Fast moving pulsars as probes of interstellar medium

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ABSTRACT

Pulsars moving through interstellar medium (ISM) produce bow shocks detected in hydrogen Hα line emission. The morphology of the bow shock nebulae allows one to probe the properties of ISM on scales ~ 0.01 pc and smaller. We performed 2D relativistic magnetohydrodynamic modeling of the pulsar bow shock and simulated the corresponding Hα emission morphology. We find that even a mild spatial inhomogeneity of ISM density, δρ/ρ ~ 1, leads to significant variations of the shape of the shock seen in Hα line emission. We successfully reproduce the morphology of the Guitar Nebula. We infer quasi-periodic density variations in the warm component of ISM with characteristic length of ~ 0.1 pc. Structures of this scale might be also responsible for the formation of the fine features seen at the forward shock of Tycho supernova remnant (SNR) in X-rays. Formation of such short periodic density structures in the warm component of ISM is puzzling, and bow-shock nebulae provide unique probes to study this phenomenon.

Key words: radiation mechanisms: thermal – hydrodynamics – stars: neutron – pulsars: individual: B2224+65

1 INTRODUCTION

Pulsars produce ultra-relativistic winds that create Pulsar Wind Nebulae (PWNe, Rees & Gunn 1974; Gaensler & Slane 2006; Kargaltsev & Pavlov 2008; Kargaltsev et al. 2015; Reynolds et al. 2017). Many fast moving pulsars quickly escape the host supernova remnant (for a recent review see Kargaltsev et al. 2017a). The proper speed of such pulsars vary in a wide range from a few kilometers per second to more than a thousand kilometers per second. Typically, these velocities are much larger than the sound speeds in the interstellar medium (ISM), c_s,ism ~ 10 – 100 km s⁻¹. The interaction of the fast mowing pulsar wind with the ISM produces a bow-shock nebula with an extended tail (see Barkov et al. 2019a; Olmi & Bucciantini 2019).

In the apex part of the PWN the ISM ram pressure confines the pulsar wind producing two shocks – forward shock in the ISM and the reverse/termination shock in the pulsar wind; the two shock are separated by the contact discontinuity (CD). This picture is similar to the structure formed by interaction of the Solar wind with the Local ISM (see Zank 1999; Pogorelov et al. 2017, for review).

Early works on the structure of the bow shock nebulae (e.g. Bucciantini et al. 2005) predicted a smooth morphology. In contrast, the observations show large morphological variations (Kargaltsev et al. 2017b), both of the PWN part (regions encompassing the shocked pulsar wind) and the shape of the bow shock (in the ISM part of the interaction region). Especially puzzling is the Guitar Nebula, which shows what can be called “closed in the back” morphology: shocks, delineated by the Hα emission, bend/curve towards the tail, not the head part.

Variations of the PWN part of the interacting flow are likely due to the internal dynamics of the shocked pulsar wind: mass loading (Morlino et al. 2015; Olmi et al. 2018), as well as fluid and current-driven instabilities (Barkov et al. 2019a) can induce larger variations in the shape of the confined pulsar wind. (We note here that so far no simulations were able to catch the long term dynamics in the tails, on scales much longer than the stand-off distances.)

One of the goals of the present study is to investigate whether the variations of the PWN part of the flow can affect the shape of the forward shock (e.g., as discussed by van Kerkwijk & Ingle 2008). Our simulations indicate that the internal dynamics of the PWN part does not affect the bow shock in any appreciable way. The basic reason is that the mildly relativistic flow within the PWN tail quickly advects downstream all the perturbations, see §4.

We then resort to external perturbations to produce the morphological variations of the Hα emission maps. We tar-
get the Guitar Nebula as the extreme example of a nontrivial morphology of the forward shock. The most surprising morphological feature of the Guitar Nebula is a sequence of “closed in the back” shocks (Vigiléus et al. 2007; Yoon & Heinz 2017; Toropina et al. 2019). Several fast moving pulsars form nebulae with Hα line emission (Brownsberger & Romani 2014), which show sequences of “closed in the back” shocks similar to those seen in the Guitar Nebula. As we demonstrate, the “closed in the back” morphology is a result of the projection effect (due to particular combination of the line of sight) and of the physical properties of the system (pulsar’s velocity and ISM density variations).

2 PHYSICS OF HYDROGEN IONISATION

A fast moving pulsar form a bow shock that heats the electrons and ions from ISM. Hot electrons ionize neutral atoms and thermalize with them. The pressure balance between the pulsar wind and the ISM gives the stand-off distance \( r_s \)

\[
r_s = \sqrt{\frac{L_w}{4\pi \rho_{\text{ISM}} v_{\text{NS}}^2}} = 4 \times 10^{16} L_{\text{W}, 36} n_{\text{ISM}, 6}^{-1/2} v_{\text{NS}, 7.5}^{-1} \text{ cm},
\]

where \( L_w \) is pulsar wind luminosity, \( \rho_{\text{ISM}} \) is the velocity of pulsar, \( \rho_{\text{ISM}} = m_p n_{\text{ISM}} \) is ISM mass and number density (here \( m_p \) is mass of proton). We use the following normalization agreement: \( \alpha \) \( 10^5 \text{ A}_1 \) cm\(^{-3}\) units. The pass-through time, which is the characteristic time for crossing the stand-off distance, is

\[
t_t = r_s / v_{\text{NS}} = 1.5 \times 10^9 L_{\text{W}, 36}^{1/2} n_{\text{ISM}, 6}^{-1/2} v_{\text{NS}, 7.5}^{-2} \text{ s}.
\]

Let us next estimate the ionization and recombination rates for hydrogen at the shock front. Following the work (Rossi et al. 1997) the ionization rate is

\[
\frac{d x_{\text{H}^+}}{dt} = n_e [c_i (1 - x_{\text{H}^+}) - c_r x_{\text{H}^+}]
\]

here \( n_e = n_p [(1 - x_{\text{H}^+}) + 0.001] \) is electron concentration, \( c_i \) and \( c_r \) are ionization and recombination rate coefficients

\[
c_i = 5.8 \times 10^{-11} \sqrt{T_0} \exp \left( -\frac{1.57 \times 10^5}{T_0} \right) \text{ cm}^3 \text{ s}^{-1}
\]

and

\[
c_r = 2.6 \times 10^{-11} \frac{1}{\sqrt{T_0}} \text{ cm}^3 \text{ s}^{-1}.
\]

In the case of a fast moving pulsar the temperature behind the bow shock can be as high as \( T \approx 2 \times 10^6 v_{\text{NS}, 7.5}^2 \) K. At such high temperatures ionization processes dominates over recombination ones by far (\( c_i \approx 10^{-7} v_{\text{NS}, 7.5} \) and \( c_r \approx 2 \times 10^{-14} v_{\text{NS}, 7.5}^{-1} \text{ cm}^3 \text{ s}^{-1} \)). The ionization time scale can be estimates as

\[
t_i = \frac{1}{c_i n_e} \approx 10^9 n_e^{-1} v_{\text{NS}, 7.5}^{-1} \text{ s}.
\]

Comparing Eq. (2) and Eq. (6) we can see \( t_i \ll t_t \), it means that the ionization is very fast and the thickness of the ionization shell should be small. In contrast, the recombination time scale \( t_r \sim 5 \times 10^7 v_{\text{NS}, 7.5} \) is much longer as compared to the pass-through time scale. Some atoms ionization process is accompanied by excitation and emission of Hα line. The most efficient condition of Hα line emission is near \( x_{\text{H}^+} \approx 0.5 \). As the ionization proceeds in a significantly more compact region compare to recombination one, it should be responsible for generation of the most distinct line emission. Thus, Hα emission is produced in a thin shell right behind the bow shock. Only for slow pulsars, with velocity below \( 10^5 \) cm s\(^{-1} \), the ionization shell thickness can be comparable with the bow shock size.

Using the algorithm suggested by Rossi et al. (1997), hydrodynamical data as density, pressure and \( x_{\text{H}^+} \) (see section 3.1) allow us to calculate the excitation rate of hydrogen and intensity of Hα line emission as

\[
\dot{e}_{\text{H}α} = n_H_n_e x_{\text{H}^+} j_{\text{H}α} \text{ erg cm}^3 \text{ s}^{-1}
\]

and emissivity coefficient

\[
j_{\text{H}α} = \frac{h^2 \sqrt{\pi f}}{k_B m_e} \exp \left( -\frac{h v_{\text{H}α}}{k_B T} \right) \text{ erg cm}^3 \text{ s}^{-1},
\]

here

\[
q_{12} = \frac{8.6 \times 10^{-6} \Omega_{12}}{T_0^{1/2} g_1} \text{ erg cm}^3 \text{ s}^{-1},
\]

and

\[
q_{21} = \frac{8.6 \times 10^{-6} \Omega_{21}}{T_0^{1/2} g_2} \text{ erg cm}^3 \text{ s}^{-1},
\]

where parameters \( g_1 \), \( g_2 \) and \( \Omega_{12} = \Omega_{21} \) are tabulated in PLUTO code\(^1\).

3 METHOD

We implement a two-step procedure to calculate the expected morphology of the Hα emission. First, as described in \S 3.1 we perform relativistic magnetohydrodynamic (MHD) simulations of a pulsar moving through inhomogeneous ISM. These calculations define the form of the bow shock and values of all relevant hydrodynamic parameters. Next, \S 3.2, we performed post-processing of the hydrodynamic data to calculate Hα emissivity maps.

3.1 Numerical Setup

We performed a number of 2D relativistic MHD simulations of the interaction of relativistic pulsar wind with external medium. The simulations were performed using a 2D geometry in Cylindrical coordinates using the PLUTO code (Mignone et al. 2007). Spatial parabolic interpolation, a 2nd order Runge-Kutta approximation in time, and an HLLC Riemann solver were used Mignone & Bodo (2005). PLUTO is a modular Godunov-type code entirely written in C and intended mainly for astrophysical applications and high Mach number flows in multiple spatial dimensions. The simulations were performed on CFCA X50 cluster of national astronomical observatory of Japan (NAOJ). The flow has been approximated as an ideal, relativistic adiabatic gas, one particle species, and polytropic index of 4/3. The size of the domain is \( R \in [0, 20a], Z \in [-3a, 100a] \) there the length unit \( a = 10^{16} \) cm (the initial ISM velocity is directed along Z-axis). To have a good resolution in the central region

\(^1\) Link http://plutocode.ph.unito.it/index.html
and long the tail zone we use a non-uniform resolution in the computational domain. For “high-resolution” simulations we adopted the following grid configuration. Number of the radial cells in a high resolution uniform grid in the central region (for \( R \in [0, 1d] \)) is \( N_{R, in} = 520 \). The outer non-uniform grid (for the region with \( R \in [1a, 20a] \)) has \( N_{R, out} = 1560 \) radial cells. In the \( Z \) direction we also adopted a non-uniform grid with three different zones. The inlet part \((Z \in [-3a, -1d])\) has \( N_{Z, out} = 520 \) non-uniform grid. The central part \((Z \in [-1a, 1d])\) has a uniform grid with \( N_{Z, in} = 1040 \) cells, and the non-uniform tail grid \((Z \in [1a, 100a])\) contains \( N_{Z, out} = 4680 \) cells. The grid parameters in “low-resolution” models are the following: \( N_{R, in} = 130 \) and \( N_{R, out} = 390 \) for the radial direction; \( N_{Z, out} = 130 \), \( N_{Z, in} = 260 \), and \( N_{Z, out} = 1170 \) for the \( Z \)-direction. See Table 1 for other parameters.

We used a so called simplified non-equilibrium cooling (SNEq) block of PLUTO code (see details in Rossi et al. 1997). This is allow to calculate the fraction of neutral atoms of hydrogen \( x_H \) in ISM taking into account ionization and recombination processes. Moreover, the radiation losses for 16 lines (like Ly \( \alpha \), He\( \alpha \), O\( \alpha \), etc.) are included in the energy equation.

3.2 \( H\alpha \) emissivity maps

To obtain \( H\alpha \) brightness maps, Eq. (7) needs to be integrated over the line of sight. These calculation were performed at the postprocessing stage on a workstation-class PC. Despite the simplicity of the numerical procedure, obtaining of high-resolution synthetic maps required quite a large amount of computations. The character of required calculations, i.e. integration over the line of sight, implies a case for effective GPU computing. We utilized PyCUDA, a CUDA API implementation for python, which allowed us to profit python visualization library matplotlib. Despite the simplicity of the numerical procedure, observation need large amount of computations. The character of required calculations, i.e. integration over the line of sight, implies a case for effective GPU computing. We utilized PyCUDA, a CUDA API implementation for python, which allowed us to profit python visualization library matplotlib. To obtain \( H\alpha \) emissivity maps by the shocked ISM matter. The shock in the ISM has a very smooth conical shape.

4 RESULTS

4.1 Overall PWN properties

We performed five runs with different spacial resolutions and model parameters to illustrate how system evolves and which parameters play a crucial role. All the relevant parameters for each run are summarized in Table 1. We name each run accordingly the following simple rule: “run resolution (hr, lr or SNEq)\( ^{2}\) – “pulsar velocity (fast or slow)” – “ISM spatial dependence (const or var2 or var3)”. All the simulations were performed in the reference frame where the pulsar is at rest, and the ISM is injected from the most left border of the computational domain (which corresponds to \( z = -3a \)). At the ISM inlet the flow velocity was set to \( v_{NS} \) and the density is determined by Eq. (11) for \( z < -3a \).

In the model hr-fast-const we inject ISM matter with constant density (i.e., \( a_p = 0 \) in Eq. (11)) and velocity set to 0.1c (see Table 1). On top panels of Fig. 1 and Fig. 2 distributions of density and pressure are shown. In these figures one can see the expected flow structure: a pulsar surrounded by the unshocked pulsar wind; at larger radii a relativistic shock wave is formed; the shocked pulsar wind and ISM matter are separated by a contact discontinuity (CD) surface; a forward shock front takes place in the ISM. On the right side from the pulsar, the shocks form a channel filled by material from the pulsar wind, which is surrounded by the shocked ISM matter. The shock in the ISM has a very smooth conical shape.

| Name                          | R   | Z   | \( v_{NS}/c \) | \( a_p \) | \( L_{W, 36} \) |
|-------------------------------|-----|-----|----------------|---------|---------------|
| hr-fast-const                 | 2080| 6240| 0.1            | 0       | 560           |
| hr-fast-var3                  | 2080| 6240| 0.1            | 0.5     | 560           |
| lr-fast-var3                  | 520 | 1560| 0.1            | 0.5     | 560           |
| lr-SNEq-slow-var3             | 520 | 1560| 0.005          | 0.5     | 1.4           |
| lr-SNEq-slow-var2             | 520 | 1560| 0.005          | 0.2     | 1.4           |

The table 1. Models parameters. Here presented name of the models, resolution in radial “R” and axial “Z” directions, neutron star speed \( v_{NS} \), density amplitude variation parameter \( a_p \) and pulsar wind power \( L_{W} \).
We see a significant growth of the Kelvin–Helmholtz (KH) instability at the CD between the shocked pulsar wind and ISM. The KH instability triggers the formation of significant perturbations in the shocked pulsar wind region, so ISM matter occasionally mixes with the shocked pulsar wind. These dense obstacles in the pulsar wind trigger the formation of additional shocks which potentially could deform the shape of the bow shock in ISM. However, the large difference (a few orders of magnitude) between the sound speed in the shocked ISM matter and the bulk speed of the shocked pulsar wind makes it almost impossible to develop a significant deformation of the shock front in ISM. We can see the confirmation of described process in the upper panels of Fig. 1 and Fig. 2.

The quasistationary phase of simulation for the model hr-fast-var3 with periodic density distribution is presented in the bottom panel of Fig. 1 and Fig. 2. Here we injected the ISM matter with variable density ($a_{\rho} = 0.5$ in Eq. (11)). A strong modulation of the shape of the ISM shock can be seen. The shape of the CD is very complicated in this simulation. We can see a series of hierarchical structures which linked to the growth of small scale vertexes triggered by KH instability. On a larger scale, one can see elongated eddies triggered by the ISM density variation. Interesting, the impact on the crosssection of the shocked pulsar wind channel is significantly smaller compared to the variation of the crosssection of forward shock in ISM.

The gas pressure, shown in Fig. 2, for the model hr-fast-const indicates on a series of more-or-less uniformly distributed shocks which are triggered in the pulsar wind channel and propagate outside in the shocked material. The exact position of the CD is difficult to localize in this figure. The model hr-fast-var3 reveals a different behavior: a series of shocks, which look like recollimation ones, can be localized just behind the high density front in the ISM.

The models with slower NS velocity require a much longer computational time, so these models were computed with a smaller computational resolution. To verify eligibility of this resolution we also performed one simulation with this resolution for a faster moving NS.

First, let’s discuss the differences and similarities between the model lr-fast-var3 (Fig. 3) and hr-fast-var3 (Fig. 1). The shape of the forward shock in ISM is essentially the same. The main difference is the structure of the CD.
surface, which in the case of lr-fast-var3 is much smoother if compared to the one obtained in hr-fast-var3 simulations. It can be explained by the influence of the numerical viscosity which suppresses the growth of the KH instability. The pressure maps (Fig. 4) show similarity of the bow shock shapes, also we see a similar shift in the position of the recollimation shocks. The difference is the absence of fine structures in the shocked pulsar wind channel. In the low resolution case, the recollimation shocks have smoother structure. As we explained in § 2 and it is shown below, the optical lines are produced at the forward shock in the ISM. The shocked pulsar wind zone is predominately filled with electron-positron pairs and should not produce any considerable amount of $\text{H}a$ emission. We therefore safely claim that the resolution does not change significantly the geometry and properties of the $\text{H}a$ emitting region.

The “slow” models feature a significantly larger density jump between the ISM and the pulsar wind (see Fig. 3). The CD is more stable and the KH instability does not disrupt the pulsar wind channel. On the other hand, the forward shock shapes are almost identical in all *-var3 models. The pressure distributions are also very similar between lr-fast-var3 and lr-SNEq-slow-var3 models.

The amplitude of the density variation has a strong impact on the shape of the bow shock in ISM (see Fig. 4). A variation of density by a factor of 3 (model lr-SNEq-slow-var3) creates a forward shock with a clear wavy shape, which is correlated to the ISM density profile: the higher density, the smaller cylindrical radius of the shock. A model with a small density variation, lr-SNEq-slow-var2, still features a wavy-shaped bow shock. The recollimation shocks in the pulsar wind channel are visible and have spatial a lag relative to the density peaks in the ISM.

In Fig. 5 we show the distribution of the radial component of speed in cylindrical coordinates for two models, lr-SNEq-slow-var3 and lr-SNEq-slow-var2. The radial speed at the shock decreases with the distance from the pulsar. On top of the decreasing trend, one can also see some speed increases in the low ISM density regions. These increases are

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Figure 2. Color maps of pressure (log-scale) for hr-fast-const and hr-fast-var3 models (see Tab. 1 for detail) in the top and bottom panels, respectively. Arrows show the velocity field, the arrow color defines the flow speed (linear scale, speed of light units).
most likely related to the change of the sound speed in ISM 
\[ c_s = \sqrt{\gamma g_p / \rho_g} \]. In the case of the considered models, even for “slow” models, the pulsar moves with record speed as compared to pulsars found in Galaxy. The “sweet spot” speed for Hα line production is about \( 10^2 \) km s\(^{-1}\). For such speed, the hydrogen ionization front is relatively thick, which makes the emission process to be efficient. In our simulations such speeds appear only on at the edge of the computational domain (see the right part of Fig. 5). As can be seen in Figs. 6 and 7 the Hα line emission appears to be the most bright in
Figure 4. Color maps of pressure (log-scale) the same models as in Fig. 3. Arrows show the velocity field, the arrow color defines the flow speed (linear scale, speed of light units).
Figure 5. Color maps of radial velocity (linear-scale, [\text{\text{km s}^{-1}}] units) for two "low-resolution" models with slower pulsar speed: lr-SNEq-slow-var3 (top panel), and lr-SNEq-fast-var2 (which has a smaller variation of ISM density, bottom panel). Arrows show the velocity field, the arrow color defines the flow speed (linear scale, speed of light units).

Figure 6. \text{\text{H\alpha}} line emission maps for model lr-SNEq-slow-var2. Viewing angle is 0.1 radian.
Figure 7. Hα line emission maps for the model lr-SNEq-slow-var3 and three different viewing angles: 0.1 radian (top panel), 0.5 (middle panel), and 1 (bottom panel).
this regime. In the case of a realistic NS proper speed, the bright $H\alpha$ region should appear close to the forward shock apex.

### 4.2 $H\alpha$ line intensity distribution

In Fig. 7 synthetic emissivity maps are shown for different viewing angles, $\theta = 0.1, 0.5,$ and $1$ radian, from the top to the bottom. The high density peaks in ISM are clearly visible as bright rings. Even a relatively small variation of the ISM density leads to a very clear change of the surface brightness, with two important factors amplifying the emission: 1) total particle density; and 2) thickness of the ionization front. If the impact of the former factor is straightforward, $(/ \propto n^2)$, addressing the latter one is more complicated. If the shock is too fast, the ionization take place almost instantly and no neutral hydrogen left to emit after shock front. If shock is too slow, hydrogen atoms do not get excited and we again do not expect any significant line emission. Therefore, to see the $H\alpha$ nebula pulsar should move with velocity $\alpha_{NS,7.5} \sim 1$.

The observational data (Brownsberger & Romani 2014) show very similar features, the brighter are regions which seem to have the smaller radius, which should correspond to high ISM density filaments crossed by the pulsar.

If ISM density has a more complicated distribution (as we see in Brownsberger & Romani 2014), one can expect a formation of structures which are more complex than ring-like structures revealed with our 2D simulations. For example, depending on the viewing angle even axial symmetric 2D distributions of emissivity can appear as bright triangles or lenses (see Fig. 7, bottom). We plan to further investigate the formation of complex structures in a future study with three-dimensional MHD simulations.

### 4.3 Guitar Nebula

The parallax to pulsar B2224+65 measured with VLBI is $2.9^{+0.2}_{-0.2}$ mas (Deller et al. 2019), which corresponds to a distance of $D = 830^{+170}_{-100}$ pc. The observations of Guitar Nebula (Chatterjee & Cordes 2002; Dolch et al. 2016) in $H\alpha$ line revealed several depressions of the forward shock with the angular distance of $15'' - 30''$ between depression. This corresponds to a linear distance of $2 - 5 \times 10^{17}$ cm (or $\sim 0.1$ pc). A comparison of the Guitar Nebula morphology with our synthetic emissivity maps (see Fig. 7) show that a modest modulation of the ISM density, $\delta n / n \approx 0.8$, can result in such shape of the forward shock. The recent observations (Dolch et al. 2016) indicate that the pulsar a few years ago passed through even more dense region with $\delta n / n \sim 2$.

### 4.4 Connection to other observations

The Herschel satellite imaging observations of nearby molecular clouds (Miville-Deschênes et al. 2010), show that filamentary structures are characterized by length-scales with a relatively narrow distribution, which spreads within a factor of two around a length of $\sim 0.1$ pc common for a wide range of column densities (Arzoumanian et al. 2011, 2019; Koch & Rosolowsky 2015; Roy et al. 2019). This observations concern only the cold component of ISM, $T \sim 10$ K. The properties of the hot ($T \sim 10^6$ K) and warm ($T \sim 10^4$ K) components of ISM are less known. As the hot and warm components are distributed in the interstellar space between the molecular clouds, their low densities make impossible obtaining of any meaningful constrains with CO line emission, as in the case of the molecular clouds. As shown above, $H\alpha$ emission from forward shocks created by fast moving pulsars are very sensitive to the density of ISM, which allows one to obtain unique information about density structure of the warm ISM. Our interpretation of the Guitar Nebula suggests that the warm component of ISM features density filaments of the same length scale as cold ISM as revealed with Herschel observations.

Interestingly, the stripe structure seen in X-rays at the forward shock wave of supernova remnant (SNR) Tycho has a similar length scale. Indeed, the characteristic angular size of $l_{gap} \sim 8''$ corresponds to linear size $\sim 0.1$ pc if we set distance to SNR to $\sim 3$ kpc (Decourchelle et al. 2001; Rosenko 2006; Eriksen et al. 2011). So far the X-ray stripes were interpreted as a result of non-linear particle acceleration in turbulent media (Bykov et al. 2011; Caprioli & Spitkovsky 2013). However, based on similarity of the linear size of X-ray features seen in Tycho and density fluctuations inferred around the Guitar Nebula, we suggest that these effects can be related and perturbations of the ISM density trigger the fine X-ray structure in Tycho SNR.

We also note that the structures we inferred in the warm ISM around the Guitar Nebula should have an origin different from the one causing the filaments observed in dense Galactic molecular clouds (e.g. Arzoumanian et al. 2011). In the case of molecular clouds the parsec-scale filaments correspond approximately to the Jeans length. The warm component of ISM has much higher temperature ($\sim 10^4$ K) and respectively much larger Jeans length, $l_J \approx c_s (\pi G \rho)^{1/2} \sim 2$ kpc, which significantly exceeds the revealed value. Thus, a new mechanism operating on the same scale, $\sim 0.1$ pc, is required to explain the formation of periodic structures in the warm ISM.

In conclusion, we show that the dynamics and morphology of nebulae around fast moving pulsars can be used as a probe of the fine sub-parsec structure of the warm ISM. In the present paper we focused on $H\alpha$ emission, however our approach can be generalized to other prominent optical lines. Numerical simulation of synthetic emission maps for several “optical” lines (from UV to IR bands) can provide unique information about the properties of ISM. Future development of such methods also can deliver independent measurements of proper pulsar velocities, ISM properties and its chemical composition.

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APPENDIX A: GPU SIMULATION OF THE Hα SYNTHETIC MAPS

To obtain Hα brightness maps the emission coefficient, which was obtained from MHD simulations, is to be integrated over line of sight at the postprocessing stage. We were interested in a very efficient algorithm, as for each MHD simulation we studied the influence of the direction of the pulsar velocity and time evolution of the synthetic maps, which is important for understanding the formation of the visual structures seen in Hα emission. We therefore adopted an approach based on the GPU computing. We used PyCUDA, a CUDA API implementation for python. The python script consists of several stages, which (i) read the data from MHD simulations; (ii) process a template for CUDA C code; (iii) copy the data to the GPU device and run block of GPU processes, and (iv) retrieve synthetic map from the device and create a visual file using matplotlib. A version of the script that includes stages (ii)-(iv), i.e. computes a synthetic map for a test array of emissivity is available at “https://github.com/dmikha/GPUmaps.git”. Below we briefly outline the key steps in this fairly simple script.

We first start with general explanation of the used algorithms. The MHD box coordinates do not match the “real world” Cartesian coordinates (where the synthetic maps are computed: XY is the plane of sky and Z axis directed along line-of-sight), and the “real world” coordinates need to be transformed to the MHD box coordinates. As the obtained coordinate do not match the nodes of the MHD grid, one needs to assign some emission coefficient at that specific location. This requires obtaining nearby nodes from the MHD array and approximating the value. To obtain the nearby nodes we used a recursive high-efficiency algorithm suitable for monotonic arrays. To determine the emission coefficient at the revealed location, we used a bilinear approximation method. This resulted in a sectionally linear dependence of the emission coefficient, thus any high-order integration methods are not justified for the integration over the line-of-sight. We therefore utilized the trapezoidal integration rule.

There are detailed tutorial how to use PyCUDA here we just provide short comments on our script. The script should include listing of the Cuda C code for the main computational block. This code is process by nvcc compiler, e.g.

```python
mod = pycuda.compiler.SourceModule(nvcc_code)
```

Before the compilation, however, one can modify the `nvcc` code with various `python` tools, e.g., to define values of some parameters:

- `nvcc` code contains
  ```
  #define s_R $s_R
  ```

- `python` script contains
  ```
  nvcc_code = nvcc_listing.substitute(s_R = N_r)
  ```

An instance of `nvcc` function callable from python can be created e.g. with

```python
create_map_cuda = mod.get_function("create_map")
```
After that one needs to transfer data to the device memory. In PyCUDA there are various ways of doing this, in our script we used two slightly different syntaxes for that. In the first case we transfer the auxiliary array with

```python
mod = SourceModule(nvcc_code)
R_d = mod.get_global('R_d')[0]
pycuda.driver.memcpy_htod(R_d, r.astype(np.float32))
```

In the second case we used the data transfer at the function call:

```python
map = np.zeros((Ny_grid,Nx_grid)).astype(np.float32)
create_map_cuda(
    pycuda.driver.Out(map),
    block=my_block, grid=my_grid)
```

Here “my_block” and “my_grid” are tuples for the processes run on GPU, and should be selected according to the capacity of the device. The remaining parts of the code are essentially usual python or C codes.

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