Extinction in the Galaxy from surface brightnesses of ESO-LV galaxies: determination of $A_R/A_B$ ratio

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ABSTRACT
A new method for the determination of the extinction in our Galaxy is proposed. The method uses surface brightnesses of external galaxies in the B and R bands. The observational data have been taken from the ESO-LV galaxy catalogue. As a first application of our model we derive the ratio of R band to B band extinction. We introduce two methods for computing the ratio which give: $0.62 \pm 0.05$ (the first method) and $0.64 \pm 0.06$ (the second method) which is in agreement with the recent literature value of 0.61. This agreement confirms the validity and efficiency of our model and is an independent verification for the standard value of the "total to selective extinction". The method of extinction determination introduced in this paper will be explored in subsequent publications.

Key words: dust, extinction - methods: statistical - Galaxy: general - galaxies: fundamental parameters

1 INTRODUCTION
Photometric parameters of external galaxies have been used several times to derive extinction in our Galaxy. Most authors use galaxy colours: de Vaucouleurs & Buta (1983), Holmberg (1974), Peterson (1970), Sandage (1973), Teerikorpi (1978). Some of them use galaxy surface brightnesses: de Vaucouleurs & Buta (1983), Holmberg (1958) while the others use galaxy absolute magnitudes: Peterson (1970), Teerikorpi (1978).

In this paper, we propose a new method of studying extinction using surface brightnesses of external galaxies in two bands. We apply this method to surface brightness data alone. An agreement between our determination and the literature value would confirm the correctness of our model and would demonstrate that the surface brightnesses of external galaxies are a good extinction indicator.

We present here two methods for estimating $A_R/A_B$. Both of them rely on the simple notion that extinction should not depend on morphological types of the galaxies, whose surface brightnesses are used for computing extinction.

2 THE SAMPLE
The ESO-LV galaxy catalogue contains many different surface brightness values for every galaxy. Three of them are given in two bands (B and R): surface brightness at half total B light radius denoted in ESO-LV as $\mu_e^B$ and $\mu_e^R$, average central surface brightness within 10 arcsec circular aperture denoted as $\mu_{0c}^B$ and $\mu_{0c}^R$ and central surface brightness in fit of generalized exponential to octants denoted as $\mu_{0e}^B$ and $\mu_{0e}^R$. We use throughout this paper surface brightness at half total B light denoted hereafter in this paper as $\mu_B$ and $\mu_R$. We preferred this measure, as the central surface brightness might be affected by saturation effects and nuclear emission, while the octant surface brightness represents extrapolated values.

Although the differences of the results obtained using these three kinds of surface brightness values are small, our analysis indicates that the sur-
face brightness at half total B light produce results with the smallest dispersions.

We use in this paper the morphological types of galaxies listed in the ESO-LV catalogue. However some of them (referred to as parametrical ones) are made by means of colour indices which, in turn, are extinction dependent. Since it is vital in this paper to use morphological types which are absolutely extinction independent we decided to reject all galaxies with such morphological types (marked in the ESO-LV catalogue with $T_{flag}$ equal to 4).

Using any astronomical sample for statistical purposes one has to specify its definition which clearly describes which objects belong to the sample. Generally galaxy samples can be apparent diameter-limited (contain galaxies which have an apparent diameter larger than a certain diameter limit) or magnitude-limited (contain galaxies which are brighter than a certain magnitude limit). The ESO-LV catalogue is diameter-limited: it contains galaxies which have visual apparent diameter in the ESO Quick Blue Survey greater or equal to 60 arcsec ($D_{org} \geq 60$ arcsec).

However, this completeness limit depends on morphological type $T$. But when we take galaxies which have $D_{org} \geq 100$ arcsec the catalogue is complete for all morphological types (Valenti 1990, 1994, Huizinga & van Albada 1992, Impey, Bothun & Malin 1988). So we decided to use in this paper galaxies which have $D_{org} \geq 100$ arcsec.

The last (minor) exclusion applied to the original ESO-LV sample was the rejection of galaxies which have excessive (probably wrong) colours: $\mu_B - \mu_R$ smaller than 0 and greater than 2.1. The exclusion rejects only 18 galaxies.

The final sample, which has been used for the analysis in this paper contains 2783 galaxies, 19 per cent of the full original ESO-LV database (which contains 15467 objects).

## 3 THE MODEL

### 3.1 The formula for extinction

As one can see in Fig. 1 the surface brightness in the B band ($\mu_B$) is strongly linearly dependent on (correlated with) surface brightness in the R band ($\mu_R$). The amplitude of this linear dependence is as high as five magnitudes (similar in both bands) and has strictly astrophysical origin (it can not be caused by extinction which is for most galaxies substantially less than one magnitude).

The Galactic extinction-free values of $\mu_B$ and $\mu_R$ should, for a given galaxy, increase due to extinction by $A_B$ and $A_B$ respectively (the amount of foreground extinction in B and R band respectively). This means that on the ($\mu_B, \mu_R$) plane extinction "moves" galaxies along the direction with slope $r$:

$$r = \frac{A_R}{A_B}$$

which describes the ratio of extinction in R band to the extinction in B band.

Taking into account the above facts we may expect that extinction-free surface brightnesses of galaxies lie on a certain straight line. The distance to this zero extinction straight line measured in the direction with slope $r$ is proportional to the Galactic extinction (see Fig. 2).

We test our assumption about the linearity of the zero extinction line by fitting straight line to the data on the ($\mu_B, \mu_R$) plane (separately for every morphological type $T$). Since the data have errors in both coordinates we have used a so called orthogonal fitting procedure (see Feigelson & Babu 1992). We found no systematic residuals from the resulting lines, confirming the validity of our linearity assumption.

If we describe the slope of the zero extinction straight line as $s^{-1}$ (we use the inverse of $s$ here for symmetry reasons - see equations 6, 7, 8, 14 and 15) we can express extinction in the B band as:

$$A_B = \frac{\mu_B - \mu_R}{1 - r s} - c$$

where $c$ is a certain constant.

Equation (2) gives a ready-to-use formula for extinction but in order to use it we have to know three parameters: $r$, $s$ and $c$. The remaining part of this paper is mainly devoted just to obtain these.

### 3.2 Selection effects

The sample used in this paper is apparent diameter ($D_{org}$) limited. We have checked, that the crucial parameters for the present analysis : $\mu_B$ and $\mu_R$ are statistically independent of apparent diameter. As we find no dependency this implies that selection effects do not influence the values of $\mu_B$ and $\mu_R$, so our formula for $A_B$, which uses just these two observational values is not influenced by selection effects (is unbiased). See Choloniewski (1991) for a general discussion of photometric selection effects.

### 3.3 Relation between $r$ and $s$

Let us define now a new variable $q$ as:

$$q = \mu_R - r \mu_B.$$  

This variable (closely related to the Q parameter defined for stars by Sharpless 1963 in a very similar context) is extinction-independent because it is equal to the vertical distance on the ($\mu_B, \mu_R$) plane between a galaxy and extinction direction line. As is clearly visible on Fig. 2 the distance to the extinction direction line is insensitive to extinction because extinction ”moves” galaxies on the ($\mu_B, \mu_R$) plane in the direction parallel to that line. So, the variable $q$ should be statistically independent on $A_B$ which implies that these two variables should be uncorrelated:

$$\rho(A_B, q) = 0$$

so their covariance should be equal to zero as well:

$$cov(A_B, q) = 0$$

When we replace in equation (5) $A_B$ and $q$ by expressions given in equations (2) and (3) respectively and apply the law of error propagation (Brandt 1970) we obtain a relation which binds the variables $r$ and $s$:

$$r \sigma^2(\mu_B) + s \sigma^2(\mu_R) = (1 + r s) \sigma^2(\mu_{B,R})$$

where $\sigma^2$ denotes the square of the standard deviation (variance). Equation (6) defines on the ($\mu_B, \mu_R$) plane a hyperbola (see Fig. 3) and may be solved with respect to $s$:

$$s = \frac{r \sigma^2(\mu_B) - cov(\mu_B, \mu_R)}{r \sigma^2(\mu_B, \mu_R) - \sigma^2(\mu_R)}$$
and $r$:

$$r = \frac{s \sigma^2(\mu_R) - \text{cov}(\mu_B, \mu_R)}{s \text{cov}(\mu_B, \mu_R) - \sigma^2(\mu_B)}. \quad (8)$$

### 3.4 Determination of $c$

Our method can provide only relative extinction, namely: extinction plus an unknown constant. So, we are free to normalize our extinction estimate to have:

$$< A_B > = 0, \quad (9)$$

what means that our extinction is the relative extinction compared to an overall mean. Equation (9) applied to equation (2) gives an expression for the constant $c$:

$$c = \frac{< \mu_B > - s < \mu_R >}{1 - r s}. \quad (10)$$

### 3.5 How to compute?

The average values (more precisely: the expectation values) present in equations (9) and (10) can be easily computed using the well known formula:

$$< x > = \frac{\sum_{i=1}^{n} x_i}{n} \quad (11)$$

where summation is done over the data in the galaxy sample.

The standard deviations and covariances present in most equations in this paper (can be computed from the following formulae:

$$\sigma^2(x) = < x^2 > - < x >^2 \quad (12)$$

$$\text{cov}(x, y) = < x y > - < x > < y > \quad (13)$$

where $x$ and $y$ denotes any variable used in this paper.

### 3.6 Statistical and geometrical context

Equations (2) and (3) define in fact a linear transformation of the "old" variables $\mu_B$ and $\mu_R$ into the "new" variables $A_B$ and $q$. The transformation is defined in such a way that the "new" variables have to be uncorrelated. The coefficients of such a linear transformation are bound by equation (6).

We solve here a general problem: how to transform linear "old" variables $x$ and $y$ into the "new" ones $X$ and $Y$:

$$X = x - s y \quad (14)$$

$$Y = y - r x \quad (15)$$

while $X$ and $Y$ are uncorrelated:

$$\rho(X, Y) = 0 \quad (16)$$

The solution of this problem, lies in the condition for $s$ and $r$:

$$r \sigma^2(X) + s \sigma^2(Y) = (1 + rs) \text{cov}(X, Y) \quad (17)$$

Equation (6) is of course the special case of this equation.

Geometrically, the linear transformation of two "old" variables into the "new" ones is equivalent to applying a certain new coordinate system, which uses two "new" axes: the $X$ "new" axis is tilted to the "old" $x$ axis by a slope $r$, while the "new" $Y$ axis is tilted to the "old" $y$ axis by a slope $s$.

Our solution is closely related to the procedure which leads to the so called *orthogonal regression line* (see Feigelson & Babu 1992). Such a line is constructed by rotating coordinates in a way to obtain uncorrelated data in "new" coordinates. Our solution can be regarded as a generalization of this procedure: we can easily reconstruct an *orthogonal regression line* by substituting in equation (17) $s = -r$.

### 4 DETERMINATION OF $A_R/A_B$: FIRST METHOD

We compute in this Section the ratio of extinction in $R$ band to the extinction in $B$ band denoted as $r$ (see equation 1) using the simple fact that extinction $A_B$ must be statistically independent on galaxy morphological type $T$.

#### 4.1 Mathematical approach

The statistical independence of $A_B$ and $T$ imply that:

$$\rho(A_B, T) = 0 \quad (18)$$

which gives:

$$\text{cov}(A_B, T) = 0. \quad (19)$$

Taking into account equation (2), using the law of error propagation (Brandt 1970) and assuming that the parameters $s$ and $c$ do not depend on $T$ we have:

$$s = \frac{\text{cov}(\mu_B, T)}{\text{cov}(\mu_R, T)} \quad (20)$$

This value for $s$ substituted into equation (8) gives the formula for $r$:

$$r = \frac{\text{cov}(\mu_B, T)}{\text{cov}(\mu_R, T)} \frac{\sigma^2(\mu_R) - \text{cov}(\mu_B, \mu_R)}{\text{cov}(\mu_B, \mu_R) - \sigma^2(\mu_B)}. \quad (21)$$

Equations (20) and (21) hold under the assumption that $s$ and $c$ do not depend on $T$. Let us check now whether this is true by computing $s$ and $c$, for different values of $r$ using equations (7) and (10). The uncertainties for $s$ and $c$ have been computed using a so called jackknife method (see Quenouille 1956, Tukey 1958 and Efron & Tibshirani 1986). The results are given on Fig. 4 and 5. It is evident that $s$ and $c$ are $T$ dependent, but for morphological type $T$ between 2.5 and 6.5, both parameters are constant for any value of $r$ in the limits of accuracy achieved here.

For this reason we computed $r$ using equation (21) taking only those galaxies for with $T$ is greater than 2.5 and less than 6.5. The resultant subsample contains 1290 objects (approximately half of the whole sample). We computed for these galaxies all the components of the equation (21):

$$\text{cov}(\mu_B, T) = 279.032 \quad (22)$$

$$\text{cov}(\mu_R, T) = 416.098 \quad (23)$$

$$\sigma^2(\mu_B) = 0.432634 \quad (24)$$

$$\sigma^2(\mu_R) = 0.530865 \quad (25)$$

$$\text{cov}(\mu_B, \mu_R) = 0.440979 \quad (26)$$

What finally gives:

$$r = 0.62 \pm 0.05 \quad (27)$$
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where the uncertainty of the result was again computed using the jackknife method.

4.2 Graphical approach

The idea which has been used here for computing $r$ ($A_B$ does not depend on $T$) can be applied in another way. Let us compute the average extinction (according to equations 2 and 7) and plot it as a function of morphological type (denoted as $<A_B>_T$) for different values of $r$ - see Fig. 6. It is clearly visible that only for $r = 0.6$ the extinction $A_B$ is not a function of morphological type $T$. So we arrive again at a similar result as expressed in equation (21).

5 DETERMINATION OF $A_R/A_B$: SECOND METHOD

We compute in this Section the ratio of extinction in R band to the extinction in B band denoted as $r$ (see equation 1) using the fact that the standard deviation of extinction $A_B$ must be the same for every galaxy morphological type $T$.

According to equation (2) and applying the law of error propagation (Brandt 1970) the value of $\sigma^2(A_B)$ can be expressed as:

$$\sigma^2(A_B) = \frac{\sigma^2(\mu_B) + s^2\sigma(\mu_B) - 2s\text{cov}(\mu_B, \mu_R)}{(1 - r)s^2}$$  \hspace{1cm} (28)

When we substitute $s$ present in equation (28) by expression given in equation (7) the resultant equation for $\sigma^2(A_B)$ depends only on $r$. When we use it for two different morphological types (say $T_1$ and $T_2$) we have:

$$\sigma_{T_1}(A_B)_r = \sigma_{T_2}(A_B)_r$$  \hspace{1cm} (29)

which can be solved with respect to $r$. Such solution is presented graphically on Fig. 7 for $T_1 = -3$ and $T_2 = 5$. The appropriate two curves intersect at $r = 0.66$ which is the solution for $r$ for this particular pair of $T$.

The values of $\sigma_T(A_B)_r$ as a function of $r$ for all morphological types ($T = -5, ..., 10$) are presented in Fig. 8. The curves intersect in many different points so we have many different "solutions" for $r$. This motivated us to compute the relative standard deviation of $\sigma_T(A_B)_r$, namely:

$$D(r) = \frac{\sigma(\sigma_T(A_B)_r)}{<\sigma_T(A_B)_r>}.\hspace{1cm} (30)$$

The minimum of the function $D(r)$ will be the solution for $r$. For two morphological types: $T = -5$ and $T = 9$ $\sigma_T(A_B)_r$ strongly differs from the mean so we decided to omit these two types in our final computations in this Section (the rejected curves are marked on Fig. 8 by a dotted lines).

The reason for existence of such two "outliers" is probably that they have different intrinsic extinction-free scatter with respect to the "zero extinction" straight line on the $(\mu_B, \mu_R)$ plane than for other galaxies: this scatter is smaller (in comparison with most galaxies) for $T = -5$ and greater for $T = 9$.

The function $D(r)$ has a minimum for:

$$r = 0.64 \pm 0.06$$  \hspace{1cm} (31)

where the uncertainty has been computed by the jackknife method (with respect to $T$).

6 DISCUSSION

The second method for determination of the $A_R/A_B$ ratio works only due to the fact that the parameter $s$ is $T$ dependent. Moreover, the stronger this dependence is, the more accurate are the results given by the second method. The opposite situation is for the first method: it works only for such galaxies for which the $s$ parameter is not $T$ dependent. So the first method can survive only due to galaxies with the same $s$ parameter which is in contradiction to the second method which feeds itself by the differences in $s$. While complementary in nature, both tests gives similar results.

Both methods have their specific disadvantages. The first one does not use the whole sample. The second one relies on the assumption that the extinction-free scatter of galaxy surface brightnesses is the same for every morphological type (what is not always true).

The value for $A_R/A_B$ which was recently published is 0.61 (Schlegel, Finkbeiner & Davis 1998). It takes into account several observational and instrumental factors and is based on formulae published by Cardelli, Clayton & Mathis 1989 and O'Donnell 1994. Our results: $0.62 \pm 0.05$ (the first method) and $0.64 \pm 0.06$ (the second method) are in very good agreement with this value. It confirms the correctness of the model introduced in this paper and practically demonstrates that two band surface brightness data of external galaxies are a good extinction indicator.

Since $A_R/A_B$ ratio is closely connected to "total to selective extinction ratio" $A_V/E(B-V)$ our result confirms its standard value which is equal to 3.1.

The method applied in this paper to obtain $A_R/A_B$ ratio relies on the assumption that extinction does not depend on morphological type. In future applications to other data (e.g., SDSS) one can use, instead of morphological type, other extinction independent quantities like: axial ratio, redshift or effective diameter.

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Figure 1. Surface brightnesses at half total B light radius of galaxies in the B and R bands for four different morphological type \( T \) intervals. The data are from a subsample of the ESO/LV galaxy catalogue which has been used for all the computations presented in this paper.
Figure 2. The idea of extinction measurements using surface brightness of galaxies in two bands: $\mu_B$ and $\mu_R$. The distance of a galaxy (marked as an asterix) to the zero extinction line measured parallel to the extinction direction line is proportional to the Galactic extinction. The $q$ parameter is extinction independent. The slope of the zero extinction line is $s^{-1}$ while the slope of the extinction direction line is $r = A_R/A_B$. 
Figure 3. The relation between the parameters $s$ and $r = A_B/A_R$ derived from the requirement that extinction $A_B$ and the parameter $q$ are statistically independent (as described in equation 6).
Figure 4. The parameter $s$ as a function of morphological type $T$ for various values of $r = A_R/A_B$. Error bars represent standard deviation ($1\sigma$).
Figure 5. The parameter $c$ as a function of morphological type $T$ for various values of $r = A_R/A_B$. Error bars represent standard deviation (1σ).
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Figure 6. The average extinction in B band $<A_B>$ as a function of morphological type $T$ for various values of $r = A_R/A_B$. The extinction should not depend on morphological type which is true only for $r=0.6$. Error bars represent standard deviation (1σ).
Figure 7. The standard deviation of extinction in B band $\sigma(A_B)$ as a function of $r = A_R/A_B$ for galaxies with $T = -3$ and $T = 5$. The point of intersection of these two curves at $r=0.66$ gives the solution for $A_R/A_B$ since $\sigma(A_B)$ should be the same for any $T$. 
Figure 8. The same as Fig. 7 but for all morphological types $T = -5, \ldots, 10). The dotted line refers to galaxies with $T = -5$ (at the bottom) and $T = 9$ (at the top). These galaxies strongly differ from the mean so they have been rejected in final computations.
Figure 9. The function $D(r)$ which describes the relative standard deviation of the $\sigma(A_B)$. The minimum of this function for $r=0.64 \pm 0.06$ represents the solution for $r = A_R/A_B$ since $\sigma(A_B)$ should be as close as possible to each other (in ideal case the same) for all morphological types.