THE QCD PERSPECTIVE ON LIFETIMES OF HEAVY-FLAVOUR HADRONS

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Abstract

Over the last few years a theoretical treatment for the weak decays of heavy-flavour hadrons has been developed that is genuinely based on QCD. Its methodology is described as it applies to total lifetimes, and the underlying theoretical issues are discussed. Theoretical expectations are compared with present data. One discrepancy emerges: the beauty baryon lifetime appears to be significantly shorter than predicted. The ramifications of those findings are analyzed in detail, and future refinements are described.

1 Introduction

1.1 Goals and Obstacles

As explained in detail elsewhere in this review, a precise measurement of the lifetimes of the various weakly decaying charm and beauty hadrons possesses great experimental value per se as well as in searches for $B^0 - \bar{B}^0$ and $D^0 - \bar{D}^0$ oscillations. Yet

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"Preliminary version; final version will form part of a Physics Report."
there exists also a strong theoretical interest in determining and interpreting those lifetimes; I want to sketch that first in rather qualitative terms and give specifics later.

Weak decays of hadrons depend on fundamental parameters of the Standard Model, in particular on the KM parameters and quark masses. It is eminently important to reliably determine their values from data. Alas – this is easier said than done theoretically (and experimentally)! For in such an endeavour we have to face the ”Dichotomy of the Two Worlds”. On the one hand there is the ”Theorists’ World” where quarks and gluons are the relevant strongly interacting entities; it is in this short-distance or Femto World where theorists like to formulate their fundamental theories. On the other hand there is the ”Real World” where hadrons constitute the relevant degrees of freedom; it is in that world where everyone (including theorists) lives and measurements are performed. To formulate predictions from the Theorists’ World in the language of the Real World and to translate findings from the Real World back into the idioms of the Theorists’ World represents the theoretical challenge one faces.

One quantitative measure for the difference between the two worlds is provided by the lifetimes of the weakly decaying hadrons carrying the same flavour. On the quark level there is obviously only a single lifetime for a given flavour. Yet in the ‘Real World’ hadrons carrying the same flavour quantum number possess different (and even vastly different) lifetimes; e.g., \( \tau(K^+)/\tau(K_S) \approx 140 \), \( \tau(D^+)/\tau(D^0) \approx 2.5 \) and \( 0.9 \leq \tau(B^-)/\tau(B_d) \leq 1.2 \). On the other hand deviations of the lifetime ratios from unity evidently decrease for an increasing heavy-flavour mass. This is as expected in a simple ‘two-component’ picture: in the limit of \( m_Q \) – the mass of the heavy-flavour quark – going to infinity the ‘Spectator Ansatz’ should hold where the lifetimes of all hadrons \( H_Q \) containing \( Q \) coincide; for finite values of \( m_Q \) there are pre-asymptotic corrections. Two lines of reasoning support this qualitative picture:

(a) The decay width of a quark \( Q \) increases very quickly with its mass \( m_Q \):

\[
\Gamma_Q \propto G_F^2 m_Q^5
\]

for \( m_Q \ll M_W \) changing into \( \Gamma_Q \propto \alpha m_Q^3/M_W^2 \) for \( m_Q \gg M_W \). Therefore for \( m_Q \) sufficiently large, its decay width is bound to exceed \( \Lambda_{QCD} \), i.e. the quark \( Q \) decays before it can hadronize. Then there is obviously a universal lifetime for such a heavy flavour and the spectator ansatz applies trivially.

(b) Analysing all non-spectator reactions explicitly, one finds their widths to increase with a smaller power of \( m_Q \) than the spectator process; thus one arrives at the spectator ansatz as the asymptotic case.

The most relevant phenomenological question then is how quickly the limit of universal lifetimes is approached.

\[\text{2The asymptotic scaling behaviour } \Gamma_Q \propto m_Q^3 \text{ (rather than } \Gamma_Q \propto m_Q) \text{ reflects the coupling of the longitudinal } W \text{ boson – the original Higgs field – to the top quark.}\]

\[\text{3For top decays this happens for } m_t \geq 130 \text{ GeV.}\]
1.2 Phenomenology: Legends with Truths

Some mechanisms had been identified very early on that generate differences in the lifetimes of hadrons \( H_Q \) with the same heavy flavour \( Q \): "Weak Annihilation" (=WA) of \( Q \) with the light valence antiquark for mesons or "W Scattering" (=WS) with the valence diquark system for baryons \(^4\). Such an analysis had first been undertaken for charm decays.

Since WA contributes to Cabibbo allowed decays of \( D^0 \), but not of \( D^+ \) mesons (in the valence quark description), it creates a difference in \( \tau(D^0) \) vs. \( \tau(D^+) \). However the WA rate is doubly suppressed relative to the spectator rate, namely by the helicity factor \( (m_q/m_c)^2 \) with \( m_q \) denoting the largest mass in the final state and by the ‘wavefunction overlap’ factor \( (f_D/m_c)^2 \) reflecting the practically zero range of the low-energy weak interactions:

\[
\Gamma_{W^{-}X}(D^0) \propto G_F^2 f_D^2 m_q^2 m_c. \tag{2}
\]

Therefore it had originally been suggested that already charm hadrons should possess approximately equal lifetimes. It then came as quite a surprise when observations showed it to be otherwise – in particular since the first data suggested a considerably larger value for \( \tau(D^+)/\tau(D^0) \) than measured today. This near-shock caused a re-appraisal of the theoretical situation; its results at that time can be summarized in four main points:

(i) One source for a lifetime difference had too quickly been discarded as insignificant. Cabibbo-allowed nonleptonic decays of \( D^+ \) – but not of \( D^0 \) – mesons produce two antiquarks in the final state that carry the same flavour:

\[ D^+ = [\bar{c}d] \rightarrow (\bar{s}u)d \]

Thus one has to allow for the interference between different quark diagrams in \( D^+ \), yet not in \( D^0 \) decays; the \( \bar{d} \) valence antiquark in \( D^+ \) mesons thus ceases to play the role of an uninvolved bystander and a difference in the \( D^+ \) vs. \( D^0 \) lifetimes will arise. While this interference had been included in descriptions of the \( D \rightarrow K\pi \) two-body modes it was ignored for total widths. For it was thought (often without stating it explicitly) that the required coherence would not be maintained between the two amplitudes when applied to inclusive transitions. This assumption was challenged in ref.\(^2\) where it was argued that even the two inclusive amplitudes remain sufficiently coherent. The interference turns out to be destructive, i.e. it prolongs \( \tau(D^+) \) over \( \tau(D^0) \), but only once the QCD radiative corrections have been included. This effect

\(^4\)A distinction is often made between W exchange in the s and in the t channel with the former case referred to as ‘weak annihilation’ and the latter as ‘W exchange’. This classification is however artificial since the two operators mix already under one-loop renormalization in QCD, as discussed later on. Both cases will summarily be referred to as WA.
is usually referred to as ‘Pauli Interference’ (=PI) although such a name would be misleading if it is interpreted as suggesting that the interference is automatically destructive.

(ii) It was argued \[3\] that the helicity suppression of the WA contribution to \(D\) decays can be vitiated. The diagram in Fig.1 contains gluon bremsstrahlung off the initial antiquark line in the \(W\)-exchange reaction; evaluating this particular diagram explicitly one finds:

\[
\Gamma^{(1)}_{W-X}(D^0) \propto (\alpha_s/\pi)G_F^2(f_D/\langle E_{\bar{q}} \rangle)^2 m_c^5
\]

with \(\langle E_{\bar{q}} \rangle\) denoting the average energy of the initial antiquark \(\bar{q}\). Using a non-relativistic wavefunction for the decaying meson one has \(\langle E_{\bar{q}} \rangle \simeq m_q\). This contribution, although of higher order in \(\alpha_s\), would dominate over the lowest order term \(\Gamma^{(0)}_{W-X}\) since helicity suppression has apparently been vitiated and the decay constant \(f_D\) is now calibrated by \(\langle E_{\bar{q}} \rangle\) with \(f_D/\langle E_{\bar{q}} \rangle \sim \mathcal{O}(1)\) rather than \(f_D/m_c \ll 1\). The spectator picture would still apply at asymptotic quark masses, since \(\Gamma^{(1)}_{W-X}/\Gamma_c \propto (f_D/\langle E_{\bar{q}} \rangle)^2 \to 0\) as \(m_c \to \infty\) due to \(f_D \propto 1/\sqrt{m_c}\). Yet if eq.(3) were indeed to hold, it would have a dramatic impact on the theoretical description of weak heavy-flavour decays: the impact of this particular pre-asymptotic correction, namely WA, would be enhanced considerably and actually be quite significant even in beauty decays. Alternatively it had been suggested \[4\] that the wavefunction of the \(D\) meson contains a \(c\bar{q}g\) component where the \(c\bar{q}\) pair forms a spin-one configuration with the gluon \(g\) balancing the spin of the \(c\bar{q}\) pair.

Both effects, namely PI and WA, work in the same direction, i.e. both enhance \(\tau(D^+)\) over \(\tau(D^0)\).

(iii) A rich structure emerges in the decays of charm baryons \[5, 6, 7\]:

- On the one hand WS contributes to the Cabibbo allowed \(\Lambda_c\) and \(\Xi^0_c\) decays; one should also keep in mind that WS is not helicity suppressed in baryon decays already to lowest order in the strong coupling. It is still reduced in size by the corresponding wavefunction overlap; yet that is at least partially off-set by WS being described by two-body phasespace versus the three-body phase space of the spectator transition; this relative enhancement in phase space can be estimated to be roughly of order \(16\pi^2\).

- PI affects the \(\Lambda_c\), \(\Xi^0_c\), and \(\Omega_c\) widths in various ways, generating destructive as well as constructive contributions! This also strongly suggests that it is very hard to make reliable numerical predictions for these baryonic lifetimes; yet the overall qualitative pattern has been predicted:

\[
\tau(\Xi^0_c) < \tau(\Lambda_c) < \tau(\Xi^+_{c})
\]  \(4a\)

\[5\]The \(1/\langle E_{\bar{q}} \rangle\) term in the amplitude derives from the propagator in the diagram.
together with
\[ \tau(\Lambda_c) < \tau(D^0) < \tau(D^+) \, . \] (4b)

(iv) Pre-asymptotic corrections might be sizeable in the lifetime ratios of beauty hadrons.

Reviews of these phenomenological descriptions can be found in \[8, 9\].

It turned out, as discussed in more detail later on, that some of the phenomenological descriptions anticipated the correct results: it is PI that provides the main engine behind the \(D^+ - D^0\) lifetime ratio; \(\Lambda_c\) is considerably shorter-lived than \(D^0\); the observed charm baryon lifetimes do obey the hierarchy stated in eq.(4).

Nevertheless the phenomenological treatments had significant shortcomings, both of a theoretical and of a phenomenological nature: (i) No agreement had emerged in the literature about how corrections in particular due to WA and WS scale with the heavy quark mass \(m_Q\). (ii) Accordingly no clear predictions could be made on the lifetime ratios among beauty hadrons, namely whether \(\tau(B^+)\) and \(\tau(B_d)\) differ by a few to several percent only, or by 20 - 30 \%, or by even more! (iii) No unequivocal prediction on \(\tau(D_s)\) or \(\tau(B_s)\) had appeared. (iv) In the absence of a systematic treatment it is easy to overlook relevant contributions, and that is actually what happened; or the absence of certain corrections had to be postulated in an ad-hoc fashion. Thus there existed an intellectual as well as practical need for a description based on a systematic theoretical framework rather than a set of phenomenological prescriptions.

1.3 From Phenomenology to Theory

In the last few years we have succeeded in showing that the non-perturbative corrections to heavy-flavour decays can be expressed through a \textit{systematic} expansion in \textit{inverse} powers of \(m_Q\). A simple analogy with nuclear \(\beta\) decay can illustrate this point. There are two effects distinguishing the decays of neutrons bound in a nucleus from the decay of free neutrons:

(a) nuclear binding effects;

(b) the impact of Pauli statistics correlating the electrons surrounding the nucleus with those emerging from \(\beta\) decay.

The typical energies of the bound electrons – \(\epsilon_{el}\) – are certainly small compared to \(E_{\text{release}}\), the energy released in the decay; let us assume – although this is not true in reality – that also the nuclear binding energies \(\epsilon_{\text{nuc}}\) were small compared to \(E_{\text{release}}\). In that case nuclear \(\beta\) decays would proceed, to a good approximation, like the decays of \textit{free} neutrons; corrections to this simple ‘spectator’ picture could be
computed via an expansion in powers of $\epsilon_{\text{nucl}} / E_{\text{release}}$ and $\epsilon_{\text{el}} / E_{\text{release}}$. In practice, however, the corrections for nuclear $\beta$ decay are incorporated by explicitly using the wave functions of the bound nucleons and electrons (obtained with the help of some fairly massive computer codes).

There arise analogous corrections to the decay rate for a quark $Q$ inside a hadron $H_Q$:

(a) interactions of the decaying quark with other partons in the hadron; this includes WA of $Q$ with the light valence antiquark for mesons or WS with the valence diquark system for baryons; they correspond to K capture of bound electrons by a heavy nucleus in the preceding example.

(b) PI effects of the decay products with other partons in the hadron; e.g.: $b\bar{u} \rightarrow c\bar{d}u\bar{u}$ or $c\bar{d} \rightarrow u\bar{d}s\bar{d}$. They prolong the lifetimes of $D^+$ and $B^-$ mesons.

The difference to the example of nuclear $\beta$ decay is quite obvious: even in the limit $m_Q \rightarrow \infty$ a non-relativistic bound-state treatment is inapplicable since the dynamical degrees of freedom of the heavy-flavour hadron $H_Q$ cannot fully be described by a hadronic wavefunction. The most reliable approach is then to evaluate weak decay rates of heavy-flavour hadrons through an expansion in powers of $\mu_{\text{had}} / m_Q$ where $\mu_{\text{had}}$ represents a hadronic scale $\leq 1$ GeV. The first few terms in this series should yield a good approximation for beauty decays; the situation for charm decays is a priori unclear (and at present remains so a posteriori as well); this will be discussed in detail later on. The vice of hadronization is then transformed into (almost) a virtue: the weak decays of heavy-flavour hadrons constitute an intriguing and novel laboratory for studying strong dynamics through their interplay with the weak forces – and this is the secondary motivation for studying them. To be more specific: the heavy-flavour mass $m_Q$ provides an expansion parameter that allows to deal with the non-perturbative dynamics of QCD in a novel way. The formalism to be employed combines the $1 / m_Q$ expansion with other elements derived from QCD proper without having to invoke a ‘deus ex machina’ – in contrast to phenomenological descriptions.

There is one concept underlying, in one form or another, all efforts to deal with hadronization, namely the notion of quark-hadron duality (henceforth referred to as duality for short). In its broadest formulation it can be stated as follows: sufficiently inclusive transition rates between hadronic systems can be calculated in terms of quarks and gluons. While the general validity of this concept has not been established in a rigorous fashion, new light has been shed on its nature, validity and applicability by $1 / m_Q$ expansions. Lifetimes obviously represent the most inclusive quantity where duality should be applicable. Nonleptonic transitions are certainly more complex than semileptonic ones; yet I will argue later that while there probably exists a quantitative difference in the degree to which duality holds in semileptonic and in

\footnote{As already mentioned, top quarks decay weakly before they can hadronize.}
nonleptonic decays, there is no qualitative one. Detailed measurements of lifetimes are then theoretically important not only to translate data on the semileptonic branching ratio into a determination of the semileptonic width, but also in their own right.

Dedicated and comprehensive studies of both charm and beauty decays are called for. The KM parameters relevant for charm decays – $V_{cs}$ and $V_{cd}$ – are well known through unitarity constraints of the 3x3 KM matrix, in contrast to the situation in beauty decays, and I consider it unlikely that charm decay studies can improve on that. On the other hand those can be used to calibrate our theoretical tools before applying them to beauty decays.

Before concluding this general introduction I want to point out a less straightforward aspect of accurate lifetime measurements: decay rate evolutions in proper time for neutral mesons will not follow a single exponential function when particle-antiparticle oscillations occur. For there exist two distinct mass eigenstates with $\Delta m \equiv m_1 - m_2 \neq 0 \neq \Delta \Gamma \equiv \Gamma_1 - \Gamma_2$. The quantity $\Delta m$ generates a deviation of the form $e^{-\Gamma t} \cos \Delta mt$ or $e^{-\Gamma t} \sin \Delta mt$; $\Delta \Gamma \neq 0$ leads to the emergence of a second exponential. The general expression reads as follows:

$$d\Gamma(B[D] \to f)/dt \propto e^{-\Gamma t} \cdot G(t)$$

$$G(t) = a + be^{-\Delta \Gamma t} + ce^{-1/2\Delta \Gamma t} \cos \Delta mt + de^{-1/2\Delta \Gamma t} \sin \Delta mt$$

where

$$a = |A(f)|^2 \left[ \frac{1}{2} (1 + q \bar{\rho}(f))^2 + Re[q \bar{\rho}(f)] \right]$$

$$b = |A(f)|^2 \left[ \frac{1}{2} \left( 1 + \frac{q}{p} \bar{\rho}(f) \right)^2 - Re[q \bar{\rho}(f)] \right]$$

$$c = |A(f)|^2 \{ 1 - \frac{q}{p} \bar{\rho}(f) \} , \quad d = 2 |A(f)|^2 Im[q \bar{\rho}(f)] , \quad \bar{\rho}(f) = \frac{\bar{A}(f)}{A(f)}$$

with $\bar{A}(f)$ and $A(f)$ denoting the amplitude for $\bar{B}[\bar{D}] \to f$ and $B[D] \to f$, respectively.

### 2 Preview of the Predictions on the Lifetime Ratios for Beauty and Charm Hadrons

In this section I summarize the numerical results and sketch the main elements of the underlying theoretical treatment in a way that can satisfy the casual reader. A more in-depth discussion of the theoretical concepts and tools will be presented in subsequent sections.
Expanding the width for the decay of a heavy-flavour hadron \( H_Q \) containing \( Q \) into an inclusive final state \( f \) through order \( 1/m_Q^3 \) one obtains \([10, 11, 12]\)

\[
\Gamma(H_Q \to f) = \frac{G_F^2 m_Q^5}{192\pi^3} |KM|^2 \left[ c^f_6 \langle H_Q|\bar{Q}Q|H_Q \rangle_{\text{norm}} + c^f_3 \frac{\langle H_Q|\bar{Q}i\sigma \cdot GQ|H_Q \rangle_{\text{norm}}}{m_Q^2} + \sum_i c^f_{6,i} \frac{\langle H_Q|\bar{Q}_i \Gamma_i q)(q \Gamma_i Q)|H_Q \rangle_{\text{norm}}}{m_Q^3} + \mathcal{O}(1/m_Q^4) \right],
\]

(6)

where the dimensionless coefficients \( c^f_i \) depend on the parton level characteristics of \( f \) (such as the ratios of the final-state quark masses to \( m_Q \)); \( KM \) denotes the appropriate combination of KM parameters, and \( \sigma \cdot G = \sigma_{\mu\nu}G_{\mu\nu} \) with \( G_{\mu\nu} \) being the gluonic field strength tensor. The last term in eq.(6) implies also the summation over the four-fermion operators with different light flavours \( q \). The expectation values of the local operators appearing on the right-hand side of eq.(6) contain the relativistic normalization of the state \( |H_Q \rangle \):

\[
\langle H_Q|O_i|H_Q \rangle_{\text{norm}} \equiv \langle H_Q|O_i|H_Q \rangle/2M_{H_Q}.
\]

(7)

It is through the quantities \( \langle H_Q|O_i|H_Q \rangle \) that the dependence on the decaying hadron \( H_Q \), and on non-perturbative forces in general, enters, instead of through wavefunctions as in nuclear \( \beta \) decay. Since these are matrix elements for on-shell hadrons \( H_Q \), one sees that \( \Gamma(H_Q \to f) \) is indeed expanded into a power series in \( \mu_{\text{had}}/m_Q < 1 \). For \( m_Q \to \infty \) the contribution from the lowest dimensional operator obviously dominates; here it is the dimension three operator \( \bar{Q}Q \). A heavy quark expansion yields:

\[
\langle H_Q|\bar{Q}Q|H_Q \rangle_{\text{norm}} = 1 + \mathcal{O}(1/m_Q^2),
\]

(8)

i.e. \( \langle H_Q|\bar{Q}Q|H_Q \rangle_{\text{norm}} = 1 \) for \( m_Q \to \infty \), reflecting the unit of heavy-flavour common to all hadrons \( H_Q \).

Eqs.(6,8) show that the leading nonperturbative corrections are of order \( 1/m_Q^2 \) rather than \( 1/m_Q \); therefore they can be expected to be rather small in beauty decays. The expectation values appearing in the \( 1/m_Q^2 \) and \( 1/m_Q^3 \) contributions can be related to other observables and their size thus be extracted in a reliable manner. Applying the master formula eq.(6) to lifetimes one arrives at the following general results:

- The leading contribution to the total decay width is given by the first term on the right-hand-side of eq.(8) that is common to all hadrons of a given heavy-flavour quantum number. For \( m_Q \to \infty \) one has thus derived – from QCD proper – the spectator picture attributing equal lifetimes to all hadrons of a given heavy-flavour! This is not a surprising result – after all without hadronization there is of course only a unique lifetime –, but it is gratifying nevertheless.
• Lifetime differences first arise at order $1/m_Q^2$ and are controlled by the expectation values of two dimension five operators, to be discussed later. These terms, which had been overlooked in the original phenomenological analyses, generate a lifetime difference between heavy-flavour baryons on one side and mesons on the other. Yet apart from small isospin or $SU(3)_{fl}$ breaking they shift the meson widths by the same amount and thus do not lead to differences among the meson lifetimes.

• Those emerge at order $1/m_Q^3$ and are expressed through the expectation values of four-fermion operators which are proportional to $f_M^2$ with $f_M$ denoting the decay constant for the meson $M$. Contributions from what is referred to as WA and PI in the original phenomenological descriptions (see the discussion in Sect.1) are systematically and consistently included. Further contributions to the baryon-meson lifetime difference also arise at this level due to WS.

• Since the transitions $b \rightarrow c \ell \nu$ or $c \rightarrow d \ell \nu$ are described by an isosinglet operator one can invoke the isospin invariance of the strong interactions to deduce for the semileptonic widths

\[
\Gamma_{SL}(B^-) = \Gamma_{SL}(B_d) \quad (9a)
\]
\[
\Gamma_{SL}(D^+) = \Gamma_{SL}(D^0) \quad (9b)
\]

and therefore

\[
\frac{\tau(B^-)}{\tau(B_d)} = \frac{BR_{SL}(B^-)}{BR_{SL}(B_d)} \quad (10a)
\]
\[
\frac{\tau(D^+)}{\tau(D^0)} = \frac{BR_{SL}(D^+)}{BR_{SL}(D^0)} \quad (10b)
\]

up to small corrections due to the KM [Cabibbo] suppressed transition $b \rightarrow u \ell \nu$ $[c \rightarrow d \ell \nu]$ which changes isospin by half a unit. The spectator picture goes well beyond eqs.(9): it assigns the same semileptonic width to all hadrons of a given heavy flavour. Yet such a property cannot be deduced on general grounds: for one had to rely on $SU(3)_{fl}$ symmetry to relate $\Gamma_{SL}(D_s)$ to $\Gamma_{SL}(D^0)$ or $\Gamma_{SL}(B_s)$ to $\Gamma_{SL}(B_d)$ and no symmetry can be invoked to relate the semileptonic widths of mesons and baryons. There is actually a WA process that generates semileptonic decays on the Cabibbo-allowed level for $D_s$ [and also for $B_c$], but not for $D^0$ and $D^+$ nor for $B_d$, $B^-$ and $B_s$ mesons: the hadrons are produced by gluon emission off the $\bar{s}$ [or the $\bar{c}$] line. Yet since the relative weight of WA is significantly reduced in meson decays, one does not expect this mechanism to change $\Gamma_{SL}(D_s)$ significantly relative to $\Gamma_{SL}(D^0)$. Actually there are corrections to the semileptonic widths arising already in order $1/m_Q^2$. On rather general grounds one predicts the expectation values $\langle P_Q|\bar{Q}Q|P_Q \rangle$ and $\langle P_Q|\bar{Q}_i \sigma \cdot GQ|P_Q \rangle$ to be largely independant of the flavour of the light antiquark in the meson and therefore

\[
\Gamma_{SL}(D_s) \simeq \Gamma_{SL}(D^0) \quad (11a)
\]
\[
\Gamma_{SL}(B_s) \simeq \Gamma_{SL}(B_d) \quad (11b)
\]
like in the naive spectator picture, but for non-trivial reasons. On the other hand, as explained later, the values of the expectation values for these operators are different when taken between baryon states and one expects

\[ \Gamma_{SL}(\Lambda_Q) > \Gamma_{SL}(P_Q) \]  

The remarks above can be summarized as follows:

\[ \Gamma(\Lambda_Q) = \Gamma([Q\bar{q}]^0) = \Gamma([Q\bar{q}]^\pm) + \mathcal{O}(1/m_Q^2) \] (12a)

\[ \Gamma(\Lambda_Q) > \Gamma([Q\bar{q}]^0) = \Gamma([Q\bar{q}]^\pm) + \mathcal{O}(1/m_Q^3) \] (12b)

\[ \Gamma(\Lambda_Q) > \Gamma([Q\bar{q}]^0) > \Gamma([Q\bar{q}]^\pm) + \mathcal{O}(1/m_Q^4) \] (12c)

Quantitative predictions for the lifetime ratios of beauty hadrons through order $1/m_b^3$ are given in Table 1 together with present data. The expectations for the lifetimes of charm hadrons are juxtaposed to the data in Table 2. The overall agreement is rather good given the theoretical and experimental uncertainties. It has to be kept in mind that the *numerical* predictions on baryon lifetimes involve quark model results for the size of the relevant expectation values. This is indicated in Tables 1 and 2 by the asterisk. There is however one obvious discrepancy: the observed $\Lambda_b$ lifetimes is significantly shorter than predicted. While – as indicated just above – predictions on baryon lifetimes have to be taken with a grain of salt, one cannot change the prediction on $\tau(\Lambda_b)/\tau(B_d)$ with complete theoretical impunity; this will be explained later on.

These numerical predictions bear out the general expectations expressed above. This will be discussed below in more detail.

For proper perspective on the role played by the heavy quark expansion some comments should be made at this point:

(i) As stated before, the early phenomenological treatments had already identified the relevant mechanisms by which lifetime differences arise, namely PI, WA and WS. Furthermore the authors of ref.[2] had argued – correctly – that PI constitutes the dominant effect. Yet it is the heavy quark expansion that provides a firm theoretical

| Observable                  | QCD (1/m_b expansion)                                                                 | Data          |
|-----------------------------|---------------------------------------------------------------------------------------|---------------|
| $\tau(B^-)/\tau(B_d)$      | $1 + 0.05(f_B/200 \text{ MeV})^2[1 + \mathcal{O}(10\%)] > 1$ (mainly due to destructive interference) | $1.04 \pm 0.05$ |
| $\bar{\tau}(B_s)/\tau(B_d)$| $1 \pm \mathcal{O}(0.01)$                                                              | $0.98 \pm 0.08$ |
| $\tau(\Lambda_b)/\tau(B_d)$| $\sim 0.9^*$                                                                          | $0.76 \pm 0.06$ |

Table 1: QCD Predictions for Beauty Lifetimes
underpinning to these expectations. In particular it clarifies – both conceptually and quantitatively – the role of WA and how its weight scales with $1/m_Q$.

(ii) Contributions of order $1/m_Q$ would dominate all other effects – if they were present! The heavy quark expansion shows unequivocally that they are absent in total rates; this has important ramifications, to be pointed out later.

(iii) Corrections of order $1/m_Q^2$ differentiate between the decays of mesons and of baryons. They had been overlooked before. The heavy quark expansion has identified them and basically determined their size.

### 3 Methodology of the Heavy Quark Expansion for Fully Integrated Rates

The weak decay of the heavy quark $Q$ inside the heavy-flavour hadron $H_Q$ proceeds within a cloud of light degrees of freedom (quarks, antiquarks and gluons) with which $Q$ and its decay products can interact strongly. It is the challenge for theorists to treat these initial and final state hadronization effects. Among the existing four Post-Voodoo theoretical technologies – QCD Sum Rules, Lattice QCD, Heavy Quark Effective Theory (HQET) and Heavy Quark Expansions – only the last one deals with inclusive decays. On the other hand it benefits, as we will see, from the results of the other three technologies, since those can determine the size of some of the relevant expectation values.$^7$

In analogy to the treatment of $e^+e^- \rightarrow hadrons$ one describes the transition rate into an inclusive final state $f$ through the imaginary part of a forward scattering

\[ \tau(D^+)/\tau(D^0) \sim 2 \quad \text{[for } f_D \simeq 200 \text{ MeV]} \]

(mainly due to destructive interference)

| Observable | QCD (1/m_c expansion) | Data |
|------------|------------------------|------|
| $\tau(D^+)/\tau(D^0)$ | $\sim 2$ | $2.547 \pm 0.043$ |
| $\tau(D_s)/\tau(D^0)$ | $1 \pm \text{few } \times 0.01$ | $1.125 \pm 0.042$ |
| $\tau(\Lambda_c)/\tau(D^0)$ | $\sim 0.5^*$ | $0.51 \pm 0.05$ |
| $\tau(\Xi_c^+)/\tau(\Lambda_c)$ | $\sim 1.3^*$ | $1.75 \pm 0.36$ |
| $\tau(\Xi_c^+)/\tau(\Xi_c^0)$ | $\sim 2.8^*$ | $3.57 \pm 0.91$ |
| $\tau(\Xi_c^+)/\tau(\Omega_c)$ | $\sim 4^*$ | $3.9 \pm 1.7$ |

Table 2: QCD Predictions for Charm Lifetimes

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$^7$It should be noted that contrary to frequent claims in the literature HQET – as it is usually and properly defined – per se does not allow to treat lifetimes or even the total semileptonic width: for those observables strongly depend on $m_Q$ whereas it is the special feature of HQET that $m_Q$ is removed from its Lagrangian.
operator evaluated to second order in the weak interactions \[10, 11, 12\]:

\[
\hat{T}(Q \to f \to Q) = i \text{Im} \int d^4x \{ \mathcal{L}_W(x) \mathcal{L}_W^\dagger(0) \}_T
\]

where \{\}_T denotes the time ordered product and \(\mathcal{L}_W\) the relevant effective weak Lagrangian expressed on the parton level. If the energy released in the decay is sufficiently large one can express the non-local operator product in eq.(13) as an infinite sum of local operators \(O_i\) of increasing dimension with coefficients \(\tilde{c}_i\) containing higher and higher powers of \(1/m_Q\). The width for \(H_Q \to f\) is then obtained by taking the expectation value of \(\hat{T}\) between the state \(H_Q\):

\[
\langle H_Q | \hat{T}(Q \to f \to Q) | H_Q \rangle \propto \Gamma(H_Q \to f) = G_F^2 |KM|^2 \sum_i \tilde{c}_i^{(f)} \langle H_Q | O_i | H_Q \rangle
\]

This master formula holds for a host of different inclusive heavy-flavour decays: semileptonic, nonleptonic and radiative transitions, KM favoured or suppressed etc. For semileptonic and nonleptonic decays treated through order \(1/m_Q^3\) it takes the form already given in eq. (6):

\[
\Gamma(H_Q \to f) = \frac{G_F^2 m_Q^5}{192\pi^3} |KM|^2 \left[ c_f^l \langle H_Q | \bar{Q}Q | H_Q \rangle_{\text{norm}} + c_f^5 \frac{\langle H_Q | \bar{Q}i\sigma \cdot GQ | H_Q \rangle}{m_Q^2} + \sum_i c_{6,i}^l \frac{\langle H_Q | (\bar{Q} \Gamma_i q)(\bar{q} \Gamma_i Q) | H_Q \rangle_{\text{norm}}}{m_Q^3} + \mathcal{O}(1/m_Q^4) \right]
\]

Integrating out the two internal loops in Figs.2 and 3 yields the operators \(\bar{Q}Q\) and \(\bar{Q}i\sigma \cdot GQ\), respectively; the black boxes represent \(\mathcal{L}_W\). Likewise the diagrams in Figs.4 generate four-quark operators; Fig.4a and 4b differ in how the light quark flavours are connected inside the hadron \(H_Q\): cutting a quark line \(q\) in Fig.2 and connecting it to the \(\bar{q}\) constituent of the \(H_Q\) meson, as shown in Fig.4a, one has a WA transition operator; cutting instead the \(\bar{q}\) line and connecting it to the \(H_Q\) constituents, see Fig.4b, leads to the four-fermion operator describing PI. Inspection of these diagrams suggests a rule-of-thumb that is borne out by explicit computations: the dimensionless coefficients \(c_f^l\) are smaller for the two-loop diagrams of Figs.2 and 3 than for the one-loop diagrams of Figs.4.

Integrating out the internal loops of Fig.3 actually generates not only the dimension-five chromomagnetic operator, but also dimension-six quark-gluon operators, namely \(\bar{Q}(iD_\mu G_{\mu\nu})\gamma_\nu Q\) and \(\bar{Q}\sigma_{\mu\nu}G_{\mu\nu}\gamma_\rho iD_\rho Q\). Using the equations of motion those can be reduced to four-quark operators. Yet their coefficients are suppressed by \(\alpha_S/\pi\) relative to the coefficient coming from Figs.4; therefore one can discard their contributions at the present level of accuracy.

\[\text{It should be kept in mind, though, that it is primarily the energy release rather than } m_Q \text{ that controls the expansion.}\]
One important general observation should be made at this point. The two operators $\bar{Q}Q$ and $\bar{Q}i\sigma \cdot GQ$ obviously do not contain any light quark or antiquark fields; those enter first on the dimension-six level. Their expectation values are, as discussed below, practically insensitive to the light degrees of freedom up to terms of order $1/m_Q^3$. The inclusive semileptonic and non-leptonic widths through order $1/m_Q^2$ are thus controlled by the decay of a single quark $Q$, yet occurring in a non-trivial background field; the spectator contribution universal to the widths for all hadrons $H_Q$ of a given heavy flavour constitutes its asymptotic part:

$$
\Gamma(H_Q) \simeq \Gamma_{\text{decay}}(H_Q) + \mathcal{O}(1/m_Q^3) , \quad (15a)
$$

$$
\Gamma_{\text{decay}}(H_Q) \simeq \Gamma_{\text{spect}}(Q) + \mathcal{O}(1/m_Q^2) . \quad (15b)
$$

However $\Gamma_{\text{decay}}$ possesses a different value for mesons and baryons, as discussed below.

The expansion introduced above will be of practical use only if its first few terms provide a good approximation and if one can determine the corresponding expectation values in a reliable manner. The latter can be achieved in two complementary ways: One relates the matrix element in question to other observables through a heavy quark expansion; or one harnesses other second-generation theoretical technologies, namely QCD sum rules, QCD simulations on the lattice and Heavy Quark Effective Theory, to calculate these quantities. The results obtained so far are listed below:

(a) Using the equations of motion one finds for the leading operator $\bar{Q}Q$:

$$
\bar{Q}Q = \bar{Q}\gamma_0 Q - \frac{\bar{Q}[(i\bar{D})^2 - (i/2)\sigma \cdot G]Q}{2m_Q^2} + g_s^2 \bar{Q}\gamma_0 t^i Q \sum_q q \gamma_0 t^i q + \mathcal{O}(1/m_Q^4) \quad (16)
$$

with the sum in the last term running over the light quarks $q$; the $t^i$ denote the colour $SU(3)$ generators. I have ignored total derivatives in this expansion since they do not contribute to the $H_Q$ expectation values. There emerge two dimension-five operators, namely $\bar{Q}(i\bar{D})^2 Q$ and $\bar{Q}\sigma \cdot GQ$ with $\bar{D}$ denoting the covariant derivative. The first one represents the square of the spatial momentum of the heavy quark $Q$ moving in the soft gluon background and thus describes its kinetic energy$^9$. The second one constitutes the chromomagnetic operator that already appeared in eq.(6). The relevant dimension-six operators can be expressed as four-fermion operators. Since $\bar{Q}\gamma_0 Q$ constitutes the Noether current for the heavy-flavour quantum number one has $\langle H_Q | \bar{Q}\gamma_0 Q | H_Q \rangle_{\text{norm}} = 1$ leading to eq.(8).

(b) The first non-perturbative corrections arise at order $1/m_Q^2$, and they can enter in two ways, namely through the chromomagnetic operator in the OPE of $\hat{T}$ or through the $1/m_Q$ expansion of the $H_Q$ expectation values of the local operator $\bar{Q}Q$.

$^9$Since it is not a Lorentz scalar, it cannot appear in eq.(6).
(c) For the chromomagnetic operator one finds

$$\langle P_Q | \bar{Q} i \sigma \cdot GQ | P_Q \rangle \text{norm} \simeq \frac{3}{2} (M_{V_Q}^2 - M_{P_Q}^2)$$  \hspace{1cm} (17a)$$

where \( P_Q \) and \( V_Q \) denote the pseudoscalar and vector mesons, respectively. For \( \Lambda_Q \) and \( \Xi_Q \) baryons one has instead

$$\langle \Lambda_Q | \bar{Q} i \sigma \cdot GQ | \Lambda_Q \rangle \simeq 0 \simeq \langle \Xi_Q | \bar{Q} i \sigma \cdot GQ | \Xi_Q \rangle$$  \hspace{1cm} (17b)$$

since the light diquark system inside \( \Lambda_Q \) and \( \Xi_Q \) carries no spin. These baryons thus represent (though only through order \( 1/m_Q^2 \)) a simpler system than the mesons where the light antiquark of course carries spin. For \( \Omega_Q \) baryons on the other hand the situation is different: for the light di-quarks \( ss \) form a spin-one configuration.

The \( \Omega_Q \) expectation value is then given by the spin-splitting in the baryon masses:

$$\langle \Omega_Q | \bar{Q} i \sigma \cdot GQ | \Omega_Q \rangle \text{norm} \simeq \frac{4}{3} [M^2(\Omega^{(3/2)}_Q) - M^2(\Omega_Q)]$$  \hspace{1cm} (17c)$$

(d) We thus have

$$\langle \Lambda_Q | \bar{Q} Q | \Lambda_Q \rangle \text{norm} = 1 - \frac{\langle (\vec{p}_Q)^2 \rangle_{\Lambda_Q}}{2m_Q^2} + \mathcal{O}(1/m_Q^3)$$  \hspace{1cm} (18a)$$

$$\langle P_Q | \bar{Q} Q | P_Q \rangle \text{norm} = 1 - \frac{\langle (\vec{p}_Q)^2 \rangle_{P_Q}}{2m_Q^2} + \frac{3}{8} \frac{M_{V_Q}^2 - M_{P_Q}^2}{m_Q^2} + \mathcal{O}(1/m_Q^3)$$  \hspace{1cm} (18b)$$

with the notation \( \langle (\vec{p}_Q)^2 \rangle_{\Lambda_Q} \equiv \langle H_Q | \bar{Q} (i \vec{D})^2 Q | H_Q \rangle \text{norm} \). The reason for the appearance of the kinetic energy term in eqs. (18) is quite transparent: The first two terms on the r.h.s. of eqs.(18) represent the mean value of the factor \( \sqrt{1 - \vec{v}^2} \) reflecting the time dilation slowing down the decay of the quark \( Q \) moving inside \( H_Q \).

(e) The value of \( \langle (\vec{p}_Q)^2 \rangle_{H_Q} \) has not been determined yet in an accurate way (details will be given later); we know, though, it has to obey the inequality \[19, 20\]

$$\langle (\vec{p}_Q)^2 \rangle_{H_Q} \geq \frac{1}{2} \langle H_Q | \bar{Q} i \sigma \cdot GQ | H_Q \rangle \text{norm} \equiv \langle \mu_G^2 \rangle_{P_Q}$$  \hspace{1cm} (19)$$

which – because of eq.(17) – is useful only for \( H_Q = P_Q \). \[10\] On the other hand, the difference in the kinetic energy of \( Q \) inside baryons and mesons can be related to the masses of charm and beauty hadrons \[21\]:

$$\langle (\vec{p}_Q)^2 \rangle_{\Lambda_Q} - \langle (\vec{p}_Q)^2 \rangle_{P_Q} \simeq \frac{2m_b m_c}{m_b - m_c} \cdot [\langle (M_B) - M_{\Lambda_b} \rangle - [\langle M_D \rangle - M_{\Lambda_c}]]$$  \hspace{1cm} (20)$$

\[10\]One should note that this inequality has now been derived in a fully field-theoretical treatment of QCD, rather than a merely quantum-mechanical one. Its proper interpretation will be discussed later.
with $\langle M_{B,D} \rangle$ denoting the ‘spin averaged’ meson masses:

$$\langle M_B \rangle \equiv \frac{1}{4}(M_B + 3M_B^*)$$  \hspace{1cm} (21)

and likewise for $\langle M_D \rangle$. In deriving eq.(20) it was implicitly assumed that the $c$ quark can be treated also as heavy; in that case $\langle (\vec{p}_c)^2 \rangle_{H_c} \simeq \langle (\vec{p}_b)^2 \rangle_{H_b}$ holds.

(f) Eqs.(17-18,20) show that the two dimension-five operators do produce differences in $P_Q$ vs. $\Lambda_Q/\Xi_Q$ vs. $\Omega_Q$ lifetimes (and semileptonic widths) of order $1/m_Q^2$; i.e., $\Gamma_{\text{decay}}(H_Q)$, as defined in eqs.(15), depends on whether $H_Q$ represents a (pseudoscalar) meson or a baryon. In the latter case it is sensitive to whether the light diquark system carries spin zero ($\Lambda_Q$, $\Xi_Q$) or one ($\Omega_Q$). The origin of such a difference is quite transparent. (i) The heavy quark $Q$ can be expected to possess a somewhat different kinetic energy inside a meson or a baryon; it can also be affected by spin-spin interactions with the light di-quarks. This difference in the heavy quark motion means that Lorentz time dilation prolongs the lifetime of the quark $Q$ to different degrees in baryons than in mesons. (ii) While there exists a spin interaction between $Q$ and the antiquark $\bar{q}$ in the meson or the $ss$ system in the $\Omega_Q$ baryon, there is no such coupling inside $\Lambda_Q$ or $\Xi_Q$.

Yet these effects do not generate a significant difference among meson lifetimes since their expectation values satisfy isospin and $SU(3)_f$ symmetry to a good degree of accuracy.

(g) Differences in meson lifetimes are generated at order $1/m_Q^3$ by dimension-six four-quark operators describing PI as well as WA; the weight of the latter is, as explained in more detail in the next section, greatly reduced. The expectation values of these operators look very similar to the one controlling $B^0 - \bar{B}^0$ oscillations:

$$\langle H_Q(p)\rangle \langle Q_L \gamma_\mu q_L | \bar{q}_L \gamma_\nu Q_L | H_Q(p) \rangle_{\text{norm}} \simeq \frac{1}{4m_{H_Q}} f_{H_Q}^2 p_\mu p_\nu$$  \hspace{1cm} (22)

with $f_{H_Q}$ denoting the decay constant for the meson $H_Q$; the so-called bag factor has been set to unity, i.e. factorization has been assumed.

(h) The situation becomes much more complex for $\Lambda_Q$ and baryon decays in general. To order $1/m_Q^3$ baryons lose the relative simplicity mentioned above: there are several different ways in which the valence quarks of the baryon can be contracted with the quark fields in the four-quark operators; furthermore WS is not helicity suppressed and thus can make a sizeable contribution to lifetime differences; also the PI effects can now be constructive as well as destructive. Finally one cannot take recourse to factorisation as a limiting case. Thus there emerge three types of numerically significant mechanisms at this order in baryon decays – in contrast to meson decays where there is a single dominant source for lifetime differences –
and their strength cannot be expressed in terms of a single observable like \( f_{HQ} \). At present we do not know how to determine the relevant matrix elements in a model-independent way. The best available guidance and inspiration is to be gleaned from quark model calculations with their inherent uncertainties. This analysis had already been undertaken in the framework of phenomenological models \([5, 6, 7]\) with the following qualitative results:

- WS contributes to \( \Lambda_Q \) and \( \Xi_Q^0 \) decays;
- a destructive interference affects \( \Lambda_Q \), \( \Xi_Q \) and \( \Omega_b \) transitions.
- a constructive interference enhances \( \Xi_Q \) and \( \Omega_c \) decays.

One thing should be obvious already at this point: with terms of different signs and somewhat uncertain size contributing to differences among baryon lifetimes one has to take even semi-quantitative predictions with a grain of salt!

The probability amplitude for WS or interference to occur is expressed in quark models by the wavefunction for \( Q \) and one of the light quarks at zero spatial separation. This quantity is then related to the meson wavefunction through the hyperfine splitting in the baryon and meson masses \([7, 14]\):

\[
\frac{|\psi_{\Lambda_Q}(0)|}{|\psi_{PQ}(0)|} \approx \frac{2m_q^* (M_{\Sigma Q} - M_{\Lambda Q})}{(M_{PQ} - m_q^*)(M_{VQ} - M_{PQ})} \tag{23}
\]

with \( m_q^* \) denoting the phenomenological constituent mass of the light quark \( q \) to be employed in these models. For the baryonic expectation values one then obtains

\[
\langle \Lambda_Q|\bar{Q}\Gamma_q Q\Gamma_q|\Lambda_Q\rangle_{\text{norm}} \sim \frac{1}{4(\mu_G^2)_{PQ}} (M_{\Sigma Q} - M_{\Lambda Q}) m_q^* F_{PQ}^2 M_{PQ} \tag{24}
\]

and likewise for \( \Xi_Q \). For the \( \Omega_Q \) one has instead

\[
\langle \Omega_Q|\bar{Q}\Gamma_q Q\Gamma_q|\Omega_Q\rangle_{\text{norm}} \sim \frac{5}{6(\mu_G^2)_{PQ}} (M_{\Sigma Q} - M_{\Lambda Q}) m_q^* F_{PQ}^2 M_{PQ}, \tag{25}
\]

reflecting the different spin substructure which already had caused the difference in eqs.(17a) vs. (17b). For simplicity one has assumed in these expressions that the hyperfine splitting in the baryon masses are universal.

To summarize this discussion:

- The non-perturbative corrections to total widths through order \( 1/m_Q^3 \) are expressed in terms of three non-trivial (types of) matrix elements: \( \langle \mu_G^2 \rangle_{HQ} \), \( \langle (\vec{p}_Q)^2 \rangle_{HQ} \) and \( \langle H_Q|(\bar{Q}\Gamma_i Q)(\bar{q}\Gamma_i Q)|H_Q \rangle \).
- The size of \( \langle \mu_G^2 \rangle_{HQ} \) is well-known for the mesons and \( \Lambda_Q \) and \( \Xi_Q \) baryons; reasonable estimates can be obtained for \( \Omega_Q \) baryons.
• A lower bound exists for the mesonic expectation value \( \langle (\vec{p}_Q)^2 \rangle_{PQ} \), but not for the baryonic one. Various arguments strongly suggest that \( \langle (\vec{p}_Q)^2 \rangle_{HQ} \) is close to its lower bound. Furthermore the quantity \( \langle (\vec{p}_Q)^2 \rangle_{\Lambda_Q} - \langle (\vec{p}_Q)^2 \rangle_{PQ} \), which is highly relevant for the difference in baryon vs. meson lifetimes (and semileptonic widths) can be extracted from mass measurements.

• The expectation value of the four-quark operators between mesons can be expressed in terms of a single quantity, namely the decay constant \( f_{PQ} \).

• However the baryonic expectation values of the four-quark operators constitute a veritable Pandora’s box, which at present is beyond theoretical control. On the other hand measurements of inclusive weak decay rates and the resulting understanding of the role played by WS and PI present us with a novel probe of the internal structure of the heavy-flavour baryons.

4 Comments on the Underlying Concepts

Before explaining these predictions in more detail and commenting on the comparison with the data in the next subsection, I want to add four general remarks and elaborate on them. While I hope they will elucidate the underlying concepts, they are not truly essential for following the subsequent discussion and the more casual reader can ignore them.

(A) On the validity of the \( 1/m_Q \) expansion in the presence of gluon emission: The WA contribution to the decay width of pseudoscalar mesons to lowest order in the strong coupling has been sketched in eq.(2). The diagram in Fig.1 contains gluon bremsstrahlung off the initial antiquark line in the \( W \)-exchange reaction; evaluating this diagram explicitly one finds, as stated before:

\[
\Gamma^{(1)}_{W^-X} \propto (\alpha_s/\pi)G_F^2(f_{H_Q}/\langle E_{\bar{q}} \rangle)^2m_Q^5
\]

with \( \langle E_{\bar{q}} \rangle \) denoting the average energy of the initial antiquark \( \bar{q} \). Since \( \Gamma^{(1)}_{W^-X} \propto 1/\langle E_{\bar{q}} \rangle^2 \), it depends very sensitively on the low-energy quantity \( \langle E_{\bar{q}} \rangle \), and its magnitude is thus quite uncertain. The presence of a \( 1/\langle E_{\bar{q}} \rangle^2 \) (or \( 1/m_Q^2 \)) term would also mean that the nonperturbative corrections to even inclusive decay widths of heavy-flavour hadrons are not ‘infrared safe’ and cannot be treated consistently through an expansion in inverse powers of the heavy quark mass \( m_Q \) only.

Fortunately the contribution stated in eq.(26) turns out to be spurious for inclusive transitions. This can best be seen by studying the imaginary part of the forward scattering amplitude \( Q\bar{q} \to f \to Q\bar{q} \) as shown in Fig.5. There are actually three poles in this amplitude indicated by the broken lines which represent different final states: one with an on-shell gluon and the other two with an off-shell gluon materializing.
as a $q\bar{q}$ pair and involving interference with the spectator decay amplitude. These processes are all of the same order in the strong coupling and therefore have to be summed over for an inclusive decay. Analysing carefully the analyticity properties of the sum of these forward scattering amplitudes one finds \cite{13, 14} that it remains finite in the limit of $\langle E_{\bar{q}} \rangle \to 0$, i.e. the amplitude for the inclusive width does not contain terms of order $1/\langle E_{\bar{q}} \rangle^2$ or even $1/\langle E_{\bar{q}} \rangle$! The contribution of WA can then be described in terms of the expectation value of a local four-fermion operator although the final state is dominated by low-mass hadronic systems.

To summarize: the quantity $f_{HQ}$ is calibrated by the large mass $m_Q$ rather than by $\langle E_{\bar{q}} \rangle$; WA is then only moderately significant even in charm decays. The $1/m_Q$ scaling thus persists to hold even in the presence of radiative corrections – but only for inclusive transitions! This caveat is not of purely academic interest. For it provides a nice toy model illustrating the qualitative difference between inclusive and exclusive transitions. The latter are represented here by the three separate cuts in Fig.5. For charm decays they correspond to the reactions $[c\bar{u}] \to s\bar{d}g$ and $[c\bar{u}] \to s\bar{d}u\bar{u}$. The two individual transition rates quite sensitively depend on a low energy scale $\langle E_{\bar{q}} \rangle$ - which considerably enhances the rate for the first transition while reducing it for the second one. Yet the dependance on $1/\langle E_{\bar{q}} \rangle$ disappears from their sum!

(B) On the fate of the corrections of order $1/m_Q$: The most important element of eq.(2) is – the one that is missing! Namely there is no term of order $1/m_Q$ in the total decay width whereas such a correction definitely exists for the mass formula: $M_{HQ} = m_Q(1 + \bar{\Lambda}/m_Q + O(1/m_Q^2))$ (and likewise for differential decay distributions). Hadronization in the initial state does generate corrections of order $1/m_Q$ to the total width, as does hadronization in the final state. Yet local colour symmetry enforces that they cancel against each other! \footnote{This can be nicely illustrated in quantum mechanical toy models: The total rate for the transition $Q \to q$ depends on the local properties of the potential, i.e. on the potential around $Q$ in a neighbourhood of size $1/m_Q$ only; the nature of the resulting spectrum for $q$ – whether it is discrete or continuous, etc. – depends of course on the long range properties of the potential, namely whether it is confining or not.} This can be understood in another more compact (though less intuitive) way as well: with the leading operator $\bar{Q}Q$ carrying dimension three only dimension-four operators can generate $1/m_Q$ corrections; yet there is no independent dimension-four operator \cite{16, 10} once the equation of motion is imposed – unless one abandons local colour symmetry thus making the operators $\bar{Q}i\gamma \cdot \partial Q$ and $\bar{Q}i\gamma \cdot BQ$ independent of each other ($B_\mu$ denotes the gluon field)! The leading non-perturbative corrections to fully integrated decay widths are then of order $1/m_Q^2$ and their size is controlled by two dimension-five operators, namely the chromomagnetic and the kinetic energy operators, as discussed above. Their contributions amount to no more than 10 percent for $B$ mesons – $(\mu_{had}/m_b)^2 \simeq O((1\, GeV/m_b)^2) \sim O(\%)$ – as borne out by the detailed analysis previewed in Table 1.
Which mass is it? For a $1/m_Q$ expansion it is of course important to understand which kind of quark mass is to be employed there, in particular since for confined quarks there exists no a priori natural choice. It had been claimed that the pole mass can and therefore should conveniently be used. Yet such claims turn out to be fallacious [17, 18]: QCD, like QED, is not Borel summable; in the high order terms of the perturbative series there arise instabilities which are customarily referred to as (infrared) renormalons representing poles in the Borel plane; they lead to an additive mass renormalization generating an irreducible uncertainty of order $\bar{\Lambda}$ in the size of the pole mass:

$$m_{\text{pole}}^{Q} = m_{Q}^{(0)} (1 + c_1 \alpha_s + c_2 \alpha_s^2 + \ldots + c_N \alpha_s^N) + O(\bar{\Lambda}) = m_{Q}^{(N)} (1 + O(\bar{\Lambda}/m_{Q}^{(N)}))$$

While this effect can safely be ignored in a purely perturbative treatment, it negates the inclusion of non-perturbative corrections $\sim O(1/m_Q^2)$, since those are then parametrically smaller than the uncertainty $\sim O(1/m_Q)$ in the definition of the pole mass. This problem can be taken care of through Wilson’s prescriptions for the operator product expansion:

$$\Gamma(H_Q \rightarrow f) = \sum_i c_i^{(f)}(\mu) \langle H_Q | O_i | H_Q \rangle (\mu)$$

where a momentum scale $\mu$ has been introduced to allow a consistent separation of contributions from Long Distance and Short Distance dynamics – $LD > \mu^{-1} > SD$ – with the latter contained in the coefficients $c_i^{(f)}$ and the former lumped into the matrix elements. The quantity $\mu$ obviously represents an auxiliary variable which drops out from the observable, in this case the decay width. In the limit $\mu \rightarrow 0$ infrared renormalons emerge in the coefficients; they cancel against ultraviolet renormalons in the matrix elements. Yet that does not mean that these infrared renormalons are irrelevant and that one can conveniently set $\mu = 0$! For to incorporate both perturbative as well as non-perturbative corrections one has to steer a careful course between ‘Scylla’ and ‘Charybdis’: while one wants to pick $\mu \ll m_Q$ so as to make a heavy quark expansion applicable, one also has to choose $\mu_{\text{had}} \ll \mu$ s.t. $\alpha_S(\mu) \ll 1$; for otherwise the perturbative corrections become uncontrollable. Wilson’s OPE allows to incorporate both perturbative and non-perturbative corrections, and this underlies also a consistent formulation of HQET; the scale $\mu$ provides an infrared cut-off that automatically freezes out infrared renormalons. For the asymptotic difference between the hadron and the quark mass one then has to write $\bar{\Lambda}(\mu) \equiv (M_{H_Q} - m_{Q}(\mu))_{m_Q \rightarrow \infty}$. This nice feature does not come for free, of course: for one has to use a ‘running’ mass $m_{Q}(\mu)$ evaluated at an intermediate scale $\mu$ which presents a technical complication. On the other hand this quantity can reliably be extracted from data [20]; furthermore it drops out from lifetime ratios, the main subject of this discussion.

On Duality: Quark-hadron duality equates transition rates calculated on the quark-gluon level with the observable ones involving the corresponding hadrons – provided one sums over a sufficient number of final states. Since the early days of the quark model this concept has been invoked in many different formulations; among other things it has never been clearly defined what constitutes a sufficient number
of final states. This lack of a precise formulation emerged since duality had never been derived from QCD in a rigorous fashion; furthermore a certain flexibility in an unproven, yet appealing intuitive concept can be of considerable heuristic benefit.

Heavy quark expansions assume the validity of duality. Nevertheless they provide new and fruitful insights into its workings. As already stated, heavy quark expansions are based on an OPE, see eq.(26), and as such are properly defined in Euclidean space. There are two possible limitations to the procedure by which duality is implemented in heavy quark expansions. (i) The size of the coefficients $c_i^{(f)}$ is controlled by short-distance dynamics. In concrete applications they are actually obtained within perturbation theory. Those computations involve integrals over all momenta. However for momenta below the scale $\mu$ a perturbative treatment is unreliable. The contributions from this regime might turn out to be numerically insignificant for the problem at hand; yet in any case one can undertake to incorporate them through the expectation values of higher-dimensional operators – the so-called condensates. This procedure is the basis of what is sometimes referred to as the ‘practical’ version of the OPE. Yet it is conceivable that there are significant short-distance contributions from non-perturbative dynamics as well; instantons provide an illustration for such a complication, although their relevance is quite unclear at present [22]. (ii) Once the OPE has been constructed and its coefficients $c_i^{(f)}$ determined there, one employs a dispersion relation to analytically continue them into Minkowski space. This means, strictly speaking, that in Minkowski space only ‘smeared’ transition rates can be predicted, i.e. transition rates averaged over some finite energy range. This situation was first analyzed in evaluating the cross section for $e^+e^- \rightarrow \text{had}$ [23]. Through an OPE QCD allows to compute

$$\langle \sigma(e^+e^- \rightarrow \text{had}; E_{c.m.}) \rangle \equiv \frac{1}{\Delta E_{sm}} \int_{E_{c.m.} - \Delta E_{sm}}^{E_{c.m.} + \Delta E_{sm}} dE_{c.m.}' \sigma(e^+e^- \rightarrow \text{had}; E_{c.m.}')$$

with $0 < \Delta E_{sm} \ll E_{c.m.}$. If the cross section happens to be a smooth function of $E_{c.m.}'$ – as it is the case well above production thresholds –, then one can effectively take the limit $\Delta E_{c.m.} \rightarrow 0$ to predict $\sigma(e^+e^- \rightarrow \text{had}; E_{c.m.})$ for a fixed c.m. energy $E_{c.m.}$. This scenario can be referred to as local duality. Yet close to a threshold, like for charm or beauty production, with its resonance structure one has to retain $\Delta E_{sm} \sim \mu_{had} \sim 0.5 - 1$ GeV. The same considerations are applied to heavy-flavour decays. Based on global duality one can predict ‘smeared’ decay rates, i.e. decay rates averaged over a finite energy interval. If the energy release is large enough the decay rate will be a smooth function of it, smearing will no longer be required and local duality emerges. How large the energy release has to be for this simplification to occur cannot be predicted (yet). For the onset of local duality is determined by terms that cannot be seen in any finite order of the $1/m_Q$ expansion. However rather general considerations lead to the following expectations: (i) This onset should be quite abrupt around some energy scale exceeding usual hadronic scales. (ii) Local duality should hold to a good degree of accuracy for beauty decays, even for nonleptonic ones.
(iii) On the other hand there is reason for concern that this onset might not have occurred for charm decays. This is sometimes referred to by saying that inclusive charm decays might receive significant contributions from (not-so-)‘distant cuts’.

5 Size of the Matrix Elements

As discussed before, see eq.(27), the size of matrix elements depends in general on the scale $\mu$ at which they are evaluated. This will be addressed separately for the different cases.

The scalar dimension-three operator $\bar{Q}Q$ can be expanded in terms of $\bar{Q}\gamma_0Q$ – with $\langle HQ|Q\gamma_0Q|HQ\rangle_{\text{norm}} = 1$ – and operators of dimension five and higher, see eq.(15).

(i) Dimension-five operators

Employing eq.(16) for $b$ and $c$ quarks one finds:

$$\langle \mu^2_G \rangle_B \equiv \langle B|\bar{b}\frac{i}{2}\sigma \cdot Gb|B\rangle_{\text{norm}} \simeq \frac{3}{4}(M_{B^*}^2 - M_B^2) \simeq 0.37 \text{ (GeV)}^2,$$

$$\langle \mu^2_G \rangle_D \equiv \langle D|\bar{c}\frac{i}{2}\sigma \cdot Gc|D\rangle_{\text{norm}} \simeq \frac{3}{4}(M_{D^*}^2 - M_D^2) \simeq 0.41 \text{ (GeV)}^2,$$

i.e., these two matrix elements that have to coincide in the infinite mass limit are already very close to each other. They are already evaluated at the scale that is appropriate for beauty and charm decays.

For the description of $B_c$ decays one needs the expectation value of the beauty as well as charm chromomagnetic operator taken between the $B_c$ state. While the $B_c^*$ and $B_c$ masses have not been measured yet, one expects the theoretical predictions \cite{34} on them to be fairly reliable. With $M(B_c^*) \simeq 6.33$ GeV and $M(B_c) \simeq 6.25$ GeV one obtains

$$\langle B_c|\bar{b}\frac{i}{2}\sigma \cdot Gb|B_c\rangle_{\text{norm}} \simeq \langle B_c|\bar{c}\frac{i}{2}\sigma \cdot Gc|B_c\rangle_{\text{norm}} \simeq 0.75 \text{ (GeV)}^2;$$

i.e., double the value as for the mesons with light antiquarks.

From eq.(20) one obtains for the difference in the kinetic energy of the heavy quark inside baryons and mesons

$$\langle (\vec{p}_b)^2 \rangle_{\Lambda_b} - \langle (\vec{p}_b)^2 \rangle_B \simeq -(0.07 \pm 0.20) \text{ (GeV)}^2$$

using available data; for charm the same value holds to this order. We see that at present one cannot tell whether the kinetic energy of the heavy quark inside baryons exceeds that inside mesons or not. The uncertainty in the mass of $\Lambda_b$ constitutes
the bottleneck; it would be quite desirable to decrease its uncertainty down to below 10 MeV or even less. For the overall size of $\langle (\vec{p}_Q)^2 \rangle_{H_Q}$ we have the following lower bounds from eq.(19):

\[
\langle (\vec{p}_b)^2 \rangle_B \geq 0.37 (GeV)^2 \tag{31}
\]

\[
\langle (\vec{p}_c)^2 \rangle_D \geq 0.40 (GeV)^2 \tag{32}
\]

To be conservative one can add a term of at most $\pm 0.15 (GeV)^2$ to the right-hand side of eqs.(31,32) reflecting the uncertainty in the scale at which the expectation values are to be evaluated \[20\]. There are various arguments as to why the size of $\langle (\vec{p}_b)^2 \rangle_B$ will not exceed this lower bound significantly. One should note that such values for $\langle (\vec{p}_b)^2 \rangle_B$ are surprisingly large: for they state that the momenta of the heavy quark inside the hadron is typically around 600 MeV. An analysis based on QCD sum rules yields a value that is consistent with the preceding discussion \[24\]:

\[
\langle (\vec{p}_b)^2 \rangle_B = (0.5 \pm 0.1) (GeV)^2 \tag{33}
\]

It can be expected that practically useful values for $\langle (\vec{p}_{c,b})^2 \rangle_{D,B}$ will be derived from lattice QCD in the forseeable future \[25\].

The leading non-perturbative corrections are thus controlled by the following parameters:

\[
\frac{\langle \mu_G^2 \rangle_B}{m_b^2} \simeq 0.016, \quad \frac{\langle (\vec{p}_b)^2 \rangle_B}{m_b^2} \sim 0.016 \tag{34a}
\]

\[
\frac{\langle \mu_G^2 \rangle_D}{m_c^2} \simeq 0.21, \quad \frac{\langle (\vec{p}_c)^2 \rangle_D}{m_c^2} \sim 0.21 \tag{34b}
\]

\[
\frac{\langle \mu_G^2 \rangle_{\Omega_c}}{m_c^2} \simeq 0.12 \tag{35}
\]

The $\Omega_c$ mass hyperfine splitting has not been measured yet. A quark model estimate of 70 MeV has been used in evaluating eq.(17b).

(ii) Dimension-six Operators

The size of the decay constants $f_B$, $f_{B_s}$, $f_D$ and $f_{D_s}$ has not been determined yet with good accuracy. Various theoretical technologies yield

\[
f_D \sim 200 \pm 30 \text{ MeV} \tag{36}
\]

\[
f_B \sim 180 \pm 40 \text{ MeV} \tag{37}
\]

\[
\frac{f_{B_s}}{f_B} \simeq 1.1 - 1.2 \tag{38}
\]

\[
\frac{f_{D_s}}{f_D} \simeq 1.1 - 1.2 \tag{39}
\]
where a higher reliability is attached to the predictions for the ratios of decay constants, i.e. eqs.(38,39), than for their absolute size, eqs.(36,37). Recent studies by WA75 and by CLEO on $D_s^+ \rightarrow \mu^+ \nu$ yielded values for $f_{D_s}$ that are somewhat larger, but still consistent with these predictions. It can be expected that lattice QCD will produce more precise results on these decay constants in the foreseeable future and that those will be checked by future measurements in a significant way. Yet there are two subtleties to be kept in mind here:

(a) The matrix element of the four-quark operator is related to the decay constant through the assumption of ‘vacuum saturation’. Such an ansatz cannot hold as an identity; it represents an approximation the validity of which has to depend on the scale at which the matrix element is evaluated. The quantities $f_B$ and $f_D$ are measured in $B \rightarrow \tau \nu$, $\mu \nu$ and $D \rightarrow \tau \nu$, $\mu \nu$, respectively and thus probed at the heavy-flavour mass, $m_B$ and $m_D$. Yet for the strong interactions controlling the size of the expectation value of the four-quark operator the heavy-flavour mass is a completely foreign parameter. If vacuum saturation makes (approximate) sense anywhere, it has to be at ordinary hadronic scales $\mu \simeq 0.5 - 1$ GeV. That means the decay constant that is observed at the heavy-flavour scale has to be evaluated at the hadronic scale $\mu$; this is achieved by the so-called hybrid renormalization to be described later.

(b) The difference in decay widths generated by the four-quark operators is stated as being proportional to $f^2_{H_Q}/m^2_Q$ with $f_{H_Q} = F_{H_Q}[1 - |\bar{\mu}|/m_Q + O(1/m^2_Q)]$; $F_{H_Q}$ represents the leading or ‘static’ term which behaves like $1/\sqrt{m_Q}$ for $m_Q \rightarrow \infty$ and therefore $F_D > F_B$. Thus asymptotically one has $F^2_{H_Q}/m^2_Q$ which indeed vanishes like $1/m^3_Q$. Yet the decay constant $f_{H_Q}$ – partly as a consequence of its usual definition via an axialvector rather than a pseudoscalar current – contains large pre-asymptotic corrections which lead to $f_D \sim f_B$. Then it is not clear which value to use for $f_{H_Q}$ when calculating width differences, the asymptotic one $-F_{H_Q}$ or the one including the pre-asymptotic corrections $-f_{H_Q}$ where numerically $F_{H_Q} > f_{H_Q}$ holds. . It can be argued that the main impact of some of the dimension-seven operators in the OPE for the meson decay width is to renormalise the $F^2_{H_Q}/m^2_Q$ term to $f^2_{H_Q}/m^2_Q$; yet this issue can be decided only through computing the contributions from dimension-seven operators. I will return to this issue when discussing beauty and charm decays specifically.

Both these considerations also apply when determining the baryonic expectation values of four-quark operators [14]. Yet – as already pointed out – that is a considerably more murky affair, as will become quite apparent in the subsequent discussions of the beauty and charm lifetimes.

While there are significant uncertainties and ambiguities in the values of the masses of beauty and charm quarks their difference which is free of renormalon contributions is tightly constrained:

$$m_b - m_c \simeq \langle M \rangle_B - \langle M \rangle_D + \langle \vec{p} \rangle^2 \cdot \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right) \simeq 3.46 \pm 0.04 \text{GeV}.$$  (40)
This value agrees very well with the one extracted from an analysis of energy spectra in semileptonic \( B \) decays \[28\]. Lifetime ratios are hardly sensitive to this difference; we list it here mainly for completeness.

## 6 Predictions on Beauty Lifetimes

Within the Standard Model there is no real uncertainty about the weak forces driving beauty decays; at the scale \( M_W \) they are given by the Lagrangian

\[
\mathcal{L}_{W}^{\Delta B = 1}(\mu = M_W) = \frac{4G_F}{\sqrt{2}} [V_{cb}\bar{c}L\gamma_\mu b_L + V_{ub}\bar{u}L\gamma_\mu b_L] \cdot [V_{ud}^*\bar{d}L\gamma_\mu u_L + V_{cs}^*\bar{s}L\gamma_\mu c_L] \tag{41}
\]

for non-leptonic transitions with an analogous expression for semileptonic ones. In eq.(41) I have ignored Cabibbo-suppressed transitions and also the \( b \to t \) coupling since here we are not interested in Penguin contributions. Radiative QCD corrections lead to a well-known renormalization at scale \( m_b \), which is often referred to as \textit{ultraviolet} renormalization:

\[
\mathcal{L}_{W}^{\Delta B = 1}(\mu = m_b) = \frac{4G_F}{\sqrt{2}} V_{cb}V_{ud}^* \{ c_1(\bar{c}L\gamma_\mu b_L)(\bar{d}L\gamma_\mu u_L) + c_2(\bar{d}L\gamma_\mu b_L)(\bar{c}L\gamma_\mu u_L) \} \tag{42}
\]

for \( b \to c \bar{u}d \) transitions and an analogous expression for \( b \to c \bar{c}s \); the QCD corrections are lumped together into the coefficients \( c_1 \) and \( c_2 \) with

\[
c_1 = \frac{1}{2}(c_+ + c_-), \quad c_2 = \frac{1}{2}(c_+ - c_-) \tag{43a}
\]

\[
c_\pm = \left[ \frac{\alpha_S(M_W^2)}{\alpha_S(m_b^2)} \right]^{\gamma_\pm}, \quad \gamma_+ = \frac{6}{33 - 2N_f} = -\frac{1}{2}\gamma_- \tag{43b}
\]

in the leading-log approximation; \( N_f \) denotes the number of active flavours. Numerically this amounts to:

\[
c_1(LL) \simeq 1.1, \quad c_2(LL) \simeq -0.23. \tag{44a}
\]

Next-to-leading-log corrections modify mainly \( c_2 \) \[27\]:

\[
c_1(LL + NLL) \simeq 1.13, \quad c_2(LL + NLL) \simeq -0.29. \tag{44b}
\]

Radiative QCD corrections thus lead to a mild enhancement of the original coupling \(- c_1 > 1 \) together with the appearance of an induced operator with a different colour flow: \( c_2 \neq 0 \). Later on we will also include the so-called ‘hybrid’ renormalization reflecting radiative corrections in the domain from \( m_b \) down to \( \mu_{had} \), the scale at which the hadronic expectation values are to be evaluated.
Such perturbative corrections affect the overall scale of the semileptonic and non-leptonic width (and thus of the semileptonic branching ratio); but by themselves they cannot generate differences among the lifetimes of beauty hadrons. Those have to be initiated by non-perturbative contributions although their numerical size depends also on perturbative corrections.

The semileptonic and non-leptonic widths of beauty hadrons $H_b$ through order $1/m_b^2$ are given by

\[ \frac{\Gamma_{SL,\text{decay}}(H_b)}{\Gamma_0} = \langle H_b | \bar{b}b | H_b \rangle_{\text{norm}} \cdot \left[ \eta_{SL} I_0(x, 0, 0) + \frac{\langle \mu_G^2 \rangle_{H_b}}{m_b^2} \left( x \frac{d}{dx} - 2 \right) I_0(x, 0, 0) \right], \]

\[ \frac{\Gamma_{NL,\text{decay}}(H_b)}{\Gamma_0} = N_C \cdot \langle H_b | \bar{b}b | H_b \rangle_{\text{norm}} \cdot \left[ \frac{A_0}{3} I_0(x, 0, 0) + \rho_{cc} I_0(x, 0, 0) + \frac{4}{3} A_2 \frac{\langle \mu_G^2 \rangle_{H_b}}{m_b^2} \left( I_0(x, x, 0) + I_2(x, x, 0) \right) \right], \]

\[ \Gamma_0 \equiv \frac{G_F^2 m_b^5}{192 \pi^3} |V(cb)|^2. \]

The following notation has been used here: $I_0$ and $I_2$ are phase-space factors, namely

\[ I_0(x, 0, 0) = (1 - x^2)(1 - 8x + x^2) - 12x^2 \log x \]

\[ I_2(x, x, 0) = (1 - x)^3, \quad x = (m_c/m_b)^2 \]

for $b \to c\bar{u}d/c\bar{l}\bar{v}$ and

\[ I_0(x, x, 0) = v(1 - 14x - 2x^2 - 12x^3) + 24x^2(1 - x^2) \log \frac{1 + v}{1 - v}, \quad v = \sqrt{1 - 4x} \]

\[ I_2(x, x, 0) = v(1 + \frac{x}{2} + 3x^2 - 3x(1 - 2x^2) \log \frac{1 + v}{1 - v}, \]

for $b \to c\bar{c}s$ transitions. The quantities $\eta_{SL}$, $\rho_{cc}$, $A_0$ and $A_2$ represent the QCD radiative corrections. More specifically one has

\[ A_2 = (c_+^2 - c_-^2), \quad A_0 = (c_+^2 + 2c_-^2) \cdot J \]

with $J$ reflecting the effect of the subleading logarithms \[30\] and

\[ \eta_{SL} \simeq 1 + \frac{2 \alpha_s}{\pi} g(m_c/m_b, m_t/m_b, 0). \]

The function $g$ can be computed numerically for arbitrary arguments \[28\] and analytically for the most interesting case $m_t = 0$ \[29\]. The allowed values for $\rho_{cc}$, which reflects the fact that QCD radiative corrections are quite sensitive to the final state quark masses, can be found in ref.\[31\].
With \( x \simeq 0.08 \) one obtains
\[
I_0(x, 0, 0)|_{x=0.08} \simeq 0.56, \quad I_2(x, 0, 0)|_{x=0.08} \simeq 0.78 \quad \text{for} \quad b \to c\bar{d}
\]
\[
I_0(x, x, 0)|_{x=0.08} \simeq 0.24, \quad I_2(x, x, 0)|_{x=0.08} \simeq 0.32 \quad \text{for} \quad b \to c\bar{s}
\]
Since these functions are normalized to unity for \( x = 0 \), one notes that the final-state quark masses reduce the available phase space quite considerably.

Some qualitative statements can illuminate the dynamical situation:

(i) As indicated in eqs.\( (15) \), \( \Gamma_{\text{SL/NL, decay}} \) differ from the naive spectator result in order \( 1/m_b^2 \).

(ii) Since \( \bar{b}i\sigma \cdot Gb \) and \( \bar{b}(i\vec{D})b \) are \( SU(3)_{FL} \) singlet operators and their expectation values are practically isospin and even \( SU(3)_{FL} \) invariant, one obtains, as stated before, \( \tau(B_d) \simeq \tau(B^-) \simeq \tau(B_s) \) through order \( 1/m_b^2 \) and likewise \( \tau(\Lambda_b) \simeq \tau(\Xi_b) \). Yet the meson and baryon lifetimes get differentiated on this level since \( \langle \mu_G^2 \rangle_B > 0 = \langle \mu_G^2 \rangle_{\Lambda_b, \Xi_b} \) and \( \langle (\vec{p}_b)^2 \rangle_B \neq \langle (\vec{p}_b)^2 \rangle_{\Lambda_b} \), see eqs.\( (20,30) \).

(iii) The semileptonic branching ratios of \( \Lambda_b \) and \( \Xi_b \) baryons remain unaffected in order \( 1/m_b^2 \) due to \( \langle \mu_G^2 \rangle_{\Lambda_b, \Xi_b} \simeq 0 \), whereas for \( B \) mesons it is (slightly) reduced \cite{10}.

Beyond order \( 1/m_b^2 \), however, this relation is changed, as discussed later on.

### 6.1 \( B^- \) vs. \( B_d \) Lifetimes

Differences in \( \tau(B^-) \) vs. \( \tau(B_d) \) are generated by the local dimension-six four-quark operators \( (\bar{b}_L\gamma_\mu q_L)(\bar{q}_L\gamma_\nu b_L) \) which explicitely depend on the light quark flavours. Such corrections are of order \( 1/m_b^3 \). Based on this scaling law one can already infer from the observed \( D \) meson lifetime ratios that the various \( B \) lifetimes will differ by no more than 10% or so. Assuming factorization for the expectation values of these four-fermion operators, i.e. applying eq.\( (22) \), one obtains

\[
\frac{\Gamma(B_d) - \Gamma(B^-)}{\Gamma(B)} \sim \frac{\Gamma_{\text{nonspec}}(B)}{\Gamma_{\text{spec}}(B)} \propto \frac{f_B^2}{m_b^3},
\]

as discussed above.

It turns out that WA can change the \( B \) lifetimes by no more than 1% or so; due to interference with the spectator reaction, they could conceivably prolong the \( B_d \) lifetime relative to the \( B^- \) lifetime, rather than reduce it. The dominant effect is clearly provided by PI, which produces a negative contribution to the \( B^- \) width:

\[
\Gamma(B^-) \simeq \Gamma_{\text{spec}}(B) + \Delta \Gamma_{\text{PI}}(B^-)
\]
\[ \Delta \Gamma_{PI}(B^-) \simeq \Gamma_0 \cdot 24\pi^2 \frac{f_B^2}{m_b^2} \left[ c_+^2 - c_-^2 + \frac{1}{N_C} (c_+^2 + c_-^2) \right] . \]  

Eq.(48b) exhibits an intriguing result: \( \Delta \Gamma_{PI}(B^-) \) is positive for \( c_+ = 1 = c_- \), i.e. PI acts constructively and thus would shorten the \( B^- \) lifetime in the absence of radiative QCD corrections \[^12\]. Including those as evaluated on the leading-log and next-to-leading-log level – \( c_+ \simeq 0.84, c_- \simeq 1.42 \) – turns PI into a destructive interference prolonging the \( B^- \) lifetime, albeit by a tiny amount only at this point. In eq.(48b) only ultraviolet renormalization has been incorporated. Hybrid renormalization \[^9\] down to the hadronic scale \( \mu_{\text{had}} \) amplifies this effect considerably; one obtains

\[ \Delta \Gamma_{PI}(B^-) \simeq \Gamma_0 \cdot 24\pi^2 \frac{f_B^2}{M_B^2} \kappa^{-4} \left[ (c_+^2 - c_-^2)\kappa^{9/4} + \frac{c_+^2 + c_-^2}{3} - \frac{1}{9} (\kappa^{9/2} - 1)(c_+^2 - c_-^2) \right] , \]

\[ \kappa \equiv \left[ \frac{\alpha_s(\mu_{\text{had}})}{\alpha_s(m_b^2)} \right]^{1/b} , \quad b = 11 - \frac{2}{3} n_F \]  

Putting everything together one finds:

\[ \frac{\tau(B^-)}{\tau(B_d)} \simeq 1 + 0.05 \cdot \frac{f_B^2}{(200 \text{ MeV})^2} , \]  

i.e. the lifetime of a charged \( B \) meson is predicted to definitely exceed that of a neutral \( B \) meson by typically several percent.

Three comments are in order for properly evaluating eq.(49):

- Although WA could conceivably prolong the \( B_d \) lifetime as stated before, its numerical significance pales by comparison to that of PI. PI acts destructive in \( B^- \) decays once radiative QCD corrections are included to the best of our knowledge. The lifetime of \( B^- \) mesons therefore has to exceed that of \( B_d \) mesons.

- While the sign of the effect is predicted in an unequivocal manner, its magnitude is not. The main uncertainty in the prediction on \( \tau(B^-)/\tau(B_d) \) is given by our present ignorance concerning the size of \( f_B \).

- Even a precise measurement of the lifetime ratio \( \tau(B^-)/\tau(B_d) \) would not automatically result in an exact determination of \( f_B \) by applying eq.(50). For corrections of order \( 1/m_b^4 \) have not (or only partially – see the discussion in the preceding subsection) been included. Unless those have been determined, one cannot extract the size of \( f_B \) even from an ideal measurement with better than a roughly 15% uncertainty.\[^{12}\] This shows that the term of ‘Pauli Interference’ should \textit{not} be construed as implying that the interference is a priori destructive.
6.2 $B_s$ Lifetimes

Very little $SU(3)_{Fl}$ breaking is anticipated between the $B_d$ and $B_s$ expectation values of the two dimension-five operators. Among the contributions from dimension-six operators WA affects $B_d$ and $B_s$ lifetimes somewhat differently due to different colour factors for these two decays. Yet it is quite irrelevant in either case. Thus one predicts the $B_d$ and the average $B_s$ lifetimes to practically coincide:

$$\tau(B_d) \simeq \bar{\tau}(B_s) \pm O(1\%) \quad (51)$$

The term ‘average $B_s$ lifetime’ is used for a reason: $B_s - \bar{B}_s$ oscillations generate two neutral beauty mesons carrying strangeness that differ in their mass as well as in their lifetime. Due basically to $m_t^2 \gg m_c^2$ one finds $\Delta m(B_s) \gg \Delta \Gamma(B_s) \neq 0$. While $\Delta m(B_s)$ can be calculated in terms of the expectation value of the local four-fermion operator $(\bar{b}_L \gamma_\mu q_L)(\bar{b}_L \gamma_\mu q_L)$ (since $m_b \ll m_t$), the situation is more complex for $\Delta \Gamma(B_s)$, since the underlying operator is nonlocal. One can however apply a heavy quark expansion; to lowest nontrivial order one obtains [33]

$$\frac{\Delta \Gamma(B_s)}{\Gamma(B_s)} \equiv \frac{\Gamma(B_{s,short}) - \Gamma(B_{s,long})}{\Gamma(B_s)} \simeq 0.18 \cdot \frac{(f_{B_s})^2}{(200 \text{ MeV})^2} \quad (52)$$

for $f_{B_s}$ not much larger than 200 MeV. Comparing eqs.(50-52) leads to the intriguing observation that the largest lifetime difference among $B^-, B_d$ and $B_s$ mesons is generated by a very subtle source: $B_s - \bar{B}_s$ oscillations! There are also two different lifetimes for neutral $B$ mesons without strangeness; yet $\Delta \Gamma(B_d)$ is suppressed by $\sim \sin^2 \theta_c$ relative to $\Delta \Gamma(B_s)$.

One can search for the existence of two different $B_s$ lifetimes by comparing $\tau(B_s)$ as measured in $B_s \rightarrow \psi\eta/\psi\phi$ on one hand and in $B_s \rightarrow l\nu X$ on the other. The former decay predominantly leads to a CP even final state and thus would, to good accuracy, reveal $\tau(B_{s,short})$; the latter exhibits the average lifetime $\bar{\tau}(B_s) = [\tau(B_{s,long}) - \tau(B_{s,short})]/2$; for semileptonic $B_s$ decays involve the CP even and odd components in a nearly equal mixture. Thus

$$\tau(B_s \rightarrow l\nu D^{(*)}) - \tau(B_s \rightarrow \psi\eta/\psi\phi) \simeq \frac{1}{2}[\tau(B_{s,long}) - \tau(B_{s,short})] \simeq 0.09 \cdot \frac{(f_{B_s})^2}{(200 \text{ MeV})^2}. \quad (53)$$

Whether an effect of this size is large enough to be ever observed in a real experiment, is doubtful. Nevertheless one should search for it even if one has sensitivity only for a 50% lifetime difference or so. For while eq.(52) represents the best presently available estimate, it is not a ‘gold-plated’ prediction. It is conceivable that the underlying computation underestimates the actual lifetime difference!

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13 This issue will be addressed in some detail in the later discussion of $D^0$ vs. $D_s$ lifetimes.
6.3 $B_c$ Lifetime

$B_c$ mesons with their two heavy constituents $- B_c = (b\bar{c})$ represent a highly intriguing system that merits special efforts to observe it. One expects a rich spectroscopy probing the inter-quark potential at distances intermediate to those that determine quarkonia spectroscopy in the charm and in the beauty system. The Isgur-Wise function for the striking channel $B_c \rightarrow l\nu\psi$ can be computed. What is most relevant for our discussion here is that its overall decays, and thus its lifetime, reflect the interplay of three classes of transitions, namely the decay of the $b$ quark, that of the $\bar{c}$ (anti)quark and WA of $b$ and $\bar{c}$:

$$\Gamma(B_c) \simeq \Gamma_{b\rightarrow c}(B_c) + \Gamma_{\bar{c}\rightarrow \bar{s}}(B_c) + \Gamma_{WA}(B_c) \quad (54)$$

While $b \rightarrow c$ and $\bar{c} \rightarrow \bar{s}$ transitions do not interfere with each other in any practical way and one can thus cleanly separate their widths, the situation is much more delicate concerning $\Gamma_{b\rightarrow c}$ and $\Gamma_{WA}$, as briefly explained later. Yet for the moment I ignore the latter although its helicity and wavefunction suppressions represented by the factors $m_b^2/m_c^2$ and $f_{B_c}^2/m_b^2$ are relatively mild ($f_{B_c} \simeq 400 - 600$ MeV [34]) and partially offset by the numerical factor $16\pi^2$ reflecting the enhancement of two-body phase space – relevant for WA – over three-body phase space appropriate for the spectator decay.

Concerning the other two transitions one would naively expect the $\bar{c}$ decay to dominate: $\Gamma_{\bar{c}\rightarrow \bar{s}}(B_c) \sim \Gamma(D^0) \simeq (4 \times 10^{-13}\text{ sec})^{-1} > \Gamma_{b\rightarrow c}(B_c) \sim \Gamma(B) \simeq (1.5 \times 10^{-12}\text{ sec})^{-1}$. It had been suggested that for a tightly bound system like $B_c$ the decay width should be expressed not in terms of the usual quark masses (whatever they are), but instead in terms of effective quark masses reduced by something like a binding energy $\mu_{BE} \sim 500$ MeV. If so, then the $B_c$ width would be reduced considerably since due to $\Gamma_Q \propto m_Q^5$ one would find a relative reduction $(\Gamma_Q + \Delta \Gamma_Q)/\Gamma_Q \sim (m_Q - \mu_{BE})^3/m_Q^5 \sim 1 - 5\mu_{BE}/m_Q$. Even more significantly, beauty decays would become more abundant than charm decays in the $B_c$ transitions since the binding energy constitutes a higher fraction of $m_c$ than of $m_b$: $\mu_{BE}/m_c > \mu_{BE}/m_b$. However this conjecture that might look quite plausible at first sight, turns out to be fallacious! For it is manifestly based on the existence of nonperturbative corrections of order $1/m_Q^3$; yet as discussed in Sect.4 those are absent in fully integrated decay rates like lifetimes due to a non-trivial cancellation between $1/m_Q$ contributions from initial-state and final-state radiation! The leading corrections arise in order $1/m_Q^3$ and they enter through the first two terms on the right-hand-side of eq.(54):

$$\Gamma_{b\rightarrow c}(B_c) = \Gamma_{b\rightarrow c,\text{decay}}(B_c) + \mathcal{O}(1/m_b^3) \quad (55a)$$

$$\Gamma_{\bar{c}\rightarrow \bar{s}}(B_c) = \Gamma_{\bar{c}\rightarrow \bar{s},\text{decay}}(B_c) + \mathcal{O}(1/m_c^3) \quad (55b)$$

The quantities $\Gamma_{b\rightarrow c,\text{decay}}(B_c)$ and $\Gamma_{\bar{c}\rightarrow \bar{s},\text{decay}}(B_c)$ are defined as in eqs.(45) and (68); i.e., the differences between $\Gamma_{\text{decay}}(B)$ and $\Gamma_{b\rightarrow c,\text{decay}}(B_c)$ enter through the different expectation values of the same operators $\bar{Q}Q$ and $\bar{Q}_c\bar{c}\sigma \cdot GQ$, and likewise for $\Gamma_{\text{decay}}(D)$.
(to be discussed in detail in Sect. 7) vs. \( \Gamma_{c \to \bar{s}, \text{decay}}(B_c) \). For the \( B_c \) meson provides a different environment for quark decay than either \( B \) or \( D \) mesons. The relevance of that difference is illustrated by eqs.\((28,29)\). Similarly one expects \( \langle (\vec{p}_b)^2 \rangle_{B_c}, \langle (\vec{p}_c)^2 \rangle_{B_c} \) to differ from \( \langle (\vec{p}_b)^2 \rangle_B, \langle (\vec{p}_c)^2 \rangle_D \).

While there are large corrections of order \( 1/m_c^2 \) in \( \Gamma_{c \to \bar{s}, \text{decay}}(B_c) \) that reduce the corresponding semileptonic branching ratio considerably, they largely cancel against each other in the total width. Therefore

\[
\Gamma_{c \to \bar{s}, \text{decay}}(B_c) \sim \Gamma_{\text{decay}}(D) \simeq \Gamma(D^0)
\]

Writing the \( \Delta B = 1 \) width of the \( B_c \) meson as a simple incoherent sum \( \Gamma_{b \to c} + \Gamma_{\text{WA}} \) actually represents an oversimplification. For there arises considerable interference between higher order \( \text{WA} \) and spectator processes. Yet for the purposes of our discussion here such effects can be ignored; they will be discussed in detail in a forthcoming publication. The \( 1/m_b^2 \) contributions to \( \Gamma_{c \to \bar{s}, \text{decay}}(B_c) \) are small, namely around a few percent. The numerical impact of \( \Gamma_{\text{WA}}(B_c \to X_{cs}) \), which is formally of order \( 1/m_b^3 \), is nevertheless sizeable due to the large decay constant, the merely mild helicity suppression, given by \( m_c^2/m_b^2 \), and the enhancement of \( \text{WA} \) by a factor \( \sim 16\pi^2 \) due to its two-body kinematics. One finds:

\[
\Gamma_{b \to c}(B_c) \geq \Gamma_{b \to c}(B).
\]

Comparing eqs.\((56,57)\) one concludes that the naive expectation turns out to be basically correct, i.e. \( \tau(B_c) \) is well below \( 10^{-12} \) sec and charm decays dominate over beauty decays with an ensuing reduction in the ‘interesting’ branching ratios like \( B_c \to l\nu\psi \) or \( B_c \to \psi\pi \).

### 6.4 Beauty Baryon Lifetimes

It is quite natural to assume that the kinetic energy of the \( b \) quark is practically the same inside the \( \Lambda_b \) and \( \Xi_b \) baryons:

\[
\langle (\vec{p}_b)^2 \rangle_{\Lambda_b} \simeq \langle (\vec{p}_b)^2 \rangle_{\Xi_b}
\]

Together with eqs.\((15,17a)\) this yields

\[
\langle \Lambda_b|\bar{b}b|\Lambda_b \rangle = \langle \Xi_b|\bar{b}b|\Xi_b \rangle + \mathcal{O}(1/m_b^3)
\]

The leading differences among the \( \Lambda_b \) and \( \Xi_b \) lifetimes then arise in order \( 1/m_b^3 \); they are generated by four-quark operators analogous to those that had already been identified in the phenomenological studies of charm baryons\[3\,\text{a},\,3\,\text{b},\,3\,\text{c}\]. Some complexities
arise, though, due to the presence of the two transitions $b \rightarrow \bar{c}ud$ and $b \rightarrow \bar{c}cs$; one finds

\[
\Gamma (\Lambda_b) = \Gamma_{\text{decay}}(\Lambda_b) + \Gamma_{\text{WS}}(\Lambda_b) - |\Delta \Gamma_{PL-}(\Lambda_b, b \rightarrow \bar{c}ud)| \quad (59a)
\]

\[
\Gamma (\Xi_b^0) = \Gamma_{\text{decay}}(\Xi_b^0) + \Gamma_{\text{WS}}(\Xi_b^0) - |\Delta \Gamma_{PL-}(\Xi_b^0, b \rightarrow \bar{c}cs)| \quad (59b)
\]

\[
\Gamma (\Xi_b^-) = \Gamma_{\text{decay}}(\Xi_b^-) - |\Delta \Gamma_{PL-}(\Xi_b^-, b \rightarrow \bar{c}ud)| - |\Delta \Gamma_{PL-}(\Xi_b^-, b \rightarrow \bar{c}cs)| \quad (59c)
\]

where

\[
\frac{\Gamma_{\text{decay}}(\Lambda_b/\Xi_b)}{\Gamma_0} = 2\Gamma_{\text{SL,decay}}(\Lambda_b/\Xi_b) + \Gamma_{\text{NL,decay}}(\Lambda_b/\Xi_b) \quad (60a)
\]

\[
\frac{\Gamma_{\text{SL,decay}}(\Lambda_b/\Xi_b)}{\Gamma_0} = \langle \Lambda_b/\Xi_b | \bar{b}b | \Lambda_b/\Xi_b \rangle_{\text{norm}} \cdot \eta_{\text{SL}} I_0(x, 0, 0) \quad (60b)
\]

\[
\frac{\Gamma_{\text{NL,decay}}(\Lambda_b/\Xi_b)}{\Gamma_0} = \langle \Lambda_b/\Xi_b | \bar{b}b | \Lambda_b/\Xi_b \rangle_{\text{norm}} \cdot A_0 \cdot [I_0(x, 0, 0) + \rho_{\text{ce}} I_0(x, x, 0)] \quad (60c)
\]

using the same notation as in eqs.(45);

\[
\Gamma_{\text{WS}}(\Lambda_b, \Xi_b) \equiv 2\tilde{\Gamma}_0 c^2_b \langle \Xi_b | \bar{b} L \gamma_\mu b L \bar{u} L \gamma_\mu u L | \Lambda_b \rangle_{\text{norm}} \quad (60d)
\]

\[
\frac{\Delta \Gamma_{PL-}(\Lambda_b, b \rightarrow \bar{c}ud)}{\Gamma_0} \equiv -c_+ (2c_+ - c_-) \langle \Lambda_b | \bar{b} L \gamma_\mu b d L \gamma_\mu d L | \Lambda_b \rangle_{\text{norm}} + \frac{2}{3} \bar{b} \gamma_\mu \gamma_5 b \bar{d} L \gamma_\mu d_L | \Lambda_b \rangle_{\text{norm}} \quad (60e)
\]

\[
\frac{\Delta \Gamma_{PL-}(\Xi_b^-, b \rightarrow \bar{c}cs)}{\Gamma_0} \equiv -c_+ (2c_+ - c_-) \langle \Xi_b^- | \bar{b} L \gamma_\mu b s L \gamma_\mu s_L | \Xi_b^- \rangle_{\text{norm}} + \frac{2}{3} \bar{b} \gamma_\mu \gamma_5 b \bar{s} L \gamma_\mu s_L | \Xi_b^- \rangle_{\text{norm}} \quad (60e)
\]

\[
\tilde{\Gamma}_0 \equiv 48\pi^2 \Gamma_0 \quad (60f)
\]

\[
\Delta \Gamma_{PL-}(\Xi_b^-, b \rightarrow \bar{c}ud) \quad (60g)
\]

\[
\Delta \Gamma_{PL-}(\Xi_b^-, b \rightarrow \bar{c}cs) \quad (60g)
\]

\[
\Delta \Gamma_{PL-}(\Xi_b^-, b \rightarrow \bar{c}ud) \text{ is obtained from eq.}(60e) \text{ by taking the expectation value between } \Xi_b^- \text{ rather than } \Lambda_b \text{ states. Those expectation values vanish for the wrong charge-flavour combination; i.e.}, \langle \Xi_b^- | \bar{b} L \gamma_\mu b d L \gamma_\mu d_L | \Xi_b^- \rangle = 0 = \langle \Lambda_b | \bar{b} L \gamma_\mu b d L \gamma_\mu d_L | \Lambda_b \rangle.
\]

\[
\Gamma_{\text{decay}} \text{ includes the naive spectator term: } \Gamma_{\text{decay}} = \Gamma_{\text{spect}} + 1/m_b^2 \text{ contributions; the latter are practically identical for } \Lambda_b \text{ and } \Xi_b, \text{ but differ for } B \text{ mesons. } \Gamma_{\text{WS}} \text{ denotes the contribution due to WS and } \Delta \Gamma_{PL-} \text{ the reduction due to destructive interference in the channels } b \rightarrow \bar{c}ud \text{ and } b \rightarrow \bar{c}cs, \text{ respectively.}
\]

On general grounds one thus obtains an inequality:

\[
\tau(\Xi_b^-) > \tau(\Lambda_b), \tau(\Xi_b^0) \quad (61)
\]

Naively one might expect $\Gamma(\Xi_b^0) > \Gamma(\Lambda_b)$ to hold, since the $b \rightarrow \bar{c}cs$ part of the width which is reduced by PI in $\Xi_b^0$ decays is smaller than the $b \rightarrow \bar{c}ud$ component which suffers from PI reduction in $\Lambda_b$ decays. On the other hand since phase space

\footnote{The channel $b \rightarrow \tau \nu q$ has been ignored here for simplicity.}

\footnote{When $|\Delta \Gamma_{PL-}(b \rightarrow \bar{c}ud)|$ is used in eq.(59c), one has, strictly speaking, to evaluate the expectation value for the state $\Xi_b$.}
is more restricted for \( b \to c\bar{c}s \) than for \( b \to c\bar{u}d \), one would likewise expect the degree of (destructive) interference to be higher for the former than the latter; it is then quite conceivable that actually \( \Gamma(\Xi_b^0) < \Gamma(\Lambda_b) \) holds. The sign of the difference in the \( \Lambda_b \) and \( \Xi_b^0 \) lifetimes therefore provides us with valuable information on the strong dynamics.

There are two complementary ways to transform these qualitative predictions into quantitative ones.

(i) One evaluates the required expectation values explicitly within a quark model, as expressed in eq.(17b, 23-25). Since the model also predicts the baryon masses in terms of its parameters, one can cross check it with the observed spectroscopy. This will become increasingly relevant in the future, yet at present provides little guidance. The expressions given in eqs.(23-25) have to be augmented by the radiative QCD corrections:

\[
\langle \Lambda_b | \bar{b} \Gamma_\mu b \bar{q} \Gamma_\nu q | \Lambda_b \rangle_{\text{norm}} \sim \frac{1}{4\langle \mu_G^2 \rangle_B} (M_{\Sigma_b} - M_{\Lambda_b}) m_q^* F_B^2 M_B \cdot \kappa^{-4} \tag{62a}
\]

\[
\langle \Omega_b | \bar{b} \Gamma_\mu b \bar{q} \Gamma_\nu q | \Omega_b \rangle_{\text{norm}} \sim \frac{1}{4\langle \mu_G^2 \rangle_B} (M_{\Sigma_b} - M_{\Omega_b}) m_q^* F_B^2 M_B \cdot \kappa^{-4} \tag{62b}
\]

with \( \kappa \) as defined in eq.(49). It should be noted that here – unlike in the case of meson decays – the sign of the PI contribution is quite robust under radiative corrections: it is proportional to \(-c_+ (2c_- - c_+)\) which is negative already for \( c_+ = 1 = c_- \). Furthermore \( \Gamma_{WS} \) is proportional to \( c_+^2 \) and thus colour-enhanced, since the baryon wavefunction is purely antisymmetric in colour space. This prescription yields lifetime differences of not more than a few percent in eq.(59) and

\[
\frac{\tau(\Lambda_b)}{\tau(B_d)} \simeq 0.9 - 0.95 . \tag{63a}
\]

(ii) As will be discussed in the next subsection, the pattern predicted for charm baryon lifetimes agrees with the observations within the experimental and theoretical uncertainties. Taking this as prima facie evidence that the heavy quark expansion through order \( 1/m_Q^3 \) applies – at least in a semi-quantitative fashion – already to charm lifetime ratios, one can extrapolate to the weight of these pre-asymptotic corrections in beauty decays using scaling like \( 1/m_Q^3 \) and \( 1/m_Q^3 \) (or \( f_M^2/m_Q^2 \)). That way one again finds that the differences in eq.(56) amount to not more than a few to several percent:

\[
\frac{\tau(\Lambda_b)}{\tau(B_d)} \simeq 0.9 . \tag{63b}
\]

Similarly one estimates \( \tau(\Xi_b) \) through a simple \( 1/m_Q^3 \) scaling behaviour from the charm baryon lifetimes:

\[
\frac{\tau(\Xi^-_b)}{\tau(\Lambda_b)} \sim 1.1 \tag{64a}
\]
\[ \frac{|\tau(\Xi_b^0) - \tau(\Lambda_b)|}{\tau(\Lambda_b)} < 0.1 \] (64b)

Obviously and not surprisingly there is some numerical fuzziness in these predictions; yet they seem to be unequivocal in stating that the \( B_d \) lifetime exceeds the \( \Lambda_b \) lifetime and the average beauty baryon lifetime by about 10 percent. However this prediction appears to be in serious (though not yet quite conclusive) disagreement with the data. If the predictions were based exclusively on adopting the quark model results for the baryonic expectation values, one could abandon them in a relatively light-hearted way: for it should not come as a shocking surprise that quark model results for baryonic matrix elements can be off-target by a substantial amount. Yet we have encountered a more serious problem here: data seem to contradict also the prediction based on an extrapolation from the observed lifetime pattern in the charm family; furthermore – as discussed next – the observed lifetime ratios of charm hadrons can be reproduced, within the expected uncertainties. This allows only one conclusion: if \( \tau(B_d) \) indeed exceeds \( \tau(\Lambda_b) \) by 25 - 30%, then a ‘theoretical price’ has to be paid, namely that

- the charm mass represents too low of a scale for allowing to go beyond merely qualitative predictions on charm baryon (or even meson) lifetimes, since it appears that corrections of order \( 1/m_c^4 \) and higher are still important;

- that the present agreement between theoretical expectations and data on charm baryon lifetimes is largely accidental and most likely would not survive in the face of more precise measurements!

At the same time an intriguing puzzle arises:

- Why are the quark model results for the relevant expectation values so much off the mark for beauty baryons? It is the deviation from unity in the lifetime ratios that is controlled by these matrix elements; finding a 30% difference rather than the expected 10% then represents a 300% error!

Some new features emerge in \( \Omega_b \) decays: since the \( ss \)-diquark system forms a spin-triplet, there are \( 1/m_b^2 \) contributions to the semileptonic and nonleptonic \( \Omega_b \) widths from the chromomagnetic operator. In order \( 1/m_b^3 \) there arises a destructive interference in the \( b \rightarrow c\bar{c}s \) channels. However a detailed discussion of \( \Omega_b \) decays seems academic at the moment.

### 6.5 Semileptonic Branching Ratios of Beauty Hadrons

As briefly discussed before, one expects the semileptonic branching ratios for \( B_d \) and \( B_s \) mesons to practically coincide, in particular since semileptonic decays probe \( \tau(B_s) \), the average \( B_s \) lifetime.
It has already been pointed out that through order $1/m_b^2$ the expected value for $BR_{SL}(\Lambda_b)$ exceeds that for $BR_{SL}(B)$ by a few percent. In order $1/m_b^3$ the nonleptonic widths of both $B_d$ and $\Lambda_b$ states receive new contributions, with $\Gamma_{NL}(\Lambda_b)$ getting further enhanced relative to $\Gamma_{NL}(B)$ as expressed in eq.(63b). Thus one predicts

$$BR_{SL}(\Lambda_b) < BR_{SL}(B_d) < BR_{SL}(B^-)$$

with the inequalities indicating differences of a few to several percent. If the present trend in the data persists, i.e. if the total $\Lambda_b$ width exceeds $\Gamma(B)$ by, say, 20 - 25 %, one would interprete this discrepancy as meaning that the nonleptonic – but not the semileptonic – width has received an unforeseen enhancement. In that case one expects $BR_{SL}(\Lambda_b)$ to fall below $BR_{SL}(B_d)$ by $\sim 20 \%$.

Putting all these observations together one concludes that the beauty lifetime averaged over $B_d$, $B^-$, $B_s$ and $\Lambda_b$ states should yield a value that is smaller – by a few percent – than $\frac{1}{2}[BR_{SL}(B_d) + BR_{SL}(B^-)] \equiv \langle BR_{SL}(B) \rangle$:

$$\langle BR_{SL}(b) \rangle < \langle BR_{SL}(B) \rangle$$

### 7 Predictions on Charm Lifetimes

Considering the wealth of rather precise experimental information available on the lifetimes of charm hadrons one feels the urge to apply heavy quark expansions to charm decays as well. Yet in doing so one has to keep in mind that such a treatment might fail here for two basic reasons, one of which has just been stated:

(i) The charm quark mass does not provide a sufficiently large scale to make the $1/m_c$ expansion converge quickly. To obtain an estimate of the size of the expansion parameter $\mu_{had}/m_c$ one can take the square root of the expression in eq.(35) representing the $1/m_c^2$ corrections:

$$\frac{\mu_{had}}{m_c} \sim \sqrt{\frac{\langle \mu_{GL}^2 \rangle_D}{m_c^2}} \simeq 0.45 .$$

This is not a small number although it is at least smaller than unity.

(ii) As discussed in Sect.4 one has to go beyond global duality and invoke local duality to predict the decay widths of real hadrons from heavy quark expansions properly defined in Euclidean space. Yet a priori it is not clear at all whether contributions from ‘distant cuts’ can be ignored since the charm quark mass does not exceed ordinary hadronic scales by a large amount. This concern is a posteriori strengthened by the following observation: Equating the observed semileptonic width of $D$ mesons with its theoretical expression through order $1/m_c^2$ (and assuming $|V_{cs}| \simeq 1$) leads to the requirement $"m_c" \simeq 1.6$ GeV. This is however a high value relative to what is
derived from charmonium spectroscopy, namely \( m_c \leq 1.4 \) GeV. A difference of 0.2 GeV in \( m_c \) might appear quite innocuous – till one realizes that the corresponding semileptonic width depending on \( m_c^5 \) changes by a factor of two when \( m_c \) is shifted by those 0.2 GeV! At this point one might suspect that corrections of higher order in \( 1/m_c \) contribute constructively boosting the theoretical value. The analysis of ref. [36] finds however that terms of order \( 1/m_c^3 \) show a tendency to decrease \( \Gamma_{SL}(D) \), and their inclusion does not help at all in reproducing the measured value of \( \Gamma_{SL}(D) \) with \( m_c \simeq 1.4 \) GeV. This suggests that contributions from ‘distant cuts’ which cannot be seen in any finite order of the \( 1/m_c \) expansion create this problem. For a different opinion on this point, see ref. [27].

I draw the following somewhat tentative conclusions from these observations:

- Predictions for the absolute size of charm hadron lifetimes cannot be trusted.
- However it is quite conceivable that lifetime ratios do not suffer from such a fundamental uncertainty due to not-so-distant cuts. I will then explore the working hypothesis that the ratio of lifetimes can be trusted in principle – though in practise only with a quite considerable grain of salt!

The quark level Lagrangian is again well known. On the Cabibbo-allowed level there is now a single non-leptonic transition described by

\[
L_\Delta^C(\mu = m_c) = \frac{4G_F}{\sqrt{2}} V_{cs} V_{ud}^* \left\{ c_1 (\bar{s}_L \gamma_\mu c_L)(\bar{u}_L \gamma_\mu d_L) + c_2 (\bar{u}_L \gamma_\mu c_L)(\bar{s}_L \gamma_\mu d_L) \right\} \tag{66}
\]

where the short-distance coefficients \( c_{1,2} \) are now evaluated at a lower\( J \)scale than in beauty decays yielding

\[
c_1(LL + NLL) \simeq 1.32, \quad c_2(LL + NLL) \simeq -0.58 \tag{67}
\]

The expressions for the semileptonic and non-leptonic decay widths of charm hadrons \( H_c \) are quite analogous to the ones for beauty hadrons (though simpler since there is only one non-leptonic decay class rather than two); through order \( 1/m_c^2 \) they are given by

\[
\frac{\Gamma_{SL, \text{decay}}(H_c)}{\Gamma_0} = \frac{\langle H_c|\bar{c}c|H_c \rangle_{\text{norm}}}{\langle H_c|\bar{c}c|H_c \rangle_{\text{norm}}} \left[ I_0(x, 0, 0) + \frac{\langle \mu_G^2 \rangle_{H_c}}{m_c^2} (x \frac{d}{dx} - 2)I_0(x, 0, 0) \right], \tag{68a}
\]

\[
\frac{\Gamma_{NL, \text{decay}}(H_c)}{\Gamma_0} = N_C \cdot \langle H_c|\bar{c}c|H_c \rangle_{\text{norm}} \left[ A_0[I_0(x, 0, 0) + \frac{\langle \mu_G^2 \rangle_{H_c}}{m_c^2} (x \frac{d}{dx} - 2)I_0(x, 0, 0)] - 8A_2 \frac{\langle \mu_G^2 \rangle_{H_c}}{m_c^2} \cdot I_2(x, 0, 0) \right]. \tag{68b}
\]
where now
\[ \Gamma_0 \equiv \frac{G_F^2 m_c^5}{192\pi^3} |V(cs)|^2, \quad x = \frac{m_s^2}{m_c^2} \]  
(68c)
and the radiative corrections lumped into \( A_0 \) and \( A_2 \) are given by the appropriate values for \( c_+ \) and \( c_- \), see eq. (67). With \( x \sim 0.012 \) one finds:
\[ I_0(x, 0, 0)|_{x=0.012} \simeq 0.91, \quad I_2(x, 0, 0)|_{x=0.012} \simeq 0.96, \]
i.e. there is much less phase space suppression than for \( b \to c \) transitions.

7.1 \( D^+ \) vs. \( D^0 \) Lifetimes

Analogously to the case of \( \tau(B^-) \) vs. \( \tau(B_d) \) the \( D^+ \) and \( D^0 \) lifetimes get first differentiated in order \( 1/m_c^3 \) when PI and WA intervene. Both affect the lifetime ratio in the same direction, namely they enhance \( \tau(D^+) \) over \( \tau(D^0) \) with the destructive interference due to PI being the dominant effect. Quantitatively one finds through order \( 1/m_c^3 \) by employing eqs.(48,49) with the appropriate substitutions and using \( f_D \sim 200 \text{ MeV} \):
\[ \frac{\tau(D^+)}{\tau(D^0)} \sim 2 \]  
(69)
For proper perspective one has to keep four observations in mind:

(i) While the expectation expressed in eq.(69) does not coincide numerically with the measured value – \( \tau(D^+)/\tau(D^0) = 2.547 \pm 0.043 \) – it agrees with it to within \( \sim 25 \% \). Such a deviation could be ascribed to \( 1/m_c^4 \) contributions ignored in eq.(69).

(ii) On the other hand, as discussed before, there is reason to doubt the reliability and thus validity of heavy quark expansions for treating nonperturbative corrections in charm decays. Yet I adopt, as already stated, the working hypothesis that a heavy quark expansion can be employed for treating the ratios of lifetimes, though not the lifetimes themselves. Such a conjecture is tested a posteriori by analysing the whole pattern of the expected and the observed lifetimes of the various charm hadrons.

(iii) Even the perturbative corrections contain sizeable numerical uncertainties. To cite but one glaring example: the size and the nature of the so-called ‘hybrid’ renormalization reflecting dynamics between the scales \( m_c \) and \( \mu_{\text{had}} \sim 0.5 - 1 \text{ GeV} \) is quite important quantitatively (they considerably enhance the destructive interference in \( D^+ \) decays.) Yet a leading-log treatment of those corrections seems woefully inadequate for dealing with such a small slice in momentum space.

(iv) One should note also the following: while WA plays only a relatively minor role in inclusive rates (generating only a 10 % or so difference in \( D^+ \) vs. \( D^0 \) lifetimes), it is likely to play a considerably more significant role in exclusive modes.

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7.2 $D_s^+$ vs. $D^0$ Lifetimes

Since the impact of WA is reduced relative to that of PI in meson decays, it is natural to compare $\tau(D_s)$ to $\tau(D^0)$ rather than to $\tau(D^+)$. Such a comparison – and of their semileptonic branching ratios – touches upon several intriguing dynamical issues. A priori $\tau(D_s)$ and $\tau(D^0)$ could differ substantially from each other due to $SU(3)_{FL}$ breaking and in particular due to a different weight of WA in the two cases. Yet the heavy quark expansion strongly suggests those two lifetimes to be close to each other.

The analysis proceeds in several steps [15]. The two operators contributing in order $1/m_c^3$ are singlets under the light quark flavours; yet $SU(3)_{FL}$ breaking could enter through their expectation values. For the chromomagnetic operator one has:

$$\frac{1}{M_D} \langle D^0 | \bar{c} i \sigma \cdot G_c | D^0 \rangle \simeq \frac{3}{2} (M_{D^*}^2 - M_{D^0}^2)$$ (70)

$$\frac{1}{M_{D_s}} \langle D_s | \bar{c} i \sigma \cdot G_c | D_s \rangle \simeq \frac{3}{2} (M_{D_s^*}^2 - M_{D_s}^2)$$ (71)

Since the measured values for the $D - D^*$ and the $D_s - D_s^*$ mass splittings are practically identical ($M_{D^*} - M_{D^0} = 142.12 \pm 0.07$ MeV and $M_{D_s^*} - M_{D_s} = 141.6 \pm 1.8$ MeV), the chromomagnetic operator cannot induce an appreciable difference between $\tau(D^0)$ and $\tau(D_s)$. [16]

Through order $1/m_c^3$ there are four distinct sources for a difference in $\Gamma(D_s)$ vs. $\Gamma(D^0)$ exceeding the 1% level: (a) The decay $D_s \to \tau \nu$; (b) PI in those Cabibbo suppressed $D_s$ decays that are driven by the quark level transition $c \to s \bar{s} u$; (c) the effects of $SU(3)_{FL}$ breaking on the expectation values of the kinetic energy operator; (d) WA in nonleptonic $D^0$ and in nonleptonic as well as semileptonic $D_s$ decays. Corrections listed under (a), (b) and (d) are generated by dim-6 operators whereas the much less familiar correction (c) is derived from a dim-5 operator.

(a) The width for the decay $D_s \to \tau \nu$ is completely determined in terms of the axial decay constant for the $D_s$ meson:

$$\Gamma(D_s) = \frac{G_F^2 m_{\tau} f_{D_s}^2 M_{D_s}}{8\pi} |V(cs)|^2 (1 - m_{\tau}^2/M_{D_s}^2)^2.$$ (72)

For $f_{D_s} \simeq 210$ MeV one gets

$$\Gamma(D_s \to \tau \nu) \simeq 0.03 \Gamma(D^0).$$ (73)

This effect necessarily reduces $\tau(D_s)$ relative to $\tau(D^0)$.

[16] The observation that the hyperfine splitting is largely independant of the flavour of the spectator (anti)quark can be understood intuitively in quark models.
(b) In $D_s$ decays PI appears in the Cabibbo-suppressed $c \rightarrow s\bar{s}u$ channel. Its weight is expressed in terms of the matrix elements of the two four-fermion operators

$$\langle D_s | (\bar{c} L \gamma_\mu s L)(\bar{s} L \gamma_\mu c L) | D_s \rangle$$

and

$$\langle D_s | (\bar{c} L \gamma_\mu \lambda^a s L)(\bar{s} L \gamma_\mu \lambda^a c L) | D_s \rangle$$

with known coefficients that are computed perturbatively (including the ‘hybrid’ renormalization of these operators down from the scale $m_c$). The effect of PI is most reliably estimated from the observed difference in the $D^+$ and $D^0$ widths. It is easy to see that the structure of the operators responsible for PI in $D_s$ decays is exactly the same as in $D^+$ decays if one replaces the $d$ quark with the $s$ quark and adds the extra factor tan$^2\theta_C$; it is then destructive as well. Thus one arrives at:

$$\delta \Gamma_{PI}(D_s) \simeq S \cdot \tan^2 \theta_C (\Gamma(D^+) - \Gamma(D^0)) \simeq -S \cdot 0.03 \Gamma(D^0)$$

(75)

where the factor $S$ has been introduced to allow for $SU(3)_{FL}$ breaking in the expectation values of the four-fermion operators. The factor $S$ is expected to exceed unity somewhat; in an factorization ansatz it is given by the ratio $f_{D_s}^2/f_D^2 \simeq 1.4$, see eq.(39). Thus

$$\delta \Gamma_{PI}(D_s) \simeq -0.04 \Gamma(D^0).$$

(76)

(c) The impact of the chromomagnetic operator on the $D_s - D^0$ lifetime ratio can be derived from the hyperfine splitting. Inserting the observed meson masses one obtains

$$\langle D^0 | \bar{c} i \frac{\sigma}{2} G c | D^0 \rangle_{\text{norm}} \simeq 0.413 \pm 0.002$$

(77a)

$$\langle D_s | \bar{c} i \frac{\sigma}{2} G c | D_s \rangle_{\text{norm}} \simeq 0.433 \pm 0.006$$

(77b)

from which one can conclude: there is – not surprisingly – very little $SU(3)_{FL}$ breaking in these matrix elements and a sizeable difference in $\tau(D_s)$ vs. $\tau(D^0)$ cannot arise from this source.

The second dim-5 operator generating $1/m_c^2$ corrections is the kinetic energy operator $\bar{c}(i\bar{D})^2 c$ where $\bar{D}$ denotes the covariant derivative. Its expectation value describes the largely non-relativistic motion of the charm quark in the gluon background field inside the charm hadron. It reflects Lorentz time dilation and thus prolongs the hadron lifetime. On general grounds one expects it to extend $\tau(D_s)$ over $\tau(D^0)$, as seen as follows. The spatial wavefunction should be more concentrated around the origin for $D_s$ than for $D^0$ mesons. This implies, via the uncertainty principle, the mean value of $(\vec{p}_c)^2$ to be larger for $D_s$ than for $D^0$; in other words the charm quark undergoes more Fermi motion as constituent of $D_s$ than of $D$ mesons. The lifetime of the charm quark is then prolonged by time dilation to a higher degree inside $D_s$ than inside $D^0$ mesons.

While the trend of this effect is quite transparent, its size is not yet clear. The relevant matrix elements $\langle D_s | \bar{c}(i\bar{D})^2 c | D_s \rangle$ and $\langle D | \bar{c}(i\bar{D})^2 c | D \rangle$ will be determined from QCD
sum rules and lattice simulations in the future. Yet in the meantime one can estimate their size from the meson masses in the charm and beauty family according to the following prescription:

\[
\langle D_s|\bar{c}(i\bar{D})^2c|D_s \rangle - \langle D|\bar{c}(i\bar{D})^2c|D \rangle \simeq \frac{2m_bm_c}{m_b-m_c}\left\{[(M_{D_s}) - (M_D)] - [(M_{B_s}) - (M_B)]\right\}
\]

where as before \(\langle M_{D,D_s,B,B_s} \rangle\) denote the spin averaged masses. Accordingly one finds

\[
\frac{\Delta \Gamma(D_s/D)}{\Gamma} \equiv \frac{\Gamma_{\text{Fermi}}(D_s) - \Gamma_{\text{Fermi}}(D)}{\Gamma} \simeq \frac{m_b}{m_c(m_b-m_c)} \times \left\{[(M_{D_s}) - (M_D)] - [(M_{B_s}) - (M_B)]\right\}.
\]

Since the \(B_s^*\) mass has not been measured yet, one cannot give a specific numerical prediction and has to content oneself with semi-quantitative statements. A 10 MeV shift in any of the terms \(\langle M \rangle\) corresponds numerically to the kinetic energy operator generating approximately a 1% change in the ratio \(\tau(D_s)/\tau(D^+)\). Invoking our present understanding of the heavy quark kinetic energy and its relationship to the hyperfine splitting one arrives at the following conjecture:

\[
\frac{\Delta \Gamma(D_s/D)|_{(a)+(b)+(c)}}{\Gamma} \sim O\left(\text{few \%}\right)
\]

(d) The mechanisms listed above in (a) - (c) taken together can be expected to extend the \(D_s\) over the \(D^0\) lifetime by at most a few percent. Comparing that estimate with the data allows us to conclude that WA cannot change \(\tau(D_s)/\tau(D^0)\) by more than 10% . It is however quite unclear how to refine this estimate at present. For the quantitative impact of WA on charm meson lifetimes in general and on the ratio \(\tau(D_s)/\tau(D^0)\) in particular is the most obscure theoretical element in the analysis. The uncertainty centers mainly on the question of how much the WA amplitude suffers from helicity suppression.

In the valence quark approximation the answer is easily given to lowest order in the strong coupling: the WA amplitude is (helicity) suppressed by the ratio \(m_q/m_c\), where \(m_q\) denotes the largest quark mass in the final state. For a proper QCD treatment one has to use the current rather than the larger constituent mass, at least for the \(1/m_c^2\) corrections. The WA amplitude is then small in \(D^0\) decays where the helicity factor reads \(m_s/m_c \sim 0.1\) and a fortiori in \(D_s\) decays where only non-strange quarks are present in the final state. The emission of semi-hard gluons that is included by summing the leading log terms in the perturbative expansion cannot circumvent this suppression \[13\]. For such gluon corrections – when properly accounted for – drive the hybrid renormalization of the corresponding four-fermion operators; however they preserve the Lorentz structure of the lowest order term and therefore do not eliminate
the helicity suppression. A helicity allowed amplitude arises in perturbation theory only at the subleading level of order $\alpha_s(m_c^2)/\pi$ and is thus expected to be numerically insignificant.

On the other hand nonperturbative dynamics can quite naturally vitiate helicity suppression and thus provide the dominant source of WA. These nonperturbative effects enter through non-factorizable contributions to the hadronic matrix elements, as analyzed in considerable detail in refs.\[41, 15\]. As such we do not know (yet), how to predict their weight from first principles. However, as shown in ref.\[41\], a detailed experimental study of the width of semileptonic decays and their lepton spectra – in particular in the endpoint region – in $D^0$ vs. $D_s$ and/or in $B^0$ vs. $B^+$ decays would allow us to extract the size of the matrix elements that control the weight of WA in all inclusive $B$ and $D$ decays. Before such data become available, we can draw only qualitative, or at best semi-quantitative conclusions: WA is not expected to affect the total lifetimes of $D_s$ and $D^0$ mesons by more than 10 - 20 %, and their ratio by less. Furthermore WA does not necessarily decrease $\tau(D_s)/\tau(D^0)$; due to its reduced amplitude its leading impact on the lifetime might be due to its interference with the spectator amplitude and thus it might even enhance $\tau(D_s)/\tau(D^0)$!

To summarize our findings on the $D_s - D^0$ lifetime ratio: (i) $SU(3)_{FL}$ breaking in the expectation values of the dim-5 operators generating the leading nonperturbative corrections of order $1/m_c^2$ can – due to ‘time dilatation’ – increase $\tau(D_s)$ by 3-5 % over $\tau(D^0)$. (ii) On the $1/m_c^3$ level there arise three additional effects. Destructive interference in Cabibbo suppressed $D_s$ decays increases $\tau(D_s)$ again by 3-5 %, whereas the single mode $D_s \rightarrow \tau\nu$ decreases it by 3 %. The three phenomena listed so far combine to yield $\tau(D_s)/\tau(D^0) \simeq 1.0 - 1.07$. (iii) Any difference over and above that has to be attributed to WA. Therefore one has to interpret the measured $D_s - D^0$ lifetime ratio as more or less direct evidence for WA to contribute no more than 10-20 % of the lifetime ratio between charm mesons. This is consistent with the indirect evidence discussed above that WA does not constitute the major effect there. (iv) These predictions can be refined in the future by two classes of more accurate data: analyzing the difference in the semileptonic spectra of charged and neutral mesons in the charm and beauty family allows to extract the size of the matrix elements that control the weight of WA; measuring the masses of $\Lambda_b$ and $B^*_s$ to better than 10 MeV allows to determine the expectation values of the kinetic energy operator.

7.3 Charm Baryon Lifetimes

The lifetimes of the weakly decaying charm baryons reflect a complex interplay of different dynamical mechanisms increasing or decreasing transition rates \[14, 1\]:

$$\Gamma(\Lambda_c^+) = \Gamma_{\text{decay}}(\Lambda_c^+) + \Gamma_{WS}(\Lambda_c^+) - |\Delta \Gamma_{PL,-}(\Lambda_c)|$$  \hspace{1cm} (81a)
\[ \Gamma(\Xi_0^c) = \Gamma_{\text{decay}}(\Xi_0^c) + \Gamma_{\text{WS}}(\Xi_0^c) + |\Delta \Gamma_{PI,+}(\Xi_0^c)| \] 

(81b)

\[ \Gamma(\Xi_+^c) = \Gamma_{\text{decay}}(\Xi_+^c) + |\Delta \Gamma_{PI,+}(\Xi_+^c)| - |\Delta \Gamma_{PI,-}(\Xi_+^c)| \] 

(81c)

The explicit expressions for \( \Gamma_{\text{decay}}, \Gamma_{\text{WS}} \) and \( \Delta \Gamma_{PI,\pm} \) are obtained from eqs.(60) by the obvious substitutions; for \( \Delta \Gamma_{PI,\pm} \) one finds

\[ \Delta \Gamma_{PI,\pm}(\Xi_c) \equiv \frac{G_F^2 |V(cs)|^2 m_c^2}{4\pi} c_{\pm}(2c_- + c_+) \langle \Xi_c | \bar{c}_L \gamma_\mu c \bar{s}_L \gamma_\mu s_L + \frac{2}{3} \bar{c} \gamma_\mu \gamma_5 c \bar{s}_L \gamma_\mu s_L \rangle |\Xi_c\rangle_{\text{norm}} \]

(82)

On rather general grounds one then concludes:

\[ \tau(\Xi_0^c) < \tau(\Xi_+^c), \quad \tau(\Xi_0^c) < \tau(\Lambda_c^+) \]

(83)

To go further requires calculating the relative weight of matrix elements of the four-quark operators. At present this can be done only by using quark wavefunctions as obtained from a potential ansatz (or from the quark model). In doing so one also has to pay proper attention to the normalization point appropriate for such an evaluation, i.e., one has to include ultraviolet as well as hybrid renormalization. One then arrives at the numbers quoted in Table 2. It has been argued \[14\] that in these expressions one should not use the ‘real’ value for the decay constant, as expressed by \( f_D \), and the observed mass splitting, but solely the leading contribution in a \( 1/m_Q^3 \) expansion: \( F_D \sim 400 \text{ MeV} \) and \( M_{\Sigma^+} - M_{\Lambda_c} \sim 400 \text{ MeV} \). One reason for that is self-consistency since the widths have been calculated through order \( 1/m_Q^3 \) only; the other – and maybe the more telling one – is based on the needs of phenomenology: for otherwise one cannot reproduce the observed magnitude of the lifetime differences. The predictions given in Table 2 agree remarkably well with the data within the experimental and theoretical uncertainties. \[16\] In the comparison of the \( \Lambda_c \) and \( D \) lifetimes one should note the following: WS and PI counteract each other in \( \Gamma(\Lambda_c) \), see eq.(81a); nevertheless \( \Gamma(\Lambda_c) \sim 2 \cdot \Gamma(D) \) can be obtained. This results from three effects: (i) WA is very much reduced in \( D \) decays. (ii) The baryonic expectation values of the four-quark operators are evaluated, as stated above, with the ‘static’ decay constant, which is considerably larger than \( f_D \) used for the mesonic expectation values. (iii) The \( 1/m_c^2 \) contributions enhance \( \Gamma(\Lambda_c) \) over \( \Gamma(D) \), as briefly discussed below.

Unfortunately there appears now a rather unpleasant ‘fly in the ointment’: as discussed above, extrapolating from these prima facie successful predictions to the lifetimes of beauty baryons leads to a less-than-successful prediction on \( \tau(\Lambda_b)/\tau(B_d) \).

The decays of \( \Omega_c \) baryons require – and deserve – a separate treatment: the \( ss \) diquarks carry spin one and the resulting spin-spin interactions of \( c \) with \( ss \) lead to new effects. To be more specific: through order \( 1/m_c^3 \) one has

\[ \Gamma(\Omega_c) = \Gamma_{\text{decay}}(\Omega_c) + |\Delta \Gamma_{PI,+}(\Omega_c)| \]

(84)

\[ ^{17} \text{One should point out that those predictions were made before these data became available; data at that time suggested considerably larger lifetime ratios.} \]
with both quantities on the right-hand-side of eq.(84) differing from the corresponding ones for \( \Lambda_c \) or \( \Xi_c \) decays. (i) Firstly, \( \langle \Omega_c | i \sigma \cdot G \Omega_c | \rangle \neq 0 = \langle \Lambda_c | i \sigma \cdot G \Lambda_c | \rangle \), see eqs.(17b,17c,35). Secondly, it is quite conceivable that these spin-spin interactions can create an appreciable difference in the kinetic energy of the \( c \) quark inside \( \Omega \) and \( \Lambda \) baryons. Thus there arise differences of order \( 1/m_c^2 \) in the total as well as in the semileptonic widths of \( \Omega_c \) and \( \Lambda_c \) baryons:

\[
\Gamma_{\text{decay}}(\Omega_c) \neq \Gamma_{\text{decay}}(\Lambda_c) \quad \Gamma_{\text{SL,decay}}(\Omega_c) \neq \Gamma_{\text{SL,decay}}(\Lambda_c)
\]

(ii) These spin-spin interactions also affect the expectation values of the dimension-six four-fermion operators that control the strength of WS and the interference effects, see eqs.(24,25), in order \( 1/m_c^3 \).

Taking everything together one estimates the \( \Omega_c \) to be the shortest lived charm baryon, as shown in Table 2. This is quite remarkable since it means that the constructive interference in \( \Omega_c \) decays outweighs the combined effect of constructive interference and WS in \( \Xi_c \) decays. It is actually not unexpected on intuitive grounds: in \( \Omega_c \) transitions interference can happen with both constituent \( s \) quarks whereas in \( \Xi_c \) baryons there is only one such \( s \) quark. This shows that inclusive weak decay rates can be harnessed to probe the internal structure of charm baryons in a novel way.

### 7.4 Semileptonic Branching Ratios of Charm Hadrons

As expected, the ratio of the semileptonic \( D^+ \) and \( D^0 \) branching ratios agrees, within the errors, with the ratio of their lifetimes:

\[
2.23 \pm 0.42 = \frac{BR_{\text{SL}}(D^+)}{BR_{\text{SL}}(D^0)} \approx \frac{\tau(D^+)}{\tau(D^0)} = 2.547 \pm 0.043
\]

Also the absolute size of, say, \( BR_{\text{SL}}(D^0) \) is of interest. If the \( D^+ - D^0 \) lifetime difference is generated mainly by a destructive interference affecting \( D^+ \) transitions, then \( D^0 \) decays should proceed in a largely normal way and the semileptonic branching ratio of \( D^0 \) mesons should be at its ‘normal’ value. In a parton model description one finds \( BR(D \to l\nu X_s) \equiv BR(c \to l\nu s) \sim 16\% \). If that number represented the proper yardstick for normal decays, one would actually conclude that \( D^+ \) decays more or less normally, whereas the nonleptonic decays of \( D^0 \) are strongly enhanced; i.e., it would imply that it is actually WA that provides the dominant mechanism for the lifetime difference. This long-standing ‘fly in the ointment’ can be removed now. For at order \( 1/m_Q^2 \) the chromomagnetic operator \( \bar{Q}i\sigma \cdot G Q \) generates an isoscalar enhancement in the nonleptonic widths of the charged and neutral mesons \([10]\). It turns out that the corresponding reduction in the semileptonic branching ratios of \( B \) mesons is rather insignificant, namely \( \sim 2\% \). Yet for charm mesons this reduction is quite large, namely around 40\%, since it is scaled up by \((m_b/m_c)^2\) and less reduced by colour
factors. The semileptonic branching ratio for $D$ mesons is thus around 9% through order $1/m_c^2$ – before the dimension-six four-fermion operators reduce $\Gamma_{NL}(D^+)$ by a factor of almost two and enhance $\Gamma_{NL}(D^0)$ by a moderate amount. As discussed above, we have to take these numbers for charm transitions with a grain of salt; on the other hand the emerging pattern in the ratio of semileptonic branching ratios as well as their absolute size is self-consistent!

It should be apparent from the discussion on $\tau(D_s)$ vs. $\tau(D^0)$ that the semileptonic widths and branching ratios of $D_s$ and $D^0$ mesons practically coincide through order $1/m_c^2$. In order $1/m_c^2$ WA can contribute to $\Gamma_{SL}(D_s)$ (as well as to $\Gamma_{NL}(D_s)$ and $\Gamma_{NL}(D^0)$). Yet from $\tau(D_s) \simeq \tau(D^0)$ one infers that the relative weight of WA is quite reduced; therefore one expects $BR_{SL}(D_s)$ to differ from $BR_{SL}(D^0)$ by not more than 10% or so in either direction:

$$1 \pm \sim \text{few}\% = \frac{\tau(D_s)}{\tau(D^0)} \sim \frac{BR_{SL}(D_s)}{BR_{SL}(D^0)} = 1 \pm \sim 10\% \quad (87)$$

The semileptonic width is not universal for all charm hadrons due to $\mathcal{O}(1/m_c^2)$ contributions even when one ignores Cabibbo suppressed channels. One actually estimates

$$\Gamma_{SL}(D)/\Gamma_{SL}(\Lambda_c)/\Gamma_{SL}(\Omega_c) \sim 1/1.5/1.2. \quad (88)$$

Therefore the ratio of semileptonic branching ratios of mesons and baryons does not faithfully reproduce the ratio of their lifetimes. Instead one expects

$$BR_{SL}(\Lambda_c) > BR_{SL}(D^0) \times \tau(\Lambda_c)/\tau(D^0) \simeq 0.5 BR_{SL}(D^0). \quad (89)$$

## 8 What Have We Learned and What Will We Learn?

Second generation theoretical technologies have been developed for treating heavy-flavour decays that are directly related to QCD without the need for invoking a ‘deus ex machina’. As far as inclusive heavy-flavour decays are concerned, the relevant technology is based on an operator expansion in inverse powers of the heavy quark mass $m_Q$. It allows to express the leading nonperturbative corrections through the expectation values of a small number of dimension-five and -six operators. Basically all such matrix elements relevant for meson decays can reliably be related to other observables; this allows to extract their size in a model-independent way. Baryons, however, possess a more complex internal structure, which becomes relevant for their decays in order $1/m_Q^3$. At present one has to rely on quark model calculations to determine the expectation values of the dimension-six operators relevant for lifetime

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18 To repeat it one last time: This is not HQET!
differences. The numerical results of such computations are of dubious reliability; accordingly predictions for lifetime ratios involving heavy-flavour baryons suffer from larger uncertainties than those involving only mesons. There is also a positive side to this, though: measuring the lifetimes (and if at all possible also the semileptonic branching ratios) of the various heavy-flavour baryons sheds new light onto their internal strong dynamics. Comprehensive studies of inclusive weak decays in the charm baryon family are particularly promising from a practical point for such studies on hadronic structure; for the pre-asymptotic corrections are much larger here than for beauty baryons.

At present there exists one rather glaring phenomenological problem for the heavy quark expansion and – not surprisingly, as just indicated – it concerns baryon decays: the observed $\Lambda_b$ lifetime is clearly shorter than predicted relative to the $B_d$ lifetime. Unless future measurements move it up significantly, one has to pay a theoretical price for that failure. As stated above, for the time being one has to employ a quark model to determine the size of the baryonic expectation values of the relevant dimension-six operators. It is not surprising per se that quark model computations can yield numerically incorrect results, in particular when sizeable cancellations occur between different contributions. The more serious problem is provided by the following aspects: the analogous treatment of charm baryons, using the same quark models, had yielded predictions that give not only the correct pattern, but also numerical values for the lifetime ratios which are surprisingly close to the data considering the uncertainties due to higher order terms in the $1/m_c$ expansion. Furthermore scaling these charm lifetime ratios up to the beauty scale, using a $1/m_Q^2$ and $1/m_Q^3$ behaviour for the pre-asymptotic corrections from the dimension-five and -six operators, respectively, also tells us that the $\Lambda_b$ lifetime should be shorter than the $B_d$ lifetime by no more than 10 %. To the degree that the observed value for $\tau(\Lambda_b)/\tau(B_d)$ falls below 0.9 one has to draw the following conclusion: one cannot trust the numerical results of quark model calculations for baryonic matrix elements – not very surprising by itself; yet furthermore and more seriously it would mean that $1/m_Q^4$ or even higher order contributions are still relevant in charm baryon decays before fading away for beauty decays. Then one had to view the apparently successful predictions on the lifetimes of charm baryons as largely coincidental – a quite sobering result!

A future discrepancy between the predictions on $\tau(B^+)/\tau(B_d)$ or $\tau(B_d)/\tau(B_s)$ and the data\textsuperscript{19} - in particular an observation that the lifetime for $B^+$ mesons is definitely shorter than for $B_d$ mesons – would have quite fundamental consequences. For the leading deviation of these ratios from unity arises at order $1/m_b^3$ and should provide a good approximation since the expansion parameter is small: $\mu_{\text{had}}/m_b \sim 0.1$. The size of this term is given by the expectation value of a four-fermion operator expressed in terms of $f_B$. A failure in this simple situation would raise very serious doubts about the validity or at least the practical relevance of the $1/m_Q$ expansion for treating even

\textsuperscript{19}It has to be stressed again that $\bar{\tau}(B_s)$ refers to the algebraic average of the two $B_s$ lifetimes.
fully inclusive nonleptonic transitions; this would leave only semileptonic transitions in the domain of their applicability. Such a breakdown of quark-hadron duality would \textit{a priori} appear as a quite conceivable and merely disappointing scenario. However such an outcome would have to be seen as quite puzzling \textit{a posteriori}; for in our analysis we have not discerned any sign indicating the existence of such a fundamental problem or a qualitative distinction between nonleptonic and semileptonic decays \cite{12, 20}. Thus even a failure would teach us a valuable, albeit sad lesson about the intricacies of the strong interactions; for the heavy quark expansion is directly and unequivocally based on QCD with the only additional assumption concerning the workings of quark-hadron duality!

The theoretical analysis of the lifetimes of heavy-flavour hadrons can be improved, refined and extended:

- \textit{improved} by a better understanding of quark-hadron duality, preferably by deriving it from QCD or at least by analysing how it operates in the lepton and photon spectra of semileptonic and radiative $B$ decays, respectively;
- \textit{refined} by a reliable determination of in particular, but not only, the baryonic expectation values of the relevant dimension-six operators;
- \textit{extended} by treating $\Xi_b$ (and $\Omega_b$) decays.

The extension will certainly have been made very soon; the refinement should be obtained in the foreseeable future, probably from a lattice simulation of QCD; when finally quark-hadron duality will be derived from QCD cannot be predicted, of course.

There is a corresponding list of future measurements that are most likely to probe and advance our understanding:

- measure $\tau(\Lambda_b)$, $\tau(\Xi_b^-)$ and $\tau(\Xi_b^0)$ \textit{separately} with good accuracy;
- confirm $\tau(B_d) \simeq \bar{\tau}(B_s)$ within an accuracy of very few percent;
- verify that $\tau(B^+) \textit{exceeds} \tau(B_d)$ by a few to several percent;
- in charm decays determine the lifetimes of $\Xi_c^{0,+}$ and $\Omega_c$ baryons with a few percent accuracy and measure $\tau(D_s)/\tau(D^0)$ with 1\% precision.

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**Figure Captions**

Figure 1: WA diagram for $D^0$ decays with gluon emission.

Figure 2: Diagram yielding the operator $\bar{Q}Q$ upon integrating out the intermediate two fermion and one antifermion lines.

Figure 3: Diagram yielding the chromomagnetic operator $\bar{Q}i\sigma \cdot GQ$.

Figure 4a: Cutting a quark line in Fig.2 to generate WA.

Figure 4b: Cutting an antiquark line in Fig.2 to obtain PI.

Figure 5: Forward scattering amplitude involving WA with its three poles.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9508408v1
This figure "fig1-2.png" is available in "png" format from:

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