New sum rules for nucleon and trinucleon total photoproduction cross-sections*

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Abstract

Two new sum rules are derived relating Dirac radii and anomalous magnetic moments of the considered strongly interacting fermions with the convergent integral over a difference of the total proton and neutron, as well as $He^3$ and $H^3$, photoproduction cross-sections.

1 INTRODUCTION

A quarter of century ago Kurt Gottfried, by considering high-energy electron-proton scattering and the nonrelativistic quark model of hadrons, found Kurt67 a sum rule relating the proton charge mean squared radius $\langle r_{Ep}^2 \rangle$ and the proton magnetic moment $\kappa_p = 1 + \mu_p$ with the integral over the total proton photoproduction cross-section $\sigma_{tot}(\nu)$ in the following form

$$\int_0^{\infty} \frac{d\nu}{\nu} \sigma_{tot}(\nu) = \frac{\pi^2 \alpha^4}{m_p^2} \left[ \frac{4}{3} m_p^2 \langle r_{Ep}^2 \rangle + 1 - \kappa_p^2 \right],$$

(1)

where $\nu$ is the energy loss in the laboratory frame, $\alpha$ is the fine structure constant and $m_p$ is the proton mass. Nowadays we know, that the Gottfried sum rule cannot be

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fulfilled as the corresponding integral diverges due to the well known rise of the total proton photoproduction cross-section at high energies.

In this contribution by a distinct way from the Gottfried approach (utilizing analytic properties of the forward Compton scattering amplitude on the proton) a proton sum rule similar to (1) can be derived, however, suffering from the same integral divergence alike as in (1) and suffering also from the unknown left-hand cut contribution.

In order to avoid both of these shortcomings we are interested in nucleon isodoublet (proton and neutron) simultaneously and derive new sum rule, relating proton Dirac radius and anomalous magnetic moments of the proton and the neutron to the integral over a difference of the total proton and neutron photoproduction cross-sections. Then, in the precision of the isotopic violation, a mutual nullification of the left-hand cut proton and neutron contributions, as well as a convergence of the corresponding integral in the sum rule, is achieved.

Because $^3\text{He}$ and $^3\text{H}$ belong to the same isodoublet, possess the spin $s=1/2$ and as a result their electromagnetic (EM) structure is also completely described by Dirac and Pauli form factors like proton and neutron, one can follow the same procedure and derive a similar sum rule, relating $^3\text{He}$ and $^3\text{H}$ Dirac radii and anomalous magnetic moments of $^3\text{He}$ and $^3\text{H}$ with the convergent integral over a difference of the total $^3\text{He}$ and $^3\text{H}$ photoproduction cross-sections.

2 SUM RULE FOR DIFFERENCE OF PROTON AND NEUTRON TOTAL PHOTOPRODUCTION CROSS-SECTIONS

As is well known, the proton and neutron are constituents of atomic nuclei with the spin $s=1/2$, the EM structure of which is completely described by two independent, e.g. Dirac $F_1(q^2)$ and Pauli $F_2(q^2)$, form factors, defined by the relation

$$\langle N| J_\mu^{EM} |N \rangle = \bar{u}(p') [\gamma_\mu F_1(q^2) + i \sigma_{\mu\nu} q^\nu 2m_N F_2(q^2)] u(p),$$

(2)

where $J_\mu^{EM} = 2/3 \bar{u} \gamma_\mu u - 1/3 \bar{d} \gamma_\mu d - 1/3 \bar{s} \gamma_\mu s$, $q^\nu = (p' - p)^\nu$ and $\bar{u}(p'), u(p)$ are the free nucleon Dirac bi-spinors. Then the asymptotic form of the very high energy elastic
electron-nucleon differential cross-section in the one-photon approximation is

\[ \frac{d\sigma^{eN \to eN}}{dq^2} = 4\pi \alpha^2 \left(\frac{q^2}{q^2} \right)^2 \left[ F_1^2(q^2) + \frac{q^2}{4m_N^2} F_2^2(q^2) \right]. \]  

(3)

Further, let us consider a very high energy peripheral electroproduction process on nucleons

\[ e^{-}(p_1) + N(p) \to e^{-}(p'_1) + X, \]  

(4)

where the produced hadronic state \( X \) is moving closely to the direction of the initial nucleon. Its matrix element in the one photon approximation takes the form

\[ M = i\sqrt{4\pi \alpha} \bar{u}(p'_1) \gamma^\mu u(p_1) < X | J^\nu | p > g_{\mu\nu} \]  

(5)

and \( m_X^2 = (p + q)^2 \).

Now, by means of the method of equivalent photons Achie69, examining the nucleon in the rest, the electron energy to be very high and the small photon momentum transfer, one can express the differential cross-section of the process (4) through integral over the total nucleon photoproduction cross-section.

Really, applying to (5) the Sudakov expansion Sud56 of the photon transferred 4-vector \( q \)

\[ q = \beta_q \tilde{p}_1 + \alpha_q \tilde{p} + q^\perp \]  

(6)

into the almost light-like vectors

\[ \tilde{p}_1 = p_1 - p^2 p / (2p_1 p), \quad \tilde{p} = p - p^2 p_1 / (2p_1 p), \]  

(7)

the Gribov representation Grib70 of the metric tensor

\[ g_{\mu\nu} = g^\perp_{\mu\nu} + \frac{2}{s} (\tilde{p}_\mu \tilde{p}_1\nu + \tilde{p}_\nu \tilde{p}_1\mu) \approx \frac{2}{s} \tilde{p}_\mu \tilde{p}_1\nu, \]  

(8)

where \( s = (p_1 + p)^2 \approx 2p_1 p \gg Q^2 = -q^2 \) and transforming the phase space volume of the final state suitably, one obtains

\[ \frac{d\sigma^{eN \to e-X}}{dq^2} = \frac{\alpha q^2}{4\pi^2} \int_{s_1}^{\infty} \int_{s_1}^{\infty} \frac{d^2 s_1}{s_1^2} \left[ q^2 + (m_e s_1 / s)^2 \right]^2 Im\tilde{A}(s_1, q) \]  

(9)

with \(-t = Q^2 = q^2, s_1 = 2qp = m_X^2 + Q^2 - m_N^2 \) and \( Im\tilde{A}(s_1, q) \) to be the imaginary part of the forward Compton scattering amplitude on the nucleon \( \tilde{A}(s_1, q) \) with only the intermediate state \( X \).
Finally, for the case of small photon momentum transfer squared in (9) and by an application of the optical theorem

\[ \text{Im} \tilde{A}(s_1, q) |_{q^2 \to 0} = \text{Im} A(s_1, q) |_{q^2 \to 0} = 4s_1 \sigma_{\gamma N \to X}^{\text{tot}}(s_1) \]  

(10)

one obtains the relation

\[ q^2 \frac{d\sigma_{e^-N\to e^-X}}{dq^2} |_{q^2 \to 0} = \frac{\alpha}{\pi} \int_{s_1^h}^{\infty} \frac{ds_1}{s_1} \sigma_{\gamma N \to X}^{\text{tot}}(s_1) \]  

(11)

between the differential cross-section of the process (4) and the total nucleon photoproduction cross-section.

Next step is an investigation of analytic properties of the almost forward Compton scattering amplitude \( \tilde{A}(s_1, q) \) in \( s_1 \)-plane. They consist in one-nucleon intermediate state pole at \( s_1 = -Q^2 \), the right-hand cut starting at the pion-nucleon threshold \( s_1 = -Q^2 + 2m_N m_\pi + m_\pi^2 \) and the \( u_1 \)-channel left-hand cut starting from \( s_1 = Q^2 - 8m_N^2 \).

Defining the path integral

\[ I = \int_{C} \frac{ds_1}{(q^2)^2} \frac{p_1^\mu p_1^\nu \tilde{A}_{\mu\nu}}{s_1^2} \]  

(12)

from the gauge invariant light-cone projection \( p_1^\mu p_1^\nu \tilde{A}_{\mu\nu} \) of the part \( \tilde{A}_{\mu\nu} \) of the total

![Figure 1: Sum rule interpretation in s_1 plane.](image)

Compton scattering tensor with photon first absorbed and then emitted along the fermion line, in \( s_1 \)-plane, as presented in Fig.1 and once closing the contour \( C \) to upper half-plane, another one to lower half-plane, the following sum rule

\[ \pi \text{Res} = \int_{\text{r.h.}}^{\infty} \frac{ds_1}{s_1^2(q^2)^2} \text{Im} \tilde{A}(s_1, q) - \int_{\text{l.h.}}^{-\infty} \frac{ds_1}{s_1^2(q^2)^2} \text{Im} \tilde{A}(s_1, q) \]  

(13)

appears with \( \text{Res} \) to be the one-nucleon intermediate state pole contribution. Then, taking into account (3) and (9) and considering proton and neutron separately, one
comes from (13) (for more detail see Appendix D in Kura81) to the sum rules for the proton

\[ 1 - F_{1p}^2(-Q^2) - \frac{Q^2}{4m_p^2} F_{2p}^2(-Q^2) = \frac{(Q^2)^2}{\pi \alpha^2} \frac{d\sigma e^{-p\to e^-X}}{dQ^2} + LCC_p \]  

(14)

and for the neutron

\[ -F_{1n}^2(-Q^2) - \frac{Q^2}{4m_n^2} F_{2n}^2(-Q^2) = \frac{(Q^2)^2}{\pi \alpha^2} \frac{d\sigma e^{-n\to e^-X}}{dQ^2} + LCC_n, \]  

(15)

where the abbreviations \(LCC_p\) and \(LCC_n\) denote the left-hand cut contributions in (13) for proton and neutron, respectively.

A similar proton sum rule to (1) can be now found taking a derivative of both sides in (14) according to \(Q^2\) at \(Q^2 \to 0\) and substituting the relation (11) for proton.

However, in order to avoid the abovementioned problems we subtract (15) from (14) and as a result (due to a small isotopic invariance violation in the EM interactions) one achieves a mutual annihilation of the left-hand cut proton and neutron contributions as follows

\[ 1 - F_{1p}^2(-Q^2) + F_{1n}^2(-Q^2) - \frac{Q^2}{4m_p^2} F_{2p}^2(-Q^2) + \frac{Q^2}{4m_n^2} F_{2n}^2(-Q^2) = \]

\[ = \frac{(Q^2)^2}{\pi \alpha^2} \left( \frac{d\sigma e^{-p\to e^-X}}{dQ^2} - \frac{d\sigma e^{-n\to e^-X}}{dQ^2} \right). \]  

(16)

Moreover, taking a derivative according to \(Q^2\) of both sides in (16) and employing the relation (9), one comes for \(Q^2 \to 0\) to the new sum rule relating Dirac mean squared radius and anomalous magnetic moments of the proton and the neutron to the integral over a difference of the total proton and neutron photoproduction cross-sections

\[ \frac{1}{3} \langle r_{1p}^2 \rangle - \mu_p^2/4m_p^2 + \mu_n^2/4m_n^2 = \frac{1}{\pi^2 \alpha} \int_{\omega_N}^{\infty} d\omega \left[ \sigma^{\gamma p\to X}(\omega) - \sigma^{\gamma n\to X}(\omega) \right] \]  

(17)

with \(\omega_N = m_e^2 + m_N^2/2m_N\), in which just a mutual cancellation of the rise of the latter cross sections for \(\omega \to \infty\) is achieved.

### 3 SUM RULE FOR He³ and H³ TOTAL PHOTO-PRODUCTION CROSS-SECTIONS

Isodoublet of nuclei \(He^3\) and \(H^3\) has, like nucleons, spin \(s=1/2\) and the elastic and inelastic scatterings of electrons on these nuclei are described by similar formulas like
nucleons. Only normalizations of the corresponding Dirac and Pauli form factors are slightly different

\[ F_{1He^3}(0) = 2; \quad F_{2He^3}(0) = \mu_{He^3}; \]
\[ F_{1H^3}(0) = 1; \quad F_{2H^3}(0) = \mu_{H^3}, \]

where \( \mu_{He^3} \) and \( \mu_{H^3} \) are anomalous magnetic moments of \( He^3 \) and \( H^3 \), respectively. So, repeating the same procedure with the latter nuclei one obtains a new sum rule of the following form

\[
\frac{1}{3} [4\langle r_{1He^3}^2 \rangle - \langle r_{1H^3}^2 \rangle] - \frac{\mu_{He^3}^2}{4m_{He^3}^2} + \frac{\mu_{H^3}^2}{4m_{H^3}^2} = \]

\[
= \frac{1}{\pi^2\alpha} \int_{\omega_N}^{\infty} d\omega \left[ \sigma_{\gamma He^3 \rightarrow X}^\text{tot}(\omega) - \sigma_{\gamma H^3 \rightarrow X}^\text{tot}(\omega) \right]
\]

relating Dirac radii and anomalous magnetic moments of \( He^3 \) and \( H^3 \) with the convergent integral over a difference of the total \( He^3 \) and \( H^3 \) photoproduction cross-sections.

### 4 CONCLUSIONS

Considering the very high energy elastic and inelastic electron-nucleon and electron trimucleon scattering with a production of a hadronic state \( X \) moving closely to the direction of initial hadrons. Then utilizing analytic properties of the forward Compton scattering amplitudes on considered strongly interacting particles, for the case of small transferred momenta new sum rules were derived, relating Dirac radii and anomalous magnetic moments of these fermions with the convergent integral over a difference of the total proton and neutron, as well as \( He^3 \) and \( H^3 \), photoproduction cross-sections.

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