Abstract
In this paper theory on collective scattering measurements of electron density fluctuations in fusion plasma is revisited. We present the first full derivation of the expression for the photocurrent beginning at the basic scattering concepts. Thereafter we derive detailed expressions for the auto- and cross-power spectra obtained from measurements. These are discussed and simple simulations made to elucidate the physical meaning of the findings. In this context, the known methods of obtaining spatial localization are discussed and appraised. Where actual numbers are applied, we utilize quantities from two collective scattering instruments: The ALTAIR diagnostic on the Tore Supra tokamak [A. Truc et al., “AL TAIR: An infrared laser scattering diagnostic on the Tore Supra tokamak,” Physica Scripta 71, 280–292, 2005] and the LOTUS diagnostic on the Wendelstein 7-AS stellarator [M. Saffman et al., “CO2 laser based two-volume collective scattering instrument for spatially localized turbulence measurements,” Rev. Sci. Instrum. 72, 2579–2592 (2001)].

I. Introduction
A. Motivation
In this paper we will revisit theoretical aspects of small-angle collective scattering of infrared light off electron density fluctuations. Our main reasons for this second look are the following:

- Working through the literature, we have found that most of the information needed for a full treatment of scattering is indeed available, but distributed among numerous authors. Further, some of these sources are not readily available. We here collect those results and present one coherent derivation from basic scattering concepts to the analytical expression for the detected photocurrent.
- It has been important to us to present understandable derivations throughout the paper. In most cases all steps are included, removing the necessity to make separate notes. One exception is section III A 2.
- The theory is reviewed from a practical point of view; the work done is in support of measurements of density fluctuations in the Wendelstein 7-AS (W7-AS) stellarator [1]. Here, we used a CO2 laser having a wavelength of 10.59 μm to make small-angle measurements [2–8].
- A number of points have been clarified, for instance the somewhat confusing term ‘antenna’ or ‘virtual local oscillator’ beam. Further, several minor corrections to the derivations in previous work have been included.
- Spatial localization of the density fluctuations measured using collective scattering is of central importance [9]. We review the methods available and discuss the pros and cons of these techniques.

The paper constitutes a synthesis between collective light scattering theory and experiment which will be useful for theoreticians and experimentalists alike in interpreting measurements.

B. Collective scattering measurements
In 1960 the first laser was demonstrated [10], which provided a stable source of monochromatic radiation. The first observation of density fluctuations in a fusion device using laser scattering was made by C. M. Surko and R. E. Slusher in the Adiabatic Toroidal Compressor (ATC) tokamak [11].

Subsequently, detection of density fluctuations using lasers has been performed in numerous machines, both applying the technique used in the ATC tokamak [12–17] and related methods, e.g. far-infrared (FIR) scattering [18–20] and phase-contrast imaging (PCI) [21–23].

Scattering using infrared light has several advantages over alternative systems: The technique is non-intrusive, i.e. it does not perturb the investigated plasma in any way. Refraction effects can be neglected due to the high frequency of the laser radiation. Further, fluctuations can be measured at all densities, the lower density limit only depending on the signal-to-noise ratio (SNR) of the acquisition electronics.

The major drawback of collective scattering is spatial localization: Direct localization, where the measurement volume is limited in size by crossing beams is only possible for extremely large wavenumbers where the fluctuation amplitude is known to be minute. However, several methods of indirect localization have been developed; one where two measurement volumes overlap in the plasma [24], one where the change of the magnetic field direction along the measurement volume is taken into account [16] and a third design which is an updated version of the crossed beam technique [2].

Summarizing the state of collective scattering diagnostics on fusion machines in 2005: A large amount of measurements has been made in these devices. The massive database strongly suggests that the density fluctuations created by turbulence cause strong transport of energy and particles out of the plasma. However, a consistent detailed picture of how the various turbulent components are correlated with global transport has not yet emerged.

C. Organization of the paper
The paper is organized as follows: In section II, we derive an expression for the detected photocurrent from first principles.
Thereafter demodulation is explained, phase separation of the detected signal is interpreted and an expression for the density fluctuations squared is presented. Finally, a simple example illustrates this density fluctuation formula for Gaussian beams. In section III, the measurement volume is treated in detail. The simple geometrical estimate is compared to a more elaborate treatment. Following this, direct and indirect localization is discussed, and general expressions for auto- and crosspower are derived. A discussion ensues, and finally simulations assist in the interpretation of localized autopower from a single measurement volume. Section IV states our main conclusions.

II. Collective Light Scattering

In this section we will investigate the theoretical aspects of scattering in detail. The main result will be the derivation of an expression for the observed photocurrent (section II D, Eq. (29)).

A classification of scattering is found in section II A, and the scattering cross section is briefly reviewed in section II B. Basic scattering theory is described in section II C, and a derivation of the incident laser power per unit area, \( r_s = \frac{\mu e^2}{4\pi m_e} \) (see section II C 1 for the definition of \( \varepsilon_s \)) is the classical electron radius and \( \xi \) is the angle between the incident and scattered power [25]. The scattering cross section \( \sigma \) per unit solid angle is then defined as

\[
\frac{d\sigma}{d\Omega} = \frac{1}{4\pi} \frac{1}{\sin^2 \xi} \sin^2 \xi.
\]

Knowing that \( d\Omega = 2\pi \sin \xi \, d\xi \) we get

\[
\sigma = 2\pi \sin \xi \int_{-1}^{1} y^2 \sin^2 \xi \, dy = 2\pi y^3 (4/3),
\]

which one could interpret as an effective size of the electron for scattering.

In this paper we describe the first 3 parts; a description of the detectors used is to be found in Ref. 2 which also contains a detailed description of the practical implementation of the localized turbulence scattering (LOTUS) diagnostic.

B. Scattering cross section

The power \( P \) per unit solid angle \( \Omega_s \) scattered at an angle \( \xi \) by an electron is given by

\[
\frac{dP}{d\Omega} = \sqrt{\frac{\mu_e}{\mu}} \frac{4\pi E_0^2}{m_e} r_s^2 \sin^2 \xi.
\]

where \( \sqrt{\frac{\mu_e}{\mu}} \) is the incident laser power per unit area.

Since the ions are much heavier than the electrons, their energetic that their momentum cannot be ignored. So the electrons do the scattering. Therefore we will restrict ourselves to consider Thomson scattering.

III. Radiation source

Our incident laser beam has a direction \( k_0 \), where \( k_0 = e_0/c \), and a wavelength \( \lambda_0 = 10.59 \mu \text{m} \). For a linearly polarized beam, the electric field is given as in Eq. (8), where \( E_0(t) = E_0 e^{i\omega_0 t} e^{i\omega_0 z} \) [29]. \( \xi_0 \) is a vector whose direction and amplitude are those of the electric field at maximum.

\[
E_0(t, r) = Re[E_0 e^{i\omega_0 t} e^{-i\xi_0 \cdot r}]
\]

Assuming Gaussian beams, the radial profile near the waist \( w \) will be of the form \( u_0(r) = e^{-r^2/w^2} \), where \( r_s \) is the perpendicular distance from the beam axis.
For very energetic electrons the relativistic corrections become significant. Inset: The incoming wave vector $k_0$ and the scattered wave vector $k_s$ are along $r$. The frequency of the laser radiation $\omega_0$ is much higher than the plasma frequency $\omega_p = \sqrt{n^2 - 1}/\omega_{ce}$. This refers to the refractive index of the plasma $n = \sqrt{1 - c^2 \omega_0^2/\omega_{pe}^2}$. This means that the scattered wave vector $k_s$ is along $r - r_f$ and the contribution to the power from the scattered field is very small [29].

$$E_s(r', t) = Re\left(E_0(r')e^{-i\omega t}\right)$$

2. Single particle scattering. For a single scatterer having index $j$ located at position $r_j$ (see Fig. 1), the scatterer radiates an electric field. This field is given in Eq. (10), where $n_j$ is along $r_j - r_f$ and approximately perpendicular to $r_j$.

$$E_j(r', t) = \frac{k_j^2 n_j \alpha_j e^{-i\omega t}}{4\pi} \left[ n_j \times \hat{E}_j(r_f) \right] \times n_j$$

The scattered field is simply the radiation field from an oscillating dipole having a moment $p$ [32].

$$E = \frac{\epsilon^2}{2\alpha_j} \left[ n \times p \right] \times n$$

Therefore, the above expression for the scattered electric field is often called the dipole approximation. It is an approximation because the equation is only valid in the nonrelativistic limit. For very energetic lasers the relativistic corrections become significant, see e.g. Ref. 25.

3. Far field approximation. Two assumptions are made:

1. The position where one measures $r'$ is far from the scattering region.

2. The opening angle of the detector is small, leading to the validity of the far field approximation [31]. This means that we can consider the scattered field from all $j$ particles in the scattering volume to have the same direction denoted $\hat{n}$ parallel to $n_j$. Therefore the scattered wave vector $k_0 = \epsilon \alpha \hat{n}$ and $k = k_0 - k_0$ is the wave vector selected by the optics, see Fig. 1.

$$E_0(r', t) = Re\left(E_0(r')e^{-i\omega t}\right)$$

$$E_s(r', t) = \frac{k_0^2 n_j \alpha_j e^{-i\omega t}}{4\pi} \left[ n_j \times \hat{E}_j(r_f) \right] \times \hat{n} e^{i\omega t}$$

In going from a single particle scattering description to more particles, we will approximate the position of the individual scatterers $r_j$ by one common vector $r$. The particles will have a density distribution $n(r, t)$. We write the scattered field as an integral over the measurement volume $V$

$$E_s(r', t) = \frac{k_0^2 n_j \alpha_j e^{-i\omega t}}{4\pi} \int_{r \in V} \left[ n_j \times \hat{E}_j(r_f) \right] \times \hat{n} e^{i\omega t}$$

D. The photocurrent

The electric field of the local oscillator (LO), see Fig. 2, beam along $n$ at the detector is given as

$$E_{LO}(r', t) = Re\left[E_0(r')e^{-i\omega t}\right]$$

where $\omega_0$ is a frequency shift and $k_{LO} = k_0 - \epsilon \alpha \hat{n}$ [29].

The incident optical power reaching the detector can be found integrating the Poynting vector over the detector area $A$ [29]

$$S(t) = \frac{1}{\hbar c} \int_A \left[ (E \times B)^{\prime} \cdot d^2r' \right]$$

$$= \frac{1}{\hbar c} \int_A \left[ |E_{LO}(r', t) + E_s(r', t)|^2 - 2 \times Re\left[ E_{LO}(r', t) \cdot E_s(r', t) \right] e^{i\omega t} \right]$$

$$= \frac{1}{\hbar c} \int_A \left[ |E_{LO}(r', t)|^2 + |E_s(r', t)|^2 + 2 \times Re\left[ E_{LO}(r', t) \cdot E_s(r', t) \right] e^{i\omega t} \right]$$

What we are interested in is the last term of the equation, namely the beating term

$$S_b(t) = \int \frac{1}{\hbar c} Re\left[ E_{LO}(r', t) \cdot E_s(r', t) \right]$$

The term containing the LO power is constant, and the contribution to the power from the scattered field is very small because its field amplitude is much smaller than that of the LO [31].

Now we define the integrand of Eq. (16) to be $i_0(r')$

$$i_0(r') = \frac{2}{\hbar c} Re\left[ E_{LO}(r', t) \cdot E_s(r', t) \right]$$

$$= \frac{2}{\hbar c} \sqrt{\hbar c} Re\left[ E_{LO}(r') \cdot E_{LO}(r') e^{i\omega t} \right]$$

Assuming a detector quantum efficiency $\eta$ leads to the photocurrent [29]

$$i_\eta(t) = \frac{\eta}{\hbar c} \int_A i_0(r')$$

The photocurrent due to an ensemble of scatterers at the detector position $r'$ (replacing $i_0$ by $i_{\eta}$, where the subscript $k$ is
the measured wavenumber) is

\[ i(t) = \frac{\hbar n}{\epsilon_0} \int_t \rho(t') d^3r' = 2 Re \left\{ \frac{1}{i \omega} \int_{\mathcal{D}} \left[ U_{LO}(t) E_{LO}(t') \right] \frac{d^3r'}{d^3r} \right\} \]

\[ = 2 Re \left\{ \frac{1}{i \omega} \int_{\mathcal{D}} e^{i \omega r'} \left[ E_{LO}(t) e^{-i \omega \mathbf{r'} \cdot \mathbf{n'}} \right] d^3r' \times \right\} \]

\[ \times \mathbf{n'} \times \mathbf{E_0} e^{i \omega \mathbf{r'} \cdot \mathbf{n'}} d^3r' \}

where we have inserted Eqs. (14) and (13) for the LO and scattered electric field, respectively. We now introduce the Fresnel-Kirchhoff diffraction formula

\[ \frac{i \omega}{\pi \epsilon_0} \int \frac{e^{-i \omega \mathbf{r'} \cdot \mathbf{n'}}}{|\mathbf{r'} - \mathbf{r}|} E_{LO}(t) e^{-i \omega \mathbf{r'} \cdot \mathbf{n'}} d^3r' = \frac{1}{i \omega} U_{LO}(t) E_{LO}(t') \]

(20)

which is the radiated field for small angles of diffraction from a known monochromatic field distribution on a diaphragm A [33]. This radiated field (the antenna or virtual LO beam [29]) propagates from the detector to the scatterers [34]. The reciprocity theorem of Helmholtz states that a point source at \( \mathbf{r} \) will produce at \( \mathbf{r'} \) the same effect as a point source of equal intensity placed at \( \mathbf{r'} \) will produce at \( \mathbf{r} \) [33]. Therefore Eq. (20) describing the field in the measurement volume (position \( \mathbf{r} \)) due to a source at the detector (position \( \mathbf{r'} \)) is equivalent to the reverse situation, where the measurement volume is the source.

In Eq. (21) we first reorganize Eq. (19) and then apply the Fresnel-Kirchhoff diffraction formula

\[ i(t) = \frac{\hbar n}{\epsilon_0} \int_{\mathcal{D}} \frac{e^{-i \omega \mathbf{r'} \cdot \mathbf{n'}}}{|\mathbf{r'} - \mathbf{r}|} \left[ E_{LO}(t) e^{-i \omega \mathbf{r'} \cdot \mathbf{n'}} \right] d^3r' \times \mathbf{n'} \times \mathbf{E_0} e^{i \omega \mathbf{r'} \cdot \mathbf{n'}} d^3r' \]

(21)

since

\[ \frac{\hbar^2 n}{4 \pi \epsilon_0} \frac{j_0}{\omega} = \frac{\hbar n}{2 \pi} \sqrt{\frac{j_0}{\omega}} \]

and

\[ [\mathbf{n'} \times \mathbf{E_0}] \times \mathbf{n'} = \mathbf{E_0} \]

(23)

The expression for the current now becomes

\[ i(t) = \frac{\hbar n}{\epsilon_0} \int_{\mathcal{D}} \frac{e^{-i \omega \mathbf{r'} \cdot \mathbf{n'}}}{|\mathbf{r'} - \mathbf{r}|} \left[ E_{LO}(t) e^{-i \omega \mathbf{r'} \cdot \mathbf{n'}} \right] d^3r' \times \mathbf{n'} \times \mathbf{E_0} e^{i \omega \mathbf{r'} \cdot \mathbf{n'}} d^3r' \]

\[ \times \mathbf{n'} \times \mathbf{E_0} e^{i \omega \mathbf{r'} \cdot \mathbf{n'}} d^3r' \}

(24)

where \( E_{LO} \) and \( E_0 \) hereafter are to be considered as scalars since the laser field and the LO field are assumed to have identical polarization.

We introduce a shorthand notation for the spatial Fourier transform

\[ U(r) = \int \frac{e^{i \omega \mathbf{r'} \cdot \mathbf{n'}}}{|\mathbf{r'} - \mathbf{r}|} U_{LO}(t) d^3r' \]

(26)

where \( U \) is called the beam profile [29, 34]. We note that

\[ \int \frac{e^{i \omega \mathbf{r'} \cdot \mathbf{n'}}}{|\mathbf{r'} - \mathbf{r}|} U_{LO}(t) d^3r' = \int \mathbf{n}(t') \mathbf{U}(\mathbf{k} - \mathbf{k'})(2\pi)^d d^k \]

\[ = \mathbf{n}(t') \mathbf{U}(\mathbf{k}) \]

(27)

Defining

\[ \gamma = \int \frac{e^{i \omega \mathbf{r'} \cdot \mathbf{n'}}}{|\mathbf{r'} - \mathbf{r}|} \mathbf{U}_{LO} d^3r' \]

(28)

Eq. (24) in its final guise is

\[ i(t) = \gamma e^{i \omega \mathbf{r'} \cdot \mathbf{n'}} (\mathbf{U}_{LO} \cdot \mathbf{n'}) - \gamma e^{-i \omega \mathbf{r'} \cdot \mathbf{n'}} (\mathbf{U}_{LO} \cdot \mathbf{n'}) \]

(29)

Note that the \( e^{i \omega \mathbf{r'} \cdot \mathbf{n'}} \) term in \( (\mathbf{U}_{LO} \cdot \mathbf{n'}) \) constitutes a spatial band pass filter (\( k \) is fixed). Three scales are involved [36]:

- Fluctuations occur at scales \( r \) much smaller than \( \lambda = 2\pi/k \Rightarrow k \ll 1 \Rightarrow e^{-i \omega \mathbf{r'} \cdot \mathbf{n'}} (= 1 \). The Fourier transform becomes the mean value of the density fluctuations, which is zero.
- Fluctuations occur at scales \( r \) similar to \( \lambda = 2\pi/k \); this leads to the main contribution to the signal.
- Fluctuations occur at scales \( r \) much larger than \( \lambda = 2\pi/k \Rightarrow k \gg 1 \Rightarrow e^{-i \omega \mathbf{r'} \cdot \mathbf{n'}} \) is highly oscillatory. The mean value will be roughly equal to that of \( e^{i \omega \mathbf{r'} \cdot \mathbf{n'}} \), which is zero.

The scattered power \( P_s \) resulting from the interference term can be written by defining a constant

\[ \xi = \frac{\hbar n}{\epsilon_0} \int_{\mathcal{D}} \mathbf{E}_{LO} \cdot \mathbf{n} \]

(30)

and replacing \( \gamma \) with this in Eq. (29)

\[ P_s(t) = \frac{\hbar n}{\epsilon_0} \int_{\mathcal{D}} |\mathbf{e}^{i \omega \mathbf{r'} \cdot \mathbf{n'}} (\mathbf{U}_{LO} \cdot \mathbf{n})| - \mathbf{e}^{-i \omega \mathbf{r'} \cdot \mathbf{n'}} (\mathbf{U}_{LO} \cdot \mathbf{n}) \]

(31)

\[ = 2 Re \{ \mathbf{E}_{LO} \cdot \mathbf{n} \} \]

If \( \mathbf{E}_0 \) and \( \mathbf{E}_0 \) are real numbers (meaning that \( \xi \) is real) we can go one step further and write

\[ P_s(t) = 2 \mathbf{E}_{LO} \cdot \mathbf{n} \]

(32)

assuming that \( \mathbf{P}_{LO} = \frac{\hbar n}{\epsilon_0} \mathbf{E}_{LO} \) (for a given \( U \), see section II G 2).

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E. Demodulation

The task now is to extract real and imaginary parts of $(n(t)U)_k$. We construct two signals that are shifted by $\pi/2$ [37]

$$j_1(t) = Re(e^{j\omega t}) = \cos(\omega t)$$

$$j_2(t) = Re(e^{j(\omega t+\pi/2)}) = \sin(\omega t)$$

(33)

Now two quantities are constructed using Eqs. (29) (divided into two equal parts) and (33)

$$i_{k1} = \frac{1}{2}j_1(t_0)$$

$$i_{k2} = \frac{1}{2}j_2(t_0)$$

$$i_{k3} = \frac{1}{2}Re[\gamma(n(t)U)_h + \gamma(n(t)U)_h - \gamma(n(t)U)_h]$$

$$i_{k4} = \frac{1}{2}Re[\gamma(n(t)U)_h]$$

(34)

Low pass filtering (LPF) of these quantities removes the terms containing the fast $2\pi\omega_a$ expression [38]. The result is that

$$i_{k,\text{complex}} = \frac{1}{2}(i_{k1} - i_{k2})$$

$$i_{k,\text{complex}} = \frac{1}{2}(Re[(n(t)U)_h] - i\cdot Im[(n(t)U)_h])$$

$$i_{k,\text{complex}} = \overline{i}(t)$$

(35)

Now we have $(n(t)U)_h$ and can analyze this complex quantity using spectral tools. The alternative to heterodyne detection is called homodyne detection. There are two advantages that heterodyne detection has compared to homodyne detection [36]:

1. The LO beam provides an amplification factor to the detected signal (see Eq. (32)).
2. It leaves the complex $(n(t)U)_h$ intact multiplied by a wave having frequency $\omega_a$, in homodyne detection the electric field complex number is transformed into a real number and the phase information is lost. The frequency sign of the scattered power tells us in which direction the fluctuations are moving.

F. Phase separation

Since the theory behind phase separation is extensively described in section 2 of Ref. 39, we will here only give a brief recapitulation of the basics.

The observed signal is interpreted as being due to a large number of 'electron bunches', each moving in a given direction. An electron bunch is defined as a collection of electrons occupying a certain region of the measurement volume $V$. This definition is motivated by the fact that even though the measurement volume includes a large number of cells $(V/\ell_a^3)$ [40] (typically $\sim 3000$ in W7-AS), the amplitude of the signal consists of both large and small values separated in time. The demodulated photocurrent $i_{k,\text{complex}}$ is a complex number; it can be written

$$i_{k,\text{complex}}(t) = \sum_{n=1}^{N_b} a_n e^{j\phi_n}$$

$$\Phi = \frac{\Omega}{2\pi}$$

(36)

where $N_b$ is the number of bunches, while $a_n$ and $\phi_n$ is the amplitude and phase of bunch number $n$, respectively. The criterion for determination of direction is

$$\Phi > 0 \Rightarrow \Phi > 0 \Rightarrow \text{fluctuations} || k$$

$$\Phi < 0 \Rightarrow \Phi < 0 \Rightarrow \text{fluctuations} || -k.$$
Combining Eqs. (43) and (39) (replacing \( n \) by \( \omega \)) we get

\[
S_\omega(k, \omega, -\omega) = S_0(k, \omega) + S_\omega(k, -\omega) = \frac{1}{\omega^2} \int \frac{I(k, \omega)}{|\langle \hat{\omega}^2 \rangle_L|^2} \, dk \, d\omega \tag{45}
\]

The term with positive frequency corresponds to density fluctuations propagating in the \( k \)-direction, while negative frequency means propagation in the opposite direction \([16]\).

The wavenumber resolution width is

\[
\Delta k^L = \left[ \frac{\langle \hat{\omega}^2 \rangle_L}{|\langle \hat{\omega}^2 \rangle_L|^2} \right]^{1/2} \tag{46}
\]

We have now arrived at the goal; replacing \( S_0(k, \omega) \) by \( S_\omega(k, \omega, -\omega) \) in the first line of Eq. (43), our final expression for the mean square density fluctuations can be calculated \([29]\). The following assumptions are made:

- Beam power expression for the density fluctuations can be calculated \([29]\). The approximations above are valid for small scattering angles.
- Forward scattering.
- Beams are focused in the measurement region with identical waist \( w \).

Furthermore, the beam profile \( U(r) \) is assumed to be

\[
U(r) = w_0(r)w_0(d^2 r) = e^{-\omega^2 z^2 + x^2 + y^2} \quad \text{for} \quad |z| < L/2
\]

\[
U(r) = 0 \quad \text{for} \quad |z| > L/2, \tag{48}
\]

where \( L \) is the measurement volume length and the beams are along \( z \).

The wavenumber resolution width \( \Delta k^L \) becomes \( 4/(\pi w^2 L) \) and we find the wavenumber resolution itself by calculating

\[
U(k) = \int \frac{\langle \hat{\omega}^2 \rangle_L}{|\langle \hat{\omega}^2 \rangle_L|^2} \, d^2 r \left[ e^{-2 i(kz/L)} \right] = 2 \sin \left( \frac{L k}{2} \right) \sqrt{\frac{2^2}{\pi^2} \int \frac{dy}{x^2} \int \frac{dx}{y^2}} \left[ \int \frac{dy}{x^2} \int \frac{dx}{y^2} \right] \tag{49}
\]

allowing us to define the transverse wavenumber resolutions \( \Delta k_x = 2w \) \((e^{-1} \text{ value}) \,[34] \) and a longitudinal wavenumber resolution \( \Delta k_z = 2\pi/L \) (line term zero) \([16]\). We further obtain an expression for the main (LO) beam power

\[
P_\text{LO} = \frac{\omega^2}{8 \mu} \int \frac{I_\text{LO}(d^2 r)}{w_0^2} \, d^2 r = \frac{\pi w^2}{4} \sqrt{\frac{\omega^2}{\mu^2}} \tag{50}
\]

\[
I_\text{LO} = \frac{\epsilon^2 P_\text{LO}}{\beta_\text{LO}} \quad \text{and} \quad P_\text{LO} = \frac{\omega^2}{8 \mu} \frac{\beta_\text{LO}}{\beta_\text{LO}} \tag{51}
\]

Using Eq. (47) for this example we get

\[
\langle \hat{\omega}^2 \rangle_L = \frac{1}{(2\pi)^2} \int \frac{I(k, \omega)}{\omega^2} \frac{1}{\omega^2} \int \frac{I(k, \omega)}{\omega^2} \, d^2 \omega \, dk \tag{47}
\]

\[
\langle \hat{\omega}^2 \rangle_L = \frac{1}{(2\pi)^2} \int \frac{I(k, \omega)}{\omega^2} \frac{1}{\omega^2} \int \frac{I(k, \omega)}{\omega^2} \, d^2 \omega \, dk \tag{51}
\]

This example concludes our section on the theory of collective light scattering. In section II D we derived the analytical expression for the photocurrent, enabling us to interpret the signal as a spatial Fourier transform of density multiplied by the beam profile. In the present section this result was used to deduce an equation for \( \Delta k \) (Eq. 47).

III. Spatial Localization

In this section we first investigate the geometry of the measurement volume (section III A). Thereafter we explore the possibilities of obtaining localized measurements; first using a simple method directly limiting the volume length (section III B) and then by assuming that the density fluctuations have certain properties (section III C).

A. The measurement volume

1. Geometrical estimate. A measurement volume is created by interference between the incoming main (M) beam (wave vector \( k_0 \)) and the local oscillator (LO) beam (wave vector \( k_1 \)). See Fig. 2.

2. Exact result. The size of the interference fringes \([38]\) is

\[
k_{\text{geom}} = \frac{2w}{\tan \frac{\theta_1}{2}} = \frac{2w}{\theta_1} \tag{52}
\]

The scattering angle determines the measured wavenumber \([16]\)

\[
k = 2k_0 \sin \frac{\theta_1}{2} \approx k_0 \theta_1 \quad \text{for} \quad \theta_1 < \frac{k_0}{k} \tag{53}
\]

The approximations above are valid for small scattering angles. Assuming that the beams have identical diameters \( 2w \), the volume length can be estimated as

\[
l_{\text{geom}} = \frac{2w}{\tan \frac{\theta_1}{2}} = \frac{4w}{\theta_1} \tag{54}
\]

The fringe number, i.e. the number of wavelengths that can be fitted into the measurement volume, is

\[
M = \frac{2w}{\pi} \approx \frac{w}{\theta_1} \tag{55}
\]

2. Exact result. The time-independent field from each of the two Gaussian beams creating a measurement volume can be written

\[
u(r, \nu, \tau, \zeta, \zeta) = \frac{-2P}{w^2 \mu} \left[ e^{-i(kz/L)} + e^{i(kz/L)} \right] e^{i\nu \tau} \tag{56}
\]

Here, \( P \) is the beam power,

\[
\nu(r, \nu, \tau, \zeta) = \frac{1 + \frac{1}{2} \nu \tau}{\tau} \tag{57}
\]

is the beam radius at \( z \) and \( \zeta \) is the Rayleigh range

\[
\zeta_0 = \frac{\pi \nu^2}{\lambda_0} \tag{58}
\]
which is the distance from the waist \( w_0 \) to where the beam radius has grown by a factor \( \sqrt{2} \). Note that we have introduced the beam waist \( w_0 \) and the Rayleigh range explicitly for the following calculations. The phase is given by

\[
\phi(z) = \arctan\left( \frac{z}{c} \right) \tag{59}
\]

We use the complete Gaussian description here instead of the simple form used in section II. An excellent treatment of the measurement volume has been given in Ref. 38; therefore we will here restrict ourselves to simply quoting the important results and approximations in sections III A 2a and III A 2b.

a. Intensity. We now want to find an expression for the interference power in the measurement volume. Since the full angle between the LO and M beams is \( \theta_c \), we will construct two new coordinate systems, rotated \( \pm \theta_c/2 \) around the \( y \)-axis. We define the constants

\[
e = \cos\left( \frac{\theta_c}{2} \right) \quad s = \sin\left( \frac{\theta_c}{2} \right)
\]

and use them to construct the two transformations from the original system

\[
x_0 = eX - sY \quad y_0 = sX + eY \quad z_0 = sX + eZ \tag{61}
\]

and

\[
x_{0\text{LO}} = eX + sY \quad y_{0\text{LO}} = sX - eY \quad z_{0\text{LO}} = -sX + eZ \tag{62}
\]

This enables us to use Eq. (56) for each beam in the rotated systems. The intensity distribution in rotated coordinates can be written

\[
|u_0 u_0^*| = \frac{2\sqrt{\lambda P_0 \omega_0^2}}{\pi w_0 (x_{0\text{LO}}/w_0)} e^{-\frac{\pi^2 x_{0\text{LO}}^2}{w_0^2}} e^{-\frac{\pi^2 sX}{w_0^2}} e^{-\frac{\pi^2 eZ}{w_0^2}} \tag{63}
\]

The intensity distribution in the original coordinate system can now be found by inserting the transformations (61) and (62) into Eq. (63). A few approximations lead to the following expression

\[
|u_0 u_0^*| = \frac{2\sqrt{\lambda P_0 \omega_0^2}}{\pi w_0} e^{-\frac{(x^2 + s^2 (1 + 3\theta_c^2 z^2/R^2))}{2w_0^2}} \tag{64}
\]

Here, the terms including \( z \) are due to beam divergence effects. Eq. (64) can be integrated over the \((x,y)\)-plane to obtain the variation of the interference power as a function of \( z \)

\[
P(z) = \int \int dx \, dy |u_0 u_0^*| = \frac{\sqrt{\lambda P_0 \omega_0^2}}{c} e^{-\frac{(x^2 + s^2 (1 + 3\theta_c^2 z^2/R^2))}{2w_0^2}} \left( 1 + \frac{c^2 z^2}{R^2} / \frac{w_0^2}{c} \right)^{-1/2} \tag{65}
\]

For small scattering angles,

\[
c \approx 1 \quad s \approx \frac{\theta_c}{2} \tag{66}
\]

meaning that the \( z \)-dependent pre-factor in Eq. (65) is close to unity for \( z \leq z_R \). Therefore the behaviour of \( P(z) \) can be gauged from the exponential function. We define the position \( z_R \) where the power has fallen to \( a \) times its maximum value

\[
P(z_R) = a P(0) \tag{67}
\]

The \( z_R \)-position is now inserted into the exponential function of Eq (65)

\[
a = e^{-\frac{z_R^2}{w_0^2}} \tag{68}
\]

The measurement volume length can now be defined as

\[
L_{\text{exact}} = 2|z_R - 1| = \frac{2w_0}{s} e^{-\frac{(1 + 3\theta_c^2 z^2/R^2)}{w_0^2}} \left( 1 - \frac{4}{\pi M} \right)^{1/2} \tag{69}
\]

The correction from the geometrical estimate (54) can be estimated by assuming that \( M \geq 2 \), this means that the correction factor

\[
\left( 1 - \frac{4}{\pi M} \right)^{1/2} \tag{70}
\]

The increase of the measurement volume length from the geometrical estimate is due to the divergence of the Gaussian beams.

As a final point, we can compare the beam divergence angle \( \theta_y \) to the scattering angle \( \theta_c \),

\[
\theta_y = \frac{\lambda}{w_0} = \frac{w_0}{z_R} = \frac{2\theta_c}{\pi M} \tag{71}
\]

A large \( M \) means that \( \theta_y < \theta_c \), so that the beams will separate as one moves away from \( z = 0 \).

b. Phase. The phase of the interference in rotated coordinates is given by

\[
\phi(z) = \phi(z_{0\text{LO}}) - \phi(z_0) \tag{72}
\]

Neglecting the \( \phi(z_{0\text{LO}}) - \phi(z_{0\text{LO}}) \) term and inserting the original coordinates, the fringe distance is

\[
\delta(z) = \frac{1}{2} \sqrt[3]{(1 + 3\theta_c^2 z^2/R^2 - 1 + 3\theta_c^2 z^2)} \approx \frac{z^2}{R^2 + 2} \tag{73}
\]

The exact expression for the fringe distance has a correction term \( \delta(z) \) compared to the geometrical estimate in Eq. (52). For example, if \( z = z_R/2 \), \( \delta \) is equal to \(-0.2\), meaning a 25% increase of the fringe distance. But of course the power in the interference pattern \( P(z) \) decreases rapidly as well.

B. Direct localization

From Eq. (54) we immediately see that spatial localization along the measurement volume can be achieved by having a large scattering angle (large \( k_i \)). We will call this method direct localization, since the measurement volume is small in the \( z \) direction.
To localize along the beams, the measurement volume length \( L_{\text{geom}} \) must be much smaller than the plasma diameter \( 2a \), where \( a \) is the minor radius of the plasma.

Assuming that \( a = 0.3 \text{ m} \), \( u = 0.01 \text{ m} \) and that we want \( L_{\text{geom}} \) to be 0.2 m, the scattering angle \( \theta_s \) is 11° (or 199 mrad). This corresponds to a wavenumber \( k \) of 1180 cm\(^{-1}\).

However, measurements show that the scattered power decreases very fast with increasing wavenumber, either as a power-law or even exponentially. This means that with our detection system, we have investigated a wavenumber range of \([14, 62] \text{ cm}^{-1}\). For this interval, the measurement volume is much longer than the plasma diameter, meaning that the measurements are integrals over the entire plasma cross section.

**C. Indirect localization**

We stated above that the measured fluctuations are line integrated quite small (of order 0.3 along the entire plasma column because the scattering angle is C. Indirect localization are integrals over the entire plasma cross section.

**1. Dual volume**

**a. Dual volume geometry.** The geometry belonging to the dual volume setup is shown in Fig. 3. The left-hand plot shows a simplified version of the optical setup and the right-hand plot shows the two volumes as seen from above. The size of the vector \( d \) connecting the two volumes is constant for a given setup, whereas the angle \( \theta_p = \arccos(\kappa_p/d) \) can be varied. The length \( d_p \) is the distance between the volumes along the major radius \( R \). The wave vectors selected by the diagnostic (\( \kappa_1 \) and \( \kappa_2 \)) and their angles with respect to \( R \) (\( \kappa_{\parallel} \) and \( \kappa_{\perp} \)) have indices corresponding to the volume number, but are identical for our diagnostic.

**b. The magnetic pitch angle.** The main component of the magnetic field is the toroidal magnetic field, \( B_T \). The small size of the magnetic field along \( R, B_T \), implies that a magnetic field line is not completely in the toroidal direction, but also has a poloidal part. The resulting angle is called the pitch angle \( \theta_p \), see Fig. 4.

\[ \theta_p = \arctan \left( \frac{B_p}{B_m} \right), \quad (74) \]

which for fixed \( z \) (as in Fig. 4) becomes

\[ \theta_p = \arctan \left( \frac{B_p}{B_m} \right). \quad (75) \]

As one moves along a measurement volume from the bottom to the top of the plasma (thereby changing \( z \)), the ratio \( B_p/B_m \) changes, resulting in a variation of the pitch angle \( \theta_p \). The central point now is that we assume that the fluctuation wavenumber parallel to the magnetic field line (\( \kappa_{\parallel} \)) is much smaller than the wavenumber perpendicular to the field line (\( \kappa_{\perp} \))

\[ \kappa_{\parallel} \ll \kappa_{\perp}, \quad (76) \]

This case is illustrated in Fig. 4, where only the \( \kappa_{\perp} \) part of the fluctuation wave vector \( \kappa \) is shown. It is clear that when \( \theta_p \) changes, the direction of \( \kappa_{\perp} \) will vary as well [16].

**c. Localized crosspower.** Below we will derive an expression for the scattered crosspower between two measurement volumes (Eq (95)). The derivation is based on work presented in Ref. 37. We will ignore constant factors and thus only do proportionality calculations to arrive at the integral. This equation will prove to be crucial for the understanding of the observed signal and the limits imposed on localization by the optical setup.

The wave vectors used for the derivation are shown in Fig. 5. The size and direction of the wave vectors \( \kappa_1 \) and \( \kappa_2 \) are allowed to differ. The positions of the measurement volumes are \( r \) (volume 1) and \( r' \) (volume 2). We assume that \( d \) is zero (see Fig. 3); effects associated with a spatial separation of the volumes are discussed after the derivation.

We introduce a few additional definitions that will prove to be useful; the difference between the two measured wave vectors \( \kappa_3 \).
This allows us to simplify Eq. (80)

\[
\langle \cdot \rangle
\]

and we assume that the local spectral density only varies along (and not across) the measurement volumes

\[
S(k, R, \omega) = S(k, Z, \omega)
\]

Inserting Eqs. (84)–(86) into Eq. (83) we arrive at

\[
I_{22}(k_1, k_2, \omega) \propto \int dZ \int d\omega S(k_2, Z, \omega) \int dx dy dz e^{i(k_2 - k_1) \cdot \rho}
\]

where we have used that

\[
k_2 = k_1 - k_2 = (k_2 \cos \beta, k_2 \sin \beta, 0)
\]

\[
\rho = x - x' = (x, y, z)
\]

Our starting point is the current spectral density (Eq. (39))

\[
I_{22}(k_1, k_2, \omega) \propto \int dx \int dx' \langle n(r, \omega)n^{*}(r', \omega) \rangle e^{i(k_2 - k_1) \cdot (r - r')}
\]

where \( \langle \cdot \rangle \) is a temporal average. Since

\[
k_2 \cdot (x - x') = k_2 \cos \beta \cdot (x - x')
\]

we can rewrite Eq. (78) using the substitution \( \rho = x - x' \) to become

\[
I_{22}(k_1, k_2, \omega) \propto \int d\rho \int d\rho^{*} \langle n(r, \omega) n^{*}(R - \rho, \omega) \rangle e^{i(k_2 - k_1) \cdot \rho}
\]

We define the local spectral density of the density fluctuations to be

\[
S(k, R, \omega) = \int d\rho \langle n(r, \omega) n^{*}(R - \rho, \omega) \rangle e^{i(k_2 - k_1) \cdot \rho},
\]

where the inverse Fourier transform yields

\[
\langle n(R, \omega) n^{*}(R - \rho, \omega) \rangle \propto \int d\omega e^{-i\omega k} S(k, R, \omega)
\]

This allows us to simplify Eq. (80)

\[
I_{22}(k_1, k_2, \omega) \propto \int d\omega \int d\omega^{*} S(k, R, \omega) U(R)(R - \rho) e^{i(k_2 - k_1) \cdot \rho}
\]

where we have assumed that the two beam profiles \( U_1 \) and \( U_1^{*} \) are identical and equal to \( U \). Further, we assume that the functional form that was used in section II, so that

\[
U(R)(R - \rho) = e^{i(k_2 - k_1) \cdot \rho} e^{-i(2\pi x' + 2\pi y' + 2\pi z)'(R - \rho)}
\]

We note that

\[
k_2 \cdot R = Xk_2 \cos \beta + Yk_2 \sin \beta
\]

and \( k_3 = 0 \) and we assume that the local spectral density only varies along (and not across) the measurement volumes

\[
S(k, R, \omega) = S(k, Z, \omega)
\]

From geometrical considerations (see Fig. 5) we find that

\[
i(k_2 - k_1) \cdot \rho = i(k_2 \cos \beta - k_2 \sin \beta) x + i(k_2 \sin \beta + k_2 \cos \beta) y - i(k_1 \cdot \rho)
\]

Since the measurement volume length \( L \) is much longer than the plasma minor radius \( a \) we find that

\[
I_{22}(k_1, k_2, \omega) \propto \int dZ \int d\omega S(k, Z, \omega) e^{i(k_2 - k_1) \cdot Z}
\]

Inserting Eqs. (89) and (90) into Eq. (87) and performing the integrations over \( x, y, z \) we arrive at

\[
I_{22}(k_1, k_2, \omega) \propto \int dZ d\omega S(k, Z, \omega) e^{i(k_2 - k_1) \cdot Z}
\]

where we have used that

\[
c_1 = k_2 \cos \beta + k_2 \sin \beta c_2 = k_2 \sin \beta + k_2 \cos \beta
\]

To perform the integration over \( k \) we assume that \( k_1 \ll k_2 \)

\[
S(k, Z, \omega) = S(k_1, Z, \omega) e^{i(k_1 \cdot Z)} \int dk = \text{im} \int d\omega
\]
This in turn indicates that the cross-field correlation length \( \tilde{L}_\perp \) measurement volumes is situated at the nadir of the triangle, Geometry concerning dual volume localization. Assuming that one of the \( (97) \), where the two volumes are borderline connected. 

\[
c_1 = \frac{k_1}{2} \cos \phi + k_2(\cos \varphi_2 - \cos \varphi_p)
\]

\[
c_2 = \frac{k_2}{2} \sin \phi + k_2(\sin \varphi_2 - \sin \varphi_p)
\]

\[
c_1^2 + c_2^2 = \frac{k_1^2}{4} + 2k_1k_2(1-\cos \varphi_2-\varphi_p)
\]

\[
+c_2k_2[\cos(\varphi_2-\phi_p) - \cos(\phi_p-\varphi_p)]
\]

so that

\[
I_{12}(\mathbf{k}_1, \mathbf{k}_2, \omega) \propto \int dZ S(\mathbf{k}_1, \mathbf{k}_2, \omega)e^{i k_1 \cdot r + k_2 \cdot r} e^{-\frac{i L_{\perp}^{(1)}(z+1)}{2}}
\]

The fluctuations are correlated for a section of the path. The fluctuations in the volumes are correlated along the entire path. For experimental settings where case 2 is true, some localization can be obtained by calculating the crosspower spectrum between the volumes. In Ref. 4 we demonstrate this technique for a situation where \( w = 4 \) mm and \( d = 29 \) mm. This along with \( L_\perp = 1 \) cm means that \( \varphi_p \approx 18^\circ \). The final issue is how to incorporate the measurement volume separation into the local spectral density \( S(\mathbf{k}_1, \mathbf{k}_2, Z, \omega) \) from Eq. (95). Assuming that we work with frequency integrated measurements we can drop \( \omega \). Further, we assume that \( Z \) is independent of the wave vector. The remaining dependency is that of \( Z, \) the vertical coordinate along the measurement volumes. For the single volume case below, \( S \) is simply assumed to be proportional to \( \delta w^2 \), see Eq. (105). In the present case, however, we need to treat the correlation between the volumes. A plausible expression for the correlation function is

\[
C_{12}(z) = \exp \left[ -\frac{\sqrt{(\delta w - \delta w_p) \delta d}}{w + \varphi_p / 2} \right].
\]

which is a Gaussian-like function. All quantities are known and independent of \( z \) except \( \delta w_p \), but we should note that \( L_{\perp} \) could depend on \( z \). The correlation function \( C_{12}(z) \) possesses the expected limits:

- \( C_{12}(z) = 1 \) for \( |\delta w - \delta w_p(z)| = 0 \)
- \( C_{12}(z) = 1 \) for \( d = 0 \)
- \( \lim_{\varphi_p \to \infty} C_{12}(z) = 1 \)
- \( \lim_{d \to \infty} C_{12}(z) = 1 \)

For actual calculations we would replace \( S \) by \( C_{12}(z) \times \delta w^2 \) in Eq. (95) and use Eq. (107) for the density fluctuation profile. For the single volume simulations in the following we do not need to include \( C_{12}(z) \).

One could argue that the pitch angle \( \phi_p \) in the two spatially separated measurement volumes is different, so that the exponential functions in Eq. (95) would have to be modified. However, the actual distance between the volumes is small and therefore the pitch angles are almost identical.

2. Single volume

The material in this section is based on work presented in Ref. 42.

a. Single volume geometry. Fig. 7 shows the geometry associated with the single volume setup. The definitions are completely analogous to the ones in Fig. 3.

\[
I_{11}(\mathbf{k}, \omega) \propto \int dZ S(\mathbf{k}, \omega)e^{-\frac{i L_{\perp}^{(1)}(z+1)}{2}}
\]

Assuming that the angles \( x \) and \( \theta_p \) are small, we can expand the function in the exponent of Eq. (100) as

\[
2k_1^2[1 - \cos(x - \theta_p)] \approx 2k_1^2[(x - \theta_p)^2]/2 = k_1^2(x - \theta_p)^2
\]
Fig. 7. Left: Schematic representation of the single volume setup (side view). Thick lines are the M beam, thin lines the LO beam, right: The single volume setup seen from above. The black dot is the measurement volume.

We introduce the instrumental selectivity function
\[ \chi = e^{-\frac{(\Delta z)^2}{\Delta z_0^2}}, \]  
(102)
where \( \Delta z = \frac{2l}{2l} = \frac{1}{\Delta z_0} \) is the transverse relative wavenumber resolution. Using this instrumental function, the scattered power can be written
\[ I_{11}(k, \omega) \propto \int dZ S(k, Z, \omega) e^{-\frac{(\Delta z)^2}{\Delta z_0^2}}, \]  
(103)
We will use this simplified equation to study how spatial resolution can be obtained indirectly. To make simulations for this purpose we need to assume a pitch angle profile and an expression for the frequency integrated local spectral density \( S(k, Z) \).

c. Modelled magnetic pitch angle. For our simulations we will take the pitch angle to be described by
\[ \theta_p(r) = \arctan \left( \frac{B}{2l} \right), \]  
(104)
an analytical profile constructed by J. H. Misguich [42], see Fig. 8. Here, \( \rho = r/a \) is the normalized minor radius coordinate, \( q_s \) is the magnetic field winding number at \( r = a \) and \( R_0 \) is the major radius of the plasma. The total pitch angle variation \( \Delta \theta_p \) is seen to be about 15°.

Fig. 8. Modelled pitch angle in degrees versus \( \rho \). We have used \( q_s = 3,3, R_0 = 2.38 \text{ m} \) and \( a = 0.75 \text{ m} \) (Tore Supra parameters, see Ref. 42).

Fig. 9. Modelled normalized density versus \( \rho \).

d. Fluctuation profiles. The frequency integrated local spectral density is assumed to be independent of the selected wave vector \( S(k, r) = S(r) = \delta n^2 \),
(105)
where we have replaced the beam coordinate \( Z \) by the radial coordinate \( r \). The normalized density profile is assumed to be
\[ \frac{n(r)}{n_0} = 0.1 + 0.9 \sqrt{1 - \rho^2}, \]  
(106)
see Fig. 9.

Further, the relative density fluctuation profile is assumed to have the following structure
\[ \frac{\delta n(r)}{n(r)} = b + c|\rho|^p, \]  
(107)
where \( b, c \) and \( p \) are fit parameters. At present we will assume the following fit parameters: \( b = 0.01, c = 0.1 \) and \( p = 3 \), see the left-hand plot of Fig. 10.

e. Simulations. Above we have introduced spatially localized expressions for all external quantities entering Eq. (103). We set the wavenumber \( k = 15 \text{ cm}^{-1} \) and the beam waist \( w = 2.7 \text{ cm} \). This means that the transverse relative wavenumber resolution \( \Delta z \) is equal to 2.8°. Fig. 11 shows \( \chi \) for \( \Delta z = 0° \) (left) and 5° (right).

Fig. 9. Modelled normalized density versus \( \rho \).

Fig. 10. Left: \( \Delta \theta_p \) versus \( \rho \), right: \( \delta n^2 \) versus \( \rho \).

Fig. 11. Left: \( \chi \) versus \( \rho \) for \( \Delta z = 0° \), right: \( \chi \) versus \( \rho \) for \( \Delta z = 5° \) (\( k = 15 \text{ cm}^{-1} \), \( w = 2.7 \text{ cm} \)).
We observe that by changing the diagnostic angle $\alpha$, the relative wavenumber resolution shown in Fig. 11. We see that the 0 case originates in the central part of the plasma, while the 5 case detects edge fluctuations.

Fig. 13 shows figures corresponding to Figs. 11 and 12, but now for a mini $x$-scan: $[-5^\circ, -2.5^\circ, 0^\circ, 2.5^\circ, 5^\circ]$. Fig. 14 shows the integrands in Fig. 13 integrated along $\rho (= L_1 \rho)$. Finally, Fig. 15 shows the effect of increasing the transverse relative wavenumber resolution $\Delta k_w$ from 2.8 to 28.0. The instrumental selectivity function (left) becomes extremely broad, leading to the total scattered power having no significant variation with $\alpha$.

What we have demonstrated with the above simulations is that localization to be possible, the following has to be true

\[ \Delta \theta_{\text{inel}} [\text{degrees}] \gg \Delta \theta_{\text{el}} [\text{degrees}] \]

(108)

Alternatively, even if this criterion is not fulfilled (case 1), some localization can be obtained using case 3: If $\theta_k$ is set so that it is outside the plasma (does not coincide with $\theta_{\text{el}}(\alpha)$ for any $\alpha$), measurements weighted towards the top and bottom of the plasma can be made [4].

IV. Conclusions

In section II, we derived an expression for the detected photocurrent from first principles. Thereafter demodulation was explained, phase separation of the detected signal was interpreted and an expression for the density fluctuations squared was presented. Finally, a simple example illustrated this density fluctuation formula for Gaussian beams.

In section III, the measurement volume was treated in detail. The simple geometrical estimate was compared to a more elaborate treatment. Following this, direct and indirect localization was discussed, and general expressions for autocular Sever power were derived. A discussion ensured, and finally simulations assisted in the interpretation of localized autocular Sever power from a single measurement volume.

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