Multiple lepton pair production in relativistic ion collisions

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Abstract

We apply the Sudakov technique to description of the multiple production of lepton pairs in peripheral collisions of ultrarelativistic heavy ions. For heavy ions with $Z_1 \alpha \sim Z_2 \alpha \ll 1$ one needs a careful treatment of the multiple Coulomb exchange between colliding ions and screening effects, whereas interaction of real or virtual lepton pairs with colliding ions can be neglected. We demonstrate that while the inclusive spectra are modified by multiple Coulomb exchange between the colliding ions, the Coulomb corrections to the momentum integrated multiplicity distributions do vanish. After transformation to the impact parameter representation the probability of $n$ lepton pair production is shown to obey the Poisson distribution. The relevant cross section is obtained.

1 Introduction

The multiplicity and the distribution of lepton pairs produced in the Coulomb fields of two colliding relativistic heavy ions are closely connected to the problem of unitarity. When heavy ions collide at relativistic velocities their Lorentz contracted electromagnetic fields are sufficiently intense to produce a large numbers of such pairs. Usually the process of lepton pairs production is considered as pair creation in the classic Coulomb potential of a charge
moving along a straight line. Such an approach allows one to investigate the impact parameter dependent total probability of the pair creation $P(b)$, which by definition is connected with the total cross section $\sigma = \int P(b)db$.

As was noticed in [2] the probability of single pair production calculated to lowest order in fine structure constant $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$ (throughout we use units for which $c = \hbar = 1$) at small impact parameters exceeds one, thus violating unitarity. This excess begins at impact parameters smaller than the Compton wavelength of the electron $\lambda_c = \frac{1}{m} = 386$ fm and at energies of practical interest (RHIC & LHC).

Allowance for the finite size of colliding nuclei doesn’t remedy the situation, because that would affect only the impact parameters comparable to the nuclei radii, which are much smaller than the Compton wavelength of the electron $R \ll \lambda_c$.

As was shown in [3] this problem can be solved by taking into account the possibility of multiple pair production, whose relative contribution grows with energy and dominates at small impact parameters. Since this early publication there has been much work has in this direction (see e.g. [4] and references therein) with the common for all statement: the probability to produce n lepton pairs in the Coulomb field of heavy ions colliding at fixed impact parameter $b$ can be approximately represented as a Poisson distribution i.e. $P(b,n) = \frac{W^n(b)}{n!}e^{-W(b)}$, where $W(b)$ is the average multiplicity of pairs at a fixed impact parameter.

Because of the somewhat controversial situation in the subject and its importance for the operation of relativistic heavy ion colliders (RHIC & LHC), in the present communication we revisit the multiple pair production based on the powerful Sudakov technique of calculation of high energy Feynman diagrams. Recently it has been applied [3] to the calculation of Coulomb corrections to single lepton pair production in relativistic heavy ion collisions. Here we extend the Sudakov approach to get the probability of multiple pair ($n \geq 2$) production in relativistic heavy ions collisions. For heavy ions with charge the numbers satisfy the conditions $Z_1\alpha \sim Z_2\alpha \ll 1$, $Z_1Z_2\alpha \gg 1$, one needs full allowance of multiple Coulomb interaction of colliding nuclei, whereas the secondary interaction of produced pairs (real or virtual) with the Coulomb fields of colliding ions can be neglected.

The paper is organized as follows. In section 2 we introduce the necessary definitions and discussed briefly the Sudakov derivation of the Born amplitude for single pair creation. Then We show how one can sum the perturbative series to get the amplitude for the process of $n$ pairs production in a compact analytical form.

In Section 3 we consider the Coulomb interaction between the colliding ions and show how it affects the amplitude and cross section of the process.
under consideration. Apart from the conventional multiple photon exchange between ions, this interaction include also the so called "screening corrections" which are the result of the ions interaction through the virtual lepton pairs and are responsible for the probability of vacuum–to–vacuum transition. In the language of the Abramovski–Gribov–Kancheli unitarity rules \[6\], the diagrams with lepton pair loops can be associated with the QED model for the pomeron, and the n-pair production amplitudes can be associated with the unitarity cuts through n exchanged pomerons. In section \[4\] we report the multiple production amplitudes in the impact representation in which the summation of the perturbation series for the n-pair production can be done in a compact closed form. Finally, using this amplitude we obtain the probability of n pair production which can be cast in the Poisson form, and the total cross section of all pair production process.

2 The amplitude of the n pairs production process.

The typical Feynman diagram (FD) describing the n lepton pairs production in the collision of relativistic nuclei with atomic numbers \(A_1, A_2\), with \(n_e\) exchanged photons between colliding nuclei as well as screening effects e.g. the insertions of \(n_s\) light by light (LBL) scattering blocks is drawn in Fig. \[1\].

Upper and lower blocks in Fig. \[1\] describes many virtual photons interac-
tion amplitudes with nuclei. They contain the complete set of \((n_e + n + 2n_s)!\) Feynman diagrams. To avoid the multiple counting in what follows, we will multiply the relevant amplitude by the factor \(1/(n_e!n_s!(2!)^{2n_s})\).

As was mentioned above we restrict ourselves to ions with charge numbers such that

\[
Z_{1,2} \gg 1, \quad Z_1 \alpha \sim Z_2 \alpha \ll 1, \quad Z_1 Z_2 \alpha > 1,
\]

which permits us to omit the multiphoton exchanges between the produced pairs and colliding ions. The dominant mechanism is a production of a single lepton pair per collision of equivalent photons, i.e., one lepton loop per two-photon ladder. The alternative mechanism of multiple pair production per collision of equivalent photons, i.e., multiple lepton loops per two-photon ladder, is suppressed by inverse powers of \(Z_1 Z_2\).

For the description of a peripheral process of \(n\) lepton pairs creation i.e. the process

\[
A_1(Z_1, p_1) + A_2(Z_2, p_2) \to A_1(Z_1, p_1') + A_2(Z_2, p_2') + e_+ e_-(r_1) + \cdots + e_+ e_-(r_n),
\]

\[r_i = q_+^i + q_-^i\] (2)

It is convenient to use the Sudakov parameterization for the four-momentum of all exchanged photons (for details see [3])

\[
k_i = \alpha_i \tilde{p}_2 + \beta_i \tilde{p}_1 + k_{i \perp}, \quad d^4k_i = \frac{s}{2d\alpha_i d\beta_i d^2k_{i \perp}},
\] (3)

\[s = (p_1 + p_2)^2, \quad s \gg p_i^2 = M_i^2 \gg m^2, \quad \tilde{p}_1 = p_1 - p_2 \frac{p_1^2}{s}, \quad \tilde{p}_2 = p_2 - p_1 \frac{p_2^2}{s},\]

\[
\tilde{p}_1^2 = \tilde{p}_2^2 = O\left(\frac{m^6}{s^2}\right), \quad \tilde{p}_1 k_{i \perp} = \tilde{p}_2 k_{i \perp} = 0, \quad s = 2p_1 p_2 = 2\tilde{p}_1 \tilde{p}_2.
\]

Here \(\tilde{p}_i\) are light-like four-vectors build from \(p_i\), \(M_i\) are the masses of colliding nuclei, \(m\) and \(s\) are the electron mass and the total center mass energy.

The denominators of intermediate states of the nucleon Green functions for upper and lower blocks are the same and have the following form

\[
s \sum_i \alpha_i - \left(\sum_i k_i\right)^2 + i0, \quad s \sum_i \beta_i - \left(\sum_i k_i\right)^2 + i0.
\] (4)

Peripheral process is characterized by small values of longitudinal Sudakov parameters \(\alpha_i, \beta_i\) and the transverse momenta of the order of electron mass

\[|\alpha_i| \sim |\beta_i| \ll 1, \quad -k_{i \perp}^2 \sim m^2.\] (5)
Further simplification follows from the form of the nominators of the exchanged photons Green function (we work in the Feynman gauge). Using the Gribov’s representation for the metric tensors

\[ g_{\mu\nu} = g_{\mu\nu}^\perp + \frac{2}{s} \left( \tilde{p}_1^\mu \tilde{p}_2^\nu + \tilde{p}_1^\nu \tilde{p}_2^\mu \right) \]  

(6)

it is easy to show that for the typical conversion of the nuclei currents \( J_\mu(p_1)J^\mu(p_2) \) only one term, which contains the scalar products of a nucleus current with a four–momentum of another nucleus, becomes relevant (with power accuracy)

\[ J_\mu(p_1)J^\mu(p_2) \approx \frac{2}{s} J_\lambda(p_1)p_1^\lambda J_\sigma(p_2)p_2^\sigma \left( 1 + O\left( \frac{m^2}{s} \right) \right) \]  

(7)

It can be seen that the quantity \( J_\mu(p_1)J^\mu(p_2)/s \) remains finite with the large values of \( s \). This fact provides great simplification of the spinor structure of the amplitude

\[ \bar{u}(p_1')\tilde{p}_2(p_1 + \chi_1 + M_1)\tilde{p}_2 \ldots (p_2 + \chi_N + M_1)\tilde{p}_2 u(p_1) \approx s^{N+1}N_1, \]  

(8)

\[ \bar{u}(p_2')\tilde{p}_1(p_2 + \eta_1 + M_2)\tilde{p}_1 \ldots (p_2 + \eta_N + M_2)\tilde{p}_1 u(p_2) \approx s^{N+1}N_2, \]

\[ N_1 = \frac{1}{s} \bar{u}(p_1')\tilde{p}_2 u(p_1), \quad N_2 = \frac{1}{s} \bar{u}(p_2')\tilde{p}_1 u(p_2). \]

Besides we have \( \sum |N_1|^2 = \sum |N_2|^2 = 2 \) for the nuclei with the spin 1/2 and \( |N_1|^2 = |N_2|^2 = 1 \) for the scalar one. Using the identity

\[ \sum_{\text{perm}} \frac{1}{\alpha_{i_1}} \frac{1}{\alpha_{i_2}} \cdots \frac{1}{\alpha_{i_N}} = \prod_{i=1}^{N} \frac{1}{\alpha_i} \sum_{j=1}^{N} \alpha_{i_j} \]  

(9)

one can be convinced that the amplitude describing the upper and lower blocks in Fig. [[ can be put in the form

\[ I_1 = N_1 \prod_{i=1}^{N} \left( \frac{s}{-s\alpha_i + i0} + \frac{s}{s\alpha_i + i0} \right), \quad I_2 = N_2 \prod_{i=1}^{N} \left( \frac{s}{-s\beta_i + i0} + \frac{s}{s\beta_i + i0} \right) \]  

(10)

with \( N = n_e + 2n_s + n - 1. \)
This expressions contain all dependence on Sudakov parameters $\alpha_i, \beta_i$ (the 4-momenta of exchanged photons in the peripheral kinematics in denominators of their Green functions can be considered as Euclidean two-vectors $k_i^2 = s\alpha_i\beta_i + k_{i\perp}^2 \approx k_{i\perp}^2 = -k_{i\perp}^2$).

At this stage the integration over Sudakov parameters can be done, because the dependence of the amplitude on $\alpha_i, \beta_i$ provides the convergence of the relevant integrals

$$
\int I_1 \prod_{i=1}^N d\alpha_i = (2\pi i)^N N_1, \quad \int I_2 \prod_{i=1}^N d\beta_i = (2\pi i)^N N_2.
$$

Let us now consider the single pair production. The amplitude of the process (1) in its lowest order (Born approximation) reads

$$
M^{(1)}_{(0)} = is(8\pi\alpha)^2 Z_1 Z_2 N_1 N_2 B_{\alpha\beta} \frac{p_1^\alpha p_2^\beta}{s q_1 q_2}, \quad B_{\alpha\beta} = \bar{v}(q_+) O_{\alpha\beta} u(q_-),
$$

$B_{\alpha\beta}$ is the Compton tensor for pair creation by two virtual photons with polarization vectors $e_1(q_1), e_2(q_2)$. \(q_{1(2)}\) are the 4–momenta of exchange photons and \(r = q_+ + q_-\). Strictly speaking the squares of these 4–vectors \(q_i^2\) do not vanish in the limit \(q_i^2 \to 0\). This fact becomes essential when one calculates the total cross section of a single pair production process. For the case of two or more pairs production (which is our case) the replacement \(q_i^2 = -q_1^2\) can safely be done.

Using the gauge invariance

$$
q_1^\alpha B_{\alpha\beta} = q_2^\beta B_{\alpha\beta} = 0
$$

one can perform the replacement

$$
\frac{B_{\alpha\beta} p_1^\alpha p_2^\beta}{s} = \frac{B_{\alpha\beta} e_1^\alpha e_2^\beta}{s_1} |q_1||q_2|, \quad e_i^\alpha = \frac{q_i^\alpha}{|q_i|}, \quad s_1 = s\alpha_2\beta_1,
$$

The quantity \(s_1\) is related to the square of the invariant mass of a pair

$$
s_1 = (q_1 + q_2)^2 = s_1 - (q_1 + q_2)^2 = (q_+ + q_-)^2.
$$

Two-dimensional vectors \(e_i\) can be interpreted as a polarization vectors of exchanged virtual photons.

Using (13) one can rewrite the Born amplitude (12) in a form

$$
M^{(1)}_{(0)} = isN_1 N_2 B(q_1, q_2),
$$

$$
B(q_1, q_2) = (8\pi\alpha)^2 Z_1 Z_2 \frac{B_{\alpha\beta} e_1^\alpha e_2^\beta}{s_1 |q_1||q_2|}.
$$
Now we are able to construct the amplitude for the process of \( n \) pairs production. Bearing in mind the expressions (11) the matrix element of two pairs production can be represented as the convolution of two Born terms from expression (16b)

\[
M^{(2)}(0) = i^2 s N_1 N_2 \int B(k, k - r_1) B(q - k, q - r_2 - k) \frac{d^2 k}{8\pi^2}.
\]

A straightforward generalization to the matrix element in the case of \( n \) pairs production reads

\[
M^{(n)}(0) = i^n s N_1 N_2 \int \prod_{i=1}^{n-1} \left( B(k_i, k_i - r_i) \frac{d^2 k_i}{8\pi^2} \right) B(h, h - r_n),
\]

\[
h = q - \sum_{i=1}^{n-1} k_i.
\]

Thus one can see that the amplitude for multiple pair production is solely determined by the convolution of the amplitudes corresponding to single pair production.

In the language of the AGK unitarity rules, this result can be interpreted as unitarity cut through all exchanged pomerons.

3 The Coulomb exchanges between ions

Let us now consider the effect of \( m \) photon exchanges between nuclei \( A_1, A_2 \). The arguments given above leads to the following matrix element for the process of \( n \) pairs production with \( m \) photons exchanges among the colliding ions

\[
M^{(n)}(m) = \frac{i^n s N_1 N_2}{m!} \int \prod_{j=1}^{m} \left( -i \alpha Z_1 Z_2 \frac{d^2 \chi_j}{\chi_j^2 + \lambda^2} \right) \left[ \prod_{i=1}^{n-1} \left( B(k_i, k_i - r_i) \frac{d^2 k_i}{8\pi^2} \right) B(k_n, k_n - r_n), \quad k_n = q - \sum_{i=1}^{n-1} k_i - \sum_{i=1}^{m} \chi_i. \right.
\]

Another effect which we take into account is the possibility of the ion–ion interaction through the LBL blocks (screening effect in Fig. 1), which in the AGK language \([4]\) is equivalent to the exchange by additional un–cut
Fig. 2: Typical kernel of the ion–ion interaction through the LBL blocks.

pomerons. It is associated with the iteration of a typical kernel

\[ L \int Y(l) d^2l, \quad (20a) \]

\[ Y(l) = \frac{(\alpha^2 Z_1 Z_2)^2}{32\pi^4} \int \frac{P}{|l_1||l_2||l_1 - l||l_1 + l|} d^2l_1 d^2l_2 \frac{d\bar{s}_1}{s_1^2}, \quad (20b) \]

\[ P = \prod_{\alpha\beta\gamma\delta} e_1^\alpha(l_1) e_2^\beta(l_1 - l) e_3^\gamma(l_2) e_4^\delta(l_2 + l), \quad (20c) \]

where we rearrange the “extra” phase volume of longitudinal Sudakov parameters \( \alpha_2, \beta_1 \) in terms of invariant mass square of LBL block and extract explicitly the boost degree of freedom of LBL block

\[ \int d\alpha_2 d\beta_1 \frac{s}{s(\alpha_2 \beta_1)^2} = \int \frac{d\beta_1}{\beta_1} \int \frac{d\bar{s}_1}{s_1^2} = L \int \frac{d\bar{s}_1}{s_1^2}, \quad (21) \]

\[ L = \ln \gamma_1 \gamma_2, \quad \bar{s}_1 = (l_1 + l_2)^2 > 4m^2. \]

The structure (20) entered the matrix element (19) in the compact form

\[ \frac{1}{n_s!} \prod_{i=1}^{n_s} \left( LY(l_i) d^2l_i \right) \quad (22) \]

Real part of (20c) (equal to one half of its s–channel discontinuity) is related with (16b)

\[ \Re \prod_{\alpha\beta\gamma\delta} e_1^\alpha(l_1) e_2^\beta(l_1 - l) e_3^\gamma(l_2) e_4^\delta(l_2 + l) = \frac{1}{2} \int B_{\alpha\beta}(l_1, r - l_1) e_1^\alpha(l_1) e_2^\beta(r - l_1) \]

\[ \cdot B_{\gamma\delta}(l_1 - l, r + l - l_1) e_3^\gamma(l_1 - l) e_4^\delta(r + l - l_1) d\Phi_r, \quad (23) \]

where \( d\Phi_r \) is the phase volume of the intermediate pair

\[ d\Phi_r = \frac{\delta^4(r - q_+ - q_-) d^3q_+ d^3q_-}{(2\pi)^2 2\varepsilon_+ 2\varepsilon_-}. \quad (24) \]

We will show later that the imaginary part of \( P \) is irrelevant either for the total cross section or for the probability of \( n \) pairs production distribution.
4 The impact parameter representation for the amplitude

The last step in building the matrix element for the process of \( n \) real pairs creation consist in transformation of the obtained above expressions in the impact–parameter representation and summation over all eikonal photons and LBL blocks. For this we introduce the identity

\[
\int \delta^2 \left( k_n - q + \sum_{i=1}^{n-1} k_i + \sum_{i=1}^{n_e} \chi_i + \sum_{i=1}^{n_v} l_m \right) d^2 k_n =
\]

\[
\frac{1}{4} \int e^{-i\rho} \exp \left[ i\rho \left( k_n + \sum_{i=1}^{n-1} k_i + \sum_{i=1}^{n_e} \chi_i + \sum_{i=1}^{n_v} l_m \right) \right] \frac{d^2 k_n}{\pi} \frac{d^2 \rho}{\pi} = 1. \quad (25)
\]

Using this expression the summation in \( n_e \) and \( n_v \) can be easily done with the result

\[
M^{(n)} = \frac{i^n \pi s}{2} N_1 N_2 \int e^{-i\rho} e^{i\Psi(\rho\lambda)} e^{-L[A(\rho)/2+i\varphi(\rho)]} \prod_{i=1}^{n} \tilde{B}(\rho, r_i) \frac{d^2 \rho}{\pi}, \quad n \geq 2,
\]

with

\[
\frac{A(\rho)}{2} + i\varphi(\rho) = \int Y(1)e^{i\rho d^2 l/\pi}, \quad \tilde{B}(\rho, r_i) = \int B(k, k - r_i) e^{i\rho q^2 k/8\pi^2}, \quad (27)
\]

\[
\Psi(\rho\lambda) = -\alpha Z_1 Z_2 \int \frac{e^{i\rho q^2} \epsilon^2 \chi}{\chi^2 + \lambda^2} \frac{d^2 \chi}{\pi} = -2\alpha Z_1 Z_2 K_0(\lambda \rho) \quad (28)
\]

where \( K_0(\lambda \rho) \) is modified Bessel function (Mac-Donald function).

The phase volume of final state which consists from the scattered nuclei and \( n \) pairs can be written in the following form

\[
d\Gamma_{n+2} = \prod_{i=1}^{n} \left( \frac{d^3 q_+ d^3 q_-}{(2\pi)^6 2\varepsilon_+ 2\varepsilon_-} \right) \frac{1}{(2\pi)^2} \frac{d^3 p'_1 d^3 p'_2}{2\varepsilon'_1 2\varepsilon'_2} \delta^4(p_1 + p_2 - p'_1 - p'_2 - \sum_{i=1}^{n} r_i)
\]

\[
= \prod_{i=1}^{n} \left( \frac{L}{(2\pi)^4} \frac{d^2 r_i}{2} ds_i d\Phi_i \right) \frac{d^2 q}{2s(2\pi)^2} \quad (29)
\]

with \( q = p'_{1+} \).

Cross section of \( n \) pairs production has the form

\[
d\sigma_n = \frac{1}{8s} \frac{\left| M_n \right|^2}{n!} d\Gamma_{n+2}. \quad (30)
\]
Statistical factor $1/n!$ is included to take into account the identity of pairs. Using the expressions (25)-(29) we get

$$ \frac{d\sigma_n}{d^2\rho} = P_n(\rho), \quad P_n(\rho) = \frac{(LA_1(\rho))^n}{n!} e^{-LA_1(\rho)}, \quad n \geq 2 \quad (31) $$

with

$$ A_1(\rho) = \frac{1}{2^5\pi^4} \int |B(\rho, r)|^2 ds_1 d^2r d\Phi_r. \quad (32) $$

It can be easily recognized that for $A(\rho)$ from (27)

$$ A(\rho) = A_1(\rho), \quad (33) $$

thus confirming the Poisson character of probability distribution in impact-parameter representation.

Let us mention that the effect of eikonal photons as well as the imaginary part of the amplitude corresponding to LBL blocks do not modify the total cross section as well as differential cross section integrated over phase volume of final nuclei. Really, integrating the square of the amplitude (26) over the phase volume one immediately obtains

$$ \int e^{i\mathbf{q}(\rho_1-\rho_2)}e^{i[\psi(\rho_1)-\psi(\rho_2)]-L[\phi(\rho_1)-\phi(\rho_2)]} f(\rho_1) f(\rho_2) \frac{d^2\mathbf{q}}{\pi} \frac{d^2\mathbf{p}_1}{\pi} \frac{d^2\mathbf{p}_2}{\pi} = \int f^2(\rho) \frac{d^2\mathbf{p}}{\pi}. \quad (34) $$

Nevertheless, the exclusive cross section is sensitive to both these factors.

Expression (32) can be simplified if one neglects the dependence of Compton tensor (14) on external photons virtualities

$$ B_{\alpha\beta}(k, r) e^\alpha e^\beta \rightarrow B_{\alpha\beta}(0, r) e^\alpha e^\beta \quad (35) $$

and use the well known relation [7]

$$ \int \sum_{4m^2} \left| B_{\alpha\beta}(0, r) e^\alpha e^\beta \right|^2 \frac{ds_1 d\Phi_r}{s_1^2} = \frac{7}{36\pi m^2}. \quad (36) $$

As a result the expression (32) can be cast in the form

$$ A(\rho) = \frac{7}{18\pi^2 m^2} (\alpha^2 Z_1 Z_2)^2 I(\rho), \quad (37) $$
\[ I(\rho) = \int_{|k_1|}^{m} \frac{\rho^{i(k_1-k_2)}}{|k_1 - r||k_2 - r|} \frac{d^2 r \ d^2 k_1 \ d^2 k_2}{\pi \ \pi \ \pi} \]

\[ = \int_{|k|}^{m} \frac{e^{i\chi \rho}}{|k - \chi||k' + \chi|} \frac{d^2 \chi \ d^2 k \ d^2 k'}{\pi \ \pi \ \pi} \]

\[ = \int e^{i\chi \rho} \ln^2 \left( \frac{m^2}{\chi^2} \right) \frac{d^2 \chi \ \pi}{\pi} \approx \frac{16}{\rho^2} \left( \ln(\rho m) + O(1) \right), \]

where we introduce the cut–off parameter \(|k| < m\) as a result of the fast decreasing of matrix element of pair production by two photons. For the case of heavy leptons production (μ or τ) the upper limit must be replaced by quantity \(Q\) which can be associated with maximal momentum transferred to nucleus without its disintegration. For the case \(\rho m \gg 1\) one has

\[ A(\rho) \approx \frac{56}{9} \left( \frac{\alpha^2 Z_1 Z_2}{\rho^2 (\rho m)^2} \right) \left( \ln(\rho m) + O(1) \right) \]

which is in the agreement with \([1]\).

Our formula (31) can be applied for the case \(n = 0\) (the probability of elastic nuclei scattering). The case \(n = 1\) needs a bit accurate consideration. The expression for \(\sigma_1\) can be cast in the following form

\[ \sigma_1 = \sigma_B + L \int A(\rho) \left( e^{-LA(\rho)} - 1 \right) d^2 \rho. \]

First term \(\sigma_B\) corresponds to the leading order of the Racah formula (see for instance \([2]\)) for the cross section in Born approximation. Inferring (39), one has to take into account the longitudinal components of momenta of exchanged photons which create the pair. The second term takes into account the unitarity corrections to the total cross section.

**Conclusions**

In conclusions let us enumerate very shortly the main results of the present work. In our paper we obtained the general form for the amplitude of \(n\) lepton pairs production, accounting for the mutual Coulomb interaction of relativistic ions. We get the probability of vacuum–vacuum transition in the closed analytic form. We confirmed that the probability of \(n\) pair production can be approximated by Poisson distribution. Although this result has been claimed before in a number of publications, the mutual interaction of ions has not been taken into account in these works. At last we have shown that the Coulomb interaction between the colliding nuclei, although important in the exclusive kinematics, does not affect the integral yield of lepton pairs.
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