Numerical Investigations of Wind Turbine Wakes under Neutral and Convective Atmospheric Stability Conditions

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Abstract. Large-eddy simulation (LES) technique combined with the actuator line model (ALM) is employed to investigate the characteristics of wind turbine wakes under both neutral and convective atmospheric boundary layer (ABL) stability regimes. Turbulence properties of the incoming wind that due to thermal stratification are collected by the precursor ABL simulation, and then are used to initialize the flow field upstream of a single wind turbine. For the neutral case, comparison with the wind tunnel experiments show an acceptable agreement in the wake velocity distribution and turbulence characteristics, which proves the accuracy of the employed numerical methods to some extent. Besides, numerical results are compared between two atmospheric conditions. Through the evaluations of the velocity deficit and turbulent intensity in the wake area, it is shown that the ABL stability has a significant effect on the wake development; and particularly, a more rapid wake recovery is observed in the convective condition. These results suggest that atmospheric thermal stability is crucial to be taken into account in the prediction of wake effects and in the design of wind farms.

1. Introduction

Wind turbine wake effect is characterized by a strong wind velocity deficit and an enhanced turbulence level in the flow region downstream of the turbine rotor. Furthermore, it is known to contribute the most energy losses and serious fatigue damages in large wind farms [1]. Therefore, it is important to investigate the properties of wakes to help the design of wind farms to mitigate their negative impacts.

As wind turbines operate in the lower part of the atmospheric boundary layer (ABL), its wake flow structure will be largely affected by the characteristics of the ABL flow such as the wind speed, wind shear and turbulence levels. Understanding and properly simulating the aerodynamics of the incoming ABL flow is necessary for accurate prediction of the wind turbine wake effect. In fact, the ABL characteristics are affected by land/sea surface characteristics as well as thermal stratification. According to the thermal condition and the dominant mechanisms of turbulence generation, the stability regimes of the ABL can be classified as stable, neutral and convective (unstable). The convective boundary layer is typically observed onshore during day time, when the terrain surface is warmer than the air. Under ABL convective conditions, heat transfer between the surface and the air produces positive buoyancy, resulting in an enhanced turbulence levels and the vertical transport of momentum and heat [2]. The stable ABL occurs usually at night, turbulence is generated by shear and destroyed by negative buoyancy and viscosity, which leads to a reduced turbulence level and a significant wind shear.
Finally, the neutral ABL represents a transitional regime between convective and stable regimes, and is observed during a relatively short period after sunset, or in windy conditions with a complete cloud cover. For a neutral ABL, there is little heating or cooling at the surface, the mean potential temperature is approximately constant with height and turbulence is mainly produced by shear in the proximity of the terrain.

Numerous experimental studies have been conducted for evaluating the effects of thermal stability on the structure of wind turbine wakes. Through performing the full-scale field measurement, Magnusson et al [3] pointed out that the velocity deficit in the wake flow area is a function of the atmospheric stratification, where a serious velocity loss could be observed for stable conditions. Chamorro et al [4] and Zhang et al [5] carried out wind tunnel tests to measure the velocity distributions downwind of the miniature wind turbines, their results consistently showed that the wakes recover faster under the convective conditions compared with the neutral cases. These experimental works are helpful to increase our knowledge of the variabilities of wind turbine wakes under different ABL thermal regimes. Nevertheless, the high cost limits the extensive applications of these experiments. One method to complement experimental measurements of wind turbine wakes are numerical simulations using computational fluid dynamics (CFD). Among all these CFD methods, large-eddy simulation (LES) technique with the advantage of providing high resolutions of spatial and temporal information, has been especially successful at revealing the fine three-dimensional structure of wind turbine wakes [6-8]. For instance, Ivanell et al [9], Porté-Agel et al [10] and Churchfield et al [11] have implemented actuator disk or line models of the turbine rotor into the ABL-LES solver, through which the development of the wake flow under various atmospheric conditions was studied.

This work intends to investigate the effect of ABL thermal stability on wind turbine wake characteristics using a recently developed LES technique. As a first step, the variability in the downstream evolution of the wind turbine wake is analyzed particularly for convective and neutral ABL stability regimes. Future work will include more complicated stratified ABL conditions. The results may constitute a database for the development and calibration of an analytical wake model that will take into account atmospheric stability.

2. Numerical Models

2.1. Governing equations

In the conventional LES studies of the ABL, neutral stability is assumed, and the spatially filtered incompressible, unsteady Navier-Stokes (N-S) equations are solved. However, when considering the atmospheric stratification, the governing equations are modified to [12]:

$$\frac{\partial \tilde{u}_i}{\partial t} = 0$$  

(1)

$$\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} = - \frac{\partial \tilde{P}}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + \delta (i \equiv g \tilde{\theta} - \bar{\theta}) + f_c \varepsilon (\tilde{u}_i - U_{gi}) + f_i$$  

(2)

$$\frac{\partial \tilde{\theta}}{\partial t} + \tilde{u}_j \frac{\partial \tilde{\theta}}{\partial x_j} = - \frac{\partial q_j}{\partial x_j}$$  

(3)

In Eqs. (1)-(3), the tilde represents a spatial filtering at scale $\tilde{\lambda}$, $\tilde{u}_i$ ($i=1, 2, 3$ corresponding to the stream-wise, cross-wind and vertical directions, respectively) is the filtered velocity. In Eq.(2), term I represents the pressure gradient, with $\tilde{P} = \tilde{p} + \frac{1}{3} \rho \tau_{kk}$ is the modified pressure including the trace part of the stress tensor; and term II represents the divergence of the stress tensor, with $\tau_{ij}$ is deviatoric part of the subgrid-scale (SGS) stress. Term III represents the Boussinesq approximation for buoyancy effects due to temperature variations in the flow, with $\tilde{\theta}$ is the resolved potential temperature, $\bar{\theta}_0$ is the reference temperature, $\langle \tilde{\theta} \rangle$ is a horizontal average, $g = 9.8 m/s$ is the acceleration due to the gravity; and term IV represents the Coriolis force due to planetary rotation, with $f_c$ is the Coriolis parameter, $U_{gi}$
is the geostrophic wind vector and \( \varepsilon_{ij3} \) is the alternating tensor. Finally, term \( \psi \) is an additional body force used to model the effect of the turbine on the flow. In Eq.(3), \( q_j = \tilde{u}_j \tilde{\theta} - \tilde{u}_j \tilde{\theta} \) represents the flux of temperature by viscous and SGS effects, and needs to be modelled as follows.

2.2. SGS model
In LES, all turbulent structures larger than the filter scale \( \tilde{\Delta} \) are resolved and the contribution of the small-scale eddies on the resolved field is parameterized using a SGS model. As the ABL flow is a high Reynolds number flow, the viscous effect can be neglected everywhere except at the terrain surface, then the SGS stress \( \tau^d_{ij} \) and the heat flux \( q_j \) can be modelled using the eddy-viscosity approach as:

\[
\tau^d_{ij} = -u_{\text{SGS}} \left( \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right)
\]

\[
q_j = - \frac{u_{\text{SGS}}}{Pr_{\text{SGS}}} \frac{\partial \tilde{\theta}}{\partial x_j}
\]

where \( Pr_{\text{SGS}} \) is the SGS Prandtl number and is assumed to be a constant [12], \( u_{\text{SGS}} \) is the SGS eddy viscosity and is given by the classical Smagorinsky model:

\[
u_{\text{SGS}} = - (C_s \tilde{\Delta})^2 \left( 2 \tilde{S}_{ij} \tilde{S}_{ij} \right)^{1/2}
\]

Here, \( C_s \) is the Smagorinsky coefficient and is set to be 0.13, \( \tilde{\Delta} \) is the filter width and is calculated as \( \left( \Delta_x \Delta_y \Delta_z \right)^{1/3}, \tilde{S}_{ij} = \left( \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right)/2 \) is the resolved strain-rate tensor.

At the bottom surface boundary, a wall model following Monin–Obukhov similarity theory is embedded, \( \tau^d_{ij} \) and \( q_j \) are directly specified using Moeng’s [13] rough surface model.

2.3. Actuator line model
In this work, actuator line method (ALM) [14] is employed to investigate the flow field around wind turbines, in which body forces are distributed radially along lines with tabulated two-dimensional airfoil characteristics representing the wind turbine blades. Compared with a full-blade geometry simulation, the ALM is considered as a good compromise between accuracy and computational cost because it requires much fewer grid points and in the meantime can reveal the properties of the flow behind a wind turbine.

Fig. 1 shows a cross-sectional airfoil element at radius \( r \) in the \((\theta, x)\) plane. As shown in Fig.1, the local velocity \( u_{\text{rel}} \) relative to the rotating blade can be expressed in terms of the tangential velocity \( u_\theta \) and the axial velocity \( u_x \), which is given by:

\[
u_{\text{rel}} = \sqrt{u_x^2 + (u_\theta - \Omega r)^2}
\]

where \( \Omega \) is the turbine angular velocity. As shown in Fig.1, \( \alpha \) is the angle of attack, defined as \( \alpha = \phi - \gamma \), where \( \phi = \tan^{-1} \left( \frac{u_\theta}{u_\theta - r\Omega} \right) \) is the angle between \( u_x \) and the rotor plane, \( \gamma \) is the local pitch angle. With the values of \( u_{\text{rel}} \) and \( \alpha \), the magnitude of lift \( L \) and drag \( D \) at each actuator segment can be given as:

\[
L = \frac{1}{2} C_L(\alpha) \rho u_{\text{rel}}^2 c dr, \quad D = \frac{1}{2} C_D(\alpha) \rho u_{\text{rel}}^2 c dr
\]

In Eq.(8), \( C_L(\alpha) \) and \( C_D(\alpha) \) represent the lift and drag coefficients, respectively, \( c \) is the chord length, \( dr \) is the actuator segment width. Then the resulting force per unit rotor area is given by:

\[
f_i = \frac{1}{2} \rho u_{\text{rel}}^2 \frac{Bc}{2\pi r} (C_L e_L + C_D e_D)
\]

where \( B \) is the number of the blades, \( r \) is the radial distance between the hub and the center of the blade element, \( e_L, e_D \) are the unite vectors in the directions of the lift and the drag, respectively.
Each actuator segment has its own value of \( f_i \), as a point force. To avoid singularities in numerical calculations, it cannot be directly applied to the flow field. Rather, it must be smoothly projected onto its surrounding computational domain as a body force field using a Gaussian projection. Therefore, at a location \((x, y, z)\) in the domain, the volume force is given as:

\[
F_i(x, y, z, t) = f_i \otimes \eta_e, \quad \text{where } \eta_e = \frac{1}{\varepsilon^2 \pi^{3/2}} \exp\left(-\frac{(r_j^2)}{\varepsilon^2}\right)
\]  

(10)

where \( r_j \) is the distance between grid point \((x_j, y_j, z_j)\) and the blade elements, \( \varepsilon \) is a parameter that serves to adjust the concentration of the regularized load, following the work of ref. [14], which is set to be twice the grid-cell size around the blade in the stream-wise direction.

3. Precursor atmospheric boundary layer simulations

The studies are performed in two distinct phases: (1) the precursor simulations of neutral and convective atmospheric wind flowing through an empty domain in the absence of any turbines; (2) wake simulations of an isolated turbine (NREL 5MW) immersed in the generated flow situations from phase (1).

Following this strategy, simulations of the atmospheric wind flow over a flat terrain are carried out at first. The computational domain size is 4.8km×2.4km×1.6km, and is discretized into a mesh of 480×240×160 hexahedral cells. All the lateral boundaries are set to be periodic. At the bottom surface, specifications of surface stress and temperature flux are discussed in Section 2.2, through calculations, a constant surface heat flux \( q_{3,\text{wall}}=+0.015\text{Km/s} \), corresponding to weak instability, is obtained for the unstable case and \( q_{3,\text{wall}}=0 \) for the neutral case. The top boundary condition is a stress/flux-free condition. Except for the setting of boundary conditions, initial conditions for the primitive variables are required. The initial velocity field is set uniformly as 8 m/s at the desired hub height along the x axis with some small divergence-free perturbations. As considering the initial temperature condition, for the unstable case, the initial potential temperature profile is prescribed to be 293K from the ground to 1500m, with an overlying inversion strength of 0.003K/m aloft. The boundary layer was simulated about 5h for the neutral case, and 4.3h for the unstable case, respectively, before it was considered to have reached a quasi-equilibrium state in which the initial transients had passed and the mean flow was undergoing gentle inertial oscillations. Statistics are performed over roughly the last 1h with a frequency of 2 Hz for two cases.

In Fig. 2, vertical profiles of horizontal- and time-averaged wind speed, turbulence intensity and kinetic shear stress are plotted for neutral and convective stability conditions. As shown in this figure, for these two cases, the incoming wind has the same wind speed at hub height but different mean wind shears and turbulence levels. Positive surface buoyancy fluxes in the convective condition creates thermal instabilities which leads to a higher turbulence level and an enhanced the turbulent mixing. As expected, under the convective condition, except for the small region near the ground, the wind speed is nearly constant with height as a result of strong turbulent mixing. The profiles of the shear stress \( -\overline{u'w'} \) showing a strong dependence on stability, the magnitude increases as the boundary layer becomes unstable, also resulting from strong turbulent mixing.
Figure 2. Vertical profiles of the time-averaged stream-wise wind velocity (left), the stream-wise turbulence intensity (middle) and the kinetic shear stress (right) under neutral and convective atmospheric conditions.

For more intuitive visualization of the structure of the turbulence in the atmospheric flow, instantaneous flow field taken from the surface to the hub height are shown in Fig. 3. The yellow elongated structures are the iso-surface of the x-directed horizontal velocity with -1.25 m/s fluctuations. Updrafts due to the temperature flux at the bottom surface are demarcated by the distributed red structures, which are the iso-surface of the vertical velocity with +1 m/s fluctuations. In the neutral case, the yellow iso-surface highlights the low-speed long streak that form within the surface boundary layer and is oriented with the flow along the x-direction, while the red iso-surface illustrates the instantaneous vertical velocity fluctuations are much smaller and more out of order compared with those yellow horizontal streaky structures, indicating that the neutral flow is no buoyancy-induced.

While in the convective case, lines of updrafts are clearly present, which can be explained by the fact that the positive buoyancy promotes vertical transport of momentum. Furthermore, it could be found that the stream-wise flow within these updrafts is slower than that outside the updrafts. These positions are of guiding significance for micro-siting work of wind turbines because when turbines are situated inside these structures, the power production will decrease. Besides, it needs to be mentioned that a horizontal gray scale contour plane at 20m above the surface is given in the convective case to illustrate the potential temperature. Lighter color indicates warmer air. As expected, the updrafts as shown in the unstable case are associated with these small pockets of warmer air.

Figure 3. Instantaneous iso-surfaces of stream-wise (yellow, -1.25m/s) and vertical (red, +1m/s) velocity fluctuations for (left) neutral and (right) convective atmospheric conditions.

4. Isolated wind turbine wake simulations

In this section, we present results from large-eddy simulations of the wake flow behind a wind turbine under different atmospheric stability conditions. The chosen turbine is NREL 5-MW, with a rotor diameter of 126 m, hub height of 90 m and the tip speed ratio (TSR) is set to be 6. It is located at $x_0$=1.2km, $y_0$=1.2km with the hub at $z_0$=0.0km, as shown in Fig.4. The single turbine simulation use the
mesh from the precursor case as the “background mesh”, then locally refined around the rotor and its wake region to ensure the computational accuracy. The initial profiles of turbulence and temperature are specified by duplicating the saved precursor flow field in the x-direction. The cases are run for roughly 6h in physical time and the statistics are performed over the final 1h.

4.1. Results under the neutral ABL condition
Firstly, wind turbine wake flow simulation under the neutral atmospheric condition is carried out with the main purpose to validate the employed numerical methods through comparing with the wind tunnel measurements. Fig. 5(a) displays the obtained profiles of the time-averaged wake deficit $\Delta u$ in the vertical direction ($z$) through the center of the turbines. The variable $\Delta u$ is calculated as $\Delta u = u_{hub} - \bar{u}$, where $u_{hub}$ is the hub height wind speed of 8 m/s and $\bar{u}$ is the time-averaged magnitude of the streamwise component of the flow. In order to validate the performance of our numerical methods, measured data from the wind-tunnel experiment [15] are included in this figure. Due to the strong wind shear in the neutrally-stratified boundary layer flow, we can see a non-axisymmetric distribution of the vertical profile in the wake area with the magnitude and the shape of these profiles are in good agreement with the wind tunnel observations. Especially, a perfectly match could be found for wake deficit value at the hub center between simulated and measured results. Normally, the wake-central velocity is used as the inflow speed to predict the energy output of the downstream wind turbine.

Similarly, Fig. 5(b) shows profiles taken at hub height laterally across the wakes. The horizontal wake profiles exhibit some asymmetry characteristic, when looking at the turbine from downwind, there is more deficit on the left side of the turbine. The same phenomenon was also observed in the numerical work of Churchfield et al. [16]. As is well known, the wake rotates due to the rotation of the turbine blades, which results in an asymmetric distribution of the wake deficit, since the upward motion raises of lower momentum from below ($y>0$), while the opposite occurs on the other side($y<0$). Apart from this reason, Coriolis force due to the planetary rotation can also bring some effect on wake asymmetry [17]. Nevertheless, with downstream development, this wake asymmetry wears off. On the whole, as shown in Fig. 5, the good agreement between the numerical calculations and the experimental results proves the accuracy of the employed methods to some extent.
Figure 5. Time-averaged wake velocity deficit profiles normalized by the hub-height average velocity. (a) Vertical profiles on the wake centreline and (b) horizontal profiles at hub height.

Fig. 6 shows contours of the turbulence intensity at different cross sections. At the near wake position \( x/D = 1 \), turbulence intensity only enhanced at the top-tip level, and remains at the free stream level in other regions. When the wake developing downstream, turbulence intensity reaches a maximum value at the position of \( x/D = 3 \), a well-known transition region between near- and far-wake regions. The maximum turbulence level occurs at the 3D downstream position instead of the position immediately downstream of the turbine rotor (such as \( x/D = 1 \)) can be partly explained by the fact that coherent vortices shedding from the blade have strong interaction with each other, stretching and merging mechanisms of these structures gradually enhance the turbulence level in the following region until reach the position of \( x=3D \). When the wake flow developing further downstream, turbulence intensity gradually decrease to the free-stream level because of the dissipation effect of the turbulence.

As can be seen in Fig. 6(b), the turbulence intensity has higher values around the upper edge of the wake and lower values around the lower edge. Besides, the turbulence levels are higher around one side \((y>0)\) and are weaker round the other side \((y<0)\). Similar behavior is observed in Fig. 5 for the velocity distributions. This is mainly due to the strong wind shear effect and the rotation of the blade, as explained above for the same trends shown in Fig. 5.

Figure 6. Contours of the turbulence intensity at different downstream cross sections.

Fig. 7 shows vertical profiles for kinematic shear stress \(-\overline{u'w'}\) and \(-\overline{v'w'}\) obtained from the simulations, respectively. These terms are responsible for the vertical and lateral kinetic energy entrainment into the wake. It is clearly seen that the turbine introduces kinematic stresses are locally much larger in magnitude than the stresses in the coming velocity field. Up to a distance of about \( x=6D \), there is a region above the turbine hub and near the top-tip with large positive stresses, in contrast to a lower region (below the hub height) with negative stress. Like in the case of the turbulence intensity, the magnitude of the positive stress reach the maximum value around the top-tip height. Again, this is due to the high levels turbulence production associated with the strong shear at that height. As the turbine...
Wake develops with downwind distance (such as $x=10D$), the relative magnitude of the kinematic stress between the upper and lower parts gradually diminishes until recovering to the initial state.

**Figure 7.** Vertical profiles of Reynolds shear stresses at different downstream cross sections.

### 4.2. Results under unstable ABL condition

Fig. 8 displays the stream-wise velocity contour in the vertical $x$-$z$ plane through the center of the turbine under neutral and convective atmospheric conditions. The results indicate that atmospheric stability significantly affects the mean velocity distribution in the turbine wakes. To be specific, in the convective boundary layer with a higher turbulence level, the wake recovers relatively rapidly as compared with the neutral case, especially a pronounced flow acceleration in the higher region of the wake. This is due to the fact that, in the unstable condition, the positive buoyancy near the ground creates thermal instabilities which enhance the vertical-convective motions and the turbulent mixing. Moreover, the wake interacts strongly with the background large-scale convective vortices in the unstable boundary layer, which also contributes to the rapid destruction of the wake shape. In contrast, in the neutral ABL flow, it takes a longer distance for the wake to recover (i.e., larger wake region), until the far downstream position ($x=10D$), the velocity at the wake center only recovers to 78% of the incoming wind speed.

Furthermore, from Fig. 8 it could be seen that for both the stable and neutral cases, the center of the wake rotation is not aligned with the stream-wise direction, but has an offset in the vertical direction, this is commonly regard to “wake meandering”. It is shown that atmospheric stability has a significant effect on the meandering characteristics of the wake. Specifically, in the near wake region, the wake meandering is much more obvious in the neutral condition; while for a given distance downwind of the turbine, wake meandering is stronger under the convective condition compared with the neutral one.

**Figure 8.** Instantaneous contours of stream-wise velocity at the vertical central plane of single wind turbine wakes for two stability conditions.

Besides the velocity deficit, the other critical aerodynamic factor in wind turbine wakes is the enhanced turbulence intensity, which is highly related to the failure of wind turbine components. Fig. 9 shows contours of the total turbulence intensity in a vertical $x$-$z$ plane through the center of the turbines. As expected, the turbulence intensity has a peak at top-tip level and exhibits a dual-peak pattern around the top-tip and bottom-tip positions, which is related to the intense production of turbulent kinetic energy.
associated with the strong shear at those locations. It is also evident that in the near wake region (x/D<5), the higher turbulence level in the incoming flow leads to also a large maximum wake turbulence level for the convective condition, compared with the neutral one. This is consistent with the observed faster recovery of the wake in the convective case. While in the far wake region, the turbulence level in the neutral case seems higher than that of the convective one. This may be explained by the fact that for the unstable case, the wake-induced shear is quickly dissipated, with the peak being less distinct; but for the neutral case, the wake-induced shear still exist and lasts for a longer distance, which leads to a relatively higher turbulence level in the far wake region.

![Instantaneous contours of turbulence intensity at the vertical central plane of single wind turbine wakes for two stability conditions.](image)

**Figure 9.** Instantaneous contours of turbulence intensity at the vertical central plane of single wind turbine wakes for two stability conditions.

5. Conclusions
Considering the atmospheric thermal effect, large-eddy simulation technique combined with the actuator line model has been used to simulate the wake of an isolated NREL 5-MW wind turbine developed in neutral and unstable turbulent boundary layer flows. A precursor atmospheric simulation is conducted firstly to collect turbulence statistics as initial conditions for subsequent wake simulation. Then, the wake simulation under the neutral atmospheric condition is carried out. The obtained results are compared with relating experimental data, a good agreement between simulated and measured results is clearly shown, which proves the reliability of the method and lays the foundation for following studies of wind turbine wake effects under different stability conditions.

Comparisons between the results obtained under neutral and convective atmospheric condition show that the atmospheric thermal stability has a significant influence on the wake shape, growth rate and recovery. In particular, the wakes recover faster under the convective condition compared with the neutral case. This enhancement in the wake recovery rate is related to the higher turbulent entrainment flux into the wake. It is also shown that atmospheric stability has a significant effect on the magnitude and spatial distribution of the turbulence intensity. Specifically, in the near wake, a higher turbulence level can be observed for the unstable case; while in the far wake, the turbulence is a little bit stronger for the neutral case.

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