A particlization procedure for the NχFD model and the evolution of the net-proton kurtosis

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Abstract. We present an analysis of the net-proton kurtosis on the crossover side of the critical point within the model of nonequilibrium chiral fluid dynamics (NχFD). The chiral order parameter is propagated explicitly and coupled to an expanding fluid of quarks and gluons to describe the dynamical situation in a heavy-ion collision. After implementing a particlization routine into this model, we study the behavior of the net-proton kurtosis as driven by the characteristic structure of the net-baryon number susceptibility in the critical region.

1. Introduction

Extreme temperatures and densities may create an extreme state of matter, the so-called quark-gluon plasma (QGP), where quarks and gluons are deconfined and chiral symmetry is restored. The transition from hadron gas to QGP is of crossover type for zero baryochemical potential, but expected to turn into a critical point and first-order phase transition for larger net-baryon densities. Main support for this assumption is obtained from effective model studies [1] or Dyson-Schwinger equations [2]. In the vicinity of a conjectured critical point, fluctuations of the order parameter or the net-quark number as calculated from thermodynamic susceptibilities exhibit characteristic peak structures [3, 4]. Experiments such as NA49 at CERN or STAR at RHIC have been trying to measure such fluctuations on an event-by-event basis. Although recent STAR measurements have reported a nonmonotonic behavior of the net-proton kurtosis as function of the beam energy [5], a thorough understanding of the dynamical processes leading to the obtained signal is still due. We address this issue using the NχFD model [6, 7, 8, 9, 10] by implementing a particlization procedure. This means that by using the Cooper-Frye formula, we find particle distributions on hypersurfaces of constant energy-density from the fluid dynamical model as described in [11]. We are then in a position to study the behavior of the experimentally accessible net-proton kurtosis during an evolution in the crossover region near a critical point [12].
2. Particlization in the $N_{\chi}FD$ model

Starting point for our study is a quark-meson model with chiral condensate $\sigma$ and dilaton field $\chi$:

$$\mathcal{L} = \frac{1}{2}(i\gamma^\mu \partial_\mu - g_0 \sigma) q + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \chi)^2 + \mathcal{L}_A - U_\sigma - U_\chi ,$$

with the constituent gluon Lagrangian $\mathcal{L}_A$, the potentials $U_\sigma$ and $U_\chi$ for the respective fields and the quark-meson coupling $g = 3.3$, determined from the constituent quark mass in vacuum. The nonequilibrium evolution during expansion and cooling after a heavy-ion collision is driven by the Langevin dynamics of the sigma field, derived from the two-particle irreducible effective action,

$$\partial_\mu \partial^\mu \sigma + \eta_\sigma \partial_t \sigma + \frac{\delta V_{\text{eff}}}{\delta \sigma} = \xi ,$$

with the mean-field potential $V_{\text{eff}}$ obtained from integrating out the quark degrees of freedom. The damping coefficient $\eta$ depends on $T$ and $\mu$ as a result of the $\sigma \leftrightarrow q\bar{q}$ pair production process. It is related to the stochastic noise field $\xi$ via

$$\langle \xi(t, \vec{x})\xi(t', \vec{x}') \rangle_\xi = \delta(\vec{x} - \vec{x}')\delta(t - t')m_\sigma \eta_\sigma \coth \left(\frac{m_\sigma}{2T}\right) .$$

To make qualitative comparisons to experimental observables, we implemented a Cooper-Frye freeze-out [13, 14], producing all non-strange particles from the UrQMD model [15, 16] along hypersurfaces of constant energy density. An example of such a hypersurface is shown in Fig. 1 for $e = 2e_0$ and the case of a crossover transition on which this study focuses on. We clearly see a smooth structure as opposed to inhomogeneities that may arise in the presence of spinodal decomposition at a first-order phase transition. Particles are produced until the total integrated energy of quark-gluon fluid and fields $\sigma, \chi$ is obtained:

$$e = e_{\text{fluid}} + \frac{1}{2} \left( \frac{\partial \sigma}{\partial t} \right)^2 + \frac{1}{2} (\nabla \sigma)^2 + U_\sigma + \frac{1}{2} \left( \frac{\partial \chi}{\partial t} \right)^2 + \frac{1}{2} (\nabla \chi)^2 + U_\chi .$$

We furthermore ensure that net-momentum, net-charge and net-baryon number are exactly conserved in each event.

3. Net-proton kurtosis and net-baryon susceptibility

We focus on a crossover evolution near the critical point of the model given by eq. (1), corresponding to a trajectory shown in Fig. 2. Here the values of $T$ and $\mu$ are calculated...
Figure 2. Event-averaged trajectory near the critical point. Although the region of enhanced susceptibility is narrow, the evolution remains there for an extended amount of time.

as volume-averages in a central region of the fireball. Initial conditions are obtained from the UrQMD transport model. Fig. 2 also shows the net-baryon susceptibility ratio $c_4/c_2$ which is related to the net-baryon kurtosis. In Fig. 3 we show this quantity along the trajectory, calculated for the corresponding values of $T$, $\mu$ and give a comparison to the net-proton kurtosis, determined from the particlization at certain points of constant energy density along the evolution. We see that the location of the minima in both curves are in clear correspondence. The two smaller maxima in $c_4/c_2$ are not visible in the kurtosis, though. This might on the one hand be due to a lack in resolution, on the other hand we may expect that in an inhomogeneous medium with varying values of $T$ and $\mu$ for each hypersurface, the dominant contribution is usually provided by regions with the minimum peak in the susceptibilities so that effects of the small peaks, which are visible in the equilibrium calculation, become washed out in a dynamic inhomogeneous setup.

4. Summary and Outlook
In the work presented here, we have extended the $N_{\chi}$FD model with a particlization procedure, an important step towards making quantitative predictions for future experiments and understanding previous results from the beam-energy scan program. With this we have come to understand the relation between susceptibilities and dynamically generated fluctuations in particle numbers as they are measured to determine the location of a possible QCD critical point. We saw that the dominant minimum in the kurtosis-related $c_4/c_2$ is reflected in a minimum around the same energy density in the net-proton kurtosis obtained from our nonequilibrium model.

In the future, we are going to include effects of sigma field fluctuations which we expect to couple to particle production, in particular (anti-)protons and pions. This could be included into the model via an interaction term $g\bar{p}\sigma p$ [17, 18, 19]. We are furthermore going to consider a subsequent hadronic afterburner.

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Figure 3. Net-proton kurtosis as function of freeze-out energy for a nonequilibrium evolution compared with the generalized susceptibilities $c_4/c_2$. Figure from [12].

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