A Resolution Enhancement of Rayleigh Wave Dispersive Imaging using Modified Phase-Shift Method

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Abstract. Multi-channel analysis of surface wave (MASW) method is an efficient tool for obtaining vertical shear wave profile. One of the key in the steps in the MASW method is generate a dispersion curves. To generate dispersion energy, the acquired shot gathers data have been transformed from the time-space (t-x) domain to the frequency-phase velocity (f-ν) domain using 2D wavefield transformation so that the dispersion curve can be determined by picking the peak of dispersion energy. In this paper, we proposed a modified phase-shift method to increase the resolution of Rayleigh wave dispersive imaging so that the accuracy of the dispersion curve determination becomes higher. This method added processing step on the conventional phase-shift method. The algorithm is written in Matlab environment. We compared this method with the conventional phase-shift and frequency-wavenumber (F-K) method and applied to synthetic and real data. We used DISECA program in Matlab to generate synthetic waveforms. Our results from the synthetic and real data show that modified phase-shift method can improve the dispersion energy imaging resolution better than the conventional phase-shift and F-K method.

1. Introduction
Multichannel analysis of surface wave (MASW) method divided into three steps, acquisition, processing, and inversion. Results of acquisition is a shot gather on time-space domain. Then, the data extracted to generate dispersion curve in processing step. The last, we invert the dispersion curve to estimate shear-wave velocity.

One of key in the steps in the MASW method is generate a dispersion curve. A shot gathers data on time-space domain transformed into frequency-phase velocity domain using 2D wavefield transformation. Many techniques was proposed: the τ – p transformation, the F-K transformation, the phase-velocity transformation, the frequency decomposition and slant-stacking, and the high-resolution linear radon transformation [1]. In this study, we used formula of power operation to modification phase-shift method. It is intended to increase the resolution of Rayleigh wave dispersive imaging. This method applied on synthetic and real data. The result compared with two other methods (conventional phase shift and F-K methods).

2. Imaging Dispersion Curve
To acquire dispersion curve in the f – ν domain, a shot gathers in the time-space (t – x) domain is necessary to go through two steps: one is to transform from the time domain to the frequency domain; the other one is to transform from the space domain to the phase velocity domain.
2.1. Frequency-Wavenumber (F-K) Method
Wavefield $u(x, t)$ is first transformed to the $f - k$ domain data $U(k, f)$ by a two-dimensional Fourier transformation:

$$U(k, f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(x, t) e^{-i(2\pi(f t + k x))} \, dt \, dx$$  

This procedure could be replaced with a one-dimensional Fourier transformation in the time domain and a one-dimensional inverse Fourier transformation in the space-domain. Then, the $f$-$v$ domain can be transformed according to the relation: $v = f / k$.

2.2. Phase-shift Method
Phase information $P(x, \omega)$ of a wavefield can be calculated by a Fourier transformation in the time domain [2]. It is expressed as:

$$P(x, \omega) = e^{-i\omega x + \varphi_0}$$  

where $\omega$ is the circular frequency, $v$ is the phase velocity and $\varphi_0$ is the initial phase. The phase is linearly correlated to offset. Integrate the phase using scanning $\varphi$:

$$U(\varphi, \omega) = \int e^{i\varphi x} P(x, \omega) \, dx = \int e^{i(\varphi - \frac{\omega}{v}) x + \varphi_0} \, dx = \sum_{i=1}^{N} e^{i(\varphi - \frac{\omega}{v}) x_i + \varphi_0}$$  

when the scanning $\varphi$ gets close to $\frac{\omega}{v}$, phase angle of complex number $e^{i(\varphi - \frac{\omega}{v}) x + \varphi_0}$ is the same in the complex plane, making the integral reaching a maximum value. At last, the $f$-$v$ domain result can be achieved according to the following relations:

$$\omega = 2\pi f$$  

$$v = \frac{\omega}{\varphi} = \frac{2\pi f}{\varphi} = \frac{2\pi}{\varphi} f$$  

2.3. Modified Phase-shift
In this method, we used algorithm based on power operation [1] to enhance resolution of phase-shift method result. The explicit formula of power operation is:

$$A_{enhanced} = A_{phase shift} * C_{norm}^\alpha$$  

where $A_{enhanced}$ is a matrix of the enhanced dispersion image with a higher resolution, $A_{phase shift}$ is a matrix of the original dispersion image generated by phase-shift method, $C_{norm}$ is the normalized coefficient matrix of $A_{phase shift}$

$$C_{norm} = \frac{A_{phase shift}}{\max(A_{phase shift})}$$  

and $\alpha$ is the exponent of the power, can be written as

$$\alpha(f) = a + \frac{b}{f}$$  

where $a$ and $b$ are two parameters that control the strength of power operation. The first parameter $a$ is the minimum exponent of the power that is applied in a high frequency range. The $b$ decides the level if the extra power in a low frequency range.
3. Examples – Synthetic Data

We used DISECA, a Matlab code to calculated synthetic dispersive waveform [3]. This program converts the dispersion curve into a dispersive waveform. The limitations of this program are only usable by the single mode. In this study, we used fundamental mode with normal dispersive model. The dispersion curve obtained by using mat_disperse program based on reflection and transmission coefficient method [4]. The synthetic waveform is applied to proposed method to generate Rayleigh-wave dispersive imaging and compared with the other two methods.

Firstly, we did forward modeling of dispersion curve using a mat_disperse program. We used six layers earth model [5]. Furthermore, the resulting dispersion curve becomes input to the program DISECA to get the dispersive waveform. Lastly, we extracted Rayleigh wave dispersive imaging using Modified Phase-shift method and compared with F-K and conventional phase-shift methods. Our results can be seen in three figures below:

![Dispersion Curve of Rayleigh Wave using F-K method](image1)

**Figure 1.** Dispersion curve of Rayleigh wave using F-K method.

Figure 1 shows the dispersion curve using F-K method. We can see that at low frequency (until 20 Hz) we lose the resolution. It will cause big uncertainty when pick the dispersion curve. So also for higher frequency (> 50 Hz), we seem to lose energy of dispersion curve. Then we used phase-shift method to extract dispersion curve, the result can be seen in figure 2:

![Dispersion Curve of Rayleigh Wave using Phase-shift method](image2)

**Figure 2.** Dispersion curve of Rayleigh wave using Phase-shift method.
Unlike F-K method, phase-shift method gives better resolution. Generally, this method provided good resolution. However, at low frequencies still provide big uncertainty. It will make us confused when pick dispersion curve. At the last, we applied modified phase-shift method to extract dispersion curve. The result can be seen in figure 3:

**Figure 3.** Dispersion curve of Rayleigh wave using Modified Phase-Shift method.

This method gives a better resolution of the two previous methods, especially in low frequencies. Although not significantly, but this method can reduce uncertainty and increase resolution at low frequencies. To validate comparison of these three methods, we picked dispersion curve in 20 Hz and plot at same figure. The result can be seen at figure 4:

**Figure 4.** Uncertainty picking of three methods at 20 Hz.

Red line, green line, and blue line is uncertainty for F-K method, Phase-shift method, and Modified Phase-shift method, respectively. Black line is the true phase velocity at frequency 20 Hz. If we compare results of three methods, blue line has a narrower than the other lines. This indicates that modified phase-shift method has smaller uncertainty than FK and phase shift methods. It can reduce pitfall when we pick dispersion curve in low frequency.
4. Examples – Real Data

Three methods were also applied to a real 24-channels shot gather obtained by using sledge hammer as source at Institut Teknologi Bandung (ITB) Campus. Interval geophones 1 m with near-offset 2 m, record length 1 second and sampling rate 1 ms. Results of extracted dispersion curve imaging using three methods above on can be seen at figures 5, 6, and 7 below:

**Figure 5.** Dispersion curve of Rayleigh wave using FK method on real data.

**Figure 6.** Dispersion curve of Rayleigh wave using Phase-shift method on real data.
Figure 7. Dispersion curve of Rayleigh wave using Modified Phase-shift method on real data.

The results show that modified phase-shift method give a better resolution than FK and phase-shift methods. It can be decrease the uncertainty when we pick the dispersion curve.

5. Conclusion
The algorithm of modified phase-shift has been compared in terms resolution in \( f-v \) domain with F-K and phase-shift methods. The results of synthetic and real data show that modified phase-shift method can be improve resolution than the other methods. It can reduce uncertainty when we pick dispersion curve. Further studies are needed to determine the parameters \( a \) and \( b \) in the modified phase-shift method.

References
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