Quark transverse momentum in hard scattering processes

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The role of transverse momentum of quarks in semi-inclusive leptoproduction will be discussed. It involves generalized distribution and fragmentation functions which depend on both the longitudinal lightcone momentum fraction of the quarks and on the transverse momentum. Constraints on these functions and relations between them arise as a consequence of the QCD equation of motion for the quark fields, Lorentz invariance and of C, P, and T invariance of the strong interactions. Experimentally one has access to the functions for instance in measurements of azimuthal asymmetries in hard processes.

Quark distribution functions and quark fragmentation functions appear in the field-theoretical description of hard processes as the parts that connect the quark and gluon lines to hadrons in the initial or final state. These parts, containing the soft physics can be considered as (connected) matrix elements of nonlocal operators built from quark and gluon fields \([1]\). The matrix elements appearing in the leading order calculation for 1-particle inclusive leptoproduction are

\[
\Phi_{ij}(p, P, S) = \frac{1}{(2\pi)^4} \int d^4 \xi \ e^{ip\cdot\xi} \langle P, S|\psi_j(0)\psi_i(\xi)|P, S\rangle,
\]  

(1)

for the distribution part of quarks with momentum \(p\) in the target hadron with momentum \(P\) and spin \(S\) and

\[
\Delta_{ij}(k, P_h, S_h) = \sum_X \frac{1}{(2\pi)^4} \int d^4 \xi \ e^{ik\cdot\xi} \langle 0|\psi_i(\xi)|P_h, X\rangle\langle P_h, X|\psi_j(0)|0\rangle,
\]

(2)

describing the decay of a quark with momentum \(k\) into a final state containing one specific hadron \(h\) with momentum \(P_h\).

In inclusive lepton-hadron scattering one finds in leading order of an expansion in \(1/Q\), where \(-q^2 = Q^2\) is the spacelike momentum transfer squared, that the relevant matrix element at high energy is \(\Phi(x) = \int dp^- d^2p_T \Phi\), in which case the nonlocality becomes a lightlike separation in \(\xi^-\). The lightlike +--direction is determined by the (up to mass effects) lightlike vector \(P\).

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The important extension when transverse momenta play a role, e.g. in processes in which there are at least two hadrons in addition to the virtual photon (or W/Z) such as 1-particle inclusive lepton-hadron scattering (ℓH → ℓhX) or Drell-Yan (AB → μ⁺μ⁻X), is the presence of transverse separations in the nonlocal matrix elements. Using constraints coming from hermiticity, parity and time reversal invariance the most general parametrization of the Dirac projections

\[ \Phi^{[\Gamma]}(x, p_T) = \int dp^- \frac{d^2 \xi}{(2\pi)^3} e^{ik\cdot\xi} \langle P, S | \psi(0) \Gamma \psi(\xi) | P, S \rangle \bigg|_{\xi^+ = 0}, \tag{3} \]

depending on \( x = p^+/P^+ \) and \( p_T \), yields in leading order for a polarized spin 1/2 hadron [2]

\[ \Phi^{[\gamma^+]} = f_1(x, p_T), \tag{4} \]

\[ \Phi^{[\gamma^+\gamma]} = \lambda g_{1L}(x, p_T) + g_{1T}(x, p_T) \frac{(p_T \cdot S_T)}{M}, \tag{5} \]

\[ \Phi^{[\sigma^+\gamma]} = S_T^i h_1(x, p_T) + \frac{\lambda p_i^T}{M} \frac{1}{h_{1L}^T(x, p_T)} - \frac{1}{M^2} \frac{p_T^i p_T^j + 2 p_T^i p_T^j S_T}{S_T} h_{1T}^j(x, p_T), \tag{6} \]

where \( \lambda \) and \( S_T^a \) are the (lightcone) helicity and the transverse spin. The \( p_T \)-integrated results, relevant in inclusive scattering, are the lightcone momentum distributions for unpolarized quarks, \( \Phi^{[\gamma^+]} = f_1(x) \), the quark helicity distribution \( \Phi^{[\gamma^+\gamma]} = \lambda g_1(x) \), and the quark transverse spin distribution \( \Phi^{[\sigma^+\gamma]} = S_T^a h_1(x) \).

In measurements of azimuthal asymmetries one becomes sensitive to \( p_T \)-weighted functions \( \Phi^{[\gamma^+\gamma]}(x) = (1/2) \int dp^- d^2 p_T p_T^2 Tr(\Phi^\Gamma) \), e.g.

\[ \frac{1}{M} \Phi^{[\gamma^+\gamma]}(x) = \frac{1}{2} \int d^2 p_T \frac{p_T^2}{2M^2} g_{1T}(x, p_T) \equiv S_T^a g_T(x), \tag{7} \]

first introduced (albeit with another name) in ref. [3]. These functions can be related to \( p_T \)-averaged higher twist correlation functions such as

\[ \Phi^{[\gamma^+\gamma]}(x) = \frac{M}{P^+} S_T^a g_T(x). \tag{8} \]

One has the relation \( g_2(x) = (g_T - g_1)(x) = d g_T^{(1)}(x)/dx \). The factor \( M/P^+ \) required by Lorentz invariance, leads to a factor \( M/Q \) in the cross sections.

In order to obtain the cross section for a 1-particle inclusive process the distribution part must be combined with a fragmentation part. At leading order for the case of summing over polarizations of the final state hadron one encounters e.g.

\[ \Delta^{[\gamma^-]}(z) = \frac{1}{4z} \int dk^+ d^2 k_T Tr(\Delta^{\gamma^-}) = D_1(z), \tag{9} \]

where \( z = P_h^-/k^- \). An example of a nonvanishing azimuthal asymmetry in scattering polarized leptons from transversely polarized nucleons is

\[
\int d^2 P_{h \perp} \left| P_{h \perp} \right| \frac{d\sigma_{LT}}{dxdydz_hd^2P_{h \perp}} = \frac{2\pi\alpha^2 s}{Q^2} \lambda_e |S_T| y(2-y) \sum_{a\bar{a}} c^2_a x_B g_{1T}^{(1)a}(x_B) D_1^a(z_h), \tag{10}
\]
where the azimuthal angles are defined with respect to the lepton scattering plane. In this expression we have used the usual scaling variables, $x_B = Q^2 / 2 P \cdot q$, $y = P \cdot q / P \cdot k$ and $z_h = P \cdot P_h / P \cdot q$ and we have included the summation over quark flavors and the weighting with the quark charges squared.

The relation between $g_2$ and $g_{1T}^{(1)}$ allows an estimate of the latter using the SLAC E143 data [4] or the Wandzura-Wilczek (WW) part of $g_2$. This estimate is shown in Fig. 1 and would lead in $\bar{e}p \rightarrow e'\pi^+X$ to an asymmetry proportional to $g_{1T}^{(1)}u / f_1^u$ which is of the order of $0.05$ [5].

A complete analysis of lepton-hadron scattering can be found in ref. [6]. E.g. the functions $h_{1L}^t$ and $h_{1T}^t$ only appear in combination with a time-reversal odd fragmentation function $H_1^- [6, 7]$. There are several theoretical aspects that have been stepsided here, such as the inclusion of diagrams dressed with gluons. In fact a whole tower of diagrams containing matrix elements with $A^+$ gluon fields in the target matrix element also contribute at leading order, precisely summing up to a gauge link needed to render the nonlocal matrix element $\Phi$ color gauge invariant. Other gluon contributions are needed to ensure electromagnetic gauge invariance at order $1/Q$ or they lead to perturbative QCD corrections.

Summarizing, we stress the fact that inclusion of transverse momenta of quarks in the formalism of nonlocal matrix elements extends the interpretability of structure functions in terms of quark distributions. The representation in terms of nonlocal quark fields also provides a natural link to models for estimating these functions.

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