An Optimal Decision Model to Determine the Location of Hubs in a Network of Communication

Fibri Rakhmawati¹, Saib Suwilo², Sutarman², M. Zarlis³

¹ Graduate School of Mathematics, University Sumatera Utara, Indonesia
² Department of Mathematics, University of Sumatera Utara, Medan, Indonesia
³ Faculty of Computer Science and Information Technology, Universitas Sumatera Utara, Medan, Indonesia

* hmvengkang@yahoo.com

Abstract. The hub location, regarded as a special variant of facility location problem, can be defined as an assignment problem of locating hubs and assigning the terminal nodes to these hubs. The objective is to minimize the cost of hub installation and the cost of routing the traffic in the communication network. There may also be capacity restrictions on the amount of traffic that can transit by hubs. This paper discusses how to model the polyhedral properties of the problems and develop a feasible neighbourhood search method to solve the model.

1. Introducing
In a transportation or communication network there would be a facility called hub. The main role of hubs could be a central distributor services to the other local server. The hub facility can spread most types of transportation services or communications.

The problem is to decide hub location can happen in most of network design, specifically when it is necessary to spread services. From the nature of a location problem, the objective is to obtain the location of hub nodes and the allocation of services to these chosen hub nodes. In standard problems when the locations of the facilities was determined then, each customer node receives service from its nearest chosen facility. However, for hub location problems, it is not necessary to assign each demand node to its nearest hub will give optimal solutions. Therefore the optimal allocations of demand centers to the located hubs must be determined. In term of applications beside in telecommunication, the hub location problem can be found in public transportation [1], packet delivery [2] and [3], and logistic planning systems [4]. Interesting review for the hub location problem can be seen in [5] and [6].

This paper focuses on hub location problem in network of telecommunication. This networks are built and designed to distribute communicable resources. On the advent of the information age there interest in the efficient design of communication networks are undoubtedly increased. These networks all have a similar intent, that is, how to carry the expected communication traffic flow from origin to destination with minimum cost. The most important factor of network design is to obtain the best layout of the components (nodes and arcs) in such a way to minimize cost while satisfying performance criterion such as no transmission delay, throughput or reliability of meeting the demand.

If one considers the communication requirement, which can be thought as a set of nodes to represent origin and destinations, the main problem is how to join the nodes together in the most efficient
manner. This problem is not an easy one; as for event a small number of nodes there would be thousands of ways of joining them together. This type of problem is called NP-complete [7].

Given that we are unable to investigate all possible ways for linking the nodes, the challenge is to get a method for finding a restricted number that will obtain a good solution. Another thing, communication network design plannings are not time critical. This means that most approaches which have been devised to solve the problems should be based on heuristics, such as, simulated annealing, tabu search, evolutionary computing [8] and [9]. In [10] they proposed a hybrid heuristic based on simulated annealing, tabu list, and improvement procedures for the uncapacitated hub location problem with single allocation. [11] developed a tabu search meta-heuristic for solving the multiple allocation p-hub median problem. In [12] they presented a hybrid genetic algorithm pruned with a diagonalization method of the intermodal hub-and-spoke network layout problem with multiple stakeholders and multi-type users. [13] developed two different meta-heuristic approaches that consist of two phases: a solution construction and a solution improvement based on local search for the intermodal terminal location problem. In [14] they provided a review for the most recent advances of solution methods for solving the hub location problem.

Exact methods are also proposed by several authors. [15] addressed a branch-and-cut algorithm for the hub location problem contained single assignment. [19] developed a Benders decomposition method for the uncapacitated hub location problem for multiple assignments in communication and transportation systems. In [16] they proposed a branch-and-price approach for the capacitated hub location problem using single assignment. In their method, Lagrangean relaxation is used to get tight lower bounds of the restricted master problem. In [17] they presented an exact algorithm capable of solving large-scale networks of the uncapacitated hub location case with multiple assignments. The algorithm utilizes Benders decomposition to a strong path-based formulation of the problem. [18] proposed also a Benders decomposition method for solving the tree of hub location problems. The exact methods are efficient only at solving relatively small network problems. Nevertheless, they are limited on searching all possible branches and obtaining an optimal solution under a reasonable computational time when the problem size is getting larger.

In many applications, it is necessary to add redundancy into the network to guarantee reliability; if one link fails it would still be possible to connect from origin to destination. In this paper we use a combination of linear Programming (LP) and heuristic based on neighbourhood search approach to tackle this problem; and these approaches can be found in Sections 4 and 5.

2. Methodology

2.1. Formulating the Network Design Problem

The design problem is aimed to meet all traffic requirements with minimum cost. Let a tree network synthesis problem which involves n nodes, and there are n \( n-2 \) possible tree structures, e.g. million possibilities for a network with as small as 10 nodes. The information needed to formulate the problem clearly is the traffic demand between each origin and destination (O – D) pair, and the linear cost function for carrying traffic on each (possible) link \((i,j)\) between nodes \(i\) and \(j\).

The following notation is needed.

\[
\begin{align*}
\text{Sets} & \quad \text{N} \quad \text{Nodes}, \\
& \quad \text{T} \quad \text{Time periods}, \\
& \quad M_k \quad \text{Modules with different kinds available for a hub located at node } k \in N \\
\end{align*}
\]

\[
\begin{align*}
\text{Traffic communication flow} & \quad f_{ij}^t \quad \text{Flow originated from node } i \in N \text{ that will go to node } j \in N \text{ at time period } t \in T \\
& \quad TFE_i \quad \text{Total flow emerged from node } i \in N \text{ at time period } t \in T
\end{align*}
\]
**TFD** \( j \)  Total flow for node \( j \in N \) at time period \( t \in T \)

**Capacities**

\( C_{mk} \)  Capacity of module type \( m \in M_k \) available for node \( k \in N \)

**Costs**

\( \alpha_k \)  Setup cost to locate a hub at node \( k \in N \) in period \( t \in T \)

\( \beta_{kj} \)  Costs to operate a hub link between hubs \( k \in N \) and \( j \in N \) in period \( t \in T \)

\( \delta_{mk} \)  Cost for installing a module of type \( m \in M_k \) at hub \( k \in N \) in period \( t \in T \)

\( \gamma_k \)  Cost to operate per unit of flow for hub \( k \in N \) in period \( t \in T \)

\( \sigma_{ij} \)  Cost for sending flow from node \( i \in N \) to node \( j \in N \) in period \( t \in T \)

**Decision variables**

\( x_{ij} \)  Binary variable to select whether there is a link between node \( i \in N \) to hub \( j \in N \) in period \( t \in T \)

\( z_{kj} \)  Binary variable to indicate whether a hub link between hubs \( k \in N \) and \( j \in N \) in period \( t \in T \)

\( v_{km} \)  Binary variable indicating whether a module with type \( m \in M_k \) is installed at hub \( k \in N \) in period \( t \in T \)

\( y_{ijk} \)  Number of communication flow from node \( i \in N \) routed from hub \( k \in N \) to hub \( j \in N \) in period \( t \in T \)

### 2.2. The Model

The objective of the problem is to find the hub location such that to minimize the overall costs. From practical point of view that the costs which could be involved are as follows: cost to link a node to a hub, cost to install a module, cost to locate a hub and cost relating to the amount of communication flow. The objective function can be expressed mathematically as follows.

\[
\text{Minimize} \sum_{i \in T} \sum_{i \in N} \alpha_i (x_{ii}^{t+1} - x_{ii}^t) + \sum_{i \in T} \sum_{k \in N} \sum_{j \in N, j \neq k} \beta_{kj} z_{kj} + \sum_{i \in T} \sum_{k \in N} \sum_{m \in M_k} \delta_{mk} v_{km}^t + \sum_{i \in T} \sum_{k \in N} \sum_{j \in N} \sum_{m \in M_k} \sigma_{ij} y_{ij}^t + \sum_{i \in T} \sum_{i \in N} \sum_{j \in N} \sum_{k \in N} \sum_{m \in M_k} TFD_{ij} y_{ij}^t
\]

(1)

Constraints to be imposed for the decision variables should be considered. The constraints are formulated in the following.

\[
\sum_{j \in N} x_{ij}^t = 1, \quad \forall i \in N, \forall t \in T
\]

(2)

\[
x_{ij}^t \leq x_{jj}^t, \quad \forall i \in N, \forall t \in T
\]

(3)

\[
x_{jj}^t \leq x_{jj}^{t-1}, \quad \forall j \in N, \forall t \in T
\]

(4)

\[
\sum_{m \in M_k} v_{km}^t \leq x_{kk}^t, \quad \forall k \in N, \forall t \in T
\]

(5)
The model is in the form of large scale mixed integer linear programs.

2.3. Neighbourhood Search

Generally, in integer programming the reduced gradient vector, which is used to detect an optimality condition, is not available, even if the problems are convex. Thus it is necessary to impose a certain condition for the local testing search steps in such a way to assure that we have obtained the “best” suboptimal integer feasible solution.

[18] has proposed a kind of quantity test to replace the pricing test in the integer programming problem. The test is run by a search through the neighbours of a given feasible point to see whether a nearby point is also feasible and could give an improvement to the objective function.

Let $[\beta]_k$ be an integer feasible point belongs to a set of neighbourhood $N([\beta]_k)$. Then we define a neighbourhood system associated with $[\beta]_k$, that is, if such an integer feasible point satisfies the following two conditions

a. If $[\beta]_j \in N([\beta]_k)$ then $[\beta]_k \in N([\beta]_j), j \neq k$.

b. $N([\beta]_k) = [\beta]_k + N(0)$.

With respect to the neighbourhood system mentioned, the proposed integerizing strategy can be written as follows.

Let a non-integer component, $x_k$, of a continuous optimal vector, $x_B$. The adjacent points of $x_k$, should be considered are $[x_k]$ and $[x_k] + 1$. If one of these points meets the constraints and y gives a minimum deterioration of the optimal objective value then we move to another component, if not we already obtained integer-feasible solution.

Let $[x_k]$ be the integer feasible point which satisfies the conditions. We could then say that if $[x_k] + 1 \in N([x_k])$ then the point $[x_k] + 1$ is either infeasible or gives an inferior value to the objective function obtained with respect to $[x_k]$. In this case $[x_k]$ can be said to be an “optimal” integer feasible solution to original the integer programming problem. Clearly, in our case, a neighbourhood search is run through proposed feasible points such that the integer feasible solution would be at the least distance from the optimal continuous solution.

3. Results and Discussion

After finding the solution of the relaxed problem, the procedure for searching a suboptimal but integer-feasible solution from an optimal continuous solution can be stated as follows.

Let $x = [x] + f$, $0 \leq f \leq 1$ be the (continuous) optimal solution of the relaxed problem, $[x]$ is the integer component of non-integer basic variable $x$ and the fractional component is $f$.

Stage 1:

Step 1. Find row $i^*$ the smallest integer infeasibility, such that $\delta_i = \min\{f_i, 1 - f_i\}$

Step 2. Do a pricing operation

$$\sum_{i \in N} TFE_i x_{ik}^t + \sum_{j \in N} \sum_{i \in N} y_{ij}^t \leq \sum_{m \in M_k} C_{km} v_{km}^t, \quad \forall k \in N, \forall t \in T \quad (6)$$

$$\sum_{j \in N, j \neq k} y_{jk}^t - \sum_{j \in N, j \neq k} y_{jk}^* = TFD_j x_{ik}^t - \sum_{j \in N} f_j x_{jk}^t, \quad \forall i, k \in N, \forall t \in T \quad (7)$$

$$z_{ij}^t \leq x_{ik}^t, \quad \forall k, j \in N, k < j, \forall t \in T \quad (8)$$

$$z_{ij}^t \leq x_{ij}^t, \quad \forall k, j \in N, k < j, \forall t \in T \quad (9)$$

$$y_{ijk}^t + y_{jik}^t \leq TFE_i z_{ij}^t, \quad \forall i, j, k \in N, k < j, \forall t \in T \quad (10)$$

$$x_{ij}^t, z_{ij}^t, v_{km}^t \in \{0, 1\}, \quad \forall i, j, k \in N, \forall m \in M_k, \forall t \in T \quad (11)$$

$$x_{ii}^t = 0, \quad \forall i \in N \quad (12)$$

$$y_{ij}^t \geq 0, \quad \forall i, j, k \in N, \forall t \in T \quad (13)$$
\[ v_i^T = e_i^T B^{-1} \]

Step 3. Calculate \( \sigma_{ij} = v_i^T a_j \)

With \( j \) corresponds to

\[
\min_j \left\{ \left[ \frac{d_j}{\sigma_{ij}} \right] \right\}
\]

Do a calculation of the movement of nonbasic \( j \) at lower bound and upper bound

Otherwise go to next non-integer nonbasic or superbasic \( j \) (if available). Eventually the column \( j^* \) is to be increased form lower bound or decreased from upper bound. If none go to next \( i^* \).

Step 4. Solve \( B a_{j^*} = a_{j^*} \) for \( a_{j^*} \).

Step 5. Conduct ratio test for the basic variables in order to maintain feasible due to the releasing of nonbasic \( j^* \) from its bounds.

Step 6. Do exchanging basis

Step 7. If row \( i^* = \emptyset \) go to Stage 2, if not Repeat from step 1.

Stage 2: Do integer search in order to improve the value integer feasible solution

4. Conclusions

The approach addressed in this paper has been shown to be efficient on a limited range of moderately sized network problem. However, the success of these investigations with a hope that it will prove to provide a meaningful tool for communication network design. The ability of the approach to find a suboptimal integer feasible solution is promising

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