Radiative Symmetry Breaking on D-branes at Non-supersymmetric Singularities

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Abstract

The possibility of radiative gauge symmetry breaking on D3-branes at non-supersymmetric orbifold singularities is examined. As an example, a simple model of D3-branes at non-supersymmetric $\mathbb{C}^3/\mathbb{Z}_6$ singularity with some D7-branes for the cancellations of R-R tadpoles in twisted sectors is analyzed in detail. We find that there are no tachyon modes in twisted sectors, and NS-NS tadpoles in twisted sectors are canceled out, though uncanceled tadpoles and tachyon modes exist in untwisted sectors. This means that this singularity background is a stable solution of string theory at tree level, though some specific compactification of six-dimensional space should be considered for a consistent untwisted sector. On D3-brane three massless “Higgs doublet fields” and three family “up-type quarks” are realized at tree level. Other fermion fields, “down-type quarks” and “leptons”, can be realized as massless modes of the open strings stretching between D3-branes and D7-branes. The Higgs doublet fields have Yukawa couplings with up-type quarks, and they also have self-couplings which give a scalar potential without flat directions. Since there is no supersymmetry, the radiative corrections may naturally develop negative Higgs mass squared and “electroweak symmetry breaking”. We explicitly calculate the open string one-loop correction to the Higgs mass squared from twisted sectors, and find that the negative value is indeed realized in this specific model.
1 Introduction

Although many phenomenological models using D-branes in string theory have been constructed (see for review Refs. [1, 2] and references therein), few attention has been paid on the dynamics of gauge symmetry breaking: the electroweak symmetry breaking, grand unified gauge symmetry breaking, and so on. In Ref. [3] the possibility of radiative electroweak symmetry breaking has been examined by the explicit string calculation in a specific “brane supersymmetry breaking” model [4]. In the model the Wilson line in a compact direction of the radius $R$ is identified as a scalar field, and there is no potential for the scalar field at tree level. Because of no supersymmetry, the potential of the scalar field can emerge at one-loop level. It has been shown by an explicit calculation in string theory that the scalar field can have finite vacuum expectation value roughly of the order of $1/R$. Though the magnitude of the negative mass squared of the scalar field depends on both the string scale and $R$ rather a complicated way, it is essentially determined by the scale $1/R$ and one-loop suppression factor. It has been pointed out, through some phenomenological discussions on the application to the electroweak symmetry breaking, that the string scale can be from TeV scale to the intermediate scale depending on the sizes of compact directions. In spite of this interesting possibility of radiative electroweak symmetry breaking, however, not so much subsequent efforts towards constructing realistic models have been made in the framework of string theory.

One of the difficult problems of this direction is that NS-NS tadpoles are not canceled out in general in non-supersymmetric models [5, 6, 7]. In some D-brane configuration R-R tadpoles have to be canceled out for consistency reasons (anomaly cancellations). In supersymmetric configurations NS-NS tadpoles are automatically canceled out, if the R-R tadpoles are canceled out, and the assuming backgrounds (background geometry and flux) are solutions of string theory. This is not the case in non-supersymmetric configurations in general and some modifications of the backgrounds are required. Some one-loop calculations may give divergent results due to NS-NS tadpoles without the modification of the background. The systematic procedure of the modification has not yet been established (see Ref. [8] for recent proposal).

In case that the string scale is in TeV region, massive string states may give sizable effects at low energies. In the low energy effective field theory, such effects can be described as some contact interactions by the exchanges of massive string states, and there is the lower bound in the string scale to a few TeV [9]. Since the size of the six-dimensional compact space should be taken appropriately large to explain the weakness of the gravitational interaction in case of TeV string scale, there is a possibility that the Kaluza-Klein modes of some fields in the standard model are light and give sizable effects at low energies [10, 11]. Since the scenario of TeV string scale predicts these effects which can be accessible in future experiments (see Ref. [12] for review), it must be very important to theoretically examine further the possibility, the electroweak symmetry breaking in this paper, in the framework of well-defined perturbative string theory.

The outline of this paper is as follows. In section 2 the system of D3-branes on non-supersymmetric $\mathbb{C}^3/\mathbb{Z}_6$ orbifold singularity is discussed in detail, and we construct a model in which all the R-R and NS-NS tadpoles in twisted sectors are canceled out and no tachyons exist in twisted sectors. This means that this non-supersymmetric $\mathbb{C}^3/\mathbb{Z}_6$
orbifold singularity is a stable solution of string theory. Three “Higgs doublet fields” and “up-type quark fields” are realized on the D3-brane as massless modes of the open string at tree level, and Higgs doublet fields have tree-level potential which has no flat direction. It is necessary for “electroweak symmetry breaking” that the one-loop radiative correction gives Higgs doublet field negative mass squared. The mass squared of Higgs fields can be obtained by calculating one-loop two point function in string world-sheet theory. In section 3 some techniques to calculate two point function in string world-sheet theory is reviewed. In section 4 the mass squared of a Higgs doublet field is calculated. Only the contribution from the twisted sectors are examined, since the contribution from the untwisted sector is model (compactification) dependent. We find that the mass squared is negative, and suggest that that the radiative “electroweak symmetry breaking” is possible in this system. Some additional comments are given at the end of this section.

2 D-branes at non-supersymmetric $\mathbb{C}^3/\mathbb{Z}_6$ singularity

We consider D3-branes at one $\mathbb{C}^3/\mathbb{Z}_6$ orbifold singularity which can be considered as one of the singularities in some six-dimensional compact space. Since we examine the properties which are determined only by the local structure of D-branes at the singularity, we consider $\mathbb{C}^3/\mathbb{Z}_6$ singularity rather than one of the singularities in some concrete compact space, say $T^6/\mathbb{Z}_6$ orbifold, for example. This is the approach which is proposed in Ref.[13].

The original massless spectrum on $n$ D3-branes is a $U(n)$ gauge multiplet of the four-dimensional $\mathcal{N} = 4$ supersymmetry, whose components are a gauge field $A^{\mu a}$ ($\mu = 0, 1, 2, 3$), six real scalar fields $X^{a\kappa}$ ($\kappa = 1, 2, \cdots, 6$) and four Weyl fermion fields $\lambda^a_{\alpha}$ ($\alpha = 1, 2, 3, 4$), where $a$ is the index of adjoint representation of $U(n)$. In the language of four-dimensional $\mathcal{N} = 1$ supersymmetry, they consist one $\mathcal{N} = 1$ gauge multiplet and three $\mathcal{N} = 1$ chiral multiplets in adjoint representation. The six real adjoint fields can be understood as brane position moduli fields. Under $SU(4)_R$ global symmetry, the six real scalar fields and four Weyl fermion fields belong to $\mathbf{6}$ and $\mathbf{4}$ representations, respectively. The transformation on six real scalars can be understood as the rotational transformation in the transverse six dimensional space. These fields correspond to the states in string world-sheet theory in the following way.

\begin{align}
A^{\mu a}(T^a)_{ij} &\sim \psi^\mu_{1/2}(\tilde{ij})_{\text{NS}}, \\
X^{a\kappa}(T^a)_{ij} &\sim \psi^{3+\kappa}_{1/2}(\tilde{ij})_{\text{NS}}, \\
\lambda^a_{\alpha}(T^a)_{ij} &\sim |s_1, s_2, s_3; \tilde{ij})_{\text{R}}\quad \text{with} \quad \prod_{r=1}^3 s_r = -1/8,
\end{align}

where NS and R denote Neveu-Schwarz sector and Ramond sector, respectively, $\psi^M_{1/2}$ with $M = 0, 1, \cdots, 9$ are creation operators of world-sheet fermions, and $\tilde{i}, j$ are Chan-Paton indices. Three quantities, $s_r = \pm 1/2$ with $r = 1, 2, 3$ in Ramond sector describe spin states in transverse six dimensional space.

The orbifold $\mathbb{Z}_N$ transformation must be a discrete subgroup of $SU(4)_R$. Namely,

$$\lambda_{\alpha} \rightarrow e^{2\pi i a_{\alpha}/N} \lambda_{\alpha}$$

with $a_{\alpha}$ as a set of integers. Number of D-branes at $\mathbb{C}^3/\mathbb{Z}_N$ singularity is a stable solution of string theory. Three “Higgs doublet fields” and “up-type quark fields” are realized on the D3-brane as massless modes of the open string at tree level, and Higgs doublet fields have tree-level potential which has no flat direction. It is necessary for “electroweak symmetry breaking” that the one-loop radiative correction gives Higgs doublet field negative mass squared. The mass squared of Higgs fields can be obtained by calculating one-loop two point function in string world-sheet theory. In section 3 some techniques to calculate two point function in string world-sheet theory is reviewed. In section 4 the mass squared of a Higgs doublet field is calculated. Only the contribution from the twisted sectors are examined, since the contribution from the untwisted sector is model (compactification) dependent. We find that the mass squared is negative, and suggest that that the radiative “electroweak symmetry breaking” is possible in this system. Some additional comments are given at the end of this section.
with \( a_1 + a_2 + a_3 + a_4 = 0 \mod N \), and

\[
Z_r \to e^{-2\pi i b_r/N} Z_r
\]

with \( b_1 = a_2 + a_3 \), \( b_2 = a_3 + a_1 \) and \( b_3 = a_1 + a_2 \), where \( Z_r \equiv (X^{2r-1} - iX^{2r})/\sqrt{2} \) are complexified scalar fields. In the string world-sheet theory, the world-sheet fermion fields transform as

\[
\psi^\mu \to \psi^\mu, \quad \psi^{(\pm)r} \to e^{\pm 2\pi i b_r/N} \psi^{(\pm)r},
\]

where \( \psi^{(\pm)r} \equiv (\psi^{2r+2} \pm i\psi^{2r+3})/\sqrt{2} \). World-sheet boson fields transform in the same way as world-sheet fermion fields. In case of \( a_4 = 0 \) and \( b_1 + b_2 + b_3 = 0 \), \( Z_N \subset SU(3) \) and \( \mathcal{N} = 1 \) SUSY remains (\( \lambda_4 \) is gaugino). Chan-Paton indices may be transformed under \( Z_N \) as

\[
|\bar{i} j\rangle \to (\gamma_3)^{ij}_{\bar{i}\bar{j}} (\gamma_3^{-1})_{\bar{j}\bar{j}},
\]

where \( \gamma_3 = \text{diag}(I_{n_0}, e^{2\pi i/N} I_{n_1}, \ldots, e^{2\pi i(N-1)/N} I_{n_{N-1}}) \) with \( n_0 + n_1 + \cdots + n_{N-1} = n \) and \( I_n \) is the \( n \times n \) unit matrix. By taking \( Z_N \) invariant states only, namely by the \( Z_N \) orbifold projection, the gauge symmetry is broken as \( U(n) \to U(n_0) \times U(n_1) \times \cdots \times U(n_{N-1}) \) and we have the following massless matter fields in bi-fundamental representations.

- **complex scalar fields:**
  \[
  \sum_{r=1}^{3} \sum_{i=0}^{N-1} (n_i, \bar{n}_{i-b_r}),
  \]

- **Weyl fermion fields:**
  \[
  \sum_{\alpha=1}^{4} \sum_{i=0}^{N-1} (n_i, \bar{n}_{i+a_\alpha}),
  \]

where \( n_i \) and \( \bar{n}_i \) mean fundamental representation and anti-fundamental representation of \( U(n_i) \), respectively.

Now we take \( n = 6 \) and consider non-supersymmetric \( Z_6 \) projection of

\[
(a_1, a_1, a_3, a_4) = (1, 1, 1, -3),
\]

\[
(b_1, b_2, b_3) = (2, 2, 2)
\]

on world-sheet fields and

\[
(n_0, n_1, \cdots, n_5) = (1, 3, 2, 0, 0, 0)
\]

on Chan-Paton indices. Note that the transformation on the six-dimensional space is \( Z_3 \), therefore, from the geometrical point of view, we are considering \( \mathbb{C}^3/Z_3 \) orbifold singularity. We have gauge symmetry of \( U(3) \times U(2) \times U(1) \) and matter of

- **Higgs doublet fields:** \( H_r \)
  \[
  3 \times (1, 2, -1),
  \]

- **left-handed quarks:** \( q_{Lr} \)
  \[
  3 \times (3, 2^*, 0),
  \]

- **right-handed quarks:** \( u^c_{Lr} \)
  \[
  3 \times (3^*, 1, +1),
  \]
where we omit to describe the charges of U(1) factors of U(3) and U(2). The origin of
three families is the three equivalent complex coordinates in six dimensional space (to be compactified) under the $\mathbb{Z}_N$ transformation. There are Yukawa couplings of

$$\mathcal{L}_Y = -g \sum_{r,s,t=1,2,3} \epsilon_{rst} \bar{u}_R^t q_L^s H^t + \text{h.c.}$$

(17)

which are originated from the superpotential among three chiral superfields in original $\mathcal{N} = 4$ theory ($g$ is the gauge coupling constant). Higgs fields follow the scalar potential of

$$V = \frac{g^2}{2} \sum_{r,s} (H^+_r T^a H_r) (H^+_s T^a H_s) + \frac{g^2}{4} \sum_{r,s} (H^+_r H_s) (H^+_s H_r)$$

(18)

which is originated from the D-term potential in the original theory ($T^a$ are generator matrices of U(2)). There is no flat direction in this potential, and Higgs doublet fields are not D-brane moduli.

The gauge anomalies or twisted R-R tadpoles on D3-brane are canceled by introducing D7-branes. We consider the following three types of D7-branes (implicitly assuming factorizable toroidal compactifications):

- D7$_1$-brane: world-volume coordinates $(x^0, x^1, x^2, x^3, x^6, x^7, x^8, x^9)$,
- D7$_2$-brane: world-volume coordinates $(x^0, x^1, x^2, x^3, x^4, x^5, x^8, x^9)$,
- D7$_3$-brane: world-volume coordinates $(x^0, x^1, x^2, x^3, x^4, x^5, x^6, x^7)$.

Namely, D7$_r$-brane is point-like in $r$-th complex plane $z_r$. Assume that $n$ D3-branes and $u$ D7$_r$-branes are on the same point ($\mathbb{C}^3/\mathbb{Z}_N$ singular point) in $z_r$. We have massless modes of the open string whose one edge is on D3-branes and another edge is on D7$_r$-branes. The boundary conditions of such open string are Neumann-Neumann type in four-dimensional space-time directions, Dirichlet-Dirichlet type in $z_r$-direction and Dirichlet-Neumann type in other directions. The massless spectrum with D3-branes and D7$_3$-branes, for example, before orbifold projection consists of two complex scalar fields in NS sector and two chiral fermions in R sector.

$$|s_1, s_2; \tilde{I}J\rangle_{\text{NS}} \quad \text{and} \quad |s_1, s_2; \tilde{I}j\rangle_{\text{NS}} \quad \text{with} \quad s_1 = s_2 = -1/2,$$

$$|s_3; \tilde{I}J\rangle_{\text{R}} \quad \text{and} \quad |s_3; \tilde{I}j\rangle_{\text{R}} \quad \text{with} \quad s_3 = 1/2,$$

(19) (20)

where $I, J$ denote Chan-Paton indices on D7$_3$-brane. There is a massless U($u$) gauge multiplet of eight-dimensional $\mathcal{N} = 1$ supersymmetry on D7$_3$-branes. In case of low string scale (TeV scale), the value of the gauge coupling of U($u$) becomes very small, and U($u$) can be considered as a global symmetry.

The $\mathbb{Z}_N$ transformation of the above open string states are

$$|s_1, s_2\rangle_{\text{NS}} \rightarrow e^{-\pi i (b_1 + b_2)/N} |s_1, s_2\rangle_{\text{NS}},$$

$$|s_3\rangle_{\text{R}} \rightarrow e^{\pi i b_3/N} |s_3\rangle_{\text{R}}.$$
Note that these phases do not belong to $\mathbb{Z}_N$, but they belong to $\mathbb{Z}_{2N}$. Chan-Paton indices may be transformed under $\mathbb{Z}_N$ as

$$
|\tilde{J}\rangle \rightarrow (\gamma_3)_{\tilde{J}^{'}\tilde{J}} (\gamma_3^{-1})_{J^{'}J},
$$

where $\gamma_3 = \text{diag}(I_{u_0}, e^{2\pi i/N} I_{u_1}, \cdots, e^{2\pi i(N-1)/N} I_{u_{N-1}})$ with $u_0 + u_1 + \cdots + u_{N-1} = u$ and $I_u$ is the $u \times u$ unit matrix. Since the phases in state transformation belong to $\mathbb{Z}_{2N}$, we need to set $\gamma_3$ differently depending on whether $b_3$ (or $b_1 + b_2 = b_3 + 2a_3$) is even or odd to have non-trivial spectra. In case of even $b_3$,

$$
\gamma_3 = \text{diag}(I_{u_0}, e^{2\pi i/N} I_{u_1}, \cdots, e^{2\pi i(N-1)/N} I_{u_{N-1}}),
$$

and in case of odd $b_3$,

$$
\gamma_3 = \text{diag}(e^{\pi i/N} I_{u_0}, e^{\pi i3/N} I_{u_1}, \cdots, e^{\pi i(2N-1)/N} I_{u_{N-1}}),
$$

where $u_0 + u_1 + \cdots + u_{N-1} = u$ and $I_u$ is the $u \times u$ unit matrix. The phases in case of even $b_3$ are $e^{2\pi im/N}$ with $m = 0, 1, \cdots, N - 1$, and the phases in case of odd $b_3$ are $e^{\pi im/N} e^{\pi i/N}$ with $m = 0, 1, \cdots, N - 1$. The resultant massless spectra are as follows. In case of even $b_3$,

$$
\text{complex scalar fields: } \sum_{i=0}^{N-1} \left[ (n_i, \bar{u}_i - \frac{1}{2}(b_1 + b_2)) + (u_i, \bar{n}_i - \frac{1}{2}(b_1 + b_2)) \right],
$$

$$
\text{Weyl fermion fields: } \sum_{i=0}^{N-1} \left[ (n_i, \bar{u}_i + \frac{1}{2}b_3) + (u_i, \bar{n}_i + \frac{1}{2}b_3) \right],
$$

and in case of odd $b_3$,

$$
\text{complex scalar fields: } \sum_{i=0}^{N-1} \left[ (n_i, \bar{u}_i - \frac{1}{2}(b_1 + b_2 + 1)) + (u_i, \bar{n}_i - \frac{1}{2}(b_1 + b_2 - 1)) \right],
$$

$$
\text{Weyl fermion fields: } \sum_{i=0}^{N-1} \left[ (n_i, \bar{u}_i + \frac{1}{2}(b_3 - 1)) + (u_i, \bar{n}_i + \frac{1}{2}(b_3 + 1)) \right].
$$

The massless states on D7$_3$-branes get projections in the same way of those on D3-branes. In case of even $b_3$, the resultant massless spectrum consists of gauge bosons of $\text{U}(u_0) \times \text{U}(u_1) \times \cdots \times \text{U}(u_{N-1})$ and

$$
\text{complex scalar fields: } \sum_{r=1}^{3} \sum_{i=0}^{N-1} (u_i, \bar{u}_{i-b_r}),
$$

$$
\text{Weyl fermion fields: } \sum_{a=1}^{4} \sum_{i=0}^{N-1} (u_i, \bar{u}_{i+a_n}).
$$

These complex scalar fields can be understood as two Wilson lines and one brane modulus.
The twisted R-R tadpole cancellation conditions in case of $\mathbb{C}^3/\mathbb{Z}_N$ singularity and $b_r = \text{even}$ for all $r = 1, 2, 3$ are

$$\left[ \prod_{r=1}^{3} 2 \sin \left( \frac{\pi k b_r}{N} \right) \right] \text{Tr} \left( (\gamma_3)^k \right) + \sum_{r=1}^{3} 2 \sin \left( \frac{\pi k b_r}{N} \right) \text{Tr} \left( (\gamma_{7r})^k \right) = 0$$

(33)

for all $k = 1, 2, \cdots, N-1$. In this formula we are considering three D7$_r$-branes ($r = 1, 2, 3$) each of which consists $u^r$ D7-branes. This conditions can be derived by calculating open string one-loop vacuum amplitudes of D3-D3 sector and D3-D7 sectors. In the closed string picture after the modular transformation, the one-loop vacuum amplitude can be understood as the sum of the amplitudes of tree-level propagation of closed string modes from D3-branes to D3-branes (D3-D3 sector) and from D3-branes to D7-branes (D3-D7 sector). The above conditions are the cancellation conditions of the contribution of the propagations of massless twisted R-R modes between D3-D3 sector and D3-D7 sectors. It can be shown that the gauge symmetry is anomaly free, if the twisted R-R tadpole cancellation conditions are satisfied.

Now, we consider again the case of $N = 6$ and $b_r = 2$ for all $r = 1, 2, 3$. The independent twisted R-R tadpole cancellation conditions are

$$3 \text{Tr} \left( (\gamma_3)^k \right) + \sum_{r=1}^{3} \text{Tr} \left( (\gamma_{7r})^k \right) = 0$$

(34)

for $k = 1, 2$. We can explicitly write these conditions as

$$\sum_{j=0}^{5} 3n_j + \sum_{r=1}^{3} u_j^r \left( e^{2\pi i/6} \right)^j = 0,$$

(35)

$$\sum_{j=0}^{2} \left\{ 3(n_j + n_{j+3}) + \sum_{r=1}^{3} (u_j^r + u_{j+3}^r) \right\} \left( e^{2\pi i/6} \right)^{2j} = 0.$$ 

(36)

We find

$$3n_j + \sum_{r=1}^{3} u_j^r = \text{const.} \quad \text{for all } j = 0, 1, \cdots, 5.$$ 

(37)

is the solution.

Consider only D7$_3$-brane for simplicity. The solution in this case is

$$(u_0^3, u_1^3, \cdots, u_5^3) = (6, 0, 3, 9, 9, 9)$$

(38)

with Eq. (13). This solution requires 36 D7$_3$-branes, and the gauge symmetry on the D7$_3$-brane is $\text{U}(6) \times \text{U}(3) \times \text{U}(9)_1 \times \text{U}(9)_2 \times \text{U}(9)_3$. The massless field contents of this model is shown in Table 1. The non-anomalous weak hypercharge $U(1)_Y$ with charge $Q_Y$ is defined as

$$Q_Y \equiv - \left( \frac{Q_3}{3} + \frac{Q_2}{2} + Q_1 \right),$$

(39)

where $Q_n$ is the U(1) charges of U(n) on D3-brane.
Table 1: Massless field contents in the model with 6 D3-branes and 36 D7\(_3\)-branes on non-supersymmetric \(\mathbb{C}^3/\mathbb{Z}_6\) orbifold singularity. Only one of many fields in D7-D7 sector is listed.

| sector | Fermi/Bose | number | U(3) | U(2) | U(1) | U(6) | U(3) | U(9)\(_1\) | U(9)\(_2\) | U(9)\(_3\) |
|--------|------------|--------|------|------|------|------|------|------------|------------|------------|
| D3-D3  | F \(q_L^r\) | 3      | 3    | 2\(^*\) | 0    | 1    | 1    | 1    | 1          | 1          |
|        | F \(\bar{u}_L^r\) | 3      | 3\(^*\) | 1    | +1   | 1    | 1    | 1    | 1          | 1          |
|        | B \(H_u^r\) | 3      | 1    | 2    | -1   | 1    | 1    | 1    | 1          |
| D3-D7  | F \(l_L\) | 1      | 3    | 1    | 0    | 1    | 3\(^*\) | 1    | 1          | 1          |
|        | B \(S\) | 1      | 1    | 1    | +1   | 1    | 1    | 9\(^*\) | 1          |
|        | B \(H_d\) | 1      | 3    | 1    | 0    | 1    | 1    | 1    | 9\(^*\)    |
| D7-D3  | F \(d_L\) | 1      | 3\(^*\) | 1    | 0    | 6    | 1    | 1    | 1          |
|        | F \(\bar{e}_L\) | 1      | 1    | 1    | -1   | 1    | 1    | 1    | 9          |
|        | B \(\Phi\) | 1      | 3\(^*\) | 1    | 0    | 1    | 1    | 9    | 1          |
|        | B \(H_\nu\) | 1      | 1    | 2\(^*\) | 0    | 1    | 1    | 1    | 9          |
| D7-D7  | F \(\bar{\nu}_L^r\) | 3      | 1    | 1    | 0    | 1    | 1    | 9    | 9\(^*\)    |

There are Yukawa couplings which come from the superpotential of the original supersymmetric theory due to the open string recombination of (D3-D3) \(\rightarrow\) (D3-D7\(_3\)) + (D7\(_3\)-D3):

\[
\Phi q_L^{r=3} l_L, \quad H_d q_L^{r=3} \bar{d}_L. \quad (40)
\]

There are scalar quartic interactions of the same origin:

\[
|H_u^{r=3} S|^2, \quad |H_u^{r=3} H_{\nu}|^2, \quad |S H_{\nu}|^2. \quad (41)
\]

Note that only one of three family states on D3-brane (only \(r = 3\) in this case) interact with D3-D7\(_3\) and D7\(_3\)-D3 states. This fact can be understood by concretely looking the recombination of string states, for example

\[
|s_1, s_2\rangle_{NS} + |s_3\rangle_R \rightarrow |s_1, s_2, s_3\rangle_R \quad (42)
\]

for D3-D7\(_3\) boson \((s_1 = s_2 = -1/2) + D7\(_3\)-D3 fermion \((s_3 = 1/2) \rightarrow\) D3-D3 fermion, and consider the transformation of each states under the \(\mathbb{Z}_N\) transformation. If we choose the symmetric solution among three kinds of D7-branes as

\[
(u_0^r, u_1^r, \cdots, u_5^r) = (2, 0, 1, 3, 3, 3) \quad \text{for all} \ r = 1, 2, 3 \quad (43)
\]

with Eq. (13), these interactions become symmetric. Asymmetric solutions would be more interesting than the symmetric solution, since three Higgs doublet fields would behave differently and obtain different one-loop corrections to their masses. In this paper we concentrate on the asymmetric solution with \(D7\(_3\)-branes only and examine the mass of the Higgs doublet field \(H_u^3\).
Although the twisted R-R tadpoles are canceled out in this model, there is no guarantee that twisted NS-NS tadpoles are also canceled out simultaneously, because of no supersymmetry. We find that the twisted NS-NS tadpoles are also canceled out by explicitly calculating the one-loop open string vacuum amplitude. After modular transformation, the amplitude can be divided into the contributions from four sectors.

\[
A_{D3-D3,R-R} = \sum_{\gamma=0}^{5} A_{D3-D3,R-R}^\gamma \\
A_{D3-D3,NS-NS} = \sum_{\gamma=0}^{5} A_{D3-D3,NS-NS}^\gamma \\
A_{D7-D7,R-R} = \sum_{\gamma=0}^{5} A_{D7-D7,R-R}^\gamma \\
A_{D7-D7,NS-NS} = \sum_{\gamma=0}^{5} A_{D7-D7,NS-NS}^\gamma
\]

where \(A_{D3-D3,R-R}^\gamma\) is the contribution from the closed string R-R sector in D3-D3 sector, and so on. The summation over \(\gamma\) comes from the \(Z_6\) projection operator

\[
P_{Z_6} = \sum_{\gamma=0}^{5} \hat{\alpha}^\gamma
\]

in world-sheet theory, where \(\hat{\alpha}\) is the \(Z_6\) operator. In the summation over \(\gamma\), the amplitudes of \(\gamma = 0, 3\) are the contributions from the untwisted sector and the amplitudes of \(\gamma = 1, 2, 4, 5\) are the contributions from twisted sectors. Since the the projection on the six-dimensional space is not by \(Z_6\) but actually by \(Z_3\), the amplitudes of \(\gamma = 0, 3\) should be considered as the contribution from the untwisted sector.

The asymptotic behavior in the limit of long distance propagation of the closed string is obtained for D3-D3 sector as follows.

\[
A_{D3-D3,R-R}^\gamma \rightarrow \int \frac{ds}{2s} \left( \frac{\pi}{s} \right)^{2} \frac{11}{6} \frac{iV_4}{\sqrt{8\pi^2\alpha'}} \left(-N_{\text{CP}}^\gamma \times 16\right) \quad \text{for } \gamma = 0, 3
\]

\[
A_{D3-D3,R-R}^\gamma \rightarrow \int \frac{ds}{2s} \frac{11}{2} \frac{iV_4}{\sqrt{8\pi^2\alpha'}} \left(-N_{\text{CP}}^\gamma \times 6\sqrt{3}\right) \quad \text{for } \gamma \neq 0, 3
\]

for R-R sector and

\[
A_{D3-D3,NS-NS}^\gamma \rightarrow \int \frac{ds}{2s} \left( \frac{\pi}{s} \right)^{2} \frac{11}{6} \frac{iV_4}{\sqrt{8\pi^2\alpha'}} \left(N_{\text{CP}}^\gamma \times 16\right) \quad \text{for } \gamma = 0
\]

\[
A_{D3-D3,NS-NS}^\gamma \rightarrow \int \frac{ds}{2s} \left( \frac{\pi}{s} \right)^{2} \frac{11}{6} \frac{iV_4}{\sqrt{8\pi^2\alpha'}} \left(N_{\text{CP}}^\gamma \times \frac{1}{4} e^s\right) \quad \text{for } \gamma = 3
\]

\[
A_{D3-D3,NS-NS}^\gamma \rightarrow \text{finite (massive modes only) for } \gamma = 1, 5
\]

\[
A_{D3-D3,NS-NS}^\gamma \rightarrow \int \frac{ds}{2s} \frac{11}{2} \frac{iV_4}{\sqrt{8\pi^2\alpha'}} \left(N_{\text{CP}}^\gamma \times 6\sqrt{3}\right) \quad \text{for } \gamma = 2, 4
\]
for NS-NS sector, where

$$N_{\gamma CP}^\gamma = (\text{Tr}(\gamma_3^\gamma))(\text{Tr}(\gamma_3^{-1}))$$

(55)

is the Chan-Paton factor. The untwisted sectors and the twisted sectors have different asymptotic behaviors. The R-R sector and NS-NS sector also have different asymptotic behaviors. The amplitudes of $\gamma = 0$ R-R and NS-NS untwisted sectors and $\gamma = 3$ R-R untwisted sectors are “ultraviolet” ($s \to \infty$) finite due to the power behavior of $(\pi/s)^2$, even though massless tadpoles exist in this sector. The amplitudes of R-R twisted sectors and $\gamma = 2, 4$ NS-NS twisted sectors linearly diverge due to the massless twisted R-R and NS-NS tadpoles. There is a tachyon mode in $\gamma = 3$ NS-NS untwisted sector which is consistent with the result in Ref. [14].

The asymptotic form of the amplitudes in D3-D7 sector can be obtained as follows.

$$A_{\gamma D3-D7,R-R} = 0$$ amplitudes are zero for $\gamma = 0, 3$, (56)

$$A_{\gamma D3-D7,R-R} \to \int \frac{ds}{2s} \frac{1}{\pi} \frac{1}{2\times 6} \left(-\bar{N}_{\gamma CP} \times 2\sqrt{3}\right)$$ for $\gamma \neq 0, 3$ (57)

for R-R sector and

$$A_{\gamma D3-D7,NS-NS} \to \text{finite (massive modes only) for } \gamma = 0,$$ (58)

$$A_{\gamma D3-D7,NS-NS} \to \int \frac{ds}{2s} \frac{1}{\pi} \frac{1}{2\times 6} \left(-\bar{N}_{\gamma CP} \times e^a\right)$$ for $\gamma = 3$, (59)

$$A_{\gamma D3-D7,NS-NS} \to \text{finite (massive modes only) for } \gamma = 1, 5,$$ (60)

$$A_{\gamma D3-D7,NS-NS} \to \int \frac{ds}{2s} \frac{1}{\pi} \frac{1}{2\times 6} \left(\bar{N}_{\gamma CP} \times 2\sqrt{3}\right)$$ for $\gamma = 2, 4$ (61)

for NS-NS sector, where

$$\bar{N}_{\gamma CP}^\gamma = (\text{Tr}(\gamma_3^\gamma))(\text{Tr}(\gamma_7^{-1}))$$

(62)

is the Chan-Paton factor. There are tadpoles in all R-R twisted sectors and $\gamma = 2, 4$ NS-NS twisted sectors. There is a tachyon mode in $\gamma = 3$ NS-NS untwisted sector.

The twisted R-R tadpoles should be canceled out between D3-D3 and D3-D7 sectors. The condition of the cancellations in the above formulae are

$$3N_{\gamma CP}^\gamma + \bar{N}_{\gamma CP}^\gamma = 0,$$ (63)

and this is nothing but the previously introduced twisted R-R tadpole cancellation conditions of Eq. (34) for the present case. It is very interesting that the twisted NS-NS tadpoles are also canceled out between D3-D3 and D3-D7 sectors with the same conditions. It is also interesting that no tachyon mode exists in twisted sector. The existence of tachyon modes in twisted sector means the instability of the singularity [15]. Therefore, the present singularity is a stable solution of string theory at tree level.

Before closing this section, we give a comment on the global definition of the model. To cancel untwisted R-R tadpoles, we have to specify the compact space (toroidal orbifolds or toroidal orientifolds, for example), and further have to introduce D7-branes, D3-branes and their anti-branes. To project out the tachyon in untwisted sector we have to take
some special orientifolds [16, 17]. This step is strongly related to concrete model buildings which we do not pursuit in this paper. The aim of this paper is to examine rather model independently whether the radiative gauge symmetry breaking is possible or not in this kinds of models of D-branes at singularities.

3 Some calculations of one-loop two point functions

We calculate two point function of the gauge boson on D9-brane. Because of the gauge invariance, the results must be zero. The following calculations is a demonstration to review the technique for superstring one-loop calculation in string world-sheet theory. The technique is directly applicable to the one-loop calculation of the masses of the Higgs doublet fields on D3-brane at $C^3/Z_6$ non-supersymmetric singularity.

Before going to the calculation in string theory, we stress the importance of the regularization of divergence, or the definition of integral, by using the example in four-dimensional field theory. Consider the one-loop correction to the mass of U(1) gauge boson by one massive Dirac fermion.

$$- \text{tr} \int \frac{d^4k}{(2\pi)^4} g_{\mu} \left( \frac{1}{m - \gamma_0 k^0} g_{\nu} \right) \frac{1}{m - \gamma_0 k^0} = -g^2 g^{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \frac{4m^2 - 2k^2}{(m^2 - k^2)^2},$$

where the external momentum is set to zero. This integral is divergent. We use the dimensional regularization, which is a gauge invariant regularization, to handle this integral.

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(m^2 - k^2)^2} = \frac{\Gamma(2 - d/2)}{(4\pi)^{d/2} \Gamma(2)} \frac{1}{(m^2)^{2-d/2}},$$

where $d = 4 - \epsilon$ with $\epsilon \to 0$ and we used the relation $z \Gamma(z) = \Gamma(z + 1)$ in the last equality.

Next, consider the the one-loop correction to U(1) gauge boson by one massless Dirac fermion.

$$- \text{tr} \int \frac{d^4k}{(2\pi)^4} g_{\mu} \left( \frac{1}{-\gamma_0 k^0} g_{\nu} \right) \frac{1}{-\gamma_0 k^0} = -2g^2 g^{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \frac{1}{-k^2}. $$

This quadratically divergent integral is regularized or rather defined as follows.

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{\delta - k^2} = \lim_{\delta \to 0} \int \frac{d^d k}{(2\pi)^d} \frac{1}{\delta - k^2} = \lim_{\delta \to 0} \frac{\Gamma(1 - d/2)}{(4\pi)^{d/2} \Gamma(1)} \frac{1}{\delta^{1-d/2}} = 0.$$
Now, we turn to the one-loop correction in string theory. Consider open superstring one-loop correction to the two point function of massless SU(N) gauge boson states on D9-brane. It is not trivially zero like the tree level contribution (the disk has three conformal killing vectors and the cylinder has one conformal killing vector). Since we consider SU(N) gauge bosons, only the planar diagrams contribute.

We try to apply the formalism which is described in the text book by Green, Schwarz and Witten[18]. There are two contributions: NS-loop and R-loop, which are corresponding to boson and fermion loops, respectively.

\[ A_{NS} = \int \frac{d^{10}k}{(2\pi)^{10}i} \text{tr} \{ \Delta \ V(1) \Delta \ V(1) \ P_{GSO} \} , \quad (69) \]
\[ A_{R} = -\int \frac{d^{10}k}{(2\pi)^{10}i} \text{tr} \{ S \ W(1) \ S \ W(1) \ P_{GSO} \} , \quad (70) \]

where \( P_{GSO} \) is the GSO projection operator,

\[ \Delta = \int_{0}^{1} x^{L_{0}-1}dx, \quad (71) \]
\[ S = G_{0} \int_{0}^{1} x^{L_{0}-1}dx = G_{0}\Delta \quad (72) \]

are propagator operators (\( L_{0} \) and \( G_{0} \) are generators in super-Virasoro algebra),

\[ V(x) = \frac{g_{O}}{\sqrt{2\alpha'}} e_{\mu} i \dot{X}^{\mu}, \quad \dot{X}^{\mu} \equiv x \frac{d}{dx} X^{\mu}(x), \quad (73) \]
\[ W(x) = g_{O} \sqrt{2} e_{\mu} \sqrt{x} \psi^{\mu}(x) \quad (74) \]

are vertex operators with zero external momentum, \( e_{\mu} \) is the polarization vector with Chan-Paton indices, and \( g_{O} \) is the open string coupling. The argument \( z = e^{-i(\sigma_{1}+i\sigma_{2})} \) of the world-sheet fields, \( X^{\mu}(z) \) and \( \psi^{\mu}(z) \), is now real: \( z = x \).

First, consider NS-loop. The straightforward calculation gives the following result.

\[ A_{NS} = g_{O}^{2} \text{tr}(e_{\mu} e_{\nu}) \eta^{\mu\nu} \frac{1}{(2\pi)^{10}} \int_{0}^{1} \frac{d\rho_{1}}{\rho_{1}} \frac{d\rho_{2}}{\rho_{2}} \theta(\rho_{1} - \rho_{2}) \]
\[ \times \left( \sqrt{\frac{\pi}{-\alpha' \ln \rho_{2}}} \right)^{10} \left\{ \frac{1}{-\ln \rho_{2}} + \sum_{l=1}^{\infty} \left( \frac{\rho_{2}/\rho_{1}}{\rho_{2}/\rho_{1}} \right)^{l} \frac{\rho_{2}/(\rho_{2}/\rho_{1})^{l}}{1 - \rho_{2}} \right\} \]
\[ \times \frac{1}{(\eta(\tau_{2}))^{8}} \frac{1}{2} \left\{ \left( \frac{\theta_{3}(\tau_{2})}{\eta(\tau_{2})} \right)^{4} - \left( \frac{\theta_{4}(\tau_{2})}{\eta(\tau_{2})} \right)^{4} \right\} , \quad (75) \]

where \( \tau_{2} \) is defined by \( \rho_{2} = \exp(2\pi i \tau_{2}) \). The factor in the first curly brackets is proportional to the correlation function \( \langle \dot{X}^{\mu}(\rho_{1}) \dot{X}^{\nu}(\rho_{2}) \rangle \) on the boundary circle. Consider the integration by \( \rho_{1} \):

\[ I_{NS} = \int_{0}^{1} \frac{d\rho_{1}}{\rho_{1}} \theta(\rho_{1} - \rho_{2}) \left\{ \frac{1}{-\ln \rho_{2}} + \sum_{l=1}^{\infty} \left( \frac{\rho_{2}/\rho_{1}}{\rho_{2}/\rho_{1}} \right)^{l} \frac{\rho_{2}/(\rho_{2}/\rho_{1})^{l}}{1 - \rho_{2}} \right\} \quad (76) \]
The first term becomes
\[ \int_0^1 \frac{d\rho_1}{\rho_1} \theta(\rho_1 - \rho_2) \frac{1}{-\ln \rho_2} = \int_{\rho_2}^1 \frac{d\rho_1}{\rho_1 - \ln \rho_2} = 1. \] (77)

The second term is a divergent quantity, and we have to define this quantity by some regularization. Consider the following regularization:
\[ \int_{\rho_2}^1 \frac{d\rho_1}{\rho_1} \sum_{l=1}^{\infty} l \frac{\rho_2/\rho_1^l + \rho_2/(\rho_2/\rho_1)^l}{1 - \rho_2^l} \rightarrow \sum_{l=1}^{\infty} l \int_{\rho_2}^1 \frac{d\rho_1}{\rho_1} \frac{(\rho_2/\rho_1)^l + \rho_2/(\rho_2/\rho_1)^l}{1 - \rho_2^l} \]
\[ = 2 \sum_{l=1}^{\infty} 1 \]
\[ \rightarrow 2 \lim_{z \rightarrow 0} \sum_{l=1}^{\infty} \frac{1}{l^2} = 2 \lim_{z \rightarrow 0} \zeta(z) = -1, \] (78)

where \( \zeta(z) \) is the Riemann zeta function and \( \zeta(0) \) is defined as \(-1/2\) by the analytic continuation. Therefore, \( I^{\text{NS}} = 0 \) and \( A^{\text{NS}} = 0 \) as required by gauge invariance.

Next, consider R-loop. The straightforward calculation (using the above zeta function regularization in part) gives the following result.

\[ A^{R} = -g_D^2 \text{tr}(e_{\mu} e_{\nu}) \eta^{\mu \nu} \frac{1}{(2\pi)^{10}} \int_0^1 \frac{d\rho_1}{\rho_1} \frac{d\rho_2}{\rho_2} \theta(\rho_1 - \rho_2) \]
\[ \times \left( \frac{\pi}{-\alpha' \ln \rho_2} \right)^{10} \left\{ \frac{1}{-\ln \rho_2} + 2 \sum_{l=1}^{\infty} l \rho_2^l \frac{(\rho_2/\rho_1)^l + (\rho_1/\rho_2)^l}{(1 - \rho_2^l)(1 + \rho_2^l)} \right\} \]
\[ \times \frac{1}{(\eta(\tau_2))^8} \cdot \frac{1}{2} \left( \frac{\theta_2(\tau_2)}{\eta(\tau_2)} \right)^4. \] (79)

The part which is proportional to \( 1/\ln \rho_2 \) in the integrant is canceled out in \( A^{\text{NS}} + A^{\text{R}} \) due to the identity \((\theta_3)^4 - (\theta_4)^4 - (\theta_2)^4 = 0\). This is the result of supersymmetry. Consider the integration by \( \rho_1 \).

\[ I^{R} = \int_0^1 \frac{d\rho_1}{\rho_1} \theta(\rho_1 - \rho_2) \left\{ \frac{1}{-\ln \rho_2} + 2 \sum_{l=1}^{\infty} l \rho_2^l \frac{(\rho_2/\rho_1)^l + (\rho_1/\rho_2)^l}{(1 - \rho_2^l)(1 + \rho_2^l)} \right\}. \] (80)

The second term is a divergent quantity, and again we have to define this quantity by some regularization. Consider the same regularization procedure in NS-loop calculation.
\[ \int_{\rho_2}^1 \frac{d\rho_1}{\rho_1} \sum_{l=1}^{\infty} l \rho_2^l \frac{(\rho_2/\rho_1)^l + (\rho_1/\rho_2)^l}{(1 - \rho_2^l)(1 + \rho_2^l)} \rightarrow 2 \sum_{l=1}^{\infty} l \int_{\rho_2}^1 \frac{d\rho_1}{\rho_1} \frac{(\rho_2/\rho_1)^l + (\rho_1/\rho_2)^l}{(1 - \rho_2^l)(1 + \rho_2^l)} \]
\[ = 2 \sum_{l=1}^{\infty} 1 \]
\[ \rightarrow 2 \lim_{z \rightarrow 0} \sum_{l=1}^{\infty} \frac{1}{l^2} = 2 \lim_{z \rightarrow 0} \zeta(z) = -1. \] (81)
Therefore, \( I^R = 0 \) and \( A^R = 0 \) as required by gauge invariance.

In the next section, we use the same definition of the divergent integrals in the calculation of mass squared of Higgs doublet field.

### 4 One-loop correction to the Higgs mass

The two point function of the Higgs doublet field can be calculated using the same technique reviewed in the previous section. We calculate one-loop two points function of \( H_3 \), one of three Higgs doublet fields, with zero external momentum. There are two contributions from D3-D3 and D3-D7 sectors. Only the planer diagrams contribute because of the conservation of Chan-Paton charges (\( U(2) \times U(1) \) charges).

In D3-D3 sector there are two planer diagrams corresponding to which boundary of annulus two open string vertex operators attach to. It is enough to calculate one of them, because the difference appears only in Chan-Paton factors: they are complex conjugate with each other. The contribution of one planar diagram in D3-D3 sector is given by

\[
A_{\text{D3-D3}} = A_{\text{NS-D3}} + A_{\text{R-D3}}
\]

with

\[
A_{\text{NS-D3}}^{\text{NS-D3}} = \int \frac{d^4k}{(2\pi)^4i} \text{tr} \left\{ \Delta V^-(1) \Delta V^+(1) P_{Z_6} P_{\text{GSO}} \right\} + (- \leftrightarrow +), \tag{83}
\]

\[
A_{\text{R-D3}}^{\text{R-D3}} = -\int \frac{d^4k}{(2\pi)^4i} \text{tr} \left\{ S W^-(1) S W^+(1) P_{Z_6} P_{\text{GSO}} \right\} + (- \leftrightarrow +), \tag{84}
\]

where

\[
V^+(x) = \frac{g_O}{\sqrt{2\alpha'}} u_i \dot{X}^+(x) \quad \text{and} \quad V^-(x) = \frac{g_O}{\sqrt{2\alpha'}} u_i^\dagger \dot{X}^-(x), \tag{85}
\]

\[
W^+(x) = g_O \sqrt{2} u \sqrt{x} \psi^+(x) \quad \text{and} \quad W^-(x) = g_O \sqrt{2} u^\dagger \sqrt{x} \psi^-(x) \tag{86}
\]

are vertex operators with zero external momentum with

\[
X^{(\pm)} \equiv \frac{1}{\sqrt{2}} (X^8 \pm iX^9) \quad \text{and} \quad \psi^{(\pm)} \equiv \frac{1}{\sqrt{2}} (\psi^8 \pm i\psi^9). \tag{87}
\]

The factor \( u \) carries Chan-Paton indices. A special care is required to correctly include the action of \( P_{Z_6} \) on Chan-Paton indices. The open string coupling constant \( g_O \) is translated to the dimensionless coupling (gauge coupling on D3-brane) by \( g = g_O/\sqrt{\alpha'} \). There are two contributions, NS-loop and R-loop, which are corresponding to boson and fermion loops, respectively. The amplitude \( A_{\text{D3-D3}} \) can be understood as mass squared, and the contribution of NS-loop (R-loop) is positive (negative). The boundary condition of the open string is Neumann-Neumann type in the direction in D3-brane world-volume, and Dirichlet-Dirichlet type in the directions of transverse six-dimensional space. No space-time momentum in the transverse six-dimensional space is allowed due to the Dirichlet-Dirichlet boundary condition.
The straightforward calculation results

\[ A_{D3-D3}^{NS} = \frac{g_0^2}{(2\pi)^6} \sum_{\gamma=0}^{5} N_{D3}^{\gamma} \int_0^1 \frac{d\rho}{\rho} \left( \sqrt{\frac{\pi}{-\alpha' \ln \rho}} \right)^4 \]

\[ \times 2\Re \left\{ -1 + (1 - e^{2\pi i \gamma/3}) \left( \frac{1}{2} - 2\pi \sum_{n=1}^{\infty} \frac{\sin(2\pi n \gamma/3)}{n} \rho^n \right) \right\} \]

\[ \times \frac{1}{2} \left[ \frac{\theta_3(\tau)}{\eta(\tau)^3} \left( \frac{\theta \left[ \begin{array}{c} 1/2 \\ \gamma/3 \end{array} \right] (-1/2|\tau)/2 \sin(\pi \gamma/3) \right) \right. \]

\[ \left. \left. - \frac{\theta_4(\tau)}{\eta(\tau)^3} \left( \frac{\theta \left[ \begin{array}{c} 1/2 \\ \gamma/3 \end{array} \right] (-1/2|\tau)/2 \sin(\pi \gamma/3) \right) \right) \right] \right) \]  \tag{88}

and

\[ A_{D3-D3}^{R} = -\frac{g_0^2}{(2\pi)^6} \sum_{\gamma=0}^{5} N_{D3}^{\gamma} \int_0^1 \frac{d\rho}{\rho} \left( \sqrt{\frac{\pi}{-\alpha' \ln \rho}} \right)^4 \]

\[ \times 2\Re \left\{ -1 - (1 - e^{2\pi i \gamma/3}) \left( 4i \sum_{n=1}^{\infty} \frac{1 - (-1)^n}{2} \sin(2\pi n \gamma/3) \rho^n \right) \right\} \]

\[ \times \frac{1}{2} \frac{\theta_2(\tau)}{\eta(\tau)^3} \left( \frac{\theta \left[ \begin{array}{c} 1/2 \\ -\gamma/3 \end{array} \right] (-1/2|\tau)/2 \sin(\pi \gamma/3) \right) \right) \]  \tag{89}

where \( \tau = it \) with \( \rho = e^{2\pi i \tau} = e^{-2\pi t} \),

\[ N_{D3}^{\gamma} \equiv 2\text{tr}(\gamma_3^{-1}) \]  \tag{90}

and

\[ \theta \left[ \begin{array}{c} \alpha \\ \beta \end{array} \right] (z|\tau) \equiv e^{2\pi i \alpha(\tau)} q^{\alpha^2/2} \prod_{n=1}^{\infty} (1 - q^n) \]

\[ \times \prod_{m=1}^{\infty} (1 + q^{m+\alpha-1/2} e^{2\pi i (\tau+\beta)}) (1 + q^{m-\alpha-1/2} e^{-2\pi i (\tau+\beta)}) \]  \tag{91}

is the generalized theta function \( (q \equiv \exp(2\pi i \tau)) \) \cite{19}. Three well-known theta functions are given by some special cases of this generalized theta function as

\[ \theta_2(\tau) = \theta \left[ \begin{array}{c} 1/2 \\ 0 \end{array} \right] (0|\tau), \quad \theta_3(\tau) = \theta \left[ \begin{array}{c} 0 \\ 0 \end{array} \right] (0|\tau), \quad \theta_4(\tau) = \theta \left[ \begin{array}{c} 0 \\ 1/2 \end{array} \right] (0|\tau). \]  \tag{92}
There are also two planar diagrams in D3-D7 sector, because there are two ways of assigning Chan-Paton indices to the open string in a loop. It is enough to calculate one of them, because the difference appears only in Chan-Paton factors: they are again complex conjugate with each other. The contribution of one planar diagram in D3-D7 sector is given by

$$A_{D3-D7} = A_{D3-D7}^N + A_{D3-D7}^R$$

with exactly the same form in Eqs. (83) and (84) for $A_{D3-D7}^N$ and $A_{D3-D7}^R$, respectively. The main difference from D3-D3 sector is the boundary condition of the open string. We have to take Neumann-Neumann boundary condition for the directions in D3-brane world-volume, Dirichlet-Dirichlet boundary condition for the transverse directions of D7-brane, and Dirichlet-Neumann boundary condition for the other directions.

The straightforward calculation gives the following results.

$$A_{D3-D7}^N = g_D^2 \frac{1}{(2\pi)^4} \frac{1}{6} \sum_{\gamma=0}^{5} N_{D7}^\gamma \int_0^1 \frac{d\rho}{\rho} \left( \sqrt{\frac{\pi}{-\alpha' \ln \rho}} \right)^4 \times 2\Re \left\{ -1 + (1 - e^{2\pi i\gamma/3}) \left( \frac{1}{2} - 2i \sum_{n=1}^{\infty} \sin \left( \frac{2\pi n \gamma}{3} \right) \frac{\rho^n}{1 - \rho^n} \right) \right\}$$

and

$$A_{D3-D7}^R = -g_D^2 \frac{1}{(2\pi)^4} \frac{1}{6} \sum_{\gamma=0}^{5} N_{D7}^\gamma \int_0^1 \frac{d\rho}{\rho} \left( \sqrt{\frac{\pi}{-\alpha' \ln \rho}} \right)^4 \times 2\Re \left\{ -1 + (1 - e^{2\pi i\gamma/3}) \left( 4i \sum_{n=1}^{\infty} \frac{1 - (-1)^n}{2} \sin \left( \frac{2\pi n \gamma}{3} \right) \frac{\rho^n}{1 - \rho^n} \right) \right\}$$

where

$$N_{D7}^\gamma \equiv 2\text{tr}(\gamma^{-1}).$$

Twisted R-R tadpole cancellation conditions are encoded in the relations of

$$N_{D7}^\gamma = -3N_{D3}^\gamma.$$
The one-loop correction to the Higgs mass squared is obtained by combining above results as follows.

\[
m^2 = \frac{(g_0/\sqrt{\alpha'})^2}{16\pi^2} \frac{2}{\alpha'} \int_0^1 \frac{d\rho}{\rho(\ln\rho)^2} \times \left\{ -\frac{1}{6} \sum_{\gamma=0}^5 \Re(N^\gamma_D) \left( Z^\gamma_{NS,\beta=0} - Z^\gamma_{NS,\beta=1} \right) + \frac{1}{6} \sum_{\gamma=0}^5 \Re(N^\gamma_D) \Re\left( \frac{1 - e^{2\pi i\gamma/3}}{2} \right) \left( Z^\gamma_{NS,\beta=0} - Z^\gamma_{NS,\beta=1} \right) - \frac{1}{6} \sum_{\gamma=0}^5 \Re(N^\gamma_D) \Re(i(1 - e^{2\pi i\gamma/3})) 2 \sum_{n=1}^{\infty} \frac{\sin(\frac{2\pi n\gamma}{3})}{\pi} \frac{\rho^n}{1 - \rho^n} \left( Z^\gamma_{NS,\beta=0} - Z^\gamma_{NS,\beta=1} \right) + \frac{1}{6} \sum_{\gamma=0}^5 \Re(N^\gamma_D) Z_{R,\beta=0} + \frac{1}{6} \sum_{\gamma=0}^5 \Re(N^\gamma_D) \Re(i(1 - e^{2\pi i\gamma/3})) 2 \sum_{n=1}^{\infty} (1 - (-1)^n) \frac{\rho^n}{1 - \rho^n} Z_{R,\beta=0} \right\} ,
\]

(98)

where the definition of \( Z_{NS,\beta=0}, Z_{NS,\beta=1} \) and \( Z_{R,\beta=0} \) are given in Appendix. The first three terms in the curly brackets in the above equation describe the one-loop contribution of bosonic modes, and the last two terms describe the contribution of fermionic modes. It is expected that the boson loop contributes positively and the fermion loop contributes negatively. As explained in section I, we consider only twisted sectors, \( \gamma = 1, 2, 4, 5 \), because of the model (compactification) dependence of the untwisted sector, \( \gamma = 0, 3 \).

Since present \( Z_6 \) transformation acts as \( Z_3 \) transformation in bosonic sector (present orbifold singularity is geometrically \( \mathbb{C}^3/Z_3 \)), the factor \( Z^\gamma_{NS,\beta=0} - Z^\gamma_{NS,\beta=1} \) is the function of \( \exp(2\pi i\gamma/3) \) and its complex conjugate as the function of \( \gamma \). Therefore, the values in case of \( \gamma = 1 \) and \( \gamma = 4 \) equal, and the values in case of \( \gamma = 2 \) and \( \gamma = 5 \) equal. On the other hand, all the three coefficients in this factor in Eq.(98) give the values of the same magnitude with opposite sign for each pair of \( \gamma = 1, 4 \) and \( \gamma = 2, 5 \). Consequently, all the boson loop contributions of twisted sectors are canceled out in this model. This is not the case in fermion loop sector, since the fermion states fully transform under \( Z_6 \).

The first term of the fermion loop contributions in Eq.(98) diverges at \( \rho \to 0 \), which corresponds to the effect of massless physical states in the loop. Since we do not follow the “\( i\epsilon \)-prescription” in the definition of the propagator operators, we have divergence of the type of \( \ln(m) \) with \( m \to 0 \) instead of some imaginary part. We neglect this divergent contribution and consider only the last term of Eq.(98). Which should give negative contribution to Higgs mass squared, since it is a fermion loop contribution.

We can make an order estimate by moving to the closed string picture and neglecting the exponentially suppressed higher-order terms in the integrant. The result is

\[
m^2 \sim -\frac{g^2}{16\pi^2} \frac{2}{\alpha'} \frac{36\sqrt{3}}{\pi} \int_0^{\infty} \frac{ds}{s} \frac{e^{-2\pi^2/s}}{1 - e^{-2\pi^2/s}} e^{-s/3} ,
\]

(99)
where \( s \equiv \pi/t \) and \( g = g_0/\sqrt{\alpha'}. \) We can have finite result due to twisted tadpole cancellations in both R-R and NS-NS sectors. The order of the “electroweak scale” is numerically obtained as

\[
v \sim \sqrt{-m^2/g^2} \simeq 10^{-2} \alpha'^{-1/2},
\]

(100)

where we used that the Higgs quartic coupling is given by the gauge coupling, Eq.(18). This is consistent with the standard understanding that the scale is given by string scale with one-loop suppression factor. This result suggests that the radiative gauge symmetry breaking on D-branes is possible in this type of models of D-branes at non-supersymmetric orbifold singularities.

At the end of this paper, we would like to give some comments on the global definition of the model. To have some consistent models with the tadpole cancellation in untwisted sector, we have to specify some concrete six-dimensional compact space, which is an orbifold or orientifold. By setting appropriate D3-branes and D7-branes at one of some orbifold fixed points in that compact space, it is possible to have standard model like massless spectrum. The tadpoles in untwisted sector can be canceled out by putting some D3-branes, D7-branes, anti-D3-branes and anti-D7-branes, at some appropriate other fixed points in that compact space. (some examples are given in Ref.[13]). Since fixed points are specially separated, no new massless mode with the gauge charge of our D3-branes appears. If the distance between fixed points can be taken very large in compare with the string scale (the size of the compact space must be large in TeV string scenario anyway), the contribution to the Higgs mass squared through the one-loop effect by these massive open string modes would be small. The tachyon modes in untwisted sector may be projected out by taking certain orientifold projections (see Ref.[19] for open descendants of the type 0B model, for example). Therefore, it would be possible to have some consistent systems without tadpoles and tachyon modes in untwisted sector with appropriate massless modes on D3-branes. It may be interesting to estimate how large the one-loop contribution of untwisted sector to the Higgs mass squared in these globally well-defined models. (In the present model only \( \gamma = 0 \) untwisted sector contributes in Eq.(98), since \( \Re(N^1_{D3}) = 0. \) It would be very interesting to explorer realistic models in this direction.

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Appendix

The definition of the functions \( Z_{\text{NS},\beta=0} \), \( Z_{\text{NS},\beta=1} \) and \( Z_{\text{R},\beta=0} \) in Eq.(98) is given as follows.

\[
Z_{\text{NS},\beta=0}^\gamma = \frac{\theta_3(\tau)}{\eta(\tau)^3} \theta \begin{bmatrix} 0 & \gamma/3 \\ 1/2 & -\gamma/3 \end{bmatrix} (0|\tau) \theta \begin{bmatrix} 0 & -1/2|\tau\rangle/2 \sin(\pi\gamma/3) \\ 1/2 & \gamma/3 \end{bmatrix} (0|\tau) \theta
\]

(101)
These are the combination of generalized theta functions which appear in open string one-loop vacuum amplitude.

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