High Frequency Vibrations Arising in System Wheel-Rail. Part I. Mathematical Modeling of the Distribution Stress in Contact Zone Wheel-Rail

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Abstract. This study is devoted to study of most used existing models of contact two solids, in relation solution of problem dynamic interaction of wheel pair and the rails. In paper also proposed and approbate a new relationship to connect force of interaction, deformation of local indentation, displacement of contacting bodies, their mechanical and geometric characteristics, as well as the parameters of condition during operation in different modes. The stress isolines at point of interaction wheel-rail on straight section of track without taking into account the defects of the contacting bodies are obtained.

1. Introduction

The modern stage of development railway transport systems is characterized an increase the speed carriages, load on axle of wheel pair, total weight of equipped rail transport. The actual and timely is adaptation of existing techniques, models and algorithms to evaluate the state of railway track and its individual elements during operation in different conditions [1-4], taking into account the reduction of time intervals between trains. An important task is to identify and analyze defects rail and wheel pairs, which influence on safety and characteristics of transport system in terms of logistics. For obtain these estimates it is necessary to know how to influence these or other parameters of contacting bodies on geometric, kinematic and dynamic functions of condition and behavior of elements upper structure truck [5-7]. To take into account many factors of condition and operation rails, it is necessary to take into account the change classical model of interaction wheel and rail. For example, the increase speed of carriage not lead significantly develop plastic deformations in contact area of two bodies, at the same time, increases the value responsible for dynamic effect of wheel and rail in the final determining ratios [8-10]. The increased requirements of engineering practice lead to complication of rheological models contact of wheel and rail, which will allow a more accurate description of the processes occurring in them. Significant stresses of interaction in the "wheel-rail" model can occur not only because high speed of carriage, but also because presence of defects on surface rails. At the same time, an important task is not to allow the re-qualification of rails from defective to acutely defective, which will lead to a stop of movement on described section of the path.

For simulate dynamic behavior a rail under action a wheel pair is proposed to use defining relations for curvilinear rods of non-thin-walled cross-section, which may have different elongation and
conditions of fixing, depending on used intermediate fasteners, equalizing joints, subplate and spruce in curved track sections [3,7,8]:

\[
\frac{\partial u}{\partial s} = (q \times \tau) + \frac{N}{k} \tau; \quad \frac{\partial q}{\partial s} = A \cdot M + B \cdot N; \quad \frac{\partial M}{\partial s} = (N \times \tau) + C \frac{\partial^2 \varphi}{\partial t^2} - \left(N^0 \times (q \times \tau)\right);
\]

\[
\frac{\partial N}{\partial s} = \rho F \frac{\partial^2 u}{\partial t^2} + f \frac{\partial u}{\partial t} - q \sin \Omega t; \quad \frac{\partial N^0}{\partial s} = -q^0,
\]

here is used the following symbol: \( t \) - time; \( s \) - longitudinal coordinate, measured along rail; \( u \) - linear displacement vector of rod axis; \( \varphi \) - vector of angular displacement of section; \( M \) and \( N \) - vectors of moments and forces in selected section; \( \tau \) - ort tangent to the axis of curvilinear rod; \( A \) and \( B \) - compliance matrix in orthogonal basis \( n \) and \( b \) (normal and binormal, relative to ort \( \tau \) in natural triangle); \( C \) - matrix of mass inertia moments reduced to length; \( \rho \) - density of rail material; \( f \) - coefficient of viscous friction; \( q \) - vector of distributed load, reduced to longitudinal axis; \( N^0 \) and \( q^0 \) - vectors of force and distributed load, previously acting on rod; \( \Omega \) - cyclic frequency of external forcing effects.

It is believed that in basis of natural triangle and the orits \( \tau, n \) and \( b \), the matrix \( A \) and \( C \) are diagonal, and values their components are determined by the following relations:

\[
a_{11} = 1/GJ_{kp}; \quad a_{22} = 1/E_{jn}; \quad a_{33} = 1/E_{jb}; \quad c_{11} = \rho \cdot (J_n + J_b); \quad c_{22} = \rho J_n; \quad c_{33} = \rho J_b,
\]

here \( G \) - shear modulus, \( J_{kp} \) - moment of inertia during torsion, \( J_n, J_b \) - inertia moments about the axes of collinear normal and binormal of natural triangle.

In order to determine the external load transmitted to rail from wheel pair is necessary to consider the contact problem and take into account local destruction of rail material and its deformation due to bending and torsion [11,12]. Standard dependences between the force of contact interaction and local indentation take into account a sufficient number of geometrical and mechanical factors that can be divided into several groups [13,14]. Each group can represent one of the following expressions, and input parameters can take different values depending on the initial conditions of contact and exposure. To compare the results obtained using the proposed approach, we choose several models containing a single functional relationship, such as, the Hertz contact model (3a), linear elastic force (3b), viscoelastic model with exponential relaxation core (3c):

\[
P = k \cdot \alpha^{3/2}, \quad P = E_{i} \cdot (\alpha - w), \quad P = E_{i} \cdot (\alpha - w) - \frac{E_{i}}{\tau} \int_{0}^{t} (\alpha - w) \cdot e^{-t'} dt',
\]

where \( \alpha \) - local indentation of rail and wheel materials, \( k \) - coefficient determining the geometry of contacting bodies and elastic characteristics their materials, \( E_{i} \) - elastic modulus of interaction area wheelset and rail, \( w \) - displacement of bottom rail foot edge, i.e. in fact, the rail deflection, \( \sigma \) - Poisson’s ratio for wheel pair, \( \tau = \eta / E_{i} \), \( \tau \) - relaxation time in case of a viscoelastic model, \( \eta \) - coefficient of viscous resistance.

Also, for comparative analysis is proposed to use several more modern models of contact, functional relationships containing several expressions, such as [8,14-20]: elastoplastic models of Kilchevsky (3) and Aleksandrov-Kadomtsev (3), model with unloading (3), model Abrata (3), linearized model with varying mechanical parameters depending on loading stage (3h), Shimash model (3), empirical model (3), three-phase energy model (3)

\[
a = \begin{cases} 
  bP^{2/3}, & dP/dt > 0, \quad P < P_b \\
  bP^{2/3} + Pd, & dP/dt > 0, \quad P > P_b \\
  bP^{2/3} + P_{max}d, & dP/dt < 0, \quad P_{max} > P_b 
\end{cases}
\]
\[
\alpha = \left\{ \begin{array}{l}
bP^{2/3}, \quad dP/dt > 0, \quad P_{\text{max}} < P, \\
(1 + \beta)c_1 + (1 - \beta)P_0, \quad dP/dt > 0, \quad P_{\text{max}} > P, \\
\end{array} \right. \\
\] \\
\] \\
\] \\
\] \\
\] \\
\]

\[
P = P_m \left[ \frac{(\alpha - \alpha_0)}{(\alpha_m - \alpha_0)} \right]^q, \quad \text{for the unloading phase} \\
P = P_m \left[ \frac{(\alpha - \alpha_0)}{(\alpha_m - \alpha_0)} \right]^{1.5}, \\
k_0 \alpha, \quad k_i = P_m^{1/3} k^{2/3}, \\
\alpha = \left( \frac{P}{k_H} \right)^{2/3} \left( 1 - \frac{\ln 2}{k_H^{2/3} k_0} \right), \quad k_H = \frac{4}{3} Q_H \sqrt{R}, \\
P = kw_0^n. \tag{3} \\
\]

At the first stage of three-phase model, the relation (3a) is fulfilled; at the second stage, \(-\alpha \leq \alpha \leq \alpha_m \), \(P(\alpha) = K_1(\alpha - \alpha_c) + k_H \alpha_c^{3/2} \), on the third - \(3(\alpha)\) here is plasticity parameter \(\lambda = 5.7, \chi = \pi, k_0^\lambda, b = ((9\pi^2(k_1 + k)^2)/16R)^{1/3}, k_1 = (1 - \sigma^2)/E_1, k = (1 - \sigma^2)/E, R_p^{-1} = R^{-1} - R_f^{-1}, P_1 = \chi^3(3R(k_1 + k)/4)^2, b_f = R_f^{-1/3}(3(k_1 + k)/4)^{2/3}, c_1 = 3\chi^{1/2}(k_1 + k)/8, R_f = (4/3(k_1 + k))P^{1/3} \chi^{-3/2}, \alpha_p(P_{\text{max}}) = (1 - \beta)P_{\text{max}}(2\chi R_p)^{-1}, K_y = 1.5k_H \sqrt{\alpha_c}, d = 1/2\chi R, k_0^\lambda - \text{smallest of plastic constants, } P_m - \text{maximum value of contact force before the start of unloading stage, } \alpha_m - \text{maximum indentation of material in interaction area, } \alpha_0 - \text{current value of indentation, } \alpha_{\text{st}} - \text{contact stiffness determined by radius of striker } R \text{ and effective contact module } Q_H, \beta = 0.33. \]

Studies have shown that optimal model for describing the relationship of power, geometric and kinematic characteristics accompanying the formation a rail when exposed to a wheel pair moving at high speeds up to 450 km / h is a model that summarizes the forces in zones of identical stresses in area of contact two bodies, also is added time summation if speed of movement composition becomes comparable with speeds of elastic waves in upper railway structure \\
\[
P(x, y, t) = \sum_{x_0} \sum_{y_0} \sum_{t_0} \left[ (\alpha_{\text{st}} - w_0) \frac{E_{\text{st}}}{A_k} + (\alpha_{\text{st}} - w_0)E_T\alpha_T \right] \frac{\dot{c} \left( (\alpha_{\text{st}} - w_0) \right)}{\alpha_T} E_T \right] \frac{\dot{c}}{\dot{t}} \frac{E_T}{G_m} \times A_m \times \frac{F_{\text{def}}}{F_{\text{def}}}, \tag{4} \\
x_0, y_0 - \text{coordinates of the beginning corresponding area of equal stress in contact zone of wheel-rail (x - measured along rail); } t_0 - \text{time of appearance wheel pair at point with zero geometric coordinates on rolling surface of rail; } \alpha_{\text{st}} - \text{local indentation at given point at certain moment in time; } w_0 - \text{displacement of rail bottom in cross section under from the action of wheel pair in its previous position; } A_k - \text{space of area the same voltage; } E_{\text{st}} - \text{modulus of elasticity at certain point of spot contact from moment the load is applied at zero point; } E_T - \text{reduced viscoelastic resistivity modulus; } \alpha_T - \text{coefficient thermal expansion of the material; } G_m - \text{median values of the front speeds of elastic waves; } E_s - \text{gradient of module elasticity, depending on the time from the beginning of loading; } r_s - \text{distance}
from zero to point in question; $A_{pr}$ - reduced area surface contact; $F_{def}$ - function of defects on rolling surface of wheel-rail, as well lateral surface of rail and flange.

**Figure 1.** The railway track of variable stiffness before the bridge.

In fig. 2 shows the zone of contact wheel-rail, obtained as a result of mathematical modeling [10, 11] and using ratio for force interaction of wheel-rail (3d), (3d). However, in the presence of defects on rolling surface of both the knee pair and rail, the ratio for contact force can vary significantly and, in general, can be represented by relation (4). In this paper, we carried out mathematical modeling of the distribution stress as a result of dynamic contact and calculated the stress levels on rail surface arising from the wheel pair's in absence of defects (Figure 2). However, these levels may increase due to: presence of chipping, corrugated and horizontal wear of rail head, presence of aluminothermy welding joints, a change in area of contact, geometric and mechanical parameters of interaction can lead to stress of 700-800 MPa and even more.

**Figure 2.** The space area of contact wheel-rail (mm2) of voltage is shown in form of isolines, solid indicates 450 MPa, a long stroke of 300 MPa, a short stroke of 150 MPa.
2. Conclusion
In this paper, based on the study of eight models dynamic contact a wheel pair and rail, the features of each of them are identify. These models do not allow to take into account the high speeds of rolling stock, the presence various defects on rolling surface wheel and rail, as well as the features of track maintenance of upper structure the railway track. In this article proposed a new model of dynamic interaction. This model takes into account the above-listed factors and allows to determine the geometric (displacement and deformation) and dynamic (contact force and normal stress) parameters of interaction wheel and rail in special point, which allow it possible to graphing isolines of desired functions with different initial conditions of interaction and construction maintenance. The proposed mathematical model can be implemented as analytically (with using boundary conditions and compatibility conditions) as well as numerically (with using modern computational tools). The proposed mathematical model can be used by organizations operating railways, when calculating condition of railway track at various sites and when planning repairs and certain types of work.

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