Semi-online Scheduling: A Survey

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Abstract. Scheduling of jobs on multiprocessing systems has been studied extensively since last five decades in two well defined algorithmic frameworks such as offline and online. In offline setting, all the information on the input jobs are known at the outset. Whereas in online setting, jobs are available one by one and each job must be scheduled irrevocably before the availability of the next job. Semi-online is an intermediate framework to address the practicability of online and offline frameworks. Semi-online scheduling is a relaxed variant of online scheduling, where an additional memory in terms of buffer or an Extra Piece of Information (EPI) is provided along with input data. The EPI may include one or more of the parameter(s) such as size of the largest job, total size of all jobs, arrival sequence of the jobs, optimum makespan value or range of job’s processing time. A semi-online scheduling algorithm was first introduced in 1997 by Kellerer et al. They envisioned semi-online scheduling as a practically significant model and obtained improved results for 2-identical machine setting. This paper surveys scholarly contributions in the design of semi-online scheduling algorithms in various parallel machine models such as identical and uniformly related by considering job’s processing formats such as preemptive and non-preemptive with the optimality criteria such as $Min-Max$ and $Max-Min$. The main focus is to present state of the art competitive analysis results of well-known semi-online scheduling algorithms in a chronological overview. The survey first introduces the online and semi-online algorithmic frameworks for the multi-processor scheduling problem with important applications and research motivation, outlines a general taxonomy for semi-online scheduling. Fifteen well-known semi-online scheduling algorithms are stated. Important competitive analysis results are presented in a chronological way by highlighting the critical ideas and intuition behind the results. An evolution time-line of semi-online scheduling setups and a classification of the references based on EPI are outlined. Finally, the survey concludes with the exploration of some of the interesting research challenges and open problems.

1 Introduction

Scheduling deals with allocation of resources to jobs in some order with application specific objectives and constraints. The concept of scheduling was introduced to address the following research question [1]: Given a list of $n$ jobs and
m(\geq 2) \) machines, what can be a sequence of executing the jobs on the machines such that all jobs are finished by latest time possible? Scheduling has now become ubiquitous in the sense that it inherently appears in all facets of daily life. Everyday, we involve ourselves in essential activities such as scheduling of meetings, setting of deadlines for projects, scheduling the maintenance periods of various tools, planning and management of events, allocating lecture halls to various courses, organizing vacations, work periods and academic curriculum etc. Scheduling finds practical applications in broad domains of computers, operations research, production, manufacturing, medical, transport and industries [17]. Widespread applicability has made scheduling an exciting area of investigation across all domains.

Scheduling of jobs on multiprocessing systems has been studied extensively over the years in well defined algorithmic frameworks of offline and online scheduling [6, 16, 17, 41, 49]. A common consideration in offline scheduling is that all information about the input jobs are known at the outset. However, in most of the current practical applications, jobs are given incrementally one by one. An irrevocable scheduling decision must be made upon receiving a job with no prior information on successive jobs [2, 12]. Scheduling in such applications is known as online scheduling. In this survey, we study a relaxed variant of online scheduling, known as semi-online scheduling, where some extra piece of information about the future jobs are known at the outset. We present the structure and organization of our survey in Figure 1.

Fig. 1. Organization of our Survey
1.1 Algorithmic Frameworks

We present three algorithmic frameworks such as offline, online and semi-online based on availability of input information in processing of a computational problem as shown in Figure 2.

- In **Offline framework**, complete input information is known at the outset. Let us consider a set \( I = \{i_1, i_2, \ldots, i_n\} \) representing all inputs of a computational problem \( X \). In offline framework, \( I \) is known prior to construct a solution for \( X \). The algorithm designed for computation of \( I \) in the offline framework is known as **offline algorithm**. An offline algorithm processes all inputs \( I \) simultaneously to produce the final output \( o \).

- In **Online framework**, the inputs are given one by one in order. Each available input must be processed immediately with no information on the successive inputs. In online framework, at the time step \( t \), the input sequence \( I_t: <i_1, i_2, \ldots, i_{t-1}, i_t> \) is known and must be processed irrevocably with no information on future input sequence \( <i_{t+1}, \ldots, i_{n-1}, i_n> \), where \( t \geq 1 \). The algorithm designed for computation of \( I \) in the online framework is known as **online algorithm**. An online algorithm produces a partial output \( o_t \) for each input \( I_t \) on the fly, where \( 1 \leq t \leq n - 1 \) before producing the final output \( o_n \).

- In **Semi-Online framework**, inputs are given one by one like online framework along with some **Extra Piece of Information (EPI)** on future inputs. At any time step \( t \), a semi-online algorithm receives input sequence \( I_t \) with an EPI and processes them irrevocably to obtain a partial output \( o_t \) on the fly, where \( 1 \leq t \leq n - 1 \) before producing the final output \( o_n \).

Semi-online is an intermediate framework to address the practicability and limitations of online and offline frameworks. In most of the current practical sce-
narios, neither all the inputs are available at the beginning nor the inputs occur exclusively in online fashion, but may occur one by one with additional information on the successive inputs. For example, an online video on demand application receives requests for downloading video files on the fly, however, it knows the highly requested video file and the largest video file among all video files before processing the current request [79]. A related model to semi-online framework is the advice model, where the EPI has been referred to as bits of advice. A comprehensive survey on advice models can be found in [92].

1.2 Semi-online Scheduling Problem

Semi-online scheduling [13] is a variant of online scheduling with an EPI on future jobs or with additional algorithmic extensions by allowing two parallel policies to operate on each incoming job. It may also include a buffer of finite length for pre-processing of a newly arrived job before the actual assignment. We now formally define the semi-online scheduling problem by presenting inputs, constraints and output as follows.

- **Inputs:**
  - A sequence $J :< J_1, J_2, ..., J_n >$ of $n$ jobs with corresponding processing time of $p_i$, where $1 \leq i \leq n$ and $p_i > 0$ are revealed one by one for processing on a list $M=(M_1, M_2, ..., M_m)$ of $m$ parallel machines, where $m \geq 2$ and $n >> m$.
  - An EPI such as arrival order of the jobs or largest processing time or upper and lower bounds on the processing time of the incoming jobs is given a priori.

- **Constraints:**
  - Each incoming job $J_i$ must be assigned irrevocably to one of the machines $M_j$ as soon as $J_i$ is given.
  - Jobs are non-preemptive, however the preemptive variant of the problem supports job splitting to execute distinct pieces of a job at non-overlapping time spans on the same or different machines.

- **Output:** Generation of a schedule, representing assignment of all jobs over $m$ machines.(we shall discuss about the output parameters and objectives in section 2.3).

1.3 Practical Applications

Here, we discuss some of the important applications, where semi-online scheduling serves as a major algorithmic framework.

- **Resource Management in Operating System** [2]: In a multi-user, time-shared operating system, it is not known at the outset the sequence of jobs or the number of jobs that would be submitted to the system. Here, jobs are given to the scheduler over time. However, it is the inherent property of the scheduler to make an educated guess about the maximum and minimum time
required to complete a resident job. The objective is to irrevocably assign the
required computer resources such as memory, processors immediately upon
the availability of a job to attain a minimum completion time.

- **Distributed Data Management** [15]: Distributed and parallel systems
  often confronted to store files of varying sizes on limited capacity remote
  servers. It is evident that files are submitted from a known source on the
  fly and each received file must be assigned immediately to one of the re-
  mote servers. The central scheduler of the system is handicapped about the
  successive submissions prior to make an irrevocable assignment. However,
  it is known for an instance that the submitting source stored the files in
  \( k \) unit capacity servers, which provides a hint for the total size of files
  to be received. The challenge is to store the files on the remote servers with
  minimum storage requirement.

- **Server Request Management or Web Caching** [40]: In a client-server
  model, it is not known in advance the number of requests that would be
  submitted to the remote servers nor the time required to process the requests.
  However, the hierarchical organizations of servers can serve as an extra piece
  of information for scheduler to cater different level of services to the requests
  with a broader objective of processing all requests as latest as possible.

- **Production and Manufacturing**: Orders from clients arrive on the fly to
  a production system. The resources such as human beings, machinery equip-
  ment(s) and manufacturing unit(s) must be allocated immediately upon rec-
  eiving each client order with no knowledge on the future orders. However,
  one could estimate the minimum or maximum time required to complete
  the order. Online arrival of the orders have high impact on the renting and
  purchasing of the high cost machines in the manufacturing units.

- **Maintenance and upgrade of industrial tools** [52]: Scheduling of var-
  ious maintenance and operational activities for modular gas turbine aircraft
  engines. The goal is to distribute different activities to the machines in such
  a way that the loads of the the machines will be balanced. The common
  practice is to maximize load of the least loaded machine.

### 1.4 Performance Measure for Semi-online Scheduling

Traditional techniques [17] for analyzing the performance of offline scheduling
algorithms are largely relied on the entire job sequence, therefore are insignifi-

cant in the performance evaluation of semi-online algorithms, which operate on
single incoming input at any given time step with minimal knowledge on the
future arrivals.

**Competitive analysis** method [8] measures the worst-case performance of a
semi-online algorithm \( ALG \) designed either for a cost minimization or maxi-

mization problem by evaluating competitive ratio\( (CR) \). For a cost minimization
problem, \( CR \) is defined as the smallest positive integer \( k(\geq 1) \), such that for all
valid sequences of inputs in the set \( I = \{i_1, i_2, ..., i_n\} \), we have
\( C_{ALG} \leq k \cdot C_{OPT} \),

where \( C_{ALG} \) is the cost obtained by semi-online algorithm \( ALG \) for any sequence
of \( I \) and \( C_{OPT} \) is the optimum cost incurred by the optimal offline algorithm.
for $I$. The \textit{Upper Bound} (UB) on the CR obtained by $ALG$ guarantees the maximum value of CR for all legal sequences of $I$. The \textit{Lower Bound} (LB) on the CR of a semi-online problem $X$ ensures that there exists an instance of $I$ such that any semi-online algorithm $ALG$ must incur a cost $C_{ALG} \geq b \cdot C_{OPT}$, where $b$ is referred to as $LB$ for $X$. The performance of $ALG$ is considered to be \textit{tight}, when $ALG$ ensures no gap between achieved $LB$ and $UB$ for the problem considered. Sometimes, the performance of $ALG$ is referred to as \textit{tight} if $C_{ALG}=k \cdot C_{OPT}$. For a cost maximization problem, CR is defined as the infimum $k$ such that for any valid input sequence of $I$, we have $k \cdot C_{ALG} \geq C_{OPT}$. The objective of a semi-online algorithm is to obtain a CR as closer as possible to 1(strictly greater than or equal to 1).

1.5 Research Motivation

Research in semi-online scheduling has been pioneered by the following non-trivial issues.

– The offline $m$-machine ($m \geq 2$) scheduling problem with makespan minimization objective has been proved to be \textit{NP-complete} by a polynomial time reduction from well-known \textit{Partition} problem \cite{7}. Let us consider an instance of scheduling $n$ jobs on $m$ parallel machines, where $n \gg m$. There are $m^n$ possible assignments of jobs. An optimum schedule can be obtained in worst case with probability $\frac{1}{m}$. Further, unavailability of prior information about the whole set of jobs poses a non-trivial challenge in the design of efficient algorithms for semi-online scheduling problems.

– Given an online scheduling problem considered in the semi-online framework, the non-trivial question raised is: \textit{What can be an additional realistic information on successive jobs that is necessary and sufficient to achieve }1\text{-competitiveness or to beat the best known bounds on the CR?}

– A semi-online algorithm is equivalent to an \textit{online algorithm with advice} in the sense that an \textit{EPI} considered in semi-online model can be encoded into bits of advice. The quantification of information into bits will help in analyzing the advice complexity of a semi-online algorithm. Any advancement in semi-online scheduling may lead to significant improvements in the best known bounds obtained by the advice models.

– Semi-online model is practically significant than the advice model as it considers feasible information on future inputs unlike bits of advice, which sometimes may constitute an unrealistic information.

1.6 Scope and Uniqueness of Our Survey

\textbf{Scope.} This paper surveys scholarly contributions in the design of semi-online scheduling algorithms in various parallel machine models such as identical and uniformly related by considering job’s processing formats such as preemptive
and non-preemptive with the optimality criteria such as Min-Max and Max-Min. The aim of the paper is to record important competitive analysis results with the exploration of novel intuitions and critical ideas in a historical chronological overview.

Uniqueness. This is a comprehensive survey article on semi-online scheduling, which describes the motivation towards semi-online scheduling research, outlines a general taxonomy, states fifteen well-known semi-online scheduling algorithms, presents state of the art contributions, explains critical ideas, overviews important results in a chronological manner, organized by EPI considered in various semi-online scheduling setups. Several non-trivial research challenges and open problems are explored for future research work. Important references are grouped together in a single article to develop basic understanding, systematic study and to update the literature on semi-online scheduling for future investigation.

2 Taxonomy of Semi-Online Scheduling

The basic terminologies, notations and definitions related to semi-online scheduling are presented in Table 1. Based on the literature study, a general taxonomy of semi-online scheduling is outlined using the three parameters(α|β|γ) based framework of Graham et al. [6] in Figure 3. Here, α represents parallel machine models, β specifies different job characteristics and γ represents optimality criteria.

Fig. 3. A General Taxonomy of Semi-Online Scheduling
| Terms          | Notations | Definitions/Descriptions/Formula                                                                 |
|---------------|-----------|--------------------------------------------------------------------------------------------------|
| Job           | $J_i$     | Program under execution, which consists of a finite number of instructions. A job is also referred to as a collection of at least one smallest indivisible sub task called *thread*. Unless specified explicitly, we assume that a job consists of single thread only. Here, we use terms job and task in the same sense. |
| Machine       | $M_j$     | An automated system capable of processing some jobs by following a set of rules. Machine can be a router, web server, robot, industrial tool, processing unit or processor, which is capable of processing the jobs. Here, we use terms machine and processor in the same sense. |
| Processing Time | $p_{ij}$ | Total time of execution of a job $J_i$ on machine $M_j$. For identical machines $p_{ij}=p_i$. |
| Largest Processing Time | $p_{max}$ | $\max\{p_i | 1 \leq i \leq n\}$ |
| Release Time  | $r_i$     | The time at which any job $J_i$ becomes available or ready for processing.                        |
| Completion Time | $c_i$     | The time at which any job $J_i$ finishes its execution                                           |
| Deadline      | $d_i$     | Latest time by which $J_i$ must be finished.                                                     |
| Load          | $l_i$     | Sum of processing times of the jobs that have been assigned to machine $M_j$.                   |
| Speed         | $S_j$     | The number of instructions processed by machine $M_j$ in unit time                              |
| Speed Ratio   | $s$       | The ratio between the speeds of two machines. For 2-machines with speeds 1 and $\frac{1}{s}$ respectively. We have speed ratio $s = \frac{1}{\frac{1}{s}} = S$ |
| Idle Time     | $\varphi$ | The duration of time at which a machine is not processing any task. During the idle time a machine is called idle. |
| Optimal Makespan | $C_{OPT}$ | $C_{OPT} = \max\{p_{max}, \frac{1}{m} \cdot \sum_{i=1}^{n} p_i\}$ |
2.1 Parallel Machine Models (α)

Parallel machine models support simultaneous execution of multiple threads of a single job or a number of jobs on m machines, where \( m \geq 2 \). Semi-online scheduling problem has been studied in parallel machine models such as identical, uniformly related (or related machines in short) and unbounded batch machines. One model differs from another based on its processing power defined in the literature [6] as follows.

- **Identical Machines (P)**: Here, all machines have equal speeds of processing any job \( J_i \). We have \( p_{ij} = p_i, \forall M_j, 1 \leq j \leq m \).
- **Related Machines (Q)**: Here, the machines operate at different speeds. For a machine \( M_j \) with speed \( S_j \), execution time of job \( J_i \) on \( M_j \) is \( p_{ij} = \frac{p_i}{S_j} \).
- **Unbounded Batch Machine (U-batch)**: A batch machine receives jobs in batches, where a batch \( U(t) \) is formed by considering all jobs that are received at time \( t \). The jobs in \( U(t) \) are processed at the same time in the sense that the completion time \( U(c_t) \) of all jobs in a batch are same. The processing time of \( U(t) \) is \( U(p_t) = \max\{p_1, p_2, ..., p_k\} \) and the completion time \( U(c_t) = t + U(p_t) \), where \( k \) is the size of \( U(t) \) i.e. the number of jobs in a batch. When the size of the batches are not bounded with any positive integers, then it is called *unbounded batch machine* with \( k = \infty \).

2.2 Job Characteristics (β)

Job characteristics describe the nature of the jobs and related characteristics to job scheduling [6, 49]. All jobs of any scheduling problem must possess at least one of the characteristics specified in set \( \beta = \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5\} \). In semi-online scheduling, a new job characteristic \( \beta_6 \) is introduced to represent extra piece of information (EPI) on future jobs. The job characteristics are presented as follows: \( \beta_1 \) specifies whether preemption or job splitting is allowed, \( \beta_2 \) specifies precedence relations or dependencies among the jobs, \( \beta_3 \) specifies release time for each job, \( \beta_4 \) specifies restrictions related to processing times of the jobs, for example, if \( \beta_4 = 1 \), that means \( p_i = 1, \forall J_i \), \( \beta_5 \) specifies deadline \( d_i \) for each job \( j_i \), indicating the execution of each \( j_i \) must be finished by time \( d_i \), otherwise an extra penalty may incur due to deadline over run. Let us throw more clarity on some of the important job characteristics as follows.

**Preemption (pmtn)** allows splitting of a job into pieces, where each piece is executed on the same or different machines in non-overlapping time spans.

**Non-preemption (N-pmtn)** ensures that once a job \( J_i \) with processing time \( p_i \) begins to execute on machine \( M_j \) at time \( t \), then \( J_i \) continues the execution on \( M_j \) until time \( t + p_i \) with no interruption in between.

**Precedence Relation** defines dependencies among the jobs by the partial order ‘\( \prec \)’ rule on the set of jobs [5]. A partial order can be defined on two jobs \( J_i \) and \( J_k \) as \( J_i \prec J_k \), which indicates execution of \( J_k \) never starts before the completion of \( J_i \). The dependencies among different jobs can be illustrated with a precedence graph \( G(p, \prec) \), where each vertex represents a job \( J_i \) and labeled
with its processing time $p_i$. A directed arc between two vertices in $G(p, <)$ i.e $J_i \rightarrow J_k$ represents $J_i \prec J_k$, where $J_i$ is referred to as predecessor of $J_k$. If there exists a cycle in the precedence graph, then scheduling is not possible for the jobs. When there is no precedence relation defined on the jobs, then they are said to be independent. We represent precedence relation among the jobs through precedence graphs by considering three jobs $J_1$, $J_2$, $J_3$ and their dependency relations as shown in Figure 4.

![Fig. 4.](image)

**Fig. 4.** (a) Cyclic dependencies among jobs. (b) $J_2 \prec J_3$ and $J_1$ is independent of $J_2$, $J_3$. (c) All jobs are independent.

**Extra Piece of Information (EPI)** is the additional information given to an online scheduling algorithm about the future jobs. Motivated by the interactive applications, a number of EPIs have been considered in the literature (see the recent surveys [92], [108]) to gain a significant performance improvement over the pure online scheduling policies [92]. We now present the definitions and notations of some well studied EPIs as follows.

- **Sum($T$).** $\sum_{i=1}^{n} p_i$ Total size of all jobs [13].
- **Max($p_{\text{max}}$).** $\max\{p_i|1 \leq i \leq n\}$ Largest processing time or largest size job [20].
- **Optimum Makespan ($Opt$).** Value of the optimum makespan is often represented by the following two general bounds [15].
  \[
  C_{\text{OPT}} \geq \frac{1}{m} \cdot \sum_{i=1}^{n} p_{ij} \quad \text{and} \quad C_{\text{OPT}} \geq p_{\text{max}}.
  \]
- **Tightly Grouped Processing time ($TGRP$).** Lower and upper bounds on the processing times of all jobs [20]. Some authors [22, 31, 45] considered either lower bound $TGRP(lb)$ or upper bound $TGRP(ub)$ on the processing times of the jobs.
- **Arrival Order of Jobs.** $p_{i+1} \leq p_i$, for $1 \leq i \leq n$ Jobs arrive, in order of non-increasing sizes ($Decr$) [21] or $r_{i+1} \geq r_i$ in order of non-decreasing release times ($Incr-r$) [70, 87].
- **Buffer($B(k)$).** A buffer $(B(k))$ is a storage unit of finite length $k(\geq 1)$, capable of storing at most $k$ jobs [13]. The weight of $B(k)$ is $w(B(k)) \leq \sum_{i=1}^{k} p_i$. Availability of buffer allows an online scheduling algorithm either to keep an incoming job temporarily in the buffer or to irrevocably assign a job to a machine in case the buffer is full [13]. Therefore, information about $k+1$ future jobs is always known prior to make an efficient scheduling decision. The following variations in the buffer length and usage of buffer
have been explored in the literature: buffer with length $k \geq 1$ i.e $B(k)$ [13], buffer with length 1 i.e $B(1)$ [13, 14] and re-ordering of buffer presented as $re\ B(k)$ [56].

- **Information on Last Job.** It is known in advance that last job has the largest processing time i.e. $p_n=p_{\text{max}}$, this EPI is denoted by LL in [26]. In [28], it is considered that several jobs arrive at the same as last job and this EPI is denoted by Sugg.

- **Inexact Partial Information.** Inexact partial information is also referred to as disturbed partial information, which deals with the scenario, where the extra piece of information available to the online algorithm is not exact. For example, the algorithm knows a nearest value of the actual $\text{Sum}$ but not the exact value. This EPI is represented as $\text{disSum}$ in [53].

- **Reassignment of Jobs** ($\text{reasgn}$). Once all jobs are assigned to the machines, again they can be reallocated to different machines with some pre-defined conditions. Several conditions on reassignment policies have been proposed in the literature [57, 62] such as reassign the last $k$ jobs, we represent as $\text{reasgn}(\text{last}(k))$, reassign arbitrary $k$ jobs i.e. $\text{reasgn}(k^*)$, reassign only the last job of all machines i.e. $\text{reasgn}(\text{last})^*$, reassign last job of any one of the machines, represented by $\text{reasgn}(\text{last})^1$.

- **Machine availability** ($\text{mchavl}$). All machines may not available initially. Machines are available on demand and the release time ($r_j$) of machine $M_j$ is known in advance [64]. Some authors have also considered the scenario where one machine is available for all jobs and other machine is available for few designated jobs [82].

- **Grade of Service (GOS) or Machine Hierarchy.** It is known a priori that machines are arranged in a hierarchical fashion to cater different levels of services to the jobs with some defined GOS [36, 46]. For example, if a GOS of $g_2$ is defined for any job $J_i$, then $J_i$ can only be assigned to machine $M_2$ and if $J_i$ has GOS of $g_1$, then it can be scheduled on any of the machines.

### 2.3 Optimality Criteria($\gamma$)

Several optimality criteria or output parameters have been investigated in the offline and online settings [17, 49]. However, in semi-online scheduling the following output parameters have been considered mostly: $\text{makespan}$ and $\text{load balancing}$.

- **Makespan** ($C_{\text{max}}$) represents completion time($c_i$) of the job that finishes last in the schedule, $C_{\text{max}}=\max\{c_i|1 \leq i \leq n\}$ or $C_{\text{max}}=\min\{l_j|1 \leq j \leq m\}$. The objective is to minimize $C_{\text{max}}$, otherwise termed as minimization of the load of highest loaded machine($\text{min-max}$).

- **Load Balancing** describes the objective to maximize the minimum machine load($\text{max-min}$) or machine cover. The scheduler assigns certain number of jobs to each machine for the processing of $n$ jobs on $m$ machines. Each job $J_i$ adds $p_i$ amount of load to the assigned machine $M_j$. The goal is to maximize the minimum load occurs on any of the machines so as to keep a balance in the incurred loads among all machines. We refer $C_{\text{min}}$ to represent $\text{max-min}$
objective. As an example, Figure 5 shows the loads of machines during the processing of a specified number of jobs.

![Timing Diagram of a Sample Schedule Showing Loads of Machines](image)

**Fig. 5.** Timing Diagram of a Sample Schedule Showing Loads of Machines

**Examples:** We present various semi-online scheduling setups based on the three fields \((\alpha|\beta|\gamma)\) classification format as shown in Table 2.

| Setup \((\alpha|\beta|\gamma)\) | Descriptions |
|-----------------------------|--------------|
| \(P_2|\text{Sum}|C_{\text{max}}\) | 2-identical machines | no preemption, total processing time | min-max. |
| \(P_2|\text{Sum}, \text{Max}|C_{\text{max}}\) | 2-identical machines | no preemption, total processing time and maximum size job | min-max. |
| \(P_m|B(k)|C_{\text{max}}\) | \(m\)-identical machines | no preemption, given a buffer of length \(k\) | min-max |
| \(Q_2|\text{Decr}|C_{\text{max}}\) | 2-uniform related machines | no preemption, jobs arrive in non-increasing order of their processing times | min-max. |
| \(Q_2|\text{pmtn}, TGRP}|C_{\text{min}}\) | 2-related machine | preemption, lower and upper bounds on the processing times of the jobs | max-min |
| \(U−\text{batch}|Max|C_{\text{max}}\) | Unbounded batch machine | no preemption, maximum size job | min-max |

3 Well-Known Semi-Online Scheduling Algorithms

For developing a basic understanding on semi-online policies, we represent fifteen well-known semi-online scheduling algorithms as follows.

- Algorithm \(H_1\) was proposed by Kellerer et al. [13] for \(P_2|B(k)|C_{\text{max}}\). Algorithm \(H_1\) assigns first \(k\) incoming jobs to the buffer \(B(k)\), where \(k \geq\)
1. When \((k+1)th\) job arrives, then a job \(J_i\) is selected from the buffer, where \(J_i \in \{J_1, J_2, \ldots, J_k, J_{k+1}\}\) and is scheduled on machine \(M_1\) such that \(l_1 + p_i \leq \frac{2}{3}(l_1 + l_2 + w(B))\). If such a \(J_i\) does not exist, then any arbitrary job is picked up from the buffer and is assigned to machine \(M_2\). Here, \(l_1, l_2\) are the loads of machines \(M_1, M_2\) respectively before assigning \(J_i\) and \(w(B) = \sum_{i=1}^{k+1} p_i\).

- **Algorithm H3** was proposed by Kellerer et al. [13] for \(P_2|\text{Sum}|C_{\text{max}}\). Algorithm \(H_3\) schedules each available job \(J_i\) on machine \(M_1\) as long as \(l_1 + p_i \leq (\frac{2}{3}) \cdot T\), where \(T = \sum_{i=1}^{n} p_i\) and \(l_1\) is the load of \(M_1\) before the assignment of \(J_i\). If \(l_1 + p_i \leq (\frac{2}{3}) \cdot T\), then algorithm \(H_3\) schedules \(J_i\) on \(M_1\) and the remaining jobs \(J_i\) are scheduled on machine \(M_2\), where \((i+1) \leq t \leq n\).

- **Algorithm Premediated List Scheduling(PLS)** is due to He and Zhang [20] for \(P_2|\text{Max}|C_{\text{max}}\). Algorithm \(PLS\) assigns each incoming job \(J_i\) to machine \(M_1\) as long as \(p_i \neq p_{\text{max}}\) and \(l_1 + p_i \leq 2 \cdot (p_{\text{max}})\), otherwise, \(J_i\) is scheduled on machine \(M_2\). Thereafter, each incoming job is scheduled on machine \(M_j\) for which \(l_j = \min\{l_1, l_2\}\).

- **Algorithm H** is due to Angelelli [22] for \(P_2|\text{Sum}|C_{\text{max}}\). Algorithm \(H\) assigns an incoming job \(J_i\) to machine \(M_j\) for which \(l_j = \max\{l_1, l_2\}\) if \(V \geq \max\{l_1 - l_2, p_i\}\), else schedules \(J_i\) on \(M_j\) for which \(l_j = \min\{l_1, l_2\}\), where \(V = T - (l_1 + l_2 + p_i)\).

- **Algorithm Ordinal** is due to Tan and He [23] for the setup \(Q_2|\text{Decr}|C_{\text{max}}\). Algorithm \(Ordinal\) schedules all jobs on machine \(M_2\) for speed ratio \(s \geq (1+\sqrt{3})\). For \(s \in [s(k-1), s(k))\), \(k \geq 1\), the sub set of jobs \(\{J_{k_i}, J_{k_i+3}|i \geq 0\}\) is scheduled on machine \(M_1\) and the sub set of jobs \(\{J_j\} \cup \{J_{k_i+3}, J_{k_i+4}, \ldots, J_{k_i+k+1}|i \geq 0\}\) is scheduled on machine \(M_2\), where \(s(k)=1\) for \(k=0\); \(s(k)=\frac{1+\sqrt{3}}{4}\) for \(k=1\) and \(s(k) = \frac{k^2-1+\sqrt{(k^2-1)^2+2k(k+1)}}{k(k+1)}\) for \(k \geq 2\).

- **Algorithm Highest Loaded Machine(HLM)** was proposed by Angelelli et al. [35] and was originally named as **algorithm H** for the setup \(P_n|\text{Sum}|C_{\text{max}}\). By observing the behavior of the algorithm, we rename it to **HLM**. Algorithm **HLM** schedules a newly arrive job either on the highest loaded machine in the set of heavily loaded machines or on the highest loaded machine in the set of slightly loaded machine.

- **Algorithm Extended Longest Processing Time(ELPT)** was proposed by Epstein and Favrholdt [42] for the setup \(Q_2|\text{Decr}|C_{\text{max}}\). Algorithm **ELPT** assigns each incoming job \(J_i\) to the fastest machine \(M_j \in \{M_1, M_2\}\) for which \(l_j + \frac{p_i}{s_j}\) is minimum, where \(S_1=1\) and \(S_2=\frac{1}{s}\) for \(s \geq 1\).

- **Algorithm Slow LPT** was proposed by Epstein and Favrholdt [42] for \(Q_2|\text{Decr}|C_{\text{max}}\). It schedules the first available job \(J_1\) to the slowest machine \(M_2\) and the next job \(J_2\) is scheduled on the fastest machine \(M_1\). It assigns the next incoming job \(J_3\) to \(M_2\) if \(s \cdot (p_1 + p_3) \leq c(s) \cdot (p_2 + p_3)\), otherwise, \(J_3\) is assigned to machine \(M_1\). Next incoming jobs are assigned to the machine \(M_j\) for which \(l_j + \frac{p_i}{s_j}\) is minimum, where \(S_1=1\) and \(S_2=\frac{1}{s}\) for \(s \geq 1\). \((c(s)\) is a function of the speed ratio interval \(s)\)

- **Algorithm Grade of Service Eligibility(GSE)** is due to Park et al. [46] for the setup \(P_2|GOS, \text{Sum}|C_{\text{max}}\). It states that upon the arrival of any job
systems resulted in a number of scheduling models. Online scheduling is one

4 Historical Overview of Semi-online Scheduling:
Important Results

In 1960’s, the curiosity to explore computational advantages of multi-processor

systems resulted in a number of scheduling models. Online scheduling is one
among such models. Graham [2] initiated the study of online scheduling of a list of $n$ jobs on $m (\geq 2)$ identical parallel machines and proposed the famous List Scheduling (LS) algorithm. Algorithm LS selects the first unscheduled job $J_i$ from the list such that all its predecessors ($J_k \prec J_i$) have been completed and schedules $J_i$ on the most lightly loaded machine. Algorithm LS achieves performance ratios of $1.5$ for $m=2$ and $2 - \frac{1}{m}$ for all $m$ by considering $C_{OPT} \geq \sum_{i=1}^{n} p_i$. In [3], Graham considered the offline setting of $m$-machine scheduling problem and proposed the seminal algorithm Largest Processing Time (LPT). Algorithm LPT first sorts the jobs in the list by non-increasing sizes and assigns them one by one to a machine that incurs smallest load after each assignment. Algorithm LPT achieves a worst-case performance ratio of $1.16$ for $m = 2$ and $1.33 - \frac{1}{3m}$ for $m \geq 2$ with the time complexity of $O(n \log n)$. These two seminal contributions of Graham served as a motivation for further investigations to address research challenges in online scheduling. Initial three decades (1966-1996) of the online scheduling research were concentrated on the improvements of the $LB$ and $UB$ on the $CR$ to achieve optimal competitiveness (please, see [16-17] for a comprehensive survey on the seminal contributions). However, no significant attention has been paid for exploring the practicability of the online scheduling model of Graham.

Motivated by real world applications, Kellerer et al. [13] proposed a novel variant of the online scheduling model by considering $EPIs$ for pre-processing of online arriving jobs and named the variant as semi-online scheduling. They conjectured that additional information on future jobs would immensely help in improving the best competitive bounds in various online scheduling setups. Following the conjecture of Keller et al., ocean of literature have been produced since last two decades in pursuance of achieving optimum competitiveness with the exploration of practically significant new $EPIs$. We now survey the critical ideas and important results given for semi-online scheduling in a historical chronological manner by classifying the results based on the $EPI$ as follows.

4.1 Early Works in Semi-online Scheduling (1997-2000)

**Buffer, Sum.** Kellerer et al. [13] envisioned semi-online scheduling as a theoretically significant and practically well performed online scheduling model. They initiated the study on semi-online scheduling by considering $Sum$ as the known $EPI$ and proposed algorithm $H_3$ for the setup $P_2|Sum|C_{max}$. Algorithm $H_3$ outperforms algorithm LS and achieves a tight bound $1.33$ for $m = 2$. To show the $LB$ $1.33$ of algorithm $H_3$, let us consider an instance of $P_2|Sum|C_{max}$, where $Sum=2$. Algorithm $H_3$ schedules each incoming job $J_i$ to machine $M_1$ until $l_1 + p_i \leq \frac{2}{3} (Sum)$ and assigns the remaining jobs to machine $M_2$. If we consider $Sum=2$, then irrespective of the input instances, the final loads $l_1, l_2$ would be $\frac{4}{3}, \frac{2}{3}$ respectively and $C_{OPT}$ would be $1$. This implies, $C_{H_3} \geq 1.33 \cdot (C_{OPT})$. The semi-online strategy of Kellerer et al. unveils that advance knowledge of $Sum$ helps any online algorithm $A$ to schedule incoming jobs to a particular machine until its load reaches upto a judiciously chosen fraction of the $Sum$ and assigns the remaining jobs to the other machine such that the ratio between $C_A$
and $C_{OPT}$ results in the improved competitive bound. They also studied semi-online scheduling with a buffer($B$) of length $k$ and proposed algorithm $H_1$ for the setup $P_2|B(k)|C_{max}$. They proved that any online scheduling algorithm with $B(k)$ achieves a CR of at least 1.33. The LB can be shown by considering an online sequence $J :< J_1/1, J_2/1, J_3/1, J_4/3 >$ of four jobs with specified processing times and $k=1$. Algorithm $H_1$ keeps the first available job $J_1$ in the buffer. Thereafter, each incoming $J_{i+1}$, where $1 \leq i \leq 3$, is either kept in the buffer or any $J_x \in \{J_i, J_{i+1}\}$ is scheduled on machine $M_1$ if $l_1 + p_x \leq \frac{2}{3} \cdot (l_1 + l_2 + w(B))$, else any $J_x$ is assigned to machine $M_2$ (Note that $w(B)=p_i + p_{i+1}$). We now have a schedule due to algorithm $H_1$ with the sequence of assignments of $J$ on the machines as follows: $J_1/1$ on $M_1$, $J_2/1$ on $M_2$, $J_4/3$ on $M_1$ and $J_3/1$ on $M_2$ such that $C_{H_1} \geq 4$, where $C_{OPT} \geq 3$. Therefore, $C_{H_1} \geq 1.33 \cdot (C_{OPT})$. A matching UB was shown to achieve a tight bound 1.33 for algorithm $H_1$.

They also studied a semi-online variant, where two parallel processors are given to virtually schedule a sequence of incoming jobs over 2-identical machines by two distinct procedures independently. Finally, the jobs are scheduled by the procedure that has incurred minimum $C_{max}$ for the entire job sequence. They obtained a tight bound 1.33 for the semi-online variant $P_2|2$-Proc|$C_{max}$. Zhang [14] studied the setup $P_2|B(1)|C_{max}$ and obtained the tight bound 1.33 with an alternate policy. The policy keeps the first job $J_1$ in the buffer and if no further jobs arrive, then $J_1$ is scheduled on machine $M_2$, else for next incoming job $J_{i+1}$, where $1 \leq i \leq n-1$, the job $J_x$ is chosen from $\{J_i, J_{i+1}\}$ such that $p_x$ is minimum (let us denote the other job as $J_y$). Now, $J_x$ is assigned to machine $M_1$ if $l_1 + p_x \leq 2 \cdot (l_1 + p_y)$, else $J_x$ is assigned to machine $M_2$. If there are no jobs to arrive further, then the last job in the buffer is assigned to machine $M_2$. The aim of the policy is to keep a larger load difference between machines $M_1$ and $M_2$ by assigning smaller jobs to $M_2$ such that at any time step, the availability and assignment of an unexpected larger job would not incur a makespan beyond $1.33 \cdot (C_{OPT})$. Angelelli [22] proposed an alternative to algorithm $H_1$ [13] as algorithm $H$ for $P_2|Sum|C_{max}$ and obtained tight bound 1.33. He analyzed the performance of algorithm $H$ by considering various ranges of lower bounds on the processing times of the jobs. Girlich et al. [18] obtained an UB 1.66 for $P_n|Sum|C_{max}$.

**TGRP, Max.** He and Zhang [20] initiated study on the setup $P_2|TGRP|C_{max}$ by assuming that all jobs have processing times within the interval of $p$ and $rp$, where $p > 0$ and $r \geq 1$. They proved that any online algorithm $A$ must have $C_A \geq \frac{r+1}{r}$ for $r \leq 2$ and $C_A \geq 1.5$ for $r > 2$. They analyzed algorithm $LS$ for $P_n|TGRP|C_{max}$ and showed that $C_{LS} \leq (1 + \frac{(m-1)(r-1)}{m}) \cdot C_{OPT}$. They obtained LB 1.33 for the setup $P_2|Max|C_{max}$ by considering the online availability of the job sequence $J :< J_1/1, J_2/1, J_3/2, J_4/2 >$, where $p_{max}=2$ is known a priori. Following the optimum policy [2] of keeping a machine free for the largest job and assigning the sequence of comparatively shorter jobs to the remaining machines, any online algorithm $A$ assigns $J_1/1$ and $J_2/1$ on $M_1$ followed by the assignments of $J_3/2$ on $M_2$ and $J_4/2$ on either $M_1$ or $M_2$ to incur $C_A \geq 4$, where $C_{OPT} \geq 3$. This implies $C_A \geq 1.33 \cdot (C_{OPT})$. They proposed algorithm PLS to
achieve a tight bound 1.33. Algorithm PLS always maintains a load difference maximum of up to \( p_{\text{max}} \) between machines \( M_1 \) and \( M_2 \) such that scheduling of the largest job on the smallest loaded machine almost equalizes the loads of both the machines.

**Opt.** Azar and Regev [15] introduced a variant of the classical bin-stretching problem, where items are available one by one in order and each available item must be packed into one of the \( m \) bins before the availability of the next item. It is known apriori that all items can be placed into \( m \) unit sized bins. The goal is to stretch the bins as minimum as possible so as to fit all items into the bins. Therefore, the bin stretching problem considered by Azar and Regev is analogous to the setup \( P_m|\text{Opt}|C_{\text{max}} \). They achieved an UB 1.625 for large \( m \). The key idea is to first define the threshold values \( \alpha \) and \( 2\alpha - 1 \) based on the value of known Opt. Then, arrange \( m \) machines in at least two distinct sets based on their loads with respect to \( \alpha \) and \( 2\alpha - 1 \). A new job is now assigned to the selected machine belongs to the chosen set. Improved rules can be defined for the selections of set and machine. Here, a non-trivial challenge is to define and characterize the threshold values.

**Decr and Preemptive Semi-online Scheduling.** Seiden et al. [21] analyzed algorithm LPT [3] with known Decr and achieved a tight bound 1.166 for \( m=2 \) and LB 1.18 for \( m=3 \). They initiated the study on \( P_m|\text{Opt}|C_{\text{max}} \) and obtained tight bound 1.366. They assumed preemption as job splitting for scheduling distinct pieces of an incoming job in the non-overlapping time slots. To understand the notion of job splitting, let us consider a sequence of jobs of unit sizes. Suppose, the required CR to be obtained is \( k \). Now, the initial \( m \) incoming jobs are split into at most two pieces each such that all pieces of a job execute in distinct time slots and all jobs are finished by time \( k \). Let \( r=\lfloor \frac{m}{k} \rfloor \), \( i \leq r \), each incoming job \( J_{m+i} \) is split and assigned to the first \( i \) machines such that each machine gets \( \frac{k}{m} \) fraction of the processing time of job \( J_{m+i} \) and its remaining fraction is assigned prior to time \( k \). We now have \( i \) highest loads of the machines represented as: \( k(1 + \frac{1}{m}) \), \( k(1 + \frac{1}{m}) \), ..., \( k(1 + \frac{1}{m}) \), which ensures non-overlapping time slots in the subsequent rounds. Similarly, the next jobs followed by the \((m + r)^{th}\) job are scheduled only in the time slots after \( k \) on at most \( r + 1 \) machines. A non-trivial challenge is to rightly choose the values of \( r \) and \( k \) such that the load to be scheduled prior to time \( k \) is at most \( k \cdot m \). The authors conjectured that the achieved tight bound 1.366 with known Decr can possibly be achieved with only known \( p_{\text{max}} \). Further, they showed that randomization in scheduling decision making does not lead to improved the CR for the setup \( P_m|\text{Opt},\text{Decr}|C_{\text{max}} \). We now present the main results obtained for semi-online scheduling in identical machines for the years 1997–2000 in table 3.

4.2 Well-known Results in Semi-online Scheduling (2001-2005)

During the years 2001-2005, semi-online scheduling was studied not only for identical machines but for uniform related machines as well. Both preemptive and non-preemptive processing formats were investigated. The concept of combined EPI and a new EPI on the last job were introduced. We present the state of the
**Table 3. Main Results for Identical Machines: 1997-2000**

| Author(s), Year | Setup(\(\alpha|\beta|\gamma\)) | Competitiveness Results |
|-----------------|----------------------------------|-------------------------|
| Kellerer et al. 1997 [13] | \(P_2|\text{Sum}|C_{\text{max}}\)\(\gamma_1\) \(P_2|B(k)|C_{\text{max}}\) \(P_2|2-\text{Proc}|C_{\text{max}}\) | 1.33 Tight for each setup |
| Zhang 1997 [14] | \(P_2|B(1)|C_{\text{max}}\) | 1.33 Tight |
| Azar and Regev 1998 [15] | \(P_n|\text{Opt}|C_{\text{max}}\) | 1.625 UB |
| Girlich et al. 1998 [18] | \(P_n|\text{Sum}|C_{\text{max}}\) | 1.66 UB |
| He and Zhang 1999 [20] | \(P_2|\text{Max}|C_{\text{max}}\) \(P_2|\text{TGRP}|r,rp,C_{\text{max}}\) \(P_n|\text{TGRP}|r,rp,C_{\text{max}}\) | 1.33 Tight with Max, 1.5 LB for \(P_2\) and \(r \geq 2\) with TGRP, \(1 + \frac{(m-1)(r-1)}{m}\) UB for \(P_n\) with TGRP. |
| Seiden et al. 2000 [21] | \(P_n|\text{pmtn},\text{Decr}|C_{\text{max}}\) \(P_2,3|\text{Deacr}|C_{\text{max}}\) | 1.366 Tight for \(m \rightarrow \infty\), 1.166 Tight for \(m = 2\), 1.18 LB for \(m = 3\) |
| Angelelli 2000 [22] | \(P_2|\text{Sum},\text{TGRP}(lb)|C_{\text{max}}\) | 1.33 Tight |

art contributions in semi-online scheduling for related machines and identical machines as follows.

**Related Machines:**

**Deacr.** Tan and He [23] proposed algorithm *Ordinal* for non-preemptive semi-online scheduling with **ordinal data** [11] and known **Deacr** for 2-uniformly related machines, where \(S_1=1\) and \(S_2 \geq 1\). They analyzed and proved competitiveness of the algorithm as an interval wise function of machines’ speed ratio \(s\). They proved the **tightness** of the algorithm in most of the intervals of \(s \in [1, \infty)\). As a main result, they produced UB \(\frac{\alpha+1}{s}\) for \(s \geq 2.732\) and LB \(\frac{\alpha+1}{s}\) for \(s \in [2.732, \infty)\). However, the LB of algorithm *Ordinal* does not match with its UB when the total length of the speed ratio interval reduces to 0.7784, where the largest gap between the intervals is at most 0.0521. Epstein and Favrholdt [30] initiated study on the setup \(Q_2|\text{pmtn},\text{Deacr}|C_{\text{max}}\) and achieved competitive ratios of \(\frac{3(s+1)}{3s+2}\) for \(1 \leq s \leq 3\) and \(2s+(s+1)\) for \(s \geq 3\). In [42], they investigated the setup \(Q_2|\text{Deacr}|C_{\text{max}}\), where \(S_1=1\) and \(S_2=\frac{1}{s}\). They expressed competitive ratios as a function of 15 speed ratio intervals. They proposed algorithm **ELPT** and achieved **tight** bound 1.28 for \(s=1.28\). They proposed algorithm **Slow-LPT** for \(s \in [1, \frac{1}{6}(1 + \sqrt{37})]\) and obtained a **tight** bound 1.28. Here, the key idea is to initially use the slowest machine and keep the fastest machine free for incoming larger jobs. They proposed algorithms **Balanced-LPT** and **Opposite-LPT** for the remaining intervals, where algorithms ELPT and Slow-LPT do not obtain tight bounds. Algorithm Balanced-LPT schedules the first job \(J_1\) on the fastest machine \(M_1\). The second job \(J_2\) is assigned to machine \(M_2\) if \(s > c(s)(p_1 + p_2)\), else job \(J_2\) is scheduled on \(M_1\), where \(c(s)=2.19\) for \(s \in [2, 2.19]\) and \(c(s)=2.57\) for \(s \in [2.35, 2.57]\). Thereafter, remaining jobs are scheduled by algorithm ELPT. Algorithm Opposite-LPT also schedules job \(J_1\) on machine \(M_1\). The second job \(J_2\) is scheduled on \(M_1\) if \(s \cdot p_2 < (p_1 + p_2) \leq c(s) \cdot s \cdot p_2\), else \(J_2\) is scheduled
on $M_2$, where $c(s)=2.35$ for $s \in [2.19, 2.35]$. Thereafter, the subsequent jobs are scheduled by the ELPT rule.

**Opt.** Epstein [33] studied semi-online scheduling for the setup $Q_2|Opt|C_{max}$, where $S_1=1$ and $S_2=\frac{1}{2}$. He proposed algorithm $SLOW$ by considering $C_{OPT}=1$ and $s \geq \sqrt{2}$. Algorithm $SLOW$ schedules an incoming job $J_i$ to machine $M_1$ if $l_2 \geq \frac{1}{s+1};$ else if $l_2 + p_j \geq \frac{C_{SL}(s)}{s}$, then job $J_i$ is assigned to machine $M_2$; else $J_i$ is scheduled on machine $M_1$, where $C_{SL}(s)=\frac{s+1}{s+1}$. Algorithm $SLOW$ performs better in the scenario, where the slowest machine $M_2$ is relatively very slow and initial jobs needs to be assigned to it for keeping the high speed machine $M_1$ relatively free for future larger jobs. For $s \leq \sqrt{2}$, Epstein proposed algorithm $FAST$ by considering $C_{OPT}=1$. Algorithm $FAST$ assigns an incoming job $J_i$ to machine $M_2$ if a job $J_j$ was earlier assigned to $M_2$ due to $l_1 < (1+\frac{1}{s}-\frac{CFA(s)}{s})$ and $(l_1+p_j) > CFA(s);$ else if $l_1 \geq (1+\frac{1}{s}-\frac{CFA(s)}{s}),$ then $J_i$ is scheduled on $M_2$; else if $(l_1+p_j) \leq CFA(s),,$ then $J_i$ is scheduled on $M_1; else J_i$ is assigned to machine $M_2,$ where $CFA(s)=\frac{2s+2}{2s+1}$ for $1 \leq s \leq \frac{1+\sqrt{17}}{4}$ and $CFA(s)=s$ for $\frac{1+\sqrt{17}}{4} \leq s \leq \sqrt{2}$. Algorithm $FAST$ performs better in the cases, where the slowest machine $M_2$ is considerably fast, thus allowing initial jobs to be scheduled on the fastest machine $M_1$. He achieved lower bounds in terms of function of defined speed ratio intervals and obtained overall $CR$ of $1.414$ and $LB$ of $1.366$.

**TGRP, Max.** He and Jiang [34] studied the setup $Q_2|pmtn, TGRP(p, xp)|C_{max}$ by considering $S_1=1$ and $S_2 \geq 1$, where $p > 0$ and job size ratio $x \geq 1$. They initiated analysis of algorithm with respect to speed ratio intervals ($s \geq 1$) and job size ratios. They achieved a *tight* bound $\frac{s^2+s+2}{s+1}$ for $s \geq 1$ and $x < 2s$. For $s \geq 1$ and $x \geq 2s$, the *tight* bound $\frac{1+2s+2s^2}{1+2s^2+3s+1}$ was obtained. Further, they investigated the setup $Q_2|pmtn, Max|C_{max}$ by considering known $p_{max}=s \geq 1$. They achieved a $CR$ of $\frac{2s^2+3s+1}{2s^2+2s+1}$ for $s \geq 1$. They explored that information on $Max$ is weaker than known *Decr* for preemptive semi-online scheduling on 2-related machine. We now present the main results obtained for semi-online scheduling on related machines for the years 2001-2005 in Table 4.

| Author(s), Year | setup($\alpha|\beta|\gamma$) | Competitiveness Results |
|------------------|-----------------------------|-------------------------|
| Tan and He 2001 [23] | $Q_2|Decr|C_{max}$ | $\frac{s+1}{s}$ Tight |
| Epstein and Favrholdt 2002 [30] | $Q_2|pmtn, Decr|C_{max}$ | $\frac{2s+2s^2}{2s^2+2s+1}$ Tight for $1 \leq s \leq 3$; $\frac{2s+2s^2}{2s^2+2s+1}$ Tight for $s \geq 3$ |
| Epstein 2003 [33] | $Q_2|Opt|C_{max}$ | $1.414$ UB, $1.366$ LB |
| He and Jiang 2004 [34] | $Q_2|pmtn, Max|C_{max}$; $Q_2|pmtn, TGRP(p, xp)|C_{max}$ | $\frac{2s^2+3s+1}{2s^2+2s+1}$ Tight with Max; $\frac{s^2+s+2}{s+1}$ Tight with TGRP. |
| Epstein and Favrholdt 2005 [42] | $Q_2|Decr|C_{max}$ | $1.28$ Tight |
Identical Machines:
Information on Last job (LL). Zhang and Ye [26] studied a semi-online variant, where it is known apriori that the last job $J_n$ is the largest one i.e. $p_n=p_{\text{max}}$. Upon availability of a job $J_i$, it is also revealed that whether $J_i$ is the last job. They proposed algorithm $A_1$ for the setup $P_2|LL|C_{\text{max}}$ and achieved a tight bound $1.414$. Algorithm $A_1$ schedules an incoming job $J_i$ on machine $M_2$ if $J_i=J_n$; else if $(l_2 + p_i) > (0.414) \cdot (l_1 + p_i)$, then $J_i$ is assigned to machine $M_1$; else $J_i$ is scheduled on $M_2$. The key idea is to reserve a machine for $J_n$ to obtain relatively minimum makespan irrespective of the size of $J_n$. They proposed algorithm List Scheduling with a waiting machine ($LSw$) for the setup $P_3|LL|C_{\text{max}}$ by keeping machine $M_3$ free for $J_n$. Algorithm $LSw$ schedules an incoming job $J_i$ on $M_3$ if $J_i=J_n$; else assigns job $J_i$ to machine $M_j \in \{M_1, M_2\}$ for which $l_j=\min\{l_1, l_2\}$. They proved a tight bound 1.5 for algorithm $LSw$. However, it would be interesting to analyze the cases, where the value of $p_n=p_{\text{max}}$ is relatively smaller or there are multiple jobs with $p_i=p_{\text{max}}$.

Combined Information. Tan and He [28] exploited the limitation of prior knowledge of $LL$ [26] by considering the following sequences $J:<J_1/1, J_2/1, J_3/2>$ and $J':<J_1/1, J_2/1, J_3/\epsilon>$, where $\epsilon >0$. They studied the semi-online variants, where two EPIs are known at the outset. They proposed the 1.2 competitive algorithm $SM$ for the setup $P_3|\text{Sum, Max}|C_{\text{max}}$. Algorithm $SM$ is designed based on the ratio between known $\text{Sum}(T)$ and $\text{Max}(p_{\text{max}})$. If $p_{\text{max}} \in \left[\frac{2T}{5}, T\right)$, then the first job $J_1$ is assigned to machine $M_2$ for which $p_1=p_{\text{max}}$ such a job is denoted as $J^1_{\text{max}}$ and other jobs are scheduled on machine $M_1$. If $p_{\text{max}} \in (0, \frac{T}{5})$, then all incoming jobs are scheduled by algorithm $LS$. If $p_{\text{max}} \in \left(\frac{T}{5}, \frac{2T}{5}\right)$, then an incoming job $J_i$ is assigned to machine $M_j \in \{M_1, M_2\}$ such that $(l_j + p_i) \in \left(\frac{2T}{5}, \frac{3T}{5}\right]$ and the successive jobs are scheduled on machine $M_{3-j}$. If $(l_j + p_i) \in \left[\frac{2T}{5}, p_{\text{max}}, \frac{3T}{5}\right]$ and if $J^1_{\text{max}}$ has not been revealed yet, then both $J_i$ and $J^1_{\text{max}}$ are assigned to $M_j$ and other jobs are scheduled on machine $M_{3-j}$. If $p_i \leq \frac{T}{5}$ or $J_i=J^1_{\text{max}}$, then $J_i$ is scheduled on $M_1$; else if $\frac{T}{5} < p_i \leq p_{\text{max}}$, then $J_i$ is scheduled on $M_2$; if two jobs have already been scheduled on $M_2$ such that $l_2 \geq \frac{2T}{5}$, then successive jobs are scheduled on $M_1$. Further, they proposed a 1.11 competitive algorithm for the setup $P_2|\text{Sum, Decr}|C_{\text{max}}$. They also showed that if $\text{Sum}$ is given, then information on $LL$ is useless and if $\text{Decr}$ is known, then knowledge of $\text{Max}$ does not substantiate to improve the best competitive bound of 1.16 [21] for $P_2|\text{Decr}|C_{\text{max}}$. Epstein [33] followed the work of [15] and achieved a tight bound 1.11 for the setup $P_2|\text{Decr, Opt}|C_{\text{max}}$. He proved the $LB$ by considering $C_{\text{OPT}}=1$ and six jobs, where the jobs $J_1$ and $J_2$ are of size $\frac{4}{3}$ each and jobs $J_3, J_4, J_5, J_6$ are of size $\frac{5}{3}$ each. If any semi-online algorithm $A$ schedules $J_1$ and $J_2$ on machine $M_1$ or on $M_2$ and schedules the remaining jobs to the other vacant machine, then we have $C_A=\frac{10}{9}$. If $J_1$ and $J_2$ are scheduled on two different machines, then by considering the size of next three jobs $(J_3', J_4', J_5')$ as $\frac{1}{3}$ each, we have $C_A=\frac{10}{9}$ (as any two jobs from $J_3', J_4', J_5'$ must be scheduled on a single machine). However, algorithm $OPT$ schedules $J_1$ and $J_2$ on one machine and assigns the remaining three jobs to the other machine to incur $C_{\text{OPT}}=1$. Therefore, we have $\frac{C_A}{C_{\text{OPT}}}=1.11$. He proposed the algorithm $\text{SIZES}$, which
schedules an incoming job $J_i$ to any $M_j \in \{M_1, M_2\}$ such that $\frac{2}{3} \leq (l_j + p_i) \leq \frac{10}{7}$, the remaining jobs are scheduled on the other machine. If $p_i \leq \frac{2}{7}$, then $J_i$ and all remaining jobs are assigned to machine $M_j$ which incurs $l_j = \min\{l_1, l_2\}$ after each assignment.

**List Model.** Yong and Shengyi [27] studied the list model of [19], which is a variant of Graham’s list scheduling model [2], where it is considered that the machines are not available at the outset. Upon availability of a job $J_i$, an algorithm may purchase a machine by incurring an unit cost. A machine is purchased such that the existing $j$ machines satisfies the following inequality: $l_j \leq T_i \leq l_{j+1}$, where $T_i = \sum_{j=1}^{i} l_j$ (total work load incurred by initial $i$ jobs).

The aim is to optimize the sum of makespan($C_{\text{max}}$) and total machine cost($m_j$). They showed that with known Max, an algorithm makes decisions on purchasing of a machine and scheduling of an incoming job $J_i$ by comparing the values of $p_i$, loads of the existing machines or total machine cost with some judiciously chosen bounds on the known $p_{\text{max}}$. They obtained an $UB$ 1.5309 and a $LB$ 1.33 with known Max. They achieved an $UB$ 1.414 and $LB$ 1.161 with known $Sum$. Further, List model can be studied to improve the existing competitive bounds by considering other well-known EPFs. We may obtain a natural variant of the list model by considering non-identical machines with well defined characteristics, which may influence the choice of an algorithm in purchasing of a machine.

**Max.** Cai [29] extended the work of [20] to obtain a tight bound ($\frac{m-2+\sqrt{(m-2)^2+8m^2}}{2m}$) for $P_m|\text{Max}|C_{\text{max}}$, where $3 \leq m \leq 17$. Further, he achieved a tight bound 1.414 for $m \to \infty$.

**TGRP.** Angelelli et al. [31] considered TGRP($ub$) < 1 and $Sum$=2 are known in advance. For 2-identical machine setup, they obtained lower bounds for various ranges of $ub$. They showed that algorithm LS is optimal for smaller $ub$ and proposed optimal algorithms for $0.5 \leq ub \leq 0.6$; $ub$=0.75 and $0.9 \leq ub < 1$. He and Dosa [43] investigated the 3-identical machine setting by considering TGRP($p, xp$), where $p > 0$ and job size ratio $x \geq 1$. They proved that algorithm LS is optimal for different intervals of $x \in [1, 1.5], [1.73, 2], [6, +\infty]$. They designed algorithms for various ranges of $x$ with improved bounds for which the gap between the competitive ratio and the lower bounds is at most 0.01417.

**Sum, Buffer.** Angellelli et al. [35] extended their previous work [22,31] and obtained an $UB$ 1.725 for $m$-identical machine with known $Sum$. Cheng et al. [44] investigated the setup $p_m|Sum|C_{\text{max}}$ by considering $Sum$=m. They followed the work of [15, 35] and obtained $UB$ 1.6 and improved $LB$ 1.5 for $m \geq 6$. Dosa et al. [37] studied the setup $P_2|B(1), Sum|C_{\text{max}}$ and obtained a tight bound 1.25. They showed that considering a $B(k)$, where $k > 1$ does not help to improve the 1.25 competitiveness. They explored that when $Sum$ is known at the outset, then the knowledge of the sizes of $k(>1)$ future jobs($k$-look ahead) does not help in improving the competitive bound. Further, they studied the setup $P_2|2-Proc|C_{\text{max}}$ with known $Sum$ and improved the tight bound 1.33 obtained in [13] to 1.2. We now present the main results obtained for semi-online scheduling on identical machines for the years 2001-2005 in table 5.
| Author(s), Year | setup(α|β|γ) | Competitiveness Results |
|----------------|----------------|-------------------------|
| Zhang and Ye 2002 [26] | $P_2|LL|C_{max}$, $P_3|LL|C_{max}$ | 1.414 Tight for $m=2$, 1.5 Tight for $m=3$. |
| Yong and Shengyi 2002 [27] | $P_m|Max|C_{max}+m$, $P_m|Sum|C_{max}+m$ | 1.53 UB and 1.33 LB with $Max$, 1.414 UB and 1.161 LB with $Sum$. |
| Tan and He 2002 [28] | $P_2|Sum, Max|C_{max}$, $P_3|Sum, Decr|C_{max}$ | 1.2 Tight with $Sum$ and $Max$, 1.11 Tight with $Sum$ and $Decr$. |
| Cai 2002 [29] | $P_m|Max|C_{max}$ | $\left(\frac{m-2+\sqrt{(m-2)^2+8m}}{2m}\right)$ Tight for $3 \leq m \leq 17$, 1.414 Tight for $m \to \infty$. |
| Angelelli et al. 2003 [31] | $P_2|Sum, TGRP(ub)|C_{max}$ | 1.2 Tight for $ub \in [0.5, 0.6], (1+\frac{ub}{k})$ Tight for $ub \in [0.75, 1]$, $0.666(1+ub)$ Tight for $ub \in [0.94, 1]$. |
| Epstein 2003 [33] | $P_2|Decr, Opt|C_{max}$ | 1.11 Tight. |
| Angelelli et al. 2004 [35] | $P_m|Sum|C_{max}$ | 1.725 UB, 1.565 LB for $m \to \infty$. |
| Dosa et al. 2004 [37] | $P_2|B(1), Sum|C_{max}$, $P_2|2-Proc, Sum|C_{max}$ | (1.25, 1.2) Tight for respective setups. |
| He and Dosa 2005 [43] | $P_3|TGRP|C_{max}$ | 1.5 Tight for $x \in (2, 2.5], \left(\frac{x^2+2}{x^2}\right)$ Tight for $x \in (2.5, 3], \left(1.66-\frac{1}{k}\right)$ Tight for $x \in (3, 6)$. |
| Cheng et al. 2005 [44] | $P_m|Sum|C_{max}$ | (1.6, 1.5) UB and LB respectively for $m \geq 6$. |

### 4.3 Advancements in Semi-online Scheduling (2006-2010)

The initial decade in semi-online scheduling research was devoted to the traditional online scheduling setups with fundamental EPIs on the future jobs. Moreover, the significance of EPI was realized with the improvement in the competitive bounds for pure online scheduling setups. During the years 2006-2010, new semi-online scheduling setups such as GOS or machine hierarchy; a variant of EPI such as inexact EPI and new policies such as job re-assignment and buffer re-ordering have been introduced. We now discuss on the important results contributed during the years 2006-2010 for semi-online scheduling on identical and related machines as follows.

**Identical Machines:**

**Sum.** Angelelli et al. [45] studied the setup $P_2|Sum, TGRP(ub)|C_{max}$ and advanced their previous work [31] for the unexplored intervals of $ub$. They showed LBs for the interval, where $ub \in [\frac{1}{k}, \frac{1}{k-1}]$ and $k \geq 2$. For an instance, a LB of $(\frac{k-1}{3}) \cdot ub + \frac{2}{3} \cdot (\frac{k+1}{k})$ was shown for $ub \in [\frac{2(k+1)}{k(2k+1)}, \frac{2k-1}{2(k-1)}]$. The LB was proved by considering two job sequences $J'$ and $J''$, where $J' = \{J_1/x, J_2/x, J_3/y, J_4/y$ and $2(k-1)$ jobs of size $ub\}$, where $x \in [0, ub]$ such that $x+y+(k-1) \cdot ub=1$ and $y \leq x \leq \frac{ub}{2}$ and $J'' = \{J_1/x, J_2/x, J_3/z$ and $2(k-1)$ jobs of size $\frac{1}{k}\}$, where...
2x + z + (k - 2) \cdot \frac{1}{k} = 1 and x < z < \frac{1}{2}. Any algorithm A has the option to schedule the first two jobs J_1 and J_2 either on the same machine or on different machines. If algorithm A schedules J_1 and J_2 on the same machine, then for the sequence J', we obtain C_A \geq 2x + (k - 1) \cdot ub. If J_1 and J_2 are scheduled on different machines, then for J'' we have C_A \geq x + (k - 1) \cdot \frac{1}{k} + z = 1 - x + \frac{1}{k}. Therefore, in both cases, we obtain C_A \geq \min\{1 - x + \frac{1}{k}, 2x + (k - 1) \cdot ub\}. We obtain C_{OPT} = 1 by assigning J_1 and J_2 to different machines for J' and by scheduling them on the same machine for J''. Therefore, we have \( \frac{C_A}{C_{OPT}} \geq \min\{1 - x + \frac{1}{k}, 2x + (k - 1) \cdot ub\} \).

By maximizing w.r.t. x, we achieve \( \frac{C_A}{C_{OPT}} \geq (\frac{k - 1}{3}) \cdot ub + \frac{2}{3} \cdot (\frac{k + 1}{k}) \). They proposed the optimal algorithm H' for ub \in [1, 2\cdot ub], which is (k \cdot ub)-competitive for ub \in \left[\frac{2(k+1)}{4k^2 + 1}, \frac{1 + 2k}{2k^2}\right]. Algorithm H' schedules an incoming job J_i on the machine M_1 if \( l_1 + p_i \leq 1 + \frac{1}{2k + 1} \); else if \( l_2 + p_i \leq 1 + \frac{1}{2k + 1} \), then \( J_i \) is scheduled on the machine M_2; else \( J_i \) is assigned to the machine \( M_j \in \{M_1, M_2\} \) such that \( l_j = \min\{l_1, l_2\} \). They also proposed a \((1 + \frac{1}{2k})\)-competitive algorithm for ub \in \left[\frac{1 + 2k}{2k^2}, \frac{2k - 1}{2k^2}\right]. \) In [50], they studied the setup \( P_3|\text{Sum}|C_{max} \) and obtained the LB \((1 + \frac{\sqrt{129} - 9}{6}) > 1.392\). An UB 1.421 was shown by a pre-processing policy of the available jobs. Here, a non-trivial challenge is to tighten or diminish the gap between the obtained LB and UB.

**Max.** Wu et al. [58] followed the work of [29] and obtained a tight bound \( 2 - \frac{1}{m-1} \) for \( m = 3, 4 \) with known Max. Sun and Huang [64] considered a variant, where all machines are not given at the outset. However, machine availability time \( r_i \) is given at the outset for each machine. W.l.o.g., it is assumed that \( r_m \geq r_{m-1} \geq ... \geq r_1 \). They obtained a LB 1.457 for \( m > 6 \). They proposed a \((2 - \frac{1}{m-1})\)-competitive algorithm, which assigns an incoming job \( J_i \) by algorithm LS unless \( p_i = p_{\text{max}} \) and \( r_{\text{min}} + l_{\text{min}} + p_i > 2 \cdot p_{\text{max}} \); otherwise \( J_i \) is scheduled on machine \( M_1 \) and the successive jobs are scheduled by algorithm LS, where \( r_{\text{min}} \) and \( l_{\text{min}} \) are the release time and load of the most lightly loaded machine respectively.

**Combined Information.** Hua et al. [48] advanced the work of [28] for 3-identical machine setting with known Sum and Max. They obtained an UB 1.4 and a LB 1.33. Wu et al. [54] tighten the gap between the obtained UB and LB of [48] and obtained a tight bound 1.33 for the setup \( P_3|\text{Sum}, \text{Max}|C_{max} \).

**GOS.** Park et al. [46] initiated the study on semi-online scheduling under GOS eligibility with known Sum. They considered that a job with \( g_i = 1 \) can only be processed by machine \( M_1 \) and if \( g_i = 2 \), then \( J_i \) can be processed by any of the machines. They proposed a 1.5-competitive semi-online algorithm for the setup \( P_3|\text{GOS}, \text{Sum}|C_{max} \). The algorithm schedules an incoming job \( J_i \) to machine \( M_1 \) if \( g_i = 1 \); else if \( g_i = 2 \) and \( l_2 + p_j \leq (\frac{3}{2}) \cdot L \), then \( J_i \) is scheduled on machine \( M_2 \); else \( J_i \) is assigned to machine \( M_1 \), where \( L = \frac{\text{Sum}}{2} \). For the same problem, Jiang et al. [47] studied the preemptive version with GOS and proposed a 1.5 competitive algorithm. For the non-preemptive case with GOS, they improved the UB from 2 obtained in [10] to 1.66. Wu and Yang [66] studied 2-identical machine case with GOS. They investigated the problem separately for known Opt and Max.
Inexact EPI. Tan and He [53] studied semi-online settings, where the value of a known EPI is given in interval or in the inexact form unlike the exact value. For some $x > 0$ and the disturbance parameter $y \geq 1$, the following EPIs were considered for the respective settings: for $P_2|\text{disOpt}|C_{\text{max}}$, it is given that $C_{\text{OPT}} \in [x, yx]$; for $P_2|\text{disSum}|C_{\text{max}}$, it is known that $\text{Sum} \in [x, yx]$ and for $P_2|\text{disMax}|C_{\text{max}}$, it is known that $p_{\text{max}} \in [x, yx]$. For $P_2|\text{disOpt}|C_{\text{max}}$, they achieved a $LB$ $1.5$ for $y \geq 1.5$ and obtained $UB$s $\frac{7y+1}{4y+2}$ for $1 \leq y \leq \frac{5+\sqrt{17}}{8}$ and $y$ for $\frac{5+\sqrt{17}}{8} \leq y < 1.5$. They proved $LB$ $1.5$ for the setup $P_2|\text{disSum}|C_{\text{max}}$, where $y \geq 1.5$. For $P_2|\text{disMax}|C_{\text{max}}$, they proved $LB$s $\frac{2y+2}{y+2}$ for $y=1.23$ and $1.5$ for $y \geq 2$. Further, they proposed the algorithm modified PLS(MPLS) and achieved an $UB$ $\frac{2\sqrt{2}}{y+2}$ for $y \in [1, 2]$ and showed its tightness for $y \in [1, \sqrt{5} - 1]$. Algorithm MPLS assigns each incoming job $J_i$ to machine $M_{i-1}$ until the arrival of any job $J_b$ for which $p_b \in [1, y]$ and $(l_1 + p_b) > 2$. Thereafter, $J_b$ and all successive jobs are scheduled by algorithm LS.

Job Reassignment. Tan and Yu [57] studied a semi-online variant, where an algorithm is allowed to re-schedule some of the already assigned jobs under certain conditions. For the setup $P_2|\text{reasgn}(\text{last}(k))|C_{\text{max}}$, they proved LB $1.5$ and showed that algorithm LS is optimal with no re-assignments. For $P_2|\text{reasgn}(\text{last})|C_{\text{max}}$, they proposed algorithm RE and obtained a tight bound $1.414$. Algorithm RE assigns an incoming job $J_i$ to the highest loaded machine $M_j$ if $l_1 \leq (\sqrt{2} + 1) \cdot (l_2 + p_i)$ and $p_i \leq \sqrt{2} \cdot l_1$; otherwise, $J_i$ is scheduled on machine $M_{s-i}$. After the assignment of all jobs, algorithm RE checks for re-assignment. If all jobs have been scheduled on the same machine $M_j$, then the job $J_n$(last job) is re-scheduled on the machine $M_{s-1}$. Let $J_{n_1}^1$ and $J_{n_2}^2$ be the last two jobs scheduled on machines $M_1$ and $M_2$ respectively. Let us consider $p_x=\max\{p_{n_1}, p_{n_2}^{2}\}$ and $p_y=\min\{p_{n_1}, p_{n_2}^{2}\}$. Algorithm RE re-assigns $J_x$ followed by $J_y$ to the $M_j$, which can obtain minimum $c_{x}$ and $c_{y}$ respectively. For $P_2|\text{reasgn}(k^*)|C_{\text{max}}$, they proposed algorithm RA and achieved a tight bound $1.33$. Algorithm RA schedules jobs $J_1$ and $J_2$ on two different machines. Let $l_1=\max\{l_1, l_2\}$. Each incoming job $J_i$, where $3 \leq i \leq n$, is scheduled on machine $M_1$ if $l_1 + p_i \leq 2 \cdot l_2$; otherwise, $J_i$ is scheduled on machine $M_2$. After the scheduling of all jobs, if $l_2 > 2 \cdot l_1$, then the job $J_{n_2}^2$ is re-scheduled on machine $M_1$. The following non-trivial questions remain open: What is the minimum number of re-assignments that is sufficient to improve the known competitive bounds? Is the re-assignment policy with EPI such as Deq, Opt, Sum or Max practically significant and helps in achieving optimal bounds on the CR?

Max-Min Objective. Tan and Wu [52] studied non-preemptive semi-online scheduling on $m$-identical machine($m \geq 3$) with $C_{\text{min}}$ objective. They proposed a $(m-1)$-competitive algorithm for the setup $P_m|\text{Sum}|C_{\text{min}}$. The idea is to keep the loads of all machines under $\frac{\text{Sum}}{2m}$. The machine $M_m$ is reserved from starting to schedule a job $J_i$, if there exists no machine $M_j$, where $1 \leq j \leq m-1$ for which $l_j$ is at most $\frac{\text{Sum}}{m}$ or $\frac{\text{Sum}}{2m}$ after the assignment of $J_i$. If there exists some machines with load at most $\frac{\text{Sum}}{m}$, then assignment of $J_i$ to $M_m$ makes $l_m > \frac{\text{Sum}}{2m}$ and if there are some machines with load at most $\frac{\text{Sum}}{2m}$, then $J_i$ and
the remaining jobs are scheduled on $M_m$. They proposed a $(m-1)$-competitive algorithm for $P_m|\text{Max}|C_{\min}$. Each incoming job is scheduled on any one of the $m-1$ machines by algorithm $LS$ until the arrival of a job $J_i$ with $p_i=p_{\max}$ or $p_i+p_{\min}\{l_1,l_2,...,l_{m-1}\}>2p_{\max}$. Such a $J_i$ is scheduled on machine $M_m$ and the successive jobs are scheduled over $m$-machines by algorithm $LS$. The idea is to maintain a load of at most $2p_{\max}$ in each machine, where the machine $M_m$ is kept idle for the largest job $J_i$ with $p_i=p_{\max}$. They obtained tight bounds 1.5 and $m-2$ for $m=3$ and $m \geq 4$ respectively with combined information on $\text{Sum}$ and $\text{Max}$. We now present the main results obtained for semi-online scheduling on identical machines for the years 2006-2010 in table 6.

Table 6. Important Results for Identical Machines: 2006-2010

| Author(s), Year | setup$(\alpha|\beta|\gamma)$ | Competitiveness Results |
|-----------------|-------------------------------|--------------------------|
| Angelelli et al. 2006 [45] | $P_2[\text{Sum}, TGRP}(ub)|C_{\max}$ | $(1 + \frac{1}{2m-1})$, Tight for $ub \in \left[ \frac{n}{2}, \frac{2(n+1)}{2(n+m-1)} \right]$ | \((\frac{n+1}{2})ub + (0.666)(\frac{n+1}{2n})\) Tight for $ub \in \left( \frac{n}{2n-1}, \frac{1}{n+1} \right)$ |
| Park et al. 2006 [46] | $P_2[GOS, \text{Sum}]C_{\max}$ | 1.5 Tight |
| Jiang et al. 2006 [47] | $P_2[GOS]C_{\max}$ | 1.66 UB, 1.5 Tight |
| Hua et al. 2006 [48] | $P_3[\text{Sum}, \text{Max}]C_{\max}$ | 1.4 UB, 1.33 LB. |
| Angelelli et al. 2007 [50] | $P_3[\text{Sum}]C_{\max}$ | 1.392 LB, 1.421 UB. |
| Tan and Wu 2007 [52] | $P_m[\text{Sum}]C_{\min}$ | $(m-1)$-competitive for $\text{Sum}$ or $\text{Max}$ |
| $P_m[\text{Max}]C_{\min}$ | $P_m[\text{Sum}, \text{Max}]C_{\min}$ | 1.5 Tight with $\text{Sum}$ and $\text{Max}$ for $m=3$, $(m-2)$ Tight for $m \geq 4$. |
| Tan and He 2007 [53] | $P_2[\text{disOpt}]C_{\max}$ | 1.5 Tight with $\text{Opt}$ or $\text{Sum}$ for $y \geq 1.5$, 1.5 Tight with $\text{Max}$ for $y \geq 2$ |
| Wu et al. 2007 [54] | $P_3[\text{Sum}, \text{Max}]C_{\max}$ | 1.33 Tight. |
| Tan and Yu 2008 [57] | $P_2[\text{reasgn}k(\text{last}k))C_{\max}$ | $(1.5, 1.33, 1.414)$ LB for respective setups |
| $P_2[\text{reasgn}k^*C_{\max}$ | $P_2[\text{reasgn}l(\text{last})]C_{\max}$ | $(2 - \frac{1}{m-1})$ Tight |
| Wu et al. 2008 [58] | $P_m[\text{Max}]C_{\max}$ | 1.457 LB for $m > 6$, $(2 - \frac{1}{m-1})$ Tight |
| Sun and Huang 2010 [64] | $P_m|\text{Opt}^{\gamma}\text{Max}|C_{\max}$ | $(1.618, 1.5)$ Tight for respective setups |

Related Machines:

Last Job. Epstein and Ye [51] followed the work of [26] and considered LL as the known EPI in their study of semi-online scheduling on 2-related machines with $\text{min-max}$ and $\text{max-min}$ optimality criteria. They considered $S_1=\frac{1}{2}$ and $S_2=1$, where $s \geq 1$. They proposed in general an algorithm for both optimality criteria
and analyzed its performance for various intervals of $s$. The algorithm schedules an incoming job $J_i$ on machine $M_1$ if $l_2 + p_i > \alpha(s) \cdot (l_1 + s \cdot p_i)$; otherwise job $J_i$ is scheduled on machine $M_2$, where $0 < \alpha(s) < \frac{1}{s}$. If $J_i = J_n$, then $J_i$ is scheduled on $M_2$. The key idea is to keep the highest speed machine $M_2$ relatively light loaded to schedule $J_n$ (largest job) on it. They obtained tight bound $2.618$ for the setup $Q_2|LL|C_{\text{min}}$. They achieved $UB$ 1.5 and $LB$ 1.465 for the setup $Q_2|LL|C_{\text{max}}$.

**Sum.** Angelelli et al. [55] studied the setup $Q_2|\text{Sum}|C_{\text{max}}$. They considered speeds $S_1 = x$, $S_2 = 1$ and $\text{Sum} = 1 + x$, where $x \geq 1$. They showed $LB$ and $UB$ as functions of $x$. They proposed algorithm $H'$ for $x \in [1, 1.28]$, which assigns an incoming job $J_i$ to machine $M_1$ if $l_1 + p_i \leq x \cdot (1 + \frac{1}{2s+1})$; otherwise $J_i$ is scheduled on machine $M_2$. They proved $\left(\frac{2x+2}{2x+1}\right)$-competitiveness of algorithm $H'$. They developed algorithm $H''$ for $x \in [1.28, 1.41]$, which assigns an incoming job $J_i$ to machine $M_1$ if $l_1 + p_i \leq x^2$; else $J_i$ is assigned to machine $M_2$. They showed that algorithm $H''$ is $x$-competitive. For $x \geq 1.41$, they designed algorithm $H'''$, which assigns an incoming job $J_i$ to machine $M_2$ if $l_2 + p_i \leq 1 + \frac{1}{x+1}$; else $J_i$ is scheduled on machine $M_1$. They proved $\left(\frac{x+2}{x+1}\right)$-competitiveness of algorithm $H'''$.

Ng et al. [60] studied the setup $Q_2|\text{Sum}|C_{\text{max}}$ by considering $S_1 = 1$ and $S_2 \geq 1$. They obtained competitive bounds as functions of intervals of $s \geq 1$, where the largest gap between the $LB$ and $UB$ is at most 0.01762. They achieved a $LB \frac{2x+2}{2x+1}$ for $s \geq \sqrt{3}$ and overall $UB$ 1.369 for $s \in [1, \infty)$. Angelelli et al. [63] investigated for the setup stated in [55, 60] by introducing a geometric representation of the scheduling problem through a planar model. They considered 2-related machine setup with speeds $S_1 = 1$, $S_2 = b$ and $\text{Sum} = b + 1$, where $b \geq 1$. They represented scheduling of jobs in planar model as a game between constructor($K$) and scheduler($H$), where $K$ submits jobs one by one and $H$ schedules a job upon its availability on a machine by following an algorithm. They illustrated the game in a plane by representing each point $(x, y)$ as the situation, where $x = l_1$ and $y = l_2$. Here, a move of $K$ corresponds to the arrival of a new job $J_i$ with $p_i > 0$ and the move of $H$ specifies, whether to move to the point $(x + p_i, y)$ or to the point $(x, y + p_i)$ from the point $(x, y)$ in the plane. The game ends after reaching the line $x + y = b + 1$. Now, the current position of the point $(x, y)$ determines the makespan incurred by the scheduler $H$. They showed a $LB$ 1.359 for $b \in [1, 3.66, 1.732]$, which they proved to be optimal for $b = 1.5$.

**Buffer.** Englert et al. [56] investigated both $m$-identical and $m$-related machines settings with a buffer of size $k \in \theta(m)(where, \theta(m)$ is a function on number of machines). They introduced the re-ordering of buffer policy which does not assign each incoming job immediately to any of the machines, rather stores the jobs in the buffer and re-order the stored input job sequence prior to construct the actual schedule so as to achieve minimum makespan. They obtained $LB$ and $UB$ 1.333, 1.465 respectively for $m$-identical machine which beats the previous best results obtained by non-reordering buffer strategies of [13, 14, 37]. For $m$-related machine setup, they obtained an $UB$ 2 with a buffer of size $m$.

**Preemptive Semi-online Scheduling.** Chassid and Epstein [59] studied preemptive semi-online scheduling on 2-related machine setup. They considered
both max-min and min-max optimality criteria with known GOS and Sum. They considered that $S_1=1$, $S_2=b$, $Sum=1$, where $b \geq 1$. They assumed that a job $J_i$ with $g_i=1$ must be processed only on machine $M_1$ and with $g_i=2$ it can be processed on any $M_j \in \{M_1, M_2\}$. They proposed 1-competitive algorithm FSA and proved its tightness for both optimality criterion with the key idea of keeping the load of machine $M_2$ at most $\frac{Sum}{b+1}$. The optimality of algorithm FSA was shown by analyzing the following two cases. Case 1: if $l_2=\frac{Sum}{b+1}$, then we have $l_1=\frac{Sum}{b+1}$ by considering $Sum=1$ and $b=1$, this implies $C_{FSA}=\frac{1}{2}$ followed by $\frac{C_{FSA}}{C_{OPT}}=1$, where $C_{OPT} \geq \frac{Sum}{2} = \frac{1}{2}$. Case 2: if $l_2 < l_1$, means machine $M_2$ has been equipped with all $J_i'$s with $g_i=2$, so the remaining $J_i'$s with $g_i=1$ have been scheduled on machine $M_1$, which eventually balances the loads of $M_1$ and $M_2$. Therefore, we obtain optimal schedules in both the cases. Ebenlendr and sglall [61] proposed an unified algorithm RatioStretch for preemptive semi-online scheduling on $m$-uniform machines($m \geq 2$). They proved that the algorithm achieves optimum approximation ratio that holds for any values of $s$ with any known EPI. They computed the ratio by linear program, where machines speeds are considered as input parameters. They established relationships among well-known semi-online setups for uniform machines and obtained competitive bounds in each setup for large $m$.

Opt. Ng et al. [60] improved the results of Epstein [33] for the speed ratio interval $s \in [1.366, 1.395]$ and obtained an $UB \frac{2s+1}{2s}$. They showed tight bound $1.366$ for overall $s \in [1, \infty)$.

Job Reassignment. Liu et al. [62] studied the setup $Q_2|GOS|C_{max}$ by considering $S_1=1$ for higher GOS machine($M_1$) and $S_2=x$ for ordinary machine($M_2$). They obtained $LBs = 1 + \frac{2a}{x+2}$ for $0 < x \leq 1$ and $1 + \frac{x+1}{x(2x+1)}$ for $x > 1$ by considering different GOS levels. They proved $LB 1 + \frac{1}{1+\frac{3}{2}}$ with re-assignment of last $k$ jobs($reasgn$($last(k)$)) and $LB \frac{(s+1)^2}{2s+3s+1}$ for re-assignment of one job from every machine($reasgn($all$)$). They proposed $\left( \frac{(x+1)^2}{x+2} \right)$-competitive algorithm $EX$-$RA$ for both types of re-assignment policies by considering $S_1=x$ and $S_2=1$, where $1 \leq x \leq 1.414$. Algorithm $EX$-$RA$ schedules the jobs $J_1$ and $J_2$ on different machines such that $l_1=\max\{p_1, p_2\}$ and $l_2=\min\{p_1, p_2\}$. For each incoming job $J_i(3 \leq i \leq n)$, if $l_1 + \frac{p_i}{2} \leq (x+1) \cdot l_2$, then job $J_i$ is scheduled on machine $M_1$; otherwise $J_i$ is assigned to machine $M_2$. After the scheduling of job $J_n$, if $l_2 \leq (x+1) \cdot l_1$, then we have $C_{EX$-$RA} = \max\{l_1, l_2\}$; otherwise the second last job of machine $M_2$ is re-scheduled on machine $M_1$ and $l_1, l_2$ is updated to obtain the final $C_{EX$-$RA} = \max\{l_1, l_2\}$. Cao and Liu [65] followed the re-assignment policies of [57, 62] for 2-related machine setup. They considered re-assignment of last job of each machine and obtained overall competitive ratio of $\min\{\sqrt{s+1}, \frac{2s+1}{2} \}$ for different speed ratio(s) intervals. We now present the main results obtained for semi-online scheduling on uniform related machines for the years 2006-2010 in Table 7.
### Table 7. Main Results for Related Machines: 2006-2010

| Author(s), Year | setup(α|β|γ) | Competitiveness Results |
|----------------|-----------|------------------------|
| Epstein and Ye 2007 [51] | Q₂[LL|Cmin, Q₂[LL|Cmax | 1.5 UB and 1.465 LB for Cmax, 2.618 Tight for Cmin. |
| Angelelli et al. 2008 [55] | Q₂[Sum|Cmax | 1.33 Tight for x = 1, x Tight for x ∈ (1.28, 1.366), \( \frac{x^2}{x+1} \) Tight for x ≥ 1.732 |
| Englert et al. 2008 [56] | Pₘ|reB(k)|Cmax, Qₘ|reB(k)|Cmax | (1.333, 1.465) LB and UB respectively for Pₘ with k ∈ \( \emptyset(m) \), (2 - \( \frac{1}{m-k+1} \)) Tight for Pₘ with k ∈ [1, \( \frac{m+1}{2} \)], 2 Tight for Qₘ with k ∈ m |
| Chassid and Epstein 2008 [59] | Q₂[pmtn,GOS,Sum|Cmin, Q₂[pmtn,GOS,Sum|Cmax | 1 Tight for both setups |
| Ng et al. 2009 [60] | Q₂[Opt|Cmax, Q₂[Sum|Cmax | (1.366, 1.369) Tight with Opt or Sum respectively. |
| Ebenlendr and Sgall 2009 [61] | Q₂[pmtn,Sum|Cmax, Q₂[pmtn,Max|Cmax, Q₂[pmtn,Decr|Cmax | 1.138 Tight with S₁ = 1.414, S₂ = S₃ = 1 and known Sum, 1.252 Tight with S₁ = 2, S₂ = S₃ = 1.732 and known Max, 1.52 Tight with known Decr. |
| Liu et al. 2009 [62] | Q₂[GOS|Cmax, Q₂[reasgn(last(k))]|Cmax, Q₂[reasgn(last)]*|Cmax | (1 + \( \frac{x+1}{2x+1} \)) LB with GOS for x > 1, (1 + \( \frac{1}{x+1} \)) LB with reasgn(last(k)), (\( \frac{x(x+1)^2}{x+2} \)) Tight with both re-assignment policies for 1 ≤ x ≤ 1.414. |
| Angelelli et al. 2010 [63] | Q₂[Sum|Cmax | 1.359 LB with b = 1.5. |
| Cao and Liu 2010 [65] | Q₂[reasgn(last)]|Cmax | (\( \sqrt{s+1} \)) Tight for 1 ≤ s < 1.618, \( \frac{s+1}{x} \) Tight for s ≥ 1.618 |

### 4.4 Recent Works in Semi-online Scheduling

The recent era of semi-online scheduling has been dominated by non-preemptive scheduling in identical machines with multiple grades of service levels (GOS) or machine hierarchy. Semi-online scheduling in unbounded batch machine has been introduced. Several instances of related machines have been studied for various unexplored speed ratio intervals. Job rejection and reassignment policies have been introduced for various setups of related machines. We now present an overview of the state of the art in semi-online scheduling for unbounded batch machine, uniform related machines and identical machines as follows.

**Unbounded Batch Machine**: Yuan et al. [68] introduced semi-online scheduling in single unbounded batch machine to improve the 1.618 competitive bound obtained by pure online strategies in [25,32]. They considered that at any time...
step $t$, we are given with $p_t$ and $r_t$ of job $J_t$, where $J_t$ is the largest job that will arrive after time $t$. They obtained tight bound 1.382 with known $p_t$ by considering at most two batches. With given $r_t$, they achieved $LB$ 1.442 and $UB$ 1.5 by constructing at most three batches. With known $r_t$, they proposed an algorithm, which constructs at most two batches. The algorithm resets the value of $r_t = \max\{r_t, \alpha \cdot (p_t)\}$, then forms the first batch ($U(t_1)$) by considering all jobs that are available by time $r_t$, and schedules them irrevocably on the machine, where $r_t$ is the release time of the first largest job and $\alpha = 0.618$. The second batch $U(t_2)$ is formed by considering all jobs that are received at the time step $t_2 = r_t + p_t$, then the value of $r_t$ is reset to $\max\{r_t, \alpha \cdot (p_t)\}$ prior to schedule all jobs of batch $U(t_2)$. They obtained a matching $UB$ 1.618 to that of pure online strategies. It is now a non-trivial challenge to beat the 1.618 competitive bound by forming at most 2 batches with known $r_t$.

**Related Machines:**

**Buffer.** Epstein et al. [95] investigated the setup $Q_m | reB(k) | C_{\text{min}}$ and proposed a $m$-competitive algorithm, where $m \geq 2$ and $k=m+1$. The algorithm keeps initial $m+1$ incoming jobs in the buffer. After arrival of the $(m+2)^{th}$ job until availability of the $n^{th}$ job, each time the smallest job $J_i$ is selected from $m+2$ available jobs and is scheduled by algorithm $LS$, while not considering the machine speeds. When there is no jobs to arrive and the buffer contains $m+1$ jobs such that $p_1 \leq p_2 \leq \ldots \leq p_m \leq p_{m+1}$, the algorithm schedules the jobs in any of the following rules.

1. Schedule $J_1$ by $LS$ rule and schedule the jobs $J_i$, where $2 \leq i \leq (m + 1)$ to the corresponding machine $M_i$, respectively, where $1 \leq j \leq m$.
2. Schedule the jobs by rule 1, but migrate $J_i$ to the machine $M_m$ for some $2 \leq i \leq m$.
3. Schedule $J_{i+1}$ to $M_k$ for $1 \leq i \leq m$ and schedule $J_1$ to a machine $M_k$ such that $2 \leq k \leq (m – 1)$.

Interestingly, the algorithm ignores the machine speeds until the arrival of all jobs, and then the relative order of the machines speeds are considered for making scheduling decision. Further, they studied the setup $Q_2 | B(1) | C_{\text{min}}$, where $S_1=1$ and $S_2 \geq 1$ and proposed a $\frac{2e+1}{2e}$ competitive algorithm. The algorithm keeps the first job $J_1$ in the buffer, thereafter on the arrival of each incoming $J_i$, $2 \leq i \leq n$, it is assumed that $p_x = \min\{p_{i-1}, p_i\}$ and $p_y = \max\{p_{i-1}, p_i\}$. Now, job $J_x$ is scheduled on machine $M_1$ if $\frac{2a+1}{2e} \geq \frac{a+1}{e}$, otherwise $J_x$ is assigned to machine $M_2$. The goal is to schedule the smaller jobs to the slowest machine and relatively larger jobs to the fastest machine so as to maximize the minimum work load incurred on a machine. Lan et al. [76] studied the setup $Q_m | B(k) | C_{\text{max}}$ and achieved a tight bound $(2 - \frac{1}{m} + \epsilon)$ with $k=m$ and $m \geq 2$, where $\epsilon > 0$.

**Job Rejection.** Min et al. [96] initiated the study on semi-online scheduling in 2-uniform machine with job rejection policy by considering $S_1=1$, $S_2 \geq 1$. The rejection policy describes a scenario, where an incoming job $J_t$ can either be assigned to a machine or can be rejected permanently by incorporating a
penalty of \( x_i \). The objective of any semi-online algorithm is to incur a minimum value for the sum of makespan with sum of all penalties. The algorithm is given beforehand with two parallel processors for making scheduling policies. Finally, the best policy is opted for actual assignment of the jobs. Min et al. proposed a semi-online algorithm with the following rules for scheduling of each incoming job: Upon availability of a new job \( J_i \), processor 1 rejects \( J_i \), if \( x_i \leq \alpha \cdot p_i \), where \( \alpha = \frac{1}{2} \); else schedules \( J_i \) by algorithm LS. On the other hand, processor 2 rejects \( J_i \), if \( x_i \leq \beta \cdot p_i \), where \( \beta = \frac{3}{2} \); else schedules \( J_i \) by algorithm LS. After the assignments of all jobs, one of the policies that has yielded a minimum objective value is opted by the algorithm for actual scheduling of the jobs. The algorithm achieves tight bounds \( \frac{2s+1}{s+1} \) for \( 1 \leq s \leq 1.618 \); and \( \frac{s+1}{s} \) for \( s > 1.618 \).

**Max.** Cai and Yang [97] investigated the setup studied in [69] and obtained lower bounds \( \text{Opt} \) of any semi-online algorithm achieves a CR of at least the defined \( LB \). In [89], the authors considered the setup studied in [69] and obtained lower bounds in terms of an algebraic function \( r(s) \) for the following unexplored speed ratio intervals.

\[
r(s) = \begin{cases} 
\frac{s+2}{s+1}, & \text{for } 1.414 \leq s \leq 2 \\
\frac{3s+2}{2s+2}, & \text{for } 2 \leq s \leq 2.732 \\
\frac{s+1}{s}, & \text{for } s \geq 2.732 
\end{cases}
\]

The idea is to schedule the first largest job on machine \( M_2 \) and to schedule the remaining jobs by algorithm LS.
In [91], they studied for the interval \( s \in [1.710, 1.732] \) and achieved tight bounds of \( \frac{2s+10}{3s+7} \) for \( s=1.7258 \) and \( \frac{s+11}{2} \) for \( 1.725 \leq s \leq 1.732 \) respectively. The obtained results draw an insight that a single algebraic function cannot formulate the tightness of the LB.

**Sum.** Dosa et al. [69] investigated the setup \( Q_2|\text{Sum}|C_{\text{max}} \) by considering \( S_1=1, S_2 \geq 1 \) and \( \text{Sum}=3s \cdot (1+s) \). They achieved tight bounds for the unexplored speed ratio interval \( 1 \leq s < 1.2808 \), which are presented by an algebraic function \( r(s) \) as follows.

\[
r(s) = \begin{cases} 
1 + \frac{1}{3s}, & \text{for } s \in [1, 1.071] \\
1 + \frac{3s}{4s+6}, & \text{for } s \in [1.071, 1.0868] \\
1 + \frac{1}{2s+7}, & \text{for } s \in [1.0868, 1.2808]
\end{cases}
\]

They proposed an algorithm by considering various time interval ranges as safe sets for scheduling decision making. The algorithm involves three subroutines as described below.

**Subroutine 1 Master**

1. Upon the arrival of a new job \( J_i \), run subroutine Slave.
2. If \( J_i=J_1 \), then run subroutine \( \text{CoalA} \) from starting; else continue \( \text{CoalA} \) from the breaking point of the last call for scheduling \( J_{i-1} \).
3. If no more jobs to arrive, then stop; else move to step 1.

**Subroutine 2 Slave**

1. Schedule \( J_i \) on machine \( M_j \), if the value of \( l_j + p_i \) is within the time interval range \( [2s, 3s + 1] \) for \( j=1 \) or \( [3s^2 - 1, 3s^2 + s] \) for \( j=2 \); thereafter, remaining jobs are scheduled on machine \( M_{3-j} \) and stop.
2. Schedule \( J_i \) on \( M_j \), if \( l_j \leq T_{j1} \) and \( l_j + p_j > T_{0j} \), where \( T_1 = 3s + 1 - 3s^2 \), \( T_2 = s \), \( T_{01} = 3s + 1 \) and \( T_{02} = 3s^2 + s \); thereafter, schedule the remaining jobs to machine \( M_{3-j} \) and stop.
3. Schedule \( J_i \) on \( M_j \) if the value of \( l_j + p_i \) is within the time interval range \( [s - 1, 3s + 1 - 3s^2] \) for \( j=1 \) or the value within the range \( [3s^2 - s - 2, s] \) for \( j=2 \); thereafter, schedule the remaining jobs on \( M_{3-j} \) until \( l_{3-j} < 2s \) for \( j=2 \) or \( l_{3-j} < 3s^2 - 1 \) for \( j=1 \); otherwise run the subroutine \( \text{Slave} \) once more.

**Subroutine 3 CoalA**

1. Schedule \( J_i \) on \( M_j \) until \( l_j + p_i < s - 1 \).
2. Schedule \( J_i \) on machine \( M_1 \), if \( p_i < 3s^2 - s - 2 \) and \( l_1 < 3s^2 - 2 \).
3. Schedule \( J_i \) on machine \( M_2 \), if \( l_2 + p_i < 3s^2 - s - 2 \).
4. If \( J_i \) is assigned to \( M_2 \), then the remaining jobs are scheduled on \( M_2 \) as long as \( l_2 \leq 3s^2 - 1 \); thereafter, schedule the next job on \( M_1 \) and the remaining jobs on \( M_2 \); Stop.

The algorithm was shown to be tight with \( CR \) of \( 1 + \frac{1}{7s} \) for \( s \in [1, 1.071] \). A similar algorithm with slight modification in the time interval ranges of the safe
sets was proposed for \( s \in [1.071, 1.08] \).

**GOS.** Hou and Kang [98] investigated semi-online hierarchical scheduling on \( m \)-uniform machine setup with max-min and min-max objectives. They considered \( m \) machines in two hierarchies, where in hierarchy 1, we have \( k \) machines, each with speed \( S_1 \geq 1 \) and \( g_j = 1 \), capable of executing all jobs. In hierarchy 2, we have rest \( m - k \) machines, each with speed \( S_2 = 1 \) and \( g_j = 2 \), capable of executing the jobs having \( g_j = 2 \). For \( Q_m|GOS|C_{min} \), they proved that no online algorithm can be possible with a bounded CR. They investigated the setup \( Q_m|\text{pmtn}, GOS|C_{min} \) and obtained UB of \( \frac{2k_s + m - k}{k_s} \) for \( 0 < s < \infty \) by applying the fractional assignment policy, where each incoming job \( J_i \) can be splitted arbitrarily among the machines. For \( Q_m|\text{pmtn}, GOS|C_{max} \), they achieved UB of \( \frac{(k_s + m - k)^2}{k_s^2 + k_s(m - k) + (m - k)^2} \) for \( 0 < s < \infty \). For \( Q_m|\text{pmtn}, GOS, \text{Sum}|C_{max} \), they proposed a 1-competitive algorithm. The idea is to schedule the jobs having \( g_j = 1 \) evenly on the machines having \( g_j = 1 \) and schedule the jobs having \( g_j = 2 \) on the machines with \( g_j = 2 \) as long as the loads of the machines are under a given threshold value. Lu and Liu [81] studied three variants of \( Q_2|GOS|C_{max} \) with known Opt or Sum or Max by considering \( S_1 = 1 \), \( S_2 = s > 0 \), \( g_j = 1 \) (for processing job \( J_i \) exclusively on machine \( M_1 \)) and \( g_j = 2 \) (for making \( J_i \) eligible for processing in any one of the \( M_j \in \{ M_1, M_2 \} \)). They proposed algorithm Gos-OPT for \( Q_2|GOS, \text{Opt}|C_{max} \) and obtained UB of \( \min \{ \frac{1+2s}{1+s}, \frac{1}{s} \} \). Algorithm Gos-OPT schedules each incoming job \( J_i \) on the machine \( M_1 \) if \( g_i = 1 \); else if \( g_i = 2 \) then \( J_i \) is scheduled by the following policy: if \( s \geq \frac{1+s}{1+s} \), then \( J_i \) is scheduled on machine \( M_2 \); if \( 0 < s < \frac{1+s}{1+s} \) and \( \frac{1+2s}{s} \leq \left( \frac{1+2s}{1+s} \right) \cdot C_{OPT} \), then \( J_i \) is scheduled on \( M_1 \). They achieved a matching UB for \( Q_2|GOS, \text{Sum}|C_{max} \) with an equivalent algorithm Gos-SUM, which is equivalent to algorithm Gos-OPT, just simply replacing \( C_{OPT} \) by \( \frac{\text{Sum}}{1+s} \) in the policy. They proposed algorithm Gos-MAX for \( Q_2|GOS, \text{Max}|C_{max} \) and obtained UB \( 1 + s \) for \( 0 < s < \frac{\sqrt{5}}{2} - 1 \). Algorithm Gos-MAX schedules an incoming job \( J_i \) on \( M_1 \) if \( g_i = 1 \); else if \( g_i = 2 \), then \( J_i \) is scheduled by the following policy: if \( 0 < s \leq \frac{\sqrt{5}}{2} - 1 \), then \( J_i \) is scheduled on \( M_1 \); if \( \frac{\sqrt{5}}{2} - 1 < s \leq 1 \) and \( \frac{1+2s}{s} \leq \left( \frac{1+2s}{\sqrt{5}} \right) \left( \max \{ p_{max}, \frac{T_i}{1+s}, T^k \} \right) \), then \( J_i \) is scheduled on \( M_1 \); if \( 1 < s \leq s^* \) and \( \frac{1+2s}{s} \leq \left( \frac{1+2s}{\sqrt{5}} \right) \left( \max \{ p_{max}, \frac{T_i}{1+s}, T^k \} \right) \), then \( J_i \) is scheduled on \( M_2 \), else \( J_i \) is assigned to machine \( M_1 \); and if \( s \geq s^* \), then \( J_i \) is scheduled on \( M_2 \). They achieved UB \( \min \{ 1 + s, \frac{1+2s}{\sqrt{5}} \} \) for \( 0 < s < 1 \) and UB \( \min \{ \frac{1+\sqrt{5}+s}{2}, \frac{1+2s}{s} \} \) for \( s \geq 1 \). (Note that: \( T^k \) is referred to as sum of the sizes of the first \( k \) jobs, \( T_k^k \) represents the sum of the sizes of the set of jobs belongs to first \( k \) jobs for which \( g_i = 1 \) and \( s^* \in (1.3247, 1.3248) \)).

**TGRP.** Luo and Xu [88] followed the work of Chassid and Epstein [59] for semi-online scheduling on 2-parallel machines with Max-Min objective. They considered hierarchical scheduling to cater different levels of services to the input jobs. They investigated two semi-online variants, one with TGRP[1, b] and obtained lower bound of \( 1 + b \) for \( b \geq 1 \). In the second case, they considered TGRP[1, b], Sum and achieved lower bound of \( b \) for \( 1 \leq b < 2 \). Cao and Liu [99] studied
the setup $Q_2|TGRP|C_{\text{max}}$ by considering $S_1=1$, $S_1 \leq S_2= \forall i, p_i \in [p, x]$, where $p > 0$ and $x \geq 1$. They proved tight bound of $\min\{\frac{2s+1}{s+1}, \frac{s+1}{s}, x\}$ for algorithm LS, where $1 \leq s \leq 1.618$ and $x \geq \frac{1+s-s^2}{s}$ with $\alpha=\frac{1+s-s^2}{s^3}$. They proposed a $s$-competitive algorithm, which is tight for $1.325 \leq s \leq 1.618$ and $x \leq \frac{s-1}{s+1}$. The algorithm schedules an incoming job $J_i$ on machine $M_2$ if $l_i^2 + \frac{p_i}{s} \leq \frac{1+s-s^2}{s}$; otherwise schedules $J_i$ on machine $M_1$. Further, they designed a new algorithm that achieves the following tight bounds for the unexplored speed ratio intervals, expressed in an algebraic function $r(s)$.

$$r(s) = \begin{cases} s, & \text{for } s \in [1.206, 1.5] \text{ and } s \leq x < \min\{2s-1, \frac{2s^2-2}{1+s-s^2}\} \\ \frac{1+s}{s}, & \text{for } s \in [1, 1.28] \text{ and } \max\{2s-1, \frac{-s+\sqrt{9s^2+8\alpha}}{2s}\} < x \leq \frac{2}{s} \end{cases}$$

The new algorithm schedules the initial four incoming jobs $J_1, J_2, J_3$ and $J_4$ on machines $M_2, M_1, M_2$ and $M_1$, respectively. Thereafter, each incoming $J_i$, where $i \geq 5$ is scheduled on machine $M_2$ if $l_i^2 + \frac{p_i}{s} \leq \frac{k-1}{k+1}$, otherwise schedules $J_i$ on machine $M_1$. (Note that: $l_i$ is the load of machine $M_j$ just before the scheduling of job $J_i$.)

**Job Reassignment.** Englert et al. [93] initiated the study on online scheduling in $m$-uniform machine with job reassignment policy, where $m \geq 2$. The aim is to explore, how far the reassignment of a small number of jobs helps to improve the $CR$ of an algorithm designed for makespan minimization in pure online setup. They proposed an algorithm that achieves a $CR$ between 1.33 and 1.7992 for different speed ratio intervals with at most $m$ reassignments. The algorithm functions in two phases, where in the first phase, online arriving jobs are scheduled on $m$ machines. And in the second phase, a specified number of jobs (at most $m$) are removed from the allocated machines and are re-scheduled on different machines as per a defined set of rules. We now present a summary of the main results obtained in recent years for semi-online scheduling on related machines in table 8.

**Identical Machines:**

**GOS.** Liu et al. [73] studied the setup $P_2|GOS, TGRP|a, ba|C_{\text{max}}$, where $a > 0$ and $b > 1$. The GOS model studied here considers two machines, where one can afford higher quality in service called higher GOS machine and the other one can cater normal quality in service called lower GOS machine. Each newly available job $J_i$ reveals its $p_i$ and $g_i$, where $g_i \in \{1, 2\}$. If $g_i=1$, then $J_i$ must be executed on machine $M_1$; if $g_i=2$, then $J_i$ can be executed on any of the machines. They proposed the algorithm $B$-ONLINE by following the the policy of Park et al. [46] and obtained the following tight bounds, expressed by an algebraic function $r(s)$.

$$r(s) = \begin{cases} \frac{1+b}{2}, & \text{for } 1.785 \leq b \leq 2 \\ 1.5, & \text{for } 2 \leq b \leq 5 \\ \frac{4+b}{6}, & \text{for } 5 \leq b \leq 6 \end{cases}$$
Algorithm $B$-ONLINE works by the following policy. Initialize the parameters $l_1=0$, $l_2=0$, $P_{\text{max}}=0$, $X=0$ and $T=0$. Upon receiving a new job $J_i$, update $P_{\text{max}}=\max\{P_{\text{max}}, p_i\}$ and $X=X+\frac{p_i}{2}$. If $g_i=1$, then schedule $J_i$ on machine $M_1$ and update $T=T+p_i$. If $g_i=2$, then $J_i$ is scheduled on $M_2$ if $l_2+p_i \leq r(s) \cdot L$, where $L=\max\{X, T, P_{\text{max}}\}$; otherwise, $J_i$ is assigned to machine $M_1$.

Further, they studied the setup $P_2|\text{GOS}, TGRP[a, b], \text{Sum}|C_{\max}$ and proposed the $(\frac{1+b}{2})$-competitive optimal algorithm $B$-SUM-ONLINE for $\text{Sum} \geq \left(\frac{2b}{b-1}\right) \cdot a$ and $1 < b < 2$. Algorithm $B$-SUM-ONLINE schedules an incoming job $J_i$ on machine $M_1$ if $g_i=1$. If $g_i=2$ and $l_2+p_i \leq \left(\frac{1+b}{2}\right) \cdot L$, then $J_i$ is scheduled on $M_2$; otherwise, $J_i$ is scheduled on $M_1$. Wu et al. [77] followed the work of Liu et al. in the study of the setup $P_2|\text{GOS}, \text{Opt}|C_{\max}$ and proposed a $1.5$ competitive optimal algorithm. The algorithm schedules an incoming job $J_i$ on machine $M_1$ if $g_i=1$. If $g_i=2$ and $l_2+p_i \leq (1.5) \cdot C_{\text{OPT}}$, then $J_i$ is scheduled on $M_2$; otherwise, $J_i$ is assigned to machine $M_1$. The objective is to keep the loads of both machines under $(1.5) \cdot C_{\text{OPT}}$. Further, they considered $\text{GOS}$ with known $p_{\text{max}}$ in 2-identical machine setup and proposed $1.618$ competitive optimal algorithm $\text{Gos-Max}$. Algorithm $\text{Gos-Max}$ works as follows: upon receiving a job $J_i$, update $X=X+\frac{p_i}{2}$, where $X=0$. If $g_i=1$, schedule $J_i$ on $M_1$ and update $T=T+p_i$, where, $T=0$. If $g_i=2$ and $l_2+p_i \leq (1.618) \cdot L$, then $J_i$ is scheduled on $M_2$, where, $L=\max\{p_{\text{max}}, X, T\}$; else schedule $J_i$ on machine $M_1$.

Chen et al. [80] studied the setup $P_2|\text{GOS}, B(k)|C_{\max}$ by considering known

### Table 8. Summary of the Recent Contributions for Related Machines

| Author(s), Year | setup($\alpha|\beta|\gamma$) | Competitiveness Results |
|-----------------|-------------------------------|-------------------------|
| Epstein et al. 2011 [95] | $Q_2|\text{reB}(k)|C_{\min}$, $Q_m|\text{reB}(k)|C_{\min}$ | $\frac{2m+1}{2m+1}$ Tight for $Q_2$ and $m$ Tight for $Q_m$ |
| Cai and Yang 2011 [97] | $Q_2|\text{Max}|C_{\max}$ | $\frac{2m+2}{2m+2}$ for $s \in [1, 1.414]$; $\frac{2m+1}{2m+1}$ for $s \in [1.414, 2.732]$ |
| Dosa et al. 2011 [69] | $Q_2|\text{Opt}|C_{\max}$, $Q_2|\text{Sum}|C_{\max}$ | $\min\{1+\frac{1}{t}, 1+\frac{s}{s+1}, 1+\frac{1}{s+1}\}$ LB with $\text{Opt}$, $(1+\frac{1}{t})$ Tight with $\text{Sum}$ |
| Hou and Kang 2011 [98] | $Q_m|\text{pmtn}, \text{GOS}|C_{\min}$, $Q_m|\text{pmtn}, \text{GOS}, \text{Sum}|C_{\max}$ | $\frac{2m+1}{2m+1}$ UB for $C_{\min}$, $1$ Tight for $C_{\max}$ |
| Lan et al. 2012 [76] | $Q_m|\text{B-osn}|C_{\max}$ | $(2-\frac{1}{m}+\epsilon)$ Tight with $k=m$ |
| Lu and Liu 2013 [81] | $Q_2|\text{GOS}, \text{Opt}|C_{\max}$, $Q_2|\text{GOS}, \text{Sum}|C_{\max}$, $Q_2|\text{GOS}, \text{Max}|C_{\max}$ | $\min\{\frac{1+b}{1+b}, \frac{1}{b}\}$ Tight with $\text{Opt}$ or $\text{Sum}$, $\min\{\frac{1+\sqrt{T+1}}{T+1}, \frac{1+s}{s}\}$ Tight with $\text{Max}$ for $s \geq 1$. |
| Luo and Xu 2015 [88] | $Q_2|\text{GOS}, TGRP|C_{\min}$, $Q_2|\text{GOS}, TGRP, \text{Sum}|C_{\min}$ | $(1+b)$ LB with $TGRP$, $(b)$ LB with $TGRP$ and $\text{Sum}$. |
| Dosa et al. 2015 [89] | $Q_2|\text{Opt}|C_{\max}$ | $\frac{b+2}{b+2}$ LB for $s \in [1.3956, 1.443]$, $\min\{\frac{3}{2}, 1, 1.81+10s/213, \frac{3}{2}, 1.81+10s/213\}$ LB for $s \in [1.666, 1.725]$. |
| Cao and Liu 2016 [99] | $Q_2|\text{TGRP}|C_{\max}$ | $s$ Tight for $s \in [1.325, 1.618]$ |
| Dosa et al. 2017 [91] | $Q_2|\text{Opt}|C_{\max}$ | $\left(\frac{s+1}{s+1}\right)$ Tight for $s \approx 1.7258$, $\left(\frac{1}{s+1}\right)$ Tight for $1.725 \leq s < 1.732$. |
| Englert et al. 2018 [93] | $Q_m|\text{casgn}|C_{\max}$ | 1.7992 UB |
such that $g_i \in \{1, 2\}$ for each incoming job $J_i$. It is assumed that machine $M_1$ can execute all jobs, whereas machine $M_2$ can execute the jobs having $g_i=2$. They proposed a 1.5 competitive optimal algorithm, which always tries to keep the largest job having $g_i=2$ in the buffer and schedule it at the end. The algorithm works in two phases, wherein the first phase, all jobs having $g_i=1$ are scheduled on $M_1$ and maximum possible jobs are assigned to $M_2$ as long as the desired $CR$ holds. In the 2nd phase, the largest job in the buffer is scheduled on the smallest loaded machine. Further, they studied the setup $P_2(GOS, reasgn(k))|C_{max}$ and proposed a 1.5 competitive optimal algorithm with $k=1$. The idea is to schedule maximum number of jobs on a particular machine $M_j$ until $l_j$ reaches up to a defined threshold, then reassign the largest job scheduled on $M_j$ to the other machine $M_{3-j}$. Zhang et al. [82] improved the bounds obtained by Liu et al. in [73] with $GOS$ and $TGRP(a,ba)$ for $1 \leq b < 3$. Further, they proved that use of preemption and idle time do not improve the competitiveness of the pure online setting of hierarchical scheduling in 2-identical machines. Luo and Xu [85] improved the bounds given in [46, 77] for 2-identical machines with known $Sum$ and different $GOS$ levels such as higher $GOS$ and lower $GOS$. Chen et al. [100] extended their previous work [80] with similar idea and considered three different setups of online hierarchical scheduling in 2-identical machines. They studied the setup, where $\sum p_i$ for the jobs with $g_i=1$ is known and proved a tight bound 1.5 for algorithm $LS$. In another setup, they assumed known values of $T_1=\sum p_i=1, \forall J_i$ such that $g_i=1$ and the value of $T_2=\sum p_i=T > 0, \forall J_i$ such that $g_i=2$. They proposed the algorithm $CMF$, which achieves a tight bound of 1.33. Algorithm $CMF$ adopts the following rule: schedule an incoming job $J_i$ by its $g_i \in \{1, 2\}$ to respective $M_j \in \{M_1, M_2\}$ if $T \leq 2$. If $T > 2$, else if $g_i=1$, then $J_i$ is scheduled on machine $M_1$, else if $g_i=2$, then $J_i$ is assigned to $M_1$ if $l_1^1 + p_x \leq \frac{1+T}{2}$, else let $x=i$, schedule $J_x$ and the remaining jobs by following rule: assign $J_x$ to $M_2$ and remaining jobs to $M_1$ if $l_1^1 + p_x > \frac{2(1+T)}{3}$, else $J_x$ along with all future jobs for which $g_i=1$ are scheduled on $M_1$ and rest of the jobs are assigned to machine $M_2$. Further, they considered $B(k)$ in the first setup and obtained a tight bound of 1.33 with $k=1$.

Qi and Yuan [101] addressed the research challenge posed by Chen et al. in [80] regarding an unified approach for semi-online hierarchical scheduling with buffer or reassignment. However, they opened up a new direction by introducing $L_p$-norm load balancing($C(p)$) as an optimality criterion for semi-online hierarchical scheduling in 2-identical machine setup. Let us represent in a schedule the final loads of $M_1$ and $M_2$ by $l_1$ and $l_2$ respectively. The load vector is represented by $L=\{l_1, l_2\}$. The $L_p$-norm is denoted as $\|L_p\|$ and defined as follows:

$$\|L_p\| = \begin{cases} (l_1^p + l_2^p)^{\frac{1}{p}}, & \text{for } 1 \leq p < \infty \\ \max\{l_1, l_2\}, & \text{for } p = \infty \end{cases}$$

They argued that $L_p$-norm objective is practically more significant than makespan, as it captures the average machine loads instead of the largest load among the machines. They obtained tight bound 1.5 by separately considering $B(k)$ and $reasgn(k)$ respectively with $k=1$ for $p=\infty$. Xiao et al. [102] followed the work
of Chen et al. [100] and addressed \( C_{\min} \) objective in a setting, where \( \text{sum of the sizes of low hierarchy jobs} (T_1 = 1) \) is known and \( B(1) \) is given. They proposed the algorithm BLS, which achieves a \( \text{tight} \) bound 1.5 for \( C_{\min} \). Algorithm BLS schedules an incoming job \( J_i \) on \( M_1 \) if \( g_i = 1 \). If \( g_i = 2 \), put \( J_i \) on the buffer, (let \( B_{\max} = \max\{p_i | \text{jobs in the buffer}\}, B_{\min} = \min\{p_i | \text{jobs in the buffer}\} \)) and if \( l_2 + B_{\max} \geq \frac{k_1 + T_0 + B_{\max}}{2} \), then \( J_{\min} \) is scheduled on \( M_1 \); else, \( J_{\min} \) is assigned to machine \( M_2 \). Further, they considered the setting, where \( T_1 \) is given and \( p_{\max} \) for hierarchy 2 i.e. \( p_{\max}^2 \) is known. They obtained a \( \text{tight} \) bound 1.5 for \( C_{\min} \). Qi and Yuan [103] studied the setup \( P_2|\text{GOS}, \text{Sum}|C_p(\text{p}) \) and proposed an algorithm that achieves a \( \text{tight} \) bound 1.5 for \( p=\infty \). The algorithm schedules an incoming \( J_i \) on machine \( M_i \) if \( g_i = 1 \). If \( g_i = 2 \), and \( l_2 + p_i \leq \frac{3}{4} \cdot T \), then \( J_i \) is assigned to machine \( M_2 \). If \( g_i = 2 \) and \( l_2 + p_i > \frac{3}{4} \cdot T \), then schedules \( J_i \) by the following rule: if \( l_2 < \frac{1}{4} \cdot T \) and \( l_2 \leq \frac{T \cdot p_i}{2} \), then schedule \( J_i \) on \( M_2 \) and all future jobs on \( M_1 \). If \( l_2 < \frac{1}{4} \cdot T \) and \( l_2 > \frac{T \cdot p_i}{2} \), then schedule \( J_i \) on \( M_1 \) and all future jobs with \( g_i = 2 \) on \( M_2 \) and jobs with \( g_i = 1 \) on \( M_1 \). If \( l_2 \geq \frac{1}{4} \cdot T \), then schedule \( J_i \) along with all future jobs on machine \( M_1 \). The idea is to schedule larger number of jobs on \( M_2 \) as long as \( l_2 \) would not exceed \( l_1 \). Importantly, the algorithm handles the larger size jobs with known \( T \). Further, they studied the setup \( P_2|\text{GOS}, T_1, T_2|C_p(\text{p}) \) and obtained a \( \text{tight} \) bound of 1.33 for \( p=\infty \). In future, some interesting consideration would be semi-online hierarchical scheduling for \( L_p \)-norm optimization with other unexplored \( EPIs \) such as \( \text{Max}, \text{TGRP}, \text{Opt}, \text{Decr} \) etc. The study remains open in \( m \)-identical machine for \( m > 2 \) and related machine setups. We now present the summary of important results for semi-online scheduling in identical machines with \( \text{GOS} \) in table 9.

**TGRP.** Cao et al. [72] studied the setup \( P_2|\text{TGRP}, \text{Max}|C_{\max} \) by considering \( \text{TGRP}[a, ba] \) and \( p_{\max} = ba \), where \( a > 0 \) and \( b \geq 1 \). They obtained \( \frac{b+1}{b+2} \) \( LB \) for \( 1 \leq b < 1.33 \) and \( \frac{3b+1}{b+2} \) \( LB \) for \( b \geq 2 \). They proposed algorithm \( \text{PIJS} \), which achieves a \( \text{tight} \) bound \( \max\{\frac{4(b+1)}{3b+4}, \frac{2b}{b+1}\} \) for \( 1.33 \leq b \leq 2 \). Algorithm \( \text{PIJS} \) schedules an incoming job \( J_i \) on machine \( M_1 \) if \( l_1 + p_i \leq k \cdot \max\{q_1^i + q_2^i + \ldots + q_{\left[\frac{i-1}{2}\right]}^i, \frac{l_1 + l_2 + p_i + p_{\max}}{2}\} \); otherwise \( J_i \) is scheduled on \( M_2 \). And this continues until the arrival of the first largest job (let \( J_{\max} \)). When \( J_{\max} \) arrives, it is scheduled on machine \( M_2 \). Thereafter, each incoming \( J_i \) is scheduled on \( M_1 \), if \( l_1 + p_i \leq k \cdot \max\{q_1^i + q_2^i + \ldots + q_{\left[\frac{i-1}{2}\right]}^i, \frac{l_1 + l_2 + p_i}{2}\} \); else \( J_i \) is scheduled on machine \( M_2 \). (Note that: \( k = \max\{\frac{4(b+1)}{3b+4}, \frac{2b}{b+1}\} \), \( l_{ij}^i \) is the load of machine \( M_j \) just before the assignment of \( J_i \), and \( q_{\left[\frac{r}{2}\right]}^i \) is the \( r^{\text{th}} \) smallest job at the arrival of \( J_i \) i.e. \( \{q_{1}^i, q_{2}^i, \ldots, q_{r}^i\} \) such that \( p_1^i \leq p_2^i \leq \ldots \leq p_r^i \).) The idea given in this study reveals that when \( p_{\max} \) is known in advance, it is better to assign \( J_{\max} \) at the outset. Cao and Wan [84] studied the setup \( P_2|\text{TGRP}[1, b], \text{Decr}|C_{\max} \). They showed that algorithm \( \text{LS} \) achieves a \( \text{tight} \) bound 1.16 for \( 1 \leq b \leq 1.5 \). With only known \( \text{TGRP}(ub) \), they obtained \( LB \) 1.16, which matches the \( UB \) given by Seiden in [21], where, \( ub \geq 1.5 \).

**Arrival Order of Jobs.** Li et al. [70] studied the scenario where an incoming job \( J_i \) requests an order to the scheduler with its release time \( r_i \) and processing
Table 9. Summary of the Recent Works on Identical Machines with GOS

| Author(s), Year | setup(α/β/γ) | Competitiveness Results |
|-----------------|----------------|-------------------------|
| Liu et al. 2011 [73] | P_2|GOS, TGRP|C_{max} \ P_2|GOS, TGRP, Sum|C_{max} | max(\frac{1+b}{2}, 1.5, \frac{1+b}{2}) Tight with GOS and TGRP for 1 < b < 6, \frac{1+b}{2} Tight with GOS, TGRP and Sum for 1 < b < 2. |
| Wu et al. 2012 [77] | P_2|GOS, Opt|C_{max} \ P_2|GOS, Max|C_{max} | (1.5, 1.618) Tight for respective setups. |
| Chen et al. 2013 [80] | P_2|GOS, B(k)|C_{max} \ P_2|GOS, reasgn(k)|C_{max} | 1.5 Tight with k = 1 for both setups |
| Zhang et al. 2013 [82] | P_2|GOS, TGRP|C_{max} | P_2|GOS, TGRP|C_{max} | 1.66 Tight with b ≥ 3 for N-pmntn, 1.5 Tight with b ≥ 2 for pmtn. |
| Luo and Xu 2014 [85] | P_2|GOS, Sum|C_{max} | (1.5, 1.53, 1.33) Tight for Sum with (higher GOS or lower GOS or both) respectively. |
| Chen et al. 2015 [100] | P_2|GOS, Sum|C_{max} \ P_2|GOS, B(1)|C_{max} | 1.33 Tight for respective setups |
| Qi and Yuan 2016 [101] | P_2|GOS, B(1)|C_{max} \ P_2|GOS, reasgn(1)|C_{max} | 1.5 Tight for both setups for p=∞ |
| Xiao et al. 2019 [102] | P_2|GOS, T_1, B(1)|C_{min} \ P_2|GOS, T_1, B(1)|C_{min} | 1.5 Tight for both setups |
| Qi and Yuan 2019 [103] | P_2|GOS, Sum|C_{max} | 1.5 Tight for p=∞ |

Then, the scheduler service the order by non-preemptively schedule the job with the objective to optimize the makespan. They considered that jobs are arriving by non-decreasing release times (Inc-r) and non-increasing sizes (Decr). They analyzed the performance of algorithm LS and obtained UB $\frac{3}{2} - \frac{1}{2m}$ for m-identical machine setup. Cheng et al. [78] studied the setup $P_m|\text{Decr}|C_{max}$. They analyzed algorithm LPT and proved tight bounds 1.18 for $m=3$ and 1.25 for $m > 3$. Tang and Nai [87] refined the results of Li et al. [70] and derived a new proof for the UB of algorithm LS.

**Job Reassignment.** Min et al. [71] considered the job’s assignment policy of Tan and Yu [57]. They obtained a tight bound of 1.41 for semi-online scheduling on 2-identical machine by allowing the reassignment of the last job of one machine only. Further, they considered known Sum besides the reassignment policy and improved the previous best UB of 1.33 to 1.25.

**Combined Information.** Cao et al. [74] considered several semi-online variants for scheduling on 2-identical machine with min-max objective. They proposed 1.2 competitive optimal algorithm OM with known Opt and Max. Further, they considered combined information on $(B(k), \text{Max})$, $(B(1), \text{Decr})$, $(B(1), \text{TGRP}(1,b))$ and obtained tight bounds (1.25, 1.16, 1.33) respectively. Lee and Lim [79] studied the setup $P_m|\text{Sum, Max}|C_{max}$ and achieved UBs (1.462, 1.5) for $m=4$, 5 respectively.

**Sum.** Albers and Hellwig [75] studied the setup $P_m|\text{Sum}|C_{max}$. They improved the LB $1.565$ [35] to 1.585 for $m$-identical machine setup, where $m \to \infty$. They
proposed algorithm Light Load, which is free from traditional job class policy considered in \[35, 44\] and achieves an UB 1.75. Lee and Lim [79] investigated the setup \(P_m|\text{Sum}|C_{\text{max}}\) for small number of machines. They obtained LBs of 1.442, 1.482 and 1.5 for \(m=4, 5\) and 6 respectively. An algorithm named ForwardFit-BackwardFit-ListScheduling was proposed by assuming \(\text{Sum} = m\), which achieves UBs of 1.4, 1.4615 and 1.5 for \(m=3, 4\) and 5 respectively. The algorithm prefers two conditions, where in the first condition, a load threshold is set up to the defined competitive bound to keep the loads of the machines under the threshold value. Before scheduling an incoming job, the loads of each machine is checked against the load threshold value. If the first condition fails, then the incoming job is scheduled by algorithm LS. An obvious question raised here is: How to choose the threshold value, which always guarantees that the scheduling of all jobs would yield the defined competitive bound for any \(m\)?

Kellerer et al. [90] obtained an UB 1.585 by considering \(\text{Sum} = m\), which matches the LB achieved by Albers and Hellwig in [75] for \(m\)-identical machine setup. They adopted the job class policy and classified the incoming jobs into four classes such as tiny, small, medium and large depending on their sizes, defined by the time intervals \((0, \alpha_2]\) for tiny, \((0, \alpha]\) for small, \((\alpha, \frac{1}{2}\alpha]\) for medium, \([\frac{1}{2}\alpha, 1]\) for large. Similarly, \(m\) machines were classified as tiny, small, medium, big and huge depending on their loads defined by the time intervals \((0, \alpha]\) for tiny, \((0, \alpha]\) for small, \((\alpha, \frac{1}{2}\alpha]\) for medium, \((\frac{1}{2}\alpha, 1]\) for big and \([1, \infty)\) for huge, where \(\alpha=0.585\). They proposed an algorithm, which executes in 2 phases, where in the 1\textsuperscript{st} phase, jobs are scheduled on the machines depending on the classes of jobs and machines. The 2\textsuperscript{nd} phase of the algorithm, emerges from the 1\textsuperscript{st} phase and runs two policies with respect to the current machines loads after the 1\textsuperscript{st} phase. Here, the classification of jobs and machines helps in improving the tightness in the competitive bound to \(1 + \alpha\). A natural question pops out from here is: Can an algorithm be possible for \(P_m|\text{Sum}|C_{\text{max}}\) with job class policy and \(\alpha < 0.585\)?

**Buffer.** Lan et al. [76] studied the general cases for identical machines by considering buffer as additional feature. For \(m\)-identical machines they obtained a tight bound 1.5 with a buffer of size 1.5\(m\).

**Opt.** Kellerer and Kotov [104] studied the setup considered by Azar and Regev in [15]. They improved the UB from 1.625 to 1.571 by considering the job class policy, where the incoming jobs and available machines are classified by their sizes and current loads respectively, defined by the specified time interval ranges. It was proved that a two phase algorithm with job class policy always guarantees the loads of each machine to be under 1.571 of the known Opt. Gabay et al. [105] further improved the UB to 1.5294. The current best known UB 1.5 for the setup \(P_m|\text{Opt}|C_{\text{max}}\) is due to Bohm et al. [106]. Further, they obtained UB 1.375 for \(m=3\). Gabay et al. [107] improved the LB 1.33 to 1.357 for the setup \(P_m|\text{Opt}|C_{\text{max}}\). Thus, minimizing the gap between the current best LB and UB in \(m\)-identical machine semi-online scheduling with known Opt. We now present the summary of important results obtained for semi-online scheduling in identical machines other than known GOS in table \[10\].
Table 10. Summary of the Recent Works on Identical Machines

| Author(s), Year | setup(α|β|γ) | Competitiveness Results |
|-----------------|------------|-------------------------|
| Cao et al. 2011 [72] | P_2[TGRP, Max|C_{max}] | \( \frac{2}{5} \) LB for \( 1 \leq b < 1.33, 1.33 \) LB for \( b \geq 2, \max\{\frac{b}{b+1}, \frac{b}{b+2}\} \) Tight for \( 1.33 \leq b < 2 \). |
| Li et al. 2011 [70] | P_m[Incr - r, Decr|C_{max}] | (\( \frac{1}{2} \) - \( \frac{1}{2m} \)) Tight. |
| Min et al. 2011 [71] | P_2[reasgn(last(1)^*)|C_{max}] | (1.41, 1.25) Tight for respective setups. |
| Cao et al. 2012 [74] | P_2[Opt, Max|C_{max}] | (1.2, 1.25, 1.16, 1.33) Tight for respective setups. |
| Albers and Hellwig 2012 [75] | P_m[Sum|C_{max}] | 1.585 LB, 1.75 UB |
| Lan et al. 2012 [76] | P_m[B(1.5m)|C_{max}] | 1.5 Tight |
| Cheng et al. 2012 [78] | P_m[Decr|C_{max}] | 1.18 Tight for \( m = 3 \), 1.25 Tight for \( m > 3 \). |
| Lee and Lim 2013 [79] | P_m[Sum|C_{max}, P_m[Max|C_{max}] | P_m[Sum, Max|C_{max}] | (1.4, 1.4615, 1.5) UB for \( m=3,4,5 \) with \( \sum \). (1.618, 1.667) Tight for \( m=4,5 \) with \( \max \). (1.462, 1.5) UB for \( m=4,5 \) with \( \sum \) and \( \max \). |
| Kellerser and Kotov 2013 [104] | P_m[Opt|C_{max}] | 1.571 UB. |
| Cao and Wan 2014 [84] | P_2[TGRP[1,b], Decr|C_{max}] | P_2[TGRP[ub], Decr|C_{max}] | 1.16 Tight for \( TGRP[1,b] \) and \( 1 \leq b < 1.5, 1.166 \) LB for \( TGRP[ub] \) and \( ub \geq 1.5 \). |
| Tang and Nie 2015 [87] | P_m[Incr - r, Decr|C_{max}] | (\( \frac{1}{2} \) - \( \frac{1}{2m} \)) UB. |
| Kellerser et al. 2015 [90] | P_m[Sum|C_{max}] | 1.585 Tight for \( m \rightarrow \infty \). |
| Gabay et al. 2015 [105] | P_m[Opt|C_{max}] | 1.5294 UB. |
| Bohm et al. 2017 [106] | P_m[Opt|C_{max}, P_3[Opt|C_{max}] | 1.5 UB for \( P_m \), 1.375 UB for \( P_3 \). |
| Gabay et al. 2017 [107] | P_m[Opt|C_{max}] | 1.357 LB. |

5 Emergence of Semi-online Scheduling Setups and Classification of the Related Works

Evolution Time-line for Semi-online Scheduling Setups. After making a comprehensive literature survey, we understand and explore various problem setups, research directions and research trends in semi-online scheduling. In this section, we sketch a time-line to represent the emergence of various semi-online scheduling setups as shown in figure [7].

Classification of Related Works based on EPI. Though many researchers have exhaustively studied semi-online scheduling based on either a single EPI or more than one EPIs, there is hardly any attempt to develop a taxonomy to classify the literature and related works based on EPI. Here we attempt to classify the whole literature on semi-online scheduling based on EPIs for identifying
related works for various setups. We present our classification in figure [7]. We consider the setups for identical($P$) and uniform($Q$) machines in our classification. The other parameters for various problem setups are processing formats such as non-preemptive($N$–$pmtn$) and preemptive($pmtn$) and optimality criteria such as makespan($C_{\text{max}}$) and Max-Min($C_{\text{min}}$). We also provide links to references of related works for each problem setup. For each $EPI$, we have mentioned the various setups which are studied in the literature along with their references. Our classification may help the researchers to focus on related works and explore specific research directions for future work. The future research work can also be carried out based on specific $EPI$ by choosing a particular set up from our classification.
6 Research Challenges and Open Problems

Semi-online scheduling has been extensively studied in various setups for the last two decades. Still, there are many research issues, which can lead to further investigations in this area. We conclude our survey on the important results and critical ideas for semi-online scheduling by exploring some of the non-trivial research challenges and open problems as follows.

6.1 Research Challenges

- Exploration of practically significant new EPIs that can help in improving the CR of the existing online scheduling algorithms.
- Generation, characterization and classification of the input job sequences that can resemble the real-world inputs in various semi-online scheduling setups.
- Minimize or diminish the gap between LB 1.585 and UB 1.6 for the setup $P_m|\text{Sum}|C_{\text{max}}$. 

Fig. 7. Classification of Related Works based on EPI
– Reduce the CR 1.366 for preemptive $P_m | C_{max}$ setting.
– Improvement of 1.5-competitive strategy for $P_2 | C_{max}$ problem with inexact partial information. The solution for $P_m | C_{max}$ problem is unknown in this case.
– Close or remove the gap [1.442, 1.5] between lower and upper bound for semi-online scheduling on unbounded parallel batch machine.
– Design of optimal semi-online scheduling algorithm with at most 1.5-competitiveness for scheduling on $m$-identical machines ($m \geq 2$) under GOS and known Opt.
– Exploration of optimal semi-online algorithms for $P_m | C_{max}$ and $Q_m | C_{max}$ setups with reassignment of job policy.

6.2 Open Problems

– Can EPI be used to develop a new complexity class for evaluating the performance of online algorithms?
– Does there exist an optimal semi-online algorithm with CR less than 1.33 for the setup $P_m | B(k) | C_{max}$ with $k=1$?
– Can the CR 1.2 be improved for the setup $P_2 | Sum | C_{max}$? Can a matching bound be possible for $P_m | Sum | C_{max}$? How far preemption can help in improving the results in this setting?
– How can we establish a relationship between semi-online scheduling and online scheduling with look ahead? Which one is practically significant? For an instance, is it possible to obtain a CR less than or equal to 1.33 for $P_m | Sum | C_{max}$, where $T$ is known for $k$ future jobs and $1 \leq k < n$.
– Can a tight bound be possible for $P_3 | C_{min}$, which is independent of number of machines?
– Does there exist an optimal semi-online algorithm for the setup $Q_m | C_{min}$? Already, a 1-competitive algorithm is known for $Q_2 | C_{min}$ due to [59].
– What can be an optimal semi-online policy for $P_m | C_{max}$? Can a 1-competitive semi-online algorithm be possible for this setting with known Decr [108]?
– What can be a tight bound for semi-online scheduling on uniform machines with overall speed ratio interval of [1, $\infty$)?
– Does there exist an optimal semi-online strategy for multiple unbounded parallel batch processors?
– Can EPI be helpful in improving the best competitive bounds obtained for online scheduling in various setups of unrelated parallel machines?

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