Numerical simulation of small-scale mixing processes in the upper ocean and atmospheric boundary layer

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Abstract. The processes of turbulent mixing and momentum and heat exchange occur in the upper ocean at depths up to several dozens of meters and in the atmospheric boundary layer within interval of millimeters to dozens of meters and can not be resolved by known large-scale climate models. Thus small-scale processes need to be parameterized with respect to large scale fields. This parameterization involves the so-called bulk coefficients which relate turbulent fluxes with large-scale fields gradients. The bulk coefficients are dependent on the properties of the small-scale mixing processes which are affected by the upper-ocean stratification and characteristics of surface and internal waves. These dependencies are not well understood at present and need to be clarified. We employ Direct Numerical Simulation (DNS) as a research tool which resolves all relevant flow scales and does not require closure assumptions typical of Large-Eddy and Reynolds Averaged Navier-Stokes simulations (LES and RANS). Thus DNS provides a solid ground for correct parameterization of small-scale mixing processes and also can be used for improving LES and RANS closure models. In particular, we discuss the problems of the interaction between small-scale turbulence and internal gravity waves propagating in the pycnocline in the upper ocean as well as the impact of surface waves on the properties of atmospheric boundary layer over wavy water surface.

1. Introduction
The processes of small-scale mixing in the atmospheric boundary layer and the upper ocean are of primary importance for mass, momentum and heat exchange between atmosphere and hydrosphere. The mixing occurs due to turbulent motions coupled with the gravity waves at the sea surface as well as internal gravity waves in the oceanic pycnocline. These coupling dramatically affects the dynamics of turbulence and thus needs to be taken into account in climate prognostic models [1,2].

In the state of the art models, the turbulent fluxes of momentum, energy and matter are calculated via bulk formulae which to a large extent are based on empirical coefficients. The latter are obtained from field and laboratory experiments and exhibit essential variability whose nature in relation to turbulence-wave interaction processes is not yet fully understood [3]. Of special interest, from the point of view of the present study, is the influence of density (or temperature) stratification of the atmospheric surface layer on wind-wave interactions and the air-sea fluxes in the atmospheric boundary layer and the interaction between small-scale turbulence and internal gravity waves propagating in the seasonal pycnocline in the upper ocean.

Measurements in the atmospheric boundary layer above waved water surface require special efforts. Especially complicated are measurements in wave troughs and in a thin layer...
above the water surface covering the viscous sublayer and the adjacent buffer layer. The
typical height of the region to be examined is of order of millimetres, which is usually much
smaller than the surface wave amplitude. Here, detailed properties of air flow cannot be
detected using contact measurement techniques, such as wave-following probes [4,5].
Methods based on particle image velocimetry (PIV) technique allow for measuring wind
velocity at heights over the surface of order 1 mm. This is sufficient for investigating both
viscous and buffer layers at not too strong winds, with friction velocities less than 10 cm/s
[6,7]. Precise measurements of the air temperature in the viscous layer and buffer layer above
waves are strongly needed but still remain difficult.

In laboratory studies of the interaction of small-scale turbulence with internal waves (IWs),
in the absence of mean shear, turbulent motions are usually induced by an oscillating grid
[2,8]. One of the most interesting and practically important aspects of the turbulence-IWs
interaction in the absence of mean shear is the effect of damping of IWs by turbulence on the
one hand, and the possibility of enhancement of small-scale turbulence by non-breaking IWs
on the other hand. The phenomena of IWs damping by turbulence was observed in laboratory
experiments [1, 9, 10]. Measurements of the IW amplitudes [10] showed an effective
enhancement of the decay rate of IW under the effect of turbulence which was found to be in
good agreement with the theoretical prediction [11] based on RANS-closure approach.

The effect of enhancement of small-scale turbulence by mechanically-generated, non-
breaking internal wave propagating in the pycnocline was also observed experimentally [12].
The results show that a sufficiently strong (as compared to turbulent velocity fluctuations),
non-breaking internal wave can significantly increase the kinetic energy of turbulence in the
well-mixed layer above the pycnocline. However, the obtained experimental data did not
provide enough detail to study the modification of turbulence kinetic energy spectra under the
influence of IWs.

Numerical experiments make a complementary tool to laboratory and field measurements.
Earlier numerical modelling of interaction between turbulent air flow and surface waves was
based on the Reynolds (ensemble) Averaged, Navier-Stokes (RANS) equations for stationary,
two-dimensional flow [13,14]. These RANS equations employed semi-empirical concept of
turbulent viscosity to express unknown Reynolds (turbulent) stresses through known mean-
field gradients. This approach is computationally cheap and provides a useful picture of the
mean velocity, pressure and turbulent fluxes in the air flow. However, it is not immediately
applicable to modelling air flow in the viscous sublayer and buffer region, which requires
inclusion of the molecular transports [15].

More advanced (but more computationally expensive) is Large-Eddy Simulation (LES)
based on numerical integration of three-dimensional, non-stationary, filtered Navier-Stokes
equations and capable of resolving 3D large-scale turbulent structure of the flow [16,17].
However, LES do not resolve properties of the flow close to the water surface (or solid wall).
Typically the first LES grid node in the vertical direction is located within the logarithmic
region of the boundary layer. As a remedy, one can perform wall-resolved LES [18] and
resolve the viscous sublayer; however, numerical cost strongly increases.

Direct Numerical Simulation (DNS) is free of this drawback and resolves the entire
turbulence spectrum from large eddies down to the dissipation length scale. Early studies [19,
20] employed DNS to investigate turbulent wind flow over surface waves for bulk Reynolds
number Re = (U0\lambda/\nu) (where U0 is the bulk wind flow velocity, \lambda the surface wave length, and
\nu the air molecular viscosity) less than 10^4 and wave slope ka = 0.1 (where k is the
wavenumber and a the wave amplitude). Later studies [21] considered a larger wave slope (ka
= 0.25) and nearly the same Re (≈ 10000). Although DNS provides full description of the turbulent flow at all physically significant scales, so far it has not been able to achieve Reynolds numbers of order 10^5 typical of the wind-wave interaction in laboratory experiments. Recent DNS considered Re = 15000 with maximum wave slope ka = 0.2 [22].

The interaction between internal waves (IWs) propagating through a pycnocline with a turbulent layer above the pycnocline was also studied with the use of DNS [23, 24]. The results show that if the ratio of IW amplitude vs. turbulent pulsations amplitude is sufficiently small (less than 0.5), turbulence strongly enhances IW damping. In this case, the damping rate obtained in DNS agrees well with the prediction of a semi-empirical closure approach based on the RANS equations [11]. However, for larger IW amplitude the effect of damping of IWs by turbulence is much weaker, and, in this case, the RANS-based theory overestimates the IW damping rate by the order of magnitude. DNS results also show that in the absence of IW turbulence decays, but its decay rate is reduced in the vicinity of the pycnocline where stratification effects are significant. In this case, at sufficiently late times most of turbulent energy is located in a layer close to the pycnocline center. Here turbulent eddies are collapsed in the vertical direction and acquire the “pancake” shape. Strong IWs modify turbulence dynamics, in that the turbulence kinetic energy (TKE) is significantly enhanced as compared to the TKE in the absence of IWs. As in the case without IW, most of turbulent energy is localized in the vicinity of the pycnocline center. Here the TKE spectrum is considerably enhanced in the entire wavenumber range as compared to the TKE spectrum in the absence of IWs.

In the present paper, we discuss how DNS can be employed to study the interaction between turbulence and surface and internal gravity waves in the upper ocean and marine boundary layer. In Section 2 the numerical algorithm used in DNS of turbulent, stably-stratified air flow over water waves is presented and some simulation results are discussed. Numerical algorithm and results of DNS of the dynamics of small-scale turbulence coupled with a strong internal gravity wave propagating in a pycnocline are discussed in Sec. 3. Conclusions are presented in Sec. 4.

2. DNS of turbulent air-flow over waved water surface

2.1 Numerical method

We perform direct numerical simulation of turbulent stably stratified Couette flow above waved water surface (Figure 1). A Cartesian framework is considered where x-axis is oriented along the mean wind, z-axis is directed vertically upwards and y-axis is orthogonal to the mean flow and parallel to the wave front. We prescribe two-dimensional, x-periodic water wave with amplitude a, wavelength \( \lambda \) and phase velocity c. The maximum wave slope considered in our DNS is \( ka = 2\pi a / \lambda = 0.2 \). The rectangular computational domain has the sizes \( L_x = 6\lambda, L_y = 4\lambda \) and \( L_z = \lambda \) in the x-, y-, and z-directions, respectively, and the air flow is assumed to be periodical in the x- and y-directions. DNS is performed in a reference frame moving with the wave phase velocity, c, so that the horizontal coordinate in the moving framework is \( x = x' - ct \), where \( x' \) is the coordinate in the laboratory reference frame. Then the lower boundary, representing the wave surface, is stationary in the moving reference frame. As in all previous DNS studies mentioned above, in the present DNS study we do not consider capillary ripples riding on the wave generally found in a realistic sea situation. The presence of ripples increases the effective wavy-surface slope and may cause local flow
separation which significantly complicates numerical solution. The no-slip boundary condition is prescribed at the lower boundary, so that the air flow velocity here coincides with the velocity in the water wave. The no-slip boundary condition is also prescribed at the upper horizontal plane moving in the $x$-direction with bulk velocity $U_0$. This condition provides the external source of momentum due to the viscous shear stress, which compensates the viscous dissipation in the boundary layer and makes the flow statistically stationary. The stable density stratification is specified by prescribing the potential temperature at the wavy surface as $\Theta = \Theta_0$ and the top boundary plane as $\Theta = \Theta_0 + \Delta \Theta$, where $\Delta \Theta > 0$.

Numerical algorithm is based on the integration of full, 3D Navier-Stokes equations for incompressible fluid under the Boussinesq approximation [25]:

$$\frac{\partial U_i}{\partial t} + \frac{\partial (U_j U_i)}{\partial x_j} = -\frac{\partial P}{\partial x_j} + \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j} + \delta_{ij} \frac{g}{\Theta_0} (\Theta - \Theta_{ref}),$$

(1)

$$\frac{\partial U_j}{\partial x_j} = 0,$$

(2)

$$\frac{\partial \Theta}{\partial t} + \frac{\partial (U_j \Theta)}{\partial x_j} = \frac{\nu}{Pr} \frac{\partial^2 \Theta}{\partial x_j \partial x_j},$$

(3)

where $x_i = x, y, z$, $U_i (i = x, y, z)$ are the velocity components, $P$ is pressure and $\Theta_{ref}$ is the reference temperature, $\nu$ is the kinematic air viscosity and $g$ the gravity acceleration. The Prandtl number, $Pr = \nu / \mu$ (where $\mu$ is thermal diffusivity).

The integration is performed in curvilinear coordinates $(\xi, \eta, \eta')$ which are related to the Cartesian coordinates $(x, y, z)$ by the mapping:

$$x = \xi - a \exp(-k \eta') \sin k \xi,$$

(4)

$$z = \eta + a \exp(-k \eta') \cos k \xi.$$  

(5)
This mapping transforms the lower waved boundary \( z_b(x) = a \cos k \xi(x) \) into a plane boundary at \( \eta = 0 \) [22]. It is easy to show that for small, finite wave amplitude \( a \) the shape of the boundary, \( z_b(x) \), up to the terms of order \( O(k^2 a^3) \), coincides with asymptotic solution for the Stokes wave [13]:

\[
z_b(x) = a \cos kx + \frac{1}{2} a^2 k(\cos 2kx - 1).
\]  \hspace{1cm} (6)

The governing equations (1)-(3) are rewritten in dimensionless variables normalized by the wavelength \( \lambda \), bulk velocity \( U_0 \) and temperature difference \( \Delta \Theta \), and pressure normalized by \( \rho U_0^2 \) (where \( \rho \) is the air density). The integration is performed using dimensionless curvilinear coordinates, \( \xi_1 = \xi / \lambda, \xi_2 = y / \lambda, \xi_3 = \eta / \lambda \), and Cartesian velocity components \( U_1 = U_x / U_0, U_2 = U_y / U_0, \) and \( U_3 = U_z / U_0 \). We also introduce a linear reference temperature profile, \( \Theta_{ref}(\xi_2) = \Delta \Theta / \lambda \), and perform the integration with respect to the dimensionless deviation of the temperature, \( \tilde{T} = (\Theta - \Theta_{ref}) / \Delta \Theta \).

The governing parameters in DNS are the bulk Reynolds and Richardson numbers defined as:

\[
\text{Re} = \frac{U_0 \lambda}{\nu},
\]
\[
\text{Ri} = g \frac{\Delta \Theta}{\Theta_0} \frac{\lambda}{U_0^2}.
\]  \hspace{1cm} (7)
\hspace{1cm} (8)

The Prandtl number, \( Pr = \nu / \mu \) (where \( \mu \) is thermal diffusivity), is prescribed as \( Pr = 0.7 \).

The governing equations are discretized in a rectangular domain with sizes \( 0 < \xi_1 < 6 \), \( 0 < \xi_2 < 4 \), and \( 0 < \xi_3 < 1 \) by employing a finite difference method of the second-order accuracy on a uniform staggered grid consisting of \( 360 \times 240 \times 180 \) nodes. An additional mapping is employed to compress the grid in the vertical direction near the boundaries in order to resolve the viscous boundary layer. In present DNS we prescribe relatively high Reynolds number (up to 80000) which requires sufficiently high resolution of fine scales where dissipation of turbulence kinetic energy occurs in DNS. In the considered case of boundary-layer flow, the viscous scale is \( \nu / u_\ast \), where \( u_\ast \) is the friction velocity and \( \nu \) the kinematic viscosity. This scale generally varies in DNS for different bulk Reynolds number and Richardson number. Our DNS employ the grid with mesh sizes \( \Delta \xi_1 = 1/60 \) in the streamwise and spanwise directions, \( \xi_1 \) and \( \xi_2 \), whereas in the vertical direction \( \xi_3 \) the mesh size increases from \( \Delta \xi_3^w \approx 0.0008 \) near the walls to \( \Delta \xi_3^c \approx 0.009 \) in the centre of computational domain. When normalized by the wall scale, the mesh sizes for different \( \text{Re} \) and \( \text{Ri} \) varies as \( \Delta \xi_1^+ \approx 2 \div 10 \) in the horizontal plane and from \( \Delta \xi_3^+ \approx 0.2 \div 0.6 \) near the boundaries to about \( \Delta \xi_3^c \approx 2 \div 5 \) in the centre of the domain. These grid spacings are generally regarded as sufficient to resolve fine-scale turbulent motions in DNS of turbulent boundary-layer flows [26].

The velocity field is initialized as a weakly perturbed laminar Couette flow and the initial temperature deviation field is put to zero. After a transient, at sufficiently large times, a statistically stationary flow regime is established. Then sampling of the velocity and temperature fields is performed at discrete time moments \( t_k, k = 1, \ldots, 1000 \), with interval
\[ t_{k+1} - t_k = 0.2 \]. The mean profiles are obtained by averaging over time and spanwise and streamwise coordinates.

2.2. Numerical results

DNS results show that there are two distinct flow regimes which are realized depending on the stratification strength measured by the bulk Richardson number, \( \text{Ri} \). If \( \text{Ri} \) is small enough, the flow remains turbulent throughout the domain and is qualitatively similar to that in the non-stratified case (Fig. 2).

Fig. 2. Figure 4a. Instantaneous contours of the vorticity modulus in \((x,z)\), \((y,z)\) and \((x,y)\) planes (top, middle and bottom, respectively) in statistically stationary, stably-stratified boundary-layer flow over waved surface with the wave slope \( k\ell = 0.2 \) and dimensionless phase velocity \( c/U_0 = 0.05 \), obtained in DNS for \( \text{Re} = 15000 \) and \( \text{Ri} = 0.04 \) at \( t = 1000 \).
Under this statistically-stationary, turbulent regime the mean velocity and temperature profiles are well predicted by the Monin & Obukhov asymptotic solutions [25].

On the other hand, if stratification is strong enough, and Ri is sufficiently large, turbulent fluctuations are not sustainable and die out in the bulk of the flow domain. However, under the influence of the surface wave, finite fluctuations are still present in the vicinity of the water surface (Fig. 3).

Fig. 3. Instantaneous field of the vorticity modulus in \((x,z)\) plain at \(y=0\), \((y,z)\) plain at \(x = 3\) and \((x,y)\) plane at \(z = 0.12\), obtained from DNS of the stably stratified boundary-layer flows for the bulk Reynolds number \(Re = 15000\), bulk Richardson number \(Ri = 0.08\), wave slope \(ka = 0.2\), and dimensionless phase velocity \(c/U_0 = 0.05\), at time \(t = 1000\).
The results in Figure 3 show that the fluctuations “survive” only for sufficiently large wave slope, and the amplitude of the fluctuations monotonically decreases as $R_i$ increases.

In order to investigate the origin of complicated flow structure in the vicinity of the water surface in Figure 4 we evaluated the instantaneous power spectrum, $E(k_x, k_y)$, obtained by the Fourier transform of the vorticity field in the horizontal ($x, y$) plane in Fig. 3.

Fig. 4. Instantaneous power spectrum of the vorticity field in the air flow over the waved water surface for $c/U_0 = 0.05$, $Re = 15000$, $Ri = 0.08$ and $ka = 0.2$ [the case shown in Fig. 3: ($x, y$) plane at $z = 0.12$].

The spectrum exhibits peaks at the wavenumbers $k = (2\pi, 0)$, $k_1 = (\pi, 2\pi)$, $k_2 = (4\pi, 0)$ and $k_3 = (0, 4\pi)$. The peak at $k = (2\pi, 0)$ corresponds to the two-dimensional (2D) disturbance directly induced in the air-flow by the surface wave. However, the 2D forcing alone would not be able to directly produce a three dimensional (3D) disturbance, heterogeneous in $y$-direction along the surface-wave front and characterized by the energy peaks at wavenumbers $k_1$ and $k_3$. A theoretical analysis (not presented here) shows that the 3D structure of the vorticity field in the horizontal ($x,y$)-plane in Fig. 3 and the spectral peak at the wavenumber $k_1 = (\pi, 2\pi)$ can be regarded as a consequence of the development of a secondary instability of the 2D wave-induced disturbances, caused by the interaction of the former with the oblique waves via parametric resonance. Indeed, Figure 4 shows that the parametric-resonance condition

$$k_{1s} = \frac{1}{2}k = \pi$$

is satisfied. Additional peaks at wave numbers $k_3$ and $k_4$ in the spectrum $E(k_x, k_y)$ may result from non-linear generation of second harmonics.
3. DNS of interaction of small-scale turbulence with internal waves

3.1 Numerical method and initial conditions

We consider a stably stratified fluid with a pycnocline (Fig. 5).

![Fig. 5. Schematic of the numerical experiment: x, y, z are the Cartesian coordinates; \( \rho_0 \) is the density above the pycnocline; \( \Delta \rho_0 \) the density jump across the pycnocline; g the gravity acceleration, \( N_0 \) the buoyancy frequency in the pycnocline center; \( L_0 \) the pycnocline thickness; \( z_p \) and \( z_t \) the locations of the pycnocline and the turbulent layer centers.]

Initial turbulence field is localized in a layer at some distance above the pycnocline. The first mode of the internal wave propagating along the pycnocline from left to right is also prescribed as initial condition. Periodic boundary conditions in the horizontal, \( x \) and \( y \), directions and Neumann (zero normal gradient) boundary condition in the vertical \( z \) direction are considered. The thickness of the pycnocline, \( L_0 \), and the buoyancy frequency in the middle of the pycnocline, \( N_0 = \left( -\frac{g}{\rho_0} \frac{d\rho_0}{dz} \right)^{1/2} \) (where \( g \) is the gravity acceleration and \( \rho_0(z) \) the fluid density), are chosen to define the characteristic length and time scales, \( L_0 \) and \( T_0 = 1 / N_0 \), which are used further to write the governing equations in the dimensionless form.

As in the previous case (Sec. 2) DNS solves the Navier-Stokes equations for the fluid velocity written under the Boussinesq approximation where the coordinates, time and velocity are normalized by the length, time and velocity scales, \( L_0 \), \( T_0 \) and \( U_0 = L_0 / T_0 \). The density deviation \( \rho \) is normalized by the density jump across the pycnocline, \( \Delta \rho_0 \) (Fig.1).

The Navier-Stokes equations for the fluid velocity and density are integrated in a cubic domain with sizes \( 0 \leq x \leq 40 \), \( -10 \leq y \leq 10 \) and \( 0 \leq z \leq 20 \) by employing a finite difference method of the second-order accuracy on a uniform rectangular staggered grid consisting of \( 400 \times 200 \times 200 \) nodes in the \( x \)-, \( y \)- and \( z \)- directions, respectively. The integration is advanced in time using the Adams-Bashforth method with time step \( \Delta t = 0.01 \). The Poisson equation for the pressure is solved by FFT transform over \( x \) and \( y \) coordinates, and Gaussian elimination method over \( z \) coordinate [23, 24]. The Neumann (zero normal gradient)
boundary condition is prescribed for all fields in the horizontal \((x,y)\) planes at \(z = 0\) and \(z = 20\), and periodic boundary conditions are prescribed in the longitudinal \((x)\) and transverse \((y)\) directions. In DNS we prescribe the Reynolds number to be \(Re = 20000\). This number is sufficiently large to render the viscous damping of IWs negligible.

The initial condition for the velocity and density fields is prescribed as a first mode of internal wave field with wavelength \(\lambda\) (and wavenumber \(k = 2\pi/\lambda\)) and frequency \(\omega\). The solution of the linearized Navier-Stokes equations for the progressive internal wave propagating from left to right in the \(x\)-direction [1]. DNS was performed with initial conditions corresponding to the IW fields with wavelength \(\lambda = 10\) (i.e. about ten times larger as compared to pycnocline thickness) and amplitude \(W_0 = 0.1\). Under these conditions, non-linear effects during the IW propagation in the pycnocline remain negligible. The mid- pycnocline level was prescribed at \(z = 8\). The turbulence layer center was set at \(z = 10\). The value of \(z_t\) was chosen to ensure that the effects of turbulent mixing and internal wave generation by turbulence in the pycnocline remained sufficiently small. Turbulent velocity field is initialized in DNS as a random, divergence-free field with a given power spectrum. The turbulence parameters were chosen so that the effects of turbulent mixing on the pycnocline structure and generation of internal waves by turbulence remain negligible during the considered time interval.

3.2 Numerical results

DNS was performed both in the freely-decaying turbulent-layer case (without initially induced IWs) and in the case where turbulence evolved in the presence of IWs propagating in the pycnocline. DNS results show that, in the former case (not shown here), there occurs a collapse of three-dimensional turbulence and formation of quasi-2D pancake vortex structures in the vicinity of the pycnocline. The horizontal spatial scale of these structures is considerably larger as compared to the characteristic size of 3D turbulent eddies which still survive in the non-stratified region sufficiently far from the pycnocline. In the latter region, the decay rate of the turbulent kinetic energy is enhanced as compared to the region in the vicinity of the pycnocline \((E(z = 11) \sim t^{1.6}\) as compared to \(E(z = 9) \sim t^{0.9}\)). As a result, the location of the kinetic energy maximum is shifted with time from the center of the turbulent layer at \(z_t = 10\) (at \(t = 0\)) to the level \(z = 9\), i.e. closer to the pycnocline. At sufficiently late times \((t > 400)\) most of turbulent kinetic energy is located in a layer occupied by pancake large-scale eddies in the vicinity of the pycnocline.

Strong IWs propagating in the pycnocline modify turbulence evolution in that at sufficiently late times the turbulence intensity is significantly enhanced in a thin layer in the vicinity of the pycnocline center (Fig. 6). Evaluation of the kinetic energy power spectrum in Figure 7 shows significant amplification (by the order of magnitude) of the kinetic energy spectrum under the effect of IW in the entire wavenumber \((k)\) range. The maximum peak in the IW spectrum (in the absence of turbulence) is due to the first harmonics at \(k = 2\pi/10\), the second harmonics peak (at \(k = 4\pi/10\)) being less by two orders of magnitude (Fig.7, left panel).
Fig. 6. Instantaneous distribution of the vorticity $\omega_y$ with imposed density contours (1.3, 1.5, 1.7) in the central (x,z)-plane at times $t = 100$ and $t = 400$, and density distribution in the (x,y)-plane at the pycnocline level ($z = 8$, bottom panel) at $t = 400$ obtained in DNS of turbulence layer. IW wavelength $\lambda = 10$.

$t = 400$
- red: turb. with IW, $z = 8$
- black: IW without turb., $z = 8$
- blue: turb. without IW, $z = 9$

Fig. 7. The kinetic energy power spectrum, $E(k)$, (left) and the spectrum of the $y$-velocity component, $E_y(k)$, (right) obtained in DNS with initially excited IWs at the pycnocline center level ($z = 8$) at time $t = 400$. 
4. Conclusions
We have performed direct numerical simulation (DNS) of turbulent atmospheric boundary layer over waved water surface and the interaction of small-scale turbulence with strong internal waves in the upper ocean. DNS resolves all physically important scales of the flow and thus does not require modeling of the subgrid-scale-flow effects on the large-scale, mean flow typical both of Large-Eddy Simulations (LES) and Reynolds-Averaged Navier-Stokes-equations simulations (RANS). Since subgrid closure taking into account the interaction of turbulence with internal and surface waves remains difficult, DNS provides benchmark results and solid ground for further LES/RANS modeling. DNS also reveals new physical effects related to the possibility of the enhancement of turbulent motions by the waves which affect mixing and thus need to be taken into account in large-scale prognostic models.

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