THE DECONFINING PHASE TRANSITION
AS AN AHARONOV-BOHM EFFECT *

JANOS POLONYI

Laboratoire de Physique Théorique, Université Louis Pasteur,
3, rue de l’Université, 67084 Strasbourg Cedex France,
and
Department of Atomic Physics, L. Eötvös University,
Pázmány P. Sétány 1/A 1117 Budapest, Hungary

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Abstract

A subjective and incomplete list of interesting and unique features
of the deconfinement phase transition is presented. Furthermore a
formal similarity of the density matrix of the Aharonov-Bohm system
and QCD is mentioned, as well.

1 Introduction

The first strong hints of the deconfined quarks at high temperature appeared
more than ten years ago [1], the numerical confirmation [2] followed soon.
Subsequently a large number of details has been clarified but the driving
force of the deconfinement transition, the confinement-deconfinement mech-
anism, remained elusive. A subjective and a rather sketchy list of remarks

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is presented here to indicate a unique and challenging aspect of this phase transition. Only one detail will be discussed in a slightly more detailed manner, the formal similarity between the density matrix of the Aharonov-Bohm (A-B) system and of QCD.

One can distinguish two confinement mechanisms [3], a hard and a soft one. The hard mechanism is responsible for the linear potential between static test quarks in the absence of dynamical quarks. The soft, low energy mechanism is which screens a test quark and was thought by V. Gribov to be similar to the supercritical vacuum of QED. More precisely the infrared instability of the perturbative QCD, the source of the hard confinement mechanism, leads to strong gluon interactions at large distances. Sufficiently far from a test quark the coupling constants reaches a large enough value to ignite the spontaneous creation of the quark-anti quark pairs which in turn shield the test quark charge. Most of the remarks mentioned here refers to the hard confinement mechanism which is more elementary and should be clarified before embarking the study of the soft mechanism of full QCD.

2 Unusual or unique features

1. Different degrees of freedom: We find different degrees of freedom at the two sides of the phase transition. This happens in a number of other phase transitions, the Mott or the localisation-delocalisation transitions may serve as examples. Observe that the elementary degrees of freedom are recovered among the highly excited states in these cases. This does not happen in the hadronic phase. The relevance of this obvious remark becomes clear by considering the thermal average of an observable $A$, $\langle A \rangle = Z^{-1} \sum_n \langle n | A | n \rangle e^{-E_n/T}$. In order to reproduce the thermal averages we use the color singlet asymptotic states $|n\rangle$ of the hamiltonian of the strong interactions. How can we recover the contributions of an isolated, deconfined quark to the given observable? The only way out of this problem is the modification of the Hilbert space at the phase transition.

2. Weakly or strongly coupled phase? The asymptotically free running coupling constants becomes small around the typical energy scale $p = T$ at high energies, $T > \Lambda_{QCD} \approx T_c$. Does that mean that the deconfined phase is weakly coupled at high enough temperature? The answer is known to be negative since long time [4]. The small parameter of the perturbation
expansion at high temperature stems from a non-perturbative quantity, the magnetic screening mass. This can be understood by recalling that the thermal bath breaks the Lorentz invariance. Though the typical energy scale is pushed up at high temperature $E \approx T$, the (off-shell) momentum scale in the loop integrals is effected differently by the temperature and the infrared stabilization of the long wavelength modes remains a difficult question. This is because the partition function of the high temperature 3 + 1 dimensional QCD can be approximated by a 3 dimensional (classical) Yang-Mills-Higgs system and the infrared sensitivity of the partition function increases by lowering the dimension. Thus the fate of the perturbation expansion which is based on massless gluons depends on the screening mechanism. The usual strategy of dealing with the IR divergences, the separation of the scales $T$, $gT$ and $g^2T$, can not solve this problem because $g$ does not reach small enough values, $g(m_{Planck}) \approx 1/2$.

3. Order parameter: The order parameter related to the hard confinement mechanism is the trace of the heavy quark propagator continued over complex time,

$$\omega(\vec{x}, t) = \langle 0 | \psi_\alpha(\vec{x}, t + \frac{i}{T}) \bar{\psi}_\alpha(\vec{x}, t) | 0 \rangle. \tag{1}$$

In the high temperature phase where the time extent of the Euclidean space-time is shorter than the correlation length, $1/T < \xi \approx \Lambda_{QCD}^{-1}$, the gluon field variables are correlated along the world line of the heavy quark and the order parameter develops a non-vanishing expectation value. A distinguishing feature of the deconfining transition is that its order parameter is not a canonical variable. It controls the symmetry with respect the global center gauge transformations performed at the initial or the final state of a transition amplitude. It is important to keep in mind that the center of the global gauge transformations is the fundamental group of the gluonic configuration space $Z_3 = \pi_1(SU(3)/Z_3)$. The only other known dynamical breakdown of the fundamental group symmetry is the liquid-droplet quantum phase transition.

4. Finite volume effects: The ratio of the gluonic partition functions with

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1. The center $C(G)$ of the group $G$ is a subgroup of $G$. It consists of the elements which commute with $G$, $[C, G] = 0$, e.g. $C(SU(N)) = Z_N$.

2. Consider the gauge transformation $\tilde{A}(\vec{x}) \to g(\vec{x})(\bar{\psi} + \tilde{A}(\vec{x}))g^\dagger(\vec{x})$ acting on the anti-hermitean gauge field in the temporal gauge. The global gauge transformations which commute with other gauge transformations leave $\tilde{A}(\vec{x})$ invariant.

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and without a static quark is given by the expectation value of the order parameter, \( e^{-\{F_q - F\}/T} = \langle \omega \rangle \). Since the spontaneous symmetry breaking does not occur in a finite system, \( \langle \omega \rangle = 0 \) and the static quarks always appear confined, \( F_q = \infty \), in finite volume. Where does the singular free energy density, \( F_q/V = \infty \), come from? This problem is solved by taking into account the destructive interference between the homotopy classes in the gluonic configuration space.

5. Symmetry breaking by the kinetic energy: The spontaneous symmetry breaking mechanism is operating at low energy where the order parameter is driven to a non-symmetrical value due to the degenerate minima of the potential energy. The kinetic energy might drive a spontaneous, or more precisely dynamical symmetry breaking at high energies. The dynamical breakdown of the center symmetry results from such a mechanism \([5]\). This can be understood by inspecting a quantum top, the baby version of the SU(2) Yang-Mills model. The configuration space which consists of the 3 \( \times \) 3 orthogonal matrices, \( \{ R \} = SO(3) = SU(2)/Z_2 \), is doubly connected and the wave functions are single and double valued in the integer (gluons) and the half-integer (quarks) spin subspaces, respectively. Consider now the transition amplitude \( A(R', R) = \langle R'|e^{-itH/\hbar}|R \rangle \) as the function of the final state \( R' \). Since an orientation of the top is indistinguishable from its 2\( \pi \) rotated copy the integer spin amplitude is doubly degenerate on the covering space SU(2), \( A(r', r) = A(r'', r) \) where the final points \( r', r'' \in SU(2) \) differ in a rotation by 2\( \pi \), \( r' = -r'' \) (center symmetry). Suppose that \( r' \) is closer to the initial point \( r \) than \( r'' \). Then the kinetic energy tends to suppress the propagation to \( r'' \) if the time available for the propagation is short (high temperature or energy). The result for an infinite top whose coordinate \( r \) influences infinitely many degrees of freedom (global gauge transformations) is that the propagation to \( r'' \) is totally suppressed (center symmetry breakdown). The confinement can be understood as the destructive interference in the quark propagator between the different homotopy classes. In fact, the center symmetry of the pure gluon system yields identical amplitudes in different homotopy classes. But a particle in the fundamental representation of the gauge group SU(\( N \)) propagating along the system picks up the phases \( e^{2i\pi n/N} \), \( n = 1, \ldots, N \) which add up to zero. The result is the absence of these particles in the final states. We find here another characteristic feature of the deconfinement transition: it corresponds to a transition amplitude rather than to the vacuum. This is the key to find a synthesis between the high
and the low energy scattering experiments, described in terms of the partons and the hadronic bound states, respectively. In other words, as the time of a collision process is shortened the transition matrix elements go over the “de-confined”, center symmetry broken phase and the elementary constituents (partons) appear.

6. Permanent confinement of triality \[5\]:

(i) The deconfining phase transition consists of the dynamical breakdown of the Gauss’ law and the modification of the Hilbert space for gluons,

\[
H = \begin{cases} 
H_0 & T < T_c, \\
H_0 \oplus H_{-1} \oplus H_1 & T > T_c,
\end{cases}
\]

(2)

where the subscript stands for the triality, the center charge \[3\]. Such a description of the phase transition is the resolution of the puzzle mentioned in point 1. (ii) The triality is permanently confined at any temperature. The deconfined quark seen in the numerical simulation is actually a composite particle containing a quark and its vacuum polarization cloud. The latter has a multi-valued wave functional in such a manner that the total (quark plus gluon) wave functional is single valued. The triality charge of the quark is screened by the unusual gluon state. (iii) The color-magnetic monopoles relate the rotations in the external and the color spaces. These monopoles acquire a half-integer spin in the gluonic states with multi-valued wave functional, a manner similar to the generation of the spin for skyrmions. The unusual gluonic screening cloud is the sum of states with odd and even number of monopoles. These components correspond to fermionic and bosonic exchange statistics. Thus the state of a deconfined quark is the sum of components with bosonic (odd number of monopoles) and fermionic (even number of monopoles) properties. The breakdown of the center symmetry leads to the mixing of the fermi and bose statistics for the deconfined quarks.

7. Triality-canonical ensemble: The transition between the canonical and the grand-canonical ensembles requires smooth enough dependence on the density. Due to the confinement mechanism the formal energy density diverges for non-integer baryon numbers, or non-vanishing triality charges

\[5\]The wave functional \(\Psi[\vec{A}(\vec{x})] \in H_\ell\) changes by the phase factor \(e^{2\pi \ell n/3}\) when the global center gauge transformation \(e^{2\pi n/3}\) is performed on \(\vec{A}(\vec{x})\). The multi-valued nature of the wave functional is to keep track of the global center gauge transformations, the elements of the fundamental group of the gluonic configuration space which are represented in a trivial manner on the gluon field.
It turns out that the triality-canonical ensemble predicts different center domain structure at the deconfining phase transition than the usual grand-canonical ensemble \cite{6}. This may happen because the center symmetry is broken spontaneously by the quark-antiquark sector for \( T < T_c \) and dynamically by the kinetic energy for \( T > T_c \) in the canonical ensemble. This furthermore means that the formal center symmetry is preserved in the presence of dynamical quarks and the results mentioned in this talk remain valid in the triality-canonical ensemble with dynamical quarks.

3 Density matrix for the A-B system and for gluons

**A-B system:** Consider a charged particle moving on the unit circle in periodic gauge where the wave function is periodic, \( \psi(\phi + 2\pi) = \psi(\phi) \). The Hamiltonian is \( H = (-i\partial_\phi - \Theta/2\pi)^2/2 \), where \( \Theta = 2\pi A_\phi \) stands for the magnetic flux of the circle. The eigenstates and the eigenvalues are \( \psi_n(\phi) = e^{in\phi} \), and \( E_n = (n - \Theta/2\pi)^2/2 \), respectively. The density matrix is given by \( \rho(\alpha, \beta) = Z^{-1} \sum_n e^{in(\alpha-\beta) - (n - \Theta/2\pi)^2/2T} \), where \( Z \) is the partition function, \( Z = \sum_n e^{-(n - \Theta/2\pi)^2/2T} \). Notice that the probability density \( p(\phi) = \rho(\phi, \phi) \) is real non-negative, as it should be. The periodicity of the wave functions gives \( \rho(\alpha, \alpha + 2\pi) = \rho(\alpha, \alpha) \).

Let us go into an aperiodic gauge by performing the transformation \( \psi(\phi) \rightarrow e^{-i\phi/2\pi} \psi(\phi) \). The Hamiltonian is simpler, \( H \rightarrow -\partial_\phi^2/2 \), but has the same spectrum as before because the wave functions are multi-valued, \( \psi(\phi + 2\pi) = e^{-i\phi\Theta} \psi(\phi) \). In particular, the eigenvectors are \( \psi_n(\phi) = e^{i\phi(n - \Theta/2\pi)} \). The density matrix transforms as \( \rho(\alpha, \beta) \rightarrow e^{-i(\alpha-\beta)\Theta/2\pi} \rho(\alpha, \beta) \), and becomes multi-valued, as well, \( \rho(\alpha, \alpha + 2\pi) = e^{i\Theta} \rho(\alpha, \alpha) \) which makes the construction of the probability density non-trivial. In fact, the choice of different Riemann-sheets for the two coordinate variables yields complex probability and partition function. But notice that the complex factor is the same for each contribution,

\[
Z_{\text{compl}} = \int d\phi \rho(\phi, \phi + 2\pi) = e^{i\Theta} \int d\phi \rho(\phi, \phi),
\]

and the imaginary part of the entropy is an overall constant which does not influence the thermalization and thermodynamics can be applied.
QCD: A similar argument can easily be constructed for gluons yielding the following results: (i) The multi-valued nature of the gluonic wave functional of a deconfined quark is shown by the possible non-positive or complex expectation value of the order parameter $\langle \omega \rangle$, the partition function of a quark. (ii) The density matrix for gluons and a deconfined quark is multi-valued as it happens for the A-B system in the aperiodic gauge. The change of the Riemann-sheet, $\rho(\alpha, \alpha) \rightarrow \rho(\alpha, \alpha + 2\pi)$, corresponds to the center transformation. (iii) Thus the complex part of the free energy and the entropy of a deconfined quark is a simple kinematical constant which agrees for each contribution to the partition function and does not influence the thermalization and the applicability of the rules of thermodynamics. (iv) The complex part of the deconfined quark entropy may lead to observable effects in the triality-canonical ensemble [7] which is more realistic than the grand-canonical one.

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