Hesitant Fuzzy MP Filters of R0-Algebras

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Abstract. In this paper, we introduce the notion of hesitant fuzzy (correlation) MP filters, some properties and equivalent conditions are investigated. The relationships between hesitant fuzzy MP filter and hesitant fuzzy correlation MP filter is discussed. Finally, we prove the invariance between the image and the original image of hesitant fuzzy correlation filter is studied under the premise of homomorphism.

1. Introduction
As a generalization of fuzzy set (see [1]), the concept of hesitant fuzzy sets was introduced by Torra (see [2]). The hesitant fuzzy number is more comprehensive than the traditional fuzzy element. It contains different groups with a certain degree of hesitation and has been applied in many mathematical models. Hesitant fuzzy numbers are more comprehensive than traditional fuzzy elements, It includes different groups with a certain degree of hesitation and can be used in many mathematical models (see [5-8]). Since filter theory plays an important role in studying logic algebras, many researchers combine it with mathematical approaches to discuss its generalized properties. Especially in recent years, a large number of researchers apply the theory of hesitant fuzzy sets to algebraic systems. For example, Peng Jiayin (see [9-10]) presented the theory of hesitant fuzzy filters in FI-algebra and BR0-algebras. Furthermore, Fu Xiaobo (see [11]) introduced the theory of hesitant fuzzy filters in lattice implication algebras. Liu Chunhui (see [12]) applied the relationship between hesitant fuzzy filters and hesitant fuzzy congruence in BE-algebras and obtained some important conclusions the theory of hesitant fuzzy sets.

In this paper, in order to study filter theory in R0-algebra more comprehensively, we introduce the notion of hesitant fuzzy (correlation) MP filters and get some useful properties in R0-algebra. In fact, we show that the invariance between the image and the original image of hesitant fuzzy correlation filter is studied under the premise of homomorphism.

2. Preliminaries
In this section, we cite the fundamental definitions that will be used in the sequel:

Definition 1. (see [3]) Let R be a \((\neg, \lor, \rightarrow)\)-type algebra, where \(\neg\) is a unary operation and \(\lor, \rightarrow\) are binary operations. If there is a partial ordering \(\leq\) on R, such that(R, \(\leq\)) is a bounded distributive lattice, \(\rightarrow\), \(\lor\) are infimum and supremum operations with respect to \(\leq\), \(\neg\) is an order-reversing involution with respect to \(\leq\), and the following conditions hold for any \(x, y, z \in R\):

\[
\begin{align*}
(T_1) & \quad x \rightarrow y = \neg y \rightarrow \neg x; \\
(T_2) & \quad 1 \rightarrow x = x, \quad x \rightarrow x = 1; \\
(T_3) & \quad y \rightarrow z \leq (x \rightarrow y) \rightarrow (x \rightarrow z);
\end{align*}
\]
\((T_4)\ x \to (y \to z) = y \to (x \to z)\);
\((T_5)\ x \to (y \lor z) = (x \to y) \lor (x \to z)\);
\((T_6)\ x \to (y \lor z) = ((x \to y) \lor (x \to z)) = 1.\)

**Proposition 1. (see [3])** Let \(R\) be an \(R_0\)-algebras, then \(R\) satisfies the following properties,
\(\forall x, y, z \in R\),

\((P_1)\ x \to y = 1\ if\ and\ only\ if\ x \leq y\ ;\)
\((P_2)\ x \leq y \to z\ if\ and\ only\ if\ y \leq x \to z\ ;\)
\((P_3)\ x \lor y \to z = (x \to z) \land (y \to z)\ and\ x \land y \to z = (x \to z) \lor (y \to z)\ ;\)
\((P_4)\ If\ y \leq z,\ then\ x \to y \leq x \to z,\ if\ x \leq y,\ then\ y \to z \leq x \to z;\)
\((P_5)\ x \to y \geq x' \lor y;\)
\((P_6)\ (x \to y) \lor (y \to x) = 1;\)
\((P_7)\ x \land x' \leq y \lor y' ;\)
\((P_8)\ x \to (y \to x) = 1;\)
\((P_9)\ x \to (x' \to y) = 1;\)
\((P_{10})\ x \lor y \leq ((x \to y) \to y) \land ((y \to x) \to x) ;\)
\((P_{11})\ x \to y \leq x \lor z \to y \lor z ,\ x \to y \leq x \land z \to y \land z ;\)
\((P_{12})\ x \to y \leq (x \to z) \lor (z \to y).\)

**Proposition 2. (see [3])** Let \(R\) be an \(R_0\)-algebra.
Define a new operator \(\otimes\) on \(R\) such that \(x \otimes y = (x \to y)'\), then the following properties are holds.
\(\forall x, y, z \in R\),

\((P_{13})\ (R, \otimes, 1)\ is\ a\ commutative\ monoid\ with\ the\ multiplicative\ unit\ element\ 1;\)
\((P_{14})\ If\ x \leq y,\ then\ x \otimes z \leq y \otimes z;\)
\((P_{15})\ x \otimes y \leq x \land y ;\)
\((P_{16})\ x \otimes y \leq z\ if\ and\ only\ if\ x \leq y \to z;\)
\((P_{17})\ x \otimes (y \lor z) = (x \otimes y) \lor (x \otimes z) ;\)
\((P_{18})\ x \otimes y \to z = x \to (y \to z)\ and\ x \to (y \to x \otimes y) = 1;\)
\((P_{19})\ x \otimes x' = 0.\)

**Lemma 1. (see [4])** Let \(R\) be an \(R_0\)-algebras.
\(\forall x, y, z \in R,\ then\)

\((P_{20})\ x \otimes (x \to y) \leq y;\)
\((P_{21})\ y \leq x \to (x \otimes y), x \leq y \to (x \otimes y).\)
\[
\begin{align*}
(P_{22}) & \quad x \rightarrow y \leq (y \rightarrow z) \rightarrow (x \rightarrow z); \\
(P_{23}) & \quad x \leq x \land y \rightarrow y.
\end{align*}
\]

**Definition 3. (see [1])** Let R be an R0-algebras. 
\(A : R \rightarrow [0, 1]\) is the mapping to the unit interval, then R is called a fuzzy set on R0-algebras.

**Definition 4. (see [3])** Let R be an R0-algebras. A is called a MP filters of R if satisfies the following conditions:
for all \(x, y \in R\), \(A \subseteq R\),
(1) \(1 \in A\);
(2) \(x \in A, x \rightarrow y \in A \Rightarrow y \in A\).

**Definition 5. (see [4])** Let R be an R0-algebras. A is called a Correlation MP filters of R if satisfies the following conditions:
for all \(x, y \in R\), \(A \subseteq R\),
(1) \(1 \in A\);
(2) \(\forall x, y, z \in R, \text{ if } x \rightarrow y, x \rightarrow (y \rightarrow z) \in A, \text{ then } x \rightarrow z \in A\).

**Definition 6. (see [2])** Let X be a reference set. Then a hesitant fuzzy set \(H_F\) in X is represented mathematical as: 
\(\mathcal{F} : \{x, h_F(x) : h_F(x) \in p([0,1]), x \in X\}\), where \(p([0,1])\) is the power set of [0, 1].

So, we can define a set of fuzzy sets an \(HFS\) by union of their membership functions.

**Definition 7. (see [2])** Let \(X = \{x \in X | \gamma \leq h_F(x)\}\) is called a hesitation level set of \(\mathcal{F}\), and \(\gamma \in P([0,1])\).

**Definition 8.** Hesitant fuzzy MP filter
In this section, we will introduce a new concept, which is called a hesitation fuzzy MP filter of R, and investigate its properties.

**Definition 8**. Hesitant fuzzy set \(\tilde{A}\) is called a hesitation fuzzy MP filter of R if satisfies the following conditions:
(\(F_1\)) \(h_{\tilde{A}}(1) \supseteq h_{\tilde{A}}(x);\)
(\(F_2\)) \(h_{\tilde{A}}(y) \supseteq h_{\tilde{A}}(x) \land h_{\tilde{A}}(x \rightarrow y)\), for all \(x, y \in R\).

Denote the set of all hesitation fuzzy MP filter of R by HFF(R).

**Theorem 1.** Let \(\tilde{A} \in HFF(R)\) and \(x, y \in R\). Then \((F_1)\) if \(x \leq y\), then \(h_{\tilde{A}}(x) \supseteq h_{\tilde{A}}(y)\).

**Proof.** Let \(\tilde{A} \in HFF(R)\). Then \((F_1)\) and \((F_2)\) holds. Let \(x \leq y\), then \(x \rightarrow y = 1\). And so by using the definition 8, we have \(h_{\tilde{A}}(y) \supseteq h_{\tilde{A}}(x) \land h_{\tilde{A}}(x \rightarrow y) = h_{\tilde{A}}(x)\).

**Theorem 2.** \(\tilde{A}\) is a hesitation fuzzy MP filter of R if and only if \((F_2)\) and \((F_4)\) hold, for all \(x, y, z \in R\), where
\[(F_4)\] \(h_i(x \rightarrow z) \supseteq h_i(x \rightarrow y) \cap h_i(y \rightarrow z)\).

**Proof.** Now we assume that \((F_2)\) holds.

\((F_2) \Rightarrow (F_4)\). From \((P_{22})\), we have \(x \rightarrow y \leq\)
\((y \rightarrow z) \rightarrow (x \rightarrow z)\), by using \((F_1)\) and \((F_2)\), we have \(h_i(x \rightarrow y) \subseteq h_i((y \rightarrow z) \rightarrow (x \rightarrow z))\).
\(h_i(x \rightarrow z) \supseteq h_i(y \rightarrow z) \cap h_i((y \rightarrow z) \rightarrow (x \rightarrow z))\). Therefore \(h_i(x \rightarrow z) \supseteq h_i(x \rightarrow y) \cap h_i(y \rightarrow z)\).

So \((F_4)\) holds.

\((F_2) \Rightarrow (F_4)\). From \((F_4)\), we have \(h_i(1 \rightarrow y) \supseteq h_i(1 \rightarrow x) \cap h_i(x \rightarrow y)\).

And according to \((T_i)\), we have \(h_i(1 \rightarrow y) \supseteq h_i(x) \cap h_i(x \rightarrow y)\). Therefore \((F_2)\) holds.

**Theorem 3.** \(\bar{A}\) is a hesitation fuzzy MP filter of \(R\) if and only if \((F_3)\) and \((F_5)\) hold. Where \(\langle F_3 \rangle\) \(h_i(x \otimes y) \supseteq h_i(x) \cap h_i(y), x, y \in R\).

**Proof.** Let \(\bar{A} \in HFF(R)\). By using \((F_5)\), we have \(h_i(x \otimes y) \supseteq h_i(x) \cap h_i(x \rightarrow (x \otimes y))\). According to \((P_{21})\) and \((F_5)\), we have \(h_i(x) \subseteq h_i(x \rightarrow (x \otimes y))\). Therefore \(h_i(x \otimes y) \supseteq h_i(x) \cap h_i(x \rightarrow y)\). That is \((F_5)\) holds.

Conversely, assume that \((F_3)\) and \((F_5)\) holds. Since \(x \leq 1\), then \(h_i(x) \subseteq h_i(1)\). by using \((F_5)\), we have \(h_i(x \otimes (x \rightarrow y)) \supseteq h_i(x) \cap h_i(x \rightarrow y)\). According to \(x \otimes (x \rightarrow y) \leq y\) and \((F_5)\), we have \(h_i(x) \supseteq h_i(x \rightarrow y)\). Then \(h_i(x \rightarrow y) \supseteq h_i(x) \cap h_i(x \rightarrow y)\).

**Theorem 4.** \(\bar{A}\) is a hesitation fuzzy MP filter of \(R\) if and only if \((F_6)\) and \((F_7)\) hold. Where,
\(\langle F_6 \rangle\) \(h_i(x \otimes y) = h_i(x) \cap h_i(y)\); \(\langle F_7 \rangle\) \(h_i(x \land y) = h_i(x) \cap h_i(y)\), for all \(x, y \in R\).

**Proof.** Assume that \(\bar{A} \in HFF(R)\). By using \((F_7)\), we have \(h_i(x \otimes y) \supseteq h_i(x) \cap h_i(y)\). According to \((P_{14})\), then \(x \otimes y \leq x, x \otimes y \leq y\). By using \((F_7)\), we have \(h_i(x \otimes y) \subseteq h_i(x)\). \(h_i(x \otimes y) \subseteq h_i(y)\). Hence \(h_i(x \otimes y) \subseteq h_i(x) \cap h_i(y)\).

Therefore, \(h_i(x \otimes y) = h_i(x) \cap h_i(y)\).

From \((F_7)\), we have \(h_i(x \land y) \subseteq h_i(x) \cap h_i(y)\).

According to \((P_{15}), (F_7)\) and \((F_6)\), we have \(h_i(x \land y)\), \(\supseteq h_i(x \otimes y) = h_i(x) \cap h_i(y)\). Hence \(h_i(x \land y) = h_i(x) \cap h_i(y)\).
\( h_3(x) \cap h_3(y). \)

Conversely, Assume that \((F_6)\) and \((F_7)\), \((F_8)\) and \((F_9)\) can be proved. From the theorem 3, \(\tilde{A}\) is a hesitation fuzzy MP filter of \(R\).

**Theorem 5.** \(\tilde{A}\) is a hesitation fuzzy MP filter of \(R\) if and only if \((F_8)\) holds. Where \(\tilde{A} \in \text{HFF}(R)\), \(x, y, z \in R\). If \(x \leq y \rightarrow z\),

From \((P_{10})\), we have \(x \otimes y \leq z\). According to \((F_1)\) and \((F_5)\), we have \(h_3(z) \supseteq h_3(x) \cap h_3(y)\).

Conversely, suppose that \((F_8)\) holds. By using \((P_{21})\) and \((F_8)\), we have \(h_3(x) \supseteq h_3(x \otimes y) \supseteq h_3(x) \cap h_3(y)\). From \((P_{23})\), we have \(x \leq x \wedge y \rightarrow y\). By \((F_8)\), we have \(h_3(y) \supseteq h_3(x) \cap h_3(x \wedge y)\).

All for all, when \(x \leq y\), \(h_3(x) \subseteq h_3(y)\).

Hence \((F_8)\) holds.

**Theorem 6.** \(\tilde{A}\) is a hesitation fuzzy MP filter of \(R\) if and only if \(\forall \gamma \in P([0,1])\), \(R(\tilde{A}, \gamma) \neq \emptyset\), \(R(\tilde{A}, \gamma)\) is a MP filter of \(R\).

**Proof.** Let \(\tilde{A}\) be the hesitation fuzzy MP filter of \(R\). \(\forall \gamma \in P([0,1])\), \(R(\tilde{A}, \gamma) \neq \emptyset\). Now we prove that \(R(\tilde{A}, \gamma)\) is a MP filter of \(R\). From \(R(\tilde{A}, \gamma) \neq \emptyset\), \(\exists x \in R\), then \(h_3(x) \supseteq \gamma\). Therefore, by using \((F_1)\), we have \(h_3(1) \supseteq h_3(x) \supseteq \gamma\), so \(1 \in R(\tilde{A}, \gamma)\).

Assume that \(x \in R(\tilde{A}, \gamma), x \rightarrow y \in R(\tilde{A}, \gamma)\), we have \(h_3(x) \supseteq \gamma\), \(h_3(x \rightarrow y) \supseteq \gamma\). By using \((F_2)\), we have \(h_3(y) \supseteq h_3(x) \cap h_3(x \rightarrow y) \supseteq \gamma\), So \(y \in R(\tilde{A}, \gamma)\). Hence \(R(\tilde{A}, \gamma)\) is MP filter of \(R\).

Conversely, \(\forall x \in R\), suppose that \(h_3(x) = \gamma_1\), then \(x \in R(\tilde{A}, \gamma_1)\), \(R(\tilde{A}, \gamma_1) \neq \emptyset\). Since \(R(\tilde{A}, \gamma_1)\) is a MP filter of \(R\), then \(1 \in R(\tilde{A}, \gamma_1)\). Hence \(h_3(1) \supseteq \gamma_1 = h_3(x)\).

\(\forall x, y \in R\), assume that \(\gamma_2 = h_3(x) \cap h_3(x \rightarrow y)\), we have \(x \in R(\tilde{A}, \gamma_2), x \rightarrow y \in R(\tilde{A}, \gamma_2)\). And so \(R(\tilde{A}, \gamma_2) \neq \emptyset\), then \(R(\tilde{A}, \gamma_2)\) is a MP filter of \(R\), Hence \(y \in R(\tilde{A}, \gamma_2)\).

Therefore \(h_3(y) \supseteq \gamma_2 = h_3(x) \cap h_3(x \rightarrow y)\).

From the Definition 8. \(\tilde{A}\) is a hesitation fuzzy MP filter of \(R\).

**Theorem 7.** \(\tilde{A}\) is a hesitation fuzzy MP filter of \(R\), then \(\forall x \in R\), \(R(\tilde{A}, \gamma_0)\) is a MP filter, for each \(\gamma_0 = h_3(x)\).
Proof. From \( x \in R(\tilde{A}, \gamma_0) \), \( R(\tilde{A}, \gamma_0) \neq \emptyset \), according to the theorem 6, \( R(\tilde{A}, \gamma_0) \) is a MP filter.

4. Hesitant fuzzy incidence MP filters

In this section, we will denote the concept hesitant fuzzy incidence MP filter of \( R_0 \)-algebra, discuss the related properties, and study the invariance of hesitant fuzzy incidence MP filter under \( R_0 \)-algebras homomorphism (isomorphism).

Definition 9. A hesitant fuzzy set \( \tilde{A} \) of \( R \) is called a hesitation fuzzy incidence MP filter if satisfies the following conditions:

1. \( h_\tilde{A}(x) \subseteq h_\tilde{A}(1) \):
2. \( h_\tilde{A}(x \rightarrow z) \supseteq h_\tilde{A}(x \rightarrow (y \rightarrow z)) \cap h_\tilde{A}(x \rightarrow y) \), for all \( x, y, z \in R \).

Denote the set of all hesitation fuzzy incidence MP filter of \( R \) by HFIF(\( R \)).

Theorem 8. Let \( \tilde{A} \) be a hesitation fuzzy incidence MP filter of \( R \), and then \( \tilde{A} \) is a hesitation fuzzy MP filter of \( R \).

Proof. Since \( \tilde{A} \) is a hesitant fuzzy incidence MP filter of \( R \). Then for all \( x, y \in R \), \( h_\tilde{A}(x) \subseteq h_\tilde{A}(1) \). Then for all \( x, y \in R \), \( h_\tilde{A}(x \rightarrow y) \supseteq h_\tilde{A}(x \rightarrow (y \rightarrow z)) \cap h_\tilde{A}(x \rightarrow y) \), for all \( x, y, z \in R \).

Theorem 9. Let \( \tilde{A} \) be a hesitation fuzzy incidence MP filter of \( R \), if \( x \leq y \), then \( h_\tilde{A}(x) \subseteq h_\tilde{A}(y) \), for all \( x, y \in A \).

Proof. It’s obvious.

From theorem 8 and reference [5], we can obtain the following theorem.

Theorem 10. Assume \( \tilde{A} \) is a hesitation fuzzy incidence MP filter of \( R \) if and only if \( R(\tilde{A}, \gamma) \) is a correlation MP filter of \( R \).

Proof. Necessity. \( \forall \gamma \in P([0,1]) \), assume that \( R(\tilde{A}, \gamma) \neq \emptyset \), that is \( \exists x \in R \), then \( h_\tilde{A}(1) \supseteq h_\tilde{A}(x) \supseteq \gamma \), so \( 1 \in R(\tilde{A}, \gamma) \). For \( \forall x, y, z \in R \), if \( x \rightarrow (y \rightarrow z), x \rightarrow y \in R(\tilde{A}, \gamma) \), Then \( h_\tilde{A}(x \rightarrow (y \rightarrow z)) \supseteq h_\tilde{A}(x \rightarrow y) \supseteq \gamma \). Hence \( h_\tilde{A}(x \rightarrow z) \supseteq h_\tilde{A}(x \rightarrow (y \rightarrow z)) \cap h_\tilde{A}(x \rightarrow y) \supseteq \gamma \), so \( x \rightarrow z \in R(\tilde{A}, \gamma) \). Therefore \( R(\tilde{A}, \gamma) \) is a incidence MP filter of \( R \).

Adequacy. Assume that \( \exists x \in R \), \( h_\tilde{A}(x) \supseteq h_\tilde{A}(1) \). Let \( h_\tilde{A}(x) = \gamma_0, x \in R(\tilde{A}, \gamma_0) \), then \( R(\tilde{A}, \gamma_0) \neq \emptyset \). For \( 1 \notin R(\tilde{A}, \gamma_0) \),
hence $R(\tilde{A}, \gamma)$ isn’t a hesitant fuzzy incidence MP filter. It’s contradiction.

Therefore $\forall x \in R, h_\gamma(x) \subseteq h_\gamma(1)$. $\forall x, y, z \in R$, let
\[\gamma = h_\gamma(x \rightarrow (y \rightarrow z)) \cap h_\gamma(x \rightarrow y),\]
then $R(\tilde{A}, \gamma) \neq \emptyset$, therefore $R(\tilde{A}, \gamma)$ is a hesitant fuzzy incidence MP filter.

For $x \rightarrow (y \rightarrow z)$, $x \rightarrow y \in R(\tilde{A}, \gamma)$, so $x \rightarrow z \in R(\tilde{A}, \gamma)$, therefore $h_\gamma(x \rightarrow z) \supseteq \gamma$.

Hence $\tilde{A} \in HFIF(R)$.

**Theorem 11.** Let $\tilde{A}$ be a hesitation fuzzy incidence MP filter of $R$, Define $B = \{x | x \in R \mid h_\gamma(x) = h_\gamma(1)\}$, then $B \in HFIF(R)$.

**Proof.** Obviously $1 \in B$. If \(x \rightarrow (y \rightarrow z) \in B\), $x \rightarrow y \in B$, then $h_\gamma(x \rightarrow (y \rightarrow z)) = h_\gamma(x \rightarrow y) = h_\gamma(1)$.

Because $\tilde{A} \in HFIF(R)$, So $h_\gamma(x \rightarrow (y \rightarrow z)) \supseteq h_\gamma(x \rightarrow (y \rightarrow z)) \cap h_\gamma(x \rightarrow y)$.

By the definition 9, $h_\gamma(1) \supseteq h_\gamma(x \rightarrow z)$. Then $h_\gamma(1) = h_\gamma(x \rightarrow z)$, so $x \rightarrow z \in B$.

Therefore, $B \in HFIF(R)$.

**Theorem 12.** Let $f : R_1 \rightarrow R_2$ be an onto homomorphism of R0-algebras, then:

(1) If $\tilde{A} \in HFIF(R_1)$, Then $f^{-1}(\tilde{A}) \in HFIF(R_2)$.

(2) If $\tilde{A} \in HFIF(R_1)$, and $f$ is isomorphism, then $f(\tilde{A}) \in HFIF(R_2)$.

**Proof.** (1) Because $f$ is homomorphism, So $f(1) = 1$.

Then $\forall x \in R_1$, $h_{f^{-1}(\tilde{A})}(x) = h_{\tilde{A}}(f(x)) \subseteq h_{\tilde{A}}(1) = h_{\tilde{A}}(f(1)) = h_{f^{-1}(\tilde{A})}(1)$. For $\forall x_1, x_2, x_3 \in R_1$, $B \in HFIF(R_1)$, hence
\[h_{\tilde{A}}(f(x_1) \rightarrow f(x_2)) \supseteq h_{\tilde{A}}(f(x_1) \rightarrow (f(x_1) \rightarrow f(x_2))) \cap h_{\tilde{A}}(f(x_1) \rightarrow f(x_2)).\]

And $f$ is homomorphic, hence $h_{\tilde{A}}(f(x_1 \rightarrow x_2)) \supseteq h_{\tilde{A}}(f(x_1 \rightarrow x_2)) \cap h_{\tilde{A}}(f(x_1 \rightarrow x_2)).$

Therefore $h_{f^{-1}(\tilde{A})}(x_1 \rightarrow x_2) \supseteq h_{f^{-1}(\tilde{A})}(x_1 \rightarrow x_2) \cap h_{f^{-1}(\tilde{A})}(x_1 \rightarrow x_2)$.

All in all, $f^{-1}(B) \in HFIF(R_2)$.

(2) $\forall y \in R_2$, $h_{f(\tilde{A})}(y) = \cup_{h_{\tilde{A}}(x) = y} h_{\tilde{A}}(x) \subseteq h_{\tilde{A}}(1) = \cup_{h_{\tilde{A}}(1) = y} h_{\tilde{A}}(1)$, and $\forall y_1, y_2, y_3 \in R_2$, for $f$ is isomorphic, hence $\exists x_1, x_2, x_3 \in R_1$, we have $f(x_1) = y_1$, $f(x_2) = y_2$, $f(x_3) = y_3$, hence
\[f(x_1 \rightarrow x_2) = f(x_1) \rightarrow f(x_2) = y_1 \rightarrow y_2,\]
\[f(x_1 \rightarrow x_2) = f(x_1) \rightarrow f(x_2) = y_1 \rightarrow y_2.\]

Then
\[h_{f(\tilde{A})}(y_1 \rightarrow y_2) = \cup_{f(x_1) \rightarrow y_1} h_{\tilde{A}}(x) = h_{\tilde{A}}(x_1 \rightarrow x_2) \supseteq h_{\tilde{A}}(x_1 \rightarrow x_2) \cap h_{\tilde{A}}(x_1 \rightarrow x_2) = \cup_{f(x_1) \rightarrow y_1} h_{\tilde{A}}(x) \cap h_{\tilde{A}}(y_1 \rightarrow y_2).\]
5. Conclusion
Uncertainty usually appears in many real world problem. Fuzzy sets and its extensions have provided successful results dealing with uncertainty in different problems. Filter is an important research object in algebraic algebra, and different filters correspond to different reasoning rules. Therefore, our main goal is to consider some connections among hesitant fuzzy sets and R0-algebras theory in this paper. We hope that our research may provide a powerful algebraic tool in hesitant fuzzy sets and other algebraic structure.

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