One-Loop Renormalization of Pure Yang-Mills with Lorentz Violation

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The explicit one-loop renormalizability of pure Yang-Mills theory with Lorentz violation is demonstrated. The result is consistent with multiplicative renormalization as the required counter terms are consistent with a single re-scaling of the Lorentz-violation parameters. In addition, the resulting beta functions indicate that the CPT-even Lorentz-violating terms increase with energy scale in opposition to the asymptotically free gauge coupling and CPT-odd terms. The calculations are performed at lowest-order in the Lorentz-violating terms as they are assumed small.

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I. INTRODUCTION

In practice, quantum field theories of interest are defined by Lagrangians which are chosen to obey certain symmetry requirements. It is well known that many such quantum field theories exhibit ultraviolet divergences which arise due to the structure of the theory at small distances. These divergences can sometimes be removed by a singular redefinition of the parameters defining the theory via the processes of renormalization (the Standard Model affords an example). It is often the case that the calculations involved in establishing renormalizability of a given theory involve a series of remarkable cancellations in which multiple diagrams combine to produce precisely the correct values of the counter-terms required for multiplicative renormalization. In these calculations, gauge symmetries play a crucial role in obtaining the required cancellations. This is certainly the case in standard Yang-Mills theory [1], where (internal) Lorentz symmetries are included in the gauge group [2]. The primary purpose of this paper is to investigate the explicit renormalization properties of Yang-Mills theory in the presence of Lorentz violation; more precisely, whether or not multiplicative renormalization can be applied as in the conventional case. As our main result we establish explicit one-loop renormalizability of pure Yang-Mills theory with Lorentz violation.

This work should be viewed as part of an extensive, systematic investigation of Lorentz violation and its possible implications for Planck-scale physics [3]. Our calculations are carried out in the framework of an explicit theory, the Standard Model Extension (SME), which has been formulated to include possible Lorentz-violating background couplings to Standard Model fields [4]. This formalism has also been extended to include gravity [5]. Extensive calculations using the SME have led to numerous experiments [6], which have in turn placed stringent bounds on parameters in the theory associated with electrons, photons [7], and hadrons. To date, however, the same can not be said for nonabelian sectors of the theory. More precisely, there has been no systematic investigation of the consistency of nonabelian gauge theories in the presence of Lorentz violation and there are very few specific experimental bounds on the nonabelian gauge parameters involved in the SME.
There are a number of reasons to investigate the consistency of nonabelian gauge theories in the presence of Lorentz violation, and to develop good bounds for the associated nonabelian gauge parameters. For example, many new experimental results in the neutrino sector suggest that some type of modification to the standard model will be required to explain the results. With this as motivation, the SME has been applied to the neutrino sector [8, 9] with the hope of generating a realistic model of oscillations. Lorentz violation may play an interesting role in weak interactions which contain gauge bosons that have not yet been analyzed in detail. In addition, QCD involves a strong coupling that can be large and lead to more significant radiative corrections. Finally, there are also indications based on general Renormalization Group analysis that gauge theories with nonpolynomial interactions naturally tend to violate Lorentz invariance [10].

Previous related theoretical work includes an analysis of the explicit one-loop structure of Lorentz-violating QED and the resulting running of the couplings [11]. In this case, conventional multiplicative renormalization is found to succeed and the beta functions indicate a variety of running behaviors, all controlled by the running of the charge (in contrast to QED, however, nonabelian theories are generally asymptotically free [12]). Part of this analysis has been extended to allow for a curved-space background [13], while other analysis involved finite, but undetermined radiative corrections due to CPT violation [14]. Other related work includes a study of deformed instantons in the theory [15], an analysis of the Casimir effect in the presence of Lorentz violation [16], and an analysis of gauge invariance of Lorentz-violating QED at higher-orders [17]. Some investigations into possible Lorentz-violation induced from the ghost sector of scalar QED have also been performed [18].

I. NOTATION AND CONVENTIONS

The pure Yang-Mills lagrangian with Lorentz violation is taken as the most general gauge invariant and power-counting renormalizable action

\[ S(A) = -\frac{1}{2} \int d^4x \text{Tr} \left[ F_{\mu\nu}F_{\mu\nu} + (k_F)_{\mu\nu\alpha\beta}F_{\mu\nu}F^{\alpha\beta} + (k_{AF})_{\kappa\kappa\lambda\mu}(A^\lambda F_{\mu\nu} - \frac{2}{3}igA^\lambda A^\mu A^{\nu}) \right]. \]  

(1)
The generators of the Lie Algebra defined by $A^\mu = A^\alpha t^\alpha$ are taken to satisfy

$$[t^a, t^b] = i f^{abc} t^c ,$$

where $f^{abc}$ are totally anti-symmetric structure constants. The trace of a product of these generators is normalized to

$$Tr[t^at^b] = C_2(r) \delta^{ab} ,$$

where $C_2(r)$ is the quadratic casimir of the representation $r$. In the adjoint representation used for the gauge fields, this is written $C_2(G)$. The field tensor is defined as

$$F^{\mu \nu} = -i g[D^{\mu}, D^{\nu}] ,$$

where the covariant derivative is $D^{\mu} = \partial^{\mu} + igA^{\mu}$.

The $k_F$ term is CPT even while the $k_{AF}$ term is CPT odd. The radiative corrections can therefore be calculated separately. In addition, properties of $F^{\mu \nu}$ imply that the $k_F$ parameters have the symmetries of the Riemann tensor and satisfy the Jacobi identity

$$k_F^{\lambda \mu \nu \rho} + k_F^{\lambda \nu \rho \mu} + k_F^{\lambda \rho \mu \nu} = 0 .$$

To simplify the analysis, $k_F$ is taken to be trace free

$$g_{\mu \alpha} k_F^{\mu \alpha \beta} = 0 .$$

Nontrivial trace components are more easily handled using coordinate redefinitions and can be removed from a pure Yang-Mills theory, at least at the classical level [15]. Expansion of the resulting Lagrangian and including a ghost field to integrate over gauge yields the conventional terms

$$\mathcal{L}_{\text{LI}} = -\frac{1}{4} \left( \partial^{\mu} A^{\alpha \nu} \partial_{\mu} A_{\nu}^\alpha - \partial^{\nu} A^{\alpha \mu} \partial_{\mu} A_{\nu}^\alpha + \frac{1}{4} (\partial^{\mu} A_{\nu}^\alpha)^2 \right) + gf^{abc} A^{\alpha \mu} A^{\beta \nu} \partial_{\mu} A_{\nu}^c$$

$$-\frac{1}{4} g^2 f^{\alpha \beta \gamma} f^{\delta \epsilon \zeta} A^{\alpha \mu} A^{\beta \nu} A_{\mu}^\delta A_{\nu}^\epsilon + \mathcal{L}_g ,$$

where the ghost Lagrangian is written in terms of the scalar, anticommuting field $c$

$$\mathcal{L}_g = \bar{c} ( -\partial^{\mu} \partial_{\mu} c + g \partial^{\mu} f^{abc} A^{b \mu}) c^c .$$
The Lorentz-violating terms are separated into a CPT-even piece
\[
\mathcal{L}_{\text{LVE}} = (k_F)_{\mu\nu}\alpha\beta \left[ -\partial^\mu A^{a\nu} \partial^\alpha A^{a\beta} + gf^{abc}(\partial^\mu A^{c\nu})A^{a\alpha}B^{b\beta} - \frac{1}{4} g^2 f^{abe} f^{cde} A^{a\mu} A^{b\nu} A^{c\alpha} A^{d\beta} \right],
\]
and a CPT-odd piece
\[
\mathcal{L}_{\text{LVO}} = - \frac{1}{2} (k_{AF})^\kappa \epsilon_{\kappa\lambda\mu\nu} A^{a\lambda} \partial^\mu A^{a\nu} + \frac{1}{6} g (k_{AF})^\kappa \epsilon_{\kappa\lambda\mu\nu} f^{abc} A^{a\lambda} B^{b\mu} A^{c\nu}.
\]
There will be a corresponding counter-term \( \delta_i \) for each of these terms in the above Lagrangian. Specifically, the standard counter-terms are defined as
\[
\mathcal{L}_{\text{ctLI}} = \mathcal{L}_{\text{ctLVE}} = \mathcal{L}_{\text{ctLVO}} = \mathcal{L}_{\text{ct}} = \delta \mathcal{L}.
\]

Explicit multiplicative renormalizability implies relations between the counter-terms analogous to the standard case. This occurs because the bare fields and couplings are defined in terms of the finite renormalized fields and couplings as follows: \( A_B^\mu = \sqrt{Z_3} A_0^\mu \), \( g_B = Z_2 g_0 \), \( (k_F)_B = Z_{k_F} k_F \), and \( (k_{AF})_B = Z_{k_{AF}} k_{AF} \). Success of this prescription requires the appropriate counter-term relations that will be demonstrated in this paper. The calculation will be performed first for the CPT-even terms, then for the CPT-odd terms. Only first-order terms in Lorentz violation will be retained, as they are assumed to be miniscule in magnitude. It is then possible to consider the CPT-even and CPT-odd independently since they do not mix (at lowest-order) under radiative corrections due to their different symmetry properties.
Throughout the paper, dimensional regularization is used to define the divergent integrals and standard field-theoretic techniques are used to extract the divergent terms [19].

II. FEYNMAN RULES FOR CPT-EVEN TERMS

The standard gluon propagator (in arbitrary $\xi$ gauge) is

$$\mu, a \quad \leftrightarrow \quad \nu, b = -i\delta^{ab}(g^{\mu\nu} - (1 - \xi)l^\mu l^\nu)/l^2$$  \hspace{1cm} (14)

A single insertion of $k_F$ is indicated by a dot on the gluon line

$$\mu, a \quad \leftrightarrow \quad \nu, b = -2i\delta^{ab}k_F^{\alpha\beta\mu\nu}q_\alpha q_\beta$$ \hspace{1cm} (15)

The standard three-point vertex is given by

$$= -gf^{abc}[g^{\mu\nu}(k - p)^\rho + g^{\nu\rho}(p - q)^\mu + g^{\rho\mu}(q - k)^\nu]$$  \hspace{1cm} (16)

A correction to this vertex due to the $k_F$ term is denoted using a dot

$$= 2gf^{abc}[k_F^{\alpha\mu\nu\rho}k_\alpha + k_F^{\alpha\nu\rho\mu}p_\alpha + k_F^{\alpha\rho\mu\nu}q_\alpha] = 2g(V_k)^{abc}_{\mu\nu\rho}$$ \hspace{1cm} (17)

In these expressions, the gluons are labeled using the associations $(a, \mu, k)$, $(b, \nu, p)$, and $(c, \rho, q)$. All momenta are defined going into the vertex. The corresponding four-point vertices are given by

$$= -ig^2[f^{abc}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + \text{perms}]$$ \hspace{1cm} (18)

The gluons are labeled clockwise using $(a, \mu)$, $(b, \nu)$, $(c, \rho)$, and $(d, \sigma)$ and the two permutation terms are obtained from the first term by the replacements $(b, \nu)$ $\leftrightarrow$ $(c, \rho)$ and $(b, \nu)$ $\leftrightarrow$ $(d, \sigma)$. The correction to the four-point vertex is

$$= -2ig^2[f^{abc}f^{cde}k_F^{\mu\nu\rho\sigma} + \text{perms}] = -2ig^2(V_k)^{abcd}_{\mu\nu\rho\sigma}$$ \hspace{1cm} (19)
with two permutation terms given by the same replacements indicated in the standard
four-point vertex.

The standard divergent piece of the gluon two-point function is given in our no-
tation by

$$\frac{1}{2} \begin{array}{c}
\text{tadpole} + \\
\text{dotted lines indicate ghosts,}
\end{array} \begin{array}{c}
(\frac{5}{3} + \frac{1}{2}(1 - \xi))i(q^2 g_{\mu\nu} - q^\mu q^\nu)\delta^{ab} \tilde{g}^2
\end{array}$$

which the 1/2 denotes the symmetry factor, the dotted lines indicate ghosts,

$$\tilde{g}^2 = \frac{g^2}{(4\pi)^2} C_2(G) \Gamma(2 - \frac{d}{2}) ,$$

and $C_2(G)$ is the standard quadratic Casimir element of the adjoint representation
of the algebra. The required counter-term is therefore

$$\delta_3 = (\frac{5}{3} + \frac{1}{2}(1 - \xi))\tilde{g}^2 = Z_3 - 1$$

where $A_B^\mu = \sqrt{Z_3} A^\mu$ gives the field strength renormalization.

Explicit calculation of the corresponding standard three- and four-point functions
lead to the following standard counter-terms

$$\delta^3_g = (\frac{2}{3} + \frac{3}{4}(1 - \xi)\tilde{g}^2 = Z_g - Z_3^{3/2} - 1$$

where $g_B = Z_g g$ is the charge renormalization, and

$$\delta^{4g}_1 = - (\frac{1}{3} + (1 - \xi))\tilde{g}^2 = (Z_g Z_3)^2 - 1.$$ 

All of these results are consistent with multiplicative renormalization conditions $Z_g = 1 - (11/6)\tilde{g}^2$ and $Z_3 = 1 + (5/3 + 1/2(1 - \xi))\tilde{g}^2$. Note that the gauge contribution
$\xi \neq 1$ is absorbed entirely by $Z_3$. This will turn out to be true in the Lorentz-violating
case as well.

**III. ONE-LOOP CPT-EVEN RESULTS**

Incorporation of the Lorentz-violating terms in Eq.(9) yields three topologically
distinct and nontrivial correction terms to the two-point function. The direct corrections
to the three-point vertices within the two-point function yields

$$\frac{1}{2} \begin{array}{c}
\text{tadpole} + \\
\text{dotted lines indicate ghosts,}
\end{array} \begin{array}{c}
(\frac{28}{3} - (1 - \xi))i\delta^{ab} \tilde{g}^2 k_F\delta^{\mu\nu} q_\alpha q_\beta
\end{array} .$$
There is one more nontrivial diagram that includes an insertion on an internal line

\[ \begin{align*}
&= \left(-\frac{4}{3} + 2(1 - \xi)\right)i\delta^{ab}g^2k_F^\alpha\mu^\beta\nu q_\alpha q_\beta.
\end{align*} \tag{26} \]

Note that there is only one diagram of this type as an insertion on the other internal line is not topologically distinct. Combining the above diagrams yields the required two-point counter-term

\[ \delta^{2g}_{k_F} = (4 + \frac{1}{2}(1 - \xi))g^2, \tag{27} \]

to cancel the divergent piece. This indicates that the bare couplings \((k_F)_B\) should be renormalized using the factor \((k_F)_B^{\mu\nu\rho\sigma} = Z_{k_F}(k_F)^{\mu\nu\rho\sigma}\) with

\[ Z_{k_F} = 1 + \frac{7}{3}g^2. \tag{28} \]

Note that the renormalization of the physical couplings \(k_F\) and \(g\) remain independent of the gauge choice. Re-scaling \(k_F\) in this way has implications for the required values for the Lorentz-violating three- and four-point vertices that will now be analyzed.

Calculation of the corrections to the three-point vertex yields the following results

\[ \begin{align*}
\text{cross terms} &= \left(3 - 3(1 - \xi)\right)g\tilde{g}^2(V_k)^{abc}_{\mu\nu\rho}, \tag{29} \\
\text{cross terms} &= \left(-\frac{3}{4} + \frac{3}{4}(1 - \xi)\right)g\tilde{g}^2(V_k)^{abc}_{\mu\nu\rho}, \tag{30} \\
\frac{1}{2} \quad &+ \text{cross terms} = \left(-9 + \frac{9}{4}(1 - \xi)\right)g\tilde{g}^2(V_k)^{abc}_{\mu\nu\rho}, \tag{31} \\
\frac{1}{2} \quad &+ \text{cross terms} = \left(\frac{3}{4} - \frac{3}{2}(1 - \xi)\right)g\tilde{g}^2(V_k)^{abc}_{\mu\nu\rho}. \tag{32}
\end{align*} \]

In the above diagrams, symmetry factors of 1/2 and appropriate cross-terms are indicated. The presence of several dots in a single diagram indicates a sum over
diagrams with each choice for the vertex or internal line correction. The sum of these diagrams yields a required counter term of
\[
\delta_{kF}^{3g} = (3 + \frac{3}{4}(1 - \xi))\tilde{g}^2.
\] (33)
This is in fact perfectly consistent with multiplicative renormalization condition of equation (28) predicted by the correction to the two-point function. Specifically, the above counter-term takes the form \(\delta_{k}^{3g} = Z_{g}Z_{3}^{3/2}Z_{kF} - 1\) as expected.

An analogous calculation of the four-point function (performed in \(\xi = 1\) gauge for calculational simplicity) yields the counter-term
\[
\delta_{k}^{4g} = 2\tilde{g}^2,
\] (34)
which is again consistent with the multiplicative renormalization prediction of Eq.(28) \(\delta_{kF}^{4g} = Z_{g}^{2}Z_{3}^{2}Z_{kF} - 1\).

IV. ONE-LOOP CPT-ODD RESULTS

Next, attention is turned to the CPT-odd \(k_{AF}\) terms given in Eq.(10). The Feynman rules for the quadratic terms (indicated again by corresponding dots) are given by
\[
\mu, a \quad \cdots \cdots \quad \nu, b = \delta^{ab}(k_{AF})^{\kappa}\epsilon_{\kappa\mu\beta\nu}p^{\beta},
\] (35)
where the momentum \(p\) flows from \(\mu\) to \(\nu\). The cubic term contributes the following correction to the three-point vertex
\[
= igf^{abc}(k_{AF})^{\kappa}\epsilon_{\kappa\mu\nu\rho}.
\] (36)
Note that \(k_{AF}\) has dimensions of energy and therefore decreases the power-counting divergences of all graphs by one power.

The nontrivial contributions of the CPT-odd terms to the two-point function are the same as shown in the diagrams of Eq.(25) and Eq.(26). The resulting required counter-term is given by
\[
\delta_{k_{AF}}^{2g} = (-2 + \frac{1}{2}(1 - \xi))\tilde{g}^2.
\] (37)
This suggests renormalization of the $k_{AF}$ parameter according to $(k_{AF})^\kappa_B = Z_{k_{AF}} (k_{AF})^\kappa$ with

$$Z_{k_{AF}} = 1 - \frac{11}{3} \tilde{g}^2 .$$

(38)

Note that the renormalization of the physical coupling is independent of gauge. This again has implications for the radiative corrections to the three-point function. An explicit calculation analogous to the one involving the CPT-even terms yields the required counter-term

$$\delta_{k_{AF}}^{3g} = (-3 + \frac{3}{4}(1 - \xi)) \tilde{g}^2 .$$

(39)

This is exactly equal to the prediction based on the multiplicative renormalization structure $\delta_{k_{AF}} = 1 + Z_{k_{AF}} Z_g Z_3^{3/2}$. Note that there is no divergent contribution to the four-point diagram due to the dimensionality of $k_{AF}$.

V. BETA FUNCTIONS

Tacitly assuming for the moment that our renormalization prescription can be extended to all orders, the renormalization constants $Z_{k_F}$ and $Z_{k_{AF}}$ can be used to deduce the one-loop beta functions for these parameters. Following the developments presented in [11], use is made of

$$\beta_{x_j} = \lim_{\epsilon \to 0} \left[ -\rho_{x_j} a_1^j + \sum_{k=1}^N \rho_{x_k} x_k \frac{\partial a_1^j}{\partial x_k} \right] ,$$

(40)

where $x_j$ represents an arbitrary running coupling in the theory. The parameters $\rho_{x_j}$ are determined by comparing the mass dimension of the renormalized parameters to the bare parameters in $d = 4 - 2\epsilon$ dimensions

$$x_{jB} = \mu^{\rho_{x_j} \epsilon} Z_{x_j} x_j .$$

(41)

This gives the values

$$\rho_g = 1 , \quad \rho_{k_F} = \rho_{k_{AF}} = 0 .$$

(42)

As in the QED case [11], the coupling $g$ completely controls the running of the Lorentz-violating parameters. The resulting beta functions are

$$\beta_g = -\frac{11 g^3}{3(4\pi)^2} C_2(G) ,$$

(43)
the same as the conventional case, and

\[
\beta_{k_F} = \frac{14g^2}{3(4\pi)^2}C_2(G)k_F, \quad \beta_{k_{AF}} = -\frac{22}{3} \frac{g^2}{(4\pi)^2}C_2(G)k_{AF}.
\]

(44)

Note that the Lorentz indices have been suppressed for simplicity. Introducing the parameter

\[
Q(\mu) = 1 + \frac{22g_0^2}{3(4\pi)^2}C_2(G) \ln \frac{\mu}{\mu_0},
\]

(45)

and defining the initial conditions \(x_{j0} = x_j(\mu_0)\) at the scale \(\mu_0\) allows the solutions to the renormalization group equations to be put in the form

\[
g^2(\mu) = Q^{-1}g_0^2,
\]

(46)

reproducing the conventional result of asymptotic freedom for \(g\) and

\[
k_F = (k_F)_0Q^{7/11}, \quad k_{AF} = (k_{AF})_0Q^{-1}.
\]

(47)

The CPT-odd \(k_{AF}\) term behaves analogously to \(g\), while the CPT-even parameter \(k_F\) increases with energy scale. This suggests that Lorentz-violation involving CPT-even effects in the strong interactions (where \(g_0\) can be relatively large) may increase significantly at higher-energy scales.

V. SUMMARY

In short, multiplicative renormalization is remarkably successful for the case of Yang-Mills theory with Lorentz violation. There are many opportunities for inconsistencies to arise, but they do not actually occur, at least at the one-loop level. The main numerical results can be summarized by the values for the renormalization factors given in Eqs.(28) and (38). These results yield beta functions that can be used to determine the running of the Lorentz-violating couplings under the assumption of full renormalizability of the model. This indicates that the \(k_F\) parameter increases with energy scale, while \(k_{AF}\) decreases with energy scale. The behavior of the standard coupling \(g\) remains asymptotically free, as in the conventional case. Note that the beta functions are independent of the gauge parameter \(\xi\), in accord with the results found in [11]. Our results indicate that in contrast to the QED case, the couplings...
in QCD may be significantly larger and lead to observable effects. The running of the parameters may have more of an impact due to the advantage gained by the size of $\alpha_s$ compared to $\alpha$. A detailed study will involve extending the current model to include the quark sector.

Note that diagrams involving ghosts did not appear in any of the calculations of Lorentz-violating corrections. This can be attributed to the fact that ghost sector Lorentz violation terms can only have symmetry properties that match the trace components of $k_F$ resulting in no contribution (at lowest-order) to trace-free Lorentz-violation parameters. Since the trace components of $k_F$ were neglected in this paper, the ghosts only contribute to conventional, Lorentz invariant terms and do not need modification. The required counter-term structure in the ghost sector is therefore the same as in the conventional case. As noted previously, for a pure Yang-Mills theory, trace components of $k_F$ are better handled by transforming to skewed coordinates with a nondiagonal metric. Violations in the ghost sector should naturally arise due to the resulting skewed form of the metric.

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