Constraints on long range force from perihelion precession of planets  
in a gauged $L_e - L_{\mu,\tau}$ scenario

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Abstract

The standard model particles can be gauged in an anomaly free way by three possible gauge symmetries  
namely $L_e - L_\mu$, $L_e - L_\tau$, and $L_\mu - L_\tau$. Of these, $L_e - L_\mu$ and $L_e - L_\tau$ forces can mediate between the  
Sun and the planets and change planetary orbits. It is well known that a deviation from the $1/r^2$  
Newtonian force can give rise to a perihelion advancement in the planetary orbit, for instance, as in the  
well known case of Einstein’s gravity which was tested from the observation of the perihelion advancement  
of the Mercury. We consider the Yukawa potential of $L_e - L_{\mu,\tau}$ force which arises between the Sun and  
the planets if the mass of the gauge boson is $M_{Z'} \leq \mathcal{O}(10^{-22})$eV. We derive the formula for the perihelion  
advancement for such Yukawa type fifth force. We find that perihelion advancement is proportional to  
the square of the semi major axis of the orbit for the Yukawa potential, unlike GR, where it is largest for  
the nearest planet. We take the observational limits of all planets for which the perihelion advancement  
is measured and we obtain the gauge boson coupling $g$ in the range $10^{-18}$ to $10^{-16}$ for the mass range  
$10^{-22}$eV to $10^{-18}$eV. This mass range of gauge boson can be a possible candidate of fuzzy dark matter  
whose effect can therefore be observed in the precession measurement of the planetary orbits.

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I. INTRODUCTION

It is well known that deviation from the inverse square law force between the Sun and the planets results in the perihelion precession of the planetary orbits around the Sun. One of the most prominent example is the case of the Einstein’s general relativity (GR) which predicts a deviation from Newtonian $1/r^2$ gravity. In fact, one of the famous classical tests of GR was to explain the perihelion advancement of the Mercury. There was a mismatch of about 43 arc seconds per century from the observation [1] which could not be explained from Newtonian mechanics by considering all non-relativistic effects such as perturbations from the other Solar System bodies, oblateness of the Sun, etc. GR explains the discrepancy with a prediction of contribution of $42.9799''$ Julian century [2]. However there is an uncertainty in the GR prediction which is about $10^{-3}$ arc seconds per century [2–5] for the Mercury orbit. The current most accurate detection of perihelion precession of Mercury is done by MESSENGER mission [3]. In the near future, more accurate results will come from BepiColombo mission [6]. Other planets also experience such perihelion shift, although the shifts are small since they are at larger distance from the Sun. Their orbital time periods are larger and could not be observed accurately [7, 8].

The uncertainty in GR prediction opens up the possibility to explore the existence of Yukawa type potential between the Sun and the planets leading to the fifth force which is a deviation from the inverse-square law. Massless or ultralight scalar, pseudoscalar or vector particles can mediate such fifth force between the Sun and the planets. Many recent papers constrain the fifth force originated from either scalar-tensor theories of gravity [9–11] or the dark matter components [11–13]. Fifth forces due to ultra light axions was studied in [14]. The unparticle long range force from perihelion precession of Mercury was studied in [15]. Perihelion precession of planets can also constrain the fifth force of dark matter [5]. In this paper, we consider the Yukawa type potential which arises in a gauged $L_e - L_{\mu,\tau}$ scenario and we calculate the perihelion shift of planets (Mercury, Venus, Earth, Mars, Jupiter, and Saturn) due to coupling of the ultralight vector gauge bosons with the electron current of the macroscopic objects along with the GR effect.

In standard model, we can construct three gauge symmetries $L_e - L_{\mu}, L_e - L_{\tau}, L_{\mu} - L_{\tau}$ in an anomaly free way and they can be gauged [16–19]. $L_e - L_{\mu}$ and $L_e - L_{\tau}$ [20–23] long range forces can be probed in a neutrino oscillation experiment. $L_{\mu} - L_{\tau}$ long range force cannot be probed in
neutrino oscillation experiment because Earth and Sun do not contain any muon charge. Recently, in [24], $L_\mu - L_\tau$ long range force was probed from the orbital period decay of neutron star-neutron star and neutron star-white dwarf binary systems since they contain large muon charge. However, as the Sun and the planets contain lots of electrons and the number of electrons is approximately equal to the number of baryons, we can probe $L_e - L_{\mu,\tau}$ long range force from the Solar System. The number of electrons in i’th macroscopic object (Sun or planet) is given by $N_i = M_i/m_n$, where $M_i$ is the mass of the i’th object and $m_n$ is the mass of nucleon which is roughly 1GeV. $L_e - L_{\mu,\tau}$ gauge boson is mediated between the classical electron current sources: Sun and planet as shown in FIG.1. This causes a fifth force between the planet and the Sun along with the gravitational force and contributes to the perihelion shift of the planets. The Yukawa type of potential in such a scenario is $V(r) \simeq \frac{g^2}{4\pi r} e^{-M_{Z'} r}$, where $g$ is the constant of coupling between the electron and the gauge boson and $M_{Z'}$ is the mass of the gauge boson. $M_{Z'}$ is restricted by the distance between the Sun and the planet which gives $M_{Z'} < 10^{-19}eV$. Therefore, the lower bound of the range of this force is given by $\lambda = \frac{1}{M_{Z'}} > 10^9Km$. In this mass range the vector gauge boson can also be a candidate for fuzzy dark matter (FDM) [25-27].

The paper is organised as follows. In section II, we give a detail calculation of the perihelion precession of planets due to such fifth force in the background of the Schwarzschild geometry around the Sun. In section III, we obtain constraints on the $L_e - L_{\mu,\tau}$ gauge coupling and the mass of the gauge boson for planets Mercury, Venus, Earth, Mars, Jupiter, and Saturn and we obtain the exclusion plot of $g$ versus $M_{Z'}$ for all the planets mentioned before. In section IV, we summarize our results. We use the natural system of units throughout the paper.

II. PERIHELION PRECESSION OF PLANETS DUE TO LONG RANGE YUKAWA TYPE OF POTENTIAL IN THE CURVED SPACETIME BACKGROUND

The dynamics of a Sun-planet system in presence of a Schwarzschild background and a non gravitational Yukawa type $L_e - L_{\mu,\tau}$ long range force is given by the following action:

$$S = -M_p \int \sqrt{-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} d\tau - g \int A_\mu J^\mu d\tau,$$  \hspace{1cm} (2.1)
FIG. 1: Mediation of $L_e - L_{\mu,\tau}$ vector gauge bosons between the planet and the Sun.

where . denotes the derivative with respect to the proper time $\tau$, $M_p$ is the mass of the planet, $g$ is the coupling constant which couples the classical current $J^\mu = q\dot{x}^\mu$ of the planet with the $L_e - L_{\mu,\tau}$ gauge field $A_\mu$ and $q$ is the charge due to the presence of electrons in the planet. Varying the action Eq. (2.1) we obtain the equation of motion of the planet as

$$\ddot{x}^\alpha + \Gamma^\alpha_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = \frac{gq}{M_p} g^\alpha\mu (\partial_\mu A_\nu - \partial_\nu A_\mu) \dot{x}^\nu. \quad (2.2)$$

In Appendix A, we show the detailed calculation of Eq. (2.2). For the static case $A_\mu = \{V(r), 0, 0, 0\}$, where $V(r)$ is the potential leading to long range $L_e - L_{\mu,\tau}$ Yukawa type of force. $\Gamma^\alpha_{\mu\nu}$ denotes the Christoffel symbol for the background spacetime. For the Sun-Planet system, the background is a Schwarzschild spacetime around the Sun. The christoffel symbols for the metric Eq. (2.3) are given in Appendix B. The Schwarzschild metric outside the Sun is given by

$$ds^2 = -(1 - \frac{2M}{r}) dt^2 + (1 - \frac{2M}{r})^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2, \quad (2.3)$$

where $M$ is the mass of Sun. Here we assume the Newton’s gravitational constant $G = 1$.

Hence, to obtain the solution for temporal part of the Eqs. (2.2), we write

$$\ddot{t} + \frac{2M}{r^2} \left(1 - \frac{2M}{r}\right) \dot{r}^2 = \frac{gq}{M_p} \frac{dV}{dr}.$$

$$\quad (2.4)$$
Integrating Eq. (2.4) once we get
\[ \dot{t} = \frac{E + \frac{gqV}{M_p}}{1 - \frac{2M}{r}}, \]  
(2.5)
where \( E \) is the constant of motion. \( E \) is identified as the total energy of the system per unit mass.

Similarly, the \( \phi \) part of Eq. (2.2) is
\[ \ddot{\phi} + \frac{2}{r} \dot{r} \dot{\phi} = 0, \]  
(2.6)
After integration we get
\[ \dot{\phi} = \frac{L}{r^2}. \]  
(2.7)
where \( L \) is the angular momentum of the system which is a constant of motion.

The radial part of Eq. (2.2) is
\[ \ddot{r} - \frac{M \dot{r}^2}{r^2 (1 - \frac{2M}{r})} + \frac{M}{r^2} \left( 1 - \frac{2M}{r} \right) \dot{r}^2 - r \left( 1 - \frac{2M}{r} \right) \dot{\phi}^2 = \frac{gq}{M_p} \left( 1 - \frac{2M}{r} \right) \frac{dV}{dr} \dot{r}. \]  
(2.8)
Using Eq. (2.5), and Eq. (2.7) in Eq. (2.8) we obtain
\[ \ddot{r} + \frac{M}{r^2 (1 - \frac{2M}{r})} \left( \left( E + \frac{gqV}{M_p} \right)^2 - \dot{r}^2 \right) - \frac{L^2}{r^3} \left( 1 - \frac{2M}{r} \right) = \frac{gq}{M_p} \left( E + \frac{gqV}{M_p} \right) \frac{dV}{dr}. \]  
(2.9)
Again, for a timelike particle \( g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = -1 \) and this gives
\[ \frac{\left( E + \frac{gqV}{M_p} \right)^2 - 1}{2} = \frac{\dot{r}^2}{2} + \frac{L^2}{2r^2} - \frac{ML^2}{r^3} - \frac{M}{r}. \]  
(2.10)
Using Eq. (2.10) in Eq. (2.9), we get
\[ \ddot{r} + \frac{3ML^2}{r^4} + \frac{M}{r^2} - \frac{L^2}{r^3} = \frac{gq}{M_p} \left( E + \frac{gqV}{M_p} \right) \frac{dV}{dr}. \]  
(2.11)
We can also obtain Eq. (2.11) by directly differentiate Eq. (2.10).

The potential \( V(r) \) is generated due to the presence of electrons in the Sun and it is given as
\[ V(r) \simeq -\frac{gQ}{4\pi r} e^{-M_2 r} + O(M). \]  
Note that we keep only the Yukawa term in the form of \( V(r) \) as we are interested in the leading order contribution only (see Appendix C). Hence, from Eq. (2.10) we write
\[ \frac{E^2 - 1}{2} = \frac{\dot{r}^2}{2} + \frac{L^2}{2r^2} - \frac{ML^2}{r^3} - \frac{M}{r} + \frac{g^2 N_1 N_2 E}{4\pi M_p r} e^{-M_2 r}, \]  
(2.12)
Where we have neglected $g^4$ term because the coupling is small and its contribution will be negligible. Here $Q = N_1$ is the number of electrons in the Sun and $q = N_2$ is the number of electrons in the planet. For planar motion, $L_x = L_y = 0$, and $\theta = \pi/2$. Since the orbit of the planet is stable, the total energy of the system is negative. So $E = -|E|$. Hence, we write Eq. (2.12) as

$$\frac{E^2 - 1}{2} = \frac{\dot{r}^2}{2} + \frac{L^2}{2r^2} - \frac{ML^2}{r^3} - \frac{M}{r} - \frac{g^2N_1N_2|E|}{4\pi M_p r}e^{-M_{2r}r}. \quad (2.13)$$

The first term on the right hand side of Eq. (2.13) represents the kinetic energy part, the second term is the centrifugal potential part, and the fourth term is the usual Newtonian potential. Due to general relativistic $\frac{ML^2}{r^3}$ term, there is an advancement of perihelion motion of a planet. The last term arises due to exchange of a $U(1)_{\mu-\nu,7}$ gauge bosons between electrons of a planet and the Sun. Here, $M_{2r}$ is the mass of the gauge boson.

$M_{2r}$ is constrained from the range of the potential which is basically the distance between the planet and the Sun. Using $\dot{r} = L \frac{d}{d\phi}$, we write Eq. (2.13) as

$$\frac{d^2u}{d\phi^2} + u = \frac{M}{L^2} + 3Mu^2 + \frac{g^2N_1N_2|E|}{4\pi L^2 M_p}e^{-\frac{M_{2r}}{u}} + \frac{g^2N_1N_2|E|M_{2r}'}{4\pi L^2 M_p u}e^{-\frac{M_{2r}'}{u}}. \quad (2.14)$$

Applying $\frac{d}{d\phi}$ on both sides and using the reciprocal coordinate $u = \frac{1}{r}$ we obtain from Eq. (2.14)

$$\frac{d^2u}{d\phi^2} + u = \frac{M}{L^2} + 3Mu^2 + \frac{g^2N_1N_2|E|}{4\pi L^2 M_p}e^{-\frac{M_{2r}}{u}} + \frac{g^2N_1N_2|E|M_{2r}'}{8\pi L^2 M_p u^2}e^{-\frac{M_{2r}'}{u}}. \quad (2.15)$$

Now expanding Eq. (2.15) up to the leading order of $M_{2r}'$ we get

$$\frac{d^2u}{d\phi^2} + u = \frac{M}{L^2} + 3Mu^2 + \frac{g^2N_1N_2|E|}{4\pi L^2 M_p}e^{-\frac{M_{2r}}{u}} - \frac{g^2N_1N_2|E|M_{2r}'}{8\pi L^2 M_p u^2}. \quad (2.16)$$

where for non circular orbit $\frac{d}{d\phi}\left(\frac{1}{r}\right) \neq 0$. The first term on the right hand side of Eq. (2.16) is the usual term which comes in Newton’s theory. The second term is the general relativistic term which is a perturbation of Newton’s theory. The last two terms arise due to the presence of long range Yukawa type potential in the theory.

We write Eq. (2.16) as

$$\frac{d^2u}{d\phi^2} + u = \frac{M'}{L^2} + 3Mu^2 - \frac{g^2N_1N_2|E|M_{2r}'}{8\pi L^2 M_p u^2}, \quad (2.17)$$

where $M' = M + g^2N_1N_2|E|/4\pi M_p$. 

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We assume that \( u = u_0(\phi) + \Delta u(\phi) \), where, \( u_0(\phi) \) is the solution of Newton’s theory with the effective mass \( M' \) and \( \Delta u(\phi) \) is the solution due to general relativistic correction and Yukawa potential. Thus we write

\[
\frac{d^2 u_0}{d\phi^2} + u_0 = \frac{M'}{L^2}.
\]

The solution of Eq. (2.18) is

\[
u_0 = \frac{M'}{L^2}(1 + e \cos \phi),
\]

where \( e \) is the eccentricity of the planetary orbit. The equation of motion for \( \Delta u(\phi) \) is

\[
\frac{d^2 \Delta u}{d\phi^2} + \Delta u = \frac{3MM'^2}{L^4}(1 + e^2 \cos^2 \phi + 2e \cos \phi) - \frac{g^2 N_1 N_2 |E| M_2^2 L^4}{8\pi L^2 M_p M'^2 (1 + e^2 \cos^2 \phi + 2e \cos \phi)}. \tag{2.20}
\]

The solution of Eq. (2.20) is

\[
\Delta u = \frac{3MM'^2}{L^4} \left[1 + \frac{e^2}{2} - \frac{e^2}{6} \cos 2\phi + e \phi \sin \phi \right] - \frac{g^2 N_1 N_2 |E| M_2^2 L^4}{8\pi L^2 M_p M'^2} \left[ - \frac{\cos \phi}{e(1 + e \cos \phi)} + \frac{\sin^2 \phi}{(1 - e^2)(1 + e \cos \phi)} - \frac{e}{(1 - e^2)^{3/2}} \sin \phi \cos^{-1} \left( \frac{e + \cos \phi}{1 + e \cos \phi} \right) \right]. \tag{2.21}
\]

The \( \Delta u \) increases linearly with \( \phi \) and contributes to the perihelion precession of planets. Therefore we identify only the related terms in Eq. (2.21) and rewrite \( \Delta u \) as

\[
\Delta u = \frac{3MM'^2}{L^4} e \phi \sin \phi + \frac{g^2 N_1 N_2 |E| M_2^2 L^4}{8\pi M_p M'^2} e \frac{\phi \sin \phi}{(1 - e^2)(1 + e)} \tag{2.22}
\]

Using Eq. (2.19) and Eq. (2.22) we get the total solution as

\[
u = \frac{M'}{L^2}(1 + e \cos \phi) + \frac{3MM'^2}{L^4} e \phi \sin \phi + \frac{g^2 N_1 N_2 |E| M_2^2 L^4}{8\pi M_p M'^2} e \frac{\phi \sin \phi}{(1 - e^2)(1 + e)} \tag{2.23}
\]

or,

\[
u = \frac{M'}{L^2}[1 + e \cos \phi(1 - \alpha)], \tag{2.24}
\]

where \( \alpha = \frac{3MM'}{L^4} + \frac{g^2 N_1 N_2 |E| M_2^2 L^4}{8\pi M_p M'^2} \frac{1}{(1 - e^2)(1 + e)}. \)

Under \( \phi \to \phi + 2\pi \), \( u \) is not same. Hence, the planet does not follow the previous orbit. So the motion of the planet is not periodic. The change in azimuthal angle after one precession is

\[
\Delta \phi = \frac{2\pi}{1 - \alpha} - 2\pi \approx 2\pi \alpha, \tag{2.25}
\]
The semi major axis and the orbital angular momentum are related by \( a = \frac{L^2}{M'(1-e^2)} \). Using this expression in Eq. (2.25) we get

\[
\Delta \phi = \frac{6\pi M}{a(1-e^2)} + \frac{g^2 N_1 N_2 |E|M_2^2 a^2(1-e^2)}{4m_p M'(1+e)}.
\] (2.26)

In natural system of units Eq. (2.26) is

\[
\Delta \phi = \frac{6\pi GM}{a(1-e^2)} + \frac{g^2 N_1 N_2 |E|M_2^2 a^2(1-e^2)}{4M_p (GM + \frac{g^2 N_1 N_2 |E|}{4\pi M_p})(1+e)}.
\] (2.27)

The total energy of the system per unit mass is

\[
E = -\frac{GM}{2a} - \frac{g^2 N_1 N_2 |E|}{8\pi a M_p} e^{-M_\phi a}.
\] (2.28)

In Appendix D, we discuss Eq. (2.28) in detail. The energy due to gravity is much larger than the energy due to long range Yukawa type force. The last term of Eq. (2.27) indicates that long range force, which arises due to \( U(1)_{L_e-L_{\mu,\tau}} \) gauge boson exchange between the electrons of composite objects, contributes to the perihelion advance of planets within the permissible limit.

III. CONSTRAINTS ON \( U(1)_{L_e-L_{\mu,\tau}} \) GAUGE COUPLING FOR PLANETS IN SOLAR SYSTEM

The contribution of the gauge boson must be within the excess of perihelion advance from the GR prediction, i.e. \((\Delta \phi)_{\text{obs}} - (\Delta \phi)_{\text{GR}} \geq (\Delta \phi)_{L_e-L_{\mu,\tau}}\). The first term in the right hand side of Eq. (2.27) is \((\Delta \phi)_{\text{GR}}\) and the second term is \((\Delta \phi)_{L_e-L_{\mu,\tau}}\). Putting the observed and GR values for \((\Delta \phi)\), we can constrain the \( U(1)_{L_e-L_{\mu,\tau}} \) gauge coupling constants for all the planets in our Solar System. For Mercury planet, we write

\[
\frac{g^2 N_1 N_2 |E|M_2^2 a^2(1-e^2)}{4M_p (GM + \frac{g^2 N_1 N_2 |E|}{4\pi M_p})(1+e)} \left( \frac{\text{century}}{T} \right) < 3.0 \times 10^{-3} \text{arcsecond/century},
\] (3.1)

where \(3 \times 10^{-3} \text{ arcsecond/century}\) is the uncertainty in the perihelion advancement from its GR prediction and put upper bound on the gauge coupling. \( T = 88 \) days is the orbital time period of Mercury. Similarly we can put upper bounds on \( g \) for the other planets. In this section, we constrain the \( U(1)_{L_e-L_{\mu,\tau}} \) gauge coupling from the observed perihelion advancement of the planets.
in the Solar System. We consider six planets: Mercury, Venus, Earth, Mars, Jupiter, and Saturn. The number of electrons in the Sun and the planets are same as the number of baryons. For ith object of mass $M_i$, the number of electrons is $N_i = M_i/m_n$, where $m_n$ is the mass of the nucleon. Here we take the mass of the Sun as $M = 10^{57}$ GeV. Using Eqs. (2.27), we put an upper bound on $g$ from the uncertainty of their perihelion advance. In TABLE I, we obtain the masses of the gauge bosons which are mediated between the Sun and the planets and in TABLE II, we show the constraints on the gauge coupling constants from the uncertainties [28, 29] of perihelion advance.

TABLE I: Summary of the masses, eccentricities [30] of the orbits, perihelion distances from the Sun and upper bounds on gauge boson mass $M_{Z'}$ which are mediated between the planets and Sun in our solar system.

| Planet   | Mass $M_p$(GeV) | Eccentricity (e) | Perihelion distance a (AU) | Mass of gauge boson $M_{Z'}$(eV) |
|----------|-----------------|------------------|-----------------------------|----------------------------------|
| Mercury  | $1.64 \times 10^{50}$ | 0.206            | 0.31                        | $\leq 4.31 \times 10^{-18}$      |
| Venus    | $2.43 \times 10^{51}$ | 0.007            | 0.72                        | $\leq 1.85 \times 10^{-18}$      |
| Earth    | $2.98 \times 10^{51}$ | 0.017            | 0.98                        | $\leq 1.36 \times 10^{-18}$      |
| Mars     | $3.19 \times 10^{50}$ | 0.093            | 1.38                        | $\leq 9.68 \times 10^{-19}$      |
| Jupiter  | $9.49 \times 10^{53}$ | 0.048            | 4.95                        | $\leq 2.69 \times 10^{-19}$      |
| Saturn   | $2.84 \times 10^{53}$ | 0.056            | 9.02                        | $\leq 1.48 \times 10^{-19}$      |

TABLE II: Summary of the uncertainties in the perihelion advance in arcseconds per century and upper bounds on gauge boson-electron coupling $g$ for planets in our solar system.

| Planet   | Uncertainty in perihelion advance (as/cy) | $g$ from perihelion advance |
|----------|------------------------------------------|----------------------------|
| Mercury  | $3.0 \times 10^{-3}$                     | $\leq 1.39 \times 10^{-18}$ |
| Venus    | $1.6 \times 10^{-3}$                     | $\leq 1.78 \times 10^{-18}$ |
| Earth    | $1.9 \times 10^{-4}$                     | $\leq 1.25 \times 10^{-18}$ |
| Mars     | $3.7 \times 10^{-5}$                     | $\leq 1.06 \times 10^{-18}$ |
| Jupiter  | $2.8 \times 10^{-2}$                     | $\leq 1.27 \times 10^{-17}$ |
| Saturn   | $4.7 \times 10^{-4}$                     | $\leq 6.65 \times 10^{-18}$ |
We can write from the fifth force constraint

\[ \frac{g^2 N_1 N_2}{4\pi G M m_p} < 1. \]  

(3.2)

This gives the upper bound on \( g \) as \( g < 3.54 \times 10^{-18} \) for all the planets. For \( U(1)_{L_e-L_{\mu,r}} \) vector gauge bosons mediation between the planet and the Sun, the mass of the gauge boson is \( M_{Z'} \leq \mathcal{O}(10^{-19})eV \). In the following we show the exclusion plots of gauge boson electron coupling for the six planets. The regions right to the coloured lines corresponding to every planet in FIG.2.

![FIG. 2: Plot of coupling constant \( g \) vs the mass of the gauge bosons \( M_{Z'} \) for all the planets. Violet line is for Jupiter planet, blue line is for Mercury, black is for Venus, cyan is for Saturn, green is for Earth and yellow is for Mars.](image)

are excluded. The upper bound of the gauge boson coupling is \( g \leq \mathcal{O}(10^{-18}) \). From FIG.2 it is clear that the Mars gives the strongest bound among all the planets. As we go to the lower mass region, the Yukawa potential effectively becomes Coulomb potential and will not contribute to the perihelion precession of planets. So as we go to the lower mass (<10^{-19}eV) region, we get weaker bound on \( g \). As we go to the higher mass region, the long range force theory breaks down.
IV. DISCUSSIONS

Since the Sun and the planets contain a significant number of electrons, we can put upper bounds on the gauge coupling and the mass of the gauge bosons in a gauged $L_e - L_{\mu,\tau}$ scenario which are mediated between the Sun and planet. This ultralight vector gauge bosons can contribute to the perihelion shift in addition to the GR prediction. From the perihelion shift calculation in presence of a long range Yukawa type potential, we obtain an upper bound on the gauge coupling $g \leq O(10^{-18})$ in a gauged $L_e - L_{\mu,\tau}$ scenario. The mass of the gauge bosons is constrained by the distance between the Sun and the planet which gives $M_{Z'} \leq 10^{-19}\text{eV}$. The electron-gauge boson coupling obtained from perihelion shift measurement is one order of magnitude less stringent than as obtained from the fifth force constraint. From Eqs. (2.27) we conclude that while the precession of perihelion due to GR is largely contributed by the planets close to Sun, the contribution of vector gauge bosons in perihelion precession is dominated by the outer planets. The gauge coupling in a gauged $L_e - L_{\mu,\tau}$ scenario obtained from the perihelion precession of planets is eight order of magnitude less stringent than as obtained from neutrino oscillation experiment. The future BepiColombo mission can give more accurate results for the perihelion precession of planets and then the bounds on coupling constants will become even more stronger. These gauge bosons ($M_{Z'} \leq 10^{-19}\text{eV}$) can be a possible candidate of Fuzzy dark matter and can be probed from precession measurement of planetary orbits.

Appendix A: Equation of motion of a planet in presence of a Schwarzschild background and a non gravitational Yukawa type of potential

The action which describes the motion of a planet in Schwarzschild background and a non gravitational long range Yukawa type of potential is given by Eq. (2.1).

Suppose $S_1 = M_p \int \sqrt{-g_{\mu\nu} x^\mu x^\nu} d\tau$. For this action, the Lagrangian is

$$L = M_p \sqrt{g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}}. \quad (A1)$$

Hence, the equation of motion is

$$\frac{d}{d\tau} \left( \frac{\partial L}{\partial \left( \frac{dx^\nu}{d\tau} \right)} \right) - \frac{\partial L}{\partial x^\sigma} = 0, \quad (A2)$$
or,
\[
\frac{1}{L} \frac{dL}{d\tau} g_{\mu\sigma} \frac{dx^\mu}{d\tau} = g_{\mu\sigma} \frac{d^2x^\mu}{d\tau^2} + \partial_\alpha g_{\mu\sigma} \frac{dx^\alpha}{d\tau} \frac{dx^\mu}{d\tau} - \frac{1}{2} \partial_{\sigma} g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}.
\] (A3)

Multiplying \(g^{\rho\sigma}\) we have,
\[
\frac{d^2x^\rho}{d\tau^2} + g^{\rho\sigma} \partial_\nu g_{\mu\sigma} \frac{dx^\nu}{d\tau} \frac{dx^\mu}{d\tau} - g^{\rho\sigma} \frac{1}{2} \partial_{\sigma} g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = \frac{1}{L} \frac{dL}{d\tau} \frac{dx^\rho}{d\tau}.
\] (A4)

or,
\[
\frac{d^2x^\rho}{d\tau^2} + \frac{1}{2} g^{\rho\sigma}(\partial_\nu g_{\mu\sigma} + \partial_\mu g_{\nu\sigma} - \partial_\sigma g_{\mu\nu}) \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} \frac{dx^\sigma}{d\tau} = \frac{1}{L} \left( \frac{dL}{d\tau} \right) \frac{dx^\rho}{d\tau},
\] (A5)

where, \(\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma}(\partial_\nu g_{\mu\sigma} + \partial_\mu g_{\nu\sigma} - \partial_\sigma g_{\mu\nu})\) is called the Christoffel symbol. We can choose \(\tau\) in such a way that \(\frac{dL}{d\tau} = 0\). This is called affine parametrization. So,
\[
\frac{d^2x^\rho}{d\tau^2} + \Gamma^\rho_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0.
\] (A6)

Suppose \(S_2 = gq \int A_\mu \frac{dx^\mu}{d\tau} d\tau = gq \int A_\mu dx^\mu\). Hence,
\[
\delta S_2 = gq \int \delta A_\mu dx^\mu + gq \int A_\mu \delta(dx^\mu),
\] (A8)

or,
\[
\delta S_2 = gq \int \frac{\partial A_\mu}{\partial x^\nu} \delta x^\nu dx^\mu + gq \int A_\mu d(\delta x^\mu).
\] (A9)

Using equation by parts and using the fact that the total derivative term will not contribute to the integration, we can write
\[
\delta S_2 = gq \int \frac{\partial A_\mu}{\partial x^\nu} \delta x^\nu dx^\mu - gq \int dA_\mu \delta x^\mu.
\] (A10)

or,
\[
\delta S_2 = gq \int \frac{\partial A_\mu}{\partial x^\nu} \delta x^\nu dx^\mu - gq \int \frac{\partial A_\mu}{\partial x^\nu} dx^\nu \delta x^\mu.
\] (A11)

Since \(\mu\) and \(\nu\) are dummy indices, we interchange \(\mu\) and \(\nu\) in the first term. Hence, we can write
\[
\delta S_2 = gq \int (\partial_\mu A_\nu - \partial_\nu A_\mu) dx^\mu \delta x^\mu = gq \int (\partial_\mu A_\nu - \partial_\nu A_\mu) \frac{dx^\nu}{d\tau} \delta x^\mu d\tau.
\] (A12)

Imposing the fact \(\delta S_1 + \delta S_2 = 0\) and using Eq. (A4), Eq. (A7) and Eq. (A12) we can write
\[
\ddot{x}^\rho + \Gamma^\rho_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = \frac{gq}{M_p} g^{\rho\mu}(\partial_\nu A_\mu - \partial_\mu A_\nu) \dot{x}^\nu,
\] (A13)

which matches with Eq. (2.2).
Appendix B: Christoffel symbols for the Schwarzschild metric

The Christoffel symbols for the Schwarzschild metric defined in Eq. (2.3) are
\[
\Gamma^t_{rt} = \frac{M}{r^2(1 - \frac{2M}{r})}, \quad \Gamma^r_{tt} = \frac{M}{r^2}
(1 - \frac{2M}{r}), \quad \Gamma^r_{rr} = -\frac{M}{r^2(1 - \frac{2M}{r})}, \quad \Gamma^r_{\theta\theta} = -r(1 - \frac{2M}{r})
\]
\[
\Gamma^r_{\phi\phi} = -r \sin^2 \theta \left(1 - \frac{2M}{r}\right), \quad \Gamma^\theta_{r\theta} = \frac{1}{r}, \quad \Gamma^\theta_{\phi\phi} = -\sin \theta \cos \theta, \quad \Gamma^\phi_{r\phi} = \frac{1}{r}, \quad \Gamma^\phi_{\theta\phi} = \cot \theta
\]  

(B1)

Appendix C: Equation of motion for the vector field \( A_\mu \)

The vector field \( A_\mu \) satisfies the Klein-Gordon equation
\[
\Box A_\mu = M^2 Z A_\mu.
\]  

(C1)

Now, for the static case, \( A_\mu = \{V(r), 0, 0, 0\} \). Hence,
\[
\Box V(r) = M^2 V(r).
\]  

(C2)

In the background of the Schwarzschild spacetime, Eq. (C2) becomes
\[
(1 - \frac{2M}{r}) \frac{d^2 V}{dr^2} + \frac{2}{r} \left(1 - \frac{M}{r}\right) \frac{dV}{dr} = M^2 Z V(r).
\]  

(C3)

So, in the Schwarzschild background, \( V(r) \) will not satisfy the Klein-Gordon equation. So we expand \( V(r) \) in a perturbation series where the perturbation parameter is the Schwarzschild mass \( M \) (here \( M \) is the mass of the Sun) and the leading order term is the Yukawa term. Let,
\[
V(r) = V_0(r) + MV_1(r) + \mathcal{O}(M^2),
\]  

(C4)

where
\[
V_0(r) = c e^{-\frac{M}{r}}, \quad c = \frac{g^2 N_1 N_2}{4\pi},
\]  

(C5)

such that
\[
\frac{d^2 V_0}{dr^2} + \frac{2}{r} \frac{dV_0}{dr} = M^2 Z V_0.
\]  

(C6)

Inserting Eq. (C4) in Eq. (C3), we get the equation for \( V_1(r) \)
\[
\frac{d^2 V_1}{dr^2} + \frac{2}{r} \frac{dV_1}{dr} = M^2 Z V_1 + \frac{2}{r^2} \frac{dV_0}{dr} + 2 \frac{dV_0}{dr}.
\]  

(C7)
Let,
\[ V_1(r) = \chi(r) \frac{e^{-M_Z r}}{r}. \]  \hfill (C8)

Now, Eq. (C7) becomes
\[ \frac{d^2 \chi}{dr^2} - 2M'_Z \frac{d\chi}{dr} = 2c \left( \frac{M^2_Z}{r} + \frac{1}{r^3} + \frac{M'_Z}{r^2} \right). \]  \hfill (C9)

Integrating Eq. (C9) once we get
\[ \frac{d\chi}{dr} - 2M'_Z \chi = 2c \left[ M^2_Z \ln(M'_Z r) - \frac{1}{2r^2} - \frac{M'_Z}{r} \right] + k_1, \]  \hfill (C10)

where \( k_1 \) is the integration constant. Eq. (C10) can be written as
\[ \frac{d}{dr} \left( e^{-2M'_Z r} \chi \right) = 2ce^{-2M'_Z r} \left[ M^2_Z \ln(M'_Z r) - \frac{1}{2r^2} - \frac{M'_Z}{r} \right] + k_1 e^{-2M'_Z r} \]  \hfill (C11)

From Eq. (C11), we can write
\[ e^{-2M'_Z r} \chi(r) = 2c \left[ M^2_Z \int_r^\infty e^{-M'_Z x} \ln(M'_Z x) dx - \int_\infty^r e^{-2M'_Z x} \frac{dx}{2x^2} - \int_\infty^r \frac{M'_Z e^{-2M'_Z x}}{x} dx \right] - \frac{k_1}{2M'_Z} e^{-2M'_Z r} + k_2, \]  \hfill (C12)

where \( k_2 \) is an integration constant. Doing integration by parts, Eq. (C12) becomes
\[ \chi(r) = c \left[ -M'_Z \ln(M'_Z r) + \frac{1}{r} + M'_Z e^{2M'_Z r} E_i(-2M'_Z r) \right] - \frac{k_1}{2M'_Z} + k_2 e^{2M'_Z r}, \]  \hfill (C13)

where \( E_i(x) \) is a special function called the exponential integral function which is defined as
\[ E_i(x) = -\int_{-x}^\infty \frac{e^{-t}}{t} dt. \]  \hfill (C14)

We chose \( k_2 = 0 \) as \( e^{2M'_Z r} \) diverges. We also chose \( k_1 = 0 \) as we are looking for particular integral. Hence, from Eq. (C13) we get
\[ V_1(r) = \frac{ce^{-M'_Z r}}{r} \left[ \frac{1}{r} - M'_Z \ln(M'_Z r) + M'_Z e^{2M'_Z r} E_i(-2M'_Z r) \right]. \]  \hfill (C15)

So the total solution of the potential is
\[ V(r) = \frac{ce^{-M'_Z r}}{r} \left[ 1 + \frac{M}{r} \left\{ 1 - M'_Z r \ln(M'_Z r) + M'_Z r e^{2M'_Z r} E_i(-2M'_Z r) \right\} \right] + O(M^2). \]  \hfill (C16)

We take the leading order term which is the Yukawa term in our calculation. The higher order terms are comparatively small.
Appendix D: Total energy of the binary system due to gravity and long range Yukawa type potential

The total energy of the binary system is

\[ E = T + V. \] (D1)

From virial theorem, \( < T > = -\frac{1}{2} < V > \). For gravity, \( < V > = -\frac{GM_pM}{2a} \) and for long range Yukawa force \( < V > = -\frac{g^2Qq}{4\pi a} e^{-M_pa} \). So the total energy per unit mass of the system is

\[ E = -\frac{GM}{2a} - \frac{g^2Qq}{8\pi a M_p} e^{-M_pa}, \] (D2)

which matches with Eq. (2.28).

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