On compact super-massive objects without event horizon

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Summary. — This paper aims to show the possibility of the existence of super-massive compact objects with radii less than the Schwarzschild one, which is one of the principal consequences of the authors geodesic-invariant gravitation equations (Ann. Phys. (Berlin), 17 (2008) 28). The physical interpretation of the solutions of the equations is based on the conclusion that only an aggregate space-time geometry + used reference frame has a physical sense.

PACS 04.20.20.Cv –
PACS . – 04.50.+h .

1. – Introduction

In Einstein’s theory of gravitation space-time is relative in the sense that the metric depends on the distribution of matter. However, long before the Einstein theory Poincaré showed that geometry of space depends also on the properties of measuring instruments. Only an aggregate “geometry + measuring instruments” has a physical meaning, verifiable by experience. After Minkowski this can be also said about geometry of space-time. Some results of the attempt to actualize these ideas, and a generalization of the vacuum Einstein’s gravitation equations are considered in [1]. Such approach allows to consider gravitation both as a field in flat space-time and as a space-time curvature. The equations do not contradict the existing observations data. However, the physical consequences resulting from them are radically different from the ones of general relativity at distances of the order of the Schwarzschild radius or less than that from a dot mass. It is very important that this fact provides a natural explanation of modern data of the Universe expansion. However, this fact leads also to another important physical consequence, which still has no confirmation.

Observations give evidences for the existence of supermassive compact cold objects in galactic centers [2]. Standard conditions of the equilibrium of selfgravitating degenerate Fermi-gas forbid the existence of very massive objects. For this reason they are usually identified with black holes. However lack of the evidence of the existence of an event horizon admits also other explanations of the nature of such objects.

In [3] the possibility of the existence of supermassive equilibrium configurations of the degenerate Fermi gas with radii less than the Schwarzschild one has been considered. Such objects
have no event horizon and are an alternative to the hypothesis of the existence of black holes. It is later, in [4], some observable consequences of the existence of such object in the Galaxy center have been considered.

However, it is seems that a theoretical justification of the existence of such objects is insufficiently convincing for observers as it is based on conclusions resulting from the author’s gravitation equations considered only in a brief note [5]. In the present paper, being based on the recent paper [1] a simple and a clear justification of possibility of the existence of such objects is given.

2. – Gravitation equations

Unlike electrodynamics, Einstein’s equations are not invariant with respect to a wide class of transformations of field variables in use (Christoffel symbols or metric tensor) leaving the equation of motion of test particles (geodesic lines) invariant [6]. For example, all Christoffel symbols \( \Gamma^\alpha_{\beta\gamma}(x) \) obtained by the transformations

\[
(1) \quad \Gamma^\alpha_{\beta\gamma}(x) = \Gamma^\alpha_{\beta\gamma}(x) + \delta^\alpha_{\beta} \phi_{\gamma}(x) + \delta^\alpha_{\gamma} \phi_{\beta}(x),
\]

where \( \phi_{\beta}(x) \) are an arbitrary differentiable vector-function, describe the same gravitational field because the geodesic equations

\[
(2) \quad \ddot{x}^\alpha + (\Gamma^\alpha_{\beta\gamma}(x) - c^{-1} \Gamma^\alpha_{\beta\gamma} \dot{x}^\beta \dot{x}^\gamma) \dot{x}^\beta \dot{x}^\gamma = 0
\]

remain invariant under transformation (1) in any given coordinate system. (The points denote differentiation with respect to \( t = x^0/c \), \( c \) is speed of light). However, Einstein’s equations are not invariant under such transformations because the Ricci tensor is transformed under (1) as follows

\[
(3) \quad \bar{R}_{\alpha\beta} = R_{\alpha\beta} - \phi_{\alpha\beta},
\]

where \( \phi_{\alpha\beta} = \phi_{\alpha\beta} - \phi_{\alpha} \phi_{\beta} \), and \( \phi_{\alpha\beta} \) is a covariant derivative with respect to \( x^\alpha \).

Each transformation (1) induces some mapping \( g_{\alpha\beta} \to \bar{g}_{\alpha\beta} \) of the metric tensor \( g_{\alpha\beta} \) which can be obtained by solving some partial differential equation [7], [1]. Such mappings leave the equations of motion of test particles invariant, and, consequently, all metric tensors resulting from a given \( g_{\alpha\beta} \) by a geodesic transformation describe the same gravitational field. Thus, such transformations are gauge transformations of the tensor \( g_{\alpha\beta} \). For this reason, one can suppose that in any gravitational theory, based on Einstein’s hypothesis of the motion of test particles along geodesic lines, only geodesic-invariant objects can have a physical sense.

The simplest geodesic-invariant tensor \( B^\beta_{\alpha\gamma} \) can be formed as follows [1]:

\[
(4) \quad B^\beta_{\alpha\gamma} = \Pi^\beta_{\alpha\gamma} - \Pi^\beta_{\gamma\alpha},
\]

where

\[
(5) \quad \Pi^\gamma_{\alpha\beta} = \Gamma^\gamma_{\alpha\beta} - (n + 1)^{-1} \left[ \delta^\gamma_{\alpha} \Gamma^\beta_{\gamma\alpha} + \delta^\gamma_{\beta} \Gamma^\alpha_{\gamma\alpha} \right]
\]
and 

$$\Pi^{\gamma}_{\alpha\beta} = \Gamma^{\gamma}_{\alpha\beta} - (n+1)^{-1} \left[ \delta^{\gamma}_{\alpha} \Gamma^{\beta}_{\beta} + \delta^{\gamma}_{\beta} \Gamma^{\alpha}_{\alpha} \right]$$

are the Thomas symbols for the Riemannian space-time $V_4$ and for the Minkowski space-time $E_4$ in a used coordinate system, respectively. The Thomas symbols are formed by the Christoffel symbols $\Gamma^{\gamma}_{\alpha\beta}$ and $\nabla^{\gamma}_{\alpha\beta}$ of $V_4$ and $E_4$, respectively, $\Gamma^\gamma_{\alpha\beta} = \Gamma_{\alpha\beta}$, and $\nabla^\gamma_{\alpha\beta} = \nabla_{\alpha\beta}$.

In paper [1] a theory in which geodesic mappings play a role of gauge transformations is considered. The investigation of the problem leads to the conclusion that is of interest to explore physical consequences from the following bimetric geodesic-invariant generalization of Einstein’s vacuum equations (1):

$$\nabla_{\alpha} B^{\alpha}_{\beta\gamma} - B^{\epsilon}_{\beta\gamma} B^{\epsilon}_{\epsilon\gamma} = 0.$$ 

The symbol $\nabla_{\alpha}$ denotes a covariant derivative in $E_4$ with respect to $x^\alpha$.

The derivation of basic equations, they physical interpretations, and a correct utilization are possible only owing to reconsider of a deep problem of space-time relativity with respect to measuring instruments, going back to Poincaré fundamental ideas. According to [1] both the space-time, $E_4$ and $V_4$, have physical meaning since only aggregate “space-time geometry + frame of reference” have a physical meaning. We can consider space-time both as the Minkowski one in an inertial reference frames (2), and as the Riemannian space-time with curvature other than zero – in the so-called proper reference frames of the field, the body of reference of which is formed by particles moving in this gravitational field. For purpose of this paper it is essentially only the important conclusion that an observer, which explores gravitational field of a distant massive compact object in an inertial reference frame, can consider global space-time as the Minkowski one.

In order to see better the relationship between (7) and the Einstein vacuum equation, it should be noted that we can select a gauge condition for the Christoffel symbols as follows:

$$Q^\alpha = \Gamma^\beta_{\alpha\beta} - \nabla^\alpha_{\alpha\beta} = 0.$$ 

At such covariant gauge condition eq. (7) coincides with the vacuum Einstein’s equations. (At least locally). Therefore, the observer which is located in an inertial reference frame far away from a compact object can describe gravity of the object field by the spherically-symmetric solution of the vacuum Einstein equations in the Minkowski space-time (in which $g_{\alpha\beta}(x)$ is simply a tensor field) at the additional condition $Q^\alpha = 0$. Such equations somewhat resemble the equations of classical electrodynamics for 4-potential together with some specific gauge conditions. It is that will be used in the next section.

3. – Gravitational Energy of a matter sphere

If a matter sphere is considered as formed by means of consecutive injections of thin spherically-symmetric layers of the matter from infinity [8], an equation for finding of the gravitational en-

\[\text{Greek indexes run from 0 to 3}
\]

\[\text{By inertial reference frame we mean the frame in which Newton’s first and second laws are obeyed globally.}\]
ergy of the sphere can be found by using the vacuum solution of the gravitational equations [1] for a dot source.

The differential equations of the motion of a test particle in the spherically-symmetric gravitational field of a dot mass \( M \) from the point of view of a distant observer can be found from the Lagrangian

\[
L = -mc \left[ Ar^2 + B(\dot{\theta}^2 + \sin^2 \theta \, \dot{\phi}^2) \right] - c^2C^{1/2},
\]

where \((t, r, \phi)\) are the spherical coordinates, \( m \) is the particle mass, \( A, B \) and \( C \) are the functions of the radial coordinate \( r \).

The equations of the motion of a test particle in the field of a point mass \( M \), based on the solution of the Einstein equations under the condition \( Q_\alpha = 0 \) in Minkowski space-time, are of the form [1]

\[
\dot{r}^2 = (c^2C/A)[1 - (C/E^2)(1 + \frac{r_g^2}{r^2})],
\]

\[
\dot{\phi} = c \frac{C}{r_g} \frac{f^2}{E},
\]

where \( f = (r_g^3 + r^3)^{1/3}, \) \( C = 1 - r_g/f, \) \( \dot{r} = dr/dt, \dot{\phi} = d\phi/dt, E = E/mc^2, J = J/r_gmc, E \) and \( J \) are the energy and angular momentum of the particle, \( r_g = 2GM/c^2 \) is the Schwarzschild radius of the central object, \( G \) is the gravitational constant.

If the radial distance \( r \) from the dot massive object is many larger than the Schwarzschild radius \( r_g \) of the object, physical consequences following from (7) are very close to the ones resulting from Einstein’s equations. However, they are very different when \( r \) is of the order of \( r_g \), or less than that.

From point of view of a distant observer free-falling particles move up to the center. The spherically-symmetric solution has no event horizon [1], and this fact is not a consequence of some specific coordinate system.

It follows from eq. (10) that at \( J = 0 \) the energy of a rest particle at the distance \( r \) from the center is:

\[
E = mc^2 \sqrt{C}.
\]

At the condition \( \mathcal{F} = r/r_g \ll 1 \), it yields

\[
E \approx mc^2 - \frac{GMm}{r}.
\]

If \( \mathcal{F} = r/r_g \gg 1 \), we obtain at first order to \( \mathcal{F}^{-1} \):

\[
E \approx \frac{mc^2 r_{g}^{3/2}}{2\sqrt{6G^{3/2}M^{3/2}}},
\]

In this case the gravitational energy of the test particle tends to zero when \( r \to 0 \).

Consequently, the difference between the gravitational energy of the thin matter layer of mass \( \delta m \) at the distance \( r \) from the center and the one at infinity is \( \delta mc^2 \sqrt{C} - \delta mc^2 \). The
gravitational energy of a material sphere, formed by means of consecutive additions of thin spherically-symmetric layers of the matter [8] is equal to

\[(15) \quad \mathcal{E}_g = - \int (1 - \sqrt{C}) dm, \]

where \(dm = 4\pi \rho r^2 dr\), and \(\rho\) is the mass-energy density.

If the matter sphere is an isentropic ideal fluid in equilibrium state, (15) can be transformed to more useful form. The equilibrium condition of the ideal isentropic fluid is of the form [9]

\[(16) \quad (\kappa^2 C)' = 0, \]

where prime denotes differentiation with respect to \(r\), \(\kappa = w/\rho_0 c^2\), \(\rho_0 \approx m_n n\) is matter rest-density, where \(m_n\) is the neutron mass and \(n\) is the particles number density, \(w\) is the enthalpy per unit volume. Therefore, we can use the following equation of the sphere equilibrium:

\[(17) \quad \kappa^2 C = C_0, \]

where

\[(18) \quad \kappa = 1 + \frac{P}{\rho_0 c^2}, \]

\(P\) is the pressure,

\[(19) \quad C_0 = 1 - \frac{r_g}{(r_g^2 + R^2)^{1/3}}, \]

where \(r_g = 2GM/R\) is the Schwarzschild radius of the sphere.

By using (17), eq. (15) can be written as follows:

\[(20) \quad \mathcal{E}_g = -Mc^2 + c^2 \sqrt{C_0} \int \kappa^{-1} dm. \]

For estimates we can use it in the more simple form

\[(21) \quad \mathcal{E}_g = -Mc^2 + Mc^2 \bar{\kappa}^{-1} \sqrt{C_0}, \]

where \(\bar{\kappa}\) is an averaged value of \(\kappa\) over the sphere.

As \(r_g/R \ll 1\) the functions \(C_0 \approx 1 - r_g/R\), and \(\kappa^{-1} \approx 1 - P/\rho_0 c^2\) because usually \(P/\rho_0 c^2 \ll 1\).

By using a polytropic equation of the fluid state we obtain that the gravitational energy of the object with radius \(R \gg r_g\) is

\[(22) \quad \mathcal{E}_g = -\frac{GM^2}{R} - \int PdV, \]

where \(dV\) is a volume element. The gravitational energy is very small as compared with full energy \(Mc^2\) of the object.

For the object with radius \(R < r_g\) the situation is another. If \(R/r_g \ll 1\), the value of \(C_0\) is close to zero, and the gravitational energy is close to \(-Mc^2\).
4. – Minimization of total energy

For the total energy of the uniform sphere we obtain the relationship

\[ E = E_0 + E_k + E_g = E_0 + E_k - \varepsilon (1 - \frac{1}{z} \sqrt{C_0}), \]

where \( E_0 = Mc^2 \), \( E_k \) is the internal energy of the fluid. Thus,

\[ E = \frac{E_0 + E_k}{2 - \frac{1}{z} \sqrt{C_0}}. \]

The magnitude \( C_0 \) contains the total mass of the object

\[ M = M_0 + M_k + M_g, \]

where \( M_0 \approx m_n c^2 \), \( M_k = E_k / c^2 \), \( M_g = E_g / c^2 \). If \( r_g / R \ll 1 \), \( M \approx M_0 + M_k \). If \( r_g / R \gg 1 \), then \( M \approx (M_0 + M_k) / 2 \). For this reason in (24) we can replace approximately \( M \) in \( C_0 \) by \( M = \lambda (M_0 + M_k) \), where \( \lambda \) is a parameter which lays in the range \( 1 \div 1/2 \).

For an uniform sphere of a nonrelativistic degenerate electronic or neutron ideal gas

\[ E_k \propto \frac{N^{5/3}}{R^2}, \]

where \( N \) is the total number of particles, and \( R \) is the radius of the sphere. Therefore, \( \varepsilon \) can be considered as the function of \( N \) and \( R \).

Figures 1-4 show the total energy of uniform spherical configurations of degenerate Fermi-gas, considered as an ideal isentropic fluid, as a function of \( R \) for the several values of \( N \). It is easy to verify that the parameter \( \lambda \) affect scarcely the plots.

It follows from the figures that the total energy has the minimum not only at \( N = 10^{56} \) (neutron star), but also at many larger values of \( N \). They are supermassive objects with radiuses \( R \) larger of the Schwarzschild radius. Table 1 shows the masses \( M \), radius \( R \), average density \( \rho_0 \), and the Schwarzschild radius \( r_g \) of the stable configurations determined by the plots minimum. The first three configuration are polytropic neutron configurations, and the configuration at last line is the electronic one. According to the used simple model the maximal mass of the purely neutronic configurations is approximately \( M = 10^5 \) of the Sun mass. However, electronic configurations are possible up to \( M = 10^{12} \) of the Sun mass.

| \( N \)  | \( M \)     | \( R \)    | \( \rho_0 \) | \( r_g \)  |
|-------|-----------|-----------|-------------|----------|
| \( 10^{56} \) | \( 1.66 \times 10^{32} \) | \( 1.11 \times 10^6 \) | \( 2.92 \times 10^{13} \) | \( 2.46 \times 10^4 \) |
| \( 10^{60} \) | \( 8.50 \times 10^{35} \) | \( 2.57 \times 10^7 \) | \( 2.35 \times 10^{13} \) | \( 1.26 \times 10^8 \) |
| \( 10^{63} \) | \( 8.36 \times 10^{38} \) | \( 1.86 \times 10^9 \) | \( 6.20 \times 10^{10} \) | \( 1.24 \times 10^{11} \) |
| \( 10^{66} \) | \( 8.36 \times 10^{41} \) | \( 1.14 \times 10^{12} \) | \( 2.69 \times 10^5 \) | \( 1.24 \times 10^{14} \) |
5. – Conclusion

Despite the fact that the above calculations yield rather qualitative than quantitative results, they show clearly that the according to equations of gravitation (7), which do not contradict available observation data, there are stable supermassive configurations of degenerate Fermi-gas with radiuses less than the Schwarzschild radius. Just such object can be located in the Galaxy center [4].

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