The self-consistent calculation of the edge states in bilayer quantum Hall bar

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Abstract. In this study, we present the spatial distributions of the edge channels for each layer in bilayer quantum Hall bar geometry for a wide range of applied magnetic fields. For this purpose, we employ a self-consistent Thomas-Fermi-Poisson approach to obtain the electron density distributions and related screened potential distributions. In order to have a more realistic description of the system we solve three dimensional Poisson equation numerically in each iteration step to obtain self consistency in the Thomas-Fermi-Poisson approach instead of employing a “frozen gate” approximation.

1. Introduction
Quantum Hall effect (QHE) has continuously been a center of attraction following its discovery [1] for both theoretical and experimental physicists because of the wide range of interesting physical phenomena it involves. These phenomena are mostly explained in terms of edge channels [2-5]. Bilayer two dimensional electron systems (2DES) also present interesting physical phenomena including fractional quantum Hall effect (FQHE) [6] and hysteresis effect [7,8]. We believe that in order to understand these effects one should have a reliable knowledge of the spatial distribution of edge channels in both layers. For this purpose, like many of the theoreticians who investigate QHE [9-12], we employ self-consistent Thomas-Fermi-Poisson approach (SCTFPA) to obtain the electron density distributions and related screened potential distributions. In order to simplify the calculations, many of the researchers interested in QHE use assumptions like in-plane charges or frozen gate approximation [4,5,8-12]. We believe that solutions of the Poisson equation obtained under such assumptions do not represent the correct nature of the electrostatic structure. Thus we solve three dimensional Poisson equation (3DPE) without employing such assumptions at every step during SCTFPA.

2. Model and Calculations

2.1. The geometry of bilayer structure
The geometry of the bilayer quantum Hall bar (BQHB) considered in this study is illustrated in Fig. 1. There are two gates at the top of the structure of 150 nm width. These gates are biased with a -1.7 V potential to confine the two dimensional electron gas (2DEG) in the x direction. So that the quantum Hall bar (QHB) structure is formed by using the gates instead of etching. Under the gates there is a GaAs layer of 40 nm thickness which is called the cap. Below this layer there is a Si-doped AlGaAs layer of 10 nm thickness and followed by a 40 nm thick undoped AlGaAs layer which is called a
The first 2DEG (top 2DEG) layer is located 90 nm below the surface. After that there is a GaAs layer of 30 nm thickness. At the end of this layer there is the second 2DEG (bottom 2DEG) layer at 120 nm below surface. Below this 2DEG again there is an AlGaAs spacer of 40 nm thickness followed by a Si-doped layer of 10 nm thickness, and then there is bulk GaAs. This structure actually is extended down to the substrate. However in our study the distance from the surface to the bottom is taken as 1000 nm. In the y direction the structure is taken invariant. In Fig. 1, the distance between the two gates which is denoted by w is taken 2000 nm.

Figure 1. A schematic layout of the BQHB considered in calculations.

In this study, the average electron density and the ionized donor density which generates the 2DEG are taken equal as opposed to some theoretical studies [8-13]. The donor density of the top donor layer and the average electron density of the top 2DEG are taken as 

\[ n_d^T = \bar{n}_e^T = 3.0 \times 10^{11} \text{ cm}^{-2} \]

In the density match situation, the density of the bottom donor layer and the average electron density of the bottom 2DEG are taken equal 

\[ n_d^B = \bar{n}_e^B = n_e^B = \bar{n}_e^T \]

In the density mismatch situation, the density of the bottom donor layer and the average density of the bottom 2DEG are taken as 

\[ n_d^B = \bar{n}_e^B = 0.85 \times n_e^B = 0.85 \times \bar{n}_e^T \]

2.2. 3D Poisson equation & boundary conditions

We can write 3DPE inside semiconductor material as

\[-\nabla \varepsilon \nabla \varphi = \rho\]

Here \( \varepsilon \) is the dielectric coefficient of the material and \( \rho \) is electron distribution in the 3D space. A proper solution of this equation depends on the boundary conditions which must be defined correctly. It is well known, different boundary conditions lead to different solutions of 3DPE for the same charge distribution.

The most important theoretical basis for the understanding of edge states is proposed by Chklovskii et al. [4,5]. Chklovskii et al. proposed how the incompressible strips (IS) and compressible strips (CS) are formed by the screening and edge effect. They solved the Poisson equation analytically in these studies. In order to obtain an analytical solution of PE, they make an approximation called in-plane charges assumption. In this assumption, the gates, the donors and the 2DEG are all assumed to be on
the same plane in the $z$ direction. In these studies, Chklovskii et al. did not consider electrochemical equilibrium and solved this problem using a purely electrostatic model. Lier and Gerhardts used SCTFP to achieve electrochemical equilibrium [9]. Following this study, some further theoretical studies are made with SCTFP [10-13], but all of them, used in-plane charges approximation. Various theoretical studies [14-18] aimed at reaching beyond in-plane approximation generally used Davies’ studies [19,20] where Davies obtained analytical solutions of 3DPE to determine the confining potential on the 2DEG generated by gates on the surface of the structure under periodical boundary conditions in $x$ and $y$ directions. However, in these studies the confining potential due to the gates is obtained only for once and without considering the charge distributions in the 2DEG. Changes in the charge distribution of 2DEG induce a potential at the metallic gates and thus change the potential due to the gates. Ignoring this effect is called as frozen gate approximation. Considering the fact that the distance between the gates and the 2DEG is very small, we believe that changes in the charge distribution of 2DEG can effect the confining potential due to the gates. The positions and widths of compressible and incompressible strips in a bilayer structure are very sensitive to small changes in the potential, frozen gate approximation might have a substantial effect on the distribution of the edge channels.

Therefore, we numerically solve 3DPE without introducing any approximations at each step for SCTFP [21]. The numerical solution of Poisson equation in 3D which we use in this study also has an advantage in bilayer system, because it includes all electrostatic interactions such as electron-electron interaction, interlayer charge interaction and electrostatic effects of the gates.

In this study we take $y$ direction as invariant since we are interested only in BQHB. So we are concerned with potential distribution in just $x$ and $z$ directions. The boundary conditions which we use are similar to Davies’ approach [19, 20]. In the $z$ direction, at $z = 0$ (on the surface) the gate potential is taken as boundary condition where there are gates, i.e. $\phi(x,y,0) = V_g$, and zero where there aren’t any gates. We assume that the potential completely heals at the bottom of the sample and first derivative of the potential is assumed to vanish at the bottom (which is 1000 nm from the surface of the sample), because we maintain the overall charge neutrality in the sample, this assumption is reliable. In the $x$ direction in the middle of the gates $\partial\phi/\partial x = 0$ is taken as the boundary condition due to the symmetry of the system.

### 2.3. Thomas-Fermi-Poisson approximation

Thomas-Fermi-Approximation (TFA) is a widely used method to obtain density of electron distributions in 2DEG [9-15]. TFA gives density of electron distribution as

$$ n_s(\vec{r}) = \int dE \frac{D(E)}{f(E)} \frac{|(E + \phi(\vec{r}) - \mu)|}{k_BT} $$

where $D(E)$ is density of state (DOS) of 2DEG, $f(E + \phi(\vec{r}) - \mu)/k_BT = 1/(1 + e^{(E + \phi(\vec{r}) - \mu)/k_BT})$ is the Fermi function and $\phi(\vec{r})$ is the potential function obtained from the solution of 3DPE which include potential due to the gates, donors and interaction between electrons, $\mu$ is the electrochemical potential, $k_B$ is the Boltzmann constant and $T$ is the temperature. In the absence of the magnetic field, DOS for 2DEG is $m/h^2 \pi$. For finite magnetic fields, DOS is Landau DOS

$$ D(E) = \frac{g_s}{2\pi^2} \sum_{n=0}^{\infty} \delta(E - E_n) $$

with $\ell = (h/mc^2) / (eB/m)$ magnetic length, $g_s$ is the spin degeneracy, $E_n = (n + 1/2)\hbar\omega_L$ Landau energy levels and $\omega_L = eB/mc$. The effective electron mass is denoted by $m$.

In the presence of the magnetic field we solve Poisson equation and Thomas-Fermi equation self-consistently to obtain the electron density. However the convergence is a serious problem when these equations are solved self consistently. One of the methods used to obtain convergence is Under Relaxation (UR) method. However this method is not very stable. A more stable method is Newton-Raphson (NR) method. The only disadvantage of NR method is that in order to converge this method
requires an initial value which is close to the solution. So that firstly the system is solved with UR method at $T = 0$ and $B = 0$. Using this solution as the initial distribution, the electron densities of both layers are calculated self consistently with NR method for some sufficiently high temperature and zero magnetic field. Then we set the magnetic field and repeat the NR procedure for the self consistent solution. After this, the temperature is reduced slowly repeating the NR procedure at each temperature step until the system reaches to aimed temperature ($T=1.4$ K).

3. Result and Discussion

The ratio of the electron density and the number of Landau levels per unit area is called the *filling factor* and given by $v = 2\pi \ell^2 n_e(\hat{r})$. When this ratio is a multiple of 2 then a Landau level is completely filled and an incompressible strip is formed. In Fig. 2, the filling factors are shown as a function of the spatial coordinates for magnetic field strengths between 6.5 T and 8.5 T for a BQHB. Here average electron densities of both 2DEGs are taken as equal. In Fig. 2, the incompressible strip (IS) is indicated with yellow colour corresponds to filling factor two. In a single layer quantum hall bar structure the width of the ISs in 2DEG decrease with decreasing magnetic field [21], however in Fig.2 the width of IS in top 2DEG shows an increase with decreasing magnetic field around $B \approx 7.75$ T. We believe that this anomalous behaviour is responsible for the hysteresis effect observed in BQHB. The reason of this increasing is that the screening features of the bottom 2DEG change directly because an IS appears at the centre of bottom 2DEG at this magnetic field. Although the top and the bottom 2DEGs have the same electron number, the bottom 2DEG enters to the plateau region at a lower magnetic field than the upper one. The reason of this is that the bottom 2DEG feels the confinement effect of the surface gates less than upper 2DEG. So the electrons in that layer spread a wider area and the maximum electron density in the centre remains lower than the top layer.

![Figure 2](image_url)

**Figure 2.** In match density condition, top (a) and bottom (b) 2DEGs electron distributions are shown in terms of filling factors as a function of position and magnetic field in gray scale. The color scale is given to right of the graphs. Light gray corresponds to higher filling factors. Yellow color is used to indicate ISs.

In the mismatch situation, the same investigation is done for magnetic field strengths between 6.0 T and 8.5 T. In Fig. 3, the electron distribution of the top and bottom 2DEGs are presented in terms of filling factor. Here the average electron density in the bottom layer is taken as 85% of that of the top layer. So observing of the IS in the bottom layer was rather delayed with respect to the top layer. In this situation, this layer make a screening effect for the potential which effects top 2DEG since the bottom layer consists of completely a compressible strip (CS) in a wide magnetic field interval. So around at 8.0 T, the IS which is in the middle of the top layer maintains the thickness and the location due to this flat potential.
In summary, we have calculated the spatial distributions of the edge channels for each layer in BQHB geometry using SCTFP A. We believe that we have obtained a more realistic description of the system by removing the “frozen gate” approximation.

![Figure 3](image)

**Figure 3.** In mismatch density condition, top (a) and bottom (b) 2DEGs electron distributions are shown in terms of filling factors as a function of position and magnetic field in gray scale. The color scale is given to right of the graphs. Light gray corresponds to higher filling factors. Yellow color is used to indicate ISs.

**Acknowledgements**
This work is supported by Selcuk University BAP Grant No. 07101037. We also thank to ITAP (Institute of Theoretical and Applied Physics; Turunc/Marmaris Turkey) for allowing us to use computational facilities.

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