QCD AND STRING THEORY

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This talk begins with some history and basic facts about string theory and its connections with strong interactions. Comparisons of stacks of Dirichlet branes with curved backgrounds produced by them are used to motivate the AdS/CFT correspondence between superconformal gauge theory and string theory on a product of Anti-de Sitter space and a compact manifold. The ensuing duality between semi-classical spinning strings and long gauge theory operators is briefly reviewed. Strongly coupled thermal SYM theory is explored via a black hole in 5-dimensional AdS space, which leads to explicit results for its entropy and shear viscosity. A conjectured universal lower bound on the viscosity to entropy density ratio, and its possible relation to recent results from RHIC, are discussed. Finally, some available results on string duals of confining gauge theories are briefly reviewed.

1 Introduction

String theory is well known to be the leading prospect for quantizing gravity and unifying it with other interactions\(^1,2\). One may also take a broader view of string theory as a description of string-like excitations that arise in many different physical systems, such as the superconducting flux tubes, cosmic strings, and of course the chromoelectric flux tubes in non-Abelian gauge theories, which are the subject of my talk. You could object that these string-like excitations are “emergent” rather than fundamental phenomena. We will see, however, that there is no sharp distinction between “emergent” and fundamental strings. We will exhibit examples, stemming from the AdS/CFT correspondence\(^3,4,5\), where the “emergent” and fundamental strings are dual descriptions of the same theory. Besides being of great theoretical interest, such gauge/string dualities are becoming a useful tool for studying strongly coupled gauge theories. A developing connection that is highlighted in this talk is with the new results at RHIC,\(^6\) there are indications that a rather strongly coupled Quark-Gluon Plasma (sQGP) has been observed.

2 Some early history

String Theory was born out of attempts to understand the Strong Interactions. Empirical evidence for a string-like structure of hadrons comes from arranging mesons and baryons into approximately linear Regge trajectories. Studies of \(\pi N\) scattering prompted Dolen, Horn and Schmid\(^7\) to make a duality conjecture stating that the sum over \(s\)-channel exchanges equals the sum over \(t\)-channel ones. This posed the problem of finding the analytic form of such dual amplitudes. Veneziano\(^8\) found the first, and very simple, expression for a manifestly dual 4-point amplitude:

\[ A(s, t) \sim \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s) - \alpha(t))} \]  

with an exactly linear Regge trajectory \(\alpha(s) = \alpha(0) + \alpha'/s\). Soon after, Nambu\(^9\), Nielsen\(^10\) and Susskind\(^11\) independently proposed its open string interpretation. This led to an explosion of interest in the early 70's in string theory as a description of strongly interacting particles. The basic idea is to think of a meson as an open string with a quark at one end-point and an anti-quark at another. Then various meson states arise as different excitations of such an open string. The split-
The string world sheet dynamics is governed by the Nambu-Goto area action
\[ S_{NG} = -T \int d\sigma d\tau \sqrt{-\det \partial_a X^\mu \partial_b X_\mu}, \]
where the indexes \( a, b \) take two values ranging over the \( \sigma \) and \( \tau \) directions on the world sheet. The string tension is related to the Regge slope through
\[ T = \frac{1}{2\pi \alpha'}. \]
The quantum consistency of the Veneziano model requires that the Regge intercept is
\[ \alpha(0) = 1, \]
so that the spin 1 state is massless but the spin 0 is a tachyon. But the \( \rho \) meson is certainly not massless, and there are no tachyons in the real world. This is how the string theory of strong interactions started to run into problems.

Calculation of the string zero-point energy gives
\[ \alpha(0) = \frac{d - 2}{24}. \]
Hence the model has to be defined in 26 space-time dimensions. Attempts to quantize such a string model directly in 3+1 dimensions led to tachyons and problems with unitarity. Consistent supersymmetric string theories were discovered in 10 dimensions, but their relation to the strong interactions was initially completely unclear. Most importantly, the Asymptotic Freedom of strong interactions was discovered\(^{12}\), singling out Quantum Chromodynamics (QCD) as the exact field theory of strong interactions. At this point most physicists gave up on strings as a description of strong interactions. Instead, since the graviton appears naturally in the closed string spectrum, string theory emerged as the leading hope for unifying quantum gravity with other forces\(^{13}\).

3 QCD gives strings a chance

Now that we know that a non-Abelian gauge theory is an exact description of strong interactions, is there any room left for string theory in this field? Luckily, the answer is positive. At short distances, much smaller than 1 fermi, the quark anti-quark potential is Coulombic, due to Asymptotic Freedom. At large distances the potential should be linear due to formation of a confining flux tube\(^{14}\). When these tubes are much longer than their thickness, one can hope to describe them, at least approximately, by semi-classical Nambu strings\(^{15}\). This appears to explain the existence of approximately linear Regge trajectories: a linear relation between angular momentum and mass-squared
\[ J = \alpha' m^2 + \alpha(0), \]
is provided by a semi-classical spinning relativistic string with massless quark and anti-quark at its endpoints. A semi-classical string approach to the QCD flux tubes is widely used, for example, in jet hadronization algorithms based on the Lund String Model\(^{16}\).

Semi-classical quantization around a long straight Nambu string predicts the quark anti-quark potential\(^{17}\)
\[ V(r) = Tr + \mu + \frac{\gamma}{r} + O(1/r^2). \]
The coefficient \( \gamma \) of the universal Lüscher term depends only on the space-time dimension \( d \) and is proportional to the Regge intercept: \( \gamma = -\pi(d - 2)/24 \). Recent lattice calculations of the force vs. distance for probe quarks and anti-quarks\(^{18}\) produce good agreement with this value in \( d = 3 \) and \( d = 4 \) for \( r > 0.7 \text{fm} \). Thus, long QCD strings appear to be well described by the Nambu-Goto area action. But quantization of short, highly quantum QCD strings, that could lead to a calculation of light meson and glueball spectra, is a much harder problem.

The connection of gauge theory with string theory is strengthened by ‘t Hooft’s generalization of QCD from 3 colors (\( SU(3) \) gauge group) to \( N \) colors (\( SU(N) \) gauge group)\(^{19}\). The idea is to make \( N \) large, while keeping the ‘t Hooft coupling \( \lambda = g_{YM}^2 N \)
fixed. In this limit each Feynman graph carries a topological factor $N^\chi$, where $\chi$ is the Euler characteristic of the graph. Thus, the sum over graphs of a given topology can perhaps be thought of as a sum over world sheets of a hypothetical “QCD string.” Since the spheres (string tree diagrams) are weighted by $N^2$, the tori (string one-loop diagrams) – by $N^0$, etc., we find that the closed string coupling constant is of order $N^{-1}$. Thus, the advantage of taking $N$ to be large is that we find a weakly coupled string theory. In the large $N$ limit the gauge theory simplifies in that only the planar diagrams contribute. But directly summing even this subclass of diagrams seems to be an impossible task. From the dual QCD string point of view, it is not clear how to describe this string theory in elementary terms.

Because of the difficulty of these problems, between the late 70’s and the mid-90’s many theorists gave up hope of finding an exact gauge/string duality. One notable exception is Polyakov who already in 1981 proposed that the string theory dual to a 4-d gauge theory should have a 5-th hidden dimension. In later work he refined this proposal, suggesting that the 5-d metric must be “warped.”

4 The Geometry of Dirichlet Branes

In the mid-nineties the Dirichlet branes, or D-branes for short, brought string theory back to gauge theory. The D-branes are soliton-like “membranes” of various internal dimensionalities contained in theories of closed superstrings. A Dirichlet $p$-brane (or Dp-brane) is a $p + 1$ dimensional hyperplane in $9 + 1$ dimensional space-time where strings are allowed to end. A D-brane is much like a topological defect: upon touching a D-brane, a closed string can open up and turn into an open string whose ends are free to move along the D-brane. For the end-points of such a string the $p + 1$ longitudinal coordinates satisfy the conventional free (Neumann) boundary conditions, while the $9 - p$ coordinates transverse to the Dp-brane have the fixed (Dirichlet) boundary conditions; hence the origin of the term “Dirichlet brane.” In a seminal paper Polchinski showed that a Dp-brane preserves $1/2$ of the bulk supersymmetries and carries an elementary unit of charge with respect to the $p+1$ form gauge potential from the Ramond-Ramond sector of type II superstring.

For our purposes, the most important property of D-branes is that they realize gauge theories on their world volume. The massless spectrum of open strings living on a Dp-brane is that of a maximally supersymmetric $U(1)$ gauge theory in $p+1$ dimensions. The $9 - p$ massless scalar fields present in this supermultiplet are the expected Goldstone modes associated with the transverse oscillations of the Dp-brane, while the photons and fermions provide the unique supersymmetric completion. If we consider $N$ parallel D-branes, then there are $N^2$ different species of open strings because they can begin and end on any of the D-branes. $N^2$ is the dimension of the adjoint representation of $U(N)$, and indeed we find the maximally supersymmetric $U(N)$ gauge theory in this setting.

The relative separations of the Dp-branes in the $9 - p$ transverse dimensions are determined by the expectation values of the scalar fields. We will be interested in the case where all scalar expectation values vanish, so that the $N$ Dp-branes are stacked on top of each other. If $N$ is large, then this stack is a heavy object embedded into a theory of closed strings which contains gravity. Naturally, this macroscopic object will curve space: it may be described by some classical metric and other background fields. Thus, we have two very different descriptions of the stack of Dp-branes: one in terms of the $U(N)$ supersymmetric gauge theory on its world
volume, and the other in terms of the classical charged p-brane background of the type II closed superstring theory. The relation between these two descriptions is at the heart of the connections between gauge fields and strings that are the subject of this talk.

4.1 Coincident D3-branes

Parallel D3-branes realize a 3 + 1 dimensional $U(N)$ gauge theory, which is a maximally supersymmetric “cousin” of QCD. Let us compare a stack of D3-branes with the Ramond-Ramond charged black 3-brane classical solution whose metric assumes the form \(23\):

\[
    ds^2 = H^{-1/2}(r) \left[ -f(r)(dx^0)^2 + (dx^i)^2 \right] + H^{1/2}(r) \left[ f^{-1}(r)dr^2 + r^2d\Omega_5^2 \right],
\]

where \(i = 1, 2, 3\) and

\[
    H(r) = 1 + \frac{L^4}{r^4}, \quad f(r) = 1 - \frac{r_0^4}{r^4}.
\]

Here \(d\Omega_5^2\) is the metric of a unit 5 dimensional sphere, \(S^5\).

In general, a \(d\)-dimensional sphere of radius \(L\) may be defined by a constraint

\[
    \sum_{i=1}^{d+1} (X^i)^2 = L^2
\]

on \(d+1\) real coordinates \(X^i\). It is a positively curved maximally symmetric space with symmetry group \(SO(d+1)\). Similarly, the \(d\)-dimensional Anti-de Sitter space, \(AdS_d\), is defined by a constraint

\[
    (X^0)^2 + (X^d)^2 - \sum_{i=1}^{d-1} (X^i)^2 = L^2,
\]

where \(L\) is its curvature radius. \(AdS_d\) is a negatively curved maximally symmetric space with symmetry group \(SO(2, d-2)\). There exists a subspace of \(AdS_d\) called the Poincaré wedge, with the metric

\[
    ds^2 = \frac{L^2}{z^2} \left( dz^2 - (dx^0)^2 + \sum_{i=1}^{d-2} (dx^i)^2 \right),
\]

where \(z \in [0, \infty)\). In these coordinates the boundary of \(AdS_d\) is at \(z = 0\).

The event horizon of the black 3-brane metric (6) is located at \(r = r_0\). In the extremal limit \(r_0 \to 0\) the 3-brane metric becomes

\[
    ds^2 = \left( 1 + \frac{L^4}{r^4} \right)^{-1/2} (- (dx^0)^2 + (dx^i)^2)
    + \left( 1 + \frac{L^4}{r^4} \right)^{1/2} (dr^2 + r^2d\Omega_5^2)
\]

Just like the stack of parallel, ground state D3-branes, the extremal solution preserves 16 of the 32 supersymmetries present in the type IIB theory. Introducing \(z = \frac{L^2}{r}\), one notes that the limiting form of (10) as \(r \to 0\) factorizes into the direct product of two smooth spaces, the Poincaré wedge (9) of \(AdS_5\), and \(S^5\), with equal radii of curvature \(L\). The 3-brane geometry may be thus viewed as a semi-infinite throat of radius \(L\) which for \(r \gg L\) opens up into flat \(9 + 1\) dimensional space. Thus, for \(L\) much larger than the string length scale, \(\sqrt{\alpha'}\), the entire 3-brane geometry has small curvatures everywhere and is appropriately described by the supergravity approximation to type IIB string theory.

The relation between \(L\) and \(\sqrt{\alpha'}\) may be found by equating the gravitational tension of the extremal 3-brane classical solution to \(N\) times the tension of a single D3-brane, and one finds

\[
    L^4 = g^2 \text{YM} \lambda \alpha'^{-2}.
\]

Thus, the size of the throat in string units is \(\lambda^{1/4}\). This remarkable emergence of the ‘t’ Hooft coupling from gravitational considerations is at the heart of the success of the AdS/CFT correspondence. Moreover, the requirement \(L \gg \sqrt{\alpha'}\) translates into \(\lambda \gg 1\): the gravitational approach is valid when the ‘t’ Hooft coupling is very strong and the perturbative field theoretic methods are not applicable.
5 The AdS/CFT Correspondence

Consideration of low-energy processes in the 3-brane background indicates that, in the low-energy limit, the \(AdS_5 \times S^5\) throat region \((r \ll L)\) decouples from the asymptotically flat large \(r\) region. Similarly, the \(\mathcal{N} = 4\) supersymmetric SU\((N)\) gauge theory on the stack of \(N\) D3-branes decouples in the low-energy limit from the bulk closed string theory. Such considerations prompted Maldacena to conjecture that type IIB string theory on \(AdS_5 \times S^5\), of radius \(L\) given in (11), is dual to the \(\mathcal{N} = 4\) SYM theory. The number of colors in the gauge theory, \(N\), is dual to the number of flux units of the 5-form Ramond-Ramond field strength.

It was further conjectured in that there exists a one-to-one map between gauge invariant operators in the CFT and fields (or extended objects) in AdS\(_5\). The dimension \(\Delta\) of an operator is determined by the mass of the dual field in AdS\(_5\). For example, for scalar operators one finds that \(\Delta(\Delta - 4) = m^2L^2\). Precise methods for calculating correlation functions of various operators in a CFT using its dual formulation were also formulated. They involve calculating the string theory path integral as a function of the boundary conditions in AdS\(_5\), which are imposed near \(z = 0\).

If the number of colors \(N\) is sent to infinity while \(g_{YM}^2N\) is held fixed and large, then there are small string scale corrections to the supergravity limit which proceed in powers of \(\frac{\alpha'}{L^2} = \left(g_{YM}^2N\right)^{-1/2}\). If we wish to study finite \(N\), then there are also string loop corrections in powers of \(\frac{1}{L^2} \sim N^{-2}\). As expected, taking \(N\) to infinity enables us to take the classical limit of the string theory on \(AdS_5 \times S^5\).

Immediate support for the AdS/CFT correspondence comes from symmetry considerations. The isometry group of AdS\(_5\) is SO\((2, 4)\), and this is also the conformal group in \(3 + 1\) dimensions. In addition we have the isometries of \(S^5\) which form SU\((4)\) \(\sim SO(6)\). This group is identical to the R-symmetry of the \(\mathcal{N} = 4\) SYM theory. After including the fermionic generators required by supersymmetry, the full isometry supergroup of the AdS\(_5\) \(\times S^5\) background is SU\((2, 2|4)\), which is identical to the \(\mathcal{N} = 4\) superconformal symmetry.

To formulate an AdS/CFT duality with a reduced amount of supersymmetry, we may place the stack of D3-branes at the tip of a 6-dimensional Ricci flat cone, whose base is a 5-dimensional compact Einstein space \(Y_5\). The metric of such a cone is \(dr^2 + r^2ds_5^2\); therefore, the 10-d metric produced by the D3-branes is obtained from (10) by replacing \(d\Omega_5^2\), the metric on \(S^5\), by \(ds_5^2\), the metric on \(Y_5\). In the \(r \to 0\) limit we then find the space \(AdS_5 \times Y_5\) as the candidate dual of the CFT on the D3-branes placed at the tip of the cone. The isometry group of \(Y_5\) is smaller than \(SO(6)\), but AdS\(_5\) is the “universal” factor present in the dual description of any large \(N\) CFT, making the \(SO(2, 4)\) conformal symmetry geometric.

The fact that after the compactification on \(Y_5\) the string theory is 5-dimensional supports earlier ideas on the necessity of the 5-th dimension to describe 4-d gauge theories. The \(z\)-direction is dual to the energy scale of the gauge theory: small \(z\) corresponds to the UV domain of the gauge theory, while large \(z\) to the IR.

In the AdS/CFT duality, type IIB strings are dual to the chromo-electric flux lines in the gauge theory, providing a string theoretic set-up for calculating the quark anti-quark potential. The quark and anti-quark are placed near the boundary of Anti-de Sitter space \((z = 0)\), and the fundamental string connecting them is required to obey the equations of motion following from the Nambu action. The string bends into the interior \((z > 0)\), and the maximum value of the \(z\)-coordinate increases with the separa-
tion \( r \) between quarks. An explicit calculation of the string action gives an attractive \( q \bar{q} \) potential:

\[
V(r) = -\frac{4\pi^2\sqrt{\lambda}}{\Gamma\left(\frac{3}{4}\right)^4 r}.
\]

(12)

Its Coulombic \( 1/r \) dependence is required by the conformal invariance of the theory. Historically, a dual string description was hoped for mainly in the cases of confining gauge theories, where long confining flux tubes have string-like properties. In a pleasant surprise, we have seen that a string description can be applicable to non-confining theories too, due to the presence of extra dimensions in the string theory.

5.1 Spinning Strings vs. Long Operators

A few years ago it was noted that the AdS/CFT duality becomes particularly powerful when applied to operators with large quantum numbers. One class of such single-trace “long operators” are the BMN operators\(^{29}\) that carry a large R-charge in the SYM theory and contain a finite number of impurity insertions. The R-charge is dual to a string angular momentum on the compact space \( Y_5 \). So, in the BMN limit the relevant states are short closed strings with a large angular momentum, and a small amount of vibrational excitation. Furthermore, by increasing the number of impurities the string can be turned into a large semi-classical object moving in \( AdS_5 \times Y_5 \). Comparing such objects with their dual long operators has become a very fruitful area of research\(^ {30}\). Work in this direction has also produced a great deal of evidence that the \( \mathcal{N} = 4 \) SYM theory is exactly integrable (see \( ^{31,32} \) for recent reviews).

A familiar example of a gauge theory operator with a large quantum number is a twist-2 operator carrying a large spin \( J \), \( \text{Tr} \, F_{+\mu}D_{+}^{J-2}F_{+}^{\mu} \). In QCD, such operators play an important role in studies of deep inelastic scattering\(^ {33}\). In the \( \mathcal{N} = 4 \) SYM theory, the dual of such a high-spin operator is a folded string spinning around the center of \( AdS_5 \).\(^ {34}\) In general, for a high spin, the anomalous dimension of such an operator is

\[
\Delta - (J + 2) \rightarrow f(\lambda) \ln J.
\]

(13)

Calculating the energy of the spinning folded string, we find that the AdS/CFT prediction is\(^ {34}\)

\[
f(\lambda) \rightarrow \frac{\sqrt{\lambda}}{\pi},
\]

(14)

in the limit of large ‘t Hooft coupling. For small \( \lambda \), perturbative calculations in the large \( \mathcal{N} = 4 \) SYM theory up to 3-loop order give\(^ {36}\)

\[
f(\lambda) = \frac{1}{2\pi^2}\left(\lambda - \frac{\lambda^2}{48} + \frac{11\lambda^3}{11520} + O(\lambda^4)\right)
\]

(15)

An approximate extrapolation formula, suggested in\(^ {36}\) works with about 10% accuracy:

\[
\tilde{f}(\lambda) = \frac{12}{\lambda} \left(-1 + \sqrt{1 + \lambda/12}\right)
\]

\[
= \frac{1}{2\pi^2}\left(\lambda - \frac{\lambda^2}{48} + \frac{\lambda^3}{1152} + O(\lambda^4)\right)
\]

(16)

Note that \( \tilde{f} \) has a branch cut running from \(-\infty \) to \(-12 \). Thus, the series has a finite radius of convergence, in accord with general arguments about planar gauge theory given by ‘t Hooft. The fact that the branch point is at a negative \( \lambda \) suggests that in the \( \mathcal{N} = 4 \) SYM theory the perturbative series is alternating, and that there is no problem in extrapolating from small to large \( \lambda \) along the positive real axis. It is, of course, highly desirable to find an exact formula for \( f(\lambda) \). Recent work\(^ {37}\) raises hopes that a solution of this problem is within reach.

6 Thermal Gauge Theory from Near-extremal D3-branes

6.1 Entropy

An important black hole observable is the Bekenstein-Hawking (BH) entropy, which is...
proportional to the area of the event horizon, \( S_{BH} = A_h/(4G) \). For the 3-brane solution (6), the horizon is located at \( r = r_0 \). For \( r_0 > 0 \) the 3-brane carries some excess energy \( E \) above its extremal value, and the BH entropy is also non-vanishing. The Hawking temperature is then defined by \( T^{-1} = \partial S_{BH}/\partial E \).

Setting \( r_0 \ll L \) in (10), we obtain a near-extremal 3-brane geometry, whose Hawking temperature is found to be \( T = r_0/(\pi L^2) \). The small \( r \) limit of this geometry is \( S^5 \) times a certain black hole in \( AdS_5 \). The 8-dimensional “area” of the event horizon is \( A_h = \pi^6 L^8 T^3 V_3 \), where \( V_3 \) is the spatial volume of the D3-brane (i.e. the volume of the \( x^1, x^2, x^3 \) coordinates). Therefore, the BH entropy is

\[
S_{BH} = \frac{\pi^2}{2} N^2 V_3 T^3 .
\] (17)

This gravitational entropy of a near-extremal 3-brane of Hawking temperature \( T \) is to be identified with the entropy of \( N = 4 \) supersymmetric \( U(N) \) gauge theory (which lives on \( N \) coincident D3-branes) heated up to the same temperature.

The entropy of a free \( U(N) \) \( \mathcal{N} = 4 \) supermultiplet, which consists of the gauge field, \( 6N^2 \) massless scalars and \( 4N^2 \) Weyl fermions, can be calculated using the standard statistical mechanics of a massless gas (the black body problem), and the answer is

\[
S_0 = \frac{2\pi^2}{3} N^2 V_3 T^3 .
\] (18)

It is remarkable that the 3-brane geometry captures the \( T^3 \) scaling characteristic of a conformal field theory (in a CFT this scaling is guaranteed by the extensivity of the entropy and the absence of dimensionful parameters). Also, the \( N^2 \) scaling indicates the presence of \( O(N^2) \) unconfined degrees of freedom, which is exactly what we expect in the \( \mathcal{N} = 4 \) supersymmetric \( U(N) \) gauge theory. But what is the explanation of the relative factor of \( 3/4 \) between \( S_{BH} \) and \( S_0 \)? In fact, this factor is not a contradiction but rather a prediction about the strongly coupled \( \mathcal{N} = 4 \) SYM theory at finite temperature. As we argued above, the supergravity calculation of the BH entropy, (17), is relevant to the \( \lambda \to \infty \) limit of the \( \mathcal{N} = 4 \) \( SU(N) \) gauge theory, while the free field calculation, (18), applies to the \( \lambda \to 0 \) limit. Thus, the relative factor of \( 3/4 \) is not a discrepancy: it relates two different limits of the theory. Indeed, on general field theoretic grounds, in the ‘t Hooft large \( N \) limit the entropy is given by

\[
S = \frac{2\pi^2}{3} N^2 f(\lambda) V_3 T^3 .
\] (19)

The function \( f \) is certainly not constant: Feynman graph calculations valid for small \( \lambda = g_{YM}^2 N \) give

\[
f(\lambda) = 1 - \frac{3}{2\pi^2} \lambda + \frac{3 + \sqrt{2}}{\pi^3} \lambda^{3/2} + \ldots
\] (20)

The BH entropy in supergravity, (17), is translated into the prediction that

\[
\lim_{\lambda \to \infty} f(\lambda) = \frac{3}{4}.
\] (21)

A string theoretic calculation of the leading correction at large \( \lambda \) gives

\[
f(\lambda) = \frac{3}{4} + \frac{45}{32\zeta(3)} \lambda^{-3/2} + \ldots
\] (22)

These results are consistent with a monotonic function \( f(\lambda) \) which decreases from 1 to \( 3/4 \) as \( \lambda \) is increased from 0 to \( \infty \). The 1/4 deficit compared to the free field value is a strong coupling effect predicted by the AdS/CFT correspondence.

It is interesting that similar deficits have been observed in lattice simulations of deconfined non-supersymmetric gauge theories. The ratio of entropy to its free field value, calculated as a function of the temperature, is found to level off at values around 0.8 for \( T \) beyond 3 times the deconfinement temperature \( T_c \). This is often interpreted as the effect of a sizable coupling. Indeed, for \( T = 3T_c \), the lattice estimates indicate that \( g_{YM}^2 N \approx 7 \). This challenges an old prejudice that the QGP is inherently very weakly coupled. We now turn to calculations.
of the shear viscosity where strong coupling effects are even more pronounced.

6.2 Shear Viscosity

The shear viscosity $\eta$ may be read off from the form of the stress-energy tensor in the local rest frame of the fluid where $T_{0i} = 0$:

$$T_{ij} = p\delta_{ij} - \eta (\partial_i u_j + \partial_j u_i - \frac{2}{3} \delta_{ij} \partial_k u_k), \quad (23)$$

where $u_i$ is the 3-velocity field. The viscosity can be also determined\textsuperscript{44} through the Kubo formula

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt d^3x e^{i\omega t} \langle [T_{xy}(t, \vec{x}), T_{xy}(0, 0)] \rangle \quad (24)$$

For the $\mathcal{N} = 4$ supersymmetric YM theory this 2-point function may be computed from absorption of a low-energy graviton $h_{xy}$ by the 3-brane metric\textsuperscript{24}. Using this method, it was found\textsuperscript{44} that at very strong coupling

$$\eta = \frac{\pi}{8} N^2 T^3, \quad (25)$$

which implies

$$\frac{\eta}{s} = \frac{\hbar}{4\pi} \quad (26)$$

after $\hbar$ is restored in the calculation (here $s = S/V_3$ is the entropy density). It has been proposed\textsuperscript{45} that this value is the universal lower bound on $\eta/s$. Indeed, at weak coupling $\eta/s$ is very large, $\sim \frac{1}{\text{quasi-particle mean free time}}$, and there is evidence that it decreases monotonically as the coupling is increased\textsuperscript{46}.

The appearance of $\hbar$ in (26) is reasonable on general physical grounds\textsuperscript{45}. The shear viscosity $\eta$ is of order the energy density times quasi-particle mean free time $\tau$. So, $\eta/s$ is of order of the energy of a quasi-particle times its mean free time, which is bounded from below by the uncertainty principle to be some constant times $\hbar$. The AdS/CFT correspondence fixes this constant to be $1/(4\pi)$, which is not far from some earlier estimates\textsuperscript{47}.

For known fluids (e.g. helium, nitrogen, water) $\eta/s$ is considerably higher than the proposed lower bound\textsuperscript{45}. On the other hand, the Quark-Gluon Plasma produced at RHIC is believed to have a very low $\eta/s$, within a factor of 2 of the bound (26)$\textsuperscript{48,47}$. This suggests that it is rather strongly coupled. Recently a new term, sQGP, which stands for "strongly coupled Quark-Gluon Plasma," has been coined to describe the deconfined state observed at RHIC\textsuperscript{49,50} (a somewhat different term, "Non-perturbative Quark-Gluon Plasma," was proposed in \textsuperscript{51}). As we have reviewed, the AdS/CFT correspondence is a theoretical laboratory which allows one to study analytically an extreme example of such a new state of matter: the thermal $\mathcal{N} = 4$ SYM theory at very strong 't Hooft coupling.

In a CFT, the pressure is related to the energy density by $p = 3e$. Hence, the speed of sound satisfies $c_s^2 = dp/de = \frac{1}{3}$. Recent lattice QCD calculations indicate that, while $c_s^2$ is much lower for temperatures slightly above $T_c$, it gets close to $1/3$ for $T \geq 2T_c$. Thus, for some range of temperatures starting around $2T_c$, QCD may perhaps be treated as an approximately conformal, yet non-perturbative, gauge theory. This suggests that AdS/CFT methods could indeed be useful in studying the physics of sQGP, and certainly gives strong motivation for continued experimental and lattice research.

Lattice calculations indicate that the deconfinement temperature $T_c$ is around 175 $MeV$, and the energy density is $\approx 0.7$ $GeV/fm^3$, around 6 times the nuclear energy density. RHIC has reached energy densities around 14 $GeV/fm^3$, corresponding to $T \approx 2T_c$. Furthermore, in a few years, heavy ion collisions at the LHC are expected to reach temperatures up to $5T_c$. Thus, RHIC and LHC should provide a great deal of useful information about the conjectured quasi-conformal temperature range of QCD.
7 String Duals of Confining Theories

It is possible to generalize the AdS/CFT correspondence in such a way that the quark anti-quark potential is linear at large distance. In an effective 5-dimensional approach\(^1\) the necessary metric is

\[
ds^2 = \frac{dz^2}{z^2} + a^2(z)(- (dx^0)^2 + (dx^1)^2) \tag{27}
\]

and the space must end at a maximum value of \(z\) where the “warp factor” \(a^2(z_{\text{max}})\) is finite.\(^b\) Placing widely separated probe quark and anti-quark near \(z = 0\), we find that the string connecting them bends toward larger \(z\) until it stabilizes at \(z_{\text{max}}\) where its tension is minimized at the value \(\frac{\alpha'}{2\pi}\). Thus, the confining flux tube is described by a fundamental string placed at \(z = z_{\text{max}}\) parallel to one of the \(x^1\)-directions. This establishes a duality between “emergent” chromo-electric flux tubes and fundamental strings in certain curved string theory backgrounds.

Several 10-dimensional supergravity backgrounds dual to confining gauge theories are now known, but they are somewhat more complicated than (27) in that the compact directions are “mixed” with the \(5\)-d \((x^\mu, z)\) space. Witten\(^56\) constructed a background in the universality class of non-supersymmetric pure glue gauge theory. While in this background there is no asymptotic freedom in the UV, hence no dimensional transmutation, the background has served as a simple model of confinement where many infrared observables have been calculated using the classical supergravity. For example, the lightest glueball masses have been found from normalizable fluctuations around the supergravity solution\(^37\). Their spectrum is discrete, and resembles qualitatively the results of lattice simulations in the pure glue theory.

\(^A\)A simple model of confinement\(^52\) is obtained for \(a(z) = 1/z\) in (27), i.e. the metric is a slice of AdS\(_5\) cut off at \(z_{\text{max}}\). Hadron spectra in models of this type were studied in \(^53,54,55\).

Introduction of a minimal \((\mathcal{N} = 1)\) supersymmetry facilitates construction of gauge/string dualities. As discussed earlier, a useful method is to place a stack of D-branes at the tip of a six-dimensional cone, whose base is \(Y_5\). For \(N\) D3-branes, one finds the background \(AdS_5 \times Y_5\) dual to a superconformal gauge theory. Furthermore, there exists an interesting way of breaking the conformal invariance for spaces \(Y_5\) whose topology includes an \(S^2\) factor. At the tip of the cone over \(Y\) one may add \(M\) wrapped D5-branes to the \(N\) D3-branes. The gauge theory on such a combined stack is no longer conformal; it exhibits a novel pattern of quasi-periodic renormalization group flow, called a duality cascade\(^58,59\) (for reviews, see \(^60,61\)).

To date, the most extensive study of a theory of this type has been carried out for a simple 6-d cone called the conifold, where one finds a \(\mathcal{N} = 1\) supersymmetric \(SU(N) \times SU(N+M)\) theory coupled to chiral superfields \(A_1, A_2\) in the \((N, \overline{N} + M)\) representation, and \(B_1, B_2\) in the \((\overline{N}, N + M)\) representation. In type IIB string theory, D5-branes source the 7-form field strength from the Ramond-Ramond sector, which is Hodge dual to the 3-form field strength. Therefore, the \(M\) wrapped D5-branes create \(M\) flux units of this field strength through the 3-cycle in the conifold; this number is dual to the difference between the numbers of colors in the two gauge groups. An exact non-singular supergravity solution dual to this gauge theory, incorporating the 3-form and the 5-form R-R field strengths, and their back-reaction on the geometry, has been found\(^59\). This back-reaction creates a “geometric transition” to the deformed conifold

\[
\sum_{a=1}^{4} z_a^2 = \epsilon^2, \tag{28}
\]

and introduces a “warp factor” so that the full 10-d geometry has the form

\[
ds^2 = h^{-1/2}(\tau)(-(dx^0)^2 + (dx^1)^2) + h^{1/2}(\tau) ds_5^2, \tag{29}
\]

\(\frac{\partial}{\partial x^i} = 0\) for any \(i\).
where $ds^2$ is the Calabi-Yau metric of the deformed conifold, which is known explicitly.

The field theoretic interpretation of this solution is unconventional. After a finite amount of RG flow, the $SU(N + M)$ group undergoes a Seiberg duality transformation\(^\text{62}\). After this transformation, and an interchange of the two gauge groups, the new gauge theory is $SU(\tilde{N}) \times SU(\tilde{N} + M)$ with the same matter and superpotential, and with $\tilde{N} = N - M$. The self-similar structure of the gauge theory under the Seiberg duality is the crucial fact that allows this pattern to repeat many times. If $N = (k + 1)M$, where $k$ is an integer, then the duality cascade stops after $k$ steps, and we find a $SU(M) \times SU(2M)$ gauge theory. This IR gauge theory exhibits a multitude of interesting effects visible in the dual supergravity background. One of them is confinement, which follows from the fact that the warp factor $h$ is finite and non-vanishing at the smallest radial coordinate, $\tau = 0$, which roughly corresponds to $z = z_{\text{max}}$ in an effective 5-d approach (27). This implies that the quark anti-quark potential grows linearly at large distances. Other notable IR effects are chiral symmetry breaking, and the Goldstone mechanism\(^\text{63}\). Particularly interesting is the appearance of an entire “baryonic branch” of the moduli space in the gauge theory, whose existence has been recently demonstrated also in the dual supergravity language\(^\text{64}\).

Besides providing various new insights into the IR physics of confining gauge theories, the availability of their string duals enables one to study Deep-Inelastic and hadron-hadron scattering in this new language\(^\text{52}\).

8 Summary

Throughout its history, string theory has been intertwined with the theory of strong interactions. The AdS/CFT correspondence\(^\text{3,4,5}\) succeeded in making precise connections between conformal 4-dimensional gauge theories and superstring theories in 10 dimensions. This duality leads to a multitude of dynamical predictions about strongly coupled gauge theories. When extended to theories at finite temperature, it serves as a theoretical laboratory for studying a novel state of matter: a gluonic plasma at very strong coupling. This appears to have surprising connections to the new state of matter, sQGP, observed at RHIC\(^\text{6}\).

Extensions of the AdS/CFT correspondence to confining gauge theories provide new geometrical viewpoints on such important phenomena as chiral symmetry breaking and dimensional transmutation. They allow for studying meson and glueball spectra, and high-energy scattering, in model gauge theories.

This recent progress offers new tantalizing hopes that an analytic approximation to QCD will be achieved along this route, at least for a large number of colors. But there is much work that remains to be done if this hope is to become reality: understanding the string duals of weakly coupled gauge theories remains an important open problem.

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