Scalable register initialization for quantum computing in an optical lattice

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Abstract
The Mott insulator state created by loading an atomic Bose–Einstein condensate (BEC) into an optical lattice may be used as a means to prepare a register of atomic qubits in a quantum computer. Such architecture requires a lattice commensurately filled with atoms, which corresponds to the insulator state only in the limit of zero inter-well tunnelling. We show that a lattice with spatial inhomogeneity created by a quadratic magnetic trapping potential can be used to isolate a subspace in the centre which is impervious to hole-hopping. Components of the wavefunction with more than one atom in any well can be projected out by selective measurement on a molecular photo-associative transition. Maintaining the molecular coupling can sustain a commensurately filled register for the duration of a quantum computation.

In the past decade, tremendous progress has been made creating and manipulating macroscopic quantum states of atoms in Bose–Einstein condensates [1]. Concurrently, experiments have achieved precise control of a small number (<10) of interacting atoms and demonstrated entangling operations between them [2]. A potential application that marries atomic control on the large and small scale is the implementation of quantum computation with neutral atoms. Building a scalable quantum computer requires the initialization of a many body system to a simple fiducial state with well-characterized qubits [3]. Ultimately, this must be done by either allowing the system to naturally cool to its ground state, or performing a suitable projective measurement of the system.

Several years ago it was proposed [4] to load an optical lattice from an atomic BEC. If one begins with a superfluid-like BEC and adiabatically turns on a lattice potential, the system experiences a phase transition to the Mott insulator (MI) state. This many body state is characterized by the same number of atoms in each lattice well and is the ground state when the intra-well interaction energy is much greater than the inter-well tunnelling [5]. Recent experiments [6] have demonstrated the Mott insulator phase transition in a magnetically confined optical lattice, with an average filling factor of two atoms per well. We argue here that the many body ground state of atoms in an optical lattice contains intrinsic number fluctuations
that make it an imperfect register. We show how this can be corrected in two steps, first by introducing an inhomogeneity to the lattice using a quadratic trapping potential and second by selective measurement of atomic pairs. This strategy allows the MI transition to become a robust mechanism for register initialization.

A key advantage of loading an optical lattice from a BEC is the availability of an initially high phase space density which can be frozen to the MI state with atoms occupying every lattice site. When the lattice is loaded such that only the lowest vibrational state of each lattice well is occupied, the system is well described by the Bose–Hubbard Hamiltonian:

\[ H_{\text{BH}} = \sum_j \varepsilon(j) n_j - J \left( a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j \right) + \frac{U}{2} n_j(n_j - 1). \]

Here \( a_j^\dagger \) and \( a_j \) are the bosonic creation and annihilation operators and \( n_j = a_j^\dagger a_j \) the number operators for an atom in the lowest vibrational state of lattice well \( j \). The energy offset at each lattice site is \( \varepsilon(j) \) which models a continuously varying external potential. The energies \( J \) and \( U \) are the tunnelling and on-site interaction energies respectively. In the tight-binding model, the nearest-neighbour tunnelling energy \( J \) is defined as one fourth the band width of the lowest occupied band. For tunnelling through a potential barrier given by \( V(x) = V \cos^2(kx) \), the tunnelling rate is closely approximated by \( [7] J/\hbar = 4/\sqrt{\pi\hbar} E_R (V/E_R)^{3/4} e^{-2\sqrt{\pi V/ER}} \), where the recoil energy is \( E_R = (\hbar k)^2/2m \) (\( m \) is atomic mass). The on-site interaction is a result of the ground-state collisions described by the s-wave scattering length \( a_s \) between two atoms in the motional state \( \phi(x) \) and is given by \( U = \frac{4\pi a_s^2\hbar^2}{m} \int dx |\phi(x)|^4 \).

For the homogeneous system, \( \varepsilon(j) = 0 \forall j \) of fixed extent, the behaviour of the system is uniquely described by the ratio \( U/J \), which decreases exponentially with the trap depth \( V \). While our results are applicable to higher dimensions, henceforth we assume a three-dimensional lattice with tight transverse confinement and tunnelling dynamics along one dimension only. For the homogeneous system, only commensurate fillings give rise to a MI transition. For the homogeneous lattice with \( M \) lattice sites, which is filled with a number of atoms whose variance is \( \Delta N \gg M \), the ground state of the system has a probability of approximately \( 1/M \) of being commensurately filled. Therefore this is not a robust mechanism for initializing a quantum computer. However, one should note that an adiabatic transfer mechanism between two sublevels of each atom may be used to fix nonuniform filling [8]. A caveat is that even with unit filling, the MI state still carries small but non-zero number fluctuations which provide a small residual coherence across the system that scales as the number of trapped atoms [9, 10]. Applying first-order perturbation theory in \( H_{\text{BH}} \), the ground state for \( N \) atoms in \( M \) wells in one dimension with \( N = M \) is approximately

\[ |\Psi_\alpha\rangle = \alpha(|T\rangle + 2\sqrt{N} J / U |S\rangle), \]

where the normalization constant is \( \alpha = (1 + 4N(J/U)^2)^{-1/2} \). Here the unit filled target state is \( |T\rangle = \prod_{j=1}^N a_j^\dagger |0\rangle \) and the symmetrized state, assuming periodic boundary conditions \( (j + M \equiv j) \), is \( |S\rangle = 1/\sqrt{4N} \sum_{j=1}^N (a_{j+1}^\dagger a_j^\dagger + a_j^\dagger a_{j+1}) |T\rangle \). The energy of the ground state is approximately \( E_\alpha \approx -4NJ^2/U \).

We therefore propose to use an inhomogeneous lattice with open boundaries created by a weak quadratic magnetic trap that acts to collect atoms near the centre of the trap and leaves empty wells (holes) at the edges. Several groups have studied the ground-state properties of atoms trapped in an optical lattice with Bose–Hubbard coupling in the presence of a quadratic trapping potential. A notable characteristic of these systems is the coexistence of superfluid and Mott insulator phases in one, two and three dimensions [4, 11–13]. Thus, even without commensurate filling \((N \neq kM, k \text{ integer})\), a region of a near unit filling is achievable. In this
paper, we specify the conditions in which a large register of singly-occupied lattice sites can be prepared at the centre of the external trap with small number fluctuations. For our analysis we assume a one-dimensional optical lattice, with \( N < M \), in the presence of a weak magnetic trap\(^3\) with oscillation frequency \( \omega_T \). The characteristic trap energy scale \( \delta = m/2(\pi/k)^2a_0^2 \) is defined so that \( \epsilon(j) = \delta j^2 \). We stipulate that the on-site interaction energy be larger than the trapping energy of the most externally trapped atom, or \( U > \epsilon((N - 1)/2) \), in order to inhibit multiple atom occupation in any well. The register is defined by a physical subspace \( \mathcal{R} \) comprising a number of wells \( n < N \) in the centre region of the trap. The barrier space flanking \( \mathcal{R} \) will act to suppress percolation of holes from the edges to the centre. The estimated probability for holes in \( \mathcal{R} \) due to tunnelling through the barrier is \( p_h \approx \prod_n^{(N-1)/2} (J/\delta(2j+1))^2 = J/2\delta^{N-\alpha_{\delta}}\Gamma[n/2]/\Gamma[(N/2+1)]^2 \), which is negligible provided the barrier region is sufficiently large and \( J/n\delta < 1 \).

When expanded in the Fock state basis, the ground state of the register has amplitude in those states with holes neighbouring atomic pairs, analogous to the homogeneous case. We describe a protocol which projects out these components by a null result from selective measurement, it is convenient to use the following incomplete basis over \( \mathcal{R} \):

\[
|T\rangle = \prod_{j=-\alpha}^{\alpha-1/2} a_j^\dagger |0\rangle, \quad |S_j^+\rangle = \frac{a_j^\dagger a_{j+2}^\dagger}{\sqrt{2}} |T\rangle, \quad |S_j^-\rangle = \frac{a_{j+1}^\dagger a_j}{\sqrt{2}} |T\rangle. \tag{3}
\]

For each \( j \) the states \( |S_j^\pm\rangle \) are distinguished by the two energetically distinct orientations of an atomic pair and its neighbouring hole with energies, \( E(S_j^\pm) = U(1 \mp \frac{\pi}{\omega_T}(2j-1)) \). The target state \( |T\rangle \) defines the zero of energy. The \( |T\rangle \) and the \( |S_j^\pm\rangle \) states are coupled to first order in \( H_{BH} \) and they span the reduced state in \( \mathcal{R} \) of the ground state of the total system.

In the limit of large \( n \), the dynamics of the register is restricted to the basis of equation (3). This argument is understood by comparison to the dynamics in the homogeneous system. In the latter, the state with the largest coupling from the target state is the symmetrized \( |S\rangle \) with coupling matrix element \( \langle T|H_{BH}|S\rangle = -2\sqrt{n}J \). The state \( |S\rangle \) itself couples to a symmetrized state \( |S'\rangle \) of all Fock states with a one-site separation between the atomic pair and the hole: \( |S'\rangle = 1/\sqrt{4n} \sum_i (a_i^\dagger a_{i+2} + a_{i+2}^\dagger a_i) |T\rangle \). The coupling between these states is \( \langle S'|H_{BH}|S\rangle = -3J \). The dynamics on time scales \( t < 1/J \) are therefore constrained to the subspace \( \{|T\rangle, |S\rangle\} \) when \( n \gg 1 \). In the inhomogeneous case, the degeneracy is absent between states with neighbouring pairs and holes, \( \{|S_j^\pm\rangle\} \) and states where pairs and holes are separated. Hence, the coupling to states outside the restricted subspace can only be smaller than in the homogeneous case.

For the measurement, we choose a catalysis laser that is on resonance from the ground state of two atoms in a single well to a bound state \( \nu \) of a dipole–dipole coupled molecular \( S + P \) state. The bound state is chosen such that the catalysis laser is far off resonance from other bound states and repulsive potentials, see figure 1. For our many body system, we adopt the set of many body states \( \{|M_j^\pm\rangle = (1/\sqrt{2})b_j^\dagger |\nu(\pi/2)|2\rangle |S_j^\pm\rangle\} \), where \( b_j^\dagger \) is the creation operator for a molecule in the bound state \( \nu \). These states describe \( n - 2 \) atoms trapped in the lattice and a single molecule at site \( j + (1 \mp 1)/2 \), with dipole–dipole coupling energy.

\(^3\) For convenience we choose \( N \) odd and fix the site index \( j = 0 \) at the minimum of the magnetic trap.
Figure 1. Schematic of the relevant couplings in the problem. The unit filled state $|T\rangle$ describing a target quantum register and the states $|S^\pm_j\rangle$ having one doubly occupied lattice site and a neighbouring hole are coupled to first order in $H_{\text{BH}}$. A catalysis laser resonantly couples the ground states $|S^\pm_j\rangle$ to the excited states $|M^\pm_j\rangle$ describing a bound molecule at the doubly occupied site. The bound states quickly decay and give the possibility of monitoring population in the ‘faulty’ register states $|S^\pm_j\rangle$.

$|M^\pm_j\rangle H_{dd}|M^\pm_j\rangle = \hbar\omega$. The free atomic Hamiltonian for $n$ atoms is

$H_A = \sum_j \hbar \omega_g |e_j\rangle\langle e_j|$, where $|e_j\rangle(|g_j\rangle)$ denotes the excited (ground) state for an atom at site $j$. Hereafter, we work in units with the numerical value of $\hbar$ equal to 1 meaning energies are understood as being in units of inverse time. The ‘bare’ energy Hamiltonian $H_0$ including coupling in the restricted basis of $H_{\text{BH}}$ is then:

$H_0 = H_A + H_{dd} + H_{\text{BH}}$

$= \omega_g \sum_j |e_j\rangle\langle e_j| + \sum_{j,\pm} E(S^\pm_j)|S^\pm_j\rangle\langle S^\pm_j|$

$+ (\omega_\nu + E(S^\pm_j) - U)|M^\pm_j\rangle\langle M^\pm_j| - \sqrt{2} J \sum_{j,\pm} (|S^\pm_j\rangle\langle T| + |T\rangle\langle S^\pm_j|)$. \hspace{1cm} (4)

Under the atom laser interaction, $H_{\text{AL}}$, the ground and excited state of each atom is coupled as is each many body state $|M^\pm_j\rangle$ and its corresponding ground state $|S^\pm_j\rangle$. In the rotating wave approximation, the interaction is:

$H_{\text{AL}} = \frac{\Omega_A}{2} \sum_j (e^{-i\omega_{\text{AL}} t}|e_j\rangle\langle g_j| + \text{h.c.}) + \frac{\Omega_M}{2} \sum_{j,\pm} (e^{-i\omega_{\text{AL}} t}|M^\pm_j\rangle\langle S^\pm_j| + \text{h.c.})$, \hspace{1cm} (5)

where $\Omega_A(M)$ are the atomic (molecular) Rabi frequencies, related by $\Omega_M = \sqrt{F_\nu}\Omega_A$, where $F_\nu$ is the Franck–Condon factor equaling the spatial overlap between the relative coordinate wavefunction describing two ground electronic state atoms trapped in a single lattice well and the molecular bound state $\nu$. For bound states of interest, such as the long-range bound states of the $0_g^e(P_{3/2})$ potential [14], the catalysis detuning from atomic resonance, $\Delta = \omega_L - \omega_{eg}$, is several thousands linewidths meaning the atomic saturation is low $s_A = (\Omega_A^2/2\Delta^2) \ll 1$. In this case, the excited atomic states can be adiabatically eliminated and each atom in a singly occupied well experiences a light shift equal to $V_c = \Delta s_A/2$. There are $n$ singly occupied wells in the $|T\rangle$ state and the total single atom light shift is therefore equal to $nV_c$. The $|S^\pm_j\rangle$
states have \( n - 2 \) singly occupied wells giving a corresponding light shift of \((n - 2)V_c\). The differential single atom light shift between these states is then \(2|V_c|\). The total Hamiltonian \( H_0 + H_{\text{AL}} \) in the rotating frame is

\[
H_I = \sum_{j, \pm} \left\{ (2|V_c| + E(S^\pm_j))|S^\pm_j\rangle\langle S^\pm_j| + (2|V_c| + E(S^\pm_j) - U) \right\} |M^\pm_j\rangle\langle M^\pm_j|
\]

\[
- \sqrt{2}J(|S^+_j\rangle\langle T| + |T\rangle\langle S^+_j|) + \frac{\Omega_M}{2} \left( |M^+_j\rangle\langle S^+_j| + |S^+_j\rangle\langle M^+_j| \right).
\]

(6)

Any population in the bound molecular states will decay at a rate \(\gamma_M \approx 2\Gamma\), where \(\Gamma\) is the single atom decay rate. For molecular photo-association by red detuned light, the decay products are typically ground state molecular species or 'hot' atoms meaning the atoms escape the trapped ground states described by equation (3). We, therefore, model the system according to a trace non-preserving master equation:

\[
\dot{\rho} = -i[H_I, \rho] - \gamma_M/2 \sum_{j, \pm} \left( |M^\pm_j\rangle\langle M^\pm_j| \rho + \rho |M^\pm_j\rangle\langle M^\pm_j| \right).
\]

(7)

We have ignored spontaneous emission due to decay from the single atom excited states at a rate \(s_\Delta\Gamma \ll \Gamma\) per atom. For time scales \(1/\gamma_M \ll t \ll 1/(U + 2|V_c|)\), the excited state coherences can be solved for. To second order in \(U/\gamma_M, 2|V_c|/\gamma_M\) they are

\[
\rho_{M^+_j, T} (t) = -i \frac{\Omega_M y_M/4}{(2|V_c| + E(S^+_j) - U)^2 + (\gamma_M/2)^2} \rho_{S^+_j, T} (t)
\]

\[
\rho_{M^+_j, M^+_j} (t) = \frac{\Omega_M^2}{4} \frac{1}{U^2 + (\gamma_M/2)^2} \rho_{S^+_j, S^+_j} (t)
\]

\[
\rho_{M^+_j, S^+_j} (t) = -i \frac{\Omega_M y_M/4}{U^2 + (\gamma_M/2)^2} \rho_{S^+_j, S^+_j} (t).
\]

(8)

Inserting these expressions back into the equations for the dynamics in the ground state we have

\[
\dot{\rho}_{S^+_j, T} = -i(E(S^+_j) + 2|V_c|)\rho_{S^+_j, T} + i\sqrt{2}J(\rho_{T, T} - \rho_{S^+_j, S^+_j})
\]

\[
- \frac{\Omega_M y_M/8}{(2|V_c| + E(S^+_j) - U)^2 + (\gamma_M/2)^2} \rho_{S^+_j, T}
\]

\[
\dot{\rho}_{T, T} = i\sqrt{2}J \sum_{j, \pm} (\rho_{S^+_j, T} - \rho_{T, S^+_j})
\]

\[
\dot{\rho}_{S^+_j, S^+_j} = -i\sqrt{2}J(\rho_{S^+_j, T} - \rho_{T, S^+_j}) - \frac{\Omega_M^2 y_M/4}{U^2 + (\gamma_M/2)^2} \rho_{S^+_j, S^+_j}.
\]

(9)

These equations describe the Bose–Hubbard coupled states with a decay in population of each state with an atomic pair at a rate \(2\kappa = \Omega_M^2 y_M/4(U^2 + (\gamma_M/2)^2)\), and decay of coherences between each of these states and state \(|T\rangle\) at a rate \(\kappa\).

This type of evolution characterized by measurement induced phase damping was studied extensively by Gagen and Milburn [15]. We now show that our system can satisfy the conditions for this effect and in particular can be driven to the \(|T\rangle\) state by monitoring the environment for a signature of decay from the molecular bound state.

For the inhomogeneous system, the state \(|T\rangle\) couples to \(2n\) distinguishable states \(|S^\pm_j\rangle\). However, we can define an effective Rabi frequency between the state \(|T\rangle\) and the subspace spanned by \([|S^\pm_j\rangle]\). This frequency is close to the coupling matrix element between the state \(|T\rangle\) and the state \(|S\rangle\) in the homogeneous system, namely \(2\sqrt{n}J\). The coherences between the
two subspaces decay at a rate $\kappa$, and the population in the subspace $|S_j^+\rangle$ decays at a rate $2\kappa$. The ‘good’ measurement regime as derived in [15] is then:

$$\Omega_M / \gamma_M \ll 1 < \kappa / 2\sqrt{nJ}. \quad (10)$$

The left side inequality ensures that the excited states $|M_j^\pm\rangle$ are weakly populated (equivalent to the condition for adiabatic elimination of these states). The right side inequality ensures that measurement is sufficiently strong to damp coherences on the time scale that they develop due to tunnelling.

The limiting quantity that determines the decay rate of the weakly saturated molecular states and hence the measurement strength is the Franck–Condon factor $F_\nu$. It is calculated for bound–bound transitions using the reflection approximation of Julienne [17], where we solve for the ground state relative coordinate wavefunction for two atoms in a lattice well using a pseudo-potential appropriate for $^{87}$Rb. We choose to couple to the $V_{\perp} = 22E_R$, the result is $F_\nu \approx 5 \times 10^{-7}$. Given this confinement, the on-site interaction using $a_c = 5.6$ nm is $U = 3.574$ kHz. Choosing an experimentally reasonable atomic Rabi frequency $\Omega_1 = 25\Gamma$, where $\Gamma = 2\pi \times 6.065$ MHz, we find $\kappa \approx 0.13U$. Here the atomic scattering due to the catalysis laser is $s_j\Gamma \approx 6.7 \times 10^{-8}\Gamma$ per atom and the off resonant light shift is $|V_c| = 3.87U$.

By way of example we define a one-dimensional register $R$ with 501 atoms that resides inside a lattice filled with $N = 551$ atoms. An external magnetic trapping frequency of $\omega_T = 2\pi \times 8$ Hz ensures that the last occupied well has an energy $\varepsilon((N-1)/2) = 0.9U$. We note that the probability for tunnelling of holes from the edges is negligible as $J/(n\delta) = 0.34$. In practice it is not important to know the exact number of atoms in the lattice as long as the trap strength is chosen such that, given the uncertainty in the number of atoms, the constraint $\varepsilon((N-1)/2) < U$ is always satisfied. These parameters fix the ratio $U/J = 500$ and the measurement strength is therefore $\kappa / 2\sqrt{nJ} \approx 1.5$.

When the environment is monitored, for instance by looking for photon scattering from the molecular state, the evolution of ground states can be modelled using quantum trajectories. For our simulation, the ground-state wavefunction $|\psi(t)\rangle = c_T(t)|T\rangle + \sum_{j=1}^{n_0} c_j(t)|S_j^+\rangle$ is updated according to the non-Hermitian Hamiltonian $H = H_T - i\gamma_M / 2 \sum_j \lambda_j |M_j^+\rangle\langle M_j^+|$. A quantum trajectory corresponding to a null measurement result converges to the target state $|T\rangle$ and freezes the state there as demonstrated in figure 2 (for a similar effect with ions see [16]). The preparation time scale is $t_{\text{prep}} = 1/\kappa$. The success or failure of the preparation is conclusive with failure probability $p_{\text{fail}} = 1 - \rho_{T,T}(0)$.

Real experiments will be constrained to finite detector efficiencies $\eta$. For $\eta = 0$, corresponding to nonselective measurement, the system dynamics evolve according to equation (9). If we represent the dynamics of the system as a pseudo-two-state system $|T\rangle$ and $|S\rangle$, with an average energy splitting $U + 2|V_c|$ the equations of motion for the pseudo-Bloch vector are:

$$\dot{u} = (U + 2|V_c|)u - \kappa u$$
$$\dot{v} = -\kappa v - (U + 2|V_c|)w - \sqrt{2nJ} w$$
$$\dot{w} = -\kappa (x + w) + 4\sqrt{2nJ} v$$
$$\dot{x} = -\kappa (x + w),$$

where $u = \text{Re}[\rho_{S,T}], v = \text{Im}[\rho_{S,T}], w = \rho_{S,S} - \rho_{T,T}$, and the decreasing norm is $x = \text{Tr}[\rho]$. After a period $1/2|V_c|$, the coherences approach steady state, and the target state population,
Figure 2. Target state fidelity, \( F(t) = \rho_{T,T}(t) \), for \( U/J = 500 \) and \( n = 500 \) atoms. The main plot shows a quantum trajectory for a state that begins in the ground state of the Bose–Hubbard Hamiltonian and converges to the unit filled state \( |T\rangle \) by a null result on a selective measurement. The time scale for saturating the target state for the parameters here is \( t_{\text{sat}} \approx \kappa^{-1} = 7.7/U \). The inset shows the evolution for three detector efficiencies (\( \eta = 1.0, 0.9, 0.8 \)) over a longer time scale. Also shown is the oscillatory dynamics with fundamental frequency \( U \) resulting when the measurement is turned off after the target state is reached.

Assuming \( \rho_{T,T}(0) \approx 1 \) and \( \kappa/2\sqrt{n}J > 1 \), is
\[
\rho_{T,T}^{ns}(t) = \rho_{T,T}(0) \exp\left\{ -8nJ^2\kappa t/(U + 2|V_c|^2 + \kappa^2) \right\}. \tag{12}
\]

As shown in figure 2, if the initial state is close to the target state then the decay time is long compared to \( 1/U \). For our parameters, the decay of population from the target state is predominantly suppressed due to the single atom light shift \( V_c \) which shifts the coupled states \( \{|T\rangle, |S^+\rangle\} \) out of resonance. This shift is dependent on the molecular state coupled to (through the detuning) and is therefore system dependent. However, in the strong measurement limit \( \kappa/2\sqrt{n}J \rightarrow \infty \), the system dynamics are frozen by virtue of the continuous quantum Zeno effect. In the case of finite detector efficiencies, we can express the approximate fidelity to be in the target state. Assuming a null measurement result, for times \( t > t_{\text{prep}} \), \( 1/2|V_c| \), it is: \( F(\eta, t) = \rho_{T,T}(t) = \eta + (1 - \eta)\rho_{T,T}^{ns}(t) \). In practice, high detection efficiencies may be obtained by applying a second photo-ionizing laser on resonance with the molecular bound state and monitoring the emission of ions.

It is necessary to keep the measurement on during a computation to maintain high fidelity in the unit filled state. If instead, the catalysis field is turned off after the target state is reached, the system will freely evolve according to \( H_{\text{BH}} \). For \( \delta/U \ll 1 \) and for times \( t < 1/J \) the fidelity can be calculated using the restricted basis set:
\[
F(t) = 1 - 8(J/U)^2(n - \cos(Ut)(1 + \sin(\delta(n - 1)t)/\sin(\delta t))). \tag{13}
\]

This solution compares well with exact numerical simulations. Note that the time averaged deviation from perfect fidelity is \( 1 - \langle F(t) \rangle = 8n(J/U)^2 \); which is twice as bad as the deviation if the system were left in the ground state \( |\Psi_g\rangle \). This result shows that other dissipative techniques for initializing a register in a lattice such as Raman side-band cooling [18], and phase space compression [19], if not corrected, will suffer from the same loss of fidelity as described by equation (13).
In summary, we have shown that efforts to prepare a register of atomic qubits in an optical lattice suffer from errors inherent in the underlying many body dynamics. We have introduced a protocol that addresses this issue to make the MI transition a robust mechanism for initialization. While the discussion has focused on one-dimensional dynamics, the method is also applicable to higher dimensions, which is the relevant regime for scalability.

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