WASP-92, WASP-93 and WASP-118: Transit timing variations and long-term stability of the systems

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ABSTRACT
We studied three exoplanetary systems with transiting planets: WASP-92, WASP-93 and WASP-118. Using ground-based photometric observations of WASP-92 and WASP-93 and Kepler-K2 observations of WASP-118, we redetermined the orbital and physical parameters of these planets. The precise times of all transits were determined. We constructed O-C diagrams of transits and analysed possible transit timing variations. We did not observe any significant deviation from a linear ephemeris for any of the selected exoplanets. We put upper-mass limits for other hypothetical planets in these systems. Using long-term numerical simulation, we looked for stable regions where another planet could exist for a long time. We used the maximum eccentricity method for this purpose. We discuss the influence of values of initial inclination and eccentricity on the shape and size of regions of stability.

Key words: planetary systems - eclipses - methods: numerical - planets and satellites: individual: WASP-92 b, WASP-93 b, WASP-118 b

1 INTRODUCTION
SuperWASP (SWASP) is one of the most successful ground-based photometry projects for searching for extrasolar planets (e.g. Kane et al. 2008 and Hellier et al. 2012). The survey started in 2004 and up to now (26 October 2018), SWASP has discovered 134 exoplanets. It uses two robotic telescopes, one located on the Canary Islands and another one in South Africa. Each one consists of array of eight Canon 200 mm f/1.8 lenses equipped with Andor iKon-L CCD cameras (Pollacco et al. 2006). The large field of view of the lenses gives a big sky coverage of 490 square degrees.

For this study we selected three hot Jupiters WASP-92 b, WASP-93 b and WASP-118 b discovered in 2016 and confirmed by Hay et al. (2016), who used spectral observations to determine the stellar parameters of the parent stars. Distances to these stars are given by the second GAIA Data Release (GDR2) (Gaia Collaboration et al. 2018). The ba-
sic parameters of the parent stars are listed in the Table 1. Additionally we note that Močnik et al. (2017) found \( \gamma \) Doradus pulsations in the light curve (LC) of WASP-118 with a period 1.9 day and semi-amplitude of \( \sim \) 200 ppm.

The aim of this paper was to study the possible presence of other planets in these systems. O-C diagrams of transits were constructed for this purpose and we put upper-mass limits for hypothetical planets in these systems. A second motivation of the present study was to perform simulations leading to the generation of stability maps for the systems.

In Section 2, we describe the photometric observations used in this paper. We analyse the LCs we used in Section 3. In Section 4, we search for possible transit timing variations (hereafter TTVs) and study them. Section 5 is concerned with the long-term stability of these systems. Our results are discussed in Section 6.

2 OBSERVATIONS

The transiting light curves of WASP-92 and WASP-93 used in this study were taken from the public archive of the Exoplanet Transit Database (ETD)\(^1\) (Poddaňy et al. 2010). Information about individual observers and the instruments they used is given in Table 2. The data from all observations (listed in Tables 3 and 4 and displayed on Figures 1 and 2) were used to determine the times of transits of WASP-92 b and WASP-93 b and for TTV analysis.

There were no observations of WASP-118 available in the ETD. However, this object was observed during Campaign 8 of the Kepler K2 mission (Howell et al. 2014). These observations started on 3 January 2016 and finished on 23 March 2016. We used de-trended short-cadence data (PDC-SAP\_FLUX) which are sampled every 58.8 seconds. The data are available to the public at the Mikulski Archive for Space Telescopes (MAST)\(^2\). During the observation period 19 transits of WASP-118 b were observed.

3 LIGHT CURVE ANALYSIS

As a first step of our LC analysis, we fitted the out-of-transit parts of the LCs by a second-order polynomial function to remove additional long-term trends and to normalize the LC levels.

In all cases, we used our software implementation of Mandel & Agol’s (2002) model for fitting transits. Markov chain Monte Carlo (MCMC) simulation was used to determine transit parameters. This method takes into account individual errors of observations and gives a realistic and statistically significant estimate of parameter values and errors. As a starting point for the MCMC fitting, we used the parameters of the planet given in Hay et al. (2016). We have used a quadratic model of limb darkening with coefficients from Claret et al. (2013) selected according to filter. We ran the MCMC simulation with 10\(^6\) steps.

\(^1\) http://var2.astro.cz/ETD/
\(^2\) https://archive.stsci.edu/index.html, doi:10.17909/T9TG7J

### Table 2. Overview of the telescopes and instruments/detectors used to obtain photometry of WASP-92 and WASP-93.

| Observer | Telescope | CCD       | \( N_{\text{tr}} \) |
|----------|-----------|-----------|-------------------|
| MB       | Dall-Kirkham 430/2940 | SBIG ST-11000 | 2                  |
|          | RitcheyChrétien 820/4780 | FLI PL230  | 3                  |
| FC       | Newton 200/940 | SBIG ST-8XME | 1                  |
| SF       | Cassegrain 350/3910 | QSI 583WSG | 1                  |
| EG       | RitcheyChrétien 400/3360 | SBIG ST-11000 | 1                  |
| JG       | Cassegrain 235/1645 | SBIG ST8-XME | 1                  |
| SG       | Cassegrain 300/3000 | SBIG ST-11000 | 1                  |
| DM       | Cassegrain 200/2000 | SBIG ST8-XME | 2                  |
| RN       | Cassegrain 300/3000 | MI G4-9000 | 1                  |
| MS       | Dall-Kirkham 400/2720 | SBIG ST10-XME | 1                  |
| FS       | Newton 410/1710 | Audine KAF1600 ME | 1                  |

### Table 3. Summary of the observing runs for WASP-92. Observers according to Table 2. The dates are given for the center of transit. \( N_{\text{exp}} \) – number of useful exposures, \( t_{\text{exp}} \) – exposure time.

| Observer | Date       | Filter | \( N_{\text{exp}} \) | \( t_{\text{exp}} \) (s) |
|----------|------------|--------|-------------------|-------------------|
| MB (DK)  | 20 Jul 2016| Clear  | 86                | 120               |
| MB (RC)  | 2 Aug 2016 | Clear  | 196               | 60                |
| FS       | 10 Aug 2017| R      | 145               | 90                |
| MB (RC)  | 20 Apr 2017| I      | 101               | 120               |
| MB (RC)  | 29 Jul 2017| I      | 105               | 120               |

3.1 WASP-92 and WASP-93

For the ground-based observations of WASP-92 and WASP-93, we selected those observations with the best signal-to-noise ratio as templates. These templates were fitted using the MCMC method as mentioned above.

To determine the new parameters of WASP-92 b, we have selected Marc Bretton’s observations from 29 July 2017 (on Figure 3 (top)). For WASP-92 b, the results of our LC analysis show that the radius of this exoplanet is significantly smaller than the value given by Hay et al. (2016). All
Figure 1. Transiting LCs of WASP-92 used for analysis. Detail description in Table 3.

Figure 2. Transiting LCs of WASP-93 used for analysis. Detail description in Table 4.
Figure 3. Top: Template of the transit of WASP-92 b (based on Marc Bretton’s observations from 29 July 2017). The solid line marks the best fit of the transit model with the parameters in Table 5. Middle: Template of the transit of WASP-93 b (based on Eric Girardin’s observations from 3 January 2017). The solid line marks the best fit of the transit model with the parameters in Table 6. Bottom: Template of the transit of WASP-118 b. Only every tenth data point is displayed on the figure. The solid line marks the best fit of the transit model with the parameters in Table 7.
Table 6. Parameters of exoplanet WASP-93 b. For a detailed description see Table 5.

| Parameter | Hay et al. (2016) | This paper |
|-----------|------------------|------------|
| $T_0$ [HJD] | 6079.56420±0.00045 | 6079.55280±0.00457 |
| $P$ [d]   | 2.732521±0.0000020 | 2.7325559±0.0000073 |
| $a$ [au]  | 0.04211±0.000035  | 0.04559±0.00275  |
| $a$ [$R_\star$] | 5.96±0.16  | 6.45±0.35  |
| $r_P$ [$R_\oplus$] | 17.901±0.863 | 14.472±0.568 |
| $r_P$ [$R_\star$] | 0.1080±0.0059 | 0.0873±0.0025 |
| $i$ [°]  | 81.18±0.29  | 82.27±0.58  |
| $\Delta$ | 0.01097±0.00013 | 0.00762±0.00044 |
| $t_T$ [d] | 0.0931±0.0010 | 0.0855±0.0180 |
| $b$ [$R_\star$] | 0.904±0.009 | 0.868±0.080 |
| $\chi^2$ | 213.98 | 150.74 |
| $\chi^2/n$ | 2.04 | 1.44 |

(2016) and Močnik et al. (2017) in a Table 7. As expected, our results are closer to the values given by Močnik et al. (2017) because they also used Kepler-K2 data and Hay et al. (2016) worked with different data.

4 TRANSIT TIMING VARIATIONS

We have repeated the MCMC simulation using the solution of our template LC (see Section 3) as the starting point for each individual transit interval, letting only the time of transit $T_T$ update. Values of $T_T$ were used to construct an O-C diagram.

No significant periodic variations were observed on any O-C transit diagram. The times of transits obtained were fitted with a linear function using the MCMC method. The calculated values of period and transit epoch, with their uncertainties, are listed in Tables 5, 6 and 7. O-C diagrams of transits calculated according to the new linear ephemerides are in Figure 4. O-C diagrams of ground-based observations of WASP-92 and WASP-93 are much noisier than the Kepler-K2 observation of WASP-118. For these noisy cases, any periodic TTV signal with amplitude greater than the maximum dispersion of these O-C diagrams over the observing period is unlikely to be seen.

Following Gibson et al. (2009), we put upper constraints on the mass of a potential perturbing planet in the system with our refined assumptions. We set the maximum variance on our O-C diagram as the amplitude of possible TTVs. We assumed that the orbits of both planets are circular and coplanar. The parameters of the transiting planet were taken from Hay et al. (2016). The period of a perturbing planet varied from 0.2 to 5 times the orbital period of the transiting planet.

We used Chamber’s MERCURY6 code (Chambers 1999) to produce 20000 synthetic O-C diagrams for different configurations of mass and orbital period of a hypothetical planet. We applied the Bulirsch-Stoer algorithm to solve our three-body problem. Our calculations covered the whole length of the observing period which, for the ground-based observations of WASP-92 and WASP-93 did not exceed two years, and was only three months for the Kepler-K2 observations of WASP-118.

The results of our simulations are on Figure 5. They display the upper-mass limit of a hypothetical perturbing planet as a function of the ratio of the orbital period of the transiting planet $P_0$ and that of the perturbing planet $P$.
Regions around $P/P_0 \approx 1$ were found to be unstable due to frequent close encounters of the planets. The general funnel-like shape of these limits is given by the condition of Hill stability (Barnes & Greenberg 2006). For orbits with a period ratio $P/P_0 \lesssim 0.4$, the tangential motion of the parent star caused by additional planet became the dominant source of TTV and thus the mass limit is linear (nearly constant) in logarithmic scale in this region. Its behaviour is similar to the upper-limit mass calculated from astrometric observations (Agol et al. 2005). Some mean-motion resonances (MMR) are clearly visible. The strongest MMR are inner resonances (1:3 (not visible for WASP-118), 1:2, 2:3 and 3:4) formed numerical integration of the orbits for $10^8$ revolutions of the known planet, giving a time span of approximately 1000 years. We again used the Bulirsch-Stoer algorithm and MERCURY6 code. The parameter $\epsilon$ which controls the accuracy of the integration was set to $10^{-8}$.

We adopted the parameters of giant planets from Hay et al. (2016). We have generated 200 thousand massless particles for each studied system. The semi-major axis of our test particles ranged from 0.01 AU to 5 times the semi-major axis of the giant planet. The impact of the initial eccentricity on the stability of the orbit is important. It is caused by the fact that for higher eccentric orbits there is a higher probability of a close encounter with the giant planet. The upper limit of eccentricity was 0.5 and the step for the semi-major axis was set to one hundredth of the semi-major axis of the giant planet. The upper limit of eccentricity was 0.5 and the step was 0.02. The upper limit of inclination between the orbits of the giant planet and the test particle was $50^\circ$, the step was $1^\circ$.

Figure 6 (top and middle) shows the stability maps in the $a - e$ plane for the WASP-92 system. We have selected maps for inclinations $i = 0^\circ$ and $i = 50^\circ$ as an example. The stability maps of all the studied systems for selected values of inclination ($i = 0^\circ$, $i = 10^\circ$, $i = 20^\circ$, $i = 30^\circ$, $i = 40^\circ$, $i = 50^\circ$) are available as supplementary material to this paper. The resulting stability maps look nearly identical (despite the differing x-axis scales) for all three systems. It is because these systems are very similar; all of them have a Jupiter-size planet on a circular orbit close to the parent star. The initial inclination of the orbit has minimal influence on the shape of the stability map in the $a - e$ plane. It plays some role in the case of orbits closer to the parent star and also for small values of initial eccentricity. Inner orbits are also stable up to inclinations of $30^\circ$. The highly inclined inner orbits are strongly affected by the precession caused by the known planet and therefore unstable. The presence of such a planet is also improbable from an evolutionary point of view. On the other hand, the impact of the initial eccentricity on the stability of the orbit is important. It is caused by the fact that for higher eccentric orbits there is a higher probability of a close encounter with the giant planet. The arc-like shape of the right border of the instability region is the result of close encounters between the test particle and the giant planet in the periastron of the particle orbit. There is a stable

### Table 7. Parameters of exoplanet WASP-118 b. For a detailed description see Table 5.

| Parameter | Hay et al. (2016) | Močnik et al. (2017) | This paper |
|-----------|------------------|----------------------|------------|
| $T_0$ [BJD] | $6787.81256 \pm 0.00002$ | $6787.81249 \pm 0.00002$ | $6787.81249 \pm 0.00002$ |
| $P$ [d] | $4.0460407 \pm 0.0000026$ | $4.046045 \pm 0.000043$ | $4.046045 \pm 0.000043$ |
| $a$ [au] | $0.00453 \pm 0.00048$ | $0.0045 \pm 0.00049$ | $0.0045 \pm 0.00049$ |
| $a$ [R$_\odot$] | $6.899 \pm 0.136$ | $6.678 \pm 0.060$ | $6.776 \pm 0.060$ |
| $r_p$ [R$_\odot$] | $16.141 \pm 0.404$ | $15.630 \pm 0.150$ | $14.940 \pm 0.268$ |
| $r_p$ [R$_\odot$] | $0.08707 \pm 0.00266$ | $0.08166 \pm 0.00153$ | $0.08059 \pm 0.00003$ |
| $i$ [$^\circ$] | $88.70 \pm 0.90$ | $88.24 \pm 0.14$ | $89.86 \pm 0.18$ |
| $\Delta$ | $0.00755 \pm 0.00019$ | $0.00668 \pm 0.00001$ | $0.00649 \pm 0.00001$ |
| $t_T$ [d] | $0.2002 \pm 0.019$ | $0.2046 \pm 0.0001$ | $0.2062 \pm 0.0002$ |
| $b$ [R$_\odot$] | $0.160 \pm 0.100$ | $0.206 \pm 0.015$ | $0.017 \pm 0.021$ |

* original value 7423.04483 shifted to the same epoch

For each of our extrasolar planetary systems, we performed numerical integration of the orbits for $10^8$ revolutions of the known planet, giving a time span of approximately 1000 years. We again used the Bulirsch-Stoer algorithm and MERCURY6 code. The parameter $\epsilon$ which controls the accuracy of the integration was set to $10^{-8}$.
strip of coplanar orbits with high eccentricities \((e \gtrsim 0.2)\) and semimajor axes nearly the same as the giant planet. This stable strip is a result of 1:1 resonance between the body in this region and known transiting planet. Freistetter et al. (2009) observed the same stable strip in their stability study of the TrES-2 system for mean anomaly \(M = 0^\circ\) what is the same case as our analysis. Many of the bodies in the 1:1 resonance could be captured on satellite-type orbits around the giant planet. The detailed analysis of this scenario and the study of the stability of the hypothetical satellites are beyond the scope of this paper.

In figure 6 (bottom), we present the stability map in the \(a - i\) plane for the WASP-92 system for initial value of eccentricity \(e = 0.0\).
centricity $e = 0.0$. The supplementary materials include the stability maps in the $a - i$ plane for the studied systems for selected initial values of eccentricity ($e = 0.0, e = 0.1, e = 0.2, e = 0.3, e = 0.4, e = 0.5$). The width of the stable regions is generally almost independent of the inclination, mainly for lower values of eccentricity. For higher values of eccentricity, there is a wider unstable region for the lower inclinations. And for inclinations around 25 – 30°, and high eccentricities (0.4 and 0.5), the stable region starts to occur inside the large unstable region. This island of stability looks slightly different in each of the individual systems. These regions of stability could be caused by Kozai resonance. The Kozai mechanism affects orbits with large eccentricities and inclinations (Thomas & Morbidelli 1996). It is strongest for the WASP-118 system (Figure 7).

6 DISCUSSION AND CONCLUSIONS

We redetermined the basic parameters of three exoplanets WASP-92 b, WASP-93 b and WASP-118 b. We used ground-based observations of WASP-92 and WASP-93 available at the public archive ETD. WASP-118 was observed during Campaign 8 of Kepler-K2 mission. We also determined accurate times of all transits of these planets to analyse for possible TTVs. We concluded that our values of planetary parameters for WASP-92 b and WASP-93 b are in generally consistent agreement with previous results of Hay et al. (2016), and with Močnik et al. (2017) for WASP-118 b.

The next aim of this paper was to discuss the possibility of the presence of other planets in these systems. Our O-C diagrams showed no significant periodic variations. We put upper limits on the masses of potential planets in these systems which could generate a TTV signal, though the amplitude of such a signal would be lower than the observed maximum deviation of the data on our O-C diagram. We found that earth-mass planets could exist in these three systems near to MMR. High quality observations over a long period would be needed to better specify the mass limits of potential perturbing planets.

Finally, we investigated the dynamical stability of these systems in order to identify stable regions where additional planets could exist for a long time (hundreds of years). Our results for all three systems are similar, despite differing values of the semi-major axes. The differences in the shapes of the stable regions are negligible in many cases. We found that a hypothetical planet could exist relatively close to the transiting one, mainly on near-circular and near-coplanar orbits. Raising the eccentricity of the orbit causes the unstable region around the transiting planet to start to grow and for extreme cases of eccentricity ($e = 0.5$) orbits with a semi-major axis up to two or three times that of the transiting planet are found to be unstable. However, the inner border of this region changes very slowly with rising eccentricity. These inner orbits are stable for low values of orbital inclination (up to $20 - 30^\circ$), but the existence of another planet in this region is really questionable due to its vicinity to the parent star. But in general, the inclination has minimal influence on the shape of the regions of stability.

From a dynamic point of view, the possibility of there being other planets in these systems is high. There exists a wide range of orbits on which such planets could exist for a long time, though our analysis of times of transits excludes

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**Figure 7.** The island of stability in WASP-118 system caused by the Kozai mechanism. *Top:* Stability plot in $a - e$ plane for $i = 20^\circ$, $i = 30^\circ$. *Bottom:* Stability plot in the $a - i$ plane for $e = 0.4$, $e = 0.5$. 

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the presence of massive planets near to the transiting exoplanet. However additional massive planets could exist on orbits further out. The presence of planets with earth-mass (or lower mass) near the transiting ones or near the mean-motion resonance is still possible.

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SUPPORTING INFORMATION

Additional Supporting Information may be found in the on-line version of this article:

Figures. Stability plots in the $a-\varepsilon$ and $a-i$ plane for different values of $i$ and $\varepsilon$ showing the maximum eccentricity for the WASP-92, WASP-93 and WASP-118 systems.

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Figure: Stability plot in the $a - e$ plane for different values of $i$ showing the maximum eccentricity for the WASP-92 system. From top left to bottom right: $i = 0^\circ$, $i = 10^\circ$, $i = 20^\circ$, $i = 30^\circ$, $i = 40^\circ$, $i = 50^\circ$. 

Figure: Stability plot in the $a - i$ plane for different values of $e$ showing the maximum eccentricity for the WASP-92 system. From top left to bottom right: $e = 0.0$, $e = 0.1$, $e = 0.2$, $e = 0.3$, $e = 0.4$, $e = 0.5$. 
Figure: Stability plot in the $a - e$ plane for different values of $i$ showing the maximum eccentricity for the WASP-93 system. From top left to bottom right: $i = 0^\circ$, $i = 10^\circ$, $i = 20^\circ$, $i = 30^\circ$, $i = 40^\circ$, $i = 50^\circ$. 
Figure: Stability plot in the $a - i$ plane for different values of $e$ showing the maximum eccentricity for the WASP-93 system. From top left to bottom right: $e = 0.0$, $e = 0.1$, $e = 0.2$, $e = 0.3$, $e = 0.4$, $e = 0.5$. 
Figure: Stability plot in the $a - e$ plane for different values of $i$ showing the maximum eccentricity for the WASP-118 system. From top left to bottom right: $i = 0^\circ$, $i = 10^\circ$, $i = 20^\circ$, $i = 30^\circ$, $i = 40^\circ$, $i = 50^\circ$. 
Figure: Stability plot in the $a - i$ plane for different values of $e$ showing the maximum eccentricity for the WASP-118 system. From top left to bottom right: $e = 0.0$, $e = 0.1$, $e = 0.2$, $e = 0.3$, $e = 0.4$, $e = 0.5$. 