Evolutionary snowdrift game with loners

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The effects of an additional strategy or character called loner in the snowdrift game are studied in a well-mixed population or fully-connected network and in a square lattice. The snowdrift game, which is a possible alternative to the prisoner’s dilemma game in studying cooperative phenomena in competing populations, consists of two types of strategies, C (cooperators) and D (defectors). In a fully-connected network, it is found that either C lives with D or the loners take over the whole population. In a square lattice, three possible situations are found: a uniform C-population, C lives with D, and the coexistence of all three characters. The presence of loners is found to enhance cooperation in a square lattice by enhancing the payoff of cooperators. The results are discussed in terms of the effects in restricting a player to compete only with his nearest neighbors in a square lattice, as opposed to competing with all players in a fully-connected network.

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I. INTRODUCTION

The evolutionary prisoner’s dilemma game (PDG) [1, 2, 3] and the snowdrift game (SG) [4] have become standard paradigms for studying the possible emergence of cooperative phenomena in a competitive setting. Physicists find such emergent phenomena fascinating, as similar cooperative effects are also found in interacting systems in physics that can be described by some minimal models, e.g. models of interacting spin systems. These games are also essential in the understanding of coexistence of (and competition between) egoistic and altruistic behavior that appear in many complex systems in biology, sociology and economics. The basic PDG [4, 5, 6] consists of two players deciding simultaneously whether to cooperate (C) or to defect (D). If one plays C and the other plays D, the cooperator pays a cost of $S = -c$ while the defector receives the highest payoff $T = b$ ($b > c > 0$). If both play C, each player receives a payoff of $R = b - c > 0$. If both play D, the payoff is $P = 0$. Thus, the PDG is characterized by the ordering of the four payoffs $T > R > P > S$, with $2R > T + S$. In a single round of the game, it is obvious that defection is a better action in a fully connected (well-fixed) population, regardless of the opponents’ decisions. Modifications on the basic PDG are, therefore, proposed in order to induce cooperations and to explain the wide-spread cooperative behavior observed in the real world. These modifications include, for example, the iterated PDG [1, 2], spatially extended PDG [7, 8, 9, 10] and games with a third strategy [11, 12, 13, 14].

The snowdrift game (SG), which is equivalent to the hawk-dove or chicken game [4, 15], is a model somewhat favorable for cooperation. It is best introduced using the following scenario [16]. Consider two drivers hurrying home in opposite directions on a road blocked by a snowdrift. Each driver has two possible actions – to shovel the snowdrift (cooperate (C)) or not to do anything (not-to-cooperate or “defect” (D)). If they cooperate, they could be back home earlier and each will get a reward of $b'$. Shovelling is a laborious job with a total cost of $c'$. Thus, each driver gets a net reward of $R = b' - c'/2$. If both drivers take action D, they both get stuck, and each gets a reward of $P = 0$. If only one driver takes action C and shovels the snowdrift, then both drivers can also go home. The driver taking action D (not to shovel) gets home without doing anything and hence gets a payoff $T = b'$, while the driver taking action C gets a “sucker” payoff of $S = b' - c'$. The SG refers to the case when $b' > c' > 0$, leading to the ranking of the payoffs $T > R > S > P$. This ordering of the payoffs defines the SG. Therefore, both the PDG and SG are defined by a payoff matrix of the form

\[
\begin{pmatrix}
C & D \\
R & S \end{pmatrix},
\]

and they differ only in the ordering of $P$ and $S$. It is this difference that makes cooperators persist more easily in the SG than in the PDG. In a well-mixed population, cooperators and detectors coexist. Due to the difficulty in measuring payoffs and the ordering of the payoffs accurately in real world situations where game theory is applicable [17, 18], the SD has been taken to be a possible alternative to the PDG in studying emerging cooperative phenomena [10].

The present work will focus on two aspects of current interest. In many circumstances, the connections...
in a competing population are better modelled by some networks providing limited interactions than a fully-connected network. Previous studies showed that different spatial structures might lead to different behaviors \cite{5, 19, 21}. For example, it has been demonstrated that spatial structures would promote cooperation in the PDG \cite{5}, but would suppress cooperation in the SG \cite{10}. There are other variations on the SG that resulted in improved cooperation \cite{22, 23}. Here, we explore the effects of an underlying network on the evolutionary SG in a population in which there exists an additional type of players. The latter is related to the fact that real-world systems usually consist of people who would adopt a strategy other than just C and D. For example, there may be people who do not like to participate in the competition and would rather take a small but fixed payoff. Hauert et al. studied the effects of the presence of such persons, called loners \cite{11,12}, in a generalization of the PDG called the public goods game (PGG). Motivated by these works of Hauert et al. \cite{11,12,16}, we study the effects of risk-averse loners in the evolutionary SG. In our model, evolution or adaptation is built in by allowing players to replace their character or strategy by that of a better-performing connected neighbor. We focus on both the steady state and the dynamics, and study how an underlying network structure affects the emergence of cooperation. It is found that in a fully-connected network, the C-players and D-players cannot coexist with the loners. In a square lattice, however, cooperators are easier to survive. Depending on the payoffs, there are situations in which C-players, D-players and loners can coexist.

In Sec. II the evolutionary SG with loners in a population with connections is presented. In Sec. III, we present detailed numerical results in fully-connected networks and in square lattices, and discuss the physics of the observed features. The effects of noise are also discussed. We summarize our results in Sec. IV.

II. THE MODEL

We consider an evolutionary snowdrift game in which the competitions between players are characterized by the payoff matrix

\[
\begin{pmatrix}
C & D & L \\
R & S & Q \\
T & P & Q
\end{pmatrix}.
\]

Here, each player takes on one of three possible characters or strategies: to cooperate (C), to defect (D), or to act as a loner (L). The matrix element gives the payoff to a player using a strategy listed in the left hand column when the opponent uses a strategy in the top row. In the basic SG, it is useful to assign \( R = 1 \) so that the payoffs can be characterized by a single parameter \( r = e'/2 = c'/(2b' - c') \) representing the cost-to-reward ratio \cite{10}. In terms of \( 0 < r < 1 \), we have \( T = 1 + r \), \( R = 1 \), \( S = 1 - r \), and \( P = 0 \). A competition involving a loner leads to a payoff \( Q \) for both players. Here, we explore the range of \( 0 < Q < 1 \).

Spatial networking effects and evolutions are incorporated into the SG as follows. At the beginning of the game, the players are arranged onto the nodes of a network and the character \( s(i) \) of each player is assigned randomly among the choices of C, D, and L. Our discussion will be mainly on fully-connected graphs and regular lattices. In a fully-connected network, every player is connected to all other players. In a square lattice, a player is linked only to his four nearest neighbors. Numerical studies are carried out using Monte Carlo simulations as reported in the work of Szabó et al. \cite{15} (see also Refs. \cite{12,14}). The evolution of the character of the players is governed by the following dynamics. At any time during the game, each player competes with all the players that he is linked to and hence has a payoff. A randomly chosen player \( i \) reassesses his own strategy by comparing his payoff \( P(i) \) with the payoff \( P(j) \) of a randomly chosen connected neighbor \( j \). With probability

\[
W[s(i), s(j)] = \frac{1}{1 + \exp [(P(i) - P(j))/K]},
\]

the player \( i \) adopts the strategy of player \( j \). Otherwise, the strategy of player \( i \) remains unchanged. Here \( K \) is a noise parameter \cite{12,13,14} that determines the likelihood that player \( i \) replaces his strategy when he meets someone with a higher payoff. For \( K \approx 0 \), a player \( i \) is almost certain to replace (not to replace) his strategy when he meets someone with a better (worse) payoff. For large \( K \), a player has a probability of 1/2 to replace his strategy, regardless of whether \( P(j) \) is better or worse than \( P(i) \). In a fully connected network, a player’s character may be replaced by any player in the system. In a square lattice, a player’s character can only be replaced by one of his four connected neighbors. As the game evolves, the fractions of players with the three characters also evolve. These fractions are referred to as frequencies. Depending on the parameters \( r \) and \( Q \), the cooperater frequency \( f_c \), defector frequency \( f_D \), and loner frequency \( f_L \) take on different values in the long time limit.

III. RESULTS AND DISCUSSIONS

A. Fully-connected network

We performed detailed numerical studies on our model. The number of players in the system is taken to be \( N = 10^4 \). In getting the fraction of players of different characters in the long time limit, we typically average over the results of \( 10^3 \) Monte-Carlo time steps per site (MCS), after allowing \( 5 \times 10^3 \) MCS for the system to reach the long time limit. Averages are also taken over 10 initial configurations of the same set of parameters. Figure 1 shows the results for fully connected networks.
A value of $K = 0.1$ is taken. The cooperater frequency $f_C$, defector frequency $f_D$, and loner frequency $f_L$ are obtained as a function of the cost-to-benefit ratio $r$ for three different values of the loner’s payoff $Q = 0.3$, 0.5, and 0.7 in a fully-connected network. The results of the snowdrift game without loners are also included for comparison (solid lines). (d) Phase diagram showing the two phases separated by $r_L(Q)$ in the $r$-$Q$ parameter space. The symbols show the numerical results of $r_L(Q)$ and the line gives the functional form $\sqrt{1 - Q}$.

This is in sharp contrast to the rock-scissors-paper game [24, 25, 26] on a fully connected network in which the three strategies coexist. We obtained $r_L(Q)$ numerically. The result is shown in Figure 1(d) as a curve in the $r$-$Q$ parameter space. It is found that $r_L(Q)$ follows the functional form $\sqrt{1 - Q}$, which will be explained later. The curve $r_L(Q)$ represents a phase boundary that separates the $r$-$Q$ space into two regions. The region below (above) the curve corresponds to a phase in which cooperators and defectors (only loners) coexist (exist).

We also studied the temporal evolution in both phases, i.e., for $r < r_L(Q)$ and $r > r_L(Q)$. Taking $Q = 0.5$, for example, $r_L = 1/\sqrt{2} = 0.707$. Figure 2 shows $f_C(t)$, $f_D(t)$ and $f_L(t)$ in the first $10^3$ MCS. The initial frequencies are 1/3 for all three characters. For values of $r$ deep into either phase (see Fig. 2), the transient behavior dies off rapidly and the extinct character typically vanishes after $\sim 10^2$ MCS. In the phase where C and D coexist, $f_C$ and $f_D$ oscillate slightly with time in the long time limit, due to the dynamical nature of the game. It is noted that for $r \approx r_L$, the strategies compete for a long while and the transient behavior lasts for a long time. This slowing down behavior is typical of that near a transition.

The behavior of $r_L(Q) = \sqrt{1 - Q}$ follows from the rule of character evolution. In a fully-connected network, all
C-players have the same payoff \( P(C) \) and all D-players have the same payoff \( P(D) \). These payoffs depend on \( f_C, f_D, \) and \( f_L \) at each time step. The payoff for a loner is \( NQ \) at all time, for a system with \( N \gg 1 \). For small \( K \), \( f_L \) decays exponentially with time if \( P(C) \) and \( P(D) \) are both greater than \( NQ \). In addition, the phase with only non-vanishing \( f_C \) and \( f_D \) is achieved by having \( P(C) = P(D) \). For this phase in the long time limit, \( P(C) = N(f_C + f_D (1 - r)) \) and \( P(D) = Nf_C (1 + r) \). Together with \( f_C + f_D = 1 \) (since \( f_L = 0 \) in the phase under consideration), the condition \( P(C) = P(D) \) implies \( f_C = 1 - r \) and \( f_D = r \). These results are identical to the basic SG (without loners) in a fully connected network. The validity of this solution requires \( P(C) > NQ \) (and hence \( P(D) > NQ \)), which is equivalent to \( r < \sqrt{1 - Q} \). This is exactly the phase boundary shown in Figure 1(d).

### B. Square Lattice

The behavior of the game in a square lattice is expected to be quite different, due to the restriction that a player can only interact with his connected neighbors. We carried out simulations on 100 × 100 square lattices with periodic boundary conditions. Figure 3(a)-(c) shows \( f_C(r) \), \( f_D(r) \), and \( f_L(r) \) for three different values of the loner payoff \( Q \). The results for the spatial SG (without loners) on a square lattice is also shown (solid lines in Figure 3(a) and 3(b)) for comparison. A value \( K = 0.1 \) is used. Several features should be noted. For \( r < r_L^{(SL)}(Q) \), the loners eventually vanish with \( f_C \) and \( f_D \) take on the mean values in the spatial SG without loners. This behavior is similar to that in fully connected networks. For \( r > r_L^{(SL)}(Q) \), however, the behavior is different from that in fully connected networks. Here, C, D, and L characters coexist. Above \( r_L^{(SL)}(Q) \), \( f_D \) drops with \( r \) to a finite value, leaving rooms for \( f_L \) to increase with \( r \). The cooperator frequency remains finite above \( r_L^{(SL)} \). Therefore, the cooperator frequency or the cooperative level in the system as a whole is significantly improved by the presence of loners. For \( r > r_L^{(SL)} \), increasing the payoff \( Q \) of loners leads to a higher cooperator frequency and lower defector frequency. Reading out \( r_L^{(SL)} \) for different values of \( Q \), we get the phase boundary as shown in Figure 3(d) that separates a region characterized by the coexistence of three characters and a region in which only C and D coexist. The results indicate that, due to the restriction imposed by the spatial geometry that a player can only interact with his four nearest neighbors, it takes a certain non-vanishing value of \( r \) for loners to survive even in the limit of \( Q \rightarrow 1 \). The behavior is therefore different from that in a fully connected network for which the boundary is given by \( \sqrt{1 - Q} \). Note that there exists a region of small values of \( r \) in which the steady state consists of a uniform population of C strategy (see Fig. 3(a) and Fig. 3(d)). For small \( Q \), loners are easier to survive, when compared with the fully connected case. Putting these results together, the phase diagram (see Fig. 3(d)) for a square lattice, therefore, shows three different phases. The most striking effect of the spatial structure is that cooperators now exist in every phase.

Interestingly, we found that the phase boundary \( r_L^{(SL)}(Q) \) in Figure 3(d) can be described quantitatively as follows. We assume that the survival of loners is related to the cooperator frequency. In particular, loner survival requires the cooperator frequency to drop below a certain level \( f(Q) \) and that this value is the same in a

![FIG. 3: (Color online) (a) Cooperator frequency \( f_C \), (b) defector frequency \( f_D \), (c) loner frequency \( f_L \) as a function of \( r \) for three different values of the loner payoff \( Q = 0.3, 0.5, \) and 0.7 in a square lattice. The results of the snowdrift game without loners in a square lattice are also included for comparison (solid lines). (d) Phase diagram showing the different phases in the \( r-Q \) parameter space. The dashed line shows the phase boundary obtained by an approximation as discussed in the text.](image-url)
square lattice as in a fully connected network. That is to say, we assume that loners could survive, for a given value of $Q$ and $K$, only when $f_C < \mathcal{C}^{-1}(Q) = 1 - \sqrt{1 - Q}$. Numerical results also indicate that when all loners extinct, $f_C$ and $f_D$ follow the results in a spatial SG without loners. This is shown as the solid line in Figure 3(a). Therefore, for a given value of $Q$, we can simply read out the value of $r$ such that $f_C = \mathcal{C}^{-1}(Q)$ from the results in spatial SG in a square lattice. For different values of $Q$, this procedure results in the dashed line shown in Figure 3(d) which describes the phase boundary quite well.

Figure 4 shows the temporal dependence of $f_C$, $f_D$, and $f_L$ in a square lattice for two values of $r$ at $Q = 0.5$. For $r = 0.55$ (Fig. 4(a)), which corresponds to a case in which only cooperators and defectors coexist, the number of loners decay rapidly in time, typically within 100 MCS. After the transient behavior, the cooperator and defector frequencies only oscillate slightly about their mean values. This behavior is similar to that in the C and D coexistence phase in Figure 1(d) for fully-connected networks. For $r = 0.65$ (Fig. 4(b)), which corresponds to a case with the three characters coexist, the long time behavior of $f_C$, $f_D$ and $f_L$ is oscillatory. Similar behavior has been found in the rock-scissors-paper game [24, 25, 20] and in the voluntary PDG [14]. Due to the dynamical nature of character evolution, there are continuous replacements of one character by another and this oscillatory behavior is expected.

The major difference between a square lattice and a fully-connected network is that in a fully-connected network, each player competes with all other players. As a result, there are only three payoffs in the system – one for each type of player, at each time step. The loners, for example, have a constant payoff of $4Q$, while the cooperators and defectors have payoffs that depend on $f_C(t)$ and $f_D(t)$. Once $NQ$ is higher than the payoffs of cooperators and defectors, the number of loners grows until they take over the whole population. In a square lattice, however, each player has a payoff that depends on his character and the detail of his neighborhood, i.e., the characters of his four connected neighbors. This implies that the C-players and D-players in a square lattice may have different payoffs depending on the characters of his connected neighbors. The loners have a constant payoff of $4Q$. The non-uniform payoffs among C-players and D-players in a lattice allow some C and D players to coexist with the loners, by evolving to spatial local configurations that favor their survivals.
Since the adaptive rule is related to the payoff of each character, it will be interesting to compare the payoffs in a spatial SG without and with loners. Figure 5(a) shows the mean payoffs of cooperators and defectors as a function of $r$ in a spatial SG in a square lattice without loners. The averaged payoff over all players is also shown. For small $r$, there is a phase with all C players and the payoff is 4 for each of the C players. For large $r$, there is a phase with all D players and the payoff is zero. For intermediate $r$ where C and D players coexist, the mean payoff drops gradually with $r$. In a spatial SG with loners (Fig. 5(b)), it is observed that the mean payoffs basically follow that in Figure 5(a) in the phase where loners are completely replaced. When loners can survive, the presence of these loners increases the payoffs of both the remaining cooperators and defectors. The loners themselves have a payoff of 2 in a 2D square lattice. The cooperators’ payoff is enhanced once loners survive and the increase follows the same form as the increase in the loner frequency with $r$ (compare the circles in Fig. 5(b) with the squares in Fig. 3(c) in the range of $r$ when loners survive). When loners survive, the payoff averaged over all players is significantly enhanced due to their presence. This is similar to what was found in the voluntary PDG [14].

![Figure 6](image_url)

**FIG. 6:** (Color online) (a) The cooperator frequency $f_C$ and (b) the loner frequency $f_L$ as a function of $r$ for three different values of the noise parameter $K = 0.1, 0.4$ and 1.0.

and it is the reason for having step-like features in $f_C$ and $f_D$ for small values of $K$. As the noise parameter $K$ increases, the adaptive mechanism is less dependent on the exact ordering of the payoffs. Therefore, the changes in $f_C$ with $r$ becomes more gradual as $K$ increases. Interestingly, less obvious step-like structures in $f_C$ are also observed in the spatial SG without loners in 2D lattices with a larger coordination number [16]. This is also related to the picture we just described. A lattice with more neighbors will give a higher number of neighborhood configurations and hence more values of the payoffs. More configurations also imply the number of players encountering a certain configuration is smaller. Thus, the number of players involved in a change in the ordering of the payoffs as $r$ changes is smaller. This has the effect of making the drop in $f_C$ gradual. Therefore, increasing $K$ for a given fixed coordination number is similar in effect as increasing the coordination number for fixed $K$.

### C. Effects of noise

All the results reported so far are for the case of $K = 0.1$. This corresponds to a case where the player is highly likely to replace his character when he meets a better-performing player. In Figure 6, we show the effects of the noise parameter for a fixed $Q = 0.3$. As $K$ increases, the step-like structure in $f_C$ as a function of $r$ becomes less obvious and $f_C$ is gradually suppressed in the $r \to 1$ limit. The most important effect of a 2D square lattice is that each player is restricted to interact with his four neighbors. Take a player of character $s(i)$, he will only encounter a finite number of configurations for which he is competing in. For example, his four neighbors may consist of 4 C-players; 3 C-players and 1 D-player or 1-loner, etc. Each of these configurations corresponds to a $P(i)$. In a square lattice, therefore, there will be a finite number of payoffs for a C-player, depending on the characters of the neighbors. Similarly, there are finite number of payoffs for a D-player. The loners always get a payoff of $4Q$. For $K \approx 0$, the adaptive mechanism is strictly governed by the ordering of these payoffs. The distribution of players in a square lattice will then evolve in time according to how the payoffs are ordered. In the long time limit, only a few favorable local configurations will survive and the number of players in each of these favorable configurations is high. As one increases $r$ slightly, the ordering of the finite number of payoffs may not change. Therefore, $f_C$ will not change with $r$ until we reach certain values of $r$ that the ordering of the payoffs is changed. This gives rise to the more sudden changes in $f_C$ as observed at some values of $r$ and it is the reason for having step-like features in $f_C$ and $f_D$ for small values of $K$.

### IV. SUMMARY

We studied the effects of the presence of loners in a snowdrift game with loners in fully-connected networks...
and in square lattices. In a fully-connected network, either cooperators live with defectors or loners take over the whole population. The condition for loners to take over is found to be $Q > 1 - r^2$. This result can be understood by following the payoffs of each strategy. In a fully-connected network, the strategies’ payoffs are particularly simple in that they depend only on the strategy frequencies at the time under consideration, with each type of player having the same payoff.

In a square lattice, the spatial SG with loners behave quite differently. It is found that the cooperators can survive in the fully parameter space covering $0 < r < 1$ and $0 < Q < 1$. Depending on the values of these parameters, there are three possible phases: a uniform C-player population, C-players and D-players coexist, and coexistence of the three characters. The underlying lattice thus makes the survival of cooperators easier. The presence of loners is also found to promote the presence of cooperators. There average payoff among all players is also found to be enhanced in the presence of loners. We discussed the influence of a square lattice in terms of the payoffs of the players. In a square lattice, spatial restriction is imposed on the players in that a player can only interact with the four nearest neighbors. This leads to a payoff that does not only depend on the character but also depend on the local environment in which the player is competing in. The players in the local environment, in turns, are also competing in their own local environment. This will lead to clustering or aggregation of players in the square lattice into configurations that the payoffs favored. The dependence of the frequencies on $r$ in a square lattice then reflects the change in preferred configurations as $r$ is changed.

We also studied the effects of the noise parameter in the adaptive mechanism. It is found that as the noise parameter increases, the change of the frequencies with $r$ becomes more gradual. This is related to the importance of the ordering of the many payoffs in the adaptive mechanism. As the noise parameter increases, the exact ordering of the payoffs becomes less important and the change in frequencies becomes more gradual.

In closing, we note that it will be interesting to further investigate the effects of loners in the snowdrift game in networks of other structures. Among them are the rewiring of regular lattices into a small-world network or a random network and the scale-free networks.

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