DEPENDENCE OF STELLAR MAGNETIC ACTIVITY CYCLES ON ROTATIONAL PERIOD IN A NONLINEAR SOLAR-TYPE DYNAMO

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ABSTRACT

We study the turbulent generation of large-scale magnetic fields using nonlinear dynamo models for solar-type stars in the range of rotational periods from 14 to 30 days. Our models take into account nonlinear effects of dynamical quenching of magnetic helicity, and escape of magnetic field from the dynamo region due to magnetic buoyancy. The results show that the observed correlation between the period of rotation and the duration of activity cycles can be explained in the framework of a distributed dynamo model with a dynamical magnetic feedback acting on the turbulent generation from either magnetic buoyancy or magnetic helicity. We discuss implications of our findings for the understanding of dynamo processes operating in solar-like stars.

Key words: dynamo – stars: activity – stars: solar-type

1. INTRODUCTION

Cyclic magnetic activity of the solar type is often observed among main-sequence stars with external convective envelopes (e.g., Baliunas et al. 1995; Hall et al. 2007). After Parker (1955), it is widely believed that the magnetic activity on solar-like stars results from large-scale dynamo processes driven by turbulent convection and rotation. Interpretation of the stellar magnetic activity is rather complicated because of nonlinear dynamo effects that need to be taken into account (Noyes et al. 1984). Moreover, even in the case of the solar dynamo many details are poorly known (Charbonneau 2011). In particular, the origin of the large-scale poloidal magnetic field of the Sun (the component of the field that lies in meridional planes) is not well understood.

One of the interesting questions is how observations of stellar magnetic activity can help us in understanding the key processes of the solar dynamo and vice versa (Brun et al. 2015). The dependence of magnetic activity cycles on the period of rotation is of particular interest. Observations show that the stellar magnetic activity grows almost linearly with increasing rotation rate (Vidotto et al. 2014), but the dynamo period decreases (Noyes et al. 1984). Thus, solar-like stars show an anticorrelation between the dynamo period and the amplitude of magnetic activity. It is interesting that this relationship can also be deduced from data on solar activity (Vitinsky et al. 1986). A comparative study of this relation for solar and stellar cycles was published by Soon et al. (1994). Further studies revealed that this correlation is not unique, and that several branches corresponding to different levels of magnetic activity can be identified (Saar & Brandenburg 1999; Böhm-Vitense 2007; Saar 2011).

Theoretically, the observed correlation between the amplitude and period of the dynamo cycle is expected in a kinematic regime (Noyes et al. 1984). However, the results of such linear analysis cannot be applied to the nonlinear dynamo. Theoretical arguments of Noyes et al. (1984) and, also, Tobias (1998) and Ossendrijver (1997) had shown that dynamo saturation mechanisms affect the correlation. Also, the type of dynamo scenario is important in this context. The concept of a flux-transport dynamo has been popular in the solar community (Charbonneau 2011). However, the stellar dynamo models that were constructed following this idea show a growth of the cycle period with an increase in the rotation rate (see Jouve et al. 2010; Karak et al. 2014). This contradicts the observations. The solution to this issue is not clear yet. Jouve et al. (2010) found that in the framework of the flux-transport models this issue can be resolved by assuming certain multiple-cell patterns of the meridional circulation. However, it is unclear how such circulation patterns are compatible with the angular momentum balance in stars. Pipin (2015) argued that the problem with the flux-transport models can be related to the direction of propagation of the dynamo wave in the convective zone. In the flux-transport models the dynamo wave propagates inward from high to low latitudes toward the regions with low magnetic diffusivity. This causes an increase in the dynamo period when the large-scale toroidal field is concentrated more strongly at the bottom of the convection zone. Such an effect has been found in 3D dynamo simulations by Guerrero et al. (2016). In the distributed dynamo models (Brandenburg 2005; Pipin & Kosovichev 2011b) the dynamo wave propagates outwards from high to low latitudes. This type of dynamo agrees with observations in the case of both solar (Pipin & Kosovichev 2011a; Pipin et al. 2012) and stellar cycles (Pipin 2015).

In this paper we study the influence of nonlinear dynamo saturation mechanisms on basic properties of the magnetic activity cycles in solar-like stars with different rotation rates. We restrict ourselves to slowly rotating stars with rotational periods from 14 to 30 days. In these cases the magnetic feedback on the differential rotation is not very strong compared to the saturation caused by magnetic buoyancy and magnetic helicity. From the results of Pipin (2015) it follows that the strongest variations of the latitudinal rotational shear due to the dynamo-generated magnetic activity are about 1% of the mean value for the case of a solar analog rotating with a period of 14 days. This agrees with the observational findings of Saar (2011), as well. The impact of such variations on the dynamo processes is not as strong as the impact of the other nonlinear effects. Thus, we neglect variations of the differential rotation in this study.
2. BASIC EQUATIONS

2.1. Dynamo Model

The induction vector of the large-scale magnetic field, \( \mathbf{B} \), in a highly conductive turbulent medium is governed by the equation (Krause & Rädler 1980, p. 271)

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{E} + \mathbf{U} \times \mathbf{B}).
\]

(1)

Here, \( \mathbf{U} \) is the mean flow velocity and \( \mathbf{E} = \mathbf{u} \times \mathbf{b} \) is the mean electromotive force, where \( \mathbf{u} \) and \( \mathbf{b} \) are fluctuations of the flow and magnetic field. The axisymmetric magnetic field is represented as a sum of the azimuthal (toroidal) and poloidal (meridional plane) components:

\[
\mathbf{B} = e_\phi B + \nabla \times \frac{A e_\phi}{r \sin \theta}.
\]

where \( e_\phi \) is the unit vector in the azimuthal direction, \( \theta \) is the polar angle, and \( A e_\phi \) is the vector-potential of the large-scale poloidal magnetic field. The assumption of scale separation (Krause & Rädler 1980, p. 271) results in a linear relationship between the mean electromotive force and the local large-scale magnetic field:

\[
\mathbf{E}_t = (\alpha_{ij} + \gamma_{ij}) \mathbf{B}_j - \eta_{ij} \nabla_j \mathbf{B}_i.
\]

(2)

Tensors on the right-hand side (rhs) of Equation (2) take into account the effect of rotation and magnetic field on the mean electromotive force.

The \( \alpha \) effect takes into account the kinetic and magnetic helicities in the following form:

\[
\alpha_{ij} = C_\alpha \sin^2 \theta \psi_\alpha \beta_{ij} \alpha^{(H)}_{ij} \eta_T + \alpha^{(M)}_{ij} \frac{\nabla \tau_c}{4\pi \beta^2}.
\]

(3)

where \( C_\alpha \) is a free parameter, \( \alpha^{(H)}_{ij} \) and \( \alpha^{(M)}_{ij} \) express the kinetic and magnetic helicity parts of the \( \alpha \) effect, respectively; \( \tau_c \) is the small-scale magnetic helicity, \( t \) is the typical length scale of the turbulence, and \( \eta_T \) is the turbulent diffusivity. Tensors \( \alpha^{(H)}_{ij} \) and \( \alpha^{(M)}_{ij} \) depend on the Coriolis number, \( \Omega^* = 4\pi \frac{\omega_c}{\tau_c} \), where \( \Omega_c \) is the rotational period and \( \tau_c \) is the convective turnover time. The magnetic quenching function of the hydrodynamical part of the \( \alpha \) effect is defined as

\[
\psi_\alpha = \frac{5}{128 \beta^4} \left( 16 \beta^2 - 3 - 3(4\beta^2 - 1) \frac{\arctan(2\beta)}{2\beta} \right).
\]

(4)

where \( \beta = |\mathbf{B}|/\sqrt{4\pi \mu_0 u'^2} \) and \( u' \) is the rms of the convective velocity. This so-called “algebraic” quenching describes the “instantaneous” magnetic feedback on the dynamo generation. It is assumed that the large-scale magnetic field varies much more slowly than the typical convective timescale.

In addition, there is a so-called “dynamical” quenching due to the conservation of magnetic helicity (Pouquet et al. 1975). Similarly to Kleeorin & Ruzmaikin (1982), Blackman & Brandenburg (2003), and Kleeorin et al. (2003) (see also Kitiashvili & Kosovichev 2009; Blackman & Subramanian 2013; Sokoloff et al. 2013), we model this effect through the second term of Equation (3) and the equation that governs the evolution of the helicity density of the fluctuating part of the magnetic field, \( \omega \) (Hubbard & Brandenburg 2012; Pipin et al. 2013):

\[
\frac{\partial \omega}{\partial t} = - \frac{\nabla}{R_m \tau_c} - 2\eta \mathbf{B} \cdot \mathbf{J} - \nabla \cdot \mathbf{F} \cdot \nabla \tau_c,
\]

(5)

where \( \omega \) is the total magnetic helicity density, \( \mathbf{F} \) is the diffusive flux of the magnetic helicity, \( R_m \) is the magnetic Reynolds number. In this paper we assume \( \tau_c = 0 \).

Tensors in the second and third terms of the rhs of Equation (2) contain contributions from the turbulent pumping and turbulent diffusivity respectively. The turbulent mean-field pumping is the sum of contributions due to the mean density gradient, \( \gamma^{(P)}_{ij} \) (Kichatinov 1991), the diamagnetic pumping, \( \gamma^{(D)}_{ij} \) (Krivodubskij 1987), and the mean-field magnetic buoyancy, \( \gamma^{(B)}_{ij} \) (Kichatinov & Pipin 1993), and also includes effects of the large-scale shear, \( \gamma^{(H)}_{ij} \) (Pipin 2013):

\[
\gamma_{ij} = \gamma^{(P)}_{ij} + \gamma^{(D)}_{ij} + \gamma^{(B)}_{ij} + \gamma^{(H)}_{ij}.
\]

(6)

In our study the most important pumping effect is related to the mean-field magnetic buoyancy:

\[
\gamma^{(B)}_{ij} = \frac{\alpha_{\text{MLT}} u'}{\gamma} \beta^2 K(\beta) \theta_a \epsilon_{ij}.
\]

where \( \alpha_{\text{MLT}} \) is the mixing-length parameter, \( \gamma \) is the adiabatic exponent, and \( \theta_a \) is the unit vector in the radial direction. The quenching of the magnetic buoyancy (see Kichatinov & Pipin 1993) is defined by the function \( K(\beta) \):

\[
K(\beta) = \frac{1}{16 \beta^4} \left( \frac{\beta^2 - 3}{\beta} \frac{\arctan(2\beta)}{2\beta} + \frac{(\beta^2 + 3)}{(\beta^2 + 1)} \right).
\]

A detailed formulation of the model has been published by Pipin (2015). The distributions of the turbulent parameters, such as the typical convective turnover time, \( \tau_c \), the mixing length, \( \ell \), the rms convective velocity, \( u' \), the mean density, \( \bar{n} \), and its gradient, \( N^p = \nabla \log n \), are computed using the mixing-length model of the solar convection zone of Stix (2002, p. 521). In particular, that model uses the mixing length \( \ell = \alpha_{\text{MLT}} |\nabla \theta|^{-1} \), where \( \theta = \nabla \log n \) is the inverse pressure height, and \( \alpha_{\text{MLT}} = 2 \). The turbulent diffusivity profile is given in the form \( \tau_c = C_\eta \frac{u' \tau_c}{3 \theta_c(1 + \exp(50(\tau_c - r)))} \), where the function \( \tau_c(r) = 1 + \exp(50(\tau_c - r)) \) controls quenching of the turbulent effects near the bottom of the convection zone, \( r_{ov} = 0.725 R_\odot \). The parameter \( C_\eta \) (in the range \( 0 < C_\eta < 1 \)) is a free parameter that controls the efficiency of mixing of the large-scale magnetic field by turbulence. It is used to adjust the period of the dynamo cycle. We use the same model of the convection zone for all the rotational periods considered in this paper.

At the bottom of the convection zone we apply a perfectly conducting boundary condition. At the top of the convection zone the poloidal field is smoothly matched to the external potential field, and the toroidal field is allowed to penetrate to the surface:

\[
\frac{\delta \eta}{\tau_c} B + (1 - \delta) E_{\theta} = 0,
\]

(7)

where \( \delta = 0.99 \) (Moss & Brandenburg 1992; Pipin & Kosovichev 2011b). The numerical integration is carried out
in latitude from pole to pole, and in radius from \( r_h = 0.715 R_c \) to \( r_r = 0.99 R_c \). The numerical scheme employs the pseudospectral approach for the numerical integration in latitude and second-order finite differences in radius.

3. RESULTS

The free parameters, \( C_\alpha, C_p, \) and \( R_m \), are used to calibrate the model for the best possible agreement with the observations of the solar cycle for the solar rotation period. In this study we use \( C_\alpha = 0.04, C_p = 0.05, \) and \( R_m = 10^4 \). In model M2 (Table 1) we use \( R_m = 10^6 \), which provides a better conservation of magnetic helicity, which is the primary nonlinear effect in this case. The profile of the rotation law is taken from the helioseismology inversion of Howe et al. (2011); it is fixed as well (see Pipin 2015). However, the radial profile of the Coriolis number \( \Omega_c / \Omega_\ast = 4 \pi^2 \rho / \rho_\ast \) varies with the rotational period. In our runs, we employ the following set of rotational periods:

\[
P_{\text{rot}} = [29.4, 25, 20, 16.7, 15.6, 14.3] \text{ yr}
\]  

(8)

For this set we compute the dynamo models (Equations (1) and (5)) taking into account the different saturation mechanisms, as specified in Table 1. The models that are listed in Table 1 employ the \( \alpha \)-effect parameter, \( C_\alpha = 0.04 \). This value is 20% above the dynamo instability threshold for \( P_{\text{rot}} = 25 \) days. Table 1 also shows results for the dynamo cycle period, the magnitude of the unsigned magnetic flux generated in the whole convection zone, the mean strength of the poloidal magnetic field at the surface, \( [B^{\text{pol}}] \), which we will use as a proxy of the line-of-sight magnetic field strength, and the strength of the radial polar magnetic field, \( |B_r| \), during the cycle minimum. The dynamo models are calculated until they reach a stationary phase. In the study we consider the magnetic field geometry to be antisymmetric relative to the equator. This is done by imposing a weak seed poloidal magnetic field with dipole symmetry in the initial conditions. Model M3 was explored for a set of the \( \alpha \)-effect parameter, \( C_\alpha = [0.03, 0.04, 0.06] \). Also, for model M2 we made additional runs for the rotational periods of 16.7 and 15.6 days using \( C_\alpha = 0.06 \) and \( C_p = 0.05 \) respectively to study the behavior of long-term variations in the dynamo process for different rotational periods.

The effect of the quenching mechanisms on the time–latitude evolution of the toroidal and poloidal magnetic fields is illustrated in Figures 1–3, where we show results for the rotational periods of 25 and 16.7 days for models M0, M1, and M2. For the rotation period of 25 days all models produce solar-like time–latitude magnetic butterfly diagrams. Model M0 has the strongest large-scale magnetic field, and it has the longest dynamo period of all the runs (see Table 1). It is also seen that the dynamo period in model M0 increases when the dynamo approaches the stationary state. This seems to be due to the strong concentration of the dynamo wave near the bottom of the convection zone in model M0. This is illustrated by snapshots of the magnetic fields distributions in Figure 2.

Increasing the rotation rate results in an increase in the dynamo period in model M0 because the toroidal field is amplified near the bottom of the convection zone. Simultaneously, the \( \alpha \) effect is suppressed there. This results in a spatial separation of the dynamo generation by the \( \alpha \) and \( \Omega \) effects. Deinzer et al. (1974) had shown that this causes an increase in the dynamo period. Note that in the flux-transport models the \( \alpha \) and \( \Omega \) effects are spatially separated by the design of the models. Models M1, M2, and M3 show a decrease in the dynamo period when the period of rotation decreases, because they preserve a distributed character of the dynamo process.

Increasing nonlinearity in the dynamo process can increase the complexity of its evolution. This happens in model M2 for the rotational period of 14 days. The model shows a long-term modulation with a period of about 40 years while it has two primary dynamo periods of 5.3 and 5.9 years. The modulation disappears when the kinetic helicity parameter \( C_\alpha \) increases. Similar long-term modulations were found for the rotational periods of 16.7 and 15.6 days with \( C_\alpha = 0.06 \) and \( C_p = 0.05 \) respectively. The long-term modulation in axisymmetric dynamo models with the dynamical quenching of the \( \alpha \) effect was demonstrated earlier in a number of papers (see, e.g., Covas et al. 1998; Kitiashvili & Kosovichev 2009). Model M2 illustrates an interesting possibility when the surface radial magnetic field almost disappears during the maximum of the grand cycle of the toroidal magnetic field. A situation when the toroidal field dominates on the stellar surface is also observed in stellar activity, but for a non-axisymmetric field and for a faster rotating star; see, e.g., the review by Donati & Landstreet (2009).

Variations of the total unsigned flux of the toroidal field in the convection zone for a star rotating with a period of 16 days are shown in Figure 4. It is seen that the nonlinear mechanism involved in the dynamo affects the amplitude of the dynamo-generated flux, and the range and period of the variations.
Model M2 has a long-term modulation of the magnetic activity for $C_0 = 0.06$. The same effect is found for model M2 with a rotational period of 14 days and $C_0 = 0.04$ (see Figure 3(c)). It is interesting that individual cycles are not always well recognized in the long-term modulations of the flux.

Figure 5(a) shows the dependence of the dynamo-generated unsigned magnetic flux in the convection zone on the period of rotation. It shows an increase in the generated flux with increasing rotation rate. It is seen that the conservation of magnetic helicity produces the strongest quenching of the dynamo process among all the nonlinear mechanisms.

Thus, variations of the dynamo period with the rotational period depend on the dynamo saturation mechanism. Figure 5(b) illustrates our results together with the observational results shown earlier by Böhm-Vitense (2007). The algebraic quenching of the $\alpha$ effect is a simple anzatz most widely used in various dynamo models. However, this nonlinear effect results in an increase in the dynamo period.
with increasing rotation rate. This tendency disagrees with the observations. The opposite trend is demonstrated by models M1, M2, and M3. This agrees qualitatively with the observations that show a complicated behavior of the stellar activity periods, indicating the existence of two populations of magnetically "active" and "inactive" stars (Saar & Brandenburg 1999; Böhm-Vitense 2007; Saar 2011).

It is interesting to estimate how the magnetic activity changes parameters relating to the number of spots during the magnetic cycle. The mechanism of formation of active regions is not well understood for the Sun or for other stars either. We assume that the magnetic spots are produced from the large-scale toroidal magnetic field by means of a nonlinear instability. The precise relationship between the Wolf sunspot number, \( W(t) \), and the toroidal magnetic field is still unknown. Therefore, for a qualitative analysis we employ a simple analytical relation:

\[
W = B_{\text{max}} \exp \left( -\frac{B_0}{B_{\text{max}}} \right). \tag{9}
\]

where \( B_0 = 1000 \) G, and \( B_{\text{max}} \) is the maximum strength of the toroidal field. The parameter \( B_0 \) is slightly different from the one used by Pipin et al. (2012), because here the value of \( B_{\text{max}} \) is estimated from the layer in the upper convection zone where the maximum of the dynamo wave is located. Thus, the depth of \( B_{\text{max}} \) is not fixed and it varies with time. This provides a smoother profile of \( W(t) \) for fast rotating stars compared to the previous definition. We have also applied the definition of \( W \) in terms of the three-halves law of Bracewell (1988) used by Pipin & Kosovichev (2011a), and obtained qualitatively similar results.

Examples of profiles of the Wolf number for different models are shown in Figure 6. The cycle of model M0 is substantially larger than that of other models and it is not shown. It is seen that magnetic buoyancy is responsible for the most asymmetric profile of \( W(t) \). Also, it is found that the rise time decreases when the cycle amplitude increases. This effect works in both cases for a decrease in the rotational period and...
for an increase in the amplitude of the $\alpha$-effect. A similar phenomenon is observed in solar activity (Vitinsky et al. 1986).

All models listed in Table 1, except model M2 with the rotational period of 14 days, reach a stationary state, which is characterized by a nonlinear oscillation of some fixed period. For the rotational periods of 14, 15, and 16 days we find long-term modulations in model M2 with $C_\alpha = [0.04, 0.05, 0.06]$ respectively. In these cases the basic dynamo period has not fixed value. For each time series of the theoretical Wolf number parameter $W_i$, we extract the individual cycles and compute the cycle parameters, i.e., the magnitude of the maximum, the period, the rise time, and the decay time. We do this in the same way as previously in Pipin & Kosovichev (2011a). Figure 6(c) shows variations of $W$ for the individual cycle profiles. These profiles were extracted from the time series of model M2 with the rotational period of 16 days and $C_\alpha = 0.06$. It is seen that the stronger cycles have a longer period in this set. The amplitude of the variations in period is about three years. The amplitude of $W$ varies by about twice the minimal amplitude. The cumulative probability distribution functions are constructed as follows. We sort the discrete set of cycle amplitudes into increasing order. Let $i_W$ be the number of instances in which the cycle amplitude is less than or equal to some value $W$. Then we compute the cumulative probability distribution:

$$CDF = \frac{\sum_{i=1}^{i_W} k}{N},$$

where $k = 1$ corresponds to the cycle with the minimal amplitude and $N$ is the total number of instances. In the set shown in Figure 6(c) $N = 92$. Equation (10) approximates the probability for the amplitude of $W$ to have values in the interval between $W_{\text{min}}$ and $W_i$. The accuracy of the approximation improves as $N \to \infty$. The same procedure was repeated for the period parameter. The result is shown in Figure 6(d) (red curve). From this analysis we conclude that the amplitude of $W$ demonstrates the existence of two populations of stellar cycles, which can be quantified as the “strong” and “weak” cycles. The two populations are not well recognized from the distribution of the dynamo period. Other examples of the application of Equation (10) to analysis of the dynamo cycles were presented by Pipin & Sokoloff (2011).

Waldmeier (1935, 1936) found that the steeper the ascending phase of a sunspot cycle, the larger its magnitude. Also, he found that the ascending phase is usually shorter than the descending one. Later, other relations of this type were discovered. They are summarized by Vitinsky et al. (1986). It is interesting to see how the Waldmeier relations, well known for the sunspot cycles, and the asymmetry of stellar cycles can change with the rotational period and the magnitude of the $\alpha$ effect. For this we consider models M2 and M3. For model M2 we use only runs with the long-term modulations. Similarly to Pipin & Kosovichev (2011a) we determine the mean rise and decay rates for each individual cycle in each simulated time series of $W$. The asymmetry is determined as the ratio between the rates of decay and rise. Figure 7 shows our results. First, it is seen that this ratio seems to be constant for the set of periodic dynamo models (without long-term modulations). The decay rates of the dynamo cycles vary consistently with the rise rate, with the increase in the $\alpha$-effect parameter $C_\alpha$, and with the rotation rate. The theoretical trend is consistent with observations of the solar activity cycles. Here we used the data set provided by SIDC (2010). However, the time series for model M2 for the rotational periods of 14 and 16 days, which has the long-term modulation of the magnetic cycle, are out of the trend. Figure 7(b) shows that these models have a correlation between the amplitude of the cycle and its period. That contradicts the general trend of the periodic models and the solar observations as well. It is interesting to note that populations of the weak cycles in the long-term modulations follow the anticorrelation between the amplitude and period of cycle. The effect is not well recognized in Figure 7(b) because the scatter of the cycle parameters is relatively small. We find that this scatter decreases with increasing rotation rate. Also, the periodic models show that asymmetry increases with increasing cycle magnitude (Figure 7(c)). The trend changes in stars with a high rotation rate and strong $\alpha$-effect. This is likely
because our parameterization of the cycle asymmetry is too simple and it does not take into account the possibly complicated form of $W$. For example, Figure 6(b) shows the profile of $W$, in which a sharp rise in activity ends in a plateau, and the maximum is not well defined. The same arguments should be taken into account in analysis of the sunspot cycles. The models with nonlinear long-term modulations do not show a dependence of the asymmetry on the cycle magnitude.

Figure 8 shows a comparison of our dynamo models with the results of the survey of Vidotto et al. (2014) for the absolute value of the mean line-of-sight magnetic field on the surface versus the period of rotation. We see that all the models agree qualitatively with the results of observations within the range of rotational periods investigated. Model M0 shows a stronger magnetic field than is found in the observations. This indicates the presence of additional dynamo saturation mechanisms, such as those implemented in models M1 and M2, i.e., nonlinear quenching due to the magnetic buoyancy or the magnetic helicity. As noted in Section 1, for rapidly rotating stars the magnetic feedback on the differential rotation and convection has to be taken into account as well. Models M3A and M3B show different results for different magnitudes of the $\alpha$-effect parameter, $C_\alpha$. We see that the observed scatter of the magnetic field strength can be explained by variations of $C_\alpha$, varying by a factor of two above the threshold for dynamo instability. From the results for the solar dynamo models we know that a similar effect can be produced by a small fluctuation of $\alpha$ on a timescale comparable to the length of the magnetic cycle (Moss et al. 2008).

The investigated sample of rotational periods can be related to different ages of the solar-like stars. Taking into account the slowdown of rotation with time as $t^{-1/2}$, where $t$ is the star’s age (Skumanich 1972), we deduce that age of stars in our sample can be found from the relation $t = t_0 \left( \frac{P}{P_0} \right)^2$, where $P_0 = 25$ days and $t_0 = 4.61$ Gyr are parameters of the modern Sun. Thus, the investigated sample of rotational periods covers the range of ages from 1.5 to 6.2 Gyr. However, on this timescale changes in the structure of the convection zone have to be taken into account. The percentage of disk occupied by sunspots is estimated from the empirical correlation $SSA \approx 16.7 W/10^4$ (Vitinsky et al. 1986). Figure 9 shows...
our estimation of the amplitude of SSA at the maximum and minimum (dashed curve) of the magnetic cycle with the age for model M2. It shows that the younger stars are significantly more spotty. This is a rather naive estimate that does not take into account mechanisms of sunspot formation, or changes in the convection zone and the differential rotation with the stellar age.

4. DISCUSSION AND CONCLUSIONS

We have studied the nonlinear magnetic feedback on the turbulent generation of large-scale magnetic fields by a solar-type dynamo for the range of rotation periods from 14 to 30 days. Our models take into account the nonlinear $\alpha$ effect, the balance of magnetic helicity density, and escape of magnetic field from the dynamo region due to magnetic buoyancy. We did not consider the possible magnetic feedback on the differential rotation and changes in the structure of the convection zone.

Currently, one of the most important observational tests for dynamo models of solar-like stars is the relationship between the magnetic cycle periods and the period of stellar rotation. Earlier, Noyes et al. (1984), and also Ossendrijver (1997) and Tobias (1998), had suggested that despite a seemingly linear character of the connection between the rotational period and the dynamo period in stellar magnetic activity, the relation is not trivial because the large-scale dynamo develops a nonlinear regime when the rotation rate increases. The revealed multiple populations of magnetic activity in late-type stars (Saar & Brandenburg 1999) showed that the theoretical interpretation of stellar magnetic activity can be complicated. It was suggested that the “active” population of stars operates a dynamo concentrated near the bottom of the convection zone, and the “inactive” population operates a distributed dynamo (Saar & Brandenburg 1999; Böhm-Vitense 2007). In most cases, the models of the first type show an increase in the dynamo period with increasing rotation rate (Jouve et al. 2010; Karak et al. 2014; Pipin 2015) and are not consistent with observations.

In our set of dynamo models we did not find a satisfactory explanation for the observed multiple populations in terms of the periods of stellar activity cycles. A number of possible hypotheses remain to be explored. For example, our study is limited to magnetic field configurations that are antisymmetric relative to the equator. The breaking of magnetic parity is often considered as a source of nonlinear long-term modulations of solar activity (Ivanova & Ruzmaikin 1976; Brandenburg...
The correlation between the period of rotation and the dynamo period could be affected by changes in the characteristics of the convection zone and changes in the differential rotation profile (Kitchatinov 2011). Models with long-term modulations and dynamical quenching of the $\alpha$-effect by the conservation of magnetic helicity show two populations of the activity cycles. The population of weak cycles tends to follow the standard Waldmeier rules. But these do not hold for the whole time series.

Donati & Landstreet (2009) presented a diagram of stellar magnetic activity for the period of rotation versus the stellar mass, which shows that fast rotating stars of solar mass have non-axisymmetric magnetic fields. It is not clear if the non-axisymmetric field on the fast rotating solar analogs is generated by the dynamo instability or results from the emergence of magnetic field on the surface. The non-axisymmetric field is rarely discussed in the solar-type dynamo. It presumes that the differential rotation suppresses the non-axisymmetric dynamo (Raedler 1986). However, for the $\alpha\Omega$ dynamos, the critical threshold for generation of non-axisymmetric magnetic fields is only about a factor 2 or 3 larger than that for the axisymmetric dynamo. The interaction of the axisymmetric and non-axisymmetric modes has never been studied for the supercritical regimes that could operate in fast rotating stars. Finite non-axisymmetric perturbations can also affect the axisymmetric dynamo (Pipin & Kosovichev 2015).

Future studies should show whether the observed multiple populations of magnetic cycles in stellar activity can be explored through differences between the axisymmetric and non-axisymmetric dynamo models.

On the basis of the results of our models we can rule out the widely used algebraic quenching of the $\alpha$-effect as a primary nonlinear saturation mechanism. The observed correlation between the period of rotation and the dynamo period can be explained by distributed dynamo models that include the dynamical magnetic feedback on the turbulent generation due to either the magnetic buoyancy or the magnetic helicity. Also, the dynamo models with magnetic buoyancy as the primary saturation mechanism show a weaker polar magnetic field than the other models. The case of fast rotation (Figure 3) shows the difference in evolution of the radial magnetic field between dynamo models with dynamic magnetic helicity quenching and those with magnetic buoyancy quenching. In particular, in model M1 (where magnetic buoyancy is the primary saturation mechanism) the distribution of the radial magnetic field is dominated by the harmonic mode with latitudinal wavenumber $\ell = 3$, and the dipole mode with $\ell = 1$ is almost absent. Model M2 (where magnetic helicity quenching is the primary saturation mechanism) demonstrates a complicated evolution of the radial magnetic field with a wider spectrum of latitudinal harmonic modes than in the case of quenching by magnetic buoyancy. Observations show a general tendency of increasing polar magnetic activity with decreasing stellar rotational period (Donati & Landstreet 2009). Thus, our results suggest that conservation of magnetic helicity is the primary nonlinear effect in the saturation of a large-scale dynamo.

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REFERENCES

Balintas, S. L., Donahue, R. A., Soon, W. H., et al. 1995, ApJ, 438, 269
Blackman, E. G., & Brandenburg, A. 2003, ApJL, 584, L99
Blackman, E. G., & Subramanian, K. 2013, MNRAS, 429, 1398
Bohm-Vitense, E. 2007, ApJ, 657, 486
Bracewell, R. N. 1988, MNRAS, 230, 535
Brandenburg, A. 2005, ApJ, 625, 539
Brandenburg, A., Krause, F., Meinel, R., Moss, D., & Tuominen, I. 1989, A&A, 213, 411
Brun, A., Garcia, R., Houdek, G., Nandy, D., & Pinsonneault, M. 2015, SSRv, 196, 303
Charbonneau, P. 2011, Lrsp, 2, 2
Covas, E., Tavakol, R., Tworkowski, A., & Brandenburg, A. 1998, A&A, 329, 350
Deinzer, W., von Kusserow, H.-U., & Stix, M. 1974, A&A, 36, 69
Donati, J.-F., & Landstreet, J. D. 2009, ARA&A, 47, 333
Guerrero, G., Smolarkiewicz, P. K., de Oliveira Dal Pino, E. M., et al. 2016, ApJ, 818, 104
Hall, J. C., Lockwood, G. W., & Skiff, B. A. 2007, AJ, 133, 862
Howe, R., Larson, T. P., Schou, J., et al. 2011, JPhCS, 271, 012061
Hubbard, A., & Brandenburg, A. 2012, ApJ, 748, 51
Ivanova, T. S., & Ruzmaikin, A. A. 1976, SVA, 20, 227
Jouve, L., Brown, B. P., & Brun, A. S. 2010, A&A, 509, A32
Karak, B. B., Kitchatinov, L. L., & Choudhuri, A. R. 2014, ApJ, 791, 59
Kitchatinov, L. L. 1991, A&A, 243, 483
Kitchatinov, L. L., & Pipin, V. V. 1993, A&A, 274, 647
Kitchatinov, L. L. 2011, AShNC, 2, 71
Kivshar, I. N., & Kosovichev, A. G. 2009, ApF2D, 103, 53
Kleerion, N., Kuzanyan, K., Moss, D., et al. 2003, A&A, 409, 1097
Kleerion, N. I., & Ruzmaikin, A. A. 1982, MHD, 18, 116
Krause, F., & Rädler, K.-H. 1980, Mean-Field Magnetohydrodynamics and Dynamo Theory (Berlin: Akademie-Verlag)
Krivodubskij, V. N. 1987, SVA, 33, 338
Moss, D., & Brandenburg, A. 1992, A&A, 256, 371
Moss, D., Sokoloff, D., Usoykin, I., & Tutubalin, V. 2008, SoPh, 250, 221
Noyes, R. W., Weiss, N. O., & Vaughan, A. H. 1984, ApJ, 287, 769
Osensendrijver, A. J. H. 1997, A&A, 323, 151
Parker, E. 1955, ApJ, 122, 293
Pipin, V. V. 2013, ApF2D, 107, 185
Pipin, V. V. 2015, MNRAS, 451, 1528
Pipin, V. V., & Kosovichev, A. G. 2011a, ApJ, 741, 1
Pipin, V. V., & Kosovichev, A. G. 2011b, ApJ, 727, L45
Pipin, V. V., & Kosovichev, A. G. 2015, ApJ, 813, 134
Pipin, V. V., & Sokoloff, D. D. 2011, Phys, 84, 065903
Pipin, V. V., Sokoloff, D. D., & Usoykin, I. G. 2012, A&A, 542, A26
Pipin, V. V., Sokoloff, D. D., Zhang, H., & Kuzanyan, K. M. 2013, ApJ, 768, 46
Pouquet, A., Frisch, U., & Léorat, J. 1975, JFM, 68, 769
Raedler, K.-H. 1986, AN, 307, 89
Saar, S. H. 2011, in IAU Symp. 273, The Physics of Sun and Star Spots, ed. D. Prasad Choudhary, & K. G. Strassmeier (Cambridge: Cambridge Univ. Press), 61
Saar, S. H., & Brandenburg, A. 1999, ApJ, 524, 295
SIDC 2010, Monthly Report on the International Sunspot Number, Online Catalogue, http://www.sidc.be/sunspot-data/
Skumanich, A. 1972, ApJ, 171, 565
Sokoloff, D., & Nesme-Ribes, E. 1994, A&A, 288, 293
Sokoloff, D., Zhang, H., Moss, D., et al. 2013, in IAU Symp. 294, Solar and Astrophysical Dynamos and Magnetic Activity, ed. A. G. Kosovichev, E. de Oliveira Dal Pino, & Y. Yan (Cambridge: Cambridge Univ. Press), 313
Soon, W. H., Balintas, S. L., & Zhang, Q. 1994, SoPh, 154, 385
Stix, M. 2002, The Sun: An Introduction (2nd ed.; Berlin: Springer)
Tobias, S. M. 1998, MNRAS, 296, 653
Vidotto, A. A., Gregory, S. D., Jardine, M., et al. 2014, MNRAS, 441, 2361
Vitinsky, A. I., Kopecky, M., & Kuklin, G. V. 1986, The Statistics of Sunspots (Statistika Pjatnoobrazovatelnoj Dejatelnosti Solntsa) (Moscow: Nauka)
Waldmeier, M. 1955, MZur, 14, 105
Waldmeier, M. 1936, AN, 259, 267
Weiss, N. O., & Tobias, S. M. 2016, MNRAS, 456, 2654