Convection in a dusty radio-frequency plasma under the influence of a thermal gradient

M Schwabe, L-J Hou, S Zhdanov, A V Ivlev, H M Thomas and G E Morfill
Max-Planck-Institut für extraterrestrische Physik, D-85748 Garching, Germany
E-mail: schwabe@mpe.mpg.de

New Journal of Physics 13 (2011) 083034 (17pp)
Received 3 June 2011
Published 30 August 2011
Online at http://www.njp.org/
doi:10.1088/1367-2630/13/8/083034

Abstract. Gas convection is a common phenomenon in systems under atmospheric pressure conditions with a temperature gradient. Under low pressure conditions, convection can be induced by creep flows along a surface. This has important applications in fluid physics as well as in low pressure plasmas in which a temperature gradient is present. Here, we visualize the gas dynamics in a system with and without a plasma using microparticles as tracers. Two types of gas convection have been identified from the particle motion, i.e. free (Rayleigh–Bénard) convection at high gas pressures, and convection induced by thermal creep at low pressures. The gas flow profile detected using the microparticles is compared with that obtained in a simulation using the direct simulation Monte Carlo method.
1. Introduction

Low-pressure plasmas are commonly used in industry and science. For instance, they are employed in thin-film deposition and etching in the semiconductor industry. Often nano- or micrometre-sized particles, acquiring large charges, appear in the discharge or are injected on purpose. The plasma is then called ‘complex’ or ‘dusty’ plasma and is a subject of study in its own right.

To lift the microparticles into the bulk of the discharge, a vertical temperature gradient can be maintained across the system. In systems with temperature gradients, convection is ubiquitous. One of the most common processes that can trigger convection is thermal creep. Thus, this effect needs to be understood in depth in order to take it into account in low-pressure plasmas as well as in low-pressure gases.

In our earlier work [1, 2], we observed an indicative particle movement in a complex plasma under the influence of a strong vertical temperature gradient. In this work, we build on these earlier observations in order to complement previous work on thermal creep [3, 4] using experimental techniques of complex plasma physics. We thus begin our investigation with some experiments with a complex plasma and complete these with experiments without plasma as well as with simulations of the gas flow.

The paper is organized as follows. After a general introduction to complex/dusty plasmas and to gas convection induced by temperature gradients, we present the conceptual design of the experiments as well as the experimental setup. Then we discuss the results of the experiments and compare these with a simulation. In section 6, the conclusions are presented.

1.1. Temperature-induced fluid convection

It is possible to induce flows in a fluid by maintaining a temperature gradient across the system. If a body force such as gravity is aligned with the temperature gradient, large-scale convection
rolls appear when the lift force on fluid elements, i.e. buoyancy, is greater than viscous friction. The ratio between these forces is described by the Rayleigh number [5]

$$\mathcal{R} = \Delta T \alpha g L^3 / (\nu \kappa),$$  \hspace{1cm} (1)

where \(g\) is the acceleration due to gravity, the coefficient of thermal expansion of the fluid is denoted by \(\alpha\), its kinematic viscosity by \(\nu\) and the thermal conductivity of the fluid by \(\kappa\). The temperature difference across the system is denoted by \(\Delta T\), and the system’s characteristic length is denoted by \(L\). The Rayleigh number scales with pressure as

$$\mathcal{R} \propto p^2.$$  \hspace{1cm} (2)

Heat transfer by convection takes place when heat conduction cannot counteract the temperature differences caused by single fluid elements that rise due to temperature-induced density differences. This defines a critical Rayleigh number \(\mathcal{R}_{cr}\) for the system. Let us consider the case of two parallel (infinite) horizontal plates. When the lower plate is at a higher temperature than the upper one, the critical Rayleigh number is \(\mathcal{R}_{cr} = 1708\) [5]. The critical wave number of the perturbations is \(k_{cr} L = 3.12\) [5]. In friction-dominated systems, convection can also occur, in which case the critical Rayleigh number and the corresponding critical wave number depend on the Prandtl number \(P = \nu / \kappa\) [6].

If the pressure increases at a given \(\Delta T\), first large convection rolls form when \(\mathcal{R} \geq \mathcal{R}_{cr}\) and then the convection becomes turbulent. The velocity of the gas molecules in convection rolls increases with pressure [7, 8].

On the other hand, at low pressures with \(\mathcal{R} \ll \mathcal{R}_{cr}\), heat transfer is mainly through conduction. In this regime, the mean free path of the gas atoms, \(\ell\), is important with respect to the size of the system \(L\), or in other words, the Knudsen number, \(K = \ell / L\), becomes finite.

Then, effects taking place in boundary layers with a longitudinal temperature gradient applied [9, 10] can have a strong influence on the whole system. One of these effects is thermal creep, inducing a gas flux in the direction of the temperature gradient along a wall. The mechanism of thermal creep was explained by Maxwell [11] and Reynolds [12] and is illustrated in figure 1 (after [9]).

The creep velocity of the gas along the surface is given by

$$\vec{v}_{tc} = K_{tc} \vec{V}_\parallel (\ln T_W),$$  \hspace{1cm} (3)

where \(K_{tc}\) is the coefficient of thermal creep [11], which is in the range 0.75–1.2 [13]. The kinematic viscosity is denoted by \(\nu\), and \(\vec{V}_\parallel T_W\) is the component of the temperature gradient tangential to the wall. As \(\nu\) is inversely proportional to pressure \(p\), the velocity of creep scales as

$$v_{tc} \propto 1/p.$$  \hspace{1cm} (4)

While creep itself is confined to the boundary layer (the Knudsen layer) between the gas and the wall with a size \(\sim \ell\), it is able to drive a flow of the bulk gas many mean free paths away from the wall. This is demonstrated by Sone [14]: a windmill is placed in front of a vertical plate in a vacuum chamber. The plate is heated at the bottom. The rotation of the windmill then depends on the pressure in the vacuum chamber. The regimes of free convection and of flux induced by thermal creep can be clearly distinguished.

Another very common method to visualize gas flow is to introduce tracer particles that follow the flow lines [15]. In the particle image velocimetry method, the particles are traced either by tracking them individually or by matching patterns between successive frames and
Figure 1. Sketch illustrating the mechanism that causes thermal creep along a wall in the direction of the temperature gradient. The dots symbolize the positions of the atoms and the arrows their thermal velocities. Those gas atoms that originate from the warmer part of the gas and hit a given surface dS in the wall transfer more momentum to this region than those from the cooler part. At the wall, they are reflected diffusively, and there is no contribution of the escaping atoms to the tangential momentum transfer. Thus, there is a net transfer of momentum from the gas to the wall in the direction opposite to the temperature gradient. As the wall is fixed in place, a gas flow establishes along the surface towards the hot side [9].

Inferring the velocity from the displacement. The particle tracking method proposed in [3] and [4] is used in the given study to visualize convection induced by thermal creep in low-pressure complex plasmas.

1.2. Complex plasmas

‘Complex’ or ‘dusty’ plasmas are weakly ionized plasmas in which micrometre-sized particles are embedded [16]. The microparticles charge up by collecting plasma particles; usually they acquire charges of several thousands of electrons. They can then be levitated in the plasma in the sheath or pre-sheath region of the discharge, in which strong electric fields are present. The individual microparticles can be visualized by illuminating them with a laser and by recording the reflected light. They can thus be easily used as probes of the local plasma or gas conditions [17]: for instance, to visualize gas flow. Mitic et al [3] used a complex plasma in the dc plasma apparatus PK-4 to investigate a creep-induced gas convection. Flanagan and Goree [4] show that microparticles, forming a ball levitated inside a glass box filled with plasma, move in vortices caused by creep when the glass box is heated on one side.

The application of a temperature gradient to a complex plasma is not uncommon; it is a suitable method to counteract gravity and lift the microparticles away from the anisotropic sheath into the plasma bulk [18]. The thermophoretic force on the microparticles is given by [18]

$$\vec{F}_{th} = -3.33 \frac{k_B r_d^2}{\sigma} \vec{\nabla} T,$$

where $k_B$ is Boltzmann’s constant, $r_d$ denotes the radius of the microparticles, $\sigma$ the atom–atom cross-section of the buffer gas and $\vec{\nabla} T$ the temperature gradient. As $\vec{F}_{th}$ is a conservative force, it cannot drive particle motion in a closed loop.
Figure 2. Vector field of the velocities of individual microparticles (6.8 ± 0.1 μm diameter) moving through the void in a complex plasma in argon at a pressure of 18 Pa and in a temperature gradient of 2150 K m⁻¹ during a single bubble event (uncertainty stemming from the uncertainty in the particle positions: ±2 mm s⁻¹). Every tenth particle position recorded during a time interval of 3.2 s length is superimposed. The vectors show the total velocity, the colour code the values of the horizontal (left) and vertical (right) components. The colour coding was truncated at the maximal and minimal values shown. The chamber centre is positioned at y = 0 mm. Taken from [2].

When thermophoresis approximately compensates for gravity, a central void is formed—a phenomenon typically observed under microgravity conditions [19]. It is generally agreed that the void is caused by the balance of the electrostatic force and the drag force exerted by the ions streaming outwards [20]. By adjusting the temperature gradient, it is also possible to overcompensate gravity and push the particles into the upper sheath [21].

However, in the presence of a temperature gradient, the system is not always stable. For instance, in recent experiments in which gravity was compensated for by thermophoresis, at low plasma voltages, we observed the microparticles moving through the central void upwards [1]. This instability caused the formation of bubbles and blobs in the void. Figure 2 shows the velocities of the microparticles moving through the void. The particle cloud beneath the void is located at a height y ≤ −4 mm. The region above the void is visible as a horizontal line in figure 2 at y = 8–9 mm. The experiment was performed in an argon plasma at a pressure of 18 Pa and a temperature gradient of 2150 K m⁻¹. The estimated Rayleigh number under those conditions is 5 × 10⁻³, which is six orders of magnitude beneath the critical Rayleigh number.

The upwards-directed movement might be caused by an instability in the plasma production or by a gas flow. Experimentally, it has been shown that the plasma glow is stable
Figure 3. Schematic diagram of the experimental chamber. Taken from [2].

during bubble events [2, 22], which suggests that there is no plasma instability present. Also, the magnitude of the velocities of the particles moving through the void scales inversely with pressure [1]. These results exclude a Rayleigh–Bénard convection as the main driver of the particle motion. In the following, we show that thermal creep induces a gas convection in our setup, which causes an additional force upwards, explaining the aforementioned observations.

2. Experimental design

2.1. Setup

The experiments were performed in a PK-3 Plus vacuum chamber [23]. In this plasma chamber, the discharge is ignited by two parallel electrodes of 6 cm diameter and 3 cm distance. They are surrounded by grounded aluminium guard rings of 1.5 cm width and 1.2 cm height to increase the confinement force acting on embedded microparticles. The vacuum vessel consists of a quadratic glass cuvette with an inner side length of 10 cm. Figure 3 shows a schematic diagram of the experimental chamber. Microparticles can be injected into the plasma with dispensers that are mounted in the upper guard ring. They are illuminated with a laser plane from the side, and the reflected light is recorded with a camera at an angle of 90° with a frame rate of up to 1000 fps. Here, we investigated only the movement of microparticles within the laser plane. In the following, we refer to the ‘left’ and ‘right’ parts of the chamber with respect to the view from the camera.

In order to levitate the injected microparticles, a vertical temperature gradient can be maintained within the chamber. For this purpose, the lower ground plate of the chamber can be heated with resistors attached to the plate, and the upper ground plate cooled with two fans, as described in [21]. The guard ring is heated directly via the ground plate, while the electrode is heated via the mount in its centre. The maximal vertical temperature difference between the ground plates that was investigated here is $\Delta T = (65 \pm 0.2) \text{K}$, corresponding to a mean temperature gradient of $(2170 \pm 7) \text{K m}^{-1}$.

2.2. Temperature profile

The temperature profile of the chamber was numerically simulated assuming that the electrode, the guard ring and the ground plate are at the same temperature. The geometry is shown in...
Figure 4. Distribution of the temperature gradient (colour) and temperature (contours) in the PK-3 Plus chamber filled with argon at a pressure of 20 Pa, simulated using FEMLAB 3.0’s convection and conduction module. The simulation was performed without heat sources and in the steady-state regime under the following conditions: temperature at the bottom: fixed at 95 °C, at the top: 30 °C; same temperature at the guard ring and the electrode, the gap between them neglected, no heat flux from or into the sides. The heat transfer equation is solved with a finite element method using Lagrange-quadratic elements. The rectangles shown are the fields of view corresponding to: (1)—figure 5, (2)—figure 6 and (3)—figure 7. The simulation box corresponds to a cut through the middle of the experimental chamber: 10 × 5.4 cm². Adapted from [2].

figure 4: the electrode and guard ring have a combined width of 9 cm and a height of 1.2 cm. They are separated by 3 cm. The total width of the chamber is 10 cm. Only a two-dimensional (2D) central cut through the chamber was simulated. In figure 4, it can be seen that the temperature gradient is practically uniform between the electrodes, but there are irregularities at the edges of the guard ring.

In experiments, we measured the horizontal temperature gradient with Pt-100 sensors placed in the radial centre of the guard ring as well as at the edge and in the centre of the electrodes when the temperature gradient was increased to reach the maximal vertical temperature gradient [2]. On the bottom, the guard ring was always hotter than the electrode. Approximately 2 min after the desired vertical temperature gradient was attained, the electrode reached a stable temperature, which was about 0.7 and 1.0 K smaller than that of the guard ring, for the electrode centre and border, respectively. This results in horizontal temperature gradients of 10 K m⁻¹ pointing from the border of the electrode towards the centre and of 100 K m⁻¹ pointing from the electrode border to the centre of the guard ring (in the small gap between the guard ring and the electrode, the temperature gradient might be much bigger, but particles do not enter this region). These gradients are two or three orders of magnitude smaller than the vertical temperature gradient of 2170 K m⁻¹ and are therefore neglected in our simulations.

2.3. Detecting gas convection with the help of microparticles

There are two options to eliminate the influence of the plasma on the microparticles: inject the particles without plasma in the first place or turn off the discharge while particles are levitated.
in the system. Any gas movement then exerts a drag force on the particles. For particles that are small compared to the mean free path of atom–atom collisions (typically of the order of 100 µm for low-pressure conditions) and that move slowly in comparison with the mean thermal velocity of the gas atoms, this force is given by \[ \vec{F}_{dn} = -\gamma_{Ep} m_d \vec{v}_d. \] 

Here, the mass of the microparticles and their velocity with respect to the gas are denoted by \( m_d \) and \( \vec{v}_d \), respectively. The strength of the gas drag is determined by the Epstein drag coefficient, \( \gamma_{Ep} \), which can be written as 

\[ \gamma_{Ep} = \delta_{Ep} \frac{4\pi}{3} \frac{n_a m_a u_n}{m_d} r_d^2, \]

with \( u_n = \sqrt{8k_B T_n/\pi m_n} \), \( n_a \) denoting the number density of the gas atoms and \( r_d \) denoting the radius of the microparticles. The mass of the gas atoms is denoted by \( m_n \). The coefficient \( \delta_{Ep} \) depends on the collision mechanism and was measured as \( \delta_{Ep} = 1.48 \pm 0.05 \) for melamine-formaldehyde (MF) particles in a room-temperature argon plasma [25]. Thus, the strength of the drag force is higher for smaller microparticles. In our experiments, we used 1.5 µm diameter silica and MF particles.

3. Results

3.1. Turning the plasma off

In the discharge, when the plasma is on and thermophoresis overcompensates for gravity, the particles are levitated in the upper part of the vacuum chamber near the upper plasma sheath. There, they form a horizontally extended cloud, the vertical position of which is defined by the balance between the thermophoretic force, gravity and the strong force exerted by the sheath electric field.

If the discharge is turned off, the electric force originating from the plasma sheath and the confinement electric field, acting downwards on the microparticles, vanish. The particles are thus pushed upwards by thermophoresis. Figure 5 shows the traced particle positions of 1.5 µm silica particles under these conditions. The field of view shows the upper left edge of the chamber, including the guard ring (region 1 in figure 4).

It can be seen that most particles are simply pushed upwards. Those on the side of the cloud, however, are also deflected to the left, i.e. towards the window and away from the chamber centre, and then reverse direction, as if reflected from the top guard ring. The thermophoretic force is conservative and thus cannot cause closed loops. Note that the particles retain a residual charge after plasma off, as is well known [26, 27]. The residual charge is about two orders of magnitude less than the initial charge [26]. It can even become positive, but the charge distribution is typically centred around negative values [27]. Thus, mutual repulsion might contribute to the observed motion of the microparticles. The reversal of direction closer to the window, however, cannot be attributed to this effect.

In order to completely eliminate the influence of the rest charge, we performed experiments with particles that are injected into our system without any plasma present.
Figure 5. Colour-coded traced particle positions superimposed over an original image. The particle positions were recorded during a time interval of $\Delta t = 0.7$ s after the plasma was switched off. The colour, coding the time, varies from purple via blue, green and yellow to red. The temperature difference between the top and the bottom ground plate was $\Delta T = 50$ K; the particles were silica particles with 1.5 $\mu$m diameter. They moved in argon gas at a pressure of $p = 38$ Pa. Once the plasma was switched off, the particles moved upwards under the action of thermophoresis. The particles on the side, however, moved sideways and then reversed direction. The direction of movement is indicated by the white arrow. Field of view: 27 $\times$ 13 mm$^2$, in the top left corner of the total field of view (region 1 in figure 4). On the upper left, the tip of the particle dispenser incorporated into the guard ring is visible.

3.2. Injecting particles into pure gas

Figure 6 shows the trajectories of particles injected into argon with a vertical temperature difference of $\Delta T = 45$ K at four different pressures. The particles were injected into the pure gas and the dispenser is located above the top right corner of the field of view. A constant temperature gradient was maintained between the ground plates. Many of the particles fell directly onto the guard ring below, but a significant portion was deflected to the sides. It can be seen in figure 6 that many particles are deflected to the left, i.e. away from the window and upwards, forming a vortex. Some particles are also deflected to the window and to a spot on the guard ring. This is probably due to the increase in temperature gradient caused by the edge of the guard ring (compare figure 4).

In figure 6, four panels show the particle tracks obtained in injections at four different pressures: $p \simeq 10$, 30, 100 and 12400 Pa. The Epstein drag coefficient is given by $\gamma = 7.95 \times p_{[Pa]} s^{-1}$. The number of frames that were superimposed was chosen such that the expected variation of speed with pressure is taken into account, except in the last case, where the particle velocity was very high. Most of the particles that were injected fell immediately from the dispenser onto the lower guard ring, and only a fraction of them stayed in the chamber long enough to be useable as tracer particles. The figures show superpositions of frames starting at a time when most of the particles that dropped directly onto the guard ring have disappeared from view.

It can be seen in this figure that the remaining particles move upwards. This is at least partly due to the thermophoretic force. At a temperature difference of $\Delta T = 45$ K, the strength
Figure 6. Overlapping trajectories of 1.5 µm particles injected into argon gas at four different pressures \( p \). The particles were injected at time \( t = 0 \) s from the dispenser located above the upper right corner of the shown field of view (\( 19 \times 25 \text{mm}^2 \), region 2 in figure 4). The temperature difference was maintained at \( \Delta T = 45 \) K. The duration of the shown time interval, \( \Delta t \), was scaled to take into account the decrease of velocity with pressure (except for panel (d)). The colours vary with time from blue (via green and yellow) to red. The direction of movement of the particles in panels (a)–(c) is indicated by a white arrow in panel (a) and that in panel (d) is marked separately. (a) \( p = 10 \) Pa, \( t = 0.50–0.53 \) s after injection of the particles, 33 frames superimposed; (b) \( p = 29 \) Pa, \( t = 1.0–1.1 \) s, 100 frames superimposed; (c) \( p = 98 \) Pa, \( t = 1.5–1.8 \) s, 300 frames superimposed; (d) \( p = 12400 \) Pa, \( t = 40.0–43.6 \) s, 450 frames superimposed.

of thermophoresis is more than three times that of gravity acting on the 1.5 µm particles. As mentioned above, thermophoresis does not, however, explain the vortex movement of the microparticles—it is a conservative force and cannot cause movement in loops. The magnitude of the vortex movement can be seen to decrease with pressure; figure 6(c) was recorded at a pressure of 100 Pa, and the deflection back to the right in the top of the chamber is hardly present anymore.

In figure 6(d), in contrast, this deflection has reappeared. This measurement was recorded at a pressure of 12 400 Pa. At this pressure, the Rayleigh number is of the order of the critical Rayleigh number for free convection (see table 1). We observe two large, stable vortices on both sides of the chamber, the one on the right is rotating in the clockwise direction and the one on the left in the counterclockwise direction. In contrast to the experiments under lower pressure conditions, the particles are trapped in the vortices for a long time (minutes). However, even at pressures below the critical pressure for the onset of free convection, we already observe motion in vortices (e.g. with a velocity \( v \sim 1 \) mm s\(^{-1}\) at a Rayleigh number of 12). We suspect that in our case, Rayleigh–Bénard convection sets in earlier than in the textbook case of two infinite surfaces, as it is aided by the fact that the centre of the electrode is slightly warmer than its side (see section 2.2).

Increasing the pressure even more finally leads us to the turbulent regime of free convection. Figure 7 shows particle paths traversed by particles injected into a gas at almost atmospheric pressure. The particles move in local vortices of varying size, which are stable only for fractions of a second and then disappear and reappear in other places. The contrast with the
Table 1. The conditions of the experimental runs and simulations presented in this paper. When no particle sizes and materials are given, we show the results of simulations (marked with a circle: the DSMC method presented in figure 9). In all experiments, argon was used as buffer gas. In the experiment marked with a star, the mean of the velocities measured from the traced particle positions in the region 21–24 mm from the right window and 9–12 mm above the electrode is given. In the experiment marked with a dagger, the longest particle trace shown in figure 5 was used to calculate the velocity. Turbulent convection is marked with a diamond. In that case, there were hardly any particle tracks visible in the figure during the entire considered time interval, so that only a lower estimate is given. In all other cases, the length of a typical particle path in the region 19–24 mm from the right window and 13–18 mm above the lower electrode was considered.

| Particles | Size  | Pressure | Rayleigh number R | Velocity (mm s\(^{-1}\)) | Accuracy (mm s\(^{-1}\)) | Comment |
|-----------|-------|----------|------------------|--------------------------|--------------------------|---------|
| N/A       | 5     | 0.00024  | 3500             |                          |                          | Max. velocity\(^{c}\) |
| N/A       | 5     | 0.00024  | 200              |                          |                          | 2 cm from edge\(^{c}\) |
| 1.5 MF    | 10    | 0.00097  | 34               | ±17                      |                          | See figures 6(a), 10 |
| 1.5 MF    | 20    | 0.0039   | 23               | ±1                       |                          | See figures 8, 10*   |
| 1.5 MF    | 29    | 0.0082   | 25               | ±13                      |                          | See figures 6(b), 10 |
| 1.5 Silica| 38    | 0.016    | 22               | ±2                       |                          | See figure 5 †       |
| 1.5 MF    | 98    | 0.093    | 10               | ±5                       |                          | See figures 6(c), 10 |
| 1.5 MF    | 200   | 0.4      | 6.6              | ±3                       |                          | See figure 10        |
| 1.5 MF    | 1100  | 12       | 0.9              | ±0.5                     |                          | See figure 10        |
| 1.5 MF    | 12400 | 1500     | 1.7              | ±0.9                     |                          | See figures 6(d), 10 |
| 1.5 MF    | 80000 | 62000    | ≥34              |                          |                          | See figures 7 and 10° |

stable convection vortices at lower pressures and the gas convection induced by creep at very low pressures is immediately obvious.

3.3. Velocity field

Using the data recorded at low pressures, it is possible to reconstruct the velocity field by tracing the particles from frame to frame. Figure 8 shows the resulting vortex motion in argon at a pressure of 20 Pa. The total velocity distribution is shown in the inset of this figure. The particles move with velocities of the order of a few cm s\(^{-1}\). In the following, we compare the experimental data with a simulation of the gas flow.

3.4. Flow mechanism

In summary, we detected the following flow patterns in the thermal creep regime.

- Particles move downwards along the vertical walls of the vacuum chamber.
- In the top part of the chamber, those particles that are close to the wall are deflected towards it and down along the wall.
Figure 7. Colour-coded overlapped trajectories of 1.5 µm particles injected into argon gas at pressures of $p = 8 \times 10^4$ Pa at a temperature difference of $\Delta T = 45$ K. Shown are particle positions during the time interval $t = 8.2$–$8.6$ s, with the particle injection occurring at $t = 0$ s. Time is coded with colours varying from purple to red as in figure 6. The white arrows indicate a selection of the particle movement at the selected time. The particles follow the turbulent gas motion in the Rayleigh–Bénard convection, in contrast to the non-turbulent motion shown above. Field of view: $57 \times 25$ mm$^2$, region 3 in figure 4. The right part of the figure corresponds to the field of view shown in figure 6.

Figure 8. Mean velocity field (arrows) superposed over streamlines of 1.5 µm particles injected into argon at $\Delta T = 45$ K and pressure $p = 20$ Pa. A total of 4000 frames ($t = 0.7$–$4.7$ s) were used to calculate the velocity map. Field of view: $50 \times 29$ mm$^2$, the right bottom corner is aligned with that of regions 2 and 3 in figure 4. Inset: distribution of total velocities.

- In the bottom part of the chamber, they are either deflected towards the wall or towards the centre of the chamber. The deflection towards the wall is attributed to the peak in temperature gradient caused by the guard ring edge.
- The particles that have been deflected away from the wall move a small distance into the chamber and then in a vortex motion upwards (compare figure 8).
We conclude from these observations that the gas is flowing along the vertical walls downwards due to thermal creep. When it reaches the bottom of the chamber, the gas moves into the chamber and back upwards in the region outside of the creep zone close to the walls. This leads to convective vortex motion as traced by the microparticles. In the main part of the chamber, there is no gas movement in the low pressure regime. At higher pressures, free convection occurs, and the gas motion spreads into the whole of the vacuum chamber.

4. Simulation

To further clarify the role of the thermal creep flow, we have carried out numerical simulations on rarefied gas dynamics in a simple, 2D confined geometry (but 3D in velocity). Note that by adopting the 2D geometry, we have actually assumed it as an infinitely long tube in the $z$-direction. We resort to a microscopic description of the gas dynamics, mathematically expressed by the Boltzmann equations, and adopt the standard direct simulation Monte Carlo (DSMC) method of Bird [28, 29], in which real gas molecules are represented by coarse-grain super-particles and collisions are treated by solving the time-dependent nonlinear Boltzmann equations using the Monte Carlo method.

Typically, $10^6$ super-particles, interacting via short-ranged hard sphere collisions, are simulated in a rectangular domain of $10$ cm width and $3$ cm height, which approximates the interior of the real discharge chamber. The domain is further divided into uniform cells, whose size is usually a fraction of the mean free path of gas molecules. The electrodes and chamber walls are assumed to be perfect thermal walls [28, 29], and their temperatures and temperature gradients are set according to the corresponding experimental measurements, i.e. constant temperatures of, respectively, 300 K on the upper electrode and 345 K on the lower one and a linear temperature gradient from 300 to 345 K on the side wall (in the $y$-direction).

Note that when a gas molecule with mass $m$ strikes a perfect thermal wall at a given local temperature $T_w \equiv T_w(y)$, all three components of its velocity $v$ are reset according to the biased-Maxwellian distribution. The component perpendicular to the wall surface is distributed as [28, 29]

$$P_{\perp}(v_\perp) = \frac{m}{k_BT_w}v_\perp \exp\left(\frac{-m v_\perp^2}{2k_BT_w}\right),$$

while the distribution of each parallel component as follows:

$$P_{\parallel}(v_\parallel) = \frac{m}{k_BT_w}v_\parallel \exp\left(\frac{-m v_\parallel^2}{2k_BT_w}\right).$$

Here, $k_B$ is again the Boltzmann constant.

Initially, the particles are uniformly and randomly distributed in the computational domain, and the particle velocities are given according to the Maxwellian distribution at room temperature. The procedures in the main simulation are as follows [28, 29]. Firstly, the positions and velocities are advanced one time step by directly solving Newton’s equations. Secondly, boundary conditions are applied to those particles that reach an electrode or a side wall, by sampling their new velocities according to equations (8) and (9). Those remaining in the system are sorted into cells according to their new positions.

Thirdly, the collisions in the gas are evaluated. In short, any pair of particles in a cell, regardless of their specific positions, could collide, and a number of particles within the
Figure 9. Simulation of gas flux in the PK-3 Plus chamber at 5 Pa, $\Delta T = 45$ K. Only the right half of the simulation box is shown. The arrows indicate the direction and magnitude of the gas movement and the background colour indicates its magnitude. The gas moves downwards along the wall in the right of the figure due to thermal creep. When the gas reaches the bottom of the simulation box, it is pushed towards the chamber centre and moves back upwards in a vortex. The method used was the standard DSMC method [28, 29]. A constant temperature of 300 K on the upper boundary and of 345 K on the lower boundary were assumed.

The same cell are randomly selected as collision partners according to the collision probability that is given by the kinetic theory (i.e. the collision probability is proportional to their relative velocity) [28, 29]. Momentum and energy are conserved during the collision. The new velocities are determined by further assuming that the post-collision relative velocity is uniformly distributed in a sphere whose radius, i.e. the relative speed, is given by momentum and energy conservation. (This assumption has been found to be an excellent approximation in general [28, 29].)

The thermal creep flow is induced automatically by the temperature gradient and reaches a steady state after a long time (typically a few million time steps with a time step being a fraction of the gas molecule mean free time). Statistical averages of the flows are made in an extra period after the system reaches steady state. Since in our study the thermal creep flow is typically about 1 m s$^{-1}$ or even smaller, whereas the thermal speed of gas molecule is about 400 m s$^{-1}$, it is difficult to resolve such a small flow from the large noise. Usually another $10^6$ time steps are needed to suppress the noise, and the higher the pressure, the more the time steps needed. This places a rather hard limitation on our simulation, especially at high pressures.

Figure 9 exhibits a typical flow pattern observed in the simulation, together with the speed distribution as the background. One sees that there are strong flows from the upper electrode to the lower one in the vicinity of the two side walls, where the flows have a maximum speed of about 3.5 m s$^{-1}$. Due to the existence of the two electrodes, the flows bend and form two vortices that extend further into the bulk region. At the outer edges (close to the centre) of the vortices, the flows reduce to about 0.2 m s$^{-1}$. In general, the flow decreases, in both amplitude and span, with an increase of pressure. As a result, the signal-to-noise ratio increases with pressure.
Our simulations allow us to definitively connect the microparticle motion at low pressures to the thermal creep flow and to give more details of the flow pattern and velocity distribution. In particular, the flow patterns agree well with experimental observation of microparticle trajectories at low pressures, as can be seen by comparing figures 6 and 8 with figure 9.

5. Scaling laws

The conditions of the experimental runs and simulations presented in this paper are summarized in table 1. In the following, we discuss scaling laws that can be compared with our results.

It is well known that convection patterns are characterized by comparatively simple scaling laws, see [30], which are very convenient to represent the results of the given study. It is worth noting that the scaling law depends on the convection regime, i.e. on gas pressure in our case. In the case of creep-induced convection, velocity versus Rayleigh number scaling can be derived directly in an extremely simple way. Let us rewrite relationships (1) and (3) as follows:

\[ \mathcal{R} = \mathcal{P} g L \frac{L^2}{v^2} \Delta T, \]
\[ v_{lc} = K_{lc} \frac{v}{L} \Delta T, \]

where we used \( \alpha = 1/T \) and the Prandtl number \( \mathcal{P} = v/\kappa \) and (for brevity) introduced \( T \equiv T_W \). For argon and neon, the latter is approximately constant; \( \mathcal{P} \approx 0.7 \) in our conditions. Note that the Rayleigh number scales with pressure as \( \mathcal{R} \propto p^2 \).

It follows immediately that

\[ (v \mathcal{R}^{1/2})_{lc} = \left( K_{lc}^2 \mathcal{P} g L \left( \frac{\Delta T}{T} \right)^3 \right)^{1/2} = V_{lc}, \]

where \( V_{lc} = \text{const} \) under the conditions of our experiments. This asymptotic is shown in figure 10 (the red dashed line drawn at \( V_{lc} = 2.3 \text{ cm s}^{-1} \)). Note that the slope \( (\ln v)/(\ln \mathcal{R}) = -1/2 \) fits fairly well to all known experimental data obtained in the regime of creep-induced convection [1, 3, 4]; compare the blue dashed line in figure 10. Note also that the ratio of the creep velocity \( (v_{lc}) \) to the experimentally measured velocity \( (v_{exp}) \) gives a geometric factor, accounting for the velocity field inhomogeneity.

In the case of free convection, the situation is more complicated [8, 31]. According to [8], the simplest behaviour occurs for the viscous regime where (‘fc’ stands for ‘free convection’)

\[ v_{fc} = C \frac{v}{\mathcal{P}L} \mathcal{R}^{2/3}, \quad C = \text{const}. \]

Combining this relationship with that defining the Rayleigh number (1), one comes to the conclusion that

\[ \left( \frac{v}{\mathcal{R}^{1/6}} \right)_{fc} = \left( C^2 g L \Delta T \mathcal{P} T \right)^{1/2} = V_{fc}. \]

This shows that the velocity is slowly increasing with \( \mathcal{R} \) in the regime of free convection. The unknown factor \( C \), or \( V_{fc} \), can be obtained either from simulations (see e.g. [31]) or experimentally. In our conditions, \( V_{fc} = 0.54 \text{ mm s}^{-1} \) and \( C = 2.2 \times 10^{-3} \) are the optimal choices, as the comparison with the experimental data shows (figure 10).
6. Conclusion

In conclusion, we have studied temperature gradient-induced gas convection in the PK-3 Plus vacuum chamber used for igniting radio-frequency (rf) plasma discharges [23]. We traced the motion of microparticles in the chamber to infer the gas motion. At low pressures in the range of tens of Pascal, we find a convection that we ascribe to the creep flow along the vertical chamber walls. This gas convection can have a strong influence on measurements with complex plasmas and limits the stability of microparticle clouds levitated by thermophoresis.

The measured deflection of the microparticles induced by convection first decreases at higher pressures and then reappears and becomes very strong in the regime where free convection is expected. At almost atmospheric pressure, we observed a turbulent convection in the chamber. We compared the measured velocity field with simulation results and found good agreement. In addition, we discuss scaling laws of the velocity as a function of the Rayleigh number and compare these with our experiments.
Acknowledgments

This work was supported by the Deutsches Zentrum für Luft-und Raumfahrt with funds from the Federal Ministry of economics and Technology according to a resolution of the Deutscher Bundestag under grant number 50 WP 0203.

References

[1] Schwabe M, Rubin-Zuzic M, Zhdanov S, Ivlev A, Thomas H M and Morfill G E 2009 Phys. Rev. Lett. 102 255005
[2] Schwabe M 2009 Dynamical effects in fluid complex plasmas PhD Thesis Ludwig-Maximilians-Universität
[3] Mitic S, Sütterlin R, Ivlev A V, Höfner H, Thoma M H, Zhdanov S and Morfill G E 2008 Phys. Rev. Lett. 101 235001
[4] Flanagan T M and Goree J 2009 Phys. Rev. E 80 046402
[5] Oertel H, Erhard P, Eiting D, Müller U, Sreenivasan K R and Warnatz J 2010 Prandtl—Essentials of Fluid Mechanics (Berlin: Springer)
[6] Ivlev A V, Zhdanov S K and Morfill G E 2007 Phys. Rev. Lett. 99 135004
[7] Kennelly A E and Sanborn H S 1914 Proc. Am. Phil. Soc. 53 55
[8] Moore D R and Weiss N O 1973 J. Fluid Mech. 58 289–312
[9] Sone Y 2000 Annu. Rev. Fluid Mech. 32 779–811
[10] Aoki K, Degond P and Mieussens L 2009 Commun. Comput. Phys. 6 919–54
[11] Maxwell J C 1879 Phil. Trans. R. Soc. 170 231–56
[12] Reynolds O 1879 Phil. Trans. R. Soc. 170 727–845
[13] Bakanov S P 1992 Sov. Phys.—Usp. 35 783–92
[14] Sone Y 1991 Phys. Fluids A 3 997–8
[15] Sinton D 2004 Microfluid. Nanofluid. 1 2–21
[16] Morfill G E and Ivlev A V 2009 Rev. Mod. Phys. 81 1353
[17] Fortov V E, Petrov O F, Usachev A D and Zobnin A V 2004 Phys. Rev. E 70 046415
[18] Rothermel H, Hagl T, Morfill G E, Thoma M H and Thomas H M 2002 Phys. Rev. Lett. 89 175001
[19] Nefedov A P et al 2003 New J. Phys. 5 33
[20] Goree J, Morfill G E, Tsytovich V N and Vladimirov S V 1999 Phys. Rev. E 59 7055–67
[21] Schwabe M, Rubin-Zuzic M, Zhdanov S, Thomas H M and Morfill G E 2007 Phys. Rev. Lett. 99 095002
[22] Schwabe M, Zhdanov S, Rubin-Zuzic M, Ivlev A, Thomas H and Morfill G 2011 IEEE Trans. Plasma. Sci. at press
[23] Thomas H M et al 2008 New J. Phys. 10 033036
[24] Epstein P 1924 Phys. Rev. 23 710–33
[25] Konopka U 2000 Wechselwirkungen geladener Staubteilchen in Hochfrequenzplasmen PhD Thesis Ruhr-Universität-Bochum
[26] Ivlev A V et al 2003 Phys. Rev. Lett. 90 055003
[27] Couèdel L, Mikikian M, Boufendi L and Samarian A A 2006 Phys. Rev. E 74 026403
[28] Bird C A 1994 Molecular Gas Dynamics and the Direct Simulation of Gas Flow (Oxford: Clarendon)
[29] Garcia A L and Baras F 1996 Direct simulation Monte Carlo: novel applications and new extensions Proc. 3rd Workshop on Modelling of Chemical Reaction Systems (Heidelberg, Germany)
[30] Landau L D and Lifshitz E M 1963 Fluid Mechanics (Reading, MA: Pergamon Press)
[31] Garcia O E, Bian N H, Naulin V and Rasmussen J J 2006 Phys. Scr. T122 104