Interplay between the $0^+_2$ resonance and the nonresonant continuum of the drip-line two-neutron halo nucleus $^{22}$C

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Introduction. Exploring the frontier of the nuclear chart is one of the most important subjects in nuclear physics. Properties of neutron drip-line nuclei, e.g., $^{11}$Li, $^{19}$B, and $^{22}$C, are therefore crucial for this purpose. Very recently, evidence for an unbound ground state of $^{20}$O was reported [1], which could extend the concept of drip-line nuclei to the unbound-state regions. In this situation, clarification of unbound states, i.e., resonance structures, of nuclei around the neutron drip-line will be a fascinating subject.

Another important aspect of this subject is the figure of the cross section to a resonance state. It is well known that rescattering [3], Raman scattering [4], hypernucleus formation [5], optical absorption [6], and quantum transport in a mesoscopic regions. In this situation, clarification of unbound states, i.e., resonance structures, of nuclei around the neutron drip-line will be a fascinating subject.

In this study we focus on $^{22}$C, the drip-line nucleus of carbon isotopes. By measuring the reaction cross section [8] and the neutron removal cross section [9], ground state properties of $^{22}$C have been intensively studied so far; the results strongly support the picture that $^{22}$C is an s-wave two-neutron halo nucleus, in consistent with the theoretical prediction of Ref. [10] based on a $^{20}$C+$n+n$ three-body model. The dominance of the s-wave configuration of the valence two neutrons gives large transition probability to the low-energy nonresonant $0^+$ continuum of $^{22}$C as discussed below. We can thus expect a significant interference between a possible low-lying resonance and the nonresonant continuum in the $0^+$ state, which makes a remarkable change in the shape of the resonant breakup cross section of $^{22}$C.

In this Letter, we investigate the resonance structure of $^{22}$C with the three-body cluster-orbital shell model (COSM) [11] through the breakup cross section of $^{22}$C by $^{12}$C at 250 MeV/nucleon evaluated by the continuum-discretized coupled-channels method (CDCC) [12–14]. COSM is a powerful method to describe a system consisting of a core plus valence nucleons; it has successfully been applied to studies of the ground and resonance states of $^9$He, $^7$He, and $^5$He [15–17]. One of the most important advantages of COSM is that the relative wave function between the core nucleus and each nucleon is directly obtained covering a quite wide space for radial behavior. CDCC is a sophisticated reaction model that has been applied to various breakup processes with high success. Our main purpose is to investigate how the resonance states of $^{22}$C predicted by COSM are “observed” in the breakup cross section, from the viewpoint of the Fano effect mentioned above.

Formalism. In the present COSM calculation, a $^{20}$C+$n+n$ three-body model is adopted for the $^{22}$C wave function:

$$\Phi_{I,M_I}(\eta_1, \eta_2) = \sum_{I_1j_1I_2j_2} A \left[ \phi_{I_1j_1}(\eta_1) \otimes \phi_{I_2j_2}(\eta_2) \right]_{I,M_I},$$

(1)

where $I$ and $M_I$ are the total spin of $^{22}$C and its third component, respectively, and $\eta_i (i = 1 \text{ or } 2)$ is the relative coordinate of the $i$th neutron to the center of the $^{20}$C core. $A$ represents the antisymmetrization operator for the two valence neutrons; antisymmetrization between a valence neutron and a neutron in $^{20}$C is approximately taken into account with the orthogonal condition model [18]. In Eq. (1), $\phi$ is the Gaussian basis function

$$\phi_{Ij,m_J}^h(\eta) = \varphi_i^h(\eta) \left[ \gamma_i(\hat{\eta}) \otimes \sigma_{1/2} \right]_{j,m_J},$$

(2)

where $\gamma$ is the spin $1/2$ wave function of neutron and

$$\varphi_i^h(\eta) = \sqrt{\frac{2}{\Gamma(I+3/2)b_{1/2}^h}} \frac{1}{\eta^2} \exp \left(-\frac{\eta^2}{2b_{1/2}^h} \right)$$

(3)
with $\Gamma$ the Gamma function. The range parameters $b_i$ ($i = 1$, \ldots, $i_{\text{max}}$) are chosen to lie in a geometric progression: $b_i = b_1 \gamma^{i-1}$. By diagonalizing an internal Hamiltonian $h$ of $^{22}\text{C}$ with the basis functions, one obtains eigenstates, each of which is characterized by $I$, $M_f$, and the energy index $c$, with the expansion coefficients $a_{c12}^{I_{12}I_{22}}$. In the present case, there is only one bound state in $J = 0$. All the other states are located above the $^{20}\text{C} + n + n$ three-body threshold, which are called pseudostates (PS).

Since COSM describes the $^{22}\text{C}$ wave function covering a quite large model space, the PS can be regarded to a good approximation as discretized continuum states. Then the total wave function of the $^{20}\text{C} + n + n + ^{12}\text{C}$ four-body reaction system with the total angular momentum $J$ and its third component $M$ can be expanded as

$$\Psi_{JM}(\eta_1, \eta_2, R) = \sum_{cIL} \left[ \Phi_{j}^{cL}(\eta_1, \eta_2) \otimes \chi_{cIL}(R) \right]_{JM}, \quad (4)$$

where $\chi_{cIL}(R)$ is the scattering wave of $^{22}\text{C}$ in the $(c, I)$ state relative to $^{12}\text{C}$; $L(R)$ is the corresponding relative angular momentum (coordinate).

By solving the four-body Schrödinger equation

$$[H - E] \Psi_{JM}(\eta_1, \eta_2, R) = 0, \quad (5)$$

$$H = T_R + U_{n_1}(R_1) + U_{n_2}(R_2) + U_c(R_c) + h \quad (6)$$

with the standard boundary condition of $\chi_{cIL}(R)$, one may obtain the scattering matrix to the $(c, I, L)$ channel. In Eq. (6), $T_R$ is the kinetic energy operator associated with $R$, $U_{n_i}$ ($i = 1$ or $2$) is the neutron distorting potential, and $U_c$ is the potential between the $^{20}\text{C}$ core and $^{12}\text{C}$. This framework is four-body CDCC [19, 20] incorporating the COSM wave functions, which we call COSM-CDCC below. We further adopt the prescription [21] based on the complex-scaling method (CSM) [22], the CSM smoothing method, to obtain a smooth breakup cross section $d^2\sigma/(d\Omega d\epsilon)$, i.e., the double differential breakup cross section (DDBUX). Here, $\epsilon$ is the breakup energy of the $^{20}\text{C} + n + n$ system measured from the three-body threshold and $\Omega$ is the solid angle of the center-of-mass (c.m.) of $^{22}\text{C}$ after the breakup; the corresponding polar angle is denoted by $\theta$ below. It should be noted that in Refs. [16, 17, 21] a continuum level density was shown to be correctly described in terms of the complex-scaled Green function, which enables one to obtain continuous strength functions of physics quantities using the CSM.

Numerical input. In the $^{20}\text{C} + n + n$ three-body Hamiltonian $h$, we adopt the Minnesota nucleon-nucleon interaction [23] and a Woods-Saxon potential for the $n$-$^{20}\text{C}$ system, consisting of the central and spin-orbit parts. As for the latter, we use Set B parameters of Ref. [10]; we have slightly changed $V_1$ and $V_s$ to 20.00 MeV and 10.50 MeV, respectively, so that the 1s state is unbound. In the COSM calculation, we include the single-particle configuration of each $n$ up to $l = 5$ ($l = 4$) for the $0^+$ ($2^+$) state of $^{22}\text{C}$. The radial wave function between $n$ and $^{20}\text{C}$ in each single-particle orbit is described by 10 Gaussian basis functions; we use the range parameters of $b_1 = 0.3$ fm and $\gamma = 1.5$. We assume a sub-closed shell up to 1d5/2 for neutron in the $^{20}\text{C}$ core.

As a result of diagonalization of $h$, we obtain the $0^+$ ground state at 289 keV below the $^{20}\text{C} + n + n$ threshold, which is consistent with the empirical value $420 \pm 940$ keV [24], together with 604 (1,385) PS above the threshold in the $0^+$ ($2^+$) state. The matter radius of the ground state is found to be 3.49 fm. In the CDCC calculation, we include the ground state and the 77 (164) PS for $0^+$ ($2^+$) below $\epsilon = 10$ MeV, which gives a convergence of the results shown below.

As for the distorting potentials of $n$, $^{12}\text{C}$ and $^{20}\text{C}$, we adopt microscopic single and double folding models, respectively, with the CEG07b nucleon-nucleon $G$-matrix interaction including the medium effects [25]. We use the nuclear densities of $^{12}\text{C}$ and $^{20}\text{C}$ given in Refs. [26] and [27], respectively, with a slight change in the parameters for the former. CDCC equations between $^{22}\text{C}$ and $^{12}\text{C}$ are solved up to $R = 30$ fm and the number of the partial waves is set to 600. In the CDCC calculation, we use the so-called no-recoil approximation to the $^{20}\text{C}$ core, as in the previous study of Ref. [28]; this approximation is considered to be valid when the mass of the core nucleus is much larger than the valence particle(s), which is satisfied well in the present case.

In the CSM smoothing method, we adopt the complex-scaling angle $\theta_{\text{CSM}}$ of $14^\circ$. The basis functions used in diagonalization of the scaled Hamiltonian $h^\text{CSM}$ are similar to above, except that we need finer and wider bases. We use $(i_{\text{max}}, b_1, \gamma) = (25, 0.2, 1.3), (20, 0.2, 1.3), (15, 0.3, 1.4)$ for the $s$, $d$, and other orbits of neutron, respectively.

Results and discussion. Figure 1 shows the DDBUX $d^2\sigma/(d\epsilon d\Omega)$ of $^{22}\text{C}$ by $^{12}\text{C}$ at 250 MeV/nucleon calculated by COSM-CDCC. One sees some structures in the DDBUX, expected to reflect properties of the resonance and the non-resonant continuum of $^{22}\text{C}$. In fact, COSM predicts some resonance states of $^{22}\text{C}$ and $^{21}\text{C}$ in the energy region shown in Fig. 1; the results are summarized in Table I. The next ques-
tion is thus how these resonances contribute to the DDBUX.

As a great advantage of the CSM-smoothing method, one can decompose the DDBUX into the components due to the three-body resonances (each of the $0^+_2$, $2^+_1$, and $2^+_2$ states), the binary resonance of $^{23}$C coupled with another neutron, and the nonresonant three-body continuum. Figure 2 shows the result of the decomposition of the breakup energy distribution $d\sigma/d\epsilon$, which is obtained by integrating the DDBUX over $\theta$ from $0^\circ$ to $0.1^\circ$. The upper and lower panels correspond to the $0^+$ and $2^+$ states of $^{22}$C, respectively. In each panel, the solid (dotted) line shows the total breakup cross section (contribution of the three-body nonresonant continuum). The contribution of the three-body resonance, $0^+_2$ ($2^+_1$) in the upper (lower) panel, is denoted by the dashed line. In both $I^+$ states, it is found that the contributions from the $^{21}$C binary resonance are negligibly small. Similarly, the $2^+_2$ resonance gives an inappreciable cross section.

For the $2^+$ state, one clearly sees that the peak in $d\sigma/d\epsilon$ is due to the $2^+_1$ resonance, which has the standard Breit-Wigner form. It is found that the peak around $(\epsilon, \theta) = (0.85, 0.65)$ shown in Fig. 3 is also due to the $2^+_1$ resonance. This finding is consistent with the angular dependence of a BUX corresponding to the multipolarity $\lambda$ of 2, i.e., the BUX has a maximum at nonzero $\theta$. Here, $\lambda$ denotes the multipolarity of $U_n$ (i = 1 or 2). The breakup to the $2^+$ state can thus be regarded as a standard transition process to a resonance quite isolated from the nonresonant continuum.

On the other hand, as shown in the upper panel of Fig. 2, the $0^+_2$ resonance has a peculiar form due to the Fano effect. Figure 3 shows the $\theta$ dependence of the Fano effect. The

![FIG. 2.](image2.png)

![FIG. 3.](image3.png)

TABLE I. Resonance energy $E_r$ and width $\Gamma_r$ of $^{22}$C and $^{21}$C.

| nucleus | $I^+$ | $E_r$ (MeV) | $\Gamma_r$ (MeV) | main configuration |
|---------|-------|-------------|-----------------|-------------------|
| $^{22}$C | $0^+_2$ | 1.02 | 0.52 | $(0d3/2)^2$ |
|         | $2^+_1$ | 0.86 | 0.10 | $(1s1/2)(0d3/2)$ |
|         | $2^+_2$ | 1.80 | 0.26 | $(0d3/2)^2$ |
| $^{21}$C | $3/2^+$ | 1.10 | 0.10 | $(0d3/2)$ |

FIG. 2. (Color online) Breakup energy distribution corresponding to $\theta = 0^\circ$–$0.1^\circ$ and its resonant and nonresonant components. See the text for details.

FIG. 3. (Color online) The $0^+_2$ DDBUX (upper panel) and the total $0^+$ DDBUX (lower panel). All results except for $\theta = 0.00^\circ$ are multiplied by the numbers beside the lines.
ground state. This gives a large breakup cross section to the low-energy $0^+$ nonresonant continuum, for which only the monopole ($\lambda = 0$) transition is responsible, with the same two-neutron configuration. It should be noted that if neutron has a finite value of $l$, it hardly exists in the low-energy nonresonant continuum of $^{22}$C because of the centrifugal barrier. At the same time, the small but non-negligible $(0d3/2)^2$ configuration of about 13% in the ground state of $^{22}$C brings the low-lying $0^+_2$ resonance. This is essentially due to the closely-located (1$s1/2$ and $(0d3/2$) neutron single-particle orbits of $^{22}$C. Thus, the resonant and nonresonant states with the same spin-parity $(0^+)$ strongly affect each other. This is the main reason for the sizable Fano effect found. The coexistence of the $0^+$ resonance and nonresonant continuum will rarely be realized when a core plus one neutron system is considered; an s-wave neutron cannot form a resonance, except through a compound process or a Feshbach resonance \[29\]. Therefore, the features of the resonant cross section shown in the present study are expected to be quite unique to an s-wave two-neutron halo nucleus, i.e., $^{22}$C.

It should be noted that in the present study we have no so-called momentum matching condition, which usually dictates whether or not a resonance is observed when the reaction $Q$ value is large. Moreover, since we have only one threshold, peculiar behavior of a resonance state often found in hadron physics, e.g., $f_0(980)$ \[30\], is not expected. The present result is purely due to the interplay between the $0^+_{1}$ resonance and the $0^+$ nonresonant continuum.

Experimental data of the DDBUX of $^{22}$C are highly desirable to validate the interesting behavior of the $0^+$ breakup cross section suggested here. For this purpose, one must eliminate the $2^+$ cross section from the total DDBUX. This can be performed quite easily, because the $2^+$ contribution will be described well by a standard Breit-Wigner form. To do this, however, we need experimental data with high energy resolution; they can hopefully be obtained at RIBF with utilizing the brand-new SAMURAI spectrometer.

**Summary.** We have proposed a new framework of four-body CDCC adopting COSM wave functions, COSM-CDCC, and applied it to the breakup process of $^{22}$C by $^{12}$C at 250 MeV/nucleon. We showed the $2^+_1$ resonance gives a clear peak in the DDBUX, whereas the $0^+_{1}$ resonant cross section has a remarkably different shape from the Breit-Wigner form. The latter is due to the coupling between the $0^+_{1}$ resonance and the $0^+$ nonresonant continuum, i.e., the Fano effect. The shape of the $0^+_{1}$ cross section changes drastically with the scattering angle of the c.m. of the $^{20}$C+$n+n$ system. The distinguished Fano effect found in the present study is expected to be unique to an s-wave two-neutron halo nucleus, i.e., $^{22}$C.

Experimental clarification of the sizable Fano effect on the $0^+_{1}$ resonance will be very interesting. From the theoretical side, inclusion of the recoil of the core nucleus $^{20}$C and its dynamical excitation during the breakup of $^{22}$C will be important future work. Extension of COSM-CDCC to five- and six-body breakup reaction will be a very challenging subject of nuclear reaction studies.

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