Medium polarization effects in $^{3}S_{1}$ spin-triplet pairing

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Stimulated by the still puzzling competition between spin-singlet and spin-triplet pairing in nuclei, the $^{3}S_{1}$ neutron-proton pairing is investigated in the framework of BCS theory of nuclear matter. The medium polarization effects are included in the single particle spectrum and also in the pairing interaction starting from the $G$-matrix, calculated in the Brueckner-Hartree-Fock approximation. The vertex corrections due to spin and isospin collective excitations of the medium are determined from the Bethe-Salpeter equation in the RPA limit, taking into account the tensor correlations. It is found that the self-energy corrections confine the superfluid state to very low-density, while remarkably quenching the magnitude of the energy gap, whereas the induced interaction has an attractive effect. The interplay between spin-singlet and spin-triplet pairing is discussed in nuclear matter as well as in finite nuclei.

I. INTRODUCTION

For several decades the strong experimental evidence of the spin-singlet pairing between like nucleons in nuclei has been stimulating intense theoretical activity until the recent years [1]. On the contrary, there is not yet a clear evidence for neutron-proton (np) spin-triplet pairing. This is the reason why this kind of pairing has received much less attention [2–4]. However, it is well known since long that the $T=0$ np interaction could give relevant pairing, being more attractive than the $T=1$ interaction [5]. In recent calculations on the competition between spin-singlet and spin-triplet pairings in N=Z nuclei it has been argued that the latter is hindered by the spin-orbit splitting [6–8]. However, in Ref. [6] it is pointed out that in very large N=Z nuclei ($A>140$) spin-triplet pairing condensates are favored because the spin-orbit force becomes vanishing small. The disappearance of the $S=1$, $T=0$ pairing with asymmetry in nuclei has been studied in, e.g., Ref. [9]. In those calculations no dynamical effects on pair correlations are considered, whereas it is proved that particle-vibration coupling could yield a significant contribution to the pairing gap magnitude in the spin-singlet case [10] and also in the neutron-proton spin-triplet one, even if less significant [11]. On the other hand, in the vicinity of the proton drip in heavier nuclei the spin-triplet pairing could potentially also become more important. This may be revealed by further theoretical and experimental investigations.

Studies of neutron-neutron (nn) and proton-proton (pp) pairing in nuclear matter have also addressed the medium collective excitations [12–13], which can enhance or quench the the pairing correlations according to the nuclear environment where the Cooper pairs are embedded. In the case spin-singlet nn pairing in symmetric nuclear matter the medium-induced interaction significantly enhances the gap, supporting calculations of energy gaps in $^{1}S_{0}$ neutron-neutron (nn) or proton-proton (pp) spin-singlet pairing in nuclei, where pair vibrations are included [10].

In the case of spin-triplet np pairing BCS calculations with bare interaction in nuclear matter predict sizable energy gaps of the order of 12 MeV, i.e. four times that of the spin-singlet [14]. Even if significant re-scaling is expected from the self-energy effects, the energy gap could be still large enough by anti-screening due to the induced interaction [12]. Therefore, the predicted effect of the spin-orbit energy splitting could be resized by the large spin-triplet pair correlation energy.

In this paper we discuss the $^{3}S_{1}$ spin-triplet np pairing in symmetric nuclear matter, taking into account both self-energy insertions to the quasi-particle spectrum and vertex corrections to the bare interaction due to collective excitations of the medium. The vertex corrections have been determined from the RPA version of the Bethe-Salpeter (BS) equation in the Landau limit. However the RPA does not consider the feedback of the effective interaction on the collective modes, i.e. dressing the polarization propagator with the full interaction in a self-consistent procedure [15, 16].

As leading term the Brueckner-Hartree-Fock (BHF) $G$-matrix is adopted to prevent the divergences due to hard-core of the nuclear force. The strong tensor force present in the bare interaction deeply affects the $G$-matrix so that it cannot be neglected. This entails that the tensor parameters must be included in the effective interaction, solution of the BS equation, when expressed in terms of Landau-Migdal parameters [17]. The resulting energy gap will be compared with the $^{1}S_{0}$ spin-singlet nn (or pp) gap and estimates, based on the local density approximation (LDA) will be made for the gaps in nuclei.

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II. THEORETICAL FRAMEWORK

A. Gap equation

In this section, the formalism of the BCS theory of the $^3\!S_D$ superfluid state of symmetric nuclear matter is set, including the medium polarization effects \[14, 18\]. The two coupled gap equations $(L = 0, 2)$ are written as

$$\Delta_{ST}^L(k) = -\frac{Z_F^2}{\pi} \int_0^\infty k'^2 dk' \sum_{L'} \frac{V_{ST}^{LL'}(k,k')}{\sqrt{\varepsilon_k^2 + \Delta(k')^2}} \Delta_{ST}^{L'}(k'),$$  \hspace{1cm} (1)

where

$$\Delta(k)^2 = \Delta_{ST}^0(k)^2 + \Delta_{ST}^2(k)^2,$$  \hspace{1cm} (2)

The prefactor $Z_F$ is the quasi-particle strength which takes into account the depletion of the Fermi surface \[19\]. The quasi-particle spectrum is given by

$$E_k^2 = (\varepsilon_k - \varepsilon_F)^2 + \Delta(k)^2,$$  \hspace{1cm} (3)

where $\varepsilon_k = k^2/2m^* + U_0$ is the single-particle energy in the effective mass approximation (EMA) and $U_0$ is mean field potential. $\varepsilon_F$ is the Fermi energy. In a consistent approach the gap equation has to be coupled to the conservation of the particle number

$$\rho = 4 \sum_k \frac{1}{2} \left[1 - \frac{\varepsilon_k - \varepsilon_F}{E_k}\right],$$  \hspace{1cm} (4)

The pairing force, in principle, contains all irreducible interaction diagrams, but here only the NN bare interaction and the medium polarization insertions will be considered, as displayed in Fig. 1. The bare two-particle interaction is

$$V_{jst}(k, k') = N_0^{-1} \sum_{lm, s's'z'} \langle \hat{k} | Y_{lm}^2 \hat{k'} | j j z \rangle C(lm, ss'z') C(l'm', ss'z') \langle j j z | V_{jst}^{ll'}(k, k') | j j z \rangle,$$  \hspace{1cm} (5)

where $j$, $s$ and $t$ are total angular momentum, spin and isospin.

![FIG. 1: Pairing interaction with screening: the first term in the r.h.s. is the bare interaction, the second one is the induced interaction, where the dashed bubble insertion is the series of ring diagrams.](image)

B. Induced interaction in RPA

In this section, we discuss the derivation of the vertex corrections to the pairing interaction from the BS equation in the RPA limit \[20\]

$$\mathcal{F}_{SMSM'}^T(k, k'; q) = G_{SM}^T(k, k'; q) + \sum_{MP'} \frac{\delta^4 k''}{(2\pi)^4} \times G_{SM}^T(k, k''; q) \lambda(k'', q) \mathcal{F}_{SM'SM'}^T(k'', k'; q),$$  \hspace{1cm} (6)

where the $k$, $k'$ and $q$ stand for energy-momentum and energy-momentum transfer, respectively, and $S$ and $T$ are total p-h spin (with $z$-projection $M$) and isospin, respectively. $\lambda(k, q)$ is the free polarization propagator \[21\]. The solution of the BS equation assumes an algebraic form and can be solved analytically \[22\] in the Landau limit, where energy and momentum lie on the Fermi surface and energy-momentum transfer $q = 0$. In that limit the leading term $G(k, k'; 0)$ (spin-isospin here omitted for simplicity)
depends only on the angle $\theta$ between $k$ and $k'$, expressed in terms of Landau-Migdal parameters (expanded in partial waves), and can be written

$$G(k, k') = N_0^{-1} \sum_{l} \left( F_l + F'_l \right) \tau_2 + G_l \sigma_1 \cdot \sigma_2 + G_{ll'} \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2 + \frac{q^2}{k_{l'}^2} H_{l} S_{12}(q) + \frac{q^2}{k_{l'}^2} H_{l'} S_{12}(q) \tau_1 \cdot \tau_2) P_l(\cos \theta)$$

(7)

where $2q = k - k'$ is the relative momentum and $S_{12}$ the tensor operator. $S_{12}(q) = 3 \langle \hat{S} \cdot q \rangle^2 - S^2$. The $P_l(\cos \theta)$ are the Legendre polynomials. The inclusion of the tensor Landau-Migdal parameters is motivated by the fact that the interaction contains a strong tensor component in the $^3SD_1$ channel, as already pointed out in the Introduction.

In the BS equation the choice of the driving term plays a crucial role. In principle it contains all irreducible processes of the interaction. The simplest approximation is to take the bare interaction itself, but, to prevent the divergences related to the hard-core of the nuclear force, we adopted the Brueckner $G$-matrix calculated in the Brueckner-Hartree-Fock (BHF) approximation. The relation between the $G$-matrix and the Landau-Migdal parameters is presented in the Appendix.

In order to derive the BS equation in the Landau limit, we follow closely Ref. [22]. After expanding in partial waves the p-h interaction $\mathcal{F}$ (the same for the leading term)

$$\mathcal{F}^{T}_{SM_S,SM'_S}(k, k'; 0) = N_0^{-1} \sum_{l m \ell' m'} \frac{4 \pi Y_m^*(\hat{k}) Y_{\ell'}^*(\hat{k}')}{(2l + 1)(2l' + 1)^{1/2}} \langle lm SM_S | JM | l' m' SM'_S | JM \rangle \mathcal{F}^{SJT}_{ll'},$$

(8)

the BS equation goes over into the algebraic equation for the $\mathcal{F}^{SJT}_{ll'}$ matrix elements

$$\mathcal{F}^{SJT}_{ll'} = G^{SJT}_{ll'} - \sum_{ll'} \frac{1}{2l + 1} G^{SJT}_{ll'} \mathcal{F}^{SJT}_{ll'},$$

(9)

where $J$ is the total angular momentum. The matrix elements $G^{SJT}_{ll'}$ are the coefficients of the partial-wave expansion of the leading term. Its expression in terms of the Landau-Migdal parameters is reported in the Appendix. In the case of $S = 0$, all partial-wave matrix elements are diagonal, because the tensor force does not affect the scalar Landau-Migdal parameters and we simply get the well known expression

$$\mathcal{F}^{SJT}_{ll} = \frac{G^{SJT}_{ll}}{1 + G^{SJT}_{ll}/(2l + 1)},$$

(10)

where $G^{00}_{ll} = F_l$, $G^{0J}_{ll} = F'_l$ and $J = l$.

In the case of $S = 1$, off-diagonal matrix elements also exist due to the coupling between vector and tensor Landau-Migdal parameters as shown in the Appendix. But only two different angular momenta $(l, l + 2)$ can couple because we have to couple $l$ and $S$ to good $J$. The explicit expression of the matrix elements $\mathcal{F}^{SJT}_{ll'}$ (for $l = J \pm 1$ and $l' = J \mp 1$) of the effective interaction is

$$\mathcal{F}^{SJT}_{ll'} = D^{-1} \left[ G^{SJT}_{ll} \left( 1 + \frac{G^{SJT}_{ll'}}{2l + 1} \right) - \left( \frac{G^{SJT}_{ll'}}{2l + 1} \right)^2 \right],$$

(11)

$$\mathcal{F}^{SJT}_{ll'} = D^{-1} G^{SJT}_{ll'},$$

where

$$D = (1 + \frac{G^{SJT}_{ll'}}{2l + 1}) \left( 1 + \frac{G^{SJT}_{ll'}}{2l + 1} \right) - \left( \frac{G^{SJT}_{ll'}}{2l + 1} \right)^2 \frac{(2l + 1)(2l' + 1)}{(2l + 1)(2l' + 1)}$$

(12)

Notice again that only two different angular momenta at most can couple together, i.e. $l = l'$ or $|l - l'| = 2$.

C. Induced interaction

For application to the gap equation the particle-hole (p-h) interaction must be converted into particle-particle (p-p) interaction and then the induced part must be taken out (second diagram on the r.h.s. of Fig. 1). The spin-isospin transformation is given by

$$\mathcal{F}^{\pi}_{pp}(q, P) = (-)^{1+l} \sum_{ST} (2T + 1) \left[ \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} T \right] \sum_{SM_S,SM'_S} \{SM_S,SM'_S|sm,sm'\} \mathcal{F}^{T}_{SM_S,SM'_S}(k, k'; 0),$$

(13)

where $\{SM_S,SM'_S|sm,sm'\}$ is the spin transformation bracket and $P$ is the total momentum [23]. For application to the np pairing interaction in the $^3SD_1$ channel the $\mathcal{F}^{T}_{ll}$ partial waves with $l, l' = 0, 2$ have to be projected out from the expansion of $\mathcal{F}^{T}(q, P)$. 
III. NUMERICAL RESULTS

The numerical evaluation of the medium polarization effects starts from the \( G \)-matrix calculated in the BHF approximation with the Argonne AV18 as two-body interaction and the consistent meson-exchange three body force [24].

From the \( G \)-matrix expansion of the self-energy the dispersion effects of the mean-field are included in the effective mass approximation (EMA) and the depletion of the Fermi surface is also approximated by the \( Z \)-factors [25].

The medium polarization is described by the BS equation, solved in the RPA, where the \( G^{ph} \)-matrix is the input, so that p-p short-range correlations and p-h long-range collective excitations of the nuclear matter are simultaneously treated in a unified manner.

Finally the p-p interaction induced by the medium polarization is added to the bare interaction and the gap equation is solved.

![FIG. 2: Energy gap with self-energy effects. Left: Comparison between \( ^3SD_1 \) and \( ^1S_0 \) gaps from bare interaction. Middle: \( ^3SD_1 \) gaps with single particle spectrum in the EMA (effective mass vs density in the inset). Right: Energy gap with depleted Fermi surface (\( Z \)-factor vs density in the inset).](image)

A. Self-energy corrections

As shown in Fig. 2a, the energy gap with only bare interaction gives for spin-triplet \( ^3SD_1 \) pairing a peak value the order of 12 MeV, which should be compared with the value of 3 MeV for spin-singlet \( ^1S_0 \) pairing [14]. The large difference between the two gaps is justified by the exponential dependence on the interaction strength of the solution of the BCS gap equation [21]. In fig. 2b, the mean field dispersive effect is also reported for comparison using the effective mass (see inset) in the quasi-particle spectrum, according to Eq.(3). This effect is well known [26]: the gap magnitude gets reduced and pairing density range is also shifted towards low densities, where \( m'/m \approx 1 \). Additional reduction of the gap is obtained when including the depletion of the Fermi sphere, as shown in Fig. 2c. The depletion is introduced via the \( Z \)-factor [13 [25], plotted in the inset of the figure. This quenching effect is more pronounced since pairing strength is exponentially dependent on \( Z^2 \). The two combined effects give rise to a remarkable quenching of the gap in a density range making the \( ^3SD_1 \) pairing a surface effect like the \( ^1S_0 \) one. However the peak value of \( ^3SD_1 \) energy gap is still over two times larger than the \( ^1S_0 \) with the same self-energy approximation.

B. Induced interaction

The p-p matrix elements of the \( G \)-matrix in the SD channel are calculated from the BHF approximation with the same two and three body force like the self-energy. The p-p matrix elements are transformed into p-h matrix elements, expressed in terms of Landau-Migdal parameters as shown in the Appendix. For such a purpose the Landau limit has been adopted, where the energy-momentum transfer is assumed to be vanishing. Since the SD components of the \( G \)-matrix derive from the tensor part of the bare interaction the additional \( H \) and \( H' \) Landau-Migdal parameters have been introduced in the p-h effective interaction. In Fig. 3 the SD partial-wave of the BHF Landau-Migdal parameters are plotted as a function of the density, Eqs.(A6-A10). The zero-order diagonal components are the Landau-Migdal parameters from the BHF \( G \)-matrix with no tensor force effect. The \( S = 1 \) partial-wave components are affected by the tensor Landau-Migdal parameters, but their effect is small. It follows that the off-diagonal matrix elements are even smaller. The main contribution comes from the isoscalar and isovector density fluctuations \( (S = 0) \) as expected.
FIG. 3: Leading p-h interaction $\mathcal{F}_{ii'}^{SJT}$ from G-matrix in SD channel.

| $\rho$ ($fm^{-3}$) | $k_F$ ($fm^{-1}$) | $SS$ (MeV·$fm^3$) | $DD$ (MeV·$fm^3$) | $SD$ (MeV·$fm^3$) |
|---------------------|-------------------|--------------------|-------------------|-------------------|
| 0.277               | 1.60              | -0.70              | -0.03             | 0.04              |
| 0.228               | 1.50              | -1.14              | -0.04             | 0.02              |
| 0.186               | 1.40              | -1.63              | -0.04             | 0.00              |
| 0.175               | 1.36              | -1.65              | 0.05              | -0.01             |
| 0.117               | 1.20              | -3.43              | -0.05             | -0.04             |
| 0.068               | 1.00              | -13.03             | -0.07             | -0.08             |
| 0.035               | 0.80              | -22.32             | 0.00              | -0.10             |

TABLE I: p-p induced interaction $(\mathcal{F}_{pp}^{10})_{ii'}$ in the $^3SD_1$ channel.
FIG. 4: Comparison among $^3SD_1$ gaps from RPA induced interaction and previous effects and $^1S_0$ in full calculations.

From the solution of the BS equation in the Landau limit with BHF Landau-Migdal parameters shown in Fig. 3 as input, the effective interaction is determined and transformed in the p-p representation, according to Eq.(13). The matrix elements of the $^3SD_1$ induced part (second diagram of the r.h.s. of Fig. 4) are reported in Table I. It easily seen that the dominant contribution is concentrated in the $S = 0$ isoscalar matrix element. This contribution is attractive much the same as for the spin-singlet pairing in symmetric nuclear matter [12].

C. Pairing gap from vertex corrections

The p-p effective interaction is added to the pairing interaction and the BCS equation is solved. The resulting gap vs. Fermi momentum is displayed in Fig. 4 in comparison with the preceding results. Two series of calculations have been performed: the first one (upper stars) shows the effects on induced interaction without self-energy corrections, the second one (lowest stars) is full calculations, self-energy plus induced interaction. There was a limit to the lower densities imposed by the missing convergence of the BHF calculation of $G$-matrix. This is due to the singularity of $G$-matrix in the density domain where large pair correlations are expected to occur. This drawback requires a self-consistent calculation of BCS equation and BHF calculation with quasi-particle energy spectrum, that is beyond the scope of the present investigation. The main result is that, due to the attractive nature of the new term, the self-energy quenching is reduced, but less than in the case of spin-singlet pairing. A second main result is that the shift of the peak value at low density produced by the self-energy is not affected by the induced interaction, suggesting that pairing is a surface phenomenon in finite nuclei. Finally, it is worth of noticing that the spin-triplet pairing in $^3SD_1$ channel is still much larger than the spin-singlet pairing in $^1S_0$ channel, as clearly shown in Fig. 4.

D. Average pairing in nuclei from LDA

To make contact with pairing in nuclei we have estimated the average gap in N=Z nuclei from the Thomas-Fermi density corresponding to the states around the chemical potential $\mu$ defined as follows [29]

$$<\Delta(\mu)> = \int d\vec{r} \sum_i \frac{1}{g(\mu)} \delta(\mu - \epsilon_i) |\phi_i(\vec{r})|^2 \Delta(\vec{r})$$

(14)

where $\phi_i(\vec{r})$ is the single-particle wave function with energy eigenvalue $\epsilon_i$, $\Delta(\vec{r})$ is the nuclear matter gap for the density $\rho(\vec{r})$ according to the local density approximation (LDA) and $g(\mu)$ is the level density at $\mu$. It easy to show that, in the $\hbar \to 0$ semiclassical limit [29]

$$<\Delta(\mu)> = \frac{\int d^3 \Delta(\vec{r}) \rho^{1/3}(\vec{r})}{\int d^3 \rho^{1/3}(\vec{r})}$$

(15)
TABLE II: Average gaps $\Delta_0$ (no screening) and $\Delta$ (screening) in N=Z nuclei from LDA. The density profiles are taken from Ref. [30].

| $A$ | $R(fm)$ | $\Delta_0$(MeV) | $\Delta$(MeV) |
|-----|---------|-----------------|---------------|
| 40  | 3.83    | 6.82            | 3.54          |
| 100 | 5.20    | 8.18            | 3.75          |
| 200 | 6.50    | 9.38            | 4.00          |

where we take for the density the phenomenological one parametrized in [30]. In Table II the results are reported for some N=Z nuclei. We see that screening substantially reduces the gap values which, however, remain stronger than in the $T = 1$ channel. A word of caution is, however, in order: in finite nuclei the effect of collective surface modes may be quite different from what can be simulated with LDA. So the results from the latter should be taken only as a qualitative indication.

IV. DISCUSSION AND CONCLUSIONS

In this paper the spin-triplet $^3SD_1$ pairing in symmetric nuclear matter has been discussed within the BCS theory with medium polarization effects. On one hand, the self-energy corrections reduce significantly the magnitude of the gap, shifting the peak value to low density. On the other hand, the induced interaction that is attractive almost in the full asymmetry range, partially restores a higher magnitude of the gap without additional squeezing of the density range of the superfluid phase. The induced interaction has been calculated from the RPA in the Landau limit, starting from the BHF p-h interaction. In this fashion the long-range correlations are built up on top of the short-range correlations from $G$-matrix. In this approximation the main contribution comes from the scalar density fluctuations, as expected. On the other hand, the feedback of the vertex corrections on the other spin-isospin fluctuations can only be treated within the framework of the induced interaction approach [13] that is a task of further investigation.

The gaps obtained in the present approximation, as large as 2-3 times the magnitude of the spin-singlet pairing in the $^1S_0$ channel, provide a strong indication of the importance of the medium polarization. The conclusion is that the $^3SD_1$ neutron-proton superfluid state in nuclear matter turns out to be more stable than the $^1S_0$ neutron-neutron or proton-proton superfluid state. This is in keeping with the calculations, where it is found that N=Z heavy nuclei ($A > 140$) np pair correlations are stronger than nn or pp ones [6]. Below this threshold the pairing between like nucleons is found to be the favored one, because the spin-orbit splitting in nuclei hinders np pairing, but the present nuclear-matter calculations address the problem whether the np pairing strength might be larger in the spin triplet than singlet pairing state even for nuclei below $A = 140$. It would be a timely issue to study the competition between spin-triplet pairing with medium screening effects and the spin-orbit splitting in finite nuclei, considering that tools already have been devised to face such a problem [32].

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Appendix A: Landau-Migdal parameters from BHF $G$-matrix

The microscopic derivation of the Landau-Migdal parameters from the BHF approximation is obtained converting the p-p $G$-matrix, as calculated with the Brueckner-Bethe-Goldstone equation, into the p-h representation. This procedure yields [33, 34]

\[
(F,F') = \frac{1}{16} \sum_{st} (2t \pm 1) G^{st} \tag{A1}
\]

\[
(G,G') = \frac{1}{16} \sum_{t} (2t \pm 1) (G^{1t} - G^{0t}) \tag{A2}
\]

\[
(H,H') = \frac{1}{24} \frac{k^2}{q^2} \sum_{t} (2t \pm 1) (\tilde{G}^{1t} - \tilde{G}^{0t}) \tag{A3}
\]
where $G^{st}$ denotes the $G$-matrix with spin $s$ and isospin $t$, and $\tilde{G}^{st}$ is the same with $\tilde{q}$ along the spin quantization axis. The isoscalar (isovector) Landau-Migdal parameters take the upper (lower) sign. Inverting the partial-wave expansion of the leading term we can determine the coefficients

$$G^{SJT}_{ll'} = N_0 \frac{[(2l + 1)(2l' + 1)]^{1/2}}{4\pi} \sum_{MM'} [(2l + 1)(2l' + 1)]^{1/2} \langle \text{Im } SM_T |JM \rangle \langle l'M' SM_T' |JM \rangle \times$$

$$\int dk d\hat{k'} Y_{lm}(\hat{k}) Y_{l'm'}^{*}(\hat{k'}) \langle SM_T, T |G^{SJT} (k, k') |SM_T', T \rangle,$$

as a function of the Landau-Migdal parameters. For the $S = 0$ component the calculation is straightforward, whereas for $S = 1$ it is quite tedious for the coupling between vector and tensor Landau-Migdal parameters. It can be found in the literature (see, e.g. Refs. [23,28]. Below we report the matrix elements needed for the calculation of vertex correction to the np pairing interaction in the channel $^3SD_1$. For $T = 0$ they are in the order

$$G^{00}_{00} = F_0$$

$$G^{02}_{22} = F_2$$

$$G^{10}_{10} = G_0$$

$$G^{11}_{11} = G_2 - \frac{1}{4} \left( \frac{7}{3} H_1 - 2H_2 + \frac{3}{7} H_3 \right)$$

$$G^{10}_{02} = -\frac{\sqrt{10}}{12} (3H_0 - 2H_1 + \frac{3}{5} H_2)$$

For $T = 1$ the isoscalar Landau-Migdal parameters must be replaced by the corresponding isovector ones, $(F \rightarrow F', ...)$.

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