A GENERAL MULTI-ATTRIBUTE MULTI-SCALE DECISION MAKING METHOD BASED ON DYNAMIC LINMAP FOR PROPERTY PERCEIVED SERVICE QUALITY EVALUATION

Wen-Jin ZUO¹,³, Deng-Feng Li²*, Gao-Feng YU³

¹Zhejiang College, Shanghai University of Finance and Economics, Jinhua, 321013 Zhejiang, China
²School of Management and Economics, University of Electronic Science and Technology of China, Chengdu, 611731 Sichuan, China
³School of Economics and Management, Fuzhou University, Fuzhou, 350108 Fujian, China

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Abstract. The scientific evaluation of property perceived service quality (PPSQ) needs multi-stage, multi-source and large-group perceived information, which is deemed to be the decision problem for dynamic, heterogeneous and large-scale data processing. Aiming at the problem, we propose a general multi-attribute multi-scale (MAMS) method based on the dynamic linear programming technique for multi-dimensional analysis of preference (LINMAP). In the dynamic LINMAP model, the classic MAMS matrix is introduced and extended into a general form. The dynamic LINMAP model is constructed by defining dynamic consistency and dynamic inconsistency. The time series weight is determined by Orness method. The new method adapts to the requirements of modern PPSQ. Finally, we verify the feasibility and effectiveness of dynamic LINMAP method by analyzing a PPSQ evaluation example. The new method improves the traditional PPSQ evaluation, and provides a perspective for large-scale data processing by the classic decision method.

Keywords: property perceived service quality, general MAMS method, the dynamic LINMAP model, complex evaluation data.

JEL Classification: C61, H83, M31.

Introduction

In the 1860s, property service originated in Britain. In 1908, the establishment of CBMO in USA is the first professional property industry organization. In the early 1980s, property management is introduced into mainland China (Deng, 2007). Nowadays, property service industry has become the basic industry of social production and life, and forms a huge market scale. Taking mainland China as an example, the latest statistics show that there are about 105,000 property service enterprises, with about 7112,000 employees. The annual
operating revenue is more than 350 billion Yuan, and the building area of property service reaches 16.45 billion square meter (China Property Management Institute [CPMI], 2018). As the progress of science and technology, property service industry is transforming to modern service industry (Xie, 2012). Because the evaluation information of modern property service quality is multi-source, multi-subject and multi-stage, the traditional evaluation method cannot meet the needs of the complex evaluation environment (Zuo et al., 2019). Therefore, it is of great practical significance to study the property perceived service quality (PPSQ) evaluation method in modern service environment.

The theory of perceived service quality (PSQ) is first proposed in the early 1980s (Gronroos, 1983). Since then, scholars have deeply studied the dimensions and methods of PSQ evaluation. One the one hand, the index system of service quality is based on dimension division. Service quality is divided into technology quality and function quality by Gronroos (1983) in the early stage, and then the components of service quality is expanded into seven dimensions based on the characteristics of employees, customers and services (Gronroos, 2000). U. Lehtinen and J. R. Lehtinen (1982) divide service quality into three dimensions: interactivity, tangibility and company quality. Juran (1986) put forward the five elements of service quality which included technical quality, psychological quality, time quality, relationship quality and moral quality. Hedvyall and Paltschik (1991) take the dimension of service quality as two aspects: the ability and willingness to provide services, and easy access to physical and psychological satisfaction. Ko and Pastore (2004) divide service quality into result quality, physical environment quality, project quality and interaction quality. One the other hand, models and methods are the key to PSQ evaluation. The customer PSQ theory is the foundation of evaluation method in most research results of service quality, in which the SERVQUAL method (Parasuraman et al., 1985) has the greatest impact. The method consists of 5 dimensions, and the measurement value of PSQ is obtained by through a scale containing 22 questions. Aimed at the problem of insufficient empirical test of SERVQUAL model, Cronin and Taylor (1992) propose a PSQ evaluation model based on performance, namely SERVPERF evaluation model. Furthermore, Erto and Vanacore (2002) measure hotel service quality by the probabilistic method based on the KANO model. Mazis (1975) proposes a weighted performance evaluation method, and it solved the problem that the influence of customer perception difference on customer perception service quality evaluation results are ignored in previous evaluation models. Weiner (1986) and Bitner (1990) respectively evaluate the PSQ of customers from psychological perspective, and propose an evaluation method based on attribution model. Brown et al. (1993) believed that the best evaluation method is to directly evaluate the difference between customer perceived performances and service expectations, and proposed the non-difference evaluation method based on SERVQUAL model.

In addition, such as SERVQUAL model (Wu, 2009; Huo, 2010; Han et al., 2013), entropy method (Yang & Shen, 2012), structural equation model (Huang & Li, 2013), analytic hierarchy process (Lo et al., 2013; Guo & Wang, 2014), fuzzy evaluation method (Liao & Hu, 2017; Shiu et al., 2016), rooted theory (Ming & Cao, 2019), quality function deployment method (Yao & Zhong, 2008; Lu, 2018), IVWMM (Liu & Zuo, 2019) and BP neural network (Liu et al., 2019) are used for PSQ successively. Among them, the empirical analysis method based on SERVQUAL model is the most commonly method in service quality evaluation. The PSQ
evaluation is used many fields, such as library (Ming & Cao, 2019), information network (Li, 2010), home-based care (H. Q. Liu & Q. Liu, 2012), tourism (Qin & Liu, 2015), logistics (Shi, 2017), distance education (Zhang & Cao, 2016) and archives (Deng et al., 2018).

Because PSQ and satisfaction have similar dimensions, models and methods, the relevant researches of satisfaction can take as reference for the research of PSQ evaluation method. Research on customer satisfaction begins in the 1960s (Cardozo, 1965). Mesarovic & Takahara (1972) conduct a systematic research on the satisfaction theory. Subsequently, some scholars discuss the theory of expectation difference and its influence on service performance (Olshavsky & Miller, 1972; Anderson, 1973). As a macro indicator system to measure the quality of economic output, customer satisfaction index evaluation model includes SCSB, ACSI, ECSI and CCSI (Liu, 2003). In general, the evaluators are limited to the customers in these methods. The structure of evaluation data is single, and there is only one stage of evaluation data. The existing methods cannot meet the requirements of dynamic, heterogeneous and large-scale PPSQ evaluation in modern service environment. Therefore, for solving the above problems, it is necessary to innovate the traditional methods in the theory and practice of PPSQ evaluation.

Decision making method has been widely used in many fields. However, the traditional decision models are difficult to be directly used in large-scale data processing. Some attempts have been made in the research on large group decision making (LGDM). There are two types of existing researches on LGDM: The clustering analysis is used in one type of researches, in which the number of decision makers is usually a few dozen (Chen, 2009; Xu et al., 2017; Zhang & Fan, 2011; Gou et al., 2018; Liu et al., 2018). Another type of researches introduce multi-attribute multi-scale (MAMS) matrix which is used for data processing in combination with the classic decision-making method (Zhang & Fan, 2014; Liu et al., 2016; You et al., 2017; Zhang & Fan, 2011). Although the latter has advantages in large-scale data processing, the method cannot process data with complex structure. There have been several reports on the research of dynamic LGDM methods (Cai et al., 2016; Xu et al., 2014, 2018; Xu & Wu, 2014). However, these research results generally belong to the first type mentioned above and cannot realize large-scale data fusion.

The classic LINMAP method is a famous method which simultaneously process overall preference information and itemized evaluation information (Shocker & Srinivasan, 1973). It is used for data processing of fuzzy number (Li & Yang, 2004; Bereketli et al., 2011), intuitionistic fuzzy number (Li, 2008; Li et al., 2010) and grey numbers (Razavi Hajiagha et al., 2012). It is also used to process heterogeneous information (Wan & Li, 2013), large-scale information (Zuo et al., 2019) and uncertain risk information (Song et al., 2018). However, there is no LINMAP method that can process large-scale, heterogeneous, and dynamic data simultaneously. The LINMAP method has widely applied in invest money selection (Xu et al., 2016), railway project investment (Xue et al., 2018) and supplier evaluation (Song et al., 2018). To sum up, the potential of LINMAP to process the data with complex structure needs to be further explored, and its application field needs to be further expanded.

The contributions of this paper mainly include the following three aspects: (1) We design a PPSQ method based on complex evaluation data. The new method can be used for multi-type of evaluators and multi-stage evaluation data processing. It not only enriches the
PPSQ evaluation method, but also suitable for complex data processing in modern service. (2) We propose a generalized MAMS method based on the classical MAMS method. Then the generalized MAMS method is used for large-scale heterogeneous data processing. The data processing power of the new method has been greatly improved, which provides a new perspective for decision model constructing of large-scale data processing. (3) We propose a dynamic LINMAP model based on Orness and MAMS methods. The PIS based on the general MAMS matrix is analyzed, and the weight of time series is determined by the Orness method. The dynamic LINMAP model is constructed by defining dynamic consistency and dynamic inconsistency. The new method adapts to the requirement of dynamic evaluation data processing.

The remainder of this paper is organized as follows. Section 1 illustrates the basic problem to be studied, includes some notations used in this article. Section 2 proposes the basic method and principle, includes analysis of time series weights and attributes weights, definition of dynamic consistency and dynamic inconsistency, and construction of dynamic LINMAP model. Section 3 uses the above method to analyze a PPSQ example with dynamic, heterogeneous and large-scale data. Section 4 compares the new method with the traditional LINMAP method, including sensitivity analysis, case analysis, comparison and summaries. Finally, we draw a brief conclusion and prospects the future research.

1. Description of problem

1.1. Definition of general MAMS decision-making

Decision makers are composed of users and experts. The former make itemized evaluation on different attributes of alternatives, while the latter make overall preference comparison between alternatives. The MAMS decision matrix is used for large-scale data processing in this paper, which is the focus of the problem.

**Definition 1** (Zhang & Fan 2011). If each attribute can be evaluated by the set of scale in the multi-attribute decision making, which is called the MAMS decision making. In the classic MAMS decision making, each attribute have the same number of scales.

**Definition 2.** If the number of scale set corresponding to each attribute is not completely the same in the MAMS decision-making, which is called the general MAMS decision-making.

1.2. Some notations and their connotations

For the sake of the description in this paper, there are some notations are as follows:

- \( A = \{A_1, A_2, \ldots, A_p\} \) denotes the set which is composed of \( p \) alternatives, where \( A_k \) is the \( k \)th alternative, \( k = 1, 2, \ldots, p \).
- \( \xi = (\xi_1, \xi_2, \ldots, \xi_q) \) denotes the weight vector of \( q \) stages, where \( \xi_t \) is the weight of the \( t \)th stage, \( \sum_{t=1}^{q} \xi_t = 1, \xi_t \geq 0, t = 1, 2, \ldots, q \).
- \( C_i = \{C_{i1}, C_{i2}, \ldots, C_{im}\} \) denotes the set which is composed of \( m \) attributes, where \( C_{it} \) is the \( i \)th attribute in the \( t \)th stage, \( i = 1, 2, \ldots, m \).
– \( \omega^t = (\omega^t_1, \omega^t_2, \ldots, \omega^t_m) \) denotes the corresponding weight vector of \( C^t \), where \( \omega^t_i \) is the corresponding weight of \( C^t_i \), \( \sum_{i=1}^{m} \omega^t_i = 1, \omega^t_i \geq 0 \).

– \( S^t = \{S^t_1, S^t_2, \ldots, S^t_m\} \) denotes the set which is composed of \( m \) subsets in the \( i \)th stage, where \( S^t_i \) is the \( i \)th element of \( S^t \).

– \( S^t_i = \{s^t_{i1}, s^t_{i2}, \ldots, s^t_{in_i}\} (j = 1, 2, \ldots, n^t_i) \) denotes the corresponding set of \( C^t_i \), where \( s^t_{ij} \) is the \( j \)th element in \( S^t_i \), and \( n^t_i \) is a natural number which denotes the scale number of the \( i \)th attribute in the \( t \)th stage. According to the existing researches (Berry et al., 1988; Liao & Hu, 2017; Yang & Shen, 2012; Guo, 2014), all elements in \( S^t_i \) form equidistant scale and increasing sequence.

– \( u^t_{ijk} \) denotes the standardized value of number of users who use \( s^t_{ij} \) evaluate \( A_k \) and \( C_i \) in the \( t \)th stage. Table 1 shows the dynamic general MAMS information structure.

– \( N^t_k \) denotes the number of users corresponding to \( A_k, s^t_{ij} \) and \( C_p \), where \( N^t \) denotes the number of users at a certain stage.

– \( \Omega = \{(k,l) \| A_k \geq A_l\} (k, l = 1, 2, \ldots, p) \) denotes the set where the expert thinks the alternative \( A_k \) is better than the alternative \( A_l \).

### Table 1. The dynamic general MAMS decision information of users’ evaluation

| \( C^t \) | \( C^t_1 / S^t_1 \) | \( C^t_2 / S^t_2 \) | \( \ldots \) | \( C^t_m / S^t_m \) |
|---|---|---|---|---|
| \( S^t \) | \( s^t_{11} \) | \( s^t_{12} \) | \( \ldots \) | \( s^t_{1n_1} \) | \( s^t_{21} \) | \( s^t_{22} \) | \( \ldots \) | \( s^t_{2n_2} \) | \( \ldots \) | \( s^t_{m1} \) | \( s^t_{m2} \) | \( \ldots \) | \( s^t_{mn_m} \) |
| \( A_1 \) | \( u^t_{11} \) | \( u^t_{12} \) | \( \ldots \) | \( u^t_{1n_1} \) | \( u^t_{21} \) | \( u^t_{22} \) | \( \ldots \) | \( u^t_{2n_2} \) | \( \ldots \) | \( u^t_{m1} \) | \( u^t_{m2} \) | \( \ldots \) | \( u^t_{mn_m} \) |
| \( A_2 \) | \( u^t_{112} \) | \( u^t_{122} \) | \( \ldots \) | \( u^t_{1n_2} \) | \( u^t_{212} \) | \( u^t_{222} \) | \( \ldots \) | \( u^t_{2n_2} \) | \( \ldots \) | \( \ldots \) | \( \ldots \) |
| \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) |
| \( A_p \) | \( u^t_{11p} \) | \( u^t_{12p} \) | \( \ldots \) | \( u^t_{1np} \) | \( u^t_{21p} \) | \( u^t_{22p} \) | \( \ldots \) | \( u^t_{2np} \) | \( \ldots \) | \( u^t_{mp1} \) | \( u^t_{mp2} \) | \( \ldots \) | \( u^t_{mnp} \) |

### 1.3. Key problems and their solving solutions

According to the above analysis, there are three problems as follows: How to standardize large-scale heterogeneous data effectively? How to improve the LINMAP model according to the requirements of dynamic data processing? How to solve the PPSQ problem based on complex data? The key problem is how to design a decision-making method for dynamic, heterogeneous and large-scale data processing.

We intend to solve the problems according to the following ideas: Firstly, for large-scale heterogeneous data processing, the general MAMS information structure is proposed based on the classical MAMS decision making matrix. For the elements in different scale sets, a standardized method is designed. Secondly, according to the demands of multi-stage information fusion, a dynamic LINMAP model is proposed by defining dynamic consistency and dynamic inconsistency. Lastly, the dynamic LINMAP method is used for PPSQ evaluation, which realizes the synchronous processing of multi-stage and large-scale perceived information.
2. The General MAMS decision-making method based on dynamic LINMAP

2.1. Standardization of general MAMS decision-making data

The data in general MAMS decision matrix includes the number of ordinary decision-makers, the scale values, time series weights and attribute weights. Before developing the model and method, we need to standardize these heterogeneous data.

In the general MAMS information structure, the number of decision makers is the same for each alternative and attribute. This is the premise for ensuring that the evaluation value in different alternatives and attributes are comparable. In general dynamic MAMS decision-making, the above hypothesis is still valid in the evaluation value at any stage, but it is not necessarily valid in the evaluation value between different stages. Therefore, there exists as follows:

$$\text{tin} = \sum_{j=1}^{N} \text{tt}^{ijk}.$$ (1)

The standardized formula for the number of decision-makers in the general MAMS matrix is as follows:

$$\mu^{ijk} = \frac{N^{ijk}}{N^{ij}}.$$ (2)

For the dynamic PPSQ evaluation, the questionnaires are designed in different stages, and they are usually completed by different decision-makers respectively. Therefore, there may be a different scale sets in each questionnaire, and the same scale value in different scale set may has different connotations. In addition, the scale set of different attributes in the same survey may be different.

**Example 1.** In a set of questionnaires with multi-stages, there are three scale sets and their implications as follows:

- $S_1^i = S_2^i = S_3^i = \{1,2\} = \{\text{dissatisfied, satisfied}\};$
- $S_2^i = S_3^i = \{0,1,2,3\} = \{\text{not good, indifferent, good, very good}\};$
- $S_3^i = S_3^i = \{0,1,2,3,4\} = \{\text{very dissatisfied, dissatisfied, general dissatisfied, satisfied, very satisfied}\}.$

Obviously, the scale value 1 denotes “dissatisfied” in $S_1^i$, “indifferent” in $S_2^i$ and “dissatisfied” in $S_3^i$ respectively. Furthermore, the connotation of “dissatisfied” in $S_1^i$ is different from that of $S_3^i$.

Therefore, it is necessary to standardize all the scale sets. The basic idea of scale value standardization is to transform all scale sets into the reference set. Determine the reference scale set is the key step. The reference scale set is the set with the most elements. We define it as follows:

**Definition 3.** Let $|S'_i|$ be the number of elements in the set $S'_i$. For any scale set $S'_i$ of the general MAMS decision matrix, if there exists:

$$f(S'_i) = \max_{1 \leq q \leq m} \max_{1 \leq s \leq m} |S'_i|,$$ (3)

then the solution $S'^*_i = \{s'^*_i, s'^*_{i2}, ..., s'^*_{in_i}\}$ is called reference scale set.
For any scale in the set $S_t^i = \{s_{in}^t, s_{in}^t, ..., s_{in}^t\}$, the standardized processing formula is as follows:

$$s_{ij}^t = s_{in}^t + \frac{(j-1)(s_{in}^t - s_{in}^t)}{n_t^t - 1},$$

(4)

where $s_{in}^t = s_{in}^t$, $s_{in}^t = s_{in}^t$.

**Example 2.** The data in Example 1 is still used here.

According to Eq. (3), the reference scale set $S_1^3 = \{0, 1, 2, 3, 4\}$.

According to Eq. (4), the standardized values of the above scale sets are as follows: $S_1^1 = S_2^1 = \{0, 4\}$, $S_1^2 = S_2^2 = \{0, 1.333, 2.667, 4\}$, $S_1^3 = S_2^3 = S_3^3 = \{0, 1, 2, 3, 4\}$.

As time series weights and attribute weights are both unknown in advance, we can regard them as standardized unknown numbers. We are going to talk about how to process the two types of data in next section.

### 2.2. Analyze the time series weights and attribute weights

**Definition 4** (Yager, 1988). Let $\xi_t$ be the weight in the $t$th stage, there exists $\sum_{t=1}^{q} \xi_t = 1$, $\xi_t \geq 0$. Then, the Orness measure of $\xi_t$ is denoted as follows:

$$\text{Orness} (\xi) = \frac{1}{q-1} \sum_{t=1}^{q} (q-t)\xi_t = \gamma, 0 \leq \gamma \leq 1.$$  

(5)

According to the existing method (Xu, 2009; Yu et al., 2019), the relationship between the weights of adjacent time periods can be further expressed as:

$$\xi_t - \xi_{t-1} = \gamma, 0 \leq \gamma \leq 1.$$  

(6)

where 0 ≤ $\gamma$ ≤ 1.

The analysis results of the stage weight reflect the preference difference of decision makers for the time series. The closer $\gamma$ is to 1, the more importance decision-maker attach to long-term data. On the contrary, the closer $\gamma$ is to zero, the more importance decision-maker attach to recent data. $\gamma = 0.5$ indicates that the importance of each period is the same in the opinion of decision-makers.

**Example 3.** For a three-stage decision-making, the decision-maker determine $\gamma = 0.3$ based on their experience. Combine to $q = 3$, Eqs (5) and (6), the time series weights are calculated as follows

$$\xi = (\xi^1, \xi^2, \xi^3) = (0.133, 0.333, 0.533).$$

Attribute weights reflect the importance of attributes. Decision-makers usually determine the relation on some attributes based on incomplete information. Early researches showed that the incomplete information structure can be expressed in five forms, includes weak ranking, strict ranking, ranking with multiples, interval form and so on (Podinovski, 2010; Wang et al., 2009). One or more of the above information structures can be used for the expression of the decision makers’ preferences.
2.3. Dynamic consistency and dynamic inconsistency

For the classic decision matrix, the positive ideal solution (PIS) is usually a row vector. Then, PIS is a multidimensional vector in the dynamic LINMAP method. The PIS based on general MAMS decision matrix in the \( t \)th stage is denoted as:

\[
 u^+ = (u_{ij}^+)^{m \times n_i},
\]

where \( u_{ij}^+ \) is PIS corresponding to different scale \( s_{ij} \) and attribute \( C_i \) in the \( t \)th stage. For example, \( u_{12}^3 \) is the PIS of vector \((u_{121}^3, u_{122}^3, \ldots, u_{12p}^3)\) in the third stage.

Based on the description in Section 1.2, the evaluation information of the alternative \( A_k \) from all users is denoted as:

\[
 P(A_k) = (u_{ik}^t)_{q \times m},
\]

where \( u_{ik}^t = (u_{i1k}^t, u_{i2k}^t, \ldots, u_{in_k}^t) \).

Similarly, the evaluation information of \( A_l \) from all users is denoted as:

\[
 P(A_l) = (u_{il}^t)_{q \times m},
\]

where \( u_{il}^t = (u_{i1l}^t, u_{i2l}^t, \ldots, u_{in_l}^t) \).

If there exists \( n_1^t = n_2^t = \cdots = n_m^t \), Eqs (7), (8) and (9) are multidimensional arrays, and the general MAMS decision information structure can be simplified to the classic MAMS decision matrix.

In particular, according to the basic principle of LINMAP method and the requirements of the core model in this paper, the distances defined below are all weighted Euclidean distances.

**Definition 5.** Let \( D_k^t \) be the dynamic distance square value between the evaluation value of \( A_k \) and its PIS in the \( t \)th stage. The dynamic distance square value \( D_k^t \) is denoted as:

\[
 D_k^t = \sum_{i=1}^m \alpha_i^t [d(p(A_k), u^+)^2].
\]

**Definition 6.** Let \( D_k \) be the comprehensive dynamic distance square value between the evaluation value of \( A_k \) and its PIS. The comprehensive dynamic distance square value \( D_k \) is denoted as:

\[
 D_k = \sum_{t=1}^q \xi_t \sum_{i=1}^m \alpha_i^t [d(p(A_k), u^+)^2].
\]

The above equation can be rewritten as:

\[
 D_k = \sum_{t=1}^q \xi_t \sum_{i=1}^m \sum_{j=1}^{n_i} \alpha_i^t (s_{ij}^t u_{ij}^t - s_{ij}^t u_{ij}^+)^2.
\]

Similarly, the comprehensive dynamic distance square value \( D_l \) can be written as:

\[
 D_l = \sum_{t=1}^q \xi_t \sum_{i=1}^m \sum_{j=1}^{n_i} \alpha_i^t (s_{ij}^t u_{ij}^t - s_{ij}^t u_{ij}^+)^2.
\]

If \( D_l \geq D_k \), the comprehensive dynamic distance square value of \( A_l \) is greater than one of \( A_k \). Then, the evaluation of \( A_k \) is superior to \( A_l \). Apparently, the value \((\xi_t, \alpha_i^t, u_{ij}^+ \) determines the ranking order of \( A_k \) and \( A_l \).
According to the above analysis, the dynamic consistency and the dynamic inconsistency between the alternative \( A_k \) and \( A_l \) are determined by the relation between the comprehensive dynamic distance square value \( D_k \) and \( D_l \).

**Definition 7.** Let \( B^t \) be the dynamic inconsistency of the alternative \( A_k \) and \( A_l \) in the \( t \)th stage, the equation of dynamic inconsistency is denoted as:

\[
B^t = (D^t_l - D^t_k)^- = \begin{cases} 
D^t_l - D^t_k & (D^t_l < D^t_k) \\
0 & (D^t_l \geq D^t_k)
\end{cases}
\]  \( (14) \)

The above equation can be further simplified for \( B^t = \max(0, D^t_l - D^t_k) \). Consider the overall preferences of all experts, the comprehensive dynamic inconsistency in the \( t \)th stage can be written as

\[
B^t_{\Omega} = \sum_{(k,l)\in\Omega} (D^t_l - D^t_k)^- = \sum_{(k,l)\in\Omega} \max(0, D^t_l - D^t_k).
\]  \( (15) \)

**Definition 8.** Let \( G^t \) be the dynamic consistency of the alternative \( A_k \) and \( A_l \) in the \( t \)th stage, the equation of dynamic consistency is denoted as:

\[
G^t = (D^t_l - D^t_k)^+ = \begin{cases} 
D^t_l - D^t_k & (D^t_l \geq D^t_k) \\
0 & (D^t_l < D^t_k)
\end{cases}
\]  \( (16) \)

The above equation can be further simplified for \( G^t = \max(0, D^t_l - D^t_k) \). Consider the overall preferences of all experts, the comprehensive dynamic consistency in the \( t \)th stage can be written as

\[
G^t_{\Omega} = \sum_{(k,l)\in\Omega} (D^t_l - D^t_k)^+ = \sum_{(k,l)\in\Omega} \max(0, D^t_l - D^t_k).
\]  \( (17) \)

According to Eqs (15) and (17), the difference of the comprehensive dynamic consistency and the comprehensive dynamic inconsistency is denoted as:

\[
G^t_{\Omega} - B^t_{\Omega} = \sum_{(k,l)\in\Omega} \sum_{t=1}^q (D^t_l - D^t_k)^+ - \sum_{(k,l)\in\Omega} \sum_{t=1}^q (D^t_l - D^t_k)^- = \sum_{(k,l)\in\Omega} \sum_{t=1}^q [(D^t_l - D^t_k)^+ - (D^t_l - D^t_k)^-] = \sum_{(k,l)\in\Omega} \sum_{t=1}^q [D^t_l - D^t_k].
\]  \( (18) \)

### 2.4. The dynamic LINMAP model

In the LINMAP method, the minimum inconsistency is taken as the optimal objective function in the mathematical programming model. Attribute weight and the relationship between consistency and inconsistency are added as constraints. Refer to existing research results (Shocker & Srinivasan, 1973; Li & Yang, 2004), the dynamic LINMAP model is denoted as:

\[
\begin{align*}
\min\{B^t_{\Omega}\} \\
G^t_{\Omega} - B^t_{\Omega} & \geq h \\
\sum_{i=1}^m \alpha_i^t & = 1, \alpha_i^t > \varepsilon \\
i & = 1,2,\ldots,m,t = 1,2,\ldots,q
\end{align*}
\]  \( (19) \)
where $G_\Omega^t$ and $B_\Omega^t$ denote the comprehensive dynamic consistency index and the comprehensive dynamic inconsistency index, respectively, $h > 0$ and $\epsilon > 0$ are given in advance. $h$ makes the comprehensive dynamic consistency greater than the comprehensive dynamic inconsistency. $\epsilon$ ensures that all attribute weights are greater than zero.

For any pair $(k,l) \in \Omega$, let $\lambda_{kl} = \min(0, \sum_{t=1}^{q} (D_{ij}^t - D_{kl}^t))$, it can be written as:

$$\lambda_{kl} \leq \sum_{t=1}^{q} (D_{ij}^t - D_{kl}^t), \quad (20)$$

where $\lambda_{kl} \geq 0$.

By combining Eqs (7), (8), (9), (12), (13), (18), (19) and (20), the dynamic LINMAP model is denoted as

$$\min \left\{ \sum_{(k,l) \in \Omega} \lambda_{kl} \right\}$$

$$\sum_{(k,l) \in \Omega} \left[ \sum_{t=1}^{q} \sum_{i=1}^{n_i^t} \sum_{j=1}^{m} \alpha_i^t [s_{ij}^t (u_{ij}^t - u_{ij}^t) + 2u_{ij}^t s_{ij}^t (u_{ij}^t - u_{ij}^t)] \right] \geq h$$

$$\sum_{t=1}^{q} \sum_{i=1}^{n_i^t} \sum_{j=1}^{m} \alpha_i^t [s_{ij}^t (u_{ij}^t - u_{ij}^t) + 2u_{ij}^t s_{ij}^t (u_{ij}^t - u_{ij}^t)] + \lambda_{kl} \geq 0$$

with $\xi^t \in H_1, \sum_{t=1}^{q} \xi^t = 1$

$\alpha_i^t \in H_2, \sum_{i=1}^{m} \alpha_i^t = 1$

$\lambda_{kl} > 0, (k,l) \in \Omega$

$i = 1, 2, ..., m, j = 1, 2, ..., n_i^t, k, l = 1, 2, ..., p, t = 1, 2, ..., q$

where $H_1$ and $H_2$ are determined by the method from Section 2.2.

After attribute weights, time series weights and PIS are determined, and the comprehensive dynamic distance square value between any alternative and its PIS are determined by use of Eq. (12) or (13). The ranking order of each alternative is determined according to its distance from PIS.

2.5. Decision process

**Step 1.** Standardize the number of ordinary decision-makers in different attributes, scales and stages by Eqs (1) and (2). Standardize the evaluation scales in different attributes and stages by Eqs (3) and (4).

**Step 2.** Combine with the $\gamma$ value, Eqs (5) and (6) to determine the time series weight, and analyze the attribute weight by the decision-makers’ experience.

**Step 3.** Analyze the PIS of general MAMS decision matrix by Eq. (7).

**Step 4.** Collect the overall preferences of experts and determine the preferences set $\Omega$.

**Step 5.** Determine the total dynamic consistency and the total dynamic inconsistency by Eq. (15) and (17), and construct the dynamic LINMAP model by Eq. (21).
**Step 6.** Solve Eq. (21), and determine the final attribute weight by combine with sensitivity analysis.

**Step 7.** Calculate the comprehensive dynamic distance square value by Eq. (12) or (13), and ranking order.

### 3. The PPSQ evaluation based on dynamic information

#### 3.1. Introduction to PPSQ evaluation

The basic data come from successive surveys of different public construction projects. Three surveys were conducted in 2014, 2016 and 2018 respectively. The four evaluation objects include museum ($A_1$), library ($A_2$), science museum ($A_3$) and grand theater ($A_4$).

In the dynamic PPSQ evaluation, there are two types of decision makers: experts and large-scale visitors. Three experts are invited from university, government and property management association. Property management experts make overall preference comparison on between the property service projects based on macroscopic perspective. Visitors who familiar with four public construction projects simultaneously are selected as general decision makers. Visitor evaluation adopts questionnaire survey in three stages. We used the methods of on-site interview and telephone interview to collect information. The main content of the questionnaire is that visitors evaluate different attributes of different alternatives by use of different scale values. By referring to the PPSQ evaluation practice of property service companies such as VANKE and GREENTOWN, we determine some attributes which composed of attitude ($C_1$), greening ($C_2$), cleaning ($C_3$), facilities ($C_4$) and safety ($C_5$). It should be noted that the scale set of three stages are different. The incoherent of the questionnaire survey led to the different scale set. The scale values are determined in advance by the decision-maker respectively. The survey data of three stages is summarized in Table 2.

**Table 2. Dynamic distribution of the number of visitor evaluation**

**Data from the first survey (2014)**

|   | $C_1^1$ | $C_1^2$ | $C_1^3$ | $C_1^4$ | $C_1^5$ |
|---|---------|---------|---------|---------|---------|
| $S^1$ | 1       | 2       | 1       | 2       | 1       |
| $A_1$ | 28      | 17      | 8       | 37      | 35      |
| $A_2$ | 15      | 30      | 25      | 20      | 21      |
| $A_3$ | 21      | 24      | 16      | 29      | 16      |
| $A_4$ | 19      | 26      | 11      | 34      | 18      |

**Data from the second survey (2016)**

|   | $C_2^1$ | $C_2^2$ | $C_2^3$ | $C_2^4$ | $C_2^5$ |
|---|---------|---------|---------|---------|---------|
| $S^2$ | 0       | 1       | 2       | 3       | 0       |
| $A_1$ | 16      | 30      | 25      | 25      | 14      |
| $A_2$ | 5       | 36      | 19      | 36      | 5       |
| $A_3$ | 8       | 36      | 33      | 19      | 14      |
| $A_4$ | 6       | 32      | 28      | 30      | 8       |

**Data from the third survey (2018)**

|   | $C_3^1$ | $C_3^2$ | $C_3^3$ | $C_3^4$ | $C_3^5$ |
|---|---------|---------|---------|---------|---------|
| $S^3$ | 0       | 1       | 2       | 3       | 0       |
| $A_1$ | 16      | 30      | 25      | 25      | 14      |
| $A_2$ | 5       | 36      | 19      | 36      | 5       |
| $A_3$ | 8       | 36      | 33      | 19      | 14      |
| $A_4$ | 6       | 32      | 28      | 30      | 8       |
3.2. Analysis process of dynamic PPSQ evaluation

According to the process in Section 2.5, the method in Section 2.4 and the data in Section 3.1, the PPSQ evaluation based on dynamic information is analyzed as follows.

Step 1. Standardize the visitors’ number of different scales, attributes and projects by Eqs (1) and (2). Standardize the different scale sets by Eqs (3) and (4). The results of standardized scale value are as follows:

\[ S_1 = \{1, 2\} = \{0, 4\}; \]
\[ S_2 = \{0, 1, 2, 3\} = \{0, 1, 3.33, 2.667, 4\}; \]
\[ S_3 = \{0, 1, 2, 3, 4\} = \{0, 1, 2, 3, 4\}. \]

The standardized form corresponding to Table 2 is shown in Table 3.

### Table 3. Standardization value of dynamic distribution of visitor evaluation

Data from the third survey (2018)

| C^3 | C_1^3 | C_2^3 | C_3^3 | C_4^3 | C_5^3 |
|-----|-------|-------|-------|-------|-------|
| S^3 | 0 1 2 3 4 0 1 2 3 4 0 1 2 3 4 0 1 2 3 4 | 0 1 2 3 4 0 1 2 3 4 |
| A_1 | 3 38 49 12 2 5 35 45 18 1 5 33 42 22 2 7 38 46 12 1 2 31 58 11 2 | 2 31 58 11 2 |
| A_2 | 12 51 32 7 2 5 50 38 8 3 7 49 38 9 1 10 48 36 9 1 4 48 43 9 0 | 4 48 43 9 0 |
| A_3 | 20 40 37 6 1 15 50 31 7 1 15 38 42 7 2 13 49 35 6 1 12 45 33 12 2 | 12 45 33 12 2 |
| A_4 | 7 25 59 12 1 6 37 39 20 2 7 19 56 20 2 6 28 46 22 2 9 22 49 19 5 | 9 22 49 19 5 |

Data from the first survey (2014)

| C^1 | C_1^1 | C_2^1 | C_3^1 | C_4^1 | C_5^1 |
|-----|-------|-------|-------|-------|-------|
| S^1 | 0 4 0 4 0 4 | 0 4 |
| A_1 | 0.622 0.378 0.178 0.822 0.822 0.178 0.778 0.222 0.400 0.600 |
| A_2 | 0.333 0.667 0.556 0.444 0.667 0.333 0.467 0.533 0.289 0.711 |
| A_3 | 0.467 0.533 0.356 0.644 0.822 0.178 0.356 0.644 0.711 0.289 |
| A_4 | 0.422 0.578 0.244 0.756 0.800 0.200 0.400 0.600 0.422 0.578 |

Data from the second survey (2016)

| C^2 | C_1^2 | C_2^2 | C_3^2 | C_4^2 | C_5^2 |
|-----|-------|-------|-------|-------|-------|
| S^2 | 0 1.333 2.667 4 0 1.333 2.667 4 0 1.333 2.667 4 0 1.333 2.667 4 0 1.333 2.667 4 |
| A_1 | 0.167 0.313 0.260 0.260 0.146 0.271 0.458 0.125 0.188 0.271 0.427 0.115 0.042 0.406 0.396 0.156 0.083 0.385 0.427 0.104 |
| A_2 | 0.052 0.375 0.198 0.375 0.052 0.115 0.792 0.042 0.156 0.406 0.354 0.083 0.125 0.479 0.292 0.104 0.083 0.479 0.333 0.104 |
| A_3 | 0.083 0.375 0.344 0.198 0.146 0.250 0.594 0.010 0.156 0.625 0.188 0.031 0.625 0.323 0.052 0.000 0.146 0.396 0.406 0.052 |
| A_4 | 0.065 0.333 0.292 0.313 0.083 0.104 0.781 0.031 0.135 0.271 0.427 0.167 0.052 0.271 0.542 0.135 0.042 0.271 0.438 0.250 |

End of Table 2
Step 2. According to the principal in Section 2.2 and decision-makers’ experience, we analyze the series weights and the attribute weights of PPSQ.

\[ g = 0.3 \] is determined by property management experts’ experience, and there exists \( q = 3 \) in a three stages’ survey. According to Eqs (5) and (6), three stages’ weights are determined as follows:

\[ H = (0.133, 0.333, 0.533). \]  

This example assumes that property management experts have the same preference on all attributes in different stages. Summarizing the experts’ preferences for all attributes, the attribute weights are determined as follows:

\[ H_2 = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5) \left| \omega_1 \leq 0.8 \omega_2, \omega_2 - \omega_3 \leq 0.15, \omega_2 - \omega_3 \geq 0.001, \omega_4 \leq 0.25, \omega_4 \geq 0.01, \omega_1 - \omega_2 \leq \omega_4 - \omega_5 \right. \]  

Step 3. Since the scale value is positive correlation with PPSQ, the maximum (minimum) value of scale correspond to biggest (smallest) value of PPSQ. According to the practice of property service, the reference point is determined to be half of the sum of the maximum scale value and the minimum scale value. The reference points divide scale values into two categories: the smaller scale values and the larger scale values. Since the total number of visitors who evaluate each project is the same in a certain stage, there are some principles as follows: the fewer the number of visitors using the smaller scale value, the better the property service quality and vice versa; the fewer the number of visitors using greater scale values, the lower the quality of the property service and vice versa. By combining the above rules and related data, the calculation results of PIS are shown in Table 4.

Step 4. Collect the overall preferences of property management experts and determine the preferences set \( \Omega \). Their preference order is summarized as follows:

\[ \Omega = \{(1,2), (2,3), (4,1), (4,3)\}, \]  

where \( (k,l) \) denotes the public property service project \( A_k \) is better than \( A_l \) in the opinion of experts. For example, the array \( (1, 2) \) denotes the property service project \( A_1 \) is better than \( A_2 \) in the opinion of experts.

Step 5. Determine the dynamic consistency and dynamic inconsistency, and construct the dynamic LINMAP model of the PPSQ evaluation data with 3 stages. By combining Eqs (20), (21), (25), (26), (27) and the data from Tables 3 and 4, the dynamic LINMAP model of PPSQ can be expressed as follows.
\[ \min \{ \lambda_{12} + \lambda_{23} + \lambda_{41} + \lambda_{43} \} = 1.0 \]

\[ 0.660 \omega_5^2 + 0.407 \omega_4^2 + 0.635 \omega_3^2 + 0.564 \omega_2^2 + 0.440 \omega_1^2 + 0.869 \omega_{2}^2 + 1.968 \omega_{2}^2 + 0.884 \omega_{2}^2 + 1.520 \omega_{2}^2 + 1.235 \omega_{2}^2 + 1.106 \omega_{2}^2 + 3.516 \omega_{1}^2 + 1.510 \omega_{1}^2 - 1.312 \omega_{1}^2 + 1.725 \omega_{1}^2 \geq 1.0 \]

\[ \lambda_{12} - 0.137 \omega_{5}^2 - 0.176 \omega_{4}^2 - 0.150 \omega_{3}^2 - 0.121 \omega_{2}^2 - 0.194 \omega_{1}^2 + 0.756 \omega_{1}^2 + 1.354 \omega_{2}^2 - 0.231 \omega_{2}^2 - 0.298 \omega_{2}^2 - 0.212 \omega_{2}^2 + 3.287 \omega_{1}^2 - 5.171 \omega_{1}^2 + 0.857 \omega_{1}^2 + 2.157 \omega_{1}^2 + 1.699 \omega_{1}^2 \geq 0 \]

\[ \lambda_{23} - 0.030 \omega_{5}^2 - 0.071 \omega_{4}^2 - 0.014 \omega_{3}^2 - 0.006 \omega_{2}^2 - 0.061 \omega_{1}^2 - 0.798 \omega_{1}^2 - 1.059 \omega_{2}^2 - 0.406 \omega_{2}^2 - 0.047 \omega_{2}^2 + 0.118 \omega_{2}^2 - 1.849 \omega_{1}^2 + 2.169 \omega_{1}^2 - 0.857 \omega_{1}^2 + 1.521 \omega_{1}^2 - 4.354 \omega_{1}^2 \geq 0 \]

\[ \lambda_{41} - 0.163 \omega_{5}^2 + 0.943 \omega_{4}^2 - 0.154 \omega_{3}^2 - 0.155 \omega_{3}^2 + 0.350 \omega_{2}^2 - 0.393 \omega_{2}^2 - 1.278 \omega_{2}^2 - 0.165 \omega_{2}^2 - 0.415 \omega_{2}^2 - 0.524 \omega_{2}^2 - 1.991 \omega_{1}^2 + 1.244 \omega_{1}^2 - 0.075 \omega_{1}^2 - 3.022 \omega_{1}^2 + 0.292 \omega_{1}^2 \geq 0 \]

\[ \lambda_{43} - 0.330 \omega_{5}^2 - 0.240 \omega_{4}^2 - 0.317 \omega_{3}^2 - 0.282 \omega_{2}^2 - 0.220 \omega_{2}^2 - 0.434 \omega_{2}^2 - 0.784 \omega_{2}^2 - 0.442 \omega_{2}^2 - 0.760 \omega_{2}^2 - 0.618 \omega_{2}^2 - 0.553 \omega_{2}^2 - 1.758 \omega_{1}^2 - 0.750 \omega_{1}^2 + 0.656 \omega_{1}^2 - 2.362 \omega_{1}^2 \geq 0 \]

\[ \omega_1 \geq 0.15, \omega_2 \geq 0.10, \omega_3 \geq 0.12, \omega_4 \geq 0.15, \omega_5 \leq 0.45, \omega_6 \leq 0.20, \omega_7 \leq 0.25, \omega_8 \geq 1.3 \omega_2 \]

\[ \omega_2 \geq 0.05, \omega_3 \leq 0.3, \omega_4 \geq 0.2 \omega_2 \]

\[ \omega_5 \geq 0.15, \omega_6 \geq 0.12, \omega_7 \geq 0.15, \omega_8 \leq 0.20, \omega_9 \leq 0.25, \omega_{10} \geq 1.3 \omega_2 \]

\[ \omega_2 \geq 0.05, \omega_3 \leq 0.3, \omega_4 \geq 0.2 \omega_2 \]

\[ \omega_5 \leq 0.15, \omega_6 \geq 0.10, \omega_7 \leq 0.12, \omega_8 \geq 0.15, \omega_9 \leq 0.45, \omega_{10} \leq 0.20, \omega_{11} \leq 0.25, \omega_{12} \geq 1.3 \omega_2 \]

\[ \omega_5 \geq 0.05, \omega_6 \leq 0.3, \omega_7 \geq 0.4 \omega_{10} \]

\[ \omega_5 + \omega_6 + \omega_7 + \omega_8 = 1, \omega_9 + \omega_3 + \omega_4 + \omega_5 \geq 1, \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 = 1 \]

\[ \lambda_{12} = 0, \lambda_{23} = 0, \lambda_{41} = 0, \lambda_{43} = 0 \]

(28)

**Step 6.** Calculate Eq. (28) by LINGO 11.0. When \( h = 1.0 \), the calculation result of attribute weight is determined as follows:

\[ (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5) = (0.441, 0.150, 0.100, 0.120, 0.189); \]

(29)

\[ (\omega_1^2, \omega_2^2, \omega_3^2, \omega_4^2, \omega_5^2) = (0.150, 0.100, 0.100, 0.500, 0.150); \]

(30)

\[ (\omega_1^3, \omega_2^3, \omega_3^3, \omega_4^3, \omega_5^3) = (0.450, 0.100, 0.180, 0.120, 0.150). \]

(31)

**Table 4. The PIS of different attributes, scales and stages**

| Data from the first survey (2014) | \( C_1 \) | \( C_1^1 \) | \( C_2 \) | \( C_3 \) | \( C_4 \) | \( C_5 \) |
|----------------------------------|--------|--------|--------|--------|--------|--------|
| \( S_1 \)                        | 0      | 4      | 0      | 4      | 0      | 4      |
| \( PIS \)                        | 0.333  | 0.667  | 0.178  | 0.822  | 0.667  | 0.333  |
|                                  |        |        |        |        |        |        |

| Data from the second survey (2016) | \( C_2 \) | \( C_2^1 \) | \( C_2^2 \) | \( C_3 \) | \( C_4 \) | \( C_5 \) |
|----------------------------------|--------|--------|--------|--------|--------|--------|
| \( S_2 \)                        | 0      | 1.333 | 2.667 | 4      | 0      | 1.333 | 2.667 | 4      | 0      | 1.333 | 2.667 | 4      |
| \( PIS \)                        | 0.052  | 0.313 | 0.344 | 0.375  | 0.052  | 0.104 | 0.792 | 0.125  | 0.135  | 0.271  | 0.427 | 0.167  | 0.042  | 0.271  | 0.542  | 0.156  | 0.042  | 0.271  | 0.438  | 0.250  |

| Data from the third survey (2018) | \( C_3 \) | \( C_3^1 \) | \( C_3^2 \) | \( C_3^3 \) | \( C_3^4 \) | \( C_3^5 \) |
|----------------------------------|--------|--------|--------|--------|--------|--------|
| \( S_2 \)                        | 0      | 1      | 2      | 3      | 4      | 0      | 1      | 2      | 3      | 4      | 0      | 1      | 2      | 3      | 4      | 0      | 1      | 2      | 3      | 4      |
| \( PIS \)                        | 0.029  | 0.240  | 0.567  | 0.115  | 0.019  | 0.048  | 0.337  | 0.433  | 0.192  | 0.029  | 0.048  | 0.183  | 0.538  | 0.212  | 0.019  | 0.058  | 0.269  | 0.442  | 0.212  | 0.019  | 0.212  | 0.558  | 0.183  | 0.048  |
Step 7. By use of Eq. (12) and the related data, the dynamic comprehensive value of the weighted Euclidean distance square of all property service projects are calculated as follows:

\[ D(A_1) = 0.066; \]  \hspace{1cm} (32)  
\[ D(A_2) = 0.138; \]  \hspace{1cm} (33)  
\[ D(A_3) = 0.242; \]  \hspace{1cm} (34)  
\[ D(A_4) = 0.008. \]  \hspace{1cm} (35)  

According to the analysis in Section 2.3, the smaller the comprehensive distance, the better the alternative. By combining the above rule and the comprehensive value of each project, the ranking order is determined as:

\[ A_4 \succ A_1 \succ A_2 \succ A_3. \]  \hspace{1cm} (36)  

4. Comparison and analysis

4.1. Sensitivity analysis

Because consistency is greater than inconsistency in the LINMAP model, there exists \( h > 0 \). And because \( h > 1.0 \), there is no feasible solution to Eq. (28). Therefore, we take 0.1 as the gradient and take different values between 0 and 1.0, and solve the calculation Eq. (28)–(36). Table 5 summarizes the comprehensive value and ranking order of all projects.

As can be seen from Table 5, the comprehensive value of the four projects remains the same when \( h \in [0.1, 0.8] \), and the comprehensive value of the four projects has some changes when \( h \in [0.9, 1.0] \). However, the ranking order of all projects remains still the same when \( h \in [0.1, 1.0] \).

Because there is no empirical analysis of \( h \) value in the existing LINMAP research results (Shocker & Srinivasan, 1973; Li & Yang, 2004; Bereketli et al., 2011; Li, 2008; Li et al., 2010; Wan & Li, 2013; Lv & Li, 2004), we combine the general practice and the experience of decision-makers to determine \( h = 1.0 \). Furthermore, we calculate the comprehensive value

| \( h \) | \( D(A_1) \) | \( D(A_2) \) | \( D(A_3) \) | \( D(A_4) \) | Ranking order |
|---|---|---|---|---|---|
| 0.1 | 0.070 | 0.118 | 0.208 | 0.009 | \( A_4 \succ A_1 \succ A_2 \succ A_3 \) |
| 0.2 | 0.070 | 0.118 | 0.208 | 0.009 | \( A_4 \succ A_1 \succ A_2 \succ A_3 \) |
| 0.3 | 0.070 | 0.118 | 0.208 | 0.009 | \( A_4 \succ A_1 \succ A_2 \succ A_3 \) |
| 0.4 | 0.070 | 0.118 | 0.208 | 0.009 | \( A_4 \succ A_1 \succ A_2 \succ A_3 \) |
| 0.5 | 0.070 | 0.118 | 0.208 | 0.009 | \( A_4 \succ A_1 \succ A_2 \succ A_3 \) |
| 0.6 | 0.070 | 0.118 | 0.208 | 0.009 | \( A_4 \succ A_1 \succ A_2 \succ A_3 \) |
| 0.7 | 0.070 | 0.118 | 0.208 | 0.009 | \( A_4 \succ A_1 \succ A_2 \succ A_3 \) |
| 0.8 | 0.070 | 0.118 | 0.208 | 0.009 | \( A_4 \succ A_1 \succ A_2 \succ A_3 \) |
| 0.9 | 0.069 | 0.119 | 0.208 | 0.009 | \( A_4 \succ A_1 \succ A_2 \succ A_3 \) |
| 1.0 | 0.066 | 0.138 | 0.242 | 0.008 | \( A_4 \succ A_1 \succ A_2 \succ A_3 \) |
of three stages and the ranking order of four projects respectively when \( h = 1.0 \), and the ranking orders of three stages are as follows: \( A_4 \succ A_2 \succ A_3 \succ A_1 \), \( A_4 \succ A_1 \succ A_2 \succ A_3 \) and \( A_4 \succ A_1 \succ A_3 \succ A_2 \).

4.2. PPSQ evaluation based on the traditional LINMAP method

As the LINMAP method based on the classic MAMS matrix (Zuo et al., 2019) is the closest to the new method of this paper in the traditional LINMAP researches, we take it as the representative of the traditional LINMAP method. In order to summarize the characteristics of the new method, the traditional LINMAP method is used to analyze the dynamic PPSQ example. Since the traditional LINMAP method cannot process the data from three stages at the same time, we will analyze the data of each stage respectively.

Using the evaluation data of first stage in Section 3, the traditional LINMAP model based on the first-stage PPSQ data can be denoted as

\[
\begin{align*}
\min \{ & \lambda_{12} + \lambda_{23} + \lambda_{41} + \lambda_{43} \\
& 1.106 \omega_1 + 3.516 \omega_2 + 0.150 \omega_3 + 1.312 \omega_4 - 4.725 \omega_5 \geq h \\
& \lambda_{12} + (3.287 \omega_1 - 5.171 \omega_2 + 0.857 \omega_3 + 2.157 \omega_4 + 1.699 \omega_5) \geq 0 \\
& \lambda_{23} + (-1.849 \omega_1 + 2.169 \omega_2 - 0.857 \omega_3 + 1.521 \omega_4 - 4.354 \omega_5) \geq 0 \\
& \lambda_{41} + (-1.991 \omega_1 - 1.244 \omega_2 - 0.075 \omega_3 - 3.022 \omega_4 + 0.292 \omega_5) \geq 0 \\
& \lambda_{43} + (-0.553 \omega_1 - 1.758 \omega_2 - 0.075 \omega_3 + 0.656 \omega_4 - 2.362 \omega_5) \geq 0 \\
& \lambda_{12} = 0, \lambda_{23} = 0, \lambda_{41} = 0, \lambda_{43} = 0 \\
& \omega_1 \geq 0.15, \omega_2 \geq 0.1, \omega_3 \geq 0.1, \omega_4 \geq 0.12, \omega_5 \geq 0.15, \omega_6 \leq 0.45, \omega_2 \leq 0.2 \\
& \omega_3 \leq 0.25, \omega_4 \geq 1.3 \omega_1, \omega_5 - \omega_3 \leq 0.05, \omega_5 - \omega_2 \geq 0.5 - \omega_4 \\
& \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 = 1
\end{align*}
\]

Using the evaluation data of second stage in Section 3, the traditional LINMAP model based on the second-stage PPSQ data can be denoted as

\[
\begin{align*}
\min \{ & \lambda_{13} + \lambda_{23} + \lambda_{41} + \lambda_{43} \\
& 0.869 \omega_1 + 1.968 \omega_2 + 0.884 \omega_3 + 1.520 \omega_4 + 1.235 \omega_5 \geq h \\
& \lambda_{12} + (0.756 \omega_1 + 1.354 \omega_2 - 0.231 \omega_3 - 0.298 \omega_4 - 0.212 \omega_5) \geq 0 \\
& \lambda_{23} + (-0.798 \omega_1 - 1.059 \omega_2 - 0.046 \omega_3 - 0.047 \omega_4 + 0.118 \omega_5) \geq 0 \\
& \lambda_{41} + (-0.393 \omega_1 - 1.278 \omega_2 - 0.165 \omega_3 - 0.415 \omega_4 - 0.524 \omega_5) \geq 0 \\
& \lambda_{43} + (-0.434 \omega_1 - 0.984 \omega_2 - 0.442 \omega_3 - 0.760 \omega_4 - 0.618 \omega_5) \geq 0 \\
& \lambda_{12} = 0, \lambda_{23} = 0, \lambda_{41} = 0, \lambda_{43} = 0 \\
& \omega_1 \geq 0.15, \omega_2 \geq 0.1, \omega_3 \geq 0.1, \omega_4 \geq 0.12, \omega_5 \geq 0.15, \omega_6 \leq 0.45, \omega_2 \leq 0.2 \\
& \omega_3 \leq 0.25, \omega_4 \geq 1.3 \omega_1, \omega_5 - \omega_3 \leq 0.05, \omega_5 - \omega_2 \geq 0.5 - \omega_4 \\
& \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 = 1
\end{align*}
\]

Using the evaluation data of third stage in Section 3, the traditional LINMAP model based on the third-stage PPSQ data can be denoted as:
The calculation results of Eqs (37)∼(39) are 
(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5) = (0.150, 0.115, 0.100, 0.485, 0.150),
(\omega_1^3, \omega_2^3, \omega_3^3, \omega_4^3, \omega_5^3) = (0.150, 0.100, 0.250, 0.225, 0.275)
and
(\omega_1^3, \omega_2^3, \omega_3^3, \omega_4^3, \omega_5^3) = (0.150, 0.115, 0.100, 0.485, 0.150) respectively. Then, the comprehensive score of all projects are calculated, and the final ranking results of four projects in three stages are
$A_4 > A_1 > A_2 > A_3$ and $A_4 > A_2 > A_3 > A_1$ respectively.

4.3. Comparative analysis and summary

Based on the dynamic PPSQ example analysis by use of the traditional LINMAP method and the new method, the results of ranking order are analyzed. The final results by using the two methods are summarized in Table 6.

| t = 1 | t = 2 | t = 3 | Comprehensive ranking order |
|-------|-------|-------|-----------------------------|
| $A_4 > A_2 > A_3 > A_1$ | $A_4 > A_1 > A_2 > A_3$ | $A_4 > A_1 > A_3 > A_2$ | $A_4 > A_1 > A_2 > A_3$ |
| $A_4 > A_2 > A_3 > A_1$ | $A_4 > A_1 > A_2 > A_3$ | $A_4 > A_2 > A_3 > A_1$ | $- -$ |

Except that the project $A_4$ is the best choice of all ranking results, the other projects varies greatly in the different ranking results. By analyzing these different ranking results from the above table, the conclusions and their reasons are as follows:

1) The evaluation results of each stage are different from the comprehensive evaluation results. Since the traditional LINMAP method cannot simultaneously fuse the evaluation information of different stages, we discuss the evaluation results using the new method. Obviously, the ranking order in the first and third stages is not the same as the comprehensive ranking by using the new method of this paper. The reason is that the comprehensive results reflect data fusion of three stages. The method based on multi-stage data fusion effectively solves the problem of dynamic PPSQ evaluation.
2) The evaluation results by the new method are different from those of the traditional LINMAP method. Except for the second stage, the ranking results of the comprehensive ranking based on the data of each stage are different. It reflects the difference between the two methods, and the new method is the improvement of the traditional LINMAP method. Furthermore, only the new method can process multi-stage data simultaneously, while not the traditional LINMAP method. It shows the advantages of the new method.

3) The comprehensive ranking order reflects the characteristics of these projects. The PSQ for all the projects ranked from the best to the worst: grand theater, museum, library and science museum. Combined with the field investigation, the reasons for the above ranking are analyzed as follows: The visitors in grand theater and museum are decent, and the property service pressure is low. Therefore, the PSQ is low. The proportion of youngsters' visitors in the library and science museum is high. Therefore, the PSQ is high.

Conclusions

This paper proposes a dynamic LINMAP model based on general MAMS matrix, and it is taken as a new method of PPSQ evaluation which suitable for multi-stage information fusion. On the one hand, the new method can effectively evaluation PPSQ based on complex information. The complexity of modern PPSQ evaluation is characterized by dynamic, heterogeneous and large-scale data. Dynamic LINMAP model is an extension of the classical LINMAP model. The information fusion method based on dynamic LINMAP model is an innovation of the PPSQ evaluation theory. On the other hand, using the dynamic LINMAP method to solve objective weights is the core of the new PPSQ method. The objective weights of the same attribute in different stages are not exactly the same. From Eqs (29) to (31), it can be seen that the objective weight of the same attribute changes at different evaluation stages, among which the weight of attitude and facilities changes greatly. The feasibility and stability of the new method are verified by an example analysis.

However, the threshold value is determined according to the experience of decision-makers and the existing research results, and there is not enough quantitative research on how to determine the threshold value. This is a question worthy of further study. In addition, the new method can also be used for hotel, library, hospital, government and other service fields. The new method can further expand the application field in the future research.

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