Analysis of problem solving ability of eight grade students of Santo Aloysius Sleman junior high school in mathematical learning using problem pased learning approach to inner tangent between the two circles material

J P Maran and A S J Renggi
Sanata Dharma University, Yogyakarta

E-mail: jeversonperimaran1@gmail.com

Abstract. This study aims to describe the steps of the learning process by using Problem Based Learning (PBL) approach and students’ problem solving ability on inner tangent between the two circles material after experiencing the learning process using PBL approach. The type of research used is design research. The subjects of the research were eighth grade students of Santo Aloysius Sleman junior high school. The object of the research was problem solving ability of eighth grade students of Santo Aloysius Sleman junior high school using PBL approach. The stages carried out in the study were the prior design, trial, and implementation of learning. Data analysis techniques of the research are data reduction, data presentation and conclusions or verification. In this study the researchers designed and developed the Hypothetical Learning Trajectory (HLT) with the PBL approach to inner tangent between the two circles material as the learning model. Researchers designed and developed Hypothetical Learning Trajectory (HLT) with Problem Based Learning (PBL) approach for tangent line to two circles as the learning model. HLT is made, implemented in learning to describe the steps of the learning process and analyze students' problem solving ability. The steps of the research were as follow: orient the students through inner tangent between the two circles; organize the students to learn; guide the individual’s or groups’ experience towards the learning process; develop and present the students’ work from group discussions; and conclude the results of the learning process that have been done. From the result of tests and interviews, the researchers found that all students experienced the five indicators of problem solving ability based on NCTM.

1. Introduction
Mathematics is a universal science that underlies the development of modern technology. It has an important role in various disciplines and advances human thinking [1]. Based on National Council of Teacher of Mathematics (NCTM) [2] the mathematical thinking process in learning mathematics has five main standard competencies; problem solving ability, understanding ability, connection ability, communication ability, and representation ability. In education, students' abilities are honed through problems, so students are able to improve their competencies. A statement will be a problem only if someone cannot find what the solution for the statement. The ability to solve mathematical problems is very important to be developed for students, which is emphasized in NCTM [2] which states that problem solving is an integral part of mathematics learning, so that it cannot be separated from mathematics learning. To solve the problem, students are required to be able to use the knowledge they have learned and various skills to overcome problems and solve them. As Krulik and Rudnick [3]
who said that “problem solving as the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation. The student must synthesize what he or she has learned and apply it to a new and different situation.”

From the results of observations and interviews to eighth grade students and mathematics’s teachers at SMP Santo Aloysius Sleman of Santo Aloysius Sleman, researchers obtained information that usually centered on the teacher (teacher center) and problem solving abilities of grade eight students are still lack because teachers rarely let students find concepts Mathematics by themselves and problems given are routine because students need a long time to be able to find these concepts although there is intervention from the teacher. Regarding the importance of problem solving ability, NCTM [2] said that in implementing mathematics learning in schools, teachers must pay attention to the five basic mathematical competencies. Therefore, the teacher has an important role in fostering students’ mathematical problem solving ability of learning models that are used or in evaluation of making questions. One of the learning models that can be used to hone students' problem solving abilities and can make students active, is Problem Based Learning model. Some studies on Problem Based Learning (PBL) in mathematics learning have positive results [4]. Based on research conducted by Tina Sri Sumartini [5], Problem Based Learning (PBL) learning models can improve students' mathematical problem solving abilities rather than using conventional learning, and mistakes made by students when do questions related to ability to solve Mathematical problems are; errors due to carelessness inaccuracy, errors in transforming information, errors in process skills, and errors in understanding questions.

In this study, researchers want to describe the steps in the learning process of eighth grade students of Santo Aloysius Sleman junior high school using the Problem Based Learning (PBL) approach to inner tangent between the two circles material and find out students' problem-solving ability of inner tangent between the two circles material after experiencing the learning process using the Problem Based Learning (PBL) approach.

1. Problem Solving Ability

The ability to solve mathematical problems is very important to be developed for students, which is emphasized in NCTM [2] which states that problem solving is an integral part of mathematics learning, so that it should not be separated from mathematics learning. Polya [6] describes mathematical problems in two types; problems to find and problems to prove. Problem to find is a problem that aims to find, determine, or get the value of a certain object that is unknown in the problem and give the appropriate conditions. While the problem to prove is the problem with a procedure to determine a statement that is true or false. Based on NCTM, indicators of problem solving abilities are students can identify known elements, the elements that are asked, and the adequacy of the required elements; students can formulate mathematical problems or arrange mathematical model; students can apply strategies to solve various problems (similar and new problems) inside or outside mathematics; students can explain and interpret the results according to the prior problem; and students can use mathematics meaningfully.

2. Problem Based Learning (PBL)

In Daryanto [7], described that Problem Based Learning (PBL) is a learning model that challenges students to “learn how to learn”, work in groups to find solutions to real problems. Problems had given to students before students learnt concepts or materials related to problems that must be solved. In addition, Problem Based Learning (PBL) is the use of various kinds of intelligence needed to confront real challenges, the ability to face everything new in the existing complexities [8]. PBL is a learning model with a student learning approach that can develop their own knowledge, develop higher skills and inquiry, empower students and increase self-confidence [9]. This learning model uses real life problems as students must learn and improve critical thinking skills and problem solving and gain knowledge of important concepts, where the teacher's task must focus on helping students achieve self-directed skills.
3. Problem Based Learning (PBL) Syntax
The syntax or steps in the process of Problem Based Learning (PBL) are as follows as in the table below [9]:

| Stage         | The Activities of Teacher and Student |
|---------------|---------------------------------------|
| Stage 1:      | The teacher explained the learning objectives and the facilities or logistics needed. The teacher motivated students to be involved in the real problem solving or chosen activities. |
| Stage 2:      | The teacher helped the students define and organize learning assignments related to problems that have been oriented in the previous stage. |
| Stage 3:      | The teacher encourages students to gather the appropriate information and carry out experiments to get the clarity needed to solve the problem. |
| Stage 4:      | The teacher helped the students to share assignments, plan, and prepare works that are appropriate as a result of problem solving in the form of reports, videos, or models. |
| Stage 5:      | The teacher helped students reflect or evaluate the problem solving process. |

2. Research Methods
This study aims to describe the steps in the learning process of eighth grade students of Santo Aloysius Sleman junior high school using PBL approach to inner tangent between the two circles material and find out students’ problem solving ability in inner tangent between the two circles material after following the learning process with the PBL approach. Therefore, this research is a design research. Cobb, Stephen, McClain, & Gravemeijer [10] define design research as research that aims to produce an impact on learning activities designed and to find out how learning can work. The stages carried out in design research are; (1) prior design, (2) trial, and (3) implementation of learning. This research was conducted at Santo Aloysius Sleman junior high school eighth grade which consist of 26 students as research subject. Then choose several students as chosen research subjects for the interview. The determination of subjects in this study was based on indicators of problem solving achieved by students on the test results. Problem solving indicators used in this research were problem solving indicators based on NCTM, namely students can identify elements known, elements asked, and adequacy of the elements needed; students can formulate mathematical problems or arrange a mathematical model; students can apply strategies to solve various problems (similar and new problems) in or outside mathematics; students can explain and interpret results according to the prior problem; students can use mathematics meaningfully [2]. The object of this research was problem solving ability of eighth grade students of Santo Aloysius Sleman junior high school using the Problem Based Learning approach. In this study the researchers designed and developed the Hypothetical Learning Trajectory (HLT) with PBL approach to inner tangent between the two circles material as the learning model. HLT is made, implemented in learning to describe the steps of the learning process that occurs and analyze students’ problem solving abilities. The instruments in this study are the Hypothetical Learning Trajectory (HLT), test questions, and interview guidelines. Data collection techniques carried out in this study were written tests, interviews and documentation. In this study, the data analysis techniques adopted the Salim & Forman theory namely data reduction, data presentation, and conclusion or verification [11].

3. Results and Discussion
Learning was carried out in 1 meeting on Wednesday, March 13th, 2019 for 2 hours of study (2x 40 minutes) to solve problems by applying formulas of tangent line to two circles. After the learning, the
researchers gave a test in the form of a written test on Wednesday, March 20th, 2019 and an interview on Wednesday, March 27th, 2019.

3.1. Learning Process
This research produced learning trajectory of inner tangent between the two circles. Based on the learning trajectory design that has been designed by researchers, there were two activities carried out in the learning process. The first activity was to solve problems related to inner tangent between the two circles, and the second activity was to find a general concept to determine the length of inner tangent between the two circles. These activities were carried out in several stages according to the syntax in Problem Based Learning (PBL).

Stage 1: Orient Students to Problems
At this stage the teacher gave a problem that leads to the learning goal, namely inner tangent the two circles and finds the general concept to determine the length of the inner tangent between the two circles. The problems were:

1. Pemberikan gambar ilustrasi

2. Pada gambar di bawah, panjang jari-jari AO = r, panjang jari-jari DP = \( r \) cm, dan panjang AB = 4 cm. Bagaimana cara menentukan panjang garis OP?

After giving the problem, the teacher asked the students to understand the problems given in the group.

Stage 2: Organize Students to Learn
At this stage, the students were asked to find things to do, and what knowledge must be needed, to achieve learning objectives or problem solving goals. The knowledge needed to solve the problem above is the properties of the tangent of a circle, perpendicular lines, the properties of a rectangle, and the theorem of Pythagoras.

![Figure 1. and Figure 2. Students were discussing the solution of the given problem](image)

Stage 3: Guide individual or group investigations
At this stage, the teacher encouraged the students to gather appropriate information to solve the problems. If there were students who were confused or had no ideas, the teacher would ask questions.
that guided the students to be able to solve the problem. The activities that happened in this stage are as follows:

**Table 2.** The activities that happened in this stage

| Students’ Activities | Guidance |
|----------------------|----------|
| One group tried to solve the problem by making a line by connecting A to P, so that the APB triangle was obtained, and concluded that right is on P. | Teacher: Is it true that the APB triangle is a right triangle? If it is, how it can be concluded that is a right triangle?  
Student: Yes, it is a right triangle, because the AP line is perpendicular to the PB radius.  
Teacher: How do you know that the line is perpendicular?  
Student: From properties of tangent  
Teacher: Is the AP a line tangent? Take a look at the notes about the tangents of the circle.  
Student: AP is not tangent circle.  
Teacher: Can the AP line help you to find the length of the OP line?  
Student: No.  
Teacher: Try to draw another line that can help you |
| One group tried to solve the problem by giving a name at the intersection point of line AB and line OP with Q, so they got two triangles namely AOQ triangle with right angles at O and BPQ triangle with right angle at P. This group could not find the length of the other sides of the formed triangle that haven’t been known yet. | Teacher: What is your goal to make Q?  
Students: in order to form two right triangles namely AOQ triangle and BPQ triangle.  
Teacher: What is the length of QB, QP, AQ and OQ?  
Student: Haven’t known yet  
Teacher: If there is only one known side in a triangle, can we find the other side?  
Student: We can’t  
Teacher: Try to draw another line that can help you |
| Four groups couldn’t associate the knowledge they had before with the given problem. | Teacher: What is the question of the problem?  
Student: Length of OP  
Teacher: Pay attention to the OP. Is the OP tangent to two circles?  
Student: Yes.  
Teacher: What are the properties of tangents to two circles?  
Student: it is perpendicular to the radius drawn to the point of tangency  
Teacher: Pay attention to AO and BP (radius of circle), if you make one of the lines longer and from the center of the other circle you draw a line to OP, what shape is formed? (The teacher gave time to the students to make the line). |

On problem 2, it was not difficult to solve it. Some groups of students directly wrote the formula to find the length of OP with what was obtained at number 1 and some other groups repeated the same steps with number 1 to determine the formula for finding the length of OP. Therefore, no guidance was given when students solved problem 2.
Stage 4: Develop and present the work
At this stage, the teacher asked one group to present the result of the discussion and the other group commented on it. One group was chosen because the results obtained from all groups were similar. The result of the discussion and drawing of one group when they presented the results of their discussion in front of the class, is as follow:

Table 3. Answer of Problems

| Figure 5. The solution of problem 1 | Figure 6. The solution of problem 2 |
|------------------------------------|------------------------------------|

When one group presented the result of their discussion, the other group paid attention. No questions were given to the group because all groups’ solutions were the same, and all groups stated that they understood what they had accomplished.

Stage 5: Analyze and evaluate the problem solving process
The teacher helped students to evaluate the processes they have carried out, as well as their understanding and conclusions regarding inner tangent between the two circles.

The conversation between teacher and students at this stage, is as follow;

Teacher: Pay attention to OP (point to the picture of the result of one group discussion on the blackboard). What formula do you use to find the length of OP?

Student: Use the Pythagoras formula $O'B = OP = \sqrt{A' consort{^2} - AO'^2}$

Teacher: Why is the formula for finding the length of OP the same as the formula for finding the length of $O'B$?
Student: Same because O'B is parallel with OP.
Teacher: How do you find the formula?
Student: We make an extension of AO and draw OP from point B to AO.
Teacher: After that?
Student: Then A right triangle is formed AO 'B with right at O '.
Teacher: Why is the triangle called a right triangle?
Student: Because the lines made form a large rectangle with right angles. So, we can find the length O 'B using the Pythagorean formula.
Teacher: Right. What are OP, AB, and AO?
Student: Tangent of OP, AB is the distance between the centers of two circles. AO is the total length of the radius of the two circles.
Teacher: So, OP is tangent line to two circles. Therefore, to determine the length of the tangents in the two circles, you can use the formula you have obtained earlier, \( OP = \sqrt{AB^2 - AO^2} \).

### 3.2. Problem Solving Ability

From the results of the test, the researchers grouped the students into two different groups based on the way of problem solving process; there were 21 students who solved the problem by writing down the unknown, and there were 5 students who solved the problem without writing down the known and asked questions. Based on the results of the grouping, the researcher discussed with the teacher to choose 2 students who were considered to have good communication skills. The purpose of the test and interview is to find out the problem solving abilities of students on the material that has been taught. Problem solving indicators used in this research were indicators of problem solving Based on NCTM. The results of the interview were coded with capital letters where; “P” refers to the researcher and “A” refers to the first group, and “B” refers to the second group. The problems are given in the test, as follow:

#### Table 4. Results of Analyze of Students Problem Solving Ability

| Indicators | Test Results | Interview result |
|------------|--------------|------------------|
| 1. Students can identify the elements that were known, asked, and the adequacy of the elements needed | First Subject | P: After you have got the problem, what you do you do? |
| | | A: Find what is known first |
| | | P: What is known? |
| | | A: The length of radius is 6, the length of rope c is 16cm, asked to find the length of the second and the third pulley center |
| | | P: What is AS? |
| | | A: the length of rope |

The above result test shows that the students wrote what is known so that it can be concluded that students can identify the known elements from the problem gave. The subject did not write down what was asked implicitly in the question, but the students knew what elements were asked to be seen from the student’s final conclusion of the solution. This is supported by the results of the interview on the side.
From the result of tests and interviews on the first indicator, it can be concluded that the students can identify the elements that are known, asked, and the adequacy of the elements needed in the given problem.

2. Students can formulate mathematical problems or arrange mathematical model

From the results of the test, students wrote down a mathematical formula or model that will be used to solve what was asked in the problem, namely determining the length of the rope c. This is confirmed by the results of the interview as well.

P: Why did you use the formula to determine the length of the rope c?
A: From the problem we know the length of radius 6, length of rope c 16cm, told to find the length of the pulley center points two and three. So, I use the formula to find the tangent lengths of the two circles. If I change it to find the distance, the formula will be like that.

From the result of test and interview on the second indicator, it can be concluded that the students can formulated mathematical problems or arrange mathematical model.

3. Students can implement strategies to solve various problems

The test result shows that the student used the formula as wrote above (which has been answered in the second indicator) to solve the given problem. Then, the student substituted the values of the variables which the mathematical operation is correct too. Therefore, it can be concluded that the student can implement strategies to solve the problem correctly.

4. Students can explain the result based on the prior problem

The result of the above test shows the student wrote conclusion at the end of the solution. The student wrote conclusions based on what was asked about the given problem, the distance between the center of the second and the third pulley.

Therefore, it can be concluded that the subject can explain the results according to the origin problem.

5. Students can use mathematics meaningfully

From the result of the test, the student knew the meaning of the variables used in the mathematical model created and performs mathematical operations correctly. This is supported by the result of the interview.

The result of the above test shows the student can used mathematics meaningfully.

Second Subject

1. Students can identify the elements that were known, asked, and the adequacy of the elements needed

From the results of the test, the subject did not write down the elements that were known and asked in the questions gave. But the subject implicitly knows what is knows and asked. This can be seen from the subject's worked on the
next indicators. This is also supported by the results of the interview on the first indicator, the subject knows what elements are knows and who are asked in the problem given. Therefore, From the result of tests and interviews on the first indicator, it can be concluded that the students can identify the elements that are known, asked, and the adequacy of the elements needed in the given problem.

2. Students can formulate mathematical problems or arrange mathematical model

From the results of the test, students wrote down a mathematical formula or model that will be used to answer what was asked in the problem, namely determining the length of the rope c. This is supported by the results of interviews besides.

From the result of test and interview on the second indicator, it can be concluded that the students can formulated mathematical problems or arrange mathematical model.

3. Students can implement strategies to solve various problems

From the results of the above tests, it appears that the subject used the formula as wrote above (which has been answered in the second indicator) to solve the problem gave. The subject then substituted the values of known variables into the formula correctly and performs mathematical operations correctly too. Therefore, it can be concluded that the student can implement strategies to solve the problem correctly.

4. Students can explain the result based on the prior problem

From the results of the above tests, the subject wrote conclusions at the end of the completion. The subject wrote conclusions according to what was asked about the problem gave, namely the distance between the center of pulley II and pulley III. Therefore, it can be concluded that the subject can explain the results according to the origin problem.

5. Students can use mathematics meaningfully

From the results of these tests, the subject knows the meaning of the variables used in the mathematical model created and performs mathematical operations correctly. This is supported by the results of the interview besides.

The result of the above test shows the student can used mathematics meaningfully.

Based on analysis of test and interview results, it can be concluded that all the students experienced the five problem solving ability indicators based on NCTM.

4. Conclusion

The research is conducted to eighth grade students of Santo Aloysius Sleman junior high school on inner tangent between the two circles using Problem Based Learning (PBL) approach as the learning model. HLT is made, implemented in learning to describe the steps of the learning process and analyze students’ problem solving ability. The steps of the research were as follow: orient the students through inner tangent between the two circles; organize the students to learn; guide the individual’s or groups’ experience towards the learning process; develop and present the students’ work from group discussions; and conclude the results of the learning process that have been done. From the result of
tests and interviews, the researchers found that all students experienced the five indicators of problem solving ability based on NCTM. The indicators of problem solving abilities are students can identify known elements, the elements that are asked, and the adequacy of the required elements; students can formulate mathematical problems or arrange mathematical model; students can apply strategies to solve various problems (similar and new problems) inside or outside mathematics; students can explain and interpret the results according to the prior problem; and students can use mathematics meaningfully.

References
[1] Ibrahim and Suparni 2009 Strategi Pembelajaran Matematika (Yogyakarta: Teras)
[2] National Council of Teacher of Mathematics (NCTM) 2000 Principles and Standarts to Education Research (Enschede, Netherland: National Intitute for Curriculum Development)
[3] Carson J 2007 A problem with problem solving: teaching thinking without teaching knowledge The Mathematics Educator 17 7-14
[4] Wagiran 2007 Peningkatan keaktivan mahasiswa dan reduksi miskonsepsi melalui pendekatan problem based-learning Jurnal Kependidikan 37 1-22
[5] Sumartini T S 2016 Peningkatan kemampuan pemecahan masalah matematis siswa melalui Pembelajaran Berbasis Masalah Jurnal Pendidikan Matematika STKIP Garut 5
[6] Cahyani H and Setyawati R W 2017 Pentingnya meningkatkan kemampuan pemecahan masalah melalui pbl untuk mempersiapkan generasi unggul menghadapi MEA Prosiding Seminar Nasional Matematika 151–60
[7] Daryanto 2014 Pendekatan Pembelajaran Saintifik Kurikulum 2013 (Yogyakarta: Gava Media)
[8] Rusman 2012 Model – Model Pembelajaran (Jakarta: PT RajaGrafindo Persada)
[9] Hosnan 2014 Pendekatan Saintifik dan Kontekstual dalam Pembelajaran Abad 21 (Bogor: Ghalia Indonesia)
[10] Indriani N 2017 Penelitian Desain Mengenai Keliling Lingkaran Menggunakan Pendekatan Pembelajaran Matematika Realistik Pada Siswa Kelas V SD Budya Wacana Yogyakarta undergraduate thesis
[11] Salim A and Forman A 2006 Pengantar dan Berpikir Kualitatif dalam Agus Salim, Teori dan Paradigma Penelitian Sosial (Yogyakarta: Tiara Wacana)