Minimum TR radiofrequency-pulse design for rapid gradient echo sequences

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Purpose: A framework to design radiofrequency (RF) pulses specifically to minimize the TR of gradient echo sequences is presented, subject to hardware and physiological constraints.

Methods: Single-band and multiband (MB) RF pulses can be reduced in duration using variable-rate selective excitation (VERSE) VERSE for a range of flip angles; however, minimum-duration pulses do not guarantee minimum TR because these can lead to a high specific absorption rate (SAR). The optimal RF pulse is found by meeting spatial encoding, peripheral nerve stimulation (PNS) and SAR constraints. A TR reduction for a range of designs is achieved and an application of this in an MB cardiac balanced steady-state free-precession (bSSFP) experiment is presented. Gradient imperfections and their imaging effects are also considered.

Results: Sequence TR with low-time bandwidth product (TBP) pulses, as used in bSSFP, was reduced up to 14%, and the TR when using high TBP pulses, as used in slab-selective imaging, was reduced by up to 72%. A breath-hold cardiac exam was reduced by 46% using both MB and the TR-optimal framework. The importance of RF-based correction of gradient imperfections is demonstrated. PNS was not a practical limitation.

Conclusion: The TR-optimal framework designs RF pulses for a range of pulse parameters, specifically to minimize sequence TR.

KEYWORDS
cardiac, radiofrequency pulse design, rapid imaging, simultaneous multislice (SMS)

1 INTRODUCTION

Rapid gradient echo sequences, including spoiled gradient echo (SPGR) and steady-state free precession (SSFP) are used across MRI for a wide range of clinical and research applications. These sequences typically acquire one line of k-space per radiofrequency (RF) excitation; therefore, minimizing TR is key to fast image acquisition. This is particularly relevant for cine cardiac imaging in which the TR determines the breath-hold duration. Furthermore, reducing TR for bSSFP also limits the impact of banding artifacts, which can become problematic for challenging $B_0$-shimming scenarios such as cardiac applications, especially at 3T and above. The minimum TR achievable is determined by multiple factors including the durations of image encoding gradients, peripheral nerve stimulation (PNS) predictions, and specific absorption rate (SAR). The latter is particularly limiting for applications...
requiring high flip angles (FAs) or using high-energy pulses, for instance, when multiband (MB) pulses are used for simultaneous multislice (SMS) imaging.\textsuperscript{5-10} For example, SAR is usually the limiting factor for cardiac bSSFP at 3T and above, and can prevent sequences from using FAs that would maximize contrast between myocardium and blood.\textsuperscript{11,12}

Much recent RF pulse design development has focused on minimum-duration techniques subject to hardware constraints, which are shown to benefit greatly from time-variable selection gradients designed using time-optimal variable rate selective excitation (VERSE),\textsuperscript{13} nonuniform Shinnar–Le Roux design,\textsuperscript{14} or optimal control.\textsuperscript{15} This has been particularly effective in improving MB RF pulses.\textsuperscript{16-18} However, the problem of minimizing RF pulse duration and minimizing TR are not equivalent; indeed, reducing pulse duration can actually increase the minimum achievable TR because it increases the pulse energy. Beqiri et al\textsuperscript{19} proposed a strategy for minimizing the TR of rapid sequences in the context of RF shimming with parallel transmission, using a nested optimization that directly minimizes the TR. In this work, we propose a similar strategy for the design of RF excitation pulses with time-variable selection gradient waveforms for single-channel RF transmission. Time-variable selection gradients also effect PNS calculations, which we investigate. This is applied to both single and MB pulse designs, and their application to SMS cardiac bSSFP imaging is demonstrated in vivo. This work has partially been presented as conference proceedings.\textsuperscript{20,21}

2 | THEORY

A TR of a rapid gradient echo sequence consists of an excitation and an encoding period. The image encoding gradients can be altered by changing the image resolution, read-out bandwidth, the maximum gradient amplitude, and slew rate. The duration of the encoding period $T_{enc}$ depends directly on scan-specific parameters as mentioned above, and so is considered to be outside of the scope of the optimization proposed in this work. The overall TR is then determined by the sum of $T_{enc}$ (which is fixed) and the RF pulse duration $T_{pulse}$ such that

$$TR = T_{pulse} + T_{enc}. \quad (1)$$

For an RF pulse waveform $B_1(t)$ we may define the following measure that is proportional to RF energy:

$$E_R = \int_0^{T_{pulse}} B_1(t)^2 \, dt \quad (2)$$

expressed in $\mu T^2$ ms. Assuming a steady-state sequence with constant FAs, RF energy can be related to TR and SAR using the relationship

$$SAR = \frac{E_{RF}}{TR} \quad (3)$$

where SAR is expressed in $W/kg^{-1}$, and $k$ is a conversion factor in $W/kg^{-1}, \mu T^{-2}$ relating to the efficiency of the RF chain.\textsuperscript{19} This factor is typically determined from simulations. In addition, RF power amplifiers typically have duty-cycle constraints on the fraction of “on-time,” $\delta_0$, which can be related to sequence TR as

$$\frac{T_{pulse}}{TR} \leq \delta_0 \quad (4)$$

Combining the limitations on TR from Equations (1), (3) and (4), the minimum achievable TR is then determined by:

$$TR_{min} = \max \left\{ \frac{T_{enc} + T_{pulse}}{T_{pulse}} \cdot \frac{\delta_0}{E_{RF} k \cdot SAR_{lim}} \right\} \quad (5)$$

where $SAR_{lim}$ is the regulatory set SAR-limit. A crucial observation from this equation is that RF pulse designs, which solely aim to minimize $T_{pulse}$ may not lead to the minimum sequence TR because the SAR restrictions may force a longer TR, whereas on increasing $T_{pulse}$ can induce RF duty-cycle restrictions.

For on-resonance RF pulses with fixed shape, it can be shown\textsuperscript{19} that

$$E_{RF} = \frac{\theta^2 \delta_2}{\gamma^2 \delta_1^2 T_{pulse}} \quad (6)$$

where $\theta$ is the FA, $\gamma$ is the gyromagnetic ratio, and $\delta_1$ and $\delta_2$ are the normalized integrals of the pulse waveform and the square of the pulse waveform, respectively. These dimensionless values are properties of the pulse shape and are defined as

$$\delta_1 = \frac{\int_0^{T_{pulse}} B_1^2(t) \, dt}{\max \{ |B_1(t)| \} \times T_{pulse}} \quad (7)$$

$$\delta_2 = \frac{\int_0^{T_{pulse}} B_1^2(t) \, dt}{\max \{ |B_1(t)| \} \times T_{pulse}}. \quad (8)$$
In the common case of fixed RF shapes with constant valued-selection gradients, these values do not vary with pulse duration; therefore, selecting the minimum TR is a straightforward problem that can be solved by finding the optimal $T_{\text{pulse}}$ that minimizes $TR_{\text{min}}$ via Equation (5). Recent “minimum-duration” RF-pulse design methods use time-variable gradient waveforms to maximize performance within gradient and RF constraints. In this case, $ERF$ is no longer a simple function of $T_{\text{pulse}}$ because the pulse shape parameters $\delta_1$ and $\delta_2$ themselves change as the peak amplitude changes. This is illustrated in Figure 1A-C.

2.1 Peripheral nerve stimulation

The use of time-variable selection gradients raises the question of whether PNS should be considered as a physiological constraint in addition to SAR. This was a topic in a recent article on minimizing TR for rapid phase-contrast imaging. PNS is caused by fast switching gradients and depends on the rate and strength of time-varying fields, as well as directionality with respect to anatomy. The “SAFE” model is often used in practice to predict stimulation. The model considers the first-derivative of a gradient waveform $G(t)$ as a peripheral nerve stimulus and low-pass filters this using a combination of different weightings:

$$SAFE(G(t)) = a_1 e^{-\frac{t}{\tau_1}} \int \frac{dG(t)}{dt} dt + a_2 e^{-\frac{t}{\tau_2}} \int \frac{dG(t)}{dt} dt.$$  \hspace{1cm} (9)

The threshold for PNS is assumed to have been exceeded when the SAFE model output exceeds some set limit. The coefficients used are prescribed by the vendor implementation of this method and relate to the hardware.

2.2 Proposed minimum TR design approach

The following section introduces a general optimization framework to the problem of minimum-TR RF pulse design, followed by a description of the more limited implementation used in this work.

The TR can be minimized by the appropriate choice of RF pulse waveform $B_1(t)$ and corresponding gradient $G(t)$, which together are written as control variable $x$. Assuming a fixed encoding time $T_{\text{enc}}$, the minimization of TR is achieved by minimizing the RF pulse-duration subject to hardware limitations (similar to Lee et al\cite{13}), as well as “sequence-level” constraints, which can be recast from the entries of Equation (5), and also a PNS constraint.

FIGURE 1 (A) Constant gradient pulses and (B) variable-rate selective excitation (VERSE) pulses computed for the same design parameters (flip angle = 40°, time bandwidth product [TBP] = 4, slice thickness = 5 mm). (C) The shape parameters of the VERSE’d pulses are variable as $B_{1\text{control}}$ changes, whereas the constant gradient pulses have fixed parameters. (D) The different relationships that determine minimum TR are all represented here. The shaded areas are unattainable, and all points on the specific absorption rate (SAR) curves are SAR constrained. The minimum TR is the maximum of all curves at a given pulse duration; the overall minimum TR is highlighted by a circle for each case. Eq, equation; RF, radiofrequency.
\[
\begin{align*}
\min_{x} T_{\text{pulse}} (x) \quad & \text{subject to} \quad (10) \\
\max |B_1(t)| & \leq B_1^{\text{max}} \quad (10.1) \\
\max |G(t)| & \leq G^{\text{max}} \quad (10.2) \\
\max \left| \frac{dG}{dt} \right| & \leq S^{\text{max}} \quad (10.3) \\
T_{\text{pulse}} (x) & \leq \frac{\delta_0}{1 - \delta_0} T_{\text{enc}} \quad (10.4) \\
\frac{E_{\text{RF}} (B_1(t))}{S_{\text{lim}}} (x) - T_{\text{pulse}} (x) & \leq T_{\text{enc}} \quad (10.5) \\
PNS_{\text{model}} (G(t), G_{\text{sequence}} (t)) & \leq PNS^{\text{max}} \quad (10.6)
\end{align*}
\]

where \(B_1^{\text{max}}\), \(G^{\text{max}}\), \(S^{\text{max}}\) correspond to hardware limitations on RF amplitude, gradient amplitude, and gradient slew-rate, respectively. \(PNS_{\text{model}}\) represents a function, such as SAFE, which returns a PNS evaluation based on the selection gradient \(G(t)\) and the remaining sequence gradients \(G_{\text{sequence}} (t)\), and \(PNS^{\text{max}}\) is a set threshold value. The constraint in Equation (10.4) could be considered a hardware limitation; however, because it depends on the sequence parameter \(T_{\text{enc}}\), we consider it a sequence-level constraint.

Rather than approach this as a general optimization problem in which both RF pulse and gradient waveforms are designed, we propose a simple method to optimize performance for a given initial RF pulse shape. For an input RF pulse (typically designed for a constant gradient), the time-optimal VERSE\textsuperscript{13} method is used to design a minimum-duration gradient waveform (and reshape the RF pulse waveform) subject to the hardware constraints in Equations (10.1)-(10.3). To satisfy the remaining constraints, a family of solutions is generated by altering the maximum \(B_1\) specified for the VERSE algorithm. We refer to this limit as \(B_1^{\text{control}}\) to distinguish this from the hardware-related maximum \(B_1^{\text{max}}\). From this family, the solution that minimizes \(T_{\text{pulse}}\) while satisfying constraints (10.4)-(10.6) is selected. The PNS constraint in Equation (10.6) can be assumed inactive for most practical imaging scenarios, which we will discuss later. A list of steps for our implementation is summarized in Table 1.

Although not general in scope, the advantages of this method are that we can start with an existing RF pulse waveform that has desired characteristics, and that VERSE (which is a deterministic algorithm) avoids convergence issues caused by local minima.

Figure 1 illustrates a single-band (SB) design case (\(FA = 40^\circ\), time bandwidth product [TBP] = 4) using this method for minimizing TR in a constant gradient and VERSE (time-variable gradient) case. Figure 1A shows different constant gradient pulses; Figure 1B shows different VERSE pulses computed for a range of \(B_1^{\text{control}}\). Figure 1C demonstrates that the pulse shape properties \(\delta_1\) and \(\delta_2\), relating to \(E_{\text{RF}}\), change significantly for the VERSE’d pulses as \(B_1^{\text{control}}\) is changed, and Figure 1D illustrates the application of sequence-level constraints in Equations (10.4) and (10.5). The minimum TR for each case is marked by a circle; in this case, using VERSE results in a reduction in minimum TR of approximately 20% compared with a constant gradient. To achieve this result, the required value of \(B_1^{\text{control}}\) was 7.08 \(\mu\)T, which would not have been obvious a priori. Simply minimizing pulse duration subject to hardware constraints would yield a solution with a TR over 7 ms, after sequence-level constraints are applied. This illustrates the importance of including sequence level constraints as part of the RF pulse design. The PNS constraint in Equation (10.6) is not shown; it was not a limiting factor for the optimal solution—this will be addressed later.

### Table 1: Processing steps for minimum-TR pulses

| Step | Description |
|------|-------------|
| 1. | For \(B_1^{\text{control}} = 2 \mu\)T: \(B_1^{\text{max}} \pm 20\) points |
| 2. | Run-time-optimal VERSE \((B_1^{\text{control}}, G^{\text{max}}, S^{\text{max}})\) to satisfy Equations (10.1)-(10.3) |
| 3. | Store \(E_{\text{RF}}, T_{\text{pulse}}\) |
| 4. | Run-time-optimal VERSE with optimal \(B_1^{\text{control}}\) |

Abbreviations: TR, pulse repetition time; VERSE, variable-rate selective excitation.

### 3 | METHODS

#### 3.1 | RF pulse design

This work considered both SB- and MB-pulse design and has used the time-optimal VERSE\textsuperscript{13} method and the recently proposed VERSE-MB method\textsuperscript{17}, respectively, for these design problems. Within the context of this article, these algorithms are examples of general design methods to evaluate a pulse set \(\{B_1(t), G(t)\}\) given pulse properties such as flip-angle, time-bandwidth product, and number of excited slices. The resulting TR-optimized designs as a consequence of VERSE will be referred to as “TR-optimal,” and TR-optimized constant gradient pulses will be referred to as “constant gradient.” For all examples, the limits \(G^{\text{max}} = 31\) mT/m\(^{-1}\) and \(S^{\text{max}} = 200\) mT/m\(^{-1}\)/ms\(^{-1}\) were used, reflecting the performance...
limits of the scanner used for experiments. Both design methods start with a SB constant gradient RF pulse waveform, and then compress this in time using VERSE for the SB case, and then further apply temporal modulation after VERSE for the MB case. Amplitude-only modulation was used to avoid known hardware issues with RF fidelity.26,27 The TR-optimal pulses were always compared against matched constant gradient pulse waveforms (ie, same starting pulse shape), which were also optimized to minimize the TR by adjusting their peak $B_1$ amplitude accordingly. In all designs, we constrained the gradient amplitude to start and end at zero. Quoted pulse durations include the selection gradient ramp-up and ramp-down times for TR-optimal designs but exclude them for constant gradient pulses. This is because in the latter case the ramps can be overlapped with other gradients in the pulse sequence, whereas this is not true for TR-optimal designs as the RF is generally on during this time.

Two example applications were designed for this work: (1) bSSFP with low TBP pulses (TBP = 2.13) and (2) high TBP (TBP = 4, 6, 8, 10) for imaging where spatial selectivity may be more important, such as slab-selective excitation for three-dimensional (3D) encoding. For both cases, we investigated the number of simultaneously excited slices $N_{sl}$ from 1–6 (1 being equivalent to single-slice imaging), with slice-thickness 5 mm. For $N_{sl} > 1$, interslice spacing (center-to-center) was computed to cover an imaging field of view of 100 mm in the through-slice direction. Both applications were designed for FAs from 25°–90°. All initial pulses on which the study was based were taken from the vendor pulse library.

To examine off-resonance effects on the slice-excitation profile because of the use of time-variable selection gradients, off-resonance analysis was conducted using Bloch equation simulations. Results are shown in Supporting Information Figure S1.

### 3.2 Minimum TR calculation

Equation (10) could be approached as a nonlinear optimization problem; however, because there is only a single design variable used in this work ($B_1^{\text{control}}$), we have taken the simpler approach of tabulating the relationship between $E_{\text{RF}}$ and $T_{\text{pulse}}$ for each design scenario by precomputing RF pulse designs for a range of $B_1^{\text{control}}$ from 2 $\mu$T to 13 $\mu$T (Table 1). The TR-optimal design can be found by searching for the intersection between the curves indicated by Equation (5) plotted on Figure 1D. The curves were precomputed using 20 points, after which the optimal $B_1^{\text{control}}$ was determined to higher precision by interpolation. Once determined, an RF pulse specifically corresponding to this optimal $B_1^{\text{control}}$ was designed.

The optimal solution will depend on the $T_{\text{enc}}$ prescribed. An advantage of this ad hoc tabulation approach is that the same precalculated family of pulses can be used with different readout geometries that alter $T_{\text{enc}}$, thus saving computational cost to redesign new pulses at run-time.

From earlier experience, we found PNS not to be an issue for typical imaging sequences; therefore, we did not explicitly enforce constraint [Equation (10.6)] for the design problems presented and did not find that PNS limits were violated by doing this. Instead, we have investigated the PNS behavior for a single imaging scenario between a constant gradient and a TR-optimal example.

All simulations used the International Electrotechnical Commission (IEC) 10-second local SAR limit ($\text{SAR}_{\text{lim}} = 20$ W/kg$^{-1}$)28; the factor $k$ was computed to relate pulse energy to maximum local SAR for the transmit coil used. The encoding time $T_{\text{enc}}$ was set to 1.79 ms in the low-TBP–bSSFP case, which was the optimized encoding time for the acquisition described in the methods below, and in the high TBP cases for slab-selective imaging the encoding time was 3.39 ms, which was also chosen for an optimized 3D bSSFP sequence. An RF duty-cycle limit of 50% “on time” (ie, $\delta_0 = 0.5$) was used, corresponding to the scanner used for experimental work.

### 3.3 Slice-shifting and nonideal gradient behavior

The time-variable selection gradients required by VERSE can lead to imperfection issues caused by limited temporal bandwidth of gradient systems. Such imperfections can be incorporated into a gradient impulse-response function (GIRF) model, which characterizes the gradient system response as a linear time-invariant system.29 In this work, a measured GIRF30 was approximated using a Lorentzian function with a time constant of 42 $\mu$s (Supporting Information Figure S2) and used to evaluate adapted RF pulses that correct for the gradient errors. Details of this process are given in Appendix A, and phantom images showing its imaging effects are shown in Supporting Information Figures S3 and S4. The predicted gradient distortion was also used to spatially shift slices away from isocenter, using the shift-theorem described in Conolly et al.31 The corrected RF waveform had a negligible effect on TR optimality, as the increase in RF energy was insignificant.

### 3.4 Peripheral nerve stimulation

The SAFE model was used with parameters $a_1 = 0.78$, $a_2 = 0.22$, $\tau_1 = 0.32$ ms, and $\tau_2 = 4.1$ ms. PNS is most likely induced by the gradient along the anteroposterior (AP) direction when humans are scanned in the supine position. To account for this, each gradient direction was evaluated with the SAFE model in Equation (9) and further scaled by 1,
0.83 and 0.61 for anterior→posterior (AP), left→right (LR), and foot→head (FH) direction, respectively. It is assumed, that when the weighted output of the SAFE model exceeds a preset stimulation level $P_{NS}^{max}$, PNS will occur.

To investigate whether the minimum TR would be affected because of the insertion of a time-variable gradient, we implemented the SAFE model off-line and repeated this for encoding oriented along all three physical gradients axes independently. This way, each encoding gradient (frequency, phase, slice) was simulated on each physical gradient (AP, LR, FH).

### 3.5 | Experiments

Cardiac imaging was performed using a breath-hold retrospective vectorcardiogram-gated cine sequence on a 3T Philips Achieva (Philips Healthcare) and a 32-channel receiver coil. Data were collected on a healthy volunteer (27-year-old man) following written informed consent under local ethical guidelines. MB imaging was performed using blipped-controlled aliasing in parallel imaging (CAIPI) as previously described for cine bSSFP. $N_{Sl} = 2$ RF pulses were designed for a FA of 40°, TBP = 2.13, slice thicknesses = 7 mm, and an interslice spacing (center-to-center) of 49 mm. The in-plane resolution was 1.6 × 2 mm, with 120 phase-encoding steps and 20 cardiac phases. No forms of in-plane acceleration (sensitivity encoding [SENSE] or half-Fourier) were used. $B_0$-shimming was performed using a custom slice-by-slice tool. Aliased MB images were unfolded and reconstructed with a SENSE-based algorithm in ReconFrame (GyroTools GmbH). Software issues arose with user-defined RF and gradient shapes close to the maximum SAR limit of 20 W/kg$^{-1}$ because the custom shapes were strictly forbidden from being stretched and shaped as usual trapezoids, which interfered with the scanner’s original TR minimization. These issues were not present when using the scanner’s native RF pulses. To make a fair comparison for in vivo experiments, all pulses were optimized for a limit of 18.6 W/kg$^{-1}$, which worked reliably. An additional software limitation, when incorporating the arbitrary-shaped gradient waveforms required for VERSE, meant that for the implemented constant gradient pulses, the ramp periods of the selection gradient could no longer be overlapped with the adjacent gradient objects in the pulse sequence. For in vivo comparisons using constant gradient pulses, the minimum TR was calculated accounting for this.

### 4 | RESULTS

Figure 2 shows the minimized TR for the case of low TBP (2.13) MB-pulse design, with slice thickness 5 mm for both constant and TR-optimal designs in Figure 2A,B, respectively. Relative performance between constant gradient and TR-optimal designs are shown in Figure 2C. Although both are optimized for TR, using TR-optimal designs reductions in TR of approximately 10% can be achieved for moderate FAs and number of excited slices ($N_{Sl}$), and, for example, 14.2% for $N_{Sl} = 6$ and a 90° FA.

An example is the reduction in TR of 12.3% from 4.93 ms to 4.32 ms, which can be achieved for a $N_{Sl} = 3$ and 60° excitation, the example pulses for which are shown in Figure 3A,B. However, it is notable that for the case of a 25° FA and SB excitation, there is a longer TR for the TR-optimal case. This case is further examined in Figure 3C,D, which plots this example alongside the 60°, $N_{Sl} = 3$ case. The reason for the lengthening of the TR for 25° is that this pulse is essentially limited by gradient amplitude and slew rate. Our implementation cannot improve on this because

![Figure 2](image-url)
the TR-optimal design cannot overlap the gradient ramps whereas the constant gradient could, and so the TR actually increases. The examples in Figure 3A,B are operating at the SAR limit and RF duty-cycle limit [i.e., Equations (10.5) and (10.4), respectively], whereas the examples in Figure 3C,D are operating at the SAR limit and sequence-timing limit [Equations (1) and (10.5)].

Figure 4 shows minimum TR results for TBP 4–10. It suggests that much larger improvements can be obtained if higher TBP pulses are used, particularly at higher FAs. For SB pulses, TR reduction of 10%–40% is possible. Similar performance was found for slice thicknesses up to 60 mm (not shown), making these results applicable to slab-selective imaging. For a MB (N_{Sl} > 1) sequence, a TR reduction of 40% is commonly possible and can reach 72% in the case of N_{Sl} = 6 and a 90° FA.

Figure 5 shows an example for a SB TBP = 6 example at 60° excitation. The TR can be reduced from 5.95 to 5.05 (15%) as shown in Figure 5A,B. An example with N_{Sl} = 3 reduces TR from 10.24 ms to 6.55 ms (36%) as shown in Figure 5C,D. Note that all pulses shown here are optimized for minimum TR, but a shorter TR is achieved when using time-variable gradients. A minimum-duration approach to design time-variable pulses would, for instance, use the peak B₁ from the constant gradient to design a VERSE pulse. If that were the case, the TR would increase to 7.93 ms and 13.1 ms as shown in 5B,D, respectively. The example given in Figure 5C operates at the SAR and RF duty-cycle limit [i.e., Equations (10.5) and (10.4), respectively], the remaining examples operate at the SAR and sequence-timing limit [Equations (1) and (10.5)].

Figure 6 shows cardiac bSSFP images in a healthy volunteer: T_{enc} = 1.79 ms with TBP = 2.13 pulses in the diastolic phase. On the left is a standard SB acquisition using a single 14.4-second breath-hold. The column to the right shows a N_{Sl} = 2 accelerated acquisition of this using a constant gradient pulse, acquired in an 8.4-second breath-hold. In the next column, a minimum-duration RF pulse was designed to have the same peak RF amplitude as the constant gradient pulse, leading to a short RF pulse that leads to high-RF
energy. However, this leads to a 0.9-ms increase in TR, increasing banding artifacts and the breath-hold period. Using our proposed framework, TR-optimal RF pulse and gradient waveforms allow imaging with a TR of 2.9 ms, leading to a breath-hold of 7.8 seconds. An animated version of this CINE dataset is available in Supporting Information Video S1.

Figure 7 shows an annotated analysis from the same data-set as Figure 6, near the end-systole phase. Banding artifacts
become clear at the edges of the myocardium in the constant gradient MB acquisition in the second column (red arrows) caused by the increase in TR. In the third column, an acquisition with minimum-duration VERSE leads to a non-TR-optimal acquisition with stronger artifacts on the outside of the myocardium, as well as in the blood pool within it, as is indicated by the red arrows. As shown in the fourth column, TR-optimal acquisition performs better, with some banding artifacts still visible at the top of the myocardium.

All TR-optimal pulse designs have been computed using maximum gradient amplitude and slew-rate constraints, raising the question of whether PNS ought to be considered as an additional constraint to achieving the minimum TR. Figure 8 shows example gradient waveforms for constant and TR-optimal gradient designs, their time derivatives, and predicted outputs from the SAFE model for a bSSFP example. The calculation uses repeated TR to account for nonsteady-state effects from Equation (9). This particular example roughly matched our in vivo demonstration, with TR = 2.81 ms, slice thickness = 7 mm, and in-plane resolution = 2 mm. The figure does not include phase-encoding gradients, as these vary through the sequence and in any case are not the major contributors to PNS. The shaded patches in Figure 8C account for the fact that the PNS predictions are orientation dependent because different gradient axes have different PNS sensitivity, AP and FH being the most and least sensitive directions, respectively. The biggest contributors towards PNS are the slopes on either side of the readout gradient. The SAFE-model output shows that replacing a constant gradient with a TR-optimal gradient increases the contribution of the slice-select gradient towards PNS prediction. However, it remains relatively low in comparison with the contribution from the readout gradient. In practice, for both low and high TBP experiments, we did not encounter any reasonable scenarios in which PNS would have limited the solutions obtained.

The off-resonance simulations in Supporting Information Figure S1 show that slice-shifting effects (calculated by maximizing cross correlation) are similar for both constant and TR-optimal pulses. Slice distortion (calculated by normalized root mean square error) on the other hand is worse for TR-optimal pulses than for the constant gradient pulses, and this effect is worse with increasing TBP caused by highly time-variable gradients.
DISCUSSION

Rapid gradient echo sequences such as bSSFP and SPGR are widely used in clinical MR and research applications. In the case of two-dimensional (2D) sequences, typically low-time–bandwidth–product RF pulses are used and pushed beyond their usual operating points when higher slice selectivity or MB imaging is desired. Much work has gone into optimizing RF pulses for minimum duration,\(^{31,34}\) in particular for MB excitation using time-variable–selection gradients.\(^{17,18,22}\) However, for rapid gradient echo sequences, SAR can quickly become a limiting factor; in this case, a shorter TR

(Figure 6) Cardiac multiband2 (MB2) balanced steady-state free-precession experiments using conventional MB2 \((N_{Sl} = 2)\) and two different TR- optimal pulses (time bandwidth = 2.13, flip angle = 40°, slice thickness = 7 mm, \(1.6 \times 2\) mm in-plane resolution. No in-plane acceleration). (A) Shows data acquired without MB using a 14.4-second breath-hold. (B) Using a MB2 pulse designed for minimum TR, this can be reduced to a 8.4-second breath-hold, which comes at a TR increase of 19.2%. The constant gradient MB2 pulse can be optimized in the same way as the \(N_{Sl} = 1\) pulse. (C) A minimum-duration radiofrequency (RF) pulse (non-TR-optimal) designed to have the same peak \(B_1\)-amplitude as the constant gradient MB2 pulse, results in an RF pulse shorter than the MB2 pulse; however, this leads to a 0.8-ms increase in TR caused by high-RF energy. This also leads to a 10.6-second breath-hold. (D) A TR-optimal pulse designed using our minimum TR framework results in a reduced 7.8-second breath-hold with only a 11.5% increase in TR compared with the data acquired with single-slice excitation. S, second

(Figure 7) An annotated version of Figure 6 using multiband2 (MB2; \(N_{Sl} = 2)\) showing just 1 slice, picked at the end-systole phase. (A) Shows the single-band image as a reference. Banding artifacts become most clear at the edges of the myocardium in the constant gradient MB acquisition in (B) (red arrows) caused by the increase in TR. In (C), a MB2 minimum-duration radiofrequency pulse (non-TR-optimal) acquisition leads to more detrimental artifacts at two ends of the myocardium (red arrows) as the band approaches the blood pool. (D) With TR-optimal, acquisition performs better, with minimal banding artifacts left in the myocardium.

5 | DISCUSSION
can often be achieved by lengthening the RF pulse. This is already well-understood within the context of constant gradient RF pulses that can be manipulated by stretching in time to reduce the peak amplitude, but whose shapes remain fixed. The situation is more complicated for minimum-duration RF pulses because their shape properties change as a function of their duration (Figure 1). Hence, this work proposes a method for extending the logic for minimizing TR with constant gradient \textsuperscript{19} pulses to VERSE pulses (or any pulses using time-variable–selection gradients), by employing an optimization framework that finds the optimal set of pulses to minimize TR.

Figure 2 depicts the reductions in TR of approximately 10\% that can be made for low TBP pulses (TBP = 2.13) typically used for rapid 2D imaging. This was demonstrated in vivo for MB cardiac bSSFP imaging (Figures 6 and 7) in which the TR was reduced from 3.1 ms to 2.9 ms, which helps by reducing the breath-hold duration and reducing the impact of bSSFP black-banding artifacts. These figures also show the results of applying minimum-duration pulse design directly—though the pulse is made shorter, in which case the TR increases to 4.0 ms.

A case where the TR-optimal approach failed is shown in Figure 3C,D. This is because the constant gradient is already at its highest amplitude, and the ramps cannot be overlapped with other gradients for the TR-optimal approach as the RF is transmitted concurrently. This can occur generally when the desired pulses have low FAs (ie, $B\_1$ limit is not reached) and thin slices (requiring high-excitation bandwidth). Our implementation did not allow the gradient ramps to be excluded.

**FIGURE 8** Time-resolved input and output waveforms of the SAFE model for peripheral nerve stimulation (PNS) prediction during a multi-TR sequence simulation, here only shown for one TR period. (A) Input gradient waveforms $G_{xyz}$ (for gradient direction x, y, z) for a balanced steady-state free-precession sequence, for a TR-optimal and constant gradient case, selected to match the TR of the TR-optimal case for clarity. The phase-encoding gradients are omitted for clarity. (B) Time-derivative of the gradient waveforms ($dG_{xyz}$) over time (dt), needed by the SAFE model in Equation (9). (C) SAFE model output; shaded areas represent variation expected based on different slice orientation (see text for details). The peak stimulation output relates to the readout gradient in the anteroposterior gradient direction for a supine positioned subject. The TR-optimal, time-variable selection gradient adds more to the model output than the constant gradient (green arrows). However, the readout gradient (blue arrows) has the larger effect towards PNS violation. Const, constant; GR, gradient.
from the VERSE-based design; however, the algorithm does allow this flexibility. A future implementation could be improved by allowing ramps to be excluded when necessary, such that the start and end of the gradient pulse are simply concatenated with neighboring ramp gradients, giving them the same flexibility as the constant gradient case.

The benefit of time-variable gradient waveforms for pulse design becomes more significant when higher TBP and MB factors are employed as shown in Figures 4 and 5. The proposed TR-optimal approach could allow use of higher TBP pulses for less penalty in TR. For example, using constant gradient RF pulses, a $60^\circ N_{sl} = 3$ excitation with TBP = 2 gives minimum TR = 4.93 ms, increasing to 8.46 ms for TBP 4 and 10.24 ms for TBP 6, whereas the TR-optimized designs can achieve minimum TRs of 4.32 ms, 6.25 ms, and 6.43 ms, respectively. For high TBP pulses, the TR-optimal designs also have notably different operating points. For the constant gradient design the optimal scenario is to push to a higher peak $B_1$, whereas the TR-optimal designs have a much lower peak $B_1$ than the system limit of 13 $\mu$T. Simply reducing the pulse duration by increasing the peak $B_1$ would not help here because the SAR limit would be exceeded. The benefits of high TBP slab selection have already been shown by Hargreaves et al34 for cardiac imaging or high-field MR angiography.35

With MB imaging, use of a higher TBP could aid multislab MB angiography,36 improve simultaneous multislice magnetic resonance fingerprinting,37,38 or potentially reduce leakage between slices.39 Going further, because turbo spin-echo sequences can also be SAR-limited, an interesting further investigation could adapt this framework for multi-echo sequences. The common ground between rapid gradient echo and, for example, turbo spin echo sequences is that low-SAR pulses have long duration, which increases echo times. Such a framework would be useful for MB turbo spin-echo,40 gradient- and spin-echo,41 and volumetric imaging techniques.42

A recent study proposed a convex optimization framework to shorten TR for four-dimensional phase-contrast MRI acquisition.5 VERSE was also used for shortening RF pulses; however, no SAR violation was reported. Their framework is conceptually similar to our work but differs by optimizing encoding gradients and derating these where necessary to meet PNS constraints, whereas we optimized selection pulses to meet encoding and SAR constraints. When PNS is not considered in optimization, the most time-efficient pulse will make the most use of the hardware until limited by peak-RF amplitude. We considered PNS using the SAFE model (Figure 8) and concluded that the readout gradient is always the largest contributor. This is inherent when the readout resolution is higher than the through-slice resolution, which is most often the case. Stimulation overheads can be seen on either side of the readout ramps. Thus, when stimulation becomes a limiting factor, an effective approach would be to alter the readout-encoding gradients, such as changing the physical readout direction or reducing the readout bandwidth.

Other SMS cardiac studies have used RF phase-cycling schemes to perform controlled aliasing for anatomic5,8-10,43-45 and quantitative imaging.46,47 Such methods generally require two unique RF pulses per shift in the slice direction. This study used blipped-CAIPI, as did another recent study27 however, our presented framework with the VERSE-MB approach17 is compatible with RF phase-cycling because the gradient is not further optimized after applying the CAIPI modulation. This is because once $B_1^{control}$ is set, the pulse duration becomes fixed; therefore, a CAIPI phase offset does not change RF energy per Parseval’s theorem. If the gradient is optimized after CAIPI, its optimal form will depend on the RF envelope; hence, a time-optimal solution will depend on the phase-cycling scheme.

A limitation of rapidly modulated time-variable gradients is that nonideal gradient performance may affect some types of scanner more than others. In this study, we have built on previous work that characterized these effects using an impulse-response function29 and have used an inherently lower bandwidth design approach for constructing the MB pulses.17 In addition, for this work we corrected the RF-pulse waveforms by resampling using the GIRF-predicted gradient waveform (see the Appendix A). We also used this to correct the frequency modulation that is required to shift the slices off-center. We used a Lorentzian function to approximate the GIRF measurement and found that a GIRF measurement is not required. Note that resampling the pulses does slightly change their energy-duration relationship, but this was not found to significantly change the solutions and so corrections were applied post hoc. Imaging results are shown in Supporting Information Figures S3 and S4.

The method presented for designing TR-optimal pulses can potentially be used with any minimum-duration RF-pulse–design approach, and more efficient design algorithms than the ones used here, such as NU-SLR14 or optimal control15,22 could yield better results. We opted to precalculate a number of solutions, from which the optimum $B_1^{control}$ can be identified quickly by interpolation, after which a single pulse can be designed in seconds. This reduces the need for a complex optimization at run-time, but does require a library of calculations to be performed for different parameters (FAs, TBP, etc.). An alternative would be to solve the minimization in Equation (10) directly. Going further, we did not experience TR improvement from lower limits on maximum gradient amplitude and slew rate; however, we did not rule out better solutions generally. We also did not experiment with gradient waveforms with nonzero start-and-end values. Therefore, in future work these could be introduced as control variables, which could avoid the issue seen in Figure 3C.D. More flexibility on gradient control is expected to become important as PNS becomes a limiting factor. Introducing more control variables and constraints would make our exhaustive tabulation approach less attractive. Instead, because the relation between pulse duration and RF energy (Figure 1D) is smooth
and changing gradient limits affects the pulse duration, a gradient-descent approach could be a plausible alternative.

6 | CONCLUSION

We propose a general framework and simple approach for designing TR-optimal RF pulses that minimize the overall sequence TR by considering hardware and sequence-level constraints. TR can be reduced by up to 14% and 72% when using low and high time-bandwidth profiles, respectively. The benefits are particularly strong for high FAs, MB acceleration factors, and time–bandwidth–product pulses. The flexibility of this framework was demonstrated in MB cardiac bSSFP and should benefit other applications such as quantitative imaging and MR angiography.

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REFERENCES

1. Scheffler K, Lehnhardt S. Principles and applications of balanced SSFP techniques. *Eur Radiol*. 2003;13:2409-2418.
2. Carr JC, Simonetti O, Bundi J, Li D, Pereles S, Finn JP. Cine MR angiography of the heart with segmented true fast imaging with steady-state precession. *Radiology*. 2001;219:828-834.
3. Atkinson DJ, Edelman RR. Cineangiography of the heart in a single breath hold with a segmented turboFLASH sequence. *Radiology*. 1991;178:357-360.
4. Loecher M, Magrath P, Aliotta E, Ennis DB. Time-optimized 4D phase contrast MRI with real-time convex optimization of gradient waveforms and fast excitation methods. *Magn Reson Med*. 2019;82:213-224.
5. Stäb D, Ritter CO, Breuer FA, Weng AM, Hahn D, Köstler H. CAIPIRINHA accelerated SSFP imaging. *Magn Reson Med*. 2011;65:157-164.
6. Price AN, Cordero-Grande L, Malik SJ, Hajnal JV. Simultaneous multislice imaging of the heart using multiband balanced SSFP with blipped-CAIPI. *Magn Reson Med*. 2020;83:2185-2196.
7. Bastiani M, Andersson JLR, Cordero-Grande L, et al. Automated processing pipeline for neonatal diffusion MRI in the developing Human Connectome Project. *Neuroimage*. 2019;185:750-763.
8. Ferrazzi G, Bassenge JP, Wink C, et al. Autoaccelerated multiband CAIPIRINHA with through-time encoding: proof of principle and application to cardiac tissue phase mapping. *Magn Reson Med*. 2019;81:1016-1030.
9. Landes V, Jao T, Nayak K. Practical implementation of SMS bSSFP in the Heart. *Proc Intl Soc Mag Reson Med*. 2018;26:18-20.
10. Wang Y, Shao X, Martin T, Moeller S, Yacoub E, Wang DJJ. Phase-cycled simultaneous multislice balanced SSFP imaging with CAIPIRINHA for efficient banding reduction. *Magn Reson Med*. 2016;76:1764-1774.
11. Schür M, Kozerke S, Fischer SE, Boesiger P. Cardiac SSFP Imaging at 3 Tesla. *Magn Reson Med*. 2004;51:799-806.
12. Srinivasan S, Ennis DB. Optimal flip angle for high contrast balanced SSFP cardiac cine imaging. *Magn Reson Med*. 2015;73:1095-1103.
13. Lee D, Lustig M, Grissom WA, Pauly JM. Time-optimal design for multidimensional and parallel transmit variable-rate selective excitation. *Magn Reson Med*. 2009;61:1471-1479.
14. Grissom WA, McKinnon GC, Vogel MW. Nonuniform and multidimensional Shinnar-Le Roux RF pulse design method. *Magn Reson Med*. 2012;68:690-702.
15. Rund A, Aigner CS, Kunisch K, Stollberger R. Magnetic resonance RF pulse design by optimal control with physical constraints. *IEEE Trans Med Imaging*. 2018;37:461-472.
16. Aigner CS, Clason C, Rund A, Stollberger R. Efficient high-resolution RF pulse design applied to simultaneous multi-slice excitation. *J Magn Reson*. 2016;263:33-44.
17. Abo Seada S, Price AN, Schneider T, Hajnal JV, Malik SJ. Multiband RF pulse design for realistic gradient performance. *Magn Reson Med*. 2019;81:362-376.
18. Grissom WA, Setsompop K, Hurley SA, Tsao J, Velikina JV, Samsonov AA. Advancing RF pulse design using an open-competition format: report from the 2015 ISMRM challenge. *Magn Reson Med*. 2017;78:1352-1361.
19. Beqiri A, Price AN, Padorno F, Hajnal JV, Malik SJ. Extended RF shimming: sequence-level parallel transmission optimization applied to steady-state free precession MRI of the heart. *NMR Biomed*. 2017;30:30:1-30:11.
20. Abo Seada S, Beqiri A, Price AN, Hajnal JV, Malik SJ. Minimum-TR pulse design for rapid gradient echo sequences. *Proc Intl Soc Mag Reson Med*. 2018;26:3403.
21. Abo Seada S, Price A, Hajnal J, Malik S. Cardiac bSSFP accelerated using minimum-TR multiband RF pulse design and GIRF-correction. *Proc Intl Soc Mag Reson Med*. 2019;27:3935.
22. Rund A, Aigner CS, Kunisch K, Stollberger R. Simultaneous multislice refocusing via time optimal control. *Magn Reson Med*. 2018;80:1416-1428.
23. Schmitt F, Irrlich W, Fischer H. Physiological side effects of fast gradient switching. In: *Echo-planar Imaging*. Berlin, Heidelberg: Springer. 1998:201-252.
24. Hebrank F, Gebhardt M, Lenz H. Method and MR device for simulating electrical simulations in a subject by MR stimulation. US Patent 6,169,403 B1. January 2, 2001.
25. Hebrank FX, Gebhardt M. SAFE-model—a new method for predicting peripheral nerve stimulations in MRI. *Proc Intl Soc Mag Reson Med.* 2000;8:2007.

26. Abo Seada S, Price AN, Hajnal JV, Malik SJ. Optimized amplitude modulated multiband RF pulse design. *Magn Reson Med.* 2017;78:2185-2193.

27. Bentatou Z, Troalen T, Bernard M, et al. Simultaneous multi-slice T1 mapping using MOLLI with blipped CAIPIRINA bSSFP. *Magn Reson Imaging.* 2020;127065.

28. International Electrotechnical Commission (IEC). IEC 60601-1-33:2010. Medical electrical equipment-Part 2–33: Particular requirements for the basic safety and essential performance of magnetic resonance equipment for medical diagnosis. 2015; Edition 3.2.

29. Vannesjo SJ, Haebnerlin M, Kasper L, et al. Gradient system characterization by impulse response measurements with a dynamic field camera. *Magn Reson Med.* 2013;69:583-593.

30. Papadakis NG, Wilkinson AA, Carpenter TA, Hall LD. A general method for measurement of the time integral of variant magnetic field gradients: application to 2D spiral imaging. *Magn Reson Imaging.* 1997;15:567-578.

31. Connolly S, Nishimura D, Macovski A, Glover G. Variable-rate selective excitation. *J Magn Reson.* 1988;78:440-458.

32. Ham CLG, Engels JML, van de Wiel GT, Machielsen A. Peripheral nerve stimulation during MRI: effects of high gradient amplitudes and switching rates. *J Magn Reson Imaging.* 1997;7:933-937.

33. Den Boer JA, Bourland JD, Nyenhuis JA, et al. Comparison of the threshold for peripheral nerve stimulation during gradient switching in whole body MR systems. *J Magn Reson Imaging.* 2002;15:520-525.

34. Hargreaves BA, Cunningham CH, Nishimura DG, Connolly SM. Variable-rate selective excitation for rapid MRI sequences. *Magn Reson Med.* 2004;52:590-597.

35. Schmitter S, Bock M, Jost S, Auerbach EJ, Uğurbil K, Van de Moortele P-F. Contrast enhancement in TOF cerebral angiography at 7 T using saturation and MT pulses under SAR constraints: impact of VERSE and sparse pulses. *Magn Reson Med.* 2012;68:188-197.

36. Schulz J, Boyacioglu R, Norris DG. Multiband multislab 3D time-of-flight magnetic resonance angiography for reduced acquisition time and improved sensitivity. *Magn Reson Med.* 2016;75:1662-1668.

37. Ye H, Cauley SF, Gagoski B, et al. Simultaneous multislice magnetic resonance fingerprinting (SMS-MRF) with direct-spiral slice-GRAPPA (ds-SG) reconstruction. *Magn Reson Med.* 2017;77:1966-1974.

38. Hong T, Han D, Kim M-O, Kim D-H. RF slice profile effects in magnetic resonance fingerprinting. *Magn Reson Imaging.* 2017;41:73-79.

39. Barth M, Breuer F, Koopmans P, Norris DG, Poser BA. Simultaneous multislice (SMS) imaging techniques. *Magn Reson Med.* 2016;75:63-81.

40. Norris DG, Boyacioglu R, Schulz J, Barth M, Koopmans P. Application of PINS radiofrequency pulses to reduce power deposition in RARE/turbo spin echo imaging of the human head. *Magn Reson Med.* 2014;71:44-49.

41. Oshio K, Feinberg DA. GRASE (Gradient-and Spin-Echo) imaging: a novel fast MRI technique. *Magn Reson Med.* 1991;20:344-349.

42. Gagoski BA, Bilgic B, Eichner C, et al. RARE/turbo spin echo imaging with simultaneous multislice wave-CAIPI. *Magn Reson Med.* 2015;73:929-938.

43. Stäb D, Speier P. Gradient-controlled local Larmor adjustment (GC-LOLA) for simultaneous multislice bSSFP imaging with improved banding behavior. *Magn Reson Med.* 2019;81:129-139.

44. Schmitter S, Moeller S, Wu X, et al. Simultaneous multislice imaging in dynamic cardiac MRI at 7T using parallel transmission. *Magn Reson Med.* 2017;77:1010-1020.

45. Rapacchi S, Troalen T, Bentatou Z, et al. Simultaneous multi-slice cardiac cine with Fourier-encoded self-calibration at 7 Tesla. *Magn Reson Med.* 2019;81:2576-2587.

46. Nazir MS, Neji R, Speier P, et al. Simultaneous multi slice (SMS) balanced steady state free precession first-pass myocardial perfusion cardiovascular magnetic resonance with iterative reconstruction at 1.5 T. *J Cardiovasc Magn Reson.* 2018;20:84.

47. Weingärtner S, Moeller S, Schmitter S, et al. Simultaneous multislice imaging for native myocardial T1 mapping: improved spatial coverage in a single breath-hold. *Magn Reson Med.* 2017;78:462-471.

**SUPPORTING INFORMATION**

Additional Supporting Information may be found online in the Supporting Information section.

**FIGURE S1** To examine off-resonance effects on the slice excitation quality, due to the use of time-variable selection gradients, off-resonance analysis was conducted using Bloch equation simulations. The spatial shift of a slice was found using cross-correlation between an on-resonance and off-resonance slice, and distortion evaluated using normalised root-mean-square error of the re-centered slice. (A) Shows a TBP2.13 slice profile simulated at an off-resonance of 100 Hz for a constant and TR-optimal gradient solution, and (B) shows the same for a TBP6 pulse. Both slices have a nominal thickness of 5 mm. A slight shift is noticed in both, but the TR-optimal TBP6 slice has a noticeable distortion. Figures (C) and (D) quantitatively show how slice-shifting and slice distortion vary with off-resonance frequency. The slice-shift is marginally larger for TR-optimal pulses than constant gradient versions. Constant gradient pulses also show little distortion, whilst TR-optimal show noticeable distortion which increase with TBP. This is due to the highly time-variable gradient.

**FIGURE S2** Measured and estimated Gradient Impulse Response Function (GIRF) from our system (Philips 3 T Achieva), for directions X (AP), Y(RL) and Z. An image-based procedure was used to measure the GIRF (30). The curve in black shows a Lorentzian fit with time-constant 42 ms. The GIRF effectively lowpass filters the intended gradient waveform. For experimental results the estimated GIRF was used for RF-based correction, to represent results which can be reproducible without measuring a GIRF.
explicitly. Phantom imaging results are shown in Supporting Information figures S3 and S4.

**FIGURE S3**
Slice profile measurements were conducted on a 3 T Philips Achieva (Philips Healthcare, Best, Netherlands) using a cylindrical phantom containing 100 mL of saline (9 g/L) doped with 1% gadolinium contrast agent (0.5 mmol/L Gd-DOTA, Dotarem). All pulses tested were designed for a flip-angle of 45 degrees, slice-thickness of 7 mm and an inter-slice spacing (center-to-center) of 42 mm. All slices were shifted from isocenter by \( \Delta x = 21 \) mm using the FM-shifting approach described in Connolly et al., reference (31). Slices were visualized using a 2D gradient echo sequence (TR = 50 ms, TE = 8.5 ms, 0.55 × 0.575 mm in-plane resolution) with the read-out gradient moved to the slice-select direction. Subfigures (A) and (B) show the constant and TR-optimal versions, respectively, with the addition of a predicted gradient shown in yellow. Subfigure (C) shows the GIRF corrected RF waveform. Note that the GIRF-corrected TR-optimal gradient pulse is identical to the non-GIRF corrected one. Subfigure (D) shows the reduction of sidelobes thanks to the correction, with some sidelobes remaining compared to the constant gradient case. This is possibly due to model inaccuracies.

**FIGURE S4**
The effect of using GIRF correction shown in a structural QA phantom using a MB bSSFP acquisition identical to the in-vivo data. The column in (A) shows the images for a constant gradient acquisition, the 2nd column (B) shows the TR-optimal acquisition without GIRF correction, and the 3rd column (C) the result with GIRF correction. A reduced TR in (B) and (C) with respect to (A) is noted by the change in band location. The spurious out-of-slice artifacts visible in b) are largely eliminated in (C).

**VIDEO S1** Animation of the in-vivo dataset, containing 30 dynamics, repeated 5 times for visibility.

**APPENDIX A**
The VERSE algorithm optimizes RF and gradient waveforms \( B_1^v \) and \( G^v \), while preserving their ratio with respect to the original waveforms

\[
W(s) = \frac{B_1(s)}{G(s)} = \frac{B_1^v(s)}{G^v(s)} \tag{A1}
\]

where \( s(t) \) is a k-space arc-length parameter defined as

\[
s(t) = \gamma \int_0^t G^v(\tau) d\tau. \tag{A2}
\]

The time-variable selection gradients required by VERSE-based methods can lead to increased fidelity issues caused by limited temporal bandwidth of gradient systems. Previous work has shown that such imperfections can be modeled to a reasonable degree as linear and time-invariant, hence characterized using a gradient impulse response function (GIRF). Assuming the GIRF well-characterizes the gradient performance, the demanded waveform \( G_{\text{demand}} \) will be distorted by GIRF \( H(t) \), such that the actually realized waveform is given by the convolution of the two

\[
G_{\text{actual}} = G_{\text{demand}} * H(t) \tag{A3}
\]

A consequence of this is that the condition of Equation (A1) is violated, as the RF pulse was designed for \( G_{\text{demand}} \) but experiences the distorted gradient field \( G_{\text{actual}} \) and thus the slice profile after VERSE will not be the same. When \( H(t) \) is known, it is possible to correct for the distorted gradient by altering the RF waveform. The actual k-space trajectory can be found by rephrasing Equation (A2).

\[
s_{\text{act}}(t) = \gamma \int_0^t G^v_{\text{actual}}(\tau) d\tau \tag{A4}
\]

and the ratio of amplitudes for the realized k-space trajectory \( W(s_{\text{act}}) \) can be found by resampling \( W(s) \) onto \( s_{\text{act}}(t) \). The corrected RF pulse can be found as

\[
B_1^{v,\text{corr}}(s_{\text{act}}) = G^v_{\text{act}}(s_{\text{act}}) W(s_{\text{act}}) \tag{A5}
\]

and is reparametrized to time using Equation (A4).

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