Successful D-term Inflation
with moduli

Tomohiro Matsuda

National Laboratory For High Energy Physics (KEK)
Tsukuba, Ibaraki 305, Japan

Abstract

We examine a natural extension of D-term inflation and construct a successful model. General type of the D-term potential is shown to produce successful inflation with appropriate COBE normalization. The effect of the dilaton shift at the inflation period is taken into account.
1 Introduction

At present, supersymmetry seems to provide the most likely solution to the problem of the large hierarchy between weak and GUT or Planck scales. As there is yet no direct experimental evidence for supersymmetry, it is worth turning to the early Universe for possible signatures. Many models of supersymmetry breaking involve many flat directions, which means that there exist many particles with weak scale mass and Planck scale suppressed couplings. Coherent production of such particles in the early Universe destroys the successful prediction of nucleosynthesis. As is discussed in ref.[1], this problem may be solved by a brief period of weak scale inflation. This weak scale inflation has already shown to be realized by thermal [2] or parametric resonance[3] models. On the other hand, in general supergravity theories, chaotic inflation suffers from some difficulties. The main reason is that the minimal supergravity potential has an exponential factor which prevents natural realization of chaotic inflation. For example, let us consider superpotential of this form:

\[ W = \frac{\lambda}{3} \Phi^3 \]  

where \( \Phi \) is a gauge singlet superfield in the hidden sector of the theory. The scalar potential for the inflation field is obtained from superpotential as:

\[ V \simeq e^{\frac{\phi}{M_p}} |\lambda \phi|^2 + \text{higher terms} \]  

where \( \phi \) represents the scalar component of \( \Phi \). Demanding that the e-foldings are larger than 60 \((N_e > 60)\), the resulting constraint is

\[ N_e \simeq \frac{8 \pi}{M_p^2} \int_{M_p}^{\phi_0} \frac{V}{V'} d\phi = \frac{\pi}{M_p^2} [\phi_0^2 - M_p^2] > 60 \]  

where we have assumed that chaotic inflation ends at \( \phi \sim M_p \). This means that the initial value of \( \phi \) should be larger than 4 \( \sim 5M_p \). The upper bound for \( \lambda \) is given by the constraint on the Hubble parameter which results from the bound for density fluctuations.

\[ H \leq 10^{14} GeV \]  

As a result, the upper bound for \( \lambda \) is \( e^{16} \lambda^2 \leq 10^{-13} \) i.e. \( \lambda^2 < 10^{-22} \). This bound is about \( 10^{-10} \) times smaller than the one for ordinary chaotic inflation. In addition to this
fine-tuning problem, slow-roll condition is also problematic. To avoid these difficulties, we can choose a potential as:

$$W = m\Phi^2$$  \hspace{1cm} (1.5)

Combined with cosmic strings\[4\], hybrid inflation\[5\] or non-trivial Kähler potential\[6\], we can make successful scenarios. But here we do not mention these another possibilities. Recently, it was proposed that a variant of hybrid inflation combined with a D-term potential can solve these difficulties \[7\]. (We can also find the idea of D-term inflation in \[8\].) For example, let us consider an anomalous $U(1)$ gauge group which is motivated from superstring effective theories\[9\]. The D-term potential for the anomalous $U(1)$ is then

$$V_0 = \frac{g^2}{2} \left| |\phi_+|^2 - |\phi_-|^2 + M^2 \right|^2$$

$$M^2 = \frac{TrQ_A}{192\pi^2} g^2 M_p^2.$$ \hspace{1cm} (1.6)

Here $g$ is the coupling constant for the anomalous $U(1)$ gauge group, $\phi_\pm$ is a scalar field which has charge $\pm 1$ under the anomalous $U(1)$ gauge group and $TrQ_A$ is the sum of the charge of the fields. We can calculate $M$ in some specific models\[4\] and is estimated to be $10^{-1} \sim 10^{-2}$ smaller than Planck mass. In addition to the scalar potential that is motivated from the anomalous $U(1)$ D-term, we should also include superpotential(F-term) which contains a coupling between $\phi_\pm$ and a singlet $\sigma$:

$$W = \lambda \sigma \phi_+ \phi_-$$ \hspace{1cm} (1.7)

and Kähler potential

$$K = \sigma \bar{\sigma} + \phi_\pm \bar{\phi}_\pm.$$ \hspace{1cm} (1.8)

The explicit form of the total scalar potential in the global supersymmetry limit is then

$$V = |\lambda \sigma|^2 (|\phi_+|^2 + |\phi_-|^2) + |\lambda \phi_+ \phi_-|^2$$

$$+ \frac{g^2}{2} (|\phi_+|^2 - |\phi_-|^2 + M^2)^2.$$ \hspace{1cm} (1.9)

This potential has two types of minima, one at $\phi_\pm = 0$ and $\sigma$ large but undetermined, and the other at $|\phi_-|^2 = M^2$ and $|\phi_+| = \sigma = 0$. The former is not a true minimum.
Hybrid inflation occurs for large $\sigma$ and small $\phi_{\pm}$ when the vacuum energy is dominated by the D-term:

$$V_D = \frac{g^2}{2} M^4 \equiv 3H^2 M_p^2. \quad (1.10)$$

Here, Hubble constant $H^2$ is given by $H^2 \sim g^2 M^4 / M_p^2$. In this region, the mass for $\phi_{\pm}$ is estimated to be:

$$m_{\phi_{\pm}}^2 = |\lambda|^2 \sigma^2 \pm g^2 M^2 \quad (1.11)$$

For large $\sigma$, this mass term becomes large and it drives $\phi_{\pm}$ to its minimum at $\phi_{\pm} = 0$ very quickly. After $\phi_{\pm}$ settled down to $\phi_{\pm} = 0$, the mass for $\sigma$ vanishes at the tree level, but is lifted by 1-loop correction which is induced by a mass splitting in the $\phi_{\pm}$ superpartners. The explicit form of 1-loop potential along the flat direction which is parametrized by $\sigma$ is

$$V_{1-loop} = \frac{g^2}{2} M^4 \left(1 + \frac{1}{16\pi^2} \log \frac{\lambda^2 \sigma^2}{\Lambda^2}\right) \quad (1.12)$$

Here $\Lambda$ is the renormalization scale. The mass for $\sigma$ is thus written as:

$$m_{\sigma}^2 = \frac{g^2 M^4}{16\pi^2 \sigma^2}. \quad (1.13)$$

Inflation occurs for $|\sigma| \geq \sigma_c = gM/\lambda$. At the end of inflation $m_{\phi_{\pm}}$ becomes negative and then $\phi_{\pm}$ begins to roll down the potential till it reaches to its true minimum at $\phi_{\pm} = M$. The resulting density fluctuation is

$$\frac{\delta \rho}{\rho} \sim \frac{\lambda g^2 M^5}{M_p^3 m_{\sigma}^2}. \quad (1.14)$$

To obtain a correct order of the density fluctuation $\delta \rho / \rho \sim 10^{-5}$, we should make some unnatural assumptions. We should set $M \sim M_p / 300$ which is about $10^{-2}$ times smaller than the one obtained from a naive consideration. In general one sets $TrQ_A \sim 100$ and $g^2 \sim 1/4$ then obtains $M^2 \sim M_p^2 \times 10^{-2}$, i.e. $M \sim M_p \times 10^{-1}$. Even if we set $TrQ_A = 1$, we obtain at least $M \sim M_p \times 10^{-2}$. We should also note that whenever we deal with a theory of stringy nature, we should inevitably consider the effects of moduli fields. For example, let us consider the dilaton dependence of the gauge coupling constant $g$. The simplest form of $g$ is

$$\frac{1}{g^2} \equiv f(S) = kS \quad (1.15)$$
where $k$ is an integer and $S$ is a dilaton superfield. In the following we set $k = 1$ for simplicity. This dependence is very crucial for the above observation because the D-term potential is now depends on the inverse of $S$, thus it pushes $S$ away to the infinity. The explicit form of the scalar potential is

$$V_D = \frac{1}{2S} M^4.$$  \hspace{2cm} (1.16)

On the other hand, when we consider an inflation scenario with supersymmetry we should also solve the Polonyi problem which comes from the flat directions or almost flat directions in supersymmetric theories. These flat directions or moduli are very common in supersymmetric theories especially in superstring motivated ones. A few years ago, L. Randall and S. Thomas\cite{[1]} have shown that the second weak inflation can solve this problem and in the subsequent papers it was discussed that thermal inflation\cite{[2]} or non-thermal fluctuation induced by parametric resonance\cite{[3]} can be the candidates for this type of weak inflation. We can also find an observation of the Polonyi problem for dilaton and moduli in \cite{[10]}. In this paper we propose a successful scenario for D-term inflation with a dilaton dependent gauge coupling. We also construct a successful chaotic inflation scenario with $\phi^4$ type potential in supersymmetric theories and show that a general type of an anomalous $U(1)$ D-term can naturally induce chaotic inflation. In this model the shift of dilaton at the inflation period plays a crucial role. Of course considering such effects of the moduli shift at the inflation period is very important when we consider a string motivated supergravity theory.

## 2 Successful inflation with a D-term potential

In this section we consider the case in which $g$ in the inflation period is dynamically determined. Here we do not assume that $g$ is always the same value throughout the early stage of the Universe. First, let us consider a naive extension of the above scenario for D-term inflation. As we have noted in the previous section, dilaton field $S$ is always pushed away to larger value at the early stage of inflation. The explicit form of the potential at inflation period is

$$V_D = \frac{M^4}{2S}. \hspace{2cm} (2.1)$$
If the potential for $S$ is not stabilized, $S$ remains large and we can never reach at the real world. Of course, this is a common situation for the string motivated supergravity models and we know that we cannot construct phenomenologically viable models unless we assume that the dilaton is really stabilized by some (unknown) mechanisms. The expected form of the dilaton potential is written in many variants. The simplest form will be

$$V_S = m^4 S^2 \quad (2.2)$$

or

$$V_S = m^2 M_p^2 S^2 \quad (2.3)$$

where $m$ is some intermediate scale $m_w \leq m \leq M_p$. We neglect the imaginary part of the dilaton superfield and simply assume that $S \sim ReS$ because the imaginary part is not important in the following discussions. One may expect many other terms suppressed by $n$ powers of Planck scale, but here we do not consider such complicated extensions of this scenario but instead we consider only one such example:

$$V_S = \frac{[m^3]^2}{M_p^2} S^2. \quad (2.4)$$

For our purpose the potential for large $S$ can be characterized simply by its power of the dilaton field $S$, thus here we do not write down the explicit form of the potential. It seems very natural to expect that the dilaton potential is stabilized by some mechanisms, but the precise form of the potential is still unknown. We may also expect that the dilaton potential is derived from the M-theory motivated models. One instance is recently derived in ref.[11] and shown to take the form:

$$V_S \sim m^4 \frac{|1 - |S|^2}{S + |S|}.$$  \quad (2.5)

This potential behaves like $V_S \sim m^4 S$ for large $S$. We use these potentials and see what happens to the density fluctuation. The total scalar potential for $S$ is now obtained:

$$V_{tot} = V_D + V_S. \quad (2.6)$$

At the early stage of the Universe, when $V_D$ is given by (2.1), $S$ takes the following value for each possible potentials.

$$V_S = m^4 S^2 \, ; \, S \sim \left(\frac{M}{m}\right)^{4/3} \quad (2.7)$$
\[ V_S = \frac{[m^3]^2}{M_p^2} S^2 \quad ; \quad S \sim \left( \frac{M^4 M_p^2}{m^6} \right)^{1/3} \]  
(2.8)

\[ V_S = m^4 S \quad ; \quad S \sim \left( \frac{M}{m} \right)^2 \]  
(2.9)

In any case, \( S \) becomes large in the inflation phase and it makes the density fluctuation smaller than the one derived with an assumption that \( S \) should be fixed at \( S \sim 1 \) throughout the inflation. Let us assume \( Tr Q_A \sim 10^2 \) and \( \lambda \sim 1 \) then the constraint for the density fluctuation is satisfied for \( S \sim 10^{1-2} \). This requirement is satisfied when the intermediate scale \( m \) is comparable to Planck mass \( M_p \), which means that dilaton is stabilized at very high energy scale like the string scale. The assumption that dilaton is stabilized at high energy scale is favorable from phenomenological point of view and sometimes it is utilized without mentioning the specific mechanism. However, as we have shown above, the shift of the dilaton field at inflation is important even if we assume that the dilaton potential is stabilized at high energy scale. For the second example, let us consider an alternative of D-term inflation and show that small \( m \) is also available for successful inflation. Here we consider an anomalous U(1) gauge group again. The D-term potential and superpotential are the same as before. The total scalar potential is

\[ V = |\lambda \sigma|^2 (|\phi_+|^2 + |\phi_-|^2) + |\lambda \phi_+ \phi_-|^2 \\
+ \frac{g^2}{2} (|\phi_+|^2 - |\phi_-|^2 + M^2)^2 \\
+ V_S. \]  
(2.10)

Here we consider a case when chaotic inflation starts with large \( \phi_+ \) or \( \phi_- \). This is possible when \( g \) is small (i.e. \( S \) is pushed away to very large value) and \( \phi_+ (\phi_-) \) is \( O(M_p) \) before inflation. Unlike ordinary supersymmetric chaotic inflation scenarios, this potential does not have any exponential factor that makes it difficult to incorporate chaotic inflation with \( \phi^4 \) type potential. The upper bound for the coupling constant \( g^2 \) is the same as ordinary chaotic inflation and is required to be \( g^2 \sim O(10^{-13}) \) i.e. \( S \sim 10^{13} \). In this case, \( V_D \) is

\[ V_D \sim \frac{(5M_p)^4}{2S}. \]  
(2.11)
The required value for $S$ ($S \sim 10^{13}$) is obtained if $m$ takes the following value for each possible dilaton potential.

$$V_S = m^4S^2 \ ; \ m \sim 10^8\text{Gev} \quad (2.12)$$

$$V_S = \frac{[m^3]^2}{M_p^2}S^2 \ ; \ m \sim 10^{11}\text{Gev} \quad (2.13)$$

$$V_S = m^4S \ ; \ m \sim 10^{11}\text{Gev} \quad (2.14)$$

We can relate these scales to the supersymmetry breaking scale or the scale of the fifth dimension[11]. Let us see whether the second weak inflation can be realized in these models. Generally, the second weak inflation is expected to be realized by some flat directions thus we focus attention to the question whether the second stage of inflation can be realized in some flat directions in the theory. We know that the second stage of thermal inflation can take place in common supersymmetric models[2] without any additional potential, but sometimes it is insufficient. What we want to see here is whether a minimal set of the D-term potential given above can help weak scale inflation. For this purpose, let us consider weak scale inflation driven by parametric resonance[3]. The first model described above is not suitable for this purpose, thus we consider only the second one. Here we consider a case where some flat directions in the theory couples to $\sigma$ through

$$V(X_F, \sigma) = \frac{m_{soft}^2}{2}X_F^2 + \lambda'^2X_F^2\sigma^2 + \text{higher terms.} \quad (2.15)$$

where $X_F$ represents some flat directions and $m_{soft}$ is the soft breaking mass. According to [3], these potentials (2.10) and (2.13) can produce an additional period of non-thermal inflation. The Universe expands in the ratio

$$\frac{a_c}{a_0} \sim \left(\frac{\lambda'}{g^2}\right)^{1/4}. \quad (2.16)$$

In our case, we obtain $a_c/a_0 \sim 10^3$ for $\lambda' \sim O(1)$ which is enough to solve the Polonyi problem. However, a serious problem arises when we consider the oscillation of the dilaton. As a result, the same problem will be there for the dilaton which should be solved again by thermal inflation. However, it seems very interesting that the problem is reduced to the displaced dilaton which sometimes plays specific roles. We hope that the mechanism stated above helps to solving the Polonyi problem, but for now a further stage of thermal inflation is still required.
3 Conclusion

We have examined a natural extension of D-term inflation and constructed a successful model. The effect of the dilaton shift at the inflation period is taken into account. It is also important to note that the model we have constructed requires no fine tuning of parameters to realize chaotic inflation.

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