MODES OF DISC ACCRETION ONTO BLACK HOLES

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I review our current theoretical understanding of the different possible rotating modes of accretion onto a black hole. I discuss both thick adiabatic flows and radiatively efficient thin disc solutions. I present a new self-consistent unified analytical description of two-phase thin disc–corona systems in which magneto-rotational instability is responsible for angular momentum transport in the disc and for the corona generation. Finally, I briefly discuss the role of magnetic fields in bridging the gap between accretion discs theory and jet production mechanisms.

1 Accretion discs around black holes: basics

Accretion is the physical process by which celestial bodies aggregate matter from their surroundings. In the case of compact objects, the gravitational binding energy that such matter must release for accretion to occur is a powerful source of luminosity. If the accreting material possesses angular momentum, though, gravitational contraction is impeded by a centrifugal barrier, as specific angular momentum is hard to dispose of, and the formation of a disc is unavoidable. It is such a disc that mediates the process of accretion: neighboring annuli of differentially rotating matter experience a viscous shear (of some kind) that transports angular momentum outwards and allows matter to slowly spiral in towards the center of the potential.

The variety of spectral energy distributions from accretion-powered objects is a clear sign that different accretion modes are possible. A fundamental discriminant is the radiative efficiency of the flow, defined as the ratio of the output bolometric luminosity to the rest mass energy accretion rate, $\epsilon \equiv L/\dot{M}c^2$. When radiative cooling is efficient (standard ‘Shakura-Sunyaev’ solution, hereafter SS1), the maximal energy per unit mass available is uniquely determined by the binding energy at the innermost stable orbit, and this fixes the efficiency. A general relativistic treatment reveals that $\epsilon$ increases from 0.06 (non-spinning black holes) to 0.42 (maximally rotating Kerr black holes). For radiatively efficient discs, the bolometric luminosity is characterized (and in general limited) by the so-called Eddington luminosity: the luminosity at which the radiative momentum flux from a spherically symmetric source is balanced by the gravitational force of the central object:

$$L_E = \frac{4\pi G m_p M c}{\sigma_T} \simeq 1.3 \times 10^{38} \frac{M}{M_\odot} \text{ erg s}^{-1},$$

where $m_p$ is the proton mass and $\sigma_T$ the Thompson scattering cross section.
When, on the other hand, radiative cooling is negligible, accretion is adiabatic and \( \epsilon \ll 1 \). Then, unless other mechanisms of energy redistribution are important, such as convection or conduction, the gravitational binding energy of the gas is converted into thermal energy and merely advected with the flow (advection dominated accretion flows or ADAF). However, any imbalance between enthalpy, gravitational and kinetic energy can drastically modify the nature of the flow, which can be dominated by convective motions (CDAF) or result in substantial mass outflow.

I will begin this review with radiatively efficient discs, paying special attention to the properties of the SS solution (sec. 2.1) and of its modification due to the effects of turbulent magnetic fields in the disc and the generation of a magnetic corona (sec. 2.2). After a brief discussion of optically thin radiatively efficient solutions (sec. 2.3), I will turn to adiabatic flows, both optically thin (sec. 3.1) and thick (sec. 3.2), concluding with a brief discussion of the role of convection and outflows (sec. 3.3). In sec. 4, I sketch the possible relationships between different accretion modes and MHD jet production mechanisms, before summarizing (sec. 5). Most of the subjects touched upon here are discussed thoroughly in more comprehensive reviews.

## 2 Radiatively efficient accretion discs

### 2.1 Optically thick flows: the standard solution

The SS solution represents an optically thick, geometrically thin, radiatively efficient accretion flow. In steady state, the disc structure is determined by solving simultaneously four conservation equations (of vertical momentum, mass, angular momentum and energy). If the equations of state for the pressure and opacity as functions of density and temperature are also given together with a viscosity law, the full disc structure can be calculated exactly. The flow is assumed to be geometrically thin, so that its pressure vertical scaleheight, \( H \equiv \frac{1}{2} \frac{\partial P}{\partial z} \), at a distance \( R \), is much smaller than the radial coordinate, \( H \ll R \). In fact, since there is no net motion of the gas in the vertical direction, conservation of vertical momentum reduces to the equation of hydrostatic equilibrium, from which we get \( \frac{H}{R} \simeq \frac{c_s}{v_K} \), namely the ratio of the disc height to the radius is approximately equal to the ratio of the local isothermal sound speed \( c_s = (P/\rho)^{1/2} \) to the local Keplerian velocity \( v_K = (GM/R)^{1/2} \). The mass conservation equation for steady state accretion reads \( \dot{M} = -2\pi R\Sigma v_r \), where \( \Sigma = \int_{-\infty}^{+\infty} \rho dz \simeq 2H\rho \) is the surface density and \( v_r \) is the radial velocity of the spiraling matter. Angular momentum conservation can be expressed by equaling the torque exerted by viscous stresses to the net rate of change of angular momentum. This yields \( W_{R\phi} = \frac{\dot{M}R^2}{2\pi\Sigma} J(R) \), where \( W_{R\phi} \) is the domi-
nant \((R - \phi)\) component of the stress tensor, \(\Omega = (GM/R^3)^{1/2}\) is the angular velocity and the term \(J(R) = \left[1 - \left(\frac{dR}{R}\right)^{1/2}\right]\) comes from the (Newtonian) no-torque at the inner boundary condition, and measures the rate at which the angular momentum is deposited onto the compact object. From the energy conservation equation the surface emissivity can be calculated by taking the opposite of the divergence of the flux. This yields, for the heat production rate,

\[
Q_+ = \frac{3GM\dot{M}J(R)}{8\pi R^3},
\]

which is completely independent on the yet unspecified viscosity law.

Understanding the nature of the disc viscosity, and therefore understanding how can the disc accrete, has been (and partially still is) the central problem of accretion disc theory. Observationally, the accretion rates needed to explain the luminosities we see are many orders of magnitude larger than standard microscopic viscosities could provide. But if the disc is turbulent the effective viscosity due to interacting eddies could easily be large enough. The original SS solution relies on a dimensional scaling for the turbulent viscosity coefficient, \(\nu = W_{R\phi}/\Omega = \alpha_v c_s H\), with the constant \(\alpha_v < 1\) (subsonic turbulence).

In geometrically thin accretion discs, the internally generated heat \(Q_+\) is transported vertically before being radiated at the surface. For optically thick discs, photons are transported to the surface via diffusion, and the vertical photon flux is given by \(F_0 = \frac{c}{4\pi} \frac{dP_{\text{rad}}}{dz}\). Therefore, the energy dissipation is locally balanced by radiation from the two disc surfaces, and we have \(Q_+ = F_0 = Q_+\). Such a balance is established over a thermal timescale, \(t_{\text{th}} \approx 1/\alpha_v \Omega\). In thin discs, this is much smaller than the viscous time, the time available to the accreting gas to radiate the energy released by the viscous stresses: \(t_v \approx (R/H)^2 J(R)/\alpha_v \Omega\). Indeed, we are led the conclusion that SS discs are “cool” enough in that \(kT \ll GMm_p/R\), and therefore \(H/R \ll 1\), as we had assumed.

The final expressions for the physical quantities in the disc as functions of central mass, accretion rate, viscosity parameter and radius have simple algebraic expression when both total pressure and opacity are dominated by one term only. As the free-free absorption is strongly suppressed at high temperatures, electron scattering is the main source of opacity in the inner, hotter parts of the disc, while free-free absorption dominates in the outermost, colder parts. Analogously, gas pressure dominates in the outer regions of the disc, while radiation pressure becomes predominant in the inner ones. Overall, three regions can be distinguished in a standard geometrically thin, optically thick accretion disc: a) an inner part where electron scattering determines the opac-
ity and radiation pressure is larger than gas pressure; b) a middle region where electron scattering is still more important than free-free absorption, but gas pressure dominates over radiation pressure; c) an outer region, where thermal gas pressure dominates and free-free absorption is the main source of opacity.

2.2 Beyond the standard model: Coupled magnetic disc–corona solutions

At present magneto-rotational instability (MRI) is favoured as the primary source of the turbulent viscosity needed to explain the luminosities of accreting black holes. Our knowledge of the physics of MHD turbulence in accretion discs can be used to build a self-consistent model of MRI-driven, thin accretion disc–corona systems in the following way.

Let us assume equipartition between kinetic turbulent energy and magnetic field excited by the MRI (Alfvén equal to turbulent speed, \( v_A = v_t \)). For almost incompressible flows, the coefficient of turbulent viscosity \( \nu \simeq v_t^2/3\Omega \), which implies \( Q_+ = \frac{2}{3}c_s\nu\beta\Omega = c_sP_{\text{mag}} \). Therefore, once the relationship between magnetic and disc pressure (given either by gas or radiation) is established, the accretion disc structure can be fully described.

To find such a relationship, we can assume that the magnetic field escapes from the thin disc via buoyancy, with a timescale \( t_b = H/2v_A \). Then, a crucial point is that the growth rate of MRI is influenced by the ratio of the gas to magnetic pressure: \( \sigma = \Omega c_g/v_A \), where \( c_g \) is the gas sound speed. The saturation field can be found by noting that, asymptotically, \( \sigma t_b = O(1) \); this automatically gives the desired scaling for the magnetic pressure in MRI dominated turbulent flows:

\[
P_{\text{mag}} = \alpha_0 \sqrt{P_{\text{gas}}P_{\text{tot}}},
\]

where \( \alpha_0 = (\frac{P_{\text{mag}}}{P_{\text{gas}}})^{1/2} \) is a constant, not necessarily smaller than unity.

The magnetic flux escaping in the vertical direction may dissipate a substantial fraction of the gravitational binding energy of the accreting gas outside the optically thick disc, with obvious deep implications for the spectrum of the emerging radiation. The fraction \( f \) of the total power dissipated in the low-density environment above and below the disc (in the so-called corona) is determined by the ratio of the vertical Poynting flux \( (F_P \simeq v_A P_{\text{mag}}) \) to the local heating rate \( Q_+ \). Under the assumption of equipartition between turbulent and magnetic energies, this translates into

\[
f = \frac{v_A}{c_s} = \sqrt{\frac{2}{\beta}} \simeq \sqrt{2\alpha_0} \left( 1 + \frac{P_{\text{rad}}(R)}{P_{\text{gas}}(R)} \right)^{-1/4}.
\]
The ratio of the radiation to gas pressure can be calculated by simply taking the SS solution and modifying it by reducing the locally dissipated energy by the factor $(1 - f)$. Then, for every value of the physical parameters $R$, $M$, $M$ and $\alpha$, Eq. (3) represents a nonlinear algebraic relation that determines $f$ and the full solution of the accretion disc–corona system. It turns out that for $2\alpha_0 < 1$ such a solution is unique, and has the following properties: $f$ tends to its maximum value, $\sqrt{2\alpha_0}$, when gas pressure dominates (low accretion rates), and decreases as the accretion rate increases and radiation pressure becomes more and more important. If instead $2\alpha_0 > 1$, there are no subsonic (i.e. satisfying $v_t < c_s$) solutions for gas pressure dominated regions, while in the radiation dominated part of the flow two solutions are possible for every value of the accretion rate. The first is a “standard” (unstable) one, with $f \ll 1$, while the second appears due to the feedback effect of the closure relation (3). This new solution, which is discussed in detail in [15], has $f \to 1$ (corona dominated) as the accretion rate increases and is thermally and viscously stable. It represents a new class of geometrically thin, optically thick corona-dominated accretion discs, relevant for systems accreting at a rate above the Eddington one.

### 2.3 The SLE solution: radiatively efficient optically thin flows

The first self consistent alternative to the standard solution for rotating accretion flows around black holes was discovered in an attempt to explain the hard spectrum of the classical black hole candidate Cygnus X-1. This so-called SLE solution is a two temperature one, with the protons much hotter than the electrons $T_p \gg T_e$. This is the case if the energy released by viscous dissipation was distributed equally among the carriers of mass, i.e. mostly to the protons (which radiate very inefficiently) and only a fraction $m_e/m_p = 1/1833$ to the electrons. The hot proton component then dominates the pressure and keeps the disc geometrically thick. This in turn leads to low density (which scales as $H^{-3}$ for given $\dot{m}$) and low Coulomb energy exchange between protons and electrons, that self-consistently implies $T_p \gg T_e$. The four main equation of the disc structure are the same as in the standard model, but the heating–cooling balance is now described by two separate equations for the two species, where the electrons mainly cool by up-scattering soft radiation coming from an outer cold disc or from dense cloudlets embedded in the hot flow.

Although the model is able to reproduce reasonably well observed spectra of accreting black holes, it is thermally unstable: an increase in $T_p$ would lead to disc expansion in the vertical direction, further reduction of Coulomb cooling and increase of proton temperature, leading to an instability. This instability is cured if the effect of advection is taken into account in the energy
equation, as will be briefly outlined in the following section.

3 Adiabatic accretion flows

3.1 Optically thin solutions

It was first noted by Ichimaru\cite{Ichimaru1977} that, in an optically thin, geometrically thick accretion disc such as the SLE one, the density might be so low that the ions are unable to transfer energy to the electrons in a timescale smaller than the viscous one. Then, part of the energy is advected with the proton flow and swallowed by the central black hole. The energy equation then can be rewritten in a vertically integrated form: \( q_+ - q_- = q_{\text{adv}} \equiv \Sigma v_r T \frac{\partial S}{\partial R} \), where \( S \) is the specific entropy of the gas. This kind of solution was later named ‘ion supported torus’\cite{Novikov1973}, and it was also demonstrated that it can be established only for accretion rates lower than a critical value \( \dot{m} < \dot{m}_{\text{crit}} \sim \alpha^2 \), ensuring that the density is low enough, either because the accretion rate is low, or the viscosity high. The flow is optically thin and radiatively inefficient. In recent years, much work has been devoted to the detailed study of such flows, renamed Advection Dominated Accretion Flows (ADAF) after Narayan and Yi\cite{Narayan1994} presented a full analytical self-similar 1D solution to the problem. This self-similar solution reveals the basic properties of an ADAF: a) the radial accretion time-scale is much shorter than that of a thin disc; b) sub-Keplerian rotation occurs, due to large internal pressure support; c) the flow is geometrically thick, in that the vertical scaleheight \( H \sim R \). More uncertain, but crucial, issues for determining physical and observable properties of ADAFs are the assumptions that ions and electrons interact only through Coulomb collisions\cite{Blandford1977}, or that only a tiny fraction of the turbulent energy is dissipated into the electrons\cite{Cranmer1998, Stone1998}.

3.2 Optically thick solutions

The last class of accretion modes comprises adiabatic solutions at accretion rates close to (or above) the Eddington one. In these cases, the inflow timescale in the inner part of the disc becomes smaller than the time it takes for a photon to escape from the disc, and the radiation produced is trapped by the flow and advected with it\cite{Ebisuzaki1985, Ebisuzaki1987}. The radius inside which radiation is trapped moves outwards as the accretion rate increases\cite{Ebisuzaki1987}. When it becomes of the

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There is, though, the possibility that magnetized radiation pressure dominated discs are subject to “photon bubble” instabilities\cite{Ebisuzaki1987} that lead to strong density inhomogeneities. As shown in\cite{Ebisuzaki1987}, such discs may radiate well above the Eddington limit and remain geometrically thin without being disrupted.
order of the Eddington one, the disc thickness stays moderate and a vertically integrated approximation may be retained (slim discs)\textsuperscript{31}. In the limit $m \gg 1$, though, the behaviour of the disc and the possible relevance of strong outflows still remain open issues. Once again, the problem is inherently 2D, and the simultaneous roles of convection, advection and outflows have to be assessed in order to model properly the expected SED. Unlike for the the optically thin ADAF/CDAF cases, the observable features of these optically thick solutions have been investigated in only a handful of cases so far\textsuperscript{32,33}, thus reducing the general appreciation of their importance for interpreting observations.

### 3.3 The role of convection and outflows in adiabatic flows

The ADAF solution has received a great deal of interest in the last decade because of its potential capability to explain the observational data from our Galactic Center\textsuperscript{23,24}. On the other hand, geometrically thick adiabatic flows are ideal models to test global numerical simulations of MHD turbulent accretion flows against, without having to deal with the complications of radiative transfer needed to simulate thin, radiative efficient discs. Indeed, already when the 1D approximation of self-similar ADAF theory is abandoned, and the full 2D nature of the problem is analysed, both from the theoretical point of view\textsuperscript{22} and from numerical simulations\textsuperscript{26,25}, it is clear that radiatively inefficient flows are prone to strong convective instabilities and/or powerful outflows. In general\textsuperscript{6}, convective flows are more likely at low values of the viscosity parameter, while strong outflows are generated for high values of $\alpha$. In the former case (purely convective flows), accretion is effectively stifled, with little or no mass inflow or outflow: the energy extracted in the inner part is convectively transported outward. Such a redistribution of energy among fluid elements in the accreting gas alters the purely advective nature of the flow and modifies the radial profiles of physical quantities, as the density, with profound implication for the interpretation of the observed radiation. In the latter case, systematic outflows remove mass and energy from the flow, with little accretion onto the black hole\textsuperscript{34,35}. Despite the big efforts made by several groups, both on the theory and on the simulation side, the relative importance of convection and outflow for adiabatic flows is still matter of a vigorous debate\textsuperscript{36,37}, the controversy being essentially over the capability of any hydrodynamical model supplemented with $\alpha$-like viscosity prescriptions to capture the basic physical properties of an inherently magneto-hydrodynamical system. As usual, it appears likely that such controversy will only be settled with the collection of more constraining observational data.
Figure 1: Schematic representation of the different radiatively efficient and inefficient accretion flows, for $\alpha_0 = 0.05$ (thin lines) and $\alpha_0 = 0.75$ (thick lines), as functions of accretion rate ($\dot{m} \equiv \epsilon_0 \dot{M} c^2 / L_E$, vertical axis) and surface density ($\Sigma$, horizontal axis). We assumed $\epsilon_0 = 0.08$ (Newtonian disc, non-rotating hole), and fixed $M = 10M_\odot$, $R = 15R_S$. Stable solutions are represented by solid lines, unstable ones by dot-dashed line. Radiatively inefficient (stable) modes are in green, optically thick, geometrically thin, radiatively efficient (stable) solutions in blue, with indicated the dominant contribution to the pressure and the value of the coronal fraction $f$. Unstable (radiation pressure) thin disc solutions are in red, while SLE solution are in orange. Also indicated in parenthesis are additional physical processed that are likely to be relevant for each kind of accretion mode.

4 Accretion modes and jet engines

It is a well established observational fact that accretion onto rotating astrophysical object is often accompanied by the ejection of collimated outflows. Magnetic fields are likely to be a key element for the understanding of such a phenomenon and a number of model of magnetically-driven jets from rotating accretion flows have been devised (for recent reviews see\cite{38,37}). Almost all of them link the power channeled into the jet to the intensity of the poloidal component of the magnetic field in the inner regions of the accretion flow\cite{39,40}:

$$L_{\text{jet}} \approx \left( \frac{B_p}{B_\phi} \right)^2 A \Omega R,$$

where $A$ represent the area of the acceleration region and $\Omega R$ is the rotational velocity of the field lines. Thus, the kinetic output from a magnetized, rotating accretion flow depends on two main factors: the scale-height of the magnetically dominated structure (because $B_p/B_\phi \sim H/R$) and the maximal rotational velocity attainable. The former should be large in all radiatively inefficient modes (in particular the more outflow-dominated ones at
high viscosity parameter) and in those dominated by powerful magnetic coro-
nae (at low/high accretion rates for low/high viscosities, respectively). The
latter depends on the angular momentum of the central hole (high spin favors
the generation of powerful jets) and on the inner boundary condition for the
accretion flow, and can be tackled only by a full general relativistic treatment
of the inner disc. Different combinations of the two factors may cause the
variety of radio properties of AGN.

5 Summary: a revised accretion map

Some of the properties of the different solutions can be summarized by the
diagram that places them in the surface density (Σ)–accretion rate (ṁ) plane
(at fixed distance from the central source), as shown in Fig. 1. For low values of
the viscosity parameter α0, the solutions split into two separate branches: the
optically thin and the optically thick one. Optically thin solutions exist only at
low accretion rates: they are the ADAF (thermally stable) and SLE (Shapiro-
Lightman-Eardley, thermally unstable). Also at low ṁ, a geometrically thin,
gas pressure dominated solution exists, with an optically thin corona whose
relative power is \( f \sim \sqrt{2\alpha_0} \); this solution becomes unstable (and loses its
corona) at the value of the accretion rate for which radiation pressure becomes
larger than gas pressure at the given distance from the black hole. At even
higher accretion rates only a radiatively inefficient, optically thick solution
exists ('slim disc'), which is thermally stable.

The topology of the diagram changes for high viscosity parameter: the
advective, radiatively inefficient solutions (ADAF, slim disc) becomes a single
branch, while radiatively efficient solutions form a second one. Of those, only
corona-dominated, for accretion rates above a critical value, are stable.

Fig. 1 also shows in parenthesis the additional processes, beside those taken
into account by the model, likely to be relevant for each kind of accretion mode.
They are meant to be indicative of the open theoretical issues in the field, and
will likely indicate the direction of the research in the immediate future.

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