What can the braking indices tell us about pulsars’ nature?

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Abstract

As a result of observational difficulties, braking indices of only six rotation-powered pulsars are obtained with certainty, all of which are remarkably smaller than the value (n = 3) expected for pure magnetodipole radiation model. This is still a real fundamental question not being well answered after nearly forty years of the discovery of pulsar. The main problem is that we are shamefully not sure about the dominant mechanisms that result in pulsars’ spin-down. Based on the previous works, the braking index is re-examined, with a conclusion of suggesting a constant gap potential drop for pulsars with magnetospheric activities. New constrains on model parameters from observed braking indices are presented.

Key words: dense matter, pulsars: general, stars: neutron
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1 Introduction

In pulsar emission models, we usually have the assumption

\[ \dot{\Omega} \propto \Omega^n, \]  

(1)

where \( \Omega = 2\pi/P \), \( P \) is the spin period, the index, \( n \), is usually assumed to be a constant measuring the efficiency of braking. We can then define the braking index, \( n \), and the second braking index, \( m \),

\[ n = \frac{\Omega \ddot{\Omega}}{\Omega^2}, \quad m = \frac{\Omega^2 \dddot{\Omega}}{\Omega^3}, \]  

(2)

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By investigating \( n \) and \( m \), we can get much information of pulsar’s radiation and spin-down mechanism. In those models where spin-down due to pure magnetodipole radiation is assumed, one has only \( n = 3 \).

The fairly accurate timing property of pulsars give us the opportunity to measure not only the period \( P \) and the period derivative \( \dot{P} \), but also the second period derivative \( \ddot{P} \) and even the third. However, there do exist difficulties in observation. As a result, braking indices of only six rotation-powered pulsars are obtained with certainty, i.e. PSR J1846-0258 \([ n = 2.65(1) \] \), PSR B0531+21 \([ 2.51(1), \) the Crab pulsar], PSR B1509–58 \([ 2.839(3) \] \), PSR J1119–6127 \([ 2.91(5) \] \), PSR B0540–69 \([ 2.140(9) \] \) and PSR B0833–45 \([ 1.4(2) \], the Vela pulsar], where the digits in the parentheses indicate the uncertainties of the last digits \([ \text{Livingstone et al., 2006, and references therein} \) \). All these six braking indices are smaller than the value \( n = 3 \) predicted by pure dipole magnetic field configuration, which may suggest that other spin-down torques do work besides the energy loss via dipole radiation.

Pulsar’s spin-down has been studied since its discovery, but why the braking indices are smaller than 3 still does not have a clear answer yet. In recent years, several new mechanisms are suggested to explain this discrepancy, e.g., the two-component model: spin-down due to both magnetodipole radiation and relativistic particle flow \([ \text{Xu & Qiao, 2001}; \text{Wu et al., 2003}; \text{Contopoulos & Spitkovsky, 2006} \) \), the models with changing inclination angles \([ \text{e.g. Ruderman, 2003} \) \), the models with increasing magnetic field strength \([ \text{e.g. Lin & Zhang, 2004;} \text{Lyne, 2004;} \text{Chen & Li, 2006} \) \), the models with additional torques due to accretion \([ \text{e.g. Chen & Li, 2006} \) \) and the model with field-reconnection in magnetosphere \([ \text{e.g. Contopoulos, 2006} \) \).

In this paper, we focus on the two-component model suggested by \text{Xu & Qiao (2001)}\. Assuming a constant potential drop \( (\sim 10^{12} \text{ V}) \) in the polar cap accelerating region, we have only one free parameter left. Comparing with observations, new constrains are presented. The six pulsars with measured braking indices can be divided into two groups. The charge density in the polar cap region could be \( \sim 10^3 \) times the the Goldreich-Julian charge density \( \rho_{GJ} \) \([ \text{Goldreich & Julian, 1969} \) \), much larger than the usually assumed value \( \sim \rho_{GJ} \). The derived magnetic field strength could be larger than the value that from pure magnetodipole assumption. Additionally, the change of momentum of inertia \( I \) would also affect the braking index and make it smaller than 3.

2 The model with constant gap potential drop

In the two-component models, the magnetic momentum is assumed to be \( \mu = \mu_\perp + \mu_\parallel \), where \( \mu = BR^3/2, \) \( \mu_\perp = \mu \sin \alpha, \) \( \mu_\parallel = \mu \cos \alpha \) \( \text{(Xu & Qiao, 2001)} \).
Contopoulos & Spitkovsky, 2006) and $\alpha$ is the inclination angle. The component $\mu_\perp$ produces magnetodipole radiation, while the other component $\mu_\parallel$ could have various choices of acceleration mechanisms such as inner vacuum gap with curvature radiation (VG, e.g. Ruderman & Sutherland, 1975, hereafter RS75), vacuum gap with resonant inverse Compton scattering (VG+ICS, e.g. Zhang et al., 2000), outer gap (OG, e.g. Cheng et al., 1986) and space charge limited flow (SCLF, e.g. Arons & Scharlemann, 1979).

The energy loss rate $\dot{E}$ of the two components usually have different dependences on $\Omega$. The magnetodipole component has $\dot{E}_\perp \propto \Omega^4$, which would induce a braking index of 3. The other component usually has a relatively weaker dependence on $\Omega$ (e.g. $\dot{E}_\parallel \propto \Omega^2$ for a constant gap potential drop due to the unipolar effect), which would induce a braking index less than 3 ($n = 1$ in that case). The magnetodipole radiation is dominant when $P$ is shorter while the other component becomes important when $P$ is longer. The combination of the two would explain the observed braking index between 1 and 3. In summary, there are three variables/parameters: $B$, $P$ and $\alpha$.

Here we firstly use the RS75 inner vacuum gap model for indication. The other models which can also be parameterized the same way will be discussed in §4.

The energy lose rates of dipole and unipolar are $\dot{E}_{\text{dip}} = -(2/3)c^{-3} \mu^2 \Omega^4 \sin^2 \alpha$ and $\dot{E}_{\text{uni}} = -2\pi r_{pc}^2 \rho \Phi = -c^{-1} \kappa BR^3 \Omega^2 \Phi \cos^2 \alpha$, where $r_{pc}$ is the polar cap radius, $\rho = \kappa \rho_{GJ}$ is the charge density of the polar cap region, $\rho_{GJ} = B/(cP)$ is the Goldreich-Julian charge density (Goldreich & Julian, 1969) and $\kappa$ is an uncertain coefficient. We use $\kappa$ as another free parameter, which would be constrained by the observations. Combining these two, we have

$$\dot{E} = I \dot{\Omega} = -\frac{2}{3c^3} \mu^2 \Omega^4 \eta,$$

where

$$\eta = \sin^2 \alpha + 6c^2 \kappa B^{-1} R^{-3} \Omega^{-2} \Phi \cos^2 \alpha.$$

The effective potential drop $\Phi$ of unipolar usually has a weak dependence on $\Omega$ (e.g. $\Phi \sim \Omega^{-1/7}$ in Xu & Qiao, 2001), or just a few×10^{12} V (e.g. RS75; Usov & Melrose, 1995), so we could assume a constant potential drop $\Phi = 10^{12}$ V in the polar gap region. At the same time, a potential drop of 10^{12} V is also a widely-accepted result from pulsar death-line criterion. The braking index and the second braking index then are

$$n = 3 - 2 \frac{\Omega^{-2}}{\tan^2 \alpha/f + \Omega^{-2}}$$

and

$$m = 2n^2 - n + (n - 3)(1 - n),$$

where $f = 6c^2 \kappa B^{-1} R^{-3} \Phi$. 

3
Assuming a pulsar of $1.4M_\odot$ in mass and 10 km in radius with known $P$, $\dot{P}$, $\dot{\Omega}$ and a constant potential drop $\Phi = 10^{12}$ V, we have three variable: $B$, $\alpha$ and $\rho$. Considering two constrains from Eqs. (3) and (5), we have only one free parameter. Here we use $\rho$ as the free parameter and solve out $B$ and $\alpha$, i.e. $B$ and $\alpha$ are plotted as function of $\rho$. Our results are presented in Figs. 1–4. The six pulsar could be divided into two groups (see Fig. [1]): three that has larger spin period $P$ and larger braking indices $n$, i.e. PSR J1846−0258, PSR B1509−58 and PSR J1119−6127 (Group I); three that has smaller $P$ and smaller $n$, i.e. Crab, Vela and PSR B0540−69 (Group II). The charge density $\rho$ could be around $10^3\rho GJ$ to make $B$ and $\alpha$ have reasonable values (Fig. [3]). If the braking index $n$ is close to 3, the spin down is mainly due to magnetodipole radiation. The $B$ value then is close that from the canonical formula $B_{\text{dip}} = 6.4 \times 10^{19} (P\dot{P})^{1/2}$, i.e. the dipole approximation is quite good. If the braking index $n$ is close to 1, the spin down torque is mainly from particle outflow. The $B$ value would depart from $B_{\text{dip}}$ considerably (Fig. [4]).

3 The change of momentum of inertia

In a more general case, we have

$$\dot{E} = I\dot{\Omega}\dot{\Omega} + \frac{1}{2}I\dot{\Omega}^2$$

(7)

In §2, we only consider the first term $I\dot{\Omega}\dot{\Omega}$ in Eq. (7) and omitted the second term $I\dot{\Omega}^2/2$, i.e. we approximate $I$ as a constant. In this section we calculate the effects of $I$ in two case: (i) volume conservative, i.e. the pulsar’s volume is a constant, (ii) volume non-conservative, i.e. assume that the pulsar’s volume changes, or equivalently the stellar radius $R$ changes. W show that the change of braking index is quite small in the first case but should be considered in the second one.

3.1 The volume conservative case

A rotating star, or specifically a pulsar, can be approximately as an Maclaurin rotation ellipsoid when the spin period is not too small (Zhou et al., 2004). For pulsars, the criteria is $P \gg 1$ ms, so the rotation ellipsoid approximation are available for all the six pulsar in this article. In this case the pulsar’s volume is assumed to be a constant, i.e. the pulsar is incompressible. The pulsar is a spherical when $\Omega = 0$ and is a rotation ellipsoid when $\Omega > 0$. We have

$$I = I_0(1 + \frac{1}{3}e^2),$$

(8)
where

\[ e = \frac{\Omega}{\sqrt{8\pi G \rho_* / 15}}, \]

(9)

\( \rho_* \) is the average star density and \( I_0 \) is the star’s momentum of inertia when \( \Omega = 0 \). For Crab, the fastest one in the six, \( e = 0.022 \). Here we define

\[ I = I_0 (1 + k\Omega^2), \]

(10)

where \( k = 5/(8\pi G \rho) \). Then we have

\[ \dot{I} = \frac{dI}{d\Omega} \frac{d\Omega}{dt} = 2kI_0 \Omega \dot{\Omega} \]

(11)

Assuming \( \dot{E} \) is a power-law function of \( \Omega \), we have

\[ I\Omega \dot{\Omega} + \frac{1}{2} \dot{I} \Omega^2 = k_u \Omega^{u+1}, \]

(12)

where \( u \) is a constant (usually between 1 and 3) and \( k_u \) is a coefficient. Here \( u \) equals to braking index if we omit the \( \dot{I} \Omega^2 / 2 \) term. With Eqs. (10), (11) and (12), we have

\[ \dot{\Omega} = \frac{k_u}{I_0} \frac{\Omega^u}{1 + 2k\Omega^2} \]

(13)

and

\[ \ddot{\Omega} = \frac{k_u}{I_0} \frac{\Omega^{u-1} \dot{\Omega}}{1 + 2k\Omega^2} \left( u - \frac{4k\Omega^2}{1 + 2k\Omega^2} \right). \]

(14)

Then we have the braking index

\[ n' = \frac{\ddot{\Omega}\Omega}{\dot{\Omega}^2} = u - \frac{4k\Omega^2}{1 + 2k\Omega^2}. \]

(15)

Or effectively we have the difference between the braking index when assuming a constant \( I \) and the braking index when considering \( \dot{I} \),

\[ \Delta n = n - n' = u - n' = \frac{4k\Omega^2}{1 + 2k\Omega^2}. \]

(16)

For Crab, \( k\Omega^2 = e^2 / 3 = 1.6 \times 10^{-4} \ll 1, \Delta n = 6.5 \times 10^{-4} \), which could be omitted. Since Crab is the fastest one of the six, this effect can be omitted for all the six pulsars.

3.2 The volume non-conservative case

In the above subsection we show that the volume conservative case does not have much effects on braking index, because the change of \( I \) is very small. Here we consider a more effective way, in which \( \dot{I} \) is larger, i.e. the star radius
$R$ changes. Here we assume that $I$ is a function of $\Omega$. Applying the first order approximation, we have $dI/d\Omega = \text{const}$. Here we define

$$\frac{dI}{d\Omega} = I f_1 \tag{17}$$

where $I$ is the current moment of inertia and $f_1$ is a coefficient. Together with Eq. (12), we have

$$\dot{\Omega} = \frac{k_u}{I} \frac{\Omega^u}{1 + f_1 \Omega} \tag{18}$$

and then

$$\ddot{\Omega} = \frac{k_u}{I} \frac{\Omega^{u-1} \dot{\Omega}}{1 + f_1 \Omega} (u - \frac{f_1 \Omega}{1 + f_1 \Omega}). \tag{19}$$

The braking index is

$$n = u - \frac{f_1 \Omega}{1 + f_1 \Omega}. \tag{20}$$

The value of $f_1 \Omega$ should be of the order of 1 to make braking index be obviously smaller than $u$ ($u$ is the braking index if $\dot{I} = 0$ is assumed). For Crab we have,

$$\dot{I} = \frac{dI}{d\Omega} \dot{\Omega} = I f_1 \dot{\Omega} \sim I \frac{1}{\Omega} \dot{\Omega} = 4 \times 10^{-4} I \text{ yr}^{-1}. \tag{21}$$

It means the $I$ of Crab changes $\sim 1\%$ in the past 30 years. This seems large but is possible. Actually, a prontoneutron star could have a radius $\sim 30$ km but a cooled one may have radius $\sim 10$ km (Lattimer & Prakash, 2004). The moment of inertia changes dramatically. The six pulsars which have measured braking indices are all young and may be still cooling. In addition, strain energy increases as a solid star spins down, which may also cause the decreasing of stellar volume (Xu et al., 2006). It is therefore not unreasonable if they have such values of $\dot{I}$. It is worth noting that the “observed” field increasing (e.g. Lin & Zhang, 2004) of the Crab pulsar could also arise from the shrinking of pulsars after quakes.

### 4 Conclusion and discussion

We present a calculation of pulsar’s braking index considering magnetodipole radiation plus RS75 inner vacuum gap with a constant potential drop. The six pulsars with measured braking index tends to have either small or big $\alpha$, i.e. they can be divided into two groups and there is a gap between the two group. Assumed a constant potential drop $\Phi = 10^{12}$ V, we get that the charge density $\rho$ in the polar cap region is $\sim 10^3$ times of $\rho_{\text{GJ}}$, much larger the usually assumed value $\sim \rho_{\text{GJ}}$. The dynamical implication of this assumption in pulsar electrodynamics is worth to research in the future. We also shows that the effects of $\dot{I}$ on braking index could not be omitted if $\dot{I}$ is of the order of $10^{-4} I \text{ yr}^{-1}$.
In our results, the value of $\rho \sim 10^3 \rho_{GJ}$ is much larger than $\rho_{GJ}$. This means that $e^\pm$ pair plasma could be accelerated in gaps with potential drop $\Phi$. Though RS75-type vacuum gap may result in sparking, which would be necessary for explaining drifting subpulses, it is still possible that pair plasma could be accelerated (i) above the vacuum gap and/or (ii) in the annular gap in Qiao et al. (2004). If not only VG but also ICS and/or OG exists (e.g. VG+ICS model in Zhang et al. 2000), i.e. the effective potential drop is larger than the potential drop of the inner gap, the charge density could be smaller. Although a constant potential drop of $10^{12}$ V is generally accepted, we would like to note that $\Phi$ might changes slightly with $\Omega$ (e.g. $\Phi \sim \Omega^{-1/7}$ in Xu & Qiao, 2001). The effects would change the braking index by a factor, e.g. $\Delta n \sim -1/7$ if $\Phi \sim \Omega^{-1/7}$.

The second braking indices of two pulsars are measured, i.e. Crab ($m = 10.23 \pm 0.03$) and PSR B1509-58 ($m = 18.3 \pm 2.9$) (Livingstone et al. 2006). The theoretical value form Eq. (6) is $m = 10.9$ for Crab and $m = 13.6$ for PSR B1509-58, which are not compatible with the observations within the error bar. However, this discrepancy might not be a serious problem, because the braking indices are affected by several processes such as very small glitches and changing of surrounding environments (e.g. interaction with ISM), which is not clearly understood.

Are there really two groups of pulsars? Is there really a gap? The answer could not be certain yet because we only have braking index from six pulsars. There might be some mechanism to affect the inclination angle especially while the pulsar forms in process of the supernova explosion, but this is not well understood. When the number of pulsars with measured braking indices doubled in the future, the answer to this question would be much more clear.

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Fig. 1. Braking index $n$ versus spin period $P$ for the two-component model. Here we use magnetic dipole plus unipolar with constant potential drop $\Phi = 10^{12}$ V. The magnetic field is assumed to be $B = 10^{12}$ G and the charge density is assumed to be $\rho = 10^3 \rho_{GJ}$. The lines from bottom to top correspond to $\alpha$ from $0^\circ$ to $90^\circ$ in $10^\circ$ increments. The six pulsars could be divided into two groups: three pulsars (squares, Group I) have larger spin period $P$, larger braking indices and larger $\alpha$ while the other three (circles, Group II) have relatively smaller $P$, smaller braking indices and smaller $\alpha$. There is a gap between these two groups. It should be noted that we could not get $\alpha$ directly from this graph, because $B$ is not all equals to $10^{12}$ G and $\rho$ is a parameter. This figure is for indication.

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Fig. 2. Magnetic field versus normalized charge density $\rho/\rho_{GJ}(=\kappa)$. The $B$ value of pulsars with larger $P$ (solid line, Group I) do not change much, but it changes substantially for the pulsars with smaller $P$ (dashed line, Group II).

Fig. 3. Inclination angle $\alpha$ versus normalized charge density $\rho/\rho_{GJ}(=\kappa)$. Solid line for the three pulsar with larger $P$ (Group I), dashed line for the three pulsars with smaller $P$ (Group II). There is a clear gap between this two groups. If we assume that $\alpha$ is neither too big nor too small, we have $\rho$ around $10^3\rho_{GJ}$.
Fig. 4. $B/B_{\text{dip}}$ versus normalized charge density $\rho/\rho_{\text{GJ}} (= \kappa)$. Solid line for the three pulsar with larger $P$ (Group I), dashed line for the three pulsars with smaller $P$ (Group II). If we assume $\rho$ is around $10^3 \rho_{\text{GJ}}$ (see Fig. 3), the $B$ values of Group I pulsar is close to $B_{\text{dip}}$ with in a factor of 2. The $B$ value of Group II pulsar in this case is a few times the value of $B_{\text{dip}}$. 