Computational analyses of perturbative effects on geostationary satellites: Case SGDC

A P Perroni¹, C R H Solórzano²

¹ Aeronautics Institute of Technology – ITA, São José dos Campos (SP), Brazil
² Federal University of ABC – UFABC, Santo André (SP), Brazil

E-mail: amandapperroni@gmail.com, carlos.solorzano@ufabc.edu.br

Abstract. This work aims to study the main perturbative effects on SGDC - Geostationary Satellite of Defense and Communication, result of the partnership between Telebrás and Brazilian Ministry of Defense. The name ”SGDC” corresponds to the group of satellites that will be launched with the purpose of transmitting broadband Internet to the less favored zones of Brazil and to intermediate communication between the military sectors. Through numerical simulations, analyses are carried out to determine the influence of the perturbations of the terrestrial gravitational field, the solar radiation pressure and the lunissolar gravitational force, besides checking the combination of these perturbations, evaluating which of the effects presents predominance.

1. Introduction

A geostationary orbit is the one disposed in Earth’s equatorial plane, at 35,796 km of altitude, with zero eccentricity and also with zero inclination on the Equator, a configuration that allows a satellite to remain motionless regarding that line and relative fixed position to any point on the Earth's surface. In addition, the satellite moves in the same direction of the planet and also presents the same period of 23 hours, 56 minutes and 4 seconds. The advantage of this orbit is that the satellite can cover a particular region of the planet in order to monitor it all the time - such that the SGDC would be able to control the continental region of Brazil autonomously. There are two types of perturbation that may affect a satellite’s position and velocity (and consequently its lifetime), classified as gravitational and non-gravitational. Gravitational perturbations include spherical harmonics, Earth tide, oceanic tide and attractive effects of the Sun and Moon, while non-gravitational perturbations include atmospheric drag, solar radiation pressure, magnetic forces, etc [1]. This work will analyze the effects of Earth Gravitational Field, Lunisolar Force and Solar Radiation Pressure.

2. Literature review

The satellites that operate in geostationary orbits do not always remain under the same point due to irregularities present in the terrestrial gravitational field and external forces acting on it, modifying the orbital elements and the orientation of the orbital plane – which, in turn, causes a displacement in the satellite parking position, suggesting the necessity of applying maintenance maneuvers [2]. Such action requires information of the satellite position at certain time [3]. Lagrange’s Planetary Equations allow to analyze the variation of orbital elements over time [4]. However, there are other ways that allow the calculation of perturbative forces. Among these forms, there is the application of Clohessy-Wiltshire equations [5], to expand disturbing forces and to properly apply analytical integration, whose solutions
can be implemented to accurately propagate orbital motion for a considerable period of time. Their calculations show that their approximation (instead of directly applying Lagrange’s Planetary Equations) can still support in solving the problem of performing maintenance maneuvers. Another way is the application of the Gauss Equations [6] due to the possibility of finding changes in orbital elements due to perturbative accelerations, being valid for random perturbations. The application of Cowell Method [7] for the lunisolar force and the non-homogeneity of the Earth is given for a satellite whose orbit height is equal to or greater than 1,600km. This method also allows to quantify the effect of solar radiation pressure on low, medium and high orbits [8]. According to [9], for higher orbits such as geostationary, it is identified that the solar radiation pressure acts directly on the semimajor axis, on the eccentricity and on the inclination of the orbit, and it is possible to use this force to maintain the orbit’s parameters [10]. For the terrestrial gravitational field, analyzed in an isolated manner, it is affirmed that the rates of variation of the orbital elements diminish with the height of the orbit [11], especially for the semimajor axis. In these cases, the behavior is a slow secular variation [12].

3. Formulation of the dynamical model
The Gauss Equations [13] allow to obtain the time derivatives of the orbital elements $i$, $\omega$, $\Omega$, $e$, $h$ and $\theta$, known as inclination, argument of perigee, right ascension of ascending node, eccentricity, specific angular momentum and true anomaly, respectively.

The Planetary Gauss Equations are

$$\frac{dh}{dt} = r p_s$$

$$\frac{de}{dt} = -\frac{h}{\mu} \sin(\theta) p_r + \frac{1}{\mu h} \left( \frac{h^2}{r^2} + \frac{\mu r}{2} \right) \cos(\theta) p_r - \left( \frac{r + h^2}{\mu} \right) \sin(\theta) p_s$$

$$\frac{d\theta}{dt} = \frac{h}{r^2} + \frac{1}{\mu h} \frac{h^2}{r} \cos(\theta) p_r - \left( \frac{r + h^2}{\mu} \right) \sin(\theta) p_s$$

$$\frac{d\Omega}{dt} = \frac{r}{h \sin(i)} \sin(u) p_w$$

$$\frac{d\omega}{dt} = -\frac{1}{\mu h} \frac{h^2}{r} \cos(\theta) p_r - \left( \frac{r + h^2}{\mu} \right) \sin(\theta) p_s - \frac{r \sin(u)}{h \cos(i)} p_w$$

where

$$r = \mu \left( 1 + e \cos(\theta) \right)$$

It is also possible to obtain the variation of other orbital elements, such as the case of $a$ (semimajor axis), $E$ and $M$ (eccentric and mean anomalies, respectively), through the variations of $h$ and $\theta$.

3.1 Earth Gravitational Field
In order to analyze the effects of the terrestrial gravitational field, $U_E$, attention is required to the gravitational potential

$$U_E = U - \frac{G m_E}{r}$$

where $U$ is the Earth’s gravitational potential and the second portion corresponds to the potential created by a perfectly spherical and homogeneous Earth in the mass distribution. $U$ suffers the effects of the terrestrial flatness and can, therefore, be described in series of zonal spherical harmonics [3], as given by equation (4):

$$U = -\frac{G m_E}{r} \left[ 1 - \sum_{k=1}^{\infty} \left( \frac{R_E}{r} \right)^k j_k P_k(\sin(\phi)) \right]$$
such that $R_E$ is the equatorial radius of Earth ($R_E = 6,378 \text{ km}$), $r$ is the distance between the bodies, $G$ universal gravitational constant ($G = 6.67408\times10^{-11}\text{m}^3/(\text{kg} \cdot \text{s}^2)$) and $m_E$ is the Earth’s mass ($m_E = 5.972 \times 10^{24}\text{kg}$). The $J_k$ zonal harmonics are commonly obtained through observation of a satellite movement around Earth [15] and through value estimation [14]. The $P_k(x)$ term are given by Rodrigues Formula [16]. According to [17], the perturbing function $U_E$ can be expressed as a function of harmonics and the distance between the two bodies. That is, the perturbative function $U_E$ dependent on $p$, vectors of perturbative acceleration, such that

$$\vec{p} = \nabla U_E = \frac{\partial U_E}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial U_E}{\partial \phi} \hat{\phi} + \frac{1}{r \cos \phi} \frac{\partial U_E}{\partial \theta} \hat{\theta} \quad (5)$$

However, as $U_E$ do not depend on longitude $\theta$, then there is no component in direction $\hat{\theta}$. After some algebraic manipulations, $p_{f2}$ to $p_{f6}$ can be obtained and, hence, the perturbative accelerations in the equatorial geocentric frame $\text{XYZ}$ are given by

$$\begin{bmatrix} p_{\text{Earth},\text{XYZ}} \end{bmatrix} = p_{f2} + p_{f3} + p_{f4} + p_{f5} + p_{f6} \quad (6)$$

To obtain them according to the local coordinate system of the satellite (named here as $\text{rsw}$), it is necessary to apply a rotation matrix.

3.2 Solar Radiation Pressure Perturbation

The solar radiation is composed of photons, particles without mass that travel at the speed of light. According to quantum physics, even if a photon has no mass, it has momentum and carries energy. The photosphere, visible surface of the Sun, works as a blackbody that emits radiation from the most varied electromagnetic spectrum, passing through low-energy radio waves in the visible spectrum to ultraviolet light, in addition to X-rays. By Stefan-Boltzmann law, the intensity of irradiated power $S_0$ is given by $\sigma T^4$, where $T$ is the absolute temperature of the black body and $\sigma$ is the Stefan-Boltzmann constant, such that

$$\sigma = 5.670 \times 10^{-8}\text{W/m}^2\text{K}^4 \quad (7)$$

To calculate the solar radiation pressure influence, it is necessary the magnitude of temperature $T$ on the photosphere, in order to obtain the intensity of irradiated power ($S_0$), leading to $T = 5,777\text{K}$ [13]. Hence,

$$S_0 = 5.670 \times 10^{-8}(5,777^4) = 63.15 \times 10^6\text{W/m}^2 \quad (8)$$

Let $R_0$ be the radius of the photosphere and $R_{CS}$ the distance from the center of the Sun to Earth. Then, it is possible to obtain the radiation intensity $S$, such that:

$$S = S_0 \left( \frac{R_0}{R_{CS}} \right)^2 \quad (9)$$

According to [18], $R_0 = 696,000\text{km}$ and $R_{CS} \approx 149.6 \times 10^6\text{km}$ or $R_{CS} = 1\text{AU}$. Hence, the value of the solar constant $S$ is $1,367\text{W/m}^2$, while the radiation pressure is given by

$$P_{SR} = \frac{S}{c} = \frac{1,367\text{W/m}^2}{2.998 \times 10^8\text{m/s}} = 4.56 \times 10^6\text{Pa} \quad (10)$$

And the force executed by solar radiation pressure is given by

$$\vec{F}_{SR} = -\vec{v} \frac{S}{c} C_R A_s \vec{u} \quad (11)$$

where $\vec{u}$ is equivalent to the unit vector that points from the satellite to the Sun and $C_R$ is the solar radiation pressure coefficient, which is 1 or 2: if $C_R = 1$, the surface is a black body that can absorb all moment of the incident photon; if $C_R = 2$, all the incident radiation is reflected such that the photon which arrives has its moment in the inverse direction, doubling the exercised force on the satellite. For this work, it is adopted the worst case, such that $C_R = 2$; that is, it is being considered the direct solar radiation pressure, therefore the effect of Earth’s shadow is not being accounted. The magnitude of the solar radiation pressure perturbation is
Nevertheless, equation (12) depends directly on the mass-area ratio of the satellite \( \left( \frac{A_s}{m} \right) \). That is, spacecrafts like solar sails are deeply affected by solar radiation pressure. For this work, it was adopted as the panel frontal area a value around 65m², considered an average measurement for known geostationary satellites and it is known that the mass of the satellite is 5,735kg [19].

In the ecliptic reference frame \((X'Y'Z')\), the \(Z'\)-axis is normal to the ecliptic, the \(X'\)-axis is in the direction of the vernal equinox and the unit vector \(\vec{u}\) along the Earth-Sun line is given by \(\lambda\), the ecliptic longitude of the Sun, the angle between the vernal equinox line and the Sun-Earth line, such that

\[
\vec{u} = \cos \lambda \hat{\mathbf{I}} + \sin \lambda \hat{\mathbf{J}}
\]  

Since the global coordinate system \(XYZ\) frame shares the line of the vernal equinox with reference frame \(X'Y'Z'\), both have \(X\)-axis in common, which facilitates the transformation of one axis into another, through \(\epsilon\) around \(X\), while \(\vec{u}\) vector and perturbation components are in the \(XYZ\) reference frame. On the other hand, to apply the accelerations in Gauss Planetary Equations, it is necessary to obtain these same components in the \(rsw\) reference frame.

### 3.3 Lunissolar Force

For this case, it is analyzed the sum of two effects, both originated from a perturbation of third body: Earth-Satellite-Moon and Earth-Satellite-Sun. Starting with the gravitational perturbation of the Moon, some elements are listed, such as: \(r\): position of spacecraft relative to the Earth; \(\vec{r}\): acceleration of spacecraft relative to the Earth; \(r_{L/5}\): position of Moon relative to spacecraft; \(r_L\): position of Moon relative to Earth.

The acceleration of the spacecraft relative to Earth and Moon can be rewritten as:

\[
\ddot{\vec{r}} = \mu_{Earth} \ddot{r} + \mu_{Moon} \left( \ddot{r}_{L/5} + \frac{\ddot{r}_L}{r_L} \right) 
\]  

With \(\mu_{Earth} = \mu\) and \(\mu_{Moon} = 4,903\, \text{km}^3/\text{s}^2\), where the second term of the equation (14) corresponds to the perturbative acceleration due to lunar gravity, \(p\). The distance between Moon and Earth is function of horizontal parallax [20]. The perturbative acceleration is calculated with the unit vectors of the \(rsw\) frame. Finally, the components of the perturbative acceleration of the Moon are given by:

\[
\begin{align*}
p_{Lr} &= \vec{p} \cdot \ddot{r} \\
p_{Ls} &= \vec{p} \cdot \ddot{s} \\
p_{Lw} &= \vec{p} \cdot \ddot{w}
\end{align*}
\]  

In order to calculate the gravitational perturbation of the Sun, it is followed the same approach as shown for the Moon in equations (14) and (15). Hence, the perturbing acceleration due to solar gravity is:

\[
\ddot{\vec{p}}_0 = \mu_0 \left( \frac{\ddot{r}_{O/5}}{r_{O/5}} - \frac{\ddot{r}_O}{r_O} \right)
\]  

where \(\mu_0 = 132.712 \times 10^{12} \, \text{km}^3/\text{s}^2\). Since \(\ddot{p}_0\) is calculated, the components of perturbative accelerations of Sun in the \(rsw\) frame can also be obtained:

\[
\begin{align*}
p_{Or} &= \vec{p} \cdot \ddot{r} \\
p_{Os} &= \vec{p} \cdot \ddot{s} \\
p_{Ow} &= \vec{p} \cdot \ddot{w}
\end{align*}
\]  

With equations (15) and (17), the perturbative accelerations of the Sun and Moon are summed up and replaced in equation (1), obtaining then the evolution of the classical orbital elements under the influence of the lunissolar force.
4. Results

To evaluate the effects of the disturbing forces on the satellite, each force was isolated in order to analyze its individual effect. By using algorithms created in MatLab and integrating the differential equations using the Runge-Kutta method of 5th order, adopting relative and absolute errors of the order of $1 \times 10^{-12}$, it was possible to evaluate the evolution of classical orbital elements for a given period of time. The conditions initial are obtained from STK (System Tool Kit) software and its international satellite database (June 4, 2017, at 6:00 UTC), given by:

\[ a = 42,165.80\, km; \, e = 4.8 \times 10^{-6}; \, i = 0.133^\circ; \, \Omega = 67.16^\circ; \, \omega = 342.906^\circ; \, \theta = 218.241^\circ \]

For the figures, there is the following label: ENSP – Earth Non-Sphericity Perturbation; SRP – Solar Radiation Pressure; LFP – Lunisolar Force Perturbation; LGP – Lunar Gravitational Perturbation; SGP – Solar Gravitational Perturbation;

![Figure 1. Evolution of semimajor axis $a$ (a), and inclination $i$ (b) for a period of 3 years.](image-url)
Figure 2. Evolution of right ascension of ascending node $\Omega$ (a), and argument of perigee $\omega$ (b) for a period of 3 years.
5. Conclusions and recommendations

Figure 1a shows small variations in the semimajor axis, with the maximum variation of $7.10^{-3}m$ for the time scale shown, but it is emphasized the predominant effect of Earth non-sphericity perturbation, solar radiation pressure, and lunisolar force (ENSP+SRP+LFP), when compared to the isolated effects of terrestrial, solar and lunar gravitational attraction and solar radiation pressure. Figure 1b also shows the predominant effect of lunissolar perturbation and Earth non-sphericity perturbation, solar radiation pressure, and lunisolar force (ENSP+SRP+LFP) on inclination at an annual rate of approximately $1^\circ/year$, since solar radiation pressure causes no variation in the inclination.

The evolution of the right ascension of ascending node is shown in Figure 2a, such that the simulations in the presence of the terrestrial flatness only are consistent with the theory orbital perturbations. Thus, in a first approximation, a regression on the right ascension of ascending node is expected, according to the basic model that analyzes the secular nodal regression (west) due to $J_2$. On
the other hand, the solar radiation pressure affects the right ascension of ascending node as well as it affects the eccentricity, inclination and pericenter argument for bodies with large $A/m$ ratio. For this study, as $A = 65 m^2$ and $m = 5375 kg$, the $A/m$ ratio is small, so when considering only the radiation pressure, there is no variation of the orbital elements. By increasing the value of $A/m$ ratio, the effects on eccentricity and inclination are amplified, as shown in [21]. The rate of rotation of the apsidal line, associated with the secular variations of the argument of perigee due to Earth's flattening is positive, as shown in Figure 2b. The predominant effect of terrestrial gravitational field harmonics is visualized through the variation of the argument of perigee, even in the presence of solar radiation pressure and lunissolar perturbation (Figure 2b).

Figure 3a shows the behavior of the eccentricity due to terrestrial non-sphericity and the effect of the perturbation of third body (Sun and Moon). The effects of the terrestrial non-sphericity and the Moon perturbation are predominant when compared to the effects of the Sun on eccentricity. Finally, Figure 3b also shows the effects of the solar radiation pressure on eccentricity. This perturbation adds energy to the system, and consequently, the eccentricity tends to increase during the first 6 months - such that the Sun, on the other side, reduces this energy addition coincidentally with a decrease in eccentricity. It can be observed that the maximum and minimum peaks happen almost in a semiannual way and the secular behavior of the lunissolar perturbation is also observed in Figure 3b. In practice, orbiting maintenance maneuvers are required as the objective of keeping the spacecraft in the nominal position, such as the so-called North-South maneuvers or maneuvers in the inclination, which applies impulses perpendicular to the orbital plane. As a complement, there are East-West maneuvers, in which the impulses are tangential to orbit: in this way, the net impulse is in the east direction, when they are in the direction of velocity. Analogously, the net impulse is in the west direction when impulses are in the opposite direction.

In general, in the case of solar radiation pressure, the most significant and dominant effect is observed in eccentricity. The same variation appears in the argument of perigee and depending on the set of initial conditions, the long period variations in the eccentricity may disappear, causing the eccentricity to be constant (known as "forced eccentricity" or "resonant eccentricity", which happens as a result of resonant properties). The analysis of the orbital behavior for periods of time as long as 100 years, relative to bodies that can remain in orbit in this scale of time, shows an interesting behavior, whose analysis is intended to be realized in the future and are not presented in this paper.

In the time evolution of the inclination, when compared the effects due to the perturbation of the Moon and the perturbation due to the flattening of the Earth, it is observed that the Moon produces great variations in the inclination, so that it reaches values close to 40 degrees. The presence of terrestrial flatness dampens such variations to periodic oscillations near 10 degrees - effect also observed in the analysis of the eccentricity.

References
[1] Al-Bermani M J F and Baron A S 2010 Calculation of solar radiation pressures effect and sun, moon attraction at high earth satellite J. of Kufa, v. 2, n. 1, ISSN 2077-5830
[2] Rodriguez M E P 2001 Control Orbital de Satélites Geostacionarios (Madrid: Universidad Complutense de Madrid) p 213
[3] Vallado D A 2004 Fundamentals of Astrodynamics and Applications (California: Kluwer Academic Publishers)
[4] Cook G E 1962 Luni-solar perturbations of the orbit of an earth satellite Oxford The Geophysical J. of Royal Astronomical Society, v 6, n 3, p 271-291
[5] No T S and Jung O C 2005 Analytical solution to perturbed geosynchronous orbit Acta Astronautica, v 56, n 7, p 641-651
[6] Sengupta P 2003 Satellite relative motion propagation and control in the presence of $J_2$ perturbations (Texas: Texas A&M University, Texas)
[7] Adnan M S K, Razalli R and Said M A M A 2018 Study of perturbation effect on satellite orbit using cowell's method (Nibong Tebal: University Science Malaysia)
[8] Baron A S 2013 Study the Effect of Solar Radiation Pressure at Several Satellite Orbits. *Baghdad Sci. J.* v 10, n 4, p 1253-1261

[9] Kelly P, Erwin R S, Bevilacqua R and Mazal L 2016 Solar Radiation Pressure Applications on Geostationary Satellites. *AAS GP&C Conference* (American Astronautical Society)

[10] Adams W M and Hodge W F 1965 *Influence of Solar Radiation Pressure on Orbital Eccentricity of a Gravity-Gradient-Oriented Lenticular Satellite* (Hampton: Langley Research Center)

[11] Haranas I and Pagiatakis S 2010 Satellite Motion in a Non-singular Gravitational Potential *Astrophysics and Space Science*, v 327, n 1, p 83-89

[12] Wagner C A 1966 Longitude variations of the Earth’s gravity field as sensed by the drift of three synchronous satellites *J. of Geophysical Research* v 71, n 6, p 1703-1711

[13] Curtis H D 2014 *Orbital Mechanics for Engineering Students* 2nd ed (Daytona Beach: Elsevier Butterworth-Heinemann)

[14] Kaula W M 1963 Tesseral Harmonics of the Gravitational Field and Geodetic Datum Shifts Derived from Camera Observation of Satellites *J. of Geophysical Research*, v 68, p 12

[15] Marsh J G, Lerch F J, Putney B H, Felsentreger T L, Sanchez B V, Klosko S M, Patel G B, Robbins J W, Williamson R G, Englis T E, Eddy W F, Chandler N L, Chinn D S, Kapoor S, Rachlind K E, Braatz L E and Palis E C 1989 NASA Technical Memorandum 10076 - The GEM-T2 Gravitational Model (Greenbelt: Goddard Space Flight Center)

[16] Ronveaux A and Mawhin J 2005 Rediscovering the contributions of Rodrigues on the representation of special functions *Expositiones Mathematicae* v 23, n 4, p 361-369.

[17] Schaub H and Junkins J L 2003 Analytical Mechanics of Aerospace Systems 3rd ed, v 1 (AIAA Education Series)

[18] Seidelmann P K 1992 *Explanatory Supplement to the Astronomical Almanac* (California: University Science Books)

[19] Hoyau C E 2017 VA236 Launch Kit – SGDC and KoreaSat-7 (Évry: Arianespace Service & Solutions)

[20] USA Navy 2018 *The Astronomical Almanac* (Massachussets: United States Naval Observatory)

[21] Casanova D, Petit A and Lemaître A 2015 Long-term evolution of space debris under the J2 effects, the solar radiation pressure and the solar and lunar perturbations *Celestial Mechanics and Dynamical Astronomy* v 123, n 2, p 223-238