ARTICLE
Proposal of Single Sideband Modulation Scheme Using Frequency Domain Filtering

Hiroaki Waraya* Masahiro Muraguchi
Department of Electrical Engineering, Tokyo University of Science, Japan

ARTICLE INFO
Article history
Received: 16 August 2021
Accepted: 24 August 2021
Published Online: 10 September 2021

Keywords:
SSB
Hilbert transform
Single carrier transmission
Digital filter
Roll-off rate

ABSTRACT
With the rapid development of wireless systems, the demand for frequency resources has been increasing in recent years. Therefore, it is necessary to consider the high-quality communication method that efficiently utilizes finite frequency resources. In this paper, Single Sideband 16 Pulse Amplitude Modulation (SSB 16PAM) scheme for the uplink communication is proposed. It transmits data in only Lower Sideband (LSB) without extra Hilbert components. Under Additive White Gaussian Noise (AWGN) channel environment, Bit Error Rate (BER) performance of the proposed scheme is superior by 3 dB in terms of Carrier-to-Noise Ratio (CNR) to 256 Quadrature Amplitude Modulation (256QAM) scheme with the same frequency efficiency and the same Peak-to-Average Power Ratio (PAPR).

Our proposed scheme employs the original frequency domain filter on the transmitter side to form an ideal spectrum. The configuration of its process is almost similar to Single Carrier-Frequency Division Multiple Access (SC-FDMA), moreover, half of the input data on the frequency domain is removed. The proposed frequency domain filter produces the SSB-modulated spectrum with a roll-off rate of zero without degrading the BER performance.

1. Introduction

Recently, the frequency resources of wireless systems are being depleted, so the high-priority issue for the subsequent wireless systems is to make a revolutionary modulation scheme with higher frequency efficiency and higher quality. In order to address this issue, several researchers in recent years have shown an interest in SSB modulation scheme combined with the quadrature modulation scheme.

The SSB modulation scheme sends data at half the occupied bandwidth compared to that of the Double Sideband (DSB) system. The SSB signal can be produced by combining the Hilbert transform and quadrature multiplexing, and its modulation scheme is only effective in scalar modulation \[1,2\]. In contrast, quadrature multiplexing scheme, which is a typical DSB modulation method, employs two carrier waves of the same frequency that are out of phase with each other by 90°. If the SSB modulation can be incorporated with the quadrature modulation, the spectrum efficiency might be twice as high as the conventional scheme. Unfortunately, they are not independent of each other as both modulations use the same signal processing for quadrature multiplexing. The in-phase component comprises the I-data and Hilbert transform of Q-data, and the quadrature component

*Corresponding Author:
Hiroaki Waraya,
Department of Electrical Engineering, Tokyo University of Science, Japan;
Email: 4319583@alumni.tus.ac.jp

DOI: https://doi.org/10.30564/jeisr.v3i2.3569
comprises Q-data and Hilbert transform of I-data on the receiver side. Thus, lossless demodulation cannot be performed analytically because of extra Hilbert components.

In this paper, SC-FDMA SSB 16PAM using the frequency domain filter is proposed, which does not require consideration of the problem of extra Hilbert components. It employs scalar modulation, so it can be SSB-modulated without extra Hilbert components. In addition, our proposed scheme employs the frequency domain filter on the transmitter side to form an ideal spectrum. The configuration of its process is almost similar to SC-FDMA, moreover, half of the input data on the frequency domain is removed to form SSB-modulated spectrum. The advantages of the proposed frequency domain filter are that it can form the spectrum with a roll-off rate of zero without the penalty of the BER performance [1], and that PAPR is improved over the conventional SSB modulation scheme. The proposed SC-FDMA SSB 16PAM reduces the peak power generated by the Hilbert transform and has the same PAPR as the SC-FDMA 256QAM with the same frequency efficiency.

Currently, the SSB processing is used for the downlink communication and not for the uplink communication, so the proposed scheme is expected to improve the quality of the single carrier transmission for the uplink communication. Under AWGN channel environment, BER performance of the proposed scheme is superior by 3 dB in terms of CNR to 256QAM scheme with the same frequency efficiency and the same PAPR. In addition, the same beneficial effect can be obtained with SC-FDMA SSB M-PAM modulation scheme.

2. Related Works

Several researchers have shown research results on the Quadrature SSB (Q-SSB) modulation [3-6]. It is expected that the spectrum efficiency can be doubled by multiplying the SSB modulation and the quadrature modulation. The transmission signal of Q-SSB modulation scheme is represented as follows:

\[ S_q(t) = \{I_q(t) + \tilde{Q}_q(t)\} \cos 2\pi f_c t + \{-I_q(t) + Q_q(t)\} \sin 2\pi f_c t. \]  

(1)

where \( \tilde{Q}_q(t) \) and \( I_q(t) \) are Hilbert components.

However, each of the in-phase component and the quadrature component contains the extra Hilbert component on the receiver side, and no method has been considered to analytically remove its component completely. Therefore, the Q-SSB signal cannot be properly demodulated by the normal method. The demodulation method with the strong equalizer such as the turbo equalizer can suppress the extra Hilbert component and the demodulation can be performed without significantly degrading the BER performance, but it becomes difficult to form an ideal spectrum with a roll-off rate of zero [7]. In addition, the Q-SSB modulation scheme is vulnerable to phase-error due to the effect of extra Hilbert components. The BER performance deteriorates under the fading channel environment. In order to avoid that problem, the 4ASK-based SSB modulation scheme in which only the quadrature component is used as the Hilbert component was proposed [1].

On the other hand, the SSB signal also has the drawback that the PAPR of the time domain signal is significantly deteriorated due to the Hilbert transform on the transmitter side. When a high peak signal is input to the non-linear device such as a transmitter power amplifier, the output signal is distorted. Its distortion causes the deterioration of the transmission quality and the increase of out-of-band radiation to adjacent systems. Although it is possible to reduce the distortion for large peak signals by using the device with excellent input/output characteristics, this method leads to the increased power consumption.

From the above, the main issues of the SSB modulation scheme are to solve the problem of the extra Hilbert components, to form the practical spectrum without deterioration of the BER performance, and to improve the PAPR that is degraded by the Hilbert transform.

3. SSB Modulation

This section describes how SSB modulation is applied. The details of the modulation method are explained in other articles [1,8,9]. In this paper, the detail of the spectrum transition of the SSB modulation on the transmitter side is described. Figure 1 depicts the spectrum transition of the SSB modulation [10,11]. The Hilbert transform depicted in Figure 1 is the method of delaying the phase by 90° at the positive frequency domain and advancing the phase by 90° at the negative frequency domain. As depicted in Figure 1, \( S_{\text{USB}}(t) \) and \( S_{\text{LSB}}(t) \) of the transmission signal is represented as follows:

\[ S(f) = S_{\text{I}}(f) \pm S_{\text{Q}}(f)S(f) = S_{\text{I}}(f) \pm S_{\text{Q}}(f). \]  

(2)

In (2) and Figure 1, when \( S_{\text{I}}(f) \) and \( S_{\text{Q}}(f) \) are added, SSB modulation by LSB is performed; on the contrary, when they are subtracted, SSB modulation by USB is performed.

Here, the spectrum transition as indicated in (2) is explained in detail. The SSB-modulated transmission signal \( S(t) \) using Hilbert transform is represented as
follows:

\[ S(t) = S(t) \pm S_0(t) \]

\[ = m(t) \cdot \cos 2\pi f_c t \pm \tilde{m}(t) \cdot \sin 2\pi f_s t, \] \hspace{1cm} (3)

where \( m(t) \) is the modulated signal, \( f_c \) is the carrier frequency. The relationship between \( m(t) \) and \( M(f) \) is represented as follows:

\[ m(t) \Leftrightarrow M(f) = \begin{cases} M_+(f), & f > 0 \\ M_-(f), & f < 0 \end{cases} \] \hspace{1cm} (4)

On the other hand, the relationship between \( \tilde{m}(t) \) and \( \tilde{M}(f) \) is represented as follows:

\[ \tilde{m}(t) \Leftrightarrow \tilde{M}(f) = \begin{cases} -jM_+(f), & f > 0 \\ jM_-(f), & f < 0 \end{cases} \] \hspace{1cm} (5)

From (3), (4), and (5), \( S_1(f) \) and \( S_2(f) \) are represented as follows:

\[ S_1(f) = \frac{1}{2} \{ M_+(f - f_c) + M_-(f + f_c) \} \]
\[ + \frac{1}{2} \{ M_+(f + f_c) + M_-(f - f_c) \} \] \hspace{1cm} (6)

\[ S_2(f) = \frac{1}{2} \{ -M_+(f - f_c) + M_-(f + f_c) \} \]
\[ + \frac{1}{2} \{ M_+(f + f_c) + M_-(f - f_c) \} \] \hspace{1cm} (7)

Therefore, the LSB signal \( S_{\text{LSB}}(f) \) and the USB signal \( S_{\text{USB}}(f) \) depicted in Figure 1 are represented as follows:

\[ S_{\text{LSB}}(f) = S_1(f) + S_2(f) \]
\[ S_{\text{USB}}(f) = S_1(f) - S_2(f) \] \hspace{1cm} (8)

\[ S_{\text{LSB}}(f) = S_1(f) + S_2(f) \]
\[ S_{\text{USB}}(f) = S_1(f) - S_2(f) \] \hspace{1cm} (9)

From (6), (7), (8) and (9), it can be seen that the SSB modulation scheme can transmit data at half the occupied bandwidth as compared with the DSB modulation scheme.

4. SC-FDMA

Since our proposed method assumes single-carrier transmission in uplink communication, SC-FDMA, which is important for explaining the proposed method, is described in this section. Figure 2 depicts the configuration around the Discrete Fourier Transform-Spread-Orthogonal Frequency Division Multiplexing (DFT-s-OFDM) used in SC-FDMA.

As depicted in Figure 2, the transmission signal is first mapped as a single carrier with low PAPR and the data is transformed into frequency components by Fast Fourier Transform (FFT). The signal on the frequency domain by the FFT is represented as follows:

\[ F(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} f(n) \cdot \exp \left( \frac{-j2\pi nk}{N} \right), \] \hspace{1cm} (10)

where \( k = 0, 1, ..., N-1 \), \( F[k] \) is the frequency-domain signal, and \( f(n) \) is the time-domain signal. The 4-times oversample is performed to form a spectrum with a roll-off rate of zero \(^{(1)}\). The reason why the position to insert nulls is in the middle of the symbol is that the data position in the frequency domain of the FFT block in MATLAB/Simulink is as depicted in Figure 3. The FFT size in Figure 3 is 256.

As depicted in Figure 3, the nulls in the 4-times oversample are arranged outside the original data, so that the side lobe of the transmission spectrum can be suppressed. In this paper, the processing to insert nulls on the frequency domain is called the frequency domain filtering.

The frequency domain filter is equivalent to multiplying a rectangular wave in the frequency domain, as depicted in Figure 4. The signal on the frequency domain after 4-times oversample is represented as follows:

\[ F_{\text{f}}(f) = F(f) \cdot \text{rect}(f), \] \hspace{1cm} (11)

where

\[ \text{rect}(f) = \begin{cases} 0, & N < |f| \leq 4N \\ 1, & |f| \leq N \end{cases} \] \hspace{1cm} (12)

The inverse Fourier transform of (12) is represented as

\[ \int_{-\pi}^{\pi} \text{rect}(f) \cdot \exp \left( \frac{2\pi nf}{T} \right) df = \frac{1}{\pi T} \sin \left( \frac{\pi f}{T} \right), \] \hspace{1cm} (13)

where \( N=1/2T \), and \( T \) is the sampling interval. If \( f_{\text{g}}(t) \) is the inverse Fourier transform of \( F_{\text{f}}(f) \), then

\[ f_{\text{g}}(t) = f(t) \cdot \frac{1}{\pi} \sin \left( \frac{\pi f}{T} \right) \]
\[ = \int_{-\infty}^{\infty} f(t) \cdot \frac{1}{\pi} \sin \left( \frac{\pi f}{T} \right) df. \] \hspace{1cm} (14)
As $f_R(t)$ is the discrete signal, $t = nT$. Then,

$$f_R(t) = \sum_{n=-\infty}^{\infty} f(nT) \frac{1}{\pi(t-nT)} \sin \left( \frac{\pi(t-nT)}{T} \right),$$

where $t = kT/4$; IFFT size increases to 4N, which is the size of 4-times the oversample. As indicated in (15), the time-domain signal after the frequency domain filter is represented by the sum of the sinc function. As shown in the red waveform in Figure 5, the output signal is formed so as to smoothly complement between the original signals.

Figure 6 depicts the actual time-domain signal after the frequency domain filtering. The red waveform is the signal before the frequency domain filtering, and the blue waveform is the signal after the frequency domain filtering. As indicated in Figure 6, the oversampling interpolates the part of the transient response between the original signal points, i.e., the 4-times oversample interpolates three points between two signal points.

**Figure 2.** Configuration of DFT-s-OFDM

**Figure 3.** The data position of the FFT block

**Figure 4.** Spectrum with the digital filter

**Figure 5.** Time domain signal $f_R(t)$, as in (15)

**Figure 6.** The signal after the frequency domain filtering

5. Proposed Method

In this section, our proposed SSB modulation scheme using the frequency domain filtering is described. In the proposed method, the SSB modulation processing is incorporated into the frequency domain filtering, by utilizing the complex conjugate centered on Direct Current (DC) component of the spectrum of the Amplitude-Shift Keying (ASK).

On the transmitter side, the signal $F[k]$ when FFT is applied is represented as (10). Here, assuming that the ASK signal is $f(t)$, the frequency component $F[k]$ is the complex conjugate centered on N/2 as depicted in Figure 7 since $f(t)$ does not have the quadrature component. In order to perform SSB modulation processing on the frequency domain filtering, only the data $S(k)$ with $k = 0 \sim N/2$ is used from $F(k)$ as depicted in Figure 8, and the other data are removed by multiplying by zero. Then, when $S(k)$ is converted into the time domain signal by IFFT, the original signal $f(t)$ is generated on the in-phase component and the Hilbert component $\tilde{f}(t)$ is generated on the quadrature component.

In this way, the spectrum of the SSB modulation signal is formed after the quadrature multiplexing. On the other hand, in the case of the symbol of QAM containing the quadrature component, the signal on the frequency
domain is asymmetric centered on DC so that the SSB modulation processing cannot be performed. In this paper, the frequency domain filtering is contained both the above SSB processing and 4-times oversample. The part of the proposed transmitter configuration is depicted in Figure 9.

On the receiver side, the spectrum copy is introduced as a demodulation method of Received signal $R[k]$. As can be seen from Figure 10, the spectrum copy complements the complex conjugate data of the original received signal $R[k]$ with the part removed by the transmitter side. Since the in-phase component is the original signal $f(t)$ on the receiver side, the demodulation is possible without implementing the spectrum copy that uses the quadrature component. However, the spectrum copy is needed to improve the CNR by 3dB on the receiving side. The reason for the improvement in CNR is that the proposed demodulation method is almost the same as the DB-SSB modulation scheme.[12]. The variance $\sigma^2$ is doubled and the average amplitude $E_s$ of the signal is also doubled in AWGN channel environment by implementing the spectrum copy that mixes the in-phase component and the quadrature component. Therefore, $|E_s|^2/|\sigma|^2$ is improved by 3dB.

In addition, The SSB processing by the proposed frequency domain filtering improves PAPR over the conventional SSB modulation scheme. Figure 11 compares the 4-times oversampled baseband time domain signals of the conventional SSB modulation method and the proposed SSB modulation method. Since the SSB processing by the frequency domain filtering can be performed before D/A conversion, the peak power generated by SSB modulation processing is reduced.

6. Results

In this paper, the spectrum, PAPR, BER performance of the SSB 16PAM signal using the frequency domain filtering are confirmed. Under AWGN channel environment, BER performance of the proposed scheme is superior by 3 dB in terms of CNR to DSB 256QAM scheme with the same frequency efficiency and same PAPR.

6.1 Simulation Specification

A simulation using MATLAB/Simulink was performed for our proposed method. Table 1 illustrates the simulation specifications used in this research. As illustrated in Table 1, the SSB 16PAM is compared with the DSB 256QAM. Similarly, other PAM-based SSB modulation methods are compared.
Figure 12 depicts a block diagram of the proposed system on the transmitter side. When evaluating the spectrum and BER performance, the proposed SC-FDMA SSB 16PAM depicted in (a) and SC-FDMA DSB 256QAM depicted in (b) are compared. On the other hand, when evaluating PAPR, the proposed SSB 16PAM depicted in (a) and the conventional SSB 16PAM depicted in (c) are compared. In this paper, Complementary Cumulative Distribution Function (CCDF) is used to evaluate PAPR. CCDF is represented as follows:

$$P \left( \text{PAPR} > \text{PAPR}_0 \right) = 1 - \int_{-\infty}^{\text{PAPR}_0} \varphi_a(x) \, dx \quad (16)$$

where \( \varphi_a(x) \) is the probability density function of PAPR. CCDF indicates the probability that PAPR above a certain value \( \text{PAPR}_0 \) is present in the symbol.

Figure 13 depicts a block diagram of the proposed system on the receiver side. The interleave function in Figure 13 regenerates complex conjugate components which were removed in the transmitter side. Here, the regeneration is performed by permutation of the first half of \( N \) point FFT components as shown in Figure 10.

**Table 1. Simulation Specification**

| Parameter            | Proposed system | Comparison system |
|----------------------|-----------------|-------------------|
| Primary Modulation   | (4PAM, 8PAM),   | (16QAM, 64QAM),   |
|                      | 16PAM           | 256QAM            |
| Secondary Modulation | SC-FDMA with SSB| SC-FDMA           |
| Modulation           | Modulation      |                   |
| FFT Size             | 64              | 64                |
| IFFT Size            | 256             | 256               |
| Data Rate            | 4Mbps           | 4Mbps             |
| Carrier Frequency    | 40.25 MHz       | 40 MHz            |
| Channel Model        | AWGN            | AWGN              |

### 6.2 Simulation Results

Figure 14 depicts the spectrum of the SC-FDMA SSB 16PAM and SC-FDMA DSB 256QAM. Figure 15 depicts CCDF of the proposed method. Figure 16 depicts the BER performance in the CNR of the proposed method. Figure 17 depicts the BER performance in the case of other modulation orders.
As observed from Figure 14, it can be seen that the proposed method forms the spectrum with roll-off rate of zero. In the spectrum of the proposed method, the left side of the main lobe suppresses the side lobe by the 4-times oversample, and the right side suppresses the side lobe by SSB modulation processing. Since the radiation level of the side robe is lower than 40dBm, the spectrum of the proposed method is considered to satisfy the spectrum mask that defines the upper limit of the allowed radiation level. In addition, since the spectrum bandwidths of SC-FDMA SSB 16PAM and SC-FDMA DSB 256QAM are the same and have the same data rates from Table I, it can be confirmed that those spectrum efficiencies are the same.

As observed from Figure 15, focusing on PAPR with respect to 0.01 of CCDF, it can be seen that the proposed SSB modulation method is 2dB superior to the conventional SSB modulation method that performs Hilbert transform after D/A conversion. The proposed method has PAPR equivalent to SC-FDMA DSB 256QAM by suppressing the peak signal generated by Hilbert transform, which is a drawback of SSB modulation scheme.

As observed from Figure 16, focusing on 0.001 of BER, it can be seen that the SC-FDMA SSB 16PAM has 3dB better BER performance under the AWGN environment in CNR than the SC-FDMA DSB 256QAM. The reason why the BER performance of the proposed method is superior to that of the comparison method is that the proposed method implements the spectrum copy on the receiver side. When the spectrum copy is performed on the received signal, the data is supplemented to the part on the frequency domain that was removed on the transmitting side, so the signal amplitude after the spectrum copy is doubled. In contrast, the noise applied to the receiving side is different between the in-phase component and the quadrature component, and the variance is doubled at the time of spectrum copy. Therefore, CNR of the proposed method can be improved by 3dB.

As shown in Figure 17, it is confirmed that the BER performance of the proposed SSB M-PAM methods is 3dB superior to that of the corresponding DSB M-QAM methods even if the number of the modulation order changes.

Finally, the proposed method and the Q-SSB modulation method described in Section II are compared. Since the Q-SSB modulation method using the turbo equalization contains Hilbert components in each of the in-phase component and the quadrature component, the eye pattern is significantly deteriorated and the BER performance is also deteriorated when the roll-off rate is low. However, in the proposed method, the original signal and its Hilbert component are separated into the in-phase component and the quadrature component. Therefore, it can form the practical spectrum with a roll-off rate of zero without deteriorating the BER performance.

![Figure 14. Spectrum of the proposed method](image)

![Figure 15. CCDF of the proposed method](image)

![Figure 16. BER performance of SSB 16PAM and DSB 256QAM](image)
7. Conclusions

In this Paper, 16PAM-based SSB modulation scheme using the frequency domain filtering is proposed. The proposed method prevents the deterioration of the quality due to the extra Hilbert component, and suppresses the peak power generated by the Hilbert transform. It has been confirmed that the BER performance of the SC-FDMA SSB 16PAM is superior by 3dB to the SC-FDMA DSB 256QAM for the same spectral efficiency and PAPR. Its characteristic of the proposed method is the same even if the modulation order changes. Therefore, in the case of a single-carrier transmission, it is considered that the PAM-based SSB modulation scheme is superior in terms of the quality to the QAM-based DSB modulation scheme.

References

[1] H. Waraya and M. Muraguchi, “Proposal of Single Sideband Modulation Scheme with Ideal Low Pass Filter for Wireless Communication Systems [J],” International Journal on Advances in Telecommunications, VOL. 13, NO. 3&4, pp. 33-42.

[2] H. Waraya and M. Muraguchi, “Proposal of a Quadrature SSB modulation Scheme for Wireless Communication Systems [D],” The Nineteenth International Conference on Networks (ICN2020), pp. 1-6, 2020.

[3] B. Pitakdumrongkija, H. Suzuki, S. Suyama, and K. Fukawa, “Single Sideband QPSK with Turbo Equalization for Mobile Communications [J],” 2005 IEEE 61st Vehicular Technology Conference, pp. 538-542, May 2005.

[4] Y. Jiang, Z. Zhou, M. Nanri, G. Ohta, and T. Sato, “Performance Evaluation of Four Orthogonal Single Sideband Elements Modulation Scheme in Multi-Carrier Transmission Systems [J],” 2011 IEEE 74th Vehicular Technology Conference, pp. 1-6, September 2011.

[5] Y. Jiang, Z. Zhou, M. Nanri, G. Ohta, and T. Sato, “Inter-Signal Interference Cancellation Filter for Four-Element Single Sideband Modulation [J],” 2012 75th Vehicular Technology Conference, pp. 1-5, 2012.

[6] A. M. Mustafa, Q. N. Nguyen, T. Sato, and G. Ohta, “Four Single-Sideband M-QAM Modulation using Soft Input Soft Output Equalizer over OFDM [J],” The 28th International Telecommunication Networks and Applications Conference (ITANAC 2018), pp. 1-6, November 2018.

[7] B. Pitakdumrongkija, H. Suzuki, S. Suyama, and K. Fukawa, “Coded Single-Sideband QPSK and Its Turbo Detection for Mobile Communication Systems [J],” IEEE Transactions on Vehicular Technology, VOL. 57, NO. 1, pp. 311-323, January 2008.

[8] X. Wang, M. Hanawa, and K. Nakamura, “Sideband Suppression Characteristics of Optical SSB Generation Filter with Sampled FBG Based 4-taps Optical Hilbert Transformer [J],” 15th APCC, pp. 622-625, 2009.

[9] C. C. Tseng and S. C. Pei, “Design of discrete-time fractional Hilbert transformer [J],” IEEE International Symposium on Circuits and Systems, pp. 525-528, May 2000.

[10] K. Takao, N. Hanzawa, S. Tanji, and K. Nakagawa, “Experimental Demonstration of Optically Phase-Shifted SSB Modulation with Fiber-Based Optical Hilbert Transformers [J],” OFC/NEOECC, 2007.

[11] J. G. R. C. Gomes and A. Petraglia, “A switched-capacitor DSB to SSB converter using a recursive Hilbert transformer with sampling rate reduction [J],” ISCAS 2000, pp. 315-318, 2000.

[12] S. A. Mujtaba, “A Novel Scheme for Transmitting QPSK as a Single-Sideband Signal [J],” IEEE GLOBECOM 1998, pp. 592-597, 1998.