Hints on the power corrections from current correlators in $x$-space

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Abstract

We consider an interpretation of the recent lattice data on the current-current correlators in the $x$-space. The data indicate rather striking difference between (axial) vector and (pseudo) scalar channels which goes beyond the predictions of the standard non-perturbative models. We argue that if the difference is to be explained by power corrections, there is a unique choice of the form of the correction. We discuss the emerging picture of the power corrections.

1 Introduction

We shall be concerned here with the current-current correlators in the coordinate space:

$$\Pi(x) = \langle 0 | J(x) J^\dagger(0) | 0 \rangle , \quad (1)$$

in case of the ($V \pm A$) and (pseudo) scalar currents:

$$J^{V \pm A}_\mu = \bar{q}_i \gamma_\mu (1 \pm \gamma_5) q_j , \quad J^{S \pm P} = [(m_i - m_j) \pm (m_i + m_j)] \bar{q}_i (1 \pm \gamma_5) q_j , \quad (2)$$

where $q_{i,j}$ and $m_{i,j}$ are the quark fields and masses. The two-point functions (1) obey a dispersion representation:

$$\Pi(x) = \frac{1}{4\pi^2} \int_0^\infty dt \frac{\sqrt{t}}{x} K_1(x\sqrt{t}) \text{Im}\Pi(t) , \quad (3)$$

where $\text{Im}\Pi(t)$ is related to the current induced cross section and $K_1(z)$ is the modified Bessel function, which behaves for small $z$ as:

$$K(z \to 0) \simeq \frac{1}{z} + \frac{z}{2} \ln z . \quad (4)$$

In the limit $x \to 0$, $\Pi(x)$ coincides with the free-field correlator and the main theoretical issue is how the asymptotic freedom gets violated at intermediate $x$.

From pure theoretical point of view, the use of the $x$-space is no better than the use of the momentum space, which is the traditional tool of the QCD sum rules [1, 2]. Each representation has its own advantages and inconveniences (for a recent discussion see [3]). The $x$-space approach is motivated and described in detail in Ref. [4]. In particular, the current correlators (1) are measured in the most direct way on the lattice. The importance of the lattice measurements [5, 6] is that they allow to study the correlation functions for currents with various quantum numbers, while direct experimental information is confined to only vector and axial-vector currents [7, 8]. The well-known $\tau$-decay data were widely
used for theoretical analyses both in the $Q$- and $x$-spaces (see, e.g., [3, 10, 11, 12]). Most recently, new lattice data on the $S, P$ channels were obtained [8]. The most interesting observation is that in the $S + P$ channel there are noticeable deviations from the instanton liquid model [3] while in the $V \pm A$ channels the agreement of the existing data with this model is quite good [2, 3]. Such deviations were in fact predicted in Ref. [13] where unconventional quadratic corrections, $\sim 1/Q^2$ were introduced. The primary aim of the present note is to perform a more detailed comparison of the lattice data with the model of Ref. [13]. We, indeed, find further support for the quadratic corrections. However, the overall picture is far from being complete and we are trying to analyze the data in a more generic way. The central assumption is that the violations of the parton model for the correlators at moderate $x$ are due to power-like corrections.

## 2 Current-current correlators

For the sake of completeness, we begin with a summary of theoretical expressions for the current correlators, both in the $Q$- and $x$-spaces. We will focus on the $(V \pm A)$ and $(S \pm P)$ channels since the recent lattice data [6] refer to these channels. In case of $(V \pm A)$ currents the correlator is defined as:

$$\Pi_{\mu\nu}(q) = i \int d^4x \, e^{iqx} \langle T J_\mu(x) J_\nu(0) \rangle = (q_\mu q_\nu - g_{\mu\nu} q^2) \Pi(q^2),$$

(5)

where $-q^2 \equiv Q^2 > 0$ in the Euclidean space-time. For the sake of definiteness we fix the flavor structure of the light-quark current $J_\mu$ as:

$$J^{V \pm A}_\mu = \bar{u} \gamma_\mu (1 \pm \gamma_5) d.$$  

(6)

In the chiral limit one has in the $(V + A)$ case (see, e.g., [3, 4]):

$$\Pi^{V+A}(Q^2) = \frac{1}{2\pi^2} \left\{ - \left( 1 + \frac{\alpha_s}{\pi} \right) \ln \frac{Q^2}{\nu^2} - \frac{\alpha_s}{\pi} \frac{\lambda^2}{Q^2} + \frac{\pi}{3} < \alpha_s (G_{\mu\nu})^2 > Q^4 + \frac{256\pi^3}{81} \frac{\alpha_s \langle \bar{q}q \rangle^2}{Q^6} \right\}.$$  

(7)

The corresponding relation for the $(V - A)$ case reads as:

$$\Pi^{V-A}(Q^2) = -\frac{64\pi}{9} \frac{\alpha_s \langle \bar{q}q \rangle^2}{Q^6},$$

(8)

In the $x$-space the same correlators, upon dividing by $\Pi_{\mu\nu}^{V+A}$ where $\Pi_{\mu\nu}^{V+A}$ stands for the perturbative correlator, are obtained by applying the equations collected for convenience in the Table 1.

### Table 1: Some useful Fourier transforms

| $Q$-space | $x$-space |
|-----------|-----------|
| $Q^2 \ln Q^2$ | $\frac{8}{\pi} \frac{1}{Q^4}$ |
| $\ln Q^2$ | $\frac{1}{\pi} \frac{1}{Q^4}$ |
| $\frac{1}{Q^4} \ln Q^2$ | $\frac{1}{4\pi^2} \frac{1}{Q^4}$ |
| $\frac{1}{Q^4} \ln^2 x^2$ | $\frac{8}{\pi^2} \ln^2 x^2$ |
| $\frac{1}{Q^4} \ln x^2$ | $\frac{4}{\pi^2} \ln x^2$ |
| $\frac{1}{Q^4} \ln Q^2$ | $\frac{1}{64\pi^4} \ln^2 x^2$ |
| $\frac{1}{Q^4} \ln^2 x^2$ | $\frac{8 \times 16 \pi^2}{3\pi^2} \ln^2 x^2$ |
| $\frac{1}{Q^4} \ln^4 x^2$ | $\frac{32 \times 16 \pi^2}{3\pi^2} \ln^2 x^2$ |
| $\frac{1}{Q^4} \ln^4 x^2$ | $\frac{8 \times 16 \times 24 \pi^2}{3\pi^2} \ln^2 x^2$ |
| $\frac{1}{Q^4} \ln^4 x^2$ | $\frac{496 \times 24 \pi^2}{3\pi^2} \ln^2 x^2$ |

In particular,

$$\frac{\Pi^{V+A}}{\Pi_{\mu\nu}^{V+A}} \rightarrow 1 - \frac{\alpha_s}{4\pi} \lambda^2 \cdot x^2 - \frac{\pi}{48} \langle \alpha_s (G_{\mu\nu})^2 \rangle x^4 \ln x^2 + \frac{2\pi^3}{81} \frac{\alpha_s \langle \bar{q}q \rangle^2}{x^6} \ln x^2.$$  

(9)
Note that \( \ln x^2 \) is negative since we start from small \( x \). An important technical point is that on the lattice one measures the trace over the Lorentz indices \( \mu, \nu \), see Eq (15). In the \( Q \)-space this is equivalent to considering \( Q^2 \cdot \Pi(Q^2) \) instead of \( \Pi(Q^2) \). The \( x \)-transform of the \( Q^2 \cdot \Pi(Q^2) \) is given by:

\[
\frac{Q^2 \cdot \Pi^{V+A}}{Q^2 \cdot \Pi^{pert}} \rightarrow 1 - \frac{\pi}{96} (\alpha_s(G_{\mu \nu}^a)^2) x^4 + \frac{2\pi^3}{81} \alpha_s(\bar{q}q)^2 x^6 \ln x^2.
\]

(10)

Next, we will concentrate on the currents having the quantum numbers of the pion and of \( a_0(980) \)-meson. The correlator of two pseudoscalar currents is defined as

\[
\Pi^P(Q^2) \equiv i \int d^4x \ e^{iQx} \langle T \{ J^\pi(x) J^\pi(0) \} \rangle,
\]

(11)

where

\[
J^\pi = i(m_u + m_d) \bar{u} \gamma_5 d.
\]

(12)

In the momentum space, to represent the result in terms of the running coupling, and masses it is more convenient to consider the second derivative in \( Q^2 \) of \( \Pi^P(Q^2) \) defined in Eq. (11), which obeys an homogeneous RGE:

\[
\frac{\partial^2 \Pi^P}{(\partial Q^2)^2} = \frac{3}{8\pi^2} \frac{(\bar{m}_u + \bar{m}_d)^2}{Q^2} \left\{ 1 + \frac{11}{3} \frac{\alpha_s}{\pi} - \frac{4}{3} \frac{\alpha_s}{Q^2} \lambda^2 + 2 \frac{\pi}{3} \frac{\alpha_s}{Q^4} (G_{\mu \nu}^a)^2 + 2 \cdot \frac{896\pi^3}{81} \frac{\alpha_s}{Q^6} (\bar{q}q)^2 \right\}.
\]

(13)

Here, the standard OPE terms can be found in [1, 2, 4] while the gluon-mass correction was introduced first in [4].

In what follows, we shall work with the appropriate ratio where the pure perturbative corrections are absorbed into the overall normalization and concentrate on the power corrections assuming that these corrections are responsible for the observed rather sharp variations of the correlation functions. Thus, in the \( x \)-space we have for the pion channel:

\[
\frac{\Pi^P}{\Pi^{pert}} \rightarrow 1 - \frac{\alpha_s}{2\pi} \lambda^2 x^2 + \frac{7\pi^3}{81} (\bar{q}q)^2 x^6 \ln x^2.
\]

(14)

Note that the coefficient in front of the last term in Eq. (14) differs both in the absolute value and sign from the corresponding expression in (13). The channel which is crucial for our analysis is the \( (S + P) \). In this channel:

\[
R_{P+S} \equiv \frac{1}{2} \left( \frac{\Pi^P}{\Pi^{pert}} + \frac{\Pi^S}{\Pi^{pert}} \right) \rightarrow 1 - \frac{\alpha_s}{2\pi} \lambda^2 x^2 + \frac{\pi}{96} (\alpha_s(G_{\mu \nu}^a)^2) x^4 + \frac{4\pi^3}{81} \alpha_s(\bar{q}q)^2 x^6 \ln x^2.
\]

(15)

This expression concludes the summary of the power corrections to the current correlators.

3 **Quadratic power corrections.**

One of the central points of the present note is that there are no \( \lambda^2 \) corrections to the \( V \pm A \) correlators as can be seen in Eq (10). On the other hand, these terms are present in the case of the \( (S + P) \) channels as can be seen in Eq (15). There is no such asymmetry in the \( Q \)-space, see Eqs (13), (8) and (11). Thus, the \( (V \pm A) \) correlator, as measured on the lattice, are \( \lambda^2 \)-term blind! Thus, we are coming to a kind of a theorem. Namely:

If one assumes:

1. that the \( V \pm A \) channels are described by the instanton liquid model while in the \( S + P \) channel there are considerable deviations from this model (as the lattice data seem to strongly indicate [6])

and

2. that this difference is due to some power corrections, then the power corrections can be uniquely identified as the gluon-mass corrections (see terms proportional to \( \lambda^2 \)).

1We assume that \( \alpha_s \lambda^2 \) does not run like \( (\alpha_s(G_{\mu \nu}^a)^2) \).

2Note that a reversed case is well known. Namely, there are certain terms which are seen in the small-\( x \) expansion and are not seen in the large-\( Q \) expansion [4].
Note that the $\lambda^2$ corrections are singled out for two reasons: First, since taking the trace over the Lorentz indices $\mu, \nu$ corresponds to multiplying by $Q^2$ as can be seen in the discussion of Eq. (10) and it is only a $1/Q^2$ correction which can become a polynomial as a result of multiplying $\Pi(Q^2)$ by $Q^2$. Second, in the $Q$-space there should be no log factor in front of $1/Q^2$, $1/Q^2 \cdot \ln Q^2$. These two conditions are satisfied in case of $(V \pm A)$ currents and are not fulfilled in the $(S \pm P)$ channels. The difference between the channels is that in the latter case the $\lambda^2$ correction is present in the imaginary part of the $\Pi^{S,P}(Q^2)$ [13]. Thus, if one retains only the $\lambda^2$ corrections, then, there are no violations of the parton (perturbative) picture in the $(V \pm A)$ channels for the correlator measured in [6] while the violations are present in the $(S \pm P)$ channels.

Of course, this limiting case is not necessarily describing the reality and we proceed to quantitative fits to the data [6].

4 Analysis of the data

![Figure 1: S + P channel: comparison of the lattice data from [6] with the OPE predictions for the two Sets of QCD condensate values given in Table 2. The dot-dashed curve is the prediction for SET 3 where the contribution of the $x^2$-term has been added to SET 2. The bold dashed curve is SET 3 + a fitted value of the $D = 8$ condensate contributions. The diamond curve is the prediction from the instanton liquid model of [12].](image)

In Fig. 1 we confront the OPE predictions with the lattice data on the $(S + P)$ channel obtained in [6]. The choice of the $(S + P)$ channel is motivated by the fact the single instanton contribution cancels from this channel [6] and it was predicted in Ref. [13] that the $\lambda^2$ correction will be manifested in this channel.

$^3$The instanton contribution is not large also in the $(V \pm A)$ channels. However the $\lambda^2$ terms are canceled from these channels, see the discussion in section 3, and we cannot add anything to the analysis of Ref. [13].
The theoretical curves in Fig. 1 correspond to two sets of values of the condensates given in Table 2. The first set (SET 1) corresponds to the standard SVZ values of the gluon and four-quark condensate, the latter being obtained using the vacuum saturation assumption. The second set (SET 2), corresponds to the values of the condensates obtained in [13], where the value of the gluon condensate is two times the SVZ value and the four-quark condensate exhibits a violation of the vacuum saturation, first obtained from $e^+e^-$ data in [13]. In SET 3, one also accounts the presence of the new $1/Q^2$-term first advocated in [13] and fitted from $e^+e^-$ data in [19]. Note also that in numerical fits we put $ln x^2 = -1$, the same, as, say, in Ref. [12].

Table 2: Different parameters used in the analysis of the $S + P$ data in units of GeV$^d$ ($d$ is the dimension of the operator).

| Sources     | $\langle \alpha_s G^2 \rangle$ | $\alpha_s \langle \bar{\psi}\psi \rangle^2$ | $(\alpha_s/\pi)\lambda^2$ |
|-------------|---------------------------------|---------------------------------------------|---------------------------|
| SET 1 (SVZ) | 0.04                            | 0.256                                       | 0                         |
| SET 2       | 0.07                            | $5.8 \times 10^{-4}$                       | 0                         |
| SET 3       | 0.07                            | $5.8 \times 10^{-4}$                       | $-0.12$                   |

The analysis indicates that a much better fit of the lattice data for the $S + P$ channel at moderate values of $x$ is achieved after the inclusion of the $1/Q^2$, or $x^2$ quadratic correction. A caveat is that we account only for the power corrections, not pure perturbative contributions. The reason is that the lattice data, in their present status, do not give any clear indication of the perturbative contributions. Note also that the data cannot discriminate between the values of the dimension four and six condensates entering in SET 1 and SET 2 as the effects of these two condensates tend to compensate each other. The agreement of the OPE with the lattice data at larger values of $x$ can be obtained by the inclusion of the $D = 8$ condensate with a size $+ (x/0.58)^8$ where we have used $ln^2 x^2 \approx 1 \approx ln x^2$. This value can be compared with the one $+ (3395/30855168) \langle \alpha_s G^2 \rangle^2 x^8 \approx (x/1.2)^8$, which one would obtain from the evaluation of these contributions in [21] and where a modified factorization of the gluon condensates proposed in [22] has been used. For completion, we show in Fig. 2, a fit of the lattice data in the $V + A$ channel using SET 3 values of the gluon and quark condensates and quadratic term plus a $D = 8$ contribution with the strength $(x/0.7)^8$ to be compared with the one $\langle \alpha_s G^2 \rangle^2 x^8 / 4328592 \approx (x/2.5)^8$ which one would obtain using the results in [21]. Both fits in Figs 1 and 2 might indicate that the vacuum saturation can be strongly violated for higher dimension condensates, a feature already encountered from different analysis of the $\tau$ and $e^+e^-$ data [13, 24, 25, 26]. Therefore, we would also expect analogous large deviations in the $V-A$ channel.

5 Discussions. Two-step QCD

While evaluating the emerging picture of the power corrections, one should face the possibility that the standard OPE (see, e.g., [12, 24]) is valid only at very short distances. What is even more important, the mass scale where higher terms in the OPE become numerically comparable to the lowest ones is not necessarily the scale associated with the resonances but could be considerably higher. There is accumulating evidence to support such a view:

A common difficulty encountered in determining the quadratic corrections is that they usually compete with the standard perturbative radiative corrections. In [14, 23], a suitable choice of the sum rules (e.g. ratio of moments) has been used such that the perturbative radiative corrections are eliminated to leading order and the contribution of the quadratic term becomes optimal. Moreover, the quadratic corrections corresponding to $\lambda^2 \approx -0.5 GeV^2$ do not affect in a significant way the determination of $\alpha_s$ from $\tau$-decay as has been explicitly shown in [13]. On the contrary, the quadratic term appears to decrease very slightly $\alpha_s$ from the $\tau$-decay and bring it closer to the world average value at $M_Z$. In Ref. [12] bounds were obtained on the value of $\lambda^2$ from the sum rules which have large perturbative terms. This turned possible due to a particular fixation of the of the perturbative terms in the complex $q^2$ plane. In particular, the common use of the running coupling would affect the procedure strongly [1].
Figure 2: $V + A$ channel: comparison of the lattice data from [6] with the OPE predictions for the SET 3 QCD condensates values given in Table 2 including a fitted value of the $D = 8$ contributions. The diamond curve is the prediction from the instanton liquid model of [12].

(1) A direct comparison of the OPE with the lattice data in the $(V - A)$ channel demonstrates that the convergence radius of the OPE is no larger than 0.3 fm [12, 6].

(2) Within the instanton liquid model [6], the distance between instantons is a few times larger than the size of the instantons. On the physical grounds, the OPE applies at distances smaller than the instanton size while the resonance properties are rather related to distance between the instantons (if encoded in the model at all). Respectively, neither the lattice data nor the predictions of the instanton liquid model exhibit any irregularity at the convergence radius of the OPE [12].

(3) Within the monopole-dominated-vacuum model the two scales are even more pronounced numerically: the monopole radius is about 0.06 fm while the distance between the monopoles is about 0.5 fm [24].

(4) If one replaces the local condensates of the standard OPE by their non-local counterparts (for a review see [25]), then the effect of the non-locality is strong already at $(0.1 \div 0.3)$ fm [26].

If, indeed, the validity of the standard OPE derived within the fundamental QCD is shrunk to very short distances, then improving within the standard OPE fits to the data obtained at presently available lattices might not be a proper criterion for selecting the right model.

Instead, there emerges a picture according to which the non-standard quadratic power corrections dominate the presently available “intermediate distances” of about $(0.1 \div 0.5)$ fm. There are two pieces of evidence of the quadratic corrections dominating over the whole range of intermediate distances indicated above:

(1) non-perturbative contribution to the heavy quark potential is linear at all the distances $r > 0.1$ fm (for discussion see [29]);

(2) Instanton density, as function of the instanton size $\rho$ is reproduced at all the distances $\rho > 0.1$ fm [28].
Moreover, 

(3) Introduction of the tachyonic gluon mass explains in a simple and unified way existence of the strong channel dependence of the scales of the violation of the asymptotic freedom [13].

To this list we add now a new observation:

(4) Instanton-liquid model plus the $\lambda^2$ correction gives a reasonable fit in the $(S + P)$ channel at distances $(0.1 \div 0.5) \text{ fm}$.

As for the status of the $\lambda^2$ correction it is most solid within the effective Higgs-like theories which are common within the monopole mechanism of confinement. Indeed, in the presence of a magnetically charged (effective) scalar field the symmetry of the theory is $SU(3)_{\text{color}} \times U(1)_{\text{magnetic}}$ [30]. Upon the spontaneous breaking of the magnetic $U(1)$ the gauge boson acquires a non-vanishing mass and its mass squared is the only parameter of dimension $d = 2$ consistent with the symmetry. Moreover, in exchanges between (color) charged particles the gauge-boson mass appears to be tachyonic mass as was demonstrated on the $U(1)$ example in Refs. [27, 3]. Detailed analysis of various power corrections within the Higgs-like models can be found in Refs. [27, 28, 3]. Moreover, if the monopole size is indeed as small as indicated above, then the effective Higgs-like theories can apply at all the distances $\sim (0.1 \div 0.5) \text{ fm}$.

To summarize, the $\lambda^2$ corrections introduced in [13] rather drastically improve agreement of the theoretical predictions for the current correlator in the $(S + P)$ channel with the lattice data (without affecting the other channels measured), in the moderate $x$-region, and which can be extended to higher $x$-values by including higher dimension condensates. The main uncertainty of the analysis is due to neglect of the pure perturbative corrections, not detected so far on the lattice. Further checks could be provided by measuring the current correlators in the other channels discussed in [13]. The success of the fits with $\lambda^2$ corrections included can be understood within the effective theories.

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