Horizon wave-function for single localized particles: GUP and quantum black hole decay

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A localised particle in Quantum Mechanics is described by a wave packet in position space, regardless of its energy. However, from the point of view of General Relativity, if the particle’s energy density exceeds a certain threshold, it should be a black hole. In order to combine these two pictures, we introduce a horizon wave-function determined by the particle wave-function in position space, which eventually yields the probability that the particle is a black hole. The existence of a minimum mass for black holes naturally follows, albeit not in the form of a sharp value around the Planck scale, but rather like a vanishing probability that a particle much lighter than the Planck mass be a black hole. We also show that our construction entails an effective Generalised Uncertainty Principle (GUP), simply obtained by adding the uncertainties coming from the two wave-functions associated to a particle. Finally, the decay of microscopic (quantum) black holes is also described in agreement with what the GUP predicts.

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I. INTRODUCTION AND MOTIVATION

Understanding all the physical aspects in the gravitational collapse of a compact object and how black holes form, remains one of the most intriguing challenges of contemporary theoretical physics. After the seminal papers of Oppenheimer and co-workers\cite{1}, the literature on the subject has grown immensely, but many issues are still open in General Relativity (see, e.g., Refs.\cite{2,3}, and references therein), not to mention the conceptual and technical difficulties one faces when the quantum nature of the collapsing matter is taken into account. Assuming quantum gravitational fluctuations are small, one can describe matter by means of Quantum Field Theory on the curved background space-time, an approach which has produced remarkable results, but is unlikely to be directly applicable to a self-gravitating system representing a collapsing object.

A general property of the Einstein theory is that the gravitational interaction is always attractive and we are thus not allowed to neglect its effect on the causal structure of space-time if we pack enough energy in a sufficiently small volume. This can occur, for example, if two particles (for simplicity, of negligible spatial extension and total angular momentum) collide with an impact parameter \( b \) shorter than the Schwarzschild radius corresponding to the total center-mass energy \( E \) of the system, that is \cite{31}

\[
b \lesssim 2 \ell_p \frac{E}{m_p} = R_H .
\]  

This \textit{hoop conjecture} \cite{4} has been checked and verified theoretically in a variety of situations, but it was initially formulated for black holes of (at least) astrophysical size \cite{5}, for which the very concept of a classical background metric and related horizon structure should be reasonably safe (for a review of some problems, see the bibliography in Ref.\cite{6}). Whether the concepts involved in the above conclusion can also be trusted for masses approaching the Planck size, however, is definitely more challenging. In fact, for masses in that range, quantum effects may hardly be neglected (for a recent discussion, see, e.g., Ref.\cite{7}) and it is reasonable that the picture arising from General Relativistic black holes must be replaced in order to include the possible existence of new objects, generically referred to as “quantum black holes” (see, e.g., Refs.\cite{8,9}).

The main complication in studying the Planck regime is that we do not have any experimental insight thereof, which makes it very difficult to tell whether any theory we could come up with is physically relevant. We might instead start from our established concepts and knowledge of nature, and push them beyond the present experimental limits. If we set out to do so, we immediately meet with a conceptual challenge: how can we describe a system containing both Quantum Mechanical objects (such as the elementary particles of the Standard Model) and classically defined horizons? The aim of this paper is precisely to introduce the definition of a wave function for the horizon that can be associated with any localised Quantum Mechanical particle\cite{10}. This tool will allow us to put on quantitative ground the condition that distinguishes a black hole from a regular particle. And we shall also see that our construction naturally leads to an effective Generalised Uncertainty Principle (GUP)\cite{11} for the particle position, and a decay rate for microscopic black holes.
The paper is organised as follows: in the next Section we introduce the main ideas that define the horizon wave-function associated with any localised Quantum Mechanical particle; in Section III, we then apply the general construction to the particularly simple case of a particle described by a Gaussian wave-function at rest in flat space-time, for which we explicitly obtain the probability that the particle is a black hole, we recover the GUP and a minimum measurable length, and estimate the decay rate of a black hole with mass around the Planck scale; finally, in Section IV, we comment on our findings and outline future applications.

II. HORIZON QUANTUM MECHANICS

Given a matter source, say a spherically symmetric “particle”, General Relativity and Quantum Mechanics naturally associate with it two length scales: the Schwarzschild radius and the Compton-de Broglie wavelength, respectively. We shall therefore start by briefly reviewing these concepts and then propose how to extend the former into the realm of Quantum Mechanics, where the latter is born.

A. Spherical trapping horizons

The appearance of a classical horizon is relatively easy to understand in a spherically symmetric space-time. Let us first recall that we can write a general spherically symmetric metric $g_{\mu\nu}$ as

$$ds^2 = g_{ij} dx^i \, dx^j + r^2(x^i) \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right),$$

where $r$ is the areal coordinate and $x^i = (x^1, x^2)$ are coordinates on surfaces where the angles $\theta$ and $\phi$ are constant. The location of a trapping horizon, a surface where the escape velocity equals the speed of light [32], is then determined by the equation [12]

$$0 = g^{ij} \nabla_i r \, \nabla_j r = 1 - \frac{2M}{r},$$

where $\nabla_i$ is the covector perpendicular to surfaces of constant area $A = 4\pi r^2$. The function $M = \ell_p m/m_p$ is the active gravitational (or Misner-Sharp) mass, representing the total energy enclosed within a sphere of radius $r$. For example, if we set $x^1 = t$ and $x^2 = r$, the function $m$ is explicitly given by the integral of the classical matter density $\rho = \rho(x^1)$ weighted by the flat metric volume measure,

$$m(t,r) = \frac{4\pi}{3} \int_0^r \rho(t,\bar{r}) \bar{r}^2 \, d\bar{r},$$

as if the space inside the sphere were flat. Of course, it is in general very difficult to follow the dynamics of a given matter distribution and verify the existence of surfaces satisfying Eq. (3), but we can say an horizon exists if there are values of $r$ such that

$$R_M = 2 \, M(t,r) \geq r,$$

which generalises the hoop conjecture (1) to continuous energy densities (in fact, the horizon radius saturates the above inequality, i.e. $R_H = r$).

Note the above equation does not lead to any mass threshold for the existence of a black hole, since $M$ is not limited from below in the classical theory, and the area of the trapping surface can be vanishingly small. However, if we consider a spin-less point-like source of mass $m$, Quantum Mechanics introduces an uncertainty in its spatial localisation, typically of the order of the Compton length,

$$\lambda_m \simeq \ell_p \frac{m_p}{m} = \frac{\ell_p^2}{M}.$$  

Assuming quantum physics is a more refined description of classical physics, the clash of the two lengths, $R_H$ and $\lambda_m$, implies that the former only makes sense provided it is larger than the latter,

$$R_H \gtrsim \lambda_m \quad \Rightarrow \quad m \gtrsim m_p,$$

or $M \gtrsim \ell_p$. Note that this argument employs the flat space Compton length (6), and it is likely that the particle’s self-gravity will affect it. However, it is still reasonable to assume the condition (7) holds as an order of magnitude estimate.

Overall, the common argument that quantum gravity effects should become relevant only at scales of order $m_p$ or higher now appears questionable, since the condition (7) implies that such a system can be fairly well-described in classical terms. This is indeed at the core of the idea of “classicalization” given in Ref. [13] and, before that, of gravitationally inspired GUPs [11, 14]. In particular, following the canonical steps that lead to the construction of Quantum Mechanics, the latter are usually assumed to hold as fundamental principles for the reformulation of Quantum Mechanics in the presence of gravity. Note then that gravity would reduce to a “kinematical effect” encoded by the modified commutators for the canonical variables. In the following we shall instead start from the introduction of an auxiliary wave-function that describes the horizon associated with a given localised particle, and show that a modified uncertainty relation follows consequently.

B. Horizon Wave-function

Let us first formulate the construction in a somewhat general fashion. For simplicity, we shall only consider quantum mechanical states representing spherically symmetric objects, which are both localised in space and at
rest in the chosen reference frame. The particle is consequently described by a wave-function \( \psi_S \in L^2(\mathbb{R}^3) \), which we assume can be decomposed into energy eigenstates,

\[
| \psi_S \rangle = \sum E C(E) | \psi_E \rangle ,
\]

where the sum represents the spectral decomposition in Hamiltonian eigenmodes,

\[
\tilde{H} | \psi_E \rangle = E | \psi_E \rangle ,
\]

and the actual Hamiltonian \( \tilde{H} \) needs not be specified yet [33]. The expression of the Schwarzschild radius in Eq. (1) can be inverted to obtain

\[
E = m_p \frac{R_H}{2 \ell_p} ,
\]

and we then define the (unnormalised) “horizon wave-function” as \( \psi_H(R_H) = C(m_p R_H/2 \ell_p) \), whose normalisation is fixed by assuming the scalar product

\[
\langle \psi_H | \phi_H \rangle = 4 \pi \int_0^\infty \psi_H^*(R_H) \phi_H(R_H) R_H^2 \ dR_H .
\]

We could now simply say that the normalised wave-function \( \psi_H \) yields the probability that an observer would detect a horizon of areal radius \( r = R_H \) associated with the particle in the quantum state \( \psi_S \). Such a horizon would necessarily be “fuzzy”, like is the position of the particle itself, but giving such a claim an experimental meaning does not appear very simple.

A more precise use of the horizon wave-function can however be already outlined. For example, having defined the wave-function \( \psi_H \) associated with a given \( \psi_S \), the probability density that the particle lies inside its own horizon of radius \( r = R_H \) will be given by

\[
P_{BH} = P_S(r < R_H) \ P_H(R_H) ,
\]

where

\[
P_S(r < R_H) = 4 \pi \int_0^{R_H} |\psi_S(r)|^2 r^2 \ dr
\]

is the probability that the particle is inside a sphere of radius \( r = R_H \), and

\[
P_H(R_H) = 4 \pi R_H^2 |\psi_H(R_H)|^2
\]

is the probability that the sphere of radius \( r = R_H \) is a horizon. Finally, the probability that the particle described by the wave-function \( \psi_S \) is a black hole will be obtained by integrating (12) over all possible values of the horizon radius, namely

\[
P_{BH} = \int_0^\infty P_S(r < R_H) \ dR_H .
\]

It is this final probability we now proceed to clarify with an example, along with a derivation of a GUP and some predictions for the decay of a quantum black hole.

III. GAUSSIAN PACKET AT REST IN FLAT SPACE

Assuming the space-time is flat, our construction can be exemplified by describing the massive particle at rest in the origin of the reference frame with the spherically symmetric Gaussian wave-function

\[
\psi_S(r) = e^{-\frac{r^2}{2 \lambda^2}} ,
\]

where we shall usually assume that the width \( \ell \) is given by the Compton length (6) of the particle,

\[
\ell = \lambda_m \sim \ell_p \frac{m_p}{m} .
\]

The above packet corresponds to the momentum space wave-function

\[
\psi_S(p) = e^{-\frac{p^2}{2 \Delta^2}} \frac{1}{\Delta^{3/2} \pi^{3/4}} ,
\]

where \( p^2 = \vec{p} \cdot \vec{p} \) is the square modulus of the spatial momentum, and the width

\[
\Delta = m_p \frac{\ell_p}{\ell} \approx m .
\]

For the energy of the particle, we can simply assume the relativistic mass-shell relation in flat space,

\[
E^2 = p^2 + m^2 ,
\]

and, upon inverting the expression of the Schwarzschild radius (1), we obtain the unnormalised wave-function

\[
\tilde{\psi}_H(R_H) = \frac{\ell^{3/2} m_p^2}{\pi^{3/4} \ell_p^{3/2} m_{\tilde{p}}^{3/2}} .
\]

Finally, the inner product (11) yields the normalized horizon wave-function

\[
\psi_H(R_H) = \frac{\ell^{3/2} m_p}{\pi^{3/4} \ell_p^{3/2} m_{\tilde{p}}} .
\]

Note that, since \( \langle \tilde{r}^2 \rangle \sim \ell^2 \) and \( \langle \tilde{R}_H^2 \rangle \sim \ell_p^4/\ell^2 \), we expect the particle will be inside its own horizon if \( \langle \tilde{r}^2 \rangle \ll \langle \tilde{R}_H^2 \rangle \), which precisely yields the condition (7) if \( \ell \sim m^{-1} \). This is clear, for example, in Fig. 1, where the probability \( P_H = P_H(r) \) is plotted along with the probability

\[
P_S(r) = 4 \pi r^2 |\psi_S(r)|^2 ,
\]

for \( m < m_p \) and \( m > m_p \). In the former case, the horizon is more likely found with a smaller radius than the particle’s, with the opposite occurring in the latter. In
or, writing $P_{\text{BH}}$ as a function of $m,$

$$P_{\text{BH}} = \frac{2}{\pi} \left[ \arctan \left( \frac{2m^2}{m_p^2} \right) + 2 \frac{m_p^2 (4 - m^4/m^4_p)}{m^2 (4 + m^4_p/m^4)^2} \right]$$

In Fig. 2, we show the probability density (24), for two different values of the Gaussian width $\ell.$ Since $\ell \sim m^{-1},$ it is already clear that such probability decreases for decreasing $m$ (below the Planck mass). In fact, in Fig. 3, we show the probability (25) that the particle is a black hole as a function of the Gaussian width $\ell$ (upper panel) and particle mass $m \sim \ell^{-1}$ (lower panel). From the plot of $P_{\text{BH}},$ it appears pretty obvious that the particle is most likely a black hole, $P_{\text{BH}} \simeq 1,$ if $\ell \lesssim \ell_p.$ Assuming as usual $\ell \sim m^{-1},$ we have thus derived the same condition (7), from a totally Quantum Mechanical picture.

An important remark is that we have here assumed flat space throughout the computation, which means the self-gravity of the particle has been neglected. It is very likely that such an approximation fails for large black holes with $m \gg m_p,$ although the general idea outlined in Section II B should still be valid. Of course, one could then improve the description of particles with $m \gg m_p$ by employing a curved-space mass-shell relation and suitable normal modes, rather than simple plane waves.
A. Effective GUP

For the Gaussian packet described above, it is easy to find that the usual Quantum Mechanical uncertainty in radial position is given by

$$\langle \Delta r^2 \rangle = 4 \pi \int_0^\infty |\psi_S(r)|^2 r^4 \, dr$$

$$- \left( 4 \pi \int_0^\infty |\psi_S(r)|^2 r^3 \, dr \right)^2$$

$$= \left( \frac{3 \pi - 8}{2 \pi} \right) \ell^2 .$$ \hspace{1cm} (27)

Analogously, the uncertainty in the horizon radius will be given by

$$\langle \Delta R_H^2 \rangle = 4 \pi \int_0^\infty |\psi_H(R_H)|^2 R_H^4 \, dR_H$$

$$- \left( 4 \pi \int_0^\infty |\psi_H(R_H)|^2 R_H^3 \, dR_H \right)^2$$

$$= 4 \left( \frac{3 \pi - 8}{2 \pi} \right) \ell^4 .$$ \hspace{1cm} (28)

Since

$$\langle \Delta p^2 \rangle = 4 \pi \int_0^\infty |\psi_S(p)|^2 p^4 \, dp$$

$$- \left( 4 \pi \int_0^\infty |\psi_S(p)|^2 p^3 \, dp \right)^2$$

$$= \left( \frac{3 \pi - 8}{2 \pi} \right) m_p^2 \ell^2 \equiv \Delta p^2 ,$$ \hspace{1cm} (29)

we can also write

$$\ell^2 = \left( \frac{3 \pi - 8}{2 \pi} \right) m_p^2 \Delta p^2 .$$ \hspace{1cm} (30)

Finally, by combining the uncertainty (27) with (28) linearly, we find

$$\Delta r \equiv \sqrt{\langle \Delta r^2 \rangle} + \gamma \sqrt{\langle \Delta R_H^2 \rangle}$$

$$= \left( \frac{3 \pi - 8}{2 \pi} \right) \ell_p m_p \Delta p + 2 \gamma \ell_p \frac{\Delta p}{m_p} ,$$ \hspace{1cm} (31)

where $\gamma$ is a coefficient of order one, and the result is plotted in Fig. 4 (for $\gamma = 1$). This is precisely the kind of GUP considered in Refs. [11], leading to a minimum measurable length

$$\Delta r \geq 2 \sqrt{\gamma \frac{3 \pi - 8}{\pi} \ell_p} \simeq 1.3 \sqrt{\gamma} \ell_p ,$$ \hspace{1cm} (32)

obtained for

$$\Delta p = \sqrt{\frac{3 \pi - 8}{\pi} \frac{m_p}{\gamma}} .$$ \hspace{1cm} (33)

Of course, one might consider different ways of combining the two uncertainties (27) and (28), or even avoid this step and just make a direct use of the horizon wavefunction. In this respect, the present approach appears more flexible, provided one is able to extend it to different physical systems, as we shall further discuss in the last Section.

B. Quantum black hole evaporation

The well-known result due to Hawking [15],

$$T_H = \frac{m^2}{8 \pi m} ,$$ \hspace{1cm} (34)

extrapolated to vanishingly small mass $M$ implies that $T_H$ diverges. On the other hand, one can derive modified black hole temperatures for $m \simeq m_p$ from the GUP [16, 17]. In particular, we just recall that one obtains

$$m = \frac{m_p^2}{8 \pi T} + 2 \pi \beta T ,$$ \hspace{1cm} (35)

where

$$\beta = \frac{\gamma}{4 \pi (3 \pi - 8)} > 0 ,$$ \hspace{1cm} (36)

in order to ensure the existence of a minimum mass for the black hole (see Fig. 5). This is a consistency condition with the result that $P_{BH} \simeq 1$ only for $m \gtrsim m_p$, or that one does not have a black hole for masses significantly smaller than $m_p$. In fact, from (35) we get

$$m_{\text{min}} = \sqrt{\beta} m_p , \quad T_{\text{max}} = \frac{m_p}{4 \pi \sqrt{\beta} .}$$ \hspace{1cm} (37)

Upon solving the above Eq. (35), and considering the “physical” branch (which reproduces the Hawking behav-
for $m \gg m_p$, one obtains

$$T = \frac{1}{4\pi\beta} \left( m - \sqrt{m^2 - \beta m^2_p} \right)$$

for $0 < \beta < 1$, where we expanded around $m \simeq m_p$. It is interesting to note that such expression for $T$ is still meaningful also for $\beta < 0$. These possibilities hint at a lattice micro structure of the space-time, and have been explored, e.g. in polymer quantization, and in world crystal physics [17].

Recalling now that the emission rate can be written as

$$\frac{dm}{dt} = -8\pi^3 m^2 T^4 \frac{\beta}{15 m^5 \ell_p},$$

we obtain the decay rate

$$-\frac{dm}{dt} \simeq 8\pi^3 \frac{m^2}{m_p \ell_p} + O(m - m_p),$$

for $T \simeq T_p = m_p$ (or $m \simeq m_p$), where $4 \cdot 10^{-5} < \alpha < 7 \cdot 10^{-4}$ when $0 < \beta < 1$.

It is perhaps questionable that objects with a mass of the order of $m_p$ can be described by the usual thermodynamical arguments, which stem from a (semi-)classical picture of black holes. However, the horizon wave-function for a particle was precisely conceived to describe this quantum regime, and we can now assume that the probability the black hole decays is given by the probability $P_T$ that the particle can be found outside its own horizon [34]. Of course, if the mass $m \ll m_p$, the horizon wave-function tells us the particle is most likely not a black hole to begin with, so the above interpretation must be restricted to $m \simeq m_p$ (see again Fig. 1).

We first define

$$P_S(r > R_H) = P_S(r > R_H) P_H(R_H),$$

where now

$$P_S(r > R_H) = 4\pi \int_{R_H}^{\infty} |\psi_S(r)|^2 r^2 dr.$$  \hspace{1cm} (42)

Upon integrating the above probability over all values of $R_H$, we then obtain (since $m \sim \ell^{-1}$)

$$P_T(m) = 1 - P_{BH}(m),$$

and, expanding (26) for $m \simeq m_p$,

$$P_T(m) \simeq a - b \frac{m - m_p}{m_p},$$

where $a \simeq 0.14$ and $b \simeq 0.65$ are positive constants of order one. The amount of the particle’s energy that can be found outside the horizon could thus be estimated by

$$\Delta m \simeq m P_T \simeq a m + O(m - m_p).$$

At the same time, from the time-energy uncertainty relation

$$\Delta E \Delta t \simeq m_p \ell_p,$$  \hspace{1cm} (46)

one obtains the typical emission time

$$\Delta t \simeq \frac{\ell^2}{\Delta R_H} \simeq \ell,$$  \hspace{1cm} (47)

where we used (1) and (28). Putting the two pieces together, we then find that a near Planck size black hole would emit according to

$$-\frac{\Delta m}{\Delta t} \simeq a \frac{m}{\ell} + O(m - m_p)$$

$$\simeq a \frac{m^2}{m_p \ell_p} + O(m - m_p),$$

in functional agreement with the prediction from the GUP given in Eq. (40).

It is now important to remark that there is a fairly large numerical discrepancy between the numerical coefficients in Eq. (40) and those in Eq. (48). For once, this disparity can perhaps be traced back to the fact that, with Eq. (39), we are applying the canonical formalism to a Planck mass particle, which is not completely sensible, since the particle/black hole should be in quasi equilibrium with its radiation for thermodynamical arguments to hold. The horizon wave-function, instead, knows nothing of the thermodynamics, and should have therefore a more general validity. However, we must point out that the above description of black hole evaporation relies on a totally static representation of the Quantum Mechanical particle, and is therefore to be viewed as a first attempt at modelling the decay of a quantum black hole in the
present picture. A more accurate account of the micro-
scopic structure of quantum black holes is indeed likely
to change the details (see, e.g. Refs. [27, 28]), but the fact
that this simple treatment leads to results similar to those
following from the GUP is already intriguing, and sug-
gestive that an even more accurate Quantum Mechanical
description should be possible. Finally, let us mention
that in this Planckian regime, regardless of the micro-
sopic model, it would certainly be more appropriate to
use the microcanonical formalism [29] (based on energy
conservation, a property not entailed by the GUP). Fu-
ture work will be devoted to refine the calculation in these
all of these directions.

IV. CONCLUSIONS AND OUTLOOK

We have here introduced a horizon wave-function as a
tool that allows us to effectively describe the emergence
of a horizon in a localised Quantum Mechanical system.
For the simple case of a spherically symmetric massive
particle, the horizon wave-function already supports the
existence of a minimum black hole mass, without assum-
ing a priori the existence of a minimum (fundamental)
length [14, 19] [35]. Moreover, it does so in a genuinely
Quantum Mechanical fashion, since it produces a negli-
gible probability that a particle with mass much smaller
than \( m_p \) is a black hole, rather than giving a sharp value
for the particle mass above which the transition from
particle to black hole occurs. Further, the description
of black holes that the horizon wave-function entails was
shown to be compatible with GUPs, since it yields the
same kind of uncertainty relation in phase space, and a
similar decay rate for Planck size objects.

The results presented here should be however viewed
as preliminary, as the notion of a horizon wave-function
requires a thorough generalisation before it can be ef-
fectively employed to analyse more interesting physical
problems. We already mentioned in the Introduction
that it is of particular conceptual interest to study the
possibility of black hole production in high-energy col-
lisions \([22, 23]\). Let us here recall that, along these
lines, Dvali and co-workers [13] recently went on to con-
njecture that the high-energy limit of all physically rel-
levant Quantum Field Theories involves the formation
of a (semi)classical state (to wit, black hole formation
for gravity), which should automatically suppress trans-
Planckian quantum fluctuations. This idea extends the
concept of a GUP to include gravity, as was considered,
for example in Refs. [11, 14] and implies that the mass
of microscopic black holes must be quantized, and admit
a minimum value [24] (for more general cases, see also
Ref. [25]). Beside the conceptual relevance for the in-
clusion of gravity in a description of all forces of nature,
there is also the potential phenomenological relevance
of quantum mechanical effects during the formation of trap-
ping horizons and black holes of astrophysical size.

In fact, one should not forget that the basic building
blocks of matter remain the Standard Model particles,
and that at such extreme energy regimes quantum effects
should not be easily overlooked. All of the above con-
jectures would therefore be conspicuously substantiated
if we could understand the extremely complex dynamics
of colliding Standard Model particles, including the ef-
fact of the gravitational interaction, around the Planck
scale \([23, 26]\). To this purpose, the definition of the hori-
zon wave-function for simple spherical systems must be
generalised to describe particle collisions and the inclu-
sion of angular momentum in the initial and final con-
fugurations [30]. It appears hard to complete such steps
without a more detailed model of “quantum black holes”,
in order to define the Hilbert space of the horizon wave-
function. One could, for example, incorporate the con-
jecture of Refs. [27] and [28], and describe the matter
sourcing the black hole geometry as a condensate at the
phase transition.

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[31] We shall use units with $c = k_B = 1$, and always display the Newton constant $G = \ell_p / m_p$, where $\ell_p$ and $m_p$ are the Planck length and mass, respectively, so that $\hbar = \ell_p m_p$.

[32] More technically, a trapping surface is the location where the divergence of outgoing null congruences vanishes.

[33] This is where, for instance, the self-gravity of the particle may enter.

[34] The subscript T is for tunnelling, which is reminiscent of the interpretation of the Hawking emission as a tunnelling process through the horizon [18]. Note, however, that the horizon is fuzzy in our description and not a (backreacting) classical surface.

[35] The existence of this mass threshold may have phenomenological implications in models with extra spatial dimensions [20, 21], where the fundamental (gravitational) length corresponds to energy scales potentially as low as a few TeV's.