Controllable $\pi$ junction with magnetic nanostructures

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Nowadays spin-electronics is one of the central topics in condensed matter physics [1, 2, 3]. There has been considerable interest in the spin injection, accumulation, transport, and detection in ferromagnet/normal metal (F/N) hybrid structures [4, 5, 6, 7, 8, 9]. Twenty years ago, Johnson and Silsbee demonstrated the spin injection and detection in a F/N/F structure for the first time [4]. Recently, spin accumulation has been observed at room temperature in all-metallic spin-valve geometry consisting of a F/N/F junction by Jedema et al. [8]. In their system, the spin-polarized bias current is applied at one F/N junction, and the voltage is measured at another F/N interface for the parallel (P) and antiparallel (AP) alignments of the F's magnetizations. They have observed the difference of the non-local voltages between the P and AP alignments due to spin accumulation in N. Also in a F/I/N/I/F (I indicates an insulator) structure, a clear evidence of spin accumulation in N has been shown [9]. In hybrid structures consisting of a ferromagnet and a superconductor (S), a suppression of the superconductivity due to spin accumulation in S has been studied theoretically and experimentally [10, 11, 12].

Furthermore, ferromagnetic Josephson (S/F/S) junctions have been studied actively in recent years [13, 14, 15, 16, 17, 18]. In the S/F/S junctions, the pair potential oscillates spatially due to the exchange interaction in F [12, 14]. When the pair potentials in two S's take different sign, the direction of the Josephson current is reversed compared to that in ordinary Josephson junctions. This state is called the $\pi$ state in contrast with the 0 state in ordinary Josephson junctions because the current-phase relation of the $\pi$ state is shifted by "$\pi$" compared to that of the 0 state. The observations of the $\pi$ state have been reported in various systems experimentally [13, 16, 17, 18]. The applications of the $\pi$ state to the quantum computing also have been proposed [19, 20, 21]. Another system to realize the $\pi$ state is a S/N/S junction with a voltage-control channel [22, 23]. In the system, the non-equilibrium electron distribution in N induced by the bias voltage plays an important role, and the sign reversal of the Josephson critical current as a function of the control voltage has been demonstrated [22, 23].

In this paper, we propose a new Josephson device in which the 0 and $\pi$ states are controlled electrically. In this device, spin accumulation is generated in a non-magnetic metal by the spin-polarized bias current flowing into the non-magnetic metal from a ferromagnet. In a metallic Josephson junction consisting of the spin accumulated non-magnetic metal sandwiched by two superconductors, the $\pi$ state appears due to the spin split of the electrochemical potential in the nonmagnetic metal. The magnitude of spin accumulation is proportional to the value of the spin-polarized bias current, and therefore the state of the Josephson junction is controlled by the current. Our proposal leads to an in-depth understanding of the spin-dependent phenomena in magnetic nanostructures as well as new possibilities for the application of superconducting spin-electronic devices.

We consider a magnetic nanostructure with two superconductors as shown in Fig. 1. The device consists of a nonmagnetic metal N (the width $w_N$, the thickness $d_N$) which is connected to a ferromagnetic metal F (the width $w_F$, the thickness $d_F$) at $x = 0$ and sandwiched by two superconductors S1, S2 located at $x = L$.

![FIG. 1: Structure of a controllable $\pi$ junction with magnetic nanostructures. The bias current $I$ flows from a ferromagnet (F) to the left side of a normal metal (N). The Josephson current $I_J$ flows in a superconductor/normal metal/superconductor (S1/N/S2) junction located at $x = L$.](image-url)
In this device, the electrode F plays a role as a spin-injector to the electrode N, and the S1/N/S2 junction is a metallic Josephson junction. The spin-diffusion length $\lambda_s$ in N is much longer than the length $\lambda_F$ in F for metallic-limit cases $R=0$, and we consider the structure with dimensions of $\lambda_s \ll (\lambda_N, \lambda_F)$ which is a realistic geometry. In the electrodes N and F, the electrical current with spin $\sigma$ is expressed as

$$j_{\sigma} = -(\sigma/\epsilon) \nabla \mu_{\sigma},$$

where $\sigma$ and $\mu$ are the electrical conductivity and the electrochemical potential (ECP) for spin $\sigma$, respectively. Here ECP is defined as $\mu_{\sigma} = \epsilon_{\sigma} + e\phi$, where $\epsilon_{\sigma}$ is the chemical potential of electrons with spin $\sigma$ and $\phi$ is the electric potential. From the continuity equation for charge, $\nabla \cdot (j_{\uparrow} + j_{\downarrow}) = 0$, and that for spin, $\nabla \cdot (j_{\uparrow} - j_{\downarrow}) = e\phi/(\tau_{\sigma} \partial n_{\sigma})$, where $\tau_{\sigma}$ is the electron relaxation time of an electron from spin $\sigma$ to $\bar{\sigma}$. In order to derive Eqs. (1) and (2), we take the relaxation-time approximation for the carrier density, $\partial n_{\sigma}/\partial t = -\delta n_{\sigma}/\tau_{\sigma}$, and use the relations $\sigma_{\sigma} = e^2 N_{\sigma} D_{\sigma}$ and $\delta n_{\sigma} = N_{\sigma} \epsilon_{\sigma}$, where $\delta n_{\sigma}$ and $\delta \epsilon_{\sigma}$ are the carrier density deviation from equilibrium and the shift of ECP, respectively. In addition, the detailed balance equation $N_{\uparrow} \tau_{\uparrow}^{-1} = N_{\downarrow} \tau_{\downarrow}^{-1}$ is also used. We use the notations $\sigma_N = 2\sigma_{\uparrow} N$ and $\sigma_F = \sigma_{\uparrow} F + \sigma_{\downarrow} F$ ($\sigma_{\uparrow} F \neq \sigma_{\downarrow} F$) in N and F hereafter.

At the interface between N and F, the interfacial current $I_f$ flows due to the difference of ECPs in N and F: $I_f = (G_{\sigma}/\epsilon)(\mu_{\sigma} F|_{z=0} - \mu_{\sigma} N|_{z=-0})$, where $G_{\sigma}$ is the spin-dependent interfacial conductance. We define the interfacial charge and spin currents as $I = I_{\uparrow} + I_{\downarrow}$ and $I_{\text{spin}} = I_{\uparrow} - I_{\downarrow}$, respectively. The spin-flip effect at the interface is neglected for simplicity. In the electrode N with the thickness and the contact dimensions being much smaller than the spin-diffusion length $d_N, \lambda_N, \lambda_F \ll \lambda_N$, $\mu^N_{\sigma}$ varies only in the $x$ direction.

The charge and spin current densities in N, $j = j_{\uparrow} + j_{\downarrow}$ and $j_{\text{spin}} = j_{\uparrow} - j_{\downarrow}$, are derived from Eqs. (1)–(3), and satisfy the continuity conditions at the interface: $j = I/A N$ and $j_{\text{spin}} = I_{\text{spin}}/A N$, where $A N = w_N d_N$ is the cross-sectional area of N. From these conditions, we obtain ECP in N, $\mu^N_{\sigma}(x) = \overline{\mu}_N + \sigma \delta \mu_N$, where $\overline{\mu}_N = (eI/\sigma_N A_N) x$ for $x < 0$, $\overline{\mu}_N = 0$ for $x > 0$, and $\delta \mu_N = (e\lambda_N I_{\text{spin}}/2\sigma_N A_N)e^{-|x|/\lambda_N}$. In the electrode F, the spin split of ECP, $\delta \mu_F$, decays in the $z$-direction because the thickness of F and the dimension of the interface are much larger than the spin-diffusion length in F ($d_F, w_N, w_F \gg \lambda_F$). In a similar way to the case of N, ECP in F is obtained from the continuity conditions for charge and spin currents. ECP in F is expressed as $\mu^F_{\sigma} = \overline{\mu}_F + \sigma \delta \mu_F$, where $\overline{\mu}_F = (eI/\sigma_F A_f) z + eV$ and $\delta \mu_F = (e\lambda_F (p_F I - I_{\text{spin}})/2\sigma_F A_f)e^{-z/\lambda_F}$ with the contact area $A_f = w_N w_F$, the voltage drop at the interface $V = (\overline{\mu}_F - \overline{\mu}_N)/\epsilon$, and the polarization of the current in F, $p_F = (\sigma_F - \sigma_{\uparrow} F)/\sigma_F$. The influence of the electrodes S1 and S2 on ECP in N may be neglected. When the superconducting gap in S1 and S2 is much larger than the spin split $\delta \mu_N$ at $x = L$, almost no quasiparticle is excited above the gap at low temperature. Therefore, the spin current does not flow into S1 and S2, and the behavior of ECP in N is not modified by the connection to the electrodes S1 and S2.

In order to obtain the relation between the bias current $I$ and the shift of ECP, $\delta \mu_N$, at the right side in N ($x > 0$), we substitute the obtained $\mu^N_{\sigma}$ and $\mu^F_{\sigma}$ for the expressions of $I$ and $I_{\text{spin}}$, and eliminate $V$. As a result, we obtain the relation between $I$ and $I_{\text{spin}}$, and finally we get the relation between $I$ and $\delta \mu_N$ as follows:

$$\delta \mu_N(x) = e\Re_N I \frac{P_1}{1-P_1} \left( \frac{\Re_N}{\Re_N} \right) + \frac{P_F}{1-P_F} \left( \frac{\Re_F}{\Re_F} \right) e^{-x/\lambda_N},$$

where $\Re_N = \lambda_N/(\sigma_N A_N)$ and $\Re_F = \lambda_F/(\sigma_F A_f)$ indicate the non-equilibrium resistances of N and F, respectively, $R = G^{-1} = (G_{\uparrow} + G_{\downarrow})^{-1}$ is the interfacial resistance, and $P_3 = (G_{\uparrow} - G_{\downarrow})/G$ is the polarization of the interfacial current. When the F/N interface is the tunnel junction ($R \gg \Re_N, \Re_F$), Eq. (4) reduces to a simple form.
The injected electron captures the opposite spin band in order to form a Cooper pair

$\delta \mu_N (x) = (e R N I P / 2) e^{-x / \lambda_N}$.

On the other hand, when the F/N junction is of metallic contact ($R = 0$), Eq. 4 becomes $\delta \mu_N (x) = e R N I P e^{-x / \lambda_N} / (2 \sqrt{(1 - \rho_F^2) \lambda_N})$, where $r = R_F / R_N$ is a mismatch of the resistances in F and N. Figure 2 shows the spacial variation of $\delta \mu_N (x)$ both for the tunnel- and metallic-limit cases with $P_1 = 0.4$ and $P_2 = 0.6$ [13, 24]. As shown in this figure, in the case of the metallic contact, $\delta \mu_N$ becomes larger with decreasing the resistance mismatch [8].

Next we consider how spin accumulation affects the Josephson current $I_1$ flowing through the S1/N/S2 junction located at $x = L$ (Fig. 1). In the metallic Josephson junction, the Andreev bound state plays a key role for the Josephson effect [13, 14, 15, 16, 17, 18]. In the S/F/S systems, Cooper pairs are formed by the Andreev reflection of spin-$\sigma$ electrons with the wave number $k_e^F \approx (\sqrt{2m / \hbar}) \sqrt{E_F + \sigma E_{xx}}$ and holes with $k_h^F \approx (\sqrt{2m / \hbar}) \sqrt{E_F - \sigma E_{xx}}$ at the energy $E \approx 0$. In the case that the exchange interaction is much weaker than the Fermi energy ($E_{xx} \ll E_F$), the stable state (0 or $\pi$) in the system depends on the dimensionless parameter $\alpha_F = (E_{xx} / E_F)(k_F - d_F)$, where $d_F$ is the thickness of F and $k_F$ is the Fermi wave number [17]. At $\alpha_F = 0$ the system is in the 0 state, and the first 0-$\pi$ transition occurs at $\alpha_F = \pi / 2$, and then the system is in the $\pi$ state at $\alpha_F = \pi$ [17]. Because the value of $E_{xx}$ is fixed in the S/F/S system, the 0 and $\pi$ states change periodically with the period $2 \pi (E_F / E_{xx})$ as a function of $d_F$. As a result, the $d_F$ dependence of the Josephson critical current shows a cusp structure and the critical current becomes minimum at the 0-$\pi$ transition [14, 17].

In analogy with the case of the S/F/S junction discussed above, when there is spin accumulation in N as shown in Fig. 3, the 0 or $\pi$ state is realized in the S1/N/S2 junction depending on the parameter $\alpha = (\delta \mu_N / E_F)(k_F w_N)$. In this case, the width $w_N$ is fixed, and the 0 and $\pi$ states are controlled through the value of $\delta \mu_N$ which is proportional to the bias current $I$ (see Eq. 13). The N part of the system is in the non-equilibrium state by the spin current in contrast with F in the equilibrium state of the S/F/S junction. However, one can discuss the critical current in the non-equilibrium S1/N/S2 junction in the same way as the equilibrium S/F/S junction because the critical current is dominated by the energy of the quasiparticles in N, not by the flow of the current [22, 23].

From the point of view of more detailed description, the free energy in the system is obtained by the summation of the energy of the Andreev bound states [13]. The bound state energy is calculated from the Bogoliubov-de Gennes equation [25], and the free energy is minimum for the phase difference 0 ($\pi$) for the 0 ($\pi$) state. In the S1/N/S2 junction with no spin accumulation in N ($\delta \mu_N = 0$), the bound states with the energy $E > 0$ contribute to the free energy. On the other hand, when spin accumulation exists in N, the spin-up (down) bound states with the energy $E > \delta \mu_N$ ($-\delta \mu_N$) contribute to the free energy because ECP is shifted by $\delta \mu_N$ ($-\delta \mu_N$) in N. The 0-$\pi$ transition occurs due to the shift of the energy region of the Andreev bound states which contribute to the free energy.

As an example, we consider the case that the F/N interface consists of a tunnel junction. The material parameters $P_1 = 0.4$, $\rho_N = \sigma_N^{-1} = 2 \mu \Omega \text{cm}$, $\lambda_N = 1 \mu m$, $w_N = 800 \text{ nm}$, and $d_N = 10 \text{ nm}$, which lead to $R_N = 2.5 \Omega$, are taken. The distance between F and S’s is taken to be $L = 500 \text{ nm}$. When no bias current is applied between F and N ($I = 0$), the S1/N/S2 junction is in the ordinary 0 state because there is no spin split.
of ECP ($\delta \mu_N = 0$). With increasing the bias current, the magnitude of the Josephson critical current decreases because the parameter $\alpha$ increases due to the increase of the spin split. When the bias current reaches the value $I = I_0 \approx 3\, \text{mA}$ which induces the spin split $\delta \mu_N \approx 1\, \text{meV}$ at $x = 500\, \text{nm}$, the parameter $\alpha \approx \pi/2$ and the first transition to the $\pi$ state from the 0 state occurs (the values of $E_F = 5\, \text{eV}$ and $k_F = 1\, \text{Å}^{-1}$ are taken [24]).

As a result, the magnitude of the Josephson critical current takes its minimum at $I = I_0$, and increases with increasing the bias current $I > I_0$. When the bias current attains $I = 2I_0$, the magnitude of the Josephson critical current becomes maximum because of $\alpha \approx \pi$, and decreases with increasing the bias current $I > 2I_0$. For $I = 3I_0$ corresponding to $\alpha \approx 3\pi/2$, the second transition to the 0 state from the $\pi$ state occurs.

Here we discuss the effect of spin accumulation on the superconducting gap [17]. The spin split $\delta \mu_N$ at $x = L$ in N causes the split of ECP of S's by $\delta \mu_N$ near the S/N interfaces. The spin split in S's decreases exponentially with the spin-diffusion length $\lambda_\delta$ from the interface. In the superconductors, the superconducting gap is not suppressed by spin accumulation until $\delta \mu_N$ exceeds the critical value of the spin split $\delta \mu_{NC}$ [11]. At low temperatures much lower than the superconducting critical temperature ($T \ll T_c$), the critical value of the spin split is obtained as $\delta \mu_{NC} \lesssim \Delta_0$ by solving the gap equation [11], where $\Delta_0$ is the superconducting gap for $\delta \mu_N = 0$ at $T = 0$. In the case discussed in the above paragraph, $\delta \mu_N \approx 1\, \text{meV}$ at the first 0-$\pi$ transition ($\alpha \approx \pi/2$). For example, $\Delta_0 \approx 1.5\, \text{meV}$ for niobium [28], and therefore the superconducting gap is almost not affected by spin accumulation at the first 0-$\pi$ transition. When superconductors with the higher value of $T_c$, e.g., MgB$_2$ ($T_c \approx 39\, \text{K}$) [28], or High-$T_c$ materials ($T_c$ is several 10 K's) [29], are used as the electrodes S1 and S2, the superconductivity is robust even at the second ($\delta \mu_N \approx 3\, \text{meV}$, $\alpha \approx 3\pi/2$) and higher 0-$\pi$ transitions.

In summary, we have proposed the novel Josephson device in which the 0 and $\pi$ states are controlled electrically. The spin split of the electrochemical potential is induced in the electrode N by the spin-polarized bias current flowing from F to N. The $\pi$ state appears in the S1/N/S2 junction due to the non-local spin accumulation in N. Because the magnitude of spin accumulation is proportional to the value of the spin-polarized bias current, the 0 and $\pi$ states of the Josephson junction are controlled by the current. Our proposal provides not only new possibilities for the application of superconducting spin-electronic devices but also the deeper understanding of the spin-dependent phenomena in the magnetic nanostructures.

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