A task of fundamental importance in the development of large-scale quantum processors is that of determining a robust, multi-platform standard to quantify the quality of the quantum operations. Characterizing quantum gates becomes harder as the error of the gates continues to decrease. Quantum process tomography [1, 2], a standard method for characterizing quantum gates, gives a full description of the protocol under study, but is very sensitive to state preparation and measurement (SPAM) errors, especially when the errors involved are comparable to those of the single qubit unitaries of the system. As an alternative, recent work has been devoted to the development of randomized benchmarking (RB) protocols [3–5], which have been implemented in ion traps [4, 6, 7], NMR [8], superconducting qubits [9–11] and atoms in optical lattices [12]. Although a RB protocol offers a reliable estimate of the average error per operator within a group of operators in its original conception, it does not provide a complete description of a given quantum process. However, recent studies [13, 14] have presented a different RB implementation in which the error of a particular gate is measured by interleaving it between other random gates within a sequence. Furthermore, RB has also been used to determine addressability errors and correlations in many-body quantum systems [15].

In this Letter, we present a complete RB characterization of two fixed-frequency superconducting qubits. An important feature of a RB protocol is the long sequences required to implement it. This sets additional constraints on the chosen gates, which have to be robust as a part of a long series of pulses in the same way they would in a real quantum algorithm. In our case, the realization of the RB protocol is made possible by modifying a previously developed cross-resonance (CR) two-qubit gate [16, 17] through refocusing the single-qubit dynamics to simplify the gate calibration.

Our implementation of the RB protocol is restricted to the Clifford group $C$ of operators, which is the normalizer of the Pauli group $P$ - that is, for each $C \in C$, $CPC^\dagger \in P$, with $P \in P$. Having the Clifford group as our set of operators is justified by two main reasons. First, a Clifford-based RB protocol is more readily extendable to systems with higher number of qubits, as choosing a random Clifford and decomposing it into a set of generators (elementary qubit operations) scales polynomially in the number of qubits [5, 18]. Second, as the set of generators tends to vary across systems, such a protocol offers portability between the different physical implementations of quantum processors.

For two-qubit systems, the Clifford group is generated from single-qubit unitaries and a controlled-NOT (CNOT) gate. When implementing the protocol, it is important to use a Clifford decomposition into elementary unitaries that minimizes the number of average two-qubit gates per Clifford, as these tend to have lower fidelities than single-qubit gates, especially in the case of superconducting qubits. Rather than following the procedure outlined in Ref. [5] for generating a random Clifford operation, here we use an optimized set of Clifford operations from which we randomly select elements. We divided the two-qubit Clifford group into four classes [27]: a class with 576 elements containing only single-qubit unitaries from the group $\{ I, X_{\pm \pi/2}, Y_{\pm \pi/2}, X_\pi, Y_\pi \}$, where $U_\theta$ represents a rotation of angle $\theta$ around the axis $U$, a class with 5184 elements containing single-qubit unitaries and one CNOT gate, a class with 5184 elements containing single-qubit unitaries and two CNOT gates and a class with 576 elements containing single-qubit unitaries and a SWAP gate, implemented by three CNOT gates. Therefore, the total number of elements in the two-qubit Clifford group is 11520, with an average of 1.5 CNOT gates per Clifford [28].

The experiments are performed on two single-junction transmons coupled via a superconducting coplanar waveguide resonator, which is also used for readout [19]. The qubit resonance frequencies are $\omega_1/2\pi = 3.2324$ and
Hamiltonian of the form
\[ H_D = \frac{\omega_2}{2\pi} \left( (\omega_1 \pm \delta_1) - (\omega_2 \pm \delta_2) \right) = \frac{\omega_2}{2\pi} (\omega_1 - \omega_2) \pm \frac{\omega_2}{2\pi} (\delta_1 - \delta_2) \]

where \( \omega_2/2\pi = 3.2945 \text{ GHz} \), with anharmonicities \( \delta_1 = (\omega_1^2 - \omega_1^0)/2\pi = -331 \) and \( \delta_2 = (\omega_2^2 - \omega_2^0)/2\pi = -216 \text{ MHz} \), whereas the bare resonator frequency is \( \omega_2/2\pi = 8.2855 \text{ GHz} \). The energy relaxation times of both qubits are observed to be \( T_1^{(1)} = 11.6 \mu s \) and \( T_1^{(2)} = 9.1 \mu s \), and the coherence times observed from Ramsey-fringe experiments are \( T_2^{(1)} = 7.1 \mu s \) and \( T_2^{(2)} = 5.6 \mu s \) [27]. The qubits are thermally anchored to the coldest stage of a dilution refrigerator with a nominal base temperature of 15 mK and are carefully shielded against thermal radiation [20, 21].

Implementing the Clifford group of operators in our experiments required modifying a previously demonstrated two-qubit CR gate [16]. The basis of the original CR gate [22, 23] involves the driving of the control qubit at the frequency of the target qubit. This results in a driving Hamiltonian of the form

\[ H_D = \frac{\omega_2}{2\pi} I \left( (\omega_1 \pm \delta_1) - (\omega_2 \pm \delta_2) \right) = \frac{\omega_2}{2\pi} (\omega_1 - \omega_2) \pm \frac{\omega_2}{2\pi} (\delta_1 - \delta_2) \]

where \( \{I, X, Y, Z\} \otimes 2 \) are the two-qubit Pauli operators, \( \epsilon \) is the drive amplitude, \( \mu \) is a coupling parameter that equals \( J/\Delta \) for ideal qubits, where \( J \) is the qubit-qubit coupling energy and \( \Delta \) is the frequency detuning between the qubits, \( m \) is a scalar representing the effect of spurious electromagnetic crosstalk between both qubits as well as the effect of higher energy levels, and \( \eta \) represents the magnitude of the Stark shift arising from the off-resonant driving of qubit 1. The term \( mIX \) results in Rabi-like oscillations of qubit 2, to which the term \( -\mu ZX \) contributes with a slower rotation whose sign depends on the state of qubit 1. The effect of the Hamiltonian \( H_D \) can be seen in the experiments shown in Fig. 1 (a). A CR flattop Gaussian pulse of variable length \( \tau_1 \) and Gaussian width \( \sigma = 8 \text{ ns} \) is applied to qubit 1 (control) at the frequency of qubit 2 (target). Depending on whether a \( \pi \) rotation is applied to qubit 1 prior to the CR pulse (circles) or not (triangles), different Rabi rates are observed on qubit 2. In the experiments where qubit 1 is in the excited state prior to the CR pulse, an additional \( \pi \) rotation is applied to qubit 1 at the end of the sequence in order to have the Rabi-oscillation signal from the joint readout between the same two computational states \{00\} and \{01\}. All single-qubit rotations are Gaussian-shaped pulses with a total length of 32 ns and \( \sigma = 8 \text{ ns} \). To avoid leakage into higher energy levels, all single-qubit pulses include a calibrated derivative in the other quadrature [9, 24].

The Hamiltonian in Equation 1, however, presents difficulties for implementing long sequences of Cliffofs due to the single-qubit terms. These terms could be explicitly tuned out with additional simultaneous pulses, but this would be rather demanding on the phase-locking and amplitude stability requirements of the electronics and on sequence complexity which would have an important negative impact on the measured gate fidelity. Instead, we construct a more manageable two-qubit Clifford by modifying the original pulse sequence in order to remove all terms except the one corresponding to \( ZX \). The new pulse sequence divides the CR pulse in three parts: an initial CR pulse of duration \( \tau_2 \), a \( \pi \) rotation of qubit 1, and a final CR pulse of opposite sign to the first one, also of duration \( \tau_2 \). As in the original pulse scheme, a final \( \pi \) rotation is applied to qubit 1 when pertinent. The net effect of the new pulse sequence is to effectively “echo” away the terms involving \( IX \) and \( ZI \) in \( H_D \) and, as a result, only the slower Rabi-rotation arising from \( ZX \) is observed [Fig. 1 (b)]. In the experiments presented here, the gate is realized by choosing a value of \( \tau_2 \) that leaves qubit 2 in a superposition of \{0\} and \{1\}. We will therefore call this gate \( ZX_{-\pi/2} \), the minus sign arising from the negative term involving \( ZX \) in Equation 1. The dashed line in Fig. 1 (b) at \( \tau_2 = 178 \text{ ns} \) shows the duration of each of the CR pulses for the \( ZX_{-\pi/2} \) gate for these data. The total \( ZX_{-\pi/2} \) gate length is, therefore, \( 2 \times \tau_2 + 2 \times 32 = 420 \text{ ns} \). The equivalent gate length for the original CR scheme is likewise shown in Fig. 1 (a) at \( \tau_1 = 356 \text{ ns} \).

We performed quantum process tomography on the \( 178 \text{ ns} ZX_{-\pi/2} \) gate (Fig. 2) by preparing an overcomplete set of 36 states generated by \( \{I, X_\pi, X_{\pm \pi/2}, Y_{\pm \pi/2}\} \),
applying the gate to each of them and performing state tomography. We use the Pauli basis to represent quantum process tomography through the Pauli transfer matrix $R$ [17]. The raw data is post-processed with a semidefinite algorithm [17] to take into account physicality constraints such as complete positivity and trace preservation of the process. We obtain a gate fidelity from the raw data of $F_g = 0.8830$ raw and $F_{\text{mle}} = 0.8799$ after applying a maximum likelihood algorithm. (b) Ideal Pauli transfer matrix representation of the $ZX_{-\pi/2}$ gate.

![Figure 2](image)

**FIG. 2.** (color online) Quantum process tomography of the $ZX_{-\pi/2}$ gate with $t_2 = 178 \text{ ns}$. (a) Experimentally extracted Pauli transfer matrix. The gate fidelity is $F_g = 0.8830$ raw and $F_{\text{mle}} = 0.8799$ after applying a maximum likelihood algorithm. (b) Ideal Pauli transfer matrix representation of the $ZX_{-\pi/2}$ gate.

We base our implementation of a two-qubit RB protocol on the theory described in Refs. [5] and [14]. We randomly choose a sequence $\{C_1, C_2, \ldots, C_{20}\}$ of 20 Cliffords amongst the 11520 elements of the two-qubit Clifford group. From this sequence, we then construct a series of truncations $\{m_1, m_2, \ldots, m_{20}\}$, where $m_i = \{C_1, C_2, \ldots, C_i\}$. In order to make each truncation self-inverting, a final pulse $I_i$ is appended at the end of each of them which returns the system to the $|00\rangle$ state. The state of the two qubits is then read out. A whole RB experiment, therefore, consists of a series of pulse trains $\{N_1, N_2, \ldots, N_{20}\}$, where $N_i = \{m_i, I_i\}$.

The fidelity of the $|00\rangle$ state after each train of pulses $N_i$, obtained by joint readout of the two qubits [25], can be fit to an exponential model $F(i, |00\rangle) = A\alpha^i + B$, and the average error rate per Clifford is related to $\alpha$ by the relation $r = 1 - \alpha - (1 - \alpha)/d$, where $d = 2^n$ for $n$ qubits [5]. In this model, state preparation and measurement errors are absorbed by the constants $A$ and $B$ and therefore the parameter $\alpha$ provides a SPAM-free estimation of the average error per Clifford. One of the shortcomings of this model as described, however, is that it only gives an estimation of the average error per Clifford generator.

We can, nonetheless, modify the protocol in order to estimate the error of a particular gate of interest, $\overline{C}$, also belonging to the Clifford group, as described in Refs. [13] and [14]. In the new protocol, we interleave random elements of the Clifford group between the gate under study $\overline{C}$. This is achieved by constructing similar random sequences of Cliffords as in the original implementation and then appending the gate $\overline{C}$ after each element in the sequence. A final inverting Clifford is added at the end of each truncation of the sequences.

![Figure 3](image)

**FIG. 3.** (color online) RB experiments on a two-qubit system. (a) Two-qubit pulse sequence for 5 Cliffords randomly selected from the 11520 elements of the two-qubit Clifford group. Tall Gaussians represent $\pi$ rotations, whereas short ones represent $\pi/2$ rotations. A final Clifford is added to make the sequence self-inverting. (b) Fidelity decay for the $|00\rangle$ in the standard RB protocol where the twirling is performed over the two-qubit Clifford group (circles) and in the interleaved protocol (triangles), in which each Clifford has an additional $ZX_{-\pi/2}$ gate appended at the end. The decays are fitted to an exponential model to extract an average error per Clifford of $r = 0.0936 \pm 0.0028$ (standard protocol) and a $ZX_{-\pi/2}$ error of $r_C = 0.0653 \pm 0.0014$. The arrow shows the truncation that would correspond to the pulse sequence shown in (a) and which has an average $|00\rangle$ state fidelity of over 50%.
The pulse sequence of one particular self-inverting 5-
Clifford long sequence, comprising 7 two-qubit and 23
single-qubit gates, is shown in Fig. 3 (a). In the case
of the standard RB implementation experiment, we obtain
\( \alpha = 0.8752 \pm 0.0078 \), which results in an average error per
Clifford of \( r = 0.0936 \pm 0.0058 \). For the interleaved exper-
iment, \( \alpha_C = 0.7990 \pm 0.0058 \), from which \( r_C \) can be esti-
mated as \( r_C = (d-1) \frac{(1-\alpha_C/\alpha)}{d} = 0.0653 \pm 0.0014 \) [13].
All errors here represent a 1σ confidence interval obtained
from the Jacobian of the fitting model [15]. The aver-
age error per Clifford for the single-qubit Clifford group
for each qubit was also measured and observed to be
\( r_1 = 0.0041 \pm 0.0001 \) for qubit 1 and \( r_2 = 0.0048 \pm 0.0002 \)
for qubit 2 [27].

The effective coupling strength of the two qubits, given
by the product \( \epsilon(t) \mu \) multiplying the \( ZX \) interaction in
Eq. 1, can be increased with the amplitude of the CR
driving tone \( \epsilon(t) \). Thus, larger driving amplitudes result
in a faster evolution of the system and, therefore, faster
oscillations in Fig. 1 (b) and shorter \( \tau_2 \). The ultimate
limit for the speed of the \( ZX_{-\pi/2} \) gate is determined
by the frequency of the oscillations induced by the \( ZX \)
term, the qubit-qubit detuning \( \Delta \), and the qubit anhar-
monicities. For the strongest drives, energy leakage into
other levels in the system prevent the gate from becoming
faster [16, 26].

We applied the two-qubit RB protocol to our system
for different \( ZX_{-\pi/2} \) lengths, with \( \tau_2 \) ranging from 115
to 800 ns. Fig. 4 shows the average error per Clifford
as a function of \( \tau_2 \) and the calculated coherence-limited
average error for the measured \( T_2 \) of both qubits (dashed
line) and for \( T_2^{1,2} = 2T_1^{1,2} \) (dash-dotted line). We see
that the experimentally extracted average errors fall inside
the shaded region. For the two shortest realizations of the
\( ZX_{-\pi/2} \), however, the RB experiments yielded an
average error per Clifford worse than the limit imposed
by coherence times. This was probably due to leakage
into higher qubit levels at high CR driving amplitudes
and to the spurious off-resonance driving of the control
qubit. For longer gates, the observed experimental values
indicate that our \( ZX_{-\pi/2} \) is essentially coherence limited.
We attribute the scattering of the data points in Fig. 4
to small variations in \( T_1 \) and \( T_2 \), which were observed to
move by about 1 μs up or down for both qubits over the
interval of six to twelve hours, approximately equal to
the amount of time taken to perform the RB protocol for
each \( ZX_{-\pi/2} \) gate length.

In conclusion, we have implemented a RB protocol on
two superconducting qubits by using a new implementa-
tion of an entangling two-qubit gate plus single-qubit
unitaries. The new gate pulse sequence echoes away the
terms in the gate Hamiltonian involving single-qubit ef-
ects, which tend to be non-reproducible in long gate se-
quences. The new gate implementation allows the exper-
imental realization of the two-qubit RB protocol for the
first time in superconducting qubits. Furthermore, an in-
terleaved RB experiment in which the two-qubit gate is
alternated with a random Clifford during the sequences
yielded a gate fidelity of 0.9347, which compares favor-
ably to the fidelity of 0.8799 obtained by quantum process
tomography performed on the same gate. Our results
show the importance of having a protocol insensitive to
to errors arising from SPAM, especially as the processes to
benchmark become higher and higher in fidelity. Mea-
measurements of our protocol as a function of the gate du-
ration suggest that our gate is currently limited by the
coherence time of our qubits.

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Our procedure is equivalent to that of Ref. [14] and finds the same average number of two-qubit gates per Clifford.
We show here an implementation for our two-qubit system of the RB protocol for single qubit gates described in Ref. [1]. The measurements (see Fig. 1) consist of standard RB experiments on each qubit separately ($C \otimes I$ for qubit 1, $I \otimes C$ for qubit 2) and on both qubits simultaneously ($C \otimes C$). The former experiments gives the average error per single-qubit Clifford for each of the qubits, whereas the latter contains information about the amount of spurious crosstalk present in the system. The data in Fig. 1 were averaged over 20 random sequences for each sequence length.

The decay of each single subsystem is fitted to the same exponential model as the two-qubit RB experiments, $F(i) = A \alpha^i + B$, where $i$ is the number of Cliffords. The results are summarized on Table I, where $\alpha_i$ is extracted from the RB experiments individually performed on qubit $i$, and $\delta \alpha = \alpha_{12} - \alpha_1 \alpha_{21}$, with $\alpha_{12}$, $\alpha_{1|2}$ and $\alpha_{2|1}$ obtained from fitting $p_{00} + p_{11}$, $p_{00} + p_{01}$ and $p_{00} + p_{10}$, respectively, in the simultaneous RB experiments.

| Twirl Group | Extracted metrics | Reduced $\chi^2$ |
|-------------|-------------------|-----------------|
| $C \otimes I$ | $0.9918 \pm 0.0002$ | 1.371 |
| $I \otimes C$ | $0.9904 \pm 0.0003$ | 0.377 |
| $C \otimes C$ | $0.9865 \pm 0.0003$ | 0.339 |
| $C \otimes C$ | $0.9876 \pm 0.0004$ | 0.243 |
| $C \otimes C$ | $0.9745 \pm 0.0011$ | 0.705 |
| $-\alpha$ | $0.0002 \pm 0.0018$ | - |

TABLE I. Summary of the extracted data from the single-qubit RB experiments on both qubits. Uncertainties represent 1σ confidence intervals.

The average errors per Clifford $r_i = (1 - \alpha_i)/2$ obtained from the above results are $r_1 = 0.0041 \pm 0.0001$ and $r_{1|2} = 0.0067 \pm 0.0002$ for qubit 1 and $r_2 = 0.0048 \pm 0.0002$ and $r_{2|1} = 0.0062 \pm 0.0002$ for qubit 2. The parameter $\delta \alpha$ represents the amount of correlations in the system, a measure of entangling errors. From our experiments we can see that, although there is some amount of crosstalk ($\delta r_{1|2} = r_{1|2} - r_1 = 0.0026 \pm 0.0003$ and $\delta r_{2|1} = r_{2|1} - r_2 = 0.0014 \pm 0.0004$) in our sample, the amount of entangling error is minimal. This reflects favourably on the performance of the $Z X \pi/2$ gate, which turns out to be coherence limited and not affected by systematic errors.
QUBIT PARAMETERS

The relaxation and coherence times of both qubits are quoted as the average of 120 independent measurements. For qubit 1, the average of the relaxation time measurements was $T_{1}^{(1)} = 11.6 \, \mu s$ with standard deviation $\sigma_{T_{1}}^{(1)} = 1.6 \, \mu s$ and the average of the coherence time measurements was $T_{2}^{(1)} = 7.1 \, \mu s$ with standard deviation $\sigma_{T_{2}}^{(1)} = 4.7 \, \mu s$. For qubit 2, $T_{1}^{(2)} = 9.1 \, \mu s$ with standard deviation $\sigma_{T_{1}}^{(2)} = 0.9 \, \mu s$ and $T_{2}^{(2)} = 5.6 \, \mu s$ with standard deviation $\sigma_{T_{2}}^{(2)} = 0.5 \, \mu s$. The qubits anharmonicities were $(\omega_{1}^{12} - \omega_{01}^{1})/2\pi = -331 \, MHz$ for qubit 1 and $(\omega_{2}^{12} - \omega_{01}^{2})/2\pi = -216 \, MHz$ for qubit 2.

DECOMPOSITION OF THE TWO-QUBIT CLIFFORD OPERATIONS

Defining $C_{1}$ as the group of single qubit Clifford operators (which has 24 different elements), the two-qubit Clifford group can be found with the help of the group $S_{1} = \{ I, R_{S}, R_{S}^{2} \}$ where $R_{S}$ is the Pauli transfer matrix of the unitary operator $S = \exp[-i(X + Y + Z)\pi/\sqrt{33}]$. This group is simply the rotation that exchanges all the axes of the Bloch sphere (the $x - y - z$ axis maps to $y - z - x$). To do this we note that there are four distinct classes of the two-qubit Clifford group. The first class consists of 576 elements ($24^{2}$) and represents all single qubit Clifford operations $C_{1}$ $C_{1}$ $C_{1}$ $C_{1}$

The second class has 5184 elements ($24^{2} \times 3^{2}$) and contains all combinations of the following sequence $C_{1}$ $S_{1}$ $C_{1}$ $S_{1}$ $C_{1}$ $S_{1}$ $C_{1}$ $S_{1}$

We call this the CNOT-like class. The third class also has 5184 elements ($24^{2} \times 3^{2}$) and contains all combinations of the following sequence $C_{1}$ $S_{1}$ $C_{1}$ $S_{1}$ $C_{1}$ $S_{1}$ $C_{1}$ $S_{1}$

We call this the iSWAP-like class. It should be noted that we are using a non-standard notation for the iSWAP gate. The final class is the SWAP class and consists of all 576 ($24^{2}$) combinations of the following sequence $C_{1}$ $C_{1}$ $C_{1}$ $C_{1}$

This is the optimal decomposition of the two-qubit Clifford group in terms of number of CNOTs as it can be shown that implementing a iSWAP requires two CNOTs and a SWAP requires three. Thus on average the number of CNOTs to implement a two-qubit Clifford is 1.5. The same is also true if the building block was the iSWAP as it takes two of these to make a CNOT and three to make a SWAP [2]. The number of single qubit gates depends on how the single qubit Cliffords are implemented. It can not be less than an average of 3.8 single qubit gates for each two-qubit Clifford and in general it will be more. This is because, for our system, it is simpler to tune up a finite set of generators and decompose all Clifford operations in terms of them. For this experiment the generating set was $\{ I, X_{\pm \pi/2}, Y_{\pm \pi/2}, X_{\pi}, Y_{\pi} \} \otimes 2$, and $ZX_{-\pi/2}$. Since we can simply incorporate any single qubit operations into the pre $C_{1}$ and any element form $S_{1}$ into the post rotation we use the following replacements for the two-qubit entangling gates of the above classes.

$\emptyset$ → $ZX_{-\pi/2}$

$\rightarrow$ $ZX_{-\pi/2}$ $Y_{-\pi/2}$ $ZX_{-\pi/2}$
and for the SWAP class we use the replacement

With these decompositions we find the average number of $ZX_{-\pi/2}$ is 1.5 and the number of single qubit gates is either 5.6 or 7 depending on whether the identity operations are included as a single qubit rotation. This is basically the same as that used in Ref. [3].

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