RECONSTRUCTING NONLINEAR STOCHASTIC BIAS FROM VELOCITY SPACE DISTORTIONS

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Received 1997 September 29; accepted 1998 April 17

ABSTRACT

We propose a strategy to measure the dark matter power spectrum using minimal assumptions about the galaxy distribution and the galaxy–dark matter cross-correlations. We argue that on large scales the central limit theorem generically assures Gaussianity of each smoothed density field, but not coherence. Asymptotically, the only surviving parameters on a given scale are galaxy variance $\sigma$, bias $b = \Omega^{0.6}/\beta$, and the galaxy–dark matter cross-correlation coefficient $r$. These can all be determined by measuring the quadrupole and hexadecapole velocity distortions in the power spectrum. Measuring them simultaneously may restore consistency between all $\beta$-determinations independent of galaxy type. The leading deviations from Gaussianity are conveniently parameterized by an Edgeworth expansion. In the mildly nonlinear regime, two additional parameters describe the full picture: the skewness parameter $s$ and nonlinear bias $b_2$. They can both be determined from the measured skewness combined with second-order perturbation theory or from an $N$-body simulation. By measuring the redshift distortion of the skewness, one can measure the density parameter $\Omega$ with minimal assumptions about the galaxy formation process. This formalism also provides a convenient parameterization to quantify statistical galaxy formation properties.

Subject headings: cosmology: theory — galaxies: formation — large-scale structure of universe — methods: statistical

1. INTRODUCTION

The measurement of the distribution of matter in the universe has been one of the frontier goals of modern cosmology. The correlation of galaxies has been measured in several surveys and is known to significant accuracy. The abundance of data has led to many theoretical challenges, especially for flat cold dark matter (CDM) cosmologies. In the simplest models, galaxies are considered an unbiased tracer of the mass. Several different measurements of velocities then allow us to measure the density of the total matter, using either cluster mass-to-light ratios or pairwise velocities (Peebles 1993). Under these assumptions, one obtains values of the density parameter $\Omega_0 \sim 0.2$. It is known, however, that galaxies of different types correlate in different ways (Strauss & Willick 1995). Thus, not all galaxies can simultaneously trace the mass, and it appears plausible that no galaxy type is a perfect tracer of mass. At this point, galaxy formation is not completely understood. Simulations of galaxy formation suggest that the galaxy formation process is stochastic and nonlinear (Cen & Ostriker 1992) and may introduce scale-dependent artifacts (Mann, Peacock, & Heavens 1998). This complicates the derivation of $\Omega_0$ from dynamical measurements. Peculiar velocity fields in principle allow one to measure the mass fluctuation spectrum (Strauss & Willick 1995), but even here one must assume that statistical properties of galaxies, such as the Tully-Fisher relation, do not depend on the local density of matter.

Assumptions need to be applied to relate the distribution of visible galaxies to that of the total matter. The most popular of relations has been to assume linear biasing, where the power spectrum of galaxies $P_g(k)$ is related to the matter power spectrum $P(k)$ by $P_g(k) = b^2 P(k)$, motivated by the peak biasing paradigm (Kaiser 1984). The gravitational effects of dark matter lead to velocity fields which amplify the redshift space power spectrum in the linear regime. By measuring the distortions and imposing a stronger assumption $\delta_g = b\delta$, a parameter $\beta = \Omega^{0.6}/b$ can be constrained (Kaiser 1987). Recent attempts to measure these have resulted in a confusing picture, wherein the results depend strongly on the galaxy types (Hamilton 1997), scale (Dekel 1997), and surveys. A potential explanation for this state of confusion is that the linear galaxy biasing model is too simplistic. It has been proposed that galaxy formation may be nonlocal (Heyl et al. 1995). A general model assuming only that galaxy formation is a stochastic process has been proposed by Dekel & Lahav (1997). Unfortunately, the most general stochastic distribution introduces several new free functions, which make an unambiguous measurement of the matter distribution seemingly impossible. In principle, higher order statistics (Fry 1994) can be used to break some of the degeneracies, but the application of the full theory is still in its infancy.

Nevertheless, the fact that the correlation properties are distorted in redshift space tells us that gravitational interactions are at work. The challenge is to translate these measured distortions into conventional cosmological parameters. An added complication arises from the fact that the redshift distortions are more accurately measured in the quasi-linear regime where nonlinear corrections are believed to be important (Hamilton 1997). If we are given complete freedom to create galaxies with any distribution we please and to allow these galaxies to have any form of correlation or anticorrelation with the dark matter density, can we still say something about the dark matter distribution? Can we invoke more observables to constrain the exhaustive class of galaxy distribution models? In this paper we present the framework for a formalism which allows us to parameterize all galaxy distribution freedom into a simple unified picture. In § 2 we begin by constructing a hierarchy of approximations. The central limit theorem argues that the density field of both the galaxy distribution...
and the matter distribution will tend to be Gaussian when smoothed on sufficiently large scales. Their cross-correlation coefficient, however, remains a free parameter. For general random processes, a bivariate Gaussian distribution will have an arbitrary correlation coefficient \(-1 \leq r \leq 1\). The distribution has three free parameters: two variances and a cross-correlation coefficient. We show in §3 how to determine all three using linear power spectrum distortions. The first deviation from Gaussianity is characterized by skewness. For a bivariate distribution, one has in general four skewness parameters. We show in §2 how to reduce this number systematically. Once the power spectrum of the matter has been determined, the skewness of the matter can be computed using either second-order perturbation theory (Hivon et al. 1995) or using N-body simulations. Skewness is also subject to redshift distortions, which can be measured. We show how these two additional observables allow one to constrain \( \Omega \) in §3. Individual galaxies may be a biased tracer of the local matter velocity field (Carlberg, Couchman, & Thomas 1990), but it is reasonable to expect that when smoothed on sufficiently large scales, the averaged velocity field will be unbiased. A geometric interpretation of the joint distribution function is given in §4. To ensure accuracy, any parameters derived using this formalism must ultimately be checked against an N-body simulation. Such simulations also allow one to probe further into the nonlinear regime. We will address these aspects in a future paper.

2. GALAXY DISTRIBUTION

We will analyze the density field smoothed by a top-hat window \( W^R (r) \) which is zero for \( r > R \) and \( 3/4 \pi R^3 \) elsewhere. The perturbation variable \( \delta \equiv (\rho - \bar{\rho}) / \bar{\rho} \) is convolved to obtain the smoothed density field \( \delta^R = \int d^3x' \, \delta (x') W^R (|x - x'|) \). The galaxy density field will be described with a subscript, \( g \), and we similarly derive a smoothed galaxy density field \( \delta^R_g \). Only the galaxy field is directly observable. We do not have an exact theory about how and where the galaxies formed, nor how they relate to the distribution of the dark matter. The smoothed galaxy density field \( \delta^R_g (x) \) could be a function not only of \( \delta^R (x) \), but also of its derivative, and possibly long-range influences. Even nongravitational effects may play a role. Quasars exhibit a proximity effect, which may influence galaxy formation. If we consider all these variables hidden and unknown, we must prescribe the density of the galaxies \( \delta^R_g \) as an effectively stochastic distribution relative to the dark matter \( \delta^R \).

In the most general galaxy distribution, we need to specify the joint probability distribution function (PDF) \( P(\delta^R_g, \delta^R) \). At the lowest order, we will approximate these as Gaussians with some covariance

\[
P(\delta^R_g, \delta^R) = \sqrt{\frac{ac - b^2}{\pi}} \exp \left[ -\frac{(\delta^R_g)^2}{2} + b \delta^R_g \delta^R - \frac{c(\delta^R)^2}{2} \right].
\]

The parameters \( a, b, c \) are related to the traditional quantities by a change of variables: the variance of the matter \( \sigma^2 = \langle (\delta^R)^2 \rangle \), and that of galaxies \( \sigma_{g}^2 = \langle (ac - b^2) \rangle \). The third free parameter is the covariance, given by the correlation coefficient \( r = \langle \delta^R_g \delta^R \rangle / \sigma_g \sigma = b/(ac)^{1/2} \). The traditional bias is defined as \( b_1 = \sigma_g / \sigma = (c/a)^{1/2} \).

To simplify the notation, we will from now on implicitly assume that all fields are smoothed at scale \( R \) unless otherwise specified, and drop the superscript \( R \) on all variables. Note that the distribution (eq. [1]) is a function of the smoothing scale \( R \). Thus, the variances and correlation coefficient are implicitly functions of the scale \( R \). The parameters obtained at different smoothing scales are not independent.

The analysis of this paper relies on equation (1) being a good approximation to the distribution of galaxies and dark matter when smoothed on large scales. A sufficient condition for this to hold is if the central limit theorem applies. Since we are considering the statistics of density fields averaged over some region, the central limit theorem will generally apply if the smoothing region is larger than the scale of nonlocality of galaxy formation or nonlinear gravitational clustering. There are some notable exceptions, however: (1) The dark matter distribution may be non-Gaussian even when the fluctuations are linear. This might be true, for example, in topological defect theories of structure formation (Gooding et al. 1992) where the smoothed density field on any scale may be non-Gaussian. Such theories present great challenges to many attempts to measure cosmological parameters, including, for example, cosmic microwave background measurements (Pen, Seljak, & Turok 1997). (2) The galaxy fluctuations could depend nonlinearly on very large scale effects, for example, external gravitational shear (van de Weygaert & Babul 1994). This could avoid the central limit theorem because of non-locality.

Even if mild non-Gaussianity is present, we will argue below that the problem remains tractable. Conversely, there is every reason to believe that on small scales, the distributions of both the galaxy and the dark matter fields are significantly non-Gaussian. We will show in §3 that mild non-Gaussianity actually allows us to break the degeneracy between the bias factor \( b_1 \) and \( \Omega \). We now turn to the next moment of the distribution, the skewness \( \langle \delta^R \rangle / \sigma^3 \). In principle, one needs to specify four such independent moments \( \langle \delta^3 \rangle / \sigma^3 \) for \( 0 \leq i \leq 3 \). A coordinate transformation simplifies equation (1) if we define \( \langle 2a \rangle^{1/2} \delta_g \equiv (u + v)/(1 - r)^{1/2} \). The variable \( u \) has unit variance along the joint distribution of the galaxies and dark matter, while \( v \) has variance \( w^2 \) and measures their mutual deviation. In this rotated frame, \( u \) and \( v \) are uncorrelated. In order to model all relevant terms, we apply a general Edgeworth expansion about the Gaussian (eq. [1]). The details of the expansion are reviewed in the Appendix. We recall (Kim & Strauss 1997) that the coefficients of the two first-order terms \( u, v \) are zero since the mean is by definition zero. The three second moments are absorbed into the definitions of \( u, v \). In principle, one needs four third-order Hermite polynomials to describe the joint distribution self-consistently at the next order. But we can reasonably assume that the galaxies are positively correlated with the dark matter distribution, so \( w \ll 1 \). In this case, the third-order terms can be rank ordered in powers of \( w \) as \( u^3, u^2 v, w^2, u v^2 \). As we will see below, it is necessary to retain the first two terms to model second-order perturbation theory. We then neglect the last two terms because they depend on higher powers of the small parameter \( w \) and disappear completely in the limit that biasing is determin-
The Edgeworth expansion then gives us the truncated skew distribution:

\[
P(u, v) = \frac{1 + (u^3 - 3u)s + (u^2 - 1)u\sigma_s/w^2}{2w} \times \exp \left( -\frac{u^2}{2} - \frac{v^2}{2w^2} \right).
\]  

(2)

The two new coefficients are the joint skewness parameter \( s \) and the second-order bias \( b_s \), which allows us to adjust the skewness of each distribution independently. The joint PDF will be discussed in more detail in § 4 below. The Taylor expansion for general biasing introduces a quadratic bias at the same order as second-order perturbation theory (Fry 1994):

\[
\delta_y = f(\delta) = b_0 \delta + b_2(\delta^2 - \sigma^2) + O(\delta^3).
\]  

(3)

We will show in § 4 that in the stochastic notation, \( b_2 = 2b_b/\alpha \).

We can now compute the basic relations needed for further calculations. All third-order moments are uniquely defined:

\[
\langle \delta^3 \rangle = \sigma^3 \left( \frac{1 + r}{2} \right) \left(6s - 6b_s\right),
\]

\[
\langle \delta^2 \delta \rangle = \sigma^2 \sigma_y \left( \frac{1 + r}{2} \right) \left(6s - 2b_s\right),
\]

\[
\langle \delta \delta^2 \rangle = \sigma^2 \sigma_y \left( \frac{1 + r}{2} \right) \left(6s + 2b_s\right),
\]

\[
\langle \delta^3 \rangle = \sigma^3 \left( \frac{1 + r}{2} \right) \left(6s + 6b_s\right).
\]  

(4)

For an initially Gaussian matter distribution with a power-law power spectrum \( P(k) = k^8 \), the skewness factor of the evolved matter distribution is given by \( S_s = \langle \delta^3 \rangle / \sigma^4 = -(3 + n) + 34/7 \) from second-order perturbation theory (Juszkiewicz, Bouchet, & Colombi 1993), which is exact for \( \Omega = 1 \) and depends only very weakly on \( \Omega \) (Bouchet et al. 1992). We obtain one relation

\[
\left( \frac{1 + r}{2} \right) \left(6s - 6b_s\right) = \sigma \left[ \frac{34}{7} - (3 + n) \right]
\]  

(5)

for the five unknowns \( \sigma, \sigma_y, r, s, b_s \). We will use five observational values to determine the remaining four galaxy distribution parameters, as well as the cosmological density parameter \( \Omega \). These determinations will rely on the comparison between redshift space and real space correlations. The real space correlation contains valuable information about the galaxy distribution itself, while the redshift space distortions are a consequence of the dynamics of the dark matter.

3. REDSHIFT SPACE DISTORTIONS

We first consider the measurement of redshift space distortions of the variance or power spectrum in the presence of stochastic biasing. At first order, the two second-order terms \( s, b_s \) can be neglected. The redshift space density of galaxies is affected by their velocities through the Jacobian \( \rho_s(x)dx = \rho_0[x(z)](dx/dz)dz \), and linear perturbation theory gives us in Fourier space

\[
\delta_s(k) = \delta(k) + \delta(k)\Omega^{0.6} \mu^2,
\]  

(6)

where \( \mu = \cos \theta \) determines the angle between the wavevector \( k \), and the line of sight (Kaiser 1987) and we have made the distant observer approximation. The power spectrum is the expectation value of the square of equation (6) and results in

\[
P_s(k, \mu) = P_g(k)(1 + \beta^2 \mu^4 + 2r\beta \mu^2),
\]  

(7)

where \( \beta = \Omega^{0.6}/b_1(k) \). \( P_g(k) \) is the undistorted power spectrum. We have assumed that the density field is truly linear. The skewness corrections which we discuss subsequently will also generate corrections to equation (7) (see Ballinger, Peacock, & Heavens 1996), and the two-point redshift space distortions must in reality be solved for simultaneously with the nonlinear corrections. This problem can be addressed using \( N \)-body simulations.

Equation (7) is in Fourier space, while our one-point function (eq. [1]) was carried out in real space. They are related through the integral over the window

\[
\sigma^2 = \int P(k)W(kR)^2 d^3k,
\]

\[
\sigma_y^2 = \int P_s(k)W(kR)^2 d^3k,
\]

\[
r = \frac{1}{\sigma_y \sigma} \int \delta(k)\delta(-k)W(kR)^2 d^3k.
\]  

(8)

Equation (8) implicitly defines the Fourier space bias \( b(k) \) and the correlation coefficient \( r \) in terms of the real space counterparts \( b(R) \) and \( r(R) \). The Legendre relation

\[
P_l = \frac{2l + 1}{2} \int^{-1}_{-1} P_s(k, \mu)P_l(\mu) d\mu,
\]  

(9)

where \( \varphi_l(\mu) \) is the \( l \)-th Legendre polynomial (Hamilton 1997), allows us to obtain moments of the angular dependence of equation (7). One can in principle measure both \( \beta \) and \( r \) by measuring the quadrupole distortion \( P_2 \) and the next-order distortion \( P_4 \) separately. We can then solve for \( r \) and \( \beta \) using the following relations:

\[
P_2 = \frac{(4/3)\beta + (4/7)\beta^2}{1 + (2/3)\beta + (1/5)\beta^2},
\]

\[
P_4 = \frac{(8/35)\beta^2}{1 + (2/3)\beta + (1/5)\beta^2}.
\]  

(10)

With sufficiently large data sets, one can measure \( r \) and \( \beta \) as a function of wavelength \( k \). An alternative approach would be to measure \( P_g \) from the angular correlation function, after which determination of the monopole \( P_0/P_2 \) and quadrupole \( P_2/P_4 \) terms would be sufficient. Peacock (1997) compared \( P_4 \) derived from the APM angular power spectrum to \( P_0 \) to determine \( \beta_0 = 0.4 \pm 0.12 \) by setting \( r = 1 \). Allowing \( r \) to vary will increase the inferred value of \( \beta \) for all such measurements. In this case, the relation between the actual value of \( \beta \) for a given inferred value of \( \beta_0 \) is \( \beta = (\beta_0^2 + 2\beta_0 + r^2)^{1/2} - r \). A similar increase in \( \beta \) for a given inferred \( \beta_0 \) using \( r = 1 \) holds for quadrupole measurements using equation (10). If stochasticity has been neglected, all
The skewness can be obtained from the bispectrum $B_s(k_1, k_2, k_3) \equiv \langle \delta(k_1) \delta(k_2) \delta(k_3) \rangle$. Isotropy requires $k_1 + k_2 + k_3 = 0$, using which we can compute the third moment:

$$\langle \delta^3 \rangle = \int d^3k_1 d^3k_2 \frac{1}{(2\pi)^3} \times B_s(k_1, k_2, -k_1 - k_2) W^R(k_1) W^R(k_2) W^R(|k_1 + k_2|) .$$

One can then measure the net skewness by inverting the angular bispectrum to obtain the three-dimensional bispectrum, in analogy to the power spectrum from APM (Baugh & Efstathiou 1994). The skewness of the smoothed galaxy field determines the last relation in equation (4). The skewness of the dark matter field is fixed by the variance (eq. [5]), allowing us to solve for both $s$ and $b_s$. Equations (5), (10), and (11) allowed us to solve for all parameters of the stochastic nonlinear biased galaxy distribution model. The final goal is to break the degeneracy of $\beta = \Omega^{1/6}/b$ to determine $\Omega$ and $b$ independently.

In redshift surveys, the measured skewness is already distorted by velocity distortions, but is nevertheless readily measurable (Kim & Strauss 1997). The second-order perturbation theory calculation of the skewness has recently been completed for both redshift space and real space (Juszkiewicz et al. 1993; Hivon et al. 1995). Their basic result was that in the absence of biasing, the skewness factor $S_3$ (see above) is weakly dependent on $\Omega$ and is very similar in real and redshift space in second-order theory. In these calculations, $S_3$ and $S_3^*$ typically differ by a few percent depending on $\Omega$. A similarly sized change occurs when the bias parameter is changed. Fortunately, the observations already allow very accurate determinations: for example, Kim & Strauss found $S_3 = 2.93 \pm 0.09$, where the errors are comparable to the expected effect. The actual redshift space skewness distortions quickly grow significantly larger than the second-order predictions. For a self-similar power spectrum with $n = -1$ and Gaussian filter $\sigma = 0.5$, Hivon et al. found $S_3 = 3.5$ and $S_3^* = 2.9$ using $N$-body simulations, which is a very significant effect and is many times larger than current observational errors. Second-order perturbation theory appears to systematically underestimate the redshift space skewness distortions. The real space skewness in simulations tends to be higher than in perturbation theory, while redshift space skewness tends to be lower in simulations. This trend suggests that direct $N$-body simulations are necessary to quantify the effect of redshift space distortions of skewness.

By measuring the skewness from solely angular correlation information (Gaztaña & Bernardeau 1998), as well as from the redshift space distribution, we obtain two measures of skewness which can be compared to each other, from which one can solve for $\Omega$. One could also smooth the density field with an anisotropic window function and compute the dependence of the skewness of the smoothed density field on the alignment of the window as was done for variance measurements by Bromley (1994). The formalism of Hivon et al. (1995) can then be modified using these anisotropic window functions. Unfortunately, the second-order perturbation theory redshift distorted skewness does not have a simple closed-form expression, and a numerical triple integral must be evaluated for each specific set of choices of $n, \Omega, W(k, \mu)$. Details of this procedure are given in Hivon et al. (1995).

4. COHERENT LIMIT

A finite truncation of the Edgeworth expansion may result in a PDF which is not positive everywhere. For small corrections $s \ll 1, b_s \ll w^2$, the PDF in equation (2) remains positive in all regions where the amplitude is still large, and for practical purposes the negative probabilities do not have a significant effect. But it is possible to leave the regime of small corrections. When the corrections become large, the PDF becomes negative when it still has a significant amplitude. We can, however, absorb the coefficient of $v$ into the exponent for small $b_s$:

$$P(u, v) = \frac{1 + (u^3 - 3u)s}{\sqrt{2\pi}} e^{-u^2/2} \times \exp \{-[(v - (u^2 - 1)b_s)^2/2w^2]\} \frac{w}{\sqrt{2\pi}} .$$

Equation (12) remains positive for all values of $b_s$. The rightmost exponential term becomes a Dirac delta function in the limit $w \to 0$:

$$P(u, v) = \frac{1 + (u^3 - 3u)s}{\sqrt{2\pi}} e^{-u^2/2} \delta_b[v - (u^2 - 1)b_s] .$$

We then reproduce equation (3) to leading order for the choices $b_0 = r\sigma_s/\sigma_g$ and $b_2 = 2b_s / b_g$. It is instructive to understand the nature of the two skewness parameters $s$ and $b_s$ graphically. In Figure 1 we show the joint PDF. The respective projections onto the galaxy and dark matter PDFs is shown in Figure 2. The projected PDF for the galaxies is

$$P(\delta_g) = \frac{1 + (\delta_g^3 - 3\delta_g)s_g}{\sigma_g \sqrt{2\pi}} e^{-\delta_g^2/2\sigma_g^2} \exp \left(-\frac{\delta_g^2}{2\sigma_g^2}\right) ,$$

where $s_g = [(1 + r)/2]^{1/2} (s + b_s)$ and that for the dark matter is the same with a change in sign of $b_s$. For real surveys, one can measure the residuals of the galaxy PDF.

![Figure 1](image-url)

**Fig. 1.—Joint probability distribution function contours.** The parameters for this plot are $\sigma = 1$, bias $b = 2$, correlation coefficient $r = 0.8$, skewness $s = 0.05$, and nonlinear bias $b_0 = 0.1$. The solid line is the contour at 1 central probability, while the dotted lines are at $\frac{1}{4}$ and $\frac{1}{2}$. The axes are in units of the dark matter standard deviation.
after fitting a third-order Edgeworth expansion to determine the accuracy of the fit, with proper account for noise (Kim & Strauss 1997). The same can be done for the dark matter by utilizing $N$-body simulations.

This model of skew biasing allows us to discuss the systematic errors in the measurement of pairwise velocity dispersions (Guzzo et al. 1997). Since pairwise galaxy velocities are measured in the nonlinear regime, the inferred mean galaxy velocities cannot be directly translated into mean dark matter pairwise velocity. Decoherence and nonlinear bias both introduce complex dependences in the conversion from galaxy velocity to dark matter velocity. Guzzo et al. (1997) showed that the one-dimensional pairwise velocity varies by galaxy type from 345 to 865 km s$^{-1}$. We must keep in mind that each galaxy type surely has different biasing and coherence properties. The pairwise velocities are typically measured at a separation of $1\ Mpc$. In the ideal case that galaxies and dark matter at this scale were accurately described by the third-order Edgeworth expansion, we would obtain incorrect inferences about the dark matter if we ignored the nonlinearity and stochasticity. The fact that the result depends strongly on galaxy type may be a hint that such effects must be taken into account. In actuality, the density field is strongly nonlinear on this scale, and the Edgeworth expansion may be a poor approximation to the actual joint galaxy–dark matter distribution. We would be better advised to go to a more moderately nonlinear scale of perhaps $10\ h^{-1}\ Mpc$ and apply the strategy of this paper.

5. Conclusions

The most general stochastic galaxy biasing problem contains more free parameters than can readily be measured in any galaxy redshift survey. We have shown that using only linear perturbation theory we can determine two parameters, the correlation coefficient $r$ and bias parameter $\beta$ using the quadrupole and hexadecapole distortions. This allows a reconstruction of the power spectrum $P(k)\Omega^2$ as well as determination of two galaxy formation parameters. In the plausible scenario that galaxies correlate strongly with the matter distribution, only two free additional parameters, $s, b_s$, need to be introduced to quantify the skewness. Second-order perturbation theory provides one linear constraint, and observations of the skewness of galaxies determine the second. By measuring the redshift distortions of skewness we can, in principle, determine both the true underlying dark matter power spectrum and the density parameter $\Omega$ independently. This picture has incorporated both stochastic correlation and second-order nonlinear bias. In an $N$-body simulation the same approach can be applied to directly compare specific models to observations. This also allows us to probe deeper into nonlinear scales.

I thank Ben Bromley, Uros Seljak, and Michael Strauss for helpful discussions. This work was supported in part by NASA grant NAG 7039.

APPENDIX

EDGEBORTH EXPANSION

In this Appendix I review the notation and implementation of the Edgeworth expansion (Juszkiewicz et al. 1995). Since we are considering distributions that are perturbations of a Gaussian $\phi(v) = \exp \left( -v^2/2 \right)/(2\pi)^{1/2}$, we choose to expand the PDF:

$$p(v) = \phi(v) \left[ c_0 - \frac{c_1}{1!} H_1(v) + \frac{c_2}{2!} H_2(v) + \cdots \right].$$

For a single variable using our normalization, we have $c_0 = 1$ since $\phi$ is already independently normalized. The first- and second-order coefficients are absent, $c_1 = c_2 = 0$, because we have defined $v$ to have zero mean and unit variance. $H_i$ are the Hermite polynomials, of which the first three are $H_1(v) = v, H_2(v) = v^2 - 1, H_3(v) = v^3 - 3v$. The coefficients are obtained as

$$c_i = (-1)^{i} \int_{-\infty}^{\infty} H_i(v) \phi(v) dv.$$  

We then find $c_3 = S_3$, which is used in the text.

For a distribution of two variables, we consider the simultaneous expansion in both variables relative to a unit-variance Gaussian $\phi(\mu, v) = \exp \left[ - (\mu^2 + v^2)/2 \right]/2\pi$ as

$$p(\mu, v) = \phi(\mu, v) \left[ c_{00} - \frac{c_{10}}{1!1!} H_1(\mu) H_1(v) - \frac{c_{01}}{0!1!} H_1(\mu) H_1(v) + \frac{c_{20}}{2!1!} H_2(\mu) + \frac{c_{11}}{1!1!} H_1(\mu) H_1(v) + \frac{c_{02}}{0!2!} H_2(v) + \cdots \right].$$
Each of the coefficients is analogously given by

$$c_{lm} = (-1)^{l+m} \int H_0(\mu)H_1(\nu)p(\mu, \nu)d\mu d\nu.$$  \hspace{1cm} (A4)

The normalizations again allow us to set $c_{00} = 1$, $c_{lm} = 0$ for $0 < l + m \leq 2$. In equation (2) we have $\mu = u$, $\nu = v/w$, $c_{30} = 6s$, $c_{21} = 6b_s/w$.

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