MINIMIZATION OF VISIBLY PUSHDOWN AUTOMATA IS NP-COMPLETE

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\begin{abstract}
We show that the minimization of visibly pushdown automata is NP-complete. This result is obtained by introducing immersions, that recognize multiple languages (over a usual, non-visible alphabet) using a common deterministic transition graph, such that each language is associated with an initial state and a set of final states. We show that minimizing immersions is NP-complete, and reduce this problem to the minimization of visibly pushdown automata.
\end{abstract}

1. INTRODUCTION

Visibly pushdown automata (VPA) are a natural model for the control flow of recursive programs and have tight connections with tree automata and XML schemas. They were considered for parsing algorithms \cite{LLZ11} under the name “input-driven pushdown automata”, and shown to have better space complexity than unrestricted pushdown automata. The name “visibly pushdown automata” is due to Alur and Madhusudan \cite{AM04}, who initiated their study from the perspective of program verification, and developed the theory in several directions (see \url{http://madhu.cs.illinois.edu/vpa/} for an exhaustive list of results). In particular, they showed that the class of visibly pushdown languages shares many desirable properties with the class of regular languages, like determinization, closure under boolean operations and the existence of a Myhill-Nerode congruence that defines canonical VPA \cite{AM04}. However, the existence of a canonical VPA does not help for minimization, in contrast to regular languages. Similarly to many other more complex automata models, like automata over infinite words, two-way automata, etc, VPA do not have unique minimal automata. Even worse, the canonical VPA can be exponentially larger than a minimal VPA. Therefore, the minimization problem for VPA, besides being very relevant in practice, is also very challenging.

\textit{Minimization up to partitioning.} Various minimization procedures have been proposed for some subclasses of deterministic VPA. Most of them use a partitioning of the state...
space into modules: when the VPA control is in a given module, and a call occurs, then the matching return brings the control back to the same module. The first model implementing this idea are single-entry VPA (Sevpa) [1], where each module has its own set of call symbols, and these sets are disjoint. Moreover, each module has a specific entry state: whenever a call of module \( m \) occurs, the VPA switches to the entry of \( m \). For any fixed partition of call symbols [1] shows that there is a unique minimal deterministic Sevpa, and that it can be computed in polynomial time. Multiple-entry VPA (Mevpa) [8] allow several possible states when entering the module, but the symbol pushed on the stack by a call depends only on the state, not on the call symbol. Mevpa enjoy the same properties as Sevpa in terms of minimization: the minimal Mevpa is unique and computable in polynomial time. The two models Sevpa and Mevpa are subsumed by call-driven automata (CDA) [4], for which states are partitioned into modules, and a call leads to a state that depends only on the call symbol. A restricted version of CDA, called expanded CDA (eCDA) [4], further requires that only one call symbol can enter each module. Minimization of eCDA is easy, it resembles the Myhill-Nerode construction. A minimization procedure for CDA is obtained by adapting that of eCDA, and generalizes the ones for Sevpa and Mevpa, in the sense that these ones can be retrieved from the minimization of CDA.

The drawback of all the subclasses mentioned above (Sevpa, Mevpa, CDA and eCDA) is that there exist families of languages for which the minimal VPA within the respective class is exponentially larger than some minimal VPA. Block VPA (BVPA) [4] were proposed to overcome this problem: for every VPA, there exists an equivalent BVPA of quadratic size, so VPA can be minimized approximately via BVPA minimization. BVPA differ from Sevpa in that the entry state is determined by the call symbol, but may also depend on the current state. There is a unique minimal BVPA for a given visibly pushdown language, and this BVPA can be computed in cubic time, up to some partition of the language.

So all the approaches for VPA minimization rely on a fixed partition, either of the state space, or of the language, and the difficulty of minimization relies on finding a good partition. Given a BVPA and two integers \( k \) and \( s \), knowing if there is an equivalent BVPA with \( k \) modules, each of size at most \( s \), is NP-complete [5].

The main result of this paper is that VPA minimization is inherently difficult: we show that the problem is NP-complete. We obtain our result by showing NP-hardness for the following problem about deterministic finite state automata (DFA), that can be of independent interest: given \( n \) regular languages and a bound \( N \), we ask if there exists some deterministic transition graph \( A \) of size \( N \) such that for every given language we find a DFA accepting it by choosing an initial state and a set of final states of \( A \). We refer to this problem as immersion minimization.

Further related work. As for regular languages, finding a minimal non-deterministic automaton is computationally hard, namely ExpTime-complete for non-deterministic VPA, [5] (hardness follows from the universality of non-deterministic VPA [2]). The paper [7] proposes an algorithm for computing locally minimal non-deterministic VPA, relying on a reduction to Partial Max-SAT. Results on the state complexity of VPA with respect to determinization, and various language operations are reported in the survey [12].

Some problems similar to the minimization of immersions also appear in the literature. However, to our best knowledge, no straightforward reduction exists from one of these problems to the minimization of immersions. The first problem is the minimization of non-deterministic finite automata with limited non-determinism. Whereas the minimization of
arbitrary non-deterministic finite automata is PSPACE-complete, it becomes NP-complete for automata that have a fixed number of initial states, and are otherwise deterministic [9]. Further NP-completeness results for minimization of automata with small degree of ambiguity are provided in [3]. A seemingly close problem from computational biology is the shortest common superstring problem, which asks for the shortest string containing each string from a given set as factor. This problem is known to be NP-complete [6].

Another related problem is the minimization of tree automata. Indeed, a word over a visibly pushdown alphabet can be viewed as the linearization of a tree, processed in a depth-first left-to-right traversal. This corresponds to an unranked tree, i.e., a finite ordered tree where the arity of each node is arbitrary. Several automata models exist for unranked trees, and the complexity of minimization ranges between PTIME and NP [10]. However, for each of these models, determinism does not correspond exactly to that of VPA, and minimization results do not transfer.

2. Automata

2.1. Visibly pushdown automata. A visibly pushdown alphabet \( \hat{\Sigma} = \Sigma_c \cup \Sigma_r \cup \Sigma_\ell \) is a finite set of symbols partitioned into call symbols in \( \Sigma_c \), return symbols in \( \Sigma_r \), and internal symbols in \( \Sigma_\ell \).

A visibly pushdown automaton (VPA for short) is a tuple \( \mathcal{C} = (\hat{\Sigma}, Q, I, F, \Gamma, \Delta) \) where \( \hat{\Sigma} \) is a visibly pushdown alphabet, \( Q \) is a finite set of states, \( I \subseteq Q \) and \( F \subseteq Q \) are the sets of initial, resp. final states, and \( \Gamma \) is the (finite) stack alphabet. The set \( \Delta \) has three types of transitions, depending on the type of the input symbol: call transitions \( \Delta_c \subseteq Q \times \Sigma_c \times Q \times \Gamma \) that push a symbol on the stack, return transitions \( \Delta_r \subseteq Q \times \Sigma_r \times \Gamma \times Q \) that pop a symbol from the stack, and internal transitions \( \Delta_\ell \subseteq Q \times \Sigma_\ell \times Q \) that leave the stack unchanged.

A configuration of \( \mathcal{C} \) is a pair \((q, \sigma)\) where \( q \in Q \) is the current state and \( \sigma \in \Gamma^* \) is the current stack content (the top of the stack is the rightmost symbol). A transition \((q, \sigma) \xrightarrow{a} \sigma_c (q', \sigma')\) corresponds to one of the following cases:

- \( a \in \Sigma_c \) and \( \sigma' = \sigma A \) for some \( q, q', A \) with \( (q, a, q', A) \in \Delta_c \),
- \( a \in \Sigma_r \) and \( \sigma = \sigma' A \) for some \( q, q', A \) with \( (q, a, A, q') \in \Delta_r \),
- \( a \in \Sigma_\ell \) and \( \sigma = \sigma' \) for some \( q, q' \) with \( (q, a, q') \in \Delta_\ell \).

Note that only return transitions can read the top stack symbol. The transition relation of \( \mathcal{C} \) extends to words from \( \Sigma^* \) as expected. The language accepted by \( \mathcal{C} \) is the set of words \( u \) such that \((q_0, \epsilon) \xrightarrow{u} \sigma_c (q_f, \sigma)\) with \( q_0 \in I, q_f \in F \) and \( \sigma \in \Gamma^* \). In particular, acceptance does not require that the final configuration has an empty stack. A VPA is deterministic if it has a single initial state, \( \Delta_c \) does not contain two rules \((q, a, q_1, \gamma_1)\) and \((q, a, q_2, \gamma_2)\) with \( (q_1, \gamma_1) \neq (q_2, \gamma_2) \), \( \Delta_r \) does not contain two rules \((q, a, \gamma, q_1)\) and \((q, a, \gamma, q_2)\) with \( q_1 \neq q_2 \), and \( \Delta_\ell \) do not contain two rules \((q, a, q_1)\) and \((q, a, q_2)\) with \( q_1 \neq q_2 \).

Minimization. We measure the size of a VPA by its number of states. This will be the parameter that we minimize. Another choice could be the size of the stack alphabet. The stack alphabet can be actually bounded by \(|Q| \cdot |\Sigma_c|\), as one can always choose it as \( Q \times \Sigma_c \), [4]. The problem we consider here is the following:
B₁ has initial state 1 and final states \{2\}. B₂ has initial state 3 and final states \{4\}. (a) Immersion of size 4. (b) A minimal immersion.

Figure 1. Two immersions for the languages \(L₁ = a^+\) and \(L₂ = a^*b\).
language. This is illustrated by Figure 1a, and yields an immersion with 4 states. A smaller immersion is obtained by merging the states 2, 3 in Figure 1a, as depicted in Figure 1b. The resulting immersion has 3 states, and is minimal for $L_1, L_2$. Another immersion with three states is obtained by merging the states 1, 4. The example shows that, in general, minimal immersions are not unique, as it is already the case for VPA.

**Proposition 2.2.** \textsc{MinImmersion} reduces in polynomial time to \textsc{MinVPA}.

\begin{proof}
Let $A_1, \ldots, A_n$ be DFA over the alphabet $\Sigma_i$, and let $L_i = L(A_i)$ for every $i$. We show that there exists an immersion of size $k$ for $L_1, \ldots, L_n$ if and only if there exists a deterministic VPA of size $k + 2$ for the language $K = \bigcup_{i=1}^n c_i L_i r$, where $\Sigma_c = \{c_1, \ldots, c_n\}$ and $\Sigma_r = \{r\}$.

Consider an immersion of size $k$ for $L_1, \ldots, L_n$ with finite, deterministic transition graph $A = (Q, \Sigma_r, \rightarrow)$, and sub-DFA $B_1, \ldots, B_n$ of $A$ such that $L(B_i) = L_i$, for all $1 \leq i \leq n$. Let $q_i$ and $F_i$ denote the initial state and the final states of $A$, respectively. From $A$ we immediately get a deterministic VPA $C$ for $K$ by letting $C = (\hat{\Sigma}, Q \cup \{q_0, q_f\}, \{q_0\}, \{q_f\}, \Gamma, \Delta)$, with stack alphabet $\Gamma = \{1, \ldots, n\}$, and $\Delta$ as follows:

- $\Delta_c = \{(q_0, c_i, q_i, i) \mid 1 \leq i \leq n\}$,
- $\Delta_\ell = \rightarrow$,
- $\Delta_r = \{(q, r, i, q_f) \mid q \in F_i, 1 \leq i \leq n\}$.

Conversely, assume there is some deterministic VPA $C = (\hat{\Sigma}, Q, \{q_0\}, F, \Gamma, \Delta)$ of size $k + 2$ for $K = \bigcup_{i=1}^n c_i L_i r$. This language is included in $\Sigma_c \Sigma_\ell \Sigma_r$, so we can assume that $C$ has a single final state, that we call $q_f$, and which has no outgoing transitions. Let $q_i$ denote the (unique) state of $C$ such that $(q_0, c_i, q_i, A_i) \in \Delta_c$, for some $A_i \in \Gamma$. We can also assume that $i$ is used instead of $A_i$ in these rules, as they are the only rules in $\Delta_c$. We define an immersion for the languages $L_1, \ldots, L_n$ as the transition graph $A = (Q, \Sigma_r, \rightarrow)$, where $Q_A = Q \setminus \{q_0, q_f\}$ and $\rightarrow = \Delta_\ell$. The sub-DFA $B_1, \ldots, B_n$ associated with this immersion are obtained by setting the initial state of $B_i$ to $q_i$, and setting $q \in F_i$ if $(q, r, i, q_f) \in \Delta_r$. It is clear that $B_i$ accepts precisely the words $w \in \Sigma_\ell$ such that $c_i w r \in L(C)$. So $L(B_i) = L_i$ for every $1 \leq i \leq n$.
\end{proof}

3. \textsc{MinImmersion} is NP-complete

It is clear that \textsc{MinImmersion} is in NP. We show NP-hardness by a reduction from 3-colorability. Let $G = (V, E)$ be an undirected graph with vertex set $V = \{1, \ldots, n\}$ and edge set $E \subseteq V^2 \setminus \{(i, i) \mid i \in V\}$. We ask whether there is a coloring $c : V \to \{0, 1, 2\}$ such that $c(i) \neq c(j)$, for every $(i, j) \in E$.

Before we define the DFA $A_1, \ldots, A_n$ we need some notations. Let $m = 2n(n-1)+2$. We fix a set $P = \{p_1, p_2, p_3, q_1, q_2\}$ of five distinct prime numbers $p$ such that $3n < p \leq c \cdot n$, for some suitable constant $c$, such that no $p \in P$ divides $m$. Let also $N = 3m+p_1+p_2+p_3+q_1+q_2$. Note that $6n^2 < N < 9n^2$, for $n$ sufficiently large.

\footnote{Recall that Chebyshev’s theorem says that there is always at least one prime between $n$ and $2n$.}
Notations. The alphabet used in the following for the DFA $A_i$ is $\Sigma = \{0, 1\}$. A path in some transition graph of the form $s_1 \xrightarrow{0} s_2 \xrightarrow{0} \cdots \xrightarrow{0} s_n$ will be called simply a path. Similarly, a cycle is a path as above, with $s_1 = s_n$. For any path $s_1 \xrightarrow{0} s_2 \xrightarrow{0} \cdots \xrightarrow{0} s_n$ we say that $s_n$ is 0-reachable from $s_1$, and $s_1$ is co-0-reachable from $s_n$. A $k$-cycle denotes a cycle of length $k$. A 1-transition is a transition labeled by 1.

We fix in the following a bijection between the set $\{2k + 1 : 1 \leq k \leq n(n-1)\}$ and the set of ordered pairs of vertices $\{(i, j) : i, j \in V, i \neq j\}$. Hereby we denote by $(i, j)$ the integer encoding the pair $(i, j)$ w.r.t. this fixed bijection.

We are now ready to define the DFA $A_i$, where $i \in V$ is a vertex of the given graph. The language $L_i$ of the DFA $A_i$ will be a subset of 0*10*. Informally, $A_i$ consists of an $m$-cycle (called “dispatch” cycle), such that from some of the vertices of this cycle there is a 1-transition to some $p$-cycle (called “counting” cycle) with $p \in P$. Each $p$-cycle has a designated “entry” node, and all 1-transitions into the cycle point to this node. Assuming that the vertices of the $m$-cycle are numbered successively $1, \ldots, m$, with 1 being the initial state, the DFA $A_i$ has the following transitions:

1. From vertex 1 there is a 1-transition to a $p_1$-cycle, and from vertex 2 there is a 1-transition to a $p_2$-cycle.
2. From every other even vertex there is a 1-transition to a $p_2$-cycle.
3. Each of the remaining $n(n-1)$ odd vertices is of the form $(j, k)$, according to the bijection fixed above. The transitions out of these vertices are the following:
   - Each odd vertex $(i, j)$ has a 1-transition to a $q_1$-cycle.
   - Each odd vertex $(j, i)$ with $(i, j) \notin E$, has a 1-transition to a $q_1$-cycle.
   - Each odd vertex $(j, i)$ with $(i, j) \in E$, has a 1-transition to a $q_2$-cycle.

Note that there is no 1-transition outgoing from vertices $(j, k)$ where $j \neq i$ and $k \neq i$. As already mentioned, the initial state of $A_i$ is the vertex 1 of the $m$-cycle. The final states are all the target states of the 1-transitions. Figure 2 shows an example $A_i$.

Remark 3.1. Note that any DFA accepting $(0^p)^*$ must contain a cycle of length divisible by $p$, if $p > 1$ is a prime.

Let $p$ be a prime from $P$. A vertex $s$ of a transition graph $A$ over $\Sigma = \{0, 1\}$ is called a $p$-vertex if there is a sub-DFA of $A$ for the language $1(0^p)^*$ with initial state $s$.

Lemma 3.2. Let $A$ be a minimal immersion for $L_1, \ldots, L_n$ of size at most $N$, and let $C$ be a $k$-cycle of $A$. Then exactly one of the two following cases holds:

1. $C$ contains at least one $p_1$-vertex and $k$ is divisible by $m$.
2. $k$ is divisible by some unique prime $p \in P$.

Proof. By assumption there is some sub-DFA $B_i$ of $A$ accepting $L_i$, for every $i$. Note first that, by minimality of $A$, every vertex of $A$ is either 0-reachable from the initial state of some $B_i$, or co-0-reachable from a final state of some $B_i$. This will ensure that one of the two cases in the statement of the lemma holds for any cycle.

Recall that immersions were defined as deterministic transition graphs. Using the assumption $|A| \leq N$, note that a vertex of $A$ cannot be both a $p$-vertex and a $p'$-vertex, for two different primes $p, p'$ from $P$. Then otherwise the size of $A$ would be at least $9n^2 > N$, which is a contradiction.

Let us denote a vertex $s$ of $A$ as special if $s$ is $p_1$-vertex and $s \xrightarrow{0} s'$, with $s'$ being $p_2$-vertex.
The first case in the statement corresponds to $C$ being 0-reachable from the initial state of some $B_i$. Clearly, $C$ needs to have at least one special vertex $s$. Note also that any vertex $t$ such that $s \xrightarrow{0^m} t$, where $j \geq 0$, must be special, because the length of the dispatch cycle is $m$. Assume by contradiction that $k$ is not divisible by $m$, and let $d = k \pmod{m}$. If $d$ is odd, then it follows from the previous remark that $C$ must contain a $p_1$-vertex that is at the same time a $p_2$-vertex or a $p_3$-vertex. If $d$ is even and not zero, then similarly, $C$ must contain a vertex that is at the same time a $p_2$-vertex and a $p_3$-vertex. So in both cases we obtain a contradiction to $|A| \leq N$, as already noted.

The second case is where $C$ is co-0-reachable from a final state of some $B_i$. Here, $k$ must be divisible by some $p \in P$. This prime is unique, as already observed.

We argue finally that the two cases are mutually exclusive. If $k$ were both divisible by $m$ and by $p \in P$ then $k > pm > N$, since $p$ does not divide $m$. But this is again a contradiction to $|A| \leq N$.

Assume that $A$ is a minimal immersion for $L_1, \ldots, L_n$ of size at most $N$. From Lemma 3.2 we deduce that the vertex set of $A$ is the disjoint union of two sets $V_1, V_2$, such that:

- Transitions within each $V_i$ ($i \in \{1, 2\}$) are labeled only by 0.
- Transitions from $V_1$ to $V_2$ are labeled only by 1.
- There are no transitions from $V_2$ to $V_1$.

To see this, let us ignore the 1-labeled transitions of $A$. Then we obtain a disjoint union of graphs (transitions are labeled only by 0s). Each such graph consists of a cycle, plus some
simple paths reaching the cycle. From Lemma 3.2 we know that each cycle is used either to
“dispatch” (case 1) or to “count” modulo some prime (case 2), and that the two cases are
mutually exclusive. So by the minimality of \( \mathcal{A} \) we can conclude that 1-labeled transitions of
\( \mathcal{A} \) go only from “dispatch” cycles to “counting” cycles. Again by minimality we can bound
the size and number of cycles in \( V_1 \) and \( V_2 \):

**Lemma 3.3.** Assume that \( \mathcal{A} \) is a minimal immersion for \( L_1, \ldots, L_n \) of size at most \( N \).
Then the vertex set of \( \mathcal{A} \) is the disjoint union of two sets \( V_1, V_2 \) as above, such that:

1. \( V_1 \) consists of at most three \( m \)-cycles.
2. \( V_2 \) consists of \( p \)-cycles, one for each \( p \in P \).

**Proof.** By Lemma 3.2 we know that \( V_2 \) contains at least \(|P|\) cycles, one for each \( p \in P \). By
minimality of \( \mathcal{A} \), \( V_2 \) has exactly one \( p \)-cycle, for each \( p \in P \).

Now we consider \( V_1 \). By Lemma 3.2 the cycles of \( V_1 \) have length divisible by \( m \). As
before, by minimality \( V_1 \) is a disjoint union of cycles. Each cycle \( C \) has the property that it
accepts some language \( \{ u \in 0^* : u1v \in L_i \text{ for some } v \in 0^* \} \) from one of the \( p_1 \)-vertices of
\( C \). In particular, \( C \) is equal to \( C'^j \) for some dispatch cycle \( C' \) of one of the \( L_i \) and some
\( j \geq 1 \). By minimality of \( \mathcal{A} \) we obtain that \( C = C' \) and \( j = 1 \). Finally, by the choice of \( N \),
we conclude that \( V_1 \) consists of at most three \( m \)-cycles.

From Lemma 3.3 we see that each of the sub-DFA for any of the \( L_i \) consists of one of
the \( m \)-cycles in \( V_1 \), with the \( p_1 \)-vertex as initial state, together with transitions labeled by 1
to the required \( p \)-cycles in \( V_2 \).

**Lemma 3.4.** The graph \( G \) is 3-colorable if and only if there is some minimal immersion
for \( L_1, \ldots, L_n \) of size at most \( N \).

**Proof.** Let us first assume that \( G \) is 3-colorable. Then we argue that \( \mathcal{A} \) can be built from at
most three dispatch cycles \( C_0, C_1, C_2 \), one for each color 0,1 and 2 (together with \( p \)-cycles,
one for each \( p \in P \)). Cycle \( C_\alpha \) can be used for all vertices \( i \neq j \) of color \( \alpha \), since they are
pairwise unconnected. To see this, note that vertex \( \langle i,j \rangle \) of \( C_\alpha \) is a \( q_1 \)-vertex according to
the definition of \( \mathcal{A}_i \); and \( \langle i,j \rangle \) is also \( q_1 \)-vertex according to \( L_j \), since \( \{i,j\} \notin E \).

Conversely, if \( \mathcal{A} \) has size at most \( N \) then by Lemma 3.3 there are at most three \( m \)-cycles
\( C_0, C_1, C_2 \) in \( \mathcal{A} \). We color vertex \( i \) by \( \alpha \) if the sub-DFA for \( L_i \) uses \( C_\alpha \). This coloring is proper,
because if the sub-DFA for \( L_i, L_j \) both use the same dispatch cycle, then \( \{i,j\} \notin E \) since
otherwise vertex \( \langle i,j \rangle \) would be both a \( q_1 \)- and a \( q_2 \)-vertex, contradicting Lemma 3.3.

Lemma 3.4 yields finally the claimed result, and also the proof of Theorem 2.1:

**Theorem 3.5.** \textsc{MinImmersion} is \( \text{NP-complete} \).

**Conclusions**

We have shown that the VPA minimization is intrinsically difficult, by exhibiting an \( \text{NP} \)-lower
bound. A minor modification of the construction reduces approximation of the chromatic
number to approximating the minimal size of an equivalent VPA, thus any constant-factor
approximation of VPA minimisation is \( \text{NP-hard} \). Our result raises the quest for efficient
implementations of SAT-based minimization algorithms for VPA.
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