Intuitionistic Fuzzy Set-Based Computational Method for Financial Time Series Forecasting

Kamlesh Bisht and Sanjay Kumar

Department of Mathematics, Statistics and Computer Science, G. B. Pant University of Agriculture and Technology, Pantnagar, India

ABSTRACT
Intuitionistic fuzzy sets (IFSs) have been proved to be more ideal than fuzzy sets to handle non-probabilistic uncertainty and non-determinism in the system. The present study proposes an IFS-based computational method to address the issue of non-determinism in financial time series forecasting. The proposed IFS-based forecasting method uses a simple computational algorithm to forecast without using complex computations using intuitionistic fuzzy logical relations. In order to see suitability of the proposed method in financial forecasting, it is implemented on three experimental data of the SBI share price, TAIEX and Dow Jones Industrial Average (DJIA). Root mean square error and statistical test are used in the study to confirm the out performance of the proposed IFS-based computational method of forecasting. Experimental results show that the proposed method outperforms various existing methods for forecasting SBI, TAIEX and DJIA.

ARTICLE HISTORY
Received 22 February 2019
Revised 27 April 2019
Accepted 14 May 2019

KEYWORDS
Intuitionistic fuzzy set; fuzzy time series; computational algorithm; non-determinism; financial forecasting

1. Introduction
Financial time series forecasting has been an important, challenging and intensive working area for researchers and practitioners. Prediction of stock price volatility, which translates to high risk, is important for investors to take investment decision for a better return. Statistical techniques based methods such as ARMA, ARIMA, ARCH and generalised ARCH were deployed for financial forecasting, but these methods fail to handle the uncertainty caused by the non-probabilistic and linguistic representation of financial time series data. Fuzzy set [1] based time series forecasting models proposed by Song and Chissom [2,3] and Chen [4] stand out as a key solution for financial instrument forecasting. Researchers and practitioners are more aware of fuzzy time series forecasting instead of the traditional time series forecasting method because of their capability of handling uncertainty caused by aforesaid reasons. Various researchers [5–9] have proposed numerous methods using fuzzy approach for financial time series forecasting. Machine learning, granular computing, clustering, neural networks, genetic algorithm (GA), particle swarm optimisation (PSO) and other nature-based optimisation techniques were integrated with a fuzzy approach to propose intelligent fuzzy time series methods for enhancing accuracy in financial time

CONTACT
Kamlesh Bisht kamlesh45848@gmail.com

© 2019 The Author(s). Published by Taylor & Francis Group on behalf of the Fuzzy Information and Engineering Branch of the Operations Research Society of China & Operations Research Society of Guangdong Province. This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0/), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.
series forecast [10–19]. Gupta et al. [20] proposed forecasting models for low-dimensional numerical data using automatic clustering, fuzzy relations and differential evolution. Soto et al. [21] proposed the interval type 2 fuzzy neural network-based time series forecasting model.

Fuzzy time series forecasting methods achieved great success to model non-probabilistic uncertainty in financial time series forecasting, but failed to handle non-determinism. Non-determinism in fuzzy time series forecasting may occur due to hesitation. This hesitation is caused by the use of a single function for both membership and non-membership and cannot be handled by a random probability distribution. Atanassov [22] generalised fuzzy set and defined Intuitionistic fuzzy set (IFS) to address the issue of non-determinism caused by non-stochastic factors. IFS includes two distinct functions to determine membership and non-membership grade of an element.

The application of IFS in time series forecasting was initiated by Joshi and Kumar [23–25] to include hesitation in fuzzy time series forecasting. The fuzzified IFS-based financial time series forecasting method was proposed by Kumar and Gangwar [26] to forecast SBI share price. Kumar and Gangwar [27] defined the intuitionistic fuzzy time series (IFTS) and used a Cartesian product of IFSs to propose a methodology for the intuitionistic fuzzy time series forecasting model. After that many researchers [8,28–31] proposed simple and high-order IFTS forecasting models. Sotirov et al. [32] applied IFS with intercriteria analysis to study behaviour of the modular neural network. Castillo et al. [33] proposed the IFS-based inference system for time series analysis to monitor a non-linear dynamic plant. Recently hesitant fuzzy sets [34], dual hesitant fuzzy set [35] and hesitant probabilistic fuzzy set have been explored by Bisht and Kumar [36], Bisht et al. [37], Gupta and Kumar [38] in time series forecasting to handle a particular type of non-determinism that occurs because of the reasons other than margin of errors and possible distribution occurs.

Computational approaches for financial time series analysis are not only useful to cope with huge time series data but also enhance accuracy in a forecast using lower and upper bounds of intervals. Various computational approaches are found in the literature of time series analysis. Top-down, bottom-down are few offline while a sliding window approach is an example of an online computational approach used in time series analysis. Computational approach in fuzzy time series forecasting was initiated by Singh [39–42] to avoid complicated max–min composition operations and the defuzzification process. Joshi and Kumar [43], Gangwar and Kumar [44] also proposed the clusters based computational method for the high order fuzzy time series forecasting method to forecast the SBI share price.

Even though existing computational algorithm-based forecasting models [39–44] enhance accuracy in time series forecasting, but none of these methods include non-determinacy. The motive and objective of the present study are to model non-determinacy and to develop a computational algorithm for time series forecasting. In this research paper, we propose an IFS-based computational method for financial time series forecasting to cope with non-determinism. This is important to note that the developed computational algorithm handles only non-stochastic non-determinism in financial time series forecasting. We also propose a unique method to define the universe of discourse using the greatest integer function.
The proposed computational method is a two-phase method. In the first phase, we use the construction methodology proposed by Jurio et al. [45] to construct IFS from a fuzzy set. In the second phase, a computational algorithm is developed using membership and non-membership and difference parameters. The inclusion of both membership and non-membership grades in the proposed computational algorithm makes it unique and competent to handle non-stochastic non-determinism in financial time series forecasting. Performance of the proposed IFS-based computational method is tested using three different stock databases. Error analysis and validity test are performed using different error measures and the statistical performance parameter validity test. We have also used the two-tailed $t$-test to confirm significant improvement in the performance of the proposed method over a few recent existing fuzzy and intuitionistic fuzzy time series forecasting methods.

2. Preliminaries

In this section, we discuss the fundamental definition of the fuzzy set, IFS, fuzzy time series and intuitionistic fuzzy time series in brief.

2.1. Fuzzy set and fuzzy time series

Let $U$ be the discrete and reference set. A fuzzy set $A$ on $U$ is a mathematical object of the form $A = \{< u, \mu_A(u) > | \forall u \in U \}$. Here, $\mu_A(u)$ is the membership grade of $u$ in fuzzy set $A$ and $\mu_A(u) \in [0, 1]$.

Fuzzy time series [2,3] is a collection of fuzzy sets and is defined as follows: let $Y(t)$ be subset of real numbers and be a reference set. Fuzzy sets $f_i(t)$ are defined on $Y(t)$. $F(t)$ collection of fuzzy sets $f_i(t)$ is a fuzzy time series on $Y(t)$. If $F(t)$ is caused by $F(t-1)$, represented as $F(t-1) \rightarrow F(t)$, then the fuzzy time series relationship is expressed as $F(t) = F(t-1) \ast R(t, t-1)$. Here, $\ast$ is the Max–Min operator. $R(t, t-1)$ and $R$ are fuzzy relationships between $F(t)$ and $F(t-1)$ and the union of fuzzy relations. This fuzzy time series model is called the first order model of $F(t)$.

2.2. IFS and intuitionistic fuzzy time series

Atanassov [22] generalised fuzzy set to IFS. An IFS is characterised by both membership and non-membership functions and is defined as follows. Let $U$ be the discrete and reference set. An IFS, $I$ on $U$ is the mathematical object of the form $I = \{< u, \mu_I(u), \nu_I(u) > | \forall u \in U \}$. Here, $\mu_I(u)$ and $\nu_I(u)$ are the membership grade and non-membership grade of $u$ in $I$. Both membership and non-membership lie in the unit interval $[0, 1]$ and satisfy the condition $0 \leq \mu_I(u) + \nu_I(u) \leq 1$. $\pi_I(u) = 1 - \mu_I(u) - \nu_I(u)$ defines the degree of non-determinacy of $u$ in $I$.

Kumar and Gangwar [27] defined IFTS as follows. Assume $Y(t), (t = \ldots, 0, 1, 2, \ldots)$, is the sequential collection of data over a time interval or universe of discourse and subset of real no. $R$. If $I_i(t), (i = 1, 2, \ldots)$ are the IFS defined in $Y(t)$ then the collection $\xi(t) = I_i(t)$ of $I_i(t)$ is known as intuitionistic fuzzy time series. If $I_i(t)$ is caused by $I_i(t-1) \in I_i(t-1)$, then the relationship can be expressed as $I_i(t-1) \rightarrow I_i(t)$. 
3. Proposed IFS-based computational method for financial time series forecasting

The proposed IFS-based computational method for financial time series forecasting is the two-phase method. In phase I, partitions of the universe of discourse, the construction method of IFS [45] and intuitionistic fuzzification of time series data is explained. In Phase II, a computational algorithm is developed for financial time series forecasting.

**Phase I: Construction of IFS and intuitionistic fuzzy logical relations (IFLR).**

The following various steps are included in phase I:

**Step 1**

The universe of discourse \( U \) for financial time series is defined as \([U_{\text{min}} - U_1, U_{\text{max}} + U_2]\). Here, \( U_{\text{min}} \) and \( U_{\text{max}} \) are the minimum and maximum of time series dataset, respectively. \( U_1 \) and \( U_2 \) are the positive integer defined as follows:

\[
U_1 = \begin{cases} 
D_1 & \text{if } D_1 < 50 \\
D_1 & \text{if } D_1 > 50 
\end{cases} 
\]

\[
U_2 = \begin{cases} 
50 - D_2, & \text{if } D_2 < 50 \\
100 - D_2, & \text{if } D_2 > 50 
\end{cases} 
\]

Here, \( D_1 = U_{\text{min}} - 100 \times \lfloor U_{\text{min}}/100 \rfloor \) and \( D_2 = U_{\text{max}} - 100 \times \lfloor U_{\text{max}}/100 \rfloor \), and symbol \( \lfloor \rfloor \) denotes the greatest integer.

**Step 2**

Partition the universe of discourse into 14 intervals \( u_i (i = 1, 2, \ldots, 14) \) of equal length and construct 14 triangular fuzzy sets \( A_i (i = 1, 2, 3, \ldots, 14) \) in accordance with the intervals \( u_i \) as follows:

\[
A_i = [E_{\text{min}} + (i - 1)k, E_{\text{min}} + ik, E_{\text{min}} + (i + 1)k], i = 1, 2, \ldots, 13 \\
A_i = [E_{\text{min}} + (i - 1)k, E_{\text{min}} + ik, E_{\text{min}} + ik], i = 14 
\]

**Step 3**

Apply the construction method [45] to construct 14 IFS \( I_i \) with respect to triangular fuzzy set \( A_i \).

**Step 4**

Use the following algorithm for intuitionistic fuzzification of the financial time series data.

\[
\text{for } i = 1 \text{ to } m \text{ (end of time series data)} \\
\quad \text{for } j = 1 \text{ to } n \text{ (end of intervals)} \\
\quad \quad \text{choose } \mu_{ki} = \max(\mu(x_1), \mu(x_2), \ldots, \mu(x_k), \ldots, \mu(x_{14})), 1 \leq k \leq n \\
\quad \quad \text{If } I_k \text{ is IFS correspondins to } \mu_{ki} \text{ then assign } I_k \text{ to } x_i. \\
\quad \text{end if} \\
\text{end for} \\
\]

Establish IFLR \( I_k \rightarrow I_j \). Here, \( I_k \) is the intuitionistic fuzzy production of month \( n \) and \( I_j \) is the then the intuitionistic fuzzy logical relation (IFLR) denoted. Here, \( I_k \) and \( I_j \) are intuitionistic fuzzy production of month \( n \) and \( n+1 \), respectively.
**Table 1.** Notations and their description used in the intuitionistic computational algorithm.

| Sr. no. | Notation | Description |
|---------|----------|-------------|
| 1       | $L[A_r]$ | Lower bound of the rth triangular fuzzy set |
| 2       | $M[A_r]$ | Mid value of the rth triangular fuzzy set |
| 3       | $U[A_r]$ | Upper bound of the rth triangular fuzzy set |
| 4       | $E_i$    | $i$th actual time series datum |
| 5       | $\mu_i^r$ | Membership of the $i$th time series datum in IFS $l$, |
| 6       | $v_i^r$  | Non-membership of the $i$th time series datum in IFS $l$, |
| 7       | $F_i$    | is the crisp forecasted output |

**Phase II: IFS-based computational algorithm.**

In this phase, we develop an IFS-based computational algorithm for financial time series forecasting. The proposed computational method is simple and avoids complicated computations of the method proposed by Kumar and Gangwar [27]. Lower and upper bounds of the intervals $(u_i)$, membership and non-membership grades are used to define few parameters for the computational algorithm. Notations used in the algorithm along with their description are shown in Table 1.

**Intuitionistic computational algorithm**

For $i = 4, 5, \ldots, N$ (end of the time series data)

Obtain the intuitionistic fuzzy logical relation for year $i-1$ to $i$

$\bar{l}_i \rightarrow \bar{s}_i$

$P = 0$ and $Q = 0$

Compute

$D_i = ||E_{i-1} - E_{i-2}| - |E_{i-2} - E_{i-3}||$

$Z_i = M[A_r] + D_i/2 \times (1 - \mu_i^r)$

$\bar{Z}_i = M[A_r] - D_i/2 \times (1 - \mu_i^r)$

If $Z_i \geq L[A_r]$ and $Z_i \leq U[A_r]$

Then $P = P + Z_i$ and $Q = Q + 1$

Else $P = P + 0$ and $Q = Q + 0$

If $\bar{Z}_i \geq L[A_r]$ and $\bar{Z}_i \leq U[A_r]$

Then $P = P + \bar{Z}_i$ and $Q = Q + 1$

Else $P = P + 0$ and $Q = Q + 0$

Again for ($Q = 1$)

If $L[A_r] < P < M[A_r]$ and $(P - L[A_r]) < (P - M[A_r])$

Then

$F_i = \frac{P \times (|\mu_i^r| - |v_i^r|) + L[A_r] \times (|\mu_i^r| - |v_i^r|)}{(|\mu_i^r - v_i^r|) + |\mu_i^r - v_i^r|}$

Else if $M[A_r] < P < U[A_r]$ and $(P - U[A_r]) < (P - M[A_r])$

then

$F_i = \frac{P \times (|\mu_i^r - v_i^r|) + U[A_r] \times (|\mu_i^r - v_i^r|)}{(|\mu_i^r - v_i^r|) + |\mu_i^r - v_i^r|}$

Else

$F_i = \frac{P \times (|\mu_i^r - v_i^r|) + M[A_r] \times (|\mu_i^r - v_i^r|)}{(|\mu_i^r - v_i^r|) + |\mu_i^r - v_i^r|}$

And for ($Q = 0$) or ($Q = 2$)

$F_i = \frac{P \times (|\mu_i^r - v_i^r|) + M[A_r] \times (|\mu_i^r - v_i^r|)}{Q \times (|\mu_i^r - v_i^r|) + |\mu_i^r - v_i^r|}$

Next $i$
### Table 2. Actual SBI share price from April 2008 to March 2010.

| Month   | SBI share price | Month   | SBI share price |
|---------|-----------------|---------|-----------------|
| 04-2008 | 1819.95         | 04-2009 | 1355.00         |
| 05-2008 | 1840.00         | 05-2009 | 1395.00         |
| 06-2008 | 1496.70         | 06-2009 | 1891.00         |
| 07-2008 | 1567.50         | 07-2009 | 1935.00         |
| 08-2008 | 1638.90         | 08-2009 | 1840.00         |
| 09-2008 | 1569.90         | 09-2009 | 1886.90         |
| 10-2008 | 1375.00         | 10-2009 | 2235.00         |
| 11-2008 | 1325.00         | 11-2009 | 2500.00         |
| 12-2008 | 1376.40         | 12-2009 | 2315.25         |
| 01-2009 | 1205.90         | 01-2010 | 2059.95         |
| 02-2009 | 1132.25         | 02-2010 | 2374.75         |
| 03-2009 | 1113.25         | 03-2010 | 2120.05         |

### Table 3. Intuitionistic fuzzy logical relation for the SBI share price.

\[
\begin{align*}
    l_7 & \rightarrow l_6, \\
    l_6 & \rightarrow l_4, \\
    l_4 & \rightarrow l_3, \\
    l_3 & \rightarrow l_2, \\
    l_2 & \rightarrow l_1, \\
    l_1 & \rightarrow l_1, \\
    l_1 & \rightarrow l_2, \\
    l_2 & \rightarrow l_3, \\
    l_3 & \rightarrow l_4, \\
    l_4 & \rightarrow l_5, \\
    l_5 & \rightarrow l_5, \\
    l_5 & \rightarrow l_5, \\
    l_5 & \rightarrow l_4, \\
    l_4 & \rightarrow l_4.
\end{align*}
\]

### 4. Experimental Study

In this section, we simulate the proposed IFS-based computational method by implementing on three well-known experimental financial time series dataset of the State Bank of India (SBI) share price at Bombay Stock Exchange (BSE), TAIEX and Dow Jones Industrial Average (DJIA).

#### 4.1. SBI Share Price Forecasting

SBI is the largest Indian multinational government-owned public bank sector and finance service company. SBI is one of the top 50 global banks with a balance sheet size of $33 trillion. SBI share price is a highly non-linear database and is very much suitable to test the performance of the proposed IFS-based computational method. SBI share price dataset taken in this study includes observations from April 2008 to March 2010 (Table 2).

**Phase I: Construction of IFS and IFLR.**

We observe \(U_{\text{min}}, U_{\text{max}}\) from Table 2. \(D_1 = 32.25, D_2 = 50\) are computed using expressions defined in step 1 of Section 3.1 to define the universe of discourse for SBI time series data as \([1100, 2550]\). We define 14 fuzzy sets \((A_i, i = 1, 2, 3, \ldots, 14)\) on the universe of discourse and apply the method proposed by Jurio et al. [45] to construct IFS in accordance with 14 fuzzy sets. Intuitionistic fuzzification for 1325 (SBI time series datum for month 12-2008) is explained as follows:

Since the SBI time series datum 1325 partially belongs to fuzzy sets \(A_2\) and \(A_3\), therefore, two IFSs, \(l_2 = <1325, 0.81219, 0.16919>\) and \(l_3 = <1325, 0.15282, 0.73363>\) are constructed. As the maximum membership grade of 1325 is in \(l_2\), therefore, we assign \(l_2\) to 1325. Similarly, other SBI time series data are intuitionistic fuzzified and are shown in Table 3 along with their corresponding IFLRs. Table 4 shows IFLRs of the SBI share price data.
Table 4. Forecasted SBI share price using proposed and other forecasting methods.

| Month  | Actual prices | Singh [40] | Gangwar and Kumar [44] | Bisht and Kumar [36] | Kumar and Gangwar [26] | Kumar and Gangwar [27] | Proposed |
|--------|----------------|------------|-------------------------|----------------------|------------------------|------------------------|----------|
| 04-2008 | 1819.95        | –          | –                       | 1877.657             | 1820.00                | 1856.39                | –        |
| 05-2008 | 1840.00        | –          | –                       | 1877.657             | 1820.00                | 1856.39                | –        |
| 06-2008 | 1496.70        | –          | 1500.00                 | 1877.657             | 1820.00                | 1856.39                | –        |
| 07-2008 | 1567.50        | 1499.58    | 1512.17                 | 1877.657             | 1849.41                | 1520.65                | 1514.28  |
| 08-2008 | 1638.90        | 1665.94    | 1673.14                 | 1877.657             | 1649.41                | 1617.85                | 1617.85  |
| 09-2008 | 1618.00        | 1659.90    | 1533.50                 | 1877.657             | 1649.41                | 1617.85                | 1617.85  |
| 10-2008 | 1496.70        | 1499.58    | 1500.00                 | 1877.657             | 1820.00                | 1856.39                | –        |
| 11-2008 | 1567.50        | 1512.17    | 1533.50                 | 1877.657             | 1820.00                | 1856.39                | –        |
| 12-2008 | 1638.90        | 1673.14    | 1673.14                 | 1877.657             | 1820.00                | 1856.39                | –        |
| 01-2009 | 1375.00        | 1313.75    | 1322.44                 | 1877.657             | 1494.41                | 1410.71                | 1410.71  |
| 02-2009 | 1325.00        | 1300.00    | 1300.00                 | 1877.657             | 1494.41                | 1410.71                | 1410.71  |
| 03-2009 | 1205.90        | 1136.06    | 1144.71                 | 1877.657             | 1572.92                | 1203.57                | 1203.57  |
| 04-2009 | 1355.00        | 1300.00    | –                       | 1877.657             | 1203.57                | 1203.57                | 1203.57  |
| 05-2009 | 1891.00        | 1900.00    | –                       | 1877.657             | 1928.57                | 1928.57                | 1928.57  |
| 06-2009 | 1935.00        | 1900.00    | 1900.00                 | 1877.657             | 1928.57                | 1928.57                | 1928.57  |
| 07-2009 | 1840.00        | 1900.00    | 1900.00                 | 1877.657             | 1928.57                | 1928.57                | 1928.57  |
| 08-2009 | 1886.90        | 1900.00    | 1900.00                 | 1877.657             | 1928.57                | 1928.57                | 1928.57  |
| 09-2009 | 2235.00        | 2300.00    | 2300.00                 | 1877.657             | 1928.57                | 1928.57                | 1928.57  |
| 10-2009 | 2500.00        | 2300.00    | 2300.00                 | 1877.657             | 1928.57                | 1928.57                | 1928.57  |
| 11-2009 | 2394.00        | 2300.00    | 2300.00                 | 1877.657             | 1928.57                | 1928.57                | 1928.57  |
| 12-2009 | 2374.75        | 2300.00    | 2300.00                 | 1877.657             | 1928.57                | 1928.57                | 1928.57  |
| 01-2010 | 2315.25        | 2300.00    | 2300.00                 | 1877.657             | 1928.57                | 1928.57                | 1928.57  |
| 02-2010 | 2059.95        | 2139.10    | 2131.38                 | 1877.657             | 1928.57                | 1928.57                | 1928.57  |
| 03-2010 | 2120.05        | 2100.00    | 2100.00                 | 1877.657             | 1928.57                | 1928.57                | 1928.57  |

Table 5. Comparison of the proposed method with other models in forecasting the SBI share price using RMSE, p-value.

| Model                      | RMSE | p-Value/Difference in average |
|----------------------------|------|------------------------------|
| Singh [40]                 | 59.184 | 0.098/−15.81                  |
| Gangwar and Kumar [44]     | 52.77  | 0.207/−13.511                 |
| Bisht and Kumar [36]       | 175.03 | 0.004***/−79.83               |
| Kumar and Gangwar [26]     | 165.29 | 0.001***/−81.41               |
| Kumar and Gangwar [27]     | 131.28 | 0.010***/−58.64               |
| Gautam et al. [31]         | 145   | 0.001***/−48.25               |
| Proposed model             | 47.62 | *** Denotes the significance at 1%. |

Phase II: Implementation of Computational Algorithm.

We apply the IFS-based computational algorithm to forecast the SBI time series data. Following is sample computation to forecast the SBI time series datum of month 12-2008, i.e. 1325. In the series of the share price data, the share price 1375 and 1325 come in the position of 8 and 9, respectively. Keeping the rule of the algorithm on the mind to forecast 1325, the logical relation for $i = 9$ is $l_3 \rightarrow l_2$. Initially, $P = 0$, $Q = 0$ and the value of the difference parameter ($D_9$), intuitionistic interval parameters ($Z_9$, $ZZ_9$) are as follows (Figure 1).

Other SBI time series data are also forecasted in a similar way and are shown in Table 5. This table also includes forecasted SBI share price using other traditional computational, non-computational and hesitant fuzzy-based forecasting models.
4.2. Forecasting the TAIEX

We implement the proposed IFS-based computational method to forecast TAIEX. TAIEX is a widely accepted financial time series data which is used by many researchers to verify performance and compare the experimental results of fuzzy time series forecasting models. In this study, huge TAIEX experimental data from year 1999 to 2003 are taken which include daily observations from last 3 days data of month October to December. Accuracy in TAIEX forecast is measured in terms of root mean square error (RMSE) and average RMSE. The mathematical expression of RMSE is as follows:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} (\text{forecasted}_i - \text{actual}_i)^2}{n}}$$

Table 6 shows the observed RMSE in the TAIEX forecast from 1999 to 2003 with the proposed and other existing fuzzy time series models.

4.3. Forecasting DJIA

Another financial time series data which is taken to evaluate the performance of the proposed IFS computational method is DJIA. DJIA is the second oldest US market index that shows the trading performance of 30 major public-owned US-based company during a standard trading session. We have implemented the proposed method over the closing price of DJIA from last 3 days of data of months October to December in each year.
| Methods | 1999  | 2000  | 2001  | 2002  | 2003  | Average RMSEs | p-Value / Difference in average |
|---------|-------|-------|-------|-------|-------|--------------|---------------------------------|
| Cai et al. [46] | 102.22 | 131.53 | 112.59 | 60.33 | 51.54 | 91.642 | 0.0014** / −59.46 |
| Chen [4], Yu and Hurang [47] | 120 | 176 | 148 | 101 | 74 | 123.8 | 0.001 ** / −91.61 |
| NASDAQ | 123.64 | 131.10 | 115.08 | 73.06 | 66.36 | 101.848 | 0.0003** / −69.66 |
| Dow Jones | 101.97 | 148.85 | 113.70 | 79.81 | 64.08 | 101.682 | 0.0009** / −69.50 |
| Chen and Chang [48] | 156.92 | 142.70 | 132.76 | 96.06 | 90.27 | 123.742 | 0.0002** / −91.56 |
| NASDAQ and Dow Jones | 106.34 | 130.13 | 113.33 | 72.33 | 60.29 | 96.484 | 0.0002** / −64.30 |
| Chen and Chang [49] | 116.64 | 123.62 | 123.85 | 71.98 | 58.06 | 98.83 | 0.0004** / −66.64 |
| NASDAQ and Dow Jones | 116.59 | 127.71 | 115.33 | 77.96 | 60.32 | 99.582 | 0.0002** / −67.40 |
| Chen and Kao [50] | 114.87 | 128.37 | 123.15 | 74.05 | 60.65 | 101.652 | 0.0009** / −67.69 |
| Dow Jones | 119.32 | 129.87 | 113.67 | 66.82 | 56.10 | 95.542 | 0.0002** / −71.10 |
| NASDAQ and Dow Jones | 111.7 | 129.87 | 123.12 | 71.01 | 64.08 | 101.682 | 0.0009** / −69.51 |
| Chen et al. [51] | 120.01 | 129.87 | 117.61 | 85.85 | 63.10 | 103.288 | 0.0002** / −79.10 |
| NASDAQ and Dow Jones | 106.34 | 130.13 | 113.33 | 72.33 | 60.29 | 96.484 | 0.0002** / −64.30 |
| Chen et al. [52] | 120.01 | 129.87 | 117.61 | 85.85 | 63.10 | 103.288 | 0.0002** / −71.10 |
| NASDAQ and Dow Jones | 106.34 | 130.13 | 113.33 | 72.33 | 60.29 | 96.484 | 0.0002** / −64.30 |
| Ye et al. [53] | 102.11 | 131.30 | 113.83 | 66.45 | 52.83 | 93.304 | 0.0009** / −61.12 |
| Hurang et al. [54] | 100.74 | 125.62 | 113.04 | 62.94 | 51.46 | 90.76 | 0.0008** / −58.57 |
| Yu and Hurang [47] | 109 | 255 | 130 | 84 | 56 | 126.8 | 0.029** / −94.61 |
| Yu and Hurang [47], Hurang and Yu [54] | 109 | 152 | 130 | 84 | 56 | 106.2 | 0.0014** / −74.01 |
| NASDAQ | 115.47 | 127.51 | 121.98 | 74.65 | 66.02 | 101.126 | 0.0001** / −68.94 |
| Dow Jones | 119.32 | 129.87 | 123.12 | 71.01 | 64.08 | 101.682 | 0.0009** / −69.51 |
| NASDAQ and Dow Jones | 109.3 | 127.32 | 115.37 | 64.71 | 52.84 | 92.828 | 0.0007** / −60.64 |
| NASDAQ and Dow Jones and M1b | 120.01 | 129.87 | 117.61 | 85.85 | 63.10 | 103.288 | 0.0002** / −71.10 |
| NASDAQ and Dow Jones and M1b | 116.64 | 123.62 | 123.85 | 71.98 | 58.06 | 98.83 | 0.0004** / −66.64 |
| NASDAQ and Dow Jones and M1b | 116.59 | 127.71 | 115.33 | 77.96 | 60.32 | 99.582 | 0.0002** / −67.40 |
| NASDAQ and Dow Jones and M1b | 114.87 | 128.37 | 123.15 | 74.05 | 60.65 | 101.652 | 0.0001** / −69.47 |
| NASDAQ and Dow Jones and M1b | 112.47 | 131.04 | 117.86 | 77.38 | 60.65 | 99.88 | 0.0002** / −67.69 |
| NASDAQ and Dow Jones and M1b | 110.94 | 124.52 | 114.66 | 64.79 | 53.63 | 92.518 | 0.0006** / −60.33 |
| NASDAQ and Dow Jones and M1b | 116.64 | 123.62 | 123.85 | 71.98 | 58.06 | 98.83 | 0.0004** / −66.64 |
| NASDAQ and Dow Jones and M1b | 116.59 | 127.71 | 115.33 | 77.96 | 60.32 | 99.582 | 0.0002** / −67.40 |
| NASDAQ and Dow Jones and M1b | 114.87 | 128.37 | 123.15 | 74.05 | 60.65 | 101.652 | 0.0001** / −69.47 |
| NASDAQ and Dow Jones and M1b | 112.47 | 131.04 | 117.86 | 77.38 | 60.65 | 99.88 | 0.0002** / −67.69 |
| NASDAQ and Dow Jones and M1b | 116.64 | 123.62 | 123.85 | 71.98 | 58.06 | 98.83 | 0.0004** / −66.64 |
| NASDAQ and Dow Jones and M1b | 116.59 | 127.71 | 115.33 | 77.96 | 60.32 | 99.582 | 0.0002** / −67.40 |

** Denotes the significance at 5%.
*** Denotes the significance at 1%.
Table 7. RMSEs, average RMSE, and \( p \)-value in DJIA forecast from the years 1998–2003.

| Models                                      | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | Average RMSE | \( p \)-Value/Difference in average |
|---------------------------------------------|------|------|------|------|------|------|--------------|-----------------------------------|
| Chen and Chen [49]                          | 124  | 115  | 127  | 121  | 74   | 66   | 104.5        | 0.0048\***/−70.68                  |
| Chen et al. [49]                            | 139  | 113  | 131  | 113  | 65   | 52   | 102.16       | 0.011\***/−68.35                   |
| Hsieh et al. [60]                           | 90   | 94   | 137  | 104  | 132  | 89   | 107.66       | 0.0005\***/−73.85                  |
| SVR-Polynomial [59]                         | 212  | 120  | 178  | 131  | 91   | 139  | 145.16       | 0.0019\***/−111.35                 |
| SVR-Polynomial [59]                         | 426  | 197  | 315  | 136  | 129  | 94   | 216.16       | 0.019\***/−182.35                  |
| SVR-PUK [61]                                | 477  | 578  | 586  | 1168 | 679  | 544  | 672          | 0.0016\***/−638.186                |
| Chen and Chen model with Width [19]         | 81   | 71   | 114  | 75   | 98   | 55   | 82.33        | 0.0042\***/−48.52                  |
| Chen and Chen model with Depth [19]         | 81   | 74   | 108  | 87   | 96   | 48   | 82.33        | 0.005\***/−48.52                   |
| AR (1)                                      | 183  | 269  | 169  | 195  | 168  | 247  | 205.166      | 0.0002\***/−171.35                 |
| AR (2)                                      | 178  | 262  | 167  | 177  | 170  | 249  | 200.5        | 0.0002\***/−166.68                 |
| GARCH-M [62]                                | 223  | 284  | 169  | 224  | 220  | 204  | 220.66       | 0.0001\***/−186.85                 |
| GARCH-AR (1)                                | 97   | 129  | 272  | 85   | 119  | 62   | 127.33       | 0.033\**/−93.52                    |
| GARCH-AR (2)                                | 151  | 377  | 199  | 103  | 115  | 143  | 181.33       | 0.02\**/−147.52                    |
| Entropy based model [63]                    | 69   | 76   | 95   | 101  | 102  | 60   | 83.33        | 0.0027\**/−52.02                   |
| Entropy and FFT based model [63]            | 72   | 71   | 84   | 88   | 86   | 53   | 75.66        | 0.003\***/−41.85                   |
| Gautam et al. [30]                          | 125.03 | 108  | 110.50 | 95.32 | 138.12 | 145 | 125.75       | 0.0002\***/−38                     |
| Proposed intuitionistic computational method| 36.75 | 21.90 | 28.37 | 25.09 | 46.14 | 44.63 | 33.81        |                                   |

** Denotes the significance at 5%.
*** Denotes the significance at 1%.

1998–2003. Table 7 shows the RMSE and average RMSE in forecasting DJIA index from the year 1998 to 2003 using the proposed and earlier developed forecasting models.

5. Error and statistical test analysis

In general, the performance of a time series forecasting models is measured in terms of RMSE and other error measures. Lower RMSE indicates higher accuracy in forecasted outputs. Even though reduced amount of error measures confirm out performance, but still a statistical test is necessary to see significance in accuracy in the forecasted outputs. We use the paired 2-tailed \( t \)-test for statistical performance analysis of the proposed IFS-based computational method.

In order to show performance of the proposed forecasting method over other computational and non-computational methods proposed by Kumar and Gangwar [26], Kumar and Gangwar [27], Singh [40], Gangwar and Kumar [44] and Bisht and Kumar [36] in terms of both error measure and \( p \)-value, we calculate RMSE and apply paired 2-tailed \( t \)-test at 1% and 5% confidence level. On analysing RMSE shown in Table 7, we find that the proposed method out performs other fuzzy and fuzzy cluster-based computational forecasting methods [40, 44]. Proposed IFS-based computational forecasting methods also outperform over other IFS and hesitant fuzzy set based non-computational methods [26, 27, 36]. On comparing the proposed method with other methods using \( p \)-value (Table 7), it was found equally
good as the methods of Singh [40] and Gangwar and Kumar [44] but better than methods of Kumar and Gangwar [26, 27] and Bisht and Kumar [36] in forecasting SBI share price. Figure 2 shows close association between actual and forecasted SBI share price using the proposed method.

Few the most cited models on financial forecasting for TAIEX data are chosen to compare the performance of the proposed method in TAIEX forecasting using RMSE and p-value. Less amount of RMSE (Table 6) confirms better performance of the proposed method over the various well-known and established financial time series forecasting methods in TAIEX forecast for the years 1999–2003 in terms of RMSE. Further to confirm significant improvement in TAIEX forecast using the proposed method, we apply the paired 2-tailed t-test at 1% and 5% confidence levels on RMSE in TAIEX forecast for the years 1999 to 2003. p-Value in Table 6 shows that the proposed method outperforms over the various earlier proposed forecasting method in forecast for the years 1999–2003. A close relationship between actual and forecasted TAIX from year 1999 to 2003 is shown in Figures 3–8.

Finally, we verify the performance of the proposed method using one more very popular experimental data set of DJIA. Experimental data set of DJIA include observations of months November and December from the years 1998 to 2003 and are shown in Table 7. Analysing the performance of the proposed models in terms of both RMSE and p-value (Table 7), it is found to be better than the earlier proposed models in DJIA forecasting from 1998 to 2003. Figures 9–13 show actual and forecasted DJIA from the years 1998–2003.
Figure 4. Actual and forecasted TAIEX of year 2000 using the proposed forecasting method.

Figure 5. Actual and forecasted TAIEX of year 2001 using the proposed forecasting method.

Figure 6. Actual and forecasted TAIEX of year 2002 using the proposed forecasting method.

Figure 7. Actual and forecasted TAIEX of year 2003 using the proposed forecasting method.
Figure 8. Actual and forecasted DJIA of year 1998 using the proposed forecasting method.

Figure 9. Actual and forecasted DJIA of year 1999 using the proposed forecasting method.

Figure 10. Actual and forecasted DJIA of year 2000 using the proposed forecasting method.

Figure 11. Actual and forecasted DJIA of year 2001 using the proposed forecasting method.
Figure 12. Actual and forecasted DJIA of year 2002 using the proposed forecasting method.

Figure 13. Actual and forecasted DJIA of year 2003 using the proposed forecasting method.

6. Conclusion

The present study proposes the IFS-based computational method for financial time series forecasting to address the issue of uncertainty and non-determinism that occur because of non-stochastic factors in the financial time series datum. The proposed computational algorithm uses both membership and non-membership grades of time series data to optimise forecast in the corresponding interval and avoids complex max–min composition operations on IFLRs. Suitability of the proposed IFS-based computational method in financial time series forecasting is shown by implementing it on three different experimental data set of the SBI share price, TAIEX and DJIA. The RMSE measure and paired 2 tailed t-test are used in the present study to confirm better performance of the proposed method in financial time series forecasting. Reduced RMSE and different p-values (Table 5) confirm that the proposed method forecast the SBI share price more accurately than other simple fuzzy and IF-based methods proposed by Kumar and Gangwar [26], Kumar and Gangwar [27], Gautam et al. [30], Bisht and Kumar [36], Singh [40] and Gangwar and Kumar [44]. Tables 6 and 7 also confirm the better performance and significant improvements in TAIEX and DJIA forecasting over various methods proposed by researchers in terms of both RMSE and p-value. The proposed method outperform other recent existing IFS-based time series forecasting methods [26, 27, 30, 36, 40, 44] but is also easy to implement in form of programme in high level programming language for huge time series data set.

In future studies, high order difference parameters can also be explored in the computational algorithm to propose new computational methods for fuzzy time series forecasting models with optimum complexity. Moreover, the computational algorithm can also be proposed for weighted intuitionistic fuzzy time series. Evolutionary computing techniques, e.g. GA, PSO and ant bee colony can be used for optimisation of weights for weighted intuitionistic fuzzy time series.
Disclosure statement
No potential conflict of interest was reported by the authors.

Notes on contributors

Kamlesh Bisht was born in 1990 in Uttarakhand, India. He received the M.Sc. degree from Kumaun University, Nainital, India in 2013. He received Ph.D. degree in Mathematics from G.B. Pant University of Agriculture and Technology in 2018. He has published seven research papers on fuzzy time series forecasting in international journals.

Sanjay Kumar received the B.Sc. degree from Kumaun University, Nainital, India in 1992. He received Master and Ph.D. degrees in Mathematics from G.B. Pant University of Agriculture and Technology in 1995 and 1998, respectively. Currently, he is an Associate Professor in the Department of Mathematics, Statistics and Computer Science in College of Basic Sciences and Humanities of G. B. Pant University of Agriculture and Technology Pantnagar, India. He has published 50 research papers in international journals. His current research interests are in the field of artificial intelligence with the application of soft and non-conventional computing techniques in time series forecasting and multi-criteria decision-making problems.

ORCID
Sanjay Kumar https://orcid.org/0000-0003-3659-5387

References
[1] Zadeh LA. Fuzzy sets. Inf Control. 1965;8(3):338–353.
[2] Song Q, Chissom BS. Forecasting enrollments with fuzzy time series—part I. Fuzzy Sets Syst. 1993;54(1):1–9.
[3] Song Q, Chissom BS. Forecasting enrollments with fuzzy time series—part II. Fuzzy Sets Syst. 1994;62(1):1–8.
[4] Chen SM. Forecasting enrollments based on fuzzy time series. Fuzzy Sets Syst. 1996;81(3):311–9.
[5] Chen SM, Chu HP, Sheu TW. TAIEX forecasting using fuzzy time series and automatically generated weights of multiple factors. IEEE Trans Syst Man Cybern A Syst Hum. 2012;42(6):1485–1495.
[6] Hung KC, Lin KP. Long-term business cycle forecasting through a potential intuitionistic fuzzy least-squares support vector regression approach. Inf Sci. 2013;224:37–48.
[7] Wang L, Liu X, Pedrycz W, et al. Determination of temporal information granules to improve forecasting in fuzzy time series. Expert Syst Appl. 2014;41(6):3134–3142.
[8] Wang YN, Lei Y, Lei Y, et al. Multi-factor high-order intuitionistic fuzzy time series forecasting model. J Syst Eng Electron. 2016;27(5):1054–1062.
[9] Rubio A, Bermúdez JD, Vercher E. Improving stock index forecasts by using a new weighted fuzzy-trend time series method. Expert Syst Appl. 2017;76:12–20.
[10] Patel J, Shah S, Thakkar P, et al. Predicting stock market index using fusion of machine learning techniques. Expert Syst Appl. 2015;42(4):2162–2172.
[11] Singh P, Borah B. Forecasting stock index price based on M-factors fuzzy time series and particle swarm optimization. Int J Approx Reason. 2014;55(3):812–833.
[12] Talarposhti FM, Sadaei HJ, Enayatifar R, et al. Stock market forecasting by using a hybrid model of exponential fuzzy time series. Int J Approx Reason. 2016;70:79–98.
[13] Lee LW, Wang LH, Chen SM. Temperature prediction and TAIEX forecasting based on fuzzy logical relationships and genetic algorithms. Expert Syst Appl. 2007;33(3):539–550.
[14] Lee LW, Wang LH, Chen SM. Temperature prediction and TAIEX forecasting based on high-order fuzzy logical relationships and genetic simulated annealing techniques. Expert Syst Appl. 2008;34(1):328–336.
[15] Park JI, Lee DJ, Song CK, et al. TAIFEX and KOSPI 200 forecasting based on two-factors high-order fuzzy time series and particle swarm optimization. Expert Syst Appl. 2010;37(2):959–967.

[16] Hsu LY, Horng SJ, Kao TW, et al. Temperature prediction and TAIFEX forecasting based on fuzzy relationships and OPTPSO techniques. Expert Syst Appl. 2010;37(4):2756–2770.

[17] Fu FP, Chi K, Che WG, et al. High-order difference heuristic model of fuzzy time series based on particle swarm optimization and information entropy for stock markets. In 2010 International Conference on Computer Design and Applications; 2010:2–2–210. IEEE.

[18] Yolcu OC, Yolcu U, Egrioglu E, et al. High order fuzzy time series forecasting method based on an intersection operation. Appl Math Model. 2016;40(19-20):8750–8765.

[19] Chen MY, Chen BT. A hybrid fuzzy time series model based on granular computing for stock price forecasting. Inf Sci. 2015;294:227–241.

[20] Gupta C, Jain A, Tayal DK, et al. ClusFuDE: forecasting low dimensional numerical data using an improved method based on automatic clustering, fuzzy relationships and differential evolution. Eng Appl Artif Intell. 2018;71:175–189.

[21] Soto J, Melin P, Castillo O. A new approach for time series prediction using ensembles of IT2FNN models with optimization of fuzzy integrators. Int J Fuzzy Syst. 2018;20(3):701–728.

[22] Atanassov KT. Intuitionistic fuzzy sets. Fuzzy Sets Syst. 1986;20:87–96.

[23] Joshi BP, Kumar S. A computational method of forecasting based on intuitionistic fuzzy sets and fuzzy time series. In Proceedings of the International Conference on Soft Computing for Problem Solving (SocProS 2011) 2011:993–1000. Springer, New Delhi.

[24] Joshi BP, Kumar S. Intuitionistic fuzzy sets based method for fuzzy time series forecasting. Cybern Syst. 2012;43(1):34–47.

[25] Joshi BP, Kumar S. Fuzzy time series model based on intuitionistic fuzzy sets for empirical research in stock market. Int J Appl Evol Comput. 2012;3(4):71–84.

[26] Kumar S, Gangwar SS. A fuzzy time series forecasting method induced by intuitionistic fuzzy sets. Int J Model Simul Sci Comput. 2015;6(4):1550041.

[27] Kumar S, Gangwar SS. Intuitionistic fuzzy time series: an approach for handling nondeterminism in time series forecasting. IEEE Trans Fuzzy Syst. 2016;24(6):1270–1281.

[28] Wang YN, Lei Y, Fan X, et al. Intuitionistic fuzzy time series forecasting model based on intuitionistic fuzzy reasoning. Math Probl Eng. 2016;2016. DOI:10.1155/2016/5035160

[29] Fan X, Lei Y, Wang Y, et al. Long-term intuitionistic fuzzy time series forecasting model based on vector quantisation and curve similarity measure. IET Signal Process. 2016;10(7):805–814.

[30] Gautam SS, Singh SR. A refined method of forecasting based on high-order intuitionistic fuzzy time series data. Prog Artif Intell. 2018;7(4):339–350.

[31] Egrioglu E, Yolcu U, Bas E. Intuitionistic high-order fuzzy time series forecasting method based on pi-sigma artificial neural networks trained by artificial bee colony. Granular Comput. 2018;1–6. DOI:10.1007/s41066-018-00143-5

[32] Sotirov S, Sotirova E, Atanassova V, et al. A hybrid approach for modular neural network design using intercriteria analysis and intuitionistic fuzzy logic. Complexity. 2018. DOI:10.1155/2018/3927951

[33] Castillo O, Alanis A, Garcia M, et al. An intuitionistic fuzzy system for time series analysis in plant monitoring and diagnosis. Appl Soft Comput. 2007;7(4):1227–1233.

[34] Torra V, Narukawa Y. On hesitant fuzzy sets and decision. In 2009 IEEE International Conference on Fuzzy Systems; 2009:1378–1382. IEEE.

[35] Zhu B, Xu Z, Xia M. Dual hesitant fuzzy sets. J Appl Math. 2012;2012. DOI:10.1155/2012/879629

[36] Bisht K, Kumar S. Fuzzy time series forecasting method based on hesitant fuzzy sets. Expert Syst Appl. 2016;64:557–568.

[37] Bisht K, Joshi DK, Kumar S. Dual hesitant fuzzy set-based intuitionistic fuzzy time series forecasting. In: Ambient communications and computer systems. Singapore: Springer; 2018. p. 317–329.

[38] Gupta KK, Kumar S. Hesitant probabilistic fuzzy set based time series forecasting method. Granular Comput. 2018:1–20. DOI:10.1007/s41066-018-0126-1

[39] Singh SR. A robust method of forecasting based on fuzzy time series. Appl Math Comput. 2007;188(1):472–484.
[40] Singh SR. A simple method of forecasting based on fuzzy time series. Appl Math Comput. 2007;186(1):330–339.
[41] Singh SR. A computational method of forecasting based on fuzzy time series. Math Comput Simul. 2008;79(3):539–554.
[42] Singh SR. A computational method of forecasting based on high-order fuzzy time series. Expert Syst Appl. 2009;36(7):10551–10559.
[43] Joshi BP, Kumar S. A computational method for fuzzy time series forecasting based on difference parameters. Int J Model Simul Sci Comput. 2013;4(01):1250023.
[44] Gangwar SS, Kumar S. Partitions based computational method for high-order fuzzy time series forecasting. Expert Syst Appl. 2012;39(15):12158–12164.
[45] Jurio A, Paternain D, Bustince H, et al. A construction method of Atanassov's intuitionistic fuzzy sets for image processing. In 2010 5th IEEE International Conference Intelligent Systems; 2010:337–342. IEEE.
[46] Cai Q, Zhang D, Zheng W, et al. A new fuzzy time series forecasting model combined with ant colony optimization and auto-regression. Knowl-Based Syst. 2015;74:61–68.
[47] Yu TH, Huarng KH. A bivariate fuzzy time series model to forecast the TAIEX. Expert Syst Appl. 2008;34(4):2945–2952.
[48] Chen SM, Chang YC. Multi-variable fuzzy forecasting based on fuzzy clustering and fuzzy rule interpolation techniques. Inf Sci. 2010;180(24):4772–4783.
[49] Chen SM, Chen CD. TAIEX forecasting based on fuzzy time series and fuzzy variation groups. IEEE Trans Fuzzy Syst. 2011;19(1):1–12.
[50] Chen SM, Chen SW. Fuzzy forecasting based on two-factors second-order fuzzy-trend logical relationship groups and the probabilities of trends of fuzzy logical relationships. IEEE Trans Cybern. 2015;45(3):391–403.
[51] Chen SM, Kao PY. TAIEX forecasting based on fuzzy time series, particle swarm optimization techniques and support vector machines. Inf Sci. 2013;247:62–71.
[52] Chen SM, Manalu GM, Pan JS, et al. Fuzzy forecasting based on two-factors second-order fuzzy-trend logical relationship groups and particle swarm optimization techniques. IEEE Trans Cybern. 2013;43(3):1102–1117.
[53] Cheng SH, Chen SM, Jian WS. Fuzzy time series forecasting based on fuzzy logical relationships and similarity measures. Inf Sci. 2016;327:272–287.
[54] Huarng KH, Yu TH. The application of neural networks to forecast fuzzy time series. Phys A Stat Mech Appl. 2006;363(2):481–491.
[55] Huarng KH, Yu TH, Hsu YW. A multivariate heuristic model for fuzzy time-series forecasting. IEEE Trans Syst Man Cybern B. 2007;37(4):836–846.
[56] Ye F, Zhang L, Zhang D, et al. A novel forecasting method based on multi-order fuzzy time series and technical analysis. Inf Sci. 2016;367-368:41–57.
[57] Yu TH, Huarng KH. A neural network-based fuzzy time series model to improve forecasting. Expert Syst Appl. 2010;37(4):3366–3372.
[58] Chen SM, Phuong BD. Fuzzy time series forecasting based on optimal partitions of intervals and optimal weighting vectors. Knowl-Based Syst. 2017;118:204–216.
[59] Vapnik VN. The nature of statistical learning. Theory. 1995.
[60] Hsieh TJ, Hsiao HF, Yeh WC. Forecasting stock markets using wavelet transforms and recurrent neural networks: an integrated system based on artificial bee colony algorithm. Appl Soft Comput. 2011;11(2):2510–2525.
[61] Üstün B, Melssen WJ, Buydens LM. Facilitating the application of Support Vector Regression by using a universal Pearson VII function based kernel. Chemometr Intell Lab Syst. 2006;81(1):29–40.
[62] Engle RF, Lilien DM, Robins RP. Estimating time varying risk premia in the term structure: the ARCH-M model. Econometrica. 1987;55:391–407.
[63] Chen MY, Chen BT. Online fuzzy time series analysis based on entropy discretization and a Fast Fourier Transform. Appl Soft Comput. 2014;14:156–166.