DOPING: A NEW NON-PARAMETRIC DEPROJECTION SCHEME

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We present a new non-parametric deprojection algorithm DOPING (Deprojection of Observed Photometry using and INverse Gambit), that is designed to extract the three dimensional luminosity density distribution $\rho$, from the observed surface brightness profile of an astrophysical system such as a galaxy or a galaxy cluster, in a generalised geometry, while taking into account changes in the intrinsic shape of the system. The observable is the 2-D surface brightness distribution of the system. While the deprojection schemes presented hitherto have always worked within the limits of an assumed intrinsic geometry, in DOPING, geometry and inclination can be provided as inputs. The $\rho$ that is most likely to project to the observed brightness data is sought; the maximisation of the likelihood is performed with the Metropolis algorithm. Unless the likelihood function is maximised, $\rho$ is tweaked in shape and amplitude, while maintaining positivity, but otherwise the luminosity distribution is allowed to be completely free-form. Tests and applications of the algorithm are discussed.

Keywords: galaxies: photometry, luminosities, radii, etc.

1. Introduction

The preliminary step involved in the dynamical modelling of galaxies, concerns the deprojection of the observed surface brightness distribution into the intrinsic luminosity density, as has been practised by [1], [2], [3], among others. Deprojection though, is a non-unique problem, unless performed under very specific configurations of geometry and inclination, as discussed by [4], [5], [6], [7] and others.

Over the years, several deprojection schemes have been advanced and implemented within the purview of astronomy; these include parametric formalisms designed by [8], [9] and [10], as well as non-parametric methods, such as the Richardson-Lucy Inversion scheme developed by [11] and [12]
and a method suggested by [13]. While the parametric schemes are essentially unsatisfactory owing to the dependence of the answer on the form of the parametrisation involved, the non-parametric schemes advanced till now have suffered from the lack of transparency and in the case of the Richardson-Lucy scheme, lack of an objective convergence criterion.

Here, we present a new, robust non-parametric algorithm: Deprojection of Observed Photometry using an INverse Gambit (DOPING). DOPING does not need to assume axisymmetry but can work in a triaxial geometry with assumed axial ratios, and is able to incorporate radial variations in eccentricity. Although the code can account for changes in position angle, this facet has not been included in the version of the algorithm discussed here. Also, here we present the 1-D results obtained with DOPING but the code provides the full 3-D density distribution that projects to observed brightness map. In a future contribution, (Chakrabarty & Ferrarese, in preparation), DOPING will be applied to recover the intrinsic luminosity density of about 100 early type galaxies observed as part of the ACS Virgo Cluster Survey, as reported in [14].

The paper has been arranged as follows. The basic framework of DOP-ING is introduced in Section 2. This is followed by a short discourse on a test of the algorithm. An application to the observed data of the galaxy vcc1422 is touched upon in Section 4. Another application of DOPING is discussed in Section 5. The paper is rounded up with a summary of the results.

2. Method
The outline of the methodology of DOPING is presented below.

1. The plane of the sky ($x - y$ plane) projection of the galaxy is considered to be built of the observed isophotes that we consider to be concentric and analytically expressible in terms of $x$ and $y$, such that the $i^{th}$ isophote has an extent of $a_i$ along $\hat{x}$ (say). The inputs to DOP-ING are the brightness $I_i$ and the projected eccentricity $e_p^i$, that define the $i^{th}$ isophote, where $i \in N, i \leq N_{data}$.

2. We set up $\rho = \rho[\xi(x, y, z)]$, where $\xi$ is the ellipsoidal radius for the geometry and inclination of choice.

3. We identify pairs of $(x_i, y_i)$ that sit inside the elliptical annulus between the $i^{th}$ and the $i + 1^{th}$ isophotes.

4. At the beginning of every iterative step, the density distribution $\rho(x_i, y_i, z)$, over the line-of-sight coordinate $z$, is updated in size and
amplitude, \( \forall \), subject to the only constraints of positivity. The scales over which this updating is performed are referred to as \( s_{cl1} \) and \( s_{cl2} \).

\( (5) \) This updating is continued till the maxima in the likelihood is identified by the inbuilt Metropolis algorithm; the likelihood is maximised when the observed brightness distribution is closest to the projection of the current choice of the density. Regularisation is provided in the form of a penalty function that is set as the product of the smoothing parameter \( \alpha \) and a function of the Laplacian of \( \rho \).

\( (6) \) The spread in the models in the neighbourhood of the maximal region of the likelihood function, is used to formulate the \((\pm 1-\sigma)\) errors on the estimated density.

3. Tests

Prior to the implementation of the algorithm, it is extensively tested using analytical models. For one of these test, the results of which are presented in Figure 1, the surface brightness distribution is extracted by projecting the analytically chosen luminosity density distribution of a toy oblate galaxy. The projection is performed along the LOS coordinate \( z \), under the assumption of an intrinsic minor to major axis ratio that goes as \( \sqrt{1/(1 + r^2/r_c^2)} \), (where \( r \) is the spherical radius and the scale length \( r_c \) is 0.5'). Thus, by construction, this toy galaxy is rounder inside the inner 0.5" and outside this radius, it quickly (by 3") flattens to a disky system with eccentricity of about 0.99.

This toy galaxy is viewed edge-on (at 90°) for this test. The brightness distribution is then ported to DOPING and the deprojection is carried out under a chosen geometry+inclination configuration; the recovered luminosity density is compared to the true density of this model.

In Figure 1, the true density of this toy oblate galaxy is shown in open circles, along the photometric major (left panel) and minor (right) axes. The open triangles are used to depict the density recovered by DOPING, under the assumptions of face-on viewing angle \( i \) and triaxiality, with the LOS extent set to double the photometric major axis. Similarly, when the galaxy is viewed at \( i=20^\circ \), with the LOS extent set to half the photometric major axis, the recovered density distribution is plotted along the azimuths of 0° and 90° in crosses. In both cases, the two photometric axes are set as related in the way suggested by the projected eccentricity \( (e_p) \) data. In the former case, the value of \( \alpha \) that is used in the penalty function, is 10 times higher than in the latter case.

As expected, when the LOS extent is set longer than in the test galaxy,
it leads to a smaller density than the true density while a shorter LOS extent is betrayed in higher recovered densities. Also, when the test galaxy, modelled as triaxial, is not viewed along one of the principle axes, then as expected, isophotal twist is recovered. This results in a steeper drop in the projection of the recovered density in the case of $i=20^\circ$ (not shown here) than in the brightness data.

Like all other deprojection algorithms, DOPING also requires a seed or a trial density distribution to begin with. The parameters in the Metropolis algorithm, namely the temperature and the scale lengths $scl_1$ and $scl_2$ (see Section 2), are chosen to ensure robustness of the algorithm.

Fig. 1. Deprojection of a model brightness profile under assumptions of (i) ratio of extent along LOS to major axis coordinate=2; $i=0^\circ$ (ii) ratio of LOS extent to extent along $\hat{x}=0.5$; $i=20^\circ$. The recovered density distributions are shown at azimuths of $0^\circ$ (right panel) and $90^\circ$ (left panel), in crosses for case (ii) and open triangles for case (i). The true density of this toy galaxy is shown in open circles. The degeneracy in the deprojection exercise, as a function of inclination and geometry is brought out in this figure.

4. Applications

We demonstrate the applicability of DOPING in real galaxies by deprojecting the surface brightness profile of vcc1422 (IC3468), which is a Virgo Cluster dwarf elliptical, covered by the ACS Virgo Cluster Survey. We choose this galaxy since being about 8 magnitudes fainter at the centre
than the test galaxy considered above, it illustrates the efficacy of DOPING over a wide range of brightness. Photometrically, it is evident that this galaxy has a small central component, (a nucleus extending to about $0'' .3$), that sits on top of a more extended component. The projected ellipticity of this outer component meanders its way up from about $0.12$ at about $0''.4$ to $0''.3$ at about $1''.6$, to jiggle down to about $0.22$ at about $120''$.

An experiment was conducted to bring out the importance of including this information about the variation in $e_p$. To ease such an exercise, the contribution of the nucleus to the surface brightness measurement was subtracted and the resulting brightness profile was then deprojected under different conditions. When all this variability in the projected shape of the galaxy is incorporated into the deprojection technique, the density recovered along the photometric major axis $\hat{x}$ is depicted in open circles in Figure 2. This is compared to the density obtained under the assumption that the whole galaxy admits a single $e_p$ of $0.25$. In both cases, oblateness and edge-on viewing are assumed. Since the two profiles significantly differ, the comparison brings out the importance of including the details of the variation in the eccentricity, even for this mildly eccentric system. With a more radically varying $e_p$ profile, this difference would only increase!

DOPING has the capacity for deprojecting a multi-component system such as vcc1422; in these cases, the seed for the sought density is chosen such that it reflects the existence of all the components. Thus, in the case of vcc1422, inside $0''.3$, the seed should bear signatures of both the components, while outside it, only the contribution from the more extended of the two components is required. The result of this deprojection is shown in Figure 3. The deprojection was performed under the assumptions of oblateness and an edge-on inclination.

5. 3-D Morphology of Galaxy Clusters

A project is underway (Chakrabarty, de Philippis & Russell in preparation) to decipher the true intrinsic morphology and inclination (to the LOS) of a galaxy cluster, by deprojecting its X-ray brightness distribution under distinct assumptions about the cluster geometry and orientation; the deprojection in question is carried out by DOPING, using the measured projected eccentricity of the system ($e_p$). If available, information about the LOS extent of the cluster is also implemented. Such information is attainable for galaxy clusters since the hot gas in these systems is capable of scattering the primordial CMB photons; this is referred to as the Sunyaev-Zeldovich Effect (SZe).
Fig. 2. The recovered density distribution of the nucleus subtracted brightness distribution of vcc1422, plotted along the $\hat{x}$, obtained under the assumptions of $i=90^\circ$ and oblateness. The profile obtained from including the information about the changes in $e_p$ with $x$ is marked with open circles, while the deprojection carried out under the assumption of a constant $e_p$ of 0.25 is represented in crosses.

Fig. 3. Deprojection of the surface brightness data of the nucleated galaxy vcc1422. The recovered density is presented in the right while its projection has been overlaid on the observed brightness profile (in grey). Again, an oblate geometry and edge-on viewing were adopted.
The cluster morphology is identified as oblate, prolate or triaxial from the mutual weighing of the different deprojected density profiles that are recovered under assorted deprojection scenarios; the deprojected density distributions are sought along the $\hat{x}$ axis. The availability of the SZe measurements allows for marked tightening of the constraints that are placed on the inclination of a system from the analysis of the X-ray brightness information alone. For the triaxial systems, the SZe data can also help constrain the intrinsic axial ratios.

In Figure 4, the recovered density profiles for the Abell clusters A1651 and A1413 are depicted; our analysis indicates that A1651 is prolate while A1413 is triaxial, with intrinsic axial ratios of 0.96 and 1.64. The inclinations $i$ are found to be $8^\circ < i < 32^\circ$ for A1651 and $66^\circ < i < 71^\circ$ for A1413.

Fig. 4. Density profiles along $\hat{x}$, recovered by deprojecting the *Chandra* X-ray brightness distribution of clusters Abell 1651 (right) and Abell 1413 (left), under the assumptions of (i) oblateness and $i=90^\circ$, shown in crosses (ii) oblateness and $i=i_{\text{min}}$; in filled circles (iii) prolateness and $i=90^\circ$; in filled triangles (iv) prolateness and $i=i_{\text{min}}$; in open circles. Here $i_{\text{min}}$ is the smallest inclination allowed under oblateness, for a measured (uniform) $e_p (=\sin^{-1} e_p)$.

6. Summary

In this paper, we have introduced a new non-parametric algorithm DOP-ING that is capable of inverting observed surface brightness distributions of
galaxies and galaxy clusters, while taking into account variations in the intrinsic shapes of these systems. The potency of DOPING is discussed in the context of a test galaxy in which the eccentricity is made to change radically with radius. The code is also successfully applied to obtain the luminosity density distribution of the faint nucleated galaxy vcc1422. Lastly, a novel use is made of the capability of DOPING to deproject in general geometries, in determining the intrinsic shape and inclination of a galaxy cluster. It is envisaged that implementing a measure of the LOS extent of a cluster from Sunyaev Zeldovich measurements, will help tighten the estimates of cluster inclination and the intrinsic axial ratios of triaxial clusters.

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