Channel capacity and performance evaluation of precoded MIMO-OFDM system with large-size constellation

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ABSTRACT

Multiple input-multiple output (MIMO) is a multipath diversity exploring approach which is emerged with orthogonal frequency division multiplexing (OFDM) to produce MIMO-OFDM that is widely used in wireless communications. This paper presents a discrete Hartley transform (DHT) precoded MIMO-OFDM system over multipath frequency-selective fading channel with large-size quadrature amplitude modulation (16-QAM, 64-QAM and 256-QAM). A mathematical models for the BER and channel capacity over mutlipath fading channels are also derived in this paper. Average Bit-error-rate (BER) and channel capacity of the presented system is considered and compared with that of the traditional MIMO-OFDM. Simulation results shows that the transmission performance and channel capacity of the proposed schemes is better than that of the traditional MIMO-OFDM without a precoder.

Journal homepage: http://iaescore.com/journals/index.php/IJECE
due to the feedback link that is used for the channel. It also required further complexity and it is hard to apply when the channel is fast fading. A unitary precoder with limited feedback was designed for spatial multiplexing MIMO-OFDM [12]. In [13], a channel independent unitary precoder was presented. The main drawback of this approach is its high complexity as it is required extra transforms at the transmitter and receiver. To solve this problem, we presented in our previous work [14] an Alamouti space time (ST-X-OFDM) based on a unitary orthogonal low complexity transform which is emerging the effects of discrete Hartley transform and the effects of DFT together in single transform. However, the system in [14] does not include channel capacity, has not introduced a mathematical formula for the bit-error-rate (BER) performance and it was not generalized to MIMO with massive antennas.

To solve the diversity problem in MIMO-OFDM, a DHT precoded MIMO-OFDM with $N_t$ transmit antennas and $N_r$ receive antennas over multipath frequency-selective fading channel is presented in this paper with large-size qadrature amplitude modulation (16-QAM, 64-QAM and 256-QAM). We also present a mathematical model for the average BER and channel capacity of the DHT precoded MIMO-OFDM is also derived. The DHT precoded MIMO-OFDM system could not only reduce the transmitter complexity [14] but also enhance the channel capacity and exploit the multipath channels diversity and achieves noticeable signal-to-noise ratio (SNR) improvement in comparison to the classic MIMO-OFDM. The presented system also achieves better channel capacity than the conventional MIMO-OFDM system.

The remaining of this research paper is formulated as follows; system model and mathematical model for general $N_t$ and $N_r$ antenna diversity is presented in section 2. Numerical results and is shown in section 3. The conclusions are drawn in section 4.

2. SYSTEM MODEL

The complex baseband systems’ block diagram of MIMO system is depicted in Figure 1. In this work, $N_t$ transmit and $N_r$ receive antennas ($N_t \times N_r$) MIMO system is considered in this paper. The input data is converted into M-array QAM symbols. These mapped symbols, $S$, are assumed to be zero-mean with equal power distribution. The covariance matrix of the mapped data is given as

$$C_S = E[SS^*] = E_s I_N,$$  \hspace{1cm} (1)

where $E_s$ is the symbol power and $I_N$ is the identity matrix of $N$ dimensions. The DHT transform is then used to modulate these data symbols as

$$r_n = H_n S$$ \hspace{1cm} (2a)

$$= \sum_{m=0}^{N-1} h_{n,m} s_m.$$ \hspace{1cm} (2b)

In (2b), $H_n$ represents the $n_{th}$ row of the DHT transform, and $h_{n,m}$ represents the $n_{th}$ row, $m_{th}$ column elements of $H$. In matrix form, equation (2) is given as

$$r = HS$$ \hspace{1cm} (3)

These precoded data symbols are then further modulated by the inverse FFT (IFFT) as

$$s^{(b)} = F^H r^{(b)},$$ \hspace{1cm} (4a)

$$s^{(b)\dagger} = F^H r^{(b)*},$$ \hspace{1cm} (4b)

where $s^{(b)\dagger} = \begin{bmatrix} s_0^{(b)*}, s_2^{(b)*}, \ldots, s_{N-1}^{(b)*}, s_0^{(b)*} \end{bmatrix}^T$. $F$ is the discrete Fourier transform (DFT) and $(\cdot)^\dagger$ represents the Hermitian, complex conjugate, operation.
Figure 1. Schematic diagram of the presented scheme

The transmission diversity and performance improvement are increased as the number of antennas diversity increased. In the case of \( N_r \) transmitter antennas, the frequency domain signal is expressed as [15]:

\[
\begin{bmatrix}
    r^{(1)} & r^{(2)} & \cdots & r^{(N_r-1)} & r^{(N_r)} \\
    -r^{(2)} & r^{(1)} & \cdots & -r^{(N_r)} & r^{(N_r-1)} \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    -r^{(N_r-1)*} & r^{(N_r)*} & \cdots & r^{(1)*} & -r^{(2)*} \\
    -r^{(N_r)*} & -r^{(N_r-1)*} & \cdots & -r^{(2)*} & r^{(1)*}
\end{bmatrix}
\]  

(5)

where the row represents the space diversity (signal assigned to each antenna) whereas the column represents the time diversity (signal assigned to the every next transmission).

The received frequency domain signal at the specific \( n_r \)th antenna is expressed as

\[
Y = Gr + \Omega,
\]

(6)

where \( Y \in \mathbb{C}^{N_t \times 1}, G \in \mathbb{C}^{N_r \times N_t}, r \in \mathbb{C}^{N_t \times 1} \) and \( \Omega \in \mathbb{C}^{N_t \times 1} \)

In the above equation, the following definitions were used:

\[
r = \begin{bmatrix} r^{(1)} \; r^{(2)} \; \cdots \; r^{(N_r)} \end{bmatrix}^T,
\]

(7)

and the channel matrix \( G \) is given as

\[
G = \begin{bmatrix}
    g^{(1,n_r)} & g^{(2,n_r)} & \cdots & g^{(N_r-1,n_r)} & g^{(N_r,n_r)} \\
    -g^{(1,n_r)} & g^{(1,n_r)} & \cdots & -g^{(N_r-1,n_r)} & -g^{(N_r,n_r)} \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    -g^{(1,n_r)*} & g^{(N_r,n_r)*} & \cdots & -g^{(2,n_r)*} & g^{(2,n_r)*} \\
    -g^{(1,n_r)*} & -g^{(N_r,n_r)*} & \cdots & -g^{(2,n_r)*} & -g^{(1,n_r)*}
\end{bmatrix}
\]

(8)

The detected signal is obtained by multiplying the signal in (6) by the complex conjugate (Hermitian) of the channel matrix as

\[
\tilde{Y} = G^H Y.
\]

(9)
Substitute (6) in (9) yields

\[ \tilde{Y} = \mathbf{G}^H \mathbf{G} \mathbf{r} + \mathbf{G}^H \Omega \]  

(10)

In (10), the term \((\mathbf{G}^H \mathbf{G})\) is \(N_t \times N_t\) diagonal matrix, \(\Sigma\), where the main diagonal is given as \(\Sigma_n = \sum_{n=1}^{N_t} |g_{mm,n}^2|\), \(n = 1, 2, ..., N_t\). Hence, (10) could be rewritten as:

\[ \tilde{Y} = \Sigma \mathbf{r} + \mathbf{G}^H \Omega. \]  

(11)

The MMSE equalizer is performed as

\[ \hat{\mathbf{r}} = \Delta \tilde{Y} \]

\[ = \Delta \left( \Sigma \mathbf{r} + \mathbf{G}^H \Omega \right) \]

\[ = \Delta \Sigma \mathbf{r} + \Delta \mathbf{G}^H \Omega, \]  

(12)

where \(\Delta\) is the MMSE equalizer which is defined as:

\[ \Delta = \frac{E_s}{E_s \Sigma + \sigma_s^2 \mathbf{I}_{N_t}}, \]

\[ = \frac{\gamma_s}{\mathbf{I}_{N_t} + \gamma_s \Sigma}. \]  

(13)

The DHT transform is then used to process the equalized signal to produce the QAM-mapped symbols as

\[ q = \mathbf{H} \hat{\mathbf{r}} \]

\[ = \mathbf{H} \Delta \Sigma \mathbf{H}^H \mathbf{S} + \mathbf{H} \Delta \mathbf{G}^H \Omega \]

\[ = \mathbf{H} \frac{\gamma_s \Sigma}{\mathbf{I}_{N_t} + \gamma_s \Sigma} \mathbf{H}^H \mathbf{S} + \mathbf{H} \frac{\gamma_s}{\mathbf{I}_{N_t} + \gamma_s \Sigma} \mathbf{G}^H \Omega. \]  

(14)

The signal power, \(P_s = E[|q|^2]\), is as given as

\[ P_s = |\mathbf{H}|^2 \frac{E_s \gamma_s^2 \Sigma^2}{(\mathbf{I}_{N_t} + \gamma_s \Sigma)^2} + |\mathbf{H}|^2 \frac{E_s \gamma_s \Sigma}{(\mathbf{I}_{N_t} + \gamma_s \Sigma)^2} \]

\[ = |\mathbf{H}|^2 \frac{E_s \gamma_s \Sigma}{\mathbf{I}_{N_t} + \gamma_s \Sigma}. \]  

(15)

The noise signal, \(\varepsilon = q - \mathbf{S}\), is given as:

\[ \varepsilon = \left( \mathbf{H} \frac{\gamma_s \Sigma}{\mathbf{I}_{N_t} + \gamma_s \Sigma} \mathbf{H}^H - \mathbf{I}_{N_t} \right) \mathbf{S} + \mathbf{H} \frac{\gamma_s}{\mathbf{I}_{N_t} + \gamma_s \Sigma} \mathbf{G}^H \Omega \]

\[ = \frac{-\mathbf{H} \mathbf{S}}{\mathbf{I}_{N_t} + \gamma_s \Sigma} + \mathbf{H} \frac{\gamma_s}{\mathbf{I}_{N_t} + \gamma_s \Sigma} \mathbf{G}^H \Omega. \]  

(16)

The noise power, \(P_n = E[|\varepsilon|^2]\), is as given in

\[ P_n = |\mathbf{H}|^2 \frac{E_s}{(\mathbf{I}_{N_t} + \gamma_s \Sigma)^2} + |\mathbf{H}|^2 \frac{E_s \Sigma}{(\mathbf{I}_{N_t} + \gamma_s \Sigma)^2} \]

\[ = |\mathbf{H}|^2 \frac{E_s}{\mathbf{I}_{N_t} + \gamma_s \Sigma}. \]  

(17)

Then the average SNR of the detected signal is given as

\[ SNR = \frac{P_s}{P_n}. \]  

(18)
The channel capacity measured in (bits/s/Hz) is then calculated as

\[ C = \log_{10} (I_N + SNR). \]  

(19)

The BER, for the M-QAM signalling, is given as [16]

\[ P_{e}^{M-QAM} = 4 - 2^{\frac{\log_2 M}{M}} Q\left(\sqrt{\frac{3P_s}{P_n (M-1)}}\right). \]  

(20)

where \( m \) is number of bits in each data symbol.

3. SIMULATION RESULTS

In this section we present numerical results to assess the transmission efficiency of the precoded MIMO-OFDM and compared it with the classic MIMO-OFDM system. In this simulation \( N=1024 \) are used. For simulation simplicity we used antenna diversity as \( 2 \times 1 \) and \( 2 \times 2 \), however, the results could be generalized for any number of transmitter antennas \( (N_t) \) and receiver antennas \( (N_r) \). The multipath Rayleigh fading channel is represented by the pedestrian type B international telecommunication union (ITU) channel, its impulse response is obtained by Nyquist sampling and shown in Figure 2. This is a standard channel that is widely used in wireless communication applications. It is clear from Figure 2 that this channel has six statistically independent taps with non light of sight with maximum spread delay 43 taps.

![Figure 2. ITU pedestrian B Channel impulse response](image)

3.1. Channel Capacity

Channel capacity is considered as outstanding metrics to measure the ability of wireless broadband communications to handle maximum data rate with reliable transmission. Higher channel capacity means that the telecommunication system can handle more applications and data transmission within a certain channel bandwidth. Therefore, system with high channel capacity, that is able to meet the market and customers requirements, is highly on demand. In this section, Computer simulation is used to investigate the capacity of the DHT-precoded MIMO-OFDM over Rayleigh fading ITU channel. Figure 3 shows the capacity in bits/s/Hz of the DHT-precoded MIMO-OFDM in comparison with the classic MIMO-OFDM over ITU pedestrian B multipath channel.

![Figure 3. Channel capacity of the DHT precoded MIMO-OFDM system normalized per unit bandwidth](image)
It is obvious from Figure 3 that, for different antenna diversity, the capacity of the scheme increases as the antenna diversity increased. It is also clear that the channel capacity per unit bandwidth of the presented MIMO-OFDM is higher than the channel capacity of the classic MIMO-OFDM for all the antenna number were used (1 × 1, 2 × 1 and 2 × 2).

3.2. BER Performance

This section presents the transmission performance of DHT-precoded MIMO-OFDM and compares it with the classic MIMO-OFDM that is not using a precoder in its structure, over ITU pedestrian B multipath channel for large-size constellation (16-QAM, 64-QAM and 256-QAM data signalling. The simulation is run for two types of antenna diversity (2 × 1 as in Figure 4 and 2 × 2 as in Figure 5).

For the case of (2 × 1), Figure 4 shows that at $10^{-5}$ BER, the precoded MIMO system gained about 8 dB SNR in comparison to classic MIMO scheme for all 16-QAM, 64-QAM and 256-QAM data mapping. This confirms that the DHT precoding MIMO-OFDM can explore the channel diversity. Figure 5 shows the average BER of the DHT precoded MIMO-OFDM in comparison to the classic MIMO-OFDM for 16-QAM, 64-QAM and 256-QAM data mapping. It is obvious from Figure 5 that the SNR gain due to the use of DHT as a precoder reduced from 8 dB in the case of 2 × 1 to 3 dB in the case of 2 × 2 diversity.

The reason why this SNR gain is reduced as the antenna diversity increased can be attributed the reason that the advantage of DHT precoded MIMO-OFDM over the classic MIMO-OFDM is on exploring the frequency diversity. Thus, for exploring the advantages of the DHT-precoded MIMO-OFDM, the channel type is a key factor. Therefore the antenna diversity contribute in reducing the effects of multipath channels and...
this is the reason why the SNR gain is lower as the antennas diversity increases. This is because the channel impairment on the transmitted signals is reduced as the number of antennas increased.

4. CONCLUSION

The transmission efficiency and channel capacity of DHT precoded MIMO-OFDM has been evaluated in this paper for large-size constellation and compared with that of the classic MIMO-OFDM scheme. Mathematical models for the channel capacity and average BER of the DHT-precoded MIMO-OFDM over multipath fading channels were also derived in this paper. Two types of antenna diversity (2 × 1 and 2 × 2 were used. It has been shown that using DHT transform as a precoder in MIMO-OFDM over multipath fading channels can achieve valuable SNR gain and increase the channel capacity. This furnishing a promising system for transmission over severe multipath fading channel with large channel capacity which enables more data transmission within a certain bandwidth.

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