The Initial Mass Function as given by the fragmentation

LORENZO ZANINETTI
Dipartimento di Fisica Generale, Via Pietro Giuria 1, 10125 Torino, Italy

Received; accepted; published online

Abstract. The dichotomy between a universal mass function (IMF) and a variable IMF which depends on local physical parameters characterises observational and theoretical stellar astronomy. In this contribution the available distributions of probability are briefly reviewed. The physical nature of two of them, gamma variate and lognormal, is then explained once the framework of the fragmentation is introduced. Interpolating techniques are then applied to the sample of the first 10 pc and to the open cluster NGC6649: in both cases lognormal distribution produces the best fit. The three power law function has also been investigated and visual comparison with an artificially generated sample of 100000 stars suggests that the variations in the spectral index are simply due to the small number of stars available in the observational sample. In order to derive the sample of masses, a new formula that allows us to express the mass as a function of the absolute magnitude and (B-V) for MAIN V , GIANTS III and SUPERGIANTS I is derived.

Key words: stars: formation ; stars: statistics ; methods: data analysis ; techniques: photometric

1. Introduction

The first analytical expression adopted to explain the low mass distribution of stars, see Salpeter (1955) was a power law of the type $\xi(M) \propto M^{-\alpha}$ where $\xi(M)$ represents the probability of having a mass between $M$ and $M + dM$; this is called initial mass function, in the following IMF. In the range $10M_\odot > M \geq 1M_\odot$, the value of $\alpha = 2.35$, has changed little over the decades and a recent evaluation quotes 2.3, see Kroupa (2001). A fractional exponential law aimed at explaining the mass distribution of asteroids, stars and galaxies was then introduced by Kiang (1966a). A three power law function was later introduced by Scalo (1986) and Kroupa et al. (1993); in this case the range of existence of the segments as well as the exponent $\alpha$ are functions of the investigated environment. After 15 years, the number of segments was raised to four once a universal IMF was introduced, see Kroupa (2001). Recent developments translate the concept of IMF from our surroundings (for example the first 10 pc) to open clusters such as the Pleiades, see Moraux et al. (2004). A connection between three segment zones and three physical processes has been investigated in Elmegreen (2004); these three zones correspond to brown dwarf masses $\approx 0.02M_\odot$, to intermediate mass stars and high mass stars. The question of uniformity or variability of the initial mass function has been carefully explored, see Meusinger (1987), Kroupa (2002), Shadmehri (2004), and Elmegreen (2004).

This paper briefly reviews the physical nature of gamma-variate and lognormal distribution (Section 2.1), introduces an algorithm that deduces the mass from the two basic photometric parameters B-V and $M_V$ (Section 3), and analyses the IMF in the light of physical and non physical functions (Section 4).

2. Preliminaries

Once the histogram of the star’s masses is obtained, we can fit it with three different probability density functions (in the following pdf): two of them are well known, the lognormal and the power law, while the third is a gamma-variate which has been applied here for the first time. The lognormal distribution $f_{\text{LN}}(x; \mu, \sigma)$ is characterised by the average value $\mu$, and the variance $\sigma^2$, see Evans et al. (2000). The power law distribution has the form $p(x; \alpha) \propto x^{-\alpha}$ where $\alpha > 0$. A refinement of the power law function consists in the multiple-part power law

$$\xi(x) \propto x^{-\alpha_i}$$

with $i$ varying from 3 to 4. This operation means that the field of existence of the function is broken into $i$ independents zones.
The gamma-variate is analysed in Section 2.1.

2.1. Fragmentation

The stellar mass has been viewed as the Jeans mass in the cloud core, see for example Larson (1992), Elmegreen (1997), Elmegreen (1999), Chabrier (2003), and Bate & Bonnell (2005). Here conversely we adopt the point of view of the fragmentation that was started by Kiang (1966b) where a rigorous demonstration of the size distribution of random Voronoi segments was given. The Kiang pdf (a gamma-variate) has the form

\[ H(x; c) = \frac{c}{\Gamma(c)} x^{c-1} e^{-cx} , \]  

(2)

where \( 0 < x < \infty, c > 0 \), and \( \Gamma(c) \) is the gamma function with argument \( c \), see Kiang (1966b). This pdf is characterised by average value, \( \mu=1 \), and variance, \( \sigma^2=1/c \).

Starting from a rigorous result in 1D, Kiang (1966b) conjectured that the Voronoi cells in 2D/3D have an area/volume distribution represented by equation (2) with \( c=4/6 \). We now briefly review a useful result that can be obtained once \( x_1 \cdots x_N \) independent random variables, are introduced. Supposing that

\[ y = x_1 \times x_2 \times \cdots \times x_{N-1} \times x_N = \prod_{j=1}^{N} x_j , \]

(3)

it can be shown that for large \( N \) the pdf of \( y \) is approximately lognormal, see Hwei Hsu (1996). From this point of view the area with \( y = x_1 \times x_2 \) or the volumes \( y = x_1 \times x_2 \times x_3 \) can be fitted with a lognormal once the number 2 and 3 are considered a transition to large \( N \).

3. Mass determination from colour absolute-magnitude

The (B-V) colour can be expressed as

\[ (B - V) = K_{BV} + \frac{T_{BV}}{T} , \]  

(4)

here \( T \) is the temperature, \( K_{BV} \) and \( T_{BV} \) are two parameters that can be derived by implementing the least square method on a series of calibrated data. The bolometric correction, \( BC \), can be expressed as

\[ BC = M_{bol} - M_V = -\frac{T_{BC}}{T} - 10 \log_{10} T + K_{BC} , \]  

(5)

where \( M_{bol} \) is the absolute bolometric magnitude, \( M_V \) is the absolute visual magnitude, \( T_{BC} \) and \( K_{BC} \) are two parameters that can be derived through the general linear least square method applied to a series of calibrated data. We now justify the theoretical basis of equations (4) and (5). We can start from the definition of (B-V)

\[ (B - V) = m_B - m_V = K - 2.5 \log_{10} \frac{\int S_B I_\lambda d\lambda}{\int S_V I_\lambda d\lambda} , \]  

(6)

where \( S_\lambda \) is the sensitivity function in the region specified by the index \( \lambda \), \( K \) is a constant and \( I_\lambda \) is the energy flux reaching the earth. We now define a sensitivity function for a pseudomonochromatic BV system

\[ S_\lambda = \delta(\lambda - \lambda_t) \quad i = B, V \] ,

(7)

where \( \delta \) denotes the Dirac delta function. On inserting the energy flux as given by the Planck distribution, the following is found

\[ (B - V) = Kt - \frac{hc}{kT} (\lambda_B - \lambda_V) 2.5 \log_{10}(e) , \]  

(8)

where \( Kt \) is a constant, \( c \) is the velocity of light, \( k \) is the Boltzmann’s constant, \( h \) is the Planck’s constant, \( \lambda_B \) the wavelength of the filter B, \( \lambda_V \) the wavelength of the filter V, and \( T \) the temperature. On inserting \( \lambda_B = 4450 \AA \) and \( \lambda_V = 5500 \AA \) in equation (8), the equation (4) is theoretically derived.

A theoretical couple, \( T_{BV}=6701 K \) and \( K_{BV}=0.51 \) can be found by inserting the constants of the Planck distribution in equation (5) and calibrating the relationship on the Sun couple, \( T_{\odot}=5777 K \) and \( (B - V)_{\odot}=0.65 \). In a similar way the bolometric correction is found to be

\[ BC = M_{bol} - M_V = K'' - 2.5 \log_{10} \int_0^\infty I_\lambda d\lambda / \int S_V I_\lambda d\lambda , \]  

(9)

where \( K'' \) is another constant.

When the numerical value of the exponential in the Planck function is much greater than one, equation (5) is found with the theoretical value \( T_{BC}=28402 K \) and \( K_{BC}=42.45 \), once BC of the sun is requested, \( BC_{\odot}=-0.08 \) at \( T_{\odot}=5777 K \).

The application of equation (4) and (5) to the calibrated physical parameters for stars of the various luminosity classes has been reported in Table 1.

| Table 1. Table of coefficients from calibrated data. |
|-----------------------------------------------------|
| \hline | MAIN, V | GIANTS, III | SUPERGIANTS I |
| \hline | \( K_{BV} \) | \(-0.641 \pm 0.01 \) | \(-0.792 \pm 0.06 \) | \(-0.749 \pm 0.01 \) |
| \( T_{BV}[K] \) | \( 7360 \pm 66 \) | \( 8527 \pm 257 \) | \( 8261 \pm 67 \) |
| \( K_{BC} \) | \( 42.74 \pm 0.01 \) | \( 44.11 \pm 0.06 \) | \( 42.87 \pm 0.01 \) |
| \( T_{BC}[K] \) | \( 31556 \pm 66 \) | \( 3685 \pm 257 \) | \( 31573 \pm 67 \) |
| \( a_{LM} \) | \( 0.062 \pm 0.04 \) | \( 0.32 \pm 0.14 \) | \( 1.29 \pm 0.32 \) |
| \( b_{LM} \) | \( 3.43 \pm 0.06 \) | \( 2.79 \pm 0.23 \) | \( 2.43 \pm 0.26 \) |
| \hline |

These numerical results can be visualised in Figure 1 when (B-V) against T is considered and in Figure 2 where the relationship between BC and T is reported; see Press et al. (1992) about the fitting techniques.

Other authors analyse the form \( T=T(B-V), [\text{Fe}/H], \log_{10} g \) where \([\text{Fe}/H]\) represents the metallicity and \( g \) the surface gravity, see Sekiguchi & Fukugita (2000) or a two piecewise function for \( B-V = f(T) \), see for example Reed (1998).

The luminosity of the star is represented through the usual formula

\[ \log_{10}(L/\odot) = 0.4(4.74 - M_{bol}) \] ,

(10)
4 MASSES

Fig. 1. Least square fit of (B-V) against 1/T, MAIN SEQUENCE V. Error on (B-V)=0.025. The calibrated data, extracted from Table 15.7 in Drilling & Landolt (2000), are represented with their error, and the full line reports the suggested fit.

Fig. 2. Relationship of BC against T, MAIN SEQUENCE V. Error on BC=0.025. The stars represent the calibrated data extracted from Table 15.7 in Drilling & Landolt (2000) and the dotted line, the suggested fit.

see, for example, equation (5.120) in Lang (1999). The connection with the mass is made through the numerical relationship

$$\log_{10} \left( \frac{L}{L_\odot} \right) = a_{LM} + b_{LM} \log_{10} \left( \frac{M}{M_\odot} \right)$$

(11)

for $M > 0.2 M_\odot$.

The numerical coefficients $a_{LM}$ and $b_{LM}$, as deduced from the calibrated data in Table 3.1 of Deeming & Bowers (1984) are reported in Table I.

With the theory here adopted the mass of the star is

$$\log_{10} \frac{M}{M_\odot} = -0.4 M_V - 0.4 K_{BC} + 4.0 \ln \left( \frac{10^{T_{BC}}}{T_{BC} - K_{BV}} \right) \left( \ln 10 \right)^{-1}$$

$$+ \frac{0.4 T_{BC} \left( \frac{1}{-B - V} + K_{BV} \right)}{b_{LM}} + 1.896 - a_{LM}.$$  

(12)

From a practical point of view the percentage of reliability of our evaluation can also be introduced:

$$\epsilon = \left( 1 - \left| \frac{M_{\text{cal}} - M_{\text{num}}}{M_{\text{cal}} + M_{\text{num}}} \right| \right) \times 100,$$

(13)

where $M_{\text{cal}}$ is the calibrated value of the star mass, see Table 15.8 in Drilling & Landolt (2000) and $M_{\text{num}}$ is the numerical value here evaluated. The resulting masses are reported in Table 2.

| sp | MK Class | $M_{\text{cal}}/M_\odot$ | $M_{\text{num}}/M_\odot$ | $\epsilon$(%) |
|----|----------|-----------------|-----------------|---------|
| B0 | V        | 17.5            | 17.34            | 99.54   |
| A0 | V        | 2.9             | 3.42             | 91.7    |
| F0 | V        | 1.6             | 1.76             | 95.23   |
| G0 | V        | 1.05            | 1.13             | 96.02   |
| K0 | V        | 0.79            | 0.81             | 98.73   |
| M2 | V        | 0.4             | 0.38             | 98.38   |
| K5 | III      | 1.2             | 6.38             | 31.63   |
| M0 | III      | 1.2             | 7.15             | 28.71   |
| A0 | I        | 16.0            | 22.35            | 83.44   |
| M0 | I        | 13.0            | 27.73            | 63.82   |
| M2 | I        | 19.0            | 28.6             | 79.82   |

The formulae here derived can be used to deduce the mass of the stars once the parameters $M_V$, B-V and luminosity class are provided.

4. Masses

We now apply the developed techniques to the nearest stars and to an open cluster, NGC6649.

4.1. The masses in the first 10 pc

The completeness in masses of the sample can be evaluated in the following way. The limiting apparent magnitude $m_L^V$ is known, for example in the case of Hipparcos it is 8. The corresponding absolute limiting magnitude is computed and inserted in equation (12). The limiting mass, $M_L^L$, for stars belonging to the MAIN SEQUENCE V is

$$\log_{10} \left( \frac{M_L^L}{M_\odot} \right) = -0.1164 m_L^V + 0.2528 \ln \left( 0.1 d_1 \right) - 4.122 + 0.505 \ln \left( \frac{0.368 \times 10^{11}}{0.5 \times 10^7 (B - V) + 0.3206 \times 10^7} \right) + 0.499 (B - V).$$

(14)

where $d_1$ is the maximum distance that characterises the sample in pc. When $d_1=10$, $m_L^V=8$, and (B-V)=1.15 we obtain $M_L^L = 0.53 M_\odot$; this is the limit over which the sample in masses is complete.

Due to the fact that Hipparcos (ESA 1997) gives B-V, $m_L^V$ and parallax, we can implement our algorithm on that database. The classification in MK classes (I,III,V) is then obtained by a comparison with the interval of existence of
4.2. The masses in open clusters

The distribution of masses in open clusters can be considered a further application of the pdf here studied. We briefly review some of the typical clusters subject to investigation: IC348 (Preibisch et al. 2003; Muench et al. 2003; Luhman et al. 2003), Taurus (Luhman 2000; Briceño et al. 2002), Orion trapezium (Hillenbrand & Carpenter 2000; Luhman et al. 2000; Muench et al. 2002), Pleiades (Bouvier et al. 1998; Luhman et al. 2000), and M35 (Barrado y Navascués et al. 2001).

Table 3. Coefficients of mass distribution in the first 10 pc, of the incomplete sample, GIANTS included, $M \geq 0.2 M_\odot$, $\overline{M} = 0.61 M_\odot$ and $N_s=137$.

| pdf          | $\chi^2$ | $c$   | $\alpha$ |
|--------------|----------|-------|----------|
| Kiang        | 14.28    | 1.67  |          |
| Power law    | 17.6     | 1.85  |          |
| Lognormal    | 0.95     | 0.6   | 0.57     |

Table 4. Coefficients of mass distribution in the first 10 pc, of the complete sample, $M \geq 0.53 M_\odot$, $\overline{M} = 0.98 M_\odot$, GIANTS excluded, and $N_s=57$.

| pdf          | $\chi^2$ | $c$   | $\alpha$ |
|--------------|----------|-------|----------|
| Kiang        | 12.62    | 3.42  |          |
| Power law    | 8.7      | 2.42  |          |
| Lognormal    | 4.55     | 1.01  | 0.41     |

Table 5. Coefficients of mass distribution in NGC 6649 where $N_s=234$ and $\overline{M} = 1.04 M_\odot$.

| pdf          | $\chi^2$ | $c$   | $\alpha$ | $\mu$ | $\sigma$ |
|--------------|----------|-------|----------|-------|----------|
| Kiang        | 35.44    | 5.02  |          |       |          |
| Power law    | 248.49   |       | 3.12     |       |          |
| Lognormal    | 29.39    | 1.03  | 0.32     |       |          |
5. Conclusions

A new method represented by equation (12), allows us to deduce the mass of stars from photometric data. This method is limited in mass, see equation (14), in the same way as the Hipparcos data are complete up to \( m_v = 8 \).

In the second part of this paper the obtained mass distribution is fitted with the pdf of physical nature: the lognormal and the gamma variate. The gamma-variante is applied for the first time as a fitting function of mass distribution. When the complete sample of masses is fitted with the gamma-variante, \( c = 3.42 \), is found; a comparison should be made with \( c = 4 \), which is the value suggested by Kiang (1966b) in the case of 2D fragmentation. Careful analysis of the results of Table 5 and Table 4 indicates that the lognormal gives the best fit to the star’s masses. On the contrary the three power law function simply represents the various segments of a continuous curve. This point of view can be expressed by plotting 100000 stars generated according to lognormal data, see Figure 8.

In Figure 8, the three indices \( \alpha_i \) are now varying in a continuous way. The great variations in \( \alpha_i \), visible in Table 6 and Figure 7, can be due to the transition from a great number of stars (smooth behaviour of \( \alpha_i \)) to a small number of stars (jumps in \( \alpha_i \) or variability in the IMF).

It is now possible in the light of the limiting mass \( M^L \), see equation (14), to explain the reason for which a power law fits the masses well in the first 10 pc. The limiting mass introduces a cut in the physical sample at a value that is probably near the averaged value, and therefore only the masses of value greater than the averaged value are detected. This point of view is simulated in Figure 9, in which a real con-

---

Table 6. Three power law in NGC 6649 where \( N_s = 234 \).

| Interval          | \( \alpha \) |
|-------------------|--------------|
| \( 0.52 \leq M/M_\odot < 0.88 \) | 6.83         |
| \( 0.88 \leq M/M_\odot < 1.8 \)  | 2.55         |
| \( 1.8 \leq M/M_\odot < 3.65 \)  | 1.0          |

---

Fig. 6. \( M_V \) against (B-V) (H-R diagram) in NGC 6649.

Fig. 7. Log–Log histogram of mass distribution in NGC 6649 with a superposition of the three–part power law. Parameters as in Table 6.

Fig. 8. Log–Log histogram of 100000 masses randomly generated according to lognormal distribution (parameters as in Table 6) with a superposition of the three–part power law.

Fig. 9. Log–Log histogram of 100000 masses randomly generated according to the Kiang distribution with \( c = 4 \) and \( \overline{M} = 2 M_\odot \). On the right, \( M \geq 0.51 M_\odot \), is visible the fitting power law distribution with \( \alpha = 2.56 \). The vertical dashed line represents the limiting mass \( M^L = 0.53 M_\odot \).
tuous mass distribution as given by a gamma-variate with \( c=4 \) is reported: due to the limiting mass \( M_L \) a power law distribution is observed.

6. acknowledgements

This note amplifies a series of lessons given in the course “Fondamenti di Astrofisica” on behalf of the organiser Roberto Gallino at the Turin University. I am grateful to the referee of this paper for helpful suggestions.

References

Barrado y Navascués, D., Stauffer, J. R., Bouvier, J., & Martín, E. L. 2001, ApJ , 546, 1006
Bate, M. R., & Bonnell, I. A. 2005, MNRAS , 356, 1201
Binney, J., & Merrifield M. : 1998, Galactic Astronomy,Princeton University Press, Princeton, p. 286
Bouvier, J., Stauffer, J. R., Martin, E. L., Barrado y Navascues, D., Wallace, B., & Bejar, V. J. S. 1998, A&A, 336, 490
Briceño, C., Luhman, K. L., Hartmann, L., Stauffer, J. R., & Kirkpatrick, J. D. 2002, ApJ , 580, 317
Chabrier, G. 2003, PASP , 115, 763
Drilling, J. S. & Landolt, A. U. in N. Cox (editor): 2000, Allen’s Astrophysical Quantities, Springer, Berlin, pp. 381-396
Deeming, T., & Bowers, R. L.: 1984, Astrophysics II: Interstellar Matter and Galaxies, Jones & Bartlett Pub, Boston
Elmegreen, B. G. 1997, ApJ , 486, 944
Elmegreen, B. G. 1999, ApJ , 515, 323
Elmegreen, B. G. 2004, MNRAS , 354, 367
ESA : 1997, The Hipparcos Catalogue (ESA SP-1200), ESA, Noordwijk
Evans M., Hastings N., Peacock B. : 2000, Statistical Distributions - third edition, John Wiley & Sons Inc., New York
Hwei P. Hsu : 1996, Probability, Random variables, & Random Processes McGraw–Hill, New York
Hillenbrand, L. A., & Carpenter, J. M. 2000, ApJ , 540, 236
Kiang, T. 1966a, Z. Astrophys., 64, 426
Kiang, T. 1966b, Z. Astrophys., 64, 433
Kroupa, P. 2001, MNRAS , 322, 231
Kroupa, P. 2002, Science, 295, 82
Kroupa, P., Tout, C. A., & Gilmore, G. 1993, MNRAS , 262, 545
Lang, K. R.: 1999 Astrophysical Formulae, third edition, Springer, Berlin
Larson, R. B. 1992, MNRAS , 256, 641
Luhman, K. L. 2000, ApJ , 540, 236
Luhman, K. L., Rieke, G. H., Young, E. T., Cotera, A. S., Chen, H., Rieke, M. J., Schneider, G., & Thompson, R. I. 2000, ApJ , 540, 1016
Luhman, K. L., Stauffer, J. R., Muench, A. A., Rieke, G. H., Lada, E. A., Bouvier, J., & Lada, C. J. 2003, ApJ , 593, 1093
Meusinger, H. 1987, Astronomische Nachrichten, 308, 189
Moraux, E., Kroupa, P., & Bouvier, J. 2004, A&A, 426, 75
Muench, A. A., Lada, E. A., Lada, C. J., & Alves, J. 2002, ApJ , 573, 366
Muench, A. A., et al. 2003, AJ , 125, 2029
Preibisch, T., Stanke, T., & Zinnecker, H. 2003, A&A, 409, 147
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P.: 1992 Numerical Recipes in Fortran, Cambridge University Press, Cambridge
Reed, B. C. 1998, JRASC, 92, 36
Salpeter, E. E. 1955, ApJ , 121, 161
Scalo, J. M. 1986, Fundamentals of Cosmic Physics, 11, 1
Sekiguchi, M., & Fukugita, M. 2000, AJ , 120, 1072
Shadmehri, M. 2004, MNRAS , 354, 375
Walker, A. R., & Laney, C. D. 1987, MNRAS , 224, 61