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Pulse Radar Randomly Interrupted Transmitting and Receiving Optimization Based on Genetic Algorithm in Radio Frequency Simulation

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Abstract: The interrupted transmitting and receiving (ITR) can be used in anechoic chamber to solve the coupling between the transmitted and reflected signals. When the ITR periods are random, the fake peaks in high resolution range profile (HRRP) of ITR echo can be reduced. Then, by utilizing the piecewise sparse property of ITR echo, the HRRP is reconstructed based on compressive sensing (CS). However, the periods of ITR determine the restricted isometry property (RIP) condition and the HRRP reconstruction performance. In order to improve the HRRP reconstruction performance, the ITR periods sequence optimization method based on genetic algorithm (GA) is proposed in this paper. The correlation coefficient of the sensing matrix columns is minimized after optimization so that the RIP of sensing matrix can be well satisfied. Simulation results illustrate that the optimization method converges fast and the HRRP reconstruction performance is improved with the optimized ITR periods.

Keywords: pulse radar, interrupted transmitting and receiving (ITR), compressive sensing (CS), genetic algorithm (GA)

1. Introduction

The radio frequency simulation (RFS) [1] in anechoic chamber is widely adopted to conduct target measurement [2-3]. Because the size of anechoic chamber is small, the round-trip time of electromagnetic wave may be sub-microsecond which is generally larger than the pulse width. Therefore, when the pulse radar signal is applied in anechoic chamber for target measurement, the reflected and transmitted signals may be coupled at the receiver. The interrupted transmitting and receiving (ITR) method is proposed in [3] to divide the pulse signal into hundreds of short pulses with each short pulse width being sub-microsecond. Then, the coupling is eliminated and the target echo can be obtained for the target measurement.

By utilizing ITR, the target echo can be regarded as piecewise sparse compared with the complete pulse echo. Then, the measurement matrix of compressive sensing (CS) [4-7] is constructed according to the ITR periods. And the target HRRP can be reconstructed by CS [8]. However, the reconstruction performance of HRRP is generally determined by the restricted isometry property (RIP) condition of the sensing matrix [9]. As the sensing matrix is formed according to the ITR, different ITR parameters will result in different properties of the sensing matrix. Therefore, to improve the reconstruction performance, the randomly-ITR (R-ITR) periods sequence should be optimized so that the sensing matrix can satisfy the RIP condition well.
As the R-ITR periods sequence is composed by multiple short pulse periods, the optimization of periods sequence becomes a multivariable optimization problem. Genetic algorithm (GA) [10-11] is a favorable method to solve the multivariable optimization problem with good robustness. Thus, the R-ITR periods sequence optimization method is proposed in this paper based on GA. The minimization of the sensing matrix columns cross correlation coefficient is used as the fitness criterion in GA. Then, the R-ITR periods sequence is optimized after iteration and the RIP of sensing matrix can be better satisfied.

The remainder of this paper is organized as follows. In Section II, we give a brief introduction to the principle of R-ITR for pulse radar. The property of ITR echo and the CS-based HRRP reconstruction are discussed as well. In Section III, the optimization method of R-ITR periods sequence is presented based on the GA algorithm. In Section IV, simulations results are presented and discussed to verify the validity of the proposed optimization method. Finally, conclusions are drawn in Section V.

2. Methods

2.1. Principle of R-ITR in Anechoic Chamber

In anechoic chamber, the pulse signal is divided into multiple short pulses after ITR so that the target echo can be received when the next short pulse is transmitted. Then, the coupling can be eliminated [3]. When the ITR periods are random, the R-ITR is shown in Figure 1.

The R-ITR control signal can be expressed as

\[ p_i(t) = \sum_{n=0}^{\infty} \text{rect}\left( \frac{t - nT_y}{\tau_n} \right) \ast \delta(t - \sum_{k=1}^{n} T_y) \]  

(1)

where \( \delta(\cdot) \) is the impulse function, \( \tau_n \) is the width of the \( n \)th short pulse, \( T_y \) is the period of the \( k \)th short pulse and \( k \leq n \). \( \ast \) is conjunction operation and \( \text{rect}(\cdot) \) is

\[ \text{rect}\left( \frac{t}{\tau_n} \right) = \begin{cases} 1, & |t/\tau_n| < 0.5 \\ 0, & \text{others} \end{cases} \]  

(2)

When the period of ITR and short pulse width is unchanged and it can suppose as \( T_y = T_s \) and \( \tau_n = \tau \). Then, we have \( \sum_{i=1}^{n} T_y = nT_s \) for the \( n \)th short pulse. And the uniformly ITR control signal is

\[ p_j(t) = \text{rect}\left( \frac{t}{\tau} \right) \ast \sum_{n=0}^{\infty} \delta(t - nT_s) \]  

(3)

Therefore, the spectrum of \( p_j(t) \) can be obtained as
\[
P(f) = \tau f_s \sum_{n=-\infty}^{\infty} \sin(nf_r \tau) \delta(f-nf_s)
\]

where \( f_s = 1/T_s \), \( \sin(x) = \sin(\pi x)/\pi x \). As the R-ITR period is random, the analytical expression of the spectrum is difficult to be obtained.

The transmitted signal is supposed as linear frequency modulation (LFM)

\[
s(t) = \text{rect} \left( \frac{t}{T_p} \right) \exp(\jmath 2\pi(f_f \frac{1}{2} t^2))
\]

where \( T_p \) is the pulse width, \( f_c \) is the carrier frequency, \( \mu = B/T_p \) and \( B \) is the bandwidth.

Considering the ITR control signal in (3), the ITR echo can be obtained as

\[
s_e(t) = p_e(t) \cdot s(t)
\]

where \( s_e(t) = \sum_{n=1}^{K} \alpha_n s(t-2R_n/C) \) is target echo, \( K \) is the number of scattering centers and \( \alpha_n \) is scattering coefficient of the \( k \)th scattering centers. \( R_n \) is the distance between radar and the \( k \)th scattering center.

The de-chirp reference signal is \( s_{ref}(t) = s(t-2R_{ref}/C) \), where \( R_{ref} \) is the reference distance and \( C \) is the speed of electromagnetic wave. Then, the difference output after de-chirping is

\[
\begin{align*}
s_{de}(t) &= p_e(t) \cdot s_e(t) \cdot s_{ref}(t) \\
&= \sum_{k=1}^{K} \left( \text{rect} \left( \frac{t}{\tau} \right) \ast \sum_{n=1}^{\infty} \delta(t-nT_f) \right) \alpha_n \left( \text{rect} \left( \frac{t-2R_n/C}{T_p} \right) \exp \left( -j \frac{4\pi}{C} \mu (t-\frac{2R_{ref}}{C}) R_{n,\lambda} \right) \right) \\
&\quad \exp \left( -j \frac{4\pi}{C} f_c R_{n,\lambda}^2 \right) \exp \left( j \frac{4\pi}{C^2} R_{n,\lambda}^2 \right)
\end{align*}
\]

where \( s_{ref}(t) \) is the conjugation of \( s_{ref}(t) \), \( R_{n,\lambda} = R_n - R_{ref} \) is the distance between the scattering center and the reference point.

The HRRP of ITR echo is obtained after Fourier transform

\[
S_e(f) = \tau f_s T_p \sum_{k=1}^{K} \alpha_k \left( \sum_{n=-\infty}^{\infty} \sin(nf_f \tau) \sin(c(t-nf_s/2)) \right)
\]

In equation (8), \( nf_s \) denotes that different orders of fake peaks appear in HRRP. Then, the range interval is

\[
\Delta R = \frac{Cf_f}{2\mu}
\]

The constraints of ITR parameters are the same with [8]

\[
\begin{align*}
\tau &\leq \frac{2R_{ref}}{C} \\
\tau + \frac{2(R_n + L)}{C} &\leq T_f < \frac{CT_f}{2BL}
\end{align*}
\]

Because \( \Delta R \) may be smaller than \( L \), the real and fake peaks will be overlapped and the target peaks in the HRRP are difficult to be extracted. However, the ITR echo is sparse compared with the completed pulse, therefore, the HRRP can be reconstructed based on compressive sensing (CS).
2.2. HRRP Reconstruction Based on CS

The vector of difference-frequency output of the complete echo can be expressed as
\[ s_t = S_{ref}^t s_r \]  
(11)
where \( s_r \) is the vector of the complete echo \( s_r(t) \), \( N \) is the total sampling number in \( T_r \), \( S_{ref} = diag[s_{ref}(0), s_{ref}(1), \ldots, s_{ref}(N-1)] \) is an \( N \times N \) diagonal matrix composed of the reference signal \( s_{ref}(t) \), and \( s_r = [s_r(0), s_r(1), \ldots, s_r(N-1)]^T \) is an \( N \times 1 \) vector of the difference-frequency output.

\( S_r = [S_r(0), S_r(1), \ldots, S_r(N-1)]^T \) is the target HRRP which can be supposed as \( K \)-sparse according to the number of scattering centers. Then, \( S_r \) can be obtained after Fourier transform of (11)
\[ S_r = \Psi^{-1} S_{ref}^t s_r \]  
(12)
where \( \Psi \) is an \( N \times N \) inverse fast Fourier transform (IFFT) matrix.

Because \( p(t) = 1 \) in \( T_s \) and 0 in \( T_{s-c} \), the rows of \( P = diag[p(0), p(1), \ldots, p(N-1)] \) are 0 and can be eliminated. Therefore, \( P \) is obtained as
\[ P_1 = \begin{pmatrix}
I_{s_0} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & I_{s_0} & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & I_{s_0}
\end{pmatrix}_{M \times N} \]  
(13)
where \( I_{s_0} \) is an \( n_0 \times n_0 \) identity matrix.

Then, the ITR echo is obtained based on the measurement of \( s_r \)
\[ s_{ITR} = P_1 s_r \]  
(14)
where \( s_{ITR} \) is an \( M \times 1 \) vector.

As \( S_{ref} S_{ref}^t = I \), equation (14) can be re-written as
\[ s_{ITR} = P_1 S_{ref} \Psi s_t \]  
(15)
After adding noise \( \xi \), we have
\[ s_{sp} = s_{ITR} + \xi \]
\[ = \Phi \Psi s_t + \xi \]  
(16)
where \( \Phi = P_1 S_{ref} \) and \( s_{sp} \) is the non-zero value of ITR echo.

The target HRRP can be reconstructed by solving the optimization problem
\[ \min_{s_t} \| \tilde{S}_r - \Phi \Psi s_t \|_1 \quad s.t. \quad \| s_{sp} - \Phi \Psi s_t \|_1 \leq \xi \]  
(17)
where \( \| \cdot \|_1 \) is the \( \ell_1 \) norm, \( \tilde{S}_r \) denotes the reconstructed HRRP.

Let \( A = \Phi \Psi \) be the sensing matrix. Generally, \( A \) should satisfy \( K \)-order RIP condition. The RIP condition of sensing matrix \( A \) is different when the ITR periods sequence is different. Therefore, by optimizing the ITR periods sequence, the RIP property of the sensing matrix as well as the HRRP reconstruction performance can be improved.

2.3. ITR periods sequence optimization based on GA

Because there are hundreds of short pulses in each pulse signal, the ITR periods sequence optimization can be converted to a multivariable optimization problem. GA is an adaptive optimization algorithm which is widely used to solve the multivariable optimization problem. The multi-point search, cross-operation, mutation operation are adopted to eliminate the worst solution and save the optimum solution. After iteration, the optimum ITR periods sequence can be obtained.
The procedure of GA is presented in Figure 2.

According to Figure 2, the ITR period should be coded firstly to obtain the initialized population. Supposing the R-ITR period $T_n$ is distributed in $[0.5\mu s, 0.8\mu s]$ with the interval of $0.1\mu s$, there are four values to be chose, i.e. $0.5\mu s, 0.6\mu s, 0.7\mu s, 0.8\mu s$. So the two-bit binary can be adopted to code $T_n$, i.e. 00, 01, 10, 11. If the R-ITR periods sequence is $0.8\mu s, 0.6\mu s, 0.5\mu s, 0.6\mu s, 0.7\mu s, 0.7\mu s$ (18)

The corresponding binary codes are obtained as 11 01 00 01 10 10 (19)

Generally, the sequence length of R-ITR periods is determined by the pulse width and the range of ITR periods. Therefore, the code length increases when the pulse width is large.

The initialized population should be evaluated by the fitness function. As we discussed above, the HRRP reconstruction performance is determined by the sensing matrix. Researches show that the RIP of the sensing matrix equals to the eigenvalues of the Gram matrix of sensing matrix and the cross correlation coefficient of the sensing matrix columns [5]. When the eigenvalues of the Gram matrix of sensing matrix are close to 1, the RIP property can be satisfied well. However, as the computation complexity of the Gram matrix eigenvalues is high, the cross correlation coefficient of the sensing matrix columns is adopted to evaluate the fitness of the population.

{a} is supposed as the $i$th ($0 \leq i < N$) column of the sensing matrix $A$. Then, the maximum cross correlation coefficient [5] of the sensing matrix $A$ is defined as

$$\rho(A) = \max_{0 \leq k, j < N \& k \neq j} \frac{|a_k^H a_j|}{\|a_k\| \|a_j\|}$$

When the cross correlation coefficient is minimized, the RIP of sensing matrix can be well satisfied. Then, we have

$$f_{opt}(T_n) = \min \{\rho(A)\} \quad \text{s.t.} \quad T_n \in P$$

(21)

where $P$ is the variation range of $T_n$.

The fitness function is often supposed as the maximization function in GA, and we have

$$F(T_n) = \max \frac{1}{\rho(A)} \quad \text{s.t.} \quad T_n \in P$$

(22)

In order to illustrate the detailed procedure of GA, the algorithm flow is presented as follows.
Step 1. Population initialization. The length of the binary code is determined by the range of the ITR periods. Then, the population can be generated with different ITR periods sequence. M series ITR periods are generated in this step.

Step 2. Calculation of the fitness function. Each individual in the population represents one ITR periods sequence. Then, by constructing the sensing matrix according to $A=\Phi\Psi$, the fitness function of one ITR periods sequence can be calculated with equation (22).

Step 3. Judgement of the fitness value. The fitness function is calculated for every ITR periods sequence. If the fitness value is satisfied, the iteration stops. Otherwise, the iteration continues.

Step 4. Selection operation. The roulette wheel is used to select the individual. The elitism selection is also adopted in this part to save the optimum individual $\{T_{sn}\}$ where $1 \leq k \leq M$.

Step 5. Cross operation. The two-point crossover is utilized to conduct the cross operation. The crossover positions and individuals are randomly selected with the crossover probability of $P_c$. Then, the new individuals can be obtained.

Step 6. Mutation operation. The mutation operation is conducted based on the population obtained in step 5. The single-point mutation is performed by randomly selecting the individual and the mutation position with the mutation probability of $P_m$.

Step 7. Population updating. The elitism selection is still used to update the population. Then, the updated population is regarded as the solution and should be used to calculate the fitness function as discussed in step 3.

3. Results and discussion

3.1. Optimization Results of the Cross Correlation Coefficient

The pulse width is $T_p=20\mu s$ with bandwidth of $B=500\text{MHz}$. The distance between the radar and target is $R=45\text{m}$. The ITR period sequences are set as $0.5\mu s$-0.8$\mu s$, $0.5\mu s$-1.2$\mu s$ and $0.8\mu s$-1.11$\mu s$ with interval of 0.1$\mu s$. The $\tau/f_s$ is 0.25 so that the maximum short pulse width is $0.3\mu s$ which ensures the target echo can be received according to equation (10).

When $T_{sn}$ distributes in $[0.5\mu s, 0.8\mu s]$ and $[0.8\mu s, 1.11\mu s]$, the two-bit binary can be used to code the ITR periods sequence. When $T_{sn}$ distributes in $[0.5\mu s, 1.2\mu s]$, there are 8 kinds of ITR periods. Therefore, three-bit binary should be used to code the $T_{sn}$. The cross probability and mutation probability are $P_c=0.57$ and $P_m=0.002$. The initialization population is $M=75$.

The length of ITR period sequence is different according to the pulse width and $T_{sn}$. Thus, the fitness function is calculated by choosing the front 20$\mu s$. Then, the ITR periods are adopted to perform the optimization algorithm. According to the steps discussed in section III, the optimization results are presented in TABLE 1 to TABLE 3.

| TABLE 1 | THE ITR PERIOD SEQUENCE BEFORE AND AFTER OPTIMIZATION($T_{sn} \in [0.5\mu s, 0.8\mu s]$) |
|---------|-------------------------------------------------------------------------------------------------|
| Before optimization | 0.8 | 0.8 | 0.8 | 0.5 | 0.7 | 0.7 | 0.8 | 0.7 | 0.6 | 0.7 | 0.8 |
| | 0.7 | 0.5 | 0.8 | 0.5 | 0.7 | 0.5 | 0.6 | 0.8 | 0.5 | 0.8 | 0.7 |
| | 0.6 | 0.5 | 0.6 | 0.8 | 0.5 | 0.7 | 0.7 | 0.7 | |
| After optimization | 0.7 | 0.8 | 0.5 | 0.5 | 0.8 | 0.8 | 0.8 | 0.5 | 0.5 | 0.8 | 0.7 |
| | 0.7 | 0.5 | 0.8 | 0.5 | 0.5 | 0.6 | 0.6 | 0.5 | 0.5 | 0.5 | 0.5 |
| | 0.5 | 0.8 | 0.8 | 0.5 | 0.5 | 0.5 | 0.8 | 0.7 | 0.5 | 0.5 | 0.8 |
TABLE 2 THE ITR PERIOD SEQUENCE BEFORE AND AFTER OPTIMIZATION($T_{SN} \in [0.5\mu s,1.2\mu s]$)

|          | 0.9 | 1.2 | 1.0 | 0.7 | 1.0 | 1.0 | 1.2 | 1.1 | 0.9 |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Before   |     |     |     |     |     |     |     |     |     |
| optimization | 0.9 | 1.2 | 1.0 | 0.5 | 0.7 | 0.8 | 0.6 | 1.1 | 1.1 |
| After    |     |     |     |     |     |     |     |     |     |
| optimization | 0.5 | 0.6 | 0.6 | 0.8 | 0.9 |     |     |     |     |

|          | 1.2 | 1.2 | 1.2 | 0.5 | 0.8 | 0.5 | 0.8 | 0.5 | 0.9 |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| After    |     |     |     |     |     |     |     |     |     |
| optimization | 1.0 | 1.1 | 1.0 | 0.5 | 0.8 | 0.7 | 1.0 | 0.9 |     |

TABLE 3 THE ITR PERIOD SEQUENCE BEFORE AND AFTER OPTIMIZATION($T_{SN} \in [0.8\mu s,1.1\mu s]$)

|          | 0.9 | 0.9 | 0.8 | 1.0 | 0.9 | 0.9 | 1.1 | 1.0 | 1.0 |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Before   |     |     |     |     |     |     |     |     |     |
| optimization | 0.8 | 0.9 | 1.0 | 1.0 | 1.1 | 1.0 | 0.9 | 1.0 |     |
| After    |     |     |     |     |     |     |     |     |     |
| optimization | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 1.1 | 0.8 | 0.8 |     |
|          | 1.1 | 0.8 | 0.9 | 1.0 | 0.8 | 0.9 | 0.9 | 1.0 |     |
| After    |     |     |     |     |     |     |     |     |     |
| optimization | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 |     | 0.9 |     |     |

According to equation (22), the maximum fitness function value corresponds to the minimum correlation coefficient. The maximum value and average value of the fitness function versus the iteration are shown in Figure 3.

![Figure 3](image)

**Figure 3** The maximum and average values of the fitness function versus the iteration. (a) The maximum value of the fitness function versus the iteration. (b) The average value of the fitness function versus the iteration.

Figure 3(a) is the maximum value of the fitness function versus the iteration. When the iterations are less, the maximum fitness function value of $T_{SN} \sim [0.5\mu s,1.2\mu s]$ is bigger than that of $T_{SN} \sim [0.5\mu s,0.8\mu s]$. The reason is that there are many individuals containing small $T_{SN}$ in the initialized population when the ITR periods range is $T_{SN} \sim [0.5\mu s,1.2\mu s]$. However, the initialized population is not the optimized solution. When the iterations increase, the fitness function value of $T_{SN} \sim [0.5\mu s,0.8\mu s]$ increases and becomes bigger than that of $T_{SN} \sim [0.5\mu s,1.2\mu s]$. Besides, the maximum value of the fitness value is the smallest when $T_{SN} \sim [0.8\mu s,1.1\mu s]$.

The average value of the fitness function is presented in Figure 3(b). It can be found the curve tendency is the same with the maximum value of the fitness function in Figure 3(a). As the worst individuals are eliminated after iteration, the fitness function value increases so that the columns cross...
correlation coefficient of the sensing matrix decreases. Then, the RIP of sensing matrix can be better satisfied after GA optimization.

In order to illustrate the columns cross correlation coefficient variation of the sensing matrix, the average columns cross correlation coefficient is obtained to compare with the coefficient after optimization. The results are shown in TABLE 4.

**TABLE 4** The columns cross correlation coefficient of the sensing matrix before and after optimization

|                | $T_{sn} \in [0.5\mu s, 0.8\mu s]$ | $T_{sn} \in [0.5\mu s, 1.2\mu s]$ | $T_{sn} \in [0.8\mu s, 1.1\mu s]$ |
|----------------|-----------------------------------|-----------------------------------|-----------------------------------|
| Before optimization | 0.4131                             | 0.4085                             | 0.5869                             |
| After optimization   | 0.2498                             | 0.2500                             | 0.3083                             |

The optimization value in TABLE 4 is obtained according to equation (22). It can be found that the columns cross correlation coefficient of the initialization individuals is 0.4131 when $T_{sn} \in [0.5\mu s, 0.8\mu s]$ and 0.4085 when $T_{sn} \in [0.5\mu s, 1.2\mu s]$. The cross correlation coefficient is the maximum when $T_{sn} \in [0.8\mu s, 1.1\mu s]$. Therefore, the results are basically identical with the curves in Figure 3.

On the other hand, the columns cross correlation coefficients are decreased after the optimization, i.e. 0.2498, 0.2500 and 0.3083. Besides, the coefficient of $T_{sn} \in [0.5\mu s, 0.8\mu s]$ is the minimum. And the RIP of the sensing matrix can be satisfied better after optimization which results in the improvement of the HRRP reconstruction performance.

### 3.2. Target HRRP Reconstruction Results Based on GA

In this part, the HRRP reconstruction is conducted based on different ITR periods sequence. Considering there are five scattering centers in the target, the distance between the scattering centers is 2m. The scattering coefficient is [0.7, 0.5, 1, 0.6, 0.55]. Firstly, the reconstructed HRRP is obtained when $T_{sn} \in [0.5\mu s, 0.8\mu s]$. Secondly, the ITR periods sequence is optimized based on the proposed optimization method. The HRRP reconstruction after optimization is obtained and the results are presented in Figure 4.

![Figure 4](image-url) The HRRP reconstruction comparison when $T_{sn} \in [0.5\mu s, 0.8\mu s]$. (a) Distribution of the scattering centers. (b) HRRP reconstruction before optimization. (c) HRRP reconstruction after optimization.

Figure 4 (a) is the distribution of target scattering centers. The scattering centers from -4m to 4m is supposed as 1 to 5. Figure 4 (b) and (c) is the HRRP reconstruction before and after optimization. It can be found that the reconstructed HRRPs are basically identical to the HRRP of complete echo. Therefore, the sensing matrices obtained by the optimized and non-optimized ITR period sequences have good RIP property when $T_{sn} \in [0.5\mu s, 0.8\mu s]$. 
Figure 5 The HRRP reconstruction comparison when $T_{sn}$~[0.5μs,1.2μs]. (a) HRRP reconstruction before optimization. (b) HRRP reconstruction after optimization.

Specially, the HRRP reconstruction comparisons are presented in Figure 5 when $T_{sn}$~[0.5μs,1.2μs]. Figure 5 (a) is the reconstructed HRRP versus the HRRP of the complete echo. Figure 5 (b) is the reconstructed HRRP obtained by the optimized ITR periods sequence. It can be found that the deviation between the reconstructed HRRP and the HRRP of the complete echo is smaller in Figure 5 (b) than that in Figure 5 (a). For example, the amplitude of the scattering center 5 in the reconstructed HRRP in Figure 5 (b) is closer to the HRRP of the complete echo. Therefore, the results indicate the effectiveness of the optimization method.

Figure 6 The HRRP reconstruction comparison when $T_{sn}$~[0.8μs,1.1μs]. (a) HRRP reconstruction before optimization. (b) HRRP reconstruction after optimization.

When $T_{sn}$~[0.8μs,1.1μs], the HRRP comparison is obtained in Figure 6. Because there are many large ITR periods, the columns cross correlation coefficient are big. And the HRRP reconstruction performance is poor as shown in Figure 6 (a). However, the ITR period sequence is optimized after GA, the reconstructed HRRP is presented in Figure 6 (b). It can be found that the deviation between the reconstructed HRRP and the HRRP of the complete echo is small which indicates the good RIP of the sensing matrix after optimization.

In order to analyze the HRRP reconstruction performance, the normalized error between the reconstructed HRRP and the HRRP of complete echo is defined as

$$\text{Error} = \frac{\| S_f - \overline{S}_f \|_2}{\| S_f \|_2}$$

(23)

where $S_f$ is the HRRP of complete echo and $\overline{S}_f$ is the reconstructed HRRP.

For different ITR period sequences, the normalized errors are summarized in TABLE 5.
TABLE 5 THE NORMALIZED ERROR BETWEEN THE RECONSTRUCTED HRRP AND THE REAL HRRP BEFORE AND AFTER OPTIMIZATION

|                  | $T_s \in [0.5\mu s,0.8\mu s]$ | $T_s \in [0.5\mu s,1.2\mu s]$ | $T_s \in [0.8\mu s,1.1\mu s]$ |
|------------------|-------------------------------|-------------------------------|-------------------------------|
| Before optimization | 0.2887                        | 0.3441                        | 0.4663                        |
| After optimization | 0.2562                        | 0.2833                        | 0.3130                        |

In TABLE 5, the normalized error of HRRP after ITR periods sequence optimization is smaller than that before optimization. The normalized error is the smallest when $T_s \in [0.5\mu s,0.8\mu s]$ after optimization. Besides, the normalized error is decreased to 0.3130 when $T_s \in [0.8\mu s,1.1\mu s]$. It indicates that the GA optimization method is effective to improve the HRRP reconstruction performance.

3.3. Reconstruction Probabilities of Scattering Centers and HRRP Based on GA

When the residual error between the reconstructed and original information is small enough, the reconstruction can be assumed to be correct [7]. Therefore, the HRRP can be supposed as corrected when the peaks $P_{\text{r}}(\text{peaks})$ is smaller than the range cell and the normalized amplitude deviation $A_{\text{r}}(\text{peaks})$ is smaller than the threshold

$$\begin{cases} P_{\text{r}}(\text{peaks}) \leq R, \\ A_{\text{r}}(\text{peaks}) \leq \gamma \end{cases}$$

where $R$, is the range resolution, and we have $R = 0.3m$ as the bandwidth is 500MHz. $\gamma$ is the threshold which is set as 0.5.

The output signal to noise ratio (SNR) is defined as

$$\text{SNR} = \max\{A_{\text{r}}\} / \sqrt{|A_{\text{r}}|^2}$$

where $A_{\text{r}}$ is the peak value in target range cell, $|A_{\text{r}}|^2$ is the noise value which is determined by $\xi$.

![Figure 7](Image)

**Figure 7** Reconstruction probabilities of scattering 1, 2 and HRRP. (a) Reconstruction probabilities of scattering 1. (b) Reconstruction probabilities of scattering 2. (c) Reconstruction probabilities of HRRP.

The reconstruction probabilities of scattering 1, 2 and HRRP are presented in Figure 7. Figure 7 (a) and (b) are the reconstruction probabilities of scattering center 1 and 2 before and after optimization. Because the coefficient of scattering center 1 is bigger than scattering center 2, the reconstruction probability is better. Meanwhile, the reconstruction probability is improved after GA optimization which is in accordance with the HRRP reconstruction error in TABLE 5.

When all the scattering centers are reconstructed, the HRRP can be supposed as success. Then, the HRRP reconstruction probability is presented in Figure 7 (c). Similarly, the reconstruction probability is better after the optimization of ITR periods sequence. Besides, the best reconstruction probability can be obtained with the ITR periods of $T_s \in [0.5\mu s,0.8\mu s]$. 

4. Conclusion

The ITR periods sequence optimization method is proposed in this paper based on GA. The columns cross correlation coefficient is used to evaluate the fitness value of the population in GA. Then, the HRRP reconstruction performance is compared before and after the GA optimization. Simulation results show that the columns cross correlation coefficient is minimized after optimization which indicates the effectiveness of the proposed method. The optimization method of the ITR periods sequence can be used in the radar signal design in the future.

Abbreviations

ITR: Interrupted transmitting and receiving; R-ITR: Randomly interrupted transmitting and receiving; HRRP: High resolution range profile; CS: Compressive sensing; RIP: Restricted isometry property; GA: Genetic algorithm; RFS: Radio frequency simulation.

Declarations

Ethics approval and consent to participate
Not applicable.

Consent for publication
Not applicable.

Availability of data and material
All data generated or analyzed during this study are included in this published article.

Competing interests:
The authors declare that they have no competing interests.

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