Near-horizon limit of the charged BTZ black hole and AdS$_2$ quantum gravity

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June 17, 2008

Abstract. We show that the 3D charged Banados-Teitelboim-Zanelli (BTZ) black hole solution interpolates between two different 2D AdS spacetimes: a near-extremal, near-horizon AdS$_2$ geometry with constant dilaton and $U(1)$ field and an asymptotic AdS$_2$ geometry with a linear dilaton. Thus, the charged BTZ black hole can be considered as interpolating between the two different formulations proposed until now for AdS$_2$ quantum gravity. In both cases the theory is the chiral half of a 2D CFT and describes, respectively, Brown-Hennaux-like boundary deformations and near-horizon excitations. The central charge $c_{as}$ of the asymptotic CFT is determined by 3D Newton constant $G$ and the AdS length $l$, $c_{as} = 3l/G$, whereas that of the near-horizon CFT also depends on the $U(1)$ charge $Q$, $c_{nh} \propto lQ/\sqrt{G}$.

1 Introduction

Quantum gravity in low-dimensional anti-de Sitter(AdS) spacetime has features that make it peculiar with respect to the higher-dimensional cases. For $d = 2, 3$ the theory is a conformal field theory (CFT) describing (Brown-Hennaux-like) boundary deformations and has a central charge determined completely by Newton constant and the AdS length $[1, 2, 3, 4]$. Conversely, in $d > 4$, quantum gravity in AdS spacetimes should admit a near-horizon description in terms of BPS solitons and D-brane excitations, whose low-energy limit is an $U(N)$ gauge theory $[5, 6, 7]$.

The difference between these two descriptions is particularly evident in their application for computing the entropy of non-perturbative gravitational configurations such as black holes, black branes and BPS states. Brown-Hennaux-like boundary excitations have been used with success to give a microscopically explanation to entropy of the BTZ black hole and of two-dimensional (2D) AdS (AdS$_2$) black holes $[8, 2]$. On the other hand, D-brane excitations account correctly for the entropy of extremal and near-extremal Reissner-Nordstrom black holes in higher dimensions $[5]$.

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Moreover, the status of the AdS$_2$/CFT$_1$ correspondence \cite{2, 3, 9, 10, 11, 12, 13} remains still enigmatic. The dual CFT$_1$ has been identified both as a conformal mechanics and as a chiral half of a 2D CFT. Progress towards a better understanding of the relationship between low- and higher-dimensional AdS/CFT correspondence has been achieved in Ref. \cite{11}. It has been shown that quantum gravity on AdS$_2$ with constant electromagnetic (EM) field and dilaton can be described by the chiral half of a twisted CFT with central charge proportional to the square of the EM field.

On the other hand, there is another formulation of AdS$_2$ quantum gravity, which uses Brown-Hennaux-like boundary states in a 2D AdS spacetime endowed with a linear dilaton \cite{2}. Also in this case the Hilbert space of the theory falls into the representation of a chiral half of a CFT, but the central charge is proportional to the inverse of 2D newton constant. The results of Ref. \cite{11} raise the question about the relationship between the two different realizations of AdS$_2$ quantum gravity.

In this paper we show that a bridge between these two formulations is three-dimensional (3D) AdS-Maxwell gravity. We find that the charged BTZ black hole admits two limiting regimes (near-horizon and asymptotic) in which the black hole is described by a 2D Maxwell-dilaton theory of gravity. In the near-horizon, near-extremal regime the black hole is described by AdS$_2$ with a constant dilaton and $U(1)$ field. In the asymptotic regime the BTZ black hole is described by $AdS_2$ with a linear dilaton background and $U(1)$ field strength $F_{rr} = Q/r$.

Both regimes are in correspondence with a CFT$_1$, which can be thought as the chiral half of a 2D CFT. The central charge of the near-horizon CFT is proportional to the electric charge $Q$ of the BTZ black hole $c_{nh} = (3k/4)\sqrt{\pi/GlQ}$ where $k$ is the level of the $U(1)$ current. The central charge of the asymptotic CFT is determined completely by 3D Newton constant $G$ and the AdS length $l$: $c_{as} = 3l/G$.

We can therefore think of the charged BTZ black hole as an interpolating solution between the near-extremal, near-horizon behavior typical of BPS-like solutions in higher dimensions (e.g. Reissner-Nordstrom black hole solutions in four and five dimensions) and the asymptotic behavior typical of Brown-Henneaux-like states.

This paper is organized as follows. In section 2 we review briefly the features of the charged BTZ black hole. In sect. 3 we investigate the two limiting regimes, namely the near-horizon limit and the asymptotic $r \to \infty$ limit. In sect. 4 we describe the dimensional reduction from three to two spacetime dimensions. In sect. 5 we investigate the CFTs that describe the two different regimes and calculate the corresponding central charges. Finally, in section 6 we present our conclusions.

## 2 The charged BTZ black hole

The charged BTZ black hole solutions are a generalization of the well-known black hole solutions in $(2 + 1)$ spacetime dimensions derived by Banados, Teitelboim and Zanelli \cite{14, 15}.

They are derived from a three-dimensional theory of gravity

$$I = \frac{1}{16\pi G} \int d^3x \sqrt{-g(3)} \left( R + \frac{2}{l^2} - 4\pi G F_{\mu\nu}F^{\mu\nu} \right),$$

where $G$ is 3D Newton constant, $\frac{1}{l^2}$ is the cosmological constant ($l$ is the AdS-length)
and $F_{\mu\nu}$ is the electromagnetic field strength. We consider the BTZ black hole with zero angular momentum and use the conventions of Ref. [16].

Electrically charged black hole solutions of the action (1) are characterized by the $U(1)$ Maxwell field [14, 17],

$$F_{tr} = \frac{Q}{r},$$

where $Q$ is the electric charge. The 3D line element is given by

$$ds^2 = -f(r)dt^2 + f^{-1}dr^2 + r^2d\theta^2,$$

with metric function:

$$f(r) = -8GM + \frac{r^2}{l^2} - 8\pi GQ^2 \ln\left(\frac{r}{l}\right),$$

where $M$ is the Arnowitt-Deser-Misner (ADM) mass, and $-\infty < t < +\infty$, $0 \leq r < +\infty$, $0 \leq \theta \leq 2\pi$. The black hole has one inner ($r_-$) and outer ($r_+$), one or no horizons depending on whether

$$\Delta = 8GM - 4\pi GQ^2 [1 - 2 \ln(2Q\sqrt{\pi G})]$$

is greater than, equal to or less than zero, respectively. Although these solutions for $r \to \infty$ are asymptotically AdS, they have a power-law curvature singularity at $r = 0$, where $R \sim (8\pi GQ^2)/r^2$. This $r \to 0$ behavior of the charged BTZ black hole has to be compared with that of the uncharged one, for which $r = 0$ represents just a singularity of the causal structure.

The Hawking temperature $T_H$ associated with the outer black hole horizon is

$$T_H = \frac{r_+}{2\pi l^2} - \frac{2GQ^2}{r_+}.$$ 

According to the Bekenstein-Hawking formula, the thermodynamic entropy of a black hole is proportional to the area $A$ of the outer event horizon, $S = \frac{A}{4G}$. For the charged BTZ black hole we have

$$S = \frac{\pi r_+}{2G} = \frac{\pi l}{G} \sqrt{2GM + 2\pi GQ^2 \ln\left(\frac{r_+}{l}\right)}.$$ 

3 The near-horizon limit

We are interested in the near-horizon, near-extremal behavior of the solution (3). It is well known that in this regime asymptotically flat charged black holes in $d \geq 4$ dimensions are described by a $AdS_2 \times S^{d-2}$ geometry, i.e. a Bertotti-Robinson spacetime. The flux of the EM field stabilize the radius of the transverse sphere, so that in the near-horizon, near-extremal limit it becomes constant and given in terms of the EM charge. Let us show that this is also the case for the charged BTZ black hole.

The extremal limit $r_+ = r_- = \gamma$ of the BTZ black hole is characterized by $\Delta = 0$ in Eq. (5), so that $\gamma$ is a double zero of the metric function [11]:

$$\gamma = 2\sqrt{\pi GQl}.$$
In order to describe the near-horizon, near-extremal limit of our three-dimensional solution we perform a translation of the radial coordinate \( r \),

\[ r = \gamma + x, \]  
(9)

and expand both the metric function (3) and \( U(1) \) field (2) in powers of \( x \). We get after some manipulations

\[ f(x) = \frac{2}{l^2} x^2 - 8G\Delta M + O(x^3), \quad F_{lx} = \frac{1}{2\sqrt{\pi G} l} + O(x), \]  
(10)

where \( \Delta M = M - M(\gamma) = M - \pi Q^2(\frac{1}{2}-\ln(2Q\sqrt{\pi G})) \) is the mass above extremality. In the near-horizon, near-extremal limit the topology of the 3D solution factorizes as \( AdS_2 \times S^1 \) and the geometry becomes that of 3D Bertotti-Robinson spacetime,

\[ ds^2(3) = -(\frac{2}{l^2} x^2 - 8G\Delta M)dt^2 + (\frac{2}{l^2} x^2 - 8G\Delta M)^{-1} dx^2 + \gamma^2 d\theta^2, \quad F_{lx} = \frac{1}{2\sqrt{\pi G} l}, \]  
(11)

The mass of the excitations above extremality can be also expressed in terms of \( \Delta r_+ = r_+ - \gamma \). Up to order three in \( \Delta r_+ \) we have

\[ \Delta M = \frac{\Delta r_+^2}{4Gl^2}. \]  
(12)

The near-horizon, extremal limit of the 3D charged AdS black hole is therefore very similar to that of its higher-dimensional, asymptotically flat, cousins such as the Reissner-Nordstrom solution in four and five dimensions. In particular, our 3D solution shares with them the thermodynamical behavior. From Eqs. (6), (7), (9) one easily finds that the extremal charged BTZ black hole is a state of zero temperature and constant entropy

\[ S(\text{ext}) = \frac{\pi \gamma}{2G} = \pi \sqrt{\frac{\pi}{G} Ql}. \]  
(13)

For small excitations near extremality we get using (12)

\[ S_{ne} = \frac{\pi \gamma}{2G} + \pi \frac{\Delta r_+}{2G} = \frac{\pi \gamma}{2G} + \pi l \sqrt{\frac{\Delta M}{G}}. \]  
(14)

### 3.1 The asymptotic \( r \to \infty \) limit

It is also interesting to discuss briefly the asymptotic \( r \to \infty \) limiting case of the 3D solution (3) and its relationship with the near-horizon solution (11). In the \( r \to \infty \) limit the metric describes 3D AdS spacetime, whereas the \( U(1) \) field goes to zero as \( 1/r \). As we shall see in detail in the next section also in this regime the 3D solution admits an effective description in terms of \( AdS_2 \) endowed with a linear varying dilaton. The dilaton parametrizes the radius of the transverse one-sphere, which in the \( r \to \infty \) limit diverges. We can therefore think of the full charged BTZ solution (3) as a 3D spacetime interpolating between two regimes admitting an effective description in terms of \( AdS_2 \).
4 Dimensional reduction of the charged BTZ black hole

The two limiting regimes of the BTZ black hole can be described by an effective 2D Maxwell-Dilaton gravity model. In order to find this 2D description, we parametrize the radius of the $S^1$ sphere in the 3D solution (3) with a scalar field (the dilaton) $\phi$:

$$ds^2 = ds^2 + l^2 \phi^2 d\theta^2.$$  \hspace{1cm} (15)

where $ds^2$ is the line element of the 2D sections of the 3D spacetime covered by the $(t, r)$ coordinates and $\phi$ is a function of $t, r$ only. We will consider only electric configurations for the 3D maxwell field, i.e. we use for $F_{\mu \nu}$ the ansatz

$$F_{t\theta} = F_{r\theta} = 0.$$  \hspace{1cm} (16)

Using Eqs. (15) and (16) into the 3D action (1) one obtains, after defining the rescaled dilaton $\eta = (l/4G)\phi$, the dimensionally reduced 2D action,

$$I = \frac{1}{2} \int d^2 x \sqrt{-g} \left( R + \frac{2}{l^2} - 4\pi G F^2 \right).$$  \hspace{1cm} (17)

The field equation stemming from the action (17) are

$$R + \frac{2}{l^2} - 4\pi G F^2 = 0$$

$$\nabla_\mu (\eta F^{\mu \nu}) = 0$$

$$-\nabla_\mu \nabla_\nu \eta + \left[ \nabla^2 \eta - \frac{\eta}{l^2} + 2\pi G \eta F^2 \right] g_{\mu \nu} = 8\pi G \eta F_{\mu \beta} F^{\beta \nu}.$$  \hspace{1cm} (18)

It is important to notice that the field equations are invariant under rescaling of the dilaton by a constant. This constant mode of the dilaton is therefore classically undetermined but it can be fixed by matching the 2D with the 3D solution.

The field equations (18) admit two classes of solutions whose metric part is always a 2D AdS spacetime: 1) AdS$_2$ with linear dilaton and with electric field which vanishes asymptotically (corresponding to the asymptotic $r \to \infty$ regime of the charged BTZ black hole); 2) AdS$_2$ with constant dilaton and electric field (corresponding to the near-horizon limit of the BTZ black hole). Let us discuss separately these solutions.

4.1 AdS$_2$ with a linear dilaton

This solution of the field Eqs. (18) is just the 3D solution (3) written in a two-dimensional form,

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2, \quad F_{\mu \nu} = \frac{Q}{r} \epsilon_{\mu \nu}, \quad \eta = \bar{\eta}_0 \frac{r}{l}$$  \hspace{1cm} (19)

where $f(r)$ has exactly the same form as given by Eq. (4), $Q$ is the electric charge and $\bar{\eta}_0$ is an integration constant related to the scale symmetry of the 2D field equations. The integration constants appearing in Eq. (19) (thus defining the physical parameters of the 2D black hole) can be easily identified in terms of the physical parameters of the BTZ
black hole. The charge $Q$ and mass $M$ of the 2D black hole are the same as those of the BTZ black hole. The constant $\bar{\eta}_0$ is determined by the ansatz (15),

$$\bar{\eta}_0 = \frac{l}{4G}. \quad (20)$$

With this identification also the temperature and entropy of the 2D black hole match exactly those for the 3D black hole given by Eqs. (6) and (7). For instance, the entropy of the 2D black hole is determined by the value of the dilaton on the horizon,

$$S = 2\pi \eta_{\text{horizon}}, \quad (21)$$

which after using Eqs. (19) and (20) reproduces exactly Eq. (7).

### 4.2 AdS$_2$ with constant dilaton and electric field

One can easily realize that the field equations (18) admit a solution describing AdS$_2$ with constant dilaton and electric field. The constant value of the dilaton, which is not fixed by the 2D field equations, is determined by the ansatz (15),

$$\eta_0 = \frac{l}{2\sqrt{\pi G}} Q. \quad (22)$$

In order to have the usual normalization of the electric field and to make contact with the model investigated in Ref. [11], it is necessary to perform a Weyl transformation of the metric and a rescaling of the $U(1)$ field strength:

$$g_{\mu\nu} = \eta_0 \bar{g}_{\mu\nu}, \quad F_{\mu\nu} = \frac{l}{2\sqrt{2\pi G \eta_0}} \bar{F}_{\mu\nu}. \quad (23)$$

After this transformation the 2D action (17), modulo total derivatives, becomes

$$I = \frac{1}{2} \int d^2 x \sqrt{-\bar{g}} \left[ \eta \left( R(\bar{g}) + \frac{(\partial \eta)^2}{\eta} + \frac{2\eta}{l^2 \eta_0} \right) - \frac{l^2}{2} \bar{F}^2 \right]. \quad (24)$$

The field equations stemming from this action allow for a solution describing AdS$_2$ with constant dilaton and electric field, which is the dimensional reduction of the near-horizon solution (11)

$$ds^2 = -(\frac{2}{l^2} x^2 - k^2) dt^2 + (\frac{2}{l^2} x^2 - k^2)^{-1} dx^2, \quad \bar{F}_{\mu\nu} = 2E \epsilon_{\mu\nu},$$

$$\eta = 2l^2 E^2, \quad E^2 = \frac{1}{4l^2 \sqrt{\pi G}} Q, \quad (25)$$

where we have used Eq. (22) and $k^2 = 8G \Delta M$.

Following Ref. [11] we can linearize the term quadratic in the $U(1)$ field strength by introducing in the action an auxiliary field $h$,

$$I = \frac{1}{2} \int d^2 x \sqrt{-\bar{g}} \left[ \eta \left( R(\bar{g}) + \frac{(\partial \eta)^2}{\eta} + \frac{2\eta}{l^2 \eta_0} \right) - \frac{h^2}{l^2} + h \epsilon_{\mu\nu} \bar{F}_{\mu\nu} \right]. \quad (26)$$

The field equations for $h$ give

$$h = \frac{l^2}{2} \epsilon_{\mu\nu} \bar{F}_{\mu\nu} = -2El^2. \quad (27)$$
5 Conformal symmetry and central charges

In view of the AdS/CFT correspondence, the existence of two limiting AdS$_2$ configurations for the charged BTZ black hole imply the duality of the gravitational configuration with two different CFTs. Both CFTs have been already investigated in the literature and in both of them the conformal transformations appear as a subgroup of the 2D diffeomorphisms. However, they differ in the way the central charge of the CFT is generated. The CFT associated with the $r \to \infty$ limit, corresponding to AdS$_2$ with a linear dilaton has been investigated in Ref. [2]. In this case the central charge of the CFT is generated by the breaking of the $SL(2, \mathbb{R})$ isometry of the AdS$_2$ background due to the non-constant dilaton [18].

The CFT associated with the near-horizon limit, corresponding to AdS$_2$ with a constant electric and dilaton field has been investigated in Ref. [11]. In this case the central charge of the CFT is generated by the boundary conditions for the EM vector potential. We will discuss the two cases separately.

5.1 The $r \to \infty$ asymptotic CFT

In this case the conformal algebra is generated by the group of asymptotic symmetries (ASG) of AdS$_2$ along the lines of Ref. [2] [16]. The calculations of Refs. [2] can be easily extended to the theory described by the action (17). The only difference is the presence of the $U(1)$ field, which however, as explained in Ref [16] for the case of 3D gravity, does not change neither the conformal algebra, which is always given by a chiral half of the Virasoro algebra, nor the value of the central charge.

The $r \to \infty$ boundary conditions for the fields, which are invariant under 2D diffeomorphisms generated by killing vectors $\chi^t = l \epsilon(t) + \mathcal{O}(1/r^2)$, $\chi^r = -l r \dot{\epsilon}(t) + \mathcal{O}(1/r)$ are

\[
\begin{align*}
g_{tt} &= -\frac{r^2}{l^2} + \mathcal{O}(\ln r), \quad g_{tr} = \mathcal{O}\left(\frac{1}{r^3}\right), \\
g_{rr} &= \frac{l^2}{r^2} + \mathcal{O}\left(\frac{\ln r}{r^4}\right), \quad \eta = \mathcal{O}(r), \quad F_{tr} = \mathcal{O}\left(\frac{1}{r}\right).
\end{align*}
\]

(28)

Notice that we allow for deformations of the dilaton and EM field that are of the same order of the background solution (19). Although the boundary conditions (28) are invariant under the action of the asymptotic symmetry group, the classical solution is not. The linear dilaton and the $Q/r$ EM field break the isometry group of AdS$_2$. The breaking of the isometry group due to the linear dilaton background produces a nonvanishing central charge in the conformal algebra [18]. Conversely, the EM field does not contribute to the boundary charges, but only enters in the renormalization of the $L_0$ Virasoro operator [16].

The generators of the conformal diffeomorphisms close in the Virasoro algebra

\[
[L_m, L_n] = (m - n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}.
\]

(29)

The central charge $c$ can be computed using a canonical realization of the ASG along the lines of Refs. [2] [10] [16]. One has

\[
c = 12\tilde{\eta}_0 = \frac{3l}{G}.
\]

(30)
where we have used Eq. (20).

The eigenvalue \( l_0 \) of the Virasoro operator \( L_0 \) is related to the black hole mass \( M \). Analogously to the 3D case, the asymptotic expansion (28) gives divergent contributions to the boundary charges. A renormalization procedure [17, 16]) allows for the definition of renormalized boundary charges and in particular of renormalized mass \( M_0(r_+) \), which has to be interpreted as the total energy (gravitational and electromagnetic) inside the horizon \( r_+ \).

\[
M_0(r_+) = M + \pi Q^2 \ln \left( \frac{r_+}{l} \right).
\] (31)

The eigenvalue of \( L_0 \) is therefore

\[
l_0 = l M_0(r_+) = l [M + \pi Q^2 \ln \left( \frac{r_+}{l} \right)]
\] (32)

5.2 The near-horizon CFT

The 2D action (26) can be recast in the form of a twisted 2D CFT in which a central term in the Virasoro algebra is generated by boundary conditions for the \( U(1) \) vector potential \( A_\mu \), along the lines of Ref. [11].

Using a conformal and Lorentz gauge respectively, we fix the diffeomorphisms and \( U(1) \) gauge freedom,

\[
ds^2 = -e^{2\rho} dx^+ dx^-, \quad \partial_\mu A_\mu = 0,
\] (33)

the action (26) becomes up to total derivatives

\[
I = \frac{1}{2} \int d^2 x \left( -4\partial_- \eta \partial_+ \rho + \frac{\eta}{l^2 \eta_0} + 2\partial_- \eta \partial_+ \eta + \partial^2 \eta \partial_+ \eta - \frac{\hbar^2}{2l^2} + 4\partial_- h \rho \partial_+ \right),
\] (34)

where we have used the fact that in the gauge (33) \( A_\mu \) can be given in terms of a scalar \( a \), \( A_\mu = \epsilon_{\mu\nu} \partial_\nu a \).

As usual for gauge-fixing the classical field equations stemming from the action (34) must be supported by constraints,

\[
T_{\pm \pm} = \frac{2}{\sqrt{-g}} \frac{\delta I}{\delta g^{\pm \pm}} = -2\partial_\pm \eta \partial_\pm \rho + \partial_\pm \partial_\pm \eta - \eta^{-1} \partial_\pm \eta \partial_\pm \eta + 2\partial_\pm h \partial_\pm \rho = 0,
\] (35)

\[
J_\pm = 2\frac{\delta I}{\delta A_\pm} = \pm 2\partial_\pm h = 0.
\] (36)

The stress-energy tensor \( T_{\pm \pm} \) and the \( U(1) \) current \( J_\pm \) are (classically) holomorphic conserved and generate, respectively, residual conformal diffeomorphisms and gauge transformations.

In the conformal gauge the vacuum AdS\(_2\) solution (25) becomes

\[
ds^2 = -2l^2 \frac{dx^+ dx^-}{(x^+ - x^-)^2}, \quad A_\pm = \frac{El^2}{2\sigma},
\] (37)

where \( \sigma = (1/2)(x^+ - x^-) \) and \( h, \eta, E \) are given by Eqs. (25), (27).

Because the dilaton is constant one naively expects that we are dealing with pure 2D quantum gravity, which is known to be described by a CFT with vanishing central charge [19]. However, it has been shown in [11] that the boundary conditions for the \( U(1) \) vector
potential at the $\sigma = 0$ conformal boundary of AdS$_2$, $A_\sigma |_{\sigma=0} = 0$, is not preserved by conformal diffeomorphisms generated by $\chi^+(x^+)$ and $\chi^-(x^-)$. It must be accompanied by a gauge transformation $\omega^+(x^+) + \omega^-(x^-)$, which in the case under consideration is given by

$$\omega^\pm = \mp \frac{E l^2}{2} \partial_x \chi^\pm. \quad (38)$$

Moreover, the requirement the boundary remains at $\sigma = 0$ determines a chiral half of the conformal diffeomorphisms in terms of the second half. The resulting conformal symmetry can be realized using Dirac brackets. Conformal transformations are generated by the improved stress-energy tensor

$$\tilde{T}_{--} = T_{--} - \frac{El^2}{2} \partial_+ J_-.$$

Expanding in Laurent modes and using the transformation law of the improved stress-energy tensor

$$\delta_\chi \tilde{T}_{--} = \chi^- \partial_- \tilde{T}_{--} + 2 \partial_- \chi^- \tilde{T}_{--} + \frac{c}{12} \partial_-^3 \chi^-,$$  \quad (40)

where we allow for the existence of an anomalous term, one finds that the operators $\tilde{L}$ span the Virasoro algebra $\{29\}$. The transformation law of the original $T_{--}$ is anomaly-free, but that of the current $J_-$ may have an anomalous term proportional to its level $k_{[11]}$,

$$\delta_\omega J_- = k \partial_- \omega^-.$$

This allows us to compute the central charge $c$ of the Virasoro algebra,

$$c = 3kE^2l^4 = \frac{3}{4} k \sqrt{\frac{\pi}{G}} lQ. \quad (42)$$

### 6 Conclusion

Using the results of the previous section we can reproduce the entropy of the 2D AdS black hole (and the entropy of the charged BTZ black hole) by calculating the density of states $\rho(l_0)$ of the CFT with a given eigenvalue $l_0$. In the semiclassical limit $c >> 1$ and for large $l_0$ we have Cardy formula,

$$S = \ln \rho(l_0) = 2\pi \sqrt{\frac{cl_0}{6}}. \quad (43)$$

Using Eqs (30) and (32) we reproduce exactly the black hole entropy (7).

In principle, one should also be able to reproduce the entropy of the near-extremal black hole (14) using a similar procedure for the near-horizon twisted CFT. However, naive application of Cardy formula in this case is not possible. The 2D solution (25) has zero mass. Although the spacetime has an horizon and we may assign to it an Hawking temperature the 2D solution cannot be interpreted as a black hole. Being characterized by a constant dilaton and EM field, there is nothing to prevent maximal extension of the spacetime beyond the horizon to recover full AdS$_2$. Thus, the horizon is not an event horizon but has to be seen as an acceleration horizon. This is not the case of AdS$_2$ endowed with a linear dilaton. The dilaton is a non-constant scalar and its inverse gives
The vanishing of the mass for the near-horizon solution implies $l_0 = 0$, which in turn implies a vanishing entropy for the untwisted near-horizon CFT. However, to calculate the density of states for the twisted CFT we have to use in the Cardy formula the eigenvalues of $\tilde{L}_0$, $\tilde{l}_0$, instead of that of $L_0$, $\tilde{l}_0$. Calculation of $\tilde{l}_0$ requires careful analysis of the CFT spectrum and detailed knowledge of the effect of the twisting on the Hilbert space of the 2D CFT.

Acknowledgements
We thank G. D’Appollonio for discussions and valuable comments.

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