Locally resonant phononic crystals with multilayers cylindrical inclusions

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Abstract: Using a Finite Difference Time Doma in (FDTD) method, we investigate the formation of band gaps in the audible frequency range with the help of locally resonant two-dimensional (2D) phononic crystals. It is shown that in a phononic crystal in which the cylindrical inclusions are consisting of a hard steel core coated with a soft rubber and the matrix is a solid such as epoxy, the gap associated with the local resonance may become large by increasing the filling factor. We also consider the case of inclusions consisting of a coaxial cylindrical multi-layers consisting of several alternate shells of soft polymer and steel. Then, the former gap is divided into several smaller gaps separated by narrow transmission bands.

1. Introduction
The locally resonant sonic materials are a class of phononic crystals which have been shown to be of great interest in reflecting acoustic waves in the audible frequency range ([1-5]). First introduced by Ping Sheng and co-workers [1], these materials lead to transmission dips at frequencies almost two orders of magnitude lower than the usual Bragg gap. Typically, in such a crystal the inclusions are consisting of a hard core (such as steel) coated with a soft rubber and the matrix is a solid (such as epoxy). In section 2, we investigate theoretically, using a finite difference time domain (FDTD) method, the behavior of the low frequency gap as a function of the filling factor of the cylindrical inclusions. In section 3, we study the effect of the multilayer coating of the inclusions on the transmission spectrum.

2. Large absolute band gap in Locally Resonant Phononic Crystals (L.R.P.C.)
We consider the case of a phononic crystal consisting of an unit cell composed of an infinitely long cylinder made of a steel core coated with a thin shell of a soft rubber and embedded in an epoxy matrix (see inset of fig 1). The physical parameters of the materials are $\rho = 7780 \text{ kg/m}^3$, $v_l = 5825 \text{ m/s}$, $v_t = 3227 \text{ m/s}$ for steel; $\rho = 1180 \text{ kg/m}^3$, $v_l = 2535 \text{ m/s}$, $v_t = 1157 \text{ m/s}$ for epoxy and $\rho = 1300 \text{ kg/m}^3$, $v_l = 24 \text{ m/s}$, $v_t = 6 \text{ m/s}$ for silicone rubber. The elastic constants, and thus the velocities of sound, are very low in the coating silicon rubber. The sonic crystal is consisting of six rows of elementary units arranged on a square lattice, with lattice parameter $a = 20 \text{ mm}$. In figure 1, we report the transmission spectrum of such a phononic crystal for three different filling factors: $f = 23\%$, $f = 55\%$ and $f = 72\%$.
Each filling factor $f$ is calculated considering the following ratio:

$$f = \frac{\pi r_e^2}{a^2}$$

where $r_e$ is the outer radius of the cylinder. For all the three studied filling factors, the thickness of the polymer shell has been kept equal to $e = 0.4 \text{ mm}$. For a low filling factor ($f = 23\%$), we observe a sharp dip in the transmission coefficient at very low frequency (1.257kHz) almost two orders of magnitude below the well known Bragg frequency gap. As already described previously in the literature [2, 3], the origin of the dip can be understood as an oscillation of the steel core with respect to the matrix whereas the rubber acts as a spring linking the steel cylinder to the epoxy matrix. As seen
in figure 1, this dip has a Fano-like resonance shape but displays a small width in the transmission spectrum. Increasing the filling factor from 23% to 72%, we observe a decrease in the amplitude of the transmission signal in the frequency range [1 kHz, 4 kHz]. The increase of the filling factor has thus created a large forbidden frequency gap in the audible frequency range.

Figure 1. Transmission along the $\Gamma X$ direction through the phononic crystal where the steel core is coated with one shell of polymer and embedded in epoxy matrix for different filling fraction. In the inset, the black (grey) color corresponds to the polymer (steel) material.

To better support the above statement, we present in figure 2 the transmission spectrum and the corresponding dispersion curves along the high symmetry directions of the reduced Brillouin zone for a filling factor $f = 72\%$, near to close packing. One can see that in the frequency range [1.10 kHz, 3.50 kHz], the dispersion curves display a large absolute forbidden band in the audible frequency range.

Figure 2. Dispersion curve and transmission coefficient along $\Gamma X$ and $\Gamma M$ direction for a phononic crystal ($f = 72\%$) in which the steel core is coated with a polymer and embedded in an epoxy matrix.

3. Multi-coaxial cylindrical inclusions in L.R.P.C.

In this section, we consider the case of a phononic crystal in which the inclusions are made of a steel core cylinder covered with five layers consisting alternatively of silicone rubber and steel (see inset of figure 3). The thickness of each shell is kept equal to 0.4 mm. The filling factor of the whole cylinder is chosen in such a way as to have $f = 72\%$ in order to ensure the lowest value of the transmission as shown in section 2. The transmission spectrum through the six rods of the phononic crystal is presented in figure 3 along the $\Gamma X$ direction of the Brillouin zone. We can now observe in the frequency range [1 kHz - 6 kHz] three forbidden frequency gaps instead of one in the previous case (fig 1). A similar calculation when the core of the inclusion is covered with three layers shows the
formation of two gaps, almost in the same frequency range. One can conclude that the number of these gaps evolves in relation with the number of coated shells.

**Figure 3.** Transmission spectrum for a phononic crystal made of cylinders consisting of a steel core covered with five alternate layers of silicone rubber and steel. The filling factor is $f = 72\%$.

One direct consequence associated with the creation of new gaps in the transmission spectrum is the formation of narrow pass bands enclosed between two adjacent gaps. The number of sharp transmission bands is linked to the number of coating shells. In figure 3, we can see two narrow pass bands at frequencies (2.671 kHz and 4.871 kHz) inside a large forbidden gap from 1 kHz to 6 kHz. In figure 4, we present a FDTD calculation of the $U_y$ component of the displacement field (where $y$ is along the propagation direction) at the monochromatic frequencies 2.671 kHz and 4.871 kHz. At the lowest frequency, one can see from the schematic view that the inner core moves in opposite phase with respect to the two other steel shells. At 4.871 kHz, the inner core does not move whereas the two others shells vibrate with opposite phase. This means that these selective transmissions occur at two specific locally resonant modes of the multi-coaxial cylinder.

**Figure 4.** (color online) Map of the displacement component $U_y$ (parallel to the propagation direction) and schematic view in the $(X, Y)$ plane for monochromatic frequencies 2.671 KHz and 4.871 kHz.
4. Conclusion
In this paper we investigated using FDTD simulations the transmission through Locally Resonant Phononic Crystals. Such materials exhibit small gaps in the audible frequency range. Each unit cell of the crystal is made of a hard core covered by a soft polymer shell embedded in an epoxy matrix. We have shown that the increase of the filling factor of the inclusion leads to enlarge the relevant sonic gap. Moreover, we studied the case of multilayer cylinders consisting of an alternate succession of polymer and steel shells covering a steel core. We highlighted the presence of narrow pass bands in the forbidden low frequency gap which have been attributed to specific resonance modes of the multilayer cylinders. Such device could be used as selective filters in acoustical circuits.

References
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