Scaling laws and intermittency in homogeneous shear flow

P. Gualtieri *, C.M. Casciola*, R. Benzi †G. Amati ‡ & R. Piva*.

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1 Abstract

In this paper we discuss the dynamical features of intermittent fluctuations in homogeneous shear flow turbulence. In this flow the energy cascade is strongly modified by the production of turbulent kinetic energy related to the presence of vortical structures induced by the shear. By using direct numerical simulations, we show that the refined Kolmogorov similarity is broken and a new form of similarity is observed, in agreement to previous results obtained in turbulent boundary layers. As a consequence, the intermittency of velocity fluctuations increases with respect to homogeneous and isotropic turbulence. We find here that the statistical properties of the energy dissipation are practically unchanged with respect to homogeneous isotropic conditions, while the increased intermittency is entirely captured in terms of the new similarity law.
2 Introduction

The issue of anomalous scaling laws in turbulence has been largely addressed for the idealized case of homogeneous and isotropic turbulence once it became clear that a purely dimensional power law \[1\] can not be consistent with the intermittent nature of energy dissipation \[2\]. The Kolmogorov-Obukhov refined similarity (RKSH) \[3\] provides the key theoretical point by relating the anomalous correction of the scaling exponents of velocity structure functions directly to the statistical properties of the dissipation field. The RKSH has been subject to close scrutiny by a number of independent investigations making use of both experiments and direct numerical simulations and it may be considered a well assessed physical law. One of the early difficulty for an accurate evaluation of scaling exponents, associated to the existence of too small scaling ranges in moderate Reynolds number turbulence, has been recently overcome with the use of the Extended Self-Similarity (ESS) \[4\]. This approach has allowed accurate estimates of the exponents from moderate Reynolds number flows \[5\] and the evaluation of the RKSH, in its extended form, from DNS data \[6\].

A more recent research interest in turbulence has been focused on the assessment of RKSH in non-isotropic and non-homogeneous conditions, namely in flows characterized by strong mean shear and anisotropy. The main focus is to understand to what extent RKSH may be considered still able to capture, in such conditions, the essential features of turbulence dynamics.

In this context, recent results for a turbulent channel flow \[7\], have pointed out how the classical RKSH holds in the higher part of the logarithmic region. This result should not be surprising since the RKSH is established as a consequence of the balance between energy transfer due to non linear interactions and dissipation. In the log-layer, where the mean shear is weak, the dynamics of the flow is characterized by the inertial energy transfer. In fact, when approaching the wall, the shear becomes larger and larger and a clear failure of the RKSH is observed \[7\]. This result gives a partial answer to the point raised above since it gives evidence that, in the near wall region, the homogeneous isotropic behavior is far from being recovered.

At this point a question naturally arises about the existence of an alternative scaling law in such conditions. On the basis of physical considerations on the respective role of energy production and energy transfer a different Kolmogorov-like scaling law has been proposed in \[7\] and preliminarily checked against DNS data from the buffer region of a turbulent channel flow. The validity of this similarity law has been confirmed by a successive experimental work in a turbulent boundary layer. However, certain aspects of the problem are still not completely clear. Actually, the new similarity law has been conceived as a direct consequence of the presence of a strong shear, but it has been verified only in wall turbulence where, in principle, some other features such as the continuous variation of the local shear, the non uniform momentum flux across adjacent layers and the suppression of wall normal velocity fluctuations may play a quite significant role.
To isolate the effect of the shear we reconsider here the problem in the context of a DNS of a homogeneous shear flow in a confined box. In fact, in the rationale of dimensional analysis, this flow presents the minimum level of complexity though maintaining the essential features we want to address: thus, it is ideal to study the effect of a pure shear avoiding other concurrent effects. On the other hand, it allows to exploit homogeneity in all spatial directions so that a more complete and accurate statistical analysis can be performed, at a level of detail that could never be achieved in the very limited buffer region of a wall bounded flow.

As a counterpart, the homogeneous shear flow presents certain drawbacks related to the artificial nature of its confinement. To understand to what extent the present flow may serve as a prototype for shear-dominated flows, we do analyze in detail the dynamics of the coherent vortical structures observed in the numerical simulation. This is instrumental to qualify the flow, i.e. its dependence on the aspect ratio of the computational box, in view of the use we do of the flow itself for the evaluation of scaling laws and their assessment.

Homogeneous shear flows have been investigated in the literature in many theoretical, numerical and experimental works. Rogers and Moin [8] and Lee et. al. [9] studied the topology and mutual interactions of vorticity showing that many dynamical features are quite similar to those observed in the wall region of a turbulent boundary layer. A more detailed study on the same subject has been performed by Kida and Tanaka [10] who discussed the regeneration cycle of the streamwise vortices in a homogeneous shear flow.

Experimentally the papers by Rose [11], Champagne Harris and Corrsin [12], Tavoularis and Corrsin [13] [14], have investigated homogenous shear flows obtained in a wind tunnel, focusing on the local isotropy of small scale fluctuations. Following the Kolmogorov theory, an important question to investigate, which becomes extremely well posed in the case of the homogeneous shear flow, concerns the recovery of small scales local isotropy in turbulent flows characterized by large scale non isotropic forcing. The same kind of question has been addressed by Saddoughi and Veeravalli [15] who investigated local isotropy for the turbulent flow in a logarithmic boundary layer.

The issue of local isotropy has been also addressed more recently by Pumir and Shraiman [16] and by Pumir [17] who performed an extensive analysis of numerical simulations of statistically stationary homogeneous shear flows. Interestingly enough, most of the statistical properties found by Pumir [17] are quite close to the experimental findings of Garg and Warhaft [18] and Shen and Warhaft [19]. In the latter experiment, for the first time, an active grid was used in order to produce a homogeneous shear flow in a wind tunnel whose integral scale in the streamwise direction is rather constant.

Taking full advantage of most of these results, and in order to be able to reconsider the issue of local isotropy from a different perspective, as we plan to do in the near future, our interest here is mainly focused on the relationship between intermittency and the anisotropy induced by the mean velocity gradient at large scale. In the context of boundary layer we have shown how the increase of intermittency may be explained in terms of the phenomenology described by the new form of scaling law. In particular, as suggested
in [4], a new length scale \( L_s = \sqrt{\bar{\epsilon}/S^3} \) enters in the description of the statistical properties of turbulence, where \( \bar{\epsilon} \) is the mean rate of energy dissipation and \( S \) the mean large scale shear. For scales smaller than \( L_s \) the statistical properties of the turbulent fluctuations are similar to those observed in homogenous and isotropic flows while for scales larger than \( L_s \) the refined Kolmogorov similarity (RKSH) is broken and intermittency increases. The phenomenological theory developed for the turbulent boundary layers relies upon the mean shear strength \( S \) as the only parameter which fixes the scale \( L_s \) where a new form of RKSH should be observed. For this reason, the homogenous shear flow becomes a natural test case to understand whether or not the phenomenological theory devised for the boundary layer is sufficiently general to be applied to any turbulent flow. As we shall discuss in this paper, our numerical simulations of homogenous shear flows support rather well the new phenomenology, leading to possible important implications in our understanding of shear turbulence.

The paper is organized in the following way. In section 3 we briefly describe how the numerical simulations are performed. In section 4 we discuss the regeneration cycle of the vortical structures observed in the flow while in section 5 we address the intermittency cycle which characterizes the dynamical behavior of the system. In section 6, after a short review of the phenomenological theory proposed for the turbulent boundary layer, we describe the intermittency properties of turbulent fluctuations. In section 7, we investigate the anomalous scaling of the energy dissipation field, and, finally, in section 8 we draw our main conclusions.

### 3 Homogeneous shear flow

We consider here a turbulent flow in a confined box with an imposed mean shear \( S \) free from boundaries. The full Navier-Stokes equations are solved (DNS), after decomposing the velocity field into mean value and fluctuation

\[
\vec{v}(\vec{x}, t) = U(y)\vec{e}_1 + \bar{u}(\vec{x}, t),
\]

where \( \vec{e}_1 \) is the unit vector in the streamwise direction \( x \). The mean gradient \( S \) is in the normal direction \( y \) while \( z \) denotes the spanwise coordinate.

The Navier Stokes equations are written in terms of velocity fluctuations

\[
\begin{cases}
\vec{\nabla} \cdot \bar{u} = 0 \\
\frac{\partial \bar{u}}{\partial t} = (\bar{u} \times \vec{\zeta}) - \vec{\nabla}(p + \frac{u^2}{2}) + \nu \nabla^2 \bar{u} - Sv\vec{e}_1 - U(y)\frac{\partial \bar{u}}{\partial x},
\end{cases}
\]

where \( \vec{\zeta} \) is the vorticity, \( u \) the normal velocity, \( p \) the pressure and \( \nu \) the kinematic viscosity. To achieve efficiency and accuracy a Fourier spectral method is advisable to solve the
initial value problem for eq. (2). However, due to the mean flow, the intrinsically non-periodic term \( U(y) \partial\vec{u}/\partial x \) appears in the equations. Luckily, by using Rogallo’s technique [20], this difficulty is removed by the transformation

\[
\begin{align*}
\xi &= x - U(y)t \\
\eta &= y \\
\zeta &= z \\
\tau &= t.
\end{align*}
\]

(3)

In the new variables periodic boundary conditions can be enforced in all spatial directions allowing efficient pseudo-spectral methods to be used for spatial discretization. The time integration is performed using a third order low storage Runge-Kutta method [21] and the nonlinear terms are fully de-aliased by zero padding. Since the image of the computational box in physical space gets distorted, eq. (3), a re-meshing procedure is periodically applied to allow long time integrations. Using periodicity in the \( \xi \) direction, the computational domain is transformed back into a non skewed domain whenever the plane \( y = \lambda_y \) has moved by \( 2\lambda_x \) in the streamwise direction, i.e. every \( \Delta t_r = \lambda_x/S\lambda_y \). This procedure may introduce aliasing errors since the dynamics of wave vectors in physical space is time dependent [22], as seen from the relation between the Fourier transforms in the computational and in the physical space,

\[
\hat{u}_i(k_x, k_y, k_z, t) = \hat{\tilde{u}}_i(k_\xi, k_\eta - S_\tau k_\xi, k_\zeta, \tau).
\]

(4)

To avoid aliasing errors, the spectral components with wave numbers outside the interval \([-k_{\text{max}}, k_{\text{max}}]\) are set to zero at re-meshing. This filtering introduces a characteristic wave number defined as

\[
k_r = \frac{k_{\text{max}}}{\sqrt{1 + A^2}}
\]

(5)

where \( k_{\text{max}} = \pi N_y/2\lambda_y \) and \( A = \lambda_x/\lambda_y \) is the aspect ratio of the computational domain. In order to assess the influence of the filtering, the aspect ratio of the box and the resolution of the grid were varied. In all cases \( k_r \) was larger than the dissipation wavenumber and no appreciable alteration of the dynamics was observed.

A crucial point for our successive analysis is the ability of the system to reach statistical stationarity. Under this respect, our computations entirely confirm the results of Pumir and Shraiman [16] and Pumir [17]. Actually the evolution of the turbulent kinetic energy is described by the equation

\[
\frac{\partial}{\partial t}[u^2] + S[uv] = -\nu[\zeta^2]
\]

(6)

where the square brackets denote spatial average. This equation suggests a possible balance between the production \( S[uv] \) and the dissipation \( \nu[\zeta^2] \). This balance is not achieved for the instantaneous fields, as shown in figure 1 which reports the history of
turbulent kinetic energy in our longest calculation. Large fluctuations in energy (42%) and enstrophy (50%) are apparent in the pseudo-cyclic behavior shown in the figure. The fluctuation level in this system is quite huge, especially when compared to that of forced homogeneous isotropic turbulence, which is limited to only a few percent of the rms value (5 – 6%) for both kinetic energy and enstrophy. In fact, the pseudo-cyclic behavior corresponds to statistically stationary conditions, as follows from time averages performed over time periods much larger than the typical length of the cycle.

Clearly, DNS enables to achieve a good level of homogeneity, as shown in figure 2. In particular both the turbulent kinetic energy and the Reynolds stresses are substantially constant in the y direction.

In order to check the dynamics of the velocity gradients, the probability density function of $\partial u/\partial x$ and of $\partial u/\partial y$ have been computed, figure 3 and 4. They are often used to assess the small scale dynamics of the flow and to characterize its degree of anisotropy. The computed values of skewness and flatness are in good agreement with the experimental results at $Re_\lambda \sim 100$ of Shen and Warhaft [19] and of Ferchichi and Tavoularis [23] at $Re_\lambda = 140$.

4 Regeneration cycle of vortical structures

The energy fluctuations discussed in the previous section correspond to a regeneration cycle of the vortical structures. Most of the previous numerical simulations have described the main features of coherent structures and the vorticity statistics for the early stages of development of the flow. In particular, Rogers & Moin [8] showed typical hairpin vortex structures remarkably similar to those observed in wall bounded flows. Lee et. all. [9] discussed streamwise vortices and high and low speed streaks commenting on the similarities with the buffer region of wall bounded flows. In a more recent work Kida & Tanaka [10] proposed a regeneration mechanism for the streamwise vortices and pointed out the role of vortex sheet instability in the formation of new vorticites. The present analysis deals with the same issues, but it is focused on the statistical steady state regime of the flow.

The phenomenology is better understood by considering a few snapshots along the history of the turbulent kinetic energy and of the Reynolds stresses. In figure 5 the large energy bursts already noticed in the previous section are clearly correlated to large negative values of the Reynolds stresses. The bursts are induced by large rates of energy injection from the mean flow associated with the presence of streamwise vortices. Figure 6, corresponding to an instant at the beginning of a stage of energy growth, actually shows a large population of quasi-streamwise vortices, here visualized through the discriminant of the velocity gradient [24]. During the successive phase of energy growth the streamwise vortices, by interacting with the mean velocity field, give rise to instantaneous profiles characterized by the typical ramp and cliff pattern. The pdf of $\partial u/\partial y$ is a suitable tool
to characterize statistically the ramp and cliff structures, figure 4. The skewness of 0.82 gives reason of intense positive events of $\partial u/\partial y$ much more probable than negative ones. As for the passive scalar, the streamwise velocity recovers the mean gradient through ramps followed by cliffs [25]. The ramps correspond, on average, to streamwise velocity increasing with $y$, hence a resulting negative spanwise vorticity is expected, on average. The pdf of the spanwise vorticity, figure 7, confirms this analysis.

Ramp and cliffs are associated with thin regions of (negative) spanwise vorticity, i.e. vortex sheets, as shown in figure 8. The sheets become unstable and eventually roll up into spanwise vortices through the classical Kelvin-Helmholtz mechanism, see figure 9 where spanwise vortices are systematically located in the regions where the roll-up process is occurring. Vortex sheets and spanwise vortices are clearly responsible for the observed energy bursts by producing large negative Reynolds stress events.

Immediately after each burst, the non-linear interactions are enhanced and the original vortex structures lose their order resulting into a randomized vorticity field. This phase, shown in figure 10, reduces the level of anisotropy of the flow. However, soon afterwards, the shear term of the vorticity equation enforces a mean orientation to the structures, see figure 11, corresponding to a minimum for the energy, just before the occurrence of the successive burst. The vortical structures are now getting more and more aligned with the streamwise direction and a new cycle starts.

The regeneration cycle just analyzed here for the statistical steady state phase is very similar to that described by Kida & Tanaka [10] for the early stages of evolution of the flow from isotropic initial conditions, suggesting that the operating mechanisms are substantially identical.

5 Intermittency cycle

The increase of the turbulent kinetic energy due to the streamwise vortices can be understood, basically, in terms of the linear lift-up mechanism and the related transient growth. A non-linear mechanism is required instead to explain the saturation and the break-down of the ordered system of vortices. In spectral space, as already discussed by Pumir [17], the Fourier mode $(0, 0, \pm 1)$ gives the leading contribution to the energy growth. From the linearized equation for this mode,

$$
\frac{d\hat{u}}{dt} = -S\hat{v} - \nu\hat{u},
\frac{d\hat{v}}{dt} = -\nu\hat{v},
$$

(7)
a growing amplitude is expected whenever $S[Re(\hat{u})Re(\hat{v}) + Im(\hat{u})Im(\hat{v})] < 0$. As soon as the energy grows appreciably, the energy drained by the interactions with the other modes in favor of the smaller scales quite rapidly saturates its amplitude. Hence, the
characteristic frequency of the intermittency cycle is determined by the dynamical balance between growth of the basic mode and energy transfer.

In principle, if the mode \((0, 0, \pm 1)\) were isolated from the others during its growth, the energy due to the interaction with the mean shear would be almost entirely found in this mode. Then, the characteristic time of the system would essentially coincide with the eddy turnover time of the basic mode, hence to the spanwise size of the box \(L_z\).

In fact, the leading mode evolves in presence of many others. In this case the saturation time, i.e. the time of the effective activation of the nonlinear energy transfer, is related to the separation, in wavenumber space, between the leading mode and the small scales modes. The activation time becomes shorter and shorter as the distribution of energy among all Fourier modes approaches a continuous distribution. For the statistical steady state regime, in particular, the nonlinear energy transfer is crucial from the very beginning of the growth. Actually the energy spectrum in correspondence of the large bursting phenomena, figure 12, shows initially a growth which, though dominated by the mode \((0, 0, \pm 1)\), is spread all over the modes in the first decade. The evolution of the spectra manifest a strong energy transfer which though occurring during the entire cycle, is clearly prevailing in the phase of energy decrease. Hence the bursting frequency can not be estimated from the isolated dynamics of the sole \((0, 0, \pm 1)\) mode, and, as a consequence, there is no a priori reason for a strong dependence of the bursting period on the size of the box.

In view of a quantitative analysis of this issue, let \(E_U\) and \(E_u\) denote the kinetic energy of the basic flow and of the fluctuations, respectively. Clearly, \(E_u\) is a function of time. Two quite different cases can be distinguished, namely

\[
\begin{align*}
& a) \quad E_u \geq E_U \text{ at } t = 0 \\
& b) \quad E_u \ll E_U \text{ at } t = 0 
\end{align*}
\]

Case b) corresponds to a problem of transition to turbulence from small, though finite, disturbances and will not be considered here in further detail. We are presently interested in case a), which is more appropriate to describe statistical stationarity. Here \(t = 0\) should be understood as an arbitrary instant of time along the cyclic evolution of the system.

Given the correlation function \(C(\tau) = \langle E_u(t + \tau)E_u(t) \rangle\), let us consider the correlation time, \(\tau_C\), defined, e.g., as the smallest \(\tau\) such that \(C(\tau_C) = 1/2C(0)\). Following the discussion of section 4, \(\tau_C\) corresponds, roughly, to the characteristic time of production of large scale velocity/vorticity instability which is the primary forcing mechanism of the homogeneous shear flow. If \(\tau_C\) is almost independent or, at least, weakly dependent on \(L_z\), the dynamical behavior of the system may be considered as substantially unaffected by the finite size of the box. A rigorous analysis of the correlation time dependence on the spanwise size \(L_z\) would require a large number of highly resolved numerical simulations far beyond the present computer capabilities. In figure 13, we show a preliminary evidence that the behavior of the kinetic energy is not too sensitive to \(L_z\), by comparing
the simulations performed with \( L_z = 2\pi \) and \( L_z = 4\pi \). The characteristic time of the generation mechanism of large scale vorticity seems not to depend on \( L_z \). These results suggest that the finite size of the box does not affect in a significant way the dynamics and the statistical properties of the homogeneous shear flow, despite the importance of the basic mode, whose length scale coincides with that of the box, in the dynamics of the flow.

6 Intermittency and scaling

In the previous sections we have analyzed the influence of the shear on the dynamics of the vortical structures. We are now ready to discuss its effect on statistical features of turbulent fluctuations, such as scaling laws and intermittency.

Concerning the velocity field, a quantitative description can be given in terms of the possible scaling behavior of the structure functions, i.e. the moments of longitudinal velocity increments

\[
< \delta V^p > = < \{ \vec{u}(\vec{x} + \vec{r}, t) - \vec{u}(\vec{x}, t) \} \cdot \vec{r} >^p
\]

where angular brackets denote ensemble averaging. For homogeneous and isotropic conditions, the dimensional prediction of Kolmogorov theory (K41) provides a scaling law in terms of separation

\[
< \delta V^p > \propto \bar{\varepsilon}^{p/3} r^{p/3}
\]

where \( \bar{\varepsilon} \) denotes the mean rate of energy dissipation per unit mass. For \( p = 3 \), equation (9) is consistent with the exact result usually referred to as the four fifth law,

\[
< \delta V^3 > = -\frac{4}{5} \bar{\varepsilon} r.
\]

This equation has been widely confirmed by a number of experimental investigations which, at the same time, showed that dimensional scaling does not hold for moments different from the third. In fact, a more complex dependence of the scaling exponents is found,

\[
< \delta V^p > \propto r^{\zeta_p},
\]

with \( \zeta_p \) a nonlinear convex function of \( p \). This anomalous scaling related to intermittency, has been actively investigated over the last twenty years.
6.1 Dimensionless parameters

To understand how intermittent fluctuations are affected by the shear, it is worthwhile to consider the typical length scales involved in the process and the relevance of the shear production term over the nonlinear inertial interactions. In presence of shear, velocity fluctuations arise for two main reasons, advection of streamwise momentum across the mean gradient and non-linear mixing. Let us denote by $\delta U = Sr$ the velocity difference at scale $r$ due to the mean flow and by $\delta u$ the fluctuation at the same scale. A rough estimate for the latter is $\delta u \propto \bar{\epsilon}^{1/3}r^{1/3}$. The length scale for which turbulent fluctuations are of the same order of magnitude as the velocity increments induced by the shear, uniquely identifies the shear scale $L_s$

$$L_s = \sqrt{\frac{\bar{\epsilon}}{S^3}},$$

(12)

that separates two different sub-ranges within the inertial range. For $L_0 \gg r \gg L_s$ (with $L_0$ the integral scale), $\delta U \gg \delta u$ and the dynamics and statistics of turbulence is expected to be dominated by the momentum flux due to the Reynolds stresses, i.e. by the production of kinetic energy. For $\eta \ll r \ll L_s$, $\delta u \gg \delta U$ and the fluctuations induced by the mean shear are negligible with respect to those typically produced by the nonlinear term. In this case the dynamics of the flow is characterized by the energy transfer due to the same inertial interactions which are typical of isotropic turbulence.

In order to quantify the predominance of either mechanism over the other, we consider the dimensionless parameter, defined as the ratio of an inertial and a shear time scale,

$$S^* = \frac{Sq^2}{\bar{\epsilon}}$$

(13)

with $q^2 = \langle u_i u_i \rangle$. $S^*$ can be interpreted as the ratio of two length scales

$$S^* = \left(\frac{l_d}{L_s}\right)^{2/3}$$

(14)

where $l_d = q^3/\bar{\epsilon}$. It follows that for large values of $S^*$ the dynamics of most of the inertial scales is dominated by the mean shear.

The other dimensionless parameter needed for a complete description of a homogeneous turbulent shear flow can be given as the ratio of the dissipative and the shear time scale,

$$S^*_c = S(\nu/\bar{\epsilon})^{1/2}$$

(15)

or, equivalently, as the ratio of the Kolmogorov length and the shear scale

$$S^*_c = \left(\frac{\eta}{L_s}\right)^{2/3}$$

(16)
Since $S_c^*$ measures the separation between the shear scale and the dissipative range, a sub-range of isotropic behavior may be recovered within the inertial range when it is small.

Typical values of $S^*$ and $S_c^*$ are shown in figure 14, which is a compilation of several independent numerical and experimental simulations. The variety of behaviors observed in these simulations can easily be interpreted in terms of the location of the shear scale $L_s$, quite different from one case to the other.

6.2 Intermittency

Let us now turn to the issue of intermittency. A quantitative measure of intermittency is provided by the flatness factor of the random variable $\delta V(r)$. Figures 15 and 16 show the probability distribution function of $\delta V(r)$ for two different separations both in the shear flow and in homogeneous isotropic turbulence. For small separations in both cases the pdf is clearly non Gaussian and the flatness factor, shown in figure 17, grows as separation decreases confirming that the dimensional scaling of the structure functions is clearly violated. However, important differences are apparent. The tails for the shear flow are quite higher and the pdf is more skewed towards negative values. For the larger separation, instead, the two pdfs are much more similar, though still not Gaussian. The Gaussian behavior is recovered only for $r$ comparable with the integral scale. Globally, we find a correspondence with what was found in the turbulent channel flow [7], where at small separation the pdfs of $\delta V(r)$ in the buffer and in the log-region differs substantially, with the negative tail much higher than the positive and an increased flatness factor in the buffer.

6.3 Similarity laws

The issue of intermittency can be quantified by looking directly at the scaling properties of the structure functions. As well known, starting from Landau objection [3] a revised form of similarity law has been proposed by Kolmogorov and Obhukov [3] (K62): assuming that the dissipation field is a random variable, equation (9) is corrected to

$$< \delta V^p > \propto < \epsilon_r^{p/3} > r^{p/3},$$

where $< \epsilon_r^{q} >$ denotes the q-th moment of the local energy dissipation field $\epsilon_{loc}$ averaged over a volume of characteristic dimension $r$. Given for the dissipation field a scaling law in terms of separation,

$$< \epsilon_r^{q} > \propto r^{\tau(q)},$$

from equation (17) the scaling exponents of the structure functions are expressed as

$$\zeta(p) = \tau(p/3) + p/3$$
with \( \tau(p/3) \) a nonlinear function of \( p \) and \( \tau(1) = 0 \). Equation (19) is consistent with the intermittent nature of the random variable \( \delta V(r) \), i.e., its increasing flatness with decreasing separation, since the flatness factor, within the inertial range, turns out to be an unbounded function, diverging for small separations as \( r^{\zeta_4-2\zeta_2} \). The intermittency is related to bursty signals characterized by localized and intense fluctuations and to the presence of coherent structures and associated regions of intense gradients which give rise to a highly non uniform dissipation field.

Scaling laws in terms of separation are found only for sufficiently large values of the Reynolds number i.e., so far, only in experimental facilities.

An extension of the scaling range can be achieved by using the Extended Self Similarity (ESS) as proposed by Benzi et. al. [4], who introduced a generalized form of the scaling law (11),

\[
< \delta V^p > \propto < \delta V^3 >^{\zeta_p/\zeta_3} .
\]

Using experimental data, the scaling law (20) has been shown to hold even at low and moderate Reynolds number, i.e. in cases, such as those amenable to DNS, where the scaling (11) is not directly detectable. Most interestingly, the ratios \( \zeta_p/\zeta_3 \) have systematically been found independent of the Reynolds number and consistent with the measurements in the high Reynolds number regime. The ESS allows a generalization of the refined Kolmogorov similarity hypothesis [6], eq. (17), namely

\[
< \delta V^p > \propto < \epsilon_r^{p/3} > < \delta V^3 >^{p/3} .
\]

This equation, which we will hereafter refer to as RKSH, has been carefully checked against experimental data for homogeneous and isotropic turbulence and confirmed also in those cases where neither \( < \epsilon_r^{p/3} > \) nor the structure functions showed a scaling behavior in \( r \). The two terms in eq. (21), namely \( < \epsilon_r^{p/3} > \) and \( < \delta V^3 > \), take into account the fluctuations in the energy dissipation rate and the energy transfer through the inertial scales, respectively.

Scaling laws as discussed so far heavily rely on the assumption of isotropy. In many turbulent flows, however, isotropy is broken by the mean shear and by the boundary conditions and a large production of turbulent kinetic energy occurs due to the interaction of the mean flow with the turbulent fluctuations. We may wonder if scaling laws emerge also in such conditions, and, in case, under which respects they differ from the classical ones.

Recent numerical investigations of wall turbulence have shown how intermittency increases approaching the wall and how the scaling exponents \( \zeta_p \) are not consistent with equation (21). Actually, in the wall region, the main dynamical process is represented by the momentum transfer occurring in the wall normal direction associated with a large production of turbulent kinetic energy via the Reynolds stresses \( dU/dy < uv > \). In this case
a term of the form \( dU/dy < \delta V^2 > \) is expected to account for the new balance between production and dissipation in the Karman-Howarth equation. Based on this analysis a new form of similarity law has been proposed for the wall region in terms of the structure function of order two \[7\]

\[
< \delta V^p > \propto \frac{< \epsilon^{p/2} >}{< \epsilon >^{p/2}} < \delta V^{2>^{p/2}}.
\] (22)

As discussed in \[7\] for a turbulent channel flow, the classical RKSH, eq. (21), holds at the center of the channel in the bulk region where the momentum transfer due to the shear term is negligible with respect to inertial transfer. The new form of similarity law holds in the near wall region where high turbulent kinetic energy production occurs.

We are presently going to check the consistency of equation (22) in the homogeneous shear flow where, in the absence of solid walls, the mean shear \( S \) is constant. In this flow the effect of the shear is isolated from other concurrent effects such as the suppression of wall normal velocity fluctuations and the non uniform momentum transfer across adjacent layers in the normal direction. Moreover the statistical analysis is simplified since homogeneity is exploited in all spatial directions.

In figure 18 the similarity law (21), for \( p = 6 \), is applied to DNS data for both isotropic and homogeneous shear turbulence. It holds for all the resolved scales in the isotropic case. For the shear flow, a clear failure is observed instead. In this case two different slopes may be extracted from the data, one in the dissipative region with the trivial value of 0.99, the other, in the inertial region, with value of 0.92. Analogous results are found for \( p = 9 \), figure 19, with a single slope for isotropic turbulence. For the shear flow, the dissipative slope is 0.98 confirming the quality of our DNS while the inertial fit gives a slope of 0.88.

A similar analysis is presented in figures 20 and 21 for the new similarity law, eq. (22), in the case of the homogeneous shear flow, for \( p = 4 \) and \( p = 6 \). A unique slope can be effectively fitted all over the range of resolved scales with the same value of 1 independently of the order of the moment. The failure of the classical RKSH and the validity of the new form for the shear flow are still better appreciated in the compensated plot of equations (21) and (22) versus the separation \( r \), figure 22 and 23.

As a technical detail concerning the evaluation of the RKSH, we remark that \( L_s = 0.8 \) and that the dissipation fit is extracted from the range of separations \([0, l_{fit}]\), with \( l_{fit} = 0.2 \). According to our theoretical considerations, for \( r \ll L_s \) we should recover the classical RKSH. However, the present value of 0.15 for \( S_c^* \) implies that when \( r \ll L_s \) we also have \( r \sim \eta \), hence no room is left for the establishment of the classical form of scaling below \( L_s \).
6.4 Extended self similarity

Before closing the section, it is worth commenting on the issue of ESS scaling in shear flows. Actually, the use of equation (20) requires some caution in the present conditions, where a superposition of different laws may be present. Clearly, a blind fit through the entire available range can extract an effective slope. It is much safer, instead, to consider the local slopes in detail, to understand to what extent a power law is really observed, and, more interestingly, the range of scales over which a certain value for the exponent is found. As an example, in figures 24 and 25, the local slopes, $p = 6$ and $p = 9$, for the shear flow are contrasted with those of homogeneous isotropic turbulence. In both cases at small separation the dissipative scaling $\delta V^p \propto r^p$ is achieved confirming the good quality of the data. As we move towards inertial separations the local slopes of isotropic turbulence remain quite constant over almost one decade. For the shear flow instead a definite deviation from constant is found at approximately $r/\eta = 15$, with the shear scale falling near by ($L_s/\eta = 16$). Similarly to what found in the lower part of the logarithmic region of the turbulent channel [26], the local slopes do not display clear ESS scaling regions, presumably due to the competition of two different statistical trends and to the lack of sufficient scale separation to distinguish between the two.

6.5 An estimate of the flatness

A different estimate of the flatness is here proposed taking into account the similarity laws given by eq. (21) and (22). Coming back to the issue of intermittency, let us consider again the flatness factor of the random variable $\delta V(r)$,

$$F(r) = \frac{<\delta V^4(r)>}{<\delta V^2(r)>^2}$$.  \hspace{1cm} (23)

According to the two forms of RKSH, eqs. (21) and (22), the same flatness factor can be expressed in terms of moments of the dissipation field as

$$F_b = \frac{<\epsilon_{r}^{4/3}>}{<\epsilon_{r}^{2/3}>^2}$$ \hspace{1cm} and \hspace{1cm} $$F_w = \frac{<\epsilon_{r}^2>}{<\epsilon_{r}>^2}$$,  \hspace{1cm} (24)

respectively. The plots in figure 26 show the curves corresponding to the three equations (23) and (24). The values computed using the new RKSH, $F_w$ in eq. (24), are in excellent agreement with those evaluated from the definition (23). It should be noted that the use of the wrong form of RKSH, corresponding to expression $F_b$ in eq. (24), substantially underestimates the intermittency.

The same analysis can be performed for homogeneous and isotropic turbulence. In the plots of figure 27, we show $F$, $F_b$ and $F_w$ for turbulence measured at the center of a jet far downstream ($R_{\lambda} \sim 800$) [1]. As one can easily observe, the new form of refined similarity hypothesis fails in homogeneous isotropic conditions, as we should expect.
The ability of the new RKSH to predict the amount of intermittency in the velocity increments from the pdf of the dissipation field is, in our opinion, a strong check on the validity of the proposed form of scaling.

7 Statistical properties of the dissipation field

Given its role in both the new and the classical form of RKSH, the study of the dissipation field may give further insight in the behavior of shear turbulence. To this purpose figure 28 reports the self-scaling of the dissipation field in logarithmic scale. The solid line with slope $\alpha = 3.1$ shows clearly that for sufficiently large separations $r$, which correspond to small values of the two moments, the power law $< \epsilon_r^3 > \propto < \epsilon_r^2 >^\alpha$ is able to fit the data. The scaling is strikingly similar to that found in homogenous and isotropic turbulence, where the estimated exponent is 3. The self-scaling seems to hold for all the moments we have been able to check, and the values found for the shear flow are always in agreement with those of homogeneous and isotropic turbulence, see table [1]. This analysis suggests that relations of the form

$$< \epsilon_r^q > \propto < \epsilon_r^2 >^\alpha_q$$

with, more or less, fixed exponents are able to describe the statistics of the fluctuations of the dissipation field either in shear or isotropic turbulence. Hence, basic statistical properties of the dissipation field remain substantially unchanged in the two cases. This issue becomes even more evident when considering the second moment of the dissipation as a function of $r$. This quantity is reported in figure 29, where the triangles and the circles, corresponding to isotropic and shear turbulence respectively, indicate a substantially identical behavior. All together, these findings suggest a universal form for the pdf of the dissipation field, $p(\epsilon_r/\bar{\epsilon})$ normalized by its mean value $\bar{\epsilon}$ which is obviously dependent on the flow field.

8 Concluding remarks & comments

The direct numerical simulation of a homogeneous shear flow in a confined box has been analyzed to address the issue of scaling laws and intermittency in the simplest conceivable environment where anisotropy is prevailing. The dynamics of the vortical structures observed in the DNS is characterized by a regeneration cycle associated to a substantial fluctuation in the level of turbulent kinetic energy. The regeneration mechanism has been discussed in detail, and our analysis confirms substantially to previous descriptions [10] which, however, were restricted to the initial phase of flow development. In fact, in complete agreement with [16] and [17], we confirm that the confinement of the flow to a periodic box generates a dynamics, which, although strongly characterized by the...
regeneration cycle of the vortical structures, reaches statically stationary conditions. The characteristic time of the system seems to be quite insensitive to changes in the aspect ratio of the computational box, hence we feel that the present flow may be considered as a valid prototype for the study of shear flows.

We find an increased intermittency, as evaluated by the flatness of the velocity increments, with respect to that observed in isotropic turbulence. In isotropic turbulence, the intermittency of the velocity increments is related to the intermittent behavior of the dissipation field via the Refined Kolmogorov Similarity Hypothesis. For the shear flow, instead, we observe a clear failure of the RKSH which confirms the conclusions previously reached from data in the buffer layer of a channel flow (DNS) and successively verified with hot-wire measurements in a flat plate boundary layer.

To attempt a clarification of this point, we have considered the possible existence of a self-scaling behavior of the structure functions. Under this respect, the extended self-similarity expressing the \( p \)-th-order structure function as a power law with exponent \( \zeta(p) \) in terms of, say, the third order moment, is a well established feature of isotropic turbulence which is able to characterize the amount of intermittency in terms of the scaling exponents. For the homogeneous shear flow, as already observed in the lower part of the logarithmic region of turbulent channel flow, scaling exponents can be defined only for a limited region of scales \( r < L_s \) above which a competition between two different statistical behaviors makes it difficult to define any scaling at all. However, recent experimental results concerning the log-layer of a turbulent boundary layer at a higher Reynolds number, seem to indicate the existence of a double scaling regime \[27\]. Nonetheless, we are presently able to grasp the nature of the intermittency in the shear flow, since the failure of the RKSH is accompanied by the establishment of a new form of scaling, as proposed in \[7\]. Originally the idea behind the new scaling law was to account for the crucial role of the shear in originating a substantial production of turbulent kinetic energy via the Reynolds stress. In principle, this mechanism, directly associated to the presence of a mean shear, is independent of other details of the flow, such as the presence of nearby solid boundary or the specific dynamics of the vortical structures which sustain the turbulent fluctuations. This view is confirmed by the present results which show how the new form of RKSH holds not only for the buffer layer of wall bounded flows, as originally verified in \[7\], but also in the present homogeneous shear flow. Here the dynamics of the vortical structures, a part from a general resemblance, is in fact much different from that observed in the wall region and no wall is present to inhibit wall normal motions. Nonetheless the new scaling law is well established, confirming that its presence is the signature of a predominant effect of the shear in the proper range of scales.

Beyond the direct observation of the new scaling law, the detailed analysis presented in the paper shows how the increased intermittency of the longitudinal velocity increments in shear dominated flow is, in fact, to be ascribed to the new scaling law. Actually, the statistical properties of the energy dissipation field are substantially similar to those we find in isotropic conditions. From our data, the scaling behavior of the energy dissipation
field can not be consistent both with the observed level of intermittency of the velocity structure functions and with the existence of the classical RKSH. On the contrary, the new RKSH is able to consistently predict the level of intermittency in the velocity structure function in terms of the sole dissipation field. This confirms that the intermittency of velocity structure functions at shear dominated scales is largely affected by the instantaneous process of turbulent kinetic energy production. In other words, since the dissipation field is approximately unchanged, to compute the increased intermittency of the velocity in presence of shear we have to include explicitly the production mechanism in the scaling law.

These comments suggest that the source of intermittency in the shear flow is in principle substantially different from that operating in isotropic turbulence. To speculate a little further on this point, let us recall that, a part from the aspect ratio of the box, the confined homogeneous shear flow is defined by two dimensionless parameters, namely $S^*$ and $S^*_c$. These two parameters control the position of the shear scale $L_s$ relative to the length scale of the large eddies, $l_d$, and to the Kolmogorov scale, $\eta$, respectively. A large value of $S^*$ implies a substantial range of scales where the shear is crucial to the dynamics. On the other hand, a small value of $S^*_c$ corresponds to an extended range of scales where the dynamics is purely inertial and where the effect of shear is largely irrelevant. One should expect the classical scaling laws of homogeneous and isotropic turbulence to emerge clearly in the limit of $S^*_c$ small within the range $L_s \gg r \gg \eta$. A major contribution of the present paper is to confirm the existence of a new form of scaling law which emerges in the limit of $S^*$ large, in the range $l_d \gg r \gg L_s$. Ideally, when both $S^*$ and $1/S^*_c$ are large, one should expect a coexistence of the two laws in the two different scaling ranges. In this ideal conditions, the intermittent behavior of velocity increments is induced by the energy injection associated to the turbulent kinetic energy production mechanism and is originated in the shear dominated range, where the scaling is totally different from the classical RKSH. However, once the energy is transferred to the smaller scales where direct injection due to shear is irrelevant, the classical energy cascade mechanism takes over. The intermittency of the velocity increments at the shear dominated scales appears to smaller scales which belongs to the classical inertial range as large scale modulations. Along the classical line of reasoning, the large scale modulations, through the cascade, are reduced to the intermittent behavior of the dissipation field. This is tantamount to postulating a certain universality of the probability distribution of the dissipation field, once, in the spirit of K62 theory, universal values for the anomalous corrections to the exponents of the structure functions are assumed in the classical part of the inertial range. These concepts are consistent with our numerical experiments with the homogeneous shear flow, even though the scale separation is not sufficient to observe both scalings simultaneously.

To conclude we like to comment about the different sets of data available to us in the present analysis. As figure ?? shows, a large shear dominated range, i.e. $S^*$ large, is not frequently met in practice. An exception is the buffer region of wall bounded flows.
There, almost the entire range of available scales falls above $L_s$, which for this case is also very close to the Kolmogorov length, $\eta$, since $S_c^* \simeq 1$. These are optimal conditions to observe the new RKSH as a pure law, without interference with its classical counterpart since the contiguous purely inertial range below $L_s$ does not exist at all. The logarithmic region, both from DNS and experiments, present $L_s \simeq l_d$, i.e. $S^* \simeq 1$, and $L_s >> \eta$. Here only the classical scaling can be expected. The conditions of the present confined homogeneous shear flows are somehow intermediate between the ones of the buffer and the log region respectively. Here $L_s$ falls almost exactly in the middle of the available range of scales, a more complex situation and by far the more interesting to discuss. If the shear dominated and the purely inertial range were large enough we might have observed the simultaneous presence of the two forms of scaling law. However this cannot be the case with the Reynolds number we can reach by DNS. In practice, what we observe in the present case can be described as a sort of superposition of the two regimes, with a dominating contribution from the new RKSH.
Table 1: Scaling exponents, $\alpha_q$, of the moments $< \varepsilon^q >$ with respect to $< \varepsilon^2 >$, for different, non-integer, values of $q$, see eq. (25).

| $\alpha_q$ | Hom. | Iso. | Hom. | Shear |
|------------|------|------|------|-------|
| $\alpha_2/3$ | -0.11 | -0.11 |
| $\alpha_4/3$ | 0.22  | 0.22  |
| $\alpha_5/3$ | 0.55  | 0.55  |
| $\alpha_6/3$ | 1.00  | 1.00  |
| $\alpha_7/3$ | 1.56  | 1.57  |
| $\alpha_8/3$ | 2.10  | 2.25  |
| $\alpha_9/3$ | 3.00  | 3.10  |

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