A Module For Boosted Dark Matter Event Generation in GENIE

Joshua Berger

Pittsburgh Particle Physics, Astrophysics, and Cosmology Center,
Department of Physics and Astronomy,
University of Pittsburgh, Pittsburgh, USA

Abstract

Models that produce a flux of semi-relativistic or relativistic boosted dark matter at large neutrino detectors are well-motivated extensions beyond the minimal weakly interacting massive particle (WIMP) paradigm. Current and upcoming liquid argon time projection chamber (LArTPC) based detectors will have improved sensitivity to such models, but also require improved theoretical modeling to better understand their signals and optimize their analyses. I present the first full Monte Carlo tool for boosted dark matter interacting with nuclei in the energy regime accessible to LArTPC detectors, including the Deep Underground Neutrino Experiment (DUNE). The code uses the nuclear and strong physics modeling of the GENIE neutrino Monte Carlo event generator with particle physics modeling for dark matter. The code will be available in GENIE v3. In addition, I present a code for generating a GENIE-compatible flux of boosted dark matter coming from the Sun that is released independently.

I. INTRODUCTION

The evidence for the existence of dark matter with a large abundance in our universe is now virtually incontrovertible. On the other hand, this mysterious form of matter has thus far eluded all attempts to discover its non-gravitational interactions with the Standard Model (SM) particles. If such interactions exist, as they should in plausible explanations for the cosmic origin of dark matter, then the cross-section is constrained to be rather small. Furthermore, in order to explain the observed galactic structure, dark matter must clump into non-relativistic halos that surround the luminous disk of galaxies. As such, very sensitive detectors with extremely low keV thresholds have been designed to look for the resultant rare soft hadronic interactions where, in this regime, the scattering of hadronic matter results in coherent nuclear recoils \( [1] \).

Neutrino detectors are optimized for detecting recoils of weakly interacting particles with
small cross-sections as well. Due to the higher energy expected from atmospheric and accelerator produced neutrinos, as well as the possibility of charged current interactions, the thresholds in these detectors for hadronic recoils have been significantly higher than a few keV. This threshold precludes them from detecting hadronic recoils of halo dark matter. On the other hand, there could exist other populations of dark matter that have very different energy distributions. For example, dark matter could be produced in semi-annihilation processes $\chi + \chi \rightarrow \chi + X$ [2] or in annihilation of a heavier component of dark matter $\chi_A + \chi_A \rightarrow \chi_B + \chi_B$ [3] in an astrophysical source of concentrated dark matter. These interactions could happen with a large rate in populations of solar captured dark matter or in the galactic center [4, 5], for example. If the population of dark matter produced in one of these ways has a velocity that is an $\mathcal{O}(1)$ fraction of the speed of light then it can transfer sufficient energy to a nucleus [5] or electron [4, 6–8] to be detected in a neutrino detector. Models of this type have also been considered in the context of inelastic dark matter models [9–12] where direct detection is generally ineffective. Depending on the boost of the dark matter, the maximum energy threshold to detect the boosted dark matter population differs. Absent a large hierarchy, which is of course a possibility, the boost will be not too much larger than 1 given one of the annihilation sources above. In this regime, the typical energy transfer is in the 10s of MeV to few GeV range. These energies are unfortunately too low to be accessible by the very largest cosmic neutrino detectors such as IceCube [13], but they are within reach of atmospheric and accelerator neutrino detectors such as the Deep Underground Neutrino Experiment (DUNE) [14] and its liquid argon time projection chamber (LArTPC) [15] predecessors [16–20], as well as Super- and Hyper-Kamiokande [21, 22]. It is therefore interesting to determine the capability of these detectors to discover streams of boosted dark matter.

In order to study such prospects, it is important to have a proper calculation of the expected signal. At these recoil energies, the dominant hadronic processes are elastic nucleon recoils, “shallow” inelastic recoils dominated by baryon resonances along with diffuse contributions, and, for boosts larger than roughly 2, deep inelastic scattering off the nucleons. The first of these is rather simple to calculate and simulate under the simplifying approximation that dark matter scatters of effectively free nucleons at rest. Since the energy scale of the recoils is comparable to or not too much larger than the scale of the nuclear Fermi momentum for large nuclei of $\mathcal{O}(100 \text{ MeV})$, nuclear effects can play a significant role in the
outcome of these interactions and the assumption of free nucleons at rest is not generally valid. In the context of a water Čerenkov detector such as Super-Kamiokande, inelastic recoils are rather tricky to reconstruct and the threshold for proton recoils is high enough that nuclear effects can be neglected with some error. On the other hand, LArTPCs will have significantly lower thresholds and stronger prospects for reconstructing multi-particle events with several tracks. In order to go beyond elastic scattering and to include nuclear effects, a more sophisticated tool is required.

Since dark matter scattering shares several similarities with neutrino neutral current scattering, I have developed such a tool as an optional module within the neutrino event generator GENIE. The philosophy of the new module has been to minimally modify the existing nuclear and strong physics modeling of GENIE, while putting in the modified parton level modeling and kinematics possible in dark matter models. For situations where empirical tuning or modeling is used based on neutrino data, this could introduce additional inaccuracies in the modeling of boosted dark matter interactions. These issues are nearly unavoidable within such modeling absent a dark matter discovery and data. This tool nevertheless provides a first description of these dark matter interactions in the regimes accessible to large volume neutrino experiments.

Other neutrino event generation tools are publicly available. GENIE is the standard used by the experiments based at Fermi National Accelerator Laboratory (FNAL), which are the experiments for which this tool is most relevant at the moment. In order to ensure compatibility with existing detector simulation and analysis frameworks, I have opted to implement these models within GENIE. Furthermore, GENIE is being actively developed and offers a sufficiently flexible framework to include the necessary modifications so that dark matter can be included. It would be interesting in the future to compare the results of this tool with other tools and other models of the nuclear and strong physics, but such a comparison is beyond the scope of this work.

The Boosted Dark Matter module that I have developed implements elastic and deep inelastic boosted dark matter nucleon interactions, with resonant scattering forthcoming, as well as the somewhat simpler possibility of electron interactions for completeness. The current version allows for fermionic or scalar dark matter interacting via a vector boson mediator with quarks and electrons. The left- and right-handed charges of fermionic dark matter and the 4 light quark flavors are free parameters that can be set by the user. This is
a flexible and broad set of models to start with, though more may be implemented in future versions. The nuclear physics effects and fragmentation physics for deep inelastic scattering proceed just as for neutrino interactions in GENIE, though some of the parameters therein have been tuned specifically for neutrino scattering.

Crucial to event generation for boosted dark matter is a determination of the flux of dark matter. In addition to the module above, I have developed an application for the generation of a flux of dark matter from the sun in a format accessible to GENIE. More specifically, this tool uses the SolTrack solar position code [25] to randomly select positions of the sun over the course of the year and generate a simple N-tuple flux file with dark matter coming from those positions.

The remainder of this paper is structured as follows. In Section II I further describe the particle physics model. Section III is devoted to determining the cross-section and hadronization input to the GENIE module. I then review the installation and operation of the GENIE module in Section IV. The flux file generation application is presented in Section V. A sample complete event generation is presented in Section VI. Finally, I conclude in Section VII.

II. PARTON LEVEL MODEL OF BOOSTED DARK MATTER

The particle physics models implemented in the boosted dark matter models include two new states: a vector boson mediator and a fermionic or scalar dark matter particle. The dark matter-mediator interaction Lagrangian is given by

$$\mathcal{L}_{\chi, \text{int}} = g_{Z'} Z'_{\mu} \bar{\chi} \gamma^\mu (Q_L^\chi P_L + Q_R^\chi P_R) \chi,$$

(1)

for fermionic dark matter, where $\chi$ is the dark matter field, $Z'_{\mu}$ is the mediator field, $g_{Z'}$ is the vector boson gauge coupling, and $Q_{L,R}^\chi$ are the charges of the left- and right-handed components of dark matter respectively. For bosonic dark matter, the interaction Lagrangian is given by

$$\mathcal{L}_{\chi, \text{int}} = i Q_S^\chi g_{Z'} Z'^\mu (\chi^\dagger \partial_\mu \chi - \partial_\mu \chi^\dagger \chi),$$

(2)

where $Q_S^\chi$ is the charge of the dark matter. The mediator $Z'$ then interacts with the SM fermions as

$$\mathcal{L}_{f, \text{int}} = g_{Z'} Z'_{\mu} \bar{\psi}_f \gamma^\mu (Q_L^f P_L + Q_R^f P_R) \psi_f,$$

(3)
where \( f \) denote the SM fermions. In practice, only the flavors of fermion that contribute in GENIE are \( u, d, s, c, e \). Note that, for the purposes of the cross-sections below, it is simpler to work in terms of vector and axial couplings

\[
Q_V^f = \frac{Q_L^f + Q_R^f}{2}, \quad Q_A^f = \frac{-Q_L^f + Q_R^f}{2},
\]

though the GENIE input is taken in terms of the left- and right-handed couplings. It will further be helpful to define the \( Z' \) currents. It will be helpful to define the \( Z' \) currents such that

\[
\mathcal{L}_{f, \text{int}} = g_{Z'} Z'_\mu J^\mu_f.
\]

Given the interactions above, the input required to GENIE, aside from some bookkeeping and kinematic changes, is the differential cross-section for each relevant nuclear-level process and a treatment of the low energy hadronization model discussed below. I now discuss each of the interaction types in turn.

III. BDM INTERACTION MODELING

A. General Kinematics

In the energy regime relevant to LArTPC neutrino detectors, coherent nuclear scattering is highly suppressed and scattering is dominantly off nucleons that become unbound from the nucleus or electrons that have negligible binding energies compared to the momentum transfer. GENIE therefore requires the differential cross-section for dark matter-nucleon or dark matter-electron scattering in the rest frame of the nucleon or electron respectively.

I consider the scattering of a dark matter particle with mass \( M_\chi \) off a nucleon with mass \( M_N \) or electron with mass \( M_e \). The dark matter energy in the working reference frame is \( E_\chi \). The momenta are labeled as

\[
\chi(k) + N/\ell(p) \to \chi(k') + X(p'),
\]

where \( X \) is the final state hadronic or leptonic system, that is everything in the final state other than the outgoing dark matter. Two different classes of scattering processes will be considered below: elastic and inelastic.
In elastic scattering, \( X = N/\ell \) for hadronic and electronic scattering respectively. A single kinematic variable describes a given scattering event. For hadronic scattering, I choose the invariant square of the four-momentum transfer,

\[
Q^2 = -q^2 = t, \tag{7}
\]

where \( q = p' - p = k - k' \). This parameter ranges from

\[
0 < Q^2 < 4 \left| p_{\text{CM}} \right|^2, \tag{8}
\]

where

\[
\left| p_{\text{CM}} \right|^2 = \left( \frac{E_{\text{CM}}^2 + M^2 - M_N^2}{2 E_{\text{CM}}} \right)^2 - M_N^2, \quad E_{\text{CM}}^2 = M_X^2 + 2 E_X M_N + M_N^2. \tag{9}
\]

For electron scattering, it is convenient to work in terms of the energy loss ratio for the dark matter,

\[
y = \frac{Q^2}{2 E_X M_e} = 1 - \frac{E'_{\chi}}{E_{\chi}}, \tag{10}
\]

where \( E'_{\chi} \) is the outgoing dark matter energy. This variable ranges from

\[
0 < y < \frac{2 M_e (E_{\chi}^2 - M_{\chi}^2)}{2(M_{\chi}^2 + M_e^2 + 2 M_e E_{\chi})}. \tag{11}
\]

For inelastic scattering of any kind, which is only applicable to scattering off a nucleon, two variables are required to describe the kinematics of the dark matter recoil. These can be chosen to be \( Q^2 \) along with the invariant mass of the final state hadronic system \( W \),

\[
W^2 = p'^2. \tag{12}
\]

The invariant mass \( W \) ranges over all kinematically accessible values,

\[
M_N < W < E_{\text{CM}} - M_{\chi}. \tag{13}
\]

For a given \( W \), the momentum transfer ranges over its standard kinematic range assuming a final state \( p' \) mass of \( W \), that is

\[
\left( \left| p_{\text{CM}} \right| - \left| p'_{\text{CM}} \right| \right)^2 < Q^2 < \left( \left| p_{\text{CM}} \right| + \left| p'_{\text{CM}} \right| \right)^2, \tag{14}
\]

with

\[
\left| p'_{\text{CM}} \right|^2 = \left( \frac{E_{\text{CM}}^2 + M_{\chi}^2 - W^2}{2 E_{\text{CM}}} \right)^2 - M_{\chi}^2. \tag{15}
\]
It is more convenient to use dimensionless parameters for DIS. I work in terms of Bjorken $x$ and the energy loss ratio $y$, which are related to $Q^2$ and $W$ by

$$x = \frac{Q^2}{Q^2 + W^2 - M_N^2}, \quad y = \frac{Q^2 + W^2 - M_N^2}{2E \chi M_N}. \quad (16)$$

Note that this $y$ is the same $y$ used for electron scattering. The full range for $x$ is

$$\frac{Q^2_{\text{min}}}{Q^2_{\text{min}} + W^2_{\text{max}} - M_N^2} < x < \frac{Q^2_{\text{max}}}{Q^2_{\text{max}} + W^2_{\text{min}} - M_N^2}, \quad (17)$$

with the minimum and maximum $W$ and $Q^2$ defined in terms of the full range in Eqs. (13) and (14). At fixed $x$, the range for $y$ is given by

$$\frac{W^2_{\text{min}} - M_N^2}{2E \chi M_N (1 - x)} < y < \frac{2M_N x (E^2 \chi - M_N^2)}{E \chi (M_N^2 + x^2 M_N^2 + 2x M_N E \chi)}. \quad (18)$$

### B. Elastic Scattering

Elastic scattering in the context of BDM is defined to be scattering processes that, before final state nuclear interactions, eject a single nucleon from the nucleus. In other words, elastic scattering is the process

$$\chi + N \rightarrow \chi + N, \quad (19)$$

where $N = p, n$ is a nucleon in the target nucleus. Since the BDM interactions are defined in terms of the interactions with the quarks, form factors are required to determine the interactions of the dark matter with the nucleons. The computation of this process for neutrinos as used by GENIE is due to Ahrens et. al. [26].

The most general matrix element for the current consistent with $CP$ is given by

$$\langle N(k')| \sum_f J_f^\mu |N(k)\rangle =$$

$$\tau(k') \left( F_1^N(Q^2) \gamma^\mu + F_2^N(Q^2) \frac{i q^\nu \sigma^{\mu\nu}}{2M_N} + F_A^N(Q^2) \gamma^\mu \gamma^5 + F_P^N(Q^2) \frac{q^\mu}{2M_N} \gamma^5 \right) u(k), \quad (20)$$

where I have dropped all spin labels, $q = k - k'$, $Q^2 = -q^2$, and $\sigma^{\mu\nu}$ is as usual

$$\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]. \quad (21)$$

Note that unlike in the case of neutrino scattering, there is a contributing pseudoscalar form factor. This contribution is proportional to the mass and axial charge of the dark matter. I revisit this contribution below, after detailing the other three form factors.
To begin, I determine the form factor normalization at $Q^2 = 0$. I break up the form factors by flavor as

$$F_i^N = Q_j^u F_i^u|N| + Q_j^d F_i^d|N| + Q_j^s F_i^s|N|,$$  

(22)

where for $i = 1, 2$, $j = V$ and for $i = A, P$, $j = A$. I assume isospin symmetry, such that $F_i^{u|p} = F_i^{d|n}$ and $F_i^{d|p} = F_i^{u|n}$. For the vector form factors, the electromagnetic interactions determine the form factors at $Q^2 = 0$. In that case, $F_1^N(0)$ is simply the nucleon charge and $F_2^N(0)$ is its anomalous magnetic moment. From (22), I then find

$$1 = \frac{2}{3} F_1^{u|p}(0) - \frac{1}{3} F_1^{d|p}(0), \quad 0 = \frac{2}{3} F_1^{d|p}(0) - \frac{1}{3} F_1^{u|p}(0),$$

(23)

from which

$$F_1^{u|p}(0) = 2, \quad F_1^{d|p}(0) = 1.$$  

(24)

Similarly, for the magnetic form factor, I find

$$\mu_p - 1 = \frac{2}{3} F_2^{u|p}(0) - \frac{1}{3} F_2^{d|p}(0), \quad \mu_n - 1 = \frac{2}{3} F_2^{d|p}(0) - \frac{1}{3} F_2^{u|p}(0),$$

(25)

where $\mu_N$ is the magnetic moment of the nucleon, from which

$$F_2^{u|p}(0) = 2\mu_p + \mu_n - 1, \quad F_2^{d|p} = 2\mu_n + \mu_p - 1.$$  

(26)

The isospin triplet combination of the axial charges can be predicted from $\beta$ decay. The other combinations are trickier to constrain, but there are some reasonably consistent experimental and lattice quantum chromodynamics (QCD) determinations [27–31]. The form factors at $Q^2 = 0$ are known as the spin form factors,

$$F_A^{f|p} = \Delta f.$$  

(27)

As with the vector form factors, I take $F_A^{u|n} = F_A^{d|p}$, $F_A^{d|n} = F_A^{u|p}$, and $F_A^{s|n} = F_A^{s|p}$, as expected by isospin. The default values for the spin form factors is described below when describing the input to GENIE.

For describing the momentum dependence of the vector form factors, it is convenient to work in the Breit frame, that is the frame in which $k + k'$ has vanishing spatial component. In this frame, the vector hadronic current matrix element becomes

$$\langle J_V \rangle = \left(2 M_N [F_1 - \tau F_2] \xi^\dagger(s') \xi(s), [F_1 + F_2] \xi^\dagger(s') i\vec{q} \times \vec{\sigma} \xi(s) \right),$$

(28)
where $\xi$ is the spin wave-function along the $z$ axis. It will be convenient to define

$$G_E = F_1 - \tau F_2, \quad G_M = F_1 + F_2,$$

where $\tau = Q^2/4M_N^2$, based on the time and spatial components of Eq. (28). These correspond to the charge and magnetization distributions within the nucleon. It is found to excellent agreement in electromagnetic scattering that these form factor follow a dipole-like distribution, that is the form factor expected for a bound state of oppositely charged point particles. The wavefunction for such a system is well-known to be exponential and the form factor is the Fourier transform of this distribution, leading to

$$G_E, G_M \propto \frac{1}{(1 + Q^2/M_V^2)^2},$$

where $M_V$ needs to be determined by fitting to data. The form factors can then be solved for, yielding

$$F_1^N(Q^2) = \frac{F_1^N(0) + \tau [F_1^N(0) + F_2^N(0)]}{(1 + \tau)(1 + Q^2/M_v^2)^2}, \quad F_2^N(Q^2) = \frac{F_2^N(0)}{(1 + \tau)(1 + Q^2/M_v^2)^2}. \quad (31)$$

For the axial form factor, I also assume dipole form,

$$F_A \propto \frac{1}{(1 + Q^2/M_A^2)^2}, \quad (32)$$

even though this assumption is less well justified in the axial case [32].

Thus far, I have neglected the pseudoscalar form factor. For the isospin octet form factors corresponding to the $\pi$ and $\eta$, these can be predicted by the assumption of PCAC and the dominance of the lowest meson pole. For the pion-like isospin current, there is some data indicating that this assumption holds. The other combinations are very difficult to access within the SM. There has been some lattice study of this, with mixed results particularly for the pion.

$$\frac{F_P^{u|N} - F_P^{d|N}}{F_A^{u|N} - F_A^{d|N}} = 4 \frac{M_N^2}{M_\pi^2 + Q^2}, \quad \frac{F_P^{u|N} + F_P^{d|N} - 2 F_P^{s|N}}{F_A^{u|N} + F_A^{d|N} - 2 F_A^{s|N}} = 4 \frac{M_N^2}{M_\eta^2 + Q^2}. \quad (33)$$

I then assume that the contribution of the strange quark is small, allowing for a solution for the two non-vanishing pseudoscalar form factors.

Given the form factors, the differential cross-section in $Q^2$ can be straightforwardly calculated. The result can be written as

$$\frac{d\sigma}{dQ^2} = \sigma_0 \left[ A \pm B \frac{s - u}{M_N^2} + C \frac{(s - u)^2}{M_N^4} \right]. \quad (34)$$
as in Ref. [26], with + for dark matter and − for anti-dark matter. The prefactor $\sigma_0$ is normalized to be
\[ \sigma_0 = \frac{g_Z^4 M_N^2}{4 \pi (E_\chi^2 - M_\chi^2) (Q^2 + M_{Z'}^2)}. \] (35)
It is helpful to further subdivide the term $A$ as
\[ A = A_{11} F_1^2 + A_{22} F_2^2 + A_{12} F_1 F_2 + A_{AA} (F_A - \tau F_P)^2. \] (36)
For fermionic dark matter, the coefficients are given by
\[ A_{11} = (Q_A^\chi)^2 (\tau - 1) (\delta + \tau) + (Q_V^\chi)^2 \tau (\tau - \delta - 1), \]
\[ A_{22} = -\tau \{(Q_A^\chi)^2 (\tau - 1) (\delta + \tau) + (Q_V^\chi)^2 [\delta + (\tau - 1) \tau]\}, \]
\[ A_{12} = 2 \tau [2 (Q_A^\chi)^2 (\tau + \delta) - (Q_V^\chi)^2 (\delta - 2 \tau)], \]
\[ A_{AA} = (1 + \tau) [(Q_A^\chi)^2 (\tau + \delta) + (Q_V^\chi)^2 (\tau - \delta)] + 16 (Q_A^\chi)^2 \delta \tau^2 \left(\frac{M_N^2}{M_{Z'}^2} + \frac{1}{4 \pi}\right)^2, \]
\[ B = 8 Q_V^\chi Q_A^\chi \tau F_A (F_1 + F_2), \]
\[ C = [(Q_A^\chi)^2 + (Q_V^\chi)^2] (F_1^2 + \tau F_2^2 + F_A^2), \] (37)
with $\delta = M_\chi^2/M_N^2$. For scalar dark matter, there is no axial coupling, so the dark matter current is conserved and there is no longitudinal coupling. Furthermore, the dark matter and anti-dark matter couplings are the same as there is no interference process. This simplifies the result significantly,
\[ A = -(Q_S^\chi)^2 (\tau + \delta) [(F_1 + F_2)^2 \tau + F_A^2 (1 + \tau)], \]
\[ B = 0, \quad C = (Q_S^\chi)^2 (F_1^2 + \tau F_2^2 + F_A^2). \] (38)

C. Deep Inelastic Scattering

1. DIS Cross-section

The modeling of the deep inelastic scattering cross-section follows closely the model implemented in GENIE for neutrino scattering, explicitly spelled out by Paschos and Yu [33]. It describes the process
\[ \chi + N \rightarrow \chi + X, \] (39)
in the $W$ regime, where $X$ includes a baryon, plus any number of additional particles. I follow their results with the necessary modifications to account for the massive nature of DM and the different couplings here. The phase space variables are taken to be $x$ and $y$. 

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Deep inelastic scattering occurs when the dark matter has sufficient energy to break apart the nucleon, approximately scattering off of quarks and gluons following a parton distribution. This process can be generally parameterized by summing over all possible final states. To see how this works in practice, we write the total cross-section for scattering into all possible hadronic final states

\[
d\sigma = \sum_{X} \frac{\mathcal{M}(\chi + N \to \chi + X_1 + \cdots + X_n)^2}{4M_N\sqrt{E^2_\chi - M^2_\chi}} (2\pi)^4 d\Pi_{n+1}(k + p; k', p'_1, \ldots, p'_n),
\]

where \(d\Pi_n\) denotes the \(n\)-body Lorentz-invariant phase space volume

\[
d\Pi_{n}(P; p_1, \ldots, p_n) = \prod_i \frac{d^3p_i}{(2\pi)^3} \frac{1}{2E_i} (2\pi)^4 \delta^{(4)} \left( P - \sum_i p_i \right),
\]

with \(p_1 + \cdots + p_n = P\). The phase space integration can be broken up as

\[
d\Pi_{n+1}(k + p; k', p'_1, \ldots, p'_n) = d\Pi_2(k + p; k', p') d\Pi_n(p'; p'_1, \ldots, p'_n) (2\pi)^3 dW^2.
\]

Then, to leading order in \(g_{Z'}\), the matrix element can be written as

\[
\sum_{X} \int d\Pi_n (2\pi)^3 |\mathcal{M}|^2 = g_{Z'}^4 L^{\mu\nu} \Delta_{\mu\rho} \Delta_{\nu\sigma} W^{\rho\sigma},
\]

where the “leptonic” tensor \(L^{\mu\nu}\) is given by

\[
L^{\mu\nu} = \frac{16}{2S + 1} \sum_{\text{spins}} \langle \chi; k'| J^\mu_{\chi} | \chi; k \rangle \langle \chi; k'| J^\nu_{\chi} | \chi; k \rangle^*,
\]

where \(S\) is the spin of the dark matter, the \(Z'\) propagator is given in unitary gauge by

\[
\Delta^{\mu\nu} = \frac{g^{\mu\nu} - q^\mu q^\nu / M^2_{Z'}}{Q^2 + M^2_{Z'}},
\]

and the hadronic tensor \(W\) is defined to include the remaining pieces

\[
W^{\mu\nu} = \sum_{X,f,\text{spins}} \frac{1}{2} \int d\Pi_n (2\pi)^3 \langle X; p' | J^\mu_f | N; p \rangle \langle N; p | J^\nu_f | X; p' \rangle^*.
\]

The tensor \(L\) is given by

\[
L^{\mu\nu} = -16 \left\{ g^{\mu\nu} \left[ Q^2 (Q_X)^2 + (4M_X^2 + Q^2) (Q_X^A)^2 \right] - 2 \left[ (Q_X^V)^2 + (Q_X^A)^2 \right] (k^\mu k'^\nu + k'^\mu k^\nu) \right\}
\]

\[
\pm 4i Q_X^V Q_X^A \epsilon^{\nu\rho\sigma} k^\rho k'^\sigma \right\},
\]

\[1\] The fermionic dark matter in this model is necessarily a Dirac fermion, so it gets a spin averaging factor. This factor is omitted in the neutrino case, where it is effectively a Weyl fermion for the purposes of scattering at these energies; only the left-handed helicity is produced and the probability of it mixing into the other helicity is small.
for fermionic dark matter with \(-\) (+) corresponding to (anti-)dark matter, while for scalar dark matter

\[ L^{\mu\nu} = 16 (Q_S^2)^2 (k^\mu + k'^\mu) (k^\nu + k'^\nu). \]  

(48)

After the integration and applying conservation of energy and momentum, the hadronic tensor can only be a function of the four vectors \(p\) and \(q\), as well as the scalar masses, \(Q^2\), and \(W^2\). It is conventional to trade the last variable for \(x\) at this stage, so that the most general form of the hadronic tensor is\(^2\)

\[ W^{\mu\nu} = -g^{\mu\nu} F_1(x, Q^2) + \frac{p^\mu p'^\nu}{p \cdot q} F_2(x, Q^2) - i e^{\mu\nu\rho\sigma} \frac{p_\rho q_\sigma}{2 p \cdot q} F_3(x, Q^2) + \frac{q^\mu q'^\nu}{p \cdot q} F_4(x, Q^2) + \frac{p^\mu q'^\nu + q^\mu p'^\nu}{2 p \cdot q} F_5(x, Q^2). \]  

(49)

Putting the pieces together, integrating over the trivial phase space variables, and changing variables, I find

\[ \frac{d\sigma}{dx \, dy} = \frac{y E_x^2}{16 \pi (E_x^2 - M_\chi^2)} \frac{1}{(Q^2 + M_D^2)^2} L^{\mu\nu} \Delta_{\mu\nu} \Delta_{\rho\sigma} W^{\rho\sigma}, \]  

(50)

The final result for the cross-section for fermionic dark matter scattering is then

\[ \frac{d\sigma}{dx \, dy} = \mathcal{F} \left\{ 4y \left[ x y (Q_V^2 + Q_A^2) + \frac{M_A^2}{M_N E_x} (Q_A^2 (2 + \Pi) - Q_V^2) \right] F_1 
+ 2 \left[ 2 [1 - y] - \frac{M_N}{E_x} x y \right] (Q_V^2 + Q_A^2) - 2 \frac{M_A^2}{E_x^2} Q_V^2 - \frac{y M_N^2}{x E_x M_N} (1 - \Pi) Q_V^2 \right] F_2 
+ 4 Q_V Q_A x [2 - y] y F_3 + 8 \frac{M_A^2}{M_N E_x} \Pi Q_A^2 x y F_4 - 4 \frac{M_A^2}{M_N E_x} \Pi Q_A^2 y F_5 \right\}, \]  

(51)

where \(+\) is for dark matter and \(-\) for anti-dark matter and I define

\[ \mathcal{F} = \frac{g_Z^4 M_N E_x^3}{32 \pi (E_x^2 - M_\chi^2)} \frac{1}{(M_\chi^2 + 2 x y E_x M_N)^2}, \]  

\[ \Pi = \left( 1 + \frac{2 E_x M_N x y}{M_\chi^2} \right)^2. \]  

(52)

For scalar dark matter scattering, I find

\[ \frac{d\sigma}{dx \, dy} = \mathcal{F} \left[ -2 Q_S^2 y \left( x y + 2 \frac{M_N^2}{M_N E_x} \right) F_1 + Q_S^2 (y - 2)^2 F_2 \right]. \]  

(53)

The structure functions \(F_i\) here are given in terms of the quark PDFs by the following relations by

\[ F_1 = 4 x \sum_f (Q_V^f + Q_A^f) \left[ f_f(x, Q^2) + \hat{f}_f(x, Q^2) \right] \]  

\[ x F_3 = -8 x \sum_f Q_V^f Q_A^f \left[ f_f(x, Q^2) - \hat{f}_f(x, Q^2) \right], \]  

(54)

\(^2\) Note that this expression corrects a minor typo in Paschos and Yu: there is a non-trivial denominator in the coefficient of \(F_5\).
where $f_f$ are the parton distribution functions for quark flavor $f$, combined with the Callan-Gross relation \[ 2x F_1 = F_2, \] and the Albright-Jarlskog relations \[ F_4 = 0, \quad x F_5 = F_2. \]

2. Hadronization Modeling

By default, GENIE includes two hadronization models. The first, based on PYTHIA 6, is most accurate at relatively large energies, above $W \gtrsim 3$ GeV, with greater accuracy at larger $W$. The other is an empirical fit model by Koba, Nielsen, and Olesen more accurate for $W \lesssim 2.3$ GeV. An interpolation is used between the two models.

While the former is, in principle, independent of the incident particle and equally accurate for neutrino and dark matter scattering, the latter is specifically tuned to (anti-)neutrino scattering data. In order to adapt this model to dark matter scattering, I make the assumption that the parameterization fit to neutrino scattering applies to dark matter scattering. Absent a first principles model in the low $W$ regime, this is the best available option.

D. Electron Scattering

Electron scattering has the most straightforward description for the purposes of neutrino detectors, as the binding energy is negligible in comparison to the relevant momentum transfers. It describes the process

\[ \chi + e^- \rightarrow \chi + e^- \] \hspace{1cm} (57)

The phase space is described in terms of the variable $y$. Neutrino scattering off electrons in GENIE uses a calculation by Marciano and Parsa \[37\].

The calculation of the differential cross-section in $y$ yields

\[
\frac{d\sigma}{dy} = A \left[ (T_1 - T_2 - T_3) Q_V^x Q_V^e + (T_1 - T_2 + T_3 - T_4) Q_A^x Q_V^e + (T_1 + T_2 - T_3 - T_4) Q_V^x Q_A^e + (T_1 + T_2 + T_3 + T_4 + T_L) Q_A^x Q_A^e \pm 4 (1 - (1 - y)^2) Q_V^x Q_A^e Q_V^e Q_A^e \right],
\] \hspace{1cm} (58)
for fermionic dark matter with + and − corresponding to dark matter and anti-dark matter respectively where
\[
A = \frac{g_Z^4 E_\chi^3 M_e}{4 \pi (E_\chi^2 - M_\chi^2)} \frac{1}{[M_2^Z + 2 E_\chi M_e y]^2},
\] (59)

and

\[
T_1 = 1 + (1 - y)^2, \quad T_2 = y \frac{M_e}{E_\chi}, \quad T_3 = y \frac{M^2_\chi}{E_\chi M_e},
\]
\[
T_4 = \frac{2 M^2_2}{E_\chi^2}, \quad T_L = 2 \left[ \frac{2 y M_\chi M_e}{M^2_2} + \frac{M_\chi}{E_\chi} \right]^2.
\] (60)

For scalar dark matter, the cross-section has a simpler form,
\[
\frac{d\sigma}{dy} = A \left[ (T_1 - T_3) Q_S^{1/2} Q_V^{1/2} + (T_1 - T_2 - T_3 - T_4) Q_S^{1/2} Q_A^{1/2} \right],
\] (61)

for both dark matter and anti-dark matter with
\[
T_1 = 2 (1 - y), \quad T_2 = y \frac{M_e}{E_\chi}, \quad T_3 = y \frac{M^2_\chi}{M_e E_\chi}, \quad T_4 = 2 \frac{M^2_\chi}{E_\chi^2}.
\] (62)

IV. INSTALLATION AND OPERATION OF GENIE MODULE

The installation follows the standard installation of GENIE, outlined at http://www.genie-mc.org/. The only required difference is, when building GENIE itself,

```
shell% configure --enable-boosted-dark-matter [other-options]
shell% gmake
shell% gmake install (*optional*)
```

The module contains several useful executable applications as outlined below. I begin, however, by describing several interesting options for the user to set in $GENIE/config/CommonParameters.xml.

A. Boosted Dark Matter Options

The file $GENIE/config/CommonParameters.xml contains several common parameters useful to multiple modules in the GENIE code. The most relevant options for boosted dark matter running are described below. Other options for GENIE running are described in
All options set below should be kept the same for studying a given model point, including for spline generation, event generation and event printing.

The CommonParameters.xml file contains several param_set blocks, among which the most relevant are as follows.

- **QuasiElastic:**
  - QEL-Ma: Axial form factor mass parameter \( M_A \) in GeV. Default: 0.990.
  - QEL-Mv: Vector form factor mass parameter \( M_V \) in GeV. Default: 0.840.

- **AnomMagnMoments:**
  - AnomMagnMoment-P: Proton anomalous magnetic moment. Default: 2.7930.
  - AnomMagnMoment-N: Neutron anomalous magnetic moment. Default: -1.913042.

- **NonResBackground:**
  - UseDRJoinScheme: Suppress the low energy DIS cross-section to smoothly join with resonant scattering. Default: true.
  - Wcut: Hadronic invariant mass at which to switch to a fully DIS model. Default: 1.7.

- **BoostedDarkMatter:**
  - ZpCoupling: Default \( Z' \) coupling, which can be overrode at run time. Default: 1.
  - DarkLeftCharge: Left-handed fermionic dark matter charge under \( Z' \). Default: -1.
  - DarkRightCharge: Right-handed fermionic dark matter charge under \( Z' \). Default: 1.
  - DarkScalarCharge: Scalar dark matter charge under \( Z' \). Default: 1.
  - UpLeftCharge: Left-handed up quark charge under \( Z' \). Default: -1.
  - UpRightCharge: Right-handed up quark charge under \( Z' \). Default: 1.
  - DownLeftCharge: Left-handed down quark charge under \( Z' \). Default: -1.
  - DownRightCharge: Right-handed down quark charge under \( Z' \). Default: 1.
  - StrangeLeftCharge: Left-handed strange quark charge under \( Z' \). Default: -1.
  - StrangeRightCharge: Right-handed strange quark charge under \( Z' \). Default: 1.
  - CharmLeftCharge: Left-handed charm quark charge under \( Z' \). Default: -1.
  - CharmRightCharge: Right-handed charm quark charge under \( Z' \). Default: 1.
  - ElectronLeftCharge: Left-handed electron charge under \( Z' \). Default: -1.
  - ElectronRightCharge: Right-handed electron charge under \( Z' \). Default: 1.
  - DMEL-Mpi: Pion mass for pseudoscalar form factor in GeV. Default: 0.1349766.
DMEL-Meta: Eta meson mass for pseudoscalar form factor in GeV. Default: 0.547862.
AxialVectorSpin-u: Up quark spin form factor of the proton. Default: 0.84.
AxialVectorSpin-d: Down quark spin form factor of the proton. Default: −0.43.
AxialVectorSpin-s: Strange quark spin form factor of the proton. Default: −0.09.

The default values for the QuasiElastic, AnomMagnMoments, and NonResBackground parameters are kept at their default GENIE values. The default BoostedDarkMatter parameters are chosen as follows. The coupling is taken to be 1. Note that only a trivial rescaling of the cross-section by $g_4^2$ is required to obtain any other value of this parameter when the coupling is taken to be 1 in GENIE. The charges are chosen to give purely axial couplings for all the fermions. The pion and eta masses are taken from Ref. [38]. The spin form factors are taken from Ref. [30].

In addition, it is worth noting one particularly important parameter for DIS event generation. Hadronizer sets the hadronization model for final state hadronic system. The default choices is an interpolation between the KNO and PYTHIA models described above. See $GENIE/src/Physics/Hadronization/ for an updated selection of options.

B. Standalone Event Generation: gevgen_dm

To generate a beam of dark matter hitting the detector along the +z axis, use the application gevgen_dm. The operation works as follows, with optional arguments in square brackets

```shell
genvgen_dm -n nev
   -e energy (or energy range)
   -m mass
   -t target_pdg
   --tune tune
   [-h]
   [-r run#]
   [-g zp_coupling]
   [-z med_ratio]
   [-f flux_description]
```

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The options function as follows, with the unchanged option description adapted from [23, 24]:

- **-n nev:**
  Number of events to generate.

- **-e energy:**
  Dark matter energy or comma separated range of energies in GeV.

- **-m mass:**
  Dark matter mass in GeV.

- **-t target_pdg:**
  Target nucleus PDG code. The PDG2006 conventions are used, so that a nucleus is specified by an integer 10LZZZAAA1, with L the strange number (0 for stable nuclei), ZZZ the three digit atomic number, AAA the three digit mass number, and I the isomer number (0 for ground state nuclei). For example, $^{16}$O has a code 1000080160 and $^{40}$Ar has a code 1000180400. This option is required only if the geometry is not specified using -f.

- **--tune tune:** Specifies a generator tune. For dark matter running, there are currently two tunes, corresponding to fermionic and scalar dark matter. These tunes are GDM18_00a_00_000 and GDM18_00b_00_000 respectively.

- **-h:**
  Prints help for gevgen_dm.
• `-r run#`:
  Specifies the run number.

• `-g zp_coupling`:
  Specifies the $Z'$ gauge coupling. Default: Taken from
  `$GENIE/config/CommonParameters.xml`.

• `-z med_ratio`:
  Specifies the ratio of the mediator mass to the boosted dark matter mass. Default: 0.5.

• `-f flux_description`:
  Specifies the dark matter flux spectrum. Only required if an energy range is specified.
  The flux can be specified in one of three ways:

  - As a ‘function’.
    For example, to specify a flux of the form $x^2 + 4e^{-x}$:
    `-f 'x{*}x+4{*}exp(-x)'

  - As a ‘vector file’.
    The file should contain 2 columns corresponding to energy (in GeV) and flux (in arbitrary units).
    For example, to specify a flux described in the file `/data/fluxvec.data`:
    `-f /data/fluxvec.data'

  - As a ‘1-D histogram (TH1D) in a ROOT file’.
    The syntax is `-f /full/path/file.root,object_name`.
    For example, in order to specify that the flux is described by the ‘nue’ TH1D object
    in `/data/flux.root`:
    `-f /data/flux.root,nue'

• `-o outfile_name`: Specifies an output filename. Default: `gntp.0.ghep.root`.

• `-w`:
  Forces generation of weighted events. This option is relevant only if a dark matter
flux is specified via ‘-f’. In this context, ‘weighted’ refers to an event generation biasing in selecting an initial state (a flux dark matter and target pair at a given dark matter energy). Internal weighting schemes for generating event kinematics can still be enabled independently even if ‘w’ is not set. Don’t use this option unless you understand what the internal biasing does and how to analyze the generated sample. Default: unweighted events.

- **--seed random_number_seed:**
  Specifies a random number generation seed.

- **--cross-sections xml_file:**
  Specifies the path of an input XML file with a dark matter cross-section spline, for example as generated by `gmkspl_dm`. Default: calculate cross-sections.

- **--event-generator-list list_name:**
  Specifies the list of event generators to use. The available dark matter generators are as follows:

  - **DMEL:** Elastic dark matter scattering;
  - **DMDIS:** Deep inelastic dark matter scattering;
  - **DME:** Dark matter-electron scattering;
  - **DMH:** Combines **DMEL** and **DMDIS**;
  - **DM:** Combines **DMH** and **DME**.

  Default: **DM**.

- **--message-thresholds xml_file:**
  Specifies the level of output to print for each module. The level is controlled by an XML file allowing users to customize the threshold of each message stream. See `$GENIE/config/Messenger.xml` for the XML file structure. Default: **Messenger.xml**.

- **--unphysical-event-mask mask:**
  Specifies a 16-bit mask to allow certain types of unphysical events to be output to the event file. Default: all unphysical events rejected.
• --event-record-print-level level: Specify the amount of information to be printed in the output event file. See GHeRecord::Print() for allowed settings.

• --mc-job-status-refresh-rate rate: Allows users to customize the refresh rate of the status file.

• --cache-file root_file: Allows users to specify a ROOT file so that results of calculation cached throughout a MC job can be re-used in subsequent MC jobs.

C. Flux-based Event Generation: gevgen_fluxdm

To generate events based on a particular dark matter flux, including direction and energies, use the application gevgen_fluxdm. The operation works as follows, with optional arguments in square brackets

```
shell% gevgen_fluxdm -f flux
    -g geometry
    -M mass
    --tune tune
    [-h]
    [-r run#]
    [-c zp_coupling]
    [-v med_ratio]
    [-t top_volume_name_at_geom]
    [-m max_path_lengths_xml_file]
    [-L length_units_at_geom]
    [-D density_units_at_geom]
    [-n n_of_events]
    [-o output_event_file_prefix]
    [-F fid_cut_string]
    [-S nrays_scan]
    [-z zmin_start]
    [--seed random_number_seed]
    [--cross-sections xml_file]
```
The options function as follows, with the unchanged option description adapted from [23, 24]:

- **-f flux**
  Specifies the flux in a ROOT file containing the simple N-tuple of dark matter momentum four-vectors. The ROOT file should contain two TTree, flux and meta. The flux tree must contain branches of GSimpleNtpEntry and the meta tree must contain branches of GSimpleNtpMeta. These classes are found in $GENIE/src/Tools/Flux/GSimpleNtpFlux.cxx and more details can be found there. The solar dark matter flux generation code described below outputs ROOT files of an appropriate form for this option.

- **-g geometry**
  Specifies the detector geometry. This option can be
  
  - A ROOT file containing a ROOT/Geant4-based geometry description (TGeoManager). By default the entire input geometry will be used. Use the -t option to allow event generation only on specific geometry volumes.
  
  - A mix of target nuclei, each with a corresponding weight. This option should only be used when the beam and/or detector are sufficiently uniform. The target mix is specified as a comma-separated list of nuclear PDG codes (in the PDG2006 convention: 10LZZZAAAI) followed by their corresponding weight fractions in brackets, as in:
    
    ‘-t code1[fraction1],code2[fraction2],...’

- **-M mass:**
  Dark matter mass in GeV. Must be consistent with the dark matter four vectors specified in the flux file.
• **--tune tune**: Specifies a generator tune. For dark matter running, there are currently two tunes, corresponding to fermionic and scalar dark matter. These tunes are `GDM18_00a_00_000` and `GDM18_00b_00_000` respectively.

• **-h**:
  Prints help for `gevgen_fluxdm`.

• **-r run#**:  
  Specifies the run number.

• **-c zp_coupling**:  
  Specifies the $Z'$ gauge coupling. Default: Taken from `$GENIE/config/CommonParameters.xml`.

• **-v med_ratio**:  
  Specifies the ratio of the mediator mass to the boosted dark matter mass. Default: 0.5.

• **-t top_vol_name_at_geom**:  
  Specifies the input top volume for event generation. Default: ‘master volume’ of the input geometry resulting in neutrino events being generated over the entire geometry volume. The option can be used to simulate events at specific sub-detectors.

• **-m max_path_lengths_xml_file**:  
  Specifies an XML file with the maximum density-weighted path-lengths for each nuclear target in the input geometry. If the option is not set then, at the MC job initialization, GENIE will scan the input geometry to determine the maximum density-weighted path lengths for all nuclear targets. The computed information is used for calculating the neutrino interaction probability scale to be used in the MC job (the tiny neutrino interaction probabilities get normalized to a probability scale which is defined as the maximum possible total interaction probability, corresponding to a maximum energy neutrino in a worst-case trajectory maximizing its density-weighted path-length, summed up over all possible nuclear targets). That probability scale is also used to calculate the absolute normalization of generated sample in terms of kton*yrs. Feeding in pre-computed maximum density weighted path lengths results in faster MC job initialization and ensures that the same interaction probability scale is used across
all MC jobs in a physics production job (the geometry is scanned by a MC ray-tracing method and the calculated safe maximum density-weighted path-lengths may differ between MC jobs).

The maximum density-weighted path-lengths for a Geant4/ROOT-based detector geometry can be pre-computed using GENIE’s gmxpl utility.

- **-L length_units_at_geom:**
  Specifies the input geometry length units. Default: ‘m’. Possible options include: ‘m’, ‘cm’, ‘mm’, ...

- **-D density_units_at_geom:**
  Specifies the input geometry density units. Default: ‘kg_m3’. Possible options include: ‘kg_m3’, ‘g_cm3’, ‘clhep_def_density_unit’ (∼1.6 × 10⁻¹⁹ g/cm³), ...

- **-n n_of_events:**
  Specifies number of events to generate.

- **-o outfile_name:**
  Sets the prefix of the output event file. This allows you to override the output event file prefix. In GENIE, the output filename is built as:
  \[
  \text{prefix.run_number.event_tree_format.file_format}
  \]
  where, in gevgen_fluxdm, by default, prefix: ‘gntp’, event_tree_format: ‘ghep’, and file_format: ‘root’.

- **-F fid_cut_string:**
  Applies a fiducial cut (for now hard-coded). If the input string starts with ”-” then reverses sense (ie. anti-fiducial).

- **-S nrays_scan:**
  Number of rays to use to scan geometry for max path length. If ‘+N’: Scan the geometry using N rays generated using flux neutrino directions pulled from the input flux N-tuple. If ‘−N’: Scan the geometry using N rays ×N points on each face of a bounding box. Each ray has a uniformly distributed random inward direction.

- **-z zmin_start:**
  \(z\) from which to start flux ray in user-world coordinates. This is an optional argument.
If left unset then flux originates on the flux window [No longer attempts to determine $z$ from geometry, generally got this wrong].

- **--seed random_number_seed:**
  Specifies a random number generation seed.

- **--cross-sections xml_file:**
  Specifies the path of an input XML file with a dark matter cross-section spline, for example as generated by `gmkspl_dm`. Default: calculate cross-sections.

- **--event-generator-list list_name:**
  Specifies the list of event generators to use. The available dark matter generators are as follows:

  - **DMEL**: Elastic dark matter scattering;
  - **DMDIS**: Deep inelastic dark matter scattering;
  - **DME**: Dark matter-electron scattering;
  - **DMH**: Combines DMEL and DMDIS;
  - **DM**: Combines DMH and DME.

  Default: DM.

- **--message-thresholds xml_file:**
  Specifies the level of output to print for each module. The level is controlled by an XML file allowing users to customize the threshold of each message stream. See `$GENIE/config/Messenger.xml` for the XML file structure. Default: `Messenger.xml`.

- **--unphysical-event-mask mask:**
  Specifies a 16-bit mask to allow certain types of unphysical events to be output to the event file. Default: all unphysical events rejected.

- **--event-record-print-level level:** Specify the amount of information to be printed in the output event file. See `G HepRecord::Print()` for allowed settings.

- **--mc-job-status-refresh-rate rate:** Allows users to customize the refresh rate of the status file.
• **--cache-file root_file**: Allows users to specify a ROOT file so that results of calculation cached throughout a MC job can be re-used in subsequent MC jobs.

D. **Cross-section Spline Generation: gmkspl_dm**

To create a cross-section spline based for a given set of dark matter parameters, use the application `gmkspl_dm`. DIS cross-section calculation in particular is very time consuming, so the cross-sections can be pre-computed at a set of energies and interpolated to save a significant amount of time. This application performs this pre-computation. The operation works as follows, with optional arguments in square brackets

```
gmkspl_dm -m mass
<-t tgtpdg, -f geomfile>
--tune tune
[-g zp_couplings]
[-z med_ratios]
[<-o | --output-cross-section> xsec_xml_file_name]
[-n nknots]
[-e max_energy]
[--seed seed_number]
[--input-cross-section xml_file]
[--event-generator-list list_name]
[--message-thresholds xml_file]
```

The options function as follows, with the unchanged option description adapted from [23, 24]:

- **-m mass**:
  Dark matter mass in GeV.

- **-t tgtpdg**:
  Specifies the target PDG codes. Multiple target PDG codes can be specified as a comma separated list. The PDG2006 conventions is used (\(10LZZZAAI\)). For example, \(^{16}O\) code = 1000080160, \(^{40}Ar\) code = 1000180400.

- **-f geomfile**:
  Specifies a ROOT file containing a ROOT/GEANT4 detector geometry description.
• --tune tune:
  Specifies a generator tune. For dark matter running, there are currently two tunes, corresponding to fermionic and scalar dark matter. These tunes are GDM18_00a_00_000 and GDM18_00b_00_000 respectively.

• -g zp_coupling:
  Specifies the $Z'$ gauge coupling. Default: Taken from $GENIE/config/CommonParameters.xml$.

• -z med_ratio:
  Specifies the ratio of the mediator mass to the boosted dark matter mass. Default: 0.5.

• <-o | --output-cross-section> xsec_xml_file_name:
  Specifies the path of an output cross-section XML file. Default: ‘xsec_splines.xml’ in current directory.

• -n nknots:
  Specifies the number of knots per spline. Default: 15 knots per decade of the spline energy range and at least 30 knots overall.

• -e max_energy:
  Specifies the maximum neutrino energy in the range of each spline. Default: upper end of the validity range of the event generation thread responsible for generating the cross section spline.

• --seed seed_number:
  Specifies the random number seed for the current job. This setting will only be relevant if MC integration methods are employed for cross-section calculation.

• --input-cross-section xml_file: Specifies the path of an input XML file. An input cross-section file could be specified when it is possible to recycle previous calculations. It is sometimes possible to recycle cross-section calculations for scattering off free nucleons when calculating nuclear cross-sections.

• --cross-sections xml_file:
  Specifies the path of an input XML file with a dark matter cross-section spline, for example as generated by gmkspl_dm. Default: calculate cross-sections.
• **--event-generator-list list_name:**
  Specifies the list of event generators to use. The available dark matter generators are as follows:
  
  – **DMEL**: Elastic dark matter scattering;
  – **DMDIS**: Deep inelastic dark matter scattering;
  – **DME**: Dark matter-electron scattering;
  – **DMH**: Combines DMEL and DMDIS;
  – **DM**: Combines DMH and DME.

  Default: **DM**.

• **--message-thresholds xml_file:**
  Specifies the level of output to print for each module. The level is controlled by an XML file allowing users to customize the threshold of each message stream. See `$GENIE/config/Messenger.xml` for the XML file structure. Default: **Messenger.xml**.

V. SOLAR DARK MATTER FLUX: GENSOFLUX

In order to complete the description of boosted dark matter interacting with the nuclei in the detector, a realistic dark matter flux is required. One well-motivated potential source of posted dark matter is annihilation processes of relic dark matter that gets captured by the Sun [5]. In this section, I describe an application I have developed called **GenSolFlux** to generate a monochromatic flux of dark matter coming from the sun in the coordinate system of the detector. The code can be obtained at [https://github.com/jberger7/GenSolFlux](https://github.com/jberger7/GenSolFlux).

Several libraries exist to determine the solar position as a function of time. For these purposes, I use the library **SolTrack** [25]. The Sun’s position is given at a given latitude and longitude as an altitude angle $a$, the angle between the Sun and the horizon in the cardinal direction of the Sun, and an azimuth angle $A$, the angle along the ground of the Sun with respect to north. These angular coordinates are then converted into the azimuthal and polar spherical coordinates angles in the lab coordinate system. For the FNAL based neutrino experiments, the lab coordinate system is chosen such that the beam direction is
+z, the vertical is +y, and the coordinate system is right-handed. The direction of the dark matter momentum is then given by the vector

\[ \hat{p} = -\left( \cos(a) \sin(A + \theta_{\text{beam}}), \sin(a), \cos(a) \cos(A + \theta_{\text{beam}}) \right), \]

where \( \theta_{\text{beam}} \) is the angle of the beam direction with respect to north, i.e. due west would be \( \theta_{\text{beam}} = 3\pi/2 \). The beam angle is taken by default to be the angle of the Long Baseline Neutrino Facility beam to the Sanford Underground Research Facility where DUNE will be hosted, but an arbitrary angle can be supplied by the user.

The application uses SolTrack, available at http://soltrack.sourceforge.net, to generate a user-specified number of solar positions randomly selected over the course of 2018 or a date range specified by the user, beginning at midnight on the first date and ending at 23:59:59 on the last day. All times are taken to be in UTC, though the calculated positions are local. For each generated solar position, the dark matter four momentum vector is constructed as outlined above. The position-space location is taken to be the origin, as the GENIE-generated interactions are usually transposed to their physical location in the detector by the detector simulation. This information, along with the GENIE-accepted dark matter PDG code, are stored in a GENIE GSimpleNtpEntry that is stored in a branch of a TTree, which, along with a simple metadata TTree are written into a ROOT file. This file can then be used with the application gevgen_fluxdm described above.

I now outline the operation of the flux generation code. Since the application requires SolTrack, which is not a GENIE requirement, the application is provided as a standalone package. A simple configure and Makefile is provided. The SolTrack include and library directories need to be specified. The ROOTSYS and GENIE variables must also be correctly set. The application can be compiled using

```shell
shell% ./configure --prefix=<install_directory> \ 
> --with-soltrack-inc=<include_dir> --with-soltrack-lib=<lib_dir>
shell% make
shell% make install
```

The application requires four command line options and is run as follows

```shell
shell% gsolflux [-h] 
    -n nev
```
-e energy
-m mass
[-o outfile_name]
[-d date_range]
[-p detector_coords]
[-b beam_angle]

Here,

- **-h:**
  Prints help for gsolflux

- **-n nev:**
  Number of incident dark matter momentum vectors to generate.

- **-e energy:**
  Dark matter energy in GeV

- **-m mass:**
  Dark matter mass in GeV

- **-o outfile_name:**
  Path to write out ROOT file containing the fluxes. Default: flux.root

- **-d date_range:**
  Range of dates in the format YYYYMMDD,YYYYMMDD in which to generate events. The earliest time is the first date at 00:00:00 and the last time is the second date 23:59:59. All times are UTC. Default: The year 2018.

- **-p detector_coords:**
  Detector coordinates as latitude,longitude in degrees. Default: (44.35°, −103.75°), the approximate location of the DUNE far detector.

- **-b beam_angle:**
  Angle of the beam with respect to North in degrees. Default: 277.64°, the approximate angle of the LBNF beam with respect to North.
VI. SAMPLE OPERATION

In this section, I present a start-to-finish example of the operation of the module. The goal is to generate 10000 events of scalar dark matter striking a $^{40}$Ar target, including all available interactions. The mass of the dark matter will be 10 GeV and its energy will be 50 GeV. The flux will come from the Sun.

The first step is to edit the $\$\text{GENIE}/\text{config}/\text{CommonParameters.xml}$ file. In particular, in the $\text{BoostedDarkMatter}$ section, the desired charges and couplings should be set. In this example, I select flavor-universal axial charges, so that

\begin{verbatim}
<param type="double" name="DarkScalarCharge"> 1 </param>
<param type="double" name="UpLeftCharge"> -1 </param>
<param type="double" name="UpRightCharge"> 1 </param>
<param type="double" name="DownLeftCharge"> -1 </param>
<param type="double" name="DownRightCharge"> 1 </param>
<param type="double" name="StrangeLeftCharge"> -1 </param>
<param type="double" name="StrangeRightCharge"> 1 </param>
<param type="double" name="CharmLeftCharge"> -1 </param>
<param type="double" name="CharmRightCharge"> 1 </param>
<param type="double" name="ElectronLeftCharge"> -1 </param>
<param type="double" name="ElectronRightCharge"> 1 </param>
\end{verbatim}

Other options may be edited if desired, but these are the options that are relevant to the particle physics modeling.

The next, longest step is to generate a cross-section spline. I choose a $g_{Z'} = 1$ and a mediator mass ratio of 0.1, so that the mediator mass is $M_{Z'} = 1$ GeV. I will write this to a file $\text{dm_Ar_scalar_m10_g1_z1.xml}$. To do this, I run

```
shell% gmkspl_dm -m 10 -t 1000180400 --tune GDM18_00b_00_000 \\
> -o dm_Ar_scalar_m10_g1_z1.xml -g 1 -z 0.1 --event-generator-list DM
```

Once this process is done, events can be generated for arbitrary incoming dark matter energy.

In order to generate events with a solar flux, a flux file must first be created, say $\text{dm_flux_scalar_m10_e50.root}$. This can be done by running
shell% generate_flux -n 10000 -m 10 -e 50 -o dm_flux_scalar_m10_e50.root

This file should be copied or moved to the directory from which GENIE is begin run. Events can then be generated and written to a file dm_Ar_scalar_m10_e50_g1_z1.0.ghep.root with

shell% gevgen_fluxdm -f dm_flux_scalar_m10_e50.root -g 1000180400 -M 10 \  > --tune GDM18_00b_00_000 -o dm_Ar_scalar_m10_e50_g1_z1 -c 1 -z 0.1 \  > --cross-sections dm_Ar_scalar_m10_g1_z1.xml --event-generator-list DM

The events will be printed to stdout, but to view the events again, run

shell% gevdump -f dm_Ar_scalar_m10_e50_g1_z1.0.ghep.root

VII. DISCUSSION & CONCLUSIONS

In this paper, I have presented the first fixed target event generator for dark matter interactions with nuclei at energies above the $Q \sim 10$ MeV scale. Using the nuclear and QCD models implemented within GENIE, the module includes the modified kinematics and interactions of dark matter. Currently, elastic and deep inelastic interactions are included. A future update to the code will include baryon resonance production. Other models, such as inelastic boosted dark matter, could also be implemented in the future.

It is worth noting that the GENIE nuclear and QCD models are tuned to neutrino interactions. It is not clear the extent to which these tunings can be transposed to dark matter interactions without introducing significant deviations. The interactions of the dark matter in the regimes considered here are tricky to understand and there is no data to which the parameters can tuned. A model based on first principles could help in this regard. The current module is, however, a valuable first treatment that can be used to study the sensitivity of near-future experiments to models of boosted dark matter.

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[1] M. W. Goodman and E. Witten, *Detectability of Certain Dark Matter Candidates*, Phys. Rev. D31 (1985) 3059, [arXiv:1003.5912].

[2] F. D’Eramo and J. Thaler, *Semi-annihilation of Dark Matter*, JHEP 06 (2010) 109, [arXiv:1003.5912].

[3] G. Belanger and J.-C. Park, *Assisted freeze-out*, JCAP 1203 (2012) 038, [arXiv:1112.4491].

[4] K. Agashe, Y. Cui, L. Necib, and J. Thaler, *(In)direct Detection of Boosted Dark Matter*, JCAP 1410 (2014), no. 10 062, [arXiv:1405.7370].

[5] J. Berger, Y. Cui, and Y. Zhao, *Detecting Boosted Dark Matter from the Sun with Large Volume Neutrino Detectors*, JCAP 1502 (2015), no. 02 005, [arXiv:1410.2246].

[6] L. Necib, J. Moon, T. Wongjirad, and J. M. Conrad, *Boosted Dark Matter at Neutrino Experiments*, Phys. Rev. D95 (2017), no. 7 075018, [arXiv:1610.03486].

[7] H. Alhazmi, K. Kong, G. Mohlabeng, and J.-C. Park, *Boosted Dark Matter at the Deep Underground Neutrino Experiment*, JHEP 04 (2017) 158, [arXiv:1611.09866].

[8] Super-Kamiokande Collaboration, C. Kachulis et al., *Search for Boosted Dark Matter Interacting With Electrons in Super-Kamiokande*, Phys. Rev. Lett. 120 (2018), no. 22 221301, [arXiv:1711.05278].

[9] D. Kim, J.-C. Park, and S. Shin, *Dark Matter “Collider” from Inelastic Boosted Dark Matter*, Phys. Rev. Lett. 119 (2017), no. 16 161801, [arXiv:1612.06867].

[10] G. F. Giudice, D. Kim, J.-C. Park, and S. Shin, *Inelastic Boosted Dark Matter at Direct Detection Experiments*, Phys. Lett. B780 (2018) 543–552, [arXiv:1712.07126].

[11] A. Chatterjee, A. De Roeck, D. Kim, Z. G. Moghaddam, J.-C. Park, S. Shin, L. H. Whitehead, and J. Yu, *Searching for boosted dark matter at ProtoDUNE*, Phys. Rev. D98 (2018), no. 7 075027, [arXiv:1803.03264].
[12] D. Kim, K. Kong, J.-C. Park, and S. Shin, Boosted Dark Matter Quarrying at Surface Neutrino Detectors, *JHEP* **08** (2018) 155, [arXiv:1804.07302](https://arxiv.org/abs/1804.07302).

[13] IceCube Collaboration, R. Abbasi et al., The IceCube Data Acquisition System: Signal Capture, Digitization, and Timestamping, *Nucl. Instrum. Meth.* **A601** (2009) 294–316, [arXiv:0810.4930](https://arxiv.org/abs/0810.4930).

[14] D. Collaboration, Long-baseline neutrino facility (lbnf) and deep underground neutrino experiment (dune) conceptual design report volume 2: The physics program for dune at lbnf, [arXiv:1512.06148](https://arxiv.org/abs/1512.06148).

[15] C. Rubbia, The Liquid Argon Time Projection Chamber: A New Concept for Neutrino Detectors,  

[16] ICARUS Collaboration, S. Amerio et al., Design, construction and tests of the ICARUS T600 detector, *Nucl. Instrum. Meth.* **A527** (2004) 329–410.

[17] ArgoNeuT Collaboration, C. Anderson et al., First Measurements of Inclusive Muon Neutrino Charged Current Differential Cross Sections on Argon, *Phys. Rev. Lett.* **108** (2012) 161802, [arXiv:1111.0103](https://arxiv.org/abs/1111.0103).

[18] LArIAT Collaboration, F. Cavanna, M. Kordosky, J. Raaf, and B. Rebel, LArIAT: Liquid Argon In A Testbeam, [arXiv:1406.5560](https://arxiv.org/abs/1406.5560).

[19] LAr1-ND, ICARUS-WA104, MicroBooNE Collaboration, M. Antonello et al., A Proposal for a Three Detector Short-Baseline Neutrino Oscillation Program in the Fermilab Booster Neutrino Beam, [arXiv:1503.01520](https://arxiv.org/abs/1503.01520).

[20] MicroBooNE Collaboration, R. Acciarri et al., Design and Construction of the MicroBooNE Detector, *JINST* **12** (2017), no. 02 P02017, [arXiv:1612.05824](https://arxiv.org/abs/1612.05824).

[21] Super-Kamiokande Collaboration, Y. Fukuda et al., The Super-Kamiokande detector, *Nucl. Instrum. Meth.* **A501** (2003) 418–462.

[22] Hyper-Kamiokande Collaboration, K. Abe et al., Hyper-Kamiokande Design Report, [arXiv:1805.04163](https://arxiv.org/abs/1805.04163).

[23] GENIE Collaboration, C. Andreopoulos, The GENIE universal, object-oriented neutrino generator, *Acta Phys. Polon.* **B37** (2006) 2349–2360.

[24] C. Andreopoulos, C. Barry, S. Dytman, H. Gallagher, T. Golan, R. Hatcher, G. Perdue, and J. Yarba, The GENIE Neutrino Monte Carlo Generator: Physics and User Manual, [arXiv:1510.05494](https://arxiv.org/abs/1510.05494).
[25] M. van der Sluys, P. van Kan, and P. Sonneveld, *Cpv in the built environment*, AIP Conference Proceedings 1679 (2015), no. 1 080003, [https://aip.scitation.org/doi/pdf/10.1063/1.4931544](https://aip.scitation.org/doi/pdf/10.1063/1.4931544).

[26] L. A. Ahrens et al., *Measurement of Neutrino - Proton and anti-neutrino - Proton Elastic Scattering*, Phys. Rev. D35 (1987) 785.

[27] COMPASS Collaboration, V. Yu. Alexakhin et al., *The Deuteron Spin-dependent Structure Function g1(d) and its First Moment*, Phys. Lett. B647 (2007) 8–17, [hep-ex/0609038](https://arxiv.org/abs/hep-ex/0609038).

[28] HERMES Collaboration, A. Airapetian et al., *Precise determination of the spin structure function g(1) of the proton, deuteron and neutron*, Phys. Rev. D75 (2007) 012007, [hep-ex/0609039](https://arxiv.org/abs/hep-ex/0609039).

[29] G. Grilli di Cortona, E. Hardy, J. Pardo Vega, and G. Villadoro, *The QCD axion, precisely*, JHEP 01 (2016) 034, [arXiv:1511.02867](https://arxiv.org/abs/1511.02867).

[30] Particle Data Group Collaboration, C. Patrignani et al., *Review of Particle Physics*, Chin. Phys. C40 (2016), no. 10 100001.

[31] F. Bishara, J. Brod, B. Grinstein, and J. Zupan, *From quarks to nucleons in dark matter direct detection*, JHEP 11 (2017) 059, [arXiv:1707.06998](https://arxiv.org/abs/1707.06998).

[32] B. Bhattacharya, G. Paz, and A. J. Tropiano, *Model-independent determination of the axial mass parameter in quasielastic antineutrino-nucleon scattering*, Phys. Rev. D92 (2015), no. 11 113011, [arXiv:1510.05652](https://arxiv.org/abs/1510.05652).

[33] E. A. Paschos and J. Y. Yu, *Neutrino interactions in oscillation experiments*, Phys. Rev. D65 (2002) 033002, [hep-ph/0107261](https://arxiv.org/abs/hep-ph/0107261).

[34] C. G. Callan, Jr. and D. J. Gross, *High-energy electroproduction and the constitution of the electric current*, Phys. Rev. Lett. 22 (1969) 156–159.

[35] C. H. Albright and C. Jarlskog, *Neutrino Production of m+ and e+ Heavy Leptons. 1.*, Nucl. Phys. B84 (1975) 467–492.

[36] Z. Koba, H. B. Nielsen, and P. Olesen, *Scaling of multiplicity distributions in high-energy hadron collisions*, Nucl. Phys. B40 (1972) 317–334.

[37] W. J. Marciano and Z. Parsa, *Neutrino electron scattering theory*, J. Phys. G29 (2003) 2629–2645, [hep-ph/0403168](https://arxiv.org/abs/hep-ph/0403168).

[38] Particle Data Group Collaboration, M. Tanabashi et al., *Review of Particle Physics*, Phys. Rev. D98 (2018), no. 3 030001.