Calculation of the direction of the strain axes in the working area of a vertical milling machine

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Abstract. The article studies the issues of static elastic deformations that appear during processing on milling type machines. The method of determining the principal axes of strain is observed.

1. Introduction
The requirements for increasing the accuracy of the parts produced assume the consideration of all factors that affect the processing accuracy [1,2]. Rigidity is considered one of the most important factors in the process of cutting. In actual processing conditions it is necessary to consider the rigidity of the entire technological system which includes the machine, the device, the cutting and the adapter tools, a workpiece. The traditional ways to increase the rigidity are to increase the rigidity of the details of the technological system and the rigidity of the joints between them. Meanwhile, the rigidity of the entire machine, that is the perception of cutting forces and the resistance to deformation from these forces, is affected not only by the intrinsic and contact rigidity of its elements, but also by the geometric characteristics of the layout of the technological system.

2. The strain axes
These characteristics can be estimated by the axes of rigidity or, in terms of the theory of the resistance of materials, the axes of the main strain. It is known [3] that at each point of the shank there are three mutually perpendicular directions along which linear deformations have extreme values. These directions are called the principal axes of strain, and linear deformations along the principal directions are the principal deformations. The most important are the axis of maximum strain in the direction of which the elastic deformations are least, and the axis of minimum strain, in the direction of which the elastic deformations are greatest. The importance of these axes is dictated by the following considerations [4]:
1. If the layout of the technological system is constructed so that the resultant of forces cutting is equal to or close to the axis of maximum strain, then the deformations will be minimal, therefore, the processing errors caused by elastic deformations are also minimized.
2. The strain axes of the main deformations have the property that if the resultant of forces passes along them, then this causes only linear deformations of the shank along these axes without causing angular deformations. This also minimizes elastic deformations.
3. Approximation of the resultant force of cutting to the strain axes increases the dynamic stability of the technological system.

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3. Calculation method for determining the strain axes

When investigating the rigidity of machines an experimental approach is generally used; it is carried out by means of a full-scale or computational experiment. The main method of the experiment is the "direction finding" of the position of the centre and the strain axes by a set of measurements when the angle of action of the loading force is changed. All these measurements take a lot of time, and in the first case they also need material costs. In this article we propose a calculated approach to determining the position of the strain axes. As we know [3] under any loading of the shank the linear and angular deformations of this body form a strain tensor:

\[ T = \begin{bmatrix}
\varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\
\varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\
\varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33}
\end{bmatrix} \]  \tag{1},

where \( \varepsilon_{ij} \) – linear and angular deformations.

The values of the principal deformations can be found from the solution of the cubic equation [3]:

\[ \varepsilon^3 - I_1 \varepsilon^2 + I_2 \varepsilon + I_3 = 0 \]  \tag{2},

where \( I_1 \) — invariants of a strain tensor (1).

The components of the strain tensor (1) can be found experimentally under single loading or by finite element analysis of the technological system. In this study a three-dimensional model of a technological system was constructed. It included a carrier system of the machine, cutting and adaptive tools, a device, a workpiece. The layout of the technological system corresponded to the case of internal cavity milling [5]. By performing the calculation in the "Extended Simulation" module of Siemens NX linear and angular deformations were determined (Figure 1). By solving equation (2) the values of the main deformations were determined:

\[ \varepsilon = \begin{bmatrix}
-0.003 \\
0.0002 \\
0.002
\end{bmatrix} \]  \tag{3}.

![Fig. 1 – Deformations in the technological system](image-url)
The component $\varepsilon_1$ corresponds to the principal greatest deformation, its direction is the axis of minimum strain; component $\varepsilon_2$ corresponds to the smallest principal deformation, its direction is the axis of maximum strain.

The directions of the principal axes of deformations can be determined by calculating the direction cosines $l_i,m_i,n_i$ in the joint solution of the system of equations [6]:

\[
\begin{align*}
(\varepsilon_i - \varepsilon_{11})l_i - \varepsilon_{12}m_i - \varepsilon_{13}n_i &= 0 \\
-\varepsilon_{12}l_i + (\varepsilon_i - \varepsilon_{22})m_i - \varepsilon_{23}n_i &= 0 \\
-\varepsilon_{13}l_i + \varepsilon_{23}m_i - (\varepsilon_i - \varepsilon_{33})n_i &= 0 \\
l_i^2 + m_i^2 + n_i^2 &= 1
\end{align*}
\] (4)

According to the calculated values, the vectors forming the strain axes were constructed (Fig. 2, the cutting and back-up tools are formally not shown). One of the axes along which the deformations are greatest is the axis of minimum strain $j_{\text{min}}$, the second along which the deformations are the smallest is the axis of maximum strain $j_{\text{max}}$. It can be noted that the position of the main deformation axes corresponds to the results of the previously performed computational experiment.

Fig. 2 – Axes of strain

4. Conclusion
Thus, the application of the proposed approach makes it possible to determine with minimum effort the position of the axes of strain. With the help of this result one can choose the cutting scheme or, conversely, change the layout of the technological system in such a way as to approximate the resultant forces of cutting to the axis of maximum strain, thereby reducing the elastic deformations and increasing the accuracy of processing.

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