Magnetoexciton dispersion in GaAs-(Ga,Al)As single and coupled quantum wells

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We discuss magnetoexcitons dispersion in single and coupled $GaAs - (Ga, Al)As$ quantum wells using the Bethe-Salpeter (B-S) formalism. The B-S formalism in the case of quantum wells provides an equation for the exciton wave function which depends on two space variables plus the time variable, i.e. the B-S equation is $2+1$-dimensional equation. We compare the results for magnetoexcitons dispersion, obtained in the LLL approximations with the results calculated by solving the exact B-S equation. It is shown that the exact B-S equation has an extra term (B-S term) that does not exist in the LLL approximation. Within the framework of the variational method, we obtain that, (i) the ground-state energy of a heavy-hole magnetoexciton with a zero wave vector in $GaAs - (Ga, Al)As$ quantum wells, calculated by means of the exact B-S equation, is very close to the ground-state energy, obtained in the LLL approximation, (ii) in a strong perpendicular magnetic field the magnetoexciton dispersion (in-plane magnetoexciton mass) is determined mainly by the B-S term rather than the term that describes the electron-hole Coulomb interaction in the LLL approximation.

1 Schrödinger equation for magnetoexcitons in quantum wells

The bound states between two charged fermions, an electron from the conductive band and a hole from the valence band, in the presence of a magnetic field are called magnetoexcitons. In what follows we consider a single quantum well (SQW) and coupled quantum wells (CQW’s) made with direct-gap semiconductor that has nondegenerate and isotropic bands: $E_c(k, k_z) = E_g + \hbar^2 k^2 / 2m_e + \hbar^2 k_z^2 / 2m_e$ and $E_v(k, k_z) = \hbar^2 k^2 / 2m_v + \hbar^2 k_z^2 / 2m_v$, where $k$ is a two-dimensional (2D) wave vector, $E_g$ is the semiconductor band gap, and $m_e \ (m_v)$ is the electron (hole) effective mass. The $z$-axis is chosen to be the axis of growth of the quantum-well structure, and the constant magnetic fields is $B = (0, 0, B)$. The x-y plane has been taken to be the plane of confinement. In what follows we neglect any electron-hole correlations along the $z$-axis. This approximation takes place when the effective mass of the hole considerably exceeds that of the electron and the slow motion of the hole is separated from the fast motion of the electron. The assumption is applicable for many crystals of $A^{III}B^{V}$ type. In the presence of confinement potentials $U_{c,v}(z)$, the corresponding electron $\varphi$ and hole $\phi$ wave functions are defined by the solutions of the one-particle Schrödinger
2 Dimensional reduction in the dynamics of bulk magnetoexcitons

Strictly speaking, the excitons are bound states between two charged fermions, and therefore, the appropriate framework for the description of the bound states is the Bethe-Salpeter
(B-S) formalism [6–9]. In the absence of a magnetic field, by using a series of approximations (such as the introduction of the equal-time wave function, the assumption that the B-S kernel depends only on the difference of the relative momenta) the B-S equation for electron-hole bound states can be simplified to the well-known Schrödinger equation for the relative internal motion [10]. The existence of a magnetic field induces a coupling between the center-of-mass and the relative internal motions, because even a small transverse exciton velocity (or small transverse wave vector \( \mathbf{Q} \)) will induce an electric field in the rest frame of the exciton which will push the electron and the hole apart, so the binding energy must decrease as the transverse velocity increases. Thus, one can expect that in the presence of a magnetic field the simplification of the B-S equation to the Schrödinger equation is not trivial.

Several non-trivial effects produced by magnetic fields have been recently predicted in quantum field theories. For example, in the massless QED, by means of the lowest Landau level (LLL) approximation, the B-S equation has been reduced to the Schrödinger equation, and as a result, it was predicted that the external constant magnetic field generates an energy gap (dynamical mass) in the spectrum of massless fermions for any arbitrary weak attractive interaction between fermions [11–14]. This effect is model independent (universal), because the physical reason of this effect lies in the fact that dynamics of the LLL is essentially \( D - 2 \)-dimensional. In other words, the essence of this effect is the dimensional reduction (from \( 3 + 1 \) to \( 1 + 1 \), or \( 2 + 1 \rightarrow 0 + 1 \)) in the dynamics of fermion pairing in the presence of the constant magnetic field. Later, it was suggested that a similar effect could explain some experimental findings in the physics of high-temperature layered superconductors [15]. In what follows, we will see that the dimensional reduction in the dynamics of magnetoexcitons manifests itself in the fact that the magnetoexciton dispersion does not depend on the electron and hole masses.

We first use the B-S formalism to describe excitons in a bulk material in the presence of a strong constant magnetic field \( \mathbf{B} \) along the z-axis. After that, we apply the bulk B-S formalism to a SQW or CQW’s. The process of generalizing the bulk equations to the case of quantum-well structures is a straightforward procedure because of the assumption that there are no electron-hole correlations along the z-axis.

The basic assumption in the B-S formalism is that the electron-hole bound states are described by the B-S wave function (B-S amplitude) \( \Psi(1; 2) = \Psi(\mathbf{r}_c, \mathbf{r}_v; z_c, z_v; t_1, t_2) \), where the variables 1 and 2 represent the corresponding coordinates and the time variables. This function determines the probability amplitude to find the electron at the point \( (\mathbf{r}_c, z_c) \) at the moment \( t_1 \) and the hole at the point \( (\mathbf{r}_v, z_v) \) at the moment \( t_2 \). The B-S amplitude satisfies the following equation:

\[
\Psi(1; 2) = \int d(1', 2', 1'', 2'') G_c(1; 1') G_v(2'; 2) I \left( \begin{array}{cc} 1' & 1'' \\ 2' & 2'' \end{array} \right) \Psi(1''; 2'). \tag{4}
\]

Here \( I \) is the irreducible B-S kernel, and \( G_{c,v} \) are the electron and the hole Green’s functions. If the screening effects are taken into account by means of the high-frequency dielec-
tric constant $\epsilon_\infty$, then the irreducible kernel is given by

$$V(\mathbf{r}; z) = -\frac{4\pi e^2}{\epsilon_\infty} \int \frac{d^2 \mathbf{q}}{(2\pi)^2} \frac{dq_z}{2\pi} \frac{1}{|\mathbf{q}|^2 + q_z^2} \exp \left[i \left(\mathbf{q} \cdot \mathbf{r} + q_z z\right)\right].$$

(5)

In what follows, we use the center-of-mass $(\mathbf{R}, Z) = (\alpha_e \mathbf{r}_e + \alpha_v \mathbf{r}_v, \alpha_e z_e + \alpha_v z_v)$ and the relative $(\mathbf{r}, z) = (\mathbf{r}_e - \mathbf{r}_v, z_e - z_v)$ coordinates. The coefficients $\alpha_e = (1 - \gamma)/2$, $\alpha_v = (1 + \gamma)/2$ are expressed in terms of the parameter $\gamma = (m_v - m_e)/(m_e + m_v)$ which accounts for the difference between the electron and the hole masses. The B-S equation for the equal-time B-S amplitude in the center-of-mass and reduced coordinates assumes the form:

$$\Psi_{Q, Q_z}(\mathbf{r}, \mathbf{R}; z, Z; t, t') = \int d\mathbf{z}'d\mathbf{z}'d^2 \mathbf{r}'d^2 \mathbf{R}'dt_1dt_2$$

$$G_c(\mathbf{R} + \alpha_e \mathbf{r}, \mathbf{R}' + \alpha_e \mathbf{r}'; Z + \frac{m_v}{M_e} z + \frac{m_e}{M_e} z', t - t_1)$$

$$G_v(\mathbf{R}' - \alpha_e \mathbf{r}', \mathbf{R} - \alpha_e \mathbf{r}; Z' - \frac{m_v}{M_e} z', Z - \frac{m_e}{M_e} z; t_1 - t)$$

$$V(\mathbf{r}'; z') \Psi_{Q, Q_z}(\mathbf{r}', \mathbf{R'}; z', Z'; t_1, t_1).$$

(6)

The B-S amplitude depends on the relative internal time $t - t'$ and on the "center-of-mass" time:

$$\Psi_{Q, Q_z}(\mathbf{r}, \mathbf{R}; z, Z; t, t') = \exp \left(-\frac{iE(Q, Q_z)}{\hbar}(\alpha_c t + \alpha_v t')\right) \psi_{Q, Q_z}(\mathbf{r}, \mathbf{R}; z, Z; t - t'),$$

(7)

where $E(Q, Q_z)$ is the exciton dispersion. Introducing the time Fourier-transforms according to the rule $f(t) = \int_{-\infty}^{\infty} f(\omega) \exp(\imath \omega t) d\omega$, we transform the B-S equation into the following form:

$$\psi_{Q, Q_z}(\mathbf{r}, \mathbf{R}; z, Z; \omega) = \int d\mathbf{z}'d\mathbf{z}'d^2 \mathbf{r}'d^2 \mathbf{R}' \frac{d\Omega}{2\pi} G_c(\mathbf{R} + \alpha_e \mathbf{r}, \mathbf{R}' + \alpha_e \mathbf{r}'; Z + \alpha_v z, Z' + \alpha_v z'; \hbar \omega + \alpha_e E(Q, Q_z))$$

$$G_v(\mathbf{R}' - \alpha_e \mathbf{r}', \mathbf{R} - \alpha_e \mathbf{r}; Z' - \alpha_c z', Z - \alpha_c z; \hbar \omega - \alpha_v E(Q, Q_z))$$

$$V(\mathbf{r}'; z') \psi_{Q, Q_z}(\mathbf{r}', \mathbf{R'}; z', Z'; \Omega).$$

(8)

where $\psi_{Q, Q_z}(\mathbf{r}, \mathbf{R}; z, Z; \Omega)$ is the Fourier transform of $\psi_{Q, Q_z}(\mathbf{r}, \mathbf{R}; z, Z; t)$. Since the translation symmetry is broken by the magnetic field, the Green’s functions can be written as a product of phase factors and translation invariant parts. The phase factor depends on the gauge. In the symmetric gauge the vector potential of the magnetic field $\mathbf{A}$ is defined by $A(\mathbf{r}) = (1/2)B \times \mathbf{r}$, and the Green’s functions are [16]:

$$G_{c,v}(\mathbf{r}, \mathbf{r}'; z, z'; \omega) = \exp \left[rac{i}{\hbar c} \mathbf{r} \cdot \mathbf{A}(\mathbf{r}')\right] \tilde{G}_{c,v}(\mathbf{r} - \mathbf{r}', z - z'; \omega).$$

(9)

The broken translation symmetry requires a phase factor for the B-S amplitude:

$$\psi_{Q, Q_z}(\mathbf{r}, \mathbf{R}; z, Z; \Omega) = \exp \left[rac{i}{\hbar c} \mathbf{r} \cdot \mathbf{A}(\mathbf{R})\right] \chi_{Q, Q_z}(\mathbf{r}, \mathbf{R}; z, Z; \Omega).$$

(10)
The B-S equation \( \Box \) admits translation invariant solution of the form:

\[
\chi_{Q,Q_z}(r, R; z, Z; \omega) = \exp \left[ -i \left( Q \cdot R + Q_z Z \right) \right] \tilde{\chi}_{Q,Q_z}(r; z; \omega). \tag{11}
\]

The function \( \tilde{\chi}_{Q,Q_z}(r; z; \omega) \) satisfies the following B-S equation:

\[
\tilde{\chi}_{Q,Q_z}(r; z; \omega) = \int d z' d Z' d^2 r' d^2 R' \frac{d \theta}{2 \pi} \exp \left[ \frac{\hbar}{\beta} \left( (r + r') \cdot A(R' - R) + \gamma r \cdot A(r') \right) \right] \\
\tilde{G}_c(R - R' + \alpha_c (r - r'); Z - Z' + \alpha_v (z - z'); \hbar \omega + \alpha_c E) \\
\tilde{G}_v(R' - R + \alpha_c (r - r'); Z' - Z + \alpha_c (z - z'); \hbar \omega - \alpha_v E) \\
V(r'; z') \tilde{\chi}_{Q,Q_z}(r'; z'; \Omega). \tag{12}
\]

The substitution \( R' \rightarrow R' + R + \gamma r \) provides the following equation for the Fourier transform of the exciton wave function \( \tilde{\chi}_{Q,Q_z}(k; z; \omega) = \int d z d^2 r \exp \left[ -i(k \cdot r + k_z z) \right] \tilde{\chi}_{Q,Q_z}(r; z; \omega) \) of the exciton wave function:

\[
\tilde{\chi}_{Q,Q_z}(k - \frac{\gamma}{2} Q; k_z; \omega) = \int \frac{d p_z}{2 \pi} d^2 q d^2 p \int_{-\infty}^{\infty} \frac{d \Omega}{2 \pi} \exp \left[ -i(q + Q) \cdot R \right] \times \\
\tilde{G}_c \left( \frac{1}{2} q + k - \frac{2e}{\hbar c} A(R); k_z + \alpha_v Q_z; \hbar \omega + \alpha_c E \right) \times \\
\tilde{G}_v \left( -\frac{1}{2} q + k - \frac{2e}{\hbar c} A(R); k_z - \alpha_c Q_z; \hbar \omega - \alpha_v E \right) \times \\
V \left( p - k - \frac{2e}{\hbar c} A(R); p_z - k_z \right) \tilde{\chi}_{Q,Q_z}(p - \frac{\gamma}{2} Q; p_z; \Omega), \tag{13}
\]

where \( V(k; k_z) = - \left( 4 \pi e^2 / \varepsilon_\infty \right)^{-1} \left( k^2 + k_z^2 \right) \) and \( \tilde{G}_{c,v}(k; k_z; \hbar \omega) \) are the Fourier transforms of \( \tilde{G}_{c,v}(r; z; \hbar \omega) \).

In the effective-mass approximation the exact fermion Green’s functions \( G_{c,v} \) are replaced by the corresponding propagator of the free fermions \( G_{c,v}^{(0)} \). The translation invariant parts \( \tilde{G}_{c,v}^{(0)} \) can be decomposed over the Landau level poles:

\[
\tilde{G}_{c,v}^{(0)}(r; z; \hbar \omega) = \int \frac{d k}{(2 \pi)^2} \frac{d \hbar \omega}{2 \pi} \tilde{G}_{c,v}^{(0)}(k; k_z; \hbar \omega) \exp i (k \cdot r + k_z z), \\
\tilde{G}_{c,v}^{(0)}(k; k_z; \hbar \omega) = 2 \sum_{n=0} \left( -1 \right)^n \exp \left( -i k^2 \right) L_n \left( 2i k^2 \right) \times \\
\left( \hbar \omega - \left[ \hbar^2 k_z^2 / 2m_e + E_g + \hbar \Omega_{c,v} (n + 1/2) \right] \right) \times \left( \pi^+ \right) \times \\
\tilde{G}_{c,v}^{(0)}(k; k_z; \hbar \omega) = 2 \sum_{n=0} \left( -1 \right)^n \exp \left( -i k^2 \right) L_n \left( 2i k^2 \right) \times \\
\left( \hbar \omega + \left[ \hbar^2 k_z^2 / 2m_e + \hbar \Omega_{c,v} (n + 1/2) \right] \right) \times \left( \pi^+ \right)^{-1}. \tag{14}
\]

Here \( L_n(x) \) are the Laguerre polynomials, and \( \hbar \Omega_{c,v} = \hbar eB / cm_{c,v} \) are the electron and hole cyclotron energies. In strong magnetic fields the probability for transitions to the excited Landau levels due to the Coulomb interaction is small. Thus, the contributions to the Green’s functions from the excited Landau levels is negligible, and therefore, one can
apply the lowest Landau level (LLL) approximation, where we keep only \( n = 0 \) term in (14):

\[
\tilde{G}_c(k; k_z; \hbar \omega) \approx 2 \exp \left( -l^2 k^2 \right) \left( \hbar \omega - \left[ E_g + \hbar^2 k_z^2/2m_e + \hbar \Omega_e/2 \right] + i0^+ \right)^{-1},
\]

\[
\tilde{G}_v(k; k_z; \hbar \omega) \approx 2 \exp \left( -l^2 k^2 \right) \left( \hbar \omega + \left[ \hbar^2 k_z^2/2m_v + \hbar \Omega_v/2 \right] - i0^+ \right)^{-1}.
\]

The solution of the B-S equation in the LLL approximation can be written in the following form:

\[
\tilde{\chi}_{Q,Q_z}(k; k_z; \omega) = \exp \left[ -l^2 \left( k + \frac{\gamma}{2} Q \right)^2 - m R_0, k \right] \Phi_{Q_z}(k_z; \omega).
\]

Thus, the LLL approximation reduces the problem from \( 3 + 1 \) dimensions to \( 1 + 1 \) dimensions problem for obtaining functions \( \Phi_{Q_z}(k_z; \omega) \) and the energy \( E(Q, Q_z) \) from the following equation:

\[
\Phi_{Q_z}(k_z; \omega) = \int \frac{dp_z}{2\pi} \frac{d\Omega}{2\pi} I_Q(p_z - k_z) \Phi_{Q_z}(p_z; \Omega)
\]

\[
\left[ \frac{1}{\hbar \omega + \alpha, E - \left( E_g + \frac{h^2}{2m_e} (k_z + \alpha, Q_z)^2 + \frac{\Omega^2}{2} \right) + i0^+} + \frac{1}{\hbar \omega + \alpha, E + \frac{h^2}{2m_v} (k_z - \alpha, Q_z)^2 + \frac{\Omega^2}{2} + i0^+} \right].
\]

In the LLL approximation, the in-plane exciton dispersion is determined by the Coulomb interaction:

\[
I_Q(k_z) = \frac{4\pi e^2}{\varepsilon \infty} \int d^2r \frac{d^2q}{(2\pi)^2} \frac{\psi_{00}^2(r)}{(q^2 + k_z^2)} \exp \left[ iq.(r + R_0) \right] .
\]

Here, \( \psi_{00}(r) = \frac{1}{\sqrt{2\pi l^2}} \exp (-r^2/4l^2) \) is the ground-state wave function of a hydrogen atom in a magnetic field. The solution of (17) can be chosen in the following form:

\[
\Phi_{Q_z}(k_z, \omega) = \phi_{Q_z}(k_z) \left[ \hbar \omega + \alpha, E - \left( E_g + \frac{h^2}{2m_e} (k_z + \alpha, Q_z)^2 + \frac{\Omega^2}{2} \right) + i0^+ \right]^{-1} \times
\]

\[
\left[ \hbar \omega - \alpha, E + \left( \frac{h^2}{2m_v} (k_z - \alpha, Q_z^2 + \frac{\Omega^2}{2} \right) - i0^+ \right]^{-1},
\]

where \( \phi_{Q_z}(k_z) \) is a function to be determined. By integrating both sides of (18) over \( \omega \), we find the following equation for the exciton wave function

\[
\Phi_{Q_z}(k_z) = \int \frac{d\omega}{2\pi} \Phi_{Q_z}(k_z, \omega) = \phi_{Q_z}(k_z) / \left( E - E_g - h^2 k_z^2/2\mu - h^2 Q_z^2/2M \right)
\]

and exciton energy \( E_b(Q, Q_z) = E_g + \frac{h}{2} \hbar \Omega - E(Q, Q_z) \) (\( \Omega = \hbar e B/\mu \) is the exciton cyclotron energy):

\[
0 = \left( \frac{h^2 k_z^2}{2\mu} + \frac{h^2 Q_z^2}{2M} + E_b(Q, Q_z) \right) \Phi_{Q_z}(k_z) - \int \frac{dp_z}{2\pi} I_Q(k_z - p_z) \Phi_{Q_z}(p_z).
\]

The exciton binding energy \( E_b > 0 \) could be obtained from the solutions of (20) by means of \( E_b = E_b(Q = 0, Q_z = 0) \).

In the case when \( Q = 0 \) and \( Q_z = 0 \), eq. (20) is similar to the well-known one-dimensional Schrödinger equation for a hydrogen atom in the adiabatic approximation [17–20].
The assumptions that: (i) we neglect any electron-hole correlations along the z-axis, and (ii)
we take into account only the first electron $E_{0e}$ and hole $E_{0h}$, confinement levels with wave
functions $\varphi_{0e}(z_c)$ and $\varphi_{0h}(z_v)$, respectively, greatly simplify the description of the motion
along the z-axis. In the cases of a SQW and CQW’s, the Fourier transform of the exciton
wave function satisfies the following B-S equation:

$$\tilde{\chi}_Q(k - \frac{\gamma}{2}Q, \omega) = \int \frac{d^2q}{(2\pi)^2} \frac{d^2p}{(2\pi)^2} \frac{d^2r}{(2\pi)^2} f(\omega + \alpha_c E) G_c \left( \frac{1}{2}q + k - \frac{\gamma}{2}A(R); \hbar \omega + \alpha_c E \right) \times$$

$$V \left( p - \left[ k - \frac{\gamma}{2}A(R) \right] \right) \tilde{\chi}_Q(p - \frac{\gamma}{2}Q; \Omega),$$

(21)

where the potential $V(k) = - (2\pi e^2 f(|k|)/\varepsilon_\infty) |k|^{-1}$ depends on the quantum-well geom-
ometry through the structure factor $f(k)$.

In the LLL approximation the exact fermion Green’s functions $G_{c,v}$ are replaced by the
corresponding propagator of the free fermions $G_{c,v}^{(0)}$.

$$G_c(k; \hbar \omega) \approx 2 \exp \left( -l^2 k^2 \right) \left( \hbar \omega - [E_g + E_{0c} + \hbar \Omega_c/2] + i0^+ \right)^{-1},$$

$$G_v(k; \hbar \omega) \approx 2 \exp \left( -l^2 k^2 \right) \left( \hbar \omega + E_{0v} + \hbar \Omega_v/2 - i0^+ \right)^{-1}.$$

(22)

The solution of the B-S equation in the LLL approximation can be written in the following
form:

$$\tilde{\chi}_Q(k; \omega) = \exp \left[ -l^2 \left( k + \frac{\gamma}{2}Q \right)^2 - iR_0 k \right] \Phi_E(\omega).$$

(23)

Thus, the LLL approximation reduces the problem from $2 + 1$-dimensions to $0 + 1$-
dimension problem. The function $\Phi_E(\omega)$ energy $E(Q)$ can be obtained from the following
B-S equation:

$$\Phi_E(\omega) = -I(|Q|) \int_{-\infty}^{\infty} \frac{d\Omega}{2\pi} \Phi_E(\Omega) \times$$

$$\left( \hbar \omega + \alpha_c E - E_g - E_{0c} - \hbar \Omega_c/2 + i0^+ \right)^{-1} \left( \hbar \omega - \alpha_v E + E_{0v} + \hbar \Omega_v/2 - i0^+ \right)^{-1}. (24)$$

In the LLL approximation, the exciton dispersion is determined by the term:

$$I(Q) = \frac{2\pi e^2}{\epsilon_\infty} \int d^2r \frac{d^2q}{(2\pi)^2} \psi_0^2(r) f(|q|) \exp \left[ iq \cdot (r + R_0) \right].$$

(25)

The solution $\Phi_E(\omega)$ of (24) can be chosen in the following form:

$$\Phi_E(\omega) = \left[ \left( \hbar \omega + \alpha_v E - E_g - E_{0v} - \frac{\hbar \Omega_v}{2} + i0^+ \right) \left( \hbar \omega - \alpha_v E + E_{0v} + \frac{\hbar \Omega_v}{2} - i0^+ \right) \right]^{-1}. (26)$$
Integrating both sides of B-S equation (24) over $\omega$, we find that the exciton dispersion is determined only by the Coulomb interaction (25):

$$E(|Q|) = E_g + E_{0c} + E_{0v} + \hbar\Omega/2 - I(|Q|). \quad (27)$$

It turns out that in the LLL approximation the magnetoexciton dispersion does not depend on the electron and hole masses and is determined only by Coulomb interaction.

The LLL approximation greatly simplifies the equations, but we may ask whether the magnetoexciton dispersion will be significantly affected by the contributions from the infinity number of Landau levels with indexes $n \geq 1$ neglected in the LLL approximation. In the next Section we address this question.

### 4 Magnetoexciton dispersion in GaAs $- (Ga, Al)$As quantum wells

In the previous two Sections, we decomposed the single-particle electron (hole) Green’s function over the Landau poles and we kept only the term with index $n = 0$. This term is relatively simple, and allows us to perform all integrations in the B-S equation (13). Unfortunately, the terms with $n \geq 1$ are more complicated, and it is impossible to perform the integrations over the corresponding variables.

There exists another approach which allows us to figure out the contributions to magnetoexciton dispersion due to the Landau levels with indexes $n \geq 1$. It starts from the B-S equation (4), but rewritten in the following form [21, 22]:

$$\left( i\hbar \frac{\partial}{\partial t_1} - E_g - \frac{1}{2m_e} \left[ -i \hbar \nabla r_e + \frac{q_e}{c} A(x_e, y_e, z_e) \right]^2 - \frac{\hbar^2}{2m_e} \frac{\partial^2}{\partial x_e^2} - U_e(z_e) \right) \times$$

$$\left( i\hbar \frac{\partial}{\partial t_2} - \frac{1}{2m_e} \left[ -i \hbar \nabla r_v - \frac{q_v}{c} A(x_v, y_v, z_v) \right]^2 - \frac{\hbar^2}{2m_e} \frac{\partial^2}{\partial x_v^2} - U_v(z_v) \right) \right) \Psi(r_e, r_v; z_e, z_v; t_1, t_2)$$

$$= iV(r_e - r_v; z_e - z_v) \Psi(r_e, r_v; z_e, z_v; t_1, t_1),$$

where $V(r, z)$ is defined by (5). Since there are no electron-hole correlations along the z-axis, we separate the variables and write the B-S amplitude in the following form:

$$\Psi(r_e, z_e, t_1; r_v, z_v, t_2) = \exp \left\{ i \left[ Q \cdot R - \frac{e}{c} r_e A(R) - \frac{E}{\hbar} (\alpha_e t_1 + \alpha_v t_2) \right] \right\} \times$$

$$\tilde{\chi}_Q(r; t_1 - t_2) \varphi_0(z_e) \phi_0(z_v), \quad (28)$$

where $E \equiv E(Q)$ is the magnetoexciton dispersion. After some tedious, but straightforward calculations, we arrive at the conclusion that the Fourier transform of the B-S amplitude

$$\tilde{\chi}_Q(r; t_1 - t_2) = \int \frac{d^2 q}{(2\pi)^2} \int_{-\infty}^{+\infty} \frac{d\Omega}{2\pi} \exp \left\{ i [q \cdot r - \Omega(t_1 - t_2)] \right\} \tilde{\chi}_Q(q; \Omega). \quad (29)$$
satisfies the following equation [21, 22]:

\[
\int \frac{d^2q'}{(2\pi)^2} \int d^2r \exp \left( i(q' - q).r \right) \left[ \hbar \Omega - \Omega_c(q', Q) - \Omega^B_c(Q, q'; r) \right] \times 
\left[ \hbar \Omega - \Omega_v(q', Q) - \Omega^B_v(Q, q'; r) \right] \tilde{\chi}_Q(q'; \Omega) 
= -t \int \frac{d^2q'}{(2\pi)^2} \frac{2\pi e^2 f(|q' - q|)}{\epsilon_{\infty} |q - q|} \int_{-\infty}^{+\infty} \frac{d\Omega'}{2\pi} \tilde{\chi}_Q(q'; \Omega').
\]

(30)

Here, we use the following notations:

\[
\Omega_c(q, Q) = E_c(q + \alpha_c Q) + E_{0c} - \alpha_c E, \quad \Omega_v(q, Q) = -E_v(q - \alpha_v Q) - E_{0v} + \alpha_v E,
\]

(31)

\[
\Omega^B_c(Q, q; r) = \frac{\hbar v}{2mc_c} (B_\perp \times r).Q + \frac{\hbar v}{2mc_c} (B_\perp \times r).q + \frac{e^2 B^2}{8mc_c^2} r^2,
\]

(32)

\[
\Omega^B_v(Q, q; r) = \frac{\hbar v}{2mc_c} (B_\perp \times r).Q - \frac{\hbar v}{2mc_c} (B_\perp \times r).q + \frac{e^2 B^2}{8mc_c^2} r^2,
\]

(33)

where \( E_{c,v}(q) = E_{c,v}(q, q_z = 0) \). We are looking for the solution of Eq. (30) of the form:

\[
\tilde{\chi}_Q(q; \Omega) = \frac{g_Q(q)}{[\hbar \Omega - \Omega_c(q, Q) + \infty^+][\hbar \Omega - \Omega_v(q, Q) - \infty^+]},
\]

(34)

where \( g_Q(q) \) is a function to be determined.

We introduce the function \( \tilde{\chi}_Q(q) \), which is the Fourier transform of the equal-time B-S amplitude (or exciton wave function) \( \tilde{\chi}_Q(r) = \tilde{\chi}_Q(r; t_1 - t_2 = 0) \):

\[
\tilde{\chi}_Q(q) = \int_{-\infty}^{+\infty} \frac{d\Omega}{2\pi} \tilde{\chi}_Q(q; \Omega).
\]

(35)

By taking into account the analytic properties of \( \tilde{\chi}_Q(q; \omega) \), we obtain the following B-S equation for determining the exciton energy \( E' = E(Q) - E_g - E_{0c} - E_{0v} \) and the Fourier transform of the exciton wave function \( \tilde{\chi}_Q(q) \):

\[
\int \frac{d^2q'}{(2\pi)^2} \left[ \left( \frac{\hbar^2 Q'^2}{2m} + \frac{\hbar^2 q'^2}{2\mu} \right) \delta(q - q') + \Omega^B_c(Q, q, q') + \Omega^B_v(Q, q, q') - \frac{2\pi e^2 f(|q' - q|)}{\epsilon_{\infty} |q - q'|} \right] \times 
\tilde{\chi}_Q(q') - \int \frac{d^2q'}{(2\pi)^2} V_{B-S}(q, q'; Q, E') \tilde{\chi}_Q(q') = E' \tilde{\chi}_Q(q),
\]

(36)

In what follows, the last term in (36) will be referred as the B-S term:

\[
V_{B-S}(q, q'; Q, E') = \frac{\left[ E_v(q' + \alpha_v Q) - E_v(q - \alpha_v Q) \right]}{E' - E_v(q' + \alpha_v Q) - E_v(q - \alpha_v Q)} \Omega^B_{cv}(Q, q) q'
\]

\[
+ \frac{\left[ E_v(q' + \alpha_v Q) - E_v(q + \alpha_v Q) \right]}{E' - E_v(q + \alpha_v Q) - E_v(q - \alpha_v Q)} \Omega^B_{cv}(Q, q) q'
\]

\[
+ \frac{1}{E' - E_v(q' + \alpha_v Q) - E_v(q - \alpha_v Q)} + \frac{1}{E' - E_v(q + \alpha_v Q) - E_v(q' - \alpha_v Q)}.
\]

(37)
Here, the following notations have been used:

\[
\Omega_{c,v}^{B}(Q, q, q') = \int d^2 r \exp \left[ i (q' - q) \cdot r \right] \Omega_{c,v}^{B}(Q, q'; r),
\]

\[
\Omega_{cc}^{B}(Q, q, q') = \int d^2 r \exp \left[ i (q' - q) \cdot r \right] \Omega_{c}^{B}(Q, q'; r) \Omega_{c}^{B}(Q, q'; r).
\]

In position representation, the B-S term generates a non-local potential which depends on the energy \(E'\):

\[
V_{B-S}(r, r'; Q, E') = \int \frac{d^2 q}{(2\pi)^2} \int \frac{d^2 q'}{(2\pi)^2} V_{B-S}(q, q'; Q, E') \exp \left[ i (q \cdot r - q' \cdot r') \right].
\]

The solution of Eq. (36) can be written as

\[
\tilde{\chi}_{Q}(q) = \exp \left( -i q \cdot R_0 \right) \Psi(q - Q_0),
\]

where the function \( \Psi(q) \) satisfies the following equation:

\[
E' \Psi(q) = \frac{\hbar^2 q^2}{2\mu} \Psi(q) - i \frac{\hbar e}{2\mu c} (B_\perp \times q) \cdot \nabla q \Psi(q) - \frac{\hbar^2 \Omega}{8 \epsilon_0} \nabla^2 q \Psi(q)
\]

\[
- \frac{2 \pi e^2}{\epsilon_0} \int \frac{d^2 q'}{(2\pi)^2} \exp \left[ i (q - q') \cdot R_0 \right] \frac{|q' - q|}{q' - q} \Psi(q')
\]

\[
- \int \frac{d^2 q'}{(2\pi)^2} \exp \left[ i (q - q') \cdot R_0 \right] V_{B-S}(q + \frac{\gamma}{2} Q_0, q' + \frac{\gamma}{2} Q_0; Q, E') \Psi(q').
\]

The B-S equation (41) differs from the Schrödinger equation. If we neglect the B-S term in the right-hand side of (41), we obtain the Schrödinger equation for magnetoexcitons with the Hamiltonian (1). It can be seen that according to the Schrödinger equation, the magnetoexciton dispersion is totally determined by the Coulomb term, while according to the B-S equation, the effective potential (40) also contributes to the magnetoexciton dispersion.

Since the Bethe-Salpeter term plays an important role in determining the magnetoexciton dispersion (see the next two Sections), one may well ask a question about the physical meaning of this term. The answer is that the B-S term takes into account the contributions to the single-particle Green’s functions (14) from the Landau levels with \(n \geq 1\).

### 5 Magnetoexciton dispersion in single GaAs/Al\(_x\)Ga\(_{1-x}\)As quantum well

In this Section, we first calculate the ground-state energy of a heavy-hole magnetoexciton with a zero wave vector \((Q = 0)\), assuming a single GaAs quantum well with a thickness \(L\) sandwiched between two \(Al_xGa_{1-x}As\) layers. The electron in-plane mass \(m_e\) and the electron z-mass \(m_{ez}\) are chosen to be \(m_e = m_{ez} = 0.067m_0\), where \(m_0\) is the bare electron mass. The in-plane heavy-hole mass \(m_v\) and the hole z-mass \(m_{vz}\) are expressed in terms of the Luttinger parameters \(\gamma_1\) and \(\gamma_2\): \(m_v = m_0/(\gamma_1 + \gamma_2)\) and \(m_{vz} = m_0/(\gamma_1 - 2\gamma_2)\).
Table 1: Variational calculations of the heavy-hole exciton ground-state energies with \( Q = 0 \) for various well widths \( L \) and weak magnetic fields \( B \). The trial function (42) depends on the variational parameter \( \beta \). The energy gap is \( E_g = 1.519 \text{ eV} \). The electron and hole confinement energy levels \( E_{c0} \) and \( E_{v0} \) are calculated assuming squared-well potentials of finite depths. The \( E_{\text{var}} \)-column represents the results from the variational calculations with the following Luttinger parameters: \( \gamma_1 = 7.36 \) and \( \gamma_2 = 2.57 \) [23]. The measured ground state energies \( E_{\text{exp}} \) are reproduced from [24]. The \( E_S \)-column represents the ground-state energies calculated according to the Schrödinger equation with the Hamiltonian [1].

\[
\begin{array}{cccccccc}
L (\text{nm}) & B (T) & \beta & E_{c0} (\text{meV}) & E_{v0} (\text{meV}) & E_{\text{var}} (\text{eV}) & E_{\text{exp}} (\text{eV}) & E_S (\text{eV}) \\
4.03 & 0 & 0.786 & 100 & 26.9 & 1.6355 & 1.638 & 1.6355 \\
4.03 & 2 & 0.810 & 100 & 26.9 & 1.6356 & 1.639 & 1.6357 \\
4.03 & 4 & 0.869 & 100 & 26.9 & 1.6356 & 1.639 & 1.6357 \\
4.32 & 0 & 0.776 & 93.5 & 24.3 & 1.6262 & 1.630 & 1.6262 \\
4.32 & 2 & 0.802 & 93.5 & 24.3 & 1.6265 & 1.631 & 1.6266 \\
4.32 & 4 & 0.861 & 93.5 & 24.3 & 1.6274 & 1.632 & 1.6275 \\
7.2 & 0 & 0.734 & 51.0 & 11.0 & 1.5719 & 1.572 & 1.5720 \\
7.2 & 2 & 0.734 & 51.0 & 11.0 & 1.5730 & 1.573 & 1.5731 \\
7.2 & 4 & 0.803 & 51.0 & 11.0 & 1.5730 & 1.573 & 1.5731 \\
\end{array}
\]

It is known that the difference between the bandgap energies of GaAs and Al\(_x\)Ga\(_{1-x}\)As provides a finite potential well, confining the electron-hole pairs in the GaAs quantum well. We assume that the potentials are square-well potentials of finite depths \( V_c = 0.6 \Delta E_g(x) \) and \( V_v = 0.4 \Delta E_g(x) \), respectively. The energy-band-gap discontinuity [23] is assumed to be \( \Delta E_g(x) = (1.555x + 0.37x^2) \text{ meV} \). The confinement energy levels \( E_{c0} \) and \( E_{v0} \) are obtained by solving the following transcendental equations:

\[
\tan \left( \frac{L}{2a_B} \sqrt{\frac{m_{c}E_{c0}}{\mu E_B}} \right) = \sqrt{\frac{V_c}{E_{c0}}} - 1,
\]

\[
\tan \left( \frac{L}{2a_B} \sqrt{\frac{m_{v}E_{v0}}{\mu E_B}} \right) = \sqrt{\frac{V_v}{E_{v0}}} - 1.
\]

Here, \( E_B = \hbar^2/2\mu a_B^2 \) is the exciton Bohr energy. The structure factor \( f(k) \) is calculated by means of the following wave functions:

\[
\psi_{c,v}^0(z) = A_{c,v} \exp \left[ z \frac{L}{a_B} \sqrt{\frac{m_{c,v}E_{c,v}}{\mu E_B}} \right] \frac{1}{\mu E_B} (V_{c,v} - E_{c,v}), \quad -\infty < z < -1/2,
\]

\[
\psi_{c,v}^0(z) = B_{c,v} \cos \left[ z \frac{L}{a_B} \sqrt{\frac{m_{c,v}E_{c,v}}{\mu E_B}} \right] \frac{1}{\mu E_B} (V_{c,v} - E_{c,v}), \quad -1/2 < z < 1/2,
\]

\[
\psi_{c,v}^0(z) = A_{c,v} \exp \left[ -z \frac{L}{a_B} \sqrt{\frac{m_{c,v}E_{c,v}}{\mu E_B}} \right] \frac{1}{\mu E_B} (V_{c,v} - E_{c,v}), \quad 1/2 < z < \infty,
\]

\[
B_{c,v} = \left[ \frac{1}{2} + a_B \left( \frac{L}{\mu E_B} \sqrt{\frac{m_{c,v}E_{c,v}}{\mu E_B}} \right) \right]^{-1/2}.
\]
expression for the in-plane exciton mass can be written as by $E_{B}$

closed to those, provided by the Schrödinger equation.

that the magnetoexciton energies, calculated by applying the B-S formalism are extremely

trial function are presented in Table 1. We used more significant figures to stress on the fact

parameter magnetic fields, i.e. $\hbar \Omega << E_B$, we use a hydrogen-like trial function with a variational

Since the B-S equation (41) is rather complicated we shall obtain numerical results for the ground-state energy within the framework of the variational approach. In the case of weak

the variational approach. In the case of weak

With this trial function we calculate the following magnetoexciton energy:

where $E(\beta)$ is defined by the solution of the following equation:

With the trial function (42), the B-S contribution to the ground state is:

The dimensionless variables $E$ and $a_B$ in the right-hand side of Eq. (44) must be replaced

by $E(\beta)h\Omega/E_B$ and $a_B/l$, respectively. The results obtained by using the hydrogen-like trial function are presented in Table 1. We used more significant figures to stress on the fact that the magnetoexciton energies, calculated by applying the B-S formalism are extremely close to those, provided by the Schrödinger equation.

The magnetoexciton dispersion are determined by the Coulomb interaction and the B-S term in Eq. (41). The contribution from the Coulomb interaction to the energy of the magnetoexciton (in $E_B$ units) increases quadratically for small wave vectors $Qa_B << 1$, and can be written as $(Qa_B)^2\mu/M_C$. The hydrogen-like trial function provides the following expression for the in-plane exciton mass $M_C$:

$$
\frac{\mu}{M_C} = 32\beta^3 \left( \frac{R}{a_B} \right)^4 \int_0^\infty dx \frac{x^2 f(x \frac{l}{a_B})}{(16\beta^2 + x^2)^{3/2}}.
$$


Table 2: Variational calculations of the heavy-hole exciton ground-state energies for various well widths $L$ and strong magnetic fields $B$. The trial function (45) depends on the variational parameter $\beta$. The energy gap is $E_g = 1.519$ eV for the $L = 4.03, 4.32, 7.2,$ and 7.49-nm wells, and $E_g = 1.512$ eV for the $L = 7.5$-nm. The $E_{\text{var}}$-column represents the energies obtained by the variational method using the following Luttinger parameters: $\gamma_1 = 6.9$ and $\gamma_2 = 2.4$ [25]. The measured ground state energies $E_{\text{exp}}$ for the $L = 4.03, 4.32, 7.2,$ and 7.49-nm wells are reproduced from [24], and for the $L = 7.5$-nm well from [26]. The $E_S$-column represents the ground-state energies calculated according to the Schrödinger equation. The $M_C$ and $M_{B-S}$ are the masses calculated according to Eqs. (49) and (50).

| $L$ (nm) | $B$ (T) | $\beta$ | $E_{\text{var}}$ (eV) | $E_{\text{exp}}$ (eV) | $E_S$ (eV) | $M_C/m_0$ | $M_{B-S}/m_0$ |
|---------|---------|---------|------------------------|------------------------|-----------|-----------|--------------|
| 4.03    | 20      | 0.85    | 1.650                  | 1.644                  | 1.651     | 0.145     | 0.0025       |
| 4.03    | 18      | 0.84    | 1.648                  | 1.643                  | 1.649     | 0.127     | 0.0010       |
| 4.03    | 16      | 0.84    | 1.647                  | 1.642                  | 1.647     | 0.114     | 0.0002       |
| 4.32    | 20      | 0.84    | 1.641                  | 1.636                  | 1.642     | 0.147     | 0.0026       |
| 4.32    | 18      | 0.83    | 1.639                  | 1.635                  | 1.640     | 0.129     | 0.0011       |
| 4.32    | 16      | 0.83    | 1.638                  | 1.634                  | 1.638     | 0.116     | 0.0002       |
| 7.2     | 20      | 0.86    | 1.587                  | 1.583                  | 1.588     | 0.176     | 0.0044       |
| 7.2     | 18      | 0.84    | 1.585                  | 1.582                  | 1.586     | 0.159     | 0.0022       |
| 7.2     | 16      | 0.84    | 1.583                  | 1.581                  | 1.584     | 0.142     | 0.0007       |
| 7.49    | 20      | 0.86    | 1.584                  | 1.580                  | 1.584     | 0.178     | 0.0046       |
| 7.49    | 18      | 0.84    | 1.582                  | 1.579                  | 1.582     | 0.161     | 0.0024       |
| 7.49    | 16      | 0.84    | 1.580                  | 1.578                  | 1.580     | 0.144     | 0.0008       |
| 7.5     | 14.5    | 0.67    | 1.577                  | 1.577                  | 1.572     | 0.131     | 0.0302       |
| 7.5     | 12      | 0.64    | 1.575                  | 1.573                  | 1.570     | 0.049     | 0.0160       |
| 7.5     | 8.5     | 0.60    | 1.572                  | 1.570                  | 1.569     | 0.026     | 0.0071       |
The contribution to the exciton dispersion due to the B-S term can be evaluated analytically. We found that it also increases quadratically for small wave vectors, but for $B < 4T$, this contribution is about one tenth of $(Qa_B)^2 \mu/M_C$. Thus, in a weak magnetic field, there is no measurable difference between the results calculated by the Schrödinger equation, and these obtained by the more complicated B-S formalism. For a weak perpendicular magnetic field and small wave vectors, the Coulomb interaction dominates, which means that a hydrogen type of ground state slightly modified by the magnetic field exists.

Next, we consider the case of a strong magnetic field. In this regime we choose the trial wave function $\psi_\beta(r)$ to be similar to the corresponding ground-state wave function of a charge particle in a magnetic field, but depending on a variational parameter $\beta$:

$$\psi_\beta(r) = \frac{1}{\sqrt{2\pi \beta}} \exp \left( -\frac{r^2}{4\beta^2} \right). \quad (45)$$

Here, and in what follows, we use the exciton cyclotron energy $h\Omega$ for energy unit and magnetic length $R$ for unit length. The ground state magnetoexciton energy will be calculated by minimizing the energy functional $E'(\beta) = (E - E_g - E_{oc} - E_{oe})/h\Omega$ with respect to the variational parameter $\beta$:

$$E' = \frac{1}{4} \left( \frac{1}{\beta^2} + \beta^2 \right) + V_C(\beta) + V_{B-S}(\beta, E') + V_C(\beta, Q) + V_{B-S}(\beta, E', Q). \quad (46)$$

Note, that (i) all terms in the last equation are dimensionless (in a cyclotron energy $h\Omega$ unit), and (ii) we have written the contributions from the Coulomb interaction and from the B-S term $C(\beta)$ as a sum of $Q$-independent terms, $V_C(\beta)$ and $V_{B-S}(\beta, E')$, and $Q$-dependent terms, $V_C(\beta, Q)$ and $V_{B-S}(\beta, E', Q)$. The $Q$-dependent terms will be used to obtain the magnetoexciton dispersion. The second and the third term in $(46)$ are given by:

$$V_C(\beta) = -\frac{E_b}{h\Omega} \sqrt{\frac{2}{\pi}} \int_0^\infty dx f(x) \frac{L}{R} \exp \left( -\frac{x^2 \beta^2}{2} \right), \quad (47)$$

$$V_{B-S}(\beta, E') = \frac{e^{-4E'\beta^2}}{4E'^2} \left\{ e^{4E'/\beta} \left[ -56E'^2\beta^4 \gamma^4 + 32E'^3\beta^6 \gamma^4 + (-1 + \gamma^2)^2 \right. \right.$$

$$\left. + 4E'/\beta^2 \left[ -1 - 2 \gamma^2 + 3\gamma^4 \right] \right\}$$

$$-32E'^2 \left[ -1 + \beta^4 \gamma^2 \left[ -1 + (3 + 4E'/\beta^2(-2 + E'/\beta^2)\gamma^2) \right] \right] \exp(4E'/\beta^2). \quad (48)$$

Here, $E_b = \sqrt{\pi/2}e^2/(\epsilon_\infty R)$ is the binding energy of the two-dimensional ($L = 0, \beta = 1$) magnetoexciton, calculated according to the Schrödinger equation.

The energy of the magnetoexciton increases quadratically for small wave vectors $(QR << 1)$: $V_C(\beta, Q) = [\mu/2M_C(L, B, \beta)](QR)^2$ and $V_{B-S}(\beta, E', Q) = [\mu/2M_{B-S}(L, B, \beta)](QR)^2$. The in-plane mass $M_C(L, B, \beta)$ is due to the Coulomb interaction and does not depend on the electron or the hole mass:

$$\frac{M_{2D}}{M_C(L, B, \beta)} = \sqrt{\frac{2}{\pi}} \int_0^\infty dx f(x) \frac{L}{R} x^2 \exp \left( -\frac{x^2 \beta^2}{2} \right), \quad (49)$$
where \( M_{2D} = 2^{3/2} \epsilon_{\infty}\hbar^2/(\sqrt{\pi} e^2 R) \). The second in-plane mass, \( M_{B-S} \), has its origin in the fact that the B-S term depends on \( Q \), and for \( QR << 1 \), \( M_{B-S} \) is defined by the following equation:

\[
\frac{\mu}{2M_{B-S}(L,B,\beta)} = -\frac{4E' \beta^2 (-1+\gamma^2)}{256E^5} \left\{ e^{4E' \beta^2} \left[ 256E^5 \beta^{12} \gamma^6 + 64E'^4 \beta^{10} \gamma^4 (5 - 17 \gamma^2) \right. \right.
\]
\[
-3\beta^2 (-1 + \gamma^2)^3 - 2E' \beta^4 (-1 + \gamma^2)^2 (1 + 12 \gamma^2) - 48E'^2 \beta^6 \gamma^2 (2 - 7\gamma^2 + 5 \gamma^4)
\]
\[
+16E'^3 \left[ -2 + 2\beta^4 + \beta^8 \gamma^2 (4 - 53 \gamma^2 + 74 \gamma^4) \right]
\]
\[
-64E'^3 \beta^2 [-\beta^2 + 16E'^3 \beta^{12} \gamma^6 + 4E'^2 \beta^{10} \gamma^4 (5 - 18 \gamma^2) + \beta^6 \gamma^2 (-7 + 33 \gamma^2 - 30 \gamma^4)
\]
\[
+2E' \left[ -1 + \beta^4 + \beta^8 \gamma^2 (2 - 29 \gamma^2 + 45 \gamma^4) \right] \gamma \right\} = \gamma \right\} \frac{\gamma_0}{t} \text{is the exponential integral function (the principle value of the integral is taken).}
\]

Table 2 gives the results of our variational calculations. It can be seen that the B-S equation provides similar results for the ground-state energies as the Schrödinger equation does. Since the B-S mass is much smaller than the Coulomb mass, one can say that in strong magnetic fields the exciton dispersion for small wave vectors \( QR << 1 \) is determined by the B-S term rather than the Coulomb interaction.

### 6 Coupled quantum wells in strong magnetic fields

In this Section, we consider exactly the same double well electron-hole system as in Refs. [27, 28]. The electron layer and hole layer have finite widths, denoted below by \( L_c \) and \( L_v \), and they are separated by a distance \( D \). We assume that the electrons and holes are confined between two parallel, infinitely high potential barriers. This assumption greatly simplifies our numerical calculations of the magnetoexciton energy and the Coulomb mass, but by neglecting the existence of the finite confinement potentials, we cannot provide a more realistic value for this part of the exciton energy related to the exciton confinement along \( z \)-direction, than the sum of the well-known terms \( h^2 \pi^2/2m_{c,v} L^2_{c,v} \). Obviously, the more realistic model of a symmetric (or asymmetric) DQW with finite quantum-well widths [29,30] will cause minor corrections to our main conclusions, which are: (1) the B-S formalism provides a term, which does not exists in the Schrödinger equation, and (2) the term plays an important role in determining the magnetoexciton dispersion.

The basic features of the CQW’s magnetoexcitons are the same as that of the SQW magnetoexcitons. However, because of the separation between the electron and hole layers, the Coulomb energy and the Coulomb in-plane mass differ quantitatively from those of the SQW magnetoexciton. In other words, in strong magnetic fields, Eq. (46) holds, but the Coulomb interaction and the corresponding in-plane mass are defined as follows:

\[
V_C(\beta) = -\frac{E_0}{M_2} \sqrt{2 \pi} \int_0^{\infty} dx e^{-\frac{x^2}{2}} F \left( x, \frac{L_c}{R}, \frac{L_v}{R}, \frac{D}{R} \right),
\]
\[
\frac{M_{2D}}{M_C(L,B,\beta)} = \sqrt{2 \pi} \int_0^{\infty} dx x^2 e^{-\frac{x^2}{2}} F \left( x, \frac{L_c}{R}, \frac{L_v}{R}, \frac{D}{R} \right).
\]
Table 3: Variational calculations of the magnetoexciton energies for various strong magnetic fields \( B \), measured relatively to the \( E_g + E_{0c} + E_{0v} \) level. The trial function (45) depends on the variational parameter \( \beta \). The \( E_{\text{var}} \)-column contains the energies calculated by the variational method with the following parameters: \( m_c = 0.067m_0 \), \( m_v = 0.18m_0 \), \( \epsilon_{\infty} = 12.35 \), \( L_c = L_v = 8\,\text{nm} \), \( D = 11.5\,\text{nm} \). The \( E_S \)-column represents the magnetoexciton energies calculated according to the Schrödinger equation. \( M_C \) is the in-plane mass defined by Eq. (52). The \( M_{B-S} \) is the mass calculated according to Eq. (50).

In CQW’s, the structure factor is:

\[
F(x; \xi_c, \xi_v, d) = \frac{16\pi^4(1 - e^{-\xi_c x})(1 - e^{-\xi_v x})e^{-dx}}{\xi_c \xi_v x^2(4\pi^2 + \xi_c^2 x^2)(4\pi^2 + \xi_v^2 x^2)}.
\]

Table 3 gives the result of our numerical calculation of the magnetoexciton energy, but relatively to the \( E_g + E_{0c} + E_{0v} \) level. We used the same parameters as in Refs. [27] and [31]. It can be seen that the B-S equation provides slightly different results for the binding energy than the Schrödinger equation.

The main difference between the B-S and the Schrödinger equation is in their predictions about the in-plane magnetoexciton mass in a strong magnetic field. Unfortunately, optical experimental studies can provide information about the exciton dispersion only for \( Q \leq Q_{\text{ph}} \), where \( hQ_{\text{ph}} \) is the photon momentum. Other studies, such as the photoluminescence measurement experiments which can measure the exciton-mass dependence of the recombination time, or experimental data related to the polariton effects, can provide information about the magnetoexciton dispersion. Many of these experimental techniques [32–36] are used to measure the magnetoexciton dispersion in the presence of an in-plane magnetic field. As we mentioned above, the measurable differences between the magnetoexciton dispersions, as predicted by the B-S formalism and by the Schrödinger equation, are to be expected in strong perpendicular magnetic fields. To the best of our knowledge, there is only one paper [27] where the exciton dispersion in GaAs/Ga_{0.67}Al_{0.33}As CQW’s in a weak perpendicular magnetic field has been measured. There is a good agreement between the mass \( M_C \) and the measured mass in a weak magnetic field. Referring to the conclusion that the B-S term in a weak magnetic field has a very small contribution to the dispersion compare to the contribution due to the Coulomb interaction, one can say that there exists a good agreement between the B-S formalism and the measurements.
Next, we discuss the fact that $M_C$ increases by about 4 times if we increase the magnetic field from 4T to 10T. If the magnetoexciton dispersion in strong magnetic fields ($B > 5T$) is determined mainly by the B-S term, then the magnetoexciton mass should not increase so dramatically, and therefore, new experimental points are needed to prove or disprove the conclusions drawn by applying the B-S formalism.

7 Conclusion

We have applied the B-S formalism to the quantum-well excitons in a constant magnetic field applied along the axis of growth of the quantum-well structure. We found that (1) in the LLL approximation the B-S equations provides the same results as the Schrödinger equation; (2) beyond the LLL approximation, the B-S equation contains an extra term (B-S term). This term takes into account the transitions to the Landau levels with indexes $n \geq 1$. We applied a variational procedure to obtain the effect of the B-S term on the magnetoexciton ground-state energy and magnetoexciton mass. We used a simple hydrogen-like trial wave function in a weak magnetic field, and figured out that in a weak perpendicular magnetic field the results obtained by the B-S formalism are very close to the results calculated by means of the Schrödinger equation. In a strong magnetic field, we used a trial function similar to the wave function of a charged particle in a magnetic field. We calculated that in a strong magnetic field, the ground-state energy is very close to that obtained by means of the Schrödinger equation, but the magnetoexciton dispersion is determined by the B-S term rather than the electron-hole Coulomb term in the Schrödinger equation.

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