On Creativity of Elementary Cellular Automata

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Abstract

We map cell-state transition rules of elementary cellular automata (ECA) onto the cognitive control versus schizotypy spectrum phase space and interpret cellular automaton behaviour in terms of creativity. To implement the mapping we draw analogies between a degree of schizotypy and generative diversity of ECA rules, and between cognitive control and robustness of ECA rules (expressed via Derrida coefficient). We found that null and fixed point ECA rules lie in the autistic domain and chaotic rules are ‘schizophrenic’. There are no highly articulated ‘creative’ ECA rules. Rules closest to ‘creativity’ domains are two-cycle rules exhibiting wave-like patterns in the space-time evolution.

Keywords: creativity, cellular automata, generative diversity, Derrida coefficient

1 Introduction: On Creativity

Creativity is ubiquitous yet elusive concept. Everyone knows what it means to be creative, e.g. to be successful in problem-solving and generation of novel thoughts ¹², but few can define creativity rigorously. Substantial progress has been achieved in the fields of computational and psychological creativity. Thus, Kowaliv, Dorin and Korb studied creativity of graph-pattern generation and progressed towards outlining creativity as based on a probability of pattern emergence ⁹ ¹⁸. In this sense, a system is creative if it produces a pattern where the likelihood of emergence is small. Wiggins formalises Boden’s concept ⁵ of exploratory creativity as exploration of a conceptual space ³⁴: thought, a question could be raised — is creativity in the complexity of conceptual space or the search engine? Another computational approach to creativity is a generation of novelty via conceptual blending ²⁹ ²², and use of analog machines in evolutionary creation of cross-domain analogies ⁴.

From a psychological and neurophysiological perspective there is a great similarity between creativity and psychoticism ¹⁰ ¹¹ ² ¹⁷. The similarities include over-inclusive cognitive style, conceptual expansion, associative thinking, and lateral thinking dominating vertical (goal-oriented) thinking. In contrast to creativity, however, psychoticism shows diminished practicality ² ¹⁷. Kuszewski ¹⁹ provides plausible and psychologically feasible indicators of creativity: divergent thinking and lack of lateral inhibition; the ability to make remote associations between ideas and concepts; the ability to switch back and forth between conventional and unconventional ideations (flexibility in thinking); generation of novel ideas appropriate for actualities; willingness to take risks; and, functional non-conformity. Cognitive control of divergent thinking is a guarantee of creativity. A person with extremely divergent thinking yet unable to control will be
Figure 1: Schizotypy versus cognitive control spaces. Original scheme redrawn from Kuszewski’s paper [19].

a ‘nutter’. Those who can fit their high schizotypy traits into a rigid cognitive frame incline to genius. Thus creativity could be positioned together with autism and schizophrenia in the same ‘phase’ space (Fig 1).

To develop cellular automata (CA) analogies of Kuszewski’s scheme we assume that a cell neighbourhood configuration of a CA represents a ‘thought’, or some other elementary quantity of a mental process, and a degree of schizotypy is proportional to the diversity of global configurations generated by the CA. We can speculate that cognitive control is equivalent to robustness of CA evolution. A cellular automaton is robust if trajectory of a disturbed automaton, with some cells’ states changed externally, does not deviate, in terms of Hamming distance, too far away from a trajectory of an undisturbed automaton. The degree of deviation caused by a disturbance is measured by the Derrida coefficient.

2 Elementary cellular automata

An elementary cellular automaton (ECA) is a one-dimensional array of finite-state automata. The automata takes two states, 0 and 1, and update their states simultaneously in discrete time by the same cell-state transition function \( f : \{0, 1\}^3 \rightarrow \{0, 1\} \). Each automaton updates its state depending on its current state and states of its two closest neighbours. When referring to cell-state transition rules we use a decimal representation of the cell-state transition table [35]: See examples in [40] and extensive analysis of ECA’s rules, parameters and global transition graphs in [41]. Due to symmetries the elementary transition rules can be grouped in the 88 classes with equivalent behaviour [32, 41]. We analyse, and illustrate our discussions with, minimal decimal value rules from each equivalence class. The ECA rules are studied using two statistical measures: the Derrida coefficient and generative morphological diversity.

The Derrida plot [5] is used in the evaluation of Boolean networks [41, 15, 13, 43]. The Derrida plot provides a statistical measure of the divergence/convergence of network dynamics.
in terms of Hamming distance $H$. The distance $H$ between two binary states of equal size, $n$, is the number of sites that differ. The normalised Hamming distance is $H/n$. The Derrida plot is calculated as follows \[43\]. We randomly select a pair of initial states, $c_0^1$ and $c_0^2$, separated by a small Hamming distance of $H_0$ at time step $t = 0$. We iterate the configurations using the same cell-state transition rule for $m$ steps and measure $H$ between configurations $c_m^1$ and $c_m^2$, repeat the measurement for more samples pairs of initial configurations with the same $H_0$, and then plot normalised $H_0$ against the mean normalised value of $H$. The procedure is repeated for larger values of $H_0$.

The Derrida coefficient \[43, 7\], analogous to the Lyapunov exponent but for discrete systems, measures sensitivity to initial conditions. The Derrida coefficient is derived from the initial slope $x$ of the Derrida plot. For these results $m=1$, initial $H_0=1$, increasing by 1 for 10 samples of 3000. The Derrida coefficient is calculated as $D = \log_2(\tan(x))$. Boolean networks and cellular automata behaving “chaotically” have positive $D$, ordered dynamics have negative $D$. For Boolean networks, $D = 0$ is attributed to dynamics at the edge of order and chaos \[13\], whereas for cellular automata $D = 0$ merely indicates stability.

Generative morphological diversity $\mu$ of a ECA characterises how many different triplets, taken at time steps $t-1$, $t$ and $t+1$, of neighbourhood configurations are generated by the ECA starting from a single central cell in a state 1 \[11, 30\]. The measure is very close to the in-degree histogram proposed in \[42\]. We have chosen $3 \times 3$ cell blocks to characterise morphology of space-time configuration because a minimal block must include a cell neighbourhood (three cells), include at least two subsequent local configurations, to characterise identifiability, and sides corresponding to time and space have the same number of cells. We calculate morphological diversity $\mu$ using blocks of neighbourhood states taken at three subsequent time steps: the automaton evolves for $m$ steps list $L$ of different $3 \times 3$ blocks from its space-time configuration $c \times T$ is filled; $m$ is chosen experimentally such that $L_m = L_{m-1}$. The diversity $\mu = |L|$ is a size of list $L$.

Values of $\mu$ and $D$ for representative rules of equivalence classes are shown in Appendix, Tab. 2.

3 Creativity of ECA rules

Representative rules of the 88 equivalence classes are mapped onto $\mu$-$D$ space in Fig. 2. Space-time configurations, starting in configuration $0\cdots010\cdots0$, generated by the rules from Fig. 2 are shown in Fig. 3. A substantial number of rules occupy a domain of low values of $\mu$ yet spread more or less equally along $D$ axis. Rules showing moderate generative diversity ($\mu=20$ to 40) have Derrida coefficients around $D = 1$. Rules with highest generative diversity ($\mu=50$ to 64) have values of $D$ ranging from nearly 1 to 1.6 (Fig. 2). The increase in generative diversity is visualised in sample configurations of representative rules (Fig. 3).

Domains of ECA behavioural classes \[28\] are shown in Fig. 4. Fixed point and two-cycle classes \[36, 37\] lie in the region of low generative diversity yet fully spread along the Derrida coefficient axis. Rules with periodic behaviour occupy a part of $\mu$-$D$ space for average values of generative diversity and Derrida coefficient equal to 1. Chaotic rules are spread from moderate to maximum values of diversity and Derrida coefficient from 0.5 to 1.5. Two complex rules reside in a region of $\mu$ equals 1 and moderate and slightly above average diversity $\mu$ (Fig. 4).

Wolfram classes \[36, 37\] $W_1$ (fixed point), $W_2$ (periodic), $W_3$ (chaotic) and $W_4$ (complex) are well arranged along the generative diversity axis, apart of class $W_4$. One rule of class $W_4$ lies in the middle of class $W_3$ and another rule of class $W_4$ lies in the intersection of classes $W_2$ and $W_3$ (Fig. 4).
Figure 2: Representative rules of 88 classes are displayed on generative diversity $\mu$ versus Derrida coefficient $D$ plane.
Figure 3: Space-time configurations of representative rules of 88 classes are displayed on generative diversity $\mu$ versus Derrida coefficient $D$ plane. Each automaton, 150 cells, started its development in configuration where all cells but one are in state 0 and evolved for 150 iterations. Boundaries are periodic. Cells in state 1 are shown by black pixels, cells in state 0 are grey.
Figure 4: Domains of main behavioural classes \cite{28} in $\mu$–$D$ space. Projections of domains onto Wolfram classes \cite{36,37} W1 to W4 are shown as solid thick lines.
Figure 5: Schizotypy versus cognitive control spaces as seen via generative morphological diversity and robustness (Derrida coefficients). Interpretation of scheme Fig. [1] in terms of ECA. Examples of space-time configurations generated by autistic, creative and schizophrenic ECA rules. Configurations evolved from initially random uniform distribution of states 0 and 1. Cells in state 1 are black pixels, in state 0 are yellow/grey pixels.
From the distribution of rules (Fig. 2) and domains of behavioural classes (Fig. 4) we can speculate that — overall — the increase in behavioural complexity, as measured by generative diversity, leads to a decrease in robustness and an increase in sensitivity to initial conditions, as measure by the Derrida coefficient.

Ideally, highly articulated creative rules would appear in the upper right corner of the upper right quadrant of the $\mu$–$D$ plane, but because this corner is almost empty, we settled on rules closest to it. Such rules should have above average generative morphological diversity, and below average Derrida coefficients: $\mu > 11$ and $D < 0.53$ (we omit rule 0 from calculating averages as not posing any interest). The following equivalence classes, labelled by their representative rules, satisfy the creativity condition: 3, 5, 11, 13, 15 and 35. Equivalence classes 3 and 5 show the highest degree of robustness, which represent cognitive control, amongst the creative rules with a yet lower degree of generative diversity, representing the degree of schizotypy. Equivalence classes 11 and 13 show higher generative diversity yet lower robustness. Exemplar configurations of creative ECA rules are shown in Fig. 5. The creative ECA are characterised by propagating patterns, which strikingly resemble waves of excitation propagating in non-linear active media.

In the quadrant of low generative diversity and high robustness we observe a transition from normal ECA to Asperger’s syndrome ECA to autistic ECA (Fig. 5). Normal rules, i.e. those with $\mu$ and $D$ values closest to average, show stationary or breathing domains of intermittent coherent patterns. Rules analogous to Asperger’s syndrome show configurations densely populated with uniform, solid, domains of cells in 1 or 0. ECA interpreted as autistic evolve to fixed all-1 or all-0 global states.

Chaotic rules populate the quadrant corresponding to schizophrenia and schizotypal personality disorders (Fig. 5). The most morphologically diverse and less robust, and thus most 'schizophrenic', equivalence classes are 30, 45, 105 and 150. Rule 30 is a 'typical' chaotic rule, even used in a random number generator [39]; when enriched with memory rule 30 shows pronounced dynamics of gliders with sophisticated interaction patterns [26].

Autistic ECA show stationary domains of alike states. There are no propagating patterns in autistic ECA. The stationary non-interacting domains imitate zones of persistent nervous activity in a brain of a severely autistic person. This could be a possible sign of desynchronisation in motor cortex [33, 24, 31].

The dynamics of ECA governed by schizophrenic rules is characterised by sudden emergence and subsequent swift collapse of domains of alike states. These are reflected in triangular tesselations visible in space-time configurations (Fig. 5). Assume that a one-dimensional ECA is an abstraction of a brain, and that patterns of 1s are analogous of neurons bursting with excitation spikes. Then a creative brain produces coherent yet morphologically rich patterns of nervous activity, e.g. propagating auto-waves, while a brain with high schizophrenic disorder shows (quasi-) chaotic, incoherent and ‘spontaneous’ outburst of nervous activity. These outburst of activity imitate abnormalities in multiple parts of the brain and diminished temporal stability [16, 3, 14].

4 Discussion

Using measures of generative morphological diversity and the Derrida coefficient we classified ECA rules onto a spectrum of autistic, schizophrenic and creative personality. Four classes are shown in Tab. 1.

Autistic rules correspond to rule classes with fixed point behaviour, schizophrenic rules are chaotic and creative rules belong to a class of two-cycle behaviour. There are two types of cre-
Table 1: Four classes of CA creativity.

| Class           | Rules                          |
|-----------------|--------------------------------|
| Creative        | 3, 5, 11, 13, 15, 35           |
| Schizophrenic   | 9, 18, 22, 25, 26, 28, 30, 37, 41, 43, 45, 54, 57, 60, 62, 73, 77, 78, 90, 94, 105, 110, 122, 126, 146, 150, 154, 156 |
| Autistic savants| 1, 2, 4, 7, 8, 10, 12, 14, 19, 32, 34, 42, 50, 51, 76, 128, 136, 138, 140, 160, 162, 168, 170, 200, 204 |
| Severely autistic| 23, 24, 27, 29, 33, 36, 40, 44, 46, 56, 58, 72, 74, 104, 106, 108, 130, 132, 142, 152, 164, 172, 178, 184, 232 |

ativity: creative product and creative process [23]. The creative ECA rules discovered correspond to a creative process; space-time configurations produced by a creative rule may not be creative. Rule 54 and 110 are computationally universal [39, 6, 25, 27] but why are they not creative? Because they lack robustness, autonomous cognitive control. These rules perform computation only with strict initial conditions. The computational circuits in these rules do not emerge in their space-time configurations by themselves.

We are aware that this interpretation will appear too simplistic, and that both personality and cellular automata are profoundly complex. However, we decided to develop this naive conceptual approach to provoke new ways of thinking and discussion about the issues. We also, believe that highly articulated creative rules might be found in a richer rule-space than ECA.

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## Appendix

Table 2: Values of $\mu$ and $D$ for representative rules of equivalence classes.

| Rule | $\mu$ | $D$ |
|------|-------|-----|
| 0    | 1     | -inf|
| 1    | 10    | -0.423|
| 2    | 6     | -0.446|
| 3    | 11    | -0.017|
| 4    | 4     | -0.431|
| 5    | 11    | -0.009|
| 6    | 11    | 0.557|
| 7    | 5     | 0.303|
| 8    | 1     | -0.424|
| 9    | 19    | 0.55|
| 10   | 6     | 0.007|
| 11   | 14    | 0.304|
| 12   | 4     | -0.007|
| 13   | 14    | 0.311|
| 14   | 7     | 0.309|
| 15   | 12    | 0|
| 16   | 21    | 0.553|
| 17   | 5     | 0.317|
| 18   | 25    | 1.143|
| 19   | 2     | 0.566|
| 20   | 6     | 0.567|
| 21   | 20    | 0.553|
| 22   | 4     | 0.564|
| 23   | 10    | 0.573|
| 24   | 10    | 0.569|
| 25   | 18    | 0.782|
| 26   | 21    | 0.786|
| 27   | 10    | 0.783|
| 28   | 12    | 0.783|
| 29   | 10    | 0.569|
| 30   | 64    | 0.98|
| 31   | 4     | 0.564|
| 32   | 1     | -0.424|
| 33   | 10    | 0.552|
| 34   | 6     | -0.013|
| 35   | 11    | 0.307|
| 36   | 4     | 0.564|
| 37   | 15    | 0.306|
| 38   | 11    | 0.792|
| 39   | 1     | 0.553|
| 40   | 27    | 1.145|
| 41   | 6     | 0.313|
| 42   | 14    | 0.561|
| 43   | 4     | 0.792|
| 44   | 64    | 0.976|
| 45   | 7     | 0.567|
| 46   | 9     | 0.309|
| 47   | 8     | 0|
| 48   | 17    | 0.975|
| 49   | 56    | 0.786|
| 50   | 10    | 0.972|