Non-Supersymmetric Gauge Theories from D-Branes in Type 0 String Theory

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Abstract
We construct non-supersymmetric four dimensional gauge theories arising as effective theories of D-branes placed on various singularities in Type 0B string theory. We mostly focus on models which are conformal in the large N limit and present both examples appearing on self-dual D3-branes on orbifold singularities and examples including orientifold planes. Moreover, we derive type 0 Hanany-Witten setups with NS 5-branes intersected by D-branes and the corresponding rules for determining the massless spectra. Finally, we discuss possible duality symmetries (Seiberg-duality) for non-supersymmetric gauge theories within this framework.

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1. Introduction

In recent years we have gained new insights about non-perturbative aspects of supersymmetric gauge theories by studying them as effective low energy theories arising on D-branes in string theory. This includes for instance the derivation of the $\mathcal{N}=2$ Seiberg-Witten curve from the embedding of five-branes in M-theory [1], as well as, the Maldacena conjecture [2] stating that conformal gauge theories arising on parallel D3-branes in type IIB string theory are in the limit of large 't Hooft coupling $\lambda = g^2_{YM}N$ and large N dual to supergravity theories on an AdS$_5$-background.

In particular the Maldacena conjecture in the large N limit has been generalised to the non-supersymmetric case by placing D3-branes on non-supersymmetric singularities [3,4]. Generically, without supersymmetry on the string theory side one expects a dilaton potential to be generated by string-loop corrections, so that only in the large N limit one can expect the corresponding non-supersymmetric gauge theories to be conformal. For finite N one gets 1/N corrections and one still hopes that the exact $\beta$ functions allow at least one interacting fixed point. In a recent proposal [5] such non-supersymmetric conformal gauge theories were advertised as a different scenario to solve the hierarchy problem of scales. Instead of a supersymmetric gauge theory broken in the TeV range, one might use a non-supersymmetric gauge theory which is conformal above some scale close to the weak scale. In [6] it was claimed that the one loop quadratic divergence of the Higgs mass does not cancel in subleading orders in N.

A different approach towards non-supersymmetric gauge theories was pushed forward by Klebanov and Tseytlin [7,8,9] following an idea by Polyakov [10]. They studied the ten-dimensional tachyonic type 0B theory and found the remarkable result that non-trivial RR-flux can remove the tachyonic instability. They studied both the case of parallel electric D3-branes and the case of parallel self-dual D3-branes. In the former case one gets a SU(N) gauge theory, with six adjoint scalars, in which the one-loop renormalization group flow of the gauge coupling is correctly captured by the non-constant dilaton in the type 0B gravity solution. In the case of self-dual D3-branes, in the limit of large N and $\lambda < 100$ they found a stable AdS$_5 \times S^5$ solution, implying that in the large N limit the gauge theory is conformal. This was further supported by an explicit computation of the two loop $\beta$-function for the gauge coupling. It vanished in the large N limit but contained the expected non-zero 1/N corrections. In [11] it was pointed out that this model can also be obtained as a $\mathbb{Z}_2$ orbifold of type IIB and thus is a member of the class of non-supersymmetric models studied in
Therefore, the general arguments presented in [12] were applicable implying that the non-supersymmetric gauge theory in question is conformal to all orders in the large N limit. Meanwhile, there have been discussions of several related issues of type 0 string theories such as the infrared behaviour [13], the physics of D-branes [14,15,16,17], branes on conifold singularities [18], orientifolds of type 0B [19,20,21] and dualities [22].

In this paper we will systematically study non-supersymmetric gauge theories arising on self-dual D3-branes sitting on various singularities in type 0B. In the case of supersymmetric singularities, the non-supersymmetric theories have some common features with their corresponding supersymmetric type IIB cousins. Since the open string annulus amplitude for self-dual D3-branes is up to factor of two the same as the corresponding type IIB amplitude, all models have the same number of bosons and fermions. In this sense they are special and similar to the compact non-supersymmetric models discussed in [23]. Using the arguments given in [12], in the large N limit all correlation functions of the type 0B theories reduce to the correlation functions of the parent $\mathcal{N}=4$ gauge theory. Thus, the type 0B orbifold theories are conformal in the large N limit. Actually, it turns out that they have the same one-loop $\beta$-function as their supersymmetric cousins and the first $1/N$ corrections appear at two loops. As we mentioned before, these type 0B gauge theories are a very special subset of large N non-supersymmetric conformal gauge theories and inherit some nice features from the corresponding supersymmetric gauge theories. Due to this, we think they deserve particular interest.

As was pointed out in [24,25] and further elaborated in [19,20,21] there exist different orientifold projections in type 0B. We will also study effective gauge theories on self-dual D3-branes on orientifolds. In contrast to the pure orbifold case, generically there remain some uncancelled twisted NSNS tadpoles, which are however suppressed in the large N limit. Thus we can still construct large N conformal gauge theories now including orthogonal and symplectic gauge groups and also matter in the (anti-)symmetric representation of the unitary gauge factors. The corresponding supersymmetric cases were discussed in [26,27].

For the supersymmetric case, it is well established that D3-branes on ALE type singularities are T-dual to cyclic Hanany-Witten models [28,29,30,31,32,33]. Using the low energy effective action derived in [7], we will argue that the same holds for self-dual D3-branes in type 0B. Essentially, since in the effective theory for self-dual D3-branes the tachyon has the expectation value $\langle T \rangle = 0$, as long as the solution is stable the theory reduces to the type IIB effective theory and everything carries over.
Thus, using NS 5-branes and dyonic D-branes we construct the type 0 analogue of Hanany-Witten models. This can be done for non-conformal cases, as well. We will discuss the “$\mathcal{N}=2$” Hanany-Witten models in some more detail, including the determination of the massless spectrum and some comments about the moduli space. The ultimate goal would of course be, to gain some higher loop or non-perturbative information using the recently proposed duality of type 0A to M-theory on $S^1/(-1)^{F_s}S$ \[22\].

2. Branes on orbifold singularities in Type 0B

In this section we study non-supersymmetric gauge theories obtained by placing D3-branes on orbifold singularities \[34\] in Type 0B string theory. As was already observed in \[8\], in order to obtain a large $N$ conformal theory one needs to have an equal number of electric and magnetic D3-branes. If we did not take the same number of electric and magnetic D-branes the annulus amplitude would be non-vanishing implying a non-trivial dilaton potential. In the field theory this would lead to a non-vanishing one loop $\beta$ function even in the large $N$ limit. To begin with we briefly review the definition of Type 0B and the parent gauge theory living on parallel D3-branes.

2.1. Type 0B string theory

There are two ways of constructing type 0B string theory. One is by implementing in the superstring the projection $P = \frac{1}{2} (1 + (-1)^{F_L + F_R})$ \[35\]. In contrast to the usual GSO projection the tachyon survives and all space-time fermionic modes are projected out. The second way of constructing Type 0B is by an orbifold of type IIB by the space-time fermion number $(-1)^{F_s}$. This removes all fermions from the untwisted sector and leads to a tachyon and further massless RR fields in the twisted sector. Computing the spectrum one finds that all states in the RR sector are doubled implying that all Dp-branes ($p$ odd) are doubled, as well. In the case of D3-branes now there exist electric (D3) and magnetic (D3$'$) branes. Using the boundary state approach it was shown in \[19\] that indeed the boundary state representing a Dp brane in type IIB splits into two boundary states of type 0B. The explicit form of the boundary states was used to derive rules for open strings stretched between the various types of Dp branes. Open strings stretched between the same kind of Dp branes carry only space-time bosonic modes, whereas open strings stretched between a D3 and a D3$'$-brane carry only space-time fermionic modes.
Now we consider type 0B as the orbifold of type IIB by \((-1)^{F_s}\) and compute the annulus amplitude for 2N D3-branes in the loop channel

\[ A = \int_0^\infty \frac{dt}{t} \Tr_{\text{open}} \left[ \frac{(1+(1)^f)}{2} \frac{(1+(-1)^{F_s})}{2} e^{-2\pi t L_0} \right]. \]  

(2.1)

We allow for a non-trivial action of \((-1)^{F_s}\) on the Chan-Paton factors parametrised by the \(\gamma\) matrix\(^1\)

\[ \gamma(-1)^{F_s} = \begin{pmatrix} 1_{n,n} & 0 \\ 0 & -1_{m,m} \end{pmatrix} \]  

(2.2)

with \(n + m = 2N\). For general \(n\) and \(m\) one obtains

\[ A = \frac{V_4}{(8\pi^2\alpha')^2} \int_0^\infty \frac{dt}{t^3} \frac{1}{2} (\Tr \gamma_1)^2 \left( \frac{f_3^8(e^{-\pi t}) - f_4^8(e^{-\pi t}) - f_2^8(e^{-\pi t})}{f_1^8(e^{-\pi t})} \right) + \]

\[ \frac{1}{2} (\Tr \gamma(-1)^{F_s})^2 \left( \frac{f_3^8(e^{-\pi t}) - f_4^8(e^{-\pi t}) + f_2^8(e^{-\pi t})}{f_1^8(e^{-\pi t})} \right). \]  

(2.3)

We would like to discuss two choices of \(m\) and \(n\) in more detail.

a.) For \(n = m = N\) (2.3) reduces to

\[ A = \frac{V_4}{(8\pi^2\alpha')^2} \int_0^\infty \frac{dt}{t^3} 2N^2 \left( \frac{f_3^8(e^{-\pi t}) - f_4^8(e^{-\pi t}) - f_2^8(e^{-\pi t})}{f_1^8(e^{-\pi t})} \right) \]  

(2.4)

which is exactly the amplitude for the same number of electric and magnetic D3-branes in type 0B. Up to a factor of two this is identical with the annulus amplitude for N D3-branes in type IIB. In particular, even without supersymmetry the amplitude vanishes and one does not get any tachyonic tadpole.

b.) For \(n = 2N\) and \(m = 0\) (2.3) reduces to

\[ A = \frac{V_4}{(8\pi^2\alpha')^2} \int_0^\infty \frac{dt}{t^3} (2N)^2 \left( \frac{f_3^8(e^{-\pi t}) - f_4^8(e^{-\pi t})}{f_1^8(e^{-\pi t})} \right), \]  

(2.5)

which is nothing else than the amplitude for 2N electric D3-branes. As we mentioned above in this case one only gets space-time bosonic modes and a tachyonic tadpole in the tree channel.

From these two examples we learn that the numbers \(m\) and \(n\) in the \(\gamma(-1)^{F_s}\) matrix are exactly the number of D3 and D3’-branes in type 0B. The massless spectra living on the D3-branes in the two cases a.) and b.) are

\(^1\) This is the string theoretic analogue of the construction made in [11] in the context of the gauge theories.
a.) Gauge group $G = SU(N) \times SU(N)^2$, three complex bosons in the adjoint and four Weyl-fermions in the $(N, N) + (N, N)$ representation of $G$.

b.) Gauge group $G = SU(2N)$ with three complex bosons in the adjoint representation of $SU(2N)$.

In case a.) the one-loop $\beta$-function vanishes and as was shown explicitly in [8] the two-loop $\beta$-function vanishes only in the large $N$ limit. As was already observed in [11], since we have embedded the gauge theory into type IIB string theory with a $\gamma$ matrix of vanishing trace, the general arguments of [12] tell us that in the large $N$ limit the non-supersymmetric $SU(N) \times SU(N)$ gauge theory is conformal. In the rest of this section we present the massless spectra obtained by putting self-dual D3-branes on singularities preserving $\mathcal{N}=2$, $\mathcal{N}=1$ and $\mathcal{N}=0$ supersymmetry, respectively.

2.2. D3-branes on $\mathcal{N}=2$, $\mathbb{Z}_K$ singularities

We are considering the $\mathbb{Z}_K$ singularity described by the action

$$\Theta = \begin{cases} 
  z_1 \to \theta z_1 \\
  z_2 \to \theta^{-1} z_2 \\
  z_3 \to z_3
\end{cases} \quad \text{with } \theta = e^{2\pi i/K} \quad (2.6)$$

on the three complex coordinates transversal to $M$ D3-branes. This case has also been discussed in [30]. Note, that since we are considering non-compact models there is no need to restrict to $\mathbb{Z}_K$ actions having a crystallographic action on the $T^4$ torus and therefore every non-negative integer $K$ is allowed. As we have discussed in the last subsection, the annulus amplitudes for self-dual D3-branes in type 0B are up to factor of two identical to the annulus amplitudes for D3-branes in type IIB. This means that the twisted tadpole cancellation conditions are the same, as well. Moreover, it means that the type 0B spectrum is bose-fermi degenerated. As was shown in [37], since $\Theta$ leaves one coordinate invariant, all twisted tadpoles are of logarithmic type

$$\sum_{k=1}^{K-1} \int_0^\infty \frac{dl}{l} \text{Tr}(\gamma_{\Theta^k})\text{Tr}(\gamma_{\Theta^{-1}}). \quad (2.7)$$

These tadpoles are in one to one correspondence with the vanishing of the one-loop $\beta$ function of the gauge couplings. Even without cancelling them, one gets an anomaly free gauge theory. We will see that the same holds for the type 0B case.

\footnote{As usual the abelian $U(1)$ subgroups decouple in the infrared}
By choosing
\[ \gamma_\Theta = \text{diag}[1, \theta_{n_2}, \ldots, \theta_{n_K}^{K-1}] \] (2.8)
with \( \sum n_j = M = NK \), in the \( N=2 \) supersymmetric case the massless spectrum on the D3-branes is

vector multiplets: \[ G = \bigotimes_{j=1}^{K} \text{SU}(n_j) \] (2.9)

hypermultiplets: \[ \bigoplus_{j=1}^{K} (\square_j, \square_{j+1}) \]

with cyclic identification. Making the choice
\[ \Gamma_\Theta = \left( \begin{array}{cc} \gamma_\Theta & 0 \\ 0 & \gamma_\Theta \end{array} \right)_{2M, 2M} \] (2.10)

for the action of \( \mathbb{Z}_K \) on the Chan-Paton factors of the D3 and D3'-branes one derives the following gauge group for the type 0B model
\[ G = \bigotimes_{j=1}^{K} \text{SU}(n_j) \times \bigotimes_{j=1}^{K} \text{SU}(n_j) \] (2.11)

where the first K factors arise on the D3-branes and the second K factors on the D3'-branes. There are additional complex bosons and Weyl-fermions in the following representations
\[ \bigoplus_{j=1}^{K} \left\{ \left( \begin{array}{c} \text{Adj}_j \\ 1 \end{array} \right) + \left( \begin{array}{c} 1 \\ \text{Adj}_j \end{array} \right) + 2 \left( \begin{array}{c} \square_j \\ 1 \end{array} \right) + 2 \left( \begin{array}{c} 1 \\ \square_j \end{array} \right) \right\} + \]
\[ \bigoplus_{j=1}^{K} \left\{ 2 \left( \begin{array}{c} \square_j \\ 1 \end{array} \right) + 2 \left( \begin{array}{c} \square_{j+1} \\ 1 \end{array} \right) + \left( \begin{array}{c} \square_j \\ 1 \end{array} \right) + \left( \begin{array}{c} 1 \\ \square_{j+1} \end{array} \right) \right\} + \left( \begin{array}{c} 1 \\ \square_j \end{array} \right) + \left( \begin{array}{c} 1 \\ \square_{j+1} \end{array} \right) \right\} \] (2.12)

where the upper row in the matrix notation refers to the gauge groups on the D3-branes and the lower row refers to the gauge groups on the D3'-branes. From (2.12) it is evident that even without cancelling the tadpoles the non-chiral spectrum is free of non-abelian gauge anomalies. Requiring cancellation of the logarithmic tadpoles leads to \( n_j = M/K \) for all \( j \in \{1, \ldots, K\} \) implying indeed that the one-loop \( \beta \)-function vanishes.
2.3. D3-branes on $\mathcal{N}=1$, $\mathbb{Z}_K$ singularities

We are considering the $\mathbb{Z}_K$ singularity described by the action

$$\Theta = \begin{cases} z_1 \to \theta^1 z_1 \\ z_2 \to \theta^2 z_2 \\ z_3 \to \theta^3 z_3 \end{cases}$$

(2.13)

with $l_1 + l_2 + l_3 = 0$. The $\mathcal{N}=1$ supersymmetric spectrum of these models is

vector: $G = \bigotimes_{j=1}^{K} \text{SU}(n_j)$

(2.14)

chiral: $\bigoplus_{a=1}^{3} \bigoplus_{j=1}^{K} \left( \begin{array}{c} j \\ a \end{array} \right)$

which in general is chiral. Choosing the $\gamma$ matrices as in (2.10) leads to the following non-supersymmetric spectrum for the type 0B case.

$$G = \bigotimes_{j=1}^{K} \text{SU}(n_j) \times \bigotimes_{j=1}^{K} \text{SU}(n_j)$$

$$\bigoplus_{j=1}^{K} \bigoplus_{a=1}^{3} \left( \begin{array}{c} j \\ a \end{array} \right) + \left( \begin{array}{c} 1 \\ j \end{array} \right) + \left( \begin{array}{c} 1 \\ j \end{array} \right) + \bigoplus_{a=1}^{3} \left[ \left( \begin{array}{c} j \\ 1 \\ a \end{array} \right) + \left( \begin{array}{c} 1 \\ j \end{array} \right) \right]$$

(2.15)

The non-abelian gauge anomaly of $\text{SU}(n_j)$ is proportional to

$$\sum_{a=1}^{3} (n_{j+l_a} - n_{j-l_a})$$

(2.16)

and the one-loop $\beta$ function of the corresponding gauge coupling is

$$\beta_1 = 3n_j - \frac{1}{2} \sum_{a=1}^{3} (n_{j+l_a} + n_{j-l_a})$$

(2.17)

which agrees with the data for the $\mathcal{N}=1$ supersymmetric model in (2.14). Both the anomalies and the one-loop $\beta$ functions vanish if all tadpoles are cancelled for $n_j = N$. 

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2.4. D3-branes on $N=0$, $\mathbb{Z}_K$ singularities

Since we are interested in models without supersymmetry there is no need to consider supersymmetric singularities only. However, to have a well defined string description one has to guarantee that the model satisfies level matching. Let us examine one specific example, where we first consider the type IIB orbifold case. We start with the $\mathbb{Z}_2$ action $\Theta : z_i \rightarrow -z_i$ for all $i \in \{1, 2, 3\}$. Checking for level matching one realizes that one gets $\Delta E = \frac{1}{4}$ which cannot be compensated in a $\mathbb{Z}_2$ orbifold. Indeed, since $K = 2$ and $l_i = 1/2$, $\Theta$ does not satisfy the consistency condition $K(l_1 + l_2 + l_3) = \text{even}$. If one would naively continue and compute the open string sector one would indeed find that in the Ramond sector $\Theta$ acts like a $\mathbb{Z}_4$, as well. However, it is possible to consider the orbifold action above as a $\mathbb{Z}_4$ orbifold, $\vec{l} = \frac{1}{4}(2, 2, 2)$, with partition function

$$Z = \frac{1}{2} \left( \begin{array}{c|c} 1 & 0 \\ \hline 0 & 1 \end{array} \right) + \frac{1}{4} \left( \begin{array}{c|c|c|c} 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array} \right).$$

(2.18)

Requiring tadpole cancellation leads to the $\gamma$ matrix

$$\gamma_{\Theta} = \text{diag}[1_N, i1_N, -1_N, -i1_N]$$

(2.19)
satisfying $\gamma_{\Theta}^4 = 1$. For the D3-brane spectrum we obtain

vector : $G = \bigotimes_{j=1}^{4} \text{SU}(N)$

(2.20)
matter : $\bigoplus_{j=1}^{2} 3(\square_j, \square_{j+2})_B + \bigoplus_{j=1}^{4} 4(\square_j, \square_{j+1})_F$.

The spectrum is free of non-abelian gauge anomalies and has vanishing one-loop $\beta$ function. This chiral model was also derived in [5] using the approach of [4]. The corresponding type 0B model is

$$G = \bigotimes_{j=1}^{4} \text{SU}(N) \times \bigotimes_{j=1}^{4} \text{SU}(N)$$

(2.21)

$$\bigoplus_{j=1}^{2} \left\{ \begin{array}{c|c|c} 3 & 0 & 0 \\ \hline 0 & 3 & 0 \\ \hline 0 & 0 & 3 \end{array} \right\}_B + \bigoplus_{j=1}^{4} \left\{ \begin{array}{c|c|c} 4 & 0 & 0 \\ \hline 0 & 4 & 0 \\ \hline 0 & 0 & 4 \end{array} \right\}_F,$$

which is free of non-abelian gauge anomalies and has vanishing one-loop $\beta$ function, as well. So far all the type 0B spectra we derived had only an even number of unitary gauge groups and bifundamental matter. The generalisation to orthogonal and symplectic gauge groups and an odd number of unitary gauge factors is achieved by using type 0B orientifolds.
3. Non-susy gauge theories from orientifolds of Type 0B

As was first pointed out in [24] and elaborated further in [25,20,21,22], in the ten-dimensional type 0B string theory there exist three different orientifold projections.

i.) First, one has the usual world-sheet parity transformation $\Omega$, which in ten-dimensions leads to a model containing tachyons both in the closed and in the open string sector. The tachyon in the open string sector is due to the untwisted tadpole cancellation condition requiring the introduction of D9 and anti D9-branes. We will see that due to the weaker tadpole cancellation conditions in the non-compact case, one can get rid of the open string tachyons leading to sensible gauge theories on the D3-branes including orthogonal and symplectic gauge groups. In order to avoid tachyonic tadpoles in the annulus amplitude, we will always work with self-dual D3-branes in the following.

ii.) As was shown in [21], the dressed parity transformation $\Omega' = \Omega (-1)^{f_R}$ projects out the ten-dimensional closed string tachyon and does lead to a model necessarily containing the same number of D9 and D9$'$-branes. The latter point is simply an implication of the fact that the world-sheet fermion number operator $(-1)^{f_R}$ exchanges $D_p$ and $D_p'$-branes. In the non-compact case such models also contain only self-dual D3-branes and generically lead to an odd number of unitary gauge factors and some fermions in the antisymmetric representation.

iii.) The third kind of ten-dimensional orientifolds of type 0B uses the combination $\Omega'' = \Omega (-1)^{F_L}$ and was discussed in [22]. Since $(-1)^{F_L}$ exchanges D-branes with the corresponding anti D-brane, these models always contain open string tachyonic modes and thus they do not seem to be suitable for constructing large N conformal gauge theories.

More specifically, we are interested in orientifolds by $\omega = \Omega(-1)^{F_L}J$ and $\omega' = \Omega'(-1)^{F_L}J$, respectively, where $J$ denotes the $\mathbb{Z}_2$ transformation $z_i \rightarrow -z_i$ for all $i \in \{1, 2, 3\}$ and $F_L$ is the left-moving space time fermion number operator. Note, that $\omega$ and $\omega'$ are T-dual to the ten-dimensional $\Omega$ and $\Omega'$ discussed in i.) and ii.) above. Besides the world sheet parity transformation we are also gauging a further discrete space time symmetry $\mathbb{Z}_K$ acting as $\Theta = \frac{1}{K} (v_1, v_2, v_3)$ on the three transversal coordinates. The Klein bottle amplitude

$$K_{\omega'/\omega} = \int_0^\infty dt \frac{1}{t} \frac{1}{K} \sum_{k=0}^{K-1} \mathrm{Tr} \left( \frac{\omega'/\omega}{\Theta^k} \frac{1 + (-1)^{f_L+f_R}}{2} e^{-2\pi t(L_0+\bar{L}_0)} \right)$$

(3.1)
receives only contributions from the untwisted closed string sector and for even $K$ also from the $\Theta^{K/2}$ twisted sector.

The $\omega'$ orientifold

Since $\omega'$ contains $(-1)^{f_R}$ the Klein bottle amplitude for type 0B string theory is up to factor of two the truncation of the type IIB Klein bottle amplitude to terms containing the insertion $(-1)^{f_R}$ in the trace. However, these are exactly the terms leading to RR exchange in the tree channel interpretation of the amplitude. The actual computation of (3.1) is identical to the type IIB computations presented in [38,27].

In order to cancel the RR tadpoles in the $\omega'$ orientifold one introduces D3/D3'-branes and in case of even $K$ also D7/D7'-branes in the background. In the following we will restrict ourselves to the $K$ odd case, the generalisation to cases including D7/D7'-branes is straightforward. As we have reviewed in the section 2, the cylinder amplitude for the same number of D3 and D3'-branes is exactly twice the amplitude for D3-branes in type IIB and in particular it vanishes. Moreover, since $\omega'$ exchanges D3 with D3'-branes, the only non-vanishing contribution to the Möbius strip amplitude arises from open strings stretched between D3 and D3'-branes. This again is exactly twice the Ramond part of the Möbius amplitude for D3-branes in type IIB. Note, that unlike the annulus amplitude the Möbius amplitude is non-zero and thus spoils the Bose-Fermi degeneracy. Thus, the tadpole cancellation conditions for RR exchange are identical to the corresponding conditions in the analogous type IIB orientifold. However there always remains an uncancelled twisted NSNS tadpole, which in the string context can be cancelled by the Fischler/Susskind mechanism [39].

The twisted RR tadpole cancellation conditions for the corresponding type IIB orientifolds have been derived in [38,27]

$$\text{Tr}(\gamma_{\Theta^{2k}}) = \pm \frac{1}{\prod_{i=1}^{3} \cos \left( \frac{\pi k v}{K} \right)}$$

where the two different signs refer to the two possible actions of $\omega'$ on the Chan-Paton factors, namely whether $\gamma_{\omega'}$ is antisymmetric or symmetric. Remember, that in the compact case this ambiguity is fixed by the untwisted tadpole cancellation condition. As shown in [38], condition (3.2) always has a solution. Taking into account that $\Omega'$ exchanges D3
and D3′-branes a consistent choice of $\Gamma$ matrices in the type 0B orientifold in terms of the corresponding $\gamma$ matrices in the type IIB orientifold is

$$\Gamma_{\Theta} = \left( \begin{array}{cc} \gamma_{\Theta} & 0 \\ 0 & \gamma_{\Theta} \end{array} \right)_{2M,2M}, \quad \Gamma_{\omega'} = \left( \begin{array}{cc} 0 & \gamma_{\omega} \\ \gamma_{\omega} & 0 \end{array} \right)_{2M,2M},$$

where $M$ denotes the number of self-dual D3-branes. Note, that due to the tadpole cancellation condition (3.2) for $k \geq 1$ $\text{Tr}(\gamma_{\Theta 2k})$ is of order $O(N^0)$, whereas $\text{Tr}(\gamma_1)$ is of order $O(N)$. Thus, one expects the effect of the remaining twisted NSNS tadpoles to be suppressed in the large $N$ limit. We will indeed find, that opposed to the models in section 2 the one-loop $\beta$ function of the gauge coupling receives $1/N$ corrections.

The $\omega$ orientifold

The story for the $\omega$ orientifold is a bit different. The Klein bottle amplitude with the $\omega$ insertion in (3.1) only contains terms leading to NSNS exchange in tree channel. Here, we also introduce the same number of D3 and D3′-branes, but in contrast to the compact case there is no untwisted RR tadpole and therefore no need to introduce anti-branes, as well. Thus, we do not have to worry about extra tachyons. The Möbius amplitude contains only traces over the NS sector leading to terms in the NSNS exchange channel. Summarising, only the annulus amplitude contains twisted RR tadpoles, whereas all three amplitudes contribute to twisted NSNS tadpoles. Thus, in order to cancel the dangerous RR tadpoles one has to require

$$\text{Tr}(\gamma_{\Theta k}) = 0$$

and like the $\omega'$ case one is left with an uncanceled massless twisted NSNS tadpole. A consistent choice of $\gamma$ matrices in the $\omega$ orientifold is

$$\Gamma_{\Theta} = \left( \begin{array}{cc} \gamma_{\Theta} & 0 \\ 0 & \gamma_{\Theta} \end{array} \right)_{2M,2M}, \quad \Gamma_{\omega} = \left( \begin{array}{cc} 0 & \gamma_{\omega} \\ \gamma_{\omega} & 0 \end{array} \right)_{2M,2M}.$$  

In the following we discuss a couple of examples of both $\omega$ and $\omega'$ orientifolds.

3.1. Orientifolds on $\mathcal{N}=4$ singularities

The simplest model is taking the brane configuration from section 2.1 and gauge the world-sheet parity transformation. The corresponding type IIB model is $\mathcal{N}=4$ susy $\text{SO}(M)$
or \( \text{SP}(2M) \) gauge theory. However, due to the different action of \( \omega' \) on the Chan-Paton factors in the type 0B model one obtains the following gauge group

\[
G = \text{SU}(N)
\]  
(3.6)
equipped with the bosonic and fermionic matter

\[
3(\text{Adj})_B + 4 \left( \begin{array}{c} \Box \\ \mathbb{P} \end{array} \right)_F.
\]  
(3.7)

For symmetric \( \gamma_{\Omega'} \) one gets the symmetric representation in (3.7) instead of the antisymmetric. Computing the one-loop \( \beta \) function coefficient one realizes that it vanishes only in the large \( N \) limit

\[
b_1 = 0 N \pm \frac{16}{3}.
\]  
(3.8)

Note, that for the antisymmetric representation the model is asymptotically free. The general form of the two-loop gauge coupling coefficient \( \beta \)-function has been derived in [40]

\[
b_2 = \frac{34}{3} C_2(G)^2 - \sum_S \left[ 2C_2(S) + \frac{1}{3} C_2(G) \right] T_2(S) - 2 \sum_F \left[ C_2(F) + \frac{5}{3} C_2(G) \right] T_2(F) + Y_4,
\]  
(3.9)

where \( C_2(R) \) denotes the eigenvalue of the second order Casimir in the representation \( R \) and \( T_2(R) \) denotes the Dynkin-Index of \( R \). The contribution of the Yukawa couplings is given by

\[
Y_4 = \frac{1}{\text{dim}(G)} \text{Tr} \left[ \hat{C}_2(F) Y_I Y_I^* \right]
\]  
(3.10)

where \( Y_I \) denotes the matrix of the Yukawa couplings. Taking into account the global \( \text{SU}(4) \) symmetry and the gauge structure of \( Y_I \), as in [8], leads to \( Y_4 = 24(N^2 - 3N + 4/N) \). Hence, the \( N^2 \) term vanishes. Up to order \( g^7 \) the \( \beta \) function is given by

\[
\beta = -\frac{g^3}{(4\pi)^2} b_1 - \frac{g^5}{(4\pi)^4} b_2
\]

\[
= \mp \frac{g^3}{(4\pi)^2} \frac{16}{3} \pm \frac{g^5}{(4\pi)^4} \frac{64}{3} \left( N - \frac{3}{N} \right) + O(g^7).
\]  
(3.11)

For \( N \geq 2 \) the coefficients \( b_2 \) and \( b_1 \) have opposite signs, which at least supports the hope that there exists an interacting fixed-point for some finite value of the coupling constant \( g \). Unfortunately, (3.11) implies \( g_2^2 = O(1/N) \) telling us that for large but finite \( N \) higher loop contributions to the \( \beta \) function can not be neglected.
Taking instead the $\omega$ projection one obtains the gauge group

$$G = \text{SO}(N) \times \text{SO}(N)$$

with matter

$$3\{(\text{Adj}, 1) + (1, \text{Adj})\}_B + 4(\Box, \Box)_F.$$  

(3.12)

Choosing $\gamma_\Omega$ symmetric leads to $\text{SP}(2N)$ gauge groups. For the $\beta$ function up to two loops one obtains

$$\beta = -\frac{g^3}{(4\pi)^2}b_1 - \frac{g^5}{(4\pi)^4}b_2$$

$$= \pm \frac{g^3}{(4\pi)^2} \frac{16}{3} \mp \frac{g^5}{(4\pi)^4} \frac{64}{3} \left( N \mp \frac{1}{2} \right) + O(g^7),$$

(3.13)

where again the highest order terms vanish and $b_2$ and $b_1$ have opposite sign.

In contrast to section 2, in all type 0B orientifold models we studied the one-loop $\beta$ function vanished only in the large N limit. The string theoretic reason for the running of the gauge coupling is, that in contrast to the annulus amplitude the Möbius amplitude does not vanish. This one-loop cosmological constant induces a dilaton potential which prevents the dilaton from being a free parameter. Since, the Möbius amplitude is $1/N$ suppressed against the annulus amplitude, in the field theory this scenario induces a $1/N$ correction to the one-loop $\beta$ function.

### 3.2. Orientifolds on $N=2$ singularities

The spectrum for type IIB orientifold of the models presented in section 2.2 for odd $K = 2P+1$ was derived in [27]

$$\text{vectormultiplets : } G = \text{SO}(N) \times \text{SU}(N-2) \times \ldots \times \text{SU}(N-2P)$$

$$\text{hypermultiplets : } \bigoplus_{j=1}^{P-1} (\Box_j, \Box_{j+1}) + \Box_P.$$  

(3.15)

In the corresponding $\omega'$ orientifold the SO($N$) group becomes an SU($N$) gauge group and all the remaining SU($n_j$) groups with $n_j = N - 2j$ are doubled. The complete gauge group is

$$G = \text{SU}(N) \times \bigotimes_{j=1}^{P} \text{SU}(n_j) \times \bigotimes_{j=1}^{P} \text{SU}(n_j)$$

(3.16)
where the first $P$ factors arise on the D3-branes and the second $P$ factors on the D3$'$-branes.

The bosonic and fermionic matter is given by

\[
\left\{ (\text{Adj}_0) + \left( \begin{array}{c} \text{Adj}_P \\ 1 \end{array} \right) + \left( \begin{array}{c} 1 \\ \text{Adj}_P \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ 1 \\ 1 \end{array} \right) + 2 \left( \begin{array}{c} 1 \\ \text{Adj} \\ 1 \\ 1 \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ P \\ P \\ P \end{array} \right) \right\}_B + \\
\bigoplus_{j=1}^{P-1} \left\{ (\text{Adj}_j) + \left( \begin{array}{c} 1 \\ \text{Adj}_j \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ 1 \\ 1 \\ 1 \end{array} \right) + 2 \left( \begin{array}{c} 1 \\ \text{Adj} \\ 1 \\ 1 \\ 1 \end{array} \right) \right\}_B + \\
\left\{ 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) \right\}_F + \\
\bigoplus_{j=1}^{P-1} \left\{ 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) \right\}_F + \\
\left\{ 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) + 2 \left( \begin{array}{c} \text{Adj} \\ 0 \end{array} \right) \right\}_F.
\]

The one loop $\beta$ function coefficients are as follows

\[b_0 = \frac{20}{3}, \ b_1 = \ldots = b_{P-1} = 0, \ b_P = -\frac{2}{3}.\] (3.18)

The spectrum for the other sign in (3.2) is $G = \text{Sp}(2N) \times \text{SU}(2N + 2) \times \ldots \times \text{SU}(2N + 2P)$ with the antisymmetric representations exchanged by the symmetric ones. We skip the representation of the corresponding $\omega$ orientifold.

3.3. Orientifolds on $\mathcal{N} = 1$ singularities

In this case one can not give such a closed form for the spectra depending on the vector $\Theta = \frac{1}{\mathcal{K}}(v_1, v_2, v_3)$ with $v_1 + v_2 + v_3 = 0$. However, it is a straightforward exercise to find the solution of (3.2) and then use the explicit form of $\gamma_{\Theta}$ to determine the spectrum of the desired model. Here we would only like to discuss the simplest model, which is the $\mathbb{Z}_3$ orientifold. The gauge group for the $\omega'$ orientifold is

\[G = \text{SU}(N - 4) \times \text{SU}(N) \times \text{SU}(N)\] (3.19)

The bosonic and fermionic matter content is given by

\[3 \left\{ \left( \begin{array}{c} \text{Adj} \\ 1 \\ 1 \\ 1 \end{array} \right) + \left( \begin{array}{c} \text{Adj} \\ 1 \\ 1 \\ 1 \end{array} \right) + \left( \begin{array}{c} \text{Adj} \\ 1 \\ 1 \\ 1 \end{array} \right) \right\}_B + \\
3 \left\{ \left( \begin{array}{c} 1 \\ \text{Adj} \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \right) + \left( \begin{array}{c} 1 \\ \text{Adj} \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \right) + \left( \begin{array}{c} 1 \\ \text{Adj} \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \right) \right\}_F + \\
\left\{ \left( \begin{array}{c} 1 \\ \text{Adj} \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \right) + \left( \begin{array}{c} 1 \\ \text{Adj} \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \right) + \left( \begin{array}{c} 1 \\ \text{Adj} \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \right) \right\}_F.\] (3.20)
The model is chiral but free of non-abelian gauge anomalies. The one-loop $\beta$ function coefficients are $b_0 = -32/3$ and $b_1 = 8$.

Computing the gauge group for the $\omega$ orientifold

$$G = \text{SO}(N) \times \text{SU}(N) \times \text{SO}(N) \times \text{SU}(N),$$

we would like to emphasise that due to string consistency namely RR tadpole cancellation we get $\text{SO}(N)$ gauge factors instead of the type IIB result $\text{SO}(N-4)$. The matter spectrum is

$$3\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}_B +$$

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}_F +$$

$$3\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}_F.$$

Note, the chiral matter contents in (3.22) is free of non-abelian gauge anomalies and the one-loop $\beta$ function coefficients are $b_0 = b_2 = \pm 22/3$ and $b_1 = b_3 = \pm 1$.

### 3.4. Orientifolds on $\mathcal{N}=0$ singularities

In this section we will discuss orientifolds of both type IIB and type 0B on a non-supersymmetric $\mathbb{Z}_5$ singularity. The $\mathbb{Z}_5$ action is defined as $\Theta = \frac{1}{5}(0,1,3)$ and satisfies level matching. Solving for the twisted RR tadpole conditions yields

$$\gamma_\Theta = \text{diag}[1, \theta_N, \theta_N^2, \theta_N^3, \theta_N^4].$$

There are also tachyon and dilaton tadpoles in the NSNS exchange channel, however they are cancelled automatically since they always appear in the same combination with the RR channel tadpoles. One obtains the following gauge group of the type IIB orientifold

$$G = \text{SO}(N+2) \times \text{SU}(N) \times \text{SU}(N)$$

equipped with some bosonic and fermionic matter given by

$$\left\{ \bigoplus_{j=1}^3 2(\text{Adj}_j) + \begin{pmatrix} \square & \square \\ 1 \end{pmatrix} + \begin{pmatrix} \square & 1 \\ \square \end{pmatrix} + 2(1, \square, \square) + (1, \square, 1) + (1, 1, \square) \right\}_B +$$

$$\left\{ \begin{pmatrix} \square & \square \\ 1 \end{pmatrix} + \begin{pmatrix} \square & 1 \\ \square \end{pmatrix} + 2(1, \square, \square) + (1, \square, 1) + (1, 1, \square) + c.c. \right\}_F.$$
Since the $\mathbb{Z}_5$ leaves one of the coordinates invariant, the spectrum is non-chiral and has vanishing one-loop $\beta$ function. The derivation of the type 0B $\omega'$ orientifold is straightforward and gives the the gauge group

$$G = \text{SU}(N+2) \times \text{SU}(N) \times \text{SU}(N) \times \text{SU}(N) \times \text{SU}(N)$$

(3.26)

with matter

$$\left\{ \left( \text{Adj} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \text{cycl.} \right) \right\}_{B} +
\left\{ \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \right\}_{B} +
\left\{ \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \right\}_{B} +
\left\{ \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \right\}_{F} +
\left\{ \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \text{c.c.} \right\}_{F}.$$

(3.27)

With this non-supersymmetric example we finish our discussion of type 0B orientifold models.

4. D3-branes on $\mathcal{N}=1$ conifold singularities in type 0B

While for branes at orbifold singularities the spectrum can be derived from perturbative string calculations, this is not anymore the case placing D3-branes at transversal conifold singularities. Here the spectrum has to be determined by an educated ‘guess’ work, which of course has to pass several subsequent consistency requirements by considering various known limits. The supersymmetric type IIB D3-branes at conifold singularities, leading to $\mathcal{N}=1$ supersymmetric field theories, were first discussed in [41,42]. This discussion was then extended in [31,32,33]. We start with the 6-dimensional conifold singularity $\mathcal{C}$, which can be described by the following hypersurface in $\mathbb{C}^4$:

$$\mathcal{C} : \quad xy = uv.$$  

(4.1)

This case was recently studied in [18]. As a first generalization we consider orbifolds of the conifold singularity, namely the quotient $\mathcal{C}/\Gamma$, $\Gamma = \mathbb{Z}_K \times \mathbb{Z}_L$, i.e. $x \to e^{2\pi i/K}x$, $
\( y \rightarrow e^{-2\pi i/K}y, \ u \rightarrow e^{2\pi i/L}u, \ v \rightarrow e^{-2\pi i/L}v. \) The corresponding hypersurface in \( \mathbb{C}^4 \) is determined by the equation
\[
C_{KL} : \quad (xy)^L = (uv)^K. \tag{4.2}
\]
This singularity is probed by \( N \) ‘self-dual’ D3-branes \((m = n = N)\). The spectrum can be obtained from the related \( \mathcal{N}=1 \) supersymmetric models of D3-branes on these spaces in type IIB superstrings \[31,33\] via the \((-1)^F\) projection. The gauge group has the following form
\[
G = \bigotimes_{j=1}^{KL} (\text{SU}(N)_e \times \text{SU}(N)_e \times \text{SU}(N)_m \times \text{SU}(N)_m). \tag{4.3}
\]
The subscript ‘\( e \)’ (‘\( m \)’) stands here for electric (magnetic) gauge group due to the \( N \) D3 (D3’) branes. We label each pair of electric plus magnetic group factor \( \text{SU}(N)_e \times \text{SU}(N)_m \) by indices \( i, I = 1 \ldots K \) and \( j, J = 1 \ldots L \). Then we obtain four types of massless scalar and fermion matter fields which are bifundamental under the gauge groups indicated by the indices:
\[
\phi, \psi : \quad (A_1)_{i+1,j+1;I,J} = (\Box_{i+1,j+1}, \bigotimes_{I,J}),
\]
\[
(A_2)_{i,j;I,J} = (\Box_{i,j}, \bigotimes_{I,J}),
\]
\[
(B_1)_{i,j;I+1,J} = (\Box_{i,j}, \bigotimes_{I+1,J}),
\]
\[
(B_2)_{i,j;I+1,J+1} = (\Box_{i,j}, \bigotimes_{I+1,J+1}). \tag{4.4}
\]
The scalars are bi-fundamental under two electric or two magnetic gauge group factors. On the other hand, with respect to the indices \( e \) and \( m \), the fermions are either in the representations \((\Box_e, \Box_m)\) or \((\Box_m, \Box_e)\). In addition there are also massless fermions which are bifundamental under electric/magnetic gauge groups with identical set of indices:
\[
\psi : \quad (\Box^e_{i,j}, \Box^m_{i,j}) + (\Box^m_{i,j}, \Box^e_{i,j}) + (\Box^e_{I,J}, \Box^m_{I,J}) + (\Box^m_{I,J}, \Box^e_{I,J}). \tag{4.5}
\]
The corresponding 1-loop \( \beta \)-function takes again the same value as in the corresponding type IIB parent model.

Just as in the corresponding \( \mathcal{N}=1 \) models, there exist a special Higgs branch in this class of models \[33,34\]. Specifically, giving a vev to all \( A_2 \) type scalar fields corresponds to resolving the original conifold singularity to the well known orbifold singularity \( \mathbb{C}^3/(\mathbb{Z}_K \times \mathbb{Z}_L) \). In this way all the \( A_2 \) scalar fields will break each \( \text{SU}(N)_{ij}^e \times \text{SU}(N)_{ij}^m \) pair down to its diagonal subgroup. The remaining three types of massless fields after the Higgs mechanism are precisely the horizontal, vertical and diagonal representations which one gets on the \( \mathbb{Z}_k \times \mathbb{Z}_L \) orbifold models.
Let us briefly consider a different class of conifold singularities which can be regarded as the mirror geometries of the orbifolded conifold spaces eq.(4.2) considered before. Specifically, these six-dimensional generalized conifold singularities are defined as

\[ \mathcal{G}_{KL} : \quad xy = u^K v^L. \]  

As usual we easily determine the field theory from the underlying \( \mathcal{N}=1 \) models \[31\]. The gauge group for \( N \) ‘self-dual’ D3 branes is given by

\[ G = \bigotimes_{j=1}^{K+L} \left( \text{SU}(N)_e \times \text{SU}(N)_m \right). \]  

Without going into too many details, there are bifundamental massless fermions and scalars in the \( (\Box, \Box) \) of adjacent gauge group factors. Furthermore, depending on different choices of B-field backgrounds, there are adjoint scalar multiplets. Finally, there are as usual massless fermions in electric/magnetic bifundamentals of gauge groups with identical indices.

5. The supergravity description and T-duality

5.1. D3-branes at transversal singularities

In this section we want to discuss the general form of the supergravity solutions for D3-branes at transversal singularities in type 0 string constructions. Following the discussion in \[18\] we will assume that the background is asymptotically of the form \( M_4 \times Y_6 \), where \( M_4 \) is flat Minkowski space-time, corresponding to the D3 world volume, and \( Y_6 \) is the non-compact transversal space with a singularity at some fixed locus. In case of the six-dimensional orbifold and conifold singularities, we considered in the last section, \( Y_6 \) is given as a cone over a five-dimensional compact manifold \( X_5 \), the so-called horizon. Locally the metric on \( Y_6 \) can then be written in the form

\[ dy_6^2 = dr^2 + r^2 d\Omega_X^2, \]  

where \( r \) is the radial coordinate on \( Y_6 \). Asymptotically, for large and small \( r \) the combined, near-horizon metric of the D3-branes at \( Y_6 \) should take the form \( AdS_5 \times X_5 \). In between, the near-horizon metric is given by a warped product \( M_4 \times \tilde{Y}_6 \), where \( \tilde{Y}_6 \) is a fibration.
of $X_5$ over the radial coordinate $r$, in general being different from the original transversal space $Y_6$.

Explicitly we consider the following ansatz for the 10-dimensional metric metric

\[ ds^2 = e^{\frac{1}{2} \phi(r)} ds^2_E \]

\[ ds^2_E = \frac{\rho^{-\frac{1}{2}}}{\sqrt{N}} e^{-\frac{1}{2} \xi(\rho)} dx^\mu dx^\mu + \frac{\sqrt{N}}{16 \rho^2} e^{\frac{1}{2} \xi(\rho) - 5 \eta(\rho)} d\rho^2 + \sqrt{N} e^{\frac{1}{2} \xi(\rho) - \eta(\rho)} d\Omega^2_X, \]  

(5.2)

where $\xi(\rho)$ and $\eta(\rho)$ are functions which depend on the radial coordinate $\rho$, and $\phi(\rho)$ is the dilaton. These functions, together with four-form gauge fields have to be determined as solutions of the equation of motions of the type 0 string effective action. They depend crucially on the potential of the tachyon field which is present in the type 0 string models. This tachyon potential contains the following typical terms:

\[ h(T) = m^2 f(T) + \frac{n^2}{f(T)} \]
\[ f(T) = 1 + T + \frac{T^2}{2}. \]  

(5.3)

Here, $m$ is the number of electric D3-branes and $n$ the number of magnetic D3-branes. Like before we consider the two cases of either purely electric (or purely magnetic) D3-branes or self-dual D3-branes.

(i) Self-dual D3-branes

In this case we have $m = n = N$. The corresponding tachyon equations of motion are solved for vanishing tachyon field $T = 0$. Hence the tachyon drops out from all the remaining equations of motion which are then identical to the equations of motion of $N$ D3-branes at a transversal singularity $Y_6$ in the corresponding type IIB superstring. Namely, for self-dual D3-branes the near horizon metric is precisely of the form $AdS_5 \times X_5$, which corresponds to vanishing functions $\xi$ and $\eta$ and constant dilaton field. After a change of variables $\rho = r^{-4}$, the metric can be written as

\[ ds^2_E = \frac{r^2}{\sqrt{N}} dx^\mu dx^\mu + \frac{\sqrt{N}}{r^2} (dr^2 + r^2 d\Omega^2_X) \]  

(5.4)

This is precisely the near horizon metric of $N$ self-dual D3-branes at the six-dimensional, transversal singularity $Y_6$, i.e. the full D3-brane metric is obtained by replacing $\frac{N}{r^4}$ by the harmonic function $H = 1 + \frac{N}{r^4}$ (suppressing the factors $g_s$ and $\alpha'$).
Let us study the orbifold singularities in a little more detail. The horizon $X_5$ is just the five-sphere modded out by the relevant discrete group $\Gamma$: $X_5 = S^5/\Gamma$. Defining $S_5$ via its embedding in $R^6$ by the equation $|z_1|^2 + |z_2|^2 + |z_3|^2 = 1$, the group $\Gamma$ acts on the three complex coordinates $z_i$ precisely as discussed in section 2. Consider e.g. the simplest $N=2$ supersymmetric orbifold singularities where $\Gamma = \mathbb{Z}_K$ acts as $z_1 \rightarrow e^{2\pi i/K} z_1$, $z_2 \rightarrow e^{-2\pi i/K} z_2$, $z_3 \rightarrow z_3$. Replacing the singularity at $z_1 = z_2 = 0$ by a smooth 2-sphere, $Y_6$ is given by the product $\mathcal{C} \times ALE_{K-1}$, where the four-dimensional spaces $ALE_{K-1}$ describe the resolutions of the $A_{K-1}$ singularities and are complex two-dimensional non-compact relatives of $K3$, i.e. non-compact Ricci-flat hyper-Kähler manifolds.\footnote{All other singularities, like the $N=1$ supersymmetric orbifolds or the conifolds can be constructed as fibrations of (two) ALE spaces.} The ALE manifolds of the $A_{K-1}$ series correspond to the metrics given by the Gibbons-Hawking multi-center ansatz

\begin{equation}
    dy_4^2 = V(\vec{x})d\vec{x}^2 + V^{-1}(\vec{x})(d\tau + \vec{\omega} \cdot d\vec{x})^2
\end{equation}

with the self-duality condition $\vec{\nabla}V = \vec{\nabla} \times \vec{\omega}$, and

\begin{equation}
    V = \sum_{i=1}^{K} \frac{1}{|\vec{x} - \vec{x}_i|}.
\end{equation}

This space $M_{K-1}$ is the smooth resolution of the singular variety $xy = z^K$ in $\mathbb{C}^3$ of type $A_{K-1}$ with $\partial M_{K-1} = S^3/\mathbb{Z}_K$. The singular situation corresponds to the pol-terms coalescing: $V = \frac{K}{|\vec{x}|}$.

On the other hand, before we have constructed the transversal singularity as a simple $\mathbb{Z}_K$ orbifold. Hence, we like to briefly recall (for the case $K = 2$) that in fact the ALE-metric (5.3) and the metric of the orbifold $\mathcal{C}^2/\mathbb{Z}_K$ can be obtained from each other by a simple coordinate transformation. For $K = 2$, the two-center Gibbons-Hawking metric (5.3) takes the following explicit form

\begin{equation}
    dy_4^2 = \left( \frac{1}{R_+} + \frac{1}{R_-} \right)^{-1} \left[ \tau + \left( \frac{z_+}{R_+} + \frac{z_-}{R_-} \right) d\tan^{-1}(y/x) \right]^2 + \left( \frac{1}{r_+} + \frac{1}{r_-} \right) [dx^2 + dy^2 + dz^2],
\end{equation}
where \( z_\pm = z \pm z_0, R^2_\pm = x^2 + y^2 + z^2_\pm \). The necessary coordinate transformations are now given by

\[
\begin{align*}
    x &= \frac{1}{8} \sqrt{r^4 - 64z_0^2} \sin \theta \cos \psi, \\
    y &= \frac{1}{8} \sqrt{r^4 - 64z_0^2} \sin \theta \sin \psi, \\
    z &= \frac{r^2}{8} \cos \theta, \\
    \tau &= 2\phi.
\end{align*}
\]  
(5.8)

Using these new coordinates the metric (5.7) transform into the following expression:

\[
dy^2_4 = \frac{1}{4} r^2 [1 - \frac{64z_0^2}{r^4}] |d\psi + \cos \theta d\phi|^2 \]

\[+ [1 - \frac{64z_0^2}{r^4}]^{-1} dr^2 + \frac{1}{4} r^2 [d\theta^2 + \sin^2 \theta d\phi^2].
\]

This is precisely the Eguchi-Hanson metric. In the singular situation, \( z_0 = 0 \), this metric just describes the orbifold space \( \mathbb{C}^2/\mathbb{Z}_2 \), since the range of \( \phi \) is \( 0 \leq \phi < \pi \). Note that resolving the singularity, i.e. \( z_0 \neq 0 \), the metric of the transversal space \( Y_6 \) is not anymore of the form (5.1), but there are corrections of the form

\[
dy^2_6 = r^2 \left( \frac{dr^2}{r^2} + d\Omega^2_X + O\left(\frac{z_0^2}{r^2}\right) \right).
\]

(ii) Electric D3-branes

Next we wish to discuss briefly the supergravity solutions for entirely electric D3-branes, i.e. we have \( m = N, n = 0 \) in the tachyon potential. Now the tachyon equations of motion are solved for non vanishing tachyon field: \( T = -1 + \ldots \). As a consequence, the ten-dimensional metric is only asymptotically in the UV of the form \( AdS_5 \times S^5 \); for general values of \( \rho \) the functions \( \xi(\rho) \) and \( \eta(\rho) \) are non-trivial, and the dilaton now becomes a radius dependent function. However the solution cannot be given anymore in closed form, but they can be expanded around the asymptotic UV (\( \rho \to 0 \)) \( AdS_5 \times S^5 \) solution. Explicitly, the first terms in this expansion are given as

\[
\xi = -(\log \rho)^{-1}, \quad \eta = -(\log \rho)^{-1}, \quad \phi = \log\left(\frac{2^{13}}{27N}\right) - 2 \log(- \log \rho).
\]

(5.11)
Due to the presence of these non-trivial functions, the ten-dimensional metric looks like the metric of D3-branes now at a transversal six-dimensional space \( \tilde{Y}_6 \), which only asymptotically in the UV is given by the original singularity \( Y_6 \), but in general has the form of being a warped product over the five-dimensional horizon \( X_5 \) \((\rho = r^{-4})\):

\[
ds^2_E = e^{-\frac{1}{2}\xi(r)} \frac{r^2}{\sqrt{N}} dx^\mu dx^\mu + e^{\frac{1}{2}\xi(r)} \frac{\sqrt{N}}{r^2} d\tilde{y}_6^2, \\
d\tilde{y}_6^2 = e^{-5\eta(r)} dr^2 + e^{-\eta(r)} d\Omega^2_X.
\]

For example, the ‘warped’ Eguchi-Hanson metric, which asymptotically is identical to the resolution of the \( \mathbb{Z}_2 \) orbifold singularity, takes the following form:

\[
d\tilde{y}_4^2 = \frac{1}{4} r^2 e^{-\eta(r)} [1 - \frac{64z_0^2}{r^4}] [d\psi + \cos \theta d\phi]^2 \\
+ e^{-5\eta(r)} [1 - \frac{64z_0^2}{r^4}]^{-1} dr^2 + \frac{1}{4} r^2 e^{-\eta(r)} [d\theta^2 + \sin^2 \theta d\phi^2].
\]

Using the change of coordinates in (5.8) it is straightforward to transform this metric into a metric which asymptotically approaches the two-center Gibbons-Hawking metric in (5.7).

5.2. T-dual supergravity description – D4-branes and NS 5-branes

In the last section we have described the supergravity picture of \( N \) D3-branes sitting on a six-dimensional transversal singularity \( Y_6 \) (resp. \( \tilde{Y}_6 \)) in type 0B string theory. Now we like to discuss the T-dual picture of D4-branes which are intersected by a certain number of NS 5-branes. In order to keep the discussion simple we only consider the supergravity metric which is T-dual to D3-branes at the \( \mathbb{Z}_K \) orbifold singularity. As it is known, in this case the \( A_{K-1} \) ALE spaces are T-dual to \( K \) parallel NS 5-branes [15].

(i) Self-dual D3-branes

As discussed before, in the case of self-dual D3-branes the solutions of the type 0B supergravity field equations are the same as in the corresponding type IIB theories, namely they are given by the \( AdS_5 \times X_5 \) background geometries. Therefore, also the T-duality transformation acts precisely in the same way as in the type IIB superstring; the \( N \) self-dual D3-branes will become after the T-duality to type 0A \( N \) electric plus \( N \) magnetic D4-branes, and the \( \mathbb{Z}_K \) orbifolds are T-dualized into \( K \) parallel NS 5-branes. The T-duality is most conveniently studied using the Gibbons-Hawking ansatz (5.3) for the metric of the transversal space. Explicitly, the T-duality with respect to the \( U(1) \)-isometry generated
by the Killing vector \( \partial/\partial \tau \) (\( x_4 \)-direction) gives with the well-known Buscher formula the conformal flat metric of \( K \) parallel, extremal NS 5-branes

\[
ds^2 = V(\vec{x})(d\tau^2 + d\vec{x}^2),
\]

\[B_{0i} = \omega_i,\]

\[e^{2\phi} = V(\vec{x}),\]  \hspace{1cm} (5.14)

where the self-duality condition for the original metric is now, in the new axion-dilaton sector, assuring the condition for an axionic instanton

\[
H_{\mu\nu\rho} = \sqrt{g} \epsilon_{\mu\nu\rho}^\sigma \partial_\sigma \phi.
\]  \hspace{1cm} (5.15)

It is clear that the singular orbifold limit \( \vec{x}_i \to 0 \) corresponds to the situation where all NS 5-branes are stacked on top of each other. Finally, putting together the \( N \) D4-branes (in the 01234-directions) intersected by the \( K \) NS 5-branes (in the 012389-directions), the complete ten-dimensional metric is given by the following expression:

\[
ds^2 \sim \frac{r^2}{\sqrt{N}}[dx^\mu dx^\mu + V(\vec{x})dx_4^2] + \frac{\sqrt{N}}{r^2}[V(\vec{x})dx_5^2, dx_6^2, dx_7^2 + dx_8^2, dx_9^2].\]  \hspace{1cm} (5.16)

(ii) Electric D3-branes

The background spaces are now given by the deformed geometries which only approach in the asymptotic UV region \( AdS_5 \times X_5 \). Nevertheless the deformed transversal spaces \( \tilde{Y}_6 \) still possess the same \( U(1) \) isometry direction as before which can be used to perform the T-duality transformation. Therefore, in the T-dual picture the electric D3-branes will be transformed into electric D4-branes, and the deformed transversal singularities \( \tilde{Y}_6 \) will T-dualize into deformed five-dimensional NS branes, which only in the UV region are described by the standard NS 5-brane metric. To be explicit consider again the \( \mathbb{Z}_K \) orbifold singularity. Since its deformation by the functions \( e^{-\xi} \) and \( e^{-5\eta} \) can be best studied using polar coordinates, the T-duality transformation will be most suitably now also performed within this parametrisation. For example, T-dualizing the deformed Eguchi-Hanson metric (5.13) in the string frame with respect to the isometry direction \( \phi \), the metric for two deformed, parallel NS 5-branes can be explicitly constructed.
6. Type 0 Hanany-Witten constructions

In the last chapter on explicit supergravity solutions we have argued that as in the type IIB superstring one can T-dualize the type 0B D3-branes at transversal singularities into type 0A D4-branes intersected by a web of NS 5-branes. In case of dyonic D4-branes the solutions are in fact identical to the corresponding type IIA solutions, whereas for purely electric D4-branes the type IIA solutions are only valid in the asymptotic UV region.

Using these results we now set up the type 0 Hanany-Witten [46] rules to obtain non-supersymmetric gauge theories from D4-branes positioned into a web of NS 5-branes. Dealing with dyonic D4-branes, the non-supersymmetric Hanany-Witten rules we derive seem to be completely reliable, since we are confident that the corresponding backgrounds are indeed stable solutions of the supergravity field equations. In this case we can even hope to derive quantum corrections in gauge theories, related to the bending of the NS 5-branes, via the embedding of the brane configurations into the type 0 version of M-theory. In fact, many perturbative properties like the $\beta$-function coefficients will not be changed comparing type II constructions with dyonic type 0 constructions, as we have already seen in the previous sections. Moreover it is quite conceivable that also non-perturbative duality symmetries, like Seiberg-duality or S-duality, can be carried over from the underlying supersymmetric type II gauge theories to the non-supersymmetric type 0 gauge theories [47]. For example, Seiberg duality can be again viewed as a particular motion of the NS 5-branes.

The result that self-dual D3-branes can end on NS 5-branes is also supported by the recently discussed dualities of type 0A/B [22]. There was was conjectured that the strong coupling limit of type 0A is M-theory on $S^1/(-1)^F_s S$ where $S$ denotes the half-shift along the circle. In [22] was was argued that this implies that the strong coupling limit of type 0B on an $S^1$ is M-theory on $T^2/(-1)^F_s S$. Analogous to type IIB this suggests an $SL(2,\mathbb{Z})$ self-duality of the type 0B, under which the fundamental string is exchanged with the self-dual D1-brane. By definition a fundamental string can end on a dyonic D5-brane. Applying S-duality leads to a dyonic D1-brane ending on a NS 5-brane. By T-duality along internal directions of the NS 5-brane one arrives at a self-dual D3-brane ending on a NS 5-brane.

Let us consider the simplest NS 5-brane configuration, namely $K$ NS 5-branes in the (012389)-directions, with suspended D4-branes which stretch along the (01234)-directions. Hence this set up is precisely like the type IIA, $\mathcal{N}=2$ Hanany-Witten configurations, discussed in [1]. In case of a compact $x_4$ direction (elliptic models) the NS 5-branes are T-dual
to the $\mathcal{N}=2$, $\mathbb{Z}_K$ orbifold singularities. More precisely, the T-dual of N D3-branes at the $\mathbb{Z}_K$ orbifold singularities leads to an elliptic Hanany-Witten construction with an equal number of N D4-branes suspended between each pair of NS 5-branes. However, we wish to generalise this picture by allowing also for a different number of D4-branes in each 5-brane interval which corresponds after T-duality to having fractional branes, being partly wrapped around non-trivial cycles of the geometric singularity. In addition we will also in general allow for non-compact $x_4$ directions.

The massless spectrum simply follows from the T-duality to the self-dual D3-branes on the $\mathbb{Z}_K$ orbifold singularity. Alternatively, one can also directly start with the $\mathcal{N}=2$ supersymmetric type IIA Hanany-Witten construction and apply the space-time fermion number projection $(-1)^{F_s}$ on the massless $\mathcal{N}=2$ spectrum. At the level of the effective gauge theories this projection should be equivalent to the modding by the $\mathbb{Z}_2$ R-symmetry, which is discussed in [11]. This projection will not alter the one-loop $\beta$-function of the gauge theories.

Then, consider $K$ parallel NS 5-branes, and $n_j$ D4-branes, being suspended in the $x^4$ direction between the $j^{th}$ and $j^{th}+1$ NS 5-brane. In case of non-elliptic models, for $j = 0$ or $j = K$ the D4-branes are semi-infinite to the left of the first NS 5-brane or, respectively, to the right of the $K^{th}$ NS 5-brane. The 4-branes are located in $x^8$-$x^9$ plane at the complex coordinate $v = x^8 + ix^9$. The four-dimensional gauge group is given by

$$G = \bigotimes_{j=1}^{K-1} SU(n_j) \times \bigotimes_{j=1}^{K-1} SU(n_j). \quad (6.1)$$

(For the elliptic models there is one more pair of gauge groups.) The gauge coupling constants $g_j$ of every $SU(n_j)$ group factor are determined by the differences between the positions of the NS 5-branes:

$$\frac{1}{g_j^2} = \frac{x_{j+1}^4 - x_j^4}{g_s}, \quad (6.2)$$

where $g_s$ is the string coupling constant. Applying the fermion number projection on the corresponding $\mathcal{N}=2$, type IIA spectrum, we obtain massless states in the following
The one-loop \(\beta\)-function coefficient is given by

\[
b_1 = 2n_j - n_{j-1} - n_{j+1}.
\]

This is precisely the same number as in the \(\mathcal{N}=2\) parent models. Note, that in the spirit of [1] this is consistent with the fact that, since the supergravity theory for self-dual D3-branes is the same as in type IIB, the bending of the NS 5-branes in type 0B is the same, too. Thus, also in the type 0B Hanany-Witten models one can derive the logarithmic running of the gauge coupling constants from the bending of the NS 5-branes. Since the 5-brane background preserves \(\mathcal{N}=2\) space-time supersymmetry, the resulting \(\mathcal{N}=0\) spectrum is automatically anomaly free. We would like to mention that in the case of \(K = 2\) the two-loop \(\beta\)-function coefficient of the \(\mathcal{N}=0\) theory accidentally vanishes, as well. Since the matter representations for the bosons and fermions are different for the \(\mathcal{N}=2\) and the related \(\mathcal{N}=0\) gauge theory of course one does not expect to have vanishing \(\mathcal{N}=0\) \(\beta\)-function coefficient for higher loops, as well.

Analogous to the Coloumb moduli space in the \(\mathcal{N}=2\) gauge theories, the motion of the dyonic D4-branes in the \((89)\) direction is related to giving vacuum expectation values to the adjoint complex scalars. Since the couplings in the \(\mathcal{N}=2\) theory are the truncation of the couplings in the \(\mathcal{N}=2\) theory one has the following commuting diagram:

\[
\begin{array}{ccc}
\mathcal{N} = 2 & \langle \phi \rangle \neq 0 & \mathcal{N} = 2 \\
\text{SU}(K) & \xrightarrow{(-1)^F} & \text{U}(1)^{K-1} \\
\mathcal{N} = 0 & \langle \phi \rangle \neq 0 & \mathcal{N} = 0 \\
\text{SU}(K)^2 & \xrightarrow{(-1)^F} & \text{U}(1)^{2K-2}
\end{array}
\]
Note that there does not exist further scalars in (6.3) which could describe the breaking up of a dyonic D4-brane into a D4 and a D4′-brane. This is consistent with the fact that all arguments given for the existence of type 0B Hanany-Witten models relied on having dyonic D4-branes. In addition to the Coulomb branch there exists also a Higgs branch which corresponds to giving vev’s to the bifundamental scalar fields. In this way a pair of electric or magnetic SU(N)\(_{e,m}^2\) gauge groups is higgsed to its diagonal subgroup SU(N)\(_{e,m}\). In the corresponding Hanany-Witten brane picture this Higgs moduli space is related to the motions of the NS 5-branes, whereas in the T-dual orbifold picture the bifundamental scalar vev’s correspond to the resolution parameters (2-spheres) of the \(\mathbb{Z}_K\) orbifold singularity. Since the electric and magnetic D-branes feel these parameters, which are related to the orbifold respectively NS 5-brane background, in the same way, it is plausible that in the field theory the vev’s of the electric and magnetic scalar fields must be the same such that the electric and magnetic gauge groups are higgsed simultaneously.

It is straightforward to generalise the Hanany-Witten set up of parallel NS 5-branes to type 0 models which contain several NS 5-branes with different kind of orientations. In particular brane box-models [48] with D5-branes in the (012348) directions which fill boxes of NS 5-branes in the (012389) directions and of NS′ 5-branes in the (012345) directions follow by applying two T-dualities with respect to \(x_4\) and \(x_8\) on the transversal \(\mathbb{Z}_K \times \mathbb{Z}_L\) orbifold singularity. These gauge theories are in general chiral and potentially lead to dangerous gauge anomalies.

The orbifolded conifold singularities in eq.(4.2) are T-dual (T-duality in \(x_4\) and \(x_8\)) to diamonds of NS 5-branes [33]. By rotating the diamonds into certain directions, the \(\mathbb{Z}_K \times \mathbb{Z}_L\) brane box models are rediscovered. On the other hand, the mirror conifold geometries eq.(4.6) lead after a single T-duality in \(x_6\) to rotated NS brane configurations [31], namely to Hanany-Witten models, where electric and magnetic D4-branes, along the (01236) directions, are suspended between intervals of \(K\) NS 5-branes in the original (012389) directions and \(L\) rotated NS′ 5-branes along the (012367) directions. The specific ordering possibilities of NS and NS′ branes correspond to different choices of B-field background. For \(K = L = 1\), \(N_c\) finite D4 branes and \(N_f\) semi-infinite D4-branes, one can easily construct a (non-elliptic) non-supersymmetric QCD type of model with gauge group \(SU(N_c)_e \times SU(N_c)_m\) and \(N_f\) massless, fundamental matter fermions and scalars plus massless fermions in bifundamental gauge representations.\(^4\) As usual, the 1-loop \(\beta\)-function is inherited from the \(\mathcal{N}=1\) parent model: \(b_1 = 3N_c - N_f\). As discussed in [50], this

\(^4\) This non-supersymmetric model and its Seiberg-dual was recently discussed in [49].
class of brane models is particularly useful to study the Seiberg-duality in the corresponding \( \mathcal{N}=1 \), type IIA brane picture by moving an appropriate number of NS or NS\(^{'}\) branes through the D-branes. Since the correlation functions in the large \( N \) limit of the type 0 models agree with those of their type II parents, it is very conceivable that carrying out the same movements of NS-branes in the non-supersymmetric type 0 models corresponds to a non-perturbative large \( N \) Seiberg-duality in non-supersymmetric field theories. Indeed, one can show along these lines that there exists a dual, non-supersymmetric QCD model with \( \tilde{N}_c = N_f - N_c \) and \( \tilde{N}_f = N_f \).

7. Summary

This paper provides a systematic study of non-supersymmetric gauge theories from electric/magnetic D-branes in type 0 constructions. Since these models share a lot of properties with their \( \mathcal{N}=2,1 \) supersymmetric counterparts from type II superstrings, their non-perturbative dynamics most likely exhibits a lot of intriguing features, like certain kinds of non-perturbative duality symmetries. In addition, we expect that these models in spite of being non-supersymmetric nevertheless possess very nice renormalization properties that could make them appealing also from the phenomenological point of view. The ultimate hope would be to gain some information about the higher loop and non-perturbative behaviour by embedding the non-supersymmetric Hanany-Witten configurations into M-theory. It would be also interesting to study D3-branes on non-supersymmetric singularities in greater detail and to find out to which kind of non-supersymmetric Hanany-Witten configurations they are mapped to.

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