Extended holographic Ricci dark energy in chameleon Brans-Dicke cosmology

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In the present work we have studied some features of the generalized Brans-Dicke (BD) model in which the scalar field is allowed to couple non-minimally with matter sector. Extended holographic Ricci dark energy (EHRDE) has been considered in the above framework of BD cosmology. Some restrictions have been derived for the BD parameter $\omega$ and a stronger matter-chameleon coupling has been observed with expansion of the universe. In this framework, the equation of state parameter of EHRDE has behaved like quintom. Also, we have reconstructed the potential and coupling function for BD model for the EHRDE. It has been observed that potential function is increasing as the matter-chameleon coupling is getting stronger.

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I. INTRODUCTION

Accumulating the observational data of Supernovae Type Ia (SN Ia) by the year 1998, Riess et al. [1] in the High-redshift Supernova Search Team and Perlmutter et al. [2] in the Supernova Cosmology Project Team have independently reported that the present universe is accelerating. The source for this late-time acceleration was dubbed “dark energy” (DE), which is distinguished from ordinary matter species such as baryons and radiation, in the sense that it has a negative pressure. Reviews on DE are available in [3–9]. The SN Ia observations have shown that about 70% of the present energy of the universe consist of DE [7]. The simplest candidate for DE is the so-called cosmological constant $\Lambda$, whose energy density remains constant [10]. If the origin of DE is not the cosmological constant, then one may seek for some alternative models for DE to explain the cosmic acceleration today. There are two approaches to construct models of DE other than cosmological constant [7]: (i) To modify the right hand side of the Einstein equations by considering specific forms of energy-momentum tensor $T_{\mu\nu}$ with negative pressure; (ii) To modify the left hand
side of the Einstein equations. The representative models that belong to the first category are the so-called quintessence \([11]\), k-essence \([12]\) and perfect fluid models \([13]\). The representative models that belong to the second class are “modified gravity” \([15–18]\) models that include \(f(R)\) gravity \([19]\), scalar-tensor theories \([21]\), and braneworld models \([22]\). An extensive review on modified gravity is presented in \([20]\). Among all scalar-tensor theories of gravity, the simplest one is the so-called Brans-Dicke theory (BD) introduced by Brans and Dicke \([23]\) that modifies general relativity in accordance with Mach’s principle. Cosmological models of classical BD theory were first studied in \([24, 25]\). In the subsequent decades, various aspects of BD cosmology have been widely investigated \([26–33]\).

The present paper is aimed at working in chameleon Brans-Dicke cosmology. Thus, we discuss a bit of chameleon cosmology in this place. Khoury and Weltman \([34, 35]\) employed self-interactions of the scalar field to avoid the bounds on such a field and dubbed such scalars to be “chameleon fields” due to the way in which the field’s mass depends on the density of matter in the local environment. In regions of high density, the chameleon “blends” with its environment and becomes essentially invisible to searches for violation of equivalence principle and the fifth force \([34]\). Ref. \([36]\) have shown that chameleon scalar field can provide explicit realizations of a quintessence model where the quintessence scalar field couples directly to baryons and dark matter with gravitational strength. Ref. \([37]\) have illustrated that interacting chameleon field plays an important role in late time universe acceleration and phantom crossing. Ref. \([38]\) proposed an interacting holographic dark energy model in chameleon-tachyon cosmology by interaction between the components of the dark sectors. Based on two independent functions of the scalar field, ref. \([39]\) constructed an exact solution describing the evolution of the type Bang-to-Rip with the phantom divide line crossing in the chameleon cosmology.

Now we come to the chameleon Brans-Dicke cosmology. In the models, where a non-minimal coupling between the scalar field and matter system is considered by introducing an arbitrary functions of the scalar field, the scalar field is regarded as a chameleon field \([33]\). Ref. \([40]\) were the first to introduce this kind of chameleon-matter coupling in the BD model to achieve accelerated expansion of the universe. In a recent paper, ref. \([33]\) considered a generalized BD model with power-law form of the scale factor and the coupling functions as the inputs and allowed non-minimal coupling with the matter sector to show that accelerated expansion of the universe can be realized for a contrained range of exponents of the potential function. Treating the BD scalar field as a chameleon scalar field and taking a non-minimal coupling of the scalar field with matter, ref. \([41]\) studied cosmological implication of holographic dark energy in the BD gravity. Ref. \([42]\)
investigated the cosmological applications of interacting holographic dark energy in BD theory with chameleon scalar field which is non-minimally coupled to the matter field and observed that phantom crossing can be constructed if the model parameters are chosen suitably. For different epochs of the cosmic evolution, ref. \[43\] investigated the BD chameleon theory of gravity and obtained exact solutions of the scale factor, scalar field and potential. Ref. \[44\] discussed late-time dynamics of a chameleonic generalized BD cosmology with the power law chameleonic function $f(\phi) \propto \phi^n$, where $n$ is a real parameter motivated from string theories.

Making an approach to dark energy in the frame of quantum gravity is the well-known holographic dark energy (HDE) inspired by the holographic principle \[45\]. The HDE, whose density is $\rho_\Lambda = 3e^2M_p^2L^{-2}$ was proposed by Li \[46\]. Subsequent studies on HDE from various points of view include \[47–50\]. In the present work we are considering a special form of HDE \[51\] dubbed as “extended holographic Ricci dark energy” (EHRDE) \[52\], whose density has the form

$$\rho_\Lambda = 3M_p^2(\alpha H^2 + \beta \dot{H})$$

where, $M_p^2$ is the reduced Planck mass, $\alpha$ and $\beta$ are constants to be determined. Like ref. \[41\], we shall assume that $\rho_\Lambda$ and pressureless dark matter are conserved separately and we shall assume a non-minimal coupling between the scalar field and the matter field.

II. EHRDE IN CHAMELEON BD COSMOLOGY

We begin with the chameleon BD theory in which the scalar field is coupled non-minimally to the matter field via the action \[33\]

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left( \phi R - \frac{\omega}{\phi} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - 2V + 2f(\phi) L_m \right)$$

In equation (2), $R$ and $\phi$ denote the Ricci scalar and BD scalar field respectively. The $f(\phi)$ and $V(\phi)$ are analytic functions of the scalar field. The matter Lagrangian density, denoted by $L_m$, is coupled with $\phi$ via the function $f(\phi)$. This function allows a non-minimal coupling between the matter system and the scalar field. If $f(\phi) = 1$, we get back the BD action with potential function $\phi$ \[33\]. Varying action with respect to the metric $g_{\mu\nu}$ and $\phi$ one gets the field equations \[33\]

$$\phi G_{\mu\nu} = T^\phi_{\mu\nu} + f(\phi) T^m_{\mu\nu} \quad (3)$$

$$\quad (2\omega + 3) \Box \phi + 2(2V - V'\phi) = T^m \phi - 2f'\phi_m \quad (4)$$
where $\Box = \nabla^\mu \nabla_\mu$, where $\nabla_\mu$ represents the covariant derivative, $T^m = g^{\mu\nu}T^m_{\mu\nu}$ and the prime denotes differentiation with respect to $\phi$. In equation (3)

$$T^\phi_{\mu\nu} = \frac{\omega}{\phi} \left( \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla_\alpha \phi \nabla^\alpha \phi \right) + \left( \nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \Box \phi \right) - V(\phi) g_{\mu\nu}$$

and

$$T^m_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-gL_m})}{\delta g^{\mu\nu}}$$

Because of the explicit coupling between matter system and $\phi$ the stress tensor $T^m_{\mu\nu}$ is not divergence free.

Now we shall apply the above framework to a homogeneous and isotropic universe described by the Friedman-Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right)$$

The universe is open, closed or flat according as $k = -1, +1$ or 0. In a spatially flat universe, equations (3) and (6) yield

$$3H^2 = \frac{f}{\phi} \rho + \frac{\omega \dot{\phi}^2}{2 \phi^2} - 3H \frac{\dot{\phi}}{\phi} + \frac{V}{\phi}$$

$$3(\dot{H} + H^2) = -\frac{3\rho}{\phi(2\omega + 3)} \left[ \gamma \phi f' + \left( \omega \left( \gamma + \frac{1}{3} \right) + 1 \right) f \right] - \frac{\omega \dot{\phi}^2}{2 \phi^2} + 3H \frac{\dot{\phi}}{\phi} + \frac{1}{2\omega + 3} \left[ 3V' + (2\omega - 3) \frac{V}{\phi} \right]$$

$$(2\omega + 3)(\ddot{\phi} + 3H \dot{\phi}) - 2(2V - \phi V') = \rho \left( (1 - 3\gamma) f + 2\gamma \phi f' \right)$$

In equations (5) to (10), $\rho = \rho_m + \rho_\Lambda$ and $\gamma = \frac{\rho_\Lambda}{\rho_m + \rho_\Lambda}$ (since dark matter is pressureless, $p_m = 0$). This approach is similar to refs. [41, 42].

In the present work, we shall investigate two cases. In one case we shall assume special forms for $V$, $f$, $a$ and $\phi$ and derive conditions that strengthen matter-chameleon coupling with expansion of the universe. In another case we shall not assume any form for $V$ and $f$. Rather we shall reconstruct them for the EHRDE.

III. DISCUSSION

A. Case I

In this section we shall consider a set of ansatz for the potential function, analytic function, scalar field and scale factor. Based on them, we shall determine constraint on the BD parameter $\omega$. 
Subsequently, based on the constraints on \( \omega \) and other parameters we shall investigate the behavior of the EHRDE in BD chameleon cosmology. Following [33] we choose
\[
V(\phi) = V_0 \phi^{l_1}; \ f(\phi) = f_0 \phi^{l_2}; \ a(t) = a_0 t^n; \ \text{and} \ \phi(t) = \phi_0 t^m
\tag{11}
\]
Since \( M_p^2 = 1/8\pi G \) and in BD theory \( \phi \propto G^{-1} \) [42] we can take EHRDE in the chameleon BD as
\[
\rho_\Lambda = 3\phi(\alpha H^2 + \beta \dot{H})
\tag{12}
\]
and using conservation equation \( \dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = 0 \) we get
\[
p_\Lambda = \phi \left( (1 - 3H^2 - 2\dot{H})\alpha + \frac{\beta}{H^2} \left( \dot{H} - H(\ddot{H} + 3H\dot{H}) \right) \right)
\tag{13}
\]
Using equations (11) - (13) in (8) we have
\[
\quad \quad \quad \quad \quad \quad \quad \eta_1 t^{2m + 2 - 3n} + \eta_2 t^{2 - m + ml_1} + \eta_3 t^{ml_2} = \eta_4
\tag{14}
\]
where, \( \eta_1, \ \eta_2, \ \eta_3 \) and \( \eta_4 \) are terms involving constant terms of ansatz (11). In equation (14) we observe that the right hand side is constant with \( t \) involved in the left hand side. This is possible if the powers of \( t \) are 0. In (14) \( ml_2 \neq 0 \). However, its coefficient \( 3nf_0f_0^{l_2}(n\alpha - \beta)(3 + \omega + 3\gamma(l_2 + \omega)) \) can be set equal to 0. Since \( n\alpha - \beta = 0 \) will lead to \( p_\Lambda = 0 \), we can set \( 3 + \omega + 3\gamma(l_2 + \omega) = 0 \).
Finally, we get the restrictions
\[
\quad \quad \omega = -3(1 + \gamma l_2)\frac{1}{1 + 3\gamma}
\tag{15}
\]
\[
-2 = m(l_2 - 1) - 3n
\tag{16}
\]
\[
-2 = m(l_1 - 1)
\tag{17}
\]
The above restrictions are also found valid if we use the ansatz (11) and (12) - (13) in (9). In (15), we have for accelerated expansion of the universe \( 3\gamma + 1 > 0 \). Hence, (15) finally leads to a constraint on the BD parameter as
\[
\omega > \frac{l_2 - 3}{1 + 3\gamma}
\tag{18}
\]
In 1973 \( \omega > 5 \) was consistent with known data. By 1981 \( \omega > 30 \) was consistent with known data. In 2003, evidence derived from the Cassini-Huygens experiment shows that the value of \( \omega \) must exceed 40,000 [53]. Hence, we can rewrite
\[
\omega > \frac{l_2 - 3}{1 + 3\gamma} \gg 1
\tag{19}
Hence, \( l_2 > 4 \). From (16) and (17) we have

\[
n = \frac{2}{3} \left( \frac{l_1 - l_2}{l_1 - 1} \right); \quad \text{and} \quad m = \frac{2}{1 - l_1}
\] (20)

If \( l_1 > 1 \) then \( m < 0 \), which implies that the scalar field is a decreasing function of \( t \). Since \( n \) has to be positive, \( l_1 > l_2 \) from (20). Again, if \( l_1 < 1 \), then \( m > 0 \). Since \( l_2 > 0 \), \( ml_2 > 0 \). Thus, for \( l_1 < 1 \), coupling function \( f \propto t^{ml_2} \) increases with time i.e. matter-chameleon coupling gets stronger as the universe expands.

Choosing appropriate parameters (keeping the restrictions in mind) we have plotted the EoS parameter \( \gamma = \frac{p}{\rho} \) for different combinations of parameters in Fig. 1. The EoS parameter clearly shows a transition from \( \gamma > -1 \) (quintessence) to \( \gamma < -1 \) (phantom). Hence, the EoS parameter is found to exhibit “quintom”-like behavior. Next we plot in Fig. 2 the deceleration parameter \( q \)

\[
q = \frac{1}{2} + \frac{3p}{2\rho}
\] (21)

We observe that the deceleration parameter has a clear transition from positive to negative side. This indicates the transition from decelerated to accelerated phase of the universe.

![Graph showing EoS parameter against t](image)

**FIG. 1:** Evolution of EoS parameter \( \gamma \) against \( t \). The solid, dot-dashed and dashed lines have the parameter combinations \( \alpha = 2.3, 1.3, 1.5, \beta = 4.5, 2.5, 1.5, l_1 = 0.3, 0.5, 0.2 \) and \( l_2 = 4.5, 6, 7 \).

**B. Case II**

In this section we shall only assume \( a(t) = a_0 t^n \); and \( \phi(t) = \phi_0 t^m \). We shall use these ansatz to reconstruct the potential \( V \) and the analytic function \( f \) of the chameleon BD cosmology. On
FIG. 2: Evolution of deceleration parameter $q$ (equation (21)) against $t$. The solid, dot-dashed and dashed lines have the parameter combinations $\alpha = 2.3, 1.3, 1.5$, $\beta = 4.5, 2.5, 1.5$, $l_1 = 0.3, 0.5, 0.2$ and $l_2 = 4.5, 6, 7$. Putting these ansatz in equation (8) we get an equation that involves $V$, $f$ and remaining terms as functions of $t$

$$V = -\frac{1}{a_0^3} \rho_{m0} t^{-3n} f + \frac{\phi_0 t^{m-2}}{2} [-m^2 \omega + 6n(m + n - (n - \beta)f)]$$  \hspace{1cm} (22)

We again differentiate it with respect to $t$ and get a new equation involving $\dot{V}$ and $\dot{f}$. Using this in (9) along with the said ansatz we get the following differential equation on $f$ with $t$ as the independent variable.

$$\frac{df(t)}{dt} = \frac{3(m + 2n)}{2} f(t) = \frac{3a_0^3 \phi_0 t^{m+3n-1} (mn(t(3 + 2\omega) - 4 - 2n) + m^3(1 + \omega) - m^2(1 + 2n) - 4n^2)}{2 \left( \rho_{m0} t^2 - a_0^3(m - 2) \phi_0(n\alpha - \beta) \right)}$$

Solving (23) we get reconstructed analytic function $f(\phi)$ as

$$f(\phi) = f(\phi_0 t^m) = t^{m+3n} \left[ C_1 t^{\frac{\alpha}{2}} - \frac{2}{\rho_{m0}(8+6m+m^3)} \right]$$

$$+ \left( \frac{mn(4 + m)(3 + 2\omega) t}{2^{2m-6n+1}} \cdot \text{Hypergeometric2F1} \left[ \frac{2+3}{4-2m-6n+1}, \frac{6(n-1)+m}{2(3n+m-2)}, \frac{a_0^3(m-2)\phi_0(n\alpha - \beta)}{\rho_{m0}} \right] \right)$$

$$\times \text{Hypergeometric2F1} \left[ \frac{4+m}{4-2m-6n+1}, \frac{6n-8+m}{2(3n+m-2)}, \frac{a_0^3(m-2)\phi_0(n\alpha - \beta)}{\rho_{m0}} \right] \right)$$  \hspace{1cm} (24)
Using (24) in (22) we get the reconstructed potential function as

\[
V(\phi) = V(\phi_0 t^m) = \frac{t^{-4+m}}{2a_0^2(2+m)(4+m)\rho_{m0}} \times \\
\left( -(2 + m)(4 + m)\rho_{m0}t^2(2C_1\rho_{m0}t^{\frac{4+m}{2}} + a_0^3\phi_0(-6mn + 6n(-n + C1^{\frac{1}{2}}(m+2n)(n\alpha - \beta)) + m^2\omega)) \right) \\
+ 4(\rho_{m0}t^2 + 3a_0^3\phi_0 t^{m+3n}(n\alpha - \beta)) \times \\
mn(4 + m)(3 + 2\omega)t \HypergeoF1\left[ 1, \frac{2+m}{1-2m-6n}, 1 + \frac{2+m}{4-2m-6n}, \frac{a_0^2(m-2)\phi_0(n\alpha-\beta)t^{-2+m+3n}}{\rho_{m0}} \right] + \\
(2 + m)(-4n^2 - 2mn(2 + n) - m^2(2n + 1) + m^2(1 + \omega)) \\
\times \HypergeoF1\left[ 1, \frac{4+m}{4-2m-6n}, 1 + \frac{4+m}{4-2m-6n}, \frac{a_0^2(m-2)\phi_0(n\alpha-\beta)t^{-2+m+3n}}{\rho_{m0}} \right]
\]

(25)

FIG. 3: Reconstructed \( f(\phi) \) against \( \phi \) for various combination of parameters.

FIG. 4: Reconstructed \( V(\phi) \) against \( \phi \) for various combination of parameters.
In Figs. 3, 4 and 5 we have plotted the reconstructed $f(\phi)$ against $\phi$, reconstructed $V(\phi)$ against $\phi$ and reconstructed $V(\phi)$ against $f(\phi)$ respectively. We observe that $f(\phi) \to 0$ as $\phi \to 0$ and $V(\phi) \to 0$ as $\phi \to 0$. These indicate satisfaction of one of the sufficient conditions for realistic models. In Fig. 5 we observe that the potential function $V(\phi)$ is increasing with the coupling function $f(\phi)$. Thus, we infer that as the matter-chameleon coupling gets stronger, the potential function also increases. In Fig. 6 we have plotted $f(\phi)$ against $t$ for a range of positive values of $m$. This figure shows that $f(\phi)$ in an increasing function of the cosmic time $t$. This indicates that with the expansion of the universe, the matter-chameleon coupling gets stronger, where the coupling function has been reconstructed for EHRDE.
IV. CONCLUSION

In the present work we have studied some features of the generalized BD model in which the scalar field is allowed to couple non-minimally with matter sector. Extended holographic Ricci dark energy has been considered in the above framework of BD cosmology. The energy density has been considered in the form \( \rho = 3\phi(aH^2 + \beta \dot{H}) \) to accommodate the BD scalar field in the dark energy density. The work has been carried out in two different perspectives. Firstly, we have assumed the ansatz \( V(\phi) = V_0 \phi^{l_1} ; f(\phi) = f_0 \phi^{l_2} ; a(t) = a_0 t^n ; \) and \( \phi(t) = \phi_0 t^m \) and derived some restrictions on the BD parameter \( \omega \). Under this restriction we have investigated the equation of state parameter for the dark energy and a quintom-like behavior has been observed. Furthermore, the deceleration parameter has shown a transition from decelerated to accelerated phase of the universe. In this scenario, it has been observed that the restrictions are leading to a stronger matter-chameleon coupling with the expansion of the universe. Secondly, we have considered only \( a(t) = a_0 t^n ; \) and \( \phi(t) = \phi_0 t^m \) and reconstructed the coupling function and potential for the extended holographic Ricci dark energy in BD model. We have observed that the potential is increasing with increase in matter-chameleon coupling. Furthermore, we have seen that for any positive value of \( m \), \( f(\phi) \) is an increasing function of \( t \). This means that for this reconstruction, the coupling function is increasing i.e. the matter-chameleon coupling is getting stronger with passage of cosmic time.

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