Delay-compensating visual positioning of the mobile robot

R.A. Sevostyanov
Assistant, Saint Petersburg State University, Universitetskaya emb., 7-9, 199034, Saint Petersburg, Russia
E-mail: sevostyanov.ruslan@gmail.com

Abstract. The paper considers the task of the automatic positioning of the mobile robot in front of the external visual marker in the case of the presence of the transport delay in the control channel. As a basic regulator without the delay compensation we use mixed approach based on the visual servoing method and a special multipurpose control structure which is capable of taking into account a set of requirements to the controlled dynamics. To compensate the delay we apply the modification of the basic regulator which keeps the original transfer matrix. We also demonstrate the efficiency of the proposed approach by the results of the experiments with the computer model.

Introduction
Every physical control system has some kind of delays. If the plant is controlled by some CPU then there are delays because of the digital nature of the computer. If the control signal is passed through the network, the delay is of course bigger. Usually delays have negative effect on the control quality, sometimes even leading to the loss of stability.

Another source of the delay might be heavy computations. For example, computer vision methods, which are very popular in the robotics today, can take quite a long time, especially if the onboard CPU is not very powerful. One particular computer vision task that we will consider further is the positioning of the mobile robot in front of some visual marker – like QR-code or aruco-marker. In order to solve the positioning task in this case we can mark out visual servoing approach [1]. This approach uses image data directly in the feedback loop to minimize the error between the actual and the desired positions of some image points – for example, marker corners projections.

Generally visual servoing approach doesn’t take into account the dynamics of the robot, only kinematics – it just tells what the velocity should be to solve the task. In order to provide better efficiency of the control we need some sort of low-level regulator which knows about the dynamical model of the robot. A good option for the tasks in which there is a certain set of requirements (for example, certain reaction to the external disturbances) to the dynamics of the controlled motion might be so-called multipurpose regulator [2]. One of its advantages is that it can divide a general task into relatively simpler subtasks and solve them independently.

In this paper we propose the control structure that combines visual servoing approach, multipurpose regulator and delay compensation [3] methods in order to solve the task of the positioning of the mobile robot in front of the external visual marker considering the presence of the delay and constant external disturbance. In our approach we solve the task without the delay and then apply special transformation of the regulator that keeps the initial transfer matrix of the system.
1. Task description

Let us consider classic differential drive robot which is controlled by setting the voltages on the left and right motors – just like the tank chassis. We also consider the delay in the control and disturbance channels. Mathematical model of such robot can be described with the equations \[ \begin{align*}
\dot{v}(t) &= A v(t) + B \tau(t-h) + \tau_e(t-h), \\
\dot{\eta}(t) &= R(\eta)v(t),
\end{align*} \]
where \(v = (v, \omega)^T\) – velocity vector; \(\tau = (\tau_v, \tau_w)^T\) – control vector; \(\eta = (x, y, \phi)^T\) – position of the robot; \(\tau_e\) – external disturbance vector; \(A\) – diagonal matrix of the friction coefficients; \(B\) – diagonal matrix of the control coefficients; \(h\) – constant delay. The only nonlinear part of the system (1) is a rotation matrix

\[ R(\phi) = \begin{pmatrix} \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{pmatrix}. \]

There is a video camera rigidly mounted on the robot's hull, directed towards the robot's course angle \(\phi\). Let us assume that in the camera's field of view there is always a visual marker described by a set of the image points \(\sigma = (x_i, y_i), i = 1, N\), which are in fact the projections of the corner points \((X_i, Y_i, Z_i), i = 1, N\) of the marker in the camera's frame of reference. The task is to position the robot in front of the marker. It can be formalized as the task of minimization of the error \(q = \sigma - \sigma_d\) between actual and the desired projection \(\sigma_d\) of the marker corners. We also must compensate the constant external disturbance and the control delay. Let us consider that we can directly measure the values of \(v, \sigma, \tau_e\) and \(Z_i\).

2. Basic regulator

In addition to the system (1) let us consider the projection errors dynamics which will be useful in further regulator's synthesis:

\[ \dot{q} = L(\sigma, Z)v + d, \]
where \(d\) – external disturbance, and matrix \(L\) for each point \((x_i, y_i)\) consists of the blocks

\[ L(x_i, y_i, Z_i) = \begin{pmatrix}
 x_i/Z_i & -(1+x_i^2) \\
 y_i/Z_i & x_i y_i \\
\end{pmatrix}. \]

In order to minimize the projection error in the case when there is no delay, let us introduce the special feedback structure known as multipurpose regulator in the form

\[ \begin{align*}
\dot{z}_v &= A z_v + B \tau + K_v(v - z_v), \\
\dot{z}_q &= L(q, Z) z_v + H_q(q - z_q), \\
\dot{\psi} &= a \psi + \beta_v(q - z_v) + \beta_q(q - z_q), \\
\dot{\zeta} &= g \psi + \mu_v(v - z_v) + \mu_q(q - z_q), \\
\tau &= -K_q z_q - K_v z_v + \zeta.
\end{align*} \]

First two equations of (2) represent the asymptotic observers of the system (1). Their role is to estimate the state of the system in the case when the whole state vector can't be measured directly. Another property of the observers is that their outputs serve as the inputs to the dynamical corrector. It is described by the third and fourth equations. The corrector defines the reaction to the external disturbances. It actually can be used as a filter to minimize the reaction of the control, for example, to the periodic disturbances. Or it can be the actual corrector, minimizing the effect of the disturbance to the accuracy of the stabilization. For example, in our task its goal is to provide the convergence of the projection errors to zero even in the presence of the constant external disturbance, like side wind. It is
worth noting that the corrector can be switched on and off in the real-time – for example, for regimes without the disturbance. Finally, the last equation describes the control signal which is directly applied to the plant.

Let us discuss the tunable parameters of the regulator. Observer matrices $K_1$ and $H_q$ affect the rate of the state estimates convergence and also the reaction to the external disturbance, because, as we said before, the output of the observers is the input of the corrector. These matrices must be positive definite to ensure the asymptotic stability of the system. Matrices $K_v$ and $K_q$ affect the dynamics of the motion in the case when there is no external disturbance. Let us consider the basic control law without the corrector:

$$\tau = -K_q q - K_v v.$$ 

Let us also consider the quadratic form

$$V = \frac{1}{2} q^T q + \frac{1}{2} v^T v.$$ 

If we take the derivative of this quadratic form with respect to the system (1) we get

$$\dot{V} = -v^T A v + v^T (L^T q + \tau).$$

It is obvious that due to the Lyapunov’s theorem we provide the asymptotic stability if we get positive definite matrix $K_v$ and $K_q = L^T$. Values of the matrix $K_v$ can be more specific if we consider particular requirements to the dynamics.

Finally, let us discuss the search process of the matrices $\alpha, \beta_v, \beta_q, \gamma, \mu_v$ and $\mu_q$ of the dynamic corrector. Matrix $\alpha$ must have eigenvalues in the open left plane to ensure the stability of the corrector. Now let us consider the tf-form of the corrector:

$$F_v(s) = (s^2 + \alpha s + \beta_v + \gamma z)^{-1} \beta_v + \mu_v,$$

$$F_q(s) = (s^2 + \alpha s + \beta_q + \gamma z_q)^{-1} \beta_q + \mu_q.$$ (4)

Let us also introduce estimation error vectors $e_v = v - z_v, e_q = q - z_q$. Using (1), (2) and (3) we get closed-loop equations of the form

$$\dot{e}_v = A e_v - K_1 e_v + \tau_e,$$

$$\dot{e}_q = L(q, Z) e_v - H_q e_q + d_e,$$

$$\dot{z}_v = A z_v + K_1 e_v - BK_q z_q - BK_v z_v + BF_v(s) e_v + BF_q(s) e_q,$$

$$\dot{z}_q = L(q, Z) z_v + H_v e_q.$$ (5)

Equating right parts of (5) to zero, we get the equilibrium position:

$$0 = A e_{v0} - K_1 e_{v0} + \tau_{e0},$$

$$0 = L_0 e_{v0} - H_v e_{q0} + d_{e0},$$

$$0 = A z_{v0} + K_1 e_{v0} - BK_q z_{q0} - BK_v z_{v0} + BF_v(0) e_{v0} + BF_q(0) e_{q0},$$

$$0 = L_0 z_{v0} + H_v e_{q0},$$ (6)

where $L_0, e_{v0}, e_{q0}, z_{v0}, z_{q0}, \tau_{e0}$ and $d_{e0}$ are values, corresponding to the equilibrium position of the system. We can see from (6) that the estimation error vectors and the disturbance vectors are directly related.

From the last equation of (6) we can derive the following relationship and introduce an auxiliary matrix $T$ as follows:

$$z_{v0} = -(L_0^T L_0)^{-1} L_0 H_v e_{q0} = T e_{q0}. $$ (7)
Now let us substitute (7) in (6) and get the following:

\[ BK_q q_0 = (AT - BK_q T + BF_q (0) + BK_q )e_q_0 + (K_1 + BF_q (0))e_v_0. \]

Therefore, we can provide astatic property of the system for the constant disturbance if the corrector satisfies the conditions

\[ F_v(0) = -B^{-1}K_1, \]
\[ F_q(0) = B^{-1}(BK_q T - AT - BK_q). \]

3. Delay compensation

Now let us discuss the problem of delay compensation. The idea is quite simple: we just want to calculate the estimate of the robot's future velocity and use it as input to our controller. We can use Cauchy's formula to calculate such estimation but we have to deal with numeric integration. Instead of this we are having the prediction as an output of the dynamic system which is in fact the derivative of the Cauchy's formula. Besides this we need to have some estimation of the future position of the robot. Putting it all together we get the regulator in the transformed form:

\[ \dot{z}_p = Az_p + B\tau + \tau_e, \]
\[ \dot{z}_v = Az_v + B\tau + K_1(\lambda - z_v), \]
\[ \dot{z}_q = R(z_q)z_v, \]
\[ \dot{z}_q = L(q, Z)z_v + H_q(q - z_q), \]
\[ \psi = \alpha\psi + \beta_v(\lambda - z_v) + \beta_q(q - z_q), \]
\[ \xi = \gamma\psi + \mu_v(\lambda - z_v) + \mu_q(q - z_q), \]
\[ \lambda = z_p + e^{A(t-h)}(v - z_p(t-h)), \]
\[ \tau = -K_q z_q - K_v z_v + \xi, \]

where \( z_p \) is the auxiliary dynamic variable needed to calculate the estimation of the robot's future velocity \( \lambda \) and \( z_q \) is the estimate of the robot's future position.

Notice that all the tunable parameters are the same – we just add some equations and transform inputs but the rest remains the same – no additional tuning is needed. Such compensative transformation keeps the transformation matrix of the regulator synthesized for the system without the delay. This allows us to solve the simpler task first and then use transformed regulator to ensure the delay compensation.

4. Computer modelling

In order to test our approach we have implemented the computer model using Matlab and Simulink which integrates robot’s dynamics and also calculates the projections of the visual marker into the image plane. We used the following numerical values of the system’s parameters:

\[ K_1 = 10E_{2x2}, \quad K_v = diag([2 \ 10]), \]
\[ H = 10E_{8x8}, \quad A = a = -E_{2x2}, \]
\[ B = \gamma = E_{2x2}, \quad \beta_v = F_v(0), \]
\[ \beta_q = F_q(0), \quad \mu_v = \mu_q = 0. \]

The initial setup is shown in the fig. 1. This is the camera image that corresponds to the marker’s position 3 m ahead of the robot and robot is turned by 30 degrees. The desired position corresponds to the distance of 1 m and 0 degrees of the robot’s course angle.
Let us first apply our regulator without the corrector, disturbance and delay. As we can see from the fig. 2, the robot successfully moves towards the marker and stops when the marker projection is at the desired position.

Now let us add the constant external disturbance $\tau_e = (0.2 \ 0.3)^T$. Fig. 3 demonstrates that in this case the robot fails to achieve the exact match of the projections due to the disturbance.
In order to fight the disturbance let us apply the action of the dynamic corrector. It can be seen from the fig. 4 that the dynamics in this case is almost equal to the one without the disturbance which means that the corrector actually helps to compensate the disturbance.

Next let us again return to the case without the corrector and without the disturbance, but now let us introduce the delay $h = 0.25\text{s}$. Fig. 5 shows that in this case robot actually goes close to the goal position but then loses the stability.

Finally let us apply our delay compensation transformation of the regulator. Dynamics for this case is shown in the fig. 6. Now, again, we have almost the same motion as it was in case without the delay.
Conclusions
In this paper we presented our solution to the problem of the positioning of the mobile robot in front of the visual marker using information from the video-camera considering delay and external disturbance. Our approach is a combination of the visual servoing method, multipurpose control structure and delay compensation via the use of the future state estimation. One of the key points of the proposed regulator is the ability to satisfy a set of requirements to the dynamics via the division into easier subtasks. Another feature is that we can first solve the task without the delay and then just apply special transformation to the regulator to compensate the delay. Experiments with the computer model demonstrate the efficiency of our approach. The next step is to test it using some physical simulator and on the real robot.

References
[1] Chaumette F., Hutchinson S. Visual Servo Control: Basic Approaches // IEEE Robotics & Automation Magazine. 2006. Vol. 13, N 4. P. 82-90.
[2] Veremey E.I. Dynamical Correction of Positioning Control Laws // Proc. of the 9th IFAC Conference on Control Applications in Marine Systems (CAMS-2013). Japan, 2013. P. 31-36.
[3] Veremey E.I. Kompensiruyuschie regulyatory po vyhodu dlya LTI system s zapazdyvaniem po upravleniyu (Compensating Output Regulators for LTI Systems with Control Delay). // Sb. trudov XI mezhdunarodnoi konferentsii PMTUKT-2018 (Proceedings of XI international conference PMTUKT-2018). Voronezh: «Novaya kniga», 2018. P. 106-110.
[4] Carona R., Aguiar A.P., Gaspar J. Control of unicycle type robots: tracking, path following and point stabilization // Proceedings of IV Jornadas de Engenharia de Electronica e Telecomunicacoes e de Computadores. Lisbon: ISEL. 2008. P. 180 –185.