On SYM theory and all order Bulk Singularity Structures of BPS Strings in type II theory

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Abstract

The complete forms of the S-matrix elements of a transverse scalar field, two world volume gauge fields, and a Potential $C_{n-1}$ Ramond-Ramond (RR) form field are investigated. In order to find an infinite number of $t, s, (t+s+u)$-channel bulk singularity structures of this particular mixed open-closed amplitude, we employ all the conformal field theory techniques to $< V_{C-2} V_{\phi} V_{A^0} V_{A^0} >$, exploring all the entire correlation functions and all order $\alpha'$ contact interactions to these supersymmetric Yang-Mills (SYM) couplings. Singularity and contact term comparisons with the other symmetric analysis, $< V_{C-1} V_{\phi} V_{A} V_{A} >$ and $< V_{C-1} V_{\phi} V_{A} V_{A} >$ are also carried out in detail. Various couplings from pull-Back of branes, Myers terms and several generalized Bianchi identities should be taken into account to be able to reconstruct all order $\alpha'$ bulk singularities of type IIB (IIA) superstring theory. Finally, we make a comment on how to derive without any ambiguity all order $\alpha'$ contact terms of this S-matrix which carry momentum of RR in transverse directions.

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1 Introduction

The non-perturbative and fundamental objects in type IIA and IIB superstring theories are called D$_p$-branes that have been first recognized in [1]. Their role was highlighted in many different subjects, most specifically, by theoretical high energy physics [2]. As originally was pointed out, D$_p$-branes include $(p+1)$-dimensional world volume directions. They are hypersurfaces in all ten dimensional flat empty space that Neumann or Dirichlet boundary conditions can be applied to them.

The effective action of multiple D$_p$-branes that embeds the commutator of transverse scalar fields was derived [3]. Making use of the direct mixed open-closed scattering amplitudes with some reasonable field content, the generalization of this action to all orders in $\alpha'$ has been done [4]. Note that some anomalous couplings on D-branes as well as the phenomena branes within branes have been discovered [5] as well.

Based on doing direct Conformal Field Theory (CFT) techniques and scattering amplitude methods several other couplings are also recently discovered in [6]. It was shown that apart from three standard ways of exploring couplings in Effective Field Theory (EFT) (pull-back of branes, Myers terms and Taylor expansions), one must introduce new EFT couplings to be able to reconstruct all the S-matrix of string amplitudes.

One can mention some of the applications for these new couplings as follows. To obtain either AdS or dS solutions and to realize the entropy growth of various systems (such as $N^3$ phenomenon for M5 branes [7]), one needs to employ new couplings of string theory.

Some important remarks for supersymmetrized version of this action were introduced [8]. Parts of the action that embeds symmetric trace of non-Abelian field content have been figured out. In [9] the action for single bosonic brane was suggested and eventually in [10] the supersymmetric part of the action was derived. Notice that various remarks for the effective actions of brane anti brane systems, including their bulk structures have also been revealed in [11]. It is worth pointing out that most of RR couplings are derived through the mixed BPS open-closed string amplitudes [12]. To elaborate on all parts of the Chern-Simons, Wess-Zumino and DBI effective actions, we just refer to [13].

The organization of the paper is as follows. In [14] the method of dealing with picture changing operator has been explained, however, we find it is hard to apply its construction to higher point functions of the mixed $C_{\nu-1}$-scalar field vertices. Hence, in the next section
we explicitly construct all the closed forms of the correlators for an asymmetric picture of the S-matrix $< V_{C-2} V_{\phi^0} V_{A^0} V_{A^0} >$ where the superscripts indicate the related picture of strings. Later on, for the concreteness we demonstrate the ultimate forms of S-matrix elements in two other symmetric pictures $< V_{C-1} V_{\phi^{-1}} V_{A^0} V_{A^0} >$ and $< V_{C-1} V_{\phi^0} V_{A^{-1}} V_{A^0} >$ as well. Notations are explained in Appendix.

The comparisons for all singularity structures and all order contact terms for both symmetric pictures have been performed in [6]. In this paper we first find out the complete form of the S-matrix in asymmetric picture then carry out the comparisons for all singularity structures as well as contact terms with symmetric amplitude in detail.

We explore various new couplings from $< V_{C-2} V_{\phi^0} V_{A^0} V_{A^0} >$ amplitude and these couplings actually carry momentum of RR in transverse directions. These new terms include very explicitly $p_i \xi$ term in the ultimate form of the amplitude. We also find out the presence of an infinite number of new $t, s, (t+s+u)$-channel bulk singularity structures which carry momentum of RR in transverse directions, that is, $p^i$ terms just in an asymmetric picture of the amplitude. These terms cannot be derived from the other pictures of amplitude as there are no winding modes ($w^i$ terms) in RR vertex operator in ten dimensions of space-time. Finally we discover new contact terms in the asymmetric picture of the amplitude and these terms will be generated from an EFT point of view as well.

Further remarks are in order. All the correlation functions of $< e^{i p \cdot x(z)} \partial_{x^i}(x_1) >$ at asymmetric picture are non-zero thus one expects to generate new bulk singularity structures from $< V_{C-2} V_{\phi^0} V_{A^0} V_{A^0} >$ amplitude that cannot be appeared even in $< V_{C-1} V_{\phi^0} V_{A^{-1}} V_{A^0} >$ amplitude. Indeed by direct comparisons with [6], we show that just in an asymmetric amplitude, one is able to definitely construct all the bulk singularity structures of the S-matrix.

Thus one needs to explore all the bulk singularities in the entire S-matrix elements that clearly involve the momentum of RR in transverse directions. It is recently shown in [15] that even by knowing various generalized Bianchi identities for RR, one still is not able to discover all the bulk singularities of the simplest $< V_{C-1} V_{\phi^0} V_{A^{-1}} V_{T^0} >$ amplitude.

In a recent attempt for the mixed closed RR-BPS branes [16] it is shown that, if one computes the amplitude (including an RR in the presence of transverse scalar field) in an asymmetric picture of RR and applies all the restricted Bianchi identities of C-field to the S-matrix elements then one is certainly able to precisely discover all the bulk singularity
structures as well as all new contact terms of the string amplitude.

Therefore not only asymmetric S-matrix has the potential to generate new structures for string couplings but also has the strong potential to precisely fix all order $\alpha'$ corrections with their exact coefficients.

Hence, to be able to construct all the bulk singularity structures for the mixed RR and transverse scalar fields one needs to find the complete form of the S-matrix analysis in asymmetric picture, explore restricted Bianchi identities and then start analyzing all the singularity structures. It has clearly been observed that even the structures of some of the new string couplings can only be figured out by direct S-matrix analysis not by T-duality transformation.

2 The $< V_{C^{-2}} V_{\phi^0} V_{A^0} V_{A^0} >$ S-matrix

Let us make some general observation about amplitudes at world-sheet level. One might wonder about the deep relation between open and closed strings. This relationship has been widely understood once we try to match up all the string singularities with their effective field theory parts. Based on various BPS and non-BPS five point functions in [17], we have made a universal conjecture that explores all order $\alpha'$ corrections to SYM couplings. This conjecture has come out of exact reconstruction of all infinite $(t + s + u)$-channel tachyonic or massless singularities, and having an RR coupling with a tachyon or scalar (gauge) field plays the major role in coinciding all the singularity structures of both string and field theory.

Notations can be seen in Appendix. Note that due to the appearance of the momentum of RR in transverse directions and the fact that winding modes are not inserted in vertex of RR, one cannot apply T-duality transformation to S-matrix of $< V_C V_A V_A V_A >$ to get to complete form of $< V_C V_\phi V_A V_A >$ S-matrix elements ( further explanations are given in [18], [19]).

Our computations are consist of the following vertex operators

\begin{align}
V_{\phi}^{(0)}(x_1) &= \int dx_1 \xi i i \left( \partial X^i(x_1) + \alpha' i k \cdot \psi^i(\psi)(x_1) \right) e^{\alpha' i k \cdot X(x_1)}, \\
V_{\phi}^{(-1)}(y) &= \int dy \xi \cdot \psi(y) e^{-\phi(y)} e^{\alpha' i k \cdot X(y)}, \\
V_{A}^{(0)}(x_2) &= \int dx_2 \xi_2 a \left( \partial X^a(x_2) + \alpha' i q \cdot \psi^a(x_2) \right) e^{\alpha' i q \cdot X(x_2)},
\end{align}
\[ V_C^{(-\frac{3}{4}, -\frac{1}{2})} (z, \bar{z}) = (P_- \mathcal{C}_{(n-1)} M_p)^{\alpha \beta} \int dz \bar{d}z e^{-3\phi(z)/2} S_\alpha (z) e^{i\frac{2}{3} p \cdot X(z)} e^{-\phi(\bar{z})/2} S_\beta (\bar{z}) e^{i\frac{2}{3} p \cdot \bar{X}(z)} \]

We also use kinematical relations as
\[ k^2 = q^2 = p^2 = 0, \quad q \cdot \xi_1 = k \cdot \xi_1 = q \cdot \xi_2 = 0. \]

where the RR vertex in asymmetric picture has been found in [20] and [21]. One might also be interested in considering some of the direct results of string amplitude computations [22]. To deal with just holomorphic world-sheet fields, we have considered the so called doubling trick. To see propagators and other conventions we suggest the Appendix of the paper, [6] and the Appendix A of [13].

The S-matrix elements of a transverse scalar field, two world volume gauge fields and one closed string C-field at disk level can be explored by evaluating all the correlators
\[ < V_C^{(-\frac{3}{4}, -\frac{1}{2})} (z, \bar{z}) V_\phi^{(0)} (x_1) V_A^{(0)} (x_2) V_A^{(0)} (x_3) > \]
where all three open strings are inserted in zero picture and the closed string RR is written down in terms of its potential.

First of all we try to explore the whole S-matrix, and to simplify the analysis we divide it out to various correlators.

To investigate all fermionic correlation functions, including a number of currents and also fermion fields in different positions, one needs to employ the so called generalized Wick-like rule which has been emphasized in [23]. Let us find the compact form of the S-matrix elements as follows
\[ A^{C^{-2 \phi^0 A^0 A^0}} \sim \text{Tr} (\lambda_1 \lambda_2 \lambda_3) \int dx_1 dx_2 dx_3 dx_4 dx_5 (P_- \mathcal{C}_{(n-1)} M_p)^{\alpha \beta} I \xi_1 \xi_2 \xi_3 \xi_4 x^{-3/4} \]
\[ \times \left( (x_45/4 C_{\alpha \beta}) \left[ a_i^1 a_i^2 a_i^3 - \eta^{ab} \eta^{2} a_1^i \right] + i \alpha^i k_2 a_i a_2 a_i^{abc} \right. \]
\[ + i \alpha^i k_1 a_i a_2 a_3 a_4^{abc} (\eta^{ab} x_{23}^{-2} + a_2 a_3) - \alpha^2 k_1 a_2 a_3 a_4 a_2 a_5^{abc} + i \alpha^i k_3 a_i a_2 a_5^{abc} \]
\[ - \alpha^2 k_3 a_2 a_3 a_4 a_2 a_5^{abc} \right), \]

where \( x_{ij} = x_i - x_j \), and also
\[ I = |x_{12}|^{a_2 k_1 k_2} |x_{13}|^{a_2 k_1 k_3} |x_{14} x_{15}|^{a_2 k_1 k_2 k_3} |x_{23}|^{a_2 k_2 k_3} |x_{24} x_{25}|^{a_2 k_2 k_3} |x_{34} x_{35}|^{a_2 k_2 k_3} |x_{45}|^{a_2 k_2 k_3} |D, p, p/D, p, p/D, p, \]
\[ a_1^i = i p^i \left( x_{14} \frac{x_{54}}{x_{14} x_{15}} \right) \]
\[ a_2^a = i k_1^a \left( x_{14} \frac{x_{14}}{x_{14} x_{12}} + x_{15} \frac{x_{15}}{x_{25} x_{12}} \right) + i k_2^a \left( x_{14} \frac{x_{43}}{x_{24} x_{23}} + x_{15} \frac{x_{53}}{x_{25} x_{23}} \right), \]

\footnote{In type II we set \( \alpha' = 2 \)}
\(a_3^b = i k_1^b \left( \frac{x_{14}}{x_{34}x_{13}} + \frac{x_{15}}{x_{35}x_{13}} \right) + i k_2^b \left( \frac{x_{24}}{x_{34}x_{23}} + \frac{x_{25}}{x_{35}x_{23}} \right),\)

\(a_2^{ac} = 2^{-1} x_{45}^{-1/4} (x_{24} x_{25})^{-1} (\Gamma^{ac} C^{-1})_{\alpha\beta},\)

\(a_3^{ie} = 2^{-1} x_{45}^{-1/4} (x_{14} x_{15})^{-1} (\Gamma^{ie} C^{-1})_{\alpha\beta},\)

\(a_4^{acie} = 2^{-2} x_{45}^{3/4} (x_{14} x_{15} x_{24} x_{25})^{-1} \left\{ (\Gamma^{acie} C^{-1})_{\alpha\beta} + \alpha' m_1 \frac{Re[x_{14} x_{25}]}{x_{12} x_{45}} \right\},\)

\(a_5^{bd} = 2^{-1} x_{45}^{-1/4} (x_{34} x_{35})^{-1} (\Gamma^{bd} C^{-1})_{\alpha\beta},\)

\(a_6^{bdac} = 2^{-2} x_{45}^{3/4} (x_{34} x_{35} x_{24} x_{25})^{-1} \left\{ (\Gamma^{bdac} C^{-1})_{\alpha\beta} + \alpha' m_2 \frac{Re[x_{24} x_{35}]}{x_{23} x_{45}} + \alpha'^2 m_3 \left( \frac{Re[x_{24} x_{35}]}{x_{23} x_{45}} \right)^2 \right\},\)

\(a_7^{bdie} = 2^{-2} x_{45}^{3/4} (x_{34} x_{35} x_{14} x_{15})^{-1} \left\{ (\Gamma^{bdie} C^{-1})_{\alpha\beta} + \alpha' m_4 \frac{Re[x_{14} x_{35}]}{x_{13} x_{45}} \right\}\)

The following expressions are introduced for the above correlators.

\[ m_1 = \left( \eta^{ec} (\Gamma^{ai} C^{-1})_{\alpha\beta} - \eta^{ca} (\Gamma^{ci} C^{-1})_{\alpha\beta} \right), \]

\[ m_2 = \left( \eta^{ed} (\Gamma^{ba} C^{-1})_{\alpha\beta} - \eta^{eb} (\Gamma^{da} C^{-1})_{\alpha\beta} - \eta^{ad} (\Gamma^{be} C^{-1})_{\alpha\beta} + \eta^{ab} (\Gamma^{dc} C^{-1})_{\alpha\beta} \right), \]

\[ m_3 = \left( C^{-1} \right)_{\alpha\beta} \left( - \eta^{ab} \eta^{cd} + \eta^{ad} \eta^{eb} \right), \]

\[ m_4 = \left( \eta^{ec} (\Gamma^{bi} C^{-1})_{\alpha\beta} - \eta^{eb} (\Gamma^{di} C^{-1})_{\alpha\beta} \right). \]

(3)

One also needs to elaborate on the last correlation function, \(a_8^{bdacie}\) that has several terms as follows

\[ a_8^{bdacie} = \langle S_\alpha (x_4) : S_\beta (x_5) : \psi^c \psi^i (x_1) : \psi^e \psi^a (x_2) : \psi^d \psi^b (x_3) \rangle \]

(4)

so that further ingredients are needed

\[ a_8^{bdacie} = \left\{ (\Gamma^{bdacie} C^{-1})_{\alpha\beta} + \alpha' m_5 \frac{Re[x_{14} x_{25}]}{x_{12} x_{45}} + \alpha' m_6 \frac{Re[x_{14} x_{35}]}{x_{13} x_{45}} + \alpha' m_7 \frac{Re[x_{24} x_{35}]}{x_{23} x_{45}} \right. \]

\[ + \alpha'^2 m_8 \left( \frac{Re[x_{14} x_{25}]}{x_{12} x_{45}} \right) \left( \frac{Re[x_{24} x_{35}]}{x_{23} x_{45}} \right) + \alpha'^2 m_9 \left( \frac{Re[x_{14} x_{35}]}{x_{13} x_{45}} \right) \left( \frac{Re[x_{24} x_{35}]}{x_{23} x_{45}} \right) \]

\[ + \alpha'^2 m_{10} \left( \frac{Re[x_{24} x_{35}]}{x_{23} x_{45}} \right)^2 \right\} 2^{-3} x_{45}^{7/4} (x_{14} x_{15} x_{24} x_{25} x_{34} x_{35})^{-1} \]

where

\[ m_5 = \left( \eta^{ec} (\Gamma^{bdai} C^{-1})_{\alpha\beta} - \eta^{ca} (\Gamma^{bdci} C^{-1})_{\alpha\beta} \right), \]

\[ m_6 = \left( \eta^{ed} (\Gamma^{baci} C^{-1})_{\alpha\beta} - \eta^{eb} (\Gamma^{daci} C^{-1})_{\alpha\beta} \right), \]

\[ m_7 = \left( \eta^{ec} (\Gamma^{bdai} C^{-1})_{\alpha\beta} - \eta^{ca} (\Gamma^{baci} C^{-1})_{\alpha\beta} \right), \]

\[ m_8 = \left( \eta^{ed} (\Gamma^{baci} C^{-1})_{\alpha\beta} - \eta^{eb} (\Gamma^{daci} C^{-1})_{\alpha\beta} \right), \]

\[ m_9 = \left( \eta^{ec} (\Gamma^{bdai} C^{-1})_{\alpha\beta} - \eta^{ca} (\Gamma^{bdci} C^{-1})_{\alpha\beta} \right), \]

\[ m_{10} = \left( \eta^{ed} (\Gamma^{baci} C^{-1})_{\alpha\beta} - \eta^{eb} (\Gamma^{daci} C^{-1})_{\alpha\beta} \right). \]
Having done all the integrations properly, one would finally read off all the elements of the S-matrix and simplifying further, one finds that the amplitude is SL(2,R) invariant. The volume of conformal killing group has been removed by fixing three position of open strings at zero, one and infinity, that is, $x_1 = 0$, $x_2 = 1$, $x_3 \to \infty$.

One comes over the evaluation of all the integrals on the location of closed string RR on upper half plane. The integrals can be computed only in terms of Gamma functions where one deals with the following sort of integrations on upper half plane

$$\int d^2 \xi |1-z|^a |z|^b (z-\bar{z})^c (z+\bar{z})^d$$

where $a, b, c$ are written just in terms of Mandelstam variables. For $d = 0, 1$ and $d = 2$ the results are obtained in [24] and [13] appropriately. We introduce the following definitions for all three Mandelstam variables as

$$s = \frac{-\alpha'}{2} (k_1 + k_3)^2, \quad t = \frac{-\alpha'}{2} (k_1 + k_2)^2, \quad u = \frac{-\alpha'}{2} (k_2 + k_3)^2$$

Having done all the integrations properly, one would finally read off all the elements of the string amplitude in asymmetric picture as follows

$$A^{C-2\rho A^\rho} = A_1 + A_2 + A_3 + A_{41} + A_{42} + A_{43} + A_5$$
$$A_6 + A_7 + A_{81} + A_{82}$$
$$A_{83} + A_{84} + A_{85} + A_{86} + A_{87} + A_{88} \quad (5)$$

where

$$A_1 \sim ip\xi_1 \text{Tr} (P_-(\xi_{(n-1)}^\rho) M_p L_1 \left[ \frac{(s-u)(-t-u)}{(-u-\frac{1}{2})} \xi_3, \xi_2 + 4uk_1, \xi_3 + k_1, \xi_2 \right. \right.$$
$$-4tk_3, \xi_2 k_1, \xi_3 - 4sk_2, \xi_3 k_1, \xi_2 + \frac{2(t+s+u-2st)}{(-u-\frac{1}{2})} k_2, \xi_3 k_3, \xi_2 \right)$$

$$A_2 \sim ik_2, \xi_2 a, \xi_1 \text{Tr} (P_-(\xi_{(n-1)}^\rho) M_p \Gamma^{ac}) L_2 \left( 2utk_1, \xi_3 - 2stk_2, \xi_3 \right) \right)$$
\begin{align*}
A_3 & \sim ik_i^e \xi_i \mathrm{Tr} \left( P \cdot \mathcal{C}'_{(n-1)} M_p \Gamma^{ie} \right) L_1 \left[ \frac{-(s-u)(-t-u)}{(-u-\frac{1}{2})} \xi_3 \xi_2 - 4uk_1 \xi_3 k_1 \xi_2 + 4tk_3 \xi_2 k_1 \xi_3 + 4sk_2 \xi_3 k_1 \xi_2 - \frac{2(t+s+u-2st)}{(-u-\frac{1}{2})} k_2 \xi_3 k_3 \xi_2 \right] \\
A_{41} & \sim i \mathrm{Tr} \left( P \cdot \mathcal{C}'_{(n-1)} M_p \Gamma^{acie} \right) \xi_{2a} \xi_{1k}^e \xi_{k1}^e L_2 \left( -2utk_1 \xi_3 + 2t \xi_2 \xi_3 \right) \\
A_{42} & \sim i \xi_i^e \xi_{2a} \mathrm{Tr} \left( P \cdot \mathcal{C}'_{(n-1)} M_p \Gamma^{ai} \right) L_1 \left\{ -2tst \xi_2 \cdot \xi_3 + 2tuk_1 \cdot \xi_3 \right\} \\
A_{43} & \sim i \xi_i^e k_{2c} \mathrm{Tr} \left( P \cdot \mathcal{C}'_{(n-1)} M_p \Gamma^{ci} \right) L_1 \left\{ -4sk_1 \cdot \xi_3 k_2 \cdot \xi_3 + 4uk_1 \cdot \xi_2 k_1 \cdot \xi_3 \right\} \\
A_5 & \sim ip \cdot \xi_i k_{3d} \xi_{3b} \mathrm{Tr} \left( P \cdot \mathcal{C}'_{(n-1)} M_p \Gamma^{bd} \right) L_2 \left( -2suk_1 \cdot \xi_2 + 2t \xi_3 \xi_2 \right) \\
A_{61} & \sim -4i \mathrm{Tr} \left( P \cdot \mathcal{C}'_{(n-1)} M_p \Gamma^{bac} \right) \xi_{2a} \xi_{3b} \xi_k^e k_{2c} \xi_{k1}^e L_1 \left( -t - s - u \right) \\
A_{62} & \sim -ip \cdot \xi_i \mathrm{Tr} \left( P \cdot \mathcal{C}'_{(n-1)} M_p \Gamma^{ba} \right) tsL_2 \left\{ -u \xi_{2a} \xi_{3b} - 2k_2 \cdot \xi_3 k_3 \cdot \xi_2 a - 2k_3 \cdot \xi_2 k_2 \cdot \xi_3 b + 2 \xi_2 \cdot \xi_3 k_2 a \cdot k_3 b \right\} \\
A_{63} & \sim -ip \cdot \xi_i \mathrm{Tr} \left( P \cdot \mathcal{C}'_{(n-1)} M_p \right) L_1 \left( u \xi_2 \cdot \xi_3 + 2k_3 \cdot \xi_2 k_2 \cdot \xi_3 \right) \left( \frac{-2st + t + s + u}{(-u-\frac{1}{2})} \right) \\
A_{71} & \sim i \mathrm{Tr} \left( P \cdot \mathcal{C}'_{(n-1)} M_p \Gamma^{bdac} \right) \xi_{1i} \xi_{3b} \xi_{k3} \xi_{k4} L_2 \left( -2tsk_3 \cdot \xi_2 + 2usk_1 \cdot \xi_2 \right) \\
A_{72} & \sim i \xi_i^e \xi_1 \xi_{2a} \mathrm{Tr} \left( P \cdot \mathcal{C}'_{(n-1)} M_p \Gamma^{ai} \right) \left\{ 2us \xi_3 k_1 \cdot \xi_2 - 2tsk_3 \cdot \xi_2 k_3 \cdot \xi_2 a + 4uk_1 \cdot \xi_2 k_1 \cdot \xi_3 k_3 - 2k_2 \cdot \xi_2 k_2 \cdot \xi_3 k_3 \cdot \xi_2 a \right\} \\
A_{81} & \sim 4i \mathrm{Tr} \left( P \cdot \mathcal{C}'_{(n-1)} M_p \Gamma^{bdacie} \right) \xi_{1i} \xi_{2a} \xi_3 k_{2c} \cdot k_{3d} L_1 \left( -t - s - u \right) \\
A_{82} & \sim -isu \mathrm{Tr} \left( P \cdot \mathcal{C}'_{(n-1)} M_p \Gamma^{badai} \right) \xi_{1i} \xi_{3b} \xi_{k3} \xi_{k4} L_2 \left\{ t \xi_2 a + 2k_1 \cdot \xi_2 k_2 a \right\} \\
A_{83} & \sim -itu \mathrm{Tr} \left( P \cdot \mathcal{C}'_{(n-1)} M_p \Gamma^{bac} \right) \xi_{1i} \xi_{2a} \xi_3 k_{2c} \cdot L_2 \left\{ -s \xi_3 a - 2k_1 \cdot \xi_3 k_3 b \right\} \\
A_{84} & \sim ist \mathrm{Tr} \left( P \cdot \mathcal{C}'_{(n-1)} M_p \Gamma^{baci} \right) k_{1e} \xi_i^e \xi_{2a} L_2 \left\{ -u \xi_3 a - 2k_2 \cdot \xi_3 k_3 b \right\} \\
A_{85} & \sim ist \mathrm{Tr} \left( P \cdot \mathcal{C}'_{(n-1)} M_p \Gamma^{baci} \right) k_{1e} \xi_i^e \xi_{1k}^e L_2 \left\{ -2k_3 \cdot \xi_2 \xi_3 a + 2k_2 \cdot \xi_2 \xi_3 b \right\} \\
A_{86} & \sim isL_1 \mathrm{Tr} \left( P \cdot \mathcal{C}'_{(n-1)} M_p \Gamma^{bi} \right) \xi_{1i} \left\{ 2tk_3 \cdot \xi_2 \xi_3 b - 2t \xi_3 \cdot \xi_2 k_3 \cdot \xi_2 a - 2uk_1 \cdot \xi_2 \xi_3 b - 4k_1 \cdot \xi_2 k_2 \cdot \xi_3 k_3 b \right\} \\
A_{87} & \sim -itL_1 \mathrm{Tr} \left( P \cdot \mathcal{C}'_{(n-1)} M_p \Gamma^{ai} \right) \xi_{1i} \left\{ -2sk_2 \cdot \xi_3 \xi_2 a + 2s \xi_3 \cdot \xi_2 k_2 a + 2uk_1 \cdot \xi_3 \xi_2 a + 4k_1 \cdot \xi_3 k_3 \cdot \xi_2 k_2 a \right\} \\
A_{88} & \sim iL_1 \mathrm{Tr} \left( P \cdot \mathcal{C}'_{(n-1)} M_p \Gamma^{ie} \right) \xi_{1i} k_{1e} \left( u \xi_2 \cdot \xi_3 + 2k_3 \cdot \xi_2 k_2 \cdot \xi_3 \right) \left\{ \frac{-2st + u + s + t}{(-u-\frac{1}{2})} \right\}
\end{align*}

where the functions \( L_1, L_2 \) are

\[
L_1 = (2)^{-(2t + s + u) - 1} \pi \frac{\Gamma(-u + \frac{1}{2}) \Gamma(-s + \frac{1}{2}) \Gamma(-t + \frac{1}{2}) \Gamma(-t - s - u)}{\Gamma(-u - t + 1) \Gamma(-t - s + 1) \Gamma(-s - u + 1)},
\]
\[ L_2 = \frac{(2)^{-2(s+u)}}{\pi} \frac{\Gamma(-u)\Gamma(-s)\Gamma(-t)\Gamma(-t-s+u+\frac{1}{2})}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)} \] (6)

One can simplify the S-matrix further. Indeed the 5th term of \( A_1 \) can be precisely cancelled by the second term of \( A_{61} \). Likewise, the 5th term of \( A_3 \) is removed by the 2nd term of \( A_{86} \). Notice that the 1st and 2nd terms \( A_{12} \) are cancelled off by the 1st and 3rd terms of \( A_{87} \) accordingly. Finally the 1st and 2nd terms of \( A_{72} \) are removed by contributions of the 3rd and 1st terms of \( A_{86} \) appropriately.

Let us consider RR in terms of its field strength while keep track of scalar field in zero picture. The S-matrix elements of \( < V_{C^{-1}} V_{\phi V} A^{1} V_{A^0} > \) to all orders in \( \alpha' \) has already been found in [6] to be

\[ A^{C^{-1} \phi^1 A^{-1} A^0} = A'_1 + A'_2 + A'_3 + A'_4 + A'_5 + A'_6 + A'_7 \] (7)

where

\[ A'_1 \sim 2^{-1/2} \xi_{1i} \xi_{2a} \xi_{3b} \left[ -k_{3c} k_{1d} \text{Tr} \left( P_- H_n(n) M_p \Gamma^{bca} \right) + k_{3c} \rho^{ij} \text{Tr} \left( P_- H_n(n) M_p \Gamma^{bca} \right) \right] 4(-t - s - u) L_1, \]

\[ A'_2 \sim 2^{-1/2} \text{Tr} \left( P_- H_n(n) M_p \Gamma^{aid} \right) \xi_{1i} \xi_{2a} k_{1d}\left\{ -2k_1 \xi_3(ut) + 2k_2 \xi_3(st) \right\} L_2 \]

\[ A'_5 \sim 2^{-1/2} \text{Tr} \left( P_- H_n(n) M_p \gamma^i \right) \xi_{1i} \left\{ \xi_3 \xi_2(2ts) + 2k_1 \xi_3(2k_3 \xi_2)t - 4uk_1 \xi_3(k_1 \xi_3) + 4sk_2 \xi_3 k_1 \xi_2 \right\} L_1 \]

\[ A'_4 \sim -2^{-1/2} (st) \xi_{2a} \left\{ \xi_3 \xi_{1i} \xi_{2a} \text{Tr} \left( P_- H_n(n) M_p \Gamma^{baid} \right) - \text{Tr} \left( P_- H_n(n) M_p \Gamma^{eaid} \right) k_{1d} k_{3c} \xi_{1i} (2 \xi_3 \xi_3) \right\} L_2 \]

\[ A'_3 \sim -2^{-1/2} \xi_{1i} k_{3c} \left\{ -2k_1 \xi_3 \xi_{3b} \text{Tr} \left( P_- H_n(n) M_p \Gamma^{bci} \right) (us) + 2k_1 \xi_3 \xi_{2a} \text{Tr} \left( P_- H_n(n) M_p \Gamma^{cai} \right) (ut) \right\} L_2 \]

\[ A'_6 \sim 2^{-1/2} \rho^{ij} \xi_{1i} \left\{ 2k_3 \xi_2 \text{Tr} \left( P_- H_n(n) M_p \gamma^b \right) \xi_{3b} - 2k_2 \xi_2 \text{Tr} \left( P_- H_n(n) M_p \gamma^c \right) k_{3c} \right\} L_2 \]

\[ A'_7 \sim 2^{-1/2} \rho^{ij} \xi_{1i} \text{Tr} \left( P_- H_n(n) M_p \gamma^a \right) \xi_{2a} \left\{ 2k_1 \xi_3 (ut) - 2k_3 \xi_2 (st) \right\} L_2 \] (8)

Finally if we deal with RR in terms of its field strength and consider the picture of scalar field in (-1) picture, then the S-matrix elements \( < V_{C^{-1}} V_{\phi V} A^{1} V_{A^0} > \) to all orders in \( \alpha' \) can be explored [18] as follows

\[ A^{C^{-1} \phi^{-1} A^0 A^0} = A''_1 + A''_2 + A''_3 + A''_4 + A''_5 \] (9)

where

\[ A''_1 \sim -2^{-1/2} \xi_{1i} \xi_{2a} \xi_{3b} \left[ k_{3d} k_{2c} \text{Tr} \left( P_- H_n(n) M_p \Gamma^{bca} \right) \right] 4(-t - s - u) L_1, \]

\[ L_2 = \frac{(2)^{-2(s+u)}}{\pi} \frac{\Gamma(-u)\Gamma(-s)\Gamma(-t)\Gamma(-t-s+u+\frac{1}{2})}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)} \] (6)
\[ \mathcal{A}_2'' \sim 2^{-1/2} \text{Tr} \left( P_- \mathcal{H}^a_{(n)} M_p \Gamma^{bai} \right) \xi_{1i} \xi_{3b} k_{3d} \left\{ 2k_1 \xi_2(u) - 2k_3 \xi_2(s) \right\} L_2 \]  
\[ \mathcal{A}_3'' \sim -2^{-1/2} \text{Tr} \left( P_- \mathcal{H}^a_{(n)} M_p \Gamma^{aci} \right) \xi_{1i} \xi_{2a} k_{2c} \left\{ -2k_2 \xi_3(s) + 2k_1 \xi_3(u) \right\} L_2 \]  
\[ \mathcal{A}_4'' \sim -2^{-1/2} \left( st \right) L_2 \left\{ \xi_{3b} k_{1i} \xi_{2a} \text{Tr} \left( P_- \mathcal{H}^a_{(n)} M_p \Gamma^{bai} \right) u + 2k_2 \xi_3 k_{3d} \xi_{1i} \xi_{2a} \text{Tr} \left( P_- \mathcal{H}^a_{(n)} M_p \Gamma^{dai} \right) \right. \]  
\[ \left. + 2k_3 \xi_2 k_{2c} \xi_{1i} \xi_{3b} \text{Tr} \left( P_- \mathcal{H}^a_{(n)} M_p \Gamma^{bci} \right) - \text{Tr} \left( P_- \mathcal{H}^a_{(n)} M_p \Gamma^{dci} \right) k_{3d} k_{2c} \xi_{1i} (2\xi_2, \xi_3) \right\} \]  
\[ \mathcal{A}_5'' \sim 2^{-1/2} \text{Tr} \left( P_- \mathcal{H}^a_{(n)} M_p \gamma^i \right) \xi_{1i} \left\{ \xi_3 \xi_2 (2t) + 2k_1 \xi_3 (2k_3 \xi_2) t - 4uk_1 \xi_2 (k_1 \xi_3) \right. \]  
\[ \left. + 4sk_2 \xi_3 k_1 \xi_2 \right\} L_1 \]

All singularity structure and contact interaction comparisons for both symmetric pictures have already been done in [6]. To be able to explore all the bulk singularity structures, in the following sections we first try to compare all order \( \alpha' \) singularity structures in symmetric and asymmetric picture and afterwards we carry out the comparisons at the level of contact terms to see whether or not some new SYM couplings can be discovered.

## 3 All order singularity comparisons between
\[ < V_{C-2} V_{\phi^0} V_{A^0} V_{A^0} > \] and \[ < V_{C-1} V_{\phi^0} V_{A-1} V_{A^0} > \]

In this section we first work with \( < V_{C-2} V_{\phi^0} V_{A^0} V_{A^0} > \) S-matrix, trying to reconstruct all the singularity structures of \( < V_{C-1} V_{\phi^0} V_{A-1} V_{A^0} > \) and then start exploring new bulk singularity structures.

If we add the 1st terms of \( \mathcal{A}_3 \) and \( \mathcal{A}_{88} \) of asymmetric picture [5], then we obtain the following terms
\[ -2istk_{1e} \text{Tr} \left( P_- \mathcal{C}^a_{(n-1)} M_p \Gamma^{ce} \right) \xi_{1i} \xi_{2} \xi_{3} L_1 \]  
\[ (11) \]

Now one needs to actually consider the sum of (11) with the 2nd terms of \( \mathcal{A}_{86}, \mathcal{A}_{87} \) accordingly to gain the following term
\[ -2ist(k_1 + k_2 + k_3)_e \text{Tr} \left( P_- \mathcal{C}^a_{(n-1)} M_p \Gamma^{ce} \right) \xi_{1i} \xi_{2} \xi_{3} L_1 \]  
\[ (12) \]

If we apply the momentum conservation to (12) and use \( p \mathcal{C} = \mathcal{H} \) then we immediately reveal that, this term is exactly the first term \( \mathcal{A}_5'' \) of symmetric picture [7].

Likewise, we need to add the 2nd terms of \( \mathcal{A}_3, \mathcal{A}_{43} \) and the 3rd term \( \mathcal{A}_{72} \) to be able to derive the following term
\[ -4iu(k_1 + k_2 + k_3)_e \text{Tr} \left( P_- \mathcal{C}^a_{(n-1)} M_p \Gamma^{ie} \right) \xi_{1i} k_1 \xi_{2} k_{1} \xi_{3} L_1 \]
which is exactly the the 3rd term $A_3'$. If we also add the last terms of $A_7$, $A_8$ and the 3rd term $A_3$, we get to

$$4it(k_1 + k_2 + k_3)c\text{Tr} (P_-(\vec{\xi}c_{(n-1)}M_{p}\Gamma^{ic})\xi_1k_1\xi_3k_3\xi_2L_1$$

which is the 2nd term $A_5'$. Eventually, by adding the fourth terms of $A_3$, $A_8$ and the 1st term $A_4$, we explore the last term $A_5'$ as follows

$$4is(k_1 + k_2 + k_3)c\text{Tr} (P_-(\vec{\xi}c_{(n-1)}M_{p}\Gamma^{ic})\xi_1k_1\xi_2k_2\xi_3L_1$$

Note that the 1st term $A_5$ produces exactly the first term $A_3'$. By applying momentum conservation to the last term $A_6$, apart from reconstructing the last term $A_4'$, we generate the 2nd term $A_6'$ as well.

Applying $k_{2c} = -(k_1 + k_3 + p)c$ to both terms of $A_{41}$ and to the 2nd term $A_8$ and keeping in mind the antisymmetric property of $\epsilon$ tensor, not only both terms of $A_2'$ but also the 2nd term $A_3'$ are precisely reconstructed, while the extra terms coming from momentum conservation can be precisely cancelled off, where the 2nd term $A_8$ is also taken into account.

Having applied momentum conservation to the 3rd term $A_6$, one would find the following term

$$-2k_3\xi_2ip.\xi_1\text{Tr} (P_-(\vec{\xi}c_{(n-1)}M_{p}\Gamma^{ba})\xi_3(k_1 + p + k_3)_a$$

where the 1st term in (13) produces the 2nd term $A_4'$, its second term generates the 1st term of $A_6'$ and essentially its 3rd term will be cancelled by the 2nd term $A_5$ of asymmetric amplitude.

4 Bulk singularity structures of $< V_{C-2}V_{φ0}V_{A0}V_{A0} >$

We start to address all order bulk singularities of asymmetric amplitude. Indeed we consider the other terms in the S-matrix of (5) that have not been taken into account in $< V_{C-1}V_{φ0}V_{A-1}V_{A0} >$. To deal with those terms, we need to add the first terms of $A_1$, $A_6$ to get to the following terms

$$-2stip.\xi_1\xi_2\xi_3\text{Tr} (P_-(\vec{\xi}c_{(n-1)}M_{p})L_1$$

One also needs to relate (14) to the other terms that have been leftover in $A_1$ of (5) (all its terms except its 5th term), to eventually explore new bulk singularity structures in asymmetric picture as follows.
\[ ip_\xi \text{Tr} \left( P_\xi \mathcal{G}_{(n-1)M_p} L_1 \left[ -2st\xi_3 \xi_2 + 4uk_1 \xi_3 k_1 \xi_2 - 4tk_3 \xi_2 k_1 \xi_3 - 4sk_2 \xi_3 k_1 \xi_2 \right] \right). \] (15)

Let us produce bulk poles in an EFT.

### 4.1 An infinite number of \((s + t + u)\)-channel bulk singularities

Let us construct all these new \((s + t + u)\)-channel bulk singularity structures in an effective field theory. The expansion of \(L_1\) and its all coefficients is given in [6]. We extract the trace and just focus on the part of the expansion that involves all poles as below

\[ 8i\pi^3 \mu_p \xi_1 \epsilon^{a_0 \cdots a_p} C_{a_0 \cdots a_p} \frac{1}{(p+1)!} \frac{1}{(s+t+u)} \sum_{n,m=0}^{\infty} c_{n,m} (s^m t^n + s^n t^m) \]

\[ \left[ -2st\xi_3 - 4tk_1 \xi_3 k_1 \xi_2 - 4sk_2 \xi_3 k_1 \xi_2 + 4uk_1 \xi_2 k_1 \xi_3 \right] \text{Tr} (\lambda_1 \lambda_2 \lambda_3) \] (16)

If we take into account the following effective field theory amplitude

\[ V^i_\alpha (C_{p+1}, \phi) G^{ij}_{\alpha \beta} (\phi) V^j_\beta (\phi, \phi_1, A_2, A_3) \] (17)

and scalar field’s propagator \((\frac{(2\pi \alpha')^2}{2} D^a \phi^i D_a \phi_i)\) as well as the following coupling

\[ (2\pi \alpha') i \mu_p \int d^{p+1} \sigma \frac{1}{(p+1)!} (\epsilon^{a_0 \cdots a_p} D_{a_0} \phi_i C_{a_1 \cdots a_p}^i) \] (18)

then one would be able to find out the vertex of \(V^i_\alpha (C_{p+1}, \phi)\) as follows

\[ G^{ij}_{\alpha \beta} (\phi) = \frac{-i \delta_{\alpha \beta} \delta^{ij}}{T_p (2\pi \alpha')^2 k^2} = \frac{-i \delta_{\alpha \beta} \delta^{ij}}{T_p (2\pi \alpha')^2 (t + s + u)}, \]

\[ V^i_\alpha (C_{p+1}, \phi) = i (2\pi \alpha') \mu_p \frac{1}{(p+1)!} (\epsilon^{a_0 \cdots a_p} k_{a_0} C_{a_1 \cdots a_p}^i) \text{Tr} (\lambda_\alpha). \] (19)

Now we employ all order \(\alpha'\) two gauge field-two scalar field SYM couplings that have been derived in [18] as follows

\[ (2\pi \alpha')^4 \frac{1}{2\pi^2} T_p (\alpha')^{n+m} \sum_{m,n=0}^{\infty} (\mathcal{L}_1^{nm} + \mathcal{L}_2^{nm} + \mathcal{L}_3^{nm}), \] (20)
\[ \mathcal{L}_{1}^{nm} = - \operatorname{Tr} \left( a_{n,m} \mathcal{D}_{nm} [D \phi^i D^b \phi_i F^{ac} F_{bc}] + b_{n,m} \mathcal{D}'_{nm} [D \phi^i F^{ac} D^b \phi_i F_{bc}] + \text{h.c.} \right), \]

\[ \mathcal{L}_{2}^{nm} = - \operatorname{Tr} \left( a_{n,m} \mathcal{D}_{nm} [D \phi^i D^b \phi_i F^{ac} F_{bc}] + b_{n,m} \mathcal{D}'_{nm} [D \phi^i F^{ac} D^b \phi_i F_{bc}] + \text{h.c.} \right), \]

\[ \mathcal{L}_{3}^{nm} = \frac{1}{2} \operatorname{Tr} \left( a_{n,m} \mathcal{D}_{nm} [D \phi^i D^a \phi_i F^{bc} F_{bc}] + b_{n,m} \mathcal{D}'_{nm} [D \phi^i F^{bc} D^a \phi_i F_{bc}] + \text{h.c.} \right), \]

for which special definitions of \( \mathcal{D}_{nm}(EFGH) \), \( \mathcal{D}'_{nm}(ABCD) \) for all higher derivative operators are appeared in \[25\].

To be able to actually produce all \((t + s + u)\)-channel bulk singularities, one needs to explore \( V^{j}_\beta(\phi, \phi_1, A_2, A_3) \) from \[20\]. Some remarks are also worth pointing out. The momentum conservation \( (k_1 + k_2 + 3) = -p^a \) and \( p_a C_{i a \cdots a p} = \mp p_i C_{a \cdots a p} \) have been taken. \( k \) is the momentum of off-shell Abelian scalar field. The orderings of \( \operatorname{Tr} (\lambda_1 \lambda_2 \lambda_3) \), \( \operatorname{Tr} (\lambda_3 \lambda_1 \lambda_2 \lambda_3) \) are also considered in such a way that one is able to find \( V^{j}_\beta(\phi, \phi_1, A_2, A_3) \) as follows

\[
V^{j}_\beta(\phi, \phi_1, A_2, A_3) = \xi_1^j \frac{(2\pi\alpha')^4 T_p}{2\pi^2} \left( \frac{st}{2} \xi_2 \xi_3 + tk_1 \xi_3 \xi_2 + sk_1 \xi_2 \xi_3 - uk_1 \xi_2 \xi_3 \right) (\alpha')^{n+m} \\
\times \left( (k_3 \cdot k_1)^m (k_3 \cdot k_2)^n + (k_3 \cdot k_1)^m (k_2 \cdot k)^n + (k_1 \cdot k_2)^m (k_1 \cdot k_2)^m \right) (a_{n,m} + b_{n,m}) \operatorname{Tr} (\lambda_1 \lambda_2 \lambda_3 \lambda_3)
\]

(21)

Note that all \( a_{n,m}, b_{n,m} \) coefficients are explored in \[25\] such as

\[
a_{0,0} = -\frac{\pi^2}{6}, b_{0,0} = -\frac{\pi^2}{12}, a_{1,0} = 2\zeta(3), a_{0,1} = 0, b_{0,1} = -\zeta(3), a_{1,1} = a_{0,2} = -7\pi^4 / 90, \]

\[
a_{2,2} = (-83\pi^6 - 7560\zeta(3)^2) / 945, b_{2,2} = -(23\pi^6 - 15120\zeta(3)^2) / 1890, \]

\[
a_{2,0} = -4\pi^4 / 90, b_{1,1} = -\pi^4 / 180, b_{0,2} = -\pi^4 / 45
\]

with \( b_{n,m} \)'s become symmetric. Substituting \[21\] and \[19\] into effective field theory amplitude \[17\], we obtain all order \( \alpha' \) bulk singularity structures in the field theory side as

\[
16\pi \mu_p \frac{\epsilon^{\alpha_1 \cdots \alpha_p} \xi_1 C_{\alpha_1 \cdots \alpha_p}}{(p + 1)! (s + t + u)} \operatorname{Tr} (\lambda_1 \lambda_2 \lambda_3) \sum_{n,m=0}^{\infty} (a_{n,m} + b_{n,m}) [s^n t^n + s^n t^m] \\
\left[ 2st \xi_2 \xi_3 + 4tk_1 \xi_3 \xi_2 + 4sk_1 \xi_2 \xi_3 - 4uk_1 \xi_2 \xi_3 \right]
\]

(22)

In order to show that we have exactly explored all order \((t + s + u)\)-channel bulk singularities of asymmetric string amplitude, we just remove the common factors and start comparing at each order of \( \alpha' \) both string and field theory amplitudes \[16\] and \[22\] accordingly.
At zeroth order for \( n = m = 0 \), the EFT amplitude (22) produces the following coefficient \(-4(a_{0,0} + b_{0,0}) = \pi^2\) and the string amplitude (13) regenerates \((2\pi^2c_{0,0}) = \pi^2\). At 1st order (22) will be led to \(-4(a_{1,0} + a_{0,1} + b_{1,0} + b_{0,1})(s + t) = 0\), and string amplitude carries \(\pi^2(c_{1,0} + c_{0,1})(s + t) = \pi^2\) coefficient. At \((\alpha')^2\) order, (22) predicts the coefficient \(-4(a_{1,1} + b_{1,1})st - 2(a_{0,2} + a_{2,0} + b_{0,2} + b_{2,0})(s^2 + t^2) = \frac{\pi^4}{3}(st) + \frac{\pi^4}{3}(s^2 + t^2)\) and string amplitude is also equivalent to \(\pi^2(c_{1,1}(2st) + (c_{2,0} + c_{0,2})(s^2 + t^2))\), which is precisely the same as EFT coefficient. One can get the same results at all orders of \(\alpha'\) (see [18] for \((\alpha')^3, (\alpha')^4\) comparisons).

Hence in this section by introducing a new coupling (18) we were precisely able to obtain all order \((t + s + u)\)-channel bulk singularity structures in both string and field theory sides. Let us concentrate on the other bulk singularity structures.

### 4.2 An infinite number of \(t, s\)-channel bulk singularity structures

Let us proceed with the 2nd terms of \(A_{71}, A_{82}\) in asymmetric picture. If we apply momentum conservation \((k_3 + p)_a = -(k_1 + k_2)_a\) to those terms, we then obtain

\[
2ik_1.\xi_2suL_2\xi_3\xi_{11}k_3d\xi_3b.\xi_1 \text{Tr} (P_-\bar{\chi}'_{(n-1)}M_p\Gamma^{bdai})(p + k_3)_a
\]  

(23)

the second term in (23) has evidently zero contribution to S-matrix, given the fact that (23) is symmetric under exchanging \(k_{3d}, k_{3a}\) while simultaneously is antisymmetric under \(\epsilon\) tensor so the result for the 2nd term of (23) is zero. Its first term includes an infinite number of t-channel singularities. Indeed these t-channel singularities are the same poles that have been shown up in the 1st terms \(A''_2\) of (9) and \(A'_{3}\) of (7) and these t-channel poles have already been produced in [18] as well.

However, the point of this section is that, besides those singularities, this S-matrix includes an infinite t-channel bulk singularities where we just demonstrate them right now. It is also important to highlight that the 1st term \(A_5\) of asymmetric amplitude is a new bulk singularity structure as follows

\[
-2iusk_1.\xi_2k_3d\xi_{3b}p.\xi_1 \text{Tr} (P_-\bar{\chi}'_{(n-1)}M_p\Gamma^{bd})L_2
\]  

(24)

that has an infinite number of t-channel bulk singularity structures where we reconstruct them in the following. Note that due to symmetries (as can be seen later on), the amplitude is antisymmetric under exchanging either \(2 \leftrightarrow 3\) or \(t \leftrightarrow s\), therefore we just
regenerate all infinite t-channel bulk singularities and finally by changing all the momenta and polarizations of two gauge fields one can produce all s-channel bulk poles as well.

Inserting just singularity contributions of \((usL_2)\) expansion to \((24)\), one finds out all infinite t-channel bulk singularities of asymmetric string amplitude (where \((\pi)^{1/2}\mu_p\) is a normalization constant of the amplitude) as follows

\[
-\frac{ip\xi_1(2\pi\alpha')^2}{(p-1)!}\mu_p\xi_3\xi_2^d\epsilon_{\alpha_0\ldots\alpha_{p-2}b}C_{\alpha_0\ldots\alpha_{p-2}}\sum_{n=-1}^{\infty}\frac{1}{n!}b_n(u+s)^{n+1}(2k_1,\xi_2)
\]  

(25)

We also take into account the following sub-amplitude in an effective field theory

\[
\mathcal{A} = V_{\alpha}^i(C_{p-1}, A_3, \phi)G_{\alpha\beta}^{ij}(\phi)V_\beta^j(\phi, A_2, \phi_3)
\]  

(26)

The kinetic term of scalars \(\frac{(2\pi\alpha')^2}{2}D^a\phi^iD_a\phi_i\) has been taken to derive the following vertex and propagator

\[
V_\beta^j(\phi, A_2, \phi_3) = -2ik_1\xi_2(2\pi\alpha')^2T_\mu\xi_1^j(\lambda_1\lambda_2\lambda_3)
\]  

(27)

\[
(G^\phi)_{ij} = -\frac{i\delta^{ij}\delta_{\alpha\beta}}{(2\pi\alpha')^2T_\mu}
\]

One also needs to apply Taylor expansion of a real scalar field through the Chern-Simons action \(\left(i(2\pi\alpha')^2\mu_p\int dp^{p+1}\sigma\frac{1}{(p-1)!}\partial_iC(p-1)\wedge F\phi^i\right)\) to obtain \(V_{\alpha}^i(C_{p-1}, A_3, \phi)\) as

\[
V_{\alpha}^i(C_{p-1}, A_3, \phi) = \frac{i(2\pi\alpha')^2\mu_p}{(p-1)!}(\epsilon)^{a_0\ldots a_{p-2}}C_{a_0\ldots a_{p-2}p}^i\xi_3a_p^k3_{a_{p-1}}\text{Tr}(\lambda_3\lambda_3)
\]  

(28)

where \(V_\beta^j(\phi, A_3, \phi_1)\) was obtained from scalar fields 's kinetic term in DBI effective action, and it receives no correction as all the kinetic terms have already been fixed in the action. Therefore to produce all infinite t-channel bulk singularity structures, we just impose all order corrections to the following mixed coupling as below

\[
\sum_{n=-1}^{\infty}b_n(\alpha')^{(n+1)}\mu_p\int dp^{p+1}\sigma\frac{1}{(p-1)!}\partial_1C_{p-1}\wedge D_1\ldots D_{a_{n-1}}F D_{a_1}^1\ldots D_{a_{n+1}}^1\phi^i
\]  

(29)

Having regarded \((24)\), one could reveal all order extended vertex operator as follows

\[
V_{\alpha}^i(C_{p-1}, A_3, \phi) = \frac{i(2\pi\alpha')^2\mu_p}{(p-1)!}(\epsilon)^{a_0\ldots a_{p-2}}C_{a_0\ldots a_{p-2}p}^i\xi_3a_p^k3_{a_{p-1}}\times\text{Tr}(\lambda_3\lambda_3)\sum_{n=-1}^{\infty}b_n(\alpha'k_3k)^{n+1}
\]  

(30)
where \(\sum_{n=1}^{\infty} b_n (\alpha' k_3, k)^{n+1} = \sum_{n=1}^{\infty} b_n (u + s)^{n+1}\) is employed. Replacing (30) and (27) to (26), we are able to explore all order t-channel bulk singularities of string amplitude as follows

\[ i p \xi_1 (2 \pi \alpha')^2 \frac{(2 \pi \alpha')^2}{(p-1)!} \mu_p \xi_{3n} k_3 \sum_{n=1}^{\infty} \frac{1}{t} b_n (u + s)^{n+1} (2 \pi \alpha' k_3, \xi_2) \]  

which are precisely the bulk singularities that have been found in (25), so we were able to reconstruct them in an effective theory as well.

Finally let us deal with all s-channel bulk singularities. We need to actually consider the sum of the first term \(A_{41}\) with the 2nd term \(A_{83}\) and make use of momentum conservation along the world volume of brane \((k_1 + k_3) \epsilon = -(k_2 + p) \epsilon\) to derive the following term

\[ -2 i u t k_1 \xi_3 \mathcal{L}_2 (P \mathcal{C} (n-1) M_p \Gamma^{ac}) \xi_1, \xi_2, (k_2 + p) \epsilon L_2 \]  

the first term in (32) has no contribution to S-matrix, because the whole (32) is symmetric under exchanging \(k_{2c}, k_{2e}\) and also is antisymmetric under \(\epsilon\) tensor so the result for the first term of (32) is zero. Its second term \(p \mathcal{C} = \mathcal{H}\) involves an infinite number of s-channel singularities which are precisely the 2nd term of \(A''_3\) and these s-channel poles have already been produced in [18].

It is also worth emphasizing the important point as follows. The 1st term \(A_2\) of asymmetric amplitude (3) is also a new bulk singularity structure as follows

\[ 2 i u t k_1 \xi_3 k_2 \xi_2 a p, \xi_1 \mathcal{L}_2 (P \mathcal{C} (n-1) M_p \Gamma^{ac}) L_2 \]  

that can be re-derived in an EFT as shown for an infinite number of t-channel bulk poles of this section.

---

3On the other hand, applying the following Bianchi identity

\[ \xi_1 \xi_2 a \left( \epsilon^{a_0 \ldots a_{p-2}} p_d H^{j}_{a_0 \ldots a_{p-2}} + p i \epsilon^{a_0 \ldots a_{p-1}} H_{a_0 \ldots a_{p-1}} \right) = 0 \]

and considering the sum of the 1st term of \(A'_{2}\), the 2nd term of \(A'_{3}\) and 1st term of \(A'_{7}\) of (1) will give rise the same s-channel singularities as follows

\[ 2 i u t k_1 \xi_3 \mathcal{L}_2 (P \mathcal{H} (n-1) M_p \Gamma^{aid}) \xi_1, \xi_2, (k_2 + p) d L_2 \]

(33)
4.3 An infinite number of u-channel singularities

In this section, we add the 2nd terms $A_{62}, A_2$ of (5) and apply momentum conservation along the brane to get to the following terms

$$-itsp.\xi_1(2k_2.\xi_3)\text{Tr} (P_-(\xi_{(n-1)}M_p\Gamma^{ac})L_2\xi_{2a}(-p - k_1))$$

where the first term in (35) reconstructs the 2nd term $A'_7$ and its second term remains to be explored as an extra u-channel pole. Adding the 1st terms $A'_{71}, A_{85}$ we derive the following u-channel bulk singularities

$$2ik_3.\xi_2tsL_2\xi_{3b}k_{1e}\text{Tr} (P_-(\xi_{(n-1)}M_p\Gamma^{bcie})(p + k_1))$$

Clearly the second term in (36) has no contribution to the S-matrix at all, because the whole (36) is symmetric under exchanging $k_1d, k_1e$ and simultaneously is antisymmetric under $\epsilon$ tensor, so the result is zero. The first term in (36) includes an infinite number of new u-channel bulk singularities that would be generated in an effective field theory in the following section. Note that the 2nd term of $A_{85}$ needs to be taken as follows

$$2i\xi_2.\xi_3stL_2 \text{Tr} (P_-(\xi_{(n-1)}M_p\Gamma^{bcie})k_{1e}k_2.k_{3b})$$

which has contribution to an infinite number of u-channel gauge poles as well. We now collect all the above terms, extracting the traces and write down all u-channel poles as

$$i\mu_p\xi_1k_{1e}\left[\epsilon_{a_0\cdots a_{p-3}ebd}p_d\left(2k_3.\xi_2\xi_{3b}C_{a_0\cdots a_{p-3}}^i - 2\xi_3.\xi_2k_{3b}C_{a_0\cdots a_{p-3}}^i\right) - 2k_2.\xi_3\xi_{2a}\epsilon_{a_0\cdots a_{p-2}eb}\right]\left(2\pi\alpha'\right)^2 \frac{1}{(p - 1)!u} \times \sum_{n=-1}^{\infty} b_n(s + t)^{n+1}$$

where the following field theory amplitude must be considered

$$\mathcal{A} = V^a_\alpha(C_{p-1}, \phi_1, A)G^{ab}_{\alpha\beta}(A)V^b_\beta(A, A_2, A_3),$$

where gauge field propagator and the other vertex operator are read off

$$V^b_\beta(A, A_2, A_3) = -iT_p(2\pi\alpha')^2\text{Tr} (\lambda_2\lambda_3\lambda_\beta)\left[2k_2.\xi_3\xi^b_2 - 2k_3.\xi_2\xi^b_3 + \xi_3.\xi_2(k_3 - k_2)^b\right],$$

$$G^{ab}_{\alpha\beta}(A) = \frac{i\delta_{\alpha\beta}\delta^{ab}}{(2\pi\alpha')^2T_p u},$$

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One also needs to deal with Taylor expansion of a real scalar field to Chern-Simons coupling as we mentioned it earlier on, so that by considering its all order corrections as \cite{29}, one is able to re-derive all order vertex operator of $V^a(C_{p-1}, \phi_1, A)$ as follows

$$V^a(C_{p-1}, \phi_1, A) = \frac{i(2\pi \alpha')^2 \mu_p}{(p-1)!} (\varepsilon)^{a_0 \cdots a_{p-2} a} C_{a_0 \cdots a_{p-2} p} \xi_1 k_{a_{p-1}} \text{Tr} (\lambda_1 \lambda_2) \sum_{n=-1}^{\infty} b_n (t+s)^{n+1}.$$ 

where $k$ is now the momentum of off-shell gauge field $k^a = -(k_2 + k_3)^a = (p + k_1)^a$. Replacing the above vertex and (40) into (39), we would be able to exactly reconstruct all order u-channel gauge field poles of string amplitude (38) as well.

Finally let us make comparisons to all order $\alpha'$ contact interactions of both symmetric and asymmetric pictures of this S-matrix.

## 5 Contact terms

First of all, the 1st term $A_{62}$ of (3) precisely produces the 1st term $A'_1$ of (7). Consider $A_{81}$ and apply momentum conservation along the world volume of brane $k_{2c} = -(p+k_1+k_3)c$ to this term, by doing so, making use of the property of antisymmetric $\epsilon$ tensor and $pC = H$, one is able to produce the first contact term $A'_1$ of (7). Finally to regenerate the 2nd term of $A'_1$, one needs to apply once more the momentum conservation $k_{2_2} = -(p+k_1+k_3)c$ to $A_{61}$ and uses the antisymmetric property of $\epsilon$ tensor where this leads to a new sort of bulk contact interaction as follows

$$4i \text{Tr} (P_\varepsilon \mathcal{C}_{(n-1)} M_p \Gamma^{dbac}) \xi_2 \xi_3 \xi_1 k_3 k_1 L_1 (-t-s-u), \quad (41)$$

Ultimately, considering the sum of the 1st terms of $A_{82}, A_{83}, A_{84}$ of asymmetric picture, takes us to another sort of new contact interaction which has the following structure

$$ip_{d} \text{Tr} (P_\varepsilon \mathcal{C}_{(n-1)} M_p \Gamma^{dai}) \xi_1 \xi_2 \xi_3 u s L_2 \quad (42)$$

Note that the above term is antisymmetric under interchanging the gauge fields. The method of finding all order $\alpha'$ higher derivative corrections to BPS string amplitudes has been explained in sections 4,5 of \cite{6}. To shorten the paper we just refer the interested reader to those concrete explanations in \cite{6}. Indeed one could easily follow \cite{6} and start to generate these new contact interactions to all orders in an EFT as well. This ends our goals of getting new bulk singularity structures as well as new couplings in string theory.
Lastly, it would be interesting to see what happens to the other asymmetric higher point string amplitudes of the mixed C-field and an even number of transverse scalar fields and at least a gauge field. We hope to come over this problem and various other issues including the beautiful mathematical structures behind string theory amplitudes in near future.

**Appendix : Notations**

In this appendix we would like to mention the notations used for the momenta, positions and indices. The lowercase Greek indices take values for 10 dimensional spacetime, as

\[ \mu, \nu = 0, 1, \ldots, 9 \]  

(A.1)

The world volume indices run as \( a, b, c = 0, 1, \ldots, p \) and transverse directions of the brane are represented by \( i, j = p + 1, \ldots, 9 \). The Doubling trick can be taken as follows

\[
\tilde{X}^\mu(\overline{z}) \rightarrow D^\mu_{\nu}X^\nu(\overline{z}) \ , \quad \tilde{\psi}^\mu(\overline{z}) \rightarrow D^\mu_{\nu}\psi^\nu(\overline{z}) \ , \quad \tilde{\phi}(\overline{z}) \rightarrow \phi(\overline{z}) \ , \quad \text{and} \quad \tilde{S}_\alpha(\overline{z}) \rightarrow M_\alpha^\beta S_\beta(\overline{z}),
\]

where

\[
D = \begin{pmatrix}
-1_{9-p} & 0 \\
0 & 1_{p+1}
\end{pmatrix}, \quad \text{and} \quad M_p = \left\{ \begin{array}{ll}
\pm \frac{i}{(p+1)!}\gamma^{a_1}\gamma^{a_2}\cdots\gamma^{a_{p+1}}\epsilon_{a_1\ldots a_{p+1}} & \text{for } p \text{ even} \\
\pm \frac{1}{(p+1)!}\gamma^{a_1}\gamma^{a_2}\cdots\gamma^{a_{p+1}}\gamma^{11}\epsilon_{a_1\ldots a_{p+1}} & \text{for } p \text{ odd}
\end{array} \right.
\]

Propagators for the entire world-sheet fields \( X^\mu, \psi^\mu, \phi \) are

\[
\langle X^\mu(z)X^\nu(w) \rangle = -\frac{\alpha'}{2}\eta^{\mu\nu}\log(z-w),
\]

\[
\langle \psi^\mu(z)\psi^\nu(w) \rangle = -\frac{\alpha'}{2}\eta^{\mu\nu}(z-w)^{-1},
\]

\[
\langle \phi(z)\phi(w) \rangle = -\frac{\alpha'}{2}\log(z-w).
\]

(A.2)

We also introduce

\[
x_4 \equiv z = x + iy \ , \quad x_5 \equiv \overline{z} = x - iy
\]

(A.3)

and SL(2,R) symmetry has been fixed by choosing the following positions for 3 open strings as

\[
x_1 = 0, \quad x_2 = 1, \quad x_3 \rightarrow \infty.
\]

(A.4)
Acknowledgements

I am indebted to R. Russo, A. Tseytlin, W. Siegel, A. Sen and N. Arkani-Hamed for helpful discussions. I would like to thank M. Bianchi, C. Hull, W. Lerche, L. Alvarez-Gaume, M. Douglas, P. Vanhove, H. Steinacker, I. Bena, J. Polchinski and A. Rebhan for various valuable discussions. I would also like to thank CERN for the hospitality. Part of this work was done during the author’s 2nd post doctoral position at Queen Mary University of London (QMUL) and he warmly thanks QMUL as well as A. Brandhuber, D. Young, N. Lambert and B. Stefanski for enjoyable discussions.

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