Thermodynamic and Mechanical Analysis of a Thermomagnetic Rotary Engine

D M Fajar\textsuperscript{1,3,a}, S N Khotimah\textsuperscript{2,b}, and Khairurrijal\textsuperscript{2,c}

\textsuperscript{1} Master Program in Physics Teaching, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Jalan Ganesa 10, Bandung 40132, Indonesia
\textsuperscript{2} Department of Physics, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Jalan Ganesa 10, Bandung 40132, Indonesia
\textsuperscript{3} Natural Science Education Program, Faculty of Tarbiyah and Teacher Training, Institut Agama Islam Negeri Jember, Jalan Mataram 1, Mangli Jember 68136, Indonesia

\textsuperscript{a} dinarmaftukh@students.itb.ac.id; \textsuperscript{b} nurul@fi.itb.ac.id; \textsuperscript{c} krijal@fi.itb.ac.id

Abstract. A heat engine in magnetic system had three thermodynamic coordinates: magnetic intensity $\mathcal{H}$, total magnetization $\mathcal{M}$, and temperature $T$, where the first two of them are respectively analogous to that of gaseous system: pressure $P$ and volume $V$. Consequently, Carnot cycle that constitutes the principle of a heat engine in gaseous system is also valid on that in magnetic system. A thermomagnetic rotary engine is one model of it that was designed in the form of a ferromagnetic wheel that can rotates because of magnetization change at Curie temperature. The study is aimed to describe the thermodynamic and mechanical analysis of a thermomagnetic rotary engine and calculate the efficiencies. In thermodynamic view, the ideal processes are isothermal demagnetization, adiabatic demagnetization, isothermal magnetization, and adiabatic magnetization. The values of thermodynamic efficiency depend on temperature difference between hot and cold reservoir. In mechanical view, a rotational work is determined through calculation of moment of inertia and average angular speed. The value of mechanical efficiency is calculated from ratio between rotational work and heat received by system. The study also obtains exergetic efficiency that states the performance quality of the engine.

1. Introduction

A thermomagnetic rotary engine constitutes one form of heat engines in magnetic system. The model is thermodynamically analogous to heat engines in gaseous system. In general a heat engine in gaseous system is created in the form of a movable piston containing gas that could expand and compress continuously with repeated isothermal and adiabatic processes. One common theory used to analyze the engine is Carnot principle by assuming the system in an idealized condition. Thermodynamic coordinates that characterize the engine are pressure ($P$), volume ($V$), and temperature ($T$). For a heat engine in magnetic system these quantities are respectively analogous to magnetic intensity ($\mathcal{H}$), total magnetization ($\mathcal{M}$), and temperature ($T$) \cite{1}. By employing influence of temperature on changing in the two coordinates, both heat engines are created.

A thermomagnetic rotary engine is composed of a ferromagnetic wheel, a permanent magnet, and a heat source. It works based on a typical property of ferromagnetic materials, that is the total
magnetization drops with temperature increase, and experiences phase transition into paramagnetic when being heated until Curie temperature ($T_c$).

A thermomagnetic rotary engine presents a suitable model to demonstrate a Carnot engine besides gaseous system [2]. It also shows one model of energy conversion from heat energy to magnetic work thermodynamically and rotational work mechanically. The study is aimed to discuss thermodynamic analysis of a thermomagnetic rotary engine as Carnot cycle in gaseous system; and mechanical analysis regarding to rotational work produced by the engine. Efficiency was calculated by ratio between work and heat received by system associated with the engine performance.

2. Theory

For a heat engine in magnetic system, as explained above, the total magnetization drops with temperature increase as equations below [3].

\[
\mathcal{M} = C_c \frac{\mathcal{H}}{T - T_c}
\]

(1)

\[
\mathcal{M} = C_c \frac{\mathcal{H}}{T}
\]

(2)

Equation (1) expresses mathematical formulation of Curie-Weiss Law that explains equilibrium state of paramagnetic phase for ferromagnetic material above Curie temperature, with $T > T_c$ and $C_c$ is Curie constant. Equation (1) is derived from Curie Law that explains equilibrium state of paramagnetic material. Equation (2) expresses mathematical formulation of Curie Law. The latter is more relevant to explain Carnot cycle for magnetic system as gaseous system than equation (1). Furthermore, it covers value of temperature of hot and cold reservoir below Curie temperature.

The first law of thermodynamics that connects heat, internal energy, and magnetic work is written as follows

\[
dQ = dU - \mu_n \mathcal{H} d\mathcal{M}
\]

(3)

where $\mu_n \mathcal{H} d\mathcal{M}$ expresses a work in magnetic system. Substituting equation (2) to (3), it gets two fundamental equations as follows [1,3].

\[
dQ = C_M dT - \mu_n \mathcal{H} d\mathcal{M}
\]

(4)

or

\[
dQ = C_M dT - \mu_n \mathcal{M} d\mathcal{H}
\]

(5)

Where $C_M = \frac{dQ}{dT}$, and $C_n = \frac{dQ}{dT}$, expresses heat capacity at constant total magnetization and magnetic intensity respectively. Equation (4) and (5) formulate thermodynamic processes in the heat engine, such as isothermal and adiabatic process. The Carnot principle of a thermomagnetic rotary engine employs both processes.

In mechanical view, the mechanism of a thermomagnetic rotary engine is explained by figure 1 below.
Figure 1. Mechanism of a thermomagnetic rotary engine.

Figure 1 depicts arrangement of a ferromagnetic wheel, a permanent magnet, and a burner as heat source positioned below the wheel. Before the burner is ignited up, the resultant of magnetic attractive forces equals zero \((F_1+F_2=F_3+F_4)\) so that the wheel does not move. Point 1 located in front of magnet is then heated. When the temperature is raised up, the magnetization of point 1 weakens so that decreases. When it reaches Curie temperature, point 1 has changed into paramagnetic. Since \((F_2>F_3+F_4)\), point 2 will shift and replace position of point 1 caused by torque created by the non-zero resultant. When point 2 is heated likewise, point 3 will shift point 2, and so on. The heating and shift occur continuously so that the wheel can rotate counter clockwise.

3. Analysis

A thermomagnetic rotary engine works by converting heat received by system to change the magnetization thermodynamically and then produce wheel rotation mechanically. It is illustrated by diagram below.

Figure 2. The principle of energy conversion of a thermomagnetic rotary engine

Based one diagram above, there are three efficiencies obtained by the analysis: thermodynamic efficiency \((\eta_{\text{term}})\), mechanical efficiency \((\eta_{\text{mech}})\), and exergetic efficiency \((\eta_{\text{ex}})\).

Exergetic efficiency \((\eta_{\text{ex}})\) states how much maximum work obtained from an engine associated with environmental equilibrium \(^4\). This term is commonly discussed in engineering view to analyze performance of the engine commercially. The formulation of it is substantively complex since it engages some external quantities. However, the study refers to Karle (2001) stating that exergetic efficiency is simply calculated by ratio between mechanical and thermodynamic efficiency \([5]\).
\[ \eta_{ex} = \frac{\eta_{mek}}{\eta_{term}}. \]  

(6)

3.1 Thermodynamic Analysis

This study is limited to focus on obtaining the first two efficiencies in thermodynamic view, a thermomagnetic rotary engine has three basic components: hot reservoir \( Q_p \) (with temperature \( T_c \)), cold reservoir \( Q_d \) (with temperature \( T_d \)), and thermodynamic work \( W_{\text{term}} \). The system also experiences four processes consist of 2 isothermal and 2 adiabatic processes as Carnot cycle in gaseous system.

Suppose initially the system is in thermodynamic equilibrium in cold reservoir with temperature of \( T_d \). The four processes occur are listed below [3].

1. Adiabatic magnetization, when the temperature is raised to \( T_c \).
2. Isothermal demagnetization, when the constant temperature \( T_c \) is reached and the system absorbs heat from hot reservoir \( Q_p \).
3. Adiabatic demagnetization in opposite direction to process no. 1, when the temperature drops to \( T_d \).
4. Isothermal magnetization in opposite direction to process no. 2, when the constant temperature \( T_d \) is reached and the system emits heat to cold reservoir \( Q_d \).

The Carnot cycle graph is plotted by finding intersection between isothermal and adiabatic curves. For isothermal curve, by assuming \( T \) in equation (2) as a constant, then it obtains equation below.

\[ \mathcal{H} = a \mathcal{M} \]  

(7)

where \( a = T/C_c \) = constant and \( a > 0 \). For adiabatic curve, by rewriting Eqs. 4 and 5 and using \( dQ = 0 \), it has two equations:

\[ C_M dT = \mu_0 \mathcal{H} d\mathcal{M} \]
\[ C_p dT = \mu_0 \mathcal{M} d\mathcal{H} \]

By dividing both equations, integrating \( d\mathcal{H} \) in left side and \( d\mathcal{M} \) in right side, it obtains

\[ \mathcal{H} = b \mathcal{M}^\gamma \]  

(8)

where \( \gamma = C_p / C_M \) and \( b \) is constant.

Since \( \gamma > 1 \), the adiabatic curve forms steep arch, while the isothermal curve forms straight line. Compared to Carnot cycle of gaseous system plotted tilt from upper left to lower right, the Carnot cycle of magnetic system is plotted tilt from lower left to upper right. It is because the coordinate \( \mathcal{M} \), which is parallel with \( V \) as generalized displacement, is inversely proportional to \( T \). While \( V \) in gaseous state is proportional to \( T \).

Figure 3 compares Carnot cycle for gaseous system (figure 3.(a)) and magnetic system in a thermomagnetic rotary engine (figure 3.(b)).
Figure 3. Carnot cycle for: (a) gaseous system, and (b) magnetic system.

Figure 4. Thermodynamic processes in a thermomagnetic rotary engine.

Figure 4 depicts thermodynamic process occurs in the engine. The alphabets in figure 4 are placed from interpretation of Carnot cycle plot in figure 3.(b). The placement cannot be specifically determined since the process takes place fast and continuous. The work each process in figure 3.(b) is derived from equation (2), (3), (4) and (5).

Process A-B (adiabatic magnetization)

$$W_{AB} = C_M (T_e - T_d)$$  \hspace{1cm} (9)

Process B-C (isothermal demagnetization in temperature $T_c$)

$$W_{BC} = \frac{\mu^2}{2C_e} (m_c^2 - m_b^2)$$  \hspace{1cm} (10)

Process C-D (adiabatic demagnetization)

$$W_{CD} = C_M (T_d - T_e)$$  \hspace{1cm} (11)

Process D-A (isothermal magnetization in temperature $T_d$)

$$W_{DA} = \frac{\mu^2}{2C_d} (m_a^2 - m_b^2)$$  \hspace{1cm} (12)

$W_{\text{total}}$ expresses total work in all thermodynamic process. Since two works in adiabatic process cancel each other, it only uses two works in isothermal process. The effective works in a cycle obtains
The heat received by system \((Q_p)\) is from isothermal magnetization process. Since \(dQ = -dW\) then it obtains
\[
Q_p = -\frac{\mu_s T_c}{2C_r} \left( M_s^z - M_r^z \right)
\] (14)
The efficiency is obtained from ration between work \(W_{term}\) and heat absorbed from hot reservoir \(Q_p\).
\[
\eta_{term} = \frac{W_{term}}{Q_p}
\] (15)
It also gets \(M_s^z - M_r^z = (M_c^z - M_b^z)\) from other calculations, not derived herein, then equation (15) is formulated as common Carnot efficiency as follows [5].
\[
\eta_{term} = 1 - \frac{T_c}{T_a}
\] (16)

Based on equation (16), the thermodynamic efficiency only depends on the value and the difference between hot and cold reservoir temperature. The larger the difference, the larger the Carnot efficiency.

3.2 Mechanical Analysis
In mechanical view, the work that can be observed in the thermomagnetic rotary engine is a rotational work of wheel as consequence of attractive magnetic force. The components necessary to find are rotational work and heat from burner, then find mechanical efficiency.

The rotational work is determined through Work-Energy Principle by finding rotational kinetic energy \(E_{krot}\).
\[
W_{mek} = \Delta E_{krot} = \frac{1}{2} I \omega^2
\] (17)
Moment of inertia \(I\) is obtained from summing moment of inertia of all wheel components, such as axis, trellis, and circular ferromagnetic wire. It can also be directly determined from experiment.

The angular speed \(\omega\) is obtained from average number of rotation during a time interval. It is also influenced by the power of burner used until certain limit that depends on Curie temperature of material. Above the limit, the power of burner does not affect significantly [6]. So, it is more effective to use the largest power.

Compared to other method, Karle (2001) find mechanical efficiency by calculating power \(P_{mek}\) generated immediately when the burner ignited up. The method is conducted when the time needed for wheel starts to move since the burner signed up is relatively long. Moreover, the rotation is predicted not smooth [5].
\[
P_{mek} = F \cdot v
\] (18)
The initial force \(F\) is determined by a finite difference method by reference of starting time and temperature difference between initial and Curie temperature of material. The initial speed \(v\) is obtained from measurement.

The heat from burner cannot be directly determined but it needs additional experiments. One is by calorimetric method that is aimed to determine power of burner \(P_{burn}\). It is done by heating a vessel contains water and noting the change of heat with time in a time interval, then it finds the power of burner.
\[
P_{burn} = \left( m_v c_v + m_g c_g \right) \frac{\Delta T}{\Delta t}
\] (19)
where \( m_c c_a \) and \( m_p c_m \) express mass times specific heat of water and the vessel respectively. Then \( P_{\text{burn}} \) times period of rotation finds heat of burner \((Q_{\text{burn}})\).

The heat from burner can also be determined by using enthalpy of standard combustion data of burner material by measuring difference of final and initial mass of burner in a period. The number of mole emitted expresses the value of \( Q_{\text{burn}} \) by assuming all emission is completely burned [7].

After all, the mechanical efficiency is formulated as follows.

\[
\eta_{\text{mek}} = \frac{W_{\text{mek}}}{Q_{\text{burn}}} \tag{20}
\]

or similarly [5],

\[
\eta_{\text{mek}} = \frac{P_{\text{mek}}}{P_{\text{burn}}} \tag{21}
\]

4. Conclusion

A thermomagnetic rotary engine is a suitable model to demonstrate a Carnot engine in other systems beside gaseous system. It also presents a model of energy conversion from heat to change of magnetization thermodynamically and produce rotational work mechanically. The analysis finds three kinds of efficiency: thermodynamic efficiency \((\eta_{\text{term}})\), mechanical efficiency \((\eta_{\text{mek}})\), and exergetic efficiency \((\eta_{\text{ex}})\).

The thermodynamic efficiency only depends on temperature difference between hot and cold reservoir. While mechanical efficiency depends on Curie temperature of material, components of wheel, and power of burners. It was calculated from ratio between rotational work and heat of burner. From thermodynamic and mechanical efficiency, it also obtained exergetic efficiency that stated the performance quality of the engine. It was obtained from ratio between mechanical and thermodynamic efficiency.

The study promotes some father experiments to find practical aspects that affect the engine’s performance. It is also intended to find various applications of heat energy conversion to broad aspects.

References

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