Influence of leakage on characteristics of the vacuum transport unit based on the water-ring pump

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Abstract. The mathematical model of the liquid pumping process using the autonomous vacuum transport unit based on the water-ring pump is proposed. Such units are used, in particular, for pumping aggressive liquids. The performance characteristics of ERSTVAK's ELRS-2 water-ring vacuum pump were used in the calculation. The system of differential equations was solved numerically. An increase in the leakage coefficient leads to a raising the absolute pressure in the tank at the end of the first stage. As a result, the volume of liquid pumped in one cycle is noticeably reduced. Therefore, the average performance and efficiency of the unit is significantly reduced. The duration of one cycle changes slightly.

1. Introduction
Transportation of liquids by vacuum systems is widely used in various branches of science and technology. Transportation of the liquid without its contact with various metal surfaces of the system is required in special cases. For example, systems that provide blood circulation during heart surgery must create and maintain the necessary pressure drop. The method for calculating the parameters of such systems based on high-vacuum pumps was developed in [1]. In most cases, low-vacuum pumps can be used. Then the advantages of water-ring vacuum pumps (WRVP) are fully manifested [2]: high reliability, simple design, low noise and vibration, and the ability to pump out any gases and vapors. These advantages of WRVP allow them to be used in the chemical industry [3], the fishing industry [4], and other industries.

Mathematical models of pumping air out of the working chamber by various vacuum pumps have been published in many articles (see [5, 6] and the bibliography). However, the effect of air leaks during the cyclical operation of the unit has not been analyzed. The purpose of this article is to create a mathematical model of an autonomous vacuum system based on WRVP and study the effect of air leakage on its performance parameters.

2. Mathematical model
Let's consider a mathematical model of the unit based on WRVP, designed for pumping liquid. We consider the gas to be perfect, and the process to be isothermal.

2.1. The stages of the unit operation
The schematic diagram of the unit is shown in figure 1. The operation of the unit is cyclical. One cycle can be divided into three stages. In the first stage, valve 2 is open, and valves 4 and 7 are closed. There
is no liquid in the working tank 3. WRVP 1 pumps air out of the tank 3. In the second stage, the WRVP is turned off, valve 4 is open, and valves 2 and 7 are closed. The liquid 6 through the pipeline 5 enters the tank 3 due to the pressure difference. In the third stage, valves 2 and 4 are closed, and valve 7 is open. The liquid is drained through the lower pipe for further transportation. The unit is ready for a new cycle of operation.

It is necessary to create a mathematical model for the first and second stages. The third stage is not considered here.

![Figure 1. Schematic diagram of the unit: 1 – WRVP, 2, 4, 7 – valves; 3 – working tank; 5 – pipeline; 6 – liquid container.](image)

### 2.2. The first stage model

As is known, in General, the differential equation for pumping air from a certain container by a vacuum pump has the form (see, for example, [7]):

\[
V_0 \cdot \frac{d p}{d t} = -p \cdot f(p) + S_T + S_G ,
\]

(1)

where \(V_0\) is the volume of the vacuum system (m\(^3\)); \(p\) – pressure (Pa); \(t\) – current time (s); \(G=f(p)\) – dependence of the effective pump on pressure (m\(^3\)/s), \(S_T\) is the stream of gas caused by leakage (Pa·m\(^3\)/s); \(S_G\) is the emission stream of gas (Pa·m\(^3\)/s).

The emission stream of gas is neglected in low-vacuum pumps. We use the formula commonly used in vacuum technology for the leakage stream:

\[
S_T = (p_A - p) \cdot k \cdot f(p),
\]

(2)

where \(k\) is the empirical constant called the leakage coefficient.

Substituting (2) in (1), we get a differential equation with the initial condition

\[
V_0 \cdot \frac{d p}{d t} = G(p(t)) \cdot (k \cdot p_A - p(t) \cdot (1 + k)), \quad p(0) = p_A.
\]

(3)

We must specify the dependence of the WRVP performance on the pressure in the working tank \(G=f(p)\) in equation (3). The dependence of the spent power of the WRVP on the pressure \(N=\psi(p)\) will be required in the future. Let’s use the method of modeling pump characteristics [9, 10]. WRVP performance characteristics were calculated using the formulas (4). In this paper, in contrast to [10], cubic spline functions were used as approximating functions \(f_i(p), \psi_i(p)\).

\[
G = f(p) = \begin{cases} 
0, & p \leq p_V; \\
 f_1(p), & p_V < p < p_1; \\
 G_m, & p \geq p_1. 
\end{cases}
\]

(4)

\[
N = \psi(p) = \begin{cases} 
N_0, & p \leq p_V; \\
 \psi_1(p), & p > p_V.
\end{cases}
\]

The results of calculations are compared with experimental data from ERSTVAK [11] on figure 2. The air flow rate (pumping rate) is given under suction conditions. Values of the found empirical
parameters for the ELRS-2 pump at 1450 rpm, \(N_0 = 3.0 \text{ kW}; \) \(p_V = 3.3 \text{ kPa}; \) \(p_1 = 60.2 \text{ kPa}; \) \(G_m = 0.0691 \text{ m}^3/\text{s} = 4.15 \text{ m}^3/\text{min}.

![Figure 2](image-url). Performance characteristics of the ELRS-2 water-ring vacuum pump at 1450 rpm. \(G – \) WRVP performance (pumping speed), \(N – \) power consumed. Points – experimental data [11], lines – lines – calculation results according to the formulas (4).

### 2.3. The second stage model

The working tank is filled with liquid during the second stage. The volume of air in it decreases, and the pressure increases. The following equality is true for an isothermal process:

\[
p_0 \cdot V_0 = p(t) \cdot V(t),
\]

where \(V_0\) is the volume of the working tank (m³); \(p_0\) is the absolute pressure in the working tank after the first stage (Pa); \(V(t), p(t)\), respectively, the air volume and pressure at time \(t\).

The pressure drop that causes the fluid to move will decrease over time according to the formula (6). Therefore, the flow of the liquid will be non-stationary.

\[
\Delta p(t) = p_A - p(t) = p_A - p_0 \cdot V_0 / V(t),
\]

where \(p_A\) is the atmospheric pressure (Pa).

Let’s use the Bernoulli equation for non-stationary turbulent flow of a liquid in a pipe (see, for example, [12]). It has the following form in the case under consideration:

\[
\frac{\Delta p(t)}{\rho g} - H_0 = a_1 \frac{W^2}{2g} (1 + \zeta) + \frac{a_2}{g} \int_0^L \frac{\partial^2 W}{\partial t} \, dx,
\]

where the last summand is due to the unsteady nature of the flow; \(W\) is the average fluid velocity (m/s); \(H_0\) is the height of liquid rise in the unit (m); \(L\) is the length of pipeline (m); \(\rho\) is the fluid density (kg/m³); \(g\) is gravitational acceleration (m/s²); \(a_1, a_2\) are coefficients of non-uniformity of longitudinal velocity profile of the fluid; \(\zeta\) is the generalized coefficient of hydraulic resistance:

\[
\zeta = \lambda L / d + \Sigma \zeta_M,
\]

where \(\lambda\) is the coefficient of friction losses; \(d\) is the pipe diameter (m); \(\zeta_M\) is the coefficient of local hydraulic losses (pipeline turns, valves, etc.). In the future, we do not take them into account separately, but add 10% to the coefficient \(\lambda\), as is usually done when designing.

We assume that for large Reynolds numbers \(Re\), we can take \(a_1 \approx 1, a_2 \approx 1\), and calculate the coefficient of friction losses using the well-known Altschul formula:

\[
\lambda = 0.11 \left( \frac{\delta + \frac{68}{Re}}{\nu} \right)^{0.25}, \quad Re = \frac{W \cdot d}{\nu}, \quad \frac{\delta}{d} = \frac{A}{d},
\]

where \(A\) is the absolute roughness of the pipe; \(\nu\) is the coefficient of kinematic viscosity of the liquid.
The pressure drop changes smoothly and does not exceed atmospheric pressure, so the compressibility of the liquid can be neglected. We consider the time derivative of the speed in formula (7) unchanged along the length of the pipeline. Then, after integration in (7), we get the differential equation

\[ L \frac{dW}{dt} = \frac{1}{\rho} \left( p_A - p_0 \frac{V_0}{V(t)} \right) - gH_0 - \frac{W^2}{2} (1 + \zeta). \]  

(10)

The smallest volume of air in the working tank is found from the condition that the available head is equal to zero:

\[ \frac{1}{\rho} \left( p_A - p_0 \frac{V_0}{V_{\min}} \right) - gH_0 = 0 \Rightarrow V_{\min} = \frac{p_0V_0}{p_2}, \quad p_2 = p_A - \rho gH_0 \]  

(11)

The pressure in the chamber will reach the value \( p_2 \) at this point. Then the volume of liquid that enters the working tank in one cycle will be equal to

\[ V_1 = V_0 - V_{\min} = V_0 \cdot (1 - \frac{p_0}{p_2}). \]  

(12)

The \( V_1 \) value is used to calculate the average performance per cycle.

The volume of air in the working tank \( V \) is reduced by the volume of the incoming liquid. Whence follows the differential equation for \( V \):

\[ \frac{dV}{dt} = -Q(t), \quad W(t) = \frac{Q(t)}{S}, \]  

where \( Q(t) \) is the volume flow rate of the liquid in the pipeline \( (m^3/s) \); \( S = \pi d^2/4 \) is the cross-sectional area of the pipeline \( (m^2) \).

Write down the initial conditions for the system of equations (10), (13):

\[ V(0) = V_0, \quad Q(0) = 0. \]  

(14)

3. Results of the simulation

Mathematical modeling of the unit operation was performed for the ELRS-2 vacuum pump with the following parameter values: \( V_0 = 0.8 \text{ m}^3 \); \( L = 7 \text{ m} \); \( H_0 = 1.5 \text{ m} = \text{const} \); \( d = 0.07 \text{ m} \); \( \Delta = 0.07 \text{ mm} \); \( \nu = 10^{-5} \text{ m}^2/\text{s} \); \( \rho = 800 \text{ kg/m}^3 \).

3.1. Results of the first stage simulation

The Cauchy problem (3) was solved numerically at different values of the leakage coefficient \( k \). The calculation results are shown in figures 3 and 4.

Figure 3 shows that at the beginning of the first stage, the pressure drops rapidly, and then tends to the limit value. The higher the leakage coefficient, the higher this limit pressure. Figure 4 shows the results of the calculation using the formula (15) of the work spent by ELRS-2 on air pumping.

\[ A_1 = \int_{0}^{T_1} \phi(p(t)) \, dt. \]  

(15)

The lower the duration of the first \( T_1 \) stage, the less work will be spent. Work \( A_1 \) increases with increasing coefficient \( k \). The goal of the first stage is to reduce the pressure in the working chamber. Therefore, it is not advisable to continue working with WRVP if the pressure is already practically not reduced. The choice of \( T_1 \) value is important for determining the energy efficient mode of operation of the plant. Then the duration of the first stage is taken \( T_1 = 30 \text{ seconds} \).
3.2. Results of the second stage simulation

Cauchy's task (10), (13), (14) was also solved numerically. The calculation results for different values of the leakage coefficient are shown in figures 5 and 6. The greater the initial pressure drop, the lower the leakage coefficient. The flow rate of the liquid in the pipeline quickly reaches the maximum value, and then gradually falls.

The second stage of duration \( T_2 \) ends when the pressure in the tank reaches the value \( p_2 \). The final pressure in the tank \( p_2 \) does not depend on \( k \).

The hydraulic power when pumping liquid is calculated as follows

\[
N_n = \Delta p \cdot Q .
\]  

(16)

Where useful work for the second stage can be determined by the formula

\[
A_2 = \int_{0}^{T_2} (p_A - p(t)) \cdot Q(t) \, dt .
\]  

(17)
Then the efficiency coefficient will be equal to $\eta=100\cdot \frac{A_2}{A_1}$. Average installation performance for the first two stages $Q_{12}=V_1/(T_1+T_2)$. The values of the parameters calculated for different $k$ are listed in Table 1.

| $k$  | $p_0$ (kPa) | $T_2$ (s) | $A_2$ (kJ) | $V_1$ (m$^3$) | $Q_{12}$ (m$^3\cdot$min$^{-1}$) | $\eta$ (%) |
|------|-------------|------------|------------|---------------|-------------------------------|-------------|
| 0    | 3.3         | 29.0       | 59.4       | 0.733         | 0.745                         | 37.4        |
| 0.05 | 5.3         | 29.6       | 54.4       | 0.702         | 0.706                         | 33.3        |
| 0.10 | 9.5         | 30.1       | 47.2       | 0.672         | 0.671                         | 28.2        |
| 0.15 | 13.4        | 30.4       | 42.3       | 0.644         | 0.640                         | 25.1        |

**4. Conclusion**

An increase in the leakage coefficient from zero to 0.15 resulted in an increase in the absolute pressure in the tank at the end of the first stage from 3.3 to 13.4 kPa. As a result, the volume of liquid pumped per cycle decreased from 0.745 to 0.664 m$^3$. Therefore, the average performance decreased from 0.745 to 0.640 m$^3$/min. The unit's efficiency decreased from 37.4% to 25.1%. However, the duration of the cycle has changed slightly. The calculated parameters allow you to select the characteristics of the device for automatic control of the unit.

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