Study of a 43 day optical quasi-periodic oscillation for BL Lac S5 0716+714.

Lin Lu, Hao-Jing Zhang, and Guo-Wei Ren

1Department of Physics
Yunnan Normal University
Kunming, 650092, People’s Republic of China
2Department of Astronomy
Xiamen University
Xiamen, 361005, People’s Republic of China

Abstract

The data for BL Lac object S5 0716+714 optical B, R and I bands are collected from November 10, 2017 to May 15, 2018. The raw data consists of 21396 quasi-simultaneous multi-band points, with 7132 data points for each band. To search for the quasi-periodic oscillation (QPO) signal of S5 0716+714, the Lomb Scargle periodogram (LSP) method and the Weighted Wavelet Z-transform (WWZ) method are used, and the light curve simulation method is utilized to estimate the significance level of the QPO. For the first time, a QPO with 43-day (> 95% confidence level) of approximate moon-like oscillations is revealed using these techniques. Furthermore, further research indicates that optical radiation is produced as a result of relativistic jet. This 43-day QPO can be explained by a spiral motion of a blob with velocity $\vec{\beta}$ along the jet. In addition, this 43-day QPO is superimposed on a longer QPO, and this superposition phenomenon can explain the linear upward trend. The viewing angle of the blob undergoes periodic changes, while jet precession results in a longer QPO. When the longer QPO fluxes are increasing state, a periodic rising tendency may be observed.

Keywords: galaxies: BL Lacertae objects: individual S5 0716+714 — galaxies: jets — method: data analysis — radiation mechanisms: non-thermal

1. Introduction

Blazars are a subclass of the Active Galactic Nuclei (AGNs), which exhibit large, rapid and violent variations across electromagnetic spectrum (Hong, S. W., Xiong, D. R. & Bai, J. M. 2018). The blazars have relativistic jets at a small angle to the direction of the line of sight (Urry & Padovani 1995; Padovani et al. 2016; Xiong et al. 2020). The Super Massive Black Holes (SMBHs) with masses ranging from $10^6 M_\odot$ to $10^{10} M_\odot$ are found in the center of blazars (Gupta et al. 2019; Esposito et al. 2015). The blazars have a non-thermal continuous electromagnetic spectrum ranging from radio to high-energy $\gamma$-rays, as well as a relativistic jet with apparent superluminal speed (Urry & Padovani 1995; Angel & Stockman 1980; Xiong et al. 2017). Blazars are typically classified into two types based on their spectral properties: BL Lacertae objects (BL Lacs) and Flat Spectrum Radio Quasars (FSRQs) (Ren et al. 2021a). FSRQs have strong emission lines with equivalent widths (EWs) greater than 5 Å, whereas BL Lacs have weak or absent emission lines with EWs less than 5 Å (Urry & Padovani 1995; Xiong et al. 2017; Hong, S. W., Xiong, D. R. & Bai, J. M. 2018). In most blazars, the Spectral Energy Distribution (SED) has a double-peak structure. In general, relativistic electron synchrotron radiation produces low-energy peaks ranging from radio to soft x-rays, whereas the Inverse Compton (IC) process produces high-energy peaks ranging from hard x-rays to high-energy $\gamma$-rays (Ren et al. 2021b,a). According to the location of the low-energy peak frequencies (Hong et al. 2017; Iyida et al. 2022), Abdo et al. (2010) had classified blazars into three categories: low synchrotron peaked (LSP) ($\nu_{\text{peak}} < 10^{14} Hz$), intermediate synchrotron peaked (ISP) ($10^{14} Hz \leq \nu_{\text{peak}} \leq 10^{15} Hz$), and high synchrotron peaked (HSP) ($\nu_{\text{peak}} > 10^{15} Hz$). Blazars exhibit a variety of light curve variations in addition to the double-peak features seen on the SED. The study of light curves may help us to study some important physical parameters, from which it is possible to study the problems of blazars, such as the interior structure, the mass of the central black hole and the radius of the radiation region, etc (Urry & Padovani 1995; Angel & Stockman 1980; Xiong et al. 2017).
Especially for those blazars with QPO signals. Some very interesting phenomena will be revealed by the study of these QPOs. Previous reports in the literature stated that QPO was found in many bands of blazars. These QPOs have time scales ranging from a few minutes to several years (Zola et al. 2016; Bhatta et al. 2016; Bhatta 2017). Based on the time scale of these QPOs, they were classified into three categories: intra-day variation (IDV) refers to QPOs that occur within a day; short-term variation (STV) refers to QPOs that occur within a few days to a few weeks; and long-term variation (LTV) refers to QPOs that occur within a few months to several years (Xiong et al. 2020). Although the study of QPOs can reveal many interesting phenomena and provide some important physical parameters, the non-uniformity of the observed data makes it difficult to obtain reliable QPOs. Uncertainties, such as equipment failure or weather conditions, as well as some unavoidable human errors, contribute to the unevenness. As a result, obtaining reliable QPOs has been a major focus of time-domain astronomy research. The LSP method and the WWZ method are two well-known methods for studying QPO signals (Rejkuba et al. 2003; Foster 1996). For non-uniformly spaced time series data, the LSP and WWZ methods outperform traditional Fourier transform methods. However, these two methods may result in false QPOs. As a result, estimating the significance level of the obtained QPO signals is critical.

In the before reported literature, S5 0716+714 (with the redshift $z = 0.31$ Ajello et al. (2020)) had been classified as an intermediate synchrotron peaked (ISP) BL Lac object ($\nu_{\text{peak}} = 10^{14.6}$ Hz). As reported in some previous studies on S5 076+714, QPOs have been found to exist in multiple bands and the time scales of these QPOs range from IDV to LTV. For example, a 50 min QPO was found by Hong et al. (2017) using the ZDCF method, the LSP method and the REDFTT method, and the QPO was explained by an innermost stable orbital period from the accretion disk; based on this hypothesis, they obtained the mass of the central black hole S5 0716+714. In 2009, A. C. Gupta et al. used wavelet analysis to discover the existence of a 27-73 min QPO, which was explained by a blob or flare in the inner portion of the accretion disk, and they also obtained the central black hole mass of this source based on these assumptions. The source S5 0716+714 is likely to have IDV-scale QPOs, as can be seen (Gupta et al. 2009). Also, Heidt & Wagner (1996) found a 4-day QPO by Structure Function (SF) method and Autocorrelation analysis method, and this 4-day QPO might be caused by a rotating hotspot from the accretion disk. A longer long period of 3.3 years in the optical band was found by Raiteri et al. (2003); Liu et al. (2012) using the Discrete Fourier Transform (DFT) (Tripathi et al. 2021) method, the Discrete Correlation Function (DCF) (Zhou et al. 2021) method and the Structure Function (Rejkuba et al. 2003) method. Raiteri et al. (2003) used a geometric model to explain the origin of this long period, namely that when the viewing angle of the jet varies periodically, it causes a periodic variation in the flux Raiteri et al. (2003); Villata et al. (2002). Previous research has clearly shown that the range of QPOs for this source is relatively broad. For the other blazars, Zhou et al. (2018) was studying the $\gamma$-ray of PKS 2247-131, a QPO with approximately 34.5 days of an approximate mouth-like oscillation was found. To explain the origin of this 34.5-day QPO, Zhou et al. used a helical motion of a blob with velocity $\vec{\beta}$ along the jet. In this work, the LSP and WWZ methods are employed to analyze the QPOs in the optical B, R and I bands of S5 0716+714. The confidence level of the QPO of S5 0716+714 is reported using the Monte Carlo simulation light curve method (Timmer & Koenig 1995; Ren et al. 2021b,a). A 43-day QPO (> 95% confidence level) is found by us for the first time. The previous study on S5 0716+714 apparently did not find a 43-day QPO, and the 43-day QPO time scale is not comparable to the previously reported QPO time scale. The 43-day QPO, on the other hand, is more similar to a month-like oscillation. This result suggests that the 43-day QPO might be caused by a helical motion of a blob with velocity $\vec{\beta}$ along the jet. However, it could also be caused by the periodic motion of the hot spot on the accretion disk. To further investigate the origin of this QPO, the spectral index and color index in the observed range are analyzed.

In this paper, the optical band data for S5 0716+714 are studied, which spans 186 days (Xiong et al. 2020). A 43-day QPO (> 95% confidence level) is reported for first time. This article is organized as follows. In Section 2, LSP method and WWZ method are introduced; In section 3, The LSP method and WWZ method are used to analyze the QPO of S5 0716+714, and the confidence level of the obtained QPO is estimated using the light curve simulation method (Ren et al. 2021b,a). In section 4, the spectral index $\alpha_{BRI}$ and color index $B - R$ on the observation span are analyzed, leading to the suggestion that the 43-day QPO may have originated from a blob making a spiral motion along the jet (Zhou et al. 2018). For the rising trend shown by the raw observations, we believe that this may be the effect of short QPOs superimposed on long QPOs, i.e., the source may be a double superimposed QPO source; In section 5, some conclusions be summarized. In this paper, the cosmological parameters $H_0 = 70\text{km s}^{-1}\text{Mpc}^{-1}$, $\Omega_0 = 0.3$ and $\Omega_{\Lambda} = 0.7$ is adopted (Tan et al. 2020).
2. PERIODIC SIGNAL ANALYSIS METHOD OF BLAZARS

2.1. Lomb-Scargle Periodogram (LSP) method

The calculation of a periodic signals hidden in noise is an important target in time domain astronomy. The LSP method is able to phase correction on non-uniformly sampled time series periodogram (Lomb 1976; Scargle 1981; Scargle, J. D. 1982). Thus, the LSP method can well search for the quasi-periodically oscillated signal hidden in the noise. Now it is supposed that there is a group of time series data \(x(t_j), j = 1, 2, 3, \ldots, N\) (Tao et al. 2016), and note that this set of data is not uniformly spaced time series data. The LSP power spectrum for the time series is given by the following expression, \(P_x(\omega_j)\) is the power spectrum function with the variable of angular frequency (\(\omega_j = 2\pi f_j\)), in which \(f_j\) is the frequency of the tested QPO, in 1/day (Ren et al. 2021b,a). Its basic formulation is:

\[
P_x(\omega_j) = \frac{1}{2} \left\{ \frac{\sum_{j=1}^{N} [(x(t_j) - \bar{x}) \cos(\omega_j t_j - \tau)]}{\sum_{j=1}^{N} \cos^2[\omega_j (t_j - \tau)]} + \frac{\sum_{j=1}^{N} [(x(t_j) - \bar{x}) \sin(\omega_j t_j - \tau)]}{\sum_{j=1}^{N} \sin^2[\omega_j (t_j - \tau)]} \right\}
\]

(1)

Here, \(\bar{x} = \frac{1}{N} \sum_{j=1}^{N} x_j\) is the mean value of time series, \(N\) is the total number of time series data points, \(\tau\) is the phase correction for the time:

\[
\tan(2\omega_j \tau) = \frac{\sum_{j=1}^{N} \sin^2(2\omega_j t_j)}{\sum_{j=1}^{N} \cos^2(2\omega_j t_j)}
\]

(2)

2.2. Weighted Wavelet Z-transform (WWZ) method

In the classical time-frequency domain time series analysis, Fourier transform and Wavelet analysis are adopted. However, both methods usually require time series data to be uniformly spaced. The observation of astronomical data is usually affected by some uncertainties, which make the observed data non-equally spaced. Therefore, Fourier transform and Wavelet analysis are not applicable to the analysis of these astronomical data. In practice, manual interpolation of astronomical data into equally spaced data has been proposed to accommodate Fourier transform methods and Wavelet analysis, but this may greatly impair the authenticity of the astronomical data.

Therefore, based on the above problem, Foster proposes the WWZ method (Foster, G 1996; Foster 1996). The WWZ method can not only effectively handle non-uniformly spaced time series data, but also reflect the stability of periodic signals. The WWZ method projects the time series onto three orthogonal normalized basis vector functions. These three basis vector functions are \(\varphi_1(t_i) = 1\), \(\varphi_2(t_i) = \cos[\omega_0(t_i - \tau_0)]\), and \(\varphi_3(t_i) = \sin[\omega_0(t_i - \tau_0)]\) (i = 1, 2, 3...N). To avoid the effect of too dense data during the analysis, statistical weighting methods are used to adjust the time series data, the expression of the statistically weighted function is \(\omega_i = \exp[-c\omega_0^2(t_i - \tau_0)^2]\). Morlet wavelet is used as the mother function of WWZ (Foster, G 1996; Xie et al. 2002). The parameter \(Z\) in WWZ is defined as follows:

\[
Z = \frac{(N_{eff} - 3)V_y}{2(V_x - V_y)}
\]

(3)

Here, \(N_{eff} = \frac{\sum \exp(-2c\omega_0^2(t_i - \tau_0)^2)}{\sum \exp(-2c\omega_0^2(t_i - \tau_0)^2)}\) indicates the number of effective data points, and \(V_x = \frac{\sum \omega_i x^2(t_i)}{\sum \omega_i} - \left(\frac{\sum \omega_i x(t_i)}{\sum \omega_i}\right)^2\) indicates the number of effective data points, and \(V_y = \frac{\sum \omega_i y^2(t_i)}{\sum \omega_i} - \left(\frac{\sum \omega_i y(t_i)}{\sum \omega_i}\right)^2\) are the simulation functions of the observed data and weighted variables, respectively (Foster, G 1996; Tao et al. 2016).

3. THE QUASI-PERIODIC OSCILLATION SIGNAL SEARCH

Because the data is collected in the optical band, it is first processed with Galactic extinction coefficients \(A_B = 0.102, A_R = 0.071\) and \(A_I = 0.053\) (Xiong et al. 2020) before being analyzed. By means of the formula
\[ F = (3631 \times 10^{-0.4 \cdot \text{mag}}) \times 1000\text{mJy}, \] these collected magnitudes are converted into the corresponding flux. 3631 is the zero-point flux density and \( \text{mag} \) is the magnitudes (Xiong et al. 2020). The average value of the flux is calculated on a daily basis due to the high density of the data collected. From the light curve shown in Fig. 1, it is clear that the light curve of this source shows an increasing trend during the observation period. To quantitatively describe the activeness extent of each band in the light curves in Fig. 1, a variability index \( E \) is introduced. Its basic formulation is as follows (Fan et al. 2002; Li et al. 2009, 2016):

\[
E = \frac{F_{\text{max}} - F_{\text{min}}}{F_{\text{max}} + F_{\text{min}}}
\]  

(4)

In which \( F_{\text{max}} \) and \( F_{\text{min}} \) are the maximum and minimum flux values at each epoch, respectively. The variability index \( E \) in each epoch is calculated and the \( E \) in each band are averaged. The average variation index \( E \) in each epoch is calculated and the \( E \) in each band are averaged. The average variation index \( E \) in each band over the observation span is obtained, the average variation index is respectively \( E_B = 0.48 \), \( E_R = 0.44 \) and \( E_I = 0.41 \). These results indicate that the optical band is active in the observed range and that the activity level increases with increasing frequency.

![Figure 1](image_url)

**Figure 1.** Fig. 1 shows the daily average light curve, spanning the period from November 10, 2017 to May 15, 2018. The red lines in the figure represent the results of the linear fit analysis of the upward trend.

### 3.1. significance level estimation and Power spectrum

In this section, we follow the discrete Fourier transform to obtain an estimate of the power spectrum of the time series and periodogram (Raiteri et al. 2021). When modelled as a power law: \( P(f) \propto f^{-\alpha} + C \), in which \( P(f) \) is the power at temporal frequency \( f \) in 1/day with spectral slope \( \alpha \), \( C \) represent the Poisson noise level which is a positive (Raiteri et al. 2021; Vaughan 2005), we take \( C = 3 \), the value of \( C \) only affects the level of power, and does not affect the solution of the period (Ren et al. 2021a). In Fig. 2, power spectrum vs. frequency log-log plot is shown. As shown in the figure, the lower frequency is dominated by red noise, while the higher frequency part is dominated by white noise. This may show spurious periods in the parts of the red noise on the periodogram. Thus, it is necessary to estimate the importance of these peaks. Following the procedure of Vaughan we estimate the \( \alpha \)-value in the power-law spectrum and evaluate the significance of the peak in the periodogram. To obtain \( \alpha \)-value, we use an least square to fit the red-noise dominated part, here we fit the power spectrum for frequencies less than 0.3 (1/d). Using \( \gamma_{\varepsilon} = -2 \ln[1 - (1 - \varepsilon)^{1/M}] \), we estimated the significance of these peaks, \( \gamma_{\varepsilon} \) is the confidence limit, \( \varepsilon \) is the "false
alarm probability\textsuperscript{\textregistered}, and \( M \) is the number of examined frequencies \citep{raiteri2021, vaughan2005}. The figure shows that there is a peak in the red noise part that exceeds the confidence of 95\%. Thus, the periodogram according to Fig. 2, there may be a 43-day QPO. Next we will follow other methods for further confirmation of this QPO. By The Monte Carlo method described by Timmer & Koenig \citep{timmer1995, ren2021a, ren2021b} and the \( \alpha \) values we take \( \alpha_B = 1.92, \alpha_R = 1.97 \) and \( \alpha_I = 1.94 \), for each of B, R and I, 20000 light curves based on uniform intervals are respectively simulated using python code. Finally, the power spectrum of the light curve is separately calculated using LSP method and WWZ method. The confidence levels of the QPOs obtained by the LSP and WWZ methods are estimated from the power spectrum of the simulated light curves, respectively. For these simulated light curve, the probability density distribution (PDF) of 20,000 power are estimated for each frequency \( f_j \) using a Gaussian kernel. The probability that the power value \( x \) at a single frequency \( f_j \) exceeds observed power value \( p(f_j) \) is: \( P = Pr\{x > p(f_j)\} = \int_{p(f_j)}^{\infty} \tilde{p}(f_j)dx \) \citep{raiteri2021, nilsson2018}. In this study, the significance level curves of 95\% are used. The position of the periodic frequency that are at least above the 95\% significance level curve are used as the positions where reliable QPO frequency occur. Using the frequency, a reliable QPO result is obtained.

\begin{figure}[h]
\centering
\begin{subfigure}{0.3\textwidth}
\centering
\includegraphics[width=\textwidth]{figure2a.png}
\caption{(a) B-band}
\end{subfigure}
\begin{subfigure}{0.3\textwidth}
\centering
\includegraphics[width=\textwidth]{figure2b.png}
\caption{(b) R-band}
\end{subfigure}
\begin{subfigure}{0.3\textwidth}
\centering
\includegraphics[width=\textwidth]{figure2c.png}
\caption{(c) I-band}
\end{subfigure}
\caption{In panel (a), (b) and (c) we follow the procedure of Vaughan to analyze the power spectra of the B, R and I bands, the dashed line is the 95\% confidence curve, and the solid line is the best-fit line, as shown in the figures, the higher frequency is dominated by white noise and the lower part by red noise, and in the red noise section the 0.23(1/d) is over 95\%. This result suggests that there was perhaps a 43-day QPO during the observation period. To obtain \( \alpha \), we fit the lower frequency less than 0.3, these \( \alpha \) are \( \alpha_B = 1.92 \pm 0.22, \alpha_R = 1.97 \pm 0.2, \alpha_I = 1.94 \pm 0.22 \) respectively. The confidence levels of the linear fit is \( 3.9 \times 10^{-10}, 2.67 \times 10^{-11}, 1.8 \times 10^{-10} \) separately.}
\end{figure}

\subsection{Results of LSP and WWZ analysis and significance level estimation}

By the LSP method and the WWZ method, as well as the discussion of the significance curve estimation methods in Section 3.1. The power spectrum of the light curves in Fig. 1 are respectively obtained by the LSP method and the WWZ method, and the significance level curves for these two methods are obtained as 95\%. In Fig. 3, the results of the LSP method analysis and the corresponding significance level curves are shown. This result is located at 0.0229 (1/day), which corresponds to a QPO of 43 days with higher than 95\%. For the results of the LSP analysis we further estimate the corresponding confidence level using the false alarm probability (FAP), using the approximate formula \( P_{FAP} = Ne^{-P_i} \), \( P_{FAP} \) is FAP, \( N \) is the number of data, \( P_i \) is the spectral power \citep{ciaramella2004}. Here we analyze the position of the spectral power above 95\% confidence level. \( P_{B,FAP} = 2.59 \times 10^{-6}, P_{R,FAP} = 1.61 \times 10^{-6}, P_{I,FAP} = 1.4 \times 10^{-6} \) are obtained. In Fig. 4, the results of the WWZ method analysis and the corresponding significance level curves are shown. This result is located at 0.0229 (1/day), which corresponds to a QPO of 43 days with a higher than 95\%. It is obvious that by using three different time series analysis methods, the results are the same. So a 43-day QPO is likely to exist in the optical band of S5 0716+714. This is also the first time that these techniques have been used to find an approximate month-like oscillation QPO in this source.
For the first time, the light curves in Fig. 1 (Gupta et al. 2019, Hong et al. 2017, Ren et al. 2021b) show that the source has a tendency to become consistently brighter over the observation span. We think this is a very interesting phenomenon, and it suggests that this source is likely a BL Lac object with a doubly superimposed QPO. This implies that, in addition to the 43-day QPO, other QPOs with longer time scales may exist (Zhou et al. 2018). To investigate the origin of this 43-day QPO, the spectral index $\alpha_{BRI}$ and the color index $B - R$ of the light curve after removing the trend will be further analyzed. Normally, for non-thermal radiation processes,
the frequency $\nu$ of the observed band and the observed flux $F$ satisfy a power law relationship, and the expression is \[ \log F = -\alpha_{\text{BRI}} \log \nu + C \], and $\alpha_{\text{BRI}}$ is the spectral index, $F_\nu$ is the observed flux of the corresponding band, $C$ is a constant (Xiong et al. 2020). Combining $\nu_B = 6.2 \times 10^{14} \text{Hz}, \nu_R = 4.8 \times 10^{14} \text{Hz}, \nu_I = 3.9 \times 10^{14} \text{Hz}$ (Xiong et al. 2020) and observed flux, the spectral index $\alpha_{\text{BRI}}$ is obtained. For the color index $B - R$, we used the light curve data after removing the trend to obtain.

In general, both thermal and non-thermal components are included in the data from the multi-band observations of blazars. Thus, it is useful to distinguish whether the fluxes obtained in the observed range are dominated by thermal or non-thermal radiation to investigate the origin of QPO. Here we have assumed that if the flux values of the light curves obtained after removing the trend are dominated by non-thermal radiation processes, then based on Pogson formula and \[ \log BRI = \alpha_{\text{BRI}} \log \nu + C \], a linear relationship between the spectral index and the color index is obtained, the corresponding expression is $2.5 \log (\frac{B}{R}) \alpha_{\text{BRI}} \sim B - R$, in which $\nu_B$ and $\nu_R$ represent $B$ and $R$ bands frequency respectively (Boyle et al. 1988; Storrie-Lombardi et al. 2001). According to this relationship, the spectral index $\alpha_{\text{BRI}}$ and the color index $B - R$ should have a good linear relationship if the flux is dominated by non-thermal radiation. Considering that the frequencies in the linear relationship are \[ \nu_B = 6.2 \times 10^{14} \text{Hz}, \nu_R = 4.8 \times 10^{14} \text{Hz} \] respectively (Xiong et al. 2020). Therefore, the slope for the fit of the spectral index $\alpha_{\text{BRI}}$ to the color index $B - R$ should be approximated as 0.27.

![Graph](image)

**Figure 5.** In the top panel of Fig. 5, the results of the linear analysis of the spectral index $\alpha_{\text{BRI}}$ and the color index $B - R$ are shown, from which a good linear relationship between the two can be seen. The slope of the linear fit is 0.25, and the confidence level is $1.4 \times 10^{-6}$. In the bottom panel of Fig. 5, the variation of the spectral index $\alpha_{\text{BRI}}$ is shown, and from this result it can be seen that the spectral index $\alpha_{\text{BRI}}$ is almost always within the gray region $1.21 \pm 0.087$ during the observation period.

In the top panel of Fig. 5, the results for the linear fit of the spectral index $\alpha_{\text{BRI}}$ and the color index $B - R$ are given, and this result shows a good linear relationship between the spectral index $\alpha_{\text{BRI}}$ and the color index $B - R$ with a slope of 0.25. Thus, this result suggests that the light curve during the observation period is likely to be dominated by non-thermal radiation processes. In other words, they come mainly from the jet of blazars. After the preliminary determination of the origin of these fluxes, further analysis is required to determine the origin of this 43-day QPO. Therefore, the variation of the spectral index $\alpha_{\text{BRI}}$ during the observation period will be further analyzed. In the top bottom of Fig. 5, the variation of the spectral index $\alpha_{\text{BRI}}$ is shown. It can be seen that almost all the spectral index $\alpha_{\text{BRI}}$ are within the gray area $1.21 \pm 0.087$, where 1.21 is the mean of the spectral index and 0.087 is the standard deviation. Since the photon index $p$ in SED and the spectral index $\alpha_{\text{BRI}}$ have a linear relationship, the expression is $\alpha_{\text{BRI}} = \frac{p-1}{2} (N(E) \sim E^{-p}, N(E) \text{ display particle distribution at energy } E, p \text{ represents photon index } )$ (Kapanadze 2021). So the photon index $p$ also varies almost constant over the observation span.

The analysis of the results in Fig. 5 shows that a 43-day QPO within the observation span is likely to be generated by geometric motion in the jet. It can be well explained by a periodic spiral motion of a blob with velocity $\beta$ along the jet (Nesci et al. 2005; Zhou et al. 2018; Camenzind & Krockenberger 1992). As illustrated in the right-hand panel of Fig. 6, we consider a geometric scenario in which the viewing angle $\theta$ of a emitting blob motion as a function of
time \( t \) (Sobacchi et al. 2017). The jet axis is the \( z \)-axis, the axis pointing to the observer perpendicular to \( z \) is the \( x \)-axis, and the axis perpendicular to \( z \) and \( x \) is the \( y \)-axis. When the blob is moving with velocity \( \beta \), the velocity \( \beta \) is decomposed on three axes in the following form (Sobacchi et al. 2017):

\[
\vec{\beta} = \beta (\sin(\phi) \cos(\omega t), \sin(\phi) \sin(\omega t), \cos(\phi))
\]  

(5)

Where \( \phi \) is the angle between \( z \)-axis and \( \omega \) is the angular velocity of the spiral motion (\( \omega = \frac{2\pi}{P_{\text{obs}}} \), \( P_{\text{obs}} \) is the QPO) (Sobacchi et al. 2017; Zhou et al. 2018). The unit vector in the direction of the observer’s line of sight is \( \vec{\eta} = (\sin(\psi), 0, \cos(\psi)) \), \( \psi \) is the angle between the \( z \)-axis and the observer’s line of sight. From the above analysis, \( \vec{\beta} \cdot \vec{\eta} = \beta \cos(\theta) \) is obtained, so the following formula can be given (Sobacchi et al. 2017):

\[
\cos(\theta) = \sin(\psi) \sin(\phi) \cos(\omega t) + \cos(\psi) \cos(\phi)
\]

(6)

The relativistic beaming factor is \( \delta = 1/\Gamma(1 - \beta \cos(\theta)) \) (Sobacchi et al. 2017), \( \Gamma = 1/(1 - \beta^2)^{1/2} \) is the bulk Lorentz factor of blob, \( \beta c \) is the speed of motion. For blazars, typical values \( \phi = 2^\circ, \psi = 5^\circ \), and \( \Gamma = 8.5 \) can be taken (Zhou et al. 2018). Under the effect of special relativity, a 43-day QPO is much shorter than the real QPO, so by equation \( P_{\text{obs}} = (1 - \beta \cos(\psi) \cos(\phi)) P \) (Zhou et al. 2018), this 43-day QPO is corrected. The corrected quasi-period is \( P \approx 10.04 \) years. On the \( z \)-direction, the distance that a blob moves in a QPO is approximately \( L \approx 3.11 \) pc. Through the equation \( d_L(z) = \sqrt{\frac{c}{H_0}(1 + z)} \int_0^\infty [\Omega_\Lambda + \Omega_0 (1 + z')^3]^{-1/2} dz' \) (Venters et al. 2009), in which \( z \) is the redshift and \( c \) is the speed of light, the luminosity distance for S5 0716+714 is further analyzed by us. When \( z = 0.31, d_L \) is equal to 1612.65 Mpc, it can be seen that this distance is very large and such a large distance makes it impossible to find the spiral motion through VLBL (Zhou et al. 2018). Although analysis suggests that the 43-day QPO of S5 0716+714 is likely to originate from a spiral motion of a blob along the jet, it still cannot be excluded that it is due to other causes. For example, the 40.04-year QPO is obtained after correction, and the time scale of the corrected QPO is close to the time scale of the orbital motion for the binary black hole, which leads us to believe that it could also be generated by the orbital motion of the binary black hole model (Xie et al. 2002; Ren et al. 2021a; Gupta et al. 2019). In 2019, Gupta et al. (2019) discovered a QPO of about 71 days while studying \( \gamma \)-ray from the blazar B2 1520+31, and they used a fluctuating hot spot from the innermost stable circular orbit of the accretion disk as an explanation for its origin. So, we cannot exclude that this 43-day QPO may have been generated for the same reason. In addition to the discussion on the origin of the 43-day QPO, there is an upward trend in the observation span. Considering the irregular variations of light curves of BL Lac objects, the upward trend caused by a short QPO superimposed on a long QPO can be considered. Assuming that the fluxes of a longer QPO is in a rising phase and the observer happens to view a helical motion of a blob along the jet, then the effect of the continuous brightening trend in Fig. 1 can be seen. One possible phenomenon is that if there is a binary black hole system at this source, then the motion of the binary black hole is perturbing the jets to produce the precession (Lainela et al. 1999), this phenomenon allowed us to observe the long-term QPOs previously reported in the literature. In the left panel of Fig. 6, we assume that the long-term QPO is at least larger than our observation span. And since the entire observation span is in an upward phase, we estimate that the long-term QPO is at least 400 days. We have used the sine function \( \sin(2\pi t/T) \) to simulate a light curve with a long-term QPO, \( T \) is the estimated long-term QPO. Meanwhile, the sine function is used to model the short-term QPO after excluding the uptrend (Butuzova 2021). When the two parts are superimposed together they show an upward trend. From Fig. 6, we can clearly see that the superposition effect of long-term QPO on short-term is almost a linearly upward line over the observation span. Therefore, using a linear fit as the rising trend in Fig. 1 should be a reasonable choice. However, the longer QPO may also come from processes such as accretion disk radiation (Neschi et al. 2005). Does the approximate month-like oscillation QPO of BL Lac S5 0716+714 exist in other bands as well? Does the source also show this trend of superposition effect in other bands? Can the QPO of approximate month-like oscillation QPO find in other blazars be explained by the variational process discussed in this paper? We will pay further attention to all these issues.
In the left side of Fig. 6, the black curve is the light curve after the trend is removed, the blue curve is the long-term light curve generated by the precession of the jet over the observation span, and the red curve represents the total flux, which is the flux of long-term QPO superimposed on the flux of short-term QPO. The blob is represented by a red point in the right panel, and when the blob moves spirally to positions 1, 2, and 3, the fluxes at 1, 2, and 3 of the black curve in the left panel are generated. When the precession of the jet and the periodic motion in the blob are observed simultaneously, the flux at positions 1, 2, and 3 on the red curve are shown. In the left panel, $\phi$ is the pitch angle between the velocity of the blob and the z-axis of the jet, $\psi$ is the angle between the lines of sight of the z-axis of the jet, and $\theta$ is the angle between the velocity of the blob and the lines of sight.

5. CONCLUSIONS

In this work, we study observational data of the optical B,R and I bands of BL Lac S5 0716+714 between MJD 58067 and MJD 58253. We obtained the following main results:

(1) Using the LSP method and the WWZ method, a 43-day QPO ($> 95\%$ confidence level) is obtained for the first time in the optical band of S5 0716+714. This result also confirms the existence of an approximate month-like oscillation QPOs in the optical band of blazars.

(2) Based on the analysis of the spectral index $\alpha_{BR}$ and color index $B-R$ during the observation, we find a good interpretation of the 43-day QPO using a spiral motion of a blob with velocity $\beta$ along the jet. In the jet, the viewing angle of the emitting blob changes periodically, and the 43-day QPO is the result of a spiral motion.

(3) We find QPOs with double superposition characteristics in BL Lac S5 0716+714. In the observational span, the observer has observed exactly two periodically evolving processes at the same time. One of these two processes produced a long-term QPO, while a spiral motion of a blob along the jet produced a 43-day QPO, and when the flux of the long-term QPO is in an upward state, a periodic upward trend may be observed. According to the analysis of the origin of the long-term QPO, we guess that there may be a binary black hole system in the source that causes the jets to precession with a longer QPO.

In the study of the optical band quasi-periodic oscillations of BL Lac S5 0716+714, we find many interesting phenomena, so we will continue to pay attention to this source in our subsequent work.

ACKNOWLEDGEMENT

This work is supported by the National Nature Science Foundation of China (Grant Nos. 11063004) and the Project of Reserve Talents for the Young and Middle-aged Academic and Technical Leaders in Yunnan Province (Grant No. 2017HB020) and the High-Energy Astrophysics Science and Technology Innovation Team of Yunnan Higher School. We sincerely appreciate the data provided by Ph.D Dingrong Xiong et al. at Yunnan Observatory in pre-reported literature. (Xiong et al. 2020). The right panel in Fig. 6 in the paper is referenced from the literature of Zhou et al. (Zhou et al. 2018). And the right panel in Fig. 6 in the paper is referenced from the literature of Sobacchi et al. (Sobacchi et al. 2017). We would like to express our sincere appreciation here.

REFERENCES

Abdo, A. A., Ackermann, M., Agudo, I., et al. 2010, ApJ, 716, 30, doi: 10.1088/0004-637X/716/1/30

Ajello, M., Angioni, R., Axelsson, M., et al. 2020, ApJ, 892, 105, doi: 10.3847/1538-4357/ab791e
