Dark Energy from Quantum Uncertainty of Remote Clocks

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The observed cosmic acceleration was attributed to a mysterious dark energy in the framework of classical general relativity. The dark energy behaves very similar with vacuum energy in quantum mechanics. However, once the quantum effects are seriously taken into account, it predicts a complete wrong result and leads to a severe fine-tuning. To solve the problem, the exact meaning of time in quantum mechanics is reexamined. We abandon the standard interpretation that time is a global parameter in quantum mechanics, replace it by a quantum dynamical variable playing the role of physical clock. We find that synchronization of two spatially separated clocks cannot be precisely realized at quantum level. There is an intrinsic quantum uncertainty of remote simultaneity, which implies an apparent vacuum energy fluctuation and gives an observed dark energy density $\rho_{de} = \frac{4}{\pi} L_P^2 L_H^{-2}$ at leading order, where $L_P$ and $L_H$ are the Planck and Hubble scale cutoffs. The fraction of the dark energy is given by $\Omega_{de} = \frac{\rho_{de}}{\rho}$ at leading order approximation, which does not evolve with time, so it is “always” comparable to the critical density. This theory is consistent with current cosmic observations.

I. INTRODUCTION

The most important observational discovery of physics in the past decade is the acceleration of expanding universe [1, 2]. In the standard model of cosmology based on the classical general relativity, the mysterious driving force of the acceleration could be simply attributed to a kind of energy unseen before, called dark energy [3]. The observational studies of the dark energy shows that it is (1) almost uniformly distributed, (2) very slowly varied with time and (3) the equation of state is around $w = -1$.

If we only consider these three properties of the dark energy, it behaves very similar with the vacuum energy we already known in quantum mechanics. However, if the quantum nature of vacuum is seriously taken into account, it gives a disappointing wrong prediction to its value [4]. The quantum mechanics predicts that it is quartic divergent up to the ultraviolet cut-off. If the validity of quantum mechanics is believed up to the Planck scale $10^{19}$ GeV, the theory gives a very large prediction $(10^{19}$ GeV$)^4$, which is about $10^{120}$ times departure to the current observation value $\rho_{de} \sim (10^{-11}$ GeV$)^4$.

Compared with the small bare value, the large result would need to be cancelled almost, but not exactly. It seems almost impossible to explain the observed dark energy within the framework of conventional quantum mechanics unless the theory is severely fine-tuned. The weakness of the vacuum energy explanation gives room to other attempts to the dark energy, such as many phenomenological scalar fields dark energy models [5], but unfortunately they are also restricted in classical or semi-classical framework. These kinds of models can also reproduces the above three properties and a correct energy density within current range of observations, by carefully tuning its kinetic term and classical potential to a specific shape. In fact, even any behavior e.g. the time evolution of dark energy and the equation of state around $w = -1$ can be engineered. Actually, without quantum mechanics, a very small cosmological constant, by phenomenological, also poses no problem. So the real question of dark energy in fact concerns the inconsistent predictions between quantum mechanics and general relativity.

The dark energy problem is a crisis deeply rooted in the foundation of physics. It is known that the vacuum energy corrections to the particle mass does gravitate [6], and hence there is by now no experimental evidence showing any violation of the equivalence principle. If we trust the equivalence principle, all energies gravitate, why we do not feel the large amount of quantum vacuum energies by their gravitational effect, that is the first part of the problem. It is an obvious contradiction between quantum mechanics and the equivalence principle. If any mechanisms prohibit their gravitational effects, why it seems that the quantum vacuum leaves a small remnant gravitational effect which drives the cosmic acceleration, which is the second part of the problem. Current observations bring forward the third part of the problem: if the dark energy is a constant vacuum energy, it is comparable with the matter energy density only in a particular epoch, since the matter energy density is diluted as the universe expanding, why the current observed vacuum energy is comparable to the matter energy density or critical energy density now, which is known as the coincident problem or “why now” problem [8].

It would be a “mission impossible” to solve these three aspects of the problem, if our arguments are built upon the two foundations mentioned: (i) standard quantum mechanics and (ii) the equivalence principle of the general relativity. Remind that these two basis by now still do not reconcile with each other to develop a consistent theory of quantum gravity, so it becomes more or less understandable that these two theories would not give a consistent prediction to the observed cosmic dark energy. The observed dark energy is likely an experimental evidence for the
conflict between these two theories. As a general believe, the difficulty of reconcile the quantum mechanics and the
general relativity is deeply rooted in the very different treatment of the concept of time [9].

The notion of time is a key to the dark energy problem, this can be seen clearly from analyzing what we really
measure in those dark energy observations. Up to date, the measurement indications for the existence of dark energy
(including the supernovae Ia and Comics Microwave Background (CMB)) come from the distance measurements \( D \)
and their relation to the redshift \( z \). In fact, we have not measured the dark energy and its equation of state directly.
The two observables \((z, D)\) are independently measured. The distance of supernovae is determined by observing the
“luminosity distance”; and the CMB is again by the “angular diameter distance” measurement of the baryon acoustic
oscillation on the last scatter surface. The redshift of the supernovae and CMB relate to the frequencies or time
measurement of distant objects. Most of the data satisfying the Hubble’s law, which states a linear dependence
between the distance and redshift, is at low redshift regime. It is the high redshift observations that detect a distance
which is significantly larger than the expected value in a flat matter dominated or curvature dominated universe
with the same Hubble constant. The unusual \( D(z) \) relation at high redshift then infers the existence of dark energy
by assuming the validity of general relativity. Until now there is no other test to tell us whether the dark energy
is truly a new component of universe or simply a misunderstanding of the distant measurements, especially at high
redshift (far off distance) regime, for example the cosmic scale remote frequency or time measurement. In fact, there
is no experimental basis to state that a distant measured frequency or remote clock is exactly the same as the native
ones.

II. QUANTUM UNCERTAINTY OF REMOTE CLOCK

In the quantum mechanics, time as a global parameter is independent with where the clocks are placed on a space-
like hypersurface. But this statement is not true in all rigor when the quantum nature of clocks is taken into account.
In the spirit of relativity, time must be operationally defined by a physical clock field \( T(x) \) describing the readings
of e.g. pointer position of the clock, where \( x = (x_0, x_1, x_2, x_3) \) are merely interpreted as external space-time point
parameter of the clock field. The clock reading \( T(x) \) is an internal time measured by a local observer, while the
external parameter \( x \) can only be measured by an external observer outside the universe. The physical clock \( T(x) \)
is assumed to be a real scalar field, and satisfies a zero-mass free field action,

\[
S_T = \int d^4x \frac{1}{2} (\partial_x T)^2. \tag{1}
\]

Now considering a thought experiment comparing the quantum states of two spatially separated quantum clocks.
The two quantum states of the clocks placed at \( x \) and \( y \) are described by states \( |T(x)\rangle \) and \( |T(y)\rangle \). If the norm of
the inner product of these two quantum states equal to 1, then these two states are identical, says, these two quantum
clocks are completely synchronized. The inner product is easy to calculate according to the clock’s action Eq. (1),
when the space-like interval \(|x - y|\) is considerable, we find the asymptotic behavior

\[
\langle T(x)|T(y)\rangle \sim \frac{1}{4\pi^2 |x - y|^2}, \tag{2}
\]

which decays with the distance between two clocks. It is transparent to understand the result, for there is no prior
reason to tell us that whether the “same” clocks spatially separated are precisely synchronized, if we want to compare
two clocks distance apart, the only thing we can do is to send a zero-mass particle (light) signal taking the information
of the state of the clock A to the clock B. But the light signal smears as the travelling distance increase, as a result
that the synchronization between two quantum clocks can not be realized precisely. The distant clock time becomes
gradually uncertain as the distance increases. If we consider the clock at \( y \) is standard, then the clock at \( x \) is uncertain.
In a homogeneous, isotropic, flat and empty universe, the uncertainty can be found by calculating the spatial evolution
of the quantum clock time

\[
\int_{T(y)}^{T(x)} DT e^{-S_T} = \frac{V^2}{4\pi^2 |x - y|^2} e^{-2V(T(x) - T(y))^2 \over |x - y|^2} e^{-2V(T(x) - T(y))^2 \over 2\sigma^2} = \frac{1}{\sigma^4(2\pi)^2} e^{-2V(T(x) - T(y))^2 \over 2\sigma^2}, \tag{3}
\]

where the uncertainty \( \sigma^2 \) of a remote clock with distance \(|x - y|\) with respect to the standard clock \( y \) is given by

\[
\sigma^2 = \langle \delta T^2(x - y) \rangle = \frac{1}{V} |x - y|, \tag{4}
\]
where \( V \) is the 3-volume IR cut-off. Therefore, the simultaneity defined by physical clock \( \langle T \rangle = \text{constant} \) has an intrinsic quantum uncertainty \( \sqrt{\langle \delta T^2(x-y) \rangle} = \sqrt{\frac{1}{V} |x-y|^{1/2}} \) being proportional to the square root of the distance from the observer. Since the IR cut-off 3-volume \( V \) here is considered to be cosmic scale but not infinity, the uncertainty of simultaneity is not zero. It is a so small number that it can be ignored in our ordinary observation, while it is considerable and important when the spatial interval is at cosmic scale. By dimensional consideration, the remote simultaneity uncertainty can be written as

\[
\langle \delta t^2 \rangle \sim L_H^{-3} L_P^4 |x-y|,
\]

where \( L_H \sim V^{1/3} \) and \( L_P \) are the infrared and ultraviolet cut-offs chosen as the Hubble and Planck scale. Therefore, if we consider the time is measured by quantum mechanical clock, but a global parameter, an intrinsic quantum uncertainty of remote simultaneity is inevitable. It is worth emphasizing that the effect is different from the time dilation, however, it smears the remote time measurement.

**III. DYNAMICAL SYSTEM UNDER PHYSICAL CLOCK**

To study the impact of the physical clock to a dynamical universe system evolving with it, we consider a whole system is defined including the clock field \( S_T[T(x)] \) and the rest (to-be-measured) universe \( S_U[\varphi(x)] \) sharing the external parameter \( x \). These two systems are assumed independent and do not interact with each other, while the time evolution of the rest universe \( S_U \) is with respect to the clock field. So the action of the whole system is separable \( S = S_U + S_T \). Before studying the system, let us first briefly proof that the system \( S \) is equivalent semi-classically to the to-be-measured system \( S_U \) where the conventional parameter time is used. Without loss of generality, considering the to-be-measured system is described by \( S_U[\varphi(x)] = \int d^4x \frac{1}{2} (\partial_x \varphi)^2 - V[\varphi] \), then the partition function of the whole system is

\[
Z = \int D\varphi DTe^{-(S_U+S_T)}, \tag{6}
\]

The functional integral \( \int DT \) of physical clock as a independent subsystem can be calculated by the mean field approximation,

\[
Z \approx Z^{MF} \int D\varphi e^{-S_{eff}}, \tag{7}
\]

Up to an unimportant constant, the effective action could be yield as

\[
S_{eff}[\varphi, \frac{\delta\varphi}{\delta T}] = \int dT \frac{1}{2} \mathcal{M} \left( \frac{\delta\varphi}{\delta T} \right)^2 - V[\varphi], \tag{8}
\]

in which \( \mathcal{M} \) is a constant depending on the integration constant of the mean field value of \( T(x) \). It is easy to see that the mean field value of \( T(x) \) is a monotonically increasing function of \( x_0 \), in this sense, the quantum clock becomes classical. The effective action now reproduces the structure of action \( S_U \), only formally, the functional derivative with respect to the clock time \( T(x) \) replaces the conventional derivative with respect to the parameter time. Generally speaking, the system \( S = S_U + S_T \) corresponds to a system satisfying the timeless Wheeler-DeWitt equation, while the system \( S_{eff} \) corresponds to an emergent system from \( S \) satisfying the Schroedinger equation in which external parameter time is defined. It is worth stressing that the theory \( S \) and \( S_U \) are equivalent at semi-classical level, but they are different at quantum level. The rest of the paper is based on the system \( S = S_U + S_T \).

**A. Zero-Point Energy**

Since the notion of time now is changed, the notion of energy changes correspondingly. Energy is defined as a time shift conserved quantity, and hence formally, the conventional derivative in the energy definition \( E \sim \frac{\delta}{\delta T} \) is replaced by a functional derivative \( E \sim \frac{\delta}{\delta T} \). Note that the action \( S_T \) is quadratic in \( T \), and \( S_U \) does not explicitly contain \( T \), so \( \langle E \rangle = \frac{\delta S}{\delta T} = 0 \). This result means that the zero-point vacuum energy of the whole system \( S \) is vanished under the physical time \( T \), which explains the first part of the problem. The physical reason for that the zero-point energy \( \frac{1}{2} \sum_k \hbar \omega_k \) does not appear is transparent, since now time is the quantum fluctuating internal field \( T(x) \) but the external parameter time \( x_0 \), so the zero-point fluctuating does not appear when the observer is standing on a quantum fluctuating reference frame.
B. Vacuum Energy Fluctuations

That is not to say the vacuum is trivial, according to the uncertainty principle, an apparent energy variance emerges out of the void related to the intrinsic time uncertainty, i.e. \( \langle \delta E^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \langle E^2 \rangle = \frac{\delta T^2}{\delta T} \neq 0 \). The further the distance, the more uncertain the time, and the larger the energy variance out of the void. This behavior described quantitatively is given by

\[
\langle \delta E(x) \delta E(0) \rangle \, d^4x = \frac{\delta^2 S_{\text{eff}}}{\delta T(x) \delta T(0)} \, d^4x \\
\approx \frac{\delta^2 S}{\delta T(x) \delta T(0)} \, d^4x = \partial_x^2 \delta^4(x) \, d^4x. \tag{9}
\]

At leading order, we have approximately written the \( S_{\text{eff}} = -\ln \int D\phi DT e^{-(S_0+S_T)} \approx S \) by using the classical action, so the leading result is expressed in terms of a widthless Dirac delta function, while it actually has a non-zero width. This calculation can be performed by first rewrite the Dirac delta distribution as a limit of the Gaussian distribution, i.e. \( \delta(x) = \lim_{a \to 0} \frac{1}{a \sqrt{\pi}} e^{-\frac{x^2}{a^2}} \), doing the derivatives and finally taking the zero width limit of the Gaussian distribution back to delta distribution,

\[
\langle \delta E(x) \delta E(0) \rangle \, d^4x \approx \lim_{a \to 0} \partial_x^2 \left( \frac{1}{a \sqrt{\pi}} e^{-\frac{x^2}{a^2}} \right)^4 \, d^4x \\
= 64a^{-4} \, |x - 0|^4 \delta^4(x) \, d^4x. \tag{10}
\]

The width of the Gaussian distribution \( a \) is an ultraviolet cut-off, the most natural choice is the Planck length \( a = L_P \). If the distance \( |x - 0| \) is large, the energy fluctuation becomes considerable when it is at cosmic scale.

To regulate the result, an infrared cut-off is required, a natural choice is the Hubble length \( |x - 0| = L_H \), as the largest distance we could see, i.e. cosmic horizon. Therefore, when we fix the radius \( |x - 0| = L_H \) and integrate over \( x \), then the total energy fluctuation of the vacuum in the Hubble scale volume is yield

\[
\langle \delta E^2 \rangle \approx 64 \, \int d^4x L_p^{-4} L_H^2 \, \delta^4(x) = 64 L_p^{-4} L_H^2. \tag{11}
\]

Then an averaged vacuum energy density (averaged in the 3-ball with fixed radius \( |x - 0| = L_H \)) due to the total vacuum energy fluctuation is predicted as

\[
\rho_{\text{de}} \approx \frac{\sqrt{\langle \delta E^2 \rangle}}{4 \pi L_H^3} = \frac{6}{\pi} L_p^{-2} L_H^{-2}, \tag{12}
\]

and

\[
\Omega_{\text{de}} = \frac{\rho_{\text{de}}}{\rho_c} \approx \frac{2}{\pi} \approx 0.64, \tag{13}
\]

where \( \rho_c = \frac{3H^2}{8\pi G} \) is the critical density, \( H = L_H^{-1} \) is the Hubble’s constant, \( 8\pi G = L_P^2 \) is the Newton’s gravitational constant, and \( \Omega_{\text{de}} \) is the fraction of the effective vacuum energy. The leading order predicted \( \Omega_{\text{de}} \) is a little lower than the current best fit from the data of Planck satellite \([14]\), but still within the allowed range. This result explains the second part of the problem.

There are several important remarks of this result to emphasize. (i) We have considered the question: what a vacuum energy fluctuation is seen by a remote observer in a homogeneous, isotropic and empty flat space, when the time is defined by a physical clock field \( T(x) \). (ii) The coordinates \( x \) in the action Eq.\([1]\) are just external parameters finally be integrated out, the reason we only pick up “time” treating quantum mechanically and the spacetime coordinates treating as the external parameter is for simplicity, a more rigor and general quantum mechanical treatment is to put the space and time on an equal footing (quantum reference frame). (iii) The precise value of the results Eq.\([12]\) and \([13]\) depends on the detailed nature of the cut-offs, and at present, the numerical factors of the cut-offs are chosen as the most natural ones.

C. Coincident Problem

It is easy to verify that, since the action \( S_T \) is quadratic in \( T \), the higher order \((> 2)\) functional derivative with respect to clock time \( T \) are all vanished. As a result, the vacuum energy fluctuation does not evolve with the clock
time, i.e. $\frac{\delta(z^2)}{z^2} = \frac{\delta z}{z} = 0$, thus leading to the fraction $\Omega_{de}$ does not vary with time. It is a constant and is “always” comparable with the critical density or matter energy density. The property explains the third part of the problem.

It is worth noting that, because in our framework the notion of time is reinterpreted, so any old notion of evolution in the standard model of cosmology must be carefully reconsidered. The exact meaning of the evolution of any quantities is that their functional derivative with respect to the clock time $T$ is non-vanished. However, for example, the Hubble parameter $H(t)$, the fraction $\Omega_i(t)$ and the equation of state $w(t)$ as functions of (parameter) time have no physical meaning in our setting. So the infrared cut-off, the Hubble constant $H$ and/or Hubble length $L_H$ in Eq. (12) is really a constant that does not vary with time. The fraction of the effective vacuum energy $\Omega_{de}$ is “always” that value. And as a vacuum energy, it is uniform and constant, and moreover, we predict that the equation of state $w = -1$ strictly does not vary with time. It is a most interesting and salient feature of the framework, certain conventional concepts of time evolution are related to the phenomenon that the state of the clock varies, but the state of the to-be-measured system does not.

IV. DISTANCE-REDSHIFT RELATION

Let us assume the time uncertainty considered in flat space is still (approximated) valid in the Hubble’s expanding universe. At small redshift $z \equiv a_0/a - 1$, the distance-redshift relation is given by $H_0 D = z + \frac{1}{2} z^2 + ...$, where $H_0$ is the Hubble’s constant at $z = 0$. Since the distant frequency or redshift measurement has been reconsidered, such effect will give a modification to the distance-redshift relation. The remote time uncertainty does not change the central value of the spectral line or redshift, only broadens it and gives a non-vanishing variance, i.e. $\langle \delta z^2 \rangle \neq 0$. As a consequence, the distance-redshift relation $D(z)$ is modified at the order $O(z^2)$ by a positive contribution,

$$H_0 D = \langle z \rangle + \frac{1}{2} \left( \langle z^2 \rangle + \langle \delta z^2 \rangle \right) + ... , \quad (14)$$

in which we have used $\langle z^2 \rangle = \langle z \rangle^2 + \langle \delta z^2 \rangle$. It is the extra positive contribution coming from the remote time/simultaneity uncertainty makes the effective “dark energy” behave repulsive. The energy variance Eq. (10), which is proportional to the distance square, indicates a proportionality $\langle \delta z^2 \rangle \propto H^2 D^2$. For $\langle z \rangle^2 \approx H_0^2 D^2$, we have $\langle \delta z^2 \rangle \approx \frac{H^2 D^2}{H_0^2 D^2} = \Omega_{de}$. The linear relationship between the distant spectral line width and redshift could be observed. From Eq. (14) we have the modified distance-redshift relation

$$H_0 D = \langle z \rangle + \frac{1}{2} (1 + \Omega_{de}) \langle z \rangle^2 + ... \quad (15)$$

Therefore, in a flat universe without ordinary matter (pressureless matter and radiation), the uncertainty of remote clock induces a redshift independent deceleration parameter $q_0 = -\Omega_{de} < 0$, which makes the flat empty universe seem being dominated by “dark energy” and accelerating.

V. CONCLUSIONS

Finally, let us summarize the paper. In this paper, we preserve the accurate validity of the equivalence principle and abandon the standard interpretation of parameter time in quantum mechanics. The quantum spatial evolution makes the physical clock field fuzzy as the distance increases, leading to a quantum uncertainty of remote simultaneity. The idea of reinterpretation of time solves the dark energy problem. This theory tells us that the observed dark energy is a quantum effect connected to the quantum uncertainty of spatially separated clocks. The apparent vacuum energy fluctuation is inevitable if we use the physical clock redefining the time, and the result fits the observation well. This framework requires a modification of the standard quantum mechanics. Although the global parameters of quantum mechanics are necessary by its intrinsic structure, there is no prior reason to interpret them as time, time here is what we read from a physical clock that needs to be described quantum mechanically. The modified quantum framework requires a relational interpretation in terms of entangled state which is more natural than its standard absolute interpretation, since not only the to-be-measured system but also the measuring instruments such as the clock are both needed to be treated by quantum mechanics. In this sense, the Wheeler-DeWitt equation plays a more fundamental role than the emerged Schroedinger equation. And most importantly, this idea provides a touchstone to the longstanding difficulty of reconciliation of the inconsistency between general relativity and quantum mechanics.
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