NONCOMMUTATIVITY IN MAXWELL-CHERN-SIMONS-MATTER THEORY SIMULATES PAULI MAGNETIC COUPLING

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Abstract:
We study interactions between like charges in the noncommutative Maxwell-Chern-Simons electrodynamics minimally coupled to spinors or scalars. We demonstrate that the non-relativistic potential profiles, for only spatial noncommutativity, are nearly identical to the ones generated by a non-minimal Pauli magnetic coupling, originally introduced by Stern [12]. Although the Pauli term has crucial roles in the context of physically relevant objects such as anyons and like-charge bound states (or ”Cooper pairs”), its inception [12] (see also [13]) was ad-hoc and phenomenological in nature. On the other hand we recover similar results by extending the minimal model to the noncommutative plane, which has developed into an important generalization to ordinary spacetime in recent years. No additional input is needed besides the noncommutativity parameter.

We prove a novel result that for complex scalar matter sector, the bound states (or ”Cooper pairs” can be generated only if the Maxwell-Chern-Simons-scalar theory is embedded in noncommutative spacetime. This is all the more interesting since the Chern-Simons term does not directly contribute a noncommutative correction term in the action.

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**Introduction:**
In recent times we have learnt to live with the fact that coexistence of quantum field theory and gravity demands a drastic change in our notions of geometry, in particular from the classical spacetime continuum to a quantum fuzzy spacetime [1]. The fuzziness induces a lower bound on the localization of a spacetime point itself. The need for this length scale in quantum gravity can be justified from a semiclassical argument: localization of a particle within the Planck length requires a large amount of energy that is enough to create a black hole, which in turn can swallow the particle. This impasse is avoided by the introduction of a fuzzy spacetime, endowed with an uncertainty relation of the form

$$\Delta x^i \Delta x^j \geq \frac{1}{2} | \theta^{ij} |.$$ 

This phenomenon can be induced by a non-trivial coordinate commutation relation,

$$[x^i, x^j] = i \theta^{ij},$$

very much in analogy to the standard phase space commutation relations,

$$[x^i, p_j] = i\hbar \delta^i_j.$$ 

The noncommutativity parameter $\theta^{ij}$ plays the role of Planck’s constant $\hbar$.

These heuristic ideas have been strongly supported in string theory. Ultra-high energy scattering amplitudes suggest a modified form of Heisenberg phase space uncertainty relation that directly leads to a minimum length scale. However, the Non-Commutative (NC) spacetime scenario has received a great impetus after the seminal work of Seiberg and Witten [2], that relates gauge theories in NC spacetime to low energy limits of open string theory moving in an antisymmetric background field [3]. The inherent non-locality in NC gauge theories gives rise to a host of interesting phenomena such as UV/IR mixing [4], loss of unitarity [5] and violation of Lorentz invariance [6] to name a few.

In an alternative approach [7], one can study effects of noncommutativity in a local quantum field theoretic framework, where one exploits the Seiberg-Witten map [2]. This scheme, to be elaborated later, will be followed in our work.

The above discussion is aimed at convincing the reader that, (at least for distances short enough), NC spacetime is quite natural and physically motivated 1. In this background, study of NC extensions of well studied quantum field theories in ordinary spacetime has gained a lot of importance and the present work falls in this category, where we will demonstrate that NC effects alter the behavior of charged matter coupled to Maxwell-Chern-Simons gauge theory in interesting and non-trivial ways. In these instances, our approach of analyzing the scattering and static potential problems will provide further insights on NC effects in a more familiar (and possibly simpler) setting of non-relativistic Schrodinger equation formalism.

Let us briefly mention the relevance of Chern-Simons theory in ordinary spacetime. Arbitrary or “anyonic” statistics [8] is a very general consequence of $2+1$-dimensional dynamics

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1It is worth mentioning that a celebrated example of an NC space appears in the dynamics of charged particles confined in a plane with a large perpendicular magnetic field. At the lowest Landau level, where most of the particles will remain at low energy, the configuration space space of the particles is effectively noncommutative, with the inverse of magnetic field identified as $\theta$. The NC manifolds in open string boundaries emerge in a similar way.
since a non-trivial linking number can be attributed to the particles moving in the plane. This exotic statistics is due to the fact that the little group of the Poincare group acting on massive representations is abelian.

This phenomenon was realized in the context of Chern-Simons (CS) gauge theory [8] where CS term induces the anyonic behavior by attaching a localized magnetic flux to a point charge. However, it was pointed out [9] that it would be more realistic (for dynamical aspects) to consider the pure CS gauge theory as a descendent of the topologically massive planar electrodynamics [10] (or Maxwell-Chern-Simons (MCS) theory), in the long wavelength limit. For example, the mass term in fermionic theory in 2+1-dimensions is parity violating and sometimes it becomes convenient to introduce the CS term to represent fermions in terms of bosons [11], at least in a non-relativistic approximation.

Later a new approach to generate anyons was established by Stern [12] (see also [13]) that does not require the CS term, but introduces a generalized connection \( \sim A^{(\kappa)}_{\mu} = A_{\mu} + \kappa \epsilon_{\mu\nu\lambda} F^{\nu\lambda} \) with which the conserved \( U(1) \) current is coupled \((eJ^{\mu}A^{(\kappa)}_{\mu})\) in a gauge invariant way. This new parameter \( \kappa \) is the non-minimal Pauli magnetic coupling which is essentially phenomenological in nature. \(^2\)

In the present work, we provide a physically motivated alternative to the ad-hoc introduction of the Pauli term: Extension of the minimal interaction model in the Non-Commutative (NC) plane. We explicitly demonstrate that the electron-electron\(^3\) potential in a non-relativistic Maxwell-Chern-Simons (MCS) gauge theory of charged particles in the NC plane, is same as the potential obtained [14] in an MCS theory with non-minimal (Pauli) interaction. It is important to note that our model consists of minimal coupling only and so no phenomenological parameter is introduced. The role of the Pauli coupling \( \kappa \) is taken over by \( \theta \) - the NC parameter. It is worthwhile to observe that we study the MCS theory because, even though classically a pure Maxwell theory can be considered in 2 + 1-dimensions, radiative corrections will anyway generate a CS term in the quantum theory. This approach of studying NC effects in field theory is new and has not been explored so far.

Besides exhibiting the Pauli magnetic moment effect and anyonic interaction potential, the NC model we have studied plays an important role in the formation of ”Cooper pairs” of electron-electron bound states in the plane [15, 14]. In the context of ”Cooper pair” condensation for scalar charges, the striking result is that the bound states appear only in the NC extension and not in the ordinary MCS-Scalar theory. This is all the more intriguing since the CS term does not directly contribute in the \( O(\theta) \) corrected classical action. It appears in the one-photon exchange Möller scattering amplitude. On the other hand, for spinorial (Dirac) matter, the possibility of bound state formation is already present in the MCS-spinor theory and the NC effect introduces a correction to that. This is very similar to the effect induced by the Pauli term [14]. The vital role played by the noncommutative space in triggering the bound state formation in scalar-MCS theory is a new result.

We follow the method used in [14, 15] where one starts from the (one photon) matrix element of Möller scattering between relativistic electrons and subsequently enforces the non-relativistic limit. Fourier transform of the momentum space matrix element yields the interaction potential \([16]\). We have restricted our analysis to \( O(\theta) \) - the lowest non-trivial order in noncommutativity.

\(^2\)Note that the 2 + 1-dimensional (Dirac) \( \gamma \)-matrix algebra allows one to include a magnetic coupling without introducing spin degrees of freedom [12].

\(^3\)”Electron” is the generic name of a charged particle.
After introducing the MCS electrodynamics and its NC extensions in the context of spinor and scalar matter sectors [17, 18], we derive the Möller matrix element from which the inter-particle potential is generated in the low energy limit. We conclude with a number of exciting areas which need to be looked at in the present formalism.

Magnetic coupling effects in NC space:
The Dirac particles interacting minimally with MCS theory is

\[ L = \bar{\psi} (i\gamma^\mu D_\mu - m) \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{s}{4} \epsilon^{\mu\nu\lambda} A_\mu F_{\nu\lambda}, \tag{1} \]

where

\[ D_\mu \psi = (\partial_\mu + ieA_\mu) \psi, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \]

\( s \) denotes the coupling of the topological term. We now introduce the NC spacetime with its \( \ast \)-product (or Moyal-Weyl product):

\[ [x^\rho, x^\sigma]_\ast = i \tilde{\theta}^{\rho\sigma}, \tag{2} \]

\[ p(x) \ast q(x) = pq + i \frac{\tilde{\theta}^{\rho\sigma}}{2} \partial_\rho p \partial_\sigma q + O(\tilde{\theta}^2). \tag{3} \]

The NC generalization of (1) is,

\[ \hat{L} = \bar{\hat{\psi}} (i\gamma^\mu \hat{D}_\mu - m) \ast \hat{\psi} - \frac{1}{4} \hat{F}^{\mu\nu} \ast \hat{F}_{\mu\nu} - \frac{s}{4} \epsilon^{\mu\nu\lambda} \hat{A}_\mu \ast \hat{F}_{\nu\lambda}. \tag{4} \]

The "hatted" variables are the counterparts of the normal variables living in NC spacetime, with the following identifications,

\[ \hat{D}_\mu \ast \hat{\psi} = (\partial_\mu + ie\hat{A}_\mu \ast) \hat{\psi}; \quad \hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu - i\hat{A}_\mu \ast \hat{A}_\nu + i\hat{A}_\nu \ast \hat{A}_\mu. \]

As is well-known [3], the gauge invariance is elevated to \( \ast \)-gauge invariance in the NC plane. It was shown by Seiberg and Witten [2] that appearance of noncommutativity is dictated by the choice of regularization in the quantum theory and quite naturally the NC version of a theory should be directly related to the commutative one by a change of variables. Explicit form of this Seiberg-Witten map [2] to the lowest non-trivial order in \( \theta \) is,

\[ \hat{A}_\mu = A_\mu + \theta^{\sigma\rho} A_\rho (\partial_\sigma A_\mu - \frac{1}{2} \partial_\mu A_\sigma); \quad \hat{F}_{\mu\nu} = F_{\mu\nu} + \theta^{\rho\sigma} (F_{\mu\rho} F_{\nu\sigma} - A_\rho \partial_\sigma F_{\mu\nu}), \]

\[ \hat{\psi} = \psi - \frac{1}{2} \theta^{\mu\nu} A_\mu \partial_\nu \psi. \tag{5} \]

We have scaled \( e\tilde{\theta} \equiv \theta \). The map (5) allows us to study NC effects in the framework of commutative quantum field theory. The important feature of this map is that it preserves gauge orbits and so \( \ast \)-gauge invariance is translated in to normal gauge invariance.

Thus (4) and (5) generates the following theory in commutative spacetime,

\[ \hat{L} = \bar{\psi} (i\gamma^\mu D_\mu - m) \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} (1 + \frac{1}{2} \theta^{\alpha\beta} F_{\alpha\beta}) - \frac{s}{4} \epsilon^{\mu\nu\lambda} A_\mu F_{\nu\lambda} - \frac{1}{2\alpha} (\partial^\mu A_\mu)^2 \]
\[-\frac{1}{4}\theta^{\alpha\beta} F_{\alpha\beta} \bar{\psi} (i\gamma^\mu D_{\mu} - m) \psi - \frac{i}{2} \theta^{\alpha\beta} F_{\mu\alpha} \bar{\psi} \gamma^\mu D_{\beta} \psi, \tag{6}\]

where total derivative terms have been dropped and simplifications in the \(\theta\)-term due to the dimensionality being 2 + 1 is taken in to account. Also a gauge fixing \(\alpha\)-dependent term is put in.

In complete analogy, from the bosonic model,

\[\mathcal{L} = (D^\mu \phi)^\dagger D_{\mu} \phi - m^2 \phi^\dagger \phi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{s}{4} \epsilon^{\mu\nu\lambda} A_{\mu} F_{\nu\lambda}, \tag{7}\]

one obtains the \(O(\theta)\) NC lagrangian,

\[\hat{\mathcal{L}} = (D^\mu \phi)^\dagger D_{\mu} \phi - m^2 \phi^\dagger \phi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}(1 + \frac{1}{2} \theta^{\alpha\beta} F_{\alpha\beta}) - \frac{s}{4} \epsilon^{\mu\nu\lambda} A_{\mu} F_{\nu\lambda} - \frac{1}{2\alpha} (\partial^\mu A_{\mu})^2 - \frac{1}{4} \theta^{\alpha\beta} F_{\alpha\beta} [D^\mu \phi]^\dagger D_{\mu} \phi - m^2 \phi^\dagger \phi] + \frac{1}{2} \theta^{\alpha\beta} [F_{\alpha\mu} (D_{\beta} \phi)^\dagger D^\mu \phi + (D^\mu \phi)^\dagger D_{\beta} \phi]. \tag{8}\]

Thus in (6) and (8) we have developed the models for spinors and scalars respectively, where the NC effects appear as interaction terms. Our aim is to extract the inter-particle potential from the non-relativistic limit of the Möller scattering between two fermions at the tree level, considering single photon scattering only. A generic feature of NC extension of a field theory is that the free (quadratic) part is not modified and so one is allowed to use the propagators and free field solutions of the commutative theory. The topologically massive photon propagator in momentum space is

\[< A^\mu A^\nu > \langle k \rangle = A g^{\mu\nu} + B k^\mu k^\nu + i C \epsilon^{\mu\nu\lambda} k_\lambda \]

\[A = \frac{2}{-k^2 + s^2}; \quad B = -\frac{\alpha A}{k^2} (-\frac{s^2}{k^2} + 1 - \frac{1}{\alpha}); \quad C = -\frac{s A}{k^2}. \tag{9}\]

We first concentrate on the spinor case. In the fermion content, the \(\gamma\)-matrices satisfy the \(so(2,1)\) algebra \([\gamma^\mu, \gamma^\nu] = 2i\epsilon^{\mu\nu\lambda} \gamma_\lambda\). They represent a 2 + 1-dimensional representation of the Dirac matrices, \(i.e.\) the Pauli matrices: \(\gamma^\mu \equiv (\sigma_z, -i\sigma_x, i\sigma_y)\). The free spinor solutions are given by,

\[u(p) = \frac{1}{\sqrt{2m(E + m)}} \left( \begin{array}{c} E + m \\ -ip_x - p_y \end{array} \right); \quad \bar{u}(p) = \frac{1}{\sqrt{2m(E + m)}} (E + m - ip_x + p_y). \tag{10}\]

It is convenient to introduce the Gordon identity (in 2+1-dimensions)

\[j^\mu(p', p) \equiv \bar{u}(p') \gamma^\mu u(p) = \frac{2m}{4m^2 - k^2} [(\bar{u}(p') u(p))]((2p - k)^\mu + \frac{i}{m} \epsilon^{\mu\nu\lambda} k_\nu p_\lambda), \tag{11}\]

where \(k^\mu \equiv (p' - p)^\mu\). Since we are interested in the non-relativistic limit, interaction terms with smaller number of derivatives will dominate. This leads us to a truncated form of the interaction part:

\[\hat{\mathcal{L}}_{int} = -A^\mu \bar{\psi} \gamma_\mu \psi + \frac{m}{4} \theta^{\alpha\beta} F_{\alpha\beta} \bar{\psi} \psi. \tag{12}\]

The first term is the normal \(U(1)\) gauge interaction term whereas the \(\theta\)-term is of a Yukawa interaction type where the massive photon interacts with the fermion mass term.
Already it is apparent that non-locality, in the form of magnetic moment of the otherwise spinless fermion, will play an essential role since the \(\theta\)-contribution of the interaction depends on the factor \(\theta^{\mu\nu}k_\nu\). This is clearly reminiscent of the dipole nature of the NC Maxwell theory [19] where the spatial extent of the dipole is \(\sim\theta^{\mu\nu}k_\nu\). The connection with the phenomenological models [12, 13] with Pauli interaction \(F^{\mu\nu}\bar{\psi}\gamma_\mu\gamma_\nu\psi\) is also obvious.

The matrix element of the Möller scattering has two parts: the Coulomb term \((M_I)\) and the \(\theta\)-term \((M_{III})\), which are given below,

\[
-iM_I = (ie)^2 j^\mu(p'_1, p_1) j^\nu(p'_2, p_2) < A_\mu A_\nu > (k),
\]

\[
-iM_{III} = (ie)^2 \frac{m}{2} \theta^{\alpha\mu} k_\alpha j(p'_1, p_1) j^\nu(p'_2, p_2) < A_\mu A_\nu > (k),
\]

where \(j(p', p) \equiv \bar{u}(p')u(p)\). In the relativistic theory there will be a contribution of the exchange term which is obtained by interchanging the final state labels and keeping in mind the particle statistics. However, this is not required for our purpose since we will study the Schrödinger potential problem where taking anti-symmetric wave functions will take care of the effect of the exchange term [16]. Equivalently, one can think of the particles as distinguishable. We introduce the center of mass frame and revert to a non-relativistic notation,

\[
k^\mu \equiv (0, \vec{k}) ; \quad p'^\mu_1 \equiv (E, \vec{p}) ; \quad p'^\mu_2 \equiv (E, -\vec{p}) ; \quad \theta_\mu = \frac{1}{2} \epsilon_{\mu\nu\lambda} \theta^{\nu\lambda} \equiv (\theta_0, \vec{\theta}).
\]

A straightforward computation of the matrix elements yields,

\[
M_I = \frac{e^2}{8m^4(E + m)^2} (1 + \frac{k^2}{4m^2})^{-2} \frac{1}{k^2 + s^2} \left[2m(E + m) - (\vec{k}.\vec{p}) - i(\vec{p} \times \vec{k})\right] \frac{1}{k^2 + s^2} \left[2iE(\vec{k} \times \vec{p})\right] \frac{1}{k^2 + s^2}
\]

\[
\quad \quad + (p_1.p_2)(4 - \frac{k^2}{m^2} - \frac{4s}{m}) - (\vec{p} . \vec{k}) \left[4 - \frac{s}{m} - \frac{(\vec{p} . \vec{k})}{m} \left(1 + \frac{4s}{m}\right) \right] + \vec{k}^2,
\]

\[
M_{III} = -i \frac{e}{2} \frac{1}{k^2 + s^2} (1 + \frac{k^2}{4m^2})^{-2} \left[-(2 + \frac{s}{m}) (E(\vec{k} \times \vec{\theta}) + \theta_0(\vec{k} \times \vec{p}))\right]
\]

\[
\quad \quad -i \left(\frac{1}{m} + \frac{2s}{k^2}\right) \left[(-\vec{k} . \vec{p})(\vec{k} . \vec{\theta})\right] + (\theta_0E + (\vec{\theta} . \vec{p}))(\vec{k}^2),
\]

\[
\alpha\text{-dependent terms will not occur since conserved currents are involved. The non-relativistic limit simplifies the expressions considerably and we find,}
\]

\[
M_I = \frac{e^2}{k^2 + s^2} [(1 - \frac{s}{m} + \frac{2is}{m} \frac{k^2}{k^2} ]
\]

\[
M_{III} = -\frac{e}{2(k^2 + s^2)} \left[\theta_0(\vec{k}^2 + 2sm - 2i(\vec{k} \times \vec{p})) - 2s \frac{(\vec{\theta} . \vec{k})(\vec{p} \cdot \vec{k})}{k^2} + 2s \vec{\theta} . \vec{p} - im(2 + \frac{s}{m})(\vec{k} \times \vec{\theta})\right].
\]

Defining the Fourier transform as

\[
V(r) = \frac{1}{(2\pi)^2} \int d^2k \ e^{i\vec{k} \cdot \vec{r}} M(k),
\]
we immediately obtain the cherished form of the electron-electron potential,

$$V_I = e^2 \left[ \frac{1}{2\pi} \left( 1 - \frac{s}{m} \right) K_0(sr) - \frac{1}{\pi ms r^2} (1 - sr K_1(sr)) \right], \quad (19)$$

$$V_{II} = -\frac{e}{4\pi} \left[ \theta_0 \left\{ 2s m K_0 + \frac{2sL}{r} K_1 \right\} + \theta \left\{ 2sp K_0 + \left( \frac{ms}{p} \right) (2 + \frac{s}{m}) \frac{L}{r} + \frac{4L^2}{p r^3} - \frac{2p}{p^2} \right\} K_1 - \frac{2}{sp} \frac{L^2}{r^3} \right]. \quad (20)$$

In the above expression, we have chosen $\vec{\theta} = \theta \hat{p}$ where $\hat{p} = \frac{\vec{p}}{p}$ is the unit vector along $\vec{p}$. This simply means that we have taken the center of mass frame in such a way that $\hat{p}$ coincides with the given constant direction $\vec{\theta}$. The expression of $V_{II}$ in (20) is one of the main results of the present work.

$V_I$ is the potential in commutative spacetime reported before [15] and $V_{II}$ constitute the $O(\theta)$ correction. For reasons of unitarity [5], in the study of NC quantum field theory, one generally restricts the noncommutativity to affect only space coordinates, keeping time as a commutative parameter. Then one immediately notices that for only spatial noncommutativity, (i.e. $\vec{\theta} = 0, \theta_0 \neq 0$), $V_{II}(\theta_0)$ does not introduce any structural change (regarding $r$-dependence) in the potential and one finds the full potential for the fermions to be,

$$V(\theta_0) \mid_{\text{spinor}} = \frac{e^2}{2\pi} \left[ 1 - \frac{s}{m} - \frac{\theta_0 sm}{e} \right] K_0(sr) - \frac{e^2}{\pi ms r^2} \left[ 1 - \left( 1 - \frac{\theta_0 sm}{2e} \right) sr K_1(sr) \right]. \quad (21)$$

The computations for the bosonic case is simpler. In the low energy limit, the leading interaction terms are

$$L_{int} = (D^\mu \phi)^\dagger D_\mu \phi + \frac{m^2}{4} \theta^{\alpha\beta} F_{\alpha\beta} \phi^\dagger \phi. \quad (22)$$

In the non-relativistic limit, we get the $\theta$-contribution to be,

$$M_{II} = -\frac{2ie}{k^2 + s^2} \left\{ \theta_0 ((k \times \vec{q}) - ism) - m(k \times \vec{\theta}) - \frac{is}{k^2} ((k \cdot \vec{q})(k \cdot \vec{\theta}) - k^2 (\vec{\theta} \cdot \vec{q})) \right\}. \quad (23)$$

This yields the potential for the bosonic case for only spatial noncommutativity,

$$V(\theta_0) \mid_{\text{scalar}} = \frac{e^2}{2\pi} \left[ 1 - \frac{\theta_0 sm}{e} \right] K_0(sr) - \frac{e^2}{\pi ms r^2} \left[ 1 - \left( 1 - \frac{\theta_0 sm}{2e} \right) sr K_1(sr) \right]. \quad (24)$$

This constitutes the other main result.

Comparing the potential profiles (21) and (24), we immediately spot the crucial difference: in the expression for scalar matter in (24), the term $\sim \frac{e^2 L - 2p}{sp} K_1(sr)$ is missing. This shows that for scalars, the term that is essential in reversing the Coulomb repulsion for bound state formation, is generated only in the NC regime. This rather dramatic outcome of the NC extension is a new result.

On the other hand, in case of fermions, the CS term by itself is able to reverse the normal (logarithmic) Coulomb repulsion between electrons making it conducive for the formation of electron-electron bound states. The Pauli non-minimal coupling generates an additional contribution in the potential that is similar to the CS contribution. A similar situation prevails in NC spacetime, $\theta_0$ will be determined by stringy effects. On the other hand, one can think of $\theta_0$ as arising from a lowest Landau level scenario, in which case it will be controlled by the inverse of magnetic field.
the present case where NC effects induce an additional term in the potential that is similar to the CS contribution.

As mentioned before, exactly similar forms of inter-electron potentials have been reported before [14] in the context non-minimal Pauli coupling proposed in [12]. The parameter $\theta_0$, \(\frac{m_0}{2e}\) to be precise, is to be identified with the non-minimal coupling $\kappa$ [14], where $\kappa$ is defined in terms of the covariant derivative $D_{\mu}\psi \equiv (\partial_{\mu} + ieA_{\mu} + i\frac{\kappa}{2}\epsilon_{\mu\nu\lambda}F^{\nu\lambda})$. There is a difference between the potentials in [14] and $V(\theta_0)$ in (21) and (24), the latter two receiving a $\theta$-contribution in the angular momentum ($L$) term as well. An intriguing point is that, although there appears no explicit contribution of the CS term in the $\theta$-correction upon exploiting the Seiberg-Witten map [17], the $O(\theta_0)$ correction terms in the potential (21) and (24) are dependent on $s$ and the correction term in $K_0$ will vanish if the CS term is absent. Actually the CS term converts the photon to a massive one which plies between the charges. This establishes the fact that the desired results are obtainable in the NC extension of the MCS-charge model, instead of incorporating the Pauli term in an ad-hoc way.

For $s$ being small compared to $m$, one can take $K_0(sr) \sim -\ln(sr)$; $K_1(sr) \sim \frac{1}{sr}$, so that $V(\theta_0)$ reduces to

$$V(\theta_0) \big|_{\text{spinor}} \sim \frac{e^2}{2\pi} \left[ \frac{s}{m} + \frac{\theta_0 sm}{e} - 1 \right] \ln(sr) - \frac{e\theta_0 L}{2\pi} \frac{1}{r^2}. \tag{25}$$

Assuming $e\theta_0$ to be small we neglect the last term and subsequently can read off an approximate expression for the $S$-wave binding energy from a semi-classical analysis performed in [14]. The result is,

$$E_{n,0} \big|_{\text{spinor}} \sim \frac{e^2 s}{2\pi} \nu \ln\left[ \frac{2\pi}{e} (n + \frac{1}{2}) \sqrt{s m v} \right], \tag{26}$$

where $\nu = \frac{\theta_0 s}{2e} - \frac{1}{2} \left( \frac{1}{s} - \frac{1}{m} \right)$ and $n >> 1$ (for details see [14]). From existing estimates of the bound on $\theta$ one can get an approximate value of the binding energy. In a similar way, results for the scalar case can also be obtained with $\nu = \frac{\theta_0 m}{2e} - \frac{1}{2 s}$.

**Conclusion and future prospects:**

Let us conclude the paper with emphasizing the following point. Effects of magnetic coupling qualitatively changes the behavior of charged particles in the problem that we have considered, i.e. scalar and spinor matter coupled to Maxwell-Chern-Simons gauge fields. Especially, in case of scalars, NC effects are solely responsible for the tantalizing possibility generating Cooper pair like bound states. As was shown before [14], these effects can be generated via the introduction of a non-minimal gauge coupling. On the other hand, we have shown in the present Letter that similar effects can be simulated if one extends the interacting model (with minimal gauge coupling) to the noncommutative plane. In the light of quantum gravity and string theory results, generalization of ordinary spacetime to a noncommutative one is natural and physically motivated. It is also interesting to note that a high energy effect such as noncommutativity in spacetime can influence a low energy phenomenon in a qualitative way. Hence it appears to us that inducing the Pauli magnetic coupling effects by extending the model to noncommutative space is a better option than directly introducing a non-minimal gauge coupling at the fundamental level.

Lastly, we provide a list of some of the interesting aspects of the problem that can be studied in near future:

1. Stern [12] has shown that for a critical value of the magnetic coupling (Pauli) term, the
system reduces to that of free anyons such that the electric effects (generated by the Maxwell term) get cancelled by the Pauli term contribution. Whether such a thing occurs in our model and what is the subsequent critical value of $\theta$ is an open problem. This issue is non-trivial since the models in question, (that of [12] and ours), are not identical.

2. Effects of space-time noncommutativity ($\vec{\theta} \neq 0$) can be explicitly studied from the expression of our potential (15,16). Loss of unitarity in spacetime NC theories and subsequent theoretical bounds on partial wave amplitudes, along the lines of [20], can be studied without invoking the non-relativistic limit. As expected, the space-time noncommutativity destroys rotational invariance in the potential (18) whereas it remains intact with only spatial noncommutativity. This is because in $2 + 1$-dimensions, $\theta_0$ points along the time direction, normal to the plane [21].

3. The same formalism can be applied in $3 + 1$-dimensions where the noncommutative effects on the electrostatic potential can be studied.

4. Studies on Lorentz invariance violating interactions in a different context has been reported [22]. Generic features of these models can be compared.

5. Extension of the model to higher orders in $\theta$- the noncommutativity parameter and promoting to non-abelian gauge groups can be rewarding.

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