On the non-perturbative part of the photon structure function$^a$

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Abstract

We discuss a dispersion relation in the photon mass and show how (in principle) model-independent constraints on the parton distribution functions of the photon, notably a momentum sumrule, can be obtained. We present two sets of parametrizations, SaS 1 and 2, corresponding to two rather extreme realizations of the non-perturbative part. Inclusive electron scattering off a real photon is found to be insufficient to constrain the non-perturbative components. The additional sensitivity provided by the photon virtuality is outlined. Previous approaches to model the non-perturbative input distributions are commented upon.

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1. Introduction

Perturbative QCD predicts only the $Q^2$ evolution of the parton distribution functions (PDFs) of the photon $f_i^\gamma(x,Q^2)$ via a set of inhomogeneous differential equations of the first kind. Hence the solutions $f_i^\gamma(x,Q^2)$ require the specification of the PDFs at some $Q^2 = Q_0^2$. Two ways exist to determine these non-perturbative input distributions $f_i^\gamma(x,Q_0^2)$. The first one is analogous to the determination of hadronic PDFs: At $Q_0$ large enough to be safely within the perturbative regime ($Q_0 \sim 2 \text{ GeV}$), the parameters of the input distributions $f_i^\gamma(x,Q_0^2)$ (shapes and normalizations) are fitted \cite{1} to the experimentally measured distributions involving the PDFs of the photon, which thus far means to the available $F_2^\gamma(x,Q^2)$ data. Since these data are currently restricted to large $x$, only the $u$-valence distribution is known with some confidence. In particular, basically no constraint on the gluon distribution of the photon exists today in such an approach.

In the second approach one pretends to know the input distributions at some very low scale $Q_0 \sim 0.5 \text{ GeV}$ apart from a single, adjustable parameter. The expectation is that, at such low scales, the photon should essentially behave like a hadron and, correspondingly, the PDF of the photon could be identified with appropriate hadronic ones. The experimental evidence for this ansatz is, however, rather weak: the only data for $Q^2$ below 2 GeV$^2$ come from the TPC/2$\gamma$ measurement \cite{2} of $F_2^\gamma(x,Q^2)$ (at an average $Q^2$ of about 0.7 GeV$^2$) and consist of no more than a handful of points in a limited $x$-range; the largest $x$-bins are moreover plagued by resonance contributions. Nonetheless, basically all recent experimental analyses accept the hadron-like parametrization of $F_2^\gamma(x,Q_0^2)$ of TPC/2$\gamma$ as the non-perturbative input. The scale $Q_0$ is considered as a free parameter and fitted to their data, hence disregarding most of the potential of their own $F_2^\gamma(x,Q^2)$ data to extract the non-perturbative part. Rather, these analyses merely quantify how compatible the more recent data are with the TPC/2$\gamma$ ansatz. The actual, fitted value of $Q_0$ is, in fact, not a significant number since it is strongly correlated with the size (and shape) of the assumed non-perturbative input.

This correlation is most easily seen by decomposing the PDFs of the (real, i.e. $P^2 = 0$) photon as follows

$$f_a^\gamma(x,Q^2) - f_a^{\gamma,\text{dir}}(x,Q^2) = f_a^{\gamma,\text{PT}}(x,Q^2,Q_0^2) + f_a^{\gamma,\text{NP}}(x,Q^2,Q_0^2) ,$$ \hspace{1cm} (1)

where the second term on the LHS describes the (properly normalized $Z_3 = 1 + O(\alpha_{\text{em}})$) probability distribution of a photon to remain a photon

$$f_a^{\gamma,\text{dir}}(x,Q^2) = Z_3 \delta_{a\gamma} \delta(1-x) .$$ \hspace{1cm} (2)

Being the solution of an inhomogeneous evolution equation, the PDF of the photon can always be written as the sum of two terms, as in (1) where the first term on the RHS is a particular solution of the inhomogeneous equation with the boundary condition

$$f_a^{\gamma,\text{PT}}(x,Q_0^2,Q_0^2) = 0 .$$ \hspace{1cm} (3)

The second term is a general solution of the corresponding homogeneous evolution equation and needs a (non-perturbative) input distribution at $Q^2 = Q_0^2$:

$$f_a^{\gamma,\text{NP}}(x,Q_0^2,Q_0^2) = \tilde{f}_a^{\text{NP}}(x) .$$ \hspace{1cm} (4)

At this point it should also be pointed out that the perturbative $Q^2$ evolution is not treated correctly in the experimental analyses of the photon structure function. The scale
\(Q_0\) is fitted to the respective data on

\[
F_2^\gamma(x, Q^2) = 2x \sum_q e_q^2 \left\{ f_q^{\gamma, PT}(x, Q^2, Q_0^2) + f_q^{\gamma, NP}(x, Q^2, Q_0^2) \right\}
\]

\[
\equiv F_2^{\gamma, PT}(x, Q^2, Q_0^2) + F_2^{\gamma, NP}(x, Q^2, Q_0^2),
\tag{5}
\]

using the FKP parametrization \(\text{FKP}\) of \(F_2^{\gamma, PT}(x, Q^2, Q_0^2)\). An error in the \(Q^2\) evolution of \(F_2^{\gamma, PT}(x, Q^2)\) arises because the hadronic part is not evolved with \(Q^2\) but kept fixed at the input scale, \(F_2^{\gamma, NP}(x, Q^2, Q_0^2) = F_2^\gamma(x)|_{\text{TPC}/2\gamma}\). There is yet another error: the FKP parametrization is based on a valence approximation, and hence fails for \(x < 0.3\) \([4]\).

The second approach, namely approximating the photonic input distributions at some low \(Q_0 \approx 0.5\,\text{GeV}\) by hadronic ones, has also been pursued in theoretical analyses \([4]\). Here the input distributions are identified with those of the pion and \(Q_0\) is fixed by theoretical prejudice. In order to have an adjustable parameter, the overall normalization of the input distributions is allowed to vary. (This “K-factor” is actually fitted to high-\(Q^2\) \((Q^2 \gg Q_0^2)\) data only. In this way one “only” assumes the leading-twist formula to evolve perturbatively down to low scales but not to describe all the physics at low scales.)

It should be stressed that the estimation of the input distributions \(f_{\alpha, NP}(x, Q_0^2, Q_0^2)\) by the ones of the pion using vector-meson dominance (VMD) and the additive quark model involves quite severe assumptions. The PDFs of the \(q\overline{q}\) “bound states” of the photon need not be the same as those of real vector mesons. Moreover, the PDFs of the short-lived \(\rho\)-meson may well differ in shape from those of the long-lived pion. In addition, not only the shapes may differ. Also the relative normalizations of the various PDFs can be different, e.g. down-quark to strange-quark distributions, valence to sea to gluon distributions. Finally, implicit (but never justified) in this approach is the assumption of a momentum sumrule and a constraint on the number of valence quarks.

2. The dispersion relation in the photon mass

It will be shown here how a dispersion relation in the photon mass can be used to obtain (in principle) model-independent constraints on the PDFs of the photon, namely a momentum sumrule, as well as constraints on the valence distribution and the overall normalization.

The moments of the photonic PDFs \((g(n) = \int_0^1 dx x^{n-1} g(x))\) can be represented as a dispersion integral in the photon mass \(\sigma^2\) (\(P^2\) is the photon virtuality) \([4]\)

\[
f_{\alpha}^\gamma(n, Q^2, P^2) = \int_0^\infty \frac{d\sigma^2}{\sigma^2 + P^2} \, \rho_{\alpha}(n, Q^2, \sigma^2).
\tag{6}
\]

Rather than describing the dispersion integral as the difference between a “point-like” part (contribution from the upper limit) and a “hadronic” part (contribution from the lower limit), it is more natural to separate short-distance and long-distance parts by a scale \(Q_0\), since the weight function \(\rho_{\alpha}\) possesses the scaling-violation pattern typical of ordinary hadronic PDFs \([3]\). At large values of \(\sigma^2\), the \(\gamma \to q\overline{q}\) transition can be calculated perturbatively. At lower values of \(\sigma^2\) one enters the resonance region: non-perturbative Regge poles will contribute signalling the appearance of \(q\overline{q}\) bound states. A general ansatz for the weight function is therefore \((\rho' \equiv \frac{d\rho}{d\sigma^2})\)

\[
\rho_{\alpha}(n, Q^2, \sigma^2) = \sum_{V=V_0}^{V_{\alpha}(Q_0)} A_V^\alpha(n, Q^2) \, \delta \left( 1 - \frac{\sigma^2}{m_V^2} \right) + \sum_{V=V_0}^{V_{\alpha}(Q_0)} B_V^\alpha(n, Q^2) \, \delta' \left( 1 - \frac{\sigma^2}{m_V^2} \right) + \Theta(\sigma^2 - Q_0^2) \left\{ \alpha_{\alpha}(n, Q^2, \sigma^2) + \beta_{\alpha}(n, Q^2, \sigma^2) \right\}.
\tag{7}
\]
The coefficients $A^V_a$, $B^V_a$, $\alpha_a$ and $\beta_a$ can be determined as follows. VMD is known to well describe photon-hadron interactions over a wide range of energies, from $\sqrt{s}$ of a few GeV up to the HERA energy (200 GeV). Hence, to very good approximation, one may neglect $A^V_a$ and take

$$B^V_a(n, Q^2, Q^2_0) = \left( \frac{e}{f^V_a} \right)^2 f^\gamma_a(n, Q^2, Q^2_0).$$

(8)

In order to obtain $\alpha_a$ and $\beta_a$ one first notices that for $Q^2_0 \ll P^2 \ll Q^2$ the resonance contributions to (7) are suppressed. Moreover, in this limit the PDF of a virtual photon (i.e. the LHS of (3)) can be calculated within perturbative QCD [7]. Then one expresses these distributions as an integral of “state” distribution functions $f^\gamma_a(x, Q^2, \sigma^2)$:

$$f^\gamma_a(x, Q^2, P^2) = \int_{P^2}^{Q^2} \frac{d\sigma^2}{\sigma^2} \sum_q 2e_q^2 f^\gamma_a(x, Q^2, \sigma^2),$$

(9)

which obey the standard, homogeneous evolution equations with the boundary condition

$$f^\gamma_a(x, \sigma^2, \sigma^2) = f^\gamma_a(x) \equiv \frac{3}{2} (x^2 + (1 - x)^2) (\delta_{aa} + \delta_{ag}).$$

(10)

Equation (9) yields an expression for the sum $\alpha_a$ plus $\beta_a'$. The decomposition into $\alpha_a$ and $\beta_a$ is more difficult. Generalized VMD arguments suggest $\alpha_a \ll \beta_a$ and hence we arrive at the final expression for the PDFs of the virtual photon

$$f^\gamma_a(n, Q^2, P^2) = f^\gamma_a^{NP}(n, Q^2, P^2) + f^\gamma_a^{PT}(n, Q^2, P^2)$$

$$\equiv \sum_{V = V_0}^{V_m(Q_0)} \left( \frac{m^2_V}{m^2_V + P^2} \right)^2 \frac{4\pi\alpha_{em}}{f^V_a} f^\gamma_a(n, Q^2, Q^2_0)$$

$$+ \int_{Q^2_0}^{Q^2} \frac{\sigma^2 d\sigma^2}{(\sigma^2 + P^2)^2} \frac{\alpha_{em}}{\pi} \sum_q e_q^2 f^\gamma_a(x, Q^2, \sigma^2).$$

(11)

Equation (11) contains three constraints on the non-perturbative distributions. The first two follow from the fact that $f^\gamma_a^{NP}$ are ordinary mesonic PDFs. Hence they should respect the number of valence quarks

$$1 = f^\gamma_a^{NP}(n = 1, Q^2, Q^2_0) \equiv \int_0^1 dx f^\gamma_a^{NP}(x, Q^2, Q^2_0)$$

(12)

and obey the momentum sumrule

$$1 = \sum_{a = u, d, g} f^\gamma_a^{NP}(n = 1, Q^2, Q^2_0) \equiv \int_0^1 dx f^\gamma_a^{NP}(x, Q^2, Q^2_0).$$

(13)

The third constraint on the photonic PDFs follows from the observation that the RHS of (11) has to be $Q_0$-independent. This fixes the overall normalization and, in turn, gives a momentum sumrule for the photonic PDFs:

$$1 - Z_3 \equiv \sum_{a = u, d, g} \int_0^1 dx f^\gamma_a(x, Q^2, P^2)$$

$$= \sum_{V = V_0}^{V_m(Q_0)} \left( \frac{m^2_V}{m^2_V + P^2} \right)^2 \frac{4\pi\alpha_{em}}{f^V_a} + \int_{Q^2_0}^{Q^2} \frac{\sigma^2 d\sigma^2}{(\sigma^2 + P^2)^2} \frac{\alpha_{em}}{\pi} \left( \sum_q e_q^2 \right)$$

(14)
since the perturbative state distributions \( f_a^{\gamma q}(n, Q^2, \sigma^2) \) obey a relation analogous to (13).

This last equation is, in fact, nothing but the observation that the vacuum fluctuations of the photons probed in \( e\gamma \) interactions are precisely the ones seen in \( e^+e^- \rightarrow \text{hadrons} \). The probability per unit \( \sigma^2 \) of their occurrence is given by the cross section of the latter reaction

\[
1 - Z_3 = \int_0^{Q^2} d\sigma^2 \left( \frac{\sigma^2}{\sigma^2 + p^2} \right)^2 \frac{\sigma_{\text{tot}}(e^+e^- \rightarrow X(\sigma))}{4\pi^2\alpha_{\text{em}}} .
\]  

Using the narrow-width approximation for the resonance (low-mass) contribution and the parton-model result at high masses

\[
\sigma_{\text{tot}}(e^+e^- \rightarrow X(\sigma)) = \sum_V \frac{4\pi^2\alpha_{\text{em}}}{3\sigma^2} \left( \frac{e}{f_V^2} \right)^2 \delta(\sigma^2 - m_V^2) + \Theta(\sigma^2 - Q_0^2) N_C \left( \sum_q e_q^2 \right) \frac{4\pi\alpha_{\text{em}}}{3\sigma^2}.
\]  

the result (14) is recovered.

Once \( Q_0 \) has been determined for a given number of vector mesons, (12) tells us how \( Q_0 \) has to be changed in order to compensate the inclusion (or omission) of a vector meson. For a given number of included vector mesons, the value of \( Q_0 \) can be determined from the continuity requirement of the \( e^+e^- \) annihilation cross section or, say, the total \( \gamma p \) cross section [8].

What remains undetermined in the approach sketched above is the shape of the non-perturbative input distributions. In line with the argument that hard processes probe short time scales, the contributions from the various vector mesons should be added coherently. Also, to rather good approximation, an \( SU_3 \)-symmetric sea distribution \( s(x) \) can certainly be assumed. Then the non-perturbative input distributions \( f_a^{\gamma np}(x, Q_0^2, Q^2) \) are given in terms of three distributions, a valence distribution \( v(x) \), a gluon distribution \( g(x) \), and the sea distribution \( s(x) \). These distributions can be determined through a fit to the available (real photon) \( F_{\gamma}^2(x, Q^2) \) data, subjected to the constraints (11), (12) and (13).

Two extreme scenarios can be considered. In the first, VMD is restricted to the well-established \( \rho^0, \omega, \phi \) states. The scale \( Q_0 \) is then known to be \( Q_0 \approx 0.6 \text{ GeV} \) from an analysis of the \( \gamma p \) total cross section [8]. This “low-\( Q_0 \)” fit (SaS set 1 distribution functions; for details of the fit see [4]) is essentially a three-parameter fit, two for the shape of the valence distribution and one for the normalization of the sea. The shapes of both the gluon and sea distributions are hardly constrained by current \( F_{\gamma}^2 \) data and take on values of an educated guess. The main theoretical error of this scenario arises from the use of perturbation theory down to rather low values of \( Q^2 \).

The spirit of the second analysis is opposite: take \( Q_0 \) well within the perturbative domain (\( Q_0 = 2 \text{ GeV} \)) at the expense of parametrizing the effects of additional vector mesons (besides \( \rho^0, \omega, \phi \)) by a simple factor \( K \) to be fitted to the data

\[
\sum_{V=\rho^0,\omega,\phi} \frac{V_{m}(Q_0)}{f_V^2} f_a^{\gamma V}(x) \approx K(Q_0) \sum_{V=\rho^0,\omega,\phi} \frac{4\pi\alpha_{\text{em}}}{f_V^2} f_a^{\gamma V}(x) .
\]  

This “high-\( Q_0 \)” fit (SaS set 2 distribution functions [4]) contains two additional parameters compared to the low-\( Q_0 \) fit, one parameter characterizing the necessary hard component of the valence distributions at larger values of \( Q_0 \), and the value of \( K \).
Note that, if higher-twist effects and other uncertainties were negligible, the following dependence of $K$ on $Q_0$ should hold

$$K(Q'_0) = K(Q_0) + \frac{\sum_q e_q^2}{\pi \sum_{V=\rho,\omega,\phi} 4\pi/f_V^2} \ln \frac{Q'_0}{Q_0^2} \approx K(Q_0) + 0.770 \ln \frac{Q'_0}{Q_0}. \quad (18)$$

With $K(0.6\text{ GeV}) = 1$, (18) predicts $K(2\text{ GeV}) = 1.93$, to be compared with 2.42, the outcome of the high-$Q_0$ fit. Two facts account for almost all the discrepancy. First, the $\chi^2$ analysis suggests that $K(0.6\text{ GeV})$ should slightly exceed unity ($K = 1.17$) increasing the prediction (18) to $K(2\text{ GeV}) = 2.10$. Second, the high-$Q_0$ fit includes only data above $Q^2 = 4\text{ GeV}^2$, while the low-$Q_0$ fit includes data down to 0.71 GeV$^2$. Indeed, good agreement is found if the latter fit is also restricted to the high-$Q^2$ data. This indicates that higher-twist contributions significantly affect $F_\gamma^2$ at low $Q^2$, but also that the leading-twist evolution of PDFs is still valid down to $Q_0 = 0.6\text{ GeV}$.

Distributions involving the inclusive PDFs of the real photon are not sufficient to disentangle the non-perturbative part. The high-$Q_0$ and low-$Q_0$ SaS sets of PDFs describe the $F_\gamma^2$ data equally well [4], although the non-perturbative parts have very different shapes and normalizations. For example, the momentum fractions carried by the perturbative and non-perturbative parts are quite different for the two cases but not visible with a real photon ($P^2 = 0$) target:

$$Q_0 = 2.0\text{ GeV} : \quad 1 - Z_3 = \alpha_{em} \left\{ 1.33 + \frac{\alpha_{em}}{\pi} \left( \sum_q e_q^2 \right) \ln \frac{Q^2}{4\text{ GeV}^2} \right\}$$

$$Q_0 = 0.6\text{ GeV} : \quad 1 - Z_3 = \alpha_{em} \left\{ 0.55 + \frac{\alpha_{em}}{\pi} \left( \sum_q e_q^2 \right) \ln \frac{Q^2}{0.36\text{ GeV}^2} \right\}$$

$$\approx \alpha_{em} \left\{ 1.06 + \frac{\alpha_{em}}{\pi} \left( \sum_q e_q^2 \right) \ln \frac{Q^2}{4\text{ GeV}^2} \right\}. \quad (19)$$

Additional information can, and has to, be obtained from two sides: perturbative and non-perturbative parts lead to differences in the hadronic final states of photon-induced reactions [8], and also show a different dependence on the photon virtuality as is evident, e.g. from (14). Further studies, both theoretically and experimentally, to exploit the sensitivity to the non-perturbative part are highly desirable.

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