de Sitter/ Anti-de Sitter global monopoles

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Abstract

We consider global monopoles in asymptotic de Sitter/ Anti-de Sitter spacetime. We present the by our numerical analysis confirmed asymptotic behaviour of the metric and Goldstone field functions. We find that the appearance of horizons in this model depends strongly on the sign and value of the cosmological constant as well as on the value of the gravitational coupling. In Anti-de Sitter (AdS) space, we find that for a fixed value of the cosmological constant, global monopoles without horizons exist only up to a critical value of the gravitational coupling. Moreover, we observe (in contrast to another recent study) that the introduction of a cosmological constant can not render a positive mass of the global monopole.

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I. INTRODUCTION

In recent years topological defects [1] in asymptotic de Sitter (dS)/ Anti- de Sitter (AdS) space-time have gained renewed interest. This is mainly due to the proposed dS/CFT [2], resp. AdS/CFT [3] correspondences. These correspondences suggest a holographic duality between gravity in a d-dimensional dS, resp. AdS space and a conformal field theory (CFT) "living" on the boundary of the dS, resp. AdS spacetime and thus being d − 1-dimensional. However, dS space-time is also interesting from a cosmological point of view since it seems to be confirmed by observational data [4] that we live in a universe with positive cosmological constant.

Gravitating global monopoles in asymptotically flat space-time were first discussed in [5,6]. These topological defects were found to have a negative mass and a deficit angle depending on the vacuum expectation value (vev) of the scalar Goldstone field and the gravitational coupling. For sufficiently high enough values of the vev the solutions have an horizon [7]. These solutions were named (after their string counterparts [8]) “supermassive monopoles”.

In [9], it was found that the introduction of a cosmological constant can render the mass of the monopole positive. This was demonstrated by a figure showing the mass function as function of the radial coordinate for different choices of the cosmological constant. In our recent work, we are mainly interested in composite monopole defects [10] in dS/AdS space-time [11]. This is why we cross-checked the results of [9] and found discrepancies between our results and those in [9].

The paper is organised as follows: we give the model in Section II and discuss the asymptotic behaviour, which should be compared to that in [9], in Section III. We give our numerical results in Section IV and conclude in Section V.

II. THE MODEL

We consider the following action:

$$S = \int \left( \frac{1}{16\pi G} (R - 2\Lambda) - \frac{1}{2} \partial_\mu \xi^a \partial^\mu \xi^a - \frac{\lambda}{4} (\xi^a \xi^a - \eta^2)^2 \right) \sqrt{-g} d^4 x$$

(1)

which describes a Goldstone triplet $\xi^a$, $a = 1, 2, 3$, interacting with gravity in an asymptotically de Sitter (dS) (for the cosmological constant $\Lambda > 0$), resp. Anti-de Sitter (AdS) ($\Lambda < 0$) space-time. $G$ is Newton’s constant, $\lambda$ is the self-coupling constant of the Goldstone field and $\eta$ the vacuum expectation value (vev) of the Goldstone field.

For the metric, the spherically symmetric Ansatz in Schwarzschild-like coordinates reads:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -A^2(r)N(r)dt^2 + N^{-1}(r)dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi)^2$$

(2)

while for the Goldstone field, we choose the hedgehog Ansatz [5]:

$$\xi^a = \eta h(r) e_r^a$$

(3)

We introduce the following dimensionless variable and coupling constants:
\[ x = \eta r \quad , \quad \alpha^2 = 4\pi G \eta^2 \quad , \quad \gamma = \frac{\Lambda}{\eta^2} . \quad (4) \]

Varying (1) with respect to the metric fields gives the Einstein equations which can be combined to give two first order differential equations for \( A \) and \( \mu \):

\[ A' = \alpha^2 Ax(h')^2 \quad (5) \]

\[ \mu' = \alpha^2 \left( h^2 - 1 + x^2 \frac{\lambda}{4} (h^2 - 1)^2 + \frac{1}{2} x^2 N(h')^2 \right) \quad (6) \]

and \( N \) and \( \mu \) are related as follows:

\[ N(x) = 1 - 2\alpha^2 - 2\frac{\mu(x)}{x} - \frac{\gamma}{3} x^2 . \quad (7) \]

Note that for \( \gamma = 0 \), the existence of solutions without horizon is restricted by \( \alpha < \sqrt{\frac{1}{2}} \) [7].

Variation with respect to the matter fields yields the Euler-Lagrange equations for the Goldstone field :

\[ (x^2 ANh')' = A(2h + \lambda x^2 h(h^2 - 1)) . \quad (8) \]

The prime denotes the derivative with respect to \( x \). Note that the equations have the same structure as for the asymptotically flat space-time [5,6]. The cosmological constant just appears in the relation defining \( \mu(x) \) and \( N(x) \).

In order to solve the system of equations uniquely, we have to introduce 4 boundary conditions, which we choose to be:

\[ \mu(0) = 0 \quad , \quad h(0) = 0 \quad , \quad A(\infty) = 1 \quad , \quad h(\infty) = 1 . \quad (9) \]

The dimensionless mass of the solution is determined by the asymptotic value \( \mu(\infty) = \mu_\infty \) of the function \( \mu(x) \) and is given by \( \mu_\infty / \alpha^2 \).

### III. ASYMPTOTIC BEHAVIOUR

Expanding the functions around the origin gives:

\[ h(x \to 0) = c_1 x + O(x^3) \quad , \quad \mu(x \to 0) = -\alpha^2 x + O(x^2) \quad , \quad A(x \to 0) = A(0)(1 + O(x^2)) \quad (10) \]

where \( c_1 \) and \( A(0) \) are free parameters to be determined numerically. The asymptotic behaviour \( (x \to \infty) \) is given by:

\[ h(x >> 1) = 1 + \frac{3}{(\gamma - 3\lambda)x^2} - \frac{9(4\alpha^2(\gamma - 3\lambda) + 9\lambda)}{2(2\gamma + 3\lambda)(\gamma - 3\lambda)^2 x^4} \]

\[ - \frac{36\mu_\infty}{(5\gamma + 3\lambda)(\gamma - 3\lambda)x^5} + O\left( \frac{1}{x^6} \right) , \quad (11) \]
\[ A(x >> 1) = 1 - \frac{9\alpha^2}{(\gamma - 3\lambda)^2} \frac{1}{x^4} + \frac{36\alpha^2(4\alpha^2(\gamma - 3\lambda) + 9\lambda)}{(2\gamma + 3\lambda)(\gamma - 3\lambda)^3} \frac{1}{x^6} + \frac{2160\mu_\infty\alpha^2}{7(5\gamma + 3\lambda)(\gamma - 3\lambda)^3} \frac{1}{x^7} + O\left(\frac{1}{x^8}\right) \] (12)

and

\[ \mu(x >> 1) = \mu_\infty + \frac{9\alpha^2}{(\gamma - 3\lambda)^2} \frac{1}{x} - \frac{6\alpha^2(\gamma(2\alpha^2 + 3) - 12\gamma\lambda(\alpha^2 - 1) + 9\lambda^2(1 - 2\alpha^2))}{(2\gamma + 3\lambda)(\gamma - 3\lambda)^3} \frac{1}{x^3} - \frac{27\mu_\infty\alpha^2(\lambda - \gamma)}{(5\gamma + 3\lambda)(\gamma - 3\lambda)^2} \frac{1}{x^4} + O\left(\frac{1}{x^5}\right) \] (13)

It is worthwhile to contrast the coefficient of the \(1/x\) correction for the mass function \(\mu(x)\) appearing in the above equation (13) with its counterpart in [9], eq.(18). While in the latter the coefficient is independent on both the cosmological constant and the self-coupling of the Goldstone field, we find here a non-trivial dependence on these parameters (which is indeed confirmed by our numerical analysis). We also remark that the expansion presented in [9] is in contradiction with the figure presented in that paper. The figure of [9] seems incompatible with the fact that the first asymptotic correction to the mass is supposed to be independent of the cosmological constant.

**IV. NUMERICAL RESULTS**

We remark that without losing generality, we can choose \(\lambda = 1.0\).

**A. The \(\gamma = 0\) limit**

This limit was studied previously in great detail in [5,6] and [7]. It was found that for \(\gamma = 0\) global monopoles have a negative mass [5,6]. Thus, the global monopole has a repulsive effect on a test particle in its neighbourhood. Further, it was shown [7] that global monopoles without horizon only exist for \(\alpha < \sqrt{\frac{1}{2}}\), while for \(\alpha > \sqrt{\frac{3}{2}}\) no static solutions exist at all. The configurations for \(\sqrt{\frac{1}{2}} < \alpha < \sqrt{\frac{3}{2}}\) were called “supermassive” monopoles.

We have redone the calculations and found perfect agreement with the results in [5–7]. Especially, we remark in view of Fig. 1 that for \(\gamma = 0\), the mass of the solution is close to \(-\frac{\pi}{2}\). Since the mass of the global monopole in flat space is just \(-\pi/2\) [6] (of course in rescaled units in comparison to here), the value of \(\frac{\text{mass}}{\alpha^2} \lesssim -\frac{\pi}{2}\) is in good agreement with these results. We found in addition that the mass function at large \(x\) is always negative and becomes more and more negative for increasing \(\alpha\) [12].

**B. Anti-de Sitter (AdS) monopoles**

By solving the three equations numerically we constructed solutions for negative values of \(\gamma\). First, we checked whether the asymptotic behaviour found in (12) is correct. We
analysis, we obtain
\[ x \equiv 0 \]
However, the inner horizon tends to the one of the "supermassive" monopoles observed previously [7].

\[ \alpha = 1 \]
where \( \mu \) we show the profile of the mass function
\[ \mu(x) \]
indeed confirmed numerically that the coefficients have the given dependence on the coupling constants

Following the investigation in [9], we have then studied the dependence of the mass \( \mu_\infty/\alpha^2 \) on the cosmological constant \( \gamma \) for a fixed value of \( \alpha \). As is demonstrated in Fig. 1
for \( \alpha = 0.1 \) and \( \alpha = 1.0 \), we find that the mass increases, but stays negative for all values of the cosmological constant. In fact, as is evident from Fig. 1, the mass decreases further with the increase of \( \alpha \) for a fixed \( \gamma \). In the limit \( \gamma \rightarrow -\infty \), the mass tends to zero. We contrast the behaviour of the mass function \( \mu(x) \) for different values of \( \gamma \) with that in Fig.1 of [9]. Since the authors choose the vev of the Goldstone field to be \( \alpha = 0.01 \), while we choose it to be 1, the values of the negative valued cosmological constant in their plot corresponds to choosing \( \gamma = -10, -3 \) here. Moreover, their choice of \( G \) leads to \( \alpha = 0.0355 \). In Fig. 2, we show the profile of the mass function \( \mu(x) \) for this choice of parameters. Clearly, the mass function is negative for all \( x \). Of course, we have to add that we have integrated the equations only up to some maximal value of \( x = x_{\max} \approx 200 \). However, as can been seen for the asymptotic expansion (13), the derivative of \( \mu(x) \) at large \( x \) is always negative and thus the function continues to decrease. Because of that, local minima or even zeros of the function \( \mu(x) \) in the asymptotic region are excluded.

Fixing \( \gamma \) and varying \( \alpha \), we observe a phenomenon not previously discussed in the literature. This is demonstrated in Fig. 3 for \( \gamma = -0.1 \). Increasing \( \alpha \), we observe that a horizon starts to form and at \( \alpha = \alpha_{cr}(\gamma) \), the solution has a degenerate horizon at \( x = x_h(\alpha_{cr}) \). Thus AdS monopoles without horizon only exist for \( \alpha < \alpha_{cr} \). We find that the value of \( \alpha_{cr} \) depends
on \( \gamma \) and that it is increasing with the decrease of \( \gamma \). E.g. we find that \( \alpha_{cr}(\gamma = -0.1) \approx 1.1 \) and \( \alpha_{cr}(\gamma = -1.0) \approx 1.85 \). Note that the solution outside the horizon can not be completely described by a AdS solution of the form:

\[ N(x) = 1 - 2\alpha^2 - \frac{2\mu_\infty}{x} - \frac{\gamma}{3}x^2 \]

(14)

where \( \mu_\infty/\alpha^2 \) is the mass of the solution. The reason is that \( h(x) \equiv 1 \) is not a solution of (8).

However, \( h(x) \) is close to 1 for \( x > x_h \) and thus (14) can be thought of as an approximation.

The solution (14) has a degenerate horizon at \( x_h^a = \sqrt{(1 - 2\alpha^2)/\gamma} \) with corresponding mass
\[ \mu_h^a/\alpha^2 = 3\alpha \sqrt{((1 - 2\alpha^2)/\gamma)^3} \].

For the values of the parameters given in Fig. 3 (especially \( \alpha = 1.098 \approx \alpha_{cr} \)), we find that
\[ x_h^a = 3.756 \]
and \( \mu_h^a/\alpha^2 = -1.465 \). From our numerical analysis, we obtain \( x_h \approx 3.5 \) and \( \mu_\infty/\alpha^2 \approx -1.7 \).

Finally, to study the appearance of horizons in this model in more detail, we have fixed
\[ \alpha = 0.7 < \sqrt{1/2} \]
and \( \alpha = 0.8 > \sqrt{1/2} \), respectively, and studied the dependence of the value of the zero of \( N(x), x_h \) with \( N(x) = 0 \) in dependence on \( \gamma \). We have chosen these two values of \( \alpha \) because only for \( \alpha \geq \sqrt{1/2} \) do horizons appear in the asymptotically flat case (\( \gamma = 0 \)) [7]. Our results are shown in Fig. 4. Clearly, for \( \alpha = 0.7 \) and \( \gamma \leq 0 \) no horizons appear which confirms the results of [7], while for \( \alpha = 0.8 \) and \( \gamma \leq 0 \) we find horizons. In fact, for a specific range of \( \gamma \leq 0 \) two horizons exist. For \( \alpha = 0.8 \), we find that this is for
\[ -0.00236 \leq \gamma < 0 \]. At \( \gamma = -0.00236 \) the two horizons join and form a degenerate horizon of type shown in Fig. 3. Thus, we have
\[ \alpha_{cr}(-0.00236) = 0.8 \] which is in good agreement with the previously presented results. In the limit \( \gamma \rightarrow 0 \), the outer horizon tends to infinity, while the inner horizon tends to the one of the “supermassive” monopoles observed previously [7].
C. de Sitter monopoles

In [9] the question was addressed whether the mass of the global monopole can become positive for specific choices of the cosmological constant. It was found that for positive cosmological constants this is possible. As a check for a future publication [11] on composite monopole defects in dS/AdS space-time, we have tried to obtain the results given in [9] and found contradictions.

Choosing $\gamma$ positive a cosmological horizon appears at $x = x_c(\gamma)$ in dS space. We find that $x_c$ is a decreasing function of the cosmological constant. For increasing $\gamma$, the value of $x_c = x_0$ tends to zero as is demonstrated in Fig. 4. We find further that $(\mu_\infty(\gamma > 0) - \mu_\infty(\gamma = 0))/\alpha^2 < 0$ for all $\gamma > 0$. Since the mass curve does only alter its shape very little when choosing different $\alpha$, we conclude that in contrast to what is claimed in [9], the appearance of a cosmological constant (of either sign) can not alter the sign of the mass of the global monopole. Rather, we find that the mass gets more negative for increasing $\gamma > 0$ which can be related to the fact that the core of the monopole increases due to increased cosmological expansion.

In Fig. 2, we present the mass function for $\alpha = 0.0355$ and $\gamma = 0.073$. This should be compared to the Fig.1 in [9]. First, we remark that we are surprised that the authors of [9] have managed to find solutions which correspond to our $\gamma = 5$. We find that increasing $\gamma$ from zero to positive values, we can construct solutions only for $\gamma < \sim 0.073$. The reason is that with increasing cosmological constant the horizon which appears for the dS solutions decreases to lie closer and closer to the core of the monopole. Clearly, the mass function $\mu(x)$ is a constantly decreasing function of the coordinate $x$ and doesn’t have local extrema like in [9]. Moreover, the asymptotic values of $\mu(x)$ are always negative.

V. CONCLUSIONS

Topological defects [1] are believed to be relevant for structure formation in the universe. Global defects, i.e. defects which don’t involve gauge fields are of special interest in this context since they have a long-range scalar field. This leads to the infiniteness of energy in flat space, but however renders a strong gravitational effect when the topological defects are studied in curved space. Moreover, in the case of the global monopole, the coupling to gravity can remove the singularity present in flat space. The space-time then has a deficit angle and is not locally flat. Moreover, the mass of the monopole is negative, which was interpreted as a repulsive effect of the monopole.

While for positive cosmological constant (dS space) a horizon, the so-called “cosmological horizon” always appears independent on the gravitational coupling $\alpha$, the existence of horizons in AdS space depends strongly on the values of the cosmological and gravitational constants. For vanishing cosmological constant $\gamma = 0$, it was found previously that horizons exist only for $\alpha \geq \sqrt{1/2}$ [7]. For $\gamma < 0$ a monotonically decreasing curve in the $\gamma$-$\alpha$-plane appears which represents the solutions with one, degenerate horizon. Above this curve, solutions with two horizons exist, while below the solutions have no horizons at all.

The authors of [9] have studied global monopoles in a dS/AdS space-time and found that the inclusion of the cosmological constant can render the mass of the monopole positive. Reconsidering these solutions with a highly accurate numerical routine (see [13] for a short
description) and studying the asymptotic behaviour we come to a different conclusion: global monopoles do not acquire a positive mass in AdS or dS space-time.

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[12] Note that the mass function $m_L(x)$ used in [7] and the function $\mu(x)$ used here are defined differently. It is $m_L(x) = x\alpha^2 + \mu(x)$. Thus even if we find $\mu(x) < 0$ the mass function $m_L(x)$ stays positive asymptotically: $m_L(x >> 1) \sim x\alpha^2 > 0$.
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FIG. 1. The value of the mass $\mu_{\infty}/\alpha^2$ is given for the AdS monopoles ($\gamma < 0$) as function of $-\gamma$. We have chosen $\alpha = 0.1$, $=1.0$ and $\lambda = 1.0$.

FIG. 2. The mass function $\mu(x)$ is shown as function of $x$ for $\gamma = -10$, $-3$ and $0.073$. We have chosen $\alpha = 0.0355$, $\lambda = 1.0$. 
FIG. 3. The profile of the metric function $N(x)$ is shown for $\gamma = -0.1$, $\lambda = 1.$ and four different choices of $\alpha$, including $\alpha = 1.098 \approx \alpha_{cr}$.

FIG. 4. The value of the radial coordinate, where $N(x = x_h) = 0$ is shown as function of the cosmological constant $\gamma$ for two different values of $\alpha$ and $\lambda = 1.0$. We have chosen $\alpha$ smaller (resp. larger) than $\sqrt{1/2}$, such that for $\gamma = 0$ no (resp. one) horizon appears.