PHD Filter for Multipath Target Tracking Using Box Particle

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Abstract. This paper presents a novel box-particle multipath probability hypothesis density (BP-MPPHD) filter for the Over-the-horizon radar (OTHR) target tracking. The proposed filter combines the multipath probability hypothesis density (MPPHD) filter with box particle based on the interval analysis method to solve the problems of high computational complexity and non-linear measurement models in OTHR system. First, the OTHR measurement model is established based on the random finite set, and then the box-particle filter is used to derive the BP-MPPHD filter based on the original MPPHD filter. The simulation results show that the proposed BP-MPPHD filter can track the multipath target effectively with less computational time, compared to the particle implementation of the MPPHD filter for multipath target in OTHR system.

Keywords: Box-particle filter, PHD filter, OTHR, Tracking.

1. Introduction

Since the Over-the-horizon Radar (OTHR) exploits the ionosphere reflections the high frequency electromagnetic waves to detect and track targets, there are multiple propagation paths and multiple measurements will be generated from one target. The traditional tracking algorithms usually assume that there is only one measurement for one target. If the traditional tracking algorithms can properly use all the measurements from one target, the performance of tracking will be greatly improved.

In the literature, some traditional algorithms have been designed to solve the multipath problem, such as multipath probabilistic data association (MPDA) [1], Multiple Detection Multiple Hypothesis Tracker (MD-MHT) [2], the multipath viterbi data association (MVDA) [3] and the Multiple Detection Joint Probabilistic Data Association (MD-JPDA) Filter [4]. However, the traditional algorithms have the problem of complex data association and high computational complexity. Since the random finite set (RFS) algorithms can avoid the data association, which have attracted extensive attention recently. Many target tracking algorithms based on RFS have been proposed, such as the Bernoulli filter [5], the probability hypothesis density (PHD) filter [6-8], the cardinality balanced multitarget multi-Bernoulli (CBMeMBer) filter [9,10], and the labeled multi-Bernoulli (LMB) filter [11-14].

Recently, some RFS algorithms have been successfully applied to address the multipath target tracking problem in OTHR system. In order to deal with the problem of multipath single target tracking, the multipath Bernoulli filter based on the Bernoulli filter is proposed in [15]. For the problem of multipath multitarget tracking problem in OTHR, the first author has proposed multipath PHD (MPPHD) filter based on the standard PHD filter in [16]. Due to the measurement models are non-linear in OTHR system, particle filter method is usually used to solve the non-linear problems. However, the particle filter implementation has the problem of high computational complexity, which hard to meet the real-time tracking requirements. Recently, the box-particle filter has been proposed in [17], which uses interval analysis method to model the measurement as interval instead of the point observation. [18-20] showed that the box-particle filter can reach the similar performance compared to the particle filter with less computational time. Therefore, for the problem of the multipath multitarget tracking, this paper proposes a novel BP-MPPHD filter, which combines the advantages of MPPHD filter with interval analysis method to solve the problems of the non-linear measurement models and high computational complexity in OTHR system.
2. The Multipath PHD Filter in OTHR

2.1 Dynamic Model and Measurement Model in OTHR System

Figure 1 shows the geometry of the transmitter, receiver and target in OTHR system. The target state is defined by \( x_k = [\rho(k), b(k), \dot{\rho}(k), \dot{b}(k)] \), where \( \rho(k) = \rho_1(k) \), \( b(k) \), \( \dot{\rho}(k) \) and \( \dot{b}(k) \) denote the ground range, bearing, range rate and bearing rate respectively.

![Transmitter-receiver model in OTHR system](image)

Since the distance between the transmitter, receiver and target is large, we can presume a linear and discrete-time state equation with the form

\[
\begin{align*}
x_k &= F x_{k-1} + u_{k-1} \\
F &= \begin{bmatrix}
1 & T & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & T \\
0 & 0 & 0 & 1
\end{bmatrix}
\end{align*}
\]

where \( u_{k-1} \) is a white Gaussian noise, the state transition matrix \( F \) is

\[
F = \begin{bmatrix}
1 & T & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & T \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

where \( T \) is the sampling interval.

As shown in Fig.1, the OTHR system uses the ionosphere reflections the high frequency electromagnetic waves to detect target, it is usually assumed that there are only two ionospheres (E and F) for simplicity. Therefore, there are four possible propagation modes in OTHR system. The OTHR measurements include slant range \( R_g = r_1 + r_2 \), Doppler \( R_r \) and azimuth \( Az = \pi/2 - \theta \) of the form

\[
z_k = [R_g(\theta), R_r(\theta), Az(\theta)]', z_k \in \mathbb{R}, \text{ where } \mathbb{R} \text{ is space of slant coordinates.}
\]

Hence the measurement models can be expressed as

\[
z_k = \begin{cases}
h_1(x_k) + \omega_{k,1} \text{ if mode EE} \\
h_2(x_k) + \omega_{k,2} \text{ if mode EF} \\
h_3(x_k) + \omega_{k,3} \text{ if mode FF} \\
h_4(x_k) + \omega_{k,4} \text{ if mode FF} \\
\text{clutter otherwise}
\end{cases}
\]

where \( \omega_{k,j} \) is white Gaussian noise, \( h_j(\cdot) \) is measurement function for the \( j \)-th propagation mode

\[
h_j(x) = \begin{cases}
\frac{\rho}{4} \sqrt{\frac{\rho^2}{2} - d \sin(b) + h^2} \\
\frac{\rho}{4} \sqrt{\frac{\rho^2}{2} + d \sin(b) + h^2} \\
\frac{\rho - d \sin(b)}{4 \sqrt{\rho^2 + h^2}} \\
\frac{\rho - d \sin(b)}{4 \sqrt{\rho^2 - d \sin(b) + h^2}}
\end{cases}
\]
2.2 The RFS Measurement Model for OTHR System

The traditional tracking algorithms usually assume that there is only one measurement for one target at the same time. However, due to multipath effect one target may produce multiple measurements from different propagation paths in OTHR system. Therefore, the RFS measurement model of MPPHD filter is different from the standard PHD filter. It is necessary to give the multiple target RFS measurement model.

In OTHR system, a finite set measurement can be denoted as \( Z_k = \{ z_{k,1}, z_{k,2}, \cdots, z_{k,N_z} \} \), where \( z_{k,1}, z_{k,2}, \cdots, z_{k,N_z} \) are the received measurements. Since the OTHR uses multiple ionospheric reflection signals to detect and track targets. Thus, the multiple target RFS measurements can be given by

\[
Z_k = \Theta_{k,1}(x_k) \cup \Theta_{k,2}(x_k) \cup \Theta_{k,3}(x_k) \cup \Theta_{k,4}(x_k) \cup \Gamma_k
\]

(5)

where \( \Theta_{k,l}(x_k), l=1,\cdots,4 \) is the measurements from the \( l \)-th propagation path, \( \Gamma_k \) is the RFS of clutter. Note that the following algorithms are based on this multiple target RFS measurement model.

2.3 The MPPHD Filter

The multitarget tracking can be seen as a Bayesian filter process, which propagates the multitarget posterior density. The MPPHD filter is one of the Bayesian filter algorithms. As with standard Bayesian filter, the MPPHD filter also has two steps: the prediction and update.

The prediction process can be realized through the following equation:

\[
D_{k|x_{k-1}}(x) = \int p_{x|x}(\zeta)p_{x|x}(x|\zeta)D_{k-1|x_{k-1}}(\zeta|Z_{k-1})d\zeta
\]

\[
+ \int b_{x|x}(x|\zeta)D_{k-1|x_{k-1}}(\zeta|x_{k-1})d\zeta + \gamma_x(x)
\]

(6)

where \( p_{x|x}(\zeta) \) denotes the probability of the target still exists at the time \( k \), \( \gamma_x(\cdot) \) is the intensity function of spontaneous birth of new target, \( b_{x|x}(\cdot|\zeta) \) denotes the intensity function of the RFS of targets spawned from the previous state \( x_{k-1} \).

The update equation for the MP-PHD filter:

\[
D_{k|x}(x|Z^{(k)}) \equiv L_{Z_k}(x|Z^{(k-1)}) \cdot D_{k|x}(x|Z^{(k-1)})
\]

(7)

where

\[
L_{Z_k}(x|Z^{(k-1)}) = q_{Z_k}(x) + \sum_{\nu \in Z_k} \omega_{\nu} \cdot \sum_{w} b_{w} d_w
\]

(8)

\[
\omega_{\nu} = \frac{\prod_{\nu \in Z_k} d_{w}}{\sum_{\nu \in Z_k} \prod_{w} d_{w}}
\]

(9)

the details can be found in [16].

3. The Box-particle MPPHD Filter

Since the multipath target measurement models are non-linear, particle filter method is usually used to solve the non-linear problems. However, the particle filter implementation usually requires a high computational time, which is difficult to meet the real-time requirements in OTHR tracking system. This subsection will propose a novel BP-MPPHD filter, which combines the advantages of
particle filter MPPHD (P-MPPHD) filter and interval analysis to solve the problems of the non-linear measurement models and high computational complexity.

If the interval measurement vector is expressed as \([Z]_k = \{[z]_{i,1}, [z]_{i,2}, \cdots, [z]_{i,N_z}\}\), then the BP-MPPHD filter can be described as follows. Note that the BP-MPPHD filter prediction step is the same as the classic box-particle PHD filter, this subsection only gives the update step. The details of other steps can be found in [20].

We assume that the predicted box-particle weights \(w_{k-1}^{(i)}\) and box particles \([x^{(i)}_{k-1}]\) have been calculated at the prediction step, then according to the update equation of the MPPHD filter, its weight is updated by

\[
\tilde{w}_{k|k}^{(i)} = q_{D,4}^{u}([x^{(i)}_{k-1}]) + \sum_{\nu \in \mathcal{Z}_k} w_{\nu} \sum_{\nu \in Z_{\nu}} b_{\nu}([x^{(i)}_{k-1}]) \tilde{w}_{k|k-1}^{(i)}
\]

where

\[
\ell_{x,[i]}(x) = \frac{g_{x,[i]}([z] | [x^{(i)}_{k-1}])}{\lambda e([z])}
\]

where \(g_{x,[i]}([z] | [x^{(i)}_{k-1}])\) is the likelihood of \(i\)th propagation path.

\[
g_{x,[i]}([z] | [x]) = \int_{[z]} p_x(z - h_i([x])) dz = \frac{|[z] \cap (h_i([x]) + [\varepsilon])|}{|[\varepsilon]|}
\]

\[
d_{\nu} = \begin{cases} 
1 + q_{D,4}^{u} p_{D,4}^{u}(\sum_{j=1}^{4} \ell_{x,[j]} \cdot \tilde{w}_{k|k-1}^{\nu}) & \text{if } \nu = \{[z]_k\}, \\
q_{D,4}^{u} p_{D,4}^{u}(\sum_{j=1}^{4} \sum_{j=1}^{4} \sum_{j=1}^{4} \ell_{x,[j]} \cdot \ell_{x,[j]} \cdot \tilde{w}_{k|k-1}^{\nu}) & \text{if } \nu = \{[z]_k, [z]_2\}, \\
q_{D,4}^{u} p_{D,4}^{u}(\sum_{j=1}^{4} \sum_{j=1}^{4} \sum_{j=1}^{4} \sum_{j=1}^{4} \ell_{x,[j]} \cdot \ell_{x,[j]} \cdot \ell_{x,[j]} \cdot \ell_{x,[j]} \cdot \tilde{w}_{k|k-1}^{\nu}) & \text{if } \nu = \{[z]_k, [z]_2, [z]_3\}, \\
q_{D,4}^{u} p_{D,4}^{u}(\sum_{j=1}^{4} \sum_{j=1}^{4} \sum_{j=1}^{4} \sum_{j=1}^{4} \sum_{j=1}^{4} \ell_{x,[j]} \cdot \ell_{x,[j]} \cdot \ell_{x,[j]} \cdot \ell_{x,[j]} \cdot \ell_{x,[j]} \cdot \tilde{w}_{k|k-1}^{\nu}) & \text{if } \nu = \{[z]_k, [z]_2, [z]_3, [z]_4\}.
\end{cases}
\]

\[
b_{\nu} = \begin{cases} 
q_{D,4}^{u} p_{D,4}^{u}(\sum_{j=1}^{4} \ell_{x,[j]} \cdot \tilde{w}_{k|k-1}^{\nu}) & \text{if } \nu = \{[z]_k\}, \\
q_{D,4}^{u} p_{D,4}^{u}(\sum_{j=1}^{4} \sum_{j=1}^{4} \ell_{x,[j]} \cdot \ell_{x,[j]} \cdot \tilde{w}_{k|k-1}^{\nu}) & \text{if } \nu = \{[z]_k, [z]_2\}, \\
q_{D,4}^{u} p_{D,4}^{u}(\sum_{j=1}^{4} \sum_{j=1}^{4} \sum_{j=1}^{4} \sum_{j=1}^{4} \ell_{x,[j]} \cdot \ell_{x,[j]} \cdot \ell_{x,[j]} \cdot \tilde{w}_{k|k-1}^{\nu}) & \text{if } \nu = \{[z]_k, [z]_2, [z]_3\}, \\
q_{D,4}^{u} p_{D,4}^{u}(\sum_{j=1}^{4} \sum_{j=1}^{4} \sum_{j=1}^{4} \sum_{j=1}^{4} \sum_{j=1}^{4} \sum_{j=1}^{4} \ell_{x,[j]} \cdot \ell_{x,[j]} \cdot \ell_{x,[j]} \cdot \ell_{x,[j]} \cdot \ell_{x,[j]} \cdot \tilde{w}_{k|k-1}^{\nu}) & \text{if } \nu = \{[z]_k, [z]_2, [z]_3, [z]_4\}.
\end{cases}
\]
4. Numerical Simulations

In this subsection, the proposed BP-MPPHD filter’s performance is compared with the P-MPPHD filter through one multitarget tracking simulation scenario in OTHR system, and each filter is simulated with the same environment that described in the following.

In this tracking simulation scheme, we assume that three non-maneuvering targets are moving in a clutter environment, which appear and disappear at different times. We model the clutter as a Poisson random finite set and assume that there are five clutters in each scan. The simulation lasts 800 seconds with 20 seconds (T=20s) sampling period, and it is assumed that target 1 and target 2 appear throughout the whole tracking experiment with initial state \( x_1 = (1100\text{km}, 0.10472\text{rad}, 0.15\text{km/s}, 8.72665\text{e-05}\text{rad/s}) \) and \( x_2 = (1170\text{km}, 0.11472\text{rad}, -0.14\text{km/s}, 7.72665\text{e-05}\text{rad/s}) \) respectively, target 3 appears at the 8th scan (\( t=160\text{s} \)) and disappears at the 23th scan (\( t=460\text{s} \)) with the initial states \( x_3 = (1170\text{km}, 0.15701\text{rad}, -0.05\text{km/s}, -8.72665\text{e-05}\text{rad/s}) \). In this experiment, it is assumed that there are no spawning targets for simplicity. Each propagation modes have the same target survive probability (\( p_s =0.98 \)) and detection probability (\( p_d =0.6 \)). It models the interval measurements with an interval length \( \Delta = [\Delta \rho, \Delta b, \Delta \dot{\rho}, \Delta \dot{b}]' \), where \( \Delta \rho =60\text{m}, \Delta b=0.03^\circ, \Delta \dot{\rho} =0.01\text{m/s} \) and \( \Delta \dot{b}=0.001^\circ/\text{s} \) are the lengths of intervals in the ground range, bearing, range rate and bearing rate respectively. Other experiment parameters are same as [16]. Finally, the optimal subpattern assignment (OSPA) distance [21] is used to evaluate the tracking performance of the two algorithms. The three target true trajectory is shown in Fig. 2.

![Fig 2. The true target trajectory.](image)

![Fig 3. Average of OSPA distance for the P-MPPHD filter and BP-MPPHD filter](image)
Fig. 4. Average of estimated target number for the P-MPPHD filter and BP-MPPHD filter

| Algorithm     | Number of particles | Runtime (sec) |
|---------------|---------------------|---------------|
| P-MPPHD filter| 2500                | 351.42        |
| BP-MPPHD filter| 50                  | 121.32        |

Table 1. Comparison of runtime and the number of particles

Fig. 3 and Fig. 4 show the performance comparison results of the proposed BP-MPPHD filter and P-MPPHD filter with the estimated target number and average of the OSPA distance respectively. We can see that the performance of the BP-MPPHD filter is very similar to the P-MPPHD filter. It means that both algorithms can track the multipath target effectively in OTHR.

Table 1 shows a comparison of the computational time and the number of particles for the proposed BP-MPPHD filter and P-MPPHD filter. It demonstrates the P-MPPHD filter has used 2500 particles and the average computational time over 351s. However, the proposed BP-MPPHD filter uses 50 box particles and the average computational time 121s. It means that the proposed BP-MPPHD filter can reach the similar performance compared to the P-MPPHD filter with less computational time, and can deal with non-linear problems of multipath multitarget tracking effectively in OTHR.

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