Three-flavour neutrino oscillations in a magnetic field

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(Dated: July 27, 2022)

In this paper we study Dirac neutrino oscillations in a magnetic field in the three-flavour case. A theoretical framework developed in [1] is extended to the case of three neutrino species. The closed expressions for neutrino flavour and spin oscillations probabilities are derived. We show that the probabilities exhibit a complicated interplay of oscillation on magnetic $\mu_\nu B$ and vacuum $\Delta m^2_{ij}/2p$ frequencies. It is also shown that neutrino oscillations in an interstellar magnetic field can modify neutrino fluxes observed in neutrino telescopes.

INTRODUCTION

It is well-known that massive neutrinos have nontrivial electromagnetic properties (see [1] for a review). In particular, neutrino magnetic moments are nonzero [2, 3]. The best terrestrial upper bounds on neutrino magnetic moments on the level of $\mu_\nu < 2.8 \pm 2.9 \times 10^{-11} \mu_B$ were obtained by the GEMMA reactor neutrino experiment [18] and by the Borexino collaboration [4] from solar neutrino fluxes.

Interaction of neutrinos with nonzero magnetic moments with a transversal magnetic field leads to a phenomenon of neutrino spin precession or neutrino spin oscillations. Neutrino spin precession in the transversal magnetic field $B_\perp$ was first considered in [5], spin-flavor precession was discussed in [6]. The resonant amplification of neutrino spin oscillations in $B_\perp$ in the presence of matter was investigated in [7, 8].

In our previous paper [9] we proposed a new approach to the problem of neutrino mixing and oscillations in a magnetic field based on the exact solutions of the Dirac equation for a massive neutrino state in a magnetic field. Within the approach, a spin operator that commutes with the evolution Hamiltonian is used to classify the massive neutrino states in a magnetic field. We calculated the probabilities of neutrino flavour and spin oscillations in a magnetic field for the case of two neutrino flavours. We have shown that the probabilities exhibit an interplay of oscillations on the vacuum $\omega_{\text{vac}} = \Delta m^2/4p$ and magnetic $\omega_B = \mu_\nu B_{\perp}$ frequencies. In [10, 11] we extended some of our results to the case of three neutrino flavours. The CP-violating effects in oscillations of Majorana neutrinos in a supernova media has been studied using our approach in [12].

In this paper we continue our research of neutrino oscillations in a magnetic field and now apply our approach to the case of three neutrino flavours. The presented derivation of the explicit expressions for the probabilities of Dirac neutrino flavour and spin oscillations enable us to investigate the astrophysical neutrino flavour composition in an experiment for the three flavour case.

I. NEUTRINO FLAVOUR AND SPIN OSCILLATIONS IN A MAGNETIC FIELD

In this section we describe the approach to deriving of the neutrino oscillations probabilities in a magnetic field developed in [9] and extend it to the case of three neutrino flavours.

The wave function $\nu_i$ of a massive Dirac neutrino state that propagates in the presence of a constant and homogeneous and arbitrary orientated magnetic field can be found as the solution of the following system of equations

$$(\gamma_\mu p^\mu - m_i - \mu_i \Sigma B)\nu_i(p) = 0,$$  

where $\mu_i$ are the neutrino magnetic moments, $i = 1, 2, 3$. Note that in this paper we suppose that Dirac neutrinos do not possess the transition magnetic moments. Effects of the transition magnetic moments were studied in [13] for the two flavour case.

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Eq. 1 can be rewritten in the equivalent Hamiltonian form
\[ H_i \nu_i = E_i \nu_i, \tag{2} \]
where the Hamiltonian
\[ H_i = m_i \gamma_0 + \gamma_0 \gamma p + \mu_i \gamma_0 \Sigma B. \tag{3} \]
is introduced.

The eigenvalues of the Hamiltonian (3) are given by
\[ E_{s_i} = \sqrt{m_i^2 + p^2 + \mu_i^2 B^2 + 2 \mu_i s \sqrt{m_i^2 B^2 + p^2 B^2}}, \tag{4} \]
where \( s = \pm 1, p = |p| \) and \( B_\perp \) is a transversal (relatively to the neutrino momentum) component of the magnetic field.

Thus, stationary solutions of Eq. 1 exist and satisfy the following relation
\[ H_i |\nu_{s_i}^s(t)\rangle = E_{s_i}^s |\nu_{s_i}^s(t)\rangle. \tag{5} \]

To clarify the meaning of the spin number \( s \) introduced in (4) and classify the neutrino stationary states in a magnetic field we use the following spin operator [9]
\[ S_i = \frac{m_i}{\sqrt{m_i^2 B^2 + p^2 B^2}} \left[ \Sigma B - \frac{i}{m_i} \gamma_0 \gamma_5 [\Sigma \times p] B \right]. \tag{6} \]

One can show that the spin operator (6) commutes with the Hamiltonian
\[ [S_i, H_i] = 0, \tag{7} \]
and satisfy the following conditions
\[ S_i^2 = 1. \tag{8} \]

Thus, the stationary states (5) are solutions of Eq. 1 that are eigenstates of \( S_i \):

\[ S_i |\nu_{s_i}^s\rangle = s |\nu_{s_i}^s\rangle, s = \pm 1, \tag{9} \]
\[ \sum_s |\nu_{s_i}^s\rangle \langle \nu_{s_i}^s| = 1, \tag{10} \]
\[ \langle \nu_{s_i}^s|\nu_{s_i'}^{s'}\rangle = \delta_{ss'} \delta_{ss'}. \tag{11} \]

In order to treat the evolution of the neutrino massive states in a magnetic field we expand the wavefunctions \( \nu_i \) over the stationary states
\[ \nu_i^L(t) = c_i^+ \nu_i^+(t) + c_i^- \nu_i^-(t), \tag{12} \]
\[ \nu_i^R(t) = d_i^+ \nu_i^+(t) + d_i^- \nu_i^-(t), \tag{13} \]
where \( L, R \) are neutrino helicities, \( \nu_i^s(t) = \exp(-iE_i^s t)\nu_i^s(0) \), and the coefficients \( c_i^\pm \) and \( d_i^\pm \) do not depend on time and can be calculated using initial conditions.

We derive general expressions for the neutrino flavour and spin oscillations probabilities in a magnetic field using the common definition and straightforward expression
\[ P_{\alpha h \rightarrow \beta h'}(t) = \left| \langle \nu_{h'}^\beta(0)|\nu_{h}^\alpha(t)\rangle \right|^2 = \sum_i U_{\beta i}^* U_{\alpha i} \langle \nu_i^h(0)|\nu_i^{h'}(t)\rangle \right|^2, \tag{14} \]
where \( U \) is the mixing matrix, \( h, h' = L, R \) are helicities of neutrino initial and final states respectively, \( \alpha, \beta = \{e, \mu, \tau\} \) are neutrino flavours and \( i = \{1, 2, 3\} \). Using [12], we derive the following expressions for the probabilities of neutrino flavour and spin oscillations assuming that the initial state is an electron neutrino:
where the projection operators are introduced

\[ P_{\nu_i^h \rightarrow \nu_i^{h'}}(t) = \left| \left( |c_1^+|^2 e^{-iE_1^+ t} + |c_1^-|^2 e^{-iE_1^- t} \right) U_{11}^{*} U_{11} + \left( |c_2^+|^2 e^{-iE_2^+ t} + |c_2^-|^2 e^{-iE_2^- t} \right) U_{12}^{*} U_{12} + \left( |c_3^+|^2 e^{-iE_3^+ t} + |c_3^-|^2 e^{-iE_3^- t} \right) U_{13}^{*} U_{13} \right| ^2, \] (15)

\[ P_{\nu_i^h \rightarrow \nu_i^{h'}}(t) = \left| \left( (d_1^+)^* c_1^+ e^{-iE_1^+ t} + (d_1^-)^* c_1^- e^{-iE_1^- t} \right) U_{11}^{*} U_{11} + \left( (d_2^+)^* c_2^+ e^{-iE_2^+ t} + (d_2^-)^* c_2^- e^{-iE_2^- t} \right) U_{12}^{*} U_{12} + \left( (d_3^+)^* c_3^+ e^{-iE_3^+ t} + (d_3^-)^* c_3^- e^{-iE_3^- t} \right) U_{13}^{*} U_{13} \right| ^2. \] (16)

Below we calculate the coefficients \( c_i^+ \) and \( d_i^+ \) and generalise (15) and (16) to the case of an arbitrary flavour of initial neutrino.

Using (9), (10) and (11), one can show that

\[ \langle \nu_i^{h'}(0)|\nu_i^h(t) \rangle = \sum_s e^{-iE_s^t} \langle \nu_i^{h'}(0)|\nu_s^h \rangle \langle \nu_s^h|\nu_i^h(0) \rangle = \sum_s e^{-iE_s^t} \langle \nu_i^{h'}(0)|\tilde{P}_s|\nu_i^h(0) \rangle, \] (17)

where the projection operators are introduced

\[ P_i^\pm = |\nu_i^\pm \rangle \langle \nu_i^\pm | = \frac{1 \pm S_i}{2}. \] (18)

Thus, the amplitudes of the transitions between massive neutrino helicity states are given by the plane wave expansion of the following form

\[ \langle \nu_i^{h'}(0)|\nu_i^h(t) \rangle = \sum_s C_{is}^{h'h} e^{-iE_s^t}, \] (19)

where

\[ C_{is}^{h'h} = \langle \nu_i^{h'}(0)|P_i^n|\nu_i^h(0) \rangle. \] (20)

The coefficients \( C_{is}^{h'h} \) are related with \( c_i^+ \) and \( d_i^+ \) introduced in (12) by the following expressions

\[ |c_i^+|^2 = C_{is}^{LL}, \quad |d_i^+|^2 = C_{is}^{RR}. \] (21)

Using the plane-wave expansion (19) one can derive the general expression for the probabilities of neutrino oscillations for the case of three neutrinos as

\[ P_{\nu_i^h \rightarrow \nu_i^{h'}}(t) = \sum_{i,j,s,s'} U_{i1}^{*} U_{j1} U_{i2}^{*} U_{j2} U_{i3}^{*} U_{j3} C_{is}^{h'h} (C_{js'}^{h'h'})^{*} e^{-i(E_i^t - E_j^t) t}, \] (22)

where \( s, s' = \pm 1 \).

From (22) it can be shown that

\[ \sum_{\beta, h'} P_{\nu_{\alpha}^h \rightarrow \nu_{\beta}^{h'}}(t) = \sum_{\alpha, h} P_{\nu_{\alpha}^h \rightarrow \nu_{\beta}^{h'}}(t) = 1 \] (23)

and

\[ P_{\nu_i^h \rightarrow \nu_i^{h'}}(0) = \delta_{\alpha \beta} \delta_{hh'}. \] (24)
II. NEUTRINO OSCILLATIONS PROBABILITIES IN THE ULTRARELATIVISTIC LIMIT

The expression (22) for the probabilities of neutrino oscillations in a magnetic field can be significantly simplified for the ultrarelativistic case.

For the neutrino energy spectrum in the ultrarelativistic limit \( m_i \ll p \) we get

\[
E_i^e \approx p + \frac{m_i^2}{2p} + \frac{\mu_i B^2}{2p} + \mu_i s B_\perp.
\]  

(25)

Given that neutrino magnetic moments are of order of \( 10^{-11} \mu_B \) or smaller, it is also reasonable to assume that \( \mu_i B_\perp \ll m_i \) (see \( \frac{4}{22} \) for a detailed discussion). Thus we can approximately write neutrino energy spectrum in a magnetic field as

\[
E_i^e \approx p + \frac{m_i^2}{2p} + \mu_i s B_\perp.
\]  

(26)

Since the differences of neutrino energies and the corresponding frequencies of neutrino oscillations are

\[
E_i^e - E_j^e \approx \frac{\Delta m_{ij}^2}{2p} + (\mu_i s - \mu_j s')B_\perp,
\]  

(27)

we expect that the probabilities of neutrino oscillations in a magnetic field exhibit the inherent interplay of the oscillations on the vacuum frequencies \( \Delta m_{ij}^2/4p \) and magnetic frequencies \( \mu_i B_\perp \).

The plane-wave expansion coefficients \( C_{is}^{hh} \) also can be simplified if we account for the fact that in the ultrarelativistic limit neutrino helicity states are given by the corresponding limit of the Dirac equation solutions:

\[
|\nu_i^e(0)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \quad |\nu_i^R(0)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}.
\]  

(28)

Then from (20) we have

\[
C_{is}^{LL} \approx \frac{1}{2} \left( 1 + s \frac{m_i B_\parallel}{\sqrt{m_i^2 B_\parallel^2 + (m_i^2 + p^2) B_\perp}} \right),
\]  

(29)

\[
C_{is}^{RL} \approx \frac{s}{2} \frac{p B_\perp}{\sqrt{m_i^2 B_\parallel^2 + (m_i^2 + p^2) B_\perp^2}}.
\]  

(30)

In the derivation above we use the fixed reference frame for which \( B_\parallel = B_x, B_\perp = (B_y, 0, 0) \). Note that in the limit \( B_\perp = 0 \) the coefficients \( C_{is}^{RL} = 0 \), and the probabilities of neutrino spin oscillations \( \nu_\alpha^L \rightarrow \nu_\beta^R \) are zeros. Thus, neutrino spin oscillations are generated by the transversal magnetic field, as it should be.

Finally, in the ultrarelativistic limit we assume that \( t \approx x \), where \( x \) is the distance travelled by neutrino.

Neglecting term of order of \( m_i^2/p^2 \) and smaller, we arrive to the expressions for the neutrino flavour \( \nu_\alpha^L \rightarrow \nu_\beta^L \) and spin \( \nu_\alpha^L \rightarrow \nu_\beta^R \) oscillations probabilities:

\[
P_{\nu_\alpha^L \rightarrow \nu_\beta^L}(x) = \frac{1}{4} \sum_{i,j} U_{\beta i}^* U_{\alpha j} U_{\beta j} U_{\alpha i} \sum_{s,s'} e^{-i(E_i^e - E_j^e)x},
\]  

(31)

\[
P_{\nu_\alpha^L \rightarrow \nu_\beta^R}(x) = \frac{1}{4} \sum_{i,j} U_{\beta i}^* U_{\alpha j} U_{\beta j} U_{\alpha i} \sum_{s,s'} s s' e^{-i(E_i^e - E_j^e)x}.
\]  

(32)

These expressions can be further simplified. For the probabilities of neutrino three-flavour oscillations we get

\[
P_{\nu_\alpha^L \rightarrow \nu_\beta^L}(x) = \sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2 \cos^2(\mu_i B_\perp x)
\]  

(33)

\[+ \sum_{i>j} 2 \cos(\mu_i B_\perp x) \cos(\mu_j B_\perp x) \left[ \text{Re}(A_{ij}^\alpha) \cos \left( \frac{\Delta m_{ij}^2}{2p} x \right) + \text{Im}(A_{ij}^\alpha) \sin \left( \frac{\Delta m_{ij}^2}{2p} x \right) \right].\]
where $A_{ij}^{\alpha \beta} = U_{\beta i}^\dagger U_{\alpha j}$. Note that $\text{Im} A_{ij}^{\alpha \beta} = J \sin \delta \sum_{k, \gamma} \epsilon_{\alpha \beta \gamma} \epsilon_{ijk}$, where $\delta$ is CP-violating phase and $J \approx 0.034$ is the leptonic Jarlskog invariant (see [13]). Thus, the last term in (33) describes CP-violating effects.

It makes sense to introduce a new observable that describes the total probability of conversion into a sterile neutrino state $P_{\nu_L \rightarrow \nu_R} = P_{\nu_L \rightarrow \nu_e^R} + P_{\nu_L \rightarrow \nu_\mu^R} + P_{\nu_L \rightarrow \nu_\tau^R}$:

$$P_{\nu_L \rightarrow \nu_R}(x) = \frac{1}{4} \sum_{i,j} U_{\beta i}^\dagger U_{\alpha j} \sum_{s,s'} s s' e^{-i(E_i^L - E_j^R)x}.$$  \hspace{1cm} (34)

This expression can be further simplified. Using the unitarity condition of the mixing matrix $\sum_{i} U_{\beta i} U_{\beta j} = \delta_{ij}$ and the fact that $E_i^L - E_i^R = 2\mu_i B_\perp$, we arrive to the final expression for the probability of neutrino spin oscillations

$$P_{\nu_L \rightarrow \nu_R}(x) = \sum_{i=1}^{3} |U_{\alpha i}|^2 \sin^2 (\mu_i B_\perp x)$$ \hspace{1cm} (35)

Note that the probability of spin oscillations (35) does not depend on the value of the CP-violating phase since it depends only on absolute values of the mixing matrix entries.

### III. COSMOGENIC NEUTRINO OSCILLATIONS IN INTERSTELLAR MEDIA

As an example of neutrino evolution in a magnetic field we study propagation of cosmogenic neutrinos in the interstellar media (the case of two neutrino flavours was studied in [13]). Cosmogenic neutrinos are neutrinos that originate from interaction of ultra-high energy cosmic rays (UHECR) with the cosmic microwave background [16]. Cosmogenic neutrino fluxes typically have energies of order of $1 \text{ EeV}$ (i.e. $10^{18} \text{ eV}$) [17].

Consider neutrinos with the energy $p = 1 \text{ EeV}$ and , $B = 2.8 \mu \text{ G}$ for the interstellar magnetic field strength (see [14]). We also set the value of CP-violating phase $\delta$ to zero. For the sake of simplicity we start our analysis with the case of equal neutrino magnetic moments: $\mu_1 = \mu_2 = \mu_3 = 10^{-11} \mu_B$. The probabilities of neutrino flavour oscillations $\nu_e \rightarrow \nu_e$, $\nu_e \rightarrow \nu_\mu$, and $\nu_e \rightarrow \nu_\tau$ as functions of distance travelled by neutrino in parsec are shown in Fig. 1a and Fig. 1b. The probabilities indeed exhibit a complicated interplay of oscillations on the following vacuum and magnetic lengths:

$$L_i^B = \frac{\pi}{\mu_i B_\perp} = 2.17 \cdot \left(\frac{B}{\mu_B}\right)^{-1} \left(\frac{\mu_i}{10^{-11} \mu_B}\right)^{-1} 10^{-3} \text{ pc},$$ \hspace{1cm} (36)

$$L_{ij}^{\text{vac}} = \frac{4\pi p}{\Delta m^2_{ij}} = 5.02 \cdot \left(\frac{\Delta m^2_{ij}}{\text{eV}^2}\right)^{-1} \left(\frac{p}{\text{GeV}}\right) \cdot 10^{-13} \text{ pc}.$$ \hspace{1cm} (37)

For the choice of parameters given above, oscillations on the vacuum lengths $L_{i12}^{\text{vac}} \approx 7 \text{ pc}$ and $L_{123}^{\text{vac}} \approx 0.2 \text{ pc}$ that can be observed at smaller scales (Fig. 1a) are modulated by oscillations on the magnetic length $L_{ij}^B \approx 370 \text{ pc}$ (Fig. 2a). This behaviour reproduces phenomenon observed in [3, 13] for the case of two neutrino flavours. For the case of $\mu_1 = \mu_2 = \mu_3 = 10^{-11} \mu_B$, Eq. (33) reduces to

$$P_{\nu_L \rightarrow \nu_R}(x) = \cos^2 (\mu B_\perp x) \left[ 3 \sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2 + 2 \sum_{i>j} A_{ij}^{\alpha \beta} \cos \left(\frac{\Delta m^2_{ij}}{2p} x\right) \right] \hspace{1cm} (38)$$

Here the expression in the square brackets is the vacuum oscillations probability and it is modulated by oscillations on magnetic frequency $\mu B_\perp$.

The interplay of oscillations on the vacuum and magnetic frequencies becomes even more pronounced if neutrino magnetic moments are not equal to each other. In Fig. 2a we show the probabilities of neutrino flavour oscillations for this particular case.

Finally, in Fig. 2b we plot the neutrino spin oscillations probability $\nu_e \rightarrow \nu_R$ for different values of the neutrino magnetic moment $\mu_2$. The magnetic moment $\mu_1$ value is fixed as $10^{-11} \mu_B$. Since the last term in the sum in (35) for the case of initial electron is proportional to a quite small value $\sin^2 \theta_{13} \approx 0.022$, we can neglect the contribution of
FIG. 1. Neutrino flavour oscillations probabilities for neutrino energy $E = 1$ EeV and magnetic moments $\mu_1 = \mu_2 = \mu_3 = 10^{-11} \mu_B$.

FIG. 2. a) Neutrino flavour oscillations probabilities for neutrino energy $E = 1$ EeV and magnetic moments $\mu_1 = 10^{-11} \mu_B$, $\mu_2 = 2 \cdot 10^{-11} \mu_B$ and $\mu_3 = 3 \cdot 10^{-11} \mu_B$; b) Neutrino spin oscillations probability $\nu_e^L \rightarrow \nu^R$ for different values of magnetic moments.

In the case of equal magnetic moments $\mu_1 = \mu_2 = \mu$ neutrino spin oscillations probability is

$$P_{\nu_e^L,\nu^R}(x) = \sin^2\left(\pi x / L_B\right),$$

where the oscillations length $L_B \approx 370$ pc. For different choices of the neutrino magnetic moments oscillations probability exhibit an interplay of oscillations on two magnetic frequencies $\omega_1 = \mu_1 B_\perp$ and $\omega_2 = \mu_2 B_\perp$. Note that distance-averaged spin oscillations probability equals 1/2 regardless of choice of neutrino magnetic moments.

IV. CONCLUSION

Our previous results on neutrino oscillations in magnetic fields were extended to the case of three neutrino flavours. We have shown that the probabilities of neutrino flavour oscillations in a magnetic field exhibit a complicated interplay of oscillations on both vacuum $\omega^{vac}_{ij} = \Delta m^2_{ij}/4p$ and magnetic $\omega^B_i = \mu_i B_\perp$ frequencies, while spin oscillations probability depends only on the magnetic frequencies $\omega^B_i$. As an example of neutrino oscillations in a magnetic field we have considered oscillations of cosmogenic neutrinos in the interstellar magnetic field.

Using our findings on three flavour neutrino oscillations, we have shown that the presence of interstellar magnetic field indeed can modify neutrino fluxes observed by terrestrial neutrino telescopes. In particular, we find that in the considered Dirac case in average half of neutrinos convert into sterile neutrino state. Using three flavour oscillations probabilities one can also compute fluxes of active neutrinos at the distance $x$ from neutrino source as

$$\Phi_\alpha(x) = \sum_\beta \Phi^0_\beta P_{\nu^L_\beta \rightarrow \nu^L_\alpha}(x),$$
where $\Phi^0_\beta$ are the fluxes at the neutrino source, and calculate neutrino flavour composition observed by a neutrino telescope:

$$r_\alpha(x) = \frac{\Phi_\alpha(x)}{\sum_\beta \Phi_\beta(x)}. \quad (41)$$

It is predicted that UHE neutrino flavour composition follows pattern $\Phi^0_\alpha : \Phi^0_\mu : \Phi^0_\tau \approx 1 : 2 : 0$ [19]. Then, for the case of vacuum oscillations, neutrino flavour composition at the terrestrial neutrino telescope is $(r^{vac}_\tau, r^{vac}_\mu, r^{vac}_\mu) \approx (1/3, 1/3, 1/3)$ [20]. It is likely that in next 20 years neutrino telescopes such as Baikal-GVD, KM3NeT, P-ONE, TAMBO and IceCube-Gen2 will collect enough data to determine the flavour composition of high-energy cosmic neutrino fluxes [21]. If the measured composition contradicts the predicted pattern (1/3, 1/3, 1/3), this might be a signal of new physics, including neutrino magnetic moments. Using the probabilities of neutrino oscillations in interstellar magnetic field we can study these possible effects of nonzero neutrino magnetic moments on UHE neutrino fluxes flavour composition. However, to study neutrino propagation through cosmic scales we have to account for the effects of coherence in neutrino oscillations and generalize the probabilities [33]. We are going to continue our research on neutrino oscillations in a magnetic field and consider effects of wave packet separation in a forthcoming paper.

ACKNOWLEDGEMENTS

The work is done within the programme of the Interdisciplinary Scientific and Educational School of Moscow University “Fundamental and Applied Space Research” and is supported by the Russian Science Foundation under grant No.22-22-00384. The work of A.P. has been supported by the Foundation for the Advancement of Theoretical Physics and Mathematics “BASIS” under Grant No. 21-2-2-26-1.

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