Gravitomagnetic measurement of the angular momentum of celestial bodies

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January 6, 2022

Abstract

The asymmetry in the time delay for light rays propagating on opposite sides of a spinning body is analyzed. A frequency shift in the perceived signals is found. A practical procedure is proposed for evidencing the asymmetry, allowing for a measurement of the specific angular momentum of the rotating mass. Orders of magnitude are considered and discussed.

A well known effect of gravity on the propagation of electromagnetic signals is the time delay: for example an electromagnetic beam emitted from a source on Earth toward another planet of the solar system, and hence reflected back, undergoes a time delay during its trip (with respect to the propagation in flat space-time), due to the influence of the gravitational field of the Sun. This effect was indeed called the "fourth" test of General Relativity\[1\],\[2\], coming fourth after the three classical ones predicted by Einstein\[3\].

Though small, the time delay in the propagation of electromagnetic waves was detected by Shapiro et al.\[4\], timing radar echoes from Mercury and Venus, by means of the radio-telescopes of Arecibo and Haystack. Anderson et al.\[5\] measured the time delay of the signals transmitted by Mariner 6 and 7 orbiting around the Sun. Finally Shapiro and Reasenberg obtained more accurate results using a Viking mission that deposited a transponder on the surface of Mars: the theoretical prediction was verified within $\pm 0.1\%$\[6\]\[7\].

These measurements accounted just for the presence of a massive source, described by the Schwarzschild solution of the Einstein field equations. However, there is another correction to the time delay, due to the spin of the source, which is produced by the so called "gravitomagnetic" interaction\[8\].

The time delay in the gravitational field of the Sun can be studied in the weak field approximation\[9\]. By using Cartesian coordinates and assuming that
the $z$ axis coincides with the direction of the angular momentum of the source, the propagation of electromagnetic signals, in the equatorial plane, is described by the null world-line

$$0 = g_{tt} dt^2 + g_{xx} dx^2 + g_{yy} dy^2 + 2g_{xt} dx dt + 2g_{yt} dy dt$$ (1)

In the weak field approximation, the bending of the trajectories due to the gravitational field is neglected. This assumption, appropriately choosing the $x$ and $y$ axes, leads to a ray trajectory that is a straight line $x = b = \text{const}; b$ is of course the closest approach distance with respect to the spinning body. On using the appropriate form of the metric elements\cite{10}, the time of flight of the electromagnetic beams can be written as

$$t_f(y_1, y_2) = t_0 + t_M + t_J$$ (2)

where

$$t_0 = \frac{y_2 - y_1}{c}$$ (3)
$$t_M = \frac{2GM}{c^3} \ln \frac{y_2 + \sqrt{b^2 + y_2^2}}{y_1 + \sqrt{b^2 + y_1^2}}$$ (4)
$$t_J = \mp \frac{2GJ}{c^4 b} \left[ \frac{y_2}{\sqrt{b^2 + y_2^2}} - \frac{y_1}{\sqrt{b^2 + y_1^2}} \right]$$ (5)

In Eqs. (4) and (5) $M$ is the mass of the source and $J = Mc a$ is its angular momentum, assumed to be orthogonal to the plane of the motion.

The quantity $y_1$ is the $y$ coordinate of the source of the signals, $y_2$ is the $y$ coordinate of the receiver. The time $t_0$ is clearly the Newtonian time of flight; $t_M$ is the gravitational time delay measured by Shapiro et al., and $t_J$ is the correction to the time delay produced by the gravitomagnetic interaction with the angular momentum of the central body. The double sign in $t_J$ means that gravitomagnetism shortens the time of flight on the left and lengthens it on the right.

This asymmetry, within the solar system, is in any case small: however its systematicness lends an opportunity to reveal the effect, appropriately combining the ticks of a ‘clock’ passing behind the spinning mass. Elsewhere\cite{10} we have outlined a way to reveal this effect, based on the fact that the relative motion of source, receiver and central mass, produces a varying time delay, which shows up as a small frequency shift. The gravitomagnetic contribution to this shift is manifested as an asymmetry between right and left with respect to the central mass. Let $\nu_0$ be the proper frequency of the electromagnetic signal; in these conditions, mirroring the signal after the occultation (time reflection), then superposing corresponding records with the signal before the occultation of the clock, a beating function will result, where the magnitude of the frequency of the beats is proportional to the angular momentum of the spinning body

$$\nu_2 = 4\nu_0 \frac{\mu a}{b^2} \frac{v_0}{c}$$ (6)
and the frequency of the basic signal is shifted with respect to the flat space-time situation by an amount proportional to the mass:

\[ \nu_1 = \nu_0 \left( 1 + 4 \frac{\mu a}{b} \frac{v_0}{c} - \frac{\mu b v_0}{R^2 c} \right) \]  

(7)

In Eqs. (6) and (7) \( v_0 \) is the apparent transverse velocity of the source in the sky, \( R \) is the distance of the observer from the central body and \( \mu = GMc^2 \). To fix a few numbers, let us consider the situation in the solar system, with the Sun as the central spinning body, and an Earth bound observer. The source is a far away astronomical body (e.g. a pulsar). In this case the orders of magnitude are

\[ \mu \sim 10^3 \text{ m} \quad a \sim 10^3 \text{ m} \quad b \sim 10^9 \text{ m} \quad R \sim 10^{11} \text{ m} \quad v_0 \sim 10^4 \text{ m/s} \]

As a consequence we have

\[ \frac{4\mu a v_0}{b c} \sim 10^{-10} \]
\[ \frac{\mu b v_0}{R^2 c} \sim 10^{-14} \]
\[ 4\frac{\mu a v_0}{b^2 c} \sim 10^{-16} \]  

(8)

The frequency effect, connected with a varying time delay in the propagation of electromagnetic signals in the vicinity of a spinning massive body, is very small, however not entirely negligible, at least when a suitable procedure to detect it is used, such as the one we have just outlined, based on the possibility of producing a beat between signals after and before the occultation.

Of course many practical problems have to be considered and discussed to transform some principle formulae into an actual measurement. Other situations of physical interest can be taken into account: for instance, a more favorable observational condition (with regard to the amount of the effect) can be the one of a pulsar orbiting a neutron star. For more details, see the discussion in our paper[10]. What we can say, at this stage, is that the method we propose seems promising for detecting gravitomagnetic effects on the propagation of electromagnetic signals, and for measuring the angular momenta of celestial bodies.

References

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