Preparing Majorana zero modes on a noisy quantum processor

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The simulation of systems of interacting fermions is one of the most anticipated applications of quantum computers. The most interesting simulations will require a fault-tolerant quantum computer, and building such a device remains a long-term goal. However, the capabilities of existing noisy quantum processors have steadily improved, sparking an interest in running simulations that, while not necessarily classically intractable, may serve as device benchmarks and help elucidate the challenges to achieving practical applications on near-term devices. Systems of non-interacting fermions are ideally suited to serve these purposes. While they display rich physics and generate highly entangled states when simulated on a quantum processor, their classical tractability enables experimental results to be verified even at large system sizes that would typically defy classical simulation. In this work, we use a noisy superconducting quantum processor to prepare Majorana zero modes as eigenstates of the Kitaev chain Hamiltonian, a model of non-interacting fermions. Our work builds on previous experiments with non-interacting fermionic systems. Previous work demonstrated error mitigation techniques applicable to the special case of Slater determinants. Here, we show how to extend these techniques to the case of general fermionic Gaussian states, and demonstrate them by preparing Majorana zero modes on systems of up to 7 qubits.

I. INTRODUCTION

The simulation of systems of interacting fermions is one of the most anticipated applications of quantum computers due to its value to commercial industry and scientific research [1, 2]. The most interesting simulations will undoubtedly require a fault-tolerant quantum computer capable of executing arbitrarily long quantum programs. The effort to build such a device is underway at academic and industrial institutions, and while fault-tolerance remains a long-term goal, the capabilities of existing prototypes have steadily improved [3–7]. These improved capabilities have sparked an interest in running simulations that, while not necessarily classically intractable, may serve as device benchmarks and help elucidate the challenges to achieving practical applications on near-term devices [8–12].

Systems of non-interacting fermions are ideally suited to serve these purposes. Despite being classically tractable, they display rich physics and produce highly entangled states when simulated on a quantum processor. Because they are classically tractable, experimental results can be verified even at large system sizes that would typically defy classical simulation. Previous experimental demonstrations of such simulations include an implementation of the Hartree-Fock method on a quantum processor [12] and the preparation of Majorana zero modes [13]. In both of these experiments, the quantum states prepared and measured belong to the class of fermionic Gaussian states, of which Slater determinants are a special case. Fermionic Gaussian states refer to eigenstates of a quadratic Hamiltonian, the defining feature of a system of non-interacting fermions.

Reference [12] demonstrated that for Slater determinants, the error mitigation techniques of physically-motivated postselection of bitstrings and state purification can be used to significantly improve the fidelity of the simulation and the accuracy of measured observables. In this work, we show that these techniques can be extended to the case of general fermionic Gaussian states, and demonstrate them by improving on the preparation of Majorana zero modes performed in Reference [13]. We also apply some additional error mitigation techniques which were not used in either reference. While Reference [13] ran experiments on only 3 qubits, here our error mitigation techniques enable us to go up to 7 qubits while also obtaining more accurate results. Our experiments are performed on a superconducting qubit processor manufactured at IBM.

Majorana zero modes (MZMs) refer to zero-energy Majorana fermion modes that exhibit topological properties due to spatial separation of the modes. A prototypical system that contains MZMs is the Kitaev chain. The Hamiltonian of a Kitaev chain is

\[
H = -t \sum_{j=1}^{n} (a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j) + \sum_{j=1}^{n} \left( \Delta a_j^\dagger a_{j+1}^\dagger + \Delta^* a_{j+1} a_j \right) + \mu \sum_{j=1}^{n} \left( a_j^\dagger a_j - \frac{1}{2} \right),
\]

where \(t\) is the tunneling amplitude, \(\Delta\) is the superconducting pairing, \(\mu\) is the chemical potential, and the \(\{a_j\}\) are fermionic annihilation operators for a system of \(n\) fermionic modes. When \(|\Delta| = t > 0\) and \(\mu = 0\), the
Hamiltonian (1) takes the form

\[ H = i\hbar \sum_{j=2}^{2n-1} \gamma_j \gamma_{j+1} \]  

(2)

where we have introduced the Majorana fermion operators

\[ \gamma_{2j-1} = a_j + a_j^\dagger, \quad \gamma_{2j} = -i(a_j - a_j^\dagger). \]

Note that \( \gamma_1 \) and \( \gamma_{2n} \) do not appear in the Hamiltonian (2); these are unpaired zero-energy Majorana modes localized at the ends of the chain. The energy and separation of the modes is robust to small perturbations in \( \mu \). MZMs can theoretically be used as carriers of quantum information with a Clifford gate set which is topologically protected against errors; this fact has motivated efforts at their experimental realization [14].

II. RESULTS

A. Circuits and observable measurement

Since the Kitaev chain Hamiltonian is quadratic in the fermionic creation and annihilation operators, its eigenstates are fermionic Gaussian states which can be prepared efficiently using the algorithm given in Reference [15]. This algorithm has linear circuit depth and requires only linear qubit connectivity. It assumes the Jordan-Wigner transform is used to map fermionic operators to qubit operators. Figure 1 displays an example circuit that shows the general structure. Besides single-qubit \( Z \) rotations and \( X \) gates, the only other type of gate present is the so-called Givens rotation gate, with matrix

\[ G(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \]

(4)

On hardware for which CNOT is the native two-qubit interaction, this gate can be implemented using the decomposition shown in Figure 2, which uses two CNOT gates in addition to single-qubit rotations.

Eigenstates of a quadratic Hamiltonian are prepared by effecting a basis change that maps the fermionic creation operators \( \{a_j^\dagger\} \) to a new set of creation operators \( \{b_j^\dagger\} \) such that the Hamiltonian takes the diagonal form

\[ H = \sum_j \varepsilon_j b_j^\dagger b_j + \text{constant} \]

(5)

where \( 0 \leq \varepsilon_1 \leq \cdots \leq \varepsilon_n \). The operators \( \{b_j^\dagger\} \) also satisfy the fermionic anticommutation relations, so they can be regarded as creation operators for fermionic excitations with excitation energies given by \( \{\varepsilon_j\} \). The \( \{b_j^\dagger\} \) are linear combinations of the original creation and annihilation operators:

\[ \begin{pmatrix} b_1^\dagger \\ \vdots \\ b_n^\dagger \end{pmatrix} = W \begin{pmatrix} a_1^\dagger \\ \vdots \\ a_n^\dagger \end{pmatrix} \]

(6)

where \( W \) is an \( n \times 2n \) matrix. The matrix \( W \) can be efficiently computed from the description of the Hamiltonian and is used to produce the quantum circuit that prepares an eigenstate of the Hamiltonian.

For each state prepared, all observables of interest can be determined from the correlation matrix. The correlation matrix \( \Gamma \) of a state \( \rho \) is defined as the block matrix

\[ \Gamma = \begin{pmatrix} T & S \\ -S^* & I - TT^* \end{pmatrix} \]

(7)
We measured the correlation matrix using a protocol similar to the one used in Reference [12] to measure the one-particle reduced density matrix (1-RDM), which is the matrix $T$ in our notation. The diagonal entries of the correlation matrix are occupation numbers that can be measured straightforwardly in the computational basis. To obtain the off-diagonal entries, we measure the operators

$$a_j^+ a_k + a_k^+ a_j \mapsto X_j X_k + Y_j Y_k$$

(10)

$$-i (a_j^+ a_k - a_k^+ a_j) \mapsto X_j Y_k - Y_j X_k$$

(11)

$$a_j^+ a_k^+ + a_k a_j \mapsto X_j X_k - Y_j Y_k$$

(12)

$$-i (a_j^+ a_k^+ - a_k a_j) \mapsto -X_j Y_k - Y_j X_k$$

(13)

where we have shown how the operators map under the Jordan-Wigner transform for neighboring modes. If the correlation matrix is real, then only the first and third operators need to be measured. A sufficient condition for the correlation matrix to be real is that the circuit used to prepare the state contains only gates with real-valued matrices, up to global phase.

We measured these operators by diagonalizing them using parity-preserving operations. For example, the third operator is diagonalized by the two-qubit gate with

$$T_{jk} = \text{Tr} \left[ a_j^+ a_k \rho \right]$$

(8)

$$S_{jk} = \text{Tr} \left[ a_j^+ a_k^+ \rho \right]$$

(9)

and $I$ is the identity matrix.
FIG. 4. Majorana site correlation. The Majorana site correlation at site $j$ is defined as the operator $i c_j c_j$ where $c_j$ denotes the $j$-th Majorana operator. The data for the 6-mode Kitaev chain is shown on the left, and the data for the 7-mode Kitaev chain is shown on the right. For each system size, we include plots for several different values of $\mu$, as indicated on the plots. Each plot includes data for the ground state (ideal values indicated by the dashed black line) and the first excited state (ideal values indicated by the dashed gray line). “Raw” refers to raw data without error mitigation, and “Mit.” refers to the data after all error mitigation techniques are applied. At $\mu = 0$ the correlation is only nonzero for $j = 2n$, indicating an exclusive correlation between Majorana fermions at the ends of the chain. As $\mu$ increases this exclusive correlation breaks down. All error bars indicate a confidence interval of two standard deviations (95% confidence) as estimated from the sample covariance.

\[
\begin{pmatrix}
  \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 1 & 0 \\
 -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}}
\end{pmatrix}.
\]

(14)

Using parity-preserving gates for measurement enables the error mitigation technique of postselection on bit-string parity, described below. These gates can be implemented on hardware similarly to the Givens rotation gates by decomposing them into two CNOT gates plus single-qubit rotations.

For a given circuit realization, only operators between modes mapped to neighboring qubits can be measured directly. In order to measure operators between all pairs of modes, we generate different circuits by permuting the columns of the matrix $W$; each permutation corresponds to a relabeling of the fermionic modes that causes different pairs of modes to be mapped to neighboring qubits. This procedure enables the measurement of operators between all pairs of modes without the need to add fermionic swap gates to the circuits. This idea is described in detail in the supplementary information to Reference [12].

For each fermionic Gaussian state, a total of $\lceil n/2 \rceil$ permutations are required. For each permutation, four different basis changes are required to measure the operators (10-13), and each basis change gives rise to two circuits, one to measure pairs of qubits starting on even indices, and one for odd indices. Together with the circuit for measuring the diagonal entries, measuring the correlation matrix of a state requires $\lceil n/2 \rceil \times 8 + 1$ circuits in total. In our experiments the correlation matrix was real, so only two of the basis changes were required and the total number of circuits was $\lceil n/2 \rceil \times 4 + 1$.

B. Error mitigation

We applied a number of error mitigation techniques to improve results.

Dynamical decoupling. Dynamical decoupling is a technique to reduce phase errors stemming from time-correlated low-frequency noise by applying refocusing pulses on idle qubits [16, 17]. There are many possible pulse sequences that could be applied. We used the 4-pulse sequence

\[ X_\pi \rightarrow Y_\pi \rightarrow X_{-\pi} \rightarrow Y_{-\pi} \]

(15)

where $X_\pi$ and $X_{-\pi}$ denote $X$ gates implemented using opposite sign pulse amplitudes. We used Qiskit [18] to schedule the dynamical decoupling pulses. The software detects idle periods in the compiled circuit and in each idle period inserts one pulse sequence, distributing the 4 pulses with even temporal spacing.
Measurement error mitigation. The effect of measurement errors can be mitigated by treating it as a classical noise channel and approximately inverting it [19]. We used the software package mthree [20, 21] to perform measurement error mitigation. The error mitigation procedure converts raw bitstring counts into error-mitigated bitstring quasiprobabilities. The error mitigation does not come for free; rather, there is increased uncertainty in computed quantities.

Postselection of bitstrings. In Reference [12] which prepared the Hartree-Fock state, the ideal final state had a well-defined particle number, so measured bitstrings with the incorrect particle number could be discarded to improve results. In our case, the final state does not have a well-defined particle number, but it does have a well-defined parity. Therefore, we can still perform postselection on the bitstrings, discarding those with the incorrect parity. A technical detail is that here we actually apply postselection not on the bitstrings directly, but on the quasiprobability distribution over bitstrings returned by the measurement error mitigation procedure.

State purification. Reference [12] exploited the fact that the 1-RDM is idempotent (it is equal to its square) to perform purification of the measured 1-RDM. That is, due to experimental error, the measured 1-RDM is not idempotent, and it was projected onto the space of idempotent matrices using a procedure called McWeeny purification [22]. In our case, it is the correlation matrix that is idempotent [23]. Therefore, McWeeny purification can also be applied here to purify the measured correlation matrix. The purification is accomplished by repeating until convergence the following numerical operation to update $\Gamma$ at iteration $k$:

$$\Gamma_{k+1} = \Gamma_k^2 (3I - 2\Gamma_k).$$

C. Experiments

For our experiments, we set $t = -1$, $\Delta = 1$, and used 5 different values for $\mu$ evenly spaced between 0 and 3. For each choice of Hamiltonian parameters, we prepared 6 eigenstates: the ground state and the first and second excited states, as well as the 3 corresponding eigenstates from the opposite end of the spectrum. We executed our circuits on the ibmq_guadalupe device accessed through the IBM Quantum service. We used Qiskit [18] to compile the circuits into basis gates supported by the hardware. The Givens rotation gates were compiled using a decomposition similar to the one shown in Figure 2, requiring 2 CNOT gates for each Givens rotation. For each circuit we collected 100,000 measurement shots.

To execute each circuit, we needed to pick a line of qubits to use. Because gate errors vary over the device, the choice of qubits can have a significant impact on performance. To pick the qubits, we used the software package mapomatic [24], which attempts to minimize the expected error in executing the circuit using a subgraph isomorphism algorithm scored on gate errors reported for the device.

Reference [25] shows how to compute a fidelity witness for experiments that prepare fermionic Gaussian states. The fidelity witness gives a lower bound on the fidelity of the experimentally measured state with the ideal state and can be easily computed from the correlation matrix; see Appendix A for a review of this result. Figure 3 (top panel) shows the fidelity witness and average error in energy.

Figure 3 (bottom panel) shows the measured excitation energies of the first and second excitations above the ground state, and their symmetric hole counterparts. We show both the values obtained from the raw data and those obtained after applying error mitigation.

Figure 4 shows the measured expectation value of the Majorana site correlation $\sigma_c^0 \sigma_j^c$. Again, both raw and error-mitigated data are displayed.

III. DISCUSSION

We created Majorana zero modes on a noisy superconducting qubit processor by preparing eigenstates of the Kitaev chain Hamiltonian. The largest chain that we simulated used 7 qubits. Even at this system size, our experimental results closely matched theory and we observed clear signatures of the topological nature of the MZMs. We measured zero-energy excitations at $\mu = 0$ which were robust to small perturbations in $\mu$. We also observed the exclusive correlation between Majorana fermions at the ends of the chain at $\mu = 0$ and increasing correlations with interior sites with increasing $\mu$. In the limit of infinite chain length, the value of $\mu = 2t$ separates two topological phases, but with a finite length chain this value is smaller; at $\mu = 1.5t$ in our experiments the correlation between the first site and interior sites is
as strong as that between the ends of the chain.

The quality of our results was made possible by our utilization of error mitigation techniques. We applied dynamical decoupling pulses to idle qubits to mitigate dephasing errors. To see whether these pulses actually improved results, we ran experiments with and without dynamical decoupling (while still applying the rest of the error mitigation techniques). Figure 5 shows the fidelity witness and energy error for these experiments on the 6-mode Kitaev chain. Applying dynamical decoupling pulses does indeed yield a significant improvement, which suggests that low frequency noise on idle qubits is a major source of dephasing error. We also applied measurement error mitigation; this is a straightforward and widely applicable technique that is becoming common practice.

The other techniques we applied were physically-motivated postselection of measured bitstrings and state purification. These techniques have previously been used in the preparation of Slater determinants [12]. Here, we showed how to extend these techniques to the general case of fermionic Gaussian states so they can be used for the preparation of arbitrary eigenstates of quadratic Hamiltonians such as the Kitaev chain Hamiltonian. These techniques were highly effective at improving the fidelity and lowering the error of our simulations.

The combined application of all of these error mitigation techniques enabled us to simulate systems of up to 7 qubits, whereas a previous experiment preparing Majorana zero modes used only 3 qubits. We note that References [26, 27] also prepared topological Majorana modes on noisy quantum processors but those works took a different approach which is not directly comparable to this work. Our results build on previous experiments suggesting that error mitigation will be crucial to achieving practical applications on NISQ hardware, and they contribute to the growing library of experiments that can serve as device benchmarks as we work towards those practical applications.

Acknowledgments

The authors acknowledge the use of IBM Quantum Services for this work. This experiment was implemented and executed using Qiskit [18]. The circuit diagrams in this paper were created using (qpic) [28]. The data plots were created using Matplotliblib [29].

Data availability

The experimental data for this work is available at https://doi.org/10.5281/zenodo.6603265.

Code availability

The source code for this work is available at https://github.com/qiskit-research/qiskit-research.

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Appendix A: Fidelity witness

Reference [25] shows how to obtain a fidelity witness for experiments preparing fermionic Gaussian states. In this section we review this result and describe how we computed the fidelity witness in our experiment.
Consider an experiment that aims to prepare a known pure fermionic Gaussian target state $\rho_t$. Let $\rho_p$ denote the imperfect state that is actually prepared in the experiment. The closeness between the two states is measured by the fidelity

$$F(\rho_t, \rho_p) = \text{Tr} \left[ \left( \sqrt{\rho_t} \rho_p \sqrt{\rho_t} \right)^2 \right] = \text{Tr}[\rho_t \rho_p] \quad (A1)$$

where the last equality holds because $\rho_t$ is pure. A fidelity witness is an observable $W$ for which the value $F_W(\rho_p) = \text{Tr}[W \rho_p]$ gives a lower bound on the fidelity between $\rho_t$ and $\rho_p$. Reference [25] describes such a witness and gives an expression for the fidelity lower bound in terms of the covariance matrices $M_t$ and $M_p$ of $\rho_t$ and $\rho_p$:

$$F_W(\rho_p) = 1 + \frac{1}{4} \text{Tr}[(M_p - M_t)^T M_t] \quad (A2)$$

The covariance matrix $M$ of a state $\rho$ has entries

$$M_{jk} = \frac{i}{2} \text{Tr}[(\gamma_j \gamma_k - \gamma_k \gamma_j) \rho] \quad (A3)$$

where the $\gamma_j$ are Majorana fermion operators. To relate the covariance matrix to the correlation matrix it is convenient to use the following alternative indexing convention for the Majorana operators:

$$\gamma_j = a_j + a_j^\dagger, \quad \gamma_j + n = -i(a_j - a_j^\dagger). \quad (A4)$$

Under this convention, the covariance matrix is related to the correlation matrix $\Gamma$ by the identity

$$M = i\Omega(2\Gamma - I)\Omega^\dagger \quad (A5)$$

where $\Omega$ is the block matrix

$$\Omega = \frac{1}{\sqrt{2}} \begin{pmatrix} I & I \\ iI & -iI \end{pmatrix}. \quad (A6)$$

Using this relation, the expression for the fidelity lower bound can be written in terms of the correlation matrices $\Gamma_t$ and $\Gamma_p$ of $\rho_t$ and $\rho_p$:

$$F_W(\rho_p) = 1 - \text{Tr} \left[ (\Gamma_t - \Gamma_p)(\Gamma_t - \frac{I}{2}) \right]. \quad (A7)$$

While Reference [25] describes an efficient protocol for measuring the fidelity lower bound without needing to measure the entire covariance or correlation matrix, in our experiment we measured the entire correlation matrix and directly used Equation (A7) to compute the fidelity lower bound.