Comments on the Hydrogen Atom Spectrum in the Noncommutative Space

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There has been disagreement in the literature on whether the hydrogen atom spectrum receives any tree-level correction due to noncommutativity. Here we shall clarify the issue and show that indeed a general argument on the structure of proton as a nonelementary particle leads to the appearance of such corrections. As a showcase, we evaluate the corrections in a simple nonrelativistic quark model with a result in agreement with the previous one we had obtained by considering the electron moving in the external electric field of proton. Thus the previously obtained bound on the noncommutativity parameter, $\theta < \left(10^4 \text{GeV}\right)^{-2}$, using the Lamb shift data, remains valid.

Recently a large amount of research work has been devoted to the study of physics on noncommutative spacetimes and in particular noncommutative Moyal plane (for a review see, e.g., [1]). In these works both quantum mechanics (QM) and field theory on noncommutative spaces have been studied. Besides the theoretical interests, by comparing the results of noncommutative version of usual physical models with present data, lower bounds on the noncommutative scale $\Lambda_{NC}$ have been obtained $[2,3,4]$ as a conservative estimate, $\Lambda_{NC} \gtrsim 1 - 10 \text{ TeV}$.

In this commentary we would like to focus on the Hydrogen atom in the noncommutative space and re-analyze its spectrum. This system has previously been considered with a disagreement on the results. Here, through a more careful analysis we intend to clarify the discrepancy. In $[5]$ we have analyzed the Hydrogen atom in the noncommutative space considering the system described by a one-particle Schrodinger equation. Explicitly we considered the electron in an external Coulomb field; hence the system is described by

$$H\ket{\psi} = i\hbar \frac{\partial}{\partial t} \ket{\psi}$$

with

$$H = \frac{\hat{p}_e^2}{2m_e} + V(\hat{x}_e),$$

where $\hat{x}_e$ and $\hat{p}_e$ are the phase space coordinates of the electron and

$$\begin{align*}
[\hat{x}_e^i, \hat{x}_e^j] &= 0 , \\
[\hat{x}_e^i, \hat{p}_e^j] &= i\hbar \delta^{ij} , \\
[\hat{p}_e^i, \hat{p}_e^j] &= 0 .
\end{align*}$$

Then it is easy to see that the new coordinates $x_i = \hat{x}_i + \frac{1}{2\theta} \hat{p}_j \hat{p}_j$, $p_i = \hat{p}_i$ satisfy the usual canonical commutation relations $[6]$.

and in terms of these “canonical” coordinates, the Hamiltonian takes the familiar form of usual Hydrogen atom plus noncommutative corrections (cf. Eq.(2.5) in Ref. $[6]$).

$$V(r,p) = -\frac{Ze^2}{r} - Ze^2 \frac{\vec{L} \cdot \vec{\theta}}{4 \hbar^3} + O(\theta^2) ,$$

where $\theta_i = \epsilon_{ijk} \theta_{jk}$, $r = \sqrt{\sum_i x_i^2}$ and $\vec{L} = \vec{r} \times \vec{p}$. (The value of $|\vec{\theta}|$ is the inverse square of the noncommutative scale $\Lambda_{NC}$.) From here we concluded that there exist noncommutative corrections to the spectrum and comparing our results with the data for the Lamb-shift experiments, we obtained the bound $\Lambda_{NC} \gtrsim 10 \text{ TeV}$.

On the other hand, in a more detailed analysis, the nucleus (here the proton) which exerts the Coulomb potential should also be considered as a dynamical object. In other words, one should solve the two-body Schrodinger equation. For the noncommutative Hydrogen atom this has been done in Ref. $[6]$. There it was assumed that the proton, similar to the electron, is described by NC QED $[7,8]$. Based on this assumption and the fact that under charge conjugation $\theta_{ij}$ changes the sign $[9]$, it was shown that the noncommutativity effects will not change the spectrum of this two-body problem at the tree level (cf. Eq.(28) of Ref. $[6]$), in disagreement with the results of $[5]$ (in particular, with the above Eq. (0.2)).

In this note we argue that in fact the assumption of Ref. $[6]$ that the effective noncommutativity parameter for proton is equal to that of electron with a minus sign...
is not physically valid. Therefore, the “cancellation” of noncommutativity effects is not complete and hence our previous results on the form (0.4) for the potential with the correction term, as well as on the lower bound on $\Lambda_{NC}$ are indeed valid. The essential point is that the proton, due to the fact that it has structure and is a composite particle, cannot be described by NC QED (applicable to elementary particles). To systems such as positronium, however, the analysis of Ref. [6] is applicable to elementary particles. To systems such as composite particle, cannot be described by NC QED (appro-}
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