Optical solitons to the fractional Schrödinger-Hirota equation

Tukur Abdulkadir Sulaiman\textsuperscript{1,2}, Hasan Bulut\textsuperscript{2,3} and Sibel Sehriban Atas\textsuperscript{2}

\textsuperscript{1}Department of Mathematics, Federal University Dutse, Jigawa, Nigeria, E-mail: sulaiman.tukur@fud.edu.ng

\textsuperscript{2}Department of Mathematics, Firat University, Elazig, Turkey, E-mail: hbulut@firat.edu.tr

\textsuperscript{3}Department of Mathematics Education, Final International University, Kyrenia, Cyprus, E-mail: sibel.s.atan@gmail.com

Abstract

This study reaches the dark, bright, mixed dark-bright, and singular optical solitons to the fractional Schrödinger-Hirota equation with a truncated $M$-fractional derivative via the extended sinh-Gordon equation expansion method. Dark soliton describes the solitary waves with lower intensity than the background, bright soliton describes the solitary waves whose peak intensity is larger than the background, and the singular soliton solutions is a solitary wave with discontinuous derivatives; examples of such solitary waves include compactions, which have finite (compact) support, and peakons, whose peaks have a discontinuous first derivative. The constraint conditions for the existence of valid solutions are given. We use some suitable values of the parameters in plotting 3-dimensional surfaces to some of the reported solutions.

Keywords: M-fractional derivative; sinh-Gordon equation; Schrödinger-Hirota equation; optical soliton

AMS 2010 codes: 49K20.

1 Introduction

Nonlinear Schrödinger equations (NLSEs) can be used to describe various complex nonlinear physical phenomena arising from the different fields of nonlinear sciences, such as; optical fibers, hydrodynamics, complex acoustics, quantum hall effect, heat pulses in solids and many other nonlinear unstable aspects [1,2]. The theory of optical solitons is one of the interesting topics for the investigation of soliton propagation through nonlinear optical fibers [3]. Optical solitons are restrained electromagnetic waves that stretch in nonlinear dispersive media and allow the intensity to remain unchanged due to the balance between dispersion and nonlinearity effects [4]. Various analytical approaches for securing optical solitons and other solutions to different kind of NLSEs have been reported to the literature such as the the sine-Gordon expansion method [5–7], the first integral method [8,9], the improved Bernoulli sub-equation function method [10,11], the trial solution method [12,13],
the new auxiliary equation method [14], the extended simple equation method [15], the solitary wave ansatz method [16], the functional variable method [17], the sub-equation method [18–20] and several others [21–33]. However, in this study, the extended sinh-Gordon equation expansion method (ShGEEM) [34–38] is used in constructing family of optical soliton and other solutions to the fractional Schrödinger-Hirota [39, 40] equation with a truncated M-fractional derivative.

The fractional Schrödinger-Hirota equation with a truncated M-fractional derivative is given as

\[
\begin{align*}
&i\mathcal{D}^\alpha_{M,t} \psi + \lambda \mathcal{D}^{2\alpha}_{M,x} \psi + \delta \mathcal{D}^\alpha_{M,t} \mathcal{D}^\alpha_{M,t} \psi + \rho \psi |\psi|^2 \psi + i(a \mathcal{D}^{3\alpha}_{M,x} \psi + b |\psi|^2 \mathcal{D}^\alpha_{M,x} \psi) \\
&= ic \mathcal{D}^\alpha_{M,x} \psi + id \mathcal{D}^\alpha_{M,x} (|\psi|^2 \psi) + ie \mathcal{D}^\alpha_{M,x} (|\psi|^2) \psi, \quad 0 < \alpha < 1, \quad \beta > 0, \quad i = \sqrt{-1},
\end{align*}
\]

where \(\psi\) is a complex-valued function of \(x\) and \(t\). The coefficients \(\lambda, \delta\) and \(a\) are the group velocity and spatio-temporal and third-order dispersions terms, respectively. The parameters \(b\) and \(e\) are the nonlinear dispersion terms. The coefficients \(c\) and \(d\) are the inter-modal dispersion and the self steepening terms, respectively [39, 40].

For the past two decades, the field of fractional calculus has attracted the attention of many researchers. Nonlinear fractional partial differential equations are used to describe various nonlinear phenomena in nonlinear science. There are several definitions of fractional derivatives available in the literature, such as the Riemann-Liouville, Caputo and Grunwald-Letnikov definitions, Atangana-Baleanu derivative in Caputo sense, Atangana-Baleanu fractional derivative in Riemann-Liouville sense [41, 42], the conformable fractional derivative [43]. Atangana et. al [44] presented some new properties to the conformable fractional derivative. Recently, Sousa and Oliveira developed the new truncated M-fractional derivative [45]. This new fractional derivative generalizes the conformable derivative proposed by Khalil et. al [43].

2 The truncated M-fractional derivative

In this section, some basic definition and theorem about the new truncated M-fractional derivative are given [45].

**Definition 1.** Let \(h: [0, \infty) \rightarrow \mathbb{R}\), then the new truncated M-fractional derivative of \(h\) of order \(\alpha\) is defined as

\[
\mathcal{D}^\alpha_M \{ (h(t)) \} = \lim_{\varepsilon \to 0} \frac{h(t \mathbb{E}_\beta (\varepsilon t^{1-\alpha})) - h(t)}{\varepsilon}, \quad \forall t > 0, \quad 0 < \alpha < 1, \quad \beta > 0,
\]

where \(\mathbb{E}_\beta(\cdot)\) is a truncated Mittag-Leffler function of one parameter [45].

**Theorem 1.** Let \(0 < \alpha \leq 1, \quad \beta > 0, \quad q, \quad r \in \mathbb{R}\), and \(g, \quad h\ \alpha\)-differentiable at a point \(t > 0\). Then:

1. \(\mathcal{D}^\alpha_M \{ (g^q + rh) (t) \} = g\mathcal{D}^\alpha_M \{ g(t) \} + r\mathcal{D}^\alpha_M \{ h(t) \}\).
2. \(\mathcal{D}^\alpha_M \{ (g \cdot h) (t) \} = g(t)\mathcal{D}^\alpha_M \{ h(t) \} + h(t)\mathcal{D}^\alpha_M \{ g(t) \}\).
3. \(\mathcal{D}^\alpha_M \{ \frac{g(t)}{h(t)} \} = \frac{h(t)\mathcal{D}^\alpha_M \{ g(t) \} - g(t)\mathcal{D}^\alpha_M \{ h(t) \}}{|h(t)|^2}\).
4. \(\mathcal{D}^\alpha_M \{ c \} = 0\), where \(g(t) = c\) is a constant.
5. If \(g\) is differentiable, then \(\mathcal{D}^\alpha_M \{ g(t) \} = \frac{t^{1-\alpha}}{\Gamma(\beta+1)} \int_0^t \frac{dg(r)}{dr} dr\).
3 The extended ShGEEM

In this section, the steps of the extended sinh-Gordon equation expansion method are presented.

**Step-1:** Consider the following nonlinear fractional partial differential equation with the new truncated M-fractional derivative:

\[ P(D_{M,x}^{\alpha,\beta} \psi, \psi^2 D_{M,x}^{2\alpha,\beta} \psi, D_{M,t}^{\alpha,\beta} \psi, D_{M,x}^{\alpha,\beta} \psi, \ldots) = 0. \]  
(3.1)

Putting the fractional travelling wave transformation

\[ \psi(x,t) = \Phi(\zeta), \quad \zeta = \frac{\Gamma(\beta + 1)}{\alpha} v(x^\alpha - vt^\alpha) \]  
(3.2)

into Eq. (3.1), produces the following nonlinear ordinary differential equation (NODE):

\[ D(\Phi, \Phi', \Phi'', \Phi^2 \Phi', \ldots) = 0, \]  
(3.3)

**Step-2:** We suppose the trial solution to Eq. (3.3) to be of the form [34]

\[ \Phi(\Theta) = \sum_{k=1}^{m} [B_k \sinh(\Theta) + A_k \cosh(\Theta)]^k + A_0, \]  
(3.4)

where \( A_0, A_k, B_k \ (k = 1, 2, \ldots, m) \) are constants to be determine later and \( \Theta \) is a function of \( \zeta \) which satisfies the following ordinary differential equations:

\[ \Theta' = \sinh(\Theta). \]  
(3.5)

The value of \( m \) is determined by using the homogeneous balance principle.

Eq. (3.5) is obtained from the fractional sinh-Gordon equation given by [34]

\[ D_{M,t}^{\alpha,\beta} \psi = \gamma \sinh(\psi). \]  
(3.6)

Eq. (3.5) possesses the following solutions [34]:

\[ \sinh(\Theta) = \pm \text{csch}(\zeta) \quad \text{or} \quad \sinh(\Theta) = \pm i \text{sech}(\zeta), \]  
(3.7)

\[ \cosh(\Theta) = \pm \text{coth}(\zeta) \quad \text{or} \quad \cosh(\Theta) = \pm \text{tanh}(\zeta), \]  
(3.8)

respectively, where \( i = \sqrt{-1} \).

**Step-3:** Putting Eq. (3.4), its possible derivatives with the fixed value of \( m \) along with Eq. (3.5) into Eq. (3.3), yields an equation in powers of hyperbolic functions; \( \Theta^l \sinh(\Theta) \cosh(\Theta) \) \( (l = 0, 1 \text{ and } i, j = 0, 1, 2, \ldots) \). We collect a set of over-determined nonlinear algebraic equations in \( A_0, A_k, B_k, v, \nu \) by setting the coefficients of \( \Theta^l \sinh(\Theta) \cosh(\Theta) \) to zero.

**Step-4:** The collected set of over-determined nonlinear algebraic equations is then solved with aid of computational software to determine the values of the parameters \( A_0, A_k, B_k, v, \nu \).

**Step-5:** Based on Eqs. (3.7) and (3.8), Eq. (3.1) possesses the following forms of solutions:
\[ \Phi(\zeta) = \sum_{k=1}^{m} \left[ \pm iB_k \sech(\zeta) \pm A_k \tanh(\zeta) \right]^k + A_0, \quad (3.9) \]

\[ \Phi(\zeta) = \sum_{k=1}^{m} \left[ \pm B_k \csch(\zeta) \pm A_k \coth(\zeta) \right]^k + A_0. \quad (3.10) \]

4 Application

In this section, we present the application of the ShGEEM to the Schrödinger-Hirota equation.

Consider equation (Eq. (1.1)) given in section 1.

Substituting the fractional complex wave transformation

\[ \psi(x, t) = \Phi(\zeta)e^{i\theta}, \quad \zeta = \frac{\Gamma(\beta + 1)}{\alpha} \nu(x^\alpha - vt^\alpha), \quad \theta = \frac{\Gamma(\beta + 1)}{\alpha} (-\kappa x^\alpha + \omega t^\alpha + \Omega) \quad (4.1) \]

into Eq. (1.1), we get the following NODE and the constraint conditions:

\[ v^2(\lambda - 3\delta \nu + 3a\kappa)\Phi'' - (\omega + \lambda \kappa^2 - 3\delta \kappa \omega + a\kappa^3 + c\kappa)\Phi + (\rho + \kappa b - \kappa d)\Phi^3 = 0, \quad (4.2) \]

\[ e = \frac{(b - 3d)(\lambda - \nu \delta) - 3a(2kd + \rho)}{2(3ak + \lambda - \nu \delta)} \quad (4.3) \]

and

\[ c = \frac{a(\omega + \kappa(-8\kappa \lambda + \nu(-3 + 6\kappa \delta) + 2\delta \omega)) - 8a^2\kappa^3 - (\lambda - \nu \delta)(\nu + 2\kappa \lambda - \nu \kappa \lambda - \delta \omega)}{2a\kappa + \lambda - \nu \delta}. \quad (4.4) \]

Balancing the terms \( \Phi^3 \) and \( \Phi'' \), yields \( m = 1 \).

With \( m = 1 \), Eq. (3.4) takes the form

\[ \Phi(\Theta) = B_1 \sinh(\Theta) + A_1 \cosh(\Theta) + A_0. \quad (4.5) \]

Substituting Eq. (4.5) and it is second derivative along with Eq. (3.5), gives and equation in powers of hyperbolic functions. We collect the set of over-determined nonlinear algebraic equations as explained in the description of the method. We further simplify the set of algebraic equations to obtained the values of the parameters. To get the solutions of Eq. (1.1), we substitute the values of the parameters into Eqs. (3.9) and (3.10).

Case-1: When

\[ A_0 = 0, \quad A_1 = -\sqrt{\frac{\omega + \kappa(c + \kappa(a\kappa + \lambda) - \delta \omega)}{b\kappa - \kappa d + \rho}}, \quad B_1 = A_1, \]

\[ \nu = -\sqrt{-\frac{(\omega + \kappa(c + \kappa(a\kappa + \lambda) - \delta \omega))}{2ak - \nu \delta + \lambda}}, \]

we have the mixed dark-bright optical soliton
respectively, where \((b\kappa - kd + \rho)(\omega + \kappa(c + \kappa(a\kappa + \lambda) - \delta \omega)) > 0\) and \((3ak - \nu\delta + \lambda)(\omega + \kappa(c + \kappa(a\kappa + \lambda) - \delta \omega)) < 0\) for valid soliton.

**Case-2:** When

\[
A_0 = 0, A_1 = -\sqrt{\frac{\omega + \kappa(c + \kappa(a\kappa + \lambda) - \delta \omega)}{b\kappa - kd + \rho}}, B_1 = 0,
\]

\[
v = -\sqrt{\frac{\omega(c - 1) - \kappa(c + \kappa(a\kappa + \lambda))}{2(3ak - \nu\delta + \lambda)}},
\]

we have the dark and singular solitons

\[
\psi_2(x, t) = \pm \sqrt{\frac{\omega + \kappa(c + \kappa(a\kappa + \lambda) - \delta \omega)}{b\kappa - kd + \rho}} \times \tanh \left[ \frac{\Gamma(\beta + 1)}{\alpha} \nu(\chi^\alpha - \nu t^\alpha) \right] e^{i(\frac{\Gamma(\beta + 1)}{\alpha})(-\kappa\nu^\alpha + \omega t^\alpha + \Omega)},
\]

(4.7)

and

\[
\psi_3(x, t) = \pm \sqrt{\frac{\omega + \kappa(c + \kappa(a\kappa + \lambda) - \delta \omega)}{b\kappa - kd + \rho}} \times \coth \left[ \frac{\Gamma(\beta + 1)}{\alpha} \nu(\chi^\alpha - \nu t^\alpha) \right] e^{i(\frac{\Gamma(\beta + 1)}{\alpha})(-\kappa\nu^\alpha + \omega t^\alpha + \Omega)},
\]

(4.8)

respectively, where \((b\kappa - kd + \rho)(\omega + \kappa(c + \kappa(a\kappa + \lambda) - \delta \omega)) > 0\) and \((3ak - \nu\delta + \lambda)(\omega(\kappa\delta - 1) - \kappa(c + \kappa(a\kappa + \lambda))) > 0\) for valid solitons.

**Case-3:** When

\[
A_0 = 0, A_1 = 0, B_1 = -\sqrt{-2(\omega + \kappa(c + \kappa(a\kappa + \lambda) - \delta \omega))},
\]

\[
v = -\sqrt{\frac{\omega(c + \kappa(a\kappa + \lambda) - \delta \omega)}{3ak - \nu\delta + \lambda}},
\]

we have the bright and singular solitons

\[
\psi_4(x, t) = \pm \sqrt{\frac{2(\omega + \kappa(c + \kappa(a\kappa + \lambda) - \delta \omega))}{b\kappa - kd + \rho}} \times \text{sech} \left[ \frac{\Gamma(\beta + 1)}{\alpha} \nu(\chi^\alpha - \nu t^\alpha) \right] e^{i(\frac{\Gamma(\beta + 1)}{\alpha})(-\kappa\nu^\alpha + \omega t^\alpha + \Omega)},
\]

(4.9)
and

\[
\psi_5(x,t) = \pm \sqrt{\frac{-2(\omega + \kappa(c + \kappa(a \kappa + \lambda) - \delta \omega))}{b \kappa - \kappa \delta + \rho}} \times \csc h \left[ \frac{\Gamma(\beta + 1)}{\alpha} \nu^\alpha (x - vt)^\alpha \right] e^{i \frac{\Gamma(\beta + 1)}{\alpha} (-\kappa \nu^{\alpha + \omega t^{\alpha + \Omega})},
\]

(4.10)

respectively, where \((3ak - v\delta + \lambda)(\omega + \kappa(c + \kappa(a \kappa + \lambda) - \delta \omega)) > 0\) for valid solitons.

5 Physical representation of the reported results

In order to have clear and good understanding of the physical properties of the constructed dark, bright and singular soliton solutions, under the choice of the suitable values of parameters and good choice of the fractional value of \(\alpha\), the 3-dimensional graphs are plotted. The perspective view of the dark, bright and singular solitons can be seen from the (a) part of figs. 1, 2, and (a), (b) parts of figs. 1 and 2, respectively.

Fig. 1 The 3D surfaces of (a) Eq. (4.7) and (b) Eq. (4.8) with fractional value \(\alpha = 0.9\).

Fig. 2 The 3D surfaces of (a) Eq. (4.9) and (b) Eq. (4.10) with fractional value \(\alpha = 0.9\).
6 Conclusions

In this study, the dark, bright, mixed dark-bright and singular optical solitons to the fractional Schrödinger-Hirota equation with a truncated M-fractional derivative are successfully revealed by using the extended sinh-Gordon equation expansion method. The truncated M-fractional derivative is a generalized form of the conformable fractional derivative. The definition of the new fractional derivative is smoothly used in transforming the fractional Schrödinger-Hirota equation to nonlinear ordinary differential equation. The reported results may be useful in explaining the physical meaning of the studied nonlinear model. The extended sinh-Gordon equation expansion method is powerful technique in obtaining wave solutions to various complex fractional nonlinear model.

References

[1] H. Bulut, T.A. Sulaiman and B. Demirdag, Dynamics of soliton solutions in the chiral nonlinear Schrödinger equations, Nonlinear Dyn., 91(3) 1985-1991 (2018)
[2] T.A. Sulaiman, T. Akturk, H. Bulut and H.M. Baskonus, Investigation of various soliton solutions to the Heisenberg ferromagnetic spin chain equation, Journal of Electromagnetic Waves and Applications, 32(9) 1093-1105 (2017)
[3] M. Younis, N. Cheemaa, S.A. Mahmood and Rizvi S.T.R., On optical solitons: the chiral nonlinear Schrödinger equation with perturbation and Bohm potential, Opt Quant Electron, 48 (2016) 542
[4] G.P. Agrawal, Nonlinear fiber optics, 5'h edition. Academic Press: New York (2013)
[5] C. Cattani, T.A. Sulaiman, H.M. Baskonus and H. Bulut, On the soliton solutions to the Nizhnik-Novikov-Veselov and the Drinfel’d-Sokolov systems, Optical and Quantum Electronics, 53 (2018) 138
[6] H.M. Baskonus, H. Bulut and T.A. Sulaiman, Investigation of various travelling wave solutions to the extended (2+1)-dimensional quantum ZK equation, The European Physical Journal Plus, 132 482 (2017)
[7] H. Bulut, T.A. Sulaiman and H.M. Baskonus, On the solitary wave solutions to the longitudinal wave equation in MEE circular rod, Optical and Quantum Electronics, 50 87 (2018)
[8] M. Eslami, F.S. Khodadad, F. Nazari and H. Rezazadeh, The first integral method applied to the Bogoyavlenskii equations by means of conformable fractional derivative, Optical and Quantum Electronics, 49(12) 391 (2017)
[9] M. Eslami, M. Mirzazadeh, B.F. Vajargah and A. Biswas, Optical solitons for the resonant nonlinear Schrödinger’s equation with time-dependent coefficients by the first integral method, Optik, 125(13) 3107-3116 (2014)
[10] T.A. Sulaiman and H. Bulut, Boussinesq equations: M-fractional solitary wave solutions and convergence analysis, Journal of Ocean Engineering and Sciences, 4(1) 1-6 (2019)
[11] H. Bulut, H.A. Isik and T.A. Sulaiman, On Some Complex Aspects of the (2+1)-dimensional Broer-Kaup-Kupershmidt System, ITM Web of Conferences, 13 01019 (2017)
[12] M. Eslami, Trial solution technique to chiral nonlinear Schrodinger’s equation in (1+2)-dimensions, Nonlinear Dynamics, 85(2) 813-816 (2016)
[13] A. Biswas, M. Mirzazadeh, M. Eslami, Q. Zhou, A. Bhrawy and M. Belic, Optical solitons in nano-fibers with spatiotemporal dispersion by trial solution method, Optik, 127(18) 7250-7257 (2016)
[14] M.M.A. Khater, A.R. Seadawy and D. Lu, Optical soliton and rogue wave solutions of the ultra-short femto-second pulses in an optical fiber via two different methods and its applications, Optik, 158 434-450 (2018)
[15] D. Lu, A.R. Seadawy and M.M.A. Khater, Dispersive optical soliton solutions of the generalized Radhakrishnan-Kundu-Lakshmanan dynamical equation with power law nonlinearity and its applications, Optik, 164 54-64 (2018)
[16] A.H. Bhrawy, A.A. Alshaery, E.M. Hilal, Z. Jovanoski and A. Biswas, Bright and dark solitons in a cascaded system, Optik, 125 6162-6165 (2014)
[17] H. Rezazadeh, New solitons solutions of the complex Ginzburg-Landau equation with Kerr law nonlinearity, Optik, 167 218-227 (2018)
[18] H. Aminikhah, A.H.R. Sheikhanib and H. Rezazadeh, Sub-equation method for the fractional regularized long-wave equations with conformable fractional derivatives, Scientia Iranica B, 23(3) 1048-1054 (2016)
[19] S.T. Mohyud-Din, T. Nawaz, E. Azharb and M.A. Akbar, Fractional sub-equation method to space-timefractional Calogero-Degasperis and potential Kadomtsev-Petviashvili equations, Journal of Taibah University for Science, 11 258-263 (2017)
[20] G.W. Wang and T.Z. Xu, The Improved Fractional Sub-Equation Method and Its Applications to Nonlinear Fractional Partial Differential Equations, Romanian Reports in Physics, 66(3) 595-602 (2014)
[21] Q. Zhou and A. Biswas, Optical solitons in parity-time-symmetric mixed linear and nonlinear lattice with non-Kerr law nonlinearity, Superlattices Microstruct., 109 588-598 (2017)
[22] I. Bendahmane, H. Triki, A. Biswas, A.S. Alshomrani, Q. Zhou, S.P. Moshokoa and M. Belic, Bright, dark and W-
shaped solitons with extended nonlinear Schrödinger’s equation for odd and even higher-order terms, Superlattices Microstruct., 114 53-61 (2018)

[23] A. Sonmezoglu, M. Yao, M. Ekici, M. Mirzazadeh and Q. Zhou, Explicit solitons in the parabolic law nonlinear negative-index materials, Nonlinear Dynamics, 88(1) 595-607 (2017)

[24] R. Yilmazer and E. Bas, Explicit Solutions of Fractional Schrödinger Equation via Fractional Calculus Operators, Int. J. Open Problems Compt. Math., 5(2) 133-141 (2012)

[25] O.A. Ilhan, B. Bulut, T.A. Sulaiman and H.M. Baskonus, Dynamic of solitary wave solutions in some nonlinear pseudoparabolic models and Dodd-Bullough-Mikhailov equation, Indian Journal of Physics, 92(8) 999-1007 (2018)

[26] H. Aminikhah, A.H. Sheikhani and H. Rezazadeh, Travelling wave solutions of nonlinear systems of PDEs by using the functional variable method, Boletim da Sociedade Paranaense de Matematica, 34(2) 213-229 (2015)

[27] A.R. Seadawy, Modulation instability analysis for the generalized derivative higher order nonlinear Schrödinger equation and its the bright and dark soliton solutions, Journal of Electromagnetic Waves and Applications, 31 1353-1362 (2017)

[28] Q. Zhou, M. Ekici, M. Mirzazadeh and A. Sonmezoglu, The investigation of soliton solutions of the coupled sine-Gordon equation in nonlinear optics, J. Mod. Opt., 64(16) 1677-1682 (2017)

[29] H. Bulut, T.A. Sulaiman and H.M. Baskonus, New solitary and optical wave structures to the Korteweg-de Vries equation with dual-power law nonlinearity, Optical and Quantum Electronics, 48 564 (2016)

[30] H.M. Baskonus, T.A. Sulaiman and H. Bulut, On the novel wave behaviors to the coupled nonlinear Maccari’s system with complex structure, Optik, 131 1036-1043 (2017)

[31] H.M. Baskonus, T.A. Sulaiman and H. Bulut, New solitary wave solutions to the (2+1)-dimensional Calogero-Bogoyavlenskii-Schiff and the Kadomtsev-Petviashvili hierarchy equations, Indian Journal of Physics, 91(10) 1237-1243 (2017)

[32] M. Usman, M. Hamid, T. Zubair, R.U. Haq and W. Wang, Operational-matrix-based algorithm for differential equations of fractional order with Dirichlet boundary conditions, The European Physical Journal Plus, 134 279 (2019)

[33] M. Usman, M. Hamid, R.U. Haq and W. Wang, An efficient algorithm based on Gegenbauer wavelets for the solutions of variable-order fractional differential equations, The European Physical Journal Plus, 133 327 (2018)

[34] X. Xian-Lin and T. Jia-Shi, Travelling Wave Solutions for Konopelchenko-Dubrovsky Equation Using an Extended sinh-Gordon Equation Expansion Method, Commun. Theor. Phys., 50 1047 (2008)

[35] T.A. Sulaiman, H.M. Baskonus and H. Bulut, Optical solitons and other solutions to the conformable space-time fractional complex Ginzburg-Landau equation under Kerr law nonlinearity, Pramana-J. Phys., 91 58 (2018)

[36] T.A. Sulaiman, G. Yel and H. Bulut, M-fractional solitons and periodic wave solutions to the Hirota-Maccari system, Modern Physics Letters B, 33(5) 1950052 (2019)

[37] H.M. Baskonus, T.A. Sulaiman and H. Bulut, Dark, bright and other optical solitons to the decoupled nonlinear Schrödinger equation arising in dual-core optical fibers, Optical and Quantum Electronics, 50 165 (2018)

[38] A. Esen, T.A. Sulaiman, H. Bulut and H.M. Baskonus, Optical solitons to the space-time fractional (1+1)-dimensional coupled nonlinear Schrödinger equation, Optik, 167 150-156 (2018)

[39] I. Bernstein, E. Zerrad, Q. Zhou, A. Biswas and N. Melikechi, Dispersive optical solitons with Schrödinger-Hirota equation by traveling wave hypothesis, Optoelectron, Adv. Mater. Rapid Commun., 9(5-6) 792-797 (2015)

[40] I. Bernstein, N. Melikechi, E. Zerrad, A. Biswas and M. Belic, Dispersive optical solitons with Schrödinger-Hirota equation using undetermined coefficients, J. Comput. Theor. Nanosci., 13(8) 5288-5293 (2016)

[41] I. Podlubny, Fractional Differential Equations, Academic Press, San Diego (1999)

[42] A. Abdon and B. Dumitru, New fractional derivatives with nonlocal and non-singular kernel: Theory and application to heat transfer model, Thermal Science, 20(2) 763-769 (2016)

[43] R. Khalil, M. Al Horani, A. Yousef and M. Sababheh, A new definition of fractional derivative, Journal of

[44] A. Atangana, D. Baleanu and A. Alsaedi, New properties of conformable derivative, Open Mathematics, 13(1) 1-10 (2015)

[45] J.V.D.C. Sousa and E.C. de Oliveira, A New Truncated M-Fractional Derivative Type Unifying Some Fractional Derivative Types with Classical Properties, International Journal of Analysis and Applications, 16(1) 83-96 (2018)