The Color-Flavor Locking Phase at $T \neq 0$.

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Abstract

We study the color-flavor locked phase of QCD with three massless quarks at high chemical potential and small non-zero temperatures. We make use of the recently introduced effective action to describe such a phase. We obtain the exact order $T^2$ behaviour of the condensates and of the pressure by formally comparing the derivative from the QCD functional with symmetry breaking to that from the effective lagrangian with external sources, respecting the residual $Z_2$ invariance of the color-flavor locked phase. From these exact results, but now at a very tentative level of conjecture, we are lead to think that the phase structure of QCD at very high density consists of two superconducting phases and a symmetric one.

1 Introduction

QCD at high density has attracted a great interest lately. A recent review by Frank Wilczek[1] is entitled ”The recent excitement in high-density QCD”. We refer to this article and also to a latest review by Krishna Rajagopal[2] for clear and authoritative descriptions of the subject. Numerous authors have recently contributed to this rapidly advancing field[3].

A particularly interesting situation occurs in QCD with three massless flavors at high values of the baryonic chemical potential, where the suggested condensation pattern would imply the phenomenon of color-flavor locking (CFL), as discussed in the papers by Alford, Rajagopal and F.Wilczek[4], by T. Schäfer and F. Wilczek[5], and by M. Alford, J. Berges, and K. Rajagopal[6].
We have recently constructed the effective action describing color-flavor locking [7]. In this work we use the effective action to derive some rigorous results on the condensate behaviour for small non-vanishing temperature in the CFL phase. This we do by introducing external sources and correlating the generating functional in QCD to the generating functional for the goldstones. In doing this the $Z_2$ invariance left in the CFL phase is taken into account. We add to our exact results some tentative conjectural considerations on the phase structure of QCD at high density.

2 The effective theory

In this Section we shortly review the effective action for the color-flavor locked (CFL) phase of QCD introduced in [7]. The CFL phase is characterized by the symmetry breaking pattern

\[ G = SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \otimes U(1) \rightarrow H = SU(3)_{c+L+R} \]

from dynamical formation of condensates of the type

\[ \langle \psi^L_{ai} \psi^L_{bj} \rangle = -\langle \psi^R_{ai} \psi^R_{bj} \rangle = \gamma_1 \delta_{ai} \delta_{bj} + \gamma_2 \delta_{aj} \delta_{bi} \]

where $\psi^L(R)_{ai}$ are Weyl spinors and a sum over spinor indices is understood. The indices $a, b$ and $i, j$ refer to $SU(3)_c$ and to $SU(3)_L$ (or $SU(3)_R$) respectively. In fact $H$ contains an additional factor $Z_2$ which will play an essential role in the following.

The effective lagrangian describes the CFL phase for momenta smaller than the energy gap (the existing numerical estimates range between 10 and 100 MeV). The relevant degrees of freedom are the massless goldstones arising from the symmetry breaking. Before the gauging of $SU(3)_c$ we need 17 goldstones. We introduce coset matrix fields $X$ and $Y$ transforming under $G$ as a left-handed and a right-handed quark field respectively. We require

\[ X \rightarrow g_c X g_L^T, \quad Y \rightarrow g_c Y g_R^T \]

with $g_c \in SU(3)_c$, $g_L \in SU(3)_L$, $g_R \in SU(3)_R$. $X$ and $Y$ are $SU(3)$ matrices, breaking respectively $SU(3)_c \otimes SU(3)_L$ and $SU(3)_c \otimes SU(3)_R$. This
gives 16 goldstones. The additional one describes the breaking of the baryon number. This is done in terms of a \( U(1) \) factor transforming under \( G \) as

\[
U \to g_{U(1)} U, \quad g_{U(1)} \in U(1)
\]  

(4)

We define the following anti-hermitian and traceless currents

\[
J_X^\mu = X \partial^\mu X^\dagger, \quad J_Y^\mu = Y \partial^\mu Y^\dagger, \quad J_\phi = U \partial^\mu U^\dagger
\]  

(5)

which transform under the global group \( G \) as

\[
J_X^\mu \to g_c J_X^\mu g_c^\dagger, \quad J_Y^\mu \to g_c J_Y^\mu g_c^\dagger, \quad J_\phi \to J_\phi
\]  

(6)

Barring WZW terms \([8]\) (see ref. \([7]\) for a brief discussion), the most general rotational symmetric (at finite density Lorentz invariance is broken to \( O(3) \)) lagrangian invariant under \( G \), with at most two derivatives, is given by

\[
L = -\frac{F_T^2}{4} Tr[(J_X^0 - J_Y^0)^2] - \alpha_T \frac{F_T^2}{4} Tr[(J_X^0 + J_Y^0)^2] - \frac{f_T^2}{2} (J_\phi^0)^2 \\
+ \frac{F_S^2}{4} Tr[(\bar{J}_X - \bar{J}_Y)^2] + \alpha_S \frac{F_S^2}{4} Tr[(\bar{J}_X + \bar{J}_Y)^2] + \frac{f_S^2}{2} (\bar{J}_\phi)^2
\]  

(7)

We have required invariance under parity, that is symmetry under \( X \leftrightarrow Y \).

With the parametrization

\[
X = e^{i\bar{\Pi}_X^a T_a}, \quad Y = e^{i\bar{\Pi}_Y^a T_a}, \quad U = e^{i\bar{\phi}}, \quad a = 1, \ldots, 8
\]  

(8)

(where the \( SU(3) \) matrices \( T_a \) satisfy \( Tr[T_a T_b] = \frac{1}{2} \delta_{ab} \)) and by defining

\[
\Pi_X = \sqrt{\alpha_T} \frac{F_T}{2} (\bar{\Pi}_X + \bar{\Pi}_Y), \quad \Pi_Y = \frac{F_T}{2} (\bar{\Pi}_X - \bar{\Pi}_Y), \quad \phi = f_T \bar{\phi}
\]  

(9)

the kinetic term is

\[
L_{\text{kin}} = \frac{1}{2} (\Pi_X^a)^2 + \frac{1}{2} (\Pi_Y^a)^2 + \frac{1}{2} (\bar{\phi})^2 - \frac{v_X^2}{2} |\nabla \Pi_X^a|^2 - \frac{v_Y^2}{2} |\nabla \Pi_Y^a|^2 - \frac{v_\phi^2}{2} |\nabla \bar{\phi}|^2
\]  

(10)

where

\[
v_X^2 = \frac{\alpha_S F_S^2}{\alpha_T F_T^2}, \quad v_Y^2 = \frac{F_S^2}{F_T^2}, \quad v_\phi^2 = \frac{f_S^2}{f_T^2}
\]  

(11)
The three different types of goldstones move with different velocities, still satisfying a linear dispersion relation $E = \nu p$.

The lagrangian of eq. (7) must have a local $SU(3)_c$ invariance inherited by the color group of QCD. To make it explicit we only need to substitute usual derivatives with covariant ones

$$\partial_\mu X \rightarrow D_\mu X = \partial_\mu X - g_\mu X, \quad \partial_\mu Y \rightarrow D_\mu Y = \partial_\mu Y - g_\mu Y, \quad g_\mu \in \text{Lie } SU(3)_c$$

The currents become

$$J_\mu^X = X \partial^\mu X^\dagger + g^\mu, \quad J_\mu^Y = Y \partial^\mu Y^\dagger + g^\mu$$

giving the lagrangian

$$\mathcal{L} = -\frac{F_0^2}{4} \text{Tr}[(X \partial^0 X^\dagger - Y \partial^0 Y^\dagger)^2] - \alpha_T \frac{F_0^2}{4} \text{Tr}[(X \partial^0 X^\dagger + Y \partial^0 Y^\dagger + 2g^0)^2]$$

$$- \frac{f_\pi^2}{2} (\phi^0)^2 + \text{spatial terms and kinetic part for } g^\mu$$

We define

$$g_\mu = ig_s \frac{T_a}{2} g_\mu^a$$

where $g_s$ is the QCD coupling constant. The gluon field acquires a mass. This can easily be seen in a gauge such that $X = Y^\dagger$. This implies

$$\bar{\Pi}_X = -\bar{\Pi}_Y$$

or

$$\Pi_X = 0, \quad \Pi_Y = F_T \bar{\Pi}_X$$

This gauge is the unitary one (the bilinear term in the goldstones and in the gluon field in eq. (14) is proportional to $g^\mu \partial_\mu (\bar{\Pi}_X + \bar{\Pi}_Y)$ and cancels out). The gluon mass (for the expected velocities of order one) is given by

$$m_g^2 = \alpha_T g_s^2 \frac{F_0^2}{4}$$

The $X \leftrightarrow Y$ symmetry, in this gauge, implies $\Pi_Y \leftrightarrow -\Pi_Y$.

For goldstone energies much smaller than the gluon mass we can neglect the gluon kinetic term. The lagrangian (14) is then the hidden gauge symmetry version of the chiral lagrangian for QCD [9] (except for the contribution
of the field $\phi$). In fact, in this limit, the gluon field becomes an auxiliary field which can be eliminated through its equation of motion

$$g_\mu = -\frac{1}{2}(X\partial_\mu X^\dagger + Y\partial_\mu Y^\dagger)$$  \hspace{1cm} (19)

obtaining

$$\mathcal{L} = -\frac{F_2^2}{4}Tr[(X\partial^0 X^\dagger - Y\partial^0 Y^\dagger)^2] - \frac{f_\phi^2}{2}(J_\phi^0)^2 + \text{spatial terms}$$  \hspace{1cm} (20)

or

$$\mathcal{L} = \frac{F_2^2}{4} \left( Tr[\dot{\Sigma}\dot{\Sigma}^\dagger] - v_\phi^2 tr[\vec{\nabla}\Sigma : \vec{\nabla}\Sigma^\dagger] \right) - \frac{f_\phi^2}{2}((J_\phi^0)^2 - v_\phi^2|\vec{J}_\phi|^2)$$  \hspace{1cm} (21)

where $\Sigma = Y^\dagger X$ transforms under the group $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$ as $\Sigma \rightarrow g_R^* \Sigma g_L^T$. The goldstone $\phi$ could be interpreted, according to ref. [5], as a dibaryon state, ($udsuds$), particularly light, as had been pointed out by R. Jaffe [10] (we had called it the "deus ex machina" state). In total, after the breaking of the color group one is left with the massless photon and 9 physical Goldstone bosons transforming as the representation $1 + 8$ of the unbroken SU(3).

3 Introduction of $Z_2$-preserving breaking and of external sources

We shall introduce external sources [11] to correlate the QCD generating functional and the generating functional for the goldstones. We consider goldstone momenta smaller than the Fermi momentum (or the chemical potential $\mu$). We recall that, although the goldstones do not necessarily move at the speed of light, they satisfy a linear dispersion relation

$$E = v|\vec{p}|$$  \hspace{1cm} (22)

where $v$ is the appropriate velocity (see eq. (11)). We will need a relation between the condensates and the parameters appearing in the effective lagrangian. To this end we introduce in the QCD lagrangian a breaking term with the same structure of the condensates, that is

$$\mathcal{L}_{QCD}^{\text{break}} = M_{aibj}^L \psi^L_{ai}\psi^L_{bj} - M_{aibj}^R \psi^R_{ai}\psi^R_{bj} + \text{c.c.}$$  \hspace{1cm} (23)
where $\psi^{L(R)}_{ai}$ are Weyl spinors and a sum over spinor indices is understood. The indices $a, b$ and $i, j$ refer to $SU(3)_c$ and to $SU(3)_L$ (or $SU(3)_R$ respectively). $M_{aibj}(x)$ is a set of external scalar fields. At the same time we introduce a breaking term in the effective lagrangian

$$L_{\text{eff}}^{\text{break}} = \left(\gamma_1 M^{L}_{aibj} + \gamma_2 M^{L}_{ajbi}\right) X_{ai}X_{bj} U^2$$

$$+ \left(\gamma_1 M^{R}_{aibj} + \gamma_2 M^{R}_{ajbi}\right) Y_{ai}Y_{bj} U^2 + \text{c.c.}$$

(24)

where the $U^2$ factor takes into account that the term in eq. (23) breaks the $U(1)_V$ in such a way to preserve a $Z_2$ symmetry. To get the desired relation one has simply to differentiate the QCD generating functional and that derived from the effective lagrangian and equate the results. Since in the vacuum one has

$$\langle X_{ia} \rangle = \langle Y_{ia} \rangle = \delta_{ia}, \quad \langle U \rangle = 1$$

(25)

the contribution to $L_{\text{eff}}$ is given by

$$L_{\text{eff}}^{\text{break}} \rightarrow (\gamma_1 M^{L}_{aibj} + \gamma_2 M^{L}_{ajbi})\delta_{ai}\delta_{bj} + (\gamma_1 M^{R}_{aibj} + \gamma_2 M^{R}_{ajbi})\delta_{ai}\delta_{bj} + \text{c.c.}$$

(26)

from which

$$\langle \psi^{L}_{ai}\psi^{L}_{bj} \rangle = \gamma_1\delta_{ai}\delta_{bj} + \gamma_2\delta_{aj}\delta_{bi}$$

$$\langle \psi^{R}_{ai}\psi^{R}_{bj} \rangle = -\gamma_1\delta_{ai}\delta_{bj} - \gamma_2\delta_{aj}\delta_{bi}$$

(27)

Now let us consider the contribution to the goldstones masses of a breaking term corresponding to the following choice for the external scalar source

$$M_{aibj}^{L} = M_{aibj}^{R} = m_1\delta_{ai}\delta_{bj} + m_2\delta_{aj}\delta_{bi}$$

(28)

with $m_1$ and $m_2$ real parameters. The projection of the condensates along these directions gives

$$\tilde{M}_{aibj}^{L} \langle \psi^{L}_{ai}\psi^{L}_{bj} \rangle = m_1 u_1^L + m_2 u_2^L$$

$$\tilde{M}_{aibj}^{R} \langle \psi^{R}_{ai}\psi^{R}_{bj} \rangle = m_1 u_1^R + m_2 u_2^R$$

(29)

where

$$u_1^{L,R} = \langle \psi^{L,R}_{aa}\psi^{L,R}_{bb} \rangle, \quad u_2^{L,R} = \langle \psi^{L,R}_{ab}\psi^{L,R}_{ba} \rangle$$

(30)
and using the expression (27) we get
\[ u_{L,R}^{1,2} = \pm \left( N^2 \gamma_1 + N \gamma_2 \right) \] (31)

(for possible future uses we do this calculation for color and flavor groups SU(N)). These expressions can be easily inverted obtaining
\[ \gamma_1 = \frac{Nu_1 - u_2}{N(N^2 - 1)}, \quad \gamma_2 = \frac{Nu_2 - u_1}{N(N^2 - 1)} \] (32)

where
\[ u_i = u_i^L = -u_i^R, \quad i = 1, 2 \] (33)

The breaking term in the effective lagrangian, taken at the values (28) for the external sources, gives rise to mass terms for the goldstones. In fact, from
\[ L_{\text{masses}} \equiv \gamma_1 \tilde{M}_{aibj}^L + \gamma_2 \tilde{M}_{aibj}^L X_{ai} X_{bj} U^2 + (X \leftrightarrow Y) + c.c. \]
\[ = (m_1 \gamma_1 + m_2 \gamma_2)(Tr[X])^2 U^2 + (m_1 \gamma_2 + m_2 \gamma_1)Tr[X^2] U^2 \]
\[ + (X \leftrightarrow Y) + c.c. \] (34)
evaluating the traces at the second order in the Goldstone fields, we get
\[ (tr[X])^2 \approx (N - \frac{1}{4} \tilde{\Pi}_X^2)^2 = N^2 - \frac{N}{2} \tilde{\Pi}_X^2 \] (35)
\[ Tr[X^2] \approx N - \tilde{\Pi}_X^2 \] (36)

and analogous expressions for the Y fields. Therefore
\[ L_{\text{masses}} \equiv 4 \left( m_1 (N^2 \gamma_1 + N \gamma_2) + m_2 (N^2 \gamma_2 + N \gamma_1) \right) \]
\[ - 8 \left( m_1 (N^2 \gamma_1 + N \gamma_2) + m_2 (N^2 \gamma_2 + N \gamma_1) \right) \tilde{\phi}^2 \]
\[ - ((N \gamma_1 + 2 \gamma_2)m_1 + (2 \gamma_1 + N \gamma_2)) \left( \tilde{\Pi}_X^2 + \tilde{\Pi}_Y^2 \right) \] (37)

Using
\[ \tilde{\Pi}_X^2 + \tilde{\Pi}_Y^2 = \frac{2}{\alpha_T F_T} \Pi_X^2 + \frac{2}{F_T} \Pi_Y^2 \] (38)
and eqs. (31) and (32) we get

\[ L_{\text{masses}} = 4(m_1 u_1 + m_2 u_2) - 8\frac{m_1 u_1 + m_2 u_2}{f_f^2} \phi^2 \]

\[ - 2\frac{((N^2 - 2)u_1 + Nu_2)m_1 + ((N^2 - 2)u_2 + Nu_1)m_2}{\alpha_T F_f^2 N^2(N - 1)} \Pi_X^2 \]

\[ - 2\frac{((N^2 - 2)u_1 + Nu_2)m_1 + ((N^2 - 2)u_2 + Nu_1)m_2}{F_f^2 N^2(N - 1)} \Pi_Y^2 \]  

(39)

Therefore the goldstones masses (that is twice the coefficient of the square terms in the Goldstone fields) are given by

\[ m_\phi^2 = \frac{16}{f_f^2} (m_1 u_1 + m_2 u_2) \]

\[ m_X^2 = 4 \frac{((N^2 - 2)u_1 + Nu_2)m_1 + ((N^2 - 2)u_2 + Nu_1)m_2}{\alpha_T F_f^2 N^2(N - 1)} \]

\[ m_Y^2 = 4 \frac{((N^2 - 2)u_1 + Nu_2)m_1 + ((N^2 - 2)u_2 + Nu_1)m_2}{F_f^2 N^2(N - 1)} \]  

(40)

4 Exact behaviour of the condensates and of the pressure to the order \( T^2 \)

We can now evaluate the behaviour of the condensates in the low temperature limit up to the order \( T^2 \). For the corresponding case of chiral symmetry breaking at zero densities the authors of ref. [11] had shown that, in the infinite volume limit and for small pion masses, it is sufficient at small temperatures to take into account only the contribution from free pions. The same reasonings go on in our case. Thus, by writing the partition function in the form

\[ Tr[e^{-\beta H}] = e^{-\beta L^3 z} \]  

(41)

we get, for a single free goldstone of mass \( M \)

\[ z = \epsilon_0 - \frac{1}{2} g_0(M^2, T) \]  

(42)

where \( \epsilon_0 \) is the energy density of the ground state and

\[ g_0(M^2, T) = 2T \int \frac{d^3p}{2\pi^3} [- \ln (1 - \exp(-E/T))] \]  

(43)
with

\[ E = \sqrt{\nu^2 |\vec{p}|^2 + M^2} \]  \hspace{1cm} (44)

The previous expression can be reduced to the standard expression through the change of variables \( \vec{p}' = v\vec{p} \), obtaining

\[ g_0(M^2, T) = \frac{2T}{v^3} \int \frac{d^3 p'}{2\pi^3} \left[ -\ln (1 - \exp(-E'/T)) \right] \]  \hspace{1cm} (45)

with

\[ E' = \sqrt{|\vec{p}'|^2 + M^2} \]  \hspace{1cm} (46)

In the limit \( M^2 \to 0 \) we get

\[ g_0 = \frac{1}{v^3} \frac{\pi^2 T^4}{45} \]  \hspace{1cm} (47)

and

\[ -\frac{\partial g_0}{\partial M^2} = \frac{1}{v^3} \frac{T^2}{12} \]  \hspace{1cm} (48)

We can now derive the expression for the pressure in the massless limit

\[ P = \frac{1}{2}(N^2 - 1)g_0(m_Y^2, T) + \frac{1}{2}g_0(m_\phi^2, T) \]  \hspace{1cm} (49)

and, for \( v_Y \approx v_\phi \approx 1 \) and \( T \to 0 \) we get the simple result

\[ P \approx \frac{\pi^2}{90} N^2 T^4 \]  \hspace{1cm} (50)

The condensates can be easily evaluated in terms of the energy density \( z \)

\[ u_i(T) = -\frac{1}{4} \frac{\partial}{\partial m_i} z \]  \hspace{1cm} (51)

as it follows from eq. (23) for the chosen external scalar sources. In our case

\[ z = -4(m_1 u_1 + m_2 u_2) - \frac{1}{2}(N^2 - 1)g_0(m_Y^2, T) - \frac{1}{2}g_0(m_\phi^2, T) \]  \hspace{1cm} (52)
Therefore

\[
\begin{align*}
    u_1(T) &= u_1 - (N^2 - 1) \frac{(N^2 - 2)u_1 + Nu_2}{N^2(N - 1)} \frac{T^2}{24F_T^2v_3^3} - \frac{u_1 T^2}{6f_T^2v_\phi^3} \\
    u_2(T) &= u_2 - (N^2 - 1) \frac{(N^2 - 2)u_2 + Nu_1}{N^2(N - 1)} \frac{T^2}{24F_T^2v_3^3} - \frac{u_2 T^2}{6f_T^2v_\phi^3}
\end{align*}
\] (53)

More complicated calculations along the lines of ref. [11] would allow to calculate an additional term in the expansion in \(T^2\).

In the case of the analogous calculation made for the chiral phase one gets an insight about the possibility of a second order phase transition simply by noticing that the chiral condensate (evaluated for small \(T\)) decreases with the temperature. Of course, this conjecture should be supported by higher order calculations, or even better by a complete calculation at any order in the temperature, but it looks as a reasonable situation. The question is, if in the present situation one can get a similar insight. To this end, we begin by the observation that now the condensate is determined by two quantities, \(\gamma_1\) and \(\gamma_2\) or their combinations \(u_1\) and \(u_2\). Speaking in terms of \((u_1, u_2)\), one can look at these two variables as the components of a vector in a two-dimensional space. To look for a signal of a phase transition one should be able to decide if the length of the vector decreases or increases with the temperature. This can be done by evaluating directly the length of the vector from eq. (53), or better by using a reference frame where the vector does not rotate (by varying \(T\)). This reference frame is simply obtained by the following change of coordinates

\[
\begin{align*}
    v_1 &= u_1 + u_2, \\
    v_2 &= u_1 - u_2
\end{align*}
\] (54)

Then, from eq. (53) we get

\[
\begin{align*}
    v_1(T) &= v_1(0) \left[ 1 - \frac{(N^2 + N - 2)(N + 1)}{N^2} \frac{T^2}{24F_T^2v_3^3} - \frac{T^2}{6f_T^2v_\phi^3} \right] \\
    v_2(T) &= v_2(0) \left[ 1 - \frac{(N^2 - N - 2)(N + 1)}{N^2} \frac{T^2}{24F_T^2v_3^3} - \frac{T^2}{6f_T^2v_\phi^3} \right]
\end{align*}
\] (55)

showing that around zero temperature both components decrease for increasing \(T\).
For $N = 3$ and all the couplings of the same order of magnitude one gets

$$\frac{v_1(T)}{v_1(0)} \approx 1 - \frac{19 T^2}{54 F^2}, \quad \frac{v_2(T)}{v_2(0)} \approx 1 - \frac{13 T^2}{54 F^2}$$

(56)

where $F$ is the common value of the couplings.

So far the results are exact (except for the assumptions of equal coupling in the last formulae) to the order $T^2$. Extrapolating from these results may be very dangerous as we do not know anything on the full expressions. However one can try to imagine the possible scenarios coming from a naive extrapolation of the previous results.

An interesting possibility, suggested by the eqs. (55), is that there exist two different temperatures $T_1$ and $T_2$, $T_2 \geq T_1$, such that $v_1(T) = 0$ for $T \geq T_1$ and $v_2(T) = 0$ for $T \geq T_2$ (we will discuss later the opposite possibility).

Since $v_1 = 0$ means $u_1 = - u_2$, and

$$\gamma_1 = \frac{N}{N - 1}u_1 = - \gamma_2$$

(57)

we see that the condensate of eq. (2) becomes antisymmetric in the exchange of the color (or of the flavor) indices. In the case $N = 3$ one can write

$$\langle \psi_{a_i}^L \psi_{b_j}^L \rangle = - \langle \psi_{a_i}^R \psi_{b_j}^R \rangle = \gamma_1 \epsilon_{abm} \epsilon_{ijm}$$

(58)

The possibility that the condensate at $T = 0$ in the CFL phase may assume this form was considered in ref. [4], with the conclusion that this is disfavored by the dynamics. But here we see that it is possible that this situation can be realized at $T \neq 0$. The equality $\gamma_1 = - \gamma_2$ has a clear group-theoretical meaning since the condensate (2) behaves, under $SU(3)_c \otimes SU(3)_{L,R}$ as $(3, 3) \otimes (6, 6)$ due to the symmetry under the overall exchange of the color and flavor indices. It is easy to show that the combination $\gamma_1 - \gamma_2$ corresponds to the representation $(3, 3)$, whereas $\gamma_1 + \gamma_2$ to the representation $(6, 6)$. Therefore, $\gamma_1 = - \gamma_2$ means that only the component $(3, 3)$ survives. On the other hand, from the point of view of symmetry breaking the situation at $T_1 \leq T \leq T_2$ would not be very much different than at lower temperatures, since still we have the same general type of symmetry breaking. In our hypothesis the symmetry would be restored only at $T \geq T_2$, when both $\gamma_1$ and $\gamma_2$ vanish.

We see that, under this assumption, there would be two superconducting phases differing only in the symmetry of the Copper pairs wave function.
In one phase (say SC1 at $T \leq T_1$) the wave function is symmetric in the simultaneous exchange of the color and flavor indices. In the other phase (SC2 at $T_1 \leq T \leq T_2$), the wave function is antisymmetric in the separate exchange of color and flavor indices. It is interesting to notice that the wave function in the phase SC2 is similar to the one obtained in the case of two flavors (see ref. [4]).

Our exact results may thus lead to conjecture, after the above extrapolation, that at high density the phase structure of QCD is given by two different superconducting phases and by a symmetric phase. Increasing the temperature one goes from the SC1 phase through a sort of crossover to the SC2 phase and after that the symmetry is restored.

Although our extrapolation from the behaviour close to $T = 0$ gives indications that $v_1$ goes to zero before $v_2$, one can ask what would happen in the opposite situation. We see immediately that when $v_2 = 0$ one has $u_1 = u_2$ and

$$\gamma_1 = \frac{1}{N(N-1)} u_1 = -\gamma_2$$

Therefore, also in this case one obtains for the condensate the expression in eq. (58).

We stress again that, differently from the results at order $T^2$, which are the main object of this note, the considerations we have just added for larger $T$ are highly speculative and go much beyond the exact results we have derived.

5 Conclusions

We have used the effective action for the color-flavor locked phase of QCD with three massless quarks at high chemical potential and derived exact behaviours at order $T^2$ by formally introducing a QCD symmetry breaking term of the same structure of the condensates and a corresponding term in the effective lagrangian, and by equating the derivative from the QCD functional to that from the effective lagrangian. More complicate calculations, not done here, would allow to calculate an additional term in the expansion in $T^2$. Although it would be unreliable to extrapolate from the obtained low $T$ results, one may very tentatively imagine the existence of two superconducting phases with a crossover between them and an ensuing second order
phase transition to the symmetric phase, when increasing $T$ at a fixed large chemical potential.

References

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