The Nature of Light in an Expanding Universe

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Abstract

In this paper, we prove the existence of two degrees of freedom that govern the movement of light in an expanding universe. The use of the fractal manifold model leads to a reciprocal causality between variation of geometry and gravity, which both play a complementary role in the universe architecture. This study unravels new facts about the distribution of matter in the universe, and provides a new interpretation of Dark Matter and Dark Energy.

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I. INTRODUCTION

Everything in our world is perceived through the intermediary of light despite its enigmatic nature. Within corpuscular model framework ([5],[7],[8],[10],[14]), light looks like a flow of particles that goes in a straight line at high speed. According to the wave model, however, ([3],[6],[11],[12],[13]), there is supposed to exist a transfer medium of the wave form. Neither the wave model nor the corpuscular model is conserved in the identification of light. However, they are both valid. The problem remains unsolved since the two different experiences give two different interpretations of the nature of light. This has been the situation for more than 300 years. Worse still, the scientific community resorts to the duality of wave and corpuscle, using new principles that delimit the regions where the two aspects are contradictory [15]. When the transmission of light in involved in addressing problems where no interchange of energy between radiation and matter is involved [16], the classical wave theory that describes the propagation of light in terms of wave motion is entirely adequate. However, the origin of spectra, the interaction of light with material media through which it passes, and some other phenomena (magneto- and electro-optics) cannot be satisfactorily explained by the classical wave theory of radiation. They can be interpreted by quantum theory. The different existing scientific models fail individually to provide a comprehensive understanding which accounts for the intricacy of all details of light observation. The scientific community has been relying upon the classical or quantum theory with a view to interpreting those phenomena to which each of the two theories is best fitted. This duality sounds irrational but acceptable in some respects, and physicists have been reluctant to accept it in the absence of major outcomes which contravene the well-established/fundamental laws in physics. What would happen if the properties of an expanding universe were to allow the corpuscular aspect of light to acquire a new behavior which may explain the wave appearance of light? What would happen if the movement of light was not a wave movement? We are not interested in revisiting this hotly-debated issue. Instead, the prime purpose is tracing the real nature of light and its complications and find new postulates in an expanding space.

In this paper, we prove that in an expanding space, geodesics are curved paths that alternate successive maxima and minima, which engender the following properties of light movement:
• Any possible movement of photon in the universe is governed by two degrees of freedom: one describing the light’s direction, and the other describing the geodesic that permits the tracking of that direction.

• The gravity may affect the direction of the light. We put in evidence how gravity changes the direction of light without any local distortion of the geometry in an expanding universe.

• The geometry variation governs the movement of matter in the universe while gravity holds matter.

The plan of this paper is as follows: In a preliminary part we present the definition of fractal manifold. New principles are introduced in the third part concerning the metric that defines the temporal spatial events. In the fourth part, we prove that in an expanding space, where points are expanding, there is no geodesic given by straight lines. All geodesics are curved due to the expansion of points. We also deduce how the variation of the universe geometry bends the light. A global distribution of matter in the universe at a very large scale is then given in the fifth part. Finally, global consequences of the application of the model as well as an illustration of the universe will be introduced in the last part.

II. PRELIMINARY

We introduce basic notions about fractal manifold, and the reader will find details about this kind of mathematical objects in [1].

Definition 1 Let $\varepsilon$ be in $\mathcal{R}_f$, and $M_\varepsilon$ be an Hausdorff topological space. We say that $M_\varepsilon$ is an $\varepsilon$-manifold if for every point $x \in M_\varepsilon$, there exist a neighborhood $\Omega_\varepsilon$ of $x$ in $M_\varepsilon$, a map $\varphi_\varepsilon$, and two open sets $V^+_{\varepsilon}$ of $\prod_{i=1}^3 \Gamma^{+}_{i\varepsilon} \times \{\varepsilon\}$ and $V^-_{\varepsilon}$ of $\prod_{i=1}^3 \Gamma^{-}_{i\varepsilon} \times \{\varepsilon\}$ such that $\varphi_\varepsilon : \Omega_\varepsilon \rightarrow V^+_{\varepsilon}$, and $T_\varepsilon \circ \varphi_\varepsilon : \Omega_\varepsilon \rightarrow V^-_{\varepsilon}$ are two homeomorphisms.

Definition 2 Let $M = \bigcup_{\varepsilon \in \mathcal{R}_f} M_\varepsilon$ be an union of Hausdorff topological spaces all disjoint or all the same. We say that $M$ admits an internal structure $x$ on $P \in M$, if there exists a $\mathcal{C}^0$ parametric path

$$x : \mathcal{R}_f \rightarrow \bigcup_{\varepsilon \in \mathcal{R}_f} M_\varepsilon, \quad \varepsilon \mapsto x(\varepsilon) \in M_\varepsilon,$$  

such that $\forall \varepsilon \in \mathcal{R}_f$, $\text{Range}(x) \cap M_\varepsilon = \{x(\varepsilon)\}$, and $\exists \varepsilon' \in \mathcal{R}_f$ such that $P = x(\varepsilon') \in M_{\varepsilon'}$. 

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Definition 3 Let $M = \bigcup_{\varepsilon \in \mathcal{R}_f} M_\varepsilon$ be an union of Hausdorff topological spaces all disjoint or all the same. Let $x : \mathcal{R}_f \subset \mathbb{R} \rightarrow \bigcup_{\varepsilon \in \mathcal{R}_f} M_\varepsilon$ be an internal structure on it. We call object of $M$ the set $\text{Range}(x)$.

Definition 4 A diagonal topological space $(M, \mathcal{T}_d)$ is called fractal manifold if $M = \bigcup_{\varepsilon \in \mathcal{R}_f} M_\varepsilon$, where $\forall \varepsilon \in \mathcal{R}_f$, $M_\varepsilon$ is an $\varepsilon$-manifold, and if $\forall P \in M$, $M$ admits an internal structure $x$ on $P$ such that there exist a neighborhood $\Omega(\text{Range}(x)) = \bigcup_{\varepsilon \in \mathcal{R}_f} \Omega_\varepsilon$, with $\Omega_\varepsilon$ a neighborhood of $x(\varepsilon)$ in $M_\varepsilon$, two open sets $V_\varepsilon^+ = \bigcup_{\varepsilon \in \mathcal{R}_f} V_\varepsilon^+$ and $V_\varepsilon^- = \bigcup_{\varepsilon \in \mathcal{R}_f} V_\varepsilon^-$, where $V_\varepsilon^+$ is an open set in $\prod_{i=1}^3 \Gamma_{i\varepsilon}^+ \times \{\varepsilon\}$ for $\sigma = \pm$, and there exist two families of maps $(\varphi_\varepsilon)_{\varepsilon \in \mathcal{R}_f}$ and $(T_\varepsilon \circ \varphi_\varepsilon)_{\varepsilon \in \mathcal{R}_f}$ such that $\varphi_\varepsilon : \Omega_\varepsilon \rightarrow V_\varepsilon^+$ and $T_\varepsilon \circ \varphi_\varepsilon : \Omega_\varepsilon \rightarrow V_\varepsilon^-$ are homeomorphisms for all $\varepsilon \in \mathcal{R}_f$.

The local coordinates of objects in fractal manifold are described in the following local chart:

Definition 5 A local chart on the fractal manifold $M$ is a triplet $(\Omega, \varphi, T \circ \varphi)$, where $\Omega = \bigcup_{\varepsilon \in \mathcal{R}_f} \Omega_\varepsilon$ is an open set of $M$, $\varphi$ is a family of homeomorphisms $\varphi_\varepsilon$ from $\Omega_\varepsilon$ to an open set $V_\varepsilon^+$ of $\prod_{i=1}^3 \Gamma_{i\varepsilon}^+ \times \{\varepsilon\}$, and $T \circ \varphi$ is a family of homeomorphisms $T_\varepsilon \circ \varphi_\varepsilon$ from $\Omega_\varepsilon$ to an open set $V_\varepsilon^-$ of $\prod_{i=1}^3 \Gamma_{i\varepsilon}^- \times \{\varepsilon\}$ for all $\varepsilon \in \mathcal{R}_f$. A collection $(\Omega_i, \varphi_i, (T \circ \varphi_i)_{i \in J})$ of local charts on the fractal manifold $M$ such that $\cup_{i \in J} \Omega_i = \bigcup_{\varepsilon \in \mathcal{R}_f} M_\varepsilon = M$, where $\cup_{i \in J} \Omega_{i, \varepsilon} = M_\varepsilon$, is called an atlas. The coordinates of an object $P \in \Omega$ related to the local chart $(\Omega, \varphi, T \circ \varphi)$ are the coordinates of the object $\varphi(P)$ in $\bigcup_{\varepsilon \in \mathcal{R}_f} \prod_{i=1}^3 \Gamma_{i\varepsilon}^+ \times \{\varepsilon\}$, and of the object $T \circ \varphi(P)$ in $\bigcup_{\varepsilon \in \mathcal{R}_f} \prod_{i=1}^3 \Gamma_{i\varepsilon}^- \times \{\varepsilon\}$.

The local transformations that characterize the nature of fractal manifold is given by the following theorem:

Theorem 1 If $M$ is a fractal manifold, then $\forall n > 1$, there exist a family of homeomorphisms $\varphi_k$, and a family of translations $T_k$ for $2^{n-1} \leq k \leq 2^n - 1$, such that one has the $2^{n-1}$ diagrams given by:

\[
\begin{align*}
M \xrightarrow{\varphi_k} \bigcup_{\delta_0 \in \mathcal{R}_f} \bigcup_{\delta_1 \in \delta_{\delta_0}} \ldots \bigcup_{\delta_{n-1} \in \delta_{\delta_0} \ldots \delta_{n-1}} & \prod_{i=1}^3 \Gamma_{i\delta_{n-1}}^{\sigma_1 \ldots \sigma_{n-1}^+} \times \{\delta_{n-1}\} \times \ldots \times \{\delta_1\} \times \{\delta_0\} \\
T_k \circ \varphi_k & \bigcup_{\delta_0 \in \mathcal{R}_f} \bigcup_{\delta_1 \in \delta_{\delta_0}} \ldots \bigcup_{\delta_{n-1} \in \delta_{\delta_0} \ldots \delta_{n-1}} & \prod_{i=1}^3 \Gamma_{i\delta_{n-1}}^{\sigma_1 \ldots \sigma_{n-1}^-} \times \{\delta_{n-1}\} \times \ldots \times \{\delta_1\} \times \{\delta_0\}
\end{align*}
\]
where \( \sigma_j = \pm \) for \( j = 1, \ldots, n - 1 \).

The existence of constant internal structures associates a fractal nature to differentiable manifolds defined by homeomorphisms on the product of graphs:

**Corollary 1** Let \( g_1, g_2, g_3 \) be three differentiable functions, and \( \Gamma_{i0} \) be their associated graphs. If \( M_0 \) is a three dimensional differentiable manifold homeomorphic to the product \( \prod_{i=1}^{3} \Gamma_{i0} \), then \( M_0 \) is a fractal manifold.

### A. Elements of Fractal Manifold

An object \( P \) of a fractal manifold \( M \) is a set \( \text{Range}(x) \), where the continuous map \( x : \mathbb{R} \rightarrow M = \bigcup_{\delta_0 \in \mathbb{R}} M_{\delta_0} \) describes the evolution of one representative element \( x(\delta_0) \in M_{\delta_0} \) of \( x \).

The local representation of object in fractal manifold will be transformed from one step to another because of the natural existence of constant internal structure that transforms classical points into objects as shown in Fig.0.

![Figure 0](image)

**Figure 0** - One illustration of classical point in fractal manifold after 3 steps.

### B. Expanding Fractal Manifold

The local representation of objects in fractal manifold expands from one step to another as described in the following. The reader will find more details in [2].
Definition 6 Let $M$ be a fractal manifold, and $P$ be an object of $M$. We say that the object $P$ is expanding if its local representation at the step $n$ is strictly included in its local representation at the step $n+1$ for all $n \geq 0$.

Definition 7 A fractal manifold $M$ is said to be expanding if all object $P$ of $M$ is expanding.

Theorem 2 All fractal manifolds are expanding manifolds.

Definition 8 Let $M$ be fractal manifold, we say that $M$ is homogeneous if all objects of $M$ have same size at a given step.

Properties 1 Every object of a fractal manifold $M$ is expanding symmetrically.

In general, the transformation of objects in fractal manifold is summarized in the following expanding diagram:

![Expanding diagram of a fractal manifold](image)

**Figure 1.** Expanding diagram of a fractal manifold.
III. A NATURAL CONTRACTION

The definition of contraction exists naturally in the mathematical model of fractal manifold, indeed, the expanding diagram (Figure.1) of a fractal manifold model is constructed via invertible homeomorphisms that allow this symmetric transformation. Using the inverse homeomorphism, we will obtain a new diagram in which the number of local coordinates is reduced in the local charts, and then the number of hidden dimensions. In this diagram, the different steps are inverted by the reduction of number of hidden variables, and objects admit a local representation in which their size is reduced. The procedure of contraction involves a change of size in the opposite meaning of expansion, and we can associate the notion of contraction to the reduction of number of hidden dimensions:

**Definition 9** We call contraction in fractal manifold a symmetric transformation that reduces the number of local coordinates from one step to another.

IV. NEW PRINCIPLES IN AN EXPANDING UNIVERSE

We are willing to announce the following principles that fit our approach of cosmology using fractal manifold model:

i) The physical universe is defined by a fractal manifold and requires at the step n:

a) Four principal coordinates for the description of points in space and time: ct, x₁, x₂ and x₃, where c is the speed of light, and xⱼ, j = 1, 2, 3, are the spacial coordinates.

b) n nested hidden dimensions δᵢ, i = 1, .., n, that describe the local transformation of points with time (see [2]).

The metric that defines the distance between two space-time events at the step n is given by

\[ dσ_n^2 = g_{ij}dy^idy^j \quad i, j = 1, 2, 3, 4, \] (2)

where the \( y^i \) represent the curvilinear coordinates and the \( g_{ij} \) are symmetric functions of \( y^i \).

If the coordinates are reduced to the galilean coordinates, the only non zero coefficients of the metric are

\[ g_{11} = g_{22} = g_{33} = -\prod_{i=1}^{n} a_i(t), \quad g_{44} = +1 \] (3)
ii) In the tangent vector space of the fractal manifold, the metric of the physical space in rectilinear coordinates that defines the distance between the space-time events $Z$ and $Z + dZ$ has a reduced expression given by

$$d\sigma_n^2 = c^2 dt^2 - \left( \prod_{i=1}^{n} a_i(t) \right)^2 (dx_1^2 + dx_2^2 + dx_3^2).$$

Indeed, we know that the proper distance along a curve is measured for $j = 1, 2, 3,$ by $\left( \prod_{i=1}^{n} a_i(t) \right) x_j$ rather than $x_j$ at the step $n$ (see [2]), then the metric at the step $n$ in rectilinear coordinates will have the expression given by the formula (4). Since $\left( \prod_{i=1}^{n} a_i(t) \right)$ is convergent, the maximal metric of the physical universe (that represents the metric of the space-time with maximal expansion), is given by:

$$d\sigma_\infty^2 = \lim_{n \to \infty} d\sigma_n^2 = c^2 dt^2 - \left( \prod_{i=1}^{\infty} a_i(t) \right)^2 (dx_1^2 + dx_2^2 + dx_3^2),$$

that can be generalized to curvilinear coordinates.

iii) In a complete universe and for all reference, the geodesics are the extremum of the optimization

$$\delta \int_{z_0}^{z_1} d\sigma_n = 0,$$

where $d\sigma_n^2$ is the general metric (2). In the tangent vector space (which is a local approximation), the $d\sigma_n^2$ is given by [4].

iv) A freely moving body follows a geodesic of space time that optimize the extremum of (6), which corresponds to the initial condition of movement.

The geometry variation (expansion or contraction) modifies the metric. Indeed, the metric $d\sigma_n^2$ at the step $n$ depends on the term $\prod_{i=1}^{n} a_i(t)$, that is to say, the time and the hidden dimensions $\delta_i$, $i = 1, ..., n$.

V. GEODESICS AND LIGHT

By propagation into space, a photon follows the geodesic of the space, it follows the path which represent, the shortest distance between two points. This can be found by choosing a path which minimizes (6). The main problem resides in the nature of the expanding space itself, Indeed, in an expanding space, the metric that expresses the temporal interval between
two nearby events at a given point of space time is variable. It reveals that any given point of space is expanding, any path will expand and loose its characteristics, then it is not easy to set up equations that govern the geodesics in an expanding space. We have to fix at least one property that characterizes our geodesic to be identified after expansion. If a geodesic is a path that passes through some points in our space, and if points are expanding, from where the path will pass? How does it work? What if the path that represents the geodesic expands and increases its dimension? Many variables in the same time will complicate our investigation. If space is expanding, we can not impose to our geodesic to be static, at least it must follows the increase of distance between points. It is reasonable to fixe only the dimension of the geodesic, which means that the path that represents the geodesic is not part of the expanding space (any part of the space will expand symmetrically). We have then to determine the nature of interaction between points in an expanding space and a path of dimension one. For this purpose we introduce the following.

A. Contact in Static Space

In the physical space, if there is no interaction between two different objects, then there is no geometrical intersection, but a natural contact may exist. If we have intersection between two objects which are locally and globally different, then the objects will loose their local differences. In the following, we introduce the notion of simple and continuous contacts between classic geometrical objects in a given metric space \((E, d)\).

**Definition 10** Let \( A \) and \( B \) be two balls in a metric space \((E, d)\). We say that the balls \( A \) and \( B \) have a simple contact if

1) \( A \cap B = \emptyset \),

2) \( \exists x \in A / \inf_{y \in B} d(x, y) = 0 \),

**Definition 11** We say that a line \( L \) has a simple contact with the point \( P \) in a metric space \((E, d)\), if

1) \( P \cap L = \emptyset \)

2) \( \inf_{x \in L} d(x, P) = 0 \)

**Definition 12** We say that a curved line \( C \) has a continuous contact \( I \) with a ball \( A \) in a metric space \((E, d)\), if
Definition 13 Let us consider two balls $A$ and $B$ with a simple contact in a metric space $(E, d)$, we say that a curved line $C$ passes through the balls $A$ and $B$ if the line $C$ has a continuous contact $\mathcal{I}$ with $A$ and a continuous contact $\mathcal{J}$ with $B$ such that $\mathcal{I}$ and $\mathcal{J}$ are contiguous.

Properties 2 Let us consider two balls $A$ and $B$ with a simple contact in a metric space $(E, d)$, and a curved line $C$ that passes through the balls $A$ and $B$. If $\mathcal{I}$ is the continuous contact of the curved line $C$ with the ball $A$, and $\mathcal{J}$ is the continuous contact of the curved line $C$ with the ball $B$, then the simple contact of the balls $A$ and $B$ held at the point $P \in \mathcal{I} \cap \mathcal{J}$.

B. Contacts in an Expanding Space

The model of fractal manifold is constructed via double homeomorphisms over graphs of mean functions, and any local representation of object in fractal manifold at the step $n$ is given in $\bigcup_{\delta_0 \in \mathbb{R}_f} \bigcup_{\delta_1 \in \mathbb{R}_{\delta_1}} \cdots \bigcup_{\delta_{n-1} \in \mathbb{R}_{\delta_{n-1}}} \prod_{i=1}^{3} \Gamma_{\delta_{n-1}}^{\sigma_0 \cdots \sigma_{n-1}} \times \{\delta_{n-1}\} \times \cdots \times \{\delta_1\} \times \{\delta_0\}$ which is imbedded in $(\mathbb{R}^6, d)$, where $d$ is an usual distance.

Definition 14 We call universe point the local representation of an object $P$ in a fractal manifold $M$ at the step $n$, for all $n \geq 0$.

Definition 15 Let $P$ and $Q$ be two universe points. We say that the universe points $P$ and $Q$ have a simple contact if

1. $P \cap Q = \emptyset$,
2. $\exists x \in P \setminus Q \inf_{y \in Q} d(x, y) = 0$.

Definition 16 We say that a straight line $\mathcal{L}$ has a simple contact with the universe point $P$, if

1. $P \cap \mathcal{L} = \emptyset$,
2. $\inf_{x \in \mathcal{L}} d(x, P) = 0$

where $d(x, P) = \inf_{y \in P} d(x, y)$. 

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Definition 17  We say that a curved line \( C \) has a continuous contact \( \mathcal{I} \) with the universe point \( P \), if

\[
\begin{align*}
&i) \; P \cap C = \emptyset \\
&ii) \; \text{There exists a subset } \mathcal{I} \subset C \text{ such that } \forall x \in \mathcal{I}, \; d(x, P) = 0.
\end{align*}
\]

Definition 18  Let us consider two universe points \( P \) and \( Q \) with a simple contact, we say that a curved line \( C \) passes through the universe point \( P \) and \( Q \) if the line \( C \) has a continuous contact \( \mathcal{I} \) with \( P \) and a continuous contact \( \mathcal{J} \) with \( Q \) such that \( \mathcal{I} \) and \( \mathcal{J} \) are contiguous.

Properties 3  Let us consider two universe points \( A \) and \( B \) with a simple contact, and a curved line \( C \) that passes through the universe points \( A \) and \( B \). If \( \mathcal{I} \) is the continuous contact of the curved line \( C \) with the universe point \( A \), and \( \mathcal{J} \) is the continuous contact of the curved line \( C \) with the universe point \( B \), then the simple contact of the universe points \( A \) and \( B \) held at the point \( P \in \mathcal{I} \cap \mathcal{J} \).

Proposition 1  If \( P \) an universe point, then any non empty subset of \( P \) is expanding symmetrically.

Proof: Using definition \( \mathcal{I} \) and definition \( \mathcal{J} \) it is not difficult to conclude the result.

Proposition 2  Let \( Q \) be a universe point, \( C \) a curved line of dimension 1.

If \( Q \cap C \neq \emptyset \), then \( Q \cap C \) is an expanding subset.

Proof: If \( Q \cap C \neq \emptyset \) then there exists a set \( S \neq \emptyset \) such that \( S \subset Q \) and \( S \subset C \). If \( Q \) is expanding, then \( S \) is expanding, which concludes the proof.

C.  Geodesic of Light

In this part, we justify the following assertions announced in \([2]\):

- In an expanding space where points are expanding, there is no geodesic given by straight lines. All geodesics are curved due to the expansion of points.

- The variation of the universe geometry bends the light.

Theorem 3  In an expanding space where points are expanding, there is no geodesic given by straight lines. All geodesics are curved due to the expansion of points.
Proof: Let us consider a $C^2$ differentiable path $t \mapsto \varphi(t)$, $t_1 \leq t \leq t_2$, on an expanding space, where the metric is given by $d\sigma^2_n = dx^4_1 - \left( \prod_{i=1}^n a_i^2(t) \right) (dx_1^2 + dx_2^2 + dx_3^2)$, with $dx_4 = cdt$ and we denote for $j = 1, 2, 3, 4$, $x^j = \varphi^j(t)$, and $v^j = \frac{d\varphi^j}{dt}$. We consider the Lagrangian given in local coordinates by

$$L(\varphi(t), \frac{d\varphi}{dt}, t) = \sum_{j,k} g_{jk}v^jv^k = \sum_{j} g_{jj}v^jv^j,$$

where $g_{11} = g_{22} = g_{33} = -\left( \prod_{i=1}^n a_i^2(t) \right)$, $g_{44} = 1$ and $g_{jk} = 0$ for $j \neq k$. If $\varphi$ is a geodesic, then $\varphi$ is an extremum of (6), which yields that $\varphi$ satisfies the Euler equations

$$\frac{\partial L}{\partial x^j} - \frac{\partial}{\partial t}\left( \frac{\partial L}{\partial v^j} \right) = 0, \quad \text{for } j = 1, 2, 3, 4,$$

that is to say

$$\sum_k g_{jk} \frac{d^2\varphi^k}{dt^2} + \sum_{k,l} \frac{\partial g_{jk}}{\partial x^l} \frac{d\varphi^l}{dt} \frac{d\varphi^k}{dt} - \frac{1}{2} \sum_{k,l} \frac{\partial g_{kl}}{\partial x^j} \frac{d\varphi^l}{dt} \frac{d\varphi^k}{dt} = 0 \quad \text{for } j = 1, 2, 3, 4,$$

which gives for the metric $d\sigma^2_n$ the following four equations

$$\left( \prod_{i=1}^n a_i^2(t) \right) \frac{d^2\varphi^1}{dt^2} + \frac{\partial}{\partial x^4} \left( \prod_{i=1}^n a_i^2(t) \right) \frac{d\varphi^4}{dt} \frac{d\varphi^1}{dt} = 0 \quad (7)$$

$$\left( \prod_{i=1}^n a_i^2(t) \right) \frac{d^2\varphi^2}{dt^2} + \frac{\partial}{\partial x^4} \left( \prod_{i=1}^n a_i^2(t) \right) \frac{d\varphi^4}{dt} \frac{d\varphi^2}{dt} = 0 \quad (8)$$

$$\left( \prod_{i=1}^n a_i^2(t) \right) \frac{d^2\varphi^3}{dt^2} + \frac{\partial}{\partial x^4} \left( \prod_{i=1}^n a_i^2(t) \right) \frac{d\varphi^4}{dt} \frac{d\varphi^3}{dt} = 0 \quad (9)$$

$$\frac{d^2\varphi^4}{dt^2} + \frac{1}{2} \frac{\partial}{\partial x^4} \left( \prod_{i=1}^n a_i^2(t) \right) \sum_{j=1}^3 \left( \frac{d\varphi^j}{dt} \right)^2 = 0. \quad (10)$$

Now, suppose that we have particles moving freely (i.e. not subject to any non gravitational forces) following a geodesic given by a straight line $\mathcal{L}$ parameterized by $\varphi(t) = At + B$, with $A = (A_1, A_2, A_3, A_4)$ and $B = (B_1, B_2, B_3, B_4)$. By substituting in the previous equations (7), (8), (9), (10), we obtain for $j = 1, 2, 3$

$$\frac{\partial}{\partial x^4} \left( \prod_{i=1}^n a_i^2(t) \right) A_4 A_j = 0$$

and

$$\frac{1}{2} \frac{\partial}{\partial x^4} \left( \prod_{i=1}^n a_i^2(t) \right) (A_1^2 + A_2^2 + A_3^2) = 0.$$
Since \( \frac{\partial}{\partial x^4} \left( \prod_{i=1}^{n} a_i^2(t) \right) \) is not equal to zero, then we have \( A_1 = A_2 = A_3 = 0 \), this means that \( \mathcal{L} \) is the path of particles that move following the time coordinate, then our particles keep the same space coordinate for different time coordinates, then they are motionless, which yields a contradiction.

**Theorem 4** The variation of the universe geometry bends the light.

**Proof:** When light is emitted or absorbed, the energy of light appears in form of concentrated units, called photons. These photons are supposed to move in straight lines. Since there is no geodesic given by straight line in an expanding space (theorem), then the photons move in curved lines that we would like to determine.

Suppose that we have a geodesic of dimension 1 that passes through successive points from a source S to a point B. If the space is an homogeneous and isotropic expanding space, then all its universe points will expand symmetrically. In the purpose to visualize the light path in an illustration, we can assimilate our universe points to balls that expand symmetrically between the source S and the point B as shown in Figure 1. When the universe points expand, there is no intersection between each of them, but only simple contact. Each universe point is considered as free point to allow the deformability (expansion) of the space.

![Figure 1](image.png)

To get the final result, it is sufficient to find out the geodesic between the extremity of two successive universe points (two universe points with simple contact). Let us consider a parameterized curve \( \gamma \) which represents the geodesic of dimension 1 that passes from the source S through the universe points \( B_1 \) and \( B_2 \). Suppose that the universe points \( B_1 \) and
$B_2$ have a simple contact. From where does the geodesic pass? If $\gamma \cap B_1 \neq \emptyset$, then there exits an arc $C \subset \gamma$ such that $C$ expands symmetrically (proposition 11), which yields that $\dim C \neq 1$, then the geodesic will have a variable local dimension which is impossible. To keep constant its dimension, the geodesic $\gamma$ should not cross any part of the universe point. Then we have

$$\gamma \cap B_1 = \emptyset$$

(11)

Since the geodesic $\gamma$ must passes through the universe point $B_1$ following the shortest way, then there exists a subset $I \subset \gamma$, such that

$$\forall x \in I, \ d(x, B_1) = 0.$$  

(12)

and where $I$ is parallel to the geodesic over the expanding boundary surface of the universe point $B_1$.

The formulas (11) and (12) imply that the curved line $\gamma$ has a continuous contact $I$ with the universe point $B_1$. The geodesic $\gamma$ passes through the universe point $B_1$ via the shortest path (parallel to the geodesic of the expanding boundary surface of the universe point $B_1$) to the universe point $B_2$, then the geodesic $\gamma$ passes to the universe point $B_2$ via the simple contact. On the universe point $B_2$, the geodesic $\gamma$ has also a continuous contact $J$ with the universe point $B_2$ that is parallel to the geodesic over the boundary surface of the universe point $B_2$, such that $I$ and $J$ are contiguous (see illustration figure.2).

![Figure 2. Illustration of geodesic between two universe points](image)

If we repeat the same procedure for the next two universe points until we reach the objective B, we will obtain a geodesic that represents a continuous fluctuation following the variable diameter of the successive universe points, where their mutual simple contact are aligned in a given direction (This direction is the same direction indicated by classical
points before their expansion). Here is an illustration of the geodesic (Figure.3a) that will be followed by the light in an homogeneous universe.

![Geodesic in an homogeneous expanding space](image)

**Corollary 2** In an expanding universe, the movement of light is characterized by two degrees of freedom:

i) One degree of freedom that represents the direction of the light. This direction is only modified by gravity or by interaction with matter.

ii) Another degree of freedom that allows the light to follow the direction along the variable geodesic subsequent of geometry variation. This degree of freedom is affected only by geometry variation.

**Corollary 3** In an expanding space defined by fractal manifold, there exist an infinity of geodesics for a given rectilinear direction.

*Proof:* Since any universe point is expanding symmetrically, then there exist an infinity of continuous contact that can represent geodesics between the extremities of its diameter, which means that there exist an infinity of possible geodesics between two events.

The only difference with the general relativity is that the direction of light is bent not because of the local deformation of geometry under gravity effect, but because of the effect of gravity on photons. In the purpose to clarify this assertion and to explain how the gravity affects the light direction, we introduce the following property of movement in an expanding universe (how the direction is maintained in a curved space? How is possible the change of direction?)

**Properties 4** The direction of the light is rectilinear if the photon passes only through aligned simple contact.
Then if there is no physical contact with matter, the change of direction of the light is possible only when the photon passes through non aligned simple contacts. This is possible only if the photon changes its geodesic and takes another simple contact under the gravity action.

**Remark 1**

1) In the figure.3, the geodesic $\gamma$ may present a countable number of points in which the geodesics are not differentiable. In the proof of the last theorem, we omit to describe the geodesics by a differential equation because we were afraid to loose the real nature of geodesics if we impose a condition of differentiability that may not fit their nature.

2) Using the geodesic of the light given in figure.3, the movement of light presents a fluctuant appearance, however it is not a wave movement. This new appearance may give to the corpuscular nature of light a possible rational interpretation of the properties of light in some experiments until now reserved for the electromagnetic wave interpretation (Refraction, diffraction, interference, and polarization).

3) If we denote by $\lambda$ the distance between two successive maxima of light geodesic, then in an homogeneous space $\lambda$ is constant (Figure.3.a), however, in a non homogeneous space $\lambda$ is variable (Figure.3b).

![Figure.3b. Geodesic in a non homogeneous expanding space](image)

**D. Darkness of the Night Sky**

To understand the main reason of the darkness of the night sky, it is obvious to find the right answer on the nature of the light movement in our universe. In an expanding universe, the geodesics are curved, they follow (parallel) the local geodesics on the surface boundary of each universe point, and pass from one universe point to another via simple contacts. The
geodesics are everywhere parallel to the local geometry of the universe via universe points. This procedure makes invisible the geometry of the universe, and the light will be reflected only if it will interact with matter. The geometry of the universe is invisible because of the dynamic of the local expansion, it allows the travel of the light without any interaction with universe points, that is why the sky is dark meanwhile the light travel in it in all direction.

VI. DISTRIBUTION OF MATTER IN THE UNIVERSE

From the geodesic of the light, we deduce the following postulate:

- *There is no intersection between matter and any expanding part of the universe, there are only simple or continuous contacts between them.*

Following this postulate, the matter has only contacts with the different universe points, then it will be located outside the universe points. Here is an illustration of two dimensional location of matter in the universe (Figure.4a, Figure.4b).

![Expanding Points](image1)

![Matter](image2)

Figure.4a. Two dimensional representation of matter location in the universe

The nature of the interaction between matter and geometry variation of the space governs the real distribution of matter in the universe. The matter appears maintained everywhere in the universe by an invisible construction (Figure.4b) for all scales.
If three packed points become bigger, another point located in the center of hole formed by these balls will expand, and then the matter is always held by universe points as illustrated in Figure 5.

The universe points are disjoint, they appear to be packed and grouped together. How does it work? Points are not fixed in an expanding universe, why do they stay grouped if there is no intersection between them? The answer to this question is given by the gravity of the matter, it plays a fundamental role in the maintain of the structure of the universe.

The matter is distributed everywhere outside the universe points. With the existence of gravity everywhere as weak as it is at large scale, the matter maintains the universe points grouped together, meanwhile the movement of matter is governed by the expansion of the universe points. Universe points appear as the only responsible for the maintain of matter in its position, they appear as holding the matter. The geometry variation and the gravity play a complementary role in the architecture of the universe, a kind of reciprocal causality.

The homogeneity of the universe is possible only in the beginning of the expansion. Indeed, in the beginning of the expansion, the universe points were homogeneous and start expanding symmetrically in the same time, after that, new universe points with different sizes appear by the presence of holes between packed universe points (due to the universe point increase of size) and so on, until we obtain a non homogeneous space as we have today.
Remark 2

1) There is no reason to obtain a self similarity at different scales, the illustration of the figure.4 is only a two dimensional representation of the distribution of matter in an homogeneous universe.

2) The structure of universe points looks like the atomic structure of the matter and it seems not strange to find similarity on the way they are packed to be stable. Matter is constituted by packed atoms, however, in the universe matter is holden by packed universe points.

4) Any massive planets or stars or galaxies could create a local distortion of the space time if the universe points were not disjoint, however with the property to be disjoint and packed together under the effect of the gravity, makes their local distortion by gravity not rational.
VII. CONSEQUENCES AND UNIVERSE ARCHITECTURE AT LARGE SCALE

The existence of disjoint universe points that produce the dynamics of the universe and delimit the interaction with matter leads to crucial postulates and significant interpretations in relation with the following points:

i) Empty bubbles in space
ii) Dark energy and dark matter
iii) Shock wave
iv) Light movement
v) Global illustration of the universe architecture.

A. Empty Bubble in Space

Following the distribution of matter in an expanding universe, there must exist many regions in the universe that look like empty space (empty bubbles of different sizes) devoid of stars, gas and other normal matter. These regions, which are surrounded by matter, represent 'universe points'. Neither light nor any normal matter will interact with these universe points which are invisible. This invisibility is revealed as a result of the distortion of light due to the curved geodesics. The curved-shaped geodesics follow the continuous contact with the universe points, which means that we can see beyond such universe points. Since the velocity of light is constant, the light of the closest galaxies on the boundary surface of a universe point will be seen on picture before the light of the farthest galaxies on the back surface boundary of this universe point (The light of the closest and farthest galaxies will appear in the same picture after a while). The time difference of the appearance of the light of galaxies may inform us about the diameter of the universe points. Universe points are disjoint in the whole universe with simple contact and this property guarantees the dynamics of the universe. The universe increases in size as a result of the expansion of universe points which are grouped and packed. Their grouping resembles holding matter whereas their expansion creates its movement.
B. About Dark Energy and Dark Matter

Scientist have become aware of the fact that the universe is expanding rapidly, and the puzzle has been to explain what pushes the stars and galaxies apart, while some mysterious gravitational force keeps them from flying away from each other. Today scientists believe that they have found the answer: *Dark matter holds things together and Dark energy speeds the expansion.*

In our approach, the geometry of the universe architecture is too intricate to be deduced from angles, pictures or from any current observational techniques based on the wave nature of light. In an expanding universe, the light cannot move as a wave, there exists an infinity of expanding regions that obstruct this kind of movement[21]. Following our investigation, dark energy and dark matter are two different names for the same thing. What our approach shares 'with previous scientists' is that dark energy govern the repulsive force and dark matter held matter, which is a feature of the expansion of the universe. However, our approach adds that this is a proper characteristic of the universe points. 'Matter' is not held by dark matter; it is surrounded by universe points and held by its (matter’s) own proper mutual gravity. Indeed, the universe points surround the matter which keeps the universe points packed together via its gravity effect. The universe points, which are naturally packed, create a kind of corridor where the matter is located, and the geodesic of light passes through. This kind of corridor also represents a natural space for gravity, which connects all matter together. Accordingly, the following postulates can be deduced:

- *The geometry variation governs the movement of matter in the universe meanwhile gravity holds matter.*
- *There is no gravity inside the universe points.*

The matter’s own mutual gravity keeps the universe points grouped and packed. These universe points (as a group) hold the matter while at the same time create its movement due to their expanding nature. The matter holds itself by its proper gravity. This gravity goes through the corridors which exist between the universe points. The gravity keeps the universe points packed which appear to hold/restrain the movement of matter.
C. Shock Wave

As a consequence of the ever-changing architecture of the universe, built with an infinite number of packed universe points, any big event in the universe such as the death \[^{22}\] of massive stars or the collision between galaxies or massive planets is transmitted to the whole universe by a shock wave that is conveyed by the intermediary of the contact of the universe points. The effect of this shock wave on the matter is translated by stellar perturbations. If the universe points hold the natural matter of the whole universe, and if the dynamics of the universe points create the movement of matter, then the gravity of any massive planet or star can not deform/bend those universe points locally.

- The only possible deformation of the universe geometry is in the form of either an expansion or a contraction. If the expansion of the universe points stops, they might collapse to their center under the effect of gravity, and then the expansion might be invertible.

D. The Light Movement

Because of the nature of geodesics in an expanding universe, we can state the following:

- Any possible movement of photon in the universe is governed by two degrees of freedom: one describing the light’s direction, and the other describing the geodesic that permits the tracking of that direction.

- The gravity may affect the direction of the light. Indeed, it can re-rout the photon path from one contact point to another, which leads to a change of direction. This means that there exists a kind of cosmic mirage that forms what is known as the Einstein ring in two dimensions \[^{23}\]. An interaction between any massive corp and photon in free movement leads to the existence of minimal distance between the massive corp and photon. The photon is absorbed by the massive corp within such minimal distance. The geodesic of the photon may be rerouted to follow a new direction if it happened to be just beyond the minimal distance. However, at the minimal distance the photon will turn around the massive corp. The minimal distance (radius from the center of the massive corp) depends on the photon’s energy and the gravity of the massive corp.
E. Universe Architecture at Very Large Scale

The following summary illustrates a global (The world of very large scales) picture of our universe which may translate our understanding of its dynamics.

The space of our universe constitutes of an infinite number of packed universe points with different sizes. Outside these points is the location of the normal matter. All these points have a simple contact with each other. By expanding, they created the actual distribution of matter which is held and scattered in the universe due to the nature of the universe points which are packed together. The non-existence of physical intersection between universe points allows the deformation of the universe as well as its dynamics. The dynamics of the universe are due to the expansion of the universe points. Also, the kinetic energy of the natural matter contributes to and prolongs this dynamics. The movement of the matter in the universe depends on the gravity and the universe geodesics. There exists an infinite number of geodesics between two events in the universe. All the geodesics in the universe are continuous contacts with the universe points. The rectilinear direction in the universe is characterized by the existence of simple aligned contacts in the trajectory.

This approach concurs with the major cosmological manifestations predicted by general relativity. However, we have come up with some different interpretation: there is no distortion of the space-time locally; however the light is bent by the presence of massive bodies without a local distortion of space time, of which the Einstein rings are a logical manifestation. There is no light that travels in a straight line in an expanding space, which offers us a new opportunity to confirm the corpuscular nature of light since the wave appearance is not lost. The path of the geodesic of light in an expanding universe allows a new interpretation of some experiments that were reserved only for the wave interpretation. The shock wave is possible within a deformable architecture of universe based on universe points joined together (packed) to form an expanding universe. The gravitational wave, however, makes little sense.
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[17] The light moves following the geodesics of the space supposed to be straight lines.
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[19] as described by the general relativity

[20] From the scale of very big structures to scale of galaxies, stars and planets

[21] If light moved as a wave, then the geometry of the universe would be visible.

[22] Symmetric or anti-symmetric explosion of stars.

[23] A sphere of light in 3 dimensions that encloses the massive corp.