On the Frequency Drift of Coronal Loop’s Fast Kink Oscillation: Effects of Quasi-static Evolution in Loop Density

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Abstract

Although the fast kink oscillation, as one of a few fundamental modes in coronal seismology, has received a lot of attention over the past two decades, observations of its frequency drift remain elusive. There is evidence that this phenomenon is related to the quasi-static evolution of loop density. We therefore consider analytically the effects of a quasi-static density evolution on the fast kink oscillation of coronal loops. From the analyses, we determine explicitly the analytic dependence of the oscillation period/frequency and amplitude on the evolving density of the oscillatory loop. The findings can well reconcile several key characters in some frequency drift observations, which are not understood. Models of fast kink oscillation in the thermal dynamic loop are also established to investigate the present effects in more detail. Our findings not only show us a possible explanation for the frequency drift of the coronal loop’s fast kink oscillation, but also a full new energy transformation mechanism where the internal energy and the kinetic energy of an oscillating coronal loop can be interchanged directly by the interaction of the loop’s oscillation and its density evolution, which we suggest may provide a new clue for the energy processes associated with a thermodynamic resonator in the space magnetic plasma.

Unified Astronomy Thesaurus concepts: Solar coronal waves (1995); Active solar corona (1988); Solar coronal seismology (1994)

1. Introduction

Owing to the existence of magnetic field, the solar atmosphere is full of bright loop-like structures when observed at extreme-ultraviolet (EUV) or X-ray pass bands (Vaiana et al. 1968, 1973; Peres et al. 1982). These loop-like structures are known as coronal loops. Coronal loops, as the main building block of the corona, are of great importance for understanding the physical nature of the solar atmosphere (Peres & Vaiana 1990; Warren 1999; Aschwanden et al. 2000; Knipp 2005; Reale et al. 2011; Reale 2014). However, many details of their physical nature and dynamics remain elusive. The complicity of the corona environment and the lack of direct measurement have greatly hampered our further understanding of them (Aschwanden et al. 2001; Aschwanden 2005; Reale 2014).

Theoretically, coronal loops are made up of magnetically confined denser and hotter plasma, and can thus support modes of oscillations (Aschwanden 1987; Roberts 2008). These oscillations are closely related to the physical properties of their host coronal loops and may open a new window for detecting the inhomogeneous corona (Nakariakov & Verwichte 2005; Yuan & Van Doorssaelaere 2016a; Li & Liu 2018). Theories of magneto-acoustic oscillations in coronal loops have indicated that the fast kink mode possesses both a unique dispersion relation and zero asymptotic dispersion for a long-wave approximation, and could provide a potential diagnostic tool for determining physical properties of the inhomogeneous corona (Edwin & Roberts 1983; Roberts et al. 1984). Due to the specific role of coronal loops’ fast kink oscillations in coronal seismology, the proposal of using the fast kink oscillations to diagnose the physical properties of their host coronal loops has attracted a great deal of attention since it was first introduced (Roberts et al. 1984). Along with an increasing number of observational cases, the study of fast kink oscillations in coronal loops has been significantly advanced (Aschwanden et al. 1999; Nakariakov et al. 1999; Verwichte et al. 2004; Aschwanden & Schrijver 2011; Wang et al. 2012; Jain et al. 2015; Yuan & Van Doorsselaere 2016b; Shen et al. 2017; Pascoe et al. 2018; Shen et al. 2018; Nechaeva et al. 2019; Anfinogentov & Nakariakov 2019). Numerous detailed observations, on the other hand, have also brought new challenges to the current theory (De Moortel & Brady 2007; Guo et al. 2015; Li et al. 2017; Su et al. 2018; Li et al. 2019). Among these challenges, a prominent one is the increasingly presented oscillating coronal loops whose frequencies show significant changes, which is also known as the frequency drift (Su et al. 2018). It indicates that the frequency drift for the oscillation in fast kink mode can occur in a quiet loop (Su et al. 2018) or a quiet fiber (Li et al. 2018). It also indicates that the brightness of the oscillating structures undergoes a significant change during the occurrence of the frequency drift, which may imply changes in the loop’s thermal properties. A detailed investigation present in Table 1 and Figure 5 of Su et al. (2018) indicates that the period of fast kink oscillation has an ≈25% increase corresponding to an ≈40% increase in the loop’s density. A decrease in the period of fast kink oscillation is also observed by Li et al. (2018). It is also worth noting that the oscillation reported by Li et al. (2018) shows a significant increase of the amplitude as the period decreases, which is very unusual.

Motivated by the observations, we consider analytically the effects of coronal loops’ density evolution on their fast kink oscillations. Analytic relations between quasi-static change in the density and its corresponding impacts on the period and
amplitude of fast kink oscillations are determined. Applications of the result to the mentioned observation are performed. The result indicates that the density evolution should play an important role in the increasingly observed frequency drift cases of loop oscillations. Based on the thermal dynamic characters of oscillating loops, models of fast kink oscillations in thermodynamic coronal loops are also established. From the models, many details of the effects of the density evolution on the fast kink oscillation are also discussed and a new energy transformation mechanism is revealed. All of these findings provide a theoretical basis for the study of the increasingly observed fast kink oscillations with frequency drift in coronal loops and their corresponding dynamics.

2. Observational Characteristics

Although frequency drift is a relatively new phenomenon in the fast kink oscillations of coronal loops, it has been observed in some observations. A detailed study of the phenomenon was presented in Su et al. (2018). From the observation, we obtain several key characteristics that are crucial for our analyses. The characteristics are listed as follows: (1) the oscillations evolve smoothly in the fast kink mode; (2) the loop’s oscillation period shows a significant change; (3) the oscillating loops maintain a relatively quiet state and there is no obvious flow observed during the oscillations; and (4) the loop’s emission also shows a significant change.

As for characteristics (3) and (4), we would like to note that they are not contradictory because coronal loops can evolve quasi-statically during most of its lifetime, which has been reported in many observations (Reale 2007, 2014). An ideal picture for this process is that the loop could be heating due to some unknown reasons and then maintains a quasi-thermal-equilibrium state for a long period. It should be mentioned that even in the second state, the loop thermal properties can significantly change, which is usually observed as a global variation of loop’s emission. Based on these characteristics, we suggest that the quasi-static change in the loop’s thermal properties, especially the loop’s density should have an influence on its fast kink oscillation and this may contribute to the frequency drift of the oscillation.

3. Effects of Quasi-static Change in Loop Density

Within this research, we consider the effects of quasi-static change in loop density based on the classical fast kink oscillation mode. The fast kink oscillation of a coronal loop is derived from an idealized uniform magnetic flux tube under a cylindrical coordinate system, which can be described mathematically by a piecewise function as

$$B, \rho, P_n = \begin{cases} B_i, \rho_i, P_{Ai} & r \leq a \\ B_e, \rho_e, P_{Be} & r > a \end{cases}$$

where $B$ and $\rho$ are the magnetic field and density; $P_n$ is the total pressure of the plasma, i.e., the sum of the gas pressure ($\rho$) and the magnetic pressure ($B^2/\mu$, $\mu$ is the magnetic permeability); $a$ is the tube radius and the subscripts $i$ and $e$ are used to denote values inside and outside the tube (see the left panel in Figure 1). Based on the fact that the thermal pressure only has a tiny influence on the magnetic equilibrium for low beta plasma in the corona, the entire system is also considered to have a uniform magnetic field. Within this framework, the fast kink oscillation can be described as a standing wave with a unique and stable phase speed $c_k$ that is derived from the continuities of disturbances (see the right panels in Figure 1) of the total pressure $P_n$ and the plasma radial Lagrangian displacement $\xi_r$ in the tube boundary and satisfies

$$c_k^2 = \frac{\rho_i v_A^2 + \rho_e v_A^2}{\rho_i + \rho_e},$$

where $v_A^2 = B^2/(\mu \rho)$ is the Alfvén speed and the subscripts $i$ and $e$ are used to denote values inside and outside the tube. As a result of this, the fast kink mode of oscillation has better anti-interference performance (Edwin & Roberts 1983; Roberts et al. 1984). This also explains why the fast kink oscillation is maintained in a thermal dynamic coronal loop just as Su et al. (2018) reported.

Theoretically, a quasi-static evolution of loop density mainly has two effects on its fast kink oscillation. One effect is that the changing loop density can directly lead to a corresponding change in the phase speed of the fast kink mode, which further causes a corresponding change in the frequency of the oscillation. And, the other effect is that on the mechanical energy of the oscillation due to the interaction between the oscillation and density evolution of the oscillating loops. We notice that all of the interaction is achieved through the internal force between the loop material. It is because of this reason that the system is subject to the conservation of momentum. We note here that nonequilibrium loops, especially those with obvious jet flows, are not considered. The situation would rarely happen in reality because the kinetic processes will strongly disrupt the fast kink mode by breaking the preconditions of the mode.

4. Result

4.1. Parametric Analyses

From the two effects of quasi-static evolution in loop density, the analytical dependencies of oscillatory period and amplitude of the fast kink mode on the loop’s density changes can be determined on the basis of the special phase speed and the conservation of the momentum during the oscillation. Here, the period and amplitude of the oscillation are both time-dependent parameters because the density of the oscillatory loop evolves over time. The changing density can directly affect the phase speed of the mode, which consequentially determines the real-time period $\tau_n$ of the oscillation by equations

$$\tau_n = \frac{2\pi}{k_n c_k},$$

$$k_n = \frac{n \pi}{L},$$

where $L$ is the loop length, $k_n$ is the wavenumber of the oscillation in the $z$ direction, and the subscripts $n = 1, 2, 3$ indicate the order of harmonics in the $z$ direction and $n = 1$ indicates the fundamental mode of a fast kink oscillation. The real-time amplitude can be determined by the conservation of momentum in differential form

$$\rho_i d\psi = -\psi_i d\rho_i.$$


When dividing both sides of the equation by $dt$ and introducing the kinetic energy of the oscillating loop $E_k = \rho_i v_i^2/2$, the equation can be rewritten as

$$dE_i = -\frac{\dot{\rho}_i}{\rho_i}E_k dt,$$

where $\dot{\rho}_i$ indicates $\partial\rho_i/\partial t$, $E_i$ is the total mechanical energy of the oscillating loop, and $dE_i$ indicates the acquired or lost mechanical energy from the oscillation due to the conservation of momentum. When introducing the potential energy of the oscillation $E_p = (B k_p \xi)^2/\mu$, we have

$$E_i = E_k + E_p = \frac{(B k_p)^2 A^2}{\mu},$$

$$E_k = E_r \left[1 - \left(\frac{\xi}{A}\right)^2\right],$$

where $\xi$ is the Lagrangian displacement of the oscillation in the $r-\phi$ plane and $A$ is the amplitude of the oscillation. The combination of the two effects yields the dynamic oscillation corresponding to the evolution of the loop density as

$$\xi = A \sin\left(\int_0^t k_n c_k dt\right)$$

$$A^2 = A_0^2 e^{-\int_0^t \frac{\omega}{2} \cos\left(\int_0^t k_n c_k dt\right) dt},$$

where $A_0$ is the initial amplitude of the oscillation. From the result, the precise dependency of a real-time period of a fast kink oscillation in fundamental mode on the evolving density of the oscillatory loop can be given as

$$\tau_1 = L \frac{\sqrt{(\rho_i + \rho_e)\mu}}{B}.$$

As for the analyses, we note that the two effects, like many studies on the damping of a fast kink oscillation, are considered independently. Theoretically, this is justified when the change in the total energy does not break the continuity conditions that the mode is subjected to (Heyvaerts & Priest 1983; Ruderman & Roberts 2002; Roberts 2008). The results are thus validated for the oscillation during which the loop is oscillating in a normal fast kink mode and its density evolves quasi-statically. As for the energetic processes, we note that the effects of the

![Figure 1. Model of the oscillating coronal loop (left) and its sectional disturbances (right) of magnetic field strength $|B|$, total pressure $P_*$, density $\rho_*$, and sectional Lagrangian displacement $\xi$ in the fast kink mode. Within the right panels, the coronal loop is marked as the black circle, the disturbances of $|B|$, $P_*$, $\rho_*$ are both normalized by their maximum values, and the disturbance of $\xi$ is drawn as a series of directional arrows with the arrow length indicating the magnitude of $\xi$. Note that (1) the disturbance of $B$ has the same direction as the disturbance of $\xi$, (2) the disturbances of $\rho_*$ and $\rho_e$ are not continuous at $r = a$, the disturbance of $\rho_e$ is far less than the disturbance of $\rho_*$ in the condition of $\rho_0 \gg \rho_e$, and is therefore not shown here, the disturbance of $\rho$ at the region of $r > a$ is replaced by the max value of the disturbance of $\rho_*$ in the corresponding panel for a better display effect; and (3) both the disturbances are tiny quantities relative to their equilibrium amounts.](image-url)
The evolving density as a function of time is plotted as colored curves in each panel, and the color denotes the four arbitrary functions that we adopted. Figure 2. Schematics of the loop’s density evolution process. Left: schematics of rarefying evolution of loop density. Right: schematics of densifying evolution of coronal loop. The evolving density as a function of time is plotted as colored curves in each panel, and the color denotes the four arbitrary functions that we adopted.

Moreover, it is also should be mentioned that for a loop whose density evolution process. Left: schematics of rarefying evolution of loop density. Right: schematics of densifying evolution of coronal loop. The evolving density as a function of time is plotted as colored curves in each panel, and the color denotes the four arbitrary functions that we adopted.

Table 1
Details of the Initial Parameters for the Dynamic Fast Kink Oscillations of Thermodynamic Coronal Loops (Zimovets & Nakariakov 2015; Goddard et al. 2016; Nechaeva et al. 2019)

| \( L \) [Mm] | \( B \) [G] | \( \rho_i \) [cm\(^{-3}\)] | \( \rho_f \) [cm\(^{-3}\)] | \( \tau_0 \) [s] | \( A_0 \) [Mm] |
|---|---|---|---|---|---|
| 100 | 11.7 | \( 10^9 \) | \( 5 \times 10^7 \) | 180 | 2 |

tenuous background are not considered here, and this should be justified for a typical oscillating case because the background just has a very marginal influence on an oscillating coronal loop that is far denser than its background (see Table 1). Moreover, it is also should be mentioned that for a loop whose density is similar to its background there is no free fast mode, and its transversal displacement will be quickly damped by the lateral leakage (Priest 2014). And, we also notice that although this analysis just considers the effects of the loop’s density evolution, it does not conflict with other effects (Heyvaerts & Priest 1983; Ruderman & Roberts 2002; Antolin et al. 2015; Guo et al. 2019). As for the changes in the total mechanical energy caused by other effects, they can be compatible with the present result by additionally inserting the corresponding terms to our Equation (5).

A qualitative analysis of Equations (7) and (10) reveals that an increase (decrease) in loop density will introduce an increase (decrease) of the loop’s oscillation period but a decrease (increase) of the loop’s oscillation amplitude. The result reveals that it is not only the amount of density change but also the way it changes that has an influence on the loop’s fast kink oscillation. In order to investigate the process in detail, we model the fast kink oscillation of thermodynamic coronal loops in this work.

4.2. Modeling of Fast Kink Oscillations in Thermal Dynamic Coronal Loops

From the analyses in Section 4.1, we notice that the evolution of density should have a complex influence on the loop’s fast kink oscillation even in an ideal framework. The result reveals that it is not only the amount of density change but also the way it changes that has an influence on the loop’s fast kink oscillation. In order to investigate the process in detail, we model the fast kink oscillation of thermodynamic coronal loops in this work.
density evolved. The most basic one of the four functions is the linear function (see the green plots in Figure 2), which is mainly used for the uniform change in loop density during the transitional period between the acceleration phase and the deceleration phase of the density evolution. The other two functions are the parabolic function (red plots in Figure 2) and the inverse parabolic function (blue plots in Figure 2), which are mainly used to simulate the acceleration phase and the deceleration phase of the density evolution. A parabolic rarefying process and inverse parabolic densifying processes are suggested to be used as a second order approximation for the start-up stage of the evolution; an inverse parabolic rarefying process and a parabolic densifying process are suggested to be used as a second order approximation for the later stage of the evolution. Besides, since most heating and cooling processes are related to the dissipation process and the typical dissipation process has an exponential solution, we also investigated the density evolution of the exponential function (black plots in Figure 2) in this work. These settings may be somewhat arbitrary but we do not want to reproduce the real observation here. On the contrary, we know that the evolution of loop density is another topic that has been long hampered by the physical complicacy of the phenomena and the difficulty of inverting physical properties of the corona from limited spectral observations, and here we are trying to find new clues of the density evolution of the coronal loop from the frequency drift of the loop’s fast kink oscillation by studying the effect of the density evolution of the coronal loop on its fast kink oscillation.

The modeling result is calculated by substituting the density evolution curves (see Figure 2) into Equation (7) and plotting it in Figure 3. Within the figure, the loop displacement is plotted as a function of time, which is in analogy with the well-known time–distance diagram in many observational works. From the constructed time–displacement diagram, one can easily identify the loop’s position changes as a function of time. It indicates that both the rarefying evolution and the densifying evolution can have a significant impact on the loop’s fast kink oscillation. The accurate evolution of the real-time period/frequency for the 16 cases can be calculated according to Equation (8) and the result is shown in Figure 4. It reveals that a rarefying evolution of density can increase the oscillating frequency to $1.4f_0$ when the loop density evolves from $\rho_{i \mid t=0}$ to $0.25\rho_{i \mid t=0}$ and $0.5\rho_{i \mid t=0}$, while a densifying evolution can decrease the oscillating frequency to $0.7f_0$ and $0.5f_0$ when the loop density evolves from $\rho_{i \mid t=0}$ to $2\rho_{i \mid t=0}$ and $4\rho_{i \mid t=0}$. This is sufficient to produce the recently observed frequency drift cases (Li et al. 2018; Su et al. 2018).

Figure 3. Dynamic oscillations of coronal loops with different evolutionary schematics of density shown in Figure 2.
It also indicates that although the real-time frequency is determined by the real-time density, the total phase difference between two moments significantly depends on the way the loop density evolved, which is part of the reason that the oscillations vary from different evolution functions of density as shown in Figure 3. A detailed evolution of the phase difference between \( t = 0 \) and \( t = 2\tau_0 \) is also calculated and shown in Figure 5. The accurate phase differences between \( t = 0 \) and \( t = 2\tau_0 \) for all investigated cases are listed in Table 2. It indicates that there is a roughly 10% ∼ 20% difference between the four specific temporal evolution ways for the four groups of cases (as shown in Figure 5). Besides, it also reveals that the differences in phase difference between different ways of density evolution tend to be more pronounced as the amount of density change increases and this effect should be more distinct for a rarefying process.

Another noteworthy result of the model is that the evolution of coronal loop density has a significant influence on the amplitude of loop’s fast kink oscillation. Since the square of amplitude is proportional to the total mechanical energy of the oscillation, this implies that the total mechanical energy of the fast kink oscillation and the internal energy of the oscillating loop can directly be transformed into each other during the fast kink oscillation of a thermodynamic coronal loop by the interaction of the oscillation and density evolution of the oscillating loop. Because the mechanical energy changes due to the momentum conservation process, this is not difficult to understand. Also, because its principle is similar to a heat engine or its reverse, we refer to the present energy transformation mechanism as the heat-engine-liking or counter-heat-engine-liking mechanism.

In order to obtain a more detailed understanding of the influence of the loop’s density evolution on the amplitude of its fast kink oscillation, we have calculated the evolution of the real-time amplitude for all 16 cases (see Figure 6). To be specific, a rarefying evolution of density can increase the oscillating amplitude to 1.2 and 1.4\( \rho_0 \) when the loop density evolves from \( \rho_{i=0} \) to 0.5\( \rho_{i=0} \) and 0.25\( \rho_{i=0} \), while a densifying evolution can decrease the oscillating frequency to 0.8 and 0.7\( \rho_0 \) when the loop density evolves from \( \rho_{i=0} \) to 2\( \rho_{i=0} \) and 4\( \rho_{i=0} \) (see Table 3).

Interestingly, the plots in Figure 6 also reveal that the way the density evolves only has a very small influence on the amplitude of the oscillation. This means that the real-time amplitude of the loop’s fast kink oscillation mainly depends on the amount of the loop’s density change. We therefore investigate more cases within which the loop density at \( t = 2\tau_0 \) gradually increases from 0.25\( \rho_{i=0} \) to 4\( \rho_{i=0} \). A total of 100 cases have been investigated. The terminal amplitude (i.e., the real-time amplitude at \( t = 2\tau_0 \)) is calculated and plots as a function of the terminal density of the coronal loop (see Figure 7). These plots show very good linearity in a log–log plot where both the x-axis and y-axis are logarithmic. It further reveals that the plots in the log–log diagram (see the bottom panel of Figure 7) have a very similar slope of \(-0.25 \pm 0.01\) (obtained by a linear fitting) for different modes of evolution (distinguished by symbol colors the same as in Figure 2). This means that the real-time amplitude of fast kink oscillation may primarily depend on the loop’s real-time density by a power law as follows:

\[
\begin{align*}
R(A) &\propto R(\rho_i)^a \\
R(A) &= A(t)/A_{i=0} \\
R(\rho_i) &= \rho_i(t)/\rho_{i=0} \\
a &= -0.25 \pm 0.01.
\end{align*}
\]
The influence of the temporal interval for a certain change in loop density is also investigated. It indicates that the temporal interval of the change density, i.e., the speed of the evolution density, has only a marginal influence on the power law given by Equation (9). The power-law index as a function of temporal interval between \( \rho_i(0) \) and \( \rho_i(t) \) for the four evolution functions of density is shown in Figure 8. The plots further reveal that the amplitude depends mainly on the amount of the loop’s density change by the power law shown in Equation (9) for the present mechanism, and both the way and speed of density change would have only a marginal influence on the oscillation amplitude. This also means that the evolution of the loop’s density may have a significant influence on the mechanical energy trapped in the oscillating loop, and we thus must be very cautious when studying the damping of the oscillation with a frequency drift because the oscillation may be affected by a mix of the present effect and other damping mechanisms (Heyvaerts & Priest 1983; Ruderman & Roberts 2002; Antolin et al. 2015; Guo et al. 2019).

5. Discussion and Conclusion

It has been observed that the period or frequency of the oscillation can significantly change over time, which theoretically could be caused by the quasi-static evolution of loop density (Reale 2007; Su et al. 2018). In order to gain a further understanding of the potential explanation, a detailed analysis of the effects of the evolving density of an oscillating loop on its oscillation of fast kink mode is highly necessary.

In this study, we consider the effects of the evolution of loop density on its fast kink oscillation in the promise of both the loop quasi-static equilibrium and its oscillatory mode being maintained. The result indicates that the quasi-static evolving loop density can indeed have significant influences on its fast kink oscillation, which is sufficient to produce several key characteristics in some observational frequency drift cases of coronal loop’s fast kink oscillations. The analytic relationships of the period/frequency and amplitude of the loop oscillation in the fast kink mode on the loop’s evolving density are determined. Based on the analytic relationships, models of fast kink oscillation in a series of quasi-static thermodynamic coronal loops are established. The result indicates that the evolving loop density can have significant influences not only on the period of the loop’s fast kink oscillation, but also on the mechanical energy of the loop oscillation. From the analyses, three implications can be attributed to the frequency drift of the loop’s fast kink oscillation caused by the density evolution.

One of the implications is it provides us a possible explanation for the frequency drift phenomenon of a loop’s fast kink oscillation. The fast kink oscillation of a coronal loop has been studied over two decades and many break-throughs have been achieved in both theory and practice.
However, a lot of attention is devoted to the fast kink oscillation of a steady coronal loop. Their results mainly lie in oscillations at a steady state. There are several theoretical investigations on the temporal evolution of oscillation. They are mainly about the damping of the oscillation (Heyvaerts & Priest 1983; Ruderman & Roberts 2002; Antolin et al. 2015; Guo et al. 2019). None of them involves the temporal change of the real-time period/frequency of the fast kink oscillation. There is evidence that the loop’s density can significantly change during the frequency drift of a loop’s fast kink oscillation. Our investigation strongly indicates that the evolution of loop density should have a significant influence on the frequency of the fast kink oscillation. It naturally leads us to the conclusion that the quasi-static evolution of loop density should play an important role in the frequency drift phenomenon of a coronal loop’s fast kink oscillation, especially for that, that occurs in a coronal loop whose density significantly changes.

The other one lies in the coronal seismology (Roberts et al. 1984; Aschwanden 1987; Priest 2014). The oscillation especially its period or frequency mainly depends on the loop properties, which show us the possibility of using the frequency drift to invert the physical evolution of the oscillatory loop. For the situation where the frequency drift is mainly caused by the quasi-static evolution of loop density, the frequency drift is suggested to be used for diagnosing the loop’s density evolution according to the analytic relation

\[ R(\tau) = R(\sqrt{\rho_i + \rho_c}) \]

where the change rate of \( \sqrt{\rho_i + \rho_c} \) could be approximated as \( R(\sqrt{\rho_i + \rho_c}) \approx R(\rho_i) \) when \( \rho_i \gg \rho_c \). With the density evolution, the corresponding amplitude change can also be estimated according to Equation (9). It is important to note that this result may be different from the real evolution curve of the amplitude because this is just the effect of density evolution. As for more cases of fast kink oscillations with frequency drift, a two-step scheme is suggested as follows: (1) one can directly calculate the density evolution of a loop from the frequency drift by using Equation (10); and then (2) use the density evolution obtained in step (1) to estimate the corresponding amplitude evolution by using Equation (9). As for the obtained amplitude curve from the above estimation, we would like to point out that it should

![Figure 6.](image-url) Evolution of the real-time amplitude for the cases shown in Figure 3, where the same as Figure 2, the color of plots indicates the four arbitrary functions of density evolution.

| \( \rho_i|_{t_0} \) | \( \rho_i|_{t_0} \) | \( \rho_i|_{t_0} \) | \( \rho_i|_{t_0} \) |
|---|---|---|---|
| Rarefying | Densifying |
| A/\( A_0 \) | | | |
| Inverse parabolic | 1.41 | 1.19 | 0.84 | 0.70 |
| Linear | 1.43 | 1.18 | 0.84 | 0.72 |
| Parabolic | 1.37 | 1.20 | 0.84 | 0.72 |
| Exponential | 1.41 | 1.18 | 0.84 | 0.72 |

Note. A The amplitude is normalized by its initial value \( A_0 \).
be deduced from the real amplitude evolution curve when searching for other energy conversion mechanisms.

Finally, it also provides us with a novel mechanism to change the trapped mechanical energy of the oscillation. It is well known that the mechanical energy of fast kink oscillation can be converted into other forms and finally dissipated by a series of dissipation processes. Unlike these mechanisms, which are ultimately related to the viscosity and mainly used to explain the damping of the oscillation, our work indicates that a two-way energy conversion can be achieved directly by the interaction of the loop’s oscillation and its density evolution without resorting to the viscosity. What is more interesting is that although fast kink oscillation can be affected by both the amount and way in which the coronal loop density changes, the amplitude of the loop oscillation depends primarily on the amount of the coronal loop’s density change. It indicates that both the speed and mode of the change have only a marginal influence on the corresponding change in the oscillation amplitude. This may imply a more universal rule existing for an interaction between the vibration and the mass variation for a vibratory system.

In summary, it is widely accepted that the coronal loop is the fundamental dynamic structure in the corona and its density can significantly change especially in its early stage (Reale 2014). The dynamic evolution of loops’ thermal property inevitably affects the other processes of the dynamic loop, including its fast kink oscillation (Roberts 2008). The research on the effects of the loop’s density evolution on its fast kink oscillation is, therefore, undoubtedly needed for further understanding the details of fast kink oscillations in thermodynamic coronal loops. Here, we present a seminal study for the analytical effects of loop’s density evolution on its fast kink oscillation, which suggests the loop’s density evolution may shed light on the increasing observed frequency drift of the loop’s fast kink oscillation. Implications of the present result on the energy conversion and coronal seismology of the corresponding dynamic fast kink mode caused by thermal dynamic evolution are also discussed, from which a novel energy transformation mechanism and a novel scheme for detecting the mechanism emerge. All of these provide us with a theoretical basis for the increasingly observed frequency drift phenomenon of a loop’s fast kink oscillation. Similar theories on other parameters’ evolution are also encouraged, if necessary. It should be noted that (1) the temperature might not be an independent parameter of the density for a quasi-static coronal loop with the same boundary condition, (2) the evolution of magnetic field should have a more higher order effect than the density evolution, and (3) a similar transformation of the magnetic energy and the internal energy should occur for the oscillating loop whose magnetic field changes significantly. We hope this work can provide a new clue and a good start toward understanding the challenges of the mysterious solar corona presented by the increasingly excellent observations.

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