Analysis and Verification of the “Translation-Torsion” Model of Planetary Transmission Mechanism

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Abstract. The planetary transmission mechanism is the core component for achieving the motility of high-speed tracked vehicles. The internal excitation results in severe vibration and low operating life of high-speed planetary transmission mechanism. However, due to the complex multiple planetary gear sets coupling, the mechanism of the internal excitation on the planetary gear is still unclear. In this paper, the "translation-torsion" coupling dynamic model of multi-set planetary transmission mechanism is established, and the influence of internal excitation on the vibration characteristics of the planetary transmission mechanism is analyzed. The dynamic model is verified by the DTE (dynamic transmission error) and the input-output transfer torque relationship. The results show that the meshing frequency of each planet gear set is consistent with the spectrogram, verifying the accuracy of the model, and studying the DTE and the input speed of the planetary transmission mechanism. The results show that when the planetary transmission mechanism is in the 2nd gear, when the input speed exceeds 1000 rpm, the fluctuation of the rotational speed has little effect on the DTE, and the fluctuation of the input torque has a linear growth trend on the DTE.

1. Introduction

High-speed tracked vehicles require flexible maneuverability in complex and variable loads, and planetary transmission mechanisms are the core components to realize the mobility of high-speed tracked vehicles [1]. The planetary transmission mechanism works under a harsh working condition and complicated load, and its vibration and noise problems are also prominent, which reduces the running accuracy, transmission efficiency and service life of the planetary transmission mechanism [2]. Therefore, in-depth study of the dynamics of planetary gear transmission system, and then the optimization design of planetary gears, has important theoretical significance and engineering application value [3].

Depending on the method used in the establishment of the kinetic model and the factors considered, the dynamic model of the planetary gear set can be classified into a lumped mass model and a finite element model. The lumped mass model is more commonly used on account of the characteristics of the planetary gear set [4]. Considering different treatments of the lumped mass model, the lumped mass model is classified into a pure torsional dynamic model and a translation-torsion dynamic model.
At present, most of the dynamic modeling of planetary gear transmission system is for the simple planetary gear set or the fixed-shaft gear transmission system [5]. For high-speed, heavy haul, multi-sets compound planetary gear mechanism, the coupling relationship among the various stages is complicated and more force factors should be considered [6]. Most of the current research is on static design level. In this paper, the "translation-torsion" coupling dynamics model of multi-set planetary transmission mechanism is established, and the influence of internal excitation on the vibration characteristics of the planetary transmission mechanism is analyzed. The dynamic model was verified by the DTE and the input-output transfer torque relationship. A schematic diagram of a seven-speed planetary transmission mechanism is shown in Figure 1.

Figure 1 Schematic diagram of the planetary transmission mechanism

2. Planetary transmission mechanism dynamics model

Planetary gear sets dynamics model:

\[
\begin{align*}
    m \ddot{y}_1 + k_{p1} y_1 &= N_{sp1} \sin \theta_{s1} - N_{sp2} \sin \theta_{s2} - N_{sp3} \sin \theta_{s3} + N_{sp4} \sin \theta_{s4} \\
    m \ddot{x}_1 + k_{x1} x_1 &= -N_{sp1} \cos \theta_{s1} - N_{sp2} \cos \theta_{s2} + N_{sp3} \cos \theta_{s3} + N_{sp4} \cos \theta_{s4} \\
    I_s \ddot{\theta}_s &= T_s - N_{sp1} R_{sb} - N_{sp2} R_{sb} - N_{sp3} R_{sb} - N_{sp4} R_{sb} - N_{rb} R_{sb} \\
    m_{p1} \ddot{u}_{p1} + k_{p1u} u_{p1} &= N_{sp1} \cos \theta_{s1} - N_{rp1} \cos \theta_{r1} \\
    m_{p1} \ddot{v}_{p1} + k_{p1v} v_{p1} &= N_{sp1} \sin \theta_{s1} + N_{rp1} \sin \theta_{r1} \\
    I_{p1} \dot{\theta}_{p1} &= -T_{p1} + N_{sp1} R_{p1b} - N_{rp1} R_{p1b} \\
    m_{p2} \ddot{u}_{p2} + k_{p2u} u_{p2} &= N_{sp2} \cos \theta_{s2} - N_{ rp2} \cos \theta_{r2} \\
    m_{p2} \ddot{v}_{p2} + k_{p2v} v_{p2} &= N_{sp2} \sin \theta_{s2} + N_{rp2} \sin \theta_{r2} \\
    I_{p2} \dot{\theta}_{p2} &= -T_{p2} + N_{sp2} R_{p2b} - N_{rp2} R_{p2b} \\
    m_{p3} \ddot{u}_{p3} + k_{p3u} u_{p3} &= N_{sp3} \cos \theta_{s3} - N_{rp3} \cos \theta_{r3} \\
    m_{p3} \ddot{v}_{p3} + k_{p3v} v_{p3} &= N_{sp3} \sin \theta_{s3} + N_{rp3} \sin \theta_{r3} \\
    I_{p3} \dot{\theta}_{p3} &= -T_{p3} + N_{sp3} R_{p3b} - N_{rp3} R_{p3b} \\
    m_{p4} \ddot{u}_{p4} + k_{p4u} u_{p4} &= N_{sp4} \cos \theta_{s4} - N_{rp4} \cos \theta_{r4} \\
    m_{p4} \ddot{v}_{p4} + k_{p4v} v_{p4} &= N_{sp4} \sin \theta_{s4} + N_{rp4} \sin \theta_{r4} \\
    I_{p4} \dot{\theta}_{p4} &= -T_{p4} + N_{sp4} R_{p4b} - N_{rp4} R_{p4b} \\
    m_{c1} \ddot{y}_c + k_{c1} y_c &= N_{cp1u} \sin \varphi_1 + N_{cp2u} \sin \varphi_2 + N_{cp3u} \sin \varphi_3 + N_{cp4u} \sin \varphi_4 \\
    m_{c1} \ddot{x}_c + k_{c1x} x_c &= N_{cp1u} \cos \varphi_1 + N_{cp2u} \cos \varphi_2 + N_{cp3u} \cos \varphi_3 + N_{cp4u} \cos \varphi_4 \\
    I_c \ddot{\theta}_c &= -T_c - N_{cw} R_{cb} + N_{cp1v} R_{cb} + N_{cp2v} R_{cb} + N_{cp3v} R_{cb} + N_{cp4v} R_{cb} \\
    m_{c1} \ddot{y}_r + k_{c1y} y_r &= N_{rp1} \sin \theta_{r1} + N_{rp2} \sin \theta_{r2} - N_{rp3} \sin \theta_{r3} + N_{rp4} \sin \theta_{r4} \\
    m_{c1} \ddot{x}_r + k_{c1x} x_r &= N_{rp1} \cos \theta_{r1} - N_{rp2} \cos \theta_{r2} - N_{rp3} \cos \theta_{r3} + N_{rp4} \cos \theta_{r4} \\
    I_r \dot{\theta}_r &= T_r + N_{rr} R_{rb} + N_{rp1} R_{rb} + N_{rp2} R_{rb} + N_{rp3} R_{rb} + N_{rp4} R_{rb} \\
\end{align*}
\]
In the formula (1):

- \( m_s, m_c, m_r, m_{pi}, (i=1,2,3,4) \) —— the quality of the sun gear, planet carrier, ring gear and planet gear \( i \);
- \( I_s, I_c, I_r, I_{pi}, (i=1,2,3,4) \) —— Moment of inertia of sun gear, planet carrier, ring gear and planet gear \( i \);
- \( k_{sy}, k_{sx}, k_{cy}, k_{cx}, k_{ry}, k_{rx} \) —— Support stiffness of sun gear, planet carrier, and ring gear in \( y \) and \( x \) directions;
- \( k_{piu}, k_{piv}, (i=1,2,3,4) \) —— Support stiffness of planet gear \( i \) in \( u_i \) and \( v_i \) directions;
- \( R_{sh}, R_{ch}, R_{rh}, R_{ph}, (i=1, 2, 3, 4) \) —— The base circle radius of sun gear, planet carrier, ring gear and planet gear \( i \);
- \( \phi_i \) \( (i=1, 2, 3, 4) \) —— the angle between planet gear \( i \) and horizontal line counterclockwise

The dynamic meshing force between sun gear and planet gear \( i \), the dynamic meshing force between planet gear \( i \) and ring gear, and the force of planet carrier in \( u \) direction and \( v \) direction subjected to planet gear \( i \), respectively, the calculation formula is as follows:

\[
N_{spi} = k_{spi} \delta_{spi} \\
N_{rpi} = k_{rpi} \delta_{rpi} \\
N_{piu} = k_{piu} u_{pi} \\
N_{piv} = k_{piv} v_{pi}
\]  

In the formula:

- \( k_{spi}, k_{rpi}, (i=1, 2, 3, 4) \) —— Time-varying meshing stiffness between sun gear and planet gear \( i \), Time-varying meshing stiffness between planet gear \( i \) and ring gear, respectively;
- \( \delta_{spi}, \delta_{rpi} \) are the meshing deformation of the gear, or the DTE.

### 3. 2nd gear modeling and verification of planetary transmission mechanism

The typical 2nd gear is used to verify the dynamic model. When the planetary transmission gear is 2nd gear, the operating elements B2, B3 and C2 are closed, the power is input by the planet carrier of 1st planet gear set, as shown in Figure 2.

According to the given working condition, the input torque is 2856Nm, and the input speed is 2380rpm. The dynamic response of the planetary transmission mechanism with 2nd gear is calculated by MATLAB. In 2nd gear, the ring gear - planet gear DTE of 1st planet gear set, the sun gear - planet gear DTE of 1st planet gear set, the ring gear - planet gear DTE of 2nd planet gear set, the sun gear - planet gear DTE of 2nd planet gear set, the ring gear-planet gear DTE of 3rd planet gear set , the sun gear-planet gear DTE of 3rd planet gear set, as shown in Figure 3.

![Figure 2 Schematic diagram of 2nd gear of the planetary transmission mechanism](image-url)
The obtained DTE of the gear 2nd in a given working condition is subjected to Fourier transform to obtain a spectrogram, the frequency component \( f = 2903 \) Hz of 1st set is basically consistent with the meshing frequency \( f = 2898 \) Hz of 1st set, and the frequency component \( f = 1857 \) Hz of 2nd set is basically consistent with the meshing frequency \( f = 1856 \) Hz of 2nd set, and the frequency component \( f = 2903 \) Hz of 3rd set is basically consistent with the meshing frequency \( f = 2898 \) Hz of 3rd set, which verifies the accuracy of the model.

![Figure. 3 Gear 2nd dynamic transfer error for a given operating condition](image)

### 4. Relationship between DTE and input torque/speed

#### 4.1. The relationship between the DTE and the input torque/speed

**4.1.1. Relationship between input speed and DTE.** Keep the input torque set at 2856 Nm, and the input rotational speed is 1000 rpm, 2000 rpm, 3000 rpm, respectively. A program compiled by MATLAB is used to calculate the DTE and its RMS value of the system, and then the influence of input torque fluctuation on DTE can be obtained.

In order to further analyze the relationship between the input speed and the DTE of the planetary transmission mechanism, The RMS values of DTE of ring gear - planet gear of 2nd set, the DTE of sun gear - planet gear of 2nd set, the DTE of ring gear - planet gear of 3rd set, the DTE of sun gear - planet gear of 3rd set are analyzed, as shown in Figure 5. The RMS value of the DTE of ring gear-planet gear of 2nd set increases first and then decreases with the increase of the rotational speed. The RMS value of the DTE of ring gear-planet gear of 3rd set increases with the rotational speed. The RMS value of the DTE of sun gear-planet gear of 2nd set increases first and then decreases with the increase of the rotational speed. The RMS value of the DTE of sun gear-planet gear of 3rd set increases with the rotational speed.
It can be seen that when the input speed is between 0 and 1000 rpm, the DTE increases linearly with the increase of the rotational speed. As the rotational speed increases, the influence of the input rotational speed on the DTE remains unchanged.

4.1.2. Relationship between input torque and DTE. Keep the input speed set at 2380 rpm, and the input torque is 1000 Nm, 2000 Nm, and 3000 Nm, respectively. DTE and its RMS value can be calculated by MATLAB program, and get the influence of input torque fluctuation on DTE.

![Figure 4](image)

**Figure. 4** Diagram of the relationship between input torque and dynamic transfer error

In order to further analyze the relationship between the input torque and the DTE of the planetary transmission mechanism. The RMS values of the DTE of ring gear - planet gear of 2nd set, sun gear - planet gear of 2nd set, ring gear - planet gear of 3rd set, sun gear - planet gear of 3rd set are analyzed, as shown in Figure 5. The RMS value of the DTE of ring gear - planet gear of 2nd set increases with torque. The RMS value of ring gear- planet gear's DTE of 3rd set increases with torque. The RMS value of the DTE of sun gear-planet gear of 2nd set increases with torque. The RMS value of the DTE of sun gear-planet gear of 3rd set increases with torque.

It can be concluded from Figure. 6 that as the input torque increase, the DTE increases linearly and continuously, indicating that the increase of torque has a greater influence on the DTE.

5. Conclusion

In this paper, the dynamic model of the planetary transmission mechanism is completed, and the second gear of the planetary transmission mechanism is analyzed according to the model. The specific conclusions are as follows:

1. According to the given working condition, the input speed is 2380 rpm, and the input torque is 2856 Nm. The DTE of the 2nd gear of the planetary transmission mechanism is calculated, and the spectrum is obtained by Fourier transform. The result shows that the meshing frequency of each planet gear set is consistent with the spectrogram, which confirms the accuracy of the model.
2. The relationship between DTE and input speed with the gear 2nd is analyzed. The analysis results show that the DTE increases linearly with the increase of the rotational speed when the input speed is between 0 and 1000 rpm. With the increase of the rotational speed, the influence of the input rotational speed on the DTE remains unchanged.

3. The relationship between DTE and input torque of gear 2nd is analyzed. The results show that with the increase of input torque, the DTE increases linearly and continuously, and indicate that the increase of torque has a greater influence on the DTE.

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