Real-Time Dynamic Spectrum Management for Multi-User Multi-Carrier Communication Systems

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Abstract—Dynamic spectrum management is recognized as a key technique to tackle interference in multi-user multi-carrier communication systems and networks. However existing dynamic spectrum management algorithms may not be suitable when the available computation time and compute power are limited, i.e., when a very fast responsiveness is required. In this paper, we present a new paradigm, theory and algorithm for real-time dynamic spectrum management (RT-DSM) under tight real-time constraints. Specifically, a RT-DSM algorithm can be stopped at any point in time while guaranteeing a feasible and improved solution. This is enabled by the introduction of a novel difference-of-variables (DoV) transformation and problem reformulation, for which a primal coordinate ascent approach is proposed with exact line search via a logarithmicly scaled grid search. The concrete proposed algorithm is referred to as iterative power difference balancing (IPDB). Simulations for different realistic wireless and wireless interference limited systems demonstrate its good performance, low complexity and wide applicability under different configurations.

Index Terms—Dynamic spectrum management, interference management, multi-user, multi-carrier, real-time

I. INTRODUCTION

Interference is a key performance-limiting factor in many state-of-the-art communication systems and networks [1]–[7]. In particular, when multiple users transmit simultaneously in a common frequency bandwidth, significant interference levels can be observed among them in practical systems. This can result in large data rate reductions [6]–[8], poor spectral and energy efficiency [9]–[13], unstable behaviour due to transient interference [16]–[19], unfairness due to unbalanced interference impact [20] and other performance degradations.

Dynamic spectrum management (DSM) is recognized as an important technique to tackle these performance degradations in such interference limited systems [5]–[8], [10]–[12]. In digital subscriber line (DSL) literature, DSM is typically categorized into three levels, namely DSM 1, DSM 2 and DSM 3. DSM 1 corresponds to single user management in terms of impulse-noise control, delay parameter tuning and transmit spectrum shaping [43]. DSM 2 addresses solutions where the transmit spectra of all users are jointly managed [5], [7], [26] so as to prevent the destructive impact of interference. DSM 3 is also referred to as vectoring and consists of the application of signal coordination methods that can actively cancel interference between users [6]–[8], [24], [25]. We want to briefly highlight here that the word ‘dynamic’ in DSM does not refer to time dynamic adaptation of the spectrum management resources, but rather to the adaptation of the spectrum management resources taking the concrete physical channel conditions of the considered scenario into account.

In this paper the focus is on DSM through the management of transmit spectra for general multi-user multi-carrier systems, including wireline DSL systems (corresponding to DSM 2) as well as wireless interference limited systems. Here, the transmit spectra of all users in the system are jointly managed and optimized, where each user employs a multi-carrier transmission technique such as orthogonal frequency division multiplexing (OFDM) or discrete multitone (DMT). In the remainder of this paper, we will refer to this technique shortly by DSM, as this term is similarly used in other literature [7], [20], [29]. Furthermore we follow a standard interference channel system model where the interference is treated as additive white Gaussian (AWG) noise, which is a very common practical model in operational networks [7].

Research work on DSM has progressed significantly over the last decade. More specifically, a whole range of DSM algorithms has been proposed ranging from centralized [30]–[35], to distributed [21], [31], [42], [44], [45] and autonomous algorithms [21], [36]–[40]. Each of these has its specific properties in terms of computational complexity and level of coordination. We refer to [21], [25] and references therein for a comparison and an overview of DSM algorithms proposed in DSL literature. The DSL setting represents one relevant example of a multi-user multi-carrier interference channel. However DSM is also of interest in several wireless settings, where similar algorithms have been proposed. Examples are multicell downlink DSM or inter-cell interference coordination for heterogeneous networks [37]. DSM for multi-user multi-
channel cellular relay networks [44], and OFDM based cognitive radio systems [9], [46]. However, none of the previously proposed DSM algorithms have addressed real-time computation constraints. More specifically, when computation time and compute power are limited, there is no guarantee that a suitable solution can be found with existing DSM algorithms. This is because existing DSM algorithms typically follow an iterative approach where it is not known in advance how many iterations are required to converge to a feasible and good solution. Furthermore existing DSM algorithms typically follow a dual decomposition approach where solution feasibility and good performance is not guaranteed until after convergence. In addition, an important issue of tackling the nonconvex DSM problem in the dual domain is the possible non-zero duality gap as the number of subcarriers used in the multi-carrier transmission is finite [28], [29].

Our focus in this paper is to tackle the above issues by a new paradigm and theory of real-time dynamic spectrum management. The corresponding real-time dynamic spectrum management algorithms succeed in working under real-time constraints where the computation time and the compute power are limited. This property is highly desirable when real-time responsiveness to changes in the network is to be guaranteed, such as changing channels and noises, users joining or leaving the network, changing QoS requirements, crosstalk control, etc. To the best of our knowledge, there exists no literature on resource allocation for interference limited communication systems that addresses such real-time constraints.

To enable this new paradigm, we first propose a novel transformation, referred to as the difference-of-variables (DoV) transformation, which transforms the standard DSM problem into a problem with alternative primal variables, referred to as power difference variables. With this reformulation in hand, a first real-time DSM algorithm is proposed, which is referred to as iterative power difference balancing (IPDB). This algorithm combines the DoV problem reformulation with a solution that follows a coordinate ascent approach with exact line search via a logarithmically scaled grid search. The combination of these two ingredients results in an efficient algorithm for which the effectiveness and real-time behaviour are analyzed and evaluated for different settings.

The paper is organized as follows. Section II briefly describes the multi-user multi-carrier system model and DSM. Section III first gives a definition of real-time DSM. Then the DoV transformation and problem are proposed. Finally, the IPDB algorithm is presented. This is extended with a procedure for dealing with inequality power constraints and equalization. The performance for different wireline and wireless scenarios and settings is presented in Section IV.

II. SYSTEM MODEL AND DYNAMIC SPECTRUM MANAGEMENT

We consider a multi-user communication system with a set $\mathcal{N} = \{1, \ldots, N\}$ of $N$ communication links over a common frequency band. Each link consists of a transmitter-receiver pair, and is also referred to as a user. In addition, each user employs a multi-carrier transmission scheme, such as OFDM or DMT. We assume perfect synchronization and a cyclic prefix length longer than the maximum channel length (considering direct as well as interference channels), so that the user data are transmitted independently and in parallel on the different subcarriers, also referred to as tones. The set of $K$ tones is denoted as $\mathcal{K} = \{1, \ldots, K\}$. All users can transmit on all tones, resulting in overlapping transmit spectra and thus multi-user interference. Note that our system also includes the single-user case, i.e., with $N = 1$, as a special case.

We focus on dynamic spectrum management through multi-user multi-carrier transmit power management and optimization. No signal coordination or vectoring between transmitters or receivers is assumed. Each user thus employs a single-user decoder. This case is well in line with many practical settings where a distinct physical location or a limited communication between transmitters and receivers does not allow for signal coordination. We follow the common standard interference channel system model where the multi-user interference is treated as AWG noise. Perfect channel state information is assumed at transmitters and receivers. The achievable bit rate of user $n$ on tone $k$ is then given as follows

$$b_k^n(s_k) \triangleq \log_2 \left( 1 + \frac{s_k^n}{\sum_{m \neq n} a_{n,m}^n s_m^m + z_k^n} \right), \quad (1)$$

with $s_k = [s_k^1, \ldots, s_k^N]^T$, $s_k^n$ denoting the transmit power of user $n$ on tone $k$, $a_{n,m}^n$ denoting the normalized channel gain from user $m$ to user $n$ on tone $k$, and $z_k^n$ denoting the normalized received noise power for user $n$ on tone $k$. A signal to noise ratio (SNR) gap [47] that characterizes imperfect coding and signal modulation, and a noise margin, may be included in the normalized channel gains and noise power.

The DSM problem can then be formulated as follows

$$\begin{array}{ll}
\text{maximize} & \sum_{n \in \mathcal{N}} w_n R^n(s_k, k \in \mathcal{K}) \\
\text{subject to} & P^n(s_k, k \in \mathcal{K}) = P^n_{\text{tot}}, \forall n \in \mathcal{N} \\
& 0 \leq s_k^n \leq s_k^{\text{mask}}, \forall n \in \mathcal{N}, \forall k \in \mathcal{K},
\end{array} \quad (2)$$

with $R^n(s_k, k \in \mathcal{K})$ denoting the achievable data rate for user $n$ and its corresponding weighting $w_n$, $P^n(s_k, k \in \mathcal{K})$ denoting the total allocated (transmit) power of user $n$, the constant $P^n_{\text{tot}}$ denoting the total power budget for user $n$, and the constant $s_k^{\text{mask}}$ denoting the maximum transmit power (spectral mask) of user $n$ on tone $k$. This corresponds to a maximization of the sum of the weighted achievable data rates (with multiple tones), under per-user total power constraints and per-tone spectral masks.

The transmit spectrum of a user refers to the user’s transmit power on all the tones. These transmit spectra are the optimization variables for the DSM problem.
We want to highlight that the per-user total power constraints are expressed as equality constraints. This is further extended to inequality constraints in Section III-D.

### III. Real-Time Dynamic Spectrum Management

Real-time computation is an important challenge in practice where computation time and compute power of communication devices and systems are limited. In this section, we present a new paradigm and theory for real-time dynamic spectrum management (RT-DSM). We first introduce our definition of a RT-DSM algorithm in Section III-A. To enable the design of RT-DSM algorithms, we then propose a novel transformation, referred to as iterative power difference balancing, in Section III-B. Using this transformation, we reformulate the DSM problem in terms of power difference variables. This allows for the design of a first RT-DSM algorithm in Section III-C, referred to as a real-time dynamic spectrum management (RT-DSM) algorithm. We first introduce our definition of a RT-DSM algorithm in Section III-A. To enable the design of RT-DSM algorithms as will be shown in Section III-B, we propose a novel transformation, which denotes the set of tones for user \( n \) and tone \( k \) with power difference variables that influence \( s^n_k \).

#### A. Definition of Real-Time Dynamic Spectrum Management Algorithm

To provide a concrete label and definition of the algorithms targeted in this paper, we introduce the following definition:

**Definition 1:** [Real-time dynamic spectrum management (RT-DSM) algorithm] A real-time dynamic spectrum management algorithm sequentially updates the transmit power variables such that these satisfy all constraints after each update.

This definition implies that RT-DSM algorithms can be stopped after each update (even after a single update of any transmit power variable), and as such they are suitable for execution under very tight computation time and compute power budgets, as they can be stopped whenever one of both resources is exhausted. This guarantees fast responsiveness, and allows for real-time operation.

#### B. Difference-of-Variables (DoV) Transformation and Optimization

The original DSM problem (2) consists of a separable objective function and coupled per-user total power constraints. An important step towards the design of RT-DSM algorithms is to eliminate the per-user total power constraints. To enable this we propose to replace the primal variables \( s^n_k \) with an alternative set of primal variables \( t^n_k \) with an alternative set of primal variables \( t^n_k \) with an alternative set of primal variables \( \gamma^n_k \), where the latter will be referred to as the power difference variables. For this we propose a novel transformation of variables, referred to as the difference-of-variables (DoV) transformation:

\[
\begin{align*}
\beta^n_k &= \sum_{j \in K} \beta^n_k(j) t^n_j + P^{n,\text{tot}} \gamma^n_k, \quad n \in \mathcal{N}, k \in \mathcal{K} \\
\end{align*}
\]

with
\[
\begin{align*}
\sum_{k \in K} \beta^n_k(j) &= 0, \quad n \in \mathcal{N}, j \in \mathcal{K} \\
\sum_{k \in K} \gamma^n_k &= 1, \quad n \in \mathcal{N} \\
\beta^n_k(k) &> 0, \quad n \in \mathcal{N}, k \in \mathcal{K}
\end{align*}
\]

where \( \beta^n_k(j), \gamma^n_k \) are (fixed) arbitrary constants that can take any value satisfying constraints (5), (6) and (7). We also define the following set \( \mathcal{B}^n_k \) for later use

\[
\mathcal{B}^n_k = \{ j \in \mathcal{K} | \beta^n_k(j) \neq 0 \}
\]

which denotes the set of tones for user \( n \) and tone \( k \) with power difference variables that influence \( s^n_k \).

Using the DoV transformation (4), we obtain a reformulation of (2) as given in the following theorem:

**Theorem 1:** Applying a DoV transformation (4), satisfying constraints (5), (6) and (7), to the DSM problem (2) results in the reformulated problem (3).

**Proof:** The objective and constraints of (3) can be obtained by applying the DoV transformation to the objective and per-tone spectral mask constraints of (2). The per-user total power constraints of (2) are not present anymore in the reformulation (3). This is because the proposed DoV transformation (4) ensures that these constraints are satisfied for all values of the power difference variables \( t^n_k \). This can straightforwardly be proven as follows:

\[
\begin{align*}
\sum_{k \in K} s^n_k &= \sum_{k \in K} \left( \sum_{j \in K} \beta^n_k(j) t^n_j + P^{n,\text{tot}} \gamma^n_k \right) \\
&= \sum_{j \in K} t^n_j \left( \sum_{k \in K} \beta^n_k(j) \right) + \left( P^{n,\text{tot}} \sum_{k \in K} \gamma^n_k \right) = 0 \\
&= P^{n,\text{tot}}
\end{align*}
\]

The strength of reformulation (3) is that the coupled per-user total power constraints are eliminated, and that the reformulation is expressed in terms of the power difference variables \( t^n_k \). Because of the constraint (5), each power difference variable \( t^n_k \) adds some transmit power to some tones but subtracts the same amount of transmit power from other tones, resulting in a zero total power change operation. This is also the reason why \( t^n_k \) is referred to as a power difference variable. The above properties of the reformulated DSM problem (3) are crucial to enable the design of RT-DSM algorithms as will be shown in Section III-C.

Reformulation (3) displays coupling in both the objective as well as the constraints. However, the coupling can be of much smaller size (compared to (2)) in the sense that it couples only a subset of all tones. More specifically, the bit loading \( b^n_k \) on a given tone and the spectral mask constraints are impacted by a number of power difference variables equal to the cardinality...
C. Iterative Power Difference Balancing

Let DoV transformation consist in sequentially updating/optimizing one power difference variable at a time. The corresponding one-dimensional (1D) formulation of this problem is as follows:

maximize \( \sum_{n \in \mathcal{N}} w_n \sum_{k \in \mathcal{K}} \log_2 \left( 1 + \frac{\sum_{j \in \mathcal{J}} \beta^m_k(j) t^n_j + P^n,\text{tot}_k}{\sum_{m \neq n} a^{n,m}_k \left( \sum_{j \in \mathcal{J}} \beta^m_k(j) t^n_j + P^n,\text{tot}_m \right) + z^n_k} \right) \) 

subject to \( 0 \leq \sum_{j \in \mathcal{J}} \beta^m_k(j) t^n_j + P^n,\text{tot}_k \leq s^{n,\text{mask}}_k, \forall n \in \mathcal{N}, \forall k \in \mathcal{K} \)

We refer to this as IPDB, which stands for Iterative Power Difference Balancing.

1) Two-tone DoV transformation:

\[ s^n_k = \begin{cases} t^n_k - t^n_{k-1} + P^n,\text{tot}_k, & k > 1 \\ t^n_1 - t^n_K + P^n,\text{tot}_1, & k = 1 \end{cases} \]  

2) Three-tone DoV transformation:

\[ s^n_k = \begin{cases} -t^n_{k+1} + 2t^n_k - t^n_{k-1} + P^n,\text{tot}_k, & k > 1 & & & k < N \\ -t^n_{k+2} + 2t^n_k - t^n_{k-1} + P^n,\text{tot}_1, & k = 1 \end{cases} \]

The two-tone DoV transformation has a coupling over two consecutive tones, i.e., \( \text{card}(\mathcal{B}^n_k) = 2 \), whereas the three-tone DoV transformation \( \text{card}(\mathcal{B}^n_k) = 3 \).

C. Iterative Power Difference Balancing

Our RT-DSM algorithm design starts from the proposed reformulated problem \([5]\). As the DSM problem corresponds to an NP-hard nonconvex problem \([29]\), we propose an iterative coordinate ascent approach to tackle it, which we refer to as iterative power difference balancing. More specifically, it consists in sequentially updating/optimizing one power difference variable at a time. The corresponding one-dimensional optimization problem is given in \((9)\), where the optimization variable is the power difference variable \( t^n_k \).

To identify the coupling level, we define a set \( A^n_k \) as follows:

\[ A^n_k = \{ j \in \mathcal{K} | \beta^m_k(j) \neq 0 \} \]

which denotes the set of tones for user \( n \) and tone \( k \) with transmit powers that are influenced by power difference variable \( t^n_k \). The objective function in \((9)\) is coupled over multiple tones, depending on the cardinality of \( A^n_k \). However a proper choice of the DoV transformation results in a small coupling level, reducing the sum to only a few terms. For instance, for the two-tone DoV transformation \((10)\), this corresponds to two terms, which means that we only consider two tones in the objective function and the constraints. The constraints correspond to plain bound constraints where the bounds \( t^n_{k,\text{min}} \) and \( t^n_{k,\text{max}} \) are simple constants that depend on the other power difference variables and are kept constant in the considered iteration. By updating the power difference variables one at a time, the total power \( P^n \) is not changed because of the zero per-user total power change property \([5]\). Each update results in an improved objective function value though, guaranteeing a monotonously improving performance. We want to highlight that, in contrast to typical existing DSM algorithms, IPDB solves the problem in the primal domain instead of the dual domain, avoiding all issues related to a possible non-zero duality gap.

The one-dimensional problem \((9)\) however still corresponds to a nonconvex problem and therefore we propose to solve it with a plain one-dimensional (1D) exhaustive search, where the interval \([t^n_{k,\text{min}}, t^n_{k,\text{max}}]\) is discretized in small steps. This can be seen as an exact line search based on a 1D
grid search. Note that iterative grid-based exhaustive one-dimensional searches have been shown to be very promising in DSM literature, such as for the iterative spectrum balancing (ISB) algorithm [21, 22]. We emphasize however that these existing algorithms focus on dual solutions where the discretization is applied to the primal variables, which are transmit powers. In our case, we focus on a primal solution where we consider power difference variables. As a result, we claim that we can make the discretization coarser, because power difference variables focus on differences between tones. Taking into account the fact that the channels (direct as well as crosstalk channels) vary over tones with some degree of smoothness, the optimal transmit spectra do not differ significantly from one tone to the next, a property that has also been referred to as spectral correlation [43]. Therefore we propose to use a fine discretization for small difference values and a coarse discretization for large differences. More specifically we choose a logarithmically scaled discretization.

To obtain this we define the following sets

\[
\mathcal{F} = \{ x | 10 \log_{10}(x) = -140 + k\delta, k \in \mathbb{Z} \}
\]

\[
\mathcal{J}^+ = F \cap [\gamma_k^{n_{\min}}, \gamma_k^{n_{\max}}]
\]

\[
\mathcal{J} = \mathcal{J}^+ \cup \{0\} \cup (-\mathcal{J}^+),
\]

where \( \delta \) is a discretization variable (referred to as granularity). In the case of dual algorithms such as ISB, \( \delta = 0.5 \) dBm/Hz is typically chosen. However for IPDB, we show in Section IV-B that a coarser granularity can be chosen (e.g., 1dBm/Hz), without significantly impacting the final accuracy, which then reduces computational complexity significantly.

We note that the zero element is included in the set \( \mathcal{J} \) to maintain monotonicity.

The resulting grid-based search approach for the 1D problem corresponds to problem (13), where the feasible space consists of set \( \mathcal{J} \).

\[
\text{maximize } f_n^p(t_k^n) \quad \text{subject to } t_k^n \in \mathcal{J}
\]

The full IPDB algorithm is given in Algorithm 1. In line 1, the power difference variables and the granularity \( \delta \) are initialized. In line 2, the constants \( \gamma_k^n \) are initialized satisfying two different constraints. A straightforward choice here is \( \gamma_k^n = 1/K \), which corresponds to an equal power allocation over all tones, i.e., \( s_k^n = P_n^{\text{tot}}/K \), which typically satisfies all power constraints in (2) and (3). The repeat loop in line 3 is a loop that can go until some stopping criterion is achieved or until some real-time deadline is reached. The loop in line 4 is the per-user loop. Note that the user order is not defined and can be arbitrarily chosen. In fact this user order can also have multiple instances of the same user. Line 5 is the inner per-user iteration with a maximum of \( I \) iterations. Line 6 is the per-tone loop. Again, the tone order is not necessarily consecutive but can be arbitrary. Line 7 is the only line that involves an update of the transmit powers and corresponds to a one-dimensional power difference variable update by a 1D exhaustive grid-based search of problem (13) over the values \( \mathcal{J} \). With the DoV transformation (4) the corresponding updated transmit powers can be obtained. In line 9 the power difference variables are then reset and the constants \( \gamma_k^n \) are updated so as to keep the transmit powers fixed. Although the latter two actions are not necessary from a theoretical point of view, they are seen to improve the performance, as the values around \( t_k^n = 0 \) are discretized at a finer granularity. This can be seen as a recentering operation so as to fully benefit from the logarithmically scaled discretization. As a result a fine granularity in transmit powers can be obtained through a sum of coarse power difference variables (that are not coarse everywhere). This results in a good final solution accuracy as demonstrated in Section IV Lines 12 and 13 correspond to an inequality procedure to consider inequalities and an equalization procedure, as discussed in Section III-D and III-E respectively. Note however that these steps are not necessary and can be disregarded at this point.

We now analyze different aspects and properties of the IPDB algorithm:

1) Tunability: We want to highlight that the IPDB algorithm offers flexibility in choosing the user order, the tone order, the number of inner iterations, the granularity \( \delta \), and in the initialization of the parameters \( \gamma_k^n \). Different such choices are evaluated in Section IV-A.

2) Real-time Property: The IPDB algorithm satisfies the real-time property from Definition III-A: it can be stopped at any moment as it satisfies the constraints after every single update of the power difference variables, which have a one-to-one mapping to the transmit powers through (4). The concrete improved real-time behaviour is demonstrated in Section IV-A.

Algorithm 1 Iterative Power Difference Balancing

1: Initialize \( \delta, t_k^n \leftarrow 0, \forall n, \forall k \)
2: Initialize \( \gamma_k^n, \forall n, \forall k \) satisfying (6) and \( 0 \leq \gamma_k^n \leq \gamma_k^{n_{\max}} \)
3: repeat
4: for \( n \leftarrow \text{userOrder} \) do
5: for \( i \leftarrow 1, I \) do
6: for \( k \leftarrow \text{toneOrder} \) do
7: \( t_k^n \leftarrow \text{Solve (13)}; s_k^n \leftarrow (13), \forall k \in \mathcal{A}_k^n \)
8: end for
9: \( t_k^n \leftarrow 0, \gamma_k^n \leftarrow s_k^n / P_n^{\text{tot}}, \forall k \)
10: end for
11: end for
12: Inequality procedure Algorithm 2
13: Equalization procedure Algorithm 3
14: until convergence stop criterion

3) Complexity: The computational complexity analysis of IPDB is rather straightforward. Most of the complexity results from line 7, which corresponds to a simple 1D exhaustive grid-based search. Under a given computational and compute power budget it is easy to determine the number of updates that can be performed, which demonstrates the benefit of the real-time property of IPDB.

4) Monotonicity: Each update results in a non-decreasing feasible objective function value. As a result IPDB has an interesting monotonicity and scalability property where more computation time or compute power consistently results in a better obtained solution.
5) Convergence: As the IPDB algorithm is a coordinate search method, the convergence behaviour is inherited from such methods. The looser the coupling between the coordinate ascent variables, the faster the convergence [49]. However, it is important to highlight that because of the real-time property that ensures constraint satisfaction after each single update, and the monotonicity property that ensures a non-decreasing objective function value, it is not extremely important that full convergence is reached when performing the IPDB algorithm. Fast numerical convergence results (up to 99% and 99.9% of full performance convergence) are demonstrated in Section IV-A.

Finally, we want to highlight that although we employ a DoV transformation with differences between the transmit powers on different tones for one user, one could in principle also employ differences between the transmit powers of different users on a single tone or different tones if there are per-tone sum power constraints or total network sum power constraints, respectively.

D. Inequality constraints

In this section we consider inequality constraints for the per-user total power constraints as given by the following DSM problem

\[
\begin{align*}
\text{maximize} & \quad \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} w_n R_n^m(s_n^k, k \in \mathcal{K}) \\
\text{subject to} & \quad P_n(s_n^k, k \in \mathcal{K}) \leq P_n^{\text{tot}}, \forall n \in \mathcal{N}, \quad 0 \leq s_n^k \leq s_n^{\text{mask}}, \forall n \in \mathcal{N}, \forall k \in \mathcal{K}.
\end{align*}
\]

To extend IPDB to also cover inequality constraints, we propose an inequality procedure that allows to reduce the per-user total powers below $P_n^{\text{tot}}$ whenever this improves the weighted sum of achievable data rates. This procedure is given in Algorithm 2. We have $\alpha > 1$ and $0 < \beta < 1$. $s_n$ computes a value larger than the current value $s_n^k$ while satisfying the per-user total power constraint as well as the spectral mask constraint. $s_{\beta}$ computes a value smaller than the current value $s_n^k$. In line 4, a per-tone weighted sum of bit rates evaluation for user $n$ is performed to check if the weighted sum of bit rates can be increased by increasing or decreasing the transmit power $s_n^k$. We note that $s_k|s_n^k=s_n^\beta$ equals $s_k$ with $s_n^k$ being replaced by $s_n^\beta$. This is repeated for all tones. This inequality procedure does not violate the real-time property as the constraints of (14) are satisfied after every single update. Also the monotonicity property is not violated as each update results in a non-decreasing feasible objective function value.

**Algorithm 2 Inequality procedure user $n$**

1. for $k \leftarrow$ toneOrder do
2. \hspace{1em} $s_n \leftarrow \min(\alpha s_n^k, P_n^{\text{tot}} - \sum_{q \in K \setminus k} s_q^n, s_n^{\text{mask}})$
3. \hspace{1em} $s_{\beta} \leftarrow \beta s_n^k$
4. \hspace{1em} $s_n^k \leftarrow \arg\max_{s_n^k, s_{\beta}} \sum_{m \in \mathcal{M}} w_m b_{nk}^m(s_k|s_n^k=s_n^\beta)$
5. end for

E. Randomization and Equalization

As mentioned in Section III-C, the IPDB algorithm is tunable in the tone order, the user order, and in the initial choice of $\gamma_k$, where the latter has a one-to-one mapping with the initial transmit powers if the power difference variables $t^n_k$ are fixed. We can use randomized values, i.e., a random tone order, random user order and random initial transmit powers. Randomization in iteration orders and initial conditions has been shown to be effective in several cases in literature [50]. It is shown in Section IV that randomization indeed results in performance gains.

However, randomization also has some side effects which may not always be desirable. For instance, when randomizing the initial transmit powers, the resulting transmit spectra may have a very non-smooth behaviour in the sense that transmit powers differ significantly from one tone to the next. For instance, in Figure 1 the resulting transmit spectra are shown when applying IPDB to a 2-user scenario (corresponding to the blue and the green curve) with randomized initial transmit spectra. The transmit spectra between tones 45 and 115 display significant jumps. The reason is that by starting from very different initial transmit powers on consecutive tones IPDB can converge to very different per-tone solutions in consecutive tones. We want to highlight that the non-smooth behaviour results in a typically better overall performance and prevents convergence to a very poor solution for which a poor choice is systematically made over multiple consecutive tones.

However, such non-smooth solution behaviour is not always desirable in practice and therefore we propose an equalization procedure that can smooth it out. This procedure is given in Algorithm 3 and consists of one simple for loop over the tones. In each loop, it is checked if a spike can be detected between three (almost) consecutive tones $k$, $k + 1$ and $k + 3$. If a down spike is detected (with more than 10 dB difference), this is averaged out. If an up spike is detected (with more than 10 dB difference) this is flattened out. The reason $k + 3$ instead of $k + 2$ is considered for the third tone is that this way one-tone wide spikes as well as two-tone wide spikes can be detected. Although the above procedure is a very low-complexity operation, it has very
good equalization performance with smooth resulting transmit spectra, as demonstrated in Section IV. From an optimization point of view, the equalization procedure can be seen as a procedure that allows to obtain overall solutions where similar per-tone solutions are chosen in neighboring tones.

We want to highlight that this equalization procedure does not have to be called in all iterations but only after every so many outer iterations.

The equalization procedure can slightly violate the real-time property during execution of line 10 and lines 12 to 14, but we want to highlight that these steps are of much smaller granularity than the main computational step of IPDB, i.e. line 7 of Algorithm 1.

Although the monotonicity property may be violated whenever the equalization procedure is called, overall this seems to provide a better performance as demonstrated in Section IV.

Algorithm 3 Equalization procedure user \( n \)

1: \( p \leftarrow \sum_{k \in K} s^n_k \)
2: for \( k \leftarrow 1 \ldots K - 3 \) do
3: \( u_t \leftarrow 10 \log_{10} (s^n_k) - 10 \)
4: \( u_r \leftarrow 10 \log_{10} (s^n_{k+3}) - 10 \)
5: \( s_{db} \leftarrow 10 \log_{10} (s^n_{k+1}) + 10 \)
6: \( d_t \leftarrow 10 \log_{10} (s^n_{k+1}) + 10 \)
7: \( d_r \leftarrow 10 \log_{10} (s^n_{k+3}) + 10 \)
8: if \( s_{db} < u_t \) and \( s_{db} < u_r \) then
9: \( s_m \leftarrow (s^n_k + s^n_{k+1} + s^n_{k+3})/3 \)
10: \( s^n_k \leftarrow s_m, s^n_{k+1} \leftarrow s_m, s^n_{k+3} \leftarrow s_m \)
11: else if \( s_{db} > d_t \) and \( s_{db} > d_r \) then
12: \( s^n_{k+1} \leftarrow \min(s^n_k, s^n_{k+3}) \)
13: \( p_r \leftarrow \sum_{k \in K} s^n_k \)
14: \( s^n_k \leftarrow \frac{p_r}{p_r}, k \in K \)
15: end if
16: end for

IV. SIMULATIONS AND PERFORMANCE ANALYSIS

In this section the performance of the IPDB algorithm is evaluated for different settings and for different performance metrics. The performance will be compared with that of the popular ISB algorithm which is also a coordinate ascent grid-based search algorithm. In contrast to IPDB which operates in the primal domain, ISB is based on a combination of a dual decomposition approach with a discrete per-tone coordinate ascent grid-based search. ISB does not have the real-time property and is thus not an RT-DSM algorithm. Wireline DSL as well as wireless LTE settings will be considered.

For our wireline DSL simulations in Sections IV-A, IV-B, IV-C, we use realistic DSL simulators, which have been validated in practice and are aligned with standards. We consider 24AWG twisted copper pair lines. The maximum transmit power is 20.4 dBm for the ADSL and ADSL2+ scenarios, and 11.5 dBm for the VDSL scenarios. The SNR gap is chosen at 12.9 dB for the DSL scenarios, corresponding to a coding gain of 3 dB, a noise margin of 6 dB, and a target symbol error probability of \( 10^{-7} \). The tone spacing is 4,3125 kHz. The DMT symbol rate is 4 kHz. The weights \( w_n \) are chosen equal for all users \( n = 1 \ldots N \), namely \( w_n = 1/N \), unless specified otherwise. When the equalization procedure is activated in Algorithm 1, it is only performed after each fifth outer iteration.

A concrete wireless LTE heterogeneous network setting will be discussed in Section IV-D.

A. IPDB performance: ADSL Case

The ADSL scenario under consideration is the near-far scenario shown in Figure 2, i.e., a 2-user downstream scenario with one far-user connected to the central office (CO) and a second near-user connected to a remote terminal (RT). This near-far type of scenarios is quite common in practice and has received a lot of attention in DSL literature. The underlying optimization problem is known to display a very nonconvex behaviour for which locally optimal DSM methods can perform poorly [21]. In our simulations, the RT-connected user is given a weight of 0.1 and the CO-connected user a weight of 0.9, to prevent that the latter is being allocated a too small achievable data rate.

The performance of both IPDB and ISB are evaluated for different configurations exploiting the tunability of IPDB. The tone order corresponds to one configuration setting for which we test four choices, TO 1 and TO 2 correspond to tone orders \([1 : K]\) and \([K : -1 : 1]\), respectively. TO 3 selects TO 1 or TO 2 with probability of 50% each, each time line 6 of IPDB is entered. TO 4 corresponds to a tone order which is a fully random permutation of the tone set \([1 : K]\), and which changes each time line 6 of IPDB is entered.

A second configuration setting corresponds to the initial transmit power spectra. For this we consider two choices: (1) EP corresponds to an equal power allocation, i.e., \( \gamma^n_k = 1/K \forall k, n \), (2) RP corresponds to a random power allocation with uniformly distributed probabilities in dB scale while satisfying (6) and \( 0 \leq \gamma^n_k \leq \gamma^n_{k, \text{mask}}/P^{n, \text{tot}} \). The number of inner iterations \( I \) is fixed at 1.

We consider IPDB with and without equalization (Algorithm 3, corresponding to EQ ON and EQ OFF, respectively. Three different DoV transformations are evaluated, which correspond to 10, 15 and 16, respectively, and where \( \pi \) is a vector that corresponds to a random permutation of vector \([1, \ldots, K]\):

1) Two-tone rand DoV transformation:

\[
\gamma^n_k = \gamma^n_{\pi(k)} + P^{n, \text{tot}} \gamma^n_k
\]
2) Three-tone 2 DoV transformation:

\[ s_k^n = \begin{cases} 
2t_k^n - t_{k+1}^n - t_{k+2}^n + P_{n,\text{tot}} \gamma_k^n, & k < N - 2 \\
2t_{N-1}^n - t_N^n - t_1^n + P_{n,\text{tot}} \gamma_{N-1}^n, & k = N - 1 \\
2t_N^n - t_1^n - t_2^n + P_{n,\text{tot}} \gamma_N^n, & k = N 
\end{cases} \]

(16)

We also consider different granularities for the discrete searches, where we consider the standard 0.5 dBm/Hz \[30\]. \[31\] for the transmit powers \( s_k^n \) in ISB, and a coarser granularity of 1 dBm/Hz and 10 dBm/Hz for the power difference variables \( t_k^n \) in IPDB.

Finally, for all configurations, the results are averaged over 15 different runs to obtain an averaged performance.

1) Weighted Sum of Achievable Data Rates Performance: In Table I the weighted sum of achievable data rates performance is compared for IPDB and ISB under the different configurations.

The standard ISB configuration corresponds to the settings TO 1, EQ OFF, EP and 0.5 dBm/Hz granularity. For this standard setting, ISB has a weighted sum of achievable data rates performance of 1.5549 Mbps. This increases spectacularly when applying randomized initial transmit powers, i.e., RP with up to 1.8274 Mbps. In addition, applying the equalization procedure (EQ ON) further improves the performance up to 1.8799 Mbps, when combined with RP. This is quite surprising as the equalization procedure not only provides smoother resulting transmit spectra (as shown in Section IV-A5) but in addition also improves the achievable data rate performance. The explanation is that in the case of randomization most of the tones converge to good per-tone solutions. When combining this with the equalization procedure, this will cause the fewer poor per-tone solutions to be forced to the larger set of good per-tone solutions, resulting in a better overall performance.

The performance of IPDB is good when using the two-tone rand DoV transformation of \[15\]. For a 1 dB granularity, one can see a performance improvement of up to 22% compared to the standard ISB configuration, and up to 1% compared to the best ISB configuration with RP and EQ ON. The IPDB performance is not so good for the two-tone and threetone 2 DoV transformations of \(10\) and \(16\), respectively. However when combining these DoV transformations with the equalization procedure, their performance is improved spectacularly. Reducing the granularity from 1 dBm/Hz steps to 10 dBm/Hz steps decreases the performance, for all cases. It is interesting to notice that the tone order does not play a significant role.

In terms of achievable data rate performance, it can be summarized that the two-tone rand DoV transformation with equalization activated and 1 dBm/Hz granularity offers the best performance, and performs better than all ISB configurations.

2) Convergence Speed: Tables II and III display the convergence speed in terms of the number of outer iterations (loop corresponding to lines 3-14) to converge to 99% and 99.9% weighted achievable sum data rate performance, respectively.

The number of outer iterations varies strongly for different IPDB configurations. A very fast convergence is achieved for the two-tone rand DoV transformation. In particular, we see convergence after less than 10 outer iterations (for 99% performance) and 16 outer iterations (for 99.9% performance), when using the settings EQ OFF and 1 dBm/Hz granularity. For the EQ ON setting, this increases to only 20 outer iterations for both 1 dBm/Hz and 10 dBm/Hz granularity. The other DoV transformations require up to 200 iterations to converge. In general, a larger granularity results in a slower convergence.

ISB only requires 2 to 6 outer iterations to converge. However, we want to highlight that ISB requires a dual optimization step to satisfy the total power constraints within each outer iteration, which is not required for IPDB. The complexity per outer iteration is thus larger for ISB.

For illustration, in Figures 3 and 4 the evolution of the weighted sum of data rates and the per-user total powers is displayed as a function of the outer iterations for a single run. It can be seen that 99% and 99.9% performance are achieved after 8 and 12 outer iterations. The per-user total powers constraints are always satisfied, which demonstrates the real-time property of IPDB.

3) Complexity Comparison IPDB versus ISB: To compare the relative computational complexity of IPDB and ISB, Tables IV and V display the relative number of bit calculations \(1\) for an 99% and 99.9% accuracy, respectively, where the computational complexity of ISB is taken as a reference. Note that for the dual optimization part in ISB, the dual search is optimized with tuned settings. As IPDB is a primal algorithm, it does not have a dual optimization part, at the cost of more outer iterations. However, it can be seen that
### TABLE I

| Method | Two-tone | Two-tone rand | Three-tone 2 | ISB |
|--------|----------|---------------|--------------|-----|
|        | EQ OFF   | EQ ON         | EQ OFF       | EQ ON | EQ OFF | EQ ON       | EQ OFF | EQ ON |
|        | 1 dB     | 10 dB         | 1 dB         | 10 dB | 1 dB   | 10 dB       | 1 dB   | 10 dB |
| TO 1   | EP 1.5080 | 1.4223        | 1.8608       | 1.6542 | 1.8542 | 1.8320      | 1.8978 | 1.8519 |
|        | EP 1.8870 | 1.8247        | 1.8951       | 1.8775 | 1.5143 | 1.3780      | 1.8324 | 1.3875 |
|        | 1.5549   | 1.5549        | 1.5549       | 1.5549 | 1.8274 | 1.8799      |        |       |
| TO 2   | EP 1.5826 | 1.4626        | 1.7914       | 1.6759 | 1.7872 | 1.8358      | 1.8761 | 1.8374 |
|        | EP 1.3724 | 1.3388        | 1.7069       | 1.7689 | 1.7940 | 1.8212      | 1.8806 | 1.8507 |
|        | 1.5066   | 1.3552        | 1.8439       | 1.3298 | 1.4114 | 1.3657      | 1.9785 | 1.7705 |
| TO 3   | EP 1.5933 | 1.4459        | 1.8647       | 1.6432 | 1.8201 | 1.8345      | 1.8731 | 1.8611 |
|        | EP 1.3957 | 1.3834        | 1.7759       | 1.7380 | 1.7975 | 1.8236      | 1.8831 | 1.8503 |
|        | 1.4564   | 1.3624        | 1.7925       | 1.4202 | 1.3887 | 1.4134      | 1.8428 | 1.7149 |
| TO 4   | EP 1.4428 | 1.4422        | 1.8543       | 1.5426 | 1.8240 | 1.8308      | 1.8739 | 1.8394 |
|        | EP 1.3972 | 1.3948        | 1.8045       | 1.7071 | 1.8099 | 1.8147      | 1.8854 | 1.8620 |
|        | 1.4265   | 1.3709        | 1.8375       | 1.7113 |        |            |        |       |

**Convergence speed [Nb of outer iterations]** to converge to 99% performance for near-far CO-RT scenario of Fig. 2. Method Two-tone rand uses permutation tones on the transformation. Equalization (TO) 1 = [1:K], TO 2 = [K-1:1], TO 3 = random permutation, EP = constant equal init power, RP = random init power satisfying total power constraint. Two right most columns correspond to ISB performance.
IPDB has a much lower overall computational complexity for specific configuration settings. In particular the two-tone rand DoV transformation with 10 dBm/Hz granularity reduces complexity by a factor 20 (for 99% performance) and factor 12 (for 99.9% performance). For IPDB with the two-tone rand DoV transformation, EQ ON and 1 dBm/Hz granularity, there is a 5% to 50% complexity reduction compared to ISB.

4) Real-time Property: A main strength of IPDB is its real-time property. To compare with ISB, the number of power updates is determined that ISB maximally requires to satisfy the per-user total power constraints taking the inner dual optimization step into account. Compared to one single power difference variable (and thus power) update for IPDB, ISB requires \( \frac{1}{5} \times 10^6 \) power updates to satisfy the per-user total power constraints (in worst case). This corresponds to the worst case number of power updates (over all outer iterations) that a user needs to converge to transmit powers so as to satisfy its total power constraint. ISB thus does not qualify as a RT-DSM algorithm, in contrast to IPDB.

5) Equalization Performance: To demonstrate the equalization impact, IPDB is simulated with RP, TO 4, 1 dBm/Hz granularity, two-tone rand DoV transformation and equalization procedure Algorithm 3 activated (EQ ON) for every outer iteration which is an integer multiple of 5.

The resulting transmit spectra before and after equalization are shown for iteration 5 and 10 in Figure 5. It can be seen that after the equalization step the transmit spectra display fewer spikes. In this case, only after two equalization steps, all spikes are removed, demonstrating the effectiveness of the equalization procedure.

B. Impact of Discretization Granularity: ADSL2+ case

As mentioned in Section III-C, the discrete grid-based search granularity for the power difference variables for IPDB can be chosen coarser than for the transmit powers for ISB, i.e., 1 dBm/Hz instead of 0.5 dBm/Hz. In this section we assess the concrete impact of different granularities for an ADSL2+ scenario as given in Figure 6 in terms final transmit spectra. The downstream ADSL2+ scenario consists of 12 users with line lengths 5000m, 4000m, 3000m, 2000m, 2000m, 1000m, 4800m, 3800m, 2800m, 2300m, 1500m, and 1300m. The distances (between CO and RTs) are 0m, 0m, 1000m, 1000m, 2000m, 2000m, 0m, 0m, 1200m, 1200m, 2400m, and 2400m.

We compare three configurations with each other: a) IPDB with 1 dBm/Hz granularity, b) IPDB with 10 dBm/Hz granularity, and c) ISB with 0.5 dBm/Hz granularity. For the three configurations, we start from the same initial transmit powers (EP setting) and all three converge to similar solutions. The weighted sum achievable data rate performance corresponds to: a) 16.0072 Mbps, b) 15.9960 Mbps, c) 16.0055 Mbps.

In Figure 7 the resulting transmit spectra for user 7 are zoomed out for tones 230 to 420 for the three above configurations. It can be seen that ISB makes steps of 0.5 dBm/Hz. In contrast, the IPDB methods (for both granularities) display a smaller step variation in magnitude, which demonstrates that a much coarser granularity for IPDB does not impact the shape of the resulting transmit spectra too much. There is some level of non-smooth behaviour for IPDB though. The equalization procedure is not able to remove this, because the equalization threshold is set at 10 dBm/Hz (as shown in lines 3,4,6,7 in Algorithm 3).

C. Inequality Constraints: VDSL case

For typical DSL scenarios, all users are allocated their full available transmit power satisfying the total power constraints with equality, i.e., \( P_n = P_n^{\text{tot}} \). However, for multi-user large crosstalk settings and under specific values for the weights \( w_n \), it is possible that some users better not be allocated all available per-user total power. Here we consider a 6-user VDSL upstream scenario, with 6 CO-connected lines with line lengths 1200m, 1000m, 800m, 600m, 450m, and
300m, corresponding to users 1 to 6, respectively. For this scenario, users 4 and 5 are not allocated all available per-user total power, i.e., $P_{n}^{\text{tot}} < P_{n}$. When running the IPDB algorithm (with RP, EQ ON, 1 dBm/Hz granularity, TO 4, DoV transformation [15]), with the inequality procedure of Algorithm 2 (with $\alpha = 1.1$ and $\beta = 0.8$), the evolution of the allocated per-user total powers is shown in Figure 8. It can be seen that the per-user total power constraints are always satisfied. After 30 iterations, both users achieve their final per-user total power allocation corresponding to 23% and 1.7% of full power allocation $P_{n}^{\text{tot}}$. This demonstrates the real-time property of IPDB while considering inequality constraints.

### D. Downlink Power Control in Heterogeneous Wireless Networks

As highlighted in Section I, the proposed RT-DSM theory and IPDB algorithm can also be applied to wireless communication settings. One highly relevant problem is downlink power control in heterogeneous cellular networks where OFDMA is used within each cell and inter-cell interference is observed between different (macro, pico, femto) cells. In [57] it is explained that this consists of two subproblems, namely a user scheduling part and a power spectrum control part. For the power spectrum control part, it is shown how DSM algorithms can lead to spectacular performance gains. The IPDB algorithm can similarly be applied to this setting so as to obtain real-time inter-cell interference coordination (ICIC) for such heterogeneous networks.

The IPDB algorithm is applied here to a system with two interfering cells, one macrocell and one femtocell. Each cell has one user. The user in the macrocell is located at the cell edge at a distance of 500m from the macrocell base station and

**TABLE IV**

| Method | Two-tone | Two-tone rand | Three-tone 2 | ISB |
|--------|----------|---------------|--------------|-----|
|        | EQ OFF   | EQ ON         | EQ OFF       | EQ ON |
|        | 1 dB     | 10 dB         | 1 dB         | 10 dB |
| TO 1   | EP       | 2.5164        | 0.2739       | 0.3604 |
|        | RP       | 1.7928        | 0.3594       | 0.3991 |
| TO 2   | EP       | 1.5272        | 0.2723       | 0.3580 |
|        | RP       | 2.0256        | 0.3604       | 0.4959 |
| TO 3   | EP       | 1.6278        | 0.2731       | 0.3991 |
|        | RP       | 1.2612        | 0.3991       | 0.5668 |
| TO 4   | EP       | 0.8795        | 0.2729       | 0.3631 |
|        | RP       | 1.9321        | 0.3631       | 0.6202 |

**TABLE V**

| Method | Two-tone | Two-tone rand | Three-tone 2 | ISB |
|--------|----------|---------------|--------------|-----|
|        | EQ OFF   | EQ ON         | EQ OFF       | EQ ON |
|        | 1 dB     | 10 dB         | 1 dB         | 10 dB |
| TO 1   | EP       | 4.2908        | 0.5398       | 3.9013 |
|        | RP       | 4.5991        | 0.6071       | 3.7376 |
| TO 2   | EP       | 1.6558        | 0.4924       | 2.1677 |
|        | RP       | 4.7169        | 0.5668       | 3.2402 |
| TO 3   | EP       | 2.3030        | 0.4959       | 1.9487 |
|        | RP       | 3.9985        | 0.6448       | 3.7474 |
| TO 4   | EP       | 2.8825        | 0.5368       | 2.0686 |
|        | RP       | 4.9687        | 0.5702       | 3.6000 |

![Fig. 8. Evolution of user powers for IPDB with inequality procedure of Algorithm 2 for 6-user upstream VDSL scenario.](image-url)
20m from the femtocell base station. The user in the femtocell is located at the same location but is served by the femtocell base station. It is known that this constitutes a challenging interference limited setting. We consider a system bandwidth of 5 MHz, a subcarrier spacing of 15 kHz, a symbol rate of 14 OFDM symbols in 1ms, 300 subcarriers, a macrocell base station transmit power of 43 dBm and a femtocell base station transmit power of 15 dBm. The ITU-PED B channel model is used with a pathloss of $31.5 + 35 \log_{10}(\text{distance})$.

The resulting bit loadings are displayed in Figure 9. As the scenario deals with two edge users, the resulting transmit power allocations are OFDMA like. There is a small overlap though in tones 20-22. The goal is however not to analyze the resulting transmit spectra and bit loadings, but is to demonstrate the wide applicability of the proposed RT-DSM theory and algorithm beyond the wireline DSL setting.

V. Conclusion

We have proposed a new paradigm, theory and algorithm for RT-DSM in multi-user multi-carrier communication systems. The RT-DSM algorithm is referred to as IPDB. IPDB is suitable for operation under tight computation time and compute power constraints, i.e., when a very fast responsiveness is required. IPDB can be stopped at any moment in time during execution while guaranteeing feasibility and improved performance. The IPDB algorithm design is enabled by a novel transformation, referred to as the DoV transformation, which transforms the standard DSM problem into an alternative optimization problem with primal power difference variables. A coordinate ascent approach is proposed to tackle the reformulated primal problem with an iterative 1D exact line search via a logarithmically scaled grid search. In contrast to existing DSM algorithms that follow a dual decomposition approach, IPDB solves the DSM problem in the primal domain, avoiding any potential issues with a non-zero duality gap, which can be seen as an important benefit. IPDB is furthermore characterized by a high tunability with additional procedures for dealing with inequalities and equalization that result in improved performance and smooth transmit power spectra. In particular, the configuration with the 'two-tone rand' DoV transformation and equalization results in fast convergence, good network wide achievable data rate performance, low computational cost, and real-time execution, outperforming the existing near-optimal ISB algorithm. This has been validated with simulations under different configuration settings for different practical wireline xDSL scenarios and for a wireless

Fig. 5. Impact when equalization procedure Algorithm is activated for IPDB at outer iteration 5 and 10, before and after the equalizations. Transmit spectra of CO-user and RT-user in Figure are displayed in blue and green, respectively.
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