TESTABILITY OF TYPE I SEESAW AT THE CERN LHC: REVEALING THE EXISTENCE OF THE $B - L$ SYMMETRY

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We study the possibility to test the Type I seesaw mechanism for neutrino masses at the CERN Large Hadron Collider. The inclusion of three generations of right-handed neutrinos ($N_i$) provides an attractive option of gauging the $B - L$ accidental symmetry in the Standard Model (as well as an extended symmetry $X = Y - 5(B - L)/4$). The production mechanisms for the right-handed neutrinos through the $Z'$ gauge boson in the $U(1)_{B - L}$ and $U(1)_X$ extensions of the Standard Model are studied. We discuss the flavor combinations of the charged leptons from the decays of $N_i$ in the $\Delta L = 2$ channels. We find that the clean channels with dilepton plus jets and possible secondary vertices of the $N$ decay could provide conclusive signals at the LHC in connection with the hierarchical pattern of the light neutrino masses and mixing properties within the Type I seesaw mechanism.

I. INTRODUCTION

The small but non-zero neutrino masses lead to a deep conjecture: Majorana nature of the neutrino masses may hold the key for a fundamentally different mass generation mechanism, although Dirac masses can be generated via the Higgs mechanism by introducing right-handed neutrinos with arbitrarily small Yukawa couplings. There are three simple scenarios where one can generate Majorana masses of the neutrinos with renormalizable operators at tree level, and in agreement with the observations, the Type I [1], Type II [2], and Type III [3] seesaw mechanisms. See also Refs. [4] and [5] for the simplest neutrino mass generation mechanisms using radiative corrections.

Perhaps the simplest and best-studied mechanism for neutrino masses is the Type I seesaw, where one introduces at least two right-handed neutrinos ($N$). Adding in the corresponding large Majorana mass terms ($M$), one results in at least two light Majorana neutrinos with masses given as $m_D^2/M$. It is important to

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mention that the inclusion of three right-handed neutrinos also provides an anomaly-free formulation for a
gauged $U(1)_{B-L}$\[6\].

The non-ambiguous test of the Majorana nature of the neutrinos, and thus a possible test of the seesaw
mechanism, will be the observation of the lepton number violation processes. The neutrinoless double beta
decay is also a crucial test and one of the most sensitive probes. Since the CERN Large Hadron Collider
(LHC) is going to lead us to a new energy frontier, searching for the heavy Majorana neutrinos at the LHC
appears to be very appealing \[7, 8\]. However, due to the rather small mixing between the heavy neutrinos
and the Standard Model (SM) leptons in a minimal Type I scheme, typically of the order $|V_{\ell N}|^2 \sim m_\nu/M_N$, the predicted effects of lepton number violation are unlikely to be observable. On the other hand, if there
are other particles beyond the SM that can mediate new interactions between them, the effects may be
significantly enhanced. For instance, with the new gauge interaction $U(1)_{B-L}$, the gauge boson $Z_{B-L}$ can
be produced copiously at the LHC via its gauge interactions with the quarks. Its subsequent decay to a pair
of heavy Majorana neutrinos may lead to a large sample of events without involving the small mixing angle
suppression of $N$ \[9, 10\]. The $\Delta L = 2$ signals will directly test its Majorana nature; and the lepton flavor
combination could probe the properties of the light neutrino mass spectrum and mixing pattern.

In this paper, we investigate the possibility to test the Type I seesaw mechanism at the LHC in the context
of two simple extensions of the Standard Model where one has an extra Abelian gauge symmetry. We focus
our attention on scenarios with a $U(1)_{B-L}$ or $U(1)_X$ ($X = Y - 5(B - L)/4$), where $B$, $L$ and $Y$ stand
for Baryon number, Lepton number and weak hypercharge, respectively. In order to cancel the anomalies,
one just need to introduce three right-handed neutrinos, which are the source for the Majorana masses. In
both scenarios one has a new neutral gauge boson, $Z'$, which couples to the right-handed neutrinos. Then,
one can expect large number events for the lepton number violating events due to the production and decays
of the TeV Majorana neutrinos. The predictions of the heavy neutrino decays in each neutrino spectrum,
Normal Hierarchy (NH), Inverted Hierarchy (IH) or Quasi-Degenerate (QD), are investigated in great detail.
We find encouraging results for the LHC signatures to learn about the light neutrino properties.

This work is organized as follows: In Section II we discuss the constraints on the mass and mixing
parameters in the Type I seesaw mechanism from the current neutrino oscillation data. The predictions for
the decays of the heavy neutrinos in the different neutrino spectra are presented in Section III. In Section
IV we discuss the possibility to test Type I seesaw at the LHC through the same-sign dilepton channels.
We summarize our findings in Section V. The mixing between light and heavy neutrinos are discussed in
Appendix A. We provide the explicit expressions for these mixings in Appendix B. The minimal extensions
of the Standard Model to $U(1)_{B-L}$ and $U(1)_X$ are discussed in Appendix C.
II. TYPE I SEESAW MECHANISM AND PARAMETER CONSTRAINTS

In the case of the Type I seesaw mechanism for neutrino masses one introduces at least two SM singlets, right-handed neutrinos, $\nu_R \sim (1, 1, 0)$, in order to generate two non-zero neutrino masses. In this case the relevant Yukawa interaction and the Majorana mass term are given by

$$- \mathcal{L}_I^I = Y_D^I \bar{L}_L \nu_R + \frac{M_N}{2} v_R^T C v_R + \text{h.c.}. \quad (1)$$

Here $\tilde{H} = i\sigma_2 H^*$ and the lepton number is broken in two units due to the presence of both terms. Now, integrating out the right-handed neutrinos one finds that the mass matrix for the light neutrinos is given by

$$M_\nu = m_D M_N^{-1} m_D^T, \quad (2)$$

where $m_D = Y_D^D v_0 / \sqrt{2}$ is the Dirac mass term and $v_0$ is the Higgs vacuum expectation value. Therefore, in this framework one could understand the smallness of neutrino masses, since the mass scale $M_N$ in the above equation could be large, $M_N \gg Y_D^D v_0$. This is the so-called canonical Type I seesaw mechanism \cite{1}. The mass matrix for neutrinos is diagonalized by unitary rotations as detailed in Appendix A. The three light neutrino masses can be expressed in the following way

$$m = V_{PMNS}^\dagger M_\nu V_{PMNS}, \quad (3)$$

where $m = \text{diag}(m_1, m_2, m_3)$ and $V_{PMNS}$ can be taken as the leptonic mixing matrix for the three generation of light neutrinos \cite{11} without the loss of generality.\footnote{The $3 \times 3$ rotational matrix is not exactly unitary when there are extra Majorana neutrinos, but it is a good approximation to equal it to the traditional $V_{PMNS}$, see the formalism in the appendix.} Working in the basis where the heavy neutrino mass matrix is diagonal and using the Casas-Ibarra parametrization \cite{12} one can write $m_D$ satisfying Eq. (2) as

$$m_D = V_{PMNS} m^{1/2} \Omega M^{1/2}, \quad (4)$$

where $M = \text{diag}(M_1, M_2, M_3)$ for heavy neutrino masses, and $\Omega$ is a complex matrix which satisfies the orthogonality condition $\Omega^T \Omega = 1$. It is shown in Appendix A that using the seesaw formula and the relation between the leptonic mixing one can find a formal solution for the mixing between the SM charged leptons ($\ell = e, \mu, \tau$) and heavy neutrinos ($N = 1, 2, 3$):

$$V_{\ell N} = V_{PMNS} m^{1/2} \Omega M^{-1/2}. \quad (5)$$

Therefore, for a given form of $\Omega$, one can establish the connection between the heavy neutrino decays and the properties of the light neutrinos. The impact of the existence of the $\Omega$ matrix on the decays of heavy
neutrinos has not been studied before in collider phenomenology. Unfortunately, since the explicit form of this matrix is unknown one cannot predict the decay pattern of the heavy neutrinos with respect to the spectrum for light neutrinos. We will present a few well-motivated typical cases where one can hope to see the connection in each spectrum for light neutrinos. It is important, however, to realize that an underlying theory would pick only one specific form of $\Omega$. This (yet unknown) form would have definite prediction for the $N$ decay patterns, through which the underlying theory could be revealed.

### A. Constraints on the Physical Parameters

#### 1. Neutrino Masses and Mixings

In order to understand the constraints coming from neutrino physics let us discuss the relation between the neutrino masses and mixing. The leptonic mixing matrix is given by

$$V_{P M N S} = \begin{pmatrix}
c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\
-c_{12}s_{13}s_{23}e^{i\delta} - c_{23}s_{12} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\
s_{12}s_{23} - e^{i\delta}c_{12}c_{23}s_{13} & -c_{23}s_{12}s_{13}e^{i\delta} - c_{12}s_{23} & c_{13}c_{23}
\end{pmatrix} \times \text{diag}(e^{i\Phi_1/2}, 1, e^{i\Phi_2/2}) \quad (6)$$

where $s_{ij} = \sin \theta_{ij}$, $c_{ij} = \cos \theta_{ij}$, $0 \leq \theta_{ij} \leq \pi/2$ and $0 \leq \delta \leq 2\pi$. The phase $\delta$ is the Dirac CP phase, and $\Phi_i$ are the Majorana phases. The experimental constraints on the neutrino masses and mixing parameters, at 2$\sigma$ level [13], are

$$7.25 \times 10^{-5} \text{eV}^2 < \Delta m_{21}^2 < 8.11 \times 10^{-5} \text{eV}^2, \quad (7)$$

$$2.18 \times 10^{-3} \text{eV}^2 < |\Delta m_{31}^2| < 2.64 \times 10^{-3} \text{eV}^2, \quad (8)$$

$$0.27 < \sin^2 \theta_{12} < 0.35, \quad (9)$$

$$0.39 < \sin^2 \theta_{23} < 0.63, \quad (10)$$

$$\sin^2 \theta_{13} < 0.040, \quad (11)$$

and $\sum_i m_i < 1.2 \text{ eV}$. For a complete discussion of these constraints see reference [14]. Following the convention, we denote the case $\Delta m_{31}^2 > 0$ as the normal hierarchy (NH); $\Delta m_{31}^2 < 0$ the inverted hierarchy (IH), and the quasi-degenerate (QD) spectrum where the lightest neutrino mass is larger than $5 \times 10^{-2}$ eV. Using the above experimental constraints, one can expect to explore the allowed values for the $V_{\ell N}$ couplings and the heavy masses. From Eq. (5), we can obtain the general expressions of $\sum_N(V_{\ell N}^*)^2$ that are collected in Appendix B.
FIG. 1: $\sum_N |V_{\ell N}|^2 M_N/100 \text{ GeV}$ versus the lightest neutrino mass for NH (left) and IH (right) in Case I (degenerate $N$), assuming vanishing Majorana phases.

2. Case I: Degenerate Heavy Neutrinos

We firstly study the simplest case where the three heavy neutrinos are degenerate. This is a highly motivated scenario since it is strongly favored to generate successful resonant lepto-genesis \cite{15, 16} at the low scale. Using Eq. (A15) and assuming degenerate heavy neutrinos we obtain the relation

$$M \sum_{N=1,2,3} (V_{\ell N}^*)^2 = (V_{PMNS}^\dagger m V_{PMNS}^\dagger)_{\ell \ell} \equiv (M_{\nu})_{\ell \ell}, \ (\ell = e, \mu, \tau). \quad (12)$$

We see that one can obtain simple relations for the heavy neutrino mixings and masses in terms of the light neutrino mass matrix independent of the unknown matrix $\Omega$, which in turn is given by the parameters from the neutrino oscillation data. One can thus predict the decays of the heavy neutrinos in each spectrum for light neutrinos. Note that in this degenerate scenario, we are unable to convert the constraints of Eq. (12) to predict $\sum_N |V_{\ell N}|^2$ in general. We can predict the decays of heavy neutrinos in terms of the other oscillation parameters only when all phases vanish since in this case the modulo square of the mixings (which govern the decay rate) are equal to the square of mixings (the left-handed side of Eq. (12)).

In Ref. [17], we have shown that using the experimental constraints on the neutrino mass parameters the
elements of the neutrino mass matrix has the following properties:

\[
M_{\nu}^{ee} \ll M_{\nu}^{\mu\mu}, M_{\nu}^{\tau\tau} \quad \text{for NH,}
\]

\[
M_{\nu}^{ee} > M_{\nu}^{\mu\mu}, M_{\nu}^{\tau\tau} \quad \text{for IH,}
\] (13)

\[
M_{\nu}^{ee} \approx M_{\nu}^{\mu\mu} \approx M_{\nu}^{\tau\tau} \quad \text{for QD.}
\] (14)

Following the same approach, we plot the allowed values for the normalized couplings of each lepton flavor in this scenario in Fig. 1 as a function of the lightest neutrino mass in each spectrum, the normal hierarchy (left panel) and the inverted hierarchy (right panel), assuming vanishing Majorana phases. We see two distinctive regions in terms of the lightest neutrino mass as expected. In the case \(m_{1(3)} < 5 \times 10^{-2} \text{ eV}\), we see the characteristic features

\[
\sum_{N} |V_{\ell N}|^2 \ll \sum_{N} |V_{\mu N}|^2, \sum_{N} |V_{\tau N}|^2 \quad \text{for NH,}
\]

\[
\sum_{N} |V_{\ell N}|^2 > \sum_{N} |V_{\mu N}|^2, \sum_{N} |V_{\tau N}|^2 \quad \text{for IH.}
\]

On the other hand, for \(m_{1(3)} > 5 \times 10^{-2} \text{ eV}\), the light neutrino masses enter the QD spectrum that leads to

\[
\sum_{N} |V_{\ell N}|^2 \approx \sum_{N} |V_{\mu N}|^2 \approx \sum_{N} |V_{\tau N}|^2.
\]

Under this mass degenerate assumption, the mixing between the heavy neutrinos and the SM charged leptons simply reflect the features of the light neutrino mass matrix in the flavor basis, as seen in Eq. (12). This is an important model-prediction. It is important to emphasize that the results shown in Fig. 1 may be used to learn about the neutrino spectrum.

### 3. Case II: Non-degenerate Heavy Neutrinos

If we relax the assumption that heavy Majorana neutrinos are nearly degenerate in mass, then the complication due to the unknown matrix \(\Omega\) arises. The explicit parameterization of \(\Omega\) is presented in Appendix A, and the general expression for the relations among the parameters are given in Appendix B.

For the purpose of illustration, let us take \(\Omega\) to be a real matrix. We could gain a general sense for the mixing parameters by varying the matrix elements of \(\Omega\) in the range of \(-1 \leq w_{ij} \leq 1\). We show \(|V_{e1}|^2 M_1/100 \text{ GeV}\) in this case in Fig. 2. The predictions of \(|V_{e2}|^2 M_2/100 \text{ GeV}\) and \(|V_{e3}|^2 M_3/100 \text{ GeV}\) are almost the same. As one can see that qualitative features for both cases of NH and IH closely resemble that in Fig. 1. This is quite encouraging since the random selection of the model parameters do not seem to totally wash out the predicted features. To further explore
FIG. 2: $|V_{e1}|^2 M_1 / 100 \text{ GeV}$ versus the lightest neutrino mass for NH (left) and IH (right) with $\Omega = R_{12} R_{13} R_{23}$ and random matrix elements $-1 \leq w_{ij} \leq 1$, assuming vanishing Majorana phases.

the model implications, we must choose a specific form of the $\Omega$ matrix, which should correspond to a particular theoretical incarnation in the right-handed neutrino sector. However, a large Majorana phase could alter the predictions \cite{17} in general. We will check on this point in the next section.

**Case IIa: $\Omega = I$**

In this simple scenario, we easily obtain transparent relations for the $N_1$ mixings:

$$|V_{e1}|^2 M_1 = m_1 c_{12} c_{13}^2 \approx m_1 c_{12}^2, \quad (15)$$

$$|V_{\mu 1}|^2 M_1 = m_1 |s_{12} c_{23} + c_{12} s_{13} s_{23} e^{i\delta}|^2 \approx m_1 s_{12}^2 c_{23}^2, \quad (16)$$

$$|V_{\tau 1}|^2 M_1 = m_1 |s_{12} s_{23} - c_{12} s_{13} c_{23} e^{i\delta}|^2 \approx m_1 s_{12}^2 s_{23}^2, \quad (17)$$

and therefore $|V_{e1}|^2 > |V_{\mu 1}|^2, |V_{\tau 1}|^2$. In the case of $N_2$ mixing:

$$|V_{e2}|^2 M_2 = m_2 c_{13}^2 s_{12}^2 \approx m_2 s_{12}^2, \quad (18)$$

$$|V_{\mu 2}|^2 M_2 = m_2 |c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta}|^2 \approx m_2 c_{12}^2 c_{23}^2, \quad (19)$$

$$|V_{\tau 2}|^2 M_2 = m_2 |s_{12} s_{13} c_{23} e^{i\delta} + c_{12} s_{23}|^2 \approx m_2 c_{12}^2 s_{23}^2, \quad (20)$$
and \(|V_{e2}|^2 \approx |V_{\mu 2}|^2 \approx |V_{\tau 2}|^2\). As for the \(N_3\) mixing,

\[
|V_{e3}|^2 M_3 = m_3 s_{13}^2 \approx 0, \tag{21}
\]

\[
|V_{\mu 3}|^2 M_3 = m_3 c_{13} s_{23}^2 \approx m_3 s_{23}^2, \tag{22}
\]

\[
|V_{\tau 3}|^2 M_3 = m_3 c_{13} c_{23}^2 \approx m_3 c_{23}^2. \tag{23}
\]

and one can see \(|V_{\mu 3}|^2, |V_{\tau 3}|^2 > |V_{e3}|^2\). These features are shown in Fig. 3.

A few remarks are in order. First of all, with this choice of a diagonal matrix \(\Omega\), the mixing angle squared \(|V_{\ell i}|^2\) for \(N_i\) is always proportional to the corresponding light neutrino mass \(m_i\). Consequently, the relative fractions of the mixing to different lepton flavors are universal for both NH and IH. Secondly, the Majorana phases do not appear in \(|V_{\ell N}|^2\) due to the special structure of \(\Omega\). Thirdly, as seen in Fig. 3, the relative strength of the mixing to different lepton flavors for each \(N_i\) closely follow that for the light neutrino mass eigenstates. In fact, very much like the light neutrino mass eigenstate labeling, this should be the defining feature to label \(N_1, N_2\) and \(N_3\), if we do not like the less illuminating ordering \(M_1 < M_2 < M_3\).

**Case IIb: \(\Omega = I_{\text{off}}\)**

We choose to study yet another simple, but different form of the matrix, namely, with \(\Omega\) as an off-diagonal unity matrix. As can be shown explicitly and supported by Fig. 4, the mixing features of \(|V_{\ell 1}|^2\) and \(|V_{\ell 3}|^2\) switch places with each other in both NH and IH, while \(|V_{\ell 2}|^2\) remain the same as in \(\Omega = I\) case. If we recall the convention for the \(N_i\) labeling, this case is indistinguishable from Case IIa. In this case \(|V_{\ell N}|^2\) are also independent of Majorana phases. This similarity can be generalized to a matrix of \(\Omega\) which has only unity as entries. We would expect that the real situation could be a well-defined superposition of the three vertical panels as long as \(\Omega\) is real.

### III. Heavy Neutrino Decays and Light Neutrino Spectra

The leading decay channels for the heavy neutrinos include \(N_i \to e_j^\pm W^\mp\), \(N_i \to \nu_j Z\) and \(N_i \to \nu_j h(H)\). The amplitude for the two first channels are proportional to the mixing between the leptons and heavy neutrinos given in Eq. (5), while the last one is proportional to the Dirac-like Yukawa terms given in Eq. (4).
FIG. 3: $|V_{ei}|^2 M_i/100$ GeV, $i = 1, 2, 3$ versus the lightest neutrino mass for NH (left panels) and IH (right panels) in Case IIa ($\Omega = I$).
FIG. 4: $|V_{ei}|^2 M_i/100$ GeV, $i = 1, 2, 3$ versus the lightest neutrino mass for NH (left panels) and IH (right panels) in Case IIb ($\Omega = I_{\text{eff}}$).
A. Decay Modes of Heavy Majorana Neutrinos with mass: $M_i > M_W$

The partial decay widths of the heavy Majorana neutrinos $N_i$ are given by

$$\Gamma_{\ell W_L} \equiv \Gamma(N_i \to \ell^- W_L^+ = \Gamma(N_i \to \ell^+ W_L^-) = \frac{g^2}{64\pi M_W^2} |V_{\ell i}|^2 M_i^3 (1 - \mu_{iW})^2, \quad (24)$$

$$\Gamma_{\ell W_T} \equiv \Gamma(N_i \to \ell^- W_T^+) = \frac{g^2}{32\pi} |V_{\ell i}|^2 M_i (1 - \mu_{iW})^2, \quad (25)$$

$$\Gamma_{\nu_i Z_L} \equiv \Gamma(N_i \to \nu_i Z_L) = \frac{g^2}{64\pi M_W^2} |V_{\ell i}|^2 M_i^3 (1 - \mu_{iZ})^2, \quad (26)$$

$$\Gamma_{\nu_i Z_T} \equiv \Gamma(N_i \to \nu_i Z_T) = \frac{g^2}{32\pi c_W^2} |V_{\ell i}|^2 M_i (1 - \mu_{iZ})^2, \quad (27)$$

where $\mu_{ij} = M_j^2/M_i^2$. If $N_i$ is heavier than the Higgs bosons $h$ and $H$ (see Appendix C for the properties of the Higgs bosons in the $B - L$ extension of the SM), one has the additional channels

$$\Gamma_{\nu_i h} \equiv \Gamma(N_i \to \nu_i h) = \frac{g^2}{64\pi M_W^2} |V_{\ell i}|^2 M_i^3 (1 - \mu_{ih})^2 \cos^2 \theta_0, \quad (28)$$

$$\Gamma_{\nu_i H} \equiv \Gamma(N_i \to \nu_i H) = \frac{g^2}{64\pi M_W^2} |V_{\ell i}|^2 M_i^3 (1 - \mu_{iH})^2 \sin^2 \theta_0. \quad (29)$$

Therefore, the total width for $N_i$ is given by

$$\Gamma_{N_i} = \sum_{\ell} \left( 2\Gamma_{\ell W_L} + 2\Gamma_{\ell W_T} + \Gamma_{\nu_i Z_L} + \Gamma_{\nu_i Z_T} + \Gamma_{\nu_i h} + \Gamma_{\nu_i H} \right). \quad (30)$$

At a high mass of $M_N$, the branching ratios of the leading channels go like

$$\Gamma(\ell^- W_L^+) \approx \Gamma(\ell^+ W_L^-) \approx \Gamma(\nu Z_L) \approx \Gamma(\nu h + \nu H). \quad (31)$$

As discussed above, the lepton-flavor contents of $N$ decays will be different in each neutrino spectrum. Here, we also study this issue in great detail for cases I and II. In order to search for the events with best reconstruction, we will only consider the $N$ decay to charged leptons plus a $W^\pm$.

1. Decays in Case I: Degenerate Heavy Neutrinos

In Fig. [5] we show the impact of the neutrino masses and mixing angles on the branching fractions of the sum of the degenerate neutrinos $N_i$ ($i = 1, 2, 3$) decaying into $e, \mu, \tau$ lepton plus $W$ boson, respectively, with the left panels for the Normal Hierarchy (NH) and the right panels of the Inverted Hierarchy (IH), assuming vanishing Majorana phases. Qualitatively, it follows the relations in Eq. (14)

$$BR(\mu^\pm W^\mp), BR(\tau^\pm W^\mp) \gg BR(e^\pm W^\mp) \text{ for NH,}$$

$$BR(e^\pm W^\mp) > BR(\mu^\pm W^\mp), BR(\tau^\pm W^\mp) \text{ for IH.} \quad (32)$$
FIG. 5: Branching fractions of degenerate neutrinos $\sum_i N_i \to \ell^+ W^- + \ell^- W^+$ ($\ell = e, \mu, \tau$) for NH and IH versus lightest neutrino mass with $M_N = 300$ GeV and $M_h = 120$ GeV, assuming vanishing Majorana phases.

The branching fraction can differ by one order of magnitude in NH case; and about a factor of few in the IH spectrum. As one expects that all these channels are quite similar when the neutrino spectrum is quasi-degenerate, $m_1 \approx m_2 \approx m_3 \geq 0.05$ eV. Therefore, in this simple case one can hope that if the heavy neutrino decays are observed in future experiments one should be able to distinguish the neutrino spectrum.

2. Decays in Case II: Non-degenerate Heavy Neutrinos

For non-degenerate neutrino spectra we once again study the simple choice: Case IIa $\Omega = I$. We show the branching fractions of processes $N_i \to \ell^+ W^- + \ell^- W^+$ ($\ell = e, \mu, \tau$, $i = 1, 2, 3$) corresponding to the lightest neutrino mass for NH and IH for $M_i = 300$ GeV in Fig. 6. As noted earlier, in this simplest case all $|V_{ei}|^2$ ($\ell = e, \mu, \tau$) are proportional to $m_i$. Therefore the branching ratio of $N_i \to \ell^\pm W^\mp$ for each lepton flavor is independent of neutrino mass and thus universal for both NH and IH. Although we cannot distinguish the neutrino mass hierarchy, we still can tell the difference of the three heavy Majorana neutrinos according to different SM lepton flavors in final states of their dominant decay channels. One has

$$BR(e^\pm W^\mp) > BR(\mu^\pm W^\mp), BR(\tau^\pm W^\mp) \text{ for } N_1,$$

$$BR(e^\pm W^\mp) \approx BR(\mu^\pm W^\mp) \approx BR(\tau^\pm W^\mp) \text{ for } N_2,$$

$$BR(\mu^\pm W^\mp), BR(\tau^\pm W^\mp) \gg BR(e^\pm W^\mp) \text{ for } N_3.$$
FIG. 6: Branching fractions of process $N_i \rightarrow \ell^+W^- + \ell^-W^+ (\ell = e, \mu, \tau; i = 1, 2, 3)$ versus the lightest neutrino mass for NH and IH in Case IIa ($\Omega = I$), when $M_i = 300$ GeV and $M_h = 120$ GeV.

This follows closely to the mixing strengths of the light neutrinos in the previous section.

As discussed previously, Case IIb $\Omega = I_{\text{eff}}$ is identical to the above if we identify $N_1 \leftrightarrow N_3$. More involved case for $\Omega$ may be some form of superposition of the three decay patterns, that is to be tested experimentally by the flavor combinations.
B. Impact of Majorana Phases in Heavy Majorana Neutrino Decays

In our previous discussion we have shown that the mixings $|V_{\ell N}|^2$ are independent of Majorana phases in both Case IIa and IIb (as well for an $\Omega$ with unity as entries). In general, the $N_i$ decay rates depend on only one Majorana phase $\Phi_2$ ($\Phi_1$) when $m_{1(3)} \approx 0$ and $s_{13} = 0$ in the NH (IH) case as shown explicitly in the appendices. In Fig. 7 we show the dependence of $N_1$ decay branching fractions for general non-degenerate case on Majorana phases $\Phi_2$ and $\Phi_1$ in NH and IH, with random selection of the $\Omega$ matrix elements. The dependence of $N_2$ and $N_3$ decays on Majorana phases are almost the same as that of $N_1$. The branching fractions of $\mu^\pm W^\mp, \tau^\pm W^\mp$ ($e^\pm W^\mp$) are typically dominant over all the range of $\Phi_2$ ($\Phi_1$) in NH (IH). The dependence on the phases for the leading channels are rather weak and it is thus hard to extract the phase information from heavy Majorana neutrino decay. Some typical situations may be similar to the cases discussed in Ref. [17], and we will not pursue further for the phase effects.

C. Total Decay Width of Heavy Majorana Neutrino

To complete this section about the heavy Majorana neutrino properties, we study their total decay widths, which are proportional to $M_\nu M_N^2/M_W^2$. In Fig. 8 we plot the total width (left axis) and decay length (right axis) for $N$ versus $M_N$ under the general non-degenerate case with random selection of the $\Omega$ matrix elements.
FIG. 8: The total width and decay length of $N$ in the general non-degenerate case, when the lightest neutrino mass $10^{-4}$ eV $\leq m_{1(3)} \leq 0.4$ eV, $M_h = 120$ GeV and $\Omega = R_{12}R_{13}R_{23}$ with random selection of the matrix elements and.

elements (similar for NH and IH). There is a large spread for the possible ranges of the decay lengths, governed by the mixing parameters. Although not generally considered as long-lived for large mass, the $N$ decay lengths may be typically in the range of $\mu$m–cm, and their decays could lead to a visible displaced vertex in the detector at the LHC.

When considering a specific model-parameter setting, we plot the total width (left axis) and decay length (right axis) in Fig. 9 for $N_i$ versus $M_N$ for $M_h = 120$ GeV in NH and IH under Case IIa with $\Omega = I$. One of the generic features for all $N_i$ and both NH and IH is a typical lower limit for their lifetime (or decay length). For instance, the typical decay length for $M_N \gtrsim 600$ GeV is above $1 \mu$m. For smaller values of $M_N$, the heavy Majorana neutrinos can be long-lived in the detector scale, making the signatures detectable at the secondary vertex. In fact, this feature remains in a majority part of the parameter space. In particular, because in this case all $|V_{\ell i}|^2$ are proportional to $m_i$, the lifetimes of $N_1$ in NH and $N_3$ in IH could be infinite when neglecting the lightest neutrino mass in whole Majorana neutrino mass range. It is interesting to note that there is a clear difference between the NH and IH scenarios: the lifetime of $N_1$ in IH and $N_3$ in NH has a narrowly predicted range within one order of magnitude, about $10 \mu$m for $M_N = 400$ GeV. If this is indeed observed, it could serve as an indication to distinguish the models. The lifetimes of $N_2$ in NH and IH are almost the same.

For Case IIb with an off-diagonal $\Omega$ matrix, the lifetime features of $N_1$ and $N_3$ are also interchanged.
FIG. 9: Total width and decay length of $N_i$ ($i = 1, 2, 3$) for NH and IH in Case IIa ($\Omega = I$), when the lightest neutrino mass $10^{-4} \text{eV} \leq m_{1(3)} \leq 0.4 \text{eV}$ and $M_h = 120 \text{GeV}$.

with each other and those of $N_2$ are still the same as Case IIa.
In order to study the prediction for the lepton flavor correlations with heavy Majorana neutrino and its LNV decay processes, the ideal production channels are the Drell-Yan processes via SM gauge bosons, $pp \to W \to N\ell$, $pp \to Z \to NN$. However, the gauge couplings to $N$ are highly suppressed to the order $O(m_\nu/M_N)$ [8]. The situation is very different in the case of the minimal $B-L$ extension of the SM (see appendix C) where one can produce the heavy neutrinos through the $Z'$ in the theory.

A. Gauge Boson Properties: $Z'$

In the limit where there is no mixing between the two Abelian sector of the minimal $B-L$ extension of the SM (see appendix C, $\epsilon = 0$ in Eq. (C3)), the mass of the new gauge boson $Z'$ is given by

$$M_{Z'} = 2g_{BL}v_S.$$  \hspace{1cm} (33)

To satisfy the experimental lower bound, $M_{Z'}/g_{BL} > (5 - 10)$ TeV, it is sufficient to assume that $v_S > 2.5 - 5$ TeV. The relevant interactions to matter are given by

$$g_{BL} Z'_\mu \left( Q^q_{BL} \bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d \right) + Q^\ell_{BL} \left( \bar{e}\gamma_\mu e + \bar{\nu}_L\gamma_\mu \nu_L + \bar{\nu}_R\gamma_\mu \nu_R \right),$$  \hspace{1cm} (34)

where the $B-L$ charges are assigned to be $Q^q_{BL} = 1/3$, and $Q^\ell_{BL} = -1$.

There has been a lot of work on the heavy neutral gauge bosons. For a recent review, see Ref. [18], and recent studies of $Z'$ at the Tevatron and LHC [19]. For a recent consideration of the phenomenological aspects of the $B-L$ model, see [9]. The expressions for the possible decays of the $Z'$ are given by

$$\Gamma(Z' \to f \bar{f}) = g_{BL}^2 \frac{M_{Z'}}{12\pi} C_f (Q^f_{BL})^2 \left( 1 + 2 \frac{m_f^2}{M_{Z'}^2} \right) \beta_f,$$  \hspace{1cm} (35)

$$\Gamma(Z' \to \sum_m \nu_m \nu_m) = 3g_{BL}^2 \frac{M_{Z'}}{24\pi} C_\nu (Q^\nu_{BL})^2,$$  \hspace{1cm} (36)

$$\Gamma(Z' \to N_m N_m) = g_{BL}^2 \frac{M_{Z'}}{24\pi} C_N (Q^N_{BL})^2 \beta_i^3.$$  \hspace{1cm} (37)

where $f = \ell, q$, the couplings $C_{f,\nu,N} = 1, C_q = 3$, and $\beta_i = \sqrt{1 - 4m_i^2/M_{Z'}^2}$ is the speed of particle $i$. Note that the decay width to Majorana particles is of a threshold behavior $\beta_i^3$, and is half of that for a Dirac particle. Well above the threshold, the $Z'$ decay branching fractions take the simple ratios for the final states

$$\sum_{\ell} \ell^+ \ell^- : \sum_q q\bar{q} : \sum_m \nu_m \nu_m : N_1 N_1 = 3 : 2 : 3 : 1 : 1/2.$$  \hspace{1cm} (38)
We show in Fig. 10 the results for the case $v_S = 3$ TeV. It scales as
\[ \Gamma(\text{tot}) \approx 0.2 \, g_{BL}^2 \, M_{Z'} < 0.05 \left( \frac{M_{Z'}}{v_S} \right)^2 M_{Z'}. \]  

(39)

Notice that this $Z'$ has the property that its coupling to quarks is suppressed with respect to the couplings to leptons. As it well known the $Z'$ in Left-Right symmetric theories has different properties from the $B - L$ case studied here. Then, from the standard analysis where one uses the leptonic channels and the channels into heavy quarks one can distinguish the $B - L$ case from the rest easily.

It is important to emphasize that in the case of the $B - L$ SM, one gets an upper bound on the mass of the heavy neutrinos $M_N \leq M_{Z'}/(2\sqrt{2}g_{BL})$ (see appendix C for details).

\subsection*{B. Heavy Majorana Neutrino Production through $Z'$ mediation at the LHC}

We are interested in the production of two heavy neutrinos. Since in this model one has a dynamical mechanism for $B - L$ breaking, there is a production mechanism through the $Z'$. Then, we are interested in the mechanism
\[ pp \rightarrow Z' \rightarrow N_1N_1. \]  

(40)
The parton level cross section for this process is

\[
\frac{d\sigma(q\bar{q} \to Z' \to N_1 N_1)}{dt} = \frac{1}{32\pi s^2 N_c} \frac{2g_{BL}^2}{9} \frac{1}{(s-M_{Z'}^2)^2 + M_{Z'}^2 \Gamma_{Z'}^2} \left[ (t-M_N^2)^2 + (u-M_N^2)^2 - 2sM_N^2 \right]
\]

where \( t = (p_q - p_{N})^2 \). The total cross section versus heavy Majorana neutrino mass at the LHC is plotted in Fig. 11 assuming \( v_S = 3 \text{ TeV} \) with (a) for \( U(1)_{B-L} \) coupling and (b) for \( U(1)_X \) coupling, as given in Tables I and II. We see that the production cross sections are quite sizable, typically of the order of \( 10^{-1} - 10^2 \) fb. The cross section drops sharply after reaching the kinematical threshold \( 2M_N > M_{Z'} \).

The Majorana signals for \( \Delta L = 2 \) decay of \( N_1 \) are

\[
N_1 N_1 \to \ell^\pm \ell^\mp W^+ W^-, \quad \ell = e, \mu, \tau
\]

To confirm the important feature of lepton number violation, we demand the \( W \)’s decay hadronically. The overall branching fraction to be included becomes

\[
\text{BR}(N_1 N_1 \to \ell^\pm \ell^\mp \text{ 4 jets}) \approx 2 \cdot \left( \frac{1}{4} \right)^2 \cdot \left( \frac{6}{9} \right)^2 = \frac{1}{18}.
\]

Note that there are also accompanying clean channels like \( \ell^\pm \ell^\mp + \text{ 4 jets} \), that are not lepton-number violating and we do not include for the rest of the analysis.
We would like to reiterate that in a significant range of the parameter space of $M_N$ and mixings, the $N$ decay could lead to distinctive signatures with a decay length longer than 10 $\mu$m, resulting in secondary displaced vertices. This may yield essentially background-free signal for $N$'s. Nevertheless, we now explore the signal observability according to the different lepton flavors without relying on the displaced vertex considerations.

For our numerical analyses, we adopt the CTEQ6L1 parton distribution function [20]. We evaluate the SM backgrounds by using the automatic package Madgraph [21]. We work in the parton-level, but simulate the detector effects by the kinematical acceptance and employ the Gaussian smearing for the electromagnetic and hadronic energies [22].

C. $N_1 N_1 \rightarrow \ell^\pm \ell^\pm + 4$ jets ($\ell = e, \mu$)

We start from the cleanest channels with $e, \mu$ in the final state from $N_1$ decay. We employ the following basic acceptance cuts for the event selection [22]

$$p_T(\ell) \geq 15 \text{ GeV}, \ |\eta(\ell)| < 2.5, \ \ (44)$$

$$p_T(j) \geq 25 \text{ GeV}, \ |\eta(j)| < 3.0, \ \ (45)$$

$$\Delta R_{jj} \geq 0.3, \ \Delta R_{j\ell}, \ \Delta R_{\ell\ell} \geq 0.4 \ \ (46)$$

The rather loose cuts on the separations $\Delta R$ are designed to keep the signal events for a heavier $Z'$ and a lighter $N$ which is fast moving and thus yields collimated decay products of a lepton and two jets. We plot the minimal isolation $\Delta R_{jj}^{\min}$ of two jets and $\Delta R_{j\ell}^{\min}$ of one jet and one charged lepton for $M_{Z'} = 1$ TeV and $M_N = 100, \ 200$ GeV, respectively, in Fig. 12. One can see that for $M_N \geq 200$ GeV with $M_{Z'} = 1$ TeV the signal consists of well-isolated one pair of same-sign leptons of arbitrary $e, \mu$ flavor combinations plus four light jets.

To simulate the detector effects on the energy-momentum measurements, we smear the electromagnetic energy, the electromagnetic energy and jet energy by a Gaussian distribution whose width is parameterized as [22]

$$\frac{\Delta E}{E} = \frac{a_{\text{cal}}}{\sqrt{E/\text{GeV}}} \oplus b_{\text{cal}}, \quad a_{\text{cal}} = 10\%, \ b_{\text{cal}} = 0.7\%, \ \ (47)$$

$$\frac{\Delta E}{E} = \frac{a_{\text{had}}}{\sqrt{E/\text{GeV}}} \oplus b_{\text{had}}, \quad a_{\text{had}} = 50\%, \ b_{\text{had}} = 3\%. \ \ (48)$$

In principle, there is no genuine SM background to the lepton-number violating processes. The leading SM background to our signal is from decays of two like-sign $W$’s to leptons. For instance, the leading
reducible background to our signal is

\[ pp \to t\bar{t}W^\pm \to W^\pm W^\pm jjbb. \]  

(49)

The QCD processes \( jjjjW^\pm W^\pm, jjW^\pm W^\pm W^\mp \) are much smaller. This is estimated based on the fact that QCD \( jjW^\pm W^\pm \to jj\ell^\pm \ell^\pm E_T \) is about 15 fb. With an additional \( \alpha_s^2 \) and 6 body phase space or one more \( W \) suppression, they are much smaller than \( t\bar{t}W^\pm \). Other EW backgrounds \( WWW, WWWW, WWWWZ \) are also neglectable. Although the background rates are large to begin with, the kinematics is quite different between the signal and the backgrounds. We outline the characteristics and propose some judicious cuts as follows.

- The SM backgrounds always come with \( W \) pair decays with missing neutrinos. To suppress backgrounds, we veto the events with large missing energy \( E_T < 20 \text{ GeV} \).

- We choose the two pairs with nearly equal masses from the six dijet combinations as the two hadronic \( W \)'s and take \( W \) boson reconstruction as \(|M_{jj} - M_W| < 15 \text{ GeV}| \). The efficiency is very high.

- In order to select the correct lepton and two jets combination and reconstruct \( N_1 \), we take advantage of the feature that the two heavy neutrinos have equal masses \( M_{\ell_1j_1j_2} = M_{\ell_2j_3j_4} \). In practice, we impose \(|M_{\ell_1j_1j_2} - M_{\ell_2j_3j_4}| < M_{N_1}/25 \). This helps for the background reduction.

The production cross section of \( N_1N_1 \) signal with the basic cuts (solid curve) and all of the cuts above (dashed curve) are plotted in Fig. 13 where branching fractions for \( N_1 \) decay to charged leptons are not
FIG. 13: Production cross section of $N_1N_1$ with basic cuts and hard final states cuts. Branching fractions for $N_1$ decay to charged leptons are not included; while $W$ decays to 2 jets are included. For comparison, the background process is also included with the sequential cuts as indicated.

included; while $W$ decays to 2 jets are included. For comparison, the background process of $t\bar{t}W^\pm$ is also included with the sequential cuts as indicated. The background is suppressed substantially.

When performing the signal significance analysis, we look for the resonance in the mass distributions of $\ell jj$ and $2\ell4j$. If we look at mass window of $|M_{\ell_1j_1j_2,\ell_2j_3j_4} - M_{N_1}| < M_{N_1}/20$ and $|M_{2\ell+4j} - M_{Z'}| < M_{Z'}/30$, the background will be at a negligible level.

D. $N_1N_1 \rightarrow \tau^\pm\ell^\pm + 4$ jets

The previous section sets the stage for the analyses in the following sections. Most of the issues for event selection and detector acceptance will remain the same for the following studies. The next presentations will thus be sketchy and mainly outlining the new features, in particular the $\tau$-reconstruction and the mass resonances.

The $\tau$ lepton final state from heavy Majorana neutrino decay plays an important role in distinguishing different neutrino mass patterns. Its identification and reconstruction are different from $e, \mu$ final states because a $\tau$ decays promptly and there will always be missing neutrinos in $\tau$ decay products. In practice
when selecting events with $\tau$’s, we require a minimal missing transverse energy

$$E_T > 20 \text{ GeV}. \quad (50)$$

This will effectively separate them from the $\ell\ell jjjj$ type of signal events.

We first note that all the $\tau$’s are very energetic from the decay of a few hundred GeV $N_1$. The missing momentum will be along the direction of the charged track. We thus assume the momentum of the missing neutrinos to be reconstructed by

$$\overrightarrow{p} (\text{invisible}) = \kappa \overrightarrow{p} (\text{track}). \quad (51)$$

Identifying $\overrightarrow{p}_T (\text{invisible})$ with the measured $E_T$, we thus obtain the $\tau$ momentum by

$$\overrightarrow{p}_T(\tau) = \overrightarrow{p}_T(\ell) + \overrightarrow{E}_T, \quad p_L(\tau) = p_L(\ell) + \frac{E_T}{p_T(\ell)} p_L(\ell).$$

The $N_1$ pair kinematics is thus fully reconstructed. The reconstructed invariant masses of $M(\ell jj)$ and $M(\tau jj)$ are plotted in Fig.14. We see that $M(\tau jj)$ distribution (dotted curve) is slightly broader as anticipated. The rather narrow mass peak of the $\ell jj$ system nevertheless serves as the most distinctive kinematical feature for the signal identification. Invariant masses of $M(\tau \ell + 4j)$ are also plotted in Fig.14. Although the existence of missing energy in the signal makes the background separation more involved, the resonant mass reconstruction proves to be high efficient and the backgrounds can still be suppressed to a negligible level.

E. $N_1 N_1 \rightarrow \tau^\pm \tau^\pm + 4 \text{ jets}$

For $\tau \tau jjjj$ events with two $\tau$’s, we generalize the momenta reconstruction to

$$\overrightarrow{p} (\text{invisible}) = \kappa_1 \overrightarrow{p} (\text{track}_1) + \kappa_2 \overrightarrow{p} (\text{track}_2). \quad (52)$$

The proportionality constants $\kappa_1, \kappa_2$ can be determined from the missing energy measurement as long as the two charge tracks are linearly independent. The $N_1$ pair kinematics can be once again fully reconstructed. The reconstructed invariant masses of $M(\tau jj)$ is plotted in Fig.15. The nice mass peaks of the $\tau jj$ system at $M_{N_1}$ and the $\tau \tau jj$ system at $M_{Z'}$ make the signal stand out of the SM backgrounds.

It is important to note a difference between the leptons from the primary $N_1$ decay and from the $\tau$ decay: the latter is much softer. In Fig.16 we show the $p_T$ distribution of the softer lepton from the $N_1$ and $\tau$ decays in the events of $\ell \ell jjjj$, $\ell \tau jjjj$ and $\tau \tau jjjj$. This feature could provide additional discrimination power to separate the three different leptonic channels if needed to fit the flavor structure for a underlying theory.
FIG. 14: Reconstructed invariant mass of $M(jj\ell)$, $M(jj\tau)$ for $M_{N_1} = 300, 600$ GeV, respectively (left), and $M(\tau\ell jj)$ for $M_{Z'} = 1, 1.5$ TeV (right).

F. Measuring Branching Fractions and Probing the Neutrino Mass Patterns

So far, we have only studied the characteristic features of the signal and backgrounds for the leading channels and have not included the proper branching fractions for the individual lepton flavors. For illustration, consider first the cleanest channel, $N_1N_1 \rightarrow e^\pm e^\pm jjjj$. The number of events is written as

$$N = L \times \sigma(pp \rightarrow N_1N_1) \times 2 \frac{\text{BR}^2(N_1 \rightarrow e^+W^-)}{\frac{6}{9}} \frac{6}{9}, \quad (53)$$

where $L$ is the integrated luminosity and the factor $(6/9)$ is due the the $W$ hadronic decay. Given a sufficient number of events $N$, the mass of $N_1$ is determined by the invariant mass of lepton and jet $M_{jjj}$. We thus predict the corresponding production rate $\sigma(pp \rightarrow N_1N_1)$ for this given mass. The only unknown in the Eq. (53) is the decay branching fraction.

We present the event contours in the BR–$M_{N}$ plane in Fig. [17] for 100 fb$^{-1}$ luminosity and degenerate case with (a) $M_{Z'} = 1$ TeV and (b) $M_{Z'} = 1.5$ TeV including all the judicious cuts described earlier, with which the backgrounds are insignificant.

In Fig. [17](c) and (d), we show the event contours in the BR–$M_{N_1}$ plane, for 100 fb$^{-1}$ luminosity and non-degenerate case including all the judicious cuts described earlier. We see that the reach to a low BR can be quite encouraging.

As we presented earlier, the $N_1$ decay branching fractions and the light neutrino mass matrix are directly
correlated. Measuring the BR’s of different flavor combinations becomes crucial in understanding the neutrino mass pattern and thus the mass generation mechanism. In the degenerate case, we have the prediction for the flavor combinations

$$\text{BR}(N N \rightarrow \ell\ell WW) \approx \begin{cases} 
2 \times (23\%)^2 & \text{for NH: } (\mu^\pm + \tau^\pm)(\mu^\pm + \tau^\pm)WW, \\
2 \times (13\%)^2 & \text{for IH: } e^\pm e^\pm WW, \\
2 \times (17\%)^2 & \text{for QD: } (e^\pm + \mu^\pm + \tau^\pm)(e^\pm + \mu^\pm + \tau^\pm)WW,
\end{cases} \quad (54)$$

for $\Phi_1 = \Phi_2 = 0$, independent of the matrix $\Omega$. On the other hand, for the non-degenerate situation, the flavor prediction is like

$$\text{BR}(N N \rightarrow \ell\ell WW) \approx \begin{cases} 
2 \times (20\%)^2 & \text{for } N_1 : e^\pm e^\pm WW, \\
2 \times (17\%)^2 & \text{for } N_2 : (e^\pm + \mu^\pm + \tau^\pm)(e^\pm + \mu^\pm + \tau^\pm)WW, \\
2 \times (23\%)^2 & \text{for } N_3 : (\mu^\pm + \tau^\pm)(\mu^\pm + \tau^\pm)WW,
\end{cases} \quad (55)$$

for $\Omega = I$, independent of the neutrino mass patterns as well as $\Phi_1$, $\Phi_2$. These predictions are the consequence from the low energy oscillation experiments and this are subject to test at the LHC to confirm the theory.

In Fig. 18 we show the event contours in the $M_{Z'} - M_N$ plane, for (a) production of $\sum_i N_i$ in NH (solid curve), IH (dashed curve) and QD (dotted curve) for the degenerate case and (b) production of $N_1$ (solid...
FIG. 16: $p_T$ distribution of the softer lepton from the $N_1$ and $\tau$ decays in the events of $\ell\ell jjjj$, $\ell\tau jjjj$ and $\tau\tau jjjj$, for a $N_1$ mass 300 GeV and $Z'$ mass 1.5 TeV. curve), $N_2$ (dashed curve) and $N_3$ (dotted curve) for the non-degenerate case with 100 fb$^{-1}$ luminosity, 10 events numbers and branching fractions of heavy neutrinos predicted in Eqs. (54) and (55). The values of $M_{Z'}$ and $M_N$ on the left-hand side of the curves would give more than 10 events for 100 fb$^{-1}$ luminosity and more accessible heavy neutrino decay branching fractions at the LHC.
FIG. 17: Event contours in the BR–$M_N$ plane at the LHC with an integrated luminosity 100 $fb^{-1}$ for degenerate case $\sum_{i=1,2,3} N_i N_i \rightarrow e^+ e^- W^- W^-$ with (a) $M_{Z'} = 1$ TeV, (b) $M_{Z'} = 1.5$ TeV, and for non-degenerate case $N_1 N_1 \rightarrow e^+ e^- W^- W^-$ with (c) $M_{Z'} = 1$ TeV and (d) $M_{Z'} = 1.5$ TeV, including all the judicious cuts describe in the early sections.

V. SUMMARY

In this article we have investigated the possibility to test the so-called Type I seesaw mechanism for neutrino masses at the CERN Large Hadron Collider in the context of two simple extensions of the Standard Model where $B - L$ is part of the gauge symmetry. We have studied in great detail the predictions of the right-handed neutrino decays in each spectrum for neutrino masses showing the most optimistic scenarios where one could hope to distinguish the spectrum using the properties of the decays.

We have found the following interesting results:
• Working in the context of two simple extensions of the Standard Model with a local gauge symmetry $B - L$ or $X = Y - \frac{2}{3}(B - L)$, one can produce the heavy neutrinos through the $Z'$ gauge boson in each scenario. In both cases one has a dynamical mechanism for the generation of heavy neutrino masses, related to $M_{Z'}$.

• In the case where the heavy neutrinos are degenerate, we show the possibility to distinguish the neutrino spectrum. The branching fractions can differ by one order of magnitude in NH case with $BR(\mu^\pm W^\mp), BR(\tau^\pm W^\mp) \gg BR(e^\pm W^\mp)$, and a factor of a few in the IH spectrum with $BR(e^\pm W^\mp) > BR(\mu^\pm W^\mp), BR(\tau^\pm W^\mp)$ when the Majorana phases are ignored. As one expects, all these channels are quite similar when the neutrino spectrum is quasi-degenerate, $m_1 \approx m_2 \approx m_3 \geq 0.05$ eV.

• In the case when $\Omega$ is an identity matrix or with only unity entries generally, we find: $BR(e^\pm W^\mp) > BR(\mu^\pm W^\mp), BR(\tau^\pm W^\mp)$ for $N_1$ decay, $BR(e^\pm W^\mp) \approx BR(\mu^\pm W^\mp) \approx BR(\tau^\pm W^\mp)$ for $N_2$ and $BR(\mu^\pm W^\mp), BR(\tau^\pm W^\mp) \gg BR(e^\pm W^\mp)$ for $N_3$ in both NH and IH. The branching fractions in these cases are independent of Majorana phases.

• In general, the form of $\Omega$ governs heavy neutrino decay patterns. Future tests on the flavor combina-
tions of SM charged leptons would reveal the specific model structure.

- The above-studied $\Delta L = 2$ channels can take the search to $M_{Z'} \approx 2$ TeV at the LHC. The sensitivity to the leptonic branching fractions of $N$ decay can be about 10\%.

- In particular, in a significant part of the parameter space of $M_N$ and the mixings, the $N$ decay could lead to distinctive signatures with secondary displaced vertices, yielding essentially background-free signal for $N$'s.

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APPENDIX A: NEUTRINO MASSES AND MIXINGS

The Type I seesaw scheme introduces right-handed neutrino states, in addition to the SM matter contents. For definitiveness, we add three of them, $\nu^i_R (i = 1, 2, 3)$, which are said to be sterile since they do not carry any SM gauge quantum numbers. The SM gauge invariant and renormalizable interactions to generate neutrino masses include both Dirac as well as Majorana terms

\[- \mathcal{L}^I_\nu = 7_L Y^D_\nu \bar{H} \nu_R + \frac{1}{2} (\bar{\nu^c})_L M_N \nu_R + \text{h.c.} \]  
(A1)

where $Y^D_\nu$, $M_N$ are $3 \times 3$ matrices in the generation space, with $\bar{H} = i \sigma_2 H^*$ and $H^T = (H^+ H^0)$. Once $H$ gets the vacuum expectation value $\langle H \rangle = v_0/\sqrt{2}$, the neutrinos acquire Dirac masses $m_D = Y^D_\nu v_0/\sqrt{2}$,

\[- \mathcal{L}^m_\nu = \frac{1}{2} \left( \bar{\nu}_L m_D \nu_R + (\bar{\nu^c})_L m^T_D (\nu^c)_R + (\bar{\nu^c})_L M_N \nu_R \right) + \text{h.c.} \]  
(A2)

To diagonalize the mass matrix for neutrinos we introduce a $6 \times 6$ unitary transformation

\[ \begin{pmatrix} \nu_L \\ (\nu^c)_L \end{pmatrix} = \mathbb{N} \begin{pmatrix} \nu_L \\ (\nu^c)_L \end{pmatrix} \text{mass} , \quad \mathbb{N} = \begin{pmatrix} U & V \\ V_C & U_C \end{pmatrix} . \]  
(A3)

Then,

\[ \mathbb{N}^\dagger \begin{pmatrix} 0 & m_D \\ m^T_D & M_N \end{pmatrix} \mathbb{N}^* = \begin{pmatrix} m & 0 \\ 0 & M \end{pmatrix} , \]  
(A4)
or explicitly,

\[ V_C^\dagger m_D^T U^* + U_C^\dagger m_D V^*_C + V_C^\dagger M_N V^*_C = m, \]  
\[ U_C^\dagger m_D^T V^* + V_C^\dagger m_D U^*_C + U_C^\dagger M_N U^*_C = M, \]  
\[ V_C^\dagger m_D^T V^* + U_C^\dagger m_D U^*_C + V_C^\dagger M_N U^*_C = 0, \]

where \( m = \text{diag}(m_1, m_2, m_3) \) and \( M = \text{diag}(M_1, M_2, M_3) \) are diagonal matrices of the mass eigenvalues. In the limit \( M_N \gg m_D \) we have

\[ m \approx \frac{m_D^2}{M_N}, \quad M \approx M_N. \]

Then we have three light neutrinos and three heavy ones, all Majorana-type. Note that \( U \) and \( V \) mix the light and heavy neutrinos, respectively, into the active weak interaction eigenstates of neutrinos. The mixing elements are typically like

\[ U^2 \approx O(1), \quad V^2 \approx \frac{m}{M}. \]

and the unitarity conditions read

\[ UU^\dagger + VV^\dagger = U^\dagger U + V_C^\dagger V_C + U_C^\dagger U_C = V^\dagger V + U_C^\dagger U_C = I, \]  
\[ UV_C^\dagger + U_C V_C^\dagger = U^\dagger V + V_C^\dagger U_C = 0. \]

It can be shown that the following relations hold

\[ V_C^\dagger m_D^T - m U^T = 0, \quad m_D U_C^* - VM = 0. \]

Assuming that a \( 3 \times 3 \) matrix \( E \) diagonalizes the mass matrix of the charged leptons, we then define

\[ E^\dagger U \equiv V_{PMNS}, \quad E^\dagger V \equiv V_{LN}, \]

\[ V_{PMNS} V_{PMNS}^\dagger + V_{LN} V_{LN}^\dagger = I, \]

where \( V_{PMNS} \) and \( V_{LN} \) describe the transitions between the light neutrino and heavy neutrino to the charged leptons, respectively, via the weak charged currents. Note that the identification of \( V_{PMNS} \) to the PMNS matrix is only approximate. We then obtain an important relation among the physical quantities

\[ V_{LN}^* M V_{LN}^\dagger = -V_{PMNS}^* m V_{PMNS}^\dagger. \]

Although the masses and mixings of the light neutrinos on the right-handed side can be measured from the oscillation experiments, it is quite involved to solve for \( V_{LN} \) via this set of quadratic equations. Absorbing
the minus sign on the right side of the above equation in the definition of $V_{\ell N}$ one can write down a formal solution with the help of an auxiliary matrix

$$V_{\ell N} = V_{PMNS} m^{1/2} \Omega M^{-1/2},$$

(A16)

where $\Omega$ is an orthogonal complex matrix which can be parameterized as

$$\Omega(w_{21}, w_{31}, w_{32}) = R_{12}(w_{21}) R_{13}(w_{31}) R_{23}(w_{32}),$$

(A17)

with

$$R_{12} = \begin{pmatrix} u_{21} & -w_{21} & 0 \\ w_{21} & u_{21} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad R_{13} = \begin{pmatrix} u_{31} & 0 & -w_{31} \\ 0 & 1 & 0 \\ w_{31} & 0 & u_{31} \end{pmatrix}, \quad R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & u_{32} & -w_{32} \\ 0 & w_{32} & u_{32} \end{pmatrix}.$$  

(A18)

where $u_{ij} = \pm \sqrt{1 - w_{ij}^2}$ and $-1 \leq w_{ij} \leq 1$ when the matrix $\Omega$ is real.

APPENDIX B: EXPLICIT EXPRESSIONS FOR THE MIXINGS $V_{\ell N}$

1. Case I: Degenerate Heavy Neutrinos

From Eq. (A15), assuming degenerate heavy neutrinos, we have

$$M_N \sum_N (V_{\ell N}^*)^2 = c_{13}^2 s_{12}^2 m_2 + e^{i \phi_1} c_{12}^2 s_{13}^2 e^{-i \phi_2} m_1 + s_{13}^2 e^{i(2 \delta - \phi_2)} m_3,$$

(B1)

$$M_N \sum_N (V_{\mu N}^*)^2 = (c_{12} c_{23} - s_{12} s_{13} s_{23} e^{-i \delta})^2 m_2 + (c_{23} s_{12} + c_{12} s_{13} s_{23} e^{-i \delta})^2 e^{-i \phi_1} m_1 + c_{13}^2 s_{23}^2 e^{i \phi_2} m_3,$$

(B2)

$$M_N \sum_N (V_{\tau N}^*)^2 = (c_{12} s_{23} + c_{23} s_{12} s_{13} e^{-i \delta})^2 m_2 + (s_{12} s_{23} - c_{12} c_{23} s_{13} e^{-i \delta})^2 e^{-i \phi_1} m_1 + c_{13}^2 s_{23}^2 e^{i \phi_2} m_3.$$  

(B3)

2. Case II: Non-degenerate Heavy Neutrinos

The general expressions for the mixing between the charged leptons and heavy neutrinos, in terms of the neutrino oscillation parameters and the unknown matrix $\Omega$, are given by

$$V_{e1} \sqrt{M_1} = \sqrt{m_2 c_{13} s_{12} w_{21}} \sqrt{1 - w_{31}^2} \sqrt{(1 - w_{21}^2)(1 - w_{31}^2)} e^{i \phi_1/2} + \sqrt{m_3 s_{13} w_{31}} e^{i(\phi_2/2 - \delta)},$$

(B4)

$$V_{\mu 1} \sqrt{M_1} = \sqrt{m_2 (c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i \delta}) w_{21}} \sqrt{1 - w_{31}^2} + \sqrt{m_1 (1 - w_{21}^2)} \sqrt{(1 - w_{21}^2)(1 - w_{31}^2)} e^{i \phi_1/2} + \sqrt{m_3 c_{13} s_{23} w_{31}} e^{i \phi_2/2},$$

(B5)

$$V_{\tau 1} \sqrt{M_1} = \sqrt{m_2 (-c_{12} s_{23} - s_{12} s_{13} c_{23} e^{i \delta}) w_{21}} \sqrt{1 - w_{31}^2} + \sqrt{m_1 (1 - w_{31}^2)} \sqrt{(1 - w_{21}^2)(1 - w_{31}^2)} e^{i \phi_1/2} + \sqrt{m_3 c_{13} c_{23} w_{31}} e^{i \phi_2/2}.$$  

(B6)
We now present the two cases according to the light neutrino mass spectra, assuming $m_{1(3)} \approx 0$ and $s_{13} = 0$.

- Normal Hierarchy:

Under the good approximations $m_1 \approx 0$ and $s_{13} = 0$, one finds the following expressions

\begin{align}
M_1 |V_{c1}|^2 &\approx \sqrt{\Delta m_{23}^2} (s_{12} w_{21} \sqrt{1 - w_{31}^2})^2, \\
M_1 |V_{e1}|^2 &\approx \sqrt{\Delta m_{21}^2 c_{12}^2 w_{21} \sqrt{1 - w_{21}^2}} + \sqrt{\Delta m_{31}^2 s_{23} w_{31} e^{i \phi_2 / 2}} |, \\
M_1 |V_{\mu 1}|^2 &\approx \sqrt{\Delta m_{21}^2 c_{12}^2 w_{21} \sqrt{1 - w_{21}^2}} - \sqrt{\Delta m_{31}^2 s_{23} w_{31} e^{i \phi_2 / 2}} |, \\
M_1 |V_{\tau 1}|^2 &\approx \sqrt{\Delta m_{21}^2 c_{12}^2 w_{21} \sqrt{1 - w_{21}^2}} - \sqrt{\Delta m_{31}^2 s_{23} w_{31} e^{i \phi_2 / 2}} |.
\end{align}
• Inverted Hierarchy:

Under the approximations $m_{3} \approx 0$ and $s_{13} = 0$, we have

\[
M_{1}|V_{e1}|^{2} \approx \left|\sqrt{\Delta m_{21}^{2}} + |\Delta m_{31}^{2}| s_{12} w_{21} \sqrt{1 - w_{31}^{2}}\right|^{2},
\]

\[
M_{1}|V_{\mu 1}|^{2} \approx \left|\sqrt{\Delta m_{21}^{2}} + |\Delta m_{31}^{2}| c_{12} c_{23} w_{21} \sqrt{1 - w_{31}^{2}}\right|^{2},
\]

\[
M_{1}|V_{\tau 1}|^{2} \approx \left|\sqrt{\Delta m_{21}^{2}} + |\Delta m_{31}^{2}| s_{12} s_{23} w_{21} \sqrt{1 - w_{31}^{2}}\right|^{2},
\]

\[
M_{2}|V_{e2}|^{2} \approx \left|\sqrt{\Delta m_{21}^{2}} + |\Delta m_{31}^{2}| s_{12} (w_{21} w_{31} w_{32} + \sqrt{(1 - w_{21}^{2})(1 - w_{31}^{2})})\right|^{2},
\]

\[
M_{2}|V_{\mu 2}|^{2} \approx \left|\sqrt{\Delta m_{21}^{2}} c_{12} c_{23} (w_{21} w_{31} w_{32} + \sqrt{(1 - w_{21}^{2})(1 - w_{31}^{2})})\right|^{2} + \left|\sqrt{\Delta m_{31}^{2}} s_{12} c_{23} (w_{21} w_{31} w_{32} + \sqrt{(1 - w_{21}^{2})(1 - w_{31}^{2})})\right|^{2},
\]

\[
M_{2}|V_{\tau 2}|^{2} \approx \left|\sqrt{\Delta m_{21}^{2}} c_{12} s_{23} (w_{21} w_{31} w_{32} + \sqrt{(1 - w_{21}^{2})(1 - w_{31}^{2})})\right|^{2} - \left|\sqrt{\Delta m_{31}^{2}} s_{12} s_{23} (w_{21} w_{31} w_{32} + \sqrt{(1 - w_{21}^{2})(1 - w_{31}^{2})})\right|^{2}.
\]
In order to understand the origin of Majorana neutrino masses it is crucial to look for new scenarios where \( g_{54} \) have to be very small we work in the case where the model is based on the gauge symmetry following properties:

\[
M_3[V_{e3}]^2 \approx |\sqrt{\Delta m_{21}^2} + |\Delta m_{31}^2| |s_{12}(-w_{32} \sqrt{1-w_{21}^2} - w_{21} w_{31} \sqrt{1-w_{23}^2}) + \sqrt{\Delta m_{31}^2} |c_{12}(w_{21} w_{32} - w_{31} \sqrt{1-w_{21}^2}(1-w_{23}^2)) e^{i\phi_3/2}|^2, 
\]
\[
M_3[V_{\mu3}]^2 \approx |\sqrt{\Delta m_{21}^2} + |\Delta m_{31}^2| |c_{12}(w_{21} w_{32} - w_{31} \sqrt{1-w_{21}^2}(1-w_{23}^2)) e^{i\phi_3/2}|^2, 
\]
\[
M_3[V_{\tau3}]^2 \approx |\sqrt{\Delta m_{21}^2} + |\Delta m_{31}^2| |c_{12} s_{23}(w_{21} w_{32} - w_{31} \sqrt{1-w_{21}^2}(1-w_{23}^2)) e^{i\phi_3/2}|^2. 
\]

\( \text{(B28)} \)

\( \text{(B29)} \)

\( \text{(B30)} \)

**APPENDIX C: \( U(1)_{B-L} \) AND \( U(1)_X \) EXTENSIONS OF THE STANDARD MODEL**

It is well-known that \( B - L \) is an accidental global symmetry in the Standard Model and its origin is unknown. In order to understand the origin of Majorana neutrino masses it is crucial to look for new scenarios where \( B - L \) can be spontaneously broken. Here we focus on a simple extension of the Standard Model where \( U(1)_{B-L} \) is a local symmetry and in order to cancel the anomalies one has to introduce three right-handed neutrinos. Therefore, this model is based on the gauge symmetry \( SU(3)_C \oplus SU(2)_L \oplus U(1)_Y \oplus U(1)_{B-L} \). The matter fields have the following properties:

\[
Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \sim (3, 2, 1/6, 1/3), \quad u_R \sim (3, 1, 2/3, 1/3), \quad d_R \sim (3, 1, -1/3, 1/3), \quad (C1)
\]

\[
l_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \sim (1, 2, -1/2, -1), \quad e_R \sim (1, 1, -1, -1), \quad \text{and} \quad \nu_R \sim (1, 1, 0, -1), \quad (C2)
\]

where \( \nu_R \) are the right-handed neutrinos. Here we use the normalization where \( Q = T_3 + Y \). In order to generate the right-handed neutrino masses and break the local \( B - L \) symmetry one has to add a new scalar field \( S \sim (1, 1, 0, 2) \).

1. **Interactions and Symmetry Breaking**

In this context the Kinetic terms for the Abelian sector are given by

\[
\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{4} F'^{\mu\nu} F'^{\mu\nu} - \frac{\epsilon}{2} F^{\mu\nu} F'^{\mu\nu}, \quad (C3)
\]

where

\[
F_{\mu\nu}^a = \partial^\mu B^\nu_a - \partial^\nu B^\mu_a, \quad \text{and} \quad F'^{\mu\nu} = \partial^\mu B'^\nu - \partial^\nu B'^\mu. \quad (C4)
\]

Here \( B^\nu \) and \( B'^\nu \) are the gauge fields for \( U(1)_Y \) and \( U(1)_{B-L} \), respectively. Since the mixing between the Abelian gauge bosons have to be very small we work in the case where \( \epsilon = 0 \). The Kinetic terms for the matter fields read as:

\[
\mathcal{L}_{\text{Kinetic}} = i \overline{Q_L} \gamma^\nu D_\nu Q_L + i \overline{\nu_R} \gamma^\mu D_\mu \nu_R + i \overline{\nu} \gamma^\mu D_\mu \nu - i \overline{\nu} \gamma^\mu D_\mu \nu R,
\]

\( \text{(C5)} \)
where
\[ D_\mu \nu_R = \partial_\mu \nu_R - ig_{BL} B'_\mu \nu_R. \] (C6)

As we have explained before the Higgs sector is composed of the SM Higgs, \( H^T = (H^+, H^0) \), and an extra Higgs, \( S = S_R + i S_I \), which is needed to break \( B - L \). The relevant Lagrangian for the scalar fields is given by
\[ \mathcal{L}_{\text{Higgs}} = (D_\mu H)^\dagger (D^\mu H) + (D_\mu S)^\dagger (D^\mu S) - V(H, S) \] (C7)
where
\[ D_\mu S = \partial_\mu S + i 2g_{BL} B'_\mu S. \] (C8)

The gauge invariant Yukawa interactions of neutrinos are
\[ -\mathcal{L}_Y^\nu = Y^D_{\nu} \tilde{i}_L \tilde{H} \nu_R + \frac{Y^M_{\nu}}{2} \nu^T_R C \nu_R S + \text{h.c..} \] (C9)

Once \( S \) gets the vacuum expectation value \( \langle S \rangle \rightarrow v_S/\sqrt{2} \), \( B - L \) is broken and one gets the mass of neutral gauge boson \( Z' = Z_{B-L} \) with \( M_{Z'} = 2g_{BL} v_S \) from the second kinetic term in Eq. (C7), and the mass matrix of right-handed neutrino with \( M_N = Y^M_{\nu} v_S/\sqrt{2} \) from Eq. (C9). In order not to upset applicability of perturbative theory, we require \( Y^M_{\nu} \leq 1 \) and get an upper bound of the mass of the heavy neutrinos \( M_N \leq M_{Z'}/(2\sqrt{2}g_{BL}) \).

The scalar potential is given by
\[ V(H, S) = -m_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 - m_S^2 S^\dagger S + \lambda_S (S^\dagger S)^2 + a_S (H^\dagger H) (S^\dagger S) \] (C10)
where all parameters are real. Notice that this scalar potential has the global symmetry \( O(4)_H \otimes O(2)_S \). The minimization conditions in this case read as
\[ 0 = v_0 \left( -m_H^2 + \lambda_H v_0^2 + \frac{a_S}{2} v_S^2 \right), \]
\[ 0 = v_S \left( -m_S^2 + \lambda_S v_S^2 + \frac{a_S}{2} v_0^2 \right). \] (C11) (C12)

Notice that one can have several vacua but only the case \( v_0 \neq 0 \) and \( v_S \neq 0 \) is allowed by the experiment. Now, in order to satisfy the condition of minimum one has to satisfy the following condition:
\[ \lambda_H a_S v_0^4 + 4 \lambda_H \lambda_S v_0^2 v_S^2 + \lambda_S a_S v_S^4 > 0. \] (C13)

The potential is bounded from below when \( \lambda_H \lambda_S - a_S^2/4 > 0 \). Using the minimization conditions above one can find the solution in the phenomenological allowed case:
\[ v_0^2 = \frac{2 \left( a_S m_S^2 - 2 \lambda_S m_H^2 \right)}{a_S^2 - 4 \lambda_H \lambda_S} > 0, \] (C14)
\[ v_S^2 = \frac{2 \left( a_S m_H^2 - 2 \lambda_H m_S^2 \right)}{a_S^2 - 4 \lambda_H \lambda_S} > 0. \] (C15)

Using these conditions one can discuss different cases for the parameters in the Lagrangian. Expressing the numerators and the denominator as \( n_1 = a_S m_H^2 - 2 \lambda_H m_S^2 \), \( n_2 = a_S m_S^2 - 2 \lambda_S m_H^2 \) and \( d = a_S^2 - 4 \lambda_S \lambda_H \).

- Imposing \( v_S^2 > 0 \) one has the case \( n_1 > 0 \) and \( d > 0 \), or \( n_1 < 0 \) and \( d < 0 \).
- Imposing \( v_0^2 > 0 \) one has \( n_2 > 0 \) and \( d > 0 \), or \( n_2 < 0 \) and \( d < 0 \).
where the mixing angle is
\[
\tan 2\theta_0 = \frac{a_{S\ell_0}v_\nu}{\lambda_H v_0^2 - \lambda_S v_S^2 + a_S (v_0^2 - v_S^2)}.
\] (C18)

It is easy to check that \(S_f\) is the Goldstone boson eaten by the \(Z'\) in the theory.

### 3. Feynman Rules

We now summarize the Feynman rules for the SM with \(U(1)_{B-L}\) and \(U(1)_X\) extensions in Tables I and II respectively.
4. Z' Decays in U(1)\(_X\) Extension

The charge of U(1)\(_X\) is defined as \(X = Y - 5(B - L)/4\) and due to the mixing between U(1)\(_Y\) and U(1)\(_X\) we have the mixing matrix of neutral gauge bosons as below

\[
\begin{pmatrix}
B^\mu \\
W_3^\mu \\
B'^\mu
\end{pmatrix} = \begin{pmatrix}
c_W - s_W c' \\
s_W c_W c' - c_W s' \\
0 & s' & c'
\end{pmatrix}
\begin{pmatrix}
A^\mu \\
Z^\mu \\
Z'^\mu
\end{pmatrix} \tag{C19}
\]

where \(s_W(c_W) = \sin \theta_W(\cos \theta_W)\), \(s'(c') = \sin(\theta')(\cos \theta')\), and \(\tan(2\theta') = 2g'_1 \sqrt{g_1^2 + g_1^2} / (g_1^2 + 25g_{BL}^2 \Delta_0^2 - g_2^2 - g'_2^2)\). \(g'_1\) is a free gauge coupling to qualify the mixing between the two U(1) gauge symmetries. One can get their mass eigenvalues as

\[
M_A = 0,
\]

\[
M_{Z,Z'} = \frac{v_0}{\sqrt{2}} \sqrt{g_1^2 + g_2^2} \left[ \frac{1}{2} \left( \frac{g_1^2 + 25(\frac{v_0}{\sqrt{2}})^2 g_{BL}^2}{g_1^2 + g_2^2} + 1 \right) \sin \theta' \sqrt{g_1^2 + g_2^2} \right]^{1/2}.
\]

The expressions for the possible decays of the Z' are given by

\[
\Gamma(Z' \to f \bar{f}) = \frac{M_{Z'}}{12\pi} C_f \left[ V_f^2 \left( 1 + \frac{2m_f^2}{M_{Z'}^2} \right) + A_f^2 \left( 1 - \frac{4m_f^2}{M_{Z'}^2} \right) \right] \sqrt{1 - \frac{4m_f^2}{M_{Z'}^2}} \tag{C22}
\]

\[
\Gamma(Z' \to \nu_m \bar{\nu}_m) = \frac{3M_{Z'}}{24\pi} C_\nu (X'_\nu)^2 \tag{C23}
\]

\[
\Gamma(Z' \to N_m N_m) = \frac{M_{Z'}}{24\pi} C_N (X_N)^2 \left[ 1 - \frac{4m_N^2}{M_{Z'}^2} \right] \sqrt{1 - \frac{4m_N^2}{M_{Z'}^2}} \tag{C24}
\]

\[
\Gamma(Z' \to W^+ W^-) = \frac{\alpha M_{Z'} s'^2}{3 \tan^2\theta_W} \frac{M_{Z'}^4}{16 M_{W}^4} \left( 1 + \frac{16M_{W}^2}{M_{Z'}^2} - \frac{68M_{W}^4}{M_{Z'}^4} - \frac{48M_{W}^6}{M_{Z'}^6} \right) \sqrt{1 - \frac{4M_{W}^2}{M_{Z'}^2}} \tag{C25}
\]

\[
\Gamma(Z' \to Zh) = \frac{M_{Z'}}{48\pi} \frac{v_0 c_\theta}{M_{Z'}^2} \left[ v_0 s_\theta K - v_0 s_\theta K \right]^2 \left[ 1 + 2 \frac{5M_{Z'}^2 - M_{H}^2}{M_{Z'}^2} + \frac{(M_{Z'}^2 - M_{H}^2)^2}{M_{Z'}^2} \right] \times \sqrt{1 - \frac{2M_{Z'}^2 + M_{H}^2}{M_{Z'}^2} + \frac{(M_{Z'}^2 - M_{H}^2)^2}{M_{Z'}^2}} \tag{C26}
\]

\[
\Gamma(Z' \to ZH) = \frac{M_{Z'}}{48\pi} \frac{v_0 c_\theta}{M_{Z'}^2} \left[ v_0 s_\theta K - v_0 s_\theta K \right]^2 \left[ 1 + 2 \frac{5M_{Z'}^2 - M_{H}^2}{M_{Z'}^2} + \frac{(M_{Z'}^2 - M_{H}^2)^2}{M_{Z'}^2} \right] \times \sqrt{1 - \frac{2M_{Z'}^2 + M_{H}^2}{M_{Z'}^2} + \frac{(M_{Z'}^2 - M_{H}^2)^2}{M_{Z'}^2}} \tag{C27}
\]

where \(f = \ell, q, C_{\ell,\nu,N} = 1, C_q = 3\) and \(V_f = (X_{fL} + X_{fR})/2, A_f = (-X_{fL} + X_{fR})/2\).

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TABLE II: Feynman rules for \( Z' \) in SM with \( U(1)_X \) extension, where \( s'(c') = \sin(\theta')(\cos(\theta')) \), \( \tan(2\theta') = 2g'_1\sqrt{g''_2 + g''_1}/(g''^2 + 25g''_2\frac{v^2}{v_0^2} - g''_2 - g''_1) \) and all momenta are incoming.

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