Coherent ray-wave structured light based on (helical) Ince-Gaussian modes

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The topological evolution of classic eigenmodes including Hermite-Laguerre-Gaussian and (helical) Ince-Gaussian modes is exploited to construct coherent state modes, which unifies the representations of traveling-wave (TW) and standing-wave (SW) ray-wave structured light for the first time and realizes the TW-SW unified ray-wave geometric beam with topology of ray-trajectories splitting effect, breaking the boundary of TW and SW structured light. We experimentally generate these new modes with high purity and dynamic control by digital holography method, revealing potential applications in optical manipulation and communication.

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Fig. 1. A SW ray-wave mode is composed by two TW ray-wave modes in opposite directions. z ranges from 0 to 2zR. The lower-right inserts show corresponding phases at focus.

teristics and intriguing patterns individually. For instance, the complex SW mode composed by vector vortex TW modes was generated to simulate entangled beating and applied in optical machining [4]. This concept can also be introduced in exotic ray-wave structured light, as shown in Figs. 1, and the TW and SW ray-wave geometric beams unveil the ray-like structures of light for propagating in freespace [15, 16] and oscillating in resonator [9, 17], respectively. However, the topological connection of TW and SW ray-wave modes has never been studied in either theory or experiment. Notably, such ray-wave structure has recently extended the frontier of modern physics such as nondiffractive effect [18], topological phase [19], and quantum-classical informatics [20]. Therefore, breaking the TW-SW boundary in ray-wave structured light is significant for unveiling more general topological evolution and potential applications.

In this Letter, a unified representation for topological evolution of TW and SW structured light is proposed and we construct the novel geometric beams with splitting ray-wave structure and mixing ray-wave structure that have not been observed before. We explore the exotic ray-wave structure in generalized geometric beams by constructing the cluster of classical trajectories. The simulated and experimental results intuitively demonstrate the topological evolution of our generalized structured light.

The PWE has various analytical forms in different coordinates. In cylindrical coordinate (r, φ, z), the eigenmodes are TW LG modes and SW LG modes [21]. The TW LG modes contain spiral phase factor e^iφ, noted as LG_{p,l}, where p, l are radial, azimuthal, and longitudinal indices (p = min(n, m) and
where wave structured geometric beams with striking ray-trajectory (orbits of a planar TW geometric beam gradually split into two studied in [15–17]. But we create new modes as the transitional frequency coupling among transverse and longitudinal modes [9, 14, 23]:

Hereinafter, we will demonstrate a family of TW-SW unified ray-wave structured light. In above, we present the generalized eigenmode family for to unify TW and SW ray-wave structured light. In above sections, we present the generalized eigenmode family to unify the topological evolution of TW and SW eigenmodes, which allows us to explore more complex coherent structured mode as on-demand superposed spatial wavepacket of eigenmodes. Hereinafter, we will demonstrate a family of TW-SW unified ray-wave structured geometric beams with striking ray-trajectory splitting and merging properties that have not been observed before. We exploit the formation of SU(2) coherent state to construct such modes [9, 14, 23]:

\[
\Phi_{n,m,l}(x,y,z) = \frac{1}{2N^{1/2}} \sum_{k=0}^{N} \left( \begin{array}{c} N \\ k \end{array} \right)^{1/2} e^{iK\phi} \psi_{n,m,l}(x,y,z),
\]

(1)

where \( \phi \) is coherent phase, \((p,q,s)\) are three integers related to frequency coupling among transverse and longitudinal modes \( Q = p + q, s = -P, (P,Q) \) are a pair of coprime integers for fulfilling frequency-degenerate condition [9], \( \Psi_{n,m,l} \in \{ \psi_{n,m,l}^{\text{HG}}, \psi_{n,m,l}^{\text{LLG}} \} \). The corresponding topological evolution of spatial wavepacket for \((n,m) = (10,0)\) and \((p,q) = (Q,0)\) is shown in Figs. 2 d1-d10 and e1-e10. The SW and TW geometric beams with oscillating and propagating ray orbits were studied in [15–17]. But we create new modes as the transitional state between them, as shown in Figs. 2 c1-c4 and d1-d4, ray orbits of a planar TW geometric beam gradually split into two fold and distribute into oscillating trajectory of the circular SW geometric beam, which unify the topological evolution of TW and SW geometric beams. Besides, the generalized geometric beam also includes the evolution from a planar trajectory beam into the circular trajectory beam carrying OAM (Figs. 2 c4-c7). Furthermore, we could utilize HIG modes to extend more exotic transformation from the OAM state into a mixing planar trajectory beam (Figs. 2 c7-c10), not proposed before either.

We also explore the higher-order formation of such coherent state mode, as the results for \((n,m) = (10,3)\) and \((p,q) = (Q,0)\) demonstrated in Figs. 2 f1-f10, g1-g10 and h1-h10, where the light on each ray state changes into higher-order HLG mode formation, namely the multi-axis vortex beam [24]. Here we markedly generalize such multi-vortex geometric beam that the TW multi-HG beam can topologically evolve into SW multi-LG beam (Figs. 2 g1-g4) and can also evolve into TW multi-LG beam (Figs. 2 g4-g7) and further into multi-mixing HG beam (Figs. 2 g7-g10). The newly proposed ray-wave geometric beams (in red and blue boxes of Figs. 2) largely enrich the structured light family and inspire the tailoring of more exotic structured light.

To explore ray-wave structure with splitting ray orbits. Due to the quantum-classical correspondence nature of coherent state, the coherent state geometric beam shows intriguing ray-wave duality [9], i.e. the wave pattern of which is localized on a cluster of classical ray trajectories. For TW modes, the cluster of classical trajectories \( \{ x_{p}^{b}, y_{p}^{b}, z|a = \pm \} \) is coupled with spatial wave packet of Eq. (1) for \( \psi_{n,m,l}^{\text{LIG}} = \psi_{n,m,l}^{\text{LLG}}(x,y,z|a) \) (geometric beams based on HLG modes) [25], where \( a \) ranges from \(-\pi/4 \) to \( \pi/4 \), \( \pm \) represents two opposite directions, \( (x_{p}^{b}, y_{p}^{b}, z) \) represents a classical trajectory labelled \( s \) where spatial wave packets are located on, and \( \{ x_{p}^{b}, y_{p}^{b}, z|a = \pm \} \) a collection of such classical trajectories, where \( s = 0, 1, \cdots, Q - 1 \) labels the ray number (see detailed expressions in Supplement). Here we select a ranging from 0 to \( \pi/4 \) for TW modes, corresponding to the mode evolution from planar to circular classical orbits, as shown in Figs. 2 c4-c7 and f4-f7. Furthermore, we can construct the cluster of classical trajectories for generalized geometric beams.

SW modes can be decomposed into two TW modes, which reveals that the cluster of classical trajectories of SW geometric beams (shown in Figs. 2 c1 and f1) can be interpreted as a superposition of two clusters of classical trajectories of TW modes in opposite directions as:

\[
\begin{align*}
\{ x_{p}^{b}, y_{p}^{b}, z \}^{\text{SW}} &= \{ x_{p}^{b}, y_{p}^{b}, z | a = \pi/4 \} + \{ x_{p}^{b}, y_{p}^{b}, z | a = -\pi/4 \}.
\end{align*}
\]

Since SW modes correspond to the case \( e = 0 \), the \( \{ x_{p}^{b}, y_{p}^{b}, z | e = 0 \}^{\text{IG}} \) can be noted as a limiting case of geometric beams based on HLG modes, revealing exotic splitting orbits in evolution of SW-TW modes with \( e \) increasing from 0 to \( \infty \) (see details in Supplement), as shown in Figs. 2 c1-c4, f1-f4.

Besides, the mixing HG modes (HIG modes with \( e \to \infty \)) are essentially equal to a superposition of two HG modes with different indices, as shown in Figs. 2 c8-c10, f8-f10, where the cluster of classical trajectories coupled with spatial wave packet of Eq. (1) for \( \psi_{n,m,l}^{\text{LIG}} = \psi_{n,m,l}^{\text{HG}}(x,y,z|e) \) (geometric beams based on HIG modes), revealing exotic mixing orbits with \( e \) increasing from 0 to \( \infty \). The exotic TW-SW-unified ray-wave structures reveal the generalized ray-wave duality in generalized geometric beams. Besides, the generalized geometric beam also includes the evolution from a planar trajectory beam into the circular trajectory beam carrying OAM (Figs. 2 c4-c7). Furthermore, we could utilize HIG modes to extend more exotic transformation from the OAM state into a mixing planar trajectory beam (Figs. 2 c7-c10), not proposed before either.
Fig. 2. The topological evolutions of TW-SW unified eigenmodes (a, b), geometric beams \((n, m) = (10, 0)\) (c, d, e), and higher-order geometric beams \((n, m) = (10, 3)\) (f, g, h). Panels (a1-a10), (d1-d10), and (g1-g10) show the corresponding three-dimensional spatial wave packets. Panels (c1-c10) and (f1-f10) show the corresponding classical trajectories for geometric beams and higher-order geometric beams, respectively. \(z\) ranges from 0 to \(2\pi R\). Panels (b1-b10), (e1-e10), and (h1-h10) show the corresponding transverse intensity and phase distributions at \(z = 0\) plane. The subplots in red box exhibit the splitting ray-wave structure and the subplots in blue box exhibit the mixing ray-wave structure. (Colormap: darkness to brightness means 0 to 1 for intensity and \(-\pi\) to \(\pi\) for phase.)
beam, providing a deeper physical insight of quantum-classical correspondence (ray-wave duality).

Experimental realization. We experimentally generate these complex modes with high-purity based on the classic digital holography method by a digital micromirror device [26, 27]. Experimental results of TW-SW unified structured light are shown in Figs. 3, where rows from top to bottom are the patterns of TW-SW unified eigenmodes, multi-path geometric beams and multi-HLG higher-order geometric beams, respectively, recorded at $z = 0$ plane, corresponding to simulated results in Figs. 2.

Discussion. Our model of TW-SW-unified structured light is largely extensible. For instance, it can also be applied to more complex SU(2) coherent state corresponding to generalized ray-wave Lissajous and trochooidal wavepacket [28, 29]. We can also study its general structure in astigmatic and vectorial optical fields. In addition, other kinds of coherent superposed formations are also expected to explore, such as SU(1,1) coherent state [30], and hybrid coherent state [31]. The TW-SW unification also act as a new mechanism to extend topological structure, so as to enable novel applications. The ray-trajectory-splitting topology provides new degrees of freedom to create multi-partite classical entangled state [20], which can be employed in high-speed optical encryption and communication [5]. The multi-singularity and complex OAM evolution of the new structured light is also in need of advanced optical tweezers and trapping [3].

In summary, we propose a new theory to unify the TW and SW formations of structured light. It generalizes the family of ray-wave geometric modes based on TW-SW-unified eigenmodes (IG, HLG, and HIG modes), extending the new topological ray-wave structures as thier complex coherent states. The generalized theoretical framework has strong extensibility and applicability to construct more complex modes and to study OAM with multi-singularities, which inspires the exploration of more topological properties of novel structured modes with their advanced applications.

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Fig. 3. Experimental results of the topological evolutions of TW-SW unified eigenmodes (a), geometric beams (b), and higher-order geometric beams (c). See Visualization 1, Visualization 2, Visualization 3 for dynamic movies of (a1-a10), (b1-b10), and (c1-c10).

REFERENCES
1. A. Forbes, Opt. Photonics News 31, 24 (2020).
2. Y. Shen, X. Wang, Z. Xie, C. Min, X. Fu, Q. Liu, M. Gong, and X. Yuan, Light, Sci. & Appl. 8, 90 (2019).
3. E. Otte and C. Denz, Appl. Phys. Rev. 7, 041308 (2020).
4. E. Otte, C. Rosales-Guzmán, B. Ndagano, C. Denz, and A. Forbes, Light, Sci. & Appl. 7, 18009 (2018).
5. B. Ndagano, B. Perez-Garcia, F. S. Roux, M. McLaren, C. Rosales-Guzman, Y. Zhang, Q. Mouane, R. I. Hernandez-Aranda, T. Konrad, and A. Forbes, Nat. Phys. 13, 397 (2017).
6. M. A. Bandres and J. C. Gutiérrez-Vega, JOSA A 21, 873 (2004).
7. V. Koltay and A. Kovařík, JOSA A 31, 274 (2014).
8. Y. Shen, Z. Wan, X. Fu, Q. Liu, and M. Gong, JOSA B 35, 2940 (2018).
9. Y.-F. Chen, C. Jiang, Y.-P. Lan, and K.-F. Huang, Phys. Rev. A 69, 053807 (2004).
10. Y. Shen, X. Yang, X. Fu, and M. Gong, Appl. Opt. 57, 9543 (2018).
11. T. Allieva and M. J. Bastiaans, Opt. Lett. 30, 1461 (2005).
12. E. Abramochkin and T. Alieva, Opt. Lett. 42, 4032 (2017).
13. Y. Shen, Y. Meng, X. Fu, and M. Gong, JOSA A 36, 578 (2019).
14. Y. Shen, Z. Wang, X. Fu, D. Naidoo, and A. Forbes, Phys. Rev. A 102, 031501 (2020).
15. C.-H. Chen and C.-F. Chiueh, Opt. Express 15, 12692 (2007).
16. Y. Shen, X. Fu, and M. Gong, Opt. Express 26, 25545 (2018).
17. T.-H. Lu and C. He, Opt. Express 23, 20876 (2015).
18. A. Zannotti, C. Denz, M. A. Alonso, and M. R. Dennis, Nat. Commun. 11 (2020).
19. T. Malhotra, R. Gutiérrez-Cuevas, J. Hassett, M. Dennis, A. Vamivakas, and M. Alonso, Phys. review letters 120, 233602 (2018).
20. Y. Shen, I. Nape, X. Yang, X. Fu, M. Gong, D. Naidoo, and A. Forbes, Light, Sci. & Appl. (in press) (2021).
21. M. W. Beijersbergen, L. Allen, H. Van der Veen, and J. Woerdman, Opt. Commun. 96, 123 (1993).
22. A. E. Siegman, Lasers (University Science, Mill Valley, CA, 1986).
23. V. Bužek and T. Quang, JOSA B 6, 2447 (1989).
24. P. Tuan, Y. Hsieh, Y. Lai, K.-F. Huang, and Y.-F. Chen, Opt. Express 26, 20481 (2018).
25. Y. Chen, S. Li, Y. Hsieh, J. Tung, H. Liang, and K.-F. Huang, Opt. Lett. 44, 2649 (2019).
26. Y.-X. Ren, R.-D. Lu, and L. Gong, Annalen Der Physik 527, 447 (2015).
27. Z. Wan, Z. Wang, X. Yang, Y. Shen, and X. Fu, Opt. Express (2020).
28. Y.-F. Chen, Y. Lin, K.-F. Huang, and T.-H. Lu, Phys. Rev. A 82, 043801 (2010).
29. Z. Wan, Z. Wang, X. Yang, Y. Shen, and X. Fu, Opt. Express 28, 31043 (2020).
30. K. Wodkiewicz and J. Eberly, JOSA B 2, 458 (1985).
31. Y. Shen, X. Yang, D. Naidoo, X. Fu, and A. Forbes, Optica 7, 820 (2020).