Acoustic topological insulator by honeycomb sonic crystals with direct and indirect band gaps

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Keywords: acoustic topological insulator, honeycomb sonic crystal, indirect band gap

Abstract

We report both experimentally and numerically that a flow-free pseudospin-dependent acoustic topological insulator (ATI) is realized by two honeycomb sonic crystals with direct and indirect band gaps. By simply rotating triangular rods of the sonic crystals, the band inversion is realized, which arises from the change of the coupling strength between the triangular rods and leads to a topological phase transition. Moreover, a direct band gap is converted into an indirect band gap when the rotation angle is larger than 32.18°. By using the triangular rods with the rotation angles of 0°, 30°, and 60°, we design two topological insulators which include a topological nontrivial sonic crystal with the direct band gap (30°) and the indirect band gap (60°), respectively. In the topological insulator composed of the sonic crystal with the indirect band gap, the pseudospin-dependent edge modes also support acoustic propagation, in which the clockwise (anticlockwise) acoustic energy flux emulates pseudospin — (pseudospin+) state. Furthermore, these edge modes are topologically protected and remain high transmission after transmitting through topological waveguides with defects. The results provide diverse concepts to design ATIs with versatile applications.

1. Introduction

In the past few years, investigation into new topologically protected edge states has attracted extensive research interest in the fields of photonics [1–13] and mechanics [14–19] because of their prospective applications. Similarly to the two cases, research on acoustic topological insulator (ATI) [20–22] has also become a hot topic owing to its potential applications in a variety of important fields, such as acoustic-noise reduction, one-way acoustic transmission and acoustic communications in integrated devices.

The first type of the ATI is realized by using circulating fluids as the role of electron spin to mimic an effective magnetic field for acoustics and break the time-reversal symmetry, in which topologically protected acoustic edge modes are observed [20, 23–26]. However, the accurate control of the flow velocity in different units is still a great challenge in its practical applications. The second type of the ATI mimics analogous Floquet topological insulators by using coupled acoustic trimers with the temporal modulation of the acoustic capacitance of each cavity or strong coupling ring resonator waveguides with two acoustic modes which corresponds to clockwise and counterclockwise propagations characterized as pseudospins [27–30]. Furthermore, the valley-projected ATI is realized by sonic crystals (SCs) with the triangular lattice. By simply rotating units clockwise and counterclockwise, the sound vortex pseudospins can be obtained and the band inversion appears simultaneously [22, 31–34].
The direct band gap

We design a SC which consists of a honeycomb-lattice array

2. Accidental double Dirac cone and band inversion

We design a SC which consists of a honeycomb-lattice array (with lattice constant $a = 90$ mm) of rod-like scatterers, and the unit is schematically shown in figure 1(a). The rods have the shape of a regular triangle (with side length $l = 24$ mm), and its orientation degree of freedom, denoted by the rotation angle $\theta$, enables the sonic crystal to realize different symmetries. The triangular rods are made of polymethyl methacrylate (PMMA) to meet the sound hard boundary condition, and the distance between the triangular rods is $d = 30$ mm. Throughout this work, the finite element method based on COMSOL Multiphysics software is utilized to numerically simulate acoustic characteristics. The material parameters are adopted as follows: the density $\rho = 1180$ kg m$^{-3}$, the longitudinal wave velocity $c_l = 2730$ m s$^{-1}$, and the transversal wave velocity $c_t = 1430$ m s$^{-1}$ for PMMA; $\rho_0 = 1.25$ kg m$^{-3}$ and $c_0 = 343$ m s$^{-1}$ for air.

Figure 1(b) shows the eigenfrequencies of two two-fold degeneracy states at the band edges of the SC composed of the honeycomb-lattice array with different values of $\theta$, in which the blue and red lines represent the dipole and quadrupole modes at the Brillouin zone (BZ) center, respectively. As shown in figure 1(b), with the increase of $\theta$, the accidental double Dirac cone of the SC appears at points A and A$'$ ($\theta = \pm 20.62^\circ$). Therefore, the SC is topological trivial in the range ($-20.62^\circ, 20.62^\circ$), while the SC is topological nontrivial in both ranges ($20.62^\circ, 60^\circ$) and ($-20.62^\circ, -60^\circ$). Besides, it is noted when the value of $\theta$ is larger than 32.18$^\circ$ (points B and B$'$), the band gap is not determined by the eigenfrequency of the dipole mode at the BZ center (blue dashed lines). The direct band gap (red shadow region) is converted into the indirect band gap (blue shadow region). In the...
previous works [21, 35–38], the designed ATI generally consists of the SCs with the direct band gap, but not with the indirect band gap. Moreover, it is worth pointing out that the existence of the accidental double Dirac cone of the SC is determined by both parameters \( l/d \) and \( \theta \), which is shown in the supplementary material that is available online at stacks.iop.org/NJP/20/093027/mmmedia.

Figure 2 shows the dispersion relations of the SCs with different values of \( \theta \). As shown in figure 2(a), for \( \theta = 0^\circ \), a double two-fold degeneracy, one for the lower bands of the \( p \) type and the other for the upper bands of the \( d \) type, appears at the BZ center, which is similar to \( p \) and \( d \) orbitals of electrons. With the increase of \( \theta \), the band gap of the SC is closed for \( \theta = 20.62^\circ \) (figure 2(b)), resulting in an accidental double Dirac cone with the desired four-fold degeneracy at the BZ center. By further increasing \( \theta \), the coupling strength between triangular rods becomes strong gradually, which destroys the four-fold degeneracy and reopens the band gap. Meanwhile, the bands are inversed. As an example, in the case of \( \theta = 30^\circ \) (figure 2(c)), the bands of the \( p \) type are above those of the \( d \) type, and the band gap of the SC is still direct. The dispersion relations of the SC with \( \theta = 60^\circ \) are shown in figure 2(d). Different from the results in figures 2(a) and (c), the band gap for \( \theta = 60^\circ \) is indirect, which agrees well with the result in figure 1(b). Besides, the band is still inversed, which is similar to the result of \( \theta = 30^\circ \). The band inversion induced by changing the parameter of \( \theta \) results in a transition from a TTSC to a TNSC.

Figure 3 shows the pressure field distributions of two two-fold degenerate points at the BZ center for \( \theta = 0^\circ \), \( 30^\circ \), and \( 60^\circ \), corresponding to the black dots at the \( \Gamma \) point in figures 2(a), (c) and (d), respectively. It is obvious that a pair of dipolar modes and a pair of quadrupolar modes are obtained for both cases. For \( \theta = 0^\circ \) and \( 60^\circ \), owing to the characteristic of the \( C_{6v} \) crystal symmetry, the dipole modes which are even or odd symmetric to the axes \( x \) and \( y \), are denoted as \( p_x \) and \( p_y \), respectively. Besides, the quadrupole modes which is odd symmetric to the axes \( x \) and \( y \) or even symmetric to the axes \( x \) and \( y \) simultaneously, are denoted as \( d_{xy} \) and \( d_{x^2−y^2} \), respectively. But for \( \theta = 30^\circ \), the SC only has the \( C_{6v} \) crystal symmetry due to the breaking of mirror symmetry, and thus, the dipole \( (p_x, p_y) \) and quadrupole \( (d_{xy}, d_{x^2−y^2}) \) modes are no longer perfectly symmetric to the axes \( x \) and \( y \). However, the SCs with the \( C_{6v} \) crystal symmetry still meet the requirements of the pseudo-time-reversal symmetry. Pseudospin-1/2 for the bulk modes can be constituted by hybridizing acoustic pressure fields as \( p_c = (p_x ± ip_y)/\sqrt{2} \) and \( d_c = (d_{x^2−y^2} ± id_{xy})/\sqrt{2} \) [21, 35].

3. ATI composed of TNSC with direct band gap

3.1. Pseudospin-dependent edge modes

Figure 4(a) shows the dispersion relations of a TNSC–TTSC supercell. It is obvious that a pair of edge modes (red and blue lines) exists inside the overlapped bulk band gap of both SCs, in which the edge modes are localized at
the interface between the TNSC and the TTSC. Acoustic pseudospin-dependent edge modes can be obtained by hybridizing a symmetric mode \((S)\) and an anti-symmetric mode \((A)\) as \(S + iA\) and \(S - iA\), respectively, in which the \(S\) and \(A\) modes are displayed in figures 4(b) and (c), respectively. Besides, the slope of the dispersion band represents the group velocity of each individual pseudospin-dependent edge mode. Therefore, as shown in figure 4(a), both pseudospin-dependent edge modes have the same acoustic velocity, but propagate in opposite directions, indicating the existence of pseudospin-orbital coupling effect and one-way pseudospin-dependent acoustic propagation.

To demonstrate the existence of the pseudospin-dependent states, we simulate the pressure field distributions in a TTSC–TNSC–TTSC supercell, which is shown in the supplementary material. The results indicate that, in the absence of a background fluid flow, the acoustic energy vortex exists in the edge modes localized at the interfaces of the supercell, in which the chirality of the energy vortex corresponds to the pseudospin states [36].

3.2. Pseudospin-dependent acoustic one-way propagation

In general, it is much difficult to obtain a particular pseudospin state even with multiple incident acoustic sources in the experiment. As shown in figure 5(a), we design a cross-waveguide splitter which is divided into four parts with the TNSC locating at the upper left and right lower corners and the TTSC locating at the upper right and left lower corners. On basis of this, four input/output ports exist in the cross-waveguide splitter, which are denoted as 1, 2, 3 and 4, respectively. When acoustic wave propagates from port 1 to port 3, before the junction, the TNSC is located on the left side while the TTSC is located on the right side. But from port 2 to port 4, the configuration of the TNSC and the TTSC is contrary. In addition, when the acoustic wave passes through the junction, the spatial symmetry is inverted immediately (the TNSC on the right side and the TTSC on the left side from port 1 to port 3), and the acoustic wave is not allowed to propagate in a straight through fashion. This is because the pseudospin-dependent state is determined by the spatial symmetry of the structure, and the pseudospin-dependent edge modes in the paths with the same spatial symmetries are always preserved and

**Figure 3.** Simulated pressure field distributions of pseudospin dipole modes \(p_x\) and \(p_y\) and pseudospin quadrupole modes \(d_{xy}\) and \(d_{x^2-y^2}\) with \(\theta = 0^\circ, 30^\circ\) and \(60^\circ\) at BZ center.

**Figure 4.** (a) Dispersion relations of a supercell consisting of a TNSC \((\theta_1 = 30^\circ)\) stacked with a TTSC \((\theta_2 = 0^\circ)\). Simulated pressure field distributions of a TNSC–TTSC supercell at \(k_{||} = 0.05\) for (b) \(S\) and (c) \(A\) modes.
meanwhile the propagation of the acoustic wave is always allowed. Figures 5(b) and (c) show the pressure field distributions with inputs at ports 1 and 2, respectively, in which from port 1 to port 2 or to port 4 (or from port 2 to port 1 or to port 3), the structural spatial symmetries are the same and the propagation of the edge modes is always allowed. Such results agree well with the aforementioned theoretical analysis.

Besides, as shown by black arrows in R1–R3 of figure 5(b), the chirality of the energy vortex in the propagation path is anticlockwise (blue circular arrows), which corresponds to pseudospin +. Note that the chirality of the energy vortex remains unchanged when the acoustic wave passes through the junction of the splitter. However, the chirality of the energy vortex is clockwise (red circular arrows), corresponding to pseudospin −, which is shown as R4–R6 in figure 5(c). This indicates that the acoustic edge mode with inputs at port 1 and port 3 supports only the pseudospin + state, while the edge mode with inputs at the other two ports supports the pseudospin − state. Such a phenomenon can be explained by the slope of the dispersion band shown in figure 4(a) which corresponds to the group velocity of each individual pseudospin edge mode. Owing to their opposite slope across the whole BZ, the pseudospin + and pseudospin − edge modes can propagate only in a one-way pattern but with opposite directions. Such a propagation characteristic corresponds to an acoustic counterpart of the QSHE.

Figures 6(a) and (b) show the measured transmission spectra with inputs at port 1 and port 2, respectively, and the experimental set-up is shown in the supplementary material. It is found that the pseudospin-dependent edge modes remain with very high transmission for two cross output ports (T12, T14, T21, and T23). However, the transmissions for T13 and T24 are deeply suppressed, which are nearly 10 dB lower than those of the two cross output ports. The experimental results for this cross-waveguide splitter further demonstrate the robust one-way pseudospin-dependent propagation.
3.3. Topologically protected acoustic propagation against defects

To verify the propagation robustness of the topological edge modes, we design a topological waveguide (shown in figure 7(a)) composed of the TNSC ($\theta_1 = 30^\circ$) and the TTSC ($\theta_2 = 0^\circ$), and introduce three kinds of defects, such as a cavity, a lattice disorder and a bend, into the topological waveguide near the TNSC/TTSC interface. Figures 7(c)–(e) show the distributions of the pressure field and the energy flow in the topological waveguides with three kinds of defects, and the topological waveguide without defects are also simulated for comparison (figure 7(b)). It is found that the pseudospin $-$ edge mode immunes to these defects and can transmit through the topological waveguides with high transmission, which further verifies that the edge modes are topological protected.

The measured transmission spectra for the topological waveguides without and with three kinds of defects are shown in figure 8. Compared with the result for topological waveguide without defects (red solid line), the transmission spectra are almost unchanged with these defects especially for the frequencies around the center of the bulk band gap, which further demonstrates that the topological edge modes are almost immune to backscatterings and remain pseudospin-dependent against all types of defects.

4. ATI composed of TNCS with indirect band gap

4.1. Pseudospin-dependent edge modes

The topological edge propagation still exists in the ATI composed of the TNCS with the indirect band gap. Figure 9(a) shows the dispersion relations of a supercell composed of a TNSC ($\theta_1 = 60^\circ$) and a TTSC ($\theta_2 = 0^\circ$), in which the band gap of the TNSC ($\theta_1 = 60^\circ$) is indirect, but the band inversion still exists around the BZ center. It is found that a pair of edge modes (red and blue lines) exists inside the overlapped bulk band gap of the supercell, and the pressure field distributions of the S and A modes are shown in figures 9(b) and (c), respectively, which is similar to that in figure 4(a). Besides, there exists a tiny gap between two dispersion curves of the edge modes at the BZ center which is much more obvious than that in figure 4(a). This is because the eigenmodes of the TNSC and the TTSC are slightly different, especially for the TNSC ($\theta_1 = 60^\circ$). Therefore, the ATI which consists of the SC with the indirect band gap could also support the acoustic QSHE.

Moreover, to verify the existence of the pseudospin-dependent states in the ATI composed of the TNCS with the indirect band gap, we also present the pressure field distributions in a TTSC–TNSC–TTSC supercell, which is shown in the supplementary material. Similar to that with the direct band gap, the acoustic energy vortex exists in the edge modes localized at the interfaces of the supercell, and the chirality of the energy vortex corresponds to the pseudospin states.
4.2. Topologically protected acoustic propagation against defects

We also design the topological waveguide composed of the TNSC ($\theta_1 = 60^\circ$) and the TTSC ($\theta_2 = 0^\circ$) to verify that the edge modes are topologically protected, which is shown in figure 10(a). Figures 10(b)–(e) present the distributions of the pressure field and the energy flow in the topological waveguides without and with three types of defects, in which the defects are the same as those in figures 7(c)–(e). It is found that from figures 10(c)–(e) that the pseudospin – edge mode at the interface of the topological waveguide immunes to these defects and can get...
around these defects with a high transmission. Besides, as shown in R2–R4, the pseudospin−edge mode still exists after getting around these defects.

Figure 11 shows the measured transmission spectra for the topological waveguides without and with three types of defects. Similar to the results in figure 8, the transmission spectra almost remain unchanged with these defects, especially for the frequencies around the center of the bulk band gap. Therefore, the pseudospin-dependent edge modes are topological protected in the ATI composed of the SC with the indirect band gap.

Furthermore, it is obvious that there exists a tiny gap between two dispersion curves of the edge modes at the BZ center in figures 4(a) and 9(a), which arises from the reduction of the point group $C_{6}$ symmetry at the
interface between the TNSC and the TTSC [35]. The eigenmodes of the TNSC are not the same as those of TTSC [21], which results in a gap of 0.0367 and 0.0891 kHz in figures 4(a) and 9(a), respectively. But the edge modes are still topologically protected even with the reduction of the C₆ symmetry. Here, we can reduce the tiny gap by adjusting the rotation angles of the TNSC and the TTSC, and the gap can reach 0.7 Hz for \( \theta_1 = -33.5° \) and \( \theta_2 = 15.5° \), which is shown in the supplementary material.

5. Discussion

In conclusion, we have demonstrated the pseudospin states and robust propagation of pseudospin-dependent edge modes in a flow-free ATI which consists of two honeycomb SCs. The results show that, the band inversion can be realized by simply rotating the triangular rods of the SCs, which is attributed to the change of the coupling strength between the triangular rods and leads to a topological phase transition. Besides, when the rotation angle of the triangular rods is larger than 32.18°, the direct band gap of the SC is converted into the indirect band gap. Furthermore, by using the triangular rods with the rotation angles of 0°, 30°, and 60°, we realize two ATIs which include a TNSC with the direct band gap (30°) and the indirect band gap (60°), respectively. The experimental results verify that the pseudospin-dependent edge modes also support acoustic propagation in the ATI composed of the TNSC with the indirect band gap, and these topologically protected edge modes are almost immune to backscattering and remain pseudospin-dependent against three types of defects, which is similar to that with the indirect band gap. This work provides new routes for designing ATIs with novel functionalities and versatile applications.

Acknowledgments

This work was supported by National Natural Science Foundation of China (Grant Nos. 11774137, 11834008 and 51779107), Six Talent Peaks Project in Jiangsu Province (Grant No. GDZB-019), Jiangsu Qing Lan Project, and Scientific Research Project for Graduate Students of Universities in Jiangsu Province, China (Grant No. CXZZ13_0670).

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