Sub- and super-radiance over macroscopic distances using a perfect lens with negative refraction

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Two atoms put at the foci of a perfect lens [J.B. Pendry, Phys. Ref. Lett. 85, 3966 (2000)] are shown to exhibit perfect sub- and super-radiance even over macroscopic distances limited only by the propagation length in the free-space decay time. If the left-handed material forming the perfect lens has nearly constant negative refraction and vanishing absorption over a spectral range larger than the natural linewidth, the imaginary part of the retarded Greens-function between the two focal points is identical to the one at the same spatial position and the atoms undergo a Markovian dynamics. Collective decay rates and level shifts are calculated from the Greens-function of the Veselago-Pendry lens and limitation as well as potential applications are discussed.

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Negative refraction of electromagnetic radiation in materials with simultaneous negative dielectric positivity $\varepsilon$ and magnetic permeability $\mu$ was first predicted by Veselago [1]. These so-called left-handed media attracted much attention recently because of possible realizations in metamaterials [2, 3, 4, 5, 6] and their application for a lens without diffraction limitations [7]. An infinite parallel slab of lossless left-handed material of thickness $d$ collects all plane waves from a point source on one side of the slab not too far away from the surface in a focal point on the other side. If the medium has a refractive index of $n = -1$ the distance between the two foci is $2d$. As pointed out by Pendry [1], the lens formed by the slab is perfect in the sense that the amplitudes of evanescent waves emerging from the source are amplified in the left handed medium (LHM) and exactly reproduced at the focal point thus leading to an image not limited by diffraction. This raises the question what happens to two atoms with radiative transitions inside the frequency range of negative refraction put in the focal points of a Veselago-Pendry lens.

We here show that the imaginary part of the retarded Greens-function between the two focal points is under ideal conditions identical to the free-space Greens-function at the same positions. As a consequence there occurs perfect sub- and super-radiance of the two atoms as well as dipole-dipole shifts even over distances large compared to the transition wavelength. The strong radiative coupling persists as long as the distance between the focal points is smaller than the propagation length during the free-space radiative decay time. For larger distances retardation and memory effects become important and the two-atom system can no longer be described by a master equation.

Let us consider an infinitely extended slab of homogeneous LHM of thickness $d$ and two atoms put in two focal points of the Veselago-Pendry lens formed by the slab as shown in fig. 1. The distance between the focal points is $d(1 - n)$, $n < 0$ being the refractive index of the LHM. The atoms are two-level systems with ground states $|1\rangle$ and excited states $|2\rangle$ and common transition frequency $\omega_0$. The dipole vectors of the atoms are denoted by $d_1$ and $d_2$.

1. FIG. 1: Two atoms put into the focal points of a Veselago-Pendry lens with $n = -1$. Focal points are all pairs of positions at the two sides of the slab with distance $2d$. The spatial regions $z > 0$ (vacuum), $-d \leq z \leq 0$ (LHM), and $z < -d$ (vacuum) are denoted by the numbers 0, 1, 2 respectively.

The coupling of the atoms to the quantized radiation field is described by the interaction Hamiltonian in dipole approximation

$$H_{WW} = -\hat{d}_1 \cdot \hat{\mathbf{E}}(r_1) - \hat{d}_1 \cdot \hat{\mathbf{E}}(r_2),$$

where $\hat{\mathbf{E}}(r)$ is the operator of the electric field in the presence of the LHM. Employing the usual Born-Markov and rotating-wave approximations one can derive a master equation for the two-atom density matrix in the interaction picture

$$\dot{\rho} = -\frac{1}{2} \sum_{k,l=1}^2 \frac{\Gamma(r_k,r_l)}{2} \left[ \hat{\sigma}_k^\dagger \hat{\sigma}_k \rho + \rho \hat{\sigma}_k^\dagger \hat{\sigma}_k - 2 \hat{\sigma}_k \rho \hat{\sigma}_k^\dagger \right] + i \sum_{k,l=1}^2 \delta \omega(r_k,r_l) \left[ \hat{\sigma}_k^\dagger \hat{\sigma}_k, \rho \right].$$

Here $\hat{\sigma}_k = |1\rangle_k \langle k|/2$ are the flip operators of the $k$th atom. The rates $\Gamma(r_k,r_l)$ describe the radiative decay of the two two-level atoms. $\Gamma(r_k,r_k)$ corresponds to the single-particle decay rates of an atom at position $r_k$ and $\Gamma(r_1,r_2)$ describes the dissipative cross coupling. Both quantities are determined by the imaginary part of the
retarded Greens-tensor of the electric field at the atomic transition frequency $\mathbf{G}(\mathbf{r}, \mathbf{r}^\prime, \omega_0) = G_{\mu\nu}(\mathbf{r}, \mathbf{r}^\prime, \omega_0) \hat{\mu} \otimes \hat{\nu}$, \textit{“o”} denoting a tensorial product and $\hat{\mu}$ and $\hat{\nu}$ are unit vectors.

\[ \Gamma(\mathbf{r}_k, \mathbf{r}_l) = \frac{2\omega^2 d\mu d\nu}{\hbar \epsilon_0 c^2} \text{Im} \left[ G_{\mu\nu}(\mathbf{r}_k, \mathbf{r}_l, \omega_0) \right]. \quad (3) \]

$\delta \omega(\mathbf{r}_k, \mathbf{r}_l)$ is the single-atom Lamb shift and $\delta \omega(\mathbf{r}_1, \mathbf{r}_2)$ describes the radiative dipole-dipole shift. It is well known that the Lamb shift is not correctly described by the two-level model. Furthermore its explicit expression as given below diverges and a renormalization is needed. The Lamb shift is however of no relevance for the present discussion and will be ignored, i.e. it is assumed to be included in the bare transition frequency. In contrast the dipole-dipole shift can correctly be calculated within the present model, when the free-field part is subtracted. This is because the Veselago-Pendry lens can only lead to contributions within a finite frequency window. Subtracting the free-field contribution one finds

\[ \delta \omega = \frac{g_{\mu\nu}}{\hbar \epsilon_0 c^2} \int_0^\infty \frac{d\omega}{\omega} \omega^2 \frac{\text{Im} \left[ \Delta G_{\mu\nu}(\mathbf{r}_1, \mathbf{r}_2, \omega) \right]}{\omega - \omega_0}. \quad (4) \]

where $\Delta G_{\mu\nu}(\mathbf{r}_1, \mathbf{r}_2, \omega) = G_{\mu\nu}(\mathbf{r}_1, \mathbf{r}_2, \omega) - G_{\mu\nu}(\mathbf{r}_1, \mathbf{r}_2, \omega)$, $G_{\mu\nu}(\mathbf{r}_1, \mathbf{r}_2, \omega)$ being the components of the free-field retarded Greens-tensor.

The master equation for the two-atom system can be written in a diagonal form introducing a basis of symmetric and antisymmetric states $|1\rangle, |2\rangle$ and $|s\rangle = (|1\rangle + |2\rangle) / \sqrt{2}$ and $|a\rangle = (|1\rangle - |2\rangle) / \sqrt{2}$. This yields for the populations

\[ \rho_{22} = -2\Gamma_{11} \rho_{22}, \quad (5) \]
\[ \rho_{aa} = + (\Gamma_{11} + \Gamma_{12}) \rho_{22} - (\Gamma_{11} + \Gamma_{12}) \rho_{aa}, \quad (6) \]
\[ \rho_{ss} = + (\Gamma_{11} + \Gamma_{12}) \rho_{22} - (\Gamma_{11} + \Gamma_{12}) \rho_{ss}, \quad (7) \]
\[ \rho_{11} = + (\Gamma_{11} + \Gamma_{12}) \rho_{22} - (\Gamma_{11} + \Gamma_{12}) \rho_{11}, \quad (8) \]

where $\Gamma_{11} = \Gamma(\mathbf{r}, \mathbf{r})$ and $\Gamma_{12} = \Gamma(\mathbf{r}_1, \mathbf{r}_2)$. In addition there is a level shift of the symmetric and antisymmetric states $|s\rangle$ and $|a\rangle$ below or above the single atom energy by the dipole-dipole shift $\delta \omega$, given in eq (4).

The retarded Greens-function corresponding to a slab with a homogeneous and linear magneto-dielectric medium (fig 1) can be calculated by a plane wave decomposition. Following one finds for the two positions $\mathbf{r}$ and $\mathbf{r}^\prime$ in vacuum on the same side of the lens

\[ \mathbf{G}^{00}(\mathbf{r}, \mathbf{r}^\prime, \omega) = \frac{i}{8\pi^2} \int d^2 k_\perp \frac{1}{k_z} \left[ \left( R^{\text{TTE}} \hat{e}(k_z) e^{ik_z \mathbf{z}} + \hat{e}(-k_z) e^{-ik_z \mathbf{z}} \right) \mathbf{h}(-k_z) e^{-ik_z \mathbf{z}^\prime} \right. \]

\[ + \left. \left( R^{\text{TM}} \mathbf{h}(k_z) e^{ik_z \mathbf{z}} + \mathbf{h}(-k_z) e^{-ik_z \mathbf{z}} \right) \hat{e}(-k_z) e^{-ik_z \mathbf{z}^\prime} \right], \quad (9) \]

where $z \leq z'$ has been assumed. For $\mathbf{r}$ and $\mathbf{r}^\prime$ being in vacuum on different sides of the lens one finds

\[ \mathbf{G}^{20}(\mathbf{r}, \mathbf{r}^\prime, \omega) = \frac{i}{8\pi^2} \int d^2 k_\perp \frac{1}{k_z} \left[ T^{\text{TE}} \hat{e}(-k_z) e^{ik_z \mathbf{z}} \mathbf{h}(-k_z) e^{-ik_z \mathbf{z}^\prime} \right. \]

\[ + \left. T^{\text{TM}} \mathbf{h}(-k_z) e^{ik_z \mathbf{z}} \hat{e}(-k_z) e^{-ik_z \mathbf{z}^\prime} \right], \quad (10) \]

The superscripts 0, 1, 2 at the Greens-functions denote the zones of positions $\mathbf{r}$ and $\mathbf{r}^\prime$: $z > 0$ (vacuum), $-d \leq z < 0$ (LHM), and $z < -d$ (vacuum) respectively. We here have used the definitions $k_z^2 = \omega^2 / c^2$, $k_z = \sqrt{(k^2 - k_z^2)}$ and $d^2 k_\perp = dk_x dk_y$. Furthermore $\mathbf{K} \equiv k_z \hat{x} + k_y \hat{y} - k_z \hat{z}$ and we have introduced the orthogonal unit vectors $\hat{e} = \mathbf{k} \times \hat{z} / |\mathbf{k} \times \hat{z}|$ and $\hat{h} = \hat{e} \times \mathbf{k} / |\mathbf{k}|$, where $p = 1$ for a normal medium and $p = -1$ for a LHM. $R^{\text{TE}}, R^{\text{TM}}$ and $T^{\text{TE}}, T^{\text{TM}}$ are the reflection and transmission functions of the lens for transverse electric and transverse magnetic modes. They read

\[ R^{\text{TE}} = \frac{R_{01} + R_{12} e^{i2k_z z_d}}{1 + R_{01} R_{12} e^{i2k_z z_d}}, \quad (11) \]
\[ R^{\text{TM}} = \frac{S_{01} + S_{12} e^{i2k_z z_d}}{1 + S_{01} S_{12} e^{i2k_z z_d}}, \quad (12) \]

and correspondingly

\[ T^{\text{TE}} = \frac{2\mu k_z}{\mu k_z + k_{1z} + 1} \frac{1 + R_{01} R_{12} e^{i2k_z z_d}}{1 + R_{01} R_{12} e^{i2k_z z_d}}, \quad (13) \]
\[ T^{\text{TM}} = \frac{2\varepsilon k_z}{\varepsilon k_z - k_{1z} - 1} \frac{1 + S_{12} e^{i2k_z z_d}}{1 + S_{01} S_{12} e^{i2k_z z_d}}, \quad (14) \]

Here $k_{1z} = \sqrt{k_0^2 - k_{\perp}^2}$ and $k_{1z}^2 = \varepsilon(\omega)\mu(\omega) \omega^2 / c^2$. $R_{ij}$ and $S_{ij}$ are the reflection coefficients at the boundaries between media $i$ and $j$ for TE and TM modes respectively.

\[ R_{ij} = \frac{\mu_j k_{1z} - \mu_i k_{1z}}{\mu_j k_{1z} + \mu_i k_{1z}} \quad S_{ij} = \frac{\varepsilon_j k_{1z} - \varepsilon_i k_{1z}}{\varepsilon_j k_{1z} + \varepsilon_i k_{1z}}, \quad (15) \]

The indexes $i, j \in \{0, 1, 2\}$ denote the region outside or inside the lens, i.e. $k_0^2 = k_{\perp}^2 = k^2 = \omega^2 / c^2$ and $k_{1z}^2 = \varepsilon(\omega)\mu(\omega) \omega^2 / c^2$.

From expressions 9 and 10 one can calculate $\text{Im}[\mathbf{G}(\mathbf{r}, \mathbf{r}, \omega_0)]$ for an ideal Veselago-Pendry-lens, i.e. for infinite transversal extension and a lossless medium with $n(\omega_0) = -1$. Since in this case $R^{\text{TE}} = R^{\text{TM}} = 0$ one finds e.g. $\text{Im}[\mathbf{G}^{00}(\mathbf{r}, \mathbf{r}, \omega_0)] = (k / 6\pi) \hat{\mathbf{1}}$, i.e. exactly the free-space value. Most importantly one finds that for all points $\mathbf{r}^\prime$ in region 2 ($z' \leq -d$)

\[ \text{Im} \left[ \mathbf{G}^{20}(\mathbf{r}^\prime, \omega, \omega_0) \right] = \text{Im} \left[ \mathbf{G}^{00}(\mathbf{r}^\prime + 2d\hat{z}, \mathbf{r}, \omega_0) \right], \quad (16) \]

since $T^{\text{TE}} = T^{\text{TM}} = e^{i(k_{1z} - k_{1z}^{d})}$ and $k_{1z} = -k_z$. The latter holds because $\mathbf{k}$ points backward in a LHM. Thus with respect to the radiative decay, the second atom located
in region 2 at \( r' \), i.e. on the other side of the Veselago-Pendry lens, behaves as if it would be located in region 0 at position \( r' + 2idz \). This implies that for an atom pair in the focal points

\[
\Gamma_{12} = \Gamma_{11}. \tag{17}
\]

Thus the antisymmetric, single excited state \(|a\rangle\) has vanishing radiative decay, while the symmetric state \(|s\rangle\) decays with twice the free-space rate. I.e. there is perfect sub- and super-radiance between the two atoms.

Remarkably the existence of sub-/super-radiant states does not seem to depend on the distance between the atoms. In particular in contrast to free space the phenomenon is possible also over distances large compared to the transition wavelength. This is due to the vanishing optical length of all pathways between the two foci of an ideal Veselago-Pendry lens within the relevant spectral width. Fig.2 shows the ratio \( \Gamma_{12}/\Gamma_{11} \) as a function of the spatial shift of atom 2 from the image of atom 1 at \( x_{im} = 0, z_{im} = 0 \) in radial (x) and axial direction (z). The profile is identical to the free-space case with atom 1 being located at \( x = 0, z = 0 \). One recognizes from fig.2 that like in free space the positions of the two atoms have to be controlled to within a fraction of the transition wavelength \( \lambda \).

\[
\Gamma_{12}/\Gamma_{11}
\]

FIG. 2: Deviation from perfect sub/super-radiance as a function of spatial shift of atom 2 out of focal point of atom 1. z corresponds to axial, x to radial shift. The dipole moments of the atoms are assumed to be oriented along the x axis. \( \Gamma_{12}/\Gamma_{11} = 1 \) corresponds to perfect sub-radiance of antisymmetric state, \( \Gamma_{12}/\Gamma_{11} = 0 \) to independent atoms.

When the lens is not perfect, e.g. in the presence of losses, the ratio \( \Gamma_{12}/\Gamma_{11} \) decreases roughly exponentially with increasing distance of the atoms and the sub-/super-radiance effect disappears. This is illustrated in Fig. 3. The radiative coupling is also not perfect if the lens has only a limited transversal extension. It is not possible to give an analytical expression for the Greens-tensor of a lens consisting of a disk of finite radius \( a \). Also a numerical calculation of \( G \) for this case is quite difficult. One can however obtain an estimate of the effect if \( d \gg \lambda \) by employing a short-wavelength or ray-optics approximation. Noting that for a lossless LHM with \( n(\omega_0) = -1 \) only propagating waves with \( k_\perp \leq \omega_0/c \) contribute to \( \text{Im} \{G^{(2)}\} \), we can model the effect of a finite transverse extension of the lens by restricting the \( k_\perp \) integration in eq. (11) to values

\[
k_\perp \leq k \sqrt{\frac{1}{4} + \left(\frac{\omega}{\lambda}\right)^2}. \tag{18}\]

The corresponding result is shown in Fig. 3. It is apparent that already a moderate ratio \( a/d \) is sufficient to obtain close to 100% suppression of decay of the antisymmetric state \(|a\rangle\).

![Diagram](image)

FIG. 3: left: \( \Gamma_{12}/\Gamma_{11} \) as function of imaginary part of refractive index \( n_1 \) for \( \text{Re}[n] = -1 \) for different thicknesses \( d \) of the lens, \( d = 100\lambda/2\pi \) (solid line), \( d = 10\lambda/2\pi \) (dashed), and \( d = 1\lambda/2\pi \) (dotted). right: \( \Gamma_{12}/\Gamma_{11} \) as function of transversal radius \( a \).

If the LHM has arbitrarily small losses at the frequency of interest and if the lens has a sufficiently large transversal extension, the previous discussion suggests that sub- and super-radiance is possible for two atoms at arbitrary distance. For causality reasons this is of course not possible. Thus the question arises what is the maximum possible separation \( 2d \) of the atoms over which the effect exists. As pointed out already in the original paper by Veselago [1], a lossless negative index material is necessarily dispersive. The positivity of the electromagnetic energy in a lossless LHM requires that \( n(\omega) = 1 + \frac{i\omega}{\omega_0} \) \( n \leq 0 \) and that \( \frac{d}{d\omega} \left( \omega \text{Re} \left[ n(\omega) \right] \right) \geq 0 \), which implies for \( n(\omega_0) = -1 \):

\[
\frac{d}{d\omega} n(\omega_0) \geq \frac{1}{\omega_0}. \tag{19}\]

As a consequence of the dispersion of the refractive index, the frequency window \( \Delta \omega \) over which \( G^{(2)}(\omega) \approx G^{(0)}(\omega) \) narrows with increasing thickness of the lens. When \( \Delta \omega \) becomes comparable to the natural linewidth of the atomic transitions \( \Gamma_{11} \), the Markov approximation used in eq. 4 is no longer valid. To give an estimate when this happens, we note from eqs. (11)-(15) that for \( d \gg \lambda \) the term in \( G^{(2)} \) that is most sensitive to dispersion is the exponential factor \( e^{iK(r-r')e^{i(k_1z_1-k_2z_2)d}} \). Taking into account a linear dispersion in this exponential factor, according to \( n = -1 + \alpha(\omega - \omega_0) \), with a real value of...
\(\alpha\), while keeping the resonance values for \(T^{\text{TE}}, T^{\text{TM}}\) and \(R^{\text{TE}}, R^{\text{TM}}\), one finds for the Greens-tensor

\[
\text{Im}[G^{20}(\omega)] = \frac{k}{8\pi} \text{Re} \left[ \int_0^1 d\xi (1 + \xi^2) e^{\frac{ik\omega}{\omega_0}} e^{\frac{\alpha(\omega - \omega_0)}{\omega_0}} \right].
\]

As can be seen from Fig. 4 the spectral width \(\Delta\omega\) of the Greens-function is in this approximation of order \(\Delta\omega \approx (k_d/\alpha)^{-1}\). Since as mentioned above for a lossless LHM \(\alpha \geq 1/\omega_0\), one arrives at \(\Delta\omega \leq c/d\). This leads to an upper bound for the distance of the atoms. The requirement \(\Delta\omega \gg \Gamma_1\) leads to

\[
d \ll \frac{c}{\Gamma_1}.
\]

This condition can easily be understood. It states that the distance between the two atoms must be small enough such that the travel time of a photon from one atom to the other is small compared to the free-space radiative lifetime.

![Diagram](image)

**FIG. 4:** Im\([G^{20}(\omega)]\) following from eq. (20) for lossless LHM with \(n = -1 + \alpha(\omega - \omega_0)\) for \(\alpha = 45/\omega_0\) for \(dk_0 = 1\) (dashed), \(0.2\) (dotted). Also shown is the numerically calculated spectrum for a specific causal model for \(n(\omega)\) with resonances of \(\varepsilon(\omega)\) and \(\mu(\omega)\) below \(\omega_0\). \(n(\omega)\) was chosen such that \(\text{Re}[\eta(\omega)] = -1\) and \(\alpha = 45/\omega_0\). The central structure is well represented by the linear-dispersion approximation (20).

Furthermore a narrowing of the spectral width with increasing thickness is apparent.

The existence of the sub-radiant state \(|a\rangle = (|21\rangle - |21\rangle)/\sqrt{2}\) can be used e.g. to prepare a maximally entangled state between the two atoms in a similar way as suggested in [11] for a cavity system. Preparing the first atom in the excited state, i.e. \(|\psi_0\rangle = |21\rangle = (|s\rangle + |a\rangle)/\sqrt{2}\) and letting the system evolve leads to a 50/50 mixture of both atoms being deexcited and both atoms being in the anti-symmetric state \(|a\rangle\), which is maximally entangled.

\[
|\psi_0\rangle\langle\psi_0| \longrightarrow \frac{1}{2}|11\rangle\langle 11| + \frac{1}{2}|a\rangle\langle a|.
\]

By detecting spontaneously emitted photons it is then possible to postselect with 50% success probability a maximally entangled pair of atoms.

While the decay properties are determined only by the Greens-tensor at one frequency, the dipole-dipole shift \(\delta\omega\) depends on the whole spectrum of the dielectric function \(\varepsilon(\omega)\) and the magnetic permeability \(\mu(\omega)\). Using various single-resonance model functions for \(\varepsilon(\omega)\) and \(\mu(\omega)\), which fulfill Kramers-Kronig relations we found values of \(\delta\omega_1\) up to 0.5\(\Gamma_1\). It is not clear however what the upper value of \(\delta\omega_1/\Gamma_1\) is, as this requires an optimization over all possible (i.e. causal) material responses \(\varepsilon(\omega)\) and \(\mu(\omega)\). If \(\delta\omega_1 \gg \Gamma_1\) could be achieved, an almost perfect coherent excitation transfer between two atoms in the focal points of the Veselago-Pendry lens is possible without the use of a resonator. In the opposite case of \(\delta\omega_1 \ll \Gamma_1\) the fidelity of such a transfer process drops to 25%. A more detailed study of the dipole-dipole shift in LHM will be the subject of further studies.

In summary we have shown that two atoms put in the focal points of an ideal, i.e. lossless Veselago-Pendry lens exhibit perfect sub- and super-radiance as long as their distance is much smaller than the propagation length of light corresponding to the free-space decay time. Since the latter can be orders of magnitude larger than the wavelength, sub- and super-radiance can occur over distances large compared to the resonance wavelength.

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