The Prediction of the Dynamic Behaviour of a Structure Using Model Updating and Frequency Based Substructuring

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Abstract. Combining analytical with experimental data to predict the dynamic behaviour of an assembled structure via the frequency based substructuring (FBS) method, is a common practice in the field of structural dynamics. However, the accuracy of the dynamic behaviour predicted from the FBS method relies heavily on the quality of experimental frequency response function (FRF) of the interfaces on which, in practice, it is very difficult to obtain. In addition, the accuracy of the FBS method is highly dependent on experimental rotational degrees of freedom (DOF) which are always found to be very difficult to measure accurately. Therefore, this paper proposes a new frequency response function (FRF) coupling scheme that may uniquely address the difficulties and improve the quality of predicted results of the FBS method. The scheme is formulated based on the finite element method, model updating technique and experimental modal analysis. A simplified finite element model of a physical test substructure is developed to generate rotational FRF data by reconciling the initial FE model using the model updating technique. It was found that the scheme adopted allows generating full translational and rotational degrees of freedom data and leads to a significant improvement in the FRF coupling process between the analytical and experimental model in the FBS method.

Keywords: Structural Dynamics, Frequency Based Substructuring, Model Updating, Experimental Modal Analysis

1. Introduction

Dynamists tend to combine analytical model with experimental model than combining models derived from analytical work for the investigation of the dynamic behaviour of engineering structures. Usually, analytical models are developed from the finite element method. However, the developed FE models are often found to be not in good agreement with experimental models due to the invalid assumptions of the model properties used in the FE models [1]. The model updating method is perceived to be the most efficient method to improve the accuracy of the FE models [2-3]. Adopting the updating method, the updated FE models has better representation of the dynamic behaviour of the physical test structures.

The dynamic behaviour of a complex built-up structure which consists of several substructures usually tends to be computationally expensive and highly problematic to be determined analytically [4-5]. Therefore, dynamic substructuring methods, which allow the dynamic behaviour of the complex structure being investigated economically and efficiently, are seen to have the edge over other methods.
One of the popular dynamic substructuring methods is the frequency based substructring (FBS) method. This method has outstanding versatility in assembling the frequency response function (FRF) of substructures which can be analytically derived or experimentally measured [6].

The review, history and general framework of FBS are available in De Klerk [7]. However, the method has been reported to suffer from several issues, which mainly due to the rotational FRF of the experimental model [8-10]. This is because the rotational FRFs are very difficult to be measured and neglecting them during the process of FRF coupling might greatly affect the accuracy of the coupled FRF [11-12]. Mathematically, considered a single node system is described in matrix as follows:

\[
\begin{bmatrix}
\begin{bmatrix}
Y_{xx} & Y_{xy} & Y_{xz} & Y_{x\phi_x} & Y_{x\phi_y} & Y_{x\phi_z} \\
Y_{yx} & Y_{yy} & Y_{yz} & Y_{y\phi_x} & Y_{y\phi_y} & Y_{y\phi_z} \\
Y_{zx} & Y_{zy} & Y_{zz} & Y_{z\phi_x} & Y_{z\phi_y} & Y_{z\phi_z} \\
Y_{x\phi_x} & Y_{x\phi_y} & Y_{x\phi_z} & F_x \\
Y_{y\phi_x} & Y_{y\phi_y} & Y_{y\phi_z} & F_y \\
Y_{z\phi_x} & Y_{z\phi_y} & Y_{z\phi_z} & F_z \\
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
\phi_x \\
\phi_y \\
\phi_z \\
\end{bmatrix}
\]

(1)

where \([x, y, z]\) and \([\phi_x, \phi_y, \phi_z]\) represent the translational and rotational DOFs respectively. The abbreviation of Y, F, and M describe the FRF matrix, force-excite and moment-excite respectively. The matrix as presented in the above equation clearly shows that 75% of the single node matrix is built based on rotational information. Therefore, neglecting the rotational DOFs at the interface during the process of coupling will definitely lead to a reduction in the interface stiffness up to 75%. In other words, the neglect of the DOFs may result in a dramatic reduction in the accuracy of the prediction of FRFs.

Previous studies reveal that including the rotational DOFs in the process of coupling has led to successful and accurate coupled FRF between FE-experimental dynamic substructuring [9,13-14]. Recently, dynamicists tend to implement several modal expansion methods to derive the rotational FRF based on the measured translational FRF. However, the expanded rotational FRF relies heavily on the quality of the measured translational FRF that might contaminated with noise. Therefore this paper aims to propose a new coupling scheme that may uniquely address the difficulties and improve the quality of predicted results of the FBS method. The scheme is formulated based on the finite element method, model updating technique and experimental modal analysis.

In this work, the high dependency on the experimental data on which the FBS method relies largely, can be significantly reduced by using a simplified finite element (SFE) model which is correlated with the translational test data to obtain rotational information and a high quality derived FRF.

2. Description of the Jointed Structure

In this study, the applicability, capability, and the accuracy of the proposed scheme for coupling the finite element and test FRFs via the FBS method were demonstrated using a jointed structure consisting of a simple beam and an irregular plate structure as shown in Figure 1. The jointed structure was 200 mm in wide and 500mm in length. Both substructures were assembled by using two bolted joints. The dynamic behaviour investigation is focused on the frequency bandwidth between 0 to 2000 Hz which contains ten modes of interest. It was found that there were several bending and torsional modes at the connection interfaces.
Prior to performing the FBS analysis, the FRF of the physical jointed structure was measured for a benchmarking purpose using the experimental modal analysis by referring to the previous work presented in [6,13,15]. The structure was tested under free-free boundary conditions by using impact testing before disassembling it into two substructures, which are the beam (finite element model) and the irregular plate (experimental model). Three uniaxial accelerometers with 10mV/g sensitivity was used to acquire the response and the measured data was analysed by (Leuven Measurement System) LMS Test Lab.

The structure was suspended with soft springs to simulate free-free boundary conditions and was excited ten times to obtain an acceptable level of accuracy for the measured data. The finite element model of the beam was developed by using 3D elements. The dynamic behaviour of the FE model was calculated by using normal modes analysis.

3. The Development of the Simplified Finite Element (SFE) Model

Previous works [9-10] revealed that there was a possibility of deriving directly moment-excited FRFs from experimental models. However, in most cases, dynamicists tend to perform the modal expansion method to derive the moment-excited FRFs to provide a full FRF matrix which usually leads to successful FRF coupling [13-15].

In this work, a new coupling scheme was developed using the model updating method in the light of experimental FRF in which the model updating method was used to improve the finite element model of the physical irregular model. The updated FE model was then used to extract and derive the incomplete rotational FRF data required. It is worth noting that constructing an accurate finite element model of large structures such as a car body in white and helicopter, which usually consist of several complex components, is very difficult, challenging and time consuming. Therefore, instead of using the actual experimental model (the irregular plate structure), a simplified finite element (SFE) model is introduced in the work.

The SFE model of the test structure was developed by constructing a 2D finite element model with a much simpler structure without any curves or slopes. Figure 2 shows the developed SFE model for the irregular shape structure. The developed SFE model has multiple parameters that can be reconciled by referring to the experimentally obtained natural frequencies. By adopting the SFE, the derived FRF from the updated SFE model can be matched as close as possible to the actual experimental FRF. The updated SFE model was then used to derive a new FRF matrix which contained both translational and rotational FRF.
4. The Finite Element Model Updating for SFE Model

In structural dynamics, model updating methods are used to improve the accuracy of finite element models by improving systematically and iteratively the design parameters of the models to achieve a better correlation with the experimentally measured dynamic behavior [16-17]. In this work, the finite element model updating method was adopted to improve the initial SFE model by updating the material properties and thickness of the model to match that of the physical irregular plate. Updating the parameters was performed by using MSC NASTRAN Solution 200 with the same objective function used in [17-18].

Table 1 presents the natural frequencies obtained from three different sources which are the experimental modal analysis (EMA), initial and updated SFE models. The total errors recorded were calculated from the direct comparisons between the natural frequencies of EMA, the initial FE and the updated FE. From the table, it can be observed that the total error recorded in the initial FE was successfully reduced from 77.4% to 2.95%. Therefore, it is imperative to note that the SFE model is reliable to be used in predicting the dynamic behaviour of the physical irregular shape.

Table 1. Comparison of the natural frequencies of the measured (EMA), initial and updated SFE models.

| Mode | EMA FE Hz | Initial | Diff. (%) | FE Hz | Updated | Diff. (%) |
|------|-----------|---------|-----------|-------|---------|-----------|
| 1    | 352.19    | 368.09  | 4.51      | 351.11| 0.31    |           |
| 2    | 474.80    | 553.38  | 16.55     | 474.53| 0.06    |           |
| 3    | 876.62    | 973.25  | 11.02     | 884.01| 0.84    |           |
| 4    | 1038.00   | 1131.70 | 9.03      | 1035.40| 0.24   |           |
| 5    | 1128.00   | 1229.60 | 9.01      | 1128.40| 0.04   |           |
| 6    | 1886.70   | 2006.20 | 6.33      | 1886.60| 0.00   |           |
| 7    | 1935.20   | 2026.10 | 4.70      | 1931.40| 0.20   |           |
| 8    | 2160.50   | 2272.80 | 5.20      | 2173.60| 0.61   |           |
| 9    | 2258.40   | 2508.00 | 11.05     | 2243.70| 0.65   |           |
|      |           |         |           |       |         | 77.4      |
|      |           |         |           |       |         | 2.95      |
5. Frequency Based Substructuring using Updated SFE Model
To evaluate the capability and accuracy of the proposed coupling scheme, the FRFs derived from the finite element model of the beam and the updated SFE model of the irregular plate were coupled as shown in Figure 3. The FRFs from both finite element models were derived by using the FRF synthesis method based on the calculated natural frequencies and mode shapes. Later, the FBS method was employed to calculate the FRF of the complete jointed structure by using rigid type of connections.

![Fig. 3. The assembly of the FE model of the beam and SFE model of the irregular plate](image)

The coupled FRF calculated from the FBS method was validated and compared with the measured FRF. Figure 4 shows the comparison of both FRFs. From the figure, there are eight visible resonance peaks and seven anti-resonances of the experimental and coupled FRF within the frequency bandwidth between 0-2000 Hz. The results obtained from the comparison indicates that the proposed FRF coupling scheme has been successfully used to calculate the FRF of the jointed structure.

![Figure 4. The comparison of coupled and experimental FRF of the assembled structure](image)

It was also found that for the first four modes, the coupled FRF matches well with the measured one with having accurate patents in the resonance frequencies as well as anti-resonance. This achievement suggests that using the proposed coupling scheme can lead to a successful prediction of the coupled FRF for a low frequency bandwidth. A good correlation obtained between the predicted and measured FRF indicates that introducing the derived rotational FRF from the SFE model during the process of coupling has enhanced the flexibility of the interfaces of the model. From the engineering point of view, including the 75% of the rotational information has improved the rigidity of the interface, which has led to a successful FRF coupling process. This achievement is also in line with that of presented in the previous work [9,13,19]. Therefore, it is worth noting that the proposed FRF coupling scheme offers a great capability to accurately predict the dynamic behaviour of the jointed structure via the FBS method.
Another striking point of the comparison is that the coupled FRF has started to differ from the experimental FRF for higher modes. The discrepancies occurred were suspected from the invalid selection of coupling types which may be unable to accurately represent the physical properties of the two bolts. This is a normal phenomenon encountered in the field of structural dynamics [4-5] because the joints are very difficult to be modelled in detail [2].

6. Conclusions
The dynamic behavior of the jointed structure using the FBS method has been investigated. The new coupling scheme is proposed to effectively and efficiently couple the two substructures. The scheme is performed by implementing the model updating method for the SFE model in the light of the experimental data. The evaluation of the capability and accuracy of the proposed scheme has been presented and discussed. The results in the previous section demonstrated that the proposed scheme has been successfully used to calculate the FRF of the assembled structure. Therefore, it is worth noting that the updated SFE model has a great capability to be used for the FBS analysis. This remarkable achievement suggests that the proposed scheme is more efficient and effective over the conventional modal expansion method in order to obtain the full FRF matrix of the connection interfaces.

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