The Blazhko Effect in RR Lyrae stars: Strong observational support for the oblique pulsator model in three stars

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ABSTRACT

Using the novel data set of the MACHO project, we show that three “Blazhko Effect” RR Lyrae stars show nearly-pure amplitude modulation of a single pulsation mode. This is strong observational evidence that the “Oblique Pulsator Model” is the correct solution to this 90-year-old problem.

Subject headings: RR Lyrae — stars: fundamental parameters — stars: oscillations
1. Introduction

RR Lyrae stars are giant A and F stars which pulsate in radial modes with periods between 0.2 and 1.2 days and with peak-to-peak V amplitudes between 0.2 and 2 magnitudes. Because of their intrinsic brightness, longevity and large amplitudes, they are among the most numerous of all the classes of variable stars (in catalogues) and are important to several fields of astrophysics: as standard candles at the base of the extragalactic distance scale; as tracers of galactic dynamics and chemical evolution among the older population of stars; as test objects for stellar evolution theory of low-mass stars; and as test objects for stellar pulsation theory. The RR Lyrae stars are divided into several subclasses: RRc stars have nearly sinusoidal light curves; RRab stars have larger amplitude, sawtooth-shaped light curves. For pulsation theory and stellar evolution theory the RRd class is of particular interest since these stars pulsate simultaneously in the fundamental and first overtone radial modes, the ratio of which places a strong constraint on the stellar structure. The most intriguing subclass consists of RRab stars which show the “Blazhko Effect”. These stars appear to be fundamental mode pulsators with light curves which are modulated on a longer timescale, typically tens to hundreds of days. The effect was first noticed by Blazhko (1907) for the circumpolar RR Lyrae star RW Dra, and it is found in about 30% of all RR Lyrae stars, including the prototype of the class, RR Lyrae itself, which was been extensively studied by Shapley (1916), Walraven (1949), and Preston, Smak & Paczynski (1965). There have been many models put forth for the Blazhko Effect. Here we present strong evidence that the oblique pulsator model, in which an oblique, global magnetic field modulates the observed amplitude with the rotation of the star, is the correct model. Our dataset is novel: it has been gathered to study a completely different problem, and our data-sampling would previously have incorrectly been considered inappropriate to the solution of the Blazhko problem.
The various models that have been proposed to explain the Blazhko Effect are listed by Stellingwerf (1976) and by Smith (1995) in his monograph (see p. 109) on the RR Lyrae stars. Several of the models concentrate on beating of pulsation modes, but they suffer from the fact that most, or all, of the Blazhko stars are pulsating in the fundamental radial mode. It is necessary to resort to special pleading for non-adiabatic effects, non-linear hydrodynamics, tidal resonances, resonances between the fundamental radial mode and a higher overtone, or resonances induced by (unobserved) non-radial modes for any beating mechanism to be viable. This is because the beat periods are long, and there are no pulsation modes with frequencies near the fundamental radial mode frequency for adiabatic, spherical, non-magnetic single stars.

Another class of proposed solutions to the problem involves magnetic fields - either modulation of the pulsation amplitude caused a magnetic cycle similar to that of the sun (Detre & Szeidl 1973) or oblique pulsation similar to that of the rapidly oscillating Ap stars (Kurtz 1990). This latter model suggests that the Blazhko stars have rigid oblique magnetic fields which modulate their pulsation amplitudes with the rotation periods of the stars. This idea was vaguely suggested by Balázs (1959), Detre & Szeidl (1973) and Preston (1964). Later it was developed in some detail by Cousens (1983), who called his model the “Obliquely Oscillating Magnetic Rotator”. Independently, Kurtz (1982) proposed the same model for the rapidly oscillating Ap (roAp) stars and called it the “oblique pulsator model”. That model has been highly successful and widely used for the roAp stars, hence we will continue with the name here. The oblique pulsator model for the roAp stars has been developed in considerable detail by Shibahashi & Takata (1993) and Takata & Shibahashi (1994, 1995) who have extended the idea in an advanced form as the explanation of the Blazhko Effect in RR Lyrae stars (Takata & Shibahashi 1998).

Until recently it has not been possible to distinguish among this plethora of theories.
A first requirement to do so is for complete mathematical models for many stars of the frequencies describing the light curve modulation. Not surprisingly, traditional observing methods preclude this. Even though 90 years has passed and the problem is important, the observational demands are extreme for a single observer, or a small group of observers. The problem is well-illustrated by the work of Preston, Smak & Paczynski (1965) who observed RR Lyrae itself on 35 nights. This star has a Blazhko period of 41 d. Their data mostly cover the sharp rise of the lightcurve to its brightness maximum; very little data were obtained during the slower descending portions of the lightcurves. This was a typical observing technique at the time when frequency analysis was done by studying the timing of maxima, the so-called “O-C method”, and amplitude variations were determined from the range of minimum to maximum - observationally most easily found in the shorter rise-time portion of the non-sinusoidal lightcurve. Observations such as these send modern, powerful frequency analysis techniques, such as Fourier analysis, into a frenzy!

What is needed are complete light curves over the pulsation period (about 12 hours, typically) and the Blazhko period (months, typically). To do this means either: 1) a multisite campaign with observers stationed at sites around the globe continuously for more than one Blazhko cycle - months of work for probably half a dozen observers, all to get data for only a few stars at best; or 2) a very long-term observing project from one site to build up a sufficient data set, or 3) a revolutionary new way to tackle the problem.

Remarkably, the second method has shown some success. Kovács (1995), working with a heroic data set mostly obtained by Kanyó (1976) over a period of 6 years, has shown that the frequencies of the circumpolar RR Lyrae star RV UMa comprise an equally-spaced triplet plus its harmonics. This is close to the predicted frequency behavior for the oblique pulsator model. Pure amplitude modulation of a magnetically-distorted, fundamental radial mode in the oblique pulsator model generates a frequency multiplet with spacings
exactly equal to the rotation frequency of the star (Takata & Shibahashi 1998). Pure amplitude modulation also requires equality of the phases of the frequency multiplet in the oblique pulsator model, a condition which was not examined by Kovács, but which does not precisely obtain for RV UMa. Because of the lack of any believable magnetic measurements for the Blazhko stars, and because of a perceived need for non-radial pulsation modes, as observed in the roAp stars, Kovács was very reserved about the viability of the oblique pulsator model explanation of the Blazhko Effect, and, for that matter, all the other models, too. To make progress on this nearly century-old problem we have utilized the extensive photometric databases of the microlensing survey known as the MACHO Project.

2. MACHO Project Observations

The MACHO (MAssive Compact Halo Object) Project (Alcock et al. 1992) is an astronomical survey experiment designed to obtain multi-epoch, two-color CCD photometry of millions of stars in the Large Magellanic Cloud (LMC) (also, in the galactic bulge and Small Magellanic Cloud = SMC). The principal goal of the project is to search for massive compact objects whose presence between the observer and a background source will result in an amplification of received flux caused by gravitational lensing. The expected rate of detectable events is very low, requiring large numbers of background sources - in this case, LMC stars - to be measured over many years. The survey makes use of a dedicated 1.27-m telescope at Mount Stromlo, Australia and because of its southerly latitude is able to obtain observations of the LMC year-round. This lack of seasonal aliasing is one factor which makes these observations so suitable for studying the Blazhko effect in RR Lyrae stars.

The camera built specifically for this project (Stubbs et al. 1993) achieves a field-of-view of 0.5 square degrees by imaging at prime focus. Observations are obtained in two bandpasses simultaneously, using a dichroic beamsplitter to direct the ‘blue’
(approximately 450-630 nm) and ‘red’ (630-760 nm) light onto 2×2 mosaics of 2048×2048 pixel Loral CCD chips. Images are obtained and read out simultaneously. The 15 µm pixel size maps to 0.63 arcsec on the sky. The data were reduced using a profile-fitting photometry routine known as Sodophot, derived from DoPHOT (Schechter, Mateo & Saha 1993). This implementation employs a single starlist generated from frames obtained in good seeing.

The results reported in this paper comprise only a small fraction of all the RR Lyrae stars (Blazhko and non-Blazhko) found by the MACHO Project. In the first instance, we sought stars which were likely foreground RR Lyrae stars according to Alcock et al. (1997). Before analysis, lightcurves were brought up to date to maximise the number of Blazhko cycles observed. Many more Blazhko RR Lyrae stars have been identified (and many remain to be confirmed) based on goodness-of-fit statistics for single-cycle periods for variable stars with appropriate magnitudes and colors toward the LMC and SMC.

Typically, the dataset for a given star covers a time-span of about 2000 d and contains 750 photometric measurements (multiple observations of some fields are obtained on a given night whenever conditions allow). Fainter, LMC member stars may have fewer useful epochs of photometry. The output photometry contains flags indicating suspicion of errors due to crowding, seeing, array defects, and radiation events. Only data free from suspected errors were employed for this work. Typical photometric uncertainties are in the range 15 to 20 mmag (millimagnitudes) or 1.5 to 2% in intensity.

With only one or two points per night, it would not seem at first thought to be possible to do a definitive frequency analysis for stars with pulsation periods on the order of 12 hours. For equally-spaced data it is well known that the frequency solutions from Fourier analysis are ambiguous with equally probable solutions occurring above and below the Nyquist frequency, twice the Nyquist frequency, etc. This is simply because of beating
between the real frequencies and the sampling frequency. However, with unequally-spaced data, such as the MACHO data, the ambiguities disappear for large enough data sets. “Large enough” is easy to define: it means that the data gathered are well-spread over all phases of all frequencies present in the data. These criteria are met for many of the Blazhko stars in the MACHO Project dataset.

Some of these stars show pure amplitude modulation of their light curves over the Blazhko period. This is as predicted for the oblique pulsator model for these stars, hence it gives very strong support for this model. Other stars, which will be discussed in future publications, show pure frequency modulation - they are probably binary stars where the pulsation frequency is Doppler-shifted by the orbital motion. Most stars show both amplitude and frequency modulation. Whether this is simply a combination of oblique pulsation and orbital motion, or whether there are further complexities to the physics of some stars classified as Blazhko stars is still to be determined.

3. Analysis, Results and Discussion

We have performed frequency analyses on over 20 RR Lyrae stars from the MACHO Project data using standard Discrete Fourier Transforms (Deeming 1975; Kurtz 1985). Three of the stars show nearly the pure amplitude modulation expected from the oblique pulsator model. It can be seen in Figure 1 from the tightness of the rising branch that there is little, or no phase modulation in star 82.8410.55 (our star naming convention is field.tile.sequence - see Alcock et al. 1992). It can also be seen here and in Figure 2 that the amplitude modulation mostly affects the maximum of the lightcurve, and has a much smaller effect on the minimum.

Table 1 shows the results of our frequency analysis for star 82.8410.55. Following the
notation of Kovács (1995): We find that a stationary Fourier-sum with frequencies of the form $kf_0$, $kf_0 \pm f_m$, where $k = 1, 2, 3, 4, 5, 6, 7$; $f_0 = 1.932166 \pm 0.000045 \, d^{-1}$ and $f_m = 0.011642 \pm 0.000029 \, d^{-1}$ gives a complete fit to the data to a standard deviation per observation of 0.028 mag, which is close to our estimated error. We have chosen a zeropoint to the timescale for which the phases $\varphi(f_0 + f_m) = \varphi(f_0 - f_m)$. For pure amplitude modulation the $\varphi(f_0)$ would also be equal to these other phases. It can be seen in Table [ that this is nearly the case, although the deviation is significant by 10$\sigma$.

All of the above is consistent with the oblique pulsator model as formulated by Takata & Shibahashi (1998). In that model an oblique, dipole magnetic field modulates the amplitude of the fundamental radial mode as a function of rotation. Mathematically, the result can be described as a mode with radial and quadrupole contributions. However, this does not imply that the mode is non-radial. The mode excited is still the radial fundamental mode, but it no longer can be described by a single spherical harmonic, because of the distorting effect of the magnetic field which suppresses the pulsation surface-brightness amplitude at the magnetic equator where the photosphere has to cross field lines, while doing little to modify the amplitude at the magnetic poles where the motion is along the field lines.

Takata & Shibahashi predict that a frequency quintuplet will be generated by the amplitude modulation, but it is clear from their equations and diagrams that for many orientations the outer two frequencies of the quintuplet have much lower amplitudes, so it is not surprising that we have yet to detect them. That will take better signal-to-noise ratio data than we have, or the discovery of a star with an appropriate rotational inclination and magnetic obliquity. A similar result is seen for the roAp stars where a frequency triplet can be seen in several stars, and where very high signal-to-noise ratio data show a frequency septuplet in HR 3831 (Kurtz et al. 1997).
Similarly, the small, but significant difference in the phase $\varphi(f_0)$ and $\varphi(f_0 \pm f_m)$ is not surprising. This same effect is seen in the roAp stars, particularly HR 3831. This is explained by a deviation from pure dipolar symmetry for the magnetic field - a deviation which is known to exist in all Ap stars for which sufficiently high accuracy observations are available.

Our results for star 82.8410.55 show almost pure amplitude modulation with a period of $85.9 \pm 0.2 \, \text{d}$. We conclude that this is strong evidence in favor of the oblique pulsator model for this star, and by extension for the Blazhko Effect. There is no need to invoke non-radial modes as many other Blazhko explanations do. Only the fundamental radial mode is excited. Models which suggest more than one excited mode are ruled out by the frequencies in Table I. If $f_0 - f_m$, $f_0$ and $f_0 + f_m$ were independent modes, then we would expect to see their harmonics, since the pulsation is non-linear. We do not see this. Instead, we see the multiples $k f_0$ split by $\pm f_m$, as expected in the oblique pulsator model. Notice also that the phases of the frequency triplets for the harmonics also show nearly equal phases - indicating that the harmonics are amplitude-modulated in phase with the fundamental. Independently excited modes would not necessarily behave that way.

Kovács (1995) suggested several potential weaknesses in the oblique pulsator model: 1) There is no verification of magnetic fields in the Blazhko stars; 2) all known Blazhko stars have modulation amplitudes in the range $0.3 - 0.7 \, \text{mag}$, whereas there should be no lower limit, since the amplitude range is a function of aspect angle, and all angles are permissible; and 3) the Blazhko stars fit the standard period-amplitude relation in their high-amplitude state. We note that 1) Takata & Shibahashi’s formulation of the oblique pulsator model requires field strengths of about 1 kG. At present there are no observations which confirm or refute the presence of magnetic fields of this strength. Such observations are strongly desirable. 2) The lower limit to the modulation amplitude may be a discovery selection bias,
so that lower amplitude Blazhko stars are more difficult to detect, hence are not yet noted. We agree that conclusive proof that no Blazhko stars exist with amplitudes lower than 0.3 mag would constitute a strong challenge to the oblique pulsator model. Our large dataset should allow us to examine this. 3) In Takata & Shibahashi’s formulation of the oblique pulsator model the energy of the mode is not affected by the presence of the magnetic field, so the amplitude at maximum is equal to that which would obtain in the absence of any magnetic field, hence the standard period-amplitude relation is expected to fit.

The last three columns of Table 1 give quantitative measures which can be tested against more sophisticated predictions of the oblique pulsator model when the theory is more advanced, and against magnetic observations when they become available. The parameter $\frac{A_{+1} + A_{-1}}{A_0}$ depends on the inclination of the rotation axis of the star to the line-of-sight and the obliquity of the magnetic axis to the rotation axis. If the Blazhko stars have approximately dipolar magnetic fields, then these geometrical angles can be derived independently from magnetic observations. That $\frac{A_{+1} + A_{-1}}{A_0}$ grows for the higher harmonics is a consequence of the amplitude modulation shown in Figure 2. There is no theoretical explanation of this yet - this is the first observational description of the effect. The same holds true for the phases in the penultimate column of Table 1. We predict from the oblique pulsator Model that the time of magnetic maximum for star 82.8410.55 will coincide with the time of pulsation maximum seen in Figure 2 and given in the caption to Table 1.

We have two more stars, AC Men (6.5722.3, also known as HV 924) and 5.5376.3686, which show pure amplitude modulation. Tables 2 and 3 show the frequency structure for these stars. Figures 3–6 show the phased lightcurves. AC Men even has exact equality of the phases $\varphi(f_0) = \varphi(f_0 + f_m) = \varphi(f_0 - f_m)$. We consider the pure amplitude modulation of this, and the nearly pure amplitude modulation of stars 82.8410.55 and 5.5376.3686 very strong support for the oblique Pulsator Model for the Blazhko effect as formulated by
Takata & Shibahashi. There is a great wealth of astrophysical data yet to be extracted from our observations and frequency analyses. We have stars which show pure frequency modulation, and stars which show a combination of frequency and amplitude modulation over their Blazhko cycles. The amplitudes and phases of the determined frequencies give detailed information about the behavior of the pulsation harmonics. Understanding these astrophysically is a great challenge for non-linear pulsation theory. A taste of this is shown in Figure 2 where it can be seen that the Blazhko amplitude modulation for star 82.8410.55 occurs mostly at the maximum of the light curve. The modulation at minimum is much less pronounced. Within the oblique pulsator model this means that the minimum state is much less affected by the orientation of the field lines than the maximum state - an interesting observation, and an interesting problem.

To assist other observers in studying these stars, we provide finder charts from ‘red’ MACHO Project images for 82.8410.55, AC Men (6.5722.3), and 5.5376.3686 in Figures 7, 8, and 9, respectively.

We can look forward to a flood of more information and interpretation of the Blazhko stars as we mine the vast data resources of the MACHO Project. Finally, after 90 years, we have found the practical observing technique for these stars.

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Table 1. Frequency Solution for 82.8410.55

| Frequency | A     | \(\varphi\) | \(\frac{A_{i+1} + A_{i-1}}{A_0}\) | \(\varphi_+ - \varphi_-\) | \(\frac{\varphi_+ - \varphi_-}{\sigma}\) |
|-----------|-------|-------------|-------------------------------|-----------------|------------------|
| \(f_0 - f_m\) | 1.932194 | 54.0 ± 1.5 | 2.216 ± 0.029 | 0.000 ± 0.042 | 0.0 |
| \(f_0\) | 1.943836 | 369.8 ± 1.5 | 1.987 ± 0.004 | 0.28 ± 0.01 | -0.229 ± 0.021 | -10.8 |
| \(f_0 + f_m\) | 1.955478 | 50.3 ± 1.5 | 2.216 ± 0.031 |                           |                   |
| \(2f_0 - f_m\) | 3.876030 | 31.9 ± 1.5 | 1.741 ± 0.048 | -0.016 ± 0.071 | -0.2 |
| \(2f_0\) | 3.887672 | 149.2 ± 1.5 | 1.545 ± 0.011 | 0.42 ± 0.02 | -0.188 ± 0.036 | -5.3 |
| \(2f_0 + f_m\) | 3.899314 | 30.6 ± 1.5 | 1.725 ± 0.052 |                           |                   |
| \(3f_0 - f_m\) | 5.819866 | 30.8 ± 1.5 | 1.339 ± 0.051 | 0.105 ± 0.077 | 1.4 |
| \(3f_0\) | 5.831508 | 102.4 ± 1.5 | 1.365 ± 0.016 | 0.56 ± 0.02 | -0.027 ± 0.039 | -0.7 |
| \(3f_0 + f_m\) | 5.843150 | 26.7 ± 1.5 | 1.444 ± 0.058 |                           |                   |
| \(4f_0 - f_m\) | 7.763702 | 24.2 ± 1.5 | 1.067 ± 0.064 | 0.416 ± 0.103 | 4.0 |
| \(4f_0\) | 7.775344 | 62.6 ± 1.5 | 1.293 ± 0.025 | 0.69 ± 0.04 | 0.018 ± 0.052 | 0.3 |
| \(4f_0 + f_m\) | 7.786986 | 19.2 ± 1.5 | 1.483 ± 0.081 |                           |                   |
| \(5f_0 - f_m\) | 9.707538 | 13.2 ± 1.5 | 0.818 ± 0.116 | 0.519 ± 0.159 | 3.3 |
| \(5f_0\) | 9.719180 | 37.2 ± 1.5 | 1.191 ± 0.041 | 0.74 ± 0.07 | 0.114 ± 0.081 | 1.4 |
| \(5f_0 + f_m\) | 9.730822 | 14.2 ± 1.5 | 1.337 ± 0.109 |                           |                   |
| \(6f_0 - f_m\) | 11.651374 | 11.1 ± 1.5 | 0.481 ± 0.139 | 0.614 ± 0.250 | 2.5 |
| \(6f_0\) | 11.663016 | 22.4 ± 1.5 | 1.125 ± 0.069 | 0.83 ± 0.11 | 0.337 ± 0.130 | 2.6 |
| \(6f_0 + f_m\) | 11.674658 | 7.4 ± 1.5 | 1.095 ± 0.208 |                           |                   |
Table 1—Continued

| Frequency | A       | \( \varphi \) | \( \frac{A_{+1} + A_{-1}}{A_0} \) | \( \varphi_+ - \varphi_- \) | \( \frac{\varphi_+ + \varphi_-}{2} \) |
|-----------|---------|---------------|---------------------------------|-----------------------------|-----------------------------|
| \( 7f_0 - f_m \) | 13.595210 | 8.2 ± 1.5 | 0.424 ± 0.188 | 0.622 ± 0.293 | 2.1 |
| \( 7f_0 \) | 13.606852 | 9.4 ± 1.5 | 0.858 ± 0.162 | 1.60 ± 0.35 | 0.7 |
| \( 7f_0 + f_m \) | 13.618494 | 6.9 ± 1.5 | 1.046 ± 0.225 | | |

Note. — The star pulsates with only one frequency, \( f_0 = 1.943836 \pm 0.000006 \) \( d^{-1} \) (\( P = 12.346721 \pm 0.00038 \) hours). The amplitude of this frequency is modulated with the Blazhko frequency of \( f_m = 0.01164 \pm 0.00003 \) \( d^{-1} \) (\( P_{Blazhko} = 85.9 \pm 0.2 \) d), which, within the oblique pulsator model, we equate to the rotation frequency of the star (or twice the rotation frequency if both pulsation/magnetic poles are seen from equal aspect). The parameters in this table fit the relation:

\[
\text{mag} = A_0 + \sum_{k=1}^{n} \left\{ A_- (k) \cos [2\pi (kf_0 - f_m)(t - t_0) + \varphi_- (k)] \right\}
\]

where \( n = 7 \) and \( t_0 = \text{HJD} \ 2449854.8322 \) for our instrumental magnitudes. Predicting the quantities in the last three columns, particularly for the harmonic frequencies, presents a strong challenge to non-linear pulsation theory.
Table 2. Frequency Solution for AC Men (6.5722.3)

| Frequency | $A$ | $\varphi$ | $\frac{A_{+1} + A_{-1}}{A_0}$ | $\varphi_{+} - \varphi_{-}$ | $\frac{\varphi_{+} - \varphi_{-}}{\sigma}$ |
|-----------|-----|-----------|-----------------------------|-----------------------------|-----------------------------|
| $f_0 - f_m$ | 1.798715 | 39.7 ± 3.5 | 2.234 ± 0.088 | 0.00 ± 0.12 |
| $f_0$ | 1.806942 | 414.8 ± 3.5 | 2.195 ± 0.008 | 0.19 ± 0.01 | -0.04 ± 0.06 | -0.6 |
| $f_0 + f_m$ | 1.815169 | 39.5 ± 3.5 | 2.234 ± 0.088 |   |
| $2f_0 - f_m$ | 3.605657 | 7.9 ± 3.5 | 1.271 ± 0.450 | 0.92 ± 0.47 | 1.9 |
| $2f_0$ | 3.613884 | 175.5 ± 3.5 | 1.954 ± 0.020 | 0.18 ± 0.03 | 0.23 ± 0.24 | 0.9 |
| $2f_0 + f_m$ | 3.622111 | 23.7 ± 3.5 | 2.187 ± 0.148 |   |
| $3f_0 - f_m$ | 5.412599 | 19.2 ± 3.5 | 1.361 ± 0.182 | 0.78 ± 0.25 | 3.0 |
| $3f_0$ | 5.420826 | 116.1 ± 3.5 | 2.055 ± 0.030 | 0.33 ± 0.04 | 0.31 ± 0.13 | 2.4 |
| $3f_0 + f_m$ | 5.429053 | 19.5 ± 3.5 | 2.137 ± 0.178 |   |
| $4f_0 - f_m$ | 7.219541 | 15.8 ± 3.5 | 1.159 ± 0.224 | 1.30 ± 0.32 | 4.0 |
| $4f_0$ | 7.227768 | 69.8 ± 3.5 | 2.220 ± 0.050 | 0.44 ± 0.07 | 0.41 ± 0.16 | 2.5 |
| $4f_0 + f_m$ | 7.235995 | 14.8 ± 3.5 | 2.463 ± 0.235 |   |
| $5f_0 - f_m$ | 9.026483 | 10.1 ± 3.5 | 2.026 ± 0.346 | 0.40 ± 0.40 | 1.0 |
| $5f_0$ | 9.034710 | 44.8 ± 3.5 | 2.221 ± 0.078 | 0.60 ± 0.12 | 0.00 ± 0.21 | 0.0 |
| $5f_0 + f_m$ | 9.042937 | 16.8 ± 3.5 | 2.422 ± 0.206 |   |
| $6f_0 - f_m$ | 10.833425 | 9.7 ± 3.5 | 2.422 ± 0.360 | 0.06 ± 0.47 | 0.1 |
| $6f_0$ | 10.841652 | 31.3 ± 3.5 | 2.404 ± 0.112 | 0.69 ± 0.18 | -0.05 ± 0.25 | -0.2 |
| $6f_0 + f_m$ | 10.849879 | 11.8 ± 3.5 | 2.487 ± 0.296 |   |
| $7f_0 - f_m$ | 12.640367 | 8.3 ± 3.5 | 2.288 ± 0.418 | 0.29 ± 0.55 | 0.5 |
| $7f_0$ | 12.648594 | 17.4 ± 3.5 | 2.496 ± 0.200 | 1.03 ± 0.35 | 0.06 ± 0.32 | 0.2 |
| $7f_0 + f_m$ | 12.656821 | 9.6 ± 3.5 | 2.575 ± 0.362 |   |
Table 2—Continued

| Frequency | A     | $\varphi$ | $\frac{A_{+1}+A_{-1}}{A_0}$ | $\varphi_+ - \varphi_-$ | $\frac{\varphi_+ - \varphi_-}{\sigma}$ |
|-----------|-------|-----------|-----------------------------|--------------------------|---------------------------------|
| $(d^{-1})$ | $(\text{mmag})$ | $(\text{rad})$ |                             |                          |                                 |
| $8f_0 - f_m$ | 14.447309 | 11.0 $\pm$ 3.5 | 0.931 $\pm$ 0.314 | 0.10 $\pm$ 0.42 | 2 $\pm$ 0.2 |
| $8f_0$ | 14.455536 | 9.5 $\pm$ 3.5 | 1.342 $\pm$ 0.365 | 2.42 $\pm$ 1.00 | 0.36 $\pm$ 0.34 | 1.0 |
| $8f_0 + f_m$ | 14.463763 | 12.1 $\pm$ 3.5 | 1.034 $\pm$ 0.282 |                          |                                 |

Note. — The star pulsates with only one frequency, $f_0 = 1.806942 \pm 0.000008$ $d^{-1}$ ($P = 13.28211 \pm 0.00006$ hours). The amplitude of this frequency is modulated with the Blazhko frequency of $f_m = 0.00823 \pm 0.00004$ $d^{-1}$ ($P_{\text{Blazhko}} = 121.6 \pm 0.6$ $d$). Other parameters are the same as Table II with where $n = 8$ and $t_0 = \text{HJD 2449656.8470}$. 
Table 3. Frequency Solution for 5.5376.3686

| Frequency | A (d<sup>−1</sup>) | ϕ (mmag) | ϕ<sub>A</sub><sup>−1</sup> | ϕ<sub>A</sub> | σ/σ<sub>A</sub><sup>−1</sup> | σ/σ<sub>A</sub> | σ/σ<sub>A</sub><sup>−1</sup> | σ/σ<sub>A</sub> |
|-----------|------------------|----------|----------------|----------|-----------------|----------|----------------|----------|
| f<sub>0</sub> - f<sub>m</sub> | 1.830195 | 79.9 ± 5.4 | -0.59 ± 0.07 | -0.01 ± 0.09 | 0.0 |
| f<sub>0</sub> | 1.853549 | 296.6 ± 5.4 | -0.89 ± 0.02 | 0.56 ± 0.03 | -0.30 ± 0.05 | -6.4 |
| f<sub>0</sub> + f<sub>m</sub> | 1.876904 | 87.2 ± 5.4 | -0.60 ± 0.06 |
| 2f<sub>0</sub> - f<sub>m</sub> | 3.683744 | 24.0 ± 5.4 | 2.17 ± 0.22 | -0.02 ± 0.24 | -1.1 |
| 2f<sub>0</sub> | 3.707098 | 138.5 ± 5.4 | 2.09 ± 0.04 | 0.58 ± 0.06 | -0.07 ± 0.12 | -0.6 |
| 2f<sub>0</sub> + f<sub>m</sub> | 3.730453 | 56.6 ± 5.4 | 2.15 ± 0.10 |
| 3f<sub>0</sub> - f<sub>m</sub> | 5.537293 | 10.6 ± 5.4 | -0.65 ± 0.51 | -0.24 ± 0.53 | -0.4 |
| 3f<sub>0</sub> | 5.560647 | 88.1 ± 5.4 | -1.00 ± 0.06 | 0.52 ± 0.09 | -0.23 ± 0.27 | -0.9 |
| 3f<sub>0</sub> + f<sub>m</sub> | 5.584002 | 35.5 ± 5.4 | -0.89 ± 0.15 |
| 4f<sub>0</sub> - f<sub>m</sub> | 7.390842 | 27.0 ± 5.4 | 2.74 ± 0.20 | -0.26 ± 0.27 | -0.9 |
| 4f<sub>0</sub> | 7.414196 | 49.8 ± 5.4 | 2.02 ± 0.11 | 1.13 ± 0.20 | -0.59 ± 0.15 | -4.0 |
| 4f<sub>0</sub> + f<sub>m</sub> | 7.437551 | 29.1 ± 5.4 | 2.48 ± 0.19 |
| 5f<sub>0</sub> - f<sub>m</sub> | 9.244391 | 6.1 ± 5.4 | -0.38 ± 0.88 | -0.65 ± 0.90 | -0.7 |
| 5f<sub>0</sub> | 9.267745 | 37.8 ± 5.4 | -0.96 ± 0.14 | 0.91 ± 0.24 | -0.25 ± 0.47 | -0.5 |
| 5f<sub>0</sub> + f<sub>m</sub> | 9.291100 | 28.4 ± 5.4 | -1.03 ± 0.19 |
| 6f<sub>0</sub> - f<sub>m</sub> | 11.097940 | 15.2 ± 5.4 | 2.59 ± 0.36 | -0.12 ± 0.43 | -0.3 |
| 6f<sub>0</sub> | 11.121294 | 26.3 ± 5.4 | 2.55 ± 0.20 | 1.45 ± 0.42 | 0.02 ± 0.25 | 0.1 |
| 6f<sub>0</sub> + f<sub>m</sub> | 11.144649 | 22.9 ± 5.4 | 2.47 ± 0.24 |

Note. — The star pulsates with only one frequency, f<sub>0</sub> = 1.853549 ± 0.000009 d<sup>−1</sup> (P = 12.94813 ± 0.00006 hours). The amplitude of this frequency is modulated with the Blazhko frequency of f<sub>m</sub> = 0.02335 ± 0.00002 d<sup>−1</sup> (P<sub>Blazhko</sub> = 42.82 ± 0.04 d). Other parameters are the same as Table 1 with where n = 6 and t<sub>0</sub> = HJD 2449891.9649.
Fig. 1.— This figure shows all of our data transformed to V for the star 82.8410.55 phased with the pulsation period of 12.346721 ± 0.00038 hours. The amplitude modulation occurs primarily at maximum. The tight fit of the rising branch of the lightcurve shows the lack of phase modulation over the Blazhko cycle which is 85.9 ± 0.2 d in this star. All points are plotted twice in phase to reveal continuity.
Fig. 2.— This figure plots the same data as Figure 1, but now phased with the $85.9 \pm 0.2$ d Blazhko period. This shows clearly for the first time that the amplitude modulation is much larger at maximum than at minimum. The physics of this is, as yet, unexplained.
Fig. 3.— This figure shows all of our data transformed to V for AC Men (6.5722.3) phased with the pulsation period of $13.28211 \pm 0.00006$ hours. As for star 82.8410.55, the amplitude modulation occurs primarily at maximum. The tight fit of the rising branch of the lightcurve shows the lack of phase modulation over the Blazhko cycle which is $121.6 \pm 0.6$ d in this star. All points are plotted twice in phase to reveal continuity.
Fig. 4.— This figure plots the same data as Figure 3, but now phased with the 121.6 ± 0.6 d Blazhko period.
Fig. 5.— This figure shows all of our data transformed to V for star 5.5736.3686 phased with the pulsation period of 12.94813 ± 0.00006 hours.
Fig. 6.— This figure plots the same data as Figure 5, but now phased with the 42.82 ± 0.04 \(d\) Blazhko period.
Fig. 7.— This finder chart for the field Blazhko RR Lyrae star 82.8410.55 was obtained in the ‘red’ MACHO bandpass. The star is indicated by the crosshairs and is located at equinox J2000.0 coordinates $\alpha$ 05:32:19.0 and $\delta$ -68:51:52. North is up, east is to the left. The chart is 3 arcmin in its longest dimension.
Fig. 8.— This finder chart for the field Blazhko RR Lyrae star AC Men (6.5722.3) was obtained in the ‘red’ MACHO bandpass. The star is indicated by the crosshairs and is located at equinox J2000.0 coordinates $\alpha$ 05:15:50.1 and $\delta$ -70:36:47. North is up, east is to the left. The chart is 3 arcmin in its longest dimension.
Fig. 9.— This finder chart for the LMC Blazkho RR Lyrae star 5.5376.3686 was obtained in the ‘red’ MACHO bandpass. The star is indicated by the crosshairs and is located at equinox J2000.0 coordinates $\alpha$ 05:13:33.6 and $\delta$ -69:26:21. North is up, east is to the left. The chart is 3 arcmin in its longest dimension.