Analysis of static loading of meshing in planetary cycloid gear

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Abstract. A main feature of planetary cycloid gears is multi-pair contact of pin-rollers and teeth of cycloidal disk. The use of schemes with great number of parallel contacts requires adoption of measures to ensuring of assembly of toothed wheels by introducing into design of pliable elements or guaranteed gaps in contacts. On basis of discrete model of force interaction of gearing elements of planetary cycloid gear the article proposes a decision of problem of gaps effect in gearing to load distribution at contact pairs. Analysis of effect of output moment, resource and operating modes of gear on forces in contact is performed. Effect of stiffness of gearing elements on size of contact zone and forces acting in gearing is shown. Law of change of forces acting in gearing in operating gear, taking into account errors of its manufacturing is established. It is shown that during the gear operation there is a continuous change of forces in contact pairs, accompanied by a rearrangement of contact zones at certain angles of rotation. The presented decision can be used, including for analysis of options for ensuring acceptable values of maximum loads due to increasing accuracy of manufacturing of gearing elements or using constructions with increased pliability.

1. Introduction

One of the first works in which the static loading of planetary gear drives was studied is [1]. Numerous studies carried out in subsequent years [2 ... 9] and others showed that various factors influence the forces and stresses in the meshing, and, above all, the design and manufacturing accuracy of the mechanism. The influence of design parameters on forces and contact stresses was studied in [2] by S.K. Malhotra and M.A. Parameswaran. J.G. Blanche and D.C.H. Yang developed models for the influence of manufacturing errors on play and ripple of the cycloidal drive torque [3]. Mathematical models for calculating deformations, gaps and forces between the teeth of the cycloidal disk and the gears rollers are presented by L. Lixing et al. [4]. The work [5], devoted to the study of contact forces and stresses in cycloidal gears should be also mentioned. In [6], a method was developed for calculating tooth deformations and gaps between the teeth of a cycloidal disk and rollers. The influence of profile correction on deformations and clearances, as well as on the number of teeth that simultaneously transmit the load, was considered.
kinematic analysis of the initial clearances in the contacts was performed in [7, 8], and contact forces, stresses, and torsional stiffness in cycloidal gears were studied using the Hertz theory and the finite element method. In [9], a cycloidal speed reducer was studied taking into account the modification of the tooth profile and the gap in the output mechanism.

The objective of this paper is to analyze the influence of the gap, the stiffness of the gear elements, the output torque, the resource and the operating modes of the gearbox on the contact zones and the engagement force in the working gear.

2. Analytical solution

In a precisely manufactured planetary-pinion gear, all the pin-holes (rollers) are in simultaneous contact with the teeth of the satellite. Due to gaps without load, the movement must be transmitted by one pair of pin-tooth satellite. However, other contact pairs come into operation under load, a contact zone is formed and the load is transmitted by all contact pairs of this zone. Clearances affect a complex of interrelated quantities: backlash and kinematic error of rotation, forces acting in meshing, and total forces transmitted to the satellite support, gearing losses, torsional stiffness of the transmission.

In [10], a solution is presented for calculating the coefficient of increase in the maximum engagement force depending on the initial clearance and the angle of rotation of the satellite for a wide range of variation of these parameters. The solution was obtained for the position of the satellite, in which the opposite rollers of the pinwheel are located in the cavity and on top of the tooth of the satellite. During transmission, the position of the satellite changes and with it the distribution of the load along the contact pairs and the angle of elastic rotation of the satellite also change.

As in [10], consider the case of identical initial clearances in contact pairs. Figure 1(a) shows the initial location of the satellite and the pinwheel, while all the initial clearances in the direction normal to the profile are the same and equal to $\Delta_z$. When determining the forces in contact in a loaded gear, it is usually assumed [1] that, when the wheel is stationary, the satellite is rotated by a certain angle $\beta$ under the action of the moment applied to it. In this case, in the $i$-th contact pair, the approach $\delta_i = \beta l_i$ along the normal to the contact surfaces and in some pairs, deformation appears $\delta_i = \beta l_i - \Delta_z$. The value included in these formulas $l_i = r_{w1} \sin \theta_i$ — is a distance from the center of the satellite $O_1$ to the line of action of the force in contact going from the center of the pin to the pole of engagement $P_z$. Therefore

$$\delta_i = \beta r_{w1} \sin \theta_i - \Delta_z. \quad (1)$$

Assuming previously that the stiffness coefficient of the contact pairs $c$ is constant, the force in the contact will be assumed proportional to the deformation:

$$F_i = c \delta_i = c (\beta r_{w1} \sin \theta_i - \Delta_z). \quad (2)$$

The torque on the satellite, created by the forces acting on the teeth of the satellite:

$$T_s = \sum_i F_i l_i = \sum_i c (\beta r_{w1} \sin \theta_i - \Delta_z) r_{w1} \sin \theta_i. \quad (3)$$

Angle of rotation $\beta$ is equal to the sum of the angles for selecting the play in gear $\beta_{bac}$ and elastic rotation of the satellite $\beta_{el}$. Angle $\beta_{bac}$ can be determined by the formula $\beta_{bac} = \Delta_z / r_{w1}$. The angle of elastic rotation, taking into account contact deformations, is approximately determined by the contact pair in which the maximum force acts $c \beta_{el} r_{w1} = F_{max}$. Having for tight mesh $F_{max} = 4T_s (r_{w1} \Delta_z)^{-1}$ with $\theta = \pi / 2$ [1], is obtained:
\[
\beta_{el} = 4T_e \left( cr_{w1}^2 c_2 \right)^{-1}.
\] (4)

**Figure 1.** Gearing pattern.

General binding of the centers of curvature of the tooth and roller

\[
\delta_i = 2F_i \frac{1 - \nu^2}{\pi Eb} \left[ \ln \frac{\pi Eb \rho_{si} r_i}{2F_i (1 - \nu^2) \rho_{com}^2} + 0.815 \right] = 2F_i \frac{1 - \nu^2}{\pi Eb} \left[ \ln \frac{\pi Eb \rho_{si}^2}{2F_i (1 - \nu^2)} + 0.815 \right],
\] (5)

where \( E \) – elastic modulus; \( \nu \) – Poisson’s ratio; \( b \) – satellite width; \( \rho_{si} \) – satellite tooth curvature radius; \( r_i \) – radius of the roller (rundle); \( \rho_{com} \) – reduced radius of curvature at the contact point, \( r_i \) – epicycloid radius of curvature.

The angle \( \theta = \pi/2 \) corresponds to the convex profile of the satellite and \( \rho_{si} = r_s \sqrt{1 - \lambda^2} \), where \( r_s \) – the radius of the circumference of the location of the rollers, \( \lambda \) – the shortening factor of the epicycloid.

The actual stiffness coefficient may depend significantly on the design of the transmission, primarily the pinion wheel. Taking into account the general deformations, we take for analysis the stiffness coefficient of the contact pair in the following form:

\[
c_{k,c} = \left( k_c \pi Eb \right) \left[ 2(1 - \nu^2) \ln \frac{z_2 r_{w1} \pi Eb r_i \sqrt{1 - \lambda^2}}{8T_s (1 - \nu^2)} + 0.815 \right]^{-1},
\] (6)

where \( k_c \) – coefficient taking into account the design features of the node, \( k_c < 1 \).

Now for \( k_c \) the formula is:
The angle of the \( i \)-th rotation of the satellite and the pinwheel will rotate around the center \( O_1 \) in the same direction by an angle \( \varphi_i \). When the pinwheel rotates one angular step, \( \varphi_2 = 2\pi z_2 \), the system will again take its initial position with the only difference being that in position 2 there will be contact pair 1, and in position 3 there will be contact pair 2, etc. To study the forces arising in the system, the following algorithm is proposed.

In the initial position, the satellite rotates through an angle \( \varphi_1 \) around the center \( O_1 \) counterclockwise (Figure 1(b)) the pinwheel will rotate around the center \( O_2 \) in the same direction by an angle \( \varphi_2 \). When the pinwheel rotates one angular step, \( \varphi_2 = 2\pi z_2 \), the system will again take its initial position with the only difference being that in position 2 there will be contact pair 1, and in position 3 there will be contact pair 2, etc. To study the forces arising in the system, the following algorithm is proposed.

In the initial position, the satellite rotates through an angle \( \beta \) and, using the given dependences for each contact pair (1, 2, etc., up to \( z_2/2 + 1 \)), deformations are calculated \( \delta_i \), and if they turn out to be positive, then the forces \( F_i \), otherwise, the forces are taken equal to zero. The value \( F_i \) is calculated using equation (5) by the iteration method, taking the initial value according to formula (8). Then the system discretely, with the selected angular step \( \Delta \varphi_2 = 2\pi/(z_2 n) \), rotates an angle \( \varphi_{2j} = 2\pi j/(z_2 n) \), where \( j = 1, 2, \ldots, n \), and again for each contact pair \( \delta_i \) is calculated. The values \( \delta_i \) depend on the angle of rotation \( \beta \), for everyone \( \varphi_{2j} \) it is different, its value is not known. Therefore, one (the same for all \( \varphi_{2j} \)) value \( \beta_0 = \beta_{\text{mac}} + \beta_{\text{el}} \) (the initial approximation) is accepted and for it all is calculated \( \delta_i = \beta_i r_{w1} \sin \theta_i - \Delta z_i \), where \( \theta_i = \arctg(\sin \tau_{ci}/(\lambda - \cos \tau_{ci})) \). The angle of the \( i \)-th pin \( \tau_{ci} = 2\pi(i-1)/z_2 + \varphi_{2j} \), where \( i = 1, 2, \ldots, z_2/2 + 1 \).

Based on the found values \( \delta_i \) and \( F_i \) the vector \( T_{\alpha j} \) of estimated values of the torque on the satellite is calculated for all positions of the satellite and the pinwheel. All elements of the vector must be the same, equal to the given value \( T_0 \). Since \( \beta_0 \) the approximate value, this condition is not satisfied. Therefore, the value \( \beta \) is specified by the formula:

\[
\beta_{k+1} = \beta_k + (T_i - T_{\text{str}}) \left( cr_{w1} \sum_{i=1}^{q} \sin^2 \theta_i \right)^{-1},
\]

where

\[
k_x = \pi \left( 1 + p_{an} \right) \left[ \pi - 2 \arcsin \frac{p_{an}}{1 + p_{an}} - 2 \frac{p_{an}}{1 + p_{an}} \left[ 1 - \left( \frac{p_{an}}{1 + p_{an}} \right)^2 \right] \right]^{-1},
\]

where \( p_{an} = k \pi \Delta z_2 E b r_{w1} \left( 8 T_1 (1 - \nu^2) \left[ \ln z_2 r_{w1} \pi E b r_{2} \frac{1 - \lambda^2}{8 T_1 (1 - \nu^2)} + 0.815 \right] \right)^{-1} \) - elastic loading parameter.

The forces acting in contact

\[
F_{ix} = k_x \frac{4 T_1}{z_2 r_{w1}} \left( \frac{\sin \theta_i - p_{an} (1 - \sin \theta_i)}{p_{an} \sin \theta_i - p_{an} (1 - \sin \theta_i)} \right) \operatorname{npur} \sin \theta_i - p_{an} (1 - \sin \theta_i) \geq 0,
\]

where \( \sin \theta_i = \sin \tau_{ci} / \left( 1 - 2 \lambda \cos \tau_{ci} + \lambda^2 \right)^{1/2} \), \( \tau_{ci} \) - pin angle on the pin wheel.
where \( k, k+1 \) - iteration numbers, \( p, q \) - contact pair numbers that are the boundaries of the range in which \( \delta_i > 0 \), in \( k \) \( - \) iteration; \( T_{sr} \) - satellite torque in \( k \) \( - \) iteration

By the specified value \( \beta \) an array of values \( \beta_1 \) and an array \( \delta_1 \) are formed. If \( \delta_1 \) are negative they are replaced by zero. Next, the vector \( T_{sr+1} \) of estimated values of the torque at the iteration \( k+1 \) is calculated. Further, the iterative process continues until the required accuracy is achieved. So the final arrays are formed \( \beta, \delta, F \).

3. Interpretation of the solution

Figure 2 shows the results of calculations performed in MatCad for gears with a gear ratio of 33. Torques \( T \), recourse \( L \) and operating modes are shown in the figure, accepted \( k_t=1 \).

![Figure 2](image)

Figure 2. Forces in contact pairs in the initial gear position.

Figure 2 shows that the gaps can significantly reduce the number of contact pairs that transmit the load, and increase the forces in engagement. The requirement of high precision manufacturing gears is mandatory to ensure high performance transmission. A comparison of the maximum forces acting in the transmissions at \( T=3150 \)Nm and \( T=630 \)Nm shows that their ratio at different initial clearances remains approximately constant (2.2 ... 2.3), and the contact zones are approximately the same.

Figure 3 shows the change in the forces in the gear engagement according to option 1 and \( k_t=0.5 \) during the rotation of the satellite and the pinwheel. The initial clearances in the engagement are 0.03 mm. In figure 3 (a), the numbers of contact pairs (1 to 8) are plotted along the left axis, and the position numbers (1 to 11) are plotted along the axis directed from the observer. Pairs 1, 2 and 8 ... 18 are not loaded at any positions. When the fore-wheel rotates by an angular step, the pattern of power loading is repeated. So on pair 3 at the end of the turn the same force acts as on pair 4 at the beginning, etc. For the presented design case, between the first and second, as well as between the third and fourth positions, the structure of the contact zones changes, the number and numbers of contact pairs perceiving the load change. In figure 3 (b), one can notice a change in the behavior of the forces at the moments of the restructuring of the structure of the contact zone.
Figure 3. Change of contact forces depending on the angle $\varphi_{2j} = 2\pi j/(z_2 n)$.

A comparison of figures 2 (a) and 3 shows that a decrease in the gear stiffness coefficient ($k_e = 1$ and $k_s = 0.5$) leads to an increase in the loaded zone (from three to five contact pairs) and a decrease in the maximum force in the contact (from 2490 N to 2010 N).

4. Conclusions
The results of the analysis show that the requirements for the accuracy of the manufacture of engagement elements with high stiffness are quite high. A decrease in the stiffness of the contact pairs can significantly reduce the engagement load; however, it increases the angle of elastic rotation. The forces in the contact pairs in the running gears are constantly changing, which is accompanied by the rearrangement of the contact zones at certain rotation angles, planetary-pinion gears are sensitive to structural changes in the contact zones with increasing load.

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