Interference in dielectrics and pseudo-measurements

Todd A. Brun*
Physics, Queen Mary and Westfield College
University of London, London E1 4NS, United Kingdom
and
Stephen M. Barnett
Department of Physics and Applied Physics,
University of Strathclyde,
107 Rottenrow, Glasgow G4 0NG, United Kingdom
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Abstract

Inserting a lossy dielectric into one arm of an interference experiment acts in many ways like a measurement. If two entangled photons are passed through the interferometer, a certain amount of information is gained about which path they took, and the interference pattern in a coincidence count measurement is suppressed. However, by inserting a second dielectric into the other arm of the interferometer, one can restore the interference pattern. Two of these pseudo-measurements can thus cancel each other out. This is somewhat analogous to the proposed quantum eraser experiments.

I. INTRODUCTION

In recent years, experimenters have gained an unprecedented ability to perform experiments on single, microscopic quantum systems: individual atoms or ions in traps, photons in optical systems, electron pairs in Josephson junctions. One benefit of this increased experimental power is the ability to probe the physics of measurement itself.

Bohr formulated quantum mechanics solely in terms of the observation of quantum systems by classical measurement devices. To him, a quantum measurement must be “based on registrations obtained by means of suitable amplification devices with irreversible functioning such as, for example, permanent marks on a photographic plate, caused by the penetration of electrons into the emulsion. In this connection, it is important to realize that the quantum-mechanical formalism permits well-defined applications referring only to such closed phenomena” [1]. Thus, to measure a quantum system was to destroy it.

*Current address: Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106-4030 USA
In the experiments of the day this was certainly true; but improvements in experimental technique now make it possible to probe individual systems without destroying them, or to make repeated or even continuous measurements on a single microscopic system.

Jeffers and Barnett [2] have described an experimental set-up using a two-photon interferometer, based on earlier proposals of Franson [3] and Steinberg, Kwiat, and Chiao [4]. Passing entangled pairs of photons through this apparatus produces an interference pattern in the measurement of coincidences. If one inserts a lossy dielectric into one arm of the interferometer, this interference pattern is substantially destroyed, even though neither photon was absorbed by the dielectric. In effect, because neither photon was absorbed, one has gained information about which arm of the detector they passed through. This set-up is described in section II.A. (See figure 1.)

This situation is in many ways similar to an ordinary quantum measurement. Information is gained about the state of the system; the superposition which leads to the interference pattern is suppressed. Moreover, this works by permitting the photons to interact with an essentially classical, dissipative environment with many internal degrees of freedom (the dielectric), quite similar to a classical measuring device. What is more, in this interaction, the photon is not destroyed – one keeps only the events in which two photons were present (the coincidences).

Nevertheless, there are a number of important differences between this and a measurement. The superposition is only modified, not destroyed; by an appropriate manipulation, coherence can be retained, and the interference pattern restored. We describe in section II.B a related experiment, where by inserting a second dielectric into the other arm of the interferometer we can restore the interference pattern (though naturally the overall count rate goes down). Thus, a second of these “pseudo-measurements” can undo the effects of the first. (See figure 2.)

This experiment is similar to the proposed quantum erasers [5], in which interference is destroyed by allowing the photons to interact with an auxiliary system which stores “which-way” information. In these erasers, the interference pattern can be restored by performing an appropriate measurement on the auxiliary system and plotting coincidences.

In section III we discuss the implications of this experiment for the understanding of measurements, and its similarities to and differences from the quantum eraser.

II. THE EXPERIMENTAL SET-UP

The entangled photon pair is produced by parametric down conversion and passed through an interferometer (see figures 1 and 2). If the dielectrics are absent, one can adjust the lengths of the two arms of the interferometer so that the electromagnetic pulses arrive at the beam splitter simultaneously. An overlap between the wave packets causes them to tend to leave the beam splitter by the same port [6–8]. If one then measures coincidences in the two detectors $D_a$ and $D_b$, their rate falls off as the overlap of the wave packets improves. It is possible to tune to a dark fringe where the number of coincidences goes to zero.

The photon pair produced by parametric down-conversion may be described by the state

$$|\Psi\rangle = \int_0^{\infty} d\omega \; f_1(\omega) f_2(\omega) \hat{a}_1^\dagger(\omega) \hat{a}_2^\dagger(\Omega - \omega) |0\rangle,$$  \hspace{1cm} (1)
where $\hat{a}^\dagger_i(\omega)$ is the creation operator for a photon in arm $i$ with frequency $\omega$, and $|0\rangle$ is the vacuum state of the free-space electromagnetic field. The functions $f_1$ and $f_2$ are bandwidth functions, which we will assume to be narrow, and $\Omega$ is the frequency of the down converted photon.

If the two entangled photons pass into dielectrics instead, one replaces the operators $\hat{a}^\dagger_i$ above with polariton creation operators $\hat{c}_i^\dagger(x, \omega)$ and the electromagnetic vacuum with $|0_{\text{dial}}\rangle$, the ground state of the polaritons within the dielectric \[2,9\].

If we pass the photons through a narrow-bandwidth filter we can specify the functions $f_1$ and $f_2$ such that

$$f_1(\omega)f_2(\omega) = \exp\left[-\frac{(\omega - \Omega/2)^2}{B^2}\right],$$

(2)

where $B^2$ determines the frequency spread. It then becomes a good approximation to expand the wave vector $k(\omega)$ within the dielectric about the frequency $\Omega/2$,

$$k(\omega) = k_{\Omega/2} + \alpha(\omega - \Omega/2) + \beta(\omega - \Omega/2)^2.$$  

(3)

In a lossy medium, the constants $k_{\Omega/2}$, $\alpha$, and $\beta$ will be complex numbers; Re($\alpha$) is the inverse group velocity, and Re($\beta$) is the dispersion, while the imaginary components determine the linear and quadratic dependence of the loss.

A. The single-dielectric experiment

As shown in \[2\], the probability of detecting two photons at $D_a$ and $D_b$ at times $t_a$ and $t_b = t_a + \tau$, respectively, is

$$P(t_a, t_b) = \eta \langle \hat{E}_a^-(t_a) \hat{E}_b^-(t_b) \hat{E}_a^+(t_b) \hat{E}_b^+(t_a) \rangle,$$  

(4)

where $\eta$ is the (constant) detector efficiency, and $\hat{E}_a^+(t_a)$ is the positive-frequency component of the electric field at the detector $D_a$ at time $t_a$, and similarly for the other electric field operators. These fields will be superpositions of the fields $\hat{E}_1$ and $\hat{E}_2$ from the first and second arms of the interferometer, respectively. A 50-50 beam splitter will produce superpositions at $D_a$ and $D_b$ which are $\pi/2$ out of phase with each other.

In terms of the operators $\hat{E}_1$ and $\hat{E}_2$ this joint probability is

$$P(t_a, t_b) = \frac{\eta}{4} \langle (\hat{E}_1^- - i\hat{E}_2^-)(t_a)(\hat{E}_2^- - i\hat{E}_1^-)(t_b) \rangle \times (\hat{E}_2^+(t_b) + i\hat{E}_1^+(t_b))(\hat{E}_1^+(t_a) + i\hat{E}_2^+(t_a)) \rangle.$$  

(5)

There is only one photon in each beam of 1 and 2, so we can drop the terms proportional to $(\hat{E}_i)^2$, and are left with

$$P(t_a, t_b) = \frac{\eta}{4} \langle (\hat{E}_1^- - i\hat{E}_2^-)(t_a)(\hat{E}_2^- - i\hat{E}_1^-)(t_b) \rangle \times (\hat{E}_2^+(t_b) \hat{E}_1^+(t_a) - \hat{E}_1^+(t_b) \hat{E}_2^+(t_a)) \rangle.$$  

(6)
The electric field operator leaving the dielectric in arm 1 is given by

\[ \hat{E}_1^+(x_1, t) = C_1 \int_0^\infty d\omega_1 \hat{c}_1(0, \omega_1) e^{i(k_1(\omega_1)x_1 - \omega_1 t)} \] (7)

plus a Langevin term which vanishes when applied to \(|0_{\text{diel}}\rangle\), the ground state of the dielectric. \(C_1\) is a complex constant. The field operator in arm 2 is

\[ \hat{E}_2^+(x_2, t) = iC_2 \int_0^\infty d\omega_2 \hat{a}_2(\omega_2) e^{-i\omega_2(t - x_2/c)} , \] (8)

where \(C_2\) is a real constant. The lengths of the two arms are \(x_1\) and \(x_2\), respectively. Given the initial state \((1)\) (with \(\hat{a}_1^\dagger\), of course, replaced by \(\hat{c}_1^\dagger\)), we can calculate the term

\[ \hat{E}_2^+(x_2, t_b)\hat{E}_1^+(x_1, t_a) = iC_1C_2 \int_0^\infty d\omega_1 f_1(\omega) f_2(\omega) \]
\[ \times \int_0^\infty d\omega_1 \hat{a}_2(\omega_2)\hat{c}_1(0, \omega) \hat{c}_1^\dagger(0, \omega) \hat{a}_2^\dagger(\Omega - \omega) |0_{\text{diel}}\rangle . \] (9)

The usual delta-function commutators give

\[ \hat{a}_2(\omega_2)\hat{c}_1(0, \omega) \hat{c}_1^\dagger(0, \omega) \hat{a}_2^\dagger(\Omega - \omega) |0_{\text{diel}}\rangle = \delta(\omega - \omega_1)\delta(\omega_2 - \Omega + \omega) |0_{\text{diel}}\rangle . \] (10)

Similar expressions hold for the other terms in \((9)\). Substituting the bandwidth function \((3)\) and the expression for the wave vector \((3)\), it is then possible to do the integrals and evaluate the coincidence probability.

Let us assume that the detectors integrate over a time \(T\), and let \(T\) become very large. Then if we integrate \(t_a\) and \(t_b\) over a full range of times \([2]\), we get a total coincidence probability of

\[ P_c = \kappa \left( 1 - \exp \left[ -\frac{(x_1 \text{Im}(\alpha_1))^2}{B^{-2} + 2x_1 \text{Im}(\beta_1)} \right] \exp \left[ -\frac{\tau_r^2}{B^{-2} + 2x_1 \text{Im}(\beta_1)} \right] \right), \] (11)

where \(\tau_r\) is the time-delay between the two arms of the interferometer

\[ \tau_r = x_2/c - \text{Re}(\alpha_1)x_1 . \] (12)

\text{Re} \alpha is the inverse group velocity associated with the dielectric in arm 1, as given in \((3)\), and \(\kappa\) is an overall constant which reflects the probability of no photon being absorbed by the dielectric. By adjusting the lengths of the arms, it is possible to set \(\tau_r = 0\). In the absence of the dielectric (i.e., \(\text{Im}(\alpha) = 0\)) this is tuning to a dark fringe; the coincidence probability becomes identically zero. With the dielectric present, however, there is always a nonzero probability of a coincidence. This reduction of the interference pattern by interaction with a lossy medium is very similar to the effect of measuring one path of a two-slit experiment, or to the destruction of interference in the quantum eraser due to interaction with a system storing “which-way” information.
B. The two-dielectric experiment

Suppose now that we insert a second dielectric into the other arm of the interferometer, as in figure 2. How does this modify the probability of coincidences? The derivation is almost identical to that before. In the initial state (11) both of the electromagnetic creation operators \( \hat{a}_i(\omega_i) \) are replaced by polariton creation operators \( \hat{c}_i(0, \omega_i) \); the expression (8) for the electric field in arm 2 is replaced by an expression analogous to (7). A dispersion relation (3) holds for both arms. Carrying out the integrals as before, we find that the probability of coincidences becomes

\[
P_c = \kappa' \left(1 - \exp\left[-\frac{(x_1 \text{Im}(\alpha_1) - x_2 \text{Im}(\alpha_2))^2}{B^{-2} + x_1 \text{Im}(\beta_1) + x_2 \text{Im}(\beta_2)}\right] \times \exp\left[-\frac{\tau_r^2}{B^{-2} + x_1 \text{Im}(\beta_1) + x_2 \text{Im}(\beta_2)}\right]\right),
\]

(13)

where \( \kappa' \) is now an overall constant representing the probability of neither photon being absorbed, and \( \tau_r \) becomes

\[
\tau_r = x_2 \text{Re}(\alpha_2) - x_1 \text{Re}(\alpha_1).
\]

(14)

Here we see that the visibility now depends on the difference between the absorption in the two dielectrics at the frequency \( \Omega/2 \). By choosing the arm lengths \( x_1 \) and \( x_2 \) and the linear absorptions \( \text{Im}(\alpha_1) \) and \( \text{Im}(\alpha_2) \) appropriately, we can restore the dark fringe, resurrecting the interference pattern. Thus, adding a second "pseudo-measurement" undoes the effect of the first, just as inserting an appropriate measurement of the ancilla in the quantum eraser can restore the interference effect.

III. DISCUSSION

Great care in interpretation is required as experiments begin to probe individual quantum systems nondestructively. It was possible to reconstruct the interference pattern in section 2b, because although the interference pattern had been exponentially suppressed, it had been suppressed coherently. By selectively suppressing the other component of the wavefunction the pattern could be restored, though at a large cost in the overall count rate.

An analogy to this might be the two slit experiment. Suppose one could narrow one of the two slits, so that fewer particles could pass through it relative to the other. One would have gained some information about the system—particles are more likely to pass through slit 2 than slit 1—and the interference pattern on the screen would be greatly reduced. It could be restored, however, by narrowing slit 2 as well, reinstating the full contrast between bright and dark fringes, at the cost of a large reduction in the brightness of the whole pattern.

This type of experiment examines one piece of the process of measurement, but only one. Measurement involves the correlation of a quantum system with the many degrees of freedom of a classical object, such as a measuring device. In the case of this experiment, this role is played by the internal degrees of freedom of a lossy dielectric [4]. But more than
this is required: the information transferred to these internal degrees of freedom must be unrecoverable.

Certainly this is so if one of the photons is actually absorbed. It is in principle possible to tell whether a photon was absorbed or not; one could measure the energy absorbed by the dielectric, for example. The detailed information of the photon’s phase and correlations, however, would be lost.

In this experiment, however, we are looking at the case where the photon escapes unscathed. Though the amplitude of the component passing through the dielectric is greatly diminished, it is still present, and retains all the relevant information about the quantum state. Thus, suitable manipulations can recover this information and demonstrate the survival of its correlations.

This situation differs somewhat from the quantum eraser \[5\]. In that experiment, the photons interact with an auxiliary system with a low number of degrees of freedom. This interaction, in effect, measures which path the photons take. The entanglement with the auxiliary system destroys the interference pattern, but there is no loss of coherence; by combining the photon measurement with an appropriate measurement of the auxiliary system, it is possible to fully restore the interference fringes.

The two experiments have points in common as well, of course. In each case, an interaction is performed which captures one aspect of the measurement process, but which is reversible by a second interaction. This reversibility in both cases arises because the information which has been extracted from the quantum system is retained coherently—in the case of the dielectric in the diminished (but not destroyed) component of the wavefunction, and in the case of the quantum eraser in the entanglement with the ancilla. A measurement, in Bohr’s sense, must be irreversible to be final.

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Figure 1. The experimental set-up for the single-dielectric interferometer.

Figure 2. The experimental set-up for the two-dielectric interferometer.
Figure 1. The single-dielectric experimental setup for two-photon interference.
Figure 2. The two-dielectric experimental setup for two-photon interference.