Acoustic waves in multifractional gas mixture with the inclusion of different materials and dimensions without Phase Transformations

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Abstract. The propagation of acoustic waves in mixtures of gas and particle fractions of different materials and sizes is studied. A mathematical model is presented, the dispersion equation is obtained, dispersion curves are calculated. The influence of the particle size and the parameters of the dispersed phase for multifractional gas mixture with ice particles, aluminum and sand on dissipation and dispersion of sound waves is analyzed. A comparison with experiment is conducted.

1. Introduction
The study of acoustics and wave dynamics of multiphase media is of considerable theoretical and practical interest in view of the wide spread of such media in nature and their use in practice [1]. Basic models of wave dynamics of dispersed media and a number of results in this area are presented in [1]. Work [2] is devoted to the problems of the investigation of two-phase flows with solid particles, droplets and bubbles. In monography [3] a brief overview of the results on the study of acoustic disturbances in monodisperse gas mixtures without phase transformations is given. Influence of polydisperse composition of the gas mixture on the distribution of monochromatic disturbances in one-component mixtures of gas and particles, or vapor with droplets is studied in [4]. Peculiarities of the propagation of monochromatic waves in two-component polydisperse mixtures of gas and vapor and liquid droplets are investigated in [5]. Propagation of spherical and cylindrical waves of small amplitude in polydispersed fogs with phase transformations is considered in [6]. A general dispersion relation of the wave number to the oscillation frequency and thermal properties of the phases is obtained. The anomalous effect of nonmonotonic dependence of dissipation of weak harmonic and pulse disturbances on the mass concentration of droplets in monodisperse aerosols with heat and mass transfer is studied in [7]. Fairly complete presentation of the propagation linear theory of flat disturbances in mono-and polydisperse two-phase gas-vapor-liquid droplet mixtures is given in [8]. The propagation of acoustic waves of different geometry in two-fraction gas mixtures with particles

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of different sizes and materials, without taking phase transformations into account is studied in [9]. Peculiarities of plane, cylindrical and spherical waves of small amplitude in the gas-vapor-droplet mixtures with solid particles are analyzed in [10], [11], [12]. In this paper the propagation of plane, cylindrical, and spherical waves of small amplitude in multifractional mixtures of gas with an arbitrary number of solid particles of different sizes and substances with significantly different thermal properties is studied.

2. Basic equations and dispersion equation.

Similarly to [8], the linearized system of equations of disturbed motion of multifractional gas mixture with solid particles of different materials and sizes, in a coordinate system relative to which the undisturbed medium is at rest, can be written as

\[
\begin{align*}
\frac{\partial \rho'_i}{\partial t} + \rho_i \left( \frac{\partial v'_i}{\partial r} + \theta \frac{v'_i}{r} \right) &= 0, \\
\frac{\partial \rho'_j}{\partial t} + \rho_j \left( \frac{\partial v'_j}{\partial r} + \theta \frac{v'_j}{r} \right) &= 0, \quad (j = 2, N)
\end{align*}
\]

\[
\rho'_{10} \frac{\partial v'_i}{\partial r} + \frac{\partial p'_{10}}{\partial r} + \sum_{j=2}^{N} n_{ij} \left( 6\pi a_j \mu_j \left( v'_i - v'_j \right) + 6\alpha_j^2 \sqrt{\pi \mu_j \rho_j} \int_{-\infty}^{t} \frac{\partial}{\partial \tau} \left( v'_i - v'_j \right) d\tau \right) = 0,
\]

\[
\rho'_{j0} \frac{\partial v'_j}{\partial r} = n_{ij} \left( 6\pi a_j \mu_j \left( v'_i - v'_j \right) + 6\alpha_j^2 \sqrt{\pi \mu_j \rho_j} \int_{-\infty}^{t} \frac{\partial}{\partial \tau} \left( v'_i - v'_j \right) d\tau \right), \quad (j = 2, N)
\]

\[
\begin{align*}
\frac{\partial T'_{i0}}{\partial t} &= \alpha_{i0} \frac{\partial T'_{i0}}{\partial r} - \sum_{j=2}^{N} 2\pi n_{ij} a_j \lambda_j Nu_{ij} \left( T'_{i0} - T'_{j0} \right), \\
\frac{\partial T'_{j0}}{\partial t} &= -2\pi n_{ij} a_j \lambda_j Nu_{ij} \left( T'_{j0} - T'_{i0} \right), \quad (j = 2, N)
\end{align*}
\]

\[
\lambda_j Nu_{ij} \left( T'_{i0} - T'_{j0} \right) + \lambda_j Nu_{ij} \left( T'_{j0} - T'_{i0} \right) = 0, \quad (j = 2, N)
\]

\[
p'_i = \frac{C_i^2}{\gamma_i \alpha_{i0}} \rho'_i + \frac{p_0}{T_0} T'_{i0}.
\]

The system of equations (1) for the parameter \( \theta = 0 \) describes plane waves in a Cartesian coordinate system, when \( \theta = 1 \) - cylindrical waves in a cylindrical coordinate system, when \( \theta = 2 \) - spherical waves in a spherical coordinate system. Here and further, variables with subscript 1 refer to the carrier phase, with indexes \( 2, N \) to the particles of the dispersed phase of the radius \( a_j, (j = 2, N) \). Index 0 corresponds to the initial undisturbed state. Dashes above are used to denote disturbed parameters. Here and further \( \rho \) - reduced density, \( \rho' \) - true density, \( v \) - speed, \( \alpha \) - volume content, \( p \) - pressure, \( \mu \) - coefficient of the dynamic viscosity of the carrier medium, \( T \) - the temperature of the carrier phase, \( T_{j0} \) - the temperature in the surface \( \Sigma \) - layer of a particle of \( j \) - type, \( T_j \) - temperature of solid particles of \( j \) - type, \( Nu_{ij} \) - the dimensionless (Nusselt number) coefficient of the carrier phase heat transfer with the border of the interface of \( j \) - type particles, \( \lambda \) - the dimensionless (Nusselt number) coefficient of the heat transfer of \( j \) - type particles with the border of the interface, \( \gamma \) - coefficient of thermal conductivity.

The system of equations (1) is closed and can be used to investigate the propagation of acoustic disturbances in polydisperse mixtures of gas and solid particles of different thermal properties and dimensions in flat, spherical and cylindrical cases.

Let us introduce into the system of equations (1) the phase velocity potentials and explore the solutions of the resulting system of equations in the form of progressive waves for disturbances

\[
\eta_j = A_j \psi
\]
where
\[
\psi = \exp\left[i(Kr - \omega t)\right] - \text{for flat disturbances}
\]
\[
\psi = H_0^{(1)}(Kr) \exp(-i\omega t) - \text{for cylindrical disturbances}
\]
\[
\psi = \frac{1}{r} \exp\left[i(Kr - \omega t)\right] - \text{for spherical disturbances}
\]
\[
K_s = K + iK_{ss}, \quad C_p = \frac{\omega}{K}, \quad \sigma = 2\pi \frac{K_{ss}}{K}.
\]

Here \(A_0\) - the oscillation amplitude of parameters disturbances, \(K\) - the complex wave number, \(i\) - the imaginary unit, \(C_p\) - the phase velocity, \(K_{ss}\) - the linear attenuation coefficient, \(\sigma\) - the attenuation decrement on the wave length, \(H_0^{(1)}(z)\) - Hankel function, which is a combination of Bessel functions of the first and second kind of order zero \(J_0(z)\) and \(Y_0(z)\).

Substituting the velocity potentials and solutions of the form (2) in the system of equations (1), we obtain the following system of linear algebraic equations and from conditions of the existence of nontrivial solution of a system of linear algebraic equations the following dispersion equation is obtained
\[
\left(\frac{C_sK_s}{\omega}\right)^2 = V(\omega)D(\omega),
\]
where
\[
V(\omega) = 1 + \sum_{j=1}^{N} \frac{m_j}{1 - \omega \tau_{\tau_j}}, \quad \tau_{\tau_j}^s = \tau_{\tau_j}^r + \frac{m_j c_j}{c_{p1}} \tau_{\tau_j}^l,
\]
\[
D(\omega) = 1 + \left(\gamma_1 - 1\right) \sum_{j=1}^{N} \frac{m_j c_j}{c_{p1}} \frac{1}{1 - \omega \tau_{\tau_j}^s}.
\]

3. Results.
As an example the mixture of air and particles of aluminum of the radius \(r_a = 10^{-6} \text{ m}\), sand of the radius \(r_s = 8 \cdot 10^{-6} \text{ m}\) and ice of the radius \(r_i = 10^{-5} \text{ m}\). Fig. 1,2 illustrate the effect of inclusions mass content on the dependences of the relative speed of sound and attenuation decrement on the wavelength on the dimensionless oscillation frequency \(\omega \tau_{\tau_j}\) respectively. The calculated curves are built using the dispersion relation (10). The graphs show that with increasing mass content of particles the sound speed dispersion and dissipation of the waves increase. Accounting of the threefraction composition and difference of thermal parameters of fractions lead to the appearance of characteristic inflexions for the dependence of the relative speed of sound in the frequency field inversely proportional to the characteristic relaxation times of the phase velocities \(\tau_{\tau_a}, \tau_{\tau_b}\) and \(\tau_{\tau_i}\) (Fig. 3). As shown in Fig. 4, the difference between the sizes of inclusions and thermal parameters of particles fractions leads to appearance of three maxima for the dependence of attenuation decrement on the wavelength at the characteristic values, \(\omega \tau_{\tau_a}, \omega \tau_{\tau_i}\) and \(\omega \tau_{\tau_i} = 1\).
4. Comparison with experiment.
An experimental investigation of attenuation and dispersion of sound waves in a mixture of air with particles of aluminum of four different radii \( r_1 = 5 \times 10^{-6} \) m, \( n_1 = 9.81 \times 10^9 \) m\(^{-1} \), \( r_2 = 7.5 \times 10^{-6} \) m,
The graph shows that in the considered frequency range experimental and theoretical values agree satisfactorily. However, the experiment of Zink and Delsasso was conducted in a frequency range that does not allow to show the presence of the maximum attenuation on the wavelength and maximum dispersion.

5. Conclusion.
This paper presents a closed system of linear differential equations of motion for multifractional mixture of gas and solid particles of different sizes and thermal properties. A dispersion equation, determining the propagation of plane, spherical and cylindrical disturbances of small amplitude derived. The dispersion curves are calculated. The influence of the parameters of the dispersed phase for three-fraction gas mixture with aluminum particles, carbon black, and ice on the dissipation and dispersion of sound waves is analyzed. It is found that the difference of thermal properties and inclusions sizes significantly affects on the dynamics of weak waves in multifractional gas mixtures and it must be considered in the development of methods of acoustic diagnostics of considered environments. Good agreement of theoretical and experimental data is shown.

Acknowledgments
This work was funded by the subsidy of the Russian Government to support the Program of Competitive Growth of Kazan Federal University among World’s Leading Academic Centers.
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