Comparative analysis of methods of recovering the well flow-rate and water-cut by thermodynamic measurements

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Abstract. An important problem of oil well diagnostics is to quickly determine changes in the amount of liquid entering the well (flow rate), and the ratio of the amount of water to the amount of liquid (water cut). To calculate these parameters, thermodynamic measurements (temperature, pressure) can be used at the wellhead and downhole of the well, as well as garlands at the wellhead of sensors 10-20 meters long, with a frequency of 1 meter, for more information. The article presents a comparative analysis of algorithms for determining these parameters based on the solution of the inverse problem. The results of numerical calculations are presented.

1. Introduction

The importance of solving the forward and inverse problems for two-phase flow (liquid-gas) is determined by the fact that more than 100,000 wells are currently being operated in Russia. Oil companies need to know the performance of each well to manage oil production wells and plan production. Monitoring of individual wells is also a requirement under various regulations. Continuous monitoring of well conditions is a very complex and expensive process that requires the installation of additional special equipment. Monitoring can be performed by using a standard set of pump telemetry, such as pressure and temperature sensors, which are already installed on almost all wells.

The problem of determining well parameters has been studied by many researchers. In [6], an approach using infrared spectroscopy and physical and chemical properties of the studied mixture is proposed. Some methods rely on obtaining water cut measurements at the wellhead where the conditions are most suitable [1, 2]. One of the oldest water detection technologies measures the dielectric parameters of a liquid [3]. The flow rate of an individual well can be measured using a flow meter. However, direct measurement of multiphase flow in a well is still a difficult task [7, 8]. In [14], an effective method for estimating the flow rate from a thermogram was developed.

In this paper, we propose a method for restoring the well flow rate and water cut of products based on solving the inverse problem with additional measurement of pressure and temperature...
at the wellhead. To solve the inverse problem and evaluate the desired parameters, global optimization methods are used: genetic algorithm, particle swarm method, and simulated annealing. These methods are compared with the [11, 12, 13] pallet method. A comparative analysis of algorithms for determining these parameters is performed, and the results of numerical calculations are presented.

2. Problem statement

Knowing the pressure and temperature in the well is necessary for the development of systems for collecting and transporting petroleum products, calculating the temperature conditions of pumps, designing systems, and methods for assessing the presence of paraffin deposits and hydrate formations, and designing gas-lift wells.

The mathematical model is based on solving the equation of heat and mass transfer. When calculating the thermal properties of the water-oil mixture and structure, information about the standard characteristics and components of the oil and gas mixture [12, 13] is used. The algorithm uses analytical formulas for calculating the thermal properties of an oil, differential equations of heat and mass transfer in a two-phase mixture, as well as semi-empirical formulas for calculating the flow structure.

\[ \frac{\partial P}{\partial z} = -10^{-6} \rho_l g \cos \alpha - i_{fr}, \]  
\[ \frac{\partial t}{\partial z} = \frac{10^3}{C_l \rho_l} C_l (t + 273) \frac{\partial P}{\partial z} + 10^3 \frac{i_{fr}}{C_l \rho_l} - \pi D_i k_i \tau \frac{GC_i}{t - t_{\Gamma}}; \]  

Here \( z_d \) — the point at which the pressure drops to the degassing level \( P(z_d) = 9 \text{ MPa} \) (the degassing point).

We assume that methane gas \((CH_4)\) dissolves in the liquid and if the pressure \( P \) exceeds the degassing pressure, i.e. gas bubbles appear in the liquid (two-phase flow: water, oil, and gas), then the system of equations take the following form \( z \in (z_d, H) \):

\[ \frac{\partial P}{\partial z} = -10^{-6} \rho_l g \cos \alpha - i_{fr}, \]  
\[ \frac{\partial t}{\partial z} = \frac{10^3}{C_l \rho_l} C_l (t + 273) \frac{\partial P}{\partial z} + 10^3 \frac{i_{fr}}{C_l \rho_l} - \pi D_i k_i \tau \frac{GC_i}{t - t_{\Gamma}}; \]
\[ \frac{\partial P}{\partial z} = -10^{-6} \rho g \cos \alpha - i_{fr}, \quad (3) \]

\[ \frac{\partial t}{\partial z} = \left\{ \frac{10^4}{C_{gl}(\rho_l(1 - \beta_g) + \rho_g \beta_g)} \left[ \alpha_l(1 - \beta_g)(t + 273) + \beta_g \right] \frac{\partial P}{\partial z} - \right. \]
\[ \left. - \frac{\pi D_i K_r}{G C_{gl}}(t - t_f) - \frac{G_{og} L_D}{G C_{gl}} \frac{\partial}{\partial P} \left( \frac{\rho g \beta_g}{\rho_l(1 - \beta_g) + \rho_g \beta_g} \right) \frac{\partial P}{\partial z} + \right. \]
\[ \left. + 10^3 \frac{i_{fr}(1 - \beta_g)}{C_{gl}(1 - \varphi_g)(\rho_l(1 - \beta_g) + \rho_g \beta_g)} \right\} \div \]
\[ \left\{ 1 + \frac{G_{og} L_D}{G C_{gl}} \frac{\partial}{\partial t} \left( \frac{\rho g \beta_g}{\rho_l(1 - \beta_g) + \rho_g \beta_g} \right) \right\} \}

Here \( P \) — pressure (PA), \( t \) — temperature (C), \( z \) — current depth (m). A detailed description of the mathematical model is presented in [11].

Suppose we can measure the pressure and temperature in the well (see figure 1). In the direct problem, you need to find the distribution of pressure and temperature in the well. To solve a direct problem, we add Cauchy data to the system of equations (1), (2), (3), (4)

\[ P(0) = P_0, \quad t(0) = t_0, \quad (5) \]

and the continuity condition

\[ [P]_{z=z_d} = 0, \quad [t]_{z=z_d} = 0. \quad (6) \]

The inverse problem is to determine the flow rate \( Q_l \) (tons / day) and water cut \( S_w \) ( % ) from measurements of temperature and pressure at the wellhead:

\[ P(z_k) = P_k, \quad t(z_k) = t_k, \quad k = 1, \ldots, K. \quad (7) \]

Here \( z_k \) — the depths at which the sensor garlands are located at the wellhead. The sensor reads on the submersible pump at the downhole of the well, \( K \) — the number of sensors.

We assume that all data is measured with an error

\[ |P_k - P^\delta_k| < \delta_P, \quad |t_k - t^\delta_k| < \delta_t. \quad (8) \]

3. Inverse problem solution

We formulate the inverse problem (1), (2), (3), (4), (5), (6), and (7) as a nonlinear operator equation:

\[ A(q) = f^\delta. \quad (9) \]

Here \( q = (Q_l, S_w), f^\delta = (P^\delta_{data}, t^\delta_{data}). \)

We will solve the inverse problem by minimizing the following functional:

\[ J(Q_l, S_w) = \sum_{k=1}^{K} (P(z_k) - P^\delta_k)^2 + (t(z_k) - t^\delta_k)^2. \quad (10) \]

To solve the minimization problem (10), we apply the genetic algorithm (GA) [5], the particle swarm optimization method (PSO) [9], the simulated annealing method (SA) [4] and compare it with the pallet method (BFS) [12, 13].
4. Methods of solving the inverse problem

4.1. Genetic Algorithm

The algorithm simulates the process of natural selection, which means that those species that can adapt to changes in the environment are able to survive and reproduce and move on to the next generation. In simple words, it mimics the “survival of the fittest” to the task among individuals of the current generation. Each individual is a point in the space of possible solutions. A detailed description of the genetic algorithm in [5].

4.2. Particle Swarm Optimization

Another meta-heuristic algorithm is PSO. It simulates the behavior of a swarm of bees, or a flock of birds while searching for prey. Each element of the pack is a possible solution, and at each iteration moves according to a certain law. A detailed description of the particle swarm method in [9].

4.3. Simulated Annealing method

To find the global minimum of the objective function, a very fast method will be used to simulate annealing with a coefficient of $c$ for a faster temperature reduction of $T(j+1) = cT(j)$ (also called the quenching method).

The method of “quenching” is to find the global minimum region by means of an ordered random search, constructed by analogy with the process of formation of a crystal structure with minimum energy during quenching. A detailed description of the annealing simulation method is given in [4].

5. Numerical results

Numerical calculations were performed to determine two parameters: flow rate and water cut. Let’s fix the following parameters: well depth $H = 2000$ m, number of steps $N = 2000$, step size $h = H/N = 1$ m.

The data of the inverse problem is taken for the exact solution of $Q_l = 200$ and $S_w = 50\%$. We assume that the data measurement error is $\delta_P = 0.025$ and $\delta_t = 0.25$. Moreover, we assume that the pressure $P_0$ and temperature $t_0$ in the downhole hole are also measured with an error.

The system of nonlinear differential equations (1), (2), (3), (4) and (5) is solved numerically from the downhole to the wellhead by the fourth-order Adams method. Calculations were performed until the stop criterion $J(Q_l, S_w) < 10^{-5}$ was reached.

Calculations were performed on a PC with an Intel Core i3-5005U processor, 2 physical cores (2.00 GHz), and 4 GB of RAM.

Genetic algorithm (GA). Algorithm parameters: number of individuals $p_0 = 150$, $p_0 * T_r = 45$, $d = 1.5$, $\mu = 0.3$. The method reaches the stop criterion in 54 iterations. The numerical solution to the inverse problem is shown in figure 2.

Particle swarm optimization method (PSO). Algorithm parameters: number of particles — $NoP = 150$, coefficients for speed components: $\alpha = 0.3$, $\beta = 0.3$. 97 iterations are required to reach the stop criterion. The result of the algorithm is shown in figure 3.

Simulated annealing method (SA). The parameters of the algorithm: the cooling coefficient $c = 0.9$. SA does 108 iterations, 12 steps (changing the position). The following solution of the inverse problem is obtained $Q_l = 200.343$, $S_w = 49.976$.

Pallet method (BFS). At a given level of measurement error, lines of pressure and temperature levels are drawn. The solution to the inverse problem has many solutions that are concentrated in the sector where the corresponding level lines intersect. The result of the algorithm is shown in figure 4 and figure 5. The set of solutions obtained by the pallet method is shown in figure 4, figure 5.
6. Conclusion
All algorithms have a fairly high speed of solving the inverse problem, which allows you to evaluate the necessary parameters almost in real-time on a laptop (except for the pallet
method), which is very important for engineers to control the well. The opportunity to get these parameters using only mathematical calculations allows companies to save money on additional more complex sensors. If you need to get a single solution, you should use the method of simulated annealing, if we are interested in the parameter area, you need to use the particle swarm method, the genetic algorithm, and the pallet method. Note that you can significantly reduce the number of iterations, taking into account a priori information about the desired solution [10]. In future work, it is planned to add the recovering of the gas factor and other parameters of the existing field.

| \(J(Q_t, S_w) < 10^{-5}\) | \(J(Q_t, S_w) < 10^{-7}\) |
|----------------|----------------|
| GA | 12.5 sec | 16.7 sec | 21 min |
| PSO | 37.7 sec | 49.5 sec | 21 min |
| SA | 9.7 sec | 13.9 sec | 21 min |
| BFS | | | |

Table 1. Calculation time for different stop criteria

Acknowledgements
The work was supported by the budget of the Institute of Computational Mathematics and Mathematical Geophysics, Siberian Branch, Russian Academy of Sciences, project no. 0315-2019-0005.

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