Size Effect of Ruderman-Kittel-Kasuya-Yosida Interaction Mediated by Electrons in Nanoribbons

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We calculated the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction between the magnetic impurities mediated by electrons in nanoribbons. It was shown that the RKKY interaction is strongly dependent on the width of the nanoribbon and the transverse positions of the impurities. The transverse confinement of electrons is responsible for the above size effect of the RKKY interaction. It provides a potential way to control the RKKY interaction by changing nanostructure geometry.

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I. INTRODUCTION

Recent years there has been a renewed interest in the Ruderman-Kittel-Kasuya-Yosida (RKKY) magnetic interaction1–4 due to its important role in giant magnetoresistance in multilayer structures5 and ferromagnetism in diluted magnetic semiconductors6. More recently the controllable RKKY interaction attracted much attentions in the field of spintronics and quantum information processing.7 The two quantized states of the spin of single localized electron can be considered as a quantum bit, and the extended nature of the controllable RKKY interaction between the coupled local spins has the potential application in building large-scale spin-based quantum computing and quantum computers. Craig et al. experimentally demonstrate the gate-controllable RKKY interaction between the localized spins in two quantum dots, each in contact with two-dimensional electron gas.8 Earlier an optical technique to generate and control the RKKY interaction between charged quantum dots was proposed by using external laser field.9 The other possibility of gate-controllable RKKY interaction mediated by electrons in the presence of Rashba spin-orbit coupling or by the helical edge states in quantum spin Hall systems were also analyzed intensively.10–14 The advantage of the controllable RKKY interaction in devices has stimulated to fully investigate and understand its properties both theoretically and experimentally.

The conventional RKKY magnetic interaction between nuclear spins or between localized spins in metals is mediated by conduction electrons, where there is no any confinement on electrons. The controllable RKKY interaction mostly involves semiconductor nanostructures. In the semiconductor nanostructures, the electrons are confined at least in one or two dimensions vertically to the electron movement. In this paper, we focus on the size effect of the RKKY interaction between two localized spins induced by the transversely confined electrons in nanoribbons. It provides full understanding of the RKKY interaction in nanostructures and a potential way to control it by changing nanostructure geometry is pointed out.

The paper is organized as follows. In Sec. II, the RKKY interaction mediated by electrons confined in nanoribbons is derived. In Sec. III, we present and discuss the size effect of the RKKY interaction. Finally we conclude with a brief summary in Sec. IV.

II. FORMALISM

We consider two magnetic impurities with the localized spins $S_i$ ($i = 1, 2$) embedded in semiconductor nanoribbons. The electrons in nanoribbon are itinerant in $x$ direction and confined in the width $d$ in $y$ direction. The localized spins interact with the conduction electrons via the $s$-$d$ coupling. The Hamiltonian describing the above basic physics is written as

$$H = \sum_{i=1}^{N} \left[ \frac{\hbar^2 k_i^2}{2m^*} + U(y_i) \right] I - J \sum_{i=1,2} \sigma_i \cdot S_i$$

(1)

where the first term of the Hamiltonian describes the conduction electrons moving along $x$ direction and confined in the $y$ direction in the nanoribbon. $U(y_i)$ is assumed to be infinite square well potential. The second term is the $s$-$d$ interaction between the conduction electrons and the localized spin $S_i$. $J$ is the $s$-$d$ interaction strength. $\sigma_i$ is Pauli matrices, $m^*$ is the effective mass of the conduction electrons, and $k_i$ is the wave vector along $x$ direction.

The eigenvalue and eigenfunction of the single-particle Hamiltonian are given by

$$\varepsilon_{n,k} = \frac{\hbar^2 k^2}{2m^* d^2} + \frac{\hbar^2 k_i^2}{2m^*},$$

and

$$\Psi_{n,k,\sigma} = \Phi_n(y) \Phi_k(x) \eta_\sigma,$$

(2)

with $\Phi_n(y) = \sqrt{\frac{2}{d}} \sin(\frac{n\pi y}{d})$, $\Phi_k(x) = \frac{1}{\sqrt{2}} e^{ikx}$, where $L$ is the length of the quasi-one-dimensional system, and $\eta_\sigma$ is the spinor for electron spin.
In the second quantization representation, the Hamiltonian $H$ is written as

$$
H = \sum_{nk\sigma} \varepsilon_{nk\sigma} c_{nk\sigma}^\dagger c_{nk\sigma} - \frac{J}{r} \sum_{i=1,2} \sum_{mnk\mu
u} \Phi_m^*(y_i) \Phi_n(y_i) \times \delta - i(k-q)x_i \eta_{\mu}^+ \sigma_3 \eta_{\nu} \cdot S_i c_{mn\mu}^\dagger c_{n\nu},
$$

where $c_{nk\sigma}^\dagger$ and $c_{nk\sigma}$ are the creation and annihilation operators of the conduction electron with wave vector $k$ and subband index $n$. $x_i$ is the position of the ith localized spin in x direction.

Next we derive the RKKY interaction by using the second-order perturbation theory.\textsuperscript{15} The effective Hamiltonian of RKKY interaction is given by

$$
H_{\text{RKKY}} = \sum_{\Gamma} \left( \langle \Gamma | H_{s-d} | \Gamma \rangle \langle \Gamma | H_{s-d} | \Gamma_0 \rangle / (E_{\Gamma_0} - E_{\Gamma}) \right),
$$

where $H_{s-d}$ presents the $s-d$ coupling, $| \Gamma \rangle$ is the excited state with the energy $E_{\Gamma}$, and $| \Gamma_0 \rangle$ is the ground state with the energy $E_{\Gamma_0}$. As usual, the excited state is taken as one particle-hole excited state $| \Gamma \rangle = c_{n\eta\mu}^\dagger c_{n\eta\nu} | \Gamma_0 \rangle$, where $\mu, \nu$ satisfy $E_{\eta\mu} > E_F$ and $E_{\eta\nu} < E_F$, respectively. From Eqs. (3)-(4), we obtain the effective Hamiltonian

$$
H_{\text{RKKY}} = J_{\text{eff}}(r, y_1, y_2) S_1 \cdot S_2
$$

$$
J_{\text{eff}}(r, y_1, y_2) = \frac{4J^2}{L_x^2} \sum_{mnk\eta} \Phi_m(y_1) \Phi_m(y_2) \Phi_n(y_2) \times \cos[(k-q)x \eta_\mu \sigma_3 \eta_\nu \cdot S_i c_{mn\mu}^\dagger c_{n\nu}]
$$

where $r$ is the relative distance vector between the two localized spins given by $r = x_1 - x_2$, $f(E_{\eta\nu})$ is the Fermi-Dirac distribution function.

Eq.(6) is the main result of our present paper. The coupling strength $J_{\text{eff}}(r, y_1, y_2)$ is dependent on the transverse positions of the two local spins. It is due to the broken symmetry of lattice translation in transverse direction. In the longitudinal direction the translational symmetry hold so that the $J_{\text{eff}}(r, y_1, y_2)$ depends only on the relative distance $r$ between the two local spins in x direction. There is no magnetic coupling between the two local spins when the transverse positions satisfy $\sin[(k-q)x] = 0$. Another feature is the different subbands occupied by electrons have different contributions to the magnetic coupling. This leads to the summation of different oscillations existing.

The above formula may be generalized to the quasi-two-dimensional system and it has similarity of the size effect for the RKKY interaction.

III. RESULT AND DISCUSSION

Our numerical results of the RKKY interaction in nanoribbons at zero temperature is presented below.
electron density. With a fixed width of the nanoribbon, the larger the electron density is, the more subbands there are under the Fermi energy level. Thus the summation of the RKKY interaction strengths may be more complicated. For example in Fig. 2 the chain line in the vicinity of $k_F = 11 \text{ nm}$ shows that the RKKY interaction strength is approaching to zero while in the vicinity of $k_F = 15 \text{ nm}$ the strength turns out to have an obvious enhanced structure. Such cases of firstly-damping-then-enhancing structures can by no means take place in the conventional RKKY interaction. We ascribe this phenomenon to the coupling of subbands interactions with different periods. At last, we should point out that it is not necessary as shown in Fig. 2 that the strength amplitude is increasing with the electron density. We found that in certain situations, the interaction of subbands could weaken RKKY interaction strength in our model.

The dependence of the RKKY interaction on the width $d$ is more clearly demonstrated in Fig. 3. In this case, we fixed the electron density $n_e$, and the positions of the two impurities $y_1$ and $y_2$. Corresponding to the different widths $d = 20 \text{ nm}$, $50 \text{ nm}$, and $80 \text{ nm}$, there are one, two, and three occupied subbands under Fermi energy, respectively. For the situation $d = 20 \text{ nm}$, the numerical results goes back to the conventional RKKY interactions. As to $d = 50 \text{ nm}$, the final RKKY interaction strength is the summation of four terms concerning both intrasubbands and intersubbands interactions with each term represents an independent oscillation mode and decaying tendency. A more complicated situation of the RKKY interaction originated from nine term contributions takes place when $d = 80 \text{ nm}$. No matter how complex the situation is, the trend of the decaying oscillation still holds although the RKKY interaction strength may not have regular period.

Another important feature of the RKKY interaction in nanoribbons is strongly dependent on the transverse position of the impurities as shown in Fig. 4 and Fig. 5. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$.

FIG. 2: Indirect RKKY magnetic interaction strength versus the distance between two localized spins along $x$ direction at several different electron densities with fixed width $d = 80 \text{ nm}$.

FIG. 3: Indirect RKKY magnetic interaction strength versus the distance between two localized spins at several different width of nanoribbons under the same electron density $2.0 \times 10^{11} \text{ cm}^{-2}$.

FIG. 4: Indirect RKKY magnetic interaction strength versus the distance between two localized spins with nanoribbon width $d = 40 \text{ nm}$, electron density $n_e = 2.0 \times 10^{11} \text{ cm}^{-2}$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$. We fixed one impurity at $y_1 = 15 \text{ nm}$, and change the location of another impurity $y_2$.
location $y_2$ of the other impurity we conclude as follow. In Fig. 4 where $d = 40$ nm, only one subband is occupied under the Fermi Level. Like previous situation, it coincides with the conventional RKKY interaction, which has strictly periodical conformity, i.e. the location $y_2$ of the other impurity only affects the amplitude of the RKKY interaction strength, but has no influence on the periods of the oscillation. Moreover, the system possesses a transverse symmetry for $y_2$ due to our using the infinite quantum well as the transverse confinement on the nanoribbon in model building. This means for example that $y_2 = 10$ nm and $y_2 = 30$ nm share an identical figure. Same to $y_2 = 15$ nm and $y_2 = 25$ nm. However, in other situations when there are more subbands under the Fermi Level for instance $d = 50$ nm, see Fig. 5 the existence of subbands interactions bring about the transverse symmetry breaking of $y_2$. This can also be verified by one of the curves, see the dash line corresponding to $y_2 = 25$ nm in Fig. 5 which settles in the very middle of the nanoribbon. The figure is special in that it conforms with the conventional RKKY interaction strength. This result can be attributed to the special location of $y_2$ which leads to only one subband interaction term to be nonzero in the summation. This subband contribution alone is a conventional RKKY interaction form. Similar results are also spotted in other nanoribbon systems with different width.

IV. SUMMARY

We derived the RKKY interaction mediated by electrons in nanoribbon. Our theoretical results demonstrate that the RKKY interaction is strongly dependent on the width of the nanoribbon and the positions of impurities in transverse direction. The transverse confinement of electrons is responsible to the size effect of the RKKY interaction. It provides the potential way to control the RKKY interaction between the local spins by tuning geometry.

V. ACKNOWLEDGMENT

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