Evidence for a smooth superconductor to normal state transition for nonzero applied magnetic field in Sr$_{0.9}$La$_{0.1}$CuO$_2$

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The effect of the magnetic field on the critical behavior of Sr$_{0.9}$La$_{0.1}$CuO$_2$ is investigated near the zero field superconductor to insulator transition at $T_c$. We present and analyze field cooled magnetization data, revealing for $0 \leq \mu_0 H \lesssim 5$ T remarkable consistency with a magnetic field induced finite size effect. It is traced back to the fact that the correlation length $\xi$ cannot grow beyond the limiting length scale $L_H$ set by the magnetic field, where at temperature $T_p(H)$, $\xi(T) = L_H$. Thus, in sufficiently homogeneous samples and nonzero $H$ the transition from the superconducting to the normal state turns out to be smooth and the appropriately scaled magnetization data fall near $T_p(H)$ on a universal curve. Consistent with the generic behavior of optimally doped cuprates we also show that the pressure effect on $T_c$ is negligibly small, while the negative value of the relative volume change under pressure mirrors that of the anisotropy.

In this study we present and analyze magnetization data of the infinite-layer compound Sr$_{0.9}$La$_{0.1}$CuO$_2$. Since near the zero field transition thermal fluctuations are expected to dominate [1–3] and in sufficiently high fields these fluctuations become effectively one dimensional [4], whereupon the effect of fluctuations increases with increasing magnetic field, it appears unavoidable to account for thermal fluctuations. Indeed, invoking the scaling theory of critical phenomena it is shown that the data are inconsistent with the traditional mean-field interpretation. On the contrary, we observe agreement with a magnetic field induced finite size effect. Indeed, when the magnetic field increases, the density of vortex lines becomes greater, but this cannot continue indefinitely, the limit is roughly set on the proximity of vortex lines by the overlapping of their cores. Because of the resulting limiting length scale $L_H$ for a field applied parallel to the c-axis the correlation length $\xi_{ab}$ cannot grow beyond [3,5–7]

$$L_H = \sqrt{\Phi_0/aH},$$

with $a \simeq 3.12$ [7]. It is comparable to the average distance between vortex lines and implies that $\xi$ cannot grow beyond $L_H$ and with that there is a magnetic field induced finite size effect. This implies that thermodynamic quantities like the magnetization, magnetic penetration depth, specific heat etc. are smooth functions of temperature near $T_p(H)$, where the correlation length cannot grow beyond $\xi(T_p) = L_H$. This scenario holds true when the magnetization data $M(T, H)$ collapses near $T_p(H)$ on a single curve when plotted as $M/(TH^{1/2})$ vs. $(T/T_c - 1)/(1 - T_p(H)/T_c)$. We observe that the magnetization data falls for $0 \leq \mu_0 H \lesssim 5$ T within experimental error on a single curve by adjusting $T_p(H)$. From the resulting field dependence of $T_p$ we deduce that the critical amplitude of the correlation length the estimate $\xi_{ab} \simeq 35$Å. Noting that $L_H$ decreases with increasing field strength it sets unavoidably the limiting length scale for sufficiently high fields, the effect of the inhomogeneity induced counterpart can be eliminated. In the present case, both the magnetic field induced shift of the peak location and peak height in $dM/dT$ clearly reveal that for $\mu_0 H \gtrsim 0.1$T the magnetic field induced finite size effect allows to probe the homogeneous parts of the grains with dimension $\gtrsim 814$Å. The observed consistency with the magnetic field induced finite size effect then implies that whenever the thermal fluctuation dominated regime is accessible, sufficiently homogeneous type II superconductors do not undergo a continuous phase transition in a nonzero magnetic field, e.g. to a state with zero resistance. Furthermore, we show that the pressure effect on the magnetization does not provide estimates for the shift of the transition temperature only, but involves the change of the volume and anisotropy as well, in analogy to MgB$_2$ [8]. Consistent with the generic behavior of optimally doped cuprates we find that in Sr$_{0.9}$La$_{0.1}$CuO$_2$ the pressure effect on $T_c$ is negligibly small, while the negative value of the relative volume change mirrors that of the anisotropy.
FIG. 1. a) Field cooled magnetization $M$ of a Sr$_{0.9}$La$_{0.1}$CuO$_2$ powder sample vs. $T$ for various applied magnetic fields. From the left to the right: $\mu_0 H = 5, 4, 3, 2.5, 1.75, 1.5, 1.25, 1, 0.8, 0.6, 0.5, 0.4, 0.3, 0.2, \text{ and } 0.1$ T; b) $dM/dT$ vs. $T$ for some data shown in Fig.1a.

In Fig.1a we displayed our ac-field-cooled magnetization data in terms of $M$ vs. $T$. Some respective $dM/dT$ vs. $T$ are shown in Fig.1b. For a detailed description of the sample preparation we refer to Kim et al. [9]. The ac-field-cooled magnetization measurements were performed with a Quantum Design (PPMS) magnetometer at temperatures ranging from 5 to 50K. The ac-frequency was set to 1000Hz and the amplitude to 0.5mT. To identify the temperature regime where critical fluctuations play an essential role it is instructive to compare the generic features of the data with the predictions of Abrikosov’s mean-field treatment [10] whereupon near the upper critical field $H_{c2}$ the magnetization is given by

$$4\pi M(T) = -\frac{1}{(2\kappa^2 - 1)\beta_A} (H_{c2}(T) - H), \quad (2)$$

and in turn

$$4\pi \frac{dM(T)}{dT} = -\frac{1}{(2\kappa^2 - 1)\beta_A} \frac{dH_{c2}(T)}{dT}, \quad \frac{dH_{c2}(T)}{dT} = -\frac{\Phi_0}{2\pi\xi_0^2T_{c2}(H)}. \quad (3)$$

$k = \lambda/\xi$ denotes the Ginzburg-Landau parameter, $\beta_A = 1.16$ for a hexagonal vortex lattice, $\xi = \xi_0 (1 - T/T_{c2}(H))^{-1/2}$ the correlation length and $T_{c2}(H)$ the mean-field transition temperature at the upper critical field critical field $H_{c2}(T)$. Thus, this approximation predicts a continuous phase transition in an applied field along the line $H_{c2}(T) = \phi_0/(2\pi\xi_0^2) (1 - T/T_{c0})$, where $T_{c0}$ is the zero field mean-field transition temperature. In Fig.2 we compare the essential predictions with the magnetization data for $\mu_0 H = 0.1$T. Although the magnetization appears to be consistent with a linear slope below some temperature “$T_{c2}(H)$” and becomes very small above, the data for $dM/dT$ deviates from the resulting discontinuity. According to Fig.1b this holds true for higher fields as well. Furthermore, the absence of a field dependence of $dM/dT$ below “$T_{c2}(H)$” is not confirmed either. In principle, the rounding of the transition could be due to an inhomogeneity or/magnetic field induced finite size effect, or the variation of superconducting properties from grain to grain. However, since the rounding of the transition increases with the applied magnetic field (Fig.1b) and the magnetic field induced limiting length scale $L_H$ (Eq.(1)) decreases one suspects that the rounding stems from the magnetic field induced finite size effect. In this case there is no sharp superconductor to normal state transition in an applied magnetic field, as predicted by the mean-field approximation. To verify this educated guess we take thermal fluctuations, the magnetic field and the inhomogeneity induced finite size effect into account.
When the rounding of the transition stems from a magnetic field or inhomogeneity induced finite size effect, the correlation length $\xi$ cannot grow beyond the limiting length $L_{H,I}$, where [1–3,7,12]

$$\xi (T_p) = \xi_0 \left| t_p/t_c \right|^{-\nu} = L_{H,I}, \quad t = 1 - T_p/T_c, \quad \nu \simeq 2/3.$$  \(4\)

$L_I$ denotes the limiting length of the homogeneous domains of the sample. Note that $\nu \simeq 2/3$ holds in the charged and uncharged (3D-XY) universality class [2,11]. In superconductors, exposed to a magnetic field $H$, there is the aforementioned additional limiting length scale $L_H = \sqrt{\Phi_0/|aH|}$ (Eq. (1)), related to the average distance between vortex lines. Indeed, as the magnetic field increases, the density of vortex lines becomes greater, but this cannot continue indefinitely, the limit is roughly set on the proximity of vortex lines by the overlapping of their cores. Because of these limiting length scales the phase transition is rounded and occurs smoothly. Consequently, the thermodynamic quantities like the magnetization, magnetic penetration depth, specific heat etc. are smooth functions of temperature near $T_p$. To uncover the scaling properties of the magnetization in this regime and to estimate the magnetic field dependence of $T_p$, we invoke the scaling properties of the free energy per unit volume in the fluctuation dominated regime. For a field applied with an angle $\delta$ from the $c$-axis it reads [1–3,7,12]

$$f = \frac{Q k_B T}{\xi_{ab}\xi_c} G(z), \quad z = \frac{H \xi_{ab}}{\Phi_0} \epsilon = \left( \cos^2 (\delta) + \frac{1}{\gamma^2} \sin^2 (\delta) \right)^{1/2},$$

where $Q$ is a universal constant, $G(z)$ a universal function of its argument, $\xi_{ab,c}$ the correlation lengths parallel to the $ab$- and $c$-axis, respectively, and $\gamma = \xi_{ab}/\xi_c$ the anisotropy. For the magnetization per unit volume we obtain then the scaling relation

$$m = -\frac{\partial f}{\partial H} = -\frac{Q k_B T H^{1/2}}{\Phi_0^{3/2}} \gamma \xi_{ab}^2 \frac{1}{z^{1/2}} \frac{dG}{dz}.$$  \(6\)

In single crystals and oriented grains for $H$ applied parallel to the $c$-axis ($\delta = 0$) the magnetization reduces for sufficiently large anisotropy $\gamma$ to

$$m = -\frac{Q k_B T H^{1/2}}{\Phi_0^{3/2}} \gamma \xi_{ab}^2 \frac{dG}{dz} \quad z = \frac{H \xi_{ab}^2}{\Phi_0},$$

while in powder samples it reduces to

$$m = -\frac{Q k_B \gamma T H^{1/2}}{\Phi_0^{3/2}} \left\langle \frac{1}{z^{1/2}} \frac{dG}{dz} \right\rangle \quad z = \frac{H \xi_{ab}^2}{\Phi_0} |\cos (\delta)|,$$

where $\langle ... \rangle = \int_0^{2\pi} ... d\delta$. Thus, when thermal fluctuations dominate and the magnetic field induced finite size effect sets the limiting length ($L_H < L_I$), data as shown in Figs.1 and 2 should collapse on a single curve when plotted as

![Figure 2](image-url)
Thus, down to 0.1T magnetic field the scaling regime is seen to parallel to the magnetic field with diameter \( L \). This reflects the fact that the fluctuations of a bulk superconductor in sufficiently high magnetic fields become effectively one dimensional, as noted by Lee and Shenoy \[4\]. Here a bulk superconductor behaves like an array of rods parallel to the magnetic field with diameter \( L_H \), while the scaling relation (6) holds for sufficiently low fields where three dimensional fluctuations dominate. On the other hand, with decreasing magnetic field the scaling regime is seen to increase. Thus, down to 0.1T the magnetic field appears to set the limiting length scale so that \( L_f \) \( \geq L_{H=0.1T} \approx 814\,\text{Å} \).

To substantiate the evidence for the dominant role of thermal fluctuations and to strengthen the discrimination between the inhomogeneity and magnetic field induced finite size effect further, we consider the temperature dependence of \( d (m/T) /dT \). From Eqs.(5) and (6) we obtain,

\[
\frac{d (m/T)}{dT} = - \frac{Q k_B \gamma}{\Phi_0} \frac{1}{\xi_{ab}^2} \frac{d \xi_{ab}}{dT} \left( \frac{d G}{dz} + z \frac{d^2 G}{dz^2} \right),
\]

Due to the magnetic field induced finite size effect \( \xi_{ab} \) cannot grow beyond \( \xi_{ab} = L_H \) so that \( d \xi_{ab} /dT = 0 \) and with that \( d (m/T) /dT = 0 \) at \( T_P \). Indeed, at \( T_P \) the scaling variable adopts the value \( z_p = \epsilon /a \) so that \( d G /dz \) and \( 2z d^2 G /dz^2 \) are temperature and field independent. In powder samples with sufficiently large anisotropy this occurs at \( z_p \approx (\epsilon) /a = 2a /\pi \). Noting that \( \gamma = \xi_{ab} /\xi_{ab} \) the peak height scales then as

\[
\left| \frac{d (m/T)}{dT} \right|_{T_P} \propto H^{(\nu-1) / (2\nu)}.
\]

This differs fundamentally from the inhomogeneity induced finite size effect, where \( (1/\xi_{ab}^2) \frac{d (\xi_{ab})}{dT} \big|_{T_P} \propto L_f^{1 / \nu-1} \) and \( z \) adopts the value \( z = \epsilon H L_f^2 /\Phi_0 \). Thus, the inhomogeneity induced finite size effect exhibits a weak field dependence arising from the change of \( d G /dz \) and \( z dG /dz \). In Fig.4 we displayed our estimates for \( d (m/T) /dT \big|_{T_P} \). For comparison we included \( d (m/T) /dT \big|_{T_P} \propto H^{-1/4} \), corresponding to Eq.(10) with \( \nu = 2/3 \). The quantitative agreement for small fields confirms that the limiting length is set by the magnetic field and not by inhomogeneities in the grains of our sample. Accordingly, we have shown that in an applied magnetic field \( \text{Sr}_{0.9}\text{La}_{0.1}\text{CuO}_2 \) does not undergo a sharp phase transition from the superconducting to the normal state up to at least 5T. Additional stringent features not accounted for by the mean-field treatment emerge from Figs.1b and 2. \( d M/dT \) and with that \( d (M/T) /dT \) and \( d (M/T) /dT \) are seen to fall below and above \( T_P \). Furthermore, this fall is field dependent. According to Eq.(9) the initial fall reflects the temperature dependence of \( (1/\xi_{ab}^2) \frac{d (\xi_{ab})}{dT} = \text{sgn} \left( 1 - T/T_c \right) \left( 1 - T/T_c \right)^{\nu-1} / (\nu T_c) \), while the field dependence stems from \( d G /dz \propto -2z dG /dz \). Thus, the fall of \( d M/dT \) and \( d (M/T) /dT \) below and above \( T_P \) and its field dependence confirm the dominant role of 3D-XY fluctuations, while the rounding of the transition reveals

\[\text{FIG. 3.} \ M/(TH^{1/2}) \text{ vs.} t/t_p (H) = (T/T_c - 1) / (1 - T_p (H) /T_c) \text{ for various magnetic fields} \ H \text{ with} T_c = 43K \text{ and the listed estimates for} T_p (H) \text{ for the data shown in Fig.1.}\]
the magnetic field induced finite size effect. In principle, the scaling function \(G(z)\) should have a singularity at some value \(z_m\) of the scaling variable below \(T_p(H)\) describing the vortex melting transition, but this singularity is not addressed here.

\[ T_p(H) = T_c \left( 1 - \left( \frac{aH\xi_{ab0}}{\Phi_0} \right)^{1/2\nu} \right). \]  

The solid line in Fig.5 is this relation with \(T_c = 43K\), \((a\xi_{ab0}/\Phi_0)^{3/4} = 0.044\), \(a = 3.12\) and \(\nu = 2/3\), yielding for the critical amplitude of the c-axis correlation length the estimate \(\xi_{c0} \approx 3.6\AA\). The criterion for 3D superconductivity below \(T_c\) is \(\xi_c = \xi_{c0} \left( 1 - T/T_c \right)^{-\nu} > c/\sqrt{2} \approx 2.4\AA\), where \(c \approx 3.4\AA\) is the c-axis lattice constant. This reveals that Sr\(_{0.9}\)La\(_{0.1}\)CuO\(_2\) is even at low temperature a 3D superconductor, as previously suggested [9].

Finally we turn to the pressure effect on the magnetization. Close to the zero field transition temperature Eq.(8) reduces to [1–3,7,12]
\[ \frac{M}{VH^{1/2}T_c^\gamma} = -\frac{QCk_B}{\phi_0^{3/2}}, \quad C = \left\langle \frac{\cos(\delta)^2\,dG}{z^{1/2}} \right\rangle_{T\to T_c, H\neq 0}, \] (12)

where \( C \) is a universal constant. As the pressure effect on the magnetization at fixed magnetic field is concerned it implies that the relative shifts of the magnetization \( M \), volume \( V \), anisotropy \( \gamma \) and \( T_c \) are not independent but close to \( T_c \) related by

\[ \frac{\Delta M}{M} = \frac{\Delta V}{V} + \frac{\Delta \gamma}{\gamma} + \frac{\Delta T_c}{T_c}. \] (13)

On that condition it is impossible to extract these changes from the temperature dependence of the magnetization. However, supposing that close to criticality the magnetization data scale within experimental error as

\[ i^j M(T) = j^j M(aT), \] (14)

where \( i^j M \) denotes the magnetization for different pressures, the universal relation (13) reduces to

\[ -\frac{\Delta T_c}{T_c} = \frac{\Delta V}{V} + \frac{\Delta \gamma}{\gamma} = 1 - a. \] (15)

Hence, when Eq.(14) holds true, the pressure and isotope effect on \( T_c \) mirrors that of the anisotropy \( \gamma = \xi_{ab}/\xi_c = \lambda_{ab}/\lambda_c \) and of the volume [8]. In Fig.6 we displayed the field cooled magnetization data for various hydrostatic pressure up to 8.45 kbar. The hydrostatic pressure was generated in a copper-beryllium piston cylinder clamp designed for magnetization measurements. The sample was mixed with Fluorient FC 77 (pressure transmitting medium). The pressure was measured in situ by monitoring the \( T_c \) shift of a small piece of In \((T_c(p=0) \approx 3.4K)\) placed within the cell. Apparently, the data collapses within experimental error on a single curve so that Eq.(14) with \( a \approx 1 \) applies. On the other hand, given the bulk modulus \( B = 1170 \) kbar [14] there is the volume change \( \Delta V/V \approx -P/1170 \) with \( P \) in kbar. It implies with Eq.(15) and in the absence of a significant pressure effect on \( T_c \) that the volume reduction mirrors essentially the increase of the anisotropy because

\[ \frac{\Delta V}{V} = -\frac{\Delta \gamma}{\gamma} \approx -\frac{P \text{ (kbar)}}{1170}. \] (16)

FIG. 6. Field cooled \((5 \times 10^{-4}T)\) magnetization of a \( \text{Sr}_{0.9}\text{La}_{0.1}\text{CuO}_2 \) powder sample vs. \( T \) near \( T_c \) for various hydrostatic pressures.

Together with the experimental fact that in both, electron and hole doped cuprates, the pressure effect on \( T_c \) depends strongly on the dopant concentration and \( dT_c/dP \) nearly vanishes close to optimum [15], \( \Delta V/V \approx -\Delta \gamma/\gamma \) appears to be a generic property of optimally doped cuprate superconductors. With that it shows that the anisotropy is essential towards an understanding of superconductivity in these materials.

We have shown that in \( \text{Sr}_{0.9}\text{La}_{0.1}\text{CuO}_2 \) thermal fluctuations do not alter the zero field thermodynamic properties near \( T_c \) only, but invalidate the assumption of an upper critical field \( H_{c2} \) over a rather extended temperature range.
Indeed, the correlation length $\xi_{ab}$ increases strongly when $T_c$ is approached. However, for nonzero magnetic field $H$ there is the limiting length scale $L_H = \sqrt{\Phi_0/aH}$. It is comparable to the average distance between vortex lines and implies that $\xi$ cannot grow beyond $L_H$ and with that there is a magnetic field induced finite size effect. Since $L_H$ decreases with increasing field strength it sets the limiting length scale for sufficiently high fields. In the present case, both the magnetic field induced shift of the peak location and peak height in $dM/dT$ clearly revealed that for $\mu_0 H \gtrsim 0.1$T the magnetic field induced finite size effect allows to probe the homogeneous parts of the grains with dimension $814\,\text{Å}$. As a result, whenever the thermal fluctuation dominated regime is accessible, homogeneous type II superconductors do not undergo in nonzero magnetic field a continuous phase transition, e.g. to a state with zero resistance. Furthermore, we have shown that the pressure effect on the magnetization does not provide estimates for the shift of the transition temperature only, but involves the change of the volume and anisotropy as well. Consistent with the generic behavior of optimally doped cuprates we found that in $\text{Sr}_{0.9}\text{La}_{0.1}\text{CuO}_2$ the pressure effect on $T_c$ is negligibly small, while the negative value of the relative volume change mirrors that of the anisotropy.

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