Verification for Reliable Product Lines

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Abstract—Many product lines are critical, and therefore reliability is a vital part of their requirements. Reliability is a probabilistic property. We therefore propose a model for feature-aware discrete-time Markov chains as a basis for verifying probabilistic properties of product lines, including reliability. We compare three verification techniques: The enumerative technique uses PRISM, a state-of-the-art symbolic probabilistic model checker; on each product. The parametric technique exploits our recent advances in parametric model checking. Finally, we propose a new bounded technique that performs a single bounded verification for the whole product line, and thus takes advantage of the common behaviours of the product line. Experimental results confirm the advantages of the last two techniques.

Index Terms—Non-Functional Requirements, Software Product Lines, Markov models, Model Checking, Probability

I. INTRODUCTION

Software Product Line Engineering (SPLE) aims at developing a large number of software systems that share a common and managed set of features. In the past years, it has been an active area in both research and industry. SPLE aims at improving productivity and reducing the time, effort and cost required to develop a family of products (also called variants). The key point to achieve this goal is to manage the variability among various products of a Software Product Line (SPL). SPLE mainly relies on model-based techniques by which variable features and behaviours are specified. The models are used to derive numerous products, each of which contains a specific set of features.

Each product of the SPL has to satisfy its functional and non-functional requirements. A common approach to assess these requirements consists in building the system and testing it. However, if the system fails to meet the requirements, costly iterations are needed to improve it. This problem is even more crucial in SPLE, where a huge number of products have to be designed. Analysis techniques for single systems therefore are expensive to apply. This is the reason why researchers have recently focused on designing new quality assurance techniques dedicated to SPLs. In particular, model checking is an automated verification technique that systematically explores the whole state space of a model in search of errors. In the last years, several approaches lifted model checking to SPLs [?], [?] (see more in Section VII). However, most of them consider qualitative properties on the sequencing of events only. They ignore important non-functional aspects of systems like reliability, availability, performance, and resource usage.

Moreover, today’s software are embedded in a wide variety of systems like networks and aircrafts, that run in environments where uncontrolled events may occur randomly and affect the system. For example, a TCP transmission might be perturbed by hardware failures in a network node; a pump motor has to run faster when the flow of water goes beyond a certain threshold. Further, some systems use randomness for their own functioning, like the USB protocol. Markov models are widespread formalisms to model probabilistic properties of systems whose behaviour is driven by random events. Still, they cannot cope with variability. A possible approach to modeling stochastic product lines is to specify them with parametric equations; resolving these equations can be difficult, however. There is thus a need for (1) behavioural models able to represent both variability and randomness, and (2) automated techniques to efficiently verify all the modelled products against probabilistic properties.

In this paper, we propose an approach to model and verify fully observable stochastic SPLs. We enrich Markov chains with an explicit notion of variability. This allows us to derive a new formalism that combine representations of SPL behaviour with Markov models. In particular, we introduce Featured Discrete-Time Markov Chains (FDTMCs), an extension of discrete-time Markov chains that allows to describe a full product line in a single model, thus sharing the common parts. In FDTMCs, transitions are associated with a probability profile that, given a product, returns the probability that the transition is executed in this product. The definition of probability profile is intended to be flexible, and allows features to arbitrarily modify the transition probability.

Next, we present and discuss three verification methods to determine which products modelled in an FDTMC (fail to) satisfy a given non-functional property. We focus more particularly on properties such as reliability that can be expressed in Probabilistic Computation Tree Logic (PCTL) [?]. A first solution to achieve this goal is to derive, for each product, the discrete-time Markov chain modelling this product from the FDTMC and to apply single-system algorithms to model check it. This enumerative technique thus performs one ex-
ploration per product. We also propose to reduce FDTMC model checking to parametric Markov-model checking, where the parameters encode the features. We can then feed the parametric model into a parametric model checker, which, in a single exploration, yields a parametric expression that encodes the value of the desired properties. Then, for each product, the checker has to evaluate this expression by replacing the parameters by the values corresponding to the features of the products. Our last proposition is a novel algorithm that exploits probability profiles to determine all the products satisfying the property in a single exploration, which benefits from the compact structure of FDTMC to reduce the cost of verification. This approach applies a bounded search through the state space of an FDTMC model, and is able to calculate approximative results considering the satisfaction of a given property within a desired precision.

To evaluate the efficiency of the three approaches, we carried out two series of experiments based on systematically extended technical examples. The results show that the enumerative approach is inefficient compared to the other two techniques, especially as the number of products grows.

The remainder of the paper is structured as follows. Section II recapitulates background on SPL model checking. In Section III we recap probability theory and give its featured version. In Section IV we introduce FDTMCs, FMDP, and present our compositional modelling approach. The three verification techniques are discussed in Section V. Section VI provides experimental results. We overview related work in Section VII and sketch future work in Section VIII

II. FOUNDATIONS

Variability in SPLs is commonly captured into features [?]. A feature (or variation point [?]) is a unit of difference that can differentiate two variants of an SPL. It can model optional components, functionalities of the system, but also cross-cutting behaviour. Dependencies between features may exist, and these are commonly modeled in a feature diagram (FD) [?]. Many different languages are used now for this purpose [?]. A product, sometimes called configuration, variant [?], or model [?], [?], gives a value to each feature of its FD. A product $p$ is valid for $d$, noted $p \models d$, if the values of its features satisfy all the constraints expressed in the FD. In this paper, we abstract from the syntax of FDs and define directly the semantics of an FD $d$ as its set of features (its signature [?]) $\Sigma_d$ and the set of its valid products (its product line [?]) $\{d\}$. We can restrict a product $p$ to a sub-signature $\Sigma'$, noted $p|_{\Sigma'}$, which selects only the value of the features of $\Sigma'$. In other words, FD form an institution [?].

As an increasing number of SPLs are developed, quality assurance techniques for them become vital and are actively studied. FDs being unable to express behaviour, several approaches for SPL model checking have emerged during the recent years (see more in Section VII). Here we follow the principles of Featured Transition Systems (FTS) [?]. Assume we have an adequate model type for single products; then the semantics of the “featured” variant of this model is a function giving for each product of the product line, the corresponding model. For instance, Transition Systems (TS) are a well-accepted model type for the qualitative behaviour of a system, and thus the semantics of FTS maps each product to a TS. At the syntactic level, the dependance on the products is moved as much as possible inside the model, so that the common parts can be described only once. Thus, an FTS is a state machine where transitions are annotated with feature expressions, i.e. formulae defined over the features. Formally, an FTS is a tuple $F \in (S, s_0, Act, trans, AP, L, d, \gamma)$ where $S$ is a set of states, $s_0$ is the initial state, $Act$ is a set of actions, $trans \subseteq S \times Act \times S$ is a set of transitions, $AP$ is a set of atomic propositions, $L : S \rightarrow 2^{AP}$ labels a state with the propositions that it satisfies, $d$ is a feature diagram and $\gamma : trans \rightarrow ([d] \rightarrow \{\top, \bot\})$ associates a transition with a feature expression, i.e. a formula that encodes the set of products able to execute the transition. The dependance on the products has thus been moved inside, to the transitions.

An FTS model-checking algorithm takes the feature information into account while looking for errors. It is thus able to keep track of the products able to execute the behaviour currently analysed, and will only examine once the common parts. Feature expressions constitute an intuitive and flexible way to represent variability inside behavioural models. In this work, we will re-use the principles of such encoding for modelling behavioural variability in stochastic systems.

Example. We illustrate the concept of FTS by means of the mine pump controller [?], [?]. The objective of this software is to control a pump that drains water from a mine. When the level of water goes beyond a certain threshold, it should turn the pump on, except when there is methane in the mine, since the motor might cause an explosion. Figure 1 presents an FTS modelling the (simplified) controller SPL. Two features influence the behaviour of the controller: the presence of a WaterSensor ($W$), which reports the water level, and MethaneAlarm ($A$), which detects the presence or absence of methane. The controller starts in its state ready. Then, it can reach the state run when the level of water is so high that the pump has to be turned on. As specified by the associated feature expression, a product $p$ can execute this transition $t$ iff $\gamma(t)(p) = T$, that is, iff feature $W$ is enabled in $p$. Whenever the level of water is low or there is methane, the pump must be turned off. The controller reacts accordingly and reaches state stopped, respectively emergency, provided that the controller is equipped with feature $W$, respectively $A$. 

![Mine pump controller FTS](image-url)
Therefore, the behaviour of the controller is determined not only by its features, but also by uncontrolled events (that is, water level and methane presence) which occur randomly. Even though FTSSs are convenient for modelling behavioural variability, they cannot represent randomness. On the contrary, stochastic models cannot cope with variability. Both abilities are required to model and check non-functional quantitative properties like “what is the probability that eventually the pump is not running even though the water level is high?” on all the products of an SPL. The purpose of this paper is to provide a formal framework for extending stochastic models with variability information.

III. FEATURED MARKOV MODELS

Markov processes are meant to model systems where random events occur. They have been used to analyse quantitative properties in a variety of areas including economics, networks, and software. Throughout the section, we recapitulate the fundamental definitions related to Markov processes, and enrich them with an explicit notion of variability along the principles sketched above.

Markov processes, and more generally any kind of stochastic process, are defined over a probability space and a measurable space. A probability space is a triple $(\Omega, F, P)$ where $\Omega$ is a finite set of outcomes, $F \subseteq 2^\Omega$ is a set of events, each of which is a set of outcomes, that forms a $\sigma$-algebra, i.e., $F$ is non-empty and closed under complementation and countable unions. $P : F \rightarrow [0, 1]$ is a probability measure function that assigns probability to events. The probability of a countable union of disjoint sets must be the sum of their probabilities, and the probability of $\Omega$ must be 1. In our minepump example, the outcomes are the combination of water level (low, normal, and high) with the presence or absence of methane (methane and no_methane, respectively); an example of event is “there is methane and the water level is not low”, that is, $\{(\text{high, methane}), (\text{normal, methane})\}$. The state will be represented by a measurable space: it is a couple $\langle S, \Sigma \rangle$ where $S$ is a set and $\Sigma \subseteq 2^S$ is a a $\sigma$-algebra. $S$ is called the state space.

Let $T$ be a totally ordered set, called time. A stochastic process is a set $\{X_t \in S | t \in T\}$ of indexed, $S$-valued random variables. Random variables are functions $X : \Omega \rightarrow S$ that maps an outcome to a state. $X_t$ represents the state of the process at time $t$. Given that we consider SPLs instead of single systems, we define that this state also depends on the features of the process. For example, if the outcome $\{(\text{high, methane})\}$ occurs while the controller is in state ready, it should reach state run if it has feature $W$ or state emergency if it has feature A; otherwise it remains in state ready. The value of $X_t$ is thus impacted by variability. Accordingly, we revise the definition of random variable, and define it as a function $X : \Omega \rightarrow ([d] \rightarrow S)$ that, given an outcome $\omega$ and a variant $p$, returns the state reached by $p$ following the occurrence of $\omega$. This revised definition leads us to the following extended definition of stochastic process.

**Definition 1** Let $(\Omega, F, P)$ be a probability space, $(S, \Sigma)$ be a measurable space, $T$ a totally ordered set, and $d$ a feature diagram. A featured stochastic process is a set $\{X_t \in S[d] | t \in T\}$ where for any $t$, $X_t$ is a random variable.

According to this definition, the state reached by the process at time $t$ is determined by (1) the outcome and (2) the product, i.e. the combination of features of the process.

Markov processes are a particular type of stochastic processes that satisfy the so-called Markov property. Intuitively, this property, often referred to as the memoryless property, implies that the probability of reaching a state at a future point of time can be determined knowing only its current state. Since we revised the definition of random variables and stochastic process, we extend the definition of the Markov property as well, and obviously we require that the property holds for each product.

**Definition 2** A featured Markov process is a variability-aware stochastic process $\{X_t \in S[d] | t \in T\}$ such that $\forall p \in [d] \bullet \forall t_1 < t_2 \in T \bullet s \in S$ we have

$$
Pr(X_{t_2}(p) = s \mid X_{u}(p) = s_u, \forall u \leq t_1) = Pr(X_{t_2}(p) = s | X_{t_1}(p) = s_1)
$$

Assume furthermore that time $T$ is equipped with an addition.

**Definition 3** A featured Markov process is time-homogeneous if

$$
Pr(X_{t_1+t_2}(p) = s | X_0(p) = s') = Pr(X_{t_2}(p) = s | X_0(p) = s')
$$

When the time is discrete, it is enough to take $t_2 = 1$.

There exist many types of Markov models: First, we can model time as continuous ($T = \mathbb{R}_{\geq 0}$) or discrete ($T = \mathbb{N}$). Second, we can assume that the process is purely stochastic (Markov chain) or partially under control of an opponent (Markov decision process). Third, rewards can be added to the model (Markov reward model), etc. Specialized algorithms have been developed for each type of model [1], [2]. In the rest of this paper, we will focus on the two most widely used models, namely time-homogeneous discrete-time Markov chains (DTMC) and time-homogeneous discrete-time Markov decision process (MDP). For each, we will derive its featured extension (FDTMC and FMDP). We leave for future work the algorithmic treatment of other featured stochastic processes, but their semantics is already defined in this section.

IV. FEATURED DISCRETE-TIME MARKOV PROCESSES

The above definition of featured Markov processes is the semantic foundation for the definition of formalisms to model probabilistic requirements in SPLs. Here, we specialize it to Featured Discrete-Time Markov Chains (FDTMCs), a formalism that extends Discrete-Time Markov Chains (DTMCs) to allow modelling both variability and stochasticity. In this section, we first briefly illustrate how classical DTMCs model the behaviour of a stochastic environment. Then we present the syntax and the semantics of FDTMC.
A. Modelling random events with DTMCs

DTMCs are a particular type of Markov processes where (1) the state space of the system is finite, and (2) time elapses at discrete steps. DTMCs can be regarded as transition systems where transitions are annotated with a probability value that describes the likelihood of their occurrence. These probabilities satisfy the usual probability axioms. In particular, for a given state, the probabilities of its outgoing transitions must sum to 1.

A example DTMCs is shown in Figure 2. It models the evolution of methane evolution in the mine in which the aforementioned pump is installed. Initially, there is no methane. At the next discrete point of time, either there is methane (probability 0.125) or not (probability 0.875). Then the methane, if present, will disappear spontaneously in 75% of the cases. Note that the probability of staying in the same state (self-loop) can be deduced from the other outgoing transitions, since they have to sum up to 1; we will thus omit them in the rest of the paper. DTMC are adequate to model systems that evolve spontaneously, without influence from the outside world.

![Fig. 2. The DTMC modelling methane evolution](image)

B. FDTMC: A featured stochastic formalism

Although DTMCs are convenient to represent stochasticity, they cannot cope with variability. To represent stochastic behaviour in SPLs, we enrich them in the same way as we enhanced transition systems with feature expressions to obtain the FTS formalism [1, 2]. The original intent of feature expressions is to specify that features can add or remove transitions. In DTMCs, however, the addition or removal of a transition leaving a given state may modify the probability of its other outgoing transitions. More generally, features can modify the probability distribution of any transition. The only restriction is the satisfaction of the probability axioms in all the products. Therefore, the probability of a given transition is not a fixed value anymore. To represent this new kind of variability, we propose to annotate transitions with a probability profile \([d] \rightarrow [0, 1]\), by a function \(\Pi : S \times S \rightarrow ([d] \rightarrow [0, 1])\) that associates to each transition a product with a probability value. Intuitively, \(\Pi(s, s')(p)\) is the probability of occurrence of the transition \((s, s')\) in the product \(p\). In the figures, a profile is drawn as several arrows, one per probability value, and each arrow has a guard that describes the set of products that yield this probability value.

**Definition 4** A Featured Discrete-Time Markov Chain (FDTMC) is a tuple \((S, s_0, d, \Pi, A, L)\) where:

- \(S\) is a finite, non-empty set of states;
- \(s_0 \in S\) is the initial state;
- \(d\) is a feature diagram;
- \(\Pi : S \times S \rightarrow ([d] \rightarrow [0, 1])\) is the transition probability function, which assigns a probability profile to each transition. Equivalently, for each starting state and each product, it gives a probability distribution on the target states. Any probability profile must satisfy the probability axiom for all the products:

\[
\forall p \in [d], \forall s \in S, \sum_{s' \in S} \Pi(s, s')(p) = 1
\]

- \(A\) is a set of atomic predicates;
- \(L : S \rightarrow 2^A\) labels each state by the set of predicates that holds there.

An FDTMC is a concise representation for a family of DTMCs, that is, one per valid product. The FDTMC modelling a particular variant \(p\) is obtained by projecting the probability profile of each transition onto \(p\). The transition probability function of the resulting DTMC is defined as \(P : S \times S \rightarrow [0, 1] : P(s, s') = \Pi(s, s')(p)\). It generalises DTMC since, when \([d]\) is a singleton, a FDTMC is simply a DTMC.

All usual operations on probabilities can be extended by considering them as a function of the product line. For instance, the product of two (independent) probability profiles \(\Pi\) and \(\Pi'\) is defined as \((\Pi \otimes \Pi')(p) = \Pi(p).\Pi'(p)\).

A path of an FDTMC is a sequence of its states. Given that transition probability depends on features, the execution probability of a path does as well. Let \(s \in S\) be a state of an FDTMC, and \(Paths(s)\) the set of paths starting from \(s\). Due to the Markov property, the probability that a finite path \(p = \rho[0], \rho[1], \ldots, \rho[n] \in Paths(s)\) is executed is given by the product of the probability profiles of its transitions:

\[
\Pi(p) = \Pi(\rho[0], \rho[1]) \otimes \cdots \otimes \Pi(\rho[n-1], \rho[n]).
\]

This constitutes the base of the unique family of probability measure on paths, by the usual cylinder set construction [4].

For instance, the FDTMC of Figure 3 (where self-loops are omitted) represents a mine that could be equipped by a natural ventilation system, by creating well-placed air entrances. This feature is static: it is selected at the construction of the mine. It cannot prevent the apparition of methane, but it will help dissipating it.

![Fig. 3. Methane evolution in a mine with optional ventilation](image)

FDTMC is a fundamental formalism, which is not meant to be used by engineers. Because of the flexibility of probability profiles, FDTMC may be hard to write down. Many states may be needed to entirely represent the system and its environment.
Moreover, many transitions have a variable probability value that depends on the features of the system. When the number of features and transitions grows, it becomes increasingly harder to have a comprehensive view on the stochastic behaviour of every variant. Therefore we introduce ways to decompose the description. A problem that we meet is that (F)DTMC are meant to describe closed systems. Therefore we can decompose a system into (F)DTMCs only if the subsystems are completely independent and do not communicate. Therefore, we introduce Markov Decision Processes (MDP) as an intermediary to decompose our systems.

C. FMDP: a Composable Stochastic Model

FMDP is a model adequate to represent composable, communicating, stochastic, featured systems. They generalise MDP, FTS and FDTMC. In this paper, we use FMDPs only to create the final FDTMC that will be model-checked. However, they are of independent interest, and there are tools that deal with MDPs directly [?, ?].

Definition 5 A Featured Discrete-Time Markov Decision Process (FMDP) is a tuple \( \langle S, s_0, Act, d, P, A, L \rangle \) where:

- \( S \) is a finite, non-empty set of states;
- \( s_0 \in S \) is the initial state;
- \( Act \) is a finite set of actions;
- \( d \) is a feature diagram;
- \( P : S \times Act \times S \to ([d] \to [0, 1]) \) is the transition probability function, which assigns a probability profile to each transition. Equivalently, for each starting state, each action and each product, it gives either a probability distribution on the target states, or always 0 to indicate that the action is not enabled. It must thus satisfy the consistency axiom:

\[
\forall p \in [d], \forall s \in S, \forall a \in Act, \sum_{s' \in S} P(s, a, s')(p) = 1 \text{ or } 0
\]  

(1)

- \( A \) is a set of atomic predicates;
- \( L : S \to 2^P \) labels each state by the set of predicates that hold there.

When actions are always enabled, the FMDP is complete:

Definition 6 A FMDP is complete if

\[
\forall p \in [d], \forall s \in S, \forall a \in Act, \sum_{s' \in S} P(s, a, s')(p) = 1
\]

When \( P \) returns either 0 or 1, a FMDP reduces to a deterministic FTS. When there is a unique action (that we call tick), a complete FMDP reduces to a FDTMC. When \( d \) has a unique product, a FMDP reduces to a MDP.

For instance, the FMDP of Fig. [3] (where self-loops are omitted) represents the evolution of the water level in the mine. The water can raise spontaneously (due to flooding) but it can also decrease spontaneously (due to evaporation and infiltration). Running the pump will favour decrease of the water level. Note that here the guards contains dynamic predicates (written in lowercase) and not static features.

\[\text{Fig. 4. Water evolution in a mine with a pump}\]

While most authors (e.g. [?]) give to MDP a semantics under a probabilistic infinite-memory scheduler that chooses the actions, we will not do so here, since our goal is to eliminate this non-determinism by composing the processes.

D. Composition

To make FDTMC descriptions more manageable, we use classical composition operators:

- the synchronized product, borrowed from PRISM reactive modules: actions can be synchronized in the style of CSP, and each process can read the predicates of other processes in its guardeds, but can only change its own state. We model these guards in the actions. The probabilistic choices of each component are independent.
- the observer product is asymmetric: the second process can immediately observe the predicates of the first one, but not conversely. If we add the converse, we would create a causality loop. The controllers are usually modelled as observers: it is assumed that their reaction is much faster than the environmental evolution, and can be considered instantaneous.

For instance, the FDTMC of the mine pump can be separated into an FDTMC (i.e. without inputs) for methane (Figure 3), an MDP (i.e. without features) for water (Figure 4), and an FTS (i.e. without probabilities) for the controller (Figure 1). Let us assume that, in a system without ventilation (\( \neg V \)), there is no methane and the water level is normal, whereas the controller is ready. At the next discrete point of time, the probability to reach state with methane and high water, is given by \( \frac{1}{32} \). In the FTS, there are two outgoing transitions of ready that are labelled by an event including this outcome: one leads to run and is labelled by event high and feature expression \( W \land \neg A \); the other leads to emergency and is labelled by event methane and feature expression \( A \). Hence, the system will reach state run if it is equipped with feature \( W \) but not \( A \), will reach state emergency if feature \( A \) is enabled, and will stay in state ready otherwise.

When composing featured systems, we must also compose their FD. We use the operator \( d_1 \land d_2 \) defined by \( [d_1 \land d_2] = \{ p \in \text{Mod}(\Sigma_1 \cup \Sigma_2) \text{ such that } p|_{\Sigma_1} \in [d_1], p|_{\Sigma_2} \in [d_2] \} \).

Definition 7 The synchronized product of two FMDPs \( M_1 = (S_1, i_1, Act_1 \times 2^{A_1}, d_1, P_1, A_1, L_1) \) and \( M_2 = (S_2, i_2, Act_2 \times 2^{A_2}, d_2, P_2, A_2, L_2) \) (with \( A_1, A_2 \) disjoint) is the FMDP \( M = M_1 || M_2 \) given by:
• \( S = S_1 \times S_2 \)
• \( s_0 = (i_1, i_2) \)
• \( Act = Act_1 \cup Act_2 \)
• \( d = d_1 \land d_2 \)
• \( A = A_1 \cup A_2 \)
• \( L((s_1, s_2)) = L_1(s_1) \cup L_2(s_2) \)

If \( a \in Act_1 \cap Act_2 \) then:
\[
P((s_1, s_2), a, (s'_1, s'_2))(p) = P_1(s_1, (a, L_2(s_2)), s'_1(p|\Sigma), P_2(s_2, (a, L_1(s_1)), s'_2(p|\Sigma))
\]
If \( a \in Act_1 \setminus Act_2 \) then:
\[
P((s_1, s_2), a, (s'_1, s'_2))(p) = P_1(s_1, (a, L_2(s_2)), s'_1(p|\Sigma), s_2 = s'_2)
\]
If \( a \in Act_2 \setminus Act_1 \) then:
\[
P((s_1, s_2), a, (s'_1, s'_2))(p) = (s_1 = s'_1).P_2(s_2, (a, L_1(s_1)), s'_2(p|\Sigma))
\]

Above, \((s = s')\) is a function that returns 1 when \(s, s'\) are equal, and 0 otherwise.

**Theorem 1** If \( M_1, M_2 \) are complete FMDPs, then their synchronized product \( M_1 || M_2 \) is also a complete FMDP.

This definition can also be used between two FDTMC, by considering them as single-action complete FMDP. Then, they synchronize on their common unique action, and their synchronized product represents their execution, synchronized on time steps, but probabilistically independent. The theorem above shows that the result is again a single-action complete FMDP, i.e. a FDTMC.

The product of FMDPs without shared actions represents their interleaved execution.

In the observer product, the actions that drive the observer will again be the sets of the atomic predicates of the observer, but they are now read immediately (hence the prime in the definition of transitions).

**Definition 8** The observer product of a FMDP \( M = (S_1, i_1, Act, d_1, P_1, A_1, L_1) \) with a FMDP \( M_2 = (S_2, i_2, Act_2, d_2, P_2, A_2, L_2) \) (with \( A_1, A_2 \) disjoint) is \( M = M_1 \otimes M_2 \) given by:

- \( S = S_1 \times S_2 \)
- \( i = (i_1, i_2) \)
- \( Act = Act_1 \cup Act_2 \)
- \( d = d_1 \land d_2 \)
- \( P((s_1, s_2), a, (s'_1, s'_2))(p) = P_1(s_1, (a, L_2(s_2)), s'_1(p|\Sigma), P_2(s_2, (a, L_1(s_1)), s'_2(p|\Sigma)) \)
- \( A = A_1 \cup A_2 \)
- \( L((s_1, s_2)) = L_1(s_1) \cup L_2(s_2) \)

Note that the definition above does not always yield a proper FMDP: the probabilities could sum to a number strictly between 0 and 1.

**Theorem 2** Let \( M_1, M_2 \) be FMDPs with \( Act_2 = 2^{A_1} \) as above. Then, \( M_1 \otimes M_2 \) is consistent for axiom \( \Box \) if \( M_2 \) is complete.

Further, we have:

**Theorem 3** Let \( M_1, M_2 \) be FMDPs with \( Act_2 = 2^{A_1} \) as above. Then, \( M_1 \otimes M_2 \) is a complete FMDP if \( M_1, M_2 \) are complete.

In particular, a FDTMC observed by a complete FMDP is again a FDTMC.

These operators are only a basis for a usable language. We plan to unify fPromela [?] and Probema [?] to obtain a modelling language easy to use (at least for Promela modellers).

The FDTMC modelling the minepump SPL is obtained by computing the synchronized composition of the the FDTMC for methane and the FMDP for water, while the deterministic completed FTS representing the controller is composed as an observer. The theorems above show that the result is indeed a FDTMC. It has three boolean features \((\wedge, \land, \lor)\), 8 products, and 24 states. For comparison, the Linux product line has about 10 000 features.

V. Verification of probabilistic properties

By combining Markov models with variability information, we pave the way for automating the verification of probabilistic properties in SPLs. We also need to provide a language to express those properties; here we use PCTL [?], that allows to express reliability properties. We want to assess the satisfaction of such properties for all the valid products of a given SPL. An example of property to verify is “Which products guarantee that the probability that the pump eventually runs in presence of methane, is less than 0.1?”. PCTL formulae are defined according to the following syntax:

\[
\Phi ::= true \mid a \mid \Phi \land \Phi \mid \neg \Phi \mid P_J(\Psi)
\]

\[
\Psi ::= X\Phi \mid \Phi_1 U^J \Phi_2 \mid \Phi_1 U^J \Phi_2
\]

where \( p \in [0, 1] \), \( J \) is an interval \( \subseteq [0, 1] \), \( t \in \mathbb{N} \cup \{\infty\} \), and \( a \in A \) is an atomic proposition. Formulae generated from \( \Phi \) are referred to as state formulae and since can be evaluated to either true or false in every state of a product. \( P \) is named the probability operator and \( P_J(\Psi) \) specifies that the probability that \( \Psi \) is satisfied from a given state must be within interval \( J \). Formulae generated from \( \Psi \) are named path formulae and their truth is to be evaluated for each execution path. The temporal operator \( X \) is called Next, and \( X\Phi \) means that \( \Phi \) must hold in the next state. \( U \) is called Until. Intuitively, a path satisfies \( \Phi_1 U \Phi_2 \) iff \( \Phi_2 \) holds in some state in the path and \( \Phi_1 \) holds in every preceding state. \( U^t \) is a bounded variant of the until. A path satisfies \( \Phi_1 U^J \Phi_2 \) iff it satisfies \( \Phi_1 U \Phi_2 \) in at most \( t \) steps. We propose three methods to verify FDTMC against PCTL formulae.

A. Enumerative Model Checking

Our first method, called enumerative, checks an FDTMC using standard DTMC verification algorithms. To that aim, it computes the projection of the probability profiles of the FDTMC to every product, and then model checks the resulting DTMCs individually. The computation of the projection requires apply the probability profile of every transition of the FDTMC. When the FDTMC is modelled as a product of
components, one may instead compute the projection of each component, and then build back the same product of the resulting MDPs (without features). Further, if no features appear in a component, like the DTMC of the example, the projection does nothing. The correctness of this method is guaranteed by the fact that the projection operator is distributive over the synchronized and observer product.

**Theorem 4** Let $M, N$ be FMDPs. Then $\forall p \in \{d\}, M_p \parallel N_p = (M \parallel \{d\})_p$ and $M_p \parallel \parallel N_p = (M \parallel \parallel N)_p$.

An undeniable advantage of the enumerative approach is the possibility to reuse the most efficient state-of-the-art tools for single-systems, with their numerous optimisations. In particular, matrix analytic methods [?], [?] cannot be applied on FDTMCs that given in these models, probability distributions cannot be represented as a two-dimensional stochastic matrix; it would be a matrix of profiles. Another advantage is that this method is appropriate for product sampling-based verification, where only a (small) subset of the products are verified. The enumerative approach, however, does not benefit from the commonalities between the products and thus performs more redundant verifications as the number of products increases.

**B. Parametric Model Checking**

In this second method, we propose to convert an FDTMC into a parametric DTMC where the parameters are the features, and reuse existing methods for model checking parametric DTMCs.

In FDTMC, probability distributions are represented by the aforementioned probability profiles. We defined a probability profile as a function that associates a product $p$ with a probability value. We can encode it as a parametric expression where the parameters are the features. For this purpose, we sum up the values that it can return, weighted by a parametric expression corresponding to the product $p$. Thus, $\epsilon(p)$ is given by

$$\epsilon(p) = b_1 \times \cdots \times b_k \times b_{k+1} \times \cdots \times b_{k+j},$$

where $b_i = \{ f_i, 1 - f_{i-1} \}$, $i = k + 1 \ldots k + j$.

This parametric expression is equal to 1 if we assign the value 1 to all the features of $p$ and the value 0 to all the others. For any other product, the expression is equal to 0.

Using the above encoding, we represent the profile as the parametric expression

$$\sum_{p_i \in \{d\}} \epsilon(p_i) \Pi(p_i).$$

When considering a particular product $p$, every term of this sum except one is equal to 0; the remaining term gives the probability. Note that if several products share the same probability value, we can drastically simplify the above sum. In particular, if the probability value depends only on a feature $f$, then we can rewrite Equation 4 as $f \times \alpha_1 + (1 - f) \times \alpha_0$ where $\alpha_1$ (resp. $\alpha_0$) is the probability of $t$ when the feature $f$ is enabled (resp. disabled).

By applying the above encoding, we reduce FDTMC verification to parametric DTMC verification, where the parameters model the presence or absence of boolean features. By doing so, we can benefit from efficient parametric model-checkers like PARAM [?] and our tool [?]. Given a parametric model, these tools return an expression containing parameters that encodes the probability we want to compute. To determine the actual probability that a given product satisfies the property, we replace, in the expression, each feature by 1 if it belongs to the product, or by 0 otherwise. The $\parallel$ operator then yields a parametric inequality, that we need to transform in a boolean formula. In the worst case, this can be done by evaluating the expression once per valid product, at the cost of an exponential time.

An advantage of this algorithm is that it performs only one exploration to compute the expression. However, this expression becomes increasingly complex as the number of (feature-dependent) transitions to explore grows.

**C. Feature-Aware Bounded Search**

Our third method relies on a novel algorithm that explores the FDTMC and keeps track of the variability and probability information obtained during the search. Its first step is to decompose the PCTL formula into its *parse tree*, i.e. a tree where each node is a state subformula of the original formula, such that the root is the formula itself, the leaves are atomic propositions, and child nodes form the direct subformulae of their parent. Similarly to CTL model checking algorithm for FTS [?], we compute the satisfaction sets of all the subformulae bottom-up. The satisfaction set of a state formula $\Phi$ is the set $Sat(\Phi) \subseteq S \times \{d\}$ such that $(s, p) \in S at(\Phi)$ iff $p$ satisfies $\Phi$ starting from $s_j$, noted $p \in [[s \models \Phi]]$. Therefore, $s \models \Phi$ can be regarded as a Boolean formula that encodes the set of products for which $s$ satisfies $\Phi$. The satisfaction rules of PCTL formulae are given as follows.

**Definition 9** Let $M$ be an FDTMC over an FD $d$, $s \in S$ one of its states. Then the satisfiability of a PCTL state formula by $s$ is calculated according to the following rules:

$$s, p \models \top \iff \top$$
$$s, p \models a \iff a \in L(s)$$
$$s, p \models \Phi_1 \land \Phi_2 \iff s, p \models \Phi_1 \land s, p \models \Phi_2$$
$$s, p \models \neg \Phi \iff \neg(s, p \models \Phi)$$
$$s, p \models P_d(\Psi) \iff \Pi(s \models \Psi)(p) \in J$$

where $\Pi(s \models \Psi)$ is a probability profile defined as

$$\Pi(s \models \Psi)(p) = \Pi(\{\rho \in Paths(s) \mid \rho, p \models \Psi\}).$$
The satisfaction of path formulae is defined as follows:

\[ \rho, p \models X \Phi \iff \rho[1], p \models \Phi \]
\[ \rho, p \models \Phi_1 U \Phi_2 \iff \exists j \cdot (\rho[j], p \models \Phi_2 \land \forall 0 \leq k < j \cdot \rho[k], p \models \Phi_1) \]
\[ \rho, p \models \Phi_1 U^t \Phi_2 \iff \exists j \leq t \cdot (\rho[j], p \models \Phi_2 \land \forall 0 \leq k < j \cdot \rho[k], p \models \Phi_1) \]

with \( \rho = \rho[0], \rho[1], \ldots \)

Since we have to use a probability profile for each state for computing the \( P_j \) operator, we also encode satisfaction sets this way, using only 0 or 1 as results: \( 1_{s \models \Phi} \) with

\[ 1_{s \models \Phi}(p) = \begin{cases} 1, & s, p \models \Phi \\ 0, & \text{otherwise.} \end{cases} \]

Apart from the probability operator, the computation of the satisfaction sets follows the same procedure as in [7]. A solution to determine for which products a state \( s \) satisfies \( \Phi \) is to compute the probability profile \( \Pi(s \models \Psi) \). If \( \Psi = X \Phi \), then this profile is computed by:

\[ \Pi(s \models X \Phi) = \sum_{s' \in S} \Pi(s, s') \otimes 1_{s' \models \Phi}. \]

Indeed, for each product \( p \), the probability that \( s \) satisfies \( X \Phi \) is equal to the probability that, in \( p, s \) reaches a state satisfying \( \Phi \) in one transition.

When \( \Psi = \Phi_1 U^t \Phi_2 \), the probability profile is computed by solving the following recursive equations:

\[ \Pi(s \models \Phi_1 U^0 \Phi_2) = 1_{s \models \Phi_2} \] \hspace{1cm} (5)
\[ \Pi(s \models \Phi_1 U^t \Phi_2) = 1_{s \models \Phi_2} \otimes \]
\[ \left( 1 - 1_{s \models \Phi_2} \right) \otimes 1_{s \models \Phi_1} \]
\[ \sum_{s' \in S} \left( \Pi(s, s') \otimes \Pi(s' \models \Phi_1 U^{t-1} \Phi_2) \right) \] \hspace{1cm} (7)
\[ \sum_{s' \in S} \left( \Pi(s, s') \otimes \Pi(s' \models \Phi_1 U^{t-1} \Phi_2) \right) \] \hspace{1cm} (8)

where \( i > 0 \). Indeed, according to the until operator, the probability that in a product \( p, s \) satisfies \( \Phi_1 U \Phi_2 \) in zero step is 1 if \( s \) satisfies \( \Phi_2 \), and 0 otherwise. The probability that \( s \) satisfies \( \Phi_1 U \Phi_2 \) in \( i > 0 \) steps in \( p \) is the probability that it satisfies the formula 0 steps or in \( j \) steps, with \( 0 < j \leq i \). To satisfy the formula in \( j \) steps in \( p, s \) must (1) not satisfy \( \Phi_2 \) in \( p \) (otherwise it satisfies the formula in zero step), (2) satisfy \( \Phi_1 \) in \( p \) (otherwise it cannot satisfy the formula at all), and (3) reach a direct successor \( s' \) that satisfies the formula in at most \( i - 1 \) steps.

The correctness of the above equations directly follow from the semantics of the bounded until, the definition of projection, and the expansion laws of the bounded until operator [7]. In the case of unbounded until, the probability value is classically obtained by removing the superscripts in the equations and solving the resulting system of linear equations [2]. But we can also obtain the solution by iterating these equations for an increasingly high bound \( k \). We obtain lower approximations of the desired probability values, that increase with \( k \) and tend to the exact value.

We iterate these equations by performing a bounded exploration on the FDTMC. Algorithm [1] presents this exploration procedure. Following the principles of FTS model checking [7], the algorithm ensures that a given path is visited only once. The idea of this algorithm is to start from the set of states that satisfy \( \Phi_2 \) for at least one product. Then we perform a backward exploration to discover new paths that satisfy \( \Phi_1 U^k \Phi_2 \). For each state found along these paths, we record a variable \( x_s \), a probability profile that is a lower approximation of \( \Pi(s \models \Phi_1 U \Phi_2) \) and will eventually reach a value above \( \Pi(s \models \Phi_1 U^k \Phi_2) \); therefore it tends to \( \Pi(s \models \Phi_1 U \Phi_2) \) with higher \( k \).

First, for each state \( s \), we record the set of products for which \( s \) satisfies \( \Phi_2 \) (Line 3). As explained above, this can be encoded as the probability profile \( 1_{s \models \Phi_2} \). If there is at least one such product, for every possible predecessor \( s' \) of \( s \) that can reach one of those products, we push the transition from \( s' \) to \( s \) together with a number 1 that indicates that the algorithm analyses this transition as part of a path of length 1 (Lines 4–6). Next, we iteratively analyse the transitions on the stack in order to explore paths of greater lengths. Let \( (s, i, s') \) be the top element of the stack (Line 9). If \( i \) exceeds \( k \), we skip the element since our approximation bound is reached. Then we apply the equations to recalculate \( x_s \) using the new probability values found for \( s \) (Line 11). Note that by doing so, we might go above \( \Pi(s \models \Phi_1 U^k \Phi_2) \) since \( x_s \) might already include paths longer than the current path, giving thus an even better approximation of \( \Pi(s \models \Phi_1 U \Phi_2) \). If one of the values changed, we may have to update the value of every predecessor of \( s \). To that aim, we add a new triplet \((s'', i+1, s')\) on stack where \( s'' \) is a predecessor of \( s' \). The algorithm always terminates since there is a finite of paths bounded by \( k \). The correctness is ensured by Equations (5) and (8).

For the bounded until, the algorithm is similar, but it works in \( k \) phases. It uses two profiles: \( x_s \) contains the previous iteration: \( x_s = \Pi(s \models \Phi_1 U^k \Phi_2) \), and computes a new profile \( x'_s = \Pi(s \models \Phi_1 U^{k+1} \Phi_2) \). The “stack” is first emptied of \( i \)-edges before dealing with \( i + 1 \) edges, which amounts to a breadth-first search.

The advantage of this bounded method is that it checks all the products in one exploration. Unlike the parametric method, our feature-aware search does not require to evaluate a rational expression for each of the products. Instead, for each PCTL state formula, it returns a Boolean formula encoding which products satisfy it.

VI. EXPERIMENTS

In this section, we report the results obtained by evaluating the performance of the three FDTMC verification techniques in terms of verification time. We consider two technical case studies as our benchmarks, which we systematically extend to obtain larger models. All the models are available on [http://info.fundp.ac.be/~pys/fdtmc/](http://info.fundp.ac.be/~pys/fdtmc/).

The first case study is an abstract model of failure-prone systems. In this model, the system has to go through successive degradation states to eventually reach an absorbing failure
state. In every degradation state, however, instead of going to the next degradation state, the system may completely break and reach a second absorbing failure state. In every degradation state, the system may also partially recover and reach the previous degradation state. Apart from the initial state and the two absorbing states, the probability of the transitions leaving each state depends on the presence of specific features. The model is extended by adding new degradation states and features.

The second case study is an abstract model of a service provider system that gives the opportunity to its users to invoke different services. The execution of a service is modelled by a sequence of states. During such executions, the system may fail and suddenly reach a failure (absorbing) state. After any service execution, the system may keep executing more services or may go to an absorbing successful-termination node. Each service requires a specific feature to be started, hence the variability within such a system. Unlike the first model, the behaviour of the features are completely independent in this model. We enlarge the model by gradually adding new services, which also increases the number of features.

For both examples, we checked that the probability that system reaches a failure state is below 0.1. This reachability property can be expressed in PCTL as \( \mathbb{P}_{<0.1}(\cdot \land \text{failure}) \). All benchmarks were run on a Dual Intel(R) Xeon(R) CPU E5530 @2.40GHz with 8Gb of RAM, equipped with GNU Linux Ubuntu server 11.04 64bit. To perform the enumerative verification, all the DTMCs modelling a specific product are first derived from the FDTMC and then verified one-by-one by using the PRISM model checker\(^1\)[?] and then evaluate the resulting expression by using JEP Java library\(^2\). For the bounded approach, we use a prototype we developed from scratch. The latter is available on http://info.fundp.ac.be/~pys/fdtmc/ as well.

The total verification time of the enumerative approach is the sum of the model-checking times to verify each single product by PRISM. We excluded the time to produce individual DTMCs as well as the time taken for creating PRISM input files. As for the parametric approach, the verification time is obtained by summing the parametric verification time and the time spent to evaluate the expression for each single product. Since the bounded approach verifies all products together, its verification time equals to the total time taken by the prototype tool. In all the experiments, we set the bound of the algorithm such that the maximum precision error is always less than \(10^{-3}\).

Figure 5 shows the verification times for the failure-recovery case study. In this case, the number of features \(f\) grows from 2 to 16. It turns out that the bounded approach outperforms the others in almost every case. The verification time of the enumerative approach grows exponentially with the number of features, as expected. We also observed that the parametric approach suffers from the growing complexity of the rational function.

Table reports the time each of the verification techniques takes to verify the models of the service provider case study. The results show that the enumerative approach takes a longer time, while the other two techniques exhibit a similar performance. On contrary to the first case, the parametric approach outperforms the bounded technique.

The results of our experiments suggest that the enumerative approach is increasingly inefficient as the number of features (and hence of products) increases. Still, if a verification task only deals with a small number of products, the enumerative approach is a reasonable choice. The cost of the parametric approach is highly dependent on the complexity of the rational function.

\(^1\)http://www.prismmodelchecker.org/
\(^2\)http://www.singularsys.com/jep/
function that is produced by the parametric model checker. This complexity varies depending on the topology of the verified model. In the first case, where feature-dependent transitions occur sequentially, the verification time grows faster than in the second case, where the features are scattered around the model. Our third algorithm exhibits a good performance in both experiments. In the second case, it remains competitive in spite of the high efficiency of the parametric approach.

Our theory is that the parametric approach performs better in models where feature-dependent transitions do often not occur in sequence, that is, when there is a limited number of feature interactions. On the contrary, if many of these sequences occur in the model then the size of the function returned by the parametric algorithm will sharply grow. In such cases, our feature-aware bounded search should instead be used.

VII. RELATED WORK

Analyzing non-functional properties of software systems has received an increasing interest during the last years. However, there are only a few work discussing this issue for SPLs [7], [8]. The recent research carried out addressed the problem at feature models or available source codes. Our approach instead focuses on the use of behavioural models. Feature models are suitable to specify and organize variability of SPLs, but they are not enough expressive and precise for quality analysis. On the other hand, analyzing the source code of an SPL is only possible after the implementation, and is not applicable at the early stage of development [9]. Ghezzi and Molzam Sharifloo [10], as well as Nunes [11] propose to use parametric model checking to check PCTL formulae on all the variants of an SPL. Yet, their modelling formalisms do not include an explicit notion of features and they do not propose alternative verification algorithms.

Given the increasing popularity of product lines in various areas including critical systems, SPL verification methods are actively studied, although most of them do not consider non-functional properties. The work surrounding FTS is the most related to ours [12], [13], [14], [15]. Several alternative to FTS exist. Larsen et al. [16] show that I/O automata are convenient for modelling product lines as open systems. Asirelli et al. [17] equip modal transition systems with a logic able to express constraints on variable behaviour. Gruler et al. [18] extend the process algebra CCS with variability operators, which allow to model alternative choices between two processes. Li et al. [19] model both the base systems and optional features as finite state machines that connect to each other. Apel et al. [20] specify features as separate units that can be composed; each feature defines safety properties that are subsequently verified using single-system model-checkers.

Outside the context of software product lines, we find the notions of Interval Markov chain [21] and Constraint Markov chain [22]. The former is a generalization of Markov chains where execution probability of transitions are given by probability intervals rather than constant values. Interval Markov chains thus concisely models a potentially infinite set of Markov chains. Then, it is possible to determine whether or not all those Markov chains satisfy a given PCTL property. Constraint Markov chains are a generalization of Interval Markov chains, where probability distributions are defined by parameters. The values that a parameter can take are determined by linear constraints between the parameters (e.g. $\alpha + \beta = 0.7$). This kind of parameterization is more general than that of FDTMC with Boolean features as presented in this paper. By extending FDTMC with numeric features [23], we obtain a formalism as expressive as Constraint Markov chains. Moreover, to the best of our knowledge there exists no algorithm for checking Constraint Markov chains against properties expressed in PCTL.

VIII. CONCLUSION

In this paper, we tackled the verification of non-functional requirements in software product lines. We extended the probability theory with notions of variability, and applied this extension to define DTMCs enriched with variability operators, aka FDTMCs. We showed that when the inner behavior of a software is not stochastic, we can model a stochastic SPL as the composition of an FTS modelling the system and a set of DTMCs modelling the stochasticity of the environment. We discussed and experimented three methods to model check FDTMCs, including a novel algorithm that directly exploits variability information contained in the model. Benchmarks suggest that enumerative verifications of the whole product line take a long time w.r.t. the other two methods.

In the future, we plan to develop a complete tool chain that will provide assistance in both the specification and verification of stochastic product lines. We will support specification of FDTMCs either via a graphical user interface or through the use of high-level textual languages. Moreover, our tool will make use of the compositional modeling approach presented in this paper. To achieve the verification, we plan to implement and optimize our feature-aware bounded algorithm within the tool and will link parametric model checkers to the tool.

A natural direction for our future work is the extension of the proposed approach to other kinds of Markov models, for example continuous time Markov chains, and Markov reward models, which are widely used to reason about other kinds of non-functional requirements such as performance and energy-consumption. The enrichment of Markov processes as presented in Section III provides a formal framework from which these new models can be derived. Moreover, our compositional

| Features | Enumerative | Parametric | Bounded |
|----------|-------------|------------|---------|
| 2        | 4,207       | 0.237      | 0.111   |
| 4        | 6,932       | 0.267      | 0.306   |
| 6        | 14,57       | 0.336      | 0.314   |
| 8        | 29,224      | 0.408      | 0.519   |
| 10       | 57,215      | 0.53       | 0.676   |
| 12       | 119,544     | 0.572      | 1.636   |
| 14       | 227,91      | 0.99       | 1.931   |
| 16       | 466,185     | 1.126      | 2.966   |
modelling method can be applied to these models as well. Still, the major challenge remains the design of efficient verification algorithms.

Also, we want to deepen the current approach to answer optimizations problems. Nowadays, optimizing non-functional requirements is an important challenge in a wide range of areas. For instance, we could consider the problem of finding the most reliable and economic products among a product line. Since an FDTMC can be regarded as a parametric model with Boolean variables (namely the features) only, addressing this problem leads us in the particular domain of Boolean linear optimization. As an alternative to model checking and algebraic methods, we will also investigate simulation-based methods for computing probabilities and rewards [?].