Effect of Air Injection on the Internal Flow Characteristics in the Draft Tube of a Francis Turbine Model

Seung-Jun Kim¹, Yong Cho² and Jin-Hyuk Kim¹,3,∗

¹ Carbon Neutral Technology R & D Department, Korea Institute of Industrial Technology, Cheonan-si 31056, Korea; kimsj617@kitech.re.kr
² K-Water Institute, Korea Water Resources Corporation, Daejeon 34045, Korea; ycho@kwater.or.kr
³ Industrial Technology (Green Process and Energy System Engineering), Korea University of Science and Technology, Daejeon 34113, Korea

* Correspondence: jinhyuk@kitech.re.kr; Tel.: +82-41-589-8447

Abstract: Under low flow-rate conditions, a Francis turbine exhibits precession of a vortex rope with pressure fluctuations in the draft tube. These undesirable flow phenomena can lead to deterioration of the turbine performance as manifested by torque and power output fluctuations. In order to suppress the rope with precession and a swirl component in the tube, the use of anti-swirl fins was investigated in a previous study. However, vortex rope generation still occurred near the cone of the tube. In this study, unsteady-state Reynolds-averaged Navier–Stokes analyses were conducted with a scale-adaptive simulation shear stress transport turbulence model. This model was used to observe the effects of the injection in the draft tube on the unsteady internal flow and pressure phenomena considering both active and passive suppression methods. The air injection affected the generation and suppression of the vortex rope and swirl component depending on the flow rate of the air. In addition, an injection level of 0.5%Q led to a reduction in the maximum unsteady pressure characteristics.

Keywords: Francis turbine; draft tube; air injection; vortex rope; internal flow characteristics; unsteady pressure; numerical analyses

1. Introduction

Solar and wind power generation processes, which generate renewable energies, depend on weather conditions, and hence, the prediction of a power generation output is difficult. The ability of power systems to maintain the balance between supply and demand is made difficult by these renewable energies with intermittent power generation [1].

Regarding flexibility in the power system, hydroelectric power generation with high load increase and decreased cycle characteristics contributes to electric power system stability and frequency adjustment. Due to changes in the operating environments, the power-generation process performs under off-design conditions rather than responsible for the peak load with high power demand and supply of electrical energy. Off-design conditions involving low flow rates generate complex flow in the flow passage, thereby leading to vibrations, noise, and hence unstable operation of the power system. Such unstable operating conditions of hydroelectric power generation cause frequent failure, which can lead to a reduction in the service lifespan and failure of the system [2–4].

Francis turbines, which are applied in hydroelectric power generation, are characterized by the precession of a vortex rope with pressure fluctuations in the draft tube (DT) under low flow-rate conditions. These fluctuations occur at frequencies close to the natural frequency of a turbine system, thereby leading to a resonance, which results in deterioration of the turbine performance as manifested in torque and output fluctuations [2,5,6].

In order to suppress vortex rope generation and to reduce the swirl component, the use of anti-swirl fins in the DT was investigated in a previous study. However, despite the
application of such fins, a vortex rope with precession was generated near the cone of the DT, and therefore, an additional method of rope suppression is required [4].

Several studies have focused on suppressing the flow instabilities associated with the vortex rope in the DT of a Francis turbine. Platonov et al. [7] conducted an experimental study on the impact of air injection on the vortex flow pattern and the pressure pulsation under maximum load conditions. The air was injected into the spiral casing of a Francis turbine model, and pressure pulsations were reduced by almost 50% due to the injection. Skripkin et al. [8] studied the influence of the injected gas on the structure and characteristics of the swirling flow in the DT geometry. The gas injection changed the pressure recovery coefficient and modified the vortex core precession frequency. Muntean et al. [9] investigated (via experiments) the unsteady pressure characteristics associated with flow instabilities in the DT of the Francis turbine. The air was injected into the DT cone wall, and the dynamic behavior was improved by air injection at 0.7QBEP. Chirkov et al. [10] used three-phase RANS models for numerical simulation in the DT of a Francis turbine with air injection through the center of the runner cone. An injection at a flow rate of 0.5% effectively eliminated cavitation in the DT for both the part and full load conditions. Yu et al. [11] simulated three-dimensional unsteady internal flow in the Francis turbine model with air admission from the main shaft center. The aeration with a suitable air volume fraction depressed the vertical flow and alleviated the pressure fluctuation in the DT. Li et al. [12] conducted a three-dimensional unsteady simulation to calculate pressure fluctuations in the DT of a Francis turbine with air admission. The pressure fluctuation caused by the vortex rope in the DT was reduced by the air admission. Foroutan and Yavuzkurt [13] carried out unsteady simulations using a DES model with a simplified DT of a Francis turbine to investigate a vortex rope control technique using water injection through the runner crown cone. The axial flow momentum was increased, and the stagnant region was removed at the center of the DT by water injection at partial load. Juposhti et al. [14] numerically investigated the mitigation of rating the vortex rope with axial water injection using a scale-adaptive simulation shear stress transport (SAS-SST) turbulence model in the DT of a Francis turbine. The axial momentum by water injection could control the formation of vortex rope and reduced the pressure fluctuations in the DT.

Thus, various related studies have focused on air or water injection aimed at suppressing vortex rope generation in the DT of the Francis turbine as the active suppression methods. However, the effects of air injection in the DT with the application of anti-swirl fins considering both active and passive suppression methods on internal flow and vortex suppression characteristics have not been systematically elucidated with low flow-rate conditions in the Francis turbine model.

In this study, unsteady-state Reynolds-averaged Navier–Stokes (RANS) equations using the SAS-SST turbulence model were employed with a three-phase (water, vapor, and air) flow analysis. This analysis was used to investigate the effects of air injection on the internal flow characteristics of a DT with anti-swirl fins for a Francis turbine under low flow-rate conditions. The unsteady internal flow and pressure fluctuation characteristics were observed in the presence of a vortex rope via the application of air injection (at different flow rates) from the cone of the runner. This study compares the injection-induced magnitudes and locations of the ropes. Furthermore, the unsteady pressure fluctuations induced by the ropes with precession resulting from the injection were investigated via unsteady pressure measurements conducted on the wall of the DT.

2. Specifications of the Francis Turbine Model

The effects of air injection on the internal flow characteristics in the DT were investigated using a Francis turbine model with a specific speed of 270-class (m, kW, min\(^{-1}\)). The specific speed of the model was calculated from Equation (1). This model was employed in a previous study where the effects of anti-swirl fins were evaluated [2]. Table 1 shows the specifications (including the energy coefficient, discharge factor, and speed factor) of
the Francis turbine model. These coefficients and factors were calculated from Equations (2)–(4) and are defined by IEC Standard 60193 [15].

\[
N_s = \frac{N \sqrt{p}}{H^2} \quad (1)
\]

\[
E_{nD} = \frac{8H}{n^2D^2} \quad (2)
\]

\[
Q_{ED} = \frac{Q}{D^2E_{0.5}} \quad (3)
\]

\[
n_{ED} = \frac{nD}{\sqrt{8H}} \quad (4)
\]

Table 1. Specifications of the Francis turbine model.

| Specifications                  | Values |
|---------------------------------|--------|
| Energy coefficient, \(E_{nD}\) (-) | 4.35   |
| Discharge coefficient, \(Q_{ED}\) (-) | 0.33   |
| Speed factor, \(n_{ED}\) (-)     | 0.48   |
| Diameter of runner outlet, \(D_2\) (m) | 0.35   |
| Number of stay vanes            | 20     |
| Number of guide vanes           | 20     |
| Number of runner blades         | 12     |

The fins were composed of two short and two long fins. The two short fins were arranged facing each other, with the same length of 0.7\(D_2\), and the two long fins also faced each other, being 1.09 and 1.3\(D_2\), as shown in Figure 1. Regarding the low flow-rate condition, a guide vane angle of 16° \((Q = 0.78 \, Q_{BEP})\) was selected for the numerical analyses based on the development of a vortex rope in a previous study [2].

![Figure 1. Location of the anti-swirl fins in the DT of the Francis turbine model.](image)

3. Numerical Analysis Methods

In the unsteady-state calculations, the ANSYS CFX-19.1 commercial software was used to analyze the three-dimensional internal flow field of the Francis turbine model [16]. The numerical grids stay vanes, guide vanes, and runner were produced using TurboGrid, whereas the numerical grids of the spiral casing and DT were generated using the ICEM-CFD functions. The boundary conditions were set using CFD-Pre for the numerical analysis, and for solving the governing equations and for conduction post process of
the numerical results, CFX-Solver and CFX-Post functions were used, respectively. The unsteady-state RANS equations for the incompressible flow of the model were calculated from the governing equations, which were discretized with a finite volume method. To ensure the physical boundaries and to the capture transitional boundary layers correctly, the discretization of the transient and advection schemes were solved via second-order backward Euler and high-resolution schemes, for which a high resolution is a bounded second-order upwind biased discretization.

Figure 2 shows the numerical grids of the Francis turbine model. The spiral casing and DT in the computational domain were constructed with tetrahedral-type and prism-type grids, whereas the stay vanes, guide vanes, and runner blades were constructed with hexahedral-type grids. By adding a prism-type grids on the DT wall, the wall function process was applied with $y^+ \leq 80$. In the runner blade geometry, the fillets of the runner blade on the hub and shroud were simplified. The O-type grids were used on the runner blade, and the $y^+$ value was kept below five. In order to select the optimal grid among the three observed grids, the grid convergence index method was used to estimate the numerical uncertainty resulting from the discretization error [17–19]. Table 2 shows the $GCI_{fine}$ values of the efficiency (a value of 0.22% calculated for optimal grids of $14.74 \times 10^6$) [20].

![O-type grid](image_url)

**Figure 2.** Numerical grids of the Francis turbine model.

**Table 2.** Discretization error calculations for the Francis turbine model.

| Title | \( \phi = \text{Efficiency} \) |
|-------|-------------------------------|
| \( N_1, N_2, N_3 \) | \( 14.74 \times 10^6, 7.05 \times 10^6, 3.59 \times 10^6 \) |
| \( r_21 \) | 1.28 |
| \( r_32 \) | 1.25 |
| \( \phi_1 \) | 1 |
| \( \phi_2 \) | 0.9952 |
| \( \phi_3 \) | 0.9808 |
| \( p \) | 5.20 |
| \( GCI_{fine} \) | 0.22% |

As the boundary conditions of the unsteady-state calculation, the total pressure and static pressure were set by considering the upper and lower reservoirs at the inlet and outlet, respectively. Three-phase flows using water, vapor, and air at 25 °C were considered the working fluid. In addition, the cavitation characteristics were considered using the Rayleigh–Plesset model that describes the growth and collapse of vapor bubbles from a liquid as a homogeneous model [21]. The water saturation pressure was set to 3169.9 Pa,
and the mean diameter of the cavitation bubble was established as $2.0 \times 10^{-6}$. The SAS-SST turbulence model, which is suitable for accurate predictions of the influence exerted by flow separation phenomena, was used [22,23]. To connect the interface between the rotating and stationary domains, transient rotor-stator conditions were applied. The steady-state analysis results were applied to the unsteady-state analysis as the initial value for improving convergence and for decreasing the computational time. Widmer et al. [24] investigated the time resolution by analyzing local vortex formation, pressure distribution, and frequency of the rotating stall, and a good agreement was found at resolutions between 1° and 5° per time step. Therefore, the time step was set to 0.0002272 s, and a total time of 0.4364 s corresponded to intervals of 1.5° over a total of eight runner revolutions in this study. The number of loops coefficient was set to five in order to improve the convergence of the unsteady-state calculations. The Courant number is a dimensionless value for the grid stability of a transient simulation. The Courant number indicates how many cells of the mesh are passed by the fluid at one time step, and the average Courant number should be a value lower than one [16,25].

$$\text{Courant number} = \frac{u\Delta t}{\Delta x}$$  \hspace{1cm} (5)

where $u$, $\Delta t$, and $\Delta x$ indicate the fluid speed, time step, and the mesh size, respectively. The average Courant number was calculated at about 0.12 with the applied grids and time step in this study. Thus, the Courant number was confirmed to have a satisfaction of less than one.

Time-averaged values were calculated from the unsteady-state analyses. To avoid the initial numerical noise, these values were used for the results in the study on values corresponding to the period of the runner’s last three revolutions after a total of five revolutions.

In order to investigate the unsteady pressure characteristics caused by the vortex rope with precession and internal flow pattern, pressure measurement points were selected (see Figure 3). These points were applied at regular intervals of $0.3D_2$ from $0.1D_2$ on the wall of the DT, and the points (p1–p4 in Figure 3) were located at the same height on the wall. Regarding the air-injection flow rates in the DT, rates of 0.1%Q and 0.5%Q for the total flow rate (Q) associated with the Francis turbine model were applied from the cone of the runner (see Figure 3). The rates employed were based on those used in previous studies [10–12,26].

![Figure 3](image-url)  
Figure 3. Pressure measurement points in the DT with the air-injection inlet of the Francis turbine model.
4. Results and Discussion

4.1. Validation of the Numerical Analysis Results

In order to validate the numerical analysis results of the Francis turbine model, the steady and unsteady-state RANS results were compared with the experimental results (see Figure 4). The efficiencies were normalized using the maximum value of the experimental results. The experimental results of the model turbine performance test were provided by the K-water in Convergence Institute in Korea [27]. To apply the uncertainty of the performance test to the results, the system ($f_s$), random ($f_r$), and total ($f_t$) uncertainties defined by IEC Standard 60193 were calculated from Equations (6)–(8) [15]. Values of ±0.224% and ±0.06% were calculated for $f_s$ and $f_r$, respectively, and a total uncertainty of ±0.245% was calculated from these two values.

\[ f_t = \pm \left( (f_s)^2 + (f_r)^2 \right)^{1/2} \]  
\[ f_s = \pm \left( (f_{cal})^2 + (f_h)^2 + (f_{ks})^2 + (f_j)^2 \right)^{1/2} \]  
\[ f_r = \pm \left( (f_{kr})^2 + (f_l)^2 \right)^{1/2} \]

where the subscripts cal, h, ks, and j in the system uncertainty indicate systematic error in the calibration, additional systematic error, errors due to physical phenomena and influence quantities in a systematic component, and errors in physical properties, respectively. The subscripts kr and l in the random uncertainty indicate errors due to physical phenomena and influence quantities in a random component, and random error, respectively [15]. The numerical and experimental results of the Francis turbine model concur with the efficiency tendency based on the flow rate, especially for the results corresponding to a guide vane angle of 16°. Therefore, the results of the numerical analysis were considered valid.

Figure 4. Comparison of the numerical and the experimental results obtained for the Francis turbine model.

4.2. Internal Flow Characteristics Relative to the Air Injection in the Draft Tube

Figure 5 shows the efficiencies determined from the unsteady-state calculations with and without the anti-swirl fins and air injection in the DT at an observed guide vane angle of 16°. Therefore, the results of the numerical analysis were considered valid.
The efficiencies were normalized using the maximum value of the experimental results using the same methods employed for the efficiency curve shown in Figure 4. For the condition without air injection, the efficiency decreased with the application of anti-swirl fins and the efficiency decreased (in general) with increasing flow rate of the injection.

Figure 5. Comparison of efficiencies obtained with and without air injection and anti-swirl fins in the DT.

The performance characteristics based on the flow rates of air injection in the DT (see Figure 6) were investigated by considering the head losses of the runner and the DT of the Francis turbine model. The head losses of the runner and the DT were calculated from Equations (9) and (10), respectively [28].

\[
H_{\text{loss}} = \frac{\Delta p_{\text{total}}}{\rho g} \tag{9}
\]

\[
H_{\text{loss runner}} = \frac{\Delta p_{\text{total}} - T\omega}{\rho g} \tag{10}
\]

where \(\Delta p_{\text{total}}, \rho, g, T, \omega, \) and \(Q\) represent the total pressure difference associated with each component of the Francis turbine model, the water density, the acceleration due to gravity, the torque of the runner, the angular velocity, and the flow rate of the turbine, respectively.

The loss distribution corresponding to the head loss of the runner and the DT with an air injection of 0.1%Q was similar to that of the condition without air injection; an injection of 0.1%Q had only a moderate effect on the turbine performance. However, the head loss of the runner decreased with an air injection of 0.5%Q, whereas the head loss of the DT increased. The efficiency decrease in the Francis turbine model (see efficiency comparison in Figure 5) is attributed to an increase in the head loss of the DT (rather than the decreased head loss of the runner).

Figure 7 shows the vortex rope characteristics revealed by the iso-surface of pressure distribution in the DT during the last one runner revolution. The investigated pressure of the iso-surface was considered as the water saturation pressure corresponding to the water level of the lower reservoir. The condition without air injection and with an anti-swirl fin (Figure 7a) generated a visible vortex rope. This rope was longer (than that generated under other conditions) along the flow direction in the DT during the revolution of the runner. As shown in Figure 7b, when anti-swirl fins were applied in the DT without air injection, rope generation in the flow direction was largely suppressed. In addition, low-pressure regions were generated near the fins due to the resistance induced by the swirl component.
in the DT under low flow-rate conditions. An air injection of 0.1%Q (see Figure 7c) led to no suppression of vortex rope generation, but this injection affected the generation of a long rope along the flow direction. However, at an injection level of 0.5%Q (see Figure 7d), the visible vortex rope in the DT disappeared and only low-pressure regions developed near the anti-swirl fins. Therefore, the influence of air injection on vortex rope generation and suppression was affected by the flow rate of the injection in the DT.

Figure 5. Comparison of efficiencies obtained with and without air injection and anti-swirl fins in the DT.

The performance characteristics based on the flow rates of air injection in the DT (see Figure 6) were investigated by considering the head losses of the runner and the DT of the Francis turbine model. The head losses of the runner and the DT were calculated from Equations (9) and (10) [28].

\[ H_{\text{total}} = \Delta \rho \frac{g}{Q} \]  
\[ H_{\text{total}} = \Delta \rho \frac{g}{Q} - T \omega \]  

where \( \Delta \rho \), \( g \), \( T \), \( \omega \), and \( Q \) represent the total pressure difference associated with each component of the Francis turbine model, the water density, the acceleration due to gravity, the torque of the runner, the angular velocity, and the flow rate of the turbine, respectively.

The loss distribution corresponding to the head loss of the runner and the DT with an air injection of 0.1%Q was similar to that of the condition without air injection; an injection of 0.1%Q had only a moderate effect on the turbine performance. However, the head loss of the runner decreased with an air injection of 0.5%Q, whereas the head loss of the DT increased. The efficiency decrease in the Francis turbine model (see efficiency comparison in Figure 5) is attributed to an increase in the head loss of the DT (rather than the decreased head loss of the runner).

Figure 7 shows the vortex rope characteristics revealed by the iso-surface of pressure distribution in the DT during the last one runner revolution. The investigated pressure of the iso-surface was considered as the water saturation pressure corresponding to the water level of the lower reservoir. The condition without air injection and with an anti-swirl fin (Figure 7a) generated a visible vortex rope. This rope was longer (than that generated under other conditions) along the flow direction in the DT during the revolution of the runner. As shown in Figure 7b, when anti-swirl fins were applied in the DT without air injection, rope generation in the flow direction was largely suppressed. In addition, low-pressure regions were generated near the fins due to the resistance induced by the swirl component in the DT under low flow-rate conditions. An air injection of 0.1%Q (see Figure 7c) led to no suppression of vortex rope generation, but this injection affected the generation of a long rope along the flow direction. However, at an injection level of 0.5%Q (see Figure 7d), the visible vortex rope in the DT disappeared and only low-pressure regions developed near the anti-swirl fins. Therefore, the influence of air injection on vortex rope generation and suppression was affected by the flow rate of the injection in the DT.

Figure 6. Head loss distribution of the runner and DT associated with the Francis turbine model.

Figure 7. Cont.
The flow phenomena occurring in the DT, owing to the application of air injection and anti-swirl fins, were investigated by comparing the streamline distributions of the time-averaged velocity, as shown in Figure 8. As shown in Figure 8a, without injection and the fins, very complex flow characteristics occurred with the swirl component in the dotted red circle in the DT. Application of the fins (see Figure 8b) led to a suppression of the vortex rope in the flow direction, but the swirl component was observed in the DT. As shown in Figure 8c, a complex flow and a swirl component were generated despite an air injection of 0.1%Q in the DT. An injection of 0.5%Q led to complex flow characteristics in the DT (see Figure 8d), but the swirl component was eliminated. These flow phenomena
can be confirmed qualitatively by examining the vortex rope generation characteristics, as shown in Figure 7.

![Streamline distributions with velocity—Travg in the DT with and without air injection and anti-swirl fins](image)

(a) Without air injection and anti-swirl fins (b) Without air injection with anti-swirl fins

(c) With air injection of 0.1%Q (d) With air injection of 0.5%Q

**Figure 8.** Streamline distributions with velocity—Travg in the DT with and without air injection and anti-swirl fins.

The absolute and relative flow angle distributions along the spanwise direction from the hub (0) to the shroud (1) of the runner outlet were investigated, as shown in Figure 9. The flow angle values were normalized using each maximum angle value. Similar angle distribution characteristics were observed, except for the air injection of 0.5%Q condition (Figure 9a). At 0.5%Q, for the injection at the runner cone with a relatively large flow rate, the absolute flow angle increased near the runner hub as the flow characteristics changed. Usually, the relative flow angle is similar to the angle of the runner outlet. However, due to the effect induced by an injection of 0.5%Q, the flow angle characteristics changed slightly from the hub to the mid-span of the runner, as shown in Figure 9b.
Figure 9. Flow angle distributions along the spanwise direction at the outlet of the runner.

Figure 10 compares the time-averaged velocity distributions (based on the flow rates of air injection) in the axial and circumferential directions at the observed line of 0.6\(D_2\) from the inlet of the DT (see Figure 3). The abscissa indicates the measurement location relative to the diameter from the wall (0) to the wall (1) of the DT. The velocities were normalized using each maximum velocity. In the axial velocity distribution (see Figure 10a), the range of backflow associated with this velocity increased with increasing flow rate of air injection in the DT. The increase in the backflow range of the axial velocities at an air injection of 0.5\%Q may have led to an increase in the head loss of the DT and a deterioration of the performance (as presented in Figures 5 and 6). Regarding the circumferential velocity distribution shown in Figure 10b, the sign of the velocity indicates the direction of the velocity. The maximum circumferential velocities at the wall of the DT varied with the application or lack of anti-swirl fins. Similar maximum velocities occurred at the wall regardless of the air-injection flow rate. In addition, for a measurement range of 0.2–0.8, the circumferential velocity decreased with an air injection of 0.5\%Q. Thus, the effects of air injection on vortex rope suppression were manifested in the influence of a 0.5\%Q injection on the velocity characteristics.
The swirl strength in the DT that leads to vortex rope generation was analyzed using the swirl number, $S$, which was calculated from Equation (11). The $S$ values for the observed line of $0.6D_2$ (as presented in Figure 3) based on the flow rate of air injection are shown in Figure 11 [3,29–31].

$$ S = \frac{\int_0^R C_m r C_u r^2 dr}{R \int_0^R C_m^2 r^2 dr} $$  \hspace{1cm} (11) 

where $C_m$ and $C_u$ denote the axial and circumferential velocity, respectively. In this study, the time-averaged axial and circumferential velocities were used to calculate the swirl number. The maximum value of each swirl number distribution occurred near the center of the DT for the condition without air-injection and with an anti-swirl fin. When fins were employed, the maximum number shifted slightly away from the center of the DT. An air injection of 0.1%Q had no effect of vortex rope suppression, but had an effect on vortex rope generation with the highest swirl number (in contrast to the condition without air injection and with an anti-swirl fin). Thus, at 0.1%Q, the rope generated along the flow...
direction in the DT is longer than that generated at other injection levels. As shown in Figure 7c, it can be seen that a relatively small flow rate of air injection such as 0.1%Q is not sufficient to suppress the vortex rope and swirl component. The maximum swirl number at an injection of 0.5%Q was relatively far from the center of the DT and was quite low due to the suppression of vortex rope generation. This can be quantitatively confirmed as the reason for rope suppression at an injection level of 0.5%Q.

4.3. Unsteady Pressure Characteristics Relative to the Air Injection in the Draft Tube

The unsteady pressure characteristics based on the flow rate of air injection in the DT (see Figure 12) were investigated via fast Fourier transformation (FFT) analysis of the pressures during a total of eight revolutions of the runner. The unsteady pressures were measured at 0.1D₂, 0.3D₂, 0.6D₂, and 0.9D₂ along the flow direction from the inlet of the DT, and the maximum pressure values were selected from four measurement points (p1–p4) at each height, as indicated in Figure 3. The magnitudes were normalized by the maximum observed pressure, and the frequencies were normalized by the rotational frequency (fₙ) of the runner. In the location of 0.1D₂, all of the observed conditions exhibited the highest normalized frequency (0.25fₙ) in the low-frequency region, corresponding to a vortex rope in the DT. The frequency of a vortex rope with precession commonly corresponds to 0.2 to 0.4 times the rotational frequency [2,32,33]. The magnitude observed for the condition without air injection and with an anti-swirl fin was larger than those observed for other conditions, but this magnitude decreased at an injection level of 0.5%Q. In addition, the first blade passing frequency (BPF) occurred under all observed conditions.
Figure 12. Cont.
In the location corresponding to $0.3D_2$, the magnitudes increased generally at a normalized frequency of $0.25f_n$ with progression in the flow direction. The magnitude obtained for the condition without air injection and with anti-swirl fins was higher than that obtained for the location of $0.1D_2$. Additionally, the first BPFs disappeared under all of the observed conditions. Similar trends were observed for the location of $0.6D_2$, where (compared with that observed under air injection and anti-swirl fins) a higher magnitude occurred for the no air injection and no fin conditions. However, the magnitude associated with an injection of $0.1%Q$ is $\approx 14\%$ higher than that obtained for an injection with anti-swirl fins. This results from the fact that a low flow rate of air injection affects the generation of the long vortex rope with a high swirl number distribution along the flow direction (see Figures 7 and 11). Consequently, highly unsteady pressure characteristics were generated. In the location of $0.9D_2$, the highest magnitude occurred for the condition without air injection and with an anti-swirl fin. This magnitude decreased significantly (by $\approx 55\%$) when an injection of $0.5%Q$ was employed, thereby revealing the effects of the air injection utilized in this study.

5. Conclusions

The effects of air injection in the DT (with anti-swirl fins) of a Francis turbine model on the unsteady internal flow characteristics at low flow-rate conditions were investigated by considering both active and passive suppression methods for the vortex rope in DT. This investigation utilized unsteady-state RANS analyses that employed three-phase flow analysis. For the total flow rate of the model, injection levels of $0.1%Q$ and $0.5%Q$ were applied for a comparison of the vortex rope suppression effects. The head losses increased with an increasing flow rate of the air injection in the DT and the efficiencies decreased accordingly.

The vortex rope generation and suppression effects induced by air injection at specified flow rates were confirmed via an iso-surface of pressure distributions. Moreover, the swirl components in the DT were confirmed via the streamline distributions of velocity. The internal flow characteristics at an injection level of $0.5%Q$, which were effective at suppressing the rope, were confirmed by comparing the absolute and relative flow angle distributions at the outlet of the runner, and the axial and circumferential velocities on the observed line in the DT. Furthermore, the swirl strength decreased at this injection level, as confirmed through the swirl number calculation that affected the generation of the rope. In addition, FFT analyses confirmed that the largest unsteady pressures occurred under the condition without air injection and with an anti-swirl fin. The observed magnitudes increased gradually (in general) along the flow direction in the DT. Additionally, the maximum magnitude decreased by $\approx 55\%$ at an injection level of $0.5%Q$. 

![Normalized unsteady pressure characteristics along the flow direction in the DT.](image-url)
The vortex rope remained near the DT cone despite the application of the anti-swirl fins. Therefore, in order to reduce the swirl number and unsteady pressure induced by the rope, air injection, which affected the generation and suppression effects of the vortex rope, was employed. The flow rate of the injection was important in this regard, i.e., the selection of an appropriate flow-rate range for air injection in the DT was essential for suppressing rope generation. In addition, based on the results of both active and passive suppression methods, the unsteady internal flow phenomena caused by optimizing the geometrical parameters of the anti-swirl fins in the DT will be observed for suppressing the swirl component and vortex rope in a future study.

**Author Contributions:** Conceptualization, validation, investigation, data curation, and resources, S.-J.K., Y.C. and J.-H.K.; writing—original draft preparation, S.-J.K. and J.-H.K.; writing—review and editing, supervision, and project administration, J.-H.K.; funding acquisition, Y.C. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was funded by the Korea Agency for Infrastructure Technology Advancement under the Ministry of Land, Infrastructure, and Transport (grant number 21IFIP-B128593-05).

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Conflicts of Interest:** The authors declare no conflict of interest.

**References**

1. Kim, S.J.; Choi, Y.S.; Cho, Y.; Choi, J.W.; Kim, J.H. Effect of runner blade thickness on flow characteristics of a Francis turbine model at low flowrates. *J. Fluids Eng.* **2020**, *142*, 031104. [CrossRef]

2. Kim, S.J.; Choi, Y.S.; Cho, Y.; Choi, J.W.; Hyun, J.J.; Joo, W.G.; Kim, J.H. Effect of fins on the internal flow characteristics in the draft tube of a Francis turbine model. *Energies* **2020**, *13*, 2806. [CrossRef]

3. Nishi, M.; Liu, S. An outlook on the draft-tube-surge study. *Int. J. Fluid Mach. Syst.* **2013**, *6*, 33–48. [CrossRef]

4. Eichhorn, M.; Taruffi, A.; Bauer, C. Expected load spectra of prototype Francis turbines in low-load operation using numerical simulations and site measurements. *J. Phys. Conf. Ser.* **2017**, *813*, 012052. [CrossRef]

5. Feng, J.J.; Li, W.F.; Wu, H.; Lu, J.L.; Liao, W.L.; Luo, X.Q. Investigation on pressure fluctuation in a Francis turbine with improvement measures. *IOP Conf. Ser. Earth Environ. Sci.* **2014**, *22*, 032006. [CrossRef]

6. Favre, A.; Müller, A.; Landry, C.; Yamamoto, K.; Avellan, F. Study of the vortex-induced pressure excitation source in a Francis turbine draft tube by particle image velocimetry. *Exp. Fluids* **2015**, *56*, 215. [CrossRef]

7. Platonov, D.; Minakov, A.; Dekterev, D.; Maslennikova, A. An experimental investigation of the air injection effect on the vortex structure and pulsation characteristics in the Francis turbine. *Int. J. Fluid Mach. Syst.* **2020**, *13*, 103–113. [CrossRef]

8. Skripkin, S.G.; Kuibin, P.A.; Shitok, S.I. The effect of air injection on the parameters of swirling flow in a Turbine-99 draft tube model. *Tech. Phys. Lett.* **2015**, *41*, 638–640. [CrossRef]

9. Muntean, S.; Susan-Resiga, R.F.; Campian, V.C.; Dumbrava, C.; Cuzmos, A. In Situ unsteady pressure measurements on the draft tube cone of the Francis turbine with air injection over an extended operating range. *UPB Sci. Bull. Ser. D* **2014**, *76*, 173–180.

10. Chirkov, D.; Scherbakov, P.; Skorospelov, V.; Cherny, S.; Zakharov, A. Numerical simulation of air injection in Francis turbine. *IOP Conf. Ser. Earth Environ. Sci.* **2019**, *240*, 022043. [CrossRef]

11. Yu, A.; Luo, X.W.; Ji, B. Numerical simulation and analysis of the internal flow in a Francis turbine with air admission. *IOP Conf. Ser. Mater. Sci. Eng.* **2015**, *72*, 042047. [CrossRef]

12. Li, W.F.; Feng, J.J.; Wu, H.; Lu, J.L.; Liao, W.L.; Luo, X.Q. Numerical investigation of pressure fluctuation reducing in draft tube of Francis turbines. *Int. J. Fluid Mach. Syst.* **2015**, *8*, 202–208.

13. Foroutan, H.; Yavuzkurt, S. Flow in the simplified draft tube of a Francis turbine operating at partial load-Part II: Control of the vortex rope. *J. Appl. Mech.* **2014**, *81*, 061010. [CrossRef]

14. Juposhti, H.J.; Maddahian, R.; Cervantes, M.J. Optimization of axial water injection to mitigate the rotating vortex rope in a Francis turbine. *Renew. Energy* **2021**, *175*, 214–231. [CrossRef]

15. International Electrotechnical Commission. *Hydraulic Turbines, Storage Pumps and Pump-Turbines—Model Acceptance Tests;* Standard No. IEC 60193; IEC: Geneva, Switzerland, 1999.

16. ANSYS CFX-19.1. *ANSYS CFX-Solver Theory Guide;* ANSYS Inc.: Canonsburg, PA, USA, 2018.

17. Richardson, L.F. IX. The approximate arithmetical solution by finite differences of physical problems involving differential equations, with an application to the stresses in a masonry dam. *Philos. Trans. R. Soc. London. Ser. A* **1911**, *210*, 307–357.

18. Richardson, L.F.; Gaunt, J.A. VIII. The deferred approach to the limit. *Philos. Trans. R. Soc. London. Ser. A* **1927**, *226*, 299–361.
19. Celik, I.B.; Ghia, U.; Roach, P.J.; Freitas, C.J.; Coleman, H.; Raad, P.E.; Coleman, H.P. Procedure for Estimation and Reporting of Uncertainty Due to Discretization in CFD Applications. J. Fluids Eng. 2008, 130, 078001.

20. Kim, S.J.; Choi, Y.S.; Cho, Y.; Choi, J.W.; Hyun, J.J.; Joo, W.G.; Kim, J.H. Analysis of the numerical grids of a Francis turbine model through grid convergence index method. KSFM J. Fluid Mach. 2020, 23, 16–22. (In Korean) [CrossRef]

21. Zwart, P.J.; Gerber, A.G.; Belamri, T. A two-phase flow model for predicting cavitating dynamics. In Proceedings of the Fifth International Conference on Multiphase Flow, Yokohama, Japan, 30 May–3 June 2004.

22. Egorov, Y.; Menter, F. Development and application of SST-SAS turbulence model in the DESIDER project. In Advances in Hybrid RANS-LES Modelling; Springer Science & Business Media: Heidelberg, Germany, 2008; pp. 261–270.

23. Menter, F.R.; Egorov, Y. The scale-adaptive simulation method for unsteady turbulent flow predictions. Part 1 Theory Model Description. Flow Turbul. Combust. 2010, 85, 113–138.

24. Widmer, C.; Staubli, T.; Ledergerber, N. Unstable characteristics and rotating stall in turbine brake operation of pump-turbines. J. Fluids Eng. 2011, 133, 041101. [CrossRef]

25. Moritz, R.A.C. Transient CFD-Analysis of a High Head Francis Turbine. Master’s thesis, Norwegian University of Science and Technology, Trondheim, Norway, 2014.

26. Kim, S.J.; Cho, Y.; Choi, J.W.; Hyun, J.J.; Kim, S.W.; Kim, J.H. Flow Characteristics according to the air-injection in the draft tube of a Francis turbine model. KSFM J. Fluid Mach. 2021, 24, 24–31. (In Korean) [CrossRef]

27. Korea Agency for Infrastructure Technology Advancement. Report Development of Construction Technology for Medium Sized Hydropower Plant; Report No. 20IFIP-B128593-04; Korea Agency for Infrastructure Technology Advancement: Anyang-si, Korea, 2020.

28. Chen, Z.; Singh, P.M.; Choi, Y.D. Francis turbine blade design on the basis of port area and loss analysis. Energies. 2016, 9, 164. [CrossRef]

29. Senoo, Y.; Kawaguchi, N.; Nagata, T. Swirl flow in conical diffusers. Bull. JSME 1978, 21, 112–119. [CrossRef]

30. Gupta, A.K.; Lilley, D.G.; Syred, N. Swirl Flows; Abacus Press: Tunbridge Wells, UK, 1984.

31. Favrel, A.; Pereira Jr, J.G.; Landry, C.; Müller, A.; Nicolet, C.; Avellan, F. New insight in Francis turbine cavitating vortex rope: Role of the runner outlet flow swirl number. J. Hydraul. Res. 2018, 56, 367–379. [CrossRef]

32. Nicolet, C.; Zobeiri, A.; Maruzewski, P.; Avellan, F. Experimental investigations on upper part load vortex rope pressure fluctuations in francis turbine draft tube. Int. J. Fluid Mach. Syst. 2011, 4, 179–190. [CrossRef]

33. Kim, S.J.; Suh, J.W.; Choi, Y.S.; Park, J.; Park, N.H.; Kim, J.H. Inter-blade vortex and vortex rope characteristics of a pump-turbine in turbine mode under low flow rate conditions. Water 2019, 11, 2554. [CrossRef]