Magnetic moments of the octet, low-lying charm, and low-lying bottom baryons in a nuclear medium

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Abstract

We study the magnetic moments of the octet, low-lying charm, and low-lying bottom baryons with nonzero light quarks in symmetric nuclear matter. This is the first study of estimating the medium modifications of magnetic moments for these low-lying charm and bottom baryons.

1. Introduction

In Ref. [1], we studied the properties of the octet, low-lying charm, and low-lying bottom baryons with nonzero light $u$ and $d$ quarks in symmetric nuclear matter. The Lorentz-scalar effective masses and vector potentials were calculated by the quark-meson coupling (QMC) model, which was invented by Guichon [2]. The model has successfully been applied for various studies, the properties of finite (hyper)nuclei [3, 4, 6, 7, 8, 9, 10, 11, 12], hadron properties in medium [13, 14, 15, 16, 17, 18], nuclear reactions [19, 20, 21, 22, 23, 24, 25, 26, 27], and neutron star structure [28, 29, 30, 31, 32]. (See Refs. [33, 34, 35] for reviews.)

Self-consistent exchange of the Lorentz-scalar-isoscalar $\sigma$-, Lorentz-vector-isoscalar $\omega$-, and Lorentz-vector-isovector $\rho$-meson fields, coupled directly to the confined, relativistically treated light $u$ and $d$ quarks, is the key mechanism of the model. This mechanism, though simple, enables to achieve the novel saturation properties of nuclear matter. All the relevant coupling constants between the light quarks and the $\sigma$-, $\omega$-, and $\rho$-meson fields in any hadrons, are the same as those in the nucleon, once the coupling constants are determined by the fit to the nuclear matter saturation properties.

The physics behind of this simple picture bases on the fact that the light-quark condensates respond faster than those of the strange and heavier quarks as nuclear density increases [32, 33, 34]. This is associated with dynamical symmetry breaking and partial restoration of chiral symmetry in nuclear medium, which are mainly driven by the reduction in the light-quark condensates. The QMC model incorporates these facts phenomenologically by the approximation that the $\sigma$-, $\omega$-, and $\rho$-meson fields couple directly only to the light quarks, but neither to the strange quark nor heavier quarks.

One of the main purposes of studying the in-medium properties of heavy baryons with nonzero light quarks is, to understand dynamical symmetry breaking, its (partial) restoration, and the roles of the light quarks in medium. These phenomena can provide us with important information on the origin of (dynamical) masses of hadrons and “normal” (not ”dark“) matter, which we can observe directly in our universe. Despite the importance, only a few studies have been made for the properties of heavy baryons with nonzero light quarks in a nuclear medium [38, 39, 40].

Extending the studies made in Ref. [1], we study here the magnetic moments and transition magnetic moments of the octet, low-lying charm, and low-lying bottom baryons with nonzero light quarks in...
symmetric nuclear matter. Although the octet baryon magnetic moments in symmetric nuclear matter were studied in Ref. [11], no studies exist for those of the charm or bottom baryons with nonzero light quarks in a nuclear medium, except for those in free space [42, 43, 44, 45, 46, 47, 48].

Concerning the magnetic moments of heavy baryons with $c$ and/or $b$ quarks, some ambiguities arise in constructing the flavor-spin wave functions [43]. These are associated with the so-called “quark order”, originating from the fact that the spin, isospin, $SU(3)$ flavor symmetry, and the Pauli principle cannot help much. The possible different quark orders in the flavor-spin wave functions yield different results for the calculated magnetic moments. Based on the conclusion drawn for the quark order in Ref. [45], we study the magnetic moments and the transition magnetic moments of the octet, low-lying charm, and low-lying bottom baryons with nonzero light quarks in symmetric nuclear matter. Furthermore, we discuss possible ambiguities originating from the MIT bag model artifact for the transition magnetic moments. This is the first study of estimating the medium modifications of magnetic moments for these low-lying charm and bottom baryons.

2. The QMC model

In this section we outline the QMC model following Refs. [35, 36]. Since the Hartree-Fock treatment in the QMC model gives very similar results with those of the Hartree [49], we use the Hartree approximation to be consistent with Ref. [1]. (See Ref. [31] for the Hartree-Fock treatment in the QMC model applied for studying the neutron star structure with hyperons.)

Using the Born-Oppenheimer approximation, a relativistic effective Lagrangian density for a hypernucleus in the QMC model is given by [3, 4, 7, 10, 12, 18],

\[
\mathcal{L}_{QMC}^{HY} = \mathcal{L}_{QMC}^N + \mathcal{L}_{QMC}^Y,
\]

\[
\mathcal{L}_{QMC}^N = -\overline{\psi}_N(\vec{r}) [i\gamma \cdot \partial - m_0^*(\sigma) - (g_2^N \omega(\vec{r}) + g_2^N \tau^N_3 b(\vec{r}) + \frac{e}{2}(1 + \tau^N_3) A(\vec{r})) \gamma_0] \psi_N(\vec{r})
\]

\[
-\frac{1}{2}[(\nabla \sigma(\vec{r}))^2 + m_\sigma^2 \sigma(\vec{r})^2] + \frac{1}{2}[(\nabla \omega(\vec{r}))^2 + m_\omega^2 \omega(\vec{r})^2]
\]

\[
+ \frac{1}{2}[(\nabla b(\vec{r}))^2 + m_b^2 b(\vec{r})^2] + \frac{1}{2}(\nabla A(\vec{r}))^2,
\]

\[
\mathcal{L}_{QMC}^Y = -\overline{\psi}_Y(\vec{r}) [i\gamma \cdot \partial - m_0^Y(\sigma) - (g_2^Y \omega(\vec{r}) + g_2^Y \tau^Y_3 b(\vec{r}) + eQ_Y A(\vec{r})) \gamma_0] \psi_Y(\vec{r}),
\]

\[
(Y = \Lambda, \Sigma^{0,\pm}, \Xi^{0,+,0,-}, \Lambda^{0,\pm}, \Sigma^{0,+,0,-}_c, \Xi^{0,+,0,-}_c, \Lambda^{0,\pm}_b, \Sigma^{0,+,0,-}_b, \Xi^{0,+,0,-}_b),
\]

where, the quasi-particles moving in single-particle orbits are three-quark clusters with the quantum numbers of a nucleon, a strange, or a bottom hyperon when expanded to the same order in velocity [3, 4, 7, 10, 12, 18]. In the above $\psi_N(\vec{r})$ [$\psi_Y(\vec{r})$] is the nucleon [hyperon (strange, charm or bottom baryon)] field. The mean-meson fields represented by, $\sigma$, $\omega$, and $b$ are the Lorentz-scalar-isoscalar, Lorentz-vector-isoscalar, and third component of Lorentz-vector-iso-vector fields, respectively, while $A$ is the Coulomb field. Therefore, the quantities in medium will be denoted by an asterisk, *.

The coupling constants of the hyperon appearing in Eq. (3) are, $g_2^N = (n_q/3)g_{2w}$, and $g_2^Y = g_2^\rho = g_2^\rho_0$, with $n_q$ being the number of valence light quarks in the hyperon $Y$ ($n_q = 3$ for $N$), where $g_{2w}$ and $g_2^\rho$ appearing in Eq. (2) are the $\omega$-$N$ and $\rho$-$N$ coupling constants, respectively. $I_{2Y}^3$ and $Q_Y$ are the third component of the hyperon isospin operator and its electric charge in units of the positron charge, $e$, respectively. The couplings between the meson fields and quarks, as already mentioned, reflect the fact that the light-quark condensates respond faster than those of the strange and heavier quarks as baryon (nuclear) density increases.

The field dependent $\sigma$-$N$ [$\sigma$-$Y$] coupling strength for the nucleon $N$ [hyperon $Y$], $g_2^N(\sigma)$ [$g_2^Y(\sigma)$] implicitly in Eq. (2) [Eq. (3)], is defined by

\[
m_{N,Y}(\sigma) = m_{N,Y} - g_{N,Y}^N(\sigma) \sigma(\vec{r}), \quad (Y = \Lambda, \Sigma, \Xi, \Lambda^{+}, \Sigma^{0,+,0,-}_c, \Xi^{0,+,0,-}_c, \Lambda^{0,\pm}_b, \Sigma^{0,+,0,-}_b, \Xi^{0,+,0,-}_b),
\]

where $m_{N}$ [$m_Y$] is the free nucleon [hyperon] mass. Note that, the dependence of the coupling strengths on the scalar field $\sigma$ must be calculated self-consistently within the quark model [2, 3, 7, 10, 11, 18] (the
MIT bag model in the present case). This characterizes the QMC model from quantum hadrodynamics (QHD) [50, 51], and from other naive symmetry-based approaches. Namely, although in such approaches, \( \sigma_{g}^{N}(\sigma)/\sigma_{g}^{N}(\sigma) \) may be \( 2/3 \) or \( 1/3 \) depending on the number of the light quarks \( n_{q} \) in the hyperon \( Y \) in free space (\( \sigma = 0 \)), this may not be true any more in a nuclear medium. (Even in free space this is not true, since the bag radii of the nucleon and hyperon are not exactly the same [52].)

For the later convenience, we define \( C_{N,Y}(\sigma) \equiv S_{N,Y}(\sigma)/S_{N,Y}(\sigma = 0) \), and \( S_{N,Y}(\sigma) \) in connection with \( m_{N,Y}^{*} \) by denoting \( q(= u, d) \) the light quarks,

\[
\frac{dm_{N,Y}^{*}(\sigma)}{d\sigma} = -n_{q} g_{\sigma}^{g} \int_{bag} d^{3}y \bar{\psi}(\bar{y}) \psi(q) \equiv -n_{q} g_{\sigma}^{g} S_{N,Y}(\sigma) = -[n_{q} g_{\sigma}^{g} S_{N,Y}(\sigma = 0)] \left( \frac{S_{N,Y}(\sigma)}{[n_{q} g_{\sigma}^{g} S_{N,Y}(\sigma = 0)]} \right) \]

\[
\equiv -[n_{q} g_{\sigma}^{g} S_{N,Y}(\sigma = 0)] C_{N,Y}(\sigma) \equiv \frac{d\sigma}{d\sigma} [g_{\sigma}^{N,Y}(\sigma) \sigma] ,
\]

(5)

where \( g_{\sigma}^{g} \) is the light-quark-\( \sigma \) coupling constant, and \( \psi_{q} \) is the light-quark ground state wave function in \( N \) or \( Y \) immersed in a nuclear medium. The \( \sigma-N \) and \( \sigma-Y \) coupling constants are defined by,

\[
g_{\sigma}^{N,Y} \equiv g_{\sigma}^{N,Y}(\sigma = 0) \equiv n_{q} g_{\sigma}^{g} S_{N,Y}(\sigma = 0).
\]

(6)

Note that, the values of \( S_{N}(\sigma) \) and \( S_{Y}(\sigma) \) are different, since the light-quark ground state wave functions in \( N \) and \( Y \) are different in free space as well as in medium. Since the light quarks in any hadrons are expected to feel the same scalar and vector mean fields as those in the nucleon, one can systematically study the hadron properties in medium using the same light-quark-meson coupling constants, which are constrained by the nuclear matter saturation properties. This is one of the big advantages of the QMC model.

Next, we consider the rest frame of symmetric nuclear matter, a spin and isospin saturated infinitely large system with only strong interaction. (See Refs. [1] [2] [32] for details.) The Dirac equations for the quarks and antiquarks in nuclear matter, in a bag of a hadron, \( h \), \( q = u \) or \( d \), and \( Q = s, c \) or \( b \), hereafter), are given by \( (x = (t, \vec{x}) \) and for \( |\vec{x}| \leq \text{bag radius} \) [14, 17, 18, 19],

\[
\left[ i\gamma^{v} \partial_{x} - (m_{q} - V_{\sigma}^{q}) \mp \gamma^{3} \left( V_{\sigma}^{q} + \frac{1}{2} V_{\rho}^{q} \right) \right] \begin{pmatrix} \psi_{u}(x) \\ \psi_{d}(x) \end{pmatrix} = 0, \quad (7)
\]

\[
\left[ i\gamma^{v} \partial_{x} - (m_{q} - V_{\sigma}^{q}) \mp \gamma^{3} \left( V_{\sigma}^{q} - \frac{1}{2} V_{\rho}^{q} \right) \right] \begin{pmatrix} \psi_{u}(x) \\ \psi_{d}(x) \end{pmatrix} = 0, \quad (8)
\]

\[
[i\gamma^{v} \partial_{x} - m_{Q}] \psi_{Q}(x) = 0, \quad [i\gamma^{v} \partial_{x} - m_{Q}] \psi_{Q}(x) = 0, \quad (9)
\]

where, the mean field potentials are defined by, \( V_{\sigma}^{q} \equiv g_{\sigma}^{q} \sigma \), \( V_{\rho}^{q} \equiv g_{\rho}^{q} \omega \), and \( V_{\rho}^{q} \equiv g_{\rho}^{q} \rho \), with \( g_{\rho}^{q} \), \( g_{\omega}^{q} \), and \( g_{\rho}^{q} \) being the corresponding quark-meson coupling constants. We assume \( SU(2) \) symmetry, \( m_{u,a} = m_{d,a} = m_{q,q} \equiv m_{q} - V_{\sigma}^{q} \), and thus \( m_{u}^{*} \) is dominated by the Lorentz-scalar mean-field potential as baryon density increases, and can be negative. Note that, \( m_{Q} = m_{Q}^{*} \), since the \( \sigma \) filed does not couple to the heavier quarks \( Q = s, c, b \). Furthermore, since \( \rho \)-meson mean field becomes zero, \( V_{\rho}^{q} = 0 \) in Eqs. (7) and (8) in symmetric nuclear matter in the Hartree approximation, we will ignore it.

The same mean fields \( \sigma \) and \( \omega \) for the quarks in Eqs. (7) and (8), satisfy self-consistently the following equations at the nucleon level, with \( m_{N}^{*}(\sigma) \) to be calculated by Eq. (13):

\[
\omega = \frac{g_{\omega}^{q} \rho_{B}}{m_{\omega}^{2}} = \frac{g_{\omega}^{q}}{m_{\omega}^{2}(2\pi)^{3}} \int d^{3}k \theta(k_{F} - |\vec{k}|), \quad (10)
\]

\[
\sigma = \frac{g_{\sigma}^{N} C_{N}(\sigma) \rho_{s}}{m_{\sigma}^{2}} = \frac{g_{\sigma}^{N}}{m_{\sigma}^{2}} C_{N}(\sigma) \frac{4}{(2\pi)^{3}} \int d^{3}k \theta(k_{F} - |\vec{k}|) \frac{m_{N}^{*}(\sigma)}{\sqrt{m_{N}^{2}(\sigma) + k^{2}}}, \quad (11)
\]

where \( k_{F} \) is the nucleon Fermi momentum.
Because of the underlying quark structure of the nucleon used to calculate $m_N^*(\sigma)$ in the nuclear medium, $C_N(\sigma)$ decreases as $\sigma$ increases, whereas in the usual point-like nucleon, $C_N(\sigma) = 1$. It is this variation of $C_N(\sigma)$ (or equivalently $\sigma$-dependence of the coupling as $g_N^*(\sigma(p_B))$), that yields a novel saturation mechanism for nuclear matter. The important dynamics which originates from the quark structure of the nucleon, is included in $C_N(\sigma)$. This $C_N(\sigma)$ also yields three-body or density dependent effective forces at the nucleon level \cite{32,33}. As a consequence, the QMC model gives the nuclear incompressibility of $K \simeq 280$ MeV with $m_q = 5$ MeV and free nucleon bag radius 0.8 fm \cite{35}. The value is in contrast to a naive version of QHD \cite{53,54}, that results in much larger value, $K \simeq 500$ MeV, where the empirically extracted value falls in the range $K = 200 - 300$ MeV. (See Ref. \cite{54} for details.)

Once the self-consistency equation for the $\sigma$ field Eq. (11) is solved, one can calculate the total energy per nucleon:

$$E^{\text{tot}}/A = \frac{4}{(2\pi)^3\rho_B} \int d^3k \theta(k_F - |\vec{k}|) \sqrt{m_N^2(\sigma) + \vec{k}^2} + \frac{m_q^2\sigma^2}{2\rho_B} + \frac{g_{\pi}\rho_B}{2m_\omega^2}. \quad (12)$$

The parameters appearing in the Lagrangian density Eqs. (1)- (3) and also above are, $m_\omega = 783$ MeV, $m_\rho = 770$ MeV, $m_\pi = 550$ MeV and $c^2/4\pi = 1/137.036$ \cite{35}. The coupling constants, $g_{\pi}^S \equiv g_\sigma$, $g_{\omega}^S \equiv g_\omega$, and $g_{\rho}^N \equiv g_\rho$, at the nucleon level are determined by the fit to the binding energy of 15.7 MeV at the saturation density $\rho_0 = 0.15$ fm$^{-3}$ ($k_F^0 = 1.305$ fm$^{-1}$) for symmetric nuclear matter, as well as $g_\rho$, to the symmetry energy of 35 MeV. The quark-meson coupling constants determined, and the current quark mass values (inputs) are listed in Table I. The corresponding coupling constants at the nucleon level are,

Table 1: Current quark mass values (inputs), quark-meson coupling constants and the bag constant $B_p$ \cite{1}, obtained with the inputs, free nucleon bag radius $R_N = 0.8$ fm, and empirical values $E^{\text{tot}}/A - m_N = -15.7$ MeV at the saturation density $\rho_0 = 0.15$ fm$^{-3}$, and the symmetry energy, 35 MeV.

| $m_{u,d}$ | 5 MeV | $g_{\pi}^S$ | 5.69 |
| $m_s$ | 250 MeV | $g_{\omega}^S$ | 2.72 |
| $m_c$ | 1270 MeV | $g_{\rho}^N$ | 9.33 |
| $m_b$ | 4200 MeV | $B_p^{1/4}$ | 170 MeV |

$g_{\pi}^S/4\pi = (g_{\pi}^N)^2/4\pi = 5.39$ (see Eq. (11) with $S_N(0) = 0.4827$, while Ref. \cite{1} mistakenly gave the value for finite nuclei), $g_{\omega}^S/4\pi = (g_{\omega}^N)^2/4\pi = (3g_{\pi}^S)^2/4\pi = 5.30$, and $g_{\rho}^N/4\pi = (g_{\rho}^N)^2/4\pi = (g_{\rho}^S)^2/4\pi = 6.93$.

The mass of a hadron $h$ in symmetric nuclear matter, $m_h^*$ (free mass is $m_h$), is calculated by

$$m_h^* = \sum_{j=q,u,d} \frac{n_j \Omega_j^* - z_h}{R_h^*} + \frac{4}{3\pi R_h^* B_p} \left[ \frac{dm_h^*}{dR_h^*} = 0, \right] \quad (13)$$

where $\Omega_j^* = \Omega_q^* = (x_j^*)^2 + (R_h^* m_q^*)^{1/2}$ ($q = u, d$), with $m_q^* = m_q - g_{\pi}^S \sigma = m_q - V_\sigma^q$, $\Omega_q^* = \Omega_{q,Q}^* = (x_q^*)^2 + (R_h^* m_Q^*)^{1/2}$ ($Q = s, c, b$), and $x_{q,Q}^*$ are the lowest mode bag eigenfrequencies, $B_p$ is the bag constant (independent of density), $n_q, Q \equiv \{n_{\bar{q},Q}\}$ are the lowest mode valence quark [antiquark] numbers of each quark flavor $q = (u, d)$, $Q = (s, c, b)$ in the hadron $h$, while $z_h$ parametrizes the sum of the center-of-mass and gluon fluctuation effects, which is independent of density \cite{34}. The bag constant $B_p = (170$ MeV$)^4$ is determined by the free nucleon mass $m_N = 939$ MeV, free nucleon bag radius $R_N = 0.8$ fm, and $m_q = 5$ MeV, which are considered to be the standard inputs in the QMC model \cite{35}. Recall that, the quark-meson coupling constants, $g_{\pi}^S$, $g_{\omega}^S$ and $g_{\rho}^S$, have already been determined by the nuclear matter saturation properties, we can use the same coupling constants which are empirically constrained, for the light quarks in any hadrons.

The ground state wave function of the quark $q$ or $Q$ in the hadron $h$ immersed in the nuclear medium satisfies the boundary condition at the bag surface,

$$j_0(x_{q,Q}^*) = \rho_{q,Q}^h \cdot j_1(x_{q,Q}^*), \quad (14)$$
where \( j_{0,1} \) are the spherical Bessel functions, and

\[
\beta^h_q = \sqrt{\frac{\Omega^*_q - m_q^2 R_h^*}{\Omega^*_q + m_q^2 R_h^*}}, \quad \beta^h_Q = \sqrt{\frac{\Omega^*_Q - m_Q R_h^*}{\Omega^*_Q + m_Q R_h^*}}.
\]

The ground state quark wave functions \( \psi^B_q, Q(\vec{r}) \) in a baryon \( B \) in symmetric nuclear matter are given by replacing \( h \to B \) in the above,

\[
\psi^B_{q, Q}(\vec{r}) = N^B_{q, Q} \left( \frac{j_0(x_{q, Q}^* r / R_B^*)}{i \beta^B_{q, Q} \vec{\sigma} \cdot \vec{r} j_1(x_{q, Q}^* r / R_B^*)} \right) \frac{\chi_s}{\sqrt{4\pi}},
\]

with

\[
(N^B_{q, Q})^{-2} = 2(R_B^*)^3 j_0^2(x_{q, Q}^*) \left[ \Omega^*_{q, Q}(\Omega^*_{q, Q} - 1) + m_q^2 R_B^*/2 \right] / x_{q, Q}^* \]

where \( r = |\vec{r}|, \hat{r} = \vec{r}/r, m_q = m_Q \) as already mentioned, and \( \chi_s \) is the Pauli spinor.

3. Baryon magnetic moments in symmetric nuclear matter

In Ref. [1] we already obtained the MIT bag model wave functions in symmetric nuclear matter for the octet, low-lying charm, and low-lying bottom baryons with nonzero light quarks. Below, we calculate the magnetic moments of these baryons and transition magnetic moments in symmetric nuclear matter.

First, we discuss the magnetic moment of an octet baryon \( B, \mu_B, \) in free space (vacuum). For the octet baryons \( B(q_1, q_2, q_3) \) specifying by this the quark order, the familiar SU(6) flavor-spin wave functions are constructed based on the isospin, spin, and Pauli principle, and consistent with the "1-2 quark order" for the quarks \( q_1 \) and \( q_2 \), namely, the first two quarks \( q_1 \) and \( q_2 \) are the closest in mass [45]. Good examples may be the wave functions of \( \Lambda \) and \( \Sigma^0 \) baryons. The first two quarks \( (q_1, q_2) = (u, d) \), are antisymmetric in the \( \Lambda \), while symmetric in the \( \Sigma^0 \). Similar arguments may not necessarily be applicable for the baryons with \( c \) and/or \( b \) quarks with nonzero light quarks, in particular, for the baryons such as \( B(q_1, q_2, q_3) \) \( (q_1 = q, q_2 = Q \neq q_3 = Q) \), since isospin symmetry and Pauli principle cannot help. As discussed in Ref. [45], different assignments for the quarks \( q_1, q_2 \) and \( q_3 \) in \( B(q_1, q_2, q_3) \) \( (q_{1,2,3} = q, Q) \) are possible in some cases consistently with the Pauli principle, and the different assignments give different results for the calculated magnetic moments. In this study we assume the "1-2 quark order" also for heavy baryons, based on the conclusions of Ref. [45], namely, the quarks \( q_1 \) and \( q_2 \) are taken to be the closest in mass.

The magnetic moment \( \mu_B \) of the baryon \( B(q_1, q_2, q_3) \) in an impulse approximation (independent quark picture), is expressed in terms of the \( j \)-th quark magnetic moments \( \mu_j (j = 1, 2, 3) \):

\[
\mu_B = \frac{1}{3} (2\mu_1 + 2\mu_2 - \mu_3).
\]

We give explicit expressions for the baryon magnetic moments and the transition magnetic moments in the second column in Table 2.

Next, as an example, we discuss the magnetic moment of a light quark \( q \) in a baryon \( B \) in the MIT bag model. For a heavy quark \( Q \) in \( B \), one may replace \( q \to Q \) with the corresponding quantities. The free space magnetic moment of a light quark \( \mu_q \) with the charge \( e_q \) in \( B \) is given by,

\[
\mu_q \equiv e_q q = e_q \left[ (N^B_q)^2 \int_0^{R_B} dr r^2 \frac{2r}{3} j_0(x_q r / R_B) \beta^B_q j_1(x_q r / R_B) \right],
\]

where, \( R_B \) is the bag radius of the baryon \( B \). With the expressions given in Table 2 it is straightforward to calculate the magnetic moments of those baryons in free space as well as in symmetric nuclear matter.

For the transition magnetic moments in free space, \( (\Sigma^0 \to \Lambda, \Sigma^+ \to \Lambda^+, \Sigma^0 \to \Lambda_0) \), denoted respectively by \( \mu_{\Sigma^0 \to \Lambda}, \mu_{\Sigma^+ \to \Lambda^+}, \mu_{\Sigma^0 \to \Lambda_0} \), some discussions are in order. In a rigorous calculation in the MIT bag model [51], the bag radius difference for the initial and final baryons arises. This means that the same flavor spectator quark wave functions in the initial and final baryons are different. Also, the integral
Table 2: Magnetic moments and transition magnetic moments in free space and in symmetric nuclear matter. The free space ones are given in the third columns in nuclear magneton, $\epsilon/2m_N$ (e the positron charge and $m_N$ the free nucleon mass, 939 MeV), while the in-medium to free space ratios, $\mu_B^\prime(\rho_B)/\mu_B$ ($\mu_B \equiv \mu_B(\rho_B = 0)$) and $|\mu_B^\prime(\rho_B)/\mu_B^\prime| (\mu_B^\prime \equiv \mu_B^\prime(\rho_B = 0)$), for $\rho_B = (\rho_0, 2\rho_0, 3\rho_0)$ with $\rho_0 = 0.15$ fm$^{-3}$, are given in the (fourth, fifth, sixth) columns. The expressions in the second column are calculated using the flavor-spin wave functions with the "1-2-quark order" for the baryon $B(q_1, q_2, q_3)$, namely, the quarks $q_1$ and $q_2$ are taken to be the closest in mass \[5\].

| $B(q_1, q_2, q_3)$ | $\mu_B$ (expression) | $\mu_B$ | $\mu_B^\prime(\rho_0)/\mu_B$ | $\mu_B^\prime(2\rho_0)/\mu_B$ | $\mu_B^\prime(3\rho_0)/\mu_B$ |
|--------------------|-------------------------|--------|-----------------------------|-----------------------------|-----------------------------|
| $p(uud)$           | $(4\mu_s - \mu_d)/3$    | 1.535  | 1.077                       | 1.103                       | 1.111                       |
| $n(ddu)$           | $(4\mu_d - \mu_u)/3$    | -1.023 | 1.077                       | 1.103                       | 1.111                       |
| $\Lambda(uds)$     | $\mu_s$                | -0.429 | 0.997                       | 0.991                       | 0.985                       |
| $\Sigma^+(uds)$    | $(4\mu_u - \mu_s)/3$    | 1.557  | 1.086                       | 1.133                       | 1.162                       |
| $\Sigma^0(uds)$    | $(2\mu_u + 2\mu_d - \mu_s)/3$ | 0.499  | 1.067                       | 1.102                       | 1.123                       |
| $\Sigma^-(uds)$    | $(4\mu_d - \mu_s)/3$    | -0.560 | 1.121                       | 1.189                       | 1.231                       |
| $\Xi^- (ssd)$      | $(4\mu_s - \mu_u)/3$    | -0.929 | 1.035                       | 1.055                       | 1.067                       |
| $\Xi^- (ssd)$      | $(4\mu_s - \mu_u)/3$    | -0.405 | 0.956                       | 0.927                       | 0.907                       |
| $\Lambda^+_c(ude)$ | $\mu_c$                | 0.423  | 0.999                       | 0.998                       | 0.996                       |
| $\Sigma^+_c(uuc)$  | $(4\mu_u - \mu_c)/3$    | 1.378  | 1.115                       | 1.179                       | 1.219                       |
| $\Sigma^0_c(udc)$  | $(2\mu_u + 2\mu_d - \mu_c)/3$ | 0.238  | 1.166                       | 1.261                       | 1.319                       |
| $\Sigma^0_c(ddc)$  | $(4\mu_d - \mu_c)/3$    | -0.903 | 1.087                       | 1.136                       | 1.167                       |
| $\Xi^+_c(usc)$     | $(2\mu_u + 2\mu_s - \mu_c)/3$ | 0.291  | 1.258                       | 1.413                       | 1.515                       |
| $\Xi^0_c(dsc)$     | $(2\mu_d + 2\mu_s - \mu_c)/3$ | -0.899 | 1.046                       | 1.072                       | 1.089                       |
| $\Lambda^0_c(ubd)$ | $\mu_b$                | -0.073 | 1.000                       | 1.000                       | 1.000                       |
| $\Sigma^0_c(uub)$  | $(4\mu_u - \mu_b)/3$    | 1.675  | 1.111                       | 1.175                       | 1.214                       |
| $\Sigma^0_c(ubd)$  | $(2\mu_u + 2\mu_d - \mu_b)/3$ | 0.437  | 1.107                       | 1.167                       | 1.205                       |
| $\Sigma^0_c(ddb)$  | $(4\mu_d - \mu_b)/3$    | -0.801 | 1.117                       | 1.183                       | 1.224                       |
| $\Xi^0_c(uub)$     | $(2\mu_u + 2\mu_s - \mu_b)/3$ | 0.499  | 1.177                       | 1.283                       | 1.351                       |
| $\Xi^0_c(ddb)$     | $(2\mu_d + 2\mu_s - \mu_b)/3$ | -0.694 | 1.063                       | 1.099                       | 1.123                       |

| Transition | $|\mu_{BB'}| (\text{expression})$ | $|\mu_{BB'}| (\text{expression})$ | $|\mu_{BB'}(\rho_0)/\mu_{BB'}|$ | $|\mu_{BB'}(2\rho_0)/\mu_{BB'}|$ | $|\mu_{BB'}(3\rho_0)/\mu_{BB'}|$ |
|-----------|-------------------------------|-------------------------------|-----------------------------|-----------------------------|-----------------------------|
| $\Sigma^0 \rightarrow \Lambda$ | $(\mu_u - \mu_d)/\sqrt{3}$ | 0.868                         | 1.085                       | 1.129                       | 1.154                       |
| $\Sigma^+_c \rightarrow \Lambda^+_c$ | $(\mu_u - \mu_d)/\sqrt{3}$ | 0.899                         | 1.086                       | 1.128                       | 1.151                       |
| $\Sigma^0_c \rightarrow \Lambda^0_c$ | $(\mu_u - \mu_d)/\sqrt{3}$ | 0.983                         | 1.095                       | 1.143                       | 1.169                       |
upper limit is restricted to the common bag radius to be shown in Eq. (19), and the defects for the spectator quark wave function overlaps arise. However, we ignore these subtle points due to the MIT bag model, and approximate the spectator quark wave function overlaps to be unity. By this approximation, the moduli of the free space transition magnetic moments (signs are not known) may be calculated by,

$$|\mu_{\Sigma^0\Lambda}| = |\mu_{\Sigma^+\Lambda^+}| = |\mu_{\Sigma^0\Lambda_0}| = \frac{1}{\sqrt{3}} |\mu_u - \mu_d| = \frac{1}{\sqrt{3}} |e_u \tilde{\eta}_u - e_d \tilde{\eta}_d|,$$  

where \( \tilde{\eta}_{u,d} \) in the last in Eq. (20) for \( B = (\Sigma^0, \Sigma^+, \Sigma^0_{\bar{u}}) \rightarrow B' = (\Lambda, \Lambda^+, \Lambda^0_{\bar{u}}) \) are defined by:

$$\tilde{\eta}_q \equiv (N_q B'_q / N_q B_q) \int_0^{\min(R_B, R_{B'})} \frac{d r^2 r}{3} \times \left[ j_0(x'_q r / R_{B'}) \beta_{B'}^q j_1(x_q r / R_B) + \beta_{B'}^q j_1(x'_q r / R_{B'}) j_0(x_q r / R_B) \right].$$

The above expression may raise ambiguities due to the so-called MIT bag model artifact, similar to those discussed in the weak-interaction vector charge calculation. Namely, a naive integration over the common bag radius leads to a violation of the Ademoll-Gatto theorem \[55\] as discussed in Ref. \[56\]. Some discussions on this issue will be given later.

In the third column in Table \[2\] we give the free space magnetic moments and transition magnetic moments calculated with the inputs given in Table \[1\]. The ratios of the in-medium to free space are given for three baryon densities, \( (\rho_0, 2\rho_0, 3\rho_0) \) in the (fourth, fifth, sixth) columns in Table \[2\]. One can notice that the most of the ratios become larger as the baryon density increases, except for \( \Lambda, \Xi^- \), and \( \Lambda^+ \).

To see easier, we show the density dependence of the magnetic moments of the octet baryons in Fig. \[1\] the low-lying charmed baryons in Fig. \[2\] the low-lying bottom baryons in Fig. \[3\] and the transition magnetic moments in Fig. \[4\]. In each figure the ratios of the in-medium to free space are shown in the left panel, while in the right panel it is shown the bare density dependence for that has the largest medium modification among all in the left panel, as well as the corresponding quark contributions.

Figure 1: Density dependence of the octet baryon magnetic moment ratios, the in-medium to free space (left panel), and that of the \( \Sigma^- \) magnetic moment, which has the largest medium modification in the left panel, and the corresponding \( d \) and \( s \) quark magnetic moments (right panel).

First, we discuss the magnetic moments of the octet baryons shown in Fig. \[1\]. As is known, the MIT bag model underestimates the octet baryon magnetic moments in free space \[62,63\], and the expression for the magnetic moment roughly proportional to the bag radius. (For example, see Table 7.1 in Ref. \[64\] for the bag radius dependence of the calculated octet baryon magnetic moments.) Thus, we estimate the medium modifications of magnetic moments by taking the ratios, the in-medium to free space. The calculated density dependence of the ratios (left panel) is very similar to that shown in Ref. \[41\] (they used different version of the QMC model). In the right panel we show the \( \Sigma^- \) magnetic moment, which has the largest medium modification among all in the left panel, as well as the corresponding quark
contributions. One can see the $s$ quark magnetic moment is only slightly modified in medium, showing a small linear increase as density increases. This very small density dependence is expected, since the $s$ quark does couple to any meson fields in the model, and the modification comes from the change in the bag radius $R^*_\Sigma$.

Next, we discuss the charm sector baryon magnetic moments shown in Fig. 2. One can easily notice that the medium modification of the $\Xi^+_c$ magnetic moment is the largest in the left panel, while that of $\Lambda^+_c$ is the smallest (slight decrease). The former is enhanced in the ratio by the small magnitude of the free space $\Xi^+_c$ magnetic moment as shown in Table 2. As can be confirmed for the $\Xi^+_c$ in the right panel, the small increase of the $u$ quark magnetic moment is enhanced in the ratio. As expected, nearly no medium modifications can be seen for the $s$ and $c$ quark magnetic moments in $\Xi^+_c$.

For the bottom sector baryon magnetic moments shown in Fig. 3 the $\Xi^0_b$ magnetic moment shows the largest medium modification in the left panel. Similar to the case of $\Xi^+_c$, the magnitude of the free space $\Xi^0_b$ magnetic moment is small, and the in-medium modification is due to the $u$ quark magnetic moment modification as shown in the right panel. Aside from $\Xi^0_b$, one may expect that a similar argument may apply for the $\Sigma^0_b$ magnetic moment from Table 2. However, $\mu_{\Sigma^0_b} = (\mu_u + 2\mu_d - \mu_b)/3$ and since $\mu_u = -2\mu_d$ in the present treatment based on the SU(2) symmetry, it gives $\mu_{\Sigma^0_b} = (\mu_u - \mu_b)/3$ (most modification...
U(3) symmetry model [57] (equivalent to the SU(3) symmetry model of Refs. [58, 59] for this case), by size of the ambiguity is the same order as in Eq. (23).

| cases in the left panel. The larger medium modification of modulus values are shown in the right panel. The density dependence seems to be similar for all the three
The density dependence of the ratios, the in-medium to free space is shown in the left panel, while the
To have a better idea on the bag radius difference, we estimate the differences in free space and at
Finally, we show in Fig. 4 the transition magnetic moments for $B = (\Sigma^0, \Sigma^+_c, \Sigma^0_b) \rightarrow B' = (\Lambda, \Lambda^+_c, \Lambda^0_b)$.

Figure 4: Density dependence of the transition magnetic moment ratios, the in-medium to free space (left panel), and the moduli of the bare values in medium (right panel).

from $\mu_u/3$, and thus the effect of medium modification from the $u$ quark is smaller than that for $\Xi^0_0$ (most modification from $2\mu_u/3$).

The density dependence of the ratios, the in-medium to free space is shown in the left panel, while the modulus values are shown in the right panel. The density dependence seems to be similar for all the three cases in the left panel. The larger medium modification of $|\mu_{\Sigma^0_b\Lambda^0_b}|$, which can be seen in the left panel, may be attributed to the larger bag radii for the $(\Sigma^0_b \rightarrow \Lambda^0_b)$, or the larger integral upper limit (common bag radius) $R^{*}_{\Lambda_b}$, than those of $R^{*}_{\Lambda}$ for $(\Sigma \rightarrow \Lambda)$, and $R^{*}_{\Lambda^+_c}$ for $(\Sigma^+_c \rightarrow \Lambda^+_c)$. Since this feature may be associated with the MIT bag model artifact, we should focus on the ratios, the in-medium to free space shown in the left panel, as will be explained. We discuss this issue next.

We focus on the ratios of the in-medium to free space transition magnetic moments, since the ratios can reduce the possible MIT bag model originated ambiguities as follows. Let us denote the true values in medium as $\mu^{\text{true}}_{BB'}(\rho_B)$ and free space as $\mu^{\text{true}}_{BB'}(0)$, respectively, and the corresponding errors by $\epsilon^{*}(\rho_B)$ and $\epsilon(0)$. Then, the ratio of the in-medium to free space can be estimated by,

$$ \frac{\mu^{\text{true}}_{BB'}(\rho_B)}{\mu^{\text{true}}_{BB'}(0)} = \frac{\mu^{\text{true}}_{BB'}(\rho_B)(1 \pm \epsilon^{*}(\rho_B))}{\mu^{\text{true}}_{BB'}(0)(1 \pm \epsilon(0))}, $$

$$ \approx \frac{\mu^{\text{true}}_{BB'}(\rho_B)}{\mu^{\text{true}}_{BB'}(0)}(1 \pm \epsilon^{*}(\rho_B) \mp \epsilon(0)) = \frac{\mu^{\text{true}}_{BB'}(\rho_B)}{\mu^{\text{true}}_{BB'}(0)}[1 \pm (\epsilon^{*}(\rho_B) - \epsilon(0))], $$

where $0 < \epsilon^{*}(\rho_B), \epsilon(0) << 1$ are assumed. The signs in front of them in the first line in Eq. (22) are expected to be the same, since $\epsilon^{*}(\rho_B)$ varies smoothly as $\rho_B \rightarrow 0$, and $\epsilon^{*}(\rho_B) \rightarrow \epsilon(0)$ to give the ratio unity. Then, $|\epsilon^{*}(\rho_B) - \epsilon(0)|$ becomes smaller in Eq. (22).

To have a better idea on the bag radius difference, we estimate the differences in free space and at $\rho_0$ [1],

$$ \left( \frac{R_B(0) - R^{*}_B(\rho_0)}{R^{*}_B(0)} \right) < 0.045, (0.045), $$

where, inequality holds for all the three cases $B = (\Sigma^0, \Sigma^+_c, \Sigma^0_b) \rightarrow B' = (\Lambda, \Lambda^+_c, \Lambda^0_b)$, with $R_B(0) - R^{*}_B(0) > 0$, as well as $R^{*}_B(\rho_0) - R^{*}_B(\rho_0) > 0$. Furthermore, since the bag model result for the diagonal magnetic moment $\mu_B(\mu_B^*)$ roughly proportional to the bag radius $R_B$ (and $R_B^*$) [62], we can expect the size of the ambiguity is the same order as in Eq. (23).

In fact, it was studied the octet baron magnetic moments and transition magnetic moments in the U(3) symmetry model [57] (equivalent to the SU(3) symmetry model of Refs. [58, 59] for this case), by
including up to the first order of mass splitting interaction. Then, the following relations were obtained,

\[
\mu_{\Sigma^0} = \frac{1}{2} [\mu_{\Sigma^+} + \mu_{\Sigma^-}], \\
\mu_{\Lambda^0} = \frac{1}{2\sqrt{3}} [\mu_{\Sigma^0} + 3\mu_{\Lambda} - 2\mu_{\Xi^0} - 2\mu_n].
\]

(24)  

(25)  

We examine the above relations in the present results of the MIT bag model in free space. For Eq. (24), we get (l.h.s., r.h.s.) = (0.499, 0.499) using the values in Table 2, while for Eq. (25) we get (l.h.s, r.h.s.) = (0.868, 0.900). For the latter, the deviation may be estimated as (0.900-0.868)/0.868 = 0.037. On the other hand, using the experimental data for the octet baryon magnetic moments [61] and \( |\mu_{\Sigma^0}^\text{Expt.}| = 1.59 \) [61] with \( |\mu_{\Sigma^0}^\text{Expt.}| = (1/2)[\mu_{\Sigma^+}^\text{Expt.} + \mu_{\Sigma^-}^\text{Expt.}] \), we get, (l.h.s, r.h.s.) = (1.59, 1.483) for Eq. (25). The deviation is estimated by (1.59-1.483)/1.483 = 0.072. Thus, including the SU(3) symmetry breaking mass splitting interaction up to the first order, the deviation from the relation is larger when we use the experimental results, than that using the MIT bag model results. Based on these estimates, the ambiguities arising from the possible MIT bag model artifact such as the bag radius difference, are not expected to affect the estimates made for the transition magnetic moments in the present status of experimental precision.

Although the estimated medium modifications for the magnetic moments and the transition magnetic moments at nuclear matter saturation density (\( \rho_0 \)) may not directly be reflected on some experimental results, the modifications in the higher density region near \( 3\rho_0 \), may affect the studies for such as heavy ion collisions, and structure and reactions occur in the inner cores of magnetars, neutron stars, and compact stars. In the cores of such high density compact objects one can expect the appearance of charm and bottom baryons, although no charm quark-matter star is expected [65], using the flat space-time equation of states (EOS). However, it was discussed that the use of EOS computed in the curved space-time of neutron stars, may yield the higher central energy densities and masses—about 16.9% for an idealistic neutron star case—than that calculated using the flat-space EOS [66]. This favors to resolve the ”hyperon puzzle” of neutron star, although, within the QMC model, the ”hyperon puzzle” has already been resolved [26-29, 31, 32]. It will also increase the possibility of charm and bottom baryon appearance in the cores of neutron stars, magnetars, and compact stars than those estimated using the flat space-time EOS. Furthermore, dominant charm hadron contributions to neutrino fluence in ultrahigh-energy neutrino production by newborn magnetars are suggested [67].

4. Summary and conclusion

We have studied the medium modifications of magnetic moments and transition magnetic moments of the octet, low-lying charm, and low-lying bottom baryons with nonzero light quarks in symmetric nuclear matter. This is the first study of estimating the medium modifications of magnetic moments and transition magnetic moments for these low-lying charm and low-lying bottom baryons. In the estimates we have assumed the "1-2-quark order" for the charm and bottom baryon flavor-spin wave functions as is practiced for the octet baryon wave functions, namely, the quarks 1 and 2 to be the closest in mass.

Since it is known that the MIT bag model underestimates the free space octet baryon magnetic moments, and also it is expected for the charm and bottom baryons, we take the ratios of the in-medium to free space magnetic moments in the estimates. The observed maximum modifications for the (strange, charm, bottom) baryon sectors are, respectively for the \( (\Sigma^-, \Sigma^0_c, \Xi^0_b) \) baryons, and the corresponding modifications are about (12, 26, 18)% at normal nuclear matter density (\( \rho_0 = 0.15 \text{ fm}^{-3} \)), and about (23, 52, 35)% at \( 3\rho_0 \).

As for the medium modifications of the transition magnetic moments \( \mu_{BB'} \) with \( B = (\Sigma^0, \Sigma^+_c, \Sigma^0_b) \rightarrow B' = (\Lambda, \Lambda^+_c, \Lambda^0_b) \), the modifications at \( \rho_0 \) are about (9, 9, 10)% , while at \( 3\rho_0 \), they are about (15, 15, 17)% . Concerning the possible ambiguities arising from the MIT bag model artifact, we have discussed the difference of the initial and final baryon bag radii in free space as well as in medium, based on the evidence observed for the violation of the Ademollo-Gato theorem in the weak-interaction vector charge calculation. Furthermore, we have studied the possible impact of the ambiguities on the estimated results based on the SU(3) (U(3)) symmetry relations using the calculated results and the experimental data. It turned out that such ambiguities are not expected to affect the estimates made in this study, in the present status of experimental precision.
The medium modifications of baryon magnetic moments estimated in this study can certainly serve as new inputs for the studies of magnetar and neutron star structure, in particular, for the systems with very high baryon density with zero temperature, and extremely strong magnetic field.

As an extension, we plan to study the medium modifications of the weak-interaction axial charges of the octet, low-lying charm, and low-lying bottom baryons with nonzero light quarks.

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