ELECTRON-POSITRON-PHOTON PLASMA AROUND A COLLAPSING STAR

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We describe electron-positron pairs creation around an electrically charged star core collapsing to an electromagnetic black hole (EMBH), as well as pairs annihilation into photons. We use the kinetic Vlasov equation formalism for the pairs and photons and show that a regime of plasma oscillations is established around the core. As a byproduct of our analysis we can provide an estimate for the thermalization time scale.

1. Dynamics of Dyadosphere

Dyadosphere was first introduced in Ref. 1 as the region surrounding an electromagnetic black hole (EMBH) in which the electromagnetic field strength exceeds the critical value $E_c$ for electron-positron pair creation via the mechanism à la Heisenberg-Euler-Schwinger.\textsuperscript{2,3} The relevance of the dyadosphere around an EMBH, for the astrophysics of gamma-ray busts has been discussed in Refs. 1, 4–6 (the external radius of dyadosphere will be denoted by $r_{ds}$). In those papers the pair production in dyadosphere has been described as an electrostatic problem: instantaneously a massive body collapses to an EMBH whose charge is large enough that the electric field strength $E$ exceeds $E_c$ and the Schwinger process is triggered in the entire dyadosphere; moreover the pairs are produced at rest and remain at rest during the whole history of their production; finally they instantaneously thermalize to a plasma configuration (see Fig. 1). These ansatz, formulated for the sake of simplicity, allow one to estimate the number density of pairs produced as well as the energy density deposited on the pairs in a straightforward manner. We relax the hypothesis that the large electric field is instantaneously built up and take the following dynamical point of view:

1. A spherically symmetric star core endowed with electric, say positive, charge $Q$, collapses. We assume that the electromagnetic field strength $E$ on the surface of the core is amplified to $E_c$ during the collapse and the Schwinger process begins.
(2) The pairs produced by the vacuum polarization progressively screen the electromagnetic field of the core, thus reducing its strength. Furthermore the charges (electrons and positrons) are accelerated by the Lorentz force in the electromagnetic field. Finally particles and antiparticles annihilate into photons.

An enormous amount of pairs \( N \sim \frac{Q e r}{\lambda_C} \), where \( \lambda_C \) is the Compton length of the electron) is produced, as claimed in Refs. 1, 4–6, if the core charge is not annihilated by the charge of the accelerated electrons during the gravitational collapse (see Ref. 7). Therefore it is useful to study the...
dynamics of the electron-positron-photon plasma in the electric field of the core in some details. This will be the main object of the next section. As a byproduct of the analysis we obtain an estimate for the time scale needed for the thermalization of the system.

In Ref. 8 it was suggested that the exact solution of Einstein-Maxwell equations describing the gravitational collapse of a thin charged shell can be used as a simplified analytical model for the gravitational collapse of a charged core; it was also discussed in some details the amplification of electromagnetic field strength on the surface of the core. Here we briefly review some of the results of Ref. 8. The region of space-time external to the core is Reissner-Nordström with line element
\[
\begin{align*}
  ds^2 &= -f dt^2 + f^{-1} dr^2 + r^2 d\Omega^2
\end{align*}
\]
in Schwarzschild-like coordinate \((t, r, \theta, \phi)\), where
\[
  f = f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2};
\]
\(M\) is the total energy of the core as measured at infinity and \(Q\) is its total charge. Let us label with \(R\) and \(T\) the radial and time-like coordinate of the shell, then the equation of motion of the core is (cfr. Ref. 8)
\[
\frac{dR}{dT} = -\frac{f(R)}{H(R)} \sqrt{H^2(R) - f(R)}
\]
where \(H(R) = \frac{M}{M_0} - \frac{M^2 + Q^2}{2M_0 r}\); \(M_0\) is the rest mass of the shell. The analytical solution of Eq. (2) was found in Ref. 8 in the form \(T = T(R)\). According to a static observer \(O\) placed at the event \(x_0 = (R, T(R), \theta_0, \phi_0)\) the core collapses with speed given by
\[
V^* = -\frac{dR^*}{dT^*} = \sqrt{1 - \frac{f(R)}{H^2(R)}} \leq 1
\]
where \(dR^* = f^{-1/2} dR\) and \(dT^* = f^{1/2} dT\) are spatial and temporal proper distances as measured by \(O\). In Fig. 2 we plot \(V^*\) as a function of \(R\) for a core with \(M = M_0 = 20M_0\) and \(\xi = \frac{Q}{M} = 10^{-3}, 10^{-2}, 10^{-1}\). Recall that dyadosphere radius is given by
\[
r_{ds} = \sqrt{\frac{\epsilon Q h}{m_e c^5}},
\]
where \(c\) is the speed of light; \(e\) and \(m_e\) are electron charge and mass respectively. Then note that \(V_{ds}^* \equiv V^*|_{R=r_{ds}} \simeq 0.2 c\) for \(\xi = 0.1\).

\(^{a}\)The condition \(M_0 = M\) corresponds to a shell starting its collapse at rest at infinity.
Figure 2. Collapse velocity of a charged star of mass $M_0 = M = 20M_\odot$ as measured by static observers as a function of the radial coordinate of the star surface. As the charge is not too large ($\xi \lesssim 0.1$) there is not much difference between collapse velocities of stars with different charge. Dyadosphere radii for different charge to mass ratios ($\xi = 10^{-3}, 10^{-2}, 10^{-1}$) are indicated in the plot together with the corresponding velocity.

2. Plasma oscillations and screening

We now turn to the pair creation taking place during the gravitational collapse. The gravitational fields of the core is considered classical; the gravitational effects of the electron-positrons-photons plasma are neglected.

The most detailed framework for studying electromagnetic vacuum polarization and particle-antiparticle scattering around an electromagnetic collapsing core is quantum electrodynamics in the classical external electromagnetic field of the core on the Reissner-Nordström space-time around the core itself. Of course a number of approximations is needed in order to make the problem be tractable. Let us discuss such approximations.

(Homogeneity) First of all, the static Reissner-Nordström space-time region external to the collapsing core is naturally splitted in space (hypersurfaces orthogonal to the static Killing field) and time. In the local frames associated with static observers, the electromagnetic field of the core is purely electric. Moreover, we will see that the length scale $L$ over which the electric field as well as the particle number densities vary, is much larger than the length scale $l$ which is...
characteristic of the electron–positron motion. Thus we can divide
dyadosphere into small regions $D_i$

$$D_i : r_i \leq r \leq r_{i+1} = r_i + \varepsilon;$$

$$r_+ \leq r_i \leq r_{ds} \quad \varepsilon \lesssim \ell; \quad (5)$$
such that for any $i$ the system formed by the electric field and the pairs can be considered homogeneous in $D_i$.

(Flat space-time) For, in geometric units, the electron charge $e$ is much larger than the electron mass $m_e$, the gravitational acceleration is negligible with respect to electric acceleration for sufficiently large electric field strengths (even much less than $E_c$), therefore we will neglect the curvature of space-time and use the local frame of a static observer as a globally inertial frame of the Minkowski space-time.

(Mean field) The number of pairs is so high that a semiclassical formalism and mean field approach can be used, in which the total electromagnetic field (core electromagnetic field and screen field due to pairs) is considered to be classical, while the electron-positron field is quantized. It has been shown that, if we neglect scattering between particles, the semiclassical evolution of the homogeneous system in a flat space-time is well described by a Boltzmann-Vlasov-Maxwell system of partial differential equations, where the electrons and positrons are described by a distribution function $f_e = f_e(t, p)$ in the phase space, where $t$ is the inertial time and $p$ the 3–momentum of electrons. Finally we use the method presented in Ref. 13 to simplify such a Boltzmann-Vlasov-Maxwell system.

Let us summarize results in Ref. 13: we obtained the following system of ordinary differential equations which simultaneously describes the creation and evolution of electron-positron pairs in a strong electric field as well as the annihilation of pairs into photons:

$$\begin{align*}
\frac{d}{dt} n_e &= S(E) - 2n_e^2 \sigma_1 \rho_e^{-1} \| \pi_e \| + 2n_e^2 \sigma_2 \\
\frac{d}{dt} n_\gamma &= 4n_e^2 \sigma_1 \rho_e^{-1} \| \pi_e \| - 4n_e^2 \sigma_2 \\
\frac{d}{dt} \rho_e &= e n_e E \rho_e^{-1} \| \pi_e \| + \frac{1}{2} e J_p(E) - 2n_e \rho_e \sigma_1 \rho_e^{-1} \| \pi_e \| + 2n_\gamma \rho_\gamma \sigma_2 \\
\frac{d}{dt} \rho_\gamma &= 4n_e \rho_e \sigma_1 \rho_e^{-1} \| \pi_e \| - 4n_e \rho_\gamma \sigma_2 \\
\frac{d}{dt} \pi_e &= e n_e E - 2n_e \pi_e \sigma_1 \rho_e^{-1} \| \pi_e \| \\
\frac{d}{dt} E &= -2en_e \rho_e^{-1} \| \pi_e \| - J_p(E)
\end{align*} \quad (6)$$

where $n_e$ ($n_\gamma$) is the electron (photon) number-density, $\rho_e$ ($\rho_\gamma$) is the electron (photon) energy-density, $\pi_e$ is the density of electron radial momentum and $E$ the electric field strength. Finally $S(E)$ is the Schwinger prob-
ability rate of pair creation, $j_p (E)$ is the polarization current-density, $\sigma_{1,2}$ are total cross sections for the processes $e^+ e^- \leftrightarrow \gamma \gamma$ and the corresponding terms describe probability rates of pair annihilation into photons and vice versa. System (6) was numerically integrated in Ref. 13.

Here it has to be integrated once for each of the regions $D_i$ (see (5)) with initial conditions

$$n_e = n_\gamma = \rho_e = \rho_\gamma = \pi_{e||} = 0; \quad E_0 = \frac{Q}{\tau_C^2}. \quad (7)$$

Let us recall the main results of the numerical integration. The system undergoes plasma oscillations:

1. the electric field oscillates with lower and lower amplitude around 0;
2. electrons and positrons oscillates back and forth in the radial direction with ultrarelativistic velocity;
3. the oscillating charges are trapped in a thin shell whose radial dimension is given by the elongation $\Delta l = |l - l_0|$ of the oscillations, where $l_0$ is the radial coordinate of the centre of oscillation and

$$l = \int_0^t \frac{\pi_{e||}}{\rho_e} dt. \quad (8)$$

Note that $\frac{\pi_{e||}}{\rho_e} \equiv v$ is the radial mean velocity of charges (we plot the elongation $\Delta l$ as a function of time in Fig. 3);
4. the lifetime $\Delta t$ of the oscillation is of the order of $10^2 - 10^4 \tau_C$ (see Fig. 3).
5. in the time $\Delta t$ the system thermalizes in the sense that both number and energy equipartition between electron–positron pairs and photon are approached.

In Fig. 4 we plot electrons mean velocity $v$ as a function of the elongation during the first half period of oscillation, which shows precisely the oscillatory behaviour.

3. Conclusions

In a paper under preparation\(^7\) we are examining the conditions under which the charge of the collapsing core is not annihilated due to vacuum polarization as a consequence of the above plasma oscillations.

Note that $e^+ e^- \leftrightarrow \gamma \gamma$ scatterings is marginal at early times ($t \ll \Delta t$) since the cross sections $\sigma_{1,2}$ are negligible in the beginning of pair production.\(^{13}\) However at late times ($t \gtrsim \Delta t$) the system is expected to
Figure 3. Electrons elongation as function of time in the case $r = \frac{1}{3}r_{ds}$. The oscillations are damped in a time of the order of $10^3\tau_C$.

relax to a plasma configuration of thermal equilibrium.\textsuperscript{13} Thus a regime of thermalized electrons-positrons-photons plasma begins in which the system can be described by hydrodynamic equations. It is shown in Refs. 12, 14 that the equations of hydrodynamic imply the expansion of the system. In “brief” the system reaches the ultrarelativistic velocities required in a realistic model for GRBs. It is worthy to remark that the time scale of hydrodynamic evolution ($t \sim 0.1s$) is, in any case, much larger than the time scale $\Delta t$ needed for thermalization.

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Figure 4. Electrons mean velocity as a function of the elongation during the first half oscillation. The plot summarize the oscillatory behaviour: as the electrons move, the mean velocity grows up from 0 to the speed of light and then falls down at 0 again.

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