Classification of vector lines and fields by geometric and numerical methods

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Abstract. Geometric and numerical methods of classification of vector lines and vector fields are proposed. The division of vector lines of arbitrary vector fields into potential and spiral lines is carried out. The division of vortex vector fields into layered and spiral vector fields is carried out.

It is shown that any vector field can be represented as a superposition of layered vector fields.

Conditions of existence of so-called forceless spiral vector fields are established.

The mathematical conditions to be satisfied by the magnetic fields of magnetic clouds of the solar wind are formulated.

Introduction
A significant part of all modern courses of continuum mechanics is devoted to a detailed description of the mathematical apparatus used in these courses. This shows the deep relationship of mathematics with applied disciplines. The reason for the existence of this relationship is that the general properties of mathematical objects used in the description of natural phenomena, as a rule, have their physical counterparts, and vice versa. In this paper, we discuss the classification of vector lines and vector fields, which, among other things, allows us to establish the «mathematical» conditions for the existence forceless magnetic fields of magnetic clouds of the solar wind.

The current classification of vector fields is based on the analysis of the consequences of the use of rotation and divergence operators on these fields.

This analysis allows us to divide arbitrary vector fields into potential and vortex fields, as well as to distinguish among them solenoid vector fields [1-2].

Considering that the basis of this classification is the use of differential operators of rotation and divergence, this method of classification can be characterized as an analytical method of classification.

However, in this analytical method of classification remain unclaimed some important geometric characteristics of vector lines and vector fields. The proposed additional classification of vector lines and vector fields is based on the possibility of effective use of these geometric characteristics.

This additional geometric classification allows to divide vector lines of arbitrary vector fields into potential and spiral ones, and also to divide vortex vector fields into layered and spiral ones.

If the geometric classification of vector lines is based directly on their qualitative topological difference from each other, the geometric classification of layered and spiral vector fields is based on the qualitative topological difference of their vector lines.
It is very likely that the main advantage of layered vector fields is the ability to represent any vector field in the form of their superposition. While, most likely, the main advantage of spiral vector fields is the possibility of selecting among them a set of vector fields, vector lines of which are parallel to the vector lines of their vortex fields.

Vector fields of this type are used in various branches of physics and in space magnetohydrodynamics. Because of their physical counterparts are the so-called a forceless magnetic field [3].

The impetus for the use of these fields in space magnetohydrodynamics is the practical importance of studying the magnetic clouds of the solar wind. Since the forceless magnetic fields of the magnetic clouds of the solar wind significantly affect the processes of the earth's high-latitude geomagnetic activity [4].

1 Potential vector fields
Potential or gradient fields $\mathbf{a}^{\text{pot}}$ are vector fields that meet the condition of the form

$$\mathbf{a}^{\text{pot}} = \nabla \Phi,$$  \hspace{1cm} (1.1)

where $\Phi$ is a differentiable scalar function.

The expression (1.1) can be rewritten as

$$\mathbf{a}^{\text{pot}} = \left| \nabla \Phi \right| \mathbf{e}^{\text{pot}} = A^{\text{pot}} \mathbf{e}^{\text{pot}}$$  \hspace{1cm} (1.2)

where $A^{\text{pot}} = \left| \nabla \Phi \right|$ - module of the vector $\mathbf{a}^{\text{pot}}$, and $\mathbf{e}^{\text{pot}}$ - directing unit vector or ort of the vector $\mathbf{a}^{\text{pot}}$.

Potential of a vector field $\mathbf{a}^{\text{pot}}$ can be determined as well as the vortex-free vector field that satisfies the condition

$$\text{rot} \ \mathbf{a}^{\text{pot}} \equiv \text{rot}(A^{\text{pot}} \mathbf{e}^{\text{pot}}) = 0.$$  \hspace{1cm} (1.3)

This paper proposes an alternative definition of potential vector fields. This alternative definition is based on the number of scalar functions that must be specified to define a potential vector field $\mathbf{a}^{\text{pot}}$.

According to this alternative definition, potential vector fields $\mathbf{a}^{\text{pot}}$ are arbitrary vector fields $\mathbf{a}$, to set which it is enough to set one scalar function $\Phi$.

2 Potential vector lines
Let us recall that vector lines are lines, tangents to which are parallel to vectors of the considered vector field [1].

It follows from this definition, that to set a family of vector lines $L^{\text{pot}}$ of potential field $\mathbf{a}^{\text{pot}}$ enough to set the field of its unit vectors

$$\mathbf{e}^{\text{pot}} = \nabla \Phi / |\nabla \Phi|.$$  \hspace{1cm} (2.1)

It is essential that the vector fields $\nabla \Phi$ and $\mathbf{e}^{\text{pot}}$ are two autonomous vector fields that have an identical set of vector lines $L^{\text{pot}}$. It is also significant that the unit field $\mathbf{e}^{\text{pot}}$, associated with the potential vector field $\nabla \Phi$ does not have to be a potential vector field $\mathbf{a}^{\text{pot}}$. Because the rotor of this field may not be zero

$$\text{rot} \ \mathbf{e}^{\text{pot}} \neq 0.$$  \hspace{1cm} (2.2)

However, despite this, below for the name of vector lines $L^{\text{pot}}$ describing vector fields $\mathbf{a}^{\text{pot}}$ and vector fields $\mathbf{e}^{\text{pot}}$ will be used the general term – potential vector lines $L^{\text{pot}}$.

In other words, in this case, a potential vector lines $L^{\text{pot}}$ are vector lines of potential vector fields $\mathbf{a}^{\text{pot}}$.

Potential vector lines $L^{\text{pot}}$ is a vector line to set which it is enough to set one scalar function $\Phi$.

3 Layered vector fields
By definition, layered vector field $\mathbf{a}^{\text{lay}}$ is the vortex vector field $\mathbf{a}^{\text{rot}}$ with potential vector lines $L^{\text{pot}}$.

It follows from this definition that this field must meet the conditions of the form

$$\mathbf{a}^{\text{lay}} = A^{\text{lay}} \mathbf{e}^{\text{lay}}$$  \hspace{1cm} (3.1)

and

$$\text{rot} \ \mathbf{a}^{\text{lay}} \neq 0.$$  \hspace{1cm} (3.2)
It is possible to give alternative definitions of layered vector fields \( \mathbf{a}^{lay} \).

For example, a layered vector field \( \mathbf{a}^{lay} \) can be defined as a vortex vector field \( \mathbf{a}^{rot} \) whose vector lines are orthogonal to the vector lines of its vortex field \( \text{rot} \ \mathbf{a}^{lay} \).

This means that [2]
\[
\mathbf{a}^{lay} \cdot \text{rot} \ \mathbf{a}^{lay} = 0.
\] (3.3)

In addition, a layered vector field \( \mathbf{a}^{lay} \) can be defined as a vortex vector field \( \mathbf{a}^{rot} \), which can be written as [2]
\[
\mathbf{a}^{lay} = \Psi \nabla \Phi.
\] (3.4)

Finally, the layered vector field \( \mathbf{a}^{lay} \) can be defined as a vortex vector field \( \mathbf{a}^{rot} \), for which it is necessary and sufficient to set two scalar functions \( \Phi \) and \( \Psi \).

4 Spiral vector fields

There is a theorem [2] proving that any vector field \( \mathbf{a} \) can be represented as
\[
\mathbf{a} = \Psi \Phi + \nabla F,
\] (4.1)
where \( \Psi, \Phi \) and \( F \) are differentiable scalar functions.

Where does it follow that any vector field \( \mathbf{a} \) can be written as a superposition of layered \( \mathbf{a}^{lay} = \Psi \nabla \Phi \) and potential \( \mathbf{a}^{pot} = \nabla F \) fields, that is, in the form
\[
\mathbf{a} = \mathbf{a}^{lay} + \mathbf{a}^{pot}.
\] (4.2)

Below is the name of an arbitrary vector field \( \mathbf{a} \) for which fair conditions are
\[
\mathbf{a} \cdot \text{rot} \ \mathbf{a} \neq 0,
\] (4.3)
\[
\text{rot} \ \mathbf{a} \neq 0,
\] (4.4)
it is proposed to use the term – spiral vector fields \( \mathbf{a}^{spir} \).

Spiral vector fields \( \mathbf{a}^{spir} \) can be defined as vortex vector fields for which it is necessary and sufficient to set three scalar functions \( \Psi, \Phi \) and \( F \).

We briefly discuss now an important distinguishing feature of the unit fields \( \mathbf{e}^{spir}_l \) of spiral vector fields \( \mathbf{a}^{spir} \). To set this attribute, you must first rewrite expression (4.2) as
\[
\mathbf{a}^{spir} = \mathbf{a}^{lay} + \mathbf{a}^{pot},
\] (4.5)
and then divide both parts of the expression (4.5) by the module \( |\mathbf{a}^{spir}| \) of the vector \( \mathbf{a}^{spir} \).

By doing this, we get
\[
\mathbf{e}^{spir} = A^{lay} \mathbf{e}^{lay}_l + A^{pot} \mathbf{e}^{pot}, \quad (\mathbf{e}^{lay}_l \neq \mathbf{e}^{pot})
\] (4.6)
where \( \mathbf{e}^{spir} \) is the unit vector of the spiral vector field \( \mathbf{a}^{spir} = A^{spir} \mathbf{e}^{spir} \), and \( \mathbf{e}^{lay}_l \neq \mathbf{e}^{pot} \) are the unit vectors of the vectors \( \mathbf{a}^{lay}_l = (|\mathbf{a}^{lay}_l|/|\mathbf{a}^{spir}|) \mathbf{e}^{lay}_l \) and \( \mathbf{a}^{pot} = (|\mathbf{a}^{pot}|/|\mathbf{a}^{spir}|) \mathbf{e}^{pot} \).

Thus, it is possible to consider that a unit vector field \( \mathbf{e}^{spir} \) of spiral vector fields \( \mathbf{a}^{spir} \) always is the superposition of layered vector fields.

5 Spiral vector lines

In the previous paragraph, it was found that in order to set a unit vector field \( \mathbf{e}^{spir} \), you must first be set two layered vector fields \( A^{lay} \mathbf{e}^{lay}_l \) and \( A^{pot} \mathbf{e}^{pot} \).

Where does it follow that to set a unit field \( \mathbf{e}^{spir} \) must take into account the specifics of the method of setting unit fields \( \mathbf{e}^{lay}_l \) and \( \mathbf{e}^{pot} \).

It was established above, to set each unit field \( \mathbf{e}^{lay}_l = \nabla \Phi/|\nabla \Phi| \) and \( \mathbf{e}^{pot} = \nabla F/|\nabla F| \) is necessary and sufficient to set a one scalar function, in this case the functions \( \Phi \) and \( F \).

Knowing the unit vector field \( \mathbf{e}^{spir} \) it is not difficult to construct a family of vector lines \( \{L^{spir}(\mathbf{e}^{spir})\} \). It is obvious that to find the family of vector lines \( \{L^{spir}(\mathbf{e}^{spir})\} \), as well as to set the unit vector field \( \mathbf{e}^{spir} \), you must to set two scalar functions \( \Phi \) and \( F \).

By definition, spiral vector lines \( L^{spir} \) are vector lines, for which you need to set two scalar functions.
It is also essential that spiral vortex vector fields $\mathbf{a}^{spir}$ can also be defined as vortex vector fields with spiral vector lines $L^{spir}$.

6 Conjugate layered vector fields

It is easy to show that any potential vector field $\nabla F$ can always be represented as a superposition of two layered vector fields.

To do this, it is enough to use a vector identity having the form

$$\nabla F_{1,2} = \nabla (F_1 \cdot F_2) \equiv F_1 \nabla F_2 + F_2 \nabla F_1,$$

where $F_{1,2}$ are differentiable scalar functions.

It is obvious that for each fixed function $F_{1,2}$ there are infinitely many functions $F_1$ and $F_2$ satisfying the conditions $F_1 \cdot F_2 = F_{1,2}$.

It follows that any fixed potential field $\nabla F_{1,2}$ can be represented as a superposition of any number of pairs of layered vector fields $F_1 \nabla F_2$ and $F_2 \nabla F_1$.

By definition, the conjugate vector fields are any two layered vector fields $F_1 \nabla F_2$ and $F_2 \nabla F_1$, the superposition of which degenerates into the potential of vector field $\nabla F_{1,2}$.

Given the validity of the expression (6.1), it is not difficult to formally divide all layered vector fields $\mathbf{a}^{lay} = \nabla \Phi \nabla \Psi$ into two types of conjugate vector fields. Formally assuming that one of these types is vector field $\Psi \nabla \Phi$ and the other $\Phi \nabla \Psi$.

It is easy to check the validity of expressions of the form

$$\text{rot} \left( F_1 \nabla F_2 \right) = \nabla F_1 \times \nabla F_2,$$

$$\text{rot} \left( F_2 \nabla F_1 \right) = \nabla F_2 \times \nabla F_1.$$

Whence it follows that

$$\nabla F_1 \times \nabla F_2 = -\nabla F_2 \times \nabla F_1.$$

It is easy to see that the expression (6.4) is no more than a well-known vector identity.

7 Layered vector field as the basis vector subspace of the space of vector fields

In mathematics, the basis of vector space is understood as an ordered set of vectors.

It is assumed that any vector of vector space can be represented in the form of a linear combination of vectors from this set.

In this paper, the set of layered vector fields $\mathbf{a}^{lay}$ is interpreted not as the basis of the space of arbitrary vector fields $\mathbf{a}$, but as some basic subspace of this space.

It is assumed that any vector field $\mathbf{a}$ can be represented as a superposition of layered vector fields $\mathbf{a}^{lay}$.

The easiest way to illustrate the validity of this assumption is as follows.

First we need to use the possibility of representing an arbitrary vector field $\mathbf{a}$ as $\mathbf{a} = \nabla \nabla \Phi + \nabla F$. Then we need to use the representation of the potential field $\nabla F$ as a superposition of conjugate layered vector fields $F_1 \nabla F_2$ and $F_2 \nabla F_1$.

By doing this, we get

$$\mathbf{a} = \nabla \nabla \Phi + F_1 \nabla F_2 + F_2 \nabla F_1 \quad \text{for} \quad (F = F_1 \cdot F_2)$$

It follows from the expression (7.1) that an arbitrary vector field $\mathbf{a}$ can always be represented as a superposition of one layered $\nabla \nabla \Phi$ and two conjugate layered $F_1 \nabla F_2$ and $F_2 \nabla F_1$ vector fields.

8 Forceless spiral vector fields

By definition forceless vector fields $\mathbf{a}_{force}$ are vector fields $\mathbf{a}$ parallel to fields $\text{rot} \mathbf{a}$.
Forceless vector fields \( \mathbf{a}_{\text{noforce}} \) can also be defined as vector fields that meet the conditions of the form
\[
\mathbf{a}_{\text{noforce}}^{\text{spir}} \times \text{rot} \mathbf{a}_{\text{noforce}}^{\text{spir}} = 0, \quad (8.1)
\]
\[
\text{rot} \mathbf{a}_{\text{noforce}}^{\text{spir}} \neq 0. \quad (8.2)
\]

Significantly that forceless vector fields \( \mathbf{a}_{\text{noforce}} \) can only be a spiral vector field \( \mathbf{a}_{\text{spir}} \). Since neither potential \( \mathbf{a}_{\text{pot}} = \nabla F \) nor layered \( \mathbf{a}_{\text{lay}} = \nabla \nabla \Phi \) vector fields can not simultaneously satisfy the conditions (8.1) and (8.2).

The fact is that the potential vector fields \( \mathbf{a}_{\text{pot}} \) can not satisfy the condition (8.2) because their vortex fields \( \text{rot} \mathbf{a}_{\text{pot}} \) are zero (\( \text{rot} \mathbf{a}_{\text{pot}} = 0 \)).

While layered vector fields \( \mathbf{a}_{\text{lay}} \) cannot satisfy the conditions (8.1) because \( \mathbf{a}_{\text{lay}} \times \text{rot} \mathbf{a}_{\text{lay}} \neq 0 \) (due to the orthogonality of vectors \( \Psi \nabla \Phi \) and \( \nabla \Psi \times \nabla \Phi \)).

9. The conditions for the existence forceless spiral vector fields

It can be shown that the conditions of existence of forceless spiral vector fields \( \mathbf{a}_{\text{noforce}}^{\text{spir}} \) have the form
\[
\left| \mathbf{a}_{\text{noforce}}^{\text{spir}} \right| = \left| \nabla F_{\text{noforce}} \cdot \mathbf{i}_{\text{noforce}}^{\text{spir}} \right|, \quad (9.1)
\]
\[
\left| \Psi_{\text{noforce}} \cdot \nabla \Phi_{\text{noforce}} \right| = \left| \nabla F_{\text{noforce}} \cdot \mathbf{i}_{\text{noforce}}^{\text{lay}} \right|, \quad (9.2)
\]
\[
\mathbf{i}_{\text{noforce}}^{\text{spir}} = \nabla \Psi_{\text{noforce}} \times \nabla \Phi_{\text{noforce}} / \left| \nabla \Psi_{\text{noforce}} \times \nabla \Phi_{\text{noforce}} \right|, \quad (9.3)
\]
where \( \mathbf{i}_{\text{noforce}}^{\text{spir}} \) - or of the vector \( \text{rot} \mathbf{a}_{\text{noforce}}^{\text{spir}} \cdot \mathbf{i}_{\text{noforce}}^{\text{lay}} \) - or of the vector \( \nabla \Phi_{\text{noforce}} \).

This means that the module \( \left| \mathbf{a}_{\text{noforce}}^{\text{spir}} \right| \) of the forceless spiral vector field \( \mathbf{a}_{\text{noforce}}^{\text{spir}} \) must be equal to module of the projection \( \left| \nabla F_{\text{noforce}} \cdot \mathbf{i}_{\text{noforce}}^{\text{spir}} \right| \) of the potential field \( \nabla F_{\text{noforce}} \) on the direction of the vector \( \mathbf{a}_{\text{noforce}}^{\text{spir}} \).

While the module \( \left| \Psi_{\text{noforce}} \nabla \Phi_{\text{noforce}} \right| \) vortex component \( \Psi_{\text{noforce}} \nabla \Phi_{\text{noforce}} \) of a forceless spiral vector field \( \mathbf{a}_{\text{noforce}}^{\text{spir}} \) must be equal to module of the projection \( \left| \nabla F_{\text{noforce}} \cdot \mathbf{i}_{\text{noforce}}^{\text{lay}} \right| \) of the potential field \( \nabla F_{\text{noforce}} \) on the direction of the vector \( \nabla \Phi_{\text{noforce}} \).

The result is that the existence of a forceless spiral vector field \( \mathbf{a}_{\text{noforce}}^{\text{spir}} \) entirely depends on will or will not be able the corresponding projection of the potential field \( \nabla F_{\text{noforce}} \) to delete the vortex component \( \Psi_{\text{noforce}} \nabla \Phi_{\text{noforce}} \) of vector field \( \mathbf{a}_{\text{noforce}}^{\text{spir}} \).

It is also essential that the vectors \( \mathbf{a}_{\text{noforce}}^{\text{spir}}, \text{rot} \mathbf{a}_{\text{noforce}}^{\text{spir}}, \Psi_{\text{noforce}} \nabla \Phi_{\text{noforce}} \) and \( \nabla F_{\text{noforce}} \) should be coplanar vectors.

10 The main geometric difference between divergent and divergent-free vector fields

The concept of divergence-free field, in essence, is a refinement of the concept of a solenoid vector field. Because this concept clarifies that a solenoid vector field is a vector field whose divergence is zero.

By definition, divergent-free vector fields \( \mathbf{a}_{\text{nodiv}} \) are arbitrary vector fields \( \mathbf{a} \), satisfying the condition of the form
\[
div \mathbf{a} = 0. \quad (10.1)
\]

Accordingly, divergent vector fields \( \mathbf{a}_{\text{div}} \) are arbitrary vector fields \( \mathbf{a} \), satisfying the condition of the form
\[
div \mathbf{a} \neq 0. \quad (10.2)
\]

Note that we can show [2] that the vector field \( \mathbf{a} \) is a divergence-free field \( \mathbf{a}_{\text{nodiv}} \) if and only if the flux of the vector through an arbitrary inner surface is zero.

The main geometric difference between divergent \( \mathbf{a}_{\text{div}} \) and divergence-free \( \mathbf{a}_{\text{nodiv}} \) vector fields is that the families of vector lines of divergence-free vector fields have a tubular structure.
Recall that vector tubes are called closed vector surfaces formed by vector lines passing through the points of a closed contour \( L \).

It is essential that the value of the vector flux through the cross section of vector tubes of divergence-free vector fields \( \mathbf{a}_{\text{nodiv}} \) remains constant along these tubes.

Therefore, vector tubes of divergence-free vector fields cannot end inside the field [1,2].

11 The relationship of a divergence-free potential and layered vector fields and harmonic functions

In mathematics, the harmonic functions \( \Phi^\Delta \) are called scalar functions \( \Phi \), satisfying the condition of the form [1]

\[
\text{Div}(\nabla \Phi) = 0
\]

(11.1)

If we consider the expression (11.1) as a definition of harmonic functions, we can talk about the existence of a relationship between harmonic functions and divergent-free potential vector fields \( \mathbf{a}^\text{pot}_{\text{nodiv}} = \nabla \Phi \).

It was found above that to set an arbitrary potential field \( \mathbf{a}^\text{pot} \), it is necessary and sufficient to set one scalar function \( \Phi \). In this paragraph it is shown that to set a divergence-free vector field \( \mathbf{a}^\text{pot}_{\text{nodiv}} \) is necessary and sufficient to set a one scalar harmonic function \( \Phi^\Delta \).

In the applied aspect, you can say that each divergence-free \( \text{div} \mathbf{E} = 0 \) electric field \( \mathbf{E} = -\nabla \varphi \), where \( \varphi \) is the electric potential, can be put into correspondence with harmonic electric potential \( \Phi^\Delta \). While each vortex-free \( \text{rot} \mathbf{H} = 0 \) magnetic field \( \mathbf{H} = \text{rot} \mathbf{A} \), where \( \mathbf{A} \) is the vector magnetic potential, can be supplied in conformance scalar harmonic magnetic potential \( A^\Delta \).

Let us now discuss the geometrical method for allocating a among the multitude of harmonic functions \( \Phi^\Delta \) of a subset of the so-called orthbigarmonic functions \( \Phi^{\pm \Delta} \).

By definition, the orthbigarmonic function \( \Phi^{\pm \Delta} \) is a scalar function whose values are equal to the product of two harmonic functions \( \Phi^{+\Delta} \) and \( \Phi^{-\Delta} \), the vector lines of the gradient fields of which \( \nabla \Phi^{+\Delta} \) and \( \nabla \Phi^{-\Delta} \) are orthogonal to each other.

This means that in this case the expressions of the form

\[
\Phi^{\pm \Delta} = \Phi^{+\Delta} \cdot \Phi^{-\Delta},
\]

(11.2)

\[
\nabla \Phi^{+\Delta} \cdot \nabla \Phi^{-\Delta} = 0
\]

(11.3)

are fair.

It is easy to verify that the divergence-free gradient field \( \mathbf{a}^\text{pot}_{\text{nodiv}} = \nabla \Phi^{\pm \Delta} \) of the orthbigarmonic function \( \Phi^{\pm \Delta} \) is a superposition of the divergence-free conjugate layered vector fields \( \mathbf{a}^{+\text{lay}}_{\text{nodiv}} = \Phi^{+\Delta} \nabla \Phi^{-\Delta} \) and \( \mathbf{a}^{-\text{lay}}_{\text{nodiv}} = \Phi^{-\Delta} \nabla \Phi^{+\Delta} \).

So orthbigarmonic function \( \Phi^{\pm \Delta} \) formally valid to consider as orthbiharmonic potential conjugate layered vector fields \( \mathbf{a}^{+\text{lay}}_{\text{nodiv}} \) and \( \mathbf{a}^{-\text{lay}}_{\text{nodiv}} \).

12 Mathematical conditions to be satisfied by magnetic fields of magnetic clouds of the solar wind

It is well known that the "mathematical" vector field \( \mathbf{a} \), considered as a physical object, can have a different physical meaning. For example, this field can be a velocity field \( \mathbf{v} \), vortex field \( \mathbf{\omega} \), electric field \( \mathbf{E} \) and magnetic field \( \mathbf{H} \).

In addition, the "mathematical" vector field \( \mathbf{a} \) can be formally assigned physical properties.

For example, in the case where the "mathematical" vector field \( \mathbf{a} \) is associated with the "physical" magnetic field \( \mathbf{H} \), it makes sense to say that this "mathematical" vector field, considered as a physical object, may have the property of freezing.

This suggests that the "mathematical" vector fields \( \mathbf{a} \) may have a hypothetical dualistic mathematical – physical nature.
However, it is obvious that any "physical" vector field must satisfy the restrictions imposed by mathematics on this field, considered as a mathematical object. Therefore, it makes sense to say that any physical vector field has a hypothetical dualistic physical - mathematical nature.

This dualistic physical - mathematical nature of vector fields should be taken into account in the study of any physical phenomena, in the mathematical description of which vector-valued functions are used.

For example, this dualistic nature should be taken into account when studying the magnetic fields of magnetic clouds of the solar wind. More specifically, in the study of these magnetic fields is necessary to consider that from the point of view of mathematics they are divergent-free forceless spiral vector fields.

It follows that the vector lines of forceless magnetic fields, considered as forceless spiral mathematical fields $\mathbf{a}^{\text{spir noforce}}$, must coincide with the lines of intersection of the surfaces of the level $\Psi = \text{const}$ and $\Phi = \text{const}$ scalar functions $\Psi$ and $\Phi$, which determine the vortex component $\Psi \nabla \Phi$ field $\mathbf{a}^{\text{spir noforce}}$.

In addition, note that the size of the module $|\mathbf{a}^{\text{spir noforce}}|$ of a spiral field must be equal to the size of the module $|\nabla F_{\text{noforce}} \cdot \mathbf{i}^{\text{spir noforce}}|$ of projection of the harmonic potential $\nabla F_{\text{noforce}}$ on the direction $\mathbf{i}^{\text{spir noforce}}$ of vector field $\mathbf{a}^{\text{spir noforce}}$.

Note that you must also take into account the limitation imposed on a spiral vector field $\mathbf{a}^{\text{spir noforce}}$ by the divergence-free property.

This restriction has the form

$$\text{div} \left( |\nabla F_{\text{noforce}} \cdot \mathbf{i}^{\text{spir noforce}}| \cdot \mathbf{i}^{\text{spir noforce}} \right) = 0. \quad (12.1)$$

where $F_{\text{noforce}} \equiv A^\Delta$, here $A^\Delta$ is the scalar harmonic magnetic potential.

**Summary**

Geometric and numerical methods for classifying vector lines and vortex vector fields are discussed.

The division of vector lines into potential and spiral, and vortex vector fields into layered and spiral is carried out.

It is shown that to each layered field can be assigned an associated the conjugate layered vector field.

It is shown that any vector field can be represented as a superposition of one layered field and two conjugate layered vector fields.

It is shown that the set of layered vector fields can be considered as a basic subspace of the space of vector fields.

The conditions for the existence forceless spiral vector fields are established.

The mathematical conditions to be satisfied by the magnetic fields of magnetic clouds of the solar wind are established.

**References**

[1] L. I. Sedov. Mechanics of continua. vol. 1, Nauka, Moscow, 1973, pp.535.
[2] N. E. Cochin. Vector calculus and the beginning of tensor calculus. Publishing USSR Academy of Sciences, Moscow, 1961, pp.426.
[3] J. Shercliff. Course of magnetic hydrodynamics. Moscow, Mir, 1967, 320 pp.
[4] N. A. Barkhatov, A. E. Levitin, E. A. Revunov, A. B. Vinogradov. Magnetic clouds solar wind as a cause of high-latitude geomagnetic activity. “Physics of Auroral Phenomena”, Proc. XXXVIII Annual Seminar, Apatity, pp. 83-86.