Unequal intensity splitting can reduce back action in interferometers

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Typical two-path interferometers are intensity-balanced because this maximizes the visibility of their interference patterns. Unbalancing the interferometer can be advised when back action on the object whose position is monitored is to be reduced. Variations of the intensity splitting ratios in two-path interferometers are analyzed in order to determine optimal interferometric performance while minimizing back action: it turns out that homodyning-like schemes perform best.

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I. INTRODUCTION

Typical two-path interferometers are balanced: the intensity in probe and reference arm are the same. This maximizes the visibility of the interference pattern because only for balanced illumination complete destructive interference is observed [1]. Also, this tends to simplify the setup, a single balanced beam splitter to split and reunite the light paths can be used, such as in the conventional Michelson-Morley setup. Since recent technological progress allows us to monitor very sensitive systems, e.g. nano-mechanical oscillators integrated into optical setups [2, 3], increasingly, the back action [4] on the object whose characteristics are monitored has to be taken into account. Here we will only consider probing of mirror positions through elastic scattering, yet, even in ideal setups radiation pressure back action cannot be avoided [4, 5].

Intuitively, it is clear that unbalancing an interferometer: sending less light towards the probed mirror, will reduce the interference fringe visibility and reduce the back action of the light onto the mirror (only intensity-unbalancing is considered here, not using unequal path lengths). Whereas the first effect is unwelcome, the second maybe desired and we therefore want to consider the tradeoff between best visibility and least back action on the object. It should, perhaps, be emphasized that these considerations do not apply to gravitational wave detection in that the quadripolar nature of gravity waves requires probing in both arms of the interferometer [6, 7].

It is known that the visibility of the interference pattern remains high when moderate power splitting imbalances are used in an interferometer [1]. It is therefore plausible that an imbalance will allow us to reduce the back action of the light onto an object while retaining some visibility. To quantify the back action we will consider radiation pressure, proportional to the light’s intensity $\hat{I}$, and its fluctuations. These intensity fluctuations tend to randomize the mirror’s position and momentum distributions.

We will treat the light fields quantum-optically but only consider coherent light fields because this yields our considerations simple, and, yet, relevant and general. ‘Simplicity’ of calculations arises from the fact that coherent states are quasi-classical, ‘relevance’ is due to the fact that coherent states are the most important light states used in interferometry. Partly this is due to the fact that other light states are hard to synthesize but also because they are so fragile ([8, 9] and references therein) that only balanced setups perform well, see e.g. [8].

Our treatment is also ‘general’ since coherent states are fluctuation minimized (the cycle-averaged fluctuation powers of any noise-minimized monochromatic light fields are the same as those of coherent states [1, 7, 10]). Our analysis can therefore be carried over to non-classical light states such as squeezed and squeezed-coherent states. Such non-classical states may allow for greater interferometric resolution power than coherent states but their cycle-averaged fluctuations are the same and it is therefore straightforward to adopt our discussions accordingly [6, 7, 10].

FIG. 1: An unbalanced interferometer is illuminated with light in a Glauber-coherent state in port $\hat{a}_0$ whereas port $\hat{b}_0$ is in vacuum. The first beam splitter $\hat{B}$ mixes the modes with mixing angle $\theta_1$. This is followed by a fixed reflecting mirror $R$ in the upper path and a sensitive moving mirror $\hat{M}$ in the lower path. Movement of $\hat{M}$ can shift the phase $\phi$ of the reflected light. Finally, the two modes are mixed again at the second beam splitter and fed into the detectors $D$.

II. RESOLUTION POWER OF UNBALANCED INTERFEROMETERS

To describe the light modes traversing the interferometer we use creation $\hat{a}_0^\dagger/\hat{b}_0^\dagger$ and annihilation $\hat{a}_0/\hat{b}_0$ operators for the upper / lower paths, see Fig. 1 which obey bosonic commutation relations, e.g. $[\hat{a}, \hat{a}_0^\dagger] = 1$. An un-
balanced beam splitter can be described by the unitary matrix, $B$, parameterized by the mixing angle $\theta$ which mixes the light modes according to

$$\hat{B}(\theta) = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}. \quad(1)$$

For an unbalanced interferometer with only coherent light input we can, without loss of generality, assume that only one input channel $(\hat{a}_0)$ is used, see Fig. 1. The input light state is a product of a coherent state with amplitude $\alpha$ in each of the two paths by the first beam splitter according to

$$\hat{M}(\phi) = \begin{pmatrix} e^{-i\phi} & 0 \\ 0 & 1 \end{pmatrix}, \quad(3)$$

where $\phi$ parameterizes the phase delay due to path length variations when the mirror $M$ is moved, see Fig. 1. The reference arm $R$ is assumed fixed and $\hat{M}(\phi)$ thus described by multiplication with unity. After superposition of the interferometer modes at the second beam splitter the entire setup, displayed in Fig. 1, gives rise to the mode transformations

$$\hat{B}(\theta_2)\hat{M}(\phi)\hat{B}(\theta_1) \begin{pmatrix} \hat{a}_0 \\ \hat{b}_0 \end{pmatrix}. \quad(4)$$

The output modes expressed in terms of the input modes then have the form

$$\hat{a}_3 = \cos(\theta_2) \left( \cos(\theta_1)\hat{a}_0 + \sin(\theta_1)\hat{b}_0 \right) + e^{-i\phi} \sin(\theta_2) \left( -\sin(\theta_1)\hat{a}_0 + \cos(\theta_1)\hat{b}_0 \right), \quad(5)$$

and

$$\hat{b}_3 = -\sin(\theta_2) \left( \cos(\theta_1)\hat{a}_0 + \sin(\theta_1)\hat{b}_0 \right) + e^{-i\phi} \cos(\theta_2) \left( -\sin(\theta_1)\hat{a}_0 + \cos(\theta_1)\hat{b}_0 \right). \quad(6)$$

To extract the maximal interferometric signal we form the difference of the output intensities detected by the detectors $D_b$ and $D_a$, that is, we consider the observable

$$\hat{O} = \hat{b}_3^\dagger\hat{b}_3 - \hat{a}_3^\dagger\hat{a}_3.$$

In our case its expectation value has the form

$$\mathcal{O} = \alpha^2 \left[ 4 \sin(\theta_2) \cos(\theta_1) \cos(\phi) + 1 + 2 \cos(\theta_1)^2 - 4 \cos(\theta_2)^2 \cos(\phi) + 2 \cos(\theta_2)^2 \right]. \quad(7)$$

and its standard deviation is $\Delta \mathcal{O} = |\alpha|$. Since the ratio of standard deviation and maximal signal gradient yields the angular resolution $\Delta \phi$, we find

$$\Delta \phi = \Delta \mathcal{O} / \left| \partial \mathcal{O} / \partial \phi \right| = \frac{1}{|\alpha \sin(2\theta_1) \sin(2\theta_2) \sin(\phi)|}. \quad(8)$$

This confirms that best interferometric resolution is achieved for a balanced interferometer, $\theta_1 = \theta_2 = \pi/4$, where the optimal working point is $\phi^* = \pi/2$ and large intensities are desired to minimize $\Delta \phi^* = 1/|\alpha| [10].$

### III. Resolution Power Traded Off Against Back Action

When we consider the intensity in the probe arm as the back action quantity we want to minimize, while maximizing the angular resolution power $1/\Delta \phi$, we are led to consider the intensity based performance ratio

$$\rho_I = \frac{1}{\Delta \phi} \mathcal{I}_{b_1} = \frac{|\sin(2\theta_1) \sin(2\theta_2) \sin(\phi)|}{|\sin(\theta_1)|^2}. \quad(9)$$

For a balanced interferometer $\theta_1 = \theta_2 = \pi/4$ operating at the working point $\phi = \phi^* = \pi/2$ we find $\rho_I = 2/|\alpha|$. Clearly the operation of an interferometer as a balanced interferometer is not optimal when it comes to avoiding back action. The intensity based performance ratio $\rho_I$ becomes maximized when we choose a balanced second beam mixer $\theta_1 = \pi/4$ and a nearly transparent first beam splitter $\theta_1 \sim 0$. In this case $\rho_I$ becomes formally unbound, indicating better performance. Note, however, that reducing the laser power to zero also has the effect of formally increasing $\rho_I$ beyond any bound. This leads us to conclude that $\rho_I$ might not be the most suitable measure for the quantification of interferometric performance, but it indicates that the trend towards better performance is given by an imbalance that reduces the flow of light towards the probed object.

We now turn to a discussion of the fluctuation based performance ratio

$$\rho_{\Delta I} = \frac{1}{\Delta \phi \Delta \mathcal{I}_{b_1}} = \frac{|\sin(2\theta_1) \sin(2\theta_2) \sin(\phi)|}{|\sin(\theta_1)|}. \quad(10)$$

In the balanced interferometer case this yields $\rho_{\Delta I} = \sqrt{2}$. For arbitrary mixing angles but assuming that the same beam splitter is used twice (e.g. in a Michelson-Morley setup) $\theta_1 = \theta_2 = \arctan(1/\sqrt{2}) \approx 35.3^\circ$ yields the best result. In this case $\rho_{\Delta I} = \sqrt{3} \sin(2\arctan(1/\sqrt{2})) \approx 1.54 \approx 1.09\sqrt{2}$, where the last expression shows a nine percent increase in performance over the balanced interferometer case.

Best performance is achieved for an interferometer in which the beam mixer is balanced, $\theta_2 = \pi/4$, and the beam splitter is nearly transparent, $\theta_1 \approx 0$. In this case an expansion in $\theta_1$ yields $\rho_{\Delta I} \approx (2 - \theta_1^2) \sin(2\theta_2) \sin(\phi)$. At the working point $\phi^*$ performance can reach $\rho_{\Delta I} = 2$ which is by $\sqrt{2}$ better than the balanced case.
IV. LOSSES IN THE INTERFEROMETER AND IMPERFECT DETECTORS

If losses occur at the moving mirror $M$ because radiation is absorbed or scattered into other modes this would have to be included through the mixing in of additional vacuum modes. In the case of coherent states the losses and associated mixing in of vacuum modes can be modelled by an amplitude attenuation factor $e^{-\kappa}$ with a positive $\kappa$. This means we can simply substitute the phase shifting transformation $\hat{M}(\phi)$ by

$$\hat{M}(\phi) = \begin{pmatrix} e^{-i\phi - \kappa} & 0 \\ 0 & 1 \end{pmatrix}. \quad (11)$$

Clearly this brakes unitarity of the state evolution and cannot be applied to states other than coherent states. In this case the expression for the angular resolution $\Delta \phi$, compare eq. [3], becomes

$$\Delta \phi(\kappa) = \frac{\sqrt{\cos(2\theta_1)(e^{2\kappa} - 1) + e^{2\kappa} + 1}}{\sqrt{2} \sin(2\theta_1) \sin(2\theta_2) \sin(\phi)} \sin(\theta_1). \quad (12)$$

At the working point $\phi^* = \pi/2$ and for a balanced beam mixer $\theta_2 = \pi/4$. This yields the fluctuation based performance ratio

$$\rho_{\Delta I} = \frac{\sqrt{2} |\sin(2\theta_1)|}{|\sin(\theta_1)\sqrt{\cos(2\theta_1)(e^{2\kappa} - 1) + e^{2\kappa} + 1}|}. \quad (13)$$

which is plotted in Fig. 2. Note that this expression for $\rho_{\Delta I}$ was derived assuming that the back action expression $\Delta I_b$ is unchanged although it may actually depend on whether light was absorbed or scattered, and how it was scattered.

If the detectors are (both equally) inefficient, i.e. only a fraction $\eta$ of the light gets detected, our results still change little. For Glauber-coherent light the performance ratio $\rho_{\Delta I}$ simply becomes $\eta \cdot \rho_{\Delta I}$.

V. CONCLUSIONS

Two-path interferometers with one movable mirror, fed with Glauber coherent light, are analyzed. The trade-off between resolution power of and the back action onto the probed mirror is analyzed. Typically best performance is attained when light is directed away from the probed object into the reference arm of the interferometer. If the setup uses the same beam splitter to split and merge the light beams a mixing angle of $\theta = \arctan(1/\sqrt{2}) \approx 35.3^\circ$ gives best results, compare Fig. 1.

In general, large imbalances, using a homodyne-like setup (weak interferometric signal superposed with strong local oscillator), yield best performance. Setups that send small amounts of light towards the probed object perform nearly equally well: the area for small values of $\theta_1$ shows very weak gradients in $\theta_1$ (see Fig. 2). Therefore some freedom remains to decide on just how little light one wants to send towards the object and how poor a phase resolution one can tolerate.

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