Dark Matter Production in an Early Matter Dominated Era

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Abstract. We investigate dark matter (DM) production in an early matter dominated era where a heavy long–lived particle decays to radiation and DM. In addition to DM annihilation into and thermal DM production from radiation, we include direct DM production from the decay of the long–lived particle. In contrast to earlier treatments the temperature dependence of the number of degrees of freedom $g_*$ in the Standard Model (SM) plasma is treated carefully. Besides the well–known cases of thermal hot and cold DM, additional regions of parameter space with the approximately correct DM relic density appear. In some of these regions the temperature dependence of $g_*$ can change the final DM density by several hundred percent. Furthermore, we analyze the effect of allowing nonvanishing initial abundances for radiation and DM. We find an upper bound on the mass of the long–lived particle if the DM annihilation cross section is below that corresponding to thermal WIMP (Weakly Interactive Massive Particle) DM in standard cosmology.

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1 Introduction

The nature of Dark Matter (DM) has been a mystery for several decades. Most proposals for its explanation [1] require new particle physics, since astrophysical and cosmological observations imply that DM consists of cold particles (which were non–relativistic at the onset of structure formation) [2, 3]. The Standard Model (SM) of particle physics does not contain any such particle, whereas many extensions of the SM do.

A widely studied class of DM candidate particles are Weakly Interacting Massive Particles (WIMPs) [2]. They can be thermally produced in the early universe. This means that at sufficiently high temperature they were in thermal equilibrium with the plasma of SM particles. However, as the universe expanded and hence cooled, their abundance dropped, until the WIMP annihilation rate equaled the Hubble expansion rate. At this temperature WIMPs decoupled (“froze out”), meaning their comoving density became essentially constant. In standard cosmology this mechanism requires a specific value for the (thermally averaged) WIMP annihilation cross section in order to reproduce the observed DM density; this cross section “happens” to be close to a typical weak cross section (hence the name). For example, supersymmetric (SUSY) extensions of the SM contain WIMP candidates [4].

Thermal WIMPs are attractive since they can be searched for in several different ways. However, neither direct [5–8] nor indirect [9–11] WIMP searches have found any signal as yet, and collider searches for particles not contained in the SM have also only yielded (a large number of) bounds but no positive evidence. This has led to renewed interest in extensions of the simple thermal WIMP scenario.

One possibility is to consider a modified expansion history of the universe. In standard cosmology one assumes that the universe became radiation dominated after the end of inflation, and stayed that way down to a temperature of about 1 eV, at which point it became dominated by (mostly dark) matter. However, string theory and other UV–complete theories suggest that there may have been an epoch of early matter domination after inflation and before Big Bang Nucleosynthesis (BBN), which must have occurred in a radiation–dominated era. This early matter domination would have been due to massive scalar particles with very weak couplings to SM particles, and hence long lifetimes. In string theory the vacuum expectation values (VEVs) of such “moduli” fields determine the couplings of the low energy theory [12, 13]. In fact, already supergravity theories where supersymmetry is broken in a hidden sector contain scalar (“Polonyi” [14]) fields with similar properties. These scalar...
fields obtained large values during inflation, if their mass was smaller than the Hubble scale during inflation [15–19]. Later these fields started to oscillate coherently, which corresponds to an ensemble of scalar particles at rest in the cosmic rest frame.

If these fields dominated the energy density of the universe, their decay produced a lot of entropy. This would have diluted the density of all particles that had been produced before. Moreover, these decays may have occurred so late that they affected the (largely) successful predictions of standard BBN; this is the cosmological moduli (or Polonyi) problem [20–24]. The detailed analysis [25, 26] showed that the reheat temperature, i.e. the highest temperature of the radiation-dominated epoch, must have been at least $\sim 4$ MeV in order not to jeopardize the success of standard BBN; this bound has more recently been confirmed in [27]. We will see below that this requires the scalar mass to be well above $10^4$ TeV. Such a large mass is problematic if this is also the scale of visible-sector superparticle masses, and one wishes to use supersymmetry to solve the hierarchy problem. (However, large superparticle masses are still acceptable for “split” Supersymmetry [28, 29].) Any realistic model assuming that DM is produced during an early epoch of matter domination by a heavy scalar [30, 31] has to respect this bound.

Early studies of the non-thermal production of DM particles focused on the reheating era at the end of inflation [32–35]; more recently, this has been analyzed in [36].

In supersymmetric or superstring theories decays of the gravitino can also cause problems with BBN (gravitino problem). Some models that solve this problem predict a period of modulus domination [37]. In other scenarios gravitino decays do not overproduce DM [38], or the gravitino is itself a stable DM candidate [39, 40]. In some supersymmetric scenarios moduli can decay to gravitinos; in this case the gravitino mass should be high enough to prevent its decay at BBN time (moduli-induced gravitino problem) [23, 38, 41]. On the other hand, if the gravitino mass is larger than that of the moduli [37, 42, 43] a solution of the moduli problem automatically solves the gravitino problem as well. In our analysis we will ignore the gravitino, implicitly assuming that one of these solutions is at work.

Generally there are two thermal DM production mechanisms. The freeze-out (FO) scenario for WIMP DM has been described above; here the relevant dynamics occurs around the freeze-out temperature, which is typically 5% of the WIMP mass. In contrast, in the freeze-in (FI) scenario dark matter is produced at temperatures above the mass of the DM particle, e.g. by the decay of a heavier particle. Here the DM annihilation cross section is so small that DM annihilation is negligible [34, 44]. These two general processes can also happen in an early matter dominated era, since the decays of the heavy scalar will generate a (subdominant) radiation component. However, other possibilities exist for DM production during such an epoch. These have been explored in [45], which in addition considered the decay of a WIMP-like visible sector particle into a lighter DM particle residing in a hidden sector.

The present analysis is based on [45]. However, we assume that DM resides in the visible sector, and do not include a dark radiation component (which is quite strongly constrained by recent cosmological data [46]). We instead focus on a more careful description of the thermal history of the universe. This includes effects due to the temperature dependence of the effective number of relativistic degrees of freedom $g_*$ (and of the analogous quantity defined via the entropy density rather than via the energy density of the radiation). Here we use the results of [47], which assumes free electroweak gauge and Higgs bosons, as appropriate for a smooth crossover electroweak transition [48–50]. Moreover, it uses results from lattice QCD around the QCD transition temperature, matched to a hadron resonance gas at lower temperatures [51, 52]. Finally, at MeV temperatures neutrino decoupling is treated using
results from [53]. Moreover, we also consider scenarios with non-vanishing radiation and DM content at the onset of the early matter dominated epoch.

The remainder of this article is organized as follows. In the next Section we describe the general formalism for computing the DM relic density in a cosmology with an early matter dominated epoch. Here we largely follow ref.[45], but with improved treatment of the radiation component. In Sec. 3 we map out regions of parameter space giving the correct DM relic density assuming initially vanishing DM and radiation content, with special emphasis on the effect of the temperature dependence of \(g_*\). In Sec. 4 we then allow non–vanishing initial radiation and DM density, before concluding in Sec. 5.

2 The General Framework for Non-Thermal Dark Matter Production

In this Section we describe the calculational framework for computing the DM relic density in a scenario with an early epoch of matter domination. In the first Subsection we define the variables we use, and the equations that determine their evolution during the early universe. Our analysis is model–independent in the sense that all relevant particle physics quantities – particle masses, the DM annihilation cross section and the decay width of the heavy scalar particle – are treated as free parameters. This discussion is based on ref.[45], but we extend it by correctly treating the temperature dependence of the number of relativistic degrees of freedom in the thermal plasma. In the second Subsection we briefly describe the numerical solution of the evolution equations, and the relation of the dimensionless quantities introduced in the first Subsection to the scaled DM relic density.

2.1 Evolution Equations

In the standard thermal DM production scenarios (both freeze–in and freeze–out) one only needs to solve a single evolution equation, namely the Boltzmann equation for the number density of the DM particles. In these scenarios all the relevant dynamics happens during the radiation dominated epoch, so the state of the universe is essentially determined uniquely by the temperature \(T\). In particular, both the Hubble parameter and the entropy density are functions of \(T\) only; moreover, the comoving entropy density is constant in this case, since the universe evolves adiabatically.

This is no longer true in the case we are interested in, where the energy density of the universe was dominated for a while by slowly decaying scalar \(\phi\) particles.\(^1\) We therefore need to track three coupled evolution equations: for the DM particles, for the \(\phi\) particles, and for the radiation content.

Following [13, 34, 45] we introduce dimensionless quantities in order to describe the evolution of the universe. To this end all dimensionful quantities are divided by appropriate powers of the “reheat temperature”, defined as

\[
T_{\text{RH}} = \sqrt{\frac{45}{\Gamma_\phi M_{\text{Pl}}} \left(\frac{45}{4\pi^2 g_*(T_{\text{RH}})}\right)^{1/4}}.
\]

Here \(M_{\text{Pl}} \simeq 1.22 \times 10^{19} \text{ GeV}\) is the Planck mass, \(\Gamma_\phi\) is the total decay width of the \(\phi\) particles, and \(g_*\) is the number of relativistic degrees of freedom in the thermal plasma, defined via the energy density of radiation [54]:

\[
\rho_R(T) = \frac{\pi^2}{30} g_*(T) T^4.
\]

\(^1\)The spin of the decaying particles is not relevant for us. However, the by far best motivated particle physics realizations of this mechanism use scalar moduli or Polonyi fields, as discussed in the Introduction.
Note that \( T_{\text{RH}} \) is a bit of an idealization: it is the temperature the universe would have if all the energy stored in \( \phi \) was instantaneously transformed into radiation at Hubble parameter \( H = \Gamma_\phi \), under the assumption that \( \phi \) particles completely dominated the energy density of the universe before their decay \([13, 27, 33, 35, 55]\). Nevertheless \( T_{\text{RH}} \) is a good estimate of the highest temperature of the radiation–dominated epoch that begins after most \( \phi \) particles have decayed. Note, however, that the thermal bath during moduli domination can be considerably hotter than \( T_{\text{RH}} \); we will come back to this point below.

The decay width of moduli (or Polonyi) fields \( \phi \) are Planck suppressed, but the precise coupling strength is model dependent. We thus write

\[
\Gamma_\phi = \alpha \frac{M_\phi^3}{M_{\text{Pl}}^2}, \quad \alpha = \frac{C}{8\pi} = \text{constant},
\]

where \( M_\phi \) is the mass of \( \phi \), and \( C \) is a constant whose value depends on the UV–complete theory. The value of \( T_{\text{RH}} \) is obtained by computing \( \Gamma_\phi \) (which requires fixing \( M_\phi \) and \( C \) or \( \alpha \)), and inserting this into eq.(2.1). Note that this is an implicit equation, since the right–hand side (rhs) depends on \( T_{\text{RH}} \) via \( g^* \). We use results from ref.[47] to compute the temperature dependence of \( g^* \).

As noted above, the success of standard BBN requires \( T_{\text{RH}} \gtrsim 4 \) MeV. This implies \( M_\phi \gtrsim 100 \) TeV for \( \alpha \sim 1 \) \([25–27]\). In Fig. 1 the dependence of the reheating temperature on the modulus mass according to eq.(2.1) is shown for different couplings \( \alpha \) between \( 10^{-4} \) and 1. Here we have assumed that only SM particles contribute to \( g^* \). If some new particles are found, as predicted e.g. by supersymmetric extensions of the SM, this figure will definitely change at higher reheating temperatures, i.e for larger \( M_\phi \). For comparison, Fig. 1 also shows results for fixed \( g^*(T_{\text{RH}}) = 10.75 \), as appropriate for the SM at temperatures before BBN but after the decoupling of muons. In this plot, with its logarithmic axes, the differences between the two sets of curves become apparent only for \( T_{\text{RH}} \) of order the QCD transition temperature \( T_c \sim 150 \) MeV or higher.\(^2\) However, this is by far not the only way in which the temperature dependence of \( g^* \) affects the final result.

Our main interest is the calculation of the Dark Matter relic density, by deriving and solving the relevant Boltzmann equations. To that end, we use \( T_{\text{RH}} \) to define the dimensionless scale parameter

\[
A \equiv aT_{\text{RH}};
\]

multiplying the scale factor in the Friedmann–Robertson–Walker metric \( a \) with \( T_{\text{RH}} \) improves the stability of the numerical solution \([13]\). This in turn allows us to define dimensionless co–moving densities for \( \phi \) particles, radiation and DM particles:

\[
\Phi \equiv \rho_\phi A^3 T_{\text{RH}}^4, \quad R \equiv \rho_R A^4 T_{\text{RH}}^4, \quad X' \equiv n_{X'} A^3 T_{\text{RH}}^3.
\]

Here \( \rho_\phi = M_\phi n_\phi \) is the energy density stored in \( \phi \) particles, and \( n_{X'} \) is the number density of the DM particles, which we call \( X' \) following ref.[45]. Note that \( R, \Phi \) and \( X' \) approach constants when \( \phi \) decays as well as the pair production and annihilation of \( X' \) particles can be neglected. Finally, we use the comoving densities to define a dimensionless comoving Hubble parameter:

\[
\tilde{H} \equiv \left( \frac{\Phi + R}{A^2} + \frac{E_{X'} X'}{T_{\text{RH}}} \right)^{1/2}.
\]

\(^2\)The curves also differ slightly for \( T_{\text{RH}} \) below the electron mass, where the actual \( g^*(T_{\text{RH}}) \) is less than 10.75. However, scenarios with such a low reheat temperature will not reproduce standard BBN.
Figure 1: Reheating temperature $T_{RH}$ as function of the mass $M_\phi$ of the particle whose energy density dominates in the early matter dominated epoch, for different coefficients $\alpha$ defined in eq.(2.3). The solid curves have been obtained including the temperature dependence of $g_*$ as predicted by the SM, whereas the dashed curves are for fixed $g_*(T_{RH}) = 10.75$.

Here $E_{X'} \approx (M_{X'}^2 + 3T^2)^{1/2}$ is the average energy per $X'$ particle; this approximation is sufficient for our purposes since the contribution of DM particles to the total energy density is always subdominant in the epoch we are interested in. $\tilde{H}$ is related to the usual (dimensionful) Hubble parameter via

$$H = \tilde{H} T_{RH}^2 A^{-3/2} c_1^{-1/2} M_{Pl}^{-1},$$

where we have introduced the constant $c_1 = 3/(8\pi)$.

As in ref.[45] we assume that $\phi$ particles can decay into $X'$ particles with effective branching ratio $B_{X'}$; the average energy per $\phi$ decay that goes into DM particles is then given by $\bar{B} M_\phi$, with

$$\bar{B} = \frac{E_{X'}B_{X'}}{M_\phi}.$$

A fraction $1 - \bar{B}$ of the $\phi$ energy will then go into SM particles, i.e. into radiation. In many cases a discrete symmetry ensures that $X'$ particles can only be produced pairwise. If $\phi \rightarrow X'X'$ is the dominant $X'$ production mode from $\phi$ decays, then $B_{X'} = 2\Gamma(\phi \rightarrow X'X')/\Gamma_\phi$.\footnote{This ansatz implicitly assumes that $X'$ particles are at least in kinetic equilibrium with the SM radiation. Note that kinetic equilibrium is much easier to attain than full chemical equilibrium.}

Note that we will mostly be interested in scenarios where $B_{X'} \ll 1$, i.e. $\bar{B} \ll 1$; the exact expression for $E_{X'}$ is then again not important.

$\phi$ particles might decay predominantly into heavy SM particles, e.g. top quarks or Higgs bosons, with masses larger than the temperature. However, these heavier SM particles will then decay almost immediately into light SM particles, i.e. into radiation. $\phi$ particles could also decay into some partners of $X'$, e.g. a pair of gluinos in supersymmetric models, which then decay almost immediately into $X'$ plus radiation. All these cases are covered by this formalism.

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In the following analysis we assume that the decay products of $\phi$ thermalize immediately, i.e. radiation always refers to a relativistic plasma in full thermal equilibrium, again following ref. [45]. This is an idealization. If $\phi$ particles decay into two body final states, these final state particles will initially have energy $M_\phi/2$, which can be much higher than $T$. These energetic SM particles could produce DM particles before they thermalize. This has been analyzed in [56], where it was shown that this source of $X'$ particles can be significant if $M_{X'}$ is relatively close to $M_\phi$. We are mostly interested in $M_{X'} \ll M_\phi$, in which case the approximation of instantaneous thermalization of the $\phi$ decay products should be applicable. Recently, it has been claimed that considering the details of thermalization process can change the maximum temperature of the universe and affect the process of DM production during and after early matter domination [57–59]. We postpone to consider these details to future studies.

The effective number of degrees of freedom $g_*$, defined via the radiation energy density as in eq.(2.2), depends on the temperature, since only particles with mass of order of or less than the temperature contribute significantly [54]. In the SM the temperature dependence is rather mild for $T \gtrsim 1$ GeV. As a first approximation one can therefore ignore terms proportional to the derivative of $g_*/dT$. This allows to derive an evolution equation for $R$ from energy conservation. The set of equations one needs to solve is then [45]:

\[
\begin{align*}
\dot{H} \frac{d\Phi}{dA} &= -c_1^{1/2} A^{1/2} \Phi, \\
\dot{H} \frac{dR}{dA} &= c_1^{1/2} A^{3/2} (1 - \bar{B}) \Phi + c_1^{1/2} M_{\text{pl}} \left[ \frac{2E_{X'} \langle \sigma v \rangle'}{A^{3/2}} \left( X'^2 - X'_{\text{EQ}}^2 \right) \right], \\
\dot{H} \frac{dX'}{dA} &= c_1^{1/2} \frac{T_{RH} B_{X'}}{M_\phi} A^{1/2} \Phi + c_1^{1/2} M_{\text{pl}} T_{RH} A^{-5/2} \langle \sigma v \rangle' \left( X'_{\text{EQ}}^2 - X'^2 \right).
\end{align*}
\]  

(2.9)

Here $c_1$ is as in eq.(2.7), and we have defined a second constant $c_\rho = \frac{\pi^3 g_*(T_{RH})}{30}$. Finally, the scaled $X'$ equilibrium number density $X'_{\text{EQ}}$ is given by

\[
X'_{\text{EQ}} \equiv \left( \frac{A}{T_{RH}} \right)^3 \frac{g_{X'} T M_{X'}^2}{2\pi^2} K_2 \left( \frac{M_{X'}}{T} \right) \rightarrow \left\{ \begin{array}{ll}
\left( \frac{A}{T_{RH}} \right)^3 \frac{\bar{g}(T)}{\pi^2} & \text{if } T \gg M_{X'} \\
\left( \frac{A}{T_{RH}} \right)^3 \frac{g_{X'}}{2\pi} \left( \frac{M_{X'} T}{2\pi} \right)^2 \exp(-M_{X'}/T) & \text{if } T \ll M_{X'}
\end{array} \right.
\]  

(2.10)

Here $g_{X'}$ counts the internal degrees of freedom of $X'$, $\bar{g} = g_{X'}(3g_{X'}/4)$ for bosonic (fermionic) $X'$, and $K_2$ is the modified Bessel function of second kind. In our numerical calculations we assume $g_{X'} = 2$, as appropriate for a spin $1/2$ Majorana (self–conjugate) fermion.

The first eq.(2.9) describes $\phi$ decays. Unfortunately it is not entirely straightforward to see in this formalism that $\Phi$ becomes constant when $\phi$ decays can be neglected. Eq.(2.1) shows that $T_{RH} \rightarrow 0$ as $\Gamma_\phi \rightarrow 0$, so eqs.(2.5) become ill–defined in this case. Note, however, that initially $\Phi$, and hence $\dot{H}$, are much bigger than unity if moduli are to dominate the universe for an extended epoch. Initially $d\Phi/dA$ is thus much less than $\Phi$. One can show that $A d\Phi/dA$ becomes of order $\Phi_I$ only when $H \simeq \Gamma_\phi$.

In the second eq.(2.9) we recognize a positive contribution to the rhs proportional to $\Phi$ stemming from $\phi$ decays, and a contribution describing the pair production from and annihilation of $X'$ particles into radiation. A similar term appears on the rhs of the third equation with opposite sign; this third equation also features a positive contribution from direct $\phi \rightarrow X'$ decays.
In order to follow the evolution of the universe more accurately we must consider the precise evolution of \( g_r \) (and related quantities) in the thermal bath. As shown in ref.[54] the evolution of the radiation component is then more easily described via the entropy density, which is given by

\[
s_R(T) = \frac{\rho_R(T) + p_R(T)}{T} = \frac{2\pi^2}{45} h_*(T) T^3. \tag{2.11}
\]

The second equation defines \( h_*(T) \), which is another measure of the effective number of relativistic degrees of freedom; in the SM, \( g_*(T) = h_*(T) \) before neutrinos decouple at MeV temperatures. In standard cosmology, the comoving entropy density is constant after the end of inflation. However, in the scenario considered here \( \phi \) decays lead to entropy production. This is described by the evolution equation

\[
\frac{ds_R}{dA} + 3Hs_R = \frac{1}{T} \left[ (1 - \tilde{B}) \Gamma_\phi \rho_\phi + 2E_{X'}(\sigma v)' \left( n_{X'}^2 - n_{X'}^{eq} \right) \right]. \tag{2.12}
\]

The factor \((1 - \tilde{B})\) in the first term of the rhs of eq.(2.12) should not be there if \( T > M_{X'} \); recall, however, that \( \tilde{B} \ll 1 \) in cases of interest, so that we make only a small mistake by including this factor. The second term describes entropy production from out–of–equilibrium annihilation of \( X' \) particles; we find that this term is always numerically insignificant. [This is true also for the last term on the rhs of the second eq.(2.9)].

This leads to the following equation describing the evolution of the temperature:

\[
\frac{dT}{dA} = \left( 1 + \frac{T}{3h_* dT} \right)^{-1} \left\{ -\frac{T}{A} + \frac{15T^6_{RH}}{2\pi^2 c_1^{1/2} M_{P1} HT^3 h_* A^{11/2}} \left( c_1^{1/2} A^{3/2} (1 - \tilde{B}) \Phi \right) \right. \\
+ c_1^{1/2} M_{P1} 2E_{X'}(\sigma v)' \left( X'^2 - X'^{eq} \right) \left\}. \tag{2.13}
\]

Note that the rhs of eq.(2.13) depends both on \( h_*(T) \) and \( (\text{via } c_p) \) on \( g_*(T) \). We will use the results of [47] for them. This equation replaces the second eq.(2.9); the first and third of these equations remain unchanged. We also need eq.(2.2) to compute the radiation density from \( T_r \), and eqs.(2.6) and (2.5) to compute the scaled Hubble parameter \( \dot{H} \). This is a closed system of equations.

As mentioned above, in the early epoch of matter domination the radiation component can be much hotter than \( T_{RH} \). If terms proportional to the derivative of \( h_* \) or \( g_* \), with respect to temperature are ignored, the evolution of the temperature for \( H \gg \Gamma_\phi \) can be computed analytically. If initially \( \rho_R = 0 \) one finds [34]:

\[
T \simeq \left( \frac{8^8}{3^{3/5}} \right)^{1/20} \left( \frac{g_*(T_{max})}{g_*(T)} \right)^{1/4} T_{\text{max}} \left( A^{-3/2} - A^{-4} \right)^{1/4}. \tag{2.14}
\]

The maximum temperature during modulus domination \( T_{\max} \) is given by [34]

\[
T_{\text{max}} = \left( \frac{3}{8} \right)^{2/5} \left( \frac{5}{\pi^3} \right)^{1/8} \left( \frac{g_*(T_{RH})^{1/2}}{g_*(T_{max})} \right)^{1/4} \left( M_{Pl} H_1 T^2_{RH} \right)^{1/4}. \tag{2.15}
\]

Using \( H_1 = \Phi_I^{1/2} T^2_{RH} / (c_1^{1/2} M_{Pl}) \) we find \( T_{\text{max}} \sim T_{RH} \Phi_I^{1/8} \), up to \( O(1) \) factors, where \( \Phi_I \) is the initial co–moving density of \( \phi \). The assumption of vanishing initial \( \rho_\phi \) is reasonable only if the matter dominated epoch lasts sufficiently long to basically erase all traces of possible
earlier radiation dominated epochs. This requires $H_I \gg \Gamma_\phi$, and hence $\sqrt{\Phi_I} \gg 1$. Hence even if $T_{RH}$ is well below the temperature $T_c$ of the QCD transition, frequently $T_{\text{max}} > T_c$, in which case an accurate treatment of the temperature dependence of $g_*$ and $h_*$ becomes important.

### 2.2 Predicted Dark Matter Abundance

As in case of standard cosmology, the dark matter annihilation cross section plays an important role. Here we will not consider specific particle physics candidates for $X'$. Instead we use the standard parametrization,

$$\langle \sigma v \rangle' = a + b v^2,$$  

(2.16)

which is applicable for particles whose final relic density is set at temperature $T < M_{X'}$. Here $a$ is nonzero only if $X'$ particles can annihilate from an $S$–wave initial state, whereas a non-vanishing $b$ can be generated also by annihilation from the $P$–wave. Thermal averaging then gives

$$\langle \sigma v \rangle' = a + 6 b T \frac{M_{X'}}{M_{\text{pl}}};$$  

(2.17)

In the following chapters we will present exact numerical solutions of the evolution equations discussed in the previous subsections. These have been obtained with the help of Mathematica. We found it convenient to rewrite the equations in terms of the logarithmic derivative $d/d(\ln A) = Ad/dA$, i.e. we multiplied eqs.(2.9) and (2.13) with $A$. We always use initial conditions

$$A = 1, \quad \Phi = \Phi_I = \frac{3 H_I^2 M_{\text{pl}}^2}{8 \pi^4 T_{RH}^4};$$  

(2.18)

in the next chapter we will follow ref.[45] in initially setting

$$R_I = X'_I = 0.$$  

(2.19)

The assumption of initial density for radiation and dark matter particles can be reasonable if $\phi$ particles completely dominate the universe for some time after inflationary reheating, so that all dependence on conditions before the $\phi$ dominated epoch is erased. Recall that in the absence of $\phi$ decays the ratio of radiation and matter densities scales like $1/A$. A possible initial radiation component can then become irrelevant if $\tilde{A} \gg 1$, where $\tilde{A}$ is the value of the dimensionless scale factor where most $\phi$ particles decay. Until this time to good approximation $H_I = \text{constant}$. The first eq.(2.9) can then be solved analytically [45]:

$$\Phi(A) = \Phi_I \exp \left[ -\frac{2}{3} \left( \frac{c_\rho}{\Phi_I} \right)^{1/2} \left( \frac{A^{3/2} - 1}{A^{3/2}} \right) \right],$$  

(2.20)

where we have used the initial value $A = 1$. Most moduli particles decay when $\Phi \simeq \Phi_I \exp(-1)$, which occurs at $A \simeq \tilde{A}$ with

$$\tilde{A} = \left[ \frac{3}{2} \left( \frac{\Phi_I}{c_\rho} \right)^{1/2} + 1 \right]^{2/3}.$$  

(2.21)

Writing

$$H_I = \gamma \Gamma_\phi,$$  

(2.22)
and using the fact that $H_I \propto \sqrt{\Phi_I}$, it is easy to see that $\tilde{A} \gg 1$ requires $\gamma \gg 1$.

In this case, and with eqs.(2.19) imposed, the final computed relic dark matter abundance will be essentially independent of the initial $\Phi_I$, or, equivalently, of $\gamma$. Note that the final scaled densities introduced in eqs.(2.5) do depend on $\Phi_I$. For example, again setting $\tilde{H} = \sqrt{\Phi_I} = \text{constant}$ a good analytical approximation for the final value of $R$ can be derived [45]:

$$R_F \equiv R(A_F) \simeq L \left( \frac{\Phi_I}{\gamma} \right)^{1/3} \Phi_I, \quad (2.23)$$

where we have defined

$$L \equiv (1 - B_{\text{eff}}) \Gamma \left( \frac{5}{3} \right) \left( \frac{3}{2} \right)^{2/3} \frac{1}{\rho_\gamma(T_{\text{now}})} \frac{\rho_{X^I}(T_{\text{now}})}{\rho_{\gamma}(T_{\text{now}})} \frac{\rho_{X^I}(T_F) g_*(T_F) h_*(T_{\text{now}}) T_F}{2 \rho_R(T_F) h_*(T_F)} \frac{T_{\text{now}}}{2T_{\text{RH}}} \frac{\Omega_\gamma h^2}{\Omega_\gamma h^2 - \Omega_{\gamma h^2}}. \quad (2.24)$$

Here $A_F$ should be so large that the comoving abundances of radiation and matter have become constant and $\phi$ decays have been completed, i.e. $A_F \gg \tilde{A}$ with $\tilde{A}$ given by eq.(2.21). On the other hand, $A_F$ should still be within the radiation dominated epoch.

Evidently $R_F \propto \Phi_I^{1/3}$. This dependence cancels once we normalize the final dark matter density to today’s energy density carried by photons, which is known very well from measurements of the Cosmic Microwave Background (CMB). We thus compute the final $X'$ relic abundance from:

$$\Omega_{DM} h^2 = \frac{\rho_{X'}(T_{\text{now}})}{\rho_\gamma(T_{\text{now}})} \Omega_\gamma h^2 = \frac{\rho_{X'}(T_F) g_*(T_F) h_*(T_{\text{now}}) T_F}{2 \rho_R(T_F) h_*(T_F)} \frac{T_{\text{now}}}{T_{\text{RH}}} \frac{\Omega_\gamma h^2}{2T_{\text{now}} T_{\text{RH}} h_*(T_F)}.$$  

(2.25)

Here $\Omega_{DM}$ is the dark matter mass density in units of the critical density, $h$ is today’s Hubble constant in units of $100 \text{ km}/(\text{s} \cdot \text{Mpc})$, and $T_F = T(A_F)$ is in the radiation dominated era, as mentioned above. In the second step we have written $\rho_\gamma = 2 \rho_R / g_*$, and used the fact that the matter density $\rho_{X'}$ scales exactly like the entropy density $s_R$ for $A \geq A_F$, i.e. after all $\phi$ decays and $X'$ annihilations ceased. Note also that $h_*$ becomes constant after electrons decoupled, i.e. for $T \ll m_\epsilon$. The present observational values of the current temperature and density of (CMB) photons cosmic microwave background (CMB), as collected by the Particle Data Group [60], are:

$$\Omega_\gamma h^2 = 2.473 \times 10^{-5}; \quad (2.26)$$

$$T_{\text{now}} = 2.7255 \text{ K} = 2.35 \times 10^{-13} \text{ GeV}.$$

We use these values in our numerical calculations. Cosmological observations also determine the total present density of non-baryonic dark matter quite accurately [60]:

$$\Omega_{DM} h^2 = 0.1186 \pm 0.002. \quad (2.27)$$

This can be used to effectively reduce the dimension of the allowed parameter space by one. However, in this paper we are more interested in mapping out the predicted relic
density as function of the relevant free parameters. These include the reheat temperature, the branching ratio for \( \phi \to X' \) decays, the \( X' \) annihilation cross section as parameterized in eq.(2.17), and the masses of the \( \phi \) and \( X' \) particles. In Sec. 4 we will in addition allow non–vanishing initial values for the radiation and \( X' \) densities.

### 3 Dark Matter Relic Density for Initially Vanishing Radiation

In this Section we show numerical results for the predicted \( X' \) relic density for vanishing initial radiation and \( X' \) densities, i.e. for initial conditions given by eqs.(2.18) and (2.19). All \( X' \) particles – and all other particles in today’s universe – then originate from \( \phi \) decay, either directly or via the radiation that originates from \( \phi \) decay.

Before presenting numerical results, it is useful to briefly discuss the different DM production mechanisms in non–thermal cosmology. Here we again closely follow ref.[45], where analytical approximations based on eqs.(2.9) were developed. As discussed in the previous Chapter, these equations ignore terms that depend on the derivative of \( g_* \) or \( h_* \) with respect to temperature. While our numerical treatment fully includes these effects, the analytical approximations remain useful as a guide to the (quite large) parameter space. We also remind the reader that, unlike ref.[45], we do not include a dark radiation component.

It should be clear that the usual thermal WIMP scenario can also be reproduced in our framework, if \( T_{RH} \) is above the conventional decoupling temperature \( T_{FO} \) defined in the radiation dominated epoch. This requires rather large reheat temperatures, and hence very large \( \phi \) masses as shown in Fig. 1, or else quite small masses for the dark matter particle \( X' \). In the latter case one would typically need additional light mediators in order to achieve a sufficiently large \( X' \) annihilation cross section. Of course, here we are mostly interested in scenarios that differ from this standard thermal WIMP scenario.

A first important observation is that for not too small \( X' \) annihilation cross section, the rhs of the third eq.(2.9) essentially vanishes over an extended range of \( A \). In the absence of \( \phi \to X' \) decays this corresponds to \( X' \) particles being in full thermal equilibrium, but in the present context this “quasi–static equilibrium” (QSE) can also be obtained through a balance between \( X' \) production from \( \phi \) decay and \( X' \) annihilation, with negligible \( X' \) production from the thermal plasma. The general QSE solution is:

\[
X'_{\text{QSE}}(A) = \left[ A^3 \frac{c_p^{1/2} B_{X'}}{c_1^{1/2} M_{X'} M_{Pl}} \Phi + X^2_{\text{EQ}} \right]^{1/2}.
\]  

(3.1)

QSE will be maintained only if the reaction rate \( n_{X'} \langle \sigma v \rangle' \) is larger than the Hubble expansion rate. This requires \( X' \geq X'_{\text{crit}} \), with

\[
X'_{\text{crit}} \equiv \frac{(n_{X'})_{\text{crit}} A^3}{T_{RH}^3} = \frac{HA^3}{(\langle \sigma v \rangle') T_{RH} T_{RH}} = \frac{HA^{3/2}}{c_1^{1/2} M_{X'} M_{Pl} T_{RH} (\langle \sigma v \rangle')}. 
\]  

(3.2)

Clearly QSE can only be achieved if \( X'_{\text{QSE}} > X'_{\text{crit}} \). This leads to a lower bound \( \langle \sigma v \rangle' \) on the thermally averaged \( X' \) annihilation cross section. This lower bound depends on the temperature. If \( T \) is much smaller than \( M_{X'} \), the term \( \propto (X'_{\text{EQ}})^2 \) in eq.(3.1) can be ignored. On the other hand, for sufficiently high temperature \( \langle \sigma v \rangle' \) can be calculated from \( X_{\text{QSE}} = X_{\text{EQ}}' \). Explicit expressions for the critical cross section can be found in [45].
One can classify different regions of parameter space according to whether the thermally averaged $X'$ annihilation cross section is above or below the critical one for $T \simeq T_{RH}$; this is called efficient and inefficient annihilation.

We first consider the case of efficient $X'$ annihilation. Let $T_{FO}$ be the $X'$ freeze–out temperature, computed in a radiation dominated universe. If $T_{FO} > T_{RH}$ then $X_{EQ} \sim 0$ for $T \lesssim T_{RH}$. In this “non–relativistic quasi static equilibrium” case the relic abundance can be approximated by

$$\Omega h^2[QSE_{nr}] \propto \frac{M_{X'}}{g_*(T_{RH})^{1/6}L^{3/4}M_{Pl}\langle\sigma v\rangle^{1/2}T_{RH}}. \quad (3.3)$$

In this case the final dark matter density depends on $\phi$ properties only via $T_{RH}$. The dependence on the annihilation cross section is as in the standard thermal WIMP scenario; however, here the relic density is also proportional to the $X'$ mass.

If $X'$ annihilation is efficient at $T_{RH}$ and $T_{FO} < T_{RH}$ we are back in the standard scenario. The relic density for non–relativistic and relativistic dark matter particles can then be estimated as

$$\Omega h^2[FO_{nr}^{rad}] \propto \frac{1}{g_*(T_{FO})^{1/2}M_{Pl}\langle\sigma v\rangle^{1/2}} \times \frac{x_{FO}'}{T_{FO}}, \quad (3.4)$$

$$\Omega h^2[FO_{r}^{rad}] \propto \frac{M_{X'}}{g_*(T_{FO})}. \quad (3.5)$$

If the annihilation of DM particles is inefficient at $T_{RH}$, the DM relic will be affected by $X'$ production during the early matter dominated era and the branching ratio for $\phi \rightarrow X'$ decay. Since most $\phi$ decays occur at $T \sim T_{RH}$ when $X'$ annihilation is assumed to be inefficient one can write

$$\Omega_{X'}h^2 = \Omega_{ann}h^2 + \Omega_{decay}h^2. \quad (3.6)$$

The contribution $\Omega_{decay}h^2$ comes from $\phi$ decays and obeys

$$\Omega_{decay}h^2 \propto L^{-3/4}B_{X'}T_{RH}M_{X'}^{-1}. \quad (3.7)$$

The second contribution to the rhs of eq.(3.6) stems from the interactions of $X'$ particles with the thermal plasma during the matter dominated epoch. Recall that we are assuming these interactions to be negligible at $T \sim T_{RH}$. However, this does not exclude the possibility that $X'$ might have been in equilibrium at higher temperatures, still in the matter dominated epoch, and decoupled at temperature $T_{FO}$ with $T_{max} > T_{FO} > T_{RH}$. This can happen only for dark matter particles that were non–relativistic at decoupling $[45, 61]$, i.e. $M_{X'} > T_{FO}$. This “modified non–relativistic freeze–out” scenario leads to

$$\Omega_{ann}h^2[FO_{nr}^{mod}] \propto \frac{g_*(T_{RH})^{1/2}T_{RH}^{4}x_{FO}'}{L^{3/4}g_*(T_{FO})M_{X'}^{3}M_{Pl}\langle\sigma v\rangle^{1/2}} \times \frac{x_{FO}'}{T_{FO}}, \quad (3.8)$$

Note that here the contribution to the relic density is again inversely proportional to the annihilation cross section, as in the case of standard thermal WIMPs. However, the dependence on $T_{RH}$ and $M_{X'}$ is quite different (and stronger) than in scenarios where $X'$ annihilation is still efficient at $T \sim T_{RH}$, c.f. eq.(3.3).
On the other hand, for sufficiently small annihilation cross section the $X'$ density never reached equilibrium. As long as this cross section is not zero, there will nevertheless be a contribution to the dark matter relic density from $X'$ pair production from the thermal plasma. This “inverse annihilation” contribution can be significant both for relativistic ($M_{X'} < T_{\text{RH}}$) and for non–relativistic ($M_{X'} > T_{\text{RH}}$) $X'$ particles:

$$\Omega_{\text{ann}}h^2[\text{IA}_{\text{nr}}] \propto \frac{g_4(T_{\text{RH}})^{3/2}T_{\text{RH}}^7M_{\text{Pl}}\langle \sigma v \rangle'}{g_*(T_\star)^3M_{X'}^5}; \quad (3.9)$$

$$\Omega_{\text{ann}}h^2[\text{IA}_{\text{r}}] \propto \frac{T_{\text{RH}}M_{X'}M_{\text{Pl}}\langle \sigma v \rangle'}{g_*(T_{\text{RH}})^{3/2}}. \quad (3.10)$$

Note that this contribution is directly proportional to the annihilation cross section (which equals the $X'$ pair production cross section); this is true also in standard cosmology if the dark matter particles never attained equilibrium, e.g. because the temperature was too low [62]. In the non–relativistic case the dependence on $T_{\text{RH}}$ and $M_{X'}$ is very strong. The production of $X'$ particles that were non–relativistic at $T_{\text{RH}}$ peaks at $T_\star \simeq 0.28M_{X'}$, when the dark matter particles were in fact semi–relativistic. Note that eq. (3.9) is valid only if $T_{\text{max}}$ is larger than $T_\star$; otherwise this contribution is exponentially suppressed. In contrast, the production of relativistic $X'$ particles peaks at $T_\star \simeq T_{\text{RH}}/2$.

Altogether one can thus distinguish seven different $X'$ production mechanisms: FO$^{\text{rad}}$$_{\text{nr}}$, FO$^{\text{rad}}$, FO$^{\text{nr}}$, IA$_r$, QSE$_{\text{nr}}$, IA$_{\text{nr}}$ and $\phi$–decay. They dominate in different regions of parameter space. Of course, these regions are smoothly connected, i.e. one can interpolate between these different regions.

Parameter regions where these different $X'$ production mechanisms are dominant are indicated in Fig. 2. Here and in the subsequent figures we use eq.(2.3) with $\alpha = 1$ to compute the total $\phi$ decay width, which in turn determines the reheat temperature via eq.(2.1). In this figure we have assumed a rather heavy $\phi$ particle, and hence a value of $T_{\text{RH}}$ well above the temperature of the QCD deconfinement transition. Moreover, we have assumed a constant ($S$–wave) $X'$ annihilation cross section, and fixed $B(\phi \rightarrow X') = 10^{-5}$. We do not consider values of $M_{X'}$ below 10 MeV, since for $M_{X'} \ll T_{\text{RH}}$ the early $\phi$–matter dominated epoch becomes essentially irrelevant. On the other hand, we restrict ourselves to $M_{X'}$ values below a few percent of $M_\phi$ since otherwise the approximation of instantaneous thermalization of $\phi$ decay products might break down, as remarked above.

We see that for $M_{X'} \lesssim T_{\text{RH}}$ one recovers the results of standard cosmology. In particular, in the top–left part of Fig. 2 one recognizes the usual WIMP strip, where $\Omega_{\text{DM}}h^2$ comes out near the desired value if $\langle \sigma v \rangle' \sim 10^{-9}$ GeV$^{-2}$. Recall that the freeze–out temperature in the radiation dominated epoch $T_{\text{FO}}$ is about $20M_{X'}$; for $\langle \sigma v \rangle' > 10^{-15}$ GeV$^{-2}$ large deviations from standard cosmology therefore become evident only for $M_{X'} \geq 10$ GeV, which corresponds to $T_{\text{FO}} \gtrsim T_{\text{RH}}$.

Still in the region of small $M_{X'}$, another region with roughly correct DM relic density can be seen for much smaller cross sections. In this “inverse annihilation” region there is sufficient $X'$ pair production from the thermal plasma, but the $X'$ density never reaches thermal equilibrium. The results of standard cosmology are now only recovered for $M_{X'} \lesssim T_{\text{RH}}$; in this part of parameter space the “inverse annihilation” mechanism can be considered to be an example of the freeze–in mechanism [44].

For larger $X'$ the standard WIMP region merges into a region where the correct relic density is obtained via thermal freeze–out in the $\phi$ matter dominated epoch. Note that this requires significantly smaller $X'$ annihilation cross section than in the WIMP region.
Figure 2: The dark matter relic density for $M_\phi = 5 \times 10^6$ GeV, corresponding to reheating temperature $T_{RH} = 848.5$ MeV, $g_\star(T_{RH}) = 73.46$, and branching ratio $B(\phi \to X') = 10^{-5}$. The dark matter mass $M_{X'}$ and the $S$–wave annihilation cross section $\langle \sigma v \rangle' = a$ are given on the $x$– and $y$–axis, respectively. The colored regions represent different bins of the final dark matter relic density, computed including the full temperature dependence of $g_\star$ and $h_\star$, whereas the solid lines are contours of constant $\Omega_{X'} h^2 = 0.12$ (deeper inside the yellow region) and 0.012, respectively, under the approximation $g_\star = h_\star = g_\star(T_{RH})$.

The reason is that here the $X'$ density keeps getting diluted by the entropy produced by $\phi$ decays; recall that the relic density is inversely proportional to the annihilation cross section in both freeze–out regions, see eqs.(3.4) and (3.8).

The latter of these equations also shows that in this region the cross section required to obtain the desired relic density scales like $M_{X'}^{-3}$. As $M_{X'}$ is increased the cross section therefore rather quickly becomes too small for $X'$ to achieve full thermal equilibrium. Recall that the Hubble parameter in the $\phi$ dominated epoch is (much) larger than in the radiation dominated epoch at the same temperature, requiring a correspondingly larger cross section to obtain equilibrium. Nevertheless in Fig. 2 the region where the DM density for $X'$ masses in the typical WIMP region (between 100 and 1000 GeV) comes out roughly correctly extends to very small cross sections, the dominant production mechanism being “inverse annihilation” or, for even smaller $\langle \sigma v \rangle'$, direct $\phi \to X'$ decays.

Finally, there is another region with roughly correct relic density in Fig. 2, where the $X'$ annihilation cross section is significantly larger than that required for thermal WIMPs in standard cosmology. Here quasi–static equilibrium between $X'$ production from $\phi$ decays and $X'$ annihilation is achieved. Eq.(3.3) shows that here the required cross section scales like $M_{X'}$. This region therefore merges with the “modified freeze–out” region for $M_{X'} \approx 100$ GeV, but allows to reproduce the correct relic density for very large $X'$ annihilation cross sections if the $X'$ mass is sufficiently large.\footnote{The possibility to obtain the correct relic density in moduli–dominated scenarios where the annihilation cross section is too large for the normal thermal WIMP scenario was to our knowledge first discussed in...}
Note that in Fig. 2 the DM relic density comes out roughly correctly for $M_{X'}$ of a few hundred GeV almost independently of the $X'$ annihilation cross section, as long as the latter does not exceed a few times $10^{-7}$ GeV$^{-2}$. This remains true [45] also for lower $T_{\text{RH}}$, corresponding to lower $\alpha$ and/or lower $M_{\phi}$; however, the $\phi$ decay region then directly merges into the QSE$_{\text{at}}$ region. Moreover, in Fig. 2 all regions with approximately correct relic density are continuous. This is no longer the case for lower $T_{\text{RH}}$, where thermal effects are important only for smaller $X'$ masses; if the effective branching ratio $B_{\text{eff}}$ is kept fixed, there is then an extended region of $X'$ masses where the $X'$ relic density is always too low, independent of the $X'$ annihilation cross section [45].

These gross features are not affected by an accurate treatment of the number of degrees of freedom in the thermal plasma. However, the solid contours in Fig. 2 show that simply taking $g_* = h_* = g_*(T_{\text{RH}})$, which seems to have been the approach used in ref.[45], can lead to sizable errors of the final DM relic density. This is further illustrated in Fig. 3, where we show the predicted DM relic density as function of $M_{X'}$. The solid, dashed and dash–dotted curves have been obtained by correctly treating the full temperature dependence of $g_*$ and $h_*$, by keeping $g_*$ and $h_*$ dependent on temperature but setting $dh_*/dT = 0$ in eq.(2.13), and by setting $g_* = h_* = g_*(T_{\text{RH}})$ everywhere, respectively.

We see that this last choice can over–predict the relic density by as much as two orders of magnitude; see the blue (top) curves for $M_{X'} \approx 3$ GeV. Here the relic density is determined by freeze–out during the $\phi$ matter dominated epoch, with $T_{\text{FO}}$ not far from the QCD transition temperature where $g_*$ and $h_*$ vary quickly. In this example, $T_{\text{RH}} = 40$ MeV is well below the QCD transition, with $g_*(T_{\text{RH}}) = 13.84$. Above the QCD deconfinement transition the actual $g_*$ is much higher, which means that the actual temperature is lower than that predicted in the approximation $g_* = h_* = g_*(T_{\text{RH}})$.

Moreover, setting the $dh_*/dT = 0$ over–predicts the relic density by about a factor of three even for large $X'$ masses, where the relic density is set by direct $\phi \rightarrow X'$ decays, which are independent of the thermal plasma. The reason is that the final physical DM density is obtained by normalizing the dimensionless comoving density $X'$ to the radiation energy density (or, equivalently, to the entropy density). Unlike in standard cosmology, the comoving entropy density is not constant during the epoch of $\phi$ matter domination; the actual temperature, or entropy density, depends on the number of degrees of freedom in the thermal plasma. Moreover, if one uses eq.(2.11) to compute the entropy density, including the $T$ dependence of $h_*$ but setting $dh_*/dT = 0$ or, equivalently, if one uses the second eq.(2.9) to describe the evolution of the radiation component, the entropy density $s_R$ will not be conserved in the radiation–dominated epoch after $\phi$ decay.

This is further illustrated by Fig. 4, where we plot the rescaled dimensionless temperature $\tilde{T} = TA/T_{\text{RH}}$ as a function of $A$ for $T_{\text{RH}} = 40$ MeV and $H_I = 10^{15}$ GeV, which determine $\Phi_I$ via $\Phi_I = 3H_I^2 M_{\tilde{p}}^2/(8\pi T_{\text{RH}}^4)$. Note that $\tilde{T}$ approaches a constant in the radiation dominated epoch if $g_*$ and $h_*$ are constant. Since we assume initial temperature $T_I = 0$, see eq.(2.19), the universe goes through the QCD transition twice in this example: once early on, during the rapid heating phase culminating at the maximal temperature estimated in eq.(2.15), and then again for much larger $A$, but (in this example) still in the $\phi$ matter dominated epoch. Since $dh_*(T)/dT \geq 0$ everywhere, the prefactor on the rhs of eq.(2.13) always tends to slow down the evolution of $T$, or $\tilde{T}$, with $A$. This implies a slower increase of $T$, and hence a reduced $T_{\text{max}}$, during reheating, but also a slower decline of $T$ when the universe undergoes the QCD transition for a second time. In particular, the simplified ref.[63].
Log$_{10}$[Ω$_h^2$] vs. $M_{X'}$ (GeV)

Figure 3: The predicted DM relic density as function of the DM mass $M_{X'}$. Different colors refer to different choices of input parameters, as indicated in the frame. The dot–dashed curves have been obtained by setting $g_\ast = h_\ast = g_\ast(T_{RH})$ everywhere. The other curves use a temperature dependent $g_\ast$ when calculating $\rho_R$, but the dashed curves have been obtained by setting $dh_\ast/dT = 0$.

treatment with $g_\ast(T) = g_\ast(T_{RH})$ will considerably overestimate the temperature, and hence thermal $X'$ production, as long as $T > T_{QCD}$, as remarked above.

Note that in the $\phi$ matter dominated epoch the radiation content of the Universe is basically determined by $\phi$ decays occurring in the previous $O(1)$ Hubble times; the radiation produced even earlier is quickly redshifted and becomes irrelevant after a few Hubble times. Hence $\rho_R(T)$, or $T$ itself, basically depends on $g_\ast$ and $h_\ast$ only at temperatures $T' \sim T$. Therefore the curves in Fig. 4 essentially coincide in the range of temperatures where $g_\ast(T) \approx g_\ast(T_{RH})$.

Finally, the curves diverge again at very large $A$, well after all $\phi$ particles have decayed. This is due to the decoupling of $e^+e^-$ pairs, which increases the photon temperature by a factor 1.4 relative to a calculation where this effect is ignored. Of course, in the case at hand one could have chosen to terminate the numerical solution of the evolution equations at a value of $A_F$ such that $T_F > m_e$ while still satisfying $T_F \ll T_{RH}$. Still, this feature shows that an accurate description of the evolution of the universe in scenarios with a $\phi$ matter dominated epoch requires a careful treatment of the temperature dependence of $g_\ast$ and $h_\ast$ over the entire range of temperatures.

The upshot of this discussion is that a simplified treatment that ignores the temperature dependence of $g_\ast$ and $h_\ast$ will produce reliable results only if the final temperature $T_F$ is chosen such that $g_\ast(T_F) \approx g_\ast(T_{RH})$, and if thermal $X'$ production mechanisms are irrelevant at all temperatures $T$ where $g_\ast(T) \neq g_\ast(T_{RH})$. The former condition can only be satisfied if $g_\ast$ remains essentially constant for an extended range of temperatures around $T_{RH}$, which in particular is not the case if $T_{RH}$ is near the QCD transition temperature. Since the “(modified) freeze–out” and the “inverse annihilation” contributions to the $X'$ relic density
depend on some range of temperatures $T > T_{\text{RH}}$ the question whether the second condition is satisfied depends on several parameters ($T_{\text{RH}}, M_{X'}, \langle \sigma v \rangle', B_{X'}$) in a rather complicated manner.

So far we have assumed the thermally averaged $X'$ cross section to be a constant. This is a good approximation for non–relativistic $X'$ particles annihilating dominantly from $S$–wave initial states. In Fig. 5 we compare this to results assuming $\langle \sigma v \rangle' = 6bT/M_{X'}$ with constant $b$. This reproduces the correct temperature dependence for non–relativistic particles annihilating from a $P$–wave initial state. Since $T/M_{X'} \simeq 0.05$ for freeze–out in the radiation–dominated epoch, $6b$ needs to be more than one order of magnitude larger than $a$ in order to obtain the correct relic density in the usual WIMP scenario. The difference between the allowed regions is much less in the green strip to the right, where thermal effects are either irrelevant ($\phi$–decay region) or peak at temperatures not far from $M_{X'}$ (inverse annihilation region); the one exception occurs in the QSE region, where the relevant temperature again satisfies $T \ll M_{X'}$. The biggest change occurs in the relativistic inverse annihilation region. In fact, using a constant ($T$–independent) annihilation cross section for $M_{X'} \ll T$ is unphysical; if $X'$ particles annihilate via the exchange of mediators whose mass exceeds $T$, one instead expects $\langle \sigma v \rangle' \propto T^2$, i.e. an even stronger $T$–dependence. The difference in slope between the green strip and the region between the dashed curves at small $X'$ masses and small cross sections therefore indicates that the treatment used in ref.[45] is not reliable here. However, since this concerns the region of parameter space that is not affected by the early $\phi$–dominated epoch, we will not pursue this issue any further.

In Figs. 6 we explore the dependence of the DM relic density on the $\phi$ mass and the effective branching ratio for $\phi \rightarrow X'$ decays. In these figures the temperature dependence of $g_s$ and $h_e$ has been treated carefully, but for simplicity we have assumed $\langle \sigma v \rangle' = a$ to be independent of temperature; the six frames correspond to different values of $a$, with fixed $M_{X'} = 100$ GeV (a typical value for a WIMP). Note that we have used eq.(2.3) with $\alpha = 1$ to compute the total $\phi$ decay width, which in turn determines the reheat temperature via eq.(2.1); hence $T_{\text{RH}}$ scales like $M_{\phi}^{3/2}$ in these figures.
Figure 5: Contours of different values of the DM relic density with mass \( M_\phi = 5 \times 10^6 \) GeV, corresponding to \( T_{\text{RH}} = 848.5 \) MeV. The dashed lines correspond to \( \Omega_{X'}/h^2 = 0.12 \) (for the lines deeper inside the yellow region) and 0.012 assuming a constant cross section \( \langle \sigma v \rangle' = a \), whereas the colored regions have been obtained assuming a constant parameter \( 6b \) in \( \langle \sigma v \rangle' = 6bT/M_{X'} \).

In frame (a) we have chosen a rather large \( X' \) annihilation cross section. Consequently the relic density is very low, unless \( M_\phi \) is rather small (so that \( T_{\text{RH}} \) is well below \( \hat{T}_{\text{FO}} \)) and \( B_{X'} \) is sizable. One is then in the QSE\(_{\text{nr}} \) region of parameter space, where the relic density scales like \( T_{\text{RH}}^{-1} \propto M_\phi^{-3/2} \), see eq.(3.3). Note that the cross section required to achieve quasi–static equilibrium scales like \( 1/B_{X'} \).

Recall that in this region of parameter space the relic density is proportional to the inverse of the \( X' \) annihilation cross section. Hence the region with too high relic density is considerably larger in frame (b), which has ten times smaller \( \langle \sigma v \rangle' \). In fact, now the relic density is in the cosmologically interesting range even in standard cosmology, which explains the large green region at large \( M_\phi \), where \( T_{\text{RH}} \geq \hat{T}_{\text{FO}} \).

In the four remaining frames the \( X' \) annihilation cross section is below that required for a thermal WIMP in standard cosmology. The final DM density will then always be too large if \( B_{X'} > 10^{-4} \); for these small cross sections, there is no mechanism to sufficiently reduce a large \( X' \) density produced directly from \( \phi \) decays. Note that even if \( \phi \) particles do not directly couple to \( X' \) particles, \( \phi \) decays into two SM particles plus two \( X' \) particles (or an \( X'\bar{X}' \) pair, if \( X' \) is not self–conjugate) will in general still occur [56]. However, the resulting branching ratio is expected to correlate with \( \langle \sigma v \rangle' \), so that a small cross section also implies a small branching ratio for these four–body modes, since both processes depend on the coupling of \( X' \) to SM particles.

\[
R_F \simeq R_I + R_F(\mu = 0) , \tag{3.11}
\]
Figure 6: Contours of different values of DM relic density in the plane spanned by the modulus mass $M_\phi$ and the $\phi \to X'$ decay branching ratio $B_{X'}$. We have fixed the dark matter mass to $M_{X'} = 100$ GeV and different thermally averaged cross sections, taken to be independent of temperature. The thermally averaged cross section $\langle \sigma v \rangle'$ in the figures is a) $10^{-6}$ GeV$^{-2}$, b) $10^{-8}$ GeV$^{-2}$, c) $10^{-9}$ GeV$^{-2}$, d) $10^{-14}$ GeV$^{-2}$, e) $10^{-20}$ GeV$^{-2}$, f) $10^{-25}$ GeV$^{-2}$, respectively. These results have been obtained using a careful treatment of the temperature dependence of $h_\sigma$ and $g_\sigma$. The colors are as in Fig. 2.
Even if $B_{X'} < 10^{-4}$, the relic density will be too large for $X'$ particles with annihilation cross section below that of standard thermal WIMPs if $M_\phi$ is too large. Recall that large $M_\phi$ imply large $T_{RH}$ and hence (too) large contribution to the $X'$ density either from inverse annihilation or, for yet larger $M_\phi$, from the standard thermal freeze–out scenario.

The former dominates in the green regions at small $B_{X'}$ in the last three frames of Fig. 6. For the chosen DM mass $M_{X'} = 100$ GeV, we see that $M_\phi \lesssim 10^7$ GeV is required, unless the $X'$ annihilation cross section is many orders of magnitude below that of thermal WIMPs. For non–relativistic $X'$ particles the annihilation cross section often scales like $M_{X'}^{-2}$. In this case we find numerically that the upper bound on $M_\phi$ scales roughly like $M_{X'}^{10/21}$, or roughly like $\sqrt{M_{X'}}$, if we keep the annihilation cross section independent of $M_{X'}$; whereas for fixed $X'$ mass, the lower bound on $M_\phi$ will scale like $\left(\langle \sigma v \rangle_{X'}^{-2/21}\right)$, which explains why the blue region in the last three frames of Fig. 6 only grows rather slowly even though the annihilation cross section is reduced by more than 10 orders of magnitude.

4 Dependence on Initial Conditions

In the previous Section we had assumed that the radiation and $X'$ densities initially vanish exactly. This is completely realistic only if $\phi$ is a (weakly coupled) inflaton decaying purely perturbatively into $X'$ particles and/or radiation. In contrast, in moduli cosmology one assumes that inflaton decay first reheats the universe as usual. However, $\phi$ attains a large value during inflation, so that eventually its density dominates the total energy density. In this case the temperature will not be zero at any time after inflaton decay. Of course, it stands to reason that if the epoch of $\phi$ domination is sufficiently long, the initial temperature will not matter, so imposing eqs.(2.19) will be a good approximation. In this Section we investigate quantitatively what impact a non–vanishing radiation content can have.

Even if at some sufficiently early time the universe is radiation dominated, $\rho_R > \rho_\phi$, eventually these two densities will become equal if $\phi$ particles are sufficiently long–lived, since the ratio $\rho_\phi/\rho_R$ increases proportional to the scale factor $a$. For a short time after this, the total radiation density will still be dominated by the “primordial” component. In this adiabatic regime (in the notation of ref.[64]) the temperature $T \propto a^{-1}$ because of entropy conservation. In the subsequent “non–adiabatic regime”, most radiation already comes from $\phi$ decay and $T \propto a^{-3/8}$ as in eq.(2.14). Note that (after inflaton decay) the temperature of the universe never increases in this scenario, as already pointed out in ref.[54].

It would be tempting to simply define our “initial” time, and “initial” scale factor, such that $\rho_{\phi,I} = \rho_{R,I}$. However, the case with initially vanishing radiation density could then not be covered. Moreover, at this initial time our dimensionless scale factor $A$ would usually not be equal to 1. We therefore prefer to define our initial conditions such that $A = 1$, and describe the initial radiation density through the dimensionless parameter

$$\mu = \frac{\rho_{R,I}}{\rho_{\phi,I}}. \tag{4.1}$$

The case covered in the previous Chapter obviously corresponds to $\mu = 0$, but very large (positive) values of $\mu$ are in principle possible. We assume that the energy density of dark matter particles is initially negligible compared to $\rho_\phi + \rho_R$. This should be a good approximation even if the initial temperature $T_I \geq M_{X'}$ and $X'$ particles were in full equilibrium,
simply because the total number of relativistic degrees of freedom should be much larger than $g_{X'}$. The Hubble parameter is then given by
\[ H_I^2 = \frac{8\pi}{3M_{Pl}^2} (\rho_{\phi,I} + \rho_{R,I}) = \frac{8\pi \Phi_I T_{RH}^3}{3M_{Pl}^2} (1 + \mu). \] (4.2)

In our numerical examples we take $H_I$ (in units of $\Gamma_\phi$) and $\mu$ as free parameters. The initial dimensionless comoving densities of scalar and radiation can then be written as:
\[ \Phi_I = \frac{3M_{Pl}^2 H_I^2}{8\pi T_{RH}^2 (1 + \mu)}, \]
\[ R_I = \mu \Phi_I. \] (4.3)

The initial temperature can therefore be defined as
\[ T_I = T_{RH} \left( \frac{30}{\pi^2 g_*(T_I)} R_I \right)^{\frac{1}{4}}. \] (4.4)

In our numerical analyses we take $g_*(T_I) = 106.75$, which is the number of degrees of freedom in the Standard Model if $T \gg m_t$ (top quark mass).

After the initial time, but before most $\phi$ particles have decayed, the dimensionless Hubble parameter $\tilde{H}$ can be estimated as
\[ \tilde{H} \simeq \Phi_I^{\frac{1}{2}} \left( 1 + \frac{\mu}{A} \right)^{\frac{1}{2}}, \] (4.5)
where we again have neglected the contribution from $X'$ particles. Evidently the second term on the rhs of eq.(4.5) becomes negligible once $A \gg \mu$; in this epoch the universe is again matter dominated. Recall, however, that $\phi$ particles do eventually decay at $A \simeq \tilde{A}$, see eq.(2.21). For $\mu \gtrsim 1$, the $\phi$ matter dominated epoch therefore occurs for
\[ \mu \ll A \lesssim \left( \frac{3}{2} \frac{\gamma}{(1 + \mu)^{1/2}} + 1 \right)^{2/3}, \] (4.6)
where $\gamma = H_I/\Gamma_\phi$, see eq.(2.22). If $\mu > 1$, an extended period of $\phi$ matter domination therefore requires $\mu^2 \ll \gamma$.

On the other hand, $\gamma$ cannot be arbitrarily large in the post–inflationary universe. We certainly need $H < M_{Pl}$ in order to treat gravity classically, see e.g. [65]. In inflationary cosmology the smallness of the density perturbations, and the upper bound on primordial gravitational waves, requires $H \lesssim 10^{-5} M_{Pl}$ during inflation [54], and hence also afterwards. We therefore adopt the bound $H_I < 10^{-5} M_{Pl}$, which implies
\[ \gamma < \frac{10^{-5}}{\alpha} \left( \frac{M_{Pl}}{M_\phi} \right)^3. \] (4.7)

The rhs of (4.7) is therefore also an upper bound on $\mu^2$ if the universe is to undergo a $\phi$ matter dominated epoch.

Since $X'$ production or annihilation has little effect on the thermal plasma, the final radiation density is simply given by where $R_F(\mu = 0)$ has been given in eq.(2.23). If $\mu^2 \ll \gamma$, the first term on the rhs of eq.(3.11) is negligible, i.e. if the universe underwent an extended period of $\phi$ matter domination, the final radiation density will come mostly from $\phi$ decays.
However, the initial conditions may affect the final DM relic density even if there is an epoch of \( \phi \) matter domination. So far we have only specified the initial radiation density in terms of \( \mu \). In complete generality the initial \( X' \) density is another free parameter. However, we wish to avoid proliferation of parameters, and therefore write the initial (co–moving, dimensionless) \( X' \) density as

\[
X'_I = \left( \frac{1}{T_{RH}} \right)^3 \frac{g_X' T_I M_{X'}}{2 \pi^2} K_2 \left( \frac{M_{X'}}{T_I} \right).
\]  

(4.8)

This is based on the Maxwell–Boltzmann distribution, but it is still a reasonably good approximation for bosons and fermions from relativistic to non–relativistic limits, as long as \( X' \) is (approximately) in full thermal equilibrium with the hot plasma. This in turn should be true if \( T_I \gtrsim M_{X'} \) unless the \( X' \) annihilation cross section is very small: equilibrium should be reached if

\[
g_{X'} (\sigma v)' \gtrsim \left( \frac{g_{*}(T_I)(1 + \mu)}{30 \mu} \right)^{3/4} \frac{1}{\sqrt{\alpha \gamma M_{X'}^2 M_{Pl}}}. \]  

(4.9)

Here we have again written \( H_I = \gamma \Gamma_{\phi} \) and used eq.(2.3) for \( \Gamma_{\phi} \). The condition (4.9) can only be violated if either \( \mu \) or \( \langle \sigma v \rangle' \) is very small. In the former case eq.(4.8) in any case predicts a very small initial \( X' \) density, so it doesn’t matter that this small number may not be correct. In the latter case interactions of \( X' \) with the thermal plasma will certainly remain negligible at later times, so we can write the final \( X' \) as sum of the initial value [which may not be given by eq.(4.8) then] and a possible contribution from direct \( \phi \to X' \) decays:

\[
X'_F \simeq X'_I + X'_{F,(Br)} \quad \text{with} \quad X'_{F,(Br)} = \frac{B_{X'} T_{RH} \Phi_I}{M_{\phi}}.
\]  

(4.10)

We are now ready to present some numerical results. In Fig. 7 we show the final DM relic density for the same \( \phi \) mass and \( B_{X'} \) as in Fig. 2. We chose two different values of the initial radiation (and \( X' \)) density, parameterized by \( \mu \): \( \mu = 10^{-5} \) (left column) and \( \mu = 1 \) (right column). Moreover, we chose three different values for the initial Hubble parameter, parameterized by \( \gamma \): \( \gamma = 10^{10} \) (first row), \( \gamma = 10^{15} \) (second row), and \( \gamma = 10^{20} \) (third row). Since \( \mu \leq 1 \), in all examples the universe is dominated by \( \phi \) matter for all \( A \) between 1 and \( \tilde{A} \) defined in eq.(2.21). Note that condition (4.6) is satisfied in all these cases.

We see that the initial conditions do not affect the final DM density if the \( X' \) annihilation cross section is sufficiently large. We saw in the previous Chapter that \( X' \) particles then achieve full thermal equilibrium with the hot plasma during the epoch of \( \phi \) matter domination; for sufficiently high \( T_{RH} \), \( X' \) will drop out of equilibrium only after all \( \phi \) particles have decayed. Adding a non–vanishing initial radiation component increases the temperature relative to the case \( \mu = 0 \), making it easier for \( X' \) to attain thermal equilibrium. Hence any scenario that leads to \( X' \) freeze–out for \( \mu = 0 \) will have \( X' \) in thermal equilibrium also for some time during \( \phi \) domination. This period of thermal equilibrium will wipe out any dependence of the final \( X' \) density on the initial conditions. For the parameters of Fig. 7 this is true for \( \langle \sigma v \rangle' \gtrsim 10^{-12} \text{ GeV}^{-2} \).
Figure 7: Contours of constant relic density for different initial conditions. The meaning of the differently colored regions is as in Fig. 2; note that we only show results where the evolution of the number of degrees of freedom with temperature has been treated carefully. We have taken $M_\phi = 5 \times 10^6$ GeV and $\alpha = 1$, leading to $T_{RH} = 848.5$ MeV, and $B_X = 10^{-5}$. The six frames are for different combinations of $\gamma$ and $\mu$: $(\gamma, \mu) = a) (10^{10}, 10^{-5})$, b) $(10^{10}, 1)$, c) $(10^{15}, 10^{-5})$, d) $(10^{15}, 1)$, e) $(10^{20}, 10^{-5})$, f) $(10^{20}, 1)$. 
Hence the initial conditions can affect the final DM density only if for \( \mu = 0 \) the latter is determined by the “inverse annihilation” or “\( \phi \) decay” mechanisms discussed in the previous chapter, see eqs. (3.7), (3.9) and (3.10). The results obtained for \( \mu = 0 \) will then only be approximately correct if the initial value \( X'_I \) is much less than the final value of \( X' \) produced during the epoch of \( \phi \) matter domination. From these equations and the initial condition (4.8) we find that the initial contribution is negligible if:

\[
\frac{\mu^{3/4}(1+\mu)^{1/4}}{\gamma^{1/2}} \ll \kappa_{\phi-\text{decay}} B_{X'} \left( \frac{\alpha M_{\phi}}{M_{\text{Pl}}} \right)^{1/2};
\]

\[
\frac{\mu^{3/4}(1+\mu)^{1/4}}{\gamma^{1/2}} \ll \kappa_{\text{IA}_\phi} \frac{\alpha^{1/2} M_{\phi}^{3/2} M_{\text{Pl}}^{1/2} T_{\text{RH}}^6 \langle \sigma v \rangle'}{M_{X'}^{1/2}};
\]

\[
\frac{\mu^{3/4}(1+\mu)^{1/4}}{\gamma^{1/2}} \ll \kappa_{\text{IA}_\phi} \alpha^{1/2} M_{\phi}^{3/2} M_{\text{Pl}}^{1/2} \langle \sigma v \rangle'.
\]  

(4.11)

The first of these inequalities applies if \( X' \) production in the epoch of \( \phi \) domination is from direct \( \phi \to X' \) decays, while the second and third inequality apply if the main \( X' \) production mechanism for \( \mu = 0 \) is inverse annihilation, with \( X' \) being non–relativistic and relativistic, respectively. In these inequalities we have only displayed the dependence on the free parameters; numerical coefficients are collected in the \( \kappa \)'s, with \( \kappa_{\phi-\text{decay}} \simeq 35 \), \( \kappa_{\text{IA}_\phi} \simeq 10^{-2} \). Altogether the initial contribution to the \( X' \) density will be negligible if the l.h.s is (much) less than the largest of the three right–hand sides.

When deriving these inequalities we have assumed that the initial temperature is larger than \( M_{X'} \), so that \( X'_I \propto T^2_I \) is not exponentially suppressed. Moreover, we have assumed that the condition (4.6) is satisfied, so that the universe underwent an extended period of \( \phi \) matter domination. Finally, we have used eq. (2.3) to compute \( \Gamma_\phi \); this is needed, since we express the initial Hubble parameter, and hence \( \Phi_I \), in terms of \( \gamma \) defined in eq. (2.22).

The first inequality (4.11) is relevant for \( M_{X'} \gtrsim 10 \text{ GeV} \) and \( \langle \sigma v \rangle' \lesssim 10^{-18} \text{ GeV}^{-2} \). For the parameters used in Figs. 7 the r.h.s amounts to about \( 2 \times 10^{-10} \). The values of the l.h.s in the six frames are of order \( 2 \times 10^{-9} \) in (a), \( 10^{-5} \) in (b), \( 6 \times 10^{-12} \) in (c), \( 3 \times 10^{-8} \) in (d), \( 2 \times 10^{-14} \) in (e), and \( 10^{-10} \) in (f). Correspondingly in the lower–right parts of the plane shown in Fig. 7 the initial contribution \( X'_I \) dominates in (a), completely dominates in (b) and (d), is subdominant but not completely negligible in (f) and can be neglected in (c) and (e). The regions with approximately correct final DM density which is dominated by \( X'_I \) are labeled as \( X'_{\text{Init}} \) in Figs. 7.

The situation is a bit more complicated in the part of parameter space where \( X' \) production is dominated by inverse annihilation if \( \mu = 0 \), since the r.h.s of the second and third inequalities (4.11) explicitly depend on the annihilation cross section and – for the second inequality – the mass of the DM particle. Let us focus on the region near the center of the plots, with \( M_{X'} \simeq 50 \text{ GeV} \) and \( \langle \sigma v \rangle' \simeq 10^{-17} \text{ GeV}^{-2} \), where the inverse annihilation process produces approximately the correct relic density for \( \mu = 0 \), with the DM particles being non–relativistic already at production. The r.h.s of the second inequality (4.11) is of order \( 10^{-10} \) here. Correspondingly in this central part of the parameter plane \( X'_I \) dominates in frames (a), (b) and (d), is negligible in (c) and (e), and contributes about equally in (f).

We thus see that for small DM annihilation cross section, one may need \( \gamma > 10^{20} \) in order to be independent of the initial conditions, even if we require the initial radiation density to be not larger than the initial \( \phi \) mass density. In contrast, for \( \mu = 0 \) the final DM relic density is independent of \( \gamma \) once \( \gamma \gtrsim 10^7 \). Note that for \( \mu = 1 \), \( T_I \sim \sqrt{\gamma} T_{\text{RH}} \). Since
BBN constraints imply $T_{RH} \geq 4$ MeV and $T_I$ should be smaller than the reheat temperature after inflation, the latter would have to be at least $10^8$ GeV if $\gamma \sim 10^{20}$. For the parameters of Fig. 7, $\gamma > 10^{20}$ with $\mu = 1$ implies $T_I \gtrsim 10^{10}$ GeV. Note, however, that possible problems from a high post–inflationary reheat temperature, e.g. overproduction of gravitinos, are alleviated by the huge amount of entropy produced during the very long epoch of $\phi$ matter domination and out–of–equilibrium decay of $\phi$ particles.

5 Summary and Conclusions

This paper treats the production of Dark Matter particles in cosmological scenarios with an early matter dominated epoch. This occurs quite naturally in inflationary cosmology if the theory contains a scalar particle $\phi$ with mass smaller than the Hubble scale during inflation and with greatly suppressed couplings to SM particles, and hence long lifetime. We improved on previous analyses of this non–thermal DM production scenario by carefully treating the temperature dependence of the number of relativistic degrees of freedom ($g_*$ and $h_*$, defined via the energy density and entropy density of radiation, respectively), and by investigating the effect of a non–vanishing initial radiation and DM density.

We found that a careful treatment of the temperature dependence of $h_*$ is very important over large regions of parameter space. This is in sharp contrast to the more commonly considered scenario of WIMP freeze–out in standard cosmology, where the $T$–dependence of $h_*$ only matters if $h_*$ varies rapidly around the freeze–out temperature; and even then the effects do not exceed the 10% level. In contrast, in the presence of an early matter–dominated period approximating $h_*$ by a constant can lead to predictions that are off by a large factor. One reason is that one always normalizes the DM density to the radiation density, or equivalently to the entropy density. In standard cosmology the comoving entropy density is basically constant after the end of inflation. This is not the case in the scenarios considered here, where (nearly) all of the entropy density is produced from $\phi$ decays. An incorrect treatment of the entropy density therefore immediately leads to a wrong prediction of the final DM density. Moreover, some production mechanisms – in particular, the production of DM particles from the thermal plasma, called “inverse annihilation” in ref.[45] – are quite sensitive to the temperature of the thermal plasma.

As noted in earlier analyses, the final relic density can be higher or lower than in standard cosmology, depending on the values of various free parameters. However, we find that even in this more generalized scenario the density of DM particles with annihilation cross section below that required of the usual thermal WIMPs will be too high, unless the branching ratio of direct $\phi \to X'$ is below $10^{-4} (M_{X'}/100 \text{ GeV})$, and the $\phi$ particles are not too heavy, $M_\phi \lesssim 10^7 \text{ GeV}(M_{X'}/100 \text{ GeV})^{2/3}$. Recall that if the $\phi$ decay width is suppressed by $M_{Pl}^2$, as in generic moduli or Polonyi models, $M_\phi < 10^7$ GeV implies a rather low reheat temperature, $T_{RH} \lesssim 1$ GeV. This bound can only be avoided if the $X'$ annihilation cross section is more than 10 orders of magnitude below that of thermal WIMPs, in which case none of the usual DM searches (direct, indirect and at colliders) is likely to yield a signal. This bound has been derived under the assumption of vanishing initial radiation and $X'$ density. Since deviating from this assumption can only increase the final DM density, it retains its validity in full generality.

We also investigated quantitatively the impact of a non–vanishing initial radiation density, parameterized by the ratio $\mu$ of initial radiation and $\phi$ matter densities; we argued that in most cases the initial density of DM particles is then also non–vanishing, and can be estimated from the equilibrium density. The initial radiation density is irrelevant if the
DM annihilation cross section is so large that DM particles attained thermal equilibrium during the period of $\phi$ matter domination (and possibly thereafter) even for $\mu = 0$. On the other hand, for small DM annihilation cross section even a small non–vanishing value of $\mu$ can have sizable effects, unless the period of early matter domination is very long. We parameterize this by the ratio $\gamma$ of the initial Hubble parameter to the total $\phi$ decay width. We find that for very small DM annihilation cross section the final DM density becomes independent of $\mu$ only if $\gamma > 10^{20} \mu^{3/2} \sqrt{1 + \mu} (10^7 \text{ GeV} / M_\phi)^2 / \left(10^9 \text{ GeV} / M_\phi\right)$, where $B_X'$ is the branching ratio for direct $\phi \to X' \to X' \to X'$ decays. Moreover, there will only be an extended period of $\phi$ matter domination if $\gamma \gg \max(1, \mu^2)$.

During the early period of $\phi$ matter domination density perturbations on scales smaller than the Hubble scale will grow linearly with the expansion parameter [54]. This enhances the perturbation spectrum at very small scales relative to standard cosmology. However, even if the DM particles are produced non–thermally, in most cases we considered they will quickly attain kinetic equilibrium with the thermal plasma. This gives them a free–streaming length which is much bigger than the size of the density perturbations that get enhanced during the early matter domination, effectively erasing these perturbations again. The same conclusion holds for very weakly coupled DM particles that did not thermalize. They would have to be produced predominantly directly from $\phi$ decay. Unless the $X'$ particles are already produced non–relativistically, e.g. $M_{X'} \simeq M_\phi / 2$ for 2–body $\phi \to X' X'$ decay, the free–streaming length of $X'$ is too large for the early “minihaloes” to survive [66, 67]. Therefore the scenario considered in this paper in almost all cases reproduces the predictions of standard CDM as far as structure formation is concerned.

However, we saw above that an early epoch of matter domination allows to reproduce the correct DM relic density for a wide range of DM annihilation cross sections, which can be both smaller or larger than that required for thermal WIMPs in standard cosmology. The annihilation cross section can in principle be inferred by observing DM annihilation in today’s universe (assuming the DM density at the point of annihilation is sufficiently well known). Moreover, the couplings of the DM particles can in principle be deduced from collider physics experiments [68–70]. One could then compute the annihilation cross section, and check whether it is compatible with the standard thermal WIMP scenario, or at least with the much larger range of cross sections that can be accommodated in our scenario. On the other hand, the $\phi$ particles are so heavy, and so weakly coupled, that they will not be produced at colliders in the foreseeable future.

In this paper we assumed that the DM particle couples directly to SM particles, allowing for the case of very weak couplings. In ref.[45] a more complicated “dark sector” was investigated, allowing for dark radiation (with temperature typically smaller than that of the visible sector) and a WIMP–like parent particle $X$ that can decay into $X'$. While we did not perform extensive numerical scans of this case, we note that an accurate treatment of the temperature dependence of $h_\star$ is as important in this case as in the somewhat simpler case we considered. Moreover, we expect the impact of a non–vanishing initial radiation density to be comparable to that in our scenario, with the possible caveat that assuming a very weakly coupled $X'$ particle to be initially in thermal equilibrium is probably less motivated than in the scenarios we consider; however, the parent $X$ particle in the scenario of ref.[45] should indeed initially have been in thermal equilibrium.

Even in our somewhat simpler scenario we had to numerically track the evolution of the $\phi$, radiation and DM densities over a very large range of Hubble parameters, or time, which is computationally rather expensive. In this paper we therefore used simple approximations for the thermally averaged DM annihilation cross sections, in most cases replacing it by
a constant. In future publications we intend to investigate specific well–motivated DM candidate particles in scenarios with an early $\phi$ matter dominated epoch, including the full energy (or temperature) dependence of the annihilation cross section.

Acknowledgments

FH thanks the organizers of “Post–Inflationary String Cosmology” workshop in Bologna, September 2017, for their hospitality and support. This work was partially supported by the TR33 “The Dark Universe” funded by the Deutsche Forschungsgemeinschaft. FH was supported by the Deutsche Akademische Austauschdienst (DAAD).

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