Temperature-independent current deficit due to induced quantum nanowire vibrations

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Abstract. We consider electronic transport through a suspended voltage-biased nanowire subject to an external magnetic field. In this paper, we show that the transverse magnetic field, which acts to induce coupling between the tunneling current and the vibrational modes of the wire, controls the current–voltage characteristics of the system in novel ways. In particular, we derive the quantum master equation for the reduced density matrix describing the nanowire vibrations. From this we find a temperature- and bias voltage-independent current deficit in the limit of high bias voltage since the current through the device is lower than its value at zero magnetic field. We also find that the corrections to the current from the back-action of the vibrating wire decay exponentially in the limit of high voltage. Furthermore, it is shown that the expression for the temperature- and bias voltage-independent current deficit holds even if the nanowire vibrational modes have been driven out of thermal equilibrium.

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1. Introduction

Nanoelectromechanical systems (NEMSs) are mesoscopic devices whose functionality depends on the possibility to induce mechanical vibrations or displacements of one or several of their components [1]. Examples of such setups are numerous and include shuttling of single electrons and Cooper pairs [2]–[6], tuning of mechanical bending vibrations of suspended nanowires [7]–[9] and mechanically mediated superconducting and magnetic proximity effects [10]–[12] to name but a few. Crucial advantages of the downscaling implied by the acronym NEMS are the high vibration frequencies and the unprecedented sensitivity to external stimuli that can be achieved. This is due in turn to the low masses of these systems and to the strong coupling between mechanical and electrical degrees of freedom at the nanometer length scale (see, e.g. [13]–[15] and references therein). Also, NEMSs border the world of quantum mechanics, which opens up the possibility to experimentally study quantum effects on the interaction between electrical and mechanical degrees of freedom in mesoscopic systems [16]–[20].

In this paper, we will consider the NEMS studied by Shekhter et al [21], who analyzed the linear conductance through a suspended voltage-biased single-walled carbon nanotube (SWNT) in the presence of a magnetic field. The main result of [21] was a prediction by the authors of a finite negative magnetoconductance at low temperatures. This result is due to a magnetic field-induced coupling of the electrons and the quantum nanomechanical degrees of freedom in the system, which leads to an effective multiconnectivity of the vibrating nanotube. More precisely, the predicted result was attributed to two effects. One is the suppression of the probability for electrons to tunnel through the nanotube in the elastic channel, where the suppression is caused by destructive quantum interference effects among multiple electron tunneling paths. The other effect is Pauli-principle restrictions on the inelastic tunneling channels. The predicted result is a low-temperature effect, because at high temperatures, where the Pauli-principle restrictions are lifted, the reduction of the probability for tunneling through the elastic channel is fully compensated by an increased probability for inelastic tunneling. Hence, in this case, the linear conductance exactly coincides with the transmission through the non-vibrating wire (see also discussion in [21]).

Here we will consider the same system as in [21], but now beyond the linear bias-voltage regime. In particular, we will show that at large enough voltages there is a current deficit, as compared to the non-vibrating wire. Furthermore, this current deficit is shown to be independent of both the temperature and the bias voltage, making this system a good candidate device for detection of quantum vibrations in nanoscale systems. Also, we find that this reduction of the current is in general not dependent on strong coupling of the nanowire vibrational modes to the thermal bath as previously reported in [22] where the current and conductance through a carbon nanotube containing an encapsulated fullerene was analyzed in the ballistic transport regime. Hence, the results presented are shown to be quite general also for oscillator distributions out of equilibrium.

2. Model

The system considered is shown in figure 1, and comprises a doubly clamped carbon nanotube suspended over a trench of width $L$, subject to a transverse magnetic field, $H$. In [21], it was shown that when the SWNT is biased by a voltage $V$ the induced mechanical oscillations of
Figure 1. Schematic outline of system under consideration. Electronic tunneling through the doubly clamped suspended SWNT under bias voltage $V$ excites quantized vibrations of the tube in the presence of a magnetic field, $H$. The resulting electromechanical coupling leads to an exponential reduction of the probability for tunneling through the elastic channel, which, together with Pauli principle restrictions on the allowed inelastic transitions, modifies the electronic transport through the system at low temperature. The result of these effects is both a temperature- and bias voltage-independent current deficit (see text) as well as a previously reported negative magnetoconductance [21]. Amplitude shown is greatly exaggerated.

the tube lead to intermediate ‘swinging states’ through which electrons can tunnel between the leads. By restricting the analysis to the fundamental mode (which gives the most important contribution) the authors showed that the system can be described by an effective Hamiltonian (equation (7) in [21]),

$$\hat{H}_{\text{eff}} = \hat{H}_{\text{leads}} + \hat{H}_{\text{osc}} + \hat{H}_T,$$

which describes charge transfer through the SWNT in the regime of non-resonant tunneling or co-tunneling in the regime of Coulomb blockade. In (1),

$$\hat{H}_{\text{leads}} = \sum_{k,\sigma = l, r} \epsilon_{\sigma, k} \hat{a}_{\sigma, k}^\dagger \hat{a}_{\sigma, k},$$

describes the electrons in the leads; $\hat{a}_{l/r, k}^\dagger$ [$\hat{a}_{l/r, k}$] are creation (annihilation) operators for electrons in state $k$ in the left/right lead with energy $\epsilon_{l/r, k}$, respectively. The second term in the Hamiltonian,

$$\hat{H}_{\text{osc}} = \hbar \omega \hat{b}^\dagger \hat{b},$$

describes the oscillating wire where $\hat{b}^\dagger$ [$\hat{b}$] is a boson operator that creates (annihilates) one vibrational quantum and $\omega = (k/m)^{1/2}$ is the frequency of the fundamental mode of oscillation with $k$ the rigidity and $m$ the effective mass of the wire (typically $\omega$ is of the order of $10^8 \text{s}^{-1}$ if $L \sim 1 \mu m$). The third term in (1) describes the interaction between the electrons and the oscillating wire,

$$\hat{H}_T = e^{i\phi (\hat{b}^\dagger + \hat{b})} \sum_{k, k'} T_{\text{eff}}(k, k') \hat{a}_{l, k}^\dagger \hat{a}_{l, k'} + \text{h.c.}$$

Here, $\phi = 4 g \pi x_0 L \Phi / \Phi_0$ is the dimensionless electron–vibron coupling strength, $\Phi_0 = h/e$ is the flux quantum, $g$ is a geometric factor of order unity and $x_0$ is the zero-point oscillation
amplitude. Finally, $T_{\text{eff}}(k, k')$ describes the coupling of the electronic states $k$ and $k'$ in the different leads due to tunneling through virtual states on the wire at zero magnetic field. The latter are supposed to be discrete due to space quantization of the electronic motion and possible Coulomb blockade energy quantization\(^1\) (see also [21]).

3. Current

To calculate the charge transport through the system for the case when the density matrix is not in thermal equilibrium, we first consider the time rate of change of the total density matrix for the system, $\dot{\sigma}(t)$, which is given by the Liouville–von Neumann equation (see [23, 24] for a similar analysis). To evaluate this we switch to the interaction picture with respect to the non-interacting Hamiltonian, $\hat{H}_0 = \hat{H}_\text{leads} + \hat{H}_\text{osc}$, for which the evolution of the density matrix is given by $i\hbar \partial_t \hat{\sigma}(t) = [\hat{H}_\text{T}(t), \hat{\sigma}]$, where $\hat{A}(t) = e^{i\hat{H}_\text{T}/\hbar} \hat{A} e^{-i\hat{H}_0/\hbar}$ is any operator in the interaction picture. Since we are interested in the energy exchange between the electrons and the oscillating wire we only need to know the evolution of the reduced density matrix, which is found by tracing out the degrees of freedom of the leads, $\hat{\rho}(t) = \text{Tr}_{\text{leads}}(\hat{\sigma}(t))$. Treating the electrons in the leads as fermionic baths whose equilibrium distributions are virtually unaffected by the charge transfer we evaluate the evolution of the density matrix to lowest order in the tunneling probability. This enables us to find the equation of motion for the reduced density matrix in the Heisenberg picture,

$$\partial_t \hat{\rho}(t) = -\frac{i}{\hbar} \left[ \hat{H}_\text{osc}, \hat{\rho}(t) \right] - \frac{1}{\hbar^2} \text{Tr} \left\{ \int_{-\infty}^{t'} dt \left[ \hat{H}_\text{T}, \left[ \hat{H}_\text{T}(t-1), \hat{\sigma}(t-1) \right] \right] \right\}. \tag{5}$$

From this we derive the stationary equation for the reduced density of the system,

$$\frac{i}{\hbar} \left[ \hat{H}_\text{osc}, \hat{\rho} \right] = |T_{\text{eff}}|^2 \text{Tr} \left[ (\hat{J}_1 + \hat{J}_2) \hat{\rho} + \hat{\rho} (\hat{J}_1^\dagger + \hat{J}_2^\dagger) \right] - e^{-i\chi \hat{x}} (\hat{J}_1 \hat{\rho} + \hat{\rho} \hat{J}_1^\dagger) e^{i\chi \hat{x}} - e^{i\chi \hat{x}} (\hat{J}_2 \hat{\rho} + \hat{\rho} \hat{J}_2^\dagger) e^{-i\chi \hat{x}}, \tag{6}$$

under the assumption made in [21] that the overlap integral $T_{\text{eff}}(k, k')$ is independent of the momenta $k$ and $k'$. In (6), $\chi = \sqrt{2} \phi / x_0$, $\hat{x}$ is the deflection operator of the oscillating wire and the operators $\hat{J}_1$ take on the form below,

$$\hat{J}_{1,2} = -\frac{v^2}{h} \int \int d\epsilon_t d\epsilon_i \int_{-\infty}^{0} d\tau e^{\pm i \hat{x} \hat{\rho}_t} e^{i \hat{H}_0 \tau} e^{-i \hat{H}_0 \tau} f_{1,2}(\epsilon_{it})(1 - f_{1,2}(\epsilon_{it}) \epsilon_{\pm} e^{\mp i \phi_{\frac{1}{2}}}). \tag{7}$$

Here, $v$ is the density of states in the leads and $f_{1,2}(\epsilon_{it})$ are the Fermi distribution for electrons in the left/right lead kept at chemical potential $\mu_{1,2} = \pm e V / 2$, respectively. Multiplying (6) by the position and momentum operator and tracing out the oscillator degree of freedom we find the following expression for the deflection, $\langle \hat{x} \rangle$, and momentum, $\langle \hat{p} \rangle$, expectation values,

$$k \langle \hat{x} \rangle = |T_{\text{eff}}|^2 h \chi \text{Tr} \left[ (\hat{J}_1 - \hat{J}_1^\dagger) \hat{\rho} \right], \tag{8a}$$

$$\langle \hat{p} \rangle = 0. \tag{8b}$$

\(^1\) We note that on-wire Coulomb interactions only result in a renormalization of the amplitude of single electron tunneling in (and out) of the wire and in quantization of its electrostatic charging energy.
Equation (8a) gives the force balance in the stationary regime between the elastic force on the wire (left-hand side) and the force induced by the charge transfer (right-hand side), where \( \hat{J}_{\pi 1} = \hat{J}_{1,2} + \hat{J}_{1,2}^\dagger \). On the other hand, a similar expression can be derived from the definition of the current operator, \( \hat{I} = (ie/\hbar)[\hat{H}, \hat{N}_l] \), where \( \hat{N}_l \) is the number operator in the left lead. From the form of the total Hamiltonian this can be expressed as \( i\hbar \omega \hat{x} + 2i\hbar \hat{p}/e \hat{I} = [\hat{H}, \hat{p}] = -i\hbar \hat{p} \).

Under the trace with the static density matrix the right-hand side of this expression vanishes, \( \langle \hat{p} \rangle = 0 \), and we find that the average mechanical deflection of the wire is proportional to the total current \( I \) through it.

Using this relationship we can thus evaluate the current from (8a). To do so we divide the operators \( \hat{J}_{\pi 1} \) into their diagonal and non-diagonal parts (subscripts d and n, respectively),

\[
\text{Tr}(\hat{J}_d \hat{p}) = \text{Tr}(\hat{J}_{d,1} \hat{\rho}_d) + \text{Tr}(\hat{J}_{n,1} \hat{\rho}_n),
\]

with respect to the eigenstates of the oscillating wire. From this analysis we find that to the zeroth order in the operators \( \hat{x} \) and \( \hat{p} \), the operators \( \hat{J}_{\pi 1} \) only have diagonal components and the expression for the force is proportional to the current, \( I_0 \), through the system (equation (8) in [21]). The higher order terms in \( \hat{x} \) and \( \hat{p} \), corresponding to the non-diagonal parts of \( \hat{J}_{\pi 1} \), are collected in the current, \( I_1 \), which, in the high bias limit, gives exponentially small corrections to total current \( I \) (see below).

\[
I = I_0 + I_1, \tag{9}
\]

\[
I_0 = \frac{G_0}{e} \sum_{n=0}^{\infty} \sum_{\ell=-n}^{\infty} P(n) |\langle n|e^{i\phi (\hat{b}^\dagger + \hat{b})}|n + \ell\rangle|^2 \times \int \text{d} \epsilon [f_1(\epsilon)(1 - f_1(\epsilon - \ell \hbar \omega)) - f_1(\epsilon)(1 - f_1(\epsilon - \ell \hbar \omega))], \tag{10}
\]

\[
I_1 = \frac{|T_{\text{eff}}|^2 e}{2} \text{Tr} \left[ \left( \hat{J}_{\pi 1} - \hat{J}_{1,2} \right) \hat{\rho}_n \right]. \tag{11}
\]

In (10), \( G_0 = 2e^2|T_{\text{eff}}|^2 \nu^2 / \hbar \) is the zero field conductance and \( P(n) \) is the probability that the fundamental mode is in quantum state \( |n\rangle \) with energy \( n\hbar \omega \).

The two terms that make up the total current \( I \) can be understood as follows. The first term, \( I_0 \), is the tunneling current between the leads that takes into account the coupling between the electronic and mechanical degrees of freedom. If the distribution of the vibrational modes are in thermal equilibrium this term is the only contribution to the current. The second term, \( I_1 \), on the other hand corresponds to the back-action on the system due to the electromechanical coupling and acts to drive the energy distribution of the vibrational modes out of thermal equilibrium. Thus, for nonzero \( I_1 \), the distribution function \( P(n) \) in (10) is in general not given by the thermal distribution.

### 3.1. Current deficit

We start the analysis of the current by first considering \( I_0 \). This term describes how the combinations of the Fermi functions in the two leads put restrictions (through the Pauli principle) on the allowed transmission channels for electrons as they exchange energy above and below the Fermi level. We start the analysis of the current by first considering \( I_0 \). This term describes how the combinations of the Fermi functions in the two leads put restrictions (through the Pauli principle) on the allowed transmission channels for electrons as they exchange energy. The higher order terms in \( \hat{x} \) and \( \hat{p} \), corresponding to the non-diagonal parts of \( \hat{J}_{\pi 1} \), are collected in the current, \( I_1 \), which, in the high bias limit, gives exponentially small corrections to total current \( I \) (see below).

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I_1 = \frac{|T_{\text{eff}}|^2 e}{2} \text{Tr} \left[ \left( \hat{J}_{\pi 1} - \hat{J}_{1,2} \right) \hat{\rho}_n \right]. \tag{11}
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with the vibrating wire. Integrating over the electronic energy this equation can be expressed as

\[ I_0 = \frac{G_0}{e} \sum_{n=0}^{\infty} \sum_{\ell=-n}^{\infty} P(n) \langle n | e^{i\phi(\hat{b}^\dagger + \hat{b})} | n + \ell \rangle^2 \left( \frac{\ell \hbar \omega - eV}{e^{\beta(\hbar \omega - eV)} - 1} - \frac{\ell \hbar \omega + eV}{e^{\beta(\hbar \omega + eV)} - 1} \right). \]  

(12)

Here, we note that similar expressions for the current have also been reported for other NEMSs with strong electron–vibron coupling (e.g. the phenomenon of Franck–Condon blockade of tunneling through molecular devices [25, 26]).

Convergence of the summation over \( \ell \) in (12) is due to the exponential decay of the matrix element \( \langle n | \exp[i \phi (\hat{b}^\dagger + \hat{b})] | n + \ell \rangle \) at large \( \ell \). Analysis shows that the average number of inelastic scattering channels, \( \bar{\ell} \), that need to be considered in this summation scales as \( \bar{\ell} \propto \phi(\bar{E}/\hbar \omega)^{1/2} \), where \( \bar{E} \) is the average energy associated with the distribution \( P(n) \). This implies that under the condition \( V \gg V_0 \) (\( eV_0 \propto \max[k_B T, \hbar \omega \ell] \)) one can neglect the \( \ell \)-dependence in the factors \( \exp[\beta(\ell \hbar \omega \pm eV)] \) for all relevant \( \ell \). The expression for the current is therefore greatly simplified in the high bias limit and the sum reduces to \( \sum_{\ell} P(n) \langle n | e^{i\phi(\hat{b}^\dagger + \hat{b})} | n + \ell \rangle^2 \bar{\ell} \). Using the completeness of a set of vibron states this expression can be directly evaluated to yield,

\[ \sum_{n=0}^{\infty} \sum_{\ell=-n}^{\infty} P(n) \langle n | e^{i\phi(\hat{b}^\dagger + \hat{b})} | n + \ell \rangle^2 \bar{\ell} = \sum_{n=0}^{\infty} \sum_{n'=0}^{\infty} P(n) \langle n | e^{i\phi(\hat{b}^\dagger + \hat{b})} \hat{b}^\dagger \hat{b} | n' \rangle \langle n' | e^{-i\phi(\hat{b}^\dagger + \hat{b})} | n \rangle - \langle n | \hat{b}^\dagger \hat{b} | n \rangle = \phi^2, \]  

(13)

which holds for any normalized distribution function \( P(n) \).

The analysis above gives a current deficit through the system at high bias voltages as compared to the current at zero magnetic field. Furthermore, the current deficit is found to be independent of both temperature and the bias voltage. To understand this one needs to consider the Pauli restrictions on the inelastic tunneling channels imposed through equation (10). From this expression one finds that for low energy electrons many of the inelastic channels, which act to compensate for the suppression of the elastic channel, are forbidden. Consequently, the current at low voltages is reduced from the non-vibrating current, \( I = G_0 V \), by an amount that is given by the extent to which the elastic channel is suppressed. As the voltage increases, more inelastic tunneling channels are opened and the differential conductance increases accordingly. In the high voltage limit, \( V \gg V_0 \), a further increase of the bias voltage will not be affected by the Pauli restrictions due to the large energy scales of the electrons, in which case the differential conductance follows that of the system at zero magnetic field. Alternatively, this can be viewed as a voltage offset that depends only on the magnetic field strength and the system’s mechanical parameters,

\[ I_0(H, V) = G_0 \left( V - \frac{\hbar \omega}{e} \phi^2 \right) = I_0(0, V - \Delta V(H)), \quad V \gg V_0, \]

\[ \Delta V(H) = 16 g^2 n^2 \frac{L^2 H^2 e}{m}. \]  

(14)

3 For self-consistency we also evaluate the scaling of the average energy, see (17b) below.
3.2. Higher order corrections to current

To evaluate the non-diagonal contribution to the current we expand the exponentials in (7) in powers of $\hat{H}_0 \tau$ and integrate over the electronic energies,

$$\hat{J}_{1,2} = -\frac{\pi^2 v^2}{\hbar} \int_{-\infty}^{0} d\tau \left( \delta(\tau) + \frac{i}{\beta \sinh(\pi \tau / \beta)} \right)^2 e^{\pm i eV\tau} \sum_{q=0}^{\infty} \frac{1}{q!} e^{\pm i \chi \hat{x}} \hat{Y} \left( \hat{H}_0, e^{\mp i \chi \hat{x}}, q \right).$$  \hspace{1cm} (15)

In (15), $\hat{Y} \left( \hat{H}_0, e^{\mp i \chi \hat{x}}, q \right) \equiv [\hat{H}_0, [\hat{H}_0, \ldots, e^{\mp i \chi \hat{x}}]]$ with $q$ indicating the number of commutators to be evaluated and $\beta = (k_B T)^{-1}$. Evaluating (11) with this expansion we find that all contributions to the current $I_1$ decay exponentially in the high bias limit as all correction terms will be of the form,

$$I_1 \propto \int_{-\infty}^{0} d\tau \frac{\tau^q}{\sinh^2(\pi \tau / \beta)} \left( \begin{array}{c} \sin(eV \tau) \\ \cos(eV \tau) \end{array} \right) \hspace{1cm} (q = \text{odd})$$

$$\propto (e^{eV} - 1)^{-1}, \hspace{1cm} \beta eV \gg 1, q \geq 2.$$  \hspace{1cm} (16)

Thus, we find that in the limit of high bias voltage, the current goes as (14), which differs from the ohmic behavior for the non-vibrating wire, $G_0 V$, by an amount that is independent of both the bias voltage and temperature as shown in figure 2. This can be understood from the fact that the increase in the current due to a further increment in the bias voltage under the conditions when $V \gg V_0$ is fulfilled is not affected by the Pauli restrictions on the electron–vibron energy exchange. Nevertheless, the current deficit at large voltage biases is a true quantum-mechanical effect on transport that originates from the Pauli restrictions, however, these restrictions only affect the tunneling probability of low energy electrons close to the Fermi level.

In contrast to the analysis of [22] where the distribution function $P(n)$ was considered to be only slightly out of equilibrium, our results for the current deficit survives independently of the form of $P(n)$, even for highly excited distributions. As an example, we have analyzed (6) and (7) separately to order $\hat{p}$, an analysis that shows that in the high bias limit the distribution function is indeed far from equilibrium as, e.g. the magnitude of the two lowest nonzero moments are 4,

$$\beta eV \gg 1,$$

$$\langle \hat{x} \rangle \sim \frac{\sqrt{2\pi} G_0 \phi x_0}{e \omega} V, \hspace{1cm} (17a)$$

$$\langle \hat{p}^2 \rangle \sim \frac{\hbar}{4 x_0^2 \omega} (eV - 2\hbar \omega \phi^2). \hspace{1cm} (17b)$$

Finally, we show the current deficit as a function of the bias voltage for realistic experimental parameters (figure 3). Here, the influence of the multiconnectivity of the electron tunneling paths in the elastic channel and the Pauli-principle restrictions on the available

4 The coefficients multiplying higher order terms in this expansion are exponentially small in the limit of high bias voltage and are thus ignored.
Figure 2. Current as a function of bias voltage calculated for two distribution function, $P_{\text{therm}}$ (dashed) corresponding to the thermal distribution, $P(n) \propto e^{-\beta \hbar \omega n}$, and the non-equilibrium distribution $P_{\text{non-eq}}$ (dotted), where $P(n) \propto \left(eV/\hbar \omega\right)/\left((eV/\hbar \omega)^2 + n^2\right)$. Note that $P_{\text{non-eq}}$ may not correspond to the real distribution function but is given as an example to reflect the results derived for $\langle \hat{p}^2 \rangle$ (see text). The current deficit is the difference between the current at zero magnetic field, $G_0 V$, and the dashed and dotted curves, respectively (note that the parameters, $\omega = 10^{10}$ s$^{-1}$, $\phi = 2$ and $T = 0.1$ K were chosen to clearly separate these two curves). Also displayed is the constant voltage offset curve, $G_0 (V - \Delta V)$.

Inelastic channels are clearly visible at low bias voltages as an increasing current deficit. Eventually, when the new inelastic tunneling channels that are added by a further increment of the bias voltage are not affected by the Pauli restrictions—which occurs at large enough bias voltages—the current deficit saturates to a constant value. This constant value of the current deficit depends on the magnetic field $H$ and the mechanical properties of the nanowire through the parameter $\phi$ as illustrated in figure 3.

4. Conclusion

The analysis followed here assumes non-resonant tunneling or co-tunneling (for the case of strong Coulomb interactions) of electrons through the wire. For the realistic example of a suspended carbon nanotube of length $L \sim 1 \mu$m, we estimate that this can be achieved for bias voltages below a few mV. For the same system, we estimate that the current offset should be of the order of a few picoamperes for magnetic fields of the order of $H \sim 20 T$ and bias voltages $V \sim 10 \mu V$ as shown in figure 3 (see also discussion in [21]). Finally the temperature range necessary for these effects to be observable requires that $T \ll eV/k_B$ (typically a few mK) to avoid smearing out of the Fermi distributions as well as back-action from the wire on the electronic system. Should this not be the case, the main result of this paper does not apply as
the current deficit depends crucially on the extent to which the Pauli principle puts restrictions on the allowed electronic tunneling channels.

As discussed above, the role of electron–electron interactions on the wire can be shown to lead only to an effective renormalization of the amplitude of single electron tunneling on (and off) the wire. In order to verify this we suggest an experimental protocol where one measures the voltage offset at finite magnetic field,

$$\Delta V(H) = \frac{I_0(0, V) - I_0(H, V)}{G_0}, \quad V \gg V_0,$$

from which the zero field conductance, $G_0$, can be deduced as $\Delta V$ is only a function of the system parameters and the magnetic field strength.

In conclusion, we have shown that for the system originally considered by Shekhter et al [21] not only will the conductance be altered due to the multiconnectivity of the electronic transport through the system, but the current will be altered as well. In particular we find that even for vibrational distributions out of thermal equilibrium the system displays a current deficit that is independent of bias voltage and temperature. This is a clear manifestation of quantum mechanical effects on transport not previously considered. Also, we have shown that the influence of internal damping in this non-resonant charge transfer process decays exponentially to all orders in the moments of the position and momentum, thus making the system considered a very good candidate for direct observation of quantum mechanical effects in mesoscopic systems.

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References

[1] Craighead H G 2000 Science 290 1532–35
[2] Shekhter R I, Galperin Y M, Gorelik L Y, Isacsson A and Jonson M 2003 J. Phys.: Condens. Matter 15 R441
[3] Shekhter R I, Gorelik L Y, Jonson M, Galperin Y M and Vinokur V M 2007 J. Comput. Theor. Nanosci. 4 860–95
[4] Erbe A, Weiss C, Zwerger W and Blick R H 2001 Phys. Rev. Lett. 87 096106
[5] Scheibe D V and Blick R H 2004 Appl. Phys. Lett. 84 4632–4
[6] Moskalenko A V, Gordeev S N, Koentjoro O F, Raithby P R, French R W, Marken F and Savel’ev S E 2008 arXiv:0810.2430
[7] Jonsson L M, Gorelik L Y, Shekhter R I and Jonson M 2007 New J. Phys. 9 90
[8] Jonsson L M, Santandrea F, Gorelik L Y, Shekhter R I and Jonson M 2008 Phys. Rev. Lett. 100 186802
[9] Sazonova V, Yaish Y, Ustunel H, Roundy D, Arias T A and McEuen P L 2004 Nature 431 284–7
[10] Isacsson A, Gorelik L Y, Shekhter R I, Galperin Y M and Jonson M 2002 Phys. Rev. Lett. 89 277002
[11] Fedorets D, Gorelik L Y, Shekhter R I and Jonson M 2005 Phys. Rev. Lett. 95 057203
[12] Gorelik L Y, Shekhter R I, Vinokur V M, Feldman D E, Kozub V I and Jonson M 2003 Phys. Rev. Lett. 91 088301
[13] Lassagne B, Garcia-Sanchez D, Aguasca A and Bachtold A 2008 Nano Lett. 8 3735–8
[14] Jensen K, Kim K and Zettle A 2008 Nat. Nanotechnol. 3 533–7
[15] Blencowe M P 2005 Contemp. Phys. 46 249–64
[16] LaHaye M D, Buu O, Camarota B and Schwab K C 2004 Science 304 74–7
[17] Naik A, Buu O, LaHaye M D, Armour A D, Clerk A A, Blencowe M P and Schwab K C 2006 Nature 443 193–6
[18] LeRoy B J, Lemay S G, Kong J and Dekker C 2004 Nature 432 371–4
[19] Knobel R G and Cleland A N 2003 Nature 424 291–3
[20] Schwab K C and Roukes M L 2005 Phys. Today 58 36–42
[21] Shekhter R I, Gorelik L Y, Glazman L I and Jonson M 2006 Phys. Rev. Lett. 97 156801
[22] Sonne G, Gorelik L Y, Shekhter R I and Jonson M 2008 Europhys. Lett. 84 27002
[23] Fedorets D, Gorelik L Y, Shekhter R I and Jonson M 2004 Phys. Rev. Lett. 92 166801
[24] Novotny T, Donarini A and Jauho A P 2003 Phys. Rev. Lett. 90 256801
[25] Koch J and von Oppen F 2005 Phys. Rev. Lett. 94 206804
[26] Braig S and Flensberg K 2003 Phys. Rev. B 68 205324