Evolution from Canonical to Millisecond Pulsar through the X-ray Accretion Stage

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Abstract. We model the evolution of canonical pulsars from the death line to millisecond pulsars through the X-ray neutron star stage of accretion from a low-mass companion. We trace this evolution in magnetic field strength starting at \( B = 10^{12} \) to \( 10^8 \) G and in period of about 1 s to milliseconds. Important factors are accretion rate and the decay rate of the magnetic field. A broad swathe is traced in the \( B - P \) plane according to the value of these factors, which represent different conditions of the binary pair. An important ingredient is the dependence of the stellar moment of inertia on rotation frequency (time).

Key words: X-ray neutron stars: evolution from canonical to millisecond pulsar

1. Background

Pulsars are believed to have formed from the collapsed cores of massive stars, and are spun up by angular momentum conservation to moderate frequencies of 30 Hz or so, like the Crab pulsar. Thereafter, they radiate their angular momentum through the strong magnetic dipole field which is fixed in the star, ever more slowly as the rotational rate decreases. Depending on the magnetic field strength and frequency, the radiation mechanism turns off and canonical pulsars disappear from the radio sky at frequencies of 1 Hz or there about, depending on the magnetic field strength. They have crossed what has been referred to as the “death line”. Millisecond pulsars are thought to originate from these radio silent stars, being spun up by accretion from a low-mass companion. Their magnetic fields are typically \( 10^8 \) to \( 10^9 \) Gauss, as compared to \( 10^{11} \) to \( 10^{13} \) Gauss for canonical pulsars. Either the field decays ohmically during the long epoch in which they are radio and X-ray silent, or the field is partially destroyed by the accretion process, or by some as yet unknown mechanism. In this intermediate accretion phase, they are X-ray neutron stars. The missing link between canonical pulsars with mean period of 0.7 s and millisecond pulsars was recently discovered, an X-ray star with period of 2.5 ms.

So far as we know, the evolution from canonical to millisecond pulsar has never been modeled. We do so here. We find essentially a continuum of evolutionary tracks according to the rate at which matter is accreted from the companion, and the rate at which the magnetic field decays. The evolutionary tracks essentially fill all the space in the B-P plane, starting at the death line at fields typical of canonical pulsars, and extending downward in field strength, broadening to fill the space on both sides of the deathline, and extending to the small periods of millisecond pulsars. All are potential tracks of some particular binary pair, since accretion rates vary by several orders of magnitude and presumably so do decay rates of the magnetic field.

2. Evolution from canonical to ms pulsars

There are two distinct aspects to developing an evolutionary framework. One has to do with the accretion process itself, which has been developed by a number of authors in the framework of classical physics. We quote some of the important results below and refer to the original literature for details (Elsner & Lamb 1977, Chakrabarty & E. H. Morgan 1998, van der Klis 2000). The other aspect has to do with the structure of the neutron star and its response to added mass, but most especially to its response to changes in rotational frequency due to the changing centrifugal forces. Typically, the moment of inertia has been computed in general relativity for a non-rotating star (Hartle 1967, Hartle & Thorne 1968). It is based on the Oppenheimer-Volkoff metric. However, for the purpose of tracing the evolution of an accreting star from ~ 1 Hz to 400 to 600 Hz we do not neglect the response of the shape, structure and composition of the star as it is spun up over this vast range of frequencies from essentially zero to values that approach the Kepler frequency. Nor do we neglect the dragging of local inertial frames. These features are included in our calculation of the tracks of neutron stars from canonical objects starting...
with large fields and very small frequencies at the “death line” to the small fields but rapid rotation of millisecond pulsars. However, the expression of the moment of inertia and a definition of the various factors that enter are too long to reproduce here. We refer instead to our derivation given in Ref. (Glendenning & Weber 1992).

The spin-up torque of the accreting matter causes a change in the star’s angular momentum according to the relation (Elsner & Lamb 1977, Lipunov 1992)

\[
\frac{dJ}{dt} = M\dot{\Omega}(r_m) - N(r_c),
\]

where \(M\) denotes the accretion rate and \((G = c = 1)\)

\[\tilde{\Omega}(r_m) = \sqrt{Mr_m}\]

(2)

is the angular momentum added to the star per unit mass of accreted matter. The quantity \(N\) stands for the magnetic plus viscous torque term,

\[N(r_c) = \kappa \mu^2 r_c^{-3},\]

with \(\mu \equiv R^3 B\) the star’s magnetic moment (\(\kappa \sim 0.1\)). We assume that the magnetic field evolves according to

\[B(t) = B(\infty) + (B(0) - B(\infty))e^{-t/t_d}\]

(4)

where \(B(\infty) = 10^8\) G, \(B(0) = 10^{12}\) G and \(t_d = 10^5\) to \(10^7\) yr (cf. Konar & Bhattacharyya 1999). However, we shall also make a comparison with a purely exponential decay. The quantities \(r_m\) and \(r_c\) denote the radius of the inner edge of the accretion disk and the co-rotating radius, respectively, and are given by \((\xi \sim 1)\)

\[r_m = \xi r_A,\]

(5)

and

\[r_c = (M\Omega^{-2})^{1/3}.\]

(6)

Accretion will be inhibited by a centrifugal barrier if the neutron star’s magnetosphere rotates faster than the Kepler frequency at the magnetosphere. Hence \(r_m < r_c\), otherwise accretion onto the star will cease. The Alfvén radius \(r_A\), where the magnetic energy density equals the kinetic energy density of the accreting matter, in Eq. is defined by

\[r_A = \left(\frac{\mu^4}{2M M^2}\right)^{1/7}.
\]

(7)

The rate of change of a star’s angular frequency \(\Omega \equiv J/I\) then follows from Eq. as

\[I(t)\frac{d\Omega(t)}{dt} = \dot{M}\tilde{\Omega} - \Omega(t)\frac{dI(t)}{dt} - \kappa \mu(t)^2 r_c(t)^{-3},\]

(8)

with the explicit time dependences as indicated. Evidently, the second term on the right-hand-side of Eq. depends linearly on \(\Omega\) to leading order, while the third terms grows quadratically with \(\Omega\). As mentioned, the change of the moment of inertia with time (or correspondingly frequency) is an essential part of the evolution and we do not neglect this term in the frequency evolution equation.

The stellar model from which the moment of inertia is computed is described as follows. Neutron star matter has a charge neutral composition of hadrons consisting of members of the baryon octet together with leptons. The properties of such matter are calculated in a covariant mean field theory as described in Refs. (Ellis, Kapusta & Olive 1991, Glendenning & Moszkowski 1991). The values of the parameters that define the coupling constants of the theory are certain fairly well constrained properties of nuclear matter and hypernuclei as described in the references; (binding energy of symmetric nuclear matter \(B/A = -16.3\) MeV, saturation density \(\rho = 0.153\) fm\(^{-3}\), compression modulus \(K = 300\) MeV, symmetry energy coefficient \(a_{sym} = 32.5\) MeV, nucleon effective mass at saturation \(m_{sat} = 0.7m\) and ratio of hyperon to nucleon couplings \(x_\sigma = 0.6, x_\omega = 0.653 = x_\rho\) that yield, together with the foregoing parameters, the correct \(A\) binding in nuclear matter (Glendenning & Moszkowski 1991). The above are representative, but by no means well determined in all cases. But the effect of the response of the moment of inertia to frequency on the time evolution of the accreting system should be approximately represented.

3. Evolutionary Tracks in the \(B - P\) Plane

As a first orientation as to our results and how they relate to known pulsars as regards their magnetic field strength and their rotational period, we show the evolutionary tracks for four different accretion rates given in units of \(\dot{M} = 10^{-10}\) solar masses per year in Fig. 1. The decay rate of the magnetic field is taken to have the value \(t_d = 10^6\) yr in each case. The initial condition is that after some indefinite era on the death line at a field strength of \(10^{12}\) Gauss, accretion from a companion commences. The X-ray neutron star gains angular momentum and its period decreases and over a longer timescale, the magnetic field decays. One can see already that a wide swathe of \(B\) and \(P\) is traced out.

In the above example, the field was assumed to decay to a finite asymptotic value of \(10^8\) Gauss. A very different assumption, namely that the field decays eventually to zero, \(B(t) = B(0)e^{-t/t_d}\), modifies only the results below the asymptotic value, as is seen by comparing Fig. 1 and 2. However, the conclusion concerning the origin of millisecond pulsars is quite different. For purely exponential decay, one would conclude that high frequency pulsars are created only in high accretion rate binaries.

In the remainder of the paper, we assume the field decays to an asymptotic value, since from the above comparison we see how exponential decay would modify the
Fig. 1. Evolutionary tracks traced by neutron stars in the X-ray accretion stage, beginning on the death line with large $B$ field and ending as millisecond stars, for various accretion rates. Here $t_d = 10^6$ yr.

picture. We show time tags on a sample track in Fig. 3 which provides some sense of time lapse. The first part of a track is traversed in short time, but the remainder ever more slowly. This shows up also in $dP/dt$ as a function of time.

For each of three accretion rates we show the dependence on three field decay rates in Figs. 4, 5 and 6. Depending on decay rate of the field and accretion rate, an X-ray neutron star may spend some time on either side of the death line, but if it accreted long enough, always ends up as a candidate for a millisecond pulsar if the magnetic field decays to an asymptotic value such as was assumed. However, if the field decays exponentially to zero, only high accretion rates would lead to millisecond pulsars. Of course, if accretion turns off at some time, the evolution is arrested.

4. Summary

We have computed the evolutionary tracks in the $B - P$ plane due to mass accretion onto neutron stars beginning at the death line with a typical field strength of $10^{12}$ Gauss, to shorter periods and low fields. According to the assumed accretion rate and decay constant for the magnetic field, the tracks indicate that the individual binaries with characteristics ranging from Z to Atoll sources will evolve along paths that cover a broad swathe in the $B - P$ plane. These include tracks of X-ray stars corresponding to low accretion rates that follow a path beyond the death line in the so-called ‘graveyard’.

We have assumed two particular forms for the law of decay of the magnetic field. (1) The field approaches an asymptotic value of $10^8$ Gauss such as is typical of millisecond pulsars. This assumption leads to a particular form for the termination of evolutionary tracks. All accretors, no
Fig. 4. Evolutionary tracks for a neutron star starting at the death line and evolving by accretion to lower field and high frequency for an accretion rate $10^{-10}$ solar masses per year, and for three values of the magnetic field decay rate $t_d$ as marked.

Fig. 5. Similar to Fig. 4 but with a different accretion rate as marked.

Fig. 6. Similar to Fig. 4 but with a different accretion rate as marked.

matter what the accretion rate, will end with millisecond periods, unless accretion ceases beforehand. (2) If instead, we had assumed a purely exponential decay, the tracks would not tend to an asymptotic value, but would continue to decrease in the strength of $B$. The tracks would still cover a broad swathe in the $B - P$ plane. But one would conclude that only the higher accretion rate binaries, particularly the Z-sources, could produce millisecond period neutron stars. If accretion continues for too long a time, the neutron star will be carried to very low fields and across the death line, or an overcritical mass will have been accreted, leading instead to a black hole.

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