Testing the homogeneity of type Ia Supernovae in near-infrared for accurate distance estimations

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ABSTRACT

Since the discovery of the accelerating expansion of the Universe more than two decades ago, Type Ia Supernovae (SNe Ia) have been extensively used as standardisable candles in the optical. However, SNe Ia have shown to be more homogeneous in the near-infrared (NIR), where the effect of dust extinction is also attenuated. In this work, we explore the possibility of using a low number of NIR observations for accurate distance estimations, given the homogeneity at these wavelengths. We found that one epoch in J and/or H band, plus good gr-band coverage, gives an accurate estimation of peak magnitudes in the J (\(J_{\text{max}}\)) and H (\(H_{\text{max}}\)) bands. The use of a single NIR epoch only introduces an additional scatter of \(\sim 0.05\) mag for epochs around the time of \(B\)-band peak magnitude (\(T_{\text{max}}\)). We also tested the effect of optical cadence and signal-to-noise ratio (S/N) in the estimation of \(T_{\text{max}}\) and its uncertainty propagation to the NIR peak magnitudes. Both cadence and S/N have a similar contribution, where we constrained the introduced scatter of each to \(<0.02\) mag in \(J_{\text{max}}\) and \(<0.01\) in \(H_{\text{max}}\). However, these effects are expected to be negligible, provided the data quality is comparable to that obtained for observations of nearby SNe (\(z \lesssim 0.1\)). The effect of S/N in the NIR was tested as well. For SNe Ia at 0.08 < \(z\) < 0.1, NIR observations with better S/N than that found in the CSP sample is necessary to constrain the introduced scatter to a minimum (\(\lesssim 0.05\) mag). These results provide confidence for our FLOWS project that is aimed at using SNe Ia with public ZTF optical light curves and few NIR epochs to map out the peculiar velocity field of the local Universe. This will allow us to determine the distribution of dark matter in our own supercluster, Laniakea, and to test the standard cosmological model by measuring the growth rate of structures, parameterised by \(fD\), and the Hubble-Lemaître constant, \(H_0\).

Key words. supernovae: general – cosmology: observations – distance scale

1. Introduction

The expansion rate of the Universe, parameterised by the Hubble-Lemaître parameter, \(H(z)\), varies across cosmic time. In the last few years, there has been tremendous effort to measure the local value, known as the Hubble-Lemaître constant (\(H_0\)), with extremely high precision (<2% uncertainty; Riess et al. 2021). Recent results have further increased the discrepancy in the value of \(H_0\) between the local distance ladder (\(H_0 = 73.04 \pm 1.04\) km s\(^{-1}\) Mpc\(^{-1}\), baseline with systematics; Riess et al. 2021) and the cosmic microwave background (CMB; \(H_0 = 67.4 \pm 0.5\) km s\(^{-1}\) Mpc\(^{-1}\); Planck Collaboration VI 2020) measurements, colloquially known as the ‘Hubble tension’, to 5\(\sigma\) (however, see Freedman et al. 2019; Huang et al. 2020; Khetan et al. 2021 for some alternative local measurements). This discrepancy possibly hints towards new physics beyond the standard cosmological model, or alternatively, unaccounted systematic effects (see Di Valentino et al. 2021 for a recent review on the Hubble tension).

In the local Universe, the recession velocities measured from galaxies are affected by a combination of the expansion of the Universe and the gravitational pull of other adjacent galaxies. The measurement of these peculiar velocities is critical for two main reasons. First, cosmological analyses with Type Ia Supernovae (SNe Ia) rely on discerning the contribution of peculiar velocities to isolate the cosmological redshift. Secondly, peculiar velocities can be used to infer the matter-density distribution...
in the local Universe (Peebles 1976), including our own supercluster, Laniakea (e.g., Tully et al. 2014). The latter provides a direct measurement of the growth-rate of structure, which can be compared to estimates from the early Universe (e.g., Linder 2005).

Current measurements of peculiar velocities often rely on methods such as the Fundamental-Plane and Tully-Fisher relations (e.g., Tully et al. 2016), which provide distances with relatively large uncertainties (with an rms of ~20–30% per galaxy) and only reach out to $z \sim 0.05$, standing in the way of the study of the peculiar velocities at further distances. Therefore, there is a lack of higher-precision methods that can also extend to further redshifts for the estimation of distances in the local Universe.

Since the discovery of the accelerating expansion of the Universe more than two decades ago (Riess et al. 1998; Perlmutter et al. 1999), SNe Ia have been extensively used as cosmological distance indicators. In the optical, their light curves can be standardized through empirical relations between their peak brightness, stretch (e.g., Rust 1974; Pskovskii 1977; Phillips 1993), and colour (Tripp 1998). In addition, SNe Ia are brighter in the optical (where detectors are larger as well), compared to other wavelengths, making them easier to observe. Therefore, cosmological analyses with SNe Ia (e.g., Betoule et al. 2014; Scolnic et al. 2018; Abbott et al. 2019) commonly focus on optical wavelengths and rely on light-curve fitters, such as SALT2 (Guy et al. 2005, 2007), for the estimation of their light-curve parameters. Moreover, SNe Ia in the optical have recently been used to estimate the growth-rate of structures (e.g., Boruah et al. 2020; Stahl et al. 2021).

SNe Ia were first proposed as distance indicators in the near-infrared (NIR) around four decades ago (Elias et al. 1981, 1985, but also see Meikle 2000), where they seem to be true ‘standard candles’ (as opposed to ‘standardisable candles’ in the optical). In other words, an estimation of the NIR peak magnitudes is all that is needed to measure distances. The NIR light curves of SNe Ia present lower intrinsic dispersion than the optical light curves and have the advantage of being less affected by dust extinction, which makes them exceptional for measuring cosmological distances (e.g., Krisciunas et al. 2004; Wood-Versey et al. 2008; Freedman et al. 2009; Barone-Nugent et al. 2012; Phillips 2012; Weyant et al. 2014; Friedman et al. 2015; Johansson et al. 2021). Moreover, the NIR light curves of SNe Ia have already been used to constrain $H_0$ to a few percent (e.g., Burns et al. 2018; Dhawan et al. 2018).

The low intrinsic dispersion of SNe Ia in the NIR raises the possibility of using them to achieve accurate cosmography by measuring peculiar velocities of local galaxies, reaching out to $z \sim 0.1$ or even beyond. However, the sample of SNe Ia observed in the NIR is currently low due to several factors: low NIR detector sensitivity in the past; SNe Ia are fainter at these wavelengths, where the sky brightness dramatically decreases the contrast for the (SN) observations, thus needing to integrate for longer; and the number of facilities with NIR instruments (compared to optical ones) is low. However, by taking advantage of the exceptional homogeneity of the SNe Ia in the NIR, we can possibly reconstruct their light curves with just a few photometric data points (e.g., Krisciunas et al. 2004), increasing the total number of observed objects.

Given the large stream of optical photometry publicly provided by the Zwicky Transient Facility (ZTF; Graham et al. 2019), which works as a precursor and testing ground for LSST, hundreds to thousands of SNe Ia are being followed-up with high-cadence (average of 2 days) $gr$-bands photometry. Thus, ZTF can provide the optical data coverage while NIR photometry can be obtained with other facilities. This work aims to test how accurately we can retrieve NIR peak magnitudes with well-covered optical light curves and few NIR epochs for distance estimations. Our results will give assurance to use SNe Ia with public ZTF $gr$-band light curves with sparse data in the NIR to reconstruct the cosmography of our local supercluster, measure the growth-rate of structure and $H_0$, and test $\Lambda$CDM and alternative cosmological models. In the future, this can be extended to use optical data from the Rubin Observatory Legacy Survey of Space and Time (LSST) and NIR data from telescopes such as the Roman Space Telescope and James Webb Space Telescope.

This paper is organised as follows: in Sect. 2, we present the sample of SNe Ia and the quality cuts used throughout this work, together with the SNoOpy fits. In Sect. 3, we describe the simulations of NIR light curves. The comparison between the fits of the simulations and our reference sample is presented in Sect. 4, while the study of systematics is presented in Sect. 5. In Sect. 6, we estimate distances using the reference sample of SNe Ia and compare them with those from the simulations. Finally, we summarise and conclude in Sect. 7. The scripts used throughout this work can be found online.

2. Data and method

In this work, we use the Carnegie Supernova Project (CSP; Hamuy et al. 2006) sample as it is one of the most comprehensive samples of SNe Ia with extensive $ugVRiYJH$ (optical to NIR) coverage and well-understood magnitude systems to date. The data from CSP-I consists of three data releases (DRs) described in Contreras et al. (2010, DR1), Stritzinger et al. (2011, DR2) and Krisiunias et al. (2017, DR3), while the data from CSP-II are described in Phillips et al. (2019) and Hsiao et al. (2019). CSP-II does not have a public DR to date, but one is in progress (Suntzeff et al., in prep.). We include all the 134 SNe Ia from CSP-I and 202 from CSP-II (the cosmology sub-sample from Phillips et al. 2019). Thus, the CSP sample we use consists of a total of 336 SNe Ia.

As CSP observations have used different filters and telescopes throughout the different campaigns, we apply S-corrections (Stritzinger et al. 2011) to work in a single magnitude system, simplifying the handling of data. This is specifically useful for those CSP-I SNe with multiple $V, Y$, or $J$ bands, and for combining SNe from CSP-I and CSP-II.

2.1. Light-curve fitter

Currently, a small number of SNe Ia light-curve fitters work in the NIR, such as SNoOpy (Burns et al. 2011), PISCOOLA (Müller-Bravo et al. 2022), and BayeSN (Mandel et al. 2022). The difference between them is that SNoOpy and BayeSN are template-based fitters (i.e. trained on well-sampled SNe Ia) while PISCOOLA uses a data-driven approach that relies on Gaussian processes (GP; Rasmussen & Williams 2006). However, for the aim of this work, as we need to reconstruct the NIR light curves with a few photometric points, PISCOOLA is not an option. On the other hand, the difference between SNoOpy and BayeSN is that the latter uses a probabilistic approach, constructing a hierarchical Bayesian model for the spectral energy distribution

1. https://www.ztf.caltech.edu/ztf-public-releases.html
2. https://github.com/HOSTFLOWS/flows_sims
of SNe Ia, therefore obviating the need of ad-hoc $K$-corrections (as is the case of SNooPy). However, BayeSN assumes a dust extinction law for SNe Ia, so it does not have the same freedom as SNooPy. Additionally, BayeSN is not public, so we choose to use SNooPy, with its latest version (v2.6), trained on the updated NIR spectral templates from Hsiao et al. (2007). However, SNooPy, as in the case of other light-curve fitters, heavily relies on optical data for accurate fitting.

To fit the SNe Ia, we use SNooPy with the *max_model* model, as we require measurements of $J_{\text{max}}$ and $H_{\text{max}}$. The resulting fits provide the following output parameters: $T_{\text{max}}$, $s_{BV}$, and $x_{\text{max}}$, where $T_{\text{max}}$ is the time of $B$-band peak magnitude, $s_{BV}$ is the ‘colour stretch’ parameter as defined in Burns et al. (2014) and $x_{\text{max}}$ represents the peak magnitude in $x$-band, for each of the observed filters. An example fit for SN 2004eo is shown in Fig. 1. We note that the multi-colour light-curve templates (optical to NIR) are driven by the values of $T_{\text{max}}$ and $s_{BV}$. For more information on why the *max_model* model is used instead of other models, such as $EBV_{\text{model2}}$, see Appendix A. All magnitudes presented are in the CSP natural system and the reported uncertainties from SNooPy fits are statistical uncertainties only.

### 2.2. Sample cuts

As not all CSP SNe are useful for the purpose of this work, we proceeded to apply some cuts to the initial sample. We only used SNe Ia labelled as ‘normal’ according to Krisciunas et al. (2017) and Ashall et al. (2020). A current re-analysis of SNe Ia performed by Phillips et al. (in prep.) has re-labelled 1991T-like SNe from the CSP sample. Thus, we use their definition henceforth. This cut removes 34 of the initial 336 SNe Ia: one 2003fg-like SN, four 1986G-like SNe, 10 1991T-like SNe, sixteen 1991bg-like SNe, and 3 peculiar SNe (labelled as ‘...’) in Krisciunas et al. (2017).

Finally, we require at least one photometric point before and after the time of $J$-band peak ($J_{\text{max}}$) and $H$-band peak ($H_{\text{max}}$), as precise measurements of $J_{\text{max}}$ and $H_{\text{max}}$ are needed. These constraints are less stringent compared to those in the optical as the light-curve fits are mainly driven by the optical bands. In addition, we require at least three photometric points in each NIR band, to have a relatively good coverage of the light-curve peak. We note that the SNe are fitted with SNooPy first to estimate $T_{\text{max}}$. This cut removes 53 of the remaining 164 SNe.

We then proceed to remove any SN without $g$, $r$, $J$, or $H$ bands, as these are strictly required for our analysis: $g$ and $r$ being the bands used by ZTF while $J$ and $H$ being the NIR bands commonly available. The $Y$ band is not included as catalogues of standard stars for this band are not available for the whole sky, which are needed for the calibration. We note that 80 SNe do not have a $g$ band (all from CSP-II), while all of them have an $r$ band. In this step, we identified and removed 138 of the remaining 302 SNe.

The next cut requires the SN to have coverage of the optical peak as the estimation of $T_{\text{max}}$ is fundamental when fitting the light curves of SNe Ia (Sects. 2.3 and 3). For this, we need to have one or more photometric points at least two days before and two days after $T_{\text{max}}$ in $B$, $g$, $V$, or $r$ bands, providing an accurate estimation of the location of the peak. Almost all the SNe have the optical decline well covered, while some do not have pre-peak observations. We note that the SNe are fitted with SNooPyPy first to estimate $T_{\text{max}}$. This cut removes 53 of the remaining 164 SNe.

Finally, we require at least one photometric point before and after the time of $J$-band peak ($J_{\text{max}}$) and $H$-band peak ($H_{\text{max}}$), as precise measurements of $J_{\text{max}}$ and $H_{\text{max}}$ are needed. These constraints are less stringent compared to those in the optical as the light-curve fits are mainly driven by the optical bands. In addition, we require at least three photometric points, in each NIR band, to have a relatively good coverage of the light-curve peak. We note that the SNe are fitted with SNooPyPy first, with the same model as before, to estimate the locations of the NIR peaks. Here, we remove 61 of the 111 remaining SNe, leaving us with a sample of 50 CSP SNe Ia, comprising our reference sample.

In the top panel of Fig. 2, we show the redshift distribution of the reference sample and the complete CSP-I + CSP-II sample. One downside of the cuts applied is that all objects with $z \gtrsim 0.05$ are removed. An object at $z = 0.1$ is approximately 1.5 mag fainter than one at $z = 0.05$, which translates into a poorer signal-to-noise ratio (S/N) of the observations, making the estimations of light-curve parameters less accurate. However, SNe at higher $z$ also have faster restframe cadences compared to those at lower $z$ (for the same observed cadence), although at $z \lesssim 0.1$, there is only a small difference. The effects of cadence and S/N are later discussed in Sects. 5.1–5.3.

In the bottom panel of Fig. 2, we show the $s_{BV}$ distribution of the reference sample and the ‘normal’ SNe Ia from the CSP-I + CSP-II sample. Both distributions span approximately the same range in $s_{BV}$, with no major differences apart from the number of objects.

The cuts from this section are summarised in Table 1, where the numbers are also split by survey (CSP-I and CSP-II). Tighter constraints could be used, although this would further reduce the size of our sample, reducing statistics as well, and change the distribution of light-curve parameters.

#### 2.3. Reference Sample

Although the data quality of CSP-I and CSP-II are almost identical, the data coverage in the NIR bands is not the same. The NIR light curves of SNe Ia from CSP-I are on average better populated than those from CSP-II. Examples of SNe from both surveys are shown in Figs. 3 and 4. This sample of 50 SNe Ia are later used as reference for the simulations in Sect. 3.

We create three different sets of reference values according to the bands used for the fits: (i) all bands; (ii) $gJH$; and (iii) $gr$ (see Table 2). We note that sets (ii) and (iii) are mainly used
as control sets to test whether there are any discrepancies in the output parameters (e.g., $T_{\text{max}}$) compared to using all bands (see Sect. 4).

### 3. Simulations

Given that we want to replicate what real observations would be (i.e. optical bands well covered with few NIR data points), the simulations consist of taking the complete $gr$-band light curves, plus $n$ epochs of coeval $J$- and $H$-band photometric points, for $n = 1, 2,$ and $3$. We note that we are sampling from the available photometry (henceforth referred to as ‘simulations’ in this paper). Combinations without repetition of the $JH$-band photometry are used for this:

$$C_n(N) = \binom{N}{n} = \frac{N!}{n!(N-n)!},$$

where $C$ is the total number of combinations and $N$ is the total amount of $JH$ epochs, for each band individually. For instance, a SN with 10 epochs of coeval $J$- and $H$-band photometric points would have $C_1(10) = 10$, $C_2(10) = 45$ and $C_3(10) = 120$ simulations for 1, 2, and 3 $JH$ epochs, respectively.

The resulting simulations are then fit, using SNoopy, as described in Sect. 2.3, and the output parameters saved for a later comparison (Sect. 4). In Fig. 5, we show an example of simulations with $n = 1$ for ASASSN-14hu, with the respective fits for...
Table 2. Bands used for the fits of the reference sample (Sect. 2.3) and simulations (Sect. 3).

| Sample     | Bands            |
|------------|------------------|
| Reference  | All bands, grJH, gr |
| Simulations| grJH, grJ, grH    |

each simulation in J and H bands. As can be seen from the fits, the values of $J_{\text{max}}$ and $H_{\text{max}}$ highly depend on the epochs of the photometry. We note that as we are dealing with few J and H photometric epochs, a proper estimation of the NIR peaks is, in principle, not possible, so we measure the time with respect to $T_{\text{max}}$, which can be accurately obtained from the complete optical gr-band light curves.

As we are aiming to reduce the scatter, ideally, we would need data around $T_{\text{max}}$. Unfortunately, as it is hard to obtain photometric data at specific epochs due to different constraints (weather, time, allocation, etc.), we have to look for a time the NIR light-curve templates have a larger gradient or slope (see Fig. 5 and Sect. 5 for further discussion). The weighted mean and the uncertainty on the weighted mean of these residuals are summarised in Table 3. Given that the uncertainties in light-curve fits are different for the simulations with only one epoch ($n = 1$), metrics (i) and (ii) are the same, while (iii) is not calculated. Also note that these metrics are defined in the restframe, i.e. epochs are corrected for time dilation using the SN redshift.

In Fig. 6, we show $J_{\text{max}}$ and $H_{\text{max}}$ residuals between the simulations and the reference sample (using all bands) as a function of metric (i), for simulations with grJH bands and $n = 1$. From this comparison, we see that the scatter in the residuals tends to be smaller around $T_{\text{max}}$, with the smallest scatter before $T_{\text{max}}$, and increases at later epochs. In general, the scatter is smaller for the H band compared to the J band. This is consistent with what has been found in other works (e.g., Dhawan et al. 2015). Additionally, we note that offsets in $J_{\text{max}}$ and $H_{\text{max}}$ tend to be larger where the NIR light-curve templates have a larger gradient or slop (see Fig. 5 and Sect. 5 for further discussion). The weighted mean and the uncertainty on the weighted mean of these residuals are summarised in Table 3. Given that the uncertainties in light-curve parameters from the reference sample and simulations are correlated (both use the same photometry), henceforth, we only use those from the latter for the statistics.

4. Analysis

Assuming that SNe Ia are standard candles in the NIR, peak NIR magnitudes can be directly used to estimate distances without further corrections (e.g., from stretch or colour). Thus, our main interest is to see how well we can measure $J_{\text{max}}$ and $H_{\text{max}}$ using just a few NIR photometric points. However, as can be seen in Fig. 5, the retrieved peak magnitudes highly depend on the time of the observations with respect to $T_{\text{max}}$. We therefore define three different metrics to understand what type of observations are best at obtaining accurate $J_{\text{max}}$ and $H_{\text{max}}$ measurements: (i) time of the closest J/H epoch with respect to $T_{\text{max}}$; (ii) mean time of the J/H epochs with respect to $T_{\text{max}}$; and (iii) difference between the earliest and latest (i.e. range) J/H epochs. Note that in the case of the simulations with only one epoch ($n = 1$), metrics (i) and (ii) are the same, while (iii) is not calculated. Also note that these metrics are defined in the restframe, i.e. epochs are corrected for time dilation using the SN redshift.

We also performed simulations for J and H bands separately to test whether having only one NIR band produces similar results compared to having two. To summarise, we carried out simulations with grJ, grH, and grJH bands (see Table 2), each with 1, 2, and 3 NIR epochs. The different sets of fits for the reference sample and simulations are summarised in Table 2.
window with a large-enough range. We noticed that a time range between $-5$ to $15$ days with respect to $T_{\text{max}}$ possess a relatively low scatter in $J_{\text{max}}$ and $H_{\text{max}}$. The weighted mean ($\Delta$) and weighted standard deviation ($\sigma$) of the residuals in this time window are $\Delta = 0.004 \text{ mag}$ and $\sigma = 0.047 \text{ mag}$, and $\Delta = -0.006 \text{ mag}$ and $\sigma = 0.053 \text{ mag}$, for $J_{\text{max}}$ and $H_{\text{max}}$, respectively.

In the case of the simulations with $gr$-band light curves, similar results are obtained, meaning that a single NIR band is as good as two NIR bands (see Fig. B.1). When comparing with the simulations with $n = 2$ and $n = 3$, the scatter in the residuals is reduced. For $n = 2$, we find a scatter in $J_{\text{max}}$ of 0.039 mag, while for $n = 3$, we find 0.030 mag. In the case of $H_{\text{max}}$, the scatter gets reduced to 0.043 mag and 0.038 mag, for $n = 2$ and $n = 3$, respectively. Although the offsets change for $n = 2, 3$, they still remain relatively small ($\lesssim 0.01$ mag). We also found that metrics (ii) and (iii) are not as relevant as metric (i) (see Fig. 7 for simulations with $n = 3$). In other words, the most important point is to have a NIR photometric epoch close to $T_{\text{max}}$. Having more photometric points helps reducing the scatter, although they are less essential for an accurate estimation of $J_{\text{max}}$ or $H_{\text{max}}$.

When comparing other light-curve parameters, such as $g_{\text{max}}$, $r_{\text{max}}$, $s_{\text{BV}}$, and $T_{\text{max}}$, between the reference sample and simulations, we found negligible differences ($\lesssim 0.02$ mag) for the first three parameters and $<0.1$ day for the last. These very small differences are produced by the NIR data. Although the fits are mainly driven by the optical light curves, the NIR also affects the estimation of $T_{\text{max}}$, and therefore, the other light-curve parameters. When using $gr$-bands and $gr$ bands as reference, we found similar results as when using all bands as reference (we note that for $gr$ bands only, $J_{\text{max}}$ and $H_{\text{max}}$ cannot be calculated).

5. Data-quality systematics

As was previously mentioned, the light-curve fits heavily rely on having a proper estimation of $T_{\text{max}}$. Any uncertainty in this parameter is propagated to others, such as the NIR peak magnitude. It follows that having small uncertainties and fast cadence is ideal, particularly in the optical bands. However, this is not always possible. In this section, we test how cadence and S/N affect the estimations of $J_{\text{max}}$ and $H_{\text{max}}$.

5.1. Cadence

To measure the effect of cadence, we started by taking the $gr$-band light curves of the reference sample and simulated different cadences by taking every two observations in each light curve, every three, every four, and so on. We make sure to ‘move’ the first observation taken so we have different starting points until every epoch before $T_{\text{max}}$ have been used (the reader should not confuse these simulations with those from Sect. 3). We note that the observer-frame cadences across the optical filters are very similar for a single SN as CSP observations were taken with multiple filters during the same nights (see Figs. 3 and 4). Our reference sample has an average cadence of $\sim 3.1$ days in $gr$, while the simulations have cadences $>3.1$ days.

In the top panel of Fig. 8, we show the difference in $T_{\text{max}}$ between the cadence simulations and reference sample ($\Delta T_{\text{max}}$). For the cadence simulations, only $gr$-bands are used and all those with restframe cadences $<10$ days are considered. Surprisingly, we can see that despite the inclusion of simulations with slow cadences (i.e. large gaps of up to ten days), the distribution of $\Delta T_{\text{max}}$ has a standard deviation of $\sim 0.5$ days, with a weighted average of 0.01 days (weighted by the uncertainty in $T_{\text{max}}$ of the simulations). By looking at the significance ($\Delta T_{\text{max}}/\sigma_{\text{Tmax}}$; bottom panel of Fig. 8), we see that most are relatively small, with an average of 0.13, while very few simulations have a significance greater than 3.0.

This accurate estimation of $T_{\text{max}}$ can be explained by having small uncertainties and a good coverage across the light curves, that is, having at least one observation before and after peak, as this helps anchor the light-curve templates. Looking at Fig. 9, we see how the weighted average and standard deviation of $\Delta T_{\text{max}}$ depend on cadence, shown as red squares with error bars. All weighted means are $<0.1$ days. For cadences $<7$ days, the standard deviations are all $<0.3$ days, while for cadences $>7$ days, they are $>0.5$ days, but less than 1 day. We note that average observer-frame cadences of ZTF (2 days) and LSST (assuming 3-days baseline cadence in each band) would have a very small effect on the estimation of $T_{\text{max}}$ while some additional scatter in $T_{\text{max}}$ is expected for cadences such as that of DES (7 days; Brout et al. 2019). We also note that the difference between observer-frame and restframe cadences is less than one day at $z < 0.1$. 

Fig. 6. $J_{\text{max}}$ (top panel) and $H_{\text{max}}$ (bottom panel) residuals, between simulations and reference samples, for coeval $J$- and $H$-band epochs and reference value. The weighted mean and uncertainty on the weighted mean in bins of two days are shown in blue. A “correction snake” and its uncertainty are calculated by fitting the residuals with GPs (red line and shaded region), while N is the total number of simulations.
Table 3. Weighted mean ($\Delta$) and uncertainty on the weighed mean ($\sigma$) of $J_{\text{max}}$ and $H_{\text{max}}$ residuals between the reference sample (Sect. 2.3) and simulations (Sect. 3).

| Phase range | $[-10, -8]$ | $[-8, -6]$ | $[-6, -4]$ | $[-4, -2]$ | $[-2, 0]$ | $[0, 2]$ | $[2, 4]$ | $[4, 6]$ | $[6, 8]$ | $[8, 10]$ | $[10, 12]$ | $[12, 14]$ | $[14, 16]$ | $[16, 18]$ |
|-------------|--------------|--------------|--------------|--------------|--------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| $\Delta_{J}(J_{\text{max}})$ | 0.054 | 0.024 | 0.019 | 0.005 | -0.005 | 0.012 | 0.009 | -0.028 | -0.042 | -0.033 | -0.008 | 0.032 | 0.041 | 0.007 |
| $\sigma_{J}(J_{\text{max}})$ | 0.011 | 0.007 | 0.006 | 0.005 | 0.009 | 0.010 | 0.008 | 0.014 | 0.027 | 0.027 | 0.036 | 0.047 | 0.041 | 0.050 |
| $\Delta_{H}(H_{\text{max}})$ | -0.012 | -0.033 | -0.014 | -0.009 | 0.005 | 0.007 | -0.005 | -0.007 | -0.024 | -0.018 | -0.012 | -0.027 | -0.022 | -0.020 |
| $\sigma_{H}(H_{\text{max}})$ | 0.013 | 0.010 | 0.011 | 0.009 | 0.011 | 0.007 | 0.009 | 0.018 | 0.008 | 0.011 | 0.010 | 0.014 | 0.019 | 0.026 |

Notes. The data from the first couple of rows are represented by blue circles with error bars in Fig. 6. The NIR bands used for the light-curve simulations are given as subscripts. For instance, $\Delta_{JH}(H_{\text{max}})$ is the weighted mean of $H_{\text{max}}$ residuals using $J$- and $H$-band light curves.

Fig. 7. $J_{\text{max}}$ residuals between simulations with three coeval $J$- and $H$-band epochs and reference sample as a function of metrics (i), (ii), and (iii) (top, middle, and bottom panels, respectively). The weighted mean and uncertainty on the weighted mean in bins of two days are shown in blue for each of the panels. The residuals in $J_{\text{max}}$ do not vary much as a function of metrics (ii) and (iii), which means that metric (i) is the most relevant for estimating $J_{\text{max}}$. The metrics are described in Sect. 4, while $N$ is the total number of simulations. The global offsets in the middle and bottom panels are driven by those simulations with phases $\leq$5 days and $\geq$30 days from the top panel.

We further test how any uncertainty in $T_{\text{max}}$ could propagate to $J_{\text{max}}$ and $H_{\text{max}}$ by multiplying $\Delta T_{\text{max}}$ with the gradient (slope) of the NIR light-curve templates (using the definition of error propagation), resulting in additional scatter. We note that the shape of the templates depends on the value of $s_{BV}$, which is obtained from the fits of the $gr$-band light curves from the reference sample. The results are shown in Fig. 10. The average offset is negligible and the scatter is $<0.02$ mag in $J$ band for NIR observations between $-10$ and $50$ days. The scatter is much smaller for $H$ band ($<0.01$ mag) than for $J$ band, as the former has a smoother light-curve shape with smaller gradient.

Fig. 8. $T_{\text{max}}$ comparison for cadence simulations. Top panel: difference in $T_{\text{max}}$ between the cadence simulations (Sect. 5.1) and reference sample ($\Delta T_{\text{max}}$). All simulations with restframe cadences $<10$ days are considered here. The weighted average $\Delta_{\text{max}}$ is 0.01 days (vertical dotted line). Bottom panel: $\Delta T_{\text{max}}$ over the uncertainty in $T_{\text{max}}$ (i.e. significance) for the cadence simulations. The average significance is 0.13 (vertical dotted line), while very few simulations have a significance greater than 3.0. The average uncertainty in $T_{\text{max}}$ (0.19 days) is much larger than the weighted average $\Delta T_{\text{max}}$. 

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(see Fig. 5). We note that these scatters are even lower around $T_{\text{max}}$ and can be considered as upper limits as cadences of up to 10 days are being considered here. We also note that these values are much smaller than the scatter found in Sect. 4 (i.e. \(-0.05\) mag), although these have to be added in quadrature.

Apart from the scatter introduced by different cadences, we can isolate the effect of coverage of the rise of the light curves, which also affects the estimations of $T_{\text{max}}$. In Fig. 11, we show the difference in $T_{\text{max}}$ between the cadence simulations and reference sample ($\Delta T_{\text{max}}$) as a function of the first observation used for the simulations. One can see that independent of the cadence of the simulations, the scatter in $\Delta T_{\text{max}}$ starts rapidly increasing for the first observations with phases of \(\geq 5\) days. This is really important as it suggests that real observations should ideally cover earlier epochs (\(<5\) days) in $gr$-bands to provide precise estimations of $T_{\text{max}}$. However, observations starting at later epochs can still be used, with the caveat of introducing additional scatter.

5.2. Signal-to-noise in the optical

The effect of S/N is important as it does not only depend on things such as the detector, length of the exposures, and so on, but also on the distance and brightness of the objects. To measure how the S/N of optical observations affects the estimation of $T_{\text{max}}$, a similar procedure is followed as before. We started by taking $gr$-band light curves of the reference sample and simulated different S/N by multiplying the flux uncertainties in each of the light curves by $x$, for $x = 2, 3, 4, 5, 6, 7$, and randomly sampling new observations using a normal distribution with the new uncertainties. We note that the uncertainties in the CSP observations are relatively small in part due to observing nearby SNe. We use the S/N in $g$ band as reference, although the S/N in $r$ band is very similar. For each SN, the median S/N is used instead of the mean as the S/N greatly changes as a function of light-curve phase.

After calculating the difference in $T_{\text{max}}$ between the S/N simulations and reference sample, we find relatively similar results compared to those in Sect. 5.1. The distribution of $\Delta T_{\text{max}}$ has a standard deviation of 0.13 days, with a weighted average of 0.02 days (top panel of Fig. 12), while the average significance in $\Delta T_{\text{max}}$ is 0.23 with almost no simulations with a significance greater than 3.0 (bottom panel of Fig. 12).

Figure 13 shows $\Delta T_{\text{max}}$ as a function of S/N. This figure shows that the scatter in $\Delta T_{\text{max}}$ slowly increases as the S/N decreases (note the logarithmic scale in the $x$-axis). Despite the decrease in S/N of the simulations, SNooPy is still able to retrieve a relatively accurate estimation of $T_{\text{max}}$. This can be explained by having decent coverage throughout the entire light curves.

We also test how the uncertainty in $T_{\text{max}}$ propagates to $J_{\text{max}}$ and $H_{\text{max}}$. The results are shown in Fig. 14. Although $J_{\text{max}}$ seems to have some slight offset, this is very small (\(<0.005\) mag). The effect of S/N only introduces an additional scatter \(\lesssim 0.02\) mag.
in the J band and <0.01 mag in the H band, being even lower around $T_{\text{max}}$ (smaller than the scatter found in Sect. 4). These are similar to the dispersion found in Sect. 5.1. As we are including simulations with S/N as low as ~10, these dispersions can also be considered as upper limits. Observations of low-z SNe Ia, such as those observed by the CSP, typically have small photometric uncertainties due to their intrinsically bright nature.

From this set of tests, we conclude that SNooPy can give an accurate estimation of $T_{\text{max}}$, and therefore, $J_{\text{max}}$ and $H_{\text{max}}$, for different cadences and S/N in gr-bands. Additionally, H band is less affected by uncertainties in the estimation of $T_{\text{max}}$ compared to J band, due the smooth and relatively flat shape of the SN light curve at these wavelengths.

### 5.3. Signal-to-noise in the NIR

The effect of S/N is also important for NIR observations of SNe Ia, especially as the S/N is lower at these wavelengths compared to the optical. To test the effect of S/N of NIR observations in the estimation of the NIR peaks, we proceed in a similar fashion as described in the previous section. However, this time the complete gr-band light curves are used together with the closest NIR epoch to $T_{\text{max}}$. The uncertainty in the NIR photometric point is then varied to simulate different S/N and the light curves are fitted to estimate $J_{\text{max}}$ and $H_{\text{max}}$.

The average S/N of our reference sample is 54 and 39 in J and H bands, respectively. These are relatively low compared to the average S/N of ~100 in g band and SNe Ia at higher redshift would have even lower values. The CSP sample contains thirteen SNe Ia with J-band data at $z > 0.08$ with an average S/N of ~21 and only eight objects with H-band data with an average S/N of ~8. There is not only a large difference in S/N between low-z ($\lesssim 0.05$) and high-z SNe ($\gtrsim 0.08$), but also between NIR bands.

Figure 15 shows the residual in $J_{\text{max}}$ between NIR S/N simulations with J-band $S/N > 21$ and the reference sample ($\Delta J_{\text{max}}$). The weighted mean and standard deviations of $\Delta J_{\text{max}}$ are 0.006 mag and 0.070 mag, respectively. The scatter is larger than the one found in Sect. 4 (~0.05 mag), which was expected. The weighted mean and standard deviations of $\Delta H_{\text{max}}$ are 0.014 mag and 0.107 mag, respectively, for simulations with $S/N > 8$. These are much larger compared to the J band and is due to the lower S/N.

In Fig. 16, we show the $\Delta J_{\text{max}}$ (top panel) and $\Delta H_{\text{max}}$ (bottom panel) as a function of $S/N$. The scatter in both bands is similar for the same S/N, although the average S/N is lower in the H band. This test shows the importance of having high S/N observations in the NIR. If the aim is having an accurate estimation of $J_{\text{max}}$ and $H_{\text{max}}$ for SNe Ia at $z \sim 0.1$, NIR observations with better S/N than those obtained by CSP must be achieved (see Fig. 16). We note that, however, several of the CSP observations were obtained with a 1 meter telescope (Swope), so using 2 meter-class telescopes or better is ideal. We also note that in the restframe, the J-band light curves of SNe Ia are intrinsically brighter than their H-band light curves, making J-band observations better suited for measuring distances at high $z$. 

### Fig. 11. Difference in $T_{\text{max}}$ between the cadence simulations (Sect. 5.1) and reference sample ($\Delta T_{\text{max}}$) as a function of the phase of the first observation. The simulations are split in three groups according to the cadence (cad): $2 < \text{cad} < 5$ days (purple crosses), $5 < \text{cad} < 8$ days (pink circles), and $8 < \text{cad} < 10$ days (gold squares). $T_{\text{max}}$ is shown as a vertical dashed line. Independent of the cadence, the scatter in $\Delta T_{\text{max}}$ starts rapidly increasing for the first observations with phases of $\geq 5$ days.

### Fig. 12. $T_{\text{max}}$ comparison for S/N simulations. Top panel: difference in $T_{\text{max}}$ between the S/N simulations (Sect. 5.2) and reference sample ($\Delta T_{\text{max}}$). The weighted average $\Delta T_{\text{max}}$ is ~0.007 days (vertical dotted line). Bottom panel: $\Delta T_{\text{max}}$ over the uncertainty in $T_{\text{max}}$ (i.e. significance) for the S/N simulations. The average significance is 0.23 (vertical dotted line), while very few simulations have a significance greater than 3.0. The average uncertainty in $T_{\text{max}}$ (0.12 days) is larger than the weighted average $\Delta T_{\text{max}}$. 

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6. Near-infrared distances

The final step in this work is to calculate the precision in the distance estimations from the simulations of Sect. 4. Assuming that SNe Ia are standard candles in the NIR, the peak apparent magnitude is the only parameter necessary to calculate distances and its uncertainty is directly propagated to the measured distance:

\[ \mu = m_{\text{max}} - M, \]  
(2)

where \( \mu \) is the distance modulus, \( m_{\text{max}} \) is the peak apparent magnitude in a NIR band (e.g., \( J_{\text{max}} \) or \( H_{\text{max}} \)) and \( M \) is the peak absolute magnitude in that same band. To calculate distances, a cosmological model needs to be fitted. For simplicity, we assume a flat ΛCDM cosmology and fix the value of \( M (J_f = M_H = -18.5 \text{ mag}) \), fitting only \( H_0 \) and the intrinsic dispersion of SNe Ia (\( \sigma_{\text{int}} \)). Only SNe at \( z > 0.01 \) are used as the contribution from peculiar velocities is relatively small at these redshifts. This reduces our reference sample to 36 objects. The resulting Hubble diagram in \( J \) band is shown in Fig. 17 (red circles). The Hubble residuals have an rms of 0.166 mag, while \( \sigma_{\text{int}} = 0.14 \text{ mag} \) was obtained.

To calculated distances with the simulations, we use the values of \( H_0 \) and \( \sigma_{\text{int}} \) obtained with the reference sample and apply the offsets found in Sect. 4:

\[ \mu = [m_{\text{max}} + \Delta(b, n, p)] - M, \]  
(3)

where the offset \( \Delta(b, n, p) \) depends on the NIR band \( (b) \), the number of epochs \( (n) \), and the phase \( (p) \) of the epoch closest to \( T_{\text{max}} \). The uncertainty associated to this offset is added in quadrature. If one assumes that SNe Ia are standard candles in the NIR, one would expect to measure the same offsets, as found in Sect. 4, in \( J_{\text{max}} \) and \( H_{\text{max}} \) for a different sample of SNe Ia. Therefore, these can be used as a correction term.

The \( J \)-band Hubble diagram for simulations with \( n = 1 \) is shown in Fig. 17 (grey circles). Using the simulations with phase between \(-5 \text{ days} \) and \(15 \text{ days} \), where a low scatter in \( J_{\text{max}} \) was found (see Sect. 4), an rms of 0.180 mag is obtained. Although the simulations have larger scatter than the reference sample, the difference is relatively small (0.014 mag).

In the case of the \( H \) band, the reference sample and the simulations have Hubble residuals rms of 0.149 mag and 0.147 mag, respectively. The rms values are very similar, being slightly smaller for the simulations (a negligible difference of 0.002 mag). Although a larger scatter was expected for the simulations, we note that sample used to build the Hubble diagram is not exactly the same as that used in Sect. 4. If the offsets found in Sect. 4 are not applied, very similar results are found, only increasing the rms in \(-0.002 \text{ mag} \) for \( J \) and \( H \) bands. Therefore, we believe that these offsets might not be necessary to apply.

As expected, the scatter in \( H \) band is lower than in the \( J \) band. In addition, for the \( H \) band \( \sigma_{\text{int}} = 0.12 \text{ mag} \), smaller than for the \( J \) band. If the simulations are used to fit \( H_0 \) and \( \sigma_{\text{int}} \) instead of the reference sample, we find the same rms values. The values of \( H_0 \) and \( \sigma_{\text{int}} \) do change, but the differences are insignificant (\( \lesssim 1.5 \sigma \)) for both the \( J \) and \( H \) bands.

By using simulations with 2 and 3 NIR epochs, the scatter is reduced to 0.178 mag and 0.168 mag for the \( J \) band and 0.138 mag and 0.127 mag for the \( H \) band, respectively. Although the reduced scatter is expected, we did not expect to have lower scatter for the simulation than for the reference sample in the \( H \) band. This could...
be caused by limitations in our analysis. For instance, SNooPy was trained on a sub-sample of the SNe Ia from CSP, which can possibly bias the estimation of light-curve parameters, artificially producing a better rms in the simulations. This will be studied in the future, as other NIR light-curve fitters or accurate theoretical light-curve models of SNe Ia become available.

The Hubble residual rms values found in this work are smaller than those found by Uddin et al. (2020), which have values of 0.183 mag in both J and H bands, for SNe Ia at $z > 0.01$. However, they did not include CSP-II SNe and the cuts applied to their sample are different to those applied in this work (Sect. 2.2). Nonetheless, we have shown here that sparse NIR observations of SNe Ia can be used to measure accurate distance and obtained comparable scatter to those found using well-sampled NIR light curves.

7. Summary and conclusions

In this work, we have explored whether it is possible to obtain accurate J- and H-band peak magnitudes from just a few NIR epochs plus good gr-band coverage. To this end, we used the CSP SN Ia sample, the most comprehensive samples with extensive optical to NIR (uBVIrYJH) coverage that exist to date. Combining CSP-I and CSP-II, we gathered a total of 336 SNe Ia, to which we applied a set of quality cuts, reducing the number to 50 SNe, comprising our reference sample. The objects were fitted with SNooPy, using the max_model model, to produce a set of reference values for $J_{\text{max}}$ and $H_{\text{max}}$, using all available bands.

A set of simulations was created by selecting $n$ (where $n = 1, 2, 3$) coeval epochs in J and H bands (the reddest available bands), using combinations without repetition, and using all the available photometry in the g and r bands. The resulting simulated gr/JH-band light-curves were then fitted with SNooPy, with the same configuration used for the reference sample, to obtain estimations of $J_{\text{max}}$ and $H_{\text{max}}$. We then proceeded to compare the NIR peak magnitudes between the simulations and reference sample, finding relatively good agreement in general (residuals $<0.01$ mag). For simulations with $n = 1$, we found that NIR epochs between $-5$ to 15 days with respect to $T_{\text{max}}$ only introduce an additional scatter of $\sim 0.05$ mag in both $J_{\text{max}}$ and $H_{\text{max}}$, with respect to our reference sample. Similar results where found when using J and H bands independently, that is, grJ and grH.

For simulations with $n = 2, 3$, we find that the scatter in the estimation of $J_{\text{max}}$ and $H_{\text{max}}$ is reduced and that the most relevant factor when estimating the NIR peak magnitudes is the time of the closest NIR epoch to $T_{\text{max}}$ (metric i).

A set of tests were performed to estimate the effect of cadence of the optical light curves in the estimation of $T_{\text{max}}$ and its uncertainty propagation to $J_{\text{max}}$ and $H_{\text{max}}$. The results show that cadences up to 10 days only introduce an additional scatter of $<0.02$ mag in $J_{\text{max}}$, being much smaller in $H_{\text{max}} (<0.01$ mag). The effect of optical S/N in the estimation of $J_{\text{max}}$ and $H_{\text{max}}$ was also tested, finding similar scatters. These scatters are smaller than those found in Sect. 4 and can be considered as upper limits if NIR around $T_{\text{max}}$ is used (phases between $-5$ to 15 days). In addition, we tested the effect of the S/N in the NIR light curves. For simulations with S/N similar or better than that of CSP SNe Ia at $z > 0.08$, we found larger scatter compared to the analysis of Sect. 4 ($>0.05$ mag). From these tests, we conclude that SNooPy is expected to retrieve accurate estimations of NIR peak magnitudes for SNe Ia out to $z \sim 0.1$ provided for NIR observations with better S/N than those obtained by CSP.

This work presents some limitations that need to be considered. For example, SNooPy was trained on a sub-sample of the SNe Ia from CSP, which can possibly bias the estimation of light-curve parameters. The use of theoretical SN Ia models to produce synthetic observations in the NIR would help in this...
The results from this work can be used by the community to develop the strategy of future surveys of SNe Ia, such as that from the Roman Space Telescope. In addition, they provide confidence for our Aarhus-Barcelona FLOWS project, which is aimed at using SNe Ia with public ZTF optical light curves and minimal (one to three) NIR epochs to map out the peculiar velocity field of the local Universe. This will allow us to determine the distribution of dark matter in our home supercluster and test the standard cosmological model by measuring the growth of structure (FD) and the local value of $H_0$.

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Appendix A: Light-curve fitter

SNooPy is a versatile light-curve fitter that works on optical and NIR bands. It incorporates different fitting models, such as \textit{max\_model} and \textit{EBV\_model2}, adapting to the needs of the science. The main difference between both is that the former fits each band independently (using a common $T_{\text{max}}$ and $s_{\text{BV}}$ for the light-curve templates), while the latter fits a dust extinction law for the host galaxy\textsuperscript{5}. Hence, the different models might not necessarily produce the same results. In this appendix, the two models mentioned above are tested.

The reference sample from Section 2.3 was fitted with SNooPy using the \textit{max\_model} and \textit{EBV\_model2} models. From these fits, we see that the bulk of the objects have a difference in $T_{\text{max}} < 0.1$ days, and there is no significant difference in the average estimation of $T_{\text{max}}$ (see Fig. A.1).

The object with the largest difference in $T_{\text{max}}$ is SN 2006X. This object is known to be a highly reddened SN (see, e.g., Wang et al. 2008; Folatelli et al. 2010), which might cause this inconsistency. However, looking at the fits of this and other SNe Ia, we see a difference between the SNooPy models. An example of the resulting fits for SN 2004eo, using the \textit{EBV\_model2} model, is presented in Fig. A.2. From the fits, one can see that the \textit{EBV\_model2} model does not produce reliable fits of the bluest ($u$) and reddest ($H$) filters (compared to the fits from Fig. 1). A similar behaviour is observed in other objects, including SN 2006X. In addition, the \textit{EBV\_model2} model produces larger uncertainties in the estimation of $T_{\text{max}}$ than the \textit{max\_model} model (see Fig. A.3). Thus, from these tests, we choose to use the \textit{max\_model} model throughout this work.

\textsuperscript{5} https://users.obs.carnegiescience.edu/cburns/SNooPyDocs/html/models.html

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Fig. A.1. $T_{\text{max}}$ residuals ($\Delta T_{\text{max}}$) between \textit{max\_model} and \textit{EBV\_model2}, using the reference sample from Section 2.3. The weighted mean of $\Delta T_{\text{max}}$ is $\sim -0.001$ mag (vertical black line).

Fig. A.2. SNooPy fits of SN 2004eo with the \textit{EBV\_model2}. Note that the \textit{EBV\_model2} model does not produce reliable fits of the bluest ($u$) and reddest ($H$) filters.

Fig. A.3. Comparison in the uncertainty of $T_{\text{max}}$ ($\sigma_{T_{\text{max}}}$) between \textit{max\_model} and \textit{EBV\_model2}, using the reference sample from Sect. 2.3. We note that the uncertainties from the latter are larger.
Appendix B: NIR offsets with single $J$ and $H$ bands

In Figure B.1, we show show the residuals in $J_{\text{max}}$ and $H_{\text{max}}$ using all bands for the reference sample, and $grJ$ and $grH$, respectively, for the simulations.

Fig. B.1. $J_{\text{max}}$ (top panel) and $H_{\text{max}}$ (bottom panel) residuals, between simulations with 1 NIR epoch, using $grJ$ and $grH$, respectively, and reference values. The weighted mean ($\Delta$) and uncertainty on the weighted mean ($\sigma$) in bins of 2 days are shown in blue. A ‘correction snake’ and its uncertainty are calculated by fitting the residuals with GPs (red line and shaded region); $N$ is the total number of simulations.