Recent developments in the constituent quark model including quark-antiquark pairs

Elena Santopinto\(^1\) and Roelof Bijker\(^2\)

\(^1\) INFN, Sezione di Genova, via Dodecaneso 33, 16164 Genova, Italy
\(^2\) ICN-UNAM, AP 70-543, 04510 Mexico DF, Mexico

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Abstract. We present the formalism for a new generation of unquenched quark models for baryons in which the effects of quark-antiquark pairs (\(u\bar{u}, d\bar{d}\), and \(s\bar{s}\)) are taken into account in an explicit form via a microscopic, QCD-inspired, quark-antiquark creation mechanism. The present approach is an extension of the flux-tube breaking model of Geiger and Isgur in which now the contributions of quark-antiquark pairs can be studied for any initial baryon and for any flavor of the \(q\bar{q}\) pair. It is shown that the inclusion of \(q\bar{q}\) pairs leads to a large contribution of orbital angular momentum to the proton spin.

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1 Introduction

One of the main goals of hadronic physics is to understand the structure of the nucleon and its excited states in terms of effective degrees of freedom and, at a more fundamental level, the emergence of these effective degrees of freedom from QCD, the underlying theory of quarks and gluons \(^1\). Despite the progress made in lattice calculations, it remains a daunting problem to solve the QCD equations in the non-perturbative region. Therefore, one has to rely on effective models of hadrons, such as the constituent quark model (CQM).

There exists a large variety of CQMs, among others the Isgur-Karl model \(^2\), the Capstick-Isgur model \(^3\), the collective model \(^4\), the hypercentral model \(^5\), the chiral boson-exchange model \(^6\) and the Bonn instanton model \(^7\). While these models display important and peculiar differences, they share the main features: the effective degrees of freedom of three constituent quarks (\(qqq\) configurations), the \(SU(6)\) spin-flavor symmetry and a long-range confining potential. All of these models reproduce the mass spectrum of baryon resonances reasonably well. At the same time, they show very similar deviations for other properties, such as for example the photcouplings. Since the photcouplings depend mostly on the spin-flavor structure, all models have the same \(SU(6)\) structure in common, show the same behavior, e.g. the photcouplings for the \(\Delta(1232)\) are underpredicted by a large amount, even though their ratio is reproduced correctly. In general, the helicity amplitudes (or transition form factors) show deviations from CQM calculations at low values of \(Q^2\). As an illustration we show in Fig. 1 the transverse electromagnetic transition form factors of the \(D_{13}(1520)\) resonance for different CQMs. The problem of missing strength at low \(Q^2\) can be attributed to the lack of explicit quark-antiquark degrees of freedom, which become more important in the outer region of the nucleon.

Additional evidence for higher Fock components in the baryon wave function (\(qqq-\bar{q}q\) configurations) comes from CQM studies of the electromagnetic and strong decay.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Transition form factors for the \(D_{13}(1520)\) resonance. Experimental data are compared with theoretical predictions from the collective \(U(7)\) model \(^4\) (dotted line) and the hypercentral model \(^5\) (solid line).}
\end{figure}
widths of $\Delta(1232)$ and $N(1440)$, the spin-orbit splitting of $\Lambda(1405)$ and $\Lambda(1520)$, the low $Q^2$ behavior of transition form factors, and the large $\eta$ decay widths of $N(1535)$, $\Lambda(1670)$ and $\Sigma(1750)$. More direct evidence for the importance of quark-antiquark components in the proton comes from measurements of the $d/\bar{u}$ asymmetry in the nucleon sea \[8,9\] and parity-violating electron scattering experiments, which report a nonvanishing strange quark contribution, albeit small, to the charge and magnetization distributions \[10\].

The role of higher Fock components in the CQM has been studied in a series of papers by Riska \et al. \[11\] in which it was shown that an appropriate admixture of the lowest $q\bar{q}$ configurations may reduce the observed discrepancies between experiment and theory for several low-lying baryon resonances. The importance of mesonic contributions to the spin and flavor structure of the nucleon is reviewed in \[8,12\]. In the CQM based approach by Geiger and Isgur, the effects of quark-antiquark pairs were included in a flux-tube breaking model based on valence-quark plus glue dominance to which $q\bar{q}$ pairs are added in perturbation \[13,14\]. The latter approach has the advantage that the effects of quark-antiquark pairs are introduced into the CQM via a QCD-inspired pair-creation mechanism, which opens the possibility to study the importance of effects of $q\bar{q}$ pairs in baryon structure in a systematic and unified way.

The aim of the this contribution is to present a generalization of the flux-tube breaking model of \[14\]. The resulting unquenched quark model is valid for any initial baryon (or baryon resonance) and for any flavor of the quark-antiquark pair (not only $s\bar{s}$ as in \[14\], but also $u\bar{u}$ and $d\bar{d}$), and can be applied to any model of baryons and mesons. As a test of the formalism, we present some results in the closure limit. Finally, we discuss an application of the unquenched quark model to the spin of the proton. In a separate contribution, we present an application to the flavor asymmetry of the nucleon sea \[15\].

### 2 Unquenched quark model

In the flux-tube model for hadrons, the quark potential model arises from an adiabatic approximation to the gluonic degrees of freedom embodied in the flux tube \[16\]. The role of quark-antiquark pairs in meson spectroscopy was studied in a flux-tube breaking model \[13\] in which the $q\bar{q}$ pair is created with the $^3P_0$ quantum numbers of the vacuum. Subsequently, it was shown by Geiger and Isgur \[17\] that a miraculous set of cancellations between apparently uncorrelated sets of intermediate states occurs in such a way that they compensate each other and do not destroy the good CQM results for the mesons. In particular, the OZI hierarchy is preserved and there is a near immunity of the long-range confining potential, since the change in the linear potential due to the creation of quark-antiquark pairs in the string can be reabsorbed into a new strength of the linear potential, i.e. in a new string tension. As a result, the net effect of the mass shifts from pair creation is smaller than the naive expectation of the order of the strong decay widths. However, it is necessary to sum over large towers of intermediate states to see that the spectrum of the mesons, after unquenching and renormalizing, is only weakly perturbed. An important conclusion is that no simple truncation of the set of meson loops is able to reproduce such results \[17\].

The extension of the flux-tube breaking model to baryons requires a proper treatment of the permutation symmetry between identical quarks. As a first step, Geiger and Isgur investigated the importance of $s\bar{s}$ loops in the proton by taking into account the contribution of the six different diagrams of Fig. 2 with $q_1q_2q_3 = uud$ and $q\bar{q} = s\bar{s}$.

![Fig. 2. Quark line diagrams for A → BC with q1q2q3 = uud and q\bar{q} = s\bar{s}](image-url)

Fig. 2. Quark line diagrams for $A \rightarrow BC$ with $q_1q_2q_3 = uud$ and $q\bar{q} = s\bar{s}$.
tions are expressed in the basis of harmonic oscillator wave functions.

The ensuing unquenched quark model is based on an adiabatic treatment of the flux-tube dynamics to which $q\bar{q}$ pairs with vacuum quantum numbers are added as a perturbation [14]. The pair-creation mechanism is inserted at the quark level and the one-loop diagrams are calculated by summing over a complete set of intermediate states. Under these assumptions, to leading order in pair creation, the baryon wave function is given by

$$|\psi_A\rangle = N \left[ |A\rangle + \sum_{BClJ} \int dk \ |BCk lJ\rangle \times \frac{\langle BCk lJ | T^\dagger | A\rangle}{M_A - E_B - E_C} \right].$$  \hspace{1cm} (1)

Here $T^\dagger$ is the $^3P_0$ quark-antiquark pair-creation operator [14]. $A$ denotes the initial baryon and $B$ and $C$ the intermediate baryons and meson, $k$ and $l$ represent the relative radial momentum and orbital angular momentum of $B$ and $C$, and $J$ is the total angular momentum $J = J_B + J_C + l$.

In general, matrix elements of an observable $\hat{O}$ can be expressed as

$$\mathcal{O} = \langle \psi_A | \hat{O} | \psi_A \rangle = \mathcal{O}_{\text{val}} + \mathcal{O}_{\text{sea}},$$  \hspace{1cm} (2)

where the first term denotes the contribution from the valence quarks

$$\mathcal{O}_{\text{val}} = N^2 \langle A | \hat{O} | A \rangle$$  \hspace{1cm} (3)

and the second term that from the $q\bar{q}$ pairs

$$\mathcal{O}_{\text{sea}} = N^2 \sum_{BClJ} \int dk \sum_{B'C'l'J'} \int dk' \frac{\langle A | T | B'C'k'l'J'\rangle}{M_A - E_B - E_C} \langle B'C'k'l'J' | \hat{O} | BCk lJ\rangle \frac{\langle BCk lJ | T^\dagger | A\rangle}{M_A - E_B - E_C}. \hspace{1cm} (4)$$

As mentioned before, we developed an algorithm based upon group-theoretical techniques to generate a complete set of intermediate states of good permutational symmetry, which makes it possible to perform the sum over intermediate states up to saturation, and not just for the first few shells as in [14]. Not only does this have a significant impact on the numerical result, but it is necessary for consistency with the OZI-rule and the success of CQMs in hadron spectroscopy.

### 3 Closure limit

The evaluation of the contribution of the quark-antiquark pairs simplifies considerably in the closure limit, which arises when the energy denominators in Eq. (1) do not depend on the quantum numbers of the intermediate states. In this case, the sum over the complete set of intermediate states can be solved by closure and the contribution of the quark-antiquark pairs to the matrix element reduces to

$$\mathcal{O}_{\text{sea}} \propto \langle A | T \hat{O} T^\dagger | A \rangle.$$  \hspace{1cm} (5)

Moreover, if the pair-creation operator does not couple to the motion of the valence quarks, the valence quarks act as spectators. In this case, the contribution of the $q\bar{q}$ pairs reduces to the expectation value of $\mathcal{O}$ between the $^3P_0$ pair states created by $T^\dagger$

$$\mathcal{O}_{\text{sea}} \propto \langle 0 | T \hat{O} T^\dagger | 0 \rangle,$$  \hspace{1cm} (6)

the so-called closure-spectator limit. Especially when combined with symmetries, the closure limit not only provides simple expressions for the relative flavor content of physical observables, but also can give further insight into the origin of cancellations between the contributions from different intermediate states.

As an example, we discuss some results for the operator

$$\Delta q = 2 \langle S_z(q) + S_z(\bar{q}) \rangle,$$  \hspace{1cm} (7)

which determines the fraction of the baryon’s spin carried by the quarks and antiquarks with flavor $u$, $d$, and $s$. First, we consider the ground state decuplet baryons with $^410[56, 0^+]_{1/2}$ of Fig. 3. Since the valence-quark configuration of the $\Delta$ resonances does not contain strange quarks, the contribution $\Delta s$ of the $s\bar{s}$ pairs to the spin is given

![Fig. 3. Ground state decuplet baryons](image)

Table 1. Relative contributions of $\Delta u$, $\Delta d$ and $\Delta s$ in the closure limit to the spin of the ground state decuplet baryons

| $qqq$     | $^410[56, 0^+]$ | $\Delta u$ | $\Delta d$ | $\Delta s$ |
|-----------|-----------------|------------|------------|------------|
| uuu       | $\Delta^{++}$   | 9          | 0          | 0          |
| uud       | $\Delta^+$      | 6          | 3          | 0          |
| udd       | $\Delta^0$      | 3          | 6          | 0          |
| ddd       | $\Delta^-$      | 0          | 9          | 0          |
| uus       | $\Sigma^{++}$   | 6          | 0          | 3          |
| uds       | $\Sigma^+$      | 3          | 3          | 3          |
| dds       | $\Sigma^{0+}$   | 0          | 6          | 3          |
| uss       | $\Sigma^0$      | 3          | 0          | 6          |
| dss       | $\Xi^{++}$      | 0          | 3          | 6          |
| sss       | $\Omega^-$      | 0          | 0          | 9          |


by the closure-spectator limit which vanishes due to the properties of the $^3P_0$ wave functions. The same holds for the contribution of $dd$ pairs to the $\Delta^{++}, \Sigma^{++}, \Xi^{*0}$ and $\Omega^-$ resonances, and that of $uu$ pairs to the $\Delta^-, \Sigma^{*-}, \Xi^*$ and $\Omega^-$ resonances. In the closure limit the relative contribution of the quark flavors from the quark-antiquark pairs to the baryon spin is the same as that from the valence quarks

$$\Delta u_{\text{sea}} : \Delta d_{\text{sea}} : \Delta s_{\text{sea}} = \Delta u_{\text{val}} : \Delta d_{\text{val}} : \Delta s_{\text{val}} .$$  (8)$$

The latter property is a consequence of the spin-flavor symmetry of the ground state baryons and holds for both the octet with quantum numbers $^{10}[56, 0^+]$ and the decuplet with $^{10}[56, 0^+]$ (see Fig. 3). Table 1 shows the relative contributions of $\Delta u$, $\Delta d$ and $\Delta s$ to the spin of the ground state decuplet baryons in the closure limit. The results for the ground state octet baryons are given in Table 2. Finally, the results for the decuplet and octet baryons are related by

$$(\Delta u + \Delta d + \Delta s)_{\text{decuplet}} = 3 (\Delta u + \Delta d + \Delta s)_{\text{octet}} .$$  (9)$$

At a qualitative level, a vanishing closure limit helps to explain the phenomenological success of CQMs. As an example, the strange content of the proton which vanishes in the closure limit, is expected to be small, in agreement with the experimental data from parity-violating electron scattering (for the most recent data see [10]).

In addition, the results in Tables 1 and 2 impose very stringent conditions on the numerical calculations, since each entry involves the sum over a complete set of intermediate states. Therefore, the closure limit provides a highly nontrivial test of the computer codes which involves both the spin-flavor sector, the permutation symmetry, the construction of a complete set of intermediate states and the implementation of the sum over all of these states.

4 Proton spin

The unquenched quark model makes it possible to study the effect of quark-antiquark pairs on the fraction of the proton spin carried by quarks. Ever since the European Muon Collaboration at CERN showed that the total quark spin constitutes a rather small fraction of the spin of the nucleon [10], there has been an enormous interest in the spin structure of the proton [20]. The most recent value for the contribution of the quark spins is $33.0 \pm 3.9 \%$ [21]. The total spin of the proton is distributed among valence and sea quarks, orbital angular momentum and gluons

$$\frac{1}{2} = \frac{1}{2} (\Delta u + \Delta d + \Delta s) + \Delta L + \Delta G ,$$  (10)$$

where

$$\Delta q = \int_0^1 dx [q_1(x) + \bar{q}_1(x) - q_1(x) - \bar{q}_1(x)]$$  (11)$$

is the fraction of the proton’s spin carried by the light quarks and antiquarks with flavor $q = u, d, s$, $\Delta L$ and $\Delta G$ represent the contributions from orbital angular momentum and gluons, respectively. There is increasing evidence that the gluon contribution is small (either positive or negative) and compatible with zero [22][23], which indicates that the missing spin of the proton must be attributed to orbital angular momentum of the quarks and antiquarks.

In the unquenced quark model, there are no explicit gluons, so the last term $\Delta G$ is absent from the outset. The fraction of the spin of the proton carried by the quarks is obtained from

$$\Delta q = 2 (S_z(q) + S_z(\bar{q})) .$$  (12)$$

We carried out a calculation in which the parameters were taken from the literature [14][24], and found that a large part of the proton spin is due to orbital angular momentum. More specifically, the $q\bar{q}$ pairs contribute about half of the proton spin, of which one quarter is due to the spin of the sea quarks and three quarters to orbital angular momentum. Similar conclusions regarding the importance of the contribution of orbital angular momentum to the proton spin were reached in studies with meson-cloud models [25] and with axial exchange currents [20].

5 Summary, conclusions and outlook

In this contribution, we have discussed the importance of quark-antiquark pairs in baryon spectroscopy. We have proposed a method based on the flux-tube breaking model that was originally introduced by Kokoski and Isgur for mesons [13] and later extended by Geiger and Isgur to

![Fig. 4. Ground state octet baryons](image-url)
study the effects of $s\bar{s}$ loops in the proton [14]. Here, we have presented a new generation of unquenched quark models for baryons by including, in addition to $s\bar{s}$ loops, the contributions of $u\bar{u}$ and $d\bar{d}$ loops as well.

As a first application, we have applied the closure limit of the model - in which all intermediate states are degenerate - to the flavor decomposition of the spin of the ground state octet and decuplet baryons. It has been found that the relative contributions of the quark flavors from the $q\bar{q}$ pairs are the same as that of the valence quarks. The closure limit not only provides a stringent test of the numerical results, but also sheds light on the physical properties of (unquenched) quark models.

Next, the unquenched quark model has been applied to the problem of the spin crisis of the proton. It was shown that the inclusion of $q\bar{q}$ pairs leads to a relatively large contribution (about 40 %) of orbital angular momentum to the proton spin. In a separate contribution, we discuss an application to the flavor asymmetry of the nucleon sea [15].

The present formalism is, obviously within the assumptions of the approach, valid for any initial baryon, any flavor of the $q\bar{q}$ pairs and any model of hadron structure. As such, it holds great promise in its ability to address in a general and systematic way a large number of open problems in the structure and spectroscopy of light baryons such as, among others, the flavor asymmetry of the nucleon sea, the spin crisis of the proton, the electromagnetic and strong couplings, the electromagnetic elastic and transition form factors of baryon resonances, their sea quark content and their flavor decomposition [27].

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