Theoretical calculations of resonant signals in the atomic-beam quantum frequency standard with laser pumping and detection

A Yu Rumyantsev\textsuperscript{1} and M V Petrenko\textsuperscript{1,2}

\textsuperscript{1} Russian Institute of Radionavigation and Time, 120 Obukhovskoy Obozony prospect, St. Petersburg, 192012, Russia
\textsuperscript{2} Ioffe Institute, 26 Politekhnicheskaya Street, St. Petersburg, 194021, Russia

E-mail: m.petrenko@mail.ioffe.ru

Abstract. We consider the theoretical model of the resonant signals formation in cesium atomic-beam QFS depending on input parameters, such as the number of atoms, the power and the linewidth of the laser radiation in detection area. On the basis of the model calculations were made and the values of fluorescence signal, which are specific for compact cesium QFS, were obtained. The influence of fundamentally irremovable noise sources on the characteristics of atomic-beam frequency standards was considered. On the basis of theoretical expressions, the values of the spectral power density of the noises were calculated and the signal-to-noise ratio was obtained. It was shown that the signal-to-noise ratio for compact cesium atomic-beam QFS does not exceed the value $5 \times 10^4$, which leads to an estimation of the best achievable value of the short-term frequency instability by the quantity $7 \times 10^{-13}$ for 1 second.

1. Introduction
At the present time the optical pumping of atoms is one of the most promising ways for increasing the characteristics of quantum devices, such as miniature quantum frequency standards (QFSs) based on a vapor cell [1, 2], quantum magnetometers [3, 4], and atomic-beam QFSs. In particular, one of the most used sources of reference frequency is the atomic-beam QFS with laser pumping and detection [5, 6]. The frequency instability of such a standard directly depends on both the quality of the resonant fluorescence signal and the amount of noise introduced by various sources into the output signal of the standard. In this paper we present the theoretical model that allows one to determine the characteristics of a resonant signal of atomic-beam QFS based on $^{133}$Cs atoms depending on input parameters such as the atomic flux, the linewidth and the power of laser radiation. In the presented model the processes occurring during the interaction of cesium atoms with pumping and detection radiation and with the microwave field in the resonator are taken into account. We also consider fundamentally irremovable noise sources that affect the fluorescence signal of atoms when they are exposed to laser radiation and, as a consequence, the final frequency instability of atomic-beam QFS.

In this paper we investigate atomic-beam QFS on $^{133}$Cs atoms, which scheme is shown in figure 1. This QFS works according to the following principle. A beam of $^{133}$Cs atoms (a) is emitted from the source (b) and propagates along the axis of the atomic-beam tube. In the pumping area (c) atoms in the presence of a static magnetic field interact with a field of laser radiation (d), which frequency is tuned to the frequency of the $F = 4 \rightarrow F' = 4$ transition of the $D_2$ line of $^{133}$Cs atoms, the radiation in this
case is linearly polarized in the direction perpendicular to the magnetic field (σ-polarization). Part of the atoms goes to the level with \( F = 3, m_f = 0 \) of the ground state depleting the level with \( F = 4 \). Then, while passing through the Ramsey cavity \((e)\) and interaction with a microwave field tuned to the hyperfine transition frequency \( F = 3, m_f = 0 \rightarrow F = 4, m_f = 0 \) (9.19263 GHz for \(^{133}\text{Cs}\)), these atoms transfer to the state \( F = 4, m_f = 0 \) with probability that depends on the frequency detuning of the microwave field from the frequency of the hyperfine transition. Further, in the detection area \((f)\) atoms in the state \( F = 4, m_f = 0 \) interact with laser radiation \((g)\), tuned to the frequency of the cyclic transition \( F = 4 \rightarrow F' = 5 \) of \( D_2 \) line, and emit fluorescence photons that are detected by the photodiode \((h)\).

![Diagram](image.png)

**Figure 1.** The scheme of atomic-beam QFS with laser pumping and detection.

2. Resonant signals calculation

Let us consider the method of formation of resonant signals in compact cesium atomic-beam QFS with laser pumping and detection. The values of the parameters used in this method, which are specific for compact cesium QFS, are presented in Table 1.

The source of the atomic beam in the atomic-beam QFS is a volume filled with cesium vapor and heated to a temperature of the order of 120 °C. The cesium atomic beam is emitted from this volume, passing through a collimator formed of microchannels, and propagates along the atomic-beam tube. Such a source \([6, 7]\) can provide a flow of \(^{133}\text{Cs}\) atoms in the detection region at level \( I = 10^{10} \) – \( 10^{11} \) atoms/s.

In the optical pumping area, according to \([8]\), in the considered model of a compact atomic-beam QFS the maximum pumping rate of \(^{133}\text{Cs}\) atoms to the \( F = 3 \) level of the ground state is realized and is equal to \( \rho_p = 0.155 \) of the total number of atoms in the beam.

In the Ramsey cavity a transition between the hyperfine sublevels \( F = 3, m_f = 0 \rightarrow F = 4, m_f = 0 \) of the \(^{133}\text{Cs}\) ground state occurs under the influence of the microwave field. The transition probability in the absence of phase shift of the microwave field between the arms of the resonator is given by the expression \([5, 6]\)

\[
P(v, \Delta, b) = \frac{4b^2}{\alpha^2} \sin^2 \frac{\alpha \tau_{res}(v)}{2} \left[ \cos \frac{\alpha \tau_{rel}(v)}{2} \cos \frac{\Omega \tau_{rel}(v)}{2} - \frac{\Omega}{\alpha} \sin \frac{\alpha \tau_{rel}(v)}{2} \sin \frac{\Omega \tau_{rel}(v)}{2} \right]^2,
\]

where \( v \) – velocity of the atom, \( \Omega \) – frequency detuning of the microwave field from the frequency of the hyperfine transition, \( b \) – the parameter of the power of the microwave field (Rabi frequency of the
magnetic dipole interaction), \( \Delta = \Omega / 2\pi - \) frequency detuning in Hz, \( \alpha = (\Omega^2 + b^2)^{1/2} \), \( \tau_{\text{res}}(v) = I_{\text{res}} / v \), \( T_{\text{res}}(v) = L_{\text{res}} / v \). When the frequency of the microwave field is equal to the frequency of the hyperfine transition, the maximum transition probability for the most probable velocity of atoms in the beam is realized for \( b_{\text{max}} = \pi / \tau_{\text{res}}(v) = 34690.2 \text{ s}^{-1} \).

**Table 1.** Parameters of the compact atomic-beam cesium QFS.

| Parameter                        | Symbol | Value |
|----------------------------------|--------|-------|
| **Atomic beam source**          |        |       |
| Source temperature, °C           | \( T_{Cs} \) | 120   |
| **Pumping area**                 |        |       |
| Power of the pumping laser, mW   | \( P_P \) | 1 – 10|
| Length of the pumping area, mm   | \( l_p \) | 5     |
| Height of the pumping area, mm   | \( z_p \) | 2     |
| **Ramsey cavity**                |        |       |
| Interaction length of an atom with microwave field, cm | \( l_{\text{res}} \) | 1     |
| Flight length of the cavity, cm  | \( L_{\text{res}} \) | 15    |
| **Detection area**               |        |       |
| Atomic flow in the detection area, atoms/s | \( I_{\text{at}} \) | \( 10^{10} – 10^{13} \) |
| Power of the detection laser, mW | \( P_{\text{las}} \) | 1 – 10|
| Detection laser linewidth, Hz    | \( \gamma_{\text{las}} \) | \( 10^5 – 10^7 \) |
| Length of the detection area, mm | \( l_d \) | 5     |
| Height of the detection area, mm | \( z_d \) | 2     |
| **Silicon photodiode**           |        |       |
| Efficiency of the fluorescence photons collection | \( \rho_{pd} \) | 0.2   |
| Photodiode quantum efficiency    | \( \eta \) | 0.95  |
| Photodiode shunt resistance, \( \Omega \) | \( R_{pd} \) | \( 10^{6} \) |
| Photodiode temperature, °C       | \( T_{pd} \) | 40    |
| Dark current, A                  | \( I_{\text{dark}} \) | \( 10^{-8} \) |
| Rejection factor of the direct laser light | \( r_{pd} \) | \( 10^{-6} \) |

Thus, atoms in the state \( F = 4, m_F = 0 \), the number of which is proportional to the pumping rate and is determined by the expression (1), enter the detection area. In the detection area the fluorescence of these atoms occurs when interacting with the laser radiation tuned to the frequency of \( F = 4 \to F' = 5 \) transition. The number of photons emitted by an atom during fluorescence is described by [9]

\[
\beta(\gamma_{\text{las}}, I_{\text{las}}, v) = \Gamma \Pi(\gamma_{\text{las}}, I_{\text{las}}) \tau_d(v),
\]

where \( I_{\text{las}} = P_{\text{las}} / (l_d z_d) \) is the power density of the laser radiation in the detection area, \( \gamma_{\text{las}} = 2 \pi v_{\text{las}} \), \( \Gamma = 3.2815 \times 10^7 \text{ s}^{-1} \) is the rate of free radiative transition \( F' = 5 \to F = 4 \) [10], \( \tau_d(v) = l_d / v \) is the spent time of an atom in the laser field, \( \Pi(\gamma_{\text{las}}, I_{\text{las}}) \) is the average population of the exited state \( F' = 5 \), which is calculated according to [9, 10, 11].

The collection of fluorescence photons in the detection area is performed by a silicon photodiode whose parameters are given in Table 1. Combining expressions (1) and (2) and using the data from Table 1, we can write the expression for the microwave resonance signal registered by the photodiode in volts:

\[
S_{\text{res}}(\gamma_{\text{las}}, I_{\text{las}}, I_{\text{at}}, \Delta, b) = e \rho_{pd} \eta \rho_{pd} P_{\text{pd}} P_{\text{las}} \int_0^\infty \beta(\gamma_{\text{las}}, I_{\text{las}}, v) P(v, \Delta, b) f_{\text{at}}(v, T_{Cs}) dv,
\]

where \( e \) is the electron charge, \( f_{\text{at}}(v, T_{Cs}) \) is Maxwell velocity distribution of the atoms in the beam for the source temperature \( T_{Cs} \).
Figure 2 shows the result of the calculations of the microwave resonance signals. Calculations have shown that the values of the recorded signals are tens of millivolts. With a decrease in the microwave power parameter, the width of the resonance reduces along with the predicted decrease in the signal amplitude. This is apparently due to the shift of the maximum of the probability of a hyperfine transition under the action of a microwave field to the region of low atomic velocities in a beam. When integrating according to formula (5), this leads to a decrease in the width of the resonant signal.

\[ S_{\text{sho}}(\gamma_{l аs}, I_{l аs}, I_{at}) = 2(2eR_{pd})^2 \eta \rho_p \rho_p I_{at} \left[ \int_0^\infty \beta(\gamma_{l аs}, I_{l аs}, v) P(v,0 \text{ Hz}, b_{\text{max}}) f_M(v, T_{Cs}) dv \right. \]

\[ \left. + \eta \rho_p \int_0^\infty \left[ \beta(\gamma_{l аs}, I_{l аs}, v) \right] P(v,0 \text{ Hz}, b_{\text{max}}) f_M(v, T_{Cs}) dv \right] \]
Here we take into account both the contribution of the photon noise associated with fluctuations in the number of fluorescence photons emitted by one atom and the shot atomic noise associated with fluctuations in the number of atoms in the beam.

One of the main parameters of the interaction of laser radiation with atoms that affect the noise of the output signal is also the spectral purity of the radiation. The laser frequency noise leads to additional fluctuations in the number of photons emitted by the atoms, which in the case of a wide laser emission line can significantly decrease the signal-to-noise ratio. According to [9, 11], the laser-frequency fluctuations induce coupled fluctuations of the fluorescence signals emitted by two different atoms of the interaction area. The spectral power density of the output signal noise in this case is expressed by the formula [9]

$$S_{\text{las}}(\gamma_{\text{las}}, I_{\text{las}}, I_{\text{at}}) = [S_{\text{res}}(\gamma_{\text{las}}, I_{\text{las}}, I_{\text{at}}, 0 \text{ Hz}, b_{\text{max}})]^2 \frac{4(3\Gamma + \gamma_{\text{las}})}{\Omega_{R}^2(\Gamma + \gamma_{\text{las}})} \times \frac{\gamma_{\text{las}}^2\Gamma}{\Omega_{R}^2(\Gamma + 4\gamma_{\text{las}}) + \frac{\Gamma}{2}(3\Gamma + \gamma_{\text{las}})(\Gamma + 2\gamma_{\text{las}})}$$

(6)

where $\Omega_{R}^2(I_{\text{at}})$ is the square of the Rabi frequency of the electric dipole interaction.

Additional noise in the output signal is made by a photodiode that collects fluorescence photons. The spectral power density of this noise can be written in the form

$$S_{pd}(I_{\text{las}}) = (eR_{pd})^2 \left( \frac{2\pi r_{pd}^2}{h\nu} + \frac{2I_{\text{dark}}}{e} + 4\frac{kT_{pd}}{e^2 R_{pd}} \right)$$

(7)

The first term describes the noise caused by the direct illumination of the photodiode by laser radiation with the $r_{pd}$ rejection factor (shielding), the second term is the noise caused by the dark current of the photodiode and the third term is the thermal noise of the photodiode current on the shunt resistance.

The signal-to-noise ratio for the output signal of the atomic-beam QFS in the 1 Hz band is written taking into account formulas (4), (5) – (7) in the form [9, 12]

$$\text{SNR}(\gamma_{\text{las}}, I_{\text{las}}, I_{\text{at}}) = \frac{J_{\text{las}}(\gamma_{\text{las}}, I_{\text{las}}, I_{\text{at}})}{\sqrt{S_{\text{shot}}(\gamma_{\text{las}}, I_{\text{las}}, I_{\text{at}}) + S_{\text{las}}(\gamma_{\text{las}}, I_{\text{las}}, I_{\text{at}}) + S_{pd}(I_{\text{las}})}}$$

(8)

The signal-to-noise ratios calculated from formula (8) are shown in Figure 4 and Figure 5. In Figure 4 it is shown that the signal-to-noise ratio increases substantially with increasing number of atoms in the beam and with a decrease in the linewidth of the laser radiation in the detection area. Calculations have shown that the use of lasers with a linewidth more than 3 MHz in atomic-beam QFSs can be considered inexpedient, since it leads to a noticeable decrease of the signal-to-noise ratio. According to the obtained data, it is necessary to investigate the possibilities of increasing the number of atoms in the beam to improve the signal-to-noise ratio. However, the method proposed in [6] for increasing the density of atoms by raising the source temperature to $130 – 150 \degree\text{C}$ leads to a significant increase in the consumption of the working material and a degradation of the angular distribution of the atomic flux [7, 13].

Figure 5 shows the behavior of the signal-to-noise ratio as a function of the power of the detection laser. Calculations have shown that with laser power increasing the signal-to-noise ratio tends to its limit value. It is due to the influence of the noise caused by the laser frequency noise, which is at high power of the laser radiation becomes dominant over the other noises. It should be noted that if the photodiode is not shielded from direct laser light with a suppression factor exceeding $10^4$, the contribution of the photodiode noise will increase with laser power increasing, which will lead to a decrease in the signal-to-noise ratio. It also follows from the results of calculations that when using detection lasers with a power of less than 5 mW, the effect of the laser linewidth on the signal-to-noise ratio becomes more critical. The signal-to-noise ratio significantly decreases when the linewidth exceeds 500 kHz.
4. Conclusion

In conclusion, let us consider the instability of the atomic-beam QFS with laser pumping and detection. The relative short-term frequency instability of the atomic-beam QFS with rectangular modulation of the frequency of the microwave field can be calculated by the formula

$$\sigma = \frac{\sqrt{2} \Delta f}{\pi \cdot \text{SNR}(\gamma_{\text{lat}}, I_{\text{lat}}, I_{\text{at}}) \cdot f_0 \sqrt{\tau}},$$

where $f_0 = 9.19263$ GHz is the frequency of the hyperfine transition, $\Delta f$ is the halfwidth of the microwave resonance ($\Delta f = 700$ Hz for $b_{\text{max}}$) and $\tau$ is the measuring time. Calculations of the signal-to-noise ratio showed that its value does not exceed $5 \times 10^4$ in the range of the parameters considered, which are specific for compact cesium atomic-beam QFSs. This leads to an estimation of the best possible value of the relative short-term frequency instability with a value of $7 \times 10^{-13}$ for 1 second.

References

[1] Ermak S V, Petrenko M V and Semenov V V 2016 Tech. Phys. Lett. 42 127
[2] Fedorov M I, Ermak S V, Petrenko M V, Pyatyshchev E N and Semenov V V 2016 J. Phys.: Conf. Ser. 769 012046
[3] Baranov A A, Ermak S V, Sagitov E A, Smolin R V and Semenov V V 2016 Tech. Phys. Lett. 42 186
[4] Sagitov E A, Ermak S V, Petrenko M V and Semenov V V 2016 J. Phys.: Conf. Ser. 769 012044
[5] Vanier J and Audoin C 1989 The quantum physics of atomic frequency standards (Philadelphia: IOP Publishung Ltd) p 453
[6] Pimenov A V and Pleshanov S A 2010 Elektronnaya Tekhnika. SVCH Tekhnika 507 16
[7] Rumyanstev A Yu, Petrenko M V, Poniaev S A, Shustrov Yu A, Kochegarov V P and Schennikov D L 2017 J. Phys.: Conf. Ser. 929 012095
[8] Avila G et al 1987 Phys. Rev. A 36 3719
[9] Dimarcq N, Giordano V, Cerez P and Theobald G 1994 Appl. Phys. B 59 135
[10] Steck D A Cesium D Line Data (http://steck.us/alkalidata)
[11] Halswanter Th, Ritsch H, Cooper J and Zoller P 1988 Phys. Rev. A 38 5652
[12] de Clercq E, Clairon A, Dahmani B, Gerard A and Aynie P 1989 Frequency standards and metrology ed. A De Marchi (Berlin, Heidelberg: Springer-Verlag) p 120
[13] Olander D R and Kruger V 1970 J. Appl. Phys. 41 2769