Witten’s cubic open string field theory on multiple $Dp$-branes

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Abstract
We study Witten’s cubic open string field theory on multiple $Dp$-branes. On multiple $Dp$-branes the string fields carry $U(N)$ group indices. Mapping the string world-sheet onto the upper half complex plane and evaluating the Polyakov string path integral of Witten’s cubic open string, we obtain a Fock space representation of one, two and three-string vertex operators. In the low energy region, the system is described by massless gauge fields and massless scalar fields, carrying $U(N)$ group indices. The two-string vertex induces mass terms for the gauge fields and the scalar fields. The cubic interaction reduces correctly to the cubic term of the non-Abelian gauge fields, cubic interaction terms of the gauge fields, and the scalar fields on $(p+1)$ dimensional space. Thus, Witten’s string field theory may be a useful tool to study the quantum dynamics of open strings on multiple $Dp$-branes.

Keywords $Dp$-brane · Fock space · Non-Abelian gauge field · Open string field theory · Matrix model

1 Introduction
The covariant open string field theory [1–3] which contains only the cubic interaction, has been proven to be a useful tool for exploring string theory corrections [4, 5], entanglement entropy in string theory [6], and the off-shell dynamics of string theory such as tachyon condensation [7–10]. The cubic string field theory of Witten on multiple $Dp$-branes is also expected to be useful for studying the quantum dynamics of string theory in a systematic manner. Therefore, it is important to develop cubic string field theory as a practical tool to study various subjects in theoretical physics. String field theories are usually constructed on the configuration space, with overlapping conditions. However, in order to apply the string field theory to particle physics, we need a Fock space representation of the theory. It is not an easy task to convert the configuration space representation of string field theory into the Fock space representation, as noted earlier in a work on the light-cone string field theory [11, 12]. In the light-cone string field theory, the correct Fock space representation of the string field theory is obtained by using the Neumann functions on the world sheets of strings, which are mapped onto the upper half-plane [13–15]. The purpose of this work is to extend the previous work on the covariant string field theory on multiple $Dp$-branes in the proper-time gauge [16] to Witten’s cubic open string field theory on multiple $Dp$-branes. Our main task is to construct the three-string vertex operator.

In the absence of $Dp$-branes, the three-string vertex operator for Witten’s cubic string field theory has been constructed in [17–21]. The Witten’s cubic string field theory has been discussed also in the presence of the $Dp$-branes in [22–26] before. But these previous works mostly focused on the tachyonic vacuum on a $D25$-brane. Refs. [27–31] discussed the effective field theory action of $Dp$-branes, using numerical method, called the level truncation. Relationships between the string field theory and matrix models are studied in [32–36] and scattering of strings from D-branes has been explored in detail in [37–41].

Two approaches to the covariant string theory exist: the first quantized theory, which is based on the Polyakov string path integral [42], and the second quantized theory, which is formulated as the cubic string field theory. We shall establish these two approaches for string theory on multiple $Dp$-branes by explicitly constructing Fock space representations of string vertex operators. The cubic string field theory provides a unique way to parameterize the world sheet diagram in terms of local flat coordinate patches which describes the free string propagation. The world sheet of three strings in
the cubic string field theory forms a conical surface with an excessive angle \( \pi \). It can be mapped onto a unit disk on which the Neumann boundary condition is imposed. Then, the unit disk is mapped again onto the upper half-plane. This procedure defines the Schwarz–Christoffel transformation from the world sheet onto the upper half-plane and fixes the Green’s function on the world sheet of three strings. Using the Green’s function on the world sheet, we can evaluate the Polyakov string path integral which is subject to the temporal boundary condition, fixed by the momenta of external string states. The Fock space representation of the three-string vertex is obtained by rewriting the Polyakov string path integral in terms of the oscillator operators. As a result, we obtain explicit expressions of the Neumann functions of the corresponding Vertex operators, i.e., the Fock space representation of the vertex operators.

After constructing the one, two, and three-string vertex operators, we calculate scattering amplitudes in the low energy region, where only massless gauge fields and massless scalar fields are excited. The one-string vertex operator corresponds to insertion of field operators with zero momenta, and the two-string vertex operator yields mass terms of the component fields as expected. From the three-string vertex operator we obtain the cubic term in the covariant non-Abelian gauge field action and the covariant coupling of the scalar field and the gauge field. If we choose higher spin excited states, we would obtain their consistent couplings from the vertex operators. We conclude with a brief summary and discussion on possible extensions and applications of the present work in various areas in string theory and related subjects.

2 Witten’s cubic open string fields on \( Dp \)-branes

Witten’s cubic open string field theory is described by a BRST invariant action, which has only a cubic interaction term

\[
S_{\text{open}} = \int \mathrm{tr} \left( \Psi \ast Q\Psi + \frac{2g}{3} \Psi \ast \Psi \ast \Psi \right),
\]

where the star product between the string field operators is defined as

\[
(\Psi_1 \ast \Psi_2)[X(\sigma)] = \int \prod_{\frac{\pi}{2} \leq \sigma \leq \pi} DX^{(1)}(\sigma) \prod_{0 \leq \sigma \leq \frac{\pi}{2}} DX^{(2)}(\sigma) \\
\prod_{\frac{\pi}{2} \leq \sigma \leq \pi} \delta \left[ X^{(1)}(\sigma) - X^{(2)}(\pi - \sigma) \right] \\
\Psi[X^{(1)}(\sigma)] \Psi[X^{(2)}(\sigma)].
\]

The star product is associative and the string field action is invariant under the BRST gauge transformation

\[
\delta \Psi = Q \ast c + \Psi \ast c - c \ast \Psi.
\]

On \( Dp \)-branes, in terms of the normal modes, the open string coordinates \( X^I, I = 0, \ldots, d \), are expanded as

\[
\begin{align*}
X^\mu(\sigma) &= \chi^\mu + 2 \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} e_n \cos (n\sigma), \quad \mu = 0, 1, \ldots, p, \\
X^i(\sigma) &= 2 \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \sin (n\sigma), \quad i = p + 1, \ldots, d.
\end{align*}
\]

The string coordinates tangential to the \( Dp \)-branes satisfy the Neumann boundary condition, and the string coordinates orthogonal to the \( Dp \)-branes satisfy the Dirichlet boundary condition. (See Fig. 1) The string field \( \Psi \) may carry the group indices:

\[
\Psi[X] = \frac{1}{\sqrt{2}} \Psi^0[X] + \Psi^a[X] T^a, \quad a = 1, \ldots, N^2 - 1,
\]

where \( \Psi^0 \) is the \( U(1) \) component and \( \Psi^a, a = 1, \ldots, N^2 - 1 \) are the \( SU(N) \) components. If we introduced three local coordinate patches, which describe propagation of three open strings, we might have depicted the string world-sheet of three-open-string interaction by Fig. 2.

To evaluate the Polyakov string path integral with temporal boundary on the string world-sheet, we need a Green’s function for the coordinate fields \( X^I, I = 0, \ldots, d - 1 \). However, it is difficult to find a Green’s function on the string world-sheet directly. Therefore, we map the string world-sheet to the upper half complex plane, where Green’s function is simple. First, we map the world-sheet onto a unit disk by a conformal transformation:

\[
\begin{align*}
\omega_1 &= e^{2\pi i} \left( \frac{1 + ie^{\xi_1}}{1 - ie^{\xi_1}} \right)^{\frac{2}{3}}, \\
\omega_2 &= \left( \frac{1 + ie^{\xi_2}}{1 - ie^{\xi_2}} \right)^{\frac{2}{3}}, \\
\omega_3 &= e^{-2\pi i} \left( \frac{1 + ie^{\xi_3}}{1 - ie^{\xi_3}} \right)^{\frac{2}{3}}.
\end{align*}
\]
where the local coordinates on the three patches are given as $\zeta_r = \xi_r + i \eta_r$, $r = 1, 2, 3$. Figure 3 Depicts the three-open-string world-sheet mapped on a unit disk ($\omega$-plane).

The interaction point $B$, where all three open strings meet, is mapped to the origin of the disk; the external strings are located at $e^{i \frac{\pi}{3}}$, $1$, $e^{-i \frac{\pi}{3}}$. Then, each local coordinate patch on the unit disk is mapped onto the upper half-plane by the following conformal transformation:

$z = -i \frac{\omega_r - 1}{\omega_r + 1}$, \( \frac{\pi}{3} \leq \arg \omega_r \leq \frac{2\pi}{3} \), $r = 1, 2, 3$. (7)

The three-open-string world-sheet mapped on the $z$-plane is described by Fig. 4 [?]. The external strings are mapped to three points on the real line

$Z_1 = \sqrt{3}$, $Z_2 = 0$, $Z_3 = -\sqrt{3}$. (8)

Having mapped the world-sheet of three strings onto the upper half-plane, we can adopt the well-known Green's functions on the upper half complex plane,

$G_N(z, z') = \ln |z - z'| + \ln |z - z'|^*$, for Neumann boundary condition,

$G_D(z, z') = \ln |z - z'| - \ln |z - z'|^*$, for Dirichlet boundary condition. (9)

In order to illustrate the procedure we shall follow, we consider the free string case first. The world-sheet of the free open string is an infinite strip, which is mapped onto the upper half plane by a simple conformal transformation

$\zeta = \ln z$, (10)

where $\zeta = \xi + i \eta$. The Green's function on the upper half complex plane in the Neumann direction (for $X^0$) and the Green's function in the Dirichlet direction (for $X^i$), respectively, are given as follows:
\[ N(\zeta, \zeta') = \ln |e^\zeta - e^{\zeta'}| - \ln |e^\zeta - e^{\zeta''}| \]
\[ = - \sum_{n \geq 1} \frac{2}{n} e^{-n|\zeta - \zeta'|} \cos(m n \rho_r) \cos(m n \rho_s') - 2 \max(\zeta, \zeta') \]
\[ D(\zeta, \zeta') = \ln |e^\zeta - e^{\zeta'}| - \ln |e^\zeta - e^{\zeta''}| \]
\[ = - \sum_{n = 1} \frac{2}{n} e^{-n|\zeta - \zeta'|} \sin m n \sin m n' . \]
\[ (11) \]

In case of interacting strings, the Green’s functions may be written as
\[ N(\rho_s, \rho_s') = \ln |z - z'| + \ln |z - z''| \]
\[ = - \delta_{\rho} \left\{ \sum_{n \geq 1} \frac{2}{n} e^{-n|\zeta - \zeta'|} \cos(m n \rho_r) \cos(m n \rho_s') - 2 \max(\zeta, \zeta') \right\} \]
\[ + 2 \sum_{n, m \geq 0} \tilde{N}^m_{nm} e^{m z_r} \cos(m n \rho_r) \sin(m n \rho_s') \]
\[ (12a) \]
\[ = - \delta_{\rho} \left\{ \sum_{n \geq 1} \frac{2}{n} e^{-n|\zeta - \zeta'|} \sin(m n \rho_r) \sin(m n \rho_s') \right\} \]
\[ + 2 \sum_{n, m \geq 0} \tilde{N}^m_{nm} e^{m z_r} \sin(m n \rho_r) \sin(m n \rho_s') \]
\[ (12b) \]

where \( \rho_s \) and \( \rho_s' \) lie in the region of the \( r \)-th and \( s \)-th local patches respectively. The general formula for the Neumann functions \( \tilde{N}^m_{nm} \) is given in the Appendix.

Taking the limit \( \zeta' \to Z_s \) (\( \eta_s \to -\infty \)) or \( \zeta' \to Z_r \) (\( \eta_r \to -\infty \)) of Eq. (12b), we find
\[ \tilde{F}_{n0} = 0, \quad \tilde{F}_{n0} = 0, \quad \text{for } n \geq 0. \]
\[ (13) \]
By differentiating Eq. (12b) with respect to \( \zeta_r \), we obtain
\[ \tilde{F}_{nm} = - \frac{1}{nm} \int_{z_r} dx_r \int_{z_r} \frac{dz_r}{2 \pi i} \frac{1}{(z - z')^2} e^{-m z_r} e^{m z_r} (z - z') e^{m z_r} \]
\[ (14) \]
Hence, it turns out that
\[ \tilde{F}_{nm} = - \tilde{N}^m_{nm} \quad \text{for } n, m \geq 1. \]
\[ (15) \]

It is interesting that we need only calculate the Neumann functions to construct the Fock space representations of the multi-string vertices on \( Dp \)-branes. Putting all this together, we may write the Fock space representation of the three-string vertex in terms of the Neumann function as
\[ e^{Z} = \operatorname{exp} \left\{ \sum_{\rho_s} \ln a_{\rho_s} \left( \frac{Z_{\rho_s}}{Z_{\rho_s} - \frac{1}{2}} \right) \prod_{\rho_s' \neq \rho_s} \left| Z_{\rho_s} - Z_{\rho_s'} \right| \right\} \]
\[ \times \left[ \left( \sum_{n \geq 1} \frac{2}{n} e^{-n|\zeta - \zeta'|} \cos(m n \rho_r) \cos(m n \rho_s') - 2 \max(\zeta, \zeta') \right) \right. \]
\[ \left. + 2 \sum_{n, m \geq 0} \tilde{N}^m_{nm} e^{m z_r} \cos(m n \rho_r) \sin(m n \rho_s') \right\} \]
\[ (16) \]

where the Schwarz–Christoffel (SC) map on the \( r \)-th local coordinate patch is defined as a series expansion
\[ e^{-Z} = \frac{a_r}{(Z_r - Z_r)^3} + \sum_{n=0} e^{n(\zeta_r - Z_r)\rho_r} . \]
\[ (17) \]

### 3 One-string vertex for string field theory on multiple \( Dp \)-branes

The identity functional \( I \) for the closed string with respect to \( \star \) may be given by an overlapping delta functional as
\[ I[X(\rho)] = \langle X(\rho) | I \rangle = \prod_{0 \leq \sigma \leq \pi} \delta(X(\alpha) - X(\beta)) \]
\[ (18) \]

This defines the one-string vertex operator. A pictorial representation of the overlapping delta functional is presented in figure.

We can choose the SC mapping from the string worldsheet to a unit disk on the \( \omega \)-plane as
\[ \omega = \left( \frac{1 + i e^\zeta}{1 - i e^\zeta} \right)^2, \quad 0 \leq \zeta \leq \pi . \]
\[ (19) \]

On the \( \omega \)-plane, the external string is located at \( \omega = 1 \). The disk can be mapped onto the \( z \)-complex plane by a well-known conformal mapping
\[ z = -i \frac{\omega - 1}{\omega + 1} = -i \left( \frac{1 + i e^\zeta}{1 - i e^\zeta} \right)^2 . \]
\[ (20) \]

The external string is mapped onto \( Z = 0 \) on the \( z \)-complex plane. It describes an open string, propagating with one end sealed by the overlapping condition. (See Fig. 5) Thanks to momentum conservation, component fields do not carry a momentum.

The Fock space representation of the one-closed-string identity (vertex) is obtained from expansion of \( e^{-Z} \) around \( Z = 0 \)
\[ e^{-Z} = -i \left( \frac{1 + \omega_Z}{1 - \omega_Z} \right) \]
\[ = \frac{2}{Z} + \frac{Z}{2} + \frac{Z^3}{8} + \frac{Z^5}{16} + \frac{5Z^7}{128} + \frac{7Z^9}{256} + O(Z^{11}) \]
\[ (21) \]

The Fock representation of the one-string vertex follows from the details of calculation given in the Appendix.
\[
|I_1\rangle = \frac{1}{2} \exp \left\{ \sum_{n,m \geq 1} N_{nm} \alpha_n^\dagger \cdot \alpha_m^\dagger - \sum_{n,m \geq 1} \tilde{N}_{nm} \alpha_n^\dagger \cdot \eta_m \right\}
\]

where
\[
N_{00} = \ln 2, \quad N_{10} = 0, \quad N_{20} = 1,
\]
\[
\tilde{N}_{00} = 0, \quad \tilde{N}_{40} = \frac{1}{2}, \quad \tilde{N}_{50} = 0, \ldots,
\]
\[
\tilde{N}_{11} = 1, \quad \tilde{N}_{12} = 0, \quad \tilde{N}_{21} = 0,
\]
\[
\tilde{N}_{22} = 1, \quad \tilde{N}_{13} = 0, \quad \tilde{N}_{23} = 0, \ldots.
\]

4 Two-string vertex for cubic string field theory on multiple Dp-branes

Two-string overlapping, which defines the two-string vertex, may be written as
\[
\langle X^{(1)}, X^{(2)} | I \rangle = \prod_{\xi \in \Sigma} \delta(X^{(1)}(\sigma) - X^{(2)}(\pi - \sigma)) \prod_{\xi \in \Sigma} \delta(X^{(2)}(\sigma) - X^{(1)}(\pi - \sigma)).
\]

The mapping from the world-sheet coordinates \( \zeta_r = \xi_r + i \eta_r \), \( r = 1, 2 \) onto the unit disk is given as follows:
\[
\omega_1 = -i \left( 1 + i e^{\xi_1} \right) = -i \left( 1 - e^{2\xi_1} + 2 e^{\xi_1} \cos \eta_1 \right),
\]
\[
\omega_2 = i \left( 1 + i e^{\xi_2} \right) = i \left( 1 - e^{2\xi_2} + 2 e^{\xi_2} \sin^2 \eta_2 \right).
\]

Two external strings (at asymptotic regions) are located at \(-i\) and \(i\) at the unit disk on the \( \omega \)-complex plane respectively. Since the ranges of local coordinates \( \eta_r, r = 1, 2 \) are confined to \([0, \pi]\), the images of string world trajectories are inside the unit disk:
\[
|\omega_r| = \frac{1 + e^{2\xi_r} - 2 e^{\xi_r} \sin \eta_r}{1 + e^{2\xi_r} + 2 e^{\xi_r} \sin^2 \eta_r}.
\]

This describes a linear coupling of two open strings on Dp-branes. This two-string vertex may be useful to study a linear coupling of open strings on Dp-branes of different kinds (Fig. 6).

Mapping onto the complex plane may be carried out as follows:
\[
\zeta_1 = -i \frac{\omega_1 - 1}{\omega_1 + 1} = \frac{i (1 + i e^{\xi_1})}{1 - i e^{\xi_1}} - 1 = \frac{2}{\zeta_1 + 1} - 1,
\]
\[
\zeta_2 = -i \frac{\omega_2 - 1}{\omega_2 + 1} = -i \frac{i (1 + i e^{\xi_2})}{1 - i e^{\xi_2}} - 1 = \frac{2}{\zeta_2 + 1} - 1.
\]

As in the case of the one-string vertex operator, if the ranges of \( \eta_r, r = 1, 2 \) are limited to \([0, \pi]\), the images of the string world-sheet cover only the upper half complex \( \zeta \)-plane.

The Fock space representation of the two-closed-string (identity) vertex follows from the general expression of the vertex operator, Eq. (16)
\[
|I[2]\rangle = \exp \left\{ \sum_{r,s} \ln 2 \left( \frac{\langle p^{(r)} \rangle^2}{2} - 1 \right) \right\} 2^{\langle p^{(1)} \rangle^2 - 1} \exp \left\{ \sum_{r,s} \left( \sum_{n,m \geq 1} \frac{1}{2} \tilde{N}_{nm} \alpha_n^{(r)\dagger} \cdot \alpha_m^{(s)\dagger} + \sum_{n \geq 1} \tilde{N}_{nm} \alpha_n^{(r)\dagger} \cdot p^{(s)} \right) \right\} \exp \left\{ -\sum_{r,s} \left( \sum_{n,m \geq 1} \frac{1}{2} \tilde{N}_{nm} \alpha_n^{(r)\dagger} \cdot \alpha_m^{(s)\dagger} \eta^{(s)}_{\eta} \right) \right\} |0\rangle.
\]

Here the Neumann functions are explicitly evaluated as
\[ \bar{N}_{00}^{12} = \ln |Z_1 - Z_2| = \ln 2, \quad \bar{N}_{10}^{11} = \bar{N}_{20}^{22} = \ln 2 \]
\[ N_{10}^{11} = -1, \quad N_{20}^{22} = 1, \quad N_{10}^{12} = -1, \quad N_{20}^{21} = 1, \]
\[ N_{10}^{11} = \frac{1}{2}, \quad N_{20}^{22} = \frac{1}{2}, \quad N_{10}^{12} = 1, \quad N_{20}^{21} = 1, \quad N_{30}^{22} = 1. \]

Using Eq. (28) and the general formula for the Neumann function \( N_{nm}^{rs} \), \( n, m \geq 1 \) given by Eq. (81), we are able to calculate \( \bar{N}_{11}^{11} \) for the Neumann function of the two-string vertex, which we will need shortly:

\[ \bar{N}_{11}^{11} = \oint_{Z_1} \frac{dz_1}{2\pi i} \oint_{Z_2} \frac{dz_2}{2\pi i} \frac{1}{(z_1 - z_2)^2} \left( \frac{2}{z_1 + 1} - 1 \right) \left( \frac{2}{z_2 + 1} - 1 \right) = 0, \] (31a)

\[ \bar{N}_{11}^{12} = \oint_{Z_1} \frac{dz_1}{2\pi i} \oint_{Z_2} \frac{dz_2}{2\pi i} \frac{1}{(z_1 - z_2)^2} \left( \frac{2}{z_1 + 1} - 1 \right) \left( \frac{2}{z_2 - 1} - 1 \right) = \frac{1}{2}, \] (31b)

\[ \bar{N}_{21}^{21} = \oint_{Z_1} \frac{dz_1}{2\pi i} \oint_{Z_2} \frac{dz_2}{2\pi i} \frac{1}{(z_1 - z_2)^2} \left( \frac{2}{z_1 + 1} - 1 \right) \left( \frac{2}{z_2 - 1} - 1 \right) = \frac{1}{2}, \] (31c)

\[ \bar{N}_{22}^{22} = \oint_{Z_1} \frac{dz_1}{2\pi i} \oint_{Z_2} \frac{dz_2}{2\pi i} \frac{1}{(z_1 - z_2)^2} \left( \frac{2}{z_1 + 1} - 1 \right) \left( \frac{2}{z_2 - 1} - 1 \right) = 0. \] (31d)

We can make use of the two-string vertex to evaluate the quadratic term in the string field action in terms of component fields. Let us choose the external string state to calculate the string field action in the low energy region as follows:

\[ \langle \psi^{(1)}, \psi^{(2)} \rangle = \left\langle 0 \prod_{r=1}^{2} \left( A_{r} p^{(r)} d_{i}^{(r)} + \varphi_{r} p^{(r)} d_{i}^{(r)} \right) \eta^{i} \right\rangle \]

\[ = \left\langle 0 \prod_{r=1}^{2} \left( A_{r} + \varphi_{r} \right) \right\rangle. \] (32)

The quadratic terms in the string field action may be written in the low energy region as

\[ S_{2} = g_{2} \left( \int \prod_{r=1}^{2} \bar{p}^{(r)} \left( \sum_{r=1}^{2} \bar{p}^{(r)} \right) \right) \left\langle 0 \prod_{r=1}^{2} \left( A_{r} + \varphi_{r} \right) \right\rangle \left| I_{2} \right| \]

\[ = g_{2} \left( \int \prod_{r=1}^{2} \bar{p}^{(r)} \left( \sum_{r=1}^{2} \bar{p}^{(r)} \right) \right) \left\langle 0 \left( A_{1} A_{2} + A_{1} \varphi_{2} \right) \right\rangle \left| I_{2} \right| \]

The quadratic terms in the gauge field may be read as

\[ g_{2} \left( \int \prod_{r=1}^{2} \bar{p}^{(r)} \left( \sum_{r=1}^{2} \bar{p}^{(r)} \right) \right) \left\langle 0 \left( A_{1} A_{2} + A_{1} \varphi_{2} \right) \right\rangle \left| I_{2} \right| \]

As we expect, in the low energy region, the quartic string field term yields a mass term for the gauge field. The gauge-scalar-field linear coupling simply vanishes:

\[ \left\langle 0 \left( A_{1} \varphi_{2} + \varphi_{1} A_{2} \right) \right\rangle \left| I_{2} \right| = 0. \] (35)

We anticipate that the quadratic term of the scalar field also reduces to a mass term for the scalar field:

\[ g_{2} \left( \int \prod_{r=1}^{2} \bar{p}^{(r)} \left( \sum_{r=1}^{2} \bar{p}^{(r)} \right) \right) \left\langle 0 \varphi_{1} \varphi_{2} \right\rangle \left| I_{2} \right| \]

\[ = g_{2} \left( \int \prod_{r=1}^{2} \bar{p}^{(r)} \left( \sum_{r=1}^{2} \bar{p}^{(r)} \right) \right) \left\langle 0 \varphi_{1} \varphi_{2} \right\rangle \left| I_{2} \right| \]

\[ = -\frac{g_{2}}{2} \left( \int \prod_{r=1}^{2} \bar{p}^{(r)} \left( \sum_{r=1}^{2} \bar{p}^{(r)} \right) \right) \left\langle 0 \varphi_{1} \varphi_{2} \right\rangle \left| I_{2} \right| \]

However, the signature of the quadratic term of the scalar is negative (if the coupling constants \( g_{2} \) is positive). This implies that the quartic string field term on multiple D-branes gives a negative mass to the scalar fields in the low energy region.
5 Three-string vertex for cubic string field theory on multiple $Dp$-branes

The cubic open string with the star product Eq. (2) defined by Witten is pictorially depicted by Fig. 7. The mapping from the world-sheet coordinates $\xi_r = \xi_i + i \eta_i, r = 1, 2, 3$ to the disk is given by Eq. (6). We employ the following conformal transformation to map the complex $\omega$-plane to the complex $z$-plane:

$$z_r = - i \frac{\omega_r - 1}{\omega_r + 1}, \quad r = 1, 2, 3. \quad (37)$$

The external strings are now located on the real line

$$Z_1 = \sqrt{3}, \quad Z_2 = 0, \quad Z_3 = -\sqrt{3}. \quad (38)$$

Each local coordinate patch is mapped onto the upper half plane for $0 \leq \eta_r \leq \pi, \quad r = 1, 2, 3.$ (See Fig. 4)

The relations between the local coordinates and $z$-complex coordinates may be manifested through expansions of $e^{-z_r}, r = 1, 2, 3$ near $z_r = Z_r$ (at the asymptotic region),

$$e^{-z_r} = \frac{a_r}{(Z_r - Z_r)^2} + \sum_{n=0}^{\infty} c_n^{(r)} (Z_r - Z_r)^n; \nonumber$$

$$a_1 = \frac{8}{3}, \quad a_2 = \frac{2}{3}, \quad a_3 = \frac{8}{3}; \quad (39)$$

$$c_0^{(1)} = \frac{2 \sqrt{3}}{3}, \quad c_1^{(1)} = \frac{5}{72}, \quad c_2^{(1)} = \frac{5 \sqrt{3}}{288}; \quad (39)$$

$$c_0^{(2)} = 0, \quad c_1^{(2)} = \frac{5}{18}, \quad c_2^{(2)} = 0; \quad (39)$$

$$c_0^{(3)} = - \frac{2 \sqrt{3}}{3}, \quad c_1^{(3)} = - \frac{5}{72}, \quad c_2^{(3)} = \frac{-5 \sqrt{3}}{288}. \quad (39)$$

These explicit expressions of expansions are useful for calculating the Neumann functions.

Calculating them explicitly involves some algebra. We evaluate Neumann functions of $\hat{N}_{00}$ using Eq. (39) and Eq. (67) given in the Appendix:

$$\hat{N}_{00}^{11} = \ln \frac{8}{3}, \quad \hat{N}_{00}^{22} = \ln \frac{2}{3}, \quad \hat{N}_{00}^{33} = \ln \frac{8}{3}, \quad (40)$$

$$\hat{N}_{00}^{12} = \frac{1}{2} \ln 3, \quad \hat{N}_{00}^{23} = \frac{1}{2} \ln 3, \quad \hat{N}_{00}^{31} = \ln 2 \sqrt{3}. \quad (40)$$

The Neumann functions of $\hat{N}_{10}$ follow from Eq. (39) and Eq. (72):

$$\hat{N}_{10}^{rs} = c_0^{(r s)}, \quad \hat{N}_{10}^{rs} = \frac{a_r}{(Z_r - Z_s)}, \quad r \neq s. \quad (41)$$

Spelling these out, we obtain:

$$\hat{N}_{10}^{11} = \frac{2 \sqrt{3}}{3}, \quad \hat{N}_{10}^{22} = 0, \quad \hat{N}_{10}^{33} = \frac{-2 \sqrt{3}}{3}, \quad (42)$$

$$\hat{N}_{10}^{12} = \frac{8}{3 \sqrt{3}}, \quad \hat{N}_{10}^{13} = \frac{-4}{3 \sqrt{3}}, \quad \hat{N}_{10}^{21} = \frac{-2}{3 \sqrt{3}}, \quad (42)$$

The Neumann functions of $\hat{N}_{0n}$ may be obtained from the general formula Eq. (81) for $\hat{N}_{mm}$ given in the Appendix:

$$\hat{N}_{0n}^{rs} = \frac{1}{nm} \oint_{Z_r} \oint_{Z_s} \frac{dz_r}{2 \pi i} \frac{dz_s}{2 \pi i} \frac{1}{(z_r - z_s)^2} e^{-m \delta_i (z_r - z_s) \delta_j (z_i)}, \quad n, m \geq 1 \quad (43)$$

For the case with $n = 1$ and $m = 1$:

$$\hat{N}_{11}^{rs} = \oint_{Z_r} \oint_{Z_s} \frac{dz_r}{2 \pi i} \frac{dz_s}{2 \pi i} \frac{1}{(z_r - z_s)^2} \left( \frac{a_r}{z_r - Z_r} \right) \left( \frac{a_s}{z_s - Z_s} \right). \quad (44)$$

When $r \neq s,$

$$\hat{N}_{11}^{rs} = \frac{a_r a_s}{(Z_r - Z_s)^2} = \frac{2^4}{3}, \quad \text{for} \quad r, s = 1, 2, 3. \quad (45)$$

When $r = s$, we may write

$$\hat{N}_{11}^{rr} = \oint_{Z_r} \oint_{Z_r} \frac{dz_r}{2 \pi i} \frac{dz_r}{2 \pi i} \frac{1}{(z_r - z_r)^2} \left( \frac{a_r}{z - Z_r} + \sum_{m=0}^{\infty} c_m^{(r)} (z_r - Z_r)^m \right)$$

$$= a_r c_1^{(r)} \quad (46)$$

Using Eq. (39) and

$$c_1^{(1)} = - \frac{5}{72}, \quad c_1^{(2)} = - \frac{5}{18}, \quad c_1^{(3)} = - \frac{5}{72} \quad (47)$$

we find

$$\hat{N}_{11}^{11} = \hat{N}_{11}^{22} = \hat{N}_{11}^{33} = - \frac{5}{27}. \quad (48)$$

With the Neumann functions for three closed strings Eq. (40), Eq. (42), and Eq. (81), the three-closed-string vertex operator may be expressed as
\[ E[1, 2, 3]|0\rangle = \exp\left\{ \sum_{r=1}^{3} \ln \frac{8}{3} \left( \frac{(p^{(r)})^2}{2} - 1 \right) \right\} \prod_{r=1}^{3} [Z_r - Z_r]^{p^{(r)} - p^{(r)}} \]

\[ \exp\left\{ \sum_{r, s} \frac{1}{2} \sum_{n, m \geq 2} N_{nm}^{rs} a_n^{(r)\dagger} a_m^{(s)\dagger} + \sum_{n \geq 1} N_{n0}^{rs} a_n^{(r)\dagger} \right\} \]

\[ \exp\left\{ - \sum_{r, s} \frac{1}{2} \sum_{n, m \geq 1} N_{nm}^{rs} a_n^{(r)\dagger} a_m^{(s)\dagger} \eta^{(r)} \right\} |0\rangle \]

(49)

6 Cubic string field theory in the zero-slope limit and matrix models

The three-string interaction may be written as

\[ S_{[3]} = \frac{2g}{3} \int \prod_{r=1}^{3} dp^{(r)} \delta \left( \sum_{r=1}^{3} p^{(r)} \right) \langle \Psi_1, \Psi_2, \Psi_3 | E[1, 2, 3]|0\rangle. \]

(50)

In the low energy regime (or in the zero-slope limit), the external string states correspond to massless gauge fields \( A^a \) or massless scalar fields \( \phi^i \). By choosing the external string state as follows:

\[ \langle \psi^{(1)}, \psi^{(2)}, \psi^{(3)} | ] = \left( 0 \prod_{r=1}^{3} (A(1)A(2)A(3)E[1, 2, 3]|0\rangle \right) \]

\[ = \left( 0 \prod_{r=1}^{3} (A^{(r)} + \varphi^{(r)}) \right)^3, \]

(51)

We can evaluate the effective interaction between the gauge fields \( A^\mu \) and the scalar fields \( \phi^i \), which describes the three-string interaction Eq. (49) and Eq. (50) in the zero-slope limit:

\[ S_{[3]} = \frac{2g}{3} \int \prod_{r=1}^{3} dp^{(r)} \delta \left( \sum_{r=1}^{3} p^{(r)} \right) \]

\[ \text{tr} \left( 0 \prod_{r=1}^{3} (A^{(r)} + \varphi^{(r)})^3 E[1, 2, 3]|0\rangle \right) \]

\[ = \frac{2g}{3} \int \prod_{r=1}^{3} dp^{(r)} \delta \left( \sum_{r=1}^{3} p^{(r)} \right) \text{tr} \left( 0 \prod_{r=1}^{3} (A^{(r)} + \varphi^{(r)}) \right) \]

\[ = g_{YM} \int \prod_{r=1}^{3} dp^{(r)} \delta \left( \sum_{r=1}^{3} p^{(r)} \right) \text{tr} (A(1)A(2)A(3)) \]

\[ = -g_{YM} \int dp^{1\cdot \times \times} i \text{tr} ( \partial_\mu A_\nu - \partial_\nu A_\mu ) \]

[\( A^\mu, A^\nu \)]

(52)

Note that the \( \phi^3 \) term vanishes because the vertex operator generates only even powers of \( a_n^{(r)\dagger} \):

\[ \text{tr} \left( 0 | \phi(1) \phi(2) \phi(3) | 0 \right) = 0. \]

(53)

6.1 Cubic gauge field interaction

The cubic gauge field interaction term can be written as

\[ S_{AAA} = \frac{2g}{3} \int \prod_{r=1}^{3} dp^{(r)} \delta \left( \sum_{r=1}^{3} p^{(r)} \right) \]

\[ \text{tr} \left( 0 | (A(1)A(2)A(3)E[1, 2, 3]|0\rangle \right) \]

\[ = \frac{2g}{3} \int \prod_{r=1}^{3} dp^{(r)} \delta \left( \sum_{r=1}^{3} p^{(r)} \right) \text{tr} \left( 0 \prod_{r=1}^{3} (A^{(r)} + \varphi^{(r)}) \right) \]

\[ = \left( \frac{2}{3} \right)^{\frac{5}{3}} g \int \prod_{r=1}^{3} dp^{(r)} \delta \left( \sum_{r=1}^{3} p^{(r)} \right) \eta^{\mu
nu} \left( p^{(1)} - p^{(2)} \right) \]

\[ \text{tr} (A(1)A(2)A(3)) \]

\[ = g_{YM} \int \prod_{r=1}^{3} dp^{(r)} \delta \left( \sum_{r=1}^{3} p^{(r)} \right) \rho^{(1\mu\nu)} \text{tr} (A(1)^\dagger A(2)A(3)) \]

\[ = -g_{YM} \int dp^{1\cdot \times \times} i \text{tr} ( \partial_\mu A_\nu - \partial_\nu A_\mu ) \]

[\( A^\mu, A^\nu \)]

(54)

6.2 Gauge-scalar field interaction

We may write the gauge-scalar cubic interaction term as follows:
\[ S_{\text{App}} = \frac{2g}{3} \int \prod_{r=1}^{3} dp^{(r)} \delta \left( \sum_{r=1}^{3} p^{(r)} \right) \text{tr} \left\{ \langle A(1) \varphi(2) \varphi(3) \right. \\
+ \varphi(1) A(2) \varphi(3) + \varphi(1) A(2) A(3) \langle E[1, 2, 3] | 0 \rangle \left. \right\} \\
= \frac{2g}{3} \int \prod_{r=1}^{3} dp^{(r)} \delta \left( \sum_{r=1}^{3} p^{(r)} \right) \\
\text{tr} \left\{ \left( A(1) \mu A(2) \varphi(3) + A(1) \varphi(2) A(3) \right) \langle E[1, 2, 3] | 0 \rangle \right\} \\
= \frac{2g}{3} \left( \frac{1}{2} \right)^{24} \frac{24}{3} \int \prod_{r=1}^{3} dp^{(r)} \delta \left( \sum_{r=1}^{3} p^{(r)} \right) \\
\left\{ \varphi(2) \varphi(3) A(1) \cdot \sum_{s} N_{10}^{rs} p^{(s)} \\
+ \varphi(1) A(2) \varphi(3) \cdot \sum_{s} N_{10}^{ss} p^{(s)} \\
+ \varphi(1) A(2) A(3) \cdot \sum_{s} N_{10}^{ss} p^{(s)} \right\} \\
= \frac{2g}{3} \left( \frac{1}{2} \right)^{24} \frac{24}{3} \int \prod_{r=1}^{3} dp^{(r)} \delta \left( \sum_{r=1}^{3} p^{(r)} \right) \\
\left\{ \varphi(2) \varphi(3) A(1) \cdot \frac{4}{3 \sqrt{3}} (p^{(2)} - p^{(1)}) \\
+ \varphi(1) A(2) \varphi(3) \cdot \frac{2}{3 \sqrt{3}} (p^{(3)} - p^{(1)}) \\
+ \varphi(1) A(2) A(3) \cdot \frac{4}{3 \sqrt{3}} (p^{(3)} - p^{(2)}) \right\} \\
= g_{YM} \int \prod_{r=1}^{3} dp^{(r)} \delta \left( \sum_{r=1}^{3} p^{(r)} \right) p^{(1)} \mu \\
\text{tr} \left( \varphi(1) [A(3)_{\mu}, \varphi(2)] \right) \right\} \\
= -g_{YM} \int dp^{+1} x i \text{tr} \left. \partial_{\mu} \varphi_{1} [A_{\mu}, \varphi] \right] \right\} \\
(55) \\
\]
world-sheet of four strings, which has two conical singular points, to the upper half complex plane. One plausible proposal would be to adopt deformation of the string world-sheet such as the four-string scattering diagram in the proper-time gauge [16]: If we deform the four-string scattering diagram to that in the proper-time gauge, the string world-sheet becomes planar, whereby the Mandelstam technique to construct a vertex operator is readily applicable.

This work can be extended in diverse directions: We may define cubic string field theory on more complex D-brane systems such as parallel D-branes (separated) [52], D1-D5 systems [53], and Non-BPS D-branes [54]. Therefore, string field theory can be used as a useful tool for exploring important subjects associated with these D-brane systems, including tachyon condensation.

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1. Integral formulas for \( \mathcal{N}_{nm}^{rs} \)

- \( \mathcal{N}_{00}^{rs} \) for \( r \neq s \):
  Taking \( z' \to Z_r \), \( (\xi' \to -\infty) \),
  \[
  N(\rho, \rho') = 2 \sum_{n \geq 0} \mathcal{N}_{nn}^{rs} e^{n \xi} \cos(n \eta) = 2 \ln |z - Z_r|.
  \] (60)

  Taking the limit where \( z \to Z_r \), \( (\xi \to -\infty) \)
  \[
  N(\rho, \rho') = 2 \mathcal{N}_{00}^{rs} = 2 \ln |Z_r - Z_s|.
  \] (61)

  Thus,
  \[
  \mathcal{N}_{00}^{rs} = |Z_r - Z_s|, \quad \text{for} \quad r \neq s.
  \] (62)

- \( \mathcal{N}_{00}^{rs} \) for \( r = s \):
  If we take the limit \( z' \to Z_r \) of Eq. (56),
  \[
  N(\rho, \rho') = 2 \mathcal{E}_{r}^{rs} + 2 \sum_{n \geq 0} \mathcal{N}_{nn}^{rs} e^{n \xi} \cos(n \eta) = 2 \ln |z - Z_r|.
  \] (63)

  Taking the limit \( z \to Z_r \) of Eq. (63) again, we find
  \[
  \mathcal{N}_{00}^{rs} = \ln |z_r - Z_r| - \xi_r.
  \] (64)

  Since near \( z_r = Z_r \), we expand \( e^{-\xi} \)
  \[
  e^{-\xi} = \frac{a_r}{(z_r - Z_r) + \sum_{n=0} \mathcal{E}_{n}^{(r)} (z_r - Z_r)^n}.
  \] (65)

  It follows that in the limit \( z_r \to Z_r \)
  \[
  \xi_r = \ln |z_r - Z_r| - \ln a_r.
  \] (66)

  Using Eq. (65) and Eq. (66) in the limit, where \( z_r \to Z_r \), we obtain
  \[
  \mathcal{N}_{00}^{rs} = \ln a_r.
  \] (67)

- \( \mathcal{N}_{nm}^{rs} \), \( n \geq 1 \): Differentiating Eq. (57) with respect to \( \xi_r \) (assume \( \xi_r \geq \xi' \)),
  \[
  \frac{\partial}{\partial \xi_r} N(\rho, \rho') = \frac{1}{2} \left( \frac{\partial \xi_r}{\partial \xi_r} \right) \left( \frac{1}{z - z'} + \frac{1}{z - \xi_r} \right)
  \] (68)

  \[
  = \mathcal{E}_{rs} \left\{ \frac{1}{2} \sum_{n \geq 0} \omega_r^n (\omega_r^{s*} + \omega_s^{r*}) + 1 \right\}
  \] (68)

  \[
  + \frac{1}{2} \sum_{n, m \geq 0} n \mathcal{N}_{nm}^{rs} \omega_r^n (\omega_s^m + \omega_s^{r*} m)
  \] (68)

  where we use
  \[
  \frac{\partial}{\partial \xi_r} \omega_r = \omega_r \frac{\partial}{\partial \omega_r} = \frac{1}{2} \left( \frac{\partial}{\partial \xi_r} - i \frac{\partial}{\partial \eta_r} \right).
  \] (69)

  Taking the limit, \( z' \to Z_r \) (\( \omega_r \to 0 \)).
\[
\delta s + \sum_{n \geq 1} n\tilde{N}_{ns}^{\omega r} = \left( \frac{\partial z}{\partial \xi r} \right) \frac{1}{z - Z_s} \quad (70)
\]

By evaluating the contour integral around \(\omega r = 0\) (\(z = Z_s\)),
\[
\oint_{\omega r = 0} d\omega r \omega r^{-n-1} \left( \delta s + \sum_{m \geq 1} \tilde{N}_{sm}^{\omega r} \omega m \right)
\]
\[
= \oint_{\omega r = 0} d\omega r \omega r^{-n-1} \left( \frac{\partial z}{\partial \xi r} \right) \frac{1}{z - Z_s}
\]
we obtain
\[
\tilde{N}_{rn}^{\omega r} = \tilde{N}_{rn}^{\omega r} = \frac{1}{n} \oint_{Z_r} \frac{dz}{2\pi i z - Z_s} e^{-m_{rs}(z)} , \quad n \geq 1. \quad (72)
\]

Here we use \(d\omega r = \omega r d\xi r\), and
\[
\frac{d\omega r}{\omega r} \frac{d\zeta}{\partial \xi r} = d\xi r \frac{d\zeta}{d\xi r} = d\zeta. \quad (73)
\]

- \(\tilde{N}_{rn}^{\omega r}, n \geq 1\): Differentiating Eq. (57) with respect to \(\xi r\)

\[
\frac{\partial}{\partial \xi r} N(\rho r, \rho' r) = -\frac{1}{2} \left( \frac{\partial \zeta}{\partial \xi r} \right) \left( \frac{1}{z - z'} + \frac{1}{z' - z''} \right)
\]
\[
= \delta s \left\{ \frac{1}{2} \sum_{n \geq 1} \omega r^{-n} (\omega r + \omega r^{*n}) + 1 \right\} + \frac{1}{2} \sum_{n,m \geq 0} m\tilde{N}_{nm}^{\omega r} \omega m (\omega r + \omega r^{*n})
\]

Here we make use of
\[
\frac{\partial}{\partial \zeta} = \omega r' \frac{\partial}{\partial \omega r} = \frac{1}{2} \left( \frac{\partial}{\partial \xi r} - i \frac{\partial}{\partial \eta r} \right). \quad (75)
\]

Taking the limit \(z \to Z_s (\omega r \to 0)\),
\[
\delta s + \sum_{m \geq 1} m\tilde{N}_{rm}^{\omega r} = \left( \frac{\partial z}{\partial \xi r} \right) \frac{1}{z' - Z_s} \quad (76)
\]

By evaluating the contour integral around \(\omega r = 0 (z' = Z_s)\),
\[
\oint_{\omega r = 0} d\omega r \omega r^{-m-1} \left( \delta s + \sum_{m \geq 1} m\tilde{N}_{sm}^{\omega r} \omega m \right)
\]
\[
= \oint_{\omega r = 0} d\omega r \omega r^{-m-1} \left( \frac{\partial z}{\partial \xi r} \right) \frac{1}{z' - Z_s}
\]
we obtain
\[
\tilde{N}_{rn}^{\omega r} = \frac{1}{m} \oint_{Z_r} \frac{dz'}{2\pi i z' - Z_r} e^{-m_{rs}(z')} , \quad m \geq 1. \quad (78)
\]

Here we use \(d\omega r' = \omega r' d\xi r'\) and
\[
d\omega r \frac{\partial z}{\partial \xi r} = d\xi r \frac{\partial z}{\partial \xi r} = dz'. \quad (79)
\]

- \(\tilde{N}_{rn}^{\omega r}\): Differentiating Eq. (68) with respect to \(\xi r\)

\[
\frac{\partial}{\partial \xi r} \frac{\partial}{\partial \xi r} N(\rho r, \rho' r) = \frac{1}{2} \left( \frac{\partial z}{\partial \xi r} \right) \left( \frac{\partial z}{\partial \xi r} \right) \frac{\partial z}{\partial \xi r} \left( \frac{1}{z - z'} + \frac{1}{z' - z''} \right)
\]
\[
= \frac{1}{2} \left( \frac{\partial z}{\partial \xi r} \right) \left( \frac{\partial z}{\partial \xi r} \right) \left( \frac{1}{(z - z')^2} \right) = \delta s \frac{1}{2} \sum_{n \geq 1} n\omega r^{-n-1} \omega r^{*n} \quad (80)
\]

Then evaluating the contour integral around \(\omega r = 0 (z' = Z_s)\),
\[
\oint_{\omega r = 0} d\omega r' \omega r^{-m-1} \omega r^{*m-1}
\]
we obtain
\[
\tilde{N}_{rn}^{\omega r} = \frac{1}{nm} \oint_{Z_r} \frac{dz'}{2\pi i (z' - z'' - z'')^2} e^{-m_{rs}(z')-m_{rs}(z'')} , \quad n, m \geq 1. \quad (81)
\]

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