Associated Higgs + jet(s) production at the LHC and CCFM gluon dynamics in a proton

A.V. Lipatov\textsuperscript{1,2}; M.A. Malyshev\textsuperscript{1}

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\textsuperscript{1}Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, 119991, Moscow, Russia

\textsuperscript{2}Joint Institute for Nuclear Research, 141980, Dubna, Moscow region, Russia

Abstract

We consider the associated production of Higgs boson and hadronic jet(s) in \( pp \) collisions at the LHC for the first time using the \( k_T \)-factorization approach. Our analysis is based on the off-shell gluon-gluon fusion subprocess, where non-zero transverse momenta of initial gluons are taken into account and covers \( H \rightarrow \gamma\gamma \), \( H \rightarrow ZZ^* \rightarrow 4l \) (with \( l = e, \mu \)) and \( H \rightarrow W^+W^- \rightarrow e^\pm\mu^\mp\nu\bar{\nu} \) decay channels. The transverse momentum dependent (TMD) gluon densities in a proton are taken from Catani-Ciafaloni-Fiorani-Marchesini evolution equation. To simulate the kinematics of the produced jets the TMD parton shower implemented into the Monte-Carlo event generator \textsc{cascade} is applied. The comparison of our results with the latest experimental data taken by the CMS and ATLAS Collaborations at \( \sqrt{s} = 8 \) and 13 TeV and conventional higher-order perturbative QCD calculations is presented. We highlight observables, which are sensitive to the TMD gluon densities in a proton.

Keywords: Higgs boson, QCD evolution, small-\( x \), TMD parton densities.

*\textsuperscript{*}e-mail: lipatov@theory.sinp.msu.ru
Recently, the ATLAS and CMS Collaborations have presented measurements \cite{1-8} of the total and differential cross sections of Higgs boson production in pp collisions at the LHC conditions, both inclusive and associated with one or more hadronic jets. These data have been taken for $H \to \gamma\gamma$, $H \to ZZ^* \to 4l$ (where $l = e$ or $\mu$) and $H \to W^+W^- \to e^\pm\mu^\mp\nu\bar{\nu}$ decay channels at the center-of-mass energies $\sqrt{s} = 8$ and 13 TeV. Such measurements allow to probe fundamental properties of Higgs boson (for example, spin and couplings to gauge bosons and fermions) and provide a testing ground for perturbative Quantum Chromodynamics (pQCD) predictions. Moreover, they can be used to investigate the gluon dynamics in a proton since the dominant mechanism of inclusive Higgs production is the gluon-gluon fusion (see, for example, \cite{9} and references therein).

The reported measurements \cite{1-8} are found to be in good agreement with the next-to-next-to-leading-order (NNLO) pQCD predictions \cite{10-15} obtained using hres \cite{16} and/or nnlops \cite{17} Monte-Carlo tools\footnote{The N^3LO predictions for gluon-gluon fusion subprocess have become available recently \cite{18} and the NLO perturbative electroweak corrections to the Higgs production have been evaluated \cite{19-23}.}. The available NNLO calculations can be even improved at low transverse momenta by the soft-gluon resummation procedure, which has been carried out up to next-to-next-to-leading logarithmic accuracy (NNLL) \cite{24,25}. An alternative description of the LHC data \cite{1-8} can be achieved in the framework of the high-energy QCD factorization \cite{26}, or $k_T$-factorization approach of QCD \cite{27}. This approach is based on the Balitsky-Fadin-Kuraev-Lipatov (BFKL) \cite{28} or Catani-Ciafaloni-Fiorani-Marchesini (CCFM) \cite{29} gluon evolution equations, which resum large logarithmic terms proportional to $\alpha_s^n \ln^n s \sim \alpha_s^n \ln^n 1/x$, important at high energies $\sqrt{s}$ (or, equivalently, at small longitudinal momentum fraction $x$ of the colliding proton carried by an interacting gluon). It can be understood since typical $x$ values are about of $x \sim m_H/\sqrt{s} \sim 0.008 - 0.015$ for Higgs mass $m_H = 125$ GeV and $\sqrt{s} = 13$ TeV. Additionally, the CCFM equation takes into account terms proportional to $\alpha_s^n \ln^n 1/(1 - x)$ and therefore can be applied for both small and large $x$ \cite{29}. The $k_T$-factorization QCD approach has certain technical advantages in the ease of including higher-order pQCD radiative corrections (namely, main part of NLO + NNLO + ... terms corresponding to real initial-state gluon emissions) in the form of transverse momentum dependent (TMD, or unintegrated) gluon density and can be used as a convenient alternative to explicit higher-order pQCD calculations. The detailed description of this approach can be found, for example, in reviews \cite{30,31}.

The $k_T$-factorization formalism was applied \cite{32-41} to the inclusive Higgs boson production at the LHC. As it was demonstrated \cite{38-41}, this approach, being supplemented with the CCFM gluon dynamics, is able to describe the data obtained at $\sqrt{s} = 8$ and 13 TeV in the diphoton, four-lepton and $H \to W^+W^- \to e^\pm\mu^\mp\nu\bar{\nu}$ decay channels even with leading-order off-shell (depending on the non-zero transverse momenta of incoming gluons) production amplitudes. Comparison with the higher-order pQCD calculations was presented \cite{41}. The sensitivity of the $k_T$-factorization predictions to the TMD gluon density specially was pointed out \cite{41-43}.

The associated production of Higgs boson and one or more hadronic jets is of special interest from different points of view. In our opinion, the most intriguing and remarkable point is connected with the distinctive feature of the $k_T$-factorization approach regarding the final state jet formation. While in the conventional (DGLAP-based) parton level pQCD the produced jets are fully determined by corresponding hard scattering amplitude, in the $k_T$-factorization scenario in addition to the quarks and/or gluons produced in the hard subprocesses (which can form the hadronic jets) there is a number of gluons radiated in the course of their non-collinear evolution, which also give rise to final state jets. So,
the measured events with the detected jets could be useful in discrimination between the two calculation schemes. Therefore, it is of interest and importance to generate the $k_T$-factorization predictions for such events and, of course, test these predictions in as many cases as possible. Closely related to this is selection of the TMD gluon densities in a proton best suited to describe the available experimental data.

In the present note we extend the previous consideration \cite{33,38,41} of inclusive Higgs production to $H+$ jet(s) events. Such calculations are performed for the first time. To correctly implement into our evaluations the kinematics of the final state jets, the method of \cite{44} is applied. This method is based on the reconstruction of CCFM evolution ladder using a TMD parton shower routine implemented into the Monte-Carlo event generator CASCADE \cite{45}.

Our main formulas were obtained in previous papers \cite{33,38,41}. However, for the reader’s convenience, let us very shortly describe the basic calculation steps. We start from the off-shell gluon fusion subprocesses:

$$g^*(k_1) + g^*(k_2) \rightarrow H(p) \rightarrow V(p_1) + V(p_2),$$  

where four-momenta of all particles are indicated in the parentheses and $V$ denotes $\gamma$, $W^\pm$ or $Z$ bosons (any of the gauge bosons can decay into leptons and/or neutrino). It is important that both initial gluons carry non-zero transverse momenta: $k_{1T}^2 = -k_{2T}^2 \neq 0$, $k_{1T}^2 = -k_{2T}^2 \neq 0$. Using the effective Lagrangian for the Higgs coupling to gluons \cite{46,47} valid in the limit of infinite top quark mass, $m_t \rightarrow \infty$, one can easily obtain the corresponding off-shell production amplitudes. The latter can be written in a form \cite{38,41} (see also \cite{39}):

$$|\mathcal{A}|^2 = \frac{1}{152\pi^4} \alpha_s^2 a_s^2 G_F^2 |\mathcal{F}|^2 \frac{s^2 + p_T^2}{(s - m_H^2)^2 + m_H^2 \Gamma_H^2} \cos^2 \phi,$$

for $H \rightarrow \gamma \gamma$ decay and

$$|\mathcal{A}|^2 = \frac{512\pi}{9} \alpha_s^2 G_F \sqrt{2} m_Z^2 C_V \frac{(s + p_T^2)(s - m_H^2)^2 + m_H^2 \Gamma_H^2}{(s - m_H^2)^2 + m_H^2 \Gamma_H^2} \cos^2 \phi \times \frac{(g_{(V)R}^4 + g_{(V)L}^4)(l_3 \cdot l_3)(l_2 \cdot l_4) + 2g_{(V)R}^2 g_{(V)R}^2(l_1 \cdot l_4)(l_2 \cdot l_3)}{[(p_T^2 - m_V^2)^2 + m_V^2 \Gamma_V^2][(p_T^2 - m_V^2)^2 + m_V^2 \Gamma_V^2]},$$

for $H \rightarrow ZZ^{*} \rightarrow 4l$ or $H \rightarrow W^+W^- \rightarrow e^+\mu^+\nu\bar{\nu}$ decays. Here $G_F$ is the Fermi coupling constant, $l_1$ and $l_3$ are the gauge bosons decay leptons four-momenta, $l_2$ and $l_4$ are their antileptons four-momenta (so that $p_1 = l_1 + l_2$ and $p_2 = l_3 + l_4$), $s = (k_1 + k_2)^2$, $p_T = k_{1T} + k_{2T}$ is the transverse momentum of the Higgs boson, $\phi$ is the azimuthal angle between the transverse momenta of initial off-shell gluons, $m_V$ and $\Gamma_V$ are the masses and decay widths of corresponding particles. The exact expressions for $\mathcal{F}$, $C_V$, left and right weak current constants $g_{(V)L}$ and $g_{(V)R}$ are listed \cite{38,41}, there all the calculation details are given. The gauge invariant off-shell production amplitudes (2) and (3) have been implemented into the parton-level Monte-Carlo event generator PEGASUS \cite{48}.

An important point of our calculations is connected with the proper determination of associated jets four-momenta. As it was noted above, the produced Higgs boson is accompanied by a number of gluons radiated in the course of the non-collinear evolution, which give rise to final jets. Similar to \cite{44}, to reconstruct one or few leading hadronic jets from all of these initial state gluon emissions we have used the anti-$k_T$ algorithm with radii $R_{\text{jet}}$ as implemented into the FASTJET tool \cite{52}. Technically, we generate a Les Houches Event file \cite{53} in the PEGASUS calculations and then process the file with a TMD shower tool implemented into the Monte-Carlo event generator CASCADE \cite{45}.
In this way we reconstruct the CCFM evolution ladder and consistently compute the cross section of associated $H + \text{jet(s)}$ production according to the experimental setup\footnote{A simplified model to implement the effects of parton showers into analytical calculations was used in early calculations\cite{33}.}. The CCFM equation seems to be the most suitable tool for our consideration because it smoothly interpolates between the small-$x$ BFKL gluon dynamics and conventional DGLAP one.

Concerning the CCFM-evolved gluon densities in a proton, in the present note we tested two different sets\footnote{A comprehensive collection of the TMD gluon densities can be found in the \textsc{tmdlib} package\cite{51}, which is a C++ library providing a framework and interface to the different parametrizations.}, namely, JH$'2013$ set 2\cite{49} and (more old) A0 set\cite{50}. The input parameters of latest gluon density, JH$'2013$ set 2, have been derived from the best description of high-precision HERA data on proton structure functions $F_2(x, Q^2)$ and $F_2^\gamma(x, Q^2)$\cite{49}. Throughout this paper, all the calculations are based on the following parameter setting. We kept $n_f = 4$ active (massless) quark flavours, set $\Lambda_{\text{QCD}} = 200(250)$ MeV and used two-loop (one-loop) QCD coupling for JH$'2013$ set 2 (A0) gluon densities. As it is often done, the renormalization scale was taken to be $\mu_R^2 = m_H^2$. The factorization scale was taken as $\mu_F^2 = \hat{s} + Q_T^2$ (where $Q_T$ is the net transverse momentum of the initial off-shell gluon pair), that is dictated mainly by the CCFM evolution algorithm (see\cite{49,50} for more information). To evaluate the theoretical uncertainties we use auxiliary gluon densities JH$'2013$ set 2$^+$ and JH$'2013$ set 2$^-$ as well as A0$^+$ and A0$^-$ instead of default gluon distribution functions. These two sets refer to the varied hard scales in the strong coupling constant $\alpha_s$ in the off-shell amplitude: ‘$^+$’ stands for $2\mu_R$, while ‘$^-$’ refers to $\mu_R/2$. Following\cite{54}, we set electroweak and Higgs bosons masses $m_Z = 91.1876$ GeV, $m_W = 80.403$ GeV and $m_H = 125.1$ GeV, their total decay widths $\Gamma_Z = 2.4952$ GeV, $\Gamma_W = 2.085$ GeV and $\Gamma_H = 4.3$ MeV and use $\sin^2 \theta_W = 0.23122$.

We start the discussion by presenting our results for associated Higgs boson and jet production in the diphoton decay channel. The latest measurements were done by the CMS\cite{1} and ATLAS\cite{5} Collaborations at the $\sqrt{s} = 13$ TeV. The applied experimental cuts are collected in Table 1. An additional requirement (the isolation criterion) is introduced for the photons in both experiments: the sum of transverse energy of particles around every photon within the radius $\Delta R = \sqrt{\Delta \eta^2 + \Delta \phi^2} = 0.3$ has to be smaller than $E_{\text{iso}} = 10$ GeV. The results of our calculations are presented on Figs. 1 and 2. Note that here we concentrated only on some of the kinematical variables among the quite large variety of those, presented by the CMS and ATLAS Collaborations: number of jets $N_{\text{jet}}$, leading jet transverse momentum $p_T^1$, and rapidity $y^1$, rapidity difference between the diphoton system and leading jet, $\Delta y^{\gamma\gamma j_1}$, azimuthal angle difference between the two leading jets, $\Delta \phi^{j_1 j_2}$ and difference between the average pseudorapidity of these jets and pseudorapidity of the diphoton system $|\eta^{j_1 j_2} - \eta^{\gamma\gamma}|$ (Zeppenfeld variable\cite{55}). We added the contributions from weak boson fusion subprocesses ($W^+W^- \rightarrow H$ and ZZ $\rightarrow H$), associated $H\gamma Z$ or $HW^\pm\gamma$ production and associated $tH$ production to our results. These contributions are essential at high transverse momenta and have been calculated in the conventional pQCD approach with the NLO accuracy (we took them from the CMS\cite{1} and ATLAS\cite{5} papers). Also we show for comparison the NNLO pQCD predictions, calculated with the \textsc{nnlops} program \cite{17} and taken from\cite{1,5}. As one can see, the measured cross sections can be, in general, reasonably well described by the $k_T$-factorization evaluations within the theoretical and experimental uncertainties. However, the predictions based on the JH$'2013$ set 2 and A0 gluon densities behave differently for some observables, especially for correlation observables, such as $\Delta y^{\gamma\gamma j_1}$ and azimuthal angle difference.
Moreover, “old” A0 one tends to underestimate the data although giving relatively larger number of jets at larger transverse momenta. Unfortunately, the current level of experimental accuracy does not allow us to favor one or another TMD gluon density in a proton. More precise future measurements of such observables could be promising to distinguish between the latter. The NNLO pQCD predictions behave similarly to the A0 results (except for distribution in $\Delta y_{\gamma\gamma j}$), though having larger normalization. The difference between the NNLO pQCD and JH’2013 set 2 results is clearly seen for $\Delta y_{\gamma\gamma j}$ and, in some sense, for $\Delta \phi_{\gamma\gamma j}$ observable. Nevertheless, our calculations demonstrate the possibility of $k_T$-factorization approach supplemented with the CCFM gluon dynamics to reasonably describe the collider data on events containing hadronic jets in final state.

Next, we turn to the $H \to ZZ^* \to 4l$ and $H \to W^+W^- \to e^\pm \mu^\mp \nu\bar{\nu}$ decay modes. In these channels, the available experimental data have been obtained by the CMS [2–4] and ATLAS [6–8] Collaborations at $\sqrt{s} = 8$ and 13 TeV. The applied experimental cuts are listed in Tables 2 and 3, respectively. Our results for several interesting observables, namely, $N_{\text{jet}}$, leading jet transverse momentum $p_T^{j_1}$, rapidity difference between the Higgs boson and leading jet, $\Delta y_{Hj_1}$, pseudorapidity and azimuthal angle difference between the leading and subleading jets, $\Delta \eta_{j_1j_2}$ and $\Delta \phi_{j_1j_2}$, invariant masses of Higgs-leading jet system $m_{Hj_1}$ and Higgs-dijet system $m_{Hj_1j_2}$, are shown in Figs. 3 — 6. As in the case of diphoton decay mode, the contributions from weak boson fusion subprocess, associated $HZ$, $HW^\pm$ and $t\bar{t}H$ production calculated in the NLO pQCD approximation and taken from [2–4, 6–8] were added to the off-shell gluon-gluon fusion. We find again that latest JH’2013 set 2 gluon distribution reasonably well describes the LHC data within the estimated theoretical uncertainties whereas A0 gluon lacks normalization. The measured data [2–4, 6–8] point on the following distinctive observables, which reveal the difference between JH’2013 set 2 and A0 gluon densities: Higgs-jet rapidity difference $\Delta y_{Hj_1}$, difference in pseudorapidity between the leading and subleading jets $\Delta \eta_{j_1j_2}$ and invariant masses $m_{Hj_1}$ and $m_{Hj_1j_2}$. The latter demonstrate much larger cross section at relatively low invariant masses for A0 predictions, which is, in fact, in better agreement with the ATLAS data (see Fig. 5). Thus, in more precise forthcoming experiments the highlighted observables, in addition to the variables for inclusive Higgs production pointed earlier [33,38,41–43], could be promising to distinguish between the different TMD gluon densities or to better constrain their parameters. Like as for $\gamma\gamma$ decay channel, we plot also the collinear NNLO pQCD results, taken from [2–4,6,8]. It can be seen again, that the A0 distributions generally follow the collinear results in shape, whereas the JH’2013 set 2 results somewhat differ from them.

To conclude, we calculated for the first time the cross sections of associated Higgs and jet(s) production at the LHC conditions using the $k_T$-factorization approach. Our consideration covers different Higgs decay channels and is mainly based on the off-shell production amplitudes for gluon-gluon fusion subprocess (implemented into the Monte-Carlo event generator PEGASUS) and CCFM-evolved TMD gluon densities in a proton. To reconstruct correctly the kinematics of the final-state hadronic jets the TMD parton shower generator CASCADE has been applied. Our predictions obtained with the recent JH’2013 set 2 gluon density agree well with the experimental data taken by the CMS and ATLAS Collaborations at $\sqrt{s} = 8$ and 13 TeV. We have found observables, which are sensitive to the TMD gluon densities in a proton. As it was expected, these are the ones related with the properties of the produced jets, for example, the rapidity difference between the Higgs boson and leading jet. Unfortunately, the current level of experimental accuracy does not allow to distinguish between the latter. However, more precise future experimental studies of the pointed observables could be promising and could allow one
| $p_T^{l_1}$, GeV | ATLAS [5] | CMS [1] |
|-----------------|-----------|---------|
| $> 0.35 m^{\gamma\gamma}/3$ | $> m^{\gamma\gamma}/3$ |
| $p_T^{l_2}$, GeV | $> 0.25 m^{\gamma\gamma}$ | $> m^{\gamma\gamma}/4$ |
| $|y^{l_1}|$ | $< 2.37$ | $< 2.5$ |
| $m^{\gamma\gamma}$, GeV | $105 - 160$ | $> 90$ |
| $R_{\text{jet}}$ | 0.4 |
| $p_T^{\text{jet}}$, GeV | $> 30$ | $> 30$ |
| $|y^{\text{jet}}|$ | $< 4.4$ | $< 4.7$ |

Table 1: Basic parameters, used for simulations in the $H \to \gamma\gamma$ decay channel.

| $p_T^{l_1}$, GeV | ATLAS [8] | CMS [4] |
|-----------------|-----------|---------|
| $> 22$ |
| $p_T^{l_2}$, GeV | $> 25$ |
| $|\eta^{l_1}|$ | $< 2.47$, excl. $1.37 < |\eta^{l_1}| < 1.52 (< 2.5)$ |
| $m^{l_1}$, GeV | $10 - 55$ |
| $R_{\text{jet}}$ | 0.4 |
| $p_T^{\text{jet}}$, GeV | $> 30$, if $|\eta^{\text{jet}}| < 2.4$, $> 30$ otherwise |
| $|\eta^{\text{jet}}|$ | $< 4.5$ |
| other cuts | $p_T^{\text{miss}} > 20$ GeV |
| | $\Delta\phi^{l_i} < 1.8$ |
| | $m_T^l \equiv \sqrt{2 p_T^{l_1} p_T^{\text{miss}} [1 - \cos \Delta\phi(p_T^{l_1}, p_T^{\text{miss}})]} > 30$ GeV |
| | $m_T^H \equiv \sqrt{2 p_T^{\ell_1} p_T^{\text{miss}} [1 - \cos \Delta\phi(p_T^{\ell_1}, p_T^{\text{miss}})]} > 60$ GeV |

Table 2: Basic parameters, used for simulations in the $H \to ZZ^* \to 4l$ decay channel. By default experimental cuts for electrons are shown. Cuts for muons are placed in brackets, if differ.

to constrain the TMD gluons. Our study demonstrates the possibility of $k_T$–factorization approach supplemented with the CCFM gluon dynamics to describe the events with large number of jets in final state. It significantly extends the previous consideration of inclusive Higgs production at the LHC.

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Table 3: Basic parameters, used for simulations in the $H \rightarrow W^+W^- \rightarrow e^\pm\mu^\mp\nu\bar{\nu}$ decay channel. By default experimental cuts for electrons are shown. Cuts for muons are placed in brackets, if differ.

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Figure 1: The differential cross sections of associated Higgs boson and jet production (in the diphoton decay channel) at $\sqrt{s} = 13$ TeV as functions of $N_{\text{jet}}$, leading jet transverse momentum and rapidity. The contributions from non-gluon fusion subprocesses and NNLOPS predictions are taken from [1, 5]. The experimental data are from CMS [1] and ATLAS [5].
Figure 2: The differential cross sections of associated Higgs boson and jet production (in the diphoton decay channel) at $\sqrt{s} = 13$ TeV as functions of the rapidity difference between the diphoton system and leading jet $\Delta y_{\gamma\gamma j1}$ and Zeppenfeld variable $|\eta_{j1j2} - \eta_{\gamma\gamma}|$. The contributions from non-gluon fusion subprocesses and NNLOPS predictions are taken from [1,5]. The experimental data are from CMS [1] and ATLAS [5].
Figure 3: The differential cross sections of associated Higgs boson and jet production (in the $H \rightarrow ZZ^*$ decay channel) at $\sqrt{s} = 8$ TeV as functions of $N_{\text{jet}}$, leading jet transverse momentum and rapidity difference between the Higgs boson and leading jet $\Delta y_{Hj1}$. The contributions from non-gluon fusion subprocesses and NNLOPS predictions are taken from [2,6]. The experimental data are from CMS [2] and ATLAS [6].
Figure 4: The differential cross sections of associated Higgs boson and jet production (in the $H \rightarrow ZZ^*$ decay channel) at $\sqrt{s} = 13$ TeV as functions of $N_{\text{jet}}$ and leading jet transverse momentum. The contributions from non-gluon fusion subprocesses and NNLOPS predictions are taken from [3, 7]. The experimental data are from CMS [3] and ATLAS [7].
Figure 5: The differential cross sections of associated Higgs boson and jet production (in the $H \to ZZ^*$ decay channel) at $\sqrt{s} = 13$ TeV as functions of the rapidity and azimuthal angle difference between the leading and subleading jets, invariant masses of Higgs-leading jet system and Higgs-dijet system. The contributions from non-gluon fusion subprocesses and NNLOPS predictions are taken from [3, 7]. The experimental data are from CMS [3] and ATLAS [7].

Figure 6: The differential cross sections of associated Higgs boson and jet production (in the $H \to W^+W^-$ decay channel) at $\sqrt{s} = 8$ (right panel) and 13 TeV (left panel) as functions of $N_{\text{jet}}$. The experimental data are from CMS [4] and ATLAS [8].