Dynamically generating arbitrary spin-orbit couplings for neutral atoms

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(Dated: January 23, 2012)

Spin-orbit coupling (SOC) can give rise to interesting physics, from spin Hall to topological insulators, normally in condensed matter systems. Recently, this topical area has extended into atomic quantum gases in searching for artificial/synthetic gauge potentials. The prospects of tunable interaction and quantum state control promote neutral atoms as nature’s quantum emulators for SOC. Y.-J. Lin et al. recently demonstrated a special form of the SOC $k_x \sigma_y$: which they interpret as an equal superposition of Rashba and Dresselhaus couplings in Bose condensed atoms [Nature (London) \textbf{471}, 83 (2011)]. This work reports an idea capable of implementing arbitrary forms of SOC by switching between two pairs of Raman laser pulses like that used by Lin et al. While one pair affects $k_x \sigma_y$ for some time, a second pair creates $k_y \sigma_x$ over other times with Raman pulses from different directions and a subsequent spin rotation into $\pm k_y \sigma_x$. With sufficient many pulses, the effective actions from different durations are small and accumulate in the same exponent despite that $k_x \sigma_y$ and $\pm k_y \sigma_x$ do not commute. Our scheme involves no added complication, and can be demonstrated within current experiments. It applies equally to bosonic or fermionic atoms.

PACS numbers: 03.75.Mn, 67.85.Fg, 67.85.Jk

Introduction. Atomic quantum gases are increasingly viewed as favored model systems for emulating condensed matter physics. Optical lattices resulting from ac Stark shifts to atomic levels, are easily implemented with coherent laser beams, which confine atoms like electrons in solid states. An interesting topic concerns strong correlations as in integer/fractional quantum Hall effect and the analogous spin Hall effect. The standard description for the former involves $U(1)$ Abelian gauge fields, which can be simulated in neutral atoms through rotation \cite{1,2} or adiabatic translations in far-off-resonant laser fields \cite{3,4,5}. Non-Abelian gauge fields, e.g., as in spin-orbit coupling (SOC) \cite{6,7,8}, enable richer possibilities like fractional quantum Hall states. As a result, active researches are targeting the implementations of (SOC) in simple atomic systems.

For atoms with multiple internal states, or (pseudo-spin) spinor degrees of freedom, SOC changes single-particle spectra and competes with density-density or spin-dependent interactions, (i.e., spin-exchange and singlet-pairing interactions). Strong correlations often lead to exotic ground states \cite{9,10}, such as the plane-wave phase and the striped phase discovered recently in pseudo spin-1/2 \cite{11,12,13} or spin-1 condensates \cite{14}. Other examples offer the triangular-latticed phase or square-latticed phase in spin-2 condensates with axisymmetric SOC \cite{15,16}. In a recent experiment, the JQI group of Spielman observed both Abelian \cite{17} and non-Abelian \cite{18} gauge fields in a pseudo spin-1/2 atomic Bose gas, albeit in a special form $\propto k_x \sigma_y$ of SOC, which is an equally weighted sum of Rashba ($\propto k_x \sigma_y - k_y \sigma_z$) and Dresselhaus ($\propto k_x \sigma_y + k_y \sigma_z$) couplings \cite{19}. More generally, a SOC form of continuous rotation symmetry, or an arbitrary weighted sum of Rashba and Dresselhaus couplings, exists in solid-state materials.

Several existing theoretical proposals are capable of implementing SOC with rotation symmetry in laser atom coupled models. For instance, in a tripod scheme \cite{7}, when one-photon resonant couplings between the three lower-energy states and a higher-energy one are allowed, two dark states emerge, although spontaneous emission is always a cause of concern in this case. D. L. Campbell et al. \cite{10} proposed an alternative scheme by cyclically coupling three or four ground or metastable internal states. With sufficient laser intensities, the above induced SOCs can possess a continuous rotation symmetry. Another scheme by Jay D. Sau et al. \cite{11} employs an effective two-dimensional periodic potential created from two laser beams and their reflected lights propagating along $\hat{x}$ and $\hat{y}$ directions in $^{40}$K atoms. In the limit of small Raman coupling, their corresponding effective SOC is of a pure Rashba type in the first Brillouin zone.

In this Letter, we describe a dynamic approach for implementing rotational symmetric SOC of arbitrary forms within a pseudo-spin 1/2 atomic system. We adopt the JQI model and start from the simple SOC they proposed and recently demonstrated \cite{12}. The key to our idea is optimal control theory applied with repeated laser pulses to rotate atomic pseudo-spins. Our idea works for both atomic fermions and bosons, and can be easily adopted to other atomic models. Thus it constitutes a powerful new direction for engineering synthetic atomic gauge potentials.

The equally weighted sum of Rashba and Dresselhaus types SOC of $k_x \sigma_y$, \cite{9}, can be rotated into a form $\propto \pm k_y \sigma_x$, by performing single atom spin rotation through a Rabi pulse. Such a coherent control idea when repeated over time, can realize $k_x \sigma_y$ and $\pm k_y \sigma_x$ types SOC in subsequent time intervals of duration $\delta t$. The resulting dynamics is then described respectively by an effec-
tive Hamiltonian with pure Rashba or Dresselhaus SOC with the first order approximation for small $\delta t$. The accompanied change of atomic momentum, can be nullified through a variety of means as we describe below step by step. We start with a review of the experiment by Y.-J. Lin et al. [9], which helps to introduce our idea.

**The JQI protocol.** Consider a $F = 1$ atomic Bose-Einstein condensate (BEC) under a bias magnetic field along $\hat{z}$ located at the intersection of two Raman laser beams propagating along $\hat{y} + \hat{z}$ and $-\hat{y} + \hat{z}$, with angular frequencies $\omega_L$ and $\omega_L + \Delta\omega_L$, respectively. The two laser beams affect two photon resonant Raman coupling ($\Omega_R$) between nearby ground Zeeman states, far detuned from the excited states. Effectively, such a coupling scheme produces an artificial magnetic field along the $x$-axis direction with the resulting Hamiltonian $H_R = \Omega_R F_x \cos(2k_L\hat{y} + \Delta\omega_L t)$, where $F_{x,y,z}$ are $3 \times 3$ spin-1 matrices, $k_L = \sqrt{2}\pi/\lambda$ with $\lambda$ is the laser wavelength, and $E_L = \hbar^2k_z^2/2m$, the unit of photon recoil energy. In explicit forms, after adiabatically eliminating excited states, the total Hamiltonian becomes

$$
\hat{H}_3 = \frac{\hbar^2k_z^2}{2m} + \left( \begin{array}{ccc} E_+ & 0 & 0 \\ 0 & E_0 & 0 \\ 0 & 0 & E_- \end{array} \right) + \frac{\Omega_R}{\sqrt{2}} \left( \begin{array}{ccc} 0 & \cos(2k_L\hat{y} + \Delta\omega_L t) & 0 \\ \cos(2k_L\hat{y} + \Delta\omega_L t) & 0 & \cos(2k_L\hat{y} + \Delta\omega_L t) \\ 0 & \cos(2k_L\hat{y} + \Delta\omega_L t) & 0 \end{array} \right),
$$

where $E_+$, $E_0$ and $E_-$ are Zeeman (eigen-) energies of $M_F = 1, 0, -1$ spin states, respectively. Under the rotating wave approximation, it turns into

$$
\hat{H}_3 = \frac{\hbar^2k_z^2}{2m} + \left( \begin{array}{ccc} E_+ & 0 & 0 \\ 0 & E_0 & 0 \\ 0 & 0 & E_- \end{array} \right) + \frac{\Omega_R}{2} F_x \cos(2k_L\hat{y} + \Delta\omega_L t) - \frac{\Omega_R}{2} F_y \sin(2k_L\hat{y} + \Delta\omega_L t).
$$

Further introduce a frame transformation: $\tilde{\psi} = e^{-i F_z \Delta\omega_L t} \psi$, where $\psi$ and $\tilde{\psi}$ are the wave functions in the laboratory and transformed frames, respectively, we arrive at the Hamiltonian

$$
\hat{H}_3 = \frac{\hbar^2k_z^2}{2m} + \left( \begin{array}{ccc} 2\hbar\omega_0 + 3\delta/2 & 0 & 0 \\ 0 & \delta/2 & 0 \\ 0 & 0 & -\delta/2 \end{array} \right) + E_0 - \delta/2 + \frac{\Omega_R}{2} F_x \cos(2k_L\hat{y} + \Delta\omega_L t) - \frac{\Omega_R}{2} F_y \sin(2k_L\hat{y}),
$$

where $\hbar\omega_Z = E_- - E_0$, $\hbar\Delta\omega_L = \hbar\omega_Z + \delta$, $E_0 - E_+ = \hbar\omega_0$, $\delta$ is detuning and $\hbar\omega_0$ is the quadratic Zeeman shift. When $\hbar\omega_0$ is sufficiently large and the Raman coupling $\Omega = \Omega_R/\sqrt{2}$ is small, we neglect the state $|M_F = 1\rangle$ and a constant term $E_0 - \delta/2$. The effective Hamiltonian for the remaining two nearly degenerate states becomes

$$
\hat{H}_2 = \frac{\hbar k_z^2}{2m} + \frac{\delta}{2} \sigma_z + \frac{\Omega}{2} \sigma_x \cos(2k_L\hat{y}) - \frac{\Omega}{2} \sigma_y \sin(2k_L\hat{y}) \\
e^{iE_k\hat{y}\sigma_z} \left( \frac{\hbar k_z^2}{2m} + \frac{\delta}{2} \sigma_z + \frac{\Omega}{2} \sigma_x + 2\frac{\hbar^2k_z^2}{2m} b_{\perp}\hat{y}\sigma_z + E_L \right) \times e^{-iE_k\hat{y}\sigma_z},
$$

where the second line shows an explicit SOC term $\propto \hat{k}_y \sigma_z$ when viewed after a unitary transformation.

**FIG. 1:** (Color online). (a) A schematic illustration of the JQI implementation for SOC [9], where a $F = 1$ atomic Bose-Einstein condensate interacts with a bias magnetic field along $\hat{z}$ and two Raman laser beams propagating along $\hat{y} + \hat{z}$ and $-\hat{y} + \hat{z}$, with angular frequencies $\omega_L$ and $\omega_L + \Delta\omega_L$, respectively. (b) LEFT: Linear Zeeman shifts of the three hyperfine spin states. MIDDLE: Zeeman shifts of the three hyperfine spin states including both linear and quadratic terms. RIGHT: Zeeman shifts in the rotating frame (with frequency $\Delta\omega_L F_z$) of the pseudo-spin pointing along $\hat{z}$.

**Dynamically generating arbitrary SOC.** Our protocol for implementing the Rashba type SOC is illustrated below in Fig. 2. It relies on our ability of being able to
switch atomic pseudo-spin from along $z$- to along $y$-axis (and vice versa) using Raman pulses. In the first half period, Raman lasers $L_1$ and $L_2$ are turned on. In the second half, $L_3$ and $L_2$ are turned on instead. $L_3$ is the same as $L_1$ except it propagates along opposite direction. At the middle point, we pulse on an extra $\pi/2$ pulse to rotate the pseudo-spin from $y$- to $z$-axis, described by the operator $\exp[-i(\sigma_x/2)\pi/2]$ in the transformed frame, or the operator $\exp[iF_z \Delta \omega_L t] \exp[-i(\sigma_x/2)\pi/2] \exp[-iF_z \Delta \omega_L t]$ in the lab frame; in the end of each period, we pulse on an $-\pi/2$ pulse for the reverse rotation. Both spin rotation pulses can be accomplished with either Raman coupling from appropriately detuned lasers or rf plus microwave coupling between the two remaining internal states.

In the first half, the system is then governed by $\hat{H}_3$, $\hat{H}_3$ becomes

$$\hat{H}_3' = \frac{\hbar^2 k^2}{2m} + \left( \begin{array}{ccc} 2\hbar \omega_q + 3\delta/2 & 0 & 0 \\ 0 & \delta/2 & 0 \\ 0 & 0 & -\delta/2 \end{array} \right) + E_0 - \delta/2$$

$$+ \frac{\Omega_R}{2} F_x \cos(2k_L \hat{z}) + \frac{\Omega_R}{2} F_y \sin(2k_L \hat{z}),$$

which completes one period of our prescribed protocol. For large $\hbar \omega_q$ and small $\Omega$, the same condition as in Ref. \[9\], the effective Hamiltonian for the reduced two-state model becomes

$$\hat{H}_2' = \frac{\hbar k^2}{2m} + \frac{\delta}{2} \sigma_z + \frac{\Omega}{2} \sigma_x \cos(2k_L \hat{z}) + \frac{\sigma_y}{2} \sin(2k_L \hat{z})$$

$$= e^{-i k_L \hat{z} \sigma_x} \left( \frac{\hbar^2 k^2}{2m} + \frac{\Omega}{2} \sigma_x - \frac{h^2 k_L k_z}{2m} \sigma_z + E_L \right)$$

$$\times e^{i k_L \hat{z} \sigma_x}. \quad (6)$$

The pair of $\pi/2$ pulse (before) and $-\pi/2$ pulse (after) affects a unitary transformation

$$e^{i(\sigma_x/2)\pi/2} \hat{H}_2 e^{-i(\sigma_x/2)\pi/2}$$

$$= \frac{\hbar k^2}{2m} + \frac{\delta}{2} \sigma_z + \frac{\Omega}{2} \sigma_x \cos(2k_L \hat{z}) + \frac{\sigma_y}{2} \sin(2k_L \hat{z})$$

$$= e^{i k_L \sigma_y} \left( \frac{\hbar^2 k^2}{2m} + \frac{\Omega}{2} \sigma_y + \frac{h^2 k_L k_z}{2m} \sigma_z + E_L \right)$$

$$\times e^{-i k_L \sigma_y}. \quad (7)$$

In suitably transformed frames, respectively with $U_1 = e^{-i k_L \hat{y} \sigma_x}$ and $U_2 = e^{i k_L \hat{y} \sigma_x}$, Eqs \[4\] and \[5\] reveal explicit SOC terms $\hat{k}_x \sigma_x$ and $\hat{k}_y \sigma_y$. They cannot, however, be simply added together in the forms above. To combine the above two halves into a single Rashba or Dresselhaus type SOC, we have to eliminate these unitary transformations. Both $U_1$ and $U_2$ corresponds to spin dependent phase shifts, they can be viewed as from the impulse of an artificial or real small magnetic field along a suitable direction and with a spatial gradient. Thus they can be nullified by real magnetic field gradients or synthetic magnetic field gradients generated from spatial dependent ac Stark shifts. For instance, $U_1$ is compensated for by a magnetic field pointing along $z$-axis and a spatial gradient ($B'$) along $y$-axis, with an adjustable impulse over $\delta t'$ where $E' \equiv -\mu B'$ is the appropriate Zeeman energy gradient. After the control pulse $\delta t$, the sign of $B'$ is changed to affect a second impulse, which then leads to the following

$$e^{-i E' \hat{y} F z \delta t'/\hbar} e^{-i \hat{H}_3 \delta t'/\hbar} e^{i E' \hat{y} F z \delta t'/\hbar}$$

$$= \exp \left\{ -i \left( \frac{\hbar^2 k^2}{2m} + (\hbar \omega_q + \delta) F_z + \hbar \omega_q F_z^2 + E_0 \right.$$

$$\left. + \frac{\Omega_R}{2} F_z + \frac{h^2 k_L k_z}{2m} F_z + E_L \right) \delta t' / \hbar \right\}, \quad (8)$$

provided $E' \delta t' = 2\hbar k_L$, where we assume $E'$ is strong enough so that we can neglect the contribution from $\hat{H}_3$ during the short pulse $\delta t'$ ($\ll \delta t$). The effective two-state
dynamics is then approximately govern by

$$\exp \left\{ -i \left( \frac{\hbar^2 \mathbf{k}^2}{2m} + \frac{\delta}{2} \sigma_z + \frac{\Omega}{2} \sigma_x + \frac{\hbar^2 k_L k_y}{2m} \sigma_y \right) \delta t / \hbar \right\},$$

(9)

apart from a overall phase term involving a constant energy in the exponent. Similarly, $U_2$ is nullified as well, resulting in

$$e^{i E' F_x \delta t / \hbar} e^{-i H_z \delta t / \hbar} e^{-i E' F_x \delta t / \hbar} \exp \left\{ -i \left( \frac{\hbar^2 \mathbf{k}^2}{2m} + (\hbar \omega_q + \delta) F_z + \hbar \omega q F_z + E_0 \right. \\
+ \frac{\Omega}{2} F_x - \frac{\hbar^2 k_L k_y}{2m} F_z + E_L \left. \right) \delta t / \hbar \right\},$$

(10)

For the special case of Rashba SOC, the suggested pulse sequence are illustrated in Fig. 2(b), where the blue and cyan ones are suitable momentum impulses for compensating the unwanted momentum recoils in the first and second half cycles respectively. The red pairs are $\pm \pi/2$ pulses for rotating the pseudo-spin. If the $\pi/2$ one precedes the $\pi/2$ pulse, we find in one period $T = 2\delta t$, the total evolution operator under two-state approximation is given by

$$U(T, 0) = e^{i(\sigma_z/2)\pi/2} \left( e^{i E' F_x \delta t / \hbar} e^{-i H_z \delta t / \hbar} e^{-i E' F_x \delta t / \hbar} \right) \cdot e^{-i i(\sigma_z/2)\pi/2} \left( e^{-i E' y F_y \delta t / \hbar} e^{-i H_z \delta t / \hbar} e^{i E' y F_y \delta t / \hbar} \right) \cdot \exp \left\{ -i \left( \frac{\hbar^2 \mathbf{k}^2}{2m} + \frac{\delta}{2} \sigma_y + \frac{\Omega}{2} \sigma_x + \frac{\hbar^2 k_L k_y}{2m} \sigma_y \right) \delta t / \hbar \right\} \cdot \exp \left\{ -i \left( \frac{\hbar^2 \mathbf{k}^2}{2m} + \frac{\delta}{4} (\sigma_y + \sigma_z) + \frac{\Omega}{2} \sigma_x + \frac{\hbar^2 k_L k_y}{2m} (k_y \sigma_z - k_z \sigma_y) \right) \delta t / \hbar \right\}.$$

(12)

According to the Floquet theorem, the quasienergy $\epsilon$ of time-periodic system is derived from $\det [U(T, 0) - e^{-i T}] = 0$. Then from Eq. 12, we can easily infer that under first order of $T$ approximation, the quasienergy of our system is the same as the spectra of that with Rashba SOC. Reversing the two red $\pm \pi/2$ pulses introduces a minus sign "−", the Rashba SOC then changes into Dresselhaus SOC. By adjusting the timing constant $\delta t$, we can extend the above discussion to SOC of arbitrary form $\beta(k_y \sigma_z - k_z \sigma_y) + \sqrt{1 - |\beta|^2} (k_y \sigma_x + k_z \sigma_y)$. The steady state of the effective system Hamiltonian is reached due to elastic atomic collisions. Although in the simplest case, one period of the control protocol is often sufficient, the actual implementation can aim at a higher precision of the effective SOC Hamiltonian by increasing the number of cycles, or simple reducing $\delta t$.

A magnetic field gradient was first used in Ref. 3 for implementing Abelian gauge fields with neutral atoms. However, since real static B-field is subjected to the Maxwell’s equations $\nabla \cdot \mathbf{B} = 0$ and $\nabla \times \mathbf{B} = 0$, one cannot simply obtain a linear gradient along one direction, e.g. a B-field like $\mathbf{B} = (B_0 - by) \hat{e}_y$ is illegitimate because of its non-vanishing divergence. The simplest linear gradient B-field, therefore needs to have two components, like that of the commonly used two dimensional quadruple field, $\mathbf{B} = bx \hat{e}_x - by \hat{e}_y$. When the system is of a reduced dimension, not including x-direction as in Ref. 3, one is then equipped with a one-dimensional B-field gradient $\mathbf{B} = -by \hat{e}_y$, which is equivalent to $\mathbf{B} = -by \hat{e}_z$ upon an axis rotation, A from precisely needed for implementing $U_1$ above.

Likewise, the above B-field gradient can be simulated using ac stark shifts from position dependent laser fields far off resonant coupled to the two states forming an atomic pseudo-spin. Assuming a one-photon resonant coupling Rabi frequency $\Omega_L(\vec{r})$ and a detuning $\Delta_L = \Omega_L - \Omega_0$, the ac Stark shift takes the form $\sim \sigma_z \Omega_L^2(\vec{r})/(2\Delta_L) \propto \sigma_z L(\vec{r})/\Delta L$. A linear spatial gradient can thus be affected with a laser intensity gradient, which can be implemented using many methods, including the use of a gradient neutral density filter. Stronger gradients arise from interfering several waves forming a standing wave, e.g., with $I_L(\vec{r}) \sim \cos^2(qy/2) = (1 + \cos qy)/2$, linear gradient is $\sim \pm qy$ around the nodal points of cos qy.

More generally, the state dependent gradients can be engineered to couple states in the same Zeeman manifold. For example, the above ac Stark shifts from one-photon coupling can be substituted with two-photon Raman coupling with suitable differential detuning, like in Bragg scattering, which then implements impulses $\propto \pm y \sigma_z$, $\propto \pm z \sigma_y$ or $\propto \pm z \sigma_z$.

**Summarizing** We present a coherent control protocol capable of realizing the Rashba type SOC in a pseudo-spin 1/2 atomic quantum gas 4. For most systems, our protocol can be implemented in one cycle, involving
two separate resonant Raman coupling. More elaborate forms are possible with multiple control pulses. When more than one control cycle is implemented, we can further enhance the precision and strength of the SOC, or the corresponding artificially created gauge potentials. In addition, the scheme we suggest is independent of quantum statistics of atoms, thus can be adopted to fermionic atoms as well. Our idea thus opens the door for dynamically implementing artificial gauge potentials in cold atomic systems based on coherent control theory.

Finally, we compare our idea with two previous schemes [10, 11]. In Ref. [10], three and four laser fields are needed, cyclically coupled to three or four internal states. Nearly pure Rashba or Dresselhaus SOC then results respectively in the limit of large intensity laser fields. It remains open to find a suitable experimental system. In Ref. [11], along each axis of $x$- $y$-two lasers with different frequency and their respective reflections are needed. Only in the far-detuned and small Raman coupling $\Omega_R$ limit, Rashba or Dresselhaus SOC can be implemented, which results in a relatively small SOC, proportional to $\Omega_R$. By simply turning on several pulses, and making use of the coherent control, we can dynamically generate arbitrary SOC for neutral atoms.

This work is supported by the NSFC (Contracts No. 91121005 and No. 11004116). L.Y. is supported by the NKBRSF of China and by the research program 2010THZO of Tsinghua University.

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