Assessment of learning progression on mathematical problem solving of students using open approach

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ABSTRACT: The purpose of this research was to assess the progress of mathematical problem solving skills of students at Bannamphrae School in Chiang Mai in which the Open Approach was employed for class management. This research study was a quantitative research. The scope of the study included 23 fifth-grade students at Bannamphrae School in Chiang Mai, who had been selected through the purposive random sampling from the population covering 520 students in total. All of the students were in the fifth to sixth grades and were from 18 schools nationwide, which were participating in the Project on Advanced-Thinking Skills Development on Mathematics in the Northeast. The instrument used in the study was the mathematics problem solving test, which was comprised of 6 main items and 9 minor items. The data was analyzed via Conquest 2.0 software to discover the progress of their mathematical problem solving abilities through the developed framework of learning progress including 4 levels ranging from 0 to 3. The research revealed that the students at Bannamphrae School had shown different levels of progress with respect to mathematics problem solving as follows: 10 students were at Level 0 (Need Improvement), 6 students at Level 1 (Fair), 2 students at Level 2 (Good), and 6 students were at Level 3 (Very Good).

1. Background

In the 21st century, the world is dramatically changing. Mathematics, therefore, plays an important role in the development of human thinking. Learning mathematics focuses on learners. Through lesson study and the Open Approach, the learners become the center of learning by providing students with creative and systematic thinking and updated knowledge. The students become able to analyze problems or situations thoroughly and carefully. Moreover, they develop skills that will allow them to make predictions, plans, and decisions, as well as properly solve problems in their daily lives. What was needed for the students to become more advanced was basic knowledge, which included memorizing basic rules; understanding the principles or theories, which are used to solve simple problems and more complex ones; and appropriately understanding and interpreting problems and situations so that when seeking to find the answers, a distorted perception of the problem could be avoided. Moreover, it was necessary for them to write explanations and justifications for the answers they had discovered by themselves, as well as to validate the answers and the accuracy of their
calculations. In order to find the answers, they also needed to use numbers, mathematical symbols, or other means, such as figures and diagrams.[5]

Problem solving was an important focus of mathematics instruction. This has resulted in educators around the world turning to mathematical problem solving at all levels of the mathematics curriculum.[4] In Thailand, the Core Curriculum for Basic Education (BE 2551) was set up with the aim that learning management is to provide learners with knowledge, skills, competence, and desirable characteristics according to Learning Standards, which are based on the principle that learners are most important given the belief that everyone is able to make progress in learning and developing himself/herself. In learning management, students should be encouraged to naturally develop themselves to reach their full potential. Most importantly, individual differences and brain development should be taken into account.[6] However, what has happened in the classroom has not matched the purpose of this course; that is, learning management generally focuses on learning achievement, while the importance of the learning process is overlooked. This does not help the learners develop their competence.[1] Regarding the statistics of the results of the National Primary Education (O-NET) on mathematics in academic year 2016, sixth grade students, ninth grade students, and twelfth grade students had average scores of 43.47, 32.40, and 26.59, respectively, all of which were below the standard of 50 percent.[7] Moreover, according to the International Student Assessment 2015 (PISA 2015), in which 72 countries participated, Thailand received the 54th highest rating among the participating countries and had an average score of 421 points in mathematics, which is significantly lower than the international average of 500 points.[3]

Most mathematics teachers in Thailand use Direct Instruction as a teaching method, and as a result, students cannot express their own mathematical solutions.[1] Maitree Inprasit, et al. [5] argued that in order to promote effective learning reform, not only is there the need to change the teaching methods, but innovation is also needed. At present, Open Approach consists of a new teaching innovation called “Lesson Study” in which teachers need to understand the concepts or ideas of the students as much as possible in order to assist the students to discover ways or ideas to solve problems under the supervision of the teachers. As a result, learning management was developed using the concept of the Open-Ended Problems developed by Professor Nobuhiko Noida of the University of Tsukuba in Japan. It is used as a guideline for the development of the instructional activities of the learning management plan in the manner of creating a situation for mathematical problems. Students solve the open-ended problems through an investigative process and problem-solving. The students can create and develop new mathematical knowledge including the development of their thinking skills based on the different answers. Thereby, the different abilities of the students can be supported. The students can also collaborate in solving the open-ended problems because each student’s response to the activity will be different depending upon each individual's ability. In addition, this method, can empower the students to reach their full potential. Hence, the students will be led to discover mathematical formulas and rules of their own. The emphasis of the Open Approach does not focus upon the correct answer, but instead, focuses on different ways of searching for the answers.

Therefore, the researcher is interested in examining how to assess the students’ progress in learning mathematical problem solving in the classroom. Furthermore, in order to analyze the data received as a guideline to further develop learning management in the field of mathematics, the researcher is also interested in assessing the progress of the students, who have received Open-Approach instruction in learning mathematical problem solving.

2. Purpose of Study
To assess the progress of mathematical problem solving skills of fifth grade students on numbers and procedures using Lesson Study innovation by means of the Open Approach.

3. Concept of Study
The Investigation of the Progress of Mathematical Problem Solving of Fifth grade students using Lesson Study innovation by means of the Open Approach
The concept was as follows:

The Open Approach
- Presenting open-ended-problems
- Students’ self-learning
- Class discussions
- Connecting the students’ concepts

Learning Progression for Mathematical Problem Solving

**Figure 1. Conceptual Framework**

4. **Population and Samples**
The sample consisted of 23 fifth grade students of Bannamphrae School in the Hangdong District of Chiang Mai.

The population was 520 students nationwide from 18 schools who were participating in the Project on Advanced-Thinking Skills Development on Mathematics in the Northeast.

5. **Terms and Definitions**

5.1 *The study on learning progression on mathematical problem*
Refers to the process of showing students’ methods or strategies about problem solving.

5.2 *The assessment on the learning progression of mathematical problem solving*
Means the process of investigating the students’ learning progression beginning from the lowest level to the uppermost level through the assessment form on solving open-ended questions.

5.3 *Mathematical procedures*
Mean the progress in the following areas: 1) problem solving learning, 2) the connections between mathematics and mathematics or between mathematics and other sciences, 3) giving reasons, 4) presentations and communication, 5) creativity, 6) skills, and 7) the problem solving process. In other words, it is a process of applying mathematical knowledge, the steps of the problem solving process, the strategies, and the students’ experiences to solving mathematical problems. The process ranges from Unrecalled Memory, Basic Memory and Reproduction, Simple Skills and Concepts, to Strategic / Extended Thinking through open-ended questions.

5.4 *Open Approach*
Refers to the process of setting up situations with open-ended problems and then encouraging the students to solve the problems by themselves. The Open Approach focuses on developing the learners’ mathematical potentials through open-ended questions.[2] The mathematical thinking process, used in mathematics class through the Open Approach, is as follows:

1) Presenting open-ended problems
2) Self-learning
3) Promoting class discussions and extended thinking
4) Connecting all concepts which were touched upon in class

6. **Mathematical problem solving**
Means solving open-ended questions or mathematical situations.

6.1 *Methodology*
This was a quantitative research study. An assessment form on mathematical competence was employed to collect the data from fifth grade students at Bannamphrae School. The data was analyzed together with the data derived from the 17 other schools. The learning progression on mathematical problem solving was assessed in terms of the mathematical process and the mathematical strategies on the basis of the Structural Model.[3]

6.2 Instrument and Instrument Quality

The Research Instrument was the assessment form on learning progression, which focused upon mathematical problem solving on numbers and procedures comprising 6 major items and 9 minor items for fifth grade primary students

- Quality of individual items: the data was analyzed using ConQuest 2.0 program developed by Margaret L. Wu, Raymond J. Adams, Mark R. Wilson, and Samuel A. Haldane in 2007. Item difficulty, discrimination, and suitability were considered in relation to MNSQ and T in Weight Fit. It was found that all open-ended questions had been at a moderate level, and the MNSQ was in an acceptable range. Subsequently, all items were accepted to be used. According to the Threshold at each level of each response in the Wright Map, it was found that Items 1, 2, 4, 5, 6, 8, and 9 had a higher difficulty value, which corresponded to the Difficulty Level or Progress Map and had a range level from Easy to Difficult (the easiest level was scored 0; the hardest level was scored at 4). However, Item 3 and Item 7 did not have any increase in difficulty in relation to Difficulty Level or Progress Map. In addition, it was found that the MNSQ at each level of response on the Wright Map was within an acceptable CI range. The data indicated that this assessment form could be used as a tool to distinguish learner's learning progression based upon the assessment level in the created Progress Map.

- Quality of the test as a whole: According to the ConQuest 2.0 program developed by Margaret L. Wu, Raymond J. Adams, Mark R. Wilson, and Samuel A. Haldane in 2007, the test information has the maximum point of the y-axis at zero, which indicates that the difficulty level of the whole test is moderate to easy. The Internal consistency reliability (Cronbach's alpha coefficient; \( \alpha \)) was 0.83, which meant that the quality of the whole test has internal consistency. Regarding the quality analysis for the whole test, it revealed that EAP/PV reliability was 0.827, indicating that the quality of the whole test had exhibited a high degree of reliability.

7. Results

According to the analysis of learning progression on mathematical problem solving of students which is based on the set criteria, it can be concluded that 23 fifth grade students at Bannamphare School showed different levels of progression as follows: a) 10 students were at Level 0 (Poor), b) 6 students were at Level 1 (Fair), c) 3 students were at Level 2 (Good), and d) 4 students were at Level 3 (Very Good). This indicated that the students were progressing in learning mathematical problem solving at a poor to fair level. The following table shows 4 different levels of learning progression involved with mathematical problem solving.

| Table 1: The Levels of Progress Assessment in the Mathematical Process |
|---------------------------------------------------------------|
| **Level** | **Process** | **Progression Level Description/Example** |
|-----------|-------------|------------------------------------------|
| 3 (Very Good) | **Strategic/Extended Thinking** | Students can demonstrate a complex multi-step solution to find the right answer and/or can extend existing knowledge to other knowledge. For example, from the problem, "Find the sum of 3 consecutive numbers of odd number.", students can find the sum and write it in the following mathematical equation: \( n + (n + 2) + (n + 4) \). |
Table 1: (Cont’)

| Level   | Process                          | Progression Level Description/Example                                                                 |
|---------|----------------------------------|-------------------------------------------------------------------------------------------------------|
| 2 (Good)| Simple Skills and Concepts       | Students can apply basic knowledge to solving both single and double-stage math problems by solving more complex problems. The problem solving process used by the students mostly reflects that the students have an understanding of the principles or theories. However, the solution has not been totally completed. There may be some misconceptions. For example, students can apply the principle of addition, subtraction, multiplication and division to solve problems, such as \(2 + 3 \times 8 ÷ 4 - 3 = ?\). The multiplication and division have to be done first before proceeding to the addition and subtraction. However, the results may not be accurate due to some inappropriate or incorrect calculation. |
| 1 (Fair)| Basic Memory and Reproduction    | Students can apply basic knowledge to solve simple math problems. This is a one-step solution. It is a problem that students are familiar with, or a problem, which is similar to exercises in the textbook. For example, students can tell the properties of addition, subtraction, multiplication, or division. They can also remember the multiplication table or basic multiplication concepts from Sections 2 to 12 (\(2 \times 3 = 6; 12 \times 3 = 36\)). However, students lack the basic understanding of how to apply this principle to higher levels. It can be considered from a mathematical process that is not suitable for problem solving. This cannot lead to further discoveries because there are misconceptions about basic mathematical concepts. |
| 0 (Poor)| Unrecalled Memory                | Students can respond to some knowledge, but the responses reflect that the learner could not memorize important or necessary content as the basis for solving problems, such as using inappropriate mathematical processes or procedures. Students have no fundamental knowledge about the properties of addition, subtraction, multiplication, or division. Neither is it possible for them to solve simple problems because they cannot recognize multipliers or basic multiplication concepts from Sections 2 to 12. Subsequently, they have no mathematical basis to solve the problem at a higher level. Students do not give answers, nor do they answer questions, which are not related to open-ended questions. |
8. Discussion
In regard to the assessment of the learning progression on mathematical problem solving skills of fifth grade primary students at Bannamphrae School where Open Approach innovation was used in the instruction, the raw score and the full raw score was 36 points. The highest and the lowest scores were 19 and 1 points, respectively. The average score was 7.65 points of 23 students in total. Considering the individual student's progress in learning the math problems (EAP), there were four levels of grading based on the Wright Map at Level 0 to Level 3 as follows: a) 10 students were at Level 0 (Poor), b) 6 students at Level 1 (Fair), c) 3 students were at Level 2 (Good), and d) 4 students were at Level 3 (Very Good).

The researcher considered the raw score and the actual progress score of mathematical problem solving. Next, these were compared with the progress map, and then one step at a time, what students needed to know in order to enhance their mathematical competence was presented. Based on the actual competency level of the students, most students were found to be at a Poor or a Fair Level. This indicated that the students had been unable to remember important or essential content as a basis for solving problems, such as using inappropriate mathematical processes or procedures. No fundamental knowledge about the properties of addition, subtraction, multiplication, or division had been found. Furthermore, the students had been unable to find simple solutions to solve problems. Therefore, when carried out in proper order, the assessment based on the Progress Map will be a guideline for the development of the students’ potential. This process helps students learn important points that should be improved for effective self-development.

9. Suggestions
According to the analysis of learning progression on mathematical problem solving for fifth grade primary students, it was found that a large number of students had been at a poor or a fair level. Teachers should provide students with extra classes or methods to adjust their knowledge base, especially those having misconceptions about the lessons. For example, divisibility is one of the problems that students encounter. When the students’ notes had been analyzed, it was found that each of them had had different misconceptions. If these research results are implemented in math classes, students will know their levels, as well as their weaknesses, which need to be improved.

- The research results indicated that the assessment on learning and the lesson study had reflected the actual problems that had occurred in classes. Both of these should be conducted together with the Open Approach in order to successfully develop the learners’ potential to its highest.

- Based on the test results gained from fifth grade primary students at Bannamphrae School, the researcher believes if the results from this study were applied to junior primary students, it would be more effective given that these students would have more time to develop their self-learning than the senior primary students.

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