Non-time-orthogonal reference frames in the theory of relativity

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Abstract
A simple, though rarely considered, thought experiment on relativistic rotation is described in which internal inconsistencies in the theory of relativity seem to arise. These apparent inconsistencies are resolved by appropriate insight into the nature, and unique properties, of the non-time-orthogonal rotating frame. The analysis also explains a heretofore inexplicable experimental result.

I. Introduction

Although addressed by Einstein\cite{1,2,3} and others\cite{4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19} in the first half of the twentieth century, the relativistically rotating reference frame continues to be a research topic of interest, and, in fact, has often generated significant discussion and debate\cite{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19} (References cited in this section are not exhaustive.) Some\cite{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19}, who have considered the thought experiment described below and/or

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the Sagnac experiment, have proposed the possible existence of a preferred reference frame, which is somehow disguised within our present understanding of special relativity.

The present article does not support this view, and in the final analysis, is consonant with the special and general theories of relativity. It does, however, illustrate one way in which the extant theory does appear to be self-contradictory. This seeming internal incongruity is resolved by analysis of the non-time-orthogonal nature (i.e., time is not orthogonal to at least one spatial dimension) of the rotating frame. In the process, however, non-time-orthogonal (NTO) frames are found to exhibit certain unique and somewhat surprising characteristics. Though some of these characteristics are not what might presently be considered typically relativistic, they are not only in agreement with empirical evidence, but explain what has heretofore been considered an anomalous experimental result.

II. Rotating Frames in Thought and Practice

A. Simple Gedanken Experiment

Consider the rotating disk of Figure 1 with a rim mounted light source capable of emitting light pulses in both directions along the disk circumference. A cylindrical mirror, polished side facing inward, is mounted on the rim as well.

\[
\begin{align*}
    \omega & > 0 \\
    T & = 0 \\
    \omega & < 0 \\
    T & > 0
\end{align*}
\]

Figure 1. Thought Experiment

At time \( T = 0 \), an observer attached to the light source triggers the emission of two very short light pulses, one in the clockwise direction, the other in the counter clockwise direction. Each light wave packet is 1° long at the rim radius, i.e., it is 1/360 of the rim circumference.

In the lab frame each light pulse has speed \( c \), and each is reflected by the cylindrical mirror, such that it travels a circular route with radius equal to the disk radius \( r \). As the two light pulses are travelling, the disk is rotating c.c.w. at a high enough to produce relativistic rim velocities. Since the observer mounted on the light source is moving c.c.w. as well, the c.w. moving light pulse reaches him before the c.c.w. pulse does. From the point of view of the lab, this conclusion is inescapable, and since we are talking about separate detectable physical events, it must be true from the point of view of the observer on the disk as well.

The disk observer knows that each light signal traveled the same distance in his frame (he set up the experiment). He also knows that one of them took less time to travel that distance than the other. Hence, the only conclusion he can make is that the speed of light on the disk was greater for the c.w. travelling pulse than for the c.c.w. travelling pulse. Due to symmetry, it also seems inevitable that this conclusion holds true locally as well as globally.

\[20\] G. Sagnac, *Comptes Rendus*, 157, 708 (1913).

\[21\] E.J. Post, "Sagnac effect," *Mod. Phys.* 39, 475-493 (1967).
Note this “experiment” did not entail any wave interference measurements. It dealt simply with arrival times of short photon wave packets and had nothing to do with the wave nature of those packets.

The conundrum, of course, is that, according to relativity theory, the speed of light is invariant and always equal to $c$ in all directions, no matter what frame one is in. (Those who believe that the general theory of relativity may say otherwise are referred to the Appendix.) The problem is compounded when one considers that most analyses of relativistic rotation utilize an infinite series of local Lorentz frames instantaneously co-moving with the rotating frame at a given radius from the center of rotation. Calculations of things like spatial distance around the circumference are then made by summing (i.e., integrating) infinitesimal quantities (e.g., $dx$) from all of the local co-moving Lorentz frames.

But in Lorentz frames the speed of light is always $c$. So in effect, when one uses such frames one is assuming the local speed of light is $c$, isotropic and invariant. But, as we have illustrated in our thought experiment, this does not appear to be the case for rotating frames. And since the invariance of the speed of light is one postulate upon which the theory of relativity rests, one must immediately re-evaluate not only the suitability of such approaches, but also the very theory itself.

B. The Sagnac Experiment

The analysis of the previous section literally reeks of a preferred frame (“absolute” space), and proponents of such a thing commonly cite the Sagnac experiment (see Figure 2) as empirical proof.22

In the Sagnac experiment, a light beam is emitted radially from the center of a rotating disk and is split by a half-silvered mirror M at radius $r$. From there one part of the beam is reflected by mirrors appropriately placed on the disk such that it travels in one direction effectively around the circumference. The other half of the beam travels the same route over the same distance, but in the opposite direction. The beams then meet up again and are reflected back to the center where interference of the two beams results in a fringing, i.e., a displacement of one light wave with respect to the other.

If the speed of light on the disk were invariant, then as the rotational velocity of the disk increased, the fringe pattern would remain unchanged, similar to what one finds in the Michelson-Morley experiment. However, when this was done by Sagnac20 and others21 who have repeated his experiment, the fringe pattern did in fact change, indicating a dependence of the rotating frame speed of light on both direction and rotational speed.

The test results have experimental accuracy only to first order in $v/c = \omega r/c$, but they indicate that the speed of a light ray tangent to the circumference measured locally on the disk is equal to$^{23}$

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22 See, for example, reference 18.

23 Post (ref. 21) presents the Sagnac results in terms of the fringe shift (his equation (1)). Using (1) of the present paper, one can derive (1) in Post (use $A = \pi r^2$), and vice versa.
\[ |v_{\text{light}}| \cong c \pm \omega r, \]  

(1)

where the approximately equal sign implies accuracy to first order, and the sign in front of the last term depends on the relative direction of the rim tangent and light ray velocities. This same relationship can be easily derived using the logic of the previous section. It is obviously disconcerting, as (1) looks far more Galilean/Newtonian in nature than relativistic.

Some have attributed the Sagnac phenomenon to wave effects. Mashhoon et al.\(^{24}\), for example, consider it to be a “manifestation of the coupling of orbital angular momentum of a particle .. to rotation”. For a wave this perturbation in the Hamiltonian induces a phase shift such as that measured in the Sagnac experiment. In somewhat similar fashion, Anandan\(^{25}\) asserts “.. this effect depends only on the frequency of the beams ...”.

However, such analyses fail to answer the question raised by our thought experiment, which was not based on wave interference, but solely on arrival times of very short wave packets. As (1) is readily deduced from that thought experiment, it appears a more fundamental reconciliation with the theory of relativity is needed.

III. Transformation to the Rotating Frame

We will adopt what is presently the most widely\(^{10,11,26,27,28,29}\), though not universally\(^{17}\), accepted transformation (see (2.a-d) below) between the lab and rotating frames. This coordinate transformation, where upper case coordinates represent the inertial frame K, lower case denote the rotating frame k, and the axis of rotation is coincident with both the Z and z axes, is

\[
\begin{align*}
cT &= ct \quad (2.a) \\
R &= r \quad (2.b) \\
\Phi &= \phi + \omega t \quad (2.c) \\
Z &= z \quad (2.d)
\end{align*}
\]

\(\omega\) is the angular velocity of the disk, and \(t\), the coordinate time for the rotating system, is the proper time of a standard clock located at the origin of the rotating coordinate frame, i.e., it is equivalent to any standard clock at rest in K. Note that \(t\) is only a coordinate. It is merely a label and cannot be expected to equal proper time at any given point on the disk (except, of course, at \(r=0\)).

The metric for the rotating system can be found from the line element for the standard cylindrical coordinate system of the Minkowski space K

\[^{24}\text{Bahram Mashhoon, Richard Neutze, Mark Hannam, Geoffrey E. Stedman, “Observable frequency shifts via spin-rotation coupling”, Phys. Lett. A, 249, 161-166 (1998).}\]
\[^{25}\text{J. Anandan, “Sagnac effect in relativistic and nonrelativistic physics,” Phys. Rev. D 24(2), 338-346 (1981).}\]
\[^{26}\text{C. Moller, The Theory of Relativity (Clarendon Oxford, 1969).}\]
\[^{27}\text{L. D. Landau, and E. M. Lifshitz, The Classical Theory of Fields (Addison-Wesley, Reading, 1962), pp. 271-298.}\]
\[^{28}\text{Oyvind Gron,., "Relativistic description of a rotating disk", Am. J. of Phys. 43(10), 869-876 (1975).}\]
\[^{29}\text{Robert D. Klauber, “New perspectives on the relatively rotating disk and non-time-orthogonal reference frames”, Found. Phys. Lett. 11(5), 405-443 (1998). On page 421 Klauber lists assumptions upon which the transformation is based.}\]
\[^{30}\text{Adler, R., Bazin, M., and Schiffer, M., Introduction to General Relativity (McGraw-Hill, New York, 1975), 2nd ed., p. 121-122.}\]
\[^{31}\text{Post (reference 21) noted the transformation is determinable experimentally only to first order and suggested the presence of a factor he termed \(\gamma\) on the right hand side of (2.a) and in front of the second term on the right hand side of (2.c). He considered that this factor could be unity, the Lorentz contraction factor, or perhaps something else. Ehrenfest, Franklin, and Trocheries (references 4,5,6) considered transformations other than (2), which never proved completely satisfactory. Although Stedman (reference 19 and in private communication) contends there is latitude in the choice of transformation due to gauge freedom, he does consider the same transformation to the rotating frame used by Selleri (ref. 17).}\]
\[ ds^2 = -c^2dT^2 + dR^2 + R^2d\Phi^2 + dZ^2 \]  

(3)

Assuming \( ds \) is invariant, one can find \( dT, dR, d\Phi, \) and \( dZ \) from (2), and insert into (3) to obtain the line element, and hence the metric, of the coordinate grid in \( k \).

\[ ds^2 = -c^2 \left(1 - r^2 \omega^2/c^2\right) dt^2 + dr^2 + r^2 d\phi^2 + 2r^2 \omega d\phi dt + dz^2 \]  

(4)

\[ = g_{\alpha\beta} dx^\alpha dx^\beta \]

For a fixed point in the rotating frame (i.e., \( dr = d\phi = dz = 0 \)) with \( ds^2 = -c^2 d\tau^2 \) inserted into (4), we find the local proper time on the disk to be

\[ d\tau = \left(1 - r^2 \omega^2/c^2\right)^{1/2} dt = \left(1 - r^2 \omega^2/c^2\right)^{1/2} dT. \]  

(5)

That this is the familiar Lorentz time dilation factor (with \( v = \omega r \)), in full accord with numerous cyclotron experiments, supports the contention that (2.a-d) is indeed the correct form of the transformation.

**IV. Speed of Light in NTO Frames**

Note that the metric \( g_{\alpha\beta} \) of (4) is non-diagonal, and hence the rotating frame is non-time-orthogonal (NTO). The presence of the non-zero line element term in \( d\phi dt \) indicates that in 4D spacetime, the time axis is not orthogonal to the spatial axis for the circumferential direction. As we shall see, this has profound implications for measurement of the speed of light.

**A. Analytical Determination of the Speed of Light**

Consider the path of a photon travelling in the circumferential direction, e.g., along the disk rim. For light, \( ds = 0 \) (see (3)), and for the path considered, \( dr = d\phi = dz = 0 \). Inserting these values in (4), solving the resultant quadratic equation for \( d\phi \), dividing by \( dt \), and multiplying by \( r \), one obtains a coordinate velocity for the photon as seen from the rotating frame

\[ v_{\text{light,circum,coord}} = \frac{rd\phi}{dt} = -r \omega \pm c. \]  

(6)

To find the physical velocity one would measure with standard rods and clocks mounted on the rotating frame, we must use (5) to convert coordinate time \( dt \) in (6) to physical time on standard rotating clocks at radius \( r \). This yields

\[ v_{\text{light,circum,phys}} = \frac{rd\phi}{d\tau} = \frac{1}{\sqrt{1 - r^2 \omega^2/c^2}} \frac{rd\phi}{dt} = \frac{-r \omega \pm c}{\sqrt{1 - v^2/c^2}}, \]  

(7)

which is the exact form of the approximate (first order) relationship (1) deduced from the Sagnac experiment and our thought experiment.

Note that had we started with a time orthogonal frame, there would have been no \( d\phi dt \) type off diagonal term in (4), and hence no \( r \omega \) term in (7). (For example, take the lab frame with \( \omega = 0 \) in (7). Alternatively, set the off diagonal term to zero in (4) and follow the steps used to derive (7).)

It is important to note that the non-relativistic looking, Newtonian-like, velocity addition relationship of (7) is a direct result of non-time-orthogonality. In NTO frames light speed is neither invariant nor isotropic. In the time orthogonal frames more typically dealt with in special and general relativity, it is invariant and isotropic. Further the degree of anisotropy in the speed of light is directly correlated with the magnitude of the NTO off diagonal term in the frame’s line element.
B. Picturing NTO Frames and Light Speed

The effect of non-time-orthogonality can be visualized with the aid of Figure 3, which shows the time and spatial circumferential axes of both the lab frame $k$ and the rotating frame $K$ at radius $r$ $(=R)$. We set up our coordinates such that $\phi = z = \Phi = Z = 0$, and note that only very small changes in $\phi, \Phi, t,$ and $T$ are considered. The thinner (orthogonal) lines represent the lab frame axes (upper case); the thicker (NTO) lines, the rotating frame (lower case).

\[
\begin{align*}
\text{Figure 3. NTO vs TO Frames} & \quad \text{Figure 4. Two TO Frames}
\end{align*}
\]

Note that the effect of the time dilation factor appearing in (4), (5), and (7) is masked, because we are plotting the time axis of the rotating frame as our coordinate time $t$ (time on a clock at $r=0$). This is not the physical time on a standard clock fixed to that frame at radius $r \neq 0$. If one wishes, one can simply multiply the coordinate time $t$ by the (second order) time dilation factor at any point in the discussion of this section to obtain exact relationships for physical quantities. For the present, however, the primary emphasis is on first order effects, as reflected in relationship (1).

From the line element (4) and basic trigonometry (generalized Pythagorean theorem) one can determine the slope of the rotating frame time axis. Alternatively, set the RHS of (3) equal to RHS of the first line in (4) with $d\phi = dr = dz = dR = dZ = 0$, use (2.a) and (2.b), and solve for $Rd\Phi/cdT$.

MN is the path of a light ray and has null path length, i.e., $ds = 0$. Observe that for a given amount of coordinate time (which is the same in both $k$ and $K$, i.e., $c\Delta T = c\Delta t$), the light ray travels a certain spatial distance $l$ in $k$, but a greater spatial distance $L$ in $K$. Hence the speed of light measured in $k$ is less than that in $K$, and this corresponds with the plus sign before the $c$ in (6). For a light ray in the opposite direction (minus sign in (6)) one can show graphically (with a light ray in the second quadrant of Figure 3 at right angle to MN) that the corresponding $l$ distance is greater than $L$, and hence the velocity for that ray would be greater in $k$ than in $K$.

Given that the slope of MN is unity, $L = c\Delta T$. Dividing this by $\Delta T$, one gets the speed of light in $K$ as $c$. The $k$ time axis has slope $c/v = c/\omega r$, so

\[
l = L - c\Delta T (\omega r/c) .
\]

Dividing this by $\Delta T$, one arrives at (6) for the coordinate speed of light in $k$ (with the plus sign for $c$ since light ray MN is traveling in the direction of disk rotation). Note that larger values of $\omega$ or $r$ mean a greater degree of non-time-orthogonality and light speed discrepancy from $c$.

Figure 4, presented for completeness, depicts a co-moving inertial (Lorentz) frame $K_1$, having instantaneous velocity equal to the circumferential velocity of the rotating frame at $r$. Note that in both inertial frames $K$ and $K_1$, time is (Minkowski spacetime) orthogonal to 3D space and the speed of light ray MN equals $c$.

In general one can conclude that non-invariance and degree of anisotropy for the local, physically measurable speed of light are directly dependent on the slope of the time axis relative to the 3D space axes. All
frames for which time is orthogonal to space have isotropic light speed equal to $c$. All NTO frames have anisotropic light speed not equal to $c$.

Note that both the rotating frame and the instantaneously co-moving Lorentz frame exhibit some of the same properties. They both have the same time dilation as seen from the lab, as well as the same 4D interval $ds$ between events. However, because of differences in time orthogonality, each has different measured speeds for light. Hence, a co-moving Lorentz frame can not simply be assumed to be an appropriate local surrogate for the rotating frame.

V. Comparison with Experiment

A. Michelson-Morley Revisited

Although the analysis herein is supported by the Sagnac experiment, one might ask why then did Michelson and Morley not find the speed of light in the directions of galactic and solar orbital rotation different from that in other directions? The answer, the author submits, is that bodies in gravitational orbits follow geodesics, i.e., they are in "free fall". That is, they are in locally inertial, time orthogonal frames and therefore obey Lorentzian mechanics.

Consider a planet in orbit around a star that doesn’t rotate relative to distant stars (i.e., one solar day equals one year). An observer inside a windowless box (similar to Einstein’s enclosed gedanken elevator) on that planet could do tests (pails of water, Foucault pendulum, Coriolis effects, etc.) to determine that she is not rotating. Hence her frame, the frame of the planet, would be time orthogonal, and her experiments would also find the speed of light to be isotropic, equal to $c$, and independent of the planet’s orbital velocity.

If, however, her planet were rotating relative to distant stars at $\omega$, her measurements would detect a rotation rate of $\omega$, independent of orbital angular velocity around her own star. Her frame would then be NTO, with all of the concomitant phenomena described in Section IV. These phenomena would depend only on $\omega r$, the surface velocity of the planet relative to the Lorentz frame in which its axis of rotation is fixed. No variation in the speed of light would be found from the solar or galactic orbital velocities.

It is noteworthy that Michelson and Gale measured the Sagnac effect for the earth’s surface velocity in the 1920’s. And in order to maintain accuracy, the Global Positioning System must apply a Sagnac velocity correction to its electromagnetic signals.

B. Modern Michelson-Morley Experiment

Although the original Michelson-Morley experiment and almost all subsequent tests of similar nature were not precise enough to detect any non-null effect due to the earth surface velocity, one such test was. In 1978, Brillet and Hall found a “null” effect at the $\Delta t/t = 3\times10^{-15}$ level, ostensibly verifying standard relativity theory to high order. However, to obtain this result they subtracted out a persistent “spurious” non-null signal of amplitude $2\times10^{-13}$ at twice the apparatus rotation frequency.

In 1981 Aspen pointed out that this “spurious” signal would correspond to a test apparatus velocity of 363 m/sec. The earth surface velocity due to its rotation at the test site is 355 m/sec.

VI. Related Issues

In this section we briefly discuss several other rotating frame issues that are either prevalent in the literature or otherwise worth addressing.

32 A.A. Michelson, and H.G. Gale, “The effect of the earth’s rotation on the velocity of light, Part II,” Astrophys. J. 61, 140-145 (1925). See also A.A. Michelson, “The effect of the earth’s rotation on the velocity of light, Part I,” Astrophys. J. 61, 137-139 (1925).

33 D. W. Allan, and M. A. Weiss, “Around-the-World Relativistic Sagnac Experiment,” Science, 228, 69-70 (1985).

34 A. Brillet and J. L. Hall, “Improved laser test of the isotropy of space,” Phys. Rev. Lett., 42(9), 549-552 (1979).

35 Mark P. Haugan and Clifford M. Will, “Modern tests of special relativity,” Phys. Today, May 1987, 69-76. It is ironic that Haugan and Will cited the Brillet and Hall experiment as proof of the invariance of the speed of light.

36 H. Aspen, “Laser interferometry experiments on light speed anisotropy,” Phys. Lett., 85A(8,9), 411-414 (1981).
A. Relativistic Mass-Energy

Klauber\textsuperscript{37} uses transformation (2) to show that the mass-energy of an object fixed in the (NTO) rotating frame has typical relativistic dependence on speed, in this case $\omega r$. Given the well known cyclotron experiments, this lends further support for the correctness of transformation (2).

B. Generalized Coordinates and Light Speed

Some may argue, in the spirit of generalized coordinates common to general relativity, that the velocity of light deduced in Section IV is merely a coordinate value. That is, it is not what one would measure with standard rods and clocks, but an expression in terms of arbitrary coordinates, which may in fact be anything one chooses. By choosing the appropriate generalized coordinate system we could then “deduce” any value we like.

This argument is in fact erroneous. One can calculate physical values for distance, time, velocity or any other measurable quantity from the metric of the particular coordinate grid employed. (See the Appendix for an example.) “Physical” values are those one would measure using physical instruments such as standard rods and standard clocks. “Coordinate” values are those one calculates using the arbitrary values for length and time associated with any arbitrarily chosen grid. In effect, physical values are those values one calculates when the associated basis vector of the generalized coordinate system has unit length. Given any coordinate value associated with a non-unit basis vector, one simply calculates the equivalent (physical) value associated with a unit basis vector pointed in the same direction. Coordinate values can be any number, while physical values are unique. For details we refer the reader to Misner, Thorne, and Wheeler\textsuperscript{38}, Malvern\textsuperscript{39}, and Klauber\textsuperscript{40}.

Avoiding excessive complexity, we simply note that we have taken care that quantities in relationships such as (1) and (7) are indeed physical values. We note further that in the thought experiment of section II.A no coordinate grid whatsoever is used. The observer simply uses standard rods and clocks, which yield the same values regardless of the coordinate grid employed.

A related argument posits that the degree of orthogonality of any axis in generalized coordinates relative to any other axis is arbitrary, and hence we can choose our time axis in any direction we like. In fact, after using transformation (2), some\textsuperscript{41} assert that we must then transform to a locally time orthogonal frame. But this is effectively the same as using local Lorentz co-moving frames. This in turn necessitates invariant, isotropic light speed and gives rise to problems already discussed, as well as the discontinuity in time described in the following subsection. While in generalized coordinates we can arbitrarily define our $t$ coordinate in any of an infinite number of ways, if it is to represent the physical time nature chooses, then calculations done using it must match up with phenomena observed in the physical world.

We also note that any transformation to a rotating frame must incorporate the general form of (2.c). That is, the transformation of the azimuthal angle must include a term like $\omega t$, regardless of whether one believes other multiplicative factors (such as the second order Lorentz factor) should also be involved. When one squares the $d\Phi$ of (2.c), or any other relation with a $\omega dt$ term, and inserts the result in (3), one then ends up with off diagonal metric terms such as those of (4). Hence no matter what physically reasonable transformation\textsuperscript{42} one chooses, one must find that the rotating frame is NTO. And the primary effects of an NTO frame on observable phenomena are first order, i.e., they are independent of the presence or absence of a Lorentz factor in the transformation.

\textsuperscript{37} Ref. 29, pp. 427-429.
\textsuperscript{38} Charles W. Misner, Kip S. Thorne, and John A. Wheeler, \textit{Gravitation} (Freeman, New York, 1973), pp. 37, 821-822, and many other places throughout the text.
\textsuperscript{39} Lawrence E. Malvern, \textit{Introduction to the Mechanics of a Continuous Medium} (Prentice-Hall, Englewood Cliffs, New Jersey, 1969), Appendix I, Sec. 5, pp. 606-613.
\textsuperscript{40} Ref. 29, pp. 426-427.
\textsuperscript{41} See for example, ref. 30, pp. 124.
\textsuperscript{42} For justification of (2.a), see ref. 29, pp. 416-418.
C. Simultaneity in the Rotating Frame

Although this and the following subsection may seem counterintuitive to one cultured by a relativistic age, we suggest they deserve serious consideration as a possible way in which nature might actually work on rotating frames.

Using the co-moving local Lorentz frame methodology, one finds, due to the standard lack of agreement in simultaneity between Lorentz frames in relative motion, a quite bizarre result. (See Klauber[43].) If we consider a spatial path 360° around a given circumference, we find the clock at 360° has a different time on it than the clock at 0°, even though time remained constant all along the spatial path. This means the clock can not be synchronized with itself. It also implies that a continuous standard tape measure laid out around the circumference would not meet back up with itself at the same point in time. In other words, there would be a discontinuity in time.

Peres[44] noted this result as well, in addition to demonstrating that this methodology led to a “radial velocity of light [that is] not the same inward and outward.” He concluded, “All this is the heavy price which we are paying to make the azimuthal velocity of light ...equal to c.”

Reconciliation of analysis with reality occurs if simultaneity on a rotating disk is the same as that in the lab. Note that this is true for the time transformation of (2.a) wherein \( dt = 0 \) between two events, if \( dT = 0 \) between those same events. Hence \( (1-\omega^2 r^2/c^2)^{1/2} dt \), the time passed on standard (physical) clocks in the rotating frame, is also zero. (Note that clocks running at different rates can still agree that no time passed on either one between events, and so can share a common simultaneity.) For this definition of simultaneity, there is no discontinuity in time. A line painted around a closed path on the rotating disk then does meet back up with itself at the same point in time.

D. Length Contraction: To Be or Not to Be

The co-moving local Lorentz frames approach implies that standard rods on a rotating disk contract circumferentially. Many researchers[45] have concluded from this that the disk surface is therefore curved, thereby ostensibly resolving the famous Ehrenfest paradox[46].

However, Tartaglia’s[48] interpretation of Ehrenfest’s paradox is more insightful. He notes that in Lorentz frames each observer sees rods in the other’s frame as contracted, and “an observer on board the [rotating] disk would not perceive any curvature since in his reference frame [there is no contraction].”

Still further, if local Lorentz frames are valid surrogates for a rotating frame, then an observer on the rim of a rotating disk would likewise see the lab rods as contracted. And he would therefore conclude that the lab frame must be curved, which of course, it is not. Klauber[49] and Tartaglia both conclude that internal contradictions in the theory disappear only if there is no Lorentz contraction effect between the disk and lab frames.

Transformation (2) actually implies this. Consider a small circumferentially aligned rod of proper length \( R\Delta\Phi \) (\( \Delta\Phi \) is small) in the lab, which to a lab observer must have simultaneous endpoints, both at time \( T=0 \). Using (2.a), (2.b), and (2.c) one then finds the length of that rod as seen from the rotating disk to be \( r\Delta\phi = R\Delta\Phi \), i.e., no observed Lorentz contraction. The same logic works in reverse for a rod on the disk. Again, we emphasize that quantities discussed herein are physical, not merely coordinate, in nature.

The same conclusion may be drawn from the 4D line element \( ds \), which is invariant between frames, NTO or not, inertial or not. In general between any two frames (notation should be obvious)

\[
ds^2 = -c dt^2 + dX^2 + g_{XT}dXdT = -c dt^2 + dx^2 + g_{xt}dxdt
\]

where the off diagonal terms vanish for TO frames, and physical quantities are assumed (i.e., a coordinate grid with unit basis vectors is chosen.) Note that if the two frames share the same simultaneity, then when

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43 Ref. 29, pp. 413-415.
44 Asher Peres, “Synchronization of clocks in a rotating frame,” Phys. Rev. D, 18(6), pp. 2173-2174 (1978).
45 See ref. 3.
46 P. Ehrenfest, “Gleichfmrige Rotation starrer Kpfer und Relativittheorie,” Phys. Z. 10, 918-918 (1909).
47 See for example, ref. 9.
48 A. Tartaglia, “Lengths on rotating platforms”, Found. Phys. Lett., 12(1), 17-28 (1999).
49 Ref. 29, pp 418-419, 431-433, 437, 439.
\(dT = 0, \ dt = 0\), and therefore \(dx = dX\). The length \(dx\) of a rod in one frame equals the length \(dX\) of the same rod as seen from the other frame, i.e., there is no Lorentz contraction. (Note that if \(dT = 0\), but \(dt \neq 0\), then \(dx \neq dX\), and there is Lorentz contraction.)\(^{50}\)

The Lorentz contraction results from the Lorentz transformation, and in a sense is little more than an optical illusion. No Lorentz contracted object ever “feels” contracted. The contraction appears because of the disagreement in simultaneity between frames, which is inherent within the Lorentz transformation.

We conclude that if two frames share the same simultaneity, then there is no Lorentz contraction effect between those frames. Hence, equivalence of simultaneity for the lab and rotating frames means no Ehrenfest paradox, as well as no discontinuity in time.\(^{52}\)

E. Absolute Nature of Rotation

Translating systems display robust relativity. For such systems, absolute velocity does not exist, and there is no way to determine a preferred frame. As a result, light speed is invariant and transformations must be Lorentzian.

Rotating systems differ from translating systems in that one can determine one’s angular velocity absolutely (in the sense of Mach). The preferred frame is then the non-rotating frame, which any observer can readily identify. Further, an observer within her own frame can, in fact, do tests that unambiguously determine her circumferential velocity relative to the Lorentz frame in which her axis of rotation is fixed.

If two types of frames have such fundamental dissimilarity at their cores, is it not presumptuous to assume, as has been the general practice, that the phenomena associated with each are identical? Is it correct to simply presume that invariant light speed, Lorentz contraction, and disagreement in simultaneity can be directly extrapolated to rotating frames? We suggest that it is not, and that the proper course of action consists of building a theory of relativistic rotation from empirical data on rotating frames alone, independently of preconceived conceptions.

VII. Summary and Conclusions

The analysis of the previous section may, of course, run counter to a few long cherished ideas. But it does cleanly resolve certain major issues, and in the process leaves the essence of relativity theory intact.

Invariants like \(ds\) remain invariant. Every aspect of the theory for time orthogonal frames (the vast majority of applications) remains unchanged. Lorentz frames are still related by Lorentz transformations (with concomitant effects such as Lorentz contraction, etc.), and differential geometry continues its reign as descriptor of non-inertial systems. Neither special nor general relativity need be altered in any regard, provided of course, that NTO frames are appropriately interpreted.

All of the results obtained herein are derived from two postulates: i) transformation (2) relates rotating and non-rotating frames, and ii) the 4D line element \(ds\) is invariant. If these postulates are valid, it appears one must conclude the following.

NTO frames display non-invariant and non-isotropic local, physical speed of light, to a degree dependent on the degree of non-time-orthogonality. Lorentz frames are appropriate local surrogates for (curved or flat) TO frames, but not for (curved or flat) NTO frames. Rotating frames are truly (not superficially) NTO, and if one makes a straightforward interpretation, unfettered by preconceptions, of the most widely accepted transformation, one concludes that rotating and non-rotating frames share the same simultaneity. This in turn implies an absence of the Lorentz contraction effect between those frames.

NTO analysis for rotating frames predicts time dilation and mass energy increase with tangential speed in consonance with cyclotron experiments. It is also in full agreement with the Sagnac experiment and related

\(^{50}\) There is a little subtlety here. Calculation of the Lorentz contraction actually involves rod endpoints that appear to be different events to different observers. The present example assumes the same two events are measured by both observers. However, the conclusion remains valid. If the endpoint events look simultaneous to two different observers, then the rod length measured must be the same for each.

\(^{51}\) See ref. 29, pp. 415, 418-419, 423.

\(^{52}\) Neither Lorentz contraction nor simultaneity differences can presently be measured directly by experiment.
thought experiments. Importantly, it also explains the persistent signal found in the Michelson-Morley type Brillet and Hall experiment, which has heretofore been considered inexplicable.

A Appendix

The following discussion should be read only in the context of time orthogonal reference frames. Such frames make up the bulk of all applications, and virtually 100% of textbook problems. The conclusions drawn in this appendix are subsequently modified for NTO frames.

The speed of light in non-Lorentzian systems can be a source of confusion as it is sometimes (misleadingly) said that the speed of light in general relativity can be different than \( c \). This is true if, for example, one measures the speed of light near a massive star using a clock based on earth. (Time on such a clock is effectively the coordinate time in a Schwarzschild coordinate system.) As is well known, due to the intense gravitation field, the passage of time close to the star is dilated relative to earth time, and using the earth clock, one would indeed calculate a light speed other than \( c \). However, use of standard rods and clocks adjacent the light ray itself would result in a speed of precisely \( c \). In the language of Section VI.B, the speed of light calculated with the earth clock is a coordinate speed, whereas that measured with rods and clocks proximate to the light ray is the physical speed.

Other confusion exists for scenarios where spacetime itself expands or contracts. For example, just after the big bang, space itself was expanding much like the surface of a balloon being blown up. A photon in space (analogous to an ant on the surface of the balloon) at a different location than an observer could then move away from the observer faster than \( c \) (analogous to faster than the ant can crawl on the surface) because the space (balloon surface) between the photon and the observer is itself expanding. Yet a photon spatially coincident with an observer could never be seen by that observer to have speed greater than \( c \), and local standard rods and clocks adjacent any photon would find its speed equaling \( c \) regardless of the dynamical state of spacetime itself.

More mathematically, for a given generalized coordinate system in a non-inertial frame, we have

\[
ds^2 = g_{tt}c^2dt^2 + g_{xx}dx^2 + g_{yy}dy^2 + g_{zz}dz^2
\]

where \( g_{tt} \) is negative. Consider a ray of light passing in the \( x \) direction such that \( dy = dz = 0 \). The coordinate speed of light is found by setting \( ds = 0 \) and solving for the length in generalized coordinates (the coordinate length) that the light ray travels divided by the time in generalized coordinates (the coordinate time), i.e.,

\[
\frac{dx}{dt} = \sqrt{-\frac{g_{tt}}{g_{xx}}} c.
\]

On the other hand, the physical speed of light is the physical length divided by the physical time. Physical length (measured by standard rods) is \( \sqrt{g_{xx}}dx \), and physical time (measured by standard clocks) is \( \sqrt{-g_{tt}}dt \). So the speed of light as measured by physical instruments, regardless of the generalized coordinates chosen, is

\[
\frac{\sqrt{g_{xx}}dx}{\sqrt{-g_{tt}}dt} = c,
\]

and this is always equal to \( c \) (for time orthogonal frames.)

\[\text{See refs. 38, 39, and 40.}\]