Heavy quark-meson mass gap from spectroscopy

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Abstract

Using a quite accurate flavour-independence of $nS$-level mass differences in heavy quarkonia and a corresponding quasiclassical expression for the heavy quark binding energy, one shows that experimental data on masses of mesons with heavy quarks allow one to make the estimate $\bar{\Lambda} = 0.63 \pm 0.03$ GeV.

Introduction

The QCD dynamics makes a screen in observation of pure electroweak effects in heavy quark interactions. One of powerful tools in theoretical studies of hadrons with a single heavy quark, is HQET [1], where the heavy quark-meson mass gap $\bar{\Lambda} = m(q\bar{q}) - m_Q + O(\bar{\Lambda}/m_Q)$ has a significant meaning. In the $\bar{\Lambda}$ expression given above, $m(q\bar{q})$ denotes the heavy-light meson mass $m(q\bar{q}) = (3m_V(q\bar{q}) + m_P(q\bar{q}))/4$, averaged over the spin-dependent part of interaction and correspondingly expressed through the masses of vector and pseudoscalar states, and $m_Q$ is the heavy quark pole mass related to its "running" mass [2].

At present, $\bar{\Lambda}$ is evaluated in the following ways. The first one is the HQET sum rules giving $\bar{\Lambda} = 0.5 \pm 0.1$ GeV [3]. The second is the full QCD sum rules for heavy quarkonia [4], where the pole or "running" masses of heavy quarks are determined from the experimental data on the heavy quarkonia masses and leptonic constants, so that $\bar{\Lambda} = 0.6 - 0.7$ GeV [5]. A special way is the nonrelativistic version of QCD sum rules for bottomonium [6], where one can use a region of moderate values of the spectral function moment numbers, so that the nonperturbative contribution given by the gluon condensate can be still neglected and the Coulomb corrections can be quite accurately taken into account. Voloshin M. has found the "$\mu$-independent" $b$ quark pole mass $m^*_b = m_b - 0.56\alpha_s(\mu) = 4.639 \pm 0.002$ GeV, corresponding to $\bar{\Lambda} = 0.62 \pm 0.02$ GeV. Next, one considers the inclusive semileptonic width
of a heavy-light meson in a way allowing one to take into account hard gluon corrections to the weak current of quarks, so that one inserts light quark loops into the gluon propagator. The comparison of the calculated width with the experimental value gives the "running" mass of heavy quark. Its value depends on the $l$ number of the light quark loops. In the infinite $l$ limit, one finds $\bar{\Lambda} = 0.25$ GeV [7]. However, the latter estimate of hard corrections is generally based on the spectator mechanism assuming the neglecting of the heavy quark binding into the meson. Despite the renormalon ambiguity in the heavy quark mass determination [8], one should not straightforwardly conclude that there is a deep disagreement between the sum rule estimates of $\bar{\Lambda}$ and the evaluation in the decay analysis, since the latter is based on the consideration of isolated, but dressed heavy quark, whereas the sum rules handle with exactly defined quantities involving no additional assumptions like the spectator picture. Moreover, the sum rule analysis is made for a finite number of loops.

Further, the nonrelativistic sum rule consideration of bottomonium can give a quite accurate value of $b$ quark mass at the moment numbers, where the gluon condensate gives a negligible contribution. In the sum rules at high numbers of the spectral moments, the gluon condensate contribution determines the binding energy in the $1S$-state and, hence, the difference between the double quark mass and the level mass. However, to use experimental data on the excited level masses and leptonic constants is enough to extract the quark mass at the moderate values of the spectral moment numbers. Therefore, the information on the excitation masses can allow one to determine the heavy quark masses.

In the present paper we use the experimental regularity in the heavy quarkonium spectra, where one finds a quite accurate flavour-independence in the $S$-wave level spacing. This approximate independence results in a number of explicit relations between the quark masses, excitation numbers and binding energies [9]. These equations compose a complete system, which allows one to determine the heavy quark-meson mass gap of high accuracy

$$\bar{\Lambda} = 0.63 \pm 0.03 \text{ GeV}.$$  

1 Basic relations

Determine the heavy quark pole mass

$$m_Q = m(Q\bar{q}) - \bar{\Lambda} - \frac{\mu^2}{2m(Q\bar{q})} + O(1/m_Q^2), \quad (1)$$
where $\mu_\pi^2$ is the average square of quark momentum inside a heavy-light meson. In the following we put

$$\mu_\pi^2 = 2\langle T \rangle \mu_{Q \bar{q}} , \tag{2}$$

where $\mu_{Q \bar{q}}$ is the reduced mass of the $Q \bar{q}$ system, and $T$ is the kinetic energy of quarks. The reasonable choice of $\mu_{Q \bar{q}}$ is $\bar{\Lambda} [1, 10]$. The $m(Q \bar{q})$ values for $Q = b, c$ and $q = u, d$ are known experimentally \[1\]

$$m_B(1S) = 5.313 \text{ GeV}, \quad m_D(1S) = 1.975 \text{ GeV}, \tag{3}$$

with the accuracy better than 5 MeV.

By the Feynman - Hellmann theorem for the $Q_1 \bar{Q}_2$ system

$$\frac{dE}{d\mu_{12}} = -\frac{\langle T \rangle}{\mu_{12}}, \quad \mu_{12} = \frac{m_1 m_2}{m_1 + m_2} , \tag{4}$$

the flavour-independent spacing of heavy quarkonium levels is reached if $\langle T \rangle = T$ is the constant value. Relation (4) fixes the quark mass dependence of the binding energy in heavy quarkonia. To complete the system of equations, we use the Bohr - Sommerfeld quantization of $nS$-states at the constant $\langle T \rangle$ in the flavour-independent logarithmic potential

$$E(n) = C + T \ln \frac{n^2}{\mu_{12}} , \tag{5}$$

where $C$ is a flavour-independent constant. Eq.(5) gives the excitation number dependence of the binding energy in the heavy quarkonia. Note, that the semiclassical WKB approximation of 3-dimensional potential problem leads to the substitution of $n - 1/4$ for $n$ in (5). However, the latter substitution does not result in the better description of experimental data\[1\] (Fig.1). To isolate the $n$ dependence, we consider the ratio $\alpha(n)$

$$\alpha(n) = \frac{M(nS) - M(1S)}{M(2S) - M(1S)} = \frac{\ln n}{\ln 2} \tag{6}$$

in the current model or

$$\alpha_{\text{WKB}}(n) = \frac{\ln[(4n - 1)/3]}{\ln[7/3]} .$$

The comparison of model approximation (6) with the experimental values \[1\] and WKB modification is shown in Fig.1. One can see that the applied

\[1\] The same tendency was certainly observed in the Coulomb potential, where the Bohr–Sommerfeld equation gave exact results.
The model is more suitable for the accurate description of $M(nS)$ values with the accuracy up to 30 MeV, so that the parameter $T$ is equal to

$$T = 0.43 \pm 0.02 \text{ GeV}. \quad (7)$$

Note, that in contrast to the analysis in [9], we use the excitation masses averaged over the spin-dependent part of a potential, this procedure is more reasonable. In addition, the analysis of heavy quarkonium spectra performed in [9] under the WKB approximation results in the smaller value of $T \approx 0.37$ GeV. The mass of $nS$-level is determined by the following

$$M(nS) = m_1 + m_2 + E(n).$$

For the sake of convenience we introduce the "initial" value $n_i(\mu_{12})$ depending on the reduced mass and related to the flavour-independent constant $C$ in [9]

$$C = -T \ln \frac{n_i^2(\mu_{12})}{\mu_{12}}. \quad (8)$$

Let us use the experimental data on the heavy quarkonium spectra

$$M_T(4S) \approx 2m_B(1S), \quad M_\psi(3S) \approx 2m_D(1S),$$
which are valid with the 30 MeV accuracy. These equations can be rewritten down as:

\[ 2T \ln \frac{n_{th}(b\bar{b})}{n_i(b\bar{b})} = 2\bar{\Lambda} + \frac{\mu^2}{m_b}, \]  
\[ 2T \ln \frac{n_{th}(c\bar{c})}{n_i(c\bar{c})} = 2\bar{\Lambda} + \frac{\mu^2}{m_c}, \]

where \( n_{th}(b\bar{b}) = 4, n_{th}(c\bar{c}) = 3 \). From eq.(8) one can find

\[ \ln \frac{n_i(b\bar{b})}{n_i(c\bar{c})} = \frac{1}{2} \ln \frac{m_b}{m_c}. \]  

Combining (2), (9-11), one gets

\[ \bar{\Lambda} = \frac{m_b m_c}{m_b - m_c} \ln \left\{ \sqrt{\frac{m_b n_{th}(c\bar{c})}{m_c n_{th}(b\bar{b})}} \right\}, \]  

where the dependence on the \( T \) parameter is hidden in explicit relations for the quark masses through the heavy-light meson masses and \( \bar{\Lambda} \).

Eq.(12) can be solved numerically, and it gives:

\[ \bar{\Lambda} = 0.63 \pm 0.03 \text{ GeV}. \]  

As for the quark masses in the first order over \( 1/m_Q \), one finds

\[ m_b = 4.63 \pm 0.03 \text{ GeV}, \quad m_c = 1.18 \pm 0.07 \text{ GeV}. \]

The additional uncertainty in \( c \) quark mass estimate is related to the replacement \( \bar{\Lambda}/m(Q\bar{q}) \to \bar{\Lambda}/(m(Q\bar{q}) - \bar{\Lambda}) + O(\bar{\Lambda}^2/m_Q^2) \) in the heavy quark mass expression, i.e. it is caused by terms of the second order over the inverse heavy quark mass.

The \( \mu^2 \) parameter is equal to \( 0.54 \pm 0.08 \text{ GeV}^2 \).

The performed calculations allow one to predict the \( nS \)-level masses of \( \bar{b}c \) family below the \( BD \) pair threshold

\[ m_{B_c}(1S) = 6.37 \pm 0.04 \text{ GeV}, \quad m_{B_c}(2S) = 6.97 \pm 0.04 \text{ GeV}. \]

The 1S-level position is slightly higher than in previous estimates in the framework of potential models. This deviation is basically caused by the greater value of \( T \) parameter, but not by the difference in the quark mass

\[ \text{The analogous estimate with the additional assumption } n_i(b\bar{b}) = 1 \text{ was considered in } [12]. \]

\[ \text{The obtained result is in a good agreement with the restrictions derived in } [13]. \]
values, since the $\bar{b}c$ level masses are not very much sensitive to the quark mass variation. Using the estimate of spin-dependent splitting of $1S$-level in $\bar{b}c$ system, $m(1^{-}) - m(0^{-}) \approx 60 - 70$ MeV [1,4], one gets the mass of the basic pseudoscalar state

$$m_{B_{c}}(0^{-}) = 6.32 \pm 0.05 \text{ GeV}.$$ 

Conclusion

Using the regularity of heavy quarkonium spectra described within the quasiclassical approach, we have evaluated the heavy quark-meson mass gap \( \bar{\Lambda} = 0.63 \pm 0.03 \text{ GeV} \), as well as the pole masses of $b$ and $c$ quarks. The $B_{c}$ meson mass can be predicted $m_{B_{c}} = 6.32 \pm 0.05$ GeV.

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6
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