ON HOLOMORPHIC MAPPINGS WITH RELATIVELY $p$-COMPACT RANGE

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Abstract. Related to the concept of $p$-compact operator with $p \in [1, \infty]$ introduced by Sinha and Karn [20], this paper deals with the space $\mathcal{H}_{\mathcal{K}_p}^\infty(U, F)$ of all Banach-valued holomorphic mappings on an open subset $U$ of a complex Banach space $E$ whose ranges are relatively $p$-compact subsets of $F$. We characterize such holomorphic mappings as those whose Mujica’s linearisations on the canonical predual of $\mathcal{H}_{\mathcal{K}_p}^\infty(U)$ are $p$-compact operators. This fact allows us to make a complete study of them. We show that $\mathcal{H}_{\mathcal{K}_p}^\infty$ is a surjective Banach ideal of bounded holomorphic mappings which is generated by composition with the ideal of $p$-compact operators and contains the Banach ideal of all right $p$-nuclear holomorphic mappings. We also characterize holomorphic mappings with relatively $p$-compact ranges as those bounded holomorphic mappings which factorize through a quotient space of $\ell_p$ or as those whose transposes are quasi $p$-nuclear operators (respectively, factor through a closed subspace of $\ell_p$).

Introduction

Inspired by classical Grothendieck’s characterization of a relatively compact subset of a Banach space as a subset of the convex hull of a norm null sequence of vectors [12], Sinha and Karn [20] introduced and studied $p$-compact sets and $p$-compact operators with $p \in [1, \infty]$.

Let $E$ be a complex Banach space with closed unit ball $B_E$. Let $p \in (1, \infty)$ and let $p^*$ denote the conjugate index of $p$ given by $p^* = p/(p - 1)$. Given $p \in [1, \infty)$, $\ell_p(E)$ denotes the Banach space of all absolutely $p$-summable sequences $(x_n)$ in $E$, equipped with the norm

$$||(x_n)||_p = \left( \sum_{n=1}^{\infty} ||x_n||^p \right)^{1/p},$$

and $c_0(E)$ stands for the Banach space of all norm null sequences $(x_n)$ in $E$, endowed with the norm

$$||(x_n)||_\infty = \sup \{ ||x_n|| : n \in \mathbb{N} \}.$$

In the case $E = \mathbb{C}$, we will simply write $\ell_p$ and $c_0$.

For $p \in (1, \infty)$, the $p$-convex hull of a sequence $(x_n) \in \ell_p(E)$ is defined by

$$p\text{-conv}(x_n) = \left\{ \sum_{n=1}^{\infty} a_n x_n : (a_n) \in B_{\ell_{p^*}} \right\}.$$
Similarly, we set

\[
\begin{align*}
1\text{-conv}(x_n) &= \left\{ \sum_{n=1}^{\infty} a_n x_n : (a_n) \in B_{c_0} \right\}, \quad (x_n) \in \ell_1(E), \\
\infty\text{-conv}(x_n) &= \left\{ \sum_{n=1}^{\infty} a_n x_n : (a_n) \in B_{\ell_1} \right\}, \quad (x_n) \in c_0(E).
\end{align*}
\]

Note that \(\infty\text{-conv}(x_n)\) coincides with \(\text{abco}(\{x_n : n \in \mathbb{N}\})\), the norm-closed absolutely convex hull of the set \(\{x_n : n \in \mathbb{N}\}\) in \(E\).

Following [20], given \(p \in [1, \infty]\), a set \(K \subseteq E\) is said to be \(relatively \ p\text{-compact}\) if there is a sequence \((x_n) \in \ell_p(E)\) \((x_n) \in c_0(E)\) if \(p = \infty\) such that \(K \subseteq p\text{-conv}(x_n)\). Such a sequence is not unique but a \textit{measure of the size of the \(p\)-compact set} \(K\) is introduced in [10, p. 297] (see also [14]) defining

\[
m_p(K, E) = \begin{cases} 
\inf\{\|x_n\|_p : (x_n) \in \ell_p(E), K \subseteq p\text{-conv}(x_n)\} & \text{if } 1 \leq p < \infty, \\
\inf\{\|x_n\|_\infty : (x_n) \in c_0(E), K \subseteq p\text{-conv}(x_n)\} & \text{if } p = \infty,
\end{cases}
\]

and \(m_p(K, E) = \infty\) if \(K\) is not \(p\)-compact. We frequently will write \(m_p(K)\) instead of \(m_p(K, E)\).

A linear operator between Banach spaces \(T : E \rightarrow F\) is said to be \(p\text{-compact}\) if \(T(B_E)\) is a relatively \(p\)-compact subset of \(F\). The space of all \(p\)-compact linear operators from \(E\) to \(F\) is denoted by \(K_p(E, F)\), and \(K_p\) is a Banach operator ideal endowed with the norm \(k_p(T) = m_p(T(B_E))\) (see [20, Theorem 4.2] and [10, Proposition 3.15]).

From the cited Grothendieck’s result [12], a set \(K \subseteq E\) is relatively compact if and only if for every \(\varepsilon > 0\), there is a sequence \((x_n) \in c_0(E)\) with \(\|x_n\|_\infty \leq \sup_{x \in K} \|x\| + \varepsilon\) such that \(K \subseteq \infty\text{-conv}(x_n)\). Hence we can consider compact sets as \(\infty\)-compact sets and, in this way, compact operators can be viewed as \(\infty\)-compact operators with \(k_\infty\) being the usual operator norm.

The work of Sinha and Karn [20] motivated many papers on \(p\)-compactness in operator spaces (see [3, 8, 9, 10, 11, 14, 19], among others) and also in Lipschitz spaces [1, 2].

The extension of this theory to the polynomial and holomorphic settings was addressed in [5, 6]. In these environments, the property of \(p\)-compactness was studied from the following local point of view: a mapping \(f : U \rightarrow F\) is said to be \(locally \ p\text{-compact}\) if every point \(x \in U\) has a neighborhood \(U_x \subseteq U\) such that \(f(U_x)\) is relatively \(p\)-compact in \(F\).

The aim of this note is to study a subclass of locally \(p\)-compact holomorphic mappings, namely, \textit{holomorphic mappings with relatively \(p\)-compact range}. Notice that every such mapping is locally \(p\)-compact but the converse is not true. For instance, if \(\mathbb{D}\) denotes the open complex unit disc, the holomorphic mapping \(f : \mathbb{D} \rightarrow c_0\) defined by \(f(z) = \sum_{n=1}^{\infty} z^n e_n\), where \(e_n\) is the canonical basis of \(\ell_1\), is locally \(1\)-compact but it has not relatively compact range (see [15, Example 3.2]) and, consequently, neither relatively \(p\)-compact range for any \(p \geq 1\).

Our motivation to deal with this class of mappings also arises from the study (initiated in [15] and continued in [7, 13]) on the Banach space \(H_K^\infty(U, F)\) formed by all holomorphic mappings from \(U\) to \(F\) with relatively compact range, equipped with the supremum norm.

We now briefly describe the content of this paper. Let \(E\) and \(F\) be complex Banach spaces, \(U\) an open subset of \(E\) and \(p \in [1, \infty]\). Let \(H^\infty(U, F)\) denote the Banach space of all bounded holomorphic mappings from \(U\) into \(F\), endowed with the supremum norm. In particular, \(H^\infty(U)\) stands for \(H^\infty(U, \mathbb{C})\).

In [15], Mujica provided a linearisation method of the members of \(H^\infty(U, F)\), which will be an essential tool in our analysis of the subject. If \(\mathcal{G}^\infty(U)\) is the canonical predual of \(H^\infty(U)\)
obtained by Mujica [15] via an identification denoted $g_U$, we will establish that a bounded holomorphic mapping $f: U \to F$ has relatively $p$-compact range if and only if Mujica’s linearisation $T_f: \mathcal{G}^\omega(U) \to F$ is a $p$-compact operator. This fact has some interesting applications.

If $\mathcal{H}^\omega_{K_p}(U, F)$ denotes the space of all holomorphic mappings with relatively $p$-compact range $f: U \to F$ with the natural norm $k^\omega_p (f) = m_p(f(U))$, we will prove that $\mathcal{H}^\omega_{K_p}$ is a surjective Banach ideal of bounded holomorphic mappings which is generated by composition with the ideal $K_p$ of $p$-compact operators. This means that each mapping $f \in \mathcal{H}^\omega_{K_p}(U, F)$ admits a factorization $f = T \circ g$, where $G$ is a complex Banach space, $g \in \mathcal{H}^\omega(U, G)$ and $T \in K_p(G, F)$. Moreover, $k^\omega_p (f)$ coincides with $\inf \{k_p(T) \|g\|_\omega\}$, where the infimum is extended over all such factorizations of $f$ and, curiously, this infimum is attained at the factorization $f = T_f \circ g_U$ due to Mujica [15].

In parallelism with the linear case, we introduce the notion of right $p$-nuclear holomorphic mapping from $U$ to $F$, study its linearisation on $\mathcal{G}^\omega(U)$, analyse its ideal property and show that every right $p$-nuclear holomorphic mapping has relatively $p$-compact range.

Moreover, we characterize the members of $\mathcal{H}^\omega_{K_p}(U, F)$ as those bounded holomorphic mappings from $U$ to $F$ which factorize through a quotient space of $\ell_p^\omega$, and also as those whose transposes are quasi $p$-nuclear operators (respectively, factor through a closed subspace of $\ell_p$).

1. The results

From now on, unless otherwise stated, $E$ and $F$ will denote complex Banach spaces, $U$ will be an open subset of $E$ and $p \in [1, \infty]$.

As usual, $\mathcal{L}(E, F)$ denotes the Banach space of all bounded linear operators from $E$ to $F$ endowed with the operator canonical norm, and $E^*$ stands for the dual space of $E$. The subspaces of $\mathcal{L}(E, F)$ consisting of all compact operators and finite-rank bounded operators from $E$ to $F$ will be denoted by $K(E, F)$ and $F(E, F)$, respectively.

In this section, we will study holomorphic mappings $f: U \to F$ so that $f(U)$ is a relatively $p$-compact subset of $F$. We denote by $\mathcal{H}^\omega_{K_p}(U, F)$ the set formed by such mappings and, for each $f \in \mathcal{H}^\omega_{K_p}(U, F)$, we define $k^\omega_p (f) = m_p(f(U))$.

The space of all holomorphic mappings with relatively compact range from $U$ to $F$, denoted $\mathcal{H}^\omega_{K_p}(U, F)$, is a Banach space equipped with the supremum norm (see [13, Corollary 2.11]). On account of the following result, we will only study in this paper the case $1 \leq p < \infty$.

**Proposition 1.1.** $\mathcal{H}^\omega_{K_p}(U, F) = \mathcal{H}^\omega_{\mathcal{K}}(U, F)$ and $k^\omega_{\mathcal{K}} (f) = \|f\|_\omega$ for all $\mathcal{H}^\omega_{K_p}(U, F)$.

**Proof.** Let $f \in \mathcal{H}^\omega_{K_p}(U, F)$ and let $(y_n) \in c_0(F)$ be such that $f(U) \subseteq \omega$-conv$(y_n)$. Since $\omega$-conv$(y_n)$ is relatively compact in $F$, it follows that $f \in \mathcal{H}^\omega_{\mathcal{K}}(U, F)$ with $\|f\|_\omega \leq \|y_n\|_\omega$, and taking infimum over all such sequences $(y_n)$, we have $\|f\|_\omega \leq k^\omega_{\omega}(f)$.

Conversely, let $f \in \mathcal{H}^\omega_{\mathcal{K}}(U, F)$. By classical Grothendieck’s result, for every $\varepsilon > 0$, there exists $(y_n) \in c_0(F)$ with $\|y_n\|_\omega \leq \|f\|_\omega + \varepsilon$ such that $f(U) \subseteq \omega$-conv$(y_n)$. Hence $f \in \mathcal{H}^\omega_{\mathcal{K}}(U, F)$ and $k^\omega_{\omega}(f) \leq \|f\|_\omega$. \qed

1.1. Linearisation. Our first aim is to characterize holomorphic mappings with relatively $p$-compact range in terms of the $p$-compactness of their linearisations on the canonical predual of $\mathcal{H}^\omega(U)$.

Towards this end, we first recall the following result due to Mujica [15] concerning linearisation of holomorphic mappings on Banach spaces.
Theorem 1.2. \cite{15} Let $E$ be a complex Banach space and $U$ be an open set in $E$. Let $\mathcal{G}^\infty(U)$ denote the norm-closed linear subspace of $\mathcal{H}^\infty(U)^*$ generated by the functionals $\delta(x) \in \mathcal{H}^\infty(U)^*$ with $x \in U$, defined by $\delta(x)(f) = f(x)$ for all $f \in \mathcal{H}^\infty(U)$.

(i) The mapping $g_U: U \to \mathcal{G}^\infty(U)$ defined by $g_U(x) = \delta(x)$ is holomorphic with $\|\delta(x)\| = 1$ for all $x \in U$.

(ii) For every complex Banach space $F$ and every mapping $f \in \mathcal{H}^\infty(U,F)$, there exists a unique operator $T_f \in L(\mathcal{G}^\infty(U), F)$ such that $T_f \circ g_U = f$. Furthermore, $\|T_f\| = \|f\|_w$.

(iii) For every complex Banach space $F$, the mapping $f \mapsto T_f$ is an isometric isomorphism from $\mathcal{H}^\infty(U,F)$ onto $L(\mathcal{G}^\infty(U), F)$.

(iv) $\mathcal{H}^\infty(U)$ is isometrically isomorphic to $\mathcal{G}^\infty(U)^*$, via the mapping $J_U: \mathcal{H}^\infty(U) \to \mathcal{G}^\infty(U)^*$ given by $J_U(f)(g_U(x)) = f(x)$ for all $f \in \mathcal{H}^\infty(U)$ and $x \in U$.

(v) $B_{\mathcal{G}^\infty(U)}$ coincides with $\text{abco}(g_U(U))$. \qed

We are now ready to state the aforementioned characterization.

Theorem 1.3. Let $p \in [1, \infty)$ and $f \in \mathcal{H}^\infty(U,F)$. The following conditions are equivalent:

(i) $f$ has relatively $p$-compact range.

(ii) $T_f: \mathcal{G}^\infty(U) \to F$ is a $p$-compact linear operator.

In this case, $k^\mathcal{H}^\infty_p(f) = k_p(T_f)$. Furthermore, the mapping $f \mapsto T_f$ is an isometric isomorphism from $(\mathcal{H}^\infty_{k_p}(U,F), k^\mathcal{H}^\infty_p)$ onto $(\mathcal{K}_p(\mathcal{G}^\infty(U), F), k_p)$.

Proof. Using Theorem \cite{12} we have the following inclusions:

$$f(U) = T_f \circ g_U(U) \subseteq T_f(\text{abco}(g_U(U))) = T_f(B_{\mathcal{G}^\infty(U)}) \subseteq \text{abco}(T_f \circ g_U(U)) = \text{abco}(f(U)).$$

(i) $\Rightarrow$ (i): If $T_f \in \mathcal{K}_p(\mathcal{G}^\infty(U), F)$, then $f \in \mathcal{H}^\infty_{k_p}(U,F)$ with

$$k^\mathcal{H}^\infty_p(f) = m_p(f(U)) \leq m_p(T_f(B_{\mathcal{G}^\infty(U)})) = k_p(T_f)$$

by the first inclusion above.

(i) $\Rightarrow$ (ii): If $f \in \mathcal{H}^\infty_{k_p}(U,F)$, then $T_f \in \mathcal{K}_p(\mathcal{G}^\infty(U), F)$ with

$$k_p(T_f) = m_p(T_f(B_{\mathcal{G}^\infty(U)})) \leq m_p(\text{abco}(f(U))) = m_p(f(U)) = k^\mathcal{H}^\infty_p(f),$$

by the second inclusion above and the known fact that a set $K \subseteq F$ is $p$-compact if and only if $\text{abco}(K)$ is $p$-compact, in whose case $m_p(K) = m_p(\text{abco}(K))$.

The last assertion of the statement follows immediately from part (iii) of Theorem \cite{12} and from the above proof. \qed

1.2. Banach ideal property. Our next goal is to study the Banach ideal structure of $(\mathcal{H}^\infty_{k_p}, k^\mathcal{H}^\infty_p)$. Inspired by the notion of Banach operator ideal \cite{18}, the following type of ideals was considered in \cite{7}.

An ideal of bounded holomorphic mappings (or simply, a bounded-holomorphic ideal) is a subclass $I^{\mathcal{H}^\infty}$ of the class of bounded holomorphic mappings $\mathcal{H}^\infty$ such that for each complex Banach space $E$, each open subset $U$ of $E$ and each complex Banach space $F$, the components

$$I^{\mathcal{H}^\infty}(U,F) := I^{\mathcal{H}^\infty} \cap \mathcal{H}^\infty(U,F)$$

satisfy the following three conditions:

(11) $I^{\mathcal{H}^\infty}(U,F)$ is a linear subspace of $\mathcal{H}^\infty(U,F)$,

(12) For any $g \in \mathcal{H}^\infty(U)$ and $y \in F$, the mapping $g \cdot y: x \mapsto g(x)y$ from $U$ to $F$ is in $I^{\mathcal{H}^\infty}(U,F)$,
(13) The ideal property: if $H, G$ are complex Banach spaces, $V$ is an open subset of $H$, $h \in \mathcal{H}(V, U)$, $f \in \mathcal{H}^w(U, F)$ and $S \in \mathcal{L}(F, G)$, then $S \circ f \circ h \in \mathcal{H}^w(V, G)$.

Suppose that a function $\|\cdot\|_{\mathcal{T}^w} : \mathcal{T}^w \rightarrow \mathbb{R}^+_0$ satisfies the following three properties:

(N1) $(\mathcal{T}^w(U, F), \|\cdot\|_{\mathcal{T}^w})$ is a normed (Banach) space with $\|f\|_{\mathcal{T}^w} \leq \|f\|_{\mathcal{T}^w}$ for all $f \in \mathcal{T}^w(U, F)$,
(N2) $\|g \cdot y\|_{\mathcal{T}^w} = \|g\|_{\mathcal{T}^w} \|y\|$ for all $g \in \mathcal{H}(U, G)$ and $y \in F$,
(N3) If $H, G$ are complex Banach spaces, $V$ is an open subset of $H, h \in \mathcal{H}(V, U)$, $f \in \mathcal{T}^w(U, F)$ and $S \in \mathcal{L}(F, G)$, then $\|S \circ f \circ h\|_{\mathcal{T}^w} \leq \|S\| \|f\|_{\mathcal{T}^w}$.

Then $(\mathcal{T}^w(U, F), \|\cdot\|_{\mathcal{T}^w})$ is called a normed (Banach) bounded-holomorphic ideal.

A normed bounded-holomorphic ideal $\mathcal{T}^w$ is said to be:

(R) regular if for any $f \in \mathcal{H}(U, F)$, we have that $f \in \mathcal{T}^w(U, F)$ with $\|f\|_{\mathcal{T}^w} = \|k_F \circ f\|_{\mathcal{T}^w}$ whenever $k_F \circ f \in \mathcal{T}^w(U, F^*)$, where $k_F$ denotes the isometric linear embedding from $F$ into $F^*$.

(S) surjective if for any mapping $f \in \mathcal{H}(U, F)$, any open subset $V$ of a complex Banach space $G$ and any surjective mapping $\pi \in \mathcal{H}(V, U)$, we have that $f \in \mathcal{T}^w(U, F)$ with $\|f\|_{\mathcal{T}^w} = \|f \circ \pi\|_{\mathcal{T}^w}$ whenever $f \circ \pi \in \mathcal{T}^w(V, F)$.

Bearing in mind Theorems 1.3, 3.2 in [4] (see also Theorem 2.4 in [7]) shows that $\mathcal{H}^\omega_p$ is generated by composition with the operator ideal $\mathcal{K}_p$ (see [7, Definition 2.3]).

Corollary 1.4. Let $p \in [1, \infty)$ and $f \in \mathcal{H}^\omega(U, F)$. The following conditions are equivalent:

(i) $f : U \rightarrow F$ has relatively $p$-compact range.

(ii) $f = T \circ g$ for some complex Banach space $G$, $g \in \mathcal{H}^\omega(U, G)$ and $T \in \mathcal{K}_p(G, F)$.

In this case, we have

$$k^\omega_p(f) = \inf \{k_p(T) \|g\|_{\mathcal{L}} \mid \text{where the infimum runs over all factorizations of } f \text{ as in (ii), and this infimum is attained at } T_f \circ g_U\},$$

where $k^\omega_p(f)$ denotes the isometric linear embedding from $f$ into $F^*$.

Furthermore, the mapping $f \mapsto T_f$ is an isometric isomorphism from $(\mathcal{K}_p \circ \mathcal{H}^\omega(U, F), \|\cdot\|_{\mathcal{K}_p \circ \mathcal{H}^\omega})$ onto $(\mathcal{K}_p(\mathcal{G}^\omega(U, F), k_p))$.

The following result gathers some Banach ideal properties of $\mathcal{H}^\omega_p$.

Theorem 1.5. For each $p \in [1, \infty)$, $(\mathcal{H}^\omega_p, k^\omega_p)$ is a surjective Banach bounded-holomorphic ideal. Furthermore, the ideal $(\mathcal{H}^\omega_p(U, F), k^\omega_p)$ is regular whenever $F$ is reflexive.

Proof. In view of Corollary 1.4, Corollary 2.5 in [7] yields that $(\mathcal{H}^\omega_p, k^\omega_p)$ is a Banach bounded-holomorphic ideal. Then we only have to study its surjectivity and its regularity.

(S) Let $f \in \mathcal{H}^\omega(U, F)$ and assume that $f \circ \pi \in \mathcal{H}^\omega_p(V, F)$, where $V$ is an open subset of a complex Banach space $G$ and $\pi \in \mathcal{H}(V, U)$ is surjective. Since $f(U) = (f \circ \pi)(V)$, it is immediate that $f \in \mathcal{H}^\omega_p(U, F)$ with $k^\omega_p(f) = k^\omega_p(f \circ \pi)$. Hence $(\mathcal{H}^\omega_p, k^\omega_p)$ is surjective.

(R) Suppose now that $F$ is reflexive. Let $f \in \mathcal{H}^\omega(U, F)$ and assume that $k_F \circ f \in \mathcal{H}^\omega(U, F^*)$. We can take a sequence $(y_n)$ in $L_p(F)$ (in $c_0(F)$ if $p = 1$) such that $(k_F \circ f)(y_n)$ is a $p$-convex set.

The converse inequality follows from the condition (N3) satisfied by $(\mathcal{H}^\omega_p, k^\omega_p)$, and this completes the proof.
1.3. Factorization. We now present a factorization result for holomorphic mappings with relatively \( p \)-compact range which should be compared with [11] Proposition 2.9. 

**Corollary 1.6.** Let \( p \in [1, \infty) \) and \( f \in \mathcal{H}^\omega(U, F) \). The following conditions are equivalent:

(i) \( f : U \to F \) has relatively \( p \)-compact range.

(ii) There exist a closed subspace \( M \) in \( \ell_p \) (\( c_0 \) instead of \( \ell_p \) if \( p = 1 \)), a separable Banach space \( G \), an operator \( T \) in \( \mathcal{K}_p(\ell_p / M, G) \), a mapping \( g \) in \( \mathcal{H}^\omega_p(U, \ell_p / M) \) and an operator \( S \) in \( \mathcal{K}(G, F) \) such that \( f = S \circ T \circ g \).

In this case, \( k_p^{\mathcal{H}^\omega}(f) = \inf\{\|S\|\|k_p(T)\|\|g\|_{\infty}\} \), where the infimum is extended over all factorizations of \( f \) as in (ii).

**Proof.** We will only prove it for \( p \in (1, \infty) \). The case \( p = 1 \) is similarly obtained.

(i) \( \Rightarrow \) (ii): Suppose that \( f \in \mathcal{H}^\omega_p(U, F) \). By Theorem [13] T \( f_j \in \mathcal{K}_p(\mathcal{G}^\omega(U), F) \) with \( k_p(T_j) = k_p^{\mathcal{H}^\omega}(f) \). Applying [11] Proposition 2.9, for each \( \epsilon > 0 \), there exist a closed subspace \( M \subseteq \ell_p \), a separable Banach space \( G \), an operator \( T \in \mathcal{K}_p(\ell_p / M, G) \), an operator \( S \in \mathcal{K}(G, F) \) and an operator \( R \in \mathcal{K}(\mathcal{G}^\omega(U), \ell_p / M) \) such that \( f_j = S \circ T \circ g \) with \( \|S\|\|k_p(T)\|\|R\| \leq k_p(T_j) + \epsilon \). Moreover, \( R = T \circ g \) with \( \|g\|_{\infty} = \|R\| \) for some \( g \in \mathcal{H}^\omega_p(U, \ell_p / M) \) by [13] Corollary 2.11. Thus we obtain

\[
f = T \circ g = S \circ T \circ g \circ T \circ g = S \circ T \circ g \circ T \circ g = S \circ T \circ g,
\]

with

\[
\|S\|\|k_p(T)\|\|g\|_{\infty} = \|S\|\|k_p(T)\|\|R\| \leq k_p(T_j) + \epsilon = k_p^{\mathcal{H}^\omega}(f) + \epsilon.
\]

Since \( \epsilon \) was arbitrary, we deduce that \( \|S\|\|k_p(T)\|\|g\|_{\infty} \leq k_p^{\mathcal{H}^\omega}(f) \).

(ii) \( \Rightarrow \) (i): Assume that \( f = S \circ T \circ g \) is a factorization as in (ii). Since \( S \circ T \in \mathcal{K}_p(\ell_p / M, F) \) by the ideal property of \( \mathcal{K}_p \), Corollary [1.4] yields that \( f \in \mathcal{H}^\omega_p(U, F) \) with

\[
k_p^{\mathcal{H}^\omega}(f) \leq k_p(S \circ T)\|g\|_{\infty} \leq \|S\|\|k_p(T)\|\|g\|_{\infty},
\]

and taking infimum over all such factorizations of \( f \), we have \( k_p^{\mathcal{H}^\omega}(f) \leq \inf\{\|S\|\|k_p(T)\|\|g\|_{\infty}\} \). \( \square \)

1.4. Transposition. Let us recall that the *transpose* of a mapping \( f \in \mathcal{H}^\omega(U, F) \) is the bounded linear operator \( f^\ast: F^\ast \to \mathcal{H}^\omega(U) \) defined by

\[
f^\ast(y^\ast) = y^\ast \circ f \quad (y^\ast \in F^\ast).
\]

Moreover, \( \|f^\ast\| = \|f\|_{\infty} \) and \( f^\ast = J_{U^\ast} \circ (T^\ast)^\ast \), where \( J_U : \mathcal{H}^\omega(U) \to \mathcal{G}^\omega(U)^\ast \) is the isometric isomorphism defined in Theorem [12].

The Banach ideal \( \mathcal{K}_p \) is associated by duality with the ideal of quasi-\( p \)-nuclear operators. According to [17], for every \( p \in [1, \infty) \), an operator \( T \in \mathcal{L}(E, F) \) is said to be *quasi \( p \)-nuclear* if there is a sequence \( (x_n^\ast) \in \ell_p(E^\ast) \) such that

\[
\|T(x)\| \leq \left( \sum_{n=1}^{\infty} |x_n^\ast(x)|^p \right)^{1/p} \quad (x \in E).
\]

If \( QN_p(E, F) \) denotes the set formed by such operators, then \( QN_p \) is a Banach operator ideal equipped with the norm

\[
u_p^Q(T) = \inf \left\{ \left\| (x_n^\ast) \right\|_{\ell_p} : \|T(x)\| \leq \left( \sum_{n=1}^{\infty} |x_n^\ast(x)|^p \right)^{1/p} , \forall x \in E \right\}.
\]

By [10] Proposition 3.8, an operator \( T \in \mathcal{K}_p(E, F) \) if and only if its adjoint \( T^\ast \in QN_p(F^\ast, E^\ast) \). Moreover, \( k_p(T) = \nu_p^Q(T^\ast) \) by [11] Corollary 2.7]. A holomorphic version of this result can be stated as follows.
Theorem 1.7. Let \( p \in [1, \infty) \) and \( f \in \mathcal{H}^\infty(U, F) \). The following conditions are equivalent:

(i) \( f : U \to F \) has relatively \( p \)-compact range.

(ii) \( f^*: F^* \to \mathcal{H}^\infty(U) \) is a quasi \( p \)-nuclear operator.

In this case, \( k_p^{\mathcal{H}^\infty}(f) = \nu_p^2(f^*) \).

Proof. Applying Theorem [13] and Corollary 2.7 and [17] p. 32, respectively, we have

\[
f \in \mathcal{H}^\infty_p(U, F) \iff T_f \in \mathcal{K}_p(\mathcal{G}^\infty(U), F)
\]

\[
\iff (T_f)^* \in \mathcal{QN}_p(F^*, \mathcal{G}^\infty(U)^*)
\]

\[
\iff f^* = J_{U^1}^1 \circ (T_f)^* \in \mathcal{QN}_p(F^*, \mathcal{H}^\infty(U)).
\]

In this case, \( k_p^{\mathcal{H}^\infty}(f) = k_p(T_f) = \nu_p^2((T_f)^*) = \nu_p^2(f^*). \) □

Given \( p \in [1, \infty) \), let us recall (see [18]) that an operator \( T \in \mathcal{L}(E, F) \) is \( p \)-summing if there exists a constant \( C \geq 0 \) such that, regardless of the natural number \( n \) and regardless of the choice of vectors \( x_1, \ldots, x_n \) in \( E \), we have

\[
\left( \sum_{i=1}^n \|T(x_i)\|^p \right)^{\frac{1}{p}} \leq C \sup_{x^* \in B_{E^*}} \left( \sum_{i=1}^n |x^*(x_i)| \right)^{\frac{1}{p}}.
\]

The infimum of such constants \( C \) is denoted by \( \pi_p(T) \) and the linear space of all \( p \)-summing operators from \( E \) into \( F \) by \( \Pi_p(E, F) \).

The following result could be compared with [10] Proposition 3.13] stated in the linear setting.

Proposition 1.8. Let \( f \in \mathcal{H}^\infty(U, F) \) and \( g \in \mathcal{H}^\infty(U, F^*) \). Assume that \( T_f \in \Pi_p(\mathcal{G}^\infty(U), F) \) with \( p \in [1, \infty) \). Then \( f^* \circ g \in \mathcal{H}^\infty_p(U, \mathcal{H}^\infty(U)) \) with \( k_p^{\mathcal{H}^\infty}(f^* \circ g) \leq \pi_p(T_f) \|g\|_{\infty} \).

Proof. By Theorem [13] and Proposition [11], \( T_g \in \mathcal{K}(\mathcal{G}^\infty(U), F^*) \) with \( \|T_g\| = \|g\|_{\infty} \). Consequently, by [10] Proposition 3.13], the linear operator \((T_g)^* \circ T_g \in \mathcal{K}_p(\mathcal{G}^\infty(U), \mathcal{G}^\infty(U)^*) \) with \( k_p((T_g)^* \circ T_g) \leq \pi_p(T_g) \|T_g\| \). From the equality \( f^* \circ T_g = J_{U^1}^1 \circ (T_f)^* \circ T_g \), we infer that \( f^* \circ T_g \in \mathcal{K}_p(\mathcal{G}^\infty(U), \mathcal{H}^\infty(U)) \) with \( k_p(f^* \circ T_g) = k_p((T_f)^* \circ T_g) \) by the ideal property of \( \mathcal{K}_p \). Applying Theorem [13], there exists \( h \in \mathcal{H}^\infty_p(U, \mathcal{H}^\infty(U)) \) with \( k_p^{\mathcal{H}^\infty}(h) = k_p(T_h) \) such that \( f^* \circ T_g = T_h \). Hence \( f^* \circ g = h \) and thus \( f^* \circ g \in \mathcal{H}^\infty_p(U, \mathcal{H}^\infty(U)) \) with

\[
k_p^{\mathcal{H}^\infty}(f^* \circ g) = k_p(T_h) = k_p((T_f)^* \circ T_g) \leq \pi_p(T_f) \|T_g\| = \pi_p(T_f) \|g\|_{\infty}.
\]

\( p \)-Compact operators were characterized as those operators whose adjoints factor through a subspace of \( \ell_p \) [20] Theorem 3.2]. We now obtain a similar factorization for the transpose of a holomorphic mapping with relatively \( p \)-compact range (compare also to [10] Proposition 3.10]).

Corollary 1.9. Let \( p \in [1, \infty) \) and \( f \in \mathcal{H}^\infty(U, F) \). The following conditions are equivalent:

(i) \( f : U \to F \) has relatively \( p \)-compact range.

(ii) There exist a closed subspace \( M \subseteq \ell_p \) and operators \( R \in QN_p(F^*, M), S \in \mathcal{L}(M, \mathcal{H}^\infty(U)) \) such that \( f^* = S \circ R \).

Proof. (i) ⇒ (ii): If \( f \in \mathcal{H}^\infty_p(U, F) \), then \( T_f \in \mathcal{K}_p(\mathcal{G}^\infty(U), F) \) by Theorem [13]. By [10] Proposition 3.10], there exist a closed subspace \( M \subseteq \ell_p \) and operators \( R \in QN_p(F^*, M) \) and \( S_0 \in \mathcal{L}(M, \mathcal{G}^\infty(U)^*) \) such that \( (T_f)^* = S_0 \circ R \). Taking \( S = J_{U^1}^1 \circ S_0 \in \mathcal{L}(M, \mathcal{H}^\infty(U)) \), we have \( f^* = S \circ R \).

(ii) ⇒ (i): Assume that \( f^* = S \circ R \) being \( S \) and \( R \) as in the statement. It follows that \( (T_f)^* = J_{U^1}^1 \circ f^* = J_{U^1}^1 \circ S \circ R \), and thus \( T_f \in \mathcal{K}_p(\mathcal{G}^\infty(U), F) \) by [10] Proposition 3.10]. Hence \( f \in \mathcal{H}^\infty_p(U, F) \) by Theorem [13]. □
1.5. Inclusions. We will study the inclusion relations of holomorphic mappings with relatively \( p \)-compact range between them and with other classes of bounded holomorphic mappings.

Our first result follows immediately by applying Theorem [1.3] and the fact stated in [20] Proposition 4.3] that \( \mathcal{K}_p \subseteq \mathcal{K}_q \) whenever \( 1 \leq p \leq q \leq \infty \).

**Corollary 1.10.** If \( 1 \leq p \leq q < \infty \) and \( f \in \mathcal{H}_{\mathcal{K}_p}^p(U, F) \), then \( f \in \mathcal{H}_{\mathcal{K}_q}^q(U, F) \) and \( k_q^{\mathcal{H}^p}(f) \leq k_p^{\mathcal{H}^p}(f) \). \( \square \)

Let us recall that a mapping \( f \in \mathcal{H}^\infty(U, F) \) has finite dimensional range if the linear hull of its range is a finite dimensional subspace of \( F \). We denote by \( \mathcal{H}_{\mathcal{F}}^\infty(U, F) \) the set of all finite-rank bounded holomorphic mappings from \( U \) to \( F \). In the light of Theorem [1.5], it is clear that \( \mathcal{H}_{\mathcal{F}}^\infty(U, F) \) is a linear subspace of \( \mathcal{H}_{\mathcal{K}_p}^\infty(U, F) \).

In similarity with the linear case, it seems natural to introduce the following class of holomorphic mappings.

**Definition 1.11.** Let \( p \in [1, \infty) \). A mapping \( f \in \mathcal{H}^\infty(U, F) \) is said to be \( p \)-approximable if there exists a sequence \((f_n)\) in \( \mathcal{H}_{\mathcal{F}}^\infty(U, F) \) such that \( k_p^{\mathcal{H}^\infty}(f_n - f) \to 0 \) as \( n \to \infty \). We denote by \( \mathcal{H}_{\mathcal{F}}^\infty(U, F) \) the space of all \( p \)-approximable holomorphic mappings from \( U \) to \( F \).

**Proposition 1.12.** For \( p \in [1, \infty) \), every \( p \)-approximable holomorphic mapping from \( U \) to \( F \) has relatively \( p \)-compact range.

**Proof.** Let \( f \in \mathcal{H}_{\mathcal{F}}^\infty(U, F) \). Hence there is a sequence \((f_n)\) in \( \mathcal{H}_{\mathcal{F}}^\infty(U, F) \) such that \( k_p^{\mathcal{H}^\infty}(f_n - f) \to 0 \) as \( n \to \infty \). Since \( T_{f_n} \in \mathcal{F}(\mathcal{G}^\infty(U), F) \) by [15] Proposition 3.1, \( \mathcal{F}(\mathcal{G}^\infty(U), F) \subseteq \mathcal{K}_p(\mathcal{G}^\infty(U), F) \) by [20] Theorem 4.2 and \( k_p(T_{f_n} - T_f) = k_p(T_{f_n - f}) = k_p^{\mathcal{H}^\infty}(f_n - f) \) for all \( n \in \mathbb{N} \) by Theorems [1.2] and [1.3]. We deduce that \( T_f \in \mathcal{K}_p(\mathcal{G}^\infty(U), F) \) by [20] Theorem 4.2, and so \( f \in \mathcal{H}_{\mathcal{F}}^\infty(U, F) \) by Theorem [1.3]. \( \square \)

Given \( p \in [1, \infty) \), \( \ell_p^{weak}(E) \) denotes the Banach space of all weakly \( p \)-summable sequences \((x_n)\) in \( E \), endowed with the norm

\[
\|(x_n)\|_p^{weak} = \sup \left\{ \left( \sum_{n=1}^{\infty} |f(x_n)|^p \right)^{1/p} : f \in B_E \right\}.
\]

Let us recall (see [16]) that an operator \( T \in \mathcal{L}(E, F) \) is said to be right \( p \)-nuclear if there are sequences \((x_n^*) \in \ell_p^{weak}(E^*)\) and \((y_n) \in \ell_p(F)\) such that

\[
T(x) = \sum_{n=1}^{\infty} x_n^*(x)y_n \quad (x \in E),
\]

and the series converges in \( \mathcal{L}(E, F) \). The set of such operators, denoted \( \mathcal{N}^p(E, F) \), is a Banach space with the norm

\[
\nu^p(T) = \inf \left\{ \|(x_n^*)\|_p^{weak} \|y_n\|_p : (x_n) \in \ell_p^{weak}(E^*) \right\},
\]

where the infimum is taken over all representations of \( T \) as above.

A holomorphic variant of this class of operators can be introduced as follows.

**Definition 1.13.** Given \( p \in [1, \infty) \), a holomorphic mapping \( f : U \to F \) is said to be right \( p \)-nuclear if there exist sequences \((g_n)\) in \( \ell_p^{weak}(\mathcal{H}^\infty(U)) \) and \((y_n) \in \ell_p(F)\) such that \( f = \sum_{n=1}^{\infty} g_n \cdot y_n \) in \( (\mathcal{H}^\infty(U, F), \| \cdot \|_\infty) \). We set

\[
\nu^p(\mathcal{H}^\infty)(f) = \inf \left\{ \|(g_n)\|_p^{weak} \|y_n\|_p : \sum_{n=1}^{\infty} g_n \cdot y_n \right\}.
\]
with the infimum taken over all right $p$-nuclear holomorphic representations of $f$ as above. Let $\mathcal{H}_{N^p}^\infty(U, F)$ denote the set of all right $p$-nuclear holomorphic mappings from $U$ into $F$.

We now establish the relationships of a right $p$-nuclear holomorphic mapping $f : U \to F$ with its linearisation $T_f : \mathcal{G}^\infty(U) \to F$.

**Theorem 1.14.** Let $p \in [1, \infty)$ and $f \in \mathcal{H}^\infty(U, F)$. The following conditions are equivalent:

(i) $f : U \to F$ is right $p$-nuclear;

(ii) $T_f : \mathcal{G}^\infty(U) \to F$ is a right $p$-nuclear operator.

In this case, $\nu^{\mathcal{H}^\infty}(f) = \nu^p(T_f)$. Furthermore, the mapping $f \mapsto T_f$ is an isometric isomorphism from $(\mathcal{H}_{N^p}^\infty(U, F), \nu^{\mathcal{H}^\infty})$ onto $(N^p(\mathcal{G}^\infty(U), F), \nu^p)$.

**Proof.** (i) $\Rightarrow$ (ii): Assume that $f \in \mathcal{H}_{N^p}^\infty(U, F)$ and let $\sum_{n \geq 1} g_n \cdot y_n$ be a right $p$-nuclear holomorphic representation of $f$. By Theorem 1.2, there is a unique operator $T_f \in \mathcal{L}(\mathcal{G}^\infty(U, F))$ such that $T_f \circ g_U = f$. Similarly, for each $n \in \mathbb{N}$, there is a functional $T_{g_n} \in \mathcal{G}^\infty(U)^*$ with $||T_{g_n}|| = ||g_n||_\infty$ such that $T_{g_n} \circ g_U = g_n$. Notice that $\sum_{n=1}^{\infty} T_{g_n} \cdot y_n \in \mathcal{L}(\mathcal{G}^\infty(U), F)$ since

$$\sum_{k=1}^{m} ||T_{g_k} \cdot y_k|| = \sum_{k=1}^{m} ||T_{g_k}|| ||y_k|| = \sum_{k=1}^{m} ||g_k||_\infty ||y_k|| \leq ||(g_n)||_{p^\text{weak}} ||(y_n)||_p$$

for all $m \in \mathbb{N}$. We can write

$$f = \sum_{n=1}^{\infty} g_n \cdot y_n = \sum_{n=1}^{\infty} (T_{g_n} \circ g_U) \cdot y_n = \left(\sum_{n=1}^{\infty} T_{g_n} \cdot y_n\right) \circ g_U$$

in $(\mathcal{H}^\infty(U, F), ||\cdot||_\infty)$. Hence $T_f = \sum_{n=1}^{\infty} T_{g_n} \cdot y_n$ by Theorem 1.2, where $(T_{g_n}) \in \ell_{p^\text{weak}}(\mathcal{G}^\infty(U)^*)$ with $||T_{g_n}||_{p^\text{weak}} \leq ||(g_n)||_{p^\text{weak}}$. Therefore $T_f \in N^p(\mathcal{G}^\infty(U), F)$ with $\nu^p(T_f) \leq ||(g_n)||_{p^\text{weak}} ||(y_n)||_p$. Taking infimum over all right $p$-nuclear holomorphic representation of $f$, we deduce that $\nu^p(T_f) \leq \nu^{\mathcal{H}^\infty}(f)$.

(ii) $\Rightarrow$ (i): Suppose that $T_f \in N^p(\mathcal{G}^\infty(U), F)$ and let $\sum_{n \geq 1} \phi_n \cdot y_n$ be a right $p$-nuclear representation of $f$. By Theorem 1.2, for each $n \in \mathbb{N}$, there is a $g_n \in \mathcal{H}^\infty(U)$ such that $J_U(g_n) = \phi_n$ with $||g_n||_\infty = ||\phi_n||$. We have

$$\left\|f - \sum_{k=1}^{n} g_k \cdot y_k\right\| \leq \left\|f(x) - \sum_{k=1}^{n} g_k(x) y_k\right\| = \left\|T_f(g_U(x)) - \sum_{k=1}^{n} J_U(g_k)(g_U(x)) y_k\right\|$$

$$= \left\|T_f - \sum_{k=1}^{n} \phi_k \cdot y_k\right\| (g_U(x)) \leq \left\|T_f - \sum_{k=1}^{n} \phi_k \cdot y_k\right\| ||g_U(x)||$$

for all $x \in U$ and $n \in \mathbb{N}$. Taking supremum over all $x \in U$, we obtain

$$\left\|f - \sum_{k=1}^{n} g_k \cdot y_k\right\|_\infty \leq \left\|T_f - \sum_{k=1}^{n} \phi_k \cdot y_k\right\|$$

for all $n \in \mathbb{N}$. Hence $f = \sum_{n=1}^{\infty} g_n \cdot y_n$ in $(\mathcal{H}^\infty(U, F), ||\cdot||_\infty)$, where $(g_n) \in \ell_{p^\text{weak}}(\mathcal{H}^\infty(U))$ with $||g_n||_{p^\text{weak}} \leq ||(\phi_n)||_{p^\text{weak}}$. So $f \in \mathcal{H}_{N^p}^\infty(U, F)$ with $\nu^{\mathcal{H}^\infty}(f) \leq ||(\phi_n)||_{p^\text{weak}} ||(y_n)||_p$, and this implies that $\nu^{\mathcal{H}^\infty}(f) \leq \nu^p(T_f)$.

The last assertion in the statement follows easily from what was proved above and from Theorem 1.2. 

$\square$
Combining Theorem 1.14 firstly with [4, Theorem 3.2], and secondly with [2, Corollary 2.5], we derive the following two results.

**Corollary 1.15.** Let \( p \in [1, \infty) \) and \( f \in \mathcal{H}^\infty(U, F) \). The following conditions are equivalent:

(i) \( f : U \to F \) is right \( p \)-nuclear.

(ii) \( f = T \circ g \) for some complex Banach space \( G \), \( g \in \mathcal{H}^\infty(U, G) \) and \( T \in N^p(G, F) \).

In this case, we have

\[
\nu^p\mathcal{H}^\infty(f) = \|f\|_{\mathcal{H}^\infty} := \inf\{\nu^p(T) \|g\|_\infty\},
\]

where the infimum is taken over all factorizations of \( f \) as in (ii), and this infimum is attained at \( T_f \circ g_U \).

Furthermore, the mapping \( f \mapsto T_f \) is an isometric isomorphism from \((N^p \circ \mathcal{H}^\infty(U, F), \|\cdot\|_{N^p \circ \mathcal{H}^\infty}) \) onto \((N^p(G^\infty(U), F), \nu^p)\).

**Corollary 1.16.** For each \( p \in [1, \infty) \), \((\mathcal{H}^\infty_{N^p}, \nu^p\mathcal{H}^\infty)\) is a Banach bounded-holomorphic ideal.

The following relation is readily obtained.

**Corollary 1.17.** Let \( p \in [1, \infty) \) and \( f \in \mathcal{H}^\infty_{N^p}(U, F) \). Then \( f \in \mathcal{H}^\infty(U, F) \) with \( k^\mathcal{H}^\infty_f \leq \nu^p\mathcal{H}^\infty(f) \).

**Proof.** By Proposition 1.14 we have \( T_f \in N^p(G^\infty(U), F) \) with \( \nu^p(T_f) = \nu^p\mathcal{H}^\infty(f) \). It follows that \( T_f \in \mathcal{K}_p(G^\infty(U), F) \) with \( k^\mathcal{H}^\infty_f \leq \nu^p(T_f) \) (see [10, p. 295]). Hence \( f \in \mathcal{H}^\infty(U, F) \) with \( k^\mathcal{H}^\infty_f \leq \nu^p\mathcal{H}^\infty(f) \) by Theorem 1.3.

Given Banach spaces \( E, F, G \), let us recall that a normed operator ideal \( I \) is surjective if for every surjection \( Q \in \mathcal{L}(G, E) \) and every \( T \in \mathcal{L}(E, F) \), it follows from \( T \circ Q \in I(G, F) \) that \( T \in I(E, F) \) with \( \|T\|_I = \|T \circ Q\|_F \). The smallest surjective ideal which contains \( I \), denoted by \( I^{\text{sur}} \), is called the **surjective hull of** \( I \).

We now introduce the analogue concept in the holomorphic setting.

**Definition 1.18.** The **surjective hull of** a bounded-holomorphic ideal \( I^{\mathcal{H}^\infty} \) is the smallest surjective ideal which contains \( I^{\mathcal{H}^\infty} \), and it is denoted by \((I^{\mathcal{H}^\infty})^{\text{sur}}\).

We have seen that \( \mathcal{H}^\infty_{N^p} \) is a surjective bounded-holomorphic ideal which contains \( \mathcal{H}^\infty_{\mathcal{K}_p} \), and therefore \((\mathcal{H}^\infty_{N^p})^{\text{sur}} \subseteq \mathcal{H}^\infty_{\mathcal{K}_p} \), but we do not know if both sets are equal as it happens in the linear case (see [10, Proposition 3.11]).

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