Perturbative QCD analysis of the nucleon’s Pauli form factor $F_2(Q^2)$

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We perform a perturbative QCD analysis of the nucleon’s Pauli form factor $F_2(Q^2)$ in the asymptotically large $Q^2$ limit. We find that the leading contribution to $F_2(Q^2)$ has a $1/Q^6$ power behavior, consistent with the well-known result in the literature. Its coefficient depends on the leading- and subleading-twist light-cone wave functions of the nucleon, the latter describing the quarks with one unit of orbital angular momentum. We also derive at the logarithmic accuracy the asymptotic scaling $F_2(Q^2)/F_1(Q^2) \sim (\log^2 Q^2/L^2)/Q^2$ which describes recent Jefferson Lab data well.

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The electromagnetic form factors are fundamental observables of the nucleon containing important information about its internal nonperturbative structure. Since the first measurement in the mid 1950s, experimental studies of these observables have become an active frontier in nuclear and particle physics. More recently, with the development of novel experimental techniques, the magnetic form factor of the proton, and electric and magnetic form factors of the neutron have been measured with unprecedented precision at Jefferson Lab and other facilities around the world. In particular, the Pauli form factor of the proton $F_2$ extracted from the recoil polarizations at Jefferson Lab reveals a significant difference from previous data and theoretical expectations. The data shows that the ratio of Dirac to Pauli form factor $Q^2 F_2(Q^2)/F_1(Q^2)$ continues climbing at the largest $Q^2$’s measured and seems to scale as $\sqrt{Q^2}$. The result has sparked myriad speculations on its implication about the underlying microscopic structure of the proton.

A simple dimensional counting rule was devised in Refs. 4, 5 to determine the dominant power contribution to hadronic form factors at large momentum transfer. For the hadron helicity-conserving form factor $F_1(Q^2)$, it predicts a dominant scaling behavior $1/Q^4$. The power counting can be justified by QCD factorization theorems which separate short-distance quark-gluon interactions from soft hadron wave functions, see Fig. 1. Recently, a more sophisticated formalism has been developed to include higher-order perturbative QCD (PQCD) resummation, or Sudakov form factor, which treats contributions from the small-$x$ partons more appropriately. The procedure extends the applicability of leading-order PQCD predictions to moderate $Q^2$, but does not change the dominant power-law behavior significantly.

The literature on PQCD studies of $F_2(Q^2)$, however, is much meager. Since $F_2(Q^2)$ is related to the hadron helicity-flip amplitude, its power behavior is suppressed compared to $F_1(Q^2) \sim 1/Q^4$, and a natural expectation is then $1/Q^6$. This is confirmed by calculations where the quark masses are the mechanism for the helicity flip. Since the up and down current quark masses are negligible, the dominant mechanism for the spin flip in QCD come from the quark orbital angular momentum and the polarization of an extra gluon which must be included to maintain gauge invariance. A generalized power counting including the parton orbital angular momentum validates the expected scaling law for $F_2(Q^2)$. An actual PQCD calculation for $F_2(Q^2)$ requires the extension of the usual formalism to include the quark orbital angular momentum, a technology has not yet been systematically developed in the literature. The recent development in classification of the light-cone wave functions provides the necessary ingredient to perform these calculations.

In this paper we report on a first PQCD calculation of $F_2(Q^2)$ at asymptotically large momentum transfer $Q^2$. Modulo logarithms, we confirm that the leading contribution to $F_2(Q^2)$ goes like $1/Q^6$. The coefficient of the leading-power term depends on leading order (twist-three) and next-to-leading order (twist-four) light-cone wave functions. The latter is the probability amplitude for one of the quarks to carry one unit of orbital angular momentum. We compute the leading-order perturbative kernel, and estimate the coefficient using models for the light-cone wave functions. We comment on the role of the Sudakov form factor in regulating possible end-point singularities in the phase space integrals. We also derive with the logarithmic accuracy the asymptotic scaling $F_2(Q^2)/F_1(Q^2) \sim (\log^2 Q^2/L^2)/Q^2$ which we find to describe the data unexpectedly well.

To start with, we choose a coordinate system for the

![FIG. 1: A leading QCD diagram contributing to the nucleon’s form factors.](image-url)
process $P + \gamma^* \rightarrow P'$ such that $P$ and $P'$ form conju-
gate light-cone vectors. We let the light-cone components $P^+$ and $P^-$ be large in the asymptotic limit, $Q^2 = 2P^+P^-$. The $F_2(Q^2)$ form factor can be extracted from the helicity-flip matrix element

$$\langle P'_+|J^\mu|P_+\rangle = F_2(Q^2) u_1(P') \frac{i\sigma^{\mu\alpha}\gamma_\alpha}{2M} u_1(P),$$

where $J^\mu$ is the quark electromagnetic current and $u(P)$ is an on-shell spinor of the nucleon with momentum $P$. In the above coordinate system, the spin of the nucleon actually remains the same in the initial and final states.

A perturbative analysis of the same matrix element is done by computing leading-order Feynman diagrams with two gluon exchanges, for a typical graph see Fig. in which the initial (final) nucleon contains three quarks with longitudinal momenta $x_1 P$ ($y_i P'$) and transverse momenta $k_i$ ($k'_i$) of order $\Lambda_{QCD}$. Two hard-gluon exchanges ensure that the three quarks in the final state propagate collinearly after the injection of a large momentum transfer $q^\mu$. Since up and down quarks are light, quark helicity is approximately conserved during the hard scattering. Therefore, to produce a nucleon-spin conju-}
gate light-cone amplitudes are the same as those defined in Ref. [29]. In the following, we include the former so that the hard part has no dependence on $k'_i$, we can ig-

If the hard part has no dependence on $k'_i$, we can ignore the transverse momentum dependence in the quark creation operators and define the twist-three amplitude

$$\Phi_3(x_1, x_2, x_3) = 2 \int [d^2k] \psi_1(\kappa_1, \kappa_2, \kappa_3).$$

Then the final nucleon state can be simplified to,

$$|P_1\rangle_{1/2} = \frac{1}{24} \int \frac{[dx]}{\sqrt{x_1 x_2 x_3}} \Phi_3(x_1, x_2, x_3)$$

$$\times \epsilon^{abc} u^\dag_{\alpha\nu}(x_1) \left\{ u_{\alpha\nu}(x_2) d^\dag_{\nu\alpha}(x_3) - d^\dag_{\alpha\nu}(x_2) u_{\nu\alpha}(x_3) \right\} |0\rangle.$$ (3)

In the first quantization formalism, the second line can be replaced by the standard $SU(6)$ wave function.

Since the quarks in the initial nucleon state must have a total helicity $1/2$, only the following wave function component is relevant [27, 28] 

$$|P_2\rangle_{1/2} = \frac{1}{12} \int \frac{[dx]}{\sqrt{x_1 x_2 x_3}} \left\{ \tilde{k}_{1\perp} \psi_3 + \tilde{k}_{2\perp} \psi_4 \right\} (\kappa_1, \kappa_2, \kappa_3)$$

$$\times \epsilon^{abc} u^\dag_{\alpha\nu}(x_2) \left\{ u_{\alpha\nu}(x_1) d^\dag_{\alpha\nu}(x_3) - d^\dag_{\nu\alpha}(x_1) u_{\nu\alpha}(x_3) \right\} |0\rangle.$$ (4)

where we use the notation $\tilde{k}_{\perp} \equiv k_{\perp} - i k y$.

The spin-isospin structure of this wave function is exactly the same as that in Eq. (2). After the collinear expansion, the above wave function will be convoluted with transverse momenta of quarks. We define the twist-four amplitudes via

$$\Phi_4(x_2, x_1, x_3) = 2 \int \frac{[d^2k]}{M_{x_3}} k_3 \cdot \{ k_1 \psi_3 + k_2 \psi_4 \} (\kappa_1, \kappa_2, \kappa_3),$$

$$\Psi_4(x_1, x_2, x_3) = 2 \int \frac{[d^2k]}{M_{x_2}} k_2 \cdot \{ k_1 \psi_3 + k_2 \psi_4 \} (\kappa_1, \kappa_2, \kappa_3).$$

Apart from a gluon-potential dependent term, the above light-cone amplitudes are the same as those defined in Ref. [29]. In the following, we include the former so that the result is gauge invariant. To the order that we are working, the gluon potential terms arise from perturba-
tive diagrams with explicit gluons attached to the nucleon blobs. These contributions are generally considered as dynamically suppressed [30].

We introduce effective nucleon wave functions for the initial state after integrating over the transverse momenta weighted with a momentum factor from the hard part,

$$|P_3\rangle_{1/2}[k_1] = \frac{M}{24} \int \frac{[dx]}{\sqrt{x_1 x_2 x_3}} x_1 x_2 x_3 \psi_3(x_2, x_1, x_3)$$

$$\times \epsilon^{abc} u^\dag_{\alpha\nu}(x_2) \left\{ u_{\alpha\nu}(x_1) d^\dag_{\alpha\nu}(x_3) - d^\dag_{\nu\alpha}(x_1) u_{\nu\alpha}(x_3) \right\} |0\rangle,$$ (5)

$$|P_4\rangle_{1/2}[k_3] = \frac{M}{24} \int \frac{[dx]}{\sqrt{x_1 x_2 x_3}} x_3 x_2 x_3 \psi_4(x_1, x_2, x_3)$$

$$\times \epsilon^{abc} u^\dag_{\alpha\nu}(x_1) \left\{ u_{\alpha\nu}(x_2) d^\dag_{\alpha\nu}(x_3) - d^\dag_{\nu\alpha}(x_2) u_{\nu\alpha}(x_3) \right\} |0\rangle.$$ (6)

where $\kappa_1$ is a shorthand notation for $(x_1, k_1)$, and the integration measures for the quark mo-

menta are

$$[dx] = dx_1 dx_2 dx_3 \delta(x_1 + x_2 + x_3 - 1),$$

$$[d^2k] = d^2k_1 d^2k_2 d^2k_3 \delta^{(2)}(k_1 + k_2 + k_3).$$
where the argument on the left-hand side indicates the momentum being averaged. Averaging over $k_2$ does not yield independent information.

Using the twist-3 and -4 amplitudes, we can write the $F_2(Q^2)$ form factor in the following factorized form,

$$F_2(Q^2) = \int [dx][dy]\{x_3\Phi_4(x_1, x_2, x_3)T_\Phi\{\{x\}, \{y\}\}
+ x_1\Psi_4(x_2, x_1, x_3)T_\Psi\{\{x\}, \{y\}\}\} \Phi_3(y_1, y_2, y_3),$$

where $\{x\} = (x_1, x_2, x_3)$. Let us see how to extract the hard part, $T_i$, in Fig. 1.

We use the nomenclature and strategy of Ref. [11] by calling the top quark line 1, the middle 2, etc., without committing them to a specific flavor. Given the spin-isospin structure of the initial and final state nucleon wave functions, we assume that the first and third quarks have spin up and the second one spin down. Call the amplitudes $T_i$ when the electromagnetic current acts on particle $i$. Clearly because of symmetry, $T_3$ can be obtained from $T_1$ by a suitable exchange of variables (see below). Using the SU(6) wave function and the quark charge weighting, we find the structure of the hard part for the proton,

$$T^p(\{x\}, \{y\}) = \frac{2e_u}{3} T_1 + \frac{e_u + e_d}{3}(T_2 + T_3)
+ \frac{e_u}{3}(T'_1 + T'_3) + \frac{e_d}{3} T'_2,$$

where we have omitted the argument of $T_i$ on the right-hand side, and $T'_i$ has $y_1$ and $y_3$ interchanged. For the neutron, $u \leftrightarrow d$.

According to the above convention, we find the contribution to $T_1$ from Fig. 1

$$T^{d_{1g}}_{1\Phi}(\{x\}, \{y\}) = -\frac{M^2 C_B^2}{Q^6} (4\pi\alpha)^2 \frac{1}{x_1 x_3 y_1 y_3},$$

$$T^{d_{1g}}_{1\Phi}(\{x\}, \{y\}) = -\frac{M^2 C_B^2}{Q^6} (4\pi\alpha)^2 \frac{1}{x_1 x_3 y_1 y_3},$$

where here and everywhere $\bar{x} = 1 - x$. $C_B = 2/3$ is a color factor and the contribution from the twist-4 wave functions of the final-state nucleon has also been included. The corresponding contribution to $T_3$ can be obtained by interchanging the labels, $T^{d_{1g}}_{3\Phi} = T^{d_{1g}}_{1\Phi}(1 \leftrightarrow 3)$, $T^{d_{1g}}_{3\Phi} = T^{d_{1g}}_{1\Phi}(1 \leftrightarrow 3)$. This in fact is a general feature.

To find a complete expression for the hard part, we must calculate perturbative diagrams displayed in Fig. 2. A straightforward evaluation yields,

$$T_{1\Phi}(\{x\}, \{y\}) = \frac{M^2 C_B^2}{Q^6} (4\pi\alpha)^2 x_3 (G_{11} - G_{12}),$$

$$T_{1\Phi}(\{x\}, \{y\}) = \frac{M^2 C_B^2}{Q^6} (4\pi\alpha)^2 [-x_1 G_{11} - \bar{x}_1 G_{12}],$$

$$T_{2\Phi}(\{x\}, \{y\}) = \frac{M^2 C_B^2}{Q^6} (4\pi\alpha)^2 \times [x_3 (G_{22} - \bar{G}_{21} - \bar{G}_{22}) - \bar{x}_3 G_{21}],$$

FIG. 2: Perturbative diagrams contributing to the hard part of $F_2$. Mirror symmetric graphs have to added.

and $T_{2\Phi} = T_{2\Phi}(1 \leftrightarrow 3)$, $T_{3\Phi} = T_{1\Phi}(1 \leftrightarrow 3)$, and $T_{3\Phi} = T_{1\Phi}(1 \leftrightarrow 3)$. The functions $G_{ij}$ are defined as

$$G_{11} = \frac{1}{x_1 x_2 x_3 y_1 y_2 y_3},$$

$$G_{12} = \frac{1}{x_1 x_2 x_3 y_1 y_2 y_3} + \frac{1}{x_1 x_3 y_1 y_3} - \frac{1}{x_2 x_3 y_2 y_3},$$

$$G_{21} = \frac{1}{x_1 x_3 x_2 y_1 y_3 y_2},$$

$$G_{22} = \frac{1}{x_1 x_3 x_2 y_1 y_3 y_2} + \frac{1}{x_1 x_3 y_2 y_3 y_1},$$

and $\bar{G}_{21} = G_{21}(1 \leftrightarrow 3)$, $\bar{G}_{22} = G_{22}(1 \leftrightarrow 3)$.

To determine the normalization for $F_2(Q^2)$, we need to know the light-cone distribution amplitudes $\Phi_3$, $\Phi_4$, and $\Psi_4$, which can only be obtained by solving QCD nonperturbatively. However, the scale evolution of these amplitudes selects at the asymptotically large $Q^2$ the leading component with a fixed small-$x$ behavior. For example, the asymptotic form of $\Phi_3$ is $x_1 x_2 x_3 [11]$, whereas that of $\Phi_4$ is $x_1 x_2$. In Ref. [29] a set of phenomenological amplitudes satisfying these asymptotic constraints have been proposed on a basis of conformal expansion.

With the above wave functions, the integrals over momentum fractions $x_1$ and $y_1$ have logarithmic singularities, indicating that the factorization breaks down when one of the quarks in the wave function becomes soft [19, 20]. It has been suggested that the higher-order PQCD resummation, or the Sudakov form factor, suppresses the contribution at small-$x$ and provides an effective cut-off for the integrals at $x \sim \Lambda^2/Q^2$, where $\Lambda$ is a soft scale related to the size of the nucleon [21, 22, 23, 31, 32]. The outcome is that the $x_1$ integrations contribute an extra $Q^2$-dependent factor $\ln^2 Q^2/\Lambda^2$, compared to $F_1(Q^2)$. Physically, the end-point divergences indicate that quarks with different rapidity contribute equally to the hard scattering. Since the contribution from quarks with very large rapidity (small-
$x$) is suppressed by the Sudakov form factor, this $Q^2$-dependence reflects simply the kinematic broadening of the quark (and gluon) rapidity range with increasing nucleon momentum.

For an estimate, we use asymptotic wave functions [29],

$$\Phi_3 = 120 x_1 x_2 x_3 f_N, \quad \Phi_4 = 12 x_1 x_2 (f_N + \lambda_1),$$

$$\Psi_4 = 12 x_1 x_3 (f_N - \lambda_1),$$

(12)

with $f_N = 5.3 \cdot 10^{-3} \text{ GeV}^2$ and $\lambda_1 = -2.7 \cdot 10^{-2} \text{ GeV}^2$. With a choice of $\Lambda = (0.3 \text{ GeV})^2$, $Q^2 F_3^2(Q^2)$ is roughly 0.6 GeV$^2$ for $Q^2 = (5 - 20) \text{ GeV}^2$, about 1/3 of the Jefferson Lab data at $Q^2 = 5 \text{ GeV}^2$. Of course, to get a more realistic PQCD prediction in this regime, one must have the quark distribution amplitudes appropriate at this scale. However, from the comparison between the data and PQCD predictions for $F_1(Q^2)$, we believe that asymptotic PQCD is unlikely to be the dominant contribution to $F_2(Q^2)$ at $Q^2 = 3 - 5 \text{ GeV}^2$: one must take into account higher-order corrections and higher-twist effects.

![Figure 3](image-url)

FIG. 3: JLab data plotted in terms of the leading PQCD scaling. The low, middle, and upper data points correspond to $\Lambda = 200, 300, 400$, respectively.

Coming back to the scaling behavior of the ratio $F_2(Q^2)/F_1(Q^2)$ for which the Jefferson Lab data has stimulated much discussion in the literature. PQCD predicts the power-law scaling $1/Q^2$. With the new result for $F_2(Q^2)$, we can determine its scaling up to logarithmic accuracy. The strong coupling constant in the ratio simply cancels. The wave function evolution yields a factor of $\alpha_s^{32/(9\beta)}(Q^2)$ for $F_1(Q^2)$ and $\alpha_s^{8/(33)}(Q^2)$ for $F_2(Q^2)$ from the leading non-vanishing contribution, where $\beta = 11 - 2n_f/3$. Thus PQCD predicts that $(Q^2/\ln^2\beta) Q^2/(\Lambda^2)(F_2(Q^2)/F_1(Q^2))$ scales as a constant at large $Q^2$, $8/(9\beta) \ll 1$. Surprisingly, the Jefferson Lab data plotted this way (ignoring the small $8/(9\beta)$) exhibits little $Q^2$ variation for a range of choices of $\Lambda$ as shown in Fig. 3. Since we do not expect the asymptotic predictions for $F_{1,2}(Q^2)$ to work at these $Q^2$, the observed consistency might be a sign of precocious scaling as a consequence of delicate cancellations in the ratio.

A more detailed discussion on this issue along with more thorough phenomenological analyses will be given in a separate publication.

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