Design of problem-solving questions for measuring mathematical thinking type mathematization

A Nurzalena*, E Susanti, H Hapizah, M Meryansumayeka and A Miswanto
Mathematics Education, Sriwijaya University, Palembang, Indonesia

*annisanurzalena@student.unsri.ac.id

Abstract. Mathematical thinking is a thought process in developing a mathematical perspective that involves other mathematical abilities such as modeling, reasoning, proving, symbolization, representation, abstraction, and mathematization. This study aims to describe the design results of problem-solving question that can be used to measure mathematical thinking type mathematization. This research is design research consisting of five stages, namely: preliminary design, focus group discussions, trials, observations and interviews, and retrospective analysis. The instrument consisted of test, observation, and interview. Data analysis uses qualitative methods. Based on data analysis, the problem-solving questions that are designed can already measure mathematical thinking type mathematization. This can be theoretically seen from the results of focus group discussions, which states the questions have been based on content, construct and language. Illustrated from the results of the trial, the strategies that have been used by the subject specifically on question number 1 is by making pictures and using certain mathematical formulas. Which solution illustrates that the subject has geometrization and formalization capabilities. Whereas in problem number 2, the mathematical ability that arises is connecting and formalization because in solving problems most subjects associate certain mathematical concepts and ideas with use facts, concepts and rules of mathematics.

1. Introduction
The expected goals in mathematics learning according to the National Council of Teachers of Mathematics (NCTM) are problem solving, proving, reasoning, connections, communication, and representation [1]. Problem solving ability is the heart of mathematics [2]. This is in line with National Council of Teachers of Mathematics which states that problem solving is an integral part of mathematics learning, so that it cannot be separated from mathematics learning [1]. In line with this, Nurdiana also argues that problem-solving ability is very important in mathematics, not only for those who will later explore or study mathematics, but also for those who will apply it in other fields of study and apply it in their daily lives [3].

But in reality, students in Indonesia have not met these expectations. Based on the research of OECD [4], Winarni [5] and Mullis [6] explain that PISA and TIMSS have the same results, namely the ability of Indonesian students to solve problems is still relatively low. Clive et al states that PISA questions are a problem with the type of problem solving that strongly demands reasoning ability and students' problem solving abilities [7]. Even according to OECD, the PISA problem requires students' problem solving abilities especially for level 3 of PISA questions [8]. In addition, according to Stacey, the main factors causing the achievement of Indonesian students in the 2007 and 2011 TIMSS were low because...
of the low ability of students to solve problems [9]. Therefore it can be concluded that the low results of the PISA study and TIMSS students show the low ability of students to solve problems.

In fact, the important of problem-solving abilities are owned by students to train so that they are accustomed to dealing with problems in everyday life that are increasingly complex. With the low ability of students to solve problems, it is necessary to see how the mathematical abilities of these students. Because, to develop students' abilities in problem solving, what is needed is how mathematical thinking ability. Mathematical thinking is an important aspect of learning, especially in mathematics learning in Indonesia. The ability of students at the stage of thinking is a related factor. The development of mathematical thinking is very influential on problem solving and reasoning abilities, as interrelated components. The development of high-level mathematical thinking skills can help students solve problems. Students must be able to build and develop their own minds in connecting a definition and theorem to solve the problems they face [10].

According to Karadag, mathematical thinking is divided into seven types, namely modeling, reasoning, symbolization, representation, proving, abstraction, and mathematization [11]. But in this study researchers will only focus on mathematization. Mathematization is a process for mathematical real world [12]. Then Carla added that, mathematical is a process of translating real problems into symbolic forms of mathematics and can be interpreted as structuring or constructing a mathematical world [13]. According to Drijvers, mathematization is an activity to change the problem into a mathematical problem by symbolizing the problem and rearranging the mathematical system [14]. Based on the above opinion, it can be concluded that mathematization is a mathematical process that is adapted to the mathematical concept in solving a problem in everyday life. According to Karadag, matematization is divided into four types, namely formalization, connecting, geometrization, and optimization [11].

Mathematization is a process for mathematical thinking in solving a problem. The process of formalization involves the use, facts, concepts and procedures of mathematics [15]. Connecting is a learning activity where students can define how to solve a problem, situations, and mathematical ideas that are interconnected into the form of a mathematical model, and students can apply the knowledge gained to answer and solve a problem to another problem [16]. Geometrization is the application of geometrical concepts and techniques for non-geomatics problems. An important component of geometrization is to make a bridge between two different things to solve a problem [17]. Optimization is focuses on obtaining systematic minimum or maximum values from a function, opportunity, and other values in various cases [18]. The four types of mathematization are one component that is very important for students to think mathematically to solve the problems faced. The strategy they use can be so varied depending on how their perspective perceives a problem they encounter and how they think of a solution that will be used to solve the problem at hand.

Stacey concluded that the ability to think mathematically in solving a problem is an important goal of the school. Thus, a student needs to have mathematical thinking skills, to enable them to solve other problems [19]. To familiarize students with these mathematical abilities, students must often be faced with problems that require problem solving. So that they are accustomed to dealing with similar cases. Based on the description above, it is indicated that there are links between problem solving problems and mathematical thinking abilities. So the researchers were interested in conducting a study entitled "Design of Problem-Solving Questions for Measuring Mathematical Thinking Type Mathematization".

2. Research methodology

This research uses design research method because it aims to describe and identify the design results of problem solving questions that can be used to measure mathematical thinking type of mathematization in high school students which consists of five stages, namely: Preliminary Design, Focuss Group Discussion, Trial, Observation and Interview, Retrospective Analysis. The instrument used in this study consisted of test questions, observation sheets, interview sheets. The subject of this study consisted of 5 subjects with initially FF, AN, NT, AE and R. With the criteria of good, medium and low mathematical
abilities. The implementation of this study consisted of several stages described in the following figure 1:

![Figure 1. Stage of research implementation.](image)

3. Results and discussion

3.1. Preliminary design

Through the Preliminary Design process researchers search, read, and understand each literature from various sources regarding the criteria of problem solving, strategies to solve problem and steps in solving problem. Through various references that have been collected, then researchers see and compare several examples of problem solving question that can roughly measure mathematical thinking type mathematization as inspiration for researchers to design questions.

3.2. Focus Group Discussion

Researchers conducted a Focus Group Discussion with 2 experts, after discussing the three questions that had been made, there are some revision that must be improved. After the revision done, the questions changes as in the following table:

| No. | Question |
|-----|----------|
| 1   | A map of a city is illustrated by the following picture. Dito lives at the fourth crossroads in the Pelaju area. Ayu lives at the eighth intersection in the Pakjo area. Every day Dito will pick Ayu up to go to school together. Dito wants to find the nearest road to reach the house. The road can only be passed through east and north. How many different routes can Dito go to Ayu's house? |

| No. | Question |
|-----|----------|
| 2   | Aisyah is a class XI IPA 1 student and Rogaya is a grade XI IPA 2 student. Every month they have an obligation to pay class cash. The difference in cash that must be paid by Aisyah and Rogaya is 5 thousand. In February, the amount of each of Aisyah and Rogaya's money was Rp 600,000 and Rp 630,000. If in a given month their amount of money will be the same. So, what is the rest of each of Aisyah and Rogaya's money will be the same? |
3.3. Trial
The results of the draft are then tested on 5 research subjects with initial namely FF, AN, NT, AE and R. After observing the subject resolves the given problem, then a direct interview is conducted on the answers that have been done based on mathematical thinking type mathematization to determine and describe the type of mathematical thinking of the subject in depth.

3.4. Retrospective analysis
In problem table 5, it can be seen from the completion sheet on figure 2, if the AN subject tries to solve the problem with a trial and error strategy to sketch the route drawing one by one.

Figure 2. Results of Question Number 1 Subject AN.

Figure 2 shows a strategy that categorized as type of Geometrization because the subject associates the problem with the geometrical concept that has been studied where the subject draw each route one by one and then calculates how many possible road routes from the resulting image. visualizing problems with picture representation can help students translate abstract ideas into more tangible forms so as to facilitate students in understanding the problems easily.

There are also other strategies used by AK subjects as shown in the following figure 3.

Figure 3. Results of question number 1 subject AK.

To resolve these problems. AK subjects understand the concept of the formula to be used, where the subject has been able to distinguish when to use a combination formula and permutation. from figure 3, the researcher can conclude if AK subjects have mathematization type formalization ability because in solving these problems the subject involves mathematical rules and procedures. Through the strategies that have been used, the subject of AK uses a combination formula in solving these problems and is able to complete calculations according to the combination procedure itself.

The strategy from figure 2 and figure 4 show if the students who have not been able to think abstractly will tend to use geometry in describing what is contained in the problem. but on the contrary students who have thought abstractly tend to directly use mathematical symbols as a result of pouring their
mathematical ideas. Because students who try to solve problems with geometry or try to make a picture first generally have a thought process that still tends to the concrete thinking stage.

Figure 4. Results of question number 2 subject AE.

Figure 4 show AE subjects solve problems using principles, facts, concepts with Greatest Common Divisor rules. Subject AE firstly identified the problems in the question by making details of what was known about the problem. in solving a problem, subject AE initially tried to recall the experience of working on a similar problem beforehand by finally automatically thinking about what formulas or mathematical concepts that can be directly used in solving the faced problems.

Next, the researcher traces the completion steps that have been made by the subject AE. First, the subject makes a factor tree from each of Aisyah's and Rogaya's money which is known in the question. Furthermore, the subject calculates the Greatest Common Divisor value of the two factors and the value is equal to 30. Based on the quotation from the answer of interview, the subject AE turned out to prove the correctness of the answer. As shown in the picture below.

Figure 5. Results of question number 2 subject AE.

From figure 4 and figure 5, Subject AE has two different mathematization mathematical thinking skills in solving this problem. The first is formalization because the subject resolves the problem by using principles, facts, concepts or mathematical rules by using Greatest Common Divisor rules and the second is connecting because through Greatest Common Divisor and completion table, it turns out the subject can solve problems, situations, and mathematical ideas interconnected into the form of mathematical models, and subjects can apply the knowledge obtained to solve and solve one problem to another problem by connecting information obtained through the use of Greatest Common Divisor rules which
then the information is used to prove the correctness of the data obtained by sorting the data using a value table. Then it can be concluded if the subject AE has two mathematical thinking abilities of type formalization and connecting.

4. Conclusion
The problem solving questions that are designed can already measure mathematical thinking students’ type of mathematization. This can be theoretically seen from the results of the focus group discussion, which states that the problem has been based on content, construct and language. Illustrated from the results of the trial where all students can use the instrument well where both of these questions can bring up mathematical thinking type mathematization. It can be seen from various strategies that have been used by various subjects, in the matter of number 1 the types of mathematization that appear are geometrization and formalization. Whereas in problem number 2, the mathematical ability that arises is connecting and formalization.

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