Deep Inelastic Scattering on an Extremal RN-AdS Black Hole II: Holographic Fermi Surface

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We consider deep inelastic scattering (DIS) on a dense nucleus described as an extremal RN-AdS black hole with holographic quantum fermions in the bulk. We evaluate the 1-loop fermion contribution to the R-current on the charged black hole, and map it on scattering off a Fermi surface of a dense and large nucleus with fixed atomic number. Near the black hole horizon, the geometry is that of $\text{AdS}_2 \times \text{R}^3$ where the fermions develop a Fermi surface with anomalous dimensions. DIS scattering off these fermions yields to anomalous partonic distributions mostly at large-$x$, as well as modified hard scattering rules. The pertinent $R$-ratio for the black hole is discussed. For comparison, the structure functions and the $R$-ratio in the probe or dilute limit with no back-reaction on the geometry, are also derived.

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I. INTRODUCTION

Many years ago the EMC collaboration at CERN has revealed that DIS scattering on an iron nucleus deviates substantially from deuterium [1] contrary to established lore. Since then, many other collaborations using both electron and muon probes have confirmed this observation [2][4]. Although the nucleus is a collection of loosely bound nucleons with confined quarks, DIS scattering is much richer in a nucleus. The nuclear structure functions display shadowing at low-$x$, a depletion at intermediate-$x$, and an enhancement due mostly to Fermi motion at large-$x$.

QCD supports the idea that hadrons are composed of quarks and gluons as revealed by DIS scattering of electrons on nucleons at SLAC. The scaling laws initially reported follows from scattering on point-like object or partons. Because of asymptotic freedom, the partons interact weakly at short distances leading to relatively small scaling violations at intermediate-$x$. At low-$x$, perturbative QCD predicts a large enhancement in the nucleon structure functions due to the rapid growth of the gluons [5] that eventually saturate [6]. This observation has been confirmed at HERA [7][8].

DIS in holography at moderate-$x$ is different from weak coupling as it involves hadronic and not partonic constituents [9]. The large gauge coupling causes the charges to rapidly deplete their energy and momentum, making them invisible to hard probes. However, because the holographic limit enjoys approximate conformal symmetry, the structure functions and form factors exhibit various scaling laws including the parton-counting rules [10]. In contrast, DIS scattering at low-$x$ on a non-extremal thermal black-hole was argued to be partonic and fully saturated [11].

This paper is a follow up on our recent investigation of DIS scattering on a nucleus as an extremal RN-AdS black hole [12]. In the double limit of a large number of colors and gauge coupling, the leading contribution amounts to the Abelian part of the $R$-current being absorbed in bulk by the black-hole. After mapping at the boundary, the ensuing nuclear structure functions show strong shadowing at low-$x$. At next to leading-order, the $R$-current off a virtual hallow of charged fermionic pairs forming a holographic Fermi liquid around the black hole. The purpose of this paper is to detail DIS scattering on this dense holographic liquid as the analogue of DIS scattering on a nucleus described as a Fermi liquid. Some aspects of this liquid were initially discussed in [13].

This paper consists of several new results: 1/ an explicit derivation of the structure functions for DIS scattering on an extremal RN-AdS black hole; 2/ the characterization of these structure functions both at large-$x$ and low-$x$, with the identification of new anomalous exponents at both large-$x$ and low-$x$; 3/ an explicit derivation of the $R$-ratio for DIS scattering on the extremal black hole as a model for DIS scattering on a dense nucleus; 4/ an explicit derivation of the same structure functions in the probe fermion limit as a model for DIS scattering on a dilute nucleus; 5/ a comparative study of the $R$-ratio in the probe limit.

The organization of the paper is as follows: in section II, we briefly review the setting for the extremal RN-AdS black hole, and the key characteristics of the holographic Fermi liquid. In section III, we derive the contribution to the boundary effective action of an $R$-photon scattering off bulk quantum fermions. The result is quantum and dominant at large-$x$, thereby correcting the classical and leading contribution from the bulk black hole which is mostly supported at low-$x$. In section IV, we analyze the contribution stemming from the quantum fermions at low-$x$ and show that it is vanishingly small near the horizon. In section V, we detail our derivation of the $R$-ratio for the black hole in the dense regime. For comparison, we discuss in section VI the probe or dilute limit with the

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bulk fermions carrying a finite density in AdS without affecting the underlying geometry. The pertinent R-ratio in this regime is derived and analyzed. Our conclusions are in section V. Some useful details are found in several Appendices.

II. EXTREMAL BLACK HOLE: DENSE LIMIT

In this work we will address DIS scattering on a cold nucleus as a dual to an RN-AdS black hole following on our recent analysis [17]. Conventional DIS scattering on cold nuclei with many of the conventions used are reviewed in [18]. In holography, DIS scattering on a nucleus as an RN-AdS black hole is illustrated in Fig. 1. In the holographic limit, the leading contribution is shown in Fig. 1a with the structure functions being the absorbed parts of the R-current. To this order, the structure functions have a support only at low-x [12] (see below). At next-to-leading order, the R-current is absorbed through the virtual fermionic loop shown in Fig. 1b. This loop describes a fermionic hallow around the RN-AdS black hole that acts as a holographic Fermi liquid. Below we detail how this contribution leads to structure functions with support at large-x. This description is complementary to our recent analysis based on a generic density expansion around a trapped Fermi liquid [17].

A. The extremal and charged black hole

The RN-AdS black hole is described by effective gravity coupled to a U(1) gauge field in a 5-dimensional curved AdS space [14]

\[ S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} \left( R - 2\Lambda \right) - \frac{1}{4e^2} \int d^5x \sqrt{-g} F^2 \]  

(II.1)

The Ricci scalar is \( R \), and \( \kappa^2 = \frac{8\pi G_5}{\Lambda} = -\frac{6}{R^2} \) are the gravitational and cosmological constant. The curvature radius of the AdS space is \( R \) with line element

\[ ds^2 = \frac{r^2}{R^2} \left( -d\tau^2 + d\vec{x}^2 \right) + \frac{R^2}{r^2} dr^2 \]  

(II.2)

and warping factor

\[ f(r) = \left( 1 - \frac{r^2}{r^2} \right) \left( 1 - \frac{r_+^2}{r^2} \right) \left( 1 + \frac{r^2}{r_+^2} + \frac{r^2}{r_-^2} \right) \]  

(II.3)

with \( r_+ > r_- \) the outer-inner horizons satisfying \( f(r_{\pm}) = 0 \). The black-hole is charged and sources the R-potential

\[ A_t = \mu - \frac{Q}{r^2} \]  

(II.4)

provided that the electric charge \( Q \) and the geometrical charge \( q \) satisfy

\[ \frac{q^2 R^2}{Q^2} = \frac{4}{3} \times \frac{2\kappa^2}{4e^2} = \frac{R^2}{6\alpha}, \]  

(II.5)

where

\[ 2\kappa^2 = \frac{8\pi^2 R^3}{N_c^2}, \quad 4e^2 = \frac{64\pi^2 R}{N_c^2} \]  

(II.6)

We have defined \( \alpha = 1 \) for a U(1) R-charge, and \( \alpha = \frac{1}{3N_f} \) for a D3-D7 U(1) vector charge. The temperature of the RN-AdS black hole is

\[ T = \frac{\nu^2 f'(r_+)}{4\pi R^2} = \frac{r_+}{\pi R^2} \left( 1 - \frac{\mu^2 \pi^2 R^4 \gamma^2}{r_+^4} \right) \]  

(II.7)

with \( \gamma^2 = 1/12\pi^2\alpha \). The chemical potential \( \mu \) is fixed by the zero potential condition on the outer horizon \( A_t(r_+) = 0 \) or \( \mu = Q/r_+^2 \). At extremality where \( T = 0 \), we have \( r_+ = r_- = \pi R^2 \gamma \mu = \frac{R^2}{2\sqrt{\alpha}} \sqrt{\alpha} \).
The fermionic fields in bulk are characterized by the Dirac action in a charged AdS black hole geometry

\[ S = -\int d^5x \sqrt{-g} i(\bar{\psi}\Gamma^M D_M \psi - m\bar{\psi}\psi) \]  

(II.8)

with \( \bar{\psi} = \psi^\dagger \Gamma^2 \), and the long derivative

\[ D_M = \partial_M + \frac{1}{4} \omega_{abM} \Gamma^{ab} - ie_R A_M \]  

(II.9)

The indices \( M, N \cdots \) or \( \mu, \nu, r \cdots \) refer to the space-time indices, and \( a, b, \cdots \) to space-time indices with underline correspond to tangent space indices. Therefore, for example, \( \Gamma^a \) denotes the gamma matrices in the tangent space, \( \Gamma^M \) denotes gamma matrices in the curved space-time. They are specifically given in Appendix A.

A bulk fermion field of mass \( m \) and R-charge \( e_R \) is dual to a composite boundary field of conformal dimension \( \Delta = \frac{3}{2} + mR \). Since the horizon of the extremal charged RN-AdS black is characterized by a finite U(1) electric field, fermionic pair creation takes place through the Schwinger mechanism. As a result, the black-hole say with positive R-charge absorbs the negative part of the pairs and expel the positive part. Since AdS is hyperbolic and confining, the positive charge falls back to the surface of the black hole, accumulating into a hallow or holographic Fermi liquid.

The characteristics of the low-lying excitations of the holographic Fermi liquid for low frequencies \(|k^0| < \mu \) and low momenta \( k = |\vec{k}| \), have been discussed in \([15, 16]\). In particular, near the horizon the AdS\(_5 \) geometry factors into AdS\(_2 \times \) R\(^3 \).

**Case-1:** \( e_R^2 \tilde{\alpha} < \frac{1}{2}(mR)^2 \)

The fermions exhibit strong distortion in the AdS\(_2 \) geometry, with \([15]\)

\[ G_R^{11}(k^0, \vec{k}) = C(\vec{k}) (k^0)^{2\nu_k} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \]  

(II.10)

and

\[ \nu_k = \frac{1}{\sqrt{2\mu}} \left( k^2 - k_R^2 \right)^{\frac{1}{2}} \]  

\[ \equiv \frac{\sqrt{\tilde{\alpha}}}{\mu} \left( k^2 - \frac{\mu^2}{6\tilde{\alpha}} \left( e_R^2 \tilde{\alpha} - \frac{1}{2}(mR)^2 \right) \right)^{\frac{1}{2}} \]  

(II.11)

Note that \( k_R^2 < 0 \) in this case, i.e., for \( e_R^2 \tilde{\alpha} < \frac{1}{2}(mR)^2 \).

Throughout, we will use the block notation to refer to the fermionic retarded (Feynman) propagators

\[ G_{R,F} = \begin{pmatrix} G^{11} & G^{12} \\ G^{21} & G^{22} \end{pmatrix} \]  

(II.12)

**Case-2:** \( e_R^2 \tilde{\alpha} > \frac{1}{2}(mR)^2 \)

For \( k_R^2 > 0 \) and \( k \leq k_R \), the corresponding holographic spectral function exhibits oscillating behavior and gapless excitations, with comparable real and imaginary parts. In other words, the excitations in this oscillating region are short lived as they form and quickly fall into the extremal RN-AdS black hole.

Further arguments \([15, 16]\) show that the fermionic density diverges near the horizon causing strong back reaction. As a result, the near horizon geometry becomes a Lifshitz geometry whereby the Fermi-like volume is resolved into concentric Fermi spheres each describing heavy Fermions with narrow widths, thereby explaining the gapless like excitations. This resolution occurs only for \(|k^0|/\mu \sim e^{-N_c^2} \) and resorbs for \(|k^0|/\mu \sim N_c^0 \).

**Case-3:** \( e_R^2 \tilde{\alpha} > \frac{1}{2}(mR)^2 \)

For \( k_R^2 > 0 \) and \( k \geq k_R \), localized and long lived fermionic states emerge that are characterized by a Fermi momentum \( k_F > k_R \). In this case, the retarded propagator near the Fermi surface reads \([15, 16]\)
\[ G_R^{11}(k^0, \tilde{k}) \approx \frac{h_1}{k - k_F - \frac{1}{\nu_F} k^0 - \Pi(k^0)} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \]  \tag{II.13}

with

\[ \Pi(k^0) = \frac{\hbar_2 e^{\nu_F} (k^0)^{2\nu_k}}{\sqrt{2 \mu}} \left( k^2 - k_R^2 \right)^{\frac{3}{2}} \]

\[ \nu_k = \frac{1}{\sqrt{2 \mu}} \left( k^2 - k_R^2 \right)^{\frac{3}{2}} \]

\[ \gamma_k = \arg \left( \Gamma(-2\nu_k) \left( e^{-2\pi i \nu_k} - e^{-\frac{2\pi i \nu_k}{\nu_F}} \right) \right). \]  \tag{II.14}

The coefficients \( h_1 \sim \frac{r}{r_F^2} \) and \( h_2 \sim \left( \frac{r}{r_F^2} \right)^{1-2\nu_F} \) can be computed numerically. For \( \nu_k > \frac{1}{2}, \nu_k = \frac{1}{2} \) or \( \nu_k > \frac{1}{2} \) we have a Fermi liquid, a marginal Fermi liquid or a non-Fermi liquid, respectively. Note that the transition from a non-Fermi liquid to a Fermi liquid occurs for \( \omega \approx \omega_c \) which is fixed by the condition \( \omega_c = |\nu_F \Pi(\omega_c)| \).

A schematic description of the poles of \[ (II.13) \] is given in Fig. 2. For sufficiently large effective charge \( e_R \sqrt{\alpha} \), some of the largely damped quasi-normal modes (QNMs) of the RN-AdS black hole transmute to narrow quasi-bound states (QBS) close to the real axis for fixed \( k < k_F \). For increasing \( k \to k_F \) the narrow QBS start crossing the origin \( \omega = 0 \) turning to equally spaced holographic Fermi surfaces (here 4 Fermi surfaces) as discussed in [10].

For fermions with larger effective charge, i.e., for \( e_R^2 \alpha > \frac{1}{2}(mR)^2 \) or \( k_R^2 > 0 \), pair creation takes place near the horizon as we noted earlier. A hallmark of charged fermions at the Fermi surface with \( k_F > k_R > 0 \), that supports quasi-particles with \( G_R^{11} \) given in \[ (II.13) \]. For hard R-probes with large \( q^0 \) in the DIS kinematics, only \( G_R^{11}(k^0, \tilde{k}) \) is modified close to the horizon, since \( G_R^{11}(\omega_1, k + q) \) carries a large momentum and is mostly unmodified in the ultraviolet,

\[ \text{Im } G_R^{11}(\omega_1, k + q) \text{ Im } G_R^{11}(k^0, \tilde{k}) \to \]

\[ \text{Tr} \left( (\sigma_1(k^0 + q^0) - i\sigma_2(k_x + q_x) - \omega_1) \pi \delta((k + q)^2 + \omega_1) \right) \times \text{Im } G_R^{11}(k^0, \tilde{k}) \]  \tag{II.15}

### III. HOLOGRAPHIC STRUCTURE FUNCTIONS

The holographic structure functions on an extremal black-hole in leading order have been discussed in [17], to which we refer for further details. For completeness, the results will be summarized below, and extended to allow for the next to leading order contributions from the holographic Fermi liquid at the horizon.

#### A. Structure functions

We recall that the scattering amplitude of an R-photon of longitudinal momentum \( q^0 = (\omega, q, 0, 0) \) scattering on a black-hole at rest in the Lab frame with \( n^\mu = (1, 0, 0, 0) \), \[ (III.35) \], can be tensorially decomposed into two invariant functions \( \tilde{G}_{1,2} \) [12]

\[ \tilde{G}_{1\mu}(q) = \left( \eta_{\mu\nu} - \frac{q_\mu q_\nu}{Q^2} \right) \tilde{G}_1 + \left[ n_\mu n_\nu - n_\mu \frac{q_\nu}{Q^2} (q_\mu + n_\mu q_\nu) + n_\nu \frac{q_\mu}{Q^2} (n_\mu q_\nu + n_\nu q_\mu) + \frac{q_\mu q_\nu}{(Q^2)^2} (n_\cdot q)^2 \right] \tilde{G}_2 \]  \tag{III.16}

with \( Q^2 = q^2 \), thanks to with the current conservation and covariance. The corresponding DIS structure functions for an R-photon on a black hole are defined as

\[ \tilde{F}_1 = \frac{1}{2\pi} \text{Im} \tilde{G}_1, \]

\[ \tilde{F}_2 = -\frac{(n \cdot q)}{E_A} \text{Im} \frac{1}{2\pi} \tilde{G}_2. \]  \tag{III.17}

As in [12], the rest frame of a cold and extremal black hole will be dual to the rest frame of a cold nucleus at the boundary with fixed energy \( E_A = \frac{4}{3} A \mu \). Since the binding energy in a nucleus is small, we also have \( E_A \approx A m_N \) and therefore the chemical potential \( \mu \approx \frac{4}{3} m_N \). In our mapping, \( m_N \) and \( \mu \) are interchangeable for estimates.

A hard photon with virtual momentum \( q^0 = (\omega, q, 0, 0) \) scattering off the nucleus in the DIS kinematics satisfies \( q^2 - \omega^2 \equiv Q^2 \to \infty \) with \( \omega \simeq q \) and fixed Bjorken-x

\[ x_A = \frac{Q^2}{2E_A\omega} = \frac{xm_N}{E_A} \]  \tag{III.18}

#### B. Classical black-hole in leading order

As we noted earlier, the leading order contribution to the structure functions \[ (III.17) \] in DIS scattering is classical and of order \( N^2 \) as illustrated in Fig. 1. It does not involve scattering off the fermions near the holographic surface, which is of order \( N^0 \). In the regime \( Q \ll q \ll Q^2 \)}
the leading contribution to the structure functions vanishes, as the probe spin-1 R-field is prevented from falling to the black-hole by an induced potential barrier [11]. The R-current correlator is purely real with an exponentially vanishing imaginary part. In the regime \( q \gg Q^3 \), the barrier wanes away with the classical and leading contribution to the un-normalized structure function \( \tilde{F}_2 \) of the form [12]

\[
\tilde{F}_2(x_A, Q^2) \approx \tilde{C}_T \frac{\mu^2}{x_A} \left( \frac{x_A^2 Q^2}{\mu E_A} \right)^{\frac{3}{2}} + \tilde{C}_L \frac{E_A \mu^2}{x_A} \left( \frac{x_A^2 Q^2}{\mu E_A} \right)
\]

(III.19)

with

\[
\tilde{C}_T = \frac{N_c^2}{2^{17/3} \pi^2 T^2 (1/3) \hat{\alpha}^{5/3}}
\]

\[
\tilde{C}_L = \frac{N_c^2}{1152 \pi^4 \hat{\alpha}^2}
\]

(III.20)

with \( x_A E_A = x m_B \) and \( C_L \ll C_T \). This result was shown to hold for low-\( x \) or \( x_A \ll \sqrt{\mu E_A} / Q \), with the Callan-Gross relation-like \( \tilde{F}_2 = 2 x_A \tilde{F}_1 \). The normalized structure functions follow as [12]

\[
F_{1,2} \equiv 2 E_A V_A \tilde{F}_{1,2} = \left( \frac{12 \pi \hat{\alpha} A}{N_c \mu} \right)^2 \tilde{F}_{1,2}
\]

(III.21)

after using the black-hole equation of state. More specifically, we have \( (Q^2 = q^2 > 0) \)

\[
\frac{F_{2BH}(x,q^2)}{A} \approx \frac{C_T}{x} \left( \frac{3 x^2 q^2}{4 m_N^2} \right)^{\frac{5}{2}} + \frac{3 C_L}{4x} \left( \frac{3 x^2 q^2}{4 m_N^2} \right)
\]

(III.22)

with \( C_{T,L}/\tilde{C}_{T,L} = \pi^5 (48 \hat{\alpha})^3 / 2 \pi^3 \).

The normalization in (III.21) amounts overall to normalizing \( \tilde{F}_{1,2} \) by the density of the black-hole, canceling part of the model dependence of the equation of state. In a way, the normalized \( F_{1,2} \) are the un-normalized black-hole structure functions \( \tilde{F}_{1,2} \) per degree of freedom. (III.22) is dominated by the first contribution at low-\( x \).

We now show that the next contribution is dominated by scattering off bulk fermions at large-\( x \) from a holographic Fermi liquid close to the horizon.

### C. Quantum fermions in sub-leading order

The contribution of the sub-leading fermions to the induced effective action can be obtained through the holographic dictionary. The shift of the R-field \( A_M \rightarrow A_M + \delta M_0 + a_M \) amounts to a shift in the Dirac action density in (II.8) at the origin of the minimal coupling of the R-field

\[
-i \bar{\psi} ( -i e_R a_\mu(r,q) \Gamma^\mu ) \psi \equiv \bar{\psi} B(r;q) \psi
\]

(III.23)

In terms of (II.8), the bulk effective action for the 1-loop contribution in Fig. 1, at zero temperature reads

\[
S_F[a_\mu] = -(-i) \int \frac{d^4 q}{(2\pi)^4} \int \frac{d^4 k}{(2\pi)^4} \int dr_1 \sqrt{g(r_1)} \sqrt{g(r_2)} \text{Tr} \left( D_F(r_1, r_2; k + q) B(r_2; q) D_F(r_2, r_1; k) B(r_1; q) \right)
\]

(III.24)

This allows the re-writing of (III.24) in the form of the boundary action

\[
\int \frac{d^4 q}{(2\pi)^4} \int \frac{d^4 k}{(2\pi)^4} \int dr_1 \sqrt{g(r_1)} \sqrt{g(r_2)} \text{Tr} \left( D_F(r_1, r_2; k + q) B(r_2; q) D_F(r_2, r_1; k) B(r_1; q) \right)
\]

The routing of the momenta in (III.24) corresponds to the hard fermion with \( k + q \) and the soft fermion with \( k \).

The R-field in bulk \( a(r,q) \) relates to the R-field at the boundary \( A^{(0)}(q) \) through the bulk-to-boundary propagator \( K_A(r; q) \), which satisfies \( K_A(r \rightarrow \infty; q) = 1 \),

\[
a_\mu(r; q) = K_A(r; q) A^{(0)}(q). \]

(III.25)
\[ S_F[A^{(0)}_\mu] = \frac{1}{2} \int \frac{d^4q}{(2\pi)^4} A^{(0)}_\mu(q) A^{(0)}_\nu(-q) \]
\[ \times (-2i) \int \frac{d^4k}{(2\pi)^4} \int dr_1 dr_2 \sqrt{g(r_1)} \sqrt{g(r_2)} \text{Tr} \left( D_F(r_1, r_2; k + q) Q^\mu(r_2; q) D_F(r_2, r_1; k) Q^\nu(r_1; q) \right) \]

(III.26)

with the dressed bulk vertices

\[ Q^\mu(r; q) = -i \left( -ie_R K_A(r; q) \Gamma^\mu \right) \]
\[ \approx -e_R \frac{qR^2}{r} K_1 \left( \frac{qR^2}{r} \right) \Gamma^\mu. \quad \text{(III.27)} \]

We have approximated the bulk-to-boundary function \( K_A(r; q) \) by its vacuum contribution, with \( K_1(x) \) the modified Bessel function.

In the DIS regime \( Q \ll q \ll Q^2 \) with \( Q^2 = q^2 \), the spin-\( \frac{1}{2} \) fermion field remains localized near the boundary as a potential barrier develops in bulk, a phenomenon also observed for spin-1 boson fields \[11\]. In this regime, we will approximate the hard part of the fermion propagator by its vacuum (in AdS\(_5\)) result \[19\].

\[ D_F(r_1, r_2; k + q) = \int d\omega_1 \omega_1 G_F(r_1, r_2; \omega_1, k + q), \]

(III.28)

where

\[ G_F(r, r'; \omega_1, k + q) = \psi_\alpha(r, \omega_1) G^{\alpha\gamma}(\omega_1, k + q) \psi_\gamma(r', \omega_1) \quad \text{(III.29)} \]

with the vacuum (in AdS\(_5\)) solution \[20\]

\[ \psi_1(r, \omega_1) = \left( \frac{R^2}{r} \right)^{\frac{3}{2}} J_{mR} \left( \frac{R^2}{r} \right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]
\[ \psi_2(r, \omega_1) = 0. \quad \text{(III.30)} \]

and

\[ G^{11}_F(\omega_1, k + q) = \frac{\sigma_1(k^0 + q^0) - i\sigma_2(k_x + q_x) - \omega_1}{(k + q)^2 + \omega_1^2 - i\epsilon}, \]
\[ G^{22}_F(\omega_1, k + q) = \frac{\sigma_1(k^0 + q^0) + i\sigma_2(k_x + q_x) - \omega_1}{(k + q)^2 + \omega_1^2 - i\epsilon}. \quad \text{(III.31)} \]

The soft part of the fermion propagator can be separated into its contribution deep in the infrared which is modified by the induced holographic Fermi surface through the geometrical reduction to AdS\(_2\times\mathbb{R}^3\), and its ultraviolet completion. More specifically, near the AdS\(_2\times\mathbb{R}^3\) geometry, the infrared part of the soft the propagator is of the form

\[ D_F(r, r'; k) = \psi_\alpha(r, k^0, \tilde{k}) G^{F}_{\alpha\beta}(k^0, \tilde{k}) \psi_\beta(r', k^0, \tilde{k}) \quad \text{(III.32)} \]

with

\[ G^{22}_F(k^0, \tilde{k}) = \frac{1}{G^{11}_F(k^0, \tilde{k}; m \rightarrow -m)}. \quad \text{(III.33)} \]

Note that only \( G^{11}_F(k^0, \tilde{k}) \) has a singular or Fermi-like structure near \( k \rightarrow k_F \). Hence, we will ignore the contribution from \( G^{22}_F(k^0, \tilde{k}) \) to the current correlator. The normalizable wave functions are given in \[X.123\].

The time-ordered correlation function for the \( R \)-current follows from the functional derivative

\[ \tilde{G}^{F\mu\nu}(q) = \frac{\delta^2 S_F[A^{(0)}_\mu]}{\delta A^{(0)}_\mu(q) \delta A^{(0)}_\nu(-q)}, \quad \text{(III.34)} \]

Using the spectral form of the Feynman propagator \[\text{III.32,III.33}\], we can re-write \[\text{III.34}\] in a more compact form

\[ \tilde{G}^{F\mu\nu}(q) = 2i \int \frac{d^4k}{(2\pi)^4} \int d\omega_1 \omega_1 \text{Tr} \left( G^{\alpha\beta}_F(\omega_1, k + q) A^{\mu}_{\alpha}(\omega_1; q; k) G^{\nu\delta}_F(k^0, \tilde{k}) A^{\nu\delta}_{\alpha}(k; \omega_1) \right), \]

(III.35)
with the dressed vertices

\[
\Lambda^{\mu}_{\beta\gamma}(\omega_1; q; k) = \int dr_2 \sqrt{g(r_2)} \bar{\psi}_\beta(r_2, \omega_1) Q^\mu(r_2; q) \psi_\gamma(r_2, k),
\]

(III.36)

and

\[
\Lambda^{\nu\alpha}_{\kappa\lambda}(k; q; \omega_1) = \int dr_1 \sqrt{g(r_1)} \bar{\psi}_\beta(r_1, k) Q^\nu(r_1; q) \psi_\alpha(r_1, \omega_1).
\]

(III.37)

We recall that at zero temperature, the general Feynman and retarded propagators \( \mathcal{G}_{F,R} \) are related by the relationship

\[
\mathcal{G}_F(k^0, \vec{k}) = \text{Re} \mathcal{G}_R(k^0, \vec{k}) + i \text{sgn}(k^0) \text{Im} \mathcal{G}_R(k^0, \vec{k})
\]

(III.38)

Using (III.38) and the fact that \( \mathcal{G}_F(k^0, \vec{k}) \) is analytic in the upper complex \( k^0 \)-plane, allow for the re-writing of the imaginary part of (III.35) in the form

\[
\text{Im} \tilde{\mathcal{G}}^F_{\mu\nu}(q) = \int d^4k \left( \frac{d^4k}{(2\pi)^4} \int d\omega_1 \omega \Lambda^{\mu}_{\beta\gamma}(\omega_1; q; k) \Lambda^{\nu\alpha}_{\kappa\lambda}(k; q; \omega_1) \text{Re} Tr \left( \mathcal{G}^\alpha_{F-F}(\omega_1, k + q) \mathcal{G}^\beta_{F-R}(k^0, \vec{k}) \right) \right),
\]

\[
= \int d^4k \left( \frac{d^4k}{2\pi^2} \int d\omega_1 \omega \Lambda^{\mu}_{\beta\gamma}(\omega_1; q; k) \Lambda^{\nu\alpha}_{\kappa\lambda}(k; q; \omega_1) \text{Re} Tr \left( \text{Im} \mathcal{G}^\beta_{R-F}(\omega_1, k + q) \text{Im} \mathcal{G}^\alpha_{F-R}(k^0, \vec{k}) \right) \right),
\]

(III.39)

This result shows that for \( q^0 = 0 \), the imaginary part vanishes as it should as the effective action induced by the R-current (III.26) is real. For \( q^0 \neq 0 \) this result is clearly negative as it should, since its contribution to (III.26) amounts to a self-energy for the R-field which amounts to damped oscillations in time.

**D. Large-x near the horizon**

Using the vertex (XI.133) for momenta near \( k_F \), we can re-write (III.39) with

\[
\text{Re} I_{k^0}(\omega_1, q) = \text{Re} \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} a_+(k_0, k)^2 \text{tr} \left( \mathcal{G}^{11}_{F}(\omega_1, k + q) \mathcal{G}^{11}_{F}(k^0, \vec{k}) \right)
\]

\[
= \text{Re} \int_{-|q^0|}^{0} \frac{dk_0}{2\pi} a_+(k_0, k)^2 \text{tr} \left( (\sigma_1(k^0 + q^0) - i\sigma_2(k_x + q_x) - \omega_1)\pi \delta((k + q)^2 + \omega_1^2) \text{Im} \mathcal{G}^{11}_{R}(k^0, \vec{k}) \right)
\]

\[
\approx \int_{-|q^0|}^{0} \frac{dk_0}{2\pi} a_+(k_0, k)^2 (-1)\omega_1 \pi \delta((k + q)^2 + \omega_1^2) \frac{h_1 \text{Im} \Pi}{(k - k_F - \frac{k_0}{v_F} - \text{Re} \Pi)^2 + (\text{Im} \Pi)^2}
\]

(III.41)

(III.41) can be simplified by enforcing the delta function
\[
\text{Im } \hat{G}^F_{xx}(q) \approx C^2(\nu_{k_F}) C_0 k_F^2 \frac{\pi}{2} \int_{k_R}^{k_F} dk \int_{-|q|}^{0} \frac{dk_0}{2\pi} a_+(k_0, k)^2 I_2^2(x_k; q; k_F) \sqrt{s_k} \frac{\hbar_1 \text{Im } \Pi}{(k - k_F - \frac{k_0}{v_F} - \text{Re } \Pi)^2 + (\text{Im } \Pi)^2}
\]

with the overall constant
\[
C_G(\nu_{k_F}, z_-) = \frac{\pi}{2} z_-^{(2\nu_k+2)} C^2(\nu_{k_F}) C_0^2(\nu_{k_F}) \tilde{h}_1(\nu_{k_F}) C_\theta ,
\]

where \(\tilde{h}_1(\nu_{k_F}) \equiv z_- h_1\) is a dimensionless constant to be determined numerically, and \(z_- = \frac{R^2}{v_F^2}\). Again, for the DIS kinematics we set \(x_k = -q^2/2k \cdot q\), and \(|q|^2 \approx q_x\). We re-arranged the hypergeometric function \(_2F_1\) using the same Pfaff identity \((\text{III.49})\). Note that for the special value of \(\nu_k = mR + 1\), one can see that the \(x_k\) dependence of the integrand in \((\text{III.42})\) reduces to the one in \([20]\) before the multiplication by the trace (for our case the trace is \(\sqrt{s_k}\)). However, for general \(\nu_k\) the same partonic content as in \((\text{III.50})\) is noted.

For narrow quasi-particles, we may use the substitution \((k_\perp = k - k_F)\)

\[
\frac{\text{Im } \Pi}{(k_\perp - \frac{k_0}{v_F} - \text{Re } \Pi)^2 + (\text{Im } \Pi)^2} \rightarrow \pi \delta \left( k_\perp - \frac{k_0}{v_F} - \text{Re } \Pi \right)
\]

and undo the \(k_0^0\) integration in \((\text{III.42})\) with the result

\[
\text{Im } \hat{G}^F_{xx}(q) \approx \frac{1}{2} \tilde{C}_G(\nu_{k_F}) \int_{k_R}^{k_F} dk_\perp k_F^2 \frac{a_+(k_0, k_\perp)^2}{v_F^2 + \text{Re } \Pi'} \left( \frac{1}{q^2 z_-^{(1/2)}} \right)^{\nu_{k_F} + 1} x_k^{\nu_{k_F} + 2} (1 - x_k)^{-\frac{1}{2}} \_2F_1^2 (\tau_+, \tau_-, \tau - 1, 1 - x_k)
\]

with

\[
\text{Re } \Pi' = 2h_2 \nu_{k_F} |k_0^0|^{2\nu_{k_F} - 1} \text{Re } (e^{i\tau_{k_F}} (-1)^{2\nu_{k_F} - 1})
\]

\((\text{III.46})\)

where \(k_0^0\) in \(x_k\) is solution to the transcendental equation

\[
k_\perp \frac{|k_0^0|}{v_F^2} = h_2 |k_0^0|^{2\nu_{k_F}} \text{Re } (e^{i\tau_{k_F}} (-1)^{2\nu_{k_F}}) ,
\]

\((\text{III.47})\)

and we have defined a dimensionless constant

\[
\tilde{C}_G(\nu_{k_F}) = z_-^{2\nu_{k_F} + 2} \times C_G(\nu_{k_F}, z_-)
\]

\[
= \frac{\pi}{2} C^2(\nu_{k_F}) C_0^2(\nu_{k_F}) \tilde{h}_1(\nu_{k_F}) C_\theta .
\]

\((\text{III.48})\)

Also note that \(a_+(k_0, k_\perp) = \tau_1 z_- k_\perp + \tau_2 z_- k_0\) is a dimensionless coefficient with dimensionless constants \(\tau_{1,2} = \tilde{c}_{1,2}/z_-\).

In arriving to \((\text{III.45})\), we have made use of the Pfaff identity

\[
\_2F_1(a, b, c, z) = (1 - z)^{-a} \_2F_1 \left( a, c - b, c, \frac{z}{z - 1} \right)
\]

\((\text{III.49})\)

In the holographic Fermi liquid, the partonic distribution function is seen to develop a modified exponent. A comparison of the partonic distribution function \((\text{VI.97})\) in the probe limit, to \((\text{XIII.146})\) where the black hole is present, translates
with $2\tau_{\pm} = \tau \pm (\nu_k + 1/2)$ and the twist parameter $	au = m R + 3/2$. Near the black hole horizon, the parton distribution function develops a modified scaling law, but it is still seen to vanish at the end points $x_k = 0, 1$. In Fig. 3 we show the modified behavior of the partonic distribution function in (III.50) for fixed $q^2$, $\tau = 3$ and $\nu_k = \frac{1}{2}$ versus $x_k$ as the light-solid curve (green). The comparison is with the large-x dependence of the nucleon for weak coupling dashed curve (red), and strong coupling dark-solid curve (blue). Near the black hole horizon, the distribution function is shifted to intermediate-x. With our choice of parameters, the holographic result (III.50) reduces to $x_k^\tau (1 - x_k)^{\tau - 3/2}$, in comparison to the strong coupling result in vacuum $x_k^\tau (1 - x_k)^{\tau - 2}$, and the weak coupling result also in the vacuum $x_k^\tau (1 - x_k)^{\tau - 3}$. Remarkably, the formation of a holographic fermionic surface through the AdS$_2$ x R$_3$ reduction, is to shift the holographic partonic distribution to intermediate-x, and modify the hard scattering rule.

For our choice of DIS kinematics, the non-normalized $\tilde{F}_2$ structure function (III.17) follows from (III.45) in the form ($q_0 \approx q_L$)

$$ F_2(x_A, q) = \frac{4}{\pi} x_A^\tau \frac{E_A^2}{q^2} \Im \tilde{G}_{xx}^F(q) \quad \text{(III.51)} $$

with again $Q^2 = q^2 > 0$. Modulo the dispersion relation and the anomalous exponents that characterize the holographic fermions in the reduced AdS$_2$ x R$_3$ geometry, the results (III.45) and (III.51) are similar to the ones we derived recently in [17] using general arguments.

**IV. FERMIONIC CONTRIBUTION AT LOW-X**

In the DIS regime with $q \gg Q^2$ or low-x, the structure functions are dominated by the exchange of a Pomeron, a multigluon exchange with vacuum quantum numbers. In holography, this exchange is described either through a closed surface exchange [22] or a graviton [23] in bulk. For the latter, this regime was identified in the range $e^{-\sqrt{\lambda}} \ll x \ll 1/\sqrt{\lambda}$ where the exchange involves the string scattering amplitude. Since $x \gg e^{-\sqrt{\lambda}}$, the strings are small compared to the size of the AdS space so that the scattering amplitude is quasi-local with almost flat space signature.
A. General set up

The 10-dimensional tree-level effective action that describes the scattering of an R-photon off bulk quantum

\[ S = \int d^{10}x \sqrt{-g_{10}} (K\mathcal{V})_{t=0} \]

\[ = \frac{i}{8} \int d^{10}x \sqrt{-g_{10}} \left( 4v^a \tilde{\varphi} \Gamma_m \partial^p \varphi - g_m^a \left( \tilde{\varphi} \Gamma^M \partial_M \varphi v^a + 2v_a v_b \tilde{\varphi} \Gamma^a \partial^b \varphi \right) \right) F^{mn} F_{pn} \mathcal{V}_{t=0} \]  

(IV.52)

where \( v^a \) are the Killing vectors for the compact part of the 10-dimensional space. The forward R-current scattering amplitude follows from pertinent variation with respect to the R-field. Here \( \mathcal{K} \) refers to the kinematical factor involving the fermions \( \varphi \) and the R-field strength \( F \), and \( \mathcal{V} \) is the exchanged flat space 10-dimensional Virasoro-Shapiro string amplitude

\[ \mathcal{V} = \frac{\alpha'^3 s^2}{64} \prod_{\beta=\bar{s},\bar{t},\bar{u}} \frac{\Gamma \left( -\frac{\alpha'^2}{4} \right)}{\Gamma \left( 1 + \frac{\alpha'^2}{4} \right)} \]  

(IV.53)

as illustrated in Fig. 4. The 10-dimensional Mandelstam variables \( \bar{s}, \bar{t} \) are related to the 4-dimensional ones \( s, t \) through

\[ \alpha' \bar{s} = \alpha' s \frac{z^2}{R^2} + O \left( \frac{1}{\sqrt{\lambda}} \right) \]

(IV.54)

\[ \alpha' \bar{t} = \alpha' t \frac{z^2}{R^2} + O \left( \frac{1}{\sqrt{\lambda}} \right) \]

with the warping made explicit.

The imaginary part of the string amplitude (IV.53) is

\[ \text{Im}_t \mathcal{V} = \frac{\pi \alpha'}{4} \sum_{j=1}^{\infty} j \mathcal{O} \left( \frac{1}{\sqrt{\lambda}} \right) \delta \left( j - \frac{\alpha' \bar{s}}{4} \right) \]  

(IV.55)

with the delta-function summing over the closed string Regge trajectory. At low-x we have \( s \sim 1/x \) and \( j \sim s \sim 1/x, \) so that for \( \ln(1/x) \sqrt{\lambda} \ll 1, \)

\[ j \mathcal{O} \left( \frac{1}{\sqrt{\lambda}} \right) \sim \left( \frac{1}{x} \right) \mathcal{O} \left( \frac{1}{\sqrt{\lambda}} \right) \sim 1 \]  

(IV.56)

We now recall that the field strength \( F^{mn} \) describes the bulk-to-boundary R-field-strength with incoming momentum \( q^\mu \) and outgoing momentum \( q^\mu \), while \( \varphi \) describes the bulk fermion with incoming and outgoing momentum \( k^\mu \) in the anomalous Fermi surface. The low-x regime with \( x \ll 1 \) corresponds to the kinematical regime \( q \cdot k \gg q^2 \gg k^2 \), so that the dominant contribution in \( \mathcal{K} \) is the term with the spin contraction of the form \( (q \cdot k) \), i.e., the first term in (IV.52). Using

\[ \psi(k) \rightarrow \psi(k) \times \mathcal{Y}(v) \]  

(IV.57)

and normalizing

\[ \int_{S^5} d\hat{v} \sqrt{g_{S^5}} \bar{v}^a v_a |\mathcal{Y}(v)|^2 = c_5 R^2 \]  

(IV.58)

where \( \mathcal{Y}(v) \) is a spherical harmonic in \( S^5 \) in (IV.52), we can write down the one-loop effective action \( S_F \) for the diagram shown in Fig. 4 as

\[ S_F[A^{(0)}_\mu] \equiv n_{\mu} = \frac{i}{2} c_5 R^2 \int \frac{d^4 q}{(2\pi)^4} \int \frac{d^4 k}{(2\pi)^4} \int dr \sqrt{-g} F^{\mu n}(-q) F_{\nu m}(q) \text{Tr} (D_F(r, r, k) \Gamma_\mu(-i k)^\nu) \mathcal{V}_F |_{t=0} \]  

(IV.59)

We now choose the polarizations to be transverse with the additional axial gauge condition \( a_r = 0 \), so that the boundary-to-bulk R-field is
from the low-frequency expansion, and 

\[ a_\mu(r, \tilde{q}) = \left( \frac{R^2}{r} K_1 \left( \frac{q R^2}{r} \right) \right) n_\mu(q) e^{i q \cdot x} \]  

(IV.60)

The corresponding field strengths are

\[ F_{\mu \nu}(q) = i (q_\mu n_\nu - n_\mu q_\nu) \frac{q R^2}{r} K_1 \left( \frac{q R^2}{r} \right) e^{i q \cdot x}, \]

\[ F_{\mu \nu}(q) = n_\mu q^2 \frac{R^4}{r^3} K_0 \left( \frac{q R^2}{r} \right) e^{i q \cdot x}, \]  

(IV.61)

and their contraction is

\[ F_{\mu \nu} F_{\rho \sigma} = +n_\mu n_\nu \frac{q_4 R^6}{r^4} \left( \frac{K_0^2 \left( \frac{q R^2}{r} \right) + K_1^2 \left( \frac{q R^2}{r} \right)}{2} \right) \]

\[ +q_\mu q_\nu n_2 \frac{q_4 R^6}{r^4} K_1^2 \left( \frac{q R^2}{r} \right) \]  

(IV.62)

B. Low-x near the horizon

To analyze the low-x contribution of the fermions near the horizon, we will focus on the graviton exchange and make use of warped momenta \( \tilde{q} \) throughout this section. For small energy transfer \( \tilde{q}_0 \ll \mu \), the bulk-to-bulk propagator for transverse graviton \( h_x^y(\tilde{q}_0, \tilde{q}_x) \) can be written as

\[ G_{x y, x y}(\tilde{q}_0, \tilde{q}_x, r_1, r_2) = \phi(\tilde{q}_0, \tilde{q}_x, r_1) G_{x y, x y}^B(\tilde{q}_0, \tilde{q}_x) \phi(\tilde{q}_0, \tilde{q}_x, r_2) \]  

(IV.63)

where \( \phi(\tilde{q}_0, \tilde{q}_x, r_1) \) is the normalizable wave function of the graviton, and \( G_{x y, x y}^B \) is its boundary Green's function

\[ G_{x y, x y}^B(\tilde{q}_0, \tilde{q}_x) = \tilde{q}_0 \sigma G(\tilde{q}_0, \tilde{q}_x) \]  

(IV.64)

where \( \text{Re} G(\tilde{q}_0, \tilde{q}_x) = f(\tilde{q}_x, \mu) \) which can be determined from the low-frequency expansion, and [26]

\[ \text{Im} G(\tilde{q}_0, \tilde{q}_x) = -\frac{3 C}{(1 + \frac{\tilde{q}_0^2}{\mu^2})^\frac{1}{2}} \left( 1 + \left( 1 + \frac{\tilde{q}_0^2}{\mu^2} \right)^\frac{1}{2} \right) \]

\[ \times e_0 \left( \frac{\tilde{q}_x}{\mu} \right) \text{Im} G_{\pm} \left( \frac{\tilde{q}_0}{\mu}, \frac{\tilde{q}_x}{\mu} \right) \]  

(IV.65)

where \( C \) is a proportionality constant, \( e_0 \left( \frac{\tilde{q}_x}{\mu} \right) \) is a function to be determined from the low-frequency expansion coefficients, and

\[ G_{\pm} \left( \frac{\tilde{q}_0}{\mu}, \frac{\tilde{q}_x}{\mu} \right) = -2 \nu_{\pm} e^{-i \nu_{\pm} x} \frac{\Gamma(1 - \nu_{\pm})}{1 + \nu_{\pm}} \left( \frac{1}{2} \frac{\tilde{q}_0}{\mu} \right)^{2 \nu_{\pm}}, \]  

(IV.66)

\[ \nu_{\pm} = \frac{1}{2} \left( 5 + 2 \frac{\tilde{q}_x^2}{\mu^2} \pm 4 \left( 1 + \frac{\tilde{q}_0^2}{\mu^2} \right)^\frac{1}{2} \right). \]  

(IV.67)

Note that for zero energy and momentum transfer \( (\tilde{q}_0 = 0 \text{ and } \tilde{q}_x = 0) \), the bulk-to-bulk propagator of the graviton exchange vanishes \( \text{Im} G_{x y, x y}(0, 0, r_1, r_2) = 0 \) since \( G_{\pm}(0, 0) = 0 \). Therefore, the t-channel contribution of the graviton for the current-current correlation function or forward deeply virtual Compton scattering away from the probe limit vanishes. Its Reggeized form through higher spins (closed string exchange), vanishes as well.

V. R-RATIO FOR THE BLACK-HOLE

A. Particle and energy density at the horizon

Having assessed the structure functions both at large-x and small-x near the black hole horizon, we now need to normalize them. For that we need to evaluate the contribution of the bulk fermions near the horizon to the particle and energy densities, much like we did in the probe limit. More specifically, we define

\[ n e_R = \langle J^t \rangle (q z \ll 1) \]

\[ \epsilon = \langle T^{tt} \rangle (q z \ll 1) \]  

(V.68)

as the boundary expectation values of the time-component of the R-current and the energy momentum tensor. The expectation values follow from the holographic correspondence in the tadpole approximation in AdS as

\[ \langle J^t \rangle = -i e_R \int \frac{d^3 k}{(2 \pi)^3} \int \frac{dk^0}{2 \pi} I^t_k (q z \ll 1) \]

\[ \times \psi_1(k) \Gamma_4 \psi_1(k) \text{ImTr} G_{R, R}^{11} (k^0, \vec{k}) \]

\[ \langle T^{tt} \rangle = \int \frac{d^3 k}{(2 \pi)^3} \int \frac{dk^0}{2 \pi} I^t_k (q z \ll 1) \]

\[ \times \psi_1(k) \Gamma^t \psi_1(-ik^0) \text{ImTr} G_{R, R}^{11} (k^0, \vec{k}) \]  

(V.69)

with \( I^t_k (q z \ll 1) \approx I^K_k (q z \ll 1) \) playing the role of a spectral weight, and defined in [VI.107]. Evaluating the momentum integral near the Fermi surface, we find

\[ n = \frac{\langle J^t \rangle}{e_R} \approx C_f h_1 C_\theta \frac{k^3_0 (1 - \frac{k_0 h_1}{k F})}{\nu^2 + \text{Re} IV} \]  

(V.70)

with \( C_\theta = \frac{1}{2\pi} \) and the dimensionless constant.
\[ C_J = R^4 (m R^2 + \nu_{k_F}) \left( \frac{R^2}{R} k_F z_- + \epsilon_R \sqrt{\frac{\alpha}{3}} \right) \times \frac{a_+^2(k_0, k)}{(2\nu_k + 1)W^2} \left( 2\sqrt{3} \right)^{-2\nu_k} \]  

(V.71)

Since \( I_K = I_K' \), we have \( \epsilon = n k^0 \). Note that \( k_0 \) is the solution of the transcendental equation (III.47), which near the Fermi surface \( k \rightarrow k_F \) can be solved as \( k_0 \sim C_0 / z_- \) with the dimensionless constants

\[ C_0(\nu_{k_F}) \equiv (v_F \tilde{h}_2 \text{Im} (e^{i\gamma_F} (-1)^{2\nu_{k_F}})) \frac{1}{\nu_{k_F} + \text{Re} \Pi'} \]  

(V.72)

and \( \tilde{h}_2 = z_-^{1-2\nu_{k_F}} h_2 \). Therefore, for the dense limit near the horizon, we make the identification \( E_A \equiv V_A \epsilon = A \epsilon / n = A k^0 \).

### B. Normalized structure functions: dense regime

Having determined \( n, \epsilon \) in the dense limit near the horizon, we can now normalize the corresponding structure function (III.45) through the substitution

\[ \text{Im} \tilde{G}_{xx}(q) \rightarrow 2E_A V_A \times \text{Im} \tilde{G}_{xx}(q) = 2E_A A \text{Im} \tilde{G}_{xx}(q) \]  

(V.73)

The integral in (III.45) can be evaluated near the fermi surface \( k \rightarrow k_F \) with the result

\[ \text{Im} \tilde{G}_{xx}(q) \approx \frac{1}{2} \tilde{C}_G(\nu_{k_F}) a_+ (k_0, k_- = 0)^2 \frac{k_F^2}{v_{k_F}} \left( 1 \mp \frac{2n}{k_F} \right) \left( 1 + \frac{q^2 z_-^2}{v_{k_F}^2} \right)^{\nu_{k_F} + 1} x_{k_F}^{\nu_{k_F} + 2} (1 - x_{k_F})^{\nu_{k_F} - 2} \Gamma(\tau_+ + \tau_- - 1, 1 - x_{k_F}) , \]  

(V.74)

with \( \tilde{\alpha} = 1 \) for a U(1) R-charge, and \( \tilde{\alpha} = \frac{1}{4} N \) for a D3-D7 U(1) vector charge.

Using (III.51) together with (V.73) and (V.74), we can extract the normalized structure function of the holographic Fermi liquid \( (x_{k_F} k_0 = x m_N) \)

\[ \frac{F_2(x_A, q^2)}{A} = c_F C_{AdS2}(e_R, \tau, \tilde{\alpha}, v_F, \tilde{h}_2) \left( \frac{\mu}{q^2} \right)^{\nu_{k_F} + 2} x_{k_F}^{\nu_{k_F} + 5} (1 - x_{k_F})^{\nu_{k_F} - 2} \Gamma(\tau_+ + \tau_- - 1, 1 - x_{k_F}) , \]  

(V.76)

where we defined the dimensionless constant

\[ C_{AdS2}(e_R, \tau, \tilde{\alpha}, v_F, \tilde{h}_2) \equiv \left( \frac{1}{3 \tilde{\alpha}} \right)^{2\nu_{k_F} + 1} \left( \frac{2\nu_{k_F} + 1}{8 C_0^2} \left( 2\sqrt{3} + \frac{\nu_{k_F} - 2\sqrt{3}}{\epsilon_R} \right) \Gamma(\tau + \nu_{k_F} + \frac{3}{2}) \Gamma(\tau + \nu_{k_F} - \frac{3}{2}) \left( \frac{k_F}{\mu} \sqrt{\alpha} + \frac{1}{\sqrt{3}} \epsilon_R \sqrt{\alpha} \right) \Gamma(\tau - 1)^2 \right) , \]  

(V.77)
with

\[ \nu_{k_F}(e_R, \tau, \bar{\alpha}) = \left( \frac{1}{12} \left( \tau - \frac{3}{2} \right)^2 + \frac{k_F^2}{\mu^2} \bar{\alpha} - \frac{1}{3} e_R^2 \bar{\alpha} \right) \]

\[ C_0 = \left( v_F h_2 \sin(\gamma_F + 2 \pi \nu_{k_F}) \right)^{-1/2\nu_{k_F}} \]

\[ \gamma_F = \arg \left( \Gamma(-2\nu_{k_F}) \left( e^{-2\pi i \nu_{k_F}} - e^{-2\pi i e_R \bar{\alpha}} \right) \right) \]

(V.78)

Note that for a large effective charge \( e_R \sqrt{\bar{\alpha}} \to \infty \), we have \( \gamma_F = -2\pi \nu_{k_F} \) and \( C_0 \to 0 \) which implies that the structure function \( \text{V.76} \) vanishes in the probe limit \( \bar{\alpha} \sim \frac{N_c}{N_f} \to \infty \), which is also the regime where the backreaction from the flavor branes can be ignored.

FIG. 5: Dense R-ratio \( \text{V.79} \) for \( k_F/m_N = 3.5 \), BH \( \equiv \) black hole, F \( \equiv \) quantum fermions, see also text.

FIG. 6: Dense R-ratio \( \text{V.79} \) for \( k_F/m_N = 0.8 \), BH \( \equiv \) black hole, F \( \equiv \) quantum fermions, see also text.

C. R-ratio in the dense regime

We define the R-ratio of the nucleus in the dense limit as

\[ R_{\text{dense}}(x, q^2) = \frac{1}{\frac{F_{\text{dense}}^2(x, q^2)}{F_{\text{nucleon}}^2(x, q^2)}} \]  

(V.79)

with the nucleon structure function

\[ F_{\text{nucleon}}^2(x, q^2) = \left( \frac{\beta m_N^2}{q^2} \right) \tau^{-1} \left( 8\pi^2 (\tau - 1)^2 e_R^2 x^{\tau+1} (1 - x) \tau^{-2} + \frac{c_5 \pi^2}{2\sqrt{4\pi \lambda}} \left[ I_{0, 2\tau+3} + I_{1, 2\tau+3} \right] \frac{1}{x^2} \right), \]  

(V.80)

as derived below (see [VI.115]), and the dense structure function following

\[ F_{\text{dense}}^2(x, q^2) \approx C_T \left( \frac{3\eta q^2}{4m_N^2} \right)^{\frac{3}{4}} x^{\frac{3}{4}} + C_{\text{AdS2}} e_R^2 \left( \frac{\mu^2}{q^2} \right)^{\nu_{k_F} + 2} \frac{k_F^2}{\mu^2} \left( 1 - x_{k_F} \right)^{-\frac{5}{2}} \frac{1}{2} F_2^2 \tau_+ \tau_- \tau - 1, 1 - x_{k_F} \right). \]  

(V.81)

For the dense part, the first contribution the leading and dominant contribution [III.22] stems from DIS on the classical black hole in the bulk. The second and subleading contribution [V.76] stems from DIS scattering off the emerging holographic Fermi liquid near the black hole horizon. It is a quantum correction that is vanishingly small at small-x. The details for the nucleon structure function in the normalization are given below and summarized in [VI.115].

To quantify each of the contributions in the R-ratio, we
now need to fix the parameters entering this expression, many of which are tied by holography. We first fix the explicit holographic parameters: \( \tilde{\alpha} = N_c/4N_f = 1 \) (ratio of branes), \( 2\pi^2c_3/\sqrt{4\pi\lambda} = 0.01 \) (strong coupling) and \( e_R = 0.3 \) (charge of the probe fermions). Next, we fix the scaling parameters entering in the nucleon pdf: \( \tau = 3 \) (hard scaling law) and \( j = 0.08 \) (Pomeron intercept). The nucleon confining scale enters \( \mu/m = 0 \) (Fermi velocity), \( \mu/m_N = 1.2 \) (chemical potential) and two values for \( k_F/m_N = 0.8, 3.5 \) (Fermi momentum).

With all these parameters fixed, we show in Fig. 5 the behavior of the dense R-ratio vs. x for large Fermi momentum \( k_F/m_N = 3.5 \) and fixed momentum \( q/m_N = 1 \). The low-x part is dominated by the black hole with the emergent Fermi surface contributing only at large-x. This behavior is expected. At strong coupling, most of the partonic content is shifted at very low-x while scattering off the bulk black hole in leading order. The subleading and quantum correction due to scattering off the emergent Fermi surface only contributes at large-x for a sufficient large Fermi momentum. In Fig. 5 we show the same behavior for a smaller Fermi momentum \( k_F/m_N = 0.8 \). The contribution from the emergent Fermi surface is shadowed by the scattering off the black hole.

Remarkably, the essential features of scattering off a nucleus are seen in Fig. 5 when the emergent Fermi surface is made visible through a large Fermi momentum. The R-ratio exhibits shadowing for \( x < 0.1 \), anti-shadowing for \( 0.1 < x < 0.3 \), the EMC-like effect for \( 0.3 < x < 0.8 \) and Fermi motion for \( x > 0.8 \).

Finally, we note that since \( x_{k_F}/x = m_N/k_0 = m_N z_-/C_0 \), most of the dependence on matter (hence \( A \) through the location of the horizon \( z_- \)) comes from this contribution in the dense limit. We note that this contribution is finite for \( \tilde{\alpha} = 1 \) for a U(1) R-charge, and is vanishingly small for \( \tilde{\alpha} = N_c/N_f \) for D3-D7 probe branes at large \( N_c \) since \( \gamma_F \to 0 \) and therefore \( C_0 \to 0 \) in (V.78). For the latter, the R-ratio for dense nuclei (V.79) takes the limiting value

\[
R_{\text{dense}}(x, q^2) \to \frac{C_{\text{dense}} x^{1/3}}{C_{\text{large}} x^{\tau+1}(1-x)^{\tau-2} + C_{\text{small}} x^2} \quad (V.82)
\]

where we have defined the coefficients

\[
C_{\text{dense}} = C_T \left( \frac{3\tilde{\alpha}}{4} \right)^\frac{3}{2} \left( \frac{q^2}{\beta m_N^2} \right)^{\tau-\frac{1}{2}} \quad (V.83)
\]

for the dense part, and for the nucleon part

\[
C_{\text{large}} = 8\pi^2 (\tau - 1)^2 e_R^2
\]

\[
C_{\text{small}} = \frac{c_5}{2\sqrt{4\pi\lambda}} \frac{[I_0.2\tau+3 + I_1.2\tau+3]}{2^{2(\tau-4)} \Gamma^2(\tau-1)} \quad (V.84)
\]

We recall that \( \beta = 1/(m_N z_-)^2 \). The q-dependence does not drop in the dense limit.

### VI. DIS IN THE PROBE LIMIT: DILUTE REGIME

We now consider scattering in the probe limit, where the bulk fermions carry a density without affecting the underlying AdS\(_5\) geometry (with or without a wall), i.e. \( \frac{\mu}{\sqrt{\alpha}} \to 0 \) with \( \frac{\mu}{\sqrt{\alpha}} \times e_R \sqrt{\alpha} = \mu_c \) fixed where \( \tilde{\alpha} \sim \frac{N_c}{N_f} \gg 1 \) and \( \mu_c \) is chemical potential. This is the dilute limit case which amounts to using the free spectral form (III.29) with the substitution

\[
\text{Im } G_F^{\alpha\gamma}(\omega_1, k) \to \pi n_F(\omega_1, k) \delta(k^2 + \omega_1^2) \delta^{\alpha\gamma} \quad (VI.85)
\]

Here \( n_F(\omega_1, k) = \theta(\mu_c - (k^2 + \omega_1^2)^{1/2}) \) is the Fermi occupation factor for a fermion of momentum \( k \), mass \( \omega_1 \) and Fermi energy \( \mu_c \), and the vacuum (AdS\(_5\)) wavefunctions. For the confining case, the mass \( \omega_1 \) is quantized. This analysis complements the one we have discussed recently using generic arguments based on a density expansion of a trapped Fermi liquid [17].

#### A. Large-x

With this in mind, consider the case of scattering in the ultraviolet region of the black-hole, with the hard fermion of momentum \( k + q \) and the remaining fermion of momentum \( k \) treated in the probe approximation. This example will help clarify the relationship between our analysis and that in [20]. For that we use the vacuum propagator (III.28) for both the hard fermion and the density modified propagator (X.128) and (VI.85) for the soft fermion in (III.39),
with the physical condition \( \omega_1 + \omega_2 < q \) (i.e., a meson or virtual photon of mass \( q \) decaying into KK-fermions of masses \( \omega_1 \) and \( \omega_2 \), and

\[
I_{zv}(\omega_1, \omega_2, q) = \epsilon_R R^4 q \int_0^\infty dz \; z^2 J_{mR-1/2}(\omega_1 z) J_{mR-1/2}(\omega_2 z) K_1(qz)
\]

\[
\approx \epsilon_R R^4 \frac{2^{-(mR-1/2)}}{\Gamma(mR + 1/2)} \omega_2^{mR-\frac{1}{2}} q \int_0^\infty dz \; z^{mR-\frac{1}{2}} J_{mR-1/2}(\omega_1 z) K_1(qz)
\]

\[
\approx 2\epsilon_R R^4 (mR + 1/2) \frac{1}{q^2} \left( \frac{\omega_2}{q} \right)^{mR-\frac{1}{2}} \left( \frac{\omega_1}{q} \right)^{mR-\frac{1}{2}} \left( 1 + \frac{\omega_2^2}{q^2} \right)^{-(mR+\frac{1}{2})}
\]

(VI.90)
where we made use of the approximation

\[ J_{mR-1/2}(\omega z) \approx \frac{2^{-(mR-1/2)}}{\Gamma(mR + 1/2)} (\omega z)^{mR-\frac{1}{2}} \quad (VI.91) \]

for \( \omega z \ll 1 \). Note that without making the approximation \( \omega z \ll 1 \), the above integral \( I_{z\nu} \sim \)

\[ \text{Im} \tilde{G}_{\mu\nu}^{\omega}(q) \approx \frac{\pi}{4} \int \frac{d^3k}{(2\pi)^3} \int_{-|q|^2}^{q^2} \frac{d\omega^2}{2} \int_0^q \frac{dw^2}{2} I_z^2(\sqrt{s_k}, \omega_2, q) \text{Tr} \left( (-\tilde{k} + \tilde{q}_{\omega}) + \sqrt{s_k} \right) \gamma^\mu (-\tilde{k} + \tilde{q}_{\omega}) \gamma^\nu \right) \pi n_F \delta(k^2 + \omega_2^2), \]

\[ (VI.92) \]

where \( s_k = -(k + q)^2 \). The evaluation of the remaining \( k^0 \)-integral in \( (VI.92) \) using the last delta-function, yields

\[ n_\mu n_\nu n_m n_\nu \text{Im} \tilde{G}_{\mu\nu}^{\omega}(q) \approx \frac{\pi}{4} \int \frac{d^3k}{(2\pi)^3} \int_{-|q|^2}^{q^2} \frac{d\omega^2}{2} \int_0^q \frac{dw^2}{2} I_z^2(\sqrt{s_k}, \omega_2, q) \text{Tr} \left( (-\tilde{k} + \tilde{q}_{\omega}) + \sqrt{s_k} \right) \gamma^\mu (-\tilde{k} + \tilde{q}_{\omega}) \gamma^\nu \right) \frac{n_F(\omega_k, \tilde{k})}{2E_k}, \]

\[ (VI.93) \]

where we have assumed \( n \cdot q \approx 0 \) and \( k^2 \approx 0 \). Note that the trace in \( (VI.94) \) is the same trace evaluated in \( \text{[20]} \) for \( \omega_2 = 0 \) (see their Eq. 72 ). Using \( (III.17) \) with \( x_A = F_4(a; b; c; d; -\frac{q^2}{4}, -\frac{q^2}{4}) \quad (27, 28) \) where \( F_4 \) is the fourth Appell series of hypergeometric functions which is indeed convergent only for \( \omega_1 + \omega_2 < q \).

The integral in \( (VI.90) \) is in agreement with the R-current scattering on a dilatino in \( \text{[20]} \). Evaluating the integral in \( (VI.89) \) over \( \omega_1 \) using the delta-function \( \delta(\omega_1^2 - s_k) \), and using \( (VI.90) \) we have

\[ \text{Im} \tilde{G}_{\mu\nu}^{\omega}(q) = \frac{\pi}{4} \int \frac{d^3k}{(2\pi)^3} \int_{-|q|^2}^{q^2} \frac{d\omega^2}{2} \int_0^q \frac{dw^2}{2} I_z^2(\sqrt{s_k}, \omega_2, q) \text{Tr} \left( (-\tilde{k} + \tilde{q}_{\omega}) + \sqrt{s_k} \right) \gamma^\mu (-\tilde{k} + \tilde{q}_{\omega}) \gamma^\nu \right) \frac{n_F(\omega_k, \tilde{k})}{2E_k}, \]

\[ (VI.94) \]

We have \( x_k = -q^2/(2k \cdot q) = -(k + q)^2 \approx -q^2(1 - 1/x_k) \) and \( k^0 = E_k = (|\tilde{k}|^2 + \omega_2^2)^{\frac{1}{2}} < |q^0| \). We can reduce
for the mass range $\omega_2 \leq \Lambda$. Using (VI.96), we can re-
write the structure functions (VI.95) in terms of $x_k$ as

$$
\bar{F}_2(x_A, q^2) \approx 2^2 \pi^2 \tau^2 R^8 (\tau - 1)^2
\times \left( \frac{1}{q^2} \right) \tau^{-1} \int_0^{\Lambda^2 < q^2} \frac{d\omega_2^2}{2} \left( \omega_2^2 \right)^{\tau-2} \int \frac{d^3 k}{(2\pi)^3} n_F(\omega_2, \vec{k}) \frac{(n \cdot \vec{k})^2}{2E_k} x_k^{\tau+2} (1 - x_k)^{\tau-2},
$$

( VI.97)

with the twist parameter is $\tau = mR + 3/2$, following the approximation ($\omega_2 \ll q$)

$$
\sqrt{s_k} \frac{\omega_2}{q^2} \approx \left( \frac{1}{x_k} - 1 \right) \frac{1}{x_k} \frac{\omega_2}{q} \approx 0
$$

(VI.98)

#### B. Low-x

In contrast to the dense limit in (IV.64), the bulk-to-
bulk graviton propagator in the probe limit, is given by

$$
G_{xy, xy}(\bar{q}_0, \bar{q}_x, z, z') = \int_0^{\infty} \frac{d\omega_2}{2} z^2 J_\Delta(z\omega) z'^2 J_\Delta(z'\omega) \frac{1}{-t + \omega^2 - i\epsilon}
$$

(VI.99)

where $t = -\bar{q}^2 = \bar{q}_0^2 - \bar{q}_x^2$. Therefore, $G_{xy, xy}(\bar{q}_0 = 0, \bar{q}_x = 0, z, z') \neq 0$ does not vanish in the probe limit. In this limit, the graviton exchange Reggeizes by including higher spin-j (stringy) exchange as

$$
\mathcal{K}(j, \bar{q}_0, \bar{q}_x, z, z') = \int_0^{\infty} \frac{d\omega_2}{2} z^2 J_\Delta(j)(z\omega) z'^2 J_\Delta(j)(z'\omega) \frac{1}{-t + \omega^2 - i\epsilon}
$$

(VI.100)

Note that only in this section, we have added an extra-
t factor of $i$ in the gamma matrix in comparison to (III.31) and replaced $\omega$ by $-\omega$ to make the comparison with standard results easier. Inserting (IV.62) in (IV.59), we obtain the on-shell one-loop effective action $S_F[A_\mu^{(0)}] \equiv n_\mu = n_\mu n_\nu \text{Im} \tilde{G}^{\mu\nu}_F(q).$
\[ n_{\mu n_\nu} \text{Im} \tilde{G}^{\mu \nu}_{F} (q) \approx \int \frac{d^3 k}{(2\pi)^3} \int_{|q|}^{0} \frac{dk^0}{2\pi} \int dr \sqrt{-g} \]
\[ \times \left( n_{\mu n_\nu} \frac{q^4 R^6}{r^4} \left[ K_0^0 (q R^2/r) + K_1^0 (q R^2/r) + q_{\mu q_\nu} n^2 q^2 R^6 / r^4 - K_1^1 (q R^2/r) \right] \right) \]
\[ \times \text{Tr} \left( \text{Im} \mathcal{D}_R (r, r, \vec{k}) | \Gamma^{(i k) a} g^{\mu \nu} \right) \text{ Im} \mathcal{V}_R |_{l = 0} \]
\[ \approx C_{\lambda} \int dr \sqrt{-g} \sqrt{q^4 g} \int \frac{d^3 k}{2} \frac{n_{F} (\omega, \vec{k})}{2 E_k} \left( \frac{1}{(2\pi)^3} \right)^2 \frac{1}{2 E_k} \sum_{j=1}^{\infty} w_j \left( w_j^2 r_j^2 + 1 \right) \]
\[ \times \left( \frac{1}{q^2} \right)^{\tau - 2} \int \frac{d^3 k}{(2\pi)^3} \frac{n_{F} (\omega, \vec{k})}{2 E_k} \frac{1}{2 E_k} \int_{0}^{\infty} dw w^{2r+3} \]
\[ \times \left( \frac{(n \cdot k)^2}{q^2} \left[ K_0^0 (w_j) + K_1^0 (w_j) + \frac{1}{4 x_k^2} n^2 K_1^1 (w_j) \right] \right) \]
\[ \approx C_{\lambda} \left( \frac{1}{q^2} \right)^{\tau - 2} \int \frac{d^3 k}{(2\pi)^3} \frac{n_{F} (\omega, \vec{k})}{2 E_k} \left( \frac{(n \cdot k)^2}{q^2} \left[ I_{0,2r+3} + I_{1,2r+3} + \frac{1}{4 x_k^2} n^2 I_{1,2r+3} \right] \right) \]

(VI.102)

Here \( E_k = (|\vec{k}|^2 + \omega^2)^{1/2} < |q|, \ k^0 = E_k, \ \text{and} \ x_k = -\frac{\vec{k}^2}{2q^2} \).
Also we have set \( r_j = \frac{R \sqrt{\alpha^s}}{2 \sqrt{j}}, \ w_j = q R^2 / r_j = q^2 j \),
with \( \nu = \frac{1}{2} (n + 1) \). They are related to each other recursively \( (n - 1) I_{1,n} = (n + 1) I_{0,n} \).

\[ C_{\lambda} = \frac{\pi c_5 R^4}{2 \sqrt{4 \pi \lambda}} \Gamma^2 (m R + 1/2) \]  

(VI.103)
and defined the integrals

\[ I_{j,n} = \int_{0}^{\infty} dw w^n K_j^j (w) = 2^{n-2} \frac{\Gamma (\nu + j) \Gamma (\nu - j) \Gamma (\nu)^2}{\Gamma (2\nu)} \]  

(VI.104)

The structure functions of the nuclei at small-\( x \) in the probe limit are given by

\[ \hat{F}_2 (x_A, q^2) \approx 2 \pi C_{\lambda} \left( \frac{1}{q^2} \right)^{\tau - 1} \frac{1}{2 x_A (n \cdot P_A)^2} \int_{0}^{\Lambda^2 < q^2} \frac{d\omega}{2} (\omega^2)^{\tau - 2} \int \frac{d^3 k}{(2\pi)^3} \frac{n_{F} (\omega, \vec{k})}{2 E_k} \left( \frac{1}{q^2} \right)^{\tau - 1} \int \frac{d^3 k}{(2\pi)^3} \frac{n_{F} (\omega, \vec{k})}{2 E_k} \left( \frac{1}{4 x_k^2} \right)^{\tau - 1} \]  

(VI.105)

The effects caused by the diffusion in the radial direction on the structure functions far from the black-hole at low-
C. Normalized structure functions

To normalize the structure functions in the probe limit, we recall that the bulk density and bulk energy density follows from the holographic principle as

\[ \hat{n}(qz \ll 1) = \int_0^{\Lambda} \frac{d\omega}{2} I_{K}(qz \ll 1, \omega) \int_0^{k_F(\omega)} \frac{d^3 k}{(2\pi)^3} \]

and note that the large-x structure functions in (VI.110) are discussed in Appendix E.

\[ \hat{\epsilon}(qz \ll 1) = \int_0^{\Lambda} \frac{d\omega}{2} I_{K}(qz \ll 1, \omega) \times \int_0^{k_F(\omega)} \frac{d^3 k}{(2\pi)^3} \sqrt{k^2 + \omega^2} \] (VI.106)

where \( k_F(\omega) = \sqrt{\mu^2 - \omega^2} \), and

\[ I_{K}(qz \ll 1, \omega) = R^4 \int_0^{z_m} dz z^{2r-3}(\omega^2)^{\tau-2} \]

\[ = \frac{1}{2} \frac{R^4}{\tau - 1} (\omega^2 z_m^2)^{\tau-1} \omega^{-2} \] (VI.107)

after taking \( qzK_1(qz \ll 1) \approx 1 \). The \( \omega \)-integration in (VI.106) is carried over the bulk spectral density \( I_{K}(0, \omega) \) with an upper cut-off \( \Lambda \). In the conformal case, the cut-off is a priori arbitrary. In the conformally broken case, say a hard wall at \( z = z_m \), we can set \( z_m \Lambda = z_m m_N \) in (VI.106) and assuming \( \Lambda \) large, to pick only the nucleon ground state. A higher cutoff would include higher excited states of the nucleon. With this in mind, we can first undo the k-integration by approximating it near the Fermi surface, and then undo the \( \omega \)-integration by keeping only the leading contribution for \( \frac{1}{\beta} \equiv (z_m m_N)^2 > 1 \),

\[ \hat{n}(qz \ll 1) \approx \frac{1}{8\pi^2 (\tau - 1)\beta^{\tau-1}} k_F^3 \]

\[ \hat{\epsilon}(qz \ll 1) \approx \frac{1}{8\pi^2 (\tau - 1)\beta^{\tau-1}} k_F^3 E_F \] (VI.108)

with \( E_F = (k_F^2 + m_N^2)^{1/2} \). We now identify the bulk density \( \hat{n} = A/V_A \) as the density of a fixed target say a nucleus, with \( \Lambda \)-nucleons in a fixed volume \( V_A \) and a total energy \( E_A = AE_F \). The normalized structure functions \( F_{1,2} \) are then related to our earlier and un-normalized structure functions \( \tilde{F}_{1,2} \) through

\[ F_{1,2} \equiv 2E_A V_A \tilde{F}_{1,2} = 2AE_A \tilde{F}_{1,2} \] (VI.109)

Case-1 (Large-x):

The normalized structure functions at large-x follow by inserting (VI.105) and (VI.108) into (VI.109) with \( \beta = 1/(m_N z_m)^2 \)

\[ F_2(x, q^2) \]

\[ \frac{A}{\tau} \approx 8\pi^2 (\tau - 1)^2 c_A^2 \left( \frac{\beta m_N^2}{q^2} \right)^{\tau-1} x_F^{\tau+1} (1 - x_F)^{\tau-2} \]

\[ F_1(x, q^2) \]

\[ \frac{A}{\tau} \approx 4\pi^2 (\tau - 1)^2 c_A^2 \left( \frac{\beta m_N^2}{q^2} \right)^{\tau-1} A x_F^{\tau+1} (1 - x_F)^{\tau-2} . \] (VI.110)

We define the x-fractions

\[ x_F = \frac{q^2}{-2p_F q} \approx \frac{q^2}{2E_F \omega} = \frac{x m_N}{E_F} \]

\[ x_A = \frac{q^2}{-2p_A q} \approx -\frac{q^2}{2E_A \omega} = \frac{x_F}{A} \] (VI.111)

and note that the large-x structure functions in (VI.110) in the probe approximation obey the analogue of the Callan-Gross relation \( F_2 = 2x_A F_1 \) for a holographic and dilute nucleus. Also we note that (VI.110) are analogous to the so-called structure functions of the nucleus obtained through the so-called x-scaling of the structure functions of the nucleon.

Case-2 (Small-x):

Doing the momentum integrals in (VI.106) near \( k \to k_F \) and doing the appropriate normalization as in the large-x regime, we find
\[ \frac{F_2(x, q^2)}{A} \approx \pi C_x \left( \frac{\beta m_N^2}{q^2} \right)^{\frac{1}{2}} \frac{1}{x_F} [I_{0,2\tau+3} + I_{1,2\tau+3}] \]

\[ \frac{F_1(x, q^2)}{A} \approx \pi C_x \left( \frac{\beta m_N^2}{q^2} \right)^{\frac{1}{2}} \frac{1}{x_F} A \left( \frac{1}{4x_F} I_{1,2\tau+3} \right), \]

(VI.112)

where \( F_2^{\text{dilute}}(x, q^2) \) is given by the sum of (VI.110) for large-x and (VI.112) for small-x,

**D. R-ratio in the probe limit**

We define the R-ratio of the nucleus in the probe (dilute) limit as

\[ R_\text{dilute}(x, q^2) = \frac{1}{A} F_2^{\text{dilute}}(x, q^2) \]

(VI.113)

\[ \frac{F_2^{\text{dilute}}(x, q^2)}{A} \approx \left( \frac{\beta m_N^2}{q^2} \right)^{\frac{1}{2}} \left( 8 \pi^2 (\tau - 1)^2 e_R^2 x_F^{\tau-1} (1 - x_F)^{\tau-2} + \frac{c_5 \pi^2}{2 \sqrt{4\pi \lambda}} \frac{[I_{0,2\tau+3} + I_{1,2\tau+3}]}{2(2\tau-4)\Gamma(\tau - 1)} \frac{1}{x^2} \right) \]

(VI.114)

and \( F_2^{\text{nucleon}}(x, q^2) \) is given by

\[ \frac{F_2^{\text{nucleon}}(x, q^2)}{A} = \left( \frac{\beta m_N^2}{q^2} \right)^{\frac{1}{2}} \left( 8 \pi^2 (\tau - 1)^2 e_R^2 x^{\tau+1} (1 - x)^{\tau-2} + \frac{c_5 \pi^2}{2 \sqrt{4\pi \lambda}} \frac{[I_{0,2\tau+3} + I_{1,2\tau+3}]}{2(2\tau-4)\Gamma(\tau - 1)} \frac{1}{x^3} \right). \]

(VI.115)

Recall that \( \beta m_N^2 = 1/z_N^2 \) is related to the confining scale here, and that \( x_F E_F = x m_N \). Note that in (VI.110), \( j = 1 \) in the absence of transverse diffusion or curvature corrections. When the latter are included \( j \rightarrow 1 - O(1/\sqrt{\lambda}) \). The structure function of the proton follows from (VI.113) by setting \( k_F = 0 \) or through the substitution \( x_F \rightarrow x \). The R-ratio for the probe or dilute limit is independent of \( q^2 \). Note that the first contribution is proportional to \( e_R^2 \) while the second is proportional to \( e_R^4 \) independent of the R-charge.

In Fig. 7 we show the behavior of the dilute R-ratio (VI.113) versus \( x \) for fixed Fermi momentum \( k_F/m_N = 1 \). The holographic parameters used are fewer but consistent with those used for the dense R-ratio in Fig. 4. Specifically, we have used: \( e_R = 0.3 \) (R-charge of the bulk fermion), \( 2\pi^2 c_5/\sqrt{4\pi \lambda} = 0.01 \) (strong coupling), \( \tau = 3 \) (hard scaling exponent), \( j = 0.08 \) (Pomeron intercept). In the dilute case, the R-ratio is dominant at large-x and asymptotes 1 at small-x. Clearly visible is the EMC-like effect for \( 0.2 < x < 0.8 \) and the Fermi motion for \( x > 0.8 \).

We have checked that the overall features of Fig. 7 remain unchanged for smaller values of \( e_R \) but fixed \( k_F/m_N \) in conformity with the probe limit. This holographic behavior is very similar to the one we presented recently using general arguments [17].

**FIG. 7:** Dilute R-ratio (VI.113) for \( k_F/m_N = 1 \). See text.
VII. CONCLUSIONS

In the double limit of a large number of colors and strong coupling, DIS scattering off an extremal black hole is of order $N_c^2$ following from the absorption of the bulk $R$-current by the black hole. Through a suitable mapping onto a nucleus, the ensuing structure functions are dominated by low-$x$. Scattering off the black hole is the ultimate coherent scattering off a dense nucleus with strong shadowing as we noted in [12].

To order $N_c^0$, DIS scattering is off holographic fermions hovering around the black-hole horizon due to quantum pair creation. In this regime, the geometry is that of $\text{AdS}_2 \times \mathbb{R}^3$ with an emergent Fermi surface and anomalous scaling laws. DIS scattering off these bulk fermions show that their partonic distribution functions on the boundary exhibit anomalous exponents and modified hard scattering rules in comparison to scattering off bulk fermions in the dilute or probe limit. For both limits, DIS scattering exhibits the EMC-like effect at intermediate-$x$ and Fermi-like motion for large-$x$.

The fermionic contribution in the probe limit exhibits many similarities with our recent analysis of DIS scattering off a dilute nucleus using the reduction formula and the holographic identification in the dilute approximation [17]. The partonic content of the bulk fermions is found to shift to intermediate-$x$. Remarkably, our results both in the dense and dilute limit exhibit the essential features observed in the reported DIS scattering on real nuclei [11][2].

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IX. APPENDIX A: CONVENTIONS IN CURVED SPACE

The gamma matrices in curved and tangent space used to analyze the Dirac equation in the extremal RN-AdS black hole will be made explicit here. For that, consider the generic line element in curved space

$$ds^2 = -g_{tt}dt^2 + g_{rr}dr^2 + g_{ii}dx_i^2.$$  (IX.116)

If we refer to the indices in curved space by $\mu, \nu$ (also $t, i$) and those in the tangent space by $a, b$ (also $t, i$) then the gamma matrices are related by

$$\Gamma^\mu = \Gamma^a e_a^\mu, \quad \Gamma^r = \sqrt{g_{rr}} \Gamma^r.$$  (IX.117)

If we set the vierbeins as [21]

$$e^t = \sqrt{g_{tt}} dt, \quad e^i = \sqrt{g_{ii}} dx^i,$$  (IX.118)

then we have

$$\Gamma^t = \sqrt{g_{tt}} \Gamma^t, \quad \Gamma^i = \sqrt{g_{ii}} \Gamma^i, \quad \Gamma^r = \sqrt{g_{rr}} \Gamma^r.$$  (IX.119)

In the tangent space, the gamma matrices read [16]

$$\Gamma^t = \begin{pmatrix} i\sigma_1 & 0 \\ 0 & i\sigma_1 \end{pmatrix}, \quad \Gamma^z = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix},$$

$$\Gamma^r = \begin{pmatrix} -\sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix}, \quad \Gamma^i = \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix}.$$  (IX.120)

The non-vanishing spin connections are

$$\omega_{tz} = -f_0 e^t, \quad \omega_{tx} = f_1 e^i$$  (IX.121)

with

$$f_0 = \frac{1}{2g_{tt}} \sqrt{g_{rr}}, \quad f_1 = \frac{1}{2g_{ii}} \sqrt{g_{rr}}.$$  (IX.122)

X. APPENDIX B: SOFT SPINORS

The soft normalizable wavefunctions were constructed in [21], we reproduce them here for completeness. The Dirac equation in the AdS$_2 \times \mathbb{R}^3$ geometry is solved by the rescaled spinors

$$\psi_1(r; \vec{k}) = (-\sqrt{g_{rr}})^{-\frac{1}{2}} \begin{pmatrix} \Phi_1 \\ 0 \\ 0 \end{pmatrix} \times \frac{r_+}{\sqrt{R^2}} \times \frac{r_-}{\sqrt{R^2}}$$

$$\psi_2(r; \vec{k}) = (-\sqrt{g_{rr}})^{-\frac{1}{2}} \begin{pmatrix} 0 \\ \Phi_2 \\ 0 \end{pmatrix} \times \frac{r_+}{\sqrt{R^2}} \times \frac{r_-}{\sqrt{R^2}}$$  (X.123)

with

$$\Phi_1 = \begin{pmatrix} 0 \\ v_1 \\ v_2 \end{pmatrix} = \frac{1}{W} a_+(k_0, k) \Psi^{(0)}$$

$$\Phi_2 = \begin{pmatrix} 0 \\ \tilde{v}_1 \\ \tilde{v}_2 \end{pmatrix} = \Phi_1(\vec{k} \rightarrow -\vec{k})$$  (X.124)

and $W = -iv_+^a \sigma^a v_-, a_+(k_0, k) = \tilde{c}_1(k-k_F) + \tilde{c}_2 k_0 + \cdots$, where $\tilde{c}_{1,2} \sim \frac{R^2}{k^2}$. The explicit spinors are
\[ \Psi_\pm \approx v_\mp (r - r_\pm) \left( \frac{r_\pm}{R^2} \right)^{\pm \nu_k} \]

\[ v_\pm = \left( m R_2 \pm \nu_k \right) \left( \frac{k R}{r_\pm} - e_R \sqrt{\frac{\alpha}{3}} \right), \quad (X.125) \]

with \( R_2 = R/2\sqrt{3} \), and

\[ \nu_k = \sqrt{m_k^2 R_2^2 - \frac{\alpha}{3} e_R^2}, \quad m_k^2 \equiv m^2 + \frac{k^2 R_2^2}{r_\pm^2}. \quad (X.126) \]

Note that for pure AdS, the soft wave-functions simplify

\[ \Phi_1 = \Psi_\uparrow^{(0)} \quad \Phi_2 = \Phi_1 (\vec{k} \rightarrow -\vec{k}). \quad (X.127) \]

Finally, note that the Feynman propagator for the soft part in (III.32 III.33) is given by

\[ D_F(r, r'; k) = \psi_\alpha(r, k) \left( \int \frac{d\omega \rho_{B}^\omega (\omega, \vec{k})}{2\pi k^0 - \omega} \right) \psi_\gamma(r', k) \quad (X.128) \]

with the boundary spectral function \( \rho_{B}(\omega, \vec{k}) \), and the normalizable wave function \( \psi_\alpha(r, \vec{k}) \) for the Dirac equation in curved AdS_5.

**XI. APPENDIX C: EFFECTIVE VERTICES**

The soft-to-hard transition vertices entering in the bulk DIS amplitude involve (X.127) for the reduction to AdS_2 or (X.124) in general for the soft part, with the hard part of the wave-function given by

\[ u_1 = \left( \frac{R^2}{r} \right)^{\frac{1}{2}} J_{mR} - \frac{1}{2} \left( \frac{\omega}{r} \right) \]

\[ u_2 \equiv 0 \quad (XI.129) \]

More specifically, for pure AdS_2, the transition vertex is simply given by

\[ \Lambda_{11}^i (z_2; \omega_1; q; k) = \]

\[ C(\nu_k) \left( \frac{r}{R^2} \right)^{\nu_k + \frac{1}{2}} q \int \infty dz_2 z_2^{\frac{3}{2} + \nu_k} K_1(q z_2) J_{mR - \frac{1}{2}} (\omega_1 z_2), \quad (XI.130) \]

with

\[ C(\nu_k) = e_R R^2 (m R_2 + \nu_k) (2\sqrt{3})^{-\nu_k} \quad (XI.131) \]

In general, the transition vertices are of the form

\[ \Lambda_{11}^i (r_2; \omega_1; q; k) = -e_R \left( \frac{r \omega}{R^2} \right)^{\nu_k + \frac{1}{2}} \int dr_2 \sqrt{-g} (-g_{rr})^{-\frac{1}{2}} \sqrt{g_{\alpha\beta}} K_A(r_2; q) \left( \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right) \left( \begin{array}{c} v_1 \\ 0 \\ 0 \end{array} \right), \quad (XI.132) \]

\[ \Lambda_{11}^i (r_1; k; q_1) = -e_R \left( \frac{r \nu_k}{R^2} \right)^{\nu_k + \frac{1}{2}} \int dr_1 \sqrt{-g} (-g_{rr})^{-\frac{1}{2}} \sqrt{g_{\alpha\beta}} K_A(r_1; q) \left( \begin{array}{c} v_1 \\ 0 \\ 0 \end{array} \right) \left( \begin{array}{c} 0 \\ v_2 \\ 0 \end{array} \right), \quad (XI.132) \]

Using the gamma matrices explicitly, we can simplify the effective vertices (XI.132). More specifically, we have

\[ \Lambda_{11}^i (r_2; \omega_1; q; k) = i e_R \left( \frac{r \omega}{R^2} \right)^{\nu_k + \frac{1}{2}} \]

\[ \times \int dr_2 \sqrt{-g} (-g_{rr})^{-\frac{1}{2}} \sqrt{g_{\alpha\beta}} K_A(r_2; q) v_1, \quad (XI.133) \]
with the rest of the vertices following by symmetry with

\[ \Lambda_{11}^2(r_1; k; q; \omega_1) = -\Lambda_{11}^2(r_2; \omega_1; q; k), \]
\[ \Lambda_{22}(r_1; k; q; \omega_1) = \Lambda_{22}(r_2; \omega_1; q; k) \equiv 0, \]

(XI.134)

and all other components vanishing. Performing the change of variable \( r = R^2/z \) and setting \( z \ll z_+ \), we can re-write the integral in (XI.133) as

\[ C(\nu_k) = e_R R^2 \frac{(mR + \nu_k)}{W} (2\sqrt{3})^{-\nu_k}, \]

(XI.136)

Note that for the special value \( \nu_k = \nu_k^* = mR \), the integrand reduces to the one in (20), and can be evaluated exactly as

\[ I_z(\omega_1; q; k) = 2^{\nu_k + \frac{3}{2}} \frac{\Gamma\left(\frac{mR + \nu_k + 3}{2}\right) \Gamma\left(\frac{mR + \nu_k + 1}{2}\right)}{\Gamma\left(mR + \frac{3}{2}\right)} \]

(XI.138)

with

\[ I_z(\omega_1; q; k) = \left(\frac{r_+}{R^2}\right)^{\nu_k + \frac{3}{2}} \int_0^\infty dz_2 z_2^{\nu_k + \frac{3}{2}} K_1(qz_2) J_{mR - \frac{1}{2}}(\omega_1 z_2) \]

\[ = \left(\frac{r_+}{R^2}\right)^{\nu_k + \frac{3}{2}} C_z(\nu_k^*, q) q^{-(mR + \frac{3}{2})} \]

(XI.139)

and \( C_z(\nu_k^*) = 2^{mR + \frac{3}{2}} \Gamma\left(mR + \frac{3}{2}\right) \).

Note that for the special value \( \nu_k = \nu_k^* = mR \), the integrand reduces to the one in (20), and can be evaluated exactly as

\[ I_z(\omega_1; q; k) = \left(\frac{r_+}{R^2}\right)^{\nu_k + \frac{3}{2}} \int_0^\infty dz_2 z_2^{\nu_k + \frac{3}{2}} K_1(qz_2) J_{mR - \frac{1}{2}}(\omega_1 z_2) \]

\[ = \left(\frac{r_+}{R^2}\right)^{\nu_k + \frac{3}{2}} C_z(\nu_k^*, q) q^{-(mR + \frac{3}{2})} \]

(XI.139)

and \( C_z(\nu_k^*) = 2^{mR + \frac{3}{2}} \Gamma\left(mR + \frac{3}{2}\right) \).

**XII. APPENDIX D: LOW-X STRUCTURE FUNCTIONS WITH RADIAL DIFFUSION**

Far from the black hole and including diffusion in the radial direction, the structure functions can be written as

\[ F_2(x_A, q^2) = \int_0^\infty \frac{d\omega^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{n_F(\omega, \vec{k})}{2E_k} F_2(x_k, q^2, \omega) \]
\[ F_1(x_A, q^2) = \int_0^\infty \frac{d\omega^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{n_F(\omega, \vec{k})}{2E_k} F_1(x_k, q^2, \omega), \]

(XII.140)
\[
F_2(x_k, q^2, \omega) = \frac{g_0^2 \rho^{3/2}}{32\pi^{5/2}} \int \frac{dz dz' z z'}{z' z} P^2_\lambda(z, q^2) P_\psi(z', \omega)(z z' q^2) e^{\zeta_k(1-\rho)} \frac{\Im \omega^2(\omega z')}{\sqrt{\zeta_k}} e^{-\frac{\Im \omega^2(\omega z')}{\zeta_k}} \frac{1}{\sqrt{\zeta_k}},
\]

(XII.141)

with

\[
\begin{align*}
P^2_\lambda(z, q^2) &= (q z)^2 (K_1^2(q z) + K_0^2(q z)) \\P^1_\lambda(z, q^2) &= (q z)^2 K_1^2(q z) \\P_\psi(z', \omega) &= z'^{-3} \times z'^5 J_{m-1/2}(\omega z').
\end{align*}
\]

(XII.142)

\[
\rho \equiv 2/\sqrt{\lambda}, \quad \zeta(z, z', \lambda, q^2, x_k) \equiv \log\left(\frac{z' q^2}{\sqrt{\lambda} x_k}\right), \quad \text{and} \quad g_0^2 \equiv \frac{\kappa^2}{N} = 4\pi^2/N^2.
\]

XIII. APPENDIX E: BLACK HOLE WITH

\[e^2 \tilde{\alpha} < \frac{1}{2}(mR)^2 \text{ OR } k_R^2 < 0\]

Near the black-hole the bulk fermions get modified in the infrared as illustrated in Fig. 2. The modification

\[
\Im \mathcal{G}_R^{11}(\omega_1, k + q) \Im \mathcal{G}_R^{11}(k^0, \tilde{k}) \rightarrow \text{Tr} \left( (\sigma_1 (k^0 + q^0) - i \sigma_2 (k_x + q_x) - \omega_1) \pi \delta((k + q)^2 + \omega_1^2) \times \Im \mathcal{G}_R^{11}(k^0, \tilde{k}) \right)
\]

(XIII.143)

Here \( \mathcal{G}_R^{11} = \mathcal{G}_R \text{ diag}(0, 1) \) is given in (XII.10) for small \( \omega \). Again note the emerging non-Fermi liquid scaling for \( \nu_k < \frac{1}{2} \) with the transition to a normal Fermi liquid for \( \omega \approx \omega_c \) as discussed earlier. Using the vertex for pure \( \text{AdS}_2 \) (XI.130), we can re-write (III.39) as

\[
\Im \mathcal{G}_{x_A}(q) = q^2 C_\theta (-1) \int_0^\infty \frac{d\omega^2}{2} \int dk k^2 C^2(\nu_k) I^2_\omega(\omega_1; q, k) \Re I_{k^0}(\omega_1, q, x_A),
\]

(XIII.144)

with \( C_\theta = 1/(12\pi^2) \), and the real part of \( I_{k^0} \) is
\[ \begin{align*}
\text{Re } I_{k^0}(\omega_1, q, x_A) &= \text{Re } \int_{-\infty}^{\infty} \frac{dk^0}{2\pi} \text{Tr} \left( G_F^{11}(\omega_1, k + q) G_F^{11}(k^0, \tilde{k}) \right) \\
&= \text{Re } \int_{-|q^0|}^{0} \frac{dk^0}{2\pi} \text{Tr} \left( (\sigma_1(k^0 + q^0) - i\sigma_2(k_x + q_x) - \omega_1)\pi \delta((k + q)^2 + \omega_1^2) \text{Im } G_F^{11}(k^0, \tilde{k}) \right) \\
&\approx \int_{-\omega_c}^{0} \frac{dk^0}{2\pi} \left(-1\right) \omega_1 \pi \delta((k + q)^2 + \omega_1^2) \text{Im } C(\tilde{k})(k^0)_{12}^2. 
\end{align*} \] (XIII.145)

By first doing the integral over \( \omega_1 \) in (XIII.144) and using the delta function in (XIII.145), we finally obtain

\[ \begin{align*}
\text{Im } \tilde{G}_F^{\nu}(q) &\approx \frac{C_6}{4} \int dk \, k^2 (\nu_k) C_2^2(\nu_k) q^2 I_2^2(\sqrt{s_k}; q, k) \sqrt{s_k} \text{Im } \left( \frac{-C(\tilde{k})(-\omega_c)^{2\nu_k + 1}}{2\nu_k + 1} \right), \\
&\approx \frac{C_6}{4} \left( \frac{1}{q^2} \right)^{\nu_k + \frac{3}{2}} \\
&\times \int dk \, k^2 \, C_2^2(\nu_k) \, C_2^2(\nu_k) \, q^2 \, I_2^2(\sqrt{s_k}; q, k, 1 - x_k)^{mR - 1/2} \, F_1(\frac{mR + \nu_k + 2}{2}, \frac{mR - \nu_k + 1}{2}, \frac{1}{2}, 1 - x_k) \\
&\times \sqrt{s_k} \text{Im } \left( \frac{-C(\tilde{k})(-\omega_c)^{2\nu_k + 1}}{2\nu_k + 1} \right), 
\end{align*} \] (XIII.146)

with \( k^0 \) fixed to \( \omega_c \) in \( s_k = -(k + q)^2 \). We have defined \( x_k = -q^2/2k \cdot q, x_A = q^2/2E_Aq_x, \) and made use of the DIS kinematics to approximate \( s_k \approx -q^2(1 - 1/x_k), |q^0| \approx q_x. \)
[15] T. Faulkner, H. Liu, J. McGreevy and D. Vegh, Phys. Rev. D 83, 125002 (2011) [arXiv:0907.2694 [hep-th]].
[16] S. S. Gubser and J. Ren, Phys. Rev. D 86, 046004 (2012) [arXiv:1204.6315 [hep-th]].
[17] K. A. Mamo and I. Zahed, [arXiv:1808.01952 [hep-ph]].
[18] L. L. Frankfurt and M. I. Strikman, Phys. Rept. 160, 235 (1988). R. P. Bickerstaff and A. W. Thomas, J. Phys. G 15, 1523 (1989); M. Arneodo, Phys. Rept. 240, 301 (1994).
[19] J. H. Gao and B. W. Xiao, Phys. Rev. D 81, 035008 (2010) [arXiv:0912.3333 [hep-ph]].
[20] J. Polchinski and M. J. Strassler, JHEP 0305, 012 (2003) [hep-th/0209211].
[21] N. Iqbal, H. Liu and M. Mezei, [arXiv:1110.3814 [hep-th]]; T. Faulkner, N. Iqbal, H. Liu, J. McGreevy and D. Vegh, Phys. Rev. D 88, 045016 (2013) [arXiv:1306.6396 [hep-th]].
[22] M. Rho, S. J. Sin and I. Zahed, Phys. Lett. B 466, 199 (1999) [hep-th/9907126]. R. A. Janik and R. B. Peschanski, Nucl. Phys. B 586, 163 (2000) [hep-th/0003059]; E. Shuryak and I. Zahed, Annals Phys. 396, 1 (2018) [arXiv:1707.01885 [hep-ph]].
[23] R. C. Brower, J. Polchinski, M. J. Strassler and C. I. Tan, JHEP 0712, 005 (2007) [hep-th/0603115]; A. Ballon-Bayona, R. Carcasses Quevedo, M. S. Costa and M. Djuric, Phys. Rev. D 93, 035005 (2016) [arXiv:1508.00008 [hep-ph]].
[24] R. C. Brower, M. Djuric, I. Sarcevic and C. I. Tan, JHEP 1011, 051 (2010) [arXiv:1007.2259 [hep-ph]].
[25] N. Kovensky, G. Michalski and M. Schvellinger, JHEP 1810, 084 (2018) [arXiv:1807.11540 [hep-th]].
[26] M. Edalati, J. I. Jottar and R. G. Leigh, JHEP 1004, 075 (2010) [arXiv:1001.0779 [hep-th]].
[27] L. Y. Hung and Y. Shang, Phys. Rev. D 83, 024029 (2011) doi:10.1103/PhysRevD.83.024029 [arXiv:1007.2653 [hep-th]].
[28] D. Jorrin, N. Kovensky and M. Schvellinger, JHEP 1604, 113 (2016) [arXiv:1601.01627 [hep-th]].