To the problem of determining the deformation characteristics of jointed rock mass

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Abstracts. The article considers the results of the study of a fragment of rock mass weakened by a system of parallel joints. Using numerical modelling, its deformation characteristics at different ratios of the characteristic size of the fragment to the characteristic size of its structural element (layer) are studying. The conditions for the representativeness of a fragment are discussed. The results of numerical calculations are compared with the results of the previously proposed analytical calculation method and the conditions for their convergence are discussed.

1. Introduction

When studying the interaction of structures with rock mass, one of the most important problems is to determine its the mechanical characteristics, in particular - deformation properties. The main factor contributing to this problem is the specific structure of rocks – their fracturing, anisotropy, and heterogeneity. An important role also plays the scale effect, which is manifested in the deterioration of the mechanical characteristics of the rocks with an increase in the volume of the rock mass interacting with the engineering structure. The scale effect is particularly evident in the design and construction of hydraulic engineering facilities in mountainous areas, in particular, high-pressure dams and underground hydroelectric power stations.

Experimental determination of the deformation characteristics of the rock mass in this case is impossible in laboratory conditions, but in situ is very time-consuming and requires large financial costs. Taking this into account, both direct and indirect research methods are used to solve these problems, in which the rock mass is considered as an equivalent, conditionally continuous isotropic medium.

Direct methods include in situ testing methods, both static and dynamic. Static methods include tests performed using plate loading test, jacking test, borehole jacking, pressure chamber, etc. The main disadvantage of such tests is that it is not possible to examine large volumes of rock mass. In addition, the results of these studies have a large spread due to the jointing, heterogeneity and anisotropy of rocks, as well as the features of various types of test equipment.

Dynamic methods include geophysical methods that allow the study of large volumes of rock masses. However, the lack of correlation between the results of dynamic and static test methods used in calculations limits the use of dynamic methods.

Indirect methods for determining the deformation characteristics of a rock mass can be divided into empirical and analytical methods. Empirical methods are based on rock mass classification indices, such as RQD [1], RMR [2], GSI [3] and Q [4], in which the properties of rock mass are reflected implicitly. Using these indices and mechanical characteristics of intact rocks, various authors have proposed empirical dependencies for determining the deformation characteristics of rock mass, for example [5], [6].
In analytical methods, in most cases, the rock mass is considered as an equivalent, conditionally continuous isotropic or anisotropic medium. The deformation modules of such a medium integrally take into account the deformation characteristics of its structural elements: joints, inhomogeneities, intact pieces of rock, etc. Methods of solution of the mechanics of solid bodies are used to determine the value of the deformation modulus. In a number of works, the deformation of the rock mass is studied using computational models that reflect the behaviour of both intact rock and joints, whose elastic characteristics are modelled by the values of normal and $k_n$ and tangential $k_s$ stiffness. The use of normal and tangent stiffness $k_n$ and $k_s$ as deformation characteristics of joints was first considered in the paper [7], [8]. Later, a similar approach was used by other researchers, for example [9] [10].

This approach is also used in the work [11] to solve the problem of determining the integral effective deformation characteristics of a medium weakened by a system of plane - parallel joints. The solution is obtained using the asymptotic averaging method [12] and the dependencies derived from this method [10]. It should be noted that the method of asymptotic averaging is the only method that has a strict mathematical basis for evaluating the properties of structurally inhomogeneous media, which include of rock masses.

2. Discussion
In the paper [11], an element of rock mass dissected by a system of parallel joints is considered. The joints can be either unfilled with the products of rock crushing, or with a filler (Figure 1).

![Figure 1. Calculated element of rock mass with a joint [11].](image)

The formulas for calculating its deformation characteristics are as follows:

$$
E_\perp = \frac{E_i E}{E_i + \alpha E} = \frac{\delta k_n E}{\delta k_n + \alpha E} = \frac{l k_n E}{l k_n + E}, \quad E_\parallel \approx E,
$$

$$
G_\perp = \frac{G_{12} G}{G_{12} + \alpha G} = \frac{\delta k_s G}{\delta k_s + \alpha G} = \frac{l k_s G}{l k_s + G},
$$

$$
G_\parallel \approx G = \frac{E}{2(1 + \nu)}, \quad \nu_{H, \perp} = \nu_{H, \parallel} = \nu.
$$

where $E$ is the modulus of intact rock, $E_\perp$ and $E_\parallel$ are effective elastic modules, respectively, in the directions orthogonal and parallel to the joint plane. Similarly, $G_\perp$, $G_\parallel$ and $\nu_{\perp, H}$, $\nu_{\parallel, H}$ effective shear modules and Poisson ratio along the same directions, $k_n$ and $k_s$ - normal and tangent stiffness of joints, $\delta$ and $\alpha$ - respectively, the absolute and relative width of their opening, $l$ - the distance between the
joints (the width of the layer of undisturbed rock). Given that a rock mass weakened by a system of plane-parallel joints can be represented by a medium consisting of elements, the model of the rock mass is reduced to an effective transversal-isotropic medium with an isotropy plane parallel to the joint plane, whose deformation characteristics are determined using dependencies (1).

Under the conditions of an elastic two-dimensional problem, two series of calculations were performed. In the first, the deformation characteristics of layered fragments weakened by a system of parallel joints were studied. The fragment deformation modules were determined both analytically using dependencies (1) and numerically using the finite element method (FEM) implemented on the basis of the Z-Soil software package. The fragment joints were modeled using a contact element first proposed and described in the paper [13].

The advantage of this element is the ability to calculate the relative displacements of the joint walls, which allows, knowing the values of $k_n$ and $k_s$, to determine the normal and tangential stresses in the joint. The fragments with a size of 1×1 m² were examined. In numerical calculations, the fragments were loaded with steps in the direction of the normal to the plane of the joints. All calculations were performed for cases in which the frequency of fragment joints ($\lambda$- the number of joints per 1 m) increased – $\lambda=7$; $\lambda=9$; $\lambda=15$; $\lambda=26$, which respectively amounted to - 8, 10, 16 and 27 layers of intact material (Figure 2a).

![Figure 2](image)

Figure 2. Calculated fragment of rock mass. Layered (a) and block (b) fragment.

In accordance with the proposed [14] dependency (Figure 3) the specified values of the frequency of joints correspond to the following values of the rock fragment quality designation - $RQD$: 95%; 85%; 75%; 50%; 25%.
The layered structure is quite typical for rock mass. However, in the practice of construction, rock mass with a block structure are no less common. Taking this into account, another series of numerical calculations was performed to study the deformation of block fragments weakened by two normally oriented joint systems (Figure 2b). The second system of plane-parallel joints, similar to the first, coincided with the direction of the distributed load applied to the fragment.

The calculation methods used allow us to calculate the modulus of deformation of a rock fragment that is extracted from the rock mass, and to determine the conditions for its compliance with the criterion of quasi-continuity and quasi-homogeneity [15]. Compliance with this criterion allows us to consider the studied fragment as representative and use methods of continuum mechanics to determine its deformation characteristics. The mechanical characteristics of the rock mass required for calculation are determined by in situ tests.

In the Russian Federation, when designing structures that interact with rocks, the determination of the modulus of elasticity of the rock mass $E_m$ and its rock quality designation $RQD$ is regulated and carried out according to appropriate methods. As for the deformation characteristics of joints in rocks $k_n$ and $k_s$, there are currently no recommendations for their determination. A number of works are devoted to this issue in the special literature. In particular, rock joints deformation was studied in detail in [16], [17], [18]. Tests were carried out both on joint samples in laboratory conditions and on construction sites of real objects. In the first case, joint samples of various rocks were studied in order to determine the normal and tangent stiffness of the joint separating them. The following factors were taken into account: aperture thickness, morphology, and the uniaxial compression strength of the joint wall material. In the second case, the values of $k_n$ and $k_s$ were determined by plate jacking tests.

The conclusions and recommendations obtained in these studies were taken into account when studying the deformation of the two types of fragments mentioned above. Tangent stiffness $k_s$ was not taken into account in the calculations, since preliminary studies have shown that in the considered cases it does not affect the deformation properties of the studied fragments. The results of analytical and numerical calculations are presented in tables 1 and 2.

**Figure 3.** Relationship between RQD and mean discontinuity frequency [14].
Table 1. The results of calculations of fragments with elastic modules of structural elements $E = 1000$ MPa.

| $E$, MPa | RQD, % | $\lambda$ Discontinuity frequency | Number of layers (blocks) | $K_n$, MPa/mm | $E_{\perp}$, MPa Numerical calculations | $E_{\perp}$, MPa Analytical calculation | $E_{\perp}$, MPa Numerical calculations Layered fragment. | $E_{\perp}$, MPa Numerical calculations Block fragment. | The difference between the results of analytical and numerical calculations, % |
|----------|--------|----------------------------------|--------------------------|---------------|-------------------------------------|-------------------------------------|--------------------------------------|----------------------------------------|--------------------------------------------------|
| 1000     | 85     | 0.14                             | 8                        | 49            | 860                                 | 924                                 | 925                                  | 7.5                                    | 75                                          |
|          | 75     | 0.11                             | 10                       | 9.7           | 492                                 | 525                                 | 526                                  | 7.0                                    | 50                                          |
|          | 50     | 0.07                             | 16                       | 3.3           | 171                                 | 182                                 | 182                                  | 6.4                                    | 25                                          |
|          | 25     | 0.038                            | 27                       | 1.7           | 59                                  | 61                                  | 61                                   | 3.4                                    | 25                                          |

Table 2. The results of calculations of fragments with elastic modules of structural elements $E = 100000$ MPa.

| $E$, MPa | RQD, % | $\lambda$ Discontinuity frequency | Number of layers (blocks) | $K_n$, MPa/mm | $E_{\perp}$, MPa Numerical calculations | $E_{\perp}$, MPa Analytical calculation formula. | $E_{\perp}$, MPa Numerical calculations Layered fragment. | $E_{\perp}$, MPa Numerical calculations Block fragment. | The difference between the results of analytical and numerical calculations, % |
|----------|--------|----------------------------------|--------------------------|---------------|-------------------------------------|--------------------------------------------------|--------------------------------------|----------------------------------------|--------------------------------------------------|
| 100 000  | 85     | 0.14                             | 8                        | 55.5          | 6487                                | 7375                               | 7380                                 | 13.8                                   | 75                                          |
|          | 75     | 0.11                             | 10                       | 50            | 4762                                | 5278                               | 5279                                 | 10.8                                   | 50                                          |
|          | 50     | 0.07                             | 16                       | 28            | 1720                                | 1835                               | 1835                                 | 6.7                                    | 25                                          |
|          | 25     | 0.038                            | 27                       | 17            | 626                                 | 650                                 | 650                                  | 3.8                                    | 25                                          |

The results shown in tables 1 and 2 are interesting, first of all, because they fully agree with the criterion of quasi-continuity and quasi-homogeneity of fractured rock masses - $n$. It was indicated above that the criterion determines the conditions under which a fragment of fractured rocks is deformed as a quasi-solid, quasi-homogeneous body. Implementation of the criterion, with an accuracy of $10\%$, accepted in engineering calculations, requires compliance with the condition $- B/b = n_{10\%}$, where $B$ is the size of a fragment’s side and $b$ - the size of its structural element ($B/b$ changes within $8 – 11$) [15].

At the same time, it is necessary to take into consideration that the concrete criterion value - $n_{10\%}$ can be determined only if the deformation characteristics of joints and intact rocks, that also strongly affect the criterion, are known [15]. The results of the research presented in tables 1 and 2 confirm this. Comparison of the results of analytical and numerical calculations of fragments weakened by one system of parallel joints showed that the difference in the values of the modules of deformation of fragments in the direction normal to joints - $E_{\perp}$ at different values of RQD varies within 3-14 % depending on the values of the modulus of elasticity of layers and normal stiffness of joints. Thus, when determining the modulus of deformation of fragments with the elastic modulus of the layer - $E = 1000$ MPa, the calculation accuracy of $10\%$ was provided for $RQD = 85\%$ and $k_n = 49$ MPa / mm,
which corresponds to eight layers. At the same time, with the elastic modulus of the layer - \( E = 100000 \) MPa, the required calculation accuracy of 10% was provided for \( RQD = 75\% \) and \( k_n = 50 \) MPa / m. For large values of \( RQD \) the accuracy of numerical calculations exceeded 10%.

It should also be noted that in numerical modeling the values of the deformation modules of the layered and block fragments do not differ much (Table 1, 2) which confirms the absence of the influence of a second system of joints parallel to the action of the load on the deformation of the fragment and it can be ignored in studies. The results of the calculations also confirmed the conclusion made earlier [11] that the value of the relative modulus of deformation of the fragment \( \frac{E_\perp}{E} \) increases with increasing crack stiffness \( k_n \) and decreases with increasing values of the modulus of elasticity of structural blocks \( E \).

Based on the results of the analytical and numerical calculations described above, curves were constructed for the dependence of the relative module \( \frac{E_\perp}{E} \) on \( RQD \) (Figure 4). The calculated curves were plotted on the graph of the curve proposed in the paper [1] and constructed from the experimental points of numerous in situ tests. This field of experimental points was supplemented with experimental points of the graph of the dependence of the relative modulus of deformation \( \frac{E_\perp}{E} \) on the geological strength index \( GSI \), given in the work [19]. The conversion of \( GSI \) values to \( RQD \) values was performed using the dependency proposed in [20].

![Figure 4. Relation between deformation modulus ratio \( \frac{E_\perp}{E} \) and \( RQD \)](image)

The work [17] shows that in the joints of various rocks, the values of \( k_n \) change approximately in the range of 1.7 – 55.5 MPa / m. At the same time, the relative modulus of deformation \( \frac{E_\perp}{E} \) depends on three parameters – \( RQD, E \) and \( k_n \). Based on this, the coordinates of calculated curves points for
different values of $RQD$ and values of the elastic modules $- E = 1000$ and 100 000 MPa, were determined as follows. For each pair of $RQD$ and $E$, using dependencies (1), such normal stiffness values were found, within the limits specified above ($k_n = 1.7 – 55.5$ MPa/mm), so that the calculated curves, as accurately as possible, limited the upper and lower bounds of the field of points obtained during in situ tests (Figure 4). Using this approach, it was found that within the boundaries of the $RQD$ values for which the fragment is representative (tables 1 and 2), both curves fairly accurately limit the field of experimental points, which indicates the possibility of using dependencies (1) for preliminary determination of deformation characteristics of rocks weakened by a system of plane-parallel joints or two joint systems, normally oriented to each other, if the direction of one of the systems coincides with the direction of the load acting on the fragment.

3. Conclusion
The conducted research allows us to draw the following conclusions:

1. The obtained results of analytical and numerical calculations of the deformation of fragments weakened by the system of parallel joints confirmed the possibility and validity of using dependencies (1) to determine the deformation characteristics of rocks containing joints. In this case, the calculated fragment of the rock mass must be representative, i.e. meet the criterion of quasi-continuity and quasi-homogeneity.
2. The results of the calculations also allowed determining, that the ratio of the characteristic size of the fragment $- B$ to the size of its structural element $- b$ is not the only condition that determines the representativeness of the fragment, for calculating the deformation characteristics of which we can use dependencies (1). A representative rock fragment must also correspond to the deformation characteristics of the joint systems that weaken it and the intact rock pieces that they form.
3. Numerical simulation has shown that the dependencies (1) can be used to determine the modulus of deformation of a block fragment weakened by two systems of joints, if the direction of one of the systems coincides with the direction of the load acting on the fragment.

References
[1] Zhang L and Einstein H H Using RQD to Estimate the Deformation Modulus of Rock Masses. Int. J. Rock Mech. & Min. Sci, 41, 2004, 337-341.
[2] Bieniawski Z T Determining Rock Mass Deformability: Experience from Case Histories. Int. J. Rock Mech. & Min. Sci, 15, 1978, 237-24.
[3] Hoek E, Brown E T Practical estimates of Rock Mass strength. Int. J. Rock Mech. & Min. Sci, 1997, 34 (8), pp. 1165 – 1186.
[4] Barton N Some new Q-value correlations to assist in site characterization and tunnel design. Int. J. Rock Mech. & Min. Sci. nr. 39, 2002, pp. 185-216.
[5] Beniawski Z.T. Determination of rock mass deformability. Int. J. Rock Mech. & Min. Sci, Geomech. Abstr. 1978; 15.
[6] Sonmez H, Gokceoglu C and Ulusay R. Indirect determination of the modulus of deformation of rock masses based on the GSI system. Int. J. Rock Mech. & Min. Sci, 41, 2004, pp. 849-857.
[7] Duncan J and Goodman R Finite Element Analyses of Slopes in Jointed Rock, US Army Corp of Engineers, 1968, Rep. S63/3.
[8] Goodman R E Deformability of joints, Proceedings of the Symp. on determination in-situ modulus of deformation of rock, Denver, 1970, pp. 174-196.
[9] Yoshinaka R and Yamabe T Joint stiffness and the deformation behaviour of discontinuous rock. Int. J. Rock Mech. & Min. Sci, 23, No. 1, 1986, 19-28.
[10] Vlasov A and Merzlyakov V Averaging of deformation and strength properties in rock mechanics, M. Izd. DIA 2009, p. 208 (in Russian)
[11] Vlasov A, Zertsalov M and Vlasov D. Influence of normal and shear stiffness of fractures on deformation characteristics of rock mass. Geotechnics Fundamentals and Applications in Construction, 2019, 413-419. London: CRC Press.
[12] Bakhvalov N and Panasenko G Averaging of processes in periodic systems, Pb. H."Nauka” 1984, pp. 352 (in Russian).

[13] Goodman R, Taylor R and Brekke T., “A model for the mechanics of jointed rock,” J. Soil Mech. and Found., Engrg.Div, ASCE, vol. 99, 1968, pp. 637-660.

[14] Priest S and Hudson J. Int. J. Rock Mech. & Min. Sci & Geomechanics Abstracts, 13 (5),1976, pp. 135-148

[15] Ukhov S Rocky foundations of hydraulic structures. Pb. H. "Energy", 1975, pp. 34-41 (in Russian).

[16] Bandis S C Experimental studies of scale effects on shear strength and deformation of rock joints, Ph.D. thesis, Univ. of Leeds, 1980, pp. 385.

[17] Bandis S, Lumsden A and Barton N Fundamentals of Rock Joint Deformation. Int. J. Rock Mech. & Min. Sci & Geomechanics Abstracts, Vol. 20, No. 6, 1983, pp. 249-268.

[18] Rechitsky V Assessment of the stiffness characteristics of rock cracks according to field studies at hydraulic facilities, Hydrotechnical Construction, No. 8, M, Pb. H. “Energopress”, 1998, pp. 44 – 49 (in Russian).

[19] Hoek E. and Dederichs M. S. Empirical Estimation of Rock Mass Modulus. Int. J. Rock Mech. & Min. Sci. 43, 2006, pp. 203 – 215.

[20] Truzman M Corley D., Lipka D. Determination of Unit Tip Resistance for Drilled Shafts in Fractured Rocks using the Global Rock Mass Strength. Pan-Am CGS Geotechnical Conf., 2011.