Impact of longitudinal bulk viscous effects to heavy quark transport in a strongly magnetized hot QCD medium

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The effects of longitudinal bulk viscous pressure on the heavy quark dynamics have been estimated in a strongly magnetized quark-gluon plasma within the Fokker-Planck approach. The bulk viscous modification to the momentum distribution of bulk degrees of freedom has been obtained in the presence of a magnetic field while incorporating the realistic equation of state of the hot magnetized QCD medium. As the magnetic field breaks the isotropy of the medium, the analysis is done along the directions longitudinal and transverse to the field. The longitudinal bulk viscous contribution is seen to have sizable effects in the heavy quark momentum diffusion in the magnetized medium. The dependence of higher Landau levels and the equation of state on the viscous correction to the heavy quark transport has been explored in the analysis.

I. INTRODUCTION

The very recent Large Hadron Collider (LHC) measurements provide a first sign of the existence of a strong electromagnetic field in the heavy-ion collision by measuring the directed flow $v_1$ for charged hadrons and $D/D^0$ mesons for Pb+Pb collision at $\sqrt{s_{NN}} = 5.02$ TeV [1]. Several investigations have been done in the analysis of $v_1$ of hadrons with heavy quarks (HQs) incorporating the effects of a strong electromagnetic field [2–4]. The LHC measurements, together with the observation of Relativistic Heavy-Ion Collider (RHIC) [5], indicate that the strong electromagnetic field created at the early stages of the collision affects the dynamics of the HQs. The HQs are mostly created in the very initial stages of the heavy-ion collision and travel through the deconfined hot nuclear matter-Quark Gluon Plasma (QGP). The HQs witness the entire QGP evolution as the thermalization time of HQ is larger than the lifetime of the QGP created at RHIC and LHC. These aspects allow HQs to serve as a potential probe to characterize the properties of the QGP in the heavy-ion collisions [6–12].

II. HEAVY QUARK DYNAMICS IN MAGNETIC MEDIUM

HQs propagates through the thermal QGP medium while interacting with quarks and gluons via $2 \leftrightarrow 2$ scattering and can be described as the Brownian mo-
tation [39, 40]. As the dynamics of quarks and gluons are different in the presence of a strong magnetic field, the estimation of the quark and gluonic contribution to the HQ transport coefficients need to be done separately in the magnetized medium. The random motion of HQ in the QGP medium can be described by the evolution of momentum distribution function $f_{HQ}$ within the framework of Fokker-Planck equation,

$$\frac{\partial f_{HQ}}{\partial t} = \frac{\partial}{\partial p_i} \left[ A_i(p) f_{HQ} + \frac{\partial}{\partial p_j} \left[ B_{ij}(p) f_{HQ} \right] \right],$$

where $A_i$ and $B_{ij}$ respectively measure the HQ drag force and momentum diffusion in the medium and takes the forms as follows,

$$A_i = \langle \langle p - p' \rangle_i \rangle, \quad B_{ij} = \langle \langle p - p' \rangle_i (p - p')_j \rangle, \quad (2)$$

for the process, $HQ(p) + l(k) \rightarrow HQ(p') + l(k')$, where $l$ stands for thermal particles in the magnetized medium, with $|\mathcal{M}_{HQ\gamma/q}|^2$ as the matrix element. The thermal average can be defined as,

$$\langle \langle F \rangle \rangle = \frac{1}{d_{HQ} 2\pi} \int \frac{d\mathbf{r}}{2E_k} \int \frac{d^3p'}{(2\pi)^3 2E_{p'}} \int \frac{dp'}{2E'_{p'}} \times |\mathcal{M}_{HQ\gamma/q}|^2 (2\pi)^3 \delta^n(p + k - p' - k') f_{gq}(k) \times (1 + f_{gq}(k)) F.$$

The integration phase factor can be described from the dimensional reduction in the presence of strong magnetic field $B = B\hat{z}$ and takes the form $d\gamma = \frac{d^3k}{(2\pi)^3}$ for gluons and $d\gamma = \frac{|qfeB|}{2\pi} \sum_{l=0}^{\infty} \frac{l!}{l!} \frac{dk}{2\pi}$ for quarks in the magnetized medium, where $\mu_l = (2 - \delta_{l0})$ is the spin degeneracy of the Landau levels. Here, $d_{HQ}$ is the degeneracy of the HQ, $f_{gq}$ is the momentum distribution in the thermal medium, and $n = (2,4)$ for the quarks and gluons, respectively. Note that in the static limit $\mathbf{p} \rightarrow 0$, $B_{ij} \rightarrow K\delta_{ij}$, [32], where $K$ is the diffusion coefficient of HQ. As the magnetic field induces a spatial anisotropy in the medium, one we need to consider the HQ dynamics parallel and perpendicular to the magnetic field.

### III. BULK VISCOSITY CORRECTIONS IN MAGNETIC FIELD

#### Near-equilibrium thermal distribution function

Proper modelling of the system in the thermal equilibrium followed by the knowledge of the longitudinal bulk viscous part of the distribution function is needed for the effective description of the bulk viscous effects to HQ transport in a magnetized system. For the system not very far from local thermal equilibrium, the momentum distribution function has the form,

$$f_{gq} = f_{gq}^0 + \delta f_{gq}^r,$$

with $\delta f_{gq}/f_{gq}^0 \ll 1$. The effective fugacity quasiparticle model (EQPM) describes the thermal medium interactions via QCD equation of state (EoS) in terms of quark and gluon effective fugacities, $z_q$ and $z_g$ respectively [41]. The equilibrium EQPM distribution functions in the presence of the magnetic field $B = B\hat{z}$ have the forms,

$$f_q = f_q^0 = \frac{z_q \exp(-\beta E_q^*)}{1 + z_q \exp(-\beta E_q^*)}, \quad f_g = \frac{z_g \exp(-\beta E_g)}{1 - z_g \exp(-\beta E_g)},$$

The quark in the strongly magnetized medium follow a $1 + 1$-dimensional dynamics and the energy dispersion can be described by Landau quantization, $E_{\pm} = \sqrt{k^2 + m_q^2 + 2|qfeB|}$, where $l = 0,1,2,..$ is the order of the Landau levels of the quark of mass $m_q$ and charge $q_f$. The effective fugacity parameter modifies the single particle dispersion relation as, $\omega_q = E_q^0 + \omega_{\pm}$, and $\omega_{\pm} = E_{\pm} + \delta\omega_{\pm}$, where the modified part of the non-trivial dispersion relation, $\delta\omega_{q/g} = T^2\partial T \ln(z_q/z_g)$, can be interpreted as the quasiparticle collective excitations in the medium. We consider the recent $(2 + 1)$ flavor lattice QCD EoS in the current analysis [32].

Transport coefficients are essential inputs to describe the non-equilibrium correction to the distribution function. In one dimensional system, both shear and bulk viscosities lead to similar hydrodynamical evolution as the non-equilibrium correction to the distribution function.

The longitudinal bulk viscous pressure in the strongly magnetized medium takes the form, [33],

$$P_{||} = -\sum_{l=0}^{\infty} \sum_{l' = 0}^{\infty} \frac{\mu_l}{\pi} \frac{qfeB}{N_c} \int_{-\infty}^{\infty} dE_{z} \Delta_{\mu\nu} \delta f_{q} E_{z} \delta f_{q}^{\mu} \delta f_{q}^{\nu} - \sum_{l=0}^{\infty} \sum_{l' = 0}^{\infty} \delta\omega_{q/q} \frac{\mu_l}{\pi} \frac{qfeB}{N_c} \int_{-\infty}^{\infty} dE_{z} \Delta_{\mu\nu} \delta f_{q} E_{z} \delta f_{q}^{\mu} \delta f_{q}^{\nu},$$

where $\delta f_{q}$ is the covariant form of (dressed) quasiquark four-momentum and satisfy $k^{\mu} = k_{\parallel}^{\mu} + \delta\omega_{q/q} u^{\mu}$, with $k_{\parallel}^{\mu} = (\omega_{\parallel}^{l}, 0, 0, k_{\parallel})$. Here, the longitudinal projection operator takes the form $\Delta_{\mu\nu} \equiv g_{\mu\nu} - u^{\mu}u^{\nu}$, with $g_{\mu\nu} = \text{diag}(1,0,0,-1)$. The non-equilibrium part of the distribution function $\delta f_{q}^{\mu}$ can be obtained from the effective relativistic Boltzmann equation. The Boltzmann equation takes the form in the RTA as,

$$\frac{1}{\omega_{q}^{l} k_{\parallel}^{\mu}} \partial_{\mu} f_{q}^{0l}(x, k_{\parallel}) + F_{q}^{\mu} \delta f_{q}^{0l} = -\frac{\delta f_{q}^{0l}}{\tau_{R}},$$

where $\tau_{R}$ is the thermal relaxation time and $F_{q}^{\mu} = -\partial_{\mu}(\delta\omega_{q} u^{\mu} u^{\nu})$ is the mean field force term that arises from the conservation laws of particle density and energy momentum [44]. We employ Chapman-Enskog like iterative expansion to solve the Boltzmann equation to
describe $\delta f_q^\parallel$ and has the following form for the first order correction to distribution function,
\[
\delta f_q = \tau_R \left[ k_{\gamma} \gamma \partial_\gamma \beta + \frac{k_{\gamma}}{u_{\parallel} k_{\parallel}} \beta \partial_\parallel u_{\phi} - \beta \theta_\parallel \delta \omega_q \right] f_q^{\parallel} \tilde{p}^\parallel_q ,
\]
where $\tilde{p}^\parallel_q = 1 - p^\parallel_q$ and $\theta_\parallel = \partial_\parallel s^\parallel$ denotes the longitudinal expansion parameter of the magnetized system. Invoking the energy-momentum conservation laws, one can obtain $\tilde{\beta} = \chi_\beta \theta_\parallel$, where $\chi_\beta / \beta = c_s^2 = \frac{\partial p^\parallel / \partial \epsilon_q}{\partial p^\parallel / \partial \epsilon_q}$ is the square of speed of sound in the longitudinal direction of the magnetized medium. By substituting Eq. (8) to Eq. (3) and assuming that $\tau_R$ is independent of four-momenta, we obtain first-order equation to the longitudinal bulk pressure as,
\[
\Pi_\parallel = -\tau_R \beta_\Pi_\parallel \theta_\parallel , \tag{9}
\]
with the longitudinal bulk viscous coefficient $\beta_\Pi_\parallel$ as
\[
\beta_\Pi_\parallel = \beta \left[ \left( \frac{\chi_\beta}{\beta} \right) \left( f_q^{(0)} \right)_{31} + \delta \omega_q L_q^{(0)} \right] + 3 \left( f_q^{(1)} \right)_{42} \delta \omega_q f_q^{(0)} \tag{10}
\]
The thermodynamic integrals $J_{\parallel}^{(r)}$ and $L_{\parallel}^{(r)}$ employed in the analysis are presented in the Appendix. Employing the bulk viscous evolution equation in Eq. (9), the longitudinal bulk viscous correction to the distribution function take the form,
\[
\delta f_q^{(\parallel)}_{\text{bulk}} = \frac{-\beta}{\beta_\Pi_\parallel (u.k_{\parallel})} \left[ (u.k_{\parallel})^2 \frac{\chi_\beta}{\beta} - k_z^2 - (u.k_{\parallel}) \delta \omega_q \right] f_q^{\parallel} \tilde{p}^\parallel_q \Pi_\parallel . \tag{11}
\]
The bulk viscous correction to the momentum distribution function will give non-equilibrium corrections to the screening mass, which in turn can affect the matrix element for the HQ-thermal particle scattering processes.

Non-equilibrium correction to Debye screening

The realization of the EQPM from the charge renormalization can be done by analyzing the screening mass in the medium. The Debye mass in the magnetized QGP can be defined in terms of the EQPM distribution function and has the following form,
\[
\tilde{m}_D^2 = m_D^2 + \delta m_D^2 , \tag{12}
\]
where $\delta m_D^2$ is the shift in Debye mass due to the longitudinal bulk viscous correction, $\delta m_D^2 = \frac{4\pi}{\tau_R} \sum_f \frac{|q f eB|}{\pi} \sum_{l=0}^{\infty} \mu_l \int_0^{\infty} dk_z f_q^{l}(1 - f_q^{l})$. The Debye mass in the viscous medium can be defined in the leading order as,
\[
m_D^2 = m_D^2 + m m_D^2 , \tag{14}
\]
where $m m_D^2$ is the shift in Debye mass due to the longitudinal bulk viscous correction, $\delta m_D^2 = \frac{4\pi}{\tau_R} \sum_f \frac{|q f eB|}{\pi} \sum_{l=0}^{\infty} \mu_l \int_0^{\infty} dk_z f_q^{l}(1 - f_q^{l})$ and we have,
\[
\delta m_D^2 = \frac{4\pi \alpha_s}{3T} f_q^{(0)} \tag{15}
\]
The effective running coupling constant within the EQPM, $\alpha_{eff} = (T, z_q, z_q, eB)$ can be defined from $m_D^2 = \frac{\pi}{4 \alpha_s} m_D^2$ where $\alpha_s (z_q = 1)$. The correction to the screening mass will reflect in the effective coupling and will act as essential dynamical input in the HQ dynamics in the QGP medium.

Bulk viscous correction to the HQ drag transport

The quark contribution to the momentum diffusion of HQ in the static limit in terms of momentum transfer can be defined from Eq. (2) and has the following form,
\[
K_q^{\text{quark}} = \int d^3 q \frac{d \Gamma}{d^3 q} q^2 , \quad K_q^{\text{quark}} = \frac{1}{2} \int d^3 q \frac{d \Gamma}{d^3 q} q^2 , \tag{16}
\]
where $q = p - p'$ is the momentum transfer due to the interaction and
\[
\frac{d \Gamma}{d^3 q} = \frac{1}{h Q} \frac{1}{\left( 2\pi \right)^3} \frac{1}{2 E_q} \int d \epsilon' \frac{d \epsilon'}{2 E'_{k'}} \left| \mathcal{M}_{HQ,q} \right|^2 \times \left( 2\pi \right)^2 \delta^2(p + k - p' - k') f_q(z) \left( 1 - f_q^l(z') \right) , \tag{17}
\]
denotes the HQ-quark scattering rate per unit volume of momentum transfer with $M_{HQ}$ is the mass of HQ. The scattering rate can be defined from the retarded self-energy as follows [22],
\[
\frac{d \Gamma}{d^3 q} = \frac{8 \pi \alpha_{eff} C^{HQ}_{R}}{(2\pi)^3} \lim_{\omega \to \omega} \frac{T}{\left( Q^2 - \Re \Pi^{HQ}_{R} \left| \text{Fermion} \right|^2 + \left( \Im \Pi^{HQ}_{R} \left| \text{Fermion} \right|^2 \right) \right)} \tag{18}
\]
where $C^{HQ}_{R}$ is the Casimir factor of the HQ. Here, $Q = (\omega, q)$ with $\omega$ is the energy transfer and static limit imposes the condition $\omega \to 0$. It is important to emphasize that we consider the Landau approximation as most of the scattering processes are soft (small momentum transfer $\sim g T$). The real and imaginary part of the retarded
self-energy takes the form in the viscous medium,
\[ \text{Re} \Pi_{\text{Fermion}}^{0}(\omega, \mathbf{q}) = -\frac{g_{f}^{2}}{q_{f}^{2}} \tilde{s}(q_{\perp}), \tag{19} \]
\[ \text{Im} \Pi_{\text{Fermion}}^{0}(\omega, \mathbf{q}) = \frac{\pi \omega}{2} \tilde{s}(q_{\perp}) \left[ \delta(\omega - q_{\perp}) + \delta(\omega + q_{\perp}) \right], \tag{20} \]
where quantity \( \tilde{s}(q_{\perp}) \) can be defined as,
\[ \tilde{s}(q_{\perp}) = 4 \pi \alpha_{s} \sum_{f} \int_{0}^{\infty} \frac{dz}{z^{2}} \sum_{i=0}^{\infty} \mu_{i} \left( \frac{z}{f_{0}^{q}} \right) \exp \left( -\frac{z^{2}}{f_{0}^{q}} \right), \tag{21} \]
\[ \equiv s(q_{\perp}) + \delta s(q_{\perp}). \tag{22} \]
Note that \( \tilde{s}(q_{\perp} = 0) \) denotes the leading order contribution (quark part) to the Debye screening mass in the strongly magnetized viscous medium. Hence, the viscous correction to the \( s(q_{\perp} = 0) \) can be defined as \( \delta s(q_{\perp}) = \tilde{\delta m}_{D}^{2} \exp \left( -\frac{q_{f}^{2}}{2 f_{0}^{q}} \right). \]

The quantity \( N(T, x) \) and \( \delta N(T, x) \) takes the following form respectively,
\[ N = \frac{1}{T} \sum_{f} \int_{0}^{\infty} dq f_{0} q \exp \left( -\frac{m_{D}^{2} + \delta m_{D}^{2}}{\sqrt{2} q_{0} f_{0}} \right), \tag{26} \]
\[ \delta N = \frac{1}{T} \sum_{f} \int_{0}^{\infty} dq f_{0} q \exp \left( -\frac{m_{D}^{2} + \delta m_{D}^{2}}{\sqrt{2} q_{0} f_{0}} \right), \tag{27} \]
The vanishing longitudinal component of the quark contribution in the static limit \( \omega \rightarrow 0 \) is well explored in the Ref. \[22, 23\] and can be understood from the Eqs. \[16, 18\] and \[20\].

The bulk viscous correction to the gluonic contribution to the HQ momentum diffusion is primarily incorporated through the screening mass while defining the HQ-gluon matrix element. The matrix element for the HQ-gluon scattering process in the static limit is investigated in the Ref. \[22, 23\]. Following the same prescriptions as in the Ref. \[23\] to describe the longitudinal component of the gluonic contribution in the viscous medium, we obtain
\[ K_{\parallel}^{\text{gluon}} = K_{\parallel}^{\text{gluon}} + \delta K_{\parallel}^{\text{gluon}}, \tag{28} \]
\[ K_{\parallel}^{\text{gluon}} = \frac{4}{3 \pi} \alpha_{s} N_{c} C_{R}^{HQQ} \left[ \int_{0}^{\infty} dq \frac{q^{2}}{(q^{2} + s(q_{\perp}))^{2}} \right] \times \int_{q/2}^{\infty} d| \mathbf{k} | \left[ \left( 1 + \frac{q^{2}}{2 | \mathbf{k} |^{2}} \right) f_{0}^{g} \right] f_{0}^{g}, \tag{29} \]
\[ \delta K_{\parallel}^{\text{gluon}} = \frac{4}{3 \pi} \alpha_{s} N_{c} C_{R}^{HQQ} \left[ \int_{0}^{\infty} dq \frac{q^{2}}{(q^{2} + s(q_{\perp}))^{2}} \right] \times \int_{q/2}^{\infty} d| \mathbf{k} | \left[ \left( 1 + \frac{q^{2}}{2 | \mathbf{k} |^{2}} \right) f_{0}^{g} \right] f_{0}^{g}, \tag{30} \]
isotropic up to the leading order of $m_D/T$, whereas the quark contributions are highly anisotropic in nature. The anisotropy of the HQ momentum diffusion can be quantified by the ratio $K_\parallel/K_\perp$ in which $K_\parallel$ and $K_\perp$ denotes the total contribution (both quark and gluonic) to the longitudinal and transverse components of the HQ momentum diffusion in the QGP medium. The effect bulk viscosity to the anisotropy can be described from $R = K_\parallel/K_\perp$.

IV. DISCUSSIONS

We initiate the discussion with the temperature dependence of first order longitudinal bulk viscous coefficient of the strongly magnetized QGP medium as plotted in Fig. 2 (left panel). Within the RTA, we have $\zeta = \tau_B \beta_{||}$, where $\zeta$ is the longitudinal bulk viscosity in the magnetized medium. We observe that the first order bulk viscous coefficient increases with temperature. This observation is consistent with the results for the Boltzmann system in which $\beta_{||} \approx P$ with $P$ as the pressure of the QGP. The effects of higher Landau levels (HLLs) are more visible in the higher temperature range as the effect is suppressed by the factor $e^{-\sqrt{\eta P}/T}$. The longitudinal bulk viscous corrections to the Debye screening mass is depicted in Fig. 2 (right panel). To quantify the effects of the bulk viscous effects, we choose the expansion parameter $\theta_{||} = 1/\tau$, where $\tau$ is the proper time parameter. The Debye mass is sensitive to the viscous corrections, and we observe that the longitudinal bulk viscous contribution reduces the screening in the magnetized medium. This observation is in line with that of the Ref. [10].

The temperature dependence of the longitudinal bulk viscous correction to the gluonic contribution $K_{gluon}$ in the longitudinal direction at $|eB| = 15m_2^2$ is plotted in Fig. 2. The longitudinal bulk viscous correction enhances the gluonic contribution to the HQ diffusion coefficient. We have estimated the quark contribution in the perpendicular direction in the viscous strongly magnetized QGP. Bulk viscous correction reduces the perpendicular quark contribution to the HQ momentum diffusion, and the effects are more visible in the lower temperature regimes.

The HLLs effects to the bulk viscous corrections of the HQ momentum diffusion are more significant in the temperature regime near to the transition temperature. We observe in Fig. 3 that the anisotropy of the momen-
to extend the analysis to these aspects of the hot QCD in the presence of the strong magnetic field. We intend dynamics as inputs parameters to study HQ observables. Heavy meson elliptic flow is another experimentally measured quantity. We have studied the effects of HLLs on the bulk viscous corrections to the HQ diffusion gets enhanced. This, in turn, affects the magnetic field induced anisotropy in the HQ momentum diffusion in the medium. We have further demonstrated the effects of HLLs on the bulk viscous corrections to the HQ diffusion in the magnetized medium.

To summarize, we have estimated the longitudinal bulk viscous correction to the thermal distribution in the magnetized hot QCD medium by solving the effective Boltzmann equation within the EQPM. We have illustrated that the bulk viscous contribution reduces the Debye screening mass in the magnetized hot QCD medium. We have studied the HQ momentum diffusion in the magnetized viscous medium. The main observation is that the bulk viscous corrections suppress the quark contribution to the HQ momentum diffusion, whereas the gluonic contributions enhance it. However, recent calculations \cite{2--4} on the heavy meson elliptic flow in the presence of magnetic field take the following forms,

\begin{align}
J_{jq}^{(0)} &= T \sum_{f} N_{e} \left| \frac{q f B}{\pi} \right| \sum_{l=0}^{\infty} \sum_{s=1}^{\infty} \mu_{l} y_{l}^{3} (-1)^{s-1} z_{q}^{s} K_{1}(sy), \\
J_{jq}^{(1)} &= -T \sum_{f} N_{e} \left| \frac{q f B}{2 \pi^{2}} \right| \sum_{l=0}^{\infty} \sum_{s=1}^{\infty} \mu_{l} y_{l}^{2} (-1)^{s-1} z_{q}^{s} \times \left[ K_{2}(sy) - K_{0}(sy) \right], \\
L_{jq}^{(0)} &= -T \sum_{f} N_{e} \left| \frac{q f B}{2 \pi^{2}} \right| \sum_{l=0}^{\infty} \sum_{s=1}^{\infty} \mu_{l} y_{l} (-1)^{s-1} z_{q}^{s} \times \left[ K_{2}(sy) - 3K_{0}(sy) + 2K_{4}(sy) \right], \\
L_{jq}^{(1)} &= -T \sum_{f} N_{e} \left| \frac{q f B}{4 \pi^{2}} \right| \sum_{l=0}^{\infty} \sum_{s=1}^{\infty} \mu_{l} y_{l} (-1)^{s-1} z_{q}^{s} \times \left[ K_{3}(sy) - 5K_{1}(sy) + 4K_{5}(sy) \right] - \delta \omega_{q} T^{2} \sum_{f} N_{e} \left| \frac{q f B}{\pi} \right| \sum_{l=0}^{\infty} \sum_{s=1}^{\infty} \mu_{l} y_{l} (-1)^{s-1} z_{q}^{s} \times \left[ K_{2}(sy) - 3K_{0}(sy) + 2K_{4}(sy) \right], \\
J_{jq}^{(2)} &= -T \sum_{f} N_{e} \left| \frac{q f B}{2 \pi^{2}} \right| \sum_{l=0}^{\infty} \sum_{s=1}^{\infty} \mu_{l} y_{l} (-1)^{s-1} z_{q}^{s} \times \left[ K_{2}(sy) - K_{0}(sy) \right] + \delta \omega_{q} T \sum_{f} N_{e} \left| \frac{q f B}{\pi^{2}} \right| \sum_{l=0}^{\infty} \sum_{s=1}^{\infty} \mu_{l} y_{l} (-1)^{s-1} z_{q}^{s} \times \left[ K_{1}(sy) - K_{4}(sy) \right].
\end{align}

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Appendix A: Thermodynamic integrals in the magnetic field

Defining, \( y_{i}^{2} = \frac{1}{2} (m_{i}^{2} + 2l \ | q f B |) \) and \( K_{1,n}(sy) = \int_{0}^{\infty} \frac{dt}{(\cosh s y t)} \exp (-sy \cosh \theta) \), and following the prescriptions in Ref. \[21\], the \( J_{jq}^{(i)} \) and \( L_{jq}^{(i)} \) integrals in presence of magnetic field takes the following forms,

\begin{align}
J_{jq}^{(i)} &= T \sum_{f} N_{e} \left| \frac{q f B}{\pi} \right| \sum_{l=0}^{\infty} \sum_{s=1}^{\infty} \mu_{l} y_{l}^{3} (-1)^{s-1} z_{q}^{s} K_{1}(sy), \\
J_{jq}^{(2)} &= -T \sum_{f} N_{e} \left| \frac{q f B}{2 \pi^{2}} \right| \sum_{l=0}^{\infty} \sum_{s=1}^{\infty} \mu_{l} y_{l}^{2} (-1)^{s-1} z_{q}^{s} \times \left[ K_{2}(sy) - K_{0}(sy) \right], \\
L_{jq}^{(0)} &= -T \sum_{f} N_{e} \left| \frac{q f B}{2 \pi^{2}} \right| \sum_{l=0}^{\infty} \sum_{s=1}^{\infty} \mu_{l} y_{l} (-1)^{s-1} z_{q}^{s} \times \left[ K_{2}(sy) - 3K_{0}(sy) + 2K_{4}(sy) \right], \\
L_{jq}^{(1)} &= -T \sum_{f} N_{e} \left| \frac{q f B}{4 \pi^{2}} \right| \sum_{l=0}^{\infty} \sum_{s=1}^{\infty} \mu_{l} y_{l} (-1)^{s-1} z_{q}^{s} \times \left[ K_{3}(sy) - 5K_{1}(sy) + 4K_{5}(sy) \right] - \delta \omega_{q} T^{2} \sum_{f} N_{e} \left| \frac{q f B}{\pi} \right| \sum_{l=0}^{\infty} \sum_{s=1}^{\infty} \mu_{l} y_{l} (-1)^{s-1} z_{q}^{s} \times \left[ K_{2}(sy) - 3K_{0}(sy) + 2K_{4}(sy) \right], \\
J_{jq}^{(2)} &= -T \sum_{f} N_{e} \left| \frac{q f B}{2 \pi^{2}} \right| \sum_{l=0}^{\infty} \sum_{s=1}^{\infty} \mu_{l} y_{l} (-1)^{s-1} z_{q}^{s} \times \left[ K_{2}(sy) - K_{0}(sy) \right] + \delta \omega_{q} T \sum_{f} N_{e} \left| \frac{q f B}{\pi^{2}} \right| \sum_{l=0}^{\infty} \sum_{s=1}^{\infty} \mu_{l} y_{l} (-1)^{s-1} z_{q}^{s} \times \left[ K_{1}(sy) - K_{4}(sy) \right].
\end{align}

[1] S. Acharya et al. [ALICE Collaboration], arXiv:1910.14406 [nucl-ex].
[2] S. K. Das, S. Plumari, S. Chatterjee, J. Alam, F. Scardina and V. Greco, Phys. Lett. B 768, 260 (2017).
