Acoustics flow analysis in circular duct using sound intensity and dynamic mode decomposition

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Abstract. Sound intensity generation in hard-walled duct with acoustic flow (no mean-flow) is treated experimentally and shown graphically. In paper, numerous methods of visualization illustrating the vortex flow (2D, 3D) can graphically explain diffraction and scattering phenomena occurring inside the duct and around open end area. Sound intensity investigation in annular duct gives a physical picture of sound waves in any duct mode. In the paper, modal energy analysis are discussed with particular reference to acoustics acoustic orthogonal decomposition (AOD). The image of sound intensity fields before and above “cut-off” frequency region are found to compare acoustic modes which might resonate in duct. The experimental results show also the effects of axial and swirling flow. However acoustic field is extremely complicated, because pressures in non-propagating (cut-off) modes cooperate with the particle velocities in propagating modes, and vice versa. Measurement in cylindrical duct demonstrates also the cut-off phenomenon and the effect of reflection from open end. The aim of experimental study was to obtain information on low Mach number flows inducts in order to improve physical understanding and validate theoretical CFD and CAA models that still may be improved.

1. Introduction
The aim of our experimental research is directed to enrich theoretical knowledge about the birth of sound in turbulent flow conditions of the acoustic wave, a better understanding of the mechanisms of noise generation and to identify more effective ways of its reduction.

For low Mach-number isothermal flow we will see that aeroacoustic sound production is entirely due to mean flow velocity fluctuations, which may be described in terms of the underlying vortex dynamics. This leads to the idea of using so called vortex sound theory [1, 2]. Vortex sound theory is not only numerically efficient but also allows us to translate the very efficient vortex-dynamical description of elementary flows directly into sound production properties of these flows in real-live conditions. Our research will be conducted on the cross flow inside cylinders with a free-end. The circular cylinder has proved to be a fruitful area of fluid dynamics research, due to its combination of a simple geometry and complex, unsteady flow features. Sound intensity flow without mean flow generation in hard-walled circular duct with acoustic excitation is treated experimentally. The noise produced by laminar and turbulent flows in annular ducts is studied using experimental sound intensity (SI) which is a average value of pressure times particle velocity - a vector quantity. Along the duct sound energy is carried in various modes which will be examined and graphically illustrated.
Acoustic intensity is a very useful energetic quantity since it gives information about the propagation paths and the amount of energy radiated. From an experimental point of view time-averaged acoustic intensity ($I_{a}$, also called active intensity and written here as $I_{a}$), is often more interesting than instantaneous value $I_{a}(t)$. Sound intensity as a vector variable in separably couples the acoustic particle velocity and acoustic pressure ($I_{a} = p \cdot v$) and represents a stream of acoustic energy flowing in the field. This vector parameter of acoustic wave can be measured (rms) with special sound intensity probe and can be easily shown in a graphical form.

The acoustic particle velocity $v$ and mean pressure $p$ satisfy the time-averaged equations of continuity and momentum. For linear acoustics, in the absence of an external flow $\langle p v \rangle = \langle p' \cdot v \rangle / c_{0}^2 = I_{a} / c_{0}^2$, where $p'$ is the acoustic pressure perturbation and $I_{a}$ the acoustic intensity. Sound intensity can be directly measured and recorded as an acoustical flow field divided on normalized octave band frequencies. (normalized acoustic filters 1/1, 1/3, 1/12, 1/24 octave band). In traditional acoustic metrology, the analysis of acoustic fields mainly focuses on the distribution of pressure levels (scalar variable), however in a real acoustic field both scalar (acoustic pressure) and vector (the acoustic particle velocity) effects are closely with phase and amplitude related. The acoustic field may be described as a spatial distribution of pressure and particle velocity, their amplitude $p$ and $v$ being proportional for plane traveling waves ($p = \rho \cdot c_{0} \cdot v$), where $\rho$ is the density and $c_{0}$ is the sound speed.

The application of the sound intensity technique together with numerical methods has today improved the quality of acoustic diagnostics and made it possible to visualize energy wave phenomena (vector distribution) in a vibrating structure, or in an acoustic field around the structure. The visualization of acoustic energy flow in real-life acoustic 3D space fields can explain many peculiar energetic effects (scattering, vortex flow, shielding area, etc.), concerning the areas in which it is difficult to make numerical modeling and analysis with the CFD-FSI-CAA methods.

2. Experimental setup

Sound intensity generation in hard-walled duct with acoustic flow (no mean-flow) is treated experimentally and showed graphically. In paper, numerous methods of visualization illustrate the vortex flow (2D, 3D) as a sound intensity stream, can graphically explain diffraction and scattering phenomena occurring inside a long circular duct and with open end. The intensity distribution inside the duct is produced by the action of the sum of modal particle velocities and pressures. Along the duct sound energy is carried in various modes which will be examined and graphically illustrated.

In figure 1 we show a model of circular acoustic waveguide where investigations with sound intensity measurement were made. The 6 m long open-end duct with internal radius 0.474 m was used as a model for an acoustic waveguide. At one end it was connected to a loudspeaker, a source of broadband acoustic signals. The method employed in this paper is based on SI technique and experimental set-ups where monopole sources approximated with loudspeakers. The duct is excited with acoustic pink noise, so, the sound power along the duct is sent without mean flow. Measurements were made in frequency band 50-6800 Hz and analyzed in 1/3 and 1/12 octave band frequency. SI measurement were made on a duct without any obstacles present inside. Measurement region was placed at a distance about of 2.2 m from the end of duct.

![Sound incident of circular cross section acoustic waveguide](image-url)
The space inside the duct was scanned with sound intensity measuring the $x$, $y$ and $z$ components of sound intensity vectors. The image of the dipolar and quadrupolar sound generated by a flow inside a duct was obtained using a SI three-dimensional USP Microflown probe and our graphical SIWin post-processing software [3].

3. Description of acoustics flow inside empty duct

The most common situation to be found in the literature is for a plane travelling acoustic wave in ducts which reflects from the end to form a standing wave. In cylindrical ducts, plane waves, only, can propagate below a characteristic frequency which is a function of the duct diameter.

In our investigation several types of axisymmetric and spiral type of vortex breakdowns have been observed experimentally [4]. Vortex breakdown phenomenon occurs when the ratio of the azimuthal to axial momentum exceeds a certain threshold, while both quantities have to be of the same order of magnitude [5]. It can play a crucial role in a variety of technical applications. Understanding the cause of the vortex breakdown is therefore of great importance in order to develop appropriate control strategies.

In our experiment sound intensity field in a cylindrical duct was excited by a wide-band sound signal propagated from source approximated with a loudspeaker. In figure 2 we show some results of investigations for 1/12 octave band frequencies where the sectional streamlines show the topological flow multi-cell structure for high-order modes. With graphical form we can see the evolution process of flows in the cross-section plane. In this paper, higher order acoustic modes which are excited above cut-off frequency are considered. These modes with frequency above 817 Hz have a much more complicated pressure pattern compared to the plane wave mode below. When more than one mode has “cut-on”, these modes are superimposed upon the lower frequency wave mode and can co-exist with each other [6].

![Acoustics flow structures - axial mode](image)

**Figure 2.** Cross-sectional sound intensity streamlines fields below (up to 817 Hz) and above cut-off frequencies (from 917 Hz to 2175 Hz).

The frequency of 817 Hz is critical for the test waveguide that caused vortex breakdown phenomenon in the acoustic flow. Vortex breakdown phenomenon occurs when the ratio of the azimuthal to axial momentum exceeds a certain threshold, while both quantities have to be of the same order of magnitude. This frequency limiting case of vortex breakdown phenomenon for researched duct is well illustrated in figure 2. The vortex breakdown can play a crucial role in a variety of technical applications. Understanding the cause of the vortex breakdown is therefore of great importance in order to develop appropriate control strategies.

Sound intensity measurement and graphically imaging flow in a cylindrical duct can also well demonstrate the cut-off phenomenon and the effect of acoustic wave reflection from open-end. In open-ended ducts, waves reflected from the open end play an important part in sound transmission. The reflection properties of the opening are specified in terms of the impedance presented to each duct.
mode [7], and the sound intensity along the duct is related to the mode impedance and the forward wave amplitude. In figure 3 the sound field in the cross section at 15 cm from the end of the waveguide shows a tumble motion as the back-scattering wave reactions.

Also the termination of the duct assumes an important role on the shape of the field at the end of the waveguide (see also figure 7), and consequently the level of the noise at the outlet of the waveguide [8, 9].

![Figure 3](image1.png)

Figure 3. The sound field in the cross section at 15 cm from the end of the waveguide - tumble motion as the effect of back-scattering reactions is shown.

4. Conical baffle inside a duct

Good understanding of the evolution process of acoustic wave motion in pipes and ducts is critical to development of engineering designs with the most attractive operating to achieve an optimize low-noise design of duct systems and low noise emission characteristic. The detailed flow acoustic evaluation process of the in-duct flow structure has recently attracted attention of investigators.

Flow tones are generated in a wide variety of configurations. In essence, they arise due to favorable coupling between an inherent instability of a separated shear layer and a resonant acoustic mode of the flow system [10]. This process is one of the promising ways to obtain high circulation and rotation shape of waves.

Vertical flows in general may be characterized into two categories: the swirl motion rotating about duct axis and the tumble motion rotating about the diametric axis (figure 4C). Swirling rings isolate the velocity fields in the vicinity of the vortices and assess their contribution to the overall dynamics at the flow.

![Figure 4](image2.png)

Figure 4. Sound intensity streamlines around conical baffle at the below (A and B) and above cut-off frequency (C).

Direct measurement of the acoustic power flow around outlet of an obstacle as a conical baffle with a 127 mm hole can explain a diffraction and scattering phenomena occurring in this region. The space behind the conical baffle was studied in consideration of deformation of the acoustic field due to the presence of obstacles. Experimental results will present the distributions of sound intensity streamlines traveling along the duct. Two equivalent types of modes are observed: one type is a stationary sound
intensity pattern which fluctuates in time (figure 4A and B – frequency below cut-off); the other type is a steady sound intensity pattern which rotates (figure 4C – frequency above cut-off). The second of these appears more appropriate to describe sound generated by rotating sources - higher-order spinning modes (1631 Hz at the figure 4).

The complicated secondary flow patterns reveal strong non-linearity of flows. Changes of rotation effect. Change the flows inside duct from stable state to unstable state or from unstable to stable state. A well-defined swirl and/or tumble flow structure [4] is more stable than other large scale in-duct flows, therefore, it may break up later in the cycle, giving higher turbulence during acoustic flow along ducts.

On the figure 5 we show examples of this investigations for some 1/12 octave band frequencies where the sectional streamlines and the velocity vectors show the topological secondary flow multi-cell structure for high-order modes. The view planes for detections of swirl motions inside a duct are well defined in figure 5.

![Figure 5. Secondary flow patterns on multi-cell structure for high-order mode.](image)

**2. Multi-modal acoustics flow decomposition**

The theory of sound transmission and generation in hard walled ducts has been extended to include axial and swirling wave flow. This type of sound wave called spinning modes [7] is particularly evident in the range above cut-off frequency. These phenomena occur in the whole length of duct.

Wave system in the same mode travels in both directions along a duct. Above the cut-off frequency mode, the sound power transmitted along the duct is simply the difference between the forward- and backward-transmitted sound power associated with separate wave systems. Because of combined effects of system rotation (Coriolis force) and curvature (centrifugal force) on the acoustic flowing inside of duct, flows may temporarily oscillate in some cases [5]. When fluid flows in circular duct, it causes complex secondary flow structure. The well-known secondary flows[6] of the above-mentioned flows have pairs of clockwise and counter-clockwise-rotating vortices in a plane perpendicular to the axis of the duct (see figures 2 and 3). In the case of circular duct, flows may have even more complicated secondary flow (multi-cell) structure. The intensity distribution inside duct produced by the action of the axial and radial modes is extremely complicated because this propagating modes influence on each other.
In this paper, we also investigate how SI technique may be combined with POD and DMD techniques as a tool for searching the energy dominant modes in the acoustics flow field in the interior of ducts and pipes. We attempt to show that the tested coherent structures by SI give the same opportunities to analyze the distribution of energy in the sound field that we can get, for instance, from the PIV/POD methods. From the analysis of modes amplitude, it can be concluded which one represents the highest energy level.

Experimental analysis of the distribution of the volume of sound intensity field may be equivalent to statistical POD or MDM methods. Description of mode propagation as a sound intensity stream flow in hard walled cylindrical ducts, may be called acoustic orthogonal decomposition (AOD) and give best results when all acoustic modes are excited. For ducts with rigid boundaries, the fluid-resonant category may contribute significantly to unwanted noise. Height level of acoustic energy propagated along the duct occurs when the mode is excited above a cut-off frequency which depends on the mode eigenvalue and the duct radius. Modes excited below their cut-off frequency are evanescent and decay exponentially with distance along the duct. The dimensionless number which expresses the cut-off frequency independently of the radius is the term \( kr \), or as a Helmholtz number \( He \) \cite{11, 12}. This minimum wave number, called the cut-off value, below which the simple wave breaks down is investigated in duct and marks the boundary between high-frequency propagation and low-frequency decay of duct modes. In our duct acoustics flow investigation the cut-off value is about four.

**Figure 6.** Comparison results estimated with POD from literature (a) and sound intensity field measured inside investigated circular duct (b).

**Figure 7.** Three-dimensional distribution of equal sound energy acoustic structure (sound intensity isosurface with equal dB levels) inside duct in the 2.20 m from the open end of duct.
In figure 6 comparison literature results estimated with POD and with sound intensity field measured inside circular duct are shown. Comparison of some POD analysis results taken from the literature [13] confirms the usefulness of SI techniques to the decomposition of any modal distributions. Such SI studies can be carried out both in the flat and three-dimensional acoustic flow fields (figure 7).

According to the analysis above, it can be seen that the experimental decomposition of acoustic flow field using sound intensity (SI) and PIV/POD measurement techniques can be successfully applied in the acoustic waveguide duct.

8. Conclusions

Our experiment on acoustic waveguide model confirms that flow acoustic imaginations in real-life conditions are very complex, even for extremely simple modelling facility and for the sound field in a circular duct excited by loudspeaker used in the study (without flow field). These investigations provide a physical understanding of acoustic wave flow phenomena in real cylindrical duct where the measurements show both qualitative and quantitative flow diagnostics. The presentation of the vector distributions of real-life acoustic fields inside the duct areas, for which it is difficult to make a theoretical analysis of sound flow above the “cut-off frequencies”, shows that it is transmitted along the annular duct by higher-order “spinning modes”, not by plane waves. Properly modelled and analyzed flow-induced sound led directly into the modern aeroacoustic approach, in which theory and experiment are inseparable.

Along a duct, sound energy carries the various modes. The main objective of the study is to give an understanding of sound-energy transmission by higher-order duct modes. Sound propagation in circular ducts is deformed mainly by waves spiralling around the duct. These modes may be excited even in an axisymmetric duct by a disturbance which rotates about the duct axis due to local changes in impedance or obstacles on the way of travelling wave. The experimental results confirm the occurrence of axial and swirling flow inside investigated duct. Further research on the importance of energy interaction phenomena between axial and radial modes will focus for more realistic engineering applications.

The swirling modes may be excited in an axisymmetric duct by a disturbance which rotates about the duct axis. Rotations and swirling waves may occur between the mode amplitude and impedance, which applies to both above and below the cut-off frequencies. Therefore, the theory of sound transmission and generation in hard walled ducts has been extended to include axial and swirling mean flow. The presentation of the vector distributions of real-life acoustic fields can explain many particulars concerning the acoustic power flow, and flow dynamics inside the ducts.

The analysis of acoustic field with floating wave inside a duct show that the sound intensity technique is very useful tool to the visualization of vector acoustic phenomena.

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