Theoretical and experimental method for determining the frequency-dependent dynamic modulus of elasticity and damping characteristics of a titanium alloy OT-4

V N Paimushin¹,², V A Firsov¹, R K Gazizullin¹ and V M Shishkin³

¹Kazan National Research Technical University, 10 K.Marks Street, 420111, Kazan, Russia
²Kazan Federal University, 18 Kremlyovskaya Street, 420008, Kazan, Russia
³Vyatka State University, 36 Moscow Street, 610000, Kirov, Russia

E-mail: vpajmushin@mail.ru

Abstract. A significant reduction in the dynamic modulus of elasticity of the titanium alloy OT-4 during cyclic deformation in the frequency range of 0 ÷ 25 Hz and its further stabilization at frequencies of 25÷80 Hz on the basis of studies of damped bending vibrations of a series of test samples is shown. The averaged amplitude dependence of the logarithmic vibration decrement of this alloy under tension-compression is obtained by minimizing the quadratic discrepancy between the experimental and calculated logarithmic decrements of the oscillations of the test samples.

1. Introduction

Titanium alloys are distinguished by high specific strength and rigidity in combination with high thermal and corrosion resistance, which opens up great prospects for their use in the aviation industry, rocket production and transport engineering [1, 2]. This determines the need for an adequate determination of their elastic and damping properties that correspond to the operating conditions of the structure corresponding to the actual conditions.

The least studied mechanical characteristics of titanium and its alloys include the damping ability, which influences the dynamic strength of structural elements in cyclic deformation modes and, above all, at resonance, realized in the structure when the frequencies of its natural oscillations coincide with the frequency of external cyclic exposure. In this mode of loading, as is well known, the amplitude values of the parameters of the dynamic stress-strain state increase many times over. Correct and reliable their theoretical definition with the accuracy necessary for practical purposes requires proper consideration in the design ratios of the damping properties of materials of structures caused by internal friction.

The most reliable information about the characteristics of the elasticity of the material gives a dynamic method based on the experimental measurement of the frequency of bending vibrations of cantilever test specimens and comparing it with the calculated frequency of an ideally elastic sample [3, 4]. During testing of test specimens from titanium alloy OT-4 in the mode of damped flexural vibrations, a significant decrease and stabilization of the elastic modulus of this alloy with increasing frequency of oscillations compared to its static nominal value was established, which gives grounds to
call this modulus a frequency-dependent dynamic elastic modulus material [5] (in contrast to the well-known interpretation of the dynamic modulus of elasticity existing in the theory of energy dissipation [6, 7]).

The damping properties of a material under tension-compression are determined by the logarithmic vibration decrement \( \delta(\varepsilon_0) \), depending on the strain amplitude \( \varepsilon_0 \) [8, 9]. The dependence \( \delta(\varepsilon_0) \) can be determined by comparing the calculated and experimental logarithmic vibration decrement of a test sample with several amplitudes of oscillations of its free end [10, 11].

2. Determination of the frequency dependence of the dynamic modulus of elasticity of titanium alloy OT-4

The tests were carried out in the mode of damped bending vibrations of 15 horizontally fixed test samples with lengths \( L = 1450 \pm 750 \text{mm} \), which corresponds to the range of experimentally measured frequencies \( f = 73.3 \pm 2.68 \text{Hz} \). Static modulus of elasticity and density of OT-4 alloy are \( E = 1.1 \times 10^5 \text{MPa} \) and \( \rho = 4430 \text{kg/m}^3 \). The width and thickness of the test samples were \( b = 20 \text{mm} \); \( h = 1.95 \text{mm} \). The values of the dynamic modulus of elasticity of the alloy OT-4 in the specified frequency range of \( f \) were determined by the formula

\[
E_d = 38.3216 \rho L^3 f^2 / h^2,
\]

that follows from the known relation for finding the lowest frequency of free bending vibrations of a fixed beam [12] \( f = (1.875/L)^2 \sqrt{EI/m/(2\pi)} \), where \( I = bh^3/12 \); \( m = \rho bh \). The values of \( E_d \) obtained from (1) have some experimental scatter. To build a smoothing dependency, function \( E_d(f) = c_1 \exp(c_2 f) + c_3 \exp(c_4 f) \) was used. Parameters \( c_1, c_2, c_3, c_4 \) were determined by direct search from the condition \( F(c_1, c_2, c_3, c_4) = \sum_i (1 - E_{d,i}^* / E_{d,i})^2 = \text{min} \) taking into account the constraint \( E_d(0) = c_1 + c_3 = E \). The search for the minimum of the objective function was carried out by the Hook-Jeeves method [13, 14]. Thus, we found the dependence

\[
E_d(f) = 9.472 \times 10^4 \exp(-2.156 \times 10^4 f) + 1.528 \times 10^4 \exp(-1.238 \times 10^3 f),
\]

giving values of \( E_d \) in MPa. Figure 1 shows the dependences of \( E_d(f) \): the points by the formula (1), the line by the formula (2). There is a significant decrease in the dynamic modulus of elasticity \( E_d \) relative to the static modulus \( E \) in the frequency range \( f = 0 \div 25 \text{Hz} \) (approximately 14%) and its further practical stabilization at frequencies \( f > 25 \text{Hz} \).

\[\text{Figure 1. Dependencies } E_d(f) \]

\[\text{Figure 2. Dependence } \delta(\varepsilon_0) \]

3. Determination of the logarithmic vibration decrement of titanium alloy OT-4

To represent the dependence \( \delta(\varepsilon_0) \), which determines the damping properties of the OT-4 alloy, the function

\[
\delta(\varepsilon_0) = \alpha \varepsilon_0^\beta,
\]

is used.
is used. The definition of the dependence $\delta(e_0)$ is reduced to finding the parameters $\alpha$ and $\beta$ of function (3) so that the condition $F(\alpha, \beta) = \|\{\hat{\delta}\}^* - \{\hat{\delta}\}\| = \min$ is satisfied. Here $F(\alpha, \beta)$ is the objective function, $\{\hat{\delta}\}^*, \{\hat{\delta}\}$ are vectors containing, respectively, the experimental and calculated logarithmic vibration decrements $\delta(A)$ of the test sample with several amplitudes $A_j, \ j = 1, 2, \ldots, n$, of its free end. In [15], it was shown that the external aerodynamic damping can have a significant effect on the damping properties of the test sample. Therefore, the calculated dependence $\delta(A)$ should consist of the internal damping parameter $\delta_{i}(A)$ and the aerodynamic component of the damping $\delta_{e}(A)$ [16, 17]. To determine the logarithmic vibration decrement $\delta_{i}(A)$ and $\delta_{e}(A)$ with the amplitudes of the vibrations $A_j, \ j = 1, 2, \ldots, n$, it is proposed to use the finite element method [18–27] with modeling the test sample by beam elements. Matrices of internal and external aerodynamic damping of a beam finite element based on the Thomson–Kelvin–Voigt model [28], representing the viscous-elastic properties of the material, and Morison approximation [29, 30] for the aerodynamic linear force along the length of the test sample were obtained. Dependencies (3) obtained from experimental logarithmic decrement of oscillations $\delta^*(A_j)$ of different test samples may have some variation. Therefore, they must be determined on several test samples, averaging the results obtained thereafter. To obtain the average dependence (3), 9 test samples with lengths $L=150$–$400$ mm were selected. As a result, the amplitude dependence of the logarithmic vibrations decrement of titanium alloy OT-4 $\delta(e_0) = 0.1342e_0^{0.2895}$, presented in Figure 2, was obtained.

4. Conclusion

Using the theoretical-experimental method, we obtained the frequency dependence of the dynamic elastic modulus and the amplitude dependence of the logarithmic vibrations decrement of the OT-4 titanium alloy based on the study of damped bending vibrations of a series of test samples. The considered method can be applied to determine the dynamic characteristics of elasticity and damping properties of other structural materials.

Acknowledgments

The study was carried out by a grant from the Russian Science Foundation (project No. 14-19-00667).

References

[1] Chechulin B B, Ushkov S S, Razuevaeva I N and Goldfein V N 1977 Titanium alloys in mechanical engineering (Leningrad, Mashinostroyeniye) (In Russian)
[2] Il’in A A, Kolachev B A and Polkin I S 2009 Titanium alloys. Composition, structure, properties. Handbook (Moscow, VILS-MATI) (In Russian)
[3] Paimushin V N, Firsov V A, Gyunal I, Egorov A G and Kayumov R A 2014 Theoretical-Experimental Method for Determining the Parameters of Damping Based on the Study of Damped Flexural Vibrations of Test Specimens. 3. Identification of the Characteristics of Internal Damping Mechanics of Composite Materials 50 (5) 633-46 DOI: 10.1007/s11029-014-9451-x
[4] Paimushin V N, Firsov V A, Gyunal I and Shishkin V M 2015 Identification of the elasticity and damping characteristics of a fiberglass based on a study of dying flexural vibrations of test samples Mechanics of Composite Materials 51 (3) 285-300 DOI: 10.1007/s11029-015-9500-0.
[5] Paimushin V N, Firsov V A, Gyunal I and Shishkin V.M. 2017 Accounting for the frequency-dependent dynamic elastic modulus of Duralumin in deformation problems Journal of Applied Mechanics and Technical Physics 58 (3) 517-28 DOI: 10.1134/S0021894417030178.
[6] Postnikov V S 1969 *Internal friction in metals* (Moscow, Metallurgiya) (in Russian)

[7] Krishtal M A, Piguzov Yu V and Golovin S A 1964 *Internal friction in metals and alloys* (Moscow, Metallurgiya) (in Russian)

[8] Pisarenko G S, Yakovlev A P and Matveev V V 1971 *Vibration-absorbing properties of structural materials: A Handbook* (Kiev: Naukova Dumka) (in Russian)

[9] Blitsstein Yu M and Meshkov S I (1974) On the amplitude dependence of the dissipated energy during oscillations. *Energy scattering with mechanical system oscillations* (Kiev, Naukova Dumka) 24–34 (in Russian)

[10] Gyunal I, Paimushin V N, Firsov V A and Shishkin V M 2017 Identification of the damping properties of rigid isotropic materials by studying the damping flexural vibrations of test specimens *Mechanics of Solids* 52 (2) 200-11 DOI: 10.3103/S0025654417020108

[11] Paimushin V N, Firsov V A, Gyunal I and Shishkin V M 2016 Identification of the elastic and damping characteristics of carbon fiber-reinforced plastic based on a study of damping flexural vibrations of test specimens *Journal of Applied Mechanics and Technical Physics* 57 (4) 720-30 DOI: 10.1134/S0021894416040179.

[12] Kiselev V A 1980 *Construction mechanics. Special course. Dynamics and stability of structures* (Moscow, Stroyizdat) (in Russian)

[13] Shup T 1982 *Computer engineering problem solving: A practical guide* (Moscow, Mir) (in Russian)

[14] Attekov A V, Galkin S V and Zarubin V S 2003 *Optimization Methods* (Moscow, Moscow State Technical University named after N.E. Bauman) (in Russian)

[15] Paimushin V N, Firsov V A, Gyunal I and Egorov A G 2014 Theoretical-experimental method for determining the parameters of damping based on the study of damped flexural vibrations of test specimens 1. Experimental basis *Mechanics of Composite Materials* 50(2) 127-36. DOI: 10.1007/s11029-014-9400-8.

[16] Egorov A G, Kamalutdinov A M, Nuriev A N and Paimushin V N 2014 Theoretical-experimental Method for Determining the Parameters of Damping Based on the Study of Damped Flexural Vibrations of Test Specimens 2. Aerodynamic Component of Damping *Mechanics of Composite Materials* 50 (3) 267-78 DOI: 10.1007/s11029-014-9413-3

[17] Egorov A G, Kamalutdinov A M, Paimushin V N and Firsov V A 2016 Theoretical-experimental method of determining the drag coefficient of a harmonically oscillating thin plate *Journal of Applied Mechanics and Technical Physics* 57 (2) 275-82

[18] Zienkiewicz O C 1971 *The Finite Element Method in Engineering Science* (London, McGraw-Hill)

[19] Segerlind L J 1976 *Applied finite element analysis* (New York: John Wiley&Sons).

[20] Sultanov L U 2016 Modelling of large deformations of elastoplastic solids using FEM *IOP Conference Series: Materials Science and Engineering* 158 (1) 012088

[21] Dautov R and Lapin A 2015 Finite element approximation and iterative method solution of elliptic control problem with constraints to gradient of state *Lobachevskii Journal of Mathematics* 36 (1) 65–75 DOI: 10.1134/S1995080215010059

[22] Badriev I B, Banderov V V, Gnedenkova V L, Kalacheva N V, Korablev A I and Tagirov R R 2015 On the finite dimensional approximations of some mixed variational inequalities *Applied Mathematical Sciences* 9 (113–116) 5697-705 DOI: 10.12988/ams.2015.57480

[23] Badriev I B 1983 Difference-schemes for linear-problems of the filtering with discontinuous law *Izvestiya Vysshikh Uchebnykh Zavedenii Matematika* 5 3–12

[24] Solov’ev S I 1985 Fast methods for solving mesh schemes of the finite element method of second order accuracy for the Poisson equation in a rectangle *Izvestiya Vysshikh Uchebnykh Zavedenii Matematika* 10 71-4

[25] Sultanov L U 2016 Calculation of elastic-plastic deformations by FEM *IOP Conference Series: Materials Science and Engineering* 158 (1) 012090 DOI: 10.1088/1757-899X/158/1/012090

[26] Berezhnoi D V and Sagdatullin M K 2015 Calculation of interaction of deformable designs
taking into account friction in the contact zone by finite element method *Contemporary Engineering Sciences* **8** (21–24) 1091-8 DOI: 10.12988/ces.2015.58237

[27] Chebakova V J, Gerasimov A V and Kirpichnikov A P 2016 On the solving of one type of problems of mathematical physics *IOP Conference Series: Materials Science and Engineering* **158** (1) 012023 DOI: 10.1088/1757-899X/158/1/012023

[28] Khil’chevsky V V and Dubenets V G 1977 *Energy scattering with oscillations of thin-walled structural elements* (Kiev, Vishcha Shkola) (in Russian)

[29] Sarpkaya T 1986 Force on a circular cylinder in viscous oscillatory flow at low Keulegan-Carpenter numbers *Journal of Fluid Mechanics* **165** 61–71

[30] Keulegan G H and Carpenter L H 1958 Forces on cylinders and plates in an oscillating fluid *Journal of Research of National Bureau of Standards* **60** (5) 423-40