Thermophoresis and its effect on particle impaction on a cylinder for low and moderate Reynolds numbers

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Abstract

The effect of thermophoresis on the impaction of particles on a cylinder is investigated for different particle sizes, particle conductivities, temperature gradients and for Reynolds numbers between 100 and 1600. Simulations are performed using the Pencil Code, a high-order finite difference code. An overset-grid method is used to precisely simulate the flow around the cylinder. The ratio of particles impacting the cylinder and the number of particles inserted upstream of the cylinder is used to calculate an impaction efficiency. It is found that both the particle conductivity and the temperature gradient have a close to linear influence on the particle impaction efficiency for small particles. Higher Reynolds numbers result in higher impaction efficiency for middle-sized particles, while the impaction efficiency is smaller for smaller particles. In general, it is found that thermophoresis only has an effect on the small particles, while for larger particles the impaction efficiency is controlled by inertial impaction.

Finally, an algebraic model, developed based on fundamental principles, which describes the effect of thermophoresis is presented. The model is found to accurately predict the DNS results. As such, this model can be used to understand the mechanisms behind particle deposition due to the thermophoretic force, and, more importantly, to identify means by which the deposition rate can be reduced.

Keywords: particle deposition, thermophoresis, overset grids

1. Introduction

Particle impaction on surfaces can be found in a multitude of industrial systems, such as filters and heat exchanger surfaces. The impaction and deposition of material on these surfaces can significantly alter their performance, necessitating decreased maintenance intervals or an increased rate of replacement of...
components. In order to improve the design of surfaces exposed to particle laden flows, a thorough understanding of the underlying effects is needed. In this work, we will focus on how particles are transported to the solid surface. For a particle to deposit on the surface, it must first be transported to the surface before it has to stick to it. The latter mechanism is outside the scope of this study, and here all particles impacting on the surface will count towards particle deposition.

The transport of material to the surface is governed by the impaction efficiency, which is the ratio of the number of particles that actually come in contact with the cylinder to the number of particles that would come in contact with the cylinder if they were to move in a straight line, in the flow direction, from their point of origin. For an overview of the different approaches the reader is referred to the work of [1].

Due to its simplicity, a cylinder placed in a particle laden flow is a widespread test case used to study the impaction of particles on solid surfaces or heat exchanger tubes. A sketch of such a case is shown in figure 1 where the particles (shown in green) are inserted from a plane (red), which has the size of the projected area of the cylinder. The initial velocity of the particles is equal to the flow velocity at the insertion plane. If the particles followed the flow from left to right, without any change in velocity, all particles would hit the cylinder, leading to an impaction efficiency ($\eta$) of unity. In reality, this does not happen as the fluid is flowing around the cylinder, and particles are dragged along with it. A particle’s ability to follow the fluid is expressed as the particle’s Stokes number $St$, which is the ratio of the particle response time and the fluid time scale (details in §2.1). In general, a particle with a Stokes number above unity does not follow the flow very well, while particles with Stokes numbers below unity tend to follow the flow they are embedded in. [2] developed a correlation based on potential flow theory, which allows to calculate the impaction efficiency of particles on an isothermal cylinder. This correlation is a well established tool for predicting the isothermal impaction efficiency of large particles in a laminar flow, but does not yield correct results for small particles [3].

The actual mass accumulation rate on a cylinder is determined by the cap-
ture efficiency, which is the product of the impaction efficiency and the sticking efficiency. The sticking efficiency is the fraction of the impacting particles that actually stick to the surface instead of rebounding. If either the particles or the cylinder surface is at least partially melted, making them sticky, the sticking efficiency is close to unity. On the other hand, for cold and clean surfaces, particles will most likely bounce off the surface. The sticking efficiency is then essentially zero.

An experimental study, performed by [4], looked at the effect of mass accumulation on the capture efficiency and proposes an empirical power law for it. Moreover, they present a new fit function for the capture efficiency, which is bounded between 0 and 1. The particle Stokes numbers in said study were between 0.3 and 3. [3] investigated the impaction efficiency using Direct Numerical Simulation with an immersed boundary method and found a steep drop in impaction efficiency below a certain Stokes number, as particles become smaller and follow the flow better. Extending this work, Aarnes et al. studied the same case using overset grids, obtaining results that are deemed more accurate with significantly less computational efforts [5, 6].

The effect of thermophoresis on the capture efficiency is studied by several groups, both experimentally and numerically. [7] measured deposition of fly ash of a pulverized coal jet flame and simulated the deposition rate using CFD and found that thermophoresis increases the capture efficiency for smaller particles, and that the relative increase is higher the smaller the particles in question are. Experimental data from a pilot-scale furnace is compared to numerical results by [8], where the influence of deposition growth on the particle impaction and sticking efficiency is studied. They report that the higher surface temperature due to deposit growth, results in a reduced effect of thermophoresis and an increased sticking efficiency. At later times, the rate of shedding of material from the surface and deposition of material to the surface from the flow balance out, so no net change of mass sticking to the surface is observed [9].

In the work of [10] the effect of the thermophoretic force is studied both experimentally and numerically. In the experimental part, the deposition of material on cooled and un-cooled probes that are inserted into the particle laden flow above the burner section of a combined heat and power (CHP) plant is studied. Large-Eddy simulations are used to study the influence of the sticking model and thermophoresis on deposition rate predictions. They present a model that can take into account different sticking mechanisms by which large and small particles of different composition deposit. It is reported that thermophoresis accounts for three quarters of the observed deposition rate. [11] used unsteady RANS simulations to study the effect of thermophoresis on particle deposition and found that the thermophoretic force was the dominating deposition mechanism for small particles. To the knowledge of the current authors, Direct Numerical Simulations (DNS) have not previously been used to perform a parameter study of the effect of the thermophoretic force on the particle deposition rate on a cylinder in a cross flow. This further motivates the authors of the current study to investigate the influence of different flow conditions, such as flow Reynolds number, temperature gradient and particle attributes on the
effect of thermophoresis. This is the aim of the present study.

2. Theory

The governing fluid equations are the ones for continuity, momentum and energy. Pressure is taken into account through the ideal gas law and the Mach number is \( \sim 0.1 \), which is so low that the flow is considered as essentially incompressible.

Fluid equations

The continuity equation is given by

\[
\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho = -\rho \nabla \cdot \mathbf{u},
\]

(1)

with \( \rho \), \( t \) and \( \mathbf{u} \) being density, time and velocity, respectively. The equation governing the conservation of momentum is

\[
\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nabla \cdot (2\nu \mathbf{S}),
\]

(2)

where \( p \) is the pressure, \( \nu \) the kinematic viscosity, and

\[
\mathbf{S} = \frac{1}{2} \left( \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) - \frac{1}{3} I \nabla \cdot \mathbf{u}
\]

(3)

is the rate of strain tensor where \( I \) is the identity matrix. The energy equation is solved in the form of temperature:

\[
\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \frac{k_f}{\rho c_v} \nabla^2 T + \frac{2\nu S^2}{c_v} - (\gamma - 1) T \nabla \cdot \mathbf{u},
\]

(4)

where \( \gamma = c_p/c_v = 5/3 \) and \( c_v \) and \( c_p \) are the heat capacities at constant volume and pressure, respectively, while \( k_f \) is the thermal conductivity. The ideal gas law is used to tie pressure and density together:

\[
p = \rho r_u T,
\]

(5)

where \( r_u = c_p - c_v \) is the specific gas constant. To simplify the investigation, the kinematic viscosity, \( \nu \), is assumed to be constant since the temperature variations in the fluid are relatively small. Since the Pencil-Code is an explicit compressible code, the time step is limited by the speed of sound through the CFL number. The speed of sound is given by

\[
c_s = \sqrt{\gamma r_u T} = \sqrt{c_p(\gamma - 1)T}.
\]

One can therefore use \( c_p \) as a free parameter in order to artificially lower the speed of sound to obtain larger time steps. This is a valid approach as long as the Mach number is kept lower than 0.1 and the viscous heating of the fluid is negligible. To maintain a constant thermal diffusivity \( (D_{\text{thermal}} = k_f/(\rho c_p)) \) of the gas phase, and hence a constant Prandtl number \( (\text{Pr} = \nu/D_{\text{thermal}}) \), the conductivity \( (k_f) \) is changed proportionally to the specific heat capacity of the fluid \( (c_p) \).
2.1. Particle equations

The particles considered here are spherical and have low Biot numbers, which make them spatially isothermal. Numerically they are treated as point particles that are influenced by the fluid, but are too dilute to have any significant backreaction on the fluid. In other words, they are acted on by the flow but have no effect on it. This assumption is applicable for dilute flows, which is the focus of the current work. The particle size is described by its Stokes number

$$\text{St} = \frac{\tau_{St}}{\tau_f}, \quad (6)$$

where $\tau_{St} = \frac{S\rho d_p^2}{18 \nu}$ is the particle Stokes time and $\tau_f = \frac{D}{u}$ is the flow time scale. Here, $S = \frac{\rho_p}{\rho}$ is the density ratio between particle and fluid, $d_p$ is the particle diameter and $D$ is the diameter of the cylinder. Two forces are acting on the particle: the drag force and the thermophoretic force, while gravity is neglected for the small particles studied here. The drag force is given by:

$$F_D = \frac{m_p}{\tau_p} (u - v_p), \quad (7)$$

where $\tau_p$, $m_p$ and $v_p$ are the particle’s response time, mass and velocity, respectively. Using the Stokes time with the Schiller-Naumann correction term [12] to account for low to moderate particle Reynolds numbers, the particle response time becomes:

$$\tau_p = \frac{\tau_{St}}{f}, \quad (8)$$

where

$$f = 1 + 0.15 \text{Re}_p^{0.687} \quad (9)$$

and $\text{Re}_p = d_p |v_p - u| / \nu$ is the particle Reynolds number.

The thermophoretic force pushes particles from regions of high temperature to regions of low temperature. As such, it is similar to the Soret effect for gases. It was first observed in 1870 by [13], and it has later become widely studied both experimentally and theoretically. A theoretical analysis of the thermophoretic force can be found in the works of [14]. [15] gives an overview over the different regimes of thermophoresis, which are determined by the particles Knudsen number $\text{Kn} = \lambda / d_p$, where $\lambda$ is the mean free path of the gas. For the present study, all particles are in the continuum regime, which is defined by $\text{Kn} \ll 1$. The thermophoretic force is then calculated by

$$F_{th} = \Phi \frac{\mu^2 r_p \nabla T}{\rho T}, \quad (10)$$

where $r_p = d_p / 2$ is particle radius, $\mu = \rho \nu$ is dynamic viscosity and $\Phi$ is the thermophoretic force term. The expression for $\Phi$ is taken from [16]:

$$\Phi = -\frac{12 \pi K_{lc}}{2 + \Lambda}, \quad (11)$$
where the conductivity ratio between the particle and the gas is given by \( \Lambda = \frac{k_p}{k_f} \), while the temperature creep coefficient, \( K_{tc} \), used in this work has a value of 1.1, which is in the middle of the range reported by [17]. This rather simple model for \( \Phi \) simplifies the analysis, while still providing agreeable results when compared with the widely used approach proposed by [18]. From figure 2 we see that the largest relative difference between the simplified \( \Phi \) and the one obtained when using the approach of Talbot is less than a factor of two.

2.2. Theory

Due to their short response times, very small particles will follow the fluid almost perfectly, essentially behaving like tracer particles. For isothermal situations, [3] showed that a small fraction of these particles will nevertheless impact on the cylinder surface due to their small but finite radii.

For the non-isothermal case, where the temperature of the cylinder is lower than that of the surrounding gas, the thermophoretic force will induce a relative velocity between the particles and the fluid that transport the particles in the direction towards the cylinder. The effect of this is that a larger fraction of the particles impact on the cylinder surface. In the following we will try to quantify this effect.

For laminar flows, a fluid streamline that starts far upstream of the cylinder with a displacement \( \Delta x \) from the central line (the line that is parallel to the mean flow and go through the center of the cylinder) will move in the boundary layer of the cylinder with a radial displacement from the cylinder surface of \( \Delta r_f \neq \Delta x \) (see figure 3). Far upstream of the cylinder, the mass flow rate between a streamline and the center line of the cylinder is given by

\[
\dot{m}_u = H u_0 \rho_0 \Delta x, \tag{12}
\]

where \(\Delta x\) is the distance between the streamline and the center line and \( H \) is the height of the cylinder. Within the boundary layer of the cylinder, however, the mass flow rate between the streamline and the cylinder surface is given by

\[
\dot{m}_b = \int_0^{\Delta r_f} \rho H u_0 d \theta_{cyl} = \frac{\bar{\rho} u_0 Re^{1/2}}{2BD} (\Delta r_f)^2, \tag{13}
\]
where $\rho$ is the average fluid density within the boundary layer, and, by following [3], we have used the fact that the tangential fluid velocity within the boundary layer is given by

$$u_\theta(r_{cyl}) = \frac{u_0 Re^{1/2}}{BD} r_{cyl},$$

(14)

where $u_0$ is the far field fluid velocity, $B$ is a constant of the order of unity and $r_{cyl}$ is the normal distance from the cylinder surface. Since the fluid is not turbulent (i.e., streamlines do not cross each other), we know from mass conservation that the mass flux between the streamline and the central line upstream of the cylinder ($\dot{m}_u$) is equal to the mass flux between the streamline and the cylinder surface ($\dot{m}_b$).

The impaction efficiency for a stationary non-turbulent flow is given by

$$\eta = \frac{2\Delta x_{\text{max}}}{D}$$

(15)

where $\Delta x_{\text{max}}$ is the maximum $\Delta x$ of the particles that impact on the surface. Since streamlines of laminar flows do not cross each other, it is clear that for a given type of particles, all particles inside $\Delta x_{\text{max}}$ will impact on the cylinder surface while none of the particles outside of $\Delta x_{\text{max}}$ will impact. For Reynolds numbers above $\sim 48$, the flow will become unsteady and von Kármán eddies will occur in the wake of the cylinder. The effect of this unsteadiness on the front side impaction will be minor. Back side impaction may, however, be strongly affected by the von Kármán eddies. Since the focus of the current work is to study front side impaction, the definition of the impaction efficiency as given by Eq. (15) will also be used for unsteady flows.

By calculating the distance a particle will move in the radial direction due to the thermophoretic force during the time it is in the front side boundary layer of the cylinder, $\Delta_{\text{disp}}$, and setting this equal to $\Delta r_f$, the above equations can be solved to find the impaction efficiency for small particles. In the following, the focus will therefore be on finding $\Delta_{\text{disp}}$. Here, small particles are defined as particles that are so small that the main cause of impaction is the thermophoretic force. This is typically the case for $St \lesssim 0.1$.

By combining Eq. (7) and Eq. (10), while setting the radial component of the gas phase velocity to zero, we find the thermophoretic velocity of the particles
(in the radial direction) as
\[ v_{th} = \frac{\Phi \nu}{6\pi f} \frac{\nabla T}{T}. \] (16)

The correction term to the Stokes time, as given by \( f \) (see Eq. (9)), is always close to unity if the thermophoretic force is the main driver of the particle velocity relative to the surrounding fluid. We therefore set \( f = 1 \) for the remainder of this analysis. From Prandtl’s concept of thin boundary layers, we know that the thickness of the velocity boundary layer can be approximated by
\[ \delta_{vel} = \frac{D}{Re^{1/2} B}. \] (17)

The thermal boundary layer thickness is then given by
\[ \delta_{thermal} = \delta_{vel} Pr^{-1/3}. \] (18)

Hence, the average thermal gradient in the boundary layer becomes
\[ \nabla T \approx \frac{\Delta T}{\delta_{thermal}} = \frac{\Delta T}{D} BrRe^{1/2} Pr^{1/3}. \] (19)

The effect of the thermophoretic force on the position of the particle can now be considered as the radial displacement of the particle \( (\Delta_{disp}) \) from the position \( (x_{stream}) \) it would have without the influence of the thermophoretic force (see figure 3). This radial displacement is given by
\[ \Delta_{disp} = v_{th} \tau_{th}, \] (20)

where
\[ \tau_{th} = \frac{D}{u_\theta(\Delta_{disp})} \] (21)

is the time the particle stays within the front side boundary layer and \( u_\theta(\Delta_{disp}) \) is the tangential velocity of the flow in the boundary layer a distance \( \Delta_{disp} \) away from the surface of the cylinder (see figure 3). From Eqs. (14) and (21), we can now find the typical time the last particle that hits the front side of the cylinder stays within the cylinder boundary layer before it hits the boundary as
\[ \tau_{th} = \frac{BD^2}{u_0 Re^{1/2} \Delta_{disp}}. \] (22)

Combining Eq. (22) with Eq. (16), Eq. (19) and Eq. (20), and solving for \( \Delta_{disp} \), now yields
\[ \Delta_{disp}^2 = \Phi B^2 Pr^{1/3} D^2 \Delta T \] (23)

where we use that \( Re = u_0 D / \nu \). Having found \( \Delta_{disp} \), we proceed by setting the two mass fluxes defined in Eqs. (12) and (13) equal to each other and solve for \( \Delta x_{max} \) to find
\[ \Delta x_{max} = \frac{\pi}{\rho_0} \frac{Re^{1/2}}{2BD} \Delta_{disp}^2. \] (24)
In the above we have used the fact that $\Delta x = \Delta x_{\text{max}}$ when $\Delta r_f = \Delta_{\text{disp}}$.

From Eq. (15), we now find that for a non-negligible temperature difference, the capture efficiency for small Stokes numbers ($\text{St} \lesssim 0.1$) is given by

$$
\eta = \frac{2\Delta x_{\text{max}}}{D} = \frac{\Phi B \text{Pr}^{1/3} \Delta T \bar{\rho}}{6\pi \text{Re}^{1/2} T \rho_0} \approx \frac{2K_{tc}B \text{Pr}^{1/3}}{\text{Re}^{3/2}(2 + \Lambda)} \frac{\Delta T \bar{\rho}}{T \rho_0}.
$$

(25)

For laminar flows, (author?) [3] found that $B$ is independent of Reynolds number but varies with angular position on the cylinder surface. In particular, they found $1/B$ to be zero at the front stagnation point while the minimum value of $B$ was found to be 0.45 at a position 60 degrees further downstream. For the remainder of this paper we chose $B = 1.6$, which is a value that yields good model predictions. In order to obtain the last part of Eq. (25), we have used the simplified version of the thermophoretic force term ($\phi$), but any version can be used here.

It is clear that the above approach, yielding an impaction efficiency due to thermophoretic forces for small Stokes numbers, is strictly applicable only when the distance the particle travels within the boundary layer is less than a fraction $\alpha$ of the thickness of the boundary layer itself, i.e. when

$$
\frac{\Delta_{\text{disp}}}{\delta_{\text{thermal}}} = B^2 \sqrt{\frac{2K_{tc} \text{Pr} \Delta T}{(2 + \Lambda)T}} < \alpha.
$$

(26)

We shall later see that the critical value of $\alpha$ is somewhere between 0.5 and 1.

3. Numerical Methods

The simulations for the present work are performed using the Pencil Code, an open source, highly parallelizable code for compressible flows with a wide range of implemented methods to model different physical effects [20, 21, 22].

The effect of thermophoresis on the impaction efficiency is studied by releasing a large number of particles upstream of a cylinder in an established quasi-steady flow field. Every time step, new particles are inserted at random positions on the particle insertion plane (shown as a red line in figure 1) with a velocity that is equal to the inlet fluid velocity. The domain is two-dimensional and has a width of $6D$ and a length of $12D$, where $D$ is the diameter of the cylinder. The flow enters the domain on the left and leaves the domain through the outlet on the right of the domain. Navier-Stokes characteristic boundary conditions are applied at both inlet and outlet to ensure they are non-reflective for acoustic waves [23]. All other boundaries are periodic. To precisely represent the cylinder at low computational cost, a cylindrical overset grid is placed around the cylinder. This cylindrical grid communicates with the Cartesian background grid via its outer points. Summation-by-parts is used for derivatives on the surface of the cylinder, and a Padé filter is used to mitigate high frequency oscillations on the cylindrical grid. For details concerning the cylindrical overset grid, the reader is referred to (author?) [4, 6, 24].
The background grid is divided into 288 and 576 cells for the width and length, respectively, except for the studies at the highest Reynolds number 1600, where, the resolution is doubled. The cylindrical grid has 144 cells in the radial and 480 cells in the tangential direction for the cases with Reynolds numbers up to 400, and double the amount for the case with a Reynolds number of 1600. Grid stretching in the radial direction is used to ensure approximately matching cell sizes on the outer grid points of the cylindrical cells, where the background and the overset grids communicate. The code uses a sixth-order finite difference scheme for spatial discretization and a third-order Runge-Kutta scheme for temporal discretization. Since the cell size at the cylinder is much smaller than the general cell size of the background grid, the time step of the background grid can be a multiple of the time step of the cylindrical grid, with the cylindrical flow being updated more often. For details of the particle tracking scheme, the reader is referred to (author?) [3] or (author?) [6].

4. Simulations

The inflow has a temperature of 873 K and a density of 0.4 kg/m$^3$, corresponding to the density of air at this temperature and a pressure of 1 bar. The particles have a density of 400 kg/m$^3$, leading to a density ratio $S_\rho$ of 1000 based on the fluid density under inlet conditions. These values are chosen based on their relevance for particle deposition on super-heater tubes in thermal power plants, but the results are nevertheless generic since they are given as functions of non-dimensional numbers. The cylinder temperature is implemented as a Dirichlet boundary condition and set to a fixed value. The inflow velocity $u_0$ is set so that the flow Reynolds number $Re = u_0 D/\nu$ is 100 for the cases not studying the Reynolds number effects. For the cases studying the Reynolds number effect, the viscosity is changed to obtain different Reynolds numbers. To study the same range of Stokes numbers, the particle size is adjusted accordingly, and the thermal diffusivity is decreased to achieve a constant Prandtl number. The different aspects of the thermophoretic effect are analyzed by changing one critical parameter at the time, while holding the others constant. These critical parameters are: 1) Reynolds number, 2) Prandtl number, 3) temperature difference between fluid and cylinder and 4) conductivity ratio.

The conductivity ratio is given by

$$\Lambda = \frac{k_p}{k_f} = \frac{k_p}{D_{\text{thermal}} c_p \rho} = \frac{k_{pf}}{D_{\text{thermal}}},$$

where $k_{pf} = k_p/(c_p \rho)$. Since we keep the thermal diffusivity of the fluid constant when changing the conductivity ratio, this means that the conductivity ratio is essentially changed by changing $k_{pf}$.

For each of the parameters listed in table [1], the impaction efficiencies of particles in the Stokes number range between 0.01 and 10 are obtained from the DNS simulations. For each particle size (Stokes number), a certain number of particles have to impact the surface to get the statistics required in order
**Table 1: Range of parameters studied**

| Parameter          | Values                        |
|--------------------|-------------------------------|
| Reynolds number    | 100, 400, 1600                |
| Conductivity ratio | 1, 12, 144                    |
| $\Delta T$ [K]     | 0, 1, 3, 10, 173, 400         |

**Table 2: Independent variables describing the simulations**

| Variable  | Unit         | Description                                      |
|-----------|--------------|--------------------------------------------------|
| $\alpha_p$ | m²/s         | Thermal diffusivity of particles                 |
| $\rho_p$  | kg/m³        | Material density of particles                    |
| $d_p$     | m            | Diameter of particles                            |
| $\rho$    | kg/m³        | Material density of fluid                        |
| $D_{\text{thermal}}$ | m²/s | Thermal diffusivity of fluid                    |
| $u$       | m/s          | Velocity of fluid                                |
| $\nu$     | m²/s         | Viscosity of fluid                               |
| $D$       | m            | Diameter of cylinder                             |
| $T_f$     | K            | Far-field temperature of fluid                   |
| $T_s$     | K            | Temperature of cylinder                          |

To estimate an accurate impaction efficiency. Since we already know that the impaction efficiency decreases with Stokes number, it is therefore clear that we have to release more small than large particles. In particular, for particles with Stokes numbers $> 1$, a total of 15,000 particles are released for each particle size (Stokes number), while 200,000-400,000 particles are released for particles with $0.1 < St < 1$, and 2 million particles of each particle size are released for $St < 0.1$. The particles are inserted over several vortex shedding times to mitigate the effect the instantaneous vortex shedding could have on the results.

All simulations can be described by the $n = 10$ unique and independent variables that are listed in table 2. From the table we see that all variables involve a total of $k = 4$ different units (m, s, kg and K). Then, from the Buckingham-Pi theorem, we know that the simulations can be described by exactly $p = n - k = 6$ different dimensionless numbers. These dimensionless numbers are given in table 3. In this work, we study the effect of variations in all of these dimensionless numbers.

**Table 3: Dimensionless numbers describing the simulations**

| Dimensionless number | Description                                      |
|----------------------|--------------------------------------------------|
| $\Lambda = \frac{k_{pf}}{\rho D_{\text{thermal}}}$ | Conductivity ratio                              |
| Re = $Du/\nu$       | Reynolds number                                  |
| $\Theta = T_f/T_s$  | Temperature ratio between far-field and cylinder |
| $S_p = \frac{\rho_p}{\rho}$ | Density ratio between particle and fluid         |
| St = $S_p d_p^2 u/(18 \nu D)$ | Stokes number                                   |
| Pr = $\nu/D_{\text{thermal}}$ | Prandtl number                                  |
numbers, except for $S_p$, which is always kept constant at a value of one thousand. We know that changing $S_p$ means that another dimensionless number, $D/d_p = \sqrt{Re S_p/(ISST)}$, will change. The effect of this is that the level of the interception mode will shift. The interception mode is the mode by which very small particles intercept the cylinder when only pure impaction is accounted for. This is important to account for when comparing simulation results with different values of $S_p$, which for example is done in figure 12b of [1]. The simulations presented in the following are listed in table 4.

### 5. Results

In this section we will study the effect of the thermophoretic force on the impaction efficiency. In particular, we will look at how the impaction efficiency is affected by changes in the Reynolds number, temperature difference and conductivity ratio.

In their work, [5] used DNS to find the efficiency by which particles embedded in an isothermal flow impact on a cylinder in a cross flow. In their study they also used an overset grid, but they did not consider the thermophoretic force nor did they solve the energy equation. In the following, we will use their results as a reference case, from now on called the 'Base case'. Figure 4 compares the impaction efficiency of the 'Base case' with what is found for the same condition in the current work. From the figure, we can see that the impaction efficiency shows a slight decrease with decreasing Stokes number for Stokes numbers over 1, followed by a steep drop of impaction efficiency in the Stokes number range between 0.1 and 1. For even smaller Stokes numbers, the impaction efficiency decreases linearly with decreasing Stokes number. As expected, our recent simulations of isothermal cases both with (blue line) and without (orange line) the thermophoretic force (Eq. (10)) included yield the same impaction efficiency profile as the 'Base case'. A case with Brownian forces on the particles has also been performed, and one can see a weak effect...
of Brownian forces for the smallest Stokes numbers. This effect is quite weak, and, as we shall see later, the thermophoretic force will have a much stronger effect on the impaction efficiency even for very small temperature gradients.

In figure 5, the 'Base case' (isothermal) is compared to non-isothermal cases with thermophoresis. Simulations with a temperature difference between the inlet gas and the cylinder surface of $\Delta T = 173$ K and a particle conductivity ratio of 12 was used for the thermophoretic cases. It was shown in §2.4 that the models of Epstein and Talbot gave comparable values of $\Phi$. The two solid lines in figure 5 show the corresponding difference in impaction efficiency. For the smaller particles, the impaction efficiency predicted by the model of Talbot is only about 10% higher than the one predicted by Epstein, with the difference disappearing for larger particles. It is clear from the figure that the particle impaction is unaffected by the thermophoretic force for large Stokes numbers, while the thermophoretic force is dominating the impaction for small Stokes numbers. In contrast to what is observed for the isothermal case, the impaction efficiency becomes independent of the particle size for small particles. The theoretical prediction of the impaction efficiency for small Stokes numbers, as presented in Eq. (25), is represented by the red dotted horizontal line in the figure and one can see that it fits well with the numerical results for $St < 0.1$.

The effect of different conductivity ratios on the impaction efficiency is shown in figure 6. We see that the impaction efficiency for small Stokes numbers is higher for lower values of the conductivity ratio. This is because a small conductivity ratio yields a large thermophoretic force term ($\Phi$), which again results in a large thermophoretic force and hence high impaction efficiency. This effect can also be seen from Eq. (25), where the impaction efficiency for small Stokes numbers is seen to scale linearly with $\Phi$. When the simplified expression for $\Phi$ is used, a linear dependence on $\Phi$ means that the impaction efficiency is inversely proportional with $(2 + \Lambda)$. 

Figure 4: Comparison of $\eta$ for data from [5] with data obtained from an isothermal case with and without thermophoretic force.
For the smallest conductivity ratio (case C1), we find from the applicability test of our model, as given in Eq. (26), and visualized in figure 7, that the thermophoretic force is so strong that our model is not strictly applicable (i.e. $\Delta_{\text{disp}}/\delta_{\text{thermal}} > \alpha$ when $\alpha \lesssim 0.5$). This is probably the reason why the modelled impaction efficiency at small Stokes numbers, as represented by the horizontal dotted lines in figure 6, deviates somewhat from the simulated results (solid lines). It should be noted that a conductivity ratio of 1 is rather improbable. [25] give a value for the conductivity ratio of small char particles of $\Lambda \approx 9$. The conductivity ratio may, however, be quite different for other solids. In the following we have somewhat arbitrarily chosen to use $\Lambda = 12$ as a baseline for our simulations. By comparing how well our model reproduces the simulation results, it seems reasonable to assume that $\alpha \sim 0.5 - 1.0$. Based on Eq. (26), we can then find that for char particles ($\Lambda \sim 10$), the more stringent value of $\alpha$ ($=0.5$) results in the model being applicable as long as $\Delta T$ is less than $\sim 30\%$ of the far field temperature for fluids with $\text{Pr} = 0.7$. For the same conditions and $\alpha = 1.0$, our model is applicable for all values of $\Delta T$.

In the left hand panel of figure 8 the impaction efficiency is shown as a function of angular position of impact on the cylinder for different Stokes numbers for simulations with a conductivity ratio of $\Lambda = 12$. (The center-line is at 270 degrees.) For the largest Stokes number (St = 0.9), all impaction occurs within an angle, $\theta_{\text{max}}$, that is less than 60 degrees from the center line. This is consistent with the findings of [3] without thermophoresis. [3] showed that $\theta_{\text{max}} < 60$ degrees for St < 0.9 for pure inertial impaction (see also figure 9). However, we see that when thermophoresis is accounted for, the particles with smaller Stokes numbers impact the entire frontal surface of the cylinder. By increasing the strength of the thermophoretic force, which is here done by decreasing the conductivity ratio to unity, we see from the right hand panel of figure 8 that
Figure 6: Front side impaction efficiency over Stokes number for different conductivity ratios. This corresponds to simulations 'Base case', 0, C1 and C144 as listed in table 4. The dotted lines correspond to the impaction efficiency predicted by Eq. (25) for small Stokes numbers.

Figure 7: For the simulations marked in the grey area, the model from Eq. (25) is applicable. From lightest to darkest the grey areas correspond to $\alpha = 1.0, 0.5$ and 0.25. For those that are in the white area, however, Eq. (26) yields no applicability. The simulations with different Reynolds number and constant Prandtl number ("R400" and "R1600") are positioned at the same place as simulation "0".
the angular position of impaction becomes even more uniform for the smaller particles.

Since the gas is an ideal gas, the cylinder is surrounded by a boundary layer of densified gas with a significant temperature gradient for the cases with large temperature differences. A larger temperature difference yields a stronger thermophoretic force, which again results in a higher impaction efficiency for small particles. As can be seen from figure 10, the model results (dotted lines) fit the simulation results (solid lines) well for all temperature differences studied here (3 K < ∆T < 400 K). From figure 7 we do see, though, that ∆T = 400 K is close to the limit of the applicability of the model.

We will now continue by studying the effect of Reynolds number on the impaction efficiency. In this work, we increase the Reynolds number by decreasing the viscosity. While doing this, we must also increase the resolution in order to properly resolve the boundary layer, which is thinner for higher Reynolds num-

Figure 8: Front side impaction angle for Λ=12 (left panel) and Λ=1 (right panel). Simulations 0 and C1, respectively.

Figure 9: Front side impaction angle for Λ=12 with no thermophoresis.
Figure 10: Impaction efficiency over Stokes number for different cylinder temperatures (simulations 0, dT3, dT10 and dT400 as listed in table 4). A high temperature difference increases $\eta$ for small particles. The dotted lines correspond to the impaction efficiency predicted by Eq. (25) for small Stokes numbers.

If the thermal diffusivity is changed linearly with viscosity, the Prandtl number is kept constant. From figures 11 and 12 we see that increasing the flow Reynolds number results in higher front side impaction efficiencies for intermediate Stokes numbers in the range $0.1 < St < 1$. Qualitatively, this is consistent with the findings of [3] obtained for isothermal cases. From figure 11, which is obtained by maintaining a constant Prandtl number, we see that there is a clear but not dramatic Reynolds number effect for small Stokes numbers. This is supported by the model in Eq. (25), which shows that the impaction efficiency for small Stokes numbers should be inversely proportional to the square root of the Reynolds number. This is represented by the horizontal dotted lines in the figure, which accurately reproduce the DNS results. The slight increase in impaction efficiency when the Stokes number is decreased from 0.03 to 0.01 for Re=1600 is due to poor statistics. It can also be mentioned that, since higher Reynolds numbers are obtained by decreasing the viscosity, the particles for low Stokes numbers become quite small. Unlike for isothermal cases, particle size does not, however, play any role in the impaction efficiency for small Stokes number when their impaction is controlled by the thermophoretic force.

If the thermal conductivity is not changed when changing viscosity, the Prandtl number is decreased for increasing Reynolds numbers, which is the case for the simulations shown in figure 12. Here we see that the difference in impaction efficiency for the smaller Stokes numbers is larger than for the case with constant Prandtl number. By comparing the solid and the dotted lines, we see that the DNS results (solid lines) follows the model predictions (dotted lines) nicely.
Figure 11: Front side impaction efficiency over Stokes number for different flow Reynolds numbers. Simulations “0”, “R400” and “R1600”. Higher Reynolds numbers result in higher $\eta$ for medium Stokes numbers, while for low Stokes numbers $\eta$ is decreased. The dotted lines correspond to the impaction efficiency predicted by Eq. (25) for small Stokes numbers.

Figure 12: Front side impaction efficiency over Stokes number for different flow Reynolds numbers. The Prandtl number is inversely proportional to the Reynolds number, such that it equals 0.7, 0.175 and 0.043 for $Re =$100, 400 and 1600, respectively. Simulations “0”, “RPv400” and “RPv1600”. Higher Reynolds numbers result in higher $\eta$ for medium Stokes numbers, while for low Stokes numbers $\eta$ is decreased. The dotted lines correspond to the impaction efficiency predicted by Eq. (25) for small Stokes numbers.
6. Conclusions

The effect of thermophoresis on the impaction efficiency of particles on a cylinder is studied for different values of the Reynolds number, conductivity ratio, temperature difference and particle Stokes number. Compared to the case where thermophoresis is neglected, the impaction efficiencies become independent of Stokes number for small Stokes numbers ($\lesssim 0.1$), corresponding to small particles. For small Stokes numbers, the impaction efficiency is larger for low conductivity ratios, high temperature differences and low Reynolds numbers. Thermophoresis has an insignificant effect on particles with Stokes numbers larger than $\sim 0.5$, but is often dominating the impaction rate for $\lesssim 0.3$.

An algebraic model (see Eq. (25)) that predicts the impaction efficiency due to thermophoresis has been developed based on fundamental principles. The developed model is valid as long as the thermophoretic force is not too strong (see Eq. (26) and figure 7). Outside its range of validity, a reliable model does not yet exist. This should be the focus of future research.

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