ROBUST FRACTIONAL ORDER CONTROLLER FOR AN EXPENDABLE LAUNCH VEHICLE

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Abstract. Fractional calculus is an interdisciplinary area with multifarious applications especially in control systems due to more flexible parameters to adjust the dynamic behaviour of all the physical systems. This paper presents a fractional order controller structure for an expendable launch vehicle during ascent phase. The launch vehicle has highly flexible and unstable dynamic model, and in addition to that unpredictable control issues are caused by the sloshing of liquid propellants and the inertia problems of the engines. Under these circumstances controller with large robustness margins are required to meet deviations in model parameters which are unknown before the actual flight. The rigid body dynamics of the vehicle is considered for designing fractional order controller to overcome all these control issues. The robustness of fractional order controller is compared with existing classical controller for 20% variations in the aerodynamic coefficients. The stability margins of the open loop transfer function with all plant dynamics shows the supremacy of the proposed controller over the classical controller. The control inputs to launch vehicles, in the form of attitude commands, are affected by the wind. Hence the designed fractional controller simulated under the wind disturbance input and the result shows disturbance rejection capability of controller.

Keywords. Attitude Control, Fractional Order Controllers, Launch Vehicle

1. INTRODUCTION

Launch vehicles are used to transport and put satellites or spacecraft into space. When vehicle flies through the atmosphere it experiences aerodynamic forces and moments. Vehicle is aerodynamically stable if it can maintain its orientation under wind disturbances. Vehicle is aerodynamically unstable if its orientation diverges due to angle of attack by wind disturbances. In general, launch vehicles are aerodynamically unstable [1],[2]. The launch vehicle guidance system provides the inputs to the autopilot necessary to fly the vehicle in a predefined direction. These control inputs are generally in the form of attitude commands. The attitude control problem is to develop a controller with good tracking performance, adequate stability, rapid and well damped response to input command with moderate insensitivity to external disturbances [3]. Due to the aerodynamically unstable model of launch vehicle, the control design becomes more complicated. The attitude control problem of the launch vehicles requires an effective controller to track the recommended trajectory. The prime significance lies in building up an autopilot arrangement that accomplishes sufficient short period stability and performance. This makes utilization of the alleged time slice approach, in which the time varying mass and inertial properties are considered as constant over a short period of time. [1].
A conventional PID controller is easy to design and implement to the system. The stability of the system can be assured by properly designing the proportional controller. But it introduces a constant steady state error. The steady state error can be eliminated by the use of integral control, but it deteriorates stability of system. The derivative controller helps in predicting the future error depending on the current error rate. Thus, a suitable combination of the three basic modes can be improve all aspects of the system performance. [3] Though the researches are going on fractional Calculus over fast few centuries, the recent advancements in computational tools attracted more work on this area. The classical PID controllers are mostly used in industry due to its simplicity in design and more systematic procedures available in design of controllers. The design of Fractional PID controller can be done using frequency domain approach or time domain approach. The combination of time domain and frequency domain specifications also can be used as constraint for choosing control parameters. The most commonly used method is to design FOPID controller parameters using tuning methods such as genetic algorithm or particle swarm optimization. [9] The experimental validation of FOPID controllers performed in some industrial applications and experimental platforms such as Magnetic Levitation Systems and Inverted Pendulum, reinforces the capability of fractional order controllers. [4] Podlubny has proposed a generalisation of the PID controller as controller. He also demonstrated that the fractional-order PID (PI^λD^μ) controller has better controller action than classical PID controller [7]. In addition to the fractional controller design the concept of fractional calculus can be applied to system modelling also. Conventionally all the systems are model by ordinary differential equation of integer order. But the developments in the area of fractional calculus, helps to model the system as non-integer model [5],[6]. Since solution methods for fractional differential equations were not developed, the conventional integer models where used to represent the complex systems. The different definitions of non-integer order differentiation and integration has developed in past few decades. This can be used to apply the fractional calculus in to wide area of control applications [6].

2. LAUNCH VEHICLE MODELLING
Launch vehicle attitude dynamics is varying as the time changes. The effects of aerodynamics, actuator dynamics, engine inertia and vehicle flexibility causes the variation in attitude dynamics of launch vehicle. Referring to [1], for preliminary design of autopilot, the launch vehicle is assumed to be rigid. The simplified dynamic model of the launch vehicle in the pitch plane obtained by the equilibrium moment equations and force equations. Also assumed that the parameters of the model are time invariant for short time period under consideration.

![Figure 1. Geometrical diagram for rigid body dynamic model[2]](image)

The linearized equations of motions of the launch vehicle in the pitch plane can be derived by considering the force equation and moment equations.
The force equation,

\[ \ddot{z} = \left( \frac{T_r - D}{m} \right) \theta - \frac{L_a}{m} \alpha + \frac{T_r}{m} \delta \quad (1) \]

\( T_c \), control thrust, \( L_a \), the normal aerodynamic force, \( \alpha \) angle of attack, \( \dot{\theta} \) the actual altitude, \( T_r \), total thrust, \( D \) the drag, \( \alpha \), angle of attack and \( \delta \) rocket engine deflection angle.

The moment equations

\[ \dot{\theta} = \mu_a \alpha + \mu_c \delta \quad (2) \]

\[ \alpha = \theta + \frac{z}{U_0} + \alpha_w \quad (3) \]

The rigid body equation is given by

\[ G(s) = \frac{\theta(s)}{\delta(s)} = \frac{\left[ \mu_c s^2 + \frac{\zeta_c \mu_c + \zeta_c \zeta_d}{U_0} \right]}{s^3 + \frac{\zeta_c \mu_c + \zeta_c \zeta_d}{U_0} s^2 - \mu_a s + \frac{\mu_c \zeta_d \cos \delta_0}{U_0}} \quad (4) \]

\( U_0 \), longitudinal velocity, \( g \), the gravitational acceleration. The other parameter and their values during the time period is given in table1.

At the initial stage of the flight, the aerodynamic pressure is negligible and the equation (4) reduces to

\[ G(s) = \frac{\theta(s)}{\delta(s)} = \frac{1}{s^2} \quad (5) \]

During ascent phase of the flight, the aerodynamic pressure is considerable, the forward velocity is \( U_0 \) is very high and the rigid body equation can be represented as

\[ G(s) = \frac{\theta(s)}{\delta(s)} = \frac{\mu_c}{s^2 - \mu_a} \quad (6) \]

Though the launch vehicle is a complex flexible body structure, for preliminary analysis it is assumed that vehicle is completely rigid [1]. The sloshing and engine inertia effects are also removed and reduced mass \( m_0 \) of the vehicle is considered. The rocket deflection angle \( \delta \) is given by an electrohydraulic action having first order lag as transfer function. The actuator is first order lag transfer function with bandwidth \( K_A \), given as

\[ G_c(s) = \frac{K_A}{s + K_A} \quad (7) \]
Table 1. Rigid Body Parameters For a Typical Launch Vehicle

| Sr. No. | Parameter                                      | Symbol | Units | Time period (72-152) sec |
|---------|------------------------------------------------|--------|-------|--------------------------|
| 1       | Reduced mass of the model                      | $m_0$  | slugs | 5058                     |
| 2       | control thrust                                 | $T_c$  | lb    | 342000                   |
| 3       | the normal aerodynamic force /angle of attack  | $\mu_\alpha \ell_\alpha$ | lb/rad | 198000                   |
| 4       | the aerodynamic moment /angle of attack        | $\mu_c \ell_c$ | sec$^{-2}$ | 2.8                  |
| 5       | control moment coefficient /angle              | $\mu_c$ | sec$^{-2}$ | 4.56                  |

3. CONTROL SYSTEM DESIGN

3.1 SIMPLIFIED AUTOPILOT DESIGN

The vehicle dynamics represented by equation (6) has one poles in the right hand side and hence the system is unstable. Classical controller design method based on pole placement is used to design the autopilot for the launch vehicle. The block diagram of the simplified rigid body along with actuator and the feedback is shown in the figure 2.

![Autopilot for Simplified Rigid Body](image)

The attitude angle and rate of angle is sensed and feedback so as to make the system closed loop with PD controller. The closed loop transfer function of the system is given as

$$1 + \frac{K_A}{s^2 + \left(\frac{\mu_\alpha \ell_\alpha}{s} \right) + K_A} = 0$$

The values of $K_A$ and $\ell_\alpha$ are designed such that the dominant poles of the closed loop system has damping ratio $\xi = 0.7$ and natural frequency $\omega_n = 3.8 \text{ rad/sec}$. The step response of the systems is shown in figure 3.

![Step Response](image)

The step response shows that the systems gives a steady state error of 30% and settling time is also high 2 sec. The response of the autopilot needs to show good tracking performance and faster response to commands. In order to improve the tracking performance with reduced steady state error and faster response, PID controller is designed.
3.2 PID CONTROLLER DESIGN

The PID controller produces control signal $u(t)$ consisting of three terms—one proportional to input signal, $e(t)$, another one proportional to integral of input signal, $e(t)$, and the third one proportional to derivative of input signal, $e(t)$, in PID controller. The transfer function of PID controller,

$$u(t) = K_p e(t) + K_i \int e(t) dt + K_d \frac{d}{dt} e(t) \quad (9)$$

$$G_c(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d s \quad (10)$$

The PID controller with the system is shown in Figure 4

![Figure 3. Block diagram of rigid body with P](image)

In this design a PI controller is introduced in cascade with rigid body and actuator so that the pair of dominant closed loop poles satisfies specified time domain specifications $\xi$ and $\omega_n$. Here, the desired system is a second order system with a damping ratio, $\xi = 0.7$ and natural frequency, $\omega_n = 3$ rad/sec, and steady state error, $e_{ss} \leq 1\%$. Designed values for $K_i = 23.7354$, $K_p = 14.1143$, and $K_d = 0.35$.

![Figure 4: Step Response of the autopilot for the simplified rigid body](image)

![Figure 5: Step Response of rigid body with PID Control](image)

3.3 FRACTIONAL PID DESIGN

Fractional order PID (FOPID) controller is a generalized form of classical PID controller [4]. In this type of controller instead of taking integral or differential of error signal, non-integer order integral or differential of the error is input is calculated for finding out the control input to the plant[5]. As demonstrated in many literatures [6],[7], FOPID outperforms conventional PID due to the presence of additional design parameters. While designing FOPID controller, the order of
integration and differentiation can also be chosen as control parameters, in addition to the proportional, integral and derivative constants in the conventional PID.

The time domain equation of FOPID controller input is given as equation (11).

\[
u_{FOPID}(t) = K_p e(t) + \frac{d^{-\lambda}}{dt^{-\lambda}} K_i \int e(t) dt + \frac{d^{\mu}}{dt^{\mu}} K_d \frac{de(t)}{dt}
\]  

(11)

\[
G_{FOPID}(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu}
\]  

(12)

where \(K_p, K_i, K_d, \lambda, \mu\) are the proportional gain, integral gain, and derivative gain, order of integration and order of differentiation respectively. The equation (13) represents the closed loop characteristic equation of the system. It is a nonlinear equation hence the initial solution of controller parameters are obtained by fsolve() function in MATLAB. Further the accuracy of the controller is obtained by optimization of the system using genetic algorithm to minimize ITAE (Integral Time Absolute Error) as objective function[10].

\[
1 + \left(\frac{K_A}{s + K_A}\right)\left(\frac{\mu_c}{s^2 - \mu_A}\right)\left(\frac{K_p + K_i}{s^{\lambda}} + K_d s^{\mu}\right) = 0
\]  

(13)

PERFORMANCE EVALUATION AND ROBUSTNESS ANALYSIS

In this section the performance evaluation of three controller with rigid body dynamics is discussed. The main requirement of the attitude control system is to track the guidance command with less steady state error. While maintaining accurate tracking the actuator angle constraint also to be satisfactory. The actuator angle constraint is 50°[1].

4.1 TRACKING PERFORMANCE

The unit step response of Rigid Body with nominal parameter for three types of controllers is shown in the figure 7a. The system with PD control has large steady state error. Though the introduction of PID controller reduced the steady state error, it has overshoot in the response. The response with FOPID controller settles fast with zero steady state error. The figure 7b shows the actuator angle response with three controllers. The angle for FOPID is higher while comparing other two controller. But the actuator angle is within the permissible limit.
4.2 ROBUSTNESS TO PARAMETER VARIATIONS

Robustness to parameters variations is also important in the attitude control system of launch vehicle since the vehicle parameters are not known exactly. Hence the response of the system with off nominal values also analyzed without changing the controller parameters. The aerodynamic moment ($H_a$) is increased 35% and the control moment coefficient ($H_b$) is decreased by 25%. The simulation results shows the better response of system with FOPID control, without any drastic change in the response from the nominal response.
4.3 DISTURBANCE REJECTION

The disturbance rejection capability of the controller is analyzed by adding wind disturbance input at the input side of launch vehicle along with controller input. The response is analyzed with nominal and off nominal plant parameters. During atmospheric phase of attitude control wind velocity effects the longitudinal velocity. In all cases the tracking performance of the FOPID controller is better than the other two controllers. Though the magnitude of actuator angle is high for FOPID controller, it lies within the permissible limit.

CONCLUSION

The attitude controller for the launch vehicle, during the highly unstable atmospheric phase is designed. The unstable system is stabilized using PD controller in the feedback path. The tracking controller for the stabilized system is designed using PID controller and FOPID controller. In both cases the controllers are designed to meet the time domain specifications and the controller parameters are optimized using genetic algorithm for minimizing the error indices. The tracking performance and disturbance rejection capability of these controllers where analyzed for nominal and off nominal cases. The simulation results demonstrates that the FOPID controller has better robust behaviour than the conventional PID controller. The performance of the FOPID controller can be further modified by using optimizing weighed fitness function of both error signal and control input to launch vehicle.
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