Black hole mining in the RST model

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Abstract
We consider the possibility of mining black holes in the 1 + 1-dimensional dilaton gravity model of Russo, Susskind and Thorlacius. The model correctly incorporates Hawking radiation and back-reaction in a semiclassical expansion in $1/N$, where $N$ is the number of matter species. It is shown that the lifetime of a perturbed black hole is independent of the addition of any extra apparatus when realized by an arbitrary positive energy matter source. We conclude that mining does not occur in the RST model and comment on the implications of this for the black hole information paradox.

Keywords: 2d black hole, mining, black hole evaporation, semiclassical model

(Some figures may appear in colour only in the online journal)

1. Introduction

Hawking showed that black holes radiate quantum mechanically and eventually evaporate [1]. It is interesting to wonder if there is a consistent manner in which one might perturb a black hole to cause it to evaporate more quickly. Unruh and Wald [2, 3] have argued that radiation from black holes can be mined rapidly, potentially diminishing the lifetime of a 4-dimensional Schwarzschild black hole of mass $M$ to a time of order $M$. The ore is the thermal atmosphere of the black hole near the horizon, populated by matter and radiation modes. The mining equipment considered in [2, 3] was a box capable of reflecting these modes, and being lowered and raised from a position very close to the horizon. The output of such a mining operation is a single Hawking particle with energy of order $kT_{\text{Hawking}}$ at infinity, with $T_{\text{Hawking}}$ the Hawking temperature of the black hole. As such the process is inherently quantum, and will not show up in the classical quasinormal modes, nor as a classical nonlinear instability. It may be viewed as a quantum version of the Penrose process, since it involves a nontrivial interaction happening...
near the horizon. At leading order, the effect is invisible in the two-point function (unlike the Hawking effect), and instead energy conservation requires a higher order correlator with the black hole energy changing.

The mining operation can be repeated at will, leading \[2, 3\] to conjecture a maximal rate of one Planck energy per unit time per unit area, leading to the endpoint in a time of order \( M \). The unperturbed lifetime on the other hand is of order \( M^3 \). Such a drastic drop raises thorny questions concerning the black hole information paradox, where apparently one needs at least a scrambling time of order \( M \log M \) before quantum information may be carried off by the Hawking radiation \[4–7\]. By considering optimal semiclassical solutions that accomplish mining \[8\], suggests that such a process may at most diminish the lifetime of a black hole to of order \( M^2 \), safely greater than the scrambling time, but parametrically shorter than the unperturbed lifetime.

In the present work our goal is not to test the idea of rapid mining, but simply to consider whether any mining is possible in a model where the question is fully tractable. The original model computation of mining presented in \[3\] was motivated by a two-dimensional moving mirror in flat spacetime. The models of two-dimensional dilaton gravity provide an ideal testing ground for this particular question, which are a natural generalization of the moving mirror model. In particular, the RST model \[9\] incorporates a reflecting boundary condition, as well as a full treatment of back-reaction of the Hawking radiation on the geometry at the semiclassical level. The field content of the theory is the same as the s-wave reduction of four-dimensional Einstein gravity, though the coefficients in the action are changed, and the matter couplings are simplified to yield a soluble model. The theory has nontrivial black holes and a thermal spectrum of Hawking radiation is emitted. Accelerated frames see thermal Unruh radiation, yielding the ore for the mining operation. Rather than using a reflecting box as the mining equipment, we use the field interactions already present in the model\(^1\), and ask whether a matter waveform scattering off the black hole can yield any mining whatsoever, by computing the dependence of the black hole lifetime on the incoming flux. Although there exists a literature on energy conservation in RST \[10, 11\], the connection with mining and the computation of the lifetime for general flux is new.

The special features of the RST model allow matter back-reaction to be included maintaining the two-derivative character of the action. As shown long ago (reviewed in \[12\]), attempts to do this directly in four dimensions give four-derivative actions which yield spurious unphysical solutions. So we view the present paper as filling an important gap in the literature, where the question of mining can be thoroughly addressed.

It should be noted that black hole mining is often described as the ability of the mining apparatus to untrap modes with nonvanishing angular momentum in the thermal atmosphere of the black hole, which otherwise would have to tunnel through a potential barrier. While this consideration is important for mining efficiency, we note the original work \[3\] yields a nontrivial contribution from mining in the s-wave alone. In this case there is no angular momentum barrier, but rather the mining process amounts to a direct detection of Unruh radiation as seen by the accelerating apparatus near the horizon, and its subsequent extraction to infinity. Even in the s-wave, this would yield an enhanced rate of energy extraction from the black hole, and the present paper tests this idea in a soluble model including back-reaction.

\(^1\) Interactions between the matter fields in the RST model arise from direct scattering off the boundary, as well as indirect interactions involving the gravitational fields through the constraint equations.
2. Review of semiclassical two-dimensional dilaton gravity coupled to matter in RST model

As our basic approach is to consider the effect of arbitrary flux on a black hole in the RST model, we review the relevant elements of that model here. A useful general review of these models is [13].

2.1. Action and equations of motion

The RST action can be thought of as a one loop quantum corrected version of the dilaton gravity action due to Callan, Giddings, Harvey and Strominger (CGHS) [14]:

\[
S = \frac{1}{2\pi} \int d^2x \sqrt{-g} \left[ e^{-2\phi}(R + 4(\nabla\phi)^2 + 4\lambda^2) - \frac{1}{2} \sum_{i=1}^{N} (\nabla f_i)^2 \right] \\
- \frac{\kappa}{8\pi} \int d^2x \sqrt{-g} R \frac{1}{\sqrt{2}} R + 2\phi R.
\]

The action for the RST model is given by:

\[
S = \frac{1}{2\pi} \int d^2x \left[ e^{-2\phi}(2\partial_{\pm} \rho - 4\partial_{\pm} \phi \partial_{\pm} \phi + \lambda^2 e^{2\phi}) \right] \\
+ \frac{1}{2} \sum_{i=1}^{N} \partial_{\pm} f_i \partial_{\pm} f_i - \frac{\kappa}{\pi} \left( \partial_{\pm} \rho \partial_{\pm} \rho + \phi \partial_{\pm} \partial_{\pm} \rho \right) \] \tag{1}

in conformal gauge where the only non-vanishing components of the metric are \( g_{-+} = g_{+-} = -\frac{1}{2} e^{2\phi} \). \( \phi \) is the dilaton and \( f_i \) are matter scalar fields. The constants \( \kappa = \frac{N\hbar}{12\pi} \) and \( \lambda \) play the roles of Planck’s constant and the cosmological constant in this model. We will generally set the latter to unity for convenience. We work in null Kruskal-type coordinates. The \( \kappa \) dependent terms in (1) represent the effect of integrating out the one-loop fluctuations of matter fields, as well as the RST improvement term, responsible for maintaining the solubility of the equations of motion. We note the constraint equations in this gauge couple the matter fields to the gravitational fields \( \rho \) and \( \phi \):

\[
0 = T_{\pm \pm} = \left( e^{-2\phi} - \frac{\kappa}{4} \right) (4\partial_{\pm} \rho \partial_{\pm} \phi - 2\partial_{\pm}^2 \phi) + \frac{1}{2} \sum_{i=1}^{N} \partial_{\pm} f_i \partial_{\pm} f_i \\
- \kappa (\partial_{\pm} \rho \partial_{\pm} \rho - \partial_{\pm}^2 \rho + t_{\pm}).
\]

The functions \( t_{\pm} \) are fixed by boundary conditions on the stress energy tensor. We identify the matter contribution to the stress tensor as:

\[
T_{\pm \pm}^{\pm} = \frac{1}{2} \sum_{i=1}^{N} \partial_{\pm} f_i \partial_{\pm} f_i.
\]

The following field redefinitions lead to especially simple equations of motion:

\[
\Omega = \frac{\sqrt{\kappa}}{2} \phi + \frac{e^{-2\phi}}{\sqrt{\kappa}} \\
\chi = \sqrt{\kappa} \rho - \frac{\sqrt{\kappa}}{2} \phi + \frac{e^{-2\phi}}{\sqrt{\kappa}}.
\]
The resulting action, equations of motion and constraints are:

\[
S = \frac{1}{\pi} \int d^2x \left[ -\partial_+ \chi \partial_- \chi + \partial_+ \Omega \partial_- \Omega + \lambda^2 e^{\frac{2}{\sqrt{\kappa}}(\chi - \Omega)} + \frac{1}{2} \sum_{i=1}^{N} \partial_+ f_i \partial_- f_i \right]
\]

\[
\partial_+ \partial_- \chi = \partial_+ \partial_- \Omega = -\frac{\lambda^2}{\sqrt{\kappa}} e^{\frac{2}{\sqrt{\kappa}}(\chi - \Omega)}
\]

\[
\kappa t_\pm = -\partial_\pm \chi \partial_\pm \chi + \sqrt{\kappa} \partial_\pm^2 \chi + \partial_\pm \Omega \partial_\pm \Omega + \frac{1}{2} \sum_{i=1}^{N} \partial_\pm f_i \partial_\pm f_i.
\]  

(2)

We further fix the gauge to Kruskal gauge where \( \chi = \Omega \).

2.2. On boundaries, singularities and the apparent horizon

In calculating the effect of flux on the black hole lifetime in RST, we will be making use of the boundary of the spacetime, the linear dilaton vacuum (LDV), and the positions of the black hole singularity and apparent horizon. We now review these concepts and their relations.

The LDV solution is the analog of higher dimensional flat spacetime and is given by

\[
\Omega = \frac{x^+ x^-}{\sqrt{\kappa}} - \frac{\sqrt{\kappa}}{4} \log(-x^+ x^-).
\]

(3)

The scalar curvature is given by:

\[
R = \frac{8[1 - (\nabla \phi)^2]}{\sqrt{\kappa} e^{2\phi} \Omega'(\phi)}.
\]

(4)

This will be generically singular at the critical value \( \phi = \phi_{cr} \) where \( d\Omega/d\phi = 0 \) which we denote by the curve \( \hat{x}^+ (x^-) \).

The apparent horizon is the boundary of a trapped region. The variable \( e^{-\phi} \) can be thought of as a analog of a transverse sphere in the dimensional reduction of four-dimensional Einstein gravity to two-dimensional dilaton gravity. The quantity \( \nabla \phi \) will therefore be timelike in a trapped region. The boundary is determined by the equation \((\nabla \phi)^2 = 0\) which becomes

\[
\partial_+ \Omega = 0.
\]

In both subcritical and supercritical regimes, there is a unique curve \( \phi = \phi_{cr} \). In the former case, this is the boundary of the LDV. In the latter case, it is the black hole singularity where \( (4) \) diverges. To maintain cosmic censorship, RST observe that finite curvature at the boundary requires imposing the cosmic censorship boundary condition

\[
\partial_\pm \Omega = 0.
\]

(5)

This implies reflective boundary conditions for the energy flux

\[
T_{-\pm} = \kappa T_\pm = \left( \frac{d\hat{x}^\pm}{dx^-} \right)^2 \left( T_{+\pm} - \kappa T_\pm \right).
\]

(6)

These boundary conditions, together with the thunderpop matching condition, described below, yield self-consistent solutions to the semiclassical equations. The boundary conditions at \( \phi = \phi_{cr} \) can be generalized straightforwardly to represent more complicated objects sitting at the origin [15, 16], but even then one must retain the thunderpop
5 matching condition for black hole solutions. In the present work we restrict our analysis to the boundary conditions (6).

3. Generalizing RST to arbitrary matter flux

We review here the reflection of subcritical flux and black hole formation from supercritical flux. These cases constitute the general solutions to RST for arbitrary matter flux in the strictly subcritical and supercritical regimes. We then consider a general black hole solution with general ingoing subcritical flux.

3.1. Review of subcritical and supercritical cases

A subcritical flux obeys the following inequality [17]

\[ T_{++}^f < \frac{\kappa}{4(x^+)^2}. \]

When the flux always obeys this condition the boundary remains timelike and a black hole does not form. Such flux can be made to reflect from the boundary of the spacetime given appropriate boundary conditions as shown in figure 1. The opposite inequality will imply that the boundary curve becomes a spacelike singularity, which is identified as the black hole singularity. We refer to this latter case, where black hole formation occurs, as ‘supercritical’ (see figure 2).
For illustrative purposes, we review the manner in which purely subcritical flux is reflected in RST. In the subsequent section the same procedure will be used to reflect flux outside the event horizon of a black hole. We will assume the ingoing subcritical flux is localized in some region \( x^+ < x^- < x^+ \). The Bondi energy \( M_+(x^+) \) and the Kruskal momentum \( P_+(x^+) \) are defined as integrals along \( \mathcal{I}^- \):

\[
P_+(x^+) = \int_{x^-}^{x^+} T_{++}^f \, dx^+
\]

\[
M_+(x^+) = \int_{x^-}^{x^+} T_{++}^f x^+ \, dx^+.
\]

In fact, only these two integrals are needed since in our choice of gauge the flux is related to the double derivative of \( \Omega \):

\[
-\sqrt{\kappa} \partial_{\pm}^2 \Omega = -\kappa \tau_{\pm} + \frac{1}{2} \sum_{i=1}^{N} \partial_{\pm} f_i \partial_{\pm} f_i.
\]

In the case of subcritical flux, one can trace out the reflection of the flux from the boundary as shown in figure 1. There are three regions of interest separated by lines of constant \( x^- \), namely \( x^- = x_i^- \) and \( x^+ = x_i^+ \), defined via \( x_i^+ = \hat{x}^+(x_i^-) \) and \( x_i^- = \hat{x}^-(x_i^+) \).

The first region contains only infalling flux:

\[
\Omega^{(i)}(x^- < x_i^-) = -\frac{x^+(x^- + P_+(x^+))}{\sqrt{\kappa}} - \frac{\sqrt{\kappa}}{4} \log(-x^+ x^-) + \frac{M_+(x^+)}{\sqrt{\kappa}}.
\]
The function \( t_+ = \frac{1}{4\kappa x^+} \) is determined by demanding the solution approach the linear dilaton vacuum as \( x^- \to -\infty \). The second region corresponds to the reflection of flux, with (6) determining the outgoing flux \( T_- = T^I_- - \kappa t_- \):

\[
\Omega^{(i)}(x^- < x^- < x^- f) = -\frac{x^+ (x^- + P_+(x^+))}{\sqrt{\kappa}} - \frac{\sqrt{\kappa}}{4} \log(-x^+ x^-) + M_+(x^+) + F(x^-)
\]

\[\quad F(x^-) = \frac{\sqrt{\kappa}}{4} \log(-x^- x^+) - \frac{M(x^+)}{\sqrt{\kappa}} - \frac{\sqrt{\kappa}}{4} \log(\frac{\kappa}{4}).\]

The function \( F(x^-) \) and the shape of the boundary curve \( \hat{x}^+(x^-) \) is determined from the finite curvature conditions (5). The third region is the empty linear dilaton vacuum (where we denote the constants \( P^I_+ \equiv P(x^+_f) \) and \( M^I_+ \equiv M(x^+_f) \):)

\[
\Omega^{(iii)}(x^- > x^- f) = -\frac{x^+ (x^- + P^I_+)}{\sqrt{\kappa}} - \frac{\sqrt{\kappa}}{4} \log(-x^+ (x^- + P^I_+)).
\]

Having reviewed the manner in which subcritical flux is reflected with RST boundary conditions, as well as how black hole formation occurs, we now turn to an analysis of a general black hole solution with additional subcritical flux probing the black hole.

### 3.2. General black hole solution and thunderpop

The endpoint of Hawking radiation in RST results in a well known emission of energy referred to as the ‘thunderpop’ [17]. This renders the model nonlinearly stable by patching the solution across a mild Planck energy outgoing null shock with the linear dilaton vacuum. We take the viewpoint that the thunderpop is to be viewed as a condition to be imposed on the low energy theory to make it semiclassically consistent, representing the effect of some higher derivative interactions that descend from some unknown ultraviolet complete theory. This allows us to avoid the issues raised in [13] where the thunderpop was interpreted as a squeezed state in the semiclassical theory that becomes problematic when evolved back in time. The goal of the present section is to compute the general position of the thunderpop \( (x^+_{\text{pop}}, x^-_{\text{pop}}) \).

We will take a LDV solution with additional flux sent in from some interval in \( x^+ \) on \( \mathcal{I}^- \) corresponding to \( x^+_{\text{(front)}} < x^+ < x^+_{\text{(behind)}} \leq x^+_{\text{pop}} \) (see figure 3). We will assume the flux turns supercritical solely in a sub-interval \( x^+_{\text{(super)}} < x^+ < x^+_{\text{(super)}} \) (see figure 3). The consequent general black hole solution is given by:

\[
\Omega^{(i)}(x^+ > x^+_{\text{(super)})} = -\frac{x^- + P^I_+}{\sqrt{\kappa}} - \frac{\sqrt{\kappa}}{4} \log(-x^+ x^-) + \Omega^{(i)}_{\text{(front)}} (x^+) + \Omega_{\text{(behind)}} (x^+)
\]

\[\quad + \Omega_{\text{(super)}} (x^+),\]

where the total flux contribution has been separated into separate physically significant pieces. In general, the effect of each piece of flux is of the form:

\[
\delta \Omega (x^+) = -\frac{x^+}{\sqrt{\kappa}} \delta P_+ + \frac{\delta M_+}{\sqrt{\kappa}},
\]
where we are permitted to break the contributions into pieces and linearly sum since integrals over some interval can be split and summed over corresponding sub-intervals.

We include a general supercritical contribution $\Omega^{(\text{super})}$ which forms the black hole and we also separate the subcritical flux which is reflected outside the event horizon $\Omega^{(\text{front})}$ from the portion that passes behind $\Omega^{(\text{behind})}$ and propagates to the singularity. As the thunderpop corresponds to a null line crossing the endpoint of black hole evaporation, it can also be taken to define the event horizon (see figures 2 and 3). We will assume that no flux is sent in after the black hole has evaporated. Further, we will assume that the flux outside the event horizon is reflected in a manner that follows the RST prescription, hence our usage of the superscript (i) on $\Omega^{(\text{front})}$.

The reflected solution $\Omega^{(\text{ii})}$ outside the horizon is given by applying the RST boundary conditions. In the causal past of the thunderpop we have:

$$
\begin{align*}
\Omega^{(\text{ii})} &= -\frac{x^+(x^- + P_{(\text{front})}(x^+))}{\sqrt{\kappa}} - \frac{\sqrt{\kappa}}{4} \log(-x^+ x^-) \\
&\quad + \frac{M_{(\text{front})}(x^+)}{\sqrt{\kappa}} + F_{(\text{front})}(x^-) + \Omega_{\text{BH}}(x^+),
\end{align*}
$$

where we define:

Figure 3. General flux probing general black hole. Subcritical fluxes which remain outside and pass inside the black hole are shown in orange and purple respectively. Supercritical flux is shown in red, the event horizon is shown in blue, and the apparent horizon is shown in turquoise. Regions (i) ($x^- < x_f^-$), (ii) ($x_f^- < x^- < x_{\text{pop}}$) and (iii) ($x_{\text{pop}}^- < x^-$) are the natural generalizations from the RST prescription for subcritical reflected flux.
\(\Omega_{\text{BH}} \equiv \Omega_{(\text{behind})} + \Omega_{(\text{super})} \).

Evaluating \(\Omega^{(ii)}\) in the region to be patched to the evaporated LDV: \(x^+ > x_{\text{pop}}^+\) as well as along the thunderpop itself: \(x^- = x_{\text{pop}}^-\) results in the following simplified form for \(\Omega^{(ii)}\):

\[
\Omega^{(ii)}(x^+ > x_{\text{pop}}^+, x^- = x_{\text{pop}}^-) = -\frac{x^+(x_{\text{pop}}^+ + p_{\text{front}}^f)}{\sqrt{\kappa}} - \frac{\sqrt{\kappa}}{4} \log(-x^+(x_{\text{pop}}^- + p_{\text{front}}^f)) + \Omega_{\text{BH}}(x^+),
\]

which can be easily patched to an appropriately shifted LDV (3)

\[
\Omega^{(iii)} = -\frac{x^+(x_{\text{pop}}^- + p_{\text{front}}^f + p_{\text{behind}}^f + p_{\text{super}}^f)}{\sqrt{\kappa}} - \frac{\sqrt{\kappa}}{4} \log(-x^+(x_{\text{pop}}^- + p_{\text{front}}^f + p_{\text{behind}}^f + p_{\text{super}}^f)).
\]

The continuity of such a patching entails the following form of \(x_{\text{pop}}^-\):

\[
x_{\text{pop}}^- = -p_{\text{front}}^f + p_{\text{behind}}^f + p_{\text{super}}^f \frac{\kappa}{\kappa - 1} e^{-\kappa (M_{\text{front}}^f + M_{\text{behind}}^f + M_{\text{super}}^f)/\kappa}.
\]

We note this expression depends explicitly on the mining equipment, in the form of the flux in front of the horizon. Using the position for the apparent horizon \(\partial_+ \Omega = 0\) on (8) and noting no additional flux is sent in near the endpoint, we obtain

\[
x_{\text{pop}}^+ = -\frac{\kappa}{4} \left( x_{\text{pop}}^- + p_{\text{front}}^f + p_{\text{behind}}^f + p_{\text{super}}^f \right) \frac{e^{-\kappa (M_{\text{super}}^f + M_{\text{behind}}^f)/\kappa} - 1}{4 (p_{\text{super}}^f + p_{\text{behind}}^f)}.
\]

The determination of this general thunderpop position is the first main technical result of this paper.

### 4. Measuring lifetime with the event horizon

The black hole region can be defined as the complement of the causal past of \(\mathcal{I}^+\) [18], which is bounded in part by the event horizon. We observe that the event horizon ought to be defined by a null line which intersects the thunderpop and is therefore given in the region \(x^+ < x_{\text{pop}}^+\) by the thunderpop coordinate \(x_{\text{EH}} = x_{\text{pop}}^-(9)\). The endpoint of evaporation will be given by the thunderpop as both the event horizon and the apparent horizon meet at this point (see figures 2 and 3). The initial point of black hole formation will be the projection of the thunderpop coordinate onto the boundary curve, \(x_{\text{BH}}^+ = x_{\text{pop}}^+(\sigma^+)\) as this corresponds to the edge of the event horizon. We further project these boundary points of the event horizon onto \(\mathcal{I}^\pm\), enabling us to measure the lifetime using the asymptotically flat light-cone coordinate \(\sigma^+\):

\[
x^+ = e^{\sigma^+}.
\]

To determine the lifetime of the black hole, we need to determine where the null line of the event horizon intersects the timelike boundary curve where the RST boundary conditions are applied (5). An equation governing the boundary curve can be obtained by solving \(\partial_+ \Omega = 0\) on (7)
\[
\hat{x}^+ = -\frac{\kappa}{4(\hat{x}^- + p^{+\text{(front)}}(\hat{x}^+))}.
\]

Substituting in for \(x_{\text{EH}}\) gives
\[
x_{\text{EH}}^+ = \frac{\kappa}{4} \frac{1 - e^{-4(M^f_{+\text{(super)}} + M^f_{+\text{(behind)}})/\kappa}}{p^f_{+\text{(behind)}} + p^f_{+\text{(super)}}}.
\]

(12)

Converting (10) and (12) to the asymptotically flat coordinate (11), one arrives at the following:
\[
\Delta \sigma^+ = \log \left( \frac{e^{4(M^f_{+\text{(super)}} + M^f_{+\text{(behind)}})/\kappa} - 1}{1 - e^{-4(M^f_{+\text{(super)}} + M^f_{+\text{(behind)}})/\kappa}} \right) = \frac{4M_{\text{tot}}}{\kappa} > \frac{4M^f_{+\text{(super)}}}{\kappa},
\]

where the mass of the black hole is defined as
\[
M_{\text{tot}} = M^f_{+\text{(super)}} + M^f_{+\text{(behind)}},
\]

which implies that mining does not occur. The determination of this general lifetime is the second main technical result of this paper. The expression is monotonically increasing in the mass and grows arbitrarily large in the classical limit of small \(\kappa\). In particular, the lifetime is clearly larger than in the case where probing subcritical flux is absent. Moreover, it is manifest that this lifetime is immune to any flux which remains outside the event horizon and in fact grows in precisely the way one would expect for flux which passes behind the horizon. This shows that in a model where back-reaction is treated in a self-consistent manner, mining does not occur.

5. Conclusions

The issue of mining a black hole has been considered for the first time in a quantum model with back-reaction incorporated at leading order in a \(1/N\) expansion. This provides a useful generalization of the moving mirror model that appearing in the original paper by Unruh and Wald [3], which was used to justify the proposal that energy could be mined from Schwarzschild black holes. We conclude that with quantum back-reaction properly incorporated, no mining takes place in the model we consider.

The general philosophy of this kind of computation is that it correctly captures the qualitative physics of an s-wave reduction of four-dimensional Einstein gravity in the semiclassical limit. By modifying the model, adding in some hopefully harmless extra terms, it can be rendered exactly soluble. This then provides motivation to reconsider the mining proposal of Unruh and Wald in the context of such soluble models.

A key feature of the mining proposal is that one should be able to mine modes irrespective of their angular momentum. This is crucial in order to get rapid mining of the type further explored in a later essay by Unruh and Wald [2]. It would be interesting to generalize the considerations of the present paper to this case. A useful soluble model for this, which includes an analog of the potential barrier for such modes is the two-dimensional charged black hole, which has a soluble version. It is also interesting to generalize the cosmic censorship RST boundary conditions considered in the present work to more general energy conserving boundary conditions of the type studied in later dilaton gravity papers [16, 17, 19].

The original proposal of Unruh and Wald suggested that black holes can be mined at a rate of a Planck energy per unit Planck time. This would allow a rapidly mined black hole to
evaporate in a light crossing time. Such a rapid evaporation poses severe problems for extracting quantum information from black holes, in holographic approaches to quantum gravity. Subsequent work incorporating classical back-reaction led to the minimum mining lifetime being walked back to a time of $O(M^2)$ [8]. The conclusions of the present work support the idea that mining of Schwarzschild black holes cannot occur once quantum back-reaction is incorporated, potentially strengthening the conclusions of Brown.

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