Is the Froissart bound relevant for the total $pp$ cross section at $s=(14 \text{ TeV})^2$?

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Abstract

The Froissart bound limits the asymptotic $s \to \infty$ behavior of crosssections by $(\pi/t_0) \ln^2(s/(s_0))$ where $t_0$ is the lightest exchanged particle, or more generally the nearest singularity, in the $t$ channel. We suggest that in comparing this bound with data at energies less than those of LHC, glueball masses rather than the small pion mass should be used for $(t_0)^{1/2}$.

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The upper bound on total cross sections at asymptotic energies \([1, 2, 3, 4]\): 

\[
\sigma_{\text{tot}}(s) < (\pi/t_0)\ln^2(s/s_0)
\]

is well known. Its proof uses:

1) The unitarity bound:

\[
0 < a_l < 1 \tag{1}
\]

on the imaginary parts of partial waves in the expansion of the elastic two particle scattering amplitude:

\[
\text{Im}([f(k, \theta)]) = \Sigma(2l + 1) a_l P_l(\cos\theta) \tag{2}
\]

with \(\theta\) the scattering angle in the ”center of mass” Lorentz frame, \(k\) the three momentum of each particle, and \(W = s^{1/2}\) the total energy in the CM with \(s = W^2 = 4(k^2 + m^2)\) and \(|f|^2\) is the differential elastic cross section for momentum transfer \(t = -2k^2(1 - \cos(\theta))\).

2) Analyticity of the \(2 \rightarrow 2\) scattering amplitude in \(t\) for \(t < t_0\) with the \((t_0)^{1/2}\) the lightest mass exchangeable in the \(t\) channel. This implies that the partial wave expansion \([2]\) converges inside the ”Martin-Lehman” ellipse \([5]\) with foci +1 and -1 and with semi-major axis:

\[
\cos(\theta_0) = 1 + t_0/(2k^2) \tag{3}
\]

(The natural convergence domain of a series of Legendre polynomials is an ellipse with Foci at +1 and -1.)

3) Polynomial bounded elastic amplitude and imaginary part thereof, \(A(s, t)\):

\[
A(s, t) = 8\pi s^{1/2} f(k, \theta) < \text{when } (s \rightarrow \infty)(s/(s_0))^{1+\Delta(t)} \tag{4}
\]

We look for the maximal total cross section given by the optical theorem

\[
\max(\sigma_{\text{tot}}) = \max(2\pi/(k^2) \Sigma (2l + 1)a_l) \tag{5}
\]

Subject to \([1]\) and the inequality following from \([3]\) and \([5]\):

\[
A(s, t_0) = \Sigma a_l(s)(2l + 1)P_l(1 + t_0/(2k^2)) < c(s/s_0)^{1+\Delta(t_0)} \tag{6}
\]
Both (5) and (6) are dominated by high l waves. In the physical region, $-1 < \cos(\theta) < 1$, $|P_l| < 1$ and has l zeroes. The asymptotic behavior for large l is:

$$P_l(\cos(\theta)) \sim (\pi l \sin(\theta/2))^{-1/2} \cos[(l + 1/2)\theta]$$

(7)

For $\cos(\theta) = 1 + 2t_0/s$ or $\sin(\theta) \sim \theta = i(4t_0/s)^{1/2}$ the Legendre polynomials grow exponentially with l:

$$P_l(1 + 2t_0/s) \sim \exp((2l + 1)(t_0/s)^{1/2})$$

(8)

We omitted power-like pre-factors and used $2k^2 \sim s/2$ in the large s limit. Maximizing (5) subject to (6) is equivalent to minimizing (6) when (5) is held constant. The exponential growth of the Pl’s suggests using the lowest possible l waves. Consistent with (11) we then take

$$a_l = 1 \text{ for } l < L; \quad a_l = 0 \text{ for } l > L.$$  

(9)

The total cross section is then that of a black disc of radius $R = (L/k)$ consisting of the geometric, inelastic cross section $\pi R^2$ and matching shadow elastic:

$$\sigma_{tot} = 2\pi L^2/(k^2)$$

(10)

Substituting equation (9) for (the imaginary parts of) the partial waves in (7):

$$\exp((2L + 1)(t_0/s)^{1/2}) < A(s, t_0) < c(s/(s_0)^{1+\Delta(t_0)}$$

(11)

(The left-hand side is a geometric series. Approximating it by the last term introduces only $ln(ln(s))$ corrections.) Taking the log of (11) we find:

$$L < k/((t_0)^{1/2}) \cdot [1 + \Delta(t_0)]ln(s/(s_0))$$

(12)

Substituting in Eq. (10) we finally obtain the Froissart-Martin bound:

$$\sigma_{tot}(s) < 2\pi/(t_0)[1 + \Delta(t_0)]^2(\ln(s/(s_0))^2$$

(13)

Historically this bound, one of the very few rigorous result from S matrix theory, played an important role in excluding theories/models predicting cross-sections with power like rise
with energy. The derivation utilized in addition to the usual of the S matrix axioms also polynomial boundedness. We are not aware however of any model where this is not the case. Thus even Veneziano amplitudes suggested as the “Born Term” in “String Like”-description of hadrons have linearly rising Regge trajectories and for arbitrarily large time-like t behave as $s^{\alpha(t)}$ with $\alpha(t) \sim \alpha(0) + t$. However for any finite $t_0$ the requirement of polynomial boundedness still holds.

A power like rise in energy of $A(s,t)$ and also of $\sigma(t)$ appears to arises in QCD from the exchange of gluon ladders in the t-channel. However unitarization/"eikonalization" of such exchanges yields a logarithmically expanding black disc and total cross-sections behaving like $\ln^2(s/s_0)$. If hadronic cross-sections indeed have a $\ln^2(s)$ asymptotic behavior -as experimental data suggest- then the following question- the focus of the present work-as to what $t_0$ should appear in the F.B.- becomes meaningful.

Formally the answer is clearcut: the lightest hadron exchangeable in the t channel is the pion of mass $m=140$ MeV, and the nearest singularity in the imaginary part $A(s,t)$ is at $t_0 = (2m)^2$. (By unitarity, $A(s,t)$ involves products of two amplitudes and requires two-pion exchange). Yet we argue below that for energies $W$ less than 14 TeV (namely the LHC energy) a stronger ”interim” F.B. with the glue-ball mass being the nearest singularity, holds.

pp collisions at the LHC are equivalent to $10^{17}$ eV cosmic ray proton- fixed target collisions. Cross-sections at higher energies are difficult to obtain: The UHE cosmic ray spectrum is cut-off at $E \sim 10^{20}$ eV and the flux the cosmic rays above $10^{17}$ eV is extremely tiny making measurement of cross-sections very difficult.

For energies around LHC’s ($s = (14,000)^2 \text{GeV}^2$), existing cosmic ray data imply that the F.B. with $s_0 \sim \text{GeV}^2$ exceeds the measured value of the inelastic $pp$ cross sections of $\sim 80$ mb by $\sim 100$ so that it appears to be very far from being saturated. However the following more detailed considerations are in order.

The Froissart bound is universal applying to any pair $(a, b)$ of colliding hadrons. For $(a, b)$ different from $p$ or $\bar{p}$ we have data only for $W < 40$ GeV. At such energies the cross-sections depend on structural features which are specific to the colliding pair and vary from case to case: Two-versus three quarks in mesons and nucleons or the heavier $s$ quark in the Kaon (or $\phi$), yielding smaller $\bar{q}q$ bound states and smaller cross-sections of Kaons or of $\phi$ on protons the cross-sections of the pions which are made of non-strange quarks. Still all cross-sections
clearly display a rise. In the PDG fit

\[ \sigma^{a,b} = Z^{a,b} + B \ln^2 (s/s_0) + Y_{1}^{a,b} (1/s)^\eta_1 + Y_{2}^{a,b} (1/s)^\eta_2. \] (14)

\( \eta_{1,2} \) are the \( t = 0 \) intercepts (\( \sim 1/2 \)) of the even and odd signature Regge trajectories-the contribution of the latter flipping sign between \( \bar{a}b \) and \( ab \) and the constant \( Z^{a,b} \) represents geometric, \( a- \) and \( b \)-dependent structural features. For our purpose it is important that in this best fit to existing data, the \( B \ln^2 (s/s_0) \) term is universal as expected if the Froissart bound is saturated.

Equating the fitted \( B = 0.31 \) mb with \( \pi/t_0 \) leads to \( t_0 \sim 4.5 \) GeV\(^2 \sim 55.4\) m\(^2 \). We suggest that the ”effective” \( t_0 \) that can be used in the Froissart bound in the above energy regime is \( m_{\text{glue-ball}}^2 \)–the (squared) mass of the lightest glue-ball. The glue-balls are heavier than the low-lying \( \bar{q}q \) states. The lightest \( 0^{++} \) glue-ball mass computed \(^8\) in lattice QCD is \( \gtrsim 1400 \) MeV and this is also the mass of the lightest putative glue-ball candidate. Using it in \( t_0 \) reduces by a factor of 25 the margin with which the bound is satisfied for the same \( s_0 \).

Several considerations motivate our suggestion.

i) \( \bar{q}q (q = u \text{ or } d) \) states such as \( \pi (\rho) \), etc., have non-universal couplings–stronger to non-strange hadrons and rather weak to hadrons, such as \( \phi(1020) \), \( J/\psi \), etc., containing no \( u \) or \( d \) quarks and hence their exchange seems unlikely to control the universal Froissart bound.

ii) In ’tHooft’s large \( N_c \) limit or when all quark masses are much larger than \( \Lambda_{QCD} \), quark effects are turned off. The universal F.B. applies also to glue-ball-glue-ball scattering and in the large \( N_c \) limit \( t_0 \) should pertain to the glue-ball sector only.

iii) High energy collisions are dominated by gluon physics as expected from the enhanced Clebsch-Gordan coefficients in gluon \( \to 2 \) gluons splitting function. This is indicated by Hera and Fermi Lab data and a further dramatic enhancement expected at LHC makes it an intense \( gg \) machine.

iv) Spin-one gluon exchange dominates high energy collisions \(^9, 10\). It yields constant or, when iterated in the \( t \) channel to form gluon ladders, cross sections rising as a power of \( s \) whereas exchanging spin-half quarks or \( \bar{q}q \) mesons/”Ordinary” Regge trajectories yields the \( Y \) terms in Eq. \(^{11}\), falling like \( \sim (1/s)^{1/2} \).

iii) and iv) are valid in perturbative QCD. There, and also with ”BFKL” evolution—which builds the gluon exchange ladders in the \( t \) channel—the gluons are massless and \( 1/(t_0) \sim (1/(m_g))^2 \) yields a useless bound. Our suggested new bound is based on the as-
assumption that this high energy behavior of the glue sector persists also non-perturbatively. We then expect the $t$ channel threshold— the lowest state in “$t$ channel” cuts across the exchanged ”Gluon blob”, to have the mass of the lightest glue-ball $m_{gb}$.

More specifically the suggested stronger interim bound can be motivated as follows: The ordinary Froissart bound is derived for a $2 \rightarrow 2$ amplitude $A(s,t)$ which has:

a) a nearest (to 0) $t$ channel singularity at $t = t_0 = 4m^2$ with $m$ defined all along as the mass of the pion, and b) grows with energy like $A(s,t_0) \sim s^{1+\Delta(t_0)}$.

On first sight it seems that no such amplitudes exist: High energy amplitudes involving pion exchanges which naturally satisfy (a) with $t_0 = 4m^2$ fail to satisfy (b). Conversely, glue-ball exchange amplitudes guaranteed to satisfy (b) may not be singular at the two-pion threshold $t \sim t_0 = 4m^2$.

These issues are best illustrated in the context of Heisenberg’s argument for the F.B., predating Froissart by several years and which we briefly reproduce here in the framework of the renormalizable Yukawa model which existed already at that time:

$$L_{Yukawa} = \int d^4x \phi(x) \bar{\psi}_N(x) \psi_N(x).$$

A bare nucleon at $r = 0$ serves here as a point source for the pion’s field: $ \phi(r) = 1/(r \exp(mx))$. When boosted to high energy along the $z$ axis, say, it contracts but the (Lorentz invariant) profile still falls like $\exp(−mb)$ with impact parameter. The simplest version of the argument utilizes only energy considerations and “geometry”.

The relevant interactions are between the target bare nucleon—at impact parameter $B$ away and the contracted ”disc-like” pion field of the projectile (and vice-versa) and some interactions between the pionic fields generated by the two nucleons. The maximal impact, $B_{\text{max}}$, for which non-vanishing inelastic collisions can occur when no transverse diffusion of the pionic field energy density is allowed, is fixed as follows:

We demand that the energy fraction of the projectile residing at impact parameters $b > B_{\text{max}}$: $E \exp(−m_\pi \cdot B_{\text{max}})$ which participates in the collision, exceed some threshold $E_{\text{min}} = s_0^{1/2}$ for producing the lightest hadronic system in the $s$ channel. This yields the F.B. with $t_0$ and $s_0 \sim m^2$. It also implies a substantial part of inelastic events with just one additional light hadronic state produced with big rapidity gaps to the two nucleons.

Note that in order to saturate the F.B we have to assume that elements from the two discs overlapping in impact parameter interact—the matter how large the rapidity gap between
them.

As we next show this assumption which underlies the above simple geometric argument fails for the Yukawa interactions of Eq. (15):

To avoid self interactions we use the target’s \( \psi \) and the projectile’s \( \phi \) and vice-versa. The boost contracts the \( \phi \) disc in the projectile, leaving the Lorentz invariant (pseudo) scalar \( \phi(x,t) \) the same. This yields a scattering amplitude \( A(s,t) \) which is constant as a function of the energy \( E \) failing to satisfy (b). Indeed, this is what one expects from one \( \pi \) exchange between the nucleons. It behaves for \( s \to \infty \) as \( s^{J_{\text{exchange}}} \) and \( J = 0 \) yields asymptotic constant imaginary part of forward amplitude and cross-sections falling like \( 1/s \).

However the geometric picture is justified in QCD!

A basic feature of QCD is that gluon and gluon ladder exchanges automatically generate constant or rising cross sections. This is true for one-gluon exchange between \( q\bar{q} \) pairs and also if the scattering hadrons were elongated chromoelectric flux tubes confining the quarks at their ends. To see this let us repeat the above discussion with \( L(\text{interaction}) \sim E_1(x,t) \cdot E_2(x,t) \) the interaction effecting the scattering of particles 1 and 2. It follows from the Yang-Mills \( F^2 \) Lagrangian if magnetic and self (11 and 22) interactions are emitted.

We find that the simple geometric picture where the overlapping portions of the flux tubes have a finite probability to to interact at all energies is now justified: \[11\]. The vector nature of \( A_\mu \) -or equivalently the boost invariant Gauss law- implies that the \( z \) contraction of the chromoelectric flux tube by the Lorentz boost factor \( \gamma = (E/m) \) in the lab frame is compensated by an increase by the same factor \( \gamma \) of the chromoelectric fields \( E_x \) and \( E_y \) leading to \( A(s,t) \sim s \) and to constant cross sections.

The gluon exchange amplitudes involve the (induced) \( \bar{q} - q \) color dipole moments or, for the confined bound states, the lengths of the flux tubes and thus provide for a structure-dependent constant part of the cross section naturally accounting for the constant \( Z_{A,B} \) terms in the PDG fit of Eq. \[14\].

Faster \( s^{1+\Delta(0)} \) rising amplitudes can emerge if we iterate the gluon exchanges in the \( t \) channel–ala BFKL and /or other methods. The saturation of gluon density and related issues are extensively discussed in ”low x” literature \[12, 13, 14, 15\]. The key assumption that we need to make in order to proceed further is that the feature of gluon exchanges giving rise to \( s^{1+D} \) rising amplitudes \( A(s,t) \) at time-like momentum transfers \( t > 0 \) persists also non-perturbatively . With a glue like blob exchanged in the \( t \) channel in high energy
collisions the lowest t channel singularity, $t_0$, controlling the Froissart bound is $m_{gb}$. Also, we expect then that: $s_0 \sim m_{gb}^2$.

Thus while gluon exchanges in QCD can account for the observed total and various inclusive cross-sections these exchanges do not naturally lead to $t_0 = 4m^2$ (with $m$ the pion mass) in the F.B. To better understand this issue we recall some pre-QCD approaches to high energy scattering where pions played a direct and dominant role.

Since as emphasized above, pion exchange cannot effectively couple hadrons having a large rapidity gap– multi-peripheral models with repeated pion exchange in the $t$ channel were suggested some time ago [16]. In parton model [17] terms we have in the lab frame repeated evolution of the projectile’s partons or pions, with each splitting generating a pion slower on average by some factor $g$. After $n$ steps with $g^{-n}E < \text{GeV}$, the last pion in the ladder can strongly interact with the target. The ordering in rapidity of the successive ladder rungs allowed summing the series yielding:

$$A(s \to \infty, \ t \text{ fixed}) \sim \beta(t) \ s^{a(t)}. \quad (16)$$

The universal $a(t)$ depends only on masses and on the couplings of the $\pi\pi$ resonances produced in the multi-peripheral ladder. The approximation $a(t) = a(0) + a't$ implies a $a'\ln(s)^{-1}$ shrinking of the forward diffraction peak interpreted by Gribov as due to $\sim \ln(s)$ steps of a random walk in impact, $b$ space, and the corresponding increase by this factor of the (squared) range of interaction.

Similar physics arises when the ($t$ channel) partial wave amplitude $a(l,t)$ has a leading ”Regge” pole [18] in the complex angular momentum plane (the one with largest real part) $\alpha(t) = a(t)$ and residue $\beta(t)$ [19].

The apparently constant asymptotic cross sections or $A(s,0) \sim s$ required within this framework a ”Pomeron” trajectory of intercept $\alpha_P(t = 0) = 1$. Most particles lie on linearly rising trajectories of common slope $d\alpha/dt \sim 1/\text{GeV}^2$.

However, the highest intercepts were $\sim 0.5$ rather than the required 1, the slope of the Pomeron trajectory was much smaller than $\text{GeV}^{-2}$ and there were no spin-2 particles lying on it. Since also the specific intercept of unity, while possible, is not guaranteed, the modern ”Glue Pomeron” was adopted.

$\pi\pi$ thresholds pervade much of hadronic physics and contribute to the the $g - 2$ of the muon. Thus to find the mass of the $\rho$ meson one computes in lattice QCD the two-point
$(x, y)$ function of vector currents and fits the $|(x - y)|$ dependence with $\exp(-m_\rho|x - y|)$. For a stable $\rho$ this is indeed correct. However, after a sufficiently long time the two pions become the dominant state and the asymptotic behavior reverts to $\exp(-(2m|x - y|))$. By using $|x - y|$ values larger than $m_\rho^{-1}$ yet smaller than $\tau$, the $\rho$ decay time, one can estimate $m_\rho$. The large ratio $m_\rho/(\Gamma_\rho)$ required is provided if $N_c >> 1$. Likewise, the $\pi\pi$ admixture in $0^{++}$ glue-balls only slightly modifies the masses of the glue-balls.

However, in the present case the space-like $|b|$, the analog of $|x - y|$, really tends to infinity, making pion thresholds far more relevant for the Froissart bound.

A specific $A_c$ to be presented next has both $s^{(1+D)}$ rising amplitude and $\exp(-2m.b)$ behavior for large impacts. It does then provide a counter-example to the claim that no such amplitude exists and reinstates the original “Axiomatic” F.B. with $t_0 = 4m^2$. However the small overall coefficient of $A_c$ pushes to higher energies the onset of the original ”axiomatic” F.B and allows using our stronger, interim, bound with $t_0 = m^2_{gb}$ for energies bellow $W=14$ TeV of LHC.

The ”hybrid” amplitude of $\Pi$ consists of gluon ladders with two- pion exchange insertion. The gluon ladders bridge most of the rapidity gap yielding the desired strong high energy behavior, $A \sim s^{(1+D)}$, and the pion pair with the slow b fall-off, $D_\pi(b) \sim \exp(-(2mb))$, bridges most of the transverse coordinate space separation. Note that we need two pions to be exchanged in the $t$ channel whereas two (and multi) gluon states can couple to just one $0^{++}$ glue-ball.

The Froissart bound with $t_0 = 4m^2$ clearly arises in a $\lambda\pi^4$ theory with massive pions of mass $m$ [20, 21].

However, the pions in QCD (and in the real world!) are pseudo-Goldstone Bossons associated with the spontaneous breaking of the global axial SU(2).

The non-vanishing pion mass $m$ reflects the explicit symmetry breaking $u, d$ masses in the Lagrangian: $m^2 \sim (m_u^0 + m_d^0)$. The $(m_u^0 \sim m_d^0 \rightarrow 0)$ symmetry limit should be smooth with no physical quantity changing dramatically.

The reason is that confining QCD generates, independently of the quark masses, a mass gap which serves as an infrared cutoff and avoids infrared divergences of physical quantities. Hence cross sections cannot behave like $m^{-2}$ and even asymptotic bounds with overall coefficients of $1/m^2$ seem puzzling.

More generally any theory with no I.R. divergences the standard E.W. model or other
FIG. 1: A generic diagram contributing to the amplitude $A_c$. The wavy lines describe gluons comprising the upper and lower multi gluon ladders. The full double lines represent the pion circulating in the pion box. The latter couples to the gluonic blobs via color singlet gluon pairs at the four corners of the box.

Field theories with no vanishing mass parameters, cannot develop such divergences even when spontaneous symmetry breaking yields massless Goldstone particles—a result known as Elitzurs’ second theorem.

In view of this let us consider more closely the hybrid gluon ladder with a nested pion box to understand how— all the above arguments notwithstanding it does yield an amplitude $A_c(s, t)$ which gives rise to the standard Froissart bound with $t_0 = 4m^2$. 
The Goldstone character of the pions and the ensuing derivative couplings in the low energy effective chiral Lagrangian imply that the four coupling (to the upper and lower blobs in Fig. (gluon-pion) of the exchanged pions vanish (have ”Adler Zeroes”) when the four momentums $p$ and $p'$ of the pions vanish.

The three momentums $p_i$ and $p'_i$ vanish at the $t$ channel threshold: $t = t_0 = 4m^2$. More generally the space-like four momentums $p_\mu$ and $p'_\mu$ of the two pions corresponding to transverse separations larger than $1/m$ between the blobs in the figure, are smaller than $m_\pi = m$ and vanish in the $m \to 0$ limit. Hence the couplings of the pions at these four vertexes vanish at this point evading $1/(m^2)$ divergences . This does not remove the square root branch point singularity at $t = t_0 = 4m^2$ for finite pion mass $m$ but softens it. The hybrid amplitude $A_c$ with the double- pion exchange is supressed at the two pion threshold in the $t$ channel by $(m/M)^4$ due to the four couplings at the corners where $M \sim \text{GeV}$ is the denominator mass in the momentum -chiral perturbation- expansion. Further the introduction of (at least) two quark loops yields $\sim (2\pi N_c)^{-2}$ extra suppression. The last suppression is present over and above the first chiral suppression, applying if the two Nambu-Goldstone pions were replaced by any massive meson made of a quark and an anti-quark.

The amplitude with the two-pion exchange insertion is thus strongly suppressed relative to that of pure gluon exchange by a factor $F \sim 10^{-6}$. More conservatively we estimate that F lies in the interval $10^{-8} - 10^{-4}$. Even with this large suppression factor the $A_c$ amplitude in $s$ and $b$ space,

$$A_c(s, b) \sim F(s/s_0) \exp(-2mb)$$

(17)

dominates for large $b$ values the standard pure glue exchange amplitude:

$$A_0(s, b) \sim (s/s_0) \exp(-m_{gb}b).$$

(18)

The apparent difficulty of saturating a F.B. with a $1/m^2$ coefficient is resolved by the $(m/M)^4$ factor included in the F prefactor of $A_c$. This factor pushes the crossover between the interim and assymptotic F.B to infinite $s$ values as $m$ approaches zero .

The full $b$ profile for the collisions is the sum of the above two contributions: $A(tot) = A_0 + A_c$. The purely gluonic $A_0$ tends to concentrate at small impact parameters $1/m_{gb} \sim 0.14$ Fermi and the hybrid -Glue + pion pairs- diagram$A_c$ has a five times more extended
profile $1/2m \sim 0.7$ Fermi but a $F \sim 10^{-6+/-2}$ times smaller coefficient. The unitarity bound stating that the partial waves (or the eikonal function) $A(s,b)$ is less than unity, should be applied to the full amplitude $A(tot)$. However the very different shapes of the two contribution allow the approximation of applying this bound to each amplitude separately. Solving $A_0(s,b) \sim A_c(s,b) \sim 1$ then yields an approximate value for the transition between the region of energies where $A_0$ dominates (and consequently our “interim” F.B holds) and the region where $A_c$ dominates and the ”Asymptotic” original F.B is reinstated: $s/s_0 \sim F^{(1+2m/m_{gb})} \sim 10^{7.2+/-2.4}$ and the corresponding central $b$ value is $2.2+/-0.8$ Fermi. We note that for $s_0 \sim 20 \text{ GeV}^2$ similar to the value used in the PDG fit, the central value of the transition energy is $3 \cdot 10^8 \text{GeV}^2$ just above LHC’s energy and the corresponding inelastic black disc cross-section $\pi \cdot b^2 \sim 150 \text{ mb}$ is less than a factor two higher than that measured in cosmic ray experiments at these energies.

This brings us to the single most relevant question: Is it conceivable that we will find at LHC energies signals of such a transition?

The high rate of pp collisions allows, in principle, measuring the total pp cross-sections at the nominal highest LHC energy of 14 TeV and perhaps also at lower energies, with unprecedented accuracy. However, since even the more stringent interim F.B is not saturated at the LHC, this measurement alone, may not indicate the above suggested transition.

The blending in of the much larger and relatively transparent ”$A_c$ Disc” due to the pion box diagram could generate a sharper diffraction peak at small momentum transfers $t$. Unfortunately, the measurements required to verify this are rather difficult at the LHC set-up.

This still leaves us with the following indirect possible indication. The calculations of cross-section for UHE CR proton - Air nucleus (and even more so for He- Air) from pp cross-sections \[7\] \[22\] can be sensitive to the extended profiles \[23\]: the mutual shadowing is minimal for the extended, almost transparent, tails of the $b$ profiles in the $A_c$ amplitude. Consequently the $p$-$A$/pp cross-section ratio may be considerably larger than what computations based on Glaubers’ theory with standard single Gaussian parameterization of the nucleons $b$ space profiles would suggest. The effect is even more dramatic for He - Nucleus collisions since the compact Helium nucleus is comparable in size to the logarithmically expanding “pion” disc. It is tempting to speculate that the apparent indications in recent Auger data of component of the cosmic ray interactions occurring very high up in the atmo-
sphere discrepancy is related to the enhanced proton -nucleus cross-sections. Conversely the pp cross-section to be measured at LHC will then be lower than the values inferred from cosmic ray data using the standard analysis.  

We close with the following remarks:

I) The fact that to date, some 30-40 years after the introduction of QCD no glue ball states have been established impedes more quantitative implementation of our suggested 'interim” F.B. Finding the lightest glue-ball, presumably a 0$^{++}$ state as suggested by lattice calculations (but not proven to be so despite some theoretical efforts) would fix the mass in the prefactor of the interim Froissart bound.

Also the partial decay width $\Gamma(gb \to \pi\pi)$ would help pin down the glue-ball coupling to two pions in the amplitude $A_c$ above allowing a more precise estimate of its suppression factor F, and of the corresponding transition energy between the interim and asymptotic cross-sections.

II. several pieces of experimental data on pp and/or $\bar{p}p$ scattering at energies W up to 2TeV indicate that the Froissart bound is not saturated in pp and/or $\bar{p}p$ scattering. Thus $\sigma(\text{elastic})/\sigma(\text{tot})$ is 1/4 is significantly smaller than 1/2 as required for a black disc. Also the measured of slope of the differential cross-section at these energies exceeds that predicted in a black disc model and the differential cross-section does not have the $J_{21}(Rt^{1/2})/t$ shape predicted for a black disc of radius R. Finally a black disc yields a purely imaginary elastic amplitude whereas the measured interference with the Coulomb real amplitude yields:

$$\Re(A(s,t=0))/\Im(A(s,t=0)) \sim 0.1$$

It is therefore gratifying that also our more strict F.B is not saturated.

III. While the the growth of the proton proton crossection is is not far from saturating the interim F.B. we are very far from the super asymptotic energies where all hadronic cross-sections starting with large nucleus nucleus cross-sections and ending with small $\Upsilon - \Upsilon$ cross-sections converge to the same asymptotic value. Indeed as emphasized above the large yet relatively transparent disc due to the mixed -pion-gluon amplitude $A_c$ tend to make proton deuterium crossection grow initially faster and only at super-assymptotic energies will both p-p and p-D crossections start converging to the common large original large value of the Froissart limmit. We note that the universality of UHE hadronic scattering -expected when and if the F.B is fully and truly saturated - reflects not only in the independence of the value of the cross-sections on the incoming hadrons a and b. We also expect a universal
(a,b) independent pattern of produced particles. These are just the various glue balls in our interim F.B. case and one additional low energy pi pi pair when the diagram $A_c$ 'kicks in' and we revert to the axiomatic F.B. This is suggested by s channel cuts across the corresponding diagrams. These cut (apart from the a and b fragmentation products at +/- $Y$(max)) across just the gluonic blob or the latter and one pair of pions in the two cases respectively. Unfortunately glue-balls are likely to be broad -decaying into many pions and distinguishing the case when in addition to all the decay pions we have another soft pair of pions at mid-rapidity is impossible. The universality of final states produced from the (expanding) black disc is reminiscent of the universal spectrum of particles emitted via Hawking radiation from a black hole. In passing we recall that Heisenberg’s argument for a (saturated) Froissart bound suggested a sizeable fraction of all final states consisting of a one pion pair with large, +/- $Y$(max) gaps to the elastic or diffracted fragments of the incident protons. The ”survival probability” of such large gaps was studied in Q.C.D. in connection with the production of just one Higgs particle at mid rapidity gap accompanied only by the initial protons or some low lying diffractive excitation there-off and is rather small [33][34] The survival probability of the very light pion pair may be larger, but, just like for the forward diffraction peak will be extremely difficult to study experimentally.

IV. To put our interim, yet stronger, F.B. in prospective recall the e.m (photon exchange) contributions. To avoid Coulomb divergences consider pn scattering. At ultra high energies these are dominated by exchanges of ladders with two photons coupling to closed electron boxes leading to the production of many electron-positron pairs. The corresponding cross-sections have been calculated [32][35] and as expected from the vanishing photon mass violate the ordinary Froissart bound (with $t_0 \sim m^2$). The clear separation of strong and electro-magnetic interactions allows neglecting the latter at all foreseeable energies and the pre-factor $1/t_0$ in the F.B. remains $(2m)^{-2}$. We suggest that the relation of our interim F.B with the large glue-ball denominator mass and the original F.B. with the pion mass is analogous to that of the latter and the behavior expected when we have also the e.m. contributions. Here the analog of the massless photons are the gluons which due to the non-pertrurbative QCD effects become massive and the the analog of the $m_e \sim 1/2MeV$ electron loops in the QED case are the light $u/d$ 4-8 MeV quarks manifesting here via the pion loop in $A_c$. Finally the smallness of the EM couplings is reflected here in the rather large -chiral and $1/N_c$ suppression of the pion loop in diagrams contributing to $A_c$. While
the disparity between this and the unsuppressed purely gluonic exchanges is much smaller than for EM versus hadronic processes - the notion that an interim more restrictive F.B can apply is similar.

V. A recent treatment [36] of the "QCD Pomeron" using an "ADS-CFT-Like" correspondence suggested a Froissart bound with a $1/m_g^2$ pre-factor. Since the infinite $N_c$ limit is implicit in such approaches this is indeed expected. Also the specific form of the F.B is natural from the ADS-CDT point-of-view [37]. The black disc is analog to a black hole - which sitting at a distance of $\ln(s/s_0)$ from the conformal AdS boundary can have a similar radius and a geometric cross-section $\ln(s/s_0)^2$. This is further motivated by having, as discussed in Sec III above, emission of final state particles in ultra-high energy collisions, which is largely independent (apart from conserved Q.numbers) of the colliding particles, in analogy with to Hawking radiation being independent of how the B.H is formed.

I have been contemplating a stronger F.B. in the gluonic picture of the Pomeron for more than thirty years. Variants of this notion have already appeared sometime ago in, among others, works by L.McLerran and by E.Gotsman and (respectively) collaborators. The fact that such arguments fail in a perturbative framework has been emphasized by A.Kovner. The final impetus for this work came during my recent visit to Ohio state University in a discussion with Anna Staso. I am particularly grateful to Yuri Kovchegov for patiently listening to an earlier - rather primitive-version of this work, for his most constructive criticism and for invaluable help in improving it. I also thank C. Quigg and L. Frankfurt for drawing my attention to the Auger puzzling results.

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