1. INTRODUCTION

One of the most exciting prospects for the upcoming Planck satellite is its capability to measure the polarization anisotropies of the cosmic microwave background (CMB) over the entire sky in nine frequency channels. The potential rewards from these measurements are many and include tighter constraints on cosmological parameters, determination of the reionization history of the universe, and detection of signatures left by primordial gravitational waves generated during inflation (Planck Collaboration 2005).

Measurement of the CMB polarization signal presents a great experimental challenge as it is an order of magnitude smaller than the temperature signal and is especially susceptible to distortions due to optical systematics and foreground contaminants. Indeed, if left untreated, leakage from the much stronger temperature signal will contaminate the polarization maps. Maps and spectra will also suffer from leakage from E-mode polarization to B-mode polarization, jeopardizing the potential detection of inflationary B modes. At the resolution and sensitivity of the next generation of experiments, including the Planck mission, studies of primordial non-Gaussianity may also be sensitive to beam-induced systematics. In this paper, we present a novel technique for both assessing and removing systematic effects due to beams in temperature and polarization maps.

The Planck satellite is designed to extract essentially all of the information in the primordial temperature anisotropies and to measure the polarization anisotropies to high accuracy for 2 \(\lesssim\ell\lesssim2500\). This will be achieved by measuring the full-sky signal to an angular resolution of 5\textquoteleft, to a sensitivity of \(\Delta T/T \sim 2 \times 10^{-6}\), and over a frequency range of 30–857 GHz (Planck Collaboration 2005). The scientific performance of Planck depends, in part, on the behavior of systematic effects which may distort the signal.

A primary objective of Planck is to produce all-sky CMB maps at each frequency. The process by which the satellite’s time-ordered data (TOD) is wrapped back on to the sphere to create an image is known as map making. The map-making process becomes difficult due to a number of challenges: distortions in the beam, foreground contamination through far-side lobes, size of the data, and correlated noise effects. It is of critical importance to fully characterize the beam, and use this information during map-making to deconvolve beam effects. We have previously described a powerful map-making algorithm which implements the beam deconvolution technique for temperature measurements (Armitage & Wandelt 2004). In this paper, we will extend that description to include polarization measurements. We refer to this new technique as PReBeaM: Polarized Regularized Beam deconvolution Map making. While we focus on reconstructing the map with a uniform effective beam and realize corrections to the power spectrum as a consequence, other work by Souradeep et al. (2006) and Mitra et al. (2007) has focused on deriving corrections to the power spectrum due to asymmetric (noncircular) beam effects.

Within the Planck Collaboration, the CTP working group has developed five map-making methods and compared their results using the simulated 30 GHz data in what is known as the Trieste paper (Ashdown et al. 2009). The Trieste paper assessed the impact of beam asymmetries on the Planck spectra without attempting to treat the problem of beam asymmetry at the map-making level (an angular power spectrum correction method was developed based on simplifying assumptions). In addition to PReBeaM, another deconvolution map-making technique for Planck has been established by D. L. Harrison et al. (2008, in preparation). Both methods allow for arbitrary beam shapes and in both cases the asymmetry of the beam is parameterized by an asymmetry parameter \(m_{\text{max}}\) which can vary between 0 and \(\ell_{\text{max}}\). Our method scales computationally as \(\mathcal{O}(\ell_{\text{max}}^3m_{\text{max}})\); this is advantageous when large gains in accuracy can be achieved with small increases in \(m_{\text{max}}\). In contrast, the Harrison approximate method scales as \(\mathcal{O}(\ell_{\text{max}}^2)\), thereby incurring a fixed computational expense for arbitrarily large \(m_{\text{max}}\) and effectively setting a limit to the maximum \(\ell\) at which the analysis can
be done. Excluding TOD operations we get a speed-up of \( \ell_{\text{max}}/m_{\text{max}} \), which is of \( O(10^2) \) for the case we examine in this paper. The Harrison method takes advantage of the Planck scanning strategy to condense the full TOD into phase-binned rings, thereby achieving a significant reduction in processing time.

A complete characterization of the beam includes both the main beam and the far-side lobes. Sidelobes are located as far away as 90° from the main focal plane beam, and therefore require a large \( m_{\text{max}} \) parameter for a complete harmonic description. In Armitage & Wandelt (2004), we demonstrated the full potential of our method using far-side lobes and maps with foreground signals. Here, we show the usefulness of PReBeaM for deconvolving main-beam distortions. In fact, we find that it makes sense to use PReBeaM for main-beam effects since only a small \( m_{\text{max}} \) parameter is needed to capture the azimuthal structure of the main beam. In this way, we profit from the computational advantage of our method in the case of small \( m_{\text{max}} \), allowing for the unified treatment of main-beam and side-lobe effects.

In Section 2, we describe the deconvolution map-making algorithm for PReBeaM. The simulated data and beams are detailed in Section 3. We present results in Section 4 showing the effectiveness of PReBeaM in removing systematic effects due to beam asymmetry and we discuss computational considerations. We finish with our conclusions from this study in Section 5.

2. PReBeaM Method

First, we review the standard setup to the map-making problem for a solution of the least-squares (or maximum-likelihood) type.

The TOD generated by a detector is effectively a convolution of the true CMB sky with a beam function. If we consider the sky as a pixelized vector, it will have length \( n_{\text{pix}} \times n_{\text{pol}} \), where \( n_{\text{pol}} = 3 \) for the \( I \) (total intensity), \( Q \), and \( U \) Stokes components. The \( n_{\text{TOD}} \)-length TOD vector \( \mathbf{d} \) is the result of a matrix multiplication of the observation matrix \( \mathbf{A} \) with the sky \( \mathbf{s} \) plus the noise \( \mathbf{n} \):

\[
\mathbf{d} = \mathbf{A}s + \mathbf{n}. \tag{1}
\]

In our implementation of the maximum-likelihood solution, we refer to \( \mathbf{A} \) as the convolution operator. \( \mathbf{A} \) encodes information about both the scanning strategy and the optics of the scanning instrument. The least-squares estimate of the true sky, \( \hat{s} \), is given by the normal equation

\[
\mathbf{A}^T \mathbf{A} \hat{s} = \mathbf{A}^T \mathbf{d}, \tag{2}
\]

where \( \mathbf{A}^T \) is the transpose convolution operator. Equation (2) holds if the noise is stationary and uncorrelated in the time-ordered domain. The generalization to nonwhite noise is as follows:

\[
\mathbf{A}^T \mathbf{N}^{-1} \hat{s} = \mathbf{A}^T \mathbf{N}^{-1} \mathbf{d}, \tag{3}
\]

where \( \mathbf{N} \) is a noise covariance matrix. In this work, we consider CMB only and CMB plus white noise.

We modify the normal equation by introducing a regularization technique in order cope with the ill-conditioned nature of the coefficient matrix \( \mathbf{A}^T \mathbf{A} \). We split off the ill-conditioned part of \( \mathbf{A} \) by factoring it into two parts: \( \mathbf{A} = \mathbf{BG} \). The factor \( \mathbf{G} \) is what we refer to as the regularizer in PReBeaM. In our study, we choose \( \mathbf{G} \) to be a Gaussian smoothing matrix, defined in harmonic space as

\[
G_{\ell} = \exp \left( -\frac{\sigma^2 \ell (\ell + 1)}{2} \right),
\]

\[
G_{\ell}^{G,C} = \exp \left( -\frac{\sigma^2 \ell (\ell + 1) - 4}{2} \right), \tag{4}
\]

where \( \sigma = \text{FWHM}/\sqrt{8 \ln 2} \). The superscripts \( G \) and \( C \) refer to the gradient and curl components in the typical linear polarization decomposition.

In general, the width of the regularizer can be set so as to reconstruct the sky at any target resolution. In practice, one would not want to choose a regularizer that is smaller than or close to the size of the sample spacing as this would likely introduce sampling effects into the map. We use the width of the angle-averaged detector beam and suggest this as a rule-of-thumb for choosing the regularizer’s width. The choice of regularizer will certainly affect the noise properties of the reconstructed map and power spectrum as it acts to smooth away noise on small scales. PReBeaM estimates the beam-corrected and regularized \( a_{\text{map}} \) and the map is simply a visualization of this \( a_{\text{map}} \) solution. If the sampling points were distributed in a way that aliased two different \( a_{\text{map}} \)s, then the reconstruction would be singular. Our regularization scheme ensures that we are robust to the details of sampling as long as the sample space is finer than the scale set by the width of the regularizer.

Our modified normal equation becomes

\[
\mathbf{B}^T \mathbf{B} \hat{s} = \mathbf{B}^T \mathbf{d}, \tag{5}
\]

where we are solving for \( \mathbf{x} = \mathbf{G} \hat{s} \). In this way, we are attempting to construct an image of the sky at the angular resolution of the chosen regularizer.

In standard pixel-based optimal mapmaking (see, for example, Wright 1996; Borrill 1999; Doré et al. 2001; Natoli et al. 2001; de Gasperis et al. 2005), in which one assumes that the observing beam is spherically symmetric, \( \mathbf{A} \) is a sparsely filled pointing matrix. In the case of a single dish experiment, each row of \( \mathbf{A} \) contains only three nonzero elements. The deconvolution map-making approach does not assume spherically symmetric beams, instead allowing for arbitrary beam shapes. We achieve this added complexity primarily by solving the normal equation in spherical harmonic space in order to make use of fast and exact algorithms for the convolution and transpose convolution of two arbitrary functions on the sphere (Wandelt & Górski 2001; Challinor et al. 2000). These algorithms are described in abbreviated form in Section 2.1.

A secondary advantage of operating entirely in harmonic space is that artifacts due to pixelization (such as uneven sampling of the pixel) are completely avoided. Issues with undersampled pixels can be problematic for a pixel-based method—as in the case of pixels with less than three nondegenerate observations—and can result in pixels which are excluded from the map-making completely. Our approach is less sensitive to degenerate pixels resulting from poor sampling and does not result in excised pixels. The assumption of a band-limited signal, justified by finite resolution, regularizes the solution for degenerate pixels based on neighboring observations in the time stream. We demonstrate the difference between a pixel and harmonic-based method by comparing a PReBeaM temperature map at the Healpix (Górski et al. 2005) resolution of nside 1024 with a binned noiseless temperature map at the same resolution. A binning of the TOD from the 30 GHz channel into an nside
If we choose importance when we include foregrounds such as point sources), may introduce small-scale features such as ringing (of particular pixels. Figure 1 shows identical regions of the sky from the analogous PReBeaM map at nside 1024 contains no unobserved pixels. The 1024 pixel map results in a number of unobserved pixels, of which three are visible in this frame. The PReBeaM map contains no unobserved pixels.

Figure 1. Comparison of one segment of the sky from a binned noiseless map (left) and the PReBeaM temperature map (right), both at a Healpix resolution of nside 1024. At this resolution, the binned map contains a number of unobserved pixels, of which three are visible in this frame. The PReBeaM map contains no unobserved pixels.

where \( s_{LM} \) is the spherical harmonic representation of the sky and \( T_{mm'm''} \) is defined as the result of a convolution of a bandlimited function \( b \) with the sky \( s \). The Planck Level-S software (Reinecke et al. 2006) nomenclature refers to \( T_{mm'm''} \) as a ring set. This is written in harmonic space as

\[
T_{mm'm''} = \sum_\ell \left( \frac{1}{2} s_{\ell m}^I b_{\ell m}^I + s_{\ell m}^G b_{\ell m}^G \right) + s_{\ell m}^{C'} b_{\ell m}^{C'},
\]

(7)

where \((\theta_E, \psi)\) are fixed parameters which define the scanning geometry.

In Equation (7), \( d_{mM}^I(\theta_E) \) and \( d_{mM}^G(\theta_E) \) are related to the Wigner D-matrices by

\[
D_{m_m}^I(\phi, \theta, \psi) = e^{-im\phi} d_{m_m}^I(\theta) e^{-im\psi}. \tag{8}
\]

Analogously, the transpose convolution in harmonic space is given by

\[
\gamma_{m'm''}^P(\theta_E) = d_{m_m}^P(\theta_E) b_{m'm''}^P T_{mm'm''}, \tag{9}
\]

where \( P = I, G, C \).

2.2. PReBeaM Implementation

Now we outline the algorithmic steps taken to make a map from a TOD vector by PReBeaM.

First, we construct the right-hand side of Equation (6) in two steps: converting TOD to an \( T_{mm'm''} \) array and applying \( A^T \). \( T_{mm'm''} \) is constructed by transpose interpolating the TOD vector \( d \). The transpose interpolation of the TOD vector onto the \( T_{mm'm''} \) grid is akin to a binning step, where each element of the TOD is mapped, via interpolation weights, to several elements of the \( T_{mm'm''} \) cube according to the orientation and position of that data point in the scanning strategy. The interpolation scheme is described in greater detail in Section 2.3. Next, we transpose...
convolve the beam coefficients \( b_{\ell m} \) with \( T_{nm'm''} \) according to Equation (9).

Once the right-hand side has been computed, we use the conjugate gradient iterative method to solve Equation (2). With each iteration, the coefficient matrix \( A^T A \) is applied using the following procedure:

1. Apply the convolution operator, \( A \), to project the sky \( a_{\ell m} \) on to the convolution grid \( T_{nm'm''} \).
2. Inverse Fourier transform over the first two indices of \( T_{nm'm''} \) to get \( \Phi_{\ell, m} \) (we omit the transform over \( m'' \) as it is incorporated in the interpolation scheme).
3. Forward interpolate from \( \Phi_{\ell, m} \) to a TOD vector.
4. Transpose interpolate from the TOD vector to a new ring set \( \Phi_{\ell, m} \).
5. Fourier transform over the first two indices of \( \Phi_{\ell, m} \) to get \( \Phi_{\ell, m} \).
6. Apply the transpose convolution operator, \( A^T \), to project the ring set \( T_{nm'm''} \) back into a new sky \( a_{\ell m} \) vector.

### 2.3. Polynomial Interpolation and Zero-Padding

PReBeaM uses the same polynomial interpolation as implemented in the Level-S software used to generate the simulation TODs and as described in Reinecke et al. (2006). The objective of forward interpolation is to construct a TOD element at a particular co-latitude, longitude, and beam orientation using several elements of the ring set \( T \) and their corresponding weights. Transpose interpolation operates in exactly the opposite manner as the forward interpolation: distributing a single element in the TOD to multiple elements of the ring set. This is done using the same weights calculated for the forward interpolation. The entire operation of interpolation and transpose interpolation from ring set to TOD and back again is depicted in Figure 2.

PReBeaM also includes the option to zero-pad during the fast Fourier transform (FFT) and inverse FFT steps. This means that the working array (either \( T_{nm'm''} \) or \( \Phi_{\ell, m} \)) is enlarged and padded with zeros out to \( \ell_{\text{max}} \) or \( \ell_{\text{max}} \). This has the effect of decreasing the sampling interval. We found that the combined effects of small-order polynomial interpolation (order 1 or 3) and zero-padding of \( 2 \times \ell_{\text{max}} \) or \( 4 \times \ell_{\text{max}} \) dramatically reduced the residuals in our maps.

### 2.4. Parallelization Description

PReBeaM employs a hierarchical parallelization scheme using both shared-memory (OpenMP) and distributed-memory (MPI) types of parallelization. The map-making was performed on the NERSC computer Bassi. Bassi processors are distributed among compute nodes, with eight processors per node. OpenMP tasks occur within a node and MPI tasks occur between nodes.

We show a diagram of our hybrid parallelization scheme in Figure 3. The full TOD and pointings are divided equally between the nodes for input and storage of pointings. Within an iteration loop, four head nodes are designated to perform the convolutions. All other tasks are shared by all nodes, including the head nodes. Since convolution is minimally time consuming, we are not concerned with the small amount of time the nonhead nodes spend idling while convolution is performed. The number of head nodes needed corresponds to the number of detectors; for the low-frequency instrument (LFI) 30 GHz channel, this is four. Each of these four nodes performs the convolution of the sky with one of the four detectors. The resulting arrays are then distributed to all nodes for interpolation over the segment of data stored there and then gathered back onto the designated nodes for transpose convolution. Finally, the \( a_{\ell m} \) are summed, using MPI task mpi_reduce, into a single \( a_{\ell m} \) on a single node; this is the new estimate for the sky vector.

This particular scheme was devised so that the four distinct convolutions that must occur (one sky with four different beams) can take place simultaneously, while the pointings are distributed among as many nodes as possible for maximum speed in interpolation. Both convolution and interpolation and
Figure 4. EE, TE, and BB spectra of smoothed input map (black curve), binned map (blue curve), and PReBeaM (red curve). Note that the red and black curves are nearly on top of each other. The EE and TE spectra show the effect of temperature-to-polarization cross-coupling seen in the binned map spectra as shifts in the peaks and valleys and absent from the PReBeaM spectra. The input BB spectra is absent from the BB plot since the input B modes were zero. For comparison, we show a theoretical B-mode spectrum (dashed curve) from a cosmological model with a 10% tensor-to-scalar ratio. TT spectra are omitted since differences in the three spectra are not apparent in this representation.

(A color version of this figure is available in the online journal.)

their transpose operations make use of all processors on a node by using OpenMP directives.

3. SIMULATIONS AND BEAMS

The simulated Planck data on which PReBeaM was run were generated by the Planck CTP working group for the study of the performance and accuracy of five map-making codes summarized in the Trieste paper (Ashdown et al. 2009). Planck will spin at a rate of approximately 1 rpm, with an angle between the spin axis and the optical axis of $\sim 85^\circ$. We used the cycloidal scan strategy in which the spin axis follows a circular path with a period of six months, and the angle between the spin axis and the anti-Sun direction is $7.5^\circ$. TODs were generated for 366 days for the four 30 GHz LFI detectors. At a sampling frequency of 32.5 Hz, this corresponds to $1.028 \times 10^9$ samples per detector, for a total of over 65 Gb of data and pointings. The simulated data also included the effects of variable spin velocity and nutation (the option to include the effects of a finite sampling period was not included).

The data were simulated with elliptical beams having a geometric mean FWHM of 32.1865 and ellipticity (maximum FWHM divided by minimum FWHM) of 1.3562 and 1.3929 for each pair of horns. The widths and orientations of the beams were different; this was referred to as beam mismatch in the Trieste paper. In spherical harmonic space, the simulation beams were described up to a beam $m_{\text{max}}$ of 14. The same beams were used in PReBeaM to solve for the map, although we allowed the beam asymmetry parameter $m_{\text{max}}$ to vary.

4. RESULTS AND DISCUSSION

For this paper, we make temperature and polarization maps from simulated one-year observations of the four 30 GHz detectors of the Planck LFI. We examine two cases: CMB signal only and CMB plus uncorrelated (white) noise. Foreground signals and correlated noise properties will be examined in a future paper. The data from the 30 GHz channel was an optimal choice for this analysis. These receivers sit farthest away from the center of the focal plane and therefore have the strongest main-beam ellipticities of any Planck channel, while the low sampling rate and resolution minimize the data volume.

PReBeaM operates entirely in harmonic space, solving for and producing as output $a_{\ell m}$’s out to $\ell_{\text{max}}$ 512 and at the Healpix resolution of $n_{\text{side}}$ 512 ($\sim 7''$ pixel size). Most of the results presented in this paper were attained with an FFT zero-padding of factor 4, an interpolation order of 3, and an asymmetry parameter of $m_{\text{max}} = 4$ (we note where the parameters differ from this). To compare with the input signal, a reference map
Figure 5. Fractional difference in power spectrum for PReBeaM (red, blue, and cyan curves) and the binned map (black curve) for TT, EE, and TE. Spectra for PReBeaM are shown as a function of the number of iterations to demonstrate convergence. (A color version of this figure is available in the online journal.)

Figure 6. Absolute value of the beam $a_{\ell m}$ coefficients for $m = 0$ (solid), $m = 2$ (dotted), $m = 4$ (dashed), $m = 6$ (dash-dotted), and $m = 8$ (dashed-triple-dot). We plot $|a_{\ell m}^T|$ (left) and $|a_{\ell m}^E|$ (right) (we omit the $B$ component plot since $|a_{\ell m}^B| = |a_{\ell m}^E|$).

representing the true sky was created by smoothing the input $a_{\ell m}$ by a Gaussian beam of FWHM = 32'1865. Similarly, our regularizer $G$ (in Equation (4)) was set to have an FWHM of 32'1865 to match this smoothing. As noted in Section 3, the same data we use here have been processed by five map-making codes in Ashdown et al. (2009). We have chosen to compare our results with the analogous results from Springtide, one of the codes in this study. Springtide was chosen, out of the five codes, because it is the map-making code installed in and used by the Planck Data Processing Centers for the HFI and LFI instruments. It is sufficient to compare with Springtide only as no significant differences in accuracy were found between codes.
Figure 7. BB power spectra as a function of asymmetry parameter $m_{\text{max}}$ for $m_{\text{max}} = 2$ (blue curve), 4 (cyan curve), and 6 (red curve). The input BB spectra was zero so the smallest output BB spectra is most desirable. In this run, the PReBeaM input parameters interpolation order and zero-padding were set to 1 and 2, respectively.

(A color version of this figure is available in the online journal.)

(with similar baselines and in the absence of noise; Ashdown et al. 2009). Springtide is a destriping algorithm designed to remove low-frequency correlated noise from the TOD by fitting for, and subtracting, offsets. While Springtide does fit for offsets even if the noise is absent or uncorrelated, the results in this case are not significantly different from that which one would derive from a simple binning algorithm. Because we are using Springtide to represent all non-beam-deconvolution methods, we will refer to the Springtide maps as the binned maps.

We begin by examining the spectra in the binned map, PReBeaM map, and the smoothed input map shown in Figure 4. The effect of the beam mismatch is clearly seen where the peaks and valleys of the binned map spectra have been shifted toward higher multipoles. The detectors measure different Stokes $I$ which translates to artifacts in the polarization map. Deconvolution suppresses leakage from temperature to polarization as evidenced by the PReBeaM spectra which overlays the input spectra. This shift is expected to remain apparent in the TE spectra of non-beam-deconvolved maps even in the presence of noise because of larger temperature signal and the temperature-to-polarization cross-coupling.

The fractional difference in the angular power spectrum (defined as $(C_{\ell}^{\text{out}} - C_{\ell}^{\text{in}})/C_{\ell}^{\text{in}}$) of the input and output maps is shown in Figure 5. We show the fractional difference spectra for the TT, EE, and cross-correlation TE signals, omitting the BB spectra since $C_{\ell}^{BB}$ is zero in the simulation of the CMB map. The results for PReBeaM are shown at three intervals: the 25th, 50th, and 75th iterations. This shows the behavior of the

Figure 8. Worst-case bias in estimation of cosmological parameters due to errors in the power spectra of PReBeaM (dashed curve) and due to the errors in the power spectra of the binned map (solid curve). These plots estimate the bias due to the 30 GHz channel only, and should not be taken as representative of Planck as a whole.
power spectra as PReBeaM converges on the solution. The beam mismatch effect is also seen in Figure 5, where the fractional difference in the PReBeaM spectra lies closer to zero than the binned map spectra over the full range of multipole moments for EE and TE.

As described earlier, PReBeaM allows for variation in the asymmetry parameter $m_{\text{max}}$. We examined the performance of PReBeaM as a function of $m_{\text{max}}$, setting it to 2, 4, and 6. In conjunction with this, we plot the beam $a_{\ell m}$ coefficients as a function of the first few $m$ in Figure 6. A remarkable improvement in the power spectra was found by increasing $m_{\text{max}}$ from 2 to 4, while an increase from 4 to 6 only resulted in marginal improvements. This effect is best seen in the BB power spectra as shown in Figure 7. There is a simple explanation for the fact that the PReBeaM $a_{\ell m}$ become very close to the input $a_{\ell m}$ at $m_{\text{max}}$ 4 and therefore do not change much beyond $m_{\text{max}}$ 4: there is an order of magnitude drop in the beam $a_{\ell m}$ coefficients from $m_{\text{max}}$ 4 to 6 for $\ell$ 0 to 512 as seen in Figure 6. Thus, while the input TOD was simulated with a beam having an $m_{\text{max}}$ cutoff of 14, PReBeaM operates optimally at an $m_{\text{max}}$ of just 4, thereby allowing us to capitalize on the computational property that PReBeaM scales as $m_{\text{max}}$.

We define a quantity called $n_\sigma$ that models the expected bias due to beam asymmetry systematics to the $\chi^2$ statistic

$$n_\sigma = \frac{\sum_{\ell=2}^{\ell_{\text{max}}} |\Delta C_\ell|^2}{\sigma_{\text{Planck}}^2}.$$  \hspace{1cm} (10)

We use $n_\sigma$ to quantify the maximum, or worst-case bias beam systematics could induce in a cosmological parameter that happened to be degenerate with that parameter. The quantity $\sigma_{\text{Planck}}$ is the expected 1σ errors for the LFI 30 GHz channel, computed as the diagonal elements of the covariance matrix for the simulated input spectra, assuming a sky fraction of 0.65. The $n_\sigma$ values are plotted in Figure 8 and show that PReBeaM reduces the worst-case bias due to untreated beam systematics by 1 or 2 orders of magnitude.
We examine the resulting temperature and polarization (Q and U) maps. The output map for both PReBeaM and the binned map was subtracted from the smoothed input map at the same resolution to make the residual maps shown in Figure 9. PReBeaM residuals were plotted on the same color scale as the binned map, showing that PReBeaM attained smaller residuals for both temperature and polarization. As a final test, we run PReBeaM on TOD containing CMB signal and white noise and compare with the smoothed input CMB spectrum and the analogous results from Springtide (in this case, we refer to Springtide directly since this is not simply a binned map). The level of the uncorrelated noise is specified in the detector database and has a nominal standard deviation per sample time of $\sigma = 1350$ $\mu$K (Ashdown et al. 2009). PReBeaM achieves a noticeably superior fit to the input spectrum compared with Springtide from $\ell \sim 150$ to $\sim 250$. We acknowledge that the reconstructed power spectrum is sensitive to the choice of regularizer. Without exploring the full parameter space for this variable, we found that our choice for the regularizer fortuitously led to a recovered power spectrum free of excess noise on small scales. Assessing the relative performance of PReBeaM and Springtide in more detail would require performing Monte Carlo averages. We focus on the TE spectrum since the improvement is visible even for a single simulation. For the other spectra, PReBeaM performs as least as well as Springtide but the detailed difference are more difficult to assess without a Monte Carlo study.

4.1. Computational Considerations

The computational costs and advantages of our method should be noted. To perform a convolution up to $\ell_{\text{max}}$ requires $O(\ell_{\text{max}}^3 m_{\text{max}})$ for the general case. Since $m_{\text{max}}$ is bounded by $\ell_{\text{max}}$, the cost never scales worse than $O(\ell_{\text{max}}^4)$ and is only $O(\ell_{\text{max}}^3)$ for the symmetric beam case. By comparison, a brute force computation in pixel space would require $O(\ell_{\text{max}}^4)$. In this study, data were simulated with beams having an asymmetry parameter of $m_{\text{max}} = 14$, but maps were made using a cutoff value of $m_{\text{max}} = 4$ in PReBeaM. We have demonstrated that computational cost can be conserved while still achieving the benefits of beam deconvolution.

It was found that an increase in the zero-padding factor from 2 to 4 produced superior results over an increase in the interpolation order from one to three. An optimal run of PReBeaM will therefore include the largest zero-padding possible given machine memory constraints in conjunction with a polynomial interpolation of order one or three. This is advantageous since the time spent in an FFT is nearly negligible and affected only minimally with an increase in zero-padding. In contrast, time for interpolation scales as interpolation-order-squared and as this is a TOD-handling step, it dominates over any cost incurred by convolutions. In the case of the results shown here, interpolation steps consume more than 90% of the wall-clock time per iteration.

The results produced here were generated using 12 nodes on NERSC computer Bassi (making use of all eight processors per node) and was complete in about 29 wall-clock hours, for a total of 2797 CPU hours. The maximum task memory was 20 GB on a single node.

5. CONCLUSION

We have found that PReBeaM has outperformed the standard binned noiseless map using two measures: spectra and residual maps. We examined the fractional differences in the spectra and found markedly smaller differences in the PReBeaM spectra versus the binned map spectra across a range of multipole moments. We find that map-making codes which do not deconvolve beam asymmetries lead to significant systematics in the polarization power spectra measurements. The temperature-to-polarization cross-coupling due to beam asymmetries is manifested as shifts in the peaks and valleys of the spectra. These shifts are absent from the PReBeaM spectra. We translated the errors found in the power spectra to an estimate of the statistical significance of the errors in a parameter estimation resulting from these spectra, which we call $\sigma_x$. This analysis showed that the worst-case parameter bias due to beam-induced power spectrum systematics could be tens of sigma while PReBeaM reduces the risk of parameter bias due to beam systematics to much less than $1\sigma$. We also found the J, Q, and U component residual maps to be smaller for PReBeaM than for the binned map, implying smaller map-making errors.

We have discussed some advantages of using a harmonic-based map-making routine such as the avoidance of artifacts due to pixelization. An additional feature of our method is the direct generation of sky $a_{\ell m}$. Analysis of CMB data often requires calculation of the power spectrum or bispectrum from the sky $a_{\ell m}$. A pixel-based map-making method generates a map which must first be transformed into a set of $\tilde{a}_{\ell m}$ before computing the spectra. This extra step may introduce inaccuracies which will carry forward in the analysis done on the spectra.

We have presented here the first results from PReBeaM for a straightforward test cases of CMB only and CMB plus white noise, and including only the effects of beams in the main focal plane. However, there is great potential for using PReBeaM to remove or assess systematics due to the combination of foregrounds and beam side lobes. Systematics introduced by side lobes will appear on the largest scales, potentially impeding the detection of primordial $B$ modes on the scales where they are most likely to be measured. We have already shown for temperature measurements (Armitage & Wandelt 2004) that our deconvolution technique can be used.
to remove effects due to side lobes. Future work will examine the noise properties of PReBeA maps and will include foregrounds from extragalactic sources and diffuse Galactic emission.

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