Phase Transition in Rigid String Coupled to Kalb-Ramond Fields

M. Awada and D. Zoller
Physics Department
University of Cincinnati, Cincinnati, OH-45221

Abstract

Polyakov has argued that the QCD string should have long range order. We show that a phase transition does exist in a generalization of string theory characterized by the addition of the curvature of the world sheet (rigidity) and the long range Kalb-Ramond interactions to the Nambu-Goto action. Although rigid strings coupled to long range interactions exhibit the typical pathologies of higher derivative theories at the classical level, we comment based on previous results in rigid paths coupled to long range Coulomb interactions that both phases of the quantum theory are free of ghosts and tachyons.

* E-Mail address: moustafa@physunc.phy.uc.edu
Polyakov [1] has argued that the string theory appropriate to QCD should be one with long range correlations of the unit normal. The Nambu-Goto (NG) string theory is not the correct candidate for large N QCD as it disagrees with it at short distances. The NG string does not give rise to the parton-like behaviour observed in deep inelastic scattering at very high energies. The observed scattering amplitudes have a power fall-off behaviour contrary to the exponential fall-off behaviour of the NG string scattering amplitudes at short distances. The absence of scale and the power law behaviour at short distances suggest that the QCD string must have long range order at very high energies. Pursuing this end, Polyakov considered modifying the Nambu action by the renormalizable scale invariant curvature squared term (rigid strings). The theory closely resembles the two dimensional sigma model where the unit normals correspond to the sigma fields. In the large N approximation, there is no phase transition [2]. Polyakov suggested adding a topological term to produce a phase transition to a region of long range order. In this note, we couple rigid strings instead to long range Kalb-Ramond fields. Since spin systems in two dimensions may exhibit a phase transition with the inclusion of long range interactions, it is natural to conjecture likewise for rigid strings with long range Kalb-Ramond fields. Here we prove there is indeed a phase transition to a region of long range order in the large N approximation. Such a theory may therefore be relevant to QCD.

Higher derivative theories are typically pathological and we must address the consistency of the theory. The rigid string without long range interactions is consistent because the coupling constant of the curvature is not constant: the running coupling kills the curvature term in the infrared limit and the theory has linear Regge behaviour with no ghosts. Although we have not addressed the issue of ghosts in the rigid string with Kalb-Ramond fields, we have extensively investigated the analog for point particles [3]. This theory generalizes Feynman’s first quantized description of QED by adding the curvature of the world line to the action. This theory has two phases: A disordered phase whose large distance scale is essentially ordinary QED and the other is a new phase of QED with long range correlations of the unit tangents to the world lines. We have explicitly shown in the large N limit that both phases are free of ghosts and tachyons by calculating the spacetime propagator. We found that the ghosts are of order of the cut-off scale and in the absence of fine tuning of the coupling constants they decouple from the mass spectrum of the theory. Thus, due to the phase transition, the quantum fluctuations of the classical theory prevent one from taking the classical limit and inheriting the problems of the classical theory. We believe that the same mechanism would apply for our model of rigid strings coupled to the Kalb-Ramond fields. We will investigate this in a future article.

The gauge fixed action of the rigid string [1] coupled to the rank two antisymmetric Kalb-Ramond tensor field $\phi$ is [4]:
\[ I_{\text{gauge-fixed}} = \mu_0 \int d^2 \xi \rho + \frac{1}{2t} \int d^2 \xi \left[ \rho^{-1} (\partial^2 x)^2 + \lambda^{ab} (\partial_a x \partial_b x - \rho \delta_{ab}) \right] + I_{\text{Kalb-Ramond}} \]  

(1a)

where

\[ I_{K-R} = e_0 \int d^2 \xi \epsilon^{ab} \partial_a x^\mu \partial_b x^\nu \phi_{\mu\nu} + \frac{1}{12} \int d^4 x F_{\mu\nu\rho} F^{\mu\nu\rho} . \]  

(1b)

where \( e_0 \) is a coupling constant of dimension \( \text{length}^{-1} \), \( t \) is the curvature coupling constant which is dimensionless and \( F \) is the abelian field strength of \( \phi \). The integration of the \( \phi \) field is Gaussian. We obtain the following interacting long range Coulomb-like term that modifies the rigid string:

\[ \frac{1}{2t} \int \int d^2 \xi d^2 \xi' \sigma^{\mu\nu}(\xi) \sigma_{\mu\nu}(\xi') V(|x - x'|, a) \]  

(1c)

where \( V \) is the analog of the long range Coulomb potential:

\[ V(|x - x'|, a) = \frac{2g}{\pi} \frac{1}{|x(\xi) - x(\xi')|^2 + a^2 \rho} . \]  

(1d)

where \( \sigma^{\mu\nu}(\xi) = \epsilon^{ab} \partial_a x^\mu \partial_b x^\nu \). We have introduced the cut-off "a" to avoid the singularity at \( \xi = \xi' \) and define \( g = t \alpha_{\text{Coulomb}} = t \frac{e^2}{4\pi} \) which has dimension of \( \text{length}^{-2} \). The partition function is

\[ Z = \int D\lambda D\rho Dx \exp(-I_{\text{eff}}) . \]  

(2)

where the effective action \( I_{\text{eff}} \) is (1a) and (1c).

II-Large D analysis, absence of the Kalb-Ramond Coulomb interactions

The effective action is obtained by integrating over \( x^\nu, \nu = 1, \ldots D \) we have:

\[ I_{0eff} = \frac{1}{2t} \left[ \int d^2 \xi (\lambda^{ab} (\rho \delta_{ab}) + 2t \mu_0 \rho) + t D \text{tr} \ln A \right] \]  

(3)

where \( A \) is the operator

\[ A = \partial^2 \rho^{-1} \partial^2 - \partial_a \lambda^{ab} \partial_b . \]  

(4)

In the large D limit the stationary point equations resulting from varying \( \lambda \) and \( \rho \)
respectively are:
\[ \rho = \frac{tD}{2} trG \]  \hspace{1cm} (5a)
\[ 2t\mu_0 - \lambda^{ab}\delta_{ab} = tDtr(\rho^{-2}(-\partial^2G)) \]  \hspace{1cm} (5b)
where the world sheet Green’s function is defined by:
\[ G(\xi,\xi') = \langle \xi|(-\partial^2)A^{-1}\xi' \rangle \]  \hspace{1cm} (6)
The stationary points are:
\[ \rho(\xi) = \rho^*, \quad \lambda^{ab} = \lambda^* \delta^{ab} \]  \hspace{1cm} (7)
where \( \rho^* \) and \( \lambda^* \) are constants. Thus eq.(5a) becomes the mass gap equation:
\[ 1 = \frac{Dt}{2} \int \frac{d^2p}{(2\pi)^2} \frac{1}{\frac{p^2}{2} + m^2} \]  \hspace{1cm} (8)
where we define the mass
\[ m^2 = \rho^* \lambda^* \]  \hspace{1cm} (9)
this yields the mass gap equation,
\[ m = \Lambda e^{-\frac{D}{\rho^*}} \]  \hspace{1cm} (10a)
where \( \Lambda = \frac{1}{a} \) is an U.V. cut-off and \( m \) is now the mass associated with the propagator:
\[ \langle \partial_\mu x^\mu(p)\partial_\nu x^\nu(-p) \rangle = \frac{Dt}{2} \frac{\delta^{\mu\nu}}{p^2 + m^2} \]  \hspace{1cm} (10b)
On the other hand eq(5b) yields the string tension renormalization condition:
\[ \mu_0 = \frac{D}{8\pi} \frac{\Lambda^2}{\rho^*} \]  \hspace{1cm} (11)
eq (10a) agrees exactly with the one loop result.
III-Phase transition in the presence of Kalb-Ramond Coulomb interactions

The integration is no longer Gaussian. Thus we consider

\[ x^{\nu}(\xi) = x^{\nu}_0(\xi) + x^{\nu}_1(\xi) \]

and expand the Coulomb term (1c,d) to quadratic order in \( x^{\nu}_1(\xi) \) about the background straight line \( x^\nu_0 \). The \( x \)-integration is now Gaussian and the new effective action \( S_{eff} \) is given by (3) and (4) however with a new operator \( A \) that includes the Coulomb potential contributions. Using the stationary solution (7) we obtain:

\[
tr ln A_{new} = \int \frac{d^2p}{(2\pi)^2} ln[p^4 + p^2m^2 + p^2V_0(p) + V_1(p)]
\]

where

\[
V_0(p) = \frac{4g}{\pi} \int d^2\xi \frac{e^{ip.\xi}}{\xi^2 + a^2} = 8gK_0(a|p|)
\]

\[
V_1(p) = \frac{8g}{\pi} \int d^2\xi \frac{[e^{ip.\xi} - 1]}{(\xi^2 + a^2)^2} = \frac{8g}{a^2}(aK_1(a|p|) - 1)
\]

where \( K_n(z) \), \( n = 0, 1, .. \) is the Bessel function of the third kind and

\[
K_1(z) := -\frac{d}{dz}K_0(z).
\]

The new mass gap equation given by (5a),(6) and (12) that generalizes (8) is:

\[
1 = \frac{Dt}{2} \int \frac{d^2p}{(2\pi)^2} \frac{p^2}{p^2(p^2 + m^2) + p^2V_0(p) + V_1(p)}
\]

The critical line is defined by eq.(15) at \( m = 0 \):

\[
1 = \frac{Dt}{4\pi} \int_0^1 dy \frac{y^3}{y^4 + \eta(y^2K_0(y) + yK_1(y) - 1)}
\]

where \( \eta = 8ga^2 \) is a dimensionless coupling constant. We have made the change of variable \( y = ap \) and introduced the U.V cut-off \( \Lambda = \frac{1}{a} := \Lambda_0^* \). It is remarkable

* In fact there exist an \( \eta^* \) for which any choice of \( \Lambda a = c \) leads to phase transition as long as \( \eta < \eta^* \). We choose \( \Lambda = \Lambda_{Planck} \), therefore \( c \geq 1 \).
that eq.(16) is finite except at $g=0$ (absence of Coulomb interactions). In fact after tedious calculations one can prove that

$$\lim_{y \to 0} \left[ \frac{y^3}{y^4 + \eta (y^2 K_0(y) + y K_1(y) - 1)} \right] = 0$$

thus having infra-red finiteness.

The critical curve distinguishing the two phases in the $(t, \eta)$ plane is shown in Fig.1. The order parameter of the theory is the mass gap equation (15) where $m$ is the parameter that distinguishes that two phases. In the disordered phase $m > 0$, while in the ordered phase it is straightforward to show that $m = 0$. In the disordered phase the coupling constants $t$ and $g$ are completely fixed by dimensional transmutation in terms of the cut-off $\Lambda$ and $m$.

In conclusion, we have shown that the rigid string coupled to long range Kalb-Ramond fields has a phase transition in the $1/N$ approximation. The new phase is characterized by long range order with absence of scale and may describe QCD strings. Further questions for future investigations include the issue of mass spectrum of the theory and the QCD loop equation.

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This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-th/9404077v1
Long Range Ordered Phase, Order Parameter: \( m = 0 \)

Short Range Disordered Phase, Order Parameter: \( m > 0 \)