Robust tracking control of a robot manipulator using a passivity-based extended-state observer approach

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Abstract: This study presents a combination of the robust passivity-based controller (RPBC) and an extended state observer (ESO) for the tracking control of a three degree-of-freedom PUMA 500 robot manipulator under parameter variations and external disturbances. The dynamic model of the PUMA robot and its structural properties are analysed. The extra state in the ESO estimates the parameter variations and external disturbances in the control system. Then the RPBC cancels them in the control law. The stability of the proposed control system and the convergence of the observation errors are analysed. Simulations prove that the proposed controller is robust and tracks well under parameter variations, and external disturbances, validating the proposed control scheme.

1 Introduction
The development and use of robot manipulators has progressed rapidly in the past three decades. Robot manipulators are used in manufacturing, factories, industrial plants, and in the medical field [1]. Manufacturers, factories, and industrial plants use industrial robot manipulators on assembly lines, for pick and place tasks, welding, and painting [1], while in the medical field, robot manipulators are used for different types of surgeries such as prostate and kidney procedures [2–6]. There are many benefits to using robot manipulators. The most significant advantages of robot manipulators in the industrial field are decrease in labour costs, an increase in productivity and precision, and they can be used in hazardous environments [1], while in the medical field, the benefits are shorter hospitalisation for patients, reduced pain and discomfort, faster recovery time, smaller incisions and scarring [2–6].

There is no direct measurement for the manipulator’s end-effector’s position and orientation. Instead, the end-effector’s position is measured from the measured joints positions [1]. It is then imperative that the manipulator’s joints track accurately so that the end-effector can perform accurately. Trajectory tracking control has been studied throughout the past three decades. One of the most common control schemes used in trajectory tracking is adaptive control. Adaptive control is widely used in tracking control because it can handle time-varying system uncertainties [7–12].

Another control scheme used in tracking control is passivity-based control. All robot models have the passivity property. Passivity-based control was first introduced in [13], so that the closed-loop system becomes passive. Many control methods take advantage of this property in their control design [14–17]. In [14], the passivity-based properties were used to develop a new function approximation technique adaptive controller for the tracking performance of a robot manipulator. In [15], a passivity-based control scheme was designed for a certain class of non-holonomic mechanical systems by passive configuration decomposition. In [16], a passivity-based tracking controller was designed for robot manipulators with input torque constraints. In [17], a passivity-based robust motion controller was designed for an underactuated prosthetic leg. The drawback of passivity-based control is that it produces high chattering levels in the torque input signals [1].

In [18], ADRC’s tracking performance was compared to the tracking performance of robust passivity-based controller (RPBC) for a two-link planar robot model. In [18], RPBC did not track well when the plant’s parameters were varied by 30% of their nominal values and produced high chattering levels in the input signal, while ADRC’s tracking performance was unaffected by the parameter variations.

The main contribution of this paper is to develop a novel solution for the tracking control of a robot manipulator against parameter variations, external disturbances, and to produce practically achievable torque input signals with no chattering levels. The proposed control scheme uses a non-linear extended state observer (ESO), where the extra state in the ESO estimates the parameter uncertainties and external disturbances in the control system. Then the RPBC controller cancels them in the control law. System stability of the proposed control scheme and the convergence of the observer’s errors are analysed. The performance of the controller is verified by simulation on a three degree-of-freedom (DOF) PUMA 500 manipulator.

The paper is organised as follows. Section 2 describes the dynamic model of the manipulator and its structural properties; Section 3 describes the ESO design for the manipulator; Section 4 derives the convergence proof for the observer’s error dynamics; Section 5 presents the proposed control method; Section 6 presents the simulations and an overview, and Section 7 offers conclusions and recommendation for future work.

2 Problem formulation
2.1 Dynamic model
The testing platform is a PUMA 500 robot as shown in Fig. 1. The PUMA robot consists of three main joints and a spherical wrist, which combine for six DOF. Although the PUMA 500 robot has six DOF, only the three main joints are used when testing. The three main joints represent the waist, \(q_1\), the shoulder, \(q_2\), and elbow, \(q_3\), of the PUMA robot. The coordinate system is defined as per the Denavit-Hartenberg standard. The dynamic model for the robot is given as

\[
M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau
\]

(1)

where \(M(q)\) is the inertia (mass) matrix, \(C(q, \dot{q})\dot{q}\) is the matrix accounting for the centripetal and Coriolis effects, \(g(q)\) is the gravity vector, and \(\tau\) is the torque vector for the control inputs. The elements entries of the matrices are non-linear and are shown in the Appendix.
2.2 Structural properties

The dynamic model of (1) has the following properties:

Property 1: \( M(q) - 2C(q, q) \) is skew symmetric.

Property 2: The linearity-in-parameters property states that \( M(q)q + C(q, q)q + g(q) \) can always be factored as \( Y(q, q, q)\theta \), where \( Y \), the regressor matrix does not contain any dimensional or physical constants. All constants appear as a set of parameters forming the entries of the parameter vector \( \theta \). A minimal set of parameters exists for each kinematic configuration. The regressor's parameters are shown in the Appendix.

Property 3: The inertia matrix boundedness property guarantees that there exist constants \( M_n \) and \( M_M \) such that \( 0 < M_n \leq M \parallel \leq M_M \), where any suitable norm can be used.

3 Extended state observer design

The robot manipulator control system is subject to external disturbances and parameter uncertainties. Taking the external disturbances and parameter uncertainties into consideration, (1) can be re-written as

\[
M(q)\ddot{q} + C(q, q)q + g(q) + d = \tau
\]

where \( d \) is the disturbance of the control system. The non-linear ESO is designed to estimate \( d \) and cancel it in the control law.

Equation (2) can be re-written as

\[
\ddot{q} + M^{-1}d = M^{-1}(-Cq - g + \tau)
\]

Set \( f = -M^{-1}d \) and (3) is re-written as

\[
\dot{f} = f + M^{-1}(-Cq - g + \tau)
\]

We set \( x_1 = q \), \( x_2 = \dot{q} \), and \( f = x_3 \) as the state variables of the systems, \( f \) is the generalised disturbance and the extra state in the system. The dynamic model in (3) is re-written as

\[
\begin{align*}
x_1 &= x_2 \\
x_2 &= f + M^{-1}(-Cq - g + \tau) \\
x_3 &= h \\
y &= x_1
\end{align*}
\]

The non-linear ESO is defined in [19], and is designed for the above system as

\[
\text{ESO:} \begin{cases}
\dot{o}_1 = \frac{y - \bar{z}_1}{e} \\
\dot{\bar{z}}_1 = \bar{z}_1 + e(3\bar{o}_1 + \varphi(e)) \\
\dot{\bar{z}}_2 = \bar{z}_2 + 3\bar{o}_1 + M^{-1}(-Cq - g + \tau) \\
\bar{z}_3 &= \frac{1}{e}o_e
\end{cases}
\]

where \( y \) is the output of the system, \( o_e \) is the observer's error, \( z_1 \) and \( z_i \) are the observed states, and \( \bar{z}_i \) is the observed generalised disturbance. They are the estimates of \( x_i, x_2, \) and \( x_3 \), where \( x_i = f \). \( e \) is a design parameter in the ESO, and its range is defined in [19] as \( e \in (0, 1] \). \( \varphi(e) \) is defined in [19] as

\[
\varphi(e) = \begin{cases}
-\frac{1}{\pi^2} & o_e \in (-\infty, -\pi/2) \\
\frac{1}{\pi^2}\sin(o_e) & o_e \in (-\pi/2, \pi/2) \\
\frac{1}{\pi^2} & o_e \in (\pi/2, \infty)
\end{cases}
\]

The tracking of the actual states, \( x_i \), by the estimated states, \( \bar{z}_i \) is expected for all \( i = 1, 2, 3 \) as \( t \to \infty \). The estimate of \( d, \ddot{d} \), is equal to \(-M\bar{z}_3\).

4 Convergence of observer errors

Following the framework from [20], a Lyapunov candidate function is developed to show that the observer's errors converge to zero. The error equation, \( e \), is defined as \( x - \bar{x} \). The error dynamics equation for the ESO is defined as

\[
e = x - \bar{x}
\]

Since \( y = x_1 \), the observer error in (6) can be re-written as

\[
o_e = \frac{x_1 - \bar{z}_1}{e} \\
o_e = \frac{x_1}{e^2}
\]

substituting (9) into (6), the error dynamics equation is re-written as

\[
e = \Lambda e - (\varphi(e), 0, 0)^T
\]

with matrix \( \Lambda \) defined as

\[
\Lambda = \begin{bmatrix}
-3 & 1 & 0 \\
-3 & 0 & 1 \\
-1 & 0 & 0
\end{bmatrix}
\]

The positive definite Lyapunov candidate function is selected as

\[
V(e) = (Pe, e) + \int_{s_0}^{s} \varphi(s) ds, \quad \forall e = (e_1, e_2, e_3) \in R^3
\]

where \( \varphi \) is a function of the observer error, \( s \) is the variable of this function, and matrix \( P \) is derived from the following equation:

\[
A^TP + PA = -I
\]

leading \( P \) to be equal to
\[
P = \begin{bmatrix}
1 & -1/2 & -1 \\
-1/2 & 1 & -1/2 \\
-1 & -1/2 & 4
\end{bmatrix}
\] (14)

Pe is derived as

\[
Pe = \begin{bmatrix}
e_1 - 1/2e_2 - e_3 \\
-1/2e_1 + e_2 - 1/2e_3 \\
e_1 - 1/2e_2 + 4e_3
\end{bmatrix}
\] (15)

Equation (12) is re-written as

\[
V(e) = \langle Pe, e \rangle + \varphi(e_1)
\]
\[
V(e) = \langle Pe, e \rangle + \varphi(e_2)
\]
\[
V(e) = \langle Pe^2, \varphi(e_3) \rangle
\]
\[
V(e) = e^T Pe + \varphi(e_1)
\]

Differentiating \(V(e)\) with respect to time yields

\[
V(e) = e^T Pe + e^T Pe + \varphi(e_1)e_1
\]
\[
V(e) = -e_1^2 - e_2^2 - e_3^2 - (2e_1 - e_2 - 2e_3)e_1 + \varphi(e_1)e_1
\]
\[
V(e) = -e_1^2 - e_2^2 - e_3^2 - (5e_1 - 2e_2 - 2e_3 + \varphi(e_1))e_1
\]

From the definition of \(\varphi(e)\) in (7), we have the following:

\[
\varphi(e_1)e_1 \geq 0
\]
\[
2e_1\varphi(e_1) \leq e_1^2 + 2(\varphi(e_1))^2
\]
\[
2e_1\varphi(e_1) \leq e_1^2 + 2(\varphi(e_1))^2
\]

Combining (17) with (18), we get

\[
V(e) \leq -e_1^2 - e_2^2 - e_3^2 + \left(\frac{e_1^2}{2} + 2(\varphi(e_1))^2\right)
\]
\[
\leq -\left(\frac{e_1^2}{2} + \frac{e_2^2}{2} + \frac{e_3^2}{2} + 5(\varphi(e_1))^2\right)
\]
\[
\leq -\left(\frac{4e_1^2}{3} + \frac{e_2^2}{2} + \frac{e_3^2}{2}\right)
\]

From (19), it can be seen that \(V(e)\) is negative definite. This indicates that the terms \(e_1, e_2, e_3, e_1\varphi(e_1), e_2\varphi(e_1), e_3\varphi(e_1)\), and \((\varphi(e_1))^2\) are bounded. The derivative of \(V(e)\) is

\[
\dot{V}(e) = \left(\frac{52}{25}e_1^2 + \frac{2}{5}e_2^2 + \frac{2}{5}e_3^2 \right) - 3e_1 + e_2 + \varphi(e_1)
\]
\[
+ \left(2e_1^2 + \frac{2}{5}e_3^2 \right) - 3e_1 + e_3
\]
\[
+ \left(\frac{2}{5}e_1^2 - 2e_2^2 \right) - 3e_3
\]

This shows that \(\dot{V}(e)\) is bounded too since \(e_1, e_2, e_3, \varphi(e_1)\) were shown to be bounded earlier. This indicates that \(V(e)\) is uniformly continuous. Invoking Barbala's Lemma [21], the dynamic errors \(e_1, e_2, e_3\), converge to 0 as \(t \to \infty\), indicating that the estimate states are approximately equal to the actual states. This shows that \(z \approx \hat{z}\) and \(f \approx \hat{f}\).

\section{5 Robust passivity based control}

The proposed control method combines RPBC with the non-linear ESO. The ESO's estimated states are fed-back to the RPBC controller. By combining RPBC and the ESO, the proposed controller is robust to parameter variations and external disturbances. Since the extra state will estimate the generalised disturbance, and the RPBC controller will cancel them in the control law. For the PUMA 500 manipulator, each joint will have its own ESO.

Fig. 2 shows the block diagram for RBPC with the ESO, where \(\dot{q}^d, \dot{q}^u\) are the desired position and velocity trajectories of the manipulator's joints. \(q, \dot{q}\) are the actual position and velocity trajectories of the manipulator's joints. \(z, \ddot{z}\) are the estimated position and velocity trajectories of the manipulator's joints, while \(\ddot{z}\) is the estimated generalised disturbance.

The tracking errors for the joints position and velocity are defined as \(e_1 = \ddot{q} = z_1 - \dot{q}^d\) and \(e_2 = \ddot{q} = z_2 - \dot{q}^u\), respectively. The dynamic model for the robot manipulator is considered again as

\[
M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) + d = u
\]

where \(u\) is the control law. To construct the control law, it is assumed that only the structure of the model is known, but that the parameters are uncertain, and that the parametric uncertainties are bounded. It is also assumed that the joint positions and velocities are directly available. The control input is chosen as

\[
u = M(q)\dot{u} + C(q, \dot{q})\dot{v} + g(q) - Kr - M\ddot{z}
\]

where the variables \(v, a, r\) are given as

\[
v = \dot{q}^d - \Lambda\ddot{q}
\]
\[
a = v = \dot{q}^u - \Lambda\ddot{q}
\]
\[
r = q - \ddot{q} = \ddot{q} + A\ddot{q}
\]

where \(K\) and \(\Lambda\) are diagonal matrices of positive gains. The control law is re-written in terms of linear parameterisation of the robot dynamics and it becomes

\[
u = Y(q, q, a, v)\dot{\theta} - Kr - M\ddot{z}
\]

and the combination of (21) with (22) results in

\[
M(q)\dot{r} + C(q, \dot{q})\dot{r} + Kr = Y(\dot{\theta} - \theta) + Me\]

The term \(\dot{\theta}\) is chosen as

\[
\dot{\theta} = \theta_0 + \delta\theta
\]

where \(\theta_0\) is a fixed nominal parameter vector and \(\delta\theta\) is an additional control term [1]. The system in (24) becomes

\[
M(q)\dot{r} + C(q, \dot{q})\dot{r} + Kr = Y(q, q, a, v)(\dot{\theta} + \delta\theta) + Me
\]
where $\tilde{\theta} = \theta_0 - \theta$ is a constant vector and it represents the system's parametric uncertainty. Suppose $\rho$ is a non-negative constant such that $\rho \geq 0$ such that

$$\| \tilde{\theta} \| = \| \theta_0 - \theta \| \leq \rho$$

(28)

### 5.1 Stability analysis

The following Lyapunov candidate function is considered:

$$V = \frac{1}{2} r^T M r + \tilde{q}^T \Lambda \tilde{q}$$

(29)

Calculating $V$ derives

$$V = r^T M r + \frac{1}{2} \tilde{q}^T \Lambda \tilde{q} - r^T Y \tilde{q} + \tilde{r}^T Y \tilde{q} + r^T M e_s$$

(30)

Using the skew symmetry property and the definition of $r$ in (23), we have

$$V = -\tilde{q}^T \Lambda K \tilde{q} + r^T Y \tilde{q} + r^T M e_s$$

(31)

where

$$Q = \begin{bmatrix} \Lambda K \Lambda & 0 \\ 0 & K \end{bmatrix}$$

(32)

The additional term $\delta \theta$ can be designed as follows:

$$\delta \theta = \begin{cases} -\rho \frac{Y^T r}{\| Y^T r \|}, & \text{if } \| Y^T r \| > \epsilon \\ -\rho \frac{Y^T r}{\epsilon}, & \text{if } \| Y^T r \| \leq \epsilon \end{cases}$$

(33)

**Case 1:**

If $\| Y^T r \| > \epsilon$, $\delta \theta = -\rho(Y^T r/\| Y^T r \|)$, substituting into $r^T Y (\tilde{\theta} + \delta \theta)$, we get

$$r^T Y (\tilde{\theta} + \delta \theta) = r^T \tilde{Y} \tilde{\theta} - \frac{r^T Y Y^T r}{\| Y^T r \|}$$

(34)

Using the Cauchy-Schwarz inequality

$$r^T \tilde{Y} \tilde{\theta} - \| Y^T r \| \rho \leq \| r^T Y \| \| \tilde{\theta} \| - \| Y^T r \| \rho$$

(35)

It is noted that

$$\| r^T Y \| = \| Y^T r \|$$

(36)

From (27), it is known that

$$\| \tilde{\theta} \| \leq \rho$$

(37)

This leads to

$$r^T Y (\tilde{\theta} + \delta \theta) \leq \| Y^T r \| ( \| \tilde{\theta} \| - \rho)$$

(38)

From (22), $r$ is defined as

$$r = \tilde{q} + \Lambda \tilde{q}$$

(39)

$r^T$ is defined as

$$r^T = \tilde{q}^T + \Lambda \tilde{q}^T$$

(40)

$e_i$ was defined earlier as

$$e_i = \begin{bmatrix} \tilde{q} \\ \tilde{q} \end{bmatrix}$$

(41)

From (39), $r^T$ can be re-written as

$$r^T = e_i^T \Phi = e_i^T \omega$$

(42)

From this, $V$ is re-written as

$$V = -e_i^T Q e_i + r^T Y (\tilde{\theta} + \delta \theta) + e_i^T \omega M e_s$$

(43)

Since $e_i$, $M$, and $e_i$ are bounded, $V$ can be written as

$$V \leq -\lambda_{\text{min}}(Q) \| e_i \| \| Y \| \| Y \theta \| \| e_i \| \| \omega \| (M e_s) \| e_i \| \| \omega \| (M e_s) \| e_i \|$$

(44)

with $\| \omega \| (M e_s) \| e_i \| = \eta$. Using the Cauchy inequality for $\| e_i \| \eta$ in (44), we have

$$V \leq -\lambda_{\text{min}}(Q) \| e_i \| \| Y \| \| Y \theta \| \| e_i \| \| \omega \| (M e_s) \| e_i \| + \frac{1}{2} \lambda_{\text{max}}(Q) \eta^2$$

(45)

Equation (45) implies that $V < 0$ if

$$\| e_i \| \geq \sqrt{\frac{\eta}{2 \lambda_{\text{min}}(Q)}} = \delta_i$$

(46)

From (46), it can be seen that the closed-loop control system has the property of uniform ultimate boundedness (u.u.b) of the error trajectories with respect to $\delta_i$, the u.u.b radius. $\lambda_{\text{min}}(Q)$ is the smallest eigenvalue of $Q$. For the robot's joints, the closed-loop trajectories will enter and remain on the smallest level set of $V$ which contains a ball of radius $\delta_i$.

**Case 2:**

If $\| Y^T r \| \leq \epsilon$, $\delta \theta = -\rho(Y^T r/e)$, substituting into $r^T Y (\tilde{\theta} + \delta \theta)$, we get

$$r^T Y (\tilde{\theta} + \delta \theta) = r^T \tilde{Y} \tilde{\theta} - \frac{r^T Y Y^T r}{\| Y^T r \|}$$

(47)

Using the Cauchy-Schwarz inequality

$$r^T \tilde{Y} \tilde{\theta} - \| Y^T r \| \rho \leq \| r^T Y \| \| \tilde{\theta} \| - \| Y^T r \| \rho$$

(35)

It is noted that

$$\| r^T Y \| = \| Y^T r \|$$

(36)

From (27), it is known that

$$\| \tilde{\theta} \| \leq \rho$$

(37)

This leads to

$$r^T Y (\tilde{\theta} + \delta \theta) \leq \| Y^T r \| ( \| \tilde{\theta} \| - \rho)$$

(38)
6 Numerical simulations

Simulations were conducted for the proposed controller using the PUMA 500 robot at the Cleveland State University Controls, Robotics, and Mechatronics Laboratory. The reference trajectories, PUMA robot parameters, and controller gains were from [14], while \( \delta \), the dc gain had a value of 1 for the three links. The reference trajectories are

\[
q_1(t) = \text{diag}(10 30 10) + \lambda (6 35 6)\]

The input constants were used to convert the manipulator's control signals from N-m to \( V \) as in [14]. The input constants represent the combined effect of gear ratios, amplifier gains, and motor torque constants. The input constants are 0.0543, 0.0806, and 0.1078 V/Nm, for \( q_1 \), \( q_2 \), and \( q_3 \), respectively.

Fig. 3 shows the PUMA’s three joints tracking performance achieved by the proposed method. It can be seen that the controller tracks the three joints accurately. The root mean square (RMS) errors are 0.0142, 0.0003, and 0.0056 rad for \( q_1 \), \( q_2 \), and \( q_3 \), respectively.

Fig. 4 shows the torque input signals for the proposed control scheme. It can be seen that during the transient phase, the control signals are large, however the during the steady-state phase, the control signals are reasonable and practically achievable. The control signals also do not have any chattering levels.

Fig. 5 shows the manipulator's estimated states for \( q \) and \( \dot{q} \) compared to the manipulator's actual states. It can be seen that the estimates of \( q \) and \( \dot{q} \) are bounded, and track the actual states.

6.1 Robustness against parameter variations

The robustness of the proposed controller is tested by intentional, random perturbations in each \( \Theta \) component with a deviation of up to 30% [18], with the controller parameters remaining the same.

Fig. 6 shows the tracking performance of the three joints by the proposed control scheme with the controller gains remaining the same. It can be seen that the controller tracks the three joints...
accurately. The RMS errors are: 0.016, 0.005, and 0.0024 rad for $q_1$, $q_2$, and $q_3$, respectively.

Fig. 7 shows the torque input signals for the proposed control scheme. It can be seen again that the control signals were not affected in the presence of parameter variations. The control signals are still high during the transient phase, however during the steady-state phase, the control signals are reasonable and practically achievable with no chattering levels.

Fig. 8 shows the manipulator’s estimated states for $q$ and $\dot{q}$ compared to the manipulator’s actual states. It can be seen that the estimates of $q$ and $\dot{q}$ are bounded, and track the actual states compared to the manipulator’s actual states even in the presence of parameter variations.

6.2 Robustness against payload variation

Another test is conducted to test the robustness of the proposed controller by varying the payload. Simulations are done by increasing the weight of $m_4 - 6$ to represent the case when greater payload is being carried by the manipulator’s end-effector. The PUMA 500 has a payload of 2.5 kg [22]. In our simulations, the weight of $m_4 - 6$ is increased by 100% (equivalent to 1.17 kg), with the controller parameters remaining the same.

Fig. 9 shows the tracking performance of the three joints by the proposed control scheme with the payload variation. It can be seen again that the controller tracks the three joints accurately. The RMS errors are 0.0142, 0.0003, and 0.0056 rad for $q_1$, $q_2$, and $q_3$, respectively.

Fig. 10 shows the torque input signals. It can be seen again that the control signals were not affected with the payload variations. Again, the control signals are still high during the transient phase, however during the steady-state phase, the control signals are reasonable, practically achievable, and have no chattering levels.

Fig. 11 shows the manipulator’s estimated states for $q$ and $\dot{q}$ compared to the manipulator’s actual states. Again, the estimates of $q$ and $\dot{q}$ are bounded, and track the actual states, even in the presence of payload variations.
6.3 Robustness against disturbances

Another test is conducted to test the robustness of the proposed controller by adding a sinusoidal disturbance of \( d = 10\sin(2t) \), with the controller parameters remaining the same.

Fig. 12 shows the tracking performance of the three joints by the proposed control scheme with the sinusoidal disturbance. It can be seen again that the controller tracks the three joints accurately. The RMS errors are 0.0164, 0.005, and 0.0024 rad for \( q_1 \), \( q_2 \), and \( q_3 \), respectively.

Fig. 13 shows the torque input signals. It can be seen again that the control signals were not affected with the sinusoidal disturbance. Again, the control signals are still high during the transient phase, however during the steady-state phase, the control signals are reasonable, practically achievable, and have no chattering levels.

Fig. 14 shows the manipulator's estimated states for \( q \) and \( \dot{q} \) compared to the manipulator's actual states. Again, the estimates of \( q \) and \( \dot{q} \) are bounded, and track the actual states, even in the presence of the sinusoidal disturbance.

7 Conclusion

In this paper, we developed an observer-based feedback controller for tracking control of the PUMA 500 robot manipulator. The extra state in the ESO estimates the parameter uncertainties and external disturbances of the control system, and the RPBC controller cancels them in the control law. The convergence of the observation's errors shows that the observer's errors were bounded. Stability analysis shows that the proposed control scheme is stable. Future work will be the experimental implementation of the proposed controller. A comparative study will be conducted to compare the tracking results of the proposed scheme versus RPBC combined with a high gain observer.

8 Acknowledgment

This work was supported by the Department of Electrical Engineering and Computer Science at Cleveland State University. The authors thank Dr. Elie Shammas for his valuable input and feedback regarding the paper.
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10 Appendix

The dynamic equation of the PUMA 500 manipulator is presented, where

\[ c_i = \cos(q_i), \quad s_i = \sin(q_i), \quad c_i = \cos(q_i + q_j), \quad \text{and} \quad s_j = \sin(q_i + q_j) \]

\[ M_{1i} = \Theta_{c2}q_i^2 + 2\Theta_{c23}c_i + \Theta_{c3}^2 + \Theta_{\theta} \]

\[ M_{12} = \Theta_{c3}q_i^2 + \Theta_{s2} \]

\[ M_{13} = \Theta_{s2}q_i^2 + \Theta_{s23}c_i \]

\[ M_{12} = \Theta_{s2} + 2\Theta_{c3}c_i \]

\[ M_{23} = \Theta_{s2} + \Theta_{c3}c_i \]

\[ M_{31} = \Theta_{s2} \]

\[ M_{32} = \Theta_{s2} \]

\[ C_{11} = \dot{q}_i(\Theta_{c2}q_s + \Theta_{c23}c_s + \Theta_{s2}c_s + \Theta_{c3}s_s) \]

\[ -\dot{q}_i(\Theta_{c2}c_s + \Theta_{s23}c_s) \]

\[ C_{12} = \dot{q}_i(\Theta_{c2}q_s + \Theta_{c3}) - \dot{q}_i(\Theta_{c23}c_s + \Theta_{c3}s_s) \]

\[ + \Theta_{s2}c_s + \Theta_{c3}c_s + \Theta_{s23}s_s \]

\[ C_{13} = \dot{q}_i(\Theta_{c2}q_s + \Theta_{c3}s_s) + \Theta_{s2}c_s + \Theta_{c3}c_s \]

\[ C_{22} = \dot{q}_i \Theta_{c2} \]

\[ C_{23} = -\dot{q}_i \Theta_{c23} \]

\[ C_{31} = \dot{q}_i(\Theta_{c2}q_s + \Theta_{s23}c_s) \]

\[ C_{32} = \dot{q}_i(\Theta_{c2}c_s + \Theta_{c3}c_s + \Theta_{s23}s_s) \]

\[ C_{33} = -\dot{q}_i \Theta_{c2} \]
Parameter vectors:

\[ C_{32} = - \Theta_7 q_5 s_3 \]
\[ C_{33} = 0 \]
\[ G_i = 0 \]
\[ G_1 = - \Theta_{62} c_3 - \Theta_{6 c_2} \]
\[ G_1 = - \Theta_{6 c_3} \]

Parameter vectors:

\[ \Theta_1 = (m_5 + m_7) d_1^2 - 2m_5 d_1 d_3 + m_7 d_3^2 + I_{y3} + I_{y5} \]
\[ \Theta_2 = m_6 c_{23}^2 + (m_6 + m_7) a_{23}^2 - I_{23} + I_{y7} \]
\[ \Theta_3 = m_6 c_{23}^2 - I_{23} + I_{y7} \]
\[ \Theta_4 = c_{33} a_{32} m_3 \]
\[ \Theta_5 = c_{33} m_3 (d_3 - d_5) \]
\[ \Theta_6 = m_6 c_{23}^2 + 2m_6 a_{23} + (m_6 + m_7) a_{23}^2 + I_{23} + I_{y7} \]
\[ \Theta_7 = I_{33} + M a_{33}^2 \]
\[ \Theta_8 = d_2 m_5 (c_{23} + a_2) + a m_5 (d_3 - d_5) \]
\[ \Theta_9 = c_{33} g m_3 \]
\[ \Theta_{10} = g m_3 (c_{33} + a_2) + a g m_3 \]

Regressor:

The regressor matrix for the three main joints of the PUMA 500 robot is given below:

\[ Y(1, 1) = \dot{q}_i \]
\[ Y(1, 2) = \ddot{q}_i c_i^2 - 2 \dot{q}_i \dot{q}_i c_i s_i \]
\[ Y(1, 3) = \ddot{q}_i c_i^3 - 2 \dot{q}_i (\dot{q}_i - q_i) c_i s_i \]
\[ Y(1, 4) = 2q_1 c_1 c_2 - 2q_1 c_1 c_3 c_2 - 2q_1 (q_1 - q_3) c_2 c_3 \]
\[ Y(1, 5) = (\dot{q}_i + q_i) \dot{c}_2 + (\dot{q}_i + q_i) s_i \]
\[ Y(1, 6) = Y(1, 7) = Y(1, 9) = Y(1, 10) = 0 \]
\[ Y(1, 8) = q_i^2 c_2 + q_i s_2 \]
\[ Y(2, 1) = 0 \]
\[ Y(2, 2) = \dot{q}_i c_3 s_3 \]
\[ Y(2, 3) = \dot{q}_i c_3 s_3 \]
\[ Y(2, 4) = (c_{23} s_2 + s_{25} c_3) \dot{q}_i - (2q_1 q_3 + q_3^2) s_1 + (2q_1 + q_3)c_3 \]
\[ Y(2, 5) = q_3 s_3 \]
\[ Y(2, 6) = q_3 \]
\[ Y(2, 7) = q_3 \]
\[ Y(2, 8) = q_3 s_3 \]
\[ Y(2, 9) = - c_3 \]
\[ Y(2, 10) = - c_3 \]
\[ Y(3, 1) = Y(3, 2) = Y(3, 6) = Y(3, 8) = Y(3, 10) = 0 \]
\[ Y(3, 3) = \dot{q}_i c_3 c_2 s_3 \]
\[ Y(3, 4) = q_3 c_3 + \dot{q}_3 c_2 + \dot{q}_3 c_2 s_3 \]
\[ Y(3, 5) = q_3 s_3 \]
\[ Y(3, 7) = q_3 + q_3 \]
\[ Y(3, 9) = - c_3 \]