Achievable Spectral Efficiency of Hybrid Beamforming Massive MIMO Systems With Quantized Phase Shifters, Channel Non-Reciprocity and Estimation Errors

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ABSTRACT

This paper considers a hybrid beamforming massive multiple-input multiple-output (MIMO) system operating in time-division duplex (TDD) mode. We investigate the combined impacts of quantized phase shifters (QPSs), channel non-reciprocity and channel estimation errors on the achievable spectral efficiency (SE) of the considered system. We first introduce an equivalent hybrid beamforming structure, based on which, a generic and practical channel non-reciprocity model is built for massive MIMO systems with hybrid structure. This model jointly considers the mismatch between transceiver and receiver circuits, as well as the mutual coupling between antennas. Then we derive the generic expression of achievable SE for both the ideal phase shifters (IPSs) and QPSs cases. Furthermore, the closed-form expressions of SE are derived for IPSs and QPSs cases, respectively. The asymptotic performance in large antenna regime is also analyzed based on these expressions. Simulation results demonstrate the validity and accuracy of the derived analytical expressions. In addition, results show that the phase quantization and channel non-reciprocity both contribute to the SE degradation and the detrimental effects cannot be canceled completely even in the large-antenna regime.

INDEX TERMS

Massive MIMO, hybrid beamforming, channel non-reciprocity, quantized phase shifters, channel estimation errors.

I. INTRODUCTION

Massive multiple-input-multiple-output (MIMO) is a key enabler for the 5th generation cellular networks (5G) [1]. In massive MIMO systems, the base station (BS) is equipped with a large number of antennas to increase the spectral efficiency (SE) by beamforming and spatial multiplexing [2], [3]. However, the high power consumption and cost of radio frequency (RF) chains (analog-to-digital converters (ADC)/digital-to-analog converters (DAC), mixers etc.) make fully digital beamforming (requiring one RF chain per antenna) impractical [4]. Hence, hybrid beamforming is a more applicable architecture for massive MIMO systems [5].

In hybrid beamforming structure, digital beamforming is deployed at the baseband to provide multiplexing gains, whereas analog beamforming is implemented using low cost phase shifters to offer directivity gains [6]. Alternatively optimizing analog and digital beamformer is widely used to design hybrid beamforming [7]–[10]. Assuming perfect channel state information (CSI) at transmitters, the authors in [7] propose a SE maximal hybrid beamforming method using heuristic algorithms. Two-timescale hybrid beamforming is proposed in [8], where the analog beamformer is designed using only CSI statistics. Robust hybrid beamforming under Gaussian CSI errors is developed in [9] using semidefinite relaxation, which minimizes the outage probability. Aiming at minimizing sum mean square error (MSE), manifold optimization and general eigenvalue decomposition are used in [10] to obtain the analog and digital beamformer respectively.
Different from the above alternatively optimization schemes, the authors in [11] reconsider hybrid beamforming from the point of information theory and jointly design digital and analog beamformer to avoid information loss. Literature shows that, hybrid beamforming can achieve a SE performance comparable to fully digital beamforming with 8 to 16 times fewer RF chains, under the assumption of full CSI and ideal phase shifters (IPSs) at the BS [12].

Massive MIMO systems are typically assumed to operate in time-division duplex (TDD) mode, and thus the downlink (DL) CSI can be acquired by estimating uplink (UL) pilots, exploiting the reciprocity between the UL and DL channels within channel coherence time [15]. In reality however, the reciprocity only holds for the physical propagation channel, and is not applicable to the composite communication channel, which not only consists of the physical propagation channel, but also the circuits such as antennas, mixers, converters, etc., at both sides of the link. The mismatch between the transceiver and receiver circuits, as well as the mutual coupling between antennas lead to different gains on the transmitted and received signals [16]. Such impacts induce channel non-reciprocity, which further results in the SE degradation.

On the other hand, the implementation of continuous IPSs is quite challenging [12]. Hence the quantized phase shifters (QPSs) with discrete resolution is usually adopted in practical massive MIMO systems. However, as shown in [13], QPSs degrade the resolution of analog beamforming. Therefore, hardware-efficient and robust hybrid beamforming is required in practical massive MIMO systems to reduce the complexity and cost [14].

Channel reciprocity calibration is investigated in [16]–[20]. An internal calibration scheme is proposed in [17], where the array antennas exchange pilot and use expectation-maximization algorithm to compute the calibration coefficients. The authors in [16] develop a new mutual coupling calibration method employing the effect of mutual coupling between adjacent antennas. In [19], an inverse calibration method is proposed, which consider not only the channel non-reciprocity, but also the channel estimation errors. The authors in [18] proposed an over-the-air calibration framework which unifies existing calibration schemes. Different from [16]–[19], an relative calibration method is proposed for the TDD hybrid beamforming massive MIMO systems in [20]. However, these works don’t provide an analytical framework to evaluate the impact of channel non-reciprocity.

The performance analysis of massive MIMO systems with channel non-reciprocity and estimation errors are presented in [21]–[23]. The authors in [21], [22] derive the closed-form expressions of signal-to-interference-plus-noise ratio (SINR) for maximum ratio transmission (MRT) and zero forcing (ZF) beamformers in centralized massive MIMO systems. Only RF mismatch is considered in [21], whereas both RF mismatch and mutual coupling are taken into account in [22]. On the contrary, the authors in [23] derive the lower-bounds of the achievable rates for MRT and ZF in the distributed massive MIMO systems. However, these works all focus on the digital beamforming. The performance analysis of hybrid beamforming under channel non-reciprocity and estimation errors is still missing.

The performance analysis of analog/hybrid beamforming with QPSs is presented in [24]–[30]. In [24], the performance of phase only eigen beamforming and hybrid ZF with QPSs are presented assuming one RF chain per UE. In [25], the closed-form secrecy rate expression for analog beamforming is derived as a function of QPS resolution. Analytical expressions of SE and energy efficiency (EE) for hybrid beamforming with imperfect CSI and QPSs in massive MIMO relay networks are derived in [26]. In [27], the asymptotic expressions of SE are derived for MRT and ZF cascaded with QPSs. The achievable secrecy rate for the relay-assisted hybrid beamforming massive MIMO DL with QPSs and imperfect CSI is derived in [28]. The authors in [29] consider two implementation structure of hybrid beamforming, named as THIC, and compare their performance under different phase quantization levels. In [30], the authors derive the closed-form expressions of SE and EE for hybrid beamforming with QPSs and IPSs, respectively. However, regardless of the variety of performance analyses, none of them considers channel non-reciprocity in the analysis framework.

Both QPSs, channel non-reciprocity and estimation errors can degrade the performance of hybrid beamforming. However, most of the existing works only investigate part of these influencing factors. An in-depth analysis of the compound impacts of QPSs, channel non-reciprocity, and estimation errors on the performance of hybrid beamforming is still missing. Moreover, existing analysis frameworks cannot be directly extended to analyze the compound impacts of the three factors, due to the complex hardware non-reciprocity and hybrid beamforming structure. Motivated by this, in this paper, we analyze the achievable SE of TDD hybrid beamforming massive MIMO systems under the compound impacts of QPSs, channel non-reciprocity, and estimation errors. The main contributions of this paper can be summarized as follows:

- We introduce an equivalent hybrid beamforming structure to facilitate the analysis. With this structure, the transfer functions of transceiver/receiver circuits can be modeled using a single matrix.
- We consider a generic and practical channel non-reciprocity model based on the introduced equivalent hybrid beamforming structure, which takes both RF mismatches and mutual coupling into account.
- We derive the generic expressions of the achievable SINR and SE for both IPSs and QPSs based massive MIMO systems, in the presence of quantization level, channel non-reciprocity and estimation errors.
- We derive the closed-form expressions of the achievable SINR and SE for IPSs and QPSs based massive MIMO systems, respectively. The asymptotic performance for
A. EQUIVALENT HYBRID BEAMFORMING STRUCTURE

Let $W \in \mathbb{C}^{N \times K}$ and $F \in \mathbb{C}^{M \times N}$ denote the digital and analog beamformers, respectively. If we denote the physical propagation channel as $C \in \mathbb{C}^{M \times K}$, the received signal at the baseband of UEs $y_{DL} \in \mathbb{C}^{K \times 1}$ is thus given by

$$y_{DL} = R^H C^T T^H_2 F^T_1 W s + n,$$

(1)

where $s \in \mathbb{C}^{K \times 1}$ is the stack of signal, $n \sim \mathcal{CN}(0, \sigma^2)$ is the $K \times 1$ complex Additive White Gaussian Noise (AWGN) vector, in which the elements are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and variance $\sigma^2$. Correspondingly, the received signal at the baseband of BS in the UL transmission can be written as

$$y_{UL} = W^H R^H_2 F^H R^H_1 C T s + n.$$

(2)

large antenna regime is also analyzed based on these expressions.

- We quantify and compare the SE degradation in the presence of quantization level, channel non-reciprocity and estimation errors by simulations, and provide insights for the implementation of hybrid beamforming massive MIMO systems.

The rest of this paper is organized as following. Section II describes the system model. In Section III, we present the derivation of the generic SINR and SE expressions. The closed-form SE expressions, and also the asymptotic performance are derived in Section IV. Section V discusses the large antenna regime is also analyzed based on these expressions.

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Notations: The matrices and vectors are denoted as the bold face uppercase and lowercase letters, respectively. ($\cdot$) superscript, Hermitian transpose, conjugate, expectation, variance, covariance, and Kronecker product operator, respectively. $\mathbb{C}^{m \times n}$ stands for the set of all complex $m \times n$ matrices, and $\text{Tr}(X)$ denotes the trace of matrix $X$. diag($x_k$) denotes the diagonal matrix with its $k$-th diagonal element be $x_k$. $I_m$ represents the identity matrix with dimension $M \times M$. $\mathcal{CN}((\mu, \sigma^2))$ denotes the complex normal distribution with a mean of $\mu$ and a variance of $\sigma^2$. $U[a, b]$ denotes an uniformly distributed random variable between $a$ and $b$. Rayleigh($\sigma$) denotes Rayleigh distribution with scale parameter $\sigma$.

II. SYSTEM MODEL

We consider the DL transmission of a TDD based multi-user massive MIMO system operating at sub-6 GHz, in which a BS with $M$ antennas serves $K$ single antenna UEs simultaneously on the same time-frequency resource. As shown in Figure 1, hybrid beamforming with full-connected structure is adopted at the BS to reduce the implementation complexity and power consumption of digital signal processing [5]. Each of the $N$ RF chains is connected with all power amplifiers and antennas through an analog beamforming network composed of phase shifters. The transmitted signal goes through $M$ RF chains (DACs, mixer, etc.), analog beamforming network, and is summed up before being connected with each power amplifier and antenna. Note that at least one RF chain per UE is required to enable multi-stream transmission, we assume $K \leq N < M$.

The BS and UEs have independent transmit and receive hardware and here we denote the transfer functions of different hardware as $T^H_1 \in \mathbb{C}^{N \times N}$, $T^b_2 \in \mathbb{C}^{M \times M}$, $R^b_1 \in \mathbb{C}^{N \times N}$, $R^b_2 \in \mathbb{C}^{M \times M}$, $T^{\mu} = \text{diag}(t^{\mu}) \in \mathbb{C}^{K \times K}$ and $R^{\mu} = \text{diag}(r^{\mu}) \in \mathbb{C}^{K \times K}$. $T^b_2$ and $R^b_2$ are both diagonal matrix, and characterize the frequency response of RF chains in the transmitter and receiver at the BS, respectively. $T^H_1$ and $R^b_1$ describe the property of RF paths (including RF chains, phase shifters and power amplifiers, etc.) of the transmitter and receiver at $k$-th UE, respectively. Note that antenna coupling at the UE side is not considered, since each UE only has one antenna and they are usually far from each other.

![Illustration of a multi-user massive MIMO system with the full connected hybrid beamforming structure.](image)
Since the analog beamformer F maps the signals from N RF chains to M antennas, it has block diagonal structure. Therefore the matrix multiplications in (1) and (2) are commutative if Kronecker product is introduced, i.e., \( F T_1^H = (T_1^H \otimes I_{M/N})F \). \( R_b^T F H = F H (R_b^H \otimes I_{M/N}) \). In this regard, the received signals in DL and UL transmission turn to

\[
y_{DL} = R_u^T C^T T_2^H (T_1^H \otimes I_{M/N}) F W_s + n,
\]

(3)

\[
y_{UL} = W H F H (R_b^H \otimes I_{M/N}) R_b^T C w s + n.
\]

(4)

An illustration of equivalent hybrid beamforming structure is presented in Figure 2 to facilitate understanding.

**B. CHANNEL NON-RECIPROCITY**

In TDD systems, the physical propagation channels of DL and UL are assumed to be reciprocal within each channel coherence interval, since the same spectrum is used. However, channel is usually estimated in the digital domain at baseband, which is a composite channel including physical propagation channel C and transfer functions of hardware T^H, R_b^H, T, as shown in Figure 1 and 2. Hence the reciprocal nature is no longer applicable to the estimated channel.

According to (3) and (4), the composite DL channel \( H_{DL} \in \mathbb{C}^{K \times M} \) and the composite UL channel \( H_{UL} \in \mathbb{C}^{M \times K} \) can be explicitly written as

\[
H_{DL} = R_u^T C^T T_b^H,
\]

(5)

\[
H_{UL} = R_b^T C w u.
\]

(6)

From (5) and (6), we can see that the composite channel is obviously not reciprocal and they has the following relationship

\[
H_{DL} = AH_{UL} B,
\]

(7)

where \( A = R_b^T (T_u)^{-T} \in \mathbb{C}^{K \times K} \), \( B = (R_b^H)^{-T} T_b^H \in \mathbb{C}^{M \times M} \).

\( A, B \) capture the non-reciprocity effects of hardware in UEs and the BS, respectively. Considering that \( T_1^H, R_1^T, T_b, R_b^H \) are full rank matrices due to the mutual coupling, we rewrite the diagonal matrix \( A \) as \( A = I_K + A' \), and the full matrix \( B \) as \( B = I_M + B' \). \( A' \) can be represented as \( A' = \text{diag}(a_1', a_2', \ldots, a_K') \), where elements are assumed to be i.i.d. variables with zero mean and variance \( \sigma_{a_i'}^2 = \mathbb{E}[|a_i'|^2] \). Similarly, elements in \( B' \) are assumed to be i.i.d. variables with zero mean and variance \( \sigma_{b_i'}^2 = \mathbb{E}[|b_i'|^2] \). We can easily observe that the effective DL and UL channels are reciprocal if and only if the two matrices satisfy \( A' = 0 \) and \( B' = 0 \). Moreover, it is worth noting that \( A \) and \( B \) reflects the channel non-reciprocity levels and vary slowly in time, hence they are stable over many channel coherence times [20].

**C. CHANNEL ESTIMATION**

The channel estimates in the considered TDD massive MIMO system is obtained from UL training. In the full-connected hybrid beamforming structure, each RF chain is connected to all antennas and thus receives an inseparable sum of pilot signals from all antennas. That is to say, the baseband only observes an effective channel which is a product of analog beamformer and the channel matrix. Hence similar to [26], [27], we employ the round-robin scheme for channel estimation. At each time, we chose \( N \) BS antennas out from \( M \) and trained them using \( N \) RF chains. Repeating this process \( M/N \) times, we finally obtain the channel estimates for all the antennas.

We denote \( T_u \) as the number of symbols in the UL pilot sequences in each coherence interval. All UEs simultaneously transmit mutually orthogonal UL pilot sequences. We denote \( K \) is the number of symbols in the UL pilot sequences. The corresponding effective DL channel is simply taken as

\[
y_p = \sqrt{\tau_u p_u} H_{UL} x_p + n_p,
\]

(8)

where \( p_u \) is the transmit power of UL pilots, \( n_p \) is the AWGN matrix at the BS with i.i.d. \( \mathcal{CN}(0,1) \) elements. By right multiplying (8) with \( (H_p^T)^H \), the BS obtains

\[
y = Y_p (X_p)^H = \sqrt{\tau_u p_u} H_{UL} + \tilde{N}_p.
\]

(9)

The minimum mean-square error (MMSE) estimate of \( H_{UL} \) is then given by [32]

\[
\hat{H}_{UL} = \frac{\tau_u p_u}{\tau_u p_u + 1} H_{UL} + \frac{\tau_u p_u}{\tau_u p_u + 1} \tilde{N}_p.
\]

(10)

The corresponding effective DL channel is simply taken as \( H_{DL} = \hat{H}_{UL} \), without reciprocal calibration. Based on the property of the MMSE channel estimation [32], we can decompose the effective UL channel as

\[
H_{UL} = \hat{H}_{UL} + \Xi^T = \hat{H}_{DL} + \Xi^T,
\]

(11)

where \( \Xi = [\xi_1, \ldots, \xi_K]^T \in \mathbb{C}^{K \times M} \) accounts for the UL channel estimation errors, and is uncorrelated with \( \hat{H}_{UL} \). Moreover, \( \Xi \) has i.i.d. \( \mathcal{CN}(0, \frac{1}{\tau_u p_u + 1}) \) elements, and \( \hat{H}_{DL} \) has i.i.d. \( \mathcal{CN}(0, \frac{1}{\tau_u p_u + 1}) \) elements.
Plugging (11) into (7), the true composite DL channel can be expressed as

\[
\mathbf{H}_{DL} = A\hat{H}_{UL}^T \mathbf{B} = A \left( \hat{\mathbf{H}}_{DL} + \Xi \right) \mathbf{B},
\]  

which incorporates both the channel estimation errors and the channel non-reciprocity.

**D. HYBRID BEAMFORMER DESIGN**

The estimated DL channel \( \hat{\mathbf{H}}_{DL} \) is the only information available at the BS to design the hybrid beamformer. Since the following analysis is focused on the DL, we ignore subscripts and use \( \mathbf{H}, \hat{\mathbf{H}} \) to denote \( \mathbf{H}_{DL}, \hat{\mathbf{H}}_{DL} \) respectively for simplicity.

The digital beamformer \( \mathbf{W} \) controls both the amplitudes and phases of incoming complex symbols, whereas the analog beamformer \( \mathbf{F} \) only modifies the phase of the up-converted RF signal.

1) **ANALOG BEAMFORMER DESIGN:**

We design analog beamformer \( \mathbf{F} \) as the analog phase shifters in the RF domain. Moreover, we normalize \( \mathbf{F} \) to \( 1/\sqrt{M} \) to prevent the unlimited gains in the analog domain. Thus the analog beamformer can be expressed as

\[
\mathbf{F} = \frac{1}{\sqrt{M}} \begin{bmatrix}
\phi_{11} & \phi_{12} & \cdots & \phi_{1N} \\
\phi_{21} & \phi_{22} & \cdots & \phi_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
\phi_{M1} & \phi_{M2} & \cdots & \phi_{MN}
\end{bmatrix}.
\]  

Here we investigate two analog beamformer designs, i.e., \( \mathbf{F}_{IPS} \) and \( \mathbf{F}_{QPS} \), corresponding to IPSs and QPSs systems, respectively.

For IPSs systems, the phases of \( \mathbf{F}_{IPS} \) can be continuous, they are obtained by directly extracting phases of the conjugate transpose of the estimated DL channel \( \hat{\mathbf{H}} \). If we denote the phase of \( [\mathbf{F}_{IPS}]_{m,n} \) as \( \phi_{mn}^{IPS} \), and denote the phase of \( [\hat{\mathbf{H}}]^H_{m,n} \) as \( \phi_{mn} \), then we have

\[
\phi_{mn}^{IPS} = \begin{cases} 
\phi_{mn} & n \leq K \\
\phi_{\text{rand}} & K < n \leq N,
\end{cases}
\]  

where \( \phi_{\text{rand}} \sim U(-\pi, \pi) \) and \( \phi_{mn} \sim U(-\lambda, \lambda) \). So the \((m,n)\)-th element of the analog beamformer \( \mathbf{F}_{IPS} \) can be written as

\[
[\mathbf{F}_{IPS}]_{m,n} = \frac{1}{\sqrt{M}} e^{j\phi_{mn}^{IPS}}.
\]  

The implementation of continuous phase shifters is quite challenging, and hence QPSs are usually used in the practical massive MIMO systems. In this case, the phase of each element in \( \hat{\mathbf{H}}^H \) is sampled by \( \delta \) levels, and then quantized to its nearest neighbor based on the closest Euclidean distance. The phase of \( (m,n) \)-th element in \( \mathbf{F}_{QPS} \) can be calculated as

\[
\phi_{mn}^{QPS} = \begin{cases} 
2\pi q_{mn}^{*}/2^k & n \leq K \\
\phi_{\text{rand}} & K < n \leq N,
\end{cases}
\]  

where \( q_{mn}^{*} \) is determined by

\[
q_{mn}^{*} = \arg\min_{q \in \{0,1,\ldots,2^k-1\}} \left| \phi_{mn} - \frac{2\pi q}{2^k} \right|.
\]  

The quantization error can be computed as

\[
\varphi_{mn} = \phi_{mn} - \phi_{mn}^{QPS},
\]  

where \( \varphi_{mn} \sim U(-\lambda, \lambda) \) with \( \lambda = \pi/2^k \). So the \((m,n)\)-th element of the analog beamformer \( \mathbf{F}_{QPS} \) can be written as

\[
[\mathbf{F}_{QPS}]_{m,n} = \frac{1}{\sqrt{M}} e^{j\phi_{mn}^{QPS}}.
\]  

It is worth noting that we use phases of estimated effective DL channel to design the analog beamformer, which can align the phases of channel elements and thus reap the benefit of the large array gain in massive MIMO systems.

2) **DIGITAL BEAMFORMER DESIGN:**

The BS observes an effective channel \( \mathbf{H}_{eq} = \hat{\mathbf{H}} \) at the baseband. Hence digital beamforming is applied to \( \mathbf{H}_{eq} \). Here we adopt the matched filtering (MF) as the digital beamformer, which can be expressed as

\[
\mathbf{W}_{IPS} = \left( \mathbf{H}_{eq}^{IPS} \right)^H = \left( \hat{\mathbf{H}}^{IPS} \right)^H,
\]  

\[
\mathbf{W}_{QPS} = \left( \mathbf{H}_{eq}^{QPS} \right)^H = \left( \hat{\mathbf{H}}^{QPS} \right)^H,
\]  

for IPSs, for QPSs.  

Since matrix inversion is not involved, MF beamforming has lower computational complexity compared to others, but it can also achieves high SE in the large antenna regime [2].

**III. GENERIC SPECTRAL EFFICIENCY EXPRESSION DERIVATION**

In this section, we analyze the combined impacts of phase quantization, channel non-reciprocity and estimation errors on the performance of hybrid beamforming massive MIMO systems. Specifically, we will derive the generic analytical expressions of the achievable SINR and SE for IPSs and QPSs systems.

**A. SIGNAL MODEL**

To facilitate analysis, we let \( \mathbf{h}_k \in \mathbb{C}^{M \times 1} \) denote the channel coefficients from the BS to the \( k \)-th UE. Thus we have \( \mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_K]^T \in \mathbb{C}^{K \times M} \). Similarly, we let \( \mathbf{v}_k \in \mathbb{C}^{M \times 1} \), which is the \( k \)-th column of hybrid beamformer \( \mathbf{V} = \mathbf{F} \mathbf{W} \). Denote the overall beamforming weights for the \( k \)-th UE. Then the received signal at \( k \)-th UE is

\[
y_k = \sqrt{p_d} \mathbf{h}_k^T \mathbf{v}_k s_k + \sum_{l \neq k} \sqrt{p_d} \rho \mathbf{h}_l^T \mathbf{v}_k s_l + n_k
\]  

\[
= \sqrt{p_d} a_{kk} \rho \left( \mathbf{h}_k^T + \xi_k^T \right) \mathbf{v}_k s_k
\]  

\[
+ \sum_{l \neq k} \sqrt{p_d} a_{lk} \rho \left( \mathbf{h}_l^T + \xi_l^T \right) \mathbf{v}_l s_l + n_k,
\]  

where \( p_d \) is the transmit power of the BS, and \( \rho \) is the normalization factor which constrains the total BS power to
where
\[
\rho = \left(\sqrt{\mathbb{E} \left[ \text{Tr} \left( \mathbf{F} \mathbf{W} \mathbf{H}^H \mathbf{F} \mathbf{H} \right) \right]} \right)^{-1},
\]
(22)

Hence the desired signal can be expressed as
\[
\text{Var} \left( \sqrt{p_d \rho \mathbb{E} \left[ \mathbf{h}_k^T \mathbf{v}_k \right] s_k \right) = p_d \rho^2 \left( \mathbb{E} \left[ \mathbf{h}_k^T \mathbf{v}_k \right] \right)^2. \tag{30}
\]

Note that the achievable SE is derived for narrowband case, it can also act as an upper bound of achievable SE at individual subbands in wideband case, since the exclusive analog beamformer is designed for individual subbands. It is also worth noting that the achievable SE in (27) includes the three coexisting non-ideality, i.e., phase quantization, channel non-reciprocity and estimation errors. Hence, the combined impacts of these non-ideality can be quantified using the derived SE expression.

In the following subsections, we will derive the expressions for \( \text{Var} \left( I_{k,1} \right) \) and \( \text{Var} \left( I_{k,2} \right) \) respectively.

### B. DERIVATION OF DETECTION UNCERTAINTY

According to (25), we can decompose \( \text{Var} \left( I_{k,1} \right) \) as

\[
\text{Var} \left( I_{k,1} \right) = -p_d \rho^2 \mathbb{E} \left[ \mathbf{h}_k^T \mathbf{v}_k \right] + p_d \rho^2 \left( 1 + \sigma_{d_k}^2 \right) \times \left( \mathbb{E} \left[ \mathbf{h}_k^T \mathbf{B} \mathbf{v}_k \right] \right) ^2
\]

\[
+ \mathbb{E} \left[ \mathbf{h}_k^T \mathbf{B} \mathbf{v}_k \right] ^2, \tag{31}
\]

where

- \( \mathbb{E} \left[ \mathbf{h}_k^T \mathbf{v}_k \right] ^2 \) is the power of self interference under channel reciprocity and can be calculated as (32)\(^1\) in the Appendix A.
- \( \mathbb{E} \left[ \mathbf{h}_k^T \mathbf{B} \mathbf{v}_k \right] ^2 \) is the interference power caused by channel non-reciprocity and is given by (33) in the Appendix A.
- \( \mathbb{E} \left[ \mathbf{h}_k^T \mathbf{B} \mathbf{v}_k \right] ^2 \) is the interference power brought by channel estimation errors with channel reciprocity and can be expressed as (34) in the Appendix A.
- \( \mathbb{E} \left[ \mathbf{h}_k^T \mathbf{B} \mathbf{v}_k \right] ^2 \) is the interference power caused by both channel estimation errors and channel non-reciprocity, which can be written by (35) in the Appendix A.

### C. DERIVATION OF INTER-USER INTERFERENCE

According to (26), we can decompose \( \text{Var} \left( I_{k,2} \right) \) as

\[
\text{Var} \left( I_{k,2} \right) = p_d \rho^2 \left( 1 + \sigma_{d_k}^2 \right) \times \sum_{l \neq k} \left( \mathbb{E} \left[ \mathbf{h}_l^T \mathbf{v}_l \right] ^2 + \mathbb{E} \left[ \mathbf{h}_l^T \mathbf{B} \mathbf{v}_l \right] ^2 \right)
\]

\[
+ \mathbb{E} \left[ \mathbf{h}_l^T \mathbf{B} \mathbf{v}_l \right] ^2, \tag{36}
\]

\(^1\)Considering that the expressions of interference items take up too much space, we put them in the Appendix A to facilitate reading.
where

- \( E\left[ \hat{h}_k^T v_k \right]^2 \) is the interference power generated by \( l \)-th UE in the case of channel reciprocity and can be calculated as (37) in the Appendix A.
- \( E\left[ \hat{h}_k^T B' v_k \right]^2 \) is the interference power generated by \( l \)-th UE under channel estimation errors and channel reciprocity, and can be expressed as (38) in the Appendix A.
- \( E\left[ \tilde{h}_k^T v_k \right]^2 \) is the interference power generated by \( l \)-th UE under channel estimation errors and channel non-reciprocity, and can be written by (40) in the Appendix A.
- \( E\left[ \tilde{h}_k^T B' v_k \right]^2 \) is the interference power generated by \( l \)-th UE under channel estimation errors and channel non-reciprocity, which can be written by (41) in the Appendix A.

Thus, by substituting (30)-(40) into (27) and (28), we finally obtain the generic analytical expressions for the achievable SINR and SE. In the following section, we will further derive the specific closed-form expressions for IPSs and QPSs systems, respectively, using these generic expressions.

**IV. DERIVATION OF CLOSED-FORM EXPRESSIONS FOR SPECTRAL EFFICIENCY**

In this section, we first derive the closed-form expressions of SINR and SE for IPSs and QPSs systems respectively. Then the asymptotic performance in the large antenna regime is analyzed based on these expressions.

**A. SINR AND SE OF IDEAL PHASE SHIFTERS**

We start with IPSs systems. Define \( \sigma_h = \sqrt{\frac{\sigma_i}{\sigma_p}} \) and \( \sigma_\xi = \sqrt{\frac{\sigma_i}{\sigma_p+\Pi}} \). Then \( |h_k| \sim \text{Rayleigh}\left(\frac{\sigma_h}{\sqrt{2}}\right) \) with mean \( \sqrt{\pi} \sigma_h \) and variance \( \left(1 - \frac{\pi}{4}\right) \sigma_h^2 \). It can be easily verified that

\[
E\left[ |\hat{h}_k| \right]^4 = 2\sigma_h^4.
\]

Moreover, when \( k \leq K \), we have

\[
E\left[ |\hat{h}_k|^{IPS}\right]^4 = E\left[ |\hat{h}_k|^{IPS}v_k^*\right]^4 = \frac{\pi}{4M} \sigma_h^4.
\]

Thus, by substituting (30)-(40) into (27) and (28), we finally obtain the generic analytical expressions for the achievable SINR and SE. In the following section, we will further derive the specific closed-form expressions for IPSs and QPSs systems, respectively, using these generic expressions.

\[
E\left[ \hat{h}_k^T v_k \right]^2 = \frac{(M-1)(M-2)(M-3)\pi^2}{16M} \sigma_h^4 + \frac{(M-1)(M-2)(N+1) + 3N}{2M} \pi \sigma_h^4
\]

\[
E\left[ \hat{h}_k^T v_k \right]^2 = \frac{(M-N+1)\pi}{M} \sigma_h^4 + \frac{(M-1)(M+2N-2)\pi}{\sigma_h^4
\]

\[
E\left[ \hat{h}_k^T v_k \right]^2 = \frac{(M-1)(M-2)}{4M} \sigma_h^4 + \frac{(M-1)(M+2N-2)\pi}{\sigma_h^4
\]

\[
E\left[ \hat{h}_k^T v_k \right]^2 = \frac{(M-1)(M-2)}{4M} \sigma_h^4 + \frac{(M-1)(M+2N-2)\pi}{\sigma_h^4
\]

\[
E\left[ \hat{h}_k^T v_k \right]^2 = \frac{(M-N+1)\pi}{M} \sigma_h^4 + \frac{(M-1)(M+2N-2)\pi}{\sigma_h^4
\]

\[
E\left[ \hat{h}_k^T v_k \right]^2 = \frac{(M-N+1)\pi}{M} \sigma_h^4 + \frac{(M-1)(M+2N-2)\pi}{\sigma_h^4
\]

\[
E\left[ \hat{h}_k^T v_k \right]^2 = \frac{(M-N+1)\pi}{M} \sigma_h^4 + \frac{(M-1)(M+2N-2)\pi}{\sigma_h^4
\]

Similarly, different items of interference power can be expressed as (46)-(53), as shown at the bottom of this page, respectively, where \( R_{b_d} = \text{Cov}(b_d) \) with \( b_d =...
\[b_1^{11}, b_2^{21}, \ldots, b_{MM}^{MM}\] be the collection of diagonal elements of \(B\), and \(R_{bb}^{bb} = \text{Cov}(b_{bb}^{bb})\) with \(b_{bb}^{bb} = [b_{11}^{bb}, b_{12}^{bb}, \ldots, b_{MM}^{bb}]\) be the collection of off-diagonal elements of \(B\).

Thus, by substituting (46)-(53) into (27) and (28), we can obtain the closed-form expressions of SINR and SE for IPS systems. Note that these expressions include the joint effects of two coexisting non-ideality, i.e., channel non-reciprocity and estimation errors.

**B. SINR AND SE OF QUANTIZED PHASE SHIFTERS**

Next, we will derive the closed-form expressions of SINR and SE for QPSs systems. We start with the following observations in the case of \(k \leq K\)

\[
\mathbb{E} \left[ \hat{h}_k f_{ik} \right] = \mathbb{E} \left[ \hat{h}_k f_{ik}^2 \right] = \sqrt{\frac{\pi}{4M}} \sigma_h \text{sinc}(\lambda), \quad (54)
\]

\[
\mathbb{E} \left[ \hat{h}_k \hat{h}_k^* \hat{h}_k \hat{h}_k f_{ik} \right] = \mathbb{E} \left[ \hat{h}_k \hat{h}_k^* \hat{h}_k \hat{h}_k f_{ik} \right] = \frac{3}{4} \sqrt{\frac{\pi}{M}} \sigma_h^2 \text{sinc}(\lambda), \quad (55)
\]

\[
\mathbb{E} \left[ \hat{h}_k \hat{h}_k f_{ik} \hat{h}_k^* \hat{h}_k f_{ik} \right] = \mathbb{E} \left[ \hat{h}_k \hat{h}_k f_{ik} \hat{h}_k^* \hat{h}_k f_{ik} \right] = \frac{1}{M} \sigma_h^2 \text{sinc}(2\lambda). \quad (56)
\]

Combining with (22) and (23), the transmit power normalization factor can be expressed as \(\rho_{QPS} = \left( \mathbb{E} \left[ \text{Tr} \left( \mathbf{F}_{QPS} \mathbf{W}_{QPS}^H \mathbf{W}_{QPS} \mathbf{F}_{QPS}^H \right) \right] \right)^{-1}\) with

\[
\mathbb{E} \left[ \text{Tr} \left( \mathbf{F}_{QPS} \mathbf{W}_{QPS}^H \mathbf{W}_{QPS} \mathbf{F}_{QPS}^H \right) \right] = \frac{(M + 2N - 2)(M - 1)\pi}{4M} \sigma_h^2 \text{sinc}^2(\lambda) + (M + N - 1)\pi \frac{\sigma_h^2}{M} \text{sinc}^2(\lambda). \quad (57)
\]

Substituting (54)-(56) into (29), the average effective channel can be written as

\[
\mathbb{E} \left[ \mathbf{h}_k^H \mathbf{v}_{QPS} \right] = N \sigma_h^2 + \frac{(M - 1)\pi}{4} \sigma_h^2 \text{sinc}^2(\lambda). \quad (58)
\]

Similarly, different items of interference power can be expressed as (59)-(66), as shown at the bottom of this page, respectively. By substituting (59)-(66) into (27) and (28), we finally obtain the closed-form expressions of SINR and SE for QPSs systems. Note that these expressions include the joint effects of three co-existing non-ideality, i.e., phase quantization, channel non-reciprocity and estimation errors.

**C. ASYMPTOTIC PERFORMANCE IN LARGE ANTENNA REGIME**

In this subsection, we will derive the asymptotic performance of TDD hybrid beamforming massive MIMO systems based on the obtained closed-form expressions for SINR.

With the growth of antenna number at BS \(M\), the achievable SINR in both cases tend to be asymptotically identical and have the saturation level

\[
\lim_{M \to \infty} \text{SINR}_{IPPS} = \frac{1}{4K \pi \sigma_b^2 \left( 1 + \frac{\sigma^2}{\sigma_h^2} \right) \left( 1 + \frac{\sigma^2}{\sigma_h^2} \right) + \frac{\sigma^2}{\sigma_h^2}}.
\]

\[
\lim_{M \to \infty} \text{SINR}_{QPS} = \frac{1}{4K \pi \sigma_b^2 \left( 1 + \frac{\sigma^2}{\sigma_h^2} \right) \left( 1 + \frac{\sigma^2}{\sigma_h^2} \right) + \frac{\sigma^2}{\sigma_h^2}}.
\]

The corresponding asymptotic performance of SE can be easily obtained based on (57), (68) and (27). It can be seen

\[
\mathbb{E} \left[ \hat{h}_k^H v_k \right] = \frac{(M - 1)(M - 2)(M - 3)\pi^2}{16M} \text{sinc}^4(\lambda) + \frac{(M - 1)\left[ (N - 2)(N + 1) + 3N \right]}{2M} \pi \text{sinc}^2(\lambda) \frac{\sigma_h^4}{4M^2} + \frac{(M - 1)(M - 2)\pi}{2M} \text{sinc}^2(\lambda) \frac{\sigma_h^4}{4M^2} \text{sinc}^2(\lambda) \frac{\sigma_h^4}{4M^2} \text{sinc}^2(\lambda). \quad (59)
\]

\[
\mathbb{E} \left[ \hat{h}_k^H B_k^H v_k \right] = \frac{(M + N - 1)\pi^2}{4M^2} \frac{\sigma_h^4}{4M^2} \text{sinc}^2(\lambda) + \frac{(M - 1)\left[ (M - 2)N + 5N - 2\pi \right]}{8M^2} \sigma_h^4 \text{sinc}^2(\lambda) \frac{\sigma_h^4}{4M^2} \text{sinc}^2(\lambda) \frac{\sigma_h^4}{4M^2} \text{sinc}^2(\lambda). \quad (60)
\]

\[
\mathbb{E} \left[ \hat{h}_k^H B_k^H v_k \right] = \frac{(M + N - 1)\pi^2}{4M^2} \frac{\sigma_h^4}{4M^2} \text{sinc}^2(\lambda) + \frac{(M - 1)\left[ (M - 2)N + 5N - 2\pi \right]}{8M^2} \sigma_h^4 \text{sinc}^2(\lambda) \frac{\sigma_h^4}{4M^2} \text{sinc}^2(\lambda) \frac{\sigma_h^4}{4M^2} \text{sinc}^2(\lambda). \quad (61)
\]
that both channel non-reciprocity, estimation errors and phase quantization would increase the interference power, which is growing with $K$. In addition, we can observe that the effect of the three factors are coupling through the product term in the denominator. It is also worth noting that the effect of $\sigma_{b}^2$ can be eliminated in the large antenna regime, but the detrimental effects caused by other parameters cannot be canceled.

V. SIMULATION RESULTS

In this section, we present the simulation results of the derived analytical expressions for the considered TDD hybrid beamforming massive MIMO system under channel non-reciprocity, channel estimation errors, and different types of phase shifters.

Without loss of generality, the noise powers at UEs are taken as $\sigma_k^2 = 1, \forall k$. The channel coherence time is set to $1$ ms, which is the duration of a subframe in 3GPP LTE/LTE-A/NR networks [34], [35]. Each coherence interval is assumed to contains $T = 196$ symbols. A pilot reuse factor of one is assumed, i.e., $\tau_u = K$. The transmit power of UL pilots is set to $p_u = 0$ dB. Moreover, we let $\sigma_{b,k}^2 = \sigma_{b}^2, \forall k$. Variances of all the diagonal and off-diagonal elements of $\mathbf{B}^\prime$ are set to $\sigma_{b,k}^2$ and $\sigma_{b,od}^2$, respectively. In the sequel, SE and its degradation levels under varied parameter values are presented.

We begin by evaluating the SEs of IPSs and QPSs systems for varying signal-to-noise ratio (SNR, $p_d/\sigma_k^2$) and different channel non-reciprocity levels in Figure 3. To obtain these curves, we average the results of (27) over 10000 independent channel $\mathbf{H}$ and non-reciprocity variables $\mathbf{A}^\prime, \mathbf{B}^\prime$ realizations. It can be seen that the analytical curves act as lower bounds of simulated curves, and they have a perfect match for every case. This demonstrates the accuracy of derived analytical expressions. Hence, we will only investigate the performance of the derived analytical expressions in the following. Moreover, it is worth noting that the influence of channel non-reciprocity and phase quantization on the variation trend of SE is ignorable. Specifically, SE always tends to be stable when SNR $\geq 5$dB. In addition, as we can see, channel non-reciprocity leads to a substantial performance loss, especially in the large SNR region. For example, SE is decreased from 22 bits/s/Hz to 10 bits/s/Hz in QPSs case. It can be also observed that the curves for IPSs and QPSs systems are very similar, which means that a $\delta = 2$ resolution of QPSs has only slight impact on the performance. Thus next we will quantify the impact of phase quantization on the performance of the considered system.

In order to evaluate the performance loss, we introduce relative spectral efficiency degradation (RSED) metric, which is defined as

$$RSED = \frac{SE_{\text{ideal}} - SE}{SE_{\text{ideal}}} \times 100\%,$$

where $SE_{\text{ideal}}$ is the achieved SE with IPSs and perfect channel reciprocity, while $SE$ refers to the achieved SE in other cases. In Figure 4, SE and RSED are both plotted against the quantization level $\delta$. As can be seen, phase quantization has a significant influence on the SE. The performance loss...
caused by 1-bit resolution phase shifters is up to 22.91% when SNR = 20 dB. As expected, the loss is greatly reduced with the improving resolution, and it turns to be negligible when $\delta > 4$. This means that QPSs with resolution $\delta = 4$ is enough for hybrid beamforming massive MIMO systems. Moreover, we can see that though increasing antenna number at the BS could improving SE, it doesn’t change the degradation pattern.

Figure 3 shows that channel non-reciprocity has great effect on the achieved SE. We further quantify the performance loss caused by channel non-reciprocity in Figure 5. It can be observed that as the SNR improves, the RSED also increases until a saturation point is reached. When SNR is low, the system is noise limited, hence the effect of channel non-reciprocity is negligible. However, the performance loss is considerable in the large SNR region, as severe interference is generated by channel non-reciprocity. For example, the performance loss is 34.5% with setting SNR = 20 dB, $\sigma_a^2 = -10$ dB, $\sigma_{b_d}^2 = -10$ dB, $\sigma_{b_d}^2 = -20$ dB.

Furthermore, we depict RSED against varying levels of each channel non-reciprocity parameter individually in Figure 6. In obtaining the results, we varied one parameter, with other parameters set to 0 at the same time. When computing RSED, $\text{SE}_{\text{ideal}}$ in (69) is taken as the achieved SE for perfect channel reciprocity and QPSs with $\delta = 2$. Figure 6 illustrates contributions of different channel non-reciprocity parameters to the total performance loss share. It is worth noting that the off-diagonal elements of BS non-reciprocity matrix $B'$ has the most impact on the SE. For instance, the RSED can be as high as 75% when $\sigma_{b_d}^2 = -10$ dB. Meanwhile, SE degradation is the least sensitive to the diagonal elements of BS non-reciprocity matrix $B'$, and this conforms
to our asymptotic performance analysis in the large antenna regime.

Figure 4 and 5 show that, phase quantization and channel non-reciprocity both lead to the SE degradation. But which is a greater influence on the performance and how they interact and influence each other? In order to answer these two questions, we plot RSED against different channel non-reciprocity parameters with various phase shifter resolutions in Figure 7-9, respectively.

Figure 7 shows that phase quantization dominates the SE degradation when $\sigma_{\theta_{\text{rad}}}^2 \leq 15$ dB. For instance, RSED caused by channel non-reciprocity is only about 20% with $\sigma_{\theta_{\text{rad}}}^2 = -20$ dB. However, this value reaches 73% for the 1-bit phase shifter. On the contrary, channel non-reciprocity parameter $\sigma_{\theta_{\text{rad}}}^2$ turns to be the greater influence factor when $\sigma_{\theta_{\text{rad}}}^2 > 15$. For instance, channel non-reciprocity contributes to a 70% RSED with $\sigma_{\theta_{\text{rad}}}^2 = -10$ dB, but this value only increases 30% even with the 1-bit phase shifter. Similar trend is observed in Figure 8, but the turning point is at $\sigma_{\theta_{\text{rad}}}^2 = -4$ dB. Moreover, since the impact of diagonal elements of BS non-reciprocity matrix $B'$ is negligible, the RSED brought by phase quantization is always larger than that of $\sigma_{\theta_{\text{rad}}}^2$ in Figure 9.

Asymptotic performance analysis in Section IV.C shows that there are finite saturation levels for both QPSs and IPSs systems, which is different from the reciprocal channel case. Aiming at verifying this phenomenon, we plot SE against the number of antennas at BS $M$ for IPSs and QPSs systems in Figure 10 and 11, respectively. It can be observed that the SE in both cases saturates towards the levels derived in (67) and (68). Moreover, it is worth noting that these saturation levels are indeed hard to approach in the practical massive MIMO systems, since more than $10^5$ antennas is required at the BS.

VI. CONCLUSION

This paper investigated the compound effects of channel non-reciprocity, phase quantization, and channel estimation errors on the TDD hybrid beamforming massive MIMO systems. We derived the closed-form achievable SE expressions for the IPSs and QPSs systems, respectively. Simulation results showed a perfect match of analytical and simulated results, which indicates that our analytical expressions can be used to effectively evaluate the performance of the considered system. It was also shown that the phase quantization and channel non-reciprocity both lead to the SE degradation. Phase quantization dominates the SE degradation in the low SNR region, whereas channel non-reciprocity has greater influence in the high SNR region. Nevertheless, these detrimental effects cannot be canceled completely even in the large-antenna regime. Results also showcased that SE of hybrid beamforming could be greatly increased by improving the resolution of phase shifters. The performance was close to that of the IPSs systems with 4-bit resolution phase shifters. In addition, our analysis revealed that the asymptotic performance in the large antenna regime saturates to a finite level, due to the channel non-reciprocity.

APPENDIX I. EXPRESSIONS FOR INTERFERENCE ITEMS

$$\mathbb{E} \left[ |\hat{h}_k^T v_k|^2 \right] = \sum_{i,f,i',f'} \mathbb{E} \left[ \hat{h}_{ki} \hat{h}_{ki'}^* f_{if} f_{i'f'} \hat{h}_{k'i}^* \hat{h}_{k'i} \right]$$

$$= \frac{N^2}{M^2} \sum_i \mathbb{E} \left[ |\hat{h}_{ki}|^4 \right] \quad + \frac{N(N + 1)}{M^2} \sum_{i,l \neq i} \mathbb{E} \left[ |\hat{h}_{ki}|^2 \right] \mathbb{E} \left[ |\hat{h}_{kl}|^2 \right] \quad + \frac{4N}{M} \sum_{i,l \neq i} \mathbb{E} \left[ \hat{h}_{ki} \hat{h}_{ki}^* \hat{h}_{kl} \hat{h}_{kl}^* \right]$$
\[
\sum_{i, j, i' \neq i} \mathbb{E} \left[ |\hat{h}_k| \right] \mathbb{E} \left[ |\hat{b}_i'| \right] \mathbb{E} \left[ |\hat{b}_j'| \right] 
\]

(32)

\[
= \sum_{i, j, i' \neq i} \mathbb{E} \left[ |\hat{h}_k| \right] \mathbb{E} \left[ |\hat{b}_i'| \right] \mathbb{E} \left[ |\hat{b}_j'| \right] 
\]

(33)

\[
\sum_{i, j, i' \neq i} \mathbb{E} \left[ |\xi_k|^2 \right] \mathbb{E} \left[ |\xi_{k'}|^2 \right] 
\]

(34)

\[
\sum_{i, j, i' \neq i} \mathbb{E} \left[ |\xi_k|^2 \right] \mathbb{E} \left[ |\xi_{k'}|^2 \right] 
\]

(35)

\[
\sum_{i, j, i' \neq i} \mathbb{E} \left[ |\xi_k|^2 \right] \mathbb{E} \left[ |\xi_{k'}|^2 \right] 
\]

(36)

\[
= \sum_{i, j, i' \neq i} \mathbb{E} \left[ |\xi_k|^2 \right] \mathbb{E} \left[ |\xi_{k'}|^2 \right] 
\]

(37)

\[
= \sum_{i, j, i' \neq i} \mathbb{E} \left[ |\xi_k|^2 \right] \mathbb{E} \left[ |\xi_{k'}|^2 \right] 
\]

(38)
\[
\begin{align*}
\mathbb{E}\left[ \left| \mathbf{h}_k^* \mathbf{B} \mathbf{v}_l \right|^2 \right] \\
= \sum_{i,j,q,r,q^\prime} \mathbb{E}\left[ |h_k|^2 \right] \mathbb{E}\left[ |h_{iq}|^2 \right] \mathbb{E}\left[ |b_j|^2 \right] \\
= \frac{N}{M^2} \sum_{i,j,q} \mathbb{E}\left[ |h_k|^2 \right] \mathbb{E}\left[ |h_{iq}|^2 \right] \mathbb{E}\left[ |b_j|^2 \right] \\
+ \frac{N(N-1)}{M^2} \sum_{i,j,q,q^\prime} \mathbb{E}\left[ |h_k|^2 \right] \mathbb{E}\left[ |h_{iq}|^2 \right] \mathbb{E}\left[ |b_j|^2 \right] \\
+ \frac{1}{M} \sum_{i,j,q,q^\prime \neq q} \mathbb{E}\left[ |h_k|^2 \right] \mathbb{E}\left[ |h_{iq}|^2 \right] \mathbb{E}\left[ |h_{iq} f_{q^\prime}| \right] \mathbb{E}\left[ |b_j|^2 \right] \\
+ \frac{2(N-1)}{M} \sum_{i,j,q,q^\prime \neq q} \mathbb{E}\left[ |h_k|^2 \right] \mathbb{E}\left[ |h_{iq}|^2 \right] \mathbb{E}\left[ b_j f_{q^\prime} | \right] \mathbb{E}\left[ |b_j|^2 \right] \\
+ \frac{2}{M} \sum_{i,j,q,q^\prime \neq q} \mathbb{E}\left[ |h_k|^2 \right] \mathbb{E}\left[ |h_{iq}|^2 \right] \mathbb{E}\left[ h_{ij q} f_{q^\prime} | \right] \mathbb{E}\left[ |b_j|^2 \right]
\end{align*}
\]

(39)
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