Design of a Discrete Adaptive Equalizer for Noisy Channel using Quantum Behaved Particle Swarm Optimization Technique

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Abstract

Objectives: In this paper, we propose to develop a discrete adaptive equalizer based on Quantum behaved Particle Swarm Optimization (QPSO) technique for noisy channel. Methods/Statistical Analysis: Equalizers have to deal with harder and more complicated problems today due to crowded communication channels and increasing interference level. Findings: This work is an effort to counterbalance Inter-Symbol Interference (ISI) and other nonlinear impairments occurring in real world channels due to nonlinear devices installed in transceivers, cross talk, presence of impulsive noise, multipath propagation and the nature of physical medium itself. A simple, yet efficient optimization algorithm QPSO which belongs to a class of bare-bones PSO family is employed for this purpose. The performance of the proposed equalizer is compared with Least Mean Square (LMS) and other popular variations of PSO, namely Constant Weight Inertia (CWI-PSO) and Linear Decay Inertia (LDI-PSO) algorithms in order to investigate its efficacy. Application/Improvements: Mean Square Error (MSE) and Bit Error Rate (BER) are evaluated for each algorithm and a comparative study of the results reveal that QPSO enjoys an improved performance over other considered algorithms. The proposed discrete equalizer model can be widely used in communication system, especially mobile and satellite communication due to its effectiveness in noisy environment.

Keywords: Bit Error Rate, Discrete Channel Equalizer, Signal to Noise Ratio

1. Introduction

An efficient and reliable data transfer over noisy digital communication channels is the demand of present era, which has cropped up as a result of tremendous rise in internet and multimedia users. Inter-Symbol Interference (ISI), Additive White Gaussian Noise (AWGN), non-linear characteristics of electronic devices used in transceivers such as amplifiers, limited bandwidth and other effects of time varying channels are major causes that can seriously distort the symbols of transmitted data. In order to compensate these effects of physical channel in a digital communication system, a discrete adaptive equalizer offers most popular and reliable solution. The discrete equalizer is usually modelled on Finite Impulse Response (FIR) filter due to its linear phase characteristics and the equalization process is carried out using suitable adaptive optimization techniques. The popular gradient based LMS algorithm and its variants perform satisfactorily when the channel is assumed to be linear in nature but it fails to converge to the global minimum solution when nonlinearity arises or in case of multimodal and non-uniform objective function. For such situations plethora of adaptive equalizers using evolutionary methods have been proposed which works efficiently under non-linear and noisy conditions. Genetic Algorithm (GA), Bacteria Foraging Optimization (BFO), Artificial Immune System (AIS), Differential Evolutionary algorithm (DE), Particle Swarm Optimization (PSO), Modified Particle Swarm Optimization (MPSO), Wind Driven optimization (WDO), constant weight inertia PSO (CWI-PSO), Linear Decay Inertia PSO (LDI-PSO) are few of them.

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PSO has attained increasing popularity due to its simpleness, straightforwardness, easy coding and better accomplishment. Numerous modifications of PSO are introduced ever since the basic PSO system was proposed in 1995\textsuperscript{14}. Like other evolutionary algorithms, a PSO system searches for the optimal solution in a complex space by continuously updating the population of random solution which is initialized at the beginning of the algorithm. However, it does not involve evolution operators, some of which are cross over and mutation probability used in case of GA. The credible solution referred as particles wander in the problem space guided by their own experience and the experience of current best particles i.e. through collaboration of exploration (global search) and exploitation (local search). Therefore, it sounds to be a promising optimization tool that can pave way to cheap, fast and good performance, contrary to other evolutionary methods. However, it has a major folly that particle is confined to a finite search space in each iteration, which at times, can diminish its capability to converge to global optimum. This grievance of PSO was tried to resolve by introducing the concept of inertia weight into the original PSO\textsuperscript{15}. Constriction factor inertia PSO was reported to restrain the velocities of particles from reaching unacceptable value so as to avoid explosion\textsuperscript{16}. Several topologies of PSO to improve its performance have been suggested henceforth\textsuperscript{17–21}.

Quantum behaved PSO is based on quantum mechanics and classical trajectory philosophy\textsuperscript{22}. Contradictory to traditional PSO paradigm, search and solution space of a particular problem are not concordant in QPSO. Rather, state of the particle in search space is defined by wave function or probability function. Since, the search space is quantized, specific statistics about the particle which is essential for evaluating fitness or cost function is not recognized. Measurement of particle position is done by collapsing or transforming quantum state to classical state. Each particle explores in an orientation beginning with individual latest position towards local attractor spot situated in between particle best pbest and global best gbest position. So, QPSO is superior to standard PSO algorithm in search capability. Many researchers from different communities have successfully solved wide range of continuous, complex, multi-dimension and multi-modal optimization problems using QPSO. Few prominent applications are in field of electromagnetic devices\textsuperscript{23,24}, on-line system identification\textsuperscript{25}, image processing\textsuperscript{26}, microelectronics\textsuperscript{27} etc. Judging by its successful applications so far, we are encouraged to exploit this algorithm to design a discrete channel equalizer.

The work presented in this paper, focuses on investigating the effectiveness of QPSO technique in developing a discrete channel equalizer for a popular channel under noisy condition, which is yet an unexplored application. The QPSO based scheme is correlated to two famous models of PSO- Constant Weight Inertia (CW1-PSO) and Linear Decay Inertia (LDI-PSO) to mark the effectiveness of QPSO among other popular variations of PSO, in developing a discrete adaptive equalizer for noisy channel. It is also compared to the benchmark LMS algorithm to bring about its efficacy. The remaining paper is executed in such a way so as to deal with the structure and principle of adaptive channel equalizer in Section 2. An overview of LMS, CW1-PSO, LDI-PSO and QPSO algorithms are presented in Section 3. The algorithm to develop new discrete equalizer through QPSO is put forth in Section 4. Simulation work is carried out in section 5 followed by vigorous discussions. Section 6 concludes and summarizes the work presented in this paper.

2. Discrete Adaptive Equalizer

For reliable digital transmission system, it is crucial to combat the effect of ISI and other impairments associated with the physical medium, it is where adaptive equalizers come into picture\textsuperscript{4}. The equalization in digital communication system scenario is illustrated in Figure 1. A 3-tap FIR filter model is assumed as a channel and the block marked NL is inserted to include the nonlinear effect produced by physical channel.

The output of the channel \( a(k) \) is passed through nonlinear function block NL that produces output \( b(k) = f(a(k)) \), where \( f(\cdot) \) is any mathematical nonlinear
function. In order to realize real-world channel AWGN noise $N(k)$ with variance $\sigma^2$ is added to $b(k)$. The signal received at the receiver side is $r(k)$, which serves as input to the discrete equalizer. The equalizer adapts its internal parameters which are weights or coefficients of the transversal filter model, using appropriate optimization methodology to produce output $\hat{y}(k)$. The desired signal is obtained by delaying input signal $u(k)$ by $D$ samples and is represented as $y(k) = u(k - D)$. The error signal at the $k$th sample denoted by $e(k)$ is instrumental in adapting the weights or parameters of the discrete equalizer and is defined by following equation

$$e(k) = y(k) - \hat{y}(k); \quad 0 < k < n \quad (1)$$

where, $n$ is the number of input samples. As illustrated in Figure 1, the channel is modeled as a three tap FIR filter whose transfer function is $h(z)$. The equalizer modeled as 8-tap delay transversal filter is connected in series with the channel. Conventionally, the order of equalizer is greater than or equal to twice the order of the channel. Therefore, its transfer function is $1/h(z)$. Output of the communication channel at the $k$th instant is represented by

$$a(k) = \sum_{i=0}^{2} h_i * u(k) \quad (2)$$

where, $h_L = [h_0, h_1, h_2]$ represents the weight vector of linear channel $h(z)$ and $u(k) = [u(k), u(k-1), u(k-2), ..., u(k-n)]$ is the transmitted data sequence assumed to be independent taking values from $[1, -1]$ with an equal probability. Received input to the discrete equalizer is

$$r(k) = b(k) + N(k). \quad (3)$$

After compensating for distortion, the discrete equalizer produces an output represented by

$$\hat{y}(k) = \sum_{q=0}^{7} r(k - Q) * w(k) \quad (4)$$

where, $w = [w_0, w_1, ..., w_7]$ is the associated weight vector which is to be optimized and $Q$ is the length of equalizer. The desired signal is obtained by delaying the input signal by $m$ samples ($m = 4$ in this case) as conventionally $m$ is either $Q/2$ (for even $Q$) or $(Q+1)/2$ (for odd $Q$). Here, mean square error serves as the cost/objective function ($J$) and is given by

$$J_i(k) = \frac{1}{n} \sum_{j=1}^{n} e_{ij}^2(k); \quad 1 < i < R \quad (5)$$

where, $e_{ji}$ is the $j$th error for the $i$th particle. $R$ is the population size or number of particles and $j$ varies between 1 and number of iterations.

Main goal of the presented work is to minimize (5), representing the cost function for the problem formulated here. This is done by optimizing the weights of equalizer through QPSO algorithm. The performance of QPSO is compared with those of LMS, CWI-PSO and LDI-PSO to investigate the efficacy of QPSO based discrete equalizer.

3. Overview of the Optimization Algorithm Employed for the Equalization Task

3.1 LMS Algorithm

LMS is the benchmark algorithm for adaptive filters and is popular due to its distinctive traits like computational simplicity, guaranteed convergence and stability in stationary circumstances. This stochastic gradient method employs distinctive estimate of the gradient called as wiener solution to arrive at the optimal values of FIR filter weights used in adaptive discrete equalizer depicted in Figure 1. The update equation for the filter coefficient or weight vector $w$ at $(k + 1)$th instant is postulated as

$$w(k + 1) = w(k) + 2\mu e(k)u(k) \quad (6)$$

where, $\mu$ is the step size.

3.2 Constant Weight Inertia PSO (CWI-PSO) Algorithm

CWI-PSO proposed by Shi and Eberhart is essentially a swarm intelligence paradigm that utilizes etiquette of the swarm in searching for global optimum solution. The trajectory followed by individual particle is basically a consequence of previous best $pbest$ which resembles autobiographical memory of the individual gathered out of its own experience and global best $gbest$ analogous to publicized or group understanding which individual strives to acquire. The algorithm begins with a population of swarm consisting of $R$ individuals. Individual particle in the swarm is assumed to be infinitesimally small and volume less. $X_i(k) = [X_{i1}(k), X_{i2}(k), ..., X_{iR}(k)]$ and $V_i(k) = [V_{i1}(k), V_{i2}(k), ..., V_{iR}(k)]$ stands for the position and velocity vectors of $i$th particle respectively. These vectors are updated on dimension ‘$d$’ during the evolution of swarm population by

$$V_{id}(k + 1) = \omega * V_{id}(k) + C_i * rand(\ ) \ast (P_{id} - X_{id}(k)) + C_{i2} * rand(\ ) \ast (P_{gbid} - X_{id}(k)); 1 \leq d \leq R \quad (7)$$
3.4 QPSO Algorithm

In QPSO, a particle is represented in quantum mechanics unlike standard PSO where it is determined by \(X_i(k)\) and \(V_i(k)\) following the Newtonian mechanics. Heisenberg’s uncertainty principle claims that both the position and velocity of the particle cannot be found at the same time. Therefore, the quantum state of a particle is described by wave function or probability density function (PDF) denoted by \(\Psi(X_i,k)\) instead of position vector \(X\) and velocity vector \(V\) of standard PSO. The probability of the particle occupying position \(X\) can be known from PDF \(\Psi(X_i,k)\). The form of PDF depends upon the type of potential field, whether delta potential well or harmonic oscillator field, the particle is located in. PDF is expressed as

\[
|\Psi(X)|^2 = \frac{1}{L} \exp(-2 \|P - X\|/L)
\]

where, \(L\) is the learning inclination point (LIP) or creativity or imagination and \(P\) is the centre of delta potential well. Here, each particle formulated by (11) preserves its best position \(p_{best}\) in the entire feasible search space and compares with other existing particles to evolve \(g_{best}\) at every iteration. Random numbers \(\phi_1\) and \(\phi_2\) are generated to establish the center of gravity of delta potential well. The particles get attracted and converges to this equilibrium point described by

\[
P_d = (\varphi_{1d} P_{1d} + \varphi_{2d} P_{2d})/(\varphi_{1d} + \varphi_{2d})
\]

so that \(p_{best}\) of all particles in a swarm will arrive at unique \(g_{best}\) when \(t \to \infty\). Next knowledge seeking step is executed by evaluating the control parameter \(L\) given by

\[
L = 2 * \beta * |P - X(k)|
\]

where, \(\beta\) is an important control parameter for delta potential field. Deviation of particle’s current position from its LIP gives a measure of \(L\). The probability of finding new knowledge or solution greatly increases for high value of \(L\). At last, new position is obtained by mapping search space into solution space and is given by

\[
X_i(k + 1) = P + \beta * |P - X(k)| * \ln\left(\frac{1}{u}\right) \text{ if } u > 0.5
\]

\[
X_i(k + 1) = P - \beta * |P - X(k)| * \ln\left(\frac{1}{u}\right) \text{ if } u < 0.5
\]

where, \(u\) is a random number distributed in range [0,1]. To generate convergence, following condition must be satisfied

\[
\lim_{t \to \infty} L(t) = 0.
\]
That is, when $L \rightarrow 0$, then $x \rightarrow p$ at $t \rightarrow \alpha$. If the latest position portrays more desirable information than it is restored until termination criteria is met.

4. Realization of the Equalizer using QPSO Algorithm

The adaptive channel equalizer modelled on FIR filter is realized using the optimization algorithms considered in the previous section. The main goal of the algorithms is to change the filter weights iteratively so that the Mean Square Error (MSE) is minimised to an optimum value. The updating of weights of the equalizer using QPSO algorithm is carried out according to the steps enumerated in Table 1.

5. Simulation Results and Discussions

A Simulation example is presented in this section for the performance evaluation of the proposed discrete adaptive equalizer for noisy channel. To realize the effect of noisy environment two values of AWGN is considered, 5dB and 10dB.

Linear channel:

The channel taken for simulation is given by following transfer function

\[ h(z) = 0.3410 + 0.8760z^{-1} + 0.3410z^{-2} \]  

Nonlinear channel:

The channel is made acutely nonlinear by adopting following nonlinear mathematical function

\[ b(k) = a(k) + 0.2 \times (a(k))^2 - 0.1(a(k))^3 + 0.5 \cos(a(k)) \]  

Two vital parameters, convergence characteristics measured in terms of normalized MSE and BER are considered for performance assessment. The results obtained by QPSO are compared with those achieved by conventional LMS and two successful variants of PSO, CIW-PSO and LDI-PSO to validate the efficacy of QPSO. Parameters of all the algorithms are chosen such that the algorithm converges to the same steady state. The selected parameters corresponding to the best performance exhibited by the considered algorithm is mentioned in Table 2.

Comparison of convergence characteristics of linear and nonlinear channel at 5dB and 10dB AWGN is shown in Figure 2 and Figure 3 respectively. Numerical results of simulation are recorded in Table 3, which gives a lucid picture of performance comparison. When channel is

| Parameters     | QPSO | CIW-PSO | LDI-PSO | LMS |
|----------------|------|---------|---------|-----|
| Population size | 60   | 60      | 60      | -   |
| Iteration cycles | 200  | 200     | 200     | 200 |
| No. of input samples | 500  | 500     | 500     | 500 |
| Accl. constant, $c_1$ | 1.49 | 1.49    | 1.49    | -   |
| Accl. constant, $c_2$ | 1.49 | 1.49    | 1.49    | -   |
| Control parameter, $\beta$ | 0.54 | -       | -       | -   |
| Inertia weight, $\omega$ | -   | 0.2081  | 0.9 - 0.41 | -   |
| Step size, $\mu$ | -   | -       | -       | 0.01 |

Figure 2. Comparison of convergence plot of linear and nonlinear channels at 5dB.
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severely noisy at 5dB, QPSO achieves minimum NMSE of –5.88dB at 40th iteration for linear channel as compared to 0.184dB at 10th iteration, 0.2262dB at 167th iteration for CWI-PSO and LDI-PSO respectively. Similarly, even for nonlinear channel QPSO outperforms other algorithms by converging to –5.359dB at 26th iteration as compared to 0.349dB at 10th iteration and 0.3597dB at 153rd iteration for CWI-PSO and LDI-PSO respectively. But, LMS converges beyond 200 iterations. When channel is less noisy at 10dB, NMSE achieved by QPSO is -8.33dB at 70th iteration for linear channel and -7.039dB at 23rd iteration for nonlinear channel. Other algorithms in this work fail to be even near about these results, as can be clearly read out from Table 3.

Testing of different training algorithms is done by evaluating BER performed on 10^5 bits of input data and is shown in Figure 4 and Figure 5 at 5dB and 10dB respectively.

Comparative BER performance of various algorithms for 10^5 samples at 20dB SNR and 5dB and 10dB AWGN is summarized in Table 4.

QPSO outperforms other algorithms for both linear and nonlinear channel as the error is only 14 bits and 69 bits respectively at AWGN 10dB and 13 bits and 80 bits at AWGN 5dB. Performance improvement of QPSO is 3dB over CWI-PSO and 2dB over LDI-PSO for linear channel. It is 2dB over CWI and 1dB over LDI-PSO for nonlinear channel at 10dB AWGN. This improvement drops to 1dB

| Type of algorithm | AWGN: 5dB | AWGN: 10dB |
|------------------|-----------|------------|
|                  | Linear channel | Nonlinear channel | Linear channel | Nonlinear channel |
| QPSO             | -5.881 | 40 | -5.359 | 26 | -8.338 | 70 | -7.039 | 23 |
| CWI-PSO          | 0.184  | 10 | 0.349  | 16 | 0.1677 | 14 | 0.3251 | 16 |
| LDI-PSO          | 0.2262 | 167| 0.3597 | 153| 0.3434 | 147| 0.3313 | 115|
| LMS              | Beyond 200 iteration | Beyond 200 iteration | Beyond 200 iteration | Beyond 200 iteration |

Figure 4. Comparison of BER Plot of linear and nonlinear channels at 5dB.

Figure 5. Comparison of BER Plot of linear and nonlinear channels at 10dB.

Table 4. Comparison of BER (× 10^-5) at 20dB
over both CWI-PSO and LDI-PSO for linear channel at 5dB AWGN. For nonlinear channel the improvement is 2dB over CWI-PSO. However, it shows no improvement with respect to LDI-PSO. LMS does not work when noise level is high in the environment, as can be seen from Figure 4 and Figure 5.

In summary, QPSO comes out as a better option as compared to LMS, CIW-PSO and LDI-PSO at 5dB and 10dB which is supposedly a noisy environment. Thus, QPSO based design of discrete channel equalizer can be boldly adopted for digital communication system.

6. Conclusion

A novel FIR based channel equalizer using quantum behaved PSO is employed to develop a discrete adaptive equalizer which works efficiently, specifically under noisy channel conditions. The efficacy of this methodology is validated by comparing with benchmark LMS and two successful variations of PSO, CIW-PSO and LDI-PSO. The simulation results confirm that the presented approach paves way to build a fast and easy discrete equalizer by smart quantum depiction of PSO system. It is extremely reliable for applications exploiting real time processing and is potentially a better choice for constructing linear and non-linear discrete channel equalizers. It is also to be noted that the update equation of QPSO adopts adaptive strategy with lesser number of parameters to be optimized guiding to an improved overall performance without increase in the computation complexity.

7. References

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