Phenomenological constraints on light mixed sneutrino dark matter scenarios

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Abstract

In supersymmetric models with Dirac neutrinos, the lightest sneutrino can be a good thermal dark matter candidate when the soft sneutrino trilinear parameter is large. In this paper, we focus on scenarios where the mass of the mixed sneutrino LSP is of the order of GeV so the sneutrino dark matter is still viable complying with the limits by current and near future direct detection experiments. We investigate phenomenological constraints in the parameter space of the models, as well as the vacuum stability bound. Finally, we show that the allowed regions can be explored by measuring Higgs boson properties at future collider experiments.

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I. INTRODUCTION

On July 4th, 2012, the ATLAS and CMS collaborations of the CERN Large Hadron Collider (LHC) announced the discovery of a new particle with a mass of 125 GeV [1]. The spin and parity properties of the new particle as well as its couplings to Standard Model (SM) particles have been investigated, and proven to be consistent with the prediction of the SM. The SM has been established as a low energy effective theory that explains phenomena at energy scales below $\mathcal{O}(100)$ GeV.

Although the SM is extraordinarily successful, there are still unresolved problems. The observation of neutrino oscillations reveals that neutrinos must have finite masses and contradicts the SM, where the neutrinos are massless [2]. Cosmological observations precisely determine the energy density of dark matter (DM) in the universe while there is no candidate particle that can fulfill the dark matter abundance in the SM [3, 4]. From the theoretical viewpoint, in order to explain the observed Higgs boson mass in the framework of the SM an unnaturally huge fine-tuning between its bare mass squared and contributions from radiative corrections is required. We are obliged to construct a more fundamental theory beyond the SM to tackle these difficulties.

The problems mentioned above are solvable in supersymmetric (SUSY) extensions with right-handed neutrino chiral supermultiplets [5–20]. The couplings of the right-handed neutrinos to the left-handed counterparts provide a source of the observed neutrino masses, which are either Dirac- or Majorana-type. The hierarchy problem is avoided by introducing SUSY: The quadratically divergent SM contributions to the Higgs boson mass squared are canceled out by those from diagrams involving superparticles whose spins differ from their SM counterparts by half a unit. It is intriguing that a viable candidate for dark matter other than conventional ones is automatically introduced as a by-product in this framework: The lightest sneutrino which is mainly m of the right-handed component. When such a sneutrino is the lightest SUSY particle (LSP), the observed dark matter abundance can be explained while satisfying other experimental constraints, in sharp contrast to left-handed sneutrino LSP scenarios which are excluded by the data of direct detection of dark matter. In particular, SUSY scenarios with Dirac neutrinos and large SUSY breaking sneutrino trilinear parameters can provide a viable left-right mixed sneutrino dark matter candidate [8, 10, 11, 14, 15, 18, 20]. Sneutrino trilinear parameters of the order of other soft SUSY
breaking masses can be naturally realized in models where \( F \)-term SUSY breaking is responsible for the smallness of the neutrino Yukawa couplings and induce large mixings between the left- and right-handed sneutrino states \cite{6}. Due to the large sneutrino trilinear coupling, the lightest mixed sneutrino behaves as a weakly interacting massive particle (WIMP) and its thermal relic abundance falls in the cosmological dark matter abundance. So far, such mixed sneutrino WIMP scenarios have been screened in the light of experimental results. If the mixed sneutrino mass is of the order of 100 GeV, its thermal relic abundance can account for the observed dark matter abundance without contradicting experimental constraints. On the other hand, when the mass of the mixed sneutrino is smaller than half the mass of the discovered Higgs boson, its invisible decay rate is significantly enhanced. It has been shown that such a light sneutrino dark matter scenario is excluded in the light of the LHC results if the gaugino mass universality is imposed \cite{18}.

In this paper, we explore the GeV-mass mixed sneutrino scenarios without gaugino mass universality. We show that when the lightest neutralino mass is of the order of the mixed sneutrino mass, the thermal relic abundance of the mixed sneutrino coincides with the observed dark matter abundance. It should be emphasized that the large sneutrino trilinear coupling makes our vacuum unstable. However, the vacuum stability bound in light mixed sneutrino WIMP scenarios has been neglected in earlier works. We compute the transition rate of our vacuum to a deeper one, and show that the vacuum stability bound is not severe. Although experimental constraints are very tight, there are some regions where mixed sneutrino WIMP scenarios are viable. We show that dark matter allowed regions can be examined by precisely measuring the invisible decay rate of the observed Higgs boson at future linear colliders.

The organization of this paper is as follows: In Sec.II, the model of the mixed sneutrino dark matter is briefly reviewed. Experimental constraints on the model are summarized in Sec.III. In Sec.IV, the vacuum stability bound on our model is discussed. Sec.V is devoted to a summary.

II. MODEL

Here, we briefly review the mixed sneutrino model with lepton number conservation, which is proposed in \cite{6}. In this model, in addition to the usual matter content of the Mini-
mal Supersymmetric Standard Model (MSSM), three generations of right-handed neutrinos $\nu_{Ri}$ (sneutrinos $\tilde{\nu}_{Ri}$) are introduced. Here, $i = 1, 2, 3$ denote the generation. As a result, Dirac neutrino Yukawa interactions, soft right-handed sneutrino mass terms and soft trilinear couplings among the left-handed slepton doublet $\tilde{\ell}_i, \tilde{\nu}_{Ri}$ and the Higgs doublet with hypercharge $Y = 1/2, h_u$, which gives mass to the up-type quarks and Dirac neutrinos are added to the usual MSSM Lagrangian. The newly introduced soft terms are given by

$$\Delta L_{\text{soft}} = m_{\tilde{N}_i}^2 |\tilde{\nu}_{Ri}|^2 + A_{\tilde{\nu}_i} \tilde{\nu}_i \tilde{\nu}_{Ri}^* h_u + \text{h.c.},$$

where $m_{\tilde{N}_i}^2$ are soft right-handed sneutrino mass parameters, and $A_{\tilde{\nu}_i}$ are trilinear sneutrino $A$-parameters. In order to avoid lepton flavor violation, we have assumed that these soft parameters are diagonal in generation space. Majorana neutrino mass terms and corresponding right-handed sneutrino bilinear terms are prohibited due to lepton number conservation.

Neglecting the contribution from the Dirac neutrino masses, the sneutrino mass matrix for one generation is written as

$$M_{\tilde{\nu}}^2 = \begin{pmatrix} m_{\tilde{\nu}_L}^2 + \frac{1}{2} m_{\tilde{\nu}_Z}^2 \cos 2\beta & \frac{1}{\sqrt{2}} A_{\tilde{\nu}} v \sin \beta \\ \frac{1}{\sqrt{2}} A_{\tilde{\nu}} v \sin \beta & m_{\tilde{N}_i}^2 \end{pmatrix} ,$$

where $m_{\tilde{\nu}_L}^2$ is the soft mass parameter for the left-handed slepton doublet. The sum of the squares of the vacuum expectation values and the ratio of the vacuum expectation values are given by $v^2 = v_1^2 + v_2^2 = (246 \text{ GeV})^2$ and $\tan \beta = v_2 / v_1$, respectively. Here, $v_1$ ($v_2$) is the vacuum expectation value of the Higgs doublet with hypercharge $Y = -1/2$ ($Y = 1/2$). In this model, the $A_{\tilde{\nu}}$ is not suppressed by the smallness of the corresponding neutrino Yukawa coupling, but is of the order of other soft parameters. This large $A_{\tilde{\nu}}$ parameter gives a large mixing between the left-handed and right-handed sneutrinos,

$$\tilde{\nu}_1 = \cos \theta_{\tilde{\nu}} \tilde{\nu}_R - \sin \theta_{\tilde{\nu}} \tilde{\nu}_L , \quad \tilde{\nu}_2 = \sin \theta_{\tilde{\nu}} \tilde{\nu}_R + \cos \theta_{\tilde{\nu}} \tilde{\nu}_L ,$$

with $m_{\tilde{\nu}_1} < m_{\tilde{\nu}_2}$, and the sneutrino mixing angle $\theta_{\tilde{\nu}}$ is given by

$$\sin 2\theta_{\tilde{\nu}} = \left( \frac{\sqrt{2} A_{\tilde{\nu}} v \sin \beta}{m_{\tilde{\nu}_2}^2 - m_{\tilde{\nu}_1}^2} \right) .$$
It should be emphasized that the couplings of the lighter sneutrino to the $Z$-boson, the Higgs boson and neutralinos are suppressed by a power of the small mixing angle $\theta$, compared to those of the MSSM left-handed sneutrinos. The smallness of the sneutrino interactions plays an important role in satisfying experimental constraints as discussed in the next section. The Feynman rules for such sneutrino interactions are given by

$$Z^\mu \tilde{\nu}_1^\tau (p') \tilde{\nu}_1 (p) : -ie \frac{\sin^2 \theta}{\sin 2\theta_W} (p + p')^\mu \sin^2 \theta_\nu,$$

$$h \tilde{\nu}_1^\tau \tilde{\nu}_1 : iem_Z \frac{\sin (\alpha + \beta)}{\sin 2\theta_W} \sin^2 \theta_\nu + i \frac{2 \cos \alpha}{v \sin \beta} \sin^2 \theta_\nu \cos^2 \theta_\nu (m^2_{\tilde{\nu}_2} - m^2_{\tilde{\nu}_1}),$$

$$\tilde{\nu}_1^\tau \chi_0^i : \frac{-ig}{2\sqrt{2} \sin 2\theta_W} (\cos \theta_W N_{i2} - \sin \theta_W N_{i1}) \sin \theta_\nu (1 - \gamma_5),$$

where $e$ is the electric charge, $g$ the $SU(2)_L$ coupling constant, $m_Z$ the $Z$-boson mass and $\theta_W$ the Weinberg angle. As for SUSY parameters, $\alpha$ is the Higgs mixing angle, and the matrix $N_{ij}$ diagonalizes the neutralino mass matrix.

In the rest of this paper, for simplicity, we focus on the cases where the lighter of the tau sneutrinos is a GeV-mass thermal WIMP candidate. We assume that the lighter sneutrinos of the first two generations are too heavy to affect experimental constraints on such GeV-mass tau sneutrino WIMP scenarios.

III. EXPERIMENTAL CONSTRAINTS

Thermal WIMP candidates have been extensively tested through many experiments. In particular, if the WIMP is lighter than half of the mass of the Higgs boson and interacts with the Higgs boson, such light WIMP models can be probed also through searches for the invisible decay of the Higgs boson. We list relevant experimental constraints imposed on light tau sneutrino WIMP scenarios in Table I and comment on the constraints below.

In general, dark matter candidates must be consistent with the upper limit of the dark matter relic density $\Omega$. In our model, if the mass of the sneutrino WIMP is less than 10 GeV, sneutrinos tend to annihilate into neutrinos via neutralino exchange. For $|M_B| \ll |M_W| \simeq |\mu|$, the lightest neutralino is bino-like, and the thermal average of the sneutrino
TABLE I: Observables and experimental constraints.

| Observable | Experimental result |
|------------|---------------------|
| $\Omega h^2$ | $0.1196 \pm 0.0062$ (95% CL) [4] |
| $\sigma_{SI}^N$ | $(m_{DM}, \sigma_{SI}^N)$ constraints from LUX [21] and SuperCDMS [22] |
| $\sigma_{\text{ann}}^v$ | $(m_{DM}, \sigma_{\text{ann}}^v)$ constraint from FermiLAT [23] |
| $\Delta \Gamma(Z \to \text{inv.})$ | $< 2.0$ MeV (95% CL) [24] |
| $\text{Br}(h \to \text{inv.})$ | $< 0.29$ (95% CL) [25] |
| $m_{\tilde{\tau}_R}$ | $> 90.6$ GeV (95% CL) [26] |
| $m_{\tilde{\chi}_1^\pm}$ | $> 420$ GeV (95% CL) [26] |
| $m_{\tilde{g}}$ | $> 1.4$ TeV (95% CL) [27, 28] |

annihilation cross section is given by

$$\langle \sigma_{\text{ann}}^v \rangle = \frac{\pi \alpha^2_{\text{em}} \sin^4 \theta_\tilde{\nu} \sin^4 \theta_W \cos^4 \theta_W m_{\tilde{\chi}_1^0}^2}{256 \pi^2 \sin^4 \theta_W \cos^4 \theta_W m_{\tilde{\chi}_1^0}^2} \left(1 - \frac{m_{\tilde{\chi}_1^0}^2}{m_{\tilde{\chi}_1^0}^2}\right)^2. \quad (6)$$

The resulting thermal relic abundance of the sneutrino is approximately

$$\Omega h^2 \sim 0.1 \times \left(\frac{\sin \theta_\tilde{\nu}}{0.1}\right)^{-4} \left(\frac{m_{\tilde{\chi}_1^0}}{1 \text{ GeV}}\right)^2. \quad (7)$$

Therefore, when the sneutrino mixing angle is as small as 0.1, the relic abundance constraint requires the mass of bino-like neutralino to be as small as $\mathcal{O}(1)$ GeV. From this observation, we concentrate on the cases where both the lightest tau sneutrino mass and the bino-like neutralino mass are of the order of GeV. Such a possibility has been overlooked in earlier works.

Next, let us discuss constraints from direct detection of dark matter. For GeV-mass dark matter, the spin-independent scattering cross section is limited by the LUX and the SuperCDMS experiments [21, 22]. In our model, the scattering of sneutrinos on nucleons occurs spin-independently via $Z$-boson or Higgs boson exchange. Since the $Z$-boson coupling to the sneutrino dark matter candidate is suppressed by the square of the small mixing angle $\theta_\tilde{\nu}$ compared to that to the MSSM left-handed sneutrino, the resulting scattering cross section falls below its experimental limit. On the other hand, the coupling of the Higgs boson to the sneutrino is proportional to the large $A$-term. In the nucleon scattering cross section, the
ratio of the Higgs boson exchange contribution to the $Z$-boson counterpart is proportional to $m_{\tilde{\nu}_1}^{-2}$. Actually, the amplitude of the scattering via the Higgs boson is dominant over the one via the $Z$-boson for $m_{\tilde{\nu}_1} \sim \mathcal{O}(1) \text{ GeV}$ \cite{14}. The cross section of the scattering of the dark matter and nucleon is given by:

$$
\sigma_{SN}^{SI} = \frac{4\mu_\chi}{\pi} \frac{(Zf_p + (A - Z)f_n)^2}{A^2},
$$

(8)

where $\mu_\chi$ is the sneutrino-nucleon reduced mass, $A$ is the mass number, $Z$ is the atomic number and $f_p$ ($f_n$) is the amplitude for the proton (neutron).

As for indirect detection of dark matter, we impose the bound obtained by the FermiLAT experiment on the annihilation cross section of the sneutrino dark matter \cite{23}. In our model, however, we have found that the constraint by the indirect detection is not serious for GeV-mass sneutrino dark matter.

Let us turn to constraints from collider experiments. The upper bound of the invisible decay of the $Z$-boson is obtained at the LEP \cite{24}:

$$
\Delta \Gamma(Z \to \text{inv.}) < 2.0 \text{ MeV (95\% CL)}. \quad (9)
$$

In our model, the $Z$-boson tends to decay invisibly to a lighter mixed sneutrino pair or a lightest neutralino pair. The invisible decay width of the $Z$-boson to a pair of sneutrinos is proportional to the sneutrino mixing angle:

$$
\Gamma(Z \to \tilde{\nu}_1^* \tilde{\nu}_1) = \Gamma(Z \to \tilde{\nu}\tilde{\nu}) \sin^4 \theta_\nu \left(1 - \frac{4m_{\tilde{\nu}_2}^2}{m_Z^2}\right)^{3/2},
$$

(10)

where $\Gamma(Z \to \tilde{\nu}\tilde{\nu})$ denotes the decay width of $Z$ boson to a pair of neutrinos:

$$
\Gamma(Z \to \tilde{\nu}\tilde{\nu}) = \frac{g^2}{96\pi \cos^2 \theta_W} m_Z = 167 \text{ MeV}. \quad (11)
$$

Therefore, the sneutrino mixing angle is constrained by the result on the $Z$-boson invisible decay width.

Let us discuss experimental constraints on the Higgs boson invisible decay. The branching ratio of the Higgs invisible decay is constrained directly through the searches for $Zh \rightarrow ll + E_T^{\text{miss}}$ \cite{29, 30}, and indirectly by the best-fit analysis using the combination of all channels.
of the Higgs boson decay \cite{25}. Here, we employ the results of the best-fit constraint. In our model, the decay width of the Higgs boson to a pair of the lighter mixed sneutrino is proportional to the sneutrino mixing angle $\sin^4 \theta_{\tilde{\nu}}$:

$$
\Gamma(h \rightarrow \tilde{\nu}_1 \tilde{\nu}_1^*) = \frac{\sin^4 \theta_{\tilde{\nu}}}{16\pi m_h} \sqrt{1 - \frac{4m_{\tilde{\nu}}^2}{m_h^2}} \left| em_Z \frac{\sin(\alpha + \beta)}{\sin 2\theta_W} + \frac{2\cos \alpha}{v \sin \beta} \cos^2 \theta_{\tilde{\nu}}(m_{\tilde{\nu}_2}^2 - m_{\tilde{\nu}_1}^2) \right|^2. \tag{12}
$$

The Higgs boson can decay invisibly also to the lightest neutralino. Such a decay mode is associated with the higgsino component of the lightest neutralino. When the $\mu$-parameter is much larger than the bino mass, the contribution from this invisible decay mode to the Higgs invisible decay is much smaller than the sneutrino pair channel.

We mention experimental constraints on the masses of electroweak superparticles. The pair production of sparticles is searched for at the LEP, and the null results constrain the masses of the right-handed sleptons, and the lightest chargino as shown in \cite{31}. The LHC experiments also search for the pair productions of the sleptons and the charginos \cite{26, 32, 33}. Such pair productions are characterized by the signals for two leptons. In addition, the searches for the pair production of the lightest chargino and the next-to-lightest neutralino impose the chargino mass limit more strongly than the results of the chargino pair production. In the MSSM, the signal of the lightest chargino (next-to-lightest neutralino) is characterized by a lepton (two leptons). Therefore, the chargino neutralino pair production is associated with the signal of three leptons. In our tau sneutrino WIMP model, the lightest chargino dominantly decays to a tau with missing energy. The modes containing two taus account for half of the next-to-lightest neutralino decay width, and most of the other half is converted to missing energy without a charged track. We use the constraints on the lightest chargino mass by the searches for two or three taus. In this scenario, the mass of the lightest neutralino is close to that of the LSP, and thus the lightest neutralino is long-lived and produces displaced vertices in detectors. Since the lightest neutralino decays exclusively into a tau sneutrino and a tau neutrino, signatures of the displaced vertices are invisible. The search for the strong production of sparticles in multi-$b$-jets final states constrains the gluino mass \cite{27, 28}.

Finally, we comment on mono-photon searches at the LEP2 and LHC experiments. The LEP2 limit $\sigma(e^+e^- \rightarrow \gamma + \text{inv.}) < 15 \text{ pb}$ \cite{34} does not place severe constraints on the GeV-mass mixed sneutrino WIMP scenario \cite{14}. In our model, the processes $q\bar{q} \rightarrow \gamma \tilde{\nu}_1 \tilde{\nu}_1^*$ and
\[ q\bar{q} \rightarrow \gamma \tilde{\nu}_L\tilde{\nu}_R \] mediated by the Z-boson give rise to events with a mono-photon and missing transverse energy searched for at the LHC \cite{35}. The current LHC upper limit on the product of the vector boson coupling to quarks and that to invisible particles is at most of the order of unity, and thus not serious.

**IV. VACUUM (META-)STABILITY BOUNDS**

In the MSSM, a large trilinear soft supersymmetry breaking term is known to cause a minimum deeper than the Standard-Model-like (SML) vacuum \cite{36}. In our scenario, this is bound to be the case since the neutrino masses are attributed to their small Yukawa couplings. This is easy to see by tracing the scalar potential along the \( D \)-flat direction,

\[
|\mu^0_u| = |\tilde{\nu}_L| = |\tilde{\nu}_R| = a, \tag{13}
\]

which leads to the lowest energy,

\[
V_{\text{L.E.}} = (m_{h_u}^2 + |\mu|^2 + m_{\tilde{L}}^2 + m_{\tilde{N}}^2) a^2 - 2|A_\nu| a^3 + 3\lambda_\nu^2 a^4, \tag{14}
\]

where \( \lambda_\nu \) is the neutrino Yukawa coupling. One finds that \( V_{\text{L.E.}} < 0 \) for some \( a \) unless the sneutrino trilinear coupling fulfils the inequality,

\[
|A_\nu|^2 \leq 3(m_{h_u}^2 + |\mu|^2 + m_{\tilde{L}}^2 + m_{\tilde{N}}^2) \lambda_\nu^2, \tag{15}
\]

which is the sneutrino-sector counterpart of the “traditional” bound on \( |A_t| \) from Charge-and-Color-Breaking (CCB) minima \cite{36}. This can be re-expressed in terms of the sneutrino mass eigenvalues and mixing angle like

\[
\sin 2\theta_\nu \leq \frac{\sqrt{3} m_\nu (m_{h_u}^2 + |\mu|^2 + m_{\tilde{L}}^2 + m_{\tilde{N}}^2)^{1/2}}{m_{\tilde{\nu}_2}^2 - m_{\tilde{\nu}_1}^2}. \tag{16}
\]

This means that \( \theta_\nu > 2 \times 10^{-12} \) implies a lepton-number breaking global minimum, if one assumes that \( m_\nu \sim 1 \) eV and all the other mass parameters above are around 100 GeV. Therefore, in the range of \( \theta_\nu \) required by a viable relic density of light sneutrino DM, the SML vacuum is inevitably a local minimum with a finite lifetime.
Given the low value of $m^2_\tilde{L}$, our model can also develop an unbounded-from-below (UFB) direction, if $m^2_{h_u} + m^2_{\tilde{L}} < 0$ [37]. However, we shall not consider this direction for the reason to be explained below.

In order to judge whether the global minimum invalidates this model or not, one would need to consider two aspects: the cosmological history of the vacuum, and the lifetime of the current SML vacuum. Regarding the former, one could argue that inflation-induced scalar masses might have brought the Universe to the SML vacuum [38]. The latter then becomes the remaining criterion.

Employing a semiclassical approximation [39], one can express the false vacuum decay rate per unit volume in the form,

$$\Gamma/V = A \exp(-S[\phi]),$$  

(17)

where $A$ is a prefactor which we set to $(100 \text{ GeV})^4$ on dimensional grounds, $S$ is the Euclidean action, and $\phi$ is an O(4)-symmetric [40] stationary point of

$$S[\phi(\rho)] = 2\pi^2 \int_0^\infty d\rho \rho^3 \left[ \left| \frac{d\phi}{d\rho} \right|^2 + V(\phi) \right].$$  

(18)

The “bounce” $\phi(\rho)$ shall obey the boundary conditions,

$$\phi(\rho \to \infty) = \phi_+, \quad \frac{d\phi}{d\rho}(\rho = 0) = 0,$$  

(19)

where $\phi_+$ denotes the false vacuum. The criterion for admitting a parameter set shall be $S[\phi] > 400$ which is the requirement that the lifetime of the observable spatial volume at the SML vacuum be longer than the age of the Universe [41].

To obtain the bounce configuration $\phi(\rho)$, we use the numerical method described in Ref. [42] which works even for a scalar potential with a distant or non-existent global minimum. For a fast computation, we restrict the set $\phi$ of scalar fields to $\{h^0_d, h^0_u, \tilde{\nu}_L, \tilde{\nu}_R\}$. The other scalars are assumed to be zero along the bounce, but we do not impose on it any extra constraint such as $D$-flatness. In view of the shape of the potential, this should not preclude a tunnelling path possibly with a lower $S$. This field restriction excludes the UFB-3 direction in Ref. [43] which is a generalization of the aforementioned UFB direction [37], since
these directions would require two more non-vanishing scalar fields, e.g. a pair of down-type squarks or sleptons of a different generation from the sneutrino generation. However, such UFB paths contain intervals with non-vanishing $D$-terms which form high potential barriers. Therefore, contributions from the UFB paths to $\Gamma/V$ would be highly suppressed compared to that from a path throughout which the $D$-terms are negligible.

As a way to check the validity of our program, we compared its value of $S$ to that from CosmoTransitions\cite{44}, using the two-scalar toy model included in the package.

With the tree-level potential, plus a term proportional to $|h_u|^4$ for fitting the measured Higgs mass (see e.g.\cite{45}), one can determine the tunnelling rate by fixing $m_{\tilde{\nu}_1}$, $m_{\tilde{\nu}_2}$, $\theta_{\tilde{\nu}}$, $\mu$, $\tan \beta$, and $M_A$, the last of which is the $CP$-odd Higgs mass. To obtain the vacuum lifetime bound on $\theta_{\tilde{\nu}}$, we set $M_A = 400 \text{ GeV}$, while we choose the other parameters as in Table III. Note that the tunnelling rate is insensitive to $M_A$ for $\tan \beta \gtrsim 10$ since the $CP$-odd Higgs as well as the other extra Higgses belong mostly to $h_d$ whose components remain to be small along the bounce. (A similar discussion about the irrelevance of $M_A$ to the bounds on flavour-violating up-type trilinears is found in Ref.\cite{46}.)

The overall conclusion from the numerical computation turns out to be that the vacuum longevity constraint on $\theta_{\tilde{\nu}}$ is so loose that it allows the entire range of $\theta_{\tilde{\nu}}$ limited by $Z \to \text{inv.}$ and $h \to \text{inv.}$ For instance, any $\theta_{\tilde{\nu}} \leq 0.52$ is safe from rapid bubble nucleation for $m_{\tilde{\nu}_1} = 0.1 \text{ GeV}$. Even larger $\theta_{\tilde{\nu}}$ is allowed for higher $m_{\tilde{\nu}_1}$, since $A_{\tilde{\nu}}$ which triggers the tunnelling is proportional to $m_{\tilde{\nu}_2}^2 - m_{\tilde{\nu}_1}^2$. This trend continues up to the point $m_{\tilde{\nu}_1} \simeq 10 \text{ GeV}$ where the upper bound disappears, i.e. $S > 400$ for any $\theta_{\tilde{\nu}}$.

Apart from the above constraint at zero temperature, the vacuum stability at high temperatures is known to exclude potentially more parameter volume\cite{47,48}. For instance, Fig. 1 of Ref.\cite{48} shows that thermal effects might decrease the bound on the stop trilinear by about 20%, in the parameter space considered therein. Naively scaling the limit on the sneutrino trilinear by the same factor, one might expect to be safe provided that $\theta_{\tilde{\nu}} \leq 0.38$, which is still far above the collider bounds. We leave an explicit check of this point as a future work.
TABLE II: Parameters and reference values/scan bounds.

| Parameter | Reference value/Scan bound |
|-----------|-----------------------------|
| $\mu$     | 500 GeV                     |
| $\tan \beta$ | 10                          |
| $m_{\tilde{\nu}_2}$ | 125 GeV                    |
| $m_{\tilde{\tau}_R}$ | 120 GeV                    |
| $M_{\tilde{W}}$ | 500 GeV                     |
| $m_{\tilde{\nu}_1}$ | [0.1 GeV, 10 GeV]         |
| $\sin \theta_{\tilde{\nu}}$ | [0.01, 0.3]                |
| $M_{\tilde{B}}$ | [0.1 GeV, 20 GeV]          |

V. RESULTS

We analyze the GeV-mass region of the thermal mixed sneutrino dark matter scenarios. Since direct detections have an energy threshold, in general it is difficult to detect the GeV-mass WIMP directly. However, if the GeV-mass WIMP interacts with the Higgs boson, the search for the Higgs boson invisible decay can constrain the parameter space of the GeV-mass WIMP. In the thermal light mixed sneutrino scenarios, the Higgs invisible decay imposes the upper limit on the sneutrino mixing angle. The small mixing angle of the sneutrino requires that the mass of the lightest neutralino is of the order of 1 GeV (see Eq. (7)). On the other hand, the GUT relation $6M_{\tilde{B}} = 3M_{\tilde{W}} = M_{\tilde{g}}$, which is assumed in earlier works, and the experimental constraints on the gluino mass require the lightest neutralino mass to be $O(100$ GeV). Thus, we relax the GUT relation and focus on the GeV-mass region of the thermal mixed sneutrino dark matter and the lightest neutralino.

Model parameters and their reference values or scan bounds are summarized in Table II. We searched for the parameter set which minimizes the branching ratio of the Higgs invisible decay for a given sneutrino dark matter mass. We comment on the choices of the reference values below. To reduce the higgsino component of the lightest neutralino, the $\mu$-parameter is set to as large as 500 GeV. Then, the decay width of the Higgs boson to a pair of lightest neutralinos is adequately suppressed. The reference value of $\tan \beta = 10$ is chosen to obtain a 125 GeV Higgs boson for less hierarchical superparticle mass spectra. Our results do not strongly depend on the choice of $\tan \beta$ except for the MSSM Higgs boson properties. The heavier sneutrino mass should not be smaller than the Higgs boson mass.
in order to suppress the decay width of the Higgs boson to a pair of the lighter and heavier sneutrinos. On the other hand, since the sneutrino $A$-term, which triggers the false vacuum decay, is proportional to $m_{\tilde{\nu}_2}^2 - m_{\tilde{\nu}_1}^2$, the heavier sneutrino should be light enough. Therefore, we set $m_{\tilde{\nu}_2} = m_h = 125 \text{ GeV}$. We choose possible smallest values for the right-handed stau mass and the wino mass in the light of the LHC results about the two and three tau searches \cite{26}. The colored superparticles as well as first two generations of sleptons are assumed to be too heavy to affect our numerical results.

For our numerical computations of dark matter properties, we have implemented SUSY model files containing right-handed (s)neutrino interactions into the public code \texttt{micromegas 3.2} \cite{49}. The model files are generated with the help of the Feynman rules generation tool \texttt{LanHEP 3.1.8} \cite{50}.

Fig. 1 shows the results of our parameter scan in the $(m_{\tilde{\nu}_1}, \sin \theta_{\tilde{\nu}})$ plane. In the yellow (light-grey) region, the relic density of the sneutrino is larger than the observed dark matter relic density obtained by the Planck collaboration \cite{4}. The pink (dark-grey) region is
excluded by the Higgs boson invisible decay searches at ATLAS \cite{25}. In the allowed region, the spin independent cross section is of the order of $10^{-42} \text{cm}^2$. The constraints on dark matter direct detection rates from LUX (blue dotted line) \cite{21} and SuperCDMS (green solid line) \cite{22} are not serious for such light sneutrino masses as shown in Fig. 1.

It should be emphasized that Higgs boson invisible decay searches at future collider experiments will give a stronger constraint on such light mixed sneutrino WIMP scenarios. If the Higgs boson invisible decay branching ratio is constrained to 10% (2%), the sneutrino mixing angle $\sin \theta_{\tilde{\nu}}$ must be smaller than 0.12 (0.05). The expectations of the ATLAS and CMS collaborations on the Higgs boson invisible decay are $\text{Br} < 8.0 \%$ (95\% CL) \cite{51} and $\text{Br} < 6.4 \%$ (95\% CL) \cite{52}, respectively, at the LHC high-luminosity program with the center-of-mass energy of $\sqrt{s} = 14 \text{ TeV}$ and the luminosity of $L = 3000 \text{ fb}^{-1}$. The planned International Linear Collider (ILC) is capable of measuring the Higgs boson invisible branching ratio accurately \cite{53, 54}. Using the polarization configuration $(P_{e^-}, P_{e^+}) = (+80\%, -30\%)$ with $\sqrt{s} = 250 \text{ GeV}$ and $L = 250 \text{ fb}^{-1}$, the upper limit will reach $\text{Br}(h \rightarrow \text{inv.}) < 0.69 \%$ (95\% CL) \cite{55}. This means that the ILC is capable of excluding mixed sneutrino WIMP scenarios for $0.1 \text{ GeV} < m_{\tilde{\nu}_1} < m_h/2$.

VI. CONCLUSIONS

In supersymmetric models with Dirac neutrino masses where soft breaking trilinear sneutrino interactions are not suppressed by small neutrino Yukawa couplings, the lightest mixed sneutrino is one of the viable thermal WIMP candidates due to the non-negligible mixings between the left- and right-handed states. We have focused on the cases where the lighter of the mixed tau sneutrinos is a WIMP with mass of the order of 1 GeV, and investigated phenomenological constraints on such scenarios. We have shown that if the mass of the bino-like neutralino is also of the order of GeV, the dark matter relic abundance can be explained while adequately suppressing the invisible Higgs boson decay rate. This situation could be realized by relaxing gaugino mass universality which, if retained, would have disabled our scenario because of the severe gluino mass bound obtained at the LHC.

Special attention has been paid to the vacuum stability bound. The large trilinear soft breaking sneutrino interaction also makes a lepton number violating vacuum deeper than the MSSM-like vacuum. We have computed the relevant Euclidean action, and shown that
the lifetime of the Universe in the current phase is long enough in the allowed regions where
the dark matter and Higgs invisible decay constraints are satisfied.

Although dark matter direct detections cannot give stringent constraints on such a low
mass WIMP, we have shown that the ILC has the ability to explore the allowed region
through the Higgs invisible decay search if the mass of the mixed tau sneutrino is larger
than 0.1 GeV. Such light mixed sneutrino scenarios are good examples to show that future
linear colliders can explore model parameter regions which other experiments cannot probe.

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