Gluonic Pole Matrix Elements and Universality

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Abstract. We investigate the spectral properties of quark-quark-gluon correlators and use this to study gluonic pole matrix elements. Such matrix elements appear in principle both for distribution functions such as the Sivers function and fragmentation functions such as the Collins function. We find that for a large class of spectator models, the contribution of the gluonic pole matrix element for fragmentation functions vanishes. This result is important in the study of universality for fragmentation functions.

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INTRODUCTION

In high-energy scattering processes the structure of hadrons is accounted for using quark and gluon correlators; forward matrix elements of non-local quark and gluon operators between hadronic states. Making an expansion in the (inverse) hard scale, the leading dynamical effects come from two-field configurations at two light-like separated points, which are easily interpreted as parton densities or parton decay functions [1, 2]. These are the parton distribution functions depending on the momentum fraction \( x \) relating the parton momentum \( k = xP \) to the hadron momentum \( P \) or the fragmentation functions of partons into hadrons depending on the momentum fraction \( z \), relating the parton momentum \( k \) and the hadron momentum \( P = zk \). At sub-leading order in the hard scale or when explicitly measuring transverse momenta, other matrix elements become important such as the three-parton correlators containing parton fields at three different space-time points with light-like separations and two-parton correlators with also transverse separation (light-front correlations). These latter (light-front) correlators are described in terms of transverse momentum dependent (TMD) distribution and fragmentation functions, which are sensitive to the intrinsic transverse momenta of partons in hadrons, \( k = xP + k_T \) in a frame in which the hadron does not have transverse momentum \( (P_T = 0) \) or for fragmentation \( k = \frac{1}{z}P + k_T \). In this case one often refers to the hadron transverse momentum \( P_\perp = -zk_T \) (in a frame in which the parton does not have a transverse momentum \( (k_\perp = 0) \)).

Here, we investigate multi-parton correlators with one additional gluon in which the zero-momentum limit will be studied [3,4]. These are so-called gluonic pole matrix elements or Qiu-Sterman matrix elements, that have opposite time-reversal (T) behavior as compared to the matrix elements without the gluon. Such matrix elements involving time-reversal odd (T-odd) operator combinations are of interest because they are
essential for understanding single spin asymmetries in high energy scattering processes e.g. semi-inclusive deep inelastic scattering (SIDIS) and Drell-Yan scattering. In the collinear case T-symmetry can be used as a constraint on the parton correlators, limiting the distribution functions (DFs) to T-even ones. This constraint does not apply for the fragmentation correlator because the final state hadron is part of a jet and as such is not a plane wave, allowing both T-even and T-odd fragmentation functions (FFs). Including transverse momentum dependence, both the distribution and fragmentation correlators (Φ and Δ) are no longer constrained by T-symmetry. The reason is that the appropriate color gauge invariant operators in the correlator, are not T-invariant. The T-odd operator structure can be traced back to the color gauge link that necessarily appears in correlators to render them color gauge-invariant. But the operator structure of the correlator is also a consequence of the necessary resummation of all contributions that arise from collinear gluon polarizations, i.e. those along the hadron momentum. How this resummation takes effect is a matter of calculation \[5\]. The result is a process dependence in the path in the gauge link. After azimuthal weighting of cross sections one simply finds that the T-odd features originating from the gauge link lead to specific factors with which the T-odd functions appear in observables. For DFs this provides a mechanism leading to T-odd functions, such as the Sivers function \[6\]. Comparing T-odd effects in DFs in semi-inclusive deep inelastic scattering (SIDIS) and the Drell-Yan process one finds a relative minus sign \[7, 8\]. Similarly, comparing T-odd effects in FFs in SIDIS and electron-positron annihilation one also finds a relative minus sign, at least for the T-odd effect originating from the operator structure (gauge link) \[9\]. But, for FFs there are now in principle two mechanisms leading to T-odd functions \[9\]. However, the two mechanisms leading to T-odd functions can be distinguished. The effect coming from the hadron-jet final state not being a plane wave will not lead to process dependent factors.

In order to understand the basic features of these matrix elements we perform a spectral analysis by modeling the distribution and fragmentation functions under reasonable assumptions \[10\]. In particular we consider the differences between distribution and fragmentation functions using a spectral analysis while restricting the momentum dependence and asymptotic behavior of the vertices. In this context, the relevant gluonic pole matrix elements that we study \[10\] are Φ_G(k, k_1) and Δ_G(k, k_1) shown in Figs. 1 and 2. Of these matrix elements only the dependence on the collinear components x and x_1 in the expansion of the momenta are needed (note, the gluon momentum is parameterized as k_1 = [k_1^+, x_1, k_{1T}] in these figures). We find that while both Φ_G(x, x - x_1) and Δ_G(x, x - x_1) are nonzero, taking the limit x_1 → x, Φ_G(x, x) remains non-zero, while Δ_G(x, x) vanishes. The vanishing of the T-odd gluonic pole matrix elements is important in the study of universality of TMDPDFs and TMDFFs.

**QUARK-GLUON CORRELATORS AND GLUONIC POLES**

The quark-quark correlator depending on the collinear and transverse components of the quark momentum, k = xP + σn + k_T, where the Sudakov vector n is an arbitrary light-like four-vector n^2 = 0 that has non-zero overlap P·n with the hadron’s momentum P
\begin{align}
\Phi_{ij}(x,k_T) &= \int \frac{d(\xi \cdot P)}{(2\pi)^3} e^{ik\cdot\xi} \langle P | \Psi_j(0) \mathcal{U}_{\xi}(\xi) | P \rangle \langle P | \mathcal{U}_{\xi}(\xi) | \Psi_i(\xi) | P \rangle \bigg|_{\text{LF}}, \\
\Phi^{[\mathcal{W}]}(x,k_T) &= \int \frac{d(\xi \cdot P)}{(2\pi)^3} e^{ik\cdot\xi} \langle P | \Psi_j(0) \mathcal{U}(\xi) | P \rangle \langle P | \mathcal{U}(\xi) | \Psi_i(\xi) | P \rangle \bigg|_{\text{LC}}.
\end{align}

where LF (\xi \cdot n = 0) designates the light-front. The gauge link is the path-ordered exponential, \mathcal{U}(\eta,\xi) = \mathcal{P} \exp \left[ -ig \int_C ds A^a(s) t^a \right] along the integration path C with endpoints at \eta and \xi. Its presence in the hadronic matrix element is required by gauge-invariance. In the correlator the integration path C in the gauge link designates process-dependence. This is due to the observation that the operator structure of the correlator is also a consequence of the necessary resummation of all contributions that arise from collinear gluon polarizations, i.e. those along the hadron momentum. Collinear quark distribution functions are obtained from the TMD correlator after integration over \( k_T \),

\[ \Phi(x) = \int d^2 k_T \Phi^{[\mathcal{W}]}(x,k_T) = \int \frac{d(\xi \cdot P)}{2\pi} e^{ix \cdot P} \langle P | \Psi_j(0) \mathcal{U}^n(\xi) | \Psi_i(\xi) | P \rangle \bigg|_{\text{LC}}. \]

The non-locality is restricted to the light-cone (LC: \( \xi \cdot n = \xi_T = 0 \)) and the gauge link is unique, being the straight-line path along \( n \). In azimuthal asymmetries one needs the transverse moments contained in the correlator

\[ \Phi^{[\mathcal{W}]}(x,k_T) = \int d^2 k_T k_T^\alpha \Phi^{[\mathcal{W}]}(x,k_T). \]

The TMD correlator, expanded in distribution functions depending on \( x \) and \( k_T^2 \) contains \( T \)-even and \( T \)-odd functions, since the correlator is not \( T \)-invariant. This property is attributed to the gauge link, that depending on the process, accounts for specific initial and/or final state interactions depending on the color flow in the process. For the collinear case, the link structure becomes unique in the case of integration over \( k_T \) (Eq. [2]).

For the collinear weighted case, the transverse moments in Eq. (3) one retains a nontrivial link-dependence that prohibits the use of \( T \)-invariance as a constraint. It is possible however, to decompose the weighted quark correlators as

\[ \Phi^{[\mathcal{W}]}_\alpha(x,k_T) = \Phi^{[\mathcal{W}]}_\alpha(x) + \mathcal{C}_G^{[\mathcal{W}]} \pi \Phi^{[\mathcal{W}]}_G(x), \]

with calculable process-dependent gluonic pole factors \( \mathcal{C}_G^{[\mathcal{W}]} \) and process (link) independent correlators \( \Phi^{[\mathcal{W}]}_\alpha \) and \( \Phi^{[\mathcal{W}]}_G \). The correlator \( \Phi^{[\mathcal{W}]}_\alpha \) contains the \( T \)-even operator combination, while \( \Phi^{[\mathcal{W}]}_G \) contains the \( T \)-odd operator combination. The latter operator is precisely the soft...
limit \(x_1 \to 0\) of a quark-gluon correlator \(\Phi_G(x,x_1)\) of the type

\[
\Phi_G^c(x,x-x_1) = n_\mu \int \frac{d(\xi \cdot P)}{2\pi} \frac{d(\eta \cdot P)}{2\pi} \, e^{i x_1 (\eta \cdot P)} e^{i (x-x_1)(\xi \cdot P)} \\
\times \langle P| \bar{\psi}(0) U^n_{[0,\eta]} gG^{\mu\alpha}(\eta) U^n_{\eta,\xi} \psi(\xi) |P\rangle \bigg|_{LC} .
\] (5)

The universal T-odd distribution functions in the parameterization of \(\Phi_G(x,x)\) appear in T-odd observables such as single spin asymmetries with the specific gluonic pole factors from Eq.4.

The situation for fragmentation functions is different. The TMD fragmentation correlator depending on the collinear and transverse components of the quark momentum, \(k = \frac{1}{2} P + k_T + \sigma n\), is given by [9,5]

\[
\Delta_{ij}^{[\gamma]}(z,k_T) = \sum_x \int \frac{d(\xi \cdot P)}{(2\pi)^3} \frac{d^2 \xi}{2\pi} \, e^{i k \cdot x} \langle 0| \bar{\psi}_i(0,\xi) \psi_j(0)|P(x)\rangle \bigg|_{LF} .
\] (6)

The collinear, \(k_T\)-integrated correlator \(\Delta(z) = \int d^2 k_T \Delta^{[\gamma]}(z,k_T)\) only contains a T-even operator combination. Nevertheless one could in principle have T-even and T-odd fragmentation functions depending on \(z\) since the hadronic state \(|P(x)\rangle\) is an out-state, which is not T-invariant. In the transverse moments obtained after \(k_T\)-weighting,

\[
\Delta^\alpha_{ij}^{[\gamma]}(z) = \int d^2 k_T \, k^\alpha \Delta^{[\gamma]}(z,k_T) = \tilde{\Delta}_G^\alpha \left( \frac{1}{z} \right) + C^\alpha_{ij} \pi \Delta_G^\alpha \left( \frac{1}{z}, \frac{1}{z} \right) ,
\] (7)

the two link independent correlators \(\tilde{\Delta}_G^\alpha\) and \(\Delta_G^\alpha\) contain again a T-even and T-odd operator combination, respectively. The gluonic pole correlator is again the soft limit, \(z^{-1} = x_1 \to 0\), of the quark-gluon correlator

\[
\Delta_{Gij}^\alpha(x,x-x_1) = \sum_x \int \frac{d(\xi \cdot P)}{2\pi} \frac{d(\eta \cdot P)}{2\pi} \, e^{i x_1 (\eta \cdot P)} e^{i (x-x_1)(\xi \cdot P)} \\
\times \langle 0| \bar{\psi}_i^{n,0}(\eta) gG^{\alpha\mu}(\eta) \bar{\psi}_j^{n,0}(\xi) |P(x)\rangle \bigg|_{LC} .
\] (8)

As stated above, because of the appearance of hadronic states \(|P(x)\rangle\), each of correlators in Eq.[7] contains in principle T-even and T-odd functions. Rather than having a doubling of T-odd functions, we will show in a spectator model approach that \(\Delta_G(x,x) = 0\), which implies that T-odd fragmentation functions in the transverse moments only come from \(\tilde{\Delta}_G\), which appear with a universal strength (no gluonic pole factors). We demonstrate this starting with the collinear quark-gluon correlators, Eqs.[5] and[8] rather than the model approaches [11,12,13,14,15] that looked at the transverse momentum dependent quark correlators in Eqs.[1] and[6].

The T-odd operator parts are precisely the soft limits \((k_1 \to 0 \ \text{or} \ x_1 \ \text{and} \ z^{-1} = x_1 \to 0)\) of the gluonic pole matrix elements [9] Eqs.[4] and[7] (see Figs.[1] and[2]). As mentioned above, they arise in the decomposition of the transverse weighted quark correlators
which are the relevant operators in analyzing the azimuthal asymmetries. The process-dependent gluonic pole factors \( C_G \) are calculable and the process (link) independent correlators \( \tilde{\Phi}_G \) and \( \tilde{\Delta}_G \) contains the T-even operator combination, while \( \Phi_G \) and \( \Delta_G \) contain the T-odd operator combination. The latter one is precisely the soft limit, of the quark-gluon correlator \( \Delta_{ij}(x,x_1) \), Eq. 8.

To see this in a spectator model approach, we consider the distribution or fragmentation correlators with a spectator with mass \( M_s \). The result for the cut, but untruncated, diagrams, such as in Figs. 1 and 2 (without the gluon insertion) are of the form

\[
\Phi(x,k_T) \sim \int d(k \cdot P) \frac{F(k^2,k \cdot P)}{(k^2 - m^2 + i\varepsilon)^2} \delta ((k - P)^2 - M_s^2),
\]

where \( F(k^2,k \cdot P) \) contains the numerators of propagators and/or traces of them in the presence of Dirac Gamma matrices, as well as the vertex form factors (see for example [16]). In the above the delta function constraint in Eq. 9 has been implemented. One finds that the numerator \( F(k^2,k \cdot P) = F(x,k_T^2) \) and hence

\[
\Phi(x,k_T) \sim \frac{(1-x)^2 F(x,k_T)}{(\mu^2(x) - k_T^2)^2}, \tag{9}
\]

with \( \mu^2(x) = xM_s^2 + (1-x)m^2 - x(1-x)M^2 \). Note that \( k_T^2 = -k_T^2 \leq 0 \). The details of the numerator function depend on the details of the model, including the vertices, polarization sums, etc. These must be chosen in such a way as to not produce unphysical effects, such as a decaying proton if \( M \geq m + M_s \), thus \( m \) in Eq. 9 must represent some constituent mass in the quark propagator, rather than the bare mass. The useful feature of the result in Eq. 9 is its ability to produce reasonable valence and even sea quark distributions using the freedom in the model. The results for the fragmentation function in the spectator model is identical upon the substitution of \( x = 1/z \) [16].

We turn to the same spectral analysis of the gluonic pole correlator using the picture given in Figs. 1 and 2 for distribution and fragmentation functions respectively. Again, we only need to investigate one of the cases. Parameterizing the gluon momentum as \( k_1 = [k_1^-,x_1,k_{1T}] \), \( k_1^- = k_1 \cdot P - \frac{1}{2}x_1 M^2 \) is the first component to be integrated over [10]. Assuming that the numerator does not grow with \( k_1^- \) one can easily perform the \( k_1^- \) integrations assuming that the \( F_i \) are independent of \( k_1^- \). Taking the limit \( x_1 \to 0 \) of the basic result for the quark-gluon correlators \( \Phi_G(x,x-x_1,k_T,k_T - k_{1T}) \) we obtain the
gluonic pole correlators, for distribution functions \(0 \leq x \leq 1\) (see \cite{10} for details)

\[
\Phi_G(x,x) = -\int d^2k_T d^2k_{1T} \frac{(1-x)F_1(x,0,k_T,k_{1T})\theta(1-x)}{(\mu^2-k_T^2)(xB_1+(1-x)A_2)A_1},
\]

(10)

where \(A_i(\{m_i^2\},\{k_{iT}^2\},\{x_i\})\), and for fragmentation functions \((x = 1/z \geq 1)\)

\[
\Delta G(x,x) = 0.
\]

(11)

This result depends on the assumption that the numerator does not grow with \(k_1^-\), otherwise, one does not get the required \(x_1 \theta(x_1)\) behavior in the calculation \cite{10}. In models, terms proportional to \(k_1^- \sim k_1 \cdot P\) may easily arise from numerators of fermionic propagators \cite{17} which may easily be suppressed by form factors at the vertices. To prove a proper behavior within QCD one would need to study the fully unintegrated correlators such as e.g. in Ref. \cite{18} and show that they fall off sufficiently fast as a function of \(k_1 \cdot P\).

While our analysis is not yet the full proof that gluonic pole matrix elements vanish in the case of fragmentation, it is a step towards such a proof and the possible direction to obtain such a proof by considering the appropriate color gauge-invariant soft matrix elements. Such a proof is important as it eliminates a whole class of matrix elements parameterized in terms of T-odd fragmentation functions besides the T-odd fragmentation functions in the parameterization of the two-parton correlators.

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