Modified Particle Swarm Optimization with Chaotic Initialization Scheme for Unconstrained Optimization Problems

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ABSTRACT – Different variants of particle swarm optimization (PSO) algorithms were introduced in recent years with various improvements to tackle different types of optimization problems more robustly. However, the conventional initialization scheme tends to generate an initial population with relatively inferior solution due to the random guess mechanism. In this paper, a PSO variant known as modified PSO with chaotic initialization scheme is introduced to solve unconstrained global optimization problems more effectively, by generating a more promising initial population. Experimental studies are conducted to assess and compare the optimization performance of the proposed algorithm with four existing well-established PSO variants using seven test functions. The proposed algorithm is observed to outperform its competitors in solving the selected test problems.

INTRODUCTION

Majority of the real-world applications contain a set of parameters considered as decision variables that need to be optimized. The objective function is formulated to represent a distinct target to be accomplished by an optimization problem. In particular, the optimal combinations of decision variables can be either the lowest and highest objective function values for minimization and maximization problems, respectively. In the past decades, metaheuristic search algorithm (MSA) emerged as a promising decision-making tool to determine the optimal set of decision variables for various type of optimization problems, due to its characteristic of independent from the gradient information. MSAs can be grouped into two types based on their inspiration sources, known as swarm intelligence (SI) algorithm and evolutionary algorithm (EA). SI algorithms, including artificial bee colony (ABC) [1] and bat algorithm (BA) [2], are motivated by the cooperative behavior of animals, while the EAs, such as genetic algorithm (GA) [3] and differential evolution (DE) [4], are inspired by the Darwin’s Theory of Evolution. MSAs are widely implemented to solve various type of optimization problems with different complexity level [5-19], due to its strengths of fast convergence speed and promising global search ability.

Particle swarm optimization (PSO) algorithm emerges as one of the most popular MSAs due to its significant strengths such as simplicity implementation, high convergence rate, etc. During the searching process, the search trajectory of PSO algorithm are dependent on the collaborative behavior of the swarm members in sharing the useful information to locate the superior region of the search space. However, the conventional population initialization scheme adopted by the conventional PSO tends to generate an initial population with relatively inferior particles due to random guess mechanism [20]. Furthermore, the poor balancing of the explorative and exploitative search behavior of the conventional PSO tends to suffer with premature convergence in dealing with complex optimization problems. Extensive studies were performed in recent years to strengthen the performance of PSO in solving single-objective optimization problems (SOPs) [6, 8], multi-objectives optimization problems (MOPs) [7], constrained optimization problems (COPs) [5], etc.

Although significant numbers of PSO variants were introduced in the past decades such as those reported in [5, 8, 17, 18, 21-27], the high tendency of these PSO variants to suffer with premature convergence issue remains as an on-going challenge. There are several factors that can contribute to these undesirable issue such as the initial population with poor quality and strong dependence of particles on histrical best positions (e.g., personal best position, global best position and etc.) [28]. The initial populations of most existing PSO variants are randomly generated using uniform distributions without considering current condition of search environment. These particles might be occasionally initialized nearby local optima or solution region far away from global optimum, hence leading to premature convergence and slow convergence speed of algorithm, respectively. Conventional PSO are also found to strongly rely on the guidance of their historical best solutions such as personal best position and global best position during the search process. However, it is notable that these solutions might not be frequently updated in the middle or later stages of optimization process, therefore have high tendency of diversity loss in population. In order to address the aforementioned issues, it is necessary to design a modified initialization scheme that is able to produce initial population with better quality at the beginning of
optimization process. In addition, a more robust scheme is also designed to generate new examples that can guide particles more effectively with better solution diversity by leveraging the useful search information stored in other non-fittest particles. To this end, a modified particle swarm optimization with chaotic initialization scheme (MPSO-CIS) is introduced. The main contributions of MPSO-CIS are presented as follows:

1. A chaotic initialization scheme is designed by hybridizing the chaotic system and oppositional based learning strategies to initialize the population with improved qualities.
2. A unique exemplar derivation module (UEDM) is implemented to produce a distinctive model to substitute the self-cognitive component of each particle in searching for the promising region of the search space.
3. Performance assessments of the proposed MPSO-CIS algorithm in solving seven global optimization test problems and compared with four well-established PSO variants.

The remaining parts of this article are presented as follows. Section 2 summarizes the conventional PSO and the opposition-based learning strategy. The proposed MPSO-CIS frameworks are presented in Section 3. The performance assessments and comparisons are reported in Section 4. At last, the conclusion and future studies are summarized in Section 5.

RELATED WORK

Conventional Particle Swarm Optimization

Inspiration from the group behavior of bird flocks in locating the food sources, conventional particle swarm optimization was introduced as a powerful optimization algorithm in solving SOPs in year 1995 [29]. The search mechanisms of the conventional PSO allow each particle to search based on their best self-knowledge and the best knowledge of the whole swarm during the optimization process. Given that each particle \( n \) of the population refers to a candidate solution of a specified optimization problem, \( V_n = [V_{n,1}, ..., V_{n,d}, ..., V_{n,N}] \) and \( X_n = [X_{n,1}, ..., X_{n,d}, ..., X_{n,N}] \) define the velocity and position index with the total dimension of \( D \), and \( n \in [1,N] \) refers to the particle index with the population size of \( N \). Given the personal best position \( P_{best,n} = [P_{best, n,1}, ..., P_{best, n,d}, ..., P_{best, n,N}] \) of particle \( n \) and the global best position \( G_{best} = [G_{best,1}, ..., G_{best,d}, ..., G_{best,D}] \) of the entire population found during the optimization process, the new velocity \( V_{n,d}^{new} \) and new position \( X_{n,d}^{new} \) of each particle \( n \) in dimension \( d \) are calculated as follows:

\[
V_{n,d}^{new} = \omega V_{n,d} + c_1 r_1 (P_{best,n,d} - X_{n,d}) + c_2 r_2 (G_{best,d} - X_{n,d}) \\
X_{n,d}^{new} = X_{n,d} + V_{n,d}^{new}
\]  

(1)

where \( \omega \) refers to the inertia weight; \( c_1 \) and \( c_2 \) represent the acceleration coefficients; \( r_1 \) and \( r_2 \) denote two numbers stochastically produced from uniform distribution in range of \([0,1]\). The fitness value of the new position of particle \( n \) is assessed and its personal best position as well as the global best position are replaced if the new position achieves a better fitness value. The optimization process of the conventional PSO is replicated until the termination criteria is fulfilled. The global best position \( G_{best} \) is treated as the optimal solution to the specified problem.

Opposition-Based Learning (OBL)

The opposition-based learning (OBL) concept [30] is applied to produce a set of opposite solutions that potentially contribute to the searching process. Given the intention is to seek for solution \( x \) and the searching of solution \( x \) in opposite direction is agreed, the opposite \( x \) is calculated as follows:

\[
x = a + b - x
\]  

(3)

where \( x \) refers to a real number in range of \( a \) and \( b \). Besides that, given that \( D \) as the total number of dimensional components, the opposite number \( x = (x_1, ..., x_d, ..., x_D) \) in a multidimensional case \( P \) are calculated as follows:

\[
x_d = a_d + b_d - x_d, \hspace{1cm} d = 1, ..., D
\]  

(4)

Based on the potential solution \( P = (x_1, ..., x_d, ..., x_D) \) with \( D \) dimensional size, the potential solution is replaced by its opposite solution if the latter solution is fitter than the former solution. Otherwise, the potential solution remains as its own value. During the searching process, the search interval is recursively decreased until either the potential solution or the opposite solution is close to an existing solution for the given problem.
Metaheuristic Search Algorithms with Opposition-Based Learning Scheme

In the past decades, OBL scheme was widely modified and implemented into MSAs to enhance the performance of the optimization algorithm in dealing with optimization problems with different complexity level. In [31], a probabilistic opposition-based learning (OBL) scheme was designed into PSO to tackle noisy optimization problems effectively. A number of best particles were picked from the current swarm and the opposite swarm generated by the adopted OBL scheme, in order to preserve the swarm diversity in noisy environments. In [32], OBL scheme was implemented to an improved sine cosine algorithm (SCA) to deal with SOPs more effectively. The influence of the OBL scheme adopted in SCA algorithm was investigated and reported to improve the search accuracy, convergence rate and time complexity. In [33], an enhanced PSO algorithm known as GOPSO was proposed with the incorporation of generalized OBL (GOBL) and Cauchy mutation. The GOBL sampling scheme was adopted to convert the potential solutions from current population into another search space, in order to increase the probability in locating a number of better solutions. In [34], OBL scheme was designed into PSO algorithm to decrease the tendency of particles being stucked in local optima, and to boost the convergence speed of PSO algorithm. In [35], a gravitational search algorithm (GSA) variant is introduced with the OBL scheme to deal with combined economic and emission dispatch problems of power systems. The OBL scheme was employed to generate initial swarm with superior diversity level and to enable the generation jumping, leading to the improvement of convergence rate.

MODIFIED PARTICLE SWARM OPTIMIZATION WITH CHAOTIC INITIALIZATION SCHEME

Chaotic Initialization Scheme

A chaotic oppositional based initialization scheme (COBIS) is adopted in this work to replace the conventional population initialization scheme of PSO. At the initial stage of the optimization process, the population of MPSO-CIS is initialized with improved diversity, aiming to prevent premature convergence by leveraging the stochasticity and non-repetition characteristics of chaotic maps. Given that $\zeta_0$ and $\zeta_z$ are the preliminary condition and $z$-th sequence of chaotic variable in range of $[0, 1]$, respectively, the chaotic sequence is generated by using a modified sine map as expressed [36]:

$$\zeta_{z+1} = \sin(\rho \zeta_z), \ z = 1, ..., Z$$

(5)

where $\rho = \pi$ is a bifurcation coefficient of modified sine map; $Z$ refers to the maximum chaotic sequences. Denote $X_{\text{min}}^d$ and $X_{\text{max}}^d$ as the lowest and the highest values of design variables, respectively, where $d = 1, ..., D$. Given that $\zeta_z$ represents the final sequence of chaotic variable produced, the dimension $d$ of each chaotic swarm member $n$ denoted as $X^CS_{n,d}$, is computed as follows:

$$X^CS_{n,d} = X_{\text{min}}^d + \zeta_z \left( X_{\text{max}}^d - X_{\text{min}}^d \right)$$

(6)

where $n = 1, ..., N$. The dimension $d$ of each opposite swarm member $n$ denoted as $X^{\text{OBL}}_{n,d}$ is computed using oppositional based learning strategy as follows:

$$X^{\text{OBL}}_{n,d} = X_{\text{min}}^d + X_{\text{max}}^d - X^CS_{n,d}$$

(7)

A new population set $\phi^{\text{comb}} = \phi^{CS} \cup \phi^{\text{OBL}}$ with population size of $2N$ is formed by combining both $\phi^{CS}$ and $\phi^{\text{OBL}}$, where $\phi^{CS} = [X^CS_1, ..., X^CS_N]$ and $\phi^{\text{OBL}} = [X^{\text{OBL}}_1, ..., X^{\text{OBL}}_N]$ refer to the population sets of chaotic and opposite swarm members, respectively. After arranging all solution members in $\phi^{\text{comb}}$ from best to worst based on their objective function values, the top $N$ members of $\phi^{\text{comb}}$ with relatively superior objective function values are selected to form an initial population of MPSO-CIS denoted as $\phi^{\text{initial}} = [X_1, ..., X_N]$. The chaotic initialization scheme of MPSO-CIS is described in Figure 1.

Unique Exemplar Derivation Module

In this study, the unique exemplar derivation module (UEDM) is adopted to produce the unique exemplar of each particle $n$ to lead its search process. At the beginning stage, two exemplar candidates of particle $n$ are calculated. Given that $P_{\text{best},n1}$, $P_{\text{best},n2}$ and $P_{\text{best},n3}$ are the personal best positions of three different stochastically chosen particles with indices $n1$, $n2$ and $n3$, respectively, the first exemplar candidates $X^\text{exp}_n$ are obtained as follows:
\[ X_{n}^{\text{exp,1}} = \begin{cases} r_1 p_{\text{best,n,1}} + r_2 \left( p_{\text{best,n,2}} - p_{\text{best,n,5}} \right), & f \left( p_{\text{best,n,2}} \right) < f \left( p_{\text{best,n,5}} \right) \\ r_1 p_{\text{best,n,1}} + r_2 \left( p_{\text{best,n,3}} - p_{\text{best,n,2}} \right), & \text{Otherwise} \end{cases} \]

where \( r_1 \) and \( r_2 \) refer to two stochastic numbers produced from uniform distribution in range of \([0,1]\) and \( r_1 + r_2 = 1 \); \( f(\cdot) \) represent the fitness value of the particle. Based on Equation 8, neighborhood search is conducted around a random particle \( n \) with \( p_{\text{best,n}} \) that is located far from the global best position \( G_{\text{best}} \). Hence, \( X_{n}^{\text{exp,1}} \) tends to be explorative to avoid the stagnation of particle \( n \) by leading it to explore other regions of the search space.

Algorithm 1: Chaotic Initialization Scheme

**Input:** \( D, N, X_{d,\text{min}}, X_{d,\text{max}}, Z, \rho \)

1. Initialize \( \varphi^{CS} \leftarrow \emptyset \) and \( \varphi^{OBL} \leftarrow \emptyset \)
2. for each particle \( n \) do
3.     for each dimension \( d \) do
4.         Initialize \( \zeta_0 \in [0,1] \) and \( z = 1 \)
5.         while \( z \leq Z \) do
6.             Calculate \( \zeta_{z+1} \) using Eq. (5), \( z \leftarrow z + 1 \)
7.         end while
8.     Produce \( X_{n,\text{CS}} \) and \( X_{n,\text{OBL}} \) using Eqs. (6) and (7), respectively;
9. end for
10. \( \varphi^{CS} \leftarrow \varphi^{CS} \cup X_{n,\text{CS}} \), \( \varphi^{OBL} \leftarrow \varphi^{OBL} \cup X_{n,\text{OBL}} \)
11. end for
12. \( \varphi^{\text{Comb}} = \varphi^{CS} \cup \varphi^{OBL} \)
13. Evaluate objective function values of all \( \varphi^{\text{Comb}} \) members and sort from best to worst;
14. Select the first \( N \) members of \( \varphi^{\text{Comb}} \) to form initial population;

**Output:** \( \varphi^{\text{init}} = [X_1, \ldots, X_n] \) and the associated objective function values

**Figure 1.** Pseudocode of initialization scheme of MPSO-CIS

Given that \( p_{\text{best,n,4}} \) and \( p_{\text{best,n,5}} \) are the personal best positions of two different stochastically chosen particles with indices \( n 4 \) and \( n 5 \), respectively, the second exemplar candidates \( X_{n}^{\text{exp,2}} \) is computed as follows:

\[ X_{n}^{\text{exp,2}} = \begin{cases} r_5 G_{\text{best}} + r_2 \left( p_{\text{best,n,4}} - p_{\text{best,n,5}} \right), & f \left( p_{\text{best,n,4}} \right) < f \left( p_{\text{best,n,5}} \right) \\ r_5 G_{\text{best}} + r_2 \left( p_{\text{best,n,5}} - p_{\text{best,n,4}} \right), & \text{Otherwise} \end{cases} \]

where \( r_5 \) and \( r_2 \) refer to two stochastic numbers produced from uniform distribution in range of \([0,1]\) and \( r_5 + r_2 = 1 \). In contrast to \( X_{n}^{\text{exp,1}} \), \( X_{n}^{\text{exp,2}} \) is produced by performing neighborhood search around \( G_{\text{best}} \) that is relatively nearer to the global optimum. Therefore, the latter exemplar tends to be more exploitative in searching around the superior regions of the search space.

Given that both of the \( X_{n}^{\text{exp,1}} \) and \( X_{n}^{\text{exp,2}} \) produced randomly, two scenarios are stated to compare the fitness values of particle \( n \), \( X_{n}^{\text{exp,1}} \) and \( X_{n}^{\text{exp,2}} \), as follows:

1. If the best exemplar candidate selected between \( X_{n}^{\text{exp,1}} \) and \( X_{n}^{\text{exp,2}} \) are better than particle \( n \), the better exemplar candidate is assigned as the exemplar of particle \( n \) indicated as \( e_n \).
2. If the best exemplar candidate selected between \( X_{n}^{\text{exp,1}} \) and \( X_{n}^{\text{exp,2}} \) are worse than particle \( n \), crossover mechanism is performed to produce the third exemplar \( X_{n}^{\text{exp,3}} \) by leveraging the information of \( X_{n}^{\text{exp,1}} \) and \( X_{n}^{\text{exp,2}} \). The weightage value \( W_{n,k} \) indicates the tendency of each exemplar candidate \( k \) to contribute its information in generating each dimensional component of \( X_{n}^{\text{exp,3}} \) is computed as follows:

\[ W_{n,k} = \begin{cases} 1 + f \left( X_{n}^{\text{exp,k}} \right), & f \left( X_{n}^{\text{exp,k}} \right) \geq 0 \\ 1 + f \left( X_{n}^{\text{exp,k}} \right), & \text{Otherwise} \end{cases} \]
where \( k = 1 \) and 2. Given the \( W_{n,k} \) values, roulette wheel selection is performed to choose the exemplar candidate that contributes to each dimensional component of \( X_n^{\exp,3} \). The exemplar candidate with fitter objective function values has higher probability to contribute to compute \( X_n^{\exp,3} \), and vice versa. In order to prevent the domination of the superior exemplar candidate in contribute to the formation of \( X_n^{\exp,3} \), a stochastic dimensional index \( d_i \) is generated and the dimensional component with index of \( d_i \) is contributed by the relatively worse exemplar candidate. The best exemplar candidate among \( X_n^{\exp,1} \), \( X_n^{\exp,2} \) and \( X_n^{\exp,3} \) is allocated as the exemplar of particle \( n \) indicated as \( e_n \).

Given the fitness evaluation counter \( fes \) and the personal best positions of all particles \( P_{best} = \left[ P_{best,1}, \ldots, P_{best,n}, \ldots, P_{best,N} \right] \) in MPSO-CIS, the crossover mechanism and the unique exemplar derivation module are described in Figures 1 and 2, respectively.

### Algorithm 2: Crossover Mechanism

| Input: \( X_n^{\exp,1}, X_n^{\exp,2} \) |
|-----------------------------------------------|
| 01: Compute \( W_{n,k} \) of each exemplar candidate using Eq. (9); |
| 02: Stochastically pick dimension index \( d_i \); |
| 03: for each dimension \( d \) do |
| 04: \quad if \( d \neq d_i \), then |
| 05: \quad Perform roulette wheel selection based on \( W_{n,k} \); |
| 06: \quad \( X_n^{\exp,3} \leftarrow d \)-th variable of the selected exemplar candidate; |
| 07: \quad else if \( d = d_i \), then |
| 08: \quad \( X_n^{\exp,3} \leftarrow d_i \)-th variable of the inferior exemplar candidate; |
| 09: \quad end if |
| 10: end for |
| Output: \( X_n^{\exp,3} \) |

**Figure 2.** Pseudocode of the crossover mechanism adopted in MPSO-CIS.

### Algorithm 3: Unique Exemplar Derivation Module

| Input: Particle \( n \), \( P_{best} \), \( G_{best} \) |
|-----------------------------------------------|
| 01: Produce \( X_n^{\exp,1} \) and \( X_n^{\exp,2} \) using Eqs. (8) and (9), respectively; |
| 02: Evaluate \( f \left( X_n^{\exp,1} \right) \) and \( f \left( X_n^{\exp,2} \right) \); |
| 03: \( fes \leftarrow fes + 2 \); |
| 04: if \( \min \left[ f \left( X_n^{\exp,1} \right), f \left( X_n^{\exp,2} \right) \right] < f \left( P_{best,n} \right) \) then |
| 05: \( e_n \leftarrow \) exemplar candidate \( X_n^{\exp,1} \) or \( X_n^{\exp,2} \) with the best objective function value; |
| 06: else if \( \min \left[ f \left( X_n^{\exp,1} \right), f \left( X_n^{\exp,2} \right) \right] \geq f \left( P_{best,n} \right) \) then |
| 07: Generate \( X_n^{\exp,3} \) using crossover mechanism; */Algorithm 2*/ |
| 08: Evaluate \( f \left( X_n^{\exp,3} \right) \); |
| 09: \( fes \leftarrow fes + 1 \); |
| 10: \( e_n \leftarrow \) exemplar candidate \( X_n^{\exp,1} \), \( X_n^{\exp,2} \) or \( X_n^{\exp,3} \) with the best objective function value; |
| 11: end if |
| Output: \( e_n \) |

**Figure 3.** Pseudocode of the unique exemplar derivation module adopted in MPSO-CIS.

### Overall Framework of MPSO-CIS

The proposed MPSO-CIS calculates the new velocity of each particle \( n \) based on the exemplar \( e_n \) formulated by the unique exemplar derivation module and the global best particle \( G_{best} \). The \( d \)-th dimensional component of the new velocity and new position of each particle \( n \) are calculated as follows, respectively:
\[ V_{n,d}^{\text{new}} = \omega V_{n,d} + c_1 r_1 (e_{n,d} - X_{n,d}) + c_2 r_2 (G_{\text{best},d} - X_{n,d}) \]  \tag{11} \\
\[ X_{n,d}^{\text{new}} = X_{n,d} + V_{n,d}^{\text{new}} \]  \tag{12}

where \( \omega \) represents the inertia weight; \( n = 1, \ldots, N \) refers to the particle index; \( c_1 \) and \( c_2 \) represent the acceleration coefficients; \( r_1 \) and \( r_2 \) refer to two stochastic numbers produced from uniform distribution in range of \([0,1]\).

The overall framework of MPSO-CIS is described in Figure 4. In the early stage of the algorithm, the population is initialized via the chaotic initialization scheme. The unique exemplar derivation module is then performed to generate an exemplar \( e_n \) to contribute in calculating the new velocity and position vectors of each particle \( n \). The personal best position of particle \( n \) and the global best particle are replaced if a better particle is found. The reformulation of \( e_n \) is performed only if the particle \( n \) fails to update its personal best position for \( S \) successive times as recorded by its counter variable \( s \). The optimization process is iterated until the fitness evaluation counter \( \text{fes} \) reach the maximum fitness evaluation number \( \eta_{\text{max}} \). The global best position \( G_{\text{best}} \) produced at the end of searching process is considered as the optimal solution for a specified problem.

| Algorithm 4: MPSO-CIS |
|-----------------------|
| **Input:** \( D, N, \eta_{\text{max}} \) |
| 01: Initialize population using chaotic initialization scheme; /*Algorithm 1*/ |
| 02: while \( \text{fes} < \eta_{\text{max}} \) do |
| 03: For each particle \( n \) do |
| 04: if \( s > S \) then /*Algorithm 3*/ |
| 05: Produce \( e_n \) by triggering unique exemplar derivation module; |
| 06: \( s \leftarrow 0 \); |
| 07: end if |
| 08: Calculate \( V_{n,d}^{\text{new}} \) and \( X_{n,d}^{\text{new}} \) by using Eqs. (11) and (12), respectively; |
| 09: Evaluate \( f(X_{n}) \); |
| 10: \( \text{fes} \leftarrow \text{fes} + 1 \); |
| 11: Update \( P_{\text{best},n} \) and \( G_{\text{best}} \); |
| 12: if \( f(X_{n}) < f(P_{\text{best},n}) \) then |
| 13: \( s \leftarrow 0 \); |
| 14: else |
| 15: \( s \leftarrow s + 1 \); |
| 16: end if |
| 17: end for |
| 18: end while |
| **Output:** \( G_{\text{best}} \) |

**Figure 4.** Overall framework of MPSO-CIS.

**SIMULATION ANALYSIS OF MPSO-CIS**

**Simulation Settings**

Performance assessment of the proposed MPSO-CIS is conducted using seven global optimization test problems as described in Table 1, where \( LU \) refers to the lower and upper boundaries of the search region, and \( F_{\text{min}} \) represents the fitness value in global optimum. The optimization performance of MPSO-CIS is compared with four well-established pso variants known as: adaptive particle swarm optimization (APSO) [24], Frankenstein’s particle swarm optimization (FPSO) [25], comprehensive learning particle swarm optimization (CLPSO) [26] and feedback learning particle swarm optimization with quadratic inertia weight (FLPSO-QIW) [27]. In particular, APSO adopted parameter adaptation strategy to improve the adaptivity of the algorithm in solving various types of problems. FPSO adopted the modified population topology strategy to enable information sharing between the particles. It is notably that the strategies adopted by CLPSO and FLPSO-QIW have similarities with MPSO-CIS which also compute exemplars from non-best solutions using different learning strategies.

In this study, the inertia weight \( \omega \) of MPSO-CIS is initially set as 0.9 and recursively reduced to 0.4 at the end of the optimization process. Both of the acceleration coefficients \( c_1 \) and \( c_2 \) are set as 2.0. The threshold value of \( S \) is set as 8 proved by experimental study in offering satisfactory performance for MPSO-CIS. The maximum fitness evaluation number \( \eta_{\text{max}} \), population size \( N \) and dimensional size are set as 300000, 30, and 50, respectively, for all compared
algorithms. The simulations are conducted using MATLAB 2021b on a desktop computer utilized with Intel® Core™ i9-10900K CPU @ 3.70GHz for 25 independent runs in solving each problem.

Table 1. Global optimization benchmark functions.

| Func. | Name                  | $LU$          | $F_{\text{min}}$ |
|-------|-----------------------|---------------|------------------|
| A1    | Sphere                | $[-100, 100]^D$ | 0                |
| A2    | Schwefel 1.2          | $[-100, 100]^D$ | 0                |
| A3    | Rastrigin             | $[-5.12, 5.12]^D$ | 0                |
| A4    | Noncontinuous Rastrigin | $[-5.12, 5.12]^D$ | 0                |
| A5    | Griewank              | $[-600, 600]^D$ | 0                |
| A6    | Ackley                | $[-32, 32]^D$ | 0                |
| A7    | Weierstrass           | $[-0.5, 0.5]^D$ | 0                |

Performance Analysis of MPSO-CIS

The search accuracy and consistency of each algorithm in solving all benchmark problems are indicated by the mean fitness $F_{\text{mean}}$ and standard deviation $SD$ values and presented in Table 2. In particular, the lowest and the second-lowest $F_{\text{mean}}$ values produced for each problem are indicated in boldface and underlined, respectively. The performance comparison of the compared algorithms are summarized as $w/t/l$ and #BMF. Specifically, the proposed MPSO-CIS outperforms its competitor in $w$ function, similar performance with its competitor in $t$ function, underperform its competitor in $l$ function, and #BMF indicates the number of best mean fitness value produced by the algorithm. Furthermore, Friedman test [37] is also performed as multiple comparison between the algorithms in solving the seven test problems and the average ranking of each algorithm is reported in Table 3.

Table 2. Performance comparison of MPSO-CIS and four PSO variants.

| Func. | Criteria | APSO | FPSO | CLPSO | FLPSO-QIW | MPSO-CIS |
|-------|----------|------|------|-------|-----------|----------|
| A1    | $F_{\text{mean}}$ | 2.50E-01 | 7.03E+01 | 3.30E-48 | 2.89E-81 | 4.90E-162 |
|       | $SD$     | 1.84E-01 | 6.99E+01 | 1.27E-47 | 5.96E-81 | 2.03E-161 |
| A2    | $F_{\text{mean}}$ | 1.49E+03 | 3.45E+03 | 5.14E+03 | 2.61E+02 | 1.21E-03 |
|       | $SD$     | 4.79E+02 | 1.34E+03 | 1.01E+03 | 8.89E+01 | 3.03E-03 |
| A3    | $F_{\text{mean}}$ | 5.75E-01 | 1.83E+01 | 9.09E+01 | 2.59E+00 | 4.81E+01 |
|       | $SD$     | 6.26E-01 | 1.00E+01 | 1.07E+01 | 1.51E+00 | 1.48E+01 |
| A4    | $F_{\text{mean}}$ | 3.62E-02 | 1.61E+01 | 8.12E+01 | 5.57E+00 | 4.90E+01 |
|       | $SD$     | 3.24E-02 | 9.57E+00 | 9.77E+00 | 2.35E+00 | 1.54E+01 |
| A5    | $F_{\text{mean}}$ | 1.67E-01 | 1.88E+00 | 3.38E-11 | 5.74E-04 | 0.00E+00 |
|       | $SD$     | 8.20E-02 | 9.30E-01 | 1.71E-10 | 2.20E-03 | 0.00E+00 |
| A6    | $F_{\text{mean}}$ | 6.63E-02 | 1.81E+00 | 1.16E-14 | 3.42E-14 | 5.86E-15 |
|       | $SD$     | 2.60E-02 | 1.11E+00 | 2.57E-15 | 1.06E-14 | 1.79E-15 |
| A7    | $F_{\text{mean}}$ | 5.42E-01 | 3.33E+00 | 0.00E+00 | 1.87E-05 | 0.00E+00 |
|       | $SD$     | 1.86E-01 | 2.33E+00 | 0.00E+00 | 8.28E-05 | 0.00E+00 |

$w/t/l$ | 5/0/2 | 5/0/2 | 6/1/0 | 5/0/2 | -

#BMF | 2 | 0 | 1 | 0 | 5

Table 3. Average ranking produced by Friedman test.

| Algorithms | APSO | FPSO | CLPSO | FLPSO-QIW | MPSO-CIS |
|------------|------|------|-------|-----------|----------|
| Ranking    | 3.0000 | 4.2857 | 3.2143 | 2.2857 | 2.2143 |
| Chi-square Statistic | 7.914286 |
| $P$ value  | 9.48E-02 |

By referring to Table 2, the proposed MPSO-CIS shows the best search accuracy among its competitors by producing five best mean fitness values out of seven benchmark problems. APSO is observed to have competitive search accuracy in dealing with rastrigin and non-continuous rastrigin problems. For griewank function, MPSO-CIS is reported to be the algorithm able to locate the global optimum solution. CLPSO and FLPSO-QIW that have similar algorithmic framework
design with MPSO-CIS is reported to have competitive search accuracy in dealing with weierstrass function. CLPSO and MPSO-CIS are able to locate the global optimum solution of weierstrass function, while FLPSO-QIW demonstrate its competitive performance by producing second best mean fitness value. The competitive simulation results imply that the directional information extracted from non-best solutions are beneficial to lead the searching process more effective. According to Table 3, the compared algorithms are ranked from best to worst as MPSO-CIS, FLPSO-QIW, APSO, CLPSO and FPSO with the average ranking values of 2.2143, 2.2857, 3.0000, 3.2143 and 4.2857, respectively.

CONCLUSIONS

In this articles, a new PSO variant named as MPSO-CIS is proposed with the incorporation of chaotic initialization scheme to initialize the population. The adopted initialization scheme tends to enhance the quality and diversity of initial population. An unique exemplar derivation module is designed to produce exemplars by leveraging the useful information of non-best solutions, aiming to preserve the population diversity. In contrary to conventional PSO, the exemplars substitute the self-cognitive component of particle, aiming to accomplish better balancing of exploration and exploitation strengths of algorithm. Performance analysis report that MPSO-CIS demonstrates competitive search accuracy and consistency to the existing well-established PSO variants.

In future works, the convergence characteristics of MPSO-CIS can be investigated. The search performance of MPSO-CIS in tackling complex optimization problems with multi-objectives, constraints, and large dimensional size is worth to be investigated also. Furthermore, the feasibility of MPSO-CIS in optimizing the architecture of artificial neural networks for image processing task can be explored also.

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