From Quantum Foundations to Quantum Gravity
- An overview of the new theory -

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ABSTRACT
Spontaneous localisation is a falsifiable dynamical mechanism which modifies quantum mechanics, and explains the absence of position superpositions in the macroscopic world. However, this is an ad hoc phenomenological proposal. Adler’s theory of trace dynamics, working on a flat Minkowski space-time, derives quantum (field) theory, and spontaneous localisation, as a thermodynamic approximation to an underlying classical matrix dynamics. We describe how to incorporate gravity into trace dynamics, by using ideas from Connes’ non-commutative geometry programme. This leads us to a new quantum theory of gravity, from which we can predict spontaneous localisation, and give an estimate of the Bekenstein-Hawking entropy of a Schwarzschild black hole.

Text books on quantum mechanics often state that classical mechanics is obtained as the $\hbar \to 0$ limit of quantum mechanics. (In this limit, the Schrödinger equation goes over to the classical Hamilton-Jacobi equation). However, such a statement hides an assumption: it is implicitly assumed, based on what we observe, that position superpositions are absent in the classical world. In other words, even as the $\hbar \to 0$ limit is taken, a classical object could be in two or more locations at start of evolution (as allowed by quantum mechanics), and the Hamilton-Jacobi evolution would then imply that a classical particle would simultaneously evolve along a collection of trajectories; one trajectory per every initial location. The fact that such classical motion is never seen needs explaining, and is also the essence of the quantum measurement problem. That is, upon measurement, a pointer is never in more than one position at the same time (unlike what the Schrödinger equation predicts for the pointer) and the entangled state of the pointer and the measured quantum system collapse.
to one or the other classical outcomes. There is no universally accepted explanation as to why this should happen during the quantum-classical transition.

Here, it is important to emphasize that there is an intermediate regime between the microscopic and macroscopic, where quantum mechanics has not been experimentally tested. Simply because experiments in this intermediate regime are extremely challenging technologically, although important progress is taking place now [see e.g. the link tequantum.eu for the TEQ experiment]. This is brought out by the diagram below.

![Diagram](image)

**FIG. 1.** Tested and untested regimes of dynamics

The largest objects for which the principle of quantum linear superposition has been tested are made up of about ten thousand nucleons (molecular interferometry). The smallest objects for which classical mechanics has been tested are made up of about $10^{19}$ nucleons.
(or an order or two less, in magnitude). There is thus an enormous desert of fifteen orders in magnitude, which is untested. New physics can arise here, in such a way that the new theory agrees with quantum mechanics for small objects, and with classical mechanics for large objects. However, the new theory ensures that during passage through this desert, the principle of quantum linear superposition breaks down dynamically. Such a breakdown is not ruled out by experiments, nor is it prohibited by the extraordinary success of quantum field theory.

The Ghirardi-Rimini-Weber-Pearle theory of spontaneous localisation, proposed first during 1970s and 1980s, achieves just that, providing a unified description of quantum and classical dynamics. The basic idea behind the theory is extremely simple, and beautiful. Recall that, according to the Schrödinger equation, a quantum superposition lasts forever. Once a quantum system has been prepared in a superposition of, say two position eigenstates, it will evolve unitarily and stay in that superposed state for an infinite time. But clearly it is unphysical to talk of infinite time. Instead, GRW proposed, let us assume that the superposition of a nucleon in different position states lasts, on the average, as long as the age of the universe $\sim 10^{17}$ s. And then, it spontaneously and randomly collapses to one of those many eigenstates, with a probability given by the Born rule. Superposition is lost spontaneously [1–7].

This little change in the dynamics is enough to solve the quantum measurement problem, and to explain the absence of position superpositions in the macroscopic world. This is a consequence of quantum entanglement. Consider a bound macroscopic object [e.g. a chair] whose atoms, all put together, have $N$ nucleons. Trying to create a superposition of the chair in two states, say chair here + chair there, amounts to creating an entangled state of the $N$ nucleons. It is easy to show that such an entangled state will spontaneously collapse in a time $T/N$ to one of the two position eigenstates, where $T$ is the spontaneous collapse mean life-time of one nucleon. The GRW theory assumes $T \sim 10^{17}$ s. If we take $N \sim 10^{23}$, the superposition will collapse in a millionth of a second. Thus superpositions are not absent in the macroscopic world; rather they are extremely short-lived. On the other hand, in the microscopic quantum world, superpositions last for a very long time (instead of lasting forever). In this way, spontaneous collapse theories provide a unified description of quantum and classical dynamics. Precisely in the untested region in the above diagram, the superposition life-time is neither too large nor too small, and differences
from quantum and classical dynamics show up. Now when one takes the $h \to 0$ limit, that procedure also destroys superposition, because an extra parameter is at play: the lifetime of the superposition.

The GRW theory can be cast into a precise mathematical formulation, by expressing it as a stochastic non-linear modification of the Schrödinger equation. The non-unitary, non-linear part ensures breakdown of superposition, and its stochastic nature ensures randomness in collapse outcomes. Moreover the non-linearity is so constructed that evolution preserves norm, despite the non-unitarity. This ensures that the Born probability rule is reproduced. Also, a condition is imposed that the non-linearity should not lead to super-luminal signalling.

Whether or not dynamical collapse theories are correct can only be decided by experiment, and various ongoing experiments are pushing up the bound on the collapse time $T$. The current status is that we have two theories - quantum mechanics, and spontaneous localisation - which are both consistent with every experiment done to date. However only one of them can be correct, and that will be decided by further experiments.

Nonetheless, it is only fair to say that the theoretical structure of spontaneous collapse models has some shortcomings - overcoming these would only make these models more convincing. These models are ad hoc and phenomenological in nature, having been designed with the express purpose of solving the measurement problem, and explaining the quantum-classical transition. What is the fundamental origin of spontaneous collapse; what causes it? What is that stochastic noise which interacts with a quantum system, and introduces non-unitarity in its evolution? Why should norm be preserved despite the introduction of an external noise source? Its rather unusual in physics for stochastic effects to impact on a fundamental equation such as the Schrödinger equation. Furthermore, collapse models are non-relativistic. Generalising them to a relativistic quantum-field theoretic version has remained an unsolved problem, despite many serious efforts. All these shortcomings need to be overcome, to make spontaneous localisation into a robust physical theory that merges well with already known physical theories, such as relativistic quantum field theory. Recent developments address these issues, with rather dramatic implications, as we now describe.

Adler’s theory of trace dynamics is built on the guiding principle that quantum theory, being more fundamental than classical mechanics, should be constructed ab initio from first principles in a bottom-up fashion. Rather than our having to arrive at quantum (field)
theory by quantizing the theory’s own limit, viz. classical dynamics. Thus, we do not arrive at special relativity by relativizing Newtonian mechanics. Nor do we arrive at general relativity by relativizing Newton’s law of gravitation. The two relativity theories are built from their own new concepts and symmetry principles (universal constancy of speed of light, and interpretation of gravitation as space-time curvature). Newtonian dynamics then naturally follows as the non-relativistic limit of the relativity theory.

Trace dynamics is the classical matrix dynamics of matrices (equivalently operators) on a Minkowski space-time. A matrix describes an elementary particle or a field; the idea being that instead of using c-numbers or real numbers to describe these entities, one uses matrices. The consequences are far-reaching. Each matter or field degree of freedom is described by an operator degree of freedom, labelled say \( q \) (configuration variable). \( q \) is a function of time if it describes a particle, and of space-time if it describes a field. Thought of as a matrix, \( q \) is made of Grassmann numbers as its elements. Grassmann numbers anti-commute with each other. Such a matrix can always be written as a sum of a ‘bosonic’ matrix and a ‘fermionic’ matrix. A bosonic matrix is even grade Grassmann (matrix elements made of product of even number of Grassmann elements, so that they commute with each other), and a fermionic matrix is odd grade Grassmann (matrix elements made of product of odd number of Grassmann elements, so that these anti-commute amongst themselves). The nomenclature is natural, as bosonic / fermionic matrices are indeed used to describe bosonic / fermionic fields, in particle physics. An operator polynomial made from \( q_s \) and its time derivatives is used to construct a Lagrangian, by taking the matrix trace of this polynomial - the trace Lagrangian, as it is referred to. As in classical dynamics, time integral of the trace Lagrangian defines the action. Equations of motion are derived by extremizing the action, while varying with respect to the \( q_s \) (using the trace derivative). One arrives at Lagrange’s equations of motion, from which a Hamiltonian dynamics can also be constructed. All configuration variables as well as their canonically conjugate momenta obey arbitrary commutation relations with each other, which inevitably evolve with time, consistently with the equations of motion. Thus this is a classical dynamics because it follows from variation of the action / Lagrangian, but in a sense it is even ‘more quantum’ than quantum mechanics because the commutation relations are arbitrary (not fixed like the quantum commutation relations). There is no Planck’s constant yet - \( \hbar \) is emergent in this theory.

In spirit, trace dynamics (a classical matrix dynamics) resembles matrix models which
have been studied in the past, including in the context of string theory. The central difference
between matrix models and trace dynamics is that one does not quantise trace dynamics.
On the contrary, quantum (field) theory is derived from trace dynamics as the statistical
thermodynamics of a large number of $q$ matrices, by coarse-graining their evolution in op-
erator phase space. Thus, trace dynamics is assumed to hold at the Planck scale, and one
would like to examine what is the dynamics much below the Planck scale. This is where
statistical mechanics comes in. The system point is assumed to visit all allowed states in the
phase space, so that long time averages may equal ensemble averages. A probability distri-
bution is defined in phase space, using a suitable measure, and the equilibrium distribution
is determined by maximising the von Neumann entropy.

On the physical front, what distinguishes trace dynamics from Newtonian mechanics is the
existence of a remarkable conserved charge, which results from a global unitary invariance of
the trace Hamiltonian. This charge, known as the Adler-Millard charge, is given by the sum
over all bosonic degrees of freedom of their respective commutators $[q, p]$, minus the sum
over all the fermionic degrees of freedom, of their respective anti-commutators $\{q, p\}$. Each of
these commutators has dimensions of action, and is by itself time-dependent. Yet the Adler-
Millard charge defined from them is conserved. It turns out that at equilibrium, this charge
is equipartitioned over all the degrees of freedom - the equipartitioned value is identified with
Planck’s constant $\hbar$. It is shown that at equilibrium, the ensemble averages of the canonical
degrees of freedom obey the Heisenberg equations of motion. This is how quantum (field)
theory is derived from first principles, by starting from a well-defined matrix dynamics.
An equivalent Schrödinger functional picture can also be constructed, as in quantum field
theory.

The next significant move is to recognise that there always are statistical fluctuations
around equilibrium, such as those which are responsible for Brownian motion. Such fluctua-
tions modify the evolution equations of quantum (field) theory. In principle, the corrections
can include a non-self adjoint component as well, which causes the appearance of anti-self
adjoint corrections to the Hamiltonian.

Adler considered the role of these corrections in the context of the non-relativistic
Schrödinger equation, for matter (fermionic) degrees of freedom. This amounts to adding a
stochastic correction (including an anti-self-adjoint part) to the matter Hamiltonian. The
structure now is pretty much as in collapse models. Assuming, as in collapse models, that
norm is preserved (despite non-unitary evolution), and that superluminal signalling is not allowed, one arrives at a stochastic non-linear Schrödinger equation with the same structure as a collapse model. The theory of trace dynamics can hence explain the origin of spontaneous localisation - the latter is no longer an ad hoc proposal.

In our recent work we have addressed the unresolved issues in trace dynamics. Amongst these are the following. Trace dynamics is formulated at the Planck scale, but it assumes the space-time background to be Minkowski. It would be more natural to allow for a quantum behaviour of space-time, and to incorporate gravity, albeit not as classical gravity, but as operator gravity. We solve this problem, by bringing in the description of space-time structure from Connes’ non-commutative geometry programme. Secondly, in trace dynamics, only a non-relativistic theory of spontaneous collapse is arrived at. By bringing in gravity, we construct a relativistic theory of spontaneous localisation. Thirdly, we explain why only the fermionic (matter) degrees of freedom undergo spontaneous collapse, whereas bosonic degrees (the gravitational field say) do not. And we also explain why the norm of the evolving state vector must be preserved, despite the presence of anti-self-adjoint corrections to the Hamiltonian.

We emphasize that our primary motive behind this approach to quantum gravity was not that of incorporating gravity in trace dynamics. Rather, our goal was to arrive at a formulation of quantum (field) theory which does not refer to classical space-time. The realisation that such a formulation must exist is the single most important clue towards a quantum theory of gravity. Classical space-time is a consequence of the universe being dominated by classical macroscopic bodies. In the absence of such bodies (which in fact are a limiting case of quantum systems, thus forcing quantum theory to depend on its own limit) there will be no space-time, yet we should we able to describe quantum systems (without appealing to classical time). Thus, we do not quantize space-time; rather we get rid of space-time from quantum theory - this leads to a falsifiable candidate quantum theory of gravity, which predicts spontaneous localisation.

Our underlying physical principle/symmetry is to demand that the laws of gravitation, and of the matter sources that describe them, are invariant under general coordinate transformations of non-commuting coordinates. This takes us to the domain of Connes’ non-commutative geometry. This symmetry principle also has the flavour of trace dynamics, because the non-commuting coordinates are operators (equivalently matrices) which obey
arbitrary commutation relations amongst them.

Non-commutative geometry (NCG) provides a spectral view of gravitation and curvature, which again ties in well with trace dynamics \cite{2}. The relevant result for us, which we present here in a simplified manner, is the following. Given a Riemannian manifold describing (Euclidean) curved space-time, construct the standard Dirac operator on this space-time, and find its eigenvalues. The sum of the squares of these eigenvalues is equal, up to constants, to the Einstein-Hilbert action on that space-time! That is, denoting the Dirac operator by $D_B$, and trace of its square by $\text{Tr}[L^2_p D^2_B]$, we have

$$\text{Tr}[L^2_p D^2_B] \sim \frac{1}{L^2_p} \int d^4x \sqrt{g} R \quad (1)$$

Next, if we make the algebra of coordinates non-commutative, we no longer have the original space-time manifold, but we still have the spectral description of its curvature, as on the left hand side of the above equation. It is hence assumed that $\text{Tr}[L^2_p D^2_B]$ describes curvature of the non-commutative geometry. This celebrated spectral action, as it is called in non-commutative geometry, points to a deep connection between the Dirac operator and gravitation, and plays a crucial role in our quantum theory of gravity.

The second relevant and extremely significant result from NCG is the existence of a fundamental time parameter, which is there only in the non-commutative case, and absent in ordinary commutative geometry. This is a consequence of the so-called Tomita-Takesaki theory, and the ‘co-cycle Radon-Nikodym’ theorem. For us it suffices to note that there are a one-parameter family of inner automorphisms of the non-commutative algebra, which map elements of the algebra to other elements of the algebra; this being equivalent to a time translation. As Connes puts it, ‘non-commutative measure spaces evolve in time’. We call this Connes time, and denote it by $\tau$. When ordinary space-time is lost because of non-commutativity, Connes time emerges, and helps us to formulate quantum theory without classical time.

Because the spectral action does not depend on the existence of a space-time manifold, (and yet links to classical gravitation), it has the right properties for inclusion in trace dynamics. But with a twist. For a physicist, for something to be an action, it should be the time integral of a Lagrangian. Here, Connes time comes to our rescue, noting also that the ‘spectral action’ $\text{Tr}[L^2_p D^2_B]$ is more in tune with what we would call a trace Lagrangian in
trace dynamics. Furthermore, a trace Lagrangian should be an operator polynomial made from a configuration variable and its time derivatives. This motivated us to define a bosonic configuration variable $q_B$ as follows: $D_B \equiv (1/Lc) dq_B / d\tau$, and hence a trace Lagrangian and a trace action:

$$S_B = \frac{1}{\tau_p} \int d\tau \; Tr \left[ \frac{L_P^2}{L^2 c^2} \left( \frac{dq_B}{d\tau} \right)^2 \right]$$

(2)

Here, $L$ is a length scale associated with $q_B$, and $q_B$ is related to gravitation through the eigenvalues of $D_B$. This is how we have used NCG to incorporate gravity into trace dynamics [10].

This Lagrangian also helps us arrive at a formulation of quantum theory without classical time. To progress in that direction we must now introduce matter (fermions) and relate matter to gravity in trace dynamics, analogous to the spirit of classical general relativity. Keeping in mind that this matter ought to be quantum in nature, it is perfectly reasonable to assume (since quantum systems are not localised in space) that we should no longer make a distinction between fermionic matter and the gravitation it produces. To this end we introduced the concept of an atom of space-time-matter (STM), denoted by operator $q$, which is split into its bosonic and fermionic parts as $q = q_B + q_F$, with $q_B$ defined as above, and $q_F$ the matter (fermionic) part. The constraint on $q_F$ is that it should be possible to identify it, upon the emergence of classical space-time, as the matter degree of freedom in quantum theory, and in general relativity. Thus the STM atom carries around its own (non-commutative) geometry. An STM atom is an elementary particle plus its own space-time geometry. If we ask what is the gravitational field of an electron, we would describe the electron and its gravity together as an STM atom. At the Planck scale, the universe is populated by enormously many STM atoms, each described by its own $q$-operator, whose dynamics is described in the Hilbert space via evolution in Connes time. The fundamental action principle for an STM atom is

$$\frac{L_P}{c} \frac{S}{C_0} = \frac{1}{2} \int d\tau \; Tr \left[ \frac{L_P^2}{L^2 c^2} (\dot{q}_B + \beta_1 \dot{q}_F) (\dot{q}_B + \beta_2 \dot{q}_F) \right]$$

(3)

Here $\beta_1$ and $\beta_2$ are two constant fermionic matrices whose properties remain to be determined. This action looks similar to the action for a free point particle in classical mechanics, except that now the configuration variable does not describe just matter, but also its grav-
It is interesting that the particle description (as opposed to the description via fields) comes back in full force in our matrix dynamics. This is understandable, because classical space-time is lost, and it would not be meaningful to talk of fields when physical three-space is not there.

The equations of motion and their solutions obtained from this action are highly instructive. These are:

\[
2\dot{q}_B + (\beta_1 + \beta_2)\dot{q}_F = c_1 \tag{4}
\]
\[
\dot{q}_B (\beta_1 + \beta_2) + \beta_1\dot{q}_F\beta_2 + \beta_2\dot{q}_F\beta_1 = c_2 \tag{5}
\]

where \(c_1\) and \(c_2\) are constant bosonic and fermionic matrices, respectively. These two equations are the (matrix dynamics) precursors of the Einstein-Dirac equation and the Schrödinger-Newton equation [matter tells space-time how to curve; space-time tells matter how to move].

There is one such action term for every STM atom. It is not as if all the STM atoms together produce gravitation of the universe; rather classical space-time emerges after the fermionic parts of many entangled atoms undergo spontaneous localisation. This way, the material bodies of the universe are formed, and formed concurrently with the emergence of space-time.

At this stage the reader can follow the detailed analysis of the matrix dynamics, and emergent spontaneous localisation, by going through [10]. The discussion there also explains how the unresolved issues in trace dynamics get answered. The following figure, borrowed from [11], summarises the theory.

Quantum gravity, in its most fundamental sense at Level 0, is a matrix dynamics of STM atoms. These interact with each other via entanglement. Hence entanglement is more fundamental than quantum theory, and it is first and foremost a property of STM atoms evolving in Hilbert space. This also makes it very clear why quantum entanglement is oblivious to space and time (quantum non-locality) - because entanglement originates from Level 0 and Level I, where there is no space-time. Quantum dynamics should strictly be described at Level 0 or Level I. Describing it at level II is an approximation; this can sometimes lead to puzzles - for instance the EPR paradox arises when we try to describe quantum non-locality at Level II. There is no EPR paradox at Levels 0 and I, because there
is no space-time there, so there is no question of a space-like separation. Space-time is emergent from Hilbert space, after spontaneous localisation takes place.

Any quantum theory of gravity must also explain why superpositions of space-time geometries are absent in the classical world. Moreover, the absence of position superpositions of macroscopic bodies is a pre-requisite for the existence of classical space-time geometry. In this way of thinking it becomes apparent that the solution of the quantum measurement problem must come from a quantum theory of gravity. Since our quantum gravity predicts
spontaneous localisation of fermions, we see that the process responsible for the emergence of space-time is the same as the one that solves the measurement problem.

In order to have a relativistic theory of spontaneous collapse, it is necessary to treat time at the same footing as space. This requires that just like the position operator, time should also be treated as an operator - then there is spontaneous collapse of time as well. The loss of coordinate time as a parameter is compensated by the appearance of Connes time as the new time parameter.

Many researchers have made the case that gravity is not a fundamental force, but an emergent thermodynamic phenomenon (Sakharov, Jacobson, Padmanabhan, Verlinde, amongst others). There are underlying atoms of space-time. Adler has made the case that quantum theory is an emergent phenomenon. We agree with both these cases and we have made the case that quantum gravity itself is an emergent phenomenon, coming from the matrix dynamics of STM atoms. Space-time and its geometry, as well as the phenomenon of gravitation, emerge after the spontaneous localisation of the fermionic part of STM atoms. The thermodynamic properties of black holes testify for the emergent nature of gravity; while the random nature of outcomes in a quantum measurement testifies for the thermodynamic nature of quantum theory. In fact, the same process, viz. spontaneous localisation, explains the origin of black hole entropy, and also the collapse of the wave function in a quantum measurement.

Our underlying matrix dynamics is a deterministic and time-reversible theory; it is even linear! The apparent irreversible nature of wave function collapse, as well as of black hole evaporation, arises only because we are examining a coarse-grained version of the matrix dynamics. It would have been hard to anticipate that the sought for quantum theory of gravity will turn out to the statistical thermodynamics corresponding to the microscopic dynamics of STM atoms. In hindsight though, it seems obvious that it should be so, because both gravity and quantum theory exhibit strong thermodynamic features. Quantum gravity is to the matrix dynamics of STM atoms same as the thermodynamic properties of a gas are to the mechanical motion of its constituent microscopic molecules.

Instead of developing this story in the top-down fashion as we have done in this article, one can now also describe it in a bottom-up fashion, by starting at the most basic Level 0. We start with the action principle for STM atoms and work out their Lagrangian dynamics. The statistical mechanics of these atoms gives rise to quantum gravity, and by spontaneous
collapse, to classical general relativity with matter sources. Quantum field theory is arrived at by borrowing quantum matter from quantum gravity, and classical space-time from Level III.

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