Driven vortices in 3D layered superconductors: Dynamical ordering along the c-axis

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We study a 3D model of driven vortices in weakly coupled layered superconductors with strong pinning. Above the critical force $F_c$, we find a plastic flow regime in which pancakes in different layers are uncoupled, corresponding to a pancake gas. At a higher $F$, there is an “smectic flow” regime with short-range interlayer order, corresponding to an entangled line liquid. Later, the transverse displacements freeze and vortices become correlated along the c-axis, resulting in a transverse solid. Finally, at a force $F_s$ the longitudinal displacements freeze and we find a coherent solid of rigid lines.

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It is well-known that an external current can induce an ordering of the vortex structure in superconductors with pinning [1]. For a long time, it was believed that the high-current phase would have crystalline order. Recently, it has been found that different kinds of order are possible at high currents, depending on pinning strength and dimensionality [2–4]. This has led to numerous theoretical [2–4], experimental [4] and numerical studies [5–7].

A crystal-like structure, which could be either a perfect crystal [2] or a Bragg glass [3], is only possible in a crystal or a Bragg glass [2]. In this paper, we will present a tentative classification of the different phases that can take place in a sequence of dynamical phases upon increasing current.

We study pancake vortices in a layered superconductor, considering the long-range magnetic interactions between all the pancakes and neglecting Josephson coupling [10]. This model is adequate when the interlayer period $d$ is much smaller than the in-plane penetration length $\lambda_\parallel$ [10]. Previous simulations of driven vortices in 3D superconductors have been performed using Langevin dynamics of short-range interacting particles [11] or the driven isotropic 3D XY model [12].

The equation of motion for a pancake located in position $\mathbf{r}_i = (x_i, y_i, n_i d)$, $(\mathbf{z} \equiv c)$, is:

$$\frac{d\mathbf{r}_i}{dt} = \sum_{j \neq i} \mathbf{F}_\nu(\mathbf{r}_{ij}, z_{ij}) + \sum_p \mathbf{F}_\rho(\mathbf{r}_p) + \mathbf{F}$$

where $\rho_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ and $z_{ij} = |z_i - z_j|$ are the interplane and interplane distance between pancakes $i, j$, $\rho_p = |\mathbf{r}_i - \mathbf{r}_p|$ is the in-plane distance between the vortex $i$ and a pinning site at $\mathbf{R}_p = (z_i, \mathbf{z})$, $\eta$ is the Bardeen-Stephen friction, and $\mathbf{F} = \frac{\partial \mathbf{F}_p}{\partial \mathbf{r}} \times \mathbf{z}$ is the driving force due to an in-plane current $\mathbf{J}$. We consider a random uniform distribution of attractive pinning centers in each layer with $\mathbf{F}_p = -2a_p e^{-\left(\rho/\rho_p\right)^2} \mathbf{r}/a_p^2$, where $a_p$ is the pinning range. The magnetic interaction between pancakes $\mathbf{F}_\nu(\mathbf{r}, z) = \mathbf{F}_\rho(\mathbf{r}, z) \cdot \mathbf{r}$ is given by [8,13,14]:

$$F_{\rho}(\rho, 0) = \frac{A_v}{\rho} \left[ 1 - \frac{\lambda_\parallel}{\Lambda} \left( 1 - e^{-\rho/\lambda_\parallel} \right) \right]$$

$$F_{\rho}(\rho, z_\parallel) = \frac{\lambda_\parallel A_v}{\rho} \left[ e^{-|z_\parallel/\lambda_\parallel} - e^{-R_\parallel/\lambda_\parallel} \right].$$

Here, $R = \sqrt{z^2 + \rho^2}$ and $\Lambda = 2\lambda_\parallel / d$ is the 2D thin-film screening length. An analogous model to Eqs. (2-3) was used in [14]. We normalize length scales by $\lambda_\parallel$, energy scales by $A_v = \phi_0^2 / 4\pi^2 \Lambda$, and time is normalized by $\tau = \eta \lambda_\parallel^2 / A_v$. We consider $N_v$ pancake vortices and $N_p$ pinning centers per layer in $N_l$ rectangular layers of size $L_x \times L_y$, and the normalized vortex density is $n_v = B \lambda_\parallel^2 / \Phi_0 = (a_p/\lambda_\parallel)^2$. We consider $n_v = 0.29$ with $L_x = 16\lambda_\parallel$ and $L_y = \sqrt{3}/2L_y$, $N_v = 8$ and $N_p = 64$. We take a pinning range of $a_p = 0.2$, a large pinning strength of $A_p/A_v = 0.2$, with a high density of pinning centers $n_v = 3.125n_v$. The model of Eq.(2-3) is valid in the limit $d \ll \lambda_\parallel \ll \Lambda$. We take $d/\lambda_\parallel = 0.01$, which corresponds to BSCCO compounds [14]. Moving pancake vortices induce a total electric field $\mathbf{E} = \frac{\partial \mathbf{F}_\rho}{\partial \mathbf{r}} \times \mathbf{z}$, with $\mathbf{v} = \frac{\partial \mathbf{F}_\rho}{\partial \mathbf{r}} \cdot \mathbf{z}$. We study the dynamical regimes in the velocity-force curve at $T = 0$, solving Eq. (1) for increasing values of $\mathbf{F} = F \mathbf{y}$. We use periodic boundary conditions both in the planes and in the $z$ direction and interactions between all pancakes in all layers are considered [13]. The periodic long-range in-plane and inter-plane interaction is evaluated using Ref. [13]. The equations are integrated with a time step of $\Delta t = 0.01\tau$. 

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and averages are evaluated in 16384 integration steps after 2000 iterations for equilibration. Each simulation is started at \( F = 0 \) with a triangular vortex lattice and slowly increasing the force in steps of \( \Delta F = 0.1 \) up to values as high as \( F = 8 \).

We start with a qualitative description of the different steady states that arise as a function of increasing force. In Figure 1(a-d) we show the vortex trajectories \( \{ \mathbf{R}_n(t) \} \) for typical values of \( F \) by plotting the positions of the pancakes in five of the layers for all \( t \). In Fig.1(e-h) we show the average in-plane structure factor \( S(\mathbf{k}) = \langle \frac{1}{N} \sum_n | \sum_i \exp[i \mathbf{k} \cdot \mathbf{r}_{ni}(t)] |^2 \rangle \), with \( \mathbf{k} = (k_x, k_y) \). Above the depinning critical force \( F_c \), we find the following dynamical phases. (i) Plastic flow (\( F_p < F < F_c \)): Pancakes flow in an intricate network of “plastic” channels similar to the behavior found in 2D \( \square \). The motion in different planes is completely uncorrelated, [Fig.1(a)] and there is no signature of order in the structure factor [Fig.1(e)]. (ii) Smectic flow (\( F_s < F < F_c \)): The motion organizes in “elastic” channels that are almost parallel and separated by a distance \( \sim a_0 \), see Fig.1(b). Small and broad “smectic” peaks appear in \( S(\mathbf{k}) \) for \( \mathbf{k} \cdot \mathbf{F} = 0 \) [Fig.1(f)]. There are “acti-

![FIG. 1](image1.jpg)

FIG. 1. Vortex trajectories in the first five layers: (a) \( F = 0.6 \), (b) \( F = 1.1 \), (c) \( F = 2.0 \), (d) \( F = 3.9 \). Surface intensity plot of the averaged in-plane structure factor \( S(\mathbf{k}) \): (e) \( F = 0.6 \), (f) \( F = 1.1 \), (g) \( F = 2.0 \), (h) \( F = 3.9 \).

![FIG. 2](image2.png)

FIG. 2. (a) Velocity-force curve, left scale, black points, \( dV/dF \) (differential resistance), right scale, white points. (b) Intensity of the Bragg peaks. For smectic ordering \( S(G_1) \), \( K_y = 0 \), (\( \times \)) symbols. For longitudinal ordering \( S(G_{2,3}) \), \( K_y \neq 0 \), (\( + \)) symbols. (c) Diffusion coefficient for transverse motion \( D_x \), (\( \triangle \)), left scale. Longitudinal displacements \( \langle |\Delta y(t)|^2 \rangle \) for a given \( t \) as a function of \( F \), (\( \blacksquare \)), right scale.

The characteristic forces \( F_c, F_p, F_s, F_t \), separating the different dynamical phases are obtained from the analysis of the in-plane and out of plane structural and dynamical correlations. We show in Fig.2 the in-plane structure factor and temporal fluctuations, obtained in the same way as for 2D \( \square \). In Fig.2(a) we plot the average velocity \( V = \langle V_y(t) \rangle = \langle \frac{1}{N} \sum_n | dy(t) | \rangle \), in the direction of the force as a function of \( F \) and its corresponding derivative.
$dV/dF$ (differential resistance). The force $F_p$ corresponds to the peak in the differential resistance. We also see a small second maximum in $dV/dF$ for a force between $F_t$ and $F_s$. In Fig.2(b) we plot the magnitude of the peaks in the in-plane structure factor. We show the peak height at $G_1 = 2\pi/a_0$, corresponding to smectic ordering, and the average of the peaks corresponding to longitudinal ordering at $G_2 = \pm 2\pi/a_0(1/2, \sqrt{3}/2)$ and $G_3 = \pm 2\pi/a_0(-1/2, \sqrt{3}/2)$. We see that at $F_p$ the smectic peak rises up from zero, then at $F_t$ it reaches an almost constant value and later at $F_s$ it has a small jump. The longitudinal peak has a small finite value for forces above $F_p$, and only at $F_s$ shows a significant increment towards a large value. Comparing with the previous 2D results [5], we can make the reasonable assumption that for $F_p < F < F_t$ there is only short-range smectic order (since there is activated transverse diffusion between elastic channels, see below), for $F_t < F < F_s$ there is probably quasi-long range smectic order but short range longitudinal order, and above $F_s$ there is both transversal and longitudinal order (quasi-long-range or long-range). What is new, compared with the 2D thin film case [7], is that above a force $F_s$ there is a significant amount of longitudinal order. This may correspond either to a moving crystalline phase (if there is long-range order) or to a moving Bragg glass (if there is quasilong-range order) [5]. We have verified that, for a given $F > F_s$, there is both longitudinal and transversal order for system sizes of $N_l \times N_v = 5 \times 36, 5 \times 64, 8 \times 64, 8 \times 100, 10 \times 100$. However, a detailed finite size analysis is not possible with these few small samples. We complement our discussion of the in-plane physics with the study of the temporal fluctuations, which are shown in Fig.2(c). We calculate the transverse diffusion coefficient $D_x$ from the average quadratic transverse displacements of vortices from their center of mass position $(X_n, Y_n)$, \[ \langle (\Delta X_n(t) - \Delta X_n(0))^2 \rangle \approx D_x t. \] We find that $D_x$ is maximum at $F_p$ in coincidence with the peak in $dV/dF$. Below $F_p$ diffusion is through the intricate network of plastic channels, above $F_p$ diffusion is through activated jumps between elastic channels. $D_x$ sharply drops to zero at $F_s$, indicating that transverse displacements are localized in the transverse solid phase [5]. The drift from the center of mass of longitudinal displacements $\langle (\Delta y(t))^2 \rangle = \langle (\delta y(t) - \delta y_n(t) - \delta y_n(0) + \delta y_n(0))^2 \rangle$ is superdiffusive for $F < F_s$, similar to the results observed in 2D films [5]. For $F > F_s$ the longitudinal displacements become frozen in a constant value $\delta y(t)^2 < a_0/N_l$, as it is shown in Fig.2(c). Since in-plane displacements are localized and there are large transversal and longitudinal Bragg peaks, we call this phase a coherent solid.

Let us now discuss how the ordering along the c-axis takes place. We analyze the pair distribution function: \[ g(\rho, n) = \frac{L_x L_y}{N_c} \sum_{i,j \neq 0} \delta(\rho - \rho_{ij}) \delta_{n,n_i}. \] From $g(\rho, n)$ we define a correlation function along c-axis $C_s(n) =$ finite $C_z(n = 1)$, meaning that pancakes in neighboring planes are coupled and a “vortex line” can therefore be defined. In principle, an exponential decay $C_z(n) = \exp(-n/s^c)$ would define a correlation length for the vortex line [6]. On the other hand, long-range ordering will be given by $C_z(n \to \infty) = C_z^c > 0$. In Fig.3(a) we show $C_z(n)$ as a function of $F$ for $n = 1, 2, 3, 4$. We see that at $F_p$ there is an onset of short-range order along the c-axis with a finite $C_z(n = 1)$. At higher forces between $F_p$ and $F_t$ the other $C_z(n > 1)$ start to rise. The absence of correlations for $F < F_p$ means that pancake motion is completely random and uncorrelated between different planes. Therefore, we propose that the plastic flow regime corresponds to a pancake gas. Above $F_p$, in the smectic flow regime, it is possible to define a vortex line with short range correlations along the c-axis. Since there are in-plane jumps between elastic channels (i.e., cutting and reconnection of flux lines) we may consider this phase as an entangled line liquid. Above $F_t$, $C_z(n)$ is finite for all $n$ considered and tends to saturate upon
increasing \( n \). This indicates that vortex lines become more stiff above \( F_1 \). We also analyzed the \( c \)-axis correlation between averaged vortex densities. We first define \( \rho_v(r,n,t) = \frac{1}{N_c} \sum_i \delta(r-r_{ni}(t)) \) taking a coarse-graining scale \( \Delta r = a_0/2 \) (results do not vary much for \( \Delta r = a_0/4 \)). The regions where the average density \( \langle \rho_v(r,n) \rangle \) is large define the paths of steady state vortex motion. We can thereby calculate the overlap function of vortex trajectories between different planes as \( O_n = C_\rho(n)/C_\rho(0) \), with \( C_\rho(n) = \frac{4}{N_c} \sum_i \int \Delta r (\rho_v(r,m,0)|\rho_v(r,m+n,0)|) - 1 \). This is shown in Fig.3(b). We see that \( O_n \) also has an onset at \( F_p \). For \( F_p < F < F_1 \), we have some overlap of the elastic channels that decreases with increasing \( n \), consistent with the entangled liquid-liquid picture. More interestingly, at \( F_1 \) the overlap function \( O_n \) becomes independent of \( n \). This means that there is long-range \( c \)-axis coupling of the path of the elastic channels. When transverse displacements become localized in the \( z \)-direction, they also become locked in the \( c \)-direction. Thus, the freezing of in-plane transverse displacements occurs simultaneously with a transverse disentanglement of flux lines at \( F_1 \). A striking result is that we find \( O_n \approx 1 \) above \( F_2 \), i.e., a perfect \( c \)-axis coupling of elastic channels (within the scale \( \sim a_0/4 \)). Another interesting point to consider is the correlation of vortex velocities. If vortices in different planes move at different velocities, they will induce a Josephson voltage difference along the \( c \)-axis given by \( V_{n,n+1}(r,t) = \frac{\Phi_0}{2\pi e} \phi_{n,n+1}(r,t) \), with \( \phi_{n,n+1} \) the superconducting phase difference between planes \( n \) and \( n + 1 \). A good approximation for pancakes at \( r_{ni} \) is to write \( \phi_{n,n+1}(r,t) = \sum_i[f(r-r_{ni})-f(r-r_{n+1,i})] \) with \( f(r) \approx \arctan(x/y) \). We can therefore estimate the \( c \)-axis voltage fluctuations as \( \langle \delta^2 V_c \rangle = \sum_i \left[ \langle V_{n,n+1}(r,t) \rangle - \langle V_{n,i}(r,t) \rangle \right]^2 \Delta r \approx A \sum_i \left[ \langle V_{n,i} \rangle - \langle V_n \rangle \right]^2 \), where \( A \approx \log(L) \) or \( A \approx \log(L) \). It is clear that \( \langle \delta^2 V_c \rangle = 0 \) for pancakes moving with the same velocity in all planes. We see in Fig.3(c) that the voltage fluctuations have a maximum at \( F_p \). For \( F > F_p \), \( \langle \delta^2 V_c \rangle \) decreases, and above \( F_s \) it reaches an almost \( F \)-independent value. The fact that \( \langle \delta^2 V_c \rangle \) does not vanish above \( F_s \) is consistent with the result that \( C_z(n) < 1 \) for all values of \( F \) in Fig.3(a). In other words, while transverse displacements are strongly correlated along the \( c \)-direction for large forces [Fig.3(b)], the longitudinal displacements in different planes are weakly correlated.

In conclusion, we have clearly distinguished different dynamical phases in 3D layered superconductors considering both in-plane and \( c \)-axis ordering. The onset of short-range \( c \)-axis correlations could be studied experimentally with plasma resonance measurements. The long-range ordering along the \( c \)-axis could be studied through simultaneous measurements of \( \rho_c \) resistivity and in-plane current-voltage response.

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