Statistical Adjustment and Calibration of Complex Systems considering Multiple Outputs: Case Study of Laser-Assisted Micromachining Process

Zeinab Khalaj

Student of K. N. Toosi University, Department of Industrial Engineering, 1999143344, Tehran, Iran

zkhalaj@mail.kntu.ac.ir

Abdollah Aghaie*

Professor of K. N. Toosi University, Department of Industrial Engineering, 1999143344, Tehran, Iran

aaghaie@kntu.ac.ir

Yaser Samimi

Assistant Professor of K. N. Toosi University, Department of Industrial Engineering, 1999143344, Tehran, Iran

Y.samimi@kntu.ac.ir

Abstract

By rapid advancements in technologies, studying and simulating a real complex system tainted with uncertain parameters is too demanding. Based on the relevant literature, there are three approaches to recognize and simulate different systems: engineering, statistical, and engineering-statistical approaches. Regarding the purpose of this research, the engineering-statistical approach is applied. By considering the single output, Laser Assisted Micro-Machining (LAMM) is studied using the engineering-statistical approach. In the present study,

* Corresponding Author; Vanak Sq., pardis, St., K. N. Toosi University;
an engineering-statistical approach is applied to LAMM, studying two outputs simultaneously.

To investigate variates simultaneously, there are some complicated issues, such as calibrating variates at the same time, adjusting them concurrently, and calculating the values of parameters, with which this paper should cope. Considering the Mean Squared Prediction Error (MSPE) as the comparison index, for thrust force output, the index value was obtained as 1.48 by Kennedy and O’Hagan model, 2.47 by the model presented by Roshan and Yan, and 1.9425×10^{-4} by the proposed model. Moreover, for cutting force output, the index was obtained as 0.21 by the Kennedy and O’Hagan, 1.41 by Roshan and Yan, and 1.6×10^{-8} by the presented model. The obtained values demonstrate reasonable and comparable results for the MSPE index, compared with the models considering the outputs individually.

**Keywords:** Uncertainty Quantification, Adjustment, Calibration, Engineering-Statistical Model, Gaussian Process, Laser-Assisted Micromachining Process

1. INTRODUCTION

In the real world, we are faced with uncertainty in everyday life. By simulating complicated systems, making management decisions, evaluating the performance of systems in different situations, and then optimizing them, the uncertainty plays a critical role. To better cope with the uncertainty, different models can be defined. A model is a representation of a system, person, thing, or a proposed structure, typically on a smaller scale than the original one. According to the recent studies, three approaches are adopted to model real systems, including engineering approach, statistical approach, and engineering-statistical approach. These approaches can be described as follows.

The first approach to model a system is called the engineering approach. Engineering models, which are developed by applying some techniques such as finite element and numerical analysis, are considered physics-based and fulfill the physical interpretation of a system. Accordingly, they considerably help to understand the real system. However, to accomplish
this approach, some simplifying assumptions are considered, which make the model almost unrealistic. To put it differently, it is a time-consuming procedure to search for the reasons for the discrepancy between the model and the observed data in order to fix all the wrong assumptions. Therefore, if the developed model does not operate properly, it takes a long time to find the wrong and change it and then reformulate the model. In the following, some of the studies which are recently conducted in this field are presented in Table 1. In this field, Singh and Melkote [1]’s research is one of the most striking papers.

The second one is the statistical approach, in which an empirical model (be a GP, or a regression model) is built to capture the discrepancies. In this approach, real data are collected from the systems to estimate the unknown parameters of the model (calibration parameters). The first disadvantage of this approach is that any changes in the system result in repeating the data collection procedure, developing a new model, and collaborating it again. Moreover, the model does not respond well out of the experimental conditions, as well as lack of physical interpretation. In this regard, some relevant research is reported in Table 1. Among the conducted research, Kennedy and O'Hagan [2], and Roshan and Melkote [3] are the most influential ones for the current study. It is noteworthy that statistical inference, Bayesian approach (BA), Gaussian process (GP), maximum likelihood estimation (MLE), response surface methodology, analysis of variance (ANOVA), regression, and robust techniques are the most popular approaches in this field.
By the integration of two previous approaches, the third approach, namely the engineering-statistical approach, is developed. This approach begins with building and calibrating an engineering model (by collecting data), which fulfills the physical interpretation, then distinguishes the discrepancy and its causes using ANOVA, and tries to eliminate it through applying statistics. Afterward, the adjustment models are postulated, and eventually, the final model fulfills a physical interpretation; however, contrary to two previous approaches, this approach is not time-consuming, which is considered as a distinctive prominence feature. In this respect, relevant works done in this approach are presented in Table 1. For more information, the interested readers are referred to Yan [4], and Roshan and Yan [5], as well as Sheikh and Saghaie [6], in which computational fluid dynamics (CFD), BA and GP are applied.

It should be noted in the real world, some of the systems are too complicated to be easily modeled. In these situations, engineering approaches, including finite element and CFD, are applied to modeling the system. These models may be formed by the combination of multiple models, each of which presents a specific part of the system. For instance, such systems have more than one output variable, mostly; hence, for each variable, a different model can be used. Needless to say, these models are not concise, and there is some bias between the model values and real observations. In order to cope with the problem, statistical approaches are applied for calibrating and adjusting such complicated models, which leads to the engineering-statistical approach.

1.1. The contribution of the study

The summary of studies regarding each system modeling approach is demonstrated in Table 1. Subsequently, in this paper, an engineering-statistical approach is adopted to fulfill the drawbacks of both the statistical and engineering approaches. It is clear that although the previously proposed engineering-statistical procedures have been single output [7, 4, 6],
studying the covariance and correlation between the outputs may end up to more appropriate models to get better outputs. Therefore, instead of applying BA, which needs prior distribution as well as hard usage for multiple outputs, MLE and GP are applied.

2. METHODOLOGY

In this section, it is aimed at providing the proposed approach. As a basic model, Kennedy and O’Hagan [2] presented model (1) for a system. In this model, \( Y \) implies the output of the system, \( x = (x_1 \ldots x_p)' \) shows the vector of input variables, and \( f(x; \eta) \) plays the role of an engineering model. Moreover, \( \eta = (\eta_1 \ldots \eta_p)' \) reflects the vector of calibration parameters of the engineering model.

\[
y = \rho f(x; \eta) + \delta(x) + \epsilon.
\]

In model (1), \( \rho \) is the scale parameter, \( \delta(x) \) denotes the discrepancy function (model bias), and the random error is presented by \( \epsilon \overset{iid}{\sim} \mathcal{N}(0, \sigma^2) \). Then, by considering \( h(x) = (h_0(x) \ldots h_t(x))' \) as a set of known functions, and \( \mu = (\mu_o \ldots \mu_t)' \), they have defined a GP for the discrepancy:

\[
\delta(x) \sim \text{GP}(h(x)' \mu, \tau^2 R(.)).
\]

The covariance function is presented by \( \text{cov}(\delta(x_i), \delta(x_j)) = \tau^2 R(x_i - x_j) \), in which \( \tau^2 \) is the variance of \( \delta(x) \) and \( R \) is a \( n \times n \) correlation matrix.

Inspired Kennedy and O’Hagan’s paper, Roshan and Yan [5] considered \( \rho = 1 \) and \( \mu = 0 \); thus, model (1) and (2) were changed to model (3).

\[
y = f(x; \eta) + \delta(x) + \epsilon,
\]

\[
\delta(x) \sim \text{GP}(0, \tau^2 R(.)).
\]

As the first step of their algorithm, the model discrepancy was estimated, and its causes were detected. Next, the Gaussian correlation function can be defined as:
\[ R(x_i - x_j) = \exp \left\{ -\sum_{k=1}^{n} \theta_k (x_{ik} - x_{jk})^2 \right\}. \]  

(4)

Therefore, \( \eta \) and \( \phi = (\sigma^2, \tau^2, \theta') \) remains as unknown parameters. Then, to estimate the value of \( \eta \) and \( \phi \), \( \delta(x) \) is integrated out from the joint posterior and results in:

\[ p(\eta, \phi \mid y) \propto \frac{1}{\sqrt{2^\tau \mathbf{R} + \sigma^2 \mathbf{I}}} \exp \left\{ -\frac{1}{2} \left( y - f(\eta) \right)' \left( y - f(\eta) \right) \right\} p(\eta, \phi), \]

in which \( \mathbf{R} \) is a \( n \times n \) correlation matrix \((\mathbf{R}(i, j) = R(x_i - x_j))\), \( \mathbf{I} \) is the identity matrix \((n \times n)\), 
\( f(\eta) = (f(x_1; \eta), \ldots, f(x_n; \eta))' \), and \( y = (y_1, \ldots, y_n)' \). Subsequently, considering \( r(x) = (R(x-x_1), \ldots, R(x-x_n))' \), the discrepancy function is given by

\[ \delta(x) \sim GP\left(h(x) \mu, \tau^2 \mathbf{R}(.)\right). \]  

(6)

Roshan and Yan [5], showed that the prediction variance would be calculated by Equation 7,

\[ s^2(x) = \sigma^2_f + \sigma^2_f \hat{f}(x) \left( \hat{\mathbf{F}}' \hat{\mathbf{F}} \right)^{-1} \hat{f}(x), \]

(7)

in which \( \hat{\mathbf{F}} = \left( \hat{f}(x_1), \ldots, \hat{f}(x_n) \right)' \), and \( \hat{f}(x) \) is the gradient function of \( \hat{f}(x) \) according to \( x \):

\[ \hat{f}(x) = \hat{f}(x) \left( \frac{\hat{\beta}_1}{x_{i1}}, \hat{\beta}_2, -\hat{\beta}_3 e^{-\hat{\beta}_4 x_1}, \hat{\beta}_5 \hat{\beta}_6 e^{-\hat{\beta}_7 x_1} \right). \]  

(8)

Next, applying ANOVA, the main effects of the engineering model and the discrepancy function were studied. Then, a prior inverse-Gamma was specified for \( \sigma^2 \). In the next step, calculating the prediction variance, the parameters were estimated. By identifying the factors with significant effects on the model discrepancy, the scale adjustment model will be obtained by Equation 9. A variety of common methods on the calibration of parameters in multiple correlated responses are classified by [8].

\[ g(x; \eta, \gamma) = f(\eta, x_1, \ldots, \eta, x_p, \gamma). \]  

(9)
It should be noted the above methodology studied only single output models. In this manuscript, a methodology is proposed to study multivariate systems. Herein, all of the outputs are considered simultaneously.

In the following, the proposed model and methodology are studied. For q variables, the model is changed to:

\[
\begin{align*}
    y_i &= f_i(x; \eta) + \delta_i(x) + \epsilon_i, i \in 1, \ldots, q \\
    \delta_i(x) &\sim GP(0, \tau_i^2 R_i(.)); i = 1, \ldots, q \\
    \left(\epsilon_1, \ldots, \epsilon_q\right) &\sim MVN(0, (\sigma_1^2, \ldots, \sigma_q^2)).
\end{align*}
\]  

(10)

Optimizing \( f_i(x; \eta) \), the calibration parameters \( \eta_i \) are obtained. In this respect, an iterative algorithm is used, which updates the error covariance matrix, in every iteration. It means in every single step, the covariance matrix is replaced with a new one. This dynamic approach helps the solution to be improved much more and gets nearer to its exact value. To optimize \( f_i(x; \eta) \), the MLE method is used. Then, Gaussian Processes will be fitted for \( \delta_i(x) \), using the squared exponential kernel function with default kernel parameters. Predicting \( \delta_i(x) \) for each run, errors are estimated as:

\[
\epsilon_i = y_i - \left( f_i(x; \eta) + \delta_i(x) \right); i = 1, \ldots, q
\]  

(11)

The bias-corrected engineering models is given by

\[
\hat{y}_i = f_i(x; \eta) + \hat{\delta}_i(x)
\]  

(12)

Now, Multivariate Analysis of Variance (MANOVA) decomposition is performed on q variables, simultaneously, and the significant factors of \( f_i \) can be easily obtained. \( f_i(x_i) \) and side by side to understand the changes in main effects due to can be plotted \( \hat{y}_i(x_i) \) discrepancies, and to look at the main effect plots, making the procedure simpler. In order to
detect the factors with significant effects on the model discrepancy, the scale adjustment model will be given by (9). The procedure is stated in figure 1.

By following the above procedure, the engineering model and the real data are achieved, the model is calibrated by an MLE estimation. Calculating the errors and applying the iterative algorithm, the covariance matrix is updated until the calibrated model be adequate. Then, the discrepancy model is calculated by a GP, and the causes of it are detected by MANOVA. Calculating the errors and updating the covariance matrix, the whole procedure will be repeated to reach adequacy. The parameters and variables are stated in Table 2.

The methodology is applied to Laser-Assisted Micro Machining (LAMM), studied by [1, 7, 9]. In this problem, four variables and two outputs are considered. Roshan and Yan [5], worked on cutting force, as one of the outputs. In this study, thrust force, as another output, is studied, simultaneously as presented in section 3.

3. NUMERICAL EXAMPLE

A case study of LAMM was studied previously by some researchers; however, the engineering model was developed by Singh and Melkote [1], calibrated by Singh et al. [10], and adjusted statistically by Roshan and Yan [5]. Roshan and Yan [5], stated that the engineering model consists of a geometric model for computing strain rates, a finite element model for computing temperature distribution, a material model for computing stresses, a force model for computing forces, and an iterative algorithm to account for the machine-tool-workpiece deflection.

Regarding Table 3, four variables are considered: Nominal depth of cut ($x_1$), Speed ($x_2$), Laser power ($x_3$), and Laser location($x_4$), by 4, 2, 3, and 2 levels. Two outputs, namely cutting force and thrust force, were studied separately. However, in this paper, they are studied simultaneously. For these two variables, the model is presented by model (13).
\begin{equation}
\begin{align*}
y_i &= f_i(x; \eta) + \delta_i(x) + \varepsilon_i; i \in \{1, 2\} \\
\delta_i(x) &\sim GP\left(0, \tau_i^2 R(\cdot)\right); i = 1, 2 \\
\left(\varepsilon_i, \delta_i\right) &\sim MVN\left(0, \begin{pmatrix} \sigma_1^2 & \tau \sigma_1^2 \\ \tau \sigma_1^2 & \sigma_2^2 \end{pmatrix}\right).
\end{align*}
\end{equation}

Since the engineering models are complicated and time-consuming (14 hours), as well as expensive to be evaluated [5], $f(x; \eta)$ is replaced by a metamodel, which is considered easy-to-evaluate. The metamodel used by Singh et al. [10] for both output variables, is shown below:

$$f_i(x) = b_0 x_i^{\beta_0} \exp \left\{b_2 x_2 - b_3 x_3 \exp(-b_4 x_4)\right\}. \quad (14)$$

As the above model is nonlinear, nonlinear regression is applied for both $f_1$ and $f_2$ to achieve the parameters $b_0, \ldots, b_4$. The covariance between two forces is considered as the initial covariance and is updated frequently to achieve the optimal solution. Optimizing the engineering model, using MLE and the iterative algorithm along with considering the covariance between the outputs ($\text{cov} \left(\delta(x_i), \delta(x_j)\right) = \tau^2 R(x_i - x_j)$), the parameters of the model for thrust force are estimated as:

$\beta_{10} = 1.6062, \beta_{11} = 0.8882, \beta_{12} = 0.0006, \beta_{13} = 0.0098, \beta_{14} = 0.0019$, which were estimated by Singh et al. [10], using nonlinear regression, as $\beta_{10} = 1.605, \beta_{11} = 0.888, \beta_{12} = 0.00058, \beta_{13} = 0.009, \beta_{14} = 0.0018$. For cutting force, they are estimated as:

$\beta_{20} = 1.3581, \beta_{21} = 0.8888, \beta_{22} = 0.0014, \beta_{23} = 0.0269, \beta_{24} = 0.0034$, which were previously estimated as $\beta_{20} = 1.358, \beta_{21} = 0.888, \beta_{22} = 0.00139, \beta_{23} = 0.0268, \beta_{24} = 0.00343$ by Singh et al.[10], using nonlinear regression.

Therefore, the models for thrust force and cutting force are calibrated respectively, as below:

$$\begin{cases}
f_1(x) = 1.6062 x_1^{0.8882} \exp \left\{0.0006 x_2 - 0.0098 x_3 \exp(-0.019 x_4)\right\}, \\
f_2(x) = 1.3581 x_1^{0.8888} \exp \left\{0.0014 x_2 - 0.0269 x_3 \exp(-0.034 x_4)\right\}.\end{cases} \quad (15)$$
It is observed in Figure 2 that the model is fitted excellently, and it shows that the metamodel is proper. The actual values are shown in the vertical axis and the predicted ones on the parallel axis. Since the dots are near the bisector line, the errors can be negligible. As \( \text{cov} \left( \delta(x_i), \delta(x_j) \right) = \tau^2 R(x_i - x_j) \), using eq. (4), the covariance between the errors is calculated as:

\[
\text{cov}(e_1, e_2) = \begin{bmatrix} 0.0002 & 0.0001 \\ 0.0001 & 0.2825 \end{bmatrix}
\]  

(16)

The values of the parameters and the variables are listed in Table 4 in order to simplify studying this section.

By applying MANOVA to the engineering model, the factors with the highest level of effects on both variables were detected. This study only reported the results of Wilk’s test, since it is the most commonly used test along with well-known F approximation. Compared with \( F_\alpha \), in which \( \alpha = 0.01 \), all factors are important. The results are reported in Table 5.

As Roshan and Yan [5] have stated, two-factor interactions are not so imperative, while considering them increases the complexity of the model. Therefore, the related terms are neglected here.

Now, the discrepancy functions should be constructed. By fitting a GP, the parameters of the discrepancy function will be calculated by Equations (3, 4) as:

\[
\sigma^2 = 0.0118, \tau^2 = 1.1401, \theta' = (2.6891, 21.2868, 2.0061, 56.5802),
\]  

(17)

for thrust force, and

\[
\sigma^2 = 0.0148, \tau^2 = 0.3114, \theta' = (7.774, 27.7022, 3.1395, 265.0827),
\]  

(18)

for cutting force.
By applying MANOVA to the discrepancy function, the most effective factors on both variables are detected. The results of Wilk's test are reported in Table 6 and compared with $F_{0.01}$, all factors are insignificant. All of the factors are effective on the engineering models, and none of them are effective on the discrepancy functions. This ineffectiveness means that the responses ($y$) have been modeled as much as possible, and the remaining which are not modeled are part of $y$, which is the random error, indeed.

The main effects of $f(x)$ and $f(x) + \delta(x)$ are presented by Figure 3. It is seen that they are so close and almost covered by each other. Hence, the discrepancy function is not impressive, and it can be neglected. But, here, the model adjustment approach is continued.

The following adjusted model seems to be physically meaningful for both forces:

$$\hat{g}(x; y) = \hat{f}(x_1, x_2, x_3, x_4). \quad (19)$$

Therefore, according to Equation 19, the engineering-statistical models for the thrust force and the cutting force are adjusted, respectively.

$$\left\{ \begin{array}{l} g(x) = 1.6062x_1^{0.8882}\exp\left\{0.0006x_2 - 0.0098x_3e^{-0.0019x_4}\right\}, \\ g(x) = 1.3581x_1^{0.8888}\exp\left\{0.0014x_2 - 0.0269x_3e^{-0.0034x_4}\right\}. \end{array} \quad (20) \right.$$  

Considering $\hat{y} = \hat{f} + \hat{\delta}$, for thrust force, they are estimated as:

$$\hat{f} = 19.9315, \hat{\delta} = 0.0091, \hat{y} = 19.9315 + 0.0091 = 19.9406.$$  

And for cutting force, they are estimated as:

$$\hat{f} = 16.5787, \hat{\delta} = 0.0031, \hat{y} = 16.5787 + 0.0031 = 16.5819.$$  

Apparently, for thrust force, the mean remaining error is 0.0091, and for cutting force, the value is 0.0032.

Mean squared prediction errors seem to be a proper index for quantifying the improvement:

$$MSPE = \frac{1}{n}\sum_{i=1}^{n}\left[y_i - f(x_i)\right]^2 \quad (21)$$
For thrust force, the index was obtained 1.48 by Kennedy and O’Hagan adjustment model [2], 2.47 by the adjustment model presented by Roshan and Yan [5], and 1.9425\times10^{-4} by the current proposed model. For cutting force, the index was obtained as 0.21 by the Kennedy and O’Hagan [2], 1.41 by Roshan and Yan [5], and 1.6\times10^{-8} by the presented model.

As the above analysis shows, the errors of the proposed method are so near to zero. The reason is that the initial experiments are used again. Therefore, to validate the methodology, cross-validation and a second approach are applied. These approaches are stated in the next section.

3.1. VALIDATION

As explained before, validation is needed to be sure that the methodology works fine. Hence, to validate the methodology, the cross-validation approach is applied. In order to accomplish the cross-validation, the programming should be repeated 48 times. In each run, one observation should be omitted. Calibrating and adjusting the model for the remaining 47 observations, the results are applied to the new experiment (omitted one). Calculating the MSPE for entire experiments, the average error squares are calculated for both thrust force Relation 22 and cutting force Relation 23.

$$MSPE = \frac{1}{n}\left(\sum_{i=1}^{n} [\hat{y}_{1,i} - y_{1,i}]^2\right) = 4.0691$$

$$MSPE = \frac{1}{n}\left(\sum_{i=1}^{n} [\hat{y}_{2,i} - y_{2,i}]^2\right) = 1.5378$$

(22)

(23)

It is seen that the methodology gets solutions so near to those of the previous works, and it is validated now.

Again, as a second approach to validate the methodology, new experiments should be produced. To create new experiments, a normal distribution with mean $\bar{x}$ and variance $cov(x)$ is used to create 48 new random sets of input variables. The same procedure is done for output
variables. It means previous outputs and the covariance matrix between them are set for outputs, and for inputs Relation 24 is used.

\[
\begin{bmatrix}
16.82 & 0 & 0 & 0 \\
0 & 403.74 & 0 & 0 \\
0 & 0 & 16.82 & 0 \\
0 & 0 & 0 & 56.77 \\
\end{bmatrix}
\]

By applying the parameters discrepancy function, estimated previously, the error is calculated for both outputs Relation 25.

\[
MSPE = \frac{1}{n} \left( \sum_{i=1}^{n} \left[ \hat{y}_{1,i} - y_{1,i} \right]^2 + \sum_{i=1}^{n} \left[ \hat{y}_{2,i} - y_{2,i} \right]^2 \right) = 0.0014
\]

Again, not using the discrepancy function, the errors are calculated Relation 26.

\[
MSPE = \frac{1}{n} \left( \sum_{i=1}^{n} \left[ f_{1,i} - y_{1,i} \right]^2 + \sum_{i=1}^{n} \left[ f_{2,i} - y_{2,i} \right]^2 \right) = 170.5232
\]

It is seen that even for the created variables, the discrepancy function is proper. Thus, the methodology works better than the previous one, and new inputs and outputs validate it.

4. CONCLUSION

In this paper, by integrating the engineering and statistical approaches, a novel engineering-statistical approach was introduced for calibrating and adjusting a model for complex systems. The methodology was applied to the LAMM problem, to study the thrust force and cutting force as the outputs, simultaneously for the first time. Considering Mean Squared Prediction Errors (MSPE) as the index of model adequacy, the values were obtained \(1.9425 \times 10^{-4}\) for thrust force, and \(1.6 \times 10^{-8}\) for cutting force, by the presented model. The proposed model worked finer by the index than the models introduced previously. The errors were considerably reduced because the data from which the model is created and the data to which the model is applied are the same. Therefore, to validate the methodology, the proposed approach was applied to new data, and MSPE was gained 0.0014, which was less than the while the discrepancy
function was not applied (170.52), yet. Cross-validation was applied to all the experiments as well. For the thrust force and cutting force, MSPE was calculated 4.0691 and 1.5378, respectively. In this paper, we focused on presenting a methodology for calibration and adjustment of a multi-output model. On the other hand, the experimental designs played a crucial role in both computer experiments and physical experiments. As further works, using the space-filling design for the computer design can be a new direction for future research. Even the orthogonal space-filling design may be studied to obtain more efficient estimations.

REFERENCES

[1] Singh, R. K. and Melkote, S. N. “Force modeling in laser assisted micro-grooving including the effect of machine deflection”, ASME Journal of Manufacturing Science and Engineering, pp. 1-9 (2009).

[2] Kennedy, M. C. and O'Hagan, A. “Bayesian calibration of computer models”, Journal of Royal Statistical Society - Series B, 63, pp. 425-464 (2001).

[3] Roshan, V. J. and Melkote, S. N. “Statistical adjustments to engineering models”, Georgia Institute of Technology, pp. 1-24 (2008).

[4] Yan, H. “Statistical adjustment, calibration, and uncertainty quantification of complex computer models”, Ph.D. Thesis, H. Milton Stewart School of Industrial and Systems Engineering, Georgia Institute of Technology (2014).

[5] Roshan, V. J. and Yan, H. “Engineering-driven statistical adjustment and calibration”, Technometrics, 57(2), pp. 257-267 (2015).

[6] Sheikhi, H. and Saghaie, A. “Developing an engineering-statistical model for estimating aerodynamic coefficients of helicopter fuselage”, Chinese Journal of Aeronautics, 30(1), pp. 175-185 (2017).

[7] Rahimi, M., Shafieezadeh, A., Wood, D., Kubatko, E. J., and Dormady, N. C. “Bayesian calibration of multi-response systems via multivariate Kriging:
Methodology and geological and geotechnical case studies”, *Engineering Geology*, 260 (2019).

[8] Daniel, W., David, M., James, W. J., and François, B., “Working with dynamic crop models methods, tools and examples for agriculture and environment”, *Elsevier*, 3rd Edition (2019).

[9] Yan, G., Sun, H., and Waisman, H. “A guided bayesian inference approach for detection of multiple flaws in structures using the extended finite element method”, *Computers and Structures*, 152, pp. 27–44 (2015).

[10] Singh, R. K., Josef, V. R., and Melkote, S. N. “A statistical approach to the optimization of a laser-assisted micromachining process”, *International Journal of Advanced Manufacturing Technology*, 53, pp. 221-230 (2011).

[11] Allaire, G. “Numerical analysis and optimization: an introduction to mathematical modelling and numerical simulation”, *Oxford*, New York (2007).

[12] Nocedal, J. and Wright, S. J. “Numerical optimization”, *Springer*, New York (2006).

[13] Szabó, B.A. “the use of a priori estimates in engineering computations”, *Computer Methods in Applied Mechanics and Engineering*. 82 (1–3), pp. 139–154 (1990).

[14] Mathelin, L. and Hussaini, M. Y. “A stochastic collocation algorithm for uncertainty analysis”, *NASA Center for AeroSpace Information (CASI)*, pp. 1-16 (2003).

[15] Vepsä, A., Haapaniemi, H., Luukkanen, P., Nurkkala, P., and Saarenheimo, A. “Application of finite element model updating to a feed water pipeline of a nuclear power plant”, *Nuclear Engineering and Design*. 235(17-19), pp.1849-1865 (2005).
[16] Palumbo, G., Piccininni, A., Piglionico, V., Guglielmi, P., Sorgente D., and Tricarico, L. “Modelling residual stresses in sand-cast superduplex stainless steel”, *Journal of Materials Processing Technology;* 217, pp. 253–261 (2015).

[17] Stickler, B. and Schachinger, E., “Basic concepts in computational physics”, *Springer,* New York (2014).

[18] Sena, P. K. and Silvapulle, M. J., “An appraisal of some aspects of statistical inference under inequality constraints”, *Journal of Statistical Planning and Inference,* 107, pp. 3–43 (2002).

[19] Kumar, R., Tewari, P.C., and Khanduja, D. “Parameters optimization of fabric finishing system of a textile industry using teaching–learning-based optimization algorithm”, *International Journal of Industrial Engineering Computations,* Volume 6, Issue 2, pp. 221-234 (2018).

[20] Roussas, G. “An Introduction to measure-theoretic probability”, Second Edition, *Elsevier Inc.* USA (2014).

[21] Zio, E. “The Monte Carlo simulation method for system reliability and risk analysis”, *Springer,* London (2013).

[22] Dong, L., Xiaojing, L., and Yanhua, Y., “Investigation of uncertainty quantification method for be models using MCMC approach and application to assessment with feba data”, *Annals of Nuclear Energy,* 107, pp. 62–70 (2017).

[23] Parnianifard, A., Azfanizam, A.S., Ariffin, M.K.A., and Ismail, M.I.S. “An overview on robust design hybrid metamodeling: Advanced methodology in process optimization under uncertainty”, *International Journal of Industrial Engineering Computations,* Volume 9, Issue 1, pp. 1-32 (2018).
[24] Park, I. and Grandhi, R. V. “A bayesian statistical method for quantifying model form uncertainty and two model combination methods”, Reliability Engineering and System Safety. 129, pp. 46–56 (2014).

[25] Caiado, C.C.S. and Goldstein, M. “Bayesian uncertainty analysis for complex physical systems modelled by computer simulators with applications to tipping points”, Communications in Nonlinear Science and Numerical Simulation, pp. 1–14 (2015).

[26] Wong, S. W. K., Lum, C., Wu, L., and Zidek, J. V. “Quantifying uncertainty in lumber grading and strength prediction: a bayesian approach”, Technometrics, (Vol. 58, Issue 2), pp. 236-243 (2016).

[27] Sankararaman, S. and Mahadevan, S. “Integration of model verification, validation, and calibration for uncertainty quantification in engineering systems”, Reliability Engineering and System Safety, 138, pp. 194–209 (2015).

[28] Duru, O., Bulut, E., and Yoshida, Sh. “A fuzzy extended delphi method for adjustment of statistical time series prediction: an empirical study on dry bulk freight market case”, Expert Systems with Applications, pp. 840–848 (2012).

[29] Azzimonti, D., Bect, J., Chevalier, C., and Ginsbourger, D. “Quantifying uncertainties on excursion sets under a gaussian random field prior”, Cornell University (arXiv), (2015).

[30] Reid, N. “Statistical sufficiency”, International Encyclopedia of the Social & Behavioral Sciences (2), pp. 418–422 (2015).

[31] Junaid, M. M. and Wani, M. F. “Modelling and analysis of tool wear and surface roughness in hard turning of AISI D2 steel using response surface methodology”, International Journal of Industrial Engineering Computations, 9(1), pp. 63-74 (2018).

[32] Brandt, S. “Data analysis: statistical and computational methods for scientists and engineers”, (4), Springer, New York (2014).
[33] Korunović, N., Madić, M., Trajanović, M., and Radovanović, M. “A procedure for multi-objective optimization of tire design parameters”, *International Journal of Industrial Engineering Computations*, 6(2), pp. 199-210 (2015).

[34] Lin, H. D. and Lin, W. T. “Automated process adjustments of chip cutting operations using neural network and statistical approaches”, *Expert Systems with Applications*, 36, pp. 4338–4345 (2009).

[35] Lunardon, N. and Ronchetti, E. “Composite likelihood inference by nonparametric saddle point tests”, *Computational Statistics and Data Analysis*, pp. 80–90 (2014).

[36] Pratola, M. T. and Higdon, D. M. “Bayesian additive regression tree calibration of complex high-dimensional computer models”, *Technometrics*, 58(2), pp. 166-179 (2016).

[37] Recep, M. G., Seung-Kyum, C., and Christopher, J. S. “Uncertainty quantification and validation of 3D lattice scaffolds for computer-aided biomedical applications”, *Journal of the Mechanical Behavior of Biomedical Materials*, 71, pp. 428–440 (2017).

[38] Saikumar, R. Y., Michael, G. G., Christos, A., and Michael, D. S “Bayesian uncertainty quantification and propagation for validation of a microstructure sensitive model for prediction of fatigue crack initiation”, *Reliability Engineering & System Safety*, 164, pp. 110–123 (2017).

[39] Chen, R. B., Wang, W., and Wu, C. F. J. “Sequential designs based on bayesian uncertainty quantification in sparse representation surrogate modeling”, *Technometrics*, 59(2) pp.139-152 (2017).

[40] Neal, R. M., “Regression and classification using gaussian process priors”, *Bayesian Statistics 6* (1998).
Mondal, A., Mallick, B., Efendiev, Y., and Datta-Gupta, A. “Bayesian uncertainty quantification for subsurface inversion using a multiscale hierarchical model”, *Technometrics*, 56 (3), pp. 381-392 (2014).

Díaz-García, J. A. “On generalized multivariate analysis of variance”, *Brazilian Journal of Probability and Statistics*, 25(1), pp. 1–13(2011).

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Table 1. A summary of studies considering each approach as well as applied methods

| Authors | engineering | statistical | Engineering-statistical | Finite element | Numerical | Statistical inference | Monte Carlo | Metamodeling | BA | GP | ANOVA | MANOVA | MLE | Response surface | Regression |
|---------|-------------|-------------|--------------------------|----------------|-----------|-----------------------|-------------|---------------|----|----|-------|--------|-----|------------------|-----------|
| Allaire [11] | ● | ● | | | | | | | | | | | | | |
| Nocedal and Wright [12] | ● | ● | | | | | | | | | | | | | |
| Szabó [13] | ● | ● | | | | | | | | | | | | | |
| Mathelin L. and Hussaini [14] | ● | ● | | | | | | | | | | | | | |
| Singh and Melkote [5] | ● | ● | | | | | | | | | | | | | |
| Vepsä et al. [15] | ● | ● | | | | | | | | | | | | | |
| Palumbo et al. [16] | ● | ● | | | | | | | | | | | | | |
| Stickler and Schachinger [17] | ● | ● | | | | | | | | | | | | | |
| Sena and Silvapulle [18] | ● | ● | | | | | | | | | | | | | |
| Kumar et al. [19] | ● | ● | | | | | | | | | | | | | |
| Roussas [20] | ● | ● | | | | | | | | | | | | | |
| Zio [21] | ● | ● | | | | | | | | | | | | | |
| Dong et al. [22] | ● | ● | | | | | | | | | | | | | |
| Parnianifard et al. [23] | ● | ● | | | | | | | | | | | | | |
| Kennedy and O’Hagan [2] | ● | ● | ● | ● | | | | | | | | | | | |
| Roshan and Melkote [3] | ● | ● | ● | ● | ● | | | | | | | | | | |
| Park and Grandhi [24] | ● | ● | | | | | | | | | | | | | |
| Caiado and Goldstein [25] | ● | ● | | | | | | | | | | | | | |
| Wong et al. [26] | ● | ● | | | | | | | | | | | | | |
| Sankararaman and Mahadevan [27] | ● | ● | | | | | | | | | | | | | |
| Duru et al. [28] | ● | ● | ● | ● | | | | | | | | | | | |
| Azzimonti et al. [29] | ● | ● | ● | ● | | | | | | | | | | | |
| Reid [30] | ● | ● | | | | | | | | | | | | | |
| Junaid and Wani [31] | ● | ● | | | | | | | | | | | | | |
| Brandt [32] | ● | ● | | | | | | | | | | | | | |
| Korunović [33] | ● | ● | | | | | | | | | | | | | |
| Lin and Lin [34] | ● | ● | | | | | | | | | | | | | |
| Lunardon and Ronchetti [35] | ● | ● | | | | | | | | | | | | | |
| Pratola and Higdon [36] | ● | ● | | | | | | | | | | | | | |
| Recep et al. [37] | ● | ● | | | | | | | | | | | | | |
| Saikumar et al. [38] | ● | ● | | | | | | | | | | | | | |
| Chen et al. [39] | ● | ● | | | | | | | | | | | | | |
| Neal [40] | ● | ● | | | | | | | | | | | | | |
| Mondal et al. [41] | ● | ● | | | | | | | | | | | | | |
| Díaz-García [42] | ● | ● | | | | | | | | | | | | | |
| Rahimi et al. [7] | ● | ● | | | | | | | | | | | | | |
| Yan [4] | ● | ● | ● | ● | ● | | | | | | | | | | |
| Roshan and Yan [5] | ● | ● | ● | ● | ● | | | | | | | | | | |
| Yan et al. [9] | ● | ● | ● | ● | ● | | | | | | | | | | |
| Sheikh and Saghaie [6] | ● | ● | | | | | | | | | | | | | |
| Singh et al. [10] | ● | ● | ● | ● | ● | | | | | | | | | | |
| Current study | ● | ● | ● | ● | ● | | | | | | | | | | |
**Figure 1.** Proposed procedure

**Table 2.** Description of parameters and variables

| Parameter/ Variable | Description |
|---------------------|-------------|
| $Y = (y_1 \ldots y_q)'$ | The output variables |
| $q$ | Number of the output variables |
| $x = (x_1 \ldots x_p)'$ | The vector of input variables |
| $p$ | Number of the input variables |
| $\eta = (\eta_1 \ldots \eta_q)'$ | The vector of calibration parameters of the engineering model |
| $f(x; \eta)$ | The engineering model |
| $\rho$ | The scale parameter |
| $\delta(x)$ | The discrepancy function (model bias) |
| $\epsilon$ | The random error |
| $\sigma^2$ | The variance of the errors |
| $\mu = (\mu_0 \ldots \mu_l)'$ | The mean of the discrepancy function |
| $\tau^2$ | The variance of the discrepancy function |
| $R$ | $n \times n$ correlation matrix |
The characteristic length scale

γ  The vector of adjustment parameters of the engineering-statistical model

**Table 3.** Levels of variables based on [10]

| variable | Levels   |
|----------|----------|
| X₁       | 10 15 20 25 |
| X₂       | 10 50     |
| X₃       | 0 5 10    |
| X₄       | 100 200   |

**Figure 2.** The plot of actual vs. predicted by engineering model for thrust force (first plot) and cutting force (second plot)
### Table 4. Values of parameters and variables

| parameter/variable | description                                                                 | value                                                                 |
|--------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------|
| $Y$                | The output variables                                                       | $(y_1, y_2)'$                                                         |
| $q$                | Number of the output variables                                             | 2                                                                   |
| $x$                | e vector of input variables                                                | $(x_1, \ldots, x_4)'$                                               |
| $p$                | Number of the input variables                                              | 4                                                                   |
| $\eta$             | The vector of calibration parameters of the engineering model              | $(1.605, 0.888, 0.00058, 0.009, 0.0018)$  
  
  $(1.3581, 0.888, 0.0014, 0.0269, 0.0034)$ |
| $f(x;\eta)$        | The engineering model                                                      | $(f_1(x), f_2(x))$                                                 |
| $\rho$             | The scale parameter                                                        | 1                                                                   |
| $\delta(x)$        | The discrepancy function (model bias)                                      | $(\delta_1, \delta_2)'$                                           |
| $\sigma^2$         | The variance of the errors                                                 | $(0.0118, 0.0148)$                                                 |
| $\mu$              | The mean of the discrepancy function                                       | $(\mu_1, \mu_2)'$                                                 |
| $\tau^2$           | The variance of the discrepancy function                                   | $(1.1401, 0.3114)$                                                 |
| $R$                | $n \times n$ correlation matrix                                            | $R_1$, $R_2$                                                        |
| $\theta$           | The characteristic length scale                                           | $\theta_2' = (2.6891, 21.2868, 2.0061, 56.5802)$  
  
  $\theta_1' = (7.774, 27.7022, 3.1395, 265.0827)$ |

### Table 5. Results of general MANOVA applied on engineering function

| Factor | Test Statistic | F       | DF       | P         | $F_{0.01}$ | Being Significant |
|--------|----------------|---------|----------|-----------|------------|------------------|
|        |                |         | Num      | Denom     |            |                  |
| $X_1$  | 0.00013        | 1133.272| 6        | 78        | 0.000      | 3.04             | ✓                |
| $X_2$  | 0.40779        | 28.319  | 2        | 39        | 0.000      | 5.19             | ✓                |
| $X_3$  | 0.08162        | 48.757  | 4        | 78        | 0.000      | 3.57             | ✓                |
Table 6. Results of general MANOVA on the discrepancy function

| Factor | Test Statistic | F    | DF  | P    | $F_{0.01}$ | Being Significant |
|--------|----------------|------|-----|------|------------|-------------------|
|        |                |      | Num | Denom |            |                   |
| $X_1$  | 0.78619        | 1.662| 6   | 78   | 0.142      | 3.04              | ✔                 |
| $X_2$  | 0.99389        | 0.120| 2   | 39   | 0.887      | 5.19              | ✔                 |
| $X_3$  | 0.75552        | 2.934| 4   | 78   | 0.026      | 3.57              | ✔                 |
| $X_4$  | 0.99462        | 0.106| 2   | 39   | 0.900      | 5.19              | ✔                 |

Figure 3. The main effects of $f(x)$ and $f(x) + \delta(x)$ for (a) thrust force, (b) cutting force
Zeinab Khalaj is currently an industrial engineering PhD candidate at K. N. Toosi University of Technology, Tehran, Iran; and simultaneously a lecturer at Islamic Azad University, Karaj, Iran. Her main area of interest is statistical analysis and uncertainty quantification, as well as project management.

Abdollah Aghaie is a professor of Industrial Engineering at K. N. Toosi University of Technology in Tehran, Iran. He received his BSc from Sharif University of Technology in Tehran, MSc from New South Wales University in Sydney, Australia and PhD from Loughborough University in the U.K. His main research interests are in Modeling and Simulation, Queuing System, Quality Management and Control, Supply Chain and Data Science.

Yaser Samimi is a faculty member of the Industrial Engineering Department at the K. N. Toosi University of Technology, Tehran, Iran. He received his PhD in Industrial Engineering from the same university and holds both BSc and MSc in Industrial Engineering. His primary research interests include statistical pattern recognition, statistical process control, time series analysis and change point detection methods. He is a member of Iranian Statistical Association and Iran Institute of Industrial Engineering.