Spontaneous Symmetry Breaking and Landau Phase Transition in Horava Gravity

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Abstract

Presence of higher derivative terms in the Horava model of gravity can generate an instability in the Minkowski ground state. This in turn leads to a space dependent vacuum metric with a length scale determined by the higher derivative coupling coefficient. The translation invariance is spontaneously broken in the process. The phenomenon is interpreted as a form of Landau liquid-solid phase translation. The (metric) condensate acts as a source that modifies the Newtonian potential below the length scale but keeps it unchanged for sufficiently large distance.

Introduction and Formalism: Horava Gravity (HG) [1] has created a lot of recent interest but surprisingly an intriguing consequence of the all important higher derivative extension terms in HG has gone unnoticed: possibility of an instability in the conventional flat metric ground state leading to a richer structure of the vacuum. This is a form of the celebrated Landau liquid-solid phase transition [2] of the spacetime (only space in the present context) itself. In the present work we show that this is indeed feasible and provide a preliminary study of some of its immediate effects.

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An important lesson that we have learnt from Horava’s \cite{1} novel proposal of a UV complete model of Gravity is that Lorentz invariance is not that sacred in a theory. (See \cite{3} for a lucid exposition of the perspective regarding Lorentz invariance violation.) It is apparently admissible even for a fundamental theory of Gravity, such as the Horava model, to be Lorentz non-invariant at ultra high energy or short distance, at $\sim$ trans-Planckian regime, so long as Lorentz invariance and Einstein General Relativity (GR) is recovered at observable scales of low energy (or long distance). This has brought about a paradigm shift in the way of thinking of High Energy Physicists but is quite conventional for Condensed Matter Physicists where maintenance of Lorentz invariance is not an issue at all. Indeed, it was quite well known that higher derivative covariant terms, (generically functions of $R$, the Ricci scalar), in Einstein gravity improves the UV behavior but brings in insurmountable ghost problems \cite{4}. To overcome this in Horava Gravity (HG) only higher order spatial derivative terms are kept. These explicitly Lorentz breaking terms do not introduce ghosts and at the same time can cure the UV divergence problems. However the jury is still out on the question of whether HG is the true theory of gravity since the loss of Lorentz and diffeomorphism symmetries induces a spurious degree of freedom beside the spin 2 graviton \cite{5}. Several ways to improve the Horava model have also been suggested \cite{6}.

On the other hand, the higher derivative nature of the Horava model opens up a completely new line of thought that is of interest to us: instability in the flat metric ground state. This can lead to a Spontaneous Symmetry Breaking (SSB) as regards to translation symmetry only. \textit{This phenomenon is a liquid-solid type of phase transition in the space(time) itself a la Landau} \cite{2}. A short distance length scale is generated in the process of transition from a homogeneous ($\sim$ liquid like) phase ground state to an inhomogeneous ($\sim$ solid like) condensate phase ground state in spacetime. Hence our work strengthens the claim of HG as a viable model for Quantum Gravity because the physics below this scale is affected while GR is recovered for distances sufficiently above this scale. The more ambitious project is to construct a true spacetime crystal where translation as well as rotation symmetries are
lost. The obvious motivation is to build a suitable spacetime for Quantum Gravity which is expected to have a short distance (\(~\) Planck) scale $^3$. Fortunately we know explicitly how to proceed in Condensed Matter Physics $^{11}$, where the physics is naturally non-relativistic. Since we are in the Horava framework of gravity where Lorentz invariance is explicitly broken, formalisms exploited in (non-relativistic) Condensed Matter Physics should be applicable in the present scenario as well. We mention an earlier work by us $^{12}$ where a length scale was introduced in the metric in a higher derivative Lorentz covariant Gravity theory via SSB. However, the new non-trivial ground state broke the Lorentz invariance explicitly and furthermore the parent model has the well known ghost problem. In this sense HG serves perfectly as the laboratory to test this new idea since HG is free of ghosts and is not a Lorentz invariant theory to begin with (recall that the higher derivatives are only in the spatial sector).

In a series of pioneering works Alexander and Mctague $^{11}$ were able to construct a solid crystalline lattice (where translation and rotation symmetries are lost), from a liquid phase (where the symmetries are intact), through SSB. This analysis explained the fact that $BCC$ lattice structure is favored for metals undergoing liquid-solid transition, essentially in a model independent way. In order to achieve this the condensate has to have additional structure, \textit{i.e.} its VEV can not be a constant that we generally encounter in Particle Physics. Rather, the crucial feature of the condensate is that the Fourier transform of non-zero VEV of condensate minimizing the free energy must have support at a \textit{non-vanishing} momentum. Evidently, as is explained below, this requirement translates, in the coordinate space, to the parent lagrangian undergoing SSB, as having \textit{higher (at least fourth) order derivative terms}

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$^3$ The attempts so far made are somewhat ad-hoc: (i) Non-Commutative (NC) geometry framework $^7$ that exploits Seiberg Witten map $^8$ and incorporates NC corrections on GR. (ii) A cutoff length scale in gravity was introduced $^9$ by mapping the slice of spacetime into the phase space of quantum mechanics of fermions. The length scale and noncommutativity parameter in gravity map to the Planck’s constant in quantum mechanics. (iii) A granular spacetime is considered in $^{10}$. 

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that are quadratic in the field. The field in question is identified with the difference between the (inhomogeneous) density for solid and (constant) density of liquid, that acts as the order parameter. The free energy is expanded around the higher symmetry liquid phase and it should be mentioned that to form a proper crystal lattice one needs terms, at least of third and fourth order terms in the order parameter, in the expansion. However, the latter does not concern us in the present work as we will restrict ourselves to higher derivative quadratic terms in the HG action. The ideas developed in [11] were applied in High Energy Physics in the context of string compactification by [13].

**SSB of Higher Derivative Scalar Theory:** As a warm up exercise we consider a toy model consisting of a scalar field theory with higher spatial derivative terms,

\[ S = \int d^4 x \left[ \frac{1}{2} \phi (-\partial_0^2 + \partial_i^2) \phi - \frac{\alpha^2}{4} \phi (\partial_i^2)^2 \phi - V(\phi) \right]. \]

(1)

In fact one can think of (1) as obtained from

\[ S = \int d^4 x \left[ \frac{1}{2} \phi (-\partial_0^2 + \partial_i^2) \phi - \frac{\alpha^2}{4} \phi (\partial_0^2 + \partial_i^2)^2 \phi - V(\phi) \right] \]

by simply leaving out the time derivatives in \( \alpha^2 \) term. This is in the same spirit as obtaining the Horava model with higher order spatial Ricci scalar terms from a covariant higher derivative gravity model containing higher orders of the full Ricci scalar. Furthermore, as we will see later, after some simplifying (but not unconventional) restriction on the metric fluctuation, the Horava model will be structurally identical to this toy model. In the present preliminary analysis we drop the potential \( V(\phi) \) as does not play any role since we are concerned only with ground state solution obtained by minimizing the kinetic term. Minimizing \( V(\phi) \) restricts the solution, obtained by minimization of the kinetic energy, still further. Explicit structure of \( V(\phi) \), such as with cubic and higher order terms, becomes crucial in building the lattice in momentum space of \( \phi \) (see [11] for more details).

We are working in flat Minkowski spacetime with the metric \( \eta_{\mu\nu} \equiv diag(-1, 1, 1, 1) \). The energy is given by

\[ H = \int d^3 x \left[ \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} (\partial_i \phi)^2 + \frac{\alpha^2}{4} \phi (\partial_i^2 \phi)^2 \right]. \]

(2)
As explained above we have dropped $V(\phi)$ from further consideration.

Let us now minimize the energy. It is natural to consider the ground state to be static. Using the Fourier decomposition

$$\phi(x) = \frac{1}{(2\pi)^3} \int \phi(k)e^{ikx}dk,$$

the ground state energy can be written in the momentum space as

$$H = -\frac{1}{2}k^2\phi(\vec{k})\phi(-\vec{k}) + \frac{\alpha^2}{4}(k^2)^2\phi(\vec{k})\phi(-\vec{k}).$$

Clearly $H[k^2 = 0] = 0$ but $H[k^2 = 1/\alpha^2] = -\frac{1}{4\alpha^2}$ showing that the minimum of kinetic energy \[3\] occurs at $k^2 = \frac{1}{\alpha^2}$. Here we used the notation $|\vec{k}|^2 = k^2$.

From Figure 1, one can clearly see that the higher derivative term shifts the position of the energy minimum from $k^2 = 0$ to $k^2 = 1/\alpha^2$. Subsequently translational invariance is broken in the higher derivative theory \[11, 13\].

![Figure 1: Plot of $A(k^2) = k^2$ vs $k$ and $A(k^2) = -k^2(1 - \frac{\alpha^2}{2}k^2)$ vs $k$ where the energies are respectively given by $H(k^2) = \phi(k)\phi(-k)A(k^2)$ and $H(k^2) = \phi(k)\phi(-k)A(k^2)$. The specific form of the function $A(k^2)$ is determined by the underlying microscopic theory \[13\]. Translational symmetry is present in the first case (dashed line) as the minima of $A(k^2)$ occurs at $k^2 = 0$. Translational symmetry is broken in the second case (thick line) as the minima of $A(k^2)$ occurs at $k^2 \neq 0$.](image-url)
The condensate value of $<\phi(x)>$, (that minimizes the energy), has a nontrivial coordinate dependence which can be determined as follows:

\[
<\phi(|x|)> = \frac{1}{(2\pi)^3} \int d^3k \delta(k^2 - \frac{1}{\alpha^2}) C(\alpha k) e^{ik.x}
\]

\[
= \frac{1}{(2\pi)^2} \int d\theta \sin\theta \int dk \frac{k^2}{2k} \delta(k - \frac{1}{\alpha}) C(\alpha k) e^{ik.x}
\]

\[
= \frac{C}{2\alpha(2\pi)^2} \int_0^\pi d\theta \sin\theta e^{(i|x|\cos\theta)/\alpha}
\]

\[
= \frac{C}{(2\pi)^2} \frac{\sin(|x|/\alpha)}{|x|},
\]

where $C$ turns out to be a dimensionless number. In the above expression (4), notice that although the condensate $\phi(|x|)$ has become explicitly $x$-dependent, indicating that the translation invariance is lost, the space rotational symmetry is preserved. This is because in the Fourier integral the restriction was to integrate on the spherical surface $k^2 = 1/\alpha^2$ that does not affect rotational symmetry. For large $\alpha$ energy $H$ in (3) is minimum for $k^2 = 0$ and the condensate $<\phi(|x|)>$ in (4) becomes $|x|$-independent constant.

It is customary to consider fluctuations above the condensate by shifting the field. In the present instance this exercise will mean $\phi(x) \rightarrow \phi(x) - \phi(|x|)$ leading to a translation non-invariant (but rotation invariant) theory. There is an underlying periodic nature of the translation symmetry broken phase. The coupling coefficient $\alpha$ in the higher derivative term in the action is responsible for the inhomogeneity and is present in the resulting lagrangian. We have not provided an explicit expression for that. Hence we have succeeded in introducing a periodic structure in the deformed translation invariance through SSB and the periodicity scale can be directly linked with the coupling coefficient of the higher derivative term. We now repeat the same exercise on the Horava model of Gravity in weak field approximation.

**SSB of Higher Derivative Horava Gravity:** As we have explained above, breaking of translation invariance via SSB requires higher derivative terms in the action and this motivates us to consider higher derivative gravity theories. We take the higher derivative terms as perturbations on the linearized Horava gravity action (which reproduces Einstein’s
GR theory in the low-energy (IR) limit). As discussed before, Lorentz symmetry is already explicitly broken in Horava theory.

Our long-term goal is to generate a spacetime lattice through SSB in the way described in the previous section. However, in the present article our target is more modest: as a first step towards this objective, we intend to introduce a short distance scale ($\sim$ Planck length) in the Horava action that will break the translation invariance, without affecting the rotational symmetry. Indeed, the extension of general relativity to NC spacetimes \cite{7} also aims to achieve that but, compared to this ad-hoc procedure, our approach is much more basic and intuitive. We introduce the deformation directly in the metric as a fluctuation and the condensate plays the role of a (tensorial) order parameter in the continuum-discrete spacetime phase transition. As we have extensively discussed before, this requires the introduction of higher derivative terms in the action, $R_{ij}R_{ij}$ and $(g^{ij}R_{ij})^2$. Apart from the conventional $(g^{ij}R_{ij})$-term, the above are needed to ensure that the kinetic term is minimized for a non-zero momentum.

We start with the action $S$

$$S = \int dt \mathcal{L} = \int d^3x \sqrt{g}N(G_{ijkl}K_{ij}K_{kl} + AR + BR_{ij}R_{ij} + CR^2)$$

$$= \int d^3x \sqrt{g}N(K_{ij}K^{ij} - \lambda K^2 + AR + BR_{ij}R_{ij} + CR^2).$$

(5)

Here $g_{ij}$ is the spatial metric, $A, B, C$ are three dimension-full parameters of the theory, $N$ is the Lapse function, $R$ is the spatial Ricci scalar and $K_{ij}$ is the extrinsic curvature defined as

$$K_{ij} = \frac{1}{2N} (\partial_0 g_{ij} - \nabla_i N_j - \nabla_j N_i).$$

(6)

with $N_i(x, t)$ is the Shift vector in ADM formalism \cite{14} and the generalized De Witt metric $G_{ijkl}$ is defined as

$$G_{ijkl} = \frac{1}{2} (g^{ik}g^{jl} + g^{il}g^{jk}) - \lambda g^{ij}g^{kl}.$$

(7)

For $\lambda = 1$, $A = 1$ and $B = C = 0$ Horava gravity reduces to Einstein’s gravity.
Throughout we will work in the weak field approximation with
\[ g_{ij} = \delta_{ij} + h_{ij}, \quad N = 1 + n, \quad N_i = n_i. \] (8)

Under these perturbations (8), the expressions for the extrinsic curvature and the Ricci scalar turn out to be
\[ K_{ij} = \frac{1}{2}(\partial_0 h_{ij} - \partial_i n_j - \partial_j n_i), \quad K = \delta^{ij} K_{ij} = \frac{1}{2}(\partial_0 h - 2\partial_i n^i), \]
\[ R_{ij} = \frac{1}{2}(\partial_0 h_{jk} + \partial^k \partial_j h_{ik} - \partial^2 h_{ij} - \partial_i \partial_j h), \quad R = \partial_i \partial_j h^{ij} - \partial^2 h. \] (9)

Using the above expressions (8) and the relation
\[ \sqrt{g} R = \frac{1}{2} h_{ij} \left( -R^{ij} + \frac{1}{2} \delta^{ij} R \right) \] (10)
in the action (8) we obtain the lagrangian density \( \mathcal{L} \) of second order in \( h \):
\[ \mathcal{L} = \frac{1}{4}[\partial_0 h_{ij} \partial_0 h^{ij} - \lambda(\partial_0 h)^2 - 4(\partial_0 n_i)(\partial_j h^{ij} - \lambda \partial^j h) + (4\lambda - 2)n_i(\partial^i \partial^j n_j) - 2n_i \partial^2 n^i] 
+ \frac{A}{4} h_{ij}(\partial^2 h^{ij} - 2\partial_k \partial^i h^{jk} + 2\partial^i \partial^j h - \delta^{ij} \partial^2 h) + A n(\partial_i \partial_j h^{ij} - \partial^2 h) + C(\partial_0 \partial_j h^{ij} - \partial^2 h)(\partial_k \partial_l h^{kl} - \partial^2 h) 
+ \frac{B}{4}(\partial^k \partial_l h_{jk} - \partial^2 h_{ij} + \partial^k \partial_j h_{ik} - \partial_i \partial_j h_0(\partial_0 \partial^i h^{il} - \partial^2 h^{ij} + \partial_0 \partial^j h^{il} - \partial^i \partial^j h). \] (11)

In the above expression (8) and throughout the rest of our paper, we used the following notation:
\[ h = \delta^{ij} h_{ij}, \quad \partial^2 = \partial_i \partial^i = \delta^{ij} \partial_i \partial_j. \]

From this lagrangian (8) we obtain the conjugate momenta as
\[ p \equiv \frac{\partial \mathcal{L}}{\partial (\partial_0 n)} = 0, \quad p^i \equiv \frac{\partial \mathcal{L}}{\partial (\partial_0 n_i)} = -(\partial_j h^{ij} - \lambda \partial^j h), \] (12)
\[ \pi^{ij} \equiv \frac{\partial \mathcal{L}}{\partial (\partial_0 h_{ij})} = \frac{1}{2}(\partial_0 h^{ij} - \delta^{ij} \lambda(\partial_0 h)). \] (13)

Taking the trace of the relation (13), we can write \( \partial_0 h_{ij} \) in terms of \( \pi_{ij} \) as
\[ \partial_0 h^{ij} = 2 \left( \pi^{ij} + \frac{\lambda}{1 - 3\lambda} \delta^{ij} \pi \right). \] (14)
Using the relations (12), (13), (14) in (11) we get the Hamiltonian density,

\[ \mathcal{H} = p^i (\partial_0 n_i) + \pi^{ij} (\partial_0 h_{ij}) - \mathcal{L} \]

\[ = \pi^{ij} - \frac{\lambda}{3\lambda - 1} \pi^2 - \frac{1}{2} (2\lambda - 1) n_i (\partial^i \partial^j n_j) + \frac{1}{2} n_i \partial^2 n^i \]

\[ - \frac{1}{4} h_{ij} (\partial^2 h^{ij} - 2 \partial_k \partial^i h^{jk} + 2 \partial^i \partial^j h - \delta^{ij} \partial^2 h) - A n (\partial_i \partial_j h^{ij} - \partial^2 h) - C (\partial_i \partial_j h^{ij} - \partial^2 h) (\partial_k \partial h^{kl} - \partial^2 h) \]

\[ - \frac{B}{4} (\partial^k \partial_i h_{jk} - \partial^2 h_{ij} + \partial^k \partial_j h_{ik} - \partial_i \partial_j h) (\partial_k \partial^i h^{jl} - \partial^2 h^{ij} + \partial_k \partial^j h^{il} - \partial^i \partial^j h). \]  

Using the momentum constraints and the gauge \( n_i = 0 \), the Hamiltonian (11) now reduces to

\[ \mathcal{H} = \pi_{ij} \pi^{ij} - \frac{1}{4} h_{ij} (\partial^2 h^{ij} - 2 \partial_k \partial^i h^{jk} + 2 \partial^i \partial^j h - \delta^{ij} \partial^2 h) \]

\[ - \frac{B}{4} (\partial^k \partial_i h_{jk} - \partial^2 h_{ij} + \partial^k \partial_j h_{ik} - \partial_i \partial_j h) (\partial_k \partial^i h^{jl} - \partial^2 h^{ij} + \partial_k \partial^j h^{il} - \partial^i \partial^j h). \]  

In the static limit \( \pi_{ij} = 0 \) and with \( h_{ij} = \delta_{ij} h \) for the sake of convenience, the Hamiltonian (11) can be written in the Fourier space as \( \mathcal{H}_{\text{static}}(p) \),

\[ \mathcal{H}_{\text{static}}(p) = -\frac{p^2}{6} h(p) h(-p) \left( 1 - \frac{17B}{5} p^2 \right). \]  

The minimum of this (17) occurs at

\[ p^2 = \frac{1}{\alpha^2} = \frac{5}{34B}. \]

In a similar way as in (11) we have the explicit form of \( f(r) \) as

\[ f(r) = \frac{\sin(r/\alpha)}{r}, \]  

where \( r = |x| \) stands for the radial coordinate.

Now, to consider the SSB effects, let us consider fluctuations \( \tilde{h}_{\mu\nu} \) above the condensate,

\[ h_{ij} = \delta_{ij} f(|\vec{x}|) + \tilde{h}_{ij}(x) \]

\[ h_{00} = \tilde{h}_{00} = -2\phi \quad , \quad h_{0i} = 0, \]  

where \( f(|\vec{x}|) \) is a purely spatial function. The \( f(|\vec{x}|) \) term, which is exactly of the same form as in (11), comes from the energy minimization arguments (previously described in the
it would break the rotational invariance, and \( \tilde{h}_{\mu\nu} \) is treated as the order parameter.  

In terms of the metric (\( g \)), the lagrangian (\( \mathcal{L} \)) becomes

\[
\mathcal{L} = \frac{1}{4}[\partial_0 \tilde{h}_{ij} \partial_0 \tilde{h}^{ij} - \lambda(\partial_0 \tilde{h})^2] + \frac{A}{4} \tilde{h}_{ij}(\partial^2 \tilde{h}^{ij} - 2\partial_k \partial^i \tilde{h}^{jk} + 2\delta^{ij} \partial^2 \tilde{h})
\]

\[+ An(\partial_i \partial_j \tilde{h}^{ij} - \partial^2 \tilde{h}) + C(\partial_i \partial_j \tilde{h}^{ij} - \partial^2 h)(\partial_k \partial_l \tilde{h}^{kl} - \partial^2 h)
\]

\[+ \frac{B}{4}(\partial^j \partial_j \tilde{h}^{jk} - \partial^2 \tilde{h}^{ij} + \partial^k \partial_j \tilde{h}^{ik} - \partial_i \partial_j \tilde{h}^{ij})(\partial^i \partial_j \tilde{h}^{ij} - \partial^2 \tilde{h}^{ij} + \partial_i \partial_j \tilde{h}^{ij} - \partial^2 \tilde{h}^{ij})
\]

\[+ \frac{A}{2} \tilde{h}^{ij}(\partial_i \partial_j \tilde{f} - \delta_{ij} \partial^2 f) - 4C\tilde{h}^{ij}(\partial_i \partial_j \partial^2 f - \delta_{ij}(\partial^2 f)^2) - \frac{3B}{2} \tilde{h}^{ij}(\partial_i \partial_j \partial^2 f - \delta_{ij}(\partial^2 f)^2).
\]  

(20)

Varying this lagrangian (\( \mathcal{L} \)) with respect to the fields \( n \) and \( \tilde{h}^{ij} \) we get the following equations of motion

\[\partial_i \partial_j \tilde{h}^{ij} - \partial^2 \tilde{h} - 2\partial^2 f = 0,
\]  

(21)

\[
\frac{1}{2}(\partial_0 \partial_0 \tilde{h}_{ij} - \lambda \partial_0 \partial_0 \tilde{h}) + \frac{A}{2}(\partial^2 \tilde{h}_{ij} - 2\partial_k \partial_0 \tilde{h}^{jk} + \partial_i \partial_j \tilde{h} + \delta_{ij} \partial^2 \tilde{h})
\]

\[+ A(\partial_i \partial_j n - \delta_{ij} \partial^2 n) + 2C(\partial_i \partial_j \partial_k \partial_l \tilde{h}^{kl} - \partial^2 \partial_i \partial_j \tilde{h} - \delta_{ij} \partial^2 \partial_k \partial_l \tilde{h}^{kl} + \delta_{ij}(\partial^2 \tilde{h}))
\]

\[+ \frac{B}{2}(2\partial_i \partial_j \partial_k \partial_l \tilde{h}^{kl} - \partial^2 \partial^2 \partial_i \partial_j \tilde{h}^{jk} + \partial^2 \partial^2 \partial_j \tilde{h}^{ik} + (\partial^2)^2 \tilde{h}^{ij} - \delta_{ij} \partial^2 \partial_k \partial_l \tilde{h}^{kl} + \delta_{ij}(\partial^2 \tilde{h}))
\]

\[+ \frac{A}{2}(\partial_i \partial_j \tilde{f} - \delta_{ij} \partial^2 f) - 4C(\partial_i \partial_j \partial^2 f - \delta_{ij}(\partial^2 f)^2) - \frac{3B}{2}(\partial_i \partial_j \partial^2 f - \delta_{ij}(\partial^2 f)^2) = 0.
\]  

(22)

Note that the \( f \)-terms in (\( \mathcal{L} \)) and in (21) are subsequently acting as the components of a conserved source.

Following standard literature [15], we decompose \( \tilde{h}_{\mu\nu} \) as:

\[
\tilde{h}_{00} = -2n = -2\phi \quad , \quad \tilde{h}_{0i} = 0 \quad , \quad \tilde{h}_{ij} = 2s_{ij} - 2\psi \delta_{ij},
\]  

(23)

\footnote{It is now clear that to make a full fledged spacetime “crystal” one requires third and fourth terms in \( h_{\mu\nu} \), (as explained in [11], [13] for liquid-solid transition), in the action which will considerably complicate the model. We are not considering this additional terms in the present work.}
where \( s_{ij} \) is traceless. Using the above decomposition (23) and considering the static limit (hence dropping all time derivatives), the equations of motion (21) turn out to be:

\[
\partial^2 \psi = \frac{1}{2} \partial^2 f, \tag{24}
\]

\[
A(\partial_i \partial_j - \delta_{ij} \partial^2)(\phi - \psi) + A \partial^2 s_{ij} + \frac{A}{2} (\partial_i \partial_j - \delta_{ij} \partial^2)f + 8C \partial^2 (\partial_i \partial_j - \delta_{ij} \partial^2)\psi
-4C \partial^2 (\partial_i \partial_j - \delta_{ij} \partial^2)f + B(\partial^2)^2 s_{ij} - 3B \delta_{ij} (\partial^2)^2 \psi - \frac{3B}{2} \partial^2 (\partial_i \partial_j - \delta_{ij} \partial^2)f = 0. \tag{25}
\]

The solution for \( \psi \) is obtained from (24) as

\[
\psi(r) = -\frac{GM}{r} + \frac{1}{2} \frac{\sin \left( \frac{r}{\alpha} \right)}{r}, \tag{26}
\]

where the first term in the right hand side of (26) is the usual term as obtained in linearized Einstein Gravity and the second term is the modification due to the condensate. We now take the trace of both sides of the equation (23) to obtain the following relation between \( \phi \) and \( \psi \):

\[
\partial^2[\phi - \psi + \frac{f}{2} + \frac{3B}{4A} \partial^2 f] = 0
\implies \phi = \psi - \frac{f}{2} - \frac{3B}{4A} \partial^2 f \tag{27}
\]

Using the explicit expression of \( f(r) \) as given in (18), the expression for \( \phi \) turns out to be

\[
\phi(r) = -\frac{GM}{r} + \frac{3B}{4A\alpha^2} \frac{\sin \left( \frac{r}{\alpha} \right)}{r}. \tag{28}
\]

The modified Newtonian potential derived above in (28) is shown in Figure 2. One can see that the fluctuations die out for sufficiently large distance \( r >> \alpha \) but can have non-trivial effects for \( r \sim \alpha \).

**Conclusion:** To conclude, we have shown that higher derivative terms in the Ho- rava model of gravity can lead to an instability and a spontaneous breaking of translation symmetry of the space takes place. Hence ground state gets modified from the conventional Minkowski vacuum metric to a space dependent form. This is an example of Landau type of
Figure 2: *Plot of the usual Newtonian potential (thin line in the plot) and the modified Newtonian potential as described in previous paragraph (dashed line in the plot) vs r with \( \alpha = 2 \).*

liquid-solid phase transition occurring in the space itself. A length scale also appears in the resulting metric in a natural way. The Newtonian potential is recovered for distances large compared to this scale. Consequences of this novel form of spatial metric is worth pursuing.

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