Solving nuclear shape conundrum at HIE-ISOLDE

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Abstract.
It is the purpose of this paper to illustrate some of the low-energy nuclear physics that we want to pursue at the new HIE-ISOLDE radioactive-ion-beam facility at CERN. The University of the Western Cape leads an experimental proposal and co-authors two Letters of Intent, in collaboration with European institutions, at HIE-ISOLDE. Timely topics such as “Exploration of K-isomerism using unique high-K isomeric beams - CERN-INTC-I-101” and “Shape changes and proton-neutron pairing around the N = Z line - CERN-INTC-I-102” are addressed in these Letters of Intent. Our experimental proposal aims at performing a multi-step Coulomb-excitation of radioactive $^{70}$Se ion beams using the $^{208}$Pb($^{70}$Se,$^{70}$Se$^*$) $^{208}$Pb$^*$ reaction at a bombarding energy of 5.5 MeV/u. The physics goal is a precise measurement of the $\hbar^2/2j(j+1)$ diagonal matrix element, related to the spectroscopic quadrupole moment, in $^{70}$Se. Full simulations presented in this work show distinct angular distributions for plausible values of the spectroscopic quadrupole moment; with a predicted uncertainty of approximately ±0.1 eb. Additional diagonal and transitional matrix elements will also be obtained. These results will shed light onto the origin of rarely-found oblate shapes and shape coexistence in this region of rapidly-changing shell structure.

1. Motivation
A remarkable feature of atomic nuclei is their ability to adopt different shapes for a small cost in energy compared to their total binding energy. Deformed nuclei may adopt axially-symmetric quadrupole shapes, which can either take prolate (cigar shape) or oblate (disk shape) forms. The vast majority of nuclei present prolate shapes; with only a handful of oblate-shaped exceptions. Even doubly closed-shell nuclei, in principle spherical in their ground state, can adopt a strongly-deformed shape in an excited state when Cooper pairs are broken. These strongly-deformed shapes were originally observed in doubly-magic $^{16}$O and explained by Morinaga as 4 particles ($p_4$) with the remaining 4 holes ($h_4$), excited across the $N = Z = 8$ ($N$ for neutrons and $Z$ for protons) shell closure [1]. Mixing calculations between the spherical shell-model ground-state and deformed $2p - 2h$ and $4p - 4h$ configurations supported Morinaga’s idea [2, 3]. These findings led to the phenomenon of shape coexistence [4], now generally observed throughout the nuclear chart except for light nuclei. These strongly-deformed configurations are generally accepted to arise from the residual proton-neutron interaction [5]; the physical origin or mathematical form of which remains unknown.
Particularly, the neutron single-particle level energies calculated by the deformed nuclear shell-model for the $A \approx 70$ region (where $A = N + Z$) are shown in Fig. 1. A similar plot is expected for proton single-particle energies. The $A \approx 70$ region is one of the best testing grounds for rapidly-changing nuclear shapes because of the subshell gaps at proton and neutron numbers 34, 36 and 38. In addition, triaxial shapes, i.e., non axially-symmetric quadrupole shapes, have also been invoked in this region to explain extremely deformed prolate shapes, triaxial (or $\gamma$) vibrations and the enhancement of nuclear collectivity. Among the different shapes a nucleus can take, the rare occurrence of oblate shapes remains to be elucidated and requires special attention.

![Deformed single-particle energies for the orbitals of interest.](image)

**Figure 1.** Deformed single-particle energies for the orbitals of interest. Figure taken from [6].

A key tool for measuring nuclear deformation is the reorientation effect in Coulomb excitation measurements. This effect is a second-order perturbation that generates a time-dependent hyperfine splitting of the nuclear levels and changes the population of the different magnetic substates; hence, modifying the Coulomb-excitation cross section as a function of the magnitude and sign of the spectroscopic quadrupole moment, $Q_s$, i.e., the quadrupole shape of the nucleus in the laboratory frame [7]. The advantage of Coulomb-excitation studies is that, unlike the nuclear interaction, the electromagnetic interaction is well understood. Nuclear contributions can simply be eliminated by choosing a ‘safe’ bombarding energy well below the Coulomb barrier. Moreover, Coulomb-excitation experiments provide measurements of matrix elements. Assuming an ideal rotor, for a $J^p = 2^+$ state, $Q_s(2^+_1)$ is related to the diagonal matrix element, $\langle 2^+_1 \mid E2 \mid 2^+_1 \rangle$ by,

$$Q_s(2^+_1) = \sqrt{\frac{16\pi}{5}} \frac{1}{\sqrt{2J + 1}} \langle JJ20 \mid JJ \rangle \langle 2^+_1 \mid E2 \mid 2^+_1 \rangle = 0.75793 \langle 2^+_1 \mid E2 \mid 2^+_1 \rangle. \quad (1)$$

A reorientation-effect measurement of $^{70}$Se beams ($Z = 34$) has recently been carried out at REX-ISOLDE [8]. Such a study, when combined with recent accurate lifetime measurements using the recoil-distance Doppler shift method [9], suggests an oblate shape for the ground state of $^{70}$Se [9]. This is illustrated in Figure 2, where a positive $Q_s(2^+_1)$ value is deduced from the crossing of the Coulomb-excitation (black) sloping curve with the accurate $B(E2; 2^+_1 \rightarrow 0^+_1)$ value (red solid line) obtained from the lifetime measurement [9]. An earlier, and much shorter, lifetime measurement by Heese and collaborators suggests instead a prolate shape [10], as indicated by the overlap of the Coulomb-excitation curve and the horizontal dashed lines in
Fig. 2. Experimental and theoretical results in the light Se and Kr nuclei seems to suggest the emergence of oblate shapes as one approaches $N = Z$ [9]. However, the sudden decrease of the $B(E2; 2^+ \rightarrow 0^+)$ value from the NNDC accepted value of 44(9) W.u. [11] to the recent determination of 20.0(1.2) W.u. [9] cannot be accommodated theoretically (green asterisk in Fig. 2). A large uncertainty in the $Q_s(2^+_1)$ value prevents a more detailed comparison with theory. Aided by the higher yields and beam energies available at the new HIE-ISOLDE facility, the main goal of our experimental proposal is the accurate determination of the $(2^+_1 || \hat{E}2 || 2^+_1)$ diagonal matrix element, which, as shown in Fig. 2, currently spans in the range $-0.20 \lesssim \langle 2^+_1 || \hat{E}2 || 2^+_1 \rangle \lesssim +1.0$ eb.

![Figure 2](image_url)

**Figure 2.** The $B(E2; 2^+ \rightarrow 0^+)$ value as a function of $Q_s(2^+_1)$ for the $2^+_1$ state in $^{70}$Se. The asterisk indicates the theoretical value. The large uncertainty in the Coulomb-excitation curve prevents a conclusive $Q_s(2^+_1)$ value. Figure taken from [9].

2. Experimental Details and Full Simulations

The HIE-ISOLDE project is a major upgrade to the existing REX-ISOLDE radioactive-ion-beam (RIB) facility at CERN, with the imminent objective to provide 5.5 MeV/u post-accelerated RIBs. This bombarding energy is ideal for multi-step Coulomb-excitation studies. We have made full use of this potential and proposed the study of the $^{208}$Pb($^{70}$Se,$^{70}$Se$^*$)$^{208}$Pb$^*$ reaction at 5.5 MeV/u. Pure $^{70}$Se beams, required for Coulomb-excitation measurements, have previously been delivered at REX-ISOLDE [8] using SeCO$^+$ molecules and breaking this molecule in the electron beam ion source (EBIS) [8]. The same procedure should be available at HIE-ISOLDE. The bombarding energy of 5.5 MeV/u (or 385 MeV) is well below the Coulomb barrier, at around 427 MeV, and assuming Cline’s prescribed 5.0 fm separation between nuclear surfaces for heavy-ion reactions [12], safe for laboratory scattering angles $\theta_{lab} \leq 83^\circ$. A large Sommerfeld parameter of $\eta = 196 \gg 1$ validates the semiclassical approximation [13] and a small adiabaticity parameter of $\xi = 0.35$ enhances the population of the $2^+_1$ state in $^{70}$Se through Coulomb excitation.

The powerful particle-$\gamma$ coincidence technique will be utilised in this experiment. The de-excited $\gamma$-rays will be detected with the high-efficient MINIBALL array, composed of eight large cluster detectors and shown in Fig. 3, and the scattered $^{70}$Se ions with a forward double-sided CD-type silicon detector covering $[17^\circ, 52^\circ]$ scattering angles in the laboratory frame. Background $\gamma$ rays from the experimental hall can be suppressed by requiring a particle-$\gamma$ coincidence condition. The particle-$\gamma$ coincidence condition is satisfied by requiring a Ge hit.
and a hit in both the \( \theta \) ring and \( \phi \) sector of the Si detector within a time window. Events with Ge hits outside of this window are considered to be random coincidences and are background subtracted.

**Figure 3.** The MINIBALL HPGe \( \gamma \)-ray array (seeing from above), comprising eight large HPGe cluster detectors, at the ISOLDE facility.

**Figure 4.** Reaction kinematics for the \(^{70}\text{Se}\) ions and \(^{208}\text{Pb}\) recoils.

The state-of-art silicon detector provides kinematic characterization of scattered ions for Doppler corrections to the \( \gamma \)-rays emitted from the Coulomb-excited beam nuclei. Reaction-kinematics plots shown in the first two panels of Fig. 4 illustrate the energies of \(^{70}\text{Se}\) ions and \(^{208}\text{Pb}\) recoils as a function of scattering laboratory angles. Their distinct energies for the \([17^\circ, 52^\circ]\) angular range permits a separation in the particle energy spectra. The right panel of Fig. 4 guarantees the selection of either \(^{70}\text{Se}\) ions and \(^{208}\text{Pb}\) recoils for the \([17^\circ, 52^\circ]\) angular range.
Using matrix elements extracted from Refs. [9, 11], coupled-channel GOSIA calculations [14], based on the semiclassical approximation of Coulomb-excitation theory [13], have been performed in this work. The results are presented in Fig. 5 for the population of the $2^+_1$ state in $^{70}\text{Se}$ as a function of scattering angle in the laboratory frame and at plausible $Q_s(2^+_1)$ values given in Fig. 2. The differential cross section, shown in the left panel of Fig. 5, strongly depends on $Q_s(2^+_1)$, being stronger as the shape becomes more oblate, and peaks at $\theta_{\text{lab}} \approx 40^\circ$. The right panel of Fig. 5 shows counts per five days of beam time as a function of scattering angle, assuming a yield of $10^4$ ions/s and a 2-mg/cm$^2$ $^{208}\text{Pb}$ foil. The integrated yields for six sets of angular ranges have been summed up and illustrates the sensitivity to measure $Q_s(2^+_1)$. Five days of beam time (or fifteen shifts) will provide a measurement of $Q_s(2^+_1)$ with a $\pm 0.1$ eb statistical uncertainty given by the error bars. This uncertainty could be smaller with the increasing yields expected with the upgrade to 2 GeV proton energies planned at HIE-ISOLDE. Comparatively, a yield of $10^4$ ions/s for the previous $^{70}\text{Se}$ reorientation-effect measurement at 2.94 MeV/u yielded an area of 139(13) counts for the $2^+_1$ peak [8]. Assuming the same yield and $Q_s(2^+_1) = \pm 0.8$ eb, we approximately achieve 650 counts/day at 5.5 MeV/u; 500 counts/day for $Q_s(2^+_1) = 0$ eb.

Experimental integrated yields will be normalised to the well-known $B(E2; 2^+_1 \rightarrow 0^+_1) = 342(19)$ e$^2$f$m^4$ value in $^{70}\text{Se}$ to account for experimental unknowns in the setup, such as the systematic uncertainties in the absolute beam energy, target thickness, particle detection efficiency and dead time of the data acquisition. Uncertainties in the $\gamma$-ray efficiency, the $\phi$ asymmetry of the cluster detector and the 5.5% error in the $B(E2; 2^+_1 \rightarrow 0^+_1)$ value will also be considered; although the statistical uncertainty will likely dominate the quoted error on this measurement.

![GOSIA Simulation](image)

**Figure 5.** Simulated differential cross sections (left) and count rates per five days of beam time (right) as a function of scattering laboratory angle for the population of the $2^+_1$ state in $^{70}\text{Se}$. The dash lines in the right panel indicate the different angular ranges covered for the count-rate data points. The distinct angular distributions for plausible $Q_s(2^+_1)$ values indicate that a precise measurement of $Q_s(2^+_1)$ is feasible. The error bar accounts for an uncertainty of $\approx 0.1$ eb.

Another question lies in understanding the excitation mechanism of higher-lying levels shown in Fig. 6. With a total cross section of tens of mb, we expect to obtain information on transitional matrix elements relating the $2^+_1$, $4^+_1$, $0^+_2$, $4^+_2$ and $6^+_1$ states. About 250 counts calculated for the population of the $2^+_1$ state may provide information on the sign of $\langle 2^+_1 \mid E2 \mid 2^+_2 \rangle$ [8, 15].

![Diagram](image)
Figure 6. Theoretical (left) and experimental (right) excitation spectra and $B(E2)$ values (in units of $e^2fm^4$) for low-lying states in $^{70}$Se [9, 16]. Noticeable predictions are the large $B(E2; 2^+_2 \rightarrow 2^+_1) = 639 \ e^2fm^4$ value and the low-lying $0^+_2$ state.

3. Concluding Remarks

It is plausible that shape coexistence may be enhanced in $N \approx Z$ nuclei by the occupation of the same orbitals for protons and neutrons and the resulting neutron-proton interaction [4, 5]. In fact, low-lying excited $0^+$ states have been observed in other neutron-deficient Se [17, 18, 19] and Kr [20, 21, 22] isotopes and associated with shape coexistence. Figure 7 indicates the presence of an anomalous rotational behaviour for the $2^+_1$ state in the neutron-deficient Se, Kr and Sr isotopes. The anomalous rise at $J = 2$ is particularly pronounced for the Se isotopes; being the largest for $^{70}$Se. The general interpretation is that coexisting $0^+$ configurations mix and result in a lowering of the excitation energy for the ground state; hence, the anomalous high excitation energy of the $2^+_1$ state. Experimentally, the only candidate for a low-lying $0^+_2$ state has been suggested at a higher $\approx 2$ MeV excitation [17]; but a lower-lying $0^+$ excitation below the $2^+_2$ state could be identified in future $\beta^+$ decay measurements of $^{70}$Br. The indication of shape coexistence in $^{70}$Se may point at the existence of a low-lying $0^+$ excitation.

Figure 7. The experimental rotational parameter $h^2/2\Sigma$ for the neutron-deficient Kr (left panel), Sr (central panel) and Se (right panel) isotopes. The anomalous rise at $J = 2$ is particularly pronounced for $^{70}$Se.
The mixing of prolate-oblate shapes within rotational bands has been studied using various mean-field calculations [23, 24]. However, more realistic calculations can be expected from recent state-of-art beyond mean-field calculations, which include the interactions of multi-particle-multiple-hole excitations, and have, for instance, successfully described the structure of low-lying 0+ states in 16O in terms of self-consistent 0p−0h, 2p−2h, and 4p−4h Hartree-Fock states [25]. Similar beyond mean-field calculations will be desirable for the A ≈ 70 region.

Furthermore, neutron-deficient nuclei in the A ≈ 70 region are of astrophysical interest as these are produced in type I X-ray bursts. A better knowledge of the low-lying nuclear structure in this region is crucial for rp process network calculations and the understanding of luminosity curves through β decay and creation of elemental abundances after the burst [26]. Interestingly, comparison of the experimental Gamow-Teller strength distribution B(GT) with mean-field calculations [27, 28] suggests that nuclei β decay depending on their shape!

This experimental proposal has been defended at the INTC meeting at ISOLDE last November 2012 and the referee’s report is attached below:

CERN-INTC-2012-067, INTC-P-368, Solving the shape conundrum in 70Se.

“It is proposed to determine the sign and the magnitude of the spectroscopic quadrupole moment of the first 2+ state in 70Se, with the aim to provide a definite answer on the shape of this nucleus. The proposed method is the multi-step safe Coulomb excitation with the MINIBALL setup. Besides that, other transition matrix elements of higher-lying states can be obtained, to yield more spectroscopic information of this nucleus. This information is important to understand shape coexistence phenomena in this region, especially that discrepancies between experiment and theory are present. The physics case was found interesting and relevant. It was, however, not clear, whether the proposed experiment can provide definitive values for the E2 matrix elements from which the shape 70Se shape can be determined. Therefore, full simulations of the experiment should be performed and a Letter of Clarification should be provided clarifying this point, before the shifts can be awarded. Thus, at this point, the Committee has not recommended any shifts for the approval of the Research Board.”

The requested full simulations are shown in Figs. 4 and 5 and described in detail in section 2, and will be presented at the next INTC meeting this June 2013. The distinct angular distributions simulated in this work and shown in Fig. 5 permit a measurement of $Q_S(2^+_1)$ with an uncertainty of approximately ±0.1 eb. Additional experimental proposals will be submitted to the INTC committee at HIE-ISOLDE in the near future. Complementary reorientation-effect measurements using multi-step Coulomb-excitation reactions with stable beams are scheduled at iThemba LABS end of 2013 and beginning of 2014. Such experiments will prepare our students with hands-on and data-analysis skills for similar RIB measurements at HIE-ISOLDE. Moreover, the prospect of a RIB facility at iThemba LABS is a strong motivation for capital development at HIE-ISOLDE. J.N.O. acknowledges D. Jenkins, N. Warr and the HIE-ISOLDE collaboration for fruitful physics discussions. J.N.O. and the UWC nuclear physics group acknowledge travel funding from the South Africa-CERN (SA-CERN) collaboration.

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