Investigating and improving introductory physics students’ understanding of electric flux

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Abstract
A solid grasp of the concept of electric flux is an important pre-requisite for appropriate use of Gauss’s law in solving electrostatics problems. As part of a broader investigation focusing on improving understanding of electrostatics concepts, we investigated the conceptual difficulties of college students in a traditionally taught calculus-based introductory physics course with the concept of electric flux and then the research on student difficulties was used as a guide in the development and evaluation of a research-validated tutorial which strives to help students learn this concept better. During the investigation of difficulties and the design and validation of the guided inquiry-based tutorial, college students in a calculus-based introductory physics course were given written questions to probe the common conceptual difficulties with the electric flux related concepts, and we also interviewed a subset of those students to get an in-depth account of the reasons behind the conceptual difficulties. The guided inquiry-based learning sequences in the tutorial were also iterated several times with instructors who regularly teach these courses. Here we discuss the common student difficulties with the electric flux found in our investigations, and the development and validation of a tutorial that strives to improve student understanding. We analyse how students performed on the pre-test (administered before the electric flux tutorial but after traditional instruction in the electric flux concepts) and on the post-test (administered after students in the tutorial group had engaged with the electric flux related tutorial). The performance of students in all sections of the course was
comparable on the pre-test regardless of who taught that section. However, on the post-test, the performance of those in the sections of the course in which students engaged with the tutorial is significantly better than the section in which the tutorial was not used.

Keywords: physics education research, electric flux, tutorial, student difficulties

Introduction

This paper is the third in a series of papers focusing on improving introductory students’ understanding of electrostatics concepts [1]. One of the previous papers in this series focused on electric field and superposition principle and the other on symmetry and Gauss’s law. This paper is a bridge between them and focuses on investigating and improving introductory students’ understanding of the electric flux. The electric field and electric flux are two related but very different concepts. The net electric flux through a surface is given by the surface integral \( \Phi_E = \oint E \cdot dA \), where \( dA \) is the infinitesimal area vector and \( E \) is the electric field. The electric field is a vector quantity with units N C\(^{-1}\) while the electric flux is a scalar quantity with units Nm\(^2\) C\(^{-1}\). Despite these fundamental differences, distinguishing between electric field and flux is often challenging for high school and college students in the introductory physics courses both due to the abstractness of the physics concepts and the difficulty in the interpretation of the mathematics in this context. However, developing a solid grasp of electric flux is a pre-requisite, e.g. for being able to apply Gauss’s law appropriately to calculate the magnitude of the electric field from the information about the electric flux through a closed surface in situations in which the charge distribution has a high level of symmetry.

Investigating student common conceptual challenges with electric flux is critical for developing curricula and pedagogies to help students learn related concepts and to improve their reasoning and meta-cognitive skills [2–9]. Many investigations have focused on introductory physics students’ difficulties with electricity and magnetism and approaches to improve student learning of these concepts [10–35]. However, we are not aware of any paper that focuses exclusively on introductory students’ understanding of the electric flux except those focusing on Gauss’s law overall (in which case electric flux issues are a pre-requisite and the focus usually is on exploiting the symmetry of the charge distribution to find the electric field using Gauss’s law when possible) [17, 33, 34]. This study specifically focuses on the conceptual challenges of students with electric flux related concepts in traditionally taught calculus-based college introductory physics and then using the findings from the investigation to develop research-based tutorial that strives to help them cultivate a good grasp of electric flux related concepts.

Below, we start with the methodology for the investigation of student difficulties with the electric flux as well as the validation of the tutorial and the corresponding pre-/post-tests, followed by a discussion of student difficulties with electric flux related concepts and categorisation of the difficulties found. We then give an overview of the electric flux tutorial. Then comparison is made of the pre-/post-test data from several sections of a traditionally taught calculus-based college introductory physics course (first year college students who
were mainly engineering, math or physical science majors) in which students worked on the tutorial versus data from an ‘equivalent’ section of the course in which they did not engage with the electric flux tutorial.

**Methodology**

As noted in [1], the students who participated in this investigation were enrolled in different ‘equivalent’ sections of a second semester calculus-based college introductory physics course, mainly taken by engineering, chemistry, mathematics and physics majors. This course covers electricity and magnetism and some wave optics. This course is taken after the first introductory physics course, which covers mechanics and waves. Most of the several hundred students in different sections of this course were college freshmen who had completed high school (we do not know how many of these students had taken high school physics course but the pre-test was administered to students in all sections after traditional lecture-based instruction on electric flux in the college course). The students in this course had four hours of lecture time and one hour of recitation time. The different sections of the course were generally taught by different instructors and the recitations were taught by graduate teaching assistants. All of the sections of the course discussed in this investigation primarily had traditional lecture-based instruction in the four hours of lecture time, and in the recitations, the graduate teaching assistants fielded questions about the homework from the students and solved example problems on the material covered on the board.

As noted in [1], this course was mandatory for several hundred engineering freshmen and also for other majors, e.g. from chemistry, mathematics etc. The content covered by all the different sections of the course (both experimental and comparison groups) is the same. Each week students were asked to do homework which was from the textbook (introductory physics textbook by Halliday, Resnick and Walker) except that in the experimental group, the part of the tutorial that students did not complete in class, they were asked to complete as homework in addition to the textbook homework. In all sections of the course, each week after students submitted the homework on a particular topic, they were given a recitation quiz in the last 15–20 min of the recitation class. We note that this investigation employs a quasi-experimental design [36] in that we did not have control over many things, e.g. whether a particular student will be in the sections of the course in which the tutorial was used or the midterm and final exam given by the instructors of different sections since the instructor of each section in the same semester had full control over these things.

As noted in [1], the development of the research-validated guided inquiry-based tutorial focusing on the electric flux was carried out with the following core issues in mind: (1) the tutorial must build on students’ prior knowledge so it is important to investigate the common difficulties students have related to relevant concepts, (2) the tutorial must create an active learning environment where students have an opportunity to engage in thinking and sense making, (3) the tutorial must provide appropriate scaffolding support, guidance and feedback to students and opportunity to organise, reconstruct, and extend their knowledge. The details of the development and validation of the electric flux tutorial and the corresponding pre-test and post-test were analogous to the other two tutorials in this series [1]. Reference [1] provides details about the process of data collection before, during and after the development and validation of the questionnaire and different versions of the tutorial and pre-/post-tests on
electric flux. We note however that the process of the development and validation of the questionnaire and tutorial spanned three years and started with a cognitive task analysis from the perspective of an expert of the relevant material covered in such a course and an investigation of the common difficulties that introductory physics students in this course have with the electric flux concepts. Thus, the preliminary version of the questionnaire (which was later refined into pre-/post-test questions) and the tutorial not only used research on student difficulties as a guide but also a cognitive task analysis of the underlying concepts from an expert perspective as well as iteration with four physics instructors who teach the course regularly at each step. The cognitive task analysis from the perspective of an expert involves making a fine-grained flow chart of the concepts involved in solving a specific class of problems. Such analysis can help identify some stumbling blocks where students may have difficulty. The iteration of the material several times with four physics faculty members who had taught the relevant introductory course provided feedback that helped fine-tune the tutorial and improve its organisation and flow. Moreover, the investigation of students’ difficulties using written tests and interviews was critical for developing and validating the tutorial and the corresponding pre-test and post-test because theoretical analysis from the perspective of an expert often does not capture all of the difficulties students have with relevant concepts. The student difficulties were investigated by administering open-ended and multiple-choice questions in written form to introductory students in various sections of the course after traditional instruction in relevant concepts and via individual interviews with a subset of students. These open-ended and multiple-choice questions were validated with the help of four instructors who had taught this course several times (the questions were iterated with them to ensure that they were robust and interpreted unambiguously by experts) and with introductory physics students to ensure, e.g. that they interpreted the questions as intended. Since the validated versions of many of the open-ended questions became the pre-test and post-test questions, we will focus on student difficulties vis a vis their performance on the pre-test and post-test questions. Individual interviews were conducted using a semi-structured, think-aloud protocol to better understand the rationale for student responses before, during, and after the development of different versions of the tutorial and the corresponding pre-test and post-test. During the 15 semi-structured interviews at different stages of the development and validation, introductory students were asked to verbalise their thought processes while they answered the questions. Students read the questions about the electric flux and answered them to the best of their ability without being disturbed. We prompted them to think-aloud if they were quiet for a long time. After students had finished answering a particular question to the best of their ability, we asked them to further clarify and elaborate issues that they had not clearly addressed earlier. In summary, the development and validation of the tutorial and corresponding pre-/post-tests went through a cyclic, iterative process which included the following stages before the in class implementation in several sections of the introductory physics course:

(1) Development of the preliminary versions and refinement based on a cognitive task analysis from an expert perspective of the underlying knowledge, as well as consultation and iteration with course instructors and via research on student difficulties.

(2) Administration of the pre-test, tutorial and post-test individually to students as well as feedback from faculty members who regularly teach the relevant introductory course.

(3) Determining the impact on student learning and assessing what difficulties were not adequately addressed by the tutorial.

(4) Modifications and fine-tuning based on the feedback from the implementation and evaluation.
Discussion of student difficulties

We have placed in the appendix the pre-/post-test questions for convenience. We note that these questions require a conceptual understanding of electric flux. Moreover, the focus here is on electric flux but an overlap with the difficulties discussed (e.g. confusion due to electric field and flux) in the context of symmetry and Gauss’s law in this series [1] is inevitable since electric flux is central to Gauss’s law. However, as can be seen from the pre-/post-test questions here and in [1], the focus here is on difficulties with the electric flux concepts found in situations which are pre-requisites for invoking symmetry consideration and Gauss’s law to find the electric field [1] (i.e. in [1], the difficulties found are specifically in the context of being able to use symmetry considerations appropriately for determining if Gauss’s law can easily be used to find the electric field and then using it correctly to find the electric field if the sufficient symmetry of the charge distribution exists). Also, as noted in [1], before we discuss student difficulties found, we note that students in the sections of the class in which the tutorial was used as a pedagogical tool were less likely to have the difficulties after working on the tutorial (on the post-test) than on the pre-test (after traditional lecture-based instruction). Also, the difficulties on the pre-test and post-test were similar in nature for both the tutorial sections of the class and an equivalent comparison group consisting of students who did not work on the tutorial. The main difference between these groups was that the tutorial group students were significantly less likely to have the difficulties after working on the tutorial (the performance of students on the pre-/post-test questions is discussed later in the tutorial administration and evaluation section). Also, we first qualitatively discuss the common student difficulties with the electric flux found without separating the performance of the experimental group (tutorial group) and comparison group (non-tutorial group) on the pre-/post-tests. In other words, the pre-/post-test data discussed in a later section were graded and tabulated (in tables 1–4 presented later) for the performance on each question instead of computing the frequency of difficulties.

We also note that physics is a discipline with precise meanings for various concepts. However, we find that students sometimes had difficulty in constructing sentences related to the electric flux that were scientifically correct. In some cases, we could infer what students meant despite their imprecise language. But in other cases, it was difficult to make sense of their sentences. For example, in response to the post-test question (1), one student incorrectly claimed: ‘\( \Phi = EA \cos \theta \). All charges are pointing perpendicular to the surface so \( \Phi = 0 \).’ Further probing suggests that the student was referring to the electric field instead of charges. Similarly, in response to question (2) on the post-test, a student who provided the correct responses to parts (a) and (b), respectively, noted ‘[ball] A because charge is leaving’ and ‘[ball] C because charge is coming towards the sphere’. It was clear from the student’s response that the student was referring to the electric field lines when considering electric flux but calling them charges. Similarly, one student who correctly disagreed with the statement on the post-test question (3), drew a charge outside the cube as a counterexample and said, ‘electric field through the cube is zero because the charge passes through the cube’. The idea that this student was trying to convey was that, since the point charge is outside the cube, all the electric field lines entering the cube also leave the cube so that there is no net flux through the cube. Such incorrect use of scientific language can impede students’ ability to organise their knowledge and be able to solve problems effectively. Although it is challenging, a tutorial such as the one-on the electric flux discussed in this paper strives to help students with developing a good conceptual knowledge structure so that there is a lower probability of using imprecise language.
Finally, as noted in [1], there are many ways of categorising student difficulties and they do not have to be mutually exclusive. In order to come up with our categories below, the two researchers separately came up with the categories of the difficulties and most of their overlapping categories were kept. A few categories that were not on both researchers’ lists were discussed and some of those categories were included as separate categories after discussion. The categories discussed below have overlap with each other but we believe that this is one reasonable way to categorising the difficulties. The discarded categories were less common than those discussed below. We note that a set of related difficulties have been grouped together into four broad groups (A–D) of difficulties.

A. Difficulty interpreting different aspects of the definition of electric flux

Confusing the electric flux for a vector: A common difficulty was mistakenly thinking that the electric flux is a vector. In interviews, students with these types of responses often justified them by invoking the following facts: The expression for the electric flux involves a scalar product of two vectors. Instead of identifying \( \cos \theta \) with the angle between the electric field and the area vector, many students concluded that the electric flux is a vector because it involves \( \cos \theta \). In the interviews, students with these types of responses pointed to the fact that the electric flux can have both positive and negative signs so it must be a vector. When asked if it would make sense to say that the electric flux points at \( 30^\circ \) with respect to a given positive \( x \)-axis, students with these types of responses often avoided a direct response. The response of some of the students implied that for a physical quantity to be a vector, it is not necessary to be able to specify the exact direction for that physical quantity. Also, some students claimed that flux must be a vector since the electric field lines ‘going out’ of a closed surface contribute positively and those ‘going in’ contribute negatively to the total flux through a closed surface.

Only focusing on the number of field lines passing through a closed surface to determine the net electric flux without consideration of the angle between the electric field and the area vector at different points on the surface: The flux is proportional to the number of field lines. However, one must focus on the angle that the electric field vector at various points on the closed surface makes with the outward normal to the surface for finding the number of field lines. For example, in order to answer the post-test question (2) correctly (see the appendix), students must realise that for the balls carrying net positive charge, more field lines should point outward than inward while for the balls carrying net negative charge, more field lines should point inward than outward. In this question, for ball A, all the field lines shown are directed outward while for ball C, all the field lines are directed inward. Thus, ball A carries a net positive charge and ball C carries a net negative charge. In order to determine the ball(s) that carry a net charge of the greatest magnitude, students should note that for balls B and C, the same numbers of field lines are shown. However, four of the field lines from ball B point inward and the other four field lines point outward. Thus, they cancel each other with regard to their contributions to the net flux through the closed surface. Therefore, ball B does not have a net charge enclosed and hence has no net electric flux through it. Thus, ball C is the one which carries a net charge of the greatest magnitude. We find that although many students found the ball that carries a net positive charge correctly, finding the ball that carries the greatest magnitude of net charge was challenging for many students. A common incorrect response for question (2) on the post-test was that the net charge enclosed inside a surface is largest if the number of electric field lines penetrating the surface in a given schematic diagram is greatest. Students who made this type of mistake only focused on the number of field lines and often did not pay attention to the direction of the field lines, which is crucial for
determining the net flux through a closed surface (which is related to the net charge enclosed via Gauss’s law). Some students thought that ball B carries a net negative charge. The most common incorrect response on this question was that both ‘balls B and C’ have the same net charge enclosed because both of these have 8 field lines going inward or outward in the schematic diagram shown. The responses suggest that these students knew that there was a relation between the number of lines and the net charge enclosed. However, they didn’t realise that some of the electric field lines from ball B point inward and others point outward so that the net electric flux through ball B is zero.

B. Difficulty realising that charge enclosed can be calculated from electric flux and vice versa using Gauss’s law without knowing electric field

Assuming charge enclosed cannot be calculated from the information about the electric flux through a surface closed surface without knowing the electric field: Some students had difficulty realising that the charge enclosed inside a closed surface can be calculated from information about the electric flux through the surface without knowing the electric field. For example, in response to the post-test question (1a) (see the appendix), one can use $\Phi = Q/\varepsilon_0$ to calculate the enclosed charge $Q$ since the net electric flux $\Phi$ through the surface of the sphere is provided. However, many students used the equation $EA = Q/\varepsilon_0$ in response to this question and incorrectly reasoned that while the area is given as 0.13 m$^2$, one cannot calculate the charge enclosed because the electric field cannot be determined from the given information. Students with this type of difficulty often incorrectly assumed that it is impossible to find the net charge inside because the charge distribution is not known and the electric field cannot be determined (which they thought was a pre-requisite for calculating the charge).

Assuming electric flux through a closed surface cannot be calculated without knowing the electric field at each point on that surface: Similar to the preceding difficulty in the context of the calculation of the charge enclosed, some students had difficulty calculating the electric flux through a closed surface because they did not exploit Gauss’s law, which relates the electric flux to the net charge enclosed inside a closed surface. Students with this type of difficulty often attempted to find the electric flux in a more complicated way and got lost along way. In particular, we find that sometimes students were very focused on a particular equation relating two physical quantities and thought that the equation provides the only way to solve for the unknown. For example, some students used the equation $\Phi = \oint \vec{E} \cdot d\vec{A}$ for calculating the electric flux and claimed that the electric flux cannot be calculated without knowing the electric field at each point on a closed surface. Some students with these types of difficulties claimed that because the angle $\theta$ in the equation $\Phi_C = \oint \vec{E} \cdot d\vec{A} = \oint [E_i \cos \theta] dA$ is not known or the electric field is not uniform on the surface (and is not easy to find), it is impossible to calculate the electric flux. For example, in response to the pre-test question (1a) (see the appendix), the most common difficulty was not realising that one can calculate the electric flux through the closed surface from the knowledge of the charge enclosed with no need to know the electric field at each point on the surface. Some students incorrectly noted that explicitly knowing the electric field due to all charges at each point on the surface is required in order to calculate the electric flux through the surface and since the information about the distances of the charges is not given, it is impossible to find the electric field and hence the electric flux from the given information (we note that the surface area was provided in the problem description). For example, one interviewed student with this type of response incorrectly stated: ‘impossible because the electric field is not given’. After the interview, that student wanted to go over the correct
answer to each question and stated that he did not understand how there can be two equations for calculating the same physical quantity and why the net flux through a closed surface can be calculated using either equation \( \Phi = Q/\varepsilon_0 \) or \( \Phi = \oint E \cdot d\vec{A} \). Another student who noted that it is difficult to find the electric field in the given situation (which he incorrectly claimed was needed for finding the electric flux) stated, ‘It isn’t possible to find the net electric flux without measurements for the corner that is cut out and the degree measurements of each angle.’

**Assuming a highly symmetric surface is needed for calculating the electric flux using Gauss’s law:** Some students incorrectly claimed that whether the closed surface is symmetric or not influences whether one can find the electric flux through it using Gauss’s law. We can find the net electric flux \( \Phi_E \) through any closed surface without evaluating an integral if the charge enclosed \( Q_{\text{enclosed}} \) is known, using \( \Phi_E = Q_{\text{enclosed}}/\varepsilon_0 \). For pre-test question (1a) (see the appendix), charges +2 nC, −3 nC and −3 nC are enclosed, so the net charge within the surface is −4 nC. Thus, the electric flux is \( \Phi_E = -4 \text{ nC}/\varepsilon_0 \). We find that when answering this question, some students confused the concept of electric flux with electric field. In particular, students with this type of difficulty incorrectly claimed that since the surface is not symmetric enough, it is impossible to determine the electric flux. For example, the following student noted that the electric flux cannot be found from the information provided because the field is not uniform on the given surface: ‘I am not able to find the net flux from the information given because this shape does not have a uniform electric field.’ While some students explicitly stated that since the surface is not symmetric enough, it is impossible to determine the electric flux, others gave responses that indirectly implied something similar. Written responses and individual discussions suggest that students making these types of responses sometimes had some idea about the need for symmetry of the charge distribution for calculating some physical quantity using Gauss’s law, but they had difficulty in distinguishing whether the symmetry is required for finding the electric field or the electric flux. We note that some students also had difficulty in distinguishing between the symmetry of the charge distribution, the symmetry of the surface on which the charges are embedded and the Gaussian surface useful for finding the field using Gauss’s law.

**C. Difficulty realising that electric field can be calculated from electric flux only if charge distribution has sufficient symmetry**

**Difficulty recognising lack of symmetry in charge distribution in various situations and claiming that the electric flux is always \( \Phi = EA \):** Many students had difficulty realising that the electric flux \( \Phi = \oint \vec{E} \cdot d\vec{A} \) cannot always be written as \( \Phi_E = \vec{E} \cdot \hat{A} \) or \( \Phi = EA \) and is given by this type of expression, e.g. \( \Phi = EA \), only in situations in which there is sufficient symmetry of the charge distribution. Some students with these types of difficulties incorrectly claimed that the electric flux is given by \( \Phi_E = \vec{E} \cdot \hat{A} \) or \( \Phi = EA \) (these expressions for the electric flux are only correct in special situations in which the electric field magnitude is constant and the angle between the electric field and the area vector is constant everywhere on the surface, which was not the case in the given question). Thus, these students did not account for the symmetry of the charge distribution in determining whether \( \Phi = EA \) is valid and used this type of equation in cases in which it is not applicable. For example, in post-test question (1b) (see the appendix), the most common incorrect response was using \( E = \Phi/A \) to calculate the magnitude of the field. Some students explicitly mentioned that \( \cos \theta = 1 \) (where \( \theta \) is the angle between \( \vec{E} \) and \( \hat{A} \) and used the electric flux \( \Phi \) and the surface area \( A \) provided in the equation \( \Phi = EA \) to calculate the field). Similarly, in response to the pre-test
question (1a), students with this type of difficulty often started with the relevant equation \( \Phi = EA \) or \( \Phi = EA \cos \theta \) and claimed that the electric field and the area of the closed surface should be known to be able to calculate the electric flux. Moreover, some students who had correctly calculated the electric flux \( \Phi \) in the pre-test question (1a) (using the charge enclosed) incorrectly claimed that \( E = \Phi/A \) and calculated the magnitude of the field at point \( P \) incorrectly in the pre-test question (1b) using the information about the electric flux \( \Phi \) from question (1a) along with the area \( A \).

Assuming electric flux information can be used to calculate the electric field using Gauss’s law in cases without sufficient charge distribution symmetry: To calculate the electric field at a point on a non-symmetric closed surface which encloses a finite number of point charges distributed without a high degree of symmetry, Gauss’s law is not useful. For example, on the pre-test question (1b), students should realise that since we are given individual point charges and the closed surface is not symmetric, we cannot readily apply Gauss’s law to find the net electric field at point \( P \) on the surface. Instead, we could use Coulomb’s law if relevant information is provided. However, we would need to consider not only the charges inside the surface but also the charges outside in order to find the net electric field at a point. In particular, if we were given the distances from each individual charge to the point \( P \) and other relevant information such as angles, we can find the electric field due to each individual charge using Coulomb’s law. Then we can apply the superposition principle to the fields due to individual charges and obtain the net electric field at point \( P \). However, since we are not provided sufficient information in the problem statement (e.g. the distances and other information about direction, etc.), it is impossible to find the net electric field at that point. When answering this question, one must understand that the situations in which the electric field can be found by applying Gauss’ law are limited. In those situations with highly symmetric charge distributions, the following requirements should be satisfied: (1) we can determine the exact direction of \( E \) relative to the area vector at every point on the closed surface by symmetry (only \( \theta = 0^\circ, 180^\circ \) or \( \pm 90^\circ \) are associated with sufficiently high symmetry); (2) in some cases, we can divide the closed surface into sub-sections (for each sub-section the electric flux can be easily calculated, e.g. the side and two caps of an infinite uniform cylinder of charge) such that one of the following is true:

1. \( |\vec{E}| \) is the same everywhere on the sub-section by the symmetry of the charge distribution.
2. \( \vec{E} \) and the area vector (outward normal to the surface) are perpendicular (\( \theta = 90^\circ \)) so that there is no electric flux through that sub-section.

Some students didn’t have a functional understanding of the high symmetry requirement of the charge distribution for applying Gauss’s law to calculate the field and incorrectly claimed that it is possible to determine the electric field at a point on the surface by applying Gauss’s law even if the charge distribution was not sufficiently symmetric. For example, on the pre-test question (1b), some students incorrectly claimed that it is possible to determine the field at point \( P \) by applying Gauss’s law. The most common incorrect answer students provided in this context was that the electric field equals the electric flux divided by the area as discussed earlier.

Assuming net electric flux through a closed surface and the electric field at various points on that surface are always proportional to each other: Some students incorrectly claimed that the electric flux through a closed surface and the electric field at each point on that surface are always proportional to each other. For example, some interviewed students incorrectly claimed that if the magnitude of the electric flux through a closed surface is smaller than it is through another closed surface, the magnitude of the electric field at each point on the surface...
through which the electric flux is smaller must also be smaller. During the interview, some students claimed that if two concentric Gaussian spheres were drawn with a single point charge at the centre, the electric flux through the smaller sphere must be larger because it is closer to the positive charge at the centre and electric field is larger (some interviewed students claimed the opposite stating the flux through the larger sphere must be larger because it has a larger area). Although the net charge enclosed is the same for both surfaces which implies the net flux through both surfaces should be the same. These students were often confused about the fact that the electric flux through a surface involves consideration of both the surface area, the electric field and the angle between the field and the area vector. Some students recalled $\Phi = EA$ to justify their answers but did not take note of the fact that this equation is only true in special cases when the charge distribution has sufficient symmetry while also often focusing only on $E$ or $A$ on the right-hand side of $\Phi = EA$.

Assuming no net electric flux through a closed surface implies no electric field at any point on the surface: Some students had difficulty distinguishing between the electric field and flux since the field and flux are related physical quantities. Many students who had difficulty understanding their relation, incorrectly claimed that no net electric flux through a closed surface implies that there cannot be a non-zero electric field at any point on the surface. For example, on the post-test question (3) (see the appendix), from the knowledge of zero electric flux through the cube, we can conclude that the net charge inside the cube is zero. However, the electric field may not be zero everywhere. For example, the cube may enclose one positive charge and one negative charge of equal magnitude, which would make the electric flux through the cube zero but the electric field in general would be non-zero at various points. However, some students claimed that the electric field should be zero, because the electric field is created by the electric flux and so no flux through the cube meant no electric field anywhere on the cubic surface. For example, one student with this type of difficulty incorrectly stated: ‘Agree. No electricity with no net flux’.

Some students explicitly used $\Phi = EA$ even when sufficient symmetry did not exist for this relationship to hold to conclude that no electric flux through a closed surface implies no electric field at any point on the surface.

Assuming that the flux through a closed surface is always $\Phi = EA$ and using it with $\Phi = Q/\varepsilon_0$ to conclude that only charges inside a closed surface contributes to the field: Some students who assumed that the flux through a closed surface is always $\Phi = EA$ regardless of the symmetry of the charge distribution and also used Gauss’s law $\Phi = Q/\varepsilon_0$ incorrectly concluded that in all situations, only charges inside a closed surface contribute to the field (i.e. they incorrectly claimed that charges outside will never impact the net field on a closed surface). For example, on the post-test question (1a), since the electric flux is given and students are asked to solve for the net charge inside the sphere, we can use the relation between the electric flux and the net charge to find $Q_{\text{enclosed}} = \Phi_k \varepsilon_0 = (-26 \text{ Nm}^2 \text{ C}^{-1}) \varepsilon_0$. Some students solved for the charge enclosed correctly in post-test question (1a) but in the related question (1b), they incorrectly claimed that the electric field magnitude in post-test question (1b) must be given by $E = \Phi_k /A$ with $E = kq/r^2$. Thus, they incorrectly calculated the electric field for a point charge at the centre of the sphere using $E = kq/r^2$ assuming that the electric field magnitude is the same at every point on the surface without realising that the field they calculated is only valid for a special case. In other words, students with this type of response solved for the net charge correctly (the flux only depends on the net charge enclosed and not on where the charges are inside the closed surface) but often incorrectly used the information about the radius, area and $E = kq/r^2$ to find the electric field magnitude. Even on the pre-test question (1a), some students incorrectly noted that the electric flux is given by $\Phi_k = E \cdot A$ or $\Phi = EA$. These expressions for the electric flux are only correct in special
situations in which the electric field magnitude is constant and the angle between the electric field and the area vector is constant everywhere on the surface, which is not the case in the given question. Some students claimed that on this question, using these expressions, the flux can be evaluated by obtaining the magnitude of the electric field from Coulomb’s law using $|\vec{E}| = k|q| / r^2$ for the charges enclosed (which is not correct since the outside charges also contribute to the field) and using the area of the surface provided in the question. Students with these types of responses incorrectly assumed that only the charges inside a closed surface contribute to the electric field at a point on the surface.

Incorrectly using $\Phi = Q / \varepsilon_0$ and $\Phi = EA$ to conclude that no charge inside a closed surface implies no electric field at any point on the surface: Question (5) on the post-test (see the appendix) was extremely difficult for students and many students incorrectly agreed with the statement. Many students claimed that if there is no charge inside the Gaussian surface, the electric field must be zero everywhere inside the hollow region. Some used $\Phi = Q / \varepsilon_0$ and $\Phi = EA$ to conclude that no charge implies no flux which implies zero field at every point on the surface. For example, in order to answer the post-test question (5), students should realise that the spherical ball is non-conducting and charges are uniformly distributed on it. Without the point charge outside the sphere, regardless of whether the ball is conducting or non-conducting, the electric field inside the ball would be zero because the charges are uniformly distributed on a sphere. If the ball was a conducting ball, with the appearance of the point charge outside, the positive charges on the ball will adjust their positions so that the electric field inside the ball remains zero. However, for this non-conducting ball, with this negative point charge outside, the positions of the positive charges on the sphere will not change significantly (except for the local polarisation of the molecules that make up the sphere with charge). Thus, the electric field inside the ball is the combined field due to the hollow non-conducting ball with a uniform surface charge and the negative point charge. Since the net effect of the uniformly distributed charge on the sphere is zero, the net electric field inside is equal to the field associated with the point charge outside the sphere, and cannot be zero. Interviews and written responses suggest that the most common difficulty associated with this question was that since there is no charge enclosed in the Gaussian surface, the net flux through the surface is zero and so the electric field at all points on the Gaussian surface must be zero. Even students who explicitly noticed the presence of the point charge outside stated that the charge outside doesn’t have any influence on the field inside and hence that the field is zero due to the outside charge. The following are some typical incorrect responses:

- Agree because the point charge has to be enclosed within the sphere for it to cause an electric field.
- Agree because all charge is located outside of the Gaussian sphere. By Gauss’s law there is no electric field.
- Agree. The ball does not contain a charge and so the net electric field everywhere inside must be zero.
- The net flux inside the sphere would be zero and since there is some area to the Gaussian surface, so $\Phi_E = \oint \vec{E} \cdot d\vec{A}$ means $E = 0$.

D. Other difficulties

Assuming net electric flux through a closed surface can be non-zero even if the net electric field is zero everywhere on the surface: On the post-test question 4 (see the appendix), if the electric field is zero everywhere on the surface of a sphere, from the relation between the
electric field and flux, $\Phi_E = \oint E \cdot d\vec{A}$, one can conclude that the net electric flux through the sphere must also be zero. Therefore, students should agree with the statement they were asked to evaluate on the post-test question (4). However, some students focused on the relation between the electric flux and the net charge enclosed and incorrectly claimed that even if the electric field is zero everywhere on the sphere, the electric flux through it would not be zero so long as there is a net charge inside. They didn’t realise that if there is a net charge inside the sphere, the electric field cannot be zero everywhere on the spherical surface. Some students attempted to put one point charge inside the sphere and one point charge outside to incorrectly devise a situation in which the field will be zero at all points on the sphere and the electric flux will be zero through the surface. Their claim that in this situation both the field and flux are zero is incorrect regardless of where those two charges are placed. Furthermore, some students claimed that the flux through a closed surface only depends on the charge enclosed and not on the electric field and incorrectly generalised this statement to conclude that the field being zero everywhere on the closed surface has no impact on whether or not the net flux through the surface is zero.

Assuming net electric flux through a closed surface can be zero even if there is net charge enclosed: On the pre-test question (1c) (see the appendix), students were asked to draw a surface that contains some of the charges shown in a diagram such that the net electric flux through the surface is zero. To make the net electric flux zero through a closed surface that includes some of the charges shown, one must ensure that the net charge inside the surface is zero. The net charge inside the surface in the given situation provided in pre-test question (1c) is $-4 \, \text{nC}$. Thus, we can extend the surface such that it also encloses the point charge $+4 \, \text{nC}$. Moreover, students can choose any other hypothetical closed surface that encloses different charges that make the net charge zero. However, instead of drawing a surface that encloses zero net charge, some students claimed that a closed surface that has the electric field vector $\vec{E}$ tangent to the surface everywhere will have zero net electric flux no matter what the net charge inside is (and how many charges shown are inside). Some students who provided incorrect responses to post-test question (3) claimed that since $\Phi_E = \oint E \cdot d\vec{A} = \oint [E] \cos \theta |d\vec{A}|$, to make the net flux through the cube zero, either the electric field should be zero everywhere on the surface or the angle $\theta = 90^\circ$ everywhere on the surface. However, none of the students with these types of responses drew how charges shown should be enclosed in the closed surface to create such a situation.

Assuming electric flux through a spherical surface is always due to a ‘point’ charge at centre if the charge distribution in the region is not given explicitly: When students were asked questions without a concrete case (with a given charge distribution), they sometimes assumed that the charge distribution must be the simplest possible, namely, a point charge. Students who made such an assumption in answering questions did not state that they were considering a very special case and in general the charge distribution could be more complex. For example, a common difficulty on the post-test question (1b) (see the appendix) was assuming that there was a point charge at the centre of the sphere and using the given radius of the sphere to find the field using Coulomb’s law. Similarly, on post-test question (2d) (see the appendix) about whether we can say for certain that any of the balls contains a single point charge from the information provided, we can only conclude whether there is a net positive or negative charge inside each ball. In other words, from the information provided, there is no way for us to conclude that any of the balls contains a single point charge. In response to this question, one common incorrect response was that balls A and C contain a single point charge because all of the electric field lines are going in or out from the centre. Students assumed that there must be a point charge at the centre of each of the balls A and C. For example, one
The student stated: ‘A and C because all the flux lines seem to be directed in and out of the centre of the sphere.’ One interviewed student claimed that it is easy to tell from the figure that A and C have point charges at the centre by looking at the field lines. However, there is no way to tell from the information provided whether there is a point charge at the centre because even if the electric field lines are everywhere perpendicular to the spherical ball, the hidden charges inside could be uniformly distributed with respect to the centre of the sphere (e.g. concentric sphere) and not necessarily be localised as point charges at the centre. Also, some students incorrectly stated that ball A alone has a point charge inside because the electric field lines from ball A are symmetrical and radially outward. Again, even if the field lines are radially outward, there is no way to distinguish a point charge from a spherically symmetric charge distribution concentric with the centre of that ball (also, the given field lines do not show spherical symmetry).

Assuming charges outside a closed surface contribute to the net electric flux through the surface: The confusion between the electric field and flux led to some students claiming that charges outside a closed surface contribute to the net electric flux through the surface. On the pre-test question (1a), some students thought that charges outside the surface would also influence the electric flux. For example, in response to this question, one student stated: ‘We do not know how far away the outside charges are, so we cannot find electric flux.’

Using dimensionally incorrect equations involving flux and incorrectly using one physical quantity for another: On pre-test question (1a), some students incorrectly took the net charge enclosed $-4 \, \text{nC}$ to be the electric field and used the given area and the relation between the electric field and flux, $\Phi = EA$, to find the net flux. For example, on the pre-test question (1a), one student incorrectly stated: ‘Since $\Phi_E = |E| |\mathcal{A}| \cos \theta$, $\Phi_E = -4 \, \text{nC} \cdot 0.4 \, \text{m}^2 \cos \theta = -1.6 \, \text{C} \, \text{m}^2$.’ Another common incorrect response to this question was that $\Phi_E = Q_{\text{net}} / (\varepsilon_0 A)$, where $A$ is the total area of the surface. The students with these types of responses didn’t evaluate that the dimension of the right-hand side is not that of electric flux. Other students used dimensionally incorrect expressions that were reminiscent of Gauss’s law. For example, on the post-test question (1a), students were given the electric flux and the total area of a closed surface and asked to find the net charge enclosed inside the surface. Some students concluded that $Q = \Phi_E \cdot \varepsilon_0 / A$ or $Q = \Phi_E / A$. Other students who had difficulty distinguishing between the electric field and flux calculated the flux when asked to find the electric field.

Overview of the tutorial

The difficulties indicate that the electric flux is a challenging concept for students. Therefore, as noted earlier, we developed a tutorial that takes into account the common difficulties found and strives to help students build a good knowledge structure of flux related concepts. The entire flux tutorial can be found at http://per-central.org/items/detail.cfm?ID=12593. The tutorial keeps students actively engaged in the learning process in which various concepts build on each other. We note that the tutorial approach is different from the Paradigm approach pioneered at Oregon State University in which similar topics across different areas of physics are organised in a curriculum that keeps middle-division students actively engaged in learning [36]. We also note that before working on the electric flux tutorial, students worked on a tutorial designed to help students learn about the electric field and superposition principle [1].

The tutorial uses a guided inquiry-based approach to learning and provides support to help students interpret and visualise the concept of electric flux using diagrammatic representations, e.g. using electric field lines. It also strives to help students learn that $\vec{E}$ at a point...
due to a charge distribution can be represented with an arrow and the length of the arrow qualitatively represents the field magnitude and its direction shows the field direction. It also helps them learn that the electric field \( \vec{E} \) is defined at each point but the electric flux \( \Phi_E \) is defined through a surface area. Then, students work through several guided inquiry-based learning sequences such as the following which strive to help them solidify the concept of the electric flux:

The diagrams in figure 1 show electric field lines penetrating through two identical square sheets. The electric field lines are perpendicular to the sheet (parallel to the area vector) in both diagrams. The diagrams are drawn to the same scale:

(1) Consider the following statements from Emily and Mary:
- Mary: The net flux through both squares is zero because there are equal numbers of field lines going in and out.
- Emily: I disagree. A square is not a closed surface like a cube. A square does not have an inside and an outside. For a cube, if the same number of field lines were going in through some faces and going out through the remaining faces, the net flux would be zero. In fact, in that case, Gauss’ law tells us that there won’t be any net charge enclosed inside the cube.

Explain why Emily is correct and Mary is not.

(2) Which one of the sheets (1) or (2) above has more electric flux through it? By what factor?

(3) Can you determine the magnitude of the electric flux through either of the surfaces above from the information provided? Explain. (Hint: Are the magnitude of the electric field and the surface area given in the problem?)

After making their predictions by answering these questions to the best of their ability without help, the tutorial provides scaffolding support and strives to help students learn, e.g. that the two square sheets have the same area but sheet (II) has more field lines penetrating through it. Thus, sheet (II) has more electric flux through it. The tutorial also strives to help them learn that since we are not given the surface area, it is impossible to calculate the magnitude of the electric flux. The students are also asked to reflect upon the fact that the number of field lines penetrating a surface is only a qualitative measure of the flux through that surface. The tutorial also strives to reinforce the idea that the electric flux can be through an open or closed surface but one can relate the net flux to the net charge enclosed within a surface using Gauss’ law only for closed surfaces.

The tutorial further strives to help students learn to distinguish between the electric flux and field and understand the relationship between them. For example, students are provided an opportunity to contemplate an analogy to help them reason, e.g. about why even if there is no net charge inside a closed surface, there can be a net field at different points on the surface. The analogy they consider is between ‘the net charge enclosed and the net worth of a group’ and ‘an individual point charge enclosed and the net worth of an individual’. The following is an excerpt from a guided inquiry-based sequence that focuses on helping students reflect upon the electric flux and field using analogical reasoning:

The net worth of a group of three people is the sum of the net worths of each individual, with debts counting as negative values.

(1) If the net worth of a group of three people is $3000, the net worth of each individual **MUST** be $1000. Do you agree or disagree?

(2) If the net worth of a group of three people is zero, the net worth of each individual **must be zero**. Do you agree or disagree?
(3) If John is $3000 in debt and has no assets, Mary has $5000 in assets and carries no debts, and they have a child with $2000 in debt, what is the net worth of the family?

(4) Check to make sure your answers to questions (2) and (3) are consistent.

Students can answer question (1) by noting that the net worth of each individual is not necessarily the same (and different people can have positive or negative net worth similar to positive and negative point charges). For the second question, students must conclude that the net worth of a group is zero does not imply that the net worth of each individual is zero. Next, question (3) gives an explicit example of the general situation addressed in question (2), and students are asked to check the consistency of their responses in question (4). Then students are asked to consider various situations (with or without a net charge inside the closed surface) using the analogy as a guide (i.e. net worth of a group \( \rightarrow \) net charge enclosed, net worth of an individual \( \rightarrow \) individual point charges enclosed).

Students are also presented with a situation in which there is no net charge inside the closed surface but there are individual charges and provided guidance to learn to distinguish between the electric field and flux. The tutorial strives to help students learn, e.g. that zero electric flux through a closed surface implies zero net charge enclosed but zero net charge enclosed does not imply that there are no individual charges enclosed. The following excerpt from a guided inquiry-based sequence is an example from the tutorial that strives to help students to reason about the fact that zero electric flux through a closed surface does not necessarily imply that the electric field is zero everywhere on the surface.

The net charge enclosed within the Gaussian surface shown in figure 2 is zero.

(1) What is the net electric flux through the surface according to Gauss’s law?

(2) Use Coulomb’s law to draw three arrows labelled \( \vec{E}_1 \), \( \vec{E}_2 \) and \( \vec{E}_3 \) to show the directions of the electric field at point P above due to each of the point charges. Draw the approximate direction of the resultant electric field (vector sum of \( \vec{E}_1 \), \( \vec{E}_2 \) and \( \vec{E}_3 \) at point P) labelled \( \vec{E}_{\text{net}} \), and explain in words how you can calculate the resultant electric field \( \vec{E}_{\text{net}} \) from the individual fields. Will it be useful to choose a set of coordinate axes to calculate \( \vec{E}_{\text{net}} \)? Explain. The tails of \( \vec{E}_1 \), \( \vec{E}_2 \), \( \vec{E}_3 \) and \( \vec{E}_{\text{net}} \) should be at point P.

The net flux through the closed surface in the situation in figure 2 is zero since there is no net charge enclosed. However, the fields generated by the three individual charges are not zero and have different magnitudes and directions. The resultant net electric field \( \vec{E}_{\text{net}} \) is not zero and can be calculated if we know the displacement vectors connecting each charge to point P.
After students engage with the guided sequences that strive to help them learn the relationship between the net charge, electric flux and field, they are provided a closed surface with a net charge enclosed and asked to contemplate whether the net flux through the surface is zero. They are further told that if they think that the net flux through the closed surface given is not already zero, then they should make some changes to the situation presented to make the net flux through their closed surface zero. Then, they are asked a sequence of guiding questions with different individual charges enclosed and asked to predict the electric field at different locations on the surface. Apart from the case with no net charge inside the closed surface, students also reflect upon situations with a net charge inside the closed surface. The guided approach in the tutorial also strives to help students learn that when the net flux is not zero through a closed surface, e.g. a cube, due to the net charge enclosed, the flux through each face doesn’t have to be the same. The flux is the same through each face of the closed surface (e.g. each face of a cube) only when the surface is symmetric and the net charge inside is symmetrically distributed with respect to the different faces (there are no charges outside).

Moreover, the tutorial strives to extend students’ understanding of the electric flux and how it differs from the field, students work through examples of situations in which there is no charge inside but there are charges outside the closed surface producing a field. The following is an example (involving the situation in figure 3) of a guided inquiry-based learning sequence that strives to help students make sense of the concept of flux:

A cubic Gaussian surface with \( L = 1 \) m on a side has two horizontal faces and is in a uniform electric field of \( 50 \, \text{N} \, \text{C}^{-1} \) which is directed vertically upward (see figure 3).

1. Find the net electric flux through the cube. (Hint: If there are equal numbers of field lines going into and out of a CLOSED surface, the net flux through the surface is zero. Is the surface of the cube a closed surface?)
2. Find the numerical values of the flux through the top and bottom faces of the cube using \( \Phi_E = \int \mathbf{E} \cdot d\mathbf{A} \), where \( A \) is the area of each face and \( \theta \) is the angle between \( \mathbf{E} \) and the area vector.
3. What is the flux through each of the side faces of the cube? Explain.
4. What is the net flux through the whole cube after you added the contributions from all faces of the cube? Make sure your answer is consistent with question (1) above.
5. Is the electric field zero at all points on the surface of the cube? If not, what is its magnitude at points A and B?
6. Using Gauss’s law and your response to question (1) above for the net flux through the cube, what is the net charge enclosed within the cube?

The guided inquiry-based sequences in the tutorial strive to help students learn to distinguish between the electric field and electric flux. A common difficulty with question (5) in
the sequence without guidance was stating that the electric field is zero at point B on the side surface of the cube, although the question explicitly mentions that the cube is in a uniform electric field of $20 \text{ NC}^{-1}$. During the development of the tutorial, in the interviews and in free-response questions, some students explicitly claimed that the area vector of the side surface is perpendicular to the direction of the electric field lines, and therefore, the electric field must be zero at point B. The guided inquiry-based approach strives to reduce this kind of confusion between the electric field at a point and the contribution to the electric flux from a certain area (which was quite common after traditional instruction).

Then, the tutorial provides guidance to students with more complicated examples in which there are charges both inside and outside of the surface such as the following example:

Consider the following situation in which there are only three point charges $+q_1$, $+q_2$ and $+q_3$ near a spherical Gaussian (imaginary) surface as shown in figure 4. Point charge $+q_1$ and $+q_2$ are inside and $+q_3$ is outside the surface.

1. Use Gauss’s law to find the net electric flux through the spherical surface in terms of the charges and the permittivity of free space $\varepsilon_0$.

2. Which of the charges above do NOT contribute to the net flux through the surface? In figure 4, draw two electric field lines each, starting only from those charges that will NOT contribute to the net flux (assuming only that charge is present). Make sure these field lines penetrate the spherical surface. Based upon your drawing, explain why these charges do NOT contribute to the net flux. (Hint: Are there equal number of field lines going in and out of the closed surface due to such charges?)

3. Which of the charges in figure 4 contribute to the net electric field at point P shown on the surface? Explain.

4. Based upon your answers to (1)–(3), explain why charges that do not contribute to the net flux through a closed surface may contribute to the field at a particular point on the surface.

We find that after traditional lecture-based instruction, many students struggled with the fact that, only for a highly symmetric Gaussian surface, typically, the magnitude of the electric field is the same everywhere on the surface so that it can be readily calculated by
dividing the electric flux by the area of the surface. The tutorial strives to help reduce this difficulty. For example, students are presented with a guided inquiry-based sequence as follows:

A thin non-conducting rod of length 1 m, with a +100 nC (nano Coulomb) positive charge uniformly distributed over it, is inside three nested Gaussian surfaces as shown below. Surfaces A and B are spherical with the non-conducting rod symmetrically placed inside and surface C is irregularly shaped, as shown in figure 5.

(1) According to Gauss’ law, which surface has the maximum net electric flux passing through it, or are they all the same? What is the flux through each surface?
(2) Pₐ, Pₐ, and Pₐ are three points on surfaces A, B and C, respectively, as shown in figure 5. At which of these points is the electric field magnitude greatest, or is it the same at all three points? (Hint: Are the field lines farthest apart on surface C and is the field at various points on surface C weaker than that on surface A or B?)
(3) Is the electric field magnitude the same at every point on surface A? Explain. (Hint: Imagine dividing the non-conducting rod into point charges and finding the electric field at two different points Pₐ and Pₐ on surface A using the superposition principle.) Answer the same question for points on surface C. Explain.
(4) Can you calculate the electric field at any point on any of the surfaces without using complicated integration? Explain. Comment on why your answer makes sense even though you found the electric flux through all of the surfaces easily using Gauss’ law.
(5) From your responses to the previous parts of this question, does equal flux through two surfaces imply equal field at each point on those surfaces?
(6) Suppose Sam measures the area of the spherical surface A in figure 5 to be 2 m². Consider the following discussion:

- Sam: Now we can determine the electric field magnitude at any point on surface A since \( |\mathbf{E}| = \frac{\text{Flux}}{\text{Area}} = \frac{+100 \text{ nC}}{(\varepsilon_0 \times 2 \text{ m}^2)} \).
- Susan: I disagree! It is impossible to find \( |\mathbf{E}| \) from this information. The net flux \( \Phi_E = \oint\oint \mathbf{E} \cdot d\mathbf{A} = \oint |\mathbf{E}| \cos \theta |d\mathbf{A}| \) but the magnitude of the electric field \( |\mathbf{E}| \) is not the same everywhere on the surface nor is \( \theta = 0^\circ \) at every point on the surface. Therefore, we cannot pull \( |\mathbf{E}| \) out of the integral and \( \cos \theta \neq 1 \) everywhere. So we cannot claim that the net flux \( \Phi_E = |\mathbf{E}| \times \text{Area} \).

With whom, if either, do you agree? Explain. (Hint: To determine the validity of Sam’s statement, choose a few points on the spherical surface and comment on whether it is possible
for the field magnitude to be the same at those points? Then, discuss the validity of Susan’s statement.

These types of guided inquiry-based learning sequences strive to keep students actively engaged in the learning process and strive to help them develop a good grasp of the concept of electric flux.

Administration and evaluation of the tutorial

After the tutorial appeared to be effective in individual administration to students in the one-on-one interview situation, it was administered in several sections of the calculus-based introductory physics course after students had traditional lecture-based instruction in relevant concepts. The students were administered the pre-test in class after traditional lecture-based instruction before they worked on the tutorial. Moreover, one section of the course consisted of the comparison group (or non-tutorial group) in which students took the pre-test after traditional lecture-based instruction but then did not work on the tutorial. Instead, the post-test was administered to the students in the comparison group (non-tutorial group) as a quiz after students submitted their textbook homework problems on relevant concepts and had opportunity to ask questions about any doubts they had about those concepts.

All students had sufficient time to work through the pre-test. Next, students in the tutorial group worked through the tutorial in class in small groups and were given one week to work through the rest of the tutorial as homework if they could not complete it in class. Then, the students took the post-test in class (all students had sufficient time to take the post-test). The post-test was graded for correctness as a quiz for all the different sections of the course. The pre-test and post-test counted for a small portion of the quiz component of the course grade for the course. All questions on the post-test have equal weight. Although the pre-test was graded for completeness and not returned to students, the post-test was returned to them after grading. In addition, students were aware that the concepts discussed in the tutorial were relevant for the course and questions focusing on these concepts could appear on exams. However, overall, the grade incentive associated with these conceptual questions has been so small that we have not observed any evidence that students have made the post-test solutions available to future students in classes in which the instructor used the tutorial.

Table 1 depicts performance (on each question and also overall) on the pre-test and post-test for each section of the course in which students worked on the tutorial. As shown in the appendix, there were five questions on the post-test. We note that students in the tutorial
The group had previously worked on a tutorial on electric field and superposition principle [1] before traditional lecture-based instruction on electric flux concepts.

Table 2 shows the pre-/post-test data from a comparison group which consists of a section of the course in which the tutorial was not used. The total class time was the same in all four sections (i.e. whether or not the tutorial was administered in the section). In the tutorial sections, some of the example problems were not demonstrated by the instructor in class to save time. Instead, students worked on the tutorial. The pre-test was administered to the students in the comparison group immediately after lecture on flux concepts but the post-test was administered in the following week as part of the recitation quizzes after students had completed all homework problems on flux concepts and had the opportunity to ask questions about them. Tables 1 and 2 show that after traditional lecture-based instruction, students in both the tutorial and comparison groups performed poorly on pre-test (in other words, learning about the electric field and superposition principle earlier using tutorial [1] did not significantly impact the performance of the tutorial group on flux concepts). However, students in the comparison group did significantly worse on the post-test than the sections in which students worked on the tutorial. The \( p \) values obtained by comparing the performances of students in the tutorial and non-tutorial groups are 0.320 and \(< 0.0001\) for the pre-test and post-test, respectively.

In one type of question on the pre-/post-test, students were given an asymmetrical closed surface with discrete point charges inside and outside the surface at various locations. The area of the surface was also provided. The students were asked whether it is possible to find the net electric flux though the surface or the electric field at some point on the surface and explain their reason for their response. In other questions, students were also asked to draw a surface that encloses certain charges so that the net electric flux through the surface is zero. To answer some of these questions correctly, students must discern the relation between the net electric flux and the net charge enclosed inside the surface. They also must understand that there is no requirement for the symmetry of the charge distribution to calculate the electric

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**Table 1.** Average percentage total scores and scores on individual questions on the pre-/post-test for the tutorial group. The pre-test was administered to students after lecture-based instruction but before the flux tutorial. An additional question was included on the post-test for sections 1 and 3. \( N \) is the matched number in a given section who took both the pre-/post-tests.

| Post-test | | | | | | |
|---|---|---|---|---|---|---|
| Section | \( N \) | Pre-test Score | 1 | 2 | 3 | 4 | 5 | Total score |
| 1 | 86 | 48% | 79% | 86% | 73% | 93% | 83% | 83% |
| 2 | 62 | 49% | 78% | 90% | 83% | 98% | 86% | 86% |
| 3 | 63 | 55% | 83% | 80% | 85% | 96% | 88% | 86% |

**Table 2.** Average percentage total scores and scores on individual questions on the pre-test and post-test for the comparison group. The pre-test was administered soon after traditional lecture-based instruction. The post-test was administered in the recitation the following week after students submitted their homework on relevant concepts.

| Post-test | | | | | | |
|---|---|---|---|---|---|---|
| \( N \) | Pre-test | 1 | 2 | 3 | 4 | 5 | Total score |
| 59 | 43% | 36% | 70% | 72% | 72% | 28% | 56% |
flux through the closed surface from the net charge enclosed. Moreover, they must understand that to determine the electric field, they either should use Coulomb’s law (in which case they must know the distances of the point charges from the point where the electric field is to be calculated and angles made by the straight lines joining the charges and the point where the net field is to be calculated with respect to some axis chosen in order to apply Coulomb’s law assuming that everything in the problem lies in a plane) or recognise and exploit the symmetry of the charge distribution in order to use Gauss’s law appropriately to find the field. The average scores of students in the non-tutorial and tutorial groups on these types of questions on the post-test were 50% and 83%, respectively.

Some questions probe student understanding of the difference and the relation between the electric field and flux. In these questions, students were given statements about the electric flux and field and asked whether they agreed or disagreed with the statement and explain their reasoning or give specific examples that support their conclusion. The following are two examples:

‘If the electric flux through a cube is zero, the electric field everywhere on the surface of the cube must be zero.’

‘If the electric field is zero everywhere on the surface of a sphere, the net electric flux through the sphere must be zero.’ The first statement is false and the second statement is true. The average scores of students in the non-tutorial and tutorial groups including all questions of these types on the post-test (see the appendix) were 53% and 85%, respectively.

Table 3 shows the performance of students in the tutorial group on the pre-/post-tests partitioned into three separate groups based upon the pre-test performance (see the Range column). As can be seen from table 3, the tutorial generally helped all students especially those who performed poorly on the pre-test.
Table 4 depicts how students performed in the comparison group on the pre-/post-tests when they were divided into three separate groups based on how they performed on the pre-test. Comparison of tables 3 and 4 shows that students in low and medium pre-test ranges in the tutorial group benefitted the most from engaging with the tutorial and comparison group had poorer post-test performance than the respective tutorial group for those pre-test ranges. For the students in the highest performing pre-test range in tables 3 and 4, the performance of those in the tutorial group appears to be more stable although there are very few students in this pre-test range especially for the comparison group for making meaningful inferences.

In order to evaluate the retention of learning for different student populations at the end of the semester on electric flux concepts, we administered and analysed performance of different groups on a 25 question multiple-choice standardised conceptual test [33] in which nine questions are related to the electric flux concepts. Table 5 shows the average scores on the nine questions on electric flux concepts. In table 5, the introductory (intro) physics group without tutorial is the group of students which did not work on the tutorial (there were two such sections of the course, one of which is the one in which pre-/post-test data were discussed in tables 2 and 4). The tutorial group in table 5 consists of 4 sections of the course, and for three of these sections, the pre-test and post-test data were available and discussed earlier in tables 1 and 3. The group 'Intro Honours Students’ in table 5 is a group of introductory physics students who were in a separate section of the introductory physics course in which students solved more challenging quantitative problems, but method of instruction was traditional lecture. These honours students were generally expected to be better prepared for calculus-based introductory physics than students in the regular introductory physics sections. The test was administered after instruction in these concepts at the end of the term in all undergraduate courses except in the upper-level undergraduate E&M course, in which it was administered both in the first and last week of classes. The scores of first year physics PhD students in table 5 is for calibrating the performance of undergraduates. The last row of the table shows the average score including all nine questions. Table 5 shows

| Question # | Intro without tutorial | Intro honours students | Upper-level E and M undergraduates | First year PhD Students |
|------------|-----------------------|------------------------|-----------------------------------|------------------------|
| 1          | 56%                   | 44%                    | 42%                               | 53%                    |
| 7          | 69%                   | 73%                    | 64%                               | 68%                    |
| 8          | 37%                   | 38%                    | 52%                               | 32%                    |
| 9          | 61%                   | 53%                    | 67%                               | 68%                    |
| 12         | 45%                   | 35%                    | 43%                               | 53%                    |
| 18         | 40%                   | 28%                    | 58%                               | 39%                    |
| 20         | 52%                   | 45%                    | 55%                               | 53%                    |
| 21         | 20%                   | 43%                    | 36%                               | 53%                    |
| 23         | 34%                   | 55%                    | 42%                               | 53%                    |
| Avg        | 46%                   | 46%                    | 51%                               | 52%                    |

Table 4 depicts how students performed in the comparison group on the pre-/post-tests when they were divided into three separate groups based on how they performed on the pre-test. Comparison of tables 3 and 4 shows that students in low and medium pre-test ranges in the tutorial group benefitted the most from engaging with the tutorial and comparison group had poorer post-test performance than the respective tutorial group for those pre-test ranges. For the students in the highest performing pre-test range in tables 3 and 4, the performance of those in the tutorial group appears to be more stable although there are very few students in this pre-test range especially for the comparison group for making meaningful inferences.
that those engaged with the tutorial on the electric flux concepts significantly outperformed both the honours students and students in upper-level undergraduate courses, but not the first year PhD students. A t-test on the average score of the tutorial group with each of the other groups in table 5 shows that in this end of the semester standardised test [33], the tutorial group outperformed all groups including introductory honours students and upper-level undergraduates on the post-test, except the PhD students. Table 5 also shows that the performance of students in introductory (including the tutorial group) and upper-level courses is context dependent and on some of the questions involving symmetric charge distribution even those in the tutorial group did not perform well. This type of context dependence of student performance is found in many context in introductory physics [28–31].

Summary

This investigation is part of a series of three studies using analogous methodology that focus on investigating and improving introductory physics students’ understanding of electrostatics concepts [1]. In this paper, we discussed an investigation of the difficulties of college introductory physics students with the electric flux related concepts and how we used that research as a guide to develop a research-based tutorial which uses a guided inquiry-based learning sequences related to the electric flux. While the pre-test performance on the electric flux concepts after traditional lecture-based instruction was similar for the tutorial and comparison groups (despite the fact that the tutorial group had engaged with an earlier tutorial on electric field and superposition principle), the post-test performance of the tutorial group was significantly better. The better post-test performance suggests that fewer introductory physics students in the tutorial group manifested the difficulties described earlier. We note however that on two of the questions on the standardised test [33] that relate to the electric flux, students in the tutorial group obtained significantly less than 60% (see table 5). The performance on these questions implies that even after engaging with the tutorial, students struggled to distinguish between the electric flux and electric field in some contexts. The context dependence of student performance has been found in many previous studies (e.g. see [28–31]) and suggests that introductory physics students are not yet experts in the electric flux related concepts despite engaging with the tutorial and their knowledge structure related to these concepts is not robust enough to adapt to various contexts. Future investigation involving individual interviews will focus on why introductory students struggle with certain contexts more than others even after working on the tutorial and how the tutorial can be improved to make it even more effective. Despite these issues, the introductory students in the tutorial group performed better on average than introductory honours students and the upper-level physics students on the standardised test [33] questions related to the electric flux (see table 5). Moreover, it is encouraging that those who obtained low or medium scores on the pre-test after traditional lecture-based instruction were both likely to benefit significantly from the tutorial. In fact, improving the performance of students with low and medium pre-test scores by helping them engage with a guided inquiry-based tutorial is a step in the right direction in helping all students learn physics. Such a goal in turn is critical to prepare an additional one million STEM professionals in the US in ten years according to the recommendations of the President’s Council of Advisors on Science and Technology (PCAST) report [38].

Finally, we note that although the tutorial on the electric flux discussed here is modular (in that there is no reference in the electric flux tutorial to the tutorial on electric field and superposition principle) and it can be used independently in introductory physics classes, if
the tutorial discussed here and the other two electrostatics tutorials discussed earlier [1] are used together as a set similar to the investigation described here, introductory students’ knowledge structure pertaining to different electrostatics concepts from Coulomb’s law to Gauss’s law is likely to improve more than if only one of them is used since the tutorials are synergistic and the concepts in different tutorials build on each other. In particular, the concepts of electric field and superposition principle in the first tutorial are pre-requisites for the tutorial on the electric flux discussed here. Similarly, the tutorial on symmetry and Gauss’s law that students work on after the electric flux tutorial focuses on situations with highly symmetric charge distribution in which Gauss’s law can be used to easily find the electric field (spherical, cylindrical and planar symmetries) and strives to help students exploit the symmetry and learn to choose appropriate Gaussian surfaces in different situations to find the field using Gauss’s law.

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Appendix. Pre-/post-tests

The following information was provided for both the pre-/post-tests.

• For all problems below, leave your answers in terms of $\varepsilon_0$.

• Outward electric flux through a closed surface should be considered positive.

• All physical objects in all questions are non-conducting. Assume that all non-conductors (insulators) are non-polarisable.

PRE: (1) Consider the following sketch showing a cross-section of a closed surface with three point charges inside and three outside the surface at various locations. The surface is known to have an area of 0.4 m².

(a) If possible, find the net electric flux through the surface. If it is not possible to find the net flux from the information given, explain why it is impossible.

(b) If possible, find the net electric field at point P on the surface. If it is not possible to find the net field from the information given, explain why it is impossible.

(c) Can you draw a surface that encloses some of the charges through which the net electric flux is zero? If so, draw its cross-section on the diagram above; if not, explain why it is impossible.
POST: (1) You are told that the net electric flux through the surface of a sphere (radius = 0.10 m, area = 0.13 m², volume = 0.004 m³) is \(-26 \text{ N m}^2 \text{ C}^{-1}\). Use this information to answer the following:

(a) Either find the net charge inside the sphere or explain why it is impossible to do so.
(b) Either find the magnitude and direction of the net electric field at a point P somewhere on the surface of the sphere or explain why it is impossible to do so.

(2) The diagrams below give the pattern of the electric field lines near each of the three insulating balls that may or may not be charged. Sadly, we couldn’t measure the electric field inside the balls, so you see only the pattern outside. The full pattern is three-dimensional, of course, but these cross-sectional drawings are qualitatively correct.

(a) Which balls definitely carry a net positive charge? Explain.
(b) Which balls definitely carry a net negative charge?
(c) Which ball or balls carries a net charge of the greatest magnitude?
(d) Can we say for certain that any of the balls contain a single point charge? Explain.

(3) Consider the following statement from John: ‘If the net electrical flux through a cube is zero, the electric field everywhere on the surface of the cube must be zero.’ Do you agree or disagree? Explain.

If you disagree, draw one situation that would refute it.

(4) Consider the following statement from Mary: ‘If the electric field is zero everywhere on the surface of a sphere, the net electric flux through the sphere must be zero.’ Do you agree or disagree? Explain.

(5) Consider the following statement from Sam about the net electric field at point P inside a hollow non-polarisable non-conducting ball with uniform positive surface charge with a negative point charge near it, as shown: ‘The net electric field everywhere inside the hollow region must be zero. For example, if we draw a spherical Gaussian surface passing through point P inside the non-conducting ball, it does not enclose any net charge.’

Do you agree or disagree? Explain.

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