A Study on Some Properties of Neutrosophic Multi Topological Group

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Abstract: In this paper, we studied some properties of the neutrosophic multi topological group. For this, we introduced the definition of semi-open neutrosophic multisets, semi-closed neutrosophic multisets, neutrosophic multi regularly open set, neutrosophic multi regularly closed set, neutrosophic multi continuous mapping, and then studied the definition of a neutrosophic multi topological group and some of their properties. Moreover, since the concept of the almost topological group is very new, we introduced the definition of neutrosophic multi almost topological group. Finally, for the purpose of symmetry, we used the definition of neutrosophic multi almost continuous mapping to define neutrosophic multi almost topological group and study some of its properties.

Keywords: neutrosophic multi continuous mapping; neutrosophic multi topological group; neutrosophic multi almost continuous mapping; neutrosophic multi almost topological group

1. Introduction

Following the introduction of the fuzzy set (FS) [1], a variety of studies on generalisations of FS concepts were performed. In the sense that the theory of sets should have been a particular case of the theory of FSs, the theory of FSs is a generalisation of the classical theory of sets. Following the generalisation of FSs, many scholars used the theory of generalised FSs in a variety of fields in science and technology. Fuzzy topology (FT) was first introduced by Chang [2], and Intuitionistic fuzzy topological space (IFTS) was defined by Coker [3]. Many researchers studied topology based on neutrosophic sets (NS), such as Lupianez [4–7] and Salama et al. [8]. Kelly [9] defined the concept of bitopological space (BTS) in 1963. Kandil et al. [10] studied the topic of fuzzy bitopological space (FBTS). Some characteristics of Intuitionistic Fuzzy Bitopological Space (IFBTS) were addressed by Lee et al. [11]. Garg [12] investigated how to rank interval-valued Pythagorean FSs using a modified score function. A Pythagorean fuzzy method for order of preference by similarity to ideal solution (TOPSIS) method based on Pythagorean FSs was discussed, which took the experts’ preferences in the form of interval-valued Pythagorean fuzzy decision matrices. Moreover, different explorations of the theory of Pythagorean FSs can be seen in [13–19]. Yager [20] proposed the q-rung orthopair FSs, in which the sum of the qth powers of the membership (MS) and non-MS degrees is restricted to one [21]. Peng and Liu [22] studied the systematic transformation for information measures for q-rung orthopair FSs. Pinar and Boran [23] applied a q-rung orthopair fuzzy multi-criteria group decision-making method for supplier selection based on a novel distance measure.

Cuong et al. [24] proposed a picture FS as an extension of FS and Intuitionistic fuzzy set (IFS) that contains the concept of an element’s positive, negative, and neutral MS de-
gree. Cuong [25] investigated several picture FS characteristics and proposed distance measurements between picture FS. Phong et al. [26] investigated some picture fuzzy relation compositions. Cuong et al. [27] examined the basic fuzzy logic operators: negations, conjunctions, and disjunctions, as well as their implications on picture FSs, and also developed main operations for fuzzy inference processes in picture fuzzy systems. For picture FSs, Cuong et al. [28] demonstrated properties of an involutive picture negator and some related De Morgan fuzzy triples. Viet et al. [29] presented a picture fuzzy inference system based on MS graph, and Singh [30] studied correlation coefficients of picture FS. Garg [31] studied some picture fuzzy aggregation operations and their applications to multi-criteria decision-making. Quek et al. [32] used T-spherical fuzzy weighted aggregation operators to investigate the MADM problem. Garg [33] suggested interactive aggregation operators for T-spherical FSs and used the proposed operators to solve the MADM problem. Zeng et al. [34] studied on multi-attribute decision-making process with immediate probabilistic interactive averaging aggregation operators of T-spherical FSs and its application in the selection of solar cells. Munir et al. [35] investigated T-spherical fuzzy Einstein hybrid aggregation operators and how they could be applied in multi-attribute decision-making issues. Mahmood et al. [36] proposed the idea of a spherical FS and consequently a T-spherical FS.

Many researchers also studied FT and then generalised it in the IFS and then to the neutrosophic topology. Warren [37] studied the boundary of an FS in FT. Warren [37] studied some properties of the boundary of an FS and found that some properties are not the same as the properties of the crisp boundary of a set. Later, many authors studied the properties of the boundary of an FS. Tang [38] made heavy use of the notion of fuzzy boundary. Kharal [39] studied Frontier and Semifrontier in IFTSs. Salama et al. [40] studied generalised neutrosophic topological space (NTS), where they have discussed on properties of generalised closed sets. Azad [41] introduced the concepts of fuzzy semi-continuity (FSC), fuzzy almost continuity (FAC), and fuzzy weakly continuity (FWC) (FWC). Smarandache [42,43] suggested neutrosophic set (NS) theory, which generalised FST and IFST and incorporated a degree of indeterminacy as an independent component. Mwchahary et al. [44] studied on properties of the boundary of neutrosophic bitopological space (NBTS). Many authors studied the properties of the boundary of an FS by several methods (FS, IFS, and NS), but some of its properties are not the same as the properties of the crisp boundary of a set.

Blizard [45] traced multisets back to the very origin of numbers, arguing that in ancient times, the number was often represented by a collection of n strokes, tally marks, or units. The idea of fuzzy multiset (FMS) was introduced by Yager [46] as fuzzy bags. In the interest of brevity, we consider our attention to the basic concepts such as an open FMS, closed FMS, interior, closure, and continuity of FMSs. Yager, in [46], generalised the FS by introducing the concept of FMS (fuzzy bag), and he discussed a calculus for them in [47]. An element of an FMS can occur more than once with possibly the same or different MS values. If every element of an FMS can occur at most once, we go back to FSs [48]. In [49], Onasanya et al. defined the multi-fuzzy group (FMG), and in [50,51], the authors defined fuzzy multi-polygroups and fuzzy multi-Hv-ideals and studied their properties. In [52], Neutrosophic Multigroup (NMG) and their applications are observed. A new type of FS (FMS) was studied by Sebastian et al. [53]. This set makes use of ordered sequences of MS functions to express problems that are not covered by other extensions of FS theory, such as pixel colour. Dey et al. [54] were the first to establish the concept of multi-fuzzy complex numbers and multi-fuzzy complex sets. Over a distributive lattice, the authors [54] proposed multi fuzzy complex nilpotent matrices. Yong et al. [55] recently proposed the notion of the multi-fuzzy soft set, which is a more general fuzzy soft set, for its application to decision making.
Motivation

There is a lot of ambiguity information in the real world that crisp values cannot manage. The FS theory [1], proposed by Zadeh, is an age-old and excellent tool for dealing with uncertain information; however, it can only be used on random processes. As a result, Sebastian et al. [56] introduced FMSs, Atanassov [57] suggested the IFS theory, and Shinoj et al. [58] launched intuitionistic FMSs, all based on FS theory. The theories mentioned above have expanded in a variety of ways and have applications in a variety of fields, including algebraic structures. Some of the selected papers are those on FSs [59–61], FMSs [62–64], IFSs [65–72], and intuitionistic FMSs [73]. However, these theories are incapable of dealing with all forms of uncertainty, such as indeterminate and inconsistent data in various decision-making situations. To address this shortfall, Smarandache [74] proposed the NS theory, which makes Atanassov’s [57] theory very practical and easy to apply. In this current decade, neutrosophic environments are mainly interested by different fields of researchers. In Mathematics, much theoretical research has also been observed in the sense of neutrosophic environment. A more theoretical study will be required to build a broad framework for decision-making and to define patterns for the conception and implementation of complex networks. Deli et al. [75] and Ye [76,77] proposed the notion of neutrosophic multiset (NMS) for modelling vagueness and uncertainty in order to improve the NS theory further. From the literature survey, it was noticed that precisely the properties of the neutrosophic multi topological group (NMTG) are not performed. Now, as an update for the research in NMS, we introduced the definition of a neutrosophic semi-open set, neutrosophic semi-closed set, neutrosophic regularly open set, neutrosophic regularly closed set, neutrosophic continuous mapping, neutrosophic open mapping, neutrosophic closed mapping, neutrosophic semi-continuous mapping, neutrosophic semi-open mapping, neutrosophic semi-closed mapping. Moreover, we tried to prove some of their properties and also cited some examples. We defined the neutrosophic multi almost topological group by using the definition of neutrosophic multi almost continuous mapping and investigate some properties and theorems of a neutrosophic multi almost topological group.

2. Materials and Methods

Definition 1 ([42]). Let X be a non-empty fixed set. A neutrosophic set (NS) A is an object with the form

\[ A = \{ x \in X \mid \mu_A(x), \sigma_A(x), \gamma_A(x) \} \]

where \( T, I, F : X \rightarrow [0, 1] \) and \( 0 \leq \mu_A + \sigma_A + \gamma_A \leq 3 \) and \( \mu_A(x), \sigma_A(x), \) and \( \gamma_A(x) \) represents the degree of MS function, the degree indeterminacy, and the degree of non-MS function, respectively, of each element \( x \in X \) to set \( A \).

Definition 2 ([78]). A neutrosophic multiset (NMS) is a type of neutrosophic set (NS) in which one or more elements are repeated with the same or different neutrosophic components.

Example 1. Let \( X = \{ a, b, c \} \) then

\[ A = \{ < a, 0.6, 0.1, 0.2 >, < a, 0.5, 0.1, 0.3 >, < a, 0.4, 0.2, 0.4 >, \}
\[ \{ < b, 0.3, 0.5, 0.4 >, < b, 0.2, 0.5, 0.6 >, < b, 0.1, 0.5, 0.7 >, \}
\[ < c, 0.4, 0.5, 0.6 >, < c, 0.3, 0.5, 0.7 >, < c, 0.2, 0.6, 0.8 > \} \]

is an NMS, as the elements \( a, b, c \) are repeated.

However, \( B = \{ < a, 0.8, 0.3, 0.1 >, < b, 0.5, 0.3, 0.4 >, < c, 0.4, 0.4, 0.6 > \} \) is an NS and not an NMS.

Definition 3 ([52]). The Empty NMS is defined as \( 0_{NM} = \{ m \in X; m_{(0,1,1)} > \} \), where \( m \) can be repeated.
Definition 4 ([52]). The Whole NMS is defined as 1_{NM} = \{ m \in X; < m_{(1,0,0)} >, where m can be repeated.

Definition 5 ([52]). Let X ≠ φ, and a neutrosophic multiset (NMS) A on X can be expressed as A = \{ m \in X; (m < \tau_A(m), \sigma_A(m), \delta_A(m) >) \}, then the complement of A is defined as A^C = \{ m \in X; (m < \tau_A(m), 1 - \sigma_A(m), 1 - \delta_A(m) >) \}, where m can be repeated depending on its multiplicity, and the τ, σ, δ values may or may not be equal.

Definition 6 ([52]). Let X ≠ φ and A = \{ m \in X; (m < \tau_A(m), \sigma_A(m), \delta_A(m) >) \} and B = \{ m \in X; (m < \tau_B(m), \sigma_B(m), \delta_B(m) >) \} are NMSs. Then
(i) A \cap B = \{ m \in X; m_{< \min(\tau_A(m), \tau_B(m), \sigma_B(m), \sigma_B(m), \max(\sigma_A(m), \sigma_B(m)), \min(\delta_A(m), \delta_B(m))) >} \};
(ii) A \cup B = \{ m \in X; m_{< \max(\tau_A(m), \tau_B(m), \sigma_B(m), \tau_B(m), \min(\sigma_A(m), \tau_B(m)), \min(\delta_A(m), \delta_B(m))) >} \}.

Definition 7 ([78]). Let X ≠ φ, and a neutrosophic multiset topology (NMT) on X is a family τ_X of neutrosophic multi subsets of X if the following conditions hold:
(i) 0_{NM}, 1_{NM} ∈ τ_X;
(ii) G_1 \cap G_2 ∈ τ_X for G_1, G_2 ∈ τ_X;
(iii) \cup \{ G_{N_i} : i \in I \} ∈ τ_X.

Then (X, τ_X) is known as a neutrosophic multi topological space (NMTS), and any NMS in τ_X is called a neutrosophic multi-open set (NMOS). The element of τ_X are said to be NMOSs, an NMS F is neutrosophic multi closed set (NMCoS) if F^C is NMOS.

Definition 8 ([52]). Let X be a classical group and A be a neutrosophic multiset (NMS) on X. Then A is said to be neutrosophic multi groupoid over X if
(i) T_i^G (mn) ≥ T_i^G (m) → T_i^G (n);
(ii) I_i^G (mn) ≤ I_i^G (m) → I_i^G (n);
(iii) F_i^G (mn) ≤ F_i^G (m) → F_i^G (n), ∀ m, n ∈ X and i = 1, 2, …, P.

Moreover, A is said to be neutrosophic multi-group (NMG) over X if the neutrosophic multi groupoid satisfies the following:
(i) T_i^G (m^{-1}) ≥ T_i^G (m);
(ii) I_i^G (m^{-1}) ≤ I_i^G (m);
(iii) F_i^G (m^{-1}) ≤ F_i^G (m), ∀ m ∈ X and i = 1, 2, …, P.

Definition 9 ([52]). Let G be an NMG in a group X, and e be the identity of X. We define the NMS \mathcal{G}_e by
\mathcal{G}_e = \{ m \in X : \tau_G(m) = \tau_G(e), \sigma_G(m) = \sigma_G(e), \delta_G(m) = \delta_G(e) \}

We note for an NMG G in a group X, for every m ∈ X : \tau_G(m^{-1}) = \tau_G(m), \sigma_G(m^{-1}) = \sigma_G(m), and \delta_G(m^{-1}) = \delta_G(m). Moreover, for the identity e ∈ X : \tau_G(e) = \tau_G(m), \sigma_G(e) = \sigma_G(m), and \delta_G(e) = \delta_G(m).

3. Results

Definition 10. Let (X, τ_X) be NMTS. Then for an NMS A = \{ < x, \mu_{N_x}, \sigma_{N_x}, \delta_{N_x} > : x ∈ X \}, the neutrosophic interior of A can be defined as NM ≺ \text{Int} (A) = \{ < x, 1\text{\,min}_{N_x}, \text{\,min}_{\sigma_{N_x}}, \text{\,min}_{\delta_{N_x}} > : x ∈ X \}.

Definition 11. Let (X, τ_X) be NMTS. Then for an NMS A = \{ < x, \mu_{N_x}, \sigma_{N_x}, \delta_{N_x} > : x ∈ X \}, the neutrosophic closure of A can be defined as NM ≺ \text{Cl} (A) = \{ < x, \text{\,min}_{N_x}, \text{\,max}_{\sigma_{N_x}}, \text{\,max}_{\delta_{N_x}} > : x ∈ X \}.
**Definition 12.** Let $\mathbb{G}$ be an NMG on a group $X$. Let $\tau_X$ be a NMT on $\mathbb{G}$, then $(\mathbb{G}, \tau_X)$ is known as a neutrosophic multi topological group (NMTG) if it satisfies the given conditions:

(i) $\alpha : (\mathbb{G}, \tau_X) \times (\mathbb{G}, \tau_X) \longrightarrow (\mathbb{G}, \tau_X)$ defined by $\alpha(m, n) = mn$, $\forall \ m, n \in X$, is relatively neutrosophic multi continuous;

(ii) $\beta : (\mathbb{G}, \tau_X) \longrightarrow (\mathbb{G}, \tau_X)$ defined by $\beta(m) = m^{-1}$, $\forall \ m \in X$, is relatively neutrosophic multi continuous.

**Definition 13.** Let $\mathcal{A}$ be an NMS of an NMTS $(X, \tau_X)$, then $\mathcal{A}$ is called a neutrosophic multi semi-open set (NMSOS) of $X$ if $\exists \mathcal{B} \in \tau_X$ such that $\mathcal{A} \preceq \text{Int}(\mathcal{MN} \sim \text{Cl}(\mathcal{B}))$.

**Example 2.** Let $X = \{a, b\}$:

$$\mathcal{A} = \{< a, 0.8, 0.1, 0.2 >, < a, 0.7, 0.1, 0.3 >, < a, 0.6, 0.2, 0.4 >, < b, 0.7, 0.2, 0.3 >, < b, 0.6, 0.3, 0.4 >, < b, 0.4, 0.2, 0.5 >\};$$

$$\mathcal{B} = \{< a, 0.9, 0.1, 0.1 >, < a, 0.8, 0.1, 0.2 >, < a, 0.7, 0.2, 0.3 >, < b, 0.8, 0.2, 0.2 >, < b, 0.7, 0.2, 0.3 >, < b, 0.5, 0.2, 0.4 >\}.$$

Then $\tau = \{0_X, 1_X, \mathcal{B}\}$ is neutrosophic multi topological space.

Then $\text{Cl}(\mathcal{B}) = 1_X, \text{Int}(\mathcal{B}) = 1_X$.

Hence, $\mathcal{B}$ is NMSOS.

**Definition 14.** Let $\mathcal{A}$ be an NMS of an NMTS $(X, \tau_X)$, then $\mathcal{A}$ is called a neutrosophic multi semi-closed set (NMSCoS) of $X$ if $\exists \mathcal{B} \in \tau_X$ such that $\mathcal{MN} \sim \text{Cl}(\mathcal{MN} \sim \text{Int}(\mathcal{B})) \preceq \mathcal{A}$.

**Lemma 1.** Let $\phi : X \longrightarrow Y$ be a mapping and $\{\mathcal{A}_a\}$ be a family of NMSs of $Y$, then (i) $\phi^{-1}(\mathcal{A}_a) = \mathcal{M} \circ \phi^{-1}(\mathcal{A}_a)$ and (ii) $\phi^{-1}(\mathcal{B}_a) = \mathcal{M} \circ \phi^{-1}(\mathcal{B}_a)$.

**Proof.** Proof is straightforward. □

**Lemma 2.** Let $\mathcal{A}, \mathcal{B}$ be NMSs of $X$ and $Y$, then $1_X - \mathcal{A} \times \mathcal{B} = (\mathcal{A} \times 1_Y) \cup (1_X \times \mathcal{B})$.

**Proof.** Let $(p, q)$ be any element of $X \times Y, (1_X - \mathcal{A} \times \mathcal{B})(p, q) = \max\{1_X - \mathcal{A}(p), 1_Y - \mathcal{B}(q)\} = \max\{(\mathcal{A} \times 1_Y)(p, q), (1_X \times \mathcal{B})(p, q)\} = \{(\mathcal{A} \times 1_Y) \cup (1_X \times \mathcal{B})\}(p, q)$, for each $(p, q) \in X \times Y$. □

**Lemma 3.** Let $\phi_i : X_i \longrightarrow Y_i$ and $\mathcal{A}_i$ be NMSs of $Y_i, i = 1, 2$; we have $(\phi_1 \times \phi_2)^{-1}(\mathcal{A}_1 \times \mathcal{A}_2) = \phi_1^{-1}(\mathcal{A}_1) \times \phi_2^{-1}(\mathcal{A}_2)$.

**Proof.** For each $(p_1, p_2) \in X_1 \times X_2$, we have

$$(\phi_1 \times \phi_2)^{-1}(\mathcal{A}_1 \times \mathcal{A}_2)(p_1, p_2) = (\mathcal{A}_1 \times \mathcal{A}_2)((\phi_1(p_1), \phi_2(p_2)) = \min\{\mathcal{A}_1\phi_1(p_1), \mathcal{A}_2\phi_2(p_2)\} = \min\{\phi_1^{-1}(\mathcal{A}_1)(p_1), \phi_2^{-1}(\mathcal{A}_2)(p_2)\} = (\phi_1^{-1}(\mathcal{A}_1) \times \phi_2^{-1}(\mathcal{A}_2))(p_1, p_2).$$

□

**Lemma 4.** Let $\psi : X \longrightarrow X \times Y$ be the graph of a mapping $\phi : X \longrightarrow Y$. Then, if $\mathcal{A}, \mathcal{B}$ is NMSs of $X$ and $Y$, $\psi^{-1}(\mathcal{A} \times \mathcal{B}) = \mathcal{A} \circ \phi^{-1}(\mathcal{B})$.

**Proof.** For each $p \in X$, we have

$$\psi^{-1}(\mathcal{A} \times \mathcal{B})(p) = (\mathcal{A} \times \mathcal{B})\psi(p) = (\mathcal{A} \times \mathcal{B})(p, \phi(p)) = \min\{\mathcal{A}(p), \mathcal{B}(\phi(p))\} = (\mathcal{A} \circ \phi^{-1}(\mathcal{B}))(p).$$
Theorem 2. (i) Arbitrary union of NMSOSs is an NMSOS; (ii) and (ii) are equivalent follows from Lemma 6, since for an NMS \( A \) of an NMTS \( X, \tau_X \), such that \( 1_{NM} - NM \sim \text{Int}(A) = NM \sim \text{Cl}(1_{NM} - A) \) and \( 1_{NM} - NM \sim \text{Cl}(A) = NM \sim \text{Int}(1_{NM} - A) \).

Proof. Proof is straightforward. □

Theorem 1. The statements below are equivalent:
(i) \( A \) is an NMSCoS;
(ii) \( A^c \) is an NMOs;
(iii) \( NM \sim \text{Int}(NM \sim \text{Cl}(A)) \leq A; \)
(iv) \( NM \sim \text{Cl}(NM \sim \text{Int}(A^c)) \nsubseteq A^c. \)

Proof. (i) and (ii) are equivalent follows from Lemma 6, since for an NMS \( A \) of an NMTS \( X, \tau_X \) such that \( 1_{NM} - NM \sim \text{Int}(A) = NM \sim \text{Cl}(1_{NM} - A) \) and \( 1_{NM} - NM \sim \text{Cl}(A) = NM \sim \text{Int}(1_{NM} - A) \).

Remark 1. It is clear that every NMOS (NMSCoS) is an NMSCoS. The converse is not true.

Example 3. From Example 2, it is clear that \( B \) is a neutrosophic multi semi-open set, but \( B \) is not NMOS.

Theorem 3. If \( (X, \tau_X) \) and \( (Y, \tau_Y) \) are NMTSs, and \( X \) is a product related to \( Y \). Then the product \( A \times B \) of an NMSCoS \( A \) of \( X \) and an NMSCoS \( B \) of \( Y \) is an NMSCoS of the neutrosophic multi-product space \( X \times Y \).

Proof. Let \( P \preceq A \preceq NM \sim \text{Cl}(P) \) and \( Q \preceq B \preceq NM \sim \text{Cl}(Q) \), where \( P \in \tau_X \) and \( Q \in \tau_Y \). Then \( P \times Q \preceq A \times B \preceq NM \sim \text{Cl}(P) \times NM \sim \text{Cl}(Q) \). For NMSCoS \( P \)'s of \( X \) and \( Q \)'s of \( Y \), we have:
(a) \( \inf\{P, Q\} = \min\{\inf P, \inf Q\}; \)
(b) \( \inf\{P \times 1_{NM}\} = (\inf P) \times 1_{NM}; \)
(c) \( \inf\{1_{NM} \times Q\} = 1_{NM} \times (\inf Q). \)

It is sufficient to prove \( Nm \sim Cl(A \times B) \supseteq NM \sim Cl(A) \times NM \sim Cl(B) \). Let \( P \in \tau_X \) and \( Q \in \tau_Y \). Then
\[
NM \sim Cl(A \times B) = \inf \{ (P \times Q)^c | (P \times Q)^c \supseteq A \times B \} \\
= \inf \{ (P^c \times 1_{NM}) \cup (1_{NM} \times Q^c) | (P^c \times 1_{NM}) \cup (1_{NM} \times Q^c) \supseteq A \times B \} \\
= \inf \{ (P^c \times 1_{NM}) \cup (1_{NM} \times Q^c) | P^c \supseteq A \text{ or } Q^c \supseteq B \} \\
= \min \left[ \inf \{ (P^c \times 1_{NM}) \cup (1_{NM} \times Q^c) | P^c \supseteq A \} , \inf \{ (P^c \times 1_{NM}) \cup (1_{NM} \times Q^c) | Q^c \supseteq B \} \right]
\]
Since, \( \inf \{ (P^c \times 1_{NM}) \cup (1_{NM} \times Q^c) | P^c \supseteq A \} \supseteq \inf \{ (P^c \times 1_{NM}) | P^c \supseteq A \} \times 1_{NM} = NM \sim Cl(A) \times 1_{NM} \)
and \( \inf \{ (P^c \times 1_{NM}) \cup (1_{NM} \times Q^c) | Q^c \supseteq B \} \supseteq \inf \{ (1_{NM} \times Q^c) | Q^c \supseteq B \} \)
we have, \( NM \sim Cl(A \times B) \supseteq \min \{ NM \sim Cl(A) \times 1_{NM}, 1_{NM} \times NM \sim Cl(B) \} = NM \sim Cl(A) \times NM \sim Cl(B) \), hence the result. \( \Box \)

**Definition 15.** An NMS \( A \) of an NMTS \((X, \tau_X)\) is called a neutrosophic multi regularly open set (NMROS) of \((X, \tau_X)\) if \( NM \sim Int(NM \sim Cl(A)) = A \).

**Example 4.** Let \( X = \{ a, b \} \) and
\[
A = \{ < a, 0.4, 0.5, 0.6 >, < a, 0.3, 0.5, 0.6 >, < a, 0.2, 0.6, 0.7 >, < b, 0.5, 0.6, 0.7 >, < b, 0.3, 0.5, 0.8 > \}
\]
Then \( \tau = \{ 0_X, 1_X, A \} \) is neutrosophic multi topological space.
Clearly, \( Cl(A) = A^C, Int(Cl(A)) = A \).
Hence, \( A \) is NMROS.

**Definition 16.** An NMS \( A \) of an NMTS \((X, \tau_X)\) is called a neutrosophic multi regularly closed set (NMRCoS) of \((X, \tau_X)\) if \( NM \sim Cl(NM \sim Int(A)) = A \).

**Theorem 4.** An NMS \( A \) of NMTS \((X, \tau_X)\) is an NMRO if \( A^c \) is NMRCoS.

**Proof.** It follows from Lemma 3. \( \Box \)

**Remark 2.** It is obvious that every NMROS (NMRCoS) is an NMOS (NMCoS). The converse need not be true.

**Example 5.** Let \( X = \{ a, b \} \) and
\[
A = \{ < a, 0.8, 0.1, 0.2 >, < a, 0.7, 0.1, 0.3 >, < a, 0.6, 0.2, 0.4 >, < b, 0.7, 0.2, 0.3 >, < b, 0.6, 0.3, 0.4 >, < b, 0.4, 0.2, 0.5 > \}; \\
B = \{ < a, 0.9, 0.1, 0.1 >, < a, 0.8, 0.1, 0.2 >, < a, 0.7, 0.2, 0.3 >, < b, 0.8, 0.2, 0.2 >, < b, 0.7, 0.2, 0.3 >, < b, 0.5, 0.2, 0.4 > \}.
\]
Then \( \tau = \{ 0_X, 1_X, A \} \) is a neutrosophic multi topological space.
Then \( Cl(B) = 1_X, Int(Cl(B)) = 1_X \), which is not NMROS.

**Remark 3.** The union (intersection) of any two NMROSs (NMRCoS) need not be an NMROS (NMRCoS).
Theorem 6. (i) The closure of an NMOS is an NMRCoS; (ii) The union of any two NMROSs is an NMRCoS.

Example 6. Let $X = \{a, b\}$ and $\tau = \{0_X, 1_X, A, B, A \rightarrow B\}$ is a neutrosophic multi topological space, where

\[ A = \{ < a, 0.4, 0.5, 0.6 >, < a, 0.3, 0.5, 0.7 >, < a, 0.2, 0.6, 0.8 >, \} ; \]

\[ B = \{ < a, 0.6, 0.5, 0.4 >, < a, 0.7, 0.5, 0.3 >, < a, 0.8, 0.4, 0.2 >, \} ; \]

\[ A \cup B = \{ < a, 0.6, 0.5, 0.4 >, < a, 0.7, 0.5, 0.3 >, < a, 0.8, 0.4, 0.2 >, \} . \]

Here, $\text{Cl}(A) = B^C$, $\text{Int}(\text{Cl}(A)) = A$, and $\text{Cl}(B) = A^C$, $\text{Int}(\text{Cl}(B)) = B$. Then $\text{Cl}(A \cup B) = 1_X$. Thus, $\text{Int}(\text{Cl}(A \cup B)) = 1_X$. Hence, $A$ and $B$ is NROS, but $A \cup B$ is not NROS.

Theorem 5. (i) The intersection of any two NMROSs is an NMROS; (ii) The union of any two NMRCoSs is an NMRCoS.

Proof. (i) Let $A_1$ and $A_2$ be any two NMROSs of an NMTS $(X, \tau_X)$. Since $A_1 \cap A_2$ is NMOS (from Remark 3), we have $A_1 \cap A_2 \not\preceq NM \prec \text{Int}(NM \prec \text{Cl}(A_1 \cap A_2))$. Now, $NM \prec \text{Int}(NM \prec \text{Cl}(A_1 \cap A_2)) \not\preceq NM \prec \text{Int}(NM \prec \text{Cl}(A_1)) = A_1$ and $NM \prec \text{Int}(NM \prec \text{Cl}(A_1 \cap A_2)) \not\preceq NM \prec \text{Int}(NM \prec \text{Cl}(A_2)) = A_2$ implies that $NM \prec \text{Int}(NM \prec \text{Cl}(A_1 \cap A_2)) \not\preceq A_1 \cap A_2$, hence the theorem;

(ii) Let $A_1$ and $A_2$ be any two NMROSs of an NMTS $(X, \tau_X)$. Since $A_1 \cup A_2$ is NMOS (from Remark 3), we have $(A_1 \cup A_2) \not\preceq NM \prec \text{Cl}(NM \prec Int(A_1 \cup A_2))$. Now, $NM \prec \text{Cl}(NM \prec Int(A_1 \cup A_2)) \not\preceq NM \prec \text{Cl}(NM \prec Int(A_1)) = A_1$ and $NM \prec \text{Cl}(NM \prec Int(A_1 \cup A_2)) \not\preceq NM \prec \text{Cl}(NM \prec Int(A_2)) = A_2$ implies that $A_1 \cup A_2 \not\preceq NM \prec \text{Cl}(NM \prec Int(A_1 \cup A_2))$, hence the theorem.

Example 7. Let $X = Y = \{a, b, c\}$ and

\[ A = \{ < a, 0.4, 0.5, 0.6 >, < a, 0.3, 0.5, 0.7 >, < a, 0.2, 0.6, 0.8 >, \} ; \]

\[ B = \{ < b, 0.3, 0.5, 0.4 >, < b, 0.2, 0.5, 0.6 >, < b, 0.1, 0.5, 0.7 >, \} ; \]

\[ C = \{ < c, 0.4, 0.5, 0.6 >, < c, 0.3, 0.5, 0.7 >, < c, 0.2, 0.6, 0.8 > \} . \]
Then $\tau_X = \{0_X, 1_X, A\}$ and $\tau_Y = \{0_Y, 1_Y, B\}$ are neutrosophic multi topological spaces. Now, define a mapping $f : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ by $f(a) = f(c) = c$ and $f(b) = b$. Thus, $f$ is NCM.

**Definition 18.** Let $\phi : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ be a mapping from an NMTS $(X, \tau_X)$ to another NMTS $(Y, \tau_Y)$, then $\phi$ is called a neutrosophic multi open mapping (NMOM) if $\phi(A) \in \tau_Y$ for each $A \in \tau_X$.

**Definition 19.** Let $\phi : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ be a mapping from an NMTS $(X, \tau_X)$ to another NMTS $(Y, \tau_Y)$, then $\phi$ is said to be a neutrosophic multi closed mapping (NMCoM) if $\phi(A) \in \tau_Y$ for each $A \in \tau_X$.

**Definition 20.** Let $\phi : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ be a mapping from an NMTS $(X, \tau_X)$ to another NMTS $(Y, \tau_Y)$, then $\phi$ is called a neutrosophic multi semi-continuous mapping (NMSCM), if $\phi^{-1}(A)$ is the NMSOS of $X$, for each $A \in \tau_Y$.

**Definition 21.** Let $\phi : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ be a mapping from an NMTS $(X, \tau_X)$ to another NMTS $(Y, \tau_Y)$, then $\phi$ is called a neutrosophic multi semi-open mapping (NMSOM) if $\phi(A)$ is a SONMS for each $A \in \tau_Y$.

**Example 8.** Let $X = Y = \{a, b, c\}$ and

$$A = \left\{ \begin{array}{c} < a, 0.6, 0.1, 0.2 >, < a, 0.5, 0.1, 0.3 >, < a, 0.4, 0.1, 0.4 >, \\ < b, 0.3, 0.5, 0.4 >, < b, 0.2, 0.5, 0.6 >, < b, 0.1, 0.5, 0.7 >, \\ < c, 0.4, 0.5, 0.6 >, < c, 0.3, 0.5, 0.7 >, < c, 0.2, 0.6, 0.8 > \end{array} \right\};$$

$$B = \left\{ \begin{array}{c} < a, 0.3, 0.5, 0.4 >, < a, 0.2, 0.5, 0.6 >, < a, 0.1, 0.5, 0.7 >, \\ < b, 0.6, 0.1, 0.2 >, < b, 0.5, 0.1, 0.3 >, < b, 0.4, 0.2, 0.4 >, \\ < c, 0.4, 0.5, 0.6 >, < c, 0.3, 0.5, 0.7 >, < c, 0.2, 0.6, 0.8 > \end{array} \right\}.$$
Then $\tau_X = \{0_X, 1_X, A\}$ and $\tau_Y = \{0_Y, 1_Y, B\}$ are neutrosophic multi topological spaces.

Let us define a mapping $f : (X, \tau_X) \rightarrow (Y, \tau_Y)$ by $f(a) = f(c) = c$ and $f(b) = b$.

Thus, $f$ is NMSCM, which is not an NMCM.

**Theorem 7.** Let $X_1, X_2, Y_1$ and $Y_2$ be NMTSs such that $X_1$ is product related to $X_2$. Then, the product $\varphi_1 \times \varphi_2 : X_1 \times X_2 \rightarrow Y_1 \times Y_2$ of NMSCMs $\varphi_1 : X_1 \rightarrow Y_1$ and $\varphi_2 : X_2 \rightarrow Y_2$ is NMSCM.

**Proof.** Let $A \equiv (A_\alpha \times B_\beta)$, where $A_\alpha$’s and $B_\beta$’s are NMOSs of $Y_1$ and $Y_2$, respectively, be an NMOS of $Y_1 \times Y_2$. By using Lemma 1(i) and Lemma 3, we have

$$(\varphi_1 \times \varphi_2)^{-1}(A) = \nabla \left( \varphi_1^{-1}(A_\alpha) \times \varphi_2^{-1}(B_\beta) \right)$$

where $(\varphi_1 \times \varphi_2)^{-1}(A)$ is an NMSOS follows from Theorem 3 and Theorem 2 (i). □

**Theorem 8.** Let $X$, $X_1$ and $X_2$ be NMTSs and $p_i : X_1 \times X_2 \rightarrow X_i (i = 1, 2)$ be the projection of $X_1 \times X_2$ onto $X_i$. Then, if $\varphi : X \rightarrow X_1 \times X_2$ is an NMSCM, $p_i \varphi$ is also NMSCM.

**Proof.** For an NMOS $A$ of $X_i$, we have $(p_i \varphi)^{-1}(A) = \varphi^{-1}(p_i^{-1}(A))$. $p_i$ is an NMCM and $\varphi$ is an NMSCM, which implies that $(p_i \varphi)^{-1}(A)$ is an NMSOS of $X$. □

**Theorem 9.** Let $\varphi : X \rightarrow Y$ be a mapping from an NMTS $X$ to another NMTS $Y$. Then if the graph $\varphi : X \rightarrow X \times Y$ of $\varphi$ is NMSCM, $\varphi$ is also NMSCM.

**Proof.** From Lemma 4, $\varphi^{-1}(A) = 1_{\text{NM}} \cap \varphi^{-1}(A) = \varphi^{-1}(1_{\text{NM}} \times A)$, for each NMOS $A$ of $Y$. Since $\varphi$ is an NMSCM and $1_{\text{NM}} \times A$ is an NMOS $X \times Y$, $\varphi^{-1}(A)$ is an NMSOS of $X$ and hence $\varphi$ is an NMSCM. □

**Remark 5.** The converse of Theorem 9 is not true.

**Definition 23.** A mapping $\varphi : (X, \tau_X) \rightarrow (Y, \tau_Y)$ from an NMTS $X$ to another NMTS $Y$ is known as a neutrosophic multi almost continuous mapping (NMACM), if $\varphi^{-1}(A) \in \tau_X$ for each NMROS $A$ of $Y$.

**Example 10.** Let $X = Y = \{a, b\}$ and

$$(a) \quad A = \left\{ \begin{array}{c} < a, 0.4, 0.5, 0.5 > , < a, 0.3, 0.5, 0.6 > , < a, 0.2, 0.6, 0.7 > , \\
< b, 0.5, 0.7, 0.6 > , < b, 0.4, 0.5, 0.7 > , < b, 0.3, 0.5, 0.8 > \end{array} \right\};$$

$$(b) \quad B = \left\{ \begin{array}{c} < a, 0.5, 0.7, 0.6 > , < a, 0.4, 0.5, 0.7 > , < a, 0.3, 0.5, 0.8 > , \\
< b, 0.4, 0.5, 0.5 > , < b, 0.3, 0.5, 0.6 > , < b, 0.2, 0.6, 0.7 > \end{array} \right\}.$$  

Then $\tau_X = \{0_X, 1_X, A\}$ and $\tau_Y = \{0_Y, 1_Y, B\}$ are neutrosophic multi topological spaces.

Clearly, $\text{Cl}(B) = B^C$, $\text{Int}(\text{Cl}(B)) = B$.

Hence, $B$ is NMROS.

Now, let us define a mapping $f : (X, \tau_X) \rightarrow (Y, \tau_Y)$ by $f(a) = b$, $f(b) = a$.

Thus, $f$ is NMACM.

**Theorem 10.** Let $\varphi : (X, \tau_X) \rightarrow (Y, \tau_Y)$ be a mapping. Then the below statements are equivalent:

(a) $\varphi$ is an NMACM;
(b) $\varphi^{-1}(F)$ is an NMCoS, for each NMRCoS $F$ of $Y$;
(c) $\varphi^{-1}(A) \preceq \text{NM} \preceq \text{Int}(\varphi^{-1}(\text{NM} \preceq \text{Int}(\text{NM} \preceq \text{Cl}(A))))$, for each NMOS $A$ of $Y$;
(d) $\text{NM} \preceq \text{Cl}(\varphi^{-1}(\text{NM} \preceq \text{Cl}(\text{NM} \preceq \text{Int}(F)))) \preceq \varphi^{-1}(F)$, for each NMCoS $F$ of $Y$. 


Proof. Consider that $\phi^{-1}(A^c) = (\phi^{-1}(A))^c$, for any NMS $A$ of $Y$, (a) $\iff$ (b) follows from Theorem 4.

(a) $\implies$ (c). Since $A$ is an NMOS of $Y$, $A \precsim NM \succ Int(Cl(A))$, hence, $\phi^{-1}(A) \precsim \phi^{-1}(NM \succ Int(NM \succ Cl(A)))$. From Theorem 6 (ii), $NM \succ Int(NM \succ Cl(A))$ is an NMROS of $Y$, hence $\phi^{-1}(NM \succ Int(NM \succ Cl(A)))$ is an NMROS of $X$, Thus, $\phi^{-1}(A) \precsim \phi^{-1}(NM \succ Int(NM \succ Cl(A))) = NM \succ Int(\phi^{-1}(NM \succ Int(NM \succ Cl(A))))$.

(c) $\implies$ (a). Let $A$ be an NMROS of $Y$, then we have $\phi^{-1}(A) \precsim NM \succ Int(\phi^{-1}(NM \succ Int(NM \succ Cl(A)))) = NM \succ Int(\phi^{-1}(A))$. Thus, $\phi^{-1}(A) = NM \succ Int(\phi^{-1}(A))$. This shows that $\phi^{-1}(A)$ is an NMOS of $X$.

(b) $\iff$ (d) similarly can be proved. $\Box$

Remark 6. Clearly, an NMCM is an NMACM. The converse need not be true.

Example 11. Let $X = Y = \{a, b\}$ and

$$A = \left\{ \langle a, 0.4, 0.5, 0.5 >, \langle a, 0.3, 0.5, 0.6 >, \langle a, 0.2, 0.6, 0.7 >, \langle b, 0.4, 0.5, 0.7 >, \langle b, 0.3, 0.5, 0.8 > \right\};$$

$$B = \left\{ \langle a, 0.4, 0.5, 0.6 >, \langle a, 0.6, 0.5, 0.7 >, \langle a, 0.2, 0.6, 0.9 >, \langle b, 0.4, 0.5, 0.7 >, \langle b, 0.3, 0.5, 0.5 >, \langle b, 0.4, 0.5, 0.6 > \right\}. $$

Then, $\tau_X = \{0_X, 1_X, A\}$ and $\tau_Y = \{0_Y, 1_Y, B\}$ are neutrosophic multi topological spaces. Clearly, $Cl(B) = B^c$, $Int(Cl(B)) = B$.

Hence, $B$ is NMROS in $\tau_Y$.

Now, a mapping $f : (X, \tau_X) \to (Y, \tau_Y)$ defined by $f(a) = a, f(b) = b$. Then clearly, $f$ is NMACM but not NMCM.

Theorem 11. Neutrosophic multi semi-continuity and neutrosophic multi almost continuity are independent notions.

Definition 24. AN NMTS $(X, \tau_X)$ is called a neutrosophic multi semi-regularly space (NMSRS) if and only if the collection of all NMROSs of $X$ forms a base for NMT $\tau_X$.

Theorem 12. Let $\phi : (X, \tau_X) \to (Y, \tau_Y)$ be a mapping from an NMTS X to an NMSRS Y. Then $\phi$ is NMACM iff $\phi$ is NMCM.

Proof. From Remark 6, it suffices to prove that if $\phi$ is NMACM, then it is NMCM. Let $A \in \tau_Y$, then $A = \cup A_\alpha$, where $A_\alpha$’s are NMROSs of $Y$. Now, from Lemma 1(i), 5, and Theorem 10 (c), we obtain

$$\phi^{-1}(A) = \cup \phi^{-1}(A_\alpha) \precsim \cup NM \succ Int(\phi^{-1}(NM \succ Cl(A_\alpha))) = \cup NM \succ Int(\phi^{-1}(A_\alpha)).$$

$$\precsim NM \succ Int \cup (\phi^{-1}(A_\alpha)) = NM \succ Int(\phi^{-1}(A_\alpha)).$$

which shows that $\phi^{-1}(A_\alpha) \in \tau_X$. $\Box$

Theorem 13. Let $X_1, X_2, Y_1$ and $Y_2$ be the NMTSs, such that $Y_1$ is product related to $Y_2$. Then the product $\phi_1 \times \phi_2 : X_1 \times X_2 \to Y_1 \times Y_2$ of NMACMs $\phi_1 : X_1 \to Y_1$ and $\phi_2 : X_2 \to Y_2$ is NMCM.

Proof. Let $A = \cup (A_\alpha \times B_\beta)$, where $A_\alpha$’s and $B_\beta$’s are NMOSs of $Y_1$ and $Y_2$, respectively, be an NMOS of $Y_1$, $Y_2$. From Lemma 1(i), 3, 5, and Theorems 6, and 10 (c), we have

$$(\phi_1 \times \phi_2)^{-1}(A) = \cup \{\phi_1^{-1}(A_\alpha) \times \phi_2^{-1}(B_\beta)\}$$
Theorem 14. Let $X$, $X_1$, and $X_2$ be an NMTSs and $p_i : X_1 \times X_2 \rightarrow X_i (i = 1, 2)$ be the projection of $X_1 \times X_2$ onto $X_i$. Then if $\phi : X \rightarrow X_1 \times X_2$ is an NMACM, $p \phi$ is also an NMACM.

Proof. Since $p_i$ is NCMCM Definition 16, for any NMS $A$ of $X_i$, we have (i) $NM \sim Cl(p_i^{-1}(A)) \leq p_i^{-1}(NM \sim Cl(A))$ and (ii) $NM \sim Int(p_i^{-1}(A)) \geq p_i^{-1}(NM \sim Int(A))$. Again, since (i) each $p_i$ is an NMOs, and (ii) for any NMS $A$ of $X_1$ (a) $A \leq p_i^{-1}(p_i(A)$ and (b) $p_i^{-1}(p_i(A) = A$, we have $p_i(NM \sim Int(p_i^{-1}(A))) \leq p_i^{-1}(A) \leq A$, and hence, $p_i(NM \sim Int(p_i^{-1}(A))) \leq NM \sim Int(A)$. □

Thus, $NM \sim Int(p_i^{-1}(A)) \leq p_i^{-1}(NM \sim Int(p_i^{-1}(A))) \leq (p_i^{-1}(NM \sim Int(A)))$ establishes that $NM \sim Int(p_i^{-1}(A)) \leq p_i^{-1}(NM \sim Int(A))$. Now, for any NMOs $A$ of $X_i$,

$$\phi^{-1}(p_i \phi)^{-1}(A) = (p_i \phi)^{-1}(A) = \phi^{-1}(p_i^{-1}(A)) = NM \sim Int\left(\phi^{-1}(NM \sim Int(\phi^{-1}(p_i^{-1}(A))))\right) \leq NM \sim Int\left(\phi^{-1}(NM \sim Int(p_i^{-1}(A)))\right) \leq NM \sim Int\left(\phi^{-1}(NM \sim Int(Cl(A)))\right) \leq NM \sim Int\left(\phi^{-1}(NM \sim Int(Cl(A)))\right)

Thus, by Theorem 10 (c), $\phi \times \phi$ is NMACM.

Theorem 15. Let $X$ and $Y$ be NMTSs such that $X$ is product related to $Y$ and let $\psi : X \rightarrow Y$ be a mapping. Then, the graph $\psi : X \rightarrow X \times Y$ of $\psi$ is NMACM if $\psi$ is NMACM.

Proof. Consider that $\psi$ is an NMACM and $A$ is an NMOs of $Y$. Then, using Lemma 4 and Theorems 10 (c), we have

$$\phi^{-1}(A) = 1_{NM} \otimes \phi^{-1}(A) = \phi^{-1}(1_{NM} \otimes A) \leq NM \sim Int(\phi^{-1}(NM \sim Int(Cl(1_{NM} \otimes A))))

Thus, by Theorem 10 (c), $\phi$ is NMACM.

Conversely, let $\phi$ be an NMACM and $B = \psi (B_\alpha \times A_\beta)$, where $B_\alpha$’s and $A_\beta$’s are NMOs of $X$ and $Y$, respectively, be an NMOs of $X \times Y$.

Since $B_\alpha \otimes NM \sim Int(\phi^{-1}(NM \sim Int(Cl(A_\beta))))$ is an NMOs of $X$ contained in

$$NM \sim Int(NM \sim Cl(B_\alpha)) \otimes \phi^{-1}(NM \sim Int(Cl(A_\beta)))$$

$$B_\alpha \otimes NM \sim Int(\phi^{-1}(NM \sim Int(Cl(A_\beta))))$$
\[ \forall \, \text{NM} \sim \text{Int} \left[ \text{NM} \sim \text{Int} (\text{NM} \sim \text{Cl}(B_{\alpha})) \cap \phi^{-1} (\text{NM} \sim \text{Int} (\text{NM} \sim \text{Cl}(A_{\beta}))) \right] \]

and hence, using Lemmas 1(i), 4 and 5, and Theorems 10 (c), we have

\[
\phi^{-1}(B) = \phi^{-1}(\psi (B_{\alpha} \times A_{\beta})) \\
= \psi (B_{\alpha} \cap \phi^{-1}(A_{\beta})) \\
\leq \psi (B_{\alpha} \cap \text{NM} \sim \text{Int} (\phi^{-1} (\text{NM} \sim \text{Int} (\text{NM} \sim \text{Cl}(A_{\beta})))) ) \\
\leq \psi (\text{NM} \sim \text{Int} (\text{NM} \sim \text{Cl}(B_{\alpha}))) \times \text{NM} \sim \text{Int} (\text{NM} \sim \text{Cl}(A_{\beta})) \\
\leq \text{NM} \sim \text{Int} (\psi^{-1} (\text{NM} \sim \text{Int} (\psi (B_{\alpha} \times A_{\beta})))) \\
= \text{NM} \sim \text{Int} (\psi^{-1} (\text{NM} \sim \text{Int} (\text{NM} \sim \text{Cl}(B_{\alpha})))) \]

Thus, by Theorem 10(c), \( \psi \) is NMACM. \( \square \)

**Definition 25.** Let \( G \) be an NMG on a group \( X \). Now, if \( \tau_X \) is an NMT on \( G \), then \((G, \tau_X)\) is said to be a neutrosophic multi almost topological group (NMATG) if the given conditions are satisfied:

(i) \( a : (G, \tau_X) \times (G, \tau_X) \to (G, \tau_X) : a(m, n) = mn \) is NMACM;

(ii) \( \beta : (G, \tau_X) \to (G, \tau_X) : \beta(m) = m^{-1} \) is NMACM.

Then \((G, \tau_X)\) is known as an NMATG.

**Remark 7.** \((G, \tau_X)\) is an NMATG if the below conditions hold good:

(i) For \( g_1, g_2 \in G \) and every NMROS \( P \) containing \( g_1, g_2 \) in \( G \), \( \exists \) open neighborhoods \( R \) and \( S \) of \( g_1 \) and \( g_2 \) in \( G \) such that \( R \ast S \leq P \);

(ii) For \( g \in G \) and every \( N \) in \( G \) containing \( g \), \( \exists \) open neighborhood \( R \) of \( g \) in \( G \) so that \( R^{-1} \leq S \).

**Remark 8.** For any \( P, Q \leq G \), we denote \( P \ast Q \) by \( PQ \) and defined as \( PQ = \{gh : g \in P, h \in Q\} \) and \( P^{-1} = \{g^{-1} : g \in P\} \). If \( P = \{a\} \) for each \( a \in G \), we denote \( P \ast Q \) by \( aQ \) and \( Q \ast P \) by \( Pa \).

**Example 12.** Let, \( G = (\mathbb{Z}_3, +) \) be a classical group and

\[
A = \{< 0, 0.4, 0.5, 0.6 >, < 0, 0.3, 0.5, 0.7 >, < 0, 0.2, 0.6, 0.8 >, < 1, 0.3, 0.5, 0.4 >, < 1, 0.2, 0.5, 0.6 >, < 1, 0.1, 0.5, 0.7 >, < 2, 0.4, 0.5, 0.6 >, < 2, 0.3, 0.5, 0.7 >, < 2, 0.2, 0.6, 0.8 > \}
\]

Then \( \tau_G = \{0_G, 1_G, A\} \) is NTS and the mapping \( a : (G, \tau_G) \times (G, \tau_G) \to (G, \tau_G) : a(m, n) = mn \) and \( \beta : (G, \tau_G) \to (G, \tau_G) : \beta(m) = m^{-1} \) are NMACM. Hence, \((G, \tau_G)\) is NMATG.

**Theorem 16.** Let \((G, \tau_X)\) be an NMATG and let \( a \) be any element of \( G \). Then

(a) \( \mu_a : (G, \tau_X) \to (G, \tau_X) : \mu_a(x) = ax, \forall x \in G \), is NMACM;

(b) \( \lambda_a : (G, \tau_X) \to (G, \tau_X) : \lambda_a(x) = xa, \forall x \in G \), is NMACM.

**Proof.** (a) Let \( p \in G \) and let \( R \) be an NMROS containing \( ap \) in \( G \). By Definition 25, \( \exists \) open neighborhoods \( P, Q \) of \( a, p \) in \( G \) such that \( P \ast Q \leq R \). Especially, \( aQ \leq R \), i.e., \( \mu_a(Q) \leq R \). This proves that \( \mu_a \) is NMACM at \( p \), and hence, \( \mu_a \) is NMACM.

(b) Suppose \( p \in G \) and \( R \in \text{NMRO}(G) \) contain \( pa \). Then \( \exists \) open sets \( p \in P \) and \( a \in Q \) in \( G \) such that \( P \ast Q \leq R \). This proves \( Pa \leq R \). This shows that \( \lambda_a \) is NMACM at \( p \).

Since arbitrary element \( p \) is in \( G \), hence, \( \lambda_a \) is NMACM. \( \square \)

**Theorem 17.** Let \( \mathcal{U} \) be NMROS in a NMATG \((G, \tau_X)\). The below conditions hold good:

(a) \( m\mathcal{U} \in \text{NMROS}(G), \forall m \in G \);

(b) \( \mathcal{U}m \in \text{NMROS}(G), \forall m \in G \);
(c) \(U^{-1} \in \text{NMROS}(\mathcal{G})\).

**Proof.** (a) We first show that \(mU \in \tau_X\). Let \(p \in mU\). Then by Definition 25 of NMATGs, \(\exists\) NMOSs \(m^{-1} \in W_1\) and \(p \in W_2\) in \(\mathcal{G}\) such that \(W_1W_2 \preceq \bar{U}\). Especially, \(m^{-1}W_2 \preceq \bar{U}\). That is, equivalently, \(W_2 \preceq mLm\). This indicates that \(p \in \text{NM} \sim \text{Int}(mU)\) and thus, \(\text{NM} \sim \text{Int}(mU) = mLm\). That is \(mU \in \tau_X\). Consequently, \(mU \preceq \text{NM} \sim \text{Int}((\text{NM} \sim \text{Cl}(mU)))\).

Now, we have to prove that \(\text{NM} \sim \text{Int}(\text{NM} \sim \text{Cl}(mU)) \preceq mLm\). As \(U\) is NMOS, \(\text{NM} \sim \text{Cl}(U) \in \text{NMROS}(\mathcal{G})\). By Theorem 16, \(\mu_{m^{-1}} : (\mathcal{G}, \tau_X) \rightarrow (\mathcal{G}, \tau_X)\) is NMACM, and therefore, \(m\text{NM} \sim \text{Cl}(U)\) is NMCoS. Thus, \(\text{NM} \sim \text{Int}(\text{NM} \sim \text{Cl}(mU)) \preceq \text{NM} \sim \text{Cl}(mU)\). Since \(\text{NM} \sim \text{Int}(\text{NM} \sim \text{Cl}(mU)) \preceq \text{NM} \sim \text{Int}(\text{NM} \sim \text{Cl}(U))\), it follows that \(m^{-1}\text{NM} \sim \text{Int}(\text{NM} \sim \text{Cl}(mU)) \preceq \text{NM} \sim \text{Int}(\text{NM} \sim \text{Cl}(U))\). Thus \(mU = \text{NM} \sim \text{Int}(\text{NM} \sim \text{Cl}(mU))\). This proves that \(mU \in \text{NMROS}(\mathcal{G})\).

(b) Following the same steps as in part (1) above, we can prove that \(U\text{m} \in \text{NMROS}(\mathcal{G}), \forall m \in \mathcal{G}\).

(c) Let \(p \in U^{-1}\), then \(\exists\) open set \(p \in W \in \mathcal{G}\) such that \(W^{-1} \preceq \bar{U} \Rightarrow W \preceq U^{-1}\). Thus, \(U^{-1}\) has interior point \(p\). Thus, \(U^{-1}\) is NMOS. That is, \(U^{-1} \preceq \text{NM} \sim \text{Int}(\text{NM} \sim \text{Cl}(U^{-1}))\). Now we have to prove that \(\text{NM} \sim \text{Int}(\text{NM} \sim \text{Cl}(U^{-1})) \preceq U^{-1}\).

Since \(U\) is NMOS, \(\text{NM} \sim \text{Cl}(U)\) is NMCoS and thus \(\text{NM} \sim \text{Cl}(U^{-1})\) is CoNMS in \(\mathcal{G}\). Thus, \(\text{NM} \sim \text{Int}(\text{NM} \sim \text{Cl}(U^{-1})) \preceq \text{NM} \sim \text{Cl}(U^{-1}) \preceq \text{NM} \sim \text{Cl}(U^{-1}) \Rightarrow \text{NM} \sim \text{Int}(\text{NM} \sim \text{Cl}(U^{-1})) \preceq (\text{NM} \sim \text{Cl}(U^{-1}))^{-1} \preceq U^{-1}\). Thus, \(U^{-1} = \text{NM} \sim \text{Int}(\text{NM} \sim \text{Cl}(U^{-1}))\). This proves that \(U^{-1} \in \text{NMROS}(\mathcal{G})\). □

**Corollary 1.** Let \(Q\) be any NMCoS in an NMATG \(\mathcal{G}\). Then
(a) \(mQ \in \text{NMROS}(\mathcal{G}), \forall m \in \mathcal{G}\);
(b) \(Q^{-1} \in \text{NMROS}(\mathcal{G})\).

**Theorem 18.** Let \(U\) be any NMROS in an NMATG \(\mathcal{G}\). Then
(a) \(\text{NM} \sim \text{Cl}(U_m) = \text{NM} \sim \text{Cl}(U)m, \forall m \in \mathcal{G}\);
(b) \(\text{NM} \sim \text{Cl}(mU) = \text{mNM} \sim \text{Cl}(U), \forall m \in \mathcal{G}\);
(c) \(\text{NM} \sim \text{Cl}(U^{-1}) = \text{NM} \sim \text{Cl}(U)^{-1}\).

**Proof.** (a) Assume \(p \in \text{NM} \sim \text{Cl}(U_m)\) and consider \(q = pm^{-1}\). Let \(q \in W\) be NMOS in \(\mathcal{G}\). Then \(\exists\) NMOSs \(m^{-1} \in V_1\) and \(p \in V_2\) in \(\mathcal{G}\), such that \(V_1V_2 \preceq \text{NM} \sim \text{Int}(\text{NM} \sim \text{Cl}(W))\). By hypothesis, there is \(g \in U_m \cap V_2 \Rightarrow g^{-1} \in U \cap V_1 \Rightarrow U \cap VM \preceq \text{Int}(\text{NM} \sim \text{Cl}(W)) \Rightarrow U \cap VM \preceq \text{NM} \sim \text{Cl}(W)\). Since \(U\) is NMOS, \(U \cap W \neq \emptyset\). That is, \(m \in \text{NM} \sim \text{Cl}(U_m)\). Conversely, let \(q \in \text{NM} \sim \text{Cl}(U_m)\). Then \(q = pg\) for some \(p \in \text{NM} \sim \text{Cl}(U)\).

To prove \(\text{NM} \sim \text{Cl}(U) \preceq \text{NM} \sim \text{Cl}(U_m)\). Let \(p \in W\) be an NMOS in \(\mathcal{G}\). Then \(\exists\) NMOSs \(m \in V_1\) in \(\mathcal{G}\) and \(p \in V_2\) in \(\mathcal{G}\) so that \(V_1V_2 \preceq \text{NM} \sim \text{Int}(\text{NM} \sim \text{Cl}(W))\). Since \(p \in \text{NM} \sim \text{Cl}(U), U \cap V_2 \neq \emptyset\). There is \(g \in \mathcal{G}\). This implies \(g^{-1} \in U \cap V_1\). From Theorem 17, \(U_m\) is NMOS and thus \(U_m \cap W \neq \emptyset\), therefore \(q \in \text{NM} \sim \text{Cl}(U_m)\). Therefore \(\text{NM} \sim \text{Cl}(U_m) = \text{mNM} \sim \text{Cl}(U)^{-1}\).

(b) Following the same steps as in part (1) above, we can prove that \(\text{NM} \sim \text{Cl}(mU) = \text{mNM} \sim \text{Cl}(U)\).

(c) Since \(\text{NM} \sim \text{Cl}(U)\) is NMCoS, \(\text{NM} \sim \text{Cl}(U^{-1})\) is NMCoS in \(\mathcal{G}\). Therefore, \(U^{-1} \preceq \text{NM} \sim \text{Cl}(U)^{-1}\) this gives \(\text{NM} \sim \text{Cl}(U^{-1}) \preceq \text{NM} \sim \text{Cl}(U)^{-1}\). Next, let \(q \in \text{NM} \sim \text{Cl}(U^{-1})\). Then \(q = p^{-1}\), for some \(p \in \text{NM} \sim \text{Cl}(U)\). Let \(q \in V\) be any NMOS in \(\mathcal{G}\). Then \(\exists\) open set \(U\) in \(\mathcal{G}\) such that \(p \in \mathcal{U}\) with \(U^{-1} \preceq \text{NM} \sim \text{Int}(\text{NM} \sim \text{Cl}(U))\). Moreover, there is \(m \in \mathcal{A} \cap \mathcal{U}\) which implies \(m^{-1} \in U^{-1} \cap \text{NM} \sim \text{Int}(\text{NM} \sim \text{Cl}(V))\). That is, \(U^{-1} \cap \text{NM} \sim \text{Cl}(V)\).
Let $q \in NM \rhd Cl(U)^{-1}$. Hence, $NM \sim Cl(U)^{-1} \not\prec NM \sim Cl(U)^{-1}$. □

**Theorem 19.** Let $Q$ be NMRCo subset in an NMATG $\mathbb{G}$. Then the below assertions are true:

(a) $NM \sim Int(mQ) = aNM \sim Int(Q), \forall m \in \mathbb{G}$;
(b) $NM \sim Int(Qm) = NM \sim Int(Q)a, \forall m \in \mathbb{G}$;
(c) $NM \sim Int(Q^{-1}) = NM \sim Int(Q)^{-1}$.

**Proof.** (a) Since $Q$ is NMRCoS, $NM \sim Int(Q)$ is NMROS in $\mathbb{G}$. Consequently, $mNM \sim Int(Q) \not\sim NM \sim Int(mQ)$. Conversely, let $q \in NM \sim Int(mQ)$ be an arbitrary element. Suppose $q = mp$, for some $p \in Q$. By hypothesis, this proves $mQ$ is NMCoS, and that is $NM \sim Int(mQ)$ is NMROS in $\mathbb{G}$. Assume that $m \in U$ and $p \in V$ be NMOSs in $\mathbb{G}$, such that $UV \not\prec NM \sim Int(mQ)$. Then $mV \not\prec mQ$, which means that $mV \not\prec mNM \sim Int(Q)$. Thus, $NM \sim Int(mQ) \not\prec NM \sim Int(Q)$.

(b) Following the same steps as in part (1) above, we can prove that $NM \sim Int(Qm) \not\prec NM \sim Int(Q)m$.

(c) Since $NM \sim Int(Q)$ is NMROS, $NM \sim Int(Q)^{-1}$ is NMOS in $\mathbb{G}$. Therefore, $Q^{-1} \not\prec NM \sim Int(Q)^{-1}$ implies that $NM \sim Int(Q)^{-1} \not\prec NM \sim Int(Q)^{-1}$. Next, let $q$ be an arbitrary element of $NM \sim Int(Q)^{-1}$. Then $q = p^{-1}$, for some $p \in NM \sim Int(Q)$. Let $q \in V$ be NMOS in $\mathbb{G}$. Then $\exists$ NMOS $U$ is in $G$, such that $p \in U$ with $U^{-1} \not\prec NM \sim Cl(NM \sim Int(V))$. Moreover, there is $g \in Q \cap U$, which implies $g^{-1} \in Q^{-1} \cap NM \sim Cl(NM \sim Int(V))$. That is $Q^{-1} \cap NM \sim Cl(NM \sim Int(V)) \not\prec NM \sim Cl(U^{-1}) \not\prec NM \sim Cl(U)^{-1}$. □

**Theorem 20.** Let $U$ be any NMOS in an NMATG $\mathbb{G}$. Then

(a) $NM \sim Cl(mU) \not\prec mNM \sim Cl(U), \forall m \in \mathbb{G}$;
(b) $NM \sim Cl(Um) \not\prec NM \sim Cl(U)m, \forall m \in \mathbb{G}$;
(c) $NM \sim Cl(U^{-1}) \not\prec NM \sim Cl(U)^{-1}$.

**Proof.** (a) As $U$ is NMOS, $NM \sim Cl(U)$ is NMRCoS. From Theorem 16, $\mu_{m^{-1}}: (\mathbb{G}, \tau_X) \rightarrow (\mathbb{G}, \tau_X)$ is NMACM. Thus, $mNM \sim Cl(U)$ is NMCoS. Hence, $NM \sim Cl(mU) \not\prec mNM \sim Cl(U)$.

(b) As $U$ is NMOS, $NM \sim Cl(U)$ is NMRCoS. From Theorem 16, $\lambda_{m^{-1}}: (\mathbb{G}, \tau_X) \rightarrow (\mathbb{G}, \tau_X)$ is NMACM. Thus, $NM \sim Cl(U)m$ is NMCoS. Therefore, $NM \sim Cl(Um) \not\prec NM \sim Cl(U)m$.

(c) Since $U$ is NMOS, $NM \sim Cl(U)$ is NMRCoS, and hence, $NM \sim Cl(U)^{-1}$ is NMCoS. Consequently, $NM \sim Cl(U) \not\prec NM \sim Cl(U)^{-1}$. □

**Theorem 21.** Let $U$ be both NMSO and NMSCo subset of an NMATG $\mathbb{G}$. Then the below statements hold:

(a) $NM \sim Cl(mU) = mNM \sim Cl(U)$, for each $m \in \mathbb{G}$;
(b) $NM \sim Cl(Um) = NM \sim Cl(U)m$, for each $m \in \mathbb{G}$;
(c) $NM \sim Cl(U^{-1}) = NM \sim Cl(U)^{-1}$.

**Proof.** (a) Since $U$ is NMSOS, $NM \sim Cl(U)$ is NMRCoS, from which it implies that $NM \sim Cl(mU) \not\prec mNM \sim Cl(U)$. Further, neutrosophic multi semi-openness of $U$ gives $NM \sim Cl(U) = NM \sim Cl(NM \sim Int(U)) \not\prec mNM \sim Cl(U) = mNM \sim Cl(NM \sim Int(U))$. As $U$ is NMSCoS, $NM \sim Int(U)$ is NMROS in $\mathbb{G}$. From Theorem 20, $mNM \sim Cl(U) = mNM \sim Cl(NM \sim Int(U)) = NM \sim Cl(mNM \sim Int(U)) \not\prec NM \sim Cl(mU)$. Hence, $NM \sim Cl(mU) = mNM \sim Cl(U)$.
(b) Following the same steps as in part (1) above, we can prove that $NM \sim Cl(Um) = NM \sim Cl(U)m$.

(c) By hypothesis, this proves $NM \sim Cl(U) \text{ is NMRCoS}$ and therefore $NM \sim Cl(U)^{-1}$ is NMCoS. Consequently, $NM \sim Cl(U^{-1}) \leq NM \sim Cl(U)^{-1}$. Next, since $U$ is NMSOS, $NM \sim Cl(U) = NM \sim Cl(NM \sim Int(U)) \Rightarrow NM \sim Cl(U)^{-1} = NM \sim Cl(NM \sim Int(U))$. Moreover, as $U$ is NMSCoS, $NM \sim Int(U)$ is NMROS. From Theorem 18, $NM \sim Cl(U)^{-1} = NM \sim Cl(NM \sim Int(U)^{-1}) \leq NM \sim Cl(U^{-1})$. This shows that $NM \sim Cl(U^{-1}) = NM \sim Cl(U)^{-1}$. □

**Theorem 22.** From Theorem 21, the following statements hold:

(a) $NM \sim Int(mU) = mNM \sim Int(U)$, for each $m \in G$;
(b) $NM \sim Int(Um) = NM \sim Int(U)m$, for each $m \in G$;
(c) $NM \sim Int(U^{-1}) = NM \sim Int(U)^{-1}$.

**Proof.** (a) As $U$ is NMSCoS, $NM \sim Int(U)$ is NMROS. From Theorem 16, $\mu_{m^{-1}}: (G, \tau_G) \rightarrow (G, \tau_G)$ is NMACM. Therefore, $\mu_{m^{-1}}(NM \sim Int(U)) = mNM \sim Int(U)$ is NMOS. Thus, $mNM \sim Int(U) \leq NM \sim Int(mU)$. Next, by assumption, it implies that $NM \sim Int(U) = NM \sim Int(NM \sim Cl(U)) \Rightarrow mNM \sim Int(U) = mNM \sim Int(NM \sim Cl(U))$. As $U$ is NMSOS, $NM \sim Cl(U)$ is NMRCoS. From Theorem 19, $mNM \sim Int(NM \sim Cl(U)) = NM \sim Int(mNM \sim Cl(U)) \geq NM \sim Int(mU)$. That is, $NM \sim Int(mU) \leq mNM \sim Int(U)$. Therefore, we have, $NM \sim Int(U)m = mNM \sim Int(U)m$. Hence, it was proved.

(b) As $U$ is NMSCoS, $NM \sim Int(U)$ is NMROS. From Theorem 16, $\mu_{m^{-1}}: (G, \tau_G) \rightarrow (G, \tau_G)$ is NMACM. Thus, $\lambda^{-1}_{m^{-1}}(NM \sim Int(U)) = mNM \sim Int(U)$ is NMOS. Therefore, $NM \sim Int(U)m \leq NM \sim Int(Um)$. Next, by assumption, this proves that $NM \sim Int(U) = NM \sim Int(NM \sim Cl(U)) \Rightarrow NM \sim Int(U)m = NM \sim Int(NM \sim Cl(U)m). As U is NMSOS, NM \sim Cl(U)$ is NMCoS. From Theorem 19, $NM \sim Int(NM \sim Cl(U)m) = NM \sim Int(NM \sim Cl(U)m) \geq NM \sim Int(Um)$. That is, $NM \sim Int(Um) \leq NM \sim Int(U)m$. Therefore, $NM \sim Int(U)m = NM \sim Int(U)m$. Hence, it was proved.

(c) From assumption, this proves that $NM \sim Int(U)$ is NMROS and therefore $NM \sim Int(U)^{-1}$ is NMOS. Consequently, $NM \sim Int(U^{-1}) \leq NM \sim Int(U)^{-1}$. Next, as $U$ is NMSCoS, $NM \sim Int(U) = NM \sim Int(NM \sim Cl(U)) \Rightarrow NM \sim Int(U)^{-1} = NM \sim Int(NM \sim Cl(U)^{-1})$. Moreover, as $U$ is NMSOS, $NM \sim Cl(U)$ is NMRCoS. From Theorem 19, $NM \sim Int(U)^{-1} = NM \sim Int(NM \sim Cl(U)^{-1}) \leq NM \sim Int(U^{-1})$. This proves that $NM \sim Int(U^{-1}) = NM \sim Int(U)^{-1}$. □

**Theorem 23.** Let $A$ be NMOS in an NMATG $G$. Then $aA \leq NM \sim Int(aNM \sim Int(NM \sim Cl(A)))$ for $a \in G$.

**Proof.** Since $A$ is NMOS, so $A \leq NM \sim Int(NM \sim Cl(A)) \Rightarrow aA \leq aNM \sim Int(NM \sim Cl(A))$. From Theorem 17, $aNM \sim Int(NM \sim Cl(A))$ is NMOS (in fact, NMROS). Hence, $aA \not\leq NM \sim Int(aNM \sim Int(NM \sim Cl(A)))$. □

**Theorem 24.** Let $Q$ be any neutrosophic multi-closed subset in an NMATG $G$. Then $NM \sim Cl(aNM \sim Cl(NM \sim Int(A))) \leq aQ$ for each $a \in G$.

**Proof.** Since $Q$ is NMCoS, so $Q \geq NM \sim Cl(NM \sim Int(Q)) \Rightarrow aQ \geq aNM \sim Cl(NM \sim Int(Q))$. From Theorem 17, $aNM \sim Cl(NM \sim Int(Q))$ is NMCoS (in fact, NMRCoS). Therefore, $aQ \geq NM \sim Cl(aNM \sim Cl(NM \sim Int(A)))$. Hence, $NM \sim Cl(aNM \sim Cl(NM \sim Int(A))) \leq aQ$. □
4. Conclusions

To deal with uncertainty, the NS uses the truth membership function, indeterminacy membership function, and falsity membership function. By discovering this concept, we were able to generalise the idea of an almost topological group to an NMATG. First, we developed the definitions of NMSOS, NMSCoS, NMROS, NMRCoS, NMCM, NMOM, NMCoM, NMSCM, NMSOM, NMSCoM to propose the definition of NMATG. Some properties of NMACM were demonstrated. Finally, we defined NMATG and demonstrated some of their properties using the definition of NMACM. In this study, an NMATG is conceptualised for the environments of the NS along with some of their elementary properties and theoretic operations. Novel numerical examples are given for definitions and remarks to study NMATG. We expect that our study may spark some new ideas for the construction of the NMATG. Future work may include the extension of this work for:

1. The development of the NMATG of the neutrosophic multi-vector spaces, etc.;
2. Dealing NMATG with multi-criteria decision-making techniques.

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References

1. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353. [CrossRef]
2. Chang, C.L. Fuzzy Topological Space. *J. Math. Anal. Appl.* **1968**, *24*, 182–190. [CrossRef]
3. Coker, D. An introduction to intuitionistic fuzzy topological spaces. *Fuzzy Sets Syst.* **1997**, *88*, 81–89. [CrossRef]
4. Lupianez, F.G. On neutrosophic topology. *Int. J. Syst. Cybern.* **2008**, *37*, 797–800. [CrossRef]
5. Lupianez, F.G. Interval neutrosophic sets and topology. *Int. J. Syst. Cybern.* **2009**, *38*, 621–624. [CrossRef]
6. Lupianez, F.G. On various neutrosophic topologies. *Int. J. Syst. Cybern.* **2009**, *38*, 1009–1013.
7. Lupianez, F.G. On neutrosophic paraconsistent topology. *Int. J. Syst. Cybern.* **2010**, *39*, 598–610. [CrossRef]
8. Salama, A.A.; Smarandache, F.; Kroumov, V. Closed sets and Neutrosophic Continuous Functions. *Neutrosophic Sets Syst.* **2014**, *4*, 4–8.
9. Kelly, J.C. Bitopological spaces. *Proc. Lond. Math. Soc.* **1963**, *3*, 71–89. [CrossRef]
10. Kandil, A.; Nouth, A.A.; El-Sheikh, S.A. On fuzzy bitopological spaces. *Fuzzy Sets Syst.* **1995**, *74*, 353–363. [CrossRef]
11. Lee, S.J.; Kim, J.T. Some Properties of Intuitionistic Fuzzy Bitopological Spaces. In Proceedings of the 6th International Conference on Soft Computing and Intelligent Systems, and The 13th IEEE International Symposium on Advanced Intelligence Systems, Kobe, Japan, 20–24 November 2012; pp. 20–24. [CrossRef]
12. Garg, H. A new improved score function of an interval-valued Pythagorean fuzzy set based TOPSIS method. *Int. J. Uncertain. Quantif.* **2017**, *7*, 463–474. [CrossRef]
13. Peng, X.; Yang, Y. Some results for Pythagorean fuzzy sets. *Int. J. Intell. Syst.* **2015**, *30*, 1133–1160. [CrossRef]
14. Peng, X.; Selvachandran, G. Pythagorean fuzzy set: State of the art and future directions. *Artif. Intell. Rev.* **2017**, *52*, 1873–1927. [CrossRef]
15. Beliakov, G.; James, S. Averaging aggregation functions for preferences expressed as Pythagorean membership grades and fuzzy orthopairs. In Proceedings of the IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), Beijing, China, 6–11 July 2014; pp. 298–305. [CrossRef]
16. Dick, S.; Yager, R.R.; Yazdanbakhsh, O. On Pythagorean and complex fuzzy set operations. *IEEE Trans. Fuzzy Syst.* **2016**, *24*, 1009–1021. [CrossRef]
17. Gou, X.J.; Xu, Z.S.; Ren, P.J. The properties of continuous Pythagorean fuzzy Information. *Int. J. Intell. Syst.* **2016**, *31*, 401–424. [CrossRef]
18. He, X.; Du, Y.; Liu, W. Pythagorean fuzzy power average operators. *Fuzzy Syst. Math.* **2016**, *30*, 116–124.
19. Ejegwa, P.A. Distance and similarity measures of Pythagorean fuzzy sets. *Granul. Comput.* 2018, 5, 225–238. [CrossRef]
20. Yager, R.R. Generalized orthopair fuzzy sets. *IEEE Trans. Fuzzy Syst.* 2017, 25, 1222–1230. [CrossRef]
21. Yager, R.R.; Alajlan, N. Approximate reasoning with generalized orthopair fuzzy sets. *Inf. Fusion* 2017, 38, 65–73. [CrossRef]
22. Peng, X.; Liu, L. Information measures for q-rung orthopair fuzzy sets. *Int. J. Intell. Syst.* 2019, 34, 1795–1834. [CrossRef]
23. Pinar, A.; Boran, F.E. A q-rung orthopair fuzzy multi-criteria decision making method for supplier selection based on a novel distance measure. *Int. J. Mach. Learn. Cybern.* 2020, 11, 1749–1780. [CrossRef]
24. Cuong, B.C.; Kreinovich, V. Picture Fuzzy Sets—A new concept for computational intelligence problems. In *Proceedings of the 2013 Third World Congress on Information and Communication Technologies (WICT 2013)*, Hanoi, Vietnam, 15–18 December 2013; pp. 1–6. [CrossRef]
25. Cuong, B.C. Picture fuzzy sets. *J. Comput. Sci. Cybern.* 2014, 30, 409–420.
26. Phong, P.H.; Hieu, D.T.; Ngan, R.T.H.; Them, P.T. Some compositions of picture fuzzy relations. In *Proceedings of the 7th National Conference Fundamental and Applied Information Technology Research*, FAIR’7, Thai Nguyen, Vietnam, 19–20 June 2014; pp. 19–20.
27. Cuong, B.C.; Hai, P.V. Some fuzzy logic operators for picture fuzzy sets. In *Proceedings of the Seventh International Conference on Knowledge and Systems Engineering*, Ho Chi Minh City, Vietnam, 8–10 October 2015; pp. 132–137.
28. Cuong, B.C.; Ngan, R.T.; Hai, B.D. An involutive picture fuzzy negator on picture fuzzy sets and some De Morgan triples. In *Proceedings of the Seventh International Conference on Knowledge and Systems Engineering*, Ho Chi Minh City, Vietnam, 8–10 October 2015; pp. 126–131.
29. Viet, P.V.; Chau, H.T.M.; Hai, P.V. Some extensions of membership graphs for picture inference systems. In *Proceedings of the 2015 Seventh International Conference on Knowledge and Systems Engineering* (KSE), Ho Chi Minh City, Vietnam, 8–10 October 2015; IEEE: Piscataway, NJ, USA, 2015; pp. 192–197.
30. Singh, P. Correlation coefficients for picture fuzzy sets. *J. Intell. Fuzzy Syst.* 2015, 28, 591–604. [CrossRef]
31. Garg, H. Some picture fuzzy aggregation operators and their applications to multicriteria decision-making. *Arab. J. Sci. Eng.* 2017, 42, 5275–5290. [CrossRef]
32. Quek, S.G.; Selvachandran, G.; Munir, M.; Mahmood, T.; Ullah, K.; Son, L.H.; Pham, T.H.; Kumar, R.; Priyadarshini, I. Multi-attribute multi-perception decision-making based on generalized T-spherical fuzzy weighted aggregation operators on neutrosophic sets. *Mathematics* 2019, 7, 780. [CrossRef]
33. Garg, H.; Munir, M.; Ullah, K.; Mahmood, T.; Jan, N. Algorithm for T-spherical fuzzy multi-attribute decision making based on improved interactive aggregation operators. *Symmetry 2018*, 10, 670. [CrossRef]
34. Zeng, S.; Garg, H.; Munir, M.; Mahmood, T.; Hussain, A. A multi-attribute decision making process with immediate probabilistic interactive averaging aggregation operators of T-spherical fuzzy sets and its application in the selection of solar cells. *Energies 2019*, 12, 4436. [CrossRef]
35. Munir, M.; Kalsoom, H.; Ullah, K.; Mahmood, T.; Chu, Y.M. T-spherical fuzzy Einstein hybrid aggregation operators and their applications in multi-attribute decision making problems. *Symmetry 2020*, 12, 365. [CrossRef]
36. Mahmood, T.; Ullah, K.; Khan, Q.; Jan, N. An approach towards decision making and medical diagnosis problems using the concept of spherical fuzzy sets. *Neural Comput. Appl.* 2018, 31, 7041–7053. [CrossRef]
37. Warren, R.H. Boundary of a fuzzy set. *Indiana Univ. Math. J.* 1977, 26, 191–197. [CrossRef]
38. Tang, X. Spatial Object Modeling in Fuzzy Topological Spaces with Applications to Land Cover Change in China. Ph.D. Thesis, University of Twente, Enschede, The Netherlands, 2004.
39. Kharal, A. A Study of Frontier and Semifrontier in Intuitionistic Fuzzy Topological Spaces. *Sci. World J.* 2014, 2014, 674171. [CrossRef]
40. Salama, A.A.; Alblowi, S. Generalized neutrosophic set and generalized neutrosophic topological spaces. *Comp. Sci. Eng.* 2012, 2, 129–132. [CrossRef]
41. Azad, K.K. On Fuzzy Semi-continuity, Fuzzy Almost Continuity and Fuzzy Weakly Continuity. *J. Math. Anal. Appl.* 1981, 82, 14–32. [CrossRef]
42. Smarandache, F. Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics; University of New Mexico: Gallup, NM, USA, 2002.
43. Smarandache, F. Neutrosophic set—a generalization of the intuitionistic fuzzy set. *Int. J. Pure Appl. Math.* 2005, 24, 287–297.
44. Mwchahary, D.D.; Basumatary, B. A note on Neutrosophic Bitopological Space. *Neutrosophic Sets Syst.* 2020, 7, 365. [CrossRef]
45. Yager, R.R. On the theory of bags. *Int. J. Gen. Syst.* 1986, 13, 23–37. [CrossRef]
46. Miyamoto, S. Multiset theory. *Notre Dame J. Form. Logic* 1986, 1795–1834. [CrossRef]
47. Blizard, W. Multiset theory. *Notre Dame J. Form. Logic* 1989, 30, 36–66. [CrossRef]
48. Yager, R.R. On the theory of bags. *Int. J. Gen Syst.* 1986, 13, 23–37. [CrossRef]
49. Miyamoto, S. Fuzzy Multisets and Their Generalizations. In *Multiset Processing*; Springer: Berlin, Germany, 2001; pp. 225–235.
50. Onasanya, B.O.; Hoskova-Mayerova, S. Some Topological and Algebraic Properties of alpha-level Subsets Topology of a Fuzzy Subset. *An. St. Univ. Ovidius Constanta* 2018, 26, 213–227.
51. Onasanya, B.O.; Hoskova-Mayerova, S. Multi-fuzzy group induced by multisets. *Ital. J. Pure Appl. Math.* 2019, 41, 597–604.
52. Al Tahan, M.; Hoskova-Mayerova, S.; Davvaz, B. Fuzzy multi-polynomials. *J. Intell. Fuzzy Syst.* 2020, 38, 2337–2345. [CrossRef]
53. Al Tahan, M.; Hoskova-Mayerova, S.; Davvaz, B. Some results on (generalized) fuzzy multi-Hv-ideals of Hv-rings. *Symmetry 2019*, 11, 1376. [CrossRef]
54. Bakbak, D.; Uluçay, V.; Sahin, M. Neutrosophic Multigroups and Applications. *Mathematics* 2019, 7, 95. [CrossRef]
53. Sebastian, S.; Ramakrishnan, T.V. Multi-fuzzy sets: An extension of fuzzy sets. *Fuzzy Inf. Eng.* 2011, 1, 35–43. [CrossRef]

54. Dey, A.; Pal, M. Multi-fuzzy complex numbers and multi-fuzzy complex sets. *Int. J. Fuzzy Syst. Appl.* 2014, 4, 15–27. [CrossRef]

55. Yong, Y.; Xia, T.; Congcong, M. The multi-fuzzy soft set and its application in decision making. *Appl. Math. Model* 2013, 37, 4915–4923. [CrossRef]

56. Sebastian, S.; Ramakrishnan, T.T. Multi-Fuzzy Sets. *Int. Math. Forum* 2010, 5, 2471–2476.

57. Atanassov, K.T. Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* 1986, 20, 87–96. [CrossRef]

58. Abdullah, S.; Naeem, M. A new type of interval valued fuzzy normal subgroups of groups. *New Trends Math. Sci.* 2015, 3, 62–77.

59. Li, X.P.; Wang, G.J. (l, a)-Homomorphisms of Intuitionistic Fuzzy Groups. *Hacettepe J. Math. Stat.* 2011, 40, 663–672.

60. Palaniappan, N.; Naganathan, S.; Arjunan, K. A study on Intuitionistic L-fuzzy Subgroups. *Appl. Math. Sci.* 2009, 3, 2619–2624.

61. Sharma, P.K. Homomorphism of intuitionistic fuzzy groups. *Int. Math. Forum* 2011, 6, 3169–3178.

62. Xu, C.Y. Homomorphism of Intuitionistic Fuzzy Groups. In Proceedings of the 2007 International Conference on Machine Learning and Cybernetics, Hong Kong, China, 19–22 August 2007; pp. 178–183. [CrossRef]

63. Yuan, X.H.; Li, H.X.; Lee, E.S. On the direct product of intuitionistic fuzzy subgroups. *Adv. Fuzzy Sets Syst.* 2012, 1, 1–6.

64. Ye, S.; Ye, J. Dice Similarity Measure between Single Valued Neutrosophic Multisets and Its Application in Medical Diagnosis. *Neutrosophic Sets Syst.* 2014, 6, 49–54.

65. Yang, Y.; Xia, T.; Congcong, M. The multi-fuzzy soft set and its application in decision making. *Appl. Math. Model* 2013, 37, 4915–4923. [CrossRef]

66. Shinoj, T.K.; John, S.S. Intuitionistic multigroups and its application in medical diagnosis. *World Acad. Sci. Eng. Technol.* 2012, 6, 1418–1421.

67. Abdullah, S.; Naeem, M. A new type of interval valued fuzzy normal subgroups of groups. *New Trends Math. Sci.* 2015, 3, 62–77.

68. Li, X.P.; Wang, G.J. (l, a)-Homomorphisms of Intuitionistic Fuzzy Groups. *Hacettepe J. Math. Stat.* 2011, 40, 663–672.

69. Palaniappan, N.; Naganathan, S.; Arjunan, K. A study on Intuitionistic L-fuzzy Subgroups. *Appl. Math. Sci.* 2009, 3, 2619–2624.

70. Sharma, P.K. Homomorphism of intuitionistic fuzzy groups. *Int. Math. Forum* 2011, 6, 3169–3178.

71. Xu, C.Y. Homomorphism of Intuitionistic Fuzzy Groups. In Proceedings of the 2007 International Conference on Machine Learning and Cybernetics, Hong Kong, China, 19–22 August 2007; pp. 178–183. [CrossRef]

72. Yuan, X.H.; Li, H.X.; Lee, E.S. On the direct product of intuitionistic fuzzy subgroups. *Adv. Fuzzy Sets Syst.* 2012, 1, 1–6.

73. Shinoj, T.K.; John, S.S. Intuitionistic fuzzy multigroups. *Ann. Pure Appl. Math.* 2015, 9, 131–143.

74. Smarandache, F. *A Unifying Field in Logics; Infinite Study:* Conshohocken, PA, USA, 1998.

75. Ye, S.; Fu, J.; Ye, J. Medical Diagnosis Using Distance-Based Similarity Measures of Single Valued Neutrosophic Multisets. *Neutrosophic Sets Syst.* 2015, 7, 47–52.

76. Smarandache, F. *Neutrosophic Perspectives: Triplets, Duplets, Multisets, Hybrid Operators, Modal Logic, Hedge Algebras and Applications;* Pons Publishing House Brussels: Brussels, Belgium, 2017; p. 323.