Abstract

I discuss selected topics in contemporary Particle Physics from the point of view of the original ontological formulation of Relativistic Quantum Field Theory (RQFT), primarily due to D. Bohm and B. Hiley.

The basic platform for doing specific calculations in this paper begins systematically with modern textbook accounts and techniques. The Bohm-inspired causal RQFT (referred to as BP) is considered to be, even at this early stage of its development, a most useful and illuminating supplement to this standard RQFT.

Unfortunately (in the opinion of this author) it cannot as yet, and need not, replace the contemporary textbook RQFT. It has - for any foreseeable future - to work together with it, and thus prepare the ground for its own future independent developments. Its main role at this stage is to inspire, to give intuitive, imaginative and creative explanations of the facts exposed by experimental research that is so excellently described by standard papers and textbooks. It goes without saying that such fundamental changes in insight could inspire both theoretical and experimental research to try new paths that would be otherwise unthinkable.

I shall try to illustrate these claims by randomly picking up 3 good examples from contemporary research in Particle Physics, all having to do with the same basic theme, viz. time development of quantum systems and particularly, quantum transitions. I work entirely from the point of view of the orthodox Bohm-Hiley metatheory, but always trying to make the connection to the present day standard research that ultimately may qualify to be buried in textbooks.

PACS numbers: 11.30-j ; 11.30.Er ; 10.13.15.+g

1 Introduction

I have recently advertised [1] for practicing nuclear and particle theoreticians a novel ontological formulation of non-relativistic Quantum Mechanics (BP),
originally due to de Broglie, but independently discovered and developed by D. Bohm and his co-workers [2],[3],[4]. The discussion was illustrated with additional examples taken from contemporary Nuclear and Particle Physics.

M. R. Brown and B. Hiley discussed not so long ago the interesting possibility of considering the Heisenberg Picture (HP) as an alternative starting point [5] for developing the theory, and how in that case the original BP could be affected. Further work along these lines can be found in references [6,7]. The present paper sticks to the original Bohm-Hiley version of RQFT [2] and further developed by some members of their group, e.g. [8], which I shall refer to as the “Bohm school”. A large number of alternative ontological proposals exist to-day (some representative examples can be found in references [9],[10],[11],[12],[13]).

However, the present paper is not a review article. I feel that these alternatives have rather different fundamental assumptions, and pursue different lines of enquiry. They are therefore not considered here.

A central feature of the Bohm school is described by Bohm himself in an interview given a long time ago [14]. It refers to the meaning he assigns to the word “explanation”. This I have systematically adopted in the following sections. The discussion deals at a certain stage with the famous “gedanken” two-slit experiments, with which most of good undergraduate textbooks on Quantum Theory begin. I quote (my italics):

BOHM: (...) In this way you can explain, say, the two-slit experiment.

DAVIES: This is normally explained, of course, by proposing the interference between waves passing through the two slits.

BOHM: It’s not explained, it is merely described. If you said it was a wave, that would be an explanation. But since the electrons arrive as particles, it is no explanation. It is a sort of a metaphorical way of talking. Right? There is no explanation. We should say that quantum mechanics does not explain anything. It merely gives a formula for certain results. And I’m trying to give an explanation”.

Be it as it may, the final verdict for that we shall refer to as the Bohm Picture (BP) (to be added to the standard Schroedinger Picture (SP), the Heisenberg Picture (HP) and the Dirac Picture (DP)) is not just around the corner. The main issue for now is rather to patiently carry on with the nurturing and the step by step testing this kind of novel and promising attempts, initiated long ago by Bohm’s deep insights into the physical world.

The present particular modest contribution discusses 3 study cases of interest to the community of practicing particle physicists.

2 A short review of the Bohm Picture (BP) of Relativistic Quantum Field Theory (RQFT)

Let us shortly review some relevant and prominent features of the BP, referring the reader to the original literature for full details, e.g. [2],[3],[8].
2.1 Bohm’s field equations for Bose-Einstein fields (integer spin fields)

We begin with Bose-Einstein fields (in modern parlance: integer spin fields) in the Schrödinger representation:

\[ \{ \varphi(\vec{x}) \} \equiv \varphi_1(\vec{x}), \varphi_2(\vec{x}), \varphi_3(\vec{x}), \ldots \]  

They have well-defined classical analogs. A complex field will be considered as an ordered pair of real fields. The time-dependent Schrödinger equation is (in units that are used here we assume \( \hbar = 1 = c \), unless otherwise stated):

\[ H \Psi(\{ \varphi(\vec{x}) \}, t) = i \frac{\partial \Psi(\{ \varphi(\vec{x}) \}, t)}{\partial t} \]  

where \( H \) is the Hamiltonian in the Schrödinger representation.

The solutions are defined if some initial condition for the time-dependent wave-functional is given:

\[ \Psi(\{ \varphi(\vec{x}) \}, 0) = F(\vec{x}, ...) \]  

The Schrödinger representation for field operators is

\[ \hat{\varphi}_k(\vec{x})|\{ \varphi(\vec{x}) \} \rangle = \varphi_k(\vec{x})|\{ \varphi(\vec{x}) \} \rangle \]  

In this representation the canonically conjugate momentum to the field \( \varphi_k(\vec{x}) \) becomes

\[ \hat{\pi}_k(\vec{x}) \equiv -i \frac{\delta}{\delta \varphi_k(\vec{x})} \]  

We shall consider quantum Hamiltonians of the general form (in Schrödinger representation)

\[ H = \sum_k \int d^3\vec{x} \left[ \frac{1}{2} \frac{\delta^2}{\delta \varphi_k^2(\vec{x})} + \frac{1}{2} |\nabla \varphi_k(\vec{x})|^2 \right] + V(\{ \varphi(\vec{x}) \}) \]  

Let us now begin by briefly reviewing how the BP works in this case. The full Hamiltonian minus the quadratic part, i.e. the entire non-quadratic part, will in general be referred to as “the interaction Hamiltonian”. We start as usual [1],[2], by going over to the polar representation of the wavefunctional:

\[ \Psi(\{ \varphi(\vec{x}) \}, t) \equiv R(\{ \varphi(\vec{x}), t \})e^{iS(\{ \varphi(\vec{x}) \}, t)} \]  

where \( R \) and \( S \) are two real wavefunctionals. One inserts this into the Schrödinger equation (2) and one obtains two coupled partial differential functional equations:

\[ \frac{\partial S}{\partial t} + \frac{1}{2} \sum_k \int d^3\vec{x} \left[ \left( \frac{\delta S}{\delta \varphi_k(\vec{x})} \right)^2 + |\nabla \varphi_k(\vec{x})|^2 \right] + V + Q = 0 \]
\[ \frac{\partial}{\partial t} P(\{\varphi(\vec{x}), t\}) + \sum_k \int d^3\vec{x} \frac{\delta}{\delta \varphi_k(\vec{x})} J_k(\{\varphi(\vec{x}), t\}) = 0 \]  \hspace{1cm} (9)

These two functional equations are part of the basic quantum field dynamics in BP. However, they are not the complete story (subsection 2.3).

(i) The symbol \( Q \) stands for the Super-Quantum-Information-Potential (SQIP) and is the natural field theoretic generalisation of the Bohm many-body Quantum-Information-Potential (QIP) discussed in [1],[2]:

\[ Q(\{\varphi(\vec{x}), t\}) \equiv -\frac{1}{2} \sum_k \int d^3\vec{x} \frac{1}{R} \frac{\delta^2 R(\{\varphi(\vec{x}), t\})}{\delta \varphi_k^2(\vec{x})} \]  \hspace{1cm} (10)

We have defined

\[ P \equiv P(\{\varphi(\vec{x}), t\}) \equiv R^2(\{\varphi(\vec{x}), t\}) \equiv |\Psi(\{\varphi(\vec{x}), t\})|^2 \]  \hspace{1cm} (11)

The functional \( P \) has a double role assigned to it, just as it is the case with the non-relativistic formulation [1],[2]. First, according to equation (9), \( P \) is the locally conserved probability that the configuration of the fields at time \( t \) is \( \{\varphi(\vec{x})\} \) [2]; this is the role #1 that we shall assign to \( P \) in the present paper. Next, but not least, role #2 is that \( P \) is of paramount importance in determining the central quantity in Bohm’s ontological formulation of Quantum Theory, i.e the SQIP itself, definition (10)[2]. We shall nevertheless not insist on this most important feature of \( P \) that is a direct consequence of the time-dependent Schrödinger equations for most Hamiltonians that are relevant to known physics.

(ii) The other fundamental quantity of this formulation is the generalized current density in field space defined as:

\[ J_k(\{\varphi(\vec{x}), t\}) \equiv P \frac{\delta S}{\delta \varphi_k(\vec{x})} \]  \hspace{1cm} (12)

which strongly suggests the definition of the generalized local velocity fields \( \{\Phi(\vec{x}, t)\} \) as solutions of the coupled functional differential equations

\[ \Pi_k(\{\Phi(\vec{x}, t), t\}) \equiv \frac{\delta S}{\delta \varphi_k(\vec{x})} \{\varphi(\vec{x})=\{\varphi(\vec{x}), t\}\} \equiv \frac{1}{P} Im\{\Psi^* \frac{\delta \Psi}{\delta \varphi_k(\vec{x}, t)} \} \{\varphi(\vec{x})=\{\varphi(\vec{x}, t)\} \]  \hspace{1cm} (13)

This job is considerably simplified in practice by first Fourier transforming.

I shall consider eq(8), eq(9) and eq(13) as part of the set of fundamental equations of the Bohm formulation of an ontological causal RQFT. However, we use eq(13) in the following discussions, if needed.

I emphasize that Bohm-Hiley ontological reformulation of RQFT always treats Bose fields as continuous distributions in spacetime - basically because these quantum fields have perfectly well-defined classical analogs. The textbook
spin-0, spin-1 and spin-2 bosons, such as the Higgs, photons, gluons, electroweak bosons and gravitons [18] are, according to this viewpoint, not “particles” in any naive sense of this word, but just dynamical structural features of coupled continuous scalar, vector and symmetric tensor fields, that first become manifest when interactions with matter particles (elementary or otherwise) occur [2],[8],[16], as we shall illustrate. This is obviously not just a question of semantics!

2.2 Fourier transforms

Practical work is greatly simplified if we Fourier transform first, taking the advantages of the Poincaré invariance of relativistic field theories. Fock space techniques [19] are more expedite then the Schroedinger representation techniques in problems involving only one or two field modes. Once the time-dependent Schroedinger wavefunction is found, then we can immediately proceed to fix all the key quantities of the BP.

The important field in this paper is the spin-1 Maxwell electromagnetic field. The free field Hamiltonian is in a functional Schroedinger representation [2],[19]

\[ H_{0\text{Maxwell}} = \int d^3\vec{x} \left[ -\frac{\hbar^2}{c^2} \frac{\delta^2}{\delta \vec{A}_T(\vec{x})^2} + \nabla \times \vec{A}_T(\vec{x}) \nabla \times \vec{A}_T(\vec{x}) \right] \]  

where the symbol :: stands for “normal products” [19]. The classical vector field \( \vec{A}_T(\vec{x}, t) \) is related to the classical electric and magnetic field as follows:

\[ \vec{E}(\vec{x}, t) = -\frac{1}{c} \frac{\partial}{\partial t} \vec{A}_T(\vec{x}, t) \quad \vec{H}(\vec{x}, t) = \nabla \times \vec{A}_T(\vec{x}, t) \]  

We can transform to the Fock representation [19]

\[ \vec{A}_T(\vec{x}) = \sum_{\lambda=\pm 1} \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{e^{i\vec{k}.\vec{x}}}{\sqrt{2}|\vec{k}|} e^{i\vec{k},\lambda} a(\vec{k}, \lambda) + h.c. \]  

that must satisfy the basic commutation relations

\[ [a(\vec{k}, \lambda), a^{\dagger}(\vec{k}', \lambda')] = i\delta_{\lambda\lambda'}\delta(\vec{k} - \vec{k}') \quad \text{zero otherwise} \]  

and the conditions

\[ \vec{e}(\vec{k}, \lambda), \vec{e}(\vec{k}', \lambda') = \delta(\lambda, \lambda')\delta(\vec{k} - \vec{k}') \]  

The conventions assumed in this paper are:

\[ \vec{e}(-\vec{k}, +1) = -\vec{e}(\vec{k}, +1) \quad \vec{e}(-\vec{k}, -1) = +\vec{e}(\vec{k}, -1) \]  

\[ \vec{e}(\vec{k}, \lambda), \vec{e}(-\vec{k}, \lambda') = (-1)^{(\lambda+1)/2}\delta(\lambda, \lambda') \]
and lead to the finite zero-th Maxwell Hamiltonian:

\[
H_{0\text{Maxwell}} = \sum_{\lambda = \pm 1} \int d^3 \vec{k} \left| \vec{k} \right| a^{\dagger}(\vec{k}, \lambda) a(\vec{k}, \lambda) \quad a(\vec{k}, \lambda)|0 >= 0 \quad (21)
\]

### 2.3 Many-body Dirac wavefunctions for point-like beable particles

The relationships between fundamental classical relativity [2],[3], and the Bohm formulation of non-relativistic Quantum Field theory has a long history, going back to the fifties. There have been recent new interesting proposals by B. Hiley and co-workers on this subject [6],[7], but in the present paper we follow the original ideas, though closely linking them with contemporary textbook RQFT. Bohm and co-workers went back to basics and did make an a priori distinction between continuous beable bosonic fields (having natural classical analogs) and discrete elementary beable point-like particles, assumed to be always guided by multi-dimensional Dirac wavefunctions. I should stress at this point that the concept “beable”, as used in this paper, has the same connotations as it had in the latest published papers by Bohm and co-workers and also by J.S.Bell (who actually appears to have coined it [3]). So present day Quantum Theory is assumed in this paper to operate with basically 3 kinds of beables, all equally necessary for the self-consistency of its mathematical formalism: particles, fields and generally infinite dimensional time-dependent Schroedinger wavefunctions.

Classical Mechanics does need only the first two beables. The third kind of beables (wavefunctions) is unnecessary for its internal self-consistency and completeness; therefore it has no role to play in Classical Physics. It is therefore essential to keep in mind that the BP formulation is basically a theory of beables.

Forgetting this may lead to unnecessary confusion and endless discussions, as earlier experience shows. The concept of beables is of course totally foreign to textbook RQFT that deals solely with so-called “observables”. According to the views underlying the present paper, all “observables” are more or less complicated functions of beables, many specifically tailored to the needs of the present-day experimental practices. The inverse statement is not necessarily correct. I stress again, observables in this sense are not fundamental to the BP (subsection 3.4).

It is assumed that the definition of the Dirac Hamiltonian includes the minimal coupling prescription to all vector gauge fields (subsection 2.3). This could actually serve as an heuristic definition of what one could mean with the words “elementary particles”, when applying the BP to physically well-defined problems. For example, a “bare electron” is just a convenient mathematical abstraction in Quantum Electrodynamics, because the dimensionless fine structure constant is so small. However, it is not a gauge invariant object and so it is - by definition - unphysical. Candidates to physical quantities must be by definition gauge invariant objects. This implies among other things that an electron at rest is always accompanied by its Coulomb field in a Lorentz and gauge covari-
ant manner: it always appears and disappears together with its Coulomb field for all “observers”.

An obvious option from the ontological BP viewpoint (even if, according to its authors, it might sound somewhat ad hoc [2],[16]) is to consider leptons and quarks as present-day candidates to point-like beable “particles”, though characterized by their spatial positions only [1],[2],[3]. One should add and emphasize that they are always supposed to be actively guided by many-body beable Dirac wavefunctions, obeying the Dirac equation of motion and satisfying the principle of minimal couplings, say to $SU_c \otimes SU_{2L} \otimes U_1$ vector gauge fields[2]. This assumption is perfectly consistent with the remark that all so-called “measurements” in experimental physics ultimately boil down to “position measurements” [2],[3].

Let us be specific: assume an isolated Dirac particle having an inertial mass $m_0$ and electric charge $-e_0$:

$$[-i\vec{\alpha}.\vec{\nabla} + \beta m_0] \psi(\vec{x}) = E \psi(\vec{x})$$  \hspace{1cm} (22)

$$[-i\vec{\alpha}.\vec{\nabla} + \beta m_0] \psi_n(\vec{x}) = E \psi(\vec{x}) \quad \psi(\vec{x}) \equiv \psi^\dagger(\vec{x})\beta$$  \hspace{1cm} (23)

The equation of motion is the Dirac equation:

$$[-i\vec{\alpha}.\vec{\nabla} + \beta m_0] \Psi(\vec{x}, t) = i\hbar \frac{\partial}{\partial t} \Psi(\vec{x}, t)$$  \hspace{1cm} (24)

The (hermitean) Dirac matrices are given in the Dirac representation

$$\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \quad \gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$  \hspace{1cm} (25)

So, the additional equation of motion for this beable particle that is necessary for completing the list of quantum dynamical equations is postulated by the Bohm school to be [2],[16]:

$$\vec{v}(\{\vec{r}(t)\}, t) = \left\{ \frac{\Psi^\dagger(\vec{x}, t) \times \vec{\alpha} \times \Psi(\vec{x}, t)}{\Psi^\dagger(\vec{x}, t) \times \Psi(\vec{x}, t)} \right\}_{\vec{x}=\vec{r}(t)}$$  \hspace{1cm} (26)

This replaces the guiding condition for unrelativistic particles, but reduces to it in the non-relativistic limit. Not only is this fully consistent with this non-relativistic limit $|\vec{v}| \ll 1$ but also it is fully consistent with fundamental relativity [2,16],[17].

Consider now a new physical situation in which the beable particle finds itself in a quantum medium (i.e., the quantum vacuum) at least partly represented by fields $\{\phi(\vec{x})\}$. Then the basic ansatz (26) can be properly extended to apply to this more realistic situation, as we shall see. In this more realistic case, the modified system of equations (26) becomes a highly non-linear system of coupled ordinary differential equations, just as in the non-relativistic many-body case [1],[2], and as such one should expect the appearance of various kinds of singularities, when solving them [2,20]. Numerical solutions show for example
cases of bifurcation points even in the simplest examples, which in our context could signal a quantum transition. Also the transition to chaotic motions can be illustrated, as the number of active degrees of motion increases even slightly [2].

A somewhat similar feature occurs of course also in classical newtonian dynamics.

All of this plays a major role in the Bohm-Hiley ontological quantum theory of measurements and quantum theory of transitions in general, in which the archaic idea of a “collapse” of the wavefunction upon measurement is meaningless, and thus plays no role at all.

We emphasize that by now we are way beyond what is officially declared “meaningful”. This holds of course if one arbitrarily insists in tacitly accepting the never clearly stated, let alone proved, implicit metaphysics of standard textbook formulations (lucid discussions about this can be found in ref [2]).

The basic reason is the same as in the non-relativistic many-body case, i.e. the fundamental intrinsic quantum non-locality buried in the basic definitions (13) and (26), which cannot be given any definite meaning at all as far as textbooks are concerned.

So it is obvious that the velocities and quantum accelerations of any particle are fundamentally quantum correlated with the velocities and accelerations of all particles in the Universe, regardless of whether or not there are any classically describable potentials - in stark contradiction with that we are accustomed to believe.

Let us emphasize again some of the central ideas as far as “beable particles” are concerned:

(i) the symbol \{\vec{x}_i\}(i = 1, 2, 3...) is reserved for the set of eigenvalues of space coordinates operator in the Schroedinger representation for particle wavefunctions, which one recalls is diagonal in the spatial coordinates;

(ii) therefore, it must NOT be confused with the different symbol \{\vec{r}_i(t)\} for a collection of point-like “particle spatial positions”, which are in general time-dependent and whose ensemble averages (see the following section) have basically the same connotations as those in Classical Physics (unprecisely speaking - “observables”); note that no hidden semi-classical assumptions are involved here at all.

(iii) a warning: in the Bohmean literature one often finds the loose word “trajectory \vec{r}(t)” to mean solutions of the equations (26), given some initial and boundary conditions. This might led unaware particle theoreticians to instinctively associate this idea with their own familiar concept of “Feynman paths”.

However, the BP histories have nothing to do with the Feynman path concept.

With this warning, I shall rather use the more neutral word particle and field “histories”, instead of “trajectories”.

(iv) One must keep in mind that the infinite Dirac sea of occupied negative frequency states \mid_0^{\text{DIRAC}} does participate in principle in every single step of any solution. This is of course also true for textbooks. But fortunately this
isn’t as bad as it sounds, because of the separability, or factorization, of the total wavefunction in an infinity of finite physical linked and unlinked connected clusters [1],[2]. Thus one can verify that only very few of these particles really participate at a time in any specific case, and the rest simply drops out of sight in any meaningful and doable experiments (this occurs obviously also in standard many-body theory and RQFT, as it must of course). The famous Cluster Decomposition Principle in Quantum Field Theory and in Many-Body theory shows how and why this happens [17], [18]. Precisely the same applies to Boson QFT of subsection (2.1).

Summing up, the set of coupled non-linear ordinary differential equations for beable histories( (26) and its generalizations in section 3) together with the similar equations (13) for Bose-Einstein fields, plus all the initial and boundary conditions on the wavefunction, constitutes the complete quantum dynamical framework that replaces the fundamental laws of motion and interpretative schemes of Classical Mechanics and Field Theory, relativistic or not.

Detailed confrontation with the experimental data is the worthwhile task ahead. So, Its full range of validity and/or usefulness is ultimately a matter that only future theoretical and experimental practices can meaningfully decide.

To finish this section, let us consider an elementary illustrative exercise: imagine a single Dirac free particle with mass \( m_0 \) guided by a 4-spinor plane wavefunction.

(i) A particle guided by a positive frequency wavefunction

We begin by defining a standard reference system, say K. Let a particle of species \( n \) and rest mass \( m_{0n} \) move along K’s positive z-axis \( \vec{e}_3 \) guided by the positive energy Dirac plane wave

\[
\vec{p} = p \vec{e}_3 \quad E_n(p) = \sqrt{p^2 + m_{0n}^2} \tag{27}
\]

Let the \( \chi_\lambda(\vec{e}_3) \) be an helicity eigenstate, part of the positive frequency wavefunction \( w_{\mu \nu \lambda}(\vec{p}) \):

\[
\vec{\sigma} \cdot \vec{p} \chi_\lambda(\vec{e}_3) = 2p \lambda \chi_\lambda(\vec{e}_3) \quad \lambda = \pm \frac{1}{2} \tag{28}
\]

\[
w_{\mu \nu \lambda}^{(+)}(\vec{p}) = \sqrt{E_n(p) + m_{0n}} \left( \frac{1}{E_n(p) + m_{0n}} \right) \chi_\lambda(\vec{e}_3) \times \varphi_n \tag{29}
\]

The “internal” flavour-colour-... component of the wavefunction is called collectively \( \varphi_n \) and is supposed to be an irreducible representation of the Lie algebra \( SU_{c3} \otimes SU_{2L} \otimes U_{1Y} \).

Let us calculate the velocity \( \vec{v}(\vec{r}(t)) \) at the position \( \vec{r}(t) \) of a beable particle under guidance of this 4-spinor wavefunction:

\[
\psi^+(\vec{x}, t) \times \vec{a} \times \psi(\vec{x}, t) =
\]
\[ E(p) + m_0 \gamma_0 = \frac{1}{E(p) + m_0} \beta \left( 1 - \frac{\beta \beta}{E(p) + m_0} \right) \chi_\lambda(e^3) \times \varphi_n \] (31)

\[
\psi^\dagger(\vec{x}, t) \psi(\vec{x}, t) =
\]

\[ E(p) + m_0 = \frac{1}{E(p) + m_0} \beta \left( 1 - \frac{\beta \beta}{E(p) + m_0} \right) \chi_\lambda(e^3) \times \varphi_n \] (32)

So the beable particle velocity when it finds itself at the locality \( \vec{r}(t) \) in some inertial reference frame is given by

\[
\vec{v}(\{ \vec{r}(t) \}, t) \cdot \vec{e}_3 = \frac{1}{E(p)} \vec{p} \cdot \vec{e}_3
\] (33)

It moves parallel to the z-axis at the constant velocity \( \vec{v} \), as expected.

(ii) A particle guided by a negative frequency wavefunction

Similarly, consider a beable particle guided by the negative frequency Dirac wavefunction:

\[
\psi^{(-)}(\vec{x}, t) = w_{-n\lambda}(\vec{p}) e^{iE_n(p)t+i\vec{p} \cdot \vec{x}}
\] (34)

\[
w_{-n\lambda}(\vec{x}, t) = \sqrt{\frac{E_n(p) - m_0}{2E_n(p)}} \chi_\lambda(e^3) \times \varphi_n
\] (35)

Its velocity is then

\[
\vec{v}(\vec{r}(t), t) = \frac{\chi_\lambda(e^3) \left( 1 - \frac{\beta \beta}{E_n(p) + m_0} \right) \vec{e}_3 \times \varphi_n}{-\frac{1}{E_n(p) - m_0}}
\] (36)

which gives

\[
\vec{v}(\vec{r}(t), t) \cdot \vec{e}_3 = \frac{-\vec{p} \cdot \vec{e}_3}{E(p)}
\] (37)

and the beable particle moves antiparallel to the z-axis, i.e. with the velocity \(-\vec{v}\), as expected.

In modern standard textbooks, this would be considered to be a rather odd and archaic way of talking, although there is nothing wrong with it; it is about this stage that one shifts to the more practical abstract field theoretic.
(or many-body) descriptions, by introducing the concept of “antiparticles” (or “holes” in the Fermi sea of many-body fermion systems). This is because this is more practical, if one follows the standard way of thinking and practicing. Moreover, it is more convenient to talk in this way when directly dealing with the experimental data.

It is none the less more convenient for us, given the entirely different conceptual basis of the Bohm-Hiley causal interpretation, to keep to (13) and (26) [2].

Finally: given some initial conditions on the wavefunction, what is the probability that a definite particle spatial distribution \( \{ \vec{r}_n(t) \} \) and velocity distribution \( \{ \vec{v}_n(t) \} \), together with a field distribution \( \{ \Phi(\vec{x}, t) \} \) occurs at any time \( t \) in some inertial reference frame? The answer is:

\[
P(\{ \vec{r}_n(t) \}, \{ \Phi(\vec{x}, t) \}; t) = |\Psi(\{ \vec{r}_n(t) \}, \{ \Phi(\vec{x}, t) \}); t)|^2 \quad (38)
\]

\( \{ \vec{r}_n(t) \} \) (\( \{ \Phi(\vec{x}, t) \} \)) are particle (field) histories in this vacuum that satisfy all the relevant initial conditions on the wavefunction [1],[2].

### 2.4 Ensemble averages and connections to both standard RQFT and experimental physics

We emphasize that the BP interprets Quantum Theory as a causal metatheory of beables. Therefore, the countless counterarguments against it that can be found dispersed in the literature of the last 30 years or so are in most cases (to my mind at least) besides the point.

Nevertheless, it is quite legitimate to ask - what has the BP of RQFT to do with the standard textbook accounts and the present day experimental practices?

This was clearly thoroughly answered and explained by Bohm not only in his very first Physical Review papers on this subject in the early fifties but also in all his subsequent published work [2]. Thus only the shortest possible summary is given here.

One begins with the manifold of solutions of the above dynamical equations of motion, satisfying the appropriate boundary and initial conditions on the wavefunction. Let us next consider the desired connection to the textbooks and to the present conditions of experimental research. The suggested procedure is that one should proceed to apply standard stochastic methods and compute statistical averages \( \langle \cdot \rangle_{EA} \) over pure ensembles of such histories (i.e. over all inaccessible data on initial positions and velocities of particles and field configurations).

As an example, let us simplify the case to just one spatial dimension without losing generality:

\[
\langle X(t) \rangle_{EA} = \int dX(t) \Psi^\dagger(X(t), t) X(t) \Psi(X(t), t) \quad (39)
\]

\[
\langle P(t) \rangle_{EA} = \int dX(t) \Psi^\dagger(X(t), t) \left[ -i \frac{\partial \Psi(X(t), t)}{\partial X(t)} \right] \quad (40)
\]
\[
< E(t) >_{EA} = \int dX(t) \Psi^\dagger(X(t), t) \left[ i \frac{\partial \Psi(X(t), t)}{\partial t} \right]
\] (41)

In the general case \( \Psi(t) \) is made of sums over products of Dirac 4-spinors and the Boson fields represented in the wavefunction, that satisfy the given boundary and initial conditions. These expectation values are directly related to standard textbook prescriptions and therefore to the experimental data. Further discussions can be found in [1] and [2].

3 Case Studies

We are now ready to briefly discuss a few illustrative cases that played, and still do play, important roles in the overall development of contemporary Particle Physics.

3.1 Vacuum survival probabilities of a single positronium atom at rest

To problem of understanding and explaining how any atom could be stable (and thus exist at all) ignited the quantum revolution of the XXth century. The reason is of course known to every undergraduate today: it was simply that inevitable and fundamental classical predictions were in blatant contradiction with reality. Based on this fact, one can imagine an \textit{a posteriori} philosophizing as follows:

(i) it was fortunate that by 1913 one already had a pretty good experimental description of the behaviour of the simplest of all existing atoms - the H-atom;

(ii) it was fortunate that in 1913 no one knew about positronium atoms [21].

Let us see how the BP would explain such obvious contradictions among our “observables” and our most fundamental pre-XXth century preconceptions.

Let us imagine a single isolated positronium atom at rest somewhere in the Universe. Any isolated positronium atom, even in its \textit{ground} state, is unstable! This is not the case, however, with the H-atom, simply because of exact conservation laws, plus the still ultimately unexplained fact that a d-quark is heavier than a u-quark.

Let us look in particular to a Positronium (Ps) atom where there are no extra complications due to the strong interaction: we begin with a model Hamiltonian

\[
H_{eff} = U_0 + H_{e^+e^-} + H_{0\text{Maxwell}} + V_{0e^+e^-}
\] (42)

where the first term on the rhs \( U_0 \) is an arbitrary constant, which can be trivially renormalized away.

The use of natural units \( \hbar = 1 = c \) will be temporarily suspended in this subsection.

The next term in definition (42) is the Ps Hamiltonian (standard notation e.g. [20]):
\[ H_{e^+e^-} = -\frac{\hbar^2}{4m_0} \nabla_X^2 - \frac{\hbar^2}{m_0} \nabla_\rho^2 - \frac{\alpha}{\rho} \]  

(43)

\[ m_0 = \text{electron (positron) rest mass} \]  

(44)

\[ \rho = |\vec{x}_{e^-} - \vec{x}_{e^+}| \]  

(45)

The CM of the atom is assigned the arbitrary position

\[ \vec{X} \equiv \frac{\vec{x}_{e^-} + \vec{x}_{e^+}}{2} \]  

(46)

We consider only atomic bound states: introduce

\[ H_{0e^+e^-} \equiv H_{e^+e^-} - \frac{|\vec{p}|^2}{4m_0} \]  

(47)

\[ H_{0e^+e^-}\psi_\nu(\vec{\rho}) = \varepsilon_\nu \psi_\nu(\vec{\rho}) \]  

(48)

\[ \varepsilon_\nu \Rightarrow -|\varepsilon_{nL}| \]  

(49)

The ground state wavefunctions \( n=1 \ L=0 \ (\equiv 1S) \) are degenerate in the stated approximation (i.e. spin independent Hamiltonian).

A good quantum number that can further label these states is charge conjugation \( C \):

\[ C = (-1)^{L+S} = (-1)^S = \pm 1 \]  

(50)

Thus the spin singlet (\( C=+1 \)) and spin triplet (\( C=-1 \)) are both 1S ground states in this approximation, but even small spin-dependent residual forces can lift this degeneracy. This possibility is left out in the following discussions.

The electromagnetic field is represented by the Hamiltonian in momentum space:

\[ H_{0\text{Maxwell}} = c \sum_{\lambda=\pm 1} \int d^3\vec{p} |\vec{p}| a^\dagger(\vec{p}, \lambda) a(\vec{p}, \lambda) \]  

(51)

\[ a(\vec{p}, \lambda)\ket{0} = 0 \quad a^\dagger(\vec{p}, \lambda)\ket{0} \equiv |\vec{p}, \lambda> \]  

(52)

\[ \frac{1}{\sqrt{1 + \delta(\vec{p}_1\lambda_1, \vec{p}_2\lambda_2)}} a^\dagger(\vec{p}_2, \lambda_2) a^\dagger(\vec{p}_1, \lambda_1)\ket{0} \equiv |\vec{p}_2, \lambda_2; \vec{p}_1, \lambda_1> \]  

(53)

and so forth. The effective interaction is defined as

\[ V_{0e^+e^-} = e_0 V_{0e^+e^-}^{(1)} + e_0^2 V_{0e^+e^-}^{(2)} \]  

(54)

and can be put in the form
\[ e_0 V_{0e^+e^-}^{(1)} = \sum_{\lambda = \pm 1} \int \frac{d^3 \vec{k}}{(2\pi)^3 2|\vec{k}|} F_\lambda(\vec{k}, \vec{\rho}) [a(\vec{k}, \lambda) + a^\dagger(\vec{k}, \lambda)] \]  

(55)

\[ F_\lambda(\vec{k}, \vec{\rho}) = \frac{\hbar e_0}{2m_0 c} \sin \left( \frac{\vec{k} \cdot \vec{\rho}}{2} \right) \hat{\mathbf{d}}(\vec{k}, \lambda) \]  

(56)

and

\[ e_0^2 V_{0e^+e^-}^{(2)} = \frac{e_0^2}{2m_0 c^2} \int d^3 \vec{x} : \hat{\mathbf{A}}_T(\vec{x}) \hat{\mathbf{A}}_T^\dagger(\vec{x}) : \]  

(57)

The interaction piece (57) turns out to be spurious in our context, and will henceforth be dropped.

A non-relativistic Hamiltonian is a great formal simplification, but it is of course not an essential assumption for the purposes of this paper. The coupling to the electromagnetic field results from the standard principle of a minimal coupling [18],[19],[21]. Conservation laws then decide which of the two above mentioned degenerate states, if any, is stable against annihilation into at least 2 gammas (C=+1). The same conservation laws show that the other state is also unstable, but decaying predominantly into 3 gammas (C=-1). These so-called open channels may contribute to the S-matrix, of course.

As mentioned, the first step for implementing our program is to find the guiding beable time-dependent Schroedinger wavefunction:

\[ [H_{e^+e^-} + H_{0\text{Maxwell}} + e_0 V_{0e^+e^-}^{(1)}]|\Psi(t)\rangle = i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle \]  

(58)

The Schroedinger time-dependent wavefunctional is defined to be

\[ \Psi(\vec{\rho}, \vec{\mathbf{A}}_T(\vec{x}); t) \equiv \langle \vec{\rho}, \vec{\mathbf{A}}_T(\vec{x}) | \Psi(t) \rangle \]  

(59)

We are interested in the physical vacuum expectation value of this wavefunctional.

Unfortunately, a real understanding of the physical vacuum is quite beyond our present capabilities. One has then to proceed as usual, by adapting similar methods known to be reliable, e.g. many-body ground state methods - and hope that they somehow work also in the present context, at least up to some point! So we assume that

\[ < 0_{\text{DIRAC}}; 0 | 0_{\text{phys}} > \neq 0 \]  

(60)

\[ |0_{\text{phys}} > \leftrightarrow |0_{\text{DIRAC}} > \leftrightarrow 0_{\text{DIRAC}}; 0 > < 0_{\text{DIRAC}}; 0 | 0_{\text{phys}} > + \ldots \]  

(61)

|0 > is taken to represent the ground state of an infinite collection of non-interacting 3-D harmonic oscillators, whereas |0_{\text{DIRAC}} > represents the Dirac fermion vacuum. Explicit expressions for the perturbative wavefunction in the rest system to any order can be found e.g. in ref [21].
\[ \Psi(\vec{\rho}; \vec{A}_T(\vec{x}); t) \equiv \langle \rho; \vec{A}_T(\vec{x}) | \Psi(t) > \]  

\[ |\Psi(t) > = \sum_{N=0}^{\infty} \sum_{\nu} \sum_{\{n\}} C^{(N)}_{\nu\{n\}}(t) \exp[i\varepsilon_b t/\hbar] |\nu > \times \exp[-iE(\{n\} t/\hbar) |\{n\} > \]  

Plugging this definition into the Schroedinger equation of motion (58) one finds a recursion formula for the coefficients \( C^{(N)}_{\nu\{n\}}(t) \) for \( t \geq 0 \):

\[ C^{(0)}_{\nu\{0\}}(t) = C_{\nu\{0\}}(0) \quad N = 0 \]  

and

\[ C^{(N)}_{\nu\{n\}'}(t) = -\frac{i}{\hbar} \sum_{\nu} \sum_{\{n\}} <\nu'; \{n\}'|V^{(1)}_{\nu\{n\}}|\nu; \{n\} > \times \int_0^t dt_1 C^{(N-1)}_{\nu\{n\}'}(t_1) \times \exp[-i(\omega(\nu', E\{n\}') t_1)] \times \exp[i(\omega(\nu, \{n\}) t_1)] \quad N \geq 1 \]  

with the definition

\[ \hbar \omega(\alpha, \{m\}) \equiv -|\varepsilon_\alpha| + E\{m\} \]  

and in general

\[ <\nu'; \{n\}'|V^{(1)}_{0\{\nu\}'}|\nu; \{n\} > = \sum_{\lambda_{\nu}' = \pm 1} \int \frac{d^3k_{\nu}'}{(2\pi)^3}|k_{\nu}'| \]  

\[ \times \langle \{n\}'|a(\vec{k}_{\nu}', \lambda_{\nu}') + a^\dagger(\vec{k}_{\nu}', \lambda_{\nu}')|\{n\} > \]  

with the definition

\[ I_{\nu\nu'}(\vec{k}_{\nu}', \lambda_{\nu}') \equiv \int d^3\vec{\rho} \bar{\psi}_{\nu'}(\vec{\rho}) F_{\nu\nu'}(\vec{k}_{\nu}', \vec{\rho}) \psi_{\nu}(\vec{\rho}) \]  

Let us assume that at \( t = 0 \) the state is \( |I; 0 > \). Let us further assume that there is some probability that at some later time \( t \) it still is \( |I; 0 > \). We would like to find that probability. Then our master formula (65) leads to a prescription:

\[ C^{(0)}_{I\{0\}}(t) = c_I \]  

\[ C^{(1)}_{I\{0\}}(t) \equiv 0 \]
\[
C_{1(0)}^{(2)}(t) = \left(\frac{-i}{\hbar}\right)^2 \sum_{\nu < \nu; \{n\} \neq I, \{0\}} J_{\nu; \{n\}}(t) | \langle \nu; \{n\}|V_{0e+e-}^{(1)}|I; \{0\} \rangle |^2 \times c_I \quad (71)
\]

\[
J_{\nu; \{n\}}(t) = \int_0^t dt_1 \int_0^{t_1} dt_2 \exp\left[-i(\omega(I, \{0\}) - \omega(\nu, \{n\}))(t_2 - t_1)\right] \quad (72)
\]

Thus up to 2nd order the mode population in the initial and final must differ by one vacuum mode.

The vacuum averaged desired Ps wavefunction then becomes

\[
< \Phi(\vec{r}; \vec{x}; t) > \equiv \psi_I(\vec{r}, t) = \{c_I + C_{1(0)}^{(2)}(t) + \ldots\}\psi_I(\vec{r}, 0) \quad t \geq 0 \quad (73)
\]

One can then proceed to the desired result, which is to find the survival probability of some initial state \(\psi_I(\vec{r}, 0)\) of the atom in its rest system in vacuum. The remaining of this calculation, and its final outcome, can be found in ref [21], but that is hardly the point here. We are trying to:

(i) sketch how the BP explains the decay of an isolated Ps atom in its ground state;

(ii) find out precisely what the spacetime dependence is, as we shall need that in subsection (3.3).

Once the solution for the time-dependent wavefunction is found that satisfies the initial conditions, then the next question would be: what happens to an isolated beable “particle” (i.e. a Ps atom) in vacuum in its rest system if guided by this wavefunction (73) as time goes by, starting from some well-defined initial state, say at \(t = 0\)?

More precisely, one is asking to predict the precise time dependence of the internal vector, given definite initial conditions:

\[
\vec{r}(t) = \vec{r}_{e-}(t) - \vec{r}_{e+}(t) \quad (74)
\]

The answer to the above question is given by solving the non-relativistic edition of equation (26) [1],[2] with the definition (73) for the guiding wavefunction when \(t \geq 0\). So the answer is:

\[
\psi_I(\vec{r}, t) = \sum_{N=0}^{\infty} \psi_I^{(N)}(\vec{r}, t) \quad (75)
\]

\[
\vec{v}(\vec{r}(t), t) = \frac{1}{|\psi_I(\vec{r}(t), t)|^2}\{Im[\sum_{p=0}^{\infty} \sum_{m=0}^{p} \psi_I^{(p)}(\vec{r}, t) \nabla \psi_I^{(p-m)}(\vec{r}, t)]\}_{\vec{r} = \vec{r}(t)} \quad (76)
\]

Appropriate initial conditions have to be specified of course [1],[2] before one can find the proper solution and (if possible) confront it with textbook material and/or experimental clues).
One must not forget that conservation of probability (mandatory both in SP and the BP) is fundamental and demands that at any arbitrary time $t$:

$$|\psi_I(\vec{\rho}, t)|^2 = |\psi_I(\vec{\rho}, 0)|^2$$  \hspace{1cm} (77)

The probability that the Ps atom that was in a specified initial state (I) has survived at any time $t>0$ with the constituents in the relative position $\vec{r}(t)$ and rate of change $\vec{v}(\vec{r}(t), t) \ (81)$ is then

$$P_I(\vec{r}(t), t) = |\psi_I(\vec{r}(t), t)|^2$$  \hspace{1cm} (78)

where $\vec{r}(t)$ is a proper solution of eq (76).

Note that we thus obtain a non-relativistic approximation for the rate of change with time of the internal relative position vector of the atomic constituents, as the atom stays at rest somewhere, in agreement with our simplifying assumption about the non-relativistic motions inside the atom.

If one can imagine that the vacuum coupling is switched-off, then one would find that all Ps states would become fully stable \[1],\[2], i.e. the internal relative separation between the constituents would either not change with time for non-degenerate states, or change periodically with time if degenerate, because in this latter case one could always make linear combinations that could build up complex wavefunctions \[1],\[2].

This description is explained by the Bohm school by the balance established between the classical attractive Coulomb force acting between the $e^\pm$ constituent particles and the centrifugal quantum forces associated to the guiding wavefunction, and originating from the quantum active information potential \[1],\[2]. This replaces the classical kinetic energy of the system $e^\pm$ that played the same role in the old semi-classical Bohr atomic model. The basic physics involved in this case was discussed in ref \[1],\[2] and there is hardly any point in repeating that here.

But again: precisely what happens to any sufficiently isolated individual Ps atom, if the coupling to the quantum medium is non-existent? The BP does have a very concrete, precise and in principle checkable proposal, applicable for all times.

Whether this proposal can, or cannot, be somehow be verified in some distant future, perhaps using now unknowable experimental capabilities, is another matter altogether.

The situation is expected to change of course, if the Ps can only exist in the universal (but mostly unknown) quantum medium, being referred to here as the “vacuum”. Then, the rate with which the beable relative position vector $\vec{r}(t)$ changes with time is still given by eq. (76). If all conservation laws are obeyed, especially for energy and momentum, then the pertinent S-matrix elements (asymptotic times) can be recovered e.g. for the relevant 2 and 3 $\gamma$-decays: the original Ps atom thus becomes “metamorphosed” into $\gamma$ radiation. This is just an example of a quantum transition.

These conclusions (except of course for the Ps atom internal velocity and position vectors) formally agree with the textbooks, but the interpretations
there (if given at all) are both totally different [2],[3] and indeed inextricably linked to the Quantum Theory of Measurements. An essential difference is that in the BP this link is not placed at the very theoretical core of the theory, and thus has a completely different (and natural!) role to play (subsection 2.4).

In most practical cases, however, one is only concerned only with asymptotic times, even if in principle the theory can provide detailed information on (usually very complicated) particle histories, for all times, given of course the initial conditions. So far the rule has been that there is full numerical agreement at asymptotic times (i.e. in the S-matrix domain) with standard textbook RQFT results and predictions.

3.2 Neutrino/antineutrino flavour metamorphosis in vacuum

Suppose that in some distant supernova explosion a neutrino is released in some definite flavour state, say $\beta$. Let us further assume that it then moves in free space towards the Earth, which it reaches at local time $T$. If the distance to this supernova site is say $L$, then $L \approx T$ in the relativistic units used in this paper. This is because the neutrino moves in vacuum with almost the speed of light $c=1$.

Let us now play with the following thought: imagine that an intelligent undergraduate would like to have a genuine explanation of what really goes on behind the curtain of phenomena of neutrino flavour oscillations in vacuum (or within ordinary matter). He is now asked to make a short list of questions to which he would expect to obtain real answers:

- What happened to this free $\beta$–neutrino during the time interval $t=0$ and $t=T$ between the supernova explosion that gave birth to it and its arrival to Earth?
- How and why can there be flavour oscillating metamorphosis as the neutrino moves along some trajectory, even if the neutrino first emerged through a weak interaction process, thus in a definite flavour state $\beta$?
- Why, and precisely how, are these remarkable oscillations related to the neutrino masses?
- If a neutrino is a particle with a definite inertial mass, how come we can “see” flavour oscillations, which is typically a wave-like phenomenon?
- And so forth. The answers to these and many other similar questions can in principle at least be both quantitatively and qualitatively deduced and explained by the Bohm formulation of Quantum Field Theory. Whether the “explanation” is “right” or “wrong” is a different matter altogether. Only further predictions and difficult experimentation can decide that. We are discussing here only questions of principle.

The experimental data on neutrinos strongly suggest that their masses are very small compared to all other known leptons and quarks. We are thus dealing with highly relativistic particles.

The known neutrinos propagating in vacuum are supposed to interact very weakly with the perennial virtual heavy electroweak vacuum fluctuating modes.
The primary interaction causing these electroweak spin flips is given by the Standard Model [18]:

\[ l_{\beta\uparrow} = l_{\alpha\downarrow} + W^+ \quad l_{\beta\downarrow} = l_{\alpha\uparrow} + W^- \quad \alpha, \beta = e, \mu, \tau \quad (79) \]

with the definitions

1st lepton family: \( l_{e\uparrow} \equiv \nu_e \equiv \text{electron neutrino}; \quad l_{e\downarrow} \equiv e \equiv \text{electron}; \)

and likewise for the 2nd family (muon) and the 3rd family (tau).

The BP of Quantum Mechanics agrees with the textbook formulations that these “quantum medium” couplings must be the primary ones and the ultimate cause of the observed and famous flavour vacuum fluctuations of neutrinos.

A qualitative and quantitative natural quantum theoretic explanation of what goes on here, according to the Bohm school, would start by writing down the beable neutrino guiding wavefunction in vacuum. As a neutrino of any species \( n \) is a particle, it is natural that its guiding wavefunction should be given by a wavepacket, say:

\[ \Psi_{n\alpha}(\vec{x}, t) = \psi_n(\vec{x}, t) \times \varphi_\alpha \quad (80) \]

where the ansatz for the wavefunction is

\[
\psi(\vec{x}, t) = \sum_{n=1}^{3} \sum_{\nu=+,-} \int d^3\vec{p} C_n^{(\nu)}(\vec{p}, t) \frac{1}{\sqrt{2E_n(p)}} e^{-i(E_n t + i\vec{p}.\vec{x})} \quad t \geq 0 \quad (81)
\]

where

\[
(\vec{\alpha}(n).\vec{p} + \beta(n)m_0(n))w_n^{(\nu)}(\vec{p}, E_n(p)) = \nu E_n(p)w_n^{(\nu)}(\vec{p}, E_n(p)) \quad (82)
\]

\[
E_n(p) \equiv \sqrt{p^2 + m_0^2(n)} \quad (83)
\]

We have also a boundary and initial condition on the wavefunction

\[ \Psi(\vec{x} = \vec{L}, t = 0) = F_\beta(L)\varphi_\beta \quad t = 0 \quad (84) \]

The coefficients \( C_n^{(\nu)}(\vec{p}, t) \) in eq(85) are determined by solving the Dirac equation of motion including the initial condition:

\[
[-i\vec{\alpha}(n).\vec{\nabla} + \beta(n)m_0(n) + \sum_{\beta} V_{0S}(\alpha, \beta, W^\pm)]\Psi_{n\alpha}(\vec{x}, t) = i\frac{\partial}{\partial t}\Psi_{n\alpha}(\vec{x}, t) \quad (85)
\]

We assume that the dependence on the vacuum comes from the primary and the quantum medium (=vacuum, quantum ether,...) induced effective interaction \( V_{0S}(\mu, \nu, W^\pm) \).
This is not only suggested by the Standard Model \[18\] but to some degree also calculable. The primary couplings are between flavour lepton electroweak isodoublets \((\nu_e, e), (\nu_\mu, \mu), (\nu_\tau, \tau)\) and the very heavy \(W^\pm\) vacuum modes (components of an electroweak isovector) and symbolized by the quantum transitions \(79\). We end up with

\[
\Psi_\beta(\vec{x}, t) = \sum_{\alpha = e, \mu, \tau} \Phi_{\beta\alpha}(\vec{x}, t)\varphi_\alpha \quad t \geq 0
\]  

(86)

where the coefficients \(\Phi_{\beta\alpha}(\vec{x}, t)\) are complicated spacetime functions (i.e. space-, spin-, momentum-, energy-dependent) of the neutrino masses to be obtained by solving the specific equation \(85\).

Note that in particular the neutrino spin plays only a very modest role in all this. The paramount degrees of freedom involved here are flavour-mass and flavour flips. We are interested on the time-dependence of the wavefunction \(\Psi_\beta(\vec{x}, t)\).

It can be shown \[22\],\[23\] that the kinematical conditions here relevant are such that they allow one to make the reasonably good estimate

\[
\Phi_{\beta\alpha}(|\vec{x}| \approx L, t \approx L) = \sum_{n=1}^{3} |U^\nu_{\alpha n}|^2 \frac{e^{-i m_n^2 L/2E}}{E} 
\]

(87)

\[
U^{-1}_\nu = U^\dagger_\nu
\]

(88)

where \(E\) is a properly defined neutrino mean energy. The unitary matrix \(U^\nu\) relates by definition the \textit{flavour eigenstates} \(\varphi_\alpha\) to the \textit{mass eigenstates} \(\chi_k\):

\[
\varphi_\alpha = \sum_{k=1}^{3} |U^\nu_{\alpha k}| \chi_k \quad \alpha = \nu_e, \nu_\mu, \nu_\tau
\]

(89)

Most of our vital experimental clues are encoded in this neutrino “mapping-matrix” \(U^\nu\) (and its hadronic equivalent, the quark CKM matrix). The theoretical Job 1 is to decipher this hidden information (and similarly for quarks). After decades of world-wide very hard work, no one succeeded in doing that as yet, at least according to the opinion of the great majority of particle theoreticians.

The local probability for a neutrino, born at \(t=0\) in flavour state \(\beta\), to survive in that flavour state at time \(t>0\), at the spacetime point labelled \(\vec{x} = \vec{r}(t)\) is then according to elementary Quantum Theory as formulated by the Bohm school

\[
P_\beta(\vec{r}(t), t) = |\Psi_\beta(\vec{r}(t), t)|^2 = \sum_{\alpha} P_{\beta \rightarrow \alpha}(\vec{r}(t), t) \quad t \geq 0
\]

(90)

\[
P_{\beta \rightarrow \alpha}(\vec{r}(t), t) = |\Phi_{\beta\alpha}(\vec{r}(t), t)|^2 \quad t \geq 0
\]

(91)

Recall that the history of our neutrino beable \(\vec{r}_\alpha(t)\) with mass \(m_\alpha\) must be a solution of the Bohm equation \(92\) with the 4-spinor guiding wavefunction given by \(81\) that satisfies the appropriate initial condition:
The flavour oscillations known to exist are therefore a direct result of the interferences among sinusoidal terms originating in the guiding wavefunction (86) and that results from result (87) [22]. It can be easily shown that they depend on the mass squared differences of the three neutrino species. If the neutrinos were mass degenerate, then there would be no oscillations, contrary to the experimental results.

I would like to emphasize that from the Bohm-Hiley’s viewpoint one is really discussing here the probability that our neutrino beable is (and not merely “found if measured” in textbook parlance!) at the position $\vec{r}(t)$ and with the velocity $\vec{v}(\vec{r}(t); t)$ when the time is $t \geq 0$, having started at time $t=0$ at a distance $L$ in that channel in some definite mean energy but given flavour $\beta$ with the probability $P_\beta(\vec{r}(0) = \vec{L}, t = 0)$. What is thus interesting has very little to do with spin. On the other hand, let us remind ourselves that bona fide experiments are always carried out with large ensembles of particles; an “observation” done with a single neutrino has no statistical significance at all!

The intervention of any “flavour measuring device” in the present case has - according to textbooks at least - an inevitable consequence, that is, it boils down - in some very unclear way - to an ill-defined “collapse of the wavefunction” into some definite flavour eigenstate [3]. Then, in some vaguely specified way, this gives the Born probability (90) that immediately after the “measurement is over” the neutrino “will be found” in some definite flavour state.

The Bohm school considers all this basically quite unsatisfactory. As suggested above, the answers, suggestions and explanations given by the Bohm school are thus totally different from those we are accustomed to [2],[3], and apply equally well to individual beable particles and to ensembles.

So, there is no “collapse” of any kind, as the wavefunction is a beable just as a beable neutrino. Its guiding wavefunction, that is a solution of the equation of motion (85), actively informs in principle the beable neutrino of all the infinite potentialities of the Universe in which this particular neutrino can exist [2][3]. Any possible so-called “measurement” that “reveals” the neutrino flavour state is basically in Bohm’s view just an example of a quantum transition, ultimately initiated by collisions with virtual very heavy $W^\pm$ electroweak vacuum boson modes [2].

A more detailed and technical account of neutrino vacuum flavour oscillations along these lines will be given elsewhere [24].

3.3 Uniformly accelerated particle motions in vacuum

According to the standard point of view of modern Particle Physics we are living in a Minkowski universe. “We” means here some inertial frame, call it “the LAB (inertial) reference frame”. Imagine that the LAB frame is watching an uniformly accelerated particle, say a Ps atom. Consider then co-moving inertial
reference frames, which are here named “BODY frames”, moving along with the particle [25]. Thus at any definite proper time $\tau$ we can apply in the BODY frame at that time the theory sketched in subsection (3.1).

Let us now discuss this from the Bohm-Hiley version of RQFT, as interpreted in the present paper (incidentally, a most lucid and pedagogical discussion of the physics of Lorentz boosts was given by the same authors [26]).

The questions will be translated in a more technical language: how would the world look like from the point of view of any such BODY frame? If the atom is not under accelerations, then the answer is given in subsection 3.1. If the atom is in an uniformly accelerated motion, then the answer can be easily found as shown by J.Donnoghue and B.Holstein [27], by simply transforming the metric tensor from a LAB Minkowski frame to any co-moving BODY frame using a coordinate system appropriate to a Rindler universe [25].

The object that has our interest in this connection is the time-integrated Minkowski correlation function closely related to the definitions (71) and (72):

$$
\int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 G_{Mij}(\vec{x}_1, t_1; \vec{x}_2, t_2) \equiv \\
\frac{1}{2!} \int_{t_0}^{t} dt_2 \int_{t_0}^{t} dt_1 \{ < 0_M | \Theta(t_1 - t_2) (A_{Ti}(\vec{x}_1, t_1) A_{Tj}(\vec{x}_2, t_2)) | 0_M > + \\
+ < 0_M | \Theta(t_2 - t_1) (A_{Tj}(\vec{x}_2, t_2) A_{Ti}(\vec{x}_1, t_1)) | 0_M > \} 
$$

(93)

where $\Theta$ is the usual step function (assuming that $\eta \rightarrow 0^+$):

$$
\Theta(\tau) \equiv \frac{-1}{2\pi i} \int_{-\infty}^{+\infty} \frac{d\omega}{\omega + i\eta} e^{-i\omega \tau} 
$$

(94)

$|0_M>$ is supposed to mean “the Minkowski vacuum in any inertial reference frame”. So, by-passing irrelevant complications due to sum over polarization vectors, one finds that [27]

$$
\int_{0}^{t} dt_2 \int_{0}^{t_2} dt_1 G_{Mij}(\vec{x}_1 = 0, t_1; \vec{x}_2 = 0, t_2) = \\
= 2(\hbar c^3) \int \frac{d^3 \vec{k}}{(2\pi)^32|\vec{k}|} exp[-i|\vec{k}|c(t_2 - t_1)] 
$$

(95)

We leave this integral in this form because we are concerned only with its spacetime-dependence.

Then as proved in ref [27] much of the discussion on subsection (3.1) can be directly adapted to a Rindler metric in any co-moving BODY frame, as follows. Let us make a change from the Minkowski metric to the Rindler metric, corresponding to a boost along the z-axis producing an uniform acceleration $a$ with which the atom’s center of mass moves from the point of view of the LAB system:
\[ x = y = 0 \quad z = \frac{c^2}{a} \left( \cosh \frac{a \tau}{c} - 1 \right) \quad t = \frac{c}{a} \sinh \frac{a \tau}{c} \quad (96) \]

\[ c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 \quad (97) \]

\(\tau\) is the proper time registered by a co-moving observer. As it is shown in [27] by simply making this transformation one finds that the result (95) now becomes for a Rindler universe:

\[
\int_0^{t_2} dt_2 \int_0^{t_1} dt_1 G_{Rij}(\vec{x}_1 = 0, t_1; \vec{x}_2 = 0, t_2) = \]

\[
= 2(hc^2) \int \frac{d^3 \vec{k}}{(2\pi)^3 |\vec{k}|} \exp\{-i|\vec{k}|c\tau\} \times \left[ 1 + \frac{1}{\exp(2\pi|\vec{k}|c/a) - 1} \right] + \\
+ 2(hc^2) \int \frac{d^3 \vec{k}}{(2\pi)^3 |\vec{k}|} \frac{\exp\{i|\vec{k}|c\tau\}}{\exp(2\pi|\vec{k}|c/a) - 1} \quad (98) \]

This remarkable result reduces of course to the Minkowski result (95) if the acceleration is set to zero.

As shown in [27] this is exactly the same result that co-moving inertial observers would find, if the atom actually moved in a thermal bath of temperature

\[ T = \frac{a}{2\pi} \quad (99) \]

Summing up: from the point of view of a Minkowski LAB inertial system the vacuum is a quantum medium that to a reasonable approximation looks like an infinite sea of non-interacting quantum harmonic oscillators. However, for a co-moving BODY reference system (i.e. within a Rindler universe) the vacuum would look like a heat bath with the Unruh temperature (99)[27].

4 Summary and conclusions

I have attempted to sketch and apply in this paper a personal interpretation of the original Bohm project, in a manner that might be understandable to dedicated members of the large community of Particle physicists who may not already be familiar with it. This is simply because this particular formulation seems to me to be the most powerful, natural and credible one among its many concurrents.

After a short summary of some of the relevant background material (Section 2) the attempt is made to explain - i.e. in the Bohmean sense of this word (Section 1) - three randomly chosen, but particularly instructive, case studies (Section 3) borrowed from published papers and textbooks on Particle Physics.

Bohm’s opinion (see the quotation in Section 1) was apparently that Quantum Theory in its present official (most would perhaps add - final?) formulation,
dating back to the thirties, *explains* nothing. Nevertheless, it does give an excellent, correct and precise qualitative and quantitative *description* of what any experimenter will see, or not see, in any of his measuring apparatus, once the correct specifications are satisfied.

This is usually taken to be all there is to it.

I have argued throughout this paper for an alternative point of view due to Bohm. One ought at present to play with both conceptions: the “romantic” textbook epistemological/pragmatical version and by now the many alternative “realistic”, or “business-like” ontological versions [3].

The particular Bohm ontological version is (according to this author’s taste at least) the most satisfying one among by now countless alternative ontological versions.

Those who are not entirely happy with the standard formulations of Quantum Theory, nor e.g. with the Bohm version, will have to compromise, until the Bohm causal formulation, or perhaps some alternative one, become mature enough to stand on its own feet and ready to face head on the inevitable experimental challenges of the coming decades.

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