7$, and $ARL_0 = 200$

| $\delta$ | $R_H$ | $S_H$ | $Q_H$ | $D_H$ | $MD_H$ | $MAD_H$ | $SN_H$ | $QN_H$ |
|----------|-------|-------|-------|-------|--------|--------|--------|--------|
| 1.0      | 201.64 | 199.74 | 200.45 | 199.37 | 200.33 | 201.21 | 201.14 | 199.90 |
| 1.1      | 62.36  | 59.77  | 63.36  | 50.97  | 51.71  | 68.21  | 63.04  | 61.42  |
| 1.2      | 28.62  | 27.05  | 29.56  | 22.09  | 21.94  | 33.02  | 29.23  | 27.88  |
| 1.3      | 16.56  | 15.49  | 17.32  | 12.74  | 12.57  | 19.57  | 17.10  | 16.27  |
| 1.4      | 10.96  | 10.30  | 11.60  | 8.33   | 8.35   | 13.33  | 11.70  | 11.18  |
| 1.5      | 7.87   | 7.46   | 8.57   | 6.12   | 6.05   | 9.90   | 8.54   | 8.11   |
| 1.6      | 6.10   | 5.79   | 6.61   | 4.84   | 4.73   | 7.65   | 6.69   | 6.31   |
| 1.8      | 4.11   | 3.96   | 4.59   | 3.26   | 3.33   | 5.27   | 4.56   | 4.37   |
| 2.0      | 3.16   | 2.98   | 3.50   | 2.54   | 2.56   | 3.99   | 3.53   | 3.30   |
| 2.5      | 2.00   | 1.93   | 2.22   | 1.72   | 1.71   | 2.55   | 2.30   | 2.13   |
| 3.0      | 1.53   | 1.49   | 1.72   | 1.35   | 1.34   | 1.97   | 1.76   | 1.66   |
| 3.5      | 1.28   | 1.26   | 1.45   | 1.19   | 1.18   | 1.63   | 1.50   | 1.41   |
| 4.0      | 1.17   | 1.156  | 1.31   | 1.10   | 1.09   | 1.45   | 1.36   | 1.28   |
| $L$      | 3.288  | 3.230  | 3.210  | 3.040  | 3.030  | 3.210  | 3.130  | 3.150  |

Table 7  Average run length profiles of the HWMA dispersion chart for exponential distribution when $\lambda = 0.25$, $n = 7$, and $ARL_0 = 200$

| $\delta$ | $R_H$ | $S_H$ | $Q_H$ | $D_H$ | $MD_H$ | $MAD_H$ | $SN_H$ | $QN_H$ |
|----------|-------|-------|-------|-------|--------|--------|--------|--------|
| 1.0      | 200.69 | 200.41 | 201.63 | 200.65 | 199.48 | 200.17 | 199.75 | 201.38 |
| 1.1      | 76.40  | 75.43  | 72.66  | 64.14  | 62.04  | 80.31  | 75.31  | 74.93  |
| 1.2      | 38.26  | 36.94  | 36.11  | 30.06  | 28.28  | 41.75  | 38.71  | 37.46  |
| 1.3      | 22.66  | 22.12  | 21.91  | 17.80  | 16.54  | 25.67  | 23.34  | 23.01  |
| 1.4      | 15.18  | 14.68  | 15.22  | 11.79  | 11.22  | 17.84  | 16.16  | 15.59  |
| 1.5      | 11.20  | 10.71  | 11.04  | 8.65   | 8.09   | 13.32  | 11.93  | 11.46  |
| 1.6      | 8.56   | 8.32   | 8.53   | 6.71   | 6.39   | 10.42  | 9.24   | 8.87   |
| 1.8      | 6.78   | 5.64   | 5.89   | 4.61   | 4.42   | 7.09   | 6.29   | 5.99   |
| 2.0      | 4.31   | 4.19   | 4.45   | 3.50   | 3.32   | 5.24   | 4.75   | 4.49   |
| 2.5      | 2.66   | 2.62   | 2.77   | 2.22   | 2.16   | 3.28   | 3.03   | 2.85   |
| 3.0      | 1.99   | 1.96   | 2.10   | 1.69   | 1.64   | 2.47   | 2.27   | 2.14   |
| 3.5      | 1.65   | 1.60   | 1.73   | 1.44   | 1.41   | 2.02   | 1.85   | 1.76   |
| 4.0      | 1.44   | 1.41   | 1.53   | 1.27   | 1.25   | 1.75   | 1.63   | 1.55   |
| $L$      | 3.470  | 3.450  | 3.370  | 3.240  | 3.180  | 3.450  | 3.340  | 3.440  |

$ARL_1$ of the $S_H$ and the $D_H$ charts decrease with an increase in the sample size $n$. We observed this phenomenon for all dispersion HWMA charts that the detection ability of the charts improves as $n$ increases.

5.3 | Effect of smoothing parameter ($\lambda$)

The performance of $S_H$ and $MD_H$ charts for varying levels of sample size $\lambda = 0.03, 0.05, 0.1,$ and $0.25$, when $n = 7$ is evaluated, considering normal, logistic, and Gamma processes. Tables 13 and 14 present the ARL and EQL values of HWMA charts (together with EWMA charts). A graphical comparison of the ARL is also presented for $MD_H$ chart for Gamma distributed process in Figure 3. From Tables 12 and 13 and Figure 3, it is observed that the $ARL_1$ of the HWMA dispersion charts reduces with the decrease in $\lambda$. It indicates that the proposed HWMA dispersion charts are more sensitive in the detection of shifts for small values of $\lambda$. It is expected that for $\lambda > 0.5$, the HWMA dispersion chart will perform better for the detection of large shifts as compared to using small values of $\lambda$. As the purpose of the study is the efficient detection of small process shifts, hence we recommend the use of $\lambda < 0.25$ for the proposed HWMA dispersion charts.
5.4 Proposed HWMA charts versus EWMA charts

In this section, the performance of the proposed HWMA control charts is compared with the EWMA charts for monitoring shifts in process variability. Tables 12 and 13 present the ARL values of the HWMA and EWMA charts using the $S$ and $MD$ estimators, considering normal, logistic, and Gamma distributions at varying levels of $\lambda = 0.03, 0.05, 0.1, 0.25$. When the smoothing parameter $\lambda$ is small (i.e., $\lambda = 0.03$ or $0.05$), the proposed HWMA charts are more sensitive than the EWMA dispersion charts. For example, when $\lambda = 0.03$, considering $MD$ estimator for Gamma distributed process, the proposed $MD_H$ chart requires an average of 7 samples to detect a shift of magnitude $1.2\sigma$, while the corresponding $MD_E$ chart requires 15 samples to detect a shift of same magnitude. Similar comparisons can be drawn for other charts, considering specific choices of $\lambda$ and $n$. This shows that the proposed charts are significantly better than their respective EWMA counterparts. As the smoothing parameter $\lambda$ increases, the ARL of both HWMA and EWMA charts gets closer and closer to each other, with a slight advantage to HWMA dispersion charts. Also, it can be seen that the EQL values for HWMA charts in all cases...
TABLE 10  EQL and RARL for the HWMA dispersion charts for $\lambda = 0.05$, $n = 5$, and $ARL_0 = 200$

| Distribution   | $R_H$ | $S_H$ | $Q_H$ | $D_H$ | $MD_H$ | $MAD_H$ | $SN_H$ | $QN_H$ |
|---------------|-------|-------|-------|-------|--------|---------|--------|--------|
| Normal        | EQL   | 13.97 | 13.76 | 20.71 | 13.85  | 14.09   | 20.43  | 18.85  | 18.6   |
|               | RARL  | 1.02  | 1.00  | 1.51  | 1.01   | 1.02    | 1.48   | 1.37   | 1.35   |
| Logistic      | EQL   | 16.26 | 15.7  | 21.72 | 15.4   | 15.37   | 22.18  | 21.03  | 20.92  |
|               | RARL  | 1.06  | 1.02  | 1.41  | 1.00   | 1.00    | 1.44   | 1.37   | 1.36   |
| Student’s t   | EQL   | 19.5  | 19.28 | 19.67 | 18.12  | 17.86   | 23.58  | 22.55  | 22.13  |
|               | RARL  | 1.09  | 1.08  | 1.10  | 1.01   | 1.00    | 1.32   | 1.26   | 1.24   |
| Weibull       | EQL   | 16.08 | 16.1  | 23.67 | 15.69  | 16.09   | 23.84  | 21.95  | 22.05  |
|               | RARL  | 1.02  | 1.03  | 1.51  | 1.00   | 1.03    | 1.52   | 1.40   | 1.41   |
| Gamma         | EQL   | 18.2  | 18.19 | 23.94 | 17.26  | 17.6    | 23.96  | 23.12  | 22.29  |
|               | RARL  | 1.05  | 1.05  | 1.39  | 1.00   | 1.02    | 1.39   | 1.34   | 1.29   |
| Chi-square    | EQL   | 17.03 | 16.76 | 22.1  | 15.97  | 15.98   | 22.59  | 21.77  | 23.37  |
|               | RARL  | 1.07  | 1.05  | 1.38  | 1.00   | 1.00    | 1.41   | 1.36   | 1.46   |
| Exponential   | EQL   | 22.7  | 22.94 | 29.89 | 21.11  | 21.2    | 28.36  | 27.13  | 27.21  |
|               | RARL  | 1.08  | 1.09  | 1.42  | 1.00   | 1.00    | 1.34   | 1.29   | 1.29   |
| Log-normal    | EQL   | 46.24 | 48.37 | 41.84 | 41.46  | 37.99   | 35.77  | 34.15  | 34.31  |
|               | RARL  | 1.35  | 1.42  | 1.23  | 1.21   | 1.11    | 1.05   | 1.00   | 1.00   |

FIGURE 1  RARL comparison for dispersion HWMA control charts when $n = 5$ and $\lambda = 0.05$

is less than the EQL values of EWMA charts. It indicates that the overall performances of the HWMA charts are always better than their respective EWMA counterparts.

Moreover, Table 14 reports results of HWMA and EWMA charts based on $R$, $S$, $D$, $MD$, and $QN$ dispersion estimators, considering all the parent distributions when $n = 5$ and when $\lambda = 0.05$. HWMA dispersion charts are better than the corresponding EWMA dispersion charts, among all distributions except for the extremely skewed distribution (log-normal) where the sensitivity of the EWMA chart is higher than that of the HWMA charts. For example, when $\lambda = 0.05$, the ARL values of the HWMA chart for logistic distribution when shift size $\delta = 1.5$ for $S_H$ and $D_H$ estimators are 4.12 and 3.97, respectively, while that of EWMA chart are 6.69 and 6.57, respectively.

5.5  Proposed HWMA chart

Normal distribution:
TABLE 11 Average run length profiles of HWMA standard deviation SD chart for normal distribution when $\lambda = 0.05$ and ARL0 = 200 with varying sample size $n = 5, 7, 10, 12, \text{and } 15$

| $n$ | $L$ | $\delta$ | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.8 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
|-----|-----|---------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 5   | 1.570 | 201.34 | 23.12 | 9.2 | 5.51 | 3.89 | 3.12 | 2.58 | 2.02 | 1.69 | 1.3 | 1.15 | 1.08 | 1.05 |
| 7   | 1.530 | 200.58 | 17.35 | 6.82 | 4.08 | 3.01 | 2.4 | 2.01 | 1.6 | 1.34 | 1.04 | 1.01 | 1.00 |
| 10  | 1.364 | 200.74 | 11.87 | 4.76 | 2.93 | 2.19 | 1.8 | 1.54 | 1.25 | 1.14 | 1.02 | 1.01 | 1.00 |
| 12  | 1.638 | 200.55 | 11.57 | 4.56 | 2.98 | 2.19 | 1.77 | 1.52 | 1.23 | 1.09 | 1.01 | 1.00 | 1.00 |
| 15  | 1.600 | 199.12 | 9.75 | 4.01 | 2.52 | 1.89 | 1.53 | 1.32 | 1.12 | 1.04 | 1.01 | 1.00 | 1.00 |

FIGURE 2 Effect of sample size on the ARL of $D_H$ control chart for exponentially distributed process when $\lambda = 0.05$

For $n = 7$ and $\lambda = 0.25$ (Table 1)

- $S_H$ has the best performance as it has the lowest ARL, EQL, and RARL values among the other competing charts.
- $D_H$ is close competitor with $S_H$; they are the second best charts.
- $R_H$ and $MD_H$ charts lie between the best and worst chart performances.
- $MAD_H$, $Q_H$, $SN_H$, and $QN_H$ have the worst performances in detecting shifts.

For $n = 5$ and $\lambda = 0.05$ (Table 10)

- $S_H$, $D_H$, $R_H$, and $MD_H$ charts have the best performance compared to other charts as their ARL values are the lowest for all shift sizes.
- $SN_H$ and $QN_H$ charts lie between the best and worst chart performances.
- $Q_H$ has the highest ARL values and thus it is has the worst performance

Nonnormal symmetric distributions (logistic, and Student’s $t$):
For $n = 7$ and $\lambda = 0.25$ (Tables 2 and 3)

- $MD_H$ and $D_H$ charts for both parent distributions have the best performance in terms of their ARL values.
- The $R_H$ and $S_H$ charts are less efficient for the Student’s $t$ distribution, compared with the logistic distribution.
- The $MAD_H$, $SN_H$, and $QN_H$ are the worst performing charts in both distributions.
### TABLE 12
Average run length profiles of HWMA and EWMA standard deviation SD chart for normal, logistic, and gamma distributions with varying smoothing constant $\lambda = 0.03, 0.05, 0.1,$ and $0.25$ when $n = 7$ and $ARL_0 = 200$

| $\lambda$ | Distribution | Chart | $L$ | $\delta$ | $\delta$ | $\delta$ | $\delta$ | $\delta$ | $\delta$ | $\delta$ | $\delta$ | $\delta$ | EQL |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.03 | Normal | HWMA | 1.120 | 199.90 | 11.85 | 4.88 | 3.20 | 2.45 | 1.99 | 1.74 | 1.41 | 1.25 | 15.68 |
| | | EWMA | 0.190 | 199.37 | 25.17 | 11.79 | 7.84 | 5.84 | 4.72 | 3.98 | 3.09 | 2.57 | 22.80 |
| | Logistic | HWMA | 1.280 | 200.08 | 18.16 | 7.17 | 4.38 | 3.22 | 2.58 | 2.18 | 1.73 | 1.46 | 17.72 |
| | | EWMA | 0.199 | 200.31 | 31.20 | 14.83 | 9.63 | 7.14 | 5.77 | 4.82 | 3.74 | 3.06 | 25.74 |
| | Gamma | HWMA | 1.150 | 201.55 | 32.83 | 13.11 | 7.54 | 5.21 | 3.96 | 3.28 | 2.40 | 1.97 | 22.62 |
| | | EWMA | 0.188 | 201.92 | 36.51 | 17.09 | 11.13 | 8.35 | 6.60 | 4.42 | 3.45 | 2.89 | 28.20 |
| 0.05 | Normal | HWMA | 1.530 | 200.58 | 24.55 | 10.05 | 5.71 | 3.94 | 3.07 | 2.53 | 1.88 | 1.55 | 19.67 |
| | | EWMA | 0.295 | 201.87 | 27.26 | 11.69 | 7.14 | 5.25 | 4.14 | 3.48 | 2.66 | 2.17 | 22.73 |
| | Logistic | HWMA | 1.740 | 199.27 | 26.35 | 10.22 | 6.02 | 4.13 | 3.23 | 2.72 | 2.01 | 1.68 | 20.19 |
| | | EWMA | 0.303 | 200.51 | 32.04 | 14.74 | 9.26 | 6.85 | 5.47 | 4.60 | 3.50 | 2.89 | 25.35 |
| | Gamma | HWMA | 1.680 | 199.61 | 32.83 | 13.11 | 7.54 | 5.21 | 3.96 | 3.28 | 2.40 | 1.97 | 22.62 |
| | | EWMA | 0.294 | 200.22 | 37.82 | 17.83 | 11.08 | 8.20 | 5.36 | 4.13 | 3.33 | 2.89 | 28.09 |
| 0.10 | Normal | HWMA | 2.130 | 200.39 | 24.55 | 10.05 | 5.71 | 3.94 | 3.07 | 2.53 | 1.88 | 1.55 | 19.67 |
| | | EWMA | 0.500 | 201.73 | 27.26 | 11.69 | 7.14 | 5.25 | 4.14 | 3.48 | 2.66 | 2.17 | 21.98 |
| | Logistic | HWMA | 2.460 | 201.07 | 37.57 | 15.37 | 8.87 | 5.91 | 4.39 | 3.51 | 2.75 | 2.05 | 24.29 |
| | | EWMA | 0.521 | 201.20 | 43.86 | 14.93 | 9.16 | 6.59 | 5.12 | 4.28 | 3.24 | 2.62 | 25.21 |
| | Gamma | HWMA | 2.500 | 199.92 | 47.28 | 20.91 | 11.93 | 7.88 | 5.83 | 4.61 | 3.25 | 2.54 | 28.49 |
| | | EWMA | 0.520 | 200.79 | 44.26 | 19.31 | 11.58 | 8.18 | 6.28 | 5.22 | 3.88 | 3.12 | 28.91 |
| 0.25 | Normal | HWMA | 2.660 | 200.67 | 29.78 | 11.87 | 6.80 | 4.62 | 3.52 | 2.79 | 2.04 | 1.62 | 21.23 |
| | | EWMA | 0.950 | 201.73 | 32.34 | 12.24 | 7.03 | 4.84 | 3.64 | 3.02 | 2.25 | 1.89 | 22.09 |
| | Logistic | HWMA | 3.066 | 201.74 | 46.00 | 19.35 | 10.96 | 7.29 | 5.33 | 4.09 | 2.85 | 2.25 | 27.23 |
| | | EWMA | 1.023 | 198.27 | 45.85 | 17.99 | 9.95 | 6.73 | 4.98 | 3.99 | 2.89 | 2.31 | 26.50 |
| | Gamma | HWMA | 3.230 | 199.74 | 59.77 | 27.05 | 15.49 | 10.30 | 7.46 | 5.79 | 3.98 | 2.93 | 33.41 |
| | | EWMA | 1.068 | 199.96 | 55.95 | 24.56 | 13.68 | 8.97 | 6.60 | 5.24 | 3.73 | 2.93 | 31.46 |

For $n = 5$ and $\lambda = 0.05$ (Table 10)

- $MD_H$ chart is the best in terms of their EQL and RARL values.
- The $R_H$, $S_H$, $SN_H$, and $QN_H$ lie between the superior and inferior charts.
- The $MAD_H$ charts for both distributions have the worst performance in terms of all the run length measures.

**Skewed distributions (Weibull, chi-square, gamma, and exponential):**

For $n = 7$ and $\lambda = 0.25$ (Tables 4–7)

- The performances of $MD_H$ and $D_H$ charts are the best, as compared to all other competing charts.
- The $S_H$, $R_H$, and the $Q_H$ charts have almost the same performance among the four distributions but are more efficient for parent Weibull distribution, compared to the other distributions.
- The $MAD_H$, $SN_H$, and $Q_H$ were the least efficient charts as their ARLs, EQLs, and RARLs are high.

For $n = 5$ and $\lambda = 0.05$ (Table 9)

- The performances of $MD_H$ and $D_H$ charts are the best, as compared to all other competing charts.
- The $S_H$, $R_H$, and the $Q_H$ charts have almost the same performance among the four distributions but are more efficient for parent Weibull distribution, compared to the other distributions.
- The $MAD_H$, $SN_H$, and $Q_H$ were the least efficient charts as their ARLs, EQLs, and RARLs are high.
TB LE 13 Average run length profiles of HWMA and EWMA mean deviation from sample median MD chart for normal, logistic, and gamma distribution with varying smoothing constant $\lambda = 0.03, 0.05, 0.1,$ and $0.25$ when $n = 7$ and $ARL_0 = 200$

| $\lambda$ | Distribution | Chart | $L$ | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.8 | 2.0 | EQL |
|-----------|--------------|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0.03      | Normal       | HWMA  | 1.114 | 200.12 | 12.76 | 5.30 | 3.37 | 2.52 | 2.09 | 1.80 | 1.47 | 1.28 | 16.00 |
|           |              | EWMA  | 0.190 | 199.66 | 15.39 | 12.57 | 8.13 | 6.11 | 4.94 | 4.17 | 3.21 | 2.65 | 22.07 |
|           | Logistic     | HWMA  | 1.200 | 201.22 | 16.12 | 6.72 | 4.06 | 3.04 | 2.44 | 2.08 | 1.66 | 1.43 | 17.25 |
|           |              | EWMA  | 0.195 | 199.85 | 30.01 | 14.23 | 9.34 | 6.95 | 5.60 | 4.71 | 3.63 | 2.97 | 25.22 |
|           | Gamma        | HWMA  | 1.100 | 201.28 | 17.74 | 7.36 | 4.52 | 3.29 | 2.64 | 2.09 | 1.80 | 1.47 | 12.25 |
|           |              | EWMA  | 0.191 | 199.08 | 33.63 | 15.97 | 10.47 | 7.74 | 6.26 | 5.23 | 4.03 | 3.28 | 26.94 |
| 0.05      | Normal       | HWMA  | 1.500 | 199.35 | 17.96 | 7.13 | 4.28 | 3.09 | 2.50 | 2.09 | 1.63 | 1.40 | 14.77 |
|           |              | EWMA  | 0.294 | 201.18 | 27.13 | 12.46 | 8.06 | 5.92 | 4.74 | 3.09 | 2.65 | 2.27 | 23.27 |
|           | Logistic     | HWMA  | 1.650 | 200.84 | 24.01 | 9.46 | 5.49 | 3.84 | 3.04 | 2.56 | 2.08 | 1.66 | 19.54 |
|           |              | EWMA  | 0.300 | 201.21 | 35.84 | 17.69 | 12.57 | 8.17 | 5.47 | 4.15 | 3.38 | 2.97 | 25.03 |
|           | Gamma        | HWMA  | 1.600 | 201.16 | 27.64 | 11.95 | 6.46 | 4.42 | 3.44 | 2.87 | 2.18 | 1.80 | 20.94 |
|           |              | EWMA  | 0.299 | 201.70 | 31.46 | 14.33 | 9.14 | 6.71 | 5.41 | 4.34 | 3.44 | 2.83 | 27.11 |
| 0.10      | Normal       | HWMA  | 2.180 | 201.56 | 33.02 | 13.21 | 7.55 | 5.17 | 3.77 | 3.02 | 2.18 | 1.76 | 20.69 |
|           |              | EWMA  | 0.500 | 201.32 | 28.66 | 12.27 | 7.56 | 5.47 | 4.34 | 3.64 | 2.77 | 2.27 | 22.72 |
|           | Logistic     | HWMA  | 2.360 | 201.19 | 34.79 | 14.27 | 8.17 | 5.47 | 4.15 | 3.38 | 2.42 | 1.93 | 23.34 |
|           |              | EWMA  | 0.515 | 201.23 | 34.21 | 14.63 | 8.94 | 6.39 | 5.06 | 4.17 | 3.15 | 2.58 | 24.88 |
|           | Gamma        | HWMA  | 2.380 | 201.05 | 40.85 | 17.69 | 9.93 | 6.65 | 4.98 | 3.96 | 2.85 | 2.27 | 25.92 |
|           |              | EWMA  | 0.516 | 201.07 | 39.75 | 16.98 | 10.34 | 7.45 | 5.83 | 4.72 | 3.55 | 2.90 | 27.10 |
| 0.25      | Normal       | HWMA  | 2.722 | 201.54 | 39.11 | 13.85 | 7.55 | 5.17 | 3.77 | 3.02 | 2.18 | 1.76 | 22.38 |
|           |              | EWMA  | 0.965 | 200.59 | 35.07 | 15.30 | 7.57 | 5.18 | 3.96 | 3.22 | 2.41 | 1.94 | 22.97 |
|           | Logistic     | HWMA  | 2.920 | 200.19 | 42.13 | 17.34 | 11.69 | 6.61 | 4.93 | 3.85 | 2.71 | 2.12 | 26.06 |
|           |              | EWMA  | 0.992 | 199.23 | 42.25 | 16.43 | 9.28 | 6.29 | 4.72 | 3.80 | 2.75 | 2.23 | 25.43 |
|           | Gamma        | HWMA  | 3.030 | 200.33 | 51.71 | 21.94 | 12.57 | 8.35 | 6.05 | 4.73 | 3.33 | 2.56 | 29.55 |
|           |              | EWMA  | 1.040 | 199.58 | 65.22 | 29.60 | 17.62 | 11.80 | 8.59 | 6.75 | 4.79 | 3.71 | 36.53 |

*Extremely skewed distribution (log-normal):*

For $n = 7$ and $\lambda = 0.25$ (Table 8)

- The MAD$_H$, SN$_H$, and QN$_H$ charts have the least ARLs, thus they are the best charts in detecting shifts in the process dispersion.
- The QH$_H$, MD$_H$, and DH$_H$ charts by order come right after the best charts.
- Both S$_H$ and R$_H$ charts have the worst performance as all their run length measures are higher than other charts.

For $n = 5$ and $\lambda = 0.05$

- The SN$_H$, QN$_H$, and MAD$_H$ performances are considered superior in terms of their ARLs, EQLs, and RARLs.
- Both S$_H$ and R$_H$ charts have the worst performances as their ARL and other performance measures.

### 5.6 Effect of sample size ($n$)

- It can be observed from Table 11 and Figure 2 that the ARL$_1$ of the S$_H$ and the D$_H$ charts decrease with an increase in the sample size $n$.
- For all dispersion HWMA charts that the detection ability of the charts improves as $n$ increases.
TABLE 14 ARL for HWMA and EWMA for some dispersion charts when $\lambda = 0.05$, $n = 5$, and ARL$_0 = 200$

| Distribution | $\delta$ | HWMA $R_H$ | $S_H$ | $D_H$ | $MD_H$ | $QN_H$ | EWMA $R_E$ | $S_E$ | $D_E$ | $MD_E$ | $QN_E$ |
|--------------|---------|------------|-------|-------|--------|-------|------------|-------|-------|--------|-------|
| Normal       | 1.1     | 24.22      | 23.12 | 23.68 | 24.65  | 39.68 | 33.53      | 32.65 | 33.48 | 33.48  | 49.93 |
|              | 1.2     | 9.83       | 9.20  | 9.36  | 10.06  | 16.97 | 15.76      | 15.19 | 15.24 | 15.56  | 24.08 |
|              | 1.5     | 3.19       | 3.12  | 3.12  | 3.22   | 5.17  | 5.91       | 5.72  | 5.78  | 5.92   | 8.87  |
|              | 2.0     | 1.73       | 1.69  | 1.71  | 1.78   | 2.59  | 3.06       | 3.00  | 3.01  | 3.08   | 4.50  |
| Logistic     | 1.1     | 36.22      | 34.46 | 32.07 | 31.08  | 48.44 | 40.09      | 38.85 | 37.91 | 38.74  | 53.66 |
|              | 1.2     | 14.65      | 13.66 | 12.86 | 12.71  | 21.73 | 19.35      | 18.17 | 17.80 | 17.85  | 26.06 |
|              | 1.5     | 4.39       | 4.12  | 3.97  | 3.89   | 6.42  | 7.07       | 6.69  | 6.57  | 6.63   | 9.61  |
|              | 2.0     | 2.20       | 2.08  | 2.03  | 2.02   | 3.06  | 3.58       | 3.46  | 3.40  | 3.43   | 4.79  |
| Student's $t$| 1.1     | 51.43      | 51.30 | 44.95 | 43.88  | 52.84 | 59.04      | 59.77 | 50.97 | 51.71  | 61.42 |
|              | 1.2     | 22.10      | 22.02 | 19.13 | 18.01  | 23.58 | 29.61      | 28.86 | 27.40 | 27.04  | 33.51 |
|              | 1.5     | 2.20       | 2.08  | 2.03  | 2.02   | 3.06  | 3.58       | 3.46  | 3.40  | 3.43   | 4.79  |
| Weibull      | 1.1     | 34.08      | 34.49 | 32.45 | 33.79  | 52.18 | 39.73      | 40.35 | 38.16 | 38.18  | 52.45 |
|              | 1.2     | 14.29      | 13.86 | 13.31 | 14.01  | 23.69 | 18.59      | 18.65 | 17.74 | 17.45  | 25.01 |
|              | 1.5     | 4.39       | 4.16  | 4.30  | 6.99   | 6.65  | 6.74       | 6.44  | 6.54  | 9.24   |
|              | 2.0     | 2.12       | 2.18  | 2.08  | 2.13   | 3.30  | 3.46       | 3.46  | 3.30  | 3.32   | 4.60  |
| Gamma        | 1.1     | 42.67      | 43.64 | 38.72 | 40.37  | 52.25 | 62.36      | 59.77 | 50.97 | 51.71  | 61.42 |
|              | 1.2     | 18.27      | 18.19 | 16.56 | 17.12  | 24.16 | 28.62      | 27.05 | 22.09 | 21.94  | 27.88 |
|              | 1.5     | 5.48       | 5.48  | 4.93  | 5.05   | 7.18  | 7.87       | 7.46  | 6.12  | 6.05   | 8.11  |
|              | 2.0     | 2.57       | 2.61  | 2.38  | 2.44   | 3.29  | 3.16       | 2.98  | 2.54  | 2.56   | 3.30  |
| Chi-square   | 1.1     | 38.63      | 37.82 | 33.64 | 33.61  | 49.61 | 45.71      | 45.29 | 42.07 | 42.55  | 54.37 |
|              | 1.2     | 16.34      | 15.92 | 13.88 | 13.95  | 22.38 | 21.35      | 21.41 | 20.11 | 20.12  | 26.58 |
|              | 1.5     | 4.78       | 4.71  | 4.19  | 4.27   | 6.83  | 7.95       | 7.90  | 7.41  | 7.41   | 9.77  |
|              | 2.0     | 2.35       | 2.32  | 2.14  | 2.13   | 3.15  | 4.01       | 4.01  | 3.83  | 3.81   | 4.90  |
| Exponential  | 1.1     | 56.32      | 56.48 | 51.19 | 51.41  | 65.42 | 76.40      | 75.43 | 64.14 | 62.04  | 74.93 |
|              | 1.2     | 26.35      | 26.42 | 22.96 | 22.86  | 31.82 | 38.26      | 36.94 | 30.06 | 28.28  | 37.46 |
|              | 1.5     | 7.77       | 7.85  | 6.91  | 6.87   | 9.89  | 11.20      | 10.71 | 8.65  | 8.09   | 11.46 |
|              | 2.0     | 3.44       | 3.46  | 3.14  | 3.18   | 4.32  | 4.31       | 4.19  | 3.50  | 3.32   | 4.49  |
| Log-normal   | 1.1     | 105.55     | 108.79| 98.46 | 92.87  | 84.33 | 131.71     | 131.37| 122.06| 116.14 | 99.55 |
|              | 1.2     | 63.12      | 66.00 | 57.46 | 50.15  | 45.05 | 91.67      | 90.53 | 80.08 | 72.95  | 54.37 |
|              | 1.5     | 21.79      | 23.47 | 18.76 | 16.87  | 14.27 | 38.22      | 38.35 | 29.63 | 25.47  | 17.54 |
|              | 2.0     | 8.64       | 8.99  | 7.30  | 6.66   | 5.67  | 15.22      | 15.53 | 11.25 | 9.44   | 6.53  |

5.7 Effect of smoothing parameter ($\lambda$)

- From Tables 12 and 13 and Figure 3, it is observed that the ARL$_1$ of the HWMA dispersion charts reduces with the decrease in $\lambda$.
- The proposed HWMA dispersion charts are more sensitive in the detection of shifts for small values of $\lambda$.

5.8 Proposed HWMA charts versus EWMA charts

Tables 12 and 13 present the ARL values of the HWMA and EWMA charts using the $S$ and $MD$ estimators, considering normal, logistic, and Gamma distributions at varying levels of $\lambda = 0.03, 0.05, 0.1, 0.25$. 
• When the smoothing parameter \( \lambda \) is small (i.e., \( \lambda = 0.03 \) or 0.05), the proposed HWMA charts are more sensitive than the EWMA dispersion charts.
• As the smoothing parameter \( \lambda \) increase, the ARL of both HWMA and EWMA charts gets closer and closer to each other, with a slight advantage to HWMA dispersion charts.
• It can be seen that the EQL values for HWMA charts in all cases are less than the EQL values of EWMA charts. It indicates that the overall performances of the HWMA charts are always better than their respective EWMA counterparts.

Table 14 reports results of HWMA and EWMA charts based on \( R, S, D, MD, \) and \( QN \) dispersion estimators, considering all the parent distributions when \( n = 5 \) and when \( \lambda = 0.05 \).

• HWMA dispersion charts are better than the corresponding EWMA dispersion charts, among all distributions except for the extremely skewed distribution (log-normal) where the sensitivity of the EWMA chart is higher than that of the HWMA charts.

6 | REAL DATA APPLICATION

This section provides the real-life application of the proposed charts for monitoring the vapor pressure of the distillation process. The dataset is obtained from https://openmv.net/info/distillation-tower. A brief explanation of the distillation process and the application of the proposed charts are provided below:

6.1 | Distillation process

Distillation is an important process in chemical and industrial engineering. It separates the liquid components from a mixture based on their selective boiling and condensation, that is, vapor- and liquid-phase distribution. During this process, a distillation tower allows the liquids to distribute in multiple liquids by passing through different stages. In the first stage, time is a most important factor that allows the chemical mixtures to separate. However, distribution in later stages relies on the properties of each chemical, for example, vapor pressure, evaporation point, the properties of mixtures being separated, and the desired purity of the end distillate. In principle, these properties decide the fate of molecule to fall or rise depending on their heavy and light components followed by separation of different liquid components. A brief
A representation of distillation column (https://en.wikipedia.org/wiki/Distillation).

Explanation of the distillation process is shown in Figure 4 (https://en.wikipedia.org/wiki/Distillation). The schematic diagram of distillation unit represents a single feed that separates liquid mixtures into two produces: overhead product and bottom product. Usually, the bottom products are liquids (heavy components) while the overhead products can be either liquid or vapor or both (light components).\(^{29,30}\) Furthermore, for necessary vaporization, the reboiler is used where it works on providing heat from the bottom and a vapor is produced where it flows upward through the distillation column. Thereon, the condenser is used for cooling and condensing the vapor from the top. A liquid is produced in reflux drum and it flows downward in the distillation column to be recycled back.

### 6.2 Quality control application

This section uses the vapor pressure variable, which is a measure of the tendency of a material to change into the vapor state. It is one of the chemical properties that decide the separation of liquids into different components in the distillation process and thus it is an important variable to be monitored. The following steps were taken to apply the proposed charts on this dataset:

First, we find the distribution of the data to be used in constructing the upper control limits of the proposed HWMA charts. We used the \textit{fitdist} function in \textit{fitdistplus} package in R and tested the data for four potential distributions which are normal, gamma, Weibull and lognormal. We identified the best-fitted distribution for this data as gamma distribution with scale and rate parameters 31.08 and 0.848, respectively. Moreover, we also used the \textit{descdist} function to plot the Cullen and Frey Graph, given below in Figure 5; this graph plots the square of skewness against the kurtosis. It also indicated that gamma is the best-fitted distribution.

After distribution fitting, 50 samples size of \(n = 5\) are drawn from the data and control limits are computed for a fixed \(ARL_0 = 200\) using \(\lambda = 0.05\). These samples were standardized to have mean and standard deviation equals 0 and 1, respectively. For illustration purpose, we used \(S_H\), \(D_H\), and \(MAD_H\) control charts. The control chart coefficients for three charts are found to be \(L_S = 1.350\), \(L_D = 1.361\), and \(L_{MAD} = 1.607\).

To check the detection ability of the proposed HWMA charts, shift of \(\delta = 3\) was introduced in the standard deviation of last 10 samples. Figure 6 depicts the resulting control chart plots. As it can be seen in Figure 6, the \(S_H\) chart detected 6 out-of-control points, 7 out-of-control points were detected by \(D_H\) chart and 5 out-of-control points were detected by \(MAD_H\) chart with one false alarm.
CONCLUSION

This paper proposed and investigated a set of HWMA charts for detecting shifts in process dispersion for normal and nonnormal parent distributions, considering a wide range of dispersion estimators. The study revealed that the $S_{H}$ chart followed by the $R_{H}$ and the $MD_{H}$ charts perform better for normally distributed process while for most of the skewed and heavy symmetric parent distributions, $MD_{H}$ and $D_{H}$ charts are the best performing charts. For extremely skewed case of lognormal distribution, $SN_{H}$, $QN_{H}$, and $MAD_{H}$ charts had superior performances compared to other charts. Comparison with the EWMA dispersion charts indicated superiority of the HWMA dispersion charts, particularly when for small choices of smoothing parameter. Hence, we recommend the use of HWMA dispersion charts for efficient monitoring of process dispersion.

ORCID

Saddam Akber Abbasi https://orcid.org/0000-0003-1843-8863
Olatunde A. Adeoti https://orcid.org/0000-0003-2242-1204

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AUTHOR BIOGRAPHIES

Saddam Akbar Abbasi is currently working as an associate professor in Department of Mathematics, Statistics and Physics at Qatar University, Doha, Qatar. He received his PhD in statistics from The University of Auckland, New Zealand in 2013. Before joining Qatar University, Dr. Abbasi served as an assistant professor in King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia, for three years. His research interests include SPC, time-series analysis, profile monitoring, and nonparametric statistics.

Sara H. Nassar holds BS and MS degrees in applied statistics from Qatar University. Her research interests include quality control, statistical process control, and profile monitoring.
Mashael Mohammed Aldosari is currently enrolled in MS degree in applied statistics from Qatar University. Her research interests include quality control, statistical process control, and profile monitoring.

Olatunde A. Adeoti obtained his BSc (Hons) degree (First class honors) in mathematics from the University of Ilorin, MSc (statistics) from the University of Lagos, and PhD in statistics from the University of Ilorin, Ilorin, Nigeria. He has articles that have been published in *Quality Engineering, Quality and Reliability Engineering International, Communications in Statistics—Theory and Methods, International Journal of Quality and Reliability Management and Quality Technology*, and *Quantitative Management*. He is a member of the Nigerian Statistical Association, American Society for Quality. His research interests include Statistical Process Control, Statistical Quality Control and Management in Healthcare and Artificial Neural Network in Statistical Process Control. He is currently a senior lecturer at the Department of Statistics, Federal University of Technology, Akure, Nigeria.

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