Transport properties of the quark-gluon plasma from lattice QCD

Harvey B. Meyer

Center for Theoretical Physics
Massachusetts Institute of Technology
77 Massachusetts Ave
Cambridge, MA 02139, U.S.A.

Abstract

I review the progress made in extracting transport properties of the quark-gluon plasma from lattice QCD simulations. The information on shear and bulk viscosity, the “low-energy constants” of hydrodynamics, is encoded in the retarded correlators of $T_{\mu\nu}$, the energy-momentum tensor. Euclidean correlators, computable on the lattice, are related to the retarded correlators by an integral transform. The most promising strategy to extract shear and bulk viscosity is to study the shear and sound channel correlators where the hydrodynamic modes dominate. I present preliminary results from a comprehensive study of the gluonic plasma between 0.95$T_c$ and 4.0$T_c$.

1. Introduction

The phenomenology of heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) has revealed unexpected properties of the quark gluon plasma. Hydrodynamics calculations [1] successfully described the distribution of produced particles in these collisions [2]. This early agreement between ideal hydrodynamics and experiment has been refined in recent times. The dissipative effects of shear viscosity $\eta$ have been included in (2+1)D hydrodynamics calculations [3] and the sensitivity to initial conditions quantitatively estimated [4] for the first time. Experimentally, the elliptic flow observable $v_2$, which is sensitive to the value of $\eta$ in units of entropy density $s$, is now corrected for non-medium-generated two-particle correlations [5]. The conclusion that $\eta/s$ must be much smaller than unity has so far withstood these refinements of heavy-ion phenomenology [6]. On the theory side, it is therefore important to compute the QCD shear viscosity from first principles in order to complete the picture. Furthermore, since the heavy ion collision program at LHC will probe the quark-gluon plasma at temperatures about a factor two higher [7], it is crucial to predict the shear viscosity at $\sim 3T_c$, and to relate it to the size of elliptic flow, before experimental data is available.

In this talk I present lattice calculations of the thermal correlators of the energy-momentum tensor (EMT) in the Euclidean SU(3) pure gauge theory (i.e. quarkless QCD), and discuss methods to extract the shear and bulk viscosity from them. Computationally, the calculation is challenging enough without the inclusion of dynamical quarks, and physically, the static properties of the QGP, normalized by the number of degrees of freedom, do not depend sensitively on the flavor content away from $T_c$ [8]. In perturbation theory [9], the ratio of shear viscosity to entropy density is $O(\frac{1}{\alpha_s})$ and there is only about 30% difference between the pure gauge theory and full QCD [10] at a fixed value of $\alpha_s$ ($\eta/s$ is smaller in the pure gauge theory). The bulk viscosity $\zeta$ is
O(α²) and predicted to be much smaller than the shear viscosity in perturbation theory. As a rule of thumb, η/s ≈ 1.0 [9] and ζ/η ≈ 10^{-3} [11] at α_s = 0.25.

Since the pioneering calculation [12], there have been only few attempts to calculate the viscosities of the pure gauge theory on the lattice. Nakamura and Sakai [13] performed the first calculations at N_f ≡ (aT)^{-1} = 8, where a denotes the lattice spacing. About two years ago, the accuracy of the Euclidean correlators improved significantly [14, 15] thanks to high statistics and a more efficient two-level algorithm [16, 17]. Here I will give an update on the progress made in constraining dynamical properties of the QGP such as the viscosities. I will not describe the calculations of the electric conductivity [18] or heavy quark diffusion constant [19], except for saying that they share many features with the viscosity studies.

2. Hydrodynamics and energy-momentum tensor correlators

From the modern point of view, hydrodynamics is an effective theory that describes the slow, long-wavelength motion of a fluid. The central object is the energy momentum tensor, a symmetric rank-two tensor, whose conservation is expressed by the four continuity equations \( \partial_\nu T^{\mu\nu} = 0 \). The component \( T_{00} \) is the energy density, \( T_{ij} \) is the momentum density, which coincides with the energy flux, and the spatial components \( T_{ik} \) are the momentum fluxes. From the microscopic point of view, the matrix elements of \( T_{\mu\nu} \), viewed as a quantum operator, between any two on-shell states satisfy \( \langle \Psi | T^\mu(\eta) \Psi \rangle = 0 \).

The two macroscopic modes of fluid motion near equilibrium are the shear and sound modes. The former corresponds to the transverse diffusion of momentum. Indeed a small perturbation \( T_{03} = T_{03}(t, x_1) \) of the fluid around equilibrium gives rise to shear flow and satisfies the diffusion equation
\[
\partial_t T_{03}(t, x_1) - D_0 \partial^2_{x_1} T_{03}(t, x_1) = 0, \quad \text{(shear mode)}
\]
where the diffusion coefficient \( D \) is proportional to the shear viscosity \( \eta \), \( D = \frac{\eta}{\epsilon_T} \) (\( \epsilon \) is the energy density and \( p \) the pressure). Secondly, a sound wave propagating in the \( z \)-direction with wavelength \( \lambda = 2\pi/k \) is damped according to
\[
T_{03}(t, k) \propto e^{-\frac{1}{2}(\zeta + \pi k^2 \tau / (\epsilon + p))} \quad \text{(sound mode)}.
\]

Both the shear viscosity \( \eta \) and bulk viscosity \( \zeta \) thus contribute to the damping of sound waves. The quantities computed on the lattice are correlators of the energy-momentum tensor that depend on Euclidean time \( x_0 \) and spatial momentum \( \mathbf{q} \).

\[
C_{\rho_\mu\nu}(x_0, \mathbf{q}) = L_0^5 \int d^3x e^{i\mathbf{q}\cdot\mathbf{x}} \langle T_{\mu\nu}(x_0, \mathbf{x}) T_{\rho\nu}(0) \rangle, \quad L_0 \equiv 1/T.
\]

The so-called spectral function \( \rho(\omega, \mathbf{q}) \) is defined as \((-\pi \times)\) the imaginary part of the Fourier-space, Minkowski-time retarded correlator \( G_R(\omega, \mathbf{q}) \). I leave the temperature dependence of the functions \( C \) and \( \rho \) implicit. For \( \mu = \rho \) and \( \nu = \sigma \), it obeys a positivity condition, in our conventions \( \rho(\omega, \mathbf{p})/\omega \geq 0 \) where \( n \) is the number of time components among the four indices, and is odd in \( \omega \), \( \rho(-\omega, \mathbf{q}) = -\rho(\omega, \mathbf{q}) \). Via a Kubo-Martin Schwinger relation, this spectral function is related to the corresponding Euclidean correlator (see [19]),

\[
C_{\rho_\mu\nu}(x_0, \mathbf{q}) = L_0^5 \int_0^\infty \rho_{\rho_\mu\nu}(\omega, \mathbf{q}) \frac{\cosh \omega (\frac{1}{2} L_0 - x_0)}{\sinh \frac{\omega}{2} L_0} d\omega.
\]
A wealth of information is encoded in the spectral functions. In particular, the shear and bulk viscosities are given by Kubo formulas (see [20] for a derivation),

\[ \eta(T) = \pi \lim_{\omega \to 0} \lim_{q \to 0} \frac{\rho_{13,13}(\omega, q)}{\omega}, \quad \frac{4}{7} \zeta(T) + \eta(T) = \pi \lim_{\omega \to 0} \lim_{q \to 0} \frac{\rho_{33,33}(\omega, q)}{\omega}. \]  

They also determine the static structure of the plasma [21][22].

\[ C_{\mu \nu \rho \sigma}(r, T) \equiv \langle T_{\mu \nu}(x_0, r) T_{\rho \sigma}(x_0, 0) \rangle = \lim_{\epsilon \to 0} \int \frac{d^3 q}{(2\pi)^3} e^{iq \cdot r} \int_0^\infty d\omega \frac{e^{-\epsilon \omega} P_{\mu \nu \rho \sigma}(\omega, q, T)}{\tanh \omega/2T}. \]  

In the following, I summarize the known analytic properties of the spectral functions. The goal will be to exploit these properties to help us solve the integral equation [4] for the spectral function.

The conservation of the EMT, \( \partial_\mu T_{\mu \nu} = 0 \), implies in particular, for \( q = q\hat{r}_3 \),

\[ \omega^2 \rho_{00,00}(\omega, q) = -\omega^2 q^2 \rho_{03,03}(\omega, q) = q^2 \rho_{33,33}(\omega, q), \quad -\omega^2 \rho_{01,01}(\omega, q) = q^2 \rho_{13,13}(\omega, q). \]  

The first set of spectral functions corresponds to the sound channel and the second to the shear channel [20]. I therefore use the notation \( \rho_{00,00}(\omega, q) \equiv -\rho_{03,03}(\omega, q) \) and \( \rho_{01,01}(\omega, q) \equiv -\rho_{13,13}(\omega, q) \).

Kovtun and Starinets [23] analyzed the general tensor structure of EMT correlators. I briefly summarize their results. For a generic relativistic quantum field theory in four dimensions, there are five independent functions of \( (\omega, q) \) that determine all thermal correlators of the EMT. At \( T = 0 \), this number is reduced to two [24]. A physically motivated choice of five independent functions is: two functions corresponding to the two hydrodynamics modes, the shear mode \( G_1 \) in the notation of [23] and sound mode \( 2G_2 + 3C_L \); one function associated with the \( (T_{12}T_{12}) \) correlator and momentum in the \( \omega \) direction \( (G_2) \); finally, the correlators \( (T_{12}T_{12}) \) and \( (T_{12}T_{10}) \) (respectively proportional to \( 2C_T + C_L \) and \( C_T + 2C_L \)) determine the last two functions. As an application, it is worth noting that if \( \theta_q \) is the polar angle of \( q \),

\[ -C_{03,03}(x_0, q) = L_0 \int_0^\infty d\omega \left[ \rho_{00,00}(\omega, q) \cos^2 \theta_q + \rho_{13,13}(\omega, q) \sin^2 \theta_q \right] \frac{\cosh(\frac{\omega}{2}L_0 - x_0)}{\sinh \frac{\omega}{2}L_0}. \]  

In the future this will allow us to exploit also momenta that are not aligned along a lattice axis.

At \( T = 0 \), \( G_1 = G_2 = G_3 \) and \( C_T = C_L \), and they become functions of \( (\omega^2 - q^2) \). As a corollary, the extent to which these relations are obeyed at \( T > 0 \) provides us with a way to evaluate the importance of thermal effects on the correlation functions.

As a low-energy theory, hydrodynamics predicts the small \( k = (\omega, q) \) behavior of spectral functions in terms of a few ‘low-energy constants’. A first-order hydrodynamic expression for the shear and sound spectral functions was derived in [20]. For \( q = q\hat{r}_3 \),

\[ \frac{\rho_{13,13}(\omega, q)}{\omega} \underset{\omega \to 0}{\sim} \frac{\eta}{\pi} \frac{q^2}{\omega^2 + (\eta q^2/(e + P))^2}, \quad \frac{\rho_{33,33}(\omega, q)}{\omega} \underset{\omega \to 0}{\sim} \frac{\Gamma_\epsilon}{\pi} \frac{(e + P)q^2/\omega^2}{(e^2 - v_s^2q^2)^2 + (\Gamma_\epsilon q^2)^2}. \]  

\(^1\)For a conformal field theory, the number of independent correlators is three at \( T > 0 \), and just one at \( T = 0 \).
I introduced the speed of sound $v_s$ and the sound attenuation length $\Gamma_s = (\frac{4}{3}\eta + \zeta)/(e + P)$. Baier et al. [25] derived the dispersion relation of the sound pole to next-to-leading order accuracy for a conformal theory:

$$\omega = v_s(q)q, \quad v_s(q) = v_s \left(1 + \frac{\Gamma_s}{2q^2} \left(\tau_{\Pi} - \frac{\Gamma_s^2}{4v_s^2}\right) + O(q^4)\right). \quad (11)$$

Here $\tau_{\Pi}$ is the relaxation time for shear stress.

The Euclidean correlators and spectral functions were calculated to leading order at weak coupling in [26]. The correlators are also known to next-to-leading order at zero temperature [27, 24]. Expressing the latter results in terms of a shear channel spectral function, $\rho_{13,13}(\omega, 0)$:

$$\rho_{13,13}(\omega, 0) \omega \rightarrow \infty \sim \frac{d_A \omega^4}{10(4\pi)^2} \left[1 - \frac{5\alpha_s N_c}{9\pi}\right]. \quad (12)$$

Son and Romatschke [28] derived two sum rules for the spectral functions. The ‘bulk’ sum rule can be reexpressed in terms of sound and shear channel spectral functions,

$$2 \int_0^\infty \frac{d\omega}{\omega} \left[\rho_{13,13}(\omega, 0) - \rho_{13,13}(\omega, 0)\right]_{T=0} = (e + p) \left(1 - \frac{1}{9v_s^2} - \frac{4}{9}(e - 3p)\right). \quad (13)$$

From a practical point of view, this provides an additional constraint on the spectral functions, albeit at the price of having to determine simultaneously the $T = 0$ spectral functions. Because the symmetry group is larger at $T = 0$, the vacuum spectral functions can however be more strongly constrained by lattice data (in addition to having a longer Euclidean time extent).

In a class of strongly coupled gauge theories at large-$N_c$, exact results have been obtained for the transport coefficients. The most important result is that the shear viscosity assumes a universal value in a large class of theories, $\eta/s = 1/4\pi$ [29]. The spectral functions for the EMT have also been obtained by numerical integration in the case of the $N = 4$ SYM theory [30]. The result is compared to the hydrodynamic functional form (10) in [21]. Such a comparison teaches us up to what frequency and momentum the hydrodynamic spectral function remains a good approximation to the exact result. It turns out that up to $\omega, q \approx \pi T$, the sound channel spectral function is well described by hydrodynamics. This sort of statement necessarily depends on the details of the microscopic theory; and in a weakly coupled theory, one expects the validity of hydrodynamics to be limited to much greater wavelengths. Nevertheless I will use this observation as a guideline for the gluon plasma at $T < 4T_c$. A similar logic has also been applied in studies of Mach cones in the plasma [31].

Figure 1: Left: Euclidean sound channel correlator. Scaling of CPU-cost with $N_t$ in the deconfined phase.
relations (7) imply that in the sound channel, the numerically determined correlators of the momentum density are the most accurately determined of all. Obtaining the correlators to that accuracy is computationally demanding. Figure 1 also shows how the cost of the computation scales with $N_N^3$. For a fixed aspect ratio $N_N$, the cost therefore scales as $N_N^3$. This is in contrast with the expectation of an $N_N^1N_N^3$ scaling with a one-level algorithm.

The discretization errors affecting the Euclidean correlators calculated at finite lattice spacing were studied in perturbation theory in [35]. The study motivated in particular the choice of anisotropy $a_\tau/a_\tau \equiv \xi = 2$. Further it was shown that it is advantageous to remove the treelevel cutoff effects from the lattice correlators. I apply this technique to all the data shown in these proceedings. Finally, at treelevel the $q = 0$ correlator of $T_{0k}$ is much flatter as a function of $x_0$ than the correlator of $T_{00}$ (they become exactly independent of $x_0$ in the continuum limit). This, and the fact that $T_{0k}$ is normalized multiplicatively even on the anisotropic lattice, means that the correlators of the momentum density are the most accurately determined of all.

Relations (7) imply that in the sound channel, the numerically determined correlators $C_{00,00}$, $C_{00,03}$, $C_{03,03}$, $C_{33,03}$ and $C_{33,33}$ are all related to the same spectral function $\rho_{\text{nd}}$ modulo factors of $(\omega/q)$. Therefore a global analysis of these five correlators is the best way to constrain $\rho_{\text{nd}}$. 

### Figure 2: Sound channel spectral function at $2.32T_c$.

| $1.58T_c$ | $2.32T_c$ | free gluons | $\lambda = \infty$ SYM |
|----------|----------|-------------|---------------------|
| $(\eta + 1/2\zeta)/s$ | 0.20(3) | 0.26(3) | $\infty$ |
| $2\pi T_{\tau T}$ | 3.1(3) | 3.2(3) | $\infty$ |
| $(\eta + 1/2\zeta)/(T_{\tau T})$ | 0.40(5) | 0.51(5) | 0.17 |
| $\chi^2_{\text{dof}}$ | 1.6 | 0.2 | 0.2 |
| $q = 2\pi T/6$ | | | |
| $q = 2\pi T/3$ | | | |

Figure 2: Sound channel spectral function at $2.32T_c$. In the Table, lattice results are compared to free gluons [33] and to the strongly coupled SYM results [32][53]. Stat. errors only are given. We expect $\zeta$ to be negligible at these temperatures.

### 3. Lattice calculation

I use Monte-Carlo simulations of the anisotropic Wilson action for SU(3) gauge theory with a fixed anisotropy of $\xi = 2$, meaning that the temporal lattice spacing $a_\tau$ is half the length of the spatial lattice spacing $a_\tau$. See [32] for details of the action. The lattice spacing is related to the bare coupling (an input parameter of the simulation appearing in the lattice action) through $g_0^2 \sim 1/\log(1/a_\tau \Lambda)$. Due to the loss of continuous translation invariance on the lattice, the discretized EMT is not protected against a non-trivial $g_0$-dependent renormalization. Fully calibrating the EMT on the anisotropic lattice remains a numerical challenge. While $T_{0k}$ is normalized multiplicatively and $T_{00}$ and $T_{\tau\tau}$ can be calibrated by using thermodynamic expectation values, the spatial components present the biggest challenge.

A typical example of a Euclidean correlator computed on the lattice is shown in Fig. 1. It displays the correlator of the energy density $T_{00}$, at various non-vanishing spatial momenta. The lattice size is $16 \times 48^3$ and the temperature about $2.37T_c$. The typical size of the error bars is 3%. Obtaining the correlators to that accuracy is computationally demanding. Figure 1 also shows how the cost of the computation scales with $N_N$. Different temperatures lie approximately on a smooth curve. The dashed line corresponds to the expectation for a conformal theory, $\tau_{\text{CPU}} \propto N_N^2 N_N^3$. For a fixed aspect ratio $N_N/N_\tau$, the cost therefore scales as $N_N^3$. This is in contrast with the expectation of an $N_N^1N_N^3$ scaling with a one-level algorithm.
provided the Euclidean correlators are consistent with the Ward identities. This has to be the case in the continuum limit, but presently analyses are still based on data at finite lattice spacing. It is therefore essential to check in at least some cases that the Ward identities are satisfied to an accuracy comparable to the statistical errors. Conversely, these identities can also be used to determine some of the normalization factors of $T_{\mu\nu}$. Figure 3 compares the correlators $C_{00,03}(x_0, q)$ and $C_{00,33}(x_0, q)$ for several values of $x_0$ and $q = q_3^\rho$ (they are exactly equal in the continuum). While the normalization of $T_{03}$ is easily determined by requiring $C_{03,03}(x_0, 0) = s / T^3$, the operator $T_{33}$ requires in total four unknown normalization factors. Two of them are fixed by thermodynamics, and two have been fitted so that the two correlators on Fig. 3 agree. Since there are $O(20)$ points where the correlators are compared, this represents a non-trivial check.

In the following I present an analysis of the sound channel correlators on a $16 \times 48^4$ lattice for $x_0 \geq 1/4T$ (corresponding to $x_0 / a_T \geq 4$) and for $q \leq \pi T$. I assume for now, motivated by the inspection of the strongly coupled $N = 4$ SYM spectral functions, that hydrodynamics describes the low-frequency part of the spectral function up to that momentum.

I adopt a fit ansatz motivated by perturbation theory at high frequencies ($\rho_{\text{high}}$), and hydrodynamics at low energies ($\rho_{\text{low}}$). Specifically, $\rho_{\text{tot}} = \rho_{\text{low}} + \rho_{\text{med}} + \rho_{\text{high}}$, where

$$
\frac{\rho_{\text{low}}(\omega, q, T)}{\tanh(\omega / 2T)} = \frac{2\pi}{\omega^2 \left( \omega^2 - \omega^2 T^2 \right) \left( \sigma^2 + \omega^2 - q^2 \right) T^2} e^\pi,  \quad \omega^2 q^2 \left( \frac{\omega}{2T} \right)^2 \left( 1 + \sigma \omega^2 \right)  \quad (14)
$$

$$
\frac{\rho_{\text{med}}(\omega, q, T)}{\tanh(\omega / 2T)} = \omega^2 q^2 \left( \frac{\omega}{2T} \right)^2 \left( 1 + \sigma \omega^2 \right)  \quad (15)
$$

$$
\frac{\rho_{\text{high}}(\omega, q, T)}{\tanh(\omega / 2T)} = \omega^2 q^2 \left( \frac{\omega}{2T} \right)^2 \left( \frac{\omega}{2T} \right)^2 \left( 1 + \sigma \omega^2 \right)  \quad (16)
$$

where $\omega(q^2)$ is given by Eq. (11). Some remarks are in order. Perturbation theory and the operator product expansion can be used to systematically improve the knowledge of $\rho_{\text{high}}$. Secondly, a full second-order hydrodynamics parametrization of the spectral function at low-frequencies would significantly improve our understanding of the accuracy on the transport coefficients that can be expected from such as global fit. Thirdly, the region $\omega = O(T)$ is the one where there is least theoretical guidance. Given the present amount and quality of the data, the simple ansatz (15) used here is a reasonable choice (other functional forms could be used to test the sensitivity of
the transport coefficients to this particular choice). As the quality of the data improves, the goal is to treat this part of the spectral function ($\rho_{\text{med}}$) in a more systematic way, either by expanding it in a basis of orthogonal functions [14] or by using the Maximum Entropy Method [36].

In the present specific example I fit 7 parameters to a total of 48 data points. The parameters are $\tilde{\Gamma}_c, \sigma_1, \sigma_2, \tau_1, \ell, \sigma, M$. Figure [2] displays the reconstructed sound spectral function based on lattice data at $2.3T_c$, and the results for the fit parameters of interest are given there too.

There has been significant interest in the behavior of bulk viscosity near the phase transition [37, 38, 39]. Figure (3, left panel) displays the $C_{00,00}$ correlator as a function of $q^2$. Repeating the relation between the Euclidean correlator and the spectral function (Eq. 4) as well as the positivity property of the latter, this Figure shows that for $T = 1.01T_c$ and $q = \pi T/4$, 

$$\frac{1}{\pi T^2} \int_0^{\sqrt{2}T} \frac{d\omega}{\omega} \rho_{\text{med}}(\omega, \mathbf{q}) \ll c_v/d_sT^3 \approx 13.6(1.9) \ (c_v \text{ is the specific heat}).$$

The sound peak has therefore quasi disappeared even for a sound wavelength of $\lambda = 8/T \approx 5.8\text{fm}$. I conclude that for all practical purposes, the gluonic medium does not support sound waves near $T_c$.

4. Conclusion

I have described the calculation of transport properties of the quark gluon plasma on the lattice. For the time being the calculations are performed in the purely gluonic plasma in order to reduce the computational cost. The primary quantities computed are the Euclidean-time dependent correlators of the energy-momentum tensor. Based on these the spectral functions can be constrained, particularly with the help of information on their functional form. Hydrodynamics and perturbation theory describe (in opposite regimes) both the spatial momentum and frequency dependence of the correlators. Correlators with finite spatial momentum are then useful, because within either the shear or the sound channel they are interrelated by the energy and momentum conservation equations. Different correlators can thus be used to over-constrain the shear and sound spectral functions. Altogether, it is now possible to include 50-100 data points in a global analysis of the sound channel, and, as illustrated by Eq. 8, even more points per channel can be used if the shear and sound channels are analyzed simultaneously.

My current best guess for the viscosity of the hot matter that will be created at LHC uses the lattice result $\eta/s \approx 0.26$ for the purely gluonic plasma at $2.3T_c$, and multiplies it by the perturbative (AMY) ratio of $\eta/s$ in full QCD and in the pure gauge theory [9],

$$[\eta/s]_{\text{QGP}} \approx [\eta/s]_{\text{GP, lattice}} \cdot \left[ \frac{[\eta/s]_{\text{QGP}}}{[\eta/s]_{\text{GP}}} \right]_{\text{AMY}} \approx 0.40. \quad (17)$$

I have presented direct evidence for the disappearance of sound waves as collective excitations of the plasma in the vicinity of the deconfining phase transition. This effect leaves a strong signature on the Euclidean correlators in the pure gauge theory, and it would be very interesting to know how strong it remains in full QCD, where the transition to the QGP is a crossover. As the chiral critical point is approached at finite baryon density (assuming it exists), the suppression of sound waves becomes stronger. This has recently been studied in detail by a combination of hydrodynamics and the theory of critical phenomena [40], and even been proposed as a possible signature of the chiral critical point.

I thank the organizers of the Quark Matter 2009 conference for a very enjoyable and stimulating physics event and for the opportunity to present this work. Lattice computations were carried out on facilities of the USQCD Collaboration, which are funded by the Office of Science of the U.S. Department of Energy, as well as on the Blue Gene L rack and the desktop machines of the
Laboratory for Nuclear Science at M.I.T. This work was supported in part by funds provided by the U.S. Department of Energy under cooperative research agreement DE-FG02-94ER40818.

References

[1] P. F. Kolb, P. Huovinen, U. W. Heinz and H. Heiselberg, Phys. Lett. B 500, 232 (2001) [arXiv:hep-ph/0101137].

[2] P. Huovinen, P. F. Kolb, U. W. Heinz, P. V. Ruuskanen and S. A. Voloshin, Phys. Lett. B 503, 38 (2001) [arXiv:hep-ph/0101136].

[3] D. Teaney, J. Lauriat and E. V. Shuryak, Phys. Rev. Lett. 86, 4783 (2001) [arXiv:nucl-th/0011088].

[4] G. Policastro, D. T. Son and A. O. Starinets, Phys. Rev. Lett. 87, 4783 (2001) [arXiv:nucl-th/0011088].

[5] B. I. Abelev et al. [BRAHMS Collaboration], Nucl. Phys. A 757, 1 (2005) [arXiv:nucl-ex/0410020]; B. B. Back et al., Nucl. Phys. A 757, 28 (2005) [arXiv:nucl-ex/0410022]; J. Adams et al. [STAR Collaboration], Nucl. Phys. A 757, 102 (2005) [arXiv:nucl-ex/0510009]; K. Adcox et al. [PHENIX Collaboration], Nucl. Phys. A 757, 184 (2005) [arXiv:nucl-ex/0410003].

[6] P. Romatschke and U. Romatschke, Phys. Rev. Lett. 99, 172301 (2007) [arXiv:0706.1522 [nucl-th]]; K. Dusling and D. Teaney, Phys. Rev. C 77, 034905 (2008) [arXiv:0710.5932 [nucl-th]]; H. Song and U. W. Heinz, Phys. Lett. B 658, 279 (2008) [arXiv:0709.0743 [nucl-th]].

[7] M. Luzum and P. Romatschke, Phys. Rev. C 78, 034915 (2008) [Erratum-ibid. C 79, 039903 (2009) [arXiv:0804.4015 [nucl-th]]].

[8] M. Asakawa, T. Hatsuda and Y. Nakahara, Prog. Part. Nucl. Phys. 50, 13 (2003) (J. Phys. G 30, S231 (2004) [arXiv:hep-ph/0302064]).

[9] H. Song and U. W. Heinz, Phys. Lett. B 658, 279 (2008) [arXiv:0709.0743 [nucl-th]].

[10] P. Arnold, C. Dogan and G. D. Moore, Phys. Rev. D 74, 085021 (2006) [arXiv:hep-ph/0608012].

[11] P. Arnold and H. W. Wyld, Phys. Rev. D 35, 2518 (1987).

[12] A. Nakamura and S. Sakai, Phys. Rev. Lett. 94, 072305 (2005) [arXiv:hep-lat/0406009].

[13] H. B. Meyer, Phys. Rev. D 76, 101701 (2007) [arXiv:0704.1801 [hep-lat]].

[14] H. B. Meyer, Phys. Rev. Lett. 100, 162001 (2008) [arXiv:0710.3717 [hep-lat]].

[15] H. B. Meyer, JHEP 0301, 048 (2003) [arXiv:hep-lat/0209145].

[16] H. B. Meyer, JHEP 0401, 030 (2004) [arXiv:hep-lat/0312034].

[17] G. Aarts, C. Allton, J. Foley, S. Hands and S. Kim, Phys. Rev. Lett. 99, 022002 (2007) [arXiv:hep-lat/0703008].

[18] P. Petreczky and D. Teaney, Phys. Rev. D 73, 014508 (2006) [arXiv:hep-ph/0507318].

[19] D. Teaney, Phys. Rev. D 74, 045025 (2006) [arXiv:hep-ph/0602044].

[20] H. B. Meyer, Phys. Rev. D 79, 011502 (2009) [arXiv:0808.1950 [hep-lat]].

[21] P. Kovtun and A. O. Starinets, Phys. Rev. D 72, 086009 (2005) [arXiv:hep-th/0506184].

[22] A. A. Pivovarov, Phys. Atom. Nucl. 63, 1646 (2000) [Yad. Fiz. 63N9, 1734 (2000)] [arXiv:hep-ph/9805485].

[23] R. Baier, P. Romatschke, D. T. Son, A. O. Starinets and M. A. Stephanov, JHEP 0404, 103 (2006) [arXiv:0712.2451 [hep-th]].

[24] H. B. Meyer, JHEP 0808, 031 (2008) [arXiv:0806.3914 [hep-lat]].

[25] A. L. Kataev, N. V. Krasnikov and A. A. Pivovarov, Nucl. Phys. B 198, 508 (1982) [Erratum-ibid. B 490, 505 (1997)] [arXiv:hep-ph/9612326].

[26] P. Romatschke and D. T. Son, arXiv:0903.3949 [hep-ph].

[27] P. Kovtun, D. T. Son and A. O. Starinets, Phys. Rev. Lett. 94, 111601 (2005) [arXiv:hep-th/0405231].

[28] P. Kovtun and A. O. Starinets, Phys. Rev. Lett. 96, 131601 (2006) [arXiv:hep-th/0602059].

[29] P. M. Chesler, arXiv:0907.4503 [hep-th], these proceedings.

[30] Y. Namekawa et al. [CP-PACS Collaboration], Phys. Rev. D 64, 074507 (2001) [arXiv:hep-lat/0105012].

[31] M. A. York and G. D. Moore, arXiv:0811.0729 [hep-ph].

[32] G. Policastro, D. T. Son and A. O. Starinets, Phys. Rev. Lett. 87, 081601 (2001) [arXiv:hep-th/0104066].

[33] H. B. Meyer, JHEP 0906, 077 (2009) [arXiv:0904.1806 [hep-lat]].

[34] M. Asakawa, T. Hatsuda and Y. Nakahara, Prog. Part. Nucl. Phys. 46, 459 (2001) [arXiv:nucl-ex/0011040].

[35] D. Kharzeev and K. Tuchin, JHEP 0809, 093 (2008) [arXiv:0705.4280 [hep-ph]].

[36] H. B. Meyer, Prog. Theor. Phys. Suppl. 174, 220 (2008) [arXiv:0805.4567 [hep-lat]].

[37] K. Huebner, F. Karsch and C. Pica, Phys. Rev. D 78, 094501 (2008) [arXiv:0808.1127 [hep-lat]].

[38] Y. Minami and T. Kunihiro, arXiv:0904.2270 [hep-th].