Defying the laws of Gravity I: Model-independent reconstruction of the Universe expansion from growth data

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ABSTRACT

Using redshift space distortion data, we perform model-independent reconstructions of the growth history of matter inhomogeneity in the expanding Universe using two methods: crossing statistics and Gaussian processes. We then reconstruct the corresponding history of the Universe background expansion and fit it to type Ia supernovae data, putting constraints on $\Omega_{m,0}$, $\sigma_8$, and $\Lambda$. The results obtained are consistent with the concordance flat-$\Lambda$CDM model and General Relativity as the gravity theory given the current quality of the inhomogeneity growth data.

Key words: cosmological parameters – large-scale structure of Universe – cosmology: observations – cosmology: theory – gravitation

1 INTRODUCTION

The discovery at the end of last century of the late-time accelerated expansion rate of the Universe raised the question of its physical cause. There are two great avenues towards solving this problem: either that it is due to an unknown new physical component dubbed (physical) dark energy, or its origin lies in a modification of the laws of gravity (e.g., Sahni & Starobinsky 2000; Yoo & Watanabe 2012; Clifton et al. 2012). However, they represent particular cases of a more general situation like in scalar-tensor gravity, when both a new physical field is introduced for dark energy description and gravity is modified, too (see e.g. Boisseau et al. 2000; Copeland et al. 2006; Sahni & Starobinsky 2006).

In the concordance model the role of dark energy is played by a cosmological constant $\Lambda$ while gravity is described by Einstein’s theory of General Relativity (GR). While GR has been remarkably successful to explain observations in the Solar system (see e.g. Gammouji et al. 2006), its successful extrapolation to much larger cosmic scales remains unclear. Modified gravity models with modifications of gravity on cosmic scales are not excluded and could well be the solution to the recent acceleration of the Universe. The nature of dark energy and therefore also the correct model of gravity are burning issues of cosmology and theoretical physics in general.

The large-scale structures of the Universe are an ideal laboratory to test gravity, and to distinguish between physical dark energy and modified gravity (which may be also called geometrical dark energy as in Sahni & Starobinsky 2006). In particular, redshift-space distortion (RSD) due to galaxy peculiar velocities can be used to estimate the growth factor $f$, which is a key to understanding gravity. For a flat-FLRW Universe with dark energy as a perfect fluid with equation of state $w(z)$, the expansion history $h(z) = H(z)/H_0$ is described by

$$h^2(z) = \Omega_{m,0}(1 + z)^3 + (1 - \Omega_{m,0}) \exp \left( 3 \int_0^z \frac{1 + w(u)}{1 + u} du \right).$$

In GR, the evolution of the matter overdensity $\delta(z) = (\rho(z) - \bar{\rho}(z))/\bar{\rho}(z)$ is governed in the Newtonian approximation by

$$\ddot{\delta} + 2H\dot{\delta} = \frac{3}{2} H^2 \Omega_m \dot{\delta},$$

where dot stands for a derivative with respect to cosmic time $t$, and $\Omega_m(z) = \Omega_{m,0}(1 + z)^3/h^2(z)$ is the matter density normalized by the critical density. From eqs. (1) and (2), it is clear that changing the expansion will also affect the growth evolution. In fact, Starobinsky (1998) showed that the Universe expansion history $H(z)$ can be also reconstructed from $\delta(z)$ unambiguously in this case (the situation becomes more complicated in scalar-tensor gravity Boisseau et al. 2000).

In this paper, we aim first to reconstruct the growth history from data, and then to use it in order to deduce the expansion history (assuming GR) and to compare it with the supernovae data. We should note here that there are two different reconstructions involved here: a statistical reconstruction of the growth factor $f(z)$ from observational data on one hand, and on the other hand, the theoretical re-
construction of the background expansion $H(z)$ from $f(z)$. A similar approach was recently applied in Yin & Wei (2018). The theoretical reconstruction is described in § 2, and the statistical reconstructions together with the results are presented in § 3.

Our conclusions are drawn in § 4. We validate the method on mock data in § A.

2 METHOD

2.1 Theoretical Framework

From eq. (2), using that for any function $x(t)$,
\[
\dot{x} = H \frac{dx}{dt a} = -H \frac{dx}{dt(1+z)},
\]
one obtains
\[
\frac{d^2 \delta}{dt^2 a^2} + \left( \frac{2}{H(z)} \frac{dh}{dt a} \right) \frac{d\delta}{dt a} = \frac{3}{2} \Omega_m(z) \delta.
\]

It is convenient to introduce the growth factor
\[
f = \frac{dt a}{dt} = \Omega_m(a),
\]
where the last equality defines the growth index $\gamma$. In general, $f = f(k, z)$ and therefore $\gamma = \gamma(k, z)$ (e.g., Gannouji et al. 2009). However, in GR and for dust-like matter, $\gamma$ is $k$-independent and has weak dependence on $z$, so that $\gamma(z) \approx 0.55$, with a slight dependence on the equation of state of dark energy $w$ and the matter density parameter $\Omega_m(z)$. Note, however, that $\gamma$ may not be exactly $z$-independent for quintessence (a scalar field with a potential minimal coupled to gravity) models of dark energy as was shown in Polarski et al. (2016).

Observational data on RSD provide us with the product $f\sigma_8$, where
\[
f\sigma_8 \equiv \frac{d\sigma_8}{dt a} = \Omega_m(a),
\]
where $\sigma_8^2(z) = \frac{1}{2\pi^2} \int_0^\infty P(k, z)W_R^2(k)k^2 dk \propto \delta^2(z)
\]
is the rms of the density fluctuations smoothed over a radius $R$, usually taken to be 8 $h^{-1}\text{Mpc}$.

In Shafieloo et al. (2018) (see also L’Huillier et al. 2018), we used Pantheon and a compilation of mock data to put model-independent constraints on $(\Omega_m, \sigma_8, \gamma)$, where subscript 0 stands for the current value. However, in that paper, we treated $\gamma$ as a constant, effectively performing a consistency test of GR. In fact, one can solve the problem without assuming $\gamma = constant$. Assuming $\delta$ and $\delta'$ are known, the expansion history $h$ can be uniquely determined via (Starobinsky 1998)
\[
h^2(z) = \frac{1 + z}{\delta'(z)} \left( \frac{\delta'^2 - 3 \Omega_m(z)}{\int_0^\infty \delta'(u) \left| \frac{du}{1+u} \right|} \right),
\]
where $'$ denotes a derivative with respect to $z$ (not $ln a$).

Thus, one can obtain from RSD measurements
\[
\frac{\delta'(z)}{\delta_0} = -\frac{1}{2} \left( 1 + z \right) \frac{f\sigma_8(z)}{\sigma_8,0}, \quad \text{and}
\]
\[
\delta(z) = \delta_0 \left( 1 - \frac{1}{\delta_8,0} \int_0^z f\sigma_8(u) \frac{du}{1+u} \right).
\]

Therefore, for a given reconstruction $f\sigma_8(z)$ together with a given $(\Omega_m, \sigma_8, \gamma)$, the expansion history $h(z)$ is uniquely determined by
\[
h^2(z) = \frac{1 + z}{\delta_8,0} \left( 1 - \frac{1}{\delta_8,0} \int_0^z f\sigma_8(u) \frac{du}{1+u} \right).
\]

Therefore, reconstructing $f\sigma_8(z)$, and exploring the $(\Omega_m, \sigma_8, \gamma)$ parameter space, we can reconstruct $h(z)$ and the luminosity distance. In the following, we assume a flat-FLRW universe, which is consistent with current model-dependent (Farooq & Ratra 2013; Planck Collaboration et al. 2018) and model-independent (Rässänen 2014; Rässänen et al. 2015; L’Huillier & Shaefieloo 2017; Denissenya et al. 2018; Shafieloo et al. 2018) constraints. The luminosity distance is thus
\[
d_L(z) = \frac{c}{H_0} (1+z) \int_0^z \frac{dx}{h(x)},
\]
and the corresponding distance modulus
\[
\mu(z) = 5 \log_{10}(d_L/1 \text{ Mpc}) + 25,
\]
can then fit the reconstructed $\mu$ to SNIa data, obtain a $\chi^2$, and hence put model-independent constraints on $(\Omega_m, \sigma_8, \gamma)$.

2.2 Fit to the data

After obtaining $f\sigma_8$ from GR, we then obtain the expansion history $h$ via eq. (8), for a given choice of $(\Omega_m, \sigma_8, \gamma)$. We note that, a priori, for a given $(f\sigma_8, \Omega_m, \sigma_8, \gamma)$, there is no guarantee for $h^2(z)$ to be larger than the matter term $\Omega_m(1+z)^2$. Therefore, hereafter, we impose the positive dark energy condition that
\[
\Omega_{de}(z) = h^2(z) - \Omega_m(1+z)^2 \geq 0 \quad \forall z.
\]

Finally, calculate the $\chi^2$ for all the data:
\[
\chi^2_{tot} = \chi^2_{f\sigma_8} + \chi^2_{\text{SNIa}}.
\]

3 RESULTS

We used the same data sets as in Shafieloo et al. (2018): the Pantheon compilation (Scollnic et al. 2018), and the compilation of RSD data including: 2dFGRS (Song & Percival 2009), WiggleZ (Blake et al. 2011), 6dFGRS (Beutler et al. 2012), VIPERS (de la Torre & Peacock 2013), the SDSS Main galaxy sample (Howlett et al. 2015), 2MTF (Howlett et al. 2017), BOSS DR12 (Gil-Marín et al. 2017), FastSound (Okumura et al. 2016), and eBOSS DR14Q (Zhao et al. 2019).
Figure 1. 68% and 95% confidence area of the posterior of the cosmological parameters \((\Omega_m, 0, \sigma_8, 0)\) and the Chebyshev coefficients. The dashed lines show the \(C_0 = 1\) and \(C_1 = 0\), i.e., no deviation from the best-fit.

### 3.1 Crossing statistics

In this section, we study the effects of distorting the mean function on the final fit. In practice, this is equivalent to applying the Bayesian interpretation of the crossing statistics formalism to the reconstructed growth history (Shafieloo et al. 2011; Shafieloo 2012a,b). In this formalism, the prediction from the theory to be tested, \(\Lambda\text{CDM}+\text{GR}\) in the present case, is multiplied by some hyperfunction \(T_N(x|C_0, \ldots, C_N)\):

\[
\hat{f}_{\sigma_8}(x) = (f_{\sigma_8})^{\Lambda\text{CDM}}(x) \times T_N(x|C_0, \ldots, C_N),
\]

(16)

where

\[
T_N(x|C_0, \ldots, C_N) = \sum_l^N C_l P_l(x),
\]

(17)

\[
x = 2 \left( \frac{z}{z_{\text{max}}} - \frac{1}{2} \right),
\]

(18)

\(P_l(x)\) is the \(i\)th order Chebyshev polynomial of the first kind, and \(C_i\) are free parameters. The Chebyshev polynomials constitute an orthogonal basis for \(x \in [-1,1]\), and as such, can represent any function. The zeroth order controls the absolute scaling, the first order the tilt, and higher order in-
introduce a curvature and inflexion points. In the Bayesian interpretation of the crossing statistics, we are interested in the confidence intervals around the hyperparameters $C_i$. If the $C_i$ are consistent with $C_0 = 1, C_i = 0 (i \geq 1)$, the data have no preference for any departure from the mean function, meaning the model is consistent with the data. In case of significant deviation from $C_0 = 1, C_i = 0 (i \geq 1)$, the data suggest a preferred deviation from the mean function.

Distorting the starting $f_{\sigma_8}$, we obtain $\hat{\mu}$ and fit both $\hat{\mu}$ and $f_{\sigma_8}$ to the data. As noted by Hazra & Shafieloo (2014), going towards high orders, one might miss the effects of the lower orders. Therefore, we start by limiting to the first order, i.e., tilting the mean function. We used the emcee Monte-Carlo Markov Chain (MCMC) package to explore the parameter space $(\Omega_{m,0}, \sigma_{8,0}, C_0, C_1)$, and show the posteriors in Fig. 1. They are consistent with $C_0 = 1, C_1 = 0$, i.e., the data suggest no modifications, and are perfectly consistent with the best-fit $\Lambda$CDM+GR model. It is interesting to notice that the preferred $\Omega_{m,0}$ is rather high with respect to the Planck value, while the preferred $\sigma_{8,0}$ is low. We checked that when going to higher orders in the crossing functions, i.e., including $C_2$ and $C_3$, the contours and do not suggest further modifications (i.e., is still consistent with $C_0 = 1, C_{i>0} = 0$).

Fig. 2 shows a random selection of 16 crossing functions and their effects on the reconstructed $\mu$ and on $(\Omega_{m,0}, \sigma_{8,0})$. Their corresponding crossing parameters $C_i$ are shown as coloured points in Fig. 1. This gives the reader some intuition on how distorting the growth affects the reconstructed expansion. Small distortions from the best-fit $\Lambda$CDM+GR case can lead to significantly different $\hat{\mu}$.

### 3.2 Gaussian Process regression

We used Gaussian Process regression (GP, Rasmussen & Williams 2006) to reconstruct $f_{\sigma_8}(z)$ from the RSD measurements. GP have been increasingly used in cosmology (Holsclaw et al. 2010a,b, 2011; Shafieloo et al. 2012, 2013; Joudaki et al. 2018; L’Huillier et al. 2019) and other fields of astronomy (e.g., Iyer et al. 2019). A Gaussian process is effectively a random sampling on a function space, generalizing random numbers. GP can be used to reconstruct a smooth function $f_\ast$ at the test points $x_\ast$ given a discrete set of observations $(x_i, y_i)$ and a data covariance matrix $C$. For a given kernel, the covariance between pairs of random variables $u$ and $v$ is thus given by $K((u), (v)) = k(u, v)$, where $k(u, v)$ is the covariance kernel. The joint-distribution of the training (observed) outputs $y$ and the test (reconstructed) output $f_\ast$ is a Gaussian joint distribution given by

$$
\begin{bmatrix}
y \\
f_\ast
\end{bmatrix} \sim N \left( \begin{bmatrix} \mathbf{0} \\ \mathbf{K}(X, X) + C \end{bmatrix}, \begin{bmatrix} \mathbf{K}(X, X) & \mathbf{K}(X, X_\ast) \\ \mathbf{K}(X_\ast, X) & \mathbf{K}(X_\ast, X_\ast) \end{bmatrix} \right)
$$

where $C$ is the covariance of the data.

We use the squared exponential kernel defined as

$$
k_{\sigma_f, \ell}(x, y) = \sigma_f^2 \exp \left( -\frac{|x - y|^2}{2\ell^2} \right),
$$

where $(\sigma_f, \ell)$ are two hyperparameters controlling the amplitude and the correlation scale, and thus the deviation from the mean function. For a given $(\sigma_f^2, \ell)$, we can thus generate a number of samples of $f_{\sigma_8}$ at any redshift $z$.

In practice, we start from the best-fit $\Lambda$CDM as a mean function, use GP as a sampling of possible growth histories, and then apply the formalism from § 2.

In order to prevent fitting the noise, we impose a hard prior on $\ell \in [0.2, 1]$.

A notable difference with the work of Yin & Wei (2018) is in the GP regression itself. While they obtain the mean and one-sigma contours, we follow each individual random sampling of the function space, therefore obtaining a set of plausible (and self-consistent) couples of expansion and growth histories.

The left-hand panel of Fig. 3 shows the reconstructed
The positive dark energy condition (14) ensures that, for at least one reconstructed \( h \), yields \( \chi^2 < \chi^2_{\text{ref}} + 1 \), i.e., which is within the 1σ region of the best-fit \( \Lambda \)CDM model. In Appendix A, we demonstrate the validity of the algorithm on a simulated data set, successfully recovering the input cosmology.

It is worth noting that, since we need to assume a value for \( \Omega_{m,0} \), in order to reconstruct \( h \), we can then obtain the (uniquely defined) equation of state \( w(z) \) and growth rate \( \gamma(z) \), as

\[
\begin{align*}
  w(z) &= \frac{2(1+z)^2}{1-\Omega_m(z)} - 1, \\
  \gamma(z) &= \frac{\ln f(z)}{\ln \Omega_m(z)}.
\end{align*}
\]

The positive dark energy condition (14) ensures that \( \Omega_m(z) \geq 0 \). Due to the low quality of the current growth data, the constraints are very poor. We note that, since two consecutive integrations are needed to obtain \( h \), then in theory only one is needed to obtain \( h' \), making this method potentially less sensitive to numerical noise, provided that the quality of the \( f_\sigma_8 \) data improve substantially. Equivalently, we can reconstruct

\[
\begin{align*}
  f_{\text{de}}(z) &= \frac{\Omega_{\text{de}}(z)}{\Omega_{\text{de}}(0)} = \exp \left[ 3 \int_0^z \frac{1 + w(x)}{1 + x} \, dx \right],
\end{align*}
\]

does not involve derivatives of \( h \). Fig. 5 shows our obtained reconstructions of \( f_{\text{de}}(z) \) and \( \gamma(z) \). The quality of current data does not constrain these function beyond \( z \geq 0.1 \).

It is interesting to compare these obtained model-independent constraints on \( \Omega_{m,0}, \sigma_8 \) (Fig. 4 with those obtained in Shafieloo et al. (2018), where the approach was different. In that paper, the authors reconstructed \( h(z) \) from the Pantheon SNIa compilation via iterative smoothing, and then obtained \( f_\sigma_8(z) \) by assuming a constant \( \gamma = 0.55 \) (or varying it as a free, but constant, parameter), and obtain \( f = \Omega_m(z)^\gamma \).
4 SUMMARY & CONCLUSION

Using the latest compilation of RSD measurements, we reconstruct the growth history using two model-independent approaches, namely, crossing statistics and Gaussian processes, only assuming a flat-FLRW Universe and general relativity. We then used the method introduced by Starobinsky (1998) to reconstruct the expansion history, and fit the corresponding distance moduli to the Pantheon SNIa compilation, and finally obtained model-independent constraints on $\Omega_{m,0} - \sigma_8$ (right). In addition, it is possible to reconstruct the dark energy equation of state $w(z)$ and the growth rate $\gamma(z)$. Applying the crossing statistics formalism, i.e., multiplying the best-fit $\Lambda$CDM+GR growth by some hyperfunction, and obtained constraints on $\Omega_{m,0}, \sigma_8, C_i$, where the $C_i$ are hyperparameters of the crossing hyperfunctions, we find consistency with $C_0 = 1, C_i = 0, i \geq 1$, i.e., the data do not call for any modification to the best-fit. However, the preferred values for $\Omega_{m,0} = 0.381 \pm 0.049$ and $\sigma_8 = 0.68 \pm 0.15$ are respectively higher and lower than the Planck best-fit, although consistent with them. Using Gaussian processes gives similar results. Both approaches suggest no departure from $\Lambda$CDM+GR. However, future surveys such as the Dark Energy Spectroscopic Instrument are expected to provide more accurate measurements of the growth, and thus, to further constrain the gravity model and its parameters. This approach can be thought of as the reciprocal approach of Higdon (2010a, 2010b), which uses direct reconstructions of $h$ from the supernovae data to fit the RSD data. Both approaches can be thought of as a mutual consistency test of the data and theory: in the former paper, the reconstructed expansion histories are tested against the growth data, while here, the reconstructed growth is compared to the SNIa data.

Our analyses point towards the consistency of the reconstructed $\Lambda$CDM background evolution (via the mean function) with the growth history inside GR. While in GR, $\gamma = 0.55$ is a very good approximation as long as $\Omega_{m,0}$ is not too small (Polarski et al. (2016)), it is not valid anymore beyond GR. On the contrary, the present approach, can be applied to non-GR models provided that $G_{eff}$ is known, or can even be used to reconstruct the effective Newton constant $G_{eff}$ (L’Huillier et al. in prep), and therefore to constraining modified gravity.

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Figure 5. Model-independent reconstructions of $\Omega_m(z)$ (left) and $\gamma(z)$ (right).
In order to validate the method, we applied it to a controlled simulated realization of the data, where the input cosmology is known. We generated mock data, following the redshift distribution and errors following the data in Shafieloo et al. (2018), assuming a known cosmology of \((\Omega_m,0,\sigma_8,0) = (0.3,0.8)\). We fit a flat-\Lambda CDM universe to the total (RSD+SNIa) data, and use this best-fit \Lambda CDM model as a mean function for the GP, and use its \(\chi^2\) as a reference. Hereafter, we use subscript \text{ref} to denote the best-fit model.

We applied our pipeline, reconstruct \(f\sigma_8\) and \(h\), and calculate the \(\chi^2\) to the data (mock growth and Pantheon-like SNIa). Fig. A1 is the same as Fig. 3 but with our simulated data. In addition to the best-fit in red, the true cosmology is shown in green. Fig. A2 shows the allowed contours for \((\Omega_m,0,\sigma_8,0)\), and the true cosmology in green is inside the contours, showing the validity of the method.

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Figure A1. Reconstructions of $f\sigma_8$ (left), $h^2$ (middle), and $\mu - \mu_{\text{ref}}$ (right) for the simulated data. The shadowed area show the envelope of the reconstructions, and the red lines are those reconstructions with yield a $\chi^2$ better than $\chi^2_{\text{ref}} + 1$. The best-fit ΛCDM is shown in orange, and the true cosmology in purple.

Figure A2. Model-independent 1σ contours of $(\Omega_{m,0}, \sigma_{8,0})$. 