Radial seepage characteristics of polymer solution in micro-fractured reservoir based on a fractal network model

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Abstract. Accurate assessment of polymers’ flow behaviour is one of the key tasks for polymer flooding design in micro-fractured reservoir. In this paper, a fractal network model is developed for characterizing polymer flow in the micro-fractured reservoir based on straight capillary model, generalized Darcy’s law and constitutive equation for power-law fluids. A mathematical model for predicting the variation of pressure drop and effective permeability of polymer flow in porous media with network microstructure parameters and polymer power index is proposed. The developed model will help to calculate and characterize effective permeability alteration during polymer solutions seepage in micro-fractured media.

1. Introduction
Polymers are used in a variety of petroleum engineering applications. Polymer flooding has become one of the major EOR (enhanced oil recovery) methods in the world. The use of polymers in the injected water of a water flooding reservoir increases oil displacement efficiency by reducing the mobility of the driving phase [1-2]. Because the viscosity of polymer is related to the shear rate, there are serious defects in the direct use of Darcy’s law to simulate polymer flow in porous media [3-5]. Accurate assessment of the flow behaviour of viscous polymer solutions into the near wellbore area will provide guidance for polymer flooding optimization.

This research was designed to study the basic flow mechanisms of polymer solutions in the micro-fractured media. There has been a lot of research on underground seepage problem in porous media with micro-simulation model of capillary model [6-7], spherical particle packing model [8], lattice model [9] and pore grid model [10-11]. In recent years, Lorente et al. [12-15] introduced branch network model to study porous media seepage problems and provided a new way to model microscopic flow. However, it is found that these models are not well designed for reservoir engineering practice and not working accurately for studying near wellbore behaviour. As fluid flow has fractal feature in the microcrack network near wellbore, this study established a new model on the basis of fractal network theory to simulate polymer solution seepage.

2. Polymers used in oil displacement
Polymers that currently used in polymer flooding are partly hydrolyzed polyacrylamide (HPAM), xanthan gum (XC) and new associative polymers (NAPs) [16-18]. These several types of polymer are all non-Newtonian fluids with phenomenon of shear thinning effect when flowing in porous media.
The viscosity of polymer solutions in rock media can be characterized by the Power-law model [19], which is given by the expression.

\[ \tau = k' \dot{\gamma}^n \]  
(1)

Where, \( k' \) is flow consistency index, \( \dot{\gamma} \) is stain rate, \( n \) is power exponent.

3. Establishment of fractal network model
In this study, we consider polymers seepage in the near wellbore is equivalent to flow in branched radial network model. For each branch, we assume the network is composed of a number of smooth cylindrical channels and the thickness of channel wall is thin enough to be negligible, and all the ducts are sufficiently slender. Suppose the number of channel starting from the wellbore is \( N_0 \) (determination of this number is related with density and phase angle of perforation). Suppose that every channel is divided into \( N \) branches (e.g., \( N=2 \) in Fig. 1 and Fig.2) at the next level, and the total number of branching levels is \( m \).

![Figure 1. Radical fractal network model near wellbore](image)

![Figure 2. Sketch of two branch model](image)

If the network has fractal characteristics, then the branch number \( N \), length ratio \( \gamma \) and diameter ratio \( \beta \) remains constant. Then, for the level \( k \):

\[ l_k = l_0 \gamma^k \]  
(2)

\[ r_k = r_0 \beta^k \]  
(3)

Where, \( l_0 \) and \( r_0 \) are the length and radius of the 0th branching level; \( l_k \) and \( r_k \) are the length and radius of the \( k^{th} \) branching level.

4. Pressure drop in the near wellbore area
When polymers flow in the single capillary with radius of \( r \), the shearing stress along the channel wall is:

\[ \tau = -r \frac{dp}{2dL} \]  
(4)

Where, \( \tau \) is shear stress, \( \frac{dp}{dL} \) is the pressure gradient in the capillary.
According to Hagen-poisseuille equation [20-21], flow rate of polymer solution in the single capillary with radius of $r$ is given by:

$$ q = \frac{n\pi r^{3+1/n}}{(3n+1)(2\mu)^{1/n}} \left( \frac{\Delta p}{\Delta L} \right)^{1/n} \tag{5} $$

The flow rate in the $k$th branching level (Fig. 3) will be as follows:

$$ q_k = \frac{n\pi r_k^{3+1/n}}{(3n+1)(2\mu)^{1/n}} \left( \frac{\Delta p_k}{l_0 r^k} \right)^{1/n} \tag{6} $$

Figure 3. Sketch of pressure distribution

Suppose the injection rate in the wellbore is $Q$, then:

$$ Q = N_0 N^k q_k \tag{7} $$

By substituting equation (6) into (7) gives:

$$ \Delta p_k = \frac{2\mu Q^n (3n+1)^n}{(n\pi)^a} \frac{l_0 r^k}{N_0 n N^k r_k^{3n+1}} \tag{8} $$

Ignoring the local pressure loss of bifurcation, total pressure drop in the model is given by:

$$ \Delta p = \sum_{k=0}^{m} \Delta p_k = \frac{2\mu Q^n (3n+1)^n}{(n\pi)^a} \frac{l_0}{N_0 n r_0^{3n+1}} \frac{1-N^k}{1-N^k N^k} \tag{9} $$

Suppose the branch number $N$, $\gamma$, $\beta$, $l_0$, $r_0$ are all constant for a target reservoir, then equation (9) simply implies that the total pressure drop is a function of injection rate, apparent viscosity and power exponent $n$. Figure 1 shows the relationship between total pressure drop and the branching level under the condition of data in table 1.

| Parameter | Value |
|-----------|-------|
| $n$       | 0.4   |
| $\gamma$  | 0.5   |
| $\beta$   | 0.8   |
| $l_0$     | 0.5   |
| $r_0$     | 0.01  |
| $N$       | 2     |
| $N_0$     | 8     |
| $k$       | 6     |
| $\mu$     | 25    |

Table 1. Value of parameters.

As we can see in Fig. 4, the total pressure drop is higher with a bigger injection rate. Also, pressure drop is higher in the first level of branch, and is declining with the increasing number of branching levels.
Figure 4. Relationship of total pressure drop and the branching level

When the number of branching level \( k \) is equivalent to zero, the network only consists of \( N_0 \) tubes. Eq. 9 is then reduced to

\[
\Delta p_{k=0} = \frac{2 \mu Q^n (3n+1)^n l_0}{(n \pi)^n N_0^a r_0^{3n+1}} \tag{10}
\]

Based on equation (9) and (10), the dimensionless pressure drop of the model is obtained as:

\[
\Delta p_k = \frac{\Delta p_k}{\Delta p_{k=0}} = \gamma^k \tag{11}
\]

According to equation (11), the dimensionless pressure drop is related both with the structure of the network and the nature of the polymer solution.

5. Permeability of polymer flow in the near wellbore area

Strain rate of polymer solution flow in the single capillary is given by the expression:

\[
\dot{\gamma} = \left( \frac{1}{2 \mu r_L} \right)^{1/3} \frac{1}{r^{1/3}} \tag{12}
\]

Strain rate of polymer solution flow in the \( k \)th level can be obtained as:

\[
\dot{\gamma}_k = \left( \frac{1}{2 \mu r_L} \right)^{1/3} \frac{1}{r_k^{1/3}} \tag{13}
\]

By substituting equation (8) into (13) gives:

\[
\dot{\gamma}_k = \frac{Q(3n+1)}{n \pi r_0 N_0 N_k r_0^{3k}} \tag{14}
\]

The total strain rate in the network will be obtained as follows:

\[
\dot{\gamma} = N_0 \sum_{k=0}^{m} N_k \dot{\gamma}_k = \frac{Q(3n+1)}{n \pi r_0} \frac{1 - \left( \frac{1}{\beta^3} \right)^{n+1}}{1 - \frac{1}{\beta^3}} \tag{15}
\]

Apparent viscosity of the polymer solution in the network is given by:
\[
\mu_a = \mu \gamma^{n-1} = \mu \left[ \frac{n(3n + 1) \ln \left( \frac{1 - \left( \frac{1}{\beta} \right)^{m+1}}{1 - \frac{1}{\beta^3}} \right)}{\pi r_0} \right]^{n-1}
\] (16)

In this model, polymer’s flow in the near wellbore area is equivalent to flow in the capillary at different level of the network. According to the generalized Darcy’s law, the flow rate can be written as:

\[
Q = \frac{\Delta \rho}{\mu_a} \ln \frac{R_e}{R_w} - \frac{2\pi K_e}{R_e}
\] (17)

Where,

\[
R_e = L = \sum_{k=0}^{m} l_k = \frac{l_0}{1 - \gamma^{n+1}}
\] (18)

With equation (9) (16) (17) (18), the effective permeability of the model can be obtained as:

\[
K_e = \frac{nN_0^2r_0}{4(3n + 1) \ln \left( \frac{l_0}{R_e(1 - \gamma)} \right)} \ln \left( \frac{1 - \left( \frac{1}{\beta^3} \right)^{m+1}}{1 - \frac{1}{\beta}} \right) \left[ 1 - \left( \frac{\gamma}{N^p \beta^{2n+1}} \right)^{m+1} \right]^{n-1}
\] (19)

In equation (19), the branch number \( N \), length ratio \( \gamma \) and diameter ratio \( \beta \) reflects the nature of the reservoir. As shown in Fig.5, the effective permeability \( K_e \) decreases when the value of \( \gamma \) increases, while \( K_e \) increases when the value of increases \( \beta \). This may be explained that the larger length ratio means longer flow pass, leading to higher resistance and lower permeability. Also, a larger diameter ratio results in lower resistance and higher permeability.

![Graph showing the change of effective permeability with different length ratio and diameter ratio](image)

In order to more accurately evaluate the characteristics of the \( K_e \), the dimensionless effective permeability of polymer solution will be introduced. When the power exponent \( n \) is equal to 1, equation (19) is then reduced to:
\[ K = \frac{N_0 \sigma_0}{16 \ln \frac{l_0 (1 - \gamma^{m+1})}{R_w (1 - \gamma)} \left[ 1 - \frac{\gamma}{N \beta^2} \right]^{m+1}} \]  

The above equation reflects the permeability of Newton fluids’ flow in the porous media. Based on equation (19) and (20), the dimensionless effective permeability of polymer solution in the network is obtained as:

\[ K' = \frac{4nN_0^{n-1}}{(3n+1)} \left[ 1 - \frac{1}{N^n \beta^{3n+1}} \right]^{n-1} \left[ 1 - \frac{\gamma}{N \beta^2} \right]^{m+1} \]

Fig.6 shows that the dimensionless effective permeability increases with the increase of the power exponent n. Since n represents the non-Newtonian behaviour of polymer solution, the larger the power exponent n means the weaker non-Newtonian behaviour, leading to lower flow resistance and higher permeability.

6. Summary
(1) Based on fractal network theory, a new model is developed for studying the behavior of polymer solution seepage in micro-fractured reservoir. The modeling method can also be applied to solve other problems of underground seepage.

(2) An analytical model for calculating the pressure drop has been developed. Calculation result shows that pressure drop is higher in the first level of branch, and is declining with the increasing number of branching levels.

(3) An analytical model for calculating the effective permeability of polymer solution in the radial fractal network has been developed and is expressed as a function of the power exponents of polymer, branching number, the branching diameter ratio, branching length ratio, number of channel, the total number of branching levels, and the diameter of the 0th branching level.

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