The tidal disruption of protoplanetary accretion discs

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Accepted 1997 May 30

ABSTRACT

In this paper we revisit the problem of the tidal interaction occurring between a protostellar accretion disc and a secondary point mass following a parabolic trajectory. We model the disc response analytically and we compare our results with three-dimensional SPH simulations. Inviscid as well as viscous hydrodynamics is considered. We show that in a viscous system the response derived from inviscid considerations is predominant even for the highest estimates of an anomalous disc shear viscosity. The angular momentum lost from the disc during the encounter is derived from linear theory, for distant fly-bys, as well as the changes to the disc orientation expected in non-coplanar encounters. It is shown that the target discs can become warped and precess by a small amount during non-coplanar encounters. This small precession is shown to give rise to a relative tilt of the disc which is always more important for determining its final orientation than is the change to the orbital inclination. We discuss the implications of our results for protostellar accretion discs and planetary systems.

Key words: accretion, accretion discs – binaries: general – stars: formation – planetary systems.

1 INTRODUCTION

Low mass stars are known to form in compact groups within the dense cores of giant molecular clouds. Recent observations of nearby star-forming regions have revealed that young stellar objects (YSOs) have a binary frequency in excess of that found for field stars and roughly half of the current sample are associated with optically thick, geometrically thin, circumstellar accretion discs (see recent reviews by Mathieu 1994, Lin & Papaloizou 1996 and references therein). If the mean free path in a star-forming core is sufficiently small so as to allow close encounters between YSOs on a timescale shorter than the lifetime of a protostellar disc then the tidal interaction between a secondary object and the disc of the primary may be a dynamically significant event in the disc’s history.

The possibility that YSOs might tidally interact through their circumstellar discs has lead to recent speculation that binaries may form as the products of a capture process involving star-disc encounters (Larson 1990). However it was quickly shown that even for the most compact star-forming environments known, disc penetrating encounters leading to capture would be rare (Clarke & Pringle 1991), with the probability of such an encounter being approximately one tenth over a disc lifetime. Also observational data implies that most YSO binaries have components of similar ages (Hartigan, Strom & Strom 1994, Brandner & Zinnecker 1997), which supports local formation of the components rather than capture. We note however that more distant encounters are near certainties for protostellar accretion discs. These kinds of star-disc interactions have been extensively studied in the context of star formation. Heller (1993) investigated numerically with SPH the rate of disc tilting resulting from non-coplanar disc penetrating encounters. Clarke & Pringle (1993) investigated numerically, but with a sticky particle code, the affect of a disc penetrating encounter on the distribution of disc material. Ostriker (1994) applied linear perturbation theory to an inviscid hydrodynamical model, using a spherical harmonic expansion of the tidal potential for distant encounters, to derive an asymptotic expression for the angular momentum lost from the disc in the encounter. Korykansky & Papaloizou (1995) used a Fourier expansion in azimuthal modes, for the tidal potential, and calculated the angular momentum exchange without asymptotic assumptions. Hall, Clarke & Pringle (1996) carried out a numerical investigation of the disc response using a reduced three-body method with non-interacting particles. Due to the complexity of the tidal problem the analytical results presently available in the literature, although extensive, are also unwieldy and the numerical studies are chiefly qualitative. In this paper we present analytical and numeri-
cal (SPH) calculations of the disc response to a parabolic encounter. We present a simplified analysis isolating the most significant parts of the response which have been identified in the works listed above. We obtain explicit expressions for the angular momentum exchange (and other properties of the response) and we test these results directly against the simulations.

In Section 2 we give some of the equations basic to our subsequent considerations. In Section 3 we review concepts of the linear disc response and perform a linear mode analysis of the fluid equations. In Section 4 we derive the linear disc response for distant coplanar encounters and in Section 5 we extend this to consideration of the response for distant non-coplanar encounters. Our numerical method is outlined in Section 6 and in Section 7 we present our numerical results. In Section 8 we summarise our findings and discuss our results within the context of recent observations.

2 BASIC EQUATIONS

2.1 The accretion disc model

We consider a non–self-gravitating gas-dynamical model for the accretion disc. Processes that may give rise to internal heat generation or transport are not considered. Instead we adopt a simple polytropic relationship between gas pressure, $P$, and density, $\rho$, giving a constitutive equation of state:

$$ P = K \rho^{1+1/n}, $$

where $n$ is the polytropic index and $K$ is the polytropic constant. The associated barotropic sound speed, $c_s$, is given by

$$ c_s^2 = \frac{dP}{d\rho}. $$

The constancy of $K$ in this model requires that we assume any dissipated energy to be lost from the system. This is equivalent to assuming an efficient cooling mechanism operating in the disc.

2.1.1 Equilibrium structure

Referring to a set of cylindrical coordinates $(r, \phi, z)$ with the $z$-axis coincident with the rotation axis of the disc, the standard equation of vertical hydrostatic equilibrium in the thin disc approximation is:

$$ \frac{1}{\rho} \frac{dP}{dz} = -\Omega^2 z, \quad \Omega = \Omega(r), $$

where $\Omega$ is the Keplerian angular velocity of disc material in an initially axisymmetric disc. For a disc satisfying the polytropic equation of state $\rho(z) = \left( \frac{\Omega^2 H^2}{2K(n+1)} \right)^n (1 - z^2/H^2)^n$, we may now compute the sound speed in the disc in terms of the midplane value, $c_s$:

$$ c_s^2 = c_s^2(1 - z^2/H^2), $$

where $H = H(r)$ is the total vertical semi-thickness for which $\rho(z = \pm H) = 0$. We may now compute the sound speed in the disc in terms of the midplane value, $c_s$:

$$ c_s^2 = \frac{\Omega^2 H^2}{2K(n+1)}. $$

2.2 The secondary

Initially we shall consider the case in which the disc midplane and the orbital plane of the secondary coincide. Our results are then extended to include the case in which the two planes are mis-aligned. We take the primary and the secondary to be point masses. Neglecting any energy exchange phenomena that might occur between the secondary and the disc the energy per unit mass of the secondary referred to the non-inertial frame centred on the primary is:

$$ E = \frac{\dot{a}^2}{2} - \frac{G(M_p + M_s)}{\sigma} + \frac{\dot{q}^2 \dot{\theta}^2}{2}. \quad (3) $$

In a coplanar configuration the position of the secondary at time $t$ is $D(t) = \{\sigma(t), \theta(t), 0\}$, $M_p$ is the mass of the primary, $M_s$ is the mass of the secondary and we denote with a dot the derivative with respect to time. For a parabolic trajectory we set $E = 0$ and integrate (3) with respect to time. Referring to a set of Cartesian coordinates $(x, y, z)$ we can choose the $y$-axis to coincide with the longitude of pericentre. In this case the position of the secondary as a function of time is given parametrically:

$$ D = q[4p, (1 - 4p^2), 0], $$

in which

$$ p = \sinh \left[ \frac{1}{3} \sinh^{-1} \left( \frac{3}{4} \omega_o t \right) \right]. $$

The distance of closest approach of the secondary to the primary is denoted by $q$ and the magnitude of the angular velocity of the secondary at pericentre by $\omega_o$:

$$ \omega_o = \sqrt{2G(M_p + M_s)} \frac{q^{1/2}}{q^{1/2}}. $$

The natural origin of time corresponds to the instant of pericentre passage, i.e. $D \equiv |D| = q$ at $t = 0$.

2.3 Tidal potential for distant encounters

We consider a geometrically thin disc, initially in a state of vertical hydrostatic equilibrium and radial centrifugal equilibrium, governed by the central potential $\Psi_0$ alone. This initial state is then perturbed from equilibrium by the tidal forcing due to a secondary point mass encountering the primary/disc on a parabolic prograde orbit. The part of the
total potential that is due to the secondary, $\Psi'$, is considered to drive small perturbations of the disc’s equilibrium structure. Within the context of a fluid model we can then apply a linear perturbation analysis of the fluid equations. This will always be valid if the affect of the tide is sufficiently weak, and amounts to considering the distance of closest approach $q$ to be large when compared to the outer radius of the disc $R_o$. The total potential is $\Psi = \Psi_0 + \Psi'$, where the potential due to the primary acting at a point with position vector $r$ is given by

$$\Psi_0 = \frac{GM_s}{r}$$

and the potential due to the secondary, referred to the origin which is fixed to the primary, is given by

$$\Psi' = \frac{GM_s}{r - D} + \frac{GM_s r \cdot D}{D^3}.$$  

Expanding up to second order in terms proportional to $r/D$, and ignoring terms proportional to $(z/r)^2$, we may then write the part of the total potential due to the secondary as (Papaloizou & Terquem 1995)

$$\Psi' = \frac{GM_s}{D} \left[ 1 - \frac{|r|^2}{2D^2} + \frac{3(|r| \cdot D)^2}{2D^4} \right].$$  

3 THE DISC RESPONSE

3.1 Wave generation and propagation

For tightly wound spiral density waves propagating with a radial wave number of magnitude $|k_r|$ in a geometrically thin self-gravitating disc the standard dispersion relation (Lin & Shu 1964) is written:

$$m^2(\Omega - \Omega_p)^2 = \kappa^2 - 2\pi G\Sigma|k_r| + |k_r|^2 c_s^2.$$  

In the non–self-gravitating limit this may be expressed as

$$|k_r| = \frac{\kappa}{c_s} \sqrt{s^2 - 1},$$  

where $s$ (Binney & Tremaine 1987) gives the ratio of the forcing frequency in a frame rotating with pattern speed $\Omega_p$ to the natural radial frequency $\kappa$ (also called the epicyclic frequency):

$$s = \frac{m(\Omega - \Omega_p)}{\kappa}$$

and

$$\kappa^2 \equiv \frac{1}{r^4} \frac{d}{dr} (r^4 \Omega^2).$$

The positive integer $m$ is the azimuthal mode number. Resonances occur at $s = 0, +1, -1$ corresponding to the corotation resonance and the inner and outer Lindblad resonances respectively. When a fluid disc is subject to a perturbing force which is stationary in a frame rotating with a constant angular frequency $\Omega_p$, a stationary disturbance is set up in that frame, the disturbance having the same order of azimuthal symmetry as the perturbing force.

The radial wave number given by (3) indicates short wavelength spiral density waves which may be either inward or outward propagating. Furthermore, standard results from the analysis of the solutions of the dispersion relation (4) are that for non–self-gravitating discs there exists an evanescent zone about the corotation radius which occupies the region between the inner and outer Lindblad resonances. Inside the evanescent zone wave propagation is formally disallowed. We can further deduce that the spiral density waves become increasingly tightly wound as they travel away from the Lindblad resonances.

In the tidal problem there is a torque exerted on the disc by the secondary. The torque principally interacts with the disc through the Lindblad resonances (Goldreich & Tremaine 1979), provided that they are either in, or lie sufficiently near to the disc (Lin & Papaloizou 1993). The applied torque excites short trailing waves at the Lindblad resonances which propagate away from the evanescent zone with a wavelength $\propto H$. As long as the waves remain linear then they propagate with a conserved wave action. Inward propagating waves then become increasingly tightly wound and increase in amplitude until non-linear damping takes place and the angular momentum they transport is deposited in the disc. This is a well known process that may lead to induced accretion near the disc centre. The net angular momentum transport due to the corotation resonance is much smaller and depends on the ratio $4\pi c_s^2/\Sigma$ (Korykansky & Papaloizou 1995), evaluated at the resonant location.

Consideration of the full problem for parabolic fly–bys requires analysis of the disc response to an infinite spectrum of forcing frequencies. Recent work in this area (Ostriker 1994, Korykansky & Papaloizou 1995) has shown that a single component of the response predominates: the part of the tidal potential with $m = 2$ azimuthal symmetry applied at pericentre.

Assuming that the secondary-disc interaction occurs as an impulse near pericentre we shall treat the disc response as resulting from a non-resonant interaction. In this time-independent calculation the secondary is assumed to be fixed at pericentre so that the torque exerted on the disc can be calculated in a simple way. In practice the interaction has a finite width in the sense that a resonant perturbation may be set up at some time before pericentre passage. As a result a phase lag between the density response of the disc and the forcing potential is set up, allowing a non-vanishing net tidal torque to be applied to the disc.

We shall determine the disc response with the secondary fixed at its pericentre position using a linear mode analysis of the fluid equations and apply the calculated torque over the estimated time interval of the interaction. Korykansky & Papaloizou (1995) report that the interaction interval they observe in their numerical models is about one orbital period at the outer edge of the disc, centred at the instant of pericentre passage.

3.2 Linear mode analysis

The vertically averaged inviscid fluid equations (see Papaloizou & Lin 1995b, for a recent review) yield radial and azimuthal components of the momentum equation:

$$\frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + V_\phi \frac{\partial V_r}{\partial \phi} + \frac{V_\phi^2}{r} = -\frac{1}{\Sigma} \frac{\partial P}{\partial r},$$

$$\frac{\partial V_\phi}{\partial t} + V_r \frac{\partial V_\phi}{\partial r} + V_\phi \frac{\partial V_\phi}{\partial \phi} + V_r V_\phi = -\frac{1}{\Sigma r} \frac{\partial P}{\partial \phi} - \frac{1}{r} \frac{\partial \Psi}{\partial \phi}.$$
and the continuity equation:
\[
\frac{\partial \Sigma}{\partial t} + \frac{1}{\Sigma} \frac{\partial}{\partial r} (r \Sigma V_r) + \frac{1}{\Sigma r} \frac{\partial}{\partial \phi} (r \Sigma V_\phi) = 0.
\]

Note that within the context of the vertical averaging procedure, physical quantities are formally replaced by their vertically averaged analogs. The radial and azimuthal velocity components are denoted by \(V_r\) and \(V_\phi\) respectively. We proceed by making a standard linear analysis of the vertically averaged fluid equations such that quantities are perturbed by a small amount from their equilibrium states (e.g. \(\Sigma \rightarrow \Sigma_0 + \Sigma'\) in standard notation). Then subtracting the equations for equilibrium and neglecting quantities of second and higher orders in the perturbation one obtains the fluid equations linear in perturbed quantities:
\[
\frac{\partial V'_r}{\partial t} + \Omega \frac{\partial V'_r}{\partial \phi} - 2\Omega V'_\phi = - \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{P'}{\Sigma_0} + \Psi' \right),
\]
\[
\frac{\partial V'_\phi}{\partial t} + \Omega \frac{\partial V'_\phi}{\partial \phi} + \frac{V'_r}{r} \left[ \frac{\partial}{\partial r} (r \Omega) \right] = - \frac{1}{r} \frac{\partial}{\partial \phi} \left( \frac{P'}{\Sigma_0} + \Psi' \right),
\]
and
\[
\frac{\partial \Sigma'}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \Sigma V'_r) = 0.
\]

These equations describe the linear response of the model. We analyse for azimuthal modes such that we make replacements like \(\Sigma' \rightarrow \Sigma_0(r) \exp(i\phi)\), noting that physically meaningful results are obtained by taking the real parts. The complex amplitude functions for the time-independent forced velocity components are (Goldreich & Tremaine 1979):
\[
V_{\phi a} = \frac{1}{\Delta} \left[ \frac{m^2 \Omega^2}{r} + \frac{\kappa^2}{2 \Omega} \frac{\partial}{\partial r} \right] \Psi_a,
\]
and
\[
V_{ra} = -\frac{i}{\Delta} \left[ \frac{2m \Omega}{r} + m \Omega \frac{\partial}{\partial r} \right] \Psi_a,
\]
where \(\Delta \equiv \kappa^2 - m^2 \Omega^2\). Additionally we assume a thin disc (i.e. \(H/r\) is a small quantity) for the duration of the encounter so that we can neglect the pressure perturbation in comparison to the perturbation to the potential. The perturbation to the surface density is then determined from the perturbed continuity equation. In terms of the velocity perturbations,
\[
\Sigma_a = \frac{1}{m \Omega} \left[ \frac{m \Sigma V_{\phi a}}{r} + \frac{1}{r} \frac{\partial}{\partial r} (r \Sigma_0 V_{ra}) \right].
\]

We note that the denominator, \(\Delta\), can vanish for \(m = 1\) azimuthal modes, since for strictly Keplerian rotation \(\kappa = \Omega\). This leads to a singularity in the disc response equation which is usually dealt with by employing a dissipative phase shift in the density response (see below).

4 LINEAR DISC RESPONSE FOR DISTANT COPLANAR ENCOUNTERS

Evaluating (1) at pericentre with the plane of the secondary’s orbit coplanar with the disc midplane we have
\[
\Psi' = \frac{GM_\ast}{q} \left[ 1 + \frac{1}{4} \left( \frac{r}{q} \right)^2 (1 - 3 \cos 2\phi) \right].
\]

The axisymmetric parts of this yield no net tidal effect (Papaloizou & Pringle 1977). The non-axisymmetric part indicates a disturbance with azimuthal mode number \(m = 2\). Carrying out the potential expansion to terms of order \((r/q)^n\) yields terms describing modes with \(m = 1\) and \(m = 3\), indicating that the dominant response comes from the \(m = 2\) component with the \(m = 1\) and \(m = 3\) modes becoming increasingly significant in closer encounters. This is in agreement with the findings of Korykansky & Papaloizou (1995), those authors claim a satisfactory fit to their numerical results can be calculated by considering only the Fourier components of the perturbative potential contributing to modes \(m = 1, 2\) and \(3\); the contribution from \(m = 1\) and \(3\) modes being much smaller than that from the \(m = 2\) mode. Considering a potential perturbation
\[
\Psi' = \frac{3 GM_\ast}{4 q^3} r^2 \exp (2i\phi),
\]
the velocity and surface density perturbation amplitudes are then given by:
\[
V_{ra} = 2i \frac{GM_\ast}{\Omega q^3} r,
\]
\[
V_{\phi a} = -5 \frac{GM_\ast}{4 q^3} r,
\]
and
\[
\Sigma_a = \frac{(13 - 4n)}{4} \frac{\mu \Sigma_0}{q^3} r^{n-3}.
\]

Here \(\Sigma_0 = \Sigma_0 (R_\ast/r)^{n-1}\) in which
\[
\Sigma_0 = \mathcal{C}_n R_\ast \left[ \frac{\Omega_\ast^2 R_\ast^2}{2K(n+1)} \right]^{n-1} \left( \frac{H}{r} \right)^{2n+1}.
\]

In the above; \(\mu\) denotes the mass ratio \(M_\ast/M_p\), the rotational velocity at the outer edge of the disc is \(\Omega_\ast = \Omega_\ast (R_\ast)\) and we have considered \(H/r\) to be a constant (in practice \(H/r\) is a weak power law in \(r\), Larwood & Papaloizou 1997). We can now deduce the part of the net torque acting on the disc which gives rise to a change in the magnitude of the disc angular momentum \(J\). The rate of change of the disc’s angular momentum is then given by (Papaloizou & Pringle 1977):
\[
\frac{dJ}{dt} = - \int_{\text{disc}} \Sigma' \frac{\partial \Psi'}{\partial \phi} r \, dr \, d\phi.
\]

Notice that the linear terms in the perturbed torque are identically zero and that the net torque is second order in perturbed quantities. Angular momentum transfer will then be a correspondingly small effect in inviscid discs. In fact this part of the torque will itself only be non-vanishing if the perturbation to the density becomes phase shifted with respect to the perturbing potential such that \(\Sigma' \propto \exp (mi(\phi - \eta))\).

In order to obtain an order of magnitude estimate of the angular momentum exchange we consider the torque due to the leading non-axisymmetric term of the potential expansion outlined above. Integrating over the disc for the \(m = 2\) response, and from \(t = -\pi \Omega_\ast^{-1}\) to \(t = \pi \Omega_\ast^{-1}\), we find
\[ \Delta J \equiv \frac{J - J_0}{J_0} \]
\[ = -\frac{3\pi (13 - 4n)(7 - 2n)}{16} \mu^2 \left( \frac{R_0}{q} \right)^6 \sin 2\phi. \quad (8) \]

Note that we identify the angular momentum of the unperturbed disc, \( J_0 \), with the angular momentum content of an axisymmetric polytrope:
\[ J_0 = 2\pi \int_0^{R_0} \Sigma_0 r^2 \Omega dr. \]

In general a density perturbation is set up in phase with the potential perturbation at a finite time before pericentre passage. It is the differential rotation between the pattern of the perturbation at the outside of the disc and the secondary that results in a phase shift between the perturbing potential and the density response. The existence of a phase lag allows a net angular momentum transfer to occur between the secondary and the disc.

Our viscous numerical models also require consideration of the angular momentum exchange due to viscous dissipation. This has been derived by Papaloizou & Pringle (1977):
\[ \frac{dJ}{dt} = -\int_{\text{disc}} \frac{\Sigma_0 \Phi}{\Omega} r d r d\phi, \quad (9) \]
where \( \Phi \equiv 2r(e_{ij}e_{ij} - \frac{1}{3}e_0^2) \) defines the rate of dissipation of mechanical energy per unit mass in terms of the rate-of-strain tensor, \( e_{ij} \), of standard fluid theory. Evaluating (9) in the simple case of constant viscosity, and for the \( m = 2 \) perturbations calculated above, gives a contribution \([\Delta J]_\nu\) to the angular momentum exchange:
\[ [\Delta J]_\nu \approx -70\pi \frac{7 - 2n}{15} \frac{\nu}{R_0^2 \Omega_\nu} \mu^2 \left( \frac{R_0}{q} \right)^6. \]

Comparison of this result with equation (8) implies that for reasonable values of the polytropic index viscous torques become important when \( \eta \sim 10R^{-1} \). Where \( R \) is the Reynolds’ number \( \equiv r^2 \Omega / \nu \) evaluated at the outer edge of the disc. Since viscous timescales longer than \( \sim 10^{15} \) yr are inferred in protostellar accretion discs (Beckwith et al. 1990) we would require a dynamical phase shift that is necessarily small compared with unity. There is no reason why we should expect such a small value for \( \eta \), hence in general the viscous contribution to the process of angular momentum exchange is expected to be negligible, as is confirmed by the numerical results to be presented below.

5 EXTENSION TO DISTANT NON-COPLANAR ENCOUNTERS

Assuming the disc to be axisymmetric and the secondary-disc interaction to occur only at pericentre, we need only consider the inclination angle \( \delta \) of the vector pointing to pericentre with respect to the initial disc midplane. So if we choose the \( x \)-axis to lie along the line of nodes, then \( D \) is given by a simple rotation of the previously considered coplanar trajectory about that axis:
\[ D = q[4\rho, (1 - 4\rho^2) \cos \delta, (1 - 4\rho^2) \sin \delta]. \]

The perturbative potential may then be recalculated and decomposed into a linear sum of contributions corresponding to odd terms in \( z \) (denoted by \( \psi_o \)) and even terms in \( z \) (denoted by \( \psi_e \)):
\[ \psi' = \psi_e + \psi_o. \]

Now, to terms of first order in \( z \), equation (8) leads us to consider non-axisymmetric components
\[ \psi_o = \frac{3GM}{4} q^2 \cos^2 \delta \exp(2i\phi) \]
and
\[ \psi_e = \frac{3GM}{2} q^2 rz \sin 2\delta \exp(i\phi). \]

The non-axisymmetric part of the non-coplanar response that is even in \( z \), \( \psi_e \), carries an additional factor of \( \cos^2 \delta \) in comparison with the analogous coplanar term. Hence inclining the orbit diminishes the strength of the response found in the coplanar case, being minimised in orthogonal configurations. Thus the effectiveness of tidal truncation of the disc and the strength of non-axisymmetric waves is reduced. In addition our estimate for the angular momentum exchange is modified: \( \Delta J \to \Delta J \cos^4 \delta \).

The part of the potential that is odd in \( z \), \( \psi_o \), yields a disturbance with azimuthal mode number \( m = 1 \); this bending mode is found to warp the disc and cause its precession in circular orbit calculations (Papaloizou & Terquem 1995, Larwood et al. 1996). But note that the amplitude we compute above is two times larger than that for the analogous zero-frequency term in a circular orbit calculation with \( D = q \) (see Papaloizou & Terquem 1995). Note also that we have recovered a warping potential proportional to \( \sin 2\delta \). Ostriker (1994) showed that this result holds asymptotically for distant encounters.

5.1 Precession, warping and changes to the inclination

We consider only the warp due to the part of the perturbing potential with \( m = 1 \), i.e. the longest wavelength mode. This warp will be the longest-lived and most significant as long as the disc response is linear. Also this component will cause precessional effects.

As before we calculate the forced response assuming the perturbing potential to be fixed at its pericentre value. We suppose that our coordinate system precesses at a rate \( \omega_p \), which corresponds to the precession of the disc angular momentum vector about the orbital angular momentum vector of the secondary. In this case consideration of the linearised fluid equations in their three-dimensional form gives the response equation for the zero pattern speed forcing potential with \( m = 1 \) as (Papaloizou & Lin 1995a)
\[ \frac{d}{dr} \left( \frac{\mathcal{M}\Omega^2}{[\Omega^2 (1 - i \epsilon)^2 - \kappa^2]} \frac{dg}{dr} \right) = \frac{iI}{r}, \quad (10) \]
in which we have prescribed the complex function \( g \), defined through:
\[ -irz\Omega^2 g = \frac{P'}{\rho} + \psi_o. \]
The \( e \)-term in (10) is prescribed in order to transform the location of the singularity due to the resonant denominator. This amounts to introducing a small phase shift which is equivalent to a for a small shear viscosity, \( \nu \). The remaining symbols correspond to

\[
I = \int_{-H}^{+H} \frac{\rho z^2}{c_s^2} (\psi_0 + 2\omega_p r z \sin \delta) dz,
\]

and

\[
\mathcal{M} = \int_{-H}^{+H} \rho z^2 dz = \frac{\Sigma H^2}{2 n + 3}.
\]

We employ the function \( g \) to quantify the degree of warping manifest in our numerical calculations. This is possible since a constant value of \( g \) corresponds to a rigid tilt of the disc, for which \( r \cdot V \equiv 0 \). Then for small amplitude warps \( g = V_y'/V_0 = i \zeta/r \), where \( \zeta(r) \exp(i\psi) \) is the vertical displacement (Papaloizou & Lin 1995a, Larwood & Papaloizou 1997).

In the limit of slowly precessing small amplitude warps we may neglect the contribution to (10) from the second term in parenthesis and make use of the identity

\[
\int_{-H}^{+H} \frac{\rho z^2}{c_s^2} dz = \Sigma \Omega^{-2},
\]

derived by integrating the equation of vertical hydrostatic equilibrium (2). Then to determine \( g \) we need to integrate (10) over the disc twice. Carrying out this operation assuming \( \kappa = \Omega \) and \( \alpha \gg \alpha^* \), we find the total range in the vertical displacement \( \Delta(\zeta/r) \) to be

\[
\Delta(\zeta/r) = \frac{2n + 3}{5 - n} \alpha \mu \left( \frac{R_o}{q} \right)^3 \left( \frac{r}{H} \right)^2 \sin 2\delta. \tag{12}
\]

If shear viscosity is a small effect then bending waves propagate acoustically and should damp on the sound–crossing timescale of the disc. This can provide a test of the importance of any unquantified numerical viscous effects that might be introduced due to the vertical shearing motions present in warps (see below).

We note also that in the above calculation there is an implicit assumption that the sound-crossing time in the disc is small enough that the warp is near its steady-state value after the encounter. This is equivalent to the requirement that the interval over which the interaction occurs is not less than the sound–crossing time for the disc. If this condition is satisfied then it follows that the disc will precess approximately as a rigid body (Papaloizou & Terquem 1995) with a small frequency \( \omega_p \):

\[
\omega_p \sin \delta = \frac{\int \frac{\delta \omega_\phi}{4 \pi} r^2 \sin \phi dr d\phi}{\int \frac{\rho_0}{4 \pi} \Omega^2 dr d\phi},
\]

where the integrals are to be taken over the area of the disc midplane. Evaluating from \( r = 0 \) to \( r = R_o \) we find

\[
\frac{\omega_p}{\Omega_o} = -\frac{3}{4} \frac{\Pi - 2n}{5 - n} \left( \frac{R_o}{q} \right)^3 \cos \delta. \tag{13}
\]

At large distances of pericentre global rigid body precession is possible since the interaction interval is \( \sim 2\pi/\Omega_o \). Larwood et al. (1996) found for circular orbit binaries that when the condition for rigid body precession was marginally satisfied then the disc would precess differentially. This effect amounts to a situation in which we have the differential precession of sonically-connected annuli. For a parabolic encounter this would result in a net precession of the disc with the main contribution coming from the precession of an outer annulus of the disc.

In addition to the above, Papaloizou & Terquem (1995) showed that the \( y \)-component of the tidal torque \( T_y \) resulting from secular terms \( \Psi' \propto z \) gives rise to inclination evolution according to

\[
\frac{d\delta}{dt} \approx \frac{T_y}{J_y}.
\]

\( T_y \) is identically zero unless we can introduce a phase shift due to viscosity. Since \( \omega_p \sim T_y/J_y \), we notice that the relative importance of inclination evolution compared with precessional effects in distant encounters is measured by the ratio

\[
\frac{T_y}{T_z} \sim \alpha.
\]

Currently, values of \( \alpha \) quoted for protostellar discs are typically less than \( \sim 10^{-2} \). Therefore we should expect that as long as the disc response is linear then precessional effects are more significant in determining the final orientation of the disc than inclination changes.

6 NUMERICAL SIMULATIONS

The numerical method we employ to study the dynamics of viscous accretion discs is a version of SPH (see Monaghan 1992, and references therein) developed by Nelson & Papaloizou (1993, 1994). The formulation uses a spatially variable smoothing length, associated with each particle, defined in such a way as to ensure accurate energy conservation. Each particle’s smoothing length is defined to be half of the mean distance of the six most distant nearest neighbouring particles chosen from a list of forty–five members at each time–step.

In order to stabilize the calculations in the presence of shocks an artificial viscous pressure term is included, according to the prescription of Monaghan & Gingold (1983). Although intended to prevent particle penetration while giving positive definite dissipation, the practical implementation of the artificial viscosity introduces a shear viscosity which results in disc spreading much as in the standard theory of accretion discs (Lynden-Bell & Pringle 1974).

These standard accretion disc models were developed to study tidal interaction and have been used in previously published work (Larwood et al. 1996, Larwood & Papaloizou 1997). Similar models have been used by Artyomowicz & Lubow (1994).

6.1 Basic numerical model

The SPH disc models were constructed by generating random position data for 17500 particles, giving a zero-thickness disc of unit radius centred on the origin of the computational coordinate system (being coincident with the primary). Vertical positions were generated in a similar fashion by equally separating this initial set of particles into seven uniformly-spaced planes of approximately constant surface
density, having total vertical extent $H_0$. The particles were then allocated velocities according to a softened Keplerian potential: $\Omega = (r^2 + b^2)^{-3/4}$, the softening length $b = 0.2$ chosen to avoid the singularity of a Keplerian potential that occurs near the origin. It is then only required to determine the polytropic constant, $K$, from the equation of vertical hydrostatic equilibrium. We choose $K$ to give a total vertical semi-thickness $H_0$ at the initial outer radius, given that we employ a polytrope of index $n = 1.5$ in our numerical models (which ensures $\nu \approx$ constant, Larwood et al. 1996). Units are such that $G = M_0 = 1$, giving a time unit of $\Omega_0^{-1}$. The affect of numerical viscosity is to produce an effective shear viscosity which we calibrate against a standard constant viscosity model. The results of our calibration exercise imply

$$\alpha \sim 0.008 \times \left( \frac{R_0}{r} \right)^{1/2} \left( \frac{H}{r} \right)^{-2/3}.$$ 

Before introducing the secondary the disc models were allowed to relax for a few complete rotations solely in the gravitational field of the primary and under SPH pressure and viscous forces. This is required to establish an approximate equilibrium state of the system. Thick disc models with $H/r = 0.15$ (relevant to protostellar discs) as well as thin disc models with $H/r = 0.05$ (chosen to test discs with a significantly smaller sound speed) are considered. Due to the different thicknesses and the nature of the SPH numerical viscosity the thick and thin disc models also represent different viscosity regimes (Larwood & Papaloizou 1997), with $\nu \sim 10^{-3}$ in the thick disc case and $\nu \sim 10^{-4}$ for the thin disc models. As a result the outer radius of the disc relaxes to different values in each case. For the thin disc models we take $R_0 = 1.2$ and for the thick disc models we use $R_0 = 1.4$. The secondary is introduced at a distance of 10 units from the primary, corresponding to a time $\sim -15$, in prograde rotation with the disc. The secondary’s position is given from the computed time at each SPH integration step by the expressions derived in Section 2.2. The potential due to the secondary is then calculated from equation (4). These expressions for the secondary are used in their softened form. Parameters considered are for secondaries with unit mass ratio ($M_2 = 1$) and distances of closest approach $q = 2.4, 3.6, 4.8$ and 6.0. Both coplanar and non-coplanar cases with $\delta = 15^\circ, 30^\circ$ and $45^\circ$ are tested.

### 6.3 Determining changes to the disc orientation

As in previous work, we introduce angles $\iota$ and $\Pi$ to quantify respectively the effects of warping and of precession in our numerical data.

$$\cos \iota = \frac{J_A \cdot J_D}{|J_A||J_D|}$$

and

$$\cos \Pi = \frac{(\mathbf{J}_0 \times \mathbf{J}_D) \cdot \mathbf{u}}{|\mathbf{J}_0 \times \mathbf{J}_D||\mathbf{u}|}.$$ 

$J_D$ is the disc angular momentum calculated as the sum over all disc particle angular momenta, $J_A$ is the angular momentum within an annulus. The arbitrary reference vector $\mathbf{u}$ lies in the orbital plane of the secondary. For convenience we choose $\mathbf{u}$ such that $\Pi$ takes the initial value of $\pi/2$ radians, its subsequent evolution directly measuring the disc precession.

Retrograde precession of $J_D$ about $\mathbf{J}_0$ implies that the angle $\Pi$ decreases linearly with time, describing an arc in the orbital plane of the secondary. Applying the pericentre precession frequency for one outer–disc orbital period, we estimate the small change in the disc precession angle expected to occur in a $n = 1.5$ polytrope as $\Delta\Pi$:

$$\Delta \Pi = -\frac{12\pi}{7} \mu \left( \frac{R_0}{q} \right)^3 \cos \delta.$$ 

During the parabolic fly–by of a secondary body, the linear response of a fluid disc is to precess approximately as a rigid body (see below). The small amount of precession results in a shift of the disc midplane away from its initial configuration, providing an apparent relative tilt with respect to the initial midplane, whilst maintaining the angle between the disc and orbital angular momentum vectors. For a small change in the precession angle $\Delta\Pi$, producing a small tilt $\tau$, we expect $\tau = |\Delta\Pi| \sin \delta$.

Typical model parameters are $\mu = 1$, $q = 4R_0$ and $\delta = 30^\circ$, giving a tilt $\sim 2^\circ$, being much larger than the expected change to the inclination $\delta$.

We note that for small values, the total range in $\iota$ (being the angle between the angular momentum vector of the disc material contained within a specified cylindrical annulus and the total angular momentum vector of the disc) is equivalent to $\Delta(\zeta/r)$.

### 7 NUMERICAL RESULTS

All our disc models lost angular momentum to the secondary, resulting in a negative sign in the net angular momentum exchanges. Our thick disc models, having the larger outer radius, showed the largest loss of angular momentum; our thin disc models, having a smaller outer radius, lost less angular momentum. Non-axisymmetric waves with azimuthal mode number $m = 2$ were visibly excited in all but the most distant encounters, as we would expect given that the $m = 2$ inner Lindblad resonance with pattern speed $2\omega_0$ lies far from the disc edge when $R_0/q$ is sufficiently small. Additionally, in non-coplanar configurations, we found that the disc became warped and showed precessional effects and
inclination changes. The effect on the response of introducing a finite orbital inclination was generally to diminish the effectiveness of the secondary’s tide, although there were some indications that high inclination encounters could enhance angular momentum transfer.

7.1 Coplanar encounters

7.1.1 Angular momentum exchange

The parameters for these models are given in Table 1, along with the corresponding net angular momentum changes found in the disc models. Figure 1 shows a particle projection plot for the disc particles in model 3, taken at a time for which the secondary is close to pericentre. The angular separation between the longitude of the secondary and the major axis of the disc’s elliptical envelope is \( \sim \pi/4 \) radians. If we identify this with the dynamical phase shift, \( \eta \), then we would have a much larger phase shift than would be expected from considering dissipative effects alone. Hence the disc viscosity would not be expected to make a significant contribution to the process of angular momentum exchange in these models. In fact our results indicate that the difference in the magnitudes of the angular momentum exchanges between the thick and the thin disc models can be explained solely in terms of the different radial extents associated with the two types of model.

The magnitude of \( \eta \) at pericentre is determined by the angular velocity of the secondary in the frame rotating with the outer disc, the former varying in time. As a result models which have smaller (larger) \( q \)-values show smaller (larger) \( \eta \)-values. For the range of \( q \)-values considered here we shall take the approximate median value \( \eta = \pi/4 \) in evaluating our analytically derived expressions, noting that this gives maximal angular momentum transfer.

We present plots of the total angular momentum of the disc versus time for models 5 to 8 in Figure 2. Models 5 and 6 show an impulsive form for the angular momentum exchange which takes place over \( \sim 2\pi\Omega_0^{-1} \) time units about pericentre passage. This simple picture shows signs of breakdown in models 7 and 8, which indicate in addition a significant return of angular momentum from the secondary to the disc occurring after the initial interaction described above. This reversal in the direction of the angular momentum transfer can be understood in terms of the passage of the outer disc through the point where the \( m = 2 \) elliptical pattern and the longitude of the secondary are \( 90^\circ \) out of phase (giving zero net torque at that instant). We can also say that we expect the return of angular momentum to be more effective for parabolae with larger \( q \)-values, since in those cases \( R_o/D \) varies more slowly in time. Indeed, as the distance of closest approach becomes sufficiently large, the net angular momentum exchange should tend to zero as the disc responds adiabatically to the perturbation. In the case of a viscous disc the angular momentum exchange would tend to the small but finite value expected from viscous dissipation alone.

Our analytical expression for \( \Delta J \) does not include the affect of the return of angular momentum to the disc. We compare values derived from equation (8) with the angular momentum exchanges we determine in our numerical models, considering only the part of the interaction in which the disc initially loses angular momentum (see Figure 3). We note that an additional return of angular momentum to the disc will modify the magnitude of the resultant exchange by a factor which depends on \( q \).

As we should expect, our analytically derived expression gives the best agreement with our numerical results for the most distant encounters. However the expression slightly over-estimates these values, which is probably a consequence of choosing a phase shift that gives maximal angular momentum transfer. The numerical values for \( \Delta J \) are underestimated by the predicted values as the encounters become closer. This probably owes less to the exclusion of \( m = 1 \) and \( m = 3 \) modes from our calculations than to the onset of non-linearity in the disc response, which in some cases has been found to double the angular momentum transfer seen in similar but purely linear calculations (Korykansky & Papaloizou 1995).

Table 1. The change in the angular momentum content of the disc for model parameters used in coplanar encounters.

| model | \( H/r \) | \( q \) | \( \Delta J \) |
|-------|-------|---|---|
| 1     | 0.15  | 2.4 | \( -1.6 \times 10^{-1} \) |
| 2     | 0.15  | 3.6 | \( -3.0 \times 10^{-2} \) |
| 3     | 0.15  | 4.8 | \( -2.5 \times 10^{-3} \) |
| 4     | 0.15  | 6.0 | \( -7.6 \times 10^{-4} \) |
| 5     | 0.05  | 2.4 | \( -1.4 \times 10^{-1} \) |
| 6     | 0.05  | 3.6 | \( -1.1 \times 10^{-2} \) |
| 7     | 0.05  | 4.8 | \( -3.8 \times 10^{-4} \) |
| 8     | 0.05  | 6.0 | \( -1.9 \times 10^{-5} \) |
Figure 2. The angular momentum of the disc, in units of its initial value, versus time for models 5–8.

7.1.2 The redistribution of disc matter

In all cases with $q \geq 4R_0$, the surface density profile at equal times (examined at a total time elapsed of about 30 units) was found to be barely distinguishable from that of an unperturbed model. For $q \sim 3R_0$, there were indications that tidal truncation of the disc had occurred in much the same way as would be expected for a system involving a bound secondary (Lin & Papaloizou 1979a). This case carries a weakly non-linear component to the response with the negative angular momentum flux into the disc being sufficiently large so as to result in local non-linear dissipation near the disc edge. This behaviour of the system confirms that the break-down of linearity occurs for $q < 4R_0$ (cf. Hall et al. 1996). The affect of tidal truncation on the disc sur-

Figure 3. The magnitude of the angular momentum initially lost by the disc for models 1–8. The thick disc data is denoted by open squares, the thin disc data by open triangles and the analytical result by a solid line.

Figure 4. Surface density profiles for model 2, denoted by solid circles, and an unperturbed model, denoted by open circles. Each is taken for a total run time of about 30 units.
face density profile is represented in Figure 4. The middle part of the perturbed disc was sculpted into a more compact and uniformly distributed structure and the outer disc was abruptly cut-off. This feature will only be subject to significant subsequent evolution on the viscous timescale. In models 1 and 5 with $q \sim 2R_o$ the disc response was strongly non-linear, the surface density structure being severely disrupted and truncated. As indicated in Figure 4, a material arm of several disc radii in extent is ejected from the outer-edge of the disc and the secondary appears to capture some of the disc particles. Much of the remaining disc material is re-formed into a dense ring. The formation of a ring in both of these models is accompanied by a suppression of central mass accretion. This indicates that the ring forms by the reflection of an inwardly propagating trailing wave, carrying a negative angular momentum flux, producing an outwardly propagating leading wave carrying negative angular momentum from the inner region of the disc. The reflected wave then encounters the rear portion of its inwardly propagating counterpart and dissipation occurs. The net effect is to clear an inner annulus of material, form a ring and suppress central mass accretion.

The production of circumprimary rings in close binary encounters has been observed before in numerical simulations (Clarke & Pringle 1993). It is understandable that the formation of a ring is only found in very strong encounters since it is in these cases that the $m = 2$ spiral density waves, launched at the outer disc, are strong enough to propagate to sufficiently small radii that their radial wavelength falls below the resolution limit of any code that is used to model the disc matter. We find that the resolution at the estimated radius of reflection is marginal given the predicted radial wavelength (given by equation 7); hence we conclude that ring formation is observed in these simulations but that it is possibly explicable as a numerical effect.

Accompanying these phenomena we also find that the envelope of the disc takes on an elliptical shape, however unlike the $m = 2$ bar that is generated prior to strong interaction, this form is not centred on the primary. This feature does not disperse on a dynamical timescale which implies that the disc has become genuinely elliptical. The implication is that the disc has taken on an asymmetric structure that will only disperse on the viscous timescale. In Figure 6 we plot the side view particle projections corresponding to Figure 5.

In models 1 and 5 the secondary removed $\sim 5\%$ of the initial disc mass (i.e. about $10^5$ particles), most of which was captured. The captured material seemed to consistently manifest as a dense clump of eccentrically orbiting material (see Figure 5). Further investigation of this phenomenon shall be the subject of future studies. In model 2 with $q \sim 2.6R_o$ only $\sim 10$ particles were lost. In models 3, 4, 7 and 8, for which $q \geq 3R_o$, no material was lost from the disc.

7.2 Non-coplanar models

7.2.1 Angular momentum exchange

The data for our non-coplanar models is given in Table 2. In Figure 7 we show the angular momentum evolution of the disc for models 3, 3a, 3b and 3c. As the orbital inclination angle is increased the magnitude of the initial transfer

Figure 7. The angular momentum of the disc, in units of its initial value, versus time for models 3, 3a, 3b and 3c; represented by squares, crosses, diamonds and pluses respectively.

Figure 8. The angular momentum of the disc, in units of its initial value, versus time for models 8 and 8c; represented by squares and crosses respectively.

Figure 9. The magnitude of the angular momentum initially lost by the disc for non-coplanar models. The thick disc data is denoted by squares, the thin disc data by triangles and the analytical result by a solid line.

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Figure 5. Positional $(x, y)$ data for model 1 taken at a time $t = +13$. The same data is displayed at two different scales.

Figure 6. Positional $(x, z)$ and $(y, z)$ data for model 1 taken at a time $t = +13$. Only particles with $r < 1.2$ are plotted.
of angular momentum from the disc to the secondary decreases. Hence inclining the orbit of the secondary results in a weaker interaction. In addition the magnitude of the angular momentum returned from the secondary to the disc, compared with the net exchange in each case, is also reduced. This is because the mean distance of the secondary from the disc edge is larger at higher inclinations. We illustrate an extreme in this behaviour by examining the angular momentum evolution for models 8 and 8c in Figure (8). Although the initial transfer is largest in the coplanar model the net transfer is larger for the non-coplanar model. As for the coplanar case we compare the inferred $\Delta J$ values, determined for the initial phase of the angular momentum exchange, with our expression derived from linear theory and modified for inclined orbits of the type considered here. We present these results in Figure (9). The general behaviour of the numerical values compared with the analytical result is similar to that found in the coplanar case. In addition it appears that low inclination models give better agreement with the analysis than the high inclination models. This is probably due to the enhancement of the angular momentum flux through the non-coplanar $m = 1$ contribution to the response, which is most important at high inclinations (Papaloizou & Terquem 1995). We note also that the thin disc models appear to show larger values than the thick disc cases, this is probably due to a greater tendency to non-linearity in the response which could be more significant in the presence of the vertical shearing motions found in warps.

### 7.2.2 The redistribution of disc matter

Tidal truncation of the model discs was found to operate for non-coplanar models as for the coplanar cases, but occurring at a reduced level for larger orbital inclinations. In Figure (10) we compare the surface density profiles for models 2 and 2c, examined at a total time elapsed of 30 units from the time of the introduction of the secondary (cf. Figure 8). The surface density profile for model 2c appears to be intermediate between the coplanar case and the unperturbed model with the outer disc not as strongly cut-off as in the coplanar case but more so than in the unperturbed case (which is not cut-off at all).

### 7.2.3 Precession, warping and inclination changes

All our non-coplanar models showed a small amount of precession. In Figure (11) we give the precession angle evolution for model 3c. As for the torque which gives rise to angular momentum exchange the torque giving precession of the disc is found to assert itself essentially as an impulse at pericentre. The small amount of precession manifest in all these models was found to give a small relative tilt $\tau \sim 1^\circ - 6^\circ$, even at the largest $q$-value, in reasonable agreement with the expression derived above. We give particle projection plots of model 6c in Figure (12). It is clear that the midplane of the disc has become shifted with respect to its initial orientation.

As well as showing precessional behaviour the model discs also became warped. We compare the size of the warps found in each model at a time for which the companion has reached $r = 10$, with the equation (12), in Figure (13). The expression derived from linear theory seems reliable for distant encounters in the thick disc models. In the thick disc cases with $q < 4R_0$ the magnitude of the warp is over-estimated, consistent with non-linear damping of strong vertical shearing motions.

The size of the warps present in the thin disc models seem to obey a proportionality relation with the tidal strength of the encounter but are over-estimated by a factor of $\sim 10$. The three times larger sound-crossing time ($\equiv \int c_s^{-1}dr$) in the thin disc models prevents saturation of the warps on the encounter timescale. In Figure (14) we give the warp evolution for model 3c. A decay timescale of approximately 60 units is inferred, being about 5 sound-crossing times. A similar result was found to hold for the thin disc models. This is consistent with bending modes that propagate and damp as acoustic waves (Papaloizou & Lin 1995a).

Inclination changes to the disc were typically small in cases such that $q > 3R_0$, for $q \sim 3R_0$ and $q \sim 2R_0$ the inclination...
Figure 12. Positional data for model 1c at a time $t = +13$.

Figure 13. Positional data for model 6c, the top frame gives the unperturbed particle configuration and the bottom frame gives the positions at a time $t = +28$ after pericentre passage.
changes were more significant at $\sim 1\%$ and $\sim 10\%$ respectively. The net change to the inclination was found to be much smaller than the tilt due to precession in all cases.

8 DISCUSSION

8.1 Summary

In this paper we have presented the results of analytical calculations and hydrodynamic simulations of the tidal interaction of a point mass encountering a viscous accretion disc on a parabolic prograde fly-by. We have considered the interaction for the case when the orbital plane of the secondary coincides with the midplane of the disc as well as the case when the two planes are inclined.

We have shown that for coplanar systems the most important component of the perturbing potential that gives a net tidal effect is the $m = 2$ bar mode applied at pericentre. Further to this we have demonstrated that the affect of introducing an anomolous shear viscosity of the magnitude thought to be relevant to protostellar accretion discs is only significant for very distant encounters. The exchange of angular momentum occurring between the secondary and the accretion disc is calculated from simplified inviscid considerations by modelling the interaction as a non-resonant impulse occurring at pericentre. A net torque is found to act on the disc due to the generation of a phase lag in the density response. To calculate the disc response we apply the pericentre torque over a finite interaction interval for a fixed radius for approximately equal time intervals. Squares represent data at time $t = +31$, pluses represent data at time $t = +39$, triangles represent data at time $t = +50$, crosses represent data at time $t = +61$ and diamonds represent data at time $t = +70$.

Figure 14. The magnitude of the range in the vertical elevation at a time of about 30 units. The thick disc data for models with $q \gg 4R_o$ is denoted by open squares, thick disc data for models with $q \sim 3R_o$ by open circles, the thin disc data by asterisks and the analytical result for the thick disc models by a solid line.

Figure 15. The warp evolution shown as the inclination versus radius for approximately equal time intervals. Squares represent data at time $t = +31$, pluses represent data at time $t = +39$, triangles represent data at time $t = +50$, crosses represent data at time $t = +61$ and diamonds represent data at time $t = +70$.

Table 2. Data for model parameters used in non-coplanar encounters. $\Delta \theta$ denotes the change to the orbital inclination of the disc, all other symbols are defined in the text. Angles are expressed in radians except for $\delta$ and the quantities in parentheses which are expressed in degrees.

| model | $H/r$ | $q$ | $\delta$ | $\Delta J$ | $\Delta \theta$ | $\Delta \Pi$ | $\tau$ |
|-------|-------|-----|---------|----------|-------------|----------|------|
| 1a    | 0.15  | 2.4 | 15      | $-1.5 \times 10^{-1}$ | $1.7 \times 10^{-2}$ | $-0.19$  | $4.9 \times 10^{-2}$ (2.8) |
| 1b    | 0.15  | 2.4 | 30      | $-1.3 \times 10^{-1}$ | $2.8 \times 10^{-2}$ | $-0.17$  | $8.5 \times 10^{-2}$ (4.9) |
| 1c    | 0.15  | 2.4 | 45      | $-9.7 \times 10^{-2}$ | $2.7 \times 10^{-2}$ | $-0.15$  | $1.1 \times 10^{-1}$ (6.1) |
| 2a    | 0.15  | 3.6 | 15      | $-2.0 \times 10^{-2}$ | $4.4 \times 10^{-3}$ | $-0.11$  | $2.8 \times 10^{-2}$ (1.6) |
| 2b    | 0.15  | 3.6 | 30      | $-1.9 \times 10^{-2}$ | $6.1 \times 10^{-3}$ | $-0.10$  | $5.0 \times 10^{-2}$ (2.9) |
| 2c    | 0.15  | 3.6 | 45      | $-8.7 \times 10^{-3}$ | $5.0 \times 10^{-3}$ | $-0.08$  | $5.7 \times 10^{-2}$ (3.2) |
| 3a    | 0.15  | 4.8 | 15      | $-2.1 \times 10^{-3}$ | $3.6 \times 10^{-4}$ | $-0.08$  | $2.1 \times 10^{-2}$ (1.2) |
| 3b    | 0.15  | 4.8 | 30      | $-1.6 \times 10^{-3}$ | $4.9 \times 10^{-4}$ | $-0.07$  | $3.5 \times 10^{-2}$ (2.0) |
| 3c    | 0.15  | 4.8 | 45      | $-1.1 \times 10^{-3}$ | $2.7 \times 10^{-4}$ | $-0.05$  | $3.5 \times 10^{-2}$ (2.0) |
| 4a    | 0.15  | 6.0 | 15      | $-2.0 \times 10^{-4}$ | $0.6 \times 10^{-4}$ | $-0.06$  | $1.6 \times 10^{-2}$ (0.9) |
| 4b    | 0.15  | 6.0 | 30      | $-1.9 \times 10^{-4}$ | $0.7 \times 10^{-4}$ | $-0.05$  | $2.5 \times 10^{-2}$ (1.4) |
| 4c    | 0.15  | 6.0 | 45      | $-1.5 \times 10^{-4}$ | $0.3 \times 10^{-4}$ | $-0.04$  | $2.8 \times 10^{-2}$ (1.6) |
| 5c    | 0.05  | 2.4 | 45      | $-7.2 \times 10^{-2}$ | $2.7 \times 10^{-2}$ | $-0.15$  | $1.1 \times 10^{-1}$ (6.1) |
| 6c    | 0.05  | 3.6 | 45      | $-4.1 \times 10^{-3}$ | $1.4 \times 10^{-3}$ | $-0.07$  | $4.9 \times 10^{-2}$ (2.8) |
| 7c    | 0.05  | 4.8 | 45      | $-2.4 \times 10^{-4}$ | $0.2 \times 10^{-3}$ | $-0.05$  | $3.5 \times 10^{-2}$ (2.0) |
| 8c    | 0.05  | 6.0 | 45      | $-5.3 \times 10^{-5}$ | $0.3 \times 10^{-3}$ | $-0.04$  | $2.8 \times 10^{-2}$ (1.6) |
a $m = 1$ bending mode is excited whose action is to warp the disc, cause it to precess away from its original plane and give a small change to the inclination. The precession of the disc produces a small tilt of the disc midplane relative to its initial configuration, this effect is always more important than the inclination change in determining the final orientation of the disc. The size of the tilt is significant even for the weakest encounters considered, namely those with the distance of closest approach of order five times the outer radius of the disc, for which the exchange of angular momentum is a very small fraction of the total disc content. There is also evidence to suggest that the $m = 1$ component of the perturbing potential can become as important as the $m = 2$ component in the angular momentum exchange at high inclinations.

8.2 Consequences for protostellar accretion discs

8.2.1 Silhouette discs in the Orion Nebula

Clarke & Pringle (1991) estimated the encounter rate, $\Gamma$, for parabolic point masses encountering discs. For the densest parts of the Trapezium cluster they estimated $\Gamma \sim 0.1\text{Myr}^{-1}$ for discs with $R_o \sim 100\text{AU}$. They argued that since disc lifetimes are $\sim 1\text{Myr}$ then disc penetrating encounters are rare. We note that approximately $\Gamma \propto q^2$. The implication of this is that encounters with $q \sim 3R_o$ are the rule rather than the exception.

In our numerical models encounters with $q \sim 3R_o$ produce disc tilts $\sim 3^\circ$ and show tidal truncation such that the discs become more compact with strongly cut-off edges. Directly imaged silhouette discs in the Trapezium cluster are found to exhibit a similar property in their surface density profiles (McCaughrean & O'Dell 1996). Also the largest silhouette disc (Orion 114-426; with $R_o \sim 500\text{AU}$) which is almost edge-on to the line of sight, shows a clear radial asymmetry and vertical distortion. But note that other environmental influences cannot be ruled out as explanations for these features of the observations (O'Dell, Wen & Hu 1993, McCaughrean & O'Dell 1996).

8.2.2 Discs in eccentric binary systems

Korykansky & Papaloizou (1995) noted that it is possible to model the accretion disc response in an eccentric binary system with a succession of parabolic pericentre passages. In this picture the accretion disc suffers a series of impacts in which it loses angular momentum to the secondary, thus reducing its lifetime. The secondary would increase its eccentricity with every impact.

In coordinates based on the primary, a secondary with specific angular momentum $L$ and orbital eccentricity $e$ has a pericentre distance $q$:

$$q = \frac{1}{1 + e} \frac{L^2}{G(M_p + M_s)}.$$

Therefore at pericentre we can write the change in the eccentricity $\delta e$ in terms of the change to the specific angular momentum content of the disc $\delta J$:

$$\delta e = -\frac{2(1 + e)}{L} \delta J.$$

If we then consider the simplified case with the mass of the disc distributed in a thin ring of radius $R_o$, being in Keplerian rotation about the primary alone, then we find:

$$\delta e = -2\Delta J \left( \frac{R_o}{q} \right)^{1/2} \sqrt{\frac{1 + e}{1 + \mu}}.$$

Then considering further extreme parameters for angular momentum exchange, such as $\mu \sim 1$, $e \sim 1$ and $q \sim 2R_o$ we find that $\delta e \sim \Delta J$. Thus if we take the values for $\Delta J$ present in our models we would expect that eccentricity changes to the secondary are generally less significant. The implication of this is that an eccentric binary may remove a significant fraction of the disc's angular momentum without becoming unbound.

8.3 Consequences for planetary systems

8.3.1 The Solar system

If planets form near the midplane of an accretion disc and if protoplanetary discs can become tilted by some mechanism without exerting a significant torque upon the central star, then it is reasonable to suppose that planetary systems are able to form with non-zero inclinations to the stellar equator.

We note that it is trivial to show from standard astronomical data that the invariable plane of the Solar system is tilted by $\sim 6^\circ$ with respect to the Solar equator.

The largest tilt angles we found in our simulations were $\sim 6^\circ$ for $q \sim 2R_o$. It is therefore plausible that a unique close encounter in the early history of the Solar system could have caused the Solar obliquity. If on the other hand we were to suppose a number of more distant but coherent encounters then we must appeal to the existence of an eccentric Solar binary companion which has long since become unbound through its tidal interaction with the primitive Solar disc. However, our result of the previous Subsection indicates that unless it had an eccentricity very close to unity, a Solar binary should still be in evidence.

8.3.2 The Beta Pictoris system

A gaseous disc in which there already exists a planet located within a tidally cleared gap (Lin & Papaloizou 1979b) interacts with that planet through gravitational torques. If the outer disc becomes tilted due to a close binary encounter then the planetary orbit would not necessarily tilt rigidly with the rest of the disc. A relative inclination between the disc and the planetary orbit would be generated.

We note that the dusty circumstellar disc associated with Beta Pictoris shows an inner warp $\sim 3^\circ$ (Burrows, Krist & Stapelfeldt 1995). This has been modelled with an inclined planet orbiting interior to the disc (Mouillet et al. 1997). Asymmetries on the scale of the disc have also been noted and a stellar encounter proposed as a possible mechanism for their generation (Kalas & Jewitt 1995).

ACKNOWLEDGMENTS

This work was supported by PPARC grant GR/H/09454 and the author is supported by a PPARC studentship.
Thanks also go to Dr. Mark McCaughrean for useful discussions on the silhouette disc observations and to Professor John Papaloizou for his support and advice throughout this project. Dr. Caroline Terquem is thanked for providing instructive and insightful comments as the referee of this paper.

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