Magnetic moments of the hidden-charm strange pentaquark states

Feng Gao\textsuperscript{1} and Hao-Song Li\textsuperscript{1,2,3,4}\textsuperscript{†}
\textsuperscript{1}School of Physics, Northwest University, Xian 710127, China
\textsuperscript{2}Institute of Modern Physics, Northwest University, Xian 710127, China
\textsuperscript{3}Shaanxi Key Laboratory for theoretical Physics Frontiers, Xian 710127, China
\textsuperscript{4}Peng Huanwu Center for Fundamental Theory, Xian 710127, China

This paper calculates the magnetic moments of the hidden–charm strange pentaquark states with a quantum number of \( J^P = \frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+ \) and \( \frac{7}{2}^+ \) in the molecular, diquark–diquark–antiquark, and diquark–triquark models, respectively. Numerical results demonstrate that the magnetic moments change for a different spin-orbit coupling with the same model and when involving different models with the same angular momentum.

I. INTRODUCTION

The quark model is a successful theory, with physicists employing it to explain mesons’ and baryons’ inner structures and predict the tetraquark and pentaquark. Over the past decade, the multiquark states’ exploration has made significant progress both theoretically and experimentally, with several exotic hadronic states being experimentally observed \cite{1,2}.

In 2015, the LHCb Collaboration observed the pentaquark states in the \( J/\psi p \) invariant mass spectrum of the \( \Lambda^0_c \rightarrow J/\psi K^- p \) decays. The two candidates of the hidden–charm pentaquark are \( P_c(4380) \) and \( P_c(4450) \), respectively, whose \( J^P \) has an opposite parity with \( (\frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+, \frac{7}{2}^+) \) \cite{3}. In 2019, the \( P_c(4550) \) pentaquark state structure was confirmed and the observations revealed comprising two peaks, \( P_c(4440) \) and \( P_c(4457) \), with a statistical significance of 5.4\( \sigma \) \cite{4}. Meanwhile, the LHCb Collaboration reported a new pentaquark state observation, \( P_c(4312) \), with a statistical significance of 7.3\( \sigma \). In 2021, the LHCb Collaboration found evidence for a new structure \( P_c(4337) \) in \( B^0 \rightarrow J/\psi p \bar{p} \) decays, with a final significance of 3.1\( \sigma \) \cite{5}. The mass and width of \( P_c(4337) \) are \( 4337^{+7}_{-6} \pm 2 \) MeV and 7.5\( ^{+26}_{-13} +^{14}_{-2} \) MeV, respectively, while after an in-depth study of \( P_c(4337) \), the structure was proved to have two resonances, with masses 4454.9\( ^{+2.7}_{-2.8} \) MeV and 4467.8\( ^{+3.7}_{-3.4} \) MeV and widths 7.5\( ^{+9.7}_{-9.5} \) MeV and 5.2\( ^{+5.3}_{-5.1} \) MeV, respectively. However, the parity and angular momentum of \( P_c(4337) \) have not been determined experimentally. The predictions of \( J^P = \frac{1}{2}^- \) and \( \frac{3}{2}^- \) have been given based on the QCD sum rules \cite{19}, chiral quark model \cite{20}, and the strong decay behaviors of the \( P_{cs}(4459) \) \cite{21}.

Given that pentaquark’s magnetic moment encoding includes helpful details about the charge and magnetization distributions inside the hadrons that assists in analyzing their geometric configurations. In Ref. \cite{44}, the author

\textsuperscript{†}Electronic address: feng.gao@stumail.nwu.edu.cn
\textsuperscript{‡}Electronic address: haosongli@nwu.edu.cn

**Keywords:**
studies the magnetic moments and transition magnetic moments of the hidden-charm pentaquark states with the coupled channel effects and the D wave contributions. This study is important because magnetic moments help us understand the pentaquark’s inner structure. In this work, we calculate the magnetic moments of $P_{cs}$ based on the above three models.

The remainder of this paper is as follows. Sec.II discusses the color factor and color configuration, while Sec.III introduces the wave function of $P_{cs}$. Sec.IV calculates the magnetic moments of $P_{cs}$ in the molecular model, diquark–diquark–antiquark model, and the diquark–triquark model, and finally, Sec.V summarizes this work.

II. COLOR FACTOR AND COLOR CONFIGURATION

The quark level involves chromomagnetic interactions. Therefore, we exploit the color factor $f$ to indicate whether the color force is attractive or repulsive.

Regarding the quark-quark color interaction, the color factor $f$ is:

$$f(ik \rightarrow jl) = \frac{1}{4} \sum_{a=1}^{8} \lambda^a_{ji} \lambda^a_{lk}. \quad (1)$$

where $\lambda^a$ denotes the Gell-Mann matrices and the quark colors are labelled by $i, j, k$, and $l$. The potentials is:

$$V_{qq}(r) \approx + f\frac{\alpha_s}{r}. \quad (2)$$

Considering the quark-antiquark color interaction, the color factor $\tilde{f}$ is:

$$\tilde{f}(ik \rightarrow jl) = -\frac{1}{4} \sum_{a=1}^{8} \lambda^a_{ji} \lambda^a_{lk}. \quad (3)$$

The potentials is:

$$V_{q\bar{q}}(r) \approx + \frac{\tilde{f}\alpha_s}{r}. \quad (4)$$

In Table I we list the color factors of the multiplet in the SU(3) color representation

| Multiplet | Color Factor |
|-----------|--------------|
| $3_c \otimes \bar{3}_c$ | $-\frac{4}{3}$ $\frac{1}{6}$ |
| $1_c \otimes 8_c$ | $-\frac{3}{4}$ $\frac{1}{6}$ |
| $3_c \otimes 3_c$ | $6_c \otimes \bar{3}_c$ |
| Color Factor | $\frac{1}{7}$ $-\frac{2}{7}$ |
| $3_c \otimes 3_c \otimes 3_c$ | $1_c \otimes 8_{1c} \otimes 8_{2c} \otimes 10_c$ |
| Color Factor | $-2$ $-\frac{1}{2}$ $-\frac{1}{2}$ $1$ |
| $3_c \otimes 3_c \otimes \bar{3}_c$ | $3_{1c} \otimes 3_{2c} \otimes 6_c \otimes 15_c$ |
| Color Factor | $-\frac{4}{3}$ $-\frac{4}{3}$ $-\frac{1}{6}$ $2$ |

Color confinement implies that the physical hadrons are singlet. Under this restriction, we divide the pentaquark states into the following three categories:

1. Molecular model

Each cluster of the molecular model forms a quasibound cluster. In other words, clusters of the molecular model tend to be color singlet. We observe that $f_{1c} < f_{8c}$ in the color representation of the quark and antiquark, hence it is easier to form a singlet state than octet states. Similarly, $f_{1c} < f_{8_{1c}}/f_{8_{2c}} < f_{10_c}$ in the three-quark color representation and thus it is easier to form a singlet state than other states. Therefore, from the molecular model we have two configurations $(c\bar{c})(q_1q_2q_3)$ and $(c\bar{q}_1)(cq_2q_3)$, where $q$ denotes the $u, d, s$ quark.
2. Diquark–Diquark–antiquark model

The diquark prefers to form \( 3_c \) by comparing the color factor of \( 6_c \) and \( 3_c \). Similarly, \( 3_c \otimes 3_c \) prefers to form \( 3_c \). Hence, we have \( 3_c(D) \otimes 3_c(D) \otimes 3_c(A) \) to form a color singlet, where \( D \) and \( A \) represent the diquark and antiquark, respectively. Thus, the pentaquark configuration is \((c\bar{q}_1)(q_2q_3)(\bar{c})\), represented by the diquark–diquark–antiquark model.

3. Diquark–triquark model

The triquark involves two quarks and an antiquark, assisting in distinguishing it when utilizing the molecule model. In this case, \( f_{3c}/f_{3c} < f_{6c} < f_{3c} \) is the color representation of the triquark quark and we have \( 3_c(T) \otimes 3_c(D) \) to form a color singlet, where \( T \) represents a triquark. Thus the pentaquark configuration represented by the diquark–triquark model is \((c\bar{c}q_1)(q_2q_3)\) and \((c\bar{q}_1)(\bar{c}q_2q_3)\).

The separation of \( c \) and \( \bar{c} \) into distinct confinement volumes provides a natural suppression mechanism for the pentaquark widths. Thus we don’t consider \((\bar{c}c)(q_1q_2q_3)\) and \((\bar{c}cq_1)(q_2q_3)\).

III. WAVE FUNCTION OF HIDDEN-CHARM STRANGE PENTAQUARK STATES

In this work, we study the pentaquark states in the \(SU(3)_f\) frame. The overall wavefunction for a bounded multiquark state, while accounting for all degrees of freedom, can be written as:

\[
\psi_{wavefunction} = \phi_{flavor} \chi_{spin} \varphi_{color} \xi_{space}.
\]

Due to the Fermi statistics, the overall wavefunction above is required to be antisymmetric.

The molecular model of the pentaquark is made up of mesons and baryons. They have to be color singlet because of color confinement. The relation between the spin and flavor is \( \phi_{flavor} \chi_{spin} = \text{symmetric} \) since the color wavefunction is antisymmetric and the spatial wavefunction is symmetric in the ground state. We study the \( P_{cs} \) state in a \(SU(3)_f\) frame. There are two configurations for \( q_2q_3 \), where \( q_2q_3 \) forms the \( 3_f \) and \( 6_f \) flavor representation with the total spin \( S = 0 \) and \( 1 \), respectively. When \( q_2q_3 \) forms the \( 6_f \), it is combined with the \( q_1 \) to form the flavor representation \( 6_f \otimes 3_f = 10_f \oplus 81_f \). While, when \( q_2q_3 \) forms the \( 3_f \), it is then combined with the \( q_1 \) to form the flavor representation \( 3_f \otimes 3_f = 82_f \oplus 1_f \). After inserting \([\bar{c}c]\) and the Clebsch-Gordan coefficients, we apply the same method to the \((c\bar{q}_1)(q_2q_3)(\bar{c})\) and \((c\bar{q}_1)(\bar{c}q_2q_3)\) configurations, and we obtain the flavor wave function of \( P_{cs} \) in \( 81_f \) and \( 82_f \). The results are reported in Table II.

| model | multiplet (\( I, I_3 \)) | wave function |
|-------|-------------------|----------------|
| Molecular model | \( 8_{1f} \) | \((1,0)\) \( \frac{1}{\sqrt{6}}\{[\bar{c}(d)(c\{us\})] + (\bar{c}u)(c\{ds\})\} - \frac{1}{\sqrt{2}} (\bar{c}s)(c\{ud\}) \) |
| | \( 8_{2f} \) | \((1,0)\) \( \frac{1}{\sqrt{3}} ([\bar{c}(d)(c\{ds\})] - (\bar{c}d)(c\{us\}) \) |
| | \( 8_{1f} \) | \((1,0)\) \( \frac{1}{\sqrt{6}}\{[c\{ds\}][\bar{c} + (c\{ds\}][\bar{c}] - \frac{1}{\sqrt{2}} (\bar{c}s)(c\{ud\}) \) |
| | \( 8_{2f} \) | \((1,0)\) \( \frac{1}{\sqrt{3}} ([c\{ds\}]\bar{c} - (c\{ds\})[\bar{c}] \) |
| Diquark-diquark-antiquark model | \( 8_{1f} \) | \((1,0)\) \( \frac{1}{\sqrt{6}}\{[c\{ds\}][\bar{c} + (c\{ds\})[\bar{c}] - \frac{1}{\sqrt{2}} (\bar{c}s)(c\{ud\}) \) |
| | \( 8_{2f} \) | \((1,0)\) \( \frac{1}{\sqrt{3}} ([c\{ds\}]\bar{c} - (c\{ds\})[\bar{c}] - 2(c\{ds\})[\bar{c}] \) |
| Diquark-triquark model | \( 8_{1f} \) | \((1,0)\) \( \frac{1}{\sqrt{6}}\{[c\{ds\}][\bar{c} + (c\{ds\})[\bar{c}] - \frac{1}{\sqrt{2}} (\bar{c}s)(c\{ud\}) \) |
| | \( 8_{2f} \) | \((1,0)\) \( \frac{1}{\sqrt{3}} ([c\{ds\}]\bar{c} - (c\{ds\})[\bar{c}] - 2(c\{ds\})[\bar{c}] \) |
IV. MAGNETIC MOMENTS OF HIDDEN-CHARM STRANGE PENTAQUARK

A. Magnetic moments of the molecular model with the configuration ($\bar{c}q_1)(cq_2q_3$)

Since quarks are fundamental Dirac fermions, the operators of the total magnetic moments and the $z$-component are:

$$\hat{\mu} = Q\frac{e}{m}\hat{S}, \quad \hat{\mu}_z = Q\frac{e}{m}\hat{S}_z.$$  \hspace{1cm} (5)

As mentioned above, we do not consider the orbital excitation in the bound state, so the orbital excitation lies between the meson and baryon. The total magnetic moments formula can be written as:

$$\hat{\mu} = \hat{\mu}_B + \hat{\mu}_M + \hat{\mu}_l.$$  \hspace{1cm} (6)

where the subscripts $B$ and $M$ represent the baryon and meson, respectively, and $l$ is the orbital excitation between the meson and baryon. The magnetic moments’ specific forms can be written as:

$$\hat{\mu}_B = \sum_{i=1}^{3} g_i \hat{S}_i,$$  \hspace{1cm} (7)

$$\hat{\mu}_M = \sum_{i=1}^{2} g_i \hat{S}_i,$$  \hspace{1cm} (8)

$$\hat{\mu}_l = \mu \hat{l} = \frac{M_B \mu_B + M_M \mu_M}{M_M + M_B}.$$  \hspace{1cm} (9)

where $g_i$ is the Lande factor and $M_M$ and $M_B$ are the meson and baryon masses, respectively. The pentaquark’s ($\bar{c}q_1)(cq_2q_3$) specific magnetic moments formula in the molecular model is:

$$\mu = \langle \psi | \hat{\mu}_B + \hat{\mu}_M + \hat{\mu}_l | \psi \rangle$$

$$= \sum_{SS_z, ll_z} \langle S S_z, ll_z | J J_z \rangle^2 \left\{ \mu_I l_z + \sum_{\bar{S}_B, \bar{S}_M} \langle S_B \bar{S}_B, S_M \bar{S}_M | S S_z \rangle^2 \left[ \bar{S}_M (\mu_\bar{c} + \mu_{q_1}) \right. \right. \right. \right.$$

$$+ \sum_{\bar{S}_e} \langle S_c \bar{S}_e, S_f \bar{S}_B - \bar{S}_c | S_B \bar{S}_B \rangle^2 \left( g_{\mu_\bar{c}} \bar{S}_c + (\bar{S}_B - \bar{S}_e) (\mu_{q_2} + \mu_{q_3}) \right) \right\}.$$  \hspace{1cm} (10)

where $\psi$ represents the wave function in Table III. $S_M$, $S_B$, $S_c$ are the meson, baryon, and the diquark spin inside the baryon, respectively. $\bar{S}$ is the third spin component.

For example, the recently observed $P_{cs}(4459)$ state is supposed to be the $\bar{D}^*\Xi_c$ molecular states in the $8_{2f}$ representation with $(I, I_3) = (0, 0)$. Their flavor wave functions are:

$$|P_{cs}\rangle = \frac{1}{\sqrt{6}} \{ (\bar{c}\bar{d})(c|us\rangle) - (\bar{c}u)(c|ds\rangle) - 2(\bar{c}s)(c|ud\rangle) \}.$$  \hspace{1cm} (11)

Take $J^P = \frac{1}{2}^-$ ($\frac{1}{2}^+ \otimes 1^- \otimes 0^+$) as an example. $J_1^{P_1} \otimes J_2^{P_2} \otimes J_3^{P_3}$ are corresponding to the angular momentum and parity of baryon, meson and orbital, respectively.

$$\mu = \langle P_{cs} | \hat{\mu}_B + \hat{\mu}_M + \hat{\mu}_l | P_{cs} \rangle$$

$$= \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \left[ \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \left( \frac{1}{6} \frac{1}{2} g_{\mu_\bar{c}} + \frac{1}{6} \frac{1}{2} g_{\mu_c} + \frac{1}{6} \frac{1}{2} g_{\mu_c} + \frac{1}{6} \frac{1}{2} g_{\mu_c} \right) \right] +$$

$$\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \left[ \frac{1}{6} \frac{1}{2} g_{\mu_\bar{c}} + \frac{1}{6} \frac{1}{2} g_{\mu_c} + \frac{1}{6} \frac{1}{2} g_{\mu_c} + \frac{1}{6} \frac{1}{2} g_{\mu_c} \right] +$$

$$\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \left[ \frac{1}{6} \frac{1}{2} g_{\mu_\bar{c}} + \frac{1}{6} \frac{1}{2} g_{\mu_c} + \frac{1}{6} \frac{1}{2} g_{\mu_c} + \frac{1}{6} \frac{1}{2} g_{\mu_c} \right]$$
\[
\hat{\mu} = \frac{1}{9}(\mu_u + \mu_d + 4\mu_s + 6\mu_c - 3\mu_c).
\] (12)

In this work, we use the following constituent quark masses [45],

\[
m_u = m_d = 0.336 \text{ GeV}, \ m_s = 0.540 \text{ GeV}, \ m_c = 1.660 \text{ GeV}.
\]

The numerical results with isospin \((I, I_3) = (1, 0)\) and \((I, I_3) = (0, 0)\) are reported in Table III and IV, respectively.

TABLE III: The magnetic moments of the pentaquark states in the molecular model with the wave function \(\frac{1}{\sqrt{6}}[(\bar{c}d)(c\{us\}) + (\bar{c}u)(c\{ds\})]\) - \(\tilde{\sqrt{3}}[(\bar{s}d)(c\{ud\})]\) in \(8_{1f}\) and \(\frac{1}{\sqrt{2}}[(\bar{c}d)(c\{us\}) + (\bar{c}u)(c\{ds\})]\) in \(8_{2f}\) with isospin \((I, I_3) = (1, 0)\). They are in \(8_{1f}\) representation from \(6_f \otimes 3_f = 10_f \otimes 8_{1f}\) and \(8_{2f}\) representation from \(3_f \otimes 3_f = 1_f \otimes 8_{2f}\), respectively. The third line \(J^P_{18} \otimes J^P_{12} \otimes J^P_9\) are corresponding to the angular momentum and parity of baryon, meson and orbital, respectively. The unit is the magnetic moments of the proton.

| \((Y, I, I_3)\) | \(2S_y (J^P = \frac{1}{2}^-)\) | \(4S_y (J^P = \frac{1}{2}^-)\) | \(6S_y (J^P = \frac{1}{2}^-)\) |
|----------|----------------|----------------|----------------|
| \((0,1,0)\) | 0.263 | -0.493 | 0.735 |
| \((0,1,0)\) | -0.145 | 0.125 | -0.289 |
| \((0,1,0)\) | 0.177 | -0.551 | 0.669 |
| \((0,1,0)\) | -0.403 | 0.865 | 0.394 |

TABLE IV: The magnetic moments of the pentaquark states in the molecular model with the wave function \(\frac{1}{\sqrt{6}}[(\bar{c}d)(c\{us\}) + (\bar{c}u)(c\{ds\})]\) - \(\tilde{\sqrt{3}}[(\bar{s}d)(c\{ud\})]\) in \(8_{1f}\) and \(\frac{1}{\sqrt{2}}[(\bar{c}d)(c\{us\}) + (\bar{c}u)(c\{ds\})]\) in \(8_{2f}\) with isospin \((I, I_3) = (1, 0)\). They are in \(8_{1f}\) representation from \(6_f \otimes 3_f = 10_f \otimes 8_{1f}\) and \(8_{2f}\) representation from \(3_f \otimes 3_f = 1_f \otimes 8_{2f}\), respectively. The third line \(J^P_{18} \otimes J^P_{12} \otimes J^P_9\) are corresponding to the angular momentum and parity of baryon, meson and orbital, respectively. The unit is the magnetic moments of the proton.

| \((Y, I, I_3)\) | \(2S_y (J^P = \frac{1}{2}^-)\) | \(4S_y (J^P = \frac{1}{2}^-)\) | \(6S_y (J^P = \frac{1}{2}^-)\) |
|----------|----------------|----------------|----------------|
| \((0,1,0)\) | 0.377 | -0.067 | 0.465 |
| \((0,1,0)\) | 0.315 | -0.110 | 0.324 |

B. Magnetic moments of the diquark-diquark-antiquark model with the \((cq_{1})(q_{2q_{3}})\) configuration

In the diquark-diquark-antiquark model, there are two P-wave excitation modes inside the three-body bound state, the \(\rho\) and the \(\lambda\) excitation. The \(\rho\) mode P-wave orbital excitation lies between the diquark \((cq_{1})\) and diquark \((q_{2q_{3}})\). The \(\lambda\) mode P-wave orbital excitation lies between the \(c\) and the center of mass system of the \((cq_{1})\) and \((q_{2q_{3}})\).

The total magnetic moments formula of the diquark-diquark-antiquark model can be written as:

\[
\hat{\mu} = \hat{\mu}_H + \hat{\mu}_L + \hat{\mu}_E + \hat{\mu}_T.
\] (13)
TABLE IV: The magnetic moments of the pentaquark states in the molecular model with the wave function \( \frac{1}{\sqrt{2}}[\langle \bar{c}u \rangle(c\{ds\}) - (\bar{c}d)(c\{us\})] \) in \( 8_{1f} \) and \( \frac{1}{\sqrt{2}}[(\bar{c}d)(c\{us\}) - (\bar{c}u\rangle(c\{ds\}) - 2(\bar{c}s)(c\{ud\})] \) in \( 8_{1f} \) with isospin \( (I, I_3) = (0, 0) \). The third line \( J^P \) \( \otimes \) \( J^P \) \( \otimes \) \( J^P \) are corresponding to the angular momentum and parity of baryon, meson and orbital, respectively. The unit is the magnetic moments of the proton.

| \( (Y, I, I_3) \) | \( 8_{1f}: \frac{1}{\sqrt{2}}[\langle \bar{c}u \rangle(c\{ds\}) - (\bar{c}d)(c\{us\})] \) | \( 8_{1f}: \frac{1}{\sqrt{2}}[(\bar{c}d)(c\{us\}) - (\bar{c}u\rangle(c\{ds\}) - 2(\bar{c}s)(c\{ud\})] \) |
|-----------------|--------------------------------------|--------------------------------------|
| \( \frac{1}{2}^+ \) \( \otimes 0^- \otimes 0^+ \) | \( \frac{1}{2}^+ \) \( \otimes 1^- \otimes 0^+ \) | \( \frac{1}{2}^+ \) \( \otimes 1^- \otimes 0^+ \) |
| \( (0, 0, 0) \) | -0.201 | 0.126 | 0.117 | -0.113 | 0.263 | 0.228 | 0.352 |
| \( \frac{1}{2}^+ \) \( \otimes 0^- \otimes 0^+ \) | \( \frac{1}{2}^+ \) \( \otimes 1^- \otimes 0^+ \) | \( \frac{1}{2}^+ \) \( \otimes 1^- \otimes 0^+ \) |
| \( (0, 0, 0) \) | 0.021 | -0.076 | -0.076 | -0.046 | 0.171 | 0.145 |

where the subscripts \( H \) and \( L \) represent a heavy diquark \( (cq) \) and light diquark \( (q_2q_3) \), respectively, and \( l \) is the orbital excitation. In the diquark-diquark-antiquark model, the specific magnetic moments formula of the pentaquark \( (cq_1)(q_2q_3)\bar{c} \) is:

\[
\mu = \langle \psi | \tilde{\mu}_H + \tilde{\mu}_L + \tilde{\mu}_c + \tilde{\mu}_l | \psi \rangle = \sum_{S, J} \langle SS_z, ll_z | JJ_z \rangle \left( \sum_{S_c, S_l} \langle S_c \tilde{S}_c, S_l \tilde{S}_l | SS_z \rangle \left\{ \begin{array}{c} g_{\tilde{S}_c} \tilde{\mu}_e \\
\mu_l \end{array} \right\} + \sum_{\tilde{S}_H, \tilde{S}_L} \langle S_l \tilde{S}_H, S_l \tilde{S}_L | SS_z \rangle \left\{ \begin{array}{c} \tilde{S}_H (\mu_e + \mu_{q_1}) + \tilde{S}_L (\mu_{q_2} + \mu_{q_3}) \end{array} \right\} \right) \right) \right). \tag{14}
\]

where \( S_G \) represents the spin of \( (cq_1)(q_2q_3) \). The diquarks' masses are [46]:

\[
[u, d] = 710 \text{MeV}, \quad \{u, d\} = 909 \text{MeV}, \quad [u, s] = 948 \text{MeV}, \quad \{u, s\} = 1069 \text{MeV},
\]

\[
[c, q] = 1973 \text{MeV}, \quad \{c, q\} = 2036 \text{MeV}, \quad [c, s] = 2091 \text{MeV}, \quad \{c, s\} = 2158 \text{MeV}.
\]

The numerical results for the states with the \( \rho \) excitation mode with isospin \( (I, I_3) = (1, 0) \) and \( (I, I_3) = (0, 0) \) are presented in Table \[V\] and \[VI\] respectively. The numerical results for the states with the \( \lambda \) excitation mode with isospin \( (I, I_3) = (1, 0) \) and \( (I, I_3) = (0, 0) \) are presented in Table \[VII\] and \[VIII\] respectively.
TABLE V: The magnetic moments of the pentaquark states in the diquark-diquark-antiquark model with the wave function \(\sqrt{2}\[(ud)(us)\bar{c} + (cu)(ds)\bar{c}] - \sqrt{2}\[(cd)(us)\bar{c} - (cu)(ds)\bar{c}]\) in \(8_{1f}\) and \(\sqrt{2}\[(cd)(us)\bar{c} + (cu)(ds)\bar{c}]\) in \(8_{2f}\) with isospin \((I, I_3) = (1, 0)\). They are in \(8_{1f}\) representation from \(6_f \otimes 3_f = 10_f \oplus 8_{1f}\) and \(8_{2f}\) representation from \(3_f \otimes 3_f = 1_f \oplus 8_{2f}\), respectively. The third line \(J^{P}_{1f} \otimes J^{P}_{2f} \otimes J^{P}_{3f} \otimes J^{P}_{4f}\) are corresponding to the angular momentum and parity of \((cq_1), (q_q_3), \bar{c}\) and orbital, respectively. The \(\rho\) mode P-wave orbital excitation lies between the diquark \((cq_1)\) and diquark \((q_q_3)\). The unit is the magnetic moments of the proton.

| \(J^{P}_{1f}\) | \(s_{1f} = \frac{1}{\sqrt{2}}[(cd)(us)\bar{c} + (cu)(ds)\bar{c}] - \sqrt{2}\[(cd)(us)\bar{c} - (cu)(ds)\bar{c}]\) | \(s_{2f} = \frac{1}{\sqrt{2}}[(cd)(us)\bar{c} + (cu)(ds)\bar{c}]\) |
|---------------|---------------------------------|-----------------|
| \(0^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(0^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(0^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) |
| \(1^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(1^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(1^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) |
| \(0^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(0^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(0^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) |
| \(1^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(1^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(1^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) |

TABLE VI: The magnetic moments of the pentaquark states in the diquark-diquark-antiquark model with the wave function \(\sqrt{2}\[(cu)(ds)\bar{c} - (cd)(us)\bar{c}]\) in \(8_{1f}\) and \(\sqrt{2}\[(cd)(us)\bar{c} - (cu)(ds)\bar{c} - 2(cs)(ud)\bar{c}]\) in \(8_{2f}\) with isospin \((I, I_3) = (0, 0)\). The third line \(J^{P}_{1f} \otimes J^{P}_{2f} \otimes J^{P}_{3f} \otimes J^{P}_{4f}\) are corresponding to the angular momentum and parity of \((cq_1), (q_q_3), \bar{c}\) and orbital, respectively. The \(\rho\) mode P-wave orbital excitation lies between the diquark \((cq_1)\) and diquark \((q_q_3)\).

| \(J^{P}_{1f}\) | \(s_{1f} = \frac{1}{\sqrt{2}}[(cu)(ds)\bar{c} - (cd)(us)\bar{c}]\) | \(s_{2f} = \frac{1}{\sqrt{2}}[(cd)(us)\bar{c} - (cu)(ds)\bar{c} - 2(cs)(ud)\bar{c}]\) |
|---------------|---------------------------------|-----------------|
| \(0^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(0^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(0^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) |
| \(1^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(1^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(1^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) |
| \(0^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(0^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(0^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) |
| \(1^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(1^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(1^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) |

| \(J^{P}_{1f}\) | \(s_{1f} = \frac{1}{\sqrt{2}}[(cu)(ds)\bar{c} - (cd)(us)\bar{c}]\) | \(s_{2f} = \frac{1}{\sqrt{2}}[(cd)(us)\bar{c} - (cu)(ds)\bar{c} - 2(cs)(ud)\bar{c}]\) |
|---------------|---------------------------------|-----------------|
| \(0^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(0^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(0^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) |
| \(1^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(1^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(1^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) |
| \(0^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(0^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(0^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) |
| \(1^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(1^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) | \(1^+ \otimes 1^+ \otimes 1^+ \otimes 0^+\) |
TABLE VII: The magnetic moments of the pentaquark states in the diquark-diquark-antiquark model with the wave function \( \frac{1}{\sqrt{2}}[(cd)[us]\bar{c} + (cu)[ds]\bar{c}] - \frac{1}{\sqrt{2}}[(cs)[ud]\bar{c}] \) in \( S_{13} \) and \( \frac{1}{\sqrt{8}}[(cd)[us]\bar{c} + (cu)[ds]\bar{c}] \) in \( S_{2J} \) with isospin \((I, I_3)= (1, 0)\). The third line \( J^P_1 \otimes J^P_2 \otimes J^P_3 \otimes J^P_4 \) are corresponding to the angular momentum and parity of \((cq_1), (q_2q_3), \bar{c}\) and orbital, respectively. The \( \Lambda \) mode P-wave orbital excitation lies between the \( \bar{c} \) and the center of mass system of the \((cq_1), (q_2q_3)\). The unit is the magnetic moments of the proton.

| \((I, I_3)\) | \(S_{1J} (J^P = \frac{1}{2}^+)\) | \(S_{1J} (J^P = \frac{3}{2}^+)\) | \(S_{1J} (J^P = \frac{5}{2}^+)\) | \(S_{1J} (J^P = \frac{7}{2}^+)\) |
|----------------|----------------|----------------|----------------|----------------|
| \((0, 1)\)     | 0.514          | -0.377         | 0.368          | -0.206         |
| \((1, 0)\)     | 0.217          | 0.080          | 0.259          | 0.198          |
| \((1.277)\)    | 1.034          | 0.348          | 0.941          | 0.875          |
| \((1.495)\)    | -0.377         | 0.465          | 0.525          | 0.364          |

**TABLE VIII:** The magnetic moments of the pentaquark states in the diquark-diquark-antiquark model with the wave function \( \frac{1}{\sqrt{8}}[(cu)[ds]\bar{c} - (cd)[us]\bar{c}] \) in \( S_{13} \) and \( \frac{1}{\sqrt{8}}[(cd)[us]\bar{c} - (cu)[ds]\bar{c} - 2(cs)[ud]\bar{c}] \) in \( S_{2J} \) with isospin \((I, I_3)= (0, 0)\). The third line \( J^P_1 \otimes J^P_2 \otimes J^P_3 \otimes J^P_4 \) are corresponding to the angular momentum and parity of \((cq_1), (q_2q_3), \bar{c}\) and orbital, respectively. The \( \Lambda \) mode P-wave orbital excitation lies between the \( \bar{c} \) and the center of mass system of the \((cq_1), (q_2q_3)\).

| \((I, I_3)\) | \(S_{1J} (J^P = \frac{1}{2}^+)\) | \(S_{1J} (J^P = \frac{3}{2}^+)\) | \(S_{1J} (J^P = \frac{5}{2}^+)\) | \(S_{1J} (J^P = \frac{7}{2}^+)\) |
|----------------|----------------|----------------|----------------|----------------|
| \((0, 0)\)     | 0.459          | 0.494          | 0.612          | 0.646          |
| \((0.220)\)    | 0.462          | 0.494          | 0.612          | 0.646          |
| \((0.223)\)    | 0.462          | 0.494          | 0.612          | 0.646          |
| \((0.465)\)    | 0.459          | 0.494          | 0.612          | 0.646          |

| \((0.352)\)    | 0.459          | 0.494          | 0.612          | 0.646          |

| \((0.354)\)    | 0.459          | 0.494          | 0.612          | 0.646          |

| \((0.357)\)    | 0.459          | 0.494          | 0.612          | 0.646          |

| \((0.358)\)    | 0.459          | 0.494          | 0.612          | 0.646          |

| \((0.359)\)    | 0.459          | 0.494          | 0.612          | 0.646          |
C. Magnetic moments of the diquark-triquark model with the configuration $c_1q_1\bar{q}_2q_3$

Considering the diquark-triquark model, the total magnetic moments formula is:

$$\hat{\mu} = \hat{\mu}_D + \hat{\mu}_T + \hat{\mu}_l.$$  \hfill (15)

where the $l$ is the orbital excitation between the diquark and triquark. The magnetic moments formula of the pentaquark, $(c_1q_1)(\bar{q}_2q_3)$ in the diquark-triquark model is

$$\mu = \langle \psi | \hat{\mu}_D + \hat{\mu}_T + \hat{\mu}_l | \psi \rangle = \sum_{S_\ell, l_\ell} \langle SS_z,l| JJ_z \rangle \left[ \mu_{l\ell} + \sum_{S_D,S_T} \langle S_D\bar{S}_D,S_T\bar{S}_T| SS_z \rangle \left[ \hat{S}_D(\mu_c + \mu_{q_1}) \right. \right.$$  

$$+ \sum_{S_c} \langle S_c\bar{S}_c,S_T\bar{S}_T - \bar{S}_c\bar{S}_c| S_T\bar{S}_T \rangle \left. \left( g\mu_c\bar{S}_c + (\bar{S}_T - \bar{S}_c)(\mu_{q_2} + \mu_{q_3}) \right) \right] \right).$$  \hfill (16)

where $S_D$, $S_T$, and $S_c$ represent the diquark, triquark and the light diquark spin inside the triquark, respectively. The triquark’s masses roughly use the sum of the mass of the corresponding diquark and the antiquark. The numerical results with isospin $(I, I_3) = (1, 0)$ and $(I, I_3) = (0, 0)$ are reported in Table [X] and [XI] respectively.

I have compared the magnetic moment of $P_{cs}(4459)$ in three configurations as shown in the following Table [XII]. The magnetic moment and numerical results illustrate that molecular model is distinguishable from the other two models in $0(\frac{J^-}{P^-})$ but it is indistinguishable in $0(\frac{J^-}{P^+})$. As far as diquark-diquark-antiquark model and diquark-triquark model are concerned, they are completely indistinguishable in $0(\frac{J^-}{P^-})$ and $0(\frac{J^-}{P^+})$. In addition to this, the magnetic moment of $P_{cs}(4459)$ have been studied in other papers. In Ref. [44], the numerical value in the molecular picture was obtained as $\mu_{P_{cs}} = -0.062\mu_N$ with $0(\frac{J^-}{P^-})$ and $\mu_{P_{cs}} = 0.465\mu_N$ with $0(\frac{J^-}{P^+})$. In Ref. [17], the magnetic dipole moment of $P_{cs}(4459)$ in the molecular and diquark-diquark-antiquark pictures are extracted as $\mu_{P_{cs}} = 1.75\mu_N$ and $\mu_{P_{cs}} = 0.34\mu_N$. These numerical results differ from our results $\mu_{P_{cs}} = -0.531\mu_N$ with $0(\frac{J^-}{P^-})$ and $\mu_{P_{cs}} = -0.231\mu_N$ with $0(\frac{J^-}{P^+})$ in the molecular model and $\mu_{P_{cs}} = 0.223\mu_N$ in diquark-diquark-antiquark model because of the wavefunction and the quark mass. We compare the results in Table [XII].

V. SUMMARY

Inspired by the recently observed $P_{cs}(4459)$, we systematically calculates the magnetic moments of $P_{cs}$ with $J^P = \frac{3}{2}^+, \frac{5}{2}^+, \frac{7}{2}^+$, and $\frac{7}{2}^+$ in three models: molecular, diquark-diquark-antiquark, and diquark-triquark. Comparing the numerical results of the above three models, we observe that the magnetic moments of the states with the same quantum numbers are different. Indeed, even within the same model, the magnetic moments with different configurations are different. Next, we compare the magnetic moment of $P_{cs}(4459)$ in three configurations, which has been predicted involves an S-wave state with $I(J^P) = 0(\frac{J^-}{P^-})$ and $I(J^P) = 0(\frac{J^-}{P^+})$. The result shows that the molecular model is different from the other two models in $I(J^P) = 0(\frac{J^-}{P^-})$. These findings highlight that magnetic moments are helpful to determine their internal structures when the experimental data of $P_{cs}$ keeps accumulating, since the magnetic moments encode information about the charge distributions.

Acknowledgments

This project is supported by the National Natural Science Foundation of China under Grants No. 11905171 and No. 12047502. This work is also supported by the Natural Science Basic Research Plan in Shaanxi Province of China (Grant No. 2022JQ-025).

[1] S. K. Choi et al. [Belle]. “Observation of a narrow charmonium-like state in exclusive $B^\pm \rightarrow K^{\pm}\pi^{\pm}\pi^\mp J/\psi$ decays,” Phys. Rev. Lett. 91, 262001 (2003).
TABLE IX: The magnetic moments of the pentaquark states in the diquark-triquark model with the wave function \( \frac{1}{\sqrt{6}}[(cd)(\bar{c}u(s)) + (cu)(\bar{c}d(s))] - \frac{1}{\sqrt{3}}(s)(\bar{c}ud) \) in \( S_{1/2} \) and \( \frac{1}{\sqrt{6}}[(cd)(\bar{c}u(s)) + (cu)(\bar{c}d(s))] \) in \( S_{1/2} \) with isospin \((I, I_s) = (1, 0)\). They are in \( S_{1/2} \) representation from \( 6 \otimes 3_j = 10 \otimes S_{1/2} \) and \( S_{1/2} \) representation from \( 3_j \otimes 3_j = 1_j \otimes S_{1/2} \), respectively. The third line \( J^{P}_{1} \otimes J^{P}_{2} \otimes J^{P}_{3} \) are corresponding to the angular momentum and parity of triquark, diquark and orbital, respectively. The unit is the magnetic moments of the proton.

| \((Y, I, J_s)\) | \( J^P = \frac{1}{2}^- \) | \( J^P = \frac{3}{2}^+ \) | \( J^P = \frac{5}{2}^- \) |
|------------------|------------------|------------------|------------------|
| \( (0, 1, 0) \)  | 0.522            | -0.078           | 0.051            |
| \( 2 S_{1/2} \)  | 0.015            | 0.666            | 0.178            |
| \( 4 S_{1/2} \)  | 0.188            | 0.352            | 0.088            |
| \( 8 S_{1/2} \)  | 0.354            | 0.088            | 0.242            |

| \((Y, I, J_s)\) | \( J^P = \frac{3}{2}^+ \) | \( J^P = \frac{5}{2}^- \) | \( J^P = \frac{5}{2}^+ \) |
|------------------|------------------|------------------|------------------|
| \( (0, 1, 0) \)  | 0.577            | -0.030           | 0.098            |
| \( 4 P_{3/2} \)  | 0.152            | 0.508            | 0.157            |
| \( 6 P_{3/2} \)  | 0.157            | 0.242            | 0.370            |

| \((Y, I, J_s)\) | \( J^P = \frac{5}{2}^- \) | \( J^P = \frac{5}{2}^+ \) | \( J^P = \frac{5}{2}^+ \) |
|------------------|------------------|------------------|------------------|
| \( (0, 1, 0) \)  | 0.714            | 0.233            | 0.236            |
| \( 4 P_{5/2} \)  | 0.299            | 0.370            | 0.370            |
| \( 6 P_{5/2} \)  | 0.299            | 0.370            | 0.370            |

[2] R. Aaij et al. [LHCb], “Observation of \( J/\psi\phi \) structures consistent with exotic states from amplitude analysis of \( B^+ \rightarrow J/\psi\phi K^+ \) decays,” Phys. Rev. Lett. 118, no.2, 022003 (2017).

[3] M. Ablikim et al. [BESIII], “Evidence of Two Resonant Structures in \( e^+e^- \rightarrow \pi^+\pi^- h_c \)”, Phys. Rev. Lett. 118, no.9, 092002 (2017).

[4] R. Aaij et al. [LHCb], “Evidence for an \( \eta_c(1S)\pi^- \) resonance in \( B^0 \rightarrow \eta_c(1S)K^+\pi^- \) decays,” Eur. Phys. J. C 78, no.12, 1010 (2018).

[5] R. Aaij et al. [LHCb], “Observation of \( J/\psi p \) Resonances Consistent with Pentaquark States in \( \Lambda_b^0 \rightarrow J/\psi K^- p \) Decays,” Phys. Rev. Lett. 115, 072001 (2015).

[6] R. Aaij et al. [LHCb], “Observation of a narrow pentaquark state, \( P_c(4312)^+ \), and of two-peak structure of the \( P_c(4450)^+ \),” Phys. Rev. Lett. 122, no.22, 222001 (2019).

[7] R. Aaij et al. [LHCb], “Evidence for a new structure in the \( J/\psi p \) and \( J/\psi \bar{p} \) systems in \( B^0 \rightarrow J/\psi p \bar{p} \) decays,” arXiv:2108.04720 [hep-ex].

[8] R. Aaij et al. [LHCb], “Evidence of a \( J/\psi\Lambda \) structure and observation of excited \( \Xi^- \) states in the \( \Xi_b^+ \rightarrow J/\psi \Lambda K^- \) decay,” Sci. Bull. 66, 1278-1287 (2021).

[9] C. W. Shen, D. Röchzen, U. G. Meißner and B. S. Zou, “Exploratory study of possible resonances in heavy meson - heavy baryon coupled-channel interactions,” Chin. Phys. C 42, no.2, 023106 (2018).

[10] X. Z. Weng, X. L. Chen, W. Z. Deng and S. L. Zhu, “Hidden-charm pentaquarks and \( P_c \) states,” Phys. Rev. D 100, no.1, 016014 (2019).

[11] G. J. Wang, R. Chen, L. Ma, X. Liu and S. L. Zhu, “Magnetic moments of the hidden-charm pentaquark states,” Phys. Rev. D 94, no.9, 094018 (2016).

[12] H. Huang, J. He and J. Ping, “Looking for the hidden-charm pentaquark resonances in \( J/\psi p \) scattering,” arXiv:1904.00221 [hep-ph].
TABLE XI: The magnetic moments of the P_{c}(4459) in the molecular model, the diquark-diquark-antiquark model and the diquark-triquark model in S_{2f} representation with isospin (I, I_{3}) = (0, 0).

| P_{c}(4459) | Multiplet | Spin-orbit coupling | I(J^P) | Magnetic moment | Numerical results |
|-------------|-----------|---------------------|--------|----------------|------------------|
| Molecular model | 8_{2f} | $\frac{1}{2}^+ \otimes 1^- \otimes 0^+$ | $\frac{3}{2}^+$ | $\frac{3}{2}(6\mu_c - 3\mu_c + \mu_u + \mu_d + 4\mu_s)$ | -0.531 |
| | | | | $\frac{3}{2}(6\mu_c + 6\mu_u + \mu_c + \mu_d + 4\mu_s)$ | -0.231 |
| diquark-diquark-antiquark model | 8_{2f} | $1^+ \otimes 0^+ \otimes \frac{1}{2}^- \otimes 0^+$ | $\frac{3}{2}^+$ | $\frac{3}{2}(6\mu_c - 3\mu_c + \mu_u + \mu_d + 4\mu_s)$ | 0.223 |
| | | | | $\frac{3}{2}(6\mu_c + 6\mu_u + \mu_c + \mu_d + 4\mu_s)$ | -0.231 |
| diquark-triquark model | 8_{2f} | $\frac{1}{2}^- \otimes 1^+ \otimes 0^+$ | $\frac{3}{2}^+$ | $\frac{3}{2}(6\mu_c - 3\mu_c + \mu_u + \mu_d + 4\mu_s)$ | 0.223 |
| | | | | $\frac{3}{2}(6\mu_c + 6\mu_u + \mu_c + \mu_d + 4\mu_s)$ | -0.231 |
TABLE XII: Our results and other theoretical results for the magnetic moment of $P_{cs}(4459)$. The unit is the magnetic moments of the proton. The A, B, and C are corresponding to the molecular model, diquark-diquark-antiquark model and diquark-triquark model.

| Cases          | A     | B     | C     |
|----------------|-------|-------|-------|
| $J^P$          | $\frac{1}{2}^-$ | $\frac{3}{2}^-$ | $\frac{1}{2}^-$ | $\frac{3}{2}^-$ | $\frac{1}{2}^-$ | $\frac{3}{2}^-$ |
| Our results    | -0.531 | -0.231 | 0.223 | -0.231 | 0.223 | -0.231 |
| Ref.[44]       | -0.062 | 0.465 | -     | -     | -     | -     |
| Ref.[47]       | 1.75   | -     | 0.34  | -     | -     | -     |

[13] R. Chen, “Can the newly reported $P_{cs}(4459)$ be a strange hidden-charm $\Xi_c \bar{D}^*$ molecular pentaquark?,” Phys. Rev. D 103, no.5, 054007 (2021).
[14] Z. G. Wang, “Analysis of the $P_{cs}(4459)$ as the hidden-charm pentaquark state with QCD sum rules,” Int. J. Mod. Phys. A 36, no.10, 2150071 (2021).
[15] Z. G. Wang, “Analysis of the $P_c(4312)$, $P_c(4440)$, $P_c(4457)$ and related hidden-charm pentaquark states with QCD sum rules,” Int. J. Mod. Phys. A 35, no.01, 2050003 (2020).
[16] Z. G. Wang and Q. Xin, “Analysis of the hidden-charm pentaquark molecular states with strangeness and without strangeness via the QCD sum rules,” Chin. Phys. C 45, 123105 (2021).
[17] U. Özdem, “Magnetic dipole moments of the hidden-charm pentaquark states: $P_c(4440)$, $P_c(4577)$ and $P_{cs}(4459)$,” Eur. Phys. J. C 81, no.4, 277 (2021).
[18] K. Azizi, Y. Sarac and H. Sundu, “Investigation of $P_{cs}(4459)$ pentaquark via its strong decay to $ΛJ/Ψ$,” Phys. Rev. D 103, no.9, 094033 (2021).
[19] H. X. Chen, W. Chen, X. Liu and X. H. Liu, “Establishing the first hidden-charm pentaquark with strangeness,” Eur. Phys. J. C 81, no.5, 409 (2021).
[20] J. X. Lu, M. Z. Liu, R. X. Shi and L. S. Geng, “Understanding $P_{cs}(4459)$ as a hadronic molecule in the $Ξb$→$J/ψΛK$ decays,” Phys. Rev. D 104, no.3, 034022 (2021).
[21] F. Z. Peng, M. J. Yan, M. Sánchez Sánchez and M. P. Valderrama, “The $P_{cs}(4459)$ pentaquark from a combined effective field theory and phenomenological perspectives,” Eur. Phys. J. C 81, 666 (2021).
[22] K. Chen, B. Wang and S. L. Zhu, “Exploration of the doubly charmed molecular pentaquarks,” Phys. Rev. D 103, no.11, 116017 (2021).
[23] B. Wang, L. Meng and S. L. Zhu, “Deciphering the charged heavy quarkoniumlike states in chiral effective field theory,” Phys. Rev. D 102, 114019 (2020).
[24] B. Wang, L. Meng and S. L. Zhu, “Spectrum of the strange hidden charm molecular pentaquarks in chiral effective field theory,” Phys. Rev. D 101, no.3, 034018 (2020).
[25] X. Hu and J. Ping, “Investigation of hidden-charm pentaquarks with strangeness $S = -1$,” arXiv:2109.00972 [hep-ph].
[26] J. He, “Study of $P_c(4457)$, $P_c(4440)$, and $P_c(4312)$ in a quasipotential Bethe-Salpeter equation approach,” Eur. Phys. J. C 79, no.5, 393 (2019).
[27] R. Chen, Z. F. Sun, X. Liu and S. L. Zhu, “Strong LHCb evidence supporting the existence of the hidden-charm molecular pentaquarks,” Phys. Rev. D 100, no.1, 011502 (2019).
[28] Q. Wu and D. Y. Chen, “Production of $P_c$ states from $Λ_b$ decay,” Phys. Rev. D 100, no.11, 114002 (2019).
[29] H. X. Chen, W. Chen and S. L. Zhu, “Possible interpretations of the $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$,” Phys. Rev. D 100, no.5, 051501 (2019).
[30] M. Pavon Valderrama, “One pion exchange and the quantum numbers of the $P_c(4440)$ and $P_c(4457)$ pentaquarks,” Phys. Rev. D 100, no.9, 094028 (2019).
[31] C. Fernández-Ramírez et al. [JPAC], “Interpretation of the LHCb $P_c(4312)^+$ Signal,” Phys. Rev. Lett. 123, no.9, 092001 (2019).
[32] C. W. Xiao, J. Nieves and E. Oset, “Heavy quark spin symmetric molecular states from $\bar{D}^*(s)\Sigma(s)$ and other coupled channels in the light of the recent LHCb pentaquarks,” Phys. Rev. D 100, no.1, 014021 (2019).
[33] Y. Shimizu, Y. Yamaguchi and M. Harada, “Heavy quark spin multiplet structure of $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$,” arXiv:1904.00587 [hep-ph].
[34] M. Z. Liu, Y. W. Pan, F. Z. Peng, M. Sánchez Sánchez, L. S. Geng, A. Hosaka and M. Pavon Valderrama, “Emergence of a complete heavy-quark spin symmetry multiplet: seven molecular pentaquarks in light of the latest LHCb analysis,” Phys. Rev. Lett. 122, no.24, 242001 (2019).
[35] R. Chen, “Strong decays of the newly $P_{cs}(4459)$ as a strange hidden-charm $\Xi_c \bar{D}^*$ molecule,” Eur. Phys. J. C 81, no.2, 122 (2021).
[36] C. W. Xiao, J. J. Wu and B. S. Zou, “Molecular nature of $P_{cs}(4459)$ and its heavy quark spin partners,” Phys. Rev. D 103, no.5, 054016 (2021).
[37] R. F. Lebed, “The Pentaquark Candidates in the Dynamical Diquark Picture,” Phys. Lett. B 749, 454-457 (2015).
[38] A. Ali, I. Ahmed, M. J. Aslam, A. Parkhomenko and A. Rehman, “Interpretation of LHCb Hidden-Charm Pentaquarks within the Compact Diquark Model,” PoS ICHEP2020, 527 (2021).
[39] V. V. Anisovich, M. A. Matveev, J. Nyiri and A. N. Semenova, “Narrow pentaquarks as diquark–diquark–antiquark systems,” Mod. Phys. Lett. A 32, no.29, 1750154 (2017).
[40] P. P. Shi, F. Huang and W. L. Wang, “Hidden charm pentaquark states in a diquark model,” Eur. Phys. J. A 57, no.7, 237 (2021).
[41] M. Karliner and H. J. Lipkin, “The Constituent quark model revisited: Quark masses, new predictions for hadron masses and KN pentaquark,” arXiv:hep-ph/0307243 [hep-ph].
[42] R. Zhu and C. F. Qiao, “Pentaquark states in a diquark–triquark model,” Phys. Lett. B 756, 259-264 (2016).
[43] J. F. Giron, R. F. Lebed and S. R. Martinez, “Spectrum of hidden-charm, open-strange exotics in the dynamical diquark model,” Phys. Rev. D 104, no.5, 054001 (2021).
[44] M. W. Li, Z. W. Liu, Z. F. Sun and R. Chen, “Magnetic moments and transition magnetic moments of Pc and Ps states,” Phys. Rev. D 104, no.5, 054016 (2021).
[45] G. J. Wang, L. Meng, H. S. Li, Z. W. Liu and S. L. Zhu, “Magnetic moments of the spin-$\frac{1}{2}$ singly charmed baryons in chiral perturbation theory,” Phys. Rev. D 98, no.5, 054026 (2018).
[46] D. Ebert, R. N. Faustov and V. O. Galkin, “Masses of tetraquarks with open charm and bottom,” Phys. Lett. B 696, 241-245 (2011).
[47] U. Özdem, “Magnetic dipole moments of the hidden-charm pentaquark states: $P_c(4440)$, $P_c(4457)$ and $P_{cs}(4459)$,” Eur. Phys. J. C 81, no.4, 277 (2021).