Three-dimensional parton distribution functions $g_{1T}$ and $h_{1L}^\perp$ in the polarized proton-antiproton Drell-Yan process

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Abstract. We present predictions of the unweighted and weighted double spin asymmetries related to the transversal helicity distribution $g_{1T}$ and the longitudinal transversity distribution $h_{1L}^\perp$, two of eight leading-twist transverse momentum dependent parton distributions (TMDs) or three-dimensional parton distribution functions (3dPDFs), in the polarized proton-antiproton Drell-Yan process at typical kinematics on the Facility for Antiproton and Ion Research (FAIR). We conclude that FAIR is ideal to access the new 3dPDFs towards a detailed picture of the nucleon structure.

1 Introduction

The nucleon spin structure is an active direction under both theoretical and experimental investigations. A number of new physical quantities of the nucleon, which can provide a detailed picture of the nucleon with more detailed information in three dimensional momentum space [1,2], have been introduced. At leading twist, the quark-quark correlation matrix [3] can be decomposed into eight transverse momentum dependent parton distributions (TMDs), or we call them three-dimensional parton distribution functions (3dPDFs). Besides the three usual parton distribution functions, i.e., the unpolarized one with light-cone longitudinal momentum and transverse momentum distribution $f_1$, the helicity one with longitudinal helicity distribution $g_{1L}$, and the transversity one with transversal spin distributions $h_1$, there are five new ones. Among them the Sivers distribution $f_{1T}$ and its chiral-odd partner, the Boer-Mulders distribution $h_{1L}^\perp$, are well known for their T-odd property, i.e., they change sign under naive time reversal. The other three of the new 3dPDFs, i.e., the pretzelosity distribution $h_{1T}^\perp$, the transversal helicity distribution $g_{1T}$, and the longitudinal transversity distribution $h_{1L}^\perp$, are T-even. They can be measured through the semi-inclusive deep inelastic scattering simple. However, the single spin asymmetry related to the pretzelosity distribution $h_{1T}^\perp$ in the semi-inclusive deep inelastic scattering is rather small. For the chiral-odd distributions $h_{1T}^\perp$ and $h_{1L}^\perp$, they can be probed through the single spin asymmetries when combined with another chiral-odd distribution $h_{1}^{\perp L}$ in the pion-nucleon Drell-Yan process [7,8]. In Ref. [9], it has been shown that the polarized proton-antiproton Drell-Yan process is ideal to probe the pretzelosity distribution $h_{1T}^\perp$ and the magnitudes of the related spin asymmetries are significantly large. Thus it is natural to discuss the spin asymmetries related to the rest two 3dPDFs, $g_{1T}$ and $h_{1L}^\perp$, in the polarized proton-antiproton Drell-Yan process.

Since the proposal on measuring the transversity distributions via the polarized proton-antiproton Drell-Yan process by the polarized antiproton experiment (PAX) collaboration [10,11], there have been some other experiments [12,13,14,15,16,17,18] on the measurements of the transversity distributions. Recently, there has been new technical progress [19] towards the goal for a proton-antiproton collider with both beams polarized [20], and such plans have the potential to be realized at FAIR (Facility for Antiproton and Ion Research) in GSI Helmholtzzentrum für Schwerionenforschung. The expected antiproton beam polarizations might be $0.15 \sim 0.20$ (spin filtering with transverse target orientation) or $0.35 \sim 0.40$ (longitudinal) [21,22]. Thus the three new 3dPDFs, $h_{1T}^\perp$, $g_{1T}$, and $h_{1L}^\perp$, with important information on the quark spin and orbital correlation of the nucleon, could be measured on FAIR. Our calculation below shows that some of the double spin asymmetries related to $g_{1T}$ and $h_{1L}^\perp$ in the polarized proton-antiproton Drell-Yan process are large, and thus FAIR is an ideal facility to access the new 3dPDFs for revealing more information towards a detailed picture of the nucleon.

2 T-even 3dPDFs in the light-cone quark-diquark model

In the light-cone quark-diquark model [23], the Melosh-Wigner rotation plays an important role to understand the proton spin puzzle [24,25] due to the relativistic effect of quark transversal motions. The T-even 3dPDFs have

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been calculated \[1,2,3,20,21,25,26\]:

\[
\begin{align*}
J^{(uv)}_1(x, k_T^2) &= \frac{1}{16\pi^2} (\sin^2 \theta_0 \phi^2 + \cos^2 \theta_0 \phi^2), \\
J^{(dv)}_1(x, k_T^2) &= \frac{1}{8\pi^2} \frac{1}{3} \sin^2 \theta_0 \phi^2,
\end{align*}
\]

and

\[
\begin{align*}
J^{(uv)}(x, k_T^2) &= -\frac{1}{16\pi^2} \times \frac{1}{9} \sin^2 \theta_0 \phi^2 \mathcal{W}_V^j(x, k_T^2), \\
J^{(dv)}(x, k_T^2) &= -\frac{1}{8\pi^2} \times \frac{1}{15} \sin^2 \theta_0 \phi^2 \mathcal{W}_V^j(x, k_T^2),
\end{align*}
\]

with the notation

\[j = g_L, g_T, h_1, h_1^L, h_1^L,\]

and the superscripts “uv” and “dv” stand for the valence up and down quark distributions respectively. \[\varphi_D(D = V, S)\] is the wave function in the momentum space for the quark-diquark, and for which we can use the Brodsky-Huang-Lepage (BHL) prescription \[20,23,30\]:

\[
\varphi_D(x, k_T^2) = A_D \exp \left\{ -\frac{1}{8\alpha_D} \left[ \frac{m_q^2 + k_T^2}{x} + \frac{m_D^2 + k_T^2}{1 - x} \right] \right\},
\]

The parameters \[\alpha_D = 0.33 \text{ GeV}\] (which is the same for \[D = V, S\]), the quark mass \[m_q = 0.33 \text{ GeV}\], the diquark mass \[m_S = 0.60 \text{ GeV}\], \[m_V = 0.80 \text{ GeV}\], and \[\theta_0 = \pi/4\] are adopted for numerical calculation. \[\theta_0\] is the mixing angle that breaks the SU(6) symmetry when \[\theta_0 \neq \pi/4\]. The Melosh-Wigner rotation factors \[W_D \ (D = V, S)\] are

\[
\begin{align*}
W_{i,2}^{UL}(x, k_T^2) &= \frac{(xM_D + m_q)^2 - k_T^2}{(xM_D + m_q)^2 + k_T^2}, \\
W_{i,2}^{UL}(x, k_T^2) &= \frac{2M_N(xM_D + m_q)^2}{(xM_D + m_q)^2 + k_T^2}, \\
W_{i,1}^{UL}(x, k_T^2) &= \frac{(xM_D + m_q)^2}{(xM_D + m_q)^2 + k_T^2}, \\
W_{i,1}^{UL}(x, k_T^2) &= \frac{2M_N^2}{(xM_D + m_q)^2 + k_T^2}, \\
W_{i,1}^{UL}(x, k_T^2) &= -\frac{2M_N^2}{(xM_D + m_q)^2 + k_T^2}, \\
W_{i,1}^{UL}(x, k_T^2) &= -\frac{2M_N^2}{(xM_D + m_q)^2 + k_T^2},
\end{align*}
\]

where

\[
M_D = \sqrt{\frac{m_q^2 + k_T^2}{x} + \frac{m_D^2 + k_T^2}{1 - x}}.
\]

Using Eqs. \[11\] and \[12\], the polarized distributions can be given by the unpolarized distributions as

\[
\begin{align*}
J^{(uv)}(x, k_T^2) &= \left[ J_1^{(uv)}(x, k_T^2) - \frac{1}{2} J_1^{(dv)}(x, k_T^2) \right] \mathcal{W}_V^j(x, k_T^2), \\
&= \frac{1}{6} J_1^{(dv)}(x, k_T^2) \mathcal{W}_V^j(x, k_T^2), \\
J^{(dv)}(x, k_T^2) &= \frac{1}{3} J_1^{(dv)}(x, k_T^2) \mathcal{W}_V^j(x, k_T^2),
\end{align*}
\]

The \[g_T\] and \[h_1^L\] related asymmetries in the polarized proton-antiproton Drell-Yan process

In the polarized proton-antiproton Drell-Yan process, the cross section is \[31\]

\[
\frac{d\sigma}{dx_\perp dq_T d\Omega} = \frac{1}{4\alpha^2} \left\{ (1 + \cos^2 \theta) F_{UU} + S_{UL}S_{UL} \sin^2 \theta \cos 2\phi F_{LL}^{\cos 2\phi} \right. \\
+ |S_{UL}|S_{UL}(1 + \cos^2 \theta) \cos \phi_0 F_{TT}^{\cos \phi_0} \right. \\
+ S_{UL}S_{UL} \sin^2 \theta \left[ \cos(2\phi + \phi_0) F_{LL}^{\cos(2\phi + \phi_0)} \right. \\
+ \left. \cos(2\phi - \phi_0) F_{LL}^{\cos(2\phi - \phi_0)} \right] + \cdots
\]
where both quarks and antiquarks of all flavors are taken into account during the summation over $q$. The unit vector is defined as $\mathbf{h} \equiv q \mathbf{r}/q_T$.

Considering the charge conjugation invariance

$$ f_p^q(x, k_T^2) = f_p^\bar{q}(x, k_T^2), \quad f_\bar{p}^q(x, k_T^2) = f_\bar{p}^\bar{q}(x, k_T^2), \quad (20, 21) $$

with $p$ for proton and $\bar{p}$ for antiproton, and using the method introduced in Refs. 22, 33, we get

$$ \int dq_T F_{UU}^{\phi} = \frac{1}{N_c} \sum_q c_q^2 f_1^q(x_a) f_1^q(x_b), \quad (22) $$

$$ \int dq_T \frac{q_T^2}{8 M_N^2} F_{LL}^{\cos 2\phi} = -\frac{1}{N_c} \sum_q c_q^2 h_{1L}^1(x_a) h_{1L}^1(x_b), \quad (23) $$

$$ \int dq_T \frac{q_T^2}{2 M_N^2} F_{TL}^{\cos \phi_a} = -\frac{1}{N_c} \sum_q c_q^2 g_1^T(x_a) g_1^T(x_b), \quad (24) $$

$$ \int dq_T \frac{q_T^2}{2 M_N^2} F_{LT}^{\cos 2(\phi_a - \phi_b)} = \frac{1}{N_c} \sum_q c_q^2 h_{1L}^{1(2)}(x_a) h_{1L}^{1(2)}(x_b), \quad (25) $$

$$ \int dq_T \frac{q_T^2}{12 M_N^2} F_{LT}^{\cos(\phi_a + \phi_b)} = \frac{1}{N_c} \sum_q c_q^2 \left[ f_{1T}^{1(1)}(x_a) f_{1T}^{1(1)}(x_b) - g_{1T}^{1(1)}(x_a) g_{1T}^{1(1)}(x_b) \right], \quad (26) $$

$$ = -\frac{1}{N_c} \sum_q c_q^2 \left[ f_{1T}^{1(1)}(x_a) f_{1T}^{1(1)}(x_b) + g_{1T}^{1(1)}(x_a) g_{1T}^{1(1)}(x_b) \right], \quad (27) $$

with

$$ j^{(n)}(x) \equiv \int d\mathbf{k}_T \left( \frac{k_T^2}{2 M_N^2} \right)^n j(x, k_T^2), \quad (29) $$

for 3dPDF $j$. The unweighted double spin asymmetries are ($Q^2$ is fixed)

$$ A_{LL}^{\cos 2\phi}(x_F) = \frac{\int dq_T F_{LL}^{\cos 2\phi}(x_a, x_b, q_T)}{\int dq_T F_{UU}^{\phi}(x_a, x_b, q_T)} = \frac{N_c \int dq_T F_{LL}^{\cos 2\phi}(x_a, x_b, q_T)}{\sum_q c_q^2 f_1^q(x_a) f_1^q(x_b)}, \quad (30) $$

$$ A_{TL}^{\cos \phi_a}(x_F) = \frac{\int dq_T F_{TL}^{\cos \phi_a}(x_a, x_b, q_T)}{\int dq_T F_{UU}^{\phi}(x_a, x_b, q_T)} = \frac{N_c \int dq_T F_{TL}^{\cos \phi_a}(x_a, x_b, q_T)}{\sum_q c_q^2 g_1^T(x_a) g_1^T(x_b)}, \quad (31) $$

$$ A_{LT}^{\cos 2(\phi_a - \phi_b)}(x_F) = \frac{\int dq_T F_{LT}^{\cos 2(\phi_a - \phi_b)}(x_a, x_b, q_T)}{\int dq_T F_{UU}^{\phi}(x_a, x_b, q_T)} = \frac{N_c \int dq_T F_{LT}^{\cos 2(\phi_a - \phi_b)}(x_a, x_b, q_T)}{\sum_q c_q^2 f_{1T}^q(x_a) f_{1T}^q(x_b)}, \quad (32) $$

$$ A_{LT}^{\cos(\phi_a + \phi_b)}(x_F) = \frac{\int dq_T F_{LT}^{\cos(\phi_a + \phi_b)}(x_a, x_b, q_T)}{\int dq_T F_{UU}^{\phi}(x_a, x_b, q_T)} = \frac{N_c \int dq_T F_{LT}^{\cos(\phi_a + \phi_b)}(x_a, x_b, q_T)}{\sum_q c_q^2 f_{1T}^q(x_a) f_{1T}^q(x_b)}, \quad (33) $$

The weighted double spin asymmetries are ($Q^2$ is fixed)

$$ A_{LL}^{\cos 2\phi}(x_F) = \frac{\int dq_T W_2^2 F_{LL}^{\cos 2\phi}(x_a, x_b, q_T)}{\int dq_T W_1^2 F_{UU}^{\phi}(x_a, x_b, q_T)} = \frac{\sum_q c_q^2 h_{1L}^{1(1)}(x_a) h_{1L}^{1(1)}(x_b)}{\sum_q c_q^2 f_1^q(x_a) f_1^q(x_b)}, \quad (34) $$
The double spin asymmetries related to $g_{1T}$ and $h_{1L}^T$ are complicated and we use $g_{1T} \otimes g_{1L}$ as an approach.

In Ref. \[9\], the polarized proton-antiproton Drell-Yan process have been calculated at typical kinematics on FAIR \[10\]. In this paper, we focus on predictions of the unweighted and weighted double spin asymmetries related to $g_{1T}$ and $h_{1L}^T$ in the same process. We present numerical calculations in two different approaches as described in Ref. \[9\]:

- **Approach 1.** We use Eqs. (11) and (12) directly to calculate and only sum over the valence quark distributions.
- **Approach 2.** For the unpolarized quark and antiquark distributions, we use the CTEQ6L parametrization \[34\], and adopt a Gaussian form factor for the transverse momentum dependence which has been adopted in many phenomenological analysis \[35\]:

$$f_1(x, k_T^2) = f_1(x) \exp(-k_T^2/k_{un}^2)$$ \[40\]

with $k_{un}^2 = 0.25$ GeV$^2$. For the polarized distributions, we keep the relations in Eqs. (10) which we get in the light-cone quark-diquark model so that the Melosh-Wigner rotation factors remain as the relativistic effect of quark transversal motions.

The kinematics on FAIR are chosen as $s = 45$ GeV$^2$ and $Q^2 = 12$ GeV$^2$ \[10\]. The magnitudes of the helicity and transversity distributions in approach 2 are comparable with the global analysis results \[36,37,38,39,40\] for helicity and \[35\] for transversity at the middle $x$ region. Besides, the quark-diquark model gives a good description of the nucleon form-factors \[41,42\]. The quark-diquark model realized in approach 2 also provides reasonable descriptions of many experiments related to helicity distributions \[43\], transversity distributions \[44,45\], together with some new 3dPDFs \[45\]. The effect of the CTEQ6L parametrization, which has been well verified and constrained by many experiments concerning the unpolarized quark distributions, has been taken into account in approach 2. Thus approach 2 might give more reasonable predictions for future experiments.

We plot the unweighted and weighted asymmetries related to $g_{1T}$ and $h_{1L}^T$, as shown in Eqs. \[30\], \[31\], \[32\], \[33\], \[34\], \[35\], \[36\], \[37\], and \[38\]. The results are shown in Figs. \[1\] \[2\] \[3\] \[4\] and \[5\]. In the light-cone quark-diquark model, the ratios of $h_{1L}^{(1/2)}(x)/f_1(x)$ as shown in Ref. \[9\].
Fig. 3. The double spin asymmetries related to $h_{1L}^\perp$ and $h_1$ shown in Eqs. (32) and (36) as functions of $x_F$ for $s = 45$ GeV$^2$ and $Q^2 = 12$ GeV$^2$. The dashed curves correspond to approach 1, while the solid curves correspond to approach 2.

Fig. 4. The double spin asymmetries related to $h_{1L}^\perp$ and $h_{1T}^\perp$ shown in Eqs. (33) and (37) as functions of $x_F$ for $s = 45$ GeV$^2$ and $Q^2 = 12$ GeV$^2$. The dashed curves correspond to approach 1, while the solid curves correspond to approach 2.

Fig. 5. The double spin asymmetry related to $g_{1T}$ shown in Eq. (38) as a function of $x_F$ for $s = 45$ GeV$^2$ and $Q^2 = 12$ GeV$^2$. The dashed curve corresponds to approach 1, while the solid curve corresponds to approach 2.

4 Summary

$g_{1T}$ and $h_{1L}^\perp$, i.e., the transversal helicity and the longitudinal transversity, are two of the eight leading-twist
3-dimensional parton distributions (3dPDFs). We present predictions of the unweighted and weighted double spin asymmetries related to them in the polarized proton-antiproton Drell-Yan process at typical kinematics on FAIR respectively. We conclude that the Facility for Antiproton and Ion Research (FAIR) is ideal to access the new 3dPDFs towards a detailed picture of the nucleon structure.

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