Detection of Striped Superconductors Using Magnetic Field Modulated Josephson Effect

In a very interesting recent Letter\cite{1}, the authors suggested that a novel form of superconducting state is realized in La$_{2-x}$Ba$_x$CuO$_4$ with $x$ close to 1/8. This suggestion was based on experiments\cite{2} on this compound which found predominantly two-dimensional (2D) characters of the superconducting state, with extremely weak interplane coupling. Later this specific form of superconducting state was termed striped superconductors\cite{3}. The purpose of this Comment is to point out that the suggested form\cite{1} of the superconducting order parameter can be detected directly using magnetic field modulated Josephson effect.

The importance of charge and spin ordering notwithstanding, the most distinct feature of the state proposed in Ref. \cite{1} is that the superconducting order parameter is oscillatory with zero mean within each CuO plane. This is very similar to what happens in the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) superconducting states. It was pointed out some years ago that the order parameter structure of the FFLO states can be probed directly using magnetic field modulated Josephson effect\cite{4}. We point out that the same ideas can be applied to striped superconductors, and they can be implemented in two different but related ways.

1. Consider a Josephson junction between a striped superconductor and an ordinary superconductor with spatially uniform order parameter (but with the same internal symmetry, say d-wave), with the junction parallel to the CuO plane (such that tunneling is along the $\hat{c}$ direction). As pointed out in Ref. \cite{1}, due to the mismatch of order parameter momenta in the two superconductors, Josephson effect is suppressed. However Josephson effect can be restored by applying a magnetic field parallel to the junctions, such that the momenta mismatch is canceled by the phase modulation of Josephson coupling by the magnetic field (see Fig. 1a of Ref. \cite{3}). The condition for the restoration for the magnetic field is\cite{4}

$$B_0 = \frac{hc}{2ed}\hat{c} \times \mathbf{k},$$  \hspace{1cm} (1)

where $d$ is the thickness of the junction, and $\mathbf{k}$ is the (dominant) momentum of the Cooper pair of the striped superconductor. For the state proposed in Ref. \cite{1}, $|\mathbf{k}| = \pi/(4a)$ where $a$ is the lattice constant in the CuO plane. Assuming Josephson coupling is dominated by the top CuO layer, the direction of $\mathbf{B}$ should be parallel to the stripe direction in that layer, which is along either the $a$ or $b$ direction. To have the magnitude of $B_0$ within experimentally accessible range (say 10$T$ or below), we need the junction thickness $d \gtrsim 100nm$. In reality the junction thickness will most likely be (significantly) smaller than this, implying the desired magnitude $B_0$ is out of reach. In this case there will still be a field-dependent Josephson critical current, as long as the junction size is finite. Assuming a rectangular-shaped junction with cross section area $A$ covering exactly an integer number of periods of superconducting order parameter, we expect the critical current to be the sum of two shifted Fraunhofer patterns:

$$I_c \propto |\sin[(\pi A(B + B_0)/(\pi \Phi_0))/|]/[\sin[(\pi A(B + B_0)/(\pi \Phi_0))/]](2)$$

where $\Phi_0$ is (Cooper pair) flux quantum. The two terms correspond to contributions from the two dominant Fourier components of the order parameter; we have neglected possible contributions from (subleading) higher harmonics. In principle one can extract $B_0$ and therefore $\mathbf{k}$ from the $B$ dependence of $I_c$.

2. Due to the alternation of the stripe direction in neighboring CuO planes, Josephson coupling between the layer is suppressed for a similar reason\cite{1}. This is argued to be the source of the suppression of interplane coupling in the system of Ref. \cite{2}. Using the same reasoning as in point 1, one can in principle restore the interplane coupling, and dramatically increase bulk superconducting temperature, by applying a properly chosen magnetic field. The equation (1) still applies, except now $d$ is interlayer spacing, and $\mathbf{k}$ is momentum difference between the superconducting order parameters in neighboring CuO layers. This implies $B_0$ should be applied along the diagonal (or 110) direction, and the magnitude for restoration of interplane Josephson coupling is of order 1000$T$. While this field is too high to reach (and the system would have been destroyed by the field anyway), superconductivity may get enhanced by much lower fields when applied in the correct (or 110) direction, and this might show up in transport or other properties.

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