Dynamic Response of High-piled Wharf Pile Subjected to Underwater Explosion

Jinghan Liu¹,², Ting Tang², Zhuobin Wei², Lingfeng Li¹,² and Yuanhao Zhang¹,²

1. Naval University of Engineering, Wuhan 430033, Hubei, China
2. Naval Logistics College of PLA, Tianjin 300450, China
Email: 1226001717@qq.com

Abstract. In order to study the dynamic response of high-piled wharf pile subjected to underwater explosion, pile was simplified as an elastically supported beam. The computational model of pile was established. The correctness of the calculation model was verified by theoretical analysis. The dynamic coefficient $K_d$ was an index to the displacement and deformation of pile. In order to further investigate the dynamic response of pile, typical high-piled wharf piles subjected to shock wave load and bubble pulse load were analysed. The results show that horizontal stiffness has more impact on dynamic coefficient than torsional stiffness. The bubble pulse load poses a greater threat to pile than shock wave load. The peak load and load duration are the significant facts influencing dynamic coefficient in shock wave phase. The load duration is the significant factor influencing dynamic coefficient in bubble pulse phase.

1. Introduction
The wharf is important for maritime transportation, material transportation and logistics support. As one of the significant structural forms of wharf in China, high-piled wharf is widely located in estuaries and muddy coastal. With the frequent explosion accident and terrorist attacks, high-piled wharfs are facing security threats in peacetime and wartime. Underwater explosion is the major striking form of high-piled wharf. The pile is vulnerable to weapons attack. It is of great significance to study the dynamic response of high-piled wharf pile subjected to underwater explosion for improving the survivability and support capability of high-piled wharf design.

The underwater explosion process includes shock wave propagation stage and bubble pulse stage. In the 19th century, The U.S. and The former Soviet Union militaries firstly research the underwater explosion, especially shock wave [1,2]. The research got abundant experiment data and a mature theory, but few achievements on the underwater bubble pulse. In the 1980s, people realized that the bubble pulse was the vital damage effect that could not be ignored in underwater explosion [3]. Considering complex boundaries of the high-piled wharf pile, it is necessary to study the dynamic response of high-piled wharf subjected to both shock wave and bubble pulse.

2. Calculation Model
The structural sketch of the typical high-piled wharf is shown in figure 1. The upper part of pile is connected with wharf panel and the bottom is embedded into underwater clay [4]. The calculation beam model of pile simplifies the boundary setting as shown in figure 2. The link-banked structure and superstructure of high-piled wharf are simplified as a rigid block whose mass is $m$. The pile is simplified as Euler beam whose length is $l$. The bottom of pile is embedded into soil, which is simplified as the torsion spring and horizontal restraint in the end. The upper of pile is connected to
the rigid block, which is simplified to as horizontal spring and vertical restraint. \( K_A \) is the stiffness of torsion spring and \( K_B \) is the stiffness of horizontal spring. \( \theta_A \) is rotation angle and \( y_B \) is horizontal displacement.

\[ y(x,t) = \frac{x}{l} y_B + w(x,t) \]  

Based on the mechanics of materials and structural dynamic theory, the transverse vibration equation of the cross-section beam is presented as follow [5]:

\[
q(x,t) = EI \frac{d^4 w}{dx^4} + \rho A \frac{d^2 y}{dt^2} + m \frac{d^2 y_B}{dt^2}
\]

where \( q(x,t) \) is inertial force which is generated by transverse vibration of the beam; \( EI \) is the bending stiffness of beam; \( w \) is deflection of beam; \( \rho \) is density of beam; \( A \) is cross-section of beam.

Using separation of variables can solve the equation as follow:

\[
y(x,t) = \frac{x}{l} y_B + (C_1 \sin(\alpha x) + C_2 \cos(\alpha x) + C_3 \sinh(\alpha x) + C_4 \cosh(\alpha x))(A\sin(\omega t) + B\cos(\omega t))
\]

where \( \phi(x) \) is modal function; \( \alpha \) is modal parameter: \( \alpha = \sqrt{\frac{\omega^2 A\rho}{EI}} \); \( C_1, C_2, C_3 \) and \( C_4 \) are coefficients.

The boundary functions are shown as follow:

\[
y(B,t) = (C_2 + C_4)(A\sin(\omega t) + B\cos(\omega t))
\]

According to the equations (3)-6, \( C_1, C_2, C_3 \) and \( C_4 \) can be calculated by boundary equation.

Based the compatibility of displacements and stress at point A and B, the boundary equations are proposed as follows:

\[
y\bigg|_{x=0} = 0
\]
The coefficients exist only if determinant of the matrix is zero.

\[
\begin{vmatrix}
\frac{\eta}{l} - \frac{\eta\sin\alpha l}{l^2} & 1 & 0 \\
\alpha\cos\alpha l - \frac{\sin\alpha l}{l} & -\alpha\sin\alpha l - \frac{\cos\alpha l}{l} & 1 \\
\frac{\xi EI}{l^3}\sin\alpha l + \alpha^2\cos\alpha l & \frac{\xi EI}{l^3}\cos\alpha l - \alpha^2\sin\alpha l & \frac{\xi EI}{l^3}\sinh\alpha l - \alpha^2\cosh\alpha l \\
\frac{\xi EI}{l^3}\cosh\alpha l - \alpha^2\sinh\alpha l & \frac{\xi EI}{l^3}\sinh\alpha l - \alpha^2\cosh\alpha l & \frac{\xi EI}{l^3}\cosh\alpha l - \alpha^2\sinh\alpha l
\end{vmatrix} = 0
\]  
(11)

\(\eta\) denotes the rotational stiffness and \(\xi\) denotes the horizontal stiffness:

\[
\eta = \frac{K_d}{EI}, \quad \xi = \frac{k_d l^3}{EI}
\]

(12, 13)

3. The Modal Functions of Pile

Considering a typical high-piled wharf, the pile is 23.5 meter high with a cross section of 0.5m×0.5m. The depth of penetration is about 15m. The materials of pile adopt the reinforced concrete. The pile can be simplified as calculation model of beam. Equation (11) can be solved as \(l=8.5m\), \(EI=16927.1kNm\). The relationships between \(\alpha\) and stiffness coefficients (\(\eta\) and \(\xi\)) are shown in figure 3 and figure 4. The first three mode results are presented in figure 3.

![Figure 3. The first three mode results](image)

![Figure 4. The first modal parameter \(\alpha\)](image)

The modal parameter \(\alpha\) increase with increasing of \(\eta\) and \(\xi\) and tend to a certain value which is the modal parameter of clamped beam. The higher order vibration modes have less influence on structural response and can be ignored. The first modal parameter \(\alpha\) is shown in figure 4. It is shown that \(\alpha\) varies when the range of \(\eta\) is 0~50 and the range of \(\xi\) is 0~0.0001. The influence of \(\xi\) upon structural response is stronger than \(\eta\). The results demonstrate that horizontal stiffness factor is main controlling factor of structural factor compared to \(\eta\). Therefore, the soil at the bottom of pile has little influence on the dynamic response of pile, and the horizontal support of high-piled wharf superstructure has great influence on the dynamic response of the pile. When \(\xi \to \infty\), the calculation mode of pile is a beam that one end is fixed support and the other end is rotational spring constraint. As shown in figure 5, \(\alpha\) firstly increases rapidly and then increases smoothly with the increasing of \(\eta\). When \(\eta > 20\), the
influence of $\eta$ can be ignored and point A can be regarded as fixed support. When $\eta \to \infty$, the calculation mode of pile is a beam that one end is simply supported and the other end is horizontal spring constrained. As shown in figure 6, $\alpha$ firstly increases rapidly and then increases smoothly with the increasing of $\xi$. When $\xi > 3 \times 10^{-5}$, the influence of $\xi$ can be ignored and point B can be regarded as fixed support.

Figure 5. The relationship between $\alpha$ and $\eta$ ($\xi \to \infty$)

Figure 6. The relationship between $\alpha$ and $\xi$ ($\eta \to \infty$)

When $\eta \to \infty$, $\xi \to \infty$, the calculation mode of pile is a beam that both ends are fixed supported. The first modal parameter $\alpha = 0.556$ theoretically, coinciding with results in figure 3 and figure 4. When $\eta \to 0$, $\xi \to \infty$, the calculation mode of pile is a beam that one end is fixed supported and the other end is simply supported. The first modal parameter $\alpha = 0.462$ theoretically, coinciding with result in figure 5. When $\eta \to \infty$, $\xi \to 0$, the calculation mode of pile is a beam that one end is fixed supported and the other end is sliding bearing. The first modal parameter $\alpha = 0$ theoretically, coinciding with result in figure 6. The results from theory and model function are identical.

4. Dynamic Function of Pile Subjected to Underwater Explosion

When explosion occurs in water, the maximum displacement of high-piled wharf pile is reached in an extremely short period. During this period, damping force cannot absorb much energy from structural response. Therefore, the effect of damping is ignored in the dynamic function of pile subjected to underwater explosion. Blasting load can be simplified to transverse load which leads to the forced vibration response of the beam model. The displacement of beam is decomposed into a series of orthogonal polynomials. The dynamic function of pile subjected to underwater explosion is proposed as follow [6]:

$$y(x, t) = \sum_{i=1}^{\infty} \frac{Y_i(x)}{\rho \omega_i} \int_{0}^{l} q(x) Y_i(x) dx \int_{0}^{l} f(\tau) \sin \omega_i (t - \tau) d\tau$$

(14)

where $q(x)$ is static load function; $Y_i(x)$ is orthogonal polynomial of displacement. $f(t)$ is dynamic load function.

High-order vibration modes have little effect on dynamic response of structure. Based the regularization method for the first-order mode, dynamic function of beam model is proposed as follow.

$$y(x, t) = Y_1(x) \cdot Y_{st} \cdot \int_{0}^{l} f(\tau) \sin \omega_1 (t - \tau) d\tau$$

(15)

$$K(t) = \omega_1 \cdot \int_{0}^{l} f(\tau) \sin \omega_1 (t - \tau) d\tau$$

(16)

$$y(t) = K(t) \cdot y_{st}$$

(17)
where \( y_{st} \) is static displacement of beam under static load function \( q(x) \); \( y(t) \) is displacement function; \( K(t) \) is dynamic function.

The maximum displacement of beam under dynamic load can be expressed by dynamic function and static displacement as follow:

\[
y_{dm} = y_{sm} K_d
\]  

(18)

where \( y_{dm} \) is the maximum dynamic displacement; \( y_{sm} \) is the maximum static displacement. \( K_d \) is the maximum of dynamic function, which can reflect the displacement and deformation. The dynamic coefficient \( K_d \) is determined by structural characteristics, boundary constraints and load function.

5. Dynamic Coefficient of Pile

According to the equation (11), the first order frequency of beam can be calculated as shown in Tab.1.

| \( \eta \times 10^{-5} \) | 1 \( \times 10^{-5} \) | 5 \( \times 10^{-5} \) | 1 \( \times 10^{-4} \) | 2 \( \times 10^{-4} \) | 5 \( \times 10^{-4} \) |
|---|---|---|---|---|---|
| 1 | 2.290 | 4.297 | 5.029 | 5.471 | 5.744 |
| 5 | 2.362 | 4.570 | 5.476 | 6.064 | 6.437 |
| 10 | 2.396 | 4.701 | 5.705 | 6.383 | 6.826 |
| 20 | 2.421 | 4.801 | 5.884 | 6.648 | 7.157 |
| 40 | 2.438 | 4.866 | 6.003 | 6.828 | 7.389 |

Considering complicated boundary conditions for high-piled wharf piles, the shock wave load and bubble impulse load are complex. The loads on pile can be simplified as shown as equation (19). From equation (16) and (19), following dynamic function can be gotten shown as equation (20).

\[
f(t) = P_{max} (1 - \frac{t}{t_0})
\]

(19)

\[
K(t) = \begin{cases} 
1 + \frac{1}{\omega t_0} \sin(\omega t) - \frac{t}{t_0} \cos(\omega t) & t < t_0 \\
K(t_0) \cos(\omega(t - t_0)) + \frac{\dot{K}(t_0)}{\omega} \sin(\omega(t - t_0)) & t > t_0 
\end{cases}
\]

(20)

where \( P_{max} \) is peak value of load; \( t_0 \) is the duration of explosion load.

The dynamic coefficient \( K_d \) is obtained by solving the maximum value of dynamic function. When \( \dot{K}(t) = 0 \), \( K_d = K(t) \). If \( t < t_0 \), the dynamic coefficient of pile has been reached before \( t_0 \). This load type is impulse load. If \( t > t_0 \), the dynamic coefficient of pile will be reached after \( t_0 \). This load type is quasi-static load.

The shock wave and bubble pulse threaten the health of high-piled wharf. The peak value of shock wave is great high, but it decreases rapidly within milliseconds. Impulsive loads act on piles when bubble expands and shrinks. The peak value of bubble pulse load is about 15%~20% peak load of shock wave. The duration of impulsive load is several seconds. Based on above theory, \( P_{max} = 10P_0 \) and \( t_0 = 5 \times 10^{-3} \) during the shock wave phase. \( P_{max} = P_0 \) and \( t_0 = 1s \) during bubble pulsation phase. When \( \xi = 1 \times 10^{-4} \), The dynamic functions of pile during shock wave phase and bubble pulse phase are shown as figure 7(a)-(b).
When $\eta=10$, the dynamic functions of pile during shock wave phase and bubble pulse phase are shown as figure 8(a)-(b). The dynamic coefficient during the bubble pulse phase is higher than it during shock wave phase. With the increasing of $\eta$ and $\xi$, the dynamic coefficient increases and the dynamic function peak ahead. The results show that the displacement and deformation of pile are higher during the bubble pulse phase and more influenced by horizontal stiffness factor $\xi$.

![Diagram](image)

**Figure 7.** The relationship between $K(t)$ and $\eta$

![Diagram](image)

**Figure 8.** The relationship between $K(t)$ and $\xi$

**Table 2.** The dynamic coefficient $K_d$.

| $\eta$ | $\xi$ | shock wave phase | bubble pulse phase |
|-------|-------|------------------|-------------------|
|       | $1\times10^{-5}$ | $5\times10^{-5}$ | $1\times10^{-4}$ | $2\times10^{-4}$ | $5\times10^{-4}$ | $1\times10^{-5}$ | $5\times10^{-5}$ | $2\times10^{-4}$ | $5\times10^{-4}$ |
| 1     | 0.360  | 0.675            | 0.789            | 0.859            | 0.901            | 1.791            | 1.886            | 1.902            | 1.91              | 1.914            |
| 5     | 0.371  | 0.717            | 0.860            | 0.952            | 1.010            | 1.797            | 1.893            | 1.910            | 1.919             | 1.923            |
| 10    | 0.376  | 0.738            | 0.895            | 1.002            | 1.071            | 1.800            | 1.896            | 1.913            | 1.923             | 1.928            |
| 20    | 0.380  | 0.753            | 0.923            | 1.043            | 1.122            | 1.802            | 1.898            | 1.916            | 1.926             | 1.931            |
| 40    | 0.383  | 0.764            | 0.942            | 1.071            | 1.158            | 1.803            | 1.899            | 1.918            | 1.927             | 1.933            |
The dynamic coefficients under different boundary conditions are shown in Tab.2. The dynamic coefficient during bubble pulse phase is higher than it during shock wave phase. The results show that bubble impulsive load influences more on dynamic response of pile than shock wave load. More importantly, bubble pulse is fatal to high-piled wharf when the lower frequency of pile is lose to the frequency of bubble pulse [7]. The peak load of shock wave is far higher than bubble impulsive load and it will cause local damage of structure.

Due to the peculiarity of the boundary in the district, the shock wave reflection and diffraction could long the duration of explosion load. Figure 9 shows the dynamic function of pile under different loading durations. The dynamic function reaches the maximum before shock wave load ends and the load is impulsive load. The variation regularity of dynamic function in shock wave phase maintains unchanged, but the dynamic coefficient significantly increases. It indicates that the duration of shock wave load is the significant factor that increase dynamic coefficient. The dynamic function reached the maximum after bubble pulse load ends and the load is quasi-static load. The duration of bubble pulse load has little impact on dynamic response of pile. The dynamic coefficient increases weakly with the increasing of duration of bubble pulse load. The peak load of bubble pulse is the significant factor for the dynamic coefficient.

![Figure 9. The relationship between K(t) and t₀](image)

6. Conclusions
In this paper high-piled wharf pile was simplified as elastically supported beam. The modal function and dynamic function of pile were calculated to study the dynamic response of pile subjected to shock wave and bubble pulse. The results presented in this paper lead to the following conclusions.

1) The computational model of high-piled wharf pile was established by simplified elastically supported beam. Based on the calculation model of pile, the displacement and deformation of piles are more influenced by link-banked structure of high-piled wharf. The underwater clay has little influence on the dynamic response of pile.

2) The dynamic coefficient $K_d$ can reflect the displacement and deformation of pile. With the increasing of horizontal stiffness factor $\eta$ and torsional stiffness factor $\xi$, the dynamic function peaks ahead and $K_d$ increases. Decreasing the horizontal stiffness of link-banked structure could reduce the damage of piles.

3) The dynamic coefficient $K_d$ of pile is higher during the bubble pulse phase than during the shock wave phase. The shock wave will cause local damage of piles, but the bubble pulse has more destructive to high-piled wharf piles. Increasing load duration increases the displacement and deformation of piles. The load duration has little impact on dynamic coefficient in bubble pulse phase, but has a great influence on dynamic coefficient in shock wave phase.
7. References
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