Cosmological simulations with self-interacting dark matter – II. Halo shapes versus observations

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ABSTRACT

If dark matter has a large self-interaction scattering cross-section, then interactions among dark-matter particles will drive galaxy and cluster haloes to become spherical in their centres. Work in the past has used this effect to rule out velocity-independent, elastic cross-sections larger than \( \sigma / m \simeq 0.02 \text{ cm}^2 \text{ g}^{-1} \) based on comparisons to the shapes of galaxy cluster lensing potentials and X-ray isophotes. In this paper, we use cosmological simulations to show that these constraints were off by more than an order of magnitude because (a) they did not properly account for the fact that the observed ellipticity gets contributions from the triaxial mass distribution outside the core set by scatterings, (b) the scatter in axis ratios is large and (c) the core region retains more of its triaxial nature than estimated before. Including these effects properly shows that the same observations now allow dark matter self-interaction cross-sections at least as large as \( \sigma / m = 0.1 \text{ cm}^2 \text{ g}^{-1} \). We show that constraints on self-interacting dark matter from strong-lensing clusters are likely to improve significantly in the near future, but possibly more via central densities and core sizes than halo shapes.

Key words: methods: numerical – galaxies: haloes – dark matter.

1 INTRODUCTION

The nature of dark matter is one of the most compelling mysteries of our time. On large scales, the behaviour of dark matter is consistent with what cosmologists of yore called ‘dust’ (e.g. Tolman 1934), meaning its behaviour is consistent with being collisionless and non-relativistic (‘cold’) for the vast majority of the Universe’s history (Reid et al. 2010). This consistency has been of great interest to the particle-physics community because the most popular candidate for dark matter, the supersymmetric neutralino, displays exactly this behaviour (Steigman & Turner 1985; Griest 1988; Jungman, Kamionkowski & Griest 1996). While the supersymmetric neutralino paradigm is attractive in many ways, there are two outstanding problems with it. First, astroparticle searches have yet to turn up evidence for the existence of the neutralino, though searches are rapidly increasing their sensitivity to the particle-physics community because the most popular candidate for dark matter, the supersymmetric neutralino, displays exactly this behaviour (Steigman & Turner 1985; Griest 1988; Jungman, Kamionkowski & Griest 1996). The standard model of particle physics predicts that the neutralino is the lightest superparticle in the spectrum, making it the most likely candidate for dark matter. However, the neutralino is not the only candidate for dark matter. Other candidates include the Majorana fermion, the sterile neutrino, and the axion. Each of these candidates has its own set of predictions for the properties of dark matter, such as its mass, decay width, and relic density. The mass of dark matter is estimated to be between 100 GeV and 1 TeV, with most models predicting a mass of around 100 GeV. The decay width of dark matter is estimated to be less than 10^{-15} eV, with most models predicting a decay width of around 10^{-17} eV. The relic density of dark matter is estimated to be around 2.7 x 10^{-26} cm^3 s^{-1}, with most models predicting a relic density of around 2.2 x 10^{-26} cm^3 s^{-1}. These predictions are important for understanding the nature of dark matter and its role in the Universe. For example, the mass of dark matter is important for understanding the formation of galaxies and the large-scale structure of the Universe. The decay width of dark matter is important for understanding the stability of dark matter and the potential for dark matter to decay into observable particles. The relic density of dark matter is important for understanding the density of matter in the Universe and the fate of the Universe.

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basic class of self-interacting dark matter (SIDM) using cosmological simulations to explore its effect on dark-matter halo shapes as a function of cross-section. In a companion paper (Rocha et al. 2013) we investigate implications for dark-matter halo substructure and density profiles.

We are reinvestigating this simple SIDM model, which had been decreed "uninteresting" in several studies a decade ago, for two primary reasons. First, we suspected that the constraints that indicated that the SIDM cross-section was too small to meaningfully alter the morphology of dark-matter haloes were not as tight as claimed. Secondly, there is a wealth of new data (e.g. from near-field cosmology, lensing studies of galaxies and clusters) that may be better places to either look for SIDM or constrain its properties. In this paper and our companion paper, Rocha et al. (2013), we reevaluate past constraints on SIDM and suggest several new places to look for the effects of SIDM on halo structure.

There was a burst of work on SIDM before its untimely demise (Hogan & Dalcanton 2000; Kochanek & White 2000; Yoshida et al. 2000a,b; Dalcanton & Hogan 2001; Davé et al. 2001; Colin et al. 2002; Hennawi & Ostriker 2002). The death of isotropic, velocity-independent, elastically scattering SIDM largely came from the interpretation of three types of observations: halo evaporation in galaxy clusters (Gnedin & Ostriker 2001), cores in galaxy clusters (Yoshida et al. 2000b; Meneghetti et al. 2001) and halo shapes (Miralda-Escudé 2002). The Y2K-era constraints from the former two classes of observations were at the level of $\sigma/m \lesssim 0.3 \text{ cm}^2 \text{g}^{-1}$ and $\sigma/m \lesssim 0.1 \text{ cm}^2 \text{g}^{-1}$, respectively. However, we show in a companion paper, Rocha et al. (2013), that the evaporation and cluster-core constraints are likely overestimated.

The most stringent constraints on dark-matter models with large isotropic, elastic self-scattering cross-sections emerged from the shapes of dark-matter haloes, in particular from lens modelling of the galaxy cluster MS 2137–23 by Miralda-Escudé (2002). This massive galaxy cluster has a number of radial and tangential arcs within $\sim 200 \text{kpc}$ of the halo centre (Mellier, Fort & Kneib 1993; Miralda-Escudé 1995). Miralda-Escudé (2002) argued that self-interactions should make dark-matter haloes round within the radius $r$ where the local per-particle scattering rate equals the Hubble rate $\Gamma(r) = H_0$, or equivalently, where each dark-matter particle experiences one interaction per Hubble time. The scattering rate per particle as a function of $r$ in a halo scales in proportion to the local density and velocity dispersion,

$$\Gamma(r) \sim \rho(r)(\sigma/m)v_{\text{rms}}(r),$$

where $\rho$ is the local dark-matter mass density and $v_{\text{rms}}$ is the rms speed of dark-matter particles. Using the fact that the lens model needs to be elliptical at $70 \text{kpc}$, Miralda-Escudé (2002) set a constraint of $\sigma/m \lesssim 0.02 \text{ cm}^2 \text{g}^{-1}$ on the velocity-independent elastic scattering cross-section. This constraint is one to two orders of magnitude tighter than other typical constraints on velocity-independent scattering (Yoshida et al. 2000b; Gnedin & Ostriker 2001; Randall et al. 2008). It rendered velocity-independent scattering far too small to form cores in low surface brightness galaxies and other small galaxies (de Blok et al. 2008; Kuzio de Naray et al. 2008). This is unfortunate because the main reason SIDM was interesting at the time was that it was a mechanism to create cores in such galaxies (Spiegel & Steinhardt 2000).

The tightness of the SIDM constraints on cluster scales has meant that the focus of SIDM studies has shifted to those on velocity-dependent cross-sections, such that SIDM may significantly alter dwarf-scale or smaller dark-matter haloes while leaving cluster-mass haloes largely untouched (Feng et al. 2009; Buckley & Fox 2010; Loeb & Weiner 2011; Vogelsberger, Zavala & Loeb 2012). In recent times, such velocity-dependent interactions have arisen in hidden-sector models designed to interpret some charged-particle cosmic ray observations as evidence for dark-matter annihilation (Pospelov et al. 2008; Arkani-Hamed et al. 2009; Fox & Poppitz 2009; Feng, Kaplinghat & Yu 2010). Constraints on other hidden-sector dark-matter models have been made using X-ray isophotes of the gas in the halo of the elliptical galaxy NGC 720 (Buote et al. 2002; Feng et al. 2009; Buckley & Fox 2010; Feng et al. 2010; Ibe & Yu 2010; McDermott, Yu & Zurek 2011; Feng, Rentala & Surujon 2012).

However, as we show below, reports of the death of isotropic, velocity-independent elastic SIDM are greatly exaggerated. In this paper, we show that these earlier studies did not correctly account for the fact that the observed ellipticity (of the mass in cylinders or the projected gravitational potential) gets contributions from mass well outside the core, and the region outside the core retains its triaxiality. We also show that for ellipticity estimators that are relevant observationally, there is significant amount of scatter and the overlap between CDM and SIDM ellipticities is substantial even for $\sigma/m = 1 \text{ cm}^2 \text{g}^{-1}$. Finally, we find that in the regions where SIDM particles have suffered (on average) about one or more interactions, the residual triaxiality is larger than what has been previously estimated (Davé et al. 2001). Along with the analysis in Rocha et al. (2013), we find that studies of the central densities of dark-matter haloes are likely to yield tighter constraints on the SIDM cross-section than the morphology of the haloes.

We briefly summarize our simulations in Section 2. We present results on the three-dimensional shapes of SIDM dark-matter haloes compared to their CDM counterparts in Section 3. We reexamine the previous SIDM constraints based on halo shapes in light of our simulations in Section 4. In particular, we reexamine the Miralda-Escudé (2002) constraint in Section 4.2 and from the shapes of the X-ray isophotes of NGC 720 (Buote et al. 2002) in Section 4.3. In Section 4.4, we show how other lensing data sets may constrain SIDM in the future. We summarize the key points of this paper and present a few final thoughts in conclusion in Section 5.

2 SIMULATIONS

We modelled self-interactions by direct-simulation Monte Carlo with a scattering algorithm derived in appendix A of Rocha et al. (2013) and implemented within the GADGET-2 (Springel 2005) cosmological N-body code. Once the code passed accuracy tests, we performed cosmological simulations of CDM and SIDM with identical initial conditions for cubic boxes of $25 \text{h}^{-1}\text{ Mpc}$ on a side and $50 \text{h}^{-1}\text{ Mpc}$ on a side, each with $512^3$ particles. For the SIDM runs we explored cross-sections of $\sigma/m = 1, 0.1$ and $0.03 \text{ cm}^2 \text{g}^{-1}$, though the lowest cross-section run (with $\sigma/m = 0.03 \text{ cm}^2 \text{g}^{-1}$) provided results that were similar to CDM that we have not included them in any of the figures below. The initial conditions were generated using the MUSIC code at $z = 250$ (Hahn & Abel 2011) with a year seven Wilkinson Microwave Anisotropy cosmology (Komatsu et al. 2011): $h = 0.71$, $\Omega_m = 0.266$, $\Omega_{\Lambda} = 0.734$, $\Omega_b = 0.0449$, $n_s = 0.963$, $\sigma_8 = 0.801$. A summary of our simulation parameters, including particle mass and resolution, is provided in Table 1. We adopt a naming convention where the simulations denoted SIDM$_m$ and SIDM$_s$ have subscripts corresponding to their cross-sections in units of $\text{cm}^2 \text{g}^{-1}$. In all cases the self-interaction smoothing length as defined in Rocha et al. (2013) was set to 2.8 times the force softening.
Table 1. Summary of simulations.

| Name | $L_{\text{Box}}$ (Mpc $h^{-1}$) | $m_p$ (M$_{\odot} h^{-1}$) | $\epsilon$ (kpc $h^{-1}$) | $\sigma/m$ (cm$^2$ g$^{-1}$) |
|------|-------------------------------|-----------------|-----------------|-----------------|
| CDM  | 50                           | $6.9 \times 10^7$ | 1.0             | –               |
|      | 25                           | $8.6 \times 10^6$ | 0.4             | –               |
| SIDM$_{0.1}$ | 50                     | $6.9 \times 10^7$ | 1.0             | 0.1             |
|      | 25                           | $8.6 \times 10^6$ | 0.4             | 0.1             |
| SIDM$_1$ | 50                      | $6.9 \times 10^7$ | 1.0             | 1.0             |
|      | 25                           | $8.6 \times 10^6$ | 0.4             | 1.0             |

Note. Columns give name, simulation box size, particle mass, force resolution and interaction cross-section. We use $h=0.71$. See Rocha et al. (2013) for more details on the simulations.

We locate and characterize haloes using the publicly available Amiga Halo Finder (AHF; Knollmann & Knebe 2009) package. The total mass of a host halo $M_{\text{vir}}$ is determined as the mass within a radius $r_{\text{vir}}$ using the virial overdensity as defined in Bryan & Norman (1998). Though most of our analysis focuses on distinct (field) haloes, we also explore subhalo shapes. For these objects their masses are measured within the radius at which the radial density profile of the subhalo begins to rise again because of the presence of the host.

We present results on the shapes of haloes at $z=0$ in a radial range $r_{\text{min}}$ to $r_{\text{vir}}$, where $r_{\text{min}}$ is the minimum radius within which we trust the shape measurements. Since our two sets of simulations have different resolution, we can check the convergence of our shape estimates. For integral measures of shape (e.g. the moment-of-inertia tensor for all particles within a given radius), we find that although the density profiles look largely converged outside of the numerical-relaxation radius $r_{\text{relax}}$ defined in Power et al. (2003), the shapes do not converge until at least $r_{\text{min}} = 2r_{\text{relax}}$. This radius is roughly $r_{\text{min}} \approx 20$ kpc for the haloes in the $50 h^{-1}$ Mpc-sized simulations, with only modest dependence on halo mass and scattering cross-section, and $r_{\text{min}} \approx 10$ kpc for the $25 h^{-1}$ Mpc boxes. Below $r_{\text{min}}$ we find that the halo shapes are systematically too round. However, shapes are more robust if found in shells (either spherical or ellipsoidal) because they are less contaminated by the effects of the overly round and numerically relaxed inner regions. Shape estimates, especially integral estimates, are most reliable if there are at least $\sim 10^4$ particles within the virial radius (or tidal radius for subhaloes), consistent to what has been found in earlier work (e.g. Allgood et al. 2006; Vera-Ciro et al. 2011).

3 SIMULATED HALO SHAPES

3.1 Preliminary illustration

Before presenting a statistical comparison of CDM and SIDM halo populations, we provide a pictorial illustration of how an individual halo changes shape as we vary the cross-section. The columns of Figs 1 and 2 show surface-density maps for the same halo simulated in CDM, SIDM$_{0.1}$ and SIDM$_1$ from left to right. In Fig. 1, we

![Figure 1](https://academic.oup.com/mnras/article-abstract/430/1/105/984407/figure-1)

Figure 1. Surface density of a halo of mass $M_{\text{vir}} = 1.2 \times 10^{14}$ M$_{\odot}$ projected along the major axis of the moment-of-inertia tensor – the orientation that dominates the lensing probability. The left-hand column shows the halo for CDM, while the middle and right columns show the same halo simulated using SIDM with $\sigma/m = 0.1$ cm$^2$ g$^{-1}$ and 1.0 cm$^2$ g$^{-1}$, respectively. The bottom row shows the same information, now zoomed in on the central region. The surface density stretches logarithmically from $\approx 10^{-3}$ g cm$^{-2}$ (blue) to $\approx 10$ g cm$^{-2}$ (red). Surface densities are estimated by counts in bins.
project the halo along the major axis, which is the orientation that maximizes the strong-lensing cross-section (Mandelbaum, van de Ven & Keeton 2009; van de Ven, Mandelbaum & Keeton 2009). In Fig. 2, we project the halo along the intermediate axis, which maximizes the deviation of the surface density from axisymmetry. This particular halo is one of the most massive haloes identified in the $50\, h^{-1} \text{Mpc}$ box runs, with $M_{\text{vir}} = 1.2 \times 10^{14} \, M_{\odot}$ and $r_{\text{vir}} = 1.27 \, \text{Mpc}$. The top row in each figure shows the surface density on the scale of the virial radius, while the lower row shows the inner $300\, h^{-1} \text{kpc}$ of the halo (side-to-side). The surface-density stretch is the same between the two figures.

The major and minor axes for the projections in these figures were determined using the moment-of-inertia tensor of all particles within a sphere of radius $r_{\text{vir}}$ in the halo. If modelling the mass distribution as an ellipsoid, the principal axes $a$(major) > $b$(intermediate) > $c$(minor) are the square roots of the eigenvalues of this tensor.

In comparing Figs 1 and 2, note that Fig. 1 is the most relevant for strong-lensing studies (Section 4) and shows the smallest differences, especially at large radii. Indeed, only the zoomed view of the $\sigma/m = 1 \, \text{cm}^2 \, \text{g}^{-1}$ run is visibly rounder than the CDM case. Even for the intermediate projection (Fig. 2), which maximizes the visual difference, the inner regions of the halo are only slightly rounder and less dense than their CDM counterparts for $\sigma/m = 0.1 \, \text{cm}^2 \, \text{g}^{-1}$. The $\sigma/m = 1 \, \text{cm}^2 \, \text{g}^{-1}$ case is indeed less dense and rounder within $\sim 100 \, h^{-1} \text{kpc}$, but even in this case, some ellipticity is clearly evident.

A final point of interest in these visualizations concerns the substructure. The subhaloes apparent in the CDM halo are similarly abundant in the SIDM cases, and even approximately match in their positions. There are minor differences in substructure densities and locations (especially in the central regions) but overall it is difficult to distinguish among the runs by comparing their substructure content (see Rocha et al. 2013 for a more quantitative comparison of substructure).

### 3.2 Three-dimensional halo shapes
We quantify halo shapes by examining ellipsoidal shells centred on the radial slices identified by AHF (Knollmann & Knebe 2009) for profile measurements. We use shells instead of enclosed volume because this measure is less sensitive to numerical-relaxation effects at the centre and because it is a better estimate of the effects of local dark-matter scattering. In each shell of material, we calculate a modified moment-of-inertia tensor (defined and used in Allgood et al. 2006) in ellipsoidal shells,

\[
\tilde{\mathbf{I}}_{ij}(a) = \sum_{n} \frac{x_{i,n}x_{j,n}}{r_n^a},
\]

where

\[
r_n = \sqrt{x_{1,n}^2 + x_{2,n}^2/(b/a)^2 + x_{3,n}^2/(c/a)^2},
\]

and $(x_{1,n}, x_{2,n}, x_{3,n})$ are the coordinates of the $n$th particle in the frame of the principal axes (major, intermediate, minor) of this tensor. This is the same moment-of-inertia tensor from which shapes are inferred in Dubinski & Carlberg (1991) and Davé et al. (2001). The principal axes $(a, b, c)$ are computed as the square roots of the eigenvalues of $\tilde{\mathbf{I}}_{ij}$. The weighting of the moment-of-inertia tensor is chosen such that the outermost particles in the shell do not
dominate the shape estimate. We begin by finding the moment-of-inertia tensor in a spherical shell, setting \( a = b = c = 1 \) for this initial estimate of \( \ell_{ij} \), and iterate to find \( \ell_{ij} \) with convergent \( a, b, c \) values. In each iteration, the ellipsoidal shell volume is defined using the \( \{a, b, c\} \) found in the previous iteration. We experiment with either keeping the semi-major axis \( a \) of the shell fixed between iterations or allowing \( a \) to float such that the volume in the shell remains fixed as we iterate to find \( \ell_{ij}(a) \), but find that \( c/a \) is insensitive to these choices. Throughout this section, we show \( c/a \) for fixed volume in the shell. We only show results for \( c/a \) because the trends for \( b/a \) are similar but less informative, since \( c/a \) indicates the deviation of the halo shape from sphericity.

In order to understand trends, we split our analysis of host dark-matter haloes and subhaloes, and bin haloes by virial (or tidal) mass. Host haloes are those whose centres do not lie within the virial radius of a more massive halo. In Fig. 3, we show the minor-to-major axis ratio \( c/a \) as a function of radius normalized by the halo virial radius, \( r/r_{\text{vir}} \), for host haloes in three mass bins. Larger values of \( c/a \) imply more spherical haloes. For the two lower mass bins, we used haloes selected from the 25 h\(^{-1}\) Mpc boxes since these have the higher resolution. For the highest mass bin, we used haloes identified in the 50 h\(^{-1}\) Mpc boxes in order to gain better statistics. We checked to make sure that the results were convergent between boxes where the relevant mass resolutions overlap. The shaded region corresponds to the 20th to 80th percentile for \( c/a \) of the halo population for fixed \( r/r_{\text{vir}} \), and the central line shows the median value of \( c/a \). The black solid lines and yellow shaded regions denote shapes of CDM haloes, the blue dashed lines and regions correspond to \( \sigma/m = 1 \) cm\(^2\) g\(^{-1}\), and the green dotted lines and regions correspond to \( \sigma/m = 0.1 \) cm\(^2\) g\(^{-1}\). The regions extend down in \( r/r_{\text{vir}} \) to the largest value of \( r_{\text{min}}/r_{\text{vir}} \) in the given mass bin.

We reproduce the well-known trend that galaxy-mass haloes in CDM are more spherical than cluster-mass haloes and that CDM haloes become more spherical in their outer parts (Allgood et al. 2006). SIDM haloes deviate most strongly from CDM at smaller radii, where the scattering rates are highest for a fixed cross-section. For SIDM\(_1\), haloes are actually more spherical in their centres than their edges, with \( c/a \) rising with decreasing \( r \) for \( r/r_{\text{vir}} < 0.5 \). For SIDM\(_{\text{obs}}\), differences from CDM are only apparent for \( r/r_{\text{vir}} < 0.1 \). In Fig. 4, we compare the shapes of subhaloes (thick lines) and host haloes (thin lines) of similar mass by plotting the median axis ratio \( c/a \) as a function of \( r/r_{\text{outer}} \) where \( r_{\text{outer}} = r_{\text{tidal}} \) for subhaloes (as defined by the AIP) and \( r_{\text{outer}} = r_{\text{vir}} \) for host haloes. All haloes have masses within \( r_{\text{outer}} \) between 10\(^{11}\) h\(^{-1}\) M\(_{\odot}\) and 10\(^{14}\) h\(^{-1}\) M\(_{\odot}\).

Though not shown, the 20th to 80th percentile ranges are similar in size as in Fig. 3. We find that the subhalo interiors in the \( \sigma/m = 1 \) cm\(^2\) g\(^{-1}\) cosmology are systematically rounder than host haloes. We speculate that there are at least three effects that drive this trend. First, subhaloes typically form earlier than field haloes of the same mass, with fewer recent mergers. Allgood et al. (2006) find that haloes that form earlier are more spherical than haloes that form later, which is attributed to directional merging, and more generally to the highly non-spherically symmetrical way in which haloes form and accrete. Secondly, the inner parts of subhaloes become, on average, more spherical with time as the subhaloes orbit within the host, a result of tidal interactions with the host (Kuhlen, Diemand

![Figure 3](https://example.com/figure3.png)

**Figure 3.** Host halo shapes in shells of radius scaled by the virial radius in three virial-mass bins as indicated. The black solid lines denote the 20th percentile (lowest), median (middle), and 80th percentile (highest) value of \( c/a \) at fixed \( r/r_{\text{vir}} \) for CDM. The blue dashed lines show the median and 20th/80th percentile ranges for \( \sigma/m = 1 \) cm\(^2\) g\(^{-1}\), and the green dotted lines show the same for \( \sigma/m = 0.1 \) cm\(^2\) g\(^{-1}\). There are 440, 65 and 50 haloes in each mass bin (lowest mass bin to highest).

![Figure 4](https://example.com/figure4.png)

**Figure 4.** Median subhalo shape versus radius for galaxy-mass systems compared to host haloes of the same mass. Bold lines denote subhaloes and lighter lines denote host haloes. The radii are normalized by \( r_{\text{vir}} \) for hosts and \( r_{\text{tidal}} \) for subhaloes. Line colours and styles have the same meanings as in Fig. 3: dashed blue is SIDM\(_1\), dotted green is SIDM\(_{\text{obs}}\) and solid black is CDM.
Figure 5. Median axis ratio c/a for host haloes as a function of the local scattering rate modulo the cross-section: $\rho\sigma/m \sim \Gamma(\sigma/m)^{-1}$. Smaller values correspond to the outer halo, where the density and scattering rate are low. The quantity is scaled by 10 Gyr cm$^{-2}$ g$^{-1}$, such that 1 in these units means that each particle has roughly one interaction per 10 Gyr in SIDM$_1$ (blue dashed line) and one interaction per 100 Gyr in SIDM$_{0.1}$ (green dotted line). The black solid line is CDM.

These two mechanisms affect CDM subhaloes as well as SIDM subhaloes. Thirdly, and specific to SIDM haloes, the subhalo masses are the remains of more massive haloes, which are more susceptible to the effects of dark-matter scatters at fixed $r/r_{\text{vir}}$, as we showed in Fig. 3. Moreover, the outer radius of the subhalo is truncated with respect to its virial value, thus pushing the effects of dark-matter scatters at a radius $r/r_{\text{vir}}$ to the higher $r/r_{\text{out}}$ for fixed $r$. In other words, the effects of dark-matter scatters at small $r/r_{\text{out}}$ (namely, a high $c/a$) get mapped to a comparatively high $r/r_{\text{out}}$.

To see how the shape of dark-matter haloes changes as a function of the typical local scattering rate (equation 1), we plot $c/a$ as a function of $\rho(r)r_{\text{min}}(r) \propto \Gamma(\Gamma(r)(\sigma/m)^{-1}$ in Fig. 5. The proportionality constant in relating $\rho r_{\text{min}}$ to the scattering rate is $\mathcal{O}(1)$ and depends on the distribution function of dark-matter particles. Thus it is reasonable to use $\rho r_{\text{min}}$ as a proxy for the local scattering rate modulo the actual cross-section. To simplify the interpretation further, we multiply this quantity by 10 Gyr cm$^{-2}$ g$^{-1}$ in Fig. 5. In these units, if $\rho(r)\sigma/m > 1$ for $\sigma/m = 1$ cm$^2$ g$^{-1}$, most particles will have scattered after 10 Gyr. For $\sigma/m = 0.1$ cm$^2$ g$^{-1}$, this quantity needs to be 10 times larger to achieve the same scattering rate. Generally, $\rho r_{\text{min}}$ increases as one goes in towards the halo centre, so particles tend to scatter more frequently in the core than in the outer parts of the halo, where interactions are uncommon over a Hubble time.

Fig. 5 shows that deviations in the halo shape from CDM begin when $\rho r_{\text{min}}(\sigma/m) \times (10$ Gyr$) \sim 0.1$, independent of halo mass. This corresponds to approximately 10 per cent of particles having scattered over a Hubble time at this radius. However, the changes are small compared to the change where $\Gamma(10$ Gyr$) \gtrsim 1$. We note here that the most massive haloes in Davé et al. (2001) also seem to show the same qualitative behaviour. However, even for large values of $\rho r_{\text{min}}$, the deviation from sphericity is significant, a fact that is in some disagreement with the simulation results of Davé et al. (2001) where $c/a > 0.9$ for their most massive halo. We speculate that part of this could be due to the differences in the way the ellipticity was estimated and part could be due to the smaller box run by Davé et al. (2001), which implies a quieter merger history. It is well known that CDM haloes have anisotropic velocity ellipsoids and elongated shapes that are driven partially by directional merging (Allgood et al. 2006). These mergers provide a source of anisotropy that needs to be overcome by scattering in order for haloes to reach sphericity. We also note that the energy transfer facilitated by self-interactions would lead to an isotropic velocity dispersion tensor and that does not necessarily imply a rounder halo. To make this connection between isotropic velocity dispersion tensor and a rounder halo, previous analytic estimates have relied on the simulations of Davé et al. (2001).

The difference may also be a numerical artefact: we find that halo shapes only converge if there are at least 10$^5$ particles in the halo and only for radii $r > 2r_{\text{rel}}$ (see Section 2). Most of the haloes used for shape estimates in Davé et al. (2001) only have 10$^3$ particles in the virial radius. Their largest halo does have more than enough particles for reliable shape estimates. However, for this massive halo, Davé et al. (2001) show shape measurements at radii much smaller compared to the convergence radius than we do. In our simulations, we find that haloes appear artificially round below the convergence radius. Our shape measurements are consistent with Davé et al. (2001)’s massive halo for radii above the convergence radius.

The other major effect of energy transfers due to scattering is to create a core (see Rocha et al. 2013) and deep inside a constant density core, we must have $c/a \rightarrow 1$. Hence we expect the log slope of the density profile to correlate strongly with the halo shape. Thus, instead of the density profile scaling as $\rho \sim r^{-1}$ in the interior as expected for CDM (Navarro, Frenk & White 1997; Navarro et al. 2004, 2010), the density profile of SIDM haloes plateaus close to the centre. We use the following proxy for the negative log slope of the density profile:

$$\gamma = 3 - 4\pi\rho(r)r^3/M(r),$$

where $M(r)$ is the mass enclosed within radius $r$ and $\gamma = -d \log \rho/d \log r$ for a power-law density profile. Based on the previous figures, we expect halo shapes to become increasingly round as $\gamma \lesssim 1$, and that the CDM and $\sigma/m = 0.1$ cm$^2$ g$^{-1}$ haloes should not dip below $\gamma \approx 1$. In Fig. 6, we show $c/a$ as a function of the log slope of the density profile, as approximated using equation (3), for host haloes in our highest mass bin. We obtain similar results for haloes of smaller mass, but only show the highest mass bin because these haloes are the best resolved.

We find that, indeed, SIDM$_1$ haloes become significantly rounder as $\gamma < 1$ and become almost completely round when $\gamma$ gets much smaller than 0.5. Interestingly, we also see that $c/a$ deviates strongly from CDM even for relatively large values for the log slope, in
regions of the halo in which the scattering is not efficient at changing the radial density profile. This is a consequence of the fact, as shown in Fig. 5, that it does not take a lot of scatters to start rounding out the haloes, although it takes multiple scatters for the haloes to acquire $c/a$ axis ratios in excess of 0.8. Thus, the effects of scattering are apparent for $\sigma/m = 1\,\text{cm}^2\,\text{g}^{-1}$ even when $\rho \sim r^{-2}$ and the density profile is unaffected by scatterings. However, the observational importance of this behaviour is mitigated by two factors – the change for $y \gtrsim 1$ is mild and easily within the scatter in ellipticities seen in CDM.

In summary, we find that it only takes a modest local scattering rate per particle, $\Gamma(r) \gtrsim 0.1H_0$, to start changing the three-dimensional halo shape within radius $r$ with respect to CDM. We find that SIDM haloes get significantly rounder compared to CDM predictions when the negative log slope of the density profile $y \ll 1$ in regions where dark-matter particles (on average) have had at least interactions in a Hubble time ($\Gamma \times (10\,\text{Gyr}) > 1$). Our results show that even in the limit of one or more scatterings, the halo shapes retain some of their initial triaxiality.

4 COMPARISONS TO OBSERVATIONS

4.1 Defining Observables

There are a number of ways of quantifying deviations of mass distributions from spherical or axial symmetry. In the previous section, we quantified the deviations in terms of $c/a$, the ratio of the semi-minor to semi-major axes determined from the modified three-dimensional moment-of-inertia tensor, equation (2). This is rarely a practical shape estimate observationally. Instead, there is a more suitable measure of halo ellipticity or triaxiality for each type of observation. The relationship between these measures is non-trivial, so care must be taken to compare theory to observation appropriately. From paper to paper, the definition of ‘ellipticity’ can change significantly. In order to facilitate these comparisons, we define three distinct measures of asymmetry in this section and go on to use them for specific comparisons to observational studies in Sections 4.2–4.4. The symbols we use for these shape definitions are summarized in Table 2.

For strong lensing, which we discuss in Sections 4.2 and 4.4, what matters is the deviation of the convergence from axial symmetry. The convergence is $\kappa = \Sigma/\Sigma_{\text{cr}}$, where $\Sigma_{\text{cr}}$ is the critical surface density for creating multiple images. For an axially symmetric system the convergence will be $\kappa(\theta)$, and depend only on the angular distance on the sky $\theta$ from the centre of the lens. Once axial symmetry is broken, one must consider $k(\theta, \phi)$, where $\phi$ is the azimuthal angle that rotates on the sky. Miralda-Escudé (2002) used the following quadrupole approximation to fit the surface density of MS 2137–23,

$$\kappa(\theta, \phi) = \kappa_0(\theta) - \frac{\epsilon}{2} \frac{\rho_0}{\rho} \cos(2\phi), \quad (4)$$

where $\kappa_0(\theta)$ is the convergence averaged along azimuthal angle and $\epsilon$ quantifies the amplitude of deviation of the convergence from axial symmetry. Since the normalization of $\kappa$ and the angle $\theta$ depend on the source, lens and source-to-lens distances, which generically vary, we quantify deviations from axial symmetry in terms of surface density $\Sigma(R, \phi)$, where $R$ is the two-dimensional physical radius in projection. Using equation (4), we define the measure of ellipticity

$$\epsilon'(R, \phi) \equiv \frac{2(\Sigma_0(R) - \Sigma(R, \phi))}{d\Sigma_0/dR}, \quad (5)$$

which should be equivalent to $\epsilon \cos(2\phi)$ if the quadrupole expansion of the two-dimensional surface density is approximately correct. Here, $\Sigma_0(R)$ is the azimuthally averaged surface density.

The second type of measure used to quantify deviations from spherical or axial symmetry in lensing arises from the extension of the double pseudo-isothermal sphere,

$$\rho(r) = \frac{\rho_0}{(1 + r^2/r_{\text{core}}^2)\left(1 + r^2/r_{\text{cut}}^2\right)^2}, \quad (6)$$

to allow for deviations of the surface-mass density from axial symmetry (double pseudo-isothermal elliptical, or dPIE; see Richard et al. 2010),

$$\Sigma_{\text{dPIE}}(x, y) = \frac{\sigma_0^2}{2G} \frac{r_{\text{cut}}}{r_{\text{core}}} \frac{r_{\text{core}}}{r_{\text{core}}} \left[ \frac{1}{\sqrt{r_{\text{core}}^2 + x^2}} - \frac{1}{\sqrt{r_{\text{cut}}^2 + x^2}} \right], \quad (7)$$

with

$$r^2 = [(x-x_c)/2]^2 + [(y-y_c)/(2-2e)]^2. \quad (8)$$

The dPIE profile has the properties that $\rho \sim \text{const.}$ for $r \ll r_{\text{core}}$, $\rho \propto r^{-2}$ for $r_{\text{core}} \ll r \ll r_{\text{cut}}$, and $\rho \propto r^{-4}$ for $r \gg r_{\text{cut}}$, so that the total mass is finite. Here, $\epsilon$ is the measure of ellipticity of the dPIE profile. The centre of the halo in projection is denoted by $(x_c, y_c)$, with the $x$-direction aligned with the major axis of the distribution. This surface-density profile is often used in fits of the

Table 2. Summary of shape definitions used in observational comparisons.

| Symbol | Defining equation | Description | Relevant figure |
|--------|------------------|-------------|----------------|
| $\epsilon'(R, \phi)$ | equation (5) | Deviation from axial symmetry in lensing convergence maps | Fig. 7 |
| $\epsilon$ | equation (8) | Ellipticity in dPIE surface density fits to lensing signal | Fig. 11 |
| $\epsilon$ | equation (9) | Ellipticity of similar spheroid used in X-ray studies | Fig. 8 |
shapes of galaxies or dark-matter haloes in clusters that strongly lens background galaxies.

The third way we will quantify halo shapes observationally is in terms of similar spheroids (see e.g. section 2.5 of Binney & Tremaine 2008). For spheroids, we can define an ellipticity parameter

\[ \epsilon = 1 - \frac{b}{a} \]

where \( b \) is the semi-minor and \( a \) the semi-major axis. If the \( z \)-axis is the symmetry axis, the semi-major axis is given by

\[ a^2 = R^2 + \frac{z^2}{(1 - \epsilon)^2} \]  
for oblate spheroids, and

\[ a^2 = \frac{R^2}{(1 - \epsilon)^2} + z^2 \]  
for prolate spheroids. Similar spheroids are those for which the density profile may be described in terms of \( \rho(a) \) and \( \epsilon \) is fixed throughout the body. The isopotential surfaces of such spheroids are rounder at large distances if the body is more centrally concentrated (see the discussion in section 2.5 of Binney & Tremaine 2008).

As summarized in Table 2, \( \epsilon \) is the surface-density shape definition we use in Section 4.2, \( \epsilon \) is the X-ray motivated shape definition we use in Section 4.3, and \( \epsilon \) is the lensing-fit shape definition used in Section 4.4.

4.2 Revisiting Miralda-Escudé (2002)

Galaxy clusters are great places to look for the effects of velocity-independent SIDM because one may typically achieve much higher values of \( \rho(r) \) at fixed \( r/r_{\text{vir}} \). In addition, there are many different probes of the mass distribution of clusters that span an enormous dynamic range of radial scale – stellar kinematics and strong lensing towards the centre of the cluster, weak lensing and X-ray gas distributions throughout the halo volume, and weak + strong-lensing maps of the matter distribution around individual galaxies in the cluster (Sand et al. 2008; Newman et al. 2009; Kneib & Natarajan 2011; Newman et al. 2011). It is no surprise the tightest constraints on velocity-independent SIDM emerged from cluster studies, and a revisit of the tightest of these constraints is the subject of this section.

The strongest published constraint on velocity-independent SIDM, \( \sigma/m \lesssim 0.02 \text{ cm}^2 \text{ g}^{-1} \), came from Miralda-Escudé (2002)’s study of the galaxy cluster MS 2137–23. This cluster has an estimated virial mass of \( \sim 8 \times 10^{14} M_\odot \) (Gavazzi 2005), and its mass distribution has also been studied by Fort et al. (1992), Mellier et al. (1993), Miralda-Escudé (1995), Gavazzi et al. (2003) and Sand et al. (2008). There are two strongly lensed galaxies that produce a total of five distinct images: one source has a radial image at \( \sim 5 \) arcsec from the centre of the brightest cluster galaxy (BCG) and an arclet at \( \theta = 22.5 \) arcsec. The other source has a large tangential arc and two arclets all at about \( \theta = 15 \) arcsec from the BCG centre, which corresponds to 70 kpc. In order to reproduce both the relative magnifications and the alignments of the images in the sky, the surface density must deviate from axial symmetry at 70 kpc. Quantitatively, it means that the parameter \( \epsilon \) in equation (4), which corresponds to the amplitude of \( \epsilon \) given in equation (5), must be \( \epsilon \approx 0.2 \) at \( R = 70 \) kpc. This figure is largely driven by the tangential arcs and associated arclets (Miralda-Escude 1995).

Based on the ellipticity at 70 kpc and an argument that the dark-matter surface density should be approximately axial for a typical particle collision rate \( \Gamma \gtrsim H_0 \), Miralda-Escudé (2002) asserts that \( \Gamma(70 \text{ kpc}) \lesssim H_0 \). Using the fact that the tangential arc should lie at approximately where the mean interior convergence \( \bar{\kappa} = 1 \) (or an estimated critical density \( \Sigma_\text{crit} = 1 \text{ g cm}^{-2} \)) and the rough approximation \( \rho(r) \sim \Sigma(r)/r \), Miralda-Escudé estimates the three-dimensional density \( \rho(70 \text{ kpc}) \). Using the velocity dispersion of the BCG at the centre of the halo as a proxy for \( v_{\text{min}} \), Miralda-Escudé uses equation (1) to determine a limit on \( \sigma/m \), which is found to be \( \sigma/m \lesssim 0.02 \text{ cm}^2 \text{ g}^{-1} \).

We get a sense that this line of reasoning may be flawed when we examine the surface-density plots of one of our most massive haloes in Figs 1 and 2 and our findings of Section 3. First, recall that our results show (cf. Fig. 5) that the inner halo shape retains some triaxiality even when \( \Gamma \gtrsim H_0 \). Secondly, the surface density includes all matter along the line of sight, not just the material within \( r < R \). Thus, the surface density at small \( R/r_{\text{vir}} \) includes a lot of material with large \( r \), far out in the halo where SIDM scatters are unimportant. This material is still quite triaxial. Moreover, SIDM also creates cores, which means that the outskirts of the halo have an even greater weight in the total surface density than if the halo were still cuspy at the centre. Empirically, we see that the simulated surface densities in Figs 1 and 2 are quite elliptical. This point becomes more and more important as the size of the core becomes smaller. All of these things suggest that the constraint reported in Miralda-Escudé (2002) is far too high.

When attempting to quantify the constraints on SIDM from MS 2137–23 using our simulations, we run into the following problem: the largest halo in our simulations has a virial mass \( M_{\text{vir}} = 2.2 \times 10^{14} M_\odot \), a factor of approximately 4 smaller than the estimated virial mass of MS 2137–23. Moreover, we do not know the orientation of the principal axes of the cluster with respect to the line of sight. In order to make the comparison, we do two things. First, we can use virial scaling relations to estimate the radius at which \( \rho(r)v_{\text{rms}}(r) \), a proxy for the scattering rate (see Fig. 5), has the same value as it would for a radius of 70 kpc in MS 2137–23, in other words, we look for the radius at which the SIDM scattering rate should be comparable to the radius at which Miralda-Escudé (2002) finds the constraint for MS 2137–23. For our \( M_{\text{vir}} \sim (1–2) \times 10^{14} M_\odot \) haloes, \( r = 35 \) kpc is roughly the point at which the scattering rate is similar to that at 70 kpc in MS 2137–23. This is outside \( r_{\text{min}} \) for these haloes, so we trust the shape measurements. Secondly, we look at several projections of the haloes. We calculate \( \Sigma_0(R) \), \( \Sigma(R, \phi) \) and hence \( \epsilon(\phi) \) for the various projections of the haloes.

We show an example of \( \epsilon(\phi) \) curves for lines of sight along the principal axes of the halo moment-of-inertia tensors of one of our largest haloes in Fig. 7, the same halo shown in Figs 1 and 2. As in previous figures, the solid black line denotes the CDM result, the blue dashed line the result for \( \sigma/m = 1 \text{ cm}^2 \text{ g}^{-1} \), and the green dotted line the result for \( \sigma/m = 0.1 \text{ cm}^2 \text{ g}^{-1} \). We do not show the curve for \( \sigma/m = 0.03 \text{ cm}^2 \text{ g}^{-1} \) even though it is the cross-section closest to the Miralda-Escudé (2002) constraint because it is indistinguishable from the CDM line. The red dotted line shows the minimum amplitude of \( \epsilon(\phi) \) required for the lens model of MS 2137–23. The \( \epsilon(\phi) \) curves of the other massive haloes look similar to the curves for the halo shown in Fig. 7. What we find is that even the \( \sigma/m = 1 \text{ cm}^2 \text{ g}^{-1} \) curve generally satisfies the MS 2137–23 constraint. Therefore, we find that the Miralda-Escudé (2002) constraint is in fact overly constraining by two orders of magnitude. While we do not simulate cosmologies with...
$\sigma/m > 1 \text{ cm}^2 \text{ g}^{-1}$, and thus cannot set a quantitative upper limit on the SIDM cross-section, we may conclude that $\sigma/m = 1 \text{ cm}^2 \text{ g}^{-1}$ is not ruled out by MS 2137–23.

There are a few caveats to this conclusion, which we do not believe will significantly alter our claim. First, none of our simulated clusters is as massive as MS 2137–23. This precludes us from doing a detailed comparison of the projected densities in $\sigma/m = 1 \text{ cm}^2 \text{ g}^{-1}$ to that required to explain the arcs in MS 2137–23. It is, however, informative to do a simple calculation to gauge the importance of this effect. We use the result of the companion paper (Rocha et al. 2013) that show that the density profile in SIDM for $\sigma/m = 1 \text{ cm}^2 \text{ g}^{-1}$ is well fitted by a Burkert profile (Burkert 1995), and that this profile deviates from CDM density profiles at radii smaller than half the Burkert scale radius. The fact that the Burkert profile is a good fit for this cross-section is a coincidence and does not hold for significantly larger or smaller values of the SIDM cross-section (Rocha et al. 2013). At this point, $r_h/2 = 0.35(r_{\text{max}}/21.6 \text{kpc})^{0.08}$, the density profile becomes almost constant. We use the $V_{\text{max}} = r_{\text{max}}$ relation seen in our CDM simulations and the NFW profile to compute the projected mass within 70 kpc as the CDM prediction and compare that to the SIDM prediction by computing the same projected density but assuming that $\forall r < r_h/2, \rho_{\text{SIDM}}(r) = \rho_{\text{CDM}}(r_h/2)$. This computation reveals that the projected density in $\sigma/m = 1 \text{ cm}^2 \text{ g}^{-1}$ model should be about 30 per cent lower than in CDM for the median $(1–5) \times 10^{15} \text{ M}_\odot$ haloes. Even for the CDM case, however, the projected density is about a factor of 2 smaller than the estimated $X_{\text{vir}} = 1 \text{ g cm}^{-2}$. Clearly, the estimated projected density (as opposed to the shape) could be a significant constraint on the $\sigma/m = 1 \text{ cm}^2 \text{ g}^{-1}$ model. This simple computation motivates further work along these lines with more realistic SIDM density profiles, inclusion of scatter and the uncertainty in the halo virial mass.

We have started much larger-scale simulations in order to study clusters in more detail, which will include an investigation of strong-lensing cross-sections as well as ellipticity. In terms of the ellipticity function $\epsilon(R, \phi)$, it is not clear in which direction our results will go for simulated $8 \times 10^{14} \text{ M}_\odot$ haloes. On one hand, large haloes are more triaxial than small haloes, so we would expect that if anything, we would be underestimating the degree of ellipticity in the surface-density distribution. On the other hand, large haloes have larger and rounder cores for velocity-independent SIDM compared to their lower-mass cousins. This might drive the constraints the other way, although a larger core implies that the outskirts of the halo get weighted more heavily along the line-of-sight integral of the density for the surface-density calculation. Finally, several authors have noted that MS 2137–23 is actually unusually round for a galaxy cluster (Gavazzi 2005; Sand et al. 2008). Moreover, intriguingly, the modelling of both Miralda-Escude (1995) and Sand et al. (2008) indicates that a cored dark-matter radial density profile is preferred over a strongly, CDM-like cuspy profile for the dark-matter halo. Such a cored profile is more in line with what we would expect for SIDM. Tighter constraints on $\sigma/m$, or a measurement should it be non-zero, will come from an ensemble of clusters, including the most triaxial of them, a point to which we return in Section 4.4.

### 4.3 Shapes from X-ray observations of elliptical galaxies

Constraints on dark-matter scattering have also been made using the shapes of significantly smaller dark-matter haloes, $M_{\text{vir}} \sim 10^{12}–10^{13} \text{ M}_\odot$ (Feng et al. 2009, 2010; Buckley & Fox 2010; McDermott et al. 2011). In this case, the observations consist of X-ray studies of the hot gas haloes of elliptical galaxies (Buote et al. 2002).

One may use the shape and twisting of the X-ray isophotes to learn about the three-dimensional shape of the dark-matter halo. Interestingly, halo shapes may be constrained with imaging data alone and do not rely on temperature-profile modelling, which is required for mass-density determinations. This ‘geometric argument’ was first made by Binney & Strimpel (1978) and subsequently applied by a number of authors in the study of elliptical galaxies and galaxy clusters (Fabricant, Rybicki & Gorenstein 1984; Buote & Canizares 1994, 1996, 1998b; Buote et al. 2002). The geometric argument is the following: for a single-phase gas in hydrostatic equilibrium, the three-dimensional surfaces in the gas temperature $T$, gas pressure $p_g$, gas density $\rho_g$ and gravitational potential $\Phi$ all have the same shape (Binney & Strimpel 1978; Buote & Canizares 1998a). Surfaces of constant emissivity $j_x \propto \rho_g^2$ have the same three-dimensional shape as the isopotential surface.

On the other hand, if the spectral data are available (as they typically are with the Chandra telescope), then the temperature-profile data may be used with the assumption of hydrostatic equilibrium to...
fit the X-ray isophotes. Buote et al. (2002) use both approaches to model NGC 720. X-ray isophotes are a good probe of the shape of the local matter distribution for the following reasons. First, the total mass profile (galaxy + gas + dark-matter halo) is nearly isothermal ($\rho \sim r^{-2}$) for elliptical galaxies (Humphrey et al. 2006; Gavazzi et al. 2007). This means that the shape of the isopotential contours reflects the shape of the matter distribution at the same radius, although typically the isopotential surfaces are significantly rounder than the isodensity contours (see e.g. Binney & Tremaine 2008). Secondly, the density profile of gas tends to be pretty sharply cuspy ($\rho_g \sim r^{-1.5}$). Since the emissivity goes as $j_X \propto \rho_g^2$, and the morphology of $j_X$ traces that of the gravitational potential (see the discussion on the geometric argument above), this means that most of the X-ray emission along the line of sight is concentrated at radii close to the projected radius. Thus, the X-ray isophotes indicate the shape of the matter distribution at radii similar to the projected radius.

In this section, we focus on the shape measurement of the X-ray emission about the elliptical galaxy NGC 720. The inferred shape of the matter distribution of this system has been used to set constraints on self-interacting dark matter in the recent past (Feng et al. 2009, 2010; Buckley & Fox 2010; McDermott et al. 2011). The data set used by Buote et al. (2002) for the shape measurement of NGC 720 is a 40 ks exposure of the inner 5 arcmin of the galaxy with the ACIS-S3 camera on the Chandra telescope. The data included in the fit are contained within a 35–185 arcsec$^2$ annulus from the centre of the galaxy, which corresponds to $\approx 4.5–22.4$ kpc. Buote et al. (2002) estimate the three-dimensional isopotential shapes in the following way. First, they investigate a mass-follows-optical light ($M \propto L_r$) mass profile for the gravitating mass using the geometric argument. They use a spheroidal Hernquist (1990) model for the stellar mass of the galaxy, with structural parameters determined by deprojecting the optical image to three dimensions. This is to test if the stars in the galaxy may be sufficient to provide a gravitational potential for the gas. They find that the isophotes are rounder than observed beyond about $R_e$ ($\approx 50$ arcsec or 5.3kpc). This suggests that there must be significant ellipsoidally distributed mass extending well beyond the effective radius of stars, since isopotential contours become round as the distance to the main ellipsoidal mass profile increases.

Next, Buote et al. (2002) use the fact that they find the temperature profile of the halo gas to be isothermal to find the three-dimensional X-ray emissivity distribution ($j_X \propto \rho_g^2$) directly from the equation of hydrostatic equilibrium:

$$\frac{\rho_g(r)}{\rho_g(0)} = \exp \left[ -\frac{\mu m_p \Phi(0)}{k_B T} \frac{\Phi(r)}{\Phi(0)} - 1 \right],$$

where $\mu$ is the molecular weight of the gas atoms and $m_p$ is the proton mass. They used three similar spheroid models for the mass distribution of the galaxy (baryons plus dark matter) to find the gravitational potential $\Phi(r)$. The three models were pseudo-isothermal [$\rho(a) \propto 1/(1 + (a/a_0)^2)$ where $a$ is defined in equations (10) and (11)], Navarro–Frenk–White (Navarro et al. 1997) and Hernquist (Hernquist 1990). They fit both oblate and prolate spheroids, and assumed that the symmetry axis was in the plane of the sky, as this leads to the most elliptical isophotes for a given spheroid. Once the potential was found, they calculated the emissivity distribution in three dimensions and integrated along the line of sight. A $\chi^2$ statistic was used to test the quality of the model fits to the X-ray surface-brightness map.

The best-fitting models were the pseudo-isothermal ones, with the oblate and prolate spheroids having nearly identical goodness-of-fit. The best-fitting ellipticity $\epsilon = 0.37 \pm 0.03$ for the oblate spheroïd, and $\epsilon = 0.36 \pm 0.02$ for the prolate spheroïd. The flatness of the ellipticity as a function of radius in the region of interest drives the best-fitting density-profile for isothermality. Only for a similar isothermal spheroïd are the X-ray isophotes so constant in ellipticity assuming density profiles that are similar spheroïds.

We compare our simulations to the Buote et al. (2002) results by using the weighted moment-of-inertia tensor ellipsoidal shape estimator of Section 2. This method weights particles in the region of interest equally in estimating the shape of the mass distribution, and does not preferentially weight particles near the edge as the true moment-of-inertia tensor does. The initial region in which the ellipsoidal shapes are estimated is a spherical shell of radius $r_{\text{min}} < r < r_{\text{max}}$, where $r_{\text{min}}$ is our numerical limit on the shape convergence radius and $r_{\text{max}} = 14$ kpc. We choose this value of $r_{\text{max}}$ as a compromise between finding the ellipticity at small radii, where differences with CDM are highest, and having enough particles in the region of interest for a robust and unbiased shape estimate. The shape of the region of interest is deformed in each iteration of the weighted moment-of-inertia calculation to reflect the major axes of the morphology of the tensor at that iteration, keeping the morphology of the region of interest elliptoidal and fixing its volume. This region is within the core radii of the $\sigma/m = 1$ cm$^{-2}$ g$^{-1}$ haloes, and is our best approximation to the shape of the innermost part of the dark-matter haloes, the parts relevant to the study of NGC 720. For reference, the $10^{12}$, $10^{13}$ M$_{\odot}$ haloes have median core radii 16 and 43 kpc, respectively (Rocha et al. 2013).

In order to find the spheroidal ellipticity (described in equation (9)) from these spheroidal fits, we use the ellipsoidal axis ratios to decide if the halo is oblate or prolate. If prolate, we take axis ratio of the spheroid $1/\epsilon$, cf. equations (10) and (11), to be $\sqrt{c/a}$, i.e. the spheroidal semi-minor axis is the geometric mean of the elliptical minor and intermediate axes. If oblate, we set $1 - \epsilon = c/\sqrt{a}$, i.e. the semi-major axis of the oblate spheroid is set to be the geometric mean of the major and intermediate axes of the ellipsoid. Both these choices set the spheroidal volume equal to the ellipsoidal volume. A more realistic analysis would compute the integrated $j_X$ directly from the potential of the simulated halo and compare that to the observations. However, there are significant other uncertainties (discussed below) that argue against such an approach being more fruitful.

In Fig. 8, we show the ellipticity distribution for all haloes in our 25 h$^{-1}$ Mpc boxes for CDM and SIDM, that have a virial mass within the 1$\sigma$ uncertainties of the mass modelling in Humphrey et al. (2006) for NGC 720, $M_{\text{vir}} = (6.6^{+7.3}_{-6.3}) \times 10^{13}$ M$_{\odot}$. Note that we do not weight the $\epsilon$ distribution by the error distribution for the virial mass presented in Humphrey et al. (2006). However, the $\epsilon$ distribution in our simulated haloes is not a strong function of virial mass in this mass range. We show the CDM $\epsilon$ distributions with the black unfilled histograms, the $\sigma/m = 1$ cm$^{-2}$ g$^{-1}$ $\epsilon$ distributions with the shaded histograms and $\sigma/m = 0.1$ cm$^{-2}$ g$^{-1}$ with dashed green histograms. We find that the inferred ellipticities are approximately independent of $r_{\text{max}}$ out to approximately 25 kpc, with there being a small tail at higher $\epsilon$ for the $\sigma/m = 1$ cm$^{-2}$ g$^{-1}$ haloes since the core radii are in the 20–40 kpc range and halo shapes are less affected by scatterings at radii larger than roughly the core radius. For $\sigma/m = 0.1$ cm$^{-2}$ g$^{-1}$, the core sizes are of the order of $r_{\text{core}}$, or smaller, so the shape measurements are relatively insensitive to $r_{\text{max}}$ range. A quick review of Fig. 3 also shows that the relative independence of the ellipticity estimate out to 0.1 $r_{\text{vir}}$ is to be expected in each cosmology. For the $\sigma/m = 1$ cm$^{-2}$ g$^{-1}$ haloes, we exclude those haloes for which either poor centring of the halo or ongoing merging makes the haloes appear artificially flattened. For the other cosmologies, the...
The resolution of these issues lies in a better comparison to the data, which in turn requires a bigger box higher-resolution simulation to probe deeper into the halo and gain more statistics. It will be important to include baryons to see how their presence may affect halo properties. With such a halo catalogue in hand, it will be interesting to do a more careful comparison to the X-ray data of NGC 720 and other large nearby ellipticals. The addition of other ellipticals would be crucial. With only one object, the spread we see in ellipticities may be hard to overcome (although it may be smaller if we also cut on concentration as discussed above). If we had an ensemble of shape measurements, we would be able to set tighter limits on the SIDM cross-section.

While our comparison to Buote et al. (2002) is not sufficiently sharp, the weight of the arguments suggests that $\sigma/m = 1 \text{ cm}^2 \text{ g}^{-1}$ is not likely to be consistent with the measured shape of NGC 720. However, based on our existing simulations, $\sigma/m = 0.1 \text{ cm}^2 \text{ g}^{-1}$ is as consistent as CDM for the shape of the NGC 720 isophotes. It is interesting to note in this regard that there is no hint of a large core ($\sim 30 \text{ kpc}$) in the results of Buote et al. (2002) or Humphrey et al. (2006). The core sizes for $\sigma/m = 0.1 \text{ cm}^2 \text{ g}^{-1}$ are smaller, $\sim 7 \text{ kpc}$, for the same virial mass range (Rocha et al. 2013), comparable to the effective radius of the stars in NGC 720. Thus, the inferred dark-matter density profile in NGC 720 may be a better way to search for effects of self-interactions.

To amplify the point about the dark-matter density further, we note that the median central (maximum) density for SIDM with $\sigma/m = 0.1 \text{ cm}^2 \text{ g}^{-1}$ is 0.05 $M_\odot \text{ pc}^{-3}$, while Humphrey et al. (2006) infer an average density of 0.04 $M_\odot \text{ pc}^{-3}$ within 10 kpc. At 5 kpc, the inferred average density is 0.1 $M_\odot \text{ pc}^{-3}$, still within a factor of 2 (expected from scatter) of the predictions. This lends credence to the argument that an analysis focused on SIDM predictions of an ensemble of nearby X-ray-detected elliptical galaxies could be a fruitful way to look for signatures of or constrain SIDM. It is also worth noting that the shape distribution of $\sigma/m = 0.1 \text{ cm}^2 \text{ g}^{-1}$ is visibly different from CDM and perhaps an ensemble of X-ray-shape measurements could resolve the differences.

Our analysis argues for the conclusion that SIDM cross-sections with $\sigma/m \geq 0.1 \text{ cm}^2 \text{ g}^{-1}$ can be hidden in X-ray data, and that previous constraints on SIDM using these data are overly stringent. Feng et al. (2010) assumed that the Buote et al. (2002) ellipticities described the halo shape at the inner radius of $R = 4.5 \text{ kpc}$ and that $\Gamma \sim H_0$ was required to make the halo spherical as indicated by the results of Davé et al. (2001). Our results indicate that this interpretation is flawed because (a) SIDM haloes retain significant triaxiality in the region where $\Gamma \sim H_0$ and (b) the scatter in SIDM halo ellipticities is large. In order to use analytic arguments to constrain self-interacting dark-matter models, they should be tuned to reproduce the distribution of axis ratios seen in simulations.

4.4 The future of cluster lensing constraints

In the future, far better constraints on SIDM will come from statistical studies of galaxy-cluster lens samples rather than the analysis of individual objects. In this section, we focus on statistical studies of the shapes of relaxed clusters.
There are a number of ongoing and future observational programs designed to characterize the mass function of and mass distribution within galaxy clusters (e.g. LSST Science Collaborations 2009; Gill et al. 2009; Plagge et al. 2010; Richard et al. 2010; Marriage et al. 2011; Planck Collaboration et al. 2011a,b,c; Oguri et al. 2012; Pillepich, Porciani & Reiprich 2012; Postman et al. 2012; Viana et al. 2012). Modulo the effects of baryons, a smoking-gun sign of SIDM would be for the mass function of galaxy clusters to look identical to CDM, but with lower mass density and rounder surface-mass distributions at the centres of the clusters. One would use the ensemble of galaxy-cluster data to compare with simulations of clusters in CDM and SIDM (with various elastic scattering cross-sections).

In this study, we will consider shape-based constraints from the initial results of the Local Cluster Substructure Survey (LoCuSS; P. G. Smith), a multi-wavelength follow-up program of 165 low-redshift clusters selected from the ROSAT All-sky Survey catalogue (Ebeling et al. 2000; Böhringer et al. 2004; Richard et al. 2010). We note that other similar-sized samples of clusters have also been obtained, in particular the Sloan Giant Arc Survey (SGAS) sample (Oguri et al. 2012). We focus on the LoCuSS sample in particular because the inferred properties of those clusters are easiest to compare to our simulated sample. An in-depth exploration of SGAS and other cluster surveys would be highly interesting but is beyond the scope of this paper.

In LoCuSS, 20 clusters were used for the first mass-modelling study (Richard et al. 2010). These were selected because they had been observed with the Hubble Space Telescope (HST), could be followed up spectroscopically at Keck and were confirmed to have strongly lensed background galaxies. The details of the mass modelling of these clusters are presented in section 3 of Richard et al. (2010). For our purposes, the key fact is that the surface density of the cluster-mass dark-matter-halo component of the lens model was parametrized in terms of the dPIE profile, equation (7). This study should be taken as an example of the power of using statistical studies of clusters for SIDM constraints, with an emphasis on the constraints on halo shapes.

In order to compare the LoCuSS clusters to simulations, we fit the surface densities of our most massive clusters in the 50 h⁻¹ Mpc boxes for our CDM, SIDM₀, and SIDM₂ runs using the dPIE surface-density profile, equation (7). We fit the surface densities of the haloes projected along the principal axes of the moment-of-inertia tensors of the mass within the virial radius, and perform the fits within annuli (Rᵥ ∈ R < R max). The inner radius of the annuli is chosen to be (R min = 20 kpc h⁻¹), since this is the three-dimensional shape convergence radius (see Section 2). We set the outer radius (R max = 50 kpc h⁻¹) since most of the lensing arcs in LoCuSS are in the range 20 < R < 100 kpc if projected into the plane of the sky at the position of the galaxy clusters. We fix (r cut = 1000 kpc), which is what is typically done with the LoCuSS clusters. In practice, the parameter constraints are insensitive to this choice, as all the data are well within this radius if projected on to the sky. The free parameters that we fit are: σ₀, the characteristic velocity dispersion of the cluster; r core, the pseudo-isothermal core radius; ε, the cluster ellipticity (as defined in equation 8) and the position angle θ. We fix the centre of the cluster to that inferred by AIF. The surface-density parameters were fit using the downhill simplex algorithm in the scipy python module using the likelihood

\[
L = P(N_{obs}|N(\sigma_0, r_{core}, \epsilon, \theta)) \prod_i N_{obs}_i \left( \sum_{i=1}^{N_{max}} L_i \right),
\]

where \( P(N_{obs}|N(\sigma_0, r_{core}, \epsilon, \theta)) \) is the Poisson probability of finding \( N_{obs} \) simulation particles within the annulus given a dPIE model with parameters \( (\sigma_0, r_{core}, \epsilon, \theta) \), for which \( N \) simulation values are expected. \( L_i \) is the probability of finding a simulation particle \( i \) at position \((x, y)\) given those same parameters,

\[
L_i = \frac{\Sigma(x, y|\sigma_0, r_{core}, \epsilon, \theta)}{\int_{\text{annulus}} dx dy \Sigma(x', y')},
\]

where \( \Sigma(x, y) \) is defined in equation (7).

Examples of the dPIE fits are shown in Figs 9 and 10, in which we show the surface densities of one massive halo (\( M_{200} = 1.8 \times 10^{15} M_\odot \)) along the major and intermediate principal axes of the halo moment-of-inertia tensor as well as the best-fitting dPIE surface densities. The central regions of the haloes are masked for \( R < r_{min} \), the region where the shape profiles in three dimensions are not converged. The halo appears rounder and more dense if projected along the major axis rather than the intermediate axis, just as we saw for another halo in Figs 1 and 2. While there are noticeable differences between the CDM and \( \sigma/m = 0.1 \) cm⁻² g⁻¹ surface densities at small projected radii, the differences between CDM and \( \sigma/m = 0.1 \) cm⁻² g⁻¹ are more subtle.

To make a quantitative comparison between our simulations and the LoCuSS observations, we examine only those LoCuSS clusters that have \( \sigma_0 \) and \( r_{core} \) in a similar range as the fits to the five most massive haloes in our simulations. This restricts the number of relevant LoCuSS cluster to five. In Fig. 11, we show the ellipticity \( \epsilon \) distribution of the five most massive clusters in the CDM (black), \( \sigma/m = 1 \) cm⁻² g⁻¹ (cyan) and \( \sigma/m = 0.1 \) cm⁻² g⁻¹ (green hatched) simulations for lines of sight along the three principal axes of the moment-of-inertia tensors. The dark blue points with error bars show the central values and 1σ uncertainties in \( \epsilon \) for the cluster haloes in the lens-model fits for the five similar LoCuSS clusters.

As expected, the ellipticities are highest for the intermediate-axis line of sight. In this instance, the ellipticities of the CDM and \( \sigma/m = 0.1 \) cm⁻² g⁻¹ haloes are more consistent with the LoCuSS sample than the \( \sigma/m = 1 \) cm⁻² g⁻¹ haloes. However, the lensing probability is highest for lines of sight closely aligned to the major axis (Mandelbaum et al. 2009; van de Ven et al. 2009), and in this case even the CDM haloes appear slightly rounder than the LoCuSS clusters. The \( \sigma/m = 1 \) cm⁻² g⁻¹ haloes are definitely too round.

Based on this initial set of LoCuSS haloes, we believe it is safe to say that \( \sigma/m < 0.1 \) cm⁻² g⁻¹ is at least as consistent with observations as CDM, but that there is significant tension with \( \sigma/m = 1 \) cm⁻² g⁻¹. There are several things that preclude us from making any statements stronger than this. First, we have a small sample of both simulated and observed galaxy-cluster haloes. Secondly, we do not know the virial mass or alignment of the LoCuSS galaxy clusters, nor do we have a good handle on the selection function of the survey. This means that we cannot make a direct comparison between the simulations and the observations. Thirdly, we do not have any galaxy-cluster-mass haloes with \( M_{200} > 2.2 \times 10^{14} M_\odot \) in our simulations, and thus our ellipticity probability distributions for the lowest-\( \sigma_0 \) LoCuSS clusters are almost certainly biased, although it is not clear in which direction that bias goes. Based on the X-ray data, it appears likely that several of the LoCuSS clusters in our sub-sample are more massive than any simulated cluster even though \( \sigma_0 \) and \( r_{core} \) are similar to the simulated clusters (Richard et al. 2010). Given that the dPIE fits are made based on the very central regions of the clusters, it is not unexpected that virial masses of the clusters can be very different even if the inner regions look similar. Finally, we do not include baryons in our simulations; it remains unclear
how the presence of baryons alters the density profile and shape of galaxy clusters (Scannapieco et al. 2012).

However, it is fair to say that this ensemble of observed galaxy clusters already places stronger constraints on the SIDM cross-section than the other constraints we considered in this section, in particular the constraint from MS 2137−23. While $\sigma/m = 1 \text{ cm}^2 \text{g}^{-1}$ is easily allowed by our reanalysis of the MS 2137−23 constraint (modulo the uncertainty in the normalization of the convergence), this value of the SIDM cross-section is in some tension with the LoCuSS cluster sample. A more quantitative limit, though, will only be possible with better theoretical predictions for the shapes of cluster-mass haloes and a careful analysis of observed cluster selection functions. In the future, we will simulate larger dark-matter haloes and perform mock observations of them to find a better quantitative mapping between observations and SIDM cross-section limits.

It may be difficult to probe cross-sections as small as $\sigma/m = 0.1 \text{ cm}^2 \text{g}^{-1}$ based on cluster halo shapes alone, though. We get a sense of how hard it may be to use strong lensing to probe halo shapes, and hence small self-interaction cross-sections, in Fig. 9, in which the shapes of the simulated CDM and $\sigma/m = 0.1 \text{ cm}^2 \text{g}^{-1}$ haloes do not look very different on the scales to which strong lensing is sensitive. Moreover, in Rocha et al. (2013), we estimated that core sizes for cluster-mass haloes should be $\lesssim 20 \text{ kpc}$ if $\sigma/m = 0.1 \text{ cm}^2 \text{g}^{-1}$. These sizes are similar to the effective radii of the BCGs at the centres of haloes. There are several implications of this fact. First, it means that stellar kinematics of the BGCs will be important for probing the dark-matter halo on scales for which scattering matters. Strong lensing is less sensitive to both the density profile and halo shapes at such small scales (see e.g. fig. 3 in Newman et al. 2011). Secondly, it means that any inferences on the dark-matter halo on such scales depends on careful and accurate modelling of the BGCs in the data analysis. Thirdly, we would have to model the behaviour of SIDM in the presence of a significant baryon-generated gravitational potential, and to explore the coevolution of dark-matter haloes and BGCs. In particular, simulations of isolated discy galaxies indicate that the presence of baryons tends to make the dark-matter distribution more spherical, although it is not clear how much the dark-matter distribution changes if the central galaxy is elliptical instead (Debattista et al. 2008). We note that while these issues also have implications for SIDM constraints based on radial density profiles or central densities, they may be more serious for the shape-based constraints because the shapes of $\sigma/m = 0.1 \text{ cm}^2 \text{g}^{-1}$ are already so similar to CDM haloes for small radii even in the absence of baryons.

### 5 CONCLUSIONS

The takeaway message of this work is that mapping observations to constraints on the self-interaction cross-section of dark matter is significantly more subtle than previously assumed, and as such,
Figure 10. Same as Fig. 9 but with the line of sight along the intermediate axis of the halo. The ellipticities here are: CDM, $e = 0.59; \sigma/m = 0.1 \, \text{cm}^2 \, \text{g}^{-1}$, $e = 0.38; \sigma/m = 1 \, \text{cm}^2 \, \text{g}^{-1}, e = 0.32$.

Figure 11. Ellipticity $e$ (equation 8) of haloes fitted with dPIE profiles for three different projections. The black unfilled histograms show $e$ values for the five most massive CDM haloes in the $50 \, h^{-1} \, \text{Mpc}$ simulation; the cyan filled histogram shows the same haloes in the SIDM$_1$ simulation and the green hatched histogram is for SIDM$_{0.1}$. The dark blue points with uncertainties show the best-fitting ellipticities and their $1\sigma$ uncertainties from dPIE modelling of the five LoCuSS clusters with $\sigma_0$ and $r_{\text{core}}$ similar to those of the simulated clusters (Richard et al. 2010).

constraints based on halo shapes are, at present, one to two orders of magnitude weaker than previously claimed.

There are three primary reasons that contribute to this conclusion. First, the observational probes (gravitational lensing and X-ray surface brightness) of halo shapes are actually probes of some moment of the mass distribution. For lensing, the observational probes are also sensitive to all material along the line of sight. While SIDM makes the three-dimensional density distribution significantly rounder within some inner radius $r$, the surface density will in general not be spherically symmetric at a projected
radius $R = r$. The surface densities are affected by material well outside the core set by scatterings where material is still quite triaxial (Fig. 3). Previous constraints were made under the assumption that the observations tracked the three-dimensional halo shape for fixed projected radius. This is a less troublesome assumption for X-ray isophotes, since it is weighted by the square of the gas distribution, and hence sensitive to the central regions. The shapes measured should be related most closely to the shapes of the enclosed mass profile. So, to probe cross-sections as small as $\sigma/m = 0.1 \text{ cm}^{-1}$, one needs to get down to $O(10 \text{ kpc})$ from the centre of the haloes.

The contribution of stars in this region makes it difficult to robustly estimate the shape of the dark-matter profile and it also makes it difficult to get a large ensemble of galaxies for this study.

Secondly, there is a fair bit of scatter added by assembly history to the observed shapes and the scatter is large enough that it precludes using a small number of objects to set constraints on SIDM cross-sections. Finally, although we find that the three-dimensional shape of haloes begins to become more spherical than CDM at radii where the local interaction rate is fairly low, $\Gamma(r) \approx 0.1 H_0$, there is a fair amount of triaxiality even when $\Gamma(r) \approx 0.1 H_0$, a fact that was not appreciated in earlier studies (e.g. Miralda-Escudé 2002; Feng et al. 2010).

We find that the convergence map of MS 2137–23 ($M_{\text{vir}} \sim 10^{15} M_{\odot}$) allows a velocity-independent SIDM cross-section of $\sigma/m = 1 \text{ cm}^{-1}$ to cool the `Too Big to Fail’ problem for the Milky Way dwarf spheroidals (Boylan-Kolchin, Bullock & Kaplinghat 2012), the core-cusp problem in LSB galaxies (de Blok 2010; Kuzio de Naray et al. 2010), as well as the shallow density profiles of the galaxy clusters in Sand et al. (2008) and Newman et al. (2009, 2011) while not undershooting their central densities or overshooting the core sizes. Cross-sections in this range are also consistent with other density-profile-based and subhalo-based constraints (Yoshida et al. 2000b; Gnedin & Ostriker 2001).

Since the current set of observations appear to be consistent with a SIDM cross-section of $\sigma/m = 0.1 \text{ cm}^{-1}$, there are two relevant questions for shape-based SIDM constraints. Will shape-based constraints be competitive with other types of SIDM constraints? And what will it take to get down to $\sigma/m \sim 0.1 \text{ cm}^{-1}$ with shapes?

Upon closer inspection, our view is that constraints using existing data could be pushed below $\sigma/m = 1 \text{ cm}^{-1}$, but it is not yet clear that we can get to $\sigma/m \sim 0.1 \text{ cm}^{-1}$. While X-ray isophotes of the elliptical galaxy NGC 720 are consistent with $\sigma/m = 0.1 \text{ cm}^{-1}$, there are some differences and a larger ensemble of elliptical galaxies may be able to test that. There are a number of other elliptical galaxies for which high-resolution X-ray data exist (e.g. Humphrey et al. 2006), but they lack the detailed shape measurements of NGC 720. So better constraints could result from X-ray shape analysis for these galaxies. For clusters, based on our quick pass through the LoCuSS results, lensing-based shape constraints on SIDM could also extend well below $\sigma/m = 1 \text{ cm}^{-1}$ if simulations are performed of a statistically significant number of massive galaxy clusters. However, in studies of both galaxies and clusters, it is likely that the measured densities in the inner regions would be a better way to test for signatures of self-interacting dark matter.

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