Entropic force in the presence of black hole

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Abstract

We derive the entropic force in the presence of the Schwarzschild black hole by using the local equipartition rule and holographic principle. On the other hand, when using the Tolman temperature, one does not arrive at the Newtonian force law.
1 Introduction

Recently, Verlinde has proposed the Newtonian law of gravity as an entropic force by using the holographic principle and the equipartition rule [1]. After released his work, the dynamics of apparent horizon in the Friedmann-Robertson-Walker universe [2], the Friedmann equations [3, 4], the connection in the loop quantum gravity [5], holographic actions for black hole entropy [6], and application to holographic dark energy [7] were considered from the entropic force. Furthermore, cosmological implications were reported in [8, 9, 10, 11, 12, 13], and the extension to Coulomb force [14] and the symmetry aspect of entropy force [15] were investigated.

Explicitly, when a test particle with mass $m$ is located near a holographic screen with distance $\Delta x$, the change of entropy on the holographic screen takes the form

$$\Delta S = 2\pi m \Delta x$$

in the natural units of $\hbar = c = k_B = 1$ and $G = l_{pl}^2$. Considering that the entropy of a system depends on the distance $\Delta x$, an entropic force $F$ could be arisen from the thermodynamical conjugate of the distance

$$F \Delta x = T \Delta S$$

which is regarded as an indication that the first law of thermodynamics is satisfied on the holographic screen. Plugging (1) into (2) leads to a connection between entropic force and temperature

$$F = 2\pi m T.$$  

Let us assume that the energy $E$ is distributed on the spherical shape of holographic screen with radius $R$, where the mass $M$ is located at the origin of coordinate. Then, we introduce the equipartition rule, the equality of energy and mass, and the holographic principle, respectively, as

$$E = \frac{1}{2} NT, \quad E = M, \quad N = \frac{A}{G} = 4S$$

with the area of the holographic screen $A = 4\pi R^2$. These are combined to provide the temperature on the screen

$$T = \frac{GM}{2\pi R^2}.$$  

Substituting (5) into (3), one obtains the Newtonian force law as the entropic force

$$F = \frac{GmM}{R^2}.$$
In this work, we will derive the entropic force by replacing the mass \( M \) by the Schwarzschild black hole with mass \( M \) as

\[
dS^2_{\text{Sch}} = g_{\mu \nu} dx^\mu dx^\nu = -\left(1 - \frac{2GM}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2GM}{r}\right)} + r^2 d\Omega_2^2. \tag{7}
\]

Here the even horizon (EH) is located at \( r = r_{EH} = 2GM \) whose horizon area is given by \( A_{EH} = 4\pi r_{EH}^2 \).

## 2 Entropic force in the presence of black hole

It is well known that in the presence of the black hole, we may introduce the local temperature (Tolman temperature) and the local energy on the holographic screen. Now let us first introduce the Tolman redshift transformation on the black hole system [16]. In general, the local temperature observed by an observer at \( r > r_{EH} \) outside the Schwarzschild black hole is defined by [17]

\[
T_L(r) = \frac{T_\infty}{\sqrt{-g_{tt}}} = \frac{1}{8\pi GM} \frac{1}{\sqrt{1 - \frac{2GM}{r}}} \tag{8}
\]

where

\[
T_\infty = \frac{1}{8\pi GM} = \frac{1}{4\pi r_{EH}} \equiv T_H \tag{9}
\]

is the Hawking temperature \( T_H \) measured at infinity and the denominator of \( \sqrt{-g_{tt}} \) is the red-shifted factor. On the holographic screen at \( r = r_{EH} + l_{pl}^2/r_{EH} \) near the event horizon, this local temperature is given by \( T = 1/8\pi l_{pl}^2 \) which is independent of the black hole mass \( M \) [18]. On the other hand, for \( r \gg r_{EH} \), it reduces to the Hawking temperature \( T_H \).

Similarly, the local energy is given by

\[
E_L(r) = \frac{E_\infty}{\sqrt{-g_{tt}}} \tag{10}
\]

where the energy observed at infinity is the ADM mass \( M \)

\[
E_\infty = M = \frac{r_{EH}}{2G}. \tag{11}
\]

Eq. (10) states clearly the UV/IR scaling transformation (Tolman redshift transformation) of the energy between the bulk and the holographic screen. On the holographic screen near the horizon, we have \( E_L(r_{EH} + l_{pl}^2/r_{EH}) \propto A_{EH} \) [19].

However, it is very important to note that there is no difference between the local black hole entropy \( S_L \) near the horizon and the entropy \( S_\infty \) at infinity:

\[
S_L = \pi r_{EH}^2 = S_\infty. \tag{12}
\]
which is surely the Bekenstein-Hawking entropy for the Schwarzschild black hole. That is, the black hole entropy is invariant under the UV/IR transformation.

Now we are in a position to derive the entropic force on the holographic screen in the presence of Schwarzschild black hole. To this end, we propose the local equipartition rule

\[ E_L(r) = 2S_S(r)T_S(r), \]  

(13)

where the entropy \( S_S \) is defined on the holographic screen located at \( r > r_{EH} \)

\[ S_S(r) = \frac{\pi r^2}{G}. \]  

(14)

Then, the temperature on the screen is given by

\[ T_S(r) = \frac{GM}{2\pi r^2 \sqrt{-g_{tt}}}. \]  

(15)

Plugging (15) into (3), we obtain the entropic force as

\[ F_{BH} = 2\pi mT_S(r) = \frac{1}{\sqrt{1 - \frac{r_{EH}}{r}}} \frac{GmM}{r^2}, \]  

(16)

which shows that the mass \( m \) feels an infinitely tidal force in the limit of \( r \to r_{EH} \), while it takes the Newtonian force law at the large distance of \( r \gg r_{EH} \). This is our main result.

On the other hand, the temperature \( T_S(r) \) on the screen becomes the local temperature \( T_L(r) \) in (8) when using the Bekenstein-Hawking entropy (12) for (13). In this case, Eq.(3) takes a different form

\[ F = 2\pi mT_L(r) = \frac{m}{4GM} \frac{1}{\sqrt{1 - \frac{r_{EH}}{r}}}. \]  

(17)

which is nothing to do with the entropic force.

3 Discussions

We have obtained the entropic force in the presence of the Schwarzschild black hole by using the local equipartition rule and the holographic principle on the screen. In this case, the local energy \( E_L \) together with the entropy \( S_S \) on the screen was mainly used to derive the entropic force. When using the local (Tolman) temperature and the Bekenstein-Hawking entropy for the Schwarzschild black hole, we have failed to arrive at the Newtonian force law. At this stage, it is not clear why the latter approach is not suitable for deriving the entropic force.
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