Search for Majorana Neutrino Signal in $B_c$ Meson Rare Decay

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Abstract

We study the $B_C$ meson rare decay in order to search for the Majorana neutrino signal. It is found that the the corresponding decay rate is sensitive to the Majorana neutrino mass and mixing angles. The signal of $B_C^{\pm} \rightarrow l_1^{\pm} l_2^{\pm} M^\mp$ induced by the Majorana neutrino within the mass region $m_\pi < m_n < m_B$ may be observed at LHCb.

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I. INTRODUCTION

In the Standard Model (SM), the neutrinos are massless since there is no right-hand states. However, non-degenerated neutrino masses provide the most accepted explanation for neutrino experiments [1–8]. This is the favorite evidence for new physics beyond SM. The neutrino masses can be obtained by including the right-hand states, just like the treatment for all other fermions via Yukawa couplings with the Higgs doublet in SM. But unnaturally the hierarchy problem will become more serious due to the small mass of neutrinos. Otherwise as the right-hand neutrinos are SM gauge singlets, the Majorana mass term cannot be ruled out by the gauge invariance. In fact under the help of the Majorana mass term, it could naturally explain the smallness of the neutrino mass with the so-called see-saw mechanism [9]. The particular interest in this regard is the question about whether neutrinos are Dirac or Majorana particles. A crucial role to address this question is being played by several experiments looking for the possible existence of the lepton number violation processes.

There have been several attempts to determine the Majorana nature of neutrinos by studying the lepton number violation processes. The experimental observation of such processes may be induced by the Majorana nature of neutrinos. The neutrinoless double beta decays \(0\nu\beta\beta\) in nuclei, \((A, Z) \rightarrow (A, Z + 2) + 2e^-\), have been studied widely. By assuming that \(0\nu\beta\beta\) in nuclei are mediated by the exchange of light Majorana neutrinos, the higher precision of present experimental data has been able to set strong constrains on the effective mass, \(\langle m_{\alpha\beta} \rangle = \sum_\nu U_{\alpha\nu}U_{\beta\nu}m_\nu\), where \(\alpha, \beta = e, \mu, \tau\). The upper bound on \(\langle m_{ee} \rangle\) is 0.2 eV [10]. A global fit [11] at 99% C.L. gives \(1.1 \times 10^{-3} eV \leq \langle m_{ee} \rangle \leq 4.5 \times 10^{-3} eV\) (normal hierarchy) or \(1.2 \times 10^{-3} eV \leq \langle m_{ee} \rangle \leq 5.7 \times 10^{-3} eV\) (inverted hierarchy). From the analysis of atmospheric and solar neutrino oscillation and the tritium beta decay endpoint experiment [12, 13], the limit is \(\langle m_{\mu\mu} \rangle \leq 4.4 eV\) [14]. However, it has long been recognized that, even though the experiments are very sensitive, the extraction of the properties of the Majorana neutrinos from nuclear \(0\nu\beta\beta\) is a difficult task, because it is reliable only if the nuclear matrix elements for \(0\nu\beta\beta\) are calculated precisely.

Another way to detect the Majorana nature is to study the lepton number violation processes \(pp \rightarrow l^+l^- + X\) at LHC [15–18]. The lepton number violation processes in meson rare decays have also been investigated in refs. [19–27]. The aim of this work is to investigate the \(B_C \rightarrow llM\). This signal process can be captured at high intensity experiments such as...
and future super B factories. The $\Delta L = 2$ processes $B_C^\pm \rightarrow l^\pm l^\pm M^\mp$ can occur via Majorana neutrino exchange, and thus their experimental observation is helpful to test the Majorana nature of the neutrinos.

The paper is organized as follows. The Lagrangian of Majorana neutrinos is introduced in Section II. The formulas of the $B_C \rightarrow llM$ decays are obtained in Section III. In Section IV we give the numerical results and discussions. Finally, a short summary is given.

II. LAGRANGIAN RELATED TO MAJORANA NEUTRINO

With the same gauge group $SU(2)_L \otimes U(1)_Y$ in SM, the leptonic content in the simplest extension of the SM includes three generations of left-hand $SU(2)_L$ doublets and $n$ right-hand singlets

$$L_L = \begin{pmatrix} \nu \\ l \\ N_R \end{pmatrix}_L, \quad l_R, \quad N_R.$$  

One can write the general gauge invariant Yukawa terms with Majorana mass terms of right-hand neutrinos as

$$- \mathcal{L}_Y = f_l \bar{L}_l \phi l_R + f_\nu \bar{L}_L \tilde{\Phi} N_R + \bar{N}_R M_R N_R + h.c. \quad (2)$$

Therefore, the complete neutrino mass sector is composed of both Dirac mass which is produced via the Yukawa couplings with the Higgs doublet in the SM, and heavy Majorana mass term.

$$- \mathcal{L}_M = \bar{\nu}_L M_D N_R + \bar{N}_R^c M_R N_R + h.c.$$

$$= \begin{pmatrix} \nu_L, \bar{N}_R^c \end{pmatrix} \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu \nu_L^c \\ N_R \end{pmatrix} + h.c. \quad (3)$$

One can find the light Majorana neutrino mass are

$$M_\nu \sim -M_D M^{-1}_R M_D^T \quad (4)$$

which is called Type I see-saw mechanism. There are other proposals to naturally generate Majorana mass for neutrinos called Type II or Type III. Generally the mass terms of neutrinos with both Dirac and Majorana terms after gauge symmetry breaking can be expressed as

$$- \mathcal{L} = \bar{\nu}_L M_D N_R + \frac{1}{2} \bar{N}_R^c M_R N_R + \frac{1}{2} \bar{\nu}_L M_L \nu_L^c + h.c. \quad (5)$$
To diagnose the mass matrix

\[ M = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix}, \quad (6) \]

one need introduce mixing matrix between the gauge and mass eigenstates

\[ T = \begin{pmatrix} U_{3\times 3} & V_{3\times n} \\ X_{n\times 3} & Y_{n\times n} \end{pmatrix}. \quad (7) \]

The mixing matrix is unitary, \( TT^\dagger = T^\dagger T = 1 \). The states are redefined as follows,

\[ \nu_i L \rightarrow U_{ij} \nu_j L + V_{ik} N_{kR}, \quad (8) \]

\[ N_R \rightarrow X_{ij} \nu_j C + Y_{ik} N_{kR}. \quad (9) \]

As the \( M_N \gg M_\nu \), one has \( V \sim X \sim M_D M_N^{-1} \) and \( U^\dagger M_R + Y^\dagger M_D \simeq 0 \).

In terms of the mass eigenstates, the gauge interaction Lagrangian of the charged currents now has the following form,

\[ \mathcal{L} = -\frac{g}{\sqrt{2}} W_\mu^+ \left( \sum_{\ell=e,m=1}^3 U_{\ell m}^* \overline{\nu}_m \gamma^\mu P_L \ell \right. + \sum_{\ell=e,m'=4}^{3+n} V_{\ell m'}^* \overline{N}_{m'} \gamma^\mu P_L \ell \left. \right) + h.c. \quad (10) \]

where \( P_L = \frac{1}{2} (1 - \gamma_5) \), \( \nu_m (m = 1, 2, 3) \) and \( N_{m'} (m' = 4, \cdots, 3+n) \) are the mass eigenstates, \( U_{\ell m} \) is the mixing matrix between the light flavor and light neutrinos, and \( V_{\ell m'} \) is the mixing matrix between the light flavor and heavy neutrinos.

### III. DECAY WIDTH FOR \( B_C^\pm \rightarrow l_1^- l_2^+ M^- \)

![Feynman diagrams](image)

FIG. 1: The Feynman diagrams for \( B_C \rightarrow l_1^+ l_2^- M^- \) via Majorana neutrino mediated.

The Feynman diagrams which contribute to \( B_C^\pm \rightarrow l_1^\pm l_2^- M^\mp \) are shown in Fig. [I]. The first diagram is the annihilation which has been studied widely with the input of meson decay
constants. The second one is the emission, which is considered not enough due to the non-perturbative long distance contributions in the hadronic matrix elements. The amplitude of $B_c$ decay to pseudo-scalar contributed by annihilation diagram is expressed as

$$M^a_P = -i2G_FV_BV_Mf_{Bc}f_M \left(v_1^C \cdot p \cdot \gamma p_3 \cdot \gamma P_L v_2 \right) \sum_\nu S_\nu(p_1) + (1 \leftrightarrow 2), \quad (11)$$

where $V_B \equiv V_{cb}$, and $V_M$ is the corresponding CKM element for final meson. $p$ is the momentum of initial $B_C$ meson. $M_B$ is the mass of the meson $B_C$. $p_3$ is the final meson momentum and $p_1/p_2$ is the momentum of lepton. The $f_B$ and $f_M$ are the decay constants. For a charged or neutral pseudo-scalar meson, the decay constant is defined as

$$\langle 0 | A_\mu | P^\pm \rangle = if_P p_\mu, \quad (12)$$

$$\sqrt{2} \langle 0 | A_\mu | P^0 \rangle = if_P p_\mu, \quad (13)$$

where the operator $A_\mu$ stands the axial-vector current operator.

The full expression of propagator function $S_\nu$ is

$$S_\nu(p_1) = \frac{V_{l_1\nu}V_{l_2\nu}m_\nu}{(p_2 + p_3)^2 - m_\nu^2 + im_\nu \Gamma_\nu}. \quad (14)$$

The $V_{l_\nu}$ is the lepton mixing matrix element. With the same denotation, the amplitude for $B_C$ decay to vector (or scalar) is

$$M^a_V = i2G_F^2M_VV_BV_Vf_{Bc}f_V \left(v_1^C \cdot p \cdot \gamma p_3 \cdot \gamma P_L v_2 \right) \sum_\nu S_\nu(p_1) + (1 \leftrightarrow 2), \quad (15)$$

$$M^a_S = -i2G_F^2V_BV_Mf_{Bc}f_M \frac{m_1 - m_2}{M_M} \left(v_1^C \cdot p \cdot \gamma p_3 \cdot \gamma P_L v_2 \right) \sum_\nu S_\nu(p_1) + (1 \leftrightarrow 2). \quad (16)$$

In addition to the light neutrinos, the see-saw mechanism predicts very heavy neutrino also. The propagator functions for the two type neutrinos can be approximately expressed as

$$S(p_1) \simeq \begin{cases} \frac{V_{l_1\nu}V_{l_2\nu}m_\nu}{(p_2 + p_3)^2} & m_\nu \ll M_\pi, \\ \frac{V_{l_1\nu}V_{l_2\nu}}{m_N} & m_N \gg M_B, \end{cases} \quad (17)$$

where we still use $\nu$ to denote for the light neutrino, and $N$ for the one heavier than $m_B$.

In the meantime, there is another possibility that the neutrino mass is between the $M_B$ and the final meson mass which is denoted with $n$ ($m_\pi < m_n < m_B$). Neutrinos with such mass has been strongly constrained by direct search and cosmological observations\[29–32\] and must be sterile. The process of $B_c^\pm \rightarrow l_1^+l_2^- M^\mp$ is dominated by the annihilation diagram.
as the intermediated neutrino in s-channel can be on-shell. At this case the narrow width approximation
\[
\lim_{t \to 0} \frac{1}{(q^2 - m^2)^2 + m^2 \Gamma^2} = \frac{\pi}{m \Gamma} \delta(q^2 - m^2),
\]  
(18)
can be applied. As the \(l^\pm + X\) are the dominated decay channels, the total decay width of the neutrino can be expressed as [19],
\[
\Gamma_n = 2 \sum_l |V_{ln}|^2 \left(\frac{m_n}{m_\tau}\right)^5 \times \Gamma_\tau,
\]
(19)
where the \(m_\tau, \Gamma_\tau\) are the mass and total width of the tau-lepton. With the total decay width and narrow width approximation, one can get the decay width of \(B_C^\pm \to l_1^\pm l_2^\pm \pi^\mp\)
\[
\Gamma(l_1^\pm l_2^\pm \pi^\mp) = \frac{G_F^2 |V_{BC}|^2 f_B^2 f_\pi^2 |V_{B_\tau N}|^2 m_B m_\pi^2}{128 \pi^2 \sum_l |V_{ln}|^2 2 \Gamma_\tau \left(1 - \frac{m_\tau^2}{m_n^2}\right)^2 \left(1 - \frac{m_\pi^2}{m_B^2}\right)^2}.
\]
(20)
In general, the square of annihilation amplitudes can be written as
\[
|M_p|^2 = 8 \left(G_F^2 V_B V_M f_B f_M\right)^2 (p_1 \cdot p_2) F(p_1, p_2) + (p_1 \leftrightarrow p_2),
\]
(21)
\[
|M_S|^2 = 8 \left(G_F^2 V_B V_M f_B f_M\right)^2 \left(\frac{m_1 - m_2}{M_s}\right)^2 (p_1 \cdot p_2) F(p_1, p_2) + (p_1 \leftrightarrow p_2),
\]
(22)
\[
|M_V|^2 = 4 \left(G_F^2 V_B V_M f_B f_M\right)^2 M_V^2 \frac{\mathcal{V}(p_1, p_2)}{(p - p_1)^4} F(p_1, p_2) + (p_1 \leftrightarrow p_2),
\]
(23)
where the function \(F(p_1, p_2)\) and \(\mathcal{V}(p_1, p_2)\) are defined as,
\[
F(p_1, p_2) = \begin{cases} 
|\sum_\nu V_{1\nu} V_{2\nu} m_\nu|^2, & m_\nu \ll m_\pi, \\
|V_{1\nu} V_{2n}|^2 \frac{m_\nu}{m_\pi} \delta((p - p_1)^2 - m_n^2), & \text{on shell}, \\
|V_{1N} V_{2N}|^2 \frac{(p - p_1)^4}{m_N}, & m_N \gg m_B,
\end{cases}
\]
(24)
\[
\mathcal{V}(p_1, p_2) = 2p_1 \cdot p_2 \left[4p_1 \cdot p_3 - M_V^2 + \frac{4(p_1 \cdot p_3)^2}{M_V^2}\right] + 8p_1 \cdot p_3 p_2 \cdot p_3.
\]
(25)
In addition to the annihilation diagrams, we also consider the emission diagrams. We apply the light-cone function of mesons to calculate the hadronic amplitude. The non-perturbative effect from long-distance interaction is factorized into the light-cone function, and the leptonic number violation effect is caused by the short-distance interaction which can be calculated perturbatively. The light-cone distribution amplitude is defined as
\[
\mathcal{M}^\mu_{\beta\alpha}(k) = \int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot x} \langle 0 | \bar{q}_\alpha(x) q_\beta(0) | M \rangle,
\]
(26)
and parameterized with the twist wave functions as

\[
\mathcal{M}^B = -\frac{if_B}{4} \left\{ (\slashed{p} + m_B)\gamma^5 \phi_B(u) \right\}, \\
\mathcal{M}^\pi = -\frac{if_\pi}{4} \left\{ \gamma^5 \phi(u) - \mu_\pi \gamma^5 \left( \phi^\prime_p(u) - i\sigma_{\mu\nu}n^\nu_p \phi'_\sigma(u) \frac{\phi_\sigma(u)}{6} + i\sigma_{\mu\nu}p^\nu \phi_\sigma(u) \frac{\partial}{\partial k_T} \right) \right\}.
\]

(27)

(28)

For the wave function of \( B_c \), we take the following form in the numerical calculations\[34, 35]\,

\[
\phi_B(x) = N_B x^2 (1 - x)^2 \exp \left[ -\frac{1}{2} \left( \frac{x m_B}{\omega_B} \right)^2 \right],
\]

(29)

where \( \omega_B \) is the shape parameter, and \( N_B \) is the normalization constant. The general expression of twist-2 wave function for pion is

\[
\phi_\pi(x) = 6x(1 - x) \left( 1 + \sum_{n=1} a_n C_n^{3/2}(2x - 1) \right),
\]

(30)

where \( C_n(x) \) is the Gegenbauer polynomial. In this work, the higher twist contributions are not considered.

The amplitude of the contribution from emission Feynman diagrams can be written as

\[
\mathcal{M}^E = \int dx dy 2G_F^2 V_{ub} V_{cd} \left[ \frac{(m_{l1}l_2)}{q^2 + i\epsilon} + \sum_n \frac{V_{l1N} V_{l2N}}{m_N} \right] W^{\mu\nu} L_{\mu\nu} + (1 \leftrightarrow 2),
\]

(31)

where \( q = xp_B - yp_3 - p_2 \). \( x \) and \( y \) are the momentum fractions. The first term in Eq.(31) is the contribution of light neutrinos while the second term is that of heavy neutrinos. The leptonic and hadronic tensors are

\[
L_{\mu\nu} = \left( \bar{v}_{l1}^C \gamma_\mu \gamma_\nu P_L v_{l2} \right), \\
W^{\mu\nu} = \langle \pi | (\bar{b} \gamma^\mu (1 - \gamma^5) u) (\bar{d} \gamma^\nu (1 - \gamma^5) c) | B_c \rangle \\
= \frac{f_B f_\pi}{2N_C} (p^\mu p_\pi^\nu + p^\nu p_\pi^\mu - p \cdot p_\pi g^{\mu\nu}) \phi_B(x) \phi_\pi(y),
\]

(32)

(33)

where \( N_C \) is the color factor. One can get the emission amplitude as

\[
\mathcal{M}^E = \frac{2G_F^2 V_{ub} V_{cd} f_B f_\pi p \cdot p_\pi \phi_B \phi_\pi}{N_C} \left( \bar{v}_{l1}^C P_L v_{l2} \right) \\
\times \left[ \int dx dy \frac{(m_{l1}l_2)}{q^2 + i\epsilon} + \sum_n \frac{V_{l1N} V_{l2N}}{m_N} \right] + (1 \leftrightarrow 2).
\]

(34)

Comparing Eq.(34) with Eq.(11), one can notice that the contribution of heavy neutrino in emission diagrams is similar to that in annihilation diagrams but suppressed by the color factor \( N_C \), while in some channel also suppressed by the CKM matrix element. Thus the emission diagrams can be considered only for light neutrinos.
TABLE I: The input parameters for pseudo-scalars

| P     | \(\pi^-(\bar{u}d)\) | \(K^-(\bar{u}s)\) | \(D^- (\bar{c}d)\) | \(D_{s}^- (\bar{c}s)\) | \(B^- (\bar{u}b)\) | \(B_c^+ (\bar{c}b)\) |
|-------|----------------------|---------------------|-----------------|-----------------|----------------|----------------|
| \(f_P\) | 130.41               | 156.1               | 206             | 257.5           | 193            | 322            |
| \(M_P\) | 139.57               | 493.677             | 1869.60         | 1968.47         | 5279.17        | 6.277          |

TABLE II: The input parameters for vector mesons.

| \(V\) | \(\rho^-(\bar{u}d)\) | \(K^{*-} (\bar{u}s)\) | \(D^{*-} (\bar{c}d)\) | \(D_{s}^{*-} (\bar{c}s)\) |
|-------|----------------------|---------------------|-----------------|-----------------|
| \(f_V\) | 216                  | 220                  | 240              | 272              |
| \(m_V\) | 770                  | 891.66               | 2010.22          | 2112.3           |

IV. NUMERICAL RESULTS AND DISCUSSION

To compare the contribution from the annihilation diagram with that from emission one, we study them separately. The numerical values of the mesons’ decay constants and mass [36–39] are listed in Table. (I,II). For the light neutrinos \((m_\nu \ll m_\pi)\), we take the effective mass of the light neutrinos \(\langle m_{l_1 l_2} \rangle = |\sum_{\nu} V_{l_1 \nu} V_{l_2 \nu} m_\nu|\) as input. The contribution from the annihilation diagrams is found to be

\[
Br(B_c^z \to l_1^\pm l_2^\mp \pi^\mp) \simeq 1.4 \times 10^{-33} \left(\frac{\langle m_{l_1 l_2} \rangle}{1\text{eV}}\right)^2.
\]  

One can find the results for other channels in Table. (III) with \(\langle m_{l_1 l_2} \rangle = 1\text{eV}\).

| P     | \(\pi^-(\bar{u}d)\) | \(K^-(\bar{u}s)\) | \(D^- (\bar{c}d)\) | \(D_{s}^- (\bar{c}s)\) |
|-------|----------------------|---------------------|-----------------|-----------------|
| \(Br\) | \(1.4 \times 10^{-35}\) | \(9.6 \times 10^{-35}\) | \(6.5 \times 10^{-35}\) | \(1.8 \times 10^{-33}\) |
| \(V\) | \(\rho^-(\bar{u}d)\) | \(K^{*-} (\bar{u}s)\) | \(D^{*-} (\bar{c}d)\) | \(D_{s}^{*-} (\bar{c}s)\) |
| \(Br\) | \(6.0 \times 10^{-32}\) | \(2.3 \times 10^{-33}\) | \(3.0 \times 10^{-34}\) | \(6.4 \times 10^{-33}\) |

TABLE III: The branch ratio of \(B_c^z \to l_1^+ l_2^+ M^-\) with \(\langle m_{l_1 l_2} \rangle = 1\text{eV}\) from annihilation.

The results indicate that the branch ratio of \(B_c^z \to l_1^+ l_2^+ P^z\) is about \(10^{-33} \times |V_{P}|^2 \times \left(\frac{\langle m_{l_1 l_2} \rangle}{1\text{eV}}\right)^2\). Due to the suppression of the CKM elements \(V_{us}, V_{cd}\), the branch ratio to \(K(K^*)\) and \(D(D^*)\) are about one order smaller than \(\pi(\rho)\) and \(D_{s}(D_{s}^*)\). As the possible relevant phase between \(V_{l_1 \nu}\) and \(V_{l_2 \nu}\), the \(\langle m_{l_1 \nu l_2 \nu} \rangle\) may be very small which indicates invisible contributions.
In addition to the annihilation diagrams, the contribution from emission diagram depends on the non-perturbative parameter $\omega_B$. For $\omega_B = 1.0\, GeV$, the branch ratio of $B_c^{\pm} \to l^{\pm} l^{\pm} \pi^{\mp}$ contributed by emission diagram is

$$Br(B_c^{\pm} \to l_1^{\pm} l_2^{\pm} \pi^{\mp}) = 6.75 \times 10^{-22} \left(\frac{\langle m_{l_1 l_2}\rangle}{1\,eV}\right)^2.$$  \hspace{1cm} (36)

In Fig.(2) the result as a function of $\omega_B$ is shown. One can notice the emission diagram is dominated comparing to the annihilation diagrams for light Majorana neutrinos. However it is still below the experimental bounds.

![Graph](image.png)

**FIG. 2:** The branch ratio of $B_c^{\pm} \to l^{\pm} l^{\pm} \pi^{\mp}$ as a function of the shape parameter $\omega_B$.  

Now we consider the effect of heavy neutrino ($m_N \gg m_B$). The nature explaining to the smallness of neutrino masses in the see-saw mechanism need $m_N \gg \Lambda_{EW}$ where $\Lambda_{EW}$ is the Electro-Weak scale. The mixing element $V_{lN}$ is also very small. One can image that $V_{1N}V_{2N}/m_N \sim 1/\Lambda_{GUT}$. If $V_{1N}V_{2N}/m_N = 10^{-16}GeV^{-1}$ is taken, one can get the annihilation result

$$Br(B_c^{\pm} \to l_1^{\pm} l_2^{\pm} M^{\mp}) \simeq 3.3 \times 10^{-45} \left(\frac{|V_{1N}V_{2N}|/m_N}{10^{-16}GeV^{-1}}\right)^2.$$  \hspace{1cm} (37)

The results related to other channels with $V_{1N}V_{2N}/m_N = 10^{-16}GeV^{-1}$ are listed in Table.(IV). One can notice that the contribution of such massive neutrinos to $B_c^{\pm} \to l_1^{\pm} l_2^{\pm} M^{\mp}$ can be omitted comparing with contribution of light neutrinos. As pointed in last section, the contribution of massive neutrinos in emission diagrams is suppressed and more less.
\[ P \rightarrow \pi^-(\bar{u}d) \quad K^- (\bar{u}s) \quad D^- (\bar{c}d) \quad D_s^- (\bar{c}s) \]

\[ Br \approx 3.3 \times 10^{-45} \quad 2.1 \times 10^{-46} \quad 9.6 \times 10^{-47} \quad 2.6 \times 10^{-45} \]

\[ V \rightarrow \rho^- (\bar{u}d) \quad K^*^- (\bar{u}s) \quad D^{*-} (\bar{c}d) \quad D_s^{*-} (\bar{c}s) \]

\[ Br \approx 6.1 \times 10^{-45} \quad 3.4 \times 10^{-45} \quad 3.5 \times 10^{-46} \quad 6.6 \times 10^{-45} \]

TABLE IV: The branch ratio of \( B_C^\pm \rightarrow l_1^\pm l_2^\pm M^\mp \) with \( V_{1N}V_{2N}/m_N = 10^{-16} GeV^{-1} \). Only the annihilation diagrams are considered, since the emission diagrams are suppressed.

\[
\begin{array}{|c|c|c|c|}
\hline
&m_\nu << m_\pi & m_n \sim m_B^{*} & m_n >> m_B \\
\hline
Emission & \sim 10^{-21} & \sim 10^{-21} & < 10^{-46} \\
Annihilation & \sim 10^{-32} & < 10^{-4} & \sim 10^{-45} \\
\hline
\end{array}
\]

TABLE V: The comparing of the contribution from emission and annihilation diagrams.

At last we discuss the sterile neutrinos with mass \( m_\pi < m_n < m_B \). We take \( m_\pi = 1.77 GeV \), \( \Gamma_\pi = 2.3 \times 10^{-12} GeV \) and \( m_n = m_B/2 \). The intermediated neutrino in annihilation diagram can be on shell which will enhance such process. The emission diagram contribution has no enhancement which can be omitted. We list the approximational contribution from emission and annihilation diagrams in Table. \( \Box \) With such inputs, the branch ratio as functions of the leptonic mixing matrix elements are shown in Fig. (3). The numerical result of branch ratio for \( B_C^\pm \rightarrow l_1^\pm l_2^\pm \pi^\mp \) is

\[ Br(B_C^\pm \rightarrow l_1^\pm l_2^\pm \pi^\mp) \approx 4.39 \times 10^{-4} \times \frac{|V_{1n}V_{2n}|^2}{\sum_l |V_{ln}|^2}. \] (38)

The \( \sum_l |V_{ln}|^2 \) is a parameter relating with mixing between the leptons. The branch rations as a function of the parameter are shown in Fig. (3). One can notice that the rare decay \( B_C^\pm \rightarrow l^\pm l^\pm \pi^\mp \) may be detected on LHCb at this case. It is expected to study the property of the majorana neutrinos indirectly through such processes.

The production of \( B_c \) at LHCb have been studied widely. The gluon-gluon fusion subprocess \( gg \rightarrow B_C + X \) is the dominated production channel and much larger than the quark-antiquark annihilation subprocess \( q\bar{q} \rightarrow B_C + X \). The magnitude of the color-octet components may be estimated with the non-relativistic QCD and the contribution can be neglected comparing with color-singlet components.
As shown in Ref. [40], the corresponding cross-section is about $3nb$ at Tevatron ($\sqrt{s} = 1.96 TeV$) and $50nb$ at LHCb ($\sqrt{s} = 14 TeV$). As the desired Luminosity of LHCb is about $10 fb^{-1}$, the expect event numbers are also shown in Fig. (3). The LHC has run at 7 TeV with total Luminosity $1.11 fb^{-1}$ and the production cross-section of $B_c$ is about $22 nb$. The exclude region has been shown in Fig. (4) at 95% C.L..

![FIG. 3: The branch ratio of $B_C$ and the corresponding event number on LHCb as functions of $|V_{l1}V_{l2}|^2 / \sum_l |V_{ln}|^2$ with $m_n = m_B/2.$](image)

![FIG. 4: The solid lines stand the low bound of the parameters that is excluded by Tevatron and LHCb through $B_C^{\pm} \rightarrow l_1^{\pm} l_2^{\mp} \pi^{\mp}$ at 95% C.L.. And the dashed line is expected to be excluded by LHCb in future.](image)
V. SUMMARY

The neutrino experiments indicate very small neutrino mass. A most natural explanation to such small mass is see-saw mechanism. In this work, we study the $\Delta L = 2$ semi-leptonic decays of $B_C$ meson mediated by Majorana neutrinos and investigate the contributions from annihilation and emission diagrams for different Majorana neutrino mass. The light-cone functions of the mesons are applied to calculate the hadronic matrix element in emission diagrams. It is found that the corresponding decay widths are sensitive to the Majorana neutrino mass and the mixing angles. For a sterile neutrino with mass $m_s < m_n < m_B$, the leptonic number violating decay rates of $B_c$ can be enhanced by the annihilation diagrams and may be detectable at LHCb.

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