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Uniform spanning forests of planar graphs. (English) Zbl 1422.60022
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Summary: We prove that the free uniform spanning forest of any bounded degree proper plane graph is connected almost surely, answering a question of Benjamini, Lyons, Peres and Schramm [I. Benjamini et al., Ann. Probab. 29, No. 1, 1–65 (2001; Zbl 1016.60009)]. We provide a quantitative form of this result, calculating the critical exponents governing the geometry of the uniform spanning forests of transient proper plane graphs with bounded degrees and codegrees. We find that the same exponents hold universally over this entire class of graphs provided that measurements are made using the hyperbolic geometry of their circle packings rather than their usual combinatorial geometry.

MSC:
60D05 Geometric probability and stochastic geometry
05C10 Planar graphs; geometric and topological aspects of graph theory

Keywords:
free uniform spanning forest; critical exponents

Software:
CirclePack

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